G/G Gauged Supergroup Valued WZNW Field Theory

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Abstract

The G/G gauged supergroup valued WZNW theory is considered. It is shown that for $G = OSP(1,2)$, the $G/G$ theory tensoring a $(b, c, \beta, \gamma)$ system is equivalent to the non-critical fermionic theory. The relation between integral or half integral modeled affine superalgebra and its reduced theory, the NS or R superconformal algebra, is discussed in detail. The physical state space, i.e. the BRST semi-infinite cohomology, is calculated, for the $OSP(1,2)/OSP(1,2)$ theory.
1 Introduction

In recent years, $2D$ gravity and matrix models have aroused much interest of the string theorists [1, 2, 3, 4, 5, 6, 7, 8]. The goal is to find (non-perturbative) solutions to the string theories in space-time dimensions other than the critical ones. A striking feature of the non-critical string theory is the appearance of infinitely many copies of the physical states with non-standard ghost numbers. More recently, it has been found that the combination of the matter sector, the Liouville sector and the reparametrization ghost gives a $2D$ topological field theory, which is equivalent to the $SL(2,R)/SL(2,R)$ gauged WZNW model [9, 10, 11]. The key observation is that by adding a new term, $\partial J^{Tot,3}$, to the total energy momentum tensor, $T^{Tot}(z)$, of the $SL(2,R)/SL(2,R)$ theory, one obtains the matter and Liouville part of the non-critical strings in the form of the Hamiltonian reduction of the WZNW model. The Liouville part is essentially the remainder of the gauge field. The conformal dimensions of the ghosts also get re-adjusted when twisted by $\partial J^{Tot,3}$ term.

There are some generalizations of the above construction. One way is to consider W-gravity coupled to W-matter, and naturally one would expect a $SL(N,R)/SL(N,R)$ model [12, 13] ($W_N$ string), or a general $G/G$ theory. Another way, which is the main subject of our present paper, is to look into the non-critical fermionic string and find out its corresponding topological field theory. With respect to the latter approach, much of recent work has been focused on the Hamiltonian reduction of the super-group valued WZNW theory which gives rise to a super-Toda and (extended) superconformal field theory [14, 15, 16, 17, 18, 19, 20, 21]. However, the physical state space of the non-critical fermionic string theory has not been worked out completely (see ref. [22] for a remark on this point). In this paper, we shall show that when gluing together the super-Liouville, super-conformal matter, and the super-reparametrization ghost, one obtains a topological super-conformal field theory, which is essentially a $G/G$ model with $G = OSP(1,2)$. The physical states of the $OSP(1,2)/OSP(1,2)$ theory, identified with the BRST non-trivial states, is believed to be in one to one correspondence with that of the non-critical fermionic string. It should be possible to apply our method to the more generally extended superconformal field theory [23, 24] coupled to the generalized supergravities.

The essential ingredients of the WZNW theory is encoded in its current algebra, the Kac-Moody algebra. It is also clear that it is the structure of the algebra modules that determine the physical states in $G/G$ WZNW theory. The general structure of the affine Kac-Moody
modules has been extensively studied [25, 26, 27, 28]. In our previous paper[29], some results concerning the affine Kac-Moody modules have been generalized to the case of affine Kac-Moody superalgebras. In this paper, we try to solve a remaining issue, namely, the BRST semi-infinite cohomology of the G/G WZNW theory with G in general a supergroup, which might be relevant to our understanding of the (gauged)supergroup valued WZNW theory, as well as the role of 2d supergravity.

It is known that under Hamiltonian reduction, the representation spaces of the $\hat{SL}(N)$ algebra (resp. $\hat{OSP}(N,2)$) are equivalent to that of the $W_N$ algebra (resp. $N$-extended superconformal field theory) by imposing constraints on the currents [30, 31]. It may be the essential reason why the structure of the Virasoro module is similar to that of the $\hat{SL}(2)$ module [32, 28]. From this point of view, we can understand the correspondence between BRST states of 2D gravity and that of the $SL(2)/SL(2)$ topological field theory, which has been formulated in [3, 11]. Naturally we might expect the similarity between $W_N$ strings and $SL(N)/SL(N)$ topological field theory, as well as between $N$-extended superconformal and $OSP(N,2)/OSP(N,2)$ topological field theory.

Our paper is organized as follows. Section 2 is about gauged supergroup valued WZNW field theory. In section 3, it is shown how $OSP(1,2)/OSP(1,2)$ theory is related to the non-critical fermionic strings. The relation between different modings of the superalgebra and their counterparts, the NS and R superconformal algebra is discussed in detail in section 4. Finally, in section 5 we formulate the BRST states. In conclusion, we speculate that the BRST semi-infinite cohomology of the non-critical fermionic strings can be obtained from that of $OSP(1,2)/OSP(1,2)$ by Hamiltonian reduction approach.

2 Gauged Supergroup Valued WZNW Model

The supergroup valued WZNW action can be written as [33]

$$L_k = -k/16\pi \int d^2x \text{str}\{g^{-1}\partial_\mu gg^{-1}\partial_\nu g^{\mu\nu}\} + k/24\pi \int d^3x \text{str}\{g^{-1}\partial_\mu gg^{-1}\partial_\nu g^{\mu\nu}\} \varepsilon^{\mu\nu\rho},$$

(1)

where $g(z,\bar{z}) = e^{i\alpha(z,\bar{z})^\alpha}$ is an element of a finite dimensional Lie supergroup $G$. $\tau^\alpha$ is the generator of the corresponding Lie superalgebra $\mathcal{G}$ [34, 35, 24], which consists of the even part
$G_0$, and the odd part $G_1$. $G_0$ is by itself a Lie algebra. We shall restrict our discussion to the case that $G$ is semi-simple (for definition, see for example ref.\[36, 29\]). The (anti-)commutators satisfies

$$[\tau^\alpha, \tau^\beta] = (-1)^{d(\alpha)d(\beta)+1}[\tau^\beta, \tau^\alpha] = f^{\alpha\beta\gamma} \tau^\gamma,$$

(2)

where $f^{\alpha\beta\gamma}$ is the structure constant of $G$; $d(\alpha) = 0$ (1 resp.), when $\tau^\alpha \in G_0$ ($G_1$, resp.). What needed to point out is that $\epsilon^\alpha$ is a complex (Grassmannian resp.) number, when $d(\alpha) = 0$ (1, resp.), so that the group elements commute cyclicly among themselves inside the supertrace (see ref.\[34, 29\]),

$$\text{strgh} = \text{strhg}, \quad g, h \in G.$$

(3)

The left and right conserved currents are defined as

$$J = -k/2\partial gg^{-1}; \quad \bar{J} = -k/2g^{-1}\bar{\partial}g,$$

(4)

which satisfy, through the equation of motion,

$$\bar{\partial}J = \partial \bar{J} = 0.$$

(5)

Define

$$J^\beta = \text{str}\{\tau^\beta J\}, \quad \bar{J}^\beta = -\text{str}\{\tau^\beta \bar{J}\},$$

(6)

where the difference between the definitions of $J^\alpha$ and of $\bar{J}^\alpha$ by a minus sign “−” comes from the classical Poisson brackets (see ref.\[14\]). This also can be seen from the time reversal symmetry of WZNW theory, $z \leftrightarrow \bar{z}$, $g \leftrightarrow g^{-1}$, $J \leftrightarrow -\bar{J}$. Then we get \[33\]

$$J^\alpha(z_1)J^\beta(z_2) = \frac{f^{\alpha\beta\gamma}J^\gamma(z_2)}{z_{12}} + \frac{k\text{ str}\{\tau^\alpha\tau^\beta\}}{z_{12}},$$

$$[J_n^\alpha, J_m^\beta] = f^{\alpha\beta\gamma}J_n^\gamma + kn\text{ str}\{\tau^\alpha\tau^\beta\}\delta_{n+m,0}.$$

(7)

The supertrace is normalized in such a way that when $k$ is a positive integer, the current algebra eq.(7) has integral representation \[37, 35\]. For $G = SU(N)$, $OSP(1, 2)$ (for definition, see ref.\[36, 38, 29\]), it is the ordinary (super-)trace in the fundamental representation. The same relation holds for the antiholomorphic part.
Using the Polyakov-Wiegmann formula [39, 40],
\[ L_k(gh) = L_k(g) + L_k(h) - k/2\pi \int d^2z \text{str}(g^{-1}\partial_- g \partial_+ hh^{-1}), \]  
(8)
the gauged WZNW action [41, 42] can be written as
\[ L_k(g, A, \bar{A}) = L(g) + k/2\pi \int d^2z \text{str}(\bar{A} \partial_+ gg^{-1} - g^{-1} \partial_- g A + \bar{A} g A^{-1} - \bar{A} A) = L_k(h^{-1}g\tilde{h}) - L_k(h^{-1}\tilde{h}), \]  
(9)
where
\[ A = \partial_+ \tilde{h}h^{-1}, \quad \bar{A} = \partial_- hh^{-1}, \quad \tilde{h}, h \in H. \]  
(10)
Here \( A, \bar{A} \) are gauge fields, taking values in a subalgebra \( \mathcal{H} \) of \( G \). \( L(g, A, \bar{A}) \) is invariant under the gauge transformation
\[ A \rightarrow \lambda A\lambda^{-1} + \lambda \partial \lambda^{-1}, \quad \bar{A} \rightarrow \lambda \bar{A}\lambda^{-1} + \lambda \bar{\partial} \lambda^{-1}, \quad g \rightarrow \lambda g\lambda^{-1}, \]  
(11)
\[ \lambda \in H, \]
Following Polyakov and Wiegmann [40], changing the variables from \( A, \bar{A} \) to \( \tilde{h}, h \), we arrive at the partition function
\[ Z = \int [dg dh \tilde{h} db dc] e^{-L_k(h^{-1}g\tilde{h}) + L_{k+2\tilde{h}}(h^{-1}\tilde{h}) - L_{gh}(b,c)} \]  
(12)
Fixing the gauge at \( h = 1 \), we have
\[ Z = \int [dg d\tilde{h} db dc] e^{-L_k(g\tilde{h}) + L_{k+2\tilde{h}}(\tilde{h}) - L_{gh}(b,c)} \]  
(13)
where \( \tilde{h}_H \) is the dual Coexter number of \( H \), which is given in eq.(20) (We assume the length of the longest root of \( \mathcal{H} \) equals that of \( G \)), and
\[ L_{gh} = \text{str}(b\bar{\partial}c - \bar{b}\partial\bar{c}), \]  
(14)
is the action for the \((b, c)\) ghosts of spin 1 and 0 resp. When \( d(\alpha) = 0 \ (1, \text{resp.}) \), \((b_\alpha, c^\alpha)\) are fermionic (bosonic, resp.) spin \((1, 0)\) ghosts, and satisfy the same boundary conditions as the currents \( J^\alpha \).
\[ b = \tau^\alpha b_\alpha, \quad c = \tau^\alpha c^\beta h_{\alpha\beta}, \quad \langle b_\alpha(z_1)c^\beta(z_2) \rangle = \frac{\delta^\beta_{\bar{\beta}}}{z_{12}^2}, \]  
(15)
where $h_{\alpha\beta}$ is the inverse metric of $h^{\alpha\beta}$ \[\text{[38, 29]}\]

\[h^{\alpha\beta} = f^{\alpha\gamma}_{\rho} f^{\beta\rho}_{\gamma} (-1)^{d(\rho)}.\] (16)

Sometimes we use notations

\[c_\alpha = h_{\alpha\beta} c^\beta, \quad b_\alpha = b_\beta h^{\beta\alpha}.\] (17)

Of particular interest in our paper is the case $H = G$, which leads to the $G/G$ model. In that case there are three sectors of affine Kac-Moody superalgebra of different levels:

\[J^\alpha(z)\] with level $k$,
\[\tilde{J}^\alpha(z)\] with level $-k - 2\tilde{h}_G$,
\[J^\alpha_{gh}(z)\] with level $2\tilde{h}_G$,

where

\[J^\alpha_{gh} = f^{\alpha\beta}_{\gamma} : c^\gamma b_\beta :.\] (19)

For a Lie (super-)algebra, $\tilde{h}_G$ is the dual Coexter number, and for $G = OSP(1, 2)$, $\tilde{h}_G = 3/2$. If $G$ is a finite dimensional contragradient Lie superalgebra associated with a Cartan matrix \[\text{[35, 29]}\], it can be given by

\[\tilde{h}_G = \frac{\sum_{\alpha \in \Delta_0} |\alpha|^2 - \sum_{\alpha \in \Delta_1} |\alpha|^2}{r_G r_i^2},\] (20)

where $\Delta_i$ is the set of roots corresponding to $G_i$ \[\text{[35, 29]}\], and $r_i$ is length of the highest root, which is always in $\Delta_0$; $r_G$ is the rank of $G$.

Again there remains a twisted $N = 2$ superconformal symmetry in this model, which is the generalized form of that in ref. \[\text{[10]}\].

\[G = (J^\alpha + \tilde{J}^\alpha + \frac{1}{2} J^\alpha_{gh}) c^\beta h_{\alpha\beta}, \quad \bar{G} = \frac{\tilde{h}_G (J^\alpha - \tilde{J}^\alpha) b_\alpha (-1)^{d(\alpha)}}{k + \tilde{h}_G}.\]

\[T = \frac{\tilde{h}_G : J^\alpha J^\beta : h_{\alpha\beta}}{k + \tilde{h}_G} - \frac{\tilde{h}_G : \tilde{J}^\alpha \tilde{J}^\beta : h_{\alpha\beta}}{k + \tilde{h}_G} + : \partial c^\alpha b_\alpha :,\] (21)

\[J^{u(1)} = c^\alpha b_\alpha.\]

The central charges for the stress-energy tensors are \[\text{[29]}\]

\[c = \frac{k \text{sdim}(G)}{k + \tilde{h}_G}, \quad \bar{c} = \frac{(k + 2\tilde{h}_G) \text{sdim}(G)}{k + \tilde{h}_G}, \quad c_{gh} = -2 \text{sdim}(G),\] (22)
where $sdim(\mathcal{G}) = dim(\mathcal{G}_0) - dim(\mathcal{G}_1)$. It is easy to see that the total central charge vanishes. The OPEs between the supersymmetry generators are

\[ G(z_1)G(z_2) = 0, \ G(z_1)\bar{G}(z_2) = 0, \]

\[ G(z_1)\bar{G}(z_2) = \frac{T_{tot}^{(z_2)}}{z_{12}^2} + \frac{J^{(1)}}{z_{12}^2} + \frac{sdim(\mathcal{G})}{z_{12}^2}, \quad (23) \]

while the other super-commutators are the same as in $N = 2$ superconformal field theory. The main purpose of the present paper is to calculate the physical state space of this model. As in the standard way, we can reach that by the BRST approach. Using the above formula it is easy to prove that

\[ Q_{BRST} = \oint G \]

satisfies $Q_{BRST}^2 = 0$. And $T(z)$, $J^{tot,\alpha}$ are total $Q_{BRST}$ (anti)commutators,

\[ T(z) = \{Q_{BRST}, G(z)\}, \quad J^{tot,\alpha}(z) = \{Q_{BRST}, b^{\alpha}(z)\}, \quad (25) \]

We will compute the semi-infinite cohomology in the special case of $\mathcal{G} = OSP(1, 2)$ in section 5.

3 Noncritical Fermionic Strings and $OSP(1, 2)/OSP(1, 2)$ FT

The equivalence between 2D gravity and $SL(2)/SL(2)$ topological field theory has been worked out by the authors of ref.[10, 11, 9]. There is a one to one correspondence between the physical states in the two theories when restricting the matter sector to be in the minimal series.

The relation between 2D supergravity and $\widehat{OSP}(1, 2)$ current algebra has been studied early in ref.[43] (see also [30]). It is shown that the correlation functions of super-zweibeins possess an $\widehat{OSP}(1, 2)$ symmetry. In fact the discussion in ref.[10] on the equivalence of string theory and $SL(2)/SL(2)$ field theory can be taken over to the super case. The super conformal action can be obtained from the $OSP(1, 2)$ WZNW theory by imposing constraints on the currents [30].

3.1 Non-critical Fermionic String

The action for the fermionic string is [15, 16]

\[ S = \frac{1}{2} \int [\sqrt{h}h^{\mu\nu}\partial_\mu X \cdot \partial_\nu X + \bar{\psi} \cdot \partial X + \bar{\psi} \gamma^\mu (\partial X + \frac{1}{2} \chi \gamma^\mu \psi) \cdot \psi] d^2\xi. \quad (26) \]
Here, $X^i$ and $\psi^i$ are the world sheet scalar and Majorana fermion resp., $\chi_\mu$, the gravitino, is a world sheet Rarita-Schwinger vector-spinor, $\gamma^\mu$ the Dirac matrix in two dimensions. Exploiting the gauge invariance of the action under the reparametrization and the local supersymmetry transformation, we can reach the following “superconformal gauge”,

$$ h_{\mu\nu} = e^\varphi \delta_{\mu\nu}, \quad \chi_\mu = \gamma_\mu \chi. \quad (27) $$

In fixing the gauge, there is a Fadeev-Popov determinant involved to match the gauge volume. This determinant is calculated in ref.\[45, 46\] and is shown to be,

$$ \text{det}(L^{-1} F L) = \exp\left(\frac{10}{32\pi} S(\varphi, \chi) \int [dbd\hat{c}d\hat{d} \hat{\gamma}] \exp\{-S_{gh}^{[2,-1,3/2,-1/2]}(\hat{b}, \hat{c}, \hat{\beta}, \hat{\gamma})\}\right). \quad (28) $$

Here, $(\hat{b}, \hat{c})$ are fermionic ghosts of spin $(2, -1)$ and $(\hat{\beta}, \hat{\gamma})$ bosonic ghosts of spin $(3/2, -1/2)$,

$$ S_{gh}^{[j,1-j,i,1-i]}(b,c,\beta,\gamma) = S_f^{(j,1-j)}(b, c) + S_{\bar{b}}^{(i,1-i)}(\beta, \gamma), $$

$$ S_f^{(j,1-j)}(b, c) = \int (b\bar{c} + h.c.) $$

$$ S_{\bar{b}}^{(i,1-i)}(\beta, \gamma) = \int (\beta\bar{\gamma} + h.c.), \quad (29) $$

where, $S_f^{(j,1-j)}(b, c)$ $(S_{\bar{b}}^{(i,1-i)}(\beta, \gamma))$ is the usual action for the fermionic $(b, c)$ (bosonic $(\beta, \gamma)$) ghost of spin $(j, 1-j)$ \[47\].

The form of the super-Liouville action $S(\varphi, \chi)$, which is the supersymmetrized form of the Liouville action, is completely determined by the trace anomaly,

$$ S(\varphi, \chi) = \int \frac{1}{2} (\partial \varphi)^2 + \frac{1}{2} \bar{\theta} \chi + \frac{1}{2} (\bar{\chi} \gamma_5 \chi) e^{\varphi/2} + \mu^2 e^{\varphi}. \quad (30) $$

In terms of the super-field,

$$ \phi = \varphi + \theta \bar{\chi} + \bar{\theta} \chi + \theta \bar{\theta} f, \quad (31) $$

we shall consider the following manifestly supersymmetrically invariant action,

$$ S(\phi) = \int d^2 \xi d\theta d\bar{\theta} (\frac{1}{2} D\phi D\phi + 2\mu e^{\phi/2}), \quad (32) $$

where, $D = \partial_\theta + \theta \partial_\xi$, $\bar{D} = \partial_{\bar{\theta}} + \bar{\theta} \partial_{\bar{\xi}}$. Integrating out the odd coordinate $\theta, \bar{\theta}$, and substituting in eq.\[32\] the minimal value of the field $f$, we recover the action $S(\varphi, \chi)$ in terms of the component fields $\phi$ and $\chi$. The equation of motion for $S(\phi)$ reads,

$$ D\bar{D}\phi + \mu e^{\phi/2} = 0. \quad (33) $$

7
3.2 The Hamiltonian Reduction

Classically, eq. (33) is the constrained form of the equation of motion for the \( \text{OSP}(1,2) \) WZNW theory \[30, 14, 21\] (See eqs. (61) for the super-commutators of \( \text{OSP}(1,2) \)). The constraints are

\[
J^+ = -\bar{J}^- = 1, \quad j^+ = 0, \quad \bar{j}^- = 0. \tag{34}
\]

The constraints eq. (34) are of second class. To make a consistent quantum field theory, we shall adopt the method used by the authors in ref. [31]. We first consider the Hamiltonian reduction of the \( \text{OSP}(1,2) \) superalgebra. Upon the constraints

\[
J^+ = 1, \quad j^+ = 0, \tag{35}
\]

an irreducible representation of \( \text{OSP}(1,2) \) reduces to that of super-Virasoro algebra. By introducing a free Majorana fermion \( \psi \), the constraints eq. (35) are equivalent to the following constraints on the enlarged space \( V_{\text{OSP}(1,2)} \otimes V_{\psi} \),

\[
J^+ = -\bar{J} = 1, \quad j^+ = \sqrt{2}\psi, \quad \bar{j}^- = \sqrt{2}\bar{\psi}. \tag{36}
\]

To make the constraints eq. (36) sensible, i.e. the currents \( J^+, j^+ \) have the correct conformal dimensions, the stress-energy tensor is modified,

\[
T(z) \rightarrow T_{\text{impr}}(z) = T(z) + \partial J^3(z). \tag{37}
\]

The reduced space can be got in the standard BRST formalism, in which one should introduce fermionic ghosts \( (b, c) \) and bosonic ghosts \( (\beta, \gamma) \) with conformal isospin \( (1, 0) \) and \( (1/2, 1/2) \) resp.. The BRST operator is

\[
Q_{\text{BRST}} = \oint dz (b(J^+ - 1) + \gamma(j^+-\sqrt{2}\psi) - c\gamma^2), \tag{38}
\]

where the ghosts \( (\beta, \gamma) \) as well as Majorana fermion \( \psi \) satisfies the same boundary condition as that of \( j^\pm(z) \) on the complex plane. The total stress-energy tensor is

\[
T^{\text{tot}} = T_{\text{impr}} - \psi \partial \psi / 2 - c \partial b + (\beta \partial \gamma - \partial \beta \gamma) / 2, \tag{39}
\]

which has central charge

\[
c^{\text{tot}} = c_{\text{impr}}^{\text{OSP}(1,2)} + c^bc + c^\beta\gamma + c^\psi \\
= \frac{k}{k+3/2} - 6k - 2 - 1 + 1/2 \\
= \frac{3}{2}(1 - \frac{8(k+1)^2}{2k+3}). \tag{40}
\]
3.3 Constrained WZNW Theory

The quantum Hamiltonian reduction considered in the last subsection is equivalent to the following constrained WZNW theory,

\[ S(g, A, \psi) = S_{WZNW}(g) - \frac{1}{\pi} \int d^2 z [\bar{A}_+ (J^+ - 1) + \bar{A}_- (J^- - 1) + \bar{A} - \frac{1}{2} (\bar{j}^+ - \sqrt{2} \bar{\psi}) + \bar{A} - \frac{1}{2} (\bar{j}^- - \sqrt{2} \bar{\psi}) + \text{str}(\bar{A}gAg^{-1}) + \psi \bar{\psi}/2 + \bar{\psi} \bar{\psi}/2], \]  

(41)

where the gauge fields \( A, \bar{A} \), which are effectively the Lagrange multipliers, take values in the Borel subalgebra,

\[ A = A_{-} \frac{1}{2} \tau^{-\frac{1}{2}} + A_{-} \tau^{-}, \]

\[ \bar{A} = \bar{A}_{+} \frac{1}{2} \tau^{\frac{1}{2}} + \bar{A}_{+} \tau^{+}. \]  

(42)

\( S(g, A, \psi) \) is invariant under the following gauge transformations,

\[ \begin{align*}
  g & \to \lambda g \lambda^{-1}, \\
  \bar{A} & \to \lambda \bar{A} \lambda^{-1} + \bar{\partial} \lambda \lambda^{-1} \\
  A & \to \bar{\lambda} A \lambda^{-1} + \partial \bar{\lambda} \lambda^{-1} \\
  \psi & \to \psi + \sqrt{2} \epsilon^{\frac{1}{2}} \\
  \bar{\psi} & \to \bar{\psi} + \sqrt{2} \epsilon_{-\frac{1}{2}} \\
  \lambda & = \exp\{\epsilon^{\frac{1}{2}} \tau^{\frac{1}{2}} + \epsilon^{+} \tau^{+}\} \\
  \bar{\lambda} & = \exp\{\epsilon_{-\frac{1}{2}} \tau^{-\frac{1}{2}} + \epsilon^{-} \tau^{-}\}.
\end{align*} \]  

(43)

Because of the gauge invariance of the action \( S(g, A, \psi) \), we have to fix the gauge in order that the path integral make sense. For convenience, we choose our gauge condition to be,

\[ \bar{A} = A = 0 \]  

(44)

The change of measure \([dA d\bar{A}]\) is compensated by the introduction of the Fadeev-Popov ghost, \((b, c)\) being fermionic of spin \((1, 0)\) and \((\beta, \gamma)\) being bosonic of spin \((1/2, 1/2)\). So the gauge fixed path integral for the quantized super-Liouville theory is

\[ \int [dg][dbd\gamma][d\beta d\gamma][d\psi] \exp\{-S_{\text{twisted}}(g) - S_{gh}^{(1,0,1/2,1/2)}(b, c, \beta, \gamma) - S(\psi)\}, \]  

(45)
where, the superscript “twisted” means that the energy momentum tensor for the constrained WZNW model is improved as

\[
T(z) = T^{\text{Sugawara}}(z) + \partial J^3(z),
\]

\[
\bar{T}(\bar{z}) = \bar{T}^{\text{Sugawara}}(\bar{z}) + \bar{\partial} \bar{J}^\bar{3}(\bar{z}).
\] (46)

Combining the superconformal matter, super-Liouville and super-reparametrization ghost together, we arrive at the following path integral formalism of the non-critical string,

\[
Z = \int [dg][dbdc][d\beta d\gamma][d\psi] e^{-kS^{\text{twisted}}_{\text{WZNW}}(g) - S^{(1,0,1/2,1/2)}_{\text{gh}}(b,c,\beta,\gamma) - S(\psi)}
\]

\[
\int [d\bar{g}][d\bar{b}\bar{c}][d\bar{\beta} d\bar{\gamma}][d\bar{\psi}] e^{-\tilde{k}S^{\text{twisted}}_{\text{WZNW}}(\bar{g}) - S^{(1,0,1/2,1/2)}_{\text{gh}}(\bar{b},\bar{c},\bar{\beta},\bar{\gamma}) - S(\bar{\psi})}
\]

\[
\int [dbd\bar{c}][d\hat{\beta} d\hat{\gamma}] e^{-\hat{c}^{(2,-1/2,-1/2)}_{\text{gh}}(\hat{b},\hat{c},\hat{\beta},\hat{\gamma})}.
\] (47)

The total conformal anomaly of the theory should vanish,

\[
c^{\text{tot}} = k + 3/2 - 6k - 3 + 1/2 + \frac{\tilde{k}}{k + 3/2} - 6\tilde{k} - 3 + 1/2 - 15 = 0,
\] (48)

which leads to the consistency condition (see ref. [10] for a discussion),

\[
\tilde{k} = -k - 3
\] (49)

It is worth mentioning that we can reorganize the various ghosts appearing in the path integral, eq.(17), into the OSP(1, 2) multiplets and singlets. Recall in eq.(16) a level \( k = 3 \) OSP(1, 2) Kac-moody current algebra is defined in terms of the fermionic ghost \((b_a, c^\alpha)\) of spin \((1, 0)\) and the bosonic ghost \((\beta_\alpha, \gamma^\alpha)\) of spin \((1, 0)\), such that we have the following \(sl(2)\) isospin assignment for the ghosts \(b_a, c^\alpha, \beta_\alpha, \gamma^\alpha\),

\[
\begin{align*}
\text{ghosts} : & \quad b_-, c^+ \quad b_3, c^3 \quad b_+, c^- \\
J^3 : & \quad 1 \quad 0 \quad -1 \quad 1/2 \quad -1/2
\end{align*}
\]

Let us see what happens if the ghost Kac-Moody algebra is also twisted by \(J^{gh,3}\), i.e. the energy momentum tensor for the ghost is improved as follows,

\[
T^{gh}(z) = \partial c^\alpha b_a + \partial \gamma^\alpha \beta_\alpha + \partial J^{3,gh}
\]

\[
\bar{T}^{gh}(\bar{z}) = \bar{\partial} c^\alpha b_a + \bar{\partial} \gamma^\alpha \bar{\beta}_\alpha - \bar{\partial} J^{3,gh}
\] (50)
In such a case, the conformal spins of the ghosts will get modified as follows,

\[\text{ghosts : } (b_-, c^-) (b_3, c^2) (b_+, c^+) (\beta_{-\frac{1}{2}}, \gamma_-^{\frac{1}{2}}) (\beta_{+\frac{1}{2}}, \gamma_+^{\frac{1}{2}})\]

\[J^3 : (1, -1) (0, 0) (-1, 1) (1/2, -1/2) (-1/2, 1/2)\]

spins before twist \(\Delta: (1, 0) (1, 0) (1, 0) (1, 0) (1, 0)\)

spins after twist \(\Delta - J^3: (0, 1) (1, 0) (2, -1) (1/2, 1/2) (3/2, -1/2)\)

After the twist the action for the \(\text{OSP}(1, 2)\) ghosts can be written,

\[S_{\text{twisted}}(b_\alpha, c^\alpha) = S_{gh}^{(2,-1,3/2,-1/2)}(b_+, c^+, \beta_{1/2}, \gamma_1^{1/2}) + S_{gh}^{(1,0,1/2,1/2)}(b_3, c^3, \beta_{1/2}, \gamma_{-1/2}) + S_f^{(1,0)}(c^-, b_-).\]

Comparing eq.(51) and eq.(13), we arrive at the following conclusion,

\[\langle | \rangle_{\text{string}} = \langle | \rangle_{\text{twisted}}_{\text{OS}P(1,2)/OS(1,2)} \int [d\psi^+ d\psi^-] [d\beta d\gamma] e^{\frac{1}{2}(\psi^+ \partial \gamma - \partial \psi^+ \gamma)} S_{gh}^{(1/2, 1/2, 1/2)}(\psi^+, \psi^-, \beta, \gamma)\]

\[\psi^+ = \psi + i\tilde{\psi}, \quad \psi^- = \psi - i\tilde{\psi},\]

which is the same as to say that the noncritical fermionic string is equivalent to the twisted \(\text{OS}P(1, 2)/\text{OS}(1, 2)\) gauged WZNW model tensoring a topological field theory of the spin-(1/2) \((\psi^+, \psi^-, \beta, \gamma)\) system.

It is well known that for the gauge fixed action there exists a BRST symmetry. In our case, we come across a 2d topological conformal field theory, which means \[\text{eq}(13)\] that there is a twisted \(N=2\) superconformal algebra. The BRST charge is just one of the \(N=2\) supersymmetry charge. The \(N=2\) superconformal algebra for the \(\text{OS}P(1, 2)/\text{OS}(1, 2) \otimes (\psi^+, \psi^-, \beta, \gamma)\) theory is constructed as follows,

\[G = (J^\alpha + \tilde{J}^\alpha + \frac{1}{2} J_{gh}^\alpha) c^{\alpha} \delta_{\alpha \beta} + \psi^- \beta, \quad \bar{G} = \frac{\tilde{h}_\alpha (J^\alpha - \tilde{J}^\alpha) (\delta_{\alpha})_{-1} d^{(\alpha)}}{k+h}, \]

\[T = \frac{h_\alpha (J^\alpha - \tilde{J}^\alpha) \delta_{\alpha \beta}}{k+h}, \quad \bar{T} = \frac{\tilde{h}_\alpha (J^\alpha - \tilde{J}^\alpha) \delta_{\alpha \beta}}{k+h},\]

\[T^{gh} = \partial c^{\alpha} b_\alpha, \quad J^{(1)} = c^\beta b_\alpha + \psi^+ \tilde{\psi}^+ - 2\beta \gamma / 2,\]

\[T^{(\psi^+, \psi^-, \beta, \gamma)} = \frac{1}{2}(\partial \psi^+ \psi^- - \psi^+ \partial \psi^-) + \frac{1}{2}(\beta \partial \gamma - \partial \beta \gamma),\]

\[T^{tot} = T + \bar{T} + T^{gh} + T^{(\psi^+, \psi^-, \beta, \gamma)}.\]

The central charge for the total stress-energy tensor still vanishes.

Using the above formula it is easy to prove that

\[Q_{BRST} = \oint G\]
satisfies \( Q_{BRST}^2 = 0 \). And \( T(z), J^{tot,\alpha} \) are total \( Q_{BRST} \) (anti)commutators,

\[
T(z) = \{ Q_{BRST}, G(z) \}, \quad J^{tot,\alpha}(z) = \{ Q_{BRST}, b^\alpha(z) \}.
\] (55)

The total BRST charge can be rewritten as

\[
Q_{BRST} = Q_{BRST}^{OSP(1,2)} + Q_2,
\] (56)

where \( Q_{BRST}^{OSP(1,2)} \) is the BRST charge for the gauged \( OSP(1,2)/OSP(1,2) \) current as given in eq.(24), and \( Q_2 \) is the BRST charge for the \((\psi^+, \psi^-, \beta, \gamma)\) system,

\[
Q_2 = \oint \psi^- \beta
\] (57)

The BRST state space of \( Q_2 \) consists of only one cohomology class which is represented by the vacuum of the \((\psi^+, \psi^-, \beta, \gamma)\) system. From

\[
T_{\psi^+, \psi^-, \beta, \gamma} = \{ Q_2, (\psi^+ \partial \gamma - \partial \psi^+ \gamma)/2 \},
\] (58)

we come to the fact that the nontrivial \( Q_2 \) state must be of zero mode excitation. If the fields \( \psi^+, \psi^-, \beta, \gamma \) are periodic on the plane, there is no zero mode generators. However if they are antiperiodic, from

\[
n_{\psi^0}^- + n_{\gamma_0} = \psi_0^- \psi_0^+ + \gamma_0 \beta_0 = \{ Q_2, \psi_0^+ \gamma_0 \},
\] (59)

it can be seen that nontrivial \( Q_2 \) states must satisfy \( n_{\psi^0}^- + n_{\gamma_0} = 0 \), which leads to the choice of two vacuum states which are not connected by a finite number of zero mode actions. In both cases the only BRST state is the vacuum state. Now it is obvious that the total BRST state space is the direct product of that of the \( Q_{BRST}^{OSP(1,2)} \) and that of \( Q_2 \), i.e.

\[
H(V^{OSP(1,2)} \otimes (\psi^+, \psi^-, \beta, \gamma), Q_{BRST}) \cong H(V^{OSP(1,2)}, Q_{BRST}^{OSP(1,2)}) \otimes |vac\rangle_{\psi^+, \psi^-, \beta, \gamma}.
\] (60)

We will compute the semi-infinite cohomology \( H(V^{OSP(1,2)}, Q_{BRST}^{OSP(1,2)}) \) in section 4 after reviewing some results obtained in our previous paper about \( OSP(1,2) \).

4 R type and NS type \( OSP(1,2) \)

In (extended) superconformal field theories, there are different modings for the supersymmetry generators, the so called Neveu-Schwarz (half integral moding) or Ramond (integral moding)
sectors. Since the (extended) superconformal algebra can be considered as the reduced theory of the corresponding supergroup valued WZNW model, it deserves a special attention to consider how the different sectors in the formal case are related to the different types of the superalgebra in the later one.

In this section, we shall restrict ourselves to the case of $N=1$ superconformal algebra and the $\widehat{OSP}(1,2)$ Kac-Moody algebra, although our consideration is generalizable to the other cases.

The superalgebra $\widehat{OSP}(1,2)$ consists of the $\widehat{SL}(2)$ and the fermionic part $\{j^\pm\}$\cite{29}. The nonvanishing (anti-)commutators are

\[
\begin{align*}
\{j^+_r, j^-_s\} &= 2j^3_{r+s} + 2rk\delta_{r+s,0}; \\
\{j^\pm, j^\pm\} &= \pm 2j^\pm; \\
[j^3, j^\pm] &= \pm \frac{1}{2}j^\pm; \\
[j^\pm, j^\mp] &= -j^\pm; \\
[J^3_n, j^\pm_m] &= 2j^3_{n+m} + nk\delta_{n+m,0}; \\
[J^\pm_n, J^\pm_m] &= \pm j^\pm; \\
[d, J^n_0] &= -nJ^n_0.
\end{align*}
\]

The stress-energy tensor by Sugawara construction is

\[
T(z) = \frac{J^3J^3 + J^+ J^-/2 + J^- J^+/2 - j^+ j^-/4 + j^- j^+/4}{k + 3/2},
\]

with central charge

\[
e = \frac{k}{k + 3/2}.
\]

Two types of $\widehat{OSP}(1,2)$ algebra, which we call Ramond type and Neveu-Schwarz type $\widehat{OSP}(1,2)$ algebra resp (or in terms of ref.\cite{30}, the untwisted and twisted $\widehat{OSP}(1,2)$ current algebra, resp.), were studied in ref.\cite{29}. The fermionic generators of $\widehat{OSP}(1,2)$, i.e. $j^\pm_r$ are of integer (half integer) modes for the R (NS) type $\widehat{OSP}(1,2)$. It was shown there that, although the NS and R super Virasoro algebra are genuinely different, however, the two types of $\widehat{OSP}(1,2)$ are essentially identical via an isomorphic map, so are the representations of the two types of $\widehat{OSP}(1,2)$ superalgebra. This isomorphism is the main part of this section. The isomorphism on the superalgebras are

\[
\begin{align*}
j^n_0 \to \pm j^n_{\pm 1/2}, \\
J^3_n \to -J^3_{n\pm 1/2}, \\
J^\pm_n \to -J^\pm_{n\pm 1}, \\
J^3_n \to -J^3_n + k/2\delta_{n,0}, \\
k \to k, \\
d \to d + J^3_0.
\end{align*}
\]
For convenience we add a superscript R or NS on the generators, the modules, and the corresponding weights when we are concerned about the R or NS type $\hat{OSP}(1,2)$. From the above isomorphism eq. (64), we have

$$j^R = k/2 - j^{NS},$$

which gives the relation between the isospins of a certain state in the representations of R and NS type $\hat{OSP}(1,2)$.

However, the R and NS type $\hat{OSP}(1,2)$ current algebra, which correspond to the periodic and antiperiodic boundary conditions for the fermionic currents resp., have different behaviour on the complex plane. Physically the Virasoro algebra eq. (62) by Sugawara construction are different,

$$L^R_n = L^{NS}_n - J^{3,NS}_n + k/4\delta_{n,0}.$$  

When the Hilbert space is concerned, a representation of the $\hat{OSP}(1,2)$ algebra corresponds to two family of states, one in R sector, another in NS sector. At least at the level of current algebra, we can say that the conformal blocks are degenerated.

On the torus, there are two homotopically nontrivial cycles, the $a$ cycle and $b$ cycle resp.. Correspondingly, there are four different boundary conditions for the fermionic generators. The boundary conditions along the $\sigma$ direction are specified by the moding of the generators, while along $\tau$ direction, the boundary condition is reflected by defining the characters or the super-characters. For more detailed discussion, see ref. [29].

The structures of the Verma modules and Wakimoto modules over $\hat{OSP}(1,2)$ are very much similar to those over Virasoro and over $\hat{SL}(2)$. The modules can also be classified into cases I, II, III. Of special interest is the admissible representations (in case III), where

$$2k + 3 = \tilde{q}/q, \quad q, \tilde{q} \in \mathbb{N}, \quad q + \tilde{q} \in \text{even}, \quad gcd(q, \frac{q + \tilde{q}}{2}) = 1;$$

$$4j^R_{m,s} + 1 = m - s\frac{\tilde{q}}{q}, \quad m = 1, \ldots, \tilde{q} - 1, \quad s = 0, \ldots, q - 1, \quad m + s \in \text{odd}. \quad (67)$$

Notice that when expressed in the NS type $\hat{OSP}(1,2)$, the isospin in eq. (67) can be rewritten as, by using eq. (65),

$$4j^{NS} + 1 = (\tilde{q} - 1 - m) - (q - 1 - s)\frac{\tilde{q}}{q}. \quad (68)$$
The corresponding Verma module $M(j_{m,s}^R, k)$ has singular vector with isospin $j_{2nq\pm m,s}$. The Wakimoto module $W(j_{m,s}^R, k)$, which is realized in free fields \(^{10}^{11}^{29}\), has singular vectors with isospin $j_{2nq-m,s}, \; n > 0$, cosingular vectors with $j_{2nq-m,s}, \; n < 0$, and when modulo the submodules generated by the singular vectors, there appear singular vectors with $j_{2nq+m,s}$ in $W(j_{m,s}^R, k)$.

The free field realization of $\widehat{OSP}(1,2)$ \(^{30}^{29}\) with the Fock space $F^+$ being $W(j, k)$ is

\[
\begin{align*}
J^+ &= -\beta, \\
\check{J}^+ &= \psi^+ - \beta \psi, \\
J^3 &= -\beta \gamma + i \alpha_+ \partial \phi / 2 - \frac{1}{2} \psi \psi^+ - \epsilon / 4z, \\
\check{J}^3 &= \gamma (\psi^+ - \beta \psi) + i \alpha_+ \psi \partial \phi + (2k + 1) \partial \psi, \\
J^- &= \beta \gamma^2 - i \alpha_+ \gamma \partial \phi + \gamma \psi \psi^+ - k \partial \gamma + (k + 1) \psi \partial \psi + \epsilon \gamma / 2z, \\
\check{J}^- &= \beta \check{\gamma} + i \check{\alpha}_+ \psi \partial \phi + (2k + 1) \partial \check{\psi}, \\
T &= \beta \partial \gamma - \psi^+ \partial \psi - (\partial \phi)^2 / 2 - i \partial^2 \phi / (2 \alpha_+) - \epsilon / 8z^2
\end{align*}
\]

where $\alpha_+^2 = 2k + 3$, and $\epsilon = 0, 1$ for R type or NS type $\widehat{OSP}(1,2)$ resp.; $T(z)$ is the stress-energy tensor by Sugawara construction eq.\(^{(2)}\). From the involution $\sigma$ of $\widehat{OSP}(1,2)$,

\[
\begin{align*}
\sigma(J^+_n) &= -J^+_n, \quad \sigma(j^+_n) = \pm j^+_n, \\
\sigma(J^3_n) &= -J^3_n, \quad \sigma(k) = k, \quad \sigma(d) = d,
\end{align*}
\]

we get another free field realization,

\[
\begin{align*}
\check{J}^+ &= -\check{\beta} \check{\gamma}^2 + i \check{\alpha}_+ \check{\gamma} \partial \check{\phi} - \check{\gamma} \check{\psi} \check{\psi}^+ + \check{k} \partial \check{\gamma} - (\check{k} + 1) \check{\psi} \partial \check{\psi} - \epsilon \gamma / 2z, \\
\check{J}^+ &= -\check{\gamma} (\check{\psi}^+ - \check{\beta} \check{\psi}) - i \check{\alpha}_+ \check{\psi} \partial \check{\phi} - (2\check{k} + 1) \partial \check{\psi}, \\
\check{J}^3 &= \check{\beta} \check{\gamma} - i \check{\alpha}_+ \check{\psi} \partial \check{\phi} / 2 + \frac{1}{2} \check{\psi} \check{\psi}^+ + \epsilon / 4z, \\
\check{J}^- &= \check{\psi}^+ - \check{\beta} \check{\psi}, \\
\check{T} &= \check{\beta} \partial \check{\gamma} - \check{\psi}^+ \partial \check{\psi} - (\partial \check{\phi})^2 / 2 - i \partial^2 \check{\phi} / (2 \check{\alpha}_+) - \epsilon / 8z^2
\end{align*}
\]

The Fock space $F_{j,\alpha}$ ($\check{F}_{\check{j},\check{\alpha}}$) is generated by negative modes of these fields together with $\gamma_0$, $\psi_0$ ($\check{\beta}_0$, $\check{\psi}_0^+$). It can be verified that in fact $F_{j,\alpha}$ and $\check{F}_{\check{j},\check{\alpha}}$ are dual spaces of each other. The inner product can be defined as $(|\psi\rangle_F, |\psi\rangle_F) = 1$, and satisfies

\[
\begin{align*}
(\beta_n u, \; v) &= -(u, \check{\beta}_n v), \\
(\gamma_n u, \; v) &= (u, \check{\gamma}_n v), \\
(\psi_n u, \; v) &= -i (u, \check{\psi}_n v), \\
(\psi^+_n u, \; v) &= -i (u, \check{\psi}^+_n v), \\
(\phi_n u, \; v) &= -(u, \check{\phi}_n v), \quad \text{For} \; u \in F_{j,\alpha}, \; v \in \check{F}_{\check{j},\check{\alpha}}.
\end{align*}
\]
The inner product so defined satisfies
\[
(J_n^+ u, v) = (u, \tilde{J}_n^- v), \quad (j_n^+ u, v) = \pm i(u, \tilde{j}_n^- v),
\]
\[
(J_n^3 u, v) = (u, \tilde{J}_n^3 v).
\]  
For \( u \in F_{j,\alpha}, v \in \tilde{F}_{j,\alpha}. \)

In our following discussion, we use notation \( W(j, k) \) to denote \( F_{j,\alpha} \), and \( W^*(j, k) \) to denote \( \tilde{F}_{j,\alpha} \).

Here we show that the isomorphism between R and NS type \( \text{OSP}(1, 2) \) can be realized by a map between the free fields. We still add a superscript R, or NS to the free fields when it is in the free field realization of the R, or NS type \( \text{OSP}(1, 2) \). Let
\[
\psi_n^{+, R} \to \psi_{n+1/2}^{+, \text{NS}}, \quad \psi_n^R \to \psi_{n-1/2}^{\text{NS}}, \\
\beta_n^R \to \beta_{n+1}^{\text{NS}}, \quad \gamma_n^R \to \gamma_{n-1}^{\text{NS}},
\]
\[
\phi_n^R \to \phi_n^{\text{NS}} + \alpha_+ / 2 \delta_{n,0}
\]
i.e.
\[
\psi^{+, R} \to z^{1/2} \psi^{+, \text{NS}}, \quad \psi^R \to z^{-1/2} \psi^{\text{NS}}, \\
\beta^R \to z \beta^{\text{NS}}, \quad \gamma^R \to z^{-1} \gamma^{\text{NS}},
\]
\[
i \partial \phi^R \to i \partial \phi^{\text{NS}} + \alpha_+ / 2 z.
\]

It is consistent with the map eq. (64), and the realization of R type \( \text{OSP}(1, 2) \) in eq. (69) becomes the dual realization of NS type \( \text{OSP}(1, 2) \) in eq. (74).

Now we consider the ghost spaces. \( gh^{R,(0, \frac{1}{2})} \) denotes R type ghost space with vacuum \( |0^R \rangle \) annihilated by \( c_0^+ \) and \( c_0^{1/2} \); and \( gh^{\text{NS},(1,1)} \) denotes NS one with vacuum \( |0^{\text{NS}} \rangle \) annihilated by \( c_0^+ \).

The scripts \((0, \frac{1}{2})\) and \((1, 1)\) correspond to \((J_n^{(1)}, J_n^{(3)})\) of the ghost vacua. It can be verified that \( gh^{R,(0, \frac{1}{2})} \) and \( gh^{\text{NS},(1,1)} \) are also isomorphic, as can be seen from the following map
\[
b_n^R \to z b_n^{\text{NS}}, \quad c_n^{-, R} \to z^{-1} c_n^{+, \text{NS}}, \\
b_{n/2}^R \to z^{1/2} b_{n/2}^{\text{NS}}, \quad c_{n/2}^{-, R} \to z^{-1/2} c_{n/2}^{+, \text{NS}}, \\
b_n^3 \to z b_n^{\text{NS}}, \quad c_{n}^{3, R} \to c_{n}^{3, \text{NS}},
\]
\[
b_{n/2}^R \to -z^{-1/2} b_{n/2}^{\text{NS}}, \quad c_{n/2}^{1/2, R} \to -z^{1/2} c_{n/2}^{1/2, \text{NS}}, \\
b_n^R \to z^{-1} b_n^{\text{NS}}, \quad c_{n}^{+, R} \to z c_{n}^{-, \text{NS}},
\]  
under which the BRST operator \( \mathcal{Q}_{BRST}^R \) becomes \( -\mathcal{Q}_{BRST}^{\text{NS}} \). From the above discussion we have the following proposition

**Proposition 1**
\[
M^R(j^R, k) \cong M^{\text{NS}}(j^{\text{NS}}, k), \quad L^R(j^R, k) \cong L^{\text{NS}}(j^{\text{NS}}, k), \\
W^R(j^R, k) \cong W^{*, \text{NS}}(j^{\text{NS}}, k), \quad gh^{R,(0, \frac{1}{2})} \cong gh^{\text{NS},(1,1)},
\]  
(77)
where $j^R$ and $j^{NS}$ are related by eq.(75).

The difference between the two types of $\hat{OSP}(1,2)$ algebra becomes more obvious under the Hamiltonian reduction. Naturally, R type (NS type, resp.) will result in NS sector (R sector, resp.) super-Virasoro algebra due to the fact that they satisfy the same boundary condition on the complex plane. The twisting term $\partial J^3$ in $T_{impr}$ brings about a shift of the conformal weight, as well as the modings of the fermionic generators by half integer.

An HWS in R type $\hat{OSP}(1,2)$ with isospin $J^R$ becomes the HWS in the representation of super-Virasoro algebra with conformal weight

$$h = \frac{j^R(j^R + 1/2)}{k + 3/2} - j^R.$$  (78)

We should be more careful when considering the conformal weight of the reduced HWS which is originally an HWS of NS type $\hat{OSP}(1,2)$. Notice that here the bosonic ghosts ($\beta$, $\gamma$) and Majorana fermion $\psi$ are all antiperiodic on the complex plane. Thus the vacuum have conformal weights $-1/8$ for $(\beta$, $\gamma$) system and $1/16$ for fermion $\psi$. Precisely because that the twisting of the Majorana fermion field $\psi$ is genuine, the NS and R super-Virasoro algebras, as the results of the Hamiltonian reduction of the R type and NS type of the $\hat{OSP}(1,2)$ algebra resp., are no longer isomorphic. The conformal weight of an HWS in NS type $\hat{OSP}(1,2)$ (related to the VirasoroB algebra by Sugawara construction in eq.(72), no $\partial J^3$ term) is

$$h^{OSP(1,2),NS} = \frac{j^{NS}(j^{NS} + 1)}{k + 3/2} - c/8,$$  (79)

where $c$ is the central charge, $c = k/(k+3/2)$. Eq.(79) can also be obtained by using eqs.(65,66,78).

Now we come to the conformal weight of such an HWS in the super-Virasoro algebra,

$$h = \frac{j^{NS}(j^{NS} + 1)}{k + 3/2} - j^{NS} - \frac{k}{8k + 3/2} - 1/8 + 1/16.$$  (80)

Combining eq.(10) and eq.(67) we get the central charge for the super-Virasoro algebra,

$$c^{tot} = \frac{3}{2}(1 - \frac{2(q - \bar{q})^2}{q \bar{q}}) = c_{\bar{q},q};$$  (81)

By eqs.(67,68,78,80), we get the conformal weights for the HWS’s of the admissible representations under the Hamiltonian reduction, those in

$$h^R = \frac{(mq - (s + 1)q\bar{q})^2 - (q - \bar{q})^2}{8q\bar{q}} = \Delta_{m,s+1};$$

$$h^{NS} = \frac{(mq - sq)^2 - (q - \bar{q})^2}{8q\bar{q}} + 1/16 = \Delta_{m,s}.$$  (82)
We see that they coincide with those of the super-Virasoro algebra in the \((q, \tilde{q})\) minimal model except for those \(s = q - 1\) in the R type and \(s = 0\) in NS type \(OSP(1,2)\). Moreover, if we set \(p = q + 1\), we get the unitary part

\[
\begin{align*}
\text{c} &= \frac{3}{2} \left( 1 - \frac{8}{q(q+2)} \right), \\
\h^{R} &= \frac{(mq - (s+1)(q+2))^2 - 4}{8q(q+2)}, \\
\h^{NS} &= \frac{(mq - s(q+2))^2 - 4}{8q(q+2)} + 1/16.
\end{align*}
\]

(83)

(84)

So it is easily seen that the admissible representation \(L^{R,OSP(1,2)}_{m,s}\) reduces to \(L^{NS,Vir}_{c,q,\tilde{q},\Delta_{m,s+1}}\) and \(L^{NS,OSP(1,2)}_{m,s}\), with highest weight \(j^{NS}_{m,s} = k/2 - j^{R}_{m,s}\) to \(L^{R,Vir}_{c,q,\tilde{q},\Delta_{m,s}}\). Notice that \(L(c,q,\Delta_{m,s}) \cong L(c_{\tilde{q}},\Delta_{\tilde{q}-m,q-s})\) while their original images are not isomorphic. The Wakimoto module of the matter sector realized in the free fields, eq.(69), reduces to the Fock space \(F_{\xi,\eta}\) (in terms of ref.[22]) with

\[
\begin{align*}
\xi &= \frac{(\alpha_{+}^2 - 1)}{(2\alpha_{+})}, \\
\eta &= \frac{- (4j + 1 + \epsilon - \alpha_{+}^2)}{(2\alpha_{+})}.
\end{align*}
\]

The dual Wakimoto module of the Liouville sector in eq.(71) reduces to that with

\[
\begin{align*}
\xi &= \frac{(\alpha_{+}^2 - 1)}{(2\alpha_{+})}, \\
\eta &= \frac{(4j + 1 + \epsilon - \alpha_{+}^2)}{(2\alpha_{+})}.
\end{align*}
\]

Again the reduced Fock space \(F_{\xi,\eta}\) and \(F_{\xi,-\eta}\) are dual spaces of each other.

5 The BRST States

The physical state space under constraints can be obtained by BRST approach. To obtain the physical states in the \(OSP(1,2)/OSP(1,2)\) gauged WZNW field theory, in this section we manage to calculate the BRST cohomology with coefficient in \(OSP(1,2)\) modules. To make our discussion selfconsistent, some results in (co)homology theory are presented along with the sketch of the proofs (see also appendix for some technical details). Readers not familiar with our notations are referred to refs.[50, 22, 51] for more mathematical background.

The BRST charge is given in eq.(24), explicitly

\[
Q = \sum_n c_{\alpha,n}(J^\alpha_{-n} + J^\alpha_{-n}) - 1/2 \sum_{n,m} f^{\alpha\beta\gamma} c_{\alpha,n} c_{\beta,m} b^{\gamma}_{-n-m}.
\]

(85)

We always assume that the ghost vacuum \(|gh\rangle_0\), which has ghost number zero by our notation, is annihilated by all the positive modes of the \(b_{\alpha} , c^\alpha\) as well as the the zero modes \(b_{\alpha,0}\) unless under special declaration. The other kinds of ghost vacua, for example, the one used in ref.[50],
which is annihilated by $c_0^±$ and $c_0^+$ and is denoted by $|gh\rangle_{0, \frac{1}{2}}$ in our paper, can not be obtained from $|gh\rangle_0$ by a finite number of operations of the ghost modes, due to the fact that $c_0^{1/2}$ is bosonic. However, by fermionizing the bosonic ghosts as what was done in ref. [47], ghost vacua of different bosonic ghost numbers are interpolated by a vertex operator.

As in the standard procedure of formulating the semi-infinite cohomology, we first consider the relative complex.

$$C_{rel}^* = \{ w | w \in C^*, J_0^{tot,3} w = L_0 w = b_{3,0} w = 0 \}$$

Let

$$Q = M b_{3,0} + c_0^3 J_0^{tot,3} + \hat{Q},$$

$$M = \sum_n ( c_n^+ c_{-n} + 1/4 c_n^+ c_{-n}^± ) ,$$

and $\hat{Q}$ has no term containing $c_0^3, b_{3,0}$. When restricted to the relative complex, $Q = \hat{Q}$. We pay our attention mainly on the following relative cohomology:

$$(i) \ H_{rel}^{\hat{Q},+} (L(j_1, k_1) \otimes M(j_2, k_2)) ; \ (ii) \ H_{rel}^{\hat{Q},+} (W(j_1, k_1) \otimes W^*(j_2, k_2));$$

$$(iii) \ H_{rel}^{\hat{Q},+} (L(j_1, k_1) \otimes W(j_2, k_2)); \ (iv) \ H_{rel}^{\hat{Q},+} (L(j_1, k_1) \otimes L(j_2, k_2));$$

where we have put on the restriction that $k_1 + k_2 + 3/2 = 0$, which must be satisfied for the nilpotency of the BRST operator. We have to consider another set of $OS\hat{P}(1,2)$-modules, by abuse of language, which are called “lowest weight modules” (LWM). The vacuum vector of such a module is annihilated by $j_0^-, J^-_0$ and all $J_n^-$’s with $n > 0$. We also have LW Verma modules, Wakimoto modules and irreducible modules, which are denoted by $\tilde{M}(j, k)$, $\tilde{W}(j, k)$ and $\tilde{L}(j, k)$ respectively.

We adopt the notation $(j_2, k) \rightarrow (j_1, k)$, to denote that there exists an embedding $M(j_2, k) \rightarrow M(j_1, k)$, which is equivalent to the fact that there exists a singular vector with isospin $j_2$ in $M(j_1, k)$. If $(j_2, k) \rightarrow (j_1, k)$, define $l((j_1, k), (j_2, k)) = d$, where $d$ is the maximal number such that there exists $(j_2, k) = (j'_2, k) \rightarrow (j'_2, k) \rightarrow \ldots \rightarrow (j'_{d+1}, k) = (j_1, k)$, where $j'_i \neq j'_j$; and $l((j_1, k), (j_1, k)) = 0$. If $M(j, k)$ is in the case $\text{III}_\pm$, define $j^{±i}$ be such that $l((j, k), (j^{±i}, k)) = |i|, j^i \neq j^{−i}$. (These definitions are analogous to that in ref. [22]). Sometimes for a given level we may omit $k$ in the above definition when making no confusions.

Our results again is similar to that in 2D gravity, given in ref. [3 8]. It will be helpful to list two theorems in homology theory which are relevant to our work. One is the twisted reduction
formula (cf. reduction formula in ref. [50]), which is proved in the appendix, where the readers are referred to find some notations used in our calculation here. Another is for the double cohomology (see, for example [52, 53, 51]).

**Theorem 1** [Twisted Reduction formula] Let $G = \hat{OSP}(1, 2)$, $V$ is a $G$-module in the category $\mathcal{O}$, there is a canonical isomorphism

$$H^\infty_{rel}^{n+1}(V \otimes M(-\beta - \lambda) \otimes gh, \hat{Q}) \cong H^n(V \otimes gh_+, Q_+)[\lambda];$$

where $\beta = 2\rho = 1/2J^r_0 + 3k'$. $(j'(j) = 1, k'(k) = 1)$; $[\lambda]$ means restriction to the subspace with weight $\lambda$.

When one factor of the tensor $V_1 \otimes V_2$ is an irreducible module, we have to consider the double cohomology again and again. The following theorem will be powerful for our calculation [52, 53, 51].

**Theorem 2** Let $(c^{p,q}, d, \partial)$ be a double complex, which satisfies

$$d : c^{p,q} \to c^{p+1,q}, \partial : c^{p,q} \to c^{p,q+1}, \{d, \partial\} = 0,$$

If the two corresponding sequence both collapse at the second term. then

$$\sum_{p+q=n} H^q(H^p(c^{*,*}, d), \partial) \cong \sum_{p+q=n} H^q(H^p(c^{*,*}, \partial), d).$$

In fact our complex is rather simple, many times the following corollary is enough.

**Corollary 1** If both the cohomologies corresponding to $d$ and $\partial$ vanishes for all but one degree, then [92] holds.

First we consider the BRST states of the R type $\hat{OSP}(1, 2)$. However the BRST states of the NS type can be easily obtained by the isomorphism eqs. (64,77) between them.

**Theorem 3** (i) Let $M_j$ not in case $\text{III}_{0}^{0}$, then

$$H^\infty_{rel}^{n+1}(L(j_1, k_1) \otimes M(j_2, k_2)) \cong \begin{cases} 
\delta_{n,j_1+1/2-j_2}C, & \text{if } -1/2 - j_2 \to j_1; \\
0, & \text{otherwise} 
\end{cases}$$
(ii) for $M_j$ in case $III_0$, then

$$H_{rel}^{\infty+n+1}(L(j_1, k_1) \otimes M(j_2, k_2)) \cong \begin{cases} 
\delta_{n,0} C, & \text{if } j_1 + j_2 + \frac{1}{2} = 0; \\
\delta_{n,1} C, & \text{if } -\frac{1}{2} - j_2 \rightarrow j_1, \text{and } l(j_1, -1/2 - j_2) = 1,
\emptyset, & \text{otherwise}
\end{cases} \quad (92)$$

To prove the theorem we first prove the following lemma

**Lemma 1** (i) Let $M(j, k)$ not in case $III_0$, then

$$H^n(L(j, k) \otimes gh_-, Q_-) \cong \oplus_{j' \rightarrow j, l(j, j') = -n} C v_{j'}; \quad (93)$$

(ii) for $M(j, k)$ in case $III_0$, then

$$H^n(L(j, k) \otimes gh_-, Q_-) \cong \oplus_{j' \rightarrow j, l(j, j') = n} C v_{j'}(\delta_{n,0} + \delta_{n,-1}), \quad (94)$$

where $v_j$ is a singular vector with isospin $j$ and conformal weight \(\frac{j(j+1)}{k+2/3}\).

**Proof.** $L(j, k)$ is a resolution of Verma modules,

$$\cdots \overset{\partial}{\rightarrow} M^i \overset{\partial}{\rightarrow} M^{i+1} \overset{\partial}{\rightarrow} \cdots \overset{\partial}{\rightarrow} M^{-1} \overset{\partial}{\rightarrow} M^0 = M(j, k) \rightarrow 0 \quad (95)$$

where $M^i$ is a direct sum of Verma modules. For a Verma module $M(j, k)$, $H^n(\mathcal{G}, M(j, k), Q_-) = \delta_{n,0} C v_j$. We see that the problem in question is actually a double complex which satisfies the condition in the corollary. So

$$H^n(L(j, k) \otimes gh_-, Q_-) \cong H^n(\mathcal{H}^0(M^*, \partial) \otimes gh_-, Q_-) = H^n(\mathcal{H}^0(M^* \otimes gh_-, Q_-), \partial) \quad (96)$$

According to the fact that $Q_-, \partial$ do not change the weight of vectors (i.e., they are weight zero operators), the $\partial$ action in the right hand side (r.h.s.) of eq.(96) is trivial. This completes the proof of the lemma.

**Proof of the theorem.** By reduction formula

$$H_{rel}^{\infty+n}(L(j_1, k_1) \otimes M(j_2, k_2)) \cong H^n(L(j_1, k_1) \otimes gh_+, Q_+)\lambda, \quad (97)$$

where $\lambda = (-1/2 - j_2)(J_0^3)^{j_2} + \frac{j_2(j_2+1/2)}{k+3/2} L_0$. For the irreducible module $L(j, k)$ satisfies $L(j, k) \cong L(j, k)^*$, so by Poincaré duality theorem

$$H^n(L(j, k) \otimes gh_+, Q_+) = (H^{-n}(L(j, k) \otimes gh_-, Q_-))^*, \quad (98)$$

the theorem follows.

By theorem\(\text{3}\) we can compute the cohomology $H_{rel}^{\infty+n}(L(j_1, k_1) \otimes L(j_2, k_2))$ easily.
Theorem 4  (i) For $M(j_1, k_1)$ in the case (I),

$$H_{rel}^{\infty + n + 1}(L(j_1, k_1) \otimes L(j_2, k_2)) \cong C\delta_{n,0}\delta_{j_1 + j_2 + 1/2, 0}. \quad (99)$$

(ii) For $M(j_1, k_1)$ in the case (II),

$$H_{rel}^{\infty + n + 1}(L(j_1, k_1) \otimes L(j_2, k_2)) \cong \begin{cases} C\delta_{n,0}, & \text{if } j_1 + j_2 + 1/2 = 0; \\ C(\delta_{n,-1} + \delta_{n,1}), & \text{if } -1/2 - j_2 \rightarrow j_1, \\ \emptyset, & \text{otherwise.} \end{cases} \quad (100)$$

(iii) For $M(j_1, k_1)$ in the case (III$^0_{\pm}$),

$$H_{rel}^{\infty + n + 1}(L(j_1, k_1) \otimes L(j_2, k_2)) \cong \begin{cases} C\delta_{n,0}, & \text{if } -j_2 - 1/2 \rightarrow j_1, \\ C(\delta_{n,-1} + \delta_{n,1}), & \text{if } -1/2 - j_2 \rightarrow j_1, \\ \emptyset, & \text{otherwise.} \end{cases} \quad (101)$$

(iv) For $M(j_1, k_1)$ in the case (III$^{\pm}$),

$$H_{rel}^{\infty + n + 1}(L(j_1, k_1) \otimes L(j_2, k_2)) \cong \begin{cases} C(1 + (-1)^{n+l(j_1, -1/2 - j_2)} - \delta_{n,l(j_1, -1/2 - j_2)}), & \text{if } -j_2 - 1/2 \rightarrow j_1, \text{ and } |n| \leq l(j_1, -1/2 - j_2); \\ \emptyset, & \text{otherwise.} \end{cases} \quad (102)$$

Proof.  (i) is obvious.

(ii) and (iii).  If $M(j_1, k_1)$ in the case (II) or (III$^0_{\pm}$), the resolution of $L(j_1, k_1)$ takes the following form:

$$0 \rightarrow M_{j_1, k_1} \rightarrow M(j_1, k_1) \rightarrow L(j_1, k_1) \rightarrow 0, \quad (103)$$

where $l(j_1, j'_1) = 1$, from which the following long exact sequence can be induced

$$\cdots \rightarrow H_{rel}^{\infty + n}(M(j_1, k_1) \otimes L_2) \rightarrow H_{rel}^{\infty + n}(L(j_1, k_1) \otimes L_2) \rightarrow H_{rel}^{\infty + n + 1}(M_{j'_1, k_1} \otimes L_2) \rightarrow \rightarrow H_{rel}^{\infty + n + 1}(M(j_1, k_1) \otimes L_2) \rightarrow H_{rel}^{\infty + n + 1}(L(j_1, k_1) \otimes L_2) \rightarrow H_{rel}^{\infty + n + 2}(M_{j'_1, k_1} \otimes L_2) \rightarrow \cdots \quad (104)$$

where $L_2 = L(j_2, k_2)$.  Use theorem 3 and note that there exists a “dual relation” [29] between the structure of Verma modules at level $k$ and level $-k - 3$.  The results are easily obtained.

(iv) is the most complicated case, however the proof can go on as that of minimal model in the
Virasoro algebra \[\text{Virasoro algebra \[22\]. Without loss of generality, we assume that } k_1 + 3/2 > 0, \text{ then } L(j_1, k_1) \text{ is a resolution of the following complex}

\[
\cdots \xrightarrow{\partial_{-3}} M^i \xrightarrow{\partial} M^i+1 \xrightarrow{\partial_{i+1}} \cdots \xrightarrow{\partial_{-2}} M^{-1} \xrightarrow{\partial_{-1}} M^0 = M(j_1, k_1) \rightarrow 0,
\]

where \(M^i = M_{j_1, k_1} \oplus M_{j_1-1, k_1} \cdot (j_1^{±1}, j_1) = i\). Then we get

\[
H^{\infty+n}_{rel}(L(j_1, k_1) \otimes L(j_2, k_2)) = H^{\infty+n}_{rel}(H^0(M^i, \partial_i) \otimes L(j_2, k_2)).
\]

If \((-1/2 - j_2, k_1) \not\rightarrow (j_1, k_1)\), then by the dual relation, \((-1/2 - j_1, k_2) \not\rightarrow (j_2, k_2)\), moreover \((-1/2 - j_1^{±1}, k_2) \not\rightarrow (j_2, k_2)\). By theorem \[3\], \(H^{\infty+n}_{rel}(M^i \otimes L(j_1, k_1)) \cong \emptyset\), so by the corollary \[1\], \(H^{\infty+n}_{rel}(L(j_1, k_1) \otimes L(j_2, k_2)) \cong \emptyset\).

If \((-1/2 - j_2, k_1) \rightarrow (j_1, k_1)\), it is easily seen that the double cohomology in r.h.s. of eq.(106) both collapse at the second term (cf. \[22\]) by using theorem \[3\]. So by theorem \[3\] we have

\[
H^{\infty+n}_{rel}(L(j_1, k_1) \otimes L(j_2, k_2)) \cong \sum_{p+q=n} H^{p}(H^{\infty+q}_{rel}(M^p \otimes L(j_2, k_2)), \partial_p),
\]

from which we get eq.(102). This completes the proof of the theorem.

Even though we get some insight by calculating the BRST cohomology with coefficient in various \(\widehat{OSP}(1, 2)\)-modules, we have not yet reached the goal of our present paper, i.e. to find out the physical space of the \(\widehat{OSP}(1, 2)/OSp(1, 2)\) theory. The reason is that in the calculation of the BRST cohomology \(H(V_1 \otimes V_2 \otimes gh, Q)\), usually we take \(V_1\) to be the admissible representation of \(G\), of which there are finite number inside the “conformal grid”. However, as discussed by many authors, e.g. ref. \[42, 54\], \(V_2\) is considered to be a representation of \(G^c/G\), and the space of state can be decomposed into a direct integral of the irreducible representations of the Kac-Moody superalgebras. It is natural to consider \(V_2\) as Wakimoto modules which are the free field realization of \(G^c/G\). In the remaining of this section, we shall calculated the BRST cohomology with \(V_1\) the admissible module and \(V_2\) the Wakimoto module.

But first we compute the BRST cohomology with coefficient in the Wakimoto modules. The matter part \(W(j_1, k_1)\) is the Fock space realized in eq.(109), and the Liouville part \(W'(j_2, k_2)\) in eq.(71). And \(k + 3 = 0, \alpha_+ = \pm i \alpha_+\). We write out the diagonal part of \(G\):

\[
J_{0, \text{tot}, 3} = c_0^+ b_{+, 0} - c_0^- b_{-, 0} + \frac{1}{2} c_0^+ b_{+, 0} - \frac{1}{2} c_0^- b_{-, 0} + \sum_{n < 0} c_n^+ b_{+, n} - c_n^- b_{-, n} + \frac{1}{2} c_n^+ b_{+, n} - \frac{1}{2} c_n^- b_{-, n} c_n^+ b_{+, n} c_n^- b_{+, n} + J_0^3 + J_0^3.
\]
is the level operator. Without loss of generality, we set \( \tilde{\alpha} = i\alpha \), \( \phi^\pm = 1/\sqrt{2}(\phi \pm i\tilde{\phi}) \) and a degree on the free field

\[
\text{deg} : \quad e_n^\alpha, \gamma, \tilde{\gamma}, \psi, \tilde{\psi}, \phi_n^+, 1
\]
\[
\text{deg} : \quad b_{\alpha,n}, \beta, \tilde{\beta}, \psi^+, \tilde{\psi}^+, \phi_n^-, -1
\]

Then \( \hat{Q} \) can be rewrite as \( \hat{Q} = \hat{Q}_+ + \hat{Q}_0 \), where the degree zero operator

\[
\hat{Q}_0 = \sum_n \left( -\frac{1}{2} c_n^- \beta_n + \frac{1}{2} c_n^+ \beta_n + 1/4 c_n^+ \tilde{\psi}^+_{-n} - 1/4 c_n^- \tilde{\psi}^+_{-n} + \sum_{n \neq 0} \alpha_+ / \sqrt{2} c_n^0 \phi_n^- \right)
\]

contains only quadratic terms with opposite degrees.

We need to calculate \( H(C^*_\text{rel}, \hat{Q}_0) \). Note that

\[
\hat{L}_0 = \{ \hat{Q}_0, \hat{G}_0^0 \},
\]

where

\[
\hat{G}_0^0 = \sum_n n : (b_{-} \gamma_n - b_{+} \tilde{\gamma}_n + 2b_{\frac{1}{2}} \tilde{\psi}_n - 2b_{-\frac{1}{2}} \psi_n) + \sum_n \sqrt{2} / \alpha_+ b_{3,n} \phi_n^+.
\]

So a nontrivial \( \hat{Q}_0 \) states must be annihilated by \( \hat{L}_0 \). We can just restrict ourselves to the subspace which consists of only zero-mode excitation. However because it is a subspace of the relative complex, \( L_0 = 0 \), \( J_{\text{tot}}^{\text{rel}} = 0 \), so the subspace is nonempty only when \( \Delta_j + \Delta_{\tilde{j}} = 0 \).

To make our discussion more simple, the following relation is very useful

\[
\hat{n}_1 = \beta_0 \gamma_0 - b_{-} e_0^- = \{ \hat{Q}_0, -2b_{-} \gamma_0 \};
\]
\[
\hat{n}_2 = \tilde{\beta}_0 \tilde{\gamma}_0 - b_{+} e_0^+ = \{ \hat{Q}_0, 2b_{+} \tilde{\gamma}_0 \};
\]
\[
\hat{n}_3 = -\tilde{\psi}_0^+ \psi_0 + b_{\frac{1}{2}} \phi_0^+ = \{ \hat{Q}_0, 4b_{\frac{1}{2}} \phi_0^+ \};
\]
\[
\hat{n}_4 = -\tilde{\psi}_0^- \phi_0^- + b_{-\frac{1}{2}} e_0^- = \{ \hat{Q}_0, -4b_{-\frac{1}{2}} \phi_0^- \};
\]

(115)
Note that in the Fock space $\hat{n}_i$ is diagonalable, so a nontrivial $\hat{Q}_0$ states must satisfies $\hat{n}_i = 0$.

As discussed previously, we have to specify the Fock space vacuum. In calculating $H_{rel}^{\infty+\ast}(V_1 \otimes V_2, \hat{Q}_0)$, we first consider the case that both $V_1$ and $V_2$ are highest weight modules. The relevant Fock space vacuum is specified by the requirement that

$$\beta_n|j, j\rangle = \gamma_n|j, j\rangle = \psi_n^+|j, j\rangle = \bar{\psi}_n|j, j\rangle = 0, \quad n \geq 0$$

(116)

$$(\partial \phi)_0 \beta_n|j, j\rangle = \frac{2j}{\alpha_+} \bar{\gamma}_n|j, j\rangle, \quad (\partial \bar{\psi})_0 \beta_n|j, j\rangle = -\frac{2j}{\alpha_+} \bar{\gamma}_n|j, j\rangle.$$ 

For the ghost vacuum, we consider here the following two cases.

(i) Ghost vacuum $|gh\rangle_0$

Recall that $|gh\rangle_0$ is defined by

$$b_{\alpha, n}|gh\rangle_0 = c_{m\alpha}^n|gh\rangle_0 = 0, \quad n \geq 0, \quad m > 0,$$

so that $|gh\rangle_0$ is an $\hat{OSP}(1, 2)$ singlet. The ghost number operator is defined by

$$J_{gh}^0 = \sum_{n > 0, \alpha} (-1)^{d(\alpha) + 1} b_{\alpha, -n} c_{m\alpha}^n + \sum_{n \geq 0, \alpha} c_{m\alpha}^n b_{\alpha, n}.$$ 

(118)

So we have $J_{gh}^0|gh\rangle_0 = 0$, i.e. $|gh\rangle_0$ has ghost number zero. Note that

$$n_1 = n_\gamma + n_{c^-}, \quad n_2 = -n_\bar{\beta} + n_{c^+} - 1,$$

$$n_3 = -n_\bar{\psi} + n_{c^\frac{1}{2} + 1}, \quad n_4 = n_{\psi} + n_{c^\frac{1}{2} - 1}.$$

(119)

Keep in mind that $\beta, \gamma, \bar{\beta}, \bar{\gamma}$ are bosonic fields, while $\psi, \bar{\psi}, \bar{\psi}$, $\bar{\psi}$ are fermionic fields, we get the only solution for $\hat{n}_i = 0$ is

$$n_\gamma = n_\bar{\beta} = n_{c^-} = n_{c^\frac{1}{2}} = n_{\psi} = 0, \quad n_{c^+} = n_{\bar{\psi}} = 1.$$ 

(120)

Upon the constraint that $J_{0,\alpha, 3}^{tot} = L_0 = 0$, the $\hat{Q}_0$ cohomology state is

$$\bar{\psi}_0^+ c_0^+|j, j\rangle \otimes |gh\rangle_0,$$

when $j + \bar{j} + \frac{1}{2} = 0$. Otherwise the cohomology is empty.

(ii) Ghost vacuum $|gh\rangle_{0, \frac{1}{2}}$

Such ghost vacuum was used, e.g. in ref. [50] and is specified as follows,

$$b_{\alpha, n}|gh\rangle_{0, \frac{1}{2}} = c_{n\alpha}^0|gh\rangle_{0, \frac{1}{2}} = 0, \quad n > 0,$$

$$c_{0\alpha}^+|gh\rangle_{0, \frac{1}{2}} = c_{0\alpha}^\frac{1}{2}|gh\rangle_{0, \frac{1}{2}} = 0.$$ 

(121)
It is easy to verify that $|gh\rangle_{0,\frac{1}{2}}$ is an isospin-$\frac{1}{2}$ $OSP(1,2)$ highest weight state. If we fermionize the bosonic ghost as in ref.\cite{47} and consider the two ghost vacua at the different Bose sea level, we will find that $|gh\rangle_{0,\frac{1}{2}}$ has ghost number zero. Note that

\begin{equation}
\begin{aligned}
n_1 &= n_{\gamma} + n_{c^{-}}, & n_2 &= -n_{\tilde{\beta}} - n_{b_{+}}, \\
n_3 &= -n_{\tilde{\beta}} - n_{b_{-}}, & n_4 &= n_{\psi} + n_{c^{-}} \frac{1}{2}.
\end{aligned}
\end{equation}

The only solution for $n_i = 0$ is

\begin{equation}
n_{\gamma} = n_{\tilde{\beta}} = n_{c^{-}} = n_{c^{+}} = n_{\frac{1}{2}} = n_{\psi} = n_{c^{+}} = n_{\tilde{\psi}^{+}} = 0.
\end{equation}

Upon the condition that $j_{0}^{\text{tot},3} = L_0 = 0$, the $\hat{Q}_0$ cohomology state is

\begin{equation}
|j, \tilde{j}\rangle \otimes |gh\rangle_{0,\frac{1}{2}},
\end{equation}

when $j + \tilde{j} + 1/2 = 0$. Otherwise the cohomology is empty.

Combining the above analysis, we have the following theorem,

**Theorem 5**

(i) For ghost vacuum $|gh\rangle_0$,

\begin{equation}
H_{\text{rel}}^{\leq n+1} (W(j_1, k) \otimes W^*(j_2, -3 - k)) \cong \delta_{n,0} \delta_{j_1+j_2+1/2,0}.
\end{equation}

(ii) For ghost vacuum $|gh\rangle_{0,\frac{1}{2}}$,

\begin{equation}
H_{\text{rel}}^{\leq n} (W(j_1, k) \otimes W^*(j_2, -3 - k)) \cong \delta_{n,0} \delta_{j_1+j_2+1/2,0}.
\end{equation}

As in \cite{11}, we also consider the cohomology $H_{\text{rel}}^{\leq n} (V_1 \otimes V_2, \hat{Q}_0)$ in the Fock space, when one of, or both $V_1$, $V_2$ are lowest weight modules (LWM). The results are list in table 1. When they both are HWM (or LWM), we have $H_{\text{rel}}^{\leq n} (V_1 \otimes V_2, \hat{Q}_0) \cong H_{\text{rel}}^{\leq n} (V_1 \otimes V_2, \hat{Q})$. However, when one is LWM, another is HWM, we only have an embedding instead of an isomorphism.

From table 1, we see that when switching from our first ghost vacuum into the second ghost vacuum, there is a change of the ghost number by 1 for the BRST state. In the following discussion, we assume that the ghost vacuum is the former.

**Lemma 2**

(i) $H_{\text{rel}}^{\leq n} (L(j_1, k_1) \otimes W^*(j_2, k_2)) \neq 0$ iff $W_{-1/2-j_2,k_1}$ appears in the resolution of $L(j_1, k_1)$ in terms of Wakimoto modules.

(ii) For $W(j_1, k_1)$ not in the case III$^+ (\pm)$, if $W_{-1/2-j_2}$ appears in the resolution of $L(j_1, k_1)$, then

\begin{equation}
H_{\text{rel}}^{\leq n+1} (L(j_1, k_1) \otimes W^*(j_2, k_2)) \cong \delta_{n, \text{sign}(j_1+j_2+1/2)(j_1,-1/2-j_2)}.
\end{equation}
Proof. (i) If \( \{W^i, \partial_i\} \) is a resolution of \( L(j_1, k_1) \), using corollary 1, we get
\[
H_{\text{rel}}^{\infty + n}(L(j_1, k_1) \otimes W^*(j_2, k_2)) \cong H_{\text{rel}}^{\infty + n}(H^0(W^i, \partial_i) \otimes W^*(j_2, k_2)) \\
\cong H^{n-1}(H_{\text{rel}}^{\infty + 1}(W^i \otimes W^*(j_2, k_2)), \partial_i),
\]
from which (i) is easily to get.

(ii) Note that if the condition of (ii) holds, then it is in the degree of \( \text{sign}(j_1 + j_2 + 1/2)l(j_1, -1/2 - j_2) \). From eq. (127) we get (ii).

This completes the proof of the lemma.

**Theorem 6** (i) for \( W(j_1, k_1) \) in the case I,
\[
H_{\text{rel}}^{\infty + n + 1}(L(j_1, k_1) \otimes W^*(j_2, k_2)) \cong C\delta_{n,0} \delta_{j_1 + j_2 + 1/2,0}.
\]

(ii) for \( W(j_1, k_1) \) in the case II,
\[
H_{\text{rel}}^{\infty + n + 1}(L(j_1, k_1) \otimes W^*(j_2, k_2)) \cong \begin{cases} 
C\delta_{n,0}, & \text{if } j_1 + j_2 + 1/2 = 0; \\
C(\delta_{n,0} + \delta_{n,1}), & \text{if } -1/2 - j_2 \rightarrow j_1, \\
\emptyset, & \text{otherwise.}
\end{cases}
\]

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(iii) for $W(j_1, k_1)$ in the case $II_-$,

$$H_{rel}^{n+1}(L(j_1, k_1) \otimes W^\ast(j_2, k_2)) \cong \begin{cases} C\delta_{n,0}, & \text{if } j_1 + j_2 + 1/2 = 0; \\ C(\delta_{n,0} + \delta_{n,-1}), & \text{if } -1/2 - j_2 \rightarrow j_1, \\ \emptyset, & \text{and } l(j_1, -1/2 - j_2) = 1; \\ \emptyset, & \text{otherwise.} \end{cases}$$

(130)

(iv) for $L(j_1, k_1)$ the admissible representation of $OS\hat{P}(1, 2)$ (c.f. eq.(67)),

$$H_{rel}^{n+1}(L(j_1, k_1) \otimes W^\ast(j_2, k_2)) \cong \begin{cases} C\delta_{n,\text{sign}(j_1+j_2+1/2)(j_1,-j_2-1/2)}, & \text{if } -j_2 - 1/2 \rightarrow j_1; \\ \emptyset, & \text{otherwise.} \end{cases}$$

(131)

Proof. Combining theorem 1 and the above lemma we get the theorem.

As a summary we restrict the matter part to the admissible representation (cf. eq.(67)), the following is our main results obtained in the above discussion.

$$H_{rel}^{n+1}(L_{m,s} \otimes M_{2l\bar{q}+m,s}) \cong \delta_{n,|2l+1|};$$

$$H_{rel}^{n+1}(L_{m,s} \otimes M_{2l\bar{q}-m,s}) \cong \delta_{n,|2l|};$$

$$H_{rel}^{\infty+1}(L_{m,s} \otimes W^\ast_{2l\bar{q}+m,s}) \cong \delta_{n,(2l+1)};$$

$$H_{rel}^{\infty+1}(L_{m,s} \otimes W^\ast_{2l\bar{q}-m,s}) \cong \delta_{n,2l};$$

(132)

$$H_{rel}^{n+1}(L_{m,s} \otimes L_{2l\bar{q}+m,s}) \cong \begin{cases} C \oplus C, & \text{if } n \in \text{odd, and } |n| < |2l + 1|, \\ C, & \text{if } n = \pm(2l + 1), \\ \emptyset, & \text{otherwise;} \end{cases}$$

$$H_{rel}^{n+1}(L_{m,s} \otimes L_{2l\bar{q}-m,s}) \cong \begin{cases} C \oplus C, & \text{if } n \in \text{even, and } |n| < |2l|, \\ C, & \text{if } n = \pm2l, \\ \emptyset, & \text{otherwise,} \end{cases}$$

where the subscript stands for the HW with $4j_{m,s} + 1 = m - s(2k_i + 3), \ i = 1, 2, \ 2k_1 + 3 = \bar{q}/q, \ 2k_2 + 3 = -\bar{q}/q$.

Now that we have got the relative cohomology, the absolute cohomology is easy to get. As discussed in [5, 55, 11], we also have

$$H_{abs}^{n} \cong H_{rel}^{n} \oplus H_{rel}^{n-1},$$

(133)

where the coefficient is as that in eq.(132).
The cohomologies with coefficient being $LWM \otimes LWM$ can be very easily obtained. From the involution of the $\hat{OSP}(1,2)$, eq.(70)

\begin{align}
J^\pm_n &\to -J^\mp_n, \\
J^0_n &\to -J^3_n, \\
j^\pm_n &\to \pm j^{\mp}_n, \\
k &\to k, \\
d &\to d,
\end{align}

we get the isomorphism between $LWM$ and $HWM$

\begin{align}
M(j,k) &\to \tilde{M}(-j,k), \\
L(j,k) &\to \tilde{L}(-j,k), \\
W(j,k) &\to \tilde{W}(-j,k).
\end{align}

Moreover the ghost space $gh_0$ is invariant under such involution of $\hat{OSP}(1,2)$. So we have

\begin{align}
H^{\infty+}_2(\tilde{V}_1(-j_1,k_1) \otimes \tilde{V}_2(-j_2,k_2) \otimes gh^0, Q) &\cong H^{\infty+}_2(V_1(j_1,k_1) \otimes V_2(j_2,k_2) \otimes gh^0, Q),
\end{align}

where $\tilde{V}_i$ are LWMs while $V_i$ are corresponding HWMs under map (135).

The cohomologies with ghost vacuum $|gh\rangle_{0,\frac{1}{2}}$ is found to be isomorphic to that with ghost vacuum $|gh\rangle_{0,\frac{1}{2}}$ by a shift of ghost number 1, i.e. we have

\begin{align}
H^{\infty+}_{\text{rel}}(V_1 \otimes V_2 \otimes gh^0, \hat{Q}) &\cong H^{\infty+}_{\text{rel}}(V_1 \otimes V_2 \otimes gh^{\frac{1}{2}}, \hat{Q}).
\end{align}

In section 3, we have established the isomorphism between the R type and the NS type modules explicitly, and between the R type ghost space $gh^{R,(0,\frac{1}{2})}$ and the NS type ghost space $gh^{NS,(1,1)}$. The BRST operator can be identified by such an isomorphism. So the BRST states for the NS type $\hat{OSP}(1,2)$ can also be obtained by such a relation,

\begin{align}
H^{\infty+,*}_{\text{rel}}(V_1^{NS} \otimes V_2^{NS} \otimes gh^{NS,(1,1)}, Q) &\cong H^{\infty+,*}_R(V_1^R \otimes V_2^R \otimes gh^{R,(0,\frac{1}{2})}, Q),
\end{align}

where $V_i^R$ and $V_i^{NS}$ are isomorphic to each other, as listed in eq.(71). Notice that for the NS type $\hat{OSP}(1,2)$, the ghost vacuum $|gh\rangle_0$ annihilated by $b_0^\alpha$ can be got from $|gh\rangle_{(1,1)}$ by the action of $b_{+,0}$ on the latter one. Namely

\begin{align}
|gh\rangle_0^{NS} &= b_{+,0}|gh\rangle_{(1,1)}^{NS}.
\end{align}

So we also have

\begin{align}
H^{\infty+}_{\text{rel}}(V_1^{NS} \otimes V_2^{NS} \otimes gh^0, Q) &\cong h^n(V_1^{NS} \otimes V_2^{NS} \otimes gh^{(1,1)}, Q)
\end{align}
6 Conclusion and Speculations

The quantization of 2d supergravity has been approached by many authors through the Hamiltonian reduction of the supergroup valued WZNW model. However, in this paper, we take a somewhat different procedure. By considering the $OSP(1,2)/OSP(1,2)$ gauged WZNW model, the superconformal matter, 2d supergravity, and the super-reparametrization ghost are naturally put together to form a covariant topological conformal field theory. It remains a technical problem to find an economical way of calculating the correlation functions in our formalism, which deserves our future investigation \[57\].

The physical state space in 2D gravity has been worked out completely for $c \leq 1$ \[5, 6, 8\]. However the similar work on the 2D supergravity is far from complete \[56\]. When the matter sector is in the minimal series (in fact it is always the case ), the cohomology like this $H^{2+\ast}(L(c, \Delta) \otimes F_{\xi,\eta})$ does not appear in the literature due to the difficulty in constructing the Felder BRST cohomology for the super-Virasoro algebra. Having established the equivalence between fermionic strings and $OSP(1,2)/OSP(1,2)$ WZNW field theory, now we may come to the answers, though partly, of the problem arising in ref. \[22\]. From theorem 6, we get the following proposition, which is analogous to the results in 2D gravity \[5\].

**Proposition 2** Let $\xi = i \frac{q + \tilde{q}}{2\sqrt{qq}}$,

1. if $\eta = i \frac{2dq + mq - s\tilde{q}}{2\sqrt{qq}}$,

$$H_{rel}^{\infty +n}(L(c_{q,q}, \Delta_{m,s}) \otimes F_{\xi,\eta}) \cong \delta_{n,-2dC}; \quad (141)$$

2. if $\eta = i \frac{2dq - mq - s\tilde{q}}{2\sqrt{qq}}$,

$$H_{rel}^{\infty +n}(L(c_{q,q}, \Delta_{m,s} \otimes F_{\xi,\eta}) \cong \delta_{n,1-2dC}; \quad (142)$$

where the BRST states are in the NS (R) sector when $m + s$ is even (odd).

The problem remains on how to choose the ghost vacuum state. In the presented paper, we have analysed two kinds of ghost vacuum states, and found that the corresponding BRST states differ by ghost number one. (The method can be generalized to the construction of other different ghost vacua). The situation is similar to the so called “picture changing” operation in critical fermionic string theory \[47\]. It is promising that in this way one could establish the
exact correspondence between the correlation functions of the “fermion vertex” in noncritical
fermionic string theory and their counter parts in $OSP(1, 2)/OSP(1, 2)$ theory \[57\].

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### A Proof of the Twisted Reduction formula

We first list some notations:

category $\mathcal{O}$ is $OSP(1, 2)$-module category such that for any $OSP(1, 2)$-module $V$ in $\mathcal{O}$ satisfies

1. $V$ is $\mathbb{Z}_2$ graded which is consistent with that of $OSP(1, 2)$;
2. $V = \oplus_{\lambda \in P(V)} V_{\lambda}$, $\dim(V_{\lambda}) < \infty$;
3. $\exists \lambda_1, \lambda_2, \ldots, \lambda_n$ s.t. $P(V) \subset \cup_{i=1}^n \lambda_i + \Delta_-$.

$(\alpha, n) > 0$ iff $n > 0$ or $n = 0$, $\alpha > 0$, and vice versa, where $\alpha$, $n$ correspond to the isospin and
the conformal weight of the generators.

Let $c_\alpha = h_\alpha \beta c_\beta$, $b_\alpha = b_\beta h_\beta \alpha$,

\[
Q_+ = \sum_{(\alpha, n) < 0} c_{\alpha, n} J^\alpha_{-n} - \sum_{(\alpha, n), (-\beta, m) < 0} 1/2 f^{\alpha \beta \gamma} c_{\alpha, n} c_{\beta, m} b_\gamma^m_{-n-m}, \\
Q_- = \sum_{(\alpha, n) > 0} c_{\alpha, n} J^\alpha_{-n} - \sum_{(\alpha, n), (-\beta, m) > 0} 1/2 f^{\alpha \beta \gamma} c_{\alpha, n} c_{\beta, m} b_\gamma^m_{-n-m}.
\]

(143)

It can be verified that $Q^2 = Q_+ Q_+ + Q_+ Q_- = 0$.

Rewrite the ghost space as $gh^0 = gh^0_+ \otimes gh^0_-$, where $gh^0_+$ is generated by $c_{\alpha, n}s$ $(\alpha, n) < 0$ and $gh^0_-$ by $b_\alpha^n$, $n < 0$ and $c_{\alpha, 0}$, $\alpha < 0$. For ghost vacuum $|gh\rangle_{0, \frac{1}{2}}$, we have the following
decomposition, $gh^{0, \frac{1}{2}} = gh^{0, \frac{1}{2}}_+ \otimes gh^{0, \frac{1}{2}}_-$, where $gh^{0, \frac{1}{2}}_+$ is generated by $c_{\alpha, n}s$ $(\alpha, n) < 0$ and $gh^{0, \frac{1}{2}}_-$
generated by $b_\alpha^n$, $(\alpha, n) < 0$.

So let $V_+$, $V_-$ be modules in the category $\mathcal{O}$, then $(V_+ \otimes gh_+, Q_+)$ and $(V_- \otimes gh^0, Q_-)$ are two
differential complexes. The following formula is helpful for our proof of the twisted reduction
formula.
Proposition 3 (K"unneth formula)

\[ H^n_{rel}(V_+ \otimes V_- \otimes gh^0, Q_- + Q_+) \cong \sum_{p+q=n} H^p(V_+ \otimes gh_+, Q_+) \otimes H^q(V_- \otimes gh^0_-, Q_-)[0]. \]  

(144)

Lemma 3

\[ \dim(H^n(M(j,k) \otimes gh^0_-, Q_-)) = \delta_{n,1}, \]  

(145)

and a representation of the nontrivial \( Q_- \) state is

\[ (j^-c_{-0} + c_{-1/2,0})\]|. \]

Proof. we can define a contracting homotopy operator, 

\[ \bar{G} = \sum_{(\alpha,n)>0} J^\alpha_n h_{\alpha\beta} b^-_{\beta-n}, \]

(146)

where the operators \( \bar{J}_n^\alpha, (\alpha,n) > 0 \) are defined such that

\[
\begin{align*}
[J_n^\alpha, \bar{J}_m^\beta] &= (-1)^{d(\alpha)d(\beta)+1} [\bar{J}_m^\beta, J_n^\alpha] = \\
&= \begin{cases} 
  f_{\alpha\beta} \tilde{J}_{n+m}^\gamma, & \text{if } (\alpha + \beta, n + m) > 0; \\
  h_{\alpha\beta} n\delta_{n+m,0}, & \text{if } (\alpha + \beta, n + m) = 0; \\
  0, & \text{otherwise},
\end{cases} \\
\bar{J}_n^\alpha | &= 0.
\end{align*}
\]

(147)

It can be verified that

\[
\{Q_-, \bar{G}\} = \bar{L}_0 = \sum_{(-\alpha,n)>0} (-1)^{d(\alpha)} nb^-_{\alpha,n} c_{\alpha,n} + J^-_{\alpha,n} h_{\alpha\beta} \bar{J}_n^\beta,
\]

(148)

which satisfies

\[
\begin{align*}
[\bar{L}_0, J^-_{\alpha,n}] &= nJ^-_{\alpha,n}; \\
[\bar{L}_0, b^-_{\alpha,n}] &= nb^-_{\alpha,n}; \\
[\bar{L}_0, c_{\alpha,n}] &= -nc_{\alpha,n}; \\
[\bar{L}_0, Q_-] &= 0.
\end{align*}
\]

(149)

This proves that the nontrivial \( Q_- \) state should be \( \bar{L}_0 \) vanishing, i.e. no negative mode excitations. Now by direct computation on the subspace 

\[
\{(j^-)^m(c_{-1/2,0})^n c_{-0}|, (j^-)^m(c_{-1/2,0})^n|\},
\]

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we get the unique nontrivial $Q_-$ states

$$(j^-_{0, -0} + c_{1/2, 0})$$

up to $Q_-$ exact states.

This completes the proof of the lemma.

Now by using the above proposition and lemma, we come to the proof of the twisted reduction formula. It is analogous to the proof of the reduction formula in ref.[50] by using $f$ degree on $V_+ \otimes V_- \otimes gh^0$. More explicitly we can define

$$f\text{deg}(J_n^\alpha) = -f\text{deg}(\tilde{J}_n^\alpha) = -3n - 2\alpha,$$

$$f\text{deg}(c_n^\alpha) = -f\text{deg}(b_n^\alpha) = |3n + 2\alpha|,$$  \hspace{1cm} (150)

then $\hat{Q} = \hat{Q}_0 + \hat{Q}_\geq$, where $\hat{Q}_0 = Q_+ + Q_-$. Noticing that the subspace of the relative complex with fixed ghost number is finite dimensional, we get

$$H^{\infty, +}_\text{rel} (V_+ \otimes V_- \otimes gh^0, \hat{Q}) \cong H^{\infty, +}_\text{rel} (V_+ \otimes V_- \otimes gh^0, \hat{Q}_0),$$  \hspace{1cm} (151)

which together with eqs.(144,145) gives the twisted reduction formula eq.(89).

To compare the difference of the homology groups brought about by the different ghost vacua, we list the theorem of homotopy Lie superalgebra, theorem 2.1 in ref.[50]

$$H^n(M(\Lambda) \otimes gh_-, Q_-) \cong \delta_{n, 0}C[[|]]),$$  \hspace{1cm} (152)

The difference between eq.(145) and eq.(152) leads also the difference between the reduction formula [50] and the twisted one.

We have a conjecture here that eq.(145) can be generalized to affine Lie superalgebra $\hat{G}$ associated with a finite dimensional superalgebra $G$

$$\text{dim}(H^n(M(\Lambda) \otimes gh_-, Q_-) = \delta_{n, \text{dim}(G^+)}),$$  \hspace{1cm} (153)

where $G$ is the zero mode of $\hat{G}$, and $G^+_0$ is Borel even part of $G$; $gh^0_-$ is generated by $b_n^\alpha$s with $n < 0$ and $c_{\beta, 0}$ with $\beta$ being a negative root of $G$. 

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References

[1] E. Brezin and V.A. Kazakov, Phys. Lett. B236 (1990) 144

[2] M. Douglas and S. Shenker, Nucl. Phys. B335 (1990) 635

[3] D.J. Gross and A. Migdal, Phys. Rev. Lett. 64 (1990) 717

[4] D.J. Gross and A. Migdal, Nucl. Phys. B340 (1990) 333

[5] B.H. Lian and G.J. Zuckerman, Phys. Lett. B254 (1991) 417

[6] B.H. Lian and G.J. Zuckerman, Phys. Lett. B266 (1991) 21

[7] E. Witten, Nucl. Phys. B373 (1992) 187

[8] P. Bouwknegt, J. McCarthy and K. Pilch, Commun. Math. Phys. 145 (1992) 541

[9] O. Aharony, O. Ganor, T. Sonnenschein, S. Yankielowicz and N. Sochen, preprint TAUP-1961-92

[10] H. L. Hu and M. Yu, Phys. Lett. B289 (1992) 302

[11] H. L. Hu and M. Yu, Nucl. Phys. B391 (1993) 389

[12] O. Aharony, J. Sonnenschien and S. Yankielowicz, preprint TAUP-1977-92

[13] V. Sadov, preprint HUTP-92/A055

[14] T. Inami and K.I. Izawa, Phys. Lett. B255 (1991) 521

[15] Y.Z. Zhang, Phys. Lett. B283 (1992) 237

[16] G.W. Delius, M.T. Grisaru and P. Van Nieuwenhuizen, Nucl. Phys. B389 (1992) 25

[17] S. Komata, K. Mohri and H. Nohara, Nucl. Phys. B359 (1991) 168

[18] J. Evans and T. Hollowood, Phys. Lett. B293 (1992) 100

[19] K. Ito, J.O. Madsen and J.L. Petersen, preprint NBI-HE-92-42 (July 1992).

[20] K. Ito, J.O. Madsen and J.L. Petersen, preprint NBI-HE-92-81, to appear in the proceedings of the International Workshop on “String Theory, Quantum Gravity and the Unification of the Fundamental Interactions”, Rome, September 21-26, 1992
[21] K. Ito, Int. J. Mod. Phys. A7 (1992) 4885

[22] B.H. Lian and G.J. Zuckerman, Com. Math. Phys. 145 (1992) 561

[23] P. Di Vecchia, J.L. Petersen, M. Yu and H.B. Zheng, Phys. Lett. B174 (1986) 280

[24] W. Boucher, D. Friedan and A. Kent, Phys. Lett. B172 (1986) 316

[25] V.G. Kac and D.A. Kazhdan, Adva. Math. 34 (1979) 97

[26] V.G. Kac, Infinite Dimensional Lie Algebras. Cambridge Univ. Press, Cambridge, U. K. (1985)

[27] B.L. Feigin and E.V. Frenkel, Commun. Math. Phys. 128 (1990) 161

[28] B.L. Feigin and E.V. Frenkel, Phys. and Math. of Strings (V.G. Knizhnik memorial volume). World Scientific (1989) 271

[29] J.B. Fan and M. Yu, preprint AS-ITP-93-14, submitted to Commun. Phys. Math.

[30] M. Bershadsky and H. Ooguri, Phys. Lett. B229 (1989) 374

[31] M. Bershadsky and H. Ooguri, Commun. Math. Phys. 126 (1989) 49

[32] B.L. Feigin and D.B. Fuchs, Representations of the Virasoro Algebra. Seminar on Supermanifolds no. 5, Leites, D. (ed.)

[33] E. Witten, Commun. Math. Phys. 92 (1984) 455

[34] V.G. Kac, Adva. Math. 26 (1977) 8

[35] V.G. Kac, Adva. Math. 30 (1979) 85

[36] A. Pais and V. Rittenberg, J. Math. Phys., Vol. 16, No. 10 (1975) 2062

[37] D. Gepner and E. Witten, Nucl. Phys. B278 (1986) 493

[38] M. Scheunert, W. Nahm and V. Rittenberg, J. Math. Phys., Vol. 18, No. 1 (1977) 155

[39] A.M. Polyakov and P.B. Wiegmann, Phys. Lett. B131 (1983) 121

[40] A.M. Polyakov and P.B. Wiegmann, Phys. Lett. B141 (1983) 223

[41] D. Karabali and H.J. Schnitzer, Nucl. Phys. B329 (1990) 625
[42] K. Gawedzki and A. Kupiainen, Nucl. Phys. B\textbf{320} (1989) 649

[43] A.M. Polyakov and A.B. Zomolodchikov, Mod. Phys. Lett. A\textbf{3} (1988) 1213

[44] A.M. Polyakov, Phys. Lett. B\textbf{103} (1984) 207

[45] A.M. Polyakov, Phys. Lett. B\textbf{103} (1984) 211

[46] A.M. Polyakov, Gauge Fields and Strings. Harwood Academic Publishers (1987)

[47] D. Friedan, E. Martinec, and S. Shenker, Nucl. Phys. B\textbf{271} (1986) 93

[48] R. Dijkgraaf, E. Verlinde, and H. Verlinde, Nucl. Phys. B\textbf{352} (1991) 59

[49] M. Wakimoto, Commun. Math. Phys. \textbf{104} (1986) 405

[50] B.H. Lian and G.J. Zuckerman, Commun. Math. Phys. \textbf{135} (1991) 547

[51] P. Bouwknegt, J. McCarthy, and K. Pilch, preprint CERN-TH 6646/92, to be published in the proceedings of XXV Karpacz winter school of theoretical physics, Karpacz 17-27 February 1992

[52] P.J. Hilton and U. Stammbach, A course in homological algebra. Berlin, Heidelberg, New York: Springer 1970

[53] R. Bott and L. Tu, Differential forms in algebraic topology. Berlin, Heidelberg, New York: Springer 1982

[54] N. Ishibashi, Nucl. Phys. B\textbf{379} (1992) 199

[55] P. Bouwknegt, J. McCarthy, and K. Pilch, preprint CERN-TH 6645/92, to be published in the proceedings of the 1992 Trieste Spring School and Workshop on “String Theory”.

[56] P. Bouwknegt, J. McCarthy, and K. Pilch, Nucl. Phys. B\textbf{377} (1992) 541

[57] J.B. Fan and M. Yu, work in progress.