Based on Vibration Signal to Identify the Diesel Engine Modal Parameters under Working Condition

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Abstract. The methods to study the dynamic characteristics of diesel engine structure include finite element analysis method and experimental analysis method. It is difficult to establish an accurate finite element model, since the results of finite element analysis methods are often not accurate. For the experimental analysis method, it is mainly divided into the traditional Experimental Modal Analysis (EMA) and the Operational Modal Analysis (OMA). The EMA method always needs to measure the excitation force of the diesel engine and the vibration response of the structure simultaneously. And the influence of motion factors on the structure dynamics cannot be considered. The structure is excited by the working environment, and only analyzes the modal parameters of the structure with the vibration response. Few scholars analyze the OMA for diesel engines. This paper proposes a new method to identify the modal parameters of diesel engine based on self-excitation. In this paper, the theory of self-excitation mode parameter identification is demonstrated by derivation, and the validity of the method is verified.

1. Introduction
Traditionally, the EMA is used to obtain Modal properties of structures, through a hammer to excite the structure and obtain the modal parameters (natural frequency, modal damping and modal mode). The application of EMA has a big limitation, that needs obtain the structural excitation signal. Sometimes it's hard to get the excitation signal of the structure characterization [1].And the accuracy of Finite Element Modal(FEM) analysis is limited by the accuracy of the model. At this time, The OMA method was proposed to identify the Modal of the structure in the case of unknown input [2].

Guo et al. predicted the radiated noise from the oil pan of a diesel engine by coupling methodology of finite element analysis (FEA) and boundary element analysis (BEA), and optimized it using rib stiffeners. However, the optimized oil pan was excited by the same boundary conditions utilized for the original one, which would influence the accuracy of the structural responses after optimization.[3].Lalor N and Birth M analyzed the finite element dynamic characteristics of the diesel engine body, calculated the natural frequency and modal shape, analyzed and summarized each mode, and had a relatively directional understanding of the vibration mode[4] [5]. In addition, many diesel engine manufacturers in the world, such as MTU, TOYOTA, have also carried out extensive finite element analysis on many parts of diesel engine (body, cylinder head, bearing, connecting rod, sealing ring, etc.). In 1990, Shung. H Sung et al. used the finite element analysis software Nastran to study the
vibration and noise of a diesel engine body[6]. Seybert A.F. et al. used finite element software to conduct finite element sonic-solid coupling analysis aim at many parts of diesel engine[7].

In 1997, Southall R et al. developed Perkins V6HSDI (High Speed Direct Injection) diesel engine that met European standards by using finite element analysis technology[8]. AVL (Austria) and LMS (Belgium) have made great achievements in determining the excitation force of cylinder block by virtue of their excellent experimental capabilities. Finite element analysis technology has been applied in practical engineering design, which makes the dynamic response results obtained by computer simulation closer to the actual situation. In 2002, Baverstock L studied the structural performance of diesel engine body by means of experiment, and pointed out that the maximum stress appeared on the joint surface of main bearing cover[9].

At present, most researchers studied the dynamic characteristics of diesel engine are mainly focus on showing the working stability of diesel engine through simulation software, so as to guide the structural optimization of diesel engine. Occasionally, some scholars conduct experiments under the static condition of diesel engine with LMS or other test instruments, so as to input the obtained dynamic parameters into the simulation model or verify simulation results.

Therefore, this paper proposes a new method to identify the diesel engine modal parameters based on its self-excitation.

2. The theory of self - excitation mode parameter identification

The random subspace method is mainly based on the impulse response function of the system as the basic model. Through generalized Hankel matrix and singular value decomposition technology, the minimum realization of the system is obtained and the system matrix of the minimum order is obtained. On this basis, the modal parameters of the system are further identified[10].

2.1. Structural modal parameter feature system implementation algorithm

According to modern control theory, the structural vibration equation of diesel engine can be further written in the form of state space:

\[
y(t) = Ay(t) + Bf(t) \\
z(t) = Gy(t)
\]

(1)

Where A is the system matrix, B is the control matrix and G is the observation matrix. They jointly constitute an implementation of the system and are determined by the inherent characteristics of the system:

\[
A = \begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\]

(2)

\[
B = \begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix}
\]

(3)

According to the literature[11], the modal parameters of the system can be identified through the system matrix A:

\[
A = \Psi \Phi \Phi^{-1}
\]

(4)

\[
A = \begin{bmatrix}
\lambda_k & & \\
& \cdots & \\
& & \lambda_k
\end{bmatrix}, \Phi = [\phi_1, \phi_2, \cdots], k = 1, \ldots, N
\]

(5)

Where, \( \lambda_k \) and the natural frequency \( \omega_k \) and damping ratio \( \xi_k \) of diesel engine structure have the following relation:

\[
\lambda_k = -\xi_k \omega_k + i \sqrt{1 - \xi_k^2} \omega_k
\]

(6)
Where \( i = \sqrt{-1} \), \( \phi \) represents the mode vector of diesel engine structure. According to the above equation, if the system matrix of diesel engine structure can be obtained, the modal parameters of diesel engine can be identified by singular value decomposition of the matrix. However, the above is for a time continuous system. For actual modal analysis, since the data used is digital signal and the system is a time discrete system, it is assumed that \( t = t_0 \) when the initial condition is \( y(t_0) \), the solution of the equation of state is:

\[
y(t) = e^{At}y(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bf(\tau)d\tau
\]

Set the discrete time points are \( k = 0, 1, 2, \) and the sampling time interval \( \Delta t \), then \( t = k\Delta t \), and the solution of the equation of state can be written as the discrete form:

\[
y[(k+1)\Delta t] = e^{A\Delta t}y(k\Delta t) + e^{A\Delta t}Bf(k\Delta t)\Delta t
\]

If \( k\Delta t \) is simply denoted as \( k \), then the discrete state equation of the system can be expressed as:

\[
y(k+1) = A_1y(k) + B_1f(k)
\]

\[
z(k+1) = Gy(k+1)
\]

Among them,

\[
A_1 = e^{A\Delta t} \quad B_1 = A_1B\Delta t
\]

Substitute formula (4) into the above equation, and according to the inherent nature of the index, the following equation can be obtained:

\[
A_1 = e^{\Psi\Lambda\Psi^{-1}\Delta t} = \Psi e^{\Lambda\Delta t}\Psi^{-1}
\]

Equation (11) can be further expressed as:

\[
\Psi^{-1}A_1\Psi = e^{\Lambda\Delta t}
\]

From the above equation, it can be known that the eigenvectors of \( A_1 \) and \( A \) are the same, the eigenroots \( \mu_k \) of matrix \( A_1 \) and he eigenroots \( \lambda_k \) of matrix \( A \) satisfy: \( \lambda_k = \frac{\ln\mu_k}{\Delta t} \)

According to the above analysis, if the system matrix \( A_1 \) of discrete state equation of diesel engine structure can be obtained, the modal parameters of diesel engine structure can be obtained by singular value decomposition of the matrix.

### 2.2 The equation of state representation of the impulse response function

Z transformation of the transfer function of the system can be obtained as follows:

\[
H(z) = \sum_{k=0}^{\infty} h_k z^{-k}
\]

Where, \( z = e^{s\Delta t} \) is the Z transformation factor; \( s = \sigma + j\omega \) is the domain of complex Numbers; \( H(z) \) is the transfer function matrix in the form of Z transformation; \( h_k \) is the impulse response function of the system. The discrete equation of state of the diesel engine structure is transformed by Z, as follows:

\[
zY(z) = A_1Y(z) + B_1F(z)
\]

\[
Z(z) = GY(z)
\]

The above equation can be obtained:

\[
Z(z) = G(zI - A_1)^{-1}B_1F(z)
\]
Thus, the transfer function can be expressed as:

$$H(z) = z^{-1}G(I - z^{-1}A_1)^{-1} B_1$$  \hspace{1cm} (16)$$

As

$$(I - z^{-1}A_1)^{-1} = \sum_{k=0}^{\infty} (z^{-1}A_1)^k = \sum_{k=0}^{\infty} A_1^k z^{-k}$$  \hspace{1cm} (17)$$

Substitute into equation (16), the transfer function can be expressed as:

$$H(z) = \sum_{k=1}^{\infty} G A_1^{k-1} B_1 z^{-k}$$  \hspace{1cm} (18)$$

By comparison with equation (14), the following equation can be obtained:

$$h(k) = G A_1^{k-1} B_1$$  \hspace{1cm} (19)$$

According to the above equation, the system matrix $A_1$ of the structural state equation can be constructed by using the impulse response signal $h(k)$ of the diesel engine structure, and then the system matrix can be used to identify the modal parameters of the diesel engine structure.

2.3 Matrix construction of state equation system

System matrix $A_1$ can be obtained by constructing Hankel matrix. Hankel matrix is composed of Markov parameter matrix, and if the measured impulse response matrix is $M \times L$-order matrix $h(k)$, Hankel matrix is composed as follows:

$$H(k-1) = \begin{bmatrix}
h(k) & h(k+t_i) & \cdots & h(k+t_{\beta-1}) \\
h(k+s_i) & h(k+s_i+t_i) & \cdots & h(k+s_i+t_{\beta-1}) \\
\vdots & \vdots & \ddots & \vdots \\
h(k+s_{\alpha-1}) & h(k+s_{\alpha-1}+t_i) & \cdots & h(k+s_{\alpha-1}+t_{\beta-1})
\end{bmatrix}$$  \hspace{1cm} (20)$$

Where, $t_i$, $s_i$ are the integer corresponding to any discrete time. Since $h(k)$ is order $M \times L$, the $H(k-1)$ matrix is order $\alpha M \times \beta L$. However, due to noise pollution in measurement, $H(k-1)$ has rank deficit. Only when $\alpha, \beta$ increase to a certain degree, the rank of $H(k-1)$ is approximately equal to the order of the system. Usually, $t_i = s_i = t$, there is:

$$H(k-1) = \begin{bmatrix}
h(k) & h(k+1) & \cdots & h(k+\beta-1) \\
h(k+1) & h(k+2) & \cdots & h(k+\beta) \\
\vdots & \vdots & \ddots & \vdots \\
h(k+\alpha-1) & h(k+\alpha) & \cdots & h(k+\alpha+\beta-2)
\end{bmatrix}$$  \hspace{1cm} (21)$$

$$H_{\alpha\beta}(k-1) = PA_1^{k-1}Q$$  \hspace{1cm} (22)$$

Among $P = \begin{bmatrix}
G \\
G A_1 \\
\vdots \\
G A_1^{\alpha-1}
\end{bmatrix}$, $Q = \begin{bmatrix}
B_1 & A_1 B_1 & A_1^2 B_1 & \cdots & A_1^{\beta-1} B_1
\end{bmatrix}$, $\alpha, \beta$ is, $\frac{2n}{M} \leq \alpha \leq 2n$ and $\frac{2n}{L} \leq \beta \leq 2n$, respectively.

Let $k = 1$, then:
\[ H_{\alpha M \times \beta L}(0) = PQ \]  
(23)

Perform singular value decomposition for \( H_{\alpha M \times \beta L}(0) \):

\[ H_{\alpha M \times \beta L}(0) = U \sum V^T \]  
(24)

\[ U^T U = I, V^T V = I \]  
(25)

Where, \( U \) is the left singular vector matrix of order \( \alpha M \times 2n \); \( V \) is the right singular vector matrix of order \( \beta L \times 2n \); \( \sum \) is the \( 2n \times 2n \) order diagonal matrix, and \( \sum = \text{diag} \left[ \sigma_1, \sigma_2, \ldots, \sigma_{2n} \right] \), \( \sigma_i^2 \) is the non-zero characteristic root of \( H_{\alpha M \times \beta L}(0) \).

Introduce matrix \( H^*(0) \) such that

\[ QH^*P = I \]  
(26)

There are:

\[ H(0)H^*(0)H(0) = PQH^*PQ = PQ = H(0) \]  
(27)

Formula (23)–(27) can be obtained as follows:

\[ H^* = V \sum^{-1} U^T \]  
(28)

Set \( E_M^T = \begin{bmatrix} I_M & 0 & \cdots & 0 \end{bmatrix}, E_L^T = \begin{bmatrix} I_L & 0 & \cdots & 0 \end{bmatrix} \), where \( E_M^T \) is the matrix of order \( M \times \alpha M \) and \( E_L^T \) is the matrix of order \( L \times \beta L \). According to equation (21), it can be known that:

\[ h(k+1) = E_M^T H(k) E_L \]  
(29)

Substitute equation (22) into the above equation:

\[ h(k+1) = E_M^T PA^k Q E_L \]  
(30)

Insert equation (26) into the above equation for further arrangement:

\[ h(k+1) = E_M^T PQH^*PA^kQH^*PQ E_L = E_M^T H(0)H^*PA^kQH^*H(0)E_L \]  
(31)

\( H^*PA^kQH^* \) can be expanded as follows:

\[ H^*PA^kQH^* = (H^*PAQ)^k = \left[ H^*H(1) \right]^k \]  
(32)

By substituting equation (28) into the above equation, the following equation can be obtained:

\[ H^*PAQ = \left[ H^*H(1) \right]^k = \left[ V \sum^{-1} U^T H(1) \right]^k \]  
(33)

Substitute the above equation and (24), (25), (26) into equations (31), then:

\[ h(k+1) = E_M^T U \sum \frac{1}{2} \sum \frac{1}{2} V^T H(1) V \sum \frac{1}{2} V^T E_L \]  
(34)

By comparison with equation (19), the following equation can be obtained:

\[ A_i = \sum \frac{1}{2} U^T H(1) V \sum \frac{1}{2} \]  
(35)

\[ B_i = \sum \frac{1}{2} V^T E_L \]  
(36)

\[ G = E_M^T U \sum \frac{1}{2} \]  
(37)

According to the above analysis, the calculation of modal parameters of diesel engine structure
includes: firstly, free response characteristics are extracted from vibration response signal of diesel engine structure; However, in the process of practical analysis, the vibration of the measured signal often contains a lot of noise, the results of the self-excited modal analysis algorithm include not only the modal parameters of the structure, but also the pseudo-modal introduced by noise and other components, it is necessary to further identify the modal parameters of the diesel engine structure from the calculated results.

3. Experimental verification

In this paper, the self-excitation modal analysis experiment and the traditional modal experiment are carried out. For the validity of the self-excitation modal analysis method is studied, and the experimental object of diesel engine is TBD234V12

3.1 Self-excited experimental platform

In order to fully study the whole mode of diesel engine, 66 measuring points were selected. Figure 1. shows the measuring point layout model of the self-excitation mode experiment. Each node of the grid in the model represents the position of the sensor installed. As shown in the figure 1, 8 measuring points are evenly arranged on the cylinder head, 28 measuring points are evenly arranged on the cylinder body, 14 measuring points are symmetrically arranged on the oil sump, and 16 measuring points are symmetrically arranged on the bracket. As the theory described in section 2 and the structural properties of TBD234V12, the engine speed is set to 1500r/min with no load. The sampling frequency set 1024Hz, and the sampling time set 3 minutes. As a comparison, the traditional experimental modal analysis (EMA) method is used to analyze the modal parameters of the diesel engine structure, and the knocking excitation is applied at the input end of the diesel engine. The location of the sensors in the knocking experiment is the same as the self-excited mode experiment.

![Figure 1. experimental system of empty running self-excitation modal analysis](image)

3.2 Validity verification of identification modal parameters

| Modal Parameters | EMA | OMA |
|------------------|-----|-----|
| Natural Frequency | | |
| #1 | 26.727Hz | 24.507Hz |
| #2 | 35.352Hz | 54.524Hz |
| #3 | 55.476Hz | | |
| #4 | 94.516Hz | | |
| #5 | 120.085Hz | | |
| #6 | 135.681Hz | | |
| Damping ratio | | |
| #1 | 3.25% | 4.20% |
| #2 | 1.53% | 1.51% |
| #3 | 1.17% | | |
| #4 | 3.89% | | |
| #5 | 1.62% | | |
| #6 | 1.57% | | |
| #7 | 1.95% | | |

As the Table 1 shown, the natural frequency of the modal parameters identified by EMA method is
higher than the identified by OMA, except for the sixth modal. This is mainly because EMA is the result of diesel engine identification at rest, while OMA is the result of diesel engine at work. The modal parameter orders identified by OMA is less than the identified by EMA, This is mainly because the incentive energy of OMA is less than the incentive energy of EMA.

4. Conclusions

In this paper, the basic concept of self-excitation modal analysis method for diesel engine is presented. Then the design method of the excitation law and the identification algorithm of the modal parameters are proposed for the modal analysis method of the diesel engine. At last, the two results are analyzed and compared. The experimental results show that the self-excited modal analysis method can effectively identify the modal parameters of the diesel engine structure, and the motion state of the diesel engine will affect the modal parameters of the diesel engine structure.

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