A $2\sqrt{2k}$-approximation algorithm for minimum power $k$ edge disjoint $st$-paths

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Abstract

In minimum power network design problems we are given an undirected graph $G = (V, E)$ with edge costs $\{c_e : e \in E\}$. The goal is to find an edge set $F \subseteq E$ that satisfies a prescribed property of minimum power $p_e(F) = \sum_{v \in V} \max\{c_e : e \in F \text{ is incident to } v\}$. In the Min-Power $k$ Edge Disjoint $st$-Paths problem $F$ should contain $k$ edge disjoint $st$-paths. The problem admits a $k$-approximation algorithm, and it was an open question to achieve an approximation ratio sublinear in $k$ even for unit costs. We give a $2\sqrt{2k}$-approximation algorithm for general costs.

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1 Introduction

In network design problems one seeks a cheap subgraph that satisfies a prescribed property, often determined by pairwise connectivities or degree demands. A traditional setting is when each edge has a cost, and we want to minimize the cost of the subgraph. This setting does not capture many wireless networks scenarios, where a communication between two nodes depends on our "investment" in these nodes, like equipment and transmission energy, and the cost is the sum of these "investments". This motivates the type of problems we study here. Specifically, we consider assigning transmission ranges to the nodes of a static ad hoc wireless network so as to minimize the total power consumed, under the constraint that the bidirectional network established by the transmission ranges satisfies prescribed properties.

More formally, in minimum power network design problems we are given an undirected (simple) graph $G = (V, E)$ with (non-negative) edge costs $\{c_e \geq 0 : e \in E\}$. The goal is to find an edge subset $F \subseteq E$ of minimum total power $p_e(F) = \sum_{v \in V} \max\{c_e : e \in \delta_F(v)\}$ that satisfies a prescribed property; here $\delta_F(v)$ denotes the set of edges in $F$ incident to $v$, and a maximum taken over an empty set is assumed to be zero. Equivalently, we seek an assignment $\{a_v \geq 0 : v \in V\}$ to the nodes of minimum total value $\sum_{v \in V} a_v$, such that the activated edge set $\{e = uv \in E : c_e \leq \min\{a_u, a_v\}\}$ satisfies the prescribed property. These problems were studied already in the late 90’s, cf. [24, 26, 23, 11, 8], followed by many more. Min-power problems were also widely studied in directed graphs, usually under the assumption that to activate an edge one needs to assign power only to its tail, while heads are assigned power zero, cf. [11, 25, 17, 19]. The undirected case has an additional requirement – we want the network to be bidirected, to allow a bidirectional communication.

In the traditional edge-costs scenario, a fundamental problem in network design is the Shortest $st$-Path problem. A natural generalization and the simplest high connectivity network design problem is finding a set of $k$ disjoint $st$-paths of minimum edge cost. Here the paths may be edge disjoint – the $k$ Edge Disjoint $st$-Paths problem, or internally (node) disjoint – the $k$ Disjoint $st$-Paths problem. Both problems can be reduced to the Min-Cost $k$-Flow problem, which has a polynomial time algorithm.
Similarly, one of the most fundamental problems in the min-power setting is the MIN-Power st-Path problem. For this problem, an elegant linear time reduction to the ordinary SHORTEST st-Path problem is described by Althaus et al. [3]. Lando and Nutov [12] suggested a more general (but less time efficient) "levels reduction" that converts several min-power problems into problems with node costs. A fundamental generalization is activating a set of \( k \) edge disjoint or internally disjoint st-paths. Formally, the edge disjoint st-paths version is as follows.

**Theorem 1.** MIN-Power k-EDP admits a \( 2\sqrt{2k} \)-approximation algorithm.

Theorem 1 is based on the following combinatorial result that is of independent interest. Given a graph \( G = (V, E) \) with edge costs \( c_e \), we denote by \( c(G) = c(E) = \sum_{e \in E} c_e \) the ordinary cost of \( G \), and by \( p_e(G) = p_e(E) = \sum_{v \in V} \max\{ c_e : e \in \delta_F(v) \} \) the power cost of \( G \).
It is easy to see that $p_c(G) \leq 2c(G)$. Hajiaghayi et al. [9] proved that $c(G) \leq \sqrt{|E|}/2 \cdot p_c(G)$ for any graph $G$; this bound is tight. We will improve over this bound for inclusion minimal simple graphs that contain $k$ edge disjoint $st$-paths. Let us say that a graph $G$ is \textbf{minimally $k$-st-edge-connected} if $G$ contains $k$ edge disjoint $st$-paths but no proper subgraph of $G$ contains $k$ edge disjoint $st$-paths.

\textbf{Theorem 2.} Let $G = (V, E)$ be a minimally $k$-st-edge-connected simple graph with non-negative edge costs $\{c_e : e \in E\}$. Then $c(G) \leq \sqrt{2k} \cdot p_c(G)$.

The $2\sqrt{2k}$-approximation algorithm will simply compute a minimum cost set of $k$ edge disjoint $st$-paths, with edge costs $c_e$; this can be done in polynomial time using a min-cost $k$-flow algorithm. The approximation ratio $2\sqrt{k}$ follows from the Theorem 2 bound, and the bound $p_c(G) \leq 2c(G)$; specifically, if $F^*$ is an optimal solution to Min-Power $k$-EDP and $F$ is a min-cost set of $k$ edge disjoint paths, then $p_c(F) \leq 2c(F) \leq 2c(F^*) \leq 2\sqrt{2k} \cdot p_c(F^*)$.

A large part of research in extremal graph theory deals with determining the maximal number of edges in multigraphs and simple graphs that are edge-minimal w.r.t. some specified property. This question was widely studies for $k$-connected and $k$-edge-connected graphs. It is easy to show that a minimally $k$-edge-connected graph $G$ on $n$ nodes can be decomposed into $k$ forests and thus has at most $k(n - 1)$ edges; a graph obtained from a spanning tree by replacing each edge by $k$ parallel edges shows that this bound is tight. Mader [13] improved this bound for simple graphs – in this case $G$ has at most $k(n - k)$ edges provided that $n \geq 3k - 2$, and the complete bipartite graph $K_{k,n-k}$ shows that this bound is tight. Mader [14] also showed that this bound holds for minimally $k$-connected graphs. When $n < 3k - 2$, a minimally $k$-edge-connected or $k$-connected simple graph has at most $\frac{1}{2}(n + k)^2$ edges, see [6, 5, 7]. Similar bounds were established in [21] for graphs that are $k$-connected from a given root node to every other node (a.k.a. $k$-out-connected graphs).

To see that Theorem 2 is indeed of interest, consider the case of unit costs. Then $c(G) = |E|$ and $p_c(G) = V$. Thus already for this simple case we get the following fundamental extremal graph theory result, that to the best of our knowledge was not known before.

\textbf{Corollary 3.} Let $G = (V, E)$ be a minimally $k$-st-edge-connected (directed or undirected) simple graph. Then $|E| \leq \sqrt{2k} \cdot |V|$.

In the next section we will show that this bound is asymptotically tight.

We briefly survey some results on more general activation network design problems. Here we are given a graph $G = (V, E)$ with a pair of activation costs $\{c_e^u, c_e^v\}$ for each $uv$-edge $e \in E$; the goal is to find an edge subset $F \subseteq E$ of minimum activation cost $\tau(F) = \sum_{e \in F} \max\{c_e^u, c_e^v\}$ that satisfies a prescribed property. This generic problem was introduced by Panigrahi [22], and it includes node costs problems (when for every $v \in V$, the costs $c_e^u$ are identical for all edges $e$ incident to $v$), min-power problems (when $c_e^u = c_e^v$ for each $uv$-edge $e \in E$), and several other problems that arise in wireless networks; see a survey in [19] on this type of problems.

In the Activation $k$ Disjoint $st$-Paths problem, the $k$ paths should be internally node disjoint. This problem admits an easy 2-approximation algorithm. Based on an idea of Srinivas and Modiano [25], Alqahtani and Erlebach [1] showed that approximation ratio $\rho$ for Activation 2 Disjoint $st$-Paths implies approximation ratio $\rho$ for Activation 2 Edge Disjoint $st$-Paths. In [1] it is also claimed that Activation 2 Disjoint Paths admits a 1.5-approximation algorithm, but the proof was found to contain an error – see [20] where also a different 1.5-approximation algorithm is given. In another paper, Alqahtani and Erlebach [2] showed that the problem is polynomially solvable on graphs with bounded
treewidth. However, it is a long standing open question whether Activation 2 Disjoint st-Paths is in P or is NP-hard on general graphs, even for power costs [12, 1, 19].

2 Proof of Theorem 2 and a tight example

Let $G = (V, E)$ be a simple minimally $k$-edge-connected graph. Theorem 2 says that then $c(G) \leq \sqrt{2k} \cdot p_c(G)$ for any edge costs $\{c_e : e \in E\}$, where $c(G) = \sum_{e \in E} c_e$ and $p_c(G) = \sum_{v \in V} \max \{c_e : e \in \delta(v)\}$ (recall that $\delta(v)$ denotes the set of edges in $E$ incident to $v$). As was mentioned in Corollary 3, in the case of uniform costs this reduces to $|E| \leq \sqrt{2k} \cdot |V|$. We will need a slight generalization of this bound to any subset $U$ of $V$, as follows.

Lemma 4. Let $G = (V, E)$ be a simple minimally $k$-st-connected graph. Then for any $U \subseteq V$, $|E_U| \leq \sqrt{2k} \cdot |U|$, where $E_U$ is the set of edges in $E$ with both ends in $U$.

This lemma will be proved in the next section. For now, we will use Lemma 4 to prove Theorem 2, namely, that $c(G) \leq \sqrt{2k} \cdot p_c(G)$. Note that $c(G) \leq \sum_{xy \in E} \min \{p_c(x), p_c(y)\}$, where $p_c(v) = \max \{c(e) : e \in \delta(v)\}$. Thus, it is sufficient to prove that for any non-negative weights $\{p(v) : v \in V\}$ on the nodes, the following holds:

$$\sum_{xy \in E} \min \{p(x), p(y)\} \leq \sqrt{2k} \cdot \sum_{v \in V} p(v) . \quad (1)$$

The proof of (1) is by induction on the number $N$ of distinct $p(v)$ values. In the base case $N = 1$, all weights $p(v)$ are equal, and w.l.o.g. $p(v) = 1$ for all $v \in V$. Then (1) reduces to $|E| \leq \sqrt{2k} \cdot |V|$, which is the case $U = V$ in Lemma 4.

Assume that $N \geq 2$. Let $U = \{u \in V : p(u) = \max_{v \in V} p(v)\}$ and let $E_U$ be the set of edges with both ends in $U$. Let $\epsilon$ be the difference between the maximum weight $\max_{v \in V} p(v)$ and the second maximum weight. Let $p'$ be defined by $p'(v) = p(v) - \epsilon$ if $v \in U$ and $p'(v) = p(v)$ otherwise. Note that $|E_U| \leq \sqrt{2k} \cdot |U|$, by Lemma 4, and that $p'$ has exactly $N - 1$ distinct values, so by the induction hypothesis, (1) holds for $p'$. Thus we have:

$$\sum_{xy \in E} \min \{p(x), p(y)\} = \sum_{xy \in E} \min \{p'(x), p'(y)\} + \epsilon |E_U|$$

$$\leq \sqrt{2k} \sum_{v \in V} p'(v) + \epsilon \cdot \sqrt{2k} |U|$$

$$= \sqrt{2k} \left( \sum_{v \in V} p'(v) + \epsilon |U| \right) = \sqrt{2k} \sum_{v \in V} p(v) .$$

The first and last equalities are by the definition of $p'(v)$. The inequality is by the induction hypothesis and Lemma 4. This concludes the proof of Theorem 2, provided that we will prove Lemma 4, which we will do in the next section.

Before proving Lemma 4, let us give an example showing that for $U = V$ the bound in the lemma is asymptotically tight. Let $\ell, q$ be integer parameters, where $1 \leq \ell \leq q/2$. Let $P$ be an $st$-path with $q$ internal nodes. The length of an edge that connects two nodes $u, v$ of $P$ is the distance between them in $P$. Construct a graph $G_t$ as follows, see Fig. 1.

1. Let $F_\ell$ consist of edges of length $\ell$ and edges incident to one of $s, t$ of length at most $\ell - 1$.
   It is easy to see that $|F_\ell| = q + \ell$ and that $F_\ell$ is a union of $\ell$ edge disjoint (in fact, $\ell$ node internally disjoint) $st$-paths.
2. Let $G_\ell = (V, E_\ell)$ where $E_\ell$ is a disjoint union of $F_1, \ldots, F_\ell$. One can verify that $G_\ell$ is minimally $k$-st-connected for $k = \ell(\ell + 1)/2$ and that $|E_\ell| = q\ell + \ell(\ell + 1)/2$.

3. The graph $G_\ell$ is not simple, due to edges incident to $s$ and to $t$. To make it a simple graph, subdivide $2k - \ell = \ell^2$ edges among the $2k$ edges incident to each of $s$ and $t$; this adds $\ell^2$ nodes and $\ell^2$ edges.

Summarizing, the constructed graph $G_\ell$ has the following parameters:

\[
\begin{align*}
k & = \ell(\ell + 1)/2 \\
m & = q\ell + \ell(\ell + 1)/2 + \ell^2 \\
n & = q + \ell^2 + 2
\end{align*}
\]

For $\ell = [1/\epsilon]$ and $q = \ell^4 - \ell^2$, $m/n$ can be arbitrarily close to $\ell + 1 > \sqrt{2k}$:

\[
\frac{m}{n} = \frac{q\ell + \ell(\ell + 1)/2 + \ell^2}{q + \ell^2 + 2} > \frac{q\ell}{q + \ell^2} = \frac{\ell^2 - 1}{\ell} = \frac{(\ell - 1)(\ell + 1)}{\ell} \geq (1 - \epsilon)(\ell + 1).
\]

3 Proof of Lemma 4

We prove Lemma 4 for the directed graph obtained by orienting $k$ edge-disjoint st-paths in $G$ from $s$ to $t$; note that this directed graph is also minimally $k$-st-edge-connected. For $R \subseteq V$ let $d^\text{in}_G(R)$ and $d^\text{out}_G(R)$ be the number of edges that enter and leave $R$, respectively.

\begin{itemize}
  \item \textbf{Lemma 5.} Let $G = (V, E)$ be a minimally $k$-st-connected directed graph. There exists an ordering $s = v_1, \ldots, v_n = t$ of $V$ such that for every $i \leq n - 1$, $C_i = \{v_1, \ldots, v_i\}$ is a minimum st-cut and no edge enters $C_i$.
  
  \textbf{Proof.} For $n = 2$ the lemma is trivial so assume that $n \geq 3$. We will show that then $G$ has a node $z \in V \setminus \{s, t\}$ such that the following holds:

  \begin{enumerate}
    \item \textit{s is the tail of every edge that enters $z$ and the graph obtained from $G$ by contracting $z$ into $s$ and deleting loops is also minimally k-st-connected.}
  \end{enumerate}

  We then define the order recursively as follows. We let $G_1 = G$ and $v_1 = s$. For $i = 2, \ldots, n - 1$ we let $v_i$ be a node $z$ as above of $G_{i-1}$, and $G_i$ is obtained from $G_{i-1}$ by contracting $v_i$ into $s$. It is easy to see that the order is as required, so we only need to prove existence of $z$ as above.
\end{itemize}
Note that $d_G^\in(v) = d_G^\out(v) \geq 1$ for every $v \neq s, t$, while $d_G^\in(s) = d_G^\out(t) = k$ and $d_G^\in(t) = d_G^\out(s) = k$. By considering $G$ as a flow network with unit capacity and unit flow on every edge, we get the following.

- If $R$ is a minimum $st$-cut of $G$ (so $d_G^\in(R) = k$) then $d_G^\out(R) = 0$; this is so since for any $st$-cut $R$ the flow value equals to $d_G^\out(R) - d_G^\in(R)$.
- $G$ is acyclic, since removing a cycle does not affect the flow value, hence existence of a cycle contradicts the minimality of $G$.

The graph $G \setminus \{s, t\}$ is also acyclic and thus has a node $z$ such that no edge enters $z$, so $s$ is the tail of every edge that enters $z$. Let $G' = (V', E')$ be obtained from $G$ by contracting $z$ into $s$. It is easy to see that $G'$ is $k$-$st$-connected, so we only have to prove minimality. For that it is sufficient to prove that for any $e \in E'$ (so $e$ can be any edge of $G$ that is not an $sz$-edge) there exists a minimum $st$-cut $R$ of $G$ with $e \in \delta_G^\out(R)$ such that $z \in R$.

By the minimality of $G$, there exists a minimum $st$-cut $R$ with $e \in \delta_G^\out(R)$. If $z \in R$ then we are done. Otherwise, we claim that $R \cup \{z\}$ is also a minimum $st$-cut of $G$. Note that all edges entering $z$ are in $\delta_G^\out(R)$ (since their tail is $s$) and all edges leaving $z$ have head in $V \setminus R$ (since $G$ has no edges entering $R$). Thus $\delta_G^\out(R \cup \{z\}) = \delta_G^\in(R) - \delta_G^\in(z) + \delta_G^\out(z) = k$, concluding the proof.

Lemma 6. Let $v_1, \ldots, v_n$ be an ordering of nodes of a directed simple graph $G = (V, E)$ and let $C_i = \{v_1, \ldots, v_i\}$ for $i = 1, \ldots, n - 1$. If $d_G^\out(C_i) \leq k$ and $d_G^\in(C_i) = 0$ for all $i$ then $|E| \leq \sqrt{2kn}$.

Proof. Let us define the length $l(e)$ of an edge $e = v_iv_j$ to be $j - i$. Note that the total length of $E$ is bounded by

$$l(E) = \sum_{e \in E} l(e) = \sum_{i=1}^{n-1} d_G^\out(C_i) \leq k(n-1) < kn.$$

We claim that any simple edge set $E$ with $l(E) \leq kn$ has at most $\sqrt{2kn}$ edges. Clearly, a maximum size edge set is obtained by picking the shortest edges first, say an edge set $E_\ell$ of all edges of length at most $\ell$ and some $i \leq n - (\ell + 2)$ edges of length $\ell + 1$. Note that there are exactly $n - j$ edges of length $j$ of total length $j(n - j)$. Thus the size and the total length of $E_\ell$ are:

$$|E_\ell| = \sum_{j=1}^{\ell} (n-j) = n\ell - \ell(\ell+1)/2 = \ell \left(n - \frac{\ell+1}{2}\right)$$

$$l(E_\ell) = \sum_{j=1}^{\ell} j(n-j) = n(\ell+1)/2 - \ell(\ell+1)(2\ell+1)/6 = \ell(\ell+1)/2 \left(n - \frac{2\ell+1}{3}\right)$$

So $\ell$ is the largest integer such that $l(E_\ell) \leq kn$ and $i$ is the largest integer such that $i(\ell+1) \leq kn - l(E_\ell)$. The maximum possible number of edges is $m^* = \ell \left(n - \frac{\ell+1}{2}\right) + i$, therefore:

$$\frac{|E|}{n} \leq m^* = \ell \left(1 - \frac{\ell+1}{2n}\right) + \frac{i}{n}.$$

Since $l(E_\ell) + i(\ell+1) \leq kn$ we have:

$$\ell(\ell+1) \left(1 - \frac{2\ell+1}{3n}\right) + \frac{2i(\ell+1)}{n} \leq 2k.$$
Thus to prove that \((m^*/n)^2 \leq 2k\) it is sufficient to prove that:

\[
\left( \ell \left(1 - \frac{\ell + 1}{2n}\right) + \frac{i}{n} \right)^2 \leq \ell(\ell + 1) \left(1 - \frac{2\ell + 1}{3n}\right) + \frac{i}{n} \cdot 2(\ell + 1).
\]

Easy computations show that (recall that \(i < n\)):

\[
\left(1 - \frac{\ell + 1}{2n}\right)^2 < 1 - \frac{2\ell + 1}{3n} \quad \ell \left(1 - \frac{\ell + 1}{2n}\right) + \frac{i}{2n} < \ell + 1.
\]

Thus we get:

\[
\left( \ell \left(1 - \frac{\ell + 1}{2n}\right) + \frac{i}{n} \right)^2 = \ell^2 \left(1 - \frac{\ell + 1}{2n}\right)^2 + \frac{i}{n} \left(2\ell \left(1 - \frac{\ell + 1}{2n}\right) + \frac{i}{n} \right) < \ell(\ell + 1) \left(1 - \frac{2\ell + 1}{3n}\right) + \frac{i}{n} \cdot 2(\ell + 1),
\]

as required. ▶

Let now \(G\) and \(v_1, \ldots, v_n\) be as in Lemma 6. Note that for any node subset \(U \subseteq V\), the subsequence \(u_1, \ldots, u_{|U|}\) of \(v_1, \ldots, v_n\) of the nodes in \(U\) and the subgraph \(G[U] = (U, E_U)\) induced by \(U\) satisfy the conditions of Lemma 6, implying that \(|E_U| \leq \sqrt{2k|U|}\).

This concludes the proof of Lemma 4, and thus also the proof of Theorem 2 is complete.

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