Optimal Design of Stiffened Panels Considering Torsion of Supporting Stiffeners

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Abstract. Aiming at the optimal design and analysis of stiffened panels, this paper proposes a new algorithm considering the torsion of supporting stiffeners on axial load carrying capacity. By defining three dimensionless factors: relative bearing capacity, relative transverse elastic stiffness and relative torsional elastic stiffness, the response surface of load-bearing capacity versus two kinds of elastic stiffness is obtained. Based on genetic algorithm, a lightweight optimization model for structure layout and section parameters is established. A certain stiffened panel is optimized on this basis, and the matching design relationship between two kinds of stiffeners is summarized.

1. Introduction
Stiffened panels and shells are widely used as main load-bearing structure elements in aerospace, ship and other fields [1][2]. Buckling is the main failure form of this kind of structures, so structural stability is the main restricting factor of load-bearing capacity [3]. In the analysis of structural stability, stiffened panel is usually simplified to crossed beam system composed of longitudinal stiffeners which bear external load directly and supporting stiffeners which provide elastic support to longitudinal stiffeners. The skin is equivalent to the attached web of the reinforcement. Relevant design standards provide a simplified engineering empirical formula on bearing capacity [4]. However, it ignores the torsion of supporting stiffeners, which makes the results conservative and has great limitations. Accurate stability analysis and optimization design of stiffened panels and shells are mostly based on finite element analysis [5-7]. The finite element analysis based on explicit dynamics algorithm can obtain relatively precise load-bearing capacity which fits well with the experimental results, but it is very time-consuming and sometimes unaffordable. Hence, the engineering empirical formula which can forecast the load-bearing capacity quickly is still of great significance in the initial stage of structure design and analysis.

From the perspective of theoretical analysis, this paper improves the traditional engineering algorithm, and derives the formula for calculating axial load carrying capacity of stiffened panels considering the torsion of supporting stiffeners. On this basis, an optimization model on section parameters and structure layout of stiffened panels is established based on genetic algorithm, and the matching design relationship between longitudinal stiffeners and supporting stiffeners is summarized.

2. Theoretical analysis
2.1. Basic analytical model
Decouple longitudinal stiffeners and supporting stiffeners. When subjected to axial compression load,
supporting stiffeners will not only bend but also twist, which can provide both transverse stiffness and corner stiffness. Therefore, longitudinal stiffeners can be regarded as a multi-span compressive column supported by a series of transverse elastic supports and torsional elastic supports, as shown in figure 1.

By loading concentrated force (or torque) at the joint of the supporting stiffener, the elastic stiffness is the ratio of the concentrated force (or torque) to the deflection (or rotation angle) of the joint.

2.2. Buckling of compressive column under two kinds of elastic supports
The transverse force caused by transverse elastic supports and the bending moment caused by torsional elastic supports are both regarded as external loads, as shown in figure 2. The differential equation of the deflection curve on compressive column is

$$\frac{d^2y(x)}{dx^2} + \alpha^2 y(x) = \frac{M(x)}{EI}$$ (1)

where coefficient $\alpha^2 = P / EI$, $EI$ is the bending stiffness of longitudinal stiffeners, $P$ is axial compression load, $y(x)$ is the deflection and $M(x)$ is the bending moment on cross-section.

Consider the case that only one transverse force $Q$ applied at a distance $c$ from the right end. The function of deflection curve is

$$y(x) = \begin{cases} \frac{Q \sin \alpha c}{P \alpha \sin \alpha l} \sin \alpha x - \frac{Q c}{Pl} x & x < (l - c) \\ \frac{Q \sin \alpha (l - c)}{P \alpha \sin \alpha l} \sin \alpha (l - x) - \frac{Q (l - c)}{Pl} (l - x) & x > (l - c) \end{cases}$$ (2)

While the function of rotation angle can be described as
\[ \theta_Q = \begin{cases} \frac{Q \sin \alpha c}{P \sin al} \cos \alpha x - \frac{Q c}{Pl} & x < (l - c) \\ \frac{-Q \sin \alpha (l - c)}{P \sin al} \cos \alpha (l - x) + \frac{Q(l - c)}{Pl} & x > (l - c) \end{cases} \] (3)

Consider the case that only one bending moment \( M \) applied at a distance \( c \) from the right end. The function of deflection curve is

\[ y_M = \begin{cases} \frac{M \cos \alpha c}{P \sin al} \sin \alpha x - \frac{M}{Pl} x & x < (l - c) \\ \frac{-M \cos \alpha (l - c)}{P \sin al} \sin \alpha (l - x) + \frac{M}{Pl} (l - x) & x > (l - c) \end{cases} \] (4)

While the function of rotation angle can be described as

\[ \theta_M = \begin{cases} \frac{M \alpha \cos \alpha c}{P \sin al} \cos \alpha x - \frac{M}{Pl} & x < (l - c) \\ \frac{M \alpha \cos \alpha (l - c)}{P \sin al} \cos \alpha (l - x) - \frac{M}{Pl} & x > (l - c) \end{cases} \] (5)

According to the deflection of each joint equals the displacement of each transverse elastic supports, and the rotation angle of each joint equals the corner of each torsional elastic supports, superimpose formula (2)-(5) to obtain the linear system of equations

\[
\begin{bmatrix}
R_{m} \\
\alpha_{m}
\end{bmatrix}
= \sum \begin{bmatrix}
y_{Q} \\
y_{M}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{R_{m}}{k_{1,m}} \\
\frac{M_{m}}{k_{2,m}}
\end{bmatrix}
= \sum \begin{bmatrix}
\theta_{Q} \\
\theta_{M}
\end{bmatrix}
\] (6)

Formula (6) can also be described in the form of matrix

\[
\left[ A \right] \left[ \begin{array}{c}
R_{1} \\
M_{1} \\
L \\
R_{n} \\
M_{n}
\end{array} \right] = \left( \begin{array}{c}
0 \\
L \\
0
\end{array} \right)
\] (7)

The buckling load can be obtained in case of forcing the determinant of matrix coefficient \([ A ]\) to be zero.

Assumed the elastic supports are equally spaced and the length of each span is \( l \). According to Eular’s formula on buckling load of compressive column with two hinged ends \( P_{cr} = \pi^{2} EI / l^{2} \), defining three dimensionless factors: relative bearing capacity \( \bar{P} = P / P_{cr} \), relative transverse elastic stiffness \( \bar{k}_1 = k_1 / (\pi^2 EI / l^3) \) and relative torsional elastic stiffness \( \bar{k}_2 = k_2 / (\pi^2 EI / l) \). Now coefficient matrix \([ A ]\) is only related to \( \bar{P}, \bar{k}_1, \bar{k}_2 \) and span number \( m \), so \( |A| = 0 \) can be described as

\[
\bar{P} = \Phi(\bar{k}_1, \bar{k}_2, m)
\] (8)

For each span number \( m \), the response surface of the relative bearing capacity versus two kinds of relative elastic stiffness is obtained, as shown in figure 3. Then the buckling load of each compressive column can be easily calculated according to the response surface.
Figure 3. Response surface of the relative bearing capacity varying two kinds of relative elastic stiffness.

2.3. Stiffness of elastic supports

2.3.1. Stiffness of transverse elastic supports. Apply concentrated load \( F_i \) to the joint \( x_i \) of the supporting stiffener. The differential equation of the deflection curve on a supporting stiffener is

\[
EI_i w''(x) = Q(x) \tag{9}
\]

where \( w(x) \) is the deflection, \( Q(x) \) is the shearing force on cross-section, \( EI_i \) is bending stiffness of the supporting stiffener. Then the stiffness of transverse elastic support is the ratio of concentrated force to the deflection on the joint \( k_{1,i} = F_i / w(x_i) \).

2.3.2. Stiffness of torsional elastic supports. Apply concentrated torque \( T_i \) to the joint \( z_i \) of the supporting stiffener. According to Umansky theory, the differential equation of the rotation angle on a thin-walled stiffener with closed section is

\[
\frac{1}{\nu} E_i J_\theta \theta''(z) - GJ_d \theta'(z) = -L(z) \tag{10}
\]

where \( \theta(z) \) is the rotation angle, \( L(z) \) is the torque on cross-section, \( GJ_d \) is the free torsional stiffness, \( E_i \) is reduced elastic modulus, \( J_\theta \) is the fan-shaped moment of inertia, \( \nu \) is a parameter
reflecting the warping degree of the section. The analytical solution of equation (10) can be expressed as \( \theta = \frac{1}{\gamma} T \int g(l/b) \), where parameter \( b \) described the section dimension and function \( g(l/b) \) is related to the constrained torsional term which can be ignored when ratio \( l/b \) is large enough. Then the differential equation can be simplified as

\[
GJ \theta'(z) = L(z)
\]  

(11)

Then the stiffness of torsional elastic support is the ratio of concentrated torque to the rotation angle on the joint \( k_{z,i} = T_i / \theta(z_i) \).

3. Optimal design

3.1. Optimization model

According to the theoretical analysis above, section parameters and the number of stiffeners have a great influence on load carrying capacity. Reasonable matching design of two kinds of stiffeners can reduce structure weight greatly.

In this paper, lightweight design of stiffened panels with fixed number of longitudinal stiffeners is carried out. That is to get the minimum value of the structure weight under the condition of meeting the requirements of a given bearing capacity. In case of the quantity of design variables and the existence of discrete variables such as the number of supporting stiffeners, this paper intends to use genetic algorithm for optimal design. The mathematical expression can be described as

\[
\begin{align*}
\text{Find} & \quad a_i, b_i, n \\
\text{Min} & \quad W(a_i, b_i, n) \\
\text{s.t.} & \quad P \geq P_{\text{min}} \\
& \quad a_i \leq a_i \leq \bar{a}_i, b_i \leq b_i \leq \bar{b}_i \\
& \quad n = 2, 3, 4...
\end{align*}
\]  

(12)

where \( a_i \) and \( b_i \) are section parameters of two kinds of stiffeners, \( n \) is the number of supporting stiffeners, \( W \) is structure weight, \( P \) and \( P_{\text{min}} \) are the calculated and limited bearing capacity.

3.2. Optimization model

The optimal design of a stiffened panel with two longitudinal stiffeners is carried out. T-section longitudinal stiffeners are 2400mm length which are simply supported at both ends while box-section supporting stiffeners are 1200mm length with two fixed ends, as shown in figure 4.

![Figure 4. Structural model.](image-url)
Given a set of initial parameters (column 1 of Table 1), the bearing capacity is 141339N and the structure weight is 8.763kg. Taking this bearing capacity as the minimum critical bearing capacity, the optimization iteration is carried out respectively, for the case that having different number of supporting stiffeners. Each optimization iteration process is shown in figure 5 and the optimal results are rounded up to engineering reasonable values, as shown in Table 1.

It can be found out that the optimized structure weight in each case is significantly lower than the initial one. The width $B$ and thickness $t_1$ of longitudinal stiffeners reduce to be nearly the minimum limited values in all four cases. This is because comparing with flange, web provides a lower contribution to the moment of inertia in T-section.

Meanwhile, the dimensions of the optimized supporting stiffeners are much smaller than the initial ones. According to the response surface shown in figure 3, when the relative supporting stiffness reaches a certain value, the variation trend of the bearing capacity against the stiffness of elastic support tends to be smooth. That is, the contribution of the increase of supporting stiffeners section dimensions to the bearing capacity is obviously reduced. Therefore, the trend of optimization iteration is increasing the cross-section size of the longitudinal stiffeners and reducing the cross-section size of the supporting stiffeners.

Furthermore, when containing different number of supporting stiffeners, the optimized weight shows a great difference and the value reaches the minimum in case 2 (having two supporting stiffeners). This shows the great influence of the matching layout of two kinds of stiffeners on structure lightweight design.

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**Figure 5.** Optimization iteration process.
Table 1. Optimization Results.

|                | Initial | Case 1 | Case 2 | Case 3 | Case 4 |
|----------------|---------|--------|--------|--------|--------|
| $n$            | 3       | 1      | 2      | 3      | 4      |
| $b_1$ / mm     | 60      | 40     | 40     | 40     | 40     |
| $b_2$ / mm     | 60      | 30     | 32     | 35     | 33     |
| $H$ / mm       | 30      | 58     | 45     | 38     | 35     |
| $B$ / mm       | 50      | 42     | 40     | 40     | 40     |
| $t_1$ / mm     | 4       | 2      | 2      | 2      | 2      |
| $t_2$ / mm     | 4       | 5.9    | 4.5    | 3.9    | 3.6    |
| $P$ / kN       | 141.3   | 144.3  | 141.8  | 142.0  | 143.6  |
| $W$ / kg       | 8.763   | 6.647  | 5.504  | 5.824  | 6.381  |

4. Conclusion

This paper presents a method for calculating the axial load carrying capacity of stiffened panels considering the torsion of supporting stiffeners. The longitudinal stiffener is regarded as a multi-span compressive column supported by a series of transverse elastic supports and torsional elastic supports. By defining three dimensionless factors: the relative bearing capacity, the relative transverse elastic stiffness and the relative torsional elastic stiffness, the response surface of the bearing capacity versus two kinds of elastic supports is obtained.

Based on genetic algorithm, a lightweight optimization model for structure layout and section parameters is constructed. A certain stiffened panel is optimized on this basis. The results show that

a. In the case of having strong supporting stiffeners, the trend of optimization iteration is increasing the cross-section size of the longitudinal stiffeners and reducing the cross-section size of the supporting stiffeners.

b. When the structure contains different number of supporting stiffeners, the weight of the optimized structure has great changes. Compared with the optimization of section parameters, structure layout matching of two kinds of stiffeners has a greater impact on structure lightweight design.

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