Dynamical Theory of Phase Transitions and Topological Defect Formation in the Early Universe

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Abstract

We review the current issues of nonequilibrium phase transitions, in particular, in the early universe. Phase transitions cannot maintain thermal equilibrium and become nonequilibrium when the thermal relaxation time scale is greater than the dynamical time scale. Nonequilibrium phase transitions would have happened in certain evolution stages of the early universe because the rapid expansion quenched matter fields. We apply the recently introduced Liouville-von Neumann method, another canonical method, to nonequilibrium phase transitions in the Minkowski spacetime and find the scaling behavior of domain sizes. Topological defects are thus determined by the dynamical processes of nonequilibrium phase transitions. However, the expansion of the universe freezes domain growth in the comoving frame and thus leads to a scale-invariant domain size. We also discuss the physical implications of nonequilibrium phase transitions in the early universe.

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I. INTRODUCTION

A system undergoes a phase transition when its symmetry is broken explicitly [1, 2, 3]. The phase transition proceeds either in equilibrium or in nonequilibrium (out of equilibrium) depending on the ratio of the thermal relaxation time to the dynamical time. When the thermal relaxation time scale is shorter or longer than the dynamical time scale, the system evolves in equilibrium or out of equilibrium. In particular, a quenched system undergoes nonequilibrium phase transition when the quench rate is faster than the relaxation rate. Matter fields are believed to have undergone such nonequilibrium phase transitions in the early universe as the universe expanded and thereby temperature dropped rapidly. It is likely that such nonequilibrium phase transitions of matter fields can also be realized and observed in RHIC and LHC experiments in the near future.

The finite-temperature field theory has been the most popular approach to equilibrium phase transitions [4]. The effective potential of quantum fluctuations around a classical background provides a convenient tool to describe phase transitions. However, quantum fluctuations of long wavelength modes suffer from instability during the phase transition, become unstable and grow exponentially. This is the origin of the imaginary part of the effective action for phase transitions, which gives the decay rate of the false vacuum [5]. Thus the finite-temperature effective action does not properly take into account the dynamical processes of phase transitions.

On the other hand, nonequilibrium phase transitions have been frequently treated in the closed-time path integral defined in a complex time plane by Schwinger [6] and Keldysh [7]. Other methods are the Hartree-Fock or mean field method [8, 9, 10, 11, 12], the $1/N$-expansion method [13, 14, 15], the time-dependent variational principle [16], and etc. In this paper we use the recently developed Liouville-von Neumann (LvN) method [17, 18, 19, 20, 21]. The LvN method is a canonical method trying to solve the functional Schrödinger equation [22, 23]. The quantum LvN equation provides invariant operators that solve exactly the functional Schrödinger equation [24]. Compared with other methods, this canonical method has advantages that it can be applied directly to time-dependent systems and density operators are readily found. The mode-decomposed Hamiltonian of quenched models for phase transitions has this time-dependent feature.

We use quenched models of $\Phi^4$ theory with the sign changing mass term for nonequilib-
rium phase transitions. These quenched models have quench rates that determine explicitly time-dependent phase transition processes, an instantaneous quench model \[9, 10, 18\] and a finite quench model \[18, 25\]. In both models the field, after symmetry breaking phase transition, begins to roll from the false vacuum toward the true vacuum. During this period of phase transition, the system undergoes spinodal instability and its long wavelength modes grow exponentially. Domains grow through spinodal decomposition \[9, 10, 18, 19, 20, 21, 25, 26\]. Thus domains and topological defects are dynamically determined by the nonequilibrium field theory. Domain growth in the Minkowski spacetime exhibits the Cahn-Allen scaling behavior of classical theory. Further, unstable long wavelength modes lose quantum coherence through interaction with stable short wavelength modes and show classical correlation, thus achieve classicality. Finally, we apply this nonequilibrium field theory to the early expanding universe and study the domain growth and topological defects. Domains remain frozen in the inflation era and have scale-invariance. This affects the topological density in the early universe.

The organization of this paper is as follows: In Sec. II equilibrium phase transitions are briefly reviewed. In Sec. III we discuss the Kibble-Zurek mechanism for the dynamics of equilibrium processes. In Sec. IV some quench models are introduced for nonequilibrium phase transitions. In Sec.V we introduce nonequilibrium quantum field theory based on the functional Schrödinger equation. In Sec. VI the quantum LvN equation is used to solve the functional Schrödinger equation. In Sec. VII we find the correlation functions during the phase transitions, which determine the domain size. In Sec. VIII nonequilibrium phase transitions are studied in an expanding universe. In Sec. IX we show how the classicality of classical correlation and decoherence can be achieved for long wavelength modes.

II. EQUILIBRIUM PHASE TRANSITIONS

To understand the symmetry breaking or restoration mechanism, we consider a scalar field model with the potential

\[ V(\phi) = \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4. \]  

(1)

For a negative \( m^2 \) the symmetry of the system is broken. However, quantum fluctuations around the true vacuum may restore the symmetry when the temperature is high enough so that the thermal energy can overcome the potential energy difference between the true
and false vacua. To find the thermal contribution (correction) to the classical potential, we divide the field, $\phi = \phi_c + \phi_f$, where $\phi_c$ is the classical background field and $\phi_f$ denotes quantum fluctuations around $\phi_c$. Then the effective potential is given by

$$V_e(\phi_c) \equiv \text{Tr}(\rho V) = \frac{1}{2}(m_R^2 + \frac{\lambda_R}{24}T^2)\phi_c^2 + \frac{\lambda_R}{4!}\phi_c^4,$$

where $\rho$ is the density operator, and $m_R$ and $\lambda_R$ are renormalized coupling constants. For the broken symmetry $m^2 \to -m^2$, one has the potential

$$V_e(\phi_c) = \frac{1}{2}(T^2 - T_c^2)\phi_c^2 + \frac{\lambda_R}{4!}\phi_c^4.$$

The system thus restores the broken symmetry when $T > T_c$ [4]. In other words, the symmetry can be spontaneously broken when the temperature drops below the critical temperature $T_c$.

In the early stage of evolution after the Big Bang, the Universe would have undergone a sequence of phase transitions as the temperature dropped due to the expansion. A possible sequence of phase transitions based on particle physics is the GUT phase transition at $T_c \approx 10^{14} - 10^{16}$ Gev, the EW (electroweak) phase transition at $T_c \approx 10^2$ Gev, and Color deconfiment/Chiral phase transition at $T_c \approx 10^2$ Mev. Depending on the particle physics model, the system produces different type of topological defects [3]. The full symmetry of the system is broken to a subgroup after a phase transition. The structure of the vacuum manifold $\mathcal{M}$, the homotopy group, determines the type of topological defects: domain walls for $\pi_0(\mathcal{M}) \neq \infty$, strings for $\pi_1(\mathcal{M}) \neq \infty$, and monopoles for $\pi_2(\mathcal{M}) \neq \infty$.

### III. Kibble-Zurek Mechanism

Kibble used the principle of causality and the Ginzburg temperature to find the correlation length of the same phase (domain) in phase transitions [1]. The Ginzburg temperature is the one where the thermal energy is comparable to the free energy of a correlated region

$$k_B T_c \approx \xi^2(T_G)\Delta F(T_G),$$

so that the field can overcome the potential barrier to jump to other configurations. In this case topological defects lose stability. This temperature restricts the size of correlation length for stable defects. Topological defects are located along the boundaries of correlated
regions. Thus there is one monopole per volume $\xi^3$ and the density of monopoles is given by $\kappa/\xi^3$, and similarly one string per area $\xi^2$ and the density by $\kappa/\xi^2$.

On the other hand, the Zurek mechanism incorporates the dynamics of equilibrium processes [2]. In the adiabatic cooling (quenching) the equilibrium correlation length and the equilibrium relaxation time increase, respectively, as

$$\xi = \xi_0 |\epsilon|^{-\nu}, \quad \tau = \tau_0 |\epsilon|^{-\mu}$$

(5)

where $\epsilon = (T_c - T)/T_c$. Here $\mu$ and $\nu$ are model-dependent parameters. Also $\epsilon$ is related with the quench rate as $\epsilon = t/\tau_Q$. As temperature approaches the critical value ($T \to T_c$ or $t \to 0$), the equilibrium relaxation time becomes sufficiently longer and the process critically slows down. However, the correlation length increases indefinitely but the propagation of small fluctuations is finite; so $\tau \propto \xi/v \to \infty$. Therefore, there is a time $t^*$ when the correlation length freezes: $|t^*| = \tau(t^*)$. From the above equations the correlation length is given by

$$\xi(t^*) \approx \tau_Q^{\nu/(1+\mu)}.$$  

(6)

Thus the Kibble-Zurek mechanism determines the domain size dynamically in terms of the quench rate.

IV. NONEQUILIBRIUM PHASE TRANSITIONS

There are many cases to which nonequilibrium phase transitions can be applied but equilibrium phase transitions cannot be applied. Matter fields in a rapidly expanding universe are such an example, where the Tolman temperature drops as

$$T(t) = T_0 \frac{a_0}{a(t)},$$

(7)

where $a$ is the scale factor of the universe. Another example is the rapid quenching processes such as in the rapid cooling of quark-gluon plasma in the Heavy Ion Collision and the liquid helium $\text{He}^3$ and $\text{He}^4$, where domain walls and vortices (strings) may be formed.

As a field-theoretical model for nonequilibrium phase transitions, we consider the scalar field potential [9, 10, 18, 19, 20, 21, 25]

$$V(\phi) = \frac{1}{2} m^2(t) \phi^2 + \frac{\lambda}{4!} \phi^4,$$

(8)
where the mass $m^2$ changes signs from $m^2(-\infty) = m_i^2$ to $m^2(+\infty) = -m_f^2$. When $\tau_Q$ is the time scale for the quench, one can distinguish the adiabatic quench process, $\Delta m^2/\tau_Q \ll 1$, from the rapid quench process, $\Delta m^2/\tau_Q \gg 1$. For instance, an analytical model may be considered \[18\]

\[m^2(t) = -m^2 \tanh\left(\frac{t}{\tau_Q}\right),\]  

(9)

in which $m^2 \to +m^2$ for $t \to -\infty$ and $m^2 \to -m^2$ for $t \to +\infty$. In the limiting case $\tau_Q \to 0$, one has the instantaneous quench, $m^2 = +m^2$ for $t < 0$ and $m^2 = -m^2$ for $t > 0$. The field model can easily be generalized to an expanding universe

\[H(t) = \int d^3x \left[ \frac{\pi^2}{2a^3(t)} + a^3(t)\left(\frac{(\nabla \phi)^2}{2a^2(t)} + V(\phi)\right)\right],\]  

(10)

where $a$ is the scale factor of the FRW universe. The Minkowski spacetime is the special case of $a = 1$. In the Hartree-Fock and mean-field approximation, we divide $\phi = \phi_c + \phi_f$ to obtain the equations of motion for the classical field

\[\ddot{\phi}_c + 3\frac{\dot{a}}{a}\dot{\phi}_c - \nabla^2 \phi_c + \left(m^2(t) + \frac{\lambda}{6}\phi_c^2 + \frac{\lambda}{2}\phi_f^2\right)\phi_c = 0,\]  

(11)

and for quantum fluctuations

\[\ddot{\phi}_f + 3\frac{\dot{a}}{a}\dot{\phi}_f - \nabla^2 \phi_f + \left(m^2(t) + \frac{\lambda}{2}\phi_c^2 + \frac{\lambda}{4}\phi_f^2\right)\phi_f = 0.\]  

(12)

V. NONEQUILIBRIUM QUANTUM FIELD THEORY

The functional Schrödinger-picture provides the real-time evolution of (time-dependent) quantum systems \[22, 23\]. It is based on the functional Schrödinger equation (in unit of $\hbar = 1$)

\[i\frac{\partial}{\partial t}\Psi(\phi, t) = \hat{H}(\phi, -i\frac{\delta}{\delta \phi})\Psi(\phi, t),\]  

(13)

where $\phi$ represents a scalar or fermion field. It describes the evolution of wave functionals $\Psi(\phi, t)$ from one spacelike hypersurface $\Sigma_{t_0}$ to another $\Sigma_t$. The set of all wave functionals constitutes a Hilbert space, which has an inner product on each spacelike hypersurface $\Sigma_t$

\[\langle \Psi_1|\Psi_2 \rangle = \int \mathcal{D}[\phi]\Psi_1^*(\phi, t)\Psi_2(\phi, t).\]  

(14)

The action of operators are defined as

\[\hat{O}(\phi, \pi)|\Psi(\phi, \pi)\rangle \rightarrow \hat{O}(\phi, -i\frac{\delta}{\delta \phi})\Psi(\phi, t).\]  

(15)
Now the task is to solve Eq. (13) for the symmetry breaking potential (8). The evolution of the wave functional may be found in terms of Green function (kernel or propagator)

\[ \Psi(x, t) = \int G(x, t; x_0, t_0) \Psi(x_0, t_0) dx_0 dt_0, \]  

where the wave functional implicitly depends on space through \( \phi \). Our stratagem is to separate the Hamiltonian into a quadratic part, an exactly solvable one, and a perturbation part:

\[ \hat{H}(t) = \hat{H}_0(t) + \lambda \hat{H}_P(t). \]  

Here \( \hat{H}_0 \) includes not only the quadratic potential term but also some contribution from non-linear terms a la the Hartree-Fock or mean-field approximation. In this way the approximation becomes nonperturbative including parts of quantum corrections from the nonlinear terms. Then we may introduce a Green function for \( \hat{H}_0 \) as

\[ \left( i \frac{\partial}{\partial t} - \hat{H}_0(x, t) \right) G_0(x, t; x', t') = \delta(x - x') \delta(t - t'), \]  

and write the wave functional in terms of \( G_0 \)

\[ \Psi(x, t) = \Psi_0(x, t) + \lambda \int G_0(x, t; x', t') \hat{H}_P(x', t') \Psi(x', t') dx' dt', \]  

where \( \Psi_0 \) is a wave functional for \( \hat{H}_0 \). The wave functional \( \Psi \) can be put recursively into the righthand side of Eq. (19) to result in

\[ \Psi(1) = \Psi_0(1) + \lambda \int G_0(1, 2) \hat{H}_P(2) \Psi_0(2) + \lambda^2 \int \int G_0(1, 2) \hat{H}_P(2) G_0(2, 3) \hat{H}_P(3) \Psi_0(3) + \cdots, \]  

where \( (i) \) denotes \( (x_i, t_i) \).

The general wave functional for \( \hat{H}_0 \) can take the Gaussian wave functional

\[ \Psi_0(\phi, t) = N \exp \left[ - \int x, y (\phi(x) - \bar{\phi}(x, t))(\frac{1}{4G(x, y, t)} - i \Sigma(x, y, t)) \right] \times (\phi(y) - \bar{\phi}(y, t)) + i \int x \bar{\pi}(x, t)(\phi(x) - \bar{\phi}(x, t))]. \]  

It has nonzero expectation values of the field and momentum

\[ \langle \Psi_0 | \hat{\phi}(x) | \Psi_0 \rangle = \bar{\phi}(x, t), \quad \langle \Psi_0 | \hat{\pi}(x) | \Psi_0 \rangle = \bar{\pi}(x, t), \]  

and also the two-point correlation functions

\[ \langle \Psi_0 | \hat{\phi}(x) \hat{\phi}(y) | \Psi_0 \rangle = \bar{\phi}(x, t) \bar{\phi}(y, t) + G(x, y, t), \]

\[ \langle \Psi_0 | \hat{\pi}(x) \hat{\pi}(y) | \Psi_0 \rangle = \bar{\pi}(x, t) \bar{\pi}(y, t) + \Sigma(x, y, t). \]
VI. CANONICAL METHOD FOR NONEQUILIBRIUM PHASE TRANSITIONS

Recently, another canonical method, the so-called LvN or invariant method, has been developed based on the quantum LvN equation [17, 18, 19, 20, 21]

\[
i \frac{\partial}{\partial t} \hat{O}(\phi, -i \frac{\delta}{\delta \phi}, t) + \left[ \hat{O}(\phi, -i \frac{\delta}{\delta \phi}, t), \hat{H}(\phi, -i \frac{\delta}{\delta \phi}, t) \right] = 0. \tag{24}
\]

The idea of the LvN method for quantum mechanical systems first exploited by Lewis and Riesenfeld [24] is to solve Eq. (24) and find the solution to the Schrödinger equation as eigenstates of the operator in Eq. (24). In quantum field theory the wave functional to the Schrödinger equation is directly given by the wave functional of the operator

\[
\hat{O}(\mathbf{x}, t) \Psi_\varphi(\mathbf{x}, t) = \varphi(\mathbf{x}) \Psi_\varphi(\mathbf{x}, t). \tag{25}
\]

Note that the eigenvalue \( \varphi(\mathbf{x}) \) does not depend on time, which is a consequence of Eq. (24). In particular, for the quadratic Hamiltonian the operator satisfying Eq. (24) can be obtained explicitly. This canonical method has an advantage that quantum statistical information can be naturally incorporated into the dynamics even for nonequilibrium systems. Another merit is that it is a nonperturbative canonical method and can further be applied to fermionic and gauge systems.

We now turn to the potential (8) for nonequilibrium phase transition. As explained in Sec. V, we separate the Hamiltonian density \( \mathcal{H} \) into the quadratic part \( \mathcal{H}_0 \) and the perturbation part \( \mathcal{H}_P \):

\[
\mathcal{H}_0(t) = \frac{1}{2a^3} \pi^2 + \frac{1}{2a} (\nabla \phi)^2 + \frac{a^3}{2} \left( m^2 + \frac{\lambda}{2} \langle \phi^2 \rangle \right) \phi^2, \\
\mathcal{H}_P(t) = a^3 \left( \frac{1}{4!} \phi^4 - \frac{1}{4} \langle \phi^2 \rangle \phi^2 \right). \tag{26}
\]

It is convenient to decompose the field into Fourier modes and then to redefine them as

\[
\phi_k^{(+)}(t) = \frac{1}{\sqrt{2}} [\phi_k(t) + \phi_{-k}(t)], \quad \phi_k^{(-)}(t) = \frac{i}{\sqrt{2}} [\phi_k(t) - \phi_{-k}(t)], \tag{27}
\]

where \( \phi_k^{(\pm)} \) and \( \phi_k^{(-)} \) are the Fourier-cosine and sine modes, respectively. For simplicity reason a compact notation \( \alpha, \beta, \cdots \), will be used for \( \{k, (\pm)\} \). Then the actual Hamiltonian of symmetric state modes takes the form

\[
H(t) = \sum_{\alpha} \left[ \frac{1}{2a^3} \pi_\alpha^2 + \frac{a^3}{2} \omega_\alpha^2(t) \phi_\alpha^2 \right] + \frac{\lambda a^3}{4!} \left[ \sum_{\alpha} \phi_\alpha^4 + 3 \sum_{\alpha \neq \beta} \phi_\alpha^2 \phi_\beta^2 \right]. \tag{28}
\]
where
\[ \omega_\alpha^2(t) = m^2(t) + \frac{k^2}{a^2}. \] (29)

Thus the Hamiltonian consists of infinite number of coupled anharmonic oscillators, all of which depend on time through \( m^2(t) \). The quadratic part of the Hamiltonian becomes
\[ H_0(t) = \sum_\alpha \frac{1}{2a^3} \dot{\pi}_\alpha^2 + \frac{a^3}{2} \Omega_\alpha^2(t) \dot{\phi}_\alpha^2, \] (30)
where
\[ \Omega_\alpha^2(t) = m^2(t) + \frac{k^2}{a^2} + \frac{\lambda}{2} \sum_\beta \langle \phi_\beta^2 \rangle. \] (31)

Note that the Hamiltonian (30) simply consists of decomposed oscillators.

We first find the Green function \( G_0 \) for \( \hat{H}_0 \) and then obtain perturbatively the wave functional for the full Hamiltonian. In fact, each mode of the quadratic part \( \hat{H}_0 \) can be solved exactly in terms the time-dependent creation and annihilation operators satisfying the LvN equation. Then the wave functional for \( \hat{H}_0 \) is the product of the wave function for each mode. The time-dependent creation and annihilation operators of the \( \alpha \) mode are given by [17, 18, 19, 20, 21]
\[ \hat{a}_\alpha^\dagger(t) = -i[\varphi_\alpha(t)\hat{\pi}_\alpha - a^3\dot{\varphi}_\alpha(t)\hat{\phi}_\alpha], \quad \hat{a}_\alpha(t) = i[\varphi_\alpha^*(t)\hat{\pi}_\alpha - a^3\dot{\varphi}_\alpha^*(t)\hat{\phi}_\alpha]. \] (32)

We require these operators to satisfy the LvN equation with \( \hat{H}_\alpha \)
\[ i\frac{\partial}{\partial t}\hat{a}_\alpha^\dagger(t) + [\hat{a}_\alpha^\dagger(t), \hat{H}_\alpha(t)] = 0, \quad i\frac{\partial}{\partial t}\hat{a}_\alpha(t) + [\hat{a}_\alpha(t), \hat{H}_\alpha(t)] = 0. \] (33)

Then the auxiliary field \( \varphi_\alpha \) satisfies the mean-field equation
\[ \ddot{\varphi}_\alpha(t) + 3\frac{\dot{a}(t)}{a(t)}\dot{\varphi}_\alpha(t) + \Omega_\alpha^2(t)\varphi(t) = 0. \] (34)

Indeed, these operators satisfy the usual commutation relations at equal times
\[ [\hat{a}_\alpha(t), \hat{a}_\beta^\dagger(t)] = \delta_{\alpha\beta}, \] (35)
when the Wronskian condition meets
\[ a^3(\varphi_\alpha^*\varphi_\alpha - \varphi_\alpha^*\varphi_\alpha) = i. \] (36)

In the oscillator representation of \( \{\hat{a}_\alpha^\dagger, \hat{a}_\alpha\} \), the quadratic part takes the form
\[ \hat{H}_0(t) = \frac{a^3}{2} \sum_\alpha (\dot{\varphi}_\alpha^2 + \Omega_\alpha^2 \varphi_\alpha^2) \hat{a}_\alpha^2 + 2(\dot{\varphi}_\alpha^* \varphi_\alpha + \Omega_\alpha^2 \varphi_\alpha^* \varphi_\alpha) \hat{a}_\alpha^\dagger \hat{a}_\alpha + (\dot{\varphi}_\alpha^2 + \Omega_\alpha^2 \varphi_\alpha^2) \hat{a}_\alpha^{12}, \] (37)
and so does the perturbation part

\[ \hat{H}_p(t) = \frac{a^3}{4!} \sum_{\alpha} \sum_{k=0}^{4} \left[ \binom{4}{k} \varphi_{\alpha}^{*(4-k)} \varphi_{\alpha}^k \hat{a}_{\alpha}^{\dagger (4-k)} \hat{a}_{\alpha}^k + 3 \sum_{\alpha \neq \beta} \left( \varphi_{\alpha}^2 \hat{a}_{\alpha}^2 + 2 \varphi_{\alpha}^* \varphi_{\alpha} \hat{a}_{\alpha}^{\dagger} \hat{a}_{\alpha} + \varphi_{\alpha}^{*2} \hat{a}_{\alpha}^{12} \right) \right. \]

\[ \left. \times \left( \varphi_{\beta}^2 \hat{a}_{\beta}^2 + 2 \varphi_{\beta}^* \varphi_{\beta} \hat{a}_{\beta}^{\dagger} \hat{a}_{\beta} + \varphi_{\beta}^{*2} \hat{a}_{\beta}^{12} \right) \right]. \]

(38)

The essential point of the LvN method is that the number states of \( \hat{a}_{\alpha}^{\dagger} \) and \( \hat{a}_{\alpha} \)

\[ \hat{N}_\alpha(t)|n_\alpha, t\rangle_0 = \hat{a}_{\alpha}^{\dagger}(t)\hat{a}_{\alpha}(t)|n_\alpha, t\rangle_0 = n_\alpha|n_\alpha, t\rangle_0, \]

are exact quantum states of the time-dependent Schrödinger equation. The quantum state of the field itself is then a product of each mode state. For instance, the Gaussian vacuum state of the field is given by

\[ |0, t\rangle_0 = \prod_\alpha |0_\alpha, t\rangle_0. \]

(40)

We now find the Green function for \( \hat{H}_\alpha \)

\[ G_{\alpha\alpha}(\phi_\alpha, t; \phi'_\alpha, t') = \sum_{n_\alpha} \langle \phi_\alpha | n_\alpha, t\rangle_0 \langle n_\alpha, t' | \phi'_\alpha \rangle, \]

(41)

and the Green function for \( \hat{H}_0 \)

\[ G_0(x, t; x', t') = \prod_\alpha G_{\alpha\alpha}(\phi_\alpha, t; \phi'_\alpha, t'). \]

(42)

The wave functional for the full Hamiltonian (28) can be obtained at least perturbatively by substituting Eqs. (42) and (38) into Eq. (20).

VII. DOMAIN GROWTH IN MINKOWSKI SPACETIME

We now study how the dynamical processes of nonequilibrium phase transitions affect the domain growth and topological defect density. The quench models in Sec. IV describe such nonequilibrium processes, which can be treated exactly for the free theory and approximately for the self-interacting theory. Before the onset of the phase transitions, the mass term dominates over the last term from the quantum corrections. It is thus justified to use approximately the free theory after the onset of phase transition until it crosses the inflection point or spinodal line. This is also true for the quench process lasting for an indefinitely long period. We consider first the free theory in the Minkowski spacetime and then discuss the effect of nonlinear terms.
The free theory for the quench models is provided by the potential (8), where \( \lambda = 0 \) and \( m^2(t) \) changes signs either instantaneously or for a finite period. In the Minkowski spacetime we can apply the formalism in Sec. VI simply by letting \( a = 1 \). Before the phase transition \( (m^2 = m_i^2) \), all the modes are stable and oscillate around the true vacuum:

\[ \varphi_{0k}(t) = \frac{1}{\sqrt{2\omega_{ik}}}e^{-i\omega_{ik}t}, \quad \omega_{ik} = \sqrt{k^2 + m_i^2}. \] (43)

The two-point correlation function is the Green function at equal times

\[ G_0(x, x', t) = \langle \hat{\phi}(x, t)\hat{\phi}(x', t) \rangle_0 = G_0(x, t; x', t) \] (44)

with respect to the Gaussian vacuum or thermal equilibrium. The two-point thermal correlation function has the well-known form

\[ G_{0T}(y, x, t) = \frac{1}{4\pi^2} m_i K_1(m_i|x - y|) \frac{m_i}{2\pi^2 \sqrt{|x - y|^2 + m_i^2}} \]
\[ \times \sum_{n=1}^{\infty} K_1[m_i \sqrt{|x - y|^2 + (\beta n)^2}], \] (45)

in terms of the modified Bessel function \( K_1 \). However, after the phase transition \( (m^2 = -m_f^2) \), the true vacuum becomes the false vacuum.

First, in the instantaneous quench model, long wavelength modes \( (k < m_f) \) become unstable and exponentially grow as

\[ \varphi_{f0k}(t) = \frac{1}{\sqrt{2\omega_{ik}}} \left[ -i\frac{\omega_{ik}}{\omega_{fk}} \sinh(\omega_{fk}t) + \cosh(\omega_{fk}t) \right], \quad (\omega_{fk} = \sqrt{m_f^2 - k^2}), \] (46)

whereas short wavelength modes \( (k > m_f) \) are still stable and have the solutions

\[ \varphi_{f0k}(t) = \frac{1}{\sqrt{2\omega_{ik}}} \left[ -i\frac{\omega_{ik}}{\omega_{fk}} \sin(\omega_{fk}t) + \cos(\omega_{fk}t) \right], \quad (\omega_{fk} = \sqrt{k^2 - m_f^2}). \] (47)

The two-point thermal correlation function \( (r = |x - x'|) \) is dominated by the long wavelength modes and is given by

\[ G_{f0T}(r, t) \approx G_{0T}(0, t) \frac{\sin\left(\frac{m_f r}{2t}\right)}{\sqrt{\frac{m_f}{2t}}} \exp\left(-\frac{m_f r^2}{8t}\right). \] (48)

From Eq. (48) follows the famous Cahn-Allen scaling relation for the domain size

\[ \xi_D(t) = \sqrt{\frac{8t}{m_f}}. \] (49)
This implies that domains grow according to Eq. (49) until most of long wavelength modes cross the inflection point and sample over the true vacuum.

Second, we consider a finite quench model with a finite quench period. Such a model is mimicked by the mass term

\[ m^2(t) = m_1^2 - m_0^2 \tanh \left( \frac{t}{\tau} \right). \]  

(50)

The modes with \( k < m_f = \sqrt{m_0^2 - m_1^2} \) become unstable after the phase transition and exponentially grow, but those with \( k > m_f \) are still stable and oscillate around the false vacuum. Long after the phase transition, the unstable long wavelength modes have the asymptotic solution

\[ \varphi_{f k} = \frac{\mu_k}{\sqrt{2(m_f^2 - k^2)^{1/2}}} e^{(m_f^2 - k^2)^{1/2} t} + \frac{\nu_k}{\sqrt{2(m_f^2 - k^2)^{1/2}}} e^{-(m_f^2 - k^2)^{1/2} t}, \]  

(51)

whereas the stable short wavelength modes oscillate around the false vacuum. Here \( \mu_k \) and \( \nu_k \) depend on the quench process and satisfy the relation \( |\mu_k|^2 - |\nu_k|^2 = 1 \). The correlation function is then approximately given by

\[ G_f(0, t) \approx G_f(0, \tau) \sin \left( \frac{\sqrt{\tau \tau t} r}{m_0} \right) \frac{1}{\tau} \exp \left( -\frac{r^2}{8 \sqrt{\tau \tau t} m_0} \right). \]  

(52)

The coefficients \( \mu_k \) are obtained from the exact solutions of Eq. (34) in terms of the hypergeometric function [18]. Using the exact solutions, we obtain the two-point thermal correlation function during the quench \((-\tau < t < \tau)\)

\[ G_{m,T}(r, t) \approx G_{m,T}(0, t) \frac{\sin \left( \frac{\sqrt{\tau \tau t} r}{m_0} \right)}{\frac{1}{\sqrt{\tau \tau t} r} \exp \left( -\frac{r^2}{8 \sqrt{\tau \tau t} m_0} \right)}. \]  

(53)

Now the Cahn-Allen scaling relation reads

\[ \xi_D(t) = 2 \left( \frac{2 r t}{m_0^2} \right)^{1/4}. \]  

(54)

After the quench \((t \gg \tau)\), the two-point thermal correlation function is given by

\[ G_{f_U,T}(r, t) \approx G_{f_U,T}(0, \tilde{t}) \frac{\sin \left( \frac{\sqrt{\tau \tau t} r}{m_f^2} \right)}{\sqrt{\tau \tau t} r} \exp \left( -\frac{m_f r^2}{8 \tilde{t}} \right), \]  

(55)

where

\[ \tilde{t} = t - \frac{\tau^3}{8} (\zeta(3) - 1) (\omega_{i,k}^2 + \omega_{j,k}^2). \]  

(56)
Then the Cahn-Allen scaling relation for the domain size takes another form

$$\xi_D(t) = \sqrt{\frac{8\ell}{m_f}}. \quad (57)$$

The power of the scaling relation is the same as the instantaneous quench, except for a time-lag proportional to the cube of the quench duration $\tau$. Surprisingly, the correlation function has the pole structure at

$$\tilde{\omega}_{f,k} \tau = n \quad (n = 1, 2, 3, \ldots). \quad (58)$$

This implies larger domains for certain quench rates $\tau$ [18].

Finally, we discuss the effect of nonlinear term to the domain growth. As all wave functionals $\Psi_0$ of $\hat{H}_0$ are readily found, Eq. (20) leads to the wave functional beyond the Hartree approximation. Putting the perturbation terms (38) into Eq. (20), we first find the wave functional of the form

$$\Psi = \Psi_0^{(0)} + \sum_{n=1}^{\infty} \lambda^n \Psi_0^{(n)}, \quad (59)$$

and then calculate the Green function using the wave functional (59). An important result is that the correlation length beyond the Hartree approximation has an additional factor

$$\xi(t) = \sqrt{2n + 1/2} \xi_D(t). \quad (60)$$

This factor from the $n$th order contribution is a consequence of multiple scattering among different unstable modes [21]. The physical implication of the nonlinear effects is that the correlation length increases by $(2n + 1)^{1/2}$, where $n$ is the order of quantum contributions which depends the time for crossing the spinodal line, i.e., the period for the field rolling from the false vacuum into the true one.

**VIII. TOPOLOGICAL DEFECT DENSITY IN AN EXPANDING UNIVERSE**

As the universe expands, the temperature drops and thus provides a cooling process in a natural way. An adiabatically expanding universe has the Tolman temperature

$$T(t) = T_c \frac{a_c}{a(t)}, \quad (61)$$
where $T_c$ is the critical temperature for the phase transition and $a_c$ is the size at this moment. Then the mass term for the second order phase transition may take the form

$$m^2(t) = T_c^2 \left( \frac{a_c}{a(t)} \right)^2 - 1.$$  \hfill (62)

We now consider the free theory with the mass term (62) in the expanding universe. The Hamiltonian for the free field takes the form

$$H_0(t) = \sum_\alpha \frac{\pi^2_\alpha}{2a^3(t)} + \frac{a^3(t)}{2} \omega_\alpha^2(t) \varphi_\alpha^2.$$  \hfill (63)

where

$$\omega_\alpha^2(t) = -T_c^2 + k^2 + \frac{T_c^2a_c^2}{a^2(t)}.$$  \hfill (64)

To find exact quantum states, we follow the formalism in Sec. VI. The time-dependent creation and annihilation operators (32) are determined by the auxiliary variables now satisfying the equation

$$\ddot{\varphi}_\alpha(t) + 3\frac{\dot{a}(t)}{a(t)}\dot{\varphi}_\alpha(t) + \omega_\alpha^2(t)\varphi_\alpha(t) = 0.$$  \hfill (65)

Rewriting the auxiliary variables as $\varphi_\alpha = a^{-3/2}v_\alpha$, Eq. (65) can be written in a canonical form

$$\ddot{v}_\alpha(t) + \left( -M^2(t) + \frac{k^2 + T_c^2a_c^2}{a^2(t)} \right)v_\alpha(t) = 0,$$  \hfill (66)

where

$$M^2(t) = T_c^2 + \frac{3}{4} \left( \frac{\dot{a}}{a} \right)^2 + \frac{3}{2} \frac{\dot{a}}{a}.$$  \hfill (67)

In particular, we are interested in the unstable long wavelength modes with

$$k < \sqrt{M^2a^2 - T_c^2a_c^2}.$$  \hfill (68)

These long wavelength modes have both growing and damping solutions of the form

$$v_\alpha = \frac{c_1\alpha}{\sqrt{2\Theta_\alpha}} e^{+\int \Theta_\alpha} + \frac{c_2\alpha}{\sqrt{2\Theta_\alpha}} e^{-\int \Theta_\alpha},$$  \hfill (69)

where

$$\Theta_\alpha^2(t) = M^2(t) - \frac{k^2 + T_c^2a_c^2}{a^2(t)} - \frac{3}{4} \left( \frac{\dot{\Theta}_\alpha(t)}{\Theta_\alpha(t)} \right)^2 + \frac{1}{2} \frac{\ddot{\Theta}_\alpha(t)}{\Theta_\alpha(t)}.$$  \hfill (70)

Here $c_1\alpha$ and $c_2\alpha$ must satisfy $c_1^*\alpha c_2\alpha - c_1\alpha c_2^*\alpha = 1$ to guarantee the Wronskian condition (36). Assuming a slow variation of $\Theta_\alpha$, we obtain the adiabatic (WKB) solution with the exponent

$$\int \Theta_\alpha(t) = \int M(t) - \frac{k^2}{2} \int \frac{1}{M(t)a^2(t)} + \cdots.$$  \hfill (71)
Then the adiabatic solutions lead to the correlation function

$$G_f(x,t;x',t) \approx \frac{1}{a^3} \int \frac{d^3k}{(2\pi)^3} \left[ e^{i k \cdot (x-x')} \frac{|c_{1k}|^2}{2\sqrt{M^2 - k^2 + T^2 a^2}} \right] \times \exp \left( 2 \int M - k^2 \int \frac{1}{M a^2} \right). \quad (72)$$

Compared with the correlation function (52) in the Minkowski spacetime, the correlation function (72) has an overall decreasing factor $1/a^3$ from the expansion of the universe. Further, after integrating over momenta, the Cahn-Allen scaling relation for the domain size in the comoving frame is given by

$$\xi_D(t) = \left( 8 \int_{t_0}^t \frac{1}{Ma^2} \right)^{1/2}, \quad (73)$$

where $t_0$ is the time for most of long wavelength modes to undergo rolling motion from the false vacuum to the true vacuum. For instance, during the inflation period with $a = e^{H_0 t}$, the scaling relation becomes

$$\xi_D(t) = \left[ \frac{4}{M_0 H_0} \left( 1 - e^{-2H_0(t-t_0)} \right) \right]^{1/2} e^{-H_0 t_0}, \quad (74)$$

where

$$M_0^2 = T_c^2 + \frac{9}{4} H_0^2. \quad (75)$$

The physical implication of Eq. (74) is that the domain size in the comoving frame is frozen, $\xi_D(t) \approx \xi_D(t')$ during the inflation period, in contrast with the Minkowski spacetime where domains increase due to spinodal instability. Thus there is no Cahn-Allen scaling relation and domains remain scale-invariant in the comoving frame. However, the physical size of domains increases in proportion to the scale factor of the expanding universe

$$\xi_{PD}(t) = a(t) \xi_D. \quad (76)$$

Of course, the higher order contributions beyond the Hartree approximation increase both the comoving domain size and physical domain size by additional factor $\sqrt{2n+1}$ for the $n$-th order.

**IX. CLASSICALITY OF QUANTUM PHASE TRANSITIONS**

We investigated phase transitions within the framework of quantum field theory whereas phase transitions are studied in classical theory in most literature. Though some of characteristic features of phase transitions are well described and tested by classical theory,
quantum theory is more fundamental than classical theory. So there should be some mechanism for the emergence of classical features from quantum phase transitions. In more general context than phase transitions, this is known as the quantum-to-classical transition [27]. In the previous section we have shown that long wavelength modes play a key role in the dynamical process of phase transitions and thus determine the growth of domains. In classical theory long wavelength modes constitute just an order parameter, which describes the dynamical process. For an order parameter to achieve classicality, the quantum states of long wavelength modes must lose quantum coherence and be classically correlated along their classical trajectories [19, 28].

To answer this question we consider a simple model motivated by the potential (8). An exactly solvable model for a long wavelength mode coupled to a short wavelength mode is provided by the quadratic Hamiltonian [19]

$$H(t) = \frac{1}{2} \pi_1^2 + \frac{1}{2} \omega_1^2(t) \phi_1^2 + \frac{1}{2} \phi_2^2 + \frac{1}{2} \omega_2^2(t) \phi_2^2 + \lambda \phi_1 \phi_2. \quad (77)$$

Here $\phi_1$ and $\phi_2$ are the long and short wavelength modes, respectively. The former becomes unstable but the latter remains stable even after the phase transition. The time-dependent Schrödinger equation has a Gaussianal wave function of the form

$$\Psi_0(\phi_1, \phi_2, t) = N(t) \exp \left[ -\left\{ A_1(t) \phi_1^2 + \lambda B(t) \phi_1 \phi_2 + A_2(t) \phi_2^2 \right\} \right]. \quad (78)$$

Here $N$ is the normalization constant and the coefficients of the exponent are given by

$$A_1(t) = -i \frac{\dot{u}_1^*(t)}{2u_1^*(t)}, \quad A_2(t) = -i \frac{\dot{u}_2^*(t)}{2u_2^*(t)}, \quad B(t) = i \frac{\int u_1^*(t) u_2^*(t)}{u_1^*(t) u_2^*(t)}, \quad (79)$$

where

$$\dot{u}_1(t) + \left[ \omega_1^2(t) + \lambda^2 \left( \frac{\int u_1(t) u_2(t)}{u_1(t) u_2(t)} \right)^2 \right] u_1(t) = 0, \quad (80)$$

$$\dot{u}_2(t) + \left[ \omega_2^2(t) + \lambda^2 \left( \frac{\int u_1(t) u_2(t)}{u_1(t) u_2(t)} \right)^2 \right] u_2(t) = 0. \quad (81)$$

To model the instantaneous quench in Sec. VII, we assume $\omega_1^2(t) = -\bar{\omega}_1^2$ and $\omega_2^2(t) = \bar{\omega}_2^2$. In the weak coupling limit ($\lambda \ll \bar{\omega}_1, \bar{\omega}_2$), we find the approximate solutions

$$u_1(t) = c_1 e^{i\bar{\Omega}_1 t}, \quad u_2(t) = c_2 e^{-i\bar{\Omega}_2 t}, \quad (82)$$

where

$$\bar{\Omega}_1 = \bar{\omega}_1 - \frac{\lambda^2}{2\bar{\omega}_1} \left( \frac{\bar{\omega}_1 + i\bar{\omega}_2}{\bar{\omega}_1^2 + \bar{\omega}_2^2} \right)^2 - \frac{\lambda^4}{4\bar{\omega}_1^2} \left( \frac{1}{\bar{\omega}_2} + \frac{1}{4\bar{\omega}_1} \right) \left( \frac{\bar{\omega}_1 + i\bar{\omega}_2}{\bar{\omega}_1^2 + \bar{\omega}_2^2} \right)^4,$$

$$\bar{\Omega}_2 = \bar{\omega}_2 + \frac{\lambda^2}{2\bar{\omega}_2} \left( \frac{\bar{\omega}_1 + i\bar{\omega}_2}{\bar{\omega}_1^2 + \bar{\omega}_2^2} \right)^2 + \frac{\lambda^4}{4\bar{\omega}_2^2} \left( \frac{1}{\bar{\omega}_1} - \frac{1}{4\bar{\omega}_2} \right) \left( \frac{\bar{\omega}_1 + i\bar{\omega}_2}{\bar{\omega}_1^2 + \bar{\omega}_2^2} \right)^4. \quad (83)$$
Then the reduced density matrix for the long wavelength mode takes the form

$$\rho_R(\phi', \phi) = N^*N\sqrt{\frac{\pi}{A_1^*+A_1}}\exp[-\Gamma_C \phi'^2 - \Gamma_\Delta \phi^2 - \Gamma_M \phi_C \phi_\Delta], \quad (84)$$

where $\phi_{1,C} = (\phi' + \phi)/2$ and $\phi_{1,\Delta} = (\phi'_1 - \phi_1)/2$. To order $\lambda^4$, the coefficients of the exponent are approximated by

$$\Gamma_C = \frac{5\lambda^4 \bar{\omega}_2(\bar{\omega}_1 - \bar{\omega}_2)}{2\omega_1(\omega_1^2 + \omega_2^2)^2}, \quad (85)$$

$$\Gamma_\Delta = \frac{\lambda^2}{\omega_2(\omega_1^2 + \omega_2^2)^2}[1 + \frac{\lambda^2 \bar{\omega}_2}{(\omega_1^2 + \omega_2^2)^2}(\frac{\omega_1^4}{2\omega_1^2} - \frac{3\omega_1^2}{\omega_2} - \frac{5\omega_2^2}{2\omega_1} + \bar{\omega}_2)], \quad (86)$$

$$\Gamma_M = -2i\bar{\omega}_1[1 + \frac{\lambda^2 \bar{\omega}_2^2}{2\omega_1(\omega_1^2 + \omega_2^2)^2} + \frac{\lambda^4}{(\omega_1^2 + \omega_2^2)^4}(\frac{16}{5} - \frac{3\omega_1^2}{2\omega_1^2} - \frac{\bar{\omega}_1^2}{4\omega_1^2})]. \quad (87)$$

Roughly speaking, a system is classically correlated when its wave functions are peaked along classical trajectories or the contours of the Wigner function is close to classical ones. And it decoheres when each trajectory loses quantum coherence with its neighbors. Quantum decoherence is realized when the diagonal term $\phi_C$ of the density matrix dominates over the off-diagonal term $\phi_\Delta$. A more precise measure of quantum decoherence and classical correlation was introduced by Morikawa [29]. According to his measure, quantum decoherence is given by

$$\delta_{QD} = \frac{1}{2}\sqrt{\frac{\Gamma_C}{\Gamma_\Delta}} = \frac{\lambda}{2}\sqrt{\frac{5\bar{\omega}_2(\bar{\omega}_1 - \bar{\omega}_2)}{2\omega_1(\omega_1^2 + \omega_2^2)^2}}, \quad (88)$$

and classical correlation by

$$\delta_{CC} = \sqrt{\frac{\Gamma_\Delta^2 \Gamma_M^2}{\Gamma_C^* \Gamma_M}} = \frac{5\lambda^6}{4}\frac{|\bar{\omega}_1 - \bar{\omega}_2|}{\omega_1^2(\omega_1^2 + \omega_2^2)^2}. \quad (89)$$

The system loses quantum coherence (decoheres) when $\delta_{QD} \ll 1$ and recovers classical correlation when $\delta_{CC} \ll 1$.

We therefore find that the unstable long wavelength mode becomes classical, gaining both quantum decoherence and classical correlation due to the coupling to a stable short wavelength mode. In field models, the infinitely large number of short wavelength modes provides an environment to unstable long wavelength modes and very quickly make them classical. Quite recently, field models have been studied for the quadratic, Yukawa, and linear couplings to the environment, where quantum decoherence of long wavelength modes is observed and the decoherence time is found to be very short compared with other time scales [28]. In this way long wavelength modes become classical immediately after phase transitions, from which appears a classical order parameter [19].
X. CONCLUSION

We critically reviewed the nonequilibrium quantum phase transitions by employing the canonical method called LvN (Liouville-von Neumann) method. The nonequilibrium phase transitions are characterized by the quench time scale shorter than the thermal relaxation time scale, so that they cannot have time to relax to thermal equilibrium and evolve out of equilibrium. As such nonequilibrium phase transitions we have considered the quenched second order phase transitions modelled by the $\phi^4$-theory with an explicitly time-dependent mass that changes sign during the quench process. This model is a quantum model for the classical Ginzburg-Landau theory for spinodal decomposition.

We used the LvN method to study such time-dependent quantum field theory. The wave functionals of the Schrödinger equation carry all the quantum information of the system evolving from initial data such as thermal equilibrium or Gaussian vacuum to final nonequilibrium states. The essential idea of the LvN method is first to solve the LvN equation in terms of whose operators exact wave functionals are found for the time-dependent Schrödinger equation. In this LvN method it is easy to incorporate the thermal equilibrium because density operators are ready to be found. In particular, the exact quantum states and the Green function can be found explicitly for a quadratic potential. So our stratagem is to separate the Hamiltonian into a quadratic part of the Hartree-Fock type and a perturbation. In the momentum space and the oscillator representation, the perturbation part of our model consists only of quartic terms of the creation and/or annihilation operators. By using the Green function for the quadratic part and solving perturbatively the perturbation part, we are able to find higher order contributions to the wave functional. This method provides to go beyond the Hartree-Fock approximation.

As a consequence of the direct application of the LvN approach to the quenched second order phase transitions, we obtained some interesting results such as growth of domain sizes, topological density and decoherence of order parameter. In the instantaneous quench model, the domain sizes scale according to the Cahn-Allen scaling relation which has been observed in classical phase transitions. In the finite quench model, the domain sizes have a different scaling behavior during the quench. Surprisingly, the correlation function after the quench exhibits resonance for certain quench rates. Further, the nonlinear effect leads to a multiple-scaling relation beyond the Hartree-Fock approximation. Another interesting phenomenon
is the classicality of phase transitions due to instability of long wavelength modes. The long wavelength modes become unstable, exponentially grow due to spinodal instability and thus exhibit classical correlation. The long wavelength modes therefore achieve classicality through the coupling to a large number of stable short wavelength modes (environment). It is found that the topological defect density can be reduced by factors $(2n + 1)^{3/2}$, which is a consequence of multiple scattering at the higher order beyond the Hartree-Fock approximation.

Quantum fields in the early universe provide a natural framework for nonequilibrium phase transitions as the adiabatically expanding universe has the Tolman temperature dropping inversely proportion to the scale factor of the universe. It is widely believed that the universe would undergo a sequence of phase transitions. However, the phase transitions may not be properly treated in finite temperature field theory due to the adiabatic expansion. We apply the LvN method to the quenched second order phase transition in an expanding FRW universe. The damping effect of field due to the expansion of the universe dramatically changes the domain growth. In fact, domains in the comoving frame remain frozen and show scale-invariant behavior in the inflation era. This result strongly contrasts with the Cahn-Allen scaling relation in the Minkowski spacetime, according to which domains grow as a power of time due to spinodal instability. Thus domains growing through dynamical processes suppress formation of topological defects since topological defects form on the boundaries of domains. However, the physical sizes of domains increase in proportion to the scale factor of the universe. This implies that topological defects may not be suppressed through dynamical processes of nonequilibrium phase transitions but through the inflation of the universe.

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