Anomalous Hydrogen Recombination Line Ratios in Ultraluminous Infrared Galaxies

Kenichi Yano1,2, Shunsuke Baba3,4,7, Takao Nakagawa2, Matthew A. Malkan5, Naoki Isobe2, Mai Shirahata2, Ryosuke Doi1,2, and Vanshree Bhalotia6
1 Department of Physics, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
2 Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency, 3-1-1 Yoshinodai, Chuo-ku, Sagamihara, Kanagawa 252-5210, Japan
3 National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan
4 Graduate School of Science and Engineering, Kagoshima University, 1-21-35, Korimoto, Kagoshima 890-0065, Japan; shunsuke.baba@astrophysics.jp
5 Department of Physics and Astronomy, University of California, Los Angeles, CA 90095-1547, USA
6 Department of Physics and Astronomy, University of Hawai’i at Mānoa, 2505 Correa Rd., Honolulu, HI 96822, USA

Received 2020 December 25; revised 2021 August 16; accepted 2021 September 13; published 2021 December 6

Abstract

We conducted systematic observations of the H I Brγ (4.05 μm) and Brβ (2.63 μm) lines in 52 nearby (z < 0.3) ultraluminous infrared galaxies (ULIRGs) with AKARI. Among 33 ULIRGs wherein the lines are detected, 3 galaxies show anomalous Brβ/Brα line ratios (∼1.0), which are significantly higher than those for case B (0.565). Our observations also show that ULIRGs have a tendency to exhibit higher Brγ/Brα line ratios than those observed in Galactic H II regions. The high Brβ/Brα line ratios cannot be explained by a combination of dust extinction and case B since dust extinction reduces the ratio. We explore possible causes for the high Brγ/Brα line ratios and show that the observed ratios can be explained by a combination of an optically thick Brα line and an optically thin Brβ line. We simulated the H II regions in ULIRGs with the Cloudy code, and our results show that the high Brγ/Brα line ratios can be explained by high-density conditions, wherein the Brα line becomes optically thick. To achieve a column density large enough to make the Brα line optically thick within a single H II region, the gas density must be as high as n ~ 10^5 cm^{-3}. We therefore propose an ensemble of H II regions, in each of which the Brα line is optically thick, to explain the high Brγ/Brα line ratio.

Unified Astronomy Thesaurus concepts: Active galaxies (17); Starburst galaxies (1570); Ultraluminous infrared galaxies (1735)

1. Introduction

Ultraluminous infrared galaxies (ULIRGs) are characterized by their enormous infrared luminosities L_{IR} (8–1000 μm), which exceed 10^{12}L_{⊙} (Sanders et al. 1988). Large infrared luminosity is produced by thermal radiation from heated dust. This indicates that powerful energy sources are hidden behind dust. Since the discovery of ULIRGs in the 1980s, it has been debated whether the dominant energy source in a ULIRG is due to starburst activity and/or to an active galactic nucleus (AGN) (e.g., Sanders et al. 1988; Sanders & Mirabel 1996). However, the large amount of dust harbored in a ULIRG makes it difficult to investigate the energy source observationally, and thus it is important to determine the amount of dust extinction in a ULIRG in order to correct the observed quantities.

One of the most widely used indicators of dust extinction is the ratio of hydrogen recombination line ratios. These lines have been extensively studied and are widely used as tracers of ionized gas because hydrogen is the simplest and most abundant element in the universe (e.g., Seaton 1959; Johnson 1972; Hummer & Storey 1987; Storey & Hummer 1995). The ratio of the H I line fluxes from photoionized gas can be calculated numerically for the so-called “case B” model, in which all the Lyman-line photons are assumed to be absorbed by other hydrogen atoms, and all other H I lines are assumed to be optically thin. The line ratios calculated for case B are widely recognized to explain the observed line ratios in Galactic H II regions and in nearby starburst galaxies (e.g., Osterbrock & Ferland 2006). Comparing the observed line ratios with those from case B enables us to determine the amount of dust extinction in the H I lines. In optical-through-infrared wavelengths, the dust extinction is larger at shorter wavelengths (e.g., Draine 2003). Thus, if we determine the ratio of two H I lines, the line with the shorter wavelength is affected more by extinction than the one with the longer wavelength, hence the observed line ratio deviates from the case B prediction. Accordingly, we can evaluate dust extinction using the deviation of the observed line ratio from the case B value.

The H I line ratio that is most widely used to determine dust extinction is the ratio of the optical Hα and Hβ lines (e.g., Veilleux et al. 1995; Kim et al. 1998), i.e., the so-called Balmer decrement, because the wavelengths of these lines are easily accessible from ground-based telescopes. However, in objects such as ULIRGs, which are heavily dust-obscured, the dust extinction is so high that the optical Balmer lines trace only the outer regions of the object and therefore underestimate the extinction. To avoid this problem, we focus here on the infrared H I lines Brα (N = 5 → 4, 4.051 μm) and Brβ (N = 6 → 4, 2.626 μm), which are less affected by dust extinction compared with the optical lines. For instance, the Hβ/Hα line ratio is nearly halved from the case B result by dust extinction of A_V ~ 1 mag, whereas the Brβ/Brα line ratio is reduced by only ~4% from case B by the same extinction (Draine 2003). In contrast, dust extinction of A_V > 15 mag is expected in ULIRGs (e.g., Genzel et al. 1998); the Brβ/Brα line ratio is half that of case B with this large dust extinction. Thus, the Brβ/Brα line ratio is expected to be a good indicator of high dust extinction in ULIRGs.

To investigate the Brα and Brβ lines simultaneously, we utilized near-infrared spectroscopy from the AKARI infrared satellite (Murakami et al. 2007; Onaka et al. 2007). Owing to its unique 2.5–5.0 μm wavelength coverage, which is not completely achievable with ground-based telescopes due to...
Earth’s atmosphere, we can determine the $\mathrm{Br}/\beta/\mathrm{Br}/\alpha$ line ratio without observational bias such as aperture differences. In this study, we discuss the results of systematic observations of the $\mathrm{Br}/\beta/\mathrm{Br}/\alpha$ line ratios in ULIRGs from AKARI and report the discovery of an anomaly in the H I line ratio, which we cannot explain with case B and dust extinction. We also present the results of narrow-band imaging observations of the H $\alpha$ line flux using the Nickel 40-inch telescope at Lick Observatory, which we compare with the AKARI results. In Section 2, we present our targets, observations, and methods of data reduction. The resulting spectra and measured fluxes of the $\mathrm{Br}/\beta/\mathrm{Br}/\alpha$, $\mathrm{Br}/\beta$, and H $\alpha$ lines are presented in Section 3. The observed $\mathrm{Br}/\beta/\mathrm{Br}/\alpha$ line ratios are compared with those for case B, and we conclude that some ULIRGs show an anomalously high $\mathrm{Br}/\beta/\mathrm{Br}/\alpha$ line ratio. This cannot be explained by a combination of dust extinction and case B since dust extinction reduces the ratio. In Section 4, we discuss possible causes of the high $\mathrm{Br}/\beta/\mathrm{Br}/\alpha$ line ratio. We find that the anomaly can be explained with high-density H II regions which make the Br $\alpha$ line optically thick. This high-density model is compared with other observations of H I lines in Section 5. Possible structures of high-density H II region in ULIRGs are discussed in Section 6. We present some predictions from our results in Section 7 and summarize our study in Section 8. Throughout this paper, we assume that the universe is flat, with $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, and $H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Komatsu et al. 2011). We also assume the Milky Way dust model of Draine (2003) for the extinction curve. In that model, dust extinction at the wavelengths of the H $\alpha$, H $\beta$, Br $\alpha$, and Br $\beta$ lines are taken to be $A_{\mathrm{H}\alpha} = 0.776 A_V$, $A_{\mathrm{H}\beta} = 1.17 A_V$, $A_{\mathrm{Br}\alpha} = 3.56 \times 10^{-2} A_V$, and $A_{\mathrm{Br}\beta} = 8.19 \times 10^{-3} A_V$, respectively.

2. Observations and Data Reduction

First, we describe our AKARI observations, targets, and methods of data reduction. Then, we present details of our Nickel observations.

2.1. AKARI

2.1.1. Targets

Among the pointed observations from AKARI, we focused on those conducted during the liquid-He-cooled holding period (2006 May 8 to 2007 August 26; Murakami et al. 2007) to obtain high-quality data. Among those observations, we further focused on data obtained by the mission program “Evolution of ultraluminous infrared galaxies and active galactic nuclei” (AGNUL; P.I.: T. Nakagawa). This is the same data set as that used by Yano et al. (2016). The AGNUL program conducted systematic near-infrared spectroscopic observations of ULIRGs in the local universe. During the liquid-He-cooled holding period, 50 near-infrared grism spectroscopic observations of ULIRGs were conducted in this program. The observation log and basic information, such as redshifts and infrared luminosities of the 50 AGNUL targets, are summarized in Tables 1 and 2 of Yano et al. (2016), respectively.

In addition, we visually inspected all near-infrared spectra obtained with the InfraRed Camera (IRC) during the liquid-He-cooled holding period against the “IRC Point Source Spectral Catalogue.” We searched for possible targets to be included in the present study and found two galaxies (IRAS 09022−3615 and IRAS 10565+2448) in which the Br $\alpha$ and Br $\beta$ lines were clearly detected. They were observed by the mission program “The nature of new ULIRGs at intermediate redshift” (NULIZ; P.I.: H. HoSeong). To enlarge the sample size, we added these two targets to our sample. The observation log and basic information for the two objects are summarized in Appendix A. Altogether, we analyzed the near-infrared spectroscopic data for 52 objects using the observations described above.

2.1.2. Reduction of Spectroscopic Data

The near-infrared spectroscopic observations we analyzed were obtained with the IRC spectrograph (Onaka et al. 2007) on board the AKARI infrared satellite (Murakami et al. 2007). We used a $1 \times 1 \text{ arcmin}^2$ window to avoid source overlap. The pixel scale of the AKARI IRC was $1'' \times 1''$. We used the NG grism mode (Onaka et al. 2007) to obtain a 2.5−5.0 $\mu$m spectrum. The NG grism has a dispersion of $9.7 \times 10^{-3} \text{ pix}^{-1}$ and an effective spectral resolution of $\lambda/\delta \lambda \sim 120$ at 3.6 $\mu$m for a point source. We employed the observing mode IRC04, in which one pointing comprised eight or nine independent frames. Thus, although we assigned only one pointing for each ULIRG, we were able to eliminate the effects of cosmic-ray hits. The total net on-source exposure time was $\sim 6 \text{ min}$ for each ULIRG.

We processed the data using “IRC Spectroscopy Toolkit Version 20181203,” the standard IDL toolkit prepared for the reduction of AKARI IRC spectra (Ohyama et al. 2007; Baba et al. 2016). The process was basically the same as that used by Yano et al. (2016) but we used the latest version of the toolkit. Each frame was dark-subtracted, linearity-corrected, and flat-field corrected. Wavelength and flux calibrations were also made within the toolkit, but once a spectrum was output, we manually adjusted the wavelength calibration based on the location of the Br $\alpha$ line (Section 3.1). We take the accuracy of the final wavelength calibration to be smaller than 0.2 pixel or $\sim 2 \times 10^{-3} \mu$m.

The latest version of the toolkit, which we used in the current study, incorporates an error propagation algorithm that we have revised. In older versions, the flux error in the output spectrum contained the sum of the three components: (1) error determined from the standard deviation of the blank sky signal, (2) error due to the uncertainty of the spectral response curve, and (3) error caused by the wavelength calibration uncertainty. Component (1) is statistical error and should be included in the measurement of line fluxes and their ratios. Component (2) has two factors: (2a) uncertainty of scaling independent of wavelength, and (2b) uncertainty of curve shape change depending on wavelength. Of these, (2a) affects the measurement of line fluxes as a systematic error, and is canceled out in the calculation of line-to-line ratios. On the other hand, (2b) remains even in the line ratio estimate and should be included. For component (3), the toolkit conservatively considers the accuracy of the wavelength origin to be 1 pixel or $\sim 0.01 \mu$m by default (Ohyama et al. 2007). In this work, since we have tuned the origin as mentioned above, component (3) is actually negligible. Thus, since the purpose of this paper is the Br $\beta$/Br $\alpha$ line ratio, components (1) and (2b) should be included in the error budget, while (2a) and (3) are not. The latest version of the toolkit has been revised so that components (1), (2), and (3) can be output separately, so (3) can be excluded. However, it is difficult to disentangle sub-components (2a) and (2b). Therefore, we have included...

---

8 The catalog is publicly available at http://www.ir.isas.jaxa.jp/AKARI/Observation/update/20160425_preliminary_release.html.
components (1) and (2) (= (2a) and (2b)) in our analysis as a conservative estimate for the Brα/Brβ ratio.

We estimated the spatial extension of an object by stacking the spectrum along the dispersion direction for each source. The measured FWHM of the spatial profile is typically ~4–5 pixels, which is consistent with the size of the point-spread function of the AKARI IRC in the spectroscopic mode (Egusa et al. 2016). We adopted an aperture width of 5 pixels (=7′′) along the spatial direction for spectrum extraction for each ULIRG.

We analyzed the spectroscopic data for 52 objects, as described in Section 2.1.1. Since the eastern (E) and western (W) nuclei of IRAS 17028+5817 are resolved by the AKARI IRC, the spectra of the two nuclei were extracted separately. Thus, 53 spectra in total were obtained from the 52 observations. Figure 1 displays a sample 2.5–5.0 μm spectrum of a ULIRG.

2.2. Nickel 40-inch Telescope

For two selected targets, IRAS 10494+4424 and Mrk 273, which we find to show an anomalous Brβ/Brα ratio (Section 3.3), we performed narrow-band imaging observations of the Hα line (λ = 3 → 2, 6563 Å) with the Nickel 40-inch telescope at the Lick Observatory. In order to compare the Hα line flux with the Brackett line fluxes obtained from the AKARI observations, aperture matching becomes important. Our AKARI observations employed slitless spectroscopy, and we used an aperture width of ~7′′ to extract the near-infrared spectra. We used the narrow-band imaging observations to obtain the Hα line fluxes with the same aperture as adopted for AKARI.

We performed the Lick observations on 2014 November 26. We used the same observational method as described in Theios et al. (2016). The sky was clear, with seeing of ~2′′ FWHM. We used the Nickel Direct Imaging Camera (CDI-C2), which has 2048 × 2048 pixels, which we read out with 2 × 2 binning to yield 1024 × 1024 pixels 0′′37 on a side. The observed wavelength of the Hα line was 7167 Å at the redshift of IRAS 10494+4424 (z = 0.092) and 6811 Å at that of Mrk 273 (z = 0.037). The filters (central wavelength/FWHM in Å) that we used to measure the flux of the Hα line were 7146/80 for IRAS 10494+4424 and 6826/78 for Mrk 273 (on-band images). We used the 6826 filter for IRAS 10494+4424 and the 7146 filter for Mrk 273 to measure the flux of the underlying continuum (off-band images). We obtained three exposures in each of the Hα and continuum filters, with individual exposure times of 900 s. We dithered the telescope between exposures to mitigate the effects of hot or bad pixels in the detector. We also obtained bias and twilight-sky flat-field frames for each filter.

We reduced the data using standard IRAF procedures, including bias and flat-field corrections. We averaged the three dithered frames in each filter and subtracted the off-band from the on-band images to obtain a pure Hα + [NII] line image. The narrow-band filters were photometrically calibrated by observing standard stars and comparing them with data from the Sloan Digital Sky Survey (Ahn et al. 2012). The fluxes through the filters were consistent with each other to within 5%.

3. Results

3.1. Fluxes of Brackett Lines

We obtained 2.5–5.0 μm near-infrared spectra for 53 objects with AKARI, as discussed in the previous section. For each spectrum, the Brα line, at a rest-frame wavelength of λrest = 4.05 μm, and the Brβ line, at λrest = 2.63 μm, were fitted separately with a linear continuum and a Gaussian profile.

First, we fitted the Brα line with four free parameters: the offset and slope of the linear continuum, the normalization of the Gaussian profile, and the central wavelength. The line width was fixed at the spatial width at wavelengths near Brβ (FWHM ~4–5 pixels) since AKARI IRC employs slitless spectroscopy and the instrumental line-spread function is determined by the spatial point-spread function. We assumed that the spectral resolution was determined by the size of each object because the observations employ slitless spectroscopy and the intrinsic line widths are narrower than the Δν resolution of ~3000 km s⁻¹. The range of wavelengths used to fit continuum emission was typically ±0.08 μm around the central wavelength. The central wavelengths of the Brα lines exhibited small discrepancies from those expected from the redshifts. The discrepancy was larger than the fitting error, typically ~10⁻³ μm, but was within the wavelength calibration error of ~10⁻² μm (Ohyama et al. 2007). Therefore, we shifted the wavelengths for the entire spectrum so that the best-fit central wavelength of the Brα line matched the redshift. Next, after fitting the Brα line, we fitted the Brβ line, fixing the central wavelength as expected from the redshift and fixing the line width at the spatial width near the wavelength of Brβ as in the case of Brα; i.e., the free parameters are the offset and slope of the local continuum and the normalization of the Gaussian. We then determined the fluxes of the Brα and Brβ lines by integrating the best-fit Gaussian profiles. We show a typical result from this Gaussian fitting procedure in Figure 1.

Among the 53 objects, we were able to determine the fluxes of the Brβ line for 47 galaxies. Four of the sources (IRAS 00183–7111, IRAS 04313–1649, IRAS 10091+4704, and IRAS 23498+2423) have redshifts higher than 0.2; therefore, the Brα line does not fall within the 2.5–5.0 μm wavelength range. In addition, two sources (IRAS 21477+0502 and IRAS 23129+2548) were found to suffer from spectral overlapping with other objects. In UGC 5101, we found the continuum slope to be changing in the vicinity of the Brβ line; thus, we adopted a second-order polynomial for the shape of the continuum used to fit the Brβ line for this source.
We detected the Br$\alpha$ or Br$\beta$ lines at the 3$\sigma$ level in 33 objects. For undetected lines, we derived 3$\sigma$ upper-limit fluxes. The measured Br$\alpha$ and Br$\beta$ line fluxes ($F_{\text{Br}\alpha}$ and $F_{\text{Br}\beta}$) are summarized in Table 1, along with the 1$\sigma$ statistical errors; the 1$\sigma$ systematic errors were estimated to be $\sim$10% of the flux.

The widths of the Br$\alpha$ and Br$\beta$ lines are consistent with the limit of spectral resolution ($\Delta v \sim 3000$ km s$^{-1}$) for all targets within a fitting uncertainty of $\lesssim 100$ km s$^{-1}$. This indicates that none of the objects shows a broad component of the Br lines with an FWHM broader than $\sim 1000$ km s$^{-1}$. If the broad-line region of an AGN had contributed to the line fluxes, the hydrogen lines would have had an FWHM of a few thousand km s$^{-1}$ (Osterbrock & Ferland 2006). Thus, we conclude that the Br$\alpha$ and Br$\beta$ lines do not originate from the broad-line regions. In the AGNUL sample, Yano et al. (2016) reported a good correlation between the flux of the Br$\alpha$ line and the 3.3 $\mu$m polycyclic aromatic hydrocarbon (PAH) emission and suggested that any contribution from an AGN was not dominant for the Br$\alpha$ line. Combining these results, we conclude that the Brackett lines originate from starburst activities in all galaxies in the sample.

3.2. Flux of H$\alpha$ Line

We obtained pure H$\alpha$ + [N II] line images for IRAS 10494+4424 and Mrk 273 from the Nickel observations. Using the IRAF phot module, we performed circular-aperture photometry (7$''$ in diameter) for the images and obtained the H$\alpha$ + [N II] line fluxes. Assuming [N II]/H$\alpha$ line ratios to be 0.60 for IRAS 10494+4424 and 1.01 for Mrk 273 (Kim et al. 1998), we corrected the line ratios for the [N II] emission. The resulting H$\alpha$ line fluxes are summarized in Table 2. We discuss the H$\alpha$ line flux in comparison with the Br$\alpha$ line flux in Section 4.

3.3. Anomalous Br$\beta$/Br$\alpha$ Line Ratios

Owing to the unique 2.5–5.0 $\mu$m wavelength coverage of AKARI, the Br$\alpha$ and Br$\beta$ lines are observed simultaneously within a single spectrum. This allows us to determine accurately the Br$\beta$/Br$\alpha$ line ratio without introducing observational uncertainties such as aperture corrections. The estimated Br$\beta$/Br$\alpha$ line ratios are summarized in Table 1. On the basis of the Br$\beta$/Br$\alpha$ line ratio, we were able to determine the visual extinction ($A_V$) in the same way as for the usual Balmer decrement method. We assumed the intrinsic line flux ratio for Br$\beta$/Br$\alpha$ to be 0.565 (Osterbrock & Ferland 2006, case B with $T = 10,000$ K and low-density limit). If dust extinction affects the line fluxes, the Br$\beta$/Br$\alpha$ line ratio decreases because the Br$\beta$ line has a shorter wavelength and is attenuated more compared with the Br$\alpha$ line. Thus, we expect to observe Br$\beta$/Br$\alpha$ line ratios lower than 0.565. In Table 1, we also tabulate the values of $A_V$ inferred from the Br$\beta$/Br$\alpha$ line ratio.

The comparison of $F_{\text{Br}\alpha}$ and $F_{\text{Br}\beta}$ is shown in Figure 2. Galaxies located below the case B line in Figure 2 have a Br$\beta$/Br$\alpha$ line ratio lower than 0.565. The flux ratios for these galaxies are consistent with case B theory and dust extinction, and the $A_V$ magnitude was found to be positive for these objects. However, for four galaxies, we obtained an anomalous Br$\beta$/Br$\alpha$ line ratio, which is more than 3$\sigma$ higher than 0.565. These galaxies are located above the case B line in Figure 2; i.e., the Br$\beta$ line is enhanced relative to the Br$\alpha$ line. This is opposite to the effect of dust extinction.

We examined the spectra of the four galaxies that deviate by more than 3$\sigma$ from case B in Figure 2; they are IRAS 04074+2801, IRAS 10494+4424, IRAS 10565+2448, and Mrk 273. Among them, IRAS 04074+2801 is relatively faint, and the continuum is considerably affected by a fringe-like pattern. We found that if the wavelength range for fitting the Br$\beta$ line is widened by a factor of 1.5, the Br$\beta$ line flux decreases by $\sim 20$% for this galaxy. Thus, we excluded IRAS 04074+2801 from our discussion of the high Br$\beta$/Br$\alpha$ line ratio because of the large uncertainties in determining the continuum underlying the line. For the remaining three galaxies for which the spectra are shown in Figure 3, a change in the wavelength range does not affect the line fluxes by more than 5%. We conclude that the three galaxies (shown as blue circles in Figure 2), IRAS 10494+4424, IRAS 10565+2448, and Mrk 273, show Br$\beta$/Br$\alpha$ line ratios of 0.96 $\pm$ 0.12, 0.883 $\pm$ 0.085, and 1.086 $\pm$ 0.053, respectively, all significantly higher than the case B ratio (0.565). These line ratio anomalies are not explainable with case B theory and dust extinction, which could reduce but not increase the Br$\beta$/Br$\alpha$ line ratio. We show the spectra around the Br$\alpha$ and Br$\beta$ lines for these three galaxies in Figure 4.

We found no distinct physical properties to distinguish the three galaxies (IRAS 10494+4424, IRAS 10565+2448, and Mrk 273) with high Br$\beta$/Br$\alpha$ line ratios from the other sources. The optical classifications of these galaxies are LINER, H II galaxy, and Seyfert 2 (Veilleux et al. 1999a). The three galaxies have been observed at infrared wavelengths, as reported in several publications (e.g., Imanishi et al. 2008; Veilleux et al. 2009; Lee et al. 2012). Infrared properties, such as the strengths of their PAH emissions, were compared to those of other ULIRGs, but no significant differences were reported.

One common observational property is that the three objects have relatively low redshifts ($z \sim 0.09$ for IRAS 10494+4424 and $z \sim 0.04$ for IRAS 10565+2448 and Mrk 273) compared to the others in our sample; accordingly, the Br$\alpha$ and Br$\beta$ lines are detected with a high S/N ratio, and the Br$\beta$/Br$\alpha$ line ratio is well-determined in the three galaxies. Thus, they provide clear detections of deviations of the Br$\beta$/Br$\alpha$ line ratio from case B. This implies that anomalous Br$\beta$/Br$\alpha$ line ratios might also exist in faint galaxies for which we have not been able to verify its presence because of large uncertainties in the observed Br$\beta$/Br$\alpha$ line ratios.

To investigate whether the anomaly is actually found in faint galaxies, we averaged the near-infrared spectra of 35 galaxies with $F_{\text{Br}\alpha} < 10^{-14}$ erg s$^{-1}$ cm$^{-2}$. Each spectrum was corrected for redshift and was averaged in rest wavelength. The averaged spectrum of the 35 galaxies is shown in Figure 5. We measured the values of $F_{\text{Br}\alpha}$ and $F_{\text{Br}\beta}$ from the averaged spectrum using Gaussian fitting, obtaining $F_{\text{Br}\alpha} = (3.21 \pm 0.20) \times 10^{-15}$ erg s$^{-1}$ cm$^{-2}$ and $F_{\text{Br}\beta} = (2.30 \pm 0.16) \times 10^{-15}$ erg s$^{-1}$ cm$^{-2}$. This yields a Br$\beta$/Br$\alpha$ line ratio of 0.716 $\pm$ 0.065, which is 2.3$\sigma$ higher than the case B value of 0.565. Although the significance of this result is not high enough ($< 3\sigma$), it is opposite to the expectation that ULIRGs should show high dust extinction; i.e., the Br$\beta$/Br$\alpha$ line ratio should be lower than 0.565. We thus conclude that in future high-sensitivity observations, such as those with the James Webb Space Telescope (JWST), anomalous Br$\beta$/Br$\alpha$ line ratios may be found in faint galaxies for which we have not been able to determine the presence of the anomaly with current AKARI observations.
Table 1
Fluxes of Brackett Lines

| Object Name | $F_{\text{Br}0}$ (10^{-15} \text{ erg s}^{-1} \text{ cm}^{-2}) | $F_{\text{Br}0}/F_{\text{H}\alpha}$ | $A_V$ (mag) |
|-------------|-------------------------------------------------|-------------------------------|----------------|
| IRAS 00456−2904 | 6.80 ± 0.69 | 2.73 ± 0.83 | 0.40 ± 0.13 | 8.0 ± 7.5 |
| IRAS 00482−2721 | 4.37 ± 0.43 | <2.72 | <0.62 | >2.3 |
| IRAS 01199−2307 | 2.90 ± 0.80 | <2.20 | <0.76 | >6.9 |
| IRAS 01298−0744 | <1.86 | <4.02 | ... | ... |
| IRAS 01355−1814 | <2.56 | <2.40 | ... | ... |
| IRAS 01404−1845 | 4.02 ± 0.80 | 5.5 ± 1.0 | 1.37 ± 0.37 | −20.7 ± 6.4 |
| IRAS 01569−2939 | 4.4 ± 1.4 | <4.97 | <1.12 | >16 |
| IRAS 02480−3745 | <4.15 | <2.48 | ... | ... |
| IRAS 03209−0806 | 4.39 ± 0.49 | 2.75 ± 0.73 | 0.63 ± 0.18 | −2.4 ± 6.8 |
| IRAS 03521−0028 | <3.63 | <4.38 | ... | ... |
| IRAS 04074−2801 | <3.16 | 5.08 ± 0.93 | 1.61 | −0.24 |
| IRAS 05020−2941 | 4.7 ± 1.0 | <3.32 | <0.70 | >5.0 |
| IRAS 05189−2524 | <1.68 | <1.71 | ... | ... |
| IRAS 06035−7102 | 8.1 ± 1.0 | 3.80 ± 0.93 | 0.47 ± 0.13 | 4.2 ± 6.5 |
| IRAS 08572+3915 | 25.3 ± 2.5 | <5.94 | <0.23 | >21 |
| IRAS 08591−5248 | 2.77 ± 0.84 | 6.15 ± 0.81 | 2.22 ± 0.73 | −32.0 ± 7.7 |
| IRAS 09022−3615 | 68.8 ± 3.3 | 35.8 ± 2.2 | 0.520 ± 0.041 | 1.9 ± 1.8 |
| UGC 5101 | 18.4 ± 2.9 | <40.8 | <2.21 | >32 |
| IRAS 09463+8141 | 1.86 ± 0.51 | <2.54 | <1.36 | >21 |
| IRAS 09539+0857 | <3.36 | <2.13 | ... | ... |
| IRAS 10035+2740 | <1.90 | <3.09 | ... | ... |
| IRAS 10494−4424 & a | 10.92 ± 0.83 | 10.5 ± 1.1 | 0.96 ± 0.12 | −12.4 ± 3.0 |
| IRAS 10565+2448 & a | 35.2 ± 2.1 | 31.1 ± 2.4 | 0.883 ± 0.085 | −10.5 ± 2.3 |
| IRAS 10594−3818 | 8.4 ± 1.4 | 5.51 ± 0.93 | 0.66 ± 0.16 | −3.5 ± 5.5 |
| IRAS 11028+3130 | <3.46 | <3.36 | ... | ... |
| IRAS 11180+1623 | <4.44 | <2.04 | ... | ... |
| IRAS 11387−4116 | 4.86 ± 0.89 | <2.33 | <0.48 | >3.9 |
| IRAS 12447−3721 | 5.63 ± 0.80 | 3.54 ± 0.99 | 0.63 ± 0.20 | −2.5 ± 7.4 |
| Mkr 231 | <3.71 | <3.72 | ... | ... |
| Mkr 273 & c | 49.4 ± 1.1 | 53.7 ± 2.3 | 1.086 ± 0.053 | −15.3 ± 1.1 |
| IRAS 13469−5833 | <3.32 | <4.66 | ... | ... |
| IRAS 13539−2920 | 14.0 ± 1.2 | 5.5 ± 1.7 | 0.39 ± 0.12 | −8.5 ± 7.4 |
| IRAS 14121−0126 | 4.1 ± 1.2 | <4.59 | <1.12 | >0.16 |
| IRAS 14202−2615 | 8.03 ± 0.91 | 7.18 ± 0.68 | 0.89 ± 0.13 | −10.8 ± 3.5 |
| IRAS 14394−5332 | 9.5 ± 1.1 | 8.62 ± 0.54 | 0.91 ± 0.12 | −11.2 ± 3.0 |
| IRAS 15043−5754 | 3.31 ± 0.87 | 3.6 ± 1.0 | 1.10 ± 0.42 | −15.6 ± 8.9 |
| IRAS 16333+4630 | 4.03 ± 0.99 | <3.19 | <0.79 | >7.9 |
| IRAS 16468+5200 | <3.16 | <3.24 | ... | ... |
| IRAS 16487−5447 | 10.19 ± 0.78 | 4.2 ± 1.2 | 0.41 ± 0.12 | 7.5 ± 7.1 |
| IRAS 17028+5817 E | 3.80 ± 0.69 | 4.88 | <1.28 | >19 |
| IRAS 17028+5817 W | 6.14 ± 0.91 | 5.5 ± 1.1 | 0.90 ± 0.23 | −11.0 ± 6.0 |
| IRAS 17044−6720 | 6.6 ± 1.1 | <3.95 | <0.60 | >1.3 |
| IRAS 17068+4027 | 6.53 ± 0.90 | 3.40 ± 0.93 | 0.52 ± 0.16 | 1.9 ± 7.2 |
| IRAS 17179+5444 | <4.45 | <4.59 | ... | ... |
| IRAS 19254−7245 | 20.1 ± 1.6 | 13.8 ± 2.0 | 0.69 ± 0.11 | −4.6 ± 3.8 |
| IRAS 22088−1831 | <2.58 | <2.25 | ... | ... |
| Mkr 231 +5919 | 67.0 ± 3.3 | 41.2 ± 2.7 | 0.614 ± 0.050 | −2.0 ± 1.9 |

Notes.

a Observed flux of the Br0 line. The values differ from those reported in Yano et al. (2016) because we used a different version of the toolkit for the data reduction and revised the error estimation as reported in this paper.
b Visual extinction derived from the observed Brβ/Brα line ratio assuming case B.
c Galaxies showing clear anomalies in the Brβ/Brα line ratio.

Table 2
Observed Hα Line Flux

| Object | $F_{\text{H}\alpha}$ (10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2}) |
|--------|-------------------------------------------------|
| IRAS 10494−4424 | 1.46 ± 0.15 |
| Mkr 273 | 33.1 ± 3.3 |

3.4. Estimate of Contamination of Brackett Lines

One possible cause of the line ratio anomaly is that the Brackett lines may be blended with other features. The spectral resolution of our observations is not high (∼0.04−0.05 μm); therefore, the apparently high Brβ/Brα line ratio may be caused by contamination due to other features. We looked for
such features, with wavelengths close to those of the Brackett lines. However, there are few observations around the wavelengths of the Brackett lines (especially around the Brβ line) because the wavelength is difficult to access from the ground due to atmospheric absorption. Little information is therefore available about possible contaminant features.

We reviewed a list of infrared atomic lines provided by ISO observations (Table 4.10 of Glass 1999) and found no candidate lines near the wavelengths of the Brβ and Brα lines. Among molecular features (Table 4.10 of Cox 2000), we found one candidate line, H₂(1,0) O(2), for which the wavelength is 2.627 μm. The wavelength of this line is very close to that of the Brβ line; therefore, the flux of the Brβ line could be overestimated due to contamination by the molecular hydrogen line. We discuss below the possible effect of molecular hydrogen contamination on the Brβ/Brα line ratio for the three galaxies that show anomalous Brackett line ratios using Gaussian fitting. We fixed the width and central wavelength of the Gaussian profile and used the normalization of the Gaussian profile and the linear continuum as free parameters. Figure 6 shows the results of the Gaussian fitting of the H₂(1,0) O(3) line. We detected the H₂(1,0) O(3) line with moderate significance (>2.8σ) in all three galaxies. The measured flux is summarized in Table 3, along with the 1σ statistical error. The 1σ systematic error is estimated to be ~10% of the flux.

Using the H₂(1,0) O(3) line flux, we determined the H₂(1,0) O(2) line flux, subtracted it from the Brβ line flux, and calculated the flux ratio of the pure Brβ line to the Brα line. We summarize the results in Table 3 (F(1,0)/F(1,0)). The ratio is still more than 3σ higher than the case B value of 0.565 for IRAS 10565+2448 and Mrk 273. Thus, contamination of the Brβ line does not provide a full explanation of the anomalous line ratio at least for these galaxies. For IRAS 10494+4424, the significance of the anomaly is 2.7σ after the correction of the H₂ line. Hence, the detection of an anomalous line ratio in this ULIRG is admittedly not as robust as those in the other two, but it is still moderately significant. We thus retain this galaxy in the discussion below. We conclude that the anomalous Brβ/Brα ratio is real and that conditions intrinsic to the ionized gas itself make the ratio anomalous.

Figure 2. The Brα line flux (F_{Brα}) vs. the Brβ line flux (F_{Brβ}). The solid line shows the theoretical line ratio for case B conditions: F_{Brβ}/F_{Brα} = 0.565. The extinction vector for A_v = 10 mag is shown as a black arrow. The blue filled circles show galaxies with anomalous Brβ/Brα line ratios (more than 3σ higher than 0.565), while red filled circles represent those with the normal case B ratio. The green filled circle shows a galaxy with a high Brβ/Brα line ratio but it has been excluded from the discussion because of large uncertainties in determining the continuum underlying the lines. The red open circles represent galaxies where neither the Brα nor Brβ lines were detected. The black open square shows the averaged spectrum of 35 galaxies with F_{Brα} < 10^{-14} erg s^{-1} cm^{-2}.

Thus, we adopt the flux ratio F(1,0)/F(1,0) = 0.26 from the shock model (T = 2000 K) of Black & van Dishoeck (1987), where F(1,0) is the flux of the H₂(1,0) O(3) line.

We measured F(1,0) for the three galaxies that show anomalous Brackett line ratios using Gaussian fitting. We fixed the width and central wavelength of the Gaussian profile and used the normalization of the Gaussian profile and the linear continuum as free parameters. Figure 6 shows the results of the Gaussian fitting of the H₂(1,0) O(3) line. We detected the H₂(1,0) O(3) line with moderate significance (>2.8σ) in all three galaxies. The measured flux is summarized in Table 3, along with the 1σ statistical error. The 1σ systematic error is estimated to be ~10% of the flux.

Using the H₂(1,0) O(3) line flux, we determined the H₂(1,0) O(2) line flux, subtracted it from the Brβ line flux, and calculated the flux ratio of the pure Brβ line to the Brα line. We summarize the results in Table 3 (F(1,0)/F(1,0)). The ratio is still more than 3σ higher than the case B value of 0.565 for IRAS 10565+2448 and Mrk 273. Thus, contamination of the Brβ line does not provide a full explanation of the anomalous line ratio at least for these galaxies. For IRAS 10494+4424, the significance of the anomaly is 2.7σ after the correction of the H₂ line. Hence, the detection of an anomalous line ratio in this ULIRG is admittedly not as robust as those in the other two, but it is still moderately significant. We thus retain this galaxy in the discussion below. We conclude that the anomalous Brβ/Brα ratio is real and that conditions intrinsic to the ionized gas itself make the ratio anomalous.
can be written as \( \text{level} \). The \( \alpha \) \( \beta \) undergoes a transition to level \( \gamma \), which are the number densities of protons, electrons, and hydrogen atoms in quantum state \( \mathcal{N} \), respectively; \( \alpha_{\mathcal{N}} \) is the recombination coefficient for level \( \mathcal{N} \); and \( A_{\mathcal{N}', \mathcal{N}} \) is the Einstein A coefficient for the \( \mathcal{N}' \rightarrow \mathcal{N} \) transition. We assume case B conditions (Osterbrock & Ferland 2006), for which the Lyman lines are taken to be optically thick, so that the summation on the right-hand side ends at \( \mathcal{N}' = 2 \).

Using the cascade matrix \( C_{\mathcal{N}', \mathcal{N}} \), which is the probability that a population in level \( \mathcal{N}' \) undergoes a transition to level \( \mathcal{N} \) via all possible routes (Seaton 1959), we can rewrite Equation (1) as

\[
\frac{n_{\mathcal{N}'} A_{\mathcal{N}'} n_{\mathcal{N}'} \sum_{\mathcal{N}' = \mathcal{N}}^{\infty} \alpha_{\mathcal{N}} C_{\mathcal{N}', \mathcal{N}}}{A_{\mathcal{N}'} \sum_{\mathcal{N}' = \mathcal{N}}^{\infty} \alpha_{\mathcal{N}} C_{\mathcal{N}', \mathcal{N}}} = \frac{n_{\mathcal{N}'} A_{\mathcal{N}'} C_{\mathcal{N}', \mathcal{N}}}{A_{\mathcal{N}'} C_{\mathcal{N}', \mathcal{N}}} \tag{2}
\]

for the ratio of the level populations of states \( \mathcal{N} \) and \( \mathcal{N}' \). The cascade matrix can be written in terms of the Einstein coefficients, which do not depend on gas properties such as temperature. The recombination coefficients depend weakly on temperature; however, in Equation (3), this dependence is almost canceled out because we have the coefficients both in the numerator and the denominator. Thus, the level population depends only weakly on temperature for the case in which it is determined by the recombination process.

4. Interpretation of the Anomaly

In this section, we discuss how to interpret the high \( \text{Br}/\text{Br} \) ratio. To investigate the H II line ratios, we first consider how the level populations of the hydrogen atoms are determined by assuming three possible excitation mechanisms: recombination, collisional excitation, and resonant excitation. Then, we discuss possible explanations for the high \( \text{Br}/\text{Br} \) ratio separately for optically thin and optically thick cases. In this section, we use the Einstein coefficients from Johnson (1972) and recombination coefficients from Verter & Ferland (1996).

4.1. Excitation Mechanisms for Hydrogen Atoms

4.1.1. Recombination

As the first excitation mechanism, we discuss the recombination process. In the low-density limit, wherein collisions are negligible, the hydrogen level populations are determined by recombination and radiative transitions. In this case, the equilibrium equation for the level population of a state with principal quantum number \( \mathcal{N} \) can be written as

\[
n_p n_e \alpha_{\mathcal{N}} + \sum_{\mathcal{N}' = \mathcal{N}+1}^{\infty} n_{\mathcal{N}'} A_{\mathcal{N}', \mathcal{N}} = n_{\mathcal{N}} \sum_{\mathcal{N}' = 2}^{\infty} A_{\mathcal{N}', \mathcal{N}'}, \tag{1}
\]

where \( n_p \), \( n_e \), and \( n_{\mathcal{N}} \) are the number densities of protons, electrons, and hydrogen atoms in quantum state \( \mathcal{N} \), respectively; \( \alpha_{\mathcal{N}} \) is the recombination coefficient for level \( \mathcal{N} \); and \( A_{\mathcal{N}', \mathcal{N}} \) is the Einstein A coefficient for the \( \mathcal{N}' \rightarrow \mathcal{N} \) transition. We assume case B conditions (Osterbrock & Ferland 2006), for which the Lyman lines are taken to be optically thick, so that the summation on the right-hand side ends at \( \mathcal{N}' = 2 \).

In summary, for 3 out of 33 ULIRGs wherein we detected both the \( \text{Br} \) and \( \text{Br} \) lines, we found anomalous \( \text{Br}/\text{Br} \) line ratios. The ratios are significantly higher than the case B value even after the subtraction of possible contamination of the \( \text{H}_2 \) (1,0) O(2) line, at least in two sources, and are not explained by the effects of dust extinction. We also found that ULIRGs have a tendency to exhibit high \( \text{Br}/\text{Br} \) line ratios. As we discuss in Appendix B, the case B line ratio explains well the \( \text{Br}/\text{Br} \) line ratio in Galactic H II regions. This indicates that conditions in the H II regions in those ULIRGs with a \( \text{Br}/\text{Br} \) anomaly are entirely different from the conditions in Galactic H II regions.

Figure 3. The 2.5–5.0 \( \mu \text{m} \) near-infrared spectra of galaxies that show \( \text{Br}/\text{Br} \) line ratios significantly higher than that for case B. The best-fit Gaussian profiles for the \( \text{Br} \) and \( \text{Br} \) lines are plotted as red curves.

4.1.2. Collisional Excitation

Next, we consider the collisional excitation mechanism. In the high-density limit, wherein the level population is entirely...
determined by collisions, the level population reaches thermal equilibrium and follows the Boltzmann distribution. The ratio of level populations for this case can be written as

$$n_N \frac{g_N}{n_N'} \frac{g_{N'}}{g_N} \exp \left( \frac{E_N}{kT} \right).$$

where $g_N$ is the degeneracy of state $N$, $k$ is Boltzmann’s constant, $E_N$ is the energy of state $N$ relative to the ground state, and $T$ is the gas temperature. The critical densities for transitions $N = 1 \rightarrow 5$ and 6 are on the order of $\sim 10^{11} \text{ cm}^{-3}$ (Storey & Hummer 1995); therefore, gas densities higher than this threshold are required to achieve thermal equilibrium for the levels related to the $\text{Br}\alpha$ and $\text{Br}\beta$ line emissions.

Figure 4. Spectra around the $\text{Br}\alpha$ (right) and $\text{Br}\beta$ (left) lines for the three galaxies that show anomalous $\text{Br}\beta/\text{Br}\alpha$ line ratios. The underlying continuum has been subtracted. The best-fit Gaussian profile is plotted as a solid red curve. The residuals from the best fit are also displayed in the bottom panels using blue crosses.
The third excitation mechanism is resonant excitation. One example wherein this process becomes important is the Bowen resonance of O III lines (Osterbrock & Ferland 2006). The wavelength of the O III $2p^2 3P_2 - 3d^3 3P_0^o$ line (303.80 Å) is accidentally coincident with that of the He II Lα line (303.78 Å); therefore, the $3d^3 3P_0^o$ level of O III is pumped by the He II Lα line. This results in an enhancement of the O III lines originating from the $3d^3 3P_0^o$ state. A similar situation would occur for hydrogen lines if a line were to exist with a wavelength close to that of the He II Lα line. This results in an enhancement of the O III lines, resulting in an enhancement of the level population of the $N^o$ state.

Based on the three excitation mechanisms described above, we consider some possible explanations for the high Brβ/Brα line ratios.

4.2. Optically Thin Case

We first consider the case in which the Brackett lines are optically thin. In this case, once the level population of neutral hydrogen is determined, the line ratios of the HI lines are fixed. The emergent Brβ/Brα line ratio is then

$$\frac{F_{\text{Br} \beta}}{F_{\text{Br} \alpha}} = \frac{n_6 A_{\text{Br} \beta} h\nu_{\text{Br} \beta}}{n_5 A_{\text{Br} \alpha} h\nu_{\text{Br} \alpha}} \sim 0.440 \frac{n_6}{n_5},$$

where $n_6/n_5$ is the ratio of the populations of hydrogen atoms in levels $N = 6$ and $N = 5$. $A_{\text{line}}$ is the Einstein A coefficient for the line, $h$ is Planck’s constant, and $\nu_{\text{line}}$ is the frequency of the line. This equation shows that in order to explain a high Brβ/Brα line ratio, the level population in $N = 6$ must be enhanced relative to that in $N = 5$.

At low densities, where the recombination process is dominant, the level populations are determined by Equation (3). Assuming $T = 10,000$ K as the gas temperature, we obtain $F_{\text{Br} \beta}/F_{\text{Br} \alpha} = 0.565$, i.e., the case B ratio, by substituting Equation (3) into Equation (5). In contrast, in the high-density limit where the collisional process is dominant, the hydrogen level populations are determined by Equation (4). Again, assuming $T = 10,000$ K as the gas temperature, we obtain $F_{\text{Br} \beta}/F_{\text{Br} \alpha} = 0.522$ by substituting Equation (4) into Equation (5). If we take the limit $T \to \infty$, Equation (4) gives

$$\frac{n_6}{n_5} = \frac{g_6}{g_5} = 1.44$$

as the high-T limit, and we obtain $F_{\text{Br} \beta}/F_{\text{Br} \alpha} = 0.634$. This is the highest line ratio achievable with collisional excitation but it is still lower than the observed values. Thus, we cannot explain the high Brβ/Brα line ratios in either the high-density or the low-density limits.

In general, the level populations are affected both by the recombinations and collisional process, so the combined results are expected to lie somewhere between the above two limits. This is expressed in terms of the departure coefficient $b_{N^o}$, which is the fractional departure of the population of state $N^o$. 

![Figure 5. Averaged near-infrared spectrum of 35 galaxies with $F_{\text{Br} \alpha} < 10^{-14}$ erg s$^{-1}$ cm$^{-2}$. The best-fit Gaussian profiles for the Brα and Brβ lines are plotted as red curves.](image)

![Figure 6. Spectra around the H$_2$ (1,0) O(3) line. The underlying continuum has been subtracted. The best-fit Gaussian profile is plotted as a solid red curve. The residuals from the best fit are also displayed in the bottom panels using blue crosses.](image)
from that in thermal equilibrium, \( n^b_N \), i.e., \( n_N = b_N n^b_N \). Storey & Hummer (1995) calculated the \( b_N \) coefficients for several gas densities, and we show their results in Figure 7. Hereafter, we denote the total hydrogen number density by \( n \); i.e., we write \( n(H^0) + n(H^+) = n \), where \( n(H^0) \) and \( n(H^+) \) are the number densities of neutral and ionized hydrogen, respectively. In the ionized gas, we assume that the hydrogen atoms are fully ionized, so that \( n_e \sim n \). The level populations for \( N = 6 \) and \( N = 5 \) are affected by the collisional process for densities higher than the critical density \( n \sim 10^{11} \text{ cm}^{-3} \).

Using the \( b_N \) coefficients, we can determine the \( n_6/n_5 \) ratio for each gas density and so derive the \( \text{Br}/\text{Br} \) line ratio from Equation (5). We show these results in Figure 8. At low densities, the \( \text{Br}/\text{Br} \) line ratio is consistent with the case B ratio. When the density becomes \( n \gtrsim 10^{10} \text{ cm}^{-3} \), collisional excitation starts to contribute to the \( N = 6 \) state. This causes an enhancement of the \( \text{Br} \) line. At densities higher than \( n \gtrsim 10^{12} \text{ cm}^{-3} \), the \( N = 5 \) state also begins to be collisionally excited, and so the \( \text{Br}/\text{Br} \) line ratio approaches the ratio in thermal equilibrium.

Figure 8 indicates that the \( \text{Br}/\text{Br} \) line ratio becomes as high as \( \sim 0.75 \) at \( n \sim 10^{11} \text{ cm}^{-3} \). This ratio is within 1.6\( \sigma \) and 1.8\( \sigma \) of the observed ratios for IRAS 10565+2448 (0.88 \pm 0.09) and IRAS 10494+4424 (0.96 \pm 0.12), respectively, but it is still more than 3\( \sigma \) lower than that observed in Mrk 273 (1.09 \pm 0.05). Thus, we conclude that we cannot explain the anomaly with just the recombination and collisional processes in the optically thin case.

With resonant excitation, the \( N = 6 \) state is enhanced if a strong line with a wavelength comparable to that of the transition \( N = 5 \rightarrow 1 \) (937.8 Å) exists. As we discuss in detail in Appendix C.1, we found that if this (unknown) line has a transition probability comparable to those of forbidden lines, then the resonant process would be able to make the \( \text{Br}/\text{Br} \) line ratio anomalously high.

We reviewed the atomic and molecular data currently available to search for possible resonant lines. For the line data, we used the line list provided in the Cloudy program (Ferland et al. 1998). Cloudy is a spectral-synthesis code designed to numerically simulate an astrophysical plasma and its emissions. Extensive atomic and molecular data are collected in the code (references are available in a file distributed along with the code). We searched for possible resonant lines with a wavelength of \( \sim 937.8 \text{ Å} \) within a velocity range of \( \sim 10 \text{ km s}^{-1} \), corresponding to the thermal velocity at \( T = 10,000 \text{ K} \) (Osterbrock & Ferland 2006). We found no candidates in the Cloudy data, and thus we exclude this resonant process as a possible cause of the anomalous \( \text{Br}/\text{Br} \) line ratio.

Based on the above discussion, we conclude that we cannot explain the anomalous \( \text{Br}/\text{Br} \) line ratio if the Brackett lines are optically thin.

### 4.3. Optically Thick Case

The alternative is that the Brackett lines are optically thick and the observed line ratio deviates from Equation (5). In this case, it is possible to explain the high \( \text{Br}/\text{Br} \) line ratio if the \( \text{Br} \) line becomes optically thick and saturates, whereas the \( \text{Br} \) line remains optically thin. Herein, we discuss the optical depth effect on this ratio.

#### 4.3.1. Optical Depth of Brackett Lines

In a uniform gas, the optical depth \( \tau_{N',N} \) at the line center for the transition \( N' \rightarrow N \) at the transition \( N' \rightarrow N \) is given as

\[
\tau_{N',N} = \int_0^R \alpha_{N',N} n_N dl \sim \alpha_{N',N} N_N,
\]

where \( \alpha_{N',N} \) is the total absorption coefficient for the transition from \( N' \) to \( N \). The depth of the line is given by

\[
\log n = \log n_0 - \tau_{N',N},
\]
where \( R \) is the size of the gas, \( \alpha_{N',N} \) is the absorption cross-section of the transition \( N' \rightarrow N' \), \( n_N \) is the number density of neutral hydrogen in state \( N' \), and \( N_N \) is the column density of neutral hydrogen in state \( N' \). The optical depth of the Brackett lines is proportional to the column density of neutral hydrogen in the quantum state \( N' = 4 \).

Assuming a Gaussian profile as a line velocity profile, the absorption cross-section is related to the Einstein B coefficient, \( B_{N',N} \), by

\[
\alpha_{N',N} = \frac{hc}{4\pi^3/2} \frac{B_{N',N}}{v_{\text{Dop}}},
\]

(7)

where \( c \) is the speed of light, and \( v_{\text{Dop}} \) is the Doppler velocity half width, the distance from line center where the line profile falls to \( e^{-1} \) of its peak. If the line profile is determined solely by thermal motions, the Doppler width can be written as

\[
v_{\text{Dop}} = v_{\text{Therm}} = \sqrt{2kT/m_H},
\]

where \( m_H \) is the mass of a hydrogen atom. At \( T = 10,000 \text{ K} \), we have \( v_{\text{Therm}} \approx 13 \text{ km s}^{-1} \). If a turbulent motion with a velocity \( v_{\text{Turb}} \) affects the line width, \( v_{\text{Dop}} = \sqrt{v_{\text{Therm}}^2 + v_{\text{Turb}}^2} \).

Substituting Equation (7) into Equation (6) and assuming \( T = 10,000 \text{ K} \), we obtain the line optical depths of the \( \text{Br}\alpha \) and \( \text{Br}\beta \) lines as

\[
\tau_{\text{Br}\alpha} \sim 1.0 \left( \frac{N_4}{1.6 \times 10^{11} \text{ cm}^{-2}} \right) \left( \frac{v_{\text{Dop}}}{10 \text{ km s}^{-1}} \right)^{-1},
\]

(8)

\[
\tau_{\text{Br}\beta} \sim 0.11 \left( \frac{N_4}{1.6 \times 10^{11} \text{ cm}^{-2}} \right) \left( \frac{v_{\text{Dop}}}{10 \text{ km s}^{-1}} \right)^{-1}.
\]

(9)

Thus, assuming \( v_{\text{Dop}} \approx 10 \text{ km s}^{-1} \), for instance, we find that the \( \text{Br}\alpha \) line becomes optically thick while the \( \text{Br}\beta \) line is still optically thin when \( N_4 \approx 2 \times 10^{11} \text{ cm}^{-2} \). We further discuss the possible conditions that can produce a high \( \text{Br}\beta/\text{Br}\alpha \) line ratio on the basis of Equations (8) and (9).

### 4.3.2. Possible Conditions Producing a High Brackett Line Ratio

Herein, we assume that the high \( \text{Br}\beta/\text{Br}\alpha \) line ratio is produced within a single isolated \( \text{H\ II} \) region ionized by a single star and that what we observe is an ensemble of such \( \text{H\ II} \) regions. Within a single \( \text{H\ II} \) region, we assume that \( v_{\text{Dop}} \) is determined only by the thermal width, with \( v_{\text{Therm}} \approx 10 \text{ km s}^{-1} \), which is a typical velocity observed in nearby \( \text{H\ II} \) regions (e.g., Arthur et al. 2016). We assume that the \( \text{Br}\alpha \) line becomes optically thick within each \( \text{H\ II} \) region.

For a spherical and uniform \( \text{H\ II} \) region, we next discuss how to make the \( \text{Br}\alpha \) line optically thick using the three excitation mechanisms described in Section 4.1. First, we consider the case in which the recombination process is dominant. In this case, \( n_4 \) is determined by Equation (2), which yields \( n_4 \approx 3.6 \times 10^{-21}(n/\text{cm}^{-3})^2 \text{ cm}^{-2} \). We can thus write \( N_4 \) in terms of \( n_4 \) and \( R \) as

\[
N_4 = n_4 R \\
\sim 3.6 \times 10^{-21} \left( \frac{n}{\text{cm}^{-3}} \right)^2 \left( \frac{R}{\text{cm}} \right) \text{ cm}^{-2}.
\]

(10)

If we write \( nR = N \), where \( N \) is the total hydrogen column density, then we have

\[
N_4 = 3.6 \times 10^{-21} \left( \frac{n}{\text{cm}^{-3}} \right) \left( \frac{N}{\text{cm}^{-2}} \right) \text{ cm}^{-2}.
\]

(11)

Thus \( N_4 \) is proportional to both \( n \) and \( N \). If the \( \text{H\ II} \) region is ionized by a central star that emits a number of ionizing photons per unit time \( Q(\text{H}) \), then ionization-equilibrium at \( T = 10,000 \text{ K} \) yields

\[
Q(\text{H}) = \int \alpha_B n_e n_p dV \sim \frac{4\pi}{3} \alpha_B n^2 R^3 = \frac{4\pi}{3} \alpha_B N^3 n^{-1},
\]

\[
\sim N = 3.37 \times 10^{30} \left( \frac{Q(\text{H})}{10^{49} \text{ s}^{-1}} \right)^{1/2} \left( \frac{n}{10^8 \text{ cm}^{-3}} \right)^{1/2} \text{ cm}^{-2},
\]

(12)

where \( \alpha_B \) is the total recombination coefficient for hydrogen in case B, and \( Q(\text{H}) = 10^{49} \text{ s}^{-1} \) is a typical value for an O star (Osterbrock & Ferland 2006). Substituting Equation (12) into Equation (11), we obtain \( N_4 \) in terms of \( n \) and \( Q(\text{H}) \):

\[
N_4 = 2.35 \times 10^{11} \left( \frac{Q(\text{H})}{10^{49} \text{ s}^{-1}} \right)^{1/2} \left( \frac{n}{10^8 \text{ cm}^{-3}} \right)^{4/2} \text{ cm}^{-2}.
\]

(13)

Thus, within a single \( \text{H\ II} \) region with an ionizing source emitting \( Q(\text{H}) \approx 10^{49} \text{ s}^{-1} \), a gas density as high as \( n \approx 10^8 \text{ cm}^{-3} \) is required to achieve a column density \( N_4 \approx 2 \times 10^{11} \text{ cm}^{-2} \) that is large enough to make the \( \text{Br}\alpha \) line optically thick.

At a density \( n = 10^8 \text{ cm}^{-3} \), the collisional process is not dominant as the excitation mechanism for the population in the quantum state \( N' = 4 \) because the critical density for the \( N' = 1 \rightarrow 4 \) transition is \( \sim 10^{12} \text{ cm}^{-3} \) (Storey & Hummer 1995). This is also shown in Figure 7. The relative difference between the \( b_4 \) coefficients at \( n = 10^8 \text{ cm}^{-3} \) and at \( 10^9 \text{ cm}^{-3} \) is less than 10%, indicating that the \( \text{N} = 4 \) state is not dominantly populated by the collisional process at densities \( n \leq 10^8 \text{ cm}^{-3} \). Thus, Equation (13), in which only the recombination process is considered, is valid if we take collisional excitation into account at a density of \( n = 10^8 \text{ cm}^{-3} \).

Resonant excitation would enhance the \( \text{N} = 4 \) state if a line exists with a wavelength equal to that of the \( \text{N} = 4 \rightarrow 1 \) transition (972.5 Å). In this case, the density required to make the \( \text{Br}\alpha \) line optically thick would be lowered from the value \( n \approx 10^8 \text{ cm}^{-3} \) given by Equation (13). Based on a detailed discussion provided in Appendix C.2, we found that if a line with a wavelength of \( \sim 972.5 \text{ Å} \) and a transition probability of \( \sim 10^{-1} \text{ s}^{-1} \) exists, we should take the resonant process into consideration in determining the population of the \( \text{N} = 4 \) state. We searched for possible resonant lines with wavelengths of \( \sim 972.5 \text{ Å} \) within the velocity range \( \sim 10 \text{ km s}^{-1} \), which corresponds to the thermal velocity at \( T = 10,000 \text{ K} \) (Osterbrock & Ferland 2006) using the line list from the Cloudy code (Ferland et al. 1998) and found no candidates for the \( X_4 \) line. Thus, we conclude that the resonant process does not take place and is excluded from the excitation mechanisms for the \( \text{N} = 4 \) state.

### 4.3.3. Simulation with Cloudy

In order to investigate quantitatively the \( \text{Br}\beta/\text{Br}\alpha \) line ratio taking all excitation mechanisms into account, we used the
Cloudy code (ver. 10.00; Ferland et al. 1998) and simulated the ratio for the optically thick case. Cloudy calculates the recombination and the collisional processes altogether. Cloudy also solves the radiative transfer of lines and so can be used to investigate the effect of optical depth on the line fluxes.

To execute a simulation with Cloudy, four parameters are required: (1) the spectral shape of the incident radiation, (2) the intensity of the incident radiation, (3) the density of the surrounding gas, and (4) the criterion for stopping the calculation. We considered a single spherical H II region with a uniform gas ionized by a hot central star. For the spectral shape of the incident radiation (1), we used a blackbody radiation to simulate a typical O star (Osterbrock & Ferland 2006). For the intensity of the incident radiation (2), we specified the number of ionizing photons per unit time, $Q$ (H). We varied $Q$(H) from $10^{48}$ s$^{-1}$ to $10^{51}$ s$^{-1}$ using intervals of a decade on the assumption that the ionizing radiation is dominated by massive OB stars in starburst regions. The $Q$(H) values $10^{48}$ s$^{-1}$, $10^{49}$ s$^{-1}$, and $10^{50}$ s$^{-1}$ correspond to typical values for B stars, O5 stars, and massive O stars (O3 stars), respectively (Osterbrock & Ferland 2006). We also calculated a case with $Q$(H) = $10^{51}$ s$^{-1}$ for reference. We varied the gas density $n$ from $n = 10^2$ cm$^{-3}$ to $10^{10}$ cm$^{-3}$ using intervals of a decade. To simulate line emission from the ionized region, we adopted the electron fraction for the total gas, i.e., the degree of ionization, to be 0.1 as the stopping criterion for the calculation (4). In addition to the abovementioned parameters, we specified a spherical geometry with the inner radius of the surrounding gas, which is required in Cloudy when we use $Q$(H) as the intensity of the incident radiation, equal to $r = 10^{12}$ cm. We iterated the calculations until the difference between the line optical depths of the last two iterations became smaller than 0.20. The adopted parameters described above are summarized in Table 4. Other parameters are set to the default values of Cloudy; e.g., the line width is determined by the thermal velocity, solar abundances are adopted, and dust grains are not included in the calculations.

The results of the Cloudy simulations of the Brβ/Brα line ratio are shown in Figure 9. We find that the Brβ/Brα line ratio increases for high values of $n$ and $N$. In contrast, the ratio is close to the case B value (0.565) for low values of $n$ and $N$. The optical depth of the Brα line is proportional to $N_\alpha$, which is proportional to $n$ and $N$, as shown in Equation (11). Thus, the results are consistent with our simple estimate, indicating that a high Brβ/Brα line ratio is produced when the Brα line becomes optically thick.

We now compare the observed Brβ/Brα line ratios with the Cloudy results. Figure 9 indicates that the observed ratios in the three galaxies are consistent with $n \sim 10^7$ cm$^{-3}$, where the Brα line starts to become optically thick. In conditions with higher values of $n$, the Brβ/Brα line ratio becomes too large to match the observed anomalies. From this result, we conclude that in order to explain the observed Brβ/Brα line ratio within a single H II region, gas densities as high as $n \sim 10^8$ cm$^{-3}$ are required to achieve column densities large enough to make the Brα line optically thick.

### 5. Comparison with Other Hydrogen Recombination Lines

In Cloudy simulations, not only Brα and Brβ, but also other H I recombination lines are calculated. The intensity ratios between those lines can also be compared to observations. We here review the existing observations of other H I lines in the optical and near-infrared for the ULIRGs, in particular for the three objects in which significant anomalous Brβ/Brα ratios were found, and demonstrate that they do not contradict the predictions of our high-density model. This also explains why intensity ratio anomalies such as the one revealed in this work have not been found before. In the following subsections, we explain the reasons in detail for each H I line of interest. A basic short explanation is that even if the intensity ratio of other H I lines deviates from the case B value due to the high-density condition, that change is indistinguishable from the effect of attenuation.

As a reference model for explaining the observed Brβ/Brα line ratio, we adopt the Cloudy result calculated for $n = 10^7$ cm$^{-3}$ and $Q = 10^{49}$ s$^{-1}$. This model represents a H II region, ionized by an O3 star and surrounded by gas at high density. The parameters and important results of this reference model are summarized in Table 5. The Brβ/Brα line ratio for this model (0.940) explains well the observed values in IRAS 10494+4424, IRAS 10565+2448, and Mrk 273 of 0.96±0.12, 0.88±0.09, and 1.09±0.05, respectively. The predictions made by this model for several other H I line ratios, which are to be discussed below, are listed in Table 6. This table also shows the values for case B and the values observed in the three ULIRGs.

#### 5.1. Comparison with Hα Line

In this subsection, we compare the fluxes of the Brackett lines and the Hα line for IRAS 10494+4424, IRAS 10565+2448, and Mrk 273. For IRAS 10494+4424 and Mrk 273, we use the Hα line flux obtained from our Nickol observations (Table 2). IRAS 10565+2448 was observed with the integral field unit on the Gemini North telescope by Shih & Rupke (2010); they reported an integrated Hα line flux $F_{\text{Hα}} = (8.42 ± 0.84) \times 10^{-14}$ erg s$^{-1}$ cm$^{-2}$ for this galaxy (before extinction correction) within an aperture of $5'' \times 7''$. We adopt this flux for comparison with our results for IRAS 10565+2448.

We summarize the Hα line ratio in Table 6. Figure 10 compares the flux of the Brα, Brβ, and Hα lines. The high-density model explains well both the Hα/Brα and Brβ/Brα line ratios if the dust extinction is $A_V = 2.5$–5.0 mag; this is
The observed high Br/$\beta$ line ratio cannot be achieved by the case B model. In contrast, the extinction vector from the high-density model indicates that both the high Br/$\beta$ line ratio and the H$\beta$/H$\alpha$ line ratio are well explained by the high-density model with almost the same dust extinction. We cannot distinguish between these two models from the H$\alpha$/Br$\alpha$ line ratio.

We conclude that the high-density model is consistent with the observations of the H$\alpha$ line. The effect of dust extinction at optical wavelengths is so strong that the deviation of the H I line ratios from those of case B due to the high-density condition is easily canceled out and made unnoticeable.

5.2. Comparison with H$\beta$/H$\alpha$ Line Ratio

The H$\beta$ line ($N = 4 \rightarrow 2, 4861 \ \AA$) is also observable in the optical as well as H$\alpha$, and the H$\beta$/H$\alpha$ line ratio is one of the most intensively studied ratios among H I lines (e.g., Kim et al. 1998). We here compare the H$\beta$/H$\alpha$ line ratio with our Br$\beta$/Br$\alpha$ line ratio.

For the AGNUL sample, we use the H$\beta$/H$\alpha$ line ratio summarized in Table 3 of Yano et al. (2016). The ratios for the two galaxies added to our sample in this paper are summarized in Appendix A. The H$\beta$/H$\alpha$ line ratios for all the targets are well below the case B value of 0.348 (Table 6). This indicates that the H$\beta$/H$\alpha$ line ratio can be explained with a combination of case B and dust extinction. The inferred dust extinction in this case (A$^\text{opt}$ in the tables) is typically $\sim$2.5 mag.

A comparison of the H$\beta$/H$\alpha$ line ratio with our Br$\beta$/Br$\alpha$ line ratio is shown in Figure 11 and displays a large scatter between the two ratios. The extinction vectors in Figure 11 indicate that the H$\beta$/H$\alpha$ line ratio is strongly affected by dust extinction. In contrast, the Br$\beta$/Br$\alpha$ line ratio is almost unchanged with dust extinction of A$^\text{opt}$ $\sim$2.5 mag.

The observed high Br$\beta$/Br$\alpha$ line ratios cannot be achieved by the case B model. In contrast, the extinction vector from the high-density model indicates that both the high Br$\beta$/Br$\alpha$ line ratio and the H$\beta$/H$\alpha$ line ratio are well explained by the high-density model with dust extinction of A$^\text{opt}$ $\sim$2.5 mag.

In the high-density model, the H$\beta$/H$\alpha$ line ratio is predicted to be 0.216 (Table 6). We note that the deviation of the H$\beta$/H$\alpha$ line ratio in the high-density model is not distinguishable from the effect of dust extinction. This indicates that the deviation cannot be probed with the H$\beta$/H$\alpha$ line ratio even in conditions wherein dust extinction is very low.

From these results, we conclude that the high-density model is consistent with the observations of the H$\beta$/H$\alpha$ line ratio. We also conclude that the deviation of the H I line ratios from those
The extinction vectors for $\beta$ are shown as red circles. The black cross indicates the line ratios predicted by the high-density model, whereas the dashed arrow indicates extinction from the ratios of the high-density model with different visual extinctions. The black crosses show the line ratios for the case B model with no dust extinction, with extinction of $A_V = 5.0$ mag, and with extinction of $A_V = 2.5$ mag, and with extinction of $A_V = 5.0$ mag, respectively.

Figure 10. Hydrogen recombination line fluxes relative to the $\beta$ line. The ratios are normalized to those predicted for case B (10,000 K, low-density limit: Osterbrock & Ferland 2006). The black lines show the line ratios for the case B model with different visual extinctions. The black crosses show the line ratios predicted by the high-density model. The solid, dashed, and dotted crosses indicate the ratios with no dust extinction, with extinction of $A_V = 2.5$ mag, and with extinction of $A_V = 5.0$ mag, respectively.

5.3. Comparison with $\text{Pa}_\alpha/\text{Br}\gamma$ Line Ratio

In this subsection, we compare our results with infrared HI lines. In the infrared, the $\text{Br}\gamma$ line at 2.17 $\mu$m and the $\text{Pa}_\alpha$ line at 1.88 $\mu$m are both observable in $K$-band ($\sim$1.9–2.5 $\mu$m) observations from the ground for nearby objects ($z \sim 0.01–0.15$). These lines are relatively strong and have been observed in various studies to measure dust extinction (e.g., Goldader et al. 1995; Veilleux et al. 1999b).

The $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio predicted by the high-density model is 9.93, deviating from the case B value of 12.1 (Table 6). In the prediction, the $\text{Pa}_\alpha$ line at the shorter wavelength is weakened relative to the $\text{Br}\gamma$ line at the longer wavelength. This is the same trend as the effect of dust extinction. The $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio at 9.93 corresponds to dust extinction of $A_V = 6.60$ mag, assuming the case B ratio. This indicates that we cannot distinguish the deviation of the predicted $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio from that of case B due to the effect of dust extinction. Thus, our model is consistent with the fact that no anomaly has been reported in previous observations of the $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio.

Among our targets with anomalous $\text{Br}\beta/\beta$ line ratios, the $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio was observed in IRAS 10494+4424 by Murphy et al. (2001) and in Mrk 273 by Veilleux et al. (1999b). We cannot find observations of the $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio in IRAS 10565+2448. The $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio in Mrk 273 is 10.8 $\pm$ 1.0, which is consistent with our prediction. The $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio in IRAS 10494+4424 was reported to be 16.1 $\pm$ 1.5. This ratio indicates that the $\text{Pa}_\alpha$ line is enhanced relative to the $\text{Br}\gamma$ line compared to the case B ratio. This is opposite to the effect of dust extinction and also to the deviation of our model from the case B ratio. Thus, the ratio is explained neither by our model nor by the case B ratio with dust extinction. Murphy et al. (2001) regarded this anomaly as not significant. They claimed that the observations of the $\text{Pa}_\alpha$ and $\text{Br}\gamma$ lines in IRAS 10494+4424 were performed with different apertures, and this made the observed $\text{Pa}_\alpha/\text{Br}\gamma$ line ratio uncertain by as much as 50%. Thus, we also treat the deviation as not significant here.

Based on the above results, we conclude that our model does not contradict previous observations of the HI line ratios. The deviation of a line intensity ratio from case B due to the
high-density condition is hardly detectable in ratios other than Br/Brα in the optical and near-infrared wavelength regions.

6. Structure of High-density H II Regions

We find that we can explain the high Br/Brα line ratio with the optical depth effect. In order to make the Brα line optically thick, high-density H II regions are required in our Cloudy model. Herein, we discuss possible structures of such high-density H II regions in ULIRGs.

6.1. Two Extreme Cases

To explain the observed luminosity of the Brα line, we consider two extreme cases for the line-emitting regions: (1) an ensemble of H II regions, each ionized by a single star and (2) a giant H II region where all the ionizing stars are concentrated at the center. For each case, we discuss whether the optical depth effect can produce high Br/Brα line ratios.

6.1.1. Ensemble of H II Regions

We here consider an ensemble of H II regions, each of which is represented by the high-density model. In Table 7, we list the observed luminosity of the Brα line for each of the three galaxies with a high Br/Brα line ratio. In the high-density model, the luminosity of the Brα line produced by a single H II region is found to be $L_{\text{Br}\alpha} = 2.79 \times 10^{36}$ erg s$^{-1}$ (Equation (8)), and we estimate the number of H II regions $n_{\text{tot}}$, as shown in Table 7. From this result, we conclude that $\sim 10^{5}$ H II regions with high-density conditions are required to explain our observations.

We next estimate the expected number of H II regions along the line of sight, $n_{\text{los}}$, to determine the effect of optical depth on the line emitted from a single H II region and intercepted by other H II regions. We assume a volume-filling factor $\varepsilon \sim 10^{-3}$, which is a typical value for H II regions observed in starburst galaxies (Anantharamaiah et al. 1993). We also assume that H II regions are uniformly and spherically distributed. Using the radius $R$ of individual H II regions, the volume of the entire space in which they are distributed as an ensemble can be written as $V_{\text{tot}} = \frac{4}{3} \pi R^3$. Then, the diameter $d$ of that space becomes

$$d \sim (k_{\text{los}} \varepsilon)^{1/3} R. \quad (14)$$

However, $k_{\text{los}}$, the number of H II regions that exist on the diameter, satisfies

$$k_{\text{los}} R^3 \sim \varepsilon d R^2. \quad (15)$$

By comparing Equations (14) and (15), we find

$$k_{\text{los}} \sim \frac{1}{3} \frac{\varepsilon d}{R^2}. \quad (16)$$

Substituting $k_{\text{tot}} = 10^5$ and $\varepsilon = 10^{-6}$ into Equation (16), we obtain $k_{\text{los}} \sim 10^{-5}$. We assume the relative velocities of H II regions to be of the order of $\sim 100$ km s$^{-1}$, which is a typical line velocity observed in galaxies (e.g., Osterbrock & Ferland 2006) and is an order of magnitude higher than the thermal velocity. Then, the optical depth of the Brα line caused by the intercepting H II regions is found to be three orders of magnitude smaller than that of the H II region from which the line originates. Thus, we conclude that the line ratio produced in a single H II region is not affected by other H II regions, even if we consider an ensemble of $\sim 10^5$ H II regions.

We therefore conclude that an ensemble of H II regions, in each of which the Brα line is optically thick, can explain the high Br/Brα line ratio. This ratio is produced within each H II region, and what we observe is a collection of such H II regions. To achieve a column density large enough to make the Brα line optically thick within a single H II region, the gas density must be as high as $n \sim 10^8$ cm$^{-3}$.

6.1.2. Single Giant H II Region

We next consider another simplified model, in which the line-emitting region is not a collection of H II regions but a single giant H II region, where all the ionizing stars are concentrated at the center of a uniform gas. We investigate whether a high Br/Brα line ratio can be produced within such a giant H II region by the optical depth effect.

We have shown that ionizing sources with $Q(H)$ of the order of $\sim 10^{55}$ s$^{-1}$ are required to explain the observed luminosity of the Brα line. Thus, we also assume a central ionizing source of $Q(H) = 10^{55}$ s$^{-1}$, which corresponds to $10^5$–$10^6$ OB stars. For the turbulence velocity within the giant H II region, we assume $v_{\text{turb}} = 100$ km s$^{-1}$. In this case, Equation (8) indicates that $N_4 \sim 2 \times 10^{12}$ cm$^{-2}$ is required to make the Brα line optically thick. Substituting $N_4 = 2 \times 10^{12}$ cm$^{-2}$ and $Q(H) = 10^{55}$ s$^{-1}$ into Equation (13), we obtain $n \sim 2 \times 10^7$ cm$^{-3}$ for the gas density required to make the Brα line optically thick within the giant H II region.

To investigate the Br/Brα line ratio quantitatively for this case, we again used the Cloudy code to simulate the giant H II region. Most of the parameters are the same as those tabulated in Table 4, except that $Q(H)$ is now fixed at $10^{55}$ s$^{-1}$, and $n$ is varied at intervals of 0.5 dex up to $n = 10^8$ cm$^{-3}$, and $v_{\text{turb}} = 100$ km s$^{-1}$ is adopted. We show the Cloudy results in Figure 12. The observed Br/Brα line ratio can be explained if the gas density is within the range $n = 10^7$–$10^8$ cm$^{-3}$. The luminosity of the Brα line in these conditions is $L_{\text{Br}\alpha} \sim 2 \times 10^{41}$ erg s$^{-1}$, and we confirm that this agrees well with the observed value. We thus conclude that gas densities as high as $n \sim 10^8$ cm$^{-3}$ are also required to explain the observed Br/Brα line ratio in the extreme case in which the line-emitting region is a single giant H II region with a central ionizing source emitting $Q(H) = 10^{55}$ s$^{-1}$.

There is one caveat to the Cloudy results shown in Figure 12. With $Q(H) = 10^{55}$ s$^{-1}$, the column density of electrons within the H II region exceeds $N_4 \sim 2 \times 10^{24}$ cm$^{-2}$ for conditions with $n \geq 10^8$ cm$^{-3}$ so the H II region becomes Compton thick. The Cloudy code is not designed to simulate Compton-thick regimes (Ferland et al. 1998); therefore, the validity of the result is not guaranteed in those conditions. The process of Thomson scattering does not include any energy transfer. Thus,
we expect that the result for the emergent line ratio is still valid even if the H II region becomes Compton thick.

In summary, for both of the two extreme cases, we conclude that gas densities as high as \( n \sim 10^8 \) cm\(^{-3}\) are required to achieve a column density of neutral hydrogen large enough to make the Br\(\alpha\) line optically thick. We propose this high-density scenario as the most plausible cause of the high Br\(3\)/Br\(\alpha\) line ratio.

### 6.2. Impact of Dust Grains

The density of \( n \sim 10^8 \) cm\(^{-3}\) is in a regime denser than the densest ultracompact H\(\II\) regions in our Galaxy (e.g., Kurtz 2000; Churchwell 2002). In this regime, dust can compete with H\(\I\) for ionization photons. Then, the effective \( Q \) (H) for H\(\I\) is reduced, and a higher number density may be needed to achieve a column density sufficient to make the Br\(\alpha\) line optically thick (Equation (13)). On the other hand, it is also expected that the higher the density, the more the dust is thermally coupled with gas, and the abundance of the dust decreases due to sublimation. The overall influence of the dust includes both of these effects. Because the above Cloudy models are assumed to be dust-free, we here quantitatively evaluate how the results change when the dust is taken into account by modifying the cloudy models.

We start with the case of an ensemble of H\(\II\) regions. Cloudy simulations were performed with dust. The abundances were changed from the default to a predefined set of the interstellar medium. Both silicate and graphite grains are included with 10 size bins for each. The option to treat the dust sublimation was turned on so that the abundance of each grain species steeply decreases when its temperature is above the sublimation temperature. The Br\(3\)/Br\(\alpha\) line ratio was calculated as a function of \( n \). To examine the case of the intensest ionization, \( Q(H) \) was fixed at \( 10^{55} \) s\(^{-1}\) (corresponding to an O3 star). Other parameters are the same as in Table 4. The results are shown in Figure 13. With the same parameters as the reference model in the dust-free case \( n = 10^6 \) cm\(^{-3}\), \( Q(H) = 10^{50} \) s\(^{-1}\), the Br\(3\)/Br\(\alpha\) ratio is lowered from 0.94 to 0.46. This is due to the fact that the column density \( N \) is not much increased and Br\(\alpha\) does not become optically thick because ionizing photons are consumed by dust, and that the ratio is affected by dust extinction. The ratio of \( \sim 1 \) as observed is found at \( n = 10^7 \) cm\(^{-3}\). At this point, \( L_{\text{Br}\alpha} / L_{\text{Br}3} \) becomes \( 2.4 \times 10^{35} \) erg s\(^{-1}\), which is a factor of 12 lower than in the dust-free reference model. Thus, the estimate of \( k_{\text{ion}} \) increases up to \( 10^9 \). The estimate of \( k_{\text{ion}} \) changes by only a factor of two, and the possibility that multiple H\(\II\) regions are aligned along the line of sight and the line emission from the H\(\II\) region behind is blocked is still negligible.

Next, we move on to the case of a single giant H\(\II\) region. In the above case of the H\(\II\) region ensemble with dust, \( k_{\text{ion}} \) has increased by one order of magnitude. However, in this case, the \( Q(H) \) needed is expected to be the same as in the dust-free model, \( Q(H) = 10^{55} \) s\(^{-1}\). This is because if multiple stars are concentrated near the center, they will destroy dust grains in the same region together, and thus cancel out the dust effect more efficiently than in the ensemble case, where each star destroys the surrounding dust on its own. We thus changed the \( Q(H) \) of the above Cloudy model with dust to \( 10^{52} \) s\(^{-1}\) and calculated the Br\(3\)/Br\(\alpha\) ratio as a function of \( n \). The specification of \( v_{\text{turb}} = 100 \) km s\(^{-1}\) was also added. The results are shown in Figure 13. A ratio of \( \sim 1 \) is found at \( n = 10^7.5 \) cm\(^{-3}\). Therefore, a high-density situation is still necessary. At this point, \( L_{\text{Br}3} / L_{\text{Br}\alpha} = 1 \times 10^{42} \) erg s\(^{-1}\), which fully explains the observed line luminosities (Table 7). Hence, as predicted, in the case of a giant H\(\II\) region, the observation results can be explained by \( Q(H) = 10^{55} \) s\(^{-1}\) ( \( \sim 10^5 \) O3 stars) even if dust is taken into consideration.

The actual situation is expected to be bracketed between these two extreme cases. Therefore, we conclude that the number of massive stars needed to explain the Br\(3\)/Br\(\alpha\) anomaly may be increased by up to an order of magnitude from the estimates based on the assumption of the dust-free gas. The main conclusion (the high-density gas is needed to explain the anomalous H\(\I\) line ratio) is the same for the cases even with dust.

### 6.3. Comparison of Lifetime with Galactic Ultracompact H\(\II\) Regions

The dust-free ensemble model, about \( 10^5 \) H\(\II\) regions with gas at high densities \( n \sim 10^8 \) cm\(^{-3}\) are required to explain the high Br\(3\)/Br\(\alpha\) line ratios. Herein, we discuss the possibility of...
observing so many high-density H II regions in ULIRGs by comparison with the ultracompact H II regions in our Galaxy.

Ultracompact H II regions in our Galaxy (e.g., Kurtz 2000; Churchwell 2002) are reported to contain high-density gas up to $n \sim 10^7$ cm$^{-3}$ and to have sizes of the order of $10^{-2}$ pc (e.g., de Pree et al. 1995). The densities and sizes of these ultracompact H II regions are comparable to those of the high-density H II regions required in our model. We thus consider the high-density H II regions in our model as analogous to the ultracompact H II regions in our Galaxy.

A simple estimate of the lifetime of a H II region in an ultracompact state with a size $r \sim 10^{-2}$ pc can be obtained by dividing $r$ by the sound speed $v_s$ in the ionized material ($v_s \sim 10$ km s$^{-1}$ at $T = 10,000$ K), assuming that the H II region expands at a speed comparable to $v_s$. This yields a lifetime $r/v_s \sim 10^3$ yr for an ultracompact H II region in our Galaxy. Another estimate can be obtained from the number of ultracompact H II regions, which is estimated to be $\sim 10^3$ in our Galaxy (Churchwell 2002). Adopting the formation rate of O stars to be $\sim 10^{-2}$ stars yr$^{-1}$ in our Galaxy (de Pree et al. 1995), we get instead for the lifetime of an ultracompact H II region $\sim 10^5$ yr, some two orders of magnitude longer than that obtained from the simple expansion of a H II region. This large difference between the estimated lifetimes is recognized as the “lifetime problem,” first mentioned by Wood & Churchwell (1989), which still remains an open question (e.g., Kurtz 2000; Churchwell 2002). Herein, we simply adopt $\sim 10^5$ yr as the representative lifetime of an ultracompact H II region.

We assume that the high-density H II regions required in our high-density model have a lifetime of the same order ($\sim 10^5$ yr) as those of the ultracompact H II regions in our Galaxy. We note that the star formation rate (SFR) in a ULIRG is about two orders of magnitude higher than that in our Galaxy (Sanders et al. 1988). Scaling the number of the ultracompact H II regions observed in our Galaxy ($\sim 10^3$; Churchwell 2002) with the SFR, we thus expect the number of high-density H II regions to be on the order of $\sim 10^5$ in a ULIRG. Thus, we conclude that it is indeed possible to have $\sim 10^5$ high-density H II regions in a ULIRG, as our model predicts.

After the $\sim 10^5$ yr lifetime of the ultracompact phase, the gas density of a H II region is expected to fall below $\sim 10^3$ cm$^{-3}$ as the H II region expands (de Pree et al. 1995). The typical lifetime of an O star is on the order of $10^5$ yr (Osterbrock & Ferland 2006), which is an order of magnitude longer than that of the ultracompact H II regions. Thus, the number of H II regions with gas densities lower than $\sim 10^3$ cm$^{-3}$ is expected to be an order of magnitude larger than that of the ultracompact H II regions. However, to explain the high Br$\beta$/Br$\alpha$ line ratios, our model requires most H II regions to be in the ultracompact phase. Our results thus indicate that some mechanism is required to ensure that the Brackett lines from the H II regions in ULIRGs are dominated by those emitted from ultracompact H II regions. This problem remains when the dust is taken into account, where a larger number of denser H II regions are required if they are isolated from each other.

7. Prediction to Radio Recombination Lines

We consider the effect of high densities on radio recombination lines. Our model requires gas densities $n = 10^8$ cm$^{-3}$. At such high densities, collisional processes become important for high-$N$ states. As shown in Figure 7, hydrogen levels with low principal quantum numbers ($N \leq 15$) are not dominated by collisions, even at a density $n = 10^8$ cm$^{-3}$. In contrast, the collisional processes start to contribute significantly for states with $N \geq 20$. This indicates that hydrogen radio recombination lines emitted with transitions involving high-$N$ states are affected by the high densities our model predicts.

Peters et al. (2012) showed that at a density $n = 10^8$ cm$^{-3}$, $b_N \sim 1$ for levels with $N > 30$. For $n = 10^9$ cm$^{-3}$, where anomalous Br$\beta$/Br$\alpha$ line ratios are not found in the Cloudy simulations, only states with $N > 50$ become thermalized. This indicates that observations of radio recombination lines with low-$N$ transitions ($N < 50$) are required to probe our model predictions. However, previous observations of radio recombination lines were mainly focused on high-$N$ transitions because of difficulty of observations at the high frequencies where the transitions with $N < 50$ are located.

Radio recombination lines involving levels with $N < 50$ are now observable within the frequency range of ALMA. We thus predict that such radio recombination lines will be found to be thermalized in those galaxies with Br$\beta$/Br$\alpha$ line ratio anomalies, although the radio recombination lines are generally very weak (e.g., Izumi et al. 2016; Michiyama et al. 2020).

8. Summary

We conducted systematic observations of the H I Br$\alpha$ and Br$\beta$ lines with the AKARI IRC for 52 nearby ($z < 0.3$) ULIRGs. We detected Br$\alpha$ and Br$\beta$ lines in 33 ULIRGs. Among these, three galaxies, IRAS 10494+4424, IRAS 10565+2448, and Mrk 273, show Br$\beta$/Br$\alpha$ line ratios of $0.96 \pm 0.12$, $0.883 \pm 0.085$, and $1.086 \pm 0.053$, respectively, which are significantly higher than that for case B (0.565). We also find that ULIRGs have a tendency to exhibit higher Br$\beta$/Br$\alpha$ line ratios than those observed in Galactic H II regions. If dust extinction affects the flux of the lines, the Br$\beta$/Br$\alpha$ line ratio will decrease below 0.565 because the Br$\beta$ line has a shorter wavelength and is more attenuated than the Br$\alpha$ line. Thus, we cannot explain the high Br$\beta$/Br$\alpha$ line ratio with a combination of case B theory and dust extinction.

We investigated the cause of this anomaly and obtained the following results:

1. We explored the possibility of contamination of the Brackett lines by other lines. We identified one candidate, the H$_2$ (1,0) O(2) line, with a wavelength of 2.627 $\mu$m, that is close to the wavelength of the Br$\beta$ line (2.626 $\mu$m). We estimated the flux of the H$_2$ (1,0) O(2) line from that of another molecular hydrogen line, H$_2$ (1,0) O(3) at 2.802 $\mu$m, assuming that the line ratio corresponds to that of a 2000 K shock model (Black & van Dishoeck 1987). The expected flux of the H$_2$ (1,0) O(2) line is 5%-12% of that of the observed Br$\beta$ line. Consequently, the Br$\beta$/Br$\alpha$ line ratio is still more than 3$\sigma$ higher than that for case B in IRAS 10565+2448 and Mrk 273 even after we subtract the flux of the H$_2$ (1,0) O(2) line from that of the Br$\beta$ line. Thus, we conclude that contamination does not provide a complete explanation of the high Br$\beta$/Br$\alpha$ line ratio.

2. For the case in which the Brackett lines are optically thin, we cannot explain the high Br$\beta$/Br$\alpha$ line ratio with any of the three possible excitation mechanisms: recombination, collisional excitation, or resonant excitation.
We find that we can explain the deviation of the Br$\beta$/Br$\alpha$ line ratio from that of case B if the Br$\alpha$ line becomes optically thick while the Br$\beta$ line is still optically thin.

4. We simulated H II regions, each ionized by a single star, with the Cloudy code and found that the high Br$\beta$/Br$\alpha$ line ratio can be explained when the Br$\alpha$ line becomes optically thick. To achieve a column density large enough to make the Br$\alpha$ line optically thick within a single H II region, the gas density must be as high as $n \sim 10^8$ cm$^{-3}$.

5. We investigated the ratios of optical H I lines in the galaxies in our sample that show high Br$\beta$/Br$\alpha$ line ratios. We found that the fluxes of the optical lines are highly affected by dust extinction, and it is difficult to tell whether the line ratio contradicts case B theory. We conclude that the deviation of the H I line ratios from those of case B can be seen clearly only in the infrared H I lines because the optical lines are strongly affected by dust extinction.

6. We investigated the consistency of our high-density model with other infrared H I line observations. We compared the H I line ratios other than Br$\beta$/Br$\alpha$ with those predicted by the high-density model for the three galaxies with high Br$\beta$/Br$\alpha$ line ratios. We conclude that our model is consistent with previous observations of the Pa$\alpha$/Br$\gamma$ line ratio.

7. We consider two extreme cases for the line-emitting regions: (1) an ensemble of H II regions, each ionized by a single star and (2) a giant H II region where all the ionizing stars are concentrated at the center. For both the cases, we conclude that gas densities as high as $n \sim 10^8$ cm$^{-3}$ are required to achieve a column density of neutral hydrogen large enough to make the Br$\alpha$ line optically thick. We propose this high-density scenario as the most plausible cause of the high Br$\beta$/Br$\alpha$ line ratio. The required density may be increased by up to an order of magnitude if dust grains are taken into account.

8. Our model requires high-density H II regions with $n = 10^8$ cm$^{-3}$. This affects the high-$\lambda$ transitions of H I lines, which fall in the radio-frequency range. We predict that radio recombination lines with $\lambda < 50$ are thermalized in galaxies with high Br$\beta$/Br$\alpha$ line ratios.

We thank the anonymous referee for reading our paper carefully and sending many useful suggestions for improvement. This study is based on observations made with AKARI, a JAXA project, with the participation of ESA. We also thank the Lick Observatory staff for their assistance. This research made use of the NASA/IPAC Extragalactic Database (NED), which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Data analysis was in part carried out on the Multi-wavelength Data Analysis System operated by the Astronomy Data Center (ADC), National Astronomical Observatory of Japan. This work was supported by JSPS KAKENHI grant No. 26247030. K.Y. was supported through the Leading Graduates Schools Program, Advanced Leading Graduate Course for Photon Science (ALPS), by the Ministry of Education, Culture, Sports, Science and Technology of Japan. S.B. is supported by JSPS KAKENHI grant No. JP19J00892.

Facilities: AKARI(IRC), Nickel: 1.0 m.

Software: Cloudy (ver. 10.00; Ferland et al. 1998), IRC Spectroscopy Toolkit (Ohyama et al. 2007; Baba et al. 2016), IPython (Perez & Granger 2007), Jupyter Notebook (Klyver et al. 2016), NumPy (Harris et al. 2020), SciPy (Virtanen et al. 2020), Pandas (McKinney 2010), Matplotlib (Hunter 2007), Astropy (Astropy Collaboration et al. 2013, 2018), Lmfit (Newville et al. 2020).

Appendix A
NULIZ Targets

We summarize here the properties of the two AKARI targets added from the NULIZ program. The observation log and basic information are listed in Tables 8 and 9. The H$\beta$/H$\alpha$ line ratio used in Section 5.2 is summarized in Table 10.

Table 8
Observation Log for NULIZ Targets

| Object Name  | Observation ID | Observation Date |
|--------------|----------------|------------------|
| IRAS 09022−3615 | 3051018.1       | 2007 May 26      |
| IRAS 10565+2448 | 3051019.1       | 2007 May 28      |

Table 9
Basic Information for NULIZ Targets

| Object Name | $z^a$ | $D_L^b$ (Mpc) | $F_{25}^c$ (Jy) | $F_{60}^c$ (Jy) | $F_{100}^c$ (Jy) | $L_{IR}^d$ (10$^{12} L_{\odot}$) | Optical Class | Ref. $^f$ |
|-------------|-------|---------------|----------------|----------------|----------------|--------------------------------|--------------|---------|
| IRAS 09022−3615 | 0.060 | 266           | 1.20           | 11.6           | 11.1           | 1.64                          | H II         | 1       |
| IRAS 10565+2448 | 0.043 | 190           | 1.27           | 12.1           | 15.0           | 1.07                          | H II         | 2       |

Notes.
$^a$ Redshift.
$^b$ Luminosity distance calculated from $z$ using our adopted cosmology.
$^c$ IRAS fluxes at 25 $\mu$m ($F_{25}$), 60 $\mu$m ($F_{60}$), and 100 $\mu$m ($F_{100}$).
$^d$ Total infrared (3–1100 $\mu$m) luminosity calculated with $L_{IR} = 4\pi D_L^2 (\xi_1 \nu F_{25} + \xi_2 \nu F_{60} + \xi_3 \nu F_{100})$, where $(\xi_1, \xi_2, \xi_3) = (2.403, -0.2454, 1.6381)$ (Dale & Helou 2002).
$^e$ Optical classification of galaxies.
$^f$ References for redshifts: (1) Strauss et al. (1992); (2) Downes et al. (1993).
Appendix B

Line Ratio in the Galactic H II Regions

Our results for the high Br/Br line ratio indicate that conditions in the H II regions in the ULIRGs that exhibit the anomalies are different from those of Galactic H II regions, where the case B theory explains the line ratios well. To investigate the difference, we here discuss the applicability of the case B line ratio for the Br and Br lines in Galactic H II regions.

Because of the difficulty of observing the wavelength range where the Br and Br lines lie due to the atmospheric absorption, previous observations have been limited to those conducted in space, namely, observations with the ISO and AKARI satellites. Accordingly, we investigated observations of Galactic H II regions obtained with these two satellites.

B.1. ISO Observations

Lutz et al. (1996) obtained a 2.4–45 μm spectrum of the Galactic center with the Short Wavelength Spectrometer on board ISO. The 14″ × 21″ aperture was centered on Sgr A* to cover H II regions in the Galactic center region. They detected H I lines, including the Br and Br lines, in the wavelength range 2.5–9.0 μm. The Br/Br line ratio at the Galactic center was reported to be ~0.25, which is consistent with case B plus dust extinction (Lutz 1999). Lutz (1999) also discussed the applicability of case B in the Galactic center using the Br line and the Br line (N = 6 → 5; 7.46 μm), and a blend of the Hu line (N = 8 → 6; 7.50 μm) with the N = 11 → 7 transition (7.51 μm). They concluded that the flux ratios of these lines were consistent with the case B line ratios and that the population of the respective upper levels followed case B. Thus, the ISO results indicate that the case B line ratio is applicable to H II regions near the Galactic center.

B.2. AKARI Observations

Using AKARI near-infrared spectroscopy, Mori et al. (2014) conducted a systematic observation of 36 Galactic H II regions and provided a catalog of 2.5–5.0 μm spectra of such objects. A typical example from the cataloged spectra is shown in Figure 14.

We determined the Br and Br line fluxes for those Galactic H II regions using the 232 cataloged spectra. We fitted the Br and Br lines separately, with a Gaussian profile and a linear continuum for each spectrum. Following Mori et al. (2014), we fixed the FWHM of the Gaussian profile at 0.031 μm for spectra taken with the “Ns” slit and 0.025 μm for those taken with the “Nh” slit in order to match the spectral resolution of the slits. The central wavelengths of the lines were also fixed at 4.05 μm for the Br line and at 2.63 μm for the Br line. The range of wavelengths used for the fitting was ±0.15 μm around the central wavelength of each line. We then determined the line flux by integrating the best-fit Gaussian profile. A typical example of the Gaussian fitting is shown in Figure 14.

We show the fluxes of the Br and Br lines for Galactic H II regions obtained from the spectral catalog of Mori et al. (2014) in Figure 15. The results show that almost all the Br/Br line ratios are lower than that for case B, except for a few spectra. This indicates that the Br/Br ratios of Galactic H II regions can be explained by case B condition plus dust extinction, which is typically AV ~ 10 mag.

Combining the results of the ISO and AKARI observations, we conclude that the case B condition is valid for Galactic H II regions. Thus, the anomalous Br/Br line ratios found in some ULIRGs indicate that conditions in the H II regions in those ULIRGs differ from the case B conditions.

---

Table 10

| Object Name | F_{Hα}/F_{Brα} | A_{V}^{opt} (mag) | Reference |
|-------------|----------------|-------------------|-----------|
| IRAS 09022−3615 | 0.18 ± 0.009 | 1.90 ± 0.14 | 1 |
| IRAS 10565+2448 | 0.06 ± 0.003 | 4.83 ± 0.14 | 2 |

Notes.

* Visual extinction derived from Hα/Hβ line ratio.

* References for optical line ratio: (1) Lee et al. (2011); (2) Veilleux et al. (1995).

Figure 14. Typical example of a cataloged spectrum of a Galactic H II region by Mori et al. (2014). The spectrum was obtained from the position −04 of W31a (ID: 5200165.1) using the “Nh” slit. The best-fit Gaussian profile for the Br lines are shown as red curves.

Figure 15. The fluxes of the Br and Br lines for Galactic H II regions obtained from the spectral catalog of Mori et al. (2014). The solid line shows the theoretical line ratio for case B: F_{Hα}/F_{Brα} = 0.565. The extinction vector corresponding to A_{V} = 10 mag is shown as the black arrow.
Appendix C
Detailed Discussions on Resonant Processes

This Appendix provides detailed discussions about the effect of resonant excitations on the Brβ/Brα line ratio. First, we discuss the $N = 1 \rightarrow 6$ resonance referred to in Section 4.2. Then, we consider the $N = 1 \rightarrow 4$ resonance mentioned in Section 4.3.2.

C.1. Resonant Excitation to the $N = 6$ State

With resonant excitation, the hydrogen $N = 6$ state is enhanced if a strong line with a wavelength comparable to that of the transition $N = 6 \rightarrow 1$ (937.8 Å) exists. We denote this (unknown) line by $X_6$ and discuss its effect on the Brβ/Brα line ratio in the optically thin case described in Section 4.2.

First, we assume a case in which hydrogen is excited solely by this resonant excitation. Let $x_6$ be the number of resonance excitations per unit volume and unit time (cm$^{-3}$ s$^{-1}$). Then, the number of Brβ transitions ($N = 6 \rightarrow 4$) caused by the resonant excitation can be written as $x_6 n_{A6}/A_6 \sim 0.22 x_6$, and the number of Brα transitions ($N = 5 \rightarrow 4$) is given by $x_6 C_{6,5} C_{A} = 0.11 x_6$. Here, the $A$ and $C$ symbols are the Einstein A and cascade coefficient and cascade matrix introduced in Section 4.1.1, respectively. The Brβ/Brα line ratio is thus found to be

$$F_{Br\beta}/F_{Br\alpha} = 0.22 x_6 h \nu_{Br\beta}/0.11 x_6 h \nu_{Br\alpha} \sim 3.2.$$ 

Therefore, it is possible for resonant excitation to make the line ratio consistent with our observations, given the existence of an appropriate line transition $X_6$.

Next we discuss how strong the $X_6$ line must be in order to explain the observed anomaly. Taking the resonant excitation rate $x_6$ into account in Equation (2), we now write the level populations for $N = 3$ and 6 as

$$n_6' A_6 = n_p n_e \sum_{N=6}^{\infty} \alpha_{N'} C_{N',6} \frac{x_6}{A_6} + x_6,$$

$$\therefore \quad n_6' = n_p n_e \sum_{N=6}^{\infty} \alpha_{N'} C_{N',6} + \frac{x_6}{A_6}, \quad (C1)$$

and

$$n_5' A_5 = n_p n_e \sum_{N=5}^{\infty} \alpha_{N'} C_{N',5} + C_{6,5} x_6,$$

$$\therefore \quad n_5' = n_p n_e \sum_{N=5}^{\infty} \alpha_{N'} C_{N',5} + \frac{C_{6,5} x_6}{A_5}. \quad (C2)$$

In order to explain the observed anomaly, in which $F_{Br\beta}/F_{Br\alpha} \sim 1$, we require the ratio of the level populations to be $n_6'/n_5' \sim 2.27$. Using Equations (C1) and (C2), we find

$$n_p n_e \sum_{N=6}^{\infty} \alpha_{N'} C_{N',6} \frac{x_6}{A_6} + x_6 \leq 2.27 \left( n_p n_e \sum_{N=5}^{\infty} \alpha_{N'} C_{N',5} \frac{C_{6,5} x_6}{A_5} + \frac{C_{6,5} x_6}{A_5} \right), \quad (C3)$$

$$\therefore \quad x_6 = 2.29 \times 10^{-14} \left( \frac{n}{\text{cm}^{-3}} \right)^2 \text{cm}^{-3} \text{s}^{-1},$$

where we have approximated $n_p n_e \sim n^2$. Assuming that all photons emitted in the $X_6$ line are absorbed by hydrogen atoms, we can obtain that $x_6$ is equal to the emission rate of the $X_6$ line. We write this rate as $n \xi_X f_X A_{X_6}$, where $\xi_X$ is the abundance of the atoms emitting the $X_6$ line, relative to hydrogen; $f_X$ is the fraction of excited atoms that can radiate the $X_6$ line, relative to those in other states; and $A_{X_6}$ is the Einstein A coefficient for the $X_6$ line. From Equation (C3), we thus obtain

$$A_{X_6} = 2.29 \times 10^{-14} \xi_X f_X \left( \frac{n}{\text{cm}^{-3}} \right) \text{s}^{-1}. \quad (C4)$$

It is difficult to estimate the fraction $f_X$, so we here assume the most extreme case that it is of order unity in order to take weak lines into consideration. At a gas density $n = 10^3 \text{cm}^{-3}$ and assuming that the atoms emitting the $X_6$ line have an abundance similar to those of the metals, $\xi_X \sim 10^{-4}$, we find $A_{X_6}$ to be $\sim 10^{-7} \text{s}^{-1}$, which is close to the values for forbidden lines. This indicates that if a line exists with a wavelength $\sim 937.8$ Å and a transition probability comparable to those of forbidden lines, then the resonant process would be able to make the Brβ/Brα line ratio anomalously high.

C.2. Resonant Excitation to the $N = 4$ State

Herein, we discuss the effect of the $N = 1 \rightarrow 4$ resonant excitation on the Brβ/Brα line ratio in the optically thick case described in Section 4.3.2. We denote the possible resonant line by $X_4$ and write the rate of resonant excitation by this line as $x_4$ (cm$^{-3}$ s$^{-1}$). From Equation (2), the $N = 4$ state is populated by the recombination process at the rate\begin{equation}n_p n_e \sum_{N=4}^{\infty} \alpha_{N'} C_{N',4} \sim 6.2 \times 10^3 \text{cm}^{-3} \text{s}^{-1} \text{ at } n = 10^3 \text{cm}^{-3}.\end{equation}

This indicates that if $x_4$ is larger than $\sim 10^3 \text{cm}^{-3} \text{s}^{-1}$, the resonant process can significantly populate the $N = 4$ state when $n = 10^3 \text{cm}^{-3}$.

In the same way as for the hypothetical $N = 1 \rightarrow 6$ resonance we discussed in Section C.1, we can estimate the transition probability $A_{X_4}$ of the $X_4$ line that is required to make the resonant process dominant for the $N = 4$ population. We assume that all photons emitted by the $X_4$ line are absorbed by hydrogen such that $x_4$ is equal to the rate of emission of the $X_4$ line, $n \xi_X f_X A_{X_4}$, where $\xi_X$ is the abundance of the atoms emitting the $X_4$ line relative to hydrogen, and $f_X$ is the fraction of excited atoms which can radiate the $X_4$ line relative to those in other states. We assume $f_X$ to be of the order of unity as the most extreme case. At a gas density of $n = 10^3 \text{cm}^{-3}$ and assuming that the atoms emitting the $X_4$ line have an abundance similar to those of the metals, i.e., $\xi_X \sim 10^{-4}$, we estimate $A_{X_4}$ to be $\sim 10^{-1} \text{s}^{-1}$. Thus, if a line with a wavelength $\sim 972.5$ Å and a transition probability of $\sim 10^{-1} \text{s}^{-1}$ exists, we should take the resonant process into consideration in determining the population of the $N = 4$ state.

**ORCID iDs**

Shunsuke Baba @ https://orcid.org/0000-0002-9850-6290

Takao Nakagawa @ https://orcid.org/0000-0002-6660-9375

Matthew A. Malkan @ https://orcid.org/0000-0001-6919-1237

Vanshree Bhalotia @ https://orcid.org/0000-0002-9552-555X

**References**

Ahn, C. P., Alexandroff, R., Allende Prieto, C., et al. 2012, ApJS, 203, 21

Anantharamaih, K. R., Zhao, J.-H., Goss, W. M., & Viallefond, F. 1993, ApJ, 419, 585

Arthur, S. I., Medina, S. N. X., & Henney, W. J. 2016, MNRAS, 463, 2864
Lee, J. C., Hwang, H. S., Lee, M. G., Kim, M., & Kim, S. C. 2011, MNRAS, 414, 702
Lee, J. C., Hwang, H. S., Lee, M. G., Kim, M., & Lee, J. H. 2012, ApJ, 756, 95
Lutz, D. 1999, in ESA SP-427, The Universe as Seen by ISO, ed. P. Cox & M. Kessler (Noordwijk: ESA), 623
Lutz, D., Feuchtgruber, H., Genzel, R., et al. 1996, A&A, 315, L269
McKinney, W. 2010, in Proc. 9th Python in Science Conf., 445, ed. S. van der Walt & J. Millman (Austin, TX: SciPy), 51
Michiyama, T., Iono, D., Nakanishi, K., et al. 2020, ApJ, 895, 85
Morii, T., Onaka, T., Sakon, L. et al. 2014, ApJ, 784, 53
Murakami, H., Baba, H., Barthel, P., et al. 2007, PASJ, 59, S369
Murphy, T. W. J., Soifer, B. T., Matthews, K., Armus, L., & Kiger, J. R. 2001, AJ, 121, 97
Newville, M., Otten, R., Nelson, A., et al. 2020, lmfit/lmfit-py v1.0.1, Zenodo, doi:10.5281/zenodo.3814709
Ohyama, Y., Onaka, T., Matsuhara, H., et al. 2007, PASJ, 59, S411
Onaka, T., Matsuhara, H., Wada, T., et al. 2007, PASJ, 59, S401
Osterbrock, D. E., & Ferland, G. J. 2006, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei (2nd ed.; Sausalito, CA: Univ. Science Books)
Perez, F., & Granger, B. E. 2007, CSE, 9, 21
Peters, T., Longmore, S. N., & Dullemond, C. P. 2012, MNRAS, 425, 2352
Sanders, D. B., & Mirabel, I. F. 1996, ARA&A, 34, 749
Sanders, D. B., Soifer, B. T., Elias, J. H., et al. 1988, ApJ, 325, 74
Seaton, M. J. 1959, MNRAS, 119, 90
Shih, H.-Y., & Rupke, D. S. N. 2010, ApJ, 724, 1430
Storey, P. J., & Hummer, D. G. 1995, MNRAS, 272, 41
Strauss, M. A., Huchra, J. P., Davis, M., et al. 1992, ApJS, 83, 29
Theios, R. L., Malkan, M. A., & Ross, N. R. 2016, ApJ, 822, 45
Veilleux, S., Kim, D. C., & Sanders, D. B. 1999a, ApJ, 522, 139
Veilleux, S., Kim, D. C., & Sanders, D. B. 1999b, ApJS, 121, 261
Veilleux, S., Kim, D. C., & Sanders, D. B. 1999c, ApJ, 522, 139
Veilleux, S., Rupke, D. S. N., Kim, D. C., et al. 2009, ApJS, 182, 628
Vernier, D. A., & Ferland, G. J. 1996, ApJS, 103, 467
Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, NatMe, 17, 261
Wood, D. O. S., & Churchwell E. 1989, ApJS, 69, 831
Yano, K., Nakagawa, T., Isobe, N., & Shirahata, M. 2016, ApJ, 833, 272