Demonstration of 3-port grating phase relations

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We experimentally demonstrate the phase relations of 3-port gratings by investigating 3-port coupled Fabry-Perot cavities. Two different gratings which have the same 1st order diffraction efficiency but differ substantially in their 2nd order diffraction efficiency have been designed and manufactured. Using the gratings as couplers to Fabry-Perot cavities we could validate the results of an earlier theoretical description of the phases at a three port grating (Opt. Lett. 30, p. 1183). © 2008 Optical Society of America

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Conventional interferometers rely on splitting and recombining optical fields with partly transmissive beam splitters. When transmission through optical substrates is disadvantageous, diffractive reflection gratings can also serve as beam splitters allowing for all-reflective interferometry. As long as the grating splits an incoming beam into two outgoing beams the phase relation at the grating and hence the properties of the interferometer built thereof are analogous to the well known ones of a transmissive 2-port beam splitter. If, however, a diffractive beam splitter has more than two orders, the mirror analog and thus the simple phase relation no longer hold. Yet, a knowledge of these relations at the diffractive beam splitter is the essential premise for an understanding of multiple port interferometry. In a recent experiment a grating in 2nd order Littrow mount was used to couple light into a Fabry-Perot cavity. In this case the incoming beam was split into three outgoing beams. The phase relations at the so-called three-port grating were analyzed theoretically and the input-output relations for a Fabry-Perot cavity with a three-port coupler were derived. Theoretical investigation of the phases was solely based on energy conservation and reciprocity of the device but an experimental validation of the results has not yet occurred.

In this letter we report on an experiment that was performed to demonstrate the phase relations of optical 3-port devices. Two different gratings were designed and manufactured for this purpose, and used as couplers to Fabry-Perot interferometers.

Phase relations for 3-port gratings with equal diffraction efficiencies in the ±1st orders can be written as

\[
\begin{align*}
\phi_0 & = 0, \\
\phi_1 & = -(1/2) \arccos[(\eta_1^2 - 2\rho_0 \eta_0)/(2\rho_0 \eta_0)], \\
\phi_2 & = \arccos[-\eta_1^2/(2\rho_2 \eta_0)].
\end{align*}
\]

where \(\phi_0, \phi_1, \) and \(\phi_2\) are the phase shifts for 0th, 1st, and 2nd diffraction orders respectively. Interestingly, the coupling phases depend on the coupling amplitudes which are given by \(\eta_0, \eta_1, \) and \(\eta_2, \) again, for the 0th, 1st, and 2nd diffraction orders respectively, and by \(\rho_0\) for the normal incidence reflectivity of the grating.

Direct measurements of beam splitter phase relations are difficult. If, however, the 3-port beam splitter is used to couple light into a cavity, the cavity properties can be used to validate them. Fig. 1 shows the optical layout of a Fabry-Perot interferometer with a 3-port grating coupler. The grating is used in 2nd order Littrow mount and light from a laser source is coupled to the interferometer via the grating’s 1st order. The field amplitudes of the back reflected light (\(c_1\)) and forward reflected light (\(c_3\)) result from interference of the input field with the intra-cavity field and directly depend on the phase relations between the grating ports. In Ref. amplitude reflection coefficients for \(c_1\) and \(c_3\) as well as the amplitudes for the intra-cavity field (\(c_2\)) and the transmitted field (\(t\)) were derived and are repeated here for convenience:

\[
\begin{align*}
c_1 & = \eta_2 \exp(i\phi_2) + \eta_1^2 \exp[2i(\phi_1 + \phi)]d, \\
c_2 & = \eta_1 \exp(i\phi_1) d, \\
c_3 & = \eta_0 + \eta_1^2 \exp[2i(\phi_1 + \phi)]d, \\
t & = i\tau c_2 \exp(i\phi),
\end{align*}
\]

where the amplitude reflectance and transmittance of the...
cavity end mirror are given by $\rho_1$ and $\tau_1$ respectively. The resonance factor is given by $d = [1 - \rho_0 \rho_1 \exp(2i\phi)]^{-1}$ and the length $L$ of the cavity is expressed by the tuning parameter $\phi = \omega L/c$, where $\omega$ is the angular frequency and $c$ the speed of light.

One distinct feature of this type of grating cavity is that the grating phase relations allow for reflection coefficients (as a function of $\phi$) that are not symmetric to the detuning of the cavity. Fig. 2 shows the calculated power back reflectance $|c_1|^2$ for a cavity with coupling $\eta_1^2 = 0.1$ and an end mirror with $\rho_1 = 1$ as a function of cavity tuning $\phi$ for selected values of 2nd order diffraction efficiency $\eta_2^2$.

Fig. 2. (Color online) Calculated power back reflectance $|c_1|^2$ for a cavity with coupling $\eta_1^2 = 0.1$ and an end mirror with $\rho_1 = 1$ as a function of cavity tuning $\phi$ for selected values of 2nd order diffraction efficiency $\eta_2^2$. In all cases shown the cavity finesse is the same. For an ideal (lossless) grating the finesse depends on the 1st order diffraction efficiency $\eta_1 = [(1 - \rho_0)/2]^{1/2}$ only. For the minimal 2nd order diffraction efficiency $\eta_2, \eta_{2,\text{min}} = (1 - \rho_0)/2$ all the light is reflected back towards the laser source if the cavity is on resonance ($\phi = 0 \mod \pi$). However, for maximal 2nd order diffraction efficiency $\eta_{2,\text{max}} = (1 + \rho_0)/2$ no light is reflected back from a resonating cavity. Hence for the extremal values of $\eta_2$ the back reflected port behaves exactly like the reflection port or the transmission port of a conventional impedance matched two mirror Fabry-Perot cavity. For intermediate values of $\eta_2$ the power reflectance is no longer symmetric to the $\phi = 0$ axis and the resonance peaks are not of the usual Airy form as can be seen for the two exemplary curves $\eta_2^2 = 0.15$ and $\eta_2^2 = 0.8$, in Fig. 2.

To verify the grating behavior, two gratings with essentially the same 1st order diffraction efficiency but substantially different 2nd and hence 0th order diffraction efficiency were designed and manufactured. The gratings use a binary structure written into the top layer of a dielectric multilayer stack consisting of Ta$_2$O$_5$ and SiO$_2$ placed on a fused silica substrate. We chose a grating period of 1450 nm which corresponds to a 2nd order Littrow angle of 47.2° for the Nd:YAG laser wavelength of $\rho = 1064$ nm used. A rigorous coupled wave analysis was performed to design the grating. The ridge width is $\pi/2$ and the top layer consists of 880 nm of SiO$_2$. Fig. 3 shows the calculated diffraction efficiencies for all three diffraction orders in 2nd order Littrow mount as a function of groove depth. The gratings were produced by ultrafast high-accuracy electron beam direct writing (electron beam writer ZBA23h from Leica Microsystems Jen GmbH) and etched by means of reactive ion beam etching. The etching process was stopped after reaching a groove depth of 500 nm (G1) and 850 nm (G2) respectively.

A sketch of the experimental setup used to verify the grating phase relations is shown in Fig. 4. A beam of a diode pumped Nd:YAG non planar ring oscillator (Model Mephisto from Innolight GmbH) was spatially filtered with a triangular ring cavity. The grating (either G1 or G2) was illuminated at 2nd order Littrow angle and a cavity end mirror with $\tau_1^2 = 300$ ppm was placed parallel to the grating’s surface. The cavity length could be controlled by a piezoelectric transducer (PZT) and the three ports of interest were monitored by photodetectors.
Figs. 5 and 6 show the measured signals from the three photodetectors for linear cavity scans over one free spectral range (FSR) using G1 and G2 respectively. Also shown are the theoretical curves \(|c_1(\phi)|^2\), \(|c_2(\phi)|^2\), and \(|t(\phi)|^2\) which were obtained from Eqs. (4), (6), and (7) using measured efficiencies of the two gratings. Coupling to the cavity was measured to be identical for both gratings within the measurement accuracy of about 5% of the power meter used: \(\eta^2_1(G1) = \eta^2_1(G2) = 0.10\). For the first grating a value of \(\eta^2_2(G1) = 0.15\) and for the second one a value of \(\eta^2_2(G2) = 0.10\) was measured. The remaining values were calculated using the identities \(\eta_0^2 + \eta_1^2 + \eta_2^2 = 1\) and \(\rho_0^2 + 2\eta_1^2 = 1\). We found the calculated values within the error bars of direct measurements.

Figs. 5 and 6 show that the theoretical and measured curves agree very well. The interference at the 3-port gratings could therefore be well described by the phase relations according to Eqs. (1)–(3). The small deviations are possibly due to imperfect mode matching, and losses at the grating caused by transmission, scattering, and diffraction from periodic grating errors. As predicted, the measured intensities in the reflecting ports showed the asymmetric behavior around cavity resonances.

In conclusion, we have designed and manufactured two diffraction gratings which allowed the construction of grating-coupled Fabry-Perot cavities with the same finesse but with totally different properties of the two reflected ports, thereby confirming the phase relations that were theoretically derived earlier. Our experimental results could be fully described by phase relations based on energy conservation and reciprocity and the knowledge of the grating’s diffraction efficiencies. No further information about the gratings was required.

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