Repair and Resource Scheduling in Unbalanced Distribution Systems using Neighborhood Search

Anmar Arif, Student Member, IEEE, Zhaoyu Wang, Member, IEEE, Chen Chen, Member, IEEE, Jianhui Wang, Senior Member, IEEE,

Abstract—This paper proposes an optimization strategy to assist utility operators to recover power distribution systems after large outages. Specifically, a novel mixed-integer linear programming (MILP) model is developed for co-optimizing crews, resources, and network operations. The MILP model coordinates the damage isolation, network reconfiguration, distributed generator re-dispatch, and crew/resource logistics. We consider two different types of crews, namely, line crews for damage repair and tree crews for obstacle removal. We also model the repair resource logistic constraints. Furthermore, a new algorithm is developed for solving the distribution system repair and restoration problem (DSRRP). The algorithm starts by solving DSRRP using an assignment-based method, then a neighborhood search method is designed to iteratively improve the solution. The proposed method is validated on the modified IEEE 123-bus distribution test system.

Index Terms—Outage management, power distribution system, repair crews, routing, service restoration

NOMENCLATURE

Sets and Indices

\( m/n \) Indices for damaged components and depots
\( c \) Index for crews
\( w \) Index for depot/warehouse
\( i/j \) Indices for buses
\( k \) Index for distribution line connecting \( i \) and \( j \)
\( r \) Index for resource type
\( t \) Index for time
\( \varphi \) Index for the phase number
\( D \) Set of line crews
\( T \) Set of tree crews
\( N \) Set of damaged components and the depot
\( N(c) \) Set of components assigned to crew \( c \)
\( \Omega_{DK} \) Set of damaged lines
\( \Omega_{DK}^{(i)} \) Set of damages that need removing fallen tree first.
\( \Omega_{K(i,i)} \) Set of lines with bus \( i \) as the to bus
\( \Omega_{K(i,i)}^{(i)} \) Set of lines with bus \( i \) as the from bus
\( \Omega_{K(i)} \) Set of lines in loop \( l \)
\( \Omega_{P} \) Set of depots
\( \Omega_{V} \) Set of substations and voltage regulators
\( \Omega_{SW} \) Set of lines with switches

Parameters

\( \text{Cap}_{r}^{B} \) The capacity required to carry resource \( r \)
\( \text{Cap}_{c}^{C} \) The maximum capacity of crew \( c \)
\( \Omega_{m,r} \) The number of type \( r \) resources required to repair damaged component \( m \)
\( \text{Res}_{c,w}^{D} \) The number of type \( r \) resources that are located in depot \( w \)

\( \rho_{D}^{i} \) The cost of shedding the load at bus \( i \)
\( \rho_{SW}^{i} \) The cost of operating a switch at bus \( i \)
\( D_{m,n} \) Distance between components \( m \) and \( n \)
\( M \) Large positive number
\( P_{i,q}^{D} \) Diversified active/reactive demand at bus \( i \), phase \( \varphi \) and time \( t \)
\( P_{i,q}^{U} \) Undiversified active/reactive demand at bus \( i \) and phase \( \varphi \)
\( T_{m,c} \) The estimated time needed to repair (clear the trees at) damaged component \( m \) for line (tree) crew \( c \)
\( \tau_{m,n} \) Travel time between \( m \) and \( n \)
\( \phi_{k}^{i} \) Start/End location of crew \( c \)
\( Z_{k} \) The impedance matrix of line \( k \)
\( p_{k} \) Vector with binary entries for representing the phases of line \( k \)
\( a_{k} \) Vector representing the ratio between the primary and secondary voltages for each phase of the voltage regulator on line \( k \)
\( \delta_{w,c} \) Binary parameter equals 1 if crew \( c \) is positioned in depot \( w \)

Decision Variables

\( A_{m,c} \) Binary variable equal to 1 if component \( m \) is assigned to line/tree crew \( c \)
\( d_{m,n,c} \) A variable equals to \( D_{m,n} \) if components \( m \) and \( n \) are assigned to crew \( c \)
\( \text{Res}_{c,w}^{r} \) Amount of type \( r \) resources that crew \( c \) obtains from depot \( w \)
\( \gamma_{k,t} \) Binary variable indicates whether switch \( k \) is operated at time \( t \)
\( S_{k} \) A vector representing the apparent power of each phase for line \( k \)
\( U_{i,t} \) A vector representing the squared voltage magnitude of each phase for bus \( i \) at time \( t \)
\( X_{i,t} \) Binary variable equal to 1 if component \( m \) is in an outage area at time \( t \)
\( E_{c,m,r} \) The amount of type \( r \) resources that crew \( c \) has before repairing damaged component \( m \)
\( \alpha_{m,c} \) Arrival time of crew \( c \) at damaged component \( m \)
\( f_{m,t} \) Binary variable equal to 1 if damaged component \( m \) is repaired at time \( t \)
\( P_{i,q}^{L} \) Active/reactive load supplied at bus \( i \), phase \( \varphi \) and time \( t \)
\( P_{i,q}^{G} \) Active/reactive power generated by DG at bus \( i \), phase \( \varphi \) and time \( t \)
\( P_{i,q}^{K} \) Active/reactive power flowing on line \( k \), phase \( \varphi \) and time \( t \)
\( P_{c,w}^{D} \) A positive penalty term for the excess capacity that crew \( c \) require from depot \( w \)
\( u_{k,t} \) Binary variables indicating the status of the line \( k \) at time \( t \)
\( x_{m,n,c} \) Binary variable indicating whether crew \( c \) moves from damaged components \( m \) to \( n \)
\( y_{i,t} \) Connection status of the load at bus \( i \) and time \( t \)
\( z_{w,c} \) Binary variable equal to 1 if crew \( c \) require additional resources from depot \( w \)

I. INTRODUCTION

THE combination of an aging electrical grid and a dramatic increase in severe storms has resulted in increasing large-scale power outages. In 2016, the average outage durations for customers ranged from 27 minutes in Nebraska to 6 hours in West Virginia, while 20 hours in South Carolina due to Hurricane Matthew [1]. The year 2017 experienced 18 major weather events around the world. The 2017 outages that were caused by hurricanes Harvey, Irma, and Maria alone have cost the US around $202 billion [2].

Recently, modern outage management systems (OMS) with smart monitoring and automated control have been introduced...
to power distribution systems. OMS collects data from different sources, such as customer call, SCADA, and smart meters, to increase the situational awareness and assist the operators to schedule the repairs. Currently, utilities schedule the repair using a list of predefined restoration priorities based on previous experiences, and the network operation and the repair scheduling are split into two different processes. Our aim is to provide utilities with a better distribution system restoration decision-making process for coordinating crew scheduling, resource logistics, and network operation.

Although distribution system restoration has been long studied [3]–[5], there exist few efforts on integrating the repair scheduling with recovery operation in power distribution systems. A pre-hurricane crew mobilization mathematical model was presented in [6] for transmission networks. The authors used stochastic optimization to determine the number of crews to be mobilized to the potential damage locations. Also, the authors proposed a post-hurricane mixed integer linear programming (MILP) model to assign repair crews to damaged components without considering the travel times and repair sequence. In [7], the authors developed a stochastic program that assigns crews to substations in order to inspect and repair the damages, but the approach neglected crew routing. The authors in [8] presented a two-stage approach to decouple the crew routing and power restoration models in transmission systems. A MILP is solved in the first stage to find the priority of the damaged lines, and the routing problem is solved in the second stage using Constraint Programming. In [9], we developed a MILP that combines the distribution network operation and crew routing problems. The model was solved using a cluster-first route-second approach. Also, we developed a stochastic mixed integer linear program (SMIP) in [10] to solve the same problem with uncertainty. The problem was decomposed into two subproblems and solved using parallel progressive hedging.

Several critical factors have been neglected in the previous work on this topic. First, when scheduling the crews, one must consider the different types of crews. There are mainly two types of crews: 1) line crews who are responsible for the actual repair of grid components; and 2) tree crews who remove obstacles in the damage sites before the line crews start the repairing work. The mathematical model for optimizing the crew schedule must include both types of crews to obtain an applicable solution. Furthermore, the previous work did not include isolation of the damaged lines, which is imperative as the crews cannot repair a downed line until the power is cut off. In this paper, we improve our previous work in [9] and [10] by modeling the 3-phase operation of the distribution network and fault isolation constraints, coordinating tree and line crews, and modeling resource logistics in the distribution system repair and restoration problem (DSRRP). Due to the computational difficulty of the problem, a tri-stage algorithm is developed to solve the proposed model. The algorithm starts by solving an assignment problem, where the crews are assigned to the damaged components based on the distances between the crews and the outage locations, and the capacity of the crews. In the second stage, the DSRRP is solved with the crews dispatched to the assigned components from the first stage.

In the third stage, a neighborhood search approach [11] is used to iteratively improve the routing decisions obtained from stage two. The algorithm is used in a dynamically changing environment to handle the uncertainty of the repair time.

The rest of the paper is organized as follows. Section II develops the DSRRP mathematical formulation and Section III presents the algorithm for solving the model. The simulation results are presented in Section IV and Section V concludes this paper.

II. DISTRIBUTION NETWORK REPAIR AND RESTORATION

During extreme events, the utility receives real-time data of the condition of the network from the field devices, customer calls, and smart meters. Subsequently, the operator will have a general idea about the possible locations of the damages. Field assessors are dispatched to identify and document the exact locations of the damages. In this paper, we assume that the assessors have located the damages, and estimated the repair time and required resources. This section presents the mathematical model for coordinating line and tree crews, and the recovery operation of the network, which comes after damage assessment.

A. Objective

\[
\min \sum_{\forall t} \left( \sum_{\forall \psi} \sum_{\forall i} (1 - y_{i,t}) \rho^D P^D_{i,\psi,t} + \rho^SW \sum_{k \in \Omega_{SW}} \gamma_{k,t} \right) \quad (1)
\]

The first term in objective (1) minimizes the cost of load shedding, while the second term minimizes the cost of operating the switches. The base load shedding cost is assumed to be $14/kWh in this paper [12], and the base cost is multiplied by the load priority to obtain \( \rho^D_{i} \). The switch operation cost is set to be $8/time [13].

B. Cold load pickup

\[
P^L_{i,\psi,t} = y_{i,t} P^D_{i,\psi,t} + (y_{i,t} - y_{i,max(t-\lambda,0)}) P^U_{i,\psi,t} \quad \forall i, \psi, t \quad (2)
\]

\[
Q^L_{i,\psi,t} = y_{i,t} Q^D_{i,\psi,t} + (y_{i,t} - y_{i,max(t-\lambda,0)}) Q^U_{i,\psi,t} \quad \forall i, \psi, t \quad (3)
\]

\[
y_{i,t+1} \geq y_{i,t} \quad \forall i, t \quad (4)
\]

Constraints (2)-(3) set up the cold load pick-up (CLPU) constraint [10]. In this paper, we employ two blocks to represent CLPU as suggested in [14]. The first block is for the undiversified load \( P^U \) and the second for the diversified load \( P^D \) (i.e., the steady-state load consumption). The use of two blocks decreases the computational burden imposed by nonlinear characteristics of CLPU and provides a conservative operation assumption to guarantee supply-load balance. Define \( \lambda \) as the number of time steps required for the load to return to normal condition. The value of \( \lambda \) is equal to the CLPU duration divided by the time step. The function \( \max(t-\lambda,0) \) is used to avoid negative time steps. If at time step \( t = t_1 \), a load goes from a de-energized state \( y_{i,t_1-1} = 0 \) to an energized one \( y_{i,t_1} = 1 \), it returns to normal condition at time step \( t = t_1 + \lambda \). \( P^U_{i,\psi,t} \) is added to \( P^D_{i,\psi,t} \) before time step \( t_1 + \lambda \) to represent the undiversified load. We assume that the duration of the CLPU decaying process is one hour [14].
and the total load at pick-up time is 200% of the steady state value [15]; i.e., \( P^U_{i,p,t} \) is set to be equal to \( P^D_{i,p,t} \). Constraint (4) indicates that once a load is served it cannot be shed.

### C. Power limits

\[
0 \leq P^G_{i,p,t} \leq P^G_{i,max}, \forall i, p, t \tag{5}
\]

\[
0 \leq Q^G_{i,p,t} \leq Q^G_{i,max}, \forall i, p, t \tag{6}
\]

\[-u_{k,t} P_{k,min} \leq P^K_{k,t} \leq u_{k,t} P_{k,max} , \forall k, t \tag{7}
\]

\[-u_{k,t} Q^K_{k,min} \leq Q^K_{k,t} \leq u_{k,t} Q^K_{k,max} , \forall k, t \tag{8}
\]

Constraints (5)-(8) define the active and reactive power limits of the DGs and lines. The limits on the line-flow constraints are multiplied by \( u_{k,t} \) so that if a line is damaged or a switch is opened, there will be no power flowing on it.

### D. Power flow equations

\[
\sum_{k \in K_{\ldots}} P^T_{k,p,t} + P^G_{i,p,t} = \sum_{k \in K_{\ldots}} P^L_{k,p,t} + P^G_{i,p,t}, \forall i, p, t \tag{9}
\]

\[
\sum_{k \in K_{\ldots}} Q^T_{k,p,t} + Q^G_{i,p,t} = \sum_{k \in K_{\ldots}} Q^L_{k,p,t} + Q^G_{i,p,t}, \forall i, p, t \tag{10}
\]

\[
U_{j,t} - U_{i,t} + \bar{Z}_k S_k^* + \bar{Z}_k^* S_k \leq (2 - u_{k,t} - p_k)M, \forall k \in L_\Omega \setminus V, t \tag{11}
\]

\[
U_{j,t} - U_{i,t} + \bar{Z}_k S_k^* + \bar{Z}_k^* S_k \geq - (2 - u_{k,t} - p_k)M, \forall k \in L_\Omega \setminus V, t \tag{12}
\]

Constraints (9)-(10) are 3-phase active and reactive power node balance constraints. Constraints (11)-(12) represent Kirchhoff’s voltage law. \( S_{i,j} \in \mathbb{C}^{3 \times 3} \) is the three-phase apparent power from bus \( i \) and \( j \), and \( U_i = [\vert V_i^a \vert^2, \vert V_i^b \vert^2, \vert V_i^c \vert^2]^T \). The matrix \( \bar{Z}_{i,j} \) equals \( A \otimes Z_{i,j} \), where \( Z_{i,j} \in \mathbb{C}^{3 \times 3} \) is the impedance matrix of the line, and \( A \) is a phase shift matrix. Detailed derivation of (11) and (12) is provided in [4].

The big \( M \) method is used to decouple the voltages between lines that are disconnected or damaged. Also, if line \( k(i,j) \) is two-phase (e.g., phases \( a \) and \( c \)), then the voltage constraint is only applied to these two phases, which is realized by including \( p_k \).

The vector \( p_k \in \{0,1\}^{3 \times 1} \) represent the phases of line \( k \); e.g., for line \( k \) with phases \( a, c, p_k = [1,0,1] \).

### E. Regulators

\[-(2 - u_{k,t} - p_k)M \leq a^2_{i} U_{j,t} - U_{i,t}, \forall k \in L_\Omega \setminus V, t \tag{13}
\]

\[a^2_{i} U_{j,t} - U_{i,t} \leq (2 - u_{k,t} - p_k)M, \forall k \in L_\Omega \setminus V, t \tag{14}
\]

Constraints (13) and (14) model the relationship between the voltage magnitudes on both sides for a three-phase voltage regulator, with \( i \) as the primary side and \( j \) as the secondary side. The vector \( a_k \in \mathbb{R}^{3 \times 1} \) is the ratio between the primary and secondary winding for each phase, where the ratio is assumed to be constant [4]. Similarly to (11) and (12), the big M method is used.

### F. Reconfiguration and Isolation

\[\mathcal{X}_{i,t} U_{min} \leq U_{i,t} \leq \mathcal{X}_{i,t} U_{max} , \forall i, t \tag{15}
\]

\[-u_{k,t} M \leq U_{i,t} + U_{j,t} \leq u_{k,t} M, \forall k(i,j) \in \Omega_{DK}, t \tag{16}
\]

\[2u_{k,t} \geq \mathcal{X}_{i,t} + \mathcal{X}_{j,t}, \forall k \in \Omega_{DK}, t \tag{17}
\]

\[\mathcal{X}_{i,t} \geq \gamma_{i,t}, \forall i, t \tag{18}
\]

\[u_{k,t} = 1, \forall k \notin \{\Omega_{SW} \cup \Omega_{DK}\}, t \tag{19}
\]

\[\sum_{k \in \Omega_{K(l)}} u_{k,t} \leq |\Omega_{K(l)}| - 1, \forall l, t \tag{20}
\]

\[\gamma_{k,t} \geq u_{k,t} - u_{k,t-1}, \forall k \in \Omega_{SW}, t \tag{21}
\]

\[\gamma_{k,t} \geq u_{k,t-1} - u_{k,t}, \forall k \in \Omega_{SW}, t \tag{22}
\]

Constraint (15) ensures that the voltage is within a specified limit, and is set to equal to 0 if the bus is in an on-outage area. The voltages on the buses between damaged lines are forced to be 0 using constraint (16). Therefore, the zero voltage propagates on the rest of the network through constraints (11)-(14) until a circuit breaker (CB) or sectionalizer stops the propagation. Constraint (17) sets the values of \( \mathcal{X}_{i} \) and \( \mathcal{X}_{j} \) to be 0 if the line is damaged, and (18) forces the load to be shed if the bus is de-energized. Constraint (19) defines the default status of the lines that are not damaged or not switchable. Constraint (20) is the radiality constraint. Radiality is enforced by introducing constraints for ensuring that at least one of the lines of each possible loop in the network is open [16]. A depth-first search method is used to identify the possible loops in the network and the lines associated with them. Constraint (21)-(22) are used in order to limit the number of switching operations. We define a variable \( \gamma_{k,t} \) which is equal to 1 if the line switches its status from 0 (off) to 1 (on), or 1 (on) to 0 (off). This variable is included in the objective to minimize the number of switching operation.

### G. Routing constraints

\[
\sum_{\forall n \in N} x_{\phi^0,\phi^0} = 1, \forall c \tag{23}
\]

\[
\sum_{\forall n \in N} x_{\phi^1,\phi^1} = 1, \forall c \tag{24}
\]

\[
\sum_{\forall n \in N} x_{\phi^0,\phi^1} = 0, \forall c, m \in N \setminus \{\phi^0, \phi^1\} \tag{25}
\]

\[
\sum_{\forall n \in N} x_{\phi^0,\phi^0} = 1, \forall n \in \Omega_{DK} \tag{26}
\]

\[
\sum_{\forall n \in N} x_{\phi^1,\phi^1} = 1, \forall n \in \Omega_{DT} \tag{27}
\]

Constraint (23)-(24) guarantee that each crew starts and ends its route at the defined start (\( \phi^0 \)) and end (\( \phi^1 \)) locations. Constraint (25) is the flow conservation constraint; i.e., once a crew arrives at a damaged component, the crew moves to the next location after finishing the repairs. Constraint (26) ensures that each damaged component is repaired by only one line crews, while (27) ensures that each damaged component that needs removing a fallen trees first, is assigned to one tree crew.
H. Arrival time

\[ \alpha_{m,c} + T_{m,c} + tr_{m,n} - (1 - x_{m,n,c}) M \leq \alpha_{n,c}, \forall m \in N \setminus \{\phi_c^1\}, n \in N \setminus \{\phi_c^0, m\}, c \]  
\[ \sum_{c \in C_L} \alpha_{m,c} \geq \sum_{c \in C_T} \alpha_{m,c} + T_{m,c} \sum_{n \in N} x_{m,n,c}, \forall m \in \Omega_{DT} \]  

Constraint (28) is used to calculate the arrival time (the time when crew \( c \) starts repairing component \( m \)) for each crew at each damaged component. For a crew that travels from damaged component \( m \) to \( n \), \( \alpha_{m,c} \) equals \( \alpha_{m,c} + T_{m,c} + tr_{m,n} \). Big \( M \) is used to decouple the times to arrive at components \( m \) and \( n \) if the crew does not travel from \( m \) to \( n \). The travel times can be obtained through a geographic information system (GIS). Constraint (29) indicates that the line crews start repairing the damaged components after the tree crews clear the obstacles.

I. Resource and pickup constraints

\[ Res_D^{w,r} \geq \sum_{c \in C_L, \phi_c^w} Res_{c,\phi_c^w,r}^{c} + \sum_{c \in C_L} Res_{c,\phi_c^w,r}^{c}, \forall w, r \]  
\[ \sum_{c \in \Omega} Cap_{e,c}^{r} E_{c,m,r} \leq Cap_{r}^{c}, \forall m, c \in C_L \]  
\[ x_{m,n,c} R_{m,r} \leq E_{c,m,r}, \forall m, n, r, c \in C_L \]  
\[ M(1 - x_{m,n,c}) \leq E_{c,m,r} - R_{m,r} - E_{c,n,r} \leq M(1 - x_{m,n,c}), \forall m \in N \setminus \{\phi_c^1\}, n \in N \setminus \{\phi_c^0, m\}, c \in C_L, r \]  
\[ -M(1 - x_{w,n,c}) \leq E_{c,w,r} + Res_{c,w,r}^{p} - E_{c,n,r} \leq M(1 - x_{w,n,c}), \forall w, n, \in N \setminus \{\phi_c^0, \phi_c^1, w\}, c \in C_L, r \]  
\[ -M(1 - x_{\phi_c^0,n,c}) \leq Res_{c,\phi_c^0,r}^{c} - E_{c,n,r} \leq M(1 - x_{\phi_c^0,n,c}), \forall n \in N \setminus \{\phi_c^0\}, c \in C_L, r \]

Constraint (30) states that the total resources that the crews obtain from depot \( w \) must be less or equal to the amount of available resources in the depot. The amount of resources that a crew can carry must be limited by the crew’s capacity, which is realized by constraint (31). Constraint (32) indicates that the crews must have enough resources to repair the damaged components. Constraint (33) ensures that if a crew travels from \( m \) to \( n \), then the resources that the crew have when arriving at location \( n \) is \( E_{c,n,r} = E_{c,m,r} - R_{m,r} \). If a crew goes to depot \( w \) to pick-up supplies and travels to damaged component \( n \), then \( E_{c,n,r} = E_{c,w,r} + Res_{c,w,r}^{p} \), which is enforced by (34). Constraint (35) ensures that the number of resources that the crew has at the first damaged component is equal to the resources obtained at the starting location.

J. Restoration time

\[ \sum_{vt} f_{m,t} = 1, \forall m \in \Omega_D \]  
\[ \sum_{vt} tf_{m,t} \geq (\alpha_{m,c} + T_{m,c} \sum_{n \in N} x_{m,n,c}), \forall m \in \Omega_D \]

0 \leq \alpha_{m,c} \leq M \sum_{n \in N, c} x_{m,n,c}, \forall m \in N \setminus \{\phi_c^0, \phi_c^1\}, c \]  
\[ u_{m,t} = \sum_{r=1}^{T} f_{m,r}, \forall m \in \Omega_{DL}, t \]  
\[ \{f, x, u, y, X, \gamma\} \in \{0, 1\}, \{E, Res_c\} \geq 0 \]

Constraints (36)-(39) are used to connect the crew scheduling and power operation problems. Let \( f_{m,r} \) denote the time when the damaged component is repaired by the line crews, which equals 1 in one time interval as enforced by (36). Equation (37) determines the time when a damaged component is repaired by setting \( \sum_{\forall r} t_{f_{m,r}} \) to be greater than or equal to \( \alpha_{m,c} + T_{m,c} \) of the crew assigned to damaged component \( m \). Constraint (38) is used to set \( \alpha_{m,c} = 0 \) if crew \( c \) does not travel to component \( m \), so it would not affect constraint (37). Finally, constraint (39) indicates that the restored component becomes available after it is repaired, and remains available in all subsequent time periods. For example, if \( f_{m,t} = [0, 0, 1, 0, 0, 0] \) then \( u_{m,t} = [0, 0, 1, 1, 1, 1] \).

III. Solution Algorithm

DSRRP combines two problems, the Vehicle Routing Problem (VRP) for routing the crews [17], and distribution system operation for outage restoration. VRP is an NP-hard combinatorial optimization problem, where the computation time rises exponentially with the size of the problem. Adding distribution system operation constraints will further increase the complexity. A three-stage algorithm for solving the combined routing and distribution system operation problem is presented in this section, where the stages are: assignment, initial solution, and neighborhood search. Furthermore, to compare the developed method with current practices, a priority-based method that mimics the utilities’ scheduling procedures is developed.

A. Reoptimization algorithm

1) Assignment: by assigning the damaged components to the crews, the large VRP problem can be converted to multiple small-size Travelling Salesman Problems (TSP) [17]. The assignment problem is formulated as follows:

\[ \min \sum_{\forall m} \sum_{v} \left( \frac{1}{2} d_{m,n,c} \right) + \sum_{\forall w} \left( d_{w,m,c} + p_{c,w} \right) \]  
\[ \sum_{\forall v} A_{m,c}^{L} = 1, \forall m \in \Omega_{DK} \]  
\[ \sum_{\forall v} A_{m,c}^{T} = 1, \forall m \in \Omega_{DK} \]  
\[ \sum_{\forall r} Cap_{r} E_{c,w,r} \leq (\delta_{w,c} + z_{w,c}) Cap_{r}, \forall w, c \in C_L \]  
\[ z_{w,c} \leq \delta_{w,c}, \forall w, m, c \in C_L \]  
\[ p_{w,c} \geq z_{w,m,c} - M(1 - z_{w,c}), \forall w, m, c \in C_L \]  
\[ \sum_{\forall r} Res_{c,w,r}^{c} \leq Res_{c,w,r}^{D}, \forall w, r \]  
\[ \sum_{\forall w} Res_{c,w,r}^{c} \geq \sum_{\forall m} A_{m,c}^{L} R_{m,r}, \forall c \in C_L, r \]
The term $d_{m,n,c} \geq D_{m,n}(A^L_{m,c} + A^L_{n,c} - 1), \forall m,n,c \in C^L$ (49)

$d_{w,m,c} \geq D_{w,m}(\delta_{w,c} + A^T_{m,c} - 1), \forall w,m,c \in C^L$ (50)

$d_{m,n,c} \geq D_{m,n}(A^T_{m,c} + A^T_{n,c} - 1), \forall m,n,c \in C^T$ (51)

$d_{w,m,c} \geq D_{w,m}(\delta_{w,c} + A^T_{m,c} - 1), \forall w,m,c \in C^T$ (52)

\[ \{ A^{L/T} , z \} \in \{0,1\}, \{ P, Res^C \} \geq 0 \] (53)

The objective (41) consists of two parts: 1- minimizing the distances between the damaged components assigned to the crews; 2- limiting the number of times a crew goes back to the depot to pick up additional resources by using a penalty term. The term $d_{m,n,c}$ is divided by two since it is symmetric; i.e., $d_{m,n,c} = d_{n,m,c}$. Constraints (42)-(43) assign each damaged component to one crew. The amount of resources the crew can carry is limited by the crew’s capacity in (44). Binary variable $z_{w,c}$ is equal to 1 if a crew requires additional resources, in such case, the crew goes back to the depot to pickup more resources. Constraint (45) states that the crews can go back to the depot they started from. We set the penalty term $P_{w,c}$ to be two times of the maximum distance between the depot and the assigned damage components, as defined in (46). The big $M$ constant is added so that the penalty term equals 0 if the crew does not go back to the depot for additional resources. The crews must use the resources available in the depot as enforced by (47). Constraint (48) indicates that the number of resources the crew have should be enough to repair the assigned damaged components. Constraints (49)-(52) are used to identify the distances between the damaged components that are assigned to each crew. If components $m$ and $n$ are assigned to crew $c$, then $d_{m,n,c} \geq D_{m,n}$. Since this is a minimization problem, $d_{m,n,c} = D_{m,n}$ if $A^L_{m,c} = A^L_{n,c} = 1$.

2) Initial solution and reoptimization: after assigning each damaged component to a crew, the DSRRP is solved and the crews are dispatched to the assigned components. Subsequently, a neighborhood search method is used to improve the initial route while the crews are working. The optimization problem considered in this paper involves a dynamically changing environment due to the uncertainty of the repair time. The repair time is updated periodically either by the repair crews or the damage assessors. Therefore, we apply the neighborhood search algorithm continuously and update the routing solution as more information is obtained. The advantage of this method is that it allows the algorithm to update the solution while the repair crews are repairing the lines, therefore, loosening the time limit restriction. The pseudo-code for the proposed algorithm, referred to as the Reoptimization Algorithm, is detailed in Algorithm 1.

In Step 1, the assignment problem is solved using CPLEX [18] to obtain the binary variables $A^L_{m,c}$ and $A^T_{m,c}$. These variables are used to find $N(c)$, which is the set of damaged components assigned to crew $c$. For example, consider the set of damaged components $\Omega_{DK} = \{1,2,3,4,5\}$, if line crew 1 is assigned with damaged components 1 and 3, then $A^L_{m,c} = \{1,0,1,0,0\}$ and $N(1) = \{1,3\} \cup \Omega_P$. $N(c)$ is found for each crew in Steps 2-7. Consequently, a simplified DSRRP is solved in Step 8 by allowing the crews to only repair the assigned damaged components. In Step 10, the obtained route $x^*$ and objective $\zeta^*$ are set to be the incumbent (current best solutions) route ($\bar{x}$) and objective ($\bar{\zeta}$).

Steps 11-29 represent the neighborhood search algorithm. The algorithm selects a subset of damaged components $\bar{N}$, where $\bar{N} \subset N$, then removes the paths connected to $\bar{N}$ and sets the rest of the routes to be constant by forcing $x_{m,n,c} = \bar{x}_{m,n,c}, \forall c, m \in N \setminus \bar{N}, n \in N \setminus \bar{N}$. Afterwards, DSRRP is solved to obtain an improved solution, the process is demonstrated in Fig. 1, where $|\bar{N}| = 3$.

Steps 12 and 13 initialize a counter and the sample size (ss), respectively. In Step 15, the subset $\bar{N}$ is determined by randomly selecting ss nodes from N. The parameters ss0, h1, and h2 are constants used to tune the algorithm. The value of ss0 determines the size of the subset $\bar{N}$ in the first iteration.
The size of \( \bar{N} \) is increased after \( h_3 \) iterations with no change to the objective, and the neighborhood search algorithm is terminated after \( h_1 + h_2 \) iterations with no change to the objective. In this paper, \( ss_0 \) is set to be 3, as selecting 1 damaged component will not change the route, and selecting 2 has minimal impact on the route. The values of \( h_1 \) and \( h_2 \) were determined experimentally using several test cases, both \( h_1 \) and \( h_2 \) equal 3.

The DSRRP is solved in Step 16 with parts of the route set as constant. To obtain a faster solution, we warm-start (provide a starting point) CPLEX by using the incumbent solution. The objective value \( \zeta^* \) obtained from Step 16 is compared to the current incumbent solution \( \zeta \). If the value is improved, we set \( \zeta^* \) and \( x^* \) as the current incumbent solutions and update the counter, otherwise, the counter increases by one. The process is repeated until the counter reaches \( h_1 \), where we increase the size of the subset in Step 24. If the sample size is \( |N| \); i.e., the complete problem is solved without simplification, then the solution is optimal and the neighborhood search stops. Also, the search ends once the counter reaches \( h_1 + h_2 \), or if the time limit is reached. The crews are then dispatched to the damaged components, and the traveled paths are set as constants in the optimization problem. After that, the repair time is updated and Steps 14-26 are repeated to update the route, as shown in Fig. 2. The idea of the dynamic approach is to run Steps 14-26 while maintaining the best solution in an adaptive memory. Once the operator receives an update from the field, the neighborhood search is restarted with the newly acquired information. Whenever a crew finishes repairing the assigned damage component, the crew is provided with the current best route \( \bar{x} \).

\[ x^p = \arg \min \left\{ \sum_{p} \sum_{k \in L_p} \sum_{c \in C^L} w_p \alpha_{c,k} \mid \text{s.t.} (23)-(38) \right\} \tag{54} \]

The objective of (54) is to minimize the arrival time of the line crews at each damaged component, while prioritizing the high-priority lines through multiplying the arrival time by the weight \( w_p \). The priority-based model is similar to DSRRP, but without the power operation constraints. However, it is still difficult to solve directly in a short time using a commercial solver such as CPLEX. Therefore, the same procedure presented in Algorithm 1 is used to solve (54). After obtaining the route \( x^p \), the DSRRP problem is solved by setting \( x = x^p \); i.e., we solve \( \min \{1\} \) s.t. (2)-(40), \( x_{m,n,c} = x^p_{m,n,c}, \forall c,m,n \} \).

IV. SIMULATION AND RESULTS

The modified IEEE 123-bus distribution feeder is used as a test case for the DSRRP problem. Detailed information on the network can be found in [20]. The network is modified by including 7 dispatchable DGs and 18 new switches. The 7 DGs are rated at 300 kW and 250 kVar. It is assumed that the loads at buses 30, 48, 49, 53, 65, and 76 are critical loads. The problems of optimally allocating the resources, DGs, or switches, are out of the scope of this paper. We assume there are 3 depots, 6 line crews distributed equally between the depots, and 4 tree crews with 2 located in Depot 2 and 1 tree crew in each of the other depots. The time step in the simulation is 1 hour. The simulated problem is modeled in AMPL and solved using CPLEX 12.6.0.0 on a PC with Intel Core i7-4790 3.6 GHz CPU and 16 GB RAM.

A. DSRRP solution comparison

The repair and restoration problem is solved using four methods: 1) a cluster-first DSRRP-second (C-DSRRP) approach presented in [9], the method clusters the damaged components to the depot, then solves DSRRP; 2) the priority-based method presented in Section III-B; 3) an assignment-based method where the damaged components are assigned to the crews, then DSRRP is solved (A-DSRRP), which is similar to Steps 1-8 in Algorithm 1; 4) Reoptimization Algorithm.

Once an outage occur, the distribution network is reconfigured, and the DGs are dispatched to restore as many customers as possible, before conducting the repairs. A random event is generated on the IEEE 123-bus system, where 14 lines are damaged, four of which are damaged by trees. Fig. 3 shows the recovery operation of the distribution system to the outages before the repairs; i.e., the state of the system at time \( t = 0 \). The solution shown in Fig. 3 is obtained regardless of the solution algorithm used, as the algorithms will only affect the repair schedule and the network operation during the repairs. Before the outage, all switches are closed except 151-300 and 54-94. Since line 7-8 is damaged, the circuit breaker at the substation is opened. Sectionalizer 28-168 is switched off, forming a small microgrid, to serve the loads at buses 28 to 30. Similarly, switches 44-165, 151-300, 60-160, 60-169, 77-172, 97-174, and 97-197 are opened to form additional microgrids using the DGs in the network. The repair/tree clearing times
and required resources are given in Table I. The estimated repair time is assumed to be accurate, and the time limit is set to be 3600 seconds [21]. It is assumed that each crew can carry 30 units of resources, and the required capacities (\(Cap^k\)) for the 6 types of resources are \{3, 2.5, 2, 1, 4, 1\}. A summary of the results and performances of different solution methods is shown in Table II.

![Fig. 3. Initial state of the distribution network after 14 lines are damaged](image)

**TABLE I**

| Line | Resources (units) | Estimated repair/clearing time (hrs) |
|------|-------------------|-------------------------------------|
|      | A | B | C | D | E | F | Line Crew | Tree Crew |
| 7-8  | 1 | 2 | 0 | 1 | 0 | 0 | 2.5 | 1 |
| 15-17| 1 | 2 | 1 | 1 | 0 | 0 | 1.25 | 1 |
| 18-19| 1 | 2 | 1 | 1 | 0 | 0 | 0.5 | 1 |
| 27-33| 1 | 2 | 1 | 1 | 0 | 0 | 2.25 | 1 |
| 38-39| 1 | 2 | 1 | 1 | 0 | 0 | 1 | 0.75 |
| 54-57| 0 | 2 | 0 | 1 | 2 | 0 | 0.75 | 1 |
| 58-59| 1 | 2 | 1 | 1 | 0 | 0 | 0.5 | 1 |
| 18-163| 0 | 2 | 0 | 1 | 0 | 2 | 1.75 | 1 |
| 67-72| 0 | 2 | 0 | 1 | 0 | 0 | 4 | 1.25 |
| 76-86| 1 | 2 | 1 | 1 | 0 | 0 | 6 | 2 |
| 91-93| 0 | 2 | 0 | 1 | 2 | 0 | 1.5 | 1 |
| 93-95| 1 | 2 | 1 | 1 | 0 | 0 | 2.75 | 1 |
| 105-106| 1 | 2 | 1 | 1 | 0 | 0 | 1.75 | 1 |
| 113-114| 1 | 2 | 1 | 1 | 0 | 0 | 0.75 | 0.5 |

**TABLE II**

| Method          | Objective Value | Computation Time | kWh | Restoration Time |
|-----------------|-----------------|------------------|-----|------------------|
| A-DSRRP         | $173,613        | 411 s            | 60,749 | 14 hrs         |
| C-DSRRP         | $171,594*       | 3600 s           | 60,800 | 10 hrs          |
| Priority-based  | $168,808        | 3428 s           | 59,667 | 9 hrs           |
| Reoptimization  | $148,185        | 1135 s           | 62,436 | 9 hrs           |

*The solver did not converge to the optimal solution after 1 hour (time limit). The optimality gap is 17.74%.

The third column in Table II is the amount of energy served, and the fourth column (restoration time) is the time when all loads are restored. The assignment-based approach is the fastest but has the worst schedule, neighborhood search in the Reoptimization algorithm improved the routing solution and obtained the best objective value out of the four methods. C-DSRRP did not converge within the time limit, while the priority-based method achieved an objective value which is $20,623 larger than Reoptimization. The change in percentage of load served for each method is shown in Fig. 4. The proposed algorithm outperformed the other methods.

**B. Dynamic DSRRP**

In practice, the crew repair time is continuously changing. Moreover, the dispatch commands must be issued as fast as possible to reduce the outage duration. Therefore, the DSRRP must be solved efficiently and the solutions should be dynamically updated according to the current crew repair time. First, we assume that the crews must be dispatched to their first destination in 30 minutes. Since A-DSRRP (Steps 1-8 in Algorithm 1) converged after 411 seconds, the crews are dispatched to their first destination after running the neighborhood search algorithm for 1389 seconds. To simulate the change in repair time, it is assumed that once a crew reaches the damaged component, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is changed from 2.5 to 3 hours. The repair time is updated every 15 minutes. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time. For example, once crew 1 arrives at line 7-8, the repair time is updated to its actual value by adding a random number from the continuous uniform distribution on [-2,2] to the estimated time.

The complete route is given in Table III. The total cost is $144,113, and the total energy served is 60,764.9 kWh. Table IV shows the timeline of events after solving DSRRP, where all loads are restored after 8 hours. The initial states of the switches are shown in Fig. 3, and the subsequent switching operations are given in Table IV. Crew 6 repairs line 58-59 in time step 2, but the number of served loads does not change since line 54-57 is damaged, which forces the area to be isolated by the switches. Three lines are repaired and three switches are closed at time step 3, which leads to the recovery of loads at buses 35 to 46, and 101 to 114. Switch 18-135 is opened since lines 18-19 and 18-163 are damaged. After repairing line 7-8 in time step 4, the CB is closed and the network starts to receive power from the substation. The

![Fig. 4. Percentage of load served at each time step](image)
3-phase output of the DGs and the substation are shown in Fig. 5. The DG at bus 52 is not used in this case, as the substation can serve the loads in the area after repairing lines 7-8 and 54-57. As soon as lines 18-163, 18-19, and 27-33 are repaired, the loads at buses 18 to 27 and 31 to 33 are restored by closing the switches shown in Table IV at step 6. Repairing lines 15-17 and 67-72 restores 10 loads, and finally all loads are restored at step time 8 by repairing the remaining lines. Switch 151-300 is opened and 97-174 is closed to return the network to its original configuration, and the substation can serve all loads.

Table III

| Crew 1 | Route |
|-------|-------|
| DP 1  | 7-8   |
|       | →     |
| Crew 2 | 105-106 → DZ 1 |
| Crew 3 | 54-57 → 18-19 → DZ 1 91-93 → DZ 2 |
| Crew 4 | 113-114 → 76-80 → DZ 2 |
| Crew 5 | 38-39 → DZ 1 18-163 → DZ 3 |
| Crew 6 | 58-59 → 27-33 → DZ 3 |

Table IV

| Time step | Switch operation | Lines repaired | % Load Served |
|-----------|------------------|----------------|---------------|
| 1         | 58-59            | 28%            |
| 2         | 58-59            | 28%            |
| 3         | 38-39, 105-106   | 113-114, 50%   |
| 4         | 7-8              | 54-57, 69%    |
| 5         | 163-18           | 69%            |
| 6         | 18-19, 27-33     | 78%            |
| 7         | 15-17, 67-72     | 89%            |
| 8         | 76-86, 91-93-95  | 100%           |

Fig. 5. The 3-phase active power delivered by the DGs and at substation.

V. CONCLUSION

In this paper, a new mathematical model that combines 3-phase unbalanced distribution system operation, fault isolation and restoration, and resources coordination is developed. The model included the coordination of line and tree crews as well as equipment pickup for conducting the repairs. Furthermore, a three-stage algorithm is developed with a newly designed neighborhood search algorithm to iteratively improve the routing solution. The developed approach is able to restart when the repair time is updated, and the crews are dispatched based on the incumbent solution. Test results have shown the proposed algorithm can provide effective restoration plans within the time limit.

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