Spin-Polarized States of Nuclear Matter*

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Abstract The equations of state of spin-polarized nuclear matter and pure neutron matter are studied in the framework of the Brueckner–Hartree–Fock theory including a three-body force. The energy per nucleon $E_A(\delta)$ calculated in the full range of spin polarization $\delta = (\rho_1 - \rho_\uparrow)/\rho$ for symmetric nuclear matter and pure neutron matter fulfills a parabolic law. In both the cases the spin-symmetry energy is calculated as a function of the baryonic density along with the related quantities such as the magnetic susceptibility and the Landau parameter $G_0$. The main effect of the three-body force is to strongly reduce the degenerate Fermi gas magnetic susceptibility even more than the value with only two-body force. The equation of state is monotonically increasing with the density for all spin-aligned configurations studied here so that no any signature is found for a spontaneous transition to a ferromagnetic state.

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1 Introduction

Studies of spin-polarized nuclear and neutron matter have been mainly focused on the possible onset of a ferromagnetic transition in the neutron star core. This transition could explain in fact the high intensity magnetic fields ($10^{12}$ gauss) estimated from the timing observations in pulsars and magnetars (for a review see Ref. [1]). Besides this exciting issue the motivation for such studies can be based on a more general context of nuclear physics.

First of all, from the change of the energy per nucleon with the spin polarization one may extract a theoretical prediction for the spin symmetry energy, whose empirical value is so far quite uncertain. The nuclear matter instability against spin fluctuations is driven by the Landau parameter $G_0$, which is determined from the spin-symmetry energy. The value of this parameter is still a largely controversial topic and no agreement exists among the different approaches to the theory of nuclear matter.[2] Experimental information could come from spin giant resonances, which have not yet been clearly observed. Information could also come from heavy-ion collisions as soon as polarized heavy targets become available.

The second issue related to the study of spin-aligned states of nuclear matter is the propagation of neutrinos in neutron stars. It has been shown that the neutrino mean free path is strongly affected by the magnetic susceptibility. The latter is sizeably suppressed by the strong correlations in nuclear matter and, as a consequence, the mean free path might change sizeably and, eventually drop to zero in the presence of a ferromagnetic transition.[3,4]

There is a guess that the ferromagnetic transition could be a relativistic effect due to $\pi$-exchange. In fact all calculations based on the relativistic mean-field approach predict this transition to occur in dense matter.[5,6] On the other hand, non-relativistic approaches[7–10] do not support such a transition except Hartree–Fock calculations with phenomenological Skyrme-like forces (for a review see Ref. [4]). This aspect cannot be disconnected from the problem of the in-medium nucleon-nucleon (NN) force, which is poorly known in dense matter due to the lacking of empirical constraints far above the saturation density. However important relativistic effects can be incorporated into the effective interaction via the three-body force associated with a virtual nucleon-antinucleon excitation.[11] Moreover non-relativistic calculations including only two-body forces miss the empirical saturation point of nuclear matter.[12] So it seems worth while to investigate the spin-aligned states of nuclear and neutron matter in the non-relativistic Brueckner theory with three-body forces. They contain not only the above-mentioned relativistic contributions but also nucleonic excitations which decisively enhance the agreement between theoretical and empirical saturation densities.[13]

2 Formalism

The spin and isospin asymmetric nuclear matter (ANM) consists of spin-up neutrons ($\uparrow$), spin-down neutrons ($\downarrow$), spin-up protons ($\uparrow$), and spin-down protons ($\downarrow$) in different density states: $\rho_{\uparrow\uparrow}$, $\rho_{\uparrow\downarrow}$, $\rho_{\downarrow\uparrow}$, and $\rho_{\downarrow\downarrow}$, respectively. Therefore four parameters are required to specify a given configuration of spin and isospin ANM.

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The Fermi momenta of the four components are generally different from each other, and related to their respective densities by the following relation: \( \rho_\lambda = (1/6\pi^2)(k_F^\lambda)^3 \), where \( \lambda \) denotes the z-components of isospin and spin, i.e., \( \lambda = (\tau_z, \sigma_z) \). Instead of \( \rho_\lambda \), one can use the following four parameters to identify a given spin and isospin state, \( \beta = (\rho_n - \rho_p)/\rho, \delta_n = (\rho_{n1} - \rho_{p1})/\rho_n, \delta_p = (\rho_{p1} - \rho_{p0})/\rho_p \), where \( \rho, \rho_n, \) and \( \rho_p \) are total density, neutron density and proton density, respectively. The ratio \( \beta \) is the isospin asymmetry parameter and \( \delta_n \) and \( \delta_p \) are the spin asymmetry parameters for neutrons and protons, respectively.

The starting point of the Brueckner–Bethe–Goldstone (BBG) approach is the reaction G-matrix. The G-matrix incorporates strong short-range correlations in nuclear medium by means of the infinite ladder diagram summation of the bare NN interaction. It satisfies the Bethe–Goldstone equation. The latter can be expressed for the spin-isospin ANM in the total angular-momentum basis as follows:

\[
G^{TSJ,\lambda,\lambda'}_{LL'}(\omega, P; q, q'; \rho, \beta, \delta_n, \delta_p) = \frac{\langle \delta\lambda\lambda' (q', P) \rangle}{\omega - \epsilon_{12}^\lambda (q', P) + i\eta} G^{TSJ,\lambda,\lambda'}_{LL'}(\omega, P; q'', q'; \rho, \beta, \delta_n, \delta_p),
\]

where \( \delta\lambda\lambda' (q', P) \) are the partial wave components of the NN interaction, \( \vec{P} = \vec{k}_1 + \vec{k}_2 = \vec{k}_1' + \vec{k}_2' \) is the total momentum, \( \vec{q} = (\vec{k}_1 - \vec{k}_2)/2 \) and \( \vec{q}' = (\vec{k}_1' - \vec{k}_2')/2 \) the relative momenta of the two particles in their initial state and final state, respectively. The Pauli operator \( Q^{\lambda\lambda'} (q'', \vec{P}) \) and the energy denominator \( e_{12}^\lambda (q'', P) = \epsilon^\lambda (k_1'') + \epsilon^\lambda (k_2'') \) have been angle-averaged in order to remove the coupling between different channels, \( \alpha = \{JST\} \). It is worth while noticing that the different components of the G-matrix differ in general from each other due to the dependence of the Pauli operator and energy denominator on the spin-isospin configuration \( (\lambda, \lambda') \). The single particle energy is given by \( \epsilon^\lambda (k) = \hbar^2 k^2/2m + U^\lambda (k) \). The continuous choice for the auxiliary potential \( U^\lambda (k) \) is adopted in the present calculations since, on the one hand, it has been shown to provide a much faster convergence of the hole-line expansion than the gap choice, \( [14] \) on the other hand, it describes physically the single-particle potential felt by a nucleon in nuclear medium.

In the continuous choice, \( U^\lambda (k) \) is the real part of the on-shell mass operator, i.e.,

\[
U^\lambda (k) = \text{Re} \sum_{\vec{k}, \lambda} n_\lambda (k') \langle \vec{k}\lambda, \vec{k}'\lambda' | G[\omega = \epsilon^\lambda (k) + \epsilon^\lambda (k'), P] | \vec{k}\lambda, \vec{k}'\lambda' \rangle_A,
\]

where the subscript \( A \) denotes anti-symmetrization. For spin and isospin ANM, it is convenient to split it into two contributions as \( U^\lambda = U^\lambda + U^\lambda ', \ (\lambda' \neq \lambda) \). Each individual contribution is calculated by casting it into the partial wave expansion,

\[
U^{\sigma, \tau_z, \sigma'_z}_z (k) = \int_0^{k_F}\frac{d^3k'}{2\pi^3} \sum_{TSL} \sum_{L'L} \sum_{T'L} \sum_{S_1 S_2} \frac{i^{L-L'} L'}{2} \left[ C\left(\frac{1}{2}\sigma_z \frac{1}{2}\sigma'_{z}|SS_2\right)\right]^2 \left[ C\left(\frac{1}{2}\tau_z \frac{1}{2}\tau'_{z}|TT_2\right)\right]^2 \times \sum_{M_L} C(L'M_L SS_2 | JM_L + S_2) C(LM_L SS_2 | JM_L + S_2) Y^*_{L'M_L} (\hat{q}) Y_{LM_L} (\hat{q}) \times G^{TSJ,\lambda,\lambda'}_{LL'}(\omega, P, q, q; \rho, \beta, \delta_n, \delta_p).
\]

The summation over partial wave states is physically constrained by the selection rule \( S + T = L \) due to the Pauli principle and consequently the anti-symmetrization of the G-matrix simply implies multiplication by a factor of 2 for the allowed partial wave channels. For spin symmetric case \( (\delta_n = \delta_p = 0) \), a spin-up neutron (proton) has the same Fermi momentum as a spin-down neutron (proton) and thus the single-particle potential felt by a nucleon does not depend on the direction of its spin. The summation on the spins of the two particles in the final state and the average of that in the initial state remove the non-diagonal contributions in angular-momentum from the single-particle potential. One easily finds

\[
U^{\sigma, \tau_z, \sigma'_z}_z (k) = \frac{1}{2} \sum_{\sigma_z, \sigma'_z} U^{\sigma, \tau_z, \sigma'_z}_z (k) = \frac{1}{2} \int_0^{k_F}\frac{d^3k'}{2\pi^3} \sum_{TSL} \sum_{T'L} \left[ C\left(\frac{1}{2}\sigma_z \frac{1}{2}\tau'_{z}|TT_2\right)\right]^2 \times \sum_{M_L} C(L'M_L SS_2 | JM_L + S_2) C(LM_L SS_2 | JM_L + S_2) Y^*_{L'M_L} (\hat{q}) Y_{LM_L} (\hat{q}) \times G^{TSJ,\lambda,\lambda'}_{LL'}(\omega, P, q, q; \rho, \beta, \delta_n, \delta_p).
\]

For spin-asymmetric but isospin-symmetric nuclear matter, we have \( \beta = 0 \) and \( \delta_n = \delta_p = (\rho_1 - \rho_0)/\rho = \delta \). In this case, the single particle potential becomes

\[
U^{\sigma, \tau_z, \sigma'_z} = \int_0^{k_F}\frac{d^3k'}{2\pi^3} \sum_{TSL} \sum_{T'L} \left[ C\left(\frac{1}{2}\sigma_z \frac{1}{2}\tau'_{z}|SS_2\right)\right]^2 \times \sum_{M_L} C(L'M_L SS_2 | JM_L + S_2) C(LM_L SS_2 | JM_L + S_2) Y^*_{L'M_L} (\hat{q}) Y_{LM_L} (\hat{q}) \times G^{TSJ,\lambda,\lambda'}_{LL'}(\omega, P, q, q; \rho, \delta).
\]

The present calculations will mainly consider this spin-polarized nuclear matter as well as the spin polarized neutron matter.
3 Results and Conclusions

We performed some calculations within the BHF self-consistent approach described above. The Argonne $V_{18}$ force is adopted as bare two-body interaction. This has been implemented by a microscopic three-body force, which is described in detail in Ref. [15] together with the average procedure to transform it into an effective two-body force. In Fig. 1, the energy shift per nucleon $E_A(\delta, \rho) - E_A(0, \rho)$ in symmetric nuclear matter is plotted as a function of the square of spin polarization $\delta^2$ for a set of densities. Due to the linear dependence on $\delta^2$, it is also reported in Refs. [9] and [10] that one can write $E_A(\delta, \rho) = E_A(0, \rho) + \mathcal{E}_{\text{sym}}(\rho)\delta^2$, i.e., the spin-dependence of the energy per nucleon can be simply expressed in terms of spin-symmetry energy $\mathcal{E}_{\text{sym}}(\rho) = \left(\partial^2 E_A(\delta, \rho) / \partial \delta^2\right) / 2$ in the density range considered here.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{Upper windows: The EOS of spin-asymmetric nuclear matter as a function of spin-asymmetry at five values of density, predicted by Brueckner–Hartree–Fock calculations adopting pure $AV_{18}$ two-body force (right-upper window) and $AV_{18}$ plus the TBF (left-upper window). Left-lower window: density dependence of spin-symmetry energy for both the cases with the TBF (solid curve) and without the TBF (dash curve). Right-lower window: magnetic susceptibility $\chi/\chi_0$ (curves with symbol) and Landau parameter $G_0$ (curves without symbol) as functions of density.}
\end{figure}

The $\delta^2$-law is mainly due to the BHF approximation (two-hole line terms only). The same behavior is in fact exhibited by the energy vs. isospin too.[16] The effects of three-hole line terms are rather small when adopting the continuous choice for the auxiliary potential.[14] This choice is also adopted for the present calculations. The slope of the energy shift is monotonically increasing with density so that no signature for a ferromagnetic phase transition in symmetric nuclear matter is expected. The effect of three-body force is to enhance this slope for densities above the saturation point. This effect is more clearly shown in the plot of $\mathcal{E}_{\text{sym}}$ vs. density in Fig. 1.

The magnetic susceptibility has been also calculated from the spin-symmetry energy $\chi = \tilde{\mu}^2 / 2 \mathcal{E}_{\text{sym}}$, where $\tilde{\mu}$ is the average of neutron and proton magnetic moments (in neutron matter $\tilde{\mu}$ is the exact neutron magnetic moment). Usually one calculates the ratio of $\chi$ to $\chi_F$, $\chi_F$ being the magnetic susceptibility for a degenerate free Fermi gas. The effect of strong correlations in nuclear matter due to the two-body force is a reduction of $\chi$ with respect to $\chi_F$. This reduction increases with density up to a factor of 0.3 at $\rho = 0.8$ fm$^{-3}$. The above result is common to most Brueckner calculations.[8–11] More pronounced is the quenching due to three-body force.

Figure 2 also shows the Landau parameter $G_0$ describing the spin density fluctuations in the effective interaction. $G_0$ is simply related to the spin-symmetry energy or, equally, to the magnetic susceptibility by the relation $\chi/\chi_0 = m^*/(1 + G_0)$, where $m^*$ is the effective mass. A magnetic instability would require $G_0 < -1$, which is analogous to the condition $F_0 < -1$ for the mechanical instability giving rise to the liquid-vapour phase transition. But, the value of $G_0$ vs. density from the BHF calculation is always positive and monotonically increasing up to the highest density. The three-body force pushes up the curve of $\chi$. This result is in strong disagreement with the prediction with Skyrme forces. This is not a complete surprise since Skyrme forces are only well suited in the proximity of the empirical saturation point. Astonishing is the strong disagreement between this and the relativistic
approaches because the three-body forces contain already important relativistic effects.\[15\] The accurate knowledge of $G_0$ should lead to reliable predictions on the spin and spin-isospin giant modes as well as spin-spin part of the optical potential.\[8\]

The above calculations have been also repeated for the case of pure neutron and reported in Fig. 2. The same conclusions can be drawn as to the absence of the ferromagnetic phase transition and the quenching of the magnetic susceptibility caused by the strong correlations from the two- and three-body forces. This quenching should have a strong influence on the neutrino propagation in dense matter such as supernovae and neutron stars. In the case of the transition to a ferromagnetic state it has been shown that the mean free path could drop to zero,\[^3\] which has remarkable consequences as, for instance, on the neutron star cooling.

**Fig. 2** The same as Fig. 1, for spin-polarized neutron matter.

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