Strategy towards Mirror-fermion Signatures

George Triantaphyllou*

Institut für Theoretische Physik, Technische Universität München
James-Franck-Strasse, D-85748 Garching, GERMANY

May 24, 2022

Abstract

The existence of mirror fermions interacting strongly under a new gauge group and having masses near the electroweak scale has been recently proposed as a viable alternative to the standard-model Higgs mechanism. The main purpose of this work is to investigate which specific experimental signals are needed to clearly differentiate the mirror-fermion model from other new-physics models. In particular, the case is made for a future large lepton collider with c.o.m. energies of roughly 4 TeV or higher.

*e-mail:georg@ph.tum.de
1 Introduction

Most of the current high-energy experimental data are in good agreement with the theoretical predictions of the standard model of elementary particle physics. This model predicts however the existence of a fundamental scalar field, the Higgs particle, having a mass on the order of the electroweak scale, which has yet to be discovered. The instability of the mass of an elementary scalar field against quantum corrections nevertheless has prompted various speculations regarding the true inner structure of the Higgs sector along the years, leading to the study of new physics beyond the standard model.

It has been recently argued [1] that the standard Higgs mechanism can be seen as an effective low-energy description of a new heavy sector consisting of mirror fermions carrying quantum numbers under a gauged generation group becoming strong at around 1 TeV. Non-zero vacuum-expectation values of mirror-fermion bilinear operators can then break the electroweak symmetry dynamically at the expected energy scale.

The subsequent breaking of the mirror-generation group allows the formation of composite fermion operators which are invariant under the electroweak gauge symmetry. These operators feed masses to the standard-model fermions by mixing them with their mirror partners in a way that can suppress dangerous flavor-changing neutral currents. The resulting masses and mixings for quarks and leptons can easily accommodate current experimental results.

This model [1] presents from the theoretical side several advantages. The

“people tending to introduce new terms could name it “mirrorcolor”, and the new fermions “katoptrons” from the greek word meaning “mirror”
natural solution offered to the hierarchy problem is accompanied by an appealing
gauge-group unification including the mirror-generation group. It is quite im-
portant to stress here that the naturalness problem, far from being just a matter of
stability of the electroweak scale with respect to the Planck scale against radiative
corrections, has to do mainly with the fact that the electroweak scale is roughly
14 orders of magnitude, and not some other arbitrary number, smaller than the
gauge-coupling unification scale. The mirror model, unlike most other currently
popular new physics approaches, addresses this issue in a very satisfactory manner,
without resorting to the anthropic principle for instance. However, a deeper un-
derstanding of the mechanism that is responsible for the eventual breaking of the
strong mirror-generation group is still needed.

The gauge-coupling unification in this model is found to be not only con-
sistent with proton-decay bounds, but can also lead to a precise explanation of
the order of magnitude of the QCD and electroweak scales without any need for
fine tuning of parameters. Moreover, the introduction of mirror partners to the
known fermions restores in a certain sense the chiral symmetry missing in the stan-
dard model, and in addition could constitute a solution to the strong CP problem.

From the experimental side, electroweak precision tests could already be
providing indirect signals for physics beyond the standard model in this direction.
Of particular importance are here the almost 3 \( \sigma \) deviations of the right-handed
weak coupling of the bottom quark and the values of the \( S \) and \( T \) parameters,
which, even though still consistent with zero, can take non-negligible negative val-
ues. Contrary to most other new-physics models, these effects can be explained
within the mirror-model framework. Still missing nevertheless is a deeper theoretical understanding of why certain fermion composite operators take the particular values which make the theory consistent with these effects.

Having described the main features of the mirror model, an effort should be made to see how it can be tested phenomenologically. Of special interest is of course the search for signals which clearly differentiate it from alternative theories, and to investigate what types of high-energy facilities would be required for such an endeavor. Far from being an exhaustive or thorough study, this work tries to sketch roughly the logic which, taking into account what is feasible presently and in the years to come experimentally, can lead to a definite verification or falsification of the model. We hope to return to the specific processes described here in order to study them in more detail in the future.

2 Indirect tests

2.1 Generic features

The existence of new heavy particles can in principle influence the couplings and masses of the standard-model particles via higher-dimensional operators. The corresponding effects should be usually detectable at lower energies in anomalous decays of the known particles, or in physical quantities taking values deviating substantially from their standard-model predictions, without having to probe the heavier new-physics sector directly. The disadvantage of this program is however that various distinct theories can frequently predict similar effects. The information drawn by such analyses is useful therefore mainly to the extent of constraining the respective model parameters, and possibly ruling out whole classes of theories, but not proving
unambiguously that a particular model is correct.

As a typical example, one can quote proton decay. It was recently shown that, if mirror-fermion models are to be compatible with gauge-coupling unification, they should predict proton-decay like \( p \rightarrow e^+ \pi^0 \) with rates close to current experimental bounds, as given from the Super-Kamiokande experiment for instance. In particular, for the favored Pati-Salam gauge symmetry-breaking sequence scenario

\[
SU(4)_{PS} \times SU(2)_R \rightarrow SU(3)_C \times U(1)_Y
\]

one calculates the following approximate values for the unification scale and common gauge coupling:

\[
\Lambda_{GUT} \approx 10^{15.5} \text{ GeV}, \quad \alpha_{GUT} \approx 0.036.
\]

The value of the unification coupling quoted above is quite close to the present experimental limit of \( \alpha_{GUT} \lesssim 0.074 \) for the same unification scale. Even though detection of proton decay would be of utmost importance for the generic verification of unification schemes, it would not be able by itself to indicate which particular type of unification is favored, since there are several other frameworks (like supersymmetric unification for example) predicting similar proton lifetimes.

Another example is provided by the recently measured anomalous bottom-quark right-handed weak coupling \( \delta g^b_R \) or the negatively-centered values of the electroweak precision parameters \( S \) and \( T \), which in their turn could be related to an anomalous top-quark right-handed weak coupling. Unfortunately, this coupling is still directly unaccessible in current experiments due to the heaviness of the top quark. Even though the mirror-fermion model is one of the very few examples of theories consistent with such effects, these phenomena can only serve as
an indication and not a definite proof, since in any case the corresponding deviations of $\delta g_R$, $S$ and $T$ are smaller than 3 $\sigma$.

With regard to the $S$ parameter in particular, it should be noted that the top-quark anomalous coupling does not have to be as large as the one quoted in [1] in order to cancel the large “oblique” corrections coming from the numerous new electroweak doublets introduced in the mirror model. Since the strong mirror-generation group is broken, one can speculate that, even though it forms condensates and generates dynamical mirror-fermion masses, no vector resonances are formed, in which case the $S$ parameter is given by

$$S^0 \approx \frac{N}{6\pi},$$  \hspace{1cm} (3)

with $N$ the number of new heavy electroweak doublets. This is half as large as the estimate based on QCD-like dynamics.

Such a scenario has been studied in [3], giving a result consistent with the one of Ref. [1] for the case of roughly momentum-independent fermion self-energies. The reason for the appearance of non-QCD dynamics here is however not a “walking” gauge coupling but a broken gauge group. In the mirror-model case, the number of new weak doublets introduced is $N = 12$, so one estimates $S^0 \approx 0.64$. The presence of Majorana mirror neutrinos can make $S$ even smaller [2], since the leptonic contribution can be as small as $S_l^0 \approx -0.24$ instead of $S_l^0 \approx 0.16$ for the case of mirror neutrinos and charged leptons which are degenerate in mass. This “best-case” scenario leads to a total “oblique” contribution to the $S$-parameter given by $S^0 = S_q^0 + S_l^0 \approx 0.24$, where $S_q^0$ stands for the mirror-quark oblique contribution to the $S$ parameter.
In the mirror model, contributions of vertex corrections to the $S$-parameter coming from effective four-fermion operators can also be potentially important. In order to be within $1\,\sigma$ from the experimental limit on $S$ quoted in \cite{1}, one needs the vertex corrections to give a contribution to the $S$ parameter on the order of

$$S^{t,b} \lesssim -0.09,$$

which is about one order of magnitude smaller than the value used in \cite{1}, and in absolute value not unreasonably large. In fact, if one accepts the current experimental central value for $\delta g_b^b$, it could be produced by an anomalous top-quark coupling of about $\delta g_t^t \approx -0.03$, which is roughly equal in absolute value to $\delta g_b^b$. A small top-quark anomalous coupling can therefore easily accommodate the present experimental data.

The reduction of the needed magnitude for $\delta g_t^t$ leads in addition to the elimination of excessive fine tuning needed to keep the $T$ parameter small, since in \cite{1} this parameter is mainly affected by vertex corrections induced by $\delta g_t^t$. The isospin breaking introduced artificially within the mirror doublets in that reference in thus rendered obsolete in this scenario.

Oblique corrections to the $T$ parameter in these models are generally expected to be small since the mirror-fermion masses are dynamically generated and roughly isospin symmetric in analogy to the constituent quark masses in non-perturbative QCD, and the difference of the standard-model top- and bottom-quark masses is taken to be fed down in a gauge-invariant way. In the mirror-lepton sector things are more complicated however, since the see-saw mechanism responsible for the mass splitting between the charged mirror leptons and the mirror neutrinos
which generates the negative value of $S_0^0$ generates also positive contributions to the $T$ parameter. These would still have to be roughly canceled by the $\delta g_R^l$-contributions to accommodate current experimental limits.

Particularly useful in this respect will be direct measurements of the anomalous coupling $\delta g_R^l$ via the top-quark forward-backward asymmetry in the Next Linear Collider (NLC). Following the analysis of Ref. [7] regarding experiments at the NLC, the top quark neutral-current coupling can be constrained there at the 10% level, which should be enough to discover mirror-fermion mixing effects. This coupling could also be constrained in the planned muon collider.

Another important quantity which can deviate substantially in the mirror model from its value predicted by the standard-model is the CKM matrix element $|V_{tb}|$. In the numerical example presented in [1], it was found that

$$|V_{tb}| \approx 0.95.$$ (5)

This quantity can be made closer to unity for heavier mirror top quarks, but it cannot exceed a lot the value quoted above if one wants to reproduce the weak scale correctly. Deviations from the standard-model prediction of $|V_{tb}^{SM}| \sim 1$ would support the existence of at least one new fermion generation mixing with the known fermions in order to guarantee the unitarity of the generalized mixing matrix.

The value of $|V_{tb}|$ could be tested via virtual W-boson and top-quark decays at the Tevatron III and the NLC. At these high-energy facilities, a limit of $|V_{tb}| > 0.97$ could be obtained if the standard-model value is correct [7], providing a good testing ground for the mirror model.

A deviation of $|V_{tb}|$ from its standard-model value would provide a hint for a
mass-generating mechanism for the ordinary fermions via feed-down (a generalized “see-saw”) from the new sector. In conjunction with anomalous heavy-quark weak couplings however, it could further be an indication of mixing with at least one new fermion generation having different weak-charge assignments from the standard-model generations, and would thus lend support to the mirror-fermion framework proposed in [3]. The new-generation fermions could of course be a priori weak singlets and not directly involved in the electroweak-symmetry breaking, so it is still required to see whether they decay weakly.

2.2 Flavor-changing neutral currents

Theories introducing mirror fermions usually predict the existence of flavor-changing neutral currents (FCNC) [8]. These are expected to be particularly important for processes involving heavier quarks, since in the mirror model the masses of the known fermions are generated by their mixing with their mirror partners.

The mixing of the two lighter generations with their mirror counterparts is quite small, as can be seen in the generalized CKM matrices for quarks [1] and leptons [3]. Therefore, $K^0 - \bar{K}^0$ mixing, as well as decays like $K \rightarrow \pi \nu \bar{\nu}$, $K \rightarrow e \mu$, $\mu \rightarrow e \gamma$ and $\mu \rightarrow e \nu \bar{\nu}$ can be made to agree with present experimental limits without much effort. The same can be said about the CP-violation parameters $\epsilon$ and $\epsilon'$ in the kaon system, even though it is still conceivable that the recently reported deviation of $\epsilon'/\epsilon$ from the standard-model prediction coming from the KTeV experiment is related to such type of physics. Effects coming from the new sector can be usually suppressed by raising the mirror-fermion masses and thus decreasing somewhat the corresponding mixing.
This could not be said for the FCNC processes involving both third-generation quarks $t$ and $b$ like $B \rightarrow X_s \gamma$ or $X_s \bar{l}l$, $B_s \rightarrow l^+l^-$, or for the $B_s^0 - \bar{B}_s^0$ mixing which could be measured with some accuracy at the B-factories and the LHC-B. The same goes for the top-quark decays $t \rightarrow c\gamma, cZ^0$ which could be seen at the Tevatron III, and the off-shell $Z^0$-boson decay $Z^0* \rightarrow tc$ at the NLC. Since the mass eigenstate corresponding to the top quark has a non-negligible mirror-top-quark admixture, one could expect potentially measurable effects coming from these processes. On the other hand, processes like decays induced by mirror-lepton mixing with standard-model leptons, like $Z^0* \rightarrow \nu\nu^M$ are expected to be highly suppressed due to the small mixing of the leptons with their mirror partners and not easily detectable at experiments in the NLC for instance.

An example on the process $B \rightarrow X_s \gamma$ induced by a $W$-boson loop is presented in the following, since there exist currently precise results from the CLEO collaboration on the relevant branching ratio, and since experimental data for the other processes are not yet available or not precise enough to lead to useful model constraints. Extending the formalism of Ref.[9] to encompass mirror quarks, the inclusive rate for $B$-meson decay modeled upon the quark process $b \rightarrow s\gamma$ is given by

$$\Gamma(B \rightarrow X_s \gamma) = \frac{8}{144\pi^2}G_F^2m_b^5\alpha|V_{tb}V_{ts}^*(C(m_t) + C') + \sum_{i} V_{ib}V_{is}^*C(m_i)|^2,$$

where $G_F$ and $\alpha$ are the Fermi and fine-structure constants respectively, $m_b$ the mass of the bottom quark, and $V_{ij}$ the generalized CKM matrix elements. The subscript $i$ is running over the mirror quarks in the $W$ loop. Their masses $m_i$ bFollowing the convention of [9], from now on the mirror fermions are denoted by the symbol of their standard-model partners with a superscript “M”.

[9]
always satisfy the relation \( m_i^2 \gg m_W^2 \).

In this formalism, the functions \( C(m_i) \) and \( C' \) are given at leading order by

\[
C(m_i) = \eta^{16/23} \left( -1 - \frac{5\delta}{8} + \frac{7\delta^2}{8} - \frac{(9\delta - 3\delta^2)}{4(1 - \delta)^4} \right) + 8(\eta^{16/23} - \eta^{14/23}) \left( \frac{1 - \frac{5\delta}{8} - \frac{\delta^2}{4}}{(1 - \delta)^3} - \frac{3\delta^2 \ln \delta}{4(1 - \delta)^4} \right)
\]

\[
C' = 3 \sum_{j=1}^{8} h_j \eta^{p_j},
\]

where \( \delta = \left( \frac{m_W}{m_i} \right)^2 \), \( \eta = \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \) with \( \alpha_s \) the QCD coupling, \( m_W \) the W-boson mass and the constants \( h_j, p_j \) can be found in [9]. Recent analyses have proceeded to more precise next-to-leading-order (NLO) results [10], but they have an accuracy much higher than the one needed for the purposes of this study given the uncertainties of the mirror-fermion masses and mixing angles.

It is further assumed that the generalized CKM matrix elements encode all the information needed to study this decay in the context of the mirror model. In particular, the effects of the mirror-top-bottom matrix element \( |V_{tM_b}| \) would correspond in a certain sense to the effects stemming from the anomalous coupling \( f_{tb}^b \) in the analysis of [11]. In the standard-model, the \( b \rightarrow s\gamma \) process at the weak scale is dominated by the diagram with a top quark inside the W-boson loop due to the large CKM matrix element \( |V_{tb}| \) and the large top-quark mass.

In the mirror model however, the mixing-matrix element \( |V_{tM_b}| \) is also non-negligible. Even though the quantity \( |V_{tb}V_{ts}^*| \) is still quite larger than \( |V_{tM_b}V_{tM_s}^*| \), the diagram with a mirror top quark \( t^M \) inside the loop is enhanced due to the first term involving a large logarithm appearing in the quantity \( C(m_{tM}) \), since the mirror top quark is quite heavy. In the numerical example in [1] for instance, it has
been taken to have a mass equal to $m_{tM} \approx 810 \text{ GeV}$.

Note that there is in principle also a loop diagram involving a $Z^0$ boson contributing to this process, since there are flavor-changing neutral currents induced at tree level due to the mixing of the fermions with their mirror partners. The relevant couplings here involve the particles $(Z, b, b^M)$ and $(Z, s, b^M)$, but their are suppressed due to the small mixing terms in the corresponding mass matrix, so the contribution of this diagram is expected to be negligible. Possible additional contributions coming from scalar bound states of mirror fermions are also neglected by taking them to be heavy, since the ones with the larger couplings to the fermions of relevance here involve a heavy mirror top quark.

Of interest in the following is the ratio $R$ of the mirror-model to the standard-model decay-rate prediction, which by virtue of Eq. 6 and the preceding discussion is equal to

$$R = \frac{\Gamma_{M}(B \rightarrow X_{s}\gamma)}{\Gamma_{SM}(B \rightarrow X_{s}\gamma)} \approx \frac{|V_{tb}V_{ts}^*(C(m_t) + C') + V_{tM_{b}}V_{tM_{s}}^* C(m_{tM})|^2}{|V_{tb}^* V_{ts}^{SM*}|^2 (C(m_t) + C')^2},$$

(8)

where the superscript “$SM$” stands for the CKM matrix elements expected from the standard model, and it is assumed that the decay is dominated by diagrams with a top and a mirror-top quark inside the $W$-boson loop.

The relative interference phase $\omega$ between the products of the relevant generalized CKM matrix elements in Eq. 8 is then given by

$$\omega = \arg \left( \frac{V_{tb}V_{ts}^*}{V_{tM_{b}}V_{tM_{s}}^*} \right) = \arccos \left( \frac{\tilde{R} - 1 - \rho^2}{2\rho} \right),$$

(9)

[c]Now and in the following we will abusively refer to zero-spin fields generically as scalars even if they are strictly speaking pseudoscalars.
with
\[ R = \frac{V_{tb}^{SM}V_{ts}^{SM*}}{V_{tb}V_{ts}^*} \]
and
\[ \rho = \frac{|V_{tM}V_{tM}^*|C(m_t) + C'}{|V_{tb}V_{ts}^*|}. \]

The experimental collaboration CLEO recently reported the value
\[ B_{\text{exp}}(B \rightarrow X_s\gamma) = (3.15 \pm 0.54) \times 10^{-4} \]
for the branching ratio corresponding to this decay, where the error includes both systematic and statistical contributions. Theoretically, within the framework of the NLO calculation, it is expected that
\[ B_{\text{th}}(B \rightarrow X_s\gamma) = (3.28 \pm 0.30) \times 10^{-4}. \]

In the process of testing the mirror model, the quantity \( R \) could be also seen as the ratio of the experimental result to the theoretical prediction. Therefore, this ratio can be given by the relation
\[ R = \frac{B_{\text{exp}}}{B_{\text{th}}} = 0.96 \pm 0.26. \]

This result can be readily translated into a bounded phase \( \omega \). To provide an indicative example of how one could constrain the mirror model, we use specific numerical values for the relevant quantities, ignoring the uncertainties stemming from the mixing-matrix elements and the fermion masses.

The mixing-matrix values in the numerical example of \[ \text{[1]} \] are given by
\[ |V_{tb}^{SM}| \approx 1 \]
\[ |V_{tb}| = 0.95 \]
\[ |V_{ts}| \approx |V_{ts}^{SM}| \approx 0.038 \]
\[ |V_{TMb}| = 0.32 \]
\[ |V_{TMu}| = 0.016. \]  
(14)

Moreover, by virtue of Eq.7 one computes the values

\[ C(m_{tm}) = -0.66, \quad C(m_t) = -0.42, \quad C' = -0.52. \]  
(15)

It is interesting to see here that, due to the heaviness of the mirror-top quark, the values of the \( C \)-functions for the top and mirror-top quarks are comparable in magnitude.

Substituting the numerical quantities given above in Eq.9 constrains the interference phase \( \omega \) to be

\[ \omega \approx 57^\circ \pm 96^\circ. \]  
(16)

In principle, consistency would require to calculate the quantity \( \rho \) with the NLO \( C(m_t) \) functions. It is nevertheless assumed for simplicity that the large bottom-quark-scale uncertainty roughly drops out in the ratio of the mirror-to-standard-model quantities, and anyway the ignorance of the precise value of \( m_{tm} \) would render the NLO accuracy superfluous for the purposes of this work.

It is thus worth noting that there is presently enough experimental and theoretical input to mildly constrain some mirror-model parameters. One can conclude that potential deviations of similar quantities from the standard model predictions can be explained within the mirror model, but as stressed before cannot prove its correctness since there are alternatives ways to get similar FCNC deviations like supersymmetry or extended technicolor. Additional experimental data coming from the B-factories will further constrain the mass and mixing parameters of the mirror
model, possibly identifying on the way also novel sources of CP violation.

3 Direct tests

The unique safe method to prove or falsify the mirror model is obviously to produce directly the new heavy particles it predicts. The strongly-interacting mirror particles introduced in [1] are expected to either form scalar bound states or propagate freely, since the mirror-generation group is assumed to break when it becomes strong. In any case their masses are generally expected to be around the electroweak scale. One should therefore hope that future colliders like the LHC, the NLC and the muon collider will be able to produce these states on-shell and detect them through their decays.

3.1 Vector and fermion fields

In principle, the existence of new vector resonances should be able to differentiate a strongly interacting Higgs sector from a perturbative one, like the one in the standard model or supersymmetry. Decays of such resonances to a pair $W^+W^-$ of massive gauge bosons would indicate that the new fermions carry weak charge and are not $SU(2)_L$ singlets as some see-saw models require [13]. However, the fact that the strong mirror-generation group is broken, together with the smallness of the $S$ parameter, leads one to suspect that the theory is not confining even though mirror-fermion condensates are formed, and that no vector bound states exist like the rho in QCD or the technirho in technicolor theories.

On the contrary, one should expect elementary massive vector bosons corresponding to the broken generators of the mirror-generation group. These would
couple strongly to the mirror fermions according to the mirror-model scenario \[1\],
inducing large FCNC between them. However, it is very questionable whether these
vector bosons have widths which are narrow enough to make them experimentally
detectable. Large widths would dilute the signal of new decaying vector particles,
making their detection at the LHC conceivable by rather hard.

One should therefore search for the production of pairs of free mirror fermions.
Their weak decays would show that these are charged under \(SU(2)_L\) and not singlets,
identifying with more precision but indirectly the source of possible deviations
from the standard-model values of \(g_{t,b}^{l,b}\) and \(|V_{tb}|\). In principle, if mirror quarks do
not pair up with each-other, they could pair-up with ordinary quarks to form QCD-
singlet scalar bound states, so one should hope to see only mirror leptons propagate
freely. Mirror fermions are however expected to decay weakly fast, before they have
time to hadronize, in analogy with the top quark. The expected phenomenology
implied by the mirror model at hadronic colliders is apparently so rich that going
into a detailed quantitative discussion on the expected signals goes beyond the scope
of this study. A potentially promising process involving zero-spin bound states is
presented in the following.

3.2 Scalar fields

In this subsection, the focus will be on the search for scalar bound states of two
mirror fermions. Parity-violating decays of these states would indicate that the
new fermions are not weak singlets, in analogy with kaon decays which revealed
initially the parity-violating nature of weak interactions more than forty years ago.
Furthermore, since the couplings of the scalar bound states are expected to be
similar to the ones relevant to technipions in extended technicolor theories, a most promising scalar decay would be the one described in [14], i.e. a color-octet and neutral scalar bound state, which we name “mirror-pion” and denote here by $P_{8}^{M,0}$, having a mass equal to $M_P$ and decaying predominantly into a top-antitop quark pair due to the heaviness of the top quark.

The mirror pion is mainly produced via gluon fusion in very high energy hadronic colliders like the LHC due to the large gluon structure functions. The relevant decay widths for an $SU(N_G)$ mirror-generation group are

$$
\Gamma(P_{8}^{M,0} \rightarrow gg) = \frac{5N_G^2 N_D}{384\pi^3} \alpha_s^2(M_P) \frac{M_P^3}{v^2} \quad (17)
$$

where $v \approx 250$ GeV is the electroweak scale, $\alpha_s$ the QCD coupling, $N_D = 4$ the number of weak mirror doublets, and

$$
\Gamma(P_{8}^{M,0} \rightarrow t\bar{t}) \approx \frac{m_t^2 M_P N_D}{4\pi v^2} \left(1 - 4 \frac{m_t^2}{M_P^2}\right) \quad (18)
$$

In the equation above, $m_t$ is the ordinary top-quark mass, and a CP-conserving effective coupling of the mirror-pion to the top quark of strength $2m_t/v$ has been chosen, in analogy to QCD and extended technicolor. This should be a reasonable order-of-magnitude estimate for this coupling given the type of mirror-fermion mixing with the standard-model particles introduced in [1].

The mirror-pion decay into a top-antitop pair dominates over the other decay modes, making the cross section, which is integrated over an energy bin, roughly proportional to the two-gluon decay rate. Since the group $SU(3)_{2G}$ considered here has $N_G = 3$ instead of $N_G = 2$, one would in principle have to multiply the cross-section results of [14] by a factor of $9/4$ for the same mirror-pion and top-quark masses, accelerator c.o.m. energies and experimental cuts.
Note however that, since in the present case the strong mirror-generation group is eventually broken, one might have three distinct color-octet neutral mirror pions with different masses. Therefore, these do not interfere coherently, and they are each produced by gluon fusion with \( N_G = 1 \). This reduces the predicted signal for each mirror pion roughly by a factor of 1/4 compared to the one in [14]. Moreover, only the heaviest mirror pion should be expected to have the large coupling to the top quark assumed above due to its mixing with the mirror-top quark, and this mirror pion should be the main subject of our interest in the following.

The mirror-pion mass receives the same type of contributions as in [14], with the only difference that the extended-technicolor interactions there have to be replaced by the broken mirror-generation-group interactions in the present context. It can be therefore assumed to lie in the same mass range as in that reference, i.e. to have a mass around 350-550 GeV. For the case of masses given by \( m_t = 175 \) GeV, \( M_P = 450 \) GeV and c.o.m. energies of 14 TeV which are planned at the LHC, one should expect an integrated \( \bar{t}t \) production cross-section of about

\[
\sigma_M(p p \rightarrow P^M_8 \rightarrow \bar{t}t) \approx 10 \text{ pb}
\]  

for the mirror pion for an energy bin of \( \pm 10 \) GeV around its mass and a rapidity cut with \( Y = 2.5 \) [14].

This result should be compared with a QCD \( \bar{t}t \) background of around 70 pb’s for the same energy and rapidity cuts. For the planned LHC luminosities of about \( 10^{34} \text{ cm}^{-2}\text{s}^{-1} \), this should guarantee roughly one signal event and 7 background events per 10 seconds and thus rich statistics. The small signal-to-background ratio would nonetheless make this enhancement more difficult to observe.
In connection with the aforementioned large deviations of the quantities $|V_{tb}|$ and $g_R^t$ from their standard-model predictions and with the absence of vector resonances, such a $t - \bar{t}$ production enhancement due to a mirror pion, or other mirror-model signals at hadronic colliders, could be differentiated in principle from usual technicolor signatures or signals coming from alternative dynamical electroweak-symmetry-breaking scenarios. However, it could still be by itself rather difficult to prove that the fermions producing these signals have indeed mirror- and not standard-model-type or singlet weak-charge assignments.

To acquire this important piece of information, one needs to probe the weak charges of the new fermions directly, and the next subsection is devoted to this quite crucial issue. To end this subsection on mirror pions, one could also add that similar “higgs”-like scalar resonances would also find a very good and clean testing ground in the planned muon collider, since the corresponding processes are proportional to the square of the muon - instead of the electron - mass $m_{\mu}^2$.

### 3.3 The forward-backward asymmetry

The measurement of the forward-backward asymmetry of mirror fermions at lepton colliders due to the interference of lepton-annihilation processes into a photon and a $Z^0$ boson would be the ultimate proof of the mirror model. This asymmetry has an opposite sign from the asymmetry of standard-model-type fermions expected in 4-generation models for instance. However, since the asymmetry also changes sign going from fermion masses below the $Z^0$ peak to masses above it, mirror fermions are expected to exhibit a forward-backward asymmetry of the same (negative) sign as the one observed for the standard-model fermions, except for the (positive) one
corresponding to the top quark that should manifest itself at the NLC.

As already said, the strong mirror-generation group is broken, so one can imagine that it does not confine, although it gives dynamical masses to the mirror fermions. Thus, one could expect to observe an asymmetry of unconfined mirror fermions. This is a crucial assumption on which the analysis that follows is based. One could also try to observe an asymmetry in charged mirror-pion pair production involving charged mirror leptons, but the mirror leptons produced in the fragmentation process isotropically would, in analogy with light quarks in QCD, most likely wash-out the effect.

However, since mirror fermions in this model are strongly interacting, large radiative effects make it difficult to observe this asymmetry. A 2-TeV leptonic collider would already be probing the lower energy limits of a possible asymmetry measurement as will be seen shortly, since the first-order $\alpha_G$ corrections there, where $\alpha_G$ is the mirror-generation-group coupling, are already on the order of 24%. Even lower energies would allow the mirror-generation-group coupling, along with mirror-fermion mass-threshold effects, to more or less smear the directional information of the two mirror fermions initially produced, possibly producing pairs of oppositely charged scalar bound states in a roughly spherically-symmetric way.

It would therefore seem beyond any reasonable hope to measure any mirror-fermion forward-backward asymmetry at the NLC, unless the mirror-generation group becomes strong at energies closer to the weak scale than to 1 TeV, something which is conceivable but difficult, or some other - yet-to-be-understood - aspects of the broken mirror-generation group come into play. This collider could concentrate its effort on the also very important task of producing these new fermions,
which would at least support the existence of additional generations, if not revealing
their precise weak-charge assignments. One should anyway not exclude *a priori* the
detection of a small mirror-fermion forward-backward asymmetry at the NLC.

Obviously, what is really needed is energies high enough so that the mirror-
generation-group coupling becomes weak and mirror-fermion mass-threshold effects
small. Ideally, a leptonic collider with c.o.m. energy $E = \sqrt{s}$ of about 4-10 TeV
would be required. Such an effort would be reminiscent of the PEP-PETRA-
experiments era of the 80’s [15], but with energies three orders of magnitude higher.
It should be stressed that the planned muon collider [16] would be a perfect candi-
date for such a high-energy facility.

Since the up- and down-type members of mirror-quark weak doublets have
electromagnetic charges opposite in sign, the forward-backward asymmetry effect
would be diluted unless one chooses a mirror quark with enough mass separation from
the next lightest and the next heaviest. This mass-degeneracy lifting would reduce
heavier mirror-quark contamination and facilitate flavor identification in analogy
with the bottom- and charm-quark asymmetries in the standard model.

To avoid this difficulty, it would be preferable to measure the forward-
backward asymmetry of mirror leptons instead of mirror quarks. In such a case,
only the charged leptons would exhibit an asymmetry if the mirror neutrinos are
Majorana, i.e. if $\nu^M = \bar{\nu}^M$, which is the case in [2] and in the following. Mirror
leptons could also be experimentally easier accessible since on general grounds they
are expected to be lighter than mirror quarks. A conceivable process of considerable
interest would then be the following:

\[ \mu^+ \mu^- \rightarrow \gamma, Z^0 \rightarrow l^+ l^- . \]  \hfill (20)

Effects due to the non-relativistic nature of quarks are quite important in similar standard-model processes, where quark current masses can be much larger than the QCD scale, as is the case for the charm, bottom and top quarks. The mirror fermions considered here however have purely dynamical masses, and by the time one reaches energies where mirror-generation-group strong-coupling effects can be controlled perturbatively, the mirror fermions are to a very good approximation relativistic.

Therefore, the velocity of the mirror fermions of mass \( m \approx 200 - 600 \text{ GeV} \) is taken in the following to be \( \beta = \sqrt{1 - m^2/s} \approx 1 \). In this case, the production cross-section and the forward-backward asymmetry for the generic process \( l^+ l^- \rightarrow f^+ f^- \) via a photon and a \( Z^0 \) gauge boson for Dirac leptons \( f^\pm \) in the final state are given respectively by the following relations:

\[ \sigma(l^+ l^- \rightarrow f^+ f^-) = \frac{4 \pi N_G \alpha^2}{3s} Q_1 \]
\[ A_{FB}^0 = \frac{3Q_3}{4Q_1}, \]  \hfill (21)

where \( N_G \) is the number of “colors” carried by the final-state leptons, and the quantities \( Q_{1,3} \) are defined as \[ L \]

\[ Q_1 = \frac{1}{4}(|Q_{LL}|^2 + |Q_{RR}|^2 + |Q_{LR}|^2 + |Q_{RL}|^2) \]
\[ Q_3 = \frac{1}{4}(|Q_{LL}|^2 + |Q_{RR}|^2 - |Q_{LR}|^2 - |Q_{RL}|^2). \]  \hfill (22)

At these high energies the mirror-generation group can be considered to be unbroken, since it is in principle impossible to distinguish between the mirror
Figure 1: The scattering cross-section for the process $\mu^+\mu^- \rightarrow l^M + l^M$ as a function of the c.o.m. energy $E$. The dotted line shows the result after taking the strong mirror-generation-group effects into account.
Figure 2: The forward-backward asymmetry of the mirror leptons. The dotted line shows the decreased asymmetry due to the strong mirror-generation-group effects. The dependence of this line with energy is due to the logarithmic running of the corresponding gauge coupling.
leptons $e^M, \mu^M$ and $\tau^M$, so one has $N_G = 3$. The quantities $Q_{JK}$ introduced above are equal to

$$Q_{JK} = q^f_{J}q^f_{K} + \frac{Q^f_{J}Q^f_{K}}{s_W^2 c_W^2} \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z},$$

with $s_W^2 = 1 - c_W^2 = \sin^2 \theta_W$ the Weinberg angle, $m_Z \approx 91$ GeV and $\Gamma_Z \approx 2.5$ GeV the $Z^0$ mass and width respectively, $q^{l,f}$ the electromagnetic charges of the initial and final leptons, and

$$Q^{l,f}_{J} = I^{l,f}_{J} - s_W^2 q^{l,f}_{J},$$

the electroweak charges of the initial and final leptons of isospin $I^{l,f}_{J}$ respectively, with $J, K = L, R$.

The energies considered here are so high that the $Z^0$-boson mass and decay width could be safely neglected from the start. For the same reason, contributions to this process from scalar resonances like effective “higgses” can also be neglected since, for the muon collider for instance, one has $(m_\mu/v)^2 \ll \alpha^2$. It should be further reminded that, in the formulas above, $\alpha$ and and $s_W^2$ are energy-dependent due to the renormalization-group equations, which in the present framework take also mirror fermions into account \[2\].

Taking the mirror-model quantum-number assignments into consideration, the lepton charges are given by

$$Q^l_L = Q^f_L = -\frac{1}{2} + s_W^2, \quad Q^l_R = Q^f_R = s_W^2$$

$$q^l_L = q^f_L = q^f_R = q^f_R = -1.$$  

The smallness of the mixing of the mirror leptons with their standard-model counterparts allows one to safely neglect in this calculation the fact that mirror leptons...
are not pure mass eigenstates.

Moreover, it is easy to check that having final fermions with mirror charge assignments \[1\] instead of standard-model type ones corresponds to a \(L \leftrightarrow R\) interchange in the second subscript of the quantities \(Q_{JK}\) with \(J, K = L, R\), and a resulting change in the overall sign of \(A^0_{FB}\). The sign of the forward-backward asymmetry of the final-state fermions provides therefore a test which is unique in its importance for the experimental verification of the mirror model.

Including first-order corrections due to the strong mirror-generation-group coupling \(\alpha_G\) and neglecting possible effects due to the fact that the mirror-generation group is eventually broken, one has a production cross-section enhanced by a factor of \(\left(1 + \frac{\alpha_G(s)}{\pi}\right)\), and a corresponding reduced asymmetry result equal to

\[
A_{FB} \approx A^0_{FB} \left(1 + \frac{\alpha_G(s)}{\pi}\right)^{-1},
\]

where the running of the coupling of the mirror-generation group \(SU(3)_{2G}\) is taken to be described by the relation

\[
\alpha_G(s) = \frac{1}{1 + \frac{17}{12\pi} \ln \frac{s}{\text{TeV}^2}}.
\]

Even though the first-order correction is substantial, the precise magnitude of the second-order correction is usually debated but quite smaller \[18\], and can anyway not answer by itself the question on whether the full \(SU(3)_{2G}\)-corrected result is much smaller or larger than the first-order result. At energies of 4 TeV one finds \(\alpha_G \approx 0.44\), which is still a pretty large parameter to do perturbation theory with. The magnitude of the first-order result at energies close to 1 TeV makes clear that the numerical values presented in the following, especially for the lower energies, should be seen more as qualitative estimates than as precise predictions.
The results for the cross-section and the forward-backward asymmetry as functions of c.o.m. energy with and without first-order mirror-generation-group corrections are shown in Figures 1 and 2 respectively. Energies from 2 to 100 TeV are considered. Such high energies have already been discussed in the context of the muon collider, even though they are clearly referring to experiments in the far future. Figure 2 makes clear why the NLC energies would make such a measurement very difficult because of the large $SU(3)_{2G}$ corrections, and why the muon collider would be a very good solution.

Unlike the case of PEP-PETRA experiments, $A_{FB}^0$ is not damped by the $Z^0$-boson mass and it reaches an almost constant value. At around 10 TeV, one finds $A_{FB}^0 \approx -0.47$. At such very high energies, the forward-backward asymmetry has still a very mild energy dependence due to the renormalization of $\sin^2\theta_W$. The $SU(3)_{2G}$-corrected result has a slightly stronger energy dependence due to the mirror-generation-group coupling renormalization. For energies on the order of 10 TeV, one finds

$$A_{FB} \approx -0.42,$$

i.e. the forward-backward asymmetry is reduced by roughly 11%.

Even though for larger energies the $SU(3)_{2G}$ corrections would very slowly become smaller, the cross-section decreases fast, leaving the phenomenological analysis with less statistics. With the planned muon-collider luminosities of around $10^{34} cm^{-2} s^{-1}$ [16], a 4-TeV machine would produce about $10^3$ events per year, while a hypothetical 100-TeV machine would need luminosities roughly 3 orders of magnitude larger in order to have comparable statistics.
By using mirror leptons, one avoids any QCD corrections to the theoretical predictions. Moreover, since the energies of interest here are much higher than the $Z^0$-peak $[14]$, electroweak corrections are ignored in this first approach to such processes. For the same reason, initial-lepton polarization is not essential for this measurement. The QED corrections in their turn could be substantially reduced by an angular acceptance cut like $|\cos \theta| < \Theta$, where $\theta$ is the scattering angle between the initial and final fermions. Taking $\Theta = 0.8$ as is frequently done would however reduce (independently of the QED corrections) the cross-section by $1 - \frac{4\Theta^2}{3\Theta^3} \approx 27\%$ and the forward-backward asymmetry values by $1 - \frac{4\Theta^2}{3\Theta^3} \approx 12\%$.

Such a cut would also reduce the forwardly-peaked $W^+W^-$ standard-model background, since the $W$ bosons coming from mirror-lepton weak decay are isotropically distributed. This is the most important background process, since the mirror leptons, assumed here to be heavier than the mirror neutrinos, decay predominantly to a mirror neutrino and a $W$ boson. The mirror neutrinos are expected to decay in their turn via the weak process $\nu^M_j \rightarrow W^\pm l^{\mp}$, with a decay rate roughly equal to

$$\Gamma \approx \frac{G_F m^3_j}{8\sqrt{2}\pi}|U_{lj}|^2$$

(29)

where the index $j$ runs over the mirror neutrinos and $U_{lj}$ is an element of the lepton CKM-type mixing matrix discussed in $[2]$.

The standard-model background could be further reduced by choosing pairs of $W$-bosons which are not co-linear and co-planar, since part of the momentum of the process is carried by the mirror neutrinos. This should be done in conjunction with a hard-photon cut, to ensure that such events do not come from higher-order
standard-model interactions. Further background reduction can be achieved by a visible-energy cut, since there is missing energy from the jets carried by the mirror neutrinos. All these cuts should produce a signal-to-background ratio large enough to facilitate the analysis which would verify or falsify the mirror model.

4 Conclusions

The phenomenological “excursion” of this work, by no means claiming to have spanned the whole spectrum of conceivable tests, has shown that several experimental consequences of the strongly-interacting mirror-fermion model can be tested in various present and future high-energy facilities. Indications for the existence of mirror fermions could already be coming from the values of \( \delta g_R^b \) and the electroweak precision \( S \) and \( T \) parameters. Future measurements of \( |V_{tb}| \) and \( \delta g_R^t \) at the Tevatron III and the NLC could provide further indirect evidence for the model. Large deviations of quantities related to \( b \to s\gamma \) or other B-meson processes from their standard-model predictions could also be made consistent with or constrain such an extension of the standard model.

Direct production of new fermions and scalar bound states at hadronic colliders would offer more substantial evidence for a new strongly-interacting sector, but it would be hard to distinguish unambiguously the resulting signals from other theories like technicolor. The ultimate confirmation of the mirror model would come from a future large linear collider with c.o.m. energies around 4-10 TeV, in much the same way that the LEP/SLC experiments confirmed the standard model in the 90’s. Such a high-energy collider would be able to probe the precise chiral structure of the new fermions via their forward-backward asymmetry \( A_{FB} \), assuming of
course that they can propagate freely and are not always confined in bound states.

In the latter case one would have to resort to other methods for determining
the chirality of the new fermions, not different in spirit perhaps from the methods
used to determine the chirality of the weakly-charged electrons and protons before
the forward-backward asymmetry of standard-model fermions was measured. In
the former case however, the muon collider would be a very good candidate for
such a high-energy facility, so it would be very encouraging if it finally proved
to be technologically feasible. The discussion presented implies anyway that the
processes studied in this work deserve further detailed study within the context of
specific particle-physics experiments.

Acknowledgements
I thank P. Gambino, N. Maekawa and L. Silvestrini for useful discussions. This
research is supported by an Alexander von Humboldt Fellowship.

References

[1] G. Triantaphyllou, Technical University Munich Preprint No. TUM-HEP-
326/98, September 1998, [hep-ph/9811250].

[2] G. Triantaphyllou, to appear in Eur. Phys. Jour. C, [hep-ph/9901346].

[3] S.M. Barr, D. Chang and G. Senjanovic, Phys. Rev. Lett. 67, 2765 (1991).

[4] J.H. Field, Phys. Rev. D 58, 093010 (1998); Université de Genève Preprint
No. UGVA-DPNC 1998/09-179, September 1998, [hep-ph/9809292]; ibid Octo-
ber 1998, [hep-ph/9810288]. A.K. Grant and T. Takeuchi, UCLA Preprint No.
[5] C.T. Hill et al., Phys. Rev. D 47, 2940 (1993).

[6] T. Appelquist and G. Triantaphylou, Phys. Lett. B 278, 345 (1992).

[7] R. Frey et al., FERMILAB-CONF-97-085, April 1997, hep-ph/9704243.

[8] For a review, see J. Maalampi and M. Roos, Phys. Rep. 186, 53 (1990).

[9] M. Ciuchini et al., Phys. Lett. B 316, 127 (1993); ibid, Nucl. Phys. B 421, 41 (1994); P. Cho and B. Grinstein, Nucl. Phys. B 365, 279 (1991); erratum ibid. 427, 697 (1994); G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[10] For a recent review, see A.J. Buras, Munich Technical University Preprint TUM-HEP-316/98, June 1998, hep-ph/9806471.

[11] K. Fujikawa and A. Yamada, Phys. Rev. D 49, 5890 (1994).

[12] T. Skwarnicki, talk given at ICHEP98, July 1998, Vancouver.

[13] R.S. Chivukula et al., Phys. Rev. D 59, 075003, 1999.

[14] T. Appelquist and G. Triantaphylou, Phys. Rev. Lett. 69, 2750 (1992).

[15] We give indicatively the references P. Baringer et al., Phys. Lett. B 206, 551 (88); F. Ould-Saada et al., Zeit. Phys.C 44, 567 (89); W. Braunschweig et al., Zeit. Phys.C 48, 433 (90).
[16] C.M. Ankenbrandt et al., Fermilab Preprint No. FERMILAB-Pub-98/179, January 1999.

[17] See for instance F. del Aguila et al., Nucl. Phys. B 297, 1 (1988).

[18] S. Catani and M.H. Seymour, CERN Preprint CERN-TH/99-132, May 1999, [hep-ph/9905424](https://arxiv.org/abs/hep-ph/9905424).

[19] U. Baur, S. Keller and W.K. Sakumoto, Phys. Rev. D 57, 199 (1998).
Figure Captions

Figure 1:

The scattering cross-section for the process $\mu^+\mu^- \rightarrow l^M + l^M$ as a function of the c.o.m. energy $E$. The dotted line shows the result after taking the strong mirror-generation-group effects into account.

Figure 2:

The forward-backward asymmetry of the mirror leptons. The dotted line shows the decreased asymmetry due to the strong mirror-generation-group effects. The dependence of this line with energy is due to the logarithmic running of the corresponding gauge coupling.