Self-Interacting Dark Matter and the Excess of Small-Scale Gravitational Lenses

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Recently, Meneghetti et al. reported an excess of small-scale gravitational lenses in galaxy clusters, compared to simulations of standard cold dark matter (CDM). We propose a self-interacting dark matter (SIDM) scenario, where a population of subhalos in the clusters experiences gravothermal collapse. Using controlled N-body simulations, we show the presence of early-type galaxies in substructures accelerates gravothermal evolution and a collapsed SIDM subhalo has a steeper density profile than its CDM counterpart, leading to a larger radial galaxy-galaxy strong lensing cross section and more lens images, in better agreement with the observations. Our results indicate that strong gravitational lensing can provide a promising test of the self-interacting nature of dark matter.

Introduction. Strong gravitational lensing is characterized by the existence of giant arcs, rings, and multiple images caused by the deflection of lights by massive foreground galaxies, groups, or galaxy clusters. It provides a powerful tool for testing cosmological models, determining the mass distribution of clusters, probing substructures and dark matter properties. Recently, Meneghetti et al. reported that observed substructures in galaxy clusters are more efficient lenses than those predicted in simulations of standard cold dark matter (CDM), indicating that the former are more dense and compact. Other studies also show strong lensing clusters contain more substructures with high maximum circular velocities than predicted in CDM simulations.

In this Letter, we propose a self-interacting dark matter (SIDM) scenario to explain the observed excess. At late stages of gravothermal evolution, an SIDM halo could experience instability and collapse, resulting in a steep density profile. We design N-body simulations to model the MACS J1206.2-0847 (MACSJ1206) cluster, one of the examples studied in Meneghetti et al., and construct four benchmark cases, covering a representative range of halo concentration. The presence of early-type galaxies in the substructures could significantly accelerate the onset of the collapse. For all benchmarks, our simulated SIDM subhalos experience collapse after 6 Gyr of tidal evolution and become more dense than their CDM counterparts, assuming a self-scattering cross section per mass of $\sigma/m = 1 \text{cm}^2/\text{g}$, which is relatively conservative.

We further model strong lensing observables and compute galaxy-galaxy strong lensing (GGSL) cross sections for the benchmarks. In SIDM, the radial caustics dominates over the tangential one, and the predicted radial GGSL cross section is larger than that in CDM, with details depending on the source redshift and initial halo concentration. We will also show mock lensing images and discuss their implications for future tests. Our results indicate that SIDM is promising for explaining the excess of gravitational lenses in galaxy clusters. Intriguingly, SIDM may also explain diverse dark matter distributions in other galactic systems and the origin of supermassive black holes.

Modeling the cluster system. We model the host cluster using a spherical potential characterized by a Navarro-Frenk-White (NFW) density profile and fix its corresponding scale density and radius as $\rho_s = 1.82 \times 10^6 \text{M}_\odot/\text{kpc}^3$ and $r_s = 442 \text{kpc}$, respectively. It well reproduces the projected total mass profile of MACSJ1206. We set the apecenter of our simulated substructures to be $400 \text{kpc}$, with a tangential velocity of $1000 \text{km/s}$. During the tidal evolution, their distance to the host center oscillates in the range $140-370 \text{kpc}$. The orbit controls the significance of tidal stripping, and hence the mass loss. In our setup, the mass of the simulated substructures at 6 Gyr is $\mathcal{O}(10^{10}) \text{M}_\odot$. The strong lensing analysis focuses on substructures within 15% of the virial radius of the host cluster, which is $300 \text{kpc}$, and most of them have a mass in the range $10^{10}-10^{11} \text{M}_\odot$. Thus our simulated cluster system well represents those studied in Meneghetti et al.

For the subhalos, we use an NFW profile to model their initial dark matter distribution. We fix the initial virial halo mass to be $M_{200} = 3 \times 10^{12} \text{M}_\odot$, and choose four benchmark values for the concentration, i.e., $c_{200} = 7,49, 9.65, 12.4$ and $16.0$, corresponding to $0\sigma, 1\sigma, 2\sigma$ and $3\sigma$ higher than the cosmological median at $z = 0$. For each of the benchmarks, we convert their $(M_{200}, c_{200})$ to $(\rho_s, r_s)$ to specify the initial NFW density profile in our simulations. Note for a given set of $\rho_s$ and $r_s$, the interpretation of $c_{200}$ depends on the redshift. Consider $z = 2$, at which infall is expected to occur, the concentration of the benchmarks, from low to high, is $-1.5\sigma, -0.39\sigma, +0.73\sigma$, and $+1.8\sigma$ away from the median (z = 2) , which are representative.

We fix the initial stellar mass as $M_*= 6 \times 10^{10} \text{M}_\odot$, expected from the stellar-to-halo mass relation, and model its distribution with a truncated singular isothermal profile as in Meneghetti et al., where\[\rho_*(r) = \rho_0 r_{\text{cut}}^2 / [r^2 (r_{\text{cut}}^2 + r^2)],\]
$\rho_0$ is the density normalization factor and $r_{\text{cut}}$ is the cut-off radius. This is consistent with observations of early-type galaxies [55–58]. We take $r_{\text{cut}} = 6.23$ kpc following the size-mass relation [59], and $\rho_0 = 1.26 \times 10^7 \, \text{M}_\odot/\text{kpc}^3$. We use live particles for the subhalo and stellar components and perform both SIDM and CDM simulations. For the former, we choose $\frac{\sigma}{m} = 1 \, \text{cm}^2/\text{g}$, approximately the lower limit that could explain observations on galactic scales [28]. For comparison, we also perform CDM simulations without including stars. As we will show that the stellar component is important in producing strong lensing observables.

We use the public GADGET-2 code [60, 61], and extend it with a module modeling dark matter self-interactions [32, 43], which has been validated in both gravothermal expansion and collapse regimes with results from [30, 31, 40, 46]. We use the code SpherIC [62] to generate initial conditions for the simulated substructures. The mass of the simulated particle is $10^6 \, \text{M}_\odot$ for both subhalo and stellar components, and the softening length is 0.2 kpc. The resolution is high enough to avoid numerical artifacts concerning disruption of substructures [63, 65]. We let the simulated substructures evolve for 6 Gyr in the tidal field of MACSJ1206.

**Gravothermal collapse.** The left panel of Fig. 1 shows the evolution of the maximum circular velocity $V_{\text{max}}$ vs the total substructure mass $M_{\text{sub}}$ for the benchmarks with $c_{200} = 16.0$ (magenta) 12.4 (red) 9.65 (green) and 7.49 (blue) from our SIDM (solid thick) and CDM (solid thin) simulations, where we include both subhalo and stellar components as in [23]. The arrow on each curve denotes the direction of the evolution. The subhalos lose the majority of their mass after tidal evolution, while the stellar mass is only reduced by an $\mathcal{O}(1)$ factor. Our final total stellar and subhalo masses are consistent with those of cluster substructures from the Illustris simulations [66, 67]. For comparison, our CDM simulations without including stars are shown (dotted).

The maximum velocities of the CDM substructures decrease continuously, aside from oscillatory features due to tidal interactions. For those with stars, the final $V_{\text{max}}$ values are close to the high end of the range predicted in cosmological hydrodynamical simulations [23] (gray band), but are all below the average value from the strong lensing observations (black dashed). We also see that the $V_{\text{max}}$ values predicted in our CDM simulations without stars are still within the gray band. It implies that a large population of simulated substructures in [23] has diffuse baryon distributions and high dark matter fractions. On the other hand, our controlled simulations take an observed baryon distribution, which is compact, as an input. We will come back to this point later.

The simulated SIDM substructures follow a similar trend for most of the evolution time, but their $V_{\text{max}}$ values spike toward the measured ones at late stages, as gravothermal collapse occurs and their central densities increase. At $t = 6$ Gyr, all four SIDM benchmarks, even the one with a median concentration ($z = 0$), are denser.
than their CDM counterparts with stars, resulting in better agreement with the lensing observations [23]. The collapse occurs earlier if \( c_{200} \) is higher, leading to a higher density at 6 Gyr, as its timescale is extremely sensitive to the concentration [31]. We find the presence of secondary caustics deepens potential and accelerates gravothermal evolution, as in the isolated case [48, 65, 69]. Without stars, the collapse would not occur within 6 Gyr unless \( c_{200} \) is \( \sigma \) higher than the median, and a subhalo with median \( c_{200} \) would be nearly destroyed [43]. In the cluster environment, tidal stripping could also speed up the onset of gravothermal collapse [70–74].

**Strong lensing observables.** To further see implications for strong lensing observations, we compute GGSL cross sections for the simulated substructures. We adopt thin-lens approximation and project the mass distribution of the host cluster and substructure, assumed to be spherical, onto the lens plane, which is perpendicular to the line of sight. The distance between the substructure and the host center is fixed to be 300 kpc. We denote the angular positions as \( \theta \) and \( \beta \) on the lens and source planes, respectively. It is convenient to introduce effective lensing potential as [11, 2] :

\[
\Psi(\theta) \equiv \frac{1}{\pi} \int d^2\theta \ln |\theta - \theta'| \kappa(\theta'),
\]

where \( \kappa(\theta) = \Sigma(D_L, \theta)/\Sigma_{cr} \) is the scaled projected density and \( \Sigma_{cr} = \Sigma^2 D_S/4\pi GD_L D_{LS} \) is the critical density. \( D_L \) and \( D_S \) are lens and source angular diameter distances, respectively, and \( D_{LS} \) the distance between the two. We calculate these quantities in a flat universe with matter energy density \( \Omega_m = 0.3175 \) and \( h = 0.671 \) [75]. The lens equation is \( \beta = \theta - D_{LS} \alpha/D_S \), where \( \alpha \) is the deflection angle.

For each simulated substructure plus the host cluster, we determine the lensing potential by solving the Poisson equation \( \nabla^2 \Psi(\theta) = 2\kappa(\theta) \), where we use the fast Fourier transformation method. After obtaining \( \Psi(\theta) \), we calculate the shear matrix as \( \mathcal{A} = \partial \beta_i / \partial \theta_j = (\delta_{ij} - \Psi_{ij}) \) and the pseudo-vector shear \( \gamma = \sqrt{(\Psi_{11} - \Psi_{22})^2 + \Psi_{12}^2} \), where \( \Psi_{ij} \equiv \partial^2\Psi / \partial \theta_i \partial \theta_j \) and \( i,j \) are indices of the two spatial coordinates. The tangential and radial critical lines are contours of \( \lambda_t = 1 - \kappa - \gamma = 0 \) and \( \lambda_r = 1 - \kappa + \gamma = 0 \), respectively. We obtain their corresponding caustic lines by mapping onto the source plane using the lens equation. We compute radial and tangential GGSL cross sections defined as the area enclosed by the secondary caustic [76].

The right panel of Fig. 1 shows ratios of SIDM to CDM radial (solid) and tangential (dashed) GGSL cross sections as a function of the source redshift \( z_s \). Apparently, the SIDM substructures have larger radial cross sections than their CDM counterparts, by a factor of \( \sim 3–7 \) for \( z_s \gtrsim 1 \), and the significance increases with the concentration. The tangential cross sections are comparable for both cases. For the CDM substructures without stars, their surface density is low and the lensing effect is negligible.

Ref. [23] reports the measured GGSL cross section that sums over contributions from individual substructures is more than one order magnitude larger than the one predicted in CDM simulations. We note that the number of the observed secondary caustics is a factor of 3 larger than that from their simulations. Thus the required boost factor is \( \gtrsim 3 \) for an individual GGSL cross section. From the results shown in Fig. 1 we see the SIDM scenario is promising. In addition, as discussed above, we expect that a significant population of simulated CDM substructures in [23] has diffuse stellar distributions and they could not produce measurable strong lensing effects.

This could be a common issue related to the limitation of the AGN feedback model. For example, early-type galaxies from the IllustrisTNG simulations have more diffuse baryon distributions and higher dark matter fractions, compared to observed galaxies [77, 78]. Our work shows a compact stellar density profile is important in reproducing observed strong lenses in SIDM and CDM. Moreover, for the same initial condition in dark matter and stellar distributions, the former produces larger (radial) GGSL cross sections, due to gravothermal collapse, resulting in better agreement with the observations.

**Density profiles.** We take the benchmark with \( c_{200} = 12.4 (+2\sigma) \) and perform a detailed case study. In Fig. 2 we show its dark matter (dashed) and total (solid) density profiles with (red) and without (gray) dark matter self-interactions. After 6 Gyr of evolution, the collapse
leads to an overdense region within 1.3 kpc, and a less dense region \( r \gtrsim 1.3 \) kpc, compared to the CDM subhalo. For the CDM substructure without including stars (dotted), the density is significantly lower. The inset displays the evolution of \( V_{\text{max}} \) and \( r_{\text{max}} \) for SIDM and CDM substructures with stars, their \( r_{\text{max}} \) values decrease overall due to tidal mass loss. The SIDM one becomes further smaller at late stages, as the collapse occurs and the central density increases; see also [74].

The left panel of Fig. 3 shows the corresponding secondary radial (solid) and tangential (dashed) caustics, assuming a source at \( z_s = 3 \) for the SIDM (red) and CDM (gray) benchmarks (\( c_{200} = 12.4 \)). For SIDM, the enclosed area of the radial caustic is much larger than the tangential one. For CDM, they are comparable, but both are smaller than the area of the SIDM radial caustic. Ref. [24] focuses on the tangential caustic, because it assumes a singular isothermal mass model and the radial caustic vanishes. We show that for a collapsed SIDM substructure, the total density profile is steeper than the isothermal profile \( r^{-2} \), and hence its associated radial caustic dominates over the tangential one.

**Mock lensed images.** The left panel of Fig. 3 displays four mock sources at four representative locations; the middle (SIDM) and right (CDM) panels show their corresponding lens images, together with the critical lines mapped from the caustics using the lens equation. The innermost blue source is inside radial and tangential caustics predicted in SIDM and CDM, and it has four images in both cases [79][80]. The source in orange sits on the second fold caustic in SIDM while it only crosses the tangential caustic in CDM, thus it has one more image in the former case. Similarly, the sources in green and magenta have one more image in SIDM than in CDM. Our example explicitly illustrates that the collapsed SIDM substructure has higher capability of producing multiple images, which could be further tested statistically.

We compute the Einstein radii as \( r_E = \theta_E D_L \approx 4.3 \) kpc and \( 4.8 \) kpc for the SIDM and CDM benchmarks with stars, respectively, denoted in Fig. 2 with arrows, where \( r_E \) is defined as the radius of the circle with the same area as that enclosed by the critical line [70]. It’s not surprising that they are comparable. In our setup, the stellar component dominates the inner region and both cases have similar stellar distributions after tidal evolution. In addition, during the collapse process, the SIDM central density increases, but the total mass does not change. As indicated in Fig. 2, \( r_{\text{max}} \sim r_E \) for CDM, while \( r_{\text{max}} < r_E \) for SIDM. Thus the mass distribution induced by gravothermal collapse does not produce an appreciable change in the enclosed mass within \( r_E \). This is a unique prediction of SIDM.

**Discussion and Conclusions.** Our simulations assume a compact stellar distribution motivated by observations of early-type galaxies. Hydrodynamical simulations show that for SIDM halos with masses \( \gtrsim 10^{12} \) \( M_\odot \) stars could dominate the inner region [81][82]. Thus our assumption is well justified. In addition, the SIDM halo structure is more resilient to feedback than its CDM counterpart, because of rapid energy redistributions induced by the self-interactions; see [85][86]. We expect our overall predictions are robust, and it would be interesting to further test them with cosmological simulations. In summary, we have shown that a collapsed SIDM substructure could have a steeper density profile, resulting in a larger radial GGSL cross section and more lens images, compared to CDM, a trend favored in explaining the observed excess of strong gravitational lenses in galaxy clusters. The features of strong lensing observables predicted in the SIDM scenario could be further tested using existing data [87], and upcoming observations [88].
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