Self-consistent factorization of two-body residual interaction is proposed for arbitrary density- and current-dependent energy functionals. Following this procedure, a separable RPA (SRPA) method is constructed. SRPA considerably simplifies the calculations and demonstrates quick convergence to exact results. The method is tested for SkI3 and SkM* forces.
I. INTRODUCTION

Self-consistent nuclear models, as e.g. Skyrme-Hartree-Fock (SHF), are widely used for the description of nuclear ground state properties. They do also allow a description of excitation spectra. This is usually done within the random-phase approximation (RPA), which is meanwhile a textbook standard in nuclear physics [1, 2], for a few recent applications see [3, 4, 5]. Actually, these dynamical applications are up to now rather limited. The reason is that RPA implies the diagonalization of large matrices whose rank is determined by the size of the particle-hole 1ph space. The number of 1ph states grows huge for heavy and/or deformed nuclei. This limits the range of application for fully fledged RPA. The most applications are thus found for spherical nuclei.

The RPA problem becomes much simpler if the residual two-body interaction is reduced to a separable form, i.e.

\[ \hat{V}_{\text{res}} \rightarrow \hat{V}_{\text{res}}^{(\text{sep})} = \frac{1}{2} \sum_{k,k' = 1}^{N_{\text{sep}}} \mu_{kk'} \hat{Z}_k \hat{Z}_{k'}, \]

where \( \{ \hat{Z}_k \} \) is a set of hermitian one-body operators. The factorization changes the rank of the RPA matrix from the number of 1ph configurations to the number of basis operators \( \hat{Z}_k \) and this reduces dramatically the computational effort. The success of a separable ansatz depends, of course, on a diligent choice of the operators \( \hat{Z}_k \) and the associated strength coefficients \( \mu_{kk'} \).

Factorization of the residual interaction is widely used in nuclear theory but mainly within trivial schemes exploiting one separable term with an intuitive guess for the separable one-body operator \( \hat{Z} \). The strength constant \( \mu \) is usually fitted to reproduce available experimental data (see e.g. [6]). Obviously, accuracy and predictive power of such simple schemes are limited. Several improvements towards self-consistent schemes have been proposed during the last decades [1]-[12]. However, these schemes are not sufficiently general. Some of them are limited to analytic or simple numerical estimates [1]-[8], others are not fully self-consistent in the sense that they start from phenomenological single-particle potentials [10]-[11] or cover only particular effective forces [12].

In the present paper we propose a general self-consistent separable RPA (SRPA) approach
relevant to arbitrary density- and current-dependent functionals. The self-consistent scheme of Ref. 8 is generalized to the case of several separable operators. The operators are chosen to have maxima at different areas of the nucleus. This is crucial for an accurate reproduction of the residual interaction \( V_{\text{res}} \). A similar scheme has been successfully applied to the Kohn-Sham functional for description of collective oscillations of valence electrons in atomic clusters [8]-[15]. The nuclear case is more demanding since the SHF functional is much more involved. We will discuss the actual SRPA scheme for the case of SHF and present first successful tests for isoscalar E2 and isovector E1 giant resonances in \(^{40}\text{Ca}\) and \(^{208}\text{Pb}\) using two typical Skyrme parameterizations.

II. SRPA

A. Separable operators

In connection with nuclear density functional theory, it is preferable to sort the one-body operators \( \hat{Z}_k \) according to time parity. We thus decompose the set into hermitian time-even \( \hat{X}_k \) and time-odd \( \hat{Y}_k \) operators and the corresponding strength matrices \( \kappa_{kk'} \) and \( \eta_{kk'} \). Then the separable Hamiltonian can be written as

\[
\hat{H}_{\text{SRPA}} = \hat{h}_0 + \hat{V}^{(\text{sep})}_{\text{res}},
\]

\[
\hat{V}^{(\text{sep})}_{\text{res}} = \frac{1}{2} \sum_{kk'=1}^{N_{\text{sep}}} (\kappa_{kk'} \hat{X}_k \hat{X}_{k'} + \eta_{kk'} \hat{Y}_k \hat{Y}_{k'})
\]

where \( \hat{h}_0 \) is the mean field Hamiltonian of the ground state, \( \kappa_{kk'} \) and \( \eta_{kk'} \) are symmetric matrices. Note that the operator product is considered to be separable at the Hartree level, i.e. for an arbitrary 1ph operator \( \hat{A} \) we have

\[
[\hat{Z}_k \hat{Z}_{k'}, \hat{A}] = \hat{Z}_k \langle [\hat{Z}_{k'}, \hat{A}] \rangle + \hat{Z}_{k'} \langle [\hat{Z}_k, \hat{A}] \rangle.
\]

Furthermore, it is important to note for the following considerations that only commutators between time-even and time-odd operators have non-vanishing expectation values. That means that \( \langle [\hat{X}_k, \hat{X}_{k'}] \rangle = \langle [\hat{Y}_k, \hat{Y}_{k'}] \rangle = 0 \) while \( \langle [\hat{X}_k, \hat{Y}_{k'}] \rangle \neq 0 \).

The success of a separable ansatz depends very much on an appropriate choice of the expansion basis \( \hat{X}_k \) and \( \hat{Y}_k \). Collective modes (vibrational, giant resonances) dominate the spectral strengths and it is intuitively clear that collective operators like the various multipole
moments should play the leading role in the expansion. We will denote them as hermitian
time-even collective operators \( \hat{Q}_k \) (as e.g. a local multipole operator \( \hat{Q}_{\lambda \mu} (r) \sim r^\lambda (Y_{\lambda \mu} + Y_{\lambda \mu}^\dagger) \))
and time-odd conjugate momenta

\[
\hat{P}_k = i [\hat{H}, \hat{Q}_k]_{ph} \tag{5}
\]

where the index \( ph \) means the \( 1ph \) part of the operator. These operators can serve as
generators of a collective motion by virtue of the scaling transformation

\[
|\Psi\rangle = e^{\hat{G}} |\rangle, \quad \hat{G} = \sum_k (ip_k \hat{Q}_k - iq_k \hat{P}_k) \tag{6}
\]

where \( |\rangle \equiv |\Psi_0\rangle \) is the ground state wave function, \( \hat{G} \) is the generator, composed of various
coordinate-like \( \hat{Q}_k \) and momentum-like \( \hat{P}_k \) operators, \( p_k \) and \( q_k \) are the corresponding (time-
dependent) amplitudes, see e.g. [16, 17, 18]. The dynamical deformation (6) induces changes
of the mean-field Hamiltonian \( \hat{h} \) which can be expanded in the linear regime for the small
\( \hat{G} \) as

\[
\hat{h} \approx \hat{h}_0 + [\hat{h}_0, \hat{G}]_{ph} + [\hat{V}_{res}, \hat{G}]_{ph} \tag{7}
\]

\[
= \hat{h}_0 + [\hat{H}, \hat{G}]_{ph}.
\]

The dynamics of small-amplitude vibrations is governed by \( [\hat{H}, \hat{G}]_{ph} \) and so we optimize the
separable interaction by requiring that

\[
[\hat{H}, \hat{G}]_{ph} = [\hat{H}^{(sep)}, \hat{G}]_{ph} \tag{8}
\]

as close as possible. Since both Hamiltonians contain \( \hat{h}_0 \) in the same way, the requirement
(8) is essentially

\[
[\hat{V}_{res}, \hat{G}]_{ph} = [\hat{V}_{res}^{(sep)}, \hat{G}]_{ph}. \tag{9}
\]

Applying (8) to generators \( \hat{Q}_k \) and \( \hat{P}_k \) separately, we have
\[
[\hat{V}_{\text{res}}, \hat{P}_k]_{ph} = [\hat{V}_{\text{res}}^{(\text{sep})}, \hat{P}_k]_{ph} \\
= \sum_{kk'} \hat{X}_{k'} \kappa_{kk'} \langle [\hat{X}_{k'}', \hat{P}_k] \rangle, \quad (10a)
\]

\[
[\hat{V}_{\text{res}}, \hat{Q}_k]_{ph} = [\hat{V}_{\text{res}}^{(\text{sep})}, \hat{Q}_k]_{ph} \\
= \sum_{kk'} \hat{Y}_{k'} \eta_{kk'} \langle [\hat{Y}_{k'}', \hat{Q}_k] \rangle. \quad (10b)
\]

This shows that an optimal choice has to fulfill the condition

\[
\hat{X}_k \in \left\{ [\hat{V}_{\text{res}}, \hat{P}_k]_{ph} \right\}, \quad (11a)
\]

\[
\hat{Y}_k \in \left\{ [\hat{V}_{\text{res}}, \hat{Q}_k]_{ph} \right\}. \quad (11b)
\]

We are free to assume any linear recombination among the sets \{\hat{X}_k, \hat{Y}_k\} and thus we may choose

\[
\hat{X}_k = [\hat{V}_{\text{res}}, \hat{P}_k]_{ph}, \quad (12a)
\]

\[
\hat{Y}_k = [\hat{V}_{\text{res}}, \hat{Q}_k]_{ph}. \quad (12b)
\]

Substitution of \(\hat{V}_{\text{res}}^{(\text{sep})}\) into (12a) and (12b) has to give the identities. This results in the expressions for the inverse strength constants \(\kappa^{-1}_{k'k}\) and \(\eta^{-1}_{k'k}\) (\(\sum_k \kappa^{-1}_{k'k} \kappa_{kk'} = \delta_{kk'}, \sum_k \eta^{-1}_{kk'} \eta_{kk'} = \delta_{kk'}\)):

\[
\kappa^{-1}_{k'k} = \langle [\hat{X}_{k'}, \hat{P}_k] \rangle = \langle [[\hat{V}_{\text{res}}, \hat{P}_k]_{ph}, \hat{P}_k] \rangle, \quad (13a)
\]

\[
\eta^{-1}_{k'k} = \langle [\hat{Y}_{k'}, \hat{Q}_k] \rangle = \langle [[\hat{V}_{\text{res}}, \hat{Q}_k]_{ph}, \hat{Q}_k] \rangle. \quad (13b)
\]

The actual calculation of the commutators can be done either in wavefunction representation \[19, 20\] or in terms of explicit 1\textit{ph} matrix elements. The latter reads

\[
\langle [\hat{X}_k', \hat{P}_k] \rangle = 2 \sum_{ph} \langle p | \hat{X}_k | h \rangle \langle h | \hat{P}_k' | p \rangle, \quad (14a)
\]

\[
\langle [\hat{Y}_k', \hat{Q}_k] \rangle = 2 \sum_{ph} \langle p | \hat{Y}_k | h \rangle \langle h | \hat{Q}_k' | p \rangle. \quad (14b)
\]

Alternatively, the strength constants can be determined from the requirement

\[
\langle [\hat{P}_k, [\hat{V}_{\text{res}}, \hat{P}_k]] \rangle = \langle [\hat{P}_k, [\hat{V}_{\text{res}}^{(\text{sep})}, \hat{P}_k]] \rangle, \quad (15a)
\]

\[
\langle [\hat{Q}_k, [\hat{V}_{\text{res}}, \hat{Q}_k]] \rangle = \langle [\hat{Q}_k, [\hat{V}_{\text{res}}^{(\text{sep})}, \hat{Q}_k]] \rangle. \quad (15b)
\]
Substituting (3) and (12a) into (15a) yields

\[ \langle [\hat{P}_k, \hat{X}_{k'}] \rangle = \sum_{kk'} \langle [\hat{P}_k, \hat{X}_{k}] \rangle \kappa_{kk'} \langle [\hat{X}_{k'}, \hat{P}_{k'}] \rangle \]  

(16)

and finally Eq. (13a). A similar procedure applies to the double commutator (15b). For a physical interpretation of the requirements (15a) and (15b) mind that the similar double commutators but with the full Hamiltonian (instead of the residual interaction) correspond to \( m_3 \) and \( m_1 \) sum rules, respectively, and so represent the spring and inertia parameters \[18\] in the basis of collective generators \( \hat{Q}_k \) and \( \hat{P}_k \). The condition (13) means that collective moments \( m_3 \) and \( m_1 \) are exactly reproduced.

The basic result of this section are Eqs. (12) and (13) for the separable operators and strength constants, respectively. It is worth noting that these results can be also obtained \[21\] using the concept of nuclear self-consistency between the single-particle potential and density \[1\]. More detailed formulation of the basic double commutators in terms of the SHF functional will be given in section (II) and appendix \[B\]. The links to the vibrating potential model (VPM) \[1, 7\] are outlined in appendix \[A\]. The choice of the generators \( \hat{Q}_k \) and \( \hat{P}_k \) is yet open. An intuitive and pragmatic guess will be presented and discussed in sections (II) and \(\Pi\).

B. Conservation laws

The forms (12) are able to reproduce given conservation laws. Assume a symmetry mode whose generator \( \hat{P}_{\text{sym}} \) obeys by definition \( [\hat{H}, \hat{P}_{\text{sym}}] = 0 \). We simply have to include this mode into the set of generators, i.e. \( \hat{P}_{\text{sym}} \in \{ \hat{P}_k \} \) together with its complement \( \hat{Q}_{\text{sym}} \) given by \( [\hat{H}, \hat{Q}_{\text{sym}}] = -i \hat{P}_{\text{sym}} \). One can prove that the separable Hamiltonian (2) does also fulfill

\[ [\hat{H}_{\text{SRPA}}, \hat{P}_{\text{sym}}] = 0. \]  

(17)

From (14) we have

\[ [\hat{h}_0, \hat{P}_{\text{sym}}] = -[\hat{V}_{\text{res}}, \hat{P}_{\text{sym}}] = -\hat{X}_{\text{sym}}. \]
Then

\[
[\hat{H}_{\text{SRPA}}, \hat{P}_{\text{sym}}] = [\hat{h}_0, \hat{P}_{\text{sym}}] + \sum_{k,k'} \hat{X}_k \kappa_{kk'} \langle [\hat{X}_{k'}, \hat{P}_{\text{sym}}] \rangle
\]

\[
= -\hat{X}_{\text{sym}} + \sum_k \hat{X}_k \sum_{k'} \kappa_{kk'} \kappa_{k'sym}^{-1} \delta_k, \text{sym}
\]

\[
= -\hat{X}_{\text{sym}} + \hat{X}_{\text{sym}}
\]

which obviously yields Eq. (17). Thus all symmetry modes are recovered if properly included in the ansatz.

It is worth noting that in the case of a symmetry mode the operator of the residual interaction is

\[
\hat{X}_{\text{sym}} = -[\hat{h}_0, \hat{P}_{\text{sym}}],
\]

i.e. can be also presented as the commutator with the single-particle Hamiltonian. In this case, the residual interaction serves to restore the symmetries violated in the mean field. The similar forces were proposed in [22] to restore translational and rotational invariance.

C. Response functions

One way to solve the SRPA equations is to compute directly a desired strength function

\[
S_D(\omega) = \sum_N \left| \langle \Psi_N | \hat{D} | \Psi_0 \rangle \right|^2 (\delta(\omega - \omega_N) + \delta(\omega + \omega_N))
\]

\[
= -\frac{1}{\pi} \Im \left\{ \langle [\hat{D}, \frac{1}{\omega + i\Gamma - \mathcal{L}} \hat{D}] \rangle \right\}
\]

(19)

where \( \hat{D} \) is the operator of interest, \( \Psi_N \) and \( \omega_N \) are the wave function and energy of \( N \)-th RPA state, \( \mathcal{L} \) is the Liouvillian for small amplitude excitations. The latter is a shorthand for

\[
\mathcal{L} \hat{A} = [\hat{H}, \hat{A}]_{ph}
\]

(20)

where \( \hat{A} \) is any \( 1ph \) operator. Similarly, we introduce the unperturbed Liouvillian \( \mathcal{L}_0 \) and the Liouvillian for the residual interaction \( \mathcal{V}_{\text{res}} \) by

\[
\mathcal{L}_0 \hat{A} = [\hat{h}_0, \hat{A}], \quad \mathcal{V}_{\text{res}} \hat{A} = [\hat{V}_{\text{res}}, \hat{A}]_{ph}.
\]
For a compact formulation, we collect the operators $\hat{D}, \{\hat{X}_k, k = 1, \ldots, N_{\text{sep}}\}$ and $\{\hat{Y}_k, k = 1, \ldots, N_{\text{sep}}\}$ into the set $\{\hat{Z}_k, k = 0, \ldots, 2N_{\text{sep}}\}$ with $\hat{X}_0 = \hat{D}$. The matrices of the strength constants are joined into the matrix $\mu$ of the rank $2N_{\text{sep}} + 1$:

$$
\mu = \begin{pmatrix}
1 & 0 & 0 \\
0 & \kappa & 0 \\
0 & 0 & \eta
\end{pmatrix}.
$$

Then we define generalized response functions

$$
R_{kk'}(\omega) = \langle [\hat{Z}_k, \frac{1}{\omega + i\Gamma - \mathcal{L}}\hat{Z}_{k'}] \rangle,
$$

$$
R_{kk'}^{(0)}(\omega) = \langle [\hat{Z}_k, \frac{1}{\omega + i\Gamma - \mathcal{L}_0}\hat{Z}_{k'}] \rangle.
$$

Note that the free response function can be written easily in terms of single-particle states $p$ and $h$ as

$$
R_{kk'}^{(0)}(\omega) = \sum_{ph} \langle p|\hat{Z}_k|h\rangle \langle h|\hat{Z}_{k'}|p\rangle \left\{ \frac{1}{\omega + i\Gamma - \varepsilon_{ph}} - \frac{\Pi_k \Pi_{k'}}{\omega + i\Gamma + \varepsilon_{ph}} \right\}.
$$

where $\varepsilon_{ph}$ is the energy of the 1ph-configuration and $\Pi_k$ is the time-parity of the related operator $\hat{Z}_k$.

A relation between full and free responses is established through the following steps:

$$
R_{kk'} = \langle [\hat{Z}_k, \frac{1}{\omega + i\Gamma - \mathcal{L}_0 - V^{\text{sep}}_{\text{res}}\hat{Z}_{k'}}] \rangle
$$

$$
= \sum_{n=0}^{\infty} \langle [\hat{Z}_k, \frac{1}{\omega + i\Gamma - \mathcal{L}_0 - V^{\text{sep}}_{\text{res}}\hat{Z}_{k'}}] \rangle \frac{1}{(\omega + i\Gamma - \mathcal{L}_0)^n} \hat{Z}_{k'}^n
$$

$$
= R_{kk'}^{(0)} + \sum_{k_1k_2} R_{kk_1}^{(0)} \langle [\hat{V}_{\text{res}}^{\text{sep}}, \hat{Z}_{k_2}] \rangle R_{k_2k'}^{(0)} + \ldots
$$

$$
= R_{kk'}^{(0)} + \sum_{k_1k_2} R_{kk_1}^{(0)} \mu_{k_1k_2} R_{k_2k'}^{(0)} + \ldots .
$$

This can be recollected to the final result

$$
R_{kk'}(\omega) = \left( R_{kk'}^{(0)}(\omega) \frac{1}{1 - \mu R_{kk'}^{(0)}(\omega)} \right)
$$

Note that this operator relation deals with matrices of the rank $2(2N_{\text{sep}} + 1)$ (the additional coefficient 2 arises due to isospin). The strength function is finally

$$
S_D(\omega) = -\frac{1}{\pi} \Im \{ R_{00}(\omega) \}.
$$
The calculation of the free response function requires some caution with respect to time parity, see Eq. (23). The denominator in $R_{kk'}^{(0)}$ mixes time parities and one has to disentangle the pieces

$$\frac{1}{\omega + i\Gamma - L_0} = R_+^{(0)}(\omega) + R_-^{(0)}(\omega),$$

$$R_+^{(0)}(\omega) = \frac{L_0}{(\omega + i\Gamma)^2 - L_0^2},$$

$$R_-^{(0)}(\omega) = \frac{\omega + i\Gamma}{(\omega + i\Gamma)^2 - L_0^2},$$

where $R_+^{(0)}$ and $R_-^{(0)}$ connect operators of the same and opposite parity, respectively. One thus has to use $R_+^{(0)}$ for $\langle [\hat{X}, R^{(0)} \hat{X}] \rangle$, $\langle [\hat{Y}, R^{(0)} \hat{Y}] \rangle$, $\langle [\hat{X}, R^{(0)} \hat{D}] \rangle$, $\langle [\hat{D}, R^{(0)} \hat{D}] \rangle$ and $R_-^{(0)}$ for $\langle [\hat{X}, R^{(0)} \hat{Y}] \rangle$, $\langle [\hat{D}, R^{(0)} \hat{Y}] \rangle$. The mutually other combinations vanish.

**D. SRPA as eigenvalue problem**

As often done in RPA, one wants to determine the detailed eigenfrequencies and eigenstates of the response problem. The excitation phonon operator $\hat{C}^\dagger_N$ for the mode $N$ is determined from the RPA equations

$$[\hat{H}, \hat{C}^\dagger_N]_{ph} = \omega_N \hat{C}^\dagger_N. \tag{28}$$

The standard RPA scheme proceeds to recast that equations into the form of a matrix equation by expanding $\hat{C}^\dagger_N$ in terms of $1ph$ operators

$$\hat{C}^\dagger_N = \sum_{ph} (c^{(+)}_{N,ph} \hat{a}^+_p \hat{a}_h - c^{(-)}_{N,ph} \hat{a}^+_h \hat{a}_p). \tag{29}$$

The involved matrices can grow huge in view of the large $1ph$ spaces required for sufficient convergence of the result. The eigenvalue problem is dramatically simplified by inserting the separable Hamiltonian (2). For compact notation, we recombine again the sets $\{\hat{X}_k, \hat{Y}_k\} \rightarrow \{\hat{Z}_k, k = 1, ..., 2N_{sep}\}$ and introduce the corresponding super-matrix of strengths $(\kappa_{kk'}, \eta_{kk'}) \rightarrow \mu_{kk'}$. The RPA equations (28) then become

$$c^{(+)}_{N,ph} = \sum_{kk'} \mu_{kk'} \frac{(p|\hat{Z}_k|h) C_{N,k'}}{\omega_N - \varepsilon_{ph}} \tag{30a},$$

$$c^{(-)}_{N,ph} = -\sum_{kk'} \mu_{kk'} \frac{(h|\hat{Z}_k|p) C_{N,k'}}{\omega_N + \varepsilon_{ph}} \tag{30b}.$$
\[ C_{N,k} = \langle [\hat{Z}_k, \hat{C}_N^\dagger] \rangle \]
\[ = \sum_{ph} \left\{ (h|\hat{Z}_k|p)c_{N,ph}^{(+)} + (p|\hat{Z}_k|h)c_{N,ph}^{(-)} \right\}. \]

Eqs. (30a)-(30b) reduce the impressive number of unknowns \( c_{N,ph}^{\pm} \) to the much smaller set of unknowns \( C_{N,k} \). Such reduction (which is possible due to factorization of the residual interaction) yields an effective RPA matrix with much smaller rank.

As a next step, it is then convenient to transform \( C_{N,k} \) to unknowns
\[ \tilde{C}_{N,k} = 2 \sum_{k'} \mu_{kk'} C_{N,k'} \] (32)
which are directly connected with the amplitudes \( p_k(t) \) and \( q_k(t) \) of the scaling transformation \( \mathbb{E} \). For harmonic oscillations
\[ q_k(t) = \tilde{q}_k \cos(\omega t), \quad p_k(t) = \tilde{p}_k \sin(\omega t) \] (33)
and \( \tilde{C}_{N,k} = \tilde{q}_k \) and \( \tilde{p}_k \) for time-even and time-odd cases, respectively. The unknowns (32) regulate self-consistently contributions of the scaling operators \( \hat{Q}_k \) and \( \hat{P}_k \) into \( N \)-th RPA state. Every RPA state has its own set \( \tilde{C}_{N,k} \) (\( \tilde{q}_k \) and \( \tilde{p}_k \)) amplitudes.

Finally, after standard algebraic steps, we obtain the SRPA equation for \( \tilde{C}_{N,k} \)
\[ \sum_{k'} d_{kk'}(\omega_N) \tilde{C}_{N,k'} = 0 \] (34)
with
\[ d_{kk'}(\omega_N) = 2 \sum_{ph} \frac{(p|\hat{Z}_k|h)(h|\hat{Z}_{k'}|p)\varepsilon_{ph}}{\varepsilon_{ph}^2 - \omega_N^2} - \mu_{kk'}^{-1}. \] (35)
Eq. (34) has non-trivial solutions if
\[ \det \{d_{kk'}(\omega_N)\} = 0 \] (36)
which yields the RPA spectrum \( \omega_N \). Eq. (34) is basically a matrix of the rank \( 4N_{\text{sep}} \) (after duplication of the rank because of the isospin) which is much smaller than the necessary number of \( 1ph \) states. Typically, we can deal with \( N_{\text{sep}} = 1 - 4 \) (see section \( \text{[II]} \)) as opposed to the rank of a conventional RPA matrix which often amounts to about \( 10^3 - 10^6 \).
The normalization condition for RPA states is

$$\langle (\hat{C}_N, \hat{C}_N^\dagger) \rangle = 1$$  \hspace{1cm} (37)$$

which amounts to

$$\sum_{ph} \left( |c_{N,ph}^{(+)}|^2 - |c_{N,ph}^{(-)}|^2 \right)$$

$$= \sum_{kk'} \tilde{C}_{N,k} \tilde{C}_{N,k'} \sum_{ph} \frac{< p|\hat{Z}_k|h > < p|\hat{Z}_{k'}|h > \varepsilon_{ph}\omega_N}{(\varepsilon_{ph} - \omega_j^2)^2}$$

$$= \frac{1}{4} \sum_{kk'} \tilde{C}_{N,k} \tilde{C}_{N,k'} \frac{\partial}{\partial \omega_N} d_{kk'}(\omega_N) = 2.$$  \hspace{1cm} (38)$$

The last line of (38) represents the normalization condition in terms of derivatives of the RPA matrix elements.

The reduced probability of \(E\lambda\)-transition from the ground state to the RPA state \(\omega_N\) can be written as

$$B(E\lambda, gr \rightarrow \omega_N) = \frac{\sum_{kk'} D_{kk'}(\omega_N) A_k(\omega_N) A_{k'}(\omega_N)}{\partial_{\omega_N} \det \{d_{kk'}(\omega_N)\}}$$  \hspace{1cm} (39)$$

where \(D_{kk'}(\omega_N)\) is the algebraic supplement of the matrix element \(d_{kk'}(\omega_N)\),

$$A_k(\omega_N) = \sum_{ph} \frac{\varepsilon_{ph}(p|\hat{Z}_k|h)(h|\hat{f}_\lambda|p)}{\varepsilon_{ph}^2 - \omega_N^2}$$  \hspace{1cm} (40)$$

and \((h|\hat{f}_\lambda|p)\) is the 1ph matrix element of \(E\lambda\)-transition.

E. Connection with density functional theory

Self-consistent models are often formulated in terms of a local energy-density functional rather than through an explicit Hamiltonian (see, e.g., the nuclear Skyrme-Hartree-Fock (SHF) method [23] or the Kohn-Sham scheme in electron systems [24, 25]). So we will work out the above SRPA scheme in terms of a given energy functional

$$E = E(\{J_\alpha(r)\})$$

where

$$J_\alpha(r) = \langle \Psi | \hat{J}_\alpha(r) | \Psi \rangle$$
constitute a set of local densities and currents associated with the functional. A simple example is the situation in atoms and atomic clusters where we deal only with the local density $\rho(r) = \sum_n |\varphi_n(r)|^2$ of valence electrons. The nuclear case is much more involved. The typical set for Skyrme functional includes, for both protons and neutrons, the local density $\rho$, kinetic density $\tau$, current $j$, spin-orbit density $J$, and spin density $\sigma$ i.e.

$$J_\alpha \in \{\rho_p, \rho_n, \tau_p, \tau_n, J_p, J_n, j_p, j_n, \sigma_p, \sigma_n\},$$

(41)

see in Appendix B the detailed expressions.

The mean field Hamiltonian corresponding to the given energy functional is then determined as

$$\hat{h}(r) = \int dr \sum_\alpha \hat{J}_\alpha(r) \frac{\delta E}{\delta \hat{J}_\alpha(r)}.$$  

(42)

In the linear regime of small-amplitude deformation generated by a 1ph operator $\hat{G}$, see Eq. (43), the densities can be decomposed as

$$J_\alpha(r, t) = \bar{J}_\alpha(r) + J^G_\alpha(r, t)$$

(43)

where $\bar{J}_\alpha(r)$ is the static ground-state density and

$$J^G_\alpha(r, t) = \langle \Psi(t)|\hat{J}_\alpha(r)|\Psi(t)\rangle - \langle \Psi_0|\hat{J}_\alpha(r)|\Psi_0\rangle = \langle [\hat{J}_\alpha(r), \hat{G}(r, t)] \rangle$$

(44)

accounts for the small change through the deformation. Inserting (43) into the mean-field Hamiltonian (42) and expanding the latter in orders of $\hat{G}$, we get, in the first order, the response Hamiltonian $[\hat{V}_{res}, \hat{G}]_{ph}$ in terms of the density functional

$$[\hat{V}_{res}, \hat{G}]_{ph} = \sum_{\alpha\beta} \int dr dr' \bar{J}_\alpha(r) \frac{\delta^2 E}{\delta \bar{J}_\alpha(r) \delta J^G_\beta(r')}. $$

(45)

We now decompose the scaling state as in Eq. (46), which yields the analogous decomposition for the variation of density

$$J^G_\alpha(r, t) = -i \sum_k (p_k(t)J^{Q_k}_\alpha(r) - q_k(t)J^{P_k}_\alpha(r))$$

(46)

where, similar to Eq. (47),

$$J^{Q_k}_\alpha(r) = \langle [\hat{J}_\alpha(r), \hat{Q}_k(r)] \rangle,$$

(47a)

$$J^{P_k}_\alpha(r) = \langle [\hat{J}_\alpha(r), \hat{P}_k(r)] \rangle.$$  

(47b)
Every step proceeds much similar as for the general $\hat{G}$ because we are in the linear regime. Finally, we can specify the separable operators (12a) and (12b) as

$$\hat{X}_k(r) = \sum_{\alpha \beta} \int dr dr' \hat{J}_\alpha(r) \frac{\delta^2 E}{\delta J_\alpha(r) \delta J_\beta(r')} J^P_k(r'),$$  \hspace{1cm} (48a)$$

$$\hat{Y}_k(r) = \sum_{\alpha \beta} \int dr dr' \hat{J}_\alpha(r) \frac{\delta^2 E}{\delta J_\alpha(r) \delta J_\beta(r')} J^Q_k(r').$$  \hspace{1cm} (48b)$$

and the strength matrices (13a) and (13b) as

$$\kappa^{-1}_{kk'} = \sum_{\alpha \beta} \int dr dr' J^P_k(r) \frac{\delta^2 E}{\delta J_\alpha(r) \delta J_\beta(r')} J^{P'}_\beta(r'),$$  \hspace{1cm} (49a)$$

$$\eta^{-1}_{kk'} = \sum_{\alpha \beta} \int dr dr' J^Q_k(r) \frac{\delta^2 E}{\delta J_\alpha(r) \delta J_\beta(r')} J^{Q'}_\beta(r').$$  \hspace{1cm} (49b)$$

The latter equations demonstrate the symmetry of the strength matrices $\kappa_{kk'}$ and $\eta_{kk'}$. Note the subtle difference between separable operators and strengths. The double integrals look much the same. However, in Eqs. (48a) and (48b), they contain the current operators $\hat{J}_\alpha(r)$ and yield one-body operators as a result of the integration. Instead, Eqs. (49a) and (49b) do not include operators and so yield strength coefficients as c-numbers. For more details see Appendix B.

F. Choice of collective generators

As was mentioned above, the proper choice of generating operators $\hat{Q}_k(r)$ is crucial to achieve good convergence of the separable expansion (3) with a minimal number of separable operators. The choice is inspired by both physical and computational arguments. It should be simple and universal in the sense that it can be applied equally well to all modes and excitation channels. The main idea is that the generating operators should explore different spatial regions of the nucleus, the surface as well as the interior. This suggests that the leading scaling operator should have the form of the applied external field in the long-wave approximation, for example, $\hat{Q}^{\lambda \mu}_{k=1}(r) = r^\lambda (Y^{\lambda \mu}_{\lambda \mu}(\Omega) + \text{h.c.})$. Such a choice results in separable operators (18a) and (18b) which are most sensitive to the surface of the system. The first excitation modes of nuclear collective motion tend to vibrate predominantly in the surface region. As a result, already with this one separable term we obtain a quite good description of giant resonances. The detailed distributions depends on a subtle interplay of surface and
volume vibrations. This can be resolved by taking into account an interior of the nucleus. For this aim, the radial parts with larger powers \( r^{\lambda+p} Y_{\lambda\mu} \) and spherical Bessel functions can be used, much similar as in the local RPA [18, 20]. This results in the shift of the maxima of the operators (48a) and (48b) to the interior. Exploring different conceivable combinations, we have found a most efficient set of the generators as

\[ \hat{Q}_k(r) = R_k(r)(Y_{\lambda\mu}(\Omega) + \text{h.c.}) \quad , \quad \hat{P}_k = i[\hat{H}, \hat{Q}_k] \]  

(50)

with

\[ R_k(r) = \begin{cases} 
  r^{\lambda}, & k = 1 \\
  j^{\lambda}(q_k^r), & k = 2, 3, 4 
\end{cases} \]  

(51)

\[ q_k^r = a_k \frac{z^{\lambda}}{R_{\text{diff}}}, \quad a_2 = 0.6 \ , a_3 = 0.9 \ , a_4 = 1.2 \]

where \( R_{\text{diff}} \) is the diffraction radius of the actual nucleus and \( z^{\lambda} \) is the first root in \( j^{\lambda}(z^{\lambda}) = 0 \). The separable term with \( k = 1 \) is mainly localized at the nuclear surface while the next three terms are localized more and more in the interior. This simple set seems to be a best compromise for the description of nuclear giant resonances in light and heavy nuclei.

One may argue that one needs to explore not only different spatial regions but also the different types of the operators, associated with the variety of densities and currents (e.g. kinetic, spin-orbit). We have checked this. And indeed, this allows a perfect fine tuning of the spectra. On the other hand, it adds more separable terms. The set (50)-(51) with few and purely local generators is in our opinion the best compromise between quality of the results and the expense of the calculations.

### III. RESULTS AND DISCUSSION

We test SRPA for Skyrme forces by comparison with standard full RPA calculations. As the test cases we consider the isovector dipole and isoscalar quadrupole resonances in the two doubly magic nuclei \( ^{40}\text{Ca} \) and \( ^{208}\text{Pb} \). Most tests are performed with widely used Skyrme force SkM* [30]. The convergence of the SRPA expansion is found to be only slightly dependent on the actual force, which is demonstrated for the case of the recent parameterization SkI3 [28]. The full RPA calculations are done in a large basis which covers the 1\( ph \) excitations from all hole states to all particle states up to 30 MeV above the Fermi energy. SRPA easily
allows to extend this basis but we restrict the 1ph summations in SRPA to precisely the same active space as used in full RPA to have one-to-one comparison. The comparison is done in terms of the strength function $S_D(\omega)$ using a smoothing parameter of $\Gamma = 1\text{ MeV}$. Specifically, the Lorentz weight function was used. The value of the smoothing is quite realistic because experimental spectra are typically washed out by similar widths.

Figure 1 compares SRPA and full RPA results for SkM* forces. Three stages of SRPA expansion with increasing number of separable terms are shown: only one term $k = 1$, two terms $k = 1, 4$, and all four terms according to the labeling (51). Mind that we use in the second set the operator with $a_4 = 1.2$ since then the forces cover the extremes of both surface ($k=1$) and volume ($k=4$) sensitivity. The first stage with only $k = 1$ provides already very good description of the isoscalar E2 resonance in both nuclei. Even the low-energy quadrupole mode in $^{208}\text{Pb}$ is well reproduced. The influence of further terms is very small. The isovector E1 resonance seems to be more demanding. The first stage correctly describes the average position and width of the resonance but cannot fully cope with the detailed fragmentation pattern. Here one sees clearly the systematic improvement achieved by the further terms. The set with four terms provides a satisfying agreement. Further improvement is possible by adding more terms but the extra effort seems to be unnecessary. The agreement achieved is already more than sufficient for any practical calculations. One may even be content with a smaller sets for large scale exploratory studies.

Although the smoothing with 1 MeV provides the typical realistic shapes of strength distributions, it is worth having a quick glance at spectra with higher resolution. This is done in Fig. 2 drawn with smoothing $\Gamma =0.1\text{ MeV}$. The figure provides the detailed comparison of SRPA and exact RPA results for the most complicated cases of the isovector E1 resonance. Though we have not a perfect coincidence (which may be partly excused by numerical noise in the full RPA calculations), the results are indeed extremely close. This confirms the reliability of SRPA even at this finer scale.

Figure 3 demonstrates the quality of SRPA for another Skyrme force, namely the parameterization SkI3. It shows the toughest test case, the isovector E1 resonance in $^{208}\text{Pb}$. The full set $k=1-4$ is used. Again we see a satisfying reproduction of this ”worst case”. The other test cases (not shown here) perform even better. We also checked a few other forces and always found the same quality.

In fact, the spectrum from SkI3 looks much more realistic than that obtained from SkM*
(see lower right panel in Fig. 1. The pronounced right shoulder in SkM* is, to a large extent, an artefact from the restricted 1ph space used in these calculations. SRPA offers the opportunity to remove that restriction and to perform calculations in the full space of excitations, limited only by numerical resolution of the underlying coordinate space grid (with particle energies at least up to 600 MeV). Figure 4 compares SRPA results in the restricted and unrestricted spaces. It is obvious that the consistent inclusion of 1ph states above 30 MeV serves to diminish the unnaturally high shoulder at 16 MeV. This is a clear hint that RPA calculations converge slowly with the size of phase space. The SRPA allows to go much further in that respect.

Altogether, the tests show that the proposed set of four operators suffices for a satisfying reproduction of full RPA. The number ”four” has to be taken with a grain of salt. We have four operators of the type $\hat{Q}_k$ for protons and another four for neutrons. The set is complemented by the conjugate operators $\hat{P}_k = i[\hat{H}, \hat{Q}_k]$. This amounts to 16 operators in total and the computation of the strength function requires inversion of $17 \times 17$ matrices. The full RPA calculations, on the other hand, used matrices up to a rank of 400 and more.

IV. CONCLUSIONS

A novel self-consistent separable approach to the random phase approximation (SRPA) is proposed. SRPA employs a systematic expansion of the exact residual interaction into a sum of weighted separable terms (i.e. products of one-body operators evaluated at a Hartree level). For both, the weights and the separable operators, compact analytical expressions in terms of collective generators are derived in a self-consistent manner. The form of the separable residual interaction is optimized by the condition that the vibrating mean field as generated from collective flow reproduces the exact result. SRPA is formulated in a general way such that it can be applied to arbitrary energy functionals depending on various local densities and currents. As open point remains the choice of the relevant collective generators. Here we exploit experience gained in nuclear fluid dynamical models (local RPA) and use an efficient mix of surface and volume modes.

The general SRPA scheme is specified for the particular case of the Skyrme energy functional. The convergence of the separable expansion and performance of SRPA for giant resonances in light and heavy nuclei is investigated. The results are very encouraging.
SRPA proves to be a very good approximation to exact RPA and it requires much less numerical effort. In particular, it allows to work with huge particle-hole spaces, giving thus the chance for unrestricted RPA calculations, even in nuclei with broken symmetries.

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**APPENDIX A: COMPARISON WITH VPM**

It is interesting to compare SRPA with the Vibrating Potential Model (VPM) \[1, 7\] whose different versions are widely used in nuclear structure theory (see, for example, \[8, 9, 10, 11, 29\]). VPM deals with the mean field

\[
\hat{h}_0 = T + U_0
\]

where $U_0$ is purely local. Moreover, $U_0$ is often a density-independent phenomenological potential (oscillator, Nilsson, Woods-Saxon, ...). The vibrations of the system with the mean field (A1) are characterized by the collective deformations $q_k$ and in the framework of VPM the vibrating potential is written as

\[
U = U_0 + \delta U, \quad \delta U = \sum_k \frac{\partial U}{\partial q_k} \delta q_k = -i \sum_k X_k \delta q_k
\]

where $X_k = i \frac{\partial U}{\partial q_k}$ are single-particle operators in the r-representation, participating in VPM separable residual interactions (see e.g. \[29\]). Usually the collective momenta $P_k$ are introduced

\[
P_k = i \frac{\partial}{\partial q_k}.
\]

This allows to write

\[
X_k = i \frac{\partial U}{\partial q_k} = [U, P_k] \quad \Rightarrow \\
\hat{X}_k = \int d\mathbf{r} \hat{\rho}(\mathbf{r}) [\hat{U}, \hat{P}_k] \approx \int d\mathbf{r} \hat{\rho}(\mathbf{r}) < [\hat{U}, \hat{P}_k] >
\]
where $\hat{X}_k$ is the operator corresponding to $X_k$ and acting in the Fock space. VPM operators (A4) should be compared with the SRPA operators (12a) or (48a). For the first sight Eq. (A4) contradicts to (12a). However, there are, in fact, more similarities than differences. The equivalence of (A4) and (48a) can be demonstrated in the case of the simple energy functional

$$E = \frac{1}{2} t_0 \int dr \rho(r)^2. \quad (A5)$$

This functional gives the following mean field potential

$$U = t_0 \rho. \quad (A6)$$

VPM expression (A4) gives

$$\hat{X}_k = t_0 \int dr \hat{\rho}(r) < [\hat{\rho}(r), \hat{P}_k],$$

while SRPA expression (48a) provides

$$\hat{X}_k = \int dr dr' \hat{\rho}(r) \frac{\delta^2 E}{\delta \rho(r) \delta \rho(r')} < [\hat{\rho}(r'), \hat{P}_k] > = t_0 \int dr \hat{\rho}(r) < [\hat{\rho}(r), \hat{P}_k] >. \quad (A8)$$

Thus there is no difference between VPM and SRPA for simple two-body forces following from the functional (A5). Differences come up with non-linear terms in the mean-field potential. From this point of view SRPA seems to be more general.

**APPENDIX B: DETAILS OF SHF**

A most widely used energy functional for nuclear structure calculations is the Skyrme functional. It was originally proposed in [26] and made its breakthrough with the first fine-tuning of [27]. Since then it has been steadily further developed and constitutes today a very reliable model for nuclear structure and excitations. The functional involves the set of
local densities and currents (41) which read in general

\[ \rho_q(r) = \sum_{n \in q} \varphi_n^*(r) \varphi_n(r), \]
\[ \tau_q(r) = \sum_{n \in q} \nabla \varphi_n^*(r) \cdot \nabla \varphi_n(r), \]
\[ J_q(r) = -i \sum_{n \in q} \varphi_n^*(r) (\nabla \times \hat{\sigma}) \varphi_n(r), \]
\[ j_q(r) = -\frac{i}{2} \sum_{n \in q} [\varphi_n^*(r) \nabla \varphi_n(r) - \nabla \varphi_n^*(r) \varphi_n(r)], \]
\[ \sigma_q(r) = \sum_{n \in q} \varphi_n^*(r) \hat{\sigma} \varphi_n(r). \]

The associated operators are

\[ \hat{\rho}_q(r) = \hat{\Pi}_q \delta(\hat{r}' - r), \]
\[ \hat{\tau}_q(r) = \hat{\Pi}_q \hat{\varphi} \delta(\hat{r}' - r) \hat{\varphi}, \]
\[ \hat{J}_q(r) = \hat{\Pi}_q \delta(\hat{r}' - r) \hat{\varphi} \times \hat{\sigma}, \]
\[ \hat{j}_q(r) = \frac{1}{2} \hat{\Pi}_q \{ \hat{\varphi}, \delta(\hat{r}' - r) \}, \]
\[ \hat{\sigma}_q(r) = \hat{\Pi}_q \delta(\hat{r}' - r) \hat{\sigma} \]

where \( \hat{\Pi}_q \) is the isospin projector to \( q \in \{ p, n \} \).

We use here the Skyrme functional in the form (20)

\[ E = \int dr \left( \mathcal{E}_{\text{kin}} + \mathcal{E}_{\text{Sk}}(\rho_q, \tau_q, \sigma_q, j_q, J_q) \right) + E_C(\rho_p) - E_{\text{cm}}, \]  

(\text{B1})

\[ \mathcal{E}_{\text{kin}} = \frac{\hbar^2}{2m^*} \tau, \]  

(\text{B2})

\[ E_C = \frac{\hbar^2}{2} \int \frac{d^3r \rho_p(r) \frac{1}{|r - r'|} \rho_p(r')}{2}, \]

\[ -\frac{3}{4} \alpha^2 \frac{3}{\pi} \frac{1}{3} \int d^3r |\rho_p(r)|^\frac{3}{2}, \]  

(\text{B3})

\[ \mathcal{E}_{\text{Sk}} = +\frac{b_0}{2} \rho^2 - \frac{b_0'}{2} \sum_q \rho_q^2 + \frac{b_3}{3} \rho^{\prime 2} - \frac{b_3'}{3} \rho^2 \rho \sum_q \rho_q^2 + b_1(\rho \tau - j^2) - b_1' \sum_q (\rho_q \tau_q - j_q^2) \]

\[ -b_2 \rho \Delta \rho + \frac{b_2'}{2} \sum_q \rho_q \Delta \rho_q - b_4 (\rho \nabla \cdot J + \sigma \cdot (\nabla \times j)) \]

\[ -b_4' \sum_q [\rho_q (\nabla \cdot J_q) + \sigma_q \cdot (\nabla \times j_q)] - \tilde{b}_4 J^2 - \tilde{b}_4' \sum_q J_q^2, \]  

(\text{B4})
The sum $\sum_q$ runs over proton or neutron species. A density without index means total density, e.g. $\rho = \rho_p + \rho_n$ and similarly for the others. The parameters $b_i$ and $b'_i$ used in the above definition are chosen to give a most compact formulation of the energy functional, the corresponding mean-field Hamiltonian and residual interaction. They are related to the standard Skyrme parameters $t_i$ and $x_i$ [23, 27] by

\begin{align}
  b_0 &= t_0(1 + \frac{1}{2}x_0), \\
  b'_0 &= t_0(\frac{1}{2} + x_0), \\
  b_1 &= \frac{1}{4} \left[ t_1(1 + \frac{1}{2}x_1) + t_2(1 + \frac{1}{2}x_2) \right], \\
  b'_1 &= \frac{1}{4} \left[ t_1(\frac{1}{2} + x_1) - t_2(\frac{1}{2} + x_2) \right], \\
  b_2 &= \frac{1}{8} \left[ 3t_1(1 + \frac{1}{2}x_1) - t_2(1 + \frac{1}{2}x_2) \right], \\
  b'_2 &= \frac{1}{8} \left[ 3t_1(\frac{1}{2} + x_1) + t_2(\frac{1}{2} + x_2) \right], \\
  b_3 &= \frac{1}{4}t_3(1 + \frac{1}{2}x_3), \\
  b'_3 &= \frac{1}{4}t_3(\frac{1}{2} + x_3), \\
  b_4 &= b'_4 = \frac{1}{2}t_4.
\end{align}

Various versions exist for the spin-orbit term. The $J^2$ term is sometimes considered, sometimes not. Thus one has the choice

\begin{align}
  \tilde{b}_4 &= -\frac{1}{16}(t_1x_1 + t_2x_2), \\
  \tilde{b}'_4 &= +\frac{1}{16}(t_1 - t_2) \\
\end{align}

or

\begin{align}
  \tilde{b}_4 = 0, \quad \tilde{b}'_4 = 0.
\end{align}

The $\rho \nabla \cdot J$ term is always taken into account. Two options exist here. Conventional Skyrme forces use $b'_4 = b_4$ while a recent extension allows for more flexible isospin dependence considering $b_4$ and $b'_4$ as independent parameters [28]. The center-of-mass correction is $E_{cm} = \langle \hat{P}_{cm}^2 \rangle / 2mA$, often approximated by its diagonal terms $E_{cm} = \langle \sum_i \hat{p}_i^2 \rangle / 2mA$. The full correction is usually subtracted a posteriori and the approximate correction is expectation value of a one-body operator. In any case, there is no contribution to the residual interaction for RPA.

First variation with respect to $J_\alpha$ yields the mean-field Hamiltonian. It can be found in many publications, e.g. [20]. The step of interest is the second variation yielding the residual interaction. We obtain here
\[
\frac{\delta^2 E}{\delta \rho_q(r) \delta \rho_q'(r')} = \left( b_0 + b_3 \frac{(\alpha+1)(\alpha+2)}{3} \rho^\alpha - b'_3 \frac{\alpha(\alpha-1)}{3} \rho^{\alpha-2} \sum_q \rho_q^2 - b'_3 \frac{2\alpha}{3} \rho\alpha - b_2 \Delta \right) \delta(r - r') \\
- \left( b'_0 + b'_3 \frac{2}{3} \rho^\alpha + b'_3 \frac{2\alpha}{3} \rho^{\alpha-1} \rho_q + b'_2 \Delta \right) \delta_{qp} \delta(r - r') \\
+ \delta_{qp} \delta_{pq} \left( \frac{e^2}{|r - r'|} - \frac{1}{3} e^2 \left( \frac{3}{\pi} \right)^{2/3} \rho_p^{-2/3} \delta(r - r') \right),
\]

where \( \nu, \nu' \in \{x, y, z\} \) labels the spatial vector components and \( \epsilon_{\nu\nu'\mu} \) is the antisymmetric tensor. All combinations of derivatives not listed above vanish.

It is instructive to write the response currents in detail. The non-vanishing contributions are

\[
\rho^F_q(r) = \langle [\hat{\rho}_q, \hat{P}_k] \rangle \\
= \sum_{n \in q} \left( \varphi^*_n \hat{P}_k \varphi_n - \hat{P}_k \varphi^*_n \varphi_n \right),
\]

\[
\tau^F_q(r) = \langle [\hat{\tau}_q, \hat{P}_k] \rangle \\
= \sum_{n \in q} \left( \nabla \varphi^*_n \cdot \nabla \hat{P}_k \varphi_n - \nabla \hat{P}_k \varphi^*_n \cdot \nabla \varphi_n \right),
\]

\[
J^F_q(r) = \langle [\hat{J}_q, \hat{P}_k] \rangle \\
= -i \sum_{n \in q} \left( \varphi^*_n (\nabla \times \hat{\sigma}) \hat{P}_k \varphi_n - \hat{P}_k \varphi^*_n (\nabla \times \hat{\sigma}) \varphi_n \right),
\]

\[
J^Q_q(r) = \langle [\hat{J}_q, \hat{Q}_k] \rangle \\
= \Im \left\{ \sum_{n \in q} \left( \varphi^*_n \nabla \hat{Q}_k \varphi_n - \hat{Q}_k \varphi^*_n \nabla \varphi_n \right) \right\},
\]

\[
\sigma^Q_q(r) = \langle [\hat{\sigma}_q, \hat{Q}_k] \rangle \\
= \sum_{n \in q} \left( \varphi^*_n \hat{\sigma} \hat{Q}_k \varphi_n - \hat{Q}_k \varphi^*_n \hat{\sigma} \varphi_n \right).
\]
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**FIGURE CAPTIONS**

Figure 1: Isovector E1 and isoscalar E2 giant resonances in $^{40}$Ca and $^{208}$Pb calculated with SkM* forces in full RPA and SRPA at three levels of expansion as indicated. The strengths are weighted by Lorentz function with the averaging parameter $\Gamma = 1.0$ MeV.

Figure 2: Isovector E1 resonance in $^{40}$Ca and $^{208}$Pb calculated with SkM* forces in full RPA and SRPA with the complete set $k = 1 - 4$. The responses are weighted by Lorentz function with the small averaging parameter $\Gamma = 0.1$ MeV.

Figure 3: Isovector E1 resonance in $^{208}$Pb calculated with SkI3 forces in full RPA and SRPA with a set of four operators. The responses are weighted by Lorentz function with averaging parameter $\Gamma = 1.0$ MeV.

Figure 4: Isovector E1 resonance in $^{208}$Pb calculated in SRPA with the standard set of four operators and using the SkM* force. Two different phases spaces are compared: full line = unrestricted space of excitations, dashed line = space restricted to $ph$ energies < 30 MeV as in previous figures. The responses are weighted by Lorentz function with averaging parameter $\Gamma = 1.0$ MeV.
--- --- sep RPA

\( \omega \) [MeV]
