Reduced spin measurement back-action for a phase sensitivity ten times beyond the standard quantum limit

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Fundamental quantum noise limits the precision of quantum-based detectors, for example limiting the ultimate precision of atomic clocks, which have applications in communication, navigation and tests of fundamental physics. Collective measurements of many quantum spins can project the ensemble into an entangled, spin-squeezed state with improved quantum-limited measurement resolution. However, measurement back-action has limited previous implementations of collective measurements to only modest observed enhancements in precision. Here, we experimentally demonstrate a collective measurement with reduced measurement back-action to produce and directly observe, with no background subtraction, a spin-squeezed state with phase resolution improved by a factor of 10.5(1.5) in variance, or 10.2(6) dB, compared to the initially unentangled ensemble of \( N = 4.8 \times 10^5 \) Rb atoms. The measurement uses a cavity-enhanced probe of an optical cycling transition, mitigating back-action associated with state-changing transitions induced by the probe. This work establishes collective measurements as a powerful technique for generating useful entanglement for precision measurements.

A defining characteristic of quantum mechanics is the ability of a measurement to change the state of the system being measured. For example, a measurement of a system in a superposition of two states causes the system to project, or collapse, into one of the two discrete states. Measurements performed on an ensemble, however, can project the ensemble into an entangled state when only collective quantities are measured. In contrast, any information about the spin state of a single atom that leaks to the environment due to imperfections in the collective measurement reduces entanglement due to the collapse of the quantum spin state of individual atoms. Collective or joint measurements arise in a wide range of applications, including quantum teleportation, quantum information protocols, studies of strongly correlated quantum systems, Dicke superradiance and entanglement generation in optical, solid-state and atomic systems.

Entanglement generated by a collective measurement can be used to overcome the fundamental quantum randomness that limits a diverse set of precision measurements. Atomic sensors in particular are nearly or already limited by atomic quantum noise, so entanglement-enhanced metrology may improve some of the most precise measurements of external fields, rotations and time, and will advance searches for new physics.

Atomic sensors encode information in a quantum phase, the value of which is estimated by measuring the population of atoms in different quantum states. Quantum projection noise for an ensemble of \( N \) independent atoms limits the uncertainty in the estimate of \( \theta \) to a variance \( \Delta \theta^2 \geq \Delta \theta_{\text{SQL}}^2 = 1/N \), a limit known as the standard quantum limit (SQL) for a coherent spin state (CSS). Entanglement can be used to bypass this limitation in atomic sensors, as well as in microwave and optical fields.

A collective measurement that both resolves the quantum noise that appears in \( \theta \) and also induces sufficiently small measurement back-action can be used to subtract quantum noise from subsequent measurements of \( \theta \) (ref. 8). Although the resulting state of the ensemble is term a conditional spin-squeezed state, the reduction in noise is completely deterministic with no discarding of trials necessary. The improvement in phase estimation relative to the SQL is quantified by the observed spectroscopic enhancement, \( W^{-1} \equiv (\Delta \theta_{\text{SQL}}/\Delta \theta)^2 \). \( W^{-1} > 1 \) also guarantees that the state is entangled.

First proof-of-principle experiments using collective pre-measurements have shown that entanglement can enhance the sensitivity of an atomic ensemble. However, measurement-induced back-action, including loss of sensitivity due to decoherence and noise due to population diffusion between states, has limited the direct observations of spectroscopic enhancement to only \( W^{-1} \leq 1.4 \) (ref. 20). Such a small spectroscopic enhancement is probably not worthwhile for application in precision measurements.

In this Article, we utilize cavity-based, collective pre-measurements with reduced back-action by probing the atoms on a cycling transition to achieve a directly observed spectroscopic enhancement \( W^{-1} = 10.5(1.5) \) in an ensemble of \( N = 4.8 \times 10^5 \) Rb atoms (Fig. 1a). The techniques demonstrated here are well suited to state-of-the-art optical lattice clocks, which have even more favourable cycling transitions than the one exploited here. We emphasize that our result reflects no background subtraction or corrections for finite probe detection efficiency. Reporting the actual observed spectroscopic enhancement is critical for establishing that entangled states are more than a physical novelty and may actually be applied to enhance real measurements. In previous work, performing background subtractions would often tease out physically interesting quantum effects, while the actual spectroscopic enhancement was relatively small.

Entanglement-enhanced states have also been generated for small (\( N \leq 14 \)) collections of atomic ions using quantum-logic operations and for larger neutral atomic ensembles using atomic...
collisions\textsuperscript{27-32} or cavity-mediated optical feedback\textsuperscript{33}. Directly observed enhancements of up to $W^{-1} = 5.0(1)$, $7(1)$ and $3.6(5)$ (at $N = 8$, $4.5 \times 10^2$ and $3 \times 10^2$) have been realized using these respective approaches\textsuperscript{25,32,33}. Although the largest observed spectroscopic enhancement reported prior to our work was obtained in squeezed states generated with collisional interactions, applying this entanglement to perform enhanced measurements remains to be explored. Unless the collisional interactions can be precisely extinguished, the collisional interactions that generate the squeezing can also result in systematic errors of the measured quantity and the loss of the useful entanglement during the sensing phase. Using an optical approach, potential perturbations are only present during the pre-measurement.

**Experimental system and collective measurement**

Our experimental system\textsuperscript{34} consists of an ensemble of N pseudo-spin-1/2/\textsuperscript{34} formed by the hyperfine ground states $|\uparrow\rangle \equiv |F = 2, m_y = +2\rangle$ and $|\downarrow\rangle \equiv |F = 1, m_y = +1\rangle$ in $^{87}$Rb, separated by 6.8 GHz. The quantum state of the ensemble can be approximated as a single collective spin or Bloch vector $J \equiv \langle J \rangle$, in an abstract space defined by the collective spin operator $J = J_x \hat{x} + J_y \hat{y} + J_z \hat{z}$ (Fig. 1b; see Supplementary Section I). The spin projection operator $J_z = N_z - N/2$ can be written in terms of the measurable quantities: the total atom number $N$ and the spin-up population $\hat{N}_z = \Sigma_{i=1}^m |\uparrow_i\rangle \langle \uparrow_i|$, where $i$ labels individual atoms. The length of the vector is $J = \langle J \rangle$. For an unentangled CSS, $J = N/2$.

The quantum projection noise and standard quantum limit can be understood as arising from uncertainty in the orientation of the Bloch vector (Fig. 1b). This quantum uncertainty can be visualized as a quasi-probability distribution perpendicular to the mean vector. When the Bloch vector is oriented along $\hat{x}$, the degree of uncertainty in the orthogonal spin projections is constrained by a Heisenberg uncertainty relationship $\Delta J_x \Delta J_y \geq \hbar / 4$, where $\Delta \hat{x}$ indicates the standard deviation of repeated measurements of $X$. For a CSS of atoms $\Delta \hat{x} = \Delta \hat{y} = \Delta N_{\text{CSS}} = \sqrt{N}/2$, and so the SQL for the angle $\theta \approx J_z / J = 2N_z / N - 1$ is set by the projection noise fluctuations to $\delta J_z / \langle J \rangle = 1 / \sqrt{N}$.

The conditioned squeezed state is created by first preparing a CSS along $\hat{x}$ and then making a collective pre-measurement of $\hat{N}_z$, with measurement outcome labelled $N_{\text{up}}$, and subtracting the result from a subsequent final measurement of $\hat{N}_z$, labelled $N_{\text{fl}}$. The differential quantity $N_{\text{fl}} = N_{\text{up}}$ can possess reduced noise relative to the projection noise fluctuations $\Delta N_{\text{CSS}}$, appearing in the two separate measurements. The spin noise reduction is calculated as $R = (\Delta N_{\text{fl}} / \Delta N_{\text{CSS}})^2$. By making a collective or joint measurement, any rotation of the vector’s polar angle $\theta$ that occurs between the two measurements will still modify the differential quantity $N_{\text{fl}} = N_{\text{up}}$, leading to the desired enhancement in the estimation of applied phase shifts.

To measure the collective state population $N_z$, the atomic ensemble is coupled to the TEM$_{00}$ mode of an optical cavity. The coupling is characterized by an effective single-atom Rabi frequency $2g = 2\pi \times 894(46)$ kHz. The details of inhomogeneous coupling to the probe in our standing-wave cavity are handled as in refs 19 and 21. With no atoms present, the cavity has a resonant frequency $\omega_c$ and decay ratio $\kappa = 2\pi \times 11.8(1)$ MHz. We detune the cavity frequency from an atomic transition by $\delta = \omega_c - \omega_a = 2\pi \times 200$ MHz, where $\omega_a$ is the frequency of the atomic transition from $|\uparrow\rangle$ to an optically excited state $|e\rangle \equiv |F = 3', m_y = +3\rangle$. The radiative decay rate of $|e\rangle$ in free space is $\Gamma = 2\pi \times 6.07$ MHz. Atoms in $|\uparrow\rangle$ produce a dressed atom-cavity resonance at frequency $\omega_{ac}$ such that $\omega_{ac} - \omega_c = \sqrt{4\kappa^2 + 4g^2|N_z|^2 - \delta^2}/2$ (Fig. 1c; see Supplementary Section II). We measure $\omega_{ac}$ with a probe laser (frequency $\omega_p$) to determine $N_z$. The strength of the collective measurement is characterized by the average number of probe photons $M$, transmitted through the cavity. The probe provides collective population information corresponding to the average total number of atoms in $|\uparrow\rangle$, without providing individual atomic state information.

The information gained from a pre-measurement of $\hat{N}_z$ causes back-action on the system, as illustrated in Fig. 1d. First, the measurement reduces the collective spin projection uncertainty to $\Delta J_z = \Delta N_{\text{fl}}$, where $\Delta N_{\text{fl}}$ is the measurement imprecision. The Heisenberg uncertainty relationship requires fundamental back-action to appear in the orthogonal spin projection $\Delta J_x \geq (N/4) / \Delta N_{\text{fl}}$, referred to as anti-squeezing. Because $J_z$ is not coupled to the back-action quadrature $J_y$, the ideal measurement is intrinsically back-action evading\textsuperscript{31}.

However, real systems experience at least two additional sources of probe-induced back-action, which are also illustrated in Fig. 1d.
Both are caused by photons spontaneously scattered from the probe into free space, with the average number of scattered photons \( M_{\Delta} \) scaling linearly with the measurement strength \( M_{\Delta} \propto M_{\Delta} \). One source of back-action arises from free-space scattered photons leaking single-atom information to the environment, projecting an individual atom into \( |\uparrow\rangle \) for every free-space scattered photon. For a Bloch vector oriented along \( \hat{x} \) as we consider here, this is equivalent to a shortening of the Bloch vector length \( \beta \) such that a subsequent angular deflection \( \theta \) will produce a reduced change of the measured population \( N_{11} \).

Another source of probe-induced back-action is spontaneous Raman transitions between ground states driven by the same free-space scattering. Quantum randomness in the number of transitions between states adds noise to the measurement of \( N_{11} \) as the population diffuses amongst ground states. The added noise \( \Delta N_{1p} \) scales as \( (\Delta N_{1p})^2 \propto p M_{\Delta} \), where \( p \) is the probability for an atom to undergo a ground-state changing transition when it scatters a photon into free space. The optimum spin-noise reduction \( R \) is fundamentally limited by the need to balance the decrease in measurement noise \( \Delta N_{1m} \propto 1/\sqrt{M_{\Delta}} \) versus the increase in diffusion noise \( \Delta N_{1p} \propto \sqrt{M_{\Delta}} \). This balancing is analogous to radiation pressure back-action, which sets the SQL for measurements of mechanical position. 

The key experimental advance in this work is the elimination of state-changing transitions as a limitation on \( W \). This is achieved by creating a system in which collective coupling to the probe mode is enhanced relative to single-atom processes. Our approach uses a medium finesse \( F = 660 \) optical cavity to enhance the collective coupling, with the figure of merit \( NC \approx 6 \times 10^5 \), where \( C = 1.1(1) \times 10^{-2} \) is the single-atom cooperativity. In addition, we suppress state-changing transitions by using a \( \sigma^+ \) polarized probe light on a cycling transition. Thus, the probability \( p \) is reduced by a factor of 150 from that of our previous work. As a result, previously ignored noise sources, not yet fully understood (see Supplementary Section VI), now dominate the probe-induced back-action on measurement \( N_{11} \).

**Direct observation of spectroscopic enhancement**

In Fig. 2 we directly sense an externally applied phase shift with a resolution significantly below the SQL. We apply a small rotation \( \psi \) of the polar angle \( \theta \) using a microwave pulse, as described in Supplementary Section III. In one case, the rotation is applied to a CSS with no pre-measurement. In the second case, the rotation is applied just after the pre-measurement of \( N_{11} \), which prepares a conditional spin-squeezed state. The deflection of \( N_{11} \) is slightly smaller for the spin-squeezed state due to probe-induced collapse during the pre-measurement. However, the reduction in noise in the quantity \( N_{11} - N_{1p} \) allows the rotation angle to be estimated with an enhancement \( W^{-1} = 7.5(9) \), even though the change in \( N_{11} - N_{1p} \) is slightly smaller than in the CSS, shown by the dashed black lines. The slight reduction in signal is represented as the smaller Bloch sphere for the spin-squeezed state.

**Figure 2 | Detection of a quantum phase with entanglement-enhanced sensitivity.** We apply a small rotation \( \psi \) to the polar angle \( \theta \) of both a CSS and a spin-squeezed state, with data and representative Bloch vectors shown on the left and right, respectively. Bloch vectors are not shown to scale. Red data points show experimental trials with \( \psi = 2.3(1) \) mrad and blue data points show trials with \( \psi = 0 \). The data are represented both as histograms and Gaussian curves generated from the average and standard deviation of the measurements. The experimental timing sequence consists of probe pulses (black) and microwave rotations of the Bloch vector polar angle in radians (green). The first measurement of \( N_{11} \) is labelled \( N_{11} \) as it corresponds to the total population in \( |\uparrow\rangle \) after the microwave pulse swaps the populations, and gives the total atom number \( N = N_{11} + N_{1p} \). For the CSS, the rotation \( \psi \) is applied immediately after preparing the CSS along \( \hat{x} \). The rotation \( \psi \) appears as a change in the quantity \( N_{11} - N_{1p} \), which is normalized to the total projection noise that appears in this differential quantity. In the case of the spin-squeezed state, we perform the rotation \( \psi \) after a pre-measurement \( N_{1p} \). The rotation then appears as a change in \( N_{11} - N_{1p} \) where the projection noise largely cancels. The spin-squeezed state has a spectroscopic enhancement of \( W^{-1} = 7.5(9) \), even though the change in \( N_{11} - N_{1p} \) is statistically consistent with zero. After the microwave rotation \( \phi \), the probe-induced rotation \( \psi \) is reduced from 0.27(1) without the \( \phi \)-pulse and 0.23(1) with \( \phi \)-pulse, and blue data points show trials with \( \phi \). We apply a small rotation \( \phi \) against the increase in diffusion noise \( \Delta N_{1p} \) alone would limit \( R \). The rise in classical back-action increases \( M_{\Delta} \) of the polar angle \( \hat{r} \) is fun-

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probe-induced optical pumping between different ground states (see Supplementary Section V). Also, the inferred contribution to $r_\ell$ due to the observed classical fluctuations in probe power would only limit $R^{-1}$ to $3.2(4) \times 10^4$. The equivalent transition probability is $p \leq 4.4(8) \times 10^{-3}$. For comparison, the clock states in our previous work had a transition probability $p = 2/3$, which, in our current system, would limit $R^{-1}$ to $1.9(2)$. With population noise considerably suppressed, other sources of back-action, including optomechanical effects (Fig. 3b), appear to dominate the probe-induced back-action on $R$.

Free-space scattering also leads to a reduction in the Bloch vector length $J$ and the resulting loss of signal must be accounted for to determine the spectroscopic enhancement. To determine $J$, the polar angle of the Bloch vector $\theta$ is varied after the pre-measurement $N_{\text{tr}}$, using a microwave pulse. The population $N_\ell$ is then recorded versus the rotation angle (Fig. 3c). The fractional reduction in length of the Bloch vector is determined from the fitted contrast $C = 2J/N$ of the observed fringe. The initial contrast at $M_\ell = 0$ is $C'_\ell = 0.97(3)$ and $C$ monotonically decreases as a function of $M_\ell$, close to the limit from wavefunction collapse due to free-space scattering (Fig. 3a). We believe uncancelled inhomogeneous probe light shifts are responsible for the additional small loss of contrast (see Supplementary Section IV).

Taken together, the decrease in spin noise and loss of contrast quantify the spectroscopic enhancement of the spin-squeezed state $W^{-1} = R^{-1} C^2/C'_\ell$, as calculated in refs 18, 19 and 21. The optimum observed improvement corresponds to $W^{-1} = 10.5(1.5)$ or $10.2(6)$ dB. This value includes no measurement background noise subtraction, and thus represents the actual realized improvement in phase sensitivity.

Further confirmation that our collective measurement is near the cycling transition limit is the observed $\Delta B^2 \propto 1/N^2$ scaling of the absolute phase resolution$^{18,34}$ (Fig. 4). For comparison, the optimal phase resolution when state-changing processes are the dominant limitation on $W$ scales as $N^{-1/2}$. This more favourable scaling with $N$ is important in practical applications for which the absolute phase resolution is the key figure of merit.

**Discussion**

When the spectroscopic enhancement $W^{-1}$ is $\geq 1$, the ensemble is guaranteed to be entangled. Maximally entangled ensembles can achieve a phase estimation precision of $\Delta B^2_{\text{HL}} = 1/N^2$, known as the Heisenberg limit, which has been realized with small ensembles$^{25}$. Although our system is far from the Heisenberg limit for $N = 4.8 \times 10^5$ atoms, the absolute phase sensitivity is equivalent to $\sim 44,000$ copies of an entangled 11-atom ensemble at the Heisenberg limit. Such a comparison emphasizes the massive parallelism achievable by collective measurements to generate entanglement in neutral atom ensembles.

The optical nature of our approach, among other experimentally demonstrated$^{18-20,22,33}$ and theoretically proposed$^{38}$ approaches, offers the advantage that the probe or squeezing laser can be completely extinguished after the squeezed state is generated. In contrast, a potential challenge for squeezing generated using atomic collisions is whether the interactions that generate entanglement can be sufficiently reduced to avoid loss of accuracy and precision during the subsequent sensing period. For this reason, our approach may be compatible with a wide array of atomic sensors, and may be particularly well suited for optical lattice clocks$^{11,12}$, which have already begun to incorporate optical cavities$^{39}$. Although state-of-the-art
optical lattice clocks\textsuperscript{11,12} use smaller atom numbers and densities than are used in this work in order to avoid density-dependent systematic errors, it should be possible to achieve the same amount of enhancement demonstrated here. Approaches may include utilizing higher-finesse cavities, achieving a higher probe detection efficiency, or utilizing a three-dimensional lattice trap\textsuperscript{50} to scale up to larger atom numbers, which has the added benefit that the standard quantum limit on phase resolution would also be reduced. Finally, the low-noise, non-destructive readout method is useful for purely classical, but substantial, improvements in the measurement duty cycle of lattice clocks and other atomic sensors\textsuperscript{41}.

Straightforward technical improvements could both decrease the technical measurement noise floor and increase the total probe detection efficiency from 0(5\%) to >50\%, allowing us to reach \(N^\ast \approx 100\) in our current system with only a medium-finesse optical cavity. In addition to demonstrating a large spectroscopic enhancement relevant for state-of-the-art atomic sensors, the results here also uncover previously unimportant forms of probe-induced back-action, such as heating of the atomic motion. Further study of these forms of probe-induced back-action will likely be required to realize even further spectroscopic enhancement.

\textit{Note added in proof}: After completion of this manuscript, a paper inferring comparable spectroscopic enhancement via atomic collisions was reported\textsuperscript{42}. A modest background subtraction was applied to infer the reported value.

Received 12 February 2014; accepted 11 June 2014; published online 13 July 2014

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Figure 4 | Absolute phase sensitivity versus \(N\). Red points are the observed variance in phase estimation for spin-squeezed states, optimized with respect to measurement strength \(M_{\text{opt}}\) for different atom numbers \(N\). The data show the predicted \(1/N^2\) scaling for probing on a cycling transition\textsuperscript{36,36}, equivalent to a linear scaling of the spectroscopic enhancement \(W^{-1}\) versus \(N\), shown in the inset. The black line is a linear fit to the data. The SQL is confirmed by measuring the projection noise that appears in \(N_{\text{opt}} – N_{\text{opt}}\) (black points, each 100 trials) and observing \(1/N\) scaling of the variance. Error bars indicate 68.3\% confidence intervals.
This material is based upon work supported by the National Science Foundation (grant no. 1125844).

Author contributions
J.G.B., K.C.C., M.A.N. and J.M.W. designed and performed experiments and analysed data. Z.C. provided analytic support. J.K.T. designed the experiment and analysed data. J.G.B. wrote the manuscript and all authors provided feedback for the manuscript.

Additional information
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Competing financial interests
The authors declare no competing financial interests.