Study of Photovoltaic Cells Engineering Mathematical Model

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Abstract: The characteristic curve of photovoltaic cells is the theoretical basis of PV Power, which simplifies the existing mathematical model, eventually, obtains a mathematical model used in engineering. The characteristic curve of photovoltaic cells contains both exponential and logarithmic calculation. The exponential and logarithmic spread out through Taylor series, which includes only four arithmetic and use single chip microcontroller as the control center. The result shows that: the use of single chip microcontroller for calculating exponential and logarithmic functions, simplifies mathematical model of PV curve, also can meet the specific conditions’ requirement for engineering applications.

1. Introduction
Photovoltaic cell output characteristic curve is the foundation of PV theoretical research [1-3]. Accurate PV curve mathematical model plays a crucial role for optimization of the entire PV system. Therefore, establishing an accurate and simple mathematical model of PV characteristic curve is the basic research of photovoltaic power generation. As the PV characteristic curve is known, this study simplifies the mathematical model, and studies how to build mathematical models in single chip microcontroller on the basis of the known photovoltaic cell mathematical model. In the design of photovoltaic cell simulator, we need to consider how to make the simulator output stably of the solar cell output characteristic curve [4]. Through calculation error of simplified mathematical model and true mathematical model, optimization without sacrificing accuracy, minimizes the amount of calculation and make sure the system to calculate the required value quickly and accurately. Finally, achieves analog and output known PV curve finally.

2. Taylor series expansion of curve model
2.1. Output characteristic curve of the photovoltaic cell
PV simulator is based on the photovoltaic cell mathematical and physical model [5]. The photovoltaic cells characteristic curve:

\[ a = 0.0012 \times I_w \]  
\[ b = 0.0005 \times U_w \]  
\[ C_1 = \left(1 - \frac{I_w}{I_m}\right) e^{-\frac{U_w}{I_w}} \]  
\[ C_2 = \left(\frac{U_w}{U_{oc}} - 1\right) \ln\left(1 - \frac{I_w}{I_m}\right) \]  
\[ DI = a \times \frac{s}{s_{ref}} \times Dt + \left(\frac{s}{s_{ref}} - 1\right) \times I_w \]  
\[ DV = -b \times Dt - R_s \times DI \]  
\[ Dt = T - T_{ref} \]  
\[ I = I_w \left[1 - C_1 \left(e^{-\frac{U - DV}{C_2 E}} - 1\right)\right] + DI \]  

Obviously, mathematical model contains exponential and the logarithm. In the formula, \( U_{oc} \), \( I_m \), \( R_s \) are the output parameters of photovoltaic cells, \( S \) and \( T \) are environmental parameters, \( s_{ref} = 1000 \, \text{W/m}^2 \), \( T_{ref} = 25 \, ^\circ\text{C} \), \( U \) as independent variables, \( I \) as the dependent variable.

Equation 1 to Equation 4 only contain the output parameters of the photovoltaic cell, which only need to be calculated when the output characteristics of the photovoltaic cell parameter changes. Equation 5 to Equation 7, which include output parameters of photovoltaic cells, but also the environmental parameters, so they must be recalculated when arbitrary parameters from two parameters change. Formula 8 is the main function. When any parameter changes, it should be recalculated. Argument \( U \) varies from 0V to 60V, stepping by 0.1V. As shown in figure 1, it is recalculated function logic diagram.

![Function recomputation logic](image-url)
2.2. Taylor series expansion to exponential and logarithmic functions

Exponential function with a Taylor expansion is:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \forall x$$  \hspace{1cm} (9)

According to Taylor formula expandable depicts the block diagram in figure 2.

Logarithmic function with a Taylor expansion is:

$$\ln (x + 1) = \sum_{n=1}^{\infty} \frac{(-1)^{(x+1)}}{n}x^n, \forall x > -1$$  \hspace{1cm} (10)

According to Taylor formula expandable block diagram depicted in figure 3.

Among them, by controlling the variable comparison JD to control the accuracy, when JD is smaller, the precision gets higher, corresponding calculation speed will slow down. In figure 2, FS is the term \( \frac{x^n}{n!} \) in formula 9, i is the variable n in formula 9. In figure 3, FS was the term \( \frac{(-1)^{(x+1)}}{n}x^n \) in formula 10, I was the variables n in formula 10.

3. Optimized PV curve Experiment

3.1. MK60 introduction

Freescale's chip production MK60DN512, is a 32-bit single chip microcontroller, build-in hardware multiplier, clocked at up to 200MHz. It can fully meet the characteristic curve of single chip microcontroller performance requirements, additionally can meet the Taylor's expansion formula of programming. The microcontroller has 256KB of RAM, can fully store 48KB characteristic curve table.

3.2. The use of single chip microcontroller calculating exponential and logarithmic functions
To approximate the ideal computing speed, prevent the influence of the other block for speed, design a number of programs which are designed for testing single chip microcontroller exponential and logarithmic calculation, program flow shown in figure 4.

The program circulatingly generates a random number x, and calculates the exponential or logarithmic for x. During this process, we will continue to count the number of calculations per second, and the calculated number of times through the serial port return to the PC.

After calculating, the obtained results are shown in figure 5, 6. From the below results figures we can see: when JD is smaller, the number of exponential calculations maintain during 800 to 810; the number of logarithmic calculations maintain at 10600.

3.3 Using single chip microcontroller drawing photovoltaic cell output characteristic curve

To achieve analoging output characteristics of the photovoltaic cell, first we need to calculate PV output characteristic curve. Since the study of exponential and logarithmic calculations is calculated by Taylor series expansion, existing deviation, as shown in figure 7: actual output characteristic curve of the photovoltaic cell and approximate output characteristic curve of the photovoltaic cell.

System simulated photovoltaic curve shown in figure 8, the solid line is the mathematical model’s calculated curve. Various discrete points are system simulator operating point under different load conditions, by comparing the solid lines and the discrete points, you can clearly see the relevant parameters.
4. Verify the accuracy of the PV characteristic curve

In order to verify the photovoltaic cell output characteristic curve drew by single chip microcontroller can meet the engineering requirements, now analysis the error rate between the working points of real photovoltaic cell output characteristic curve and the working points of the approximate output characteristic curve. The results are shown in Table 1.

Table 1. List of error rate.

| Voltage value | Actual current value | Approximate current value | Error rate |
|---------------|----------------------|---------------------------|------------|
| 37.00         | 5.48                 | 5.50                      | 0.36%      |
| 37.50         | 5.47                 | 5.49                      | 0.37%      |
| 38.00         | 5.44                 | 5.46                      | 0.37%      |
| 38.50         | 5.42                 | 5.45                      | 0.55%      |
| 39.00         | 5.40                 | 5.43                      | 0.56%      |
| 39.50         | 5.38                 | 5.42                      | 0.74%      |
| 40.00         | 5.35                 | 5.40                      | 0.93%      |
| 40.50         | 5.31                 | 5.34                      | 0.56%      |
| 41.00         | 5.30                 | 5.32                      | 0.38%      |
| 41.50         | 5.12                 | 5.21                      | 1.16%      |
| 42.00         | 5.00                 | 5.17                      | 3.40%      |
| 42.50         | 4.81                 | 5.00                      | 3.95%      |
| 43.00         | 4.65                 | 4.80                      | 3.23%      |
| 43.50         | 4.42                 | 4.50                      | 1.81%      |
| 44.00         | 4.00                 | 4.22                      | 5.50%      |
| 44.50         | 3.80                 | 4.00                      | 5.26%      |
| 45.00         | 3.62                 | 3.78                      | 4.42%      |
| 45.50         | 2.10                 | 2.22                      | 5.71%      |

We can see from the analysis results of error rate: working point error rate remains 0.36% to 5.71%, meet the engineering requirements: under 10%, it shows that the use of single chip microcontroller drawing photovoltaic cell output characteristic curve is feasible.
5. Conclusions
The study is based on the mathematical model of general PV array simulator, expand by exponential and logarithmic and simplifies calculation of the mathematical model. It, how to establish mathematical models in microcontroller, can accurately simulate the output characteristic of photovoltaic cells. And analysis the calculation error between calculation mathematical model and the real mathematical model in detail, enabling the system to accurately calculate the required value. Finally, settles the use of single chip microcontroller to calculate the exponential and logarithmic calculations, and achieves the appropriate indicators, PV output characteristics curves meet the project criteria.

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7. References
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