Probabilistic foundations of quantum mechanics and quantum information

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Abstract
We discuss foundation of quantum mechanics (interpretations, superposition, principle of complementarity, locality, hidden variables) and quantum information theory.

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1 INTRODUCTION
In this paper there will be discussed foundations of quantum mechanics in connection to quantum information theory. Such a discussion is very important, since intensive development of models of quantum computing, cryptography and teleportation as well as general quantum information theory induced large interest to the very foundations of quantum theory. Many questions that have been of only theoretical (or even only philosophic) value for many years now play a fundamental role in applied and even technological investigations. For example, security of quantum cryptographic schemes and, consequently, prices of corresponding market products and technologies
depend crucially on such general quantum problems as complementary and nonlocality.

We can speak about renaissance in the study of foundations of quantum theory.

And this renaissance in quantum foundations was prepared by quantum information theory, see e.g. [1], where more than 50 talks were presented at section "Shannon meets Bohr".

In the plenary talk of K.A. Valeev at the present conference there was given a list of fundamental features of quantum mechanics (QM) that are very important for quantum computing (as well as cryptography, and teleportation).

I would like to present a similar list of distinguishing features of QM and discuss roles that those features play in our understanding of quantum reality and, in particular, quantum information theory. Formally K.A. Valeev and I discuss the same class of problems. However, our viewpoints to these problems are quite different. K.A. Valeev presented the conventional viewpoint to foundations of QM. From the conventional point of view all fundamental problems of QM (besides the problem of measurement) are already solved. For quantum computing (as well as cryptography and teleportation) this means that from the theoretical point of view this theory is well established. There are only experimental and technological problems. I do not think so!

In this paper, see also [2], I shall present nonconventional views to foundations of QM and consider some consequences for quantum information theory. I start with the list of distinguishing features of QM:

(S) This is a statistical theory of measurement.
(D) Discreteness of some fundamental physical variables.
(C) Principle of complementarity.
(NL) Nonlocality.

2 CONVENTIONAL INTERPRETATION

We start with (S). I think that everybody would agree that QM is a formalism that describes statistical properties of large ensembles of very special physical systems, so called quantum systems. QM could not say anything definite about behaviour of individual quantum systems. In quantum formalism there is nothing about trajectories of individual quantum systems. We still do not know anything about dynamics of individual electrons in atom and so on.
N. Bohr rightly pointed out that quantum formalism is not about quantum physical systems, but about our measurements on such systems. We can formulate this principle as **Principle of contextuality**. As a consequence of this principle, we have to take into account the whole physical arrangement of an experiment.

We continue to discuss (S). As a consequence of (S), it seems to be natural to assume that a quantum state should describe statistical properties of an ensemble of physical systems that were prepared under the same complex of physical conditions, **physical context**. However, by the conventional (orthodox Copenhagen) interpretation of QM a wave function provides **the complete description of an individual quantum system**. Such a viewpoint, i.e., coupling of a wave function to an individual quantum system, induces hard problems in foundations of QM. One of them is the well known **problem of measurement**, see e.g. [3].

Another (less known, but not less important) problem is the problem of the origin of **quantum randomness**. By using the conventional interpretation of QM we could not apply the conventional ensemble approach to probability theory, see e.g. [4], [2]. Typically there are used such words as “irreducible quantum randomness”, see e.g. [4]. However, such irreducible randomness could not be explained by using conventional probability theory (Kolmogorov, 1933). Typically it is said that conventional probability theory does not work in quantum physics. We should use a new quantum calculus of probabilities based on linear algebra in the Hilbert space of quantum states. In fact such a viewpoint is strongly supported by very strange (nonconventional) behaviour of quantum probabilities. It is well known that the classical rule for the addition of probabilities of alternatives:

$$P = P_1 + P_2$$

(1)

does not work in experiments with elementary particles. Instead of this rule, we have to use quantum rule:

$$P = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \theta.$$  

(2)

The classical rule for the addition of probabilities of alternatives is perturbed by so called **interference term**.

Nevertheless, many famous physicists did not want to follow to the conventional (Copenhagen) interpretation of quantum mechanics. In particular, A. Einstein supported the ensemble viewpoint to a quantum state. He
wanted to preserve the conventional ensemble viewpoint even for quantum probabilities. The origin of his views can be easily explained. In fact, in their pioneer papers M. Planck and A. Einstein used methods of classical statistical mechanics (in particular, conventional probability theory) to describe *black body radiation* and *photoelectric effect*. The only modification of classical statistical mechanics was due to the recognition of the principle (D) for the momentum variable. In some sense it was nothing than comeback to the purely corpuscular model of light (developed, for example, by Newton). However, a little bit later the purely corpuscular viewpoint to quantum systems became inconsistent with statistical data obtained in *diffraction and interference experiments*. This statistical data followed to quantum probabilistic rule (2) and not to conventional probabilistic rule (1).

To resolve this paradox between purely corpuscular behaviour of quantum systems in some experiments and purely wave behaviour in other experiments, N. Bohr proposed the principle of *complementarity*, (C). By (C) there exists incompatible complexes of experimental physical conditions (contexts) determining incompatible physical observables, e.g. position and momentum. We remark that very often (C) is identified with Heisenberg’s *uncertainty* principle. Of course, it is well known that N. Bohr proposed principle (C) after numerous discussions with W. Heisenberg on his uncertainty principle. However, (C) is essentially wider than Heisenberg’s principle of mutual perturbations inducing incompatible observables.

We remark that fathers of the orthodox Copenhagen interpretation of QM from the very beginning were strongly against the possibility to reduce quantum randomness to classical randomness. From the beginning it was claimed that it is even in principle impossible to create a kind of deterministic (classical-like) model that would reproduce quantum probabilities via e.g. uncertainty in initial conditions and other parameters. W. Heisenberg claimed this already in 1926 directly after the publication of his first paper on noncommutative representation of quantum of observables. Such a viewpoint was supported by N. Bohr in his discussions with A. Einstein on the EPR paradox. It was the conventional viewpoint that quantum mechanics is *complete* and it would be impossible to provide finer description of physical reality.

As we know at present time, such a viewpoint was not justified. There exist classical-like models reproducing quantum probabilities. We can mention e.g. *Bohmian mechanics* and *stochastic electrodynamics*. Of course, these models are not free of problems. However, I do not think that any of
these problems is essentially harder than e.g. the problem of measurement in the conventional Copenhagen approach. I think that immediate rejection of classical like prequantum models is merely a consequence of general prejudice against such models. For example, Bohmian mechanics is typically criticized for its nonlocality. However, at the same time it is widely accepted that conventional QM is nonlocal. Moreover, people totally forgot that Bell’s arguments that nowadays are used to support quantum nonlocality were originally proposed to support Bohmian mechanics. In fact, J. Bell was against locality and not against realism. Realism in the EPR-Bohm experiment (and, as a consequence, the possibility of deterministic description) was from the beginning evident for J. Bell as the direct consequence of the precise anticorrelations. J. Bell wanted to prove nonlocality of any deterministic prequantum model.\footnote{I would like to thank Shelly Goldstein who explained to me Bell’s views to reality and locality.}

\section{Contextual Ensemble Interpretation}

In the series of papers [5]-[7] I develop contextual probabilistic approach to QM. One of the main distinguishing features of this approach is demystification of quantum probabilities and, as a consequence, the whole QM. I started with (S) – understanding of the fact that QM is just a special theory of statistical measurements. Then I tried to find a condition that would specify this theory of statistical measurements among all possible theories of statistical measurements. In particular, such a condition should distinguish QM and classical statistical mechanics (both considered as theories of statistical measurements). Moreover, there should be a kind of correspondence principle that would establish transition from quantum statistics to classical statistics. The fundamental postulate of our model is the following one:

“Probabilities for all physical observables depend on complexes of experimental conditions under that measurements of observables are performed.”

We call this principle “principle of contextual probabilities.” This principle of contextual probabilities is closely related to Bohr’s principle of contextuality. However, there is very important difference between those two
principles. Bohr’s principle is a principle of *individual contextuality*. The basic notion related to Bohr’s principle of contextuality is the notion of individual *physical phenomenon*. Such a phenomenon is determined by a complex of experimental physical conditions. My principle is a principle of *statistical contextuality*. The basic notion related to my principle is the notion of contextual probability. Let us consider the two slit experiment, see also further derivation of quantum probabilistic rule (2). For N. Bohr it is important that each individual spot on the registration screen depends on the whole experimental arrangement (e.g. both slits are open or only one of them). For me it is important that the probability distribution of spots on the registration screen depends on the whole experimental arrangement. Of course, the reader could argue that there is not so much difference between individual and statistical contextuality. However, the situation is not so simple. Roughly speaking Bohr’s individual contextuality does not imply that probability spaces used in quantum theory should depend on complexes of experimental conditions. Under Bohr’s assumption on individual contextuality it is still possible (at least in principle) to consider physical observables as random variables on one fixed probability spaces: various complexes of physical conditions just determine various physical observables. In particular, after 75 years of Bohr’s individual contextualism it was still unclear why quantum probabilities follow to rules induced by the Hilbert space structure and not by the structure of conventional probability theory.

In papers [5]-[7] it was demonstrated (and it was surprising even for me) that starting with only the principle of contextual probabilities we can derive quantum probabilistic rule (2). In its general form (2) is quantum generalization of the well known formula of total probability. We discuss the contextual probabilistic interpretation of (2) on the basis of the well known two slit experiment.

In the two slit experiment rule (2) is induced by combining of statistical data obtained in three different experiments: both slits are open; only \( j \)th slit is open, \( j = 1, 2 \). The main distinguishing feature of statistical data obtained in these three experiments in the following one. By combining by (1) data obtained in experiments in that only one of slits is open we do not get the probability distribution for data obtained in the experiment in that both slits are open. On the other hand we never observe a particle that passes through both slits simultaneously - it would be observed passing the first or second slit. There is no the direct observation of particle splitting. As each particle passes only one of slits, we have the standard case of alternatives.
Thus we should use conventional rule (1) for the addition of probabilities of alternatives. This disagreement between experimental statistical data and the rule of conventional probability theory looks as a kind of paradox. The traditional solution of this paradox is the use of the wave model for elementary particles and, as a consequence, the principle of complementarity. We now perform detailed contextual analysis for the two slits experiment. We consider the following complexes of physical conditions, contexts.

\[ S = \text{both slits are open}, \quad S_j = \text{only jth slit is open}, \quad j = 1, 2. \]

In fact, probabilities in (2) are related to these three contexts. Thus 

\[ P = P_S(E) \quad \text{and} \quad P_j = P_{S \rightarrow S_j} P_{S_j}(E), \quad j = 1, 2. \]

Here we use various context-indexes. The \( P_S(E), P_{S_j}(E) \) denote probabilities of an event \( E \) with respect to various contexts. The coefficients \( P_{S \rightarrow S_j}, j = 1, 2 \), have another meaning. In general these are not probabilities of contexts \( S_j \) with respect to the context \( S \) (besides some very special, “classical”, situations), because the context \( S_j \) in general is not an event for the context \( S \). In general we could not consider a complex of physical conditions \( S_j \) as an event in the probabilistic space induced by some fixed context \( S \).

The coefficients \( P_{S \rightarrow S_j}, j = 1, 2 \), are kinds of *splitting coefficients*. To be more precise, we consider some fixed source of particles and some fixed period \( T \) during that we collect particles arriving to the registration screen. We consider an ensemble \( E \) that consists of all particles that are collected on the registration screen during the period \( T \), when both slits are open. An ensemble \( E_j \) consists of all particles that are collected on the registration screen during the period \( T \), when only \( j \)th slit is open. Suppose that \( E \) contains \( N \) particles and \( E_j \) contains \( N_j \) particles. Then splitting coefficients:

\[ P_{S \rightarrow S_j} \approx \frac{N_j}{N}, \quad (3) \]

see [5] for the details. Here we use the symbol \( \approx \), since numbers \( N \) and \( N_j \) vary from one run (of duration \( T \)) to other run (of the same duration). But if \( N \to \infty \), then fluctuations are negligibly small.

We remark (and it is important for our further considerations) that we have the following statistical alternative condition:

\[ P_{S \rightarrow S_1} + P_{S \rightarrow S_2} = 1. \quad (4) \]

The statistical alternative condition has the following meaning: the total number of particles that arrive to the registration screen when both slits are
open equals (in the average) to the sum of corresponding numbers when only one of the slits is open. So by closing e.g. the first slit we do not change the number of particles that pass the second slit (in the average). In fact, (4) gives the right description of alternative-situation in the two slit experiment. It is not related to alternative passing of slits by a particle in the experiment when both slits are open. This equation describes alternative sharing of particles between two preparation procedures: \( j \text{th slit is open, } j = 1, 2 \). However, the splitting coefficients \( P_{S \rightarrow S_j} \) would not play so important role in our considerations. The crucial role will be played by contextual probabilities \( P_{S_j}(E) \).

The conventional probability theory says that, in fact, we should have:

\[
P(E) = P(S_1)P(E/S_1) + P(S_2)P(E/S_2). \tag{5}
\]

There is assumed that complexes of physical conditions \( S_1 \) and \( S_2 \) could be considered as events with respect to probability space containing \( E \) as an event and

\[
S_1 \cap S_2 = \emptyset \text{ and } S_1 \cup S_2 = S.
\]

This is the well known \textit{formula of total probability}. In many considerations (including works of fathers of quantum mechanics, see e.g. P. Dirac and also R. Feynman) people set \( P = P(E) \) and \( P_j = P(S_j)P(E/S_j) \). Finally, they get the contradiction between conventional probabilistic rule (1) (that in fact coincides with (5)) and statistical data obtained in the interference experiments and described by quantum rule (2).

We would like to discuss physical and mathematical assumptions used in the conventional derivation of (1). The main physical assumption is that this formula is derived for one fixed context \( S \) - in the mathematical formalism - for one fixed Kolmogorov probability space; see my book [2] on extended discussions on the role of the choice of Kolmogorov’s probability space in quantum measurements. To be precise, we have to write the conventional formula of total probability as

\[
P_S(E) = P_S(S_1)P(S_1/E) + P(S_2)P_S(E/S_2), \tag{6}
\]

taking into account that all probabilities are computed with respect to the same complex of physical conditions \( S \).

As we have already remarked, in Kolmogorov probability theory it is also important that contexts \( S_1, S_2 \) can be realized as elements of the field of
events corresponding to the context $S$. Thus we would get the contradiction between classical rule (1) and quantum rule (2) only if we assume that splitting coefficients $P_{S \rightarrow S_j}$ can be interpreted as $P_S$-probabilities, $P_{S \rightarrow S_j} = P_S(S_j)$ and contextual probabilities $P_S(E)$ as conditional probabilities with respect to the context $S$, $P_{S_j}(E) = P_S(E/S_j)$. Here the conditional probabilities are given by Bayes’ formula (the additional postulate of Kolmogorov’s probability theory): $P_{S_j}(E) = P_S(E \cap S_j)/P_S(S_j)$.

But in general there are no reasons to assume that new complexes of conditions $S_j$ are “so nice” that new probability distributions are given by the Bayes’ formula. Thus, in general, the formula of total probability can be violated when we combine statistical data obtained for a few distinct contexts. In particular, it is not surprising that this formula does not hold true in statistical experiments with elementary particles, where the right hand side of this formula is perturbed by the interference term.

The following simple considerations give the derivation of quantum probabilistic transformation (2) in the classical probabilistic framework. Let $S$ and $S_j, j = 1, 2$, be three different complexes of physical conditions. We consider the transformation of probabilities induced by transitions from one complex of conditions to others:

$$S \rightarrow S_1 \text{ and } S \rightarrow S_2.$$  \hspace{1cm} (7)

We start with introducing of splitting coefficients, $P_{S \rightarrow S_j}$. These are proportional coefficients for numbers of physical systems obtained after preparations under the complexes of physical conditions $S$ and $S_j$. If (starting with the same number of particles radiated by some fixed source) we get $N$ and $N_j$ systems after $S$ and $S_j$ preparations, respectively, then $P_{S \rightarrow S_j}$ are defined by (3). We assume that splitting coefficients satisfy to statistical alternative equation (4). This is quite natural conditions: splitting (7) of the context $S$ induces just sharing of physical systems produced by a source. We have already discussed this sharing in the two slit experiment. The same situation we have in neutron interferometry for sharing of particles coming to detectors when both paths are open and when just one of the paths is open.

We introduce the measure of statistical perturbations $\delta$ induced by context transitions:

$$\delta(S \rightarrow S_j; E) = P_{S \rightarrow S_1}[P_S(E) - P_{S_1}(E)] + P_{S \rightarrow S_2}[P_S(E) - P_{S_2}(E)].$$
This quantity describes the deformation of probability distribution $P_S$ due to context transitions. By using statistical alternative equation (4) we get:

$$P_S(E) = P_{S \rightarrow S_1} P_{S_1}(E) + P_{S \rightarrow S_2} P_{S_2}(E) + \delta(S \rightarrow S_j; E).$$

(8)

Transformation (8) is the most general form of probabilistic transformations that can be induced by context transitions. We can formulate a kind of correspondence principle connecting context unstable and context stable (“classical”) transformations of probabilities:

If $S_j \rightarrow S, j = 1, 2$, i.e., $\delta(S \rightarrow S_j; E) \rightarrow 0$, then contextual probabilistic transformation (8) coincides (in the limit) with the conventional formula of total probability.

The perturbation term $\delta(S \rightarrow S_j; E)$ depends on absolute magnitudes of probabilities. It would be natural to introduce normalized coefficient of the context transition

$$\lambda(S \rightarrow S_j; E) = \frac{\delta(S \rightarrow S_j; E)}{2\sqrt{P_{S_1} P_{S_1}(E) P_{S \rightarrow S_2} P_{S_2}(E)}}.$$ 

that gives the relative measure of statistical deviations that can be induced by transitions from one complex of conditions, $S$, to others, $S_j$. Transformation (8) can be written in the form:

$$P_S(E) = \sum_{j=1,2} P_{S \rightarrow S_j} P_{S_j}(E) + 2 \sqrt{P_{S \rightarrow S_1} P_{S_1}(E) P_{S \rightarrow S_2} P_{S_2}(E)} \lambda(S \rightarrow S_j; E).$$

(9)

In fact, there are two possibilities:

1). $|\lambda(S \rightarrow S_j; E)| \leq 1$;

2). $|\lambda(S \rightarrow S_j; E)| \geq 1$.

In both cases it is convenient to introduce a new context transition parameter $\theta = \theta(S \rightarrow S_j; E)$ and represent the context transition coefficient in the form:

$$\lambda(S \rightarrow S_j; E) = \cos\theta(S \rightarrow S_j; E), \theta \in [0, \pi];$$

and

$$\lambda(S \rightarrow S_j; E) = \pm \cosh\theta(S \rightarrow S_j; E), \theta \in [0, \infty),$$

respectively.
We introduced a “phase parameter” $\theta$ by purely mathematical reasons: to get a linear representation of probabilistic transformations induced by context transitions, see section 4. However, in some experiments it could occur that such a probabilistic parameter $\theta$ has some geometric meaning. This induces the illusion of a wave associated with a particle. However, in our contextual probabilistic framework this is just a wave of probability. Such a “wave” is associated not with an individual particle, but with an ensemble of particles (in fact, with a transition from one ensemble to another).

We have two types of probabilistic transformations induced by the transition from one complex of conditions to another:

\[
P_S(E) = \sum_{j=1,2} P_{S\rightarrow S_j} P_{S_j}(E) + 2 \sqrt{P_{S\rightarrow S_1} P_{S_1}(E) P_{S\rightarrow S_2} P_{S_2}(E)} \cos \theta(S \rightarrow S_j; E).
\]

(10)

\[
P_S(E) = \sum_{j=1,2} P_{S\rightarrow S_j} P_{S_j}(E) \pm 2 \sqrt{P_{S\rightarrow S_1} P_{S_1}(E) P_{S\rightarrow S_2} P_{S_2}(E)} \cosh \theta(S \rightarrow S_j; E).
\]

(11)

We derived quantum probabilistic rule (2) in the classical probabilistic framework (in particular, without any reference to superposition of states) by taking into account context dependence of probabilities.

Relatively large statistical deviations are described by transformation (11). Such transformations do not appear in the conventional formalism of quantum mechanics. In principle, they could be described by so called hyperbolic quantum mechanics, [5].

**Conclusion.** For each fixed context (experimental arrangement), we have CLASSICAL STATISTICS. CONTEXT TRANSITION induces interference perturbation of the conventional rule for the addition of probabilistic alternatives.

## 4 LINEAR ALGEBRA FOR PROBABILITIES

One of the main distinguishing features of quantum theory is the Hilbert space calculus for probabilistic amplitudes. As we have already discussed, this calculus is typically associated with wavelike (superposition) features of quantum particles. We shall show that, in fact, the Hilbert space representation of probabilities was merely a mathematical discovery. Of course,
this discovery simplifies essentially probabilistic calculations. However, this is pure mathematics; physics is related merely to the derivation of quantum interference rule (2).

The crucial point was the derivation (at the beginning purely experimental) of transformation (2) connecting probabilities with respect to three different contexts. In fact, linear algebra can be easily derived from this transformation. Everybody familiar with the elementary geometry will see that (2) just the well known cos-theorem. This is the rule to find the third side in a triangle if we know lengths of two other sides and the angle $\theta$ between them:

$$c^2 = a^2 + b^2 - 2ab \cos \theta .$$

or if we want to have "+" before $\cos$ we use so called parallelogram law:

$$c^2 = a^2 + b^2 + 2ab \cos \theta .$$  \hspace{1cm} (12)

Here $c$ is the diagonal of the parallelogram with sides $a$ and $b$ and the angle $\theta$ between these sides. Of course, the parallelogram law is just the law of linear (two dimensional Hilbert space) algebra: for finding the length $c$ of the sum $\mathbf{c}$ of vectors $\mathbf{a}$ and $\mathbf{b}$ having lengths $a$ and $b$ and the angle $\theta$ between them.

We also can introduce complex waves by using the following elementary formula:

$$a^2 + b^2 + 2ab \cos \theta = |a + be^{i\theta}|^2 .$$  \hspace{1cm} (13)

Thus the context transitions $\mathcal{S} \rightarrow \mathcal{S}_j$ can be described by the wave of probability:

$$\varphi = \sqrt{P_{\mathcal{S} \rightarrow \mathcal{S}_1} P_{\mathcal{S}_1}(E)} + \sqrt{P_{\mathcal{S} \rightarrow \mathcal{S}_2} P_{\mathcal{S}_2}(E)} e^{i\phi(\mathcal{S} \rightarrow \mathcal{S}_j; E)} .$$

5 CONTEXTUAL PROBABILISTIC DERIVATION OF THE SUPERPOSITION PRINCIPLE IN THE TWO SLIT EXPERIMENT

We shall study in more details the possibility of contextual (purely classical) derivation of the superposition principle for complex probability amplitudes, ‘waves’, in the two slit experiment. We consider one dimensional model. It could be obtained by considering the distribution of particles on one fixed straight line, very thin strip. It is supposed that the source of particles is symmetric with respect to slits and the straight line (on the registration
screen) pass through the center of the screen. This geometry implies that coefficients $P_{S \rightarrow S_j} = 1/2, j = 1, 2$. By the symbol $E_x, x \in \mathbb{R}$, is denoted the event of the registration of a particle at the point $x$ of the straight line. We set: $p(x) = P_S(E_x)$ and $p_j(x) = P_{S_j}(E_x), j = 1, 2$. These are probabilities that a particle would arrive to the point $x$ (of the registration screen) under the complexes of physical conditions $S$ and $S_j$, respectively. By using (10) we get:

$$p(x) = \frac{1}{2} \left[ p_1(x) + p_2(x) + 2 \sqrt{p_1(x)p_2(x)\cos \theta(x)} \right].$$

This is just the special case of the general transformation of probabilities induced by context-transitions. By using (13) we represent this probability as the square of a complex amplitude, $p(x) = |\phi(x)|^2$, where

$$\phi(x) = \frac{1}{\sqrt{2}}(e^{i\theta_1(x)}\sqrt{p_1(x)} + e^{i\theta_2(x)}\sqrt{p_2(x)}),\quad (14)$$

and probabilistic phases $\theta_j(x)$ are chosen in such a way that the phase shift $\theta_1(x) - \theta_2(x) = \theta(x)$. We also introduce complex amplitudes for probabilities $p_j(x): \phi_j(x) = \frac{1}{\sqrt{2}}e^{i\theta_j(x)}\sqrt{p_j(x)}$. Here $p_j(x) = |\phi_j(x)|^2$. The complex amplitudes are said to be wave functions: $\phi(x)$ is the wave function on (the straight line of) the registration screen for both slits are open; $\phi_j(x)$ is the wave function on (the straight line of) the registration screen for $j$th slit is open.

Let us set $\xi(x) = \frac{\theta(x)}{h},$ where $h > 0$ is some scaling factor. We have:

$$\phi(x) = \frac{1}{\sqrt{2}}(e^{i\xi_1(x)}\sqrt{p_1(x)} + e^{i\xi_2(x)}\sqrt{p_2(x)})\quad \text{and} \quad \phi_j(x) = \frac{1}{\sqrt{2}}e^{i\xi_j(x)}\sqrt{p_j(x)}.$$  

By choosing $h$ as the Planck constant we get a quantum-like representation of probabilities. We recall that we did not use any kind of wave arguments. Superposition rule (14) was obtained in purely classical probabilistic (but contextual!) framework.

Suppose now that $\xi$ depends linearly on $x: \xi_j(x) = \frac{p_j x}{h}, \xi(x) = \frac{p x}{h}, p = p_1 - p_2$. Under such an assumption we shall get interference of two ‘frewaves’ corresponding to momentums $p_1$ and $p_2$. Of course, this linearity could not be extracted from our general probabilistic considerations. This is a consequence of the concrete geometry of the experiment.

We underline that we did not have in mind to create some alternative to the standard calculus of probabilities in Hilbert space. Our aim was
demystification of this calculus. We demonstrated that we could use the conventional ensemble viewpoint to probability (as it was proposed by A. Einstein) even in experiments with quantum systems. However, we should not forget about the fundamental principle of contextual probabilities.

Finally, we remark that starting with this principle we obtain not only the standard trigonometric cos-like interference, but also hyperbolic cosh-like interference, see [5] for the details. The latter calculus of (hyperbolic) probabilities could be realized (parallelly to the standard quantum calculus) in the so called hyperbolic Hilbert space, see [5]. Another way to get a linear representation for the hyperbolic probabilistic transformation () is to use the formalism of Positive Operator Valued Measures, see e.g. [8]. Thus we can say that starting with very natural and clear postulate on contextual probabilities we can get the well known POVM-calculus, [9].

However, the reader can say:

“Well, QM was demystified, by using contextual probabilities. But, finally, you reproduced the standard probabilistic calculus in the Hilbert space (in its generalized POVM-form). Could you present some practical consequences of your contextual reduction to conventional probability theory?”

This question will be discussed in the next section.

6 CONTEXTUAL VIEWPOINT TO QUANTUM INFORMATION

What is the main difference between orthodox Copenhagen interpretation of QM and contextual interpretation (Växjö interpretation, [10])?

By the Copenhagen interpretation the origin of the very special quantum statistics is in the very special behaviour of so-called quantum systems. Here a wave function describes an individual system. Superposition is superposition of states for an individual system.

By the Växjö interpretation the origin of the very special quantum statistics is in the very special complexes of physical conditions that are used to prepare ensembles of physical systems. Here a wave function (as it was claimed by Einstein) describes not an individual physical system, but an ensemble prepared under some complex of physical conditions. Superposition is not an individual property. It is related to statistical superposition in an ensemble. This is superposition of complexes of physical conditions.
The main fundamental consequence of the Växjö interpretation for quantum computing is that macroscopic classical physical systems could in principle exhibit quantum probabilistic behaviour under special complexes of physical conditions - special preparation procedures for ensembles of such systems. In particular, the statistical superposition property could be induced by ensembles of macroscopic classical physical systems. Thus so called quantum parallelism of computations could be realized by ensembles of macroscopic classical physical systems.

Of course, the Växjö viewpoint to quantum parallelism differs crucially from the Copenhagen viewpoint. By the Copenhagen interpretation after the act of quantum computation (unitary evolution) the quantum state of an individual physical system contains all possible values of a Boolean function $f$ under computation. By the Växjö interpretation all possible values of $f$ could be computed by performing computations over a large statistical ensemble of physical system. By the Växjö interpretation the origin of the huge computational power of quantum computers is not in quantum parallelism, but it is a consequence of the possibility to prepare ensembles of physical systems such that some fixed value of some parameter should be realized with probability that is close to probability 1. Therefore there is nothing mysterious that such a preparation could be in principle realized for macroscopic classical systems.

Our main prediction for quantum computing is the possibility to create macroscopic classical quantum computer.

Remark. The combination “classical quantum” looks as nonsense. However, we recall that here quantum is related only to special probabilistic behaviour of ensembles physical systems.

At the moment we could not present any concrete model of such a classical/quantum computer. However, we underline that quantum probabilistic behaviour could be found in various processes involving macroscopic systems, e.g. in economy.

Another consequence of the Växjö interpretation is the possibility to reduce quantum randomness to conventional ensemble randomness. This supports the original Einstein’s viewpoint that QM is not complete and some finer description of physical reality is in principle possible.

Finally, we remark that the contextual probabilistic approach to Bell’s inequality [2], [11]-[13], demonstrated that there is no contradiction between the local realist description and violation of Bell’s inequality by correlations calculated for special ensembles of physical systems (quantum systems). Con-
textual probabilistic analysis of Bell’s argument induced new inequalities that are not violated by quantum correlations, see [12], [13].

So if we follow to the Växjö interpretation of QM the problem of quantum nonlocality is still open! Of course, our arguments [2], [11]-[13], could not be considered as arguments against nonlocality. It may be that QM is nonlocal! However, from the contextual probabilistic viewpoint Bell’s arguments did no prove nonlocality of QM. There should be found some other arguments.

Our contextual viewpoint to quantum nonlocality has important consequences for quantum cryptography - the only quantum information scheme that (at the moment) is well developed for real applications. The possibility to represent in principle quantum correlations by classical probabilistic integrals induces some doubts in security of modern quantum information schemes.

Suppose that the classical reduction for quantum correlations would be possible. Then the only special source of security of quantum cryptographic schemes would be high (classical) sensibility of quantum systems to external perturbations.

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