Computational study of the waverider aerothermodynamics by the UST3D computer code

D S Yatsukhno
Ishlinsky Institute for Problems in Mechanics of the Russian academy of sciences, Vernadsky prospect 101(1), Moscow, 119526, Russia
E-mail: yatsukhno-ds@rambler.ru

Abstract. The paper describes the numerical simulation results for two models of the waverider. The aerodynamic characteristics as well as three-dimensional flow fields were calculated by the UST3D code. The splitting method is applied for the Navier-Stokes system integration. The solution is obtained by the time relaxation. The governing equations approximations are applied for unstructured tetrahedron grids. The UST3D code verification was performed by the first waverider model aerodynamics. The scramjet was integrated with the second waverider model. The engine energy deposition simulation was conducted by the source term addition into the energy equation. In this particular case aerothermodynamics parameters estimation is presented.

1. Introduction
The gas dynamic construction method is, in fact, the reverse aerodynamic problem. The vehicle shape is obtained by the supersonic inviscid flow field about simple bodies or their combinations. The shock wave is used for the bottom surface flow compression and lift increasing. There is no shock detaching in the absence of leading edge blunting. The upper surface is usually parallel with regard to the freestream flow and possesses a negligible drag. This idea was made real by Nonweiler for the new vehicle called waverider [1]. The latter has a higher aerodynamic efficiency than the conventional aerospace systems. However, it should be noted that waverider configuration is suitable for the specific operation conditions which define streamlines and shock shape. The waverider aerodynamics can be improved for the off-design operation points by the optimization leading edge shape optimization [2]. The viscous effects and laminar-turbulent transition can be included in the optimization processes. The overall sizes, payload and volume efficiency are optimization criteria at the same time.

An ability of the waverider to integrate with other modern aerospace systems components is of practical interest. The scramjet-integrated vehicles can be constructed using the waverider generation principles. In particular, the above mentioned method of the high-efficiency lifting surfaces shaping can be also employed for the scramjet configuration and optimization [3, 4]. Besides, waveriders configurations for the various planet atmospheres were put forward [5, 6]. Thus, the waverider application area is fairly wide. The waverider is the reliable verification and validation model due to a considerable number of the computational and experimental data.

The first part of the numerical study contains the UST3D code verification using viscous-optimized waverider aerodynamic coefficients calculations as an example. The scramjet-integrated waverider
model gas dynamic parameter evaluation under the condition of the energy source presence is given in the second part of the work.

2. Computational model
The UST3D code is based on the following governing equations [7]: the continuity equation, the Navier-Stokes equations and the energy equation. The thermal and calorific perfect gas state equations with the specific full energy relation and the Fourier law are used for the full system closing. The vector form of the system are given as

\[
\begin{align*}
\frac{\partial}{\partial t} & \left[ \begin{array}{c}
\rho \\
\rho v \\
\rho w \\
\rho E \\
\end{array} \right] + \frac{\partial}{\partial x} \left[ \begin{array}{c}
\rho u \\
\rho vu + \rho w + p \\
\rho vw + p \\
\rho vE + pv \\
\end{array} \right] + \frac{\partial}{\partial y} \left[ \begin{array}{c}
\rho u \\
\rho vu + \rho w + p \\
\rho vw + p \\
\rho vE + pv \\
\end{array} \right] + \frac{\partial}{\partial z} \left[ \begin{array}{c}
\rho u \\
\rho vu + \rho w + p \\
\rho vw + p \\
\rho vE + pv \\
\end{array} \right] = \begin{array}{c}
0 \\
0 \\
0 \\
0 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\tau_{xx} + u\tau_{xy} + v\tau_{yx} + w\tau_{zx} \\
\tau_{yy} + u\tau_{yx} + v\tau_{xy} + w\tau_{zy} \\
\tau_{zz} + u\tau_{zx} + v\tau_{xz} + w\tau_{yz} \\
\end{align*}
\]

\[
p = (\gamma - 1)\rho U = (\gamma - 1)\rho \left[ E - 0.5\left( u^2 + v^2 + w^2 \right) \right]
\]

\[
U = c_v T
\]

\[
E = \frac{p}{(\gamma - 1)\rho} + \frac{u^2 + v^2 + w^2}{2}
\]

\[
q = -\lambda \nabla T
\]

Here \( \rho \) – density; \( u, v, w \) – velocity vector projections; \( x, y, z \) – Cartesian coordinates; \( p \) – pressure; \( E \) – specific full energy; \( \lambda \) – thermal conductivity; \( T \) – temperature; \( \mu \) – dynamic viscosity; \( \gamma \) – ratio of specific heats; \( U \) – specific internal energy; \( c_v \) – specific heat capacity. Shear stresses is defined by the following relations

\[
\begin{align*}
\tau_{xx} = \mu \left( \frac{4}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial w}{\partial z} \right), \\
\tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \\
\tau_{yy} = \mu \left( \frac{4}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial w}{\partial z} \right), \\
\tau_{yz} = \tau_{zy} = \mu \left( \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} \right), \\
\tau_{zz} = \mu \left( \frac{4}{3} \frac{\partial w}{\partial z} - \frac{2}{3} \frac{\partial u}{\partial y} - \frac{2}{3} \frac{\partial v}{\partial x} \right), \\
\tau_{zx} = \tau_{xz} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)
\end{align*}
\]

The method for splitting into the physical processes was used for the mentioned system numerical integration. The general mathematical theory of such methods is described in [8]. There are significant
distinctions between classical gas dynamics splitting methods [9] and unstructured grids splitting approach. In latter case, the averaged flow parameters derivatives values are considered instead conventional finite differences. The approximations obtained by the divergence theorem given as [7]

\[
\int \frac{\partial f}{\partial \alpha} dV = \frac{1}{V_i} \int_{V_i} \frac{\partial f}{\partial \alpha} dV = \frac{1}{V_i} \int_{V_i} \hat{f} \cdot dS = \frac{1}{V_i} \sum_{j} S_j \hat{f}^{j} / n_{i,j}^{j}
\]

(7)

Here \( \alpha = x, y, z \); \( i \) – tetrahedron index; \( j \) – surface index; \( f_j^{j} = (u, v, w, p, q, \tau_{ad}) \) – flow parameters corresponding \( i \) – tetrahedron and \( j \) – surface; \( q_{ad} \) – heat flux projections. The scheme for present explicit splitting method has a first approximation order by time and space. The slip boundary conditions are applied on the vehicle surface. The wall temperature is constant.

3. Numerical simulation results

3.1. Viscous-optimized waverider model calculations

The conical-derived waverider virtual model generated for the on-design Mach number of 6 is depicted in figure 1. Table 1 contains the main geometry parameters of the vehicle. There are two groups of data obtained by the various computer codes for mentioned model. The first data involves the aerodynamic coefficients which were gained by the MAXWARP code [2] and CFL3D code [10] using structured grids. The second data is the aerodynamic characteristics calculated by UST3D code which has already been applied for the validation and verification problems [11]. The code uses unstructured grids. All computations were conducted for the three flow conditions shown in table 2 at zero angle of attack. The tetrahedron grid is shown in figure 2.

![Figure 1. Waverider virtual model [10].](image1)

![Figure 2. Unstructured grid (2515038 elements).](image2)

Table 1. Waverider reference geometry parameters.

| Parameter | Value |
|-----------|-------|
| L, cm     | 6000  |
| W, cm     | 3725  |
| \( S_{ref} \), cm² | 1.423 |
| \( S_{base} \), cm² | 105 |
Table 2. Free-stream conditions for the various Mach number.

|       | \( M_\infty = 4 \) | \( M_\infty = 6 \) | \( M_\infty = 8 \) |
|-------|------------------|------------------|------------------|
| \( p_\infty \), erg/cm\(^3\) | 17791            | 11866            | 8889             |
| \( \rho_\infty \cdot 10^4 \), g/cm\(^3\) | 0.268            | 0.179            | 0.134            |
| \( T_\infty \), K              | 231.3            |                  |                  |
| \( Re_\infty \cdot 10^4 \), cm\(^3\) | 2.156            |                  |                  |

Figures 3 (a) and 3 (b) show the lift and drag coefficients which depend on the Mach number. The planform area was used as the reference area. It should be noticed that the drag coefficient contains only inviscid part which is determined by the pressure forces. The lift-to-drag ratio is presented in figure 3 (c). The values of the all parameters are decreased with Mach number increasing. The UST3D lift and drag coefficients are lower than the corresponding structured grid codes values. Nevertheless, the three various code using the different numerical methods and grid type demonstrate generally good agreement between the corresponding calculations results.

![Figure 3](image-url)

**Figure 3.** Lift (a) and drag (b) coefficients as well as lift-to-drag ratio (c) as functions of a Mach number.

The pressure distributions at the exit flow plane for the various Mach numbers are shown in figure 4. The best visual correspondence is observed between UST3D and CFL3D results [10], which...
were obtained for Mach 4. There are somewhat pressure contours differences for on-design Mach number and Mach 8 consisting of the different compression flow degrees about leading edge. Perhaps it is due to relatively coarse unstructured grid within the leading edge. The calculations of a decay of a discontinuity were performed by the different approaches for both codes. It partly explains obtained aerodynamic coefficients results discrepancy.

Figure 4. Pressures ratio contours at exit plane for Mach 4 (a), Mach 6 (b) and Mach 8 (c).

3.2. Scramjet-integrated waverider model calculations

The second task was following. The computations of the flow over a scramjet-integrated waverider were conducted at Mach 6 for two angles of attack. The waverider model and unstructured grid are presented in figures 5–7. The distinguishing feature of the work is the taking into account the source term into the energy equation. The physical processes concerning with the chemical reactions and turbulence affection on the flow parameters are outside the framework of the paper [12, 13]. The study has a following motivation: the drag reduction and lift increase can be reached by the energy impact on the flow [14, 15]. The scramjet energy deposition capacity is defined in accordance with the following relations [15]

$$ q_{\text{max}} = 0.7 \frac{I_{\infty} f_{O_2} H_{\text{u}}}{L V_{\text{energy}}} = 280 \frac{W}{\text{cm}^3} $$

Here $I_{\infty}$ – mass flow rate; $f_{O_2} = 0.2315$ – oxygen part; $L = 34.5$ – stoichiometric coefficient; $H_{\text{u}} = 119.54 \text{ MJ/kg}$ – fuel specific enthalpy; $V_{\text{energy}} = 3000 \text{ cm}^3$ – energy release zone volume. Energy source zone location is shown in figure 8. Free-stream flow conditions and attack angles are shown in table 3.
**Figure 5.** Scramjet-integrated model sizes.

**Figure 6.** Other dimensions.

**Figure 7.** Unstructured grid (3225320 elements).
Table 3. Scramjet-integrated waverider flow conditions.

| Parameter   | Value   |
|-------------|---------|
| $p_e$, erg/cm$^3$ | 11900   |
| $\rho_e\cdot10^4$, g/cm$^3$ | 0.179   |
| $T_e$, K     | 231.3   |
| $\alpha$, °   | 0 and 5 |

Figure 8. Energy source location.

Figure 9 describes Mach number distribution for various cases. There are no considerable differences in no-energy-source flows (a, b) despite the fact that various parts of the free-stream flow fall into the air intake leading edge. The additional flow stagnation is caused by the scramjet energy deposition (c). The stagnation zone is expended with respect to the no-source operation points.

The pressure distribution is shown in figure 10. The shock wave system intensity increases with the growth of angle of attack. The counter-pressure is generated by the energy source. It results in pressure increase and its redistribution behind the energy release zone (c). The strong flow compression is observed at the blunting scramjet leading edge (a, b, c).

The temperature fields in the scramjet are depicted in figure 11. The higher temperature is observed in the stagnation regions compared with the rest of the flow. The temperature influence is manifested directly at the source location and is distributed downstream. The source capacity was selected so that temperature could provide scramjet wall suitability. The maximum scramjet temperature is approximately 2200 K. The blunted nose is another configuration part which is subjected to strong heating. The maximum nose temperature is 1750 K (the calculations results are not shown here).

The upper part pressure contours at the symmetry plane are shown in Figure 12 (b). The pressure increase within the scramjet is observed in case of source deposition.

The joint information about aerodynamic characteristics is given in table 4. Another energy source with capacity $q_{\text{max}} = 400$ Wt/cm$^3$ was also considered. There are the following energy source features. The first is the scramjet upper wall pressure growth providing lift rise. The second is the air intake shock weakening caused by the stagnation pressure decrease. The high-capacity source provides significant lift increase and drag decrease but the scramjet wall temperature in this case is approximately 3100 K (the calculations results are not shown here).
Figure 9. Mach number distribution: zero angle of attack (a), five degrees angle of attack (b) and zero angle of attack with energy source (c).

Figure 10. Pressure distribution: zero angle of attack (a), five degrees angle of attack (b) and zero angle of attack with energy source (c).
Figure 11. Temperature distribution: zero angle of attack (a), five degrees angle of attack (b) and zero angle of attack with energy source (c).

Figure 12. Upper surface (a) and corresponding pressure distributions (b).
Table 4. Aerodynamics characteristics of the scramjet-integrated vehicle.

| $p_{\infty}$, erg/cm$^3$ | $q_{\text{max}} = 0$ | $q_{\text{max}} = 280$ W/cm$^3$ | $q_{\text{max}} = 400$ W/cm$^3$ |
|-------------------------|------------------------|-------------------------------|-------------------------------|
| $C_L$                   | 0.0217                 | 0.0263 | 0.0502 |
| $C_D$                   | 0.211                  | 0.208  | 0.200  |
| $L/D$                   | 0.103                  | 0.126  | 0.251  |

4. Conclusions
The numerical study of the various hypersonic vehicles aerodynamics characteristics was conducted by the UST3D code. The good agreement between the UST3D numerical simulation results and other computer code calculations was obtained. Generally, the aerodynamic efficiency of the viscous optimized waverider obtained taking into account only pressure-based drag are not decreased considerable at the off-design operation points. The scramjet-integrated model calculations were performed with the fixed energy source and without one. The results show that the scramjet energy source allows decrease aerodynamics drag with respect to the no-source scramjet configuration. The significant increase of the lift was observed. It should be pointed that there is the additional wall heating caused by the energy source. It is necessary that energy source capacity should be selected in accordance with the scramjet chamber material thermal stability.

Acknowledgments
The work was performed within the framework of the Government program of basic research of the Russian academy of sciences (contract # AAAA-A17-117021310372-6) and partially support of the RFBR grant # 016 – 01 – 00379.

References
[1] Nonweiler T R F 1959 Aerodynamic problems of manned space vehicle Journal of the Royal Aeronautical Society 63 pp 521–528
[2] Corda S, Anderson J D 1988 Viscous Optimized Hypersonic Waveriders Designed from Axisymmetric Flow Fields AIAA 26th Aerospace Sciences Meeting January 11–14 Reno Nevada AIAA Paper 88–0369
[3] Ferguson F 1995 A Design Method for the Construction of Hypersonic Vehicle Configurations AIAA 6th International Aerospace Planes and Hypersonic Technologies Conference April 3–7 Chattanooga TN AIAA Paper 95–6009
[4] O’Neil M K L, Lewis M J 1992 Design Tradeoffs on Engine-Integrated Hypersonic Vehicles 1st Aerospace Design Conference February 3–6 Irvine CA AIAA Paper 92–1205
[5] Lunan D A 2015 Waverider, Revised Chronology 20th AIAA International Space Planes and Hypersonic Technologies Conference July 6–9 Glasgow Scotland AIAA Paper 2015 – 3529
[6] Anderson J D, Lewis M J, Kothari A P and Corda S 1990 Hypersonic Waverider for Planetary Atmospheres AIAA 28th Aerospace Sciences Meeting January 8–11 Reno Nevada AIAA Paper 90–0538
[7] Zheleznyakova A L and Surzhikov S T 2013 Application of the Method of Splitting by Physical Processes for the Computation of a Hypersonic Flow over an Aircraft Model of Complex Configuration High Temperature 51 (6) pp 816–829
[8] Marchuk G I 1988 Splitting methods (Moscow: Science) p (in Russian)
[9] Belotserkovsky O M and Davydov Yu M 1982 Method of large particles in gas dynamics. (Moscow: Science) p 392 (in Russian).
[10] Takashima N and Lewis M J 1992 Navier-Stokes computations of a Viscous Optimized Waverider AIAA 30th Aerospace Sciences Meeting January 6–9 Reno Nevada
AIAA Paper 92–0305

[11] Surzhikov S.T. 2017 Validation of computational code UST3D by the example of experimental aerodynamic data *Journal of Physics: Conference Series* **815** 012023

[12] Seleznev R K, Surzhikov S T, Shang J A 2016 Quasi-One-Dimensional Analysis of Hydrogen-Fueled Scramjet Combustors 52nd AIAA/SAE/ASEE Joint Propulsion Conference Salt Lake City *AIAA Paper* 2016–4569

[13] Seleznev R K 2017 Comparison of two-dimensional and quasi-one-dimensional models by the example of VAG experiment *Journal of Physics: Conference Series* **815** 012007

[14] Shang J S, Surzhikov S T, Yan H 2012 Hypersonic Nonequilibrium Flow Simulation Based on Kinetics Models *Frontiers in Aerospace Engineering* **1** (1) 12 p.

[15] Surzhikov S T and Shang J S 2013 Numerical Prediction of Convective and Radiative Heating of Scramjet Combustion Chamber with Hydrocarbon Fuels 51st Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition *AIAA Paper* 2013–1076

[16] Surzhikov S T and Shang J S 2013 Radiative Heat Exchange in a Hydrogen-Fueled Scramjet Combustion Chambers 51st Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition *AIAA Paper* 2013–0448

[17] Surzhikov S T, Zheleznyakova A L, Shang J S and Rivir R B 2013 Simulating Gasdynamic Interaction and Radiative Heating within Scramjets with Hydrocarbon Fuels 44th AIAA Thermophysics Conference *AIAA Paper* 2013–2642

[18] Knight D Survey of Aerodynamic Drag Reduction at High Speed by Energy Deposition *Journal of Propulsion and Power* **24** (6) pp 1153–1167

[19] Khankhasaeva Ya V, Borisov V E, Lutsky A E 2016 Energy Impact on the Flow Around Hypersonic Flying Vehicles *Physical-Chemical Kinetics in Gas Dynamics* **17**(4) (in. Russian)