SYSTEMS & CONTROL | RESEARCH ARTICLE

An algorithm to estimate parameters and states of a nonlinear maneuvering target.

S.N. Hosseini¹, M. Haeri²* and H. Khaloozadeh³

Abstract: This paper investigates the problem of unknown input estimation such as acceleration, target class, and maneuvering target tracking using a hybrid algorithm. One of the challenges of unknown input estimation is that no effective method has been presented so far that could be applied to general cases. The available methods are ineffective when the range of variation of the unknown input parameter is large. Also, the issue of determining the system class could improve the performance of the tracking algorithms in many applications. Using the Bayesian theory, the posterior distribution functions of state and parameter could be obtained concurrently. In the proposed algorithm, Liu and West and multimode filters are used for unknown parameters’ estimation, and particle filter is used to estimate the posterior density function. Parameter estimation and mode determination could be used in the resampling phase to weight the particles in accordance with the target mode. The main advantage of the adaptive parameter estimation approach is its ability to provide a quick estimation of the abruptly changing parameters from noninformative prior knowledge and to do this for multiple unknown

ABOUT THE AUTHORS

S. N. Hosseini received his B.Sc. and M.Sc. degrees both in Electrical Engineering from Islamic Azad University in 2002 and 2006. He got his Ph.D. in Electrical Engineering from Science and Research University, Tehran, Iran, in 2020. His research interests include spread spectrum FH, adaptive array processing, maneuverings target tracking algorithms, and intelligent signal processing.

M. Haeri received his B.Sc. and M.Sc. degrees both in Electrical Engineering, from Amirkarib University of Technology in 1985 and 1988, respectively. He got his Ph.D. in Electrical Engineering from the University of Saskatchewan, Canada, in 1994. His research areas include the theory and application of model predictive control, and modeling of physiological systems.

HR. Khaloozadeh received the B.Sc. degree in control engineering from Sharif University of Technology in 1990, the M.Sc. degree in control engineering from K.N. Toosi University of Technology, in 1993, and the Ph.D. degree in control engineering from Tarbiat Modares University, in 1998. His interest area is maneuvering target tracking algorithms, stochastic estimation, and time series analysis.

PUBLIC INTEREST STATEMENT

Target tracking is a critical problem in both military and civilian applications. The main purpose of a tracking system for traffic control or air defense is the estimation of target trajectories in the controlled area and their prediction into the near future. In a civilian air traffic control system, the estimated trajectories (tracks) are used to check the standard separation between pairs of targets for maintenance of safety conditions and regularity of traffic flow. In an air defense system, the estimated trajectories are generally used to perform some of the following functions: threat identification, threat evaluation, and calculation of the predicted position.

This paper investigates the problem of unknown input estimation such as acceleration, target class, and maneuvering target tracking using a hybrid algorithm. The available methods are ineffective when the range of variation of the unknown input parameter is large. Determining the system class could improve the performance of the tracking algorithms in many applications.
parameters. Simulation results show that the proposed algorithm performs better than the other input estimation and tracking methods.

**Subjects:** Mathematics & Statistics for Engineers; Systems & Control Engineering; Electrical & Electronic Engineering

**Keywords:** Maneuvering target tracking; particle filter; posterior distribution function; multimode particle filter method; state and input estimation; Liu and West filter

1. Introduction

The problems of estimating state variables of a nonlinear dynamic system and target tracking are the most important issues in the area of traffic control, robotics, surveillance, and defense systems. Generally, the target tracking means to estimate the target state from the noise observations. Input estimation is one of the state estimation methods that can be used to track maneuvering targets with nonlinear dynamics (Popoli & Blackman, 1999).

In (Khaloozadeh & Karsaz, 2008), two different approaches for tracking of targets with unknown maneuvers are investigated, including the model-based adaptive filtering and input estimation. Also, in order to overcome the problems of the input estimation method, the MIE (Modified Input Estimation) and EIE (Enhanced Input Estimation) methods are proposed in this study.

An input estimation-based method is presented in (Rahmati et al., 2012) for tracking of maneuvering targets. The proposed model is comprised of two Bayesian and Fisher models. In this method, acceleration is considered as an additional input in state equations. The main idea of the proposed method is to convert the maneuvering target problem into a no-maneuver target problem through an additive state to ensure a standard Bayesian model. By adding unknown inputs to the state vector, a higher-order state-space model is obtained and estimation of state and unknown inputs is done simultaneously (Ranhnama & Arvan, 2011).

The main issue with tracking maneuvering targets is the mismatch between the motion model and the real target dynamics. Singer published the first paper on maneuvering target tracking with human intervention, such as fighters, and presented various methods to model parametric and nonparametric motion models (Singer, 1970). According to the Singer model, the target’s acceleration could be represented by a random process with exponential autocorrelation function. In order to improve the tracking of highly maneuverable targets, the MF (Multi-parameter Fusion) Singer model is provided in (Jia et al., 2017). In this algorithm, the target’s velocity and acceleration are estimated using the fuzzy logic method. Then, the output of the fuzzy logic model is used to adjust the filter covariance matrix.

In order to describe the sudden changes in the system state space, a random variable named “system mode” is defined. In maneuvering target tracking, we aim to estimate unknown parameters (mode) with sudden changes, which are usually modeled as a finite-state Markov random process. In other words, this target parameter changes between some different modes according to the Markov chain. When the target does not have high maneuverability (high range of parameters’ changes), it has been proven that Kalman Filter (KF) can track the target with a reasonable approximation (Pulford, 2015).

In (Liu & West, 2001), a filter is presented that uses Kernel distribution to estimate the target parameter. In (Khaloozadeh & Karsaz, 2008), a modified input estimation method is proposed for maneuvering targets. This is achieved through the addition of target acceleration to the state vector. The main idea of this work is to convert the maneuvering state to the no-maneuver state by using input estimation. Then, KF is used for tracking purpose. In (Malekian Heraband & Khaloozadeh, 2016), a method based on AEKF (Augmented Extended KF) is provided for input estimation of nonlinear maneuvering targets in the presence of multiplicative noise.
In target tracking algorithms, in addition to a proper choice of observation mode, a proper model of target dynamics is also needed; because in some applications, the dynamic model of target motion is not constant in the course of time. In order to overcome this problem, various methods such as IMM (Interacting Multiple Model) have been proposed. In an IMM algorithm, a set of state estimations are performed for dynamic models and the resulted estimations are combined with appropriate weights to create a global estimation.

The IMM method is proposed in (Chen et al., 2014; Youn & Myung, 2019) and it is shown that this method performs much better than the conventional KF-based methods in the presence of uncertainties in the states. In the IMM algorithm, a number of possible models are considered for the target state along with the probability of switching among these models. An IMM could be implemented as some parallel filters, each of which corresponds to one model of the several possible models. A combination of the particle filter and IMM method could improve the performance of the tracking algorithm effectively. One of the drawbacks of the IMM method is the increased complexity with increasing the number of the target states. In other words, by increasing the number of the system working modes, the number of the filters increases as well (Jing & Vadakkepat, 2010; Nemeth et al., 2014; Zhang et al., 2018). The sequential Monte-Carlo estimation methods are used in (Li & Jilkov, 2005) for input estimation as well as the tracking. In the sequential Monte-Carlo estimation methods, a probability distribution function is described by a number of points which are samples representing that distribution. These samples are produced randomly by an algorithm. Therefore, the differences between these methods and KF lie in the random sampling approach.

The aim of the present work is to use the Bayesian theory to obtain the posterior distribution function of state and parameter simultaneously. The L&W (Liu and West) filter and IMM are used in the proposed algorithm to estimate the unknown parameters and particle filter is employed to estimate the posterior density function. This paper considers the difficult problem of joint state and parameter estimation of nonlinear and highly dynamic systems. The paper presents a sequential Monte-Carlo filter that is capable of estimating parameters in the case of tracking maneuvering targets. The main advantage of the adaptive parameter estimation approach is its ability to provide a quick estimation of the abruptly changing parameters from noninformative prior knowledge and to do this for multiple unknown parameters. Its scalability to the case of estimating multiple unknown parameters is an advantage over filters such as the IMM which is based on a multiple model implementation.

This paper is organized as follows: In Section 2, formulations of the maneuvering target tracking are provided. In Section 3, the multimode method is presented and described. The proposed algorithm is introduced in Section 4. The simulation results are presented and discussed in Section 5 to demonstrate the performance of the proposed algorithm. Finally, the conclusions are provided in Section 6.

2. Modeling of maneuvering target tracking

Since the appropriate choice of state equations has a significant effect on the performance of the tracking algorithms, in this section, we introduce the modeling of the state-space equations used in this paper. The dynamics and measurement equations are expressed as

\[
\begin{align*}
x_{k+1} &= f_k(x_k) + w_k \\
z_k &= h_k(x_k) + v_k
\end{align*}
\]

where \( x_k \) is the state vector, \( f_k \) is the known nonlinear function, \( z_k \) is the result of the measurement at the \( k^{th} \) moment, \( w_k \sim N(0, Q) \) is the process noise with Gaussian distribution and zero mean and covariance matrix \( Q \), and \( v_k(f) \sim N(0, \sigma_v^2) \) is the additive measurement noise (Fu et al., 2015).

The state-space model (1) can be defined by two stochastic processes \( x_k \) and \( z_k \). The process \( x_k \) is referred to as a hidden Markov process representing the state of interest at the discrete time \( k \), which takes values on the measurable space \( x \in \mathbb{R}^n \). The stochastic process \( z_k \) represents the
observation process which takes values on the observation space \( z \in \mathbb{R}^n \), where observations are assumed to be dependent only on the current state \( x_k \) and independent of previous states \( x_{1:k-1} \), where \( x_{1:k-1} = \{ x_1, x_2, \ldots, x_{k-1} \} \). We also assume that these stochastic processes are conditional upon the parameter vector \( \theta \), and that there exists a prior distribution, \( p(\theta) \), for the parameter vector. The general state-space model is characterized by the densities:

\[
\begin{align*}
    x_k(x_{0:k-1}, z_{1:k-1}) & \sim p(x_k|x_{k-1}, \theta) \\
    z_k(x_k, z_{1:k-1}) & \sim p(z_k|x_k, \theta)
\end{align*}
\]  

\( (2) \)

where the state model is conditional only on the previous state and the observations \( z_k \) are independent of previous observations conditional only on the state \( x_k \) at the time \( k \). Where \( z_{1:k-1} \) denotes the measurements from time 1 to time \( k-1 \). In filtering, the aim is to estimate the hidden state at the time \( k \) given a sequence of observations. This process requires the evaluation of the posterior probability density function \( p(x_k|x_{1:k-1}, \theta) \) of the hidden state vector and parameter vector conditional on the observations Using Bayesian estimation techniques it is possible to evaluate the posterior density recursively by first predicting the next state

\[
p(x_k, \theta | z_{1:k-1}) = \int p(x_k|x_{k-1}, \theta)p(x_{k-1}, \theta | z_{1:k-1})dx_{k-1}
\]  

\( (3) \)

and then updating this prediction to account for the most recent observation \( z_k \),

\[
p(x_k, \theta | z_{1:k}) = \frac{p(z_k|x_k, \theta)p(x_k, \theta | z_{1:k-1})}{p(z_k | z_{1:k-1}, \theta)}
\]  

\( (4) \)

Where

\[
p(z_k | z_{1:k-1}, \theta) = \int p(z_k | x_{k-1}, \theta)p(x_k, \theta | z_{1:k-1})dx_k
\]  

\( (5) \)

is the normalizing constant.

Determining an analytic solution for the posterior distribution \( (4) \) is generally not possible due to the normalizing constant \( (5) \) being intractable. Generally, it is necessary to create an approximation of the posterior distribution, one such approach is through sequential Monte-Carlo methods, also known as particle filters.

Particle filters present a method for approximating a distribution using a discrete set of \( N \) samples/particles with corresponding weights \( \{ x_k, \theta, \omega_k \}_{k=1}^{N} \) which create a random measure characterizing the posterior distribution \( p(x_k, \theta | z_{1:k}) \).

3. The multimode method for maneuvering target tracking

In practical scenarios, the target mode usually changes with time. For example, a fighter that is moving with a constant velocity could randomly enter the maneuvering mode. Modeling this behavior in a constant manner is impossible. Therefore, in addition to estimating the target state, it is necessary to estimate its mode as well. One of the most commonly used algorithms provided for tracking of targets whose modes could change according to a finite state random sequence is the IMM method.

It is assumed that the current state of the system could be one of the \( n \) possible modes in the mode set \( M = \{ M_1, \ldots, M_n \} \). Additionally, by default, the initial probability of the target being in mode \( M_j \) is known and equal to \( \mu_j = p(M_j) \). Also, the probability of changing from one mode to another is modeled as a first-order Markov process, with a transition matrix whose entries are as given by

\[
p_{ij} = p(M_i | M_j^{-1})
\]  

\( (6) \)

Hence, the transition matrix is equal to (Zhan et al., 2015).
\[ p = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}. \]  

(7)

4. The proposed method of unknown input estimation and maneuvering target tracking

In maneuvering targets, in addition to the system’s dynamic variables that change with time, other variables such as instantaneous angular and linear accelerations are also unknown and must be estimated. Estimating these unknown variables is one of the challenges ahead, and no effective method has been suggested so far that could be applicable in the very general cases. In the existing scenarios, when the range of variation of the unknown input variable is large, the conventional multimode method would fail to operate efficiently. Since one filter is designed for each possible value of the unknown variable, as the range of variation of the variables is extended, a large number of filters would be required, which is not possible considering all the possible modes in some cases. Therefore, in this paper, we aim to provide a method that solves the problem of estimating the unknown input variables. In this method, similar to the input estimation methods, the unknown input vector is considered to be \( \theta \). The objective is firstly to estimate this vector, and then perform the tracking. The general state-space model that is represented by probability density functions is as follows:

\[
\begin{align*}
    x_{k+1} & \sim p(x_{k+1} | x_k; \theta) \\
    z_k & \sim p(z_k | x_k; \theta)
\end{align*}
\]  

(8)

In these relations, the state prediction current observations \( z_k \) depend on the present state \( x_k \) and unknown variables \( \theta \). \( \theta \) is a vector that includes the unknown variables required to be estimated. The continuous state variables change with time and therefore, the chosen samples could be updated at each moment based on Monte-Carlo methods.

In the method presented in (Nemeth et al., 2014), the probability density function of the unknown variables is approximated as the sum of a number of Gaussian functions. In other words, we have

\[ p(\theta; z_{1:k-1}) \approx \sum_{i=1}^{N} \omega_{k-1}^i \phi(\theta | \mu_{k-1}^i, \sigma^2 V_{k-1}). \]  

(9)

In this relation, \( \phi(\theta | \mu_{k-1}^i, \sigma^2 V_{k-1}) \) is a multivariate Gaussian density function with the following mean and variance

\[
\begin{align*}
    \mu_{k-1}^i &= \gamma \bar{\theta} + (1 - \gamma) \theta \\
    V_{k-1} &= \sum_{i=1}^{N} \omega_{k-1}^i (\theta - \bar{\theta}) (\theta - \bar{\theta})^T, \\
    \bar{\theta} &= \sum_{i=1}^{N} \omega_{k-1}^i \theta
\end{align*}
\]  

(10)

where \( N \) is the number of the samples and \( \gamma \) is a constant number close to one.

This approximation method performs well when the target’s \( \theta \) does not change with time. For example, the target could be a ship or passenger airplane. As the time passes by, the estimated density function gets focused on a point. If these parameter change, then the mentioned approximation method is not able to follow these time variations. In maneuvering targets, such as fighters and drones, \( \theta \) is usually piecewise constant. For example, the angular acceleration is constant in a time period and changes in another period. In other words, there are moments at which \( \theta \) changes.

Statistically, the probability of occurrence of each changing point could be considered to be \( \beta \). In this case, the length of the expected time period during which \( \theta \) is constant would be equal to \( 1/\beta \). By these explanations, \( \theta_k \) changes with time according to the relation below.
Algorithm 1. The Stages of Running the Algorithm to Estimate the Target Parameters and Track the Maneuvering Target.

1- Setting $k = 0$
   - The initial sampling of the particles as
   - Calculating the below parameters for $i = 1, \ldots, N$
     \[ a_{i}^{0} \propto p(z_{k} | \mu_{k}, q_{k}) \]
   - Assigning the weights: $a_{0}^{i} = 1/N$
   - Selecting the particles with a probability of
     \[ \{(1 - \beta)a_{k}^{1}\}_{i=1}^{N} \text{ and } \{\beta a_{k}^{2}\}_{i=1}^{2N} \]
   - Determining the mode transition matrix: $p_{m}(c = i|z_{0})$

2-Running the algorithm for $k = 1, 2, \ldots$ and $i = 1, \ldots, N$
   - The prediction phase
     \[ m_{k-1}^{j} \sim p(m_{k-1} | m_{k-2}^{j}) \]
     \[ x_{k}^{j} \sim p(x_{k} | x_{k-1}^{j}, c, m_{k-1}^{j}) \]
     \[ a_{k}^{j} = p(z_{k} | x_{k}^{j}) g(y_{k}^{j}) \]
     \[ \dot{a}_{k}^{j} = \phi(\cdot | m_{k-1}^{j}, \omega^{2} V_{k-1}) \]
     \[ x_{k}^{j} = p(x_{k} | x_{k-1}^{j}, \dot{a}_{k}^{j}) \]
     \[ a_{k}^{j} \propto \frac{p(x_{k} | x_{k-1}^{j}, \dot{a}_{k}^{j})}{a_{k}^{j}} \]
   - Normalizing the particles' weights: $a_{k}^{j} = \frac{a_{k}^{j}}{\sum a_{k}^{j}}$
   - Resampling of particles:
     \[ N_{c} \sim \text{MNRND}(N, \hat{p}(c\{Y, Z\})) \]
     \[ p(c\{z_{k}, y_{k}\} | \{Z_{k-1}, Y_{k-1}\}) = L(c) = \sum_{j=1}^{N_{c}} a_{k}^{j} \]
     \[ \hat{x}_{k}^{c} = \sum_{j=1}^{N_{c}} a_{k}^{j} \dot{x}_{k}^{j}, \ a_{k}^{j} = \frac{a_{k}^{j}}{\sum a_{k}^{j}} \]
   - Estimating the target state and class
     \[ \hat{p}(c\{Z_{k}, Y_{k}\}) = \frac{L(c) p(c\{Z_{k}, Y_{k}\})}{\sum_{c=1}^{C} L(c) p(c\{Z_{k}, Y_{k}\})} \]
     \[ \hat{x}_{k} = \sum_{c=1}^{C} \hat{x}_{k}^{c} \hat{p}(c\{Z_{k}, Y_{k}\}) \]

3-End of loop

\[ \theta_{k} = \begin{cases} \theta_{k-1} & \text{with probability of } \beta \\ \xi_{k} & \text{with probability of } 1 - \beta \end{cases} \tag{11} \]

where $\xi_{k} \sim p_{\theta_{k-1}}(.)$ is the new value of the parameter. Accordingly, the state posterior probability function is expressed as
p(x_k, \theta_k | z_{1:k}) \propto p(z_k \mid x_k, \theta_k) p(x_k \mid x_{k-1}, \theta_k) p(\theta_k | \theta_{k-1}) + (1 - \beta) p(z_k \mid \theta) p(x_k \mid x_{k-1}, \theta_{k-1})
\delta(\theta_k - \theta_{k-1}) \omega_{k-1}^k
(12)

According to Algorithm 1, by estimating \theta, the input parameters such as linear and angular accelerations are specified.

In order to estimate the target’s class and mode, these variables could be considered as an additive variable to the state vector. Then, the target class and mode are estimated using conventional Monte-Carlo methods by employing random sampling. However, contrary to the state vector which changes with time as a Markov chain, the target’s class is constant and unchanging with time. Hence, the target’s class does not change in the prediction phase and may only change in the resampling step or at the original state of the target’s class. Since no change occurs in the prediction stage, there is a high probability that the estimated class remains constant in a wrong class and it is not possible to correct it. In order to solve this problem, a threshold stage is considered. In this stage, it is ensured that the number of samples in each class is not below a certain threshold.

Therefore, if any false classes are detected, it is possible to correct it in the next time periods, and the algorithm acts according to the correct class. In the modified algorithm presented here (Algorithm 1), N denotes the number of the particles of the filter, S denotes the number of the classes, and \( N_c \), which has a multivariate normal distribution, represents the number of the particles assigned to each class.

The existence of the class as an additive variable reduces the number of duplicate samples and increases the number of effective ones. Estimation of parameters in the particle weighting phase is highly effective, because in case the target experiences variations in velocity and acceleration or in other words, has high maneuverability (such as a fighter), the observation noise increases. Hence, particles have a higher measurement error compared to the case without maneuvers (passenger plane) and therefore, account for a larger weight. Consequently, in the resampling phase, particles with a larger error are more likely to be reproduced and less likely to be removed. On the other hand, when the target does not have maneuvers, the measurements are more precise and the search space diminishes and particles with larger measurement errors are more likely to be removed. The process of simultaneous estimation of the target class and target tracking is in accordance with Algorithm 1.

5. Simulation results and discussion

5.1. Scenario I

We assume that the dynamic equations of the target are as follows

\[ x_{k+1} = Fx_k + G(\alpha_k + w_k) \]

\[ x = \begin{bmatrix} x \\ y \end{bmatrix}, F = \begin{bmatrix} 1 & 0 & \frac{\sin \alpha T}{2} & \frac{\cos \alpha T - 1}{2} \\ 0 & 1 & \frac{1 - \cos \alpha T}{2} & \frac{\sin \alpha T}{2} \\ 0 & 0 & \cos \alpha T & \sin \alpha T \\ 0 & 0 & \sin \alpha T & \cos \alpha T \end{bmatrix}, G = \begin{bmatrix} 0.5T^2 & 0 \\ T & 0 \\ 0 & 0.5T^2 \\ 0 & T \end{bmatrix} \]

\[ w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad \alpha = \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix} \]
This system is appropriate for modeling of the target with a linear acceleration of $a_k$ and an angular acceleration of $\alpha$. The linear and angular accelerations are the parameter $\theta$ in accordance with Algorithm 1. $w_k$ is the white Gaussian noise. The observation model is also expressed as

$$ z_k = \left[ \sqrt{x_k^2 + y_k^2} \tan^{-1} \frac{y_k}{x_k} \right] + v_k, $$

where $v_k$ is the white Gaussian noise with a mean value of zero. Figure 1 shows the trajectory of a target that is moving with a zero linear acceleration. Angular acceleration is an unknown parameter that should be estimated. The variance of the noise introduced to system acceleration is assumed to be 0.001, the variance of the observation noise is 0.005. $T = 0.1$ sec, $N = 5000$ particles, and $\gamma$ and $\beta$ are assumed to be 0.96 and (0.01, 0.05), respectively.

Figure 2 shows the estimated and true angular accelerations. It is observed that in the proposed algorithm, the angular acceleration converges at a high rate even when instantaneous variations occur. In the next scenario, it is assumed that the target’s angular acceleration is known and equal to $\alpha = 0.2$, while its linear acceleration is unknown and has instantaneous variations. Figure 3 shows the true and the estimated trajectories of the target.

Figures 4 and 5 show the changes in linear acceleration in x and y dimensions. It is observed that at times when acceleration changes, the estimated acceleration converges to the true acceleration after a short time duration.

As observed in Figure 2, 4, and 5, due to the changes of the mean value and the variance of the posterior samples, the value of the unknown variable is different from its true value; however, it converges to its true value after a short period of time. Since in maneuvering targets, instantaneous changes of acceleration and velocity exist, the presented algorithm is able to reduce the tracking error.
5.2. Scenario II

We assume that two probable targets are flying from one point on planet earth to another point (2D model), with one having high maneuverability belongs to a class $c_2$ (fighter) and the other with a low maneuverability belongs to a class $c_1$ (passenger plane). The target belongs to one of the two mentioned classes, and each of those classes may be in five possible modes. The velocity constraint of each class could be stated as

$$c_1 : \vartheta \in (100, 300) \text{ m/sec}, c_2 : \vartheta \in (150, 650) \text{ m/sec}.$$
As can be observed, these two classes overlap in range \((150, 300)\). The probability functions of the velocity domain of each target could be defined as below (Nemeth et al., 2014).

\[
g(v_k^i) = \begin{cases} 
0.8 & \text{if } v_k^i \leq 100 \\
0.8 - \frac{0.7}{200}(v_k^i - 100) & \text{if } 100 < v_k^i \leq 300 \\
0.1 & \text{if } 300 < v_k^i
\end{cases}
\]


\[
g(\nu_k^2) = \begin{cases} 
0.1 & \text{if } \nu_k^2 \leq 150 \\
0.1 - \frac{0.85}{500} (\nu_k^2 - 150) & \text{if } 150 < \nu_k^2 \leq 650, \\
0.95 & \text{if } 650 < \nu_k^2,
\end{cases}
\]

where \( \vartheta = \sqrt{x^2 + y^2} \) is the velocity magnitude. The mode transition matrix in each class is defined as follows: (Nemeth et al., 2014).

\[
p = \begin{bmatrix}
0.6 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.1 & 0.6 & 0.1 & 0.1 & 0.1 \\
0.1 & 0.1 & 0.6 & 0.1 & 0.1 \\
0.1 & 0.1 & 0.1 & 0.6 & 0.1 \\
0.1 & 0.1 & 0.1 & 0.1 & 0.6
\end{bmatrix}
\]

Also, the velocity probability functions of the two classes are shown in Figure 6.

Since we have no information about the target class, the following initial values are considered for the class probability function.

\[
\hat{p}(c|Z, Y) = [0.50.5]
\]

In other words, the probability of choosing each class equals 50%. The dynamic equations are according to (9) and the equation of the received observations is as follows.

\[
z_k = h(x_k) + v_k = \sqrt{x_k^2 + y_k^2} \tan^{-1} \frac{y_k}{x_k} + v_k
\]
The process noise is Gaussian, the variance of the observation noise received by the sensors equals 0.001, the number of the samples for each class is $N_c = 5000$, $N_{thr} = 0.1N_c$, and the sampling period is assumed to be $T_s = 0.01$ sec. Figure 7 shows the target trajectory. The target has two maneuvers at different times with a constant velocity. The target class is chosen completely randomly. In one experiment, class two was selected.

Figure 8 shows the probability function of the targets’ class estimation. It is observed that the proposed algorithm estimates the target class well. Class estimation by using multiple filters that match the targets’ classes are also shown in Figure 8. It is observed that the proposed algorithm estimates the target class with much higher accuracy.

The location estimation error based on Monte-Carlo (100 times) is shown in Figure 9. As can be seen, at the times when mode starts to change, a large error exists. This error diminishes fast as time passes by. Also, the error is smaller compared to the IMM method.

The proposed algorithm has been compared, in terms of performance, to the commonly used conventional methods, according to Table 1 (added to the revised version). Hence, in the case of considering both “execution time“ and “execution accuracy” criteria, the particle filter can be an appropriate suggestion for the state estimation problems with state-dependent noise and intervention.

6. Conclusion
The aim of this paper is to estimate the unknown inputs and track the maneuvering targets using a hybrid method. In maneuvering targets, in addition to the system's dynamic variables that change with time, other variables such as instantaneous angular and linear accelerations are also unknown and must be estimated. Since one of the challenges of input estimation is to present an algorithm that makes estimations for general cases, the Bayesian theory could be employed to obtain the state and variable posterior distributions simultaneously. In the proposed algorithm, an L&W filter and a modified IMM method are used to estimate unknown parameters such as

![Figure 7. Simulated target trajectory.](image-url)
Figure 8. The probability function of determining the targets’ classes.

![Probability function of determining the targets' classes](image)

Figure 9. The location estimation error.

![Location estimation error](image)

**Table 1. Tracking results (100 Monte–Carlo simulations)**

| Estimators                          | Mean RMSE of position (m) | Total execution time (s) |
|-------------------------------------|---------------------------|--------------------------|
| The proposed modified particle filter | 3.6131                    | 1.7976                   |
| Unscented particle filter (UPF)     | 8.5862                    | 1.0209                   |
| Standard particle filter            | 6.9321                    | 41.8987                  |
instantaneous acceleration, angular acceleration, and target class, and a modified particle filter is employed for tracking purpose. Parameter estimation and mode determination could be used for particle weighting in the resampling phase, and particles could be weighted according to the target mode. This makes the search space smaller and augments the removal of particles with larger measurement errors.

Funding
The authors received no direct funding for this research.

Author details
S.N. Hosseini1
E-mail: hosseinisrn.moh@gmail.com
M. Haeri
E-mail: haeri@sharif.ir
H. Khaloozadeh3
E-mail: h_khaloozadeh@eetd.kntu.ac.ir
1 School of Mechanical, Electrical, and Computer Engineering, Islamic Azad University, Science and Research Branch, Tehran, Iran.
2 Electrical Engineering Department, Sharif University of Technology, Tehran, Iran.
3 Department of Systems and Control, K.N. Toosi University of Technology, Tehran, Iran.

Citation information
Cite this article as: An algorithm to estimate parameters and states of a nonlinear maneuvering target. S.N. Hosseini, M. Haeri & H. Khaloozadeh, Cogent Engineering (2020), 7: 1847711.

References
Chen, X., Gao, J., & Han, X. (2014). An algorithm based on interacting multiple models for maneuvering target tracking. IEEE Conference on Decision and Control (Vol. 2, pp. 405–408).
Fu, X., Shang, Y., & Yuan, H. (2015). Improved diagonal interacting multiple model algorithm for maneuvering target tracking based on H∞ filter. IET Control Theory & Application, 9(12), 1887–1892. https://doi.org/10.1049/iet-cta.2014.0685
Jia, S., Zhang, Y., & Wang, G. (2017). Highly maneuvering target tracking using multi-parameter fusion Singer model. Journal of Systems Engineering and Electronics, 28(5), 841–850. https://doi.org/10.1016/j.jsee.2017.05.03
Jing, L., & Vadakkapet, P. (2010). Interacting MCMC particle filter for tracking maneuvering target. Digital Signal Processing, (2012), 561–574. https://doi.org/10.1016/j.dsp.2009.08.011
Khaloozadeh, H., & Karsaz, A. (2008). Modified input estimation technique for tracking maneuvering targets. IET Radar Sonar & Navigation, 3(1), 30–41. https://doi.org/10.1049/iet-rsn:20080028
Li, X. R., & Jilkov, V. P. (2005). Survey of maneuvering target tracking. Part I: Dynamic models. IEEE Transactions on Aerospace and Electronic Systems, 41 (4), 1333–1366.
Liu, J., & West, M. (2001). Combined parameter and state estimation in simulation-based filtering. In Sequential Monte Carlo methods in practice (pp. 197–223).
Malekian Heraband, H., & Khaloozadeh, H. (2016). Extended input estimation method for tracking nonlinear multiplicative noises. *IET Radar Sonar & Navigation*, 4, 284–291.
Nemeth, C., Fearnhead, P., & Mihaylova, L. (2014). Sequential Monte Carlo methods for state and parameter estimation in abruptly changing environments. *IEEE Transactions on Signal Processing*, 62(5), 1245–1256. https://doi.org/10.1109/TSP.2013.2296278
Popoli, R., & Blackman, S. (1999). Design and analysis of modern tracking systems. Artech House.
Pulford, G. W. (2015). A survey of maneuvering target tracking methods. IEEE Transactions on Aerospace and Electronic Systems, 27(1).
Rahmati, H., Khaloozadeh, H., & Ayati, M. (2012). Novel approach for nonlinear maneuvering target tracking based on input estimation. Applied Mechanics and Materials, 110, 4415–4423.
Rannnamaa, S., & Arvan, M. R. (2011). Comparison of extended and unscented Kalman smoother in deriving kinematic characteristics of a high maneuver flying target. *International Conference on Modelling, Identification and Control* (pp. pp. 537–542).
Singer, R. A. (1970). Estimating optimal tracking filter performance for manned maneuvering targets. IEEE Transactions on Aerospace and Electronic Systems, AES-6(4), 473–483. https://doi.org/10.1109/TAES.1970.310128
Youn, W., & Myung, H. (2019). Robust interacting multiple model with modeling uncertainties for maneuvering target tracking. IEEE Access, 7, 65427–65442. https://doi.org/10.1109/ACCESS.2019.2915506
Zhan, K., Xu, L., & Jiang, H. (2013). Joint tracking and classification with constraints and reassignment by radar and ESM. Digital Signal Processing, 15, 561–574.
Zhang, H., Li, L., & Xie, W. (2018). Constrained multiple model particle filtering for bearings-only maneuvering target tracking. IEEE Access, 6, 51721–51734. https://doi.org/10.1109/ACCESS.2018.2869402
