Reversal Time of Jump-Noise Dynamics for Large Nucleation

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The jump-noise is a phenomenological stochastic process used to model the thermal fluctuation of magnetization in nanomagnets. In this work, the large nucleation regime of jump-noise dynamics is studied, and its reversal time is characterized from Monte Carlo simulations and analysis. Results show that the reversal time of jump-noise dynamics for large nucleation is asymptotically equal to the time constant associated with a single jump-noise scattering event from the energy minimum in the energy landscape of the magnetization. The reversal time for large nucleation depends linearly on the height of the energy barrier for large barriers. The significance of the large nucleation regime of jump-noise dynamics to phenomenologically explain the magnetoelectric switching of antiferromagnetic order parameter is also prospected.

Index Terms—Jump-noise, large nucleation, magnetization reversal

I. INTRODUCTION

The reversal time of magnetization is the time constant associated with the longitudinal relaxation of magnetization in nanomagnets under the influence of thermal effects, also known as the superparamagnetic or Néel relaxation time [1]. The reversal time provides information on the retention time for reading or the switching speed for writing in the context of a magnetic memory or a logic device [2], [3]. Estimating the reversal time correctly is crucial for steady miniaturization of any two states \( \langle t_i \rangle = \int_{t_i}^\infty \langle t \rangle \exp \left( - \frac{\lambda(m(t))dt}{\tau} \right) \).

\[ \lambda(m) = \int_{|m'|=1} S(m, m') d^2m'. \] (2)

The probability density function \( f \) of a jump to occur from \( m_i \) to \( m_{i+1} \) at time \( t_i \) is written as

\[ f(m_i, m_{i+1}|m_i) = \frac{S(m_i, m_{i+1})}{\lambda(m_i)}. \] (3)

The statistic of the jump instants \( t_i \) is given as

\[ \Pr(t_{i+1} - t_i > \tau) = \exp \left( - \int_{t_i}^{t_i+\tau} \lambda(m(t))dt \right). \] (4)

In this paper, the jump-noise and the large nucleation regime are first defined (Sec. II). Then, the reversal time extracted from Monte Carlo simulations of jump-noise dynamics in the large nucleation regime (Sec. III) is compared with that obtained from analysis (Sec. IV).

II. THE JUMP-NOISE

For a uniformly magnetized nanomagnet with magnetization \( M \) of magnitude \( M_n \), the state variable is defined by the transition probability rate function \( S \) between any two states \( \langle m_1, m_2 \rangle \) on the phase space \( \parallel m \parallel = 1 \), which is given by the formula [4]

\[ S(m_1, m_2) = B \exp \left( - \frac{1}{2\sigma^2} \| m_1 - m_2 \|^2 + \frac{e_{b0}}{2} \{ g(m_1) - g(m_2) \} \right); \quad e_{b0} = \frac{\mu_0 M_n^2 V}{kT}, \] (1)

where \( B \) and \( \sigma \) are the nucleation parameters; \( g \) is the magnetic free energy density; and \( e_{b0} \) is the energy barrier parameter, wherein \( \mu_0 \) is the vacuum permeability, \( V \) is the volume of the nanomagnet, and \( kT \) is the thermal energy. From Eq. (1), the scattering rate \( \lambda \) of a state \( m \) follows

\[ \lambda(m) = \int_{|m'|=1} S(m, m') d^2m'. \] (2)

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Equations (1)–(4) describe the jump-noise statistics. The nucleation regime is decided by the parameter \( \sigma \). A small nucleation is such that the transition probability rate from a state is appreciable only over an infinitesimal distance from the state, implying that \( \sigma \ll 1 \). A large nucleation means that...
So, the normalization constant.

The distribution is \( w(\theta) \) at time \( m \) is the same lowest energy state shown in Fig. 1. The energy density for uniaxial anisotropy could only allow precision up to three significant digits. A way will have some error because a finite sample size of 1000 applied field. The simulations are implemented in MATLAB with the help of Parallel Computing Toolbox on a server with 20-core CPU @ 2.3 GHz and 512 GB memory. The numerical simulations for large nucleation; \( \epsilon_0 = 10, B = 1 \). The reversal time obtained from Eq. (7) and from Monte Carlo simulations asymptotically converge for large \( \sigma \).

The transition probability between two equivalent energy wells against the energy barrier happens in a single random process.

The time constant of a process is associated with the slowest mode of relaxation. From Eq. (6), it is evident that transitions originating from the energy minimum have the lowest escape rate, which is expected from statistical mechanics. Therefore, the longitudinal relaxation of magnetization for very large nucleation is characterized by jump-noise scattering process from the energy minimum. The reversal time is simply reciprocal of the scattering rate (2) from the energy minimum or formally

\[
\tau \simeq \frac{1}{\lambda_0}, \quad \text{where} \quad \lambda_0 = \lambda |m = \arg \min g(m)|. \tag{7}
\]

The reversal time obtained from Eq. (7) asymptotically converges to the reversal time extracted from Monte Carlo simulations for large \( \sigma \) as shown in Fig. 2. For larger values of \( \sigma \), the statistics of the jump-noise process reduces to Eq. (6) which is independent of \( \sigma \), so the reversal time saturates. At \( \sigma \approx 1 \), there is a sharp rise in the reversal time which could be used to model critical phenomena.

The reversal time for large nucleation varies linearly with the energy barrier for large \( \epsilon_0 \) as shown in Fig 3. When \( \epsilon_0 \gg 1 \), the transition probability rate (6) behaves like a Dirac delta function centered at the energy minimum, and the normalization of the Dirac delta yields a linear \( \epsilon_0 \) term in the expression of the reversal time (7). For \( \epsilon_0 \ll 1 \), the transition probability rate (6) is uniformly equal to \( B \) on the phase space, and the reversal time \( \tau = 1/(4\pi B) \).

In contrast, for the case for small nucleation, the jump-noise dynamics averages to the classical Néel-Brown theory [9]. As a result, the reversal time varies exponentially with \( \sigma \) as shown in Fig. 4 as well as exponentially with the energy barrier (1). In this regard, the jump-noise dynamics for large nucleation exhibits a unique feature.

As mentioned in the introduction, the classical Néel-Brown theory cannot explain the fast switching speeds of AFM domain via ME effect. We theorize that the critical phenomenon of ME switching can be explained by modeling states which are farthest apart on the phase space have a non-negligible transition probability rate. On the unit sphere phase space, the transition probability rate between diametrically opposite states, if energetically favorable, is at least equal to \( \exp(-2/\sigma^2) \). Therefore, \( \sigma \sim O(1) \) for large nucleation.

III. EXTRACTION OF REVERSAL TIME

The reversal time of magnetization is extracted by performing Monte Carlo simulations on the time evolution of jump-noise induced nucleation in nanomagnets. The nanomagnets possess only uniaxial anisotropy, and there is no external applied field. The simulations are implemented in MATLAB with the help of Parallel Computing Toolbox on a server with 20-core CPU @ 2.3 GHz and 512 GB memory. The numerical methods are presented in Ref. [9], [20].

The state variable \( m \) is represented by spherical coordinates \( (\theta, \phi) \) such that \( m_x = \sin \theta \cos \phi, m_y = \sin \theta \sin \phi, m_z = \cos \theta \). We consider 1000 samples aligned along the same lowest energy state \( m_z = -1 \), without loss of generality, at time \( t = 0 \), and let the ensemble evolve with time until the ensemble equilibrates to Boltzmann distribution as shown in Fig. 1. The energy density for uniaxial anisotropy is \( g(\theta) = (1/2) \sin^2 \theta \), and the corresponding Boltzmann distribution is \( w_{eq}(\theta) = (1/Z) \exp[-\epsilon_0 g(\theta)] \sin \theta \), where \( Z \) is the normalization constant.

The reversal time \( \tau \) characterizes the longitudinal relaxation of absolute ensemble mean of the state variable as

\[
\langle |m_z| \rangle(t) \approx e^{-t/\tau}; \quad t \gg \tau. \tag{5}
\]

So, \( \tau \) can be estimated from the asymptotic value of \( \tau(\theta) = -t/\ln[\langle |m_z| \rangle] \) from simulations. The reversal time extracted this way will have some error because a finite sample size of 1000 could only allow precision up to three significant digits.

IV. RESULTS AND DISCUSSION

For very large nucleation, that is \( \sigma \gg 1 \), the first term in the transition probability rate (1) vanishes, so that

\[
S(m_1, m_2) \simeq B \exp \left[ \frac{\epsilon_0}{2} \left\{ g(m_1) - g(m_2) \right\} \right]. \tag{6}
\]
\[ \sigma = 100, B = 1. \] The reversal time varies linearly with the energy barrier for large \( \epsilon_{30} \) and saturates to \( \tau = 1/(4\pi B) \) for small \( \epsilon_{30} \).

**Fig. 3.** Reversal time of magnetization for uniaxial anisotropy for large nucleation; \( \epsilon_{30} = 10, B = 1. \) The reversal time obtained from the classical Néel-Brown theory and from Monte Carlo simulations converge for small \( \sigma \) and the nucleation is large; consequently the reversal is more due to anisotropy. At the threshold field product, \( \sigma \) nucleation should be small, and as a result the reversal is exponential dependence on the energy barrier. In future work, the large nucleation regime of jump-noise dynamics will be used to phenomenologically model the magnetoelectric switching of antiferromagnetic order parameter, an otherwise impossible phenomenon to explain classically.

**V. CONCLUSION**

The reversal time of jump-noise induced magnetization dynamics for large nucleation is asymptotically equal to the time constant associated with a single scattering event from the energy minimum. The reversal time for large nucleation depends linearly on the energy barrier for large barriers. This is in stark contrast with the classical Néel-Brown thermal activation theory, where the reversal occurs coherently over the energy barrier in infinitesimally many steps, and shows exponential dependence on the energy barrier. In future work, the large nucleation regime of jump-noise dynamics will be used to phenomenologically model the magnetoelectric switching of antiferromagnetic order parameter, an otherwise impossible phenomenon to explain classically.

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