An Updated Formalism for Line-driven Radiative Acceleration and Implications for Stellar Mass Loss

Aylecia S. Lattimer©, and Steven R. Cranmer©
Department of Astrophysical and Planetary Sciences, Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO 80309, USA
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Abstract

Radiation contributes to the acceleration of large-scale flows in various astrophysical environments because of strong opacity in the spectral lines. Quantification of the associated force is crucial to understanding these line-driven flows, and a large number of lines (due to the full set of elements and ionization stages) must be taken into account. Here we provide new calculations of the dimensionless line strengths and associated opacity-dependent force multipliers for an updated list of approximately 4.5 million spectral lines compiled from the NIST, CHIANTI, CMFGEN, and TOPbase databases. To maintain generality of application to different environments, we assume local thermodynamic equilibrium, illumination by a Planck function, and the Sobolev approximation. We compute the line forces in a two-dimensional grid of temperatures (i.e., values between 5200 and 70,000 K) and densities (varying over 11 orders of magnitude). Historically, the force multiplier function has been described by a power-law function of optical depth. We revisit this assumption by fitting alternate functions that include saturation to a constant value (Gayley’s $\bar{Q}$ parameter) in the optically thin limit. This alternate form is a better fit than the power-law form, and we use it to calculate example mass-loss rates for massive main-sequence stars. Because the power-law force multiplier does not continue to arbitrarily small optical depths, we find a sharp decrease, or quenching, of line-driven winds for stars with effective temperatures less than about 15,000 K.

Unified Astronomy Thesaurus concepts: Stellar winds (1636); Stellar mass loss (1613); Atomic physics (2063); Radiative processes (2055)

1. Introduction

The study of flows driven by photons has a long history. For example, Johnson (1925) hypothesized that helium atoms could be ejected from a star via radiation pressure, suggesting that photon-driven flows can be responsible for stellar mass loss. We now know that photon-driven outflows exist in various astrophysical environments, including accreting objects such as protostars, cataclysmic variables, and active galactic nuclei (AGNs; Owocki 2004; Puls et al. 2008; Higginbottom et al. 2014). The driving mechanism of these flows by photons is often referred to as ‘radiation pressure,’’ where the force of radiation from the spectral lines acts on the material of the outflow. The very large number of spectral lines in any given ion has a dominant effect on this pressure on the flow material (Castor 1974; Castor et al. 1975). The absorption and reemission of photons in a spectral line of frequency $\nu_0$ results in a radial transfer of momentum, driving a wind outward from the star.

Quantifying and understanding mass and energy flows is critical in the understanding of how affected objects evolve and interact with their surroundings. Massive-star winds can be directly observed in the stellar spectral energy distributions once the star is above a certain threshold luminosity (Kudritzki & Puls 2000). Ultraviolet (UV) observations showing P Cygni profiles and high flow velocities initially suggested that an extension of solar wind theory was insufficient to explain the winds of O and B stars (Cassinelli 1979). Lucy & Solomon (1970) demonstrated that the absorption and reemission of photons by 12 UV resonance lines can drive a wind in O-type stars, consequently driving mass loss in the form of the stellar wind. Castor et al. (1975, hereafter CAK) found that the large number of subordinate lines in any given ion of an atom contributes to the radiation force, proportional to the continuum flux at their specific frequency. Using 900 multiplets of C III, CAK found a mass-loss rate estimate $\sim$100 times greater than those predicted by Lucy & Solomon (1970), leading to the conclusion that the force from the lines should exceed gravity by approximately two orders of magnitude for small optical depths in O-type stars. Therefore, O stars cannot have static atmospheres, as there is no mechanism that can prevent the ejection of the surface layers of the star. This was a major advancement in the understanding of stellar winds. Models based on the theory of line-driven winds have yielded results that agree well with observations of mass-loss and terminal-flow rates (Owocki et al. 1988; Puls et al. 2008; Sundqvist et al. 2019).

Line-driven outflows are also encountered in various other environments. For example, broad absorption lines are present in the UV spectra of quasars, as well as in other wavelengths. In these cases, a blueshift of the lines suggests the presence of winds from the AGN, sometimes with velocities of up to 0.2$c$, making a line-driven disk wind a promising hydrodynamical scenario for AGN outflows (Proga 2007; Risaliti & Elvis 2010). Additionally, Kee et al. (2016) suggested that the strong line-driven winds from OB-type stars with circumstellar disks could drive ablation of the disk’s surface layers on short timescales, which could be a contributing factor to the relative rarity of O-type stars in the Galaxy (see also Kee et al. 2018a, 2018b; Kee & Kuiper 2019). By themselves, winds from massive stars can drive galactic evolution by injecting momentum, energy, and stellar material into the interstellar medium (Kudritzki & Puls 2000).

Previous authors have computed lists of spectral lines and modeled the distributions of the line strength parameters (see, for example, Abbott 1982; Shimada et al. 1994; Gormaz-Matamala et al. 2019). However, these line lists have been comprised of significantly fewer lines than those currently
available. Lucy & Solomon (1970) originally used 12 lines, CAK updated these to include 900 transitions, and Pauldrach (1987) used a list of 10,000 transitions. More recently, Gormaz-Matamala et al. (2019) used a list of \( \sim 900,000 \) lines, and Björklund et al. (2020) used a database of approximately one million lines (see also Sundqvist et al. 2019). In this work we construct a newly updated line list, comprised of 4,514,900 spectral lines. To this end, atomic data are assembled from four separate databases. We also reexamine the validity of the historical assumption of a power law to describe the line strength distribution.

This work primarily aims to provide new insight into the form of the line-force multiplier, beyond that of the CAK power-law form. This new form will also provide an alternative to computationally expensive simulations that use the full line list, such as those described in Gormaz-Matamala et al. (2019). To do this, we make some basic assumptions, such as those of local thermodynamic equilibrium (LTE) and an initial Planck function central source. These assumptions and the calculation process of dimensionless line strengths and weighting functions are described in further detail in Section 2. Section 3 describes the collection and compilation of the spectral lines. Section 4 describes the calculation of the updated force multiplier \( M(t) \), as well as the fitting of a CAK-form power law (Section 4.2) and an alternate function (Section 4.3) to the calculated values. Section 5 describes the calculation of mass-loss rates from both the CAK (Section 5.2) and alternate (Section 5.3) multiplier forms and a comparison of the two (Section 5.5). We end with a discussion of our conclusions in Section 6.

2. Radiative Acceleration of Line-driven Flows

A general way of expressing the radiative acceleration \( \mathbf{g}_{\text{rad}} \) on a parcel of gas is to take the first moment of the equation of radiative transfer. Following Hubeny & Mihalas (2014),

\[
\mathbf{g}_{\text{rad}}(r) = \frac{1}{c} \int d\nu \int d\Omega \mathbf{n} (\kappa_\nu, I_\nu - j_\nu),
\]

where \( \mathbf{n} \) is the unit vector specifying an arbitrary ray path, \( \kappa_\nu \) and \( j_\nu \) are the absorption coefficient \( (\text{cm}^2\ \text{g}^{-1}) \) and emissivity, and \( I_\nu \) is the specific intensity. In the comoving frame of a flow, it is often assumed that \( j_\nu \) is an even function of \( \mathbf{n} \) (so it is canceled out of the above moment integral) and that angle anisotropies of \( \kappa_\nu \) are sufficiently weak to allow it to be taken out of the solid-angle integral. Thus, the expression we use is

\[
\mathbf{g}_{\text{rad}}(r) = \frac{1}{c} \int \kappa_\nu F_\nu(r) d\nu,
\]

where \( F_\nu \) is the radiative flux (photon energy flux). Below, we also write the opacity \( \chi_\nu \) in units of \( \text{cm}^{-1} \) as

\[
\chi_\nu = \kappa_\nu \rho \equiv \sigma_t n,
\]

where it is sometimes useful to use the absorption coefficient \( \kappa_\nu \) or the cross section \( \sigma_t \), and we also define the mass density \( \rho \) and number density \( n \) in units of \( \text{cm}^{-3} \).

From Equation (2), we see that opacity is necessary for any acceleration due to the radiation field to occur; with zero opacity, the radiation cannot interact with the material of the flow. Spectral lines have a dramatic effect on the wind driving due to resonance that occurs when encountering continuum photons, leading to a large cross section. This effect can be strong enough that it is still magnified after being averaged over the entire continuum. Therefore, to find the total radiative force on a parcel of gas of given temperature and density, all the spectral lines that are encountered by the radiation field must be accounted for. The inclusion of as many lines as possible is imperative in developing a more complete understanding of this phenomenon.

2.1. Dimensionless Line Strengths

We follow Gayley (1995) in characterizing the distribution of spectral line strengths as a set of dimensionless ratios \( q_i \) that describe the atomic physics and dimensionless weighting factors \( \tilde{W}_i \) that describe the illumination of the atom from a given spectral energy distribution. The product \( q_i \tilde{W}_i \) represents the full ratio of radiative acceleration due to a specific line \( i \) to the acceleration on free electrons. In such a ratio of accelerations (see Equation (2)), the prefactors of \( 1/c \) cancel out, and we choose to multiply both the numerator and denominator by the mass density \( \rho \). We can write the bound line opacity as

\[
\chi_\nu = \chi_L \phi(\nu),
\]

where \( \phi(\nu) \) is the line profile function. Thus, the ratio of accelerations can be written as

\[
\frac{g_{\text{bound}}}{g_{\text{free}}} = \frac{\int \chi_\nu F_\nu d\nu}{\int \chi_L \phi(\nu) F_\nu d\nu} = \frac{\chi_L \int \phi(\nu) F_\nu d\nu}{\chi_\nu \int F_\nu d\nu}. \tag{5}
\]

For this work, we assume that the environments in question are in LTE. This assumption (also used by CAK) is often set aside when modeling line-driven winds from, e.g., massive stars, but here we retain it to maintain a level of generality concerning the environments in question. Of course, this assumption will need to be reevaluated in future applications to specific systems (see, e.g., Pauldrach 1987). Under the assumption of LTE, the quantity \( \chi_L \) is then given by

\[
\chi_L = \frac{\pi \epsilon^2}{m_e c} f_{ij} n_i (1 - e^{-h\nu_0/kT}), \tag{6}
\]

where \( f_{ij} \) is the semiclassical oscillator strength, where \( i \) and \( j \) are used to refer to the lower and upper atomic levels, respectively. The Thomson scattering opacity \( \chi_\epsilon \) is given by

\[
\chi_\epsilon = \sigma_T n_\epsilon = \left( \frac{8 \pi e^2}{3} \right) n_\epsilon, \tag{7}
\]

where \( \sigma_T \) is the Thomson scattering cross section and the classical electron radius is given by

\[
r_e = \frac{e^2}{m_e c^2}. \tag{8}
\]

Each line profile function \( \phi(\nu) \) is very narrow when integrated over the energy distribution, so we can approximate it to a Dirac delta function when evaluated at frequency \( \nu_0 \):

\[
\int \phi(\nu) F_\nu d\nu \approx F_{\nu_0}. \tag{9}
\]

We can additionally define the frequency-integrated flux \( F \) as

\[
\int F_\nu d\nu \equiv F. \tag{10}
\]
We can then define a dimensionless weighting ratio
\[
\hat{W}_i = \frac{\nu_i F_{0, \nu}}{F},
\]  
which accounts for the flux integrals in Equation (3).

In this paper, we maintain a generality of application by assuming a Planck function for the illuminating flux. We use a temperature \(T\), presumed equal to the local temperature of the gas \((T = T_{\text{rad}} = T_{\text{eff}})\), to characterize this Planck function (Noebauer & Sim 2015; Gormaz-Matamala et al. 2019). Although some authors have suggested the electron temperature is a function of the effective temperature, the radiative temperature is often taken to be equal to \(T_{\text{eff}}\) in the Planck case (see, for example, Puls et al. 2000). This assumption is commonly used for wind-driving circumstellar regions near massive-star photospheres (e.g., Drew 1989; Kee et al. 2016), in which radiative cooling in an isothermal gas efficiently maintains a nearly constant temperature \(T\), roughly equal to the stellar effective temperature \(T_{\text{eff}}\). Thus, we specify
\[
F_{\nu, 0} = \frac{2\pi h\nu_0^3}{e^{\hbar\nu_0/k_B T} - 1},
\]
and the frequency-integrated flux is given by
\[
F = \sigma T^4,
\]
where \(\sigma\) is the Stefan–Boltzmann constant. Since these are dependent on the wavelength of the transition, there is a unique weighting parameter \(\hat{W}_i\) for each line in every ion and temperature \(T\).

Using Equation (11), we can write the ratio of the bound acceleration to the free acceleration as
\[
\frac{g_{\text{bound}}}{g_{\text{free}}} = \frac{\chi_i \lambda_i \hat{W}_i}{\chi_e \nu_0} = \frac{\chi_i \lambda_i}{\chi_e} \hat{W}_i,
\]
and the dimensionless line strength parameter \(q_i\), as
\[
q_i = \frac{3 \lambda_0}{8} f_{ij} n_i (1 - e^{-\hbar\nu_0/k_B T}) \hat{W}_i.
\]

Finally, combining Equations (6), (7), and (14), we have
\[
\frac{g_{\text{bound}}}{g_{\text{free}}} = \frac{3 \lambda_0}{8} f_{ij} n_i (1 - e^{-\hbar\nu_0/k_B T}) \hat{W}_i,
\]
and we define the dimensionless line strength parameter \(q_i\), as
\[
q_i = \frac{3 \lambda_0}{8} f_{ij} n_i (1 - e^{-\hbar\nu_0/k_B T}).
\]

This is similar to Equation (9) of Gayley (1995). However, here we have included the traditional correction factor for stimulated emission.

We also define the sum of the line strengths \(\tilde{Q}\) as
\[
\tilde{Q} = \sum_i q_i \hat{W}_i.
\]

2.2. Ionization Equilibrium

In order to calculate Equation (16), we first must calculate the number density ratio \(n_i/n_e\), which is given by
\[
\frac{n_i}{n_e} = \left( \frac{n_i}{n_{\text{ion}}} \right) \left( \frac{n_{\text{ion}}}{n_{\text{el}}} \right) \left( \frac{n_{\text{el}}}{n_{\text{H}}} \right) \left( \frac{n_{\text{H}}}{n_e} \right).
\]

In LTE, the total number of particles in the lower transition level can be described as a fraction of the total number of particles in the given ionization state:
\[
\frac{n_i}{n_{\text{ion}}} = \frac{g_i e^{-(E_i - E_0)/k_B T}}{U_{\text{ion}}(T)}.
\]
Here \(U_{\text{ion}}\) is the ion-specific temperature-dependent partition function, \(g_i\) is the ground-state statistical weight, \(E_i\) is the energy of the lower level of the transition, and \(E_0\) is the ground-level energy, which was set to zero by the atomic databases used here. It should also be noted that the oscillator strength \(f_{ij}\) (as in Equation (16)) is needed only to form the product \(g_i f_{ij}\) (as in Equation (19)) and never appears alone.

The elemental abundance ratio \(n_{\text{el}}/n_{\text{H}}\) was obtained from tabulated solar abundances (Asplund et al. 2009), whereas the other two quantities in parentheses, \(n_{\text{ion}}/n_{\text{el}}\) and \(n_{\text{H}}/n_e\), were found using the Saha equation:
\[
\frac{n_{i+1}}{n_i} = 2 \frac{U_{i+1}}{U_i} \lambda_i^3 \frac{E_i}{k_B T} \exp \left[ - \frac{E_{i+1} - E_i}{k_B T} \right]
\]
where the thermal de Broglie wavelength of a free electron \(\lambda_e\) is given by
\[
\lambda_e = \frac{h}{\sqrt{2\pi m_e k_B T}}.
\]

To solve Equation (20) for the pairwise ionization fractions \(n_{i+1}/n_i\), we also need to know the electron number density \(n_e\). Therefore, we made an initial estimate for \(n_e\), which was then refined using an undercorrection technique. This was done at the end of each iteration over the Saha equation by tabulating a new estimate of \(n_e\) from the calculated ionization balance, which was then multiplied by the previous estimate. The square root of this product was then used as the estimate of \(n_e\) for the next iteration. We found that 20 iterations were sufficient to reach a stable value for \(n_e\). An example set of calculations is shown in Figure 1, which shows the final converged-upon values of \(n_e\) for our temperature range, for an example density of \(\rho = 10^{-13}\) g cm\(^{-3}\).

The initial estimate for \(n_e\) was given by
\[
n_e = 0.1 \left( \frac{\rho}{m_{\text{H}}} \right).
\]
22 values spanning $10^{-20} - 10^{-10}$ g cm$^{-3}$, a similar density range to that used by Abbott (1982). This broad range is applicable to both massive-star winds as well as other astrophysical environments that exhibit line-driven outflows.

The quantity $n_{el}$ can be found from the initial elemental abundances, here taken from Asplund et al. (2009). These are given in the form of number density ratios to the abundance of hydrogen (i.e., $n_{el}/n_{H}$). From these we find the fractional abundance by mass $\mu$ of each element:

$$\mu = \frac{A_{el}(n_{el}/n_{H})}{\sum_{el} A_{el}(n_{el}/n_{H})},$$

(23)

where $A_{el}$ is the atomic weight of the element. The number density in cm$^{-3}$ for each element can then be found from the total mass density $\rho$, the fractional abundance $\mu$, and the mass of hydrogen $m_{H}$:

$$n_{el} = \frac{\rho \mu}{A_{el} m_{H}}.$$ 

(24)

Knowing $n_{e}$ and the tabulated number density of hydrogen $n_{H}$ from Equations (23) and (24), we can then calculate the quantity $n_{H}/n_{e}$, leaving only the second quantity in Equation (18), $n_{ion}/n_{el}$. For ionization stage $I$ this is given by

$$n_{I} = \left[\frac{n_{II}}{n_{I}} + \frac{n_{III} n_{II}}{n_{III} n_{I}} + \ldots\right]^{-1},$$

(25)

where II and III represent ionization stages I and II of the element in question. Here $n_{I}$ represents $n_{ion}$ as seen in Equation (18). Each fraction in the brackets is given by the Saha equation (Equation (20)). We can similarly isolate $n_{II}/n_{el}$ and the higher ratios to find $n_{ion}/n_{el}$ for any ionization stage—and consequently $n_{ion}/n_{e}$ and $n_{I}/n_{e}$ as in Equation (18)—of any element that we consider. Figure 2 shows an example calculation of $n_{ion}/n_{el}$ for the ionization stages of oxygen. These steps were carried out for all elements from H to Ni. We considered all ionization stages for elements with atomic numbers $Z \leq 10$ and only the first 10 ionization stages for elements with $Z > 10$.

### 3. Atomic Data

#### 3.1. Partition Functions

The LTE partition functions used in Equations (19)–(20) were calculated according to the fitting procedure set forth by Cardona et al. (2010). Namely, for elements with $Z \leq 20$, we used the tabulated fit parameters in the expression

$$U = g_{1jk} + G_{jk} e^{-\varepsilon_{jk} \beta_b T} + \frac{m}{3} (n_{el}^3 - 343) e^{-E_{el}/\beta_b T},$$

(26)

where $g_{1jk}$ is the ground-state statistical weight, and $E_{nqjk}$ is given by

$$E_{nqjk} = \chi_{jk} - \frac{Z^2 \text{ Ry}}{n_{el}^q},$$

(27)

for ionization state $j$ of element $k$. Here $\chi_{jk}$ is the ionization potential, $\text{Ry}$ is the energy of one Rydberg (13.6 eV), and $Z_{eff}$ is the effective ion charge $j+1$. The effective maximum upper-level index $n_{q}$ is given by

$$n_{q} = \frac{q}{2} \left[1 + \frac{1 + 4}{q}ight],$$

(28)

with $\alpha_0$ the Bohr radius. The total number density of the gas $n_{tot}$ is given by

$$n_{tot} = n_{e} + \sum n_{el},$$

(29)

with $n_{e}$ given by the iterative undercorrection process described above, and $n_{el}$ given by Equation (24).

The quantities $m$, $G_{jk}$, and $\varepsilon_{jk}$ are drawn from Table 1 of Cardona et al. (2010). The values of $g_{1jk}$ and $\chi_{jk}$ are taken from CHIANTI for all modeled elements and ionization stages (see Section 3.2). Cardona et al. (2010) did not provide the fit parameters for the partition function for elements of $Z > 20$, and a simple fitting procedure on the available parameters was performed to empirically calculate estimates of these parameters for higher-$Z$ elements. The Cardona $\varepsilon_{jk}$ parameter correlates fairly well with ionization potential with no more than $\sim 20\%$ error, while the $m$ and $G$ parameters are well correlated with each other and weakly correlated with the ground-state statistical weight $g_{1jk}$. These fits are given by

$$\varepsilon_{jk} = \chi_{jk} (0.946 - 0.007 Z_{eff}),$$

(30)

$$m = 4 (g_{1jk})^{0.79},$$

(31)

and

$$G_{jk} = 113 m^{0.66}.$$  

(32)

These were used for all considered elements with $Z > 20$. Because these new fitting parameters were developed for the first time here, we first investigated their impact by repeating the partition function calculations with a simpler low-temperature approximation for elements with $Z > 20$: $U \approx g_{1jk}$. Doing this for the grids of temperature and density discussed below resulted in only negligible differences in the values of $U$.

#### 3.2. Database Selection

To find the total radiative force on a parcel of gas for a given temperature and density, we need to construct the most

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**Figure 2.** The ionization fraction $n_{ion}/n_{el}$ for the ionization stages of oxygen, for a fixed wind density of $\rho = 10^{-15}$ g cm$^{-3}$. 

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comprehensive line list of atomic data possible, as all spectral lines encountered by the radiation field must be accounted for. To this end, we compiled spectral line data from four sources: the National Institute of Standards and Technology (NIST; Kramida et al. 2018), version 9.0 of the CHIANTI database (Dere et al. 1997, 2019), the database of lines used by the radiative transfer code CMFGEN\(^1\) (Hillier 1990; Hillier & Miller 1998; Hillier & Lanz 2001), and the Opacity Project’s TOPbase (Cunto & Mendoza 1992; Cunto et al. 1993). The use of multiple databases was necessary, as there exist gaps in the atomic data available from each individual database.

We retrieved energy-level classifications and wavelength data for each ion. For each selected element and ionization stage, tabulations of line oscillator strengths (i.e., \(g f_i\)) were extracted, along with lower-level energies \(E_i\) and rest-frame wavelengths \(\lambda_0\). These are the parameters necessary to compute the line strength parameter \(q_i\). We did this for all elements up to Ni. For the ionization stages, we retrieved data for each element up to nine times ionized (that is, data were retrieved for ionization stages I through X). For elements with atomic number \(Z < 10\), we retrieved data for all available ionization states, up to fully ionized. In the Appendix, we summarize the process used to determine the database used for each ion, and provide a detailed breakdown of the line counts and the database used for each ion. The final line list contains 4,514,900 lines.

Figure 3(a) shows histograms for the occurrence frequency of different values of \(q_i \bar{W}_i\) for one example choice of local temperature and density. All plotted histograms were constructed using 95 logarithmically spaced bins. Stacked underneath the uppermost curve (corresponding to the total count in each bin) are individual histograms that show the contribution from each individual element. In Figure 3(b) we also show histograms computed from a subset of the CMFGEN and CHIANTI databases that includes only lines that have been observed experimentally (i.e., 589,186 lines out of the full set of 4,514,900 lines). Thus, this panel ignores the vast majority of lines (i.e., 87%) with only theoretically predicted properties. There is a notable drop-off in line strengths below \(q_i \bar{W}_i \approx 10^{-19}\) for the distribution that excludes the theoretical lines. While the contributions by low-Z elements are the same in both cases, this drop-off in line strengths represents a lack of observed lines for high-Z elements, most notably cobalt.

However, in both cases there is a significant contribution to the distribution by high-Z elements, most notably by iron, at high line strengths. For the sake of completeness, we use the line list comprised of both theoretical and observed transitions for the remainder of this work. However, for the purpose of comparison Section 4 includes calculations of the force multiplier for both the full line list and the observed-only subset.

### 4. Calculating and Fitting the Line Force Multiplier

In order to examine the dynamics of the outflows, we first calculated the line force multiplier, a measure of the strength of the radiation force, from our updated list of spectral lines and distributions of \(q_i \bar{W}_i\). We then fit and compared two functions to the resulting distributions.

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\(^1\) The atomic data used here are those that were updated by D.J. Hillier in 2016 (http://kookaburra.phyast.pitt.edu/hillier/cmfgen_files/atomic_data_15nov16.tar.gz).

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### 4.1. Calculating \(M(t)\)

The line acceleration is defined as the radiative acceleration due to electron scattering, multiplied by the line force multiplier \(M(t)\). Here, \(t\) is an optical depth parameter that is independent of the line strength. It is given by

\[
\tau = \kappa_e \rho \frac{dv}{dr} \left| \frac{dv}{dr} \right|^{-1}
\]

(33)

for expanding atmospheres (Sobolev 1957, 1960; CAK). In the case of a static atmosphere, \(t\) is equivalent to the electron scattering optical depth, while in the expanding case \(t\) is less than this depth.

In an expanding wind, we cannot simply take the sum of \(q_i \bar{W}_i = \frac{s_{\text{bound}}}{s_{\text{free}}}\) as found in Equation (15) above. The full force multiplier depends on other radiative transfer effects, such as the “self-shadowing” of the lines, due to differences in the Doppler-shifted local reference frames (Gayley 1995). Therefore, the full calculation of the force multiplier \(M(t)\) from the line list can be written as

\[
M(t) = \eta \sum_i q_i \bar{W}_i \left( 1 - e^{-\tau_i} \right),
\]

(34)

where the geometrical finite-disk factor \(\eta\) is the ratio of the true line force to that derived in the limit of purely radial photons (i.e., it is the same as the ratio \(F\) given by Equation (21) of Gayley 1995). Equation (34) assumes the Sobolev approximation and no overlapping lines, for supersonic flows. The dimensionless optical depth \(\tau_i\) of each line is defined as

\[
\tau_i = \frac{c}{v_{th}} q_i t,
\]

(35)

where \(v_{th}^2 = 2k_B T/m_p\) is the proton thermal speed for all ions of a single temperature.

\(M(t)\) corresponds to the sum over the spectral lines that contribute to the wind. Originally, \(M(t)\) was parameterized by Castor (1974) in terms of the optical depth, depending only on the structure of the wind. However, CAK proposed that \(M(t)\) could take the form of a power law, expressed as

\[
M(t) = \eta k t^{-\alpha},
\]

(36)

where \(k\) and \(\alpha\) are the fit constants of the power law. Because the finite-disk factor \(\eta\) appears in both Equations (34) and (36), we can safely neglect it when performing fits and parameterizations for quantities such as \(k\) and \(\alpha\). We re-examined the assumption presented in Equation (36) that the line force multiplier takes the form of a power law. To do this, we fit both power laws and an alternate fitting function to the values of \(M(t)\) calculated from the updated line list.

Although the CAK \(t\) parameter can be evaluated at any point in a radiatively driven outflow, here we evaluate \(M(t)\) over a logarithmic grid of \(t\) spanning from \(10^{-15}\) to 10. In the limit of \(t \rightarrow 0\) the line-force multiplier \(M(t)\) should become equal to \(\bar{Q}\). It should be noted that Abbott (1982) considered only values of \(t\) from \(10^{-9}\) to 0.1. However, we included values beyond this range in order to better examine the behavior of \(M(t)\). For example, the asymptotic behavior of \(M(t)\) in the limit of small \(t\) is not seen in the range used by Abbott (1982).

In calculating the dimensionless line strength parameter \(q_i\), the weighting function \(\bar{W}_i\), and subsequently the force multiplier \(M(t)\) as described above, we used a grid of 100...
going forward we used only the full line list that includes theoretical lines.

Figure 6 shows the sum $\tilde{Q} = \sum q_i \tilde{W}_i$, which was calculated for each temperature and density. These are compared to values from Gayley (1995), who also provided values of $\tilde{Q}$ as converted from previous works such as Abbott (1982). Note that temperatures corresponding to O and early B spectral types ($\log T$ between 4.2 and 4.6) tend to exhibit values of $\tilde{Q}$ around $10^{-5}$, independent of density, as also found by Gayley (1995). Above and below this range, there is a strong dependence of $\tilde{Q}$ on density, with $\tilde{Q}$ varying drastically at both very low temperatures ($\log T \lesssim 4.0$) and very high temperatures ($\log T \gtrsim 4.7$). This probably indicates that departures from LTE Saha ionization balance are more important to take into account for these temperatures.

4.2. CAK Power-law Fitting

We began by fitting a power law of the form proposed by CAK (Equation (36)) to the calculated values of $M(t)$. This fitting was performed using the Levenberg–Marquardt method of least-squares fitting. As is readily apparent from Figure 4, the full range of calculated $M(t)$ values cannot be described by a single power law. Therefore, we fit the initial CAK power-law form only to values of $\log(t) > -3$, in order to exclude the flat portion of the curve.

As with $\tilde{Q}$, we also compared the fitted values of $\alpha$ and $k$ to those from previous work. Figure 7 shows a comparison to Abbott (1982), Shimada et al. (1994), Gayley (1995), and Gormaz-Matamala et al. (2019), with good agreement with their values, most especially for densities that fall in the middle of our considered density range. Note that for the lowest values of $T$, our values for the power-law slope $\alpha$ never get as small as some of the plotted literature values, which are of order $\sim 0.45$. This may be explained if the power-law fits performed in other papers included portions of the flattened parts of $M(t)$. For example, Abbott (1982) calculated values of $\alpha$ and $k$ for the range $-6 \leq \log(t) \leq -1$ for a density grid similar to our own. We found that for low temperatures, the flat portions of the $M(t)$ curve begin at values of $t$ as high as $\log(t) \sim -2.5$. Inclusion of these flat portions during fitting would yield shallower slopes than those found in this work.

Despite acceptable agreement with previous works for these two parameters ($\alpha$, $k$), this preliminary power-law form of the force multiplier presents a decent fit to the calculated values of $M(t)$ for only a narrow range of $t$, namely between $\log(t) \approx -3$ and $\log(t) \approx 0$.

4.3. Alternate Fitting Function

We present an alternate fitting function in the form of a saturated power law, given by

$$M(t) = \frac{\eta \tilde{Q} k}{(k^s + \tilde{Q}^t \alpha^s)^1/s}.$$  \hspace{1cm} (37)

In this case, $\alpha$, $k$, and $s$ are fit parameters, and $\tilde{Q}$ is the calculated value as found from the line list and shown in Figure 6. The parameter $s$ is a sharpness parameter that determines how rapidly the function transitions from the low-$t$ to the high-$t$ limit. This function reduces to the CAK power-law form in the limit of large $t$, and in the limit of small $t$ reduces to
Figure 4. The force multiplier $M(t)$: (a) Varying with temperature from $T = 5200$ K to $T = 70,000$ K for a constant density of $\rho = 10^{-20}$ g cm$^{-3}$, for observed and theoretical transitions. (b) Varying with temperature from $T = 5200$ K to $T = 70,000$ K for a constant density of $\rho = 10^{-20}$ g cm$^{-3}$, for observed transitions only. (c) Varying with density from $\rho = 10^{-20}$ g cm$^{-3}$ to $\rho = 10^{-10}$ g cm$^{-3}$ for a constant temperature of $T = 5200$ K, for theoretical and observed transitions. (d) Varying with density from $\rho = 10^{-20}$ g cm$^{-3}$ to $\rho = 10^{-10}$ g cm$^{-3}$ for a constant temperature of $T = 5200$ K, for observed transitions only.

Figure 5. Ratio of the calculated force multiplier for line lists comprised of all lines to that for observed lines only, shown here for an example value of $t \approx 1$.

Figure 6. Evolution of $\tilde{Q}$ for our chosen density and temperature range, compared to values from Table 1 of Gayley (1995; black crosses) and Table 2 of Puls et al. (2000; black triangles).
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Figure 7. Comparison of CAK power-law parameters $\alpha$ (a) and $k$ (b) with values from Abbott (1982), Shimada et al. (1994), Gayley (1995), Puls et al. (2000), and Gormaz-Matamala et al. (2019), for our chosen range of densities.

$\vec{Q}$, consistent with the behavior of the calculated values of $M(t)$.

Figure 8 shows the resulting fits compared to the calculated $M(t)$ values, for an example density of $\rho = 10^{-20}$ g cm$^{-3}$. The above function was found to be flexible enough to fit the calculated $M(t)$ values quite accurately. For the full grid of parameters ($T$, $\rho$, and $t$) we computed the fractional difference $D$ between the numerical and best-fit values of $M(t)$. The median of this distribution is $\langle D \rangle = 1.95\%$, and only a tiny fraction of the model values (1.03\%) have values of $D > 30\%$.

The fits of the alternate function and the CAK power law are comparable for values of $\log t \gtrsim -4$. However, for values of $\log t \lesssim -4$, the fit achieved using the alternate function is drastically improved over that of the CAK power law. As can be seen in Figure 4, the value of $t$ at which the CAK power law begins to fail also strongly depends on temperature and density. Additionally, we did not include a high opacity cutoff in our new form of $M(t)$, such as that suggested by Gayley (1995). Upon calculation of the force multiplier out to high $t$ ($t \sim 10^{15}$), we found that $M(t)$ continues to decrease as a power law rather than as an exponential drop-off at high opacities, generally approaching the form $M(t) \propto t^{-1}$ as temperature and density increase. Therefore, we did not impose a cutoff of the force multiplier at high opacities.

Figure 9 shows the dependence of the saturated power-law fit parameters $\alpha$, $k$, and $s$ on temperature and density. Also shown is the evolution with temperature and density of $M(t)$ at $t = 1$. At high values of $t$ such as $t = 1$, the force multiplier behaves as a power law. The values of $M(t = 1)$ provide an alternate estimation of the fit parameter $k$. Comparison of Figures 9(a) and 9(b) additionally provides insight into the deviation of the fitted saturated power law (a) from the actual calculated value (b). We also include an estimate of the CAK critical-point density across the range of effective temperatures (see Section 5.4). Additionally, we indicate a locus of parameters at which the Saha ionization balance produces equal amounts of Fe III and Fe IV. The relevance of this to a proposed explanation for the phenomenon of “bistability” in line-driven winds is discussed further in Section 6.

5. Mass-loss Estimates for Massive Stars

In addition to determining a function to better describe the line force multiplier $M(t)$, we also explore the consequences of this updated form on the calculated mass-loss rates of massive stars. We do this by calculating mass-loss rates for both the traditional power-law form and our newly updated alternate form.

5.1. General Mass-loss Solution

We begin with the time-steady radial component of the momentum conservation equation. To approximate the supersonic winds of OB stars, we omit the gas pressure gradient term. Because gas pressure does not play a fundamental role in highly supersonic winds such as the ones considered in this work, we can safely neglect these terms (see, e.g., Gayley 2000; Owocki 2004). Doing this, we have

$$\frac{\partial v}{\partial r} = -\frac{GM_*}{r^2} + s_{\text{free}} + s_{\text{bound}}. \quad (38)$$
Here, \( v(r) \) is the radially dependent wind velocity, \( M_* \) is the mass of the central star, and \( G \) is the gravitational constant. The free radiative acceleration due to Thomson scattering \( g_{\text{free}} \) can be written as the Eddington factor \( \Gamma \) times the gravitational acceleration. With the stellar bolometric luminosity \( L_* \), the Eddington factor \( \Gamma \) can be written as

\[ \kappa_e \frac{L_*}{4\pi G M_*^2} \]

with the mixture-dependent Thomson scattering coefficient given by

\[ \kappa_e = \frac{\sigma_T n_e}{\rho} \approx \frac{\sigma_T}{m_H} \left( \frac{1 + X}{2} \right), \]

where \( X \) is the hydrogen mass fraction. The final approximation above is provided only for reference in the limit of full ionization (see, e.g., Mihalas 1978). In all results shown below, we use the self-consistent values of \( n_e \) computed earlier from the Saha equation. For the values of \( X \) and \( Y \), we use the bulk-composition chemical abundances of H and He given in Table 4 of Asplund et al. (2009), with \( X = 0.7154 \) and \( Y = 0.2703 \).

Using Equations (38)–(40), we can write the equation of motion as

\[ v \frac{\partial v}{\partial r} = \frac{GM_*}{r^2} \left( -1 + \Gamma + \Gamma M(t) \right). \]

As in Gayley (1995), we define the dimensionless wind acceleration factor \( w \) as

\[ w = \frac{r^2 v}{GM_* (1 - \Gamma)} \frac{dv}{dr}. \]

This allows us to write Equation (41) as

\[ F_1 = w + 1 - \frac{\Gamma}{1 - \Gamma} M(t) = 0. \]

We use this form because the CAK critical-point solution requires at least two conditions to be true for a time-steady wind:

\[ F_1 = 0 \quad \text{and} \quad F_2 = \frac{\partial F_1}{\partial w} = 0. \]

Using mass conservation, we can write the density as \( \rho = \dot{M}/(4\pi r^2v) \). Combining the definitions of the CAK \( t \) parameter and the wind acceleration factor \( w \), as given in Equations (33) and (42), respectively, we can then write \( t \) as
\( t = t_m/w \), where
\[
t_m = \frac{v_{\text{th}}cM\Gamma}{L_\alpha(1 - \Gamma)}. \tag{45}
\]
This is equivalent to Equation (52) of Gayley (1995). We now can solve Equation (44) for \( w \) and \( t_m \) to determine the mass-loss rate.

### 5.2. Mass-loss Rates for the CAK Multiplier

We begin with the traditional CAK power-law form of the force multiplier as given in Equation (36), assuming that the parameters \( \alpha \) and \( k \) are known for a given set of lines. Extending beyond CAK, we also include the finite-disk factor \( \eta \), which has a simple form for a CAK-like force multiplier when evaluated at the stellar surface:
\[
\eta \approx \frac{1}{1 + \alpha} \tag{46}
\]
(Kudritzki et al. 1989). Therefore we can write the critical-point conditions \( F_1 \) and \( F_2 \) as
\[
F_1 = w + 1 - Cw^{\alpha} = 0, \tag{47}
\]
\[
F_2 = 1 - \alpha C w^{\alpha - 1} = 0, \tag{48}
\]
where \( C = \eta \Gamma k'(1 - \Gamma)^{-1}m^{-\alpha} \). Solving \( F_2 \) for \( C \) and re-solving \( F_1 \) for \( w \), we find the analytic solutions
\[
w = \frac{\alpha}{1 - \alpha} \quad \text{and} \quad C = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}}. \tag{49}
\]
Using this solution for \( C \), we can solve for \( t_m \), which can then be converted to \( M_{\text{CAK}} \). Combining Equations (47)–(49) and recalling the definition of \( t_m \) from Equation (33), we find the mass-loss rate \( M_{\text{CAK}} \):
\[
M_{\text{CAK}} = \frac{L_\alpha(1 - \Gamma)}{v_{\text{th}}c\Gamma} \left[ \frac{\alpha^\alpha \eta \Gamma k'(1 - \Gamma)^{1-\alpha}}{(1 - \Gamma)^{1-\alpha}} \right]^{1/\alpha}. \tag{50}
\]
It is worth mentioning here that the apparent dependence of \( M_{\text{CAK}} \) on the Doppler thermal width \( v_{\text{th}} \) is in fact only a fiducial dependence, due to the definition of the \( t \) parameter (Equation (33)), which introduces \( v_{\text{th}} \) and is subsequently present in Equation (45). While a reformulation of \( t \) could eliminate this dependence, we choose to carry it through our calculations in order to maintain a level of comparability with previous works, most notably with CAK.

### 5.3. Mass-loss Rates for Updated Formalism

Next, we solve the critical-point equations for our more general form of \( M(t) \), given by Equation (37). Combining Equations (43) and (44), we find \( F_1 \) and \( F_2 \) are now given by
\[
F_1 = w + 1 - B \left[ \frac{kw^\alpha}{(kw^{\alpha} + \tilde{Q}^*_{\text{m}})^{\alpha}} \right] = 0, \tag{51}
\]
\[
F_2 = 1 - B \alpha \left[ \frac{(kw^{\alpha-1})(\tilde{Q}^*_{\text{m}}^{\alpha})}{(kw^{\alpha} + \tilde{Q}^*_{\text{m}}^{\alpha})^{\alpha+1}} \right] = 0, \tag{52}
\]
where \( B = \eta \Gamma \tilde{Q}^*/(1 - \Gamma) \). Solving Equation (51) for
\[
\tilde{Q}^*_{\text{m}} = \left( \frac{kw^\alpha B}{w + 1} \right)^{\alpha} - k^\alpha w^\alpha, \tag{53}
\]
we substitute the result into Equation (52). This gives
\[
0 = wB^\alpha + \alpha(w + 1)[(w + 1)^{\alpha} - B^\alpha]. \tag{54}
\]
In the \( \tilde{Q} \to \infty \) (\( B \gg 1 \)) limit, Equation (54) reduces to the CAK behavior, with a solution of \( w = \alpha/(1 - \alpha) \) as in Equation (49). In the opposite limit (\( B \to 0 \)), the above equation is solved only by \( w \approx -1 \), which is unphysical (see below).

For the nominal case of \( s = 1 \), Equation (54) reduces to a quadratic equation with two unique solutions for \( w \):
\[
w = -\frac{B(1 - \alpha)}{2\alpha} - 1 \pm \frac{B(1 - \alpha)}{2\alpha} \sqrt{1 + \frac{4\alpha}{B(1 - \alpha)^2}}. \tag{55}
\]
However, in the more general case of \( s = 1 \) Equation (54) must be solved numerically. In this work this is done using the Newton–Raphson method. Once the acceleration factor \( w \) is known, we can then find \( t_m \) from Equation (53), which in turn allows us to find the mass-loss rate \( M_{\text{th}} \) from Equation (45).

It is relevant that in calculating the mass-loss rates that result from both the CAK and alternate forms, we discard any sets of parameters that result in \( \Gamma \geq 1 \) (since we do not include multiple-scattering effects or super-Eddington flows). Similarly, we discard as unphysical any negative solutions of the wind acceleration factor \( w \). In Equations (54) and (55) above, the condition \( w \geq 0 \) corresponds to \( B \geq 1 \). Since both \( \eta \) and \( (1 - \Gamma) \) tend to be order-unity quantities, the condition for physically realistic solutions is \( \Gamma \geq \tilde{Q}^{-1} \). With typical values of \( \tilde{Q} \) of a few thousand, this implies that whenever \( \Gamma \) drops below \( \sim 10^{-3} \), a steady-state line-driven wind may not be possible.

### 5.4. CAK Critical Point and Stellar Parameters

Although the force multiplier \( M(t) \) is a function of both \( T \) and \( \rho \), it is possible to characterize much of the physics by evaluating \( M(t) \) at the critical point of the flow (see, e.g., CAK; Abbott 1980; Pauldrach et al. 1986; Bjorkman 1995). To do this, we need to know both the temperature and the density at the critical point. For an isothermal wind, \( T \) at the critical point is given more or less by the photospheric effective temperature. The only way to estimate the density \( \rho_{\text{crit}} \) at the critical point, though, is to have an associated “initial guess” for the full radial dependence of the plasma parameters. We provide this initial guess for a set of idealized main-sequence stellar properties (see Section 5.5) by using the modified CAK (mCAK) numerical code developed by Cranmer & Woosley (1995). This code solves the equations of mass and momentum conservation for the power-law CAK force multiplier and a standard version of the uniformly illuminated finite-disk factor \( \eta \). When considering finite sound-speed effects, it is necessary to solve simultaneous singularity and regularity conditions for the properties of the critical point (CAK).

For a sequence of stellar properties spanning effective temperatures between 5920 and 46,000 K (see below for details), we produced mCAK models with fixed line-force constants \( \alpha = 0.6 \) and \( k = 0.5 \). These models all exhibited critical points at radial distances between 1.01 and 1.02 times the photospheric stellar radius, critical wind speeds between about 50 and 120 km s\(^{-1}\) (i.e., typically about 3% of the asymptotic or terminal wind speeds of 2000–3000 km s\(^{-1}\)), and values of \( \rho_{\text{crit}} \) between \( 10^{-17} \) and \( 10^{-11} \) g cm\(^{-3}\). Figure 9 shows this trend in two-dimensional \((T, \rho)\) diagrams. A power
law of the form

\[ \rho_{\text{crit}} = (6.33 \times 10^{-16} \text{ g cm}^{-3}) \left( \frac{T}{10^4 \text{ K}} \right)^{6.2} \]  

is reasonably successful at capturing this trend as well. Figure 10 shows how the calculated parameter \( \tilde{Q} \) and the fit parameters \( \alpha, k, \) and \( s \) vary with temperature along the CAK critical-point curve, which corresponds to the black dashed line shown in Figure 9.

For the remainder of this work we use the stellar color and effective temperature sequence in Table 5 of Pecaut & Mamajek (2013)\(^2\) to calculate mass-loss rates using the methods described above. Figure 11 shows the continuous functions fit to the data from this table for both temperature–luminosity and temperature–mass relationships. These take the form of a power law and a third-order polynomial, respectively, given by

\[ \log(L/L_\odot) = 6.73 \log(T) - 25.47 \]  

and

\[ \log(M/M_\odot) = 1.29 \log(T)^3 - 15.44 \log(T)^2 + 63.02 \log(T) - 87.23. \]

For the purposes of this work, we assume that the wind temperature \( T \) remains equal to the stellar effective temperature \( T_{\text{eff}} \). These fits were done so that mass and luminosity could be calculated for any temperature along our temperature range. These fits were only calculated for the range of data that our temperature range encompasses in order to disregard behavior at the low-temperature end of the main sequence.

### 5.5. Comparison of Mass-loss Rates

Figure 12 shows a preliminary comparison of \( \dot{M}_{\text{CAK}} \) and \( \dot{M}_{\text{alt}} \). We hold steady the parameters \( \tilde{Q}, \alpha, k, \) and \( s \), with only temperature \( T \) varying, and mass and luminosity dependent on temperature as described above. There is good agreement between \( \dot{M}_{\text{CAK}} \) and \( \dot{M}_{\text{alt}} \) at high temperatures. However, there is a steep drop-off exhibited at \( \sim 12,000 \text{ K} \) in \( \dot{M}_{\text{alt}} \), whereas \( \dot{M}_{\text{CAK}} \) continues a power law described by

\[ \dot{M}_{\text{CAK}} \propto T^{8.51}. \]
Using Equation (57), this can also be written as

\[ M_{\text{CAK}} \propto \left( \frac{L_*}{L_\odot} \right)^{1.26}. \]  

(60)

This is in comparison to the common form \( M \propto L_*^{1/\alpha} \). For a value of \( \alpha = 0.7 \) as in Figure 12, this would take the form \( M \propto L_*^{42} \), whereas Equation (60) shows a slightly weaker dependence on luminosity. In Figure 12, the black curve for \( M_{\text{alt}} \) shows a strong drop-off, or quenching, which is a result of the flattening of the force multiplier at low values of \( t \).

Next we introduce varying values of \( \alpha, k, \tilde{Q}, \) and \( s \). These parameters vary according to temperature and density, as seen in Section 4.3. For the remainder of this work we consider only the version of \( M_{\text{alt}} \) in which all four parameters (\( \alpha, k, \tilde{Q}, s \)) are allowed to vary with temperature and density. Figure 13 shows the variations of \( M_{\text{alt}} \) with \( T \) and \( \rho \). Figure 14 compares the mass-loss rates resulting from the CAK and alternate forms. As in Figure 12, we see a sharp drop-off of \( M_{\text{alt}} \) in comparison to the CAK form, commonly occurring at \( \log T \approx 4.2 \). At high densities (\( \rho > 10^{-11} \text{ g cm}^{-3} \)) the departure from the CAK form is less pronounced, although still present. It should be noted that in these figures the apparent plotted end of \( M_{\text{alt}} \) between 10,000 and 20,000 K is a result of discarding any wind solution that results in a negative wind acceleration factor \( w \). Physically, this represents regions of the parameter space where the wind is quenched, a phenomenon that is not evident when \( M(t) \) is modeled as a pure power-law function of \( t \).

Also shown in Figure 14 is the photon-tiring limit, which constrains the maximum possible mass-loss rate \( M_{\text{max}} \) (Owocki & Gayley 1997). This limit is defined as the point where the kinetic energy carried away by the wind is equal to the photon energy carried by the stellar luminosity, and is also dependent on the terminal velocity of the wind. If the terminal velocity \( v_\infty \) is defined as \( v_\infty = v_\text{esc} \), then the limit \( M_{\text{max}} \) is given by

\[ M_{\text{max}} = \frac{2}{\Gamma^2} \frac{L_*}{v_\text{esc}}. \]  

(61)

with the escape velocity given by \( v_\text{esc}^2 = 2GM_*/R_* \). For this work we take the traditional value of \( \Gamma = 3 \). For lower densities (\( \rho < 10^{-16} \text{ g cm}^{-3} \)), we see that the photon-tiring limit intersects the mass-loss rate curves at approximately the same point at high temperatures where \( \Gamma \geq 1 \).

6. Discussion and Conclusions

In this work, we have constructed an updated list of atomic data, for 4,514,900 spectral lines taken from the NIST, CMFGEN, CHIANTI, and TOPbase databases. These atomic data were then used to calculate the line strength parameter \( q_i \) for each line for a density range of \( 10^{-20} \) to \( 10^{-10} \text{ g cm}^{-3} \) over a temperature range of 5200 to 70,000 K. The weighting function \( \tilde{W} \) was also calculated. These parameters were used to find the line force multiplier \( M(t) \) over a range of \( t \) from \( 10^{-15} \) to 10. The distribution of \( M(t) \) was fit using a power law as described by Equation (36), as well as an alternate function in the version of \( M_{\text{alt}} \) in which all four parameters (\( \alpha, k, \tilde{Q}, s \)) are allowed to vary with temperature and density. Figure 13 shows the variations of \( M_{\text{alt}} \) with \( T \) and \( \rho \). Figure 14 compares the mass-loss rates resulting from the CAK and alternate forms. As in Figure 12, we see a sharp drop-off of \( M_{\text{alt}} \) in comparison to the CAK form, commonly occurring at \( \log T \approx 4.2 \). At high densities (\( \rho > 10^{-11} \text{ g cm}^{-3} \)) the departure from the CAK form is less pronounced, although still present. It should be noted that in these figures the apparent plotted end of \( M_{\text{alt}} \) between 10,000 and 20,000 K is a result of discarding any wind solution that results in a negative wind acceleration factor \( w \). Physically, this represents regions of the parameter space where the wind is quenched, a phenomenon that is not evident when \( M(t) \) is modeled as a pure power-law function of \( t \).

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(61)

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Footnote:

The atomic data and other parameters calculated in the course of this work are available at https://github.com/astlv2/Rad-Winds.
the form of a saturated power law, as described by Equation (37). We found that this alternate function better describes the values of the line-force multiplier as calculated from the updated line list, especially at low values of \( t \). The residuals of this alternate function are consistently lower than those that result from the CAK form in the case of low \( t \) and are comparable for high values of \( t \). This is consistent across temperatures and densities. We included the parameter \( s \) to control the sharpness of the turnover from the power-law segment to the flat segment. \( M_{\text{alt}}(t) \) reduces to the power-law

![Figure 14. Comparison of \( M_{\text{CAK}} \) and \( M_{\text{alt}} \), by density order of magnitude. Dashed lines indicate \( M_{\text{CAK}} \), and solid lines indicate \( M_{\text{alt}} \). The lower right corner shows calculations of \( M_{\text{CAK}} \) and \( M_{\text{alt}} \) for the calculated critical-point densities. Red dashed lines indicate the photon-tiring limit.](image-url)
form in the limit of high $t$ for all values of the sharpness parameter $s$. In the limit of low $t M_{\text{ad}}(t)$ similarly reduces to the calculated value of $\dot{Q}$. Using the alternate function for $M(t)$, we also calculated mass-loss rates for the temperatures and densities in our grid, using the corresponding calculated values of $\dot{Q}$. We found that the sharpness parameter $s$ has a drastic effect on the determined mass-loss rates, especially at high temperatures. Additionally, there is a sharp drop-off in the mass-loss rates calculated from the updated form of $M(t)$ and a resulting discrepancy between it and the CAK mass-loss form. This drop-off in the mass-loss rate describes a quenching of the line-driven wind that is not present in the CAFM.

We found that the quenching of the wind typically occurs between temperatures of $10,000 \text{ K}$ and $20,000 \text{ K}$ and at luminosities of $2.5 \leq \log(L_{\odot}/L_{\odot}) \leq 2.75$. This may be a partial explanation for the discrepancy noted between empirically derived mass-loss rates and predicted values for stars of luminosities below $\sim 10^5 L_{\odot}$ (at $T \approx 36,000 \text{ K}$), often referred to in the literature as the “weak-wind problem” (Muijres et al. 2012), the calculations placed the quenching of the wind at lower luminosities and temperatures. It is also possible that these effects could be important to include when modeling the oscillations of slowly pulsating B stars, which have $T_{\text{eff}}$ values between about 10,000 and $20,000 \text{ K}$ (De Cat 2007). The interactions between their oscillations and winds remain poorly understood (e.g., Saio 2015).

Lastly, there is another physical effect that must be taken into account to fully understand how the predicted quenching effect manifests itself: collisionless decoupling between the line-driven ions and the dominant hydrogen/helium gas. This has been proposed to be important for both B-type stars (Springmann & Pauldrach 1992; Babel 1996; Krtička 2014) and some metal-enriched AGNs (Baskin & Laor 2012). In some low-density systems this decoupling can lead to frictional heating with wind temperatures far in excess of the stellar $T_{\text{eff}}$, and in others it may produce fully separated multicomponent winds with peculiar chemical abundances. It may be possible that the drastic reduction in the ion line force (which arises due to the flattening of the force multiplier) allows these systems to undergo a gentler and more gradual transition from a coupled single-fluid outflow to a quiescent hydrostatic atmosphere.

Although here we consider only the assumption of LTE, similar works have considered the effects of non-LTE (NLTE) as well (see, for example, Gormaz-Matamala et al. 2019). Puls et al. (2000) accounted for NLTE effects in the line distribution by restricting the types of lines used to those that have ground or metastable lower levels or are otherwise directly connected to others with such levels. For our purposes, it will be useful to refine the ionization balance used here by using the modified nebular approach described by others (see, for example, Abbott & Lucy 1985; Gormaz-Matamala et al. 2019). Although the assumption of a Planck function for $F(\nu)$ allowed us to maintain generality in this work, in future work it will be necessary to refine our choice of $F(\nu)$ to a more realistic distribution. For example, a self-consistent treatment of absorption in the near-star atmosphere could be applied to the phenomenon of “bistability” (e.g., Lamers & Pauldrach 1991), in which the wind sees a lower flux shortward of 91.2 nm—and a higher flux in the Balmer continuum—and this affects the relative strengths of the lines that contribute to $M(t)$. Alternatively, this bistability jump could be a result of the recombination of Fe between FeIII and FeIV, with the contribution of the FeIII lines dominating the radiative acceleration of the subsonic part of the wind (Vink et al. 1999; Vink 2000). Puls et al. (2000) similarly found that at low line strengths mass loss is dominated largely by the radiative acceleration of iron-group elements, with lighter ions playing a more important role at higher line strengths. This bistability jump is predicted to be reflected by an increase in mass loss, occurring around $\sim 20,000 \text{ K}$. Our calculations show that the relevant recombination temperature of FeIV does occur at a local maximum of the CAK k parameter (see Figure 9(a)). However, while the mass-loss rate is usually quite sensitive to $k$, we did not see any significant increase in our final calculations for $M$ at these parameters (e.g., Figure 14).

In this study we have also limited ourselves to the solar elemental abundances of Asplund et al. (2009). Other abundance patterns, such as those found in nearby galaxies with lower metallicity (Puls et al. 1996) or in certain types of chemically peculiar stars (Alecian & Stift 2019), should be explored. Additionally, we plan to explore the radial dependence of the $t$ parameter and the associated spatial variation of $M(t)$ in self-consistent models of radial outflow from stars and other luminous astrophysical sources, such as AGNs.

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Software: Python v3.7.6 (Van Rossum & Drake 2009), NumPy (Oliphant 2006; Van Der Walt et al. 2011), SciPy (Virtanen et al. 2020), matplotlib (Hunter 2007), AstroPy (Astropy Collaboration et al. 2013; Price-Whelan et al. 2018).

Appendix

Database Selection and Specific Line Counts

In cases where more than one database listed atomic data for a given ion, the database with the largest number of available transitions was used for each ionization state of each element. In general, CMFGEN was found to contain the most lines for a majority of the ions. However, where CMFGEN data were nonexistent or insufficient, the database with the next most lines was used. Generally, this was CHIANTI. For several elements and ionization states (namely N VI, N VII, Cl VIII, Cl IX, and Ni X), the necessary atomic data were not available from the databases used. Table 1 shows a breakdown of line counts $n$ by ion, with the database used for each also listed. In some databases, transitions with both a radiative decay rate and an autoionization rate were represented twice. After compiling the line list from the total available data, we discarded any such duplicate transitions.
| Ion   | $n$ | Database | Ion   | $n$ | Database | Ion   | $n$ | Database | Ion   | $n$ | Database |
|-------|-----|----------|-------|-----|----------|-------|-----|----------|-------|-----|----------|
| H I   | 435 | CM       | Na V  | 10,644 | CM | Cl IV | 8612 | CM | V III | 21 | N |
| He I  | 3857 | CM | Na VI | 10,994 | CM | Cl V | 3388 | CM | V IV | 239 | N |
| He II | 435 | CM | Na VII | 5436 | CM | Cl VI | 2377 | CM | V | 10 | N |
| Li I  | 68 | N | Na VIII | 4742 | CM | Cl VII | 1557 | CM | V VI | 4 | N |
| Li II | 134 | N | Na IX | 4201 | CH | Cl VIII | ... | ... | V VII | 7 | N |
| Li III | 2 | N | Na X | 331 | CH | Cl IX | ... | ... | V VIII | 6 | N |
| Be I  | 175 | N | Mg I | 2841 | CM | Cl X | 24 | CH | V IX | 16 | N |
| Be II | 97 | N | Mg II | 2641 | CH | Ar I | 3824 | CM | V X | 13 | N |
| Be III | 100 | N | Mg III | 4753 | CH | Ar II | 79,388 | CM | Cr I | 49,885 | CM |
| Be IV | 10 | N | Mg IV | 5706 | CM | Ar III | 6901 | CM | Cr II | 66,400 | CM |
| B I   | 96 | N | Mg V | 6377 | CM | Ar IV | 11,290 | CM | Cr III | ... | ... |
| B II  | 150 | N | Mg VI | 14,480 | CM | Ar V | 8350 | CM | Cr IV | 67,061 | CM |
| B III | 74 | N | Mg VII | 11,940 | CM | Ar VI | 5 | N | Cr V | 43,860 | CM |
| B IV | 134 | N | Mg VIII | 5820 | CM | Ar VII | 35 | CH | Cr VI | 4406 | CM |
| B V   | 58 | N | Mg IX | 5517 | CM | Ar VIII | 2743 | CH | Cr VII | 46 | CH |
| C I   | 10,204 | CM | Mg X | 26,078 | CH | Ar IX | 5691 | CH | Cr VIII | 131 | CH |
| C II  | 8017 | CM | Al I | 4985 | CM | Ar X | 4435 | CH | Cr IX | 236 | CH |
| C III | 9468 | CM | Al II | 2870 | CM | K I | 1471 | CM | Cr X | 16 | N |
| C IV  | 1297 | CM | Al III | 2665 | CM | K II | 38,603 | CM | Mn I | 164 | N |
| C V   | 2196 | CM | Al IV | 5296 | CH | K III | 220 | CM | Mn II | 49,066 | CM |
| C VI  | 1575 | CM | Al V | 6607 | CH | K IV | 18,227 | CM | Mn III | 70,218 | CM |
| N I   | 855 | CM | Al VI | 7989 | CM | K V | 7252 | CM | Mn IV | 72,374 | CM |
| N II  | 7879 | CM | Al VII | 15,486 | CM | K VI | 14,870 | CM | Mn V | 77,009 | CM |
| N III | 6710 | CM | Al VIII | 13,501 | CM | K VII | 71 | N | Mn VI | 70,116 | CM |
| N IV  | 13,886 | CM | Al IX | 5859 | CM | K VIII | 95 | N | Mn VII | 8277 | CM |
| N V   | 1296 | CM | Al X | 5041 | CM | K IX | 2758 | CH | Mn VIII | 47 | CH |
| N VI  | 2263 | CM | Si I | 2791 | CM | K X | 5700 | CH | Mn IX | 137 | CH |
| N VII | 3150 | CM | Si II | 4196 | CM | Ca I | 106 | N | Mn X | 236 | CH |
| O I   | 4145 | CM | Si III | 1328 | CM | Ca II | 238 | CH | Fe I | 141,928 | CM |
| O II  | 17,874 | CM | Si IV | 2672 | CH | Ca III | 520 | N | Fe II | 530,827 | CM |
| O III | 6516 | CM | Si V | 5354 | CH | Ca IV | 2 | N | Fe III | 136,060 | CM |
| O IV  | 7599 | CM | Si VI | 6518 | CM | Ca V | 9 | CH | Fe IV | 72,223 | CM |
| O V   | 3237 | CM | Si VII | 9364 | CM | Ca VI | 10 | CH | Fe V | 71,983 | CM |
| O VI  | 1569 | CM | Si VIII | 705 | CH | Ca VII | 86 | CH | Fe VI | 185,392 | CM |
| O VII | 3505 | CM | Si IX | 403 | CH | Ca VIII | 200 | CH | Fe VII | 86,504 | CM |
| O VIII | 1575 | CM | Si X | 5017 | CH | Ca IX | 9230 | CH | Fe VIII | 21,134 | CH |
| F I   | 119 | N | P I | 46 | N | Ca X | 2760 | CH | Fe IX | 47,085 | CH |
| F II  | 2354 | CM | P II | 217,043 | CM | Sc I | 259 | N | Fe X | 50,854 | CM |
| F III | 9725 | CM | P III | 5576 | CM | Sc II | 77,253 | CM | Co I | 118 | N |
| F IV  | 15 | N | P IV | 2537 | CM | Sc III | 687 | CM | Co II | 61,986 | CM |
| F V   | 11 | N | P V | 2700 | CH | Sc IV | 4 | N | Co III | 679,412 | CM |
| F VI  | 8415 | T | P VI | 5533 | CH | Sc V | 4 | N | Co IV | 69,425 | CM |
| F VII | 6406 | T | P VII | 3 | CH | Sc VI | ... | ... | Co V | 75,923 | CM |
| F VIII | 5614 | T | P VIII | 25 | CH | Sc VII | 15 | N | Co VI | 75,118 | CM |
| F IX  | 3488 | T | P IX | 59 | CH | Sc VIII | 16 | N | Co VII | 68,388 | CM |
| Ne I  | 2629 | CM | P X | 78 | CH | Sc IX | 15 | N | Co VIII | 88,548 | CM |
| Ne II | 5795 | CM | S I | 19,813 | CM | Sc X | ... | ... | Co IX | 12,232 | CM |
| Ne III | 2343 | CM | S II | 8527 | CM | Ti I | 490 | N | Co X | 5 | N |
| Ne IV | 9725 | CM | S III | 4543 | CM | Ti II | 93,118 | CM | Ni I | 188 | N |
| Ne V  | 13,037 | CM | S IV | 7530 | CM | Ti III | 21,722 | CM | Ni II | 51,812 | CM |
| Ne VI | 5171 | CM | S V | 3605 | CM | Ti IV | 1000 | CM | Ni III | 66,511 | CM |
| Ne VII | 5213 | CM | S VI | 1936 | CM | Ti V | 4 | N | Ni IV | 72,898 | CM |
| Ne VIII | 26,832 | CH | S VII | 73 | N | Ti VI | 11 | N | Ni V | 75,541 | CM |
| Ne IX | 216 | CH | S VIII | 54 | N | Ti VII | 1 | N | Ni VI | 79,169 | CM |
| Ne X  | 190 | CH | S IX | 51 | N | Ti VIII | 15 | N | Ni VII | 74,411 | CM |
| Na I  | 2778 | CM | S X | 57 | N | Ti IX | 14 | N | Ni VIII | 71,614 | CM |
| Na II | 5054 | CH | Cl I | 75 | N | Ti X | 43 | N | Ni IX | 79,227 | CM |
| Na III | 4368 | CH | Cl II | 52 | N | V I | 1095 | N | Ni X | ... | ... |
| Na IV | 3754 | CM | Cl III | 50 | N | V II | 1415 | N | Ni IX | ... | ... |

**Notes.** Ellipses indicate that no data were available. CMFGEN, NIST, CHIANTI, and TOPbase are abbreviated as CM, N, CH, and T, respectively.
Aylecia S. Lattimer \href{https://orcid.org/0000-0002-2004-5084}{https://orcid.org/0000-0002-2004-5084}

Steven R. Cranmer \href{https://orcid.org/0000-0002-3699-3134}{https://orcid.org/0000-0002-3699-3134}

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Aylecia S. Lattimer

Steven R. Cranmer

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