Floquet topological photonic crystals with temporally modulated media

YAO-TING WANG,1,2,* YA-WEN TSAI,3 AND WENLONG GAO4

1Department of Physics, Imperial College London, London SW7 2AZ, UK
2Department of Mathematics, Imperial College London, London SW7 2AZ, UK
3Department of Material Science and Engineering, National Tsing Hua University, Hsin-Chu 30095, Taiwan
4Department of Physics, Paderborn University, Warburger Straße 100, Paderborn 33098, Germany
* ywang14@ic.ac.uk

Abstract: We show that Floquet topological insulating states can exist in two-dimensional photonic crystals made of time-variant optical materials. By arranging the modulating phases, it facilitates effective gauge fields that give rise to topological effects. The band structures demonstrate the existence of topologically non-trivial bandgaps, thereby leading to back-scattering immune unidirectional edge states owing to bulk-edge correspondence. With these first-principle numerical results, we then verify the topological order for every Floquet band via Wilson loop approach. In the final paragraph, the possible experimental implementation for Floquet topological photonics is also discussed.

1. Introduction

Over the past few decades, the research of temporally modulated systems has been discussed but yet to be fully explored [1–9]. Due to the instability of time-varying structures, the progress of wave physics was predominately made in the nature of time-independent systems. However, the recent studies toward this subject have indicated that a time-dependent system could be stable so long as the modulation satisfies time-periodicity. The finding has attracted great attention because the research into this nascent subject of physics is thus initiated. To date, numerous fascinating phenomena have been proposed in classical wave systems, such as non-reciprocal transports [10], effective gauge fields [11–13], and wave amplifications [14]. On the other hand, in condensed matter theory the material with periodically dynamic modulation could generate novel propagating modes which travel along the boundary unidirectionally [15] and this unique state originates from the band topology carrying non-zero Chern numbers. It is worth noting that as long as this topological index remains unchanged, the wave propagation is robust against backscattering from defects and impurities. Recently, this temporally modulating scheme has been applied to classical wave systems as well as condensed-matter regime [16]. Particularly in optical systems, photonic Floquet topological insulators have been proposed and experimentally verified in periodically arranged coupled waveguides [17].

Despite the successful realization of waveguide optics, generating Floquet topological insulating states in dynamically modulated optic media remains significantly challenging as such materials do not exist in nature. Therefore, in this article a feasible way to implement time-periodic perturbation and Floquet topological photonic crystals (FPhC) is proposed. Starting from investigating Floquet band structures, topological edge states are verified by numerical calculation of Chern numbers. Then the field profiles showing robustness against defects are demonstrated. Lastly, a Floquet topological metamaterial consisting of metallic wires with temporal modulations is discussed for a viable experimental implementation.
2. Floquet band structure

In Fig. 1(a), we begin by considering a two-dimensional (2D) FPhC with hexagonal lattice in the microwave range. Each perfect electric conductor (PEC) cylinder is surrounded by three dielectrics given by

$$\varepsilon_j = \varepsilon_m [1 + \delta \cos(\Omega t + \phi_j)] ,$$

where $\varepsilon_m$, $\delta$, and $\Omega$ are dielectric constant, modulating strength, and modulating frequency, respectively. The indices $j = 1, 2, 3$ label different modulating phases. As indicated in Ref. [16], this phase arrangement between modulated dielectrics generates an effective magnetic field, which gives rise to topological bandgaps. The radius of PEC cylinder is 0.44a and the thickness of surrounding temporal dielectrics is 0.04a, where $a$ denotes lattice constant. These high filling-ratio PEC cylinders are implemented to form four-band localized modes because these modes interact more efficiently with modulated media. For TE modes, we substitute $E_z = \sum_n E_{\varepsilon,n} e^{-i(\omega t + n\Omega t)}$ into the wave equation and obtain [16]

$$\nabla \cdot \frac{1}{\mu} \nabla E_{\varepsilon,n} + \frac{(\omega + n\Omega)^2}{c^2} \left[ \varepsilon_m E_{\varepsilon,n} + \frac{\delta \varepsilon_m}{2} (e^{-i\phi} E_{\varepsilon,n-1} + e^{i\phi} E_{\varepsilon,n+1}) \right] = 0 .$$

Throughout this article, we assume the permeability of all the materials is unity, whereas the matters with magnetic response should not significantly affect the result. Numerically, as the modulation strength is weak, the harmonic index $n$ in Eq. (2) can be truncated from -2 to 2 that contributes the major composition of the total electric field. By employing FEM-based commercial software COMSOL 5.4, in Fig. 1(b) the blue dash lines represent the band structure in the absence of modulation and a four-band configuration that can be analyzed by a dipolar model is shown [18]. In this unmodulated band structure, there are two quadratic topological degeneracies at $\Gamma$ point between 1\textsuperscript{st} and 2\textsuperscript{nd} bands, and 3\textsuperscript{rd} and 4\textsuperscript{th} bands, and one linear (Dirac) degeneracy at K point between 2\textsuperscript{nd} and 3\textsuperscript{rd} bands. Due to the spin-like nature of dipolar fields $(p_x \pm ip_y)$, when a rotation-related field is applied, such as constant magnetic fields [19], Coriolis forces [20], or synthetic gauge fields [21], all the degeneracies are lifted owing to the time-reversal symmetry breaking. This can be seen through the modulated case with perturbation strength $\delta = 0.15$ in Fig. 1(b). The red solid lines in Fig. 1(b) demonstrate a quasi-frequency diagram with four isolated bands. Due to the existence of the effective gauge field, these bands lead to topologically protected edge modes when a boundary is truncated.
3. Band topology analysis

To confirm the topological order for each band, here we have numerically calculated topological orders to further identify their band topology. Since there exists no coupling between quasi-frequency structures, the topological states are therefore characterized by non-zero Chern numbers. The Chern number \(C_n\) for every band \(n\) is defined by the closed line integral of Berry connections around the first Brillouin zone (BZ), \([22,23]\)

\[
C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_1 \int_{-\pi}^{\pi} dk_2 A_{n,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta_{n,k_1}, \tag{3}
\]

where \(k = (k_1, k_2)\) is Bloch wave-vector, \(A_{n,k} = -i \langle E_{z,n} | \partial_{k_1} | E_{z,n} \rangle\) are Berry connections for the \(n\)-th band, and \(\theta_{n,k_1} = \int_{-\pi}^{\pi} dk_2 A_{n,k}\) is the evolution of the Berry phase for \(n\)th band with fixed \(k_1\) in the BZ. According to Wilson loop approach \([23–26]\), Chern numbers are equivalent to winding numbers of Berry phase evolution, which can be calculated from the eigenvalues \(w_n\) of the matrix \(W\). This matrix \(W(k_1)\), which is defined by a product of the \(N \times N\) Berry connection matrices along the Wilson loop, reads

\[
W(k_1) = \prod_{j=1}^{N} \langle E_{z,n} (k_1, k_2, j) | E_{z,n} (k_1, k_2, j+1) \rangle, \tag{4}
\]

where \(|E_{z,n}\rangle\) is Bloch function evaluated via COMSOL. With the matrix \(W(k_1)\), the corresponding eigenvalues \(w_{n,k_1}\) give rise to Berry phases, namely,

\[
\theta_{n,k_1} \equiv -\text{Im} \log(w_{n,k_1}). \tag{5}
\]

We then present the evolution of Berry phases for an effective 1D system with \(k_1\) in Fig. 2 and Chern numbers can be extracted in the following manner: within the region \(k_1 \in [-\pi, \pi]\), \(\theta \in [-\pi, \pi]\), one can map the evolution line of Berry phases onto the BZ, thereby determining the topological invariant via the number of times the Berry phases wind around the BZ. Due to time-reversal symmetry breaking, the Berry phases of the four isolated bands in Fig. 2 (red solid lines) are defined by Eq. (5). As shown in Figs. 2(a) and 2(d), the winding numbers of the 1\(^{st}\) and 4\(^{th}\) bands demonstrate that the Chern numbers are \(-1\) and \(1\). Similarly, in Figs. 2(b) and 2(c), the Berry phases of the 2\(^{nd}\) and 3\(^{rd}\) bands present zero winding numbers, thereby implying that the corresponding bands have zero Chern numbers.

![Fig. 2. Berry phase evolutions demonstrates the winding number for each band. For band 1(4), the evolution circulates once and then accumulates an additional negative(positive) \(\pi\) shift, leading to Chern number of a minus(plus) one. In contrast, Chern numbers of band 2 and 3 are both zero as there exist no circulation within Brillouin zone.](image-url)
4. Robust one-way edge modes

The emergence of topological boundary modes may be one of the most essential properties in topological materials. Based on the preceding analysis, topological one-way edge states are expected between the boundaries with distinct topology in accordance with bulk-edge correspondence. As the Chern numbers from the bottom to top bands in Fig. 1(b) are respectively given by -1,0,0,1, three corresponding bandgaps are topologically non-trivial, thereby supporting the edge states with unidirectional nature. To validate this argument, Fig. 3(a) depicts the projected band structure that presents topological edge states obtained by numerically calculating

![Projected Band Structure](image)

Fig. 3. (a) Projected band structure with truncation along x axis shows topological edge states (colored in red and green) within non-trivial bandgaps. (b) The eigen-solutions of three non-trivial edge states demonstrate the field confinement near the top and bottom PEC boundaries. At frequency equal to 3.0924, the field profiles illustrate unidirectional edge modes (yellow arrows indicate the direction of propagation) with topological protection in (c) and (d). The inset illustrates the top view of the structure and the star sign labels where the source is located.
a 1×10 supercell lattice with truncation in x direction. As expected, one-way propagating modes arise within every bandgap, and their numbers obey bulk-edge correspondence respectively. In the following, we will focus on the middle topological edge modes since the second bandgap is relatively larger than the others. Excited by a line current source with frequency equals 3.0924, the field profile demonstrates that electromagnetic waves travel along the boundary unidirectionally and propagate robustly against the defect, as shown in Fig. 3(b). In addition, when the phase arrangements flip sign, i.e. \((\phi_1, \phi_2, \phi_3) = (0, 2\pi/3, 4\pi/3)\), the wave in Fig. 3(c) propagates along the counter-clockwise direction due to the presence of negative group velocity, but the topological nature remains as it propagates around a sharp corner without reflections.

5. Possible experimental implementation

In this section, the possible experimental realization is discussed. To implement the proposed temporal dielectrics, a metamaterial scheme in microwave region may be employed as this frequency range suits the use active electrical elements such as time-variant inductors [27]. Consider a ring of conducting thin wires circulating around a PEC cylinder, as shown in Fig. 4(a). Each wire, whose radius equals 0.015a, is connected with a time-variant inductor to introduce temporal modulation. With TE polarization, the current density \(J_z\) in a lossless wire can be expressed as [27–28]

\[
d(L_m J_z) = \varepsilon_0 \alpha E_z,
\]

where \(L_m = L_0 + L_1 [1 + \delta \cos(\Omega t + \phi_j)]\). \(L_0\) and \(L_1\) are the original inductor from wires and the added time-modulated inductor, respectively. \(\alpha\) is a coefficient defined by \(\omega_p^2 (L_0 + L_1)\), where \(\omega_p\) denotes Drude plasma frequency. Also, the wave equation for electric field \(E_z\) is

\[
\nabla \cdot \left( \frac{1}{\mu} \nabla E_z - \frac{\varepsilon}{c^2} \frac{\partial^2}{\partial t^2} E_z - \mu_0 \frac{\partial}{\partial t} J_z \right) = 0.
\]

Substituting \(J_z = \sum_n J_{zn} e^{-i(\omega + n\Omega)t}\) and \(E_z = \sum_n E_{zn} e^{-i(\omega + n\Omega)t}\) into Eqs. (6) and (7), two coupled matrix differential equations are then obtained. Again, by exploiting COMSOL, Fig. 4(b) depicts Floquet band structure which exists four isolated bands with Chern numbers equal to -1,0,0,1. Similarly, these topologically non-trivial bands lead to unidirectional edge modes once a domain wall is introduced.
6. Conclusion

In conclusion, Floquet topological insulating states in a time-modulated photonic crystal have been proposed. The work starts from a FPhC with PEC cylinders surrounded by three time-variant dielectrics. The Floquet band structure is numerically calculated by COMSOL, and the Chern number analysis further proves the distinct topology for each band. We then study the emergence of unidirectional edge states through introducing a $1 \times 10$ super-cell lattice. The projected band structure demonstrates that topological edge states arise within all the Floquet bandgaps. At frequency $\omega/2\pi c = 3.0924$, the propagation exhibits topological protection since no reflection is generated when the waves hit a defect. Lastly, a possible implementation via a combination of time-variant inductors and metallic wires has been proposed. The coupling between the current density and electric field enables us to add dynamic modulations into the system, leading to Floquet topological insulating phases similar to the previous model.

Funding

Engineering and Physical Sciences Research Council (EP/T002654/1); Leverhulme Trust (Topologically protected flexural waves in thin elastic plates).

Disclosures

The authors declare that there are no conflicts of interest related to this article.

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