Interference Channels with One Cognitive Transmitter

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Abstract

This paper studies the problem of interference channels with one cognitive transmitter (ICOCT) where “cognitive” is defined from both the noncausal and causal perspectives. For the noncausal ICOCT, referred to as interference channels with degraded message sets (IC-DMS), we propose a new achievable rate region that generalizes existing achievable rate regions for IC-DMS. In the absence of the non-cognitive transmitter, the proposed region coincides with Marton’s region for the broadcast channel. Based on this result, the capacity region of a class of semi-deterministic IC-DMS is established. For the causal ICOCT, due to the complexity of the channel model, we focus primarily on the cognitive Z interference channel (ZIC), where the interference link from the cognitive transmitter to the primary receiver is assumed to be absent due to practical design considerations. Capacity bounds for such channels in different parameter regimes are obtained and the impact of such causal cognitive ability is carefully studied. In particular, depending on the channel parameters, the cognitive link may not be useful in terms of enlarging the capacity region. An optimal corner point of the capacity region is also established for the cognitive ZIC for a certain parameter regime.

I. INTRODUCTION

Cognitive radios have been proposed as an enabling technology to address the spectrum scarcity issue. Cognitive radios are capable of sensing their environment and adjusting their parameters and transmission modes in real time. Therefore, they can adaptively fill the under-utilized spaces of the wireless spectrum and greatly increase the overall spectral efficiency.

There have been recent attempts in studying the cognitive radio channel from an information theoretic point of view [1]–[4]. There, a cognitive radio channel is modeled as a two-user interference channel. One of the transmitters, the so-called cognitive transmitter, has non-causal knowledge of the other user’s transmitted messages (see Fig. 1), which is why this model is also referred to as interference channels with degraded message sets (IC-DMS). In [1], the authors combined Gelfand and Pinsker’s coding [5] with Han and Kobayashi’s simultaneous superposition code [6] to derive an achievable rate region for the general IC-DMS. In [2] and [3], the authors derived the capacity region for IC-DMS with weak interference. In [4], the capacity region for IC-DMS with strong interference was determined. Those results were extended to the Gaussian MIMO cognitive radio channels in [7].

Recently, more general coding schemes were proposed in [8] and [9], which include the results in [2] and [3] as special cases. In [8], the cognitive encoder uses rate splitting and allows the other receiver to decode part of the interference; the cognitive transmitter also cooperates by
transmitting the other user’s message, and uses the Gel’fand and Pinsker (GP) binning to cancel this known interference at its intended receiver. A similar approach was proposed independently in [9] with the difference that they introduced the superposition coding into their binning process, thus yielding improvement upon [8].

The IC-DMS model is interesting in the sense that it combines the features of both interference channel and broadcast channel. Specifically, if the cognitive user is deprived of the knowledge about the other user’s messages, it reduces to the classic interference channel; if the primary, i.e., non-cognitive user is absent from the channel model, it reduces to the general broadcast channel. However, the achievable rate regions proposed in [8] and [9] do not reduce to Marton’s region [10] for general broadcast channels, implying potential improvements are possible for the coding strategy. Notice that this situation is relevant in practice if the channel from the cognitive transmitter to the primary receiver is superior compared with that from the primary (non-cognitive) transmitter. Thus it is desirable to let the cognitive transmitter take over the primary transmitter’s responsibility instead of merely serve as a cooperative transmitter. In this work, we propose an achievable rate region for IC-DMS that generalizes the coding schemes in [8], [9] and can also reduce to Marton’s region. The proposed new achievable region helps establish the capacity region of a class of semi-deterministic IC-DMS.

While the non-causal ICOCT, i.e., IC-DMS, has been extensively studied, the causal ICOCT has been far less investigated. In the causal scenario, the cognitive transmitter adapts its transmission based on the causally received signals transmitted by the primary transmitter. This causal cognitive radio model, while more relevant and practical compared with the non-causal case, is considerably more complex than the latter because of the noisy feedback involved in the channel model. We remark that the causal ICOCT is itself a special case of an even more complex model, the so-called interference channels with generalized feedback (ICGF) in which both transmitters are causally cognitive. Two different achievable rate regions for ICGF were proposed in [11] and [12] respectively, using drastically different coding schemes. Interestingly, neither of these two regions includes the other as a subset for the general ICGF model, although for several extreme cases, the region proposed in [12] is shown to coincide with the capacity region.

For the causal cognitive radio channel, we focus on the Gaussian case and our causal ICOCT can be considered as a simplification of ICGF, by taking away the cognitive capability from one of the transmitters. Nonetheless, even with a single channel feedback, the problem is still of formidable nature. In [13], the authors imposed a degradedness assumption which leads to closed-form and relatively simple capacity inner and outer bounds. In the present work, we will instead focus on a more practical model, the so-called cognitive ZIC for the Gaussian case where the causal ICOCT is further simplified by taking away the interference link from the cognitive transmitter to the primary receiver. This Gaussian cognitive ZIC is illustrated in Fig. 3. This simplified model is largely motivated by many proposed cognitive radio schemes that require the so-called ‘interference temperature’ at the primary receiver to be sufficiently low. Thus the ZIC considered in this paper can be considered as an approximation to such cognitive radio channels where the interference imposed on the primary receiver by the cognitive transmitter is largely negligible. For the cognitive ZIC, capacity inner and outer bounds are proposed for various parameter regimes and we demonstrate that the cognitive link may not be helpful for certain parameter regimes, as far as the capacity region is concerned. A corner point on the capacity region is also established for a certain parameter regime.

This paper is organized as follows. In Section II, we consider noncausal ICOCT and propose a new achievable rate region for IC-DMS that generalizes existing results. The proposed region re-
duces to Marton’s region [10] in the absence of the primary transmitter. The proposed achievable region also allows us to establish the capacity region for a class of semi-deterministic channels. In Section III, we consider the causal cognitive ZIC. We propose several inner bounds to the capacity region for different values of channel parameters. We also introduce an outer bound to the capacity region which, together with the inner bounds, allows us to identify a capacity region corner point as well as parameter regimes for which the cognitive capability does not enlarge the capacity region. The concluding remarks are given in Section IV.

II. NONCAUSAL ICOCT

A. Channel Model

A noncausal ICOCT (or IC-DMS), is a quintuple $(\mathcal{X}_1, \mathcal{X}_2, p, \mathcal{Y}_1, \mathcal{Y}_2)$, where $\mathcal{X}_1, \mathcal{X}_2$ are two finite input alphabet sets and $\mathcal{Y}_1, \mathcal{Y}_2$ are two finite output alphabet sets, $p$ is the channel transition probability $p(y_1, y_2|x_1, x_2)$. We assume that the channels are memoryless, i.e.

$$p^n(y_1, y_2|x_1, x_2) = \prod_{i=1}^{n} p(y_{1i}, y_{2i}|x_{1i}, x_{2i})$$

where

$$x_a = (x_{a1}, \cdots, x_{an}) \in \mathcal{X}_1^n, y_a = (y_{a1}, \cdots, y_{an}) \in \mathcal{Y}_1^n$$

for $a = 1, 2$. Let $\mathcal{M}_1 = \{1, 2, \cdots, M_1\}$ and $\mathcal{M}_2 = \{1, 2, \cdots, M_2\}$ be the message sets that sender 1 (primary transmitter) and sender 2 (cognitive transmitter) will transmit, respectively. The cognitive transmitter has noncausal knowledge of user 1’s message, so there are $M_1 \cdot M_2$ codewords for $\mathbf{x}_1(i)$ and $M_1 \cdot M_2$ codewords for $\mathbf{x}_2(i, j)$.

**Definition 1:** An $(M_1, M_2, n, P_e)$ code exists for the IC-DMS, if and only if there exist two encoding functions

$$f_1 : \mathcal{M}_1 \rightarrow \mathcal{X}_1^n, \quad f_2 : \mathcal{M}_1 \times \mathcal{M}_2 \rightarrow \mathcal{X}_2^n$$

and two decoding functions

$$g_1 : \mathcal{Y}_1^n \rightarrow \mathcal{M}_1, \quad g_2 : \mathcal{Y}_2^n \rightarrow \mathcal{M}_2,$$
such that \( \max\{P_{e,1}, P_{e,2}\} \leq P_e \), where \( P_{e,1} \) and \( P_{e,2} \) denote the respective average probabilities of error at decoders 1 and 2, and are computed as

\[
P_{e,1}^{(n)} = \frac{1}{M_1 M_2} \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} P(\hat{m}_1 \neq i | x_1(i), x_2(i, j))
\]

\[
P_{e,2}^{(n)} = \frac{1}{M_1 M_2} \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} P(\hat{m}_2 \neq j | x_1(i), x_2(i, j))
\]

where \( \hat{m}_1 \) and \( \hat{m}_2 \) are the decoded message index at receiver 1 and 2 respectively.

**Definition 2:** A non-negative rate pair \((R_1, R_2)\) is achievable for the IC-DMS, if for any given \(0 < P_e < 1\) and sufficiently large \(n\), there exists a \((2^{nR_1}, 2^{nR_2}, n, P_e)\) code for the channel. The capacity region of IC-DMS is the closure of the union of all the achievable rate pairs \((R_1, R_2)\).

**B. Existing Results**

The capacity region for IC-DMS with strong interference was characterized in [4] and repeated in Proposition 1.

**Proposition 1:** [4, Theorem 1] For an IC-DMS satisfying:

\[
I(X_2; Y_2 | X_1) \leq I(X_2; Y_1 | X_1) \tag{5}
\]

\[
I(X_1, X_2; Y_1) \leq I(X_1, X_2; Y_2) \tag{6}
\]

for all joint distributions on \(X_1\) and \(X_2\), the capacity region \(C\) is the union of all rate pairs \((R_1, R_2)\) satisfying

\[
R_2 \leq I(X_2; Y_2 | X_1) \tag{7}
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y_1) \tag{8}
\]

over joint distributions \(p(x_1, x_2)p(y_1, y_2|x_1, x_2)\).

The capacity rate region for IC-DMS with weak interference was found in [2] and [3] as in Proposition 2.

**Proposition 2:** [2, Theorem 3.4] The capacity region for IC-DMS with weak interference is the convex hull of all rate pairs \((R_1, R_2)\) satisfying

\[
R_1 \leq I(U; X_1; Y_1) \tag{9}
\]

\[
R_2 \leq I(X_2; Y_2 | U, X_1) \tag{10}
\]

over all probability distributions that factor as \(p(u, x_1)p(x_2|u, x_1)p(y_1, y_2|x_1, x_2)\), with the assumption that

\[
I(X_1; Y_1) \leq I(X_1; Y_2) \tag{11}
\]

\[
I(U; Y_1 | X_1) \leq I(U; Y_2 | X_1) \tag{12}
\]

In the same paper [2], Wu et al. also proposed an achievable region for the general IC-DMS, given in Proposition 3.

**Proposition 3:** [2, Proposition 3.1] The convex hull of all rate pairs \((R_1, R_2)\) satisfying

\[
R_1 \leq I(U; X_1; Y_1) \tag{13}
\]

\[
R_2 \leq I(V; Y_2) - I(V; U, X_1) \tag{14}
\]
over all probability distributions \( p(x_1, x_2, u, v, y_1, y_2) \) that factor as
\[
p(u, x_1)p(v|u, x_1)p(x_2|v, u, x_1)p(y_1, y_2|x_1, x_2)
\]
(15)
is achievable for general IC-DMS.

More recently, Jiang and Xin proposed a general achievable rate region [8] for IC-DMS, denoted by \( \mathcal{R}_{JX} \), which includes the region proposed in [2] and [3] as special cases. In order to better compare this result with our proposed region, we provide in Proposition 4 the expressions directly derived from the error probability analysis, i.e., before the Fourier-Motzkin elimination.

**Proposition 4:** The rates \( R_1, R_2 = R_{22} + R_{20} \) are achievable if
\[
R_1 \leq I(W; Y_1 U|Q)
\]
(16)
\[
R_1 + R_{20} \leq I(WU; Y_1|Q)
\]
(17)
\[
R_{20} \leq I(U; Y_2 V|Q) - I(U; W|Q)
\]
(18)
\[
R_{22} \leq I(V; Y_2 U|Q) - I(V; W|Q)
\]
(19)
\[
R_{20} + R_{22} \leq I(UV; Y_2|Q) + I(U; V|Q) - I(U; W|Q) - I(V; W|Q)
\]
(20)
for some joint distribution that factors as
\[
p(q)p(w, x_1|q)p(u|w, q)p(v|w, q)p(x_2|u, v, w, q)p(y_1, y_2|x_1, x_2)
\]
(21)
and for which all the right-hand sides are nonnegative.

Maric et al independently proposed an achievable rate region for the IC-DMS [9], denoted by \( \mathcal{R}_{MGKS} \) which is given below.

**Proposition 5:** [9, Theorem 1] Rates \( R_1 = R_{1a} + R_{1b}, R_2 = R_{2a} + R_{2c} \) are achievable if
\[
R_1 \leq I(X_{1a}, X_{1b}; Y_1, U_{2c}|Q)
\]
(22)
\[
R_1 + R_{2c} \leq I(X_{1a}, X_{1b}, U_{2c}; Y_1|Q)
\]
(23)
\[
R_{1b} \leq I(X_{1b}; Y_1, U_{2c}|X_{1a}, Q)
\]
(24)
\[
R_{1b} + R_{2c} \leq I(X_{1b}, U_{2c}; Y_1|X_{1a}, Q)
\]
(25)
\[
R_{2a} \leq I(U_{2a}; Y_2 U_{2c}, Q) - I(U_{2a}; X_{1a}, X_{1b}|U_{2c}, Q)
\]
(26)
\[
R_2 \leq I(U_{2c}, U_{2a}; Y_2|Q) - I(U_{2c}, U_{2a}; X_{1a}, X_{1b}|Q)
\]
(27)
for some joint distribution that factors as
\[
p(q)p(x_{1a}, x_{1b}, u_{2c}, u_{2a}, x_1, x_2|q)p(y_1, y_2|x_1, x_2)
\]
(28)
and for which all the right-hand sides are nonnegative.

For the codebook generation, [9] did rate splitting for both messages \( m_1 \) and \( m_2 \). Although \( m_2 \) is split into private message \( m_{2a} \) and common message \( m_{2c} \), the sub-messages from \( m_1 \) are both private messages, namely \( m_{1a} \) and \( m_{1b} \). Also, for the Gel’fand and Pinsker binning, both \( m_{2a} \) and \( m_{2c} \) are encoded treating both \( m_{1a} \) and \( m_{1b} \) as known interference. In other words, for the binning part, \( m_{1a} \) and \( m_{1b} \) are treated as one interference. Indeed, the same rate region as that in [9] can be obtained without rate splitting for the primary user.
Without rate splitting for $m_1$, and using otherwise the same encoding scheme and following similar error analysis, one will get the achievable region as follows, denoted by $\mathcal{R}^\prime_{MGKS}$

\[
\begin{align*}
R_1 & \leq I(W; Y_1, U_{2c}|Q) \\
R_1 + R_{2c} & \leq I(W, U_{2c}; Y_1|Q) \\
R_{2a} & \leq I(U_{2a}; Y_2|U_{2c}, Q) - I(U_{2a}; W|U_{2c}, Q) \\
R_2 & \leq I(U_{2c}, U_{2a}; Y_2|Q) - I(U_{2c}, U_{2a}; W|Q)
\end{align*}
\]  

for some joint distribution that factors as

\[
p(q)p(w, u_{2c}, u_{2a}, x_1, x_2|q)p(y_1, y_2|x_1, x_2)
\]

and for which all the right-hand sides are nonnegative, where $R_2 = R_{2a} + R_{2c}$. We now show that the two regions, namely $\mathcal{R}_{MGKS}$ and $\mathcal{R}^\prime_{MGKS}$ are identical.

First, for the region $\mathcal{R}^\prime_{MGKS}$, if we set $W = (X_{1a}, X_{1b})$, we can get the same expressions of (22)-(23) and (26)-(27), with the same joint probability distribution. Since region $\mathcal{R}_{MGKS}$ has two more constraints (24)-(25), $\mathcal{R}^\prime_{MGKS} \subseteq \mathcal{R}_{MGKS}$. On the other hand, for the region $\mathcal{R}_{MGKS}$, if we set $X_{1b} = \phi$ and $X_{1a} = W$, $\mathcal{R}_{MGKS}$ is reduced to $\mathcal{R}^\prime_{MGKS}$, so $\mathcal{R}_{MGKS} \subseteq \mathcal{R}^\prime_{MGKS}$. Therefore, $\mathcal{R}_{MGKS} = \mathcal{R}^\prime_{MGKS}$.

Marton in 1979 considered the general broadcast channel model and proposed the following achievable rate region [10] which remains the largest to this date.

**Proposition 6:** [10, Theorem 2] Let $\mathcal{R}_M$ be the union of rate pairs $(R_1, R_2)$ satisfying $R_1, R_2 \geq 0$ and

\[
\begin{align*}
R_1 & \leq I(WV_1; Y_1) \\
R_2 & \leq I(WV_2; Y_2) \\
R_1 + R_2 & \leq \min\{I(W; Y_1), I(W; Y_2)\} + I(V_1; Y_1|W) \\
& \quad + I(V_2; Y_2|W) - I(V_1; V_2|W)
\end{align*}
\]

for some $(V_1, V_2, W) \rightarrow X \rightarrow (Y_1, Y_2)$. Then $\mathcal{R}_M$ is achievable for the discrete memoryless broadcast channel.

**C. A New Inner Bound**

Both [8] and [9] applied the following techniques:

1) Rate splitting $R_2$ by dividing the message $m_2$ into $m_{22}$ and $m_{20}$. Thus, the rate $R_1$ will be boosted by letting $m_{20}$ be decoded at receiver 1 which reduces the effective interference.

2) GP binning $m_2$ against $m_1$, so that this known interference will be cancelled at receiver 2, boosting the rate $R_2$.

3) User 2 (cognitive transmitter) cooperates with user 1 by transmitting message $m_1$.

However, both [8] and [9] do not have any part of $m_1$ decoded at receiver 2. While the coding scheme in [9] does involve rate splitting for $m_1$, the two split messages, $m_{1a}$ and $m_{1b}$ in [9], are both private messages and not to be decoded at receiver 2. This suggests potential for improvement since GP binning is not always optimal, i.e., interference cancellation at the receiver 2 may outperform that at the transmitter 2 by GP binning only. For example, as observed in [9], when binning against a codebook, superposition coding is optimal over GP binning when the interference rate is small [9]. Therefore, the proposed coding scheme further divides $m_1$
into private message $m_{11}$ and common message $m_{10}$ and superposition encodes $m_{11}$ on top of $m_{10}$. Additionally, since $m_{10}$ is to be completely decoded by receiver 2, binning $m_2$ against $m_{10}$ provides no improvements, thus, we only bin against $m_{11}$.

A second observation is that the coding schemes in [8] and [9] let transmitter 2 help with rate 1 through coherent transmission of the noncausally known message to receiver 1. However, if the direct link from transmitter 1 to receiver 1 is much weaker compared with that of the interference link from transmitter 2 to receiver 1, directly transmitting $m_{11}$ from transmitter 2 may be suboptimal. In the extreme case when transmitter 1 is effectively silent (due, for example to channel conditions), transmitter 2 will serve as a transmitter for a two user broadcast channel for which cross binning (e.g., Marton’s coding scheme) yields the largest achievable rates. As such, the proposed coding scheme introduces cross binning reminiscent that for the broadcast channel [14].

The above ideas lead us to Theorem 1 which gives a new achievable rate region for IC-DMS.

**Theorem 1:** The rate pair $(R_1, R_2)$ is achievable for IC-DMS, if

$$R_1 \leq I(V_{11}U_{11}V_{20}U_{10}; Y_1)$$

$$R_2 \leq I(V_{22}V_{20}; Y_2|U_{10}) - I(V_{22}V_{20}; U_{11}|U_{10})$$

$$R_1 + R_2 \leq I(V_{11}U_{11}; Y_1|V_{20}U_{10}) + I(V_{22}V_{20}U_{11}; Y_2) - I(V_{11}; V_{22}|V_{20}U_{10} - I(U_{11}; V_{22}|V_{11}V_{20}U_{10})$$

$$R_1 + R_2 \leq I(V_{11}U_{11}V_{20}U_{10}; Y_1) + I(V_{22}; Y_2|V_{20}U_{10}) - I(V_{11}; V_{22}|V_{20}U_{10})$$

$$2R_2 + R_1 \leq I(V_{11}U_{11}V_{20}; Y_1|U_{10}) + I(V_{22}; Y_2|V_{20}U_{10}) + I(V_{22}V_{20}U_{10}; Y_2) - I(V_{11}; V_{22}|V_{20}U_{10} - I(U_{11}; V_{22}|V_{11}V_{20}U_{10}) - I(V_{22}V_{20}; U_{11}|U_{10})$$

for some joint distribution that factors as

$$p(u_{10}, u_{11}, V_{10}, V_{20}, m_{20}, x_1, x_2)p(y_1, y_2|x_1, x_2)$$

and for which all the right-hand sides are nonnegative.

The above theorem is a direct consequence of applying the Fourier-Motzkin elimination to the following rate region.

**Theorem 2:** Rates $R_1 = R_{11} + R_{10}$ and $R_2 = R_{22} + R_{20}$ are achievable if

$$R_{20} \leq L_{20} - I(V_{20}; U_{11}|U_{10})$$

$$R_{11} \leq L_{11} - I(V_{11}; U_{11}|V_{20}U_{10})$$

$$R_{22} \leq L_{22} - I(V_{22}; U_{20}V_{10})$$

$$R_{11} + R_{22} \leq L_{11} + L_{22} - I(V_{11}; V_{22}|V_{20}U_{10}) - I(U_{11}; V_{11}V_{22}|V_{20}U_{10})$$

$$L_{11} \leq I(V_{11}U_{11}; Y_{1}|V_{20}U_{10}) + I(V_{11}V_{22}; U_{11}|U_{10})$$

$$L_{11} + L_{20} \leq I(V_{11}U_{11}V_{20}; Y_{1}|U_{10}) + I(V_{11}V_{20}; U_{11}|U_{10})$$

$$L_{11} + L_{20} + R_{10} \leq I(V_{11}U_{11}V_{20}; Y_{1}) + I(V_{11}V_{20}; U_{11}|U_{10})$$

$$L_{22} \leq I(V_{22}; Y_{2}|V_{20}U_{10})$$

$$L_{22} + L_{20} \leq I(V_{22}V_{20}; Y_{2}|U_{10})$$
\[ L_{22} + L_{20} + R_{10} \leq I(V_{22}V_{20}U_{10};Y_2) \]  \hfill (52)

for some joint distribution that factors as
\[ p(u_{10}, u_{11}, v_{20}, v_{22}, x_1, x_2)p(y_1, y_2|x_1, x_2) \]  \hfill (53)

and for which all the right-hand sides are nonnegative.

*Proof:* See Appendix.

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The encoding scheme is illustrated in Fig. 2. Both encoders apply rate splitting to their respective messages. Encoder 1 encodes the two sub-messages using superposition coding. Encoder 2, i.e., the cognitive transmitter’s encoding process is much more complex. It involves Gel’fand and Pinsker binning for the generation of \( v_{20}^n \), and the cross binning for the generation of \( v_{11}^n \) and \( v_{22}^n \), as used in the general broadcast channel. Specifically, encoder 2 first encodes message \( W_{20} \) into codeword \( v_{20}^n \) superimposed on \( u_{10}^n \) and applies Gel’fand and Pinsker binning against \( u_{11}^n \). Then, on top of codeword pair \( (u_{10}^n, v_{20}^n) \), it generates \( v_{11}^n \) for message \( W_{11} \) and \( v_{22}^n \) for message \( W_{22} \) and applies cross binning against each other. Encoder 2 also cooperates with the primary transmitter by transmitting codewords \( u_{11}^n \) and \( u_{10}^n \).

The achievable rate region in Theorem 2 denoted by \( R^* \), is derived based on simultaneous decoding. In Theorem 3 we introduce another region \( \tilde{R} \) based on sequential decoding, which is a subset of \( R^* \).
Theorem 3: Rates $R_1 = R_{11} + R_{10}$ and $R_2 = R_{22} + R_{20}$ are achievable if

\[
\begin{align*}
R_{20} &\leq L_{20} - I(V_{20};U_{11}|U_{10}) \quad (54) \\
R_{11} &\leq L_{11} - I(V_{11};U_{11}|V_{20}U_{10}) \quad (55) \\
R_{22} &\leq L_{22} - I(V_{22};U_{11}|V_{20}U_{10}) \quad (56)
\end{align*}
\]

\[
\begin{align*}
R_{11} + R_{22} &\leq L_{11} + L_{22} - I(V_{11};V_{22}|V_{20}U_{10}) - I(U_{11};V_{11}V_{22}|V_{20}U_{10}) \quad (57) \\
I_{20} &\leq \min\{I(V_{20};Y_1|U_{10}), I(V_{20};Y_2|U_{10})\} \quad (58) \\
I_{10} + I_{20} &\leq \min\{I(V_{20}U_{10};Y_1), I(V_{20}U_{10};Y_2)\} \quad (59) \\
L_{11} &\leq I(V_{11}U_{11};Y_1|V_{20}U_{10}) + I(V_{11}V_{20};U_{11}|U_{10}) \quad (60) \\
L_{22} &\leq I(V_{22};Y_2|V_{20}U_{10}) \quad (61)
\end{align*}
\]

for some joint distribution that factors as

\[
p(u_{10}, u_{11}, v_{11}, v_{20}, v_{22}, x_1, x_2)p(y_1, y_2|x_1, x_2) \quad (62)
\]

and for which all the right-hand sides are nonnegative.

Proof: The encoding scheme is the same as that in Theorem 2, which leads to (54)-(57). The decoders first decode messages $m_{20}$ and $m_{10}$ using simultaneous decoding, which leads to (58)-(59). After subtracting out the signals decoded in the first stage, decoder 1 proceeds to decode $m_{11}$ and decoder 2 proceeds to decode $m_{22}$, which leads to (60)-(61).

Both the above two regions $R^*$ and $R$ are convex. Therefore, no convex hull operation or time sharing is necessary. The convexity of the regions can be easily proved following the same approach as in [15, Lemma 5].

D. Special Cases

1) Strong interference: In the case of strong interference, the optimal scheme is for both user’s messages to be decoded by both receivers. Thus, by setting $V_{11} = V_{22} = U_{11} = \phi$ and $R_1 = R_{10}, R_2 = R_{20}$, $R^*$ reduces to the capacity region for IC-DMS with strong interference in Proposition 1.

2) Weak interference: By setting $V_{11} = V_{20} = U_{11} = \phi, V_{22} = X_2, U_{10} = (U, X_1)$ and $R_1 = R_{10}, R_2 = R_{22}$, and removing all the redundant conditions based on the assumption (11)-(12), $R^*$ reduces to the capacity region for IC-DMS with weak interference in Proposition 2.

3) The rate region in [2]: By setting $V_{20} = U_{10} = V_{11} = \phi, U_{11} = (U, X_1), V_{22} = V$ and $R_1 = R_{11} = L_{11}$ and $R_2 = R_{22}$, $R^*$ reduces to Wu et al’s achievable rate region for the general IC-DMS in Proposition 3.
4) \textbf{The rate region in [1]:} Our scheme in Theorem 2 is similar to the coding scheme proposed in [1] in the sense that, both users’ messages are divided into two parts: private message decoded only by the intended receivers, and common message decoded by both receivers. However, our scheme is different from that in [1] in the following ways.

- User 2’s codewords $v_{22}$ and $v_{20}$ are binned against $u_{11}$ only, while in [1], $v_{22}$ and $v_{20}$ are binned against both $u_{11}$ and $u_{10}$. Since $u_{10}$ is to be completely decoded by receiver 1, binning against $u_{10}$ provides no improvement.
- The binning of $v_{22}$ and $v_{20}$ in [1] is done independently, whereas we add dependency between them to provide potential improvements.
- In our scheme, the cognitive transmitter will cooperate with the primary user by transmitting the primary user’s messages, whereas there is no cooperation between the two users in [1].
- The coding scheme in [1] does not have the codeword $v_{11}$ as in our scheme. The function of $v_{11}$ is to potentially cancel out the interference from the message $m_{22}$ at receiver 1, thus boosting the rate $R_{11}$.

After applying the Fourier Motzkin elimination, for fixed joint distribution of those random variables, the polygon of $\mathcal{R}^*$ has constraints for four different slopes, namely, $R_1$, $R_2$, $R_1 + R_2$ and $2R_2 + R_1$, while the polygon of the region in [1] has one more slope constraint $2R_1 + R_2$. For fixed joint distribution, it turns out that the the bound for the slope $2R_2 + R_1$ of the region in [1] can be larger than that of $\mathcal{R}^*$, while the rest of the bounds are smaller than $\mathcal{R}^*$. As such, it is not easy to establish a subset relation between these two regions analytically.

5) \textbf{Jiang and Xin’s region:} Jiang and Xin’s region $\mathcal{R}_{JX}$ is different from $\mathcal{R}^*$ in that

- there is no rate splitting for $R_1$;
- the binning of $v_{22}$ and $v_{20}$ are done independently;
- there is no codeword $v_{11}$.

After setting $U_{10} = Q$, $U_{11} = W$, $V_{11} = \phi$, $V_{20} = U$, $V_{22} = V$, $R_1 = R_{11} = L_{11}$, and substituting all the $L_{22}$ and $L_{20}$ using (43)–(46), $\mathcal{R}^*$ reduces to a region, denoted by $\mathcal{R}'$, with only the following two bounds different from $\mathcal{R}_{JX}$:

\begin{align*}
R_{22} &\leq I(V;Y_2|UQ) - I(V;W|UQ) \tag{63} \\
R_{22} + R_{20} &\leq I(UV;Y_2|Q) - I(V;W|UQ) - I(U;W|Q) \tag{64}
\end{align*}

The corresponding bounds in $\mathcal{R}_{JX}$ are

\begin{align*}
R_{22} &\leq I(V;Y_2|Q) - I(V;W|Q) \tag{65} \\
R_{20} &\leq I(U;Y_2|Q) - I(U;W|Q) \tag{66} \\
R_{22} + R_{20} &\leq I(UV;Y_2|Q) + I(U;V|Q) - I(V;W|Q) - I(U;W|Q) \tag{67}
\end{align*}

According to the distribution (27), for any $(V,U,W)$ in $\mathcal{R}_{JX}$, we have

\begin{equation}
H(V|UWQ) = H(V|WQ) \tag{68}
\end{equation}

Now, we set the variables in $\mathcal{R}'$ as $V^* = (V,U), U^* = U, W^* = W$. Due to (68), it can be easily checked that (63) is equal to (65), and (64) is equal to (67). Therefore, $\mathcal{R}_{JX} \subseteq \mathcal{R}'$. Hence, $\mathcal{R}_{JX} \subseteq \mathcal{R}^*$. 

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6) **Maric et al’s region:** Maric et al’s region is different from $\mathcal{R}^*$ in that
- no part of $m_1$ is decoded by receiver 2;
- there is no codeword $v_{11}$. 

By setting $U_{10} = Q$, $U_{11} = (X_{1a}, X_{1b})$, $V_{11} = \phi$, $V_{22} = U_{2a}$, $V_{20} = U_{2c}$, $R_1 = R_{11} = L_{11}$ and substituting all $L_{22}$ and $L_{20}$ by $R_{22}$ and $R_{20}$ using (43)-(46), $\mathcal{R}^*$ reduces to exactly (22)-(23) and (26)-(27). Thus, $\mathcal{R}^*$ includes Maric et al’s region as a subset.

7) **Marton’s region:** In the absence of transmitter 1, IC-DMS reduces to the general broadcast channel. However, the achievable rate regions proposed in [8] and [9] do not reduce to Marton’s region [10]. This is due to the way binning is used in [8] and [10]: the binning is always in one direction, i.e., the primary user’s message is always treated as known interference. The new ingredient in the present work is the use of cross binning which allows us to recover Marton’s region for the broadcast channel. We now establish that $\mathcal{R}^*$ includes Marton’s achievable region as a subset in the absence of transmitter 1. Toward that end, it will be convenient if we compare the region $\mathcal{R}$ described in Theorem [3] with Marton’s region $\mathcal{R}_M$.

Setting $U_{11} = U_{10} = \phi$, $V_{20} = W$, $V_{11} = V_1$, $V_{22} = V_2$ and removing redundant constraints, the proposed region $\mathcal{R}$ becomes

$$R_{20} \leq L_{20}$$  \hspace{1cm} (69)

$$R_{11} + R_{22} \leq L_{11} + L_{22} - I(V_1; V_2|W)$$  \hspace{1cm} (70)

$$R_{10} + L_{20} \leq \min\{I(W; Y_1), I(W; Y_2)\}$$  \hspace{1cm} (71)

$$R_{11} \leq L_{11} \leq I(V_1; Y_1|W)$$  \hspace{1cm} (72)

$$R_{22} \leq L_{22} \leq I(V_2; Y_2|W)$$  \hspace{1cm} (73)

Applying the Fourier-Motzkin elimination on (69)-(73) with the definition $R_1 = R_{11} + R_{10}$ and $R_2 = R_{22} + R_{20}$, $\mathcal{R}$ becomes

$$R_1 \leq I(V_1; Y_1|W) + \min\{I(W; Y_1), I(W; Y_2)\}$$  \hspace{1cm} (74)

$$R_2 \leq I(V_2; Y_2|W) + \min\{I(W; Y_1), I(W; Y_2)\}$$  \hspace{1cm} (75)

$$R_1 + R_2 \leq \min\{I(W; Y_1), I(W; Y_2)\} + I(V_1; Y_1|W) + I(V_2; Y_2|W) - I(V_1; V_2|W)$$  \hspace{1cm} (76)

The equivalence of region (74)-(76) and $\mathcal{R}_M$ was proved by Gel’fand and Pinsker in [16]. Thus, we have shown that $\mathcal{R}$ reduces to Marton’s region $\mathcal{R}_M$ when user 1 is absent, i.e., when the IC-DMS reduces to a broadcast channel. Since $\mathcal{R} \subseteq \mathcal{R}^*$, $\mathcal{R}^*$ includes $\mathcal{R}_M$ as a subset.

We now introduce an outer bound to the capacity region for the general IC-DMS.

**Theorem 4:** An outer bound $\mathcal{R}_o$ to the capacity region of IC-DMS is the union of all rate pairs $(R_1, R_2)$ satisfying

$$R_1 \leq I(X_1 U; Y_1),$$  \hspace{1cm} (77)

$$R_2 \leq I(X_2; Y_2 | X_1),$$  \hspace{1cm} (78)

$$R_1 + R_2 \leq I(X_1 U; Y_1) + I(X_2; Y_2 | X_1 U),$$  \hspace{1cm} (79)

for some joint distribution that factors as

$$p(u, x_1, x_2)p(y_1, y_2 | x_1, x_2).$$  \hspace{1cm} (80)
Proof: We start with the rate for the cognitive user.

\[ nR_2 = H(W_2) = H(W_2 | W_1 X_1^n) \]

(81)

\[ = I(W_2; Y_2^n | W_1 X_1^n) + H(W_2 | Y_2^n W_1 X_1^n) \]

(82)

\[ \leq I(W_2; Y_2^n | W_1 X_1^n) + n \epsilon_1 \]

(83)

\[ = \sum_{i=1}^{n} I(W_2; Y_{2i} | W_1 X_1^n Y_{2i}^{i-1}) + n \epsilon_1 \]

(84)

\[ \leq \sum_{i=1}^{n} H(Y_{2i} | X_{1i}) - H(Y_{2i} | W_1 W_2 X_1^n Y_{2i}^{i-1} X_2^n) + n \epsilon_1 \]

(85)

\[ = \sum_{i=1}^{n} H(Y_{2i} | X_{1i}) - H(Y_{2i} | X_1 X_{2i}) + n \epsilon_1 \]

(86)

\[ = \sum_{i=1}^{n} I(X_{2i}; Y_{2i} | X_{1i}) + n \epsilon_1 \]

(87)

where (81) is from the independence of the messages, (83) is from Fano’s inequality, (84) is from the chain rule for mutual information, (85) is from conditioning does not increase entropy, (86) is due to the Markov chain \((W_1 W_2 X_1^n Y_{2i}^{i-1} X_2^n) \rightarrow X_{1i} X_{2i} \rightarrow Y_{2i}\) as a result of the memoryless property of the channel.

Define the random variable \(U_i = (W_1, Y_{1i}^{i-1}, Y_{2i+1}^n)\), then we have, for the primary user,

\[ nR_1 = H(W_1) \leq I(W_1; Y_1^n) + n \epsilon_2 \]

(88)

\[ = \sum_{i=1}^{n} I(W_1; Y_{1i} | Y_1^{i-1}) + n \epsilon_2 \]

(89)

\[ \leq \sum_{i=1}^{n} I(X_{1i} W_1 Y_1^{i-1} W_2 Y_{2i+1}^n; Y_{1i}) + n \epsilon_2 \]

(90)

\[ = \sum_{i=1}^{n} I(X_{1i} U_i; Y_{1i}) + n \epsilon_2 \]

(91)

\[ n(R_1 + R_2) = H(W_1) + H(W_2 | W_1) \leq I(W_1; Y_1^n) + I(W_2; Y_2^n | W_1) + n \epsilon_3 \]

(92)

\[ = \sum_{i=1}^{n} \left[ I(W_1; Y_{1i} | Y_1^{i-1}) + I(W_2; Y_{2i} | W_1 Y_1^n Y_{2i+1}^n) \right] + n \epsilon_3 \]

(93)

\[ \leq \sum_{i=1}^{n} \left[ I(X_{1i} W_1 Y_1^{i-1} Y_{1i}) + I(W_2; Y_{2i} | X_{1i} W_1 Y_1^n Y_{2i+1}^n) \right] + n \epsilon_3 \]

(94)

\[ \leq \sum_{i=1}^{n} \left[ I(X_{1i} W_1 Y_1^{i-1} Y_{1i}) + I(X_{2i} Y_1^{i-1} Y_{2i} | X_{1i} W_1 Y_1^n Y_{2i+1}^n) \right] + n \epsilon_3 \]

(95)

\[ = \sum_{i=1}^{n} \left[ I(X_{1i} W_1 Y_1^{i-1} Y_{2i+1}^n Y_{1i}) - I(Y_{2i+1}^n Y_{1i} | X_{1i} W_1 Y_1^{i-1}) + I(Y_{1i}^{i-1} Y_{2i} | X_{1i} W_1 Y_1^n Y_{2i+1}^n) + I(X_{2i} Y_{2i} | X_{1i} W_1 Y_1^n Y_{2i+1}^n Y_1^{i-1}) \right] + n \epsilon_3 \]

(96)
where (94) is due to the deterministic encoding at the primary user; (95) is because conditioning reduces entropy and the memoryless channel assumption; (97) is due to Csiszar’s identity [15, Lemma 7]. Now, we introduce a time sharing random variable $I$ that is independent of all other variables and uniformly distributed over $\{1, 2, ..., n\}$. Define $U = (U_I, I)$, $X_1 = X_{1I}$, $X_2 = X_{2I}$, $Y_1 = Y_{1I}$, $Y_2 = Y_{2I}$, then the Markov chain $U \rightarrow (X_1, X_2) \rightarrow (Y_1, Y_2)$ holds and we have

$$nR_2 \leq \sum_{i=1}^{n} I(X_{2I}; Y_{2I} | X_{1I}I) + n\epsilon_1$$
$$= nI(X_{2I}; Y_{2I} | X_{1I}I) + n\epsilon_1$$
$$= n[H(Y_{2I} | X_{1I}I) - H(Y_{2I} | X_{1I}X_{2I}I)] + n\epsilon_1$$
$$\leq n[H(Y_{2I} | X_{1II}) - H(Y_{2I} | X_{1II}X_{2II})] + n\epsilon_1$$
$$= nI(X_{2II}; Y_{2II} | X_{1II}) + n\epsilon_1$$
$$= nI(X_{2II}; Y_{2II} | X_{1II}) + n\epsilon_1$$

$$nR_1 \leq \sum_{i=1}^{n} I(X_{1I}; Y_{1I} | I) + n\epsilon_2$$
$$= nI(X_{1I}; Y_{1I} | I) + n\epsilon_2$$
$$\leq nI(X_{1I}U_I; Y_{1I}) + n\epsilon_2$$
$$= nI(X_{1I}U_I; Y_{1I}) + n\epsilon_2$$

$$n(R_1 + R_2) \leq \sum_{i=1}^{n} I(X_{1I}; Y_{1I}) + I(W_{2I}; Y_{2I} | X_{1I}U_i) + n\epsilon_3$$
$$= n[I(X_{1I}; Y_{1I}) + I(X_{2I}; Y_{2I} | X_{1I}U_I)] + n\epsilon_3$$
$$\leq n[I(X_{1I}; Y_{1I}) + I(X_{2I}; Y_{2I} | X_{1I}U_I)] + n\epsilon_3$$
$$= n[I(X_{1I}; Y_{1I}) + I(X_{2I}; Y_{2I} | X_{1I}U)] + n\epsilon_3$$

\[\text{E. Semi-deterministic IC-DMS}\]

We can now consider the semi-deterministic IC-DMS with the deterministic component for receiver 2. More specifically, the received signal at receiver 2 is a deterministic function of the input signal $X_1$ and $X_2$, i.e., $Y_2 = h(X_1, X_2)$. In other words, the channel matrix from the input $(X_1, X_2)$ to the output $Y_2$ has 0 or 1 as its entries.

For this semi-deterministic IC-DMS, if we impose an additional constraint $I(X_1; Y_1) \leq I(X_1; Y_2)$, we are able to find the capacity region using the inner bound in Theorem [11] and the outer bound in Theorem [4] as given in the following theorem.
Theorem 5: For a semi-deterministic IC-DMS, if $I(X_1; Y_1) \leq I(X_1; Y_2)$ for any input distributions, the capacity region is the union of all rate pairs $(R_1, R_2)$ satisfying

\begin{align*}
R_1 &\leq I(X_1; Y_1) \\
R_2 &\leq H(Y_2 | X_1) \\
R_1 + R_2 &\leq I(X_1; Y_1) + H(Y_2 | X_1 U)
\end{align*}

for some joint distributions that factor as

\begin{equation}
p(u, x_1, x_2)p(y_1, y_2 | x_1, x_2).
\end{equation}

Proof: Converse: By definition, we have

\begin{align*}
I(X_2; Y_2 | X_1) &= H(Y_2 | X_1) \\
I(X_2; Y_2 | X_1 U) &= H(Y_2 | X_1 U)
\end{align*}

Plug \((116)-(117)\) into Theorem 4, we get \((112)-(114)\).

Achievability: In the inequalities \((37)-(41)\) in Theorem 1, let $U_{10} = X_1$, $V_{11} = U$, $V_{22} = X_2$, $U_{11} = V_{20} = \phi$. Then, \((37)\) and \((38)\) reduce to \((112)-(113)\) directly. Also, \((40)\) reduces to

\begin{equation}
R_1 + R_2 \leq I(X_1; Y_1) + I(X_2; Y_2 | X_1) - I(U; X_2 | X_1).
\end{equation}

Due to the Markov chain \((115)\),

\begin{align*}
I(X_2; Y_2 | X_1) &= H(Y_2 | X_1) - H(Y_2 | X_1, X_2) \\
&= H(Y_2 | X_1) - H(Y_2 | X_1, X_2 U) \\
&= I(Y_2; X_2 U | X_1) \\
&= I(Y_2; U | X_1) + I(Y_2; X_2 | U X_1)
\end{align*}

Also, due to the semi-deterministic channel, we have

\begin{align*}
I(Y_2; U | X_1) &= H(U | X_1) - H(U | X_1 Y_2) \\
&= H(U | X_1) - H(U | X_1 X_2) \\
&= I(U; X_2 | X_1)
\end{align*}

Combine \((122)\) and \((125)\), \((118)\) becomes

\begin{equation}
R_1 + R_2 \leq I(X_1; Y_1) + I(X_2; Y_2 | X_1 U) = I(X_1; Y_1) + H(Y_2 | X_1 U),
\end{equation}

hence \((114)\) is achieved. Due to the additional constraint $I(X_1; Y_1) \leq I(X_1; Y_2)$, \((39)\) and \((41)\) become redundant. This completes the proof for the achievability.

The capacity region in Theorem 5 indicates that for semi-deterministic IC-DMS with constraint $I(X_1; Y_1) \leq I(X_1; Y_2)$, the optimal transmission scheme is for the primary user’s messages $m_1$ to be decoded by both receivers while the cognitive user encodes $m_2$ on top of $m_1$ using superposition coding.

III. THE COGNITIVE ZIC

In this section, we study ICOCT with causal cooperation. As explained in Section I, our focus will be on the special case of Gaussian ZIC where the interference link between the cognitive transmitter and the primary receiver is absent.
A. Channel Model

The (causal) cognitive ZIC is illustrated in Fig. 3. User 1 has message $W_1 \in \{1, 2, \ldots, 2^{nR_1}\}$ to be transmitted to receiver 1 ($Y'_1$), and user 2 has message $W_2 \in \{1, 2, \ldots, 2^{nR_2}\}$ for receiver 2 ($Y'_3$). In addition, user 2 can listen to the transmitted signal from user 1 through a noisy channel ($Y'_2$). Thus, the channel model is given by

\begin{align}
Y'_1 &= h_{11}X'_1 + Z'_1 \quad (127) \\
Y'_2 &= h_{12}X'_1 + Z'_2 \quad (128) \\
Y'_3 &= h_{13}X'_1 + h_{23}X'_2 + Z'_3 \quad (129)
\end{align}

where $h_{11}, h_{12}, h_{13}$ and $h_{23}$ are fixed real positive numbers. $Z'_1 \sim N(0, N_1), Z'_2 \sim N(0, N_2)$ and $Z'_3 \sim N(0, N_3)$ are independent Gaussian random variables. The average power constraints of the input signals are

\[ \frac{1}{n} \sum_{i=1}^{n} (x'_i)^2 \leq P'_t \quad (130) \]

where $t = 1, 2$. From (128), we have assumed implicitly perfect echo cancellation.

**Lemma 1:** Any cognitive ZIC described by (127)-(129) is equivalent, in its capacity region, to the following cognitive ZIC in standard form

\begin{align}
Y_1 &= X_1 + Z_1 \quad (131) \\
Y_2 &= KX_1 + Z_2 \quad (132) \\
Y_3 &= bX_1 + X_2 + Z_3 \quad (133)
\end{align}

where $Z_1, Z_2$ and $Z_3$ are independent zero mean, unit variance Gaussian variables, and $X_1, X_2$ are subject to respective power constraints $P_1$ and $P_2$. $K$ and $b$ are deterministic real numbers with $0 \leq K < \infty, 0 \leq b < \infty$.

The proof is through a simple scaling transformation similar to that of [17], hence omitted.
The encoding functions $f_1$ and $f_2$ for users 1 and 2 are respectively:

$$x_1 = f_1(W_1)$$  
$$x_{2i} = f_2(W_2, y_{21}, \ldots, y_{2i-1})$$

for $i = 1, 2, \ldots, n$. We only consider deterministic encoders, as nondeterministic encoders do not enlarge the capacity region (See, e.g., [18, Appendix D]).

### B. Capacity Lower Bounds

The cognitive ZIC includes the following two extreme cases: the classic ZIC (corresponding to $K = 0$) and the ZIC with degraded message sets ($K = \infty$). To simplify our notation, we define

$$\gamma(x) \triangleq \frac{1}{2} \log(1 + x).$$

For the classic ZIC, when $b \geq 1$, the capacity region, denoted by $\mathcal{R}_1$, is given below

$$R_1 \leq \gamma(P_1) \triangleq C_1$$
$$R_2 \leq \gamma(P_2) \triangleq C_2$$
$$R_1 + R_2 \leq \gamma(b^2P_1 + P_2)$$

Apparently, $\mathcal{R}_1$ is an inner bound to the capacity region for the corresponding cognitive ZIC. When $b < 1$, we do not know the whole capacity region, but the sum rate capacity is known to be:

$$R_1 + R_2 \leq \gamma(P_1) + \gamma \left( \frac{P_2}{1 + b^2P_1} \right)$$

which is achieved when user 1 is transmitting at its maximum rate, and user 2 is transmitting at a rate such that its message can be decoded at receiver 2 by treating user 1’s signal as noise.
On the other extreme, for the ZIC with degraded message sets, where user 2 has \textit{a priori} knowledge of user 1’s message, the capacity region, denoted by \( R_2 \), is the following rectangle, for all \( b \geq 0 \):

\[
R_1 \leq C_1 \quad (141)
\]
\[
R_2 \leq C_2 \quad (142)
\]

This is because for the Gaussian channel considered in this paper, user 2 can dirty paper code its own message treating user 1’s signal as known interference [19]. Therefore, this is equivalent to two parallel interference free channels. \( R_2 \) serves as a natural outer bound to the capacity region of the cognitive ZIC.

Little is known for the cognitive ZIC besides of the two extreme cases. The difficulty for the case with finite \( K \) is that it is not clear what is the optimal way to utilize the channel feedback at transmitter 2. Any information overheard through the cognitive link pertains only to message \( W_1 \), yet, unlike the noncausal case described in Section II, the absence of link from \( X_2 \) to \( Y_1 \) implies that existence of the cognitive link can not directly benefit the rate \( R_1 \) through cooperative transmission. On the other hand, since \( X_1 \) interferes receiver \( Y_3 \), encoding schemes should explore the potential of facilitating interference cancellation for the secondary user using the cognitive link. In the following, we describe several cases where we can obtain closed form capacity bounds for the cognitive ZIC and discuss their implication in terms of the impact of the “cognitive capability” on the capacity.

1) \( b^2 \geq 1 + P_2 \): In the absence of the cognitive link, this reduces to the ZIC with very strong interference. The capacity region of such ZIC coincides with the outer bound \( R_2 \) for the cognitive ZIC, suggesting that \( R_2 \) is indeed the capacity region. Notice this is the case where there is no need to utilize the channel feedback \( Y_2 \) at user 2, as far as the capacity region is concerned.

2) \( 1 \leq b^2 < 1 + P_2 \): This is a very interesting case. In the absence of the cognitive link, the capacity region is the pentagon described by \( R_1 \). On the other hand, for ZIC with degraded message sets, the capacity is a rectangle, \( R_2 \). The capacity region for the cognitive ZIC with \( 1 \leq b^2 < 1 + P_2 \) should be between these two regions. We define another region \( R_3 \) as the union of all nonnegative rate pairs \((R_1, R_2)\) such that:

\[
R_1 \leq \gamma (K^2 \alpha P_1) \quad (143)
\]
\[
R_1 \leq \gamma (P_1) \quad (144)
\]
\[
R_1 \leq \gamma (K^2 (\alpha - \beta) P_1) + \gamma \left( \frac{b^2 \beta P_1}{1 + b^2 (1 - \beta) P_1 + P_2} \right) \quad (145)
\]
\[
R_1 \leq \gamma ((1 - \beta) P_1) + \gamma \left( \frac{b^2 \beta P_1}{1 + b^2 (1 - \beta) P_1 + P_2} \right) \quad (146)
\]
\[
R_2 \leq \gamma \left( \frac{P_2}{1 + b^2 (\alpha - \beta) P_1} \right) \quad (147)
\]

where \( 0 \leq \beta \leq \alpha \leq 1 \).

\textit{Theorem 6:} The the convex hull of the union of \( R_1 \) and \( R_3 \) is achievable for the cognitive ZIC.

\textit{Proof:} We only need to prove the achievability of \( R_3 \). User 1 splits the message \( W_1 \) into two parts: the common message \( W_{12} \), which is to be decoded by all the receivers \( Y_1, Y_2 \) and \( Y_3 \);
the private message $W_{11}$, which is only to be decoded by receivers $Y_1$ and $Y_2$. User 1 randomly places the codewords of $W_{11}$ into $2^{nR_0}$ cells, indexed by $S_{11}$, and transmits $W_{11}$ using the superposition block Markov encoding [20], as shown in Fig. 5.

Fig. 5. The transmission scheme for $W_1$.

The whole transmission proceeds across $B$ blocks. In block $i$, the codeword $x_{11}^{(i)}$ includes the new message of $W_{11}^{(i)}$ and $W_{12}^{(i)}$ for the current block and the “cell index” $S_{11}^{(i-1)}$ of the $W_{11}^{(i-1)}$ in block $i-1$. User 2 always decodes the new message of $W_{11}^{(i)}$ and $W_{12}^{(i)}$ at the end of block $i$ and dirty paper code $W_2^{(i)}$ treating the previous block’s cell index $S_{11}^{(i-1)}$ as known interference. At the end of block $i$, receiver $Y_3$ first decodes $W_{12}^{(i)}$, subtracts it out, and then dirty paper decodes $W_2^{(i)}$, treating the other part of the new message, $W_{11}^{(i)}$, as noise. Therefore, the interference corresponding to $S_{11}$ is mitigated through dirty paper coding, boosting the rate of $W_2$.

In the first block, user 1 transmits a constant cell index $S_{11} = 1$, since there is no message $W_{11}$ in the previous block; in the last block, user 1 transmits constant new message $W_{11} = 1$ and $W_{12} = 1$. In all other blocks, user 1 allocates $\beta P_1$ for $W_{12}$, $0 \leq \beta \leq 1$; allocates $(\alpha - \beta)P_1$ for $W_{11}$, $\beta \leq \alpha \leq 1$, and the remaining power $(1 - \alpha)P_1$ for $S_{11}$. Since $Y_2$ always knows $S_{11}$ transmitted in the current block, the rate constraints at $Y_2$ are

\[
\begin{align*}
R_{11} & \leq \gamma(K^2(\alpha - \beta)P_1) \\
R_{12} & \leq \gamma(K^2\beta P_1) \\
R_{11} + R_{12} & \leq \gamma(K^2\alpha P_1)
\end{align*}
\]  

(148)  

(149)  

(150)

Receiver $Y_1$ also decodes $W_{12}$ first, subtracts it out, then decodes $S_{11}$. The corresponding rate constraints for reliable decoding are respectively

\[
\begin{align*}
R_{12} & \leq \gamma \left( \frac{\beta P_1}{(1-\beta)P_1 + 1} \right) \\
R_0 & \leq \gamma \left( \frac{(1 - \alpha)P_1}{1 + (\alpha - \beta)P_1} \right)
\end{align*}
\]  

(151)  

(152)

After this, $Y_1$ intersects the codewords of $W_{11}$ in bin $S_{11}$ with the ambiguity set [20] of $W_{11}$ in the previous block to find the unique codeword $W_{11}$. The uniqueness is guaranteed if

\[
R_{11} \leq \gamma((\alpha - \beta)P_1) + R_0
\]  

(153)

Combine (152) and (153), we have

\[
R_{11} \leq \gamma((1 - \beta)P_1)
\]  

(154)
The rate constraints at $Y_3$ are

\begin{align}
R_{12} & \leq \gamma \left( \frac{b^2 \beta P_1}{1 + b^2(1 - \beta)P_1 + P_2} \right) \tag{155} \\
R_2 & \leq \gamma \left( \frac{P_2}{1 + b^2(\alpha - \beta)P_1} \right) \tag{156}
\end{align}

which are the results of sequentially decoding $W_{12}$ and $W_2$. For $R_1 = R_{11} + R_{12}$, apply Fourier-Motzkin elimination on (148)-(156), we obtain the region $R_3$.

Notice that while the encoding scheme mimics that for the relay channel [20], this is for a different purpose. The reason is that the new message in each block is required to be decoded by $Y_2$ with a maximum rate $\gamma(K^2 \alpha P_1)$. In the case of $K^2 \alpha > 1$, $Y_1$ will not be able to decode this new message; the cell index transmitted in the next block is to ensure reliable decoding of the message by $Y_1$. Thus decoding of $W_1$ at $Y_1$ is accomplished in two steps, much like the way decoding is done at the receiver for the classic three node relay channel: in block $i - 1$, the received signal $Y_1$ allows the partition of message indices into $\gamma((\alpha - \beta)P_1)$ subsets, thereby allowing the construction of the ambiguity set; in block $i$ the cell index can be reliably decoded if (152) is satisfied. Uniqueness of the message index as the intersection of the ambiguity set and the cell is guaranteed if (153) is satisfied. In addition to facilitating the decoding of $W_1$ at $Y_1$, the cell index itself allows interference cancellation for the cognitive transceiver pair through dirty paper coding and decoding as it is known at the beginning of each block for transmitter 2.

Some numerical examples are given in Fig.6. We see from Fig.6 that with different values of $K$, the subset relation between $R_1$ and $R_3$ varies. To find out their precise relation, we only need to consider the corner points of the two regions as $R_1$ is a convex hull of its corner points whereas $R_3$ is a convex region. For region $R_1$, when $R_1 = C_1$,

\begin{equation}
R_2 = \frac{1}{2} \log \left( \frac{1 + b^2 P_1 + P_2}{1 + P_1} \right). \tag{157}
\end{equation}

For region $R_3$, when $R_1 = C_1$,

\begin{equation}
R_2 = \frac{1}{2} \log \left( 1 + \frac{P_2}{1 + b^2 P_1 / K^2} \right). \tag{158}
\end{equation}

In order for $R_1$ to be a subset of $R_3$, we need

\begin{equation}
\frac{1 + b^2 P_1 + P_2}{1 + P_1} \leq 1 + \frac{P_2}{1 + b^2 P_1 / K^2} \tag{159}
\end{equation}

which yields

\begin{equation}
K^2 \geq b^2 \cdot \frac{(b^2 - 1)P_1 + P_2}{1 + P_2 - b^2} \tag{160}
\end{equation}

That is, when $K$ is large enough, $R_3$ dominates $R_1$, i.e., the coding scheme where receiver 2 needs to decode both users’ messages, which is capacity achieving for the classic ZIC with strong interference, is no longer optimal here. However, when $K$ is small enough, especially when $K$ is near 1, $R_1$ will dominate $R_3$, thus the cognitive link between the two users appears to yield no rate gain for the proposed encoding scheme.
The reason why $R_1$ can sometimes outperform $R_3$ is that we apply sequential decoding at receiver 2, i.e., $Y_3$ decodes part of user 1’s new message ($W_{12}$) first, then decodes $W_2$. To improve the rate region, we can apply simultaneous decoding for $W_{12}$ and $W_2$ as in the multiple access channel, which leads to the following constraints at receiver 2:

\begin{align}
R_{12} & \leq I(W_{12}; Y_3U_2) \\
R_2 & \leq I(U_2; Y_3W_{12}) - I(U_2; S_{11}) \\
R_{12} + R_2 & \leq I(U_2W_{12}; Y_3) - I(U_2; S_{11})
\end{align}

where $W_{12}$ is part of $W_1$ to be decoded by $Y_3$; $S_{11}$ is the cell index of the message $W_{11}$, which is the other part of $W_1$; $U_2$ is an auxiliary variable for dirty paper coding $W_2$ against $S_{11}$. To evaluate (161)-(163), let $W_{12} \sim N(0, \beta P_1)$, $S_{11} \sim N(0, (1 - \alpha)P_1)$, $U_2 = X_2 + \mu S_{11}$, where $X_2 \sim N(0, P_2)$ and $\mu$ is a deterministic real number. Thus, the right hand side of (161)-(163)
can be evaluated as
\[
\zeta_1 = \frac{1}{2} \log \left( \frac{(P_2 + \mu^2 \alpha P_1)(P_2 + b^2 P_1 + 1) - (P_2 + \mu b \alpha P_1)^2}{(P_2 + \mu^2 \alpha P_1)(P_2 + b^2 \beta P_1 + 1) - (P_2 + \mu b \alpha P_1)^2} \right) \tag{164}
\]
\[
\zeta_2 = \frac{1}{2} \log \left( \frac{P_2(P_2 + b^2 \beta P_1 + 1)}{(P_2 + \mu^2 \alpha P_1)(P_2 + b^2 \beta P_1 + 1) - (P_2 + \mu b \alpha P_1)^2} \right) \tag{165}
\]
\[
\zeta_3 = \frac{1}{2} \log \left( \frac{P_2(P_2 + b^2 P_1 + 1)}{(P_2 + \mu^2 \alpha P_1)(P_2 + b^2 \beta P_1 + 1) - (P_2 + \mu b \alpha P_1)^2} \right) \tag{166}
\]
Plugging (164)-(166) into (161)-(163), we can define a new region, \( R_4 \), based on the idea of simultaneous decoding at receiver 2:
\[
\begin{align*}
R_1 &\leq \gamma(K^2 \alpha P_1) \tag{167} \\
R_1 &\leq \gamma(P_1) \tag{168} \\
R_1 &\leq \gamma(K^2(\alpha - \beta)P_1 + \gamma(\beta P_1) \tag{169} \\
R_1 &\leq \gamma/(K^2(\alpha - \beta)P_1) + \zeta_1 \tag{170} \\
R_1 &\leq \gamma((1 - \beta)P_1) + \zeta_1 \tag{171} \\
R_2 &\leq \zeta_2 \tag{172} \\
R_1 + R_2 &\leq \gamma(K^2(\alpha - \beta)P_1) + \zeta_3 \tag{173} \\
R_1 + R_2 &\leq \gamma((1 - \beta)P_1) + \zeta_3 \tag{174}
\end{align*}
\]
for all \( 0 \leq \beta \leq \alpha \leq 1 \), \( -\infty < \mu < \infty \), \( \alpha + \bar{\alpha} = 1 \) and \( \beta + \bar{\beta} = 1 \).

**Theorem 7:** Region \( R_4 \) is achievable for the cognitive ZIC.

Comparison of \( R_1 \), \( R_3 \) and \( R_4 \) is given in Fig.7. It can be seen that for all values of \( K \), \( R_4 \) is always the superset of both \( R_1 \) and \( R_3 \).

It is worth noting that the two achievable regions \( R_1 \) and \( R_3 \) have a common corner point
\[
R_1 = \gamma \left( \frac{b^2 P_1}{1 + P_2} \right), \ R_2 = C_2. \tag{175}
\]
It is conjectured that for the classic ZIC, this is indeed the corner point of the capacity region [21], [22]. It is not clear whether the existence of the cognitive link may extend this corner point using some other coding schemes for finite \( K \).

3) \( b^2 < 1 \): For the weak interference case, we only know the sum rate capacity of the classic ZIC achieved at the corner point
\[
R_1 = C_1, \ R_2 = \gamma \left( \frac{P_2}{1 + b^2 P_1} \right). \tag{176}
\]
The other corner point of the known achievable region for the ZIC is described in (175).

Let us define \( R_5 \) as the Han-Kobayashi region [6] with \( Q = \phi \) and Gaussian inputs for the classical ZIC. Then, after Fourier-Motzkin elimination, and removing redundant inequalities due to \( b < 1 \), \( R_5 \) can be expressed by
\[
\begin{align*}
R_1 &\leq \gamma(\alpha P_1) + \gamma \left( \frac{b^2(1 - \alpha)P_1}{1 + b^2 \alpha P_1 + P_2} \right) \tag{177} \\
R_2 &\leq \gamma \left( \frac{P_2}{1 + b^2 \alpha P_1} \right) \tag{178}
\end{align*}
\]
for $\alpha \in [0, 1]$. For the cognitive ZIC with weak interference, the regions $R_3$ and $R_4$ are still achievable. We can compare these three regions for different values of $K$, as plotted in Fig. [8]

When $K > 1$, $R_3$ and $R_4$ are the same and they outperform the HK region.

When $K = 1$, regions $R_3$ and $R_4$ are indistinguishable from $R_5$. In fact, we can prove that $R_3$ is indeed inequivalent to $R_5$ for $b < 1$ and $K = 1$.

**Lemma 2:** $R_3$ is equivalent to $R_5$ for $b < 1$ and $K = 1$.

**Proof:** Since $K = 1$, by removing redundant constraints, $R_3$ becomes

\[
R_1 \leq \gamma(\alpha P_1)
\]

\[
R_1 \leq \gamma((\alpha - \beta)P_1) + \gamma \left( \frac{b^2 \beta P_1}{1 + b^2(1 - \beta)P_1 + P_2} \right)
\]

\[
R_2 \leq \gamma \left( \frac{P_2}{1 + b^2(\alpha - \beta)P_1} \right)
\]
Fig. 8. Comparison of $R_3$, $R_5$ and $R_5$. (a) $P_1 = P_2 = 6, K = 2, b = 0.6$. (b) $P_1 = P_2 = 6, K = 1, b = 0.6$. (c) $P_1 = P_2 = 6, K = 0.9, b = 0.6$.

We can easily verify that since $b < 1$, for fixed $\alpha$, the right hand side of (180) is a decreasing function of $\beta$. Thus, when $\beta = 0$, (180) reaches its maximum value $\gamma(\alpha P_1)$. Therefore, (179) is redundant and can be removed. Now, for the region defined by (180)-(181), we can easily verify that for fixed $\beta$, the region increases with $\alpha$. Thus, the optimal operating point is for $\alpha = 1$, i.e., transmitter 1 uses all its power, which is intuitively true for $b < 1$. By setting $\alpha = 1$, (180)-(181) reduces to $R_5$.

When $K < 1$, $R_3$ and $R_4$ are still the same, and they are outperformed by the HK region. It is clear that, when $K \leq 1$, the idea of utilizing the cognitive link and applying dirty paper coding to boost $R_2$ is strictly suboptimal for the proposed coding scheme. Again, it is not clear if there is any other coding scheme that can improve the rate region by utilizing the cooperating link when $K \leq 1.$
C. Capacity Outer Bounds

Besides the trivial outer bound $R_2$ mentioned above, in this section, we propose a nontrivial outer bound to the capacity region of the cognitive ZIC for the case $b \leq 1$, and discuss some interesting implications from this result. Before presenting the outer bound, let us first introduce the following lemma.

**Lemma 3:** If $b \leq 1$, the capacity region of the cognitive ZIC (131)-(133) is the same as the cognitive ZIC given below:

\[
\begin{align*}
Y'_1 &= X_1 + Z_1 \\ Y'_2 &= KX_1 + Z_2 \\ Y'_3 &= bY'_1 + X_2 + \tilde{Z}_3
\end{align*}
\]

where $\tilde{Z}_3 \sim N(0, 1 - b^2)$ is independent of all the other random variables. Thus, given $X_2$,

\[
X_1 \implies Y'_1 \implies Y'_3
\]

forms a Markov chain.

**Proof:** Since $Y_1$ does not cooperate with $Y_2, Y_3$, the capacity region of the cognitive ZIC only depends on the marginal distributions $p(y_1|x_1, x_2)$ and $p(y_2, y_3|x_1, x_2)$. Further, since $Z_1, Z_2$ and $Z_3$ in the original model (131)-(133) are independent, in the modified channel, $Z_2$ and $bZ_1 + \tilde{Z}_3$ are also independent. Therefore,

\[
p(y_2', y_3'|x_1, x_2) = p(y_2, y_3|x_1, x_2).
\]

Therefore, the capacity region of the modified channel is the same as the original channel, and the Markov chain (185) follows automatically.

With the lemma above, we are ready to derive our outer bound described in the following theorem.

**Theorem 8:** Define $R_o$ to be the union of all nonnegative rate pairs $(R_1, R_2)$ satisfying

\[
\begin{align*}
R_1 &\leq I(X_1; Y_1|Y_2, UX_2) \\ R_2 &\leq I(UX_2; Y_3)
\end{align*}
\]

for all joint distributions that factor as

\[
p(u)p(x_1, x_2|u)p(y_1y_2y_3|x_1, x_2).
\]

Then $R_o$ is a capacity outer bound for the cognitive ZIC.
Proof: Define $U_i = (Y_1^{i-1}, Y_2^{i-1}, W_2, X_2^{i-1})$, and we have

\[ nR_1 = H(W_1|W_2) \]
\[ = I(W_1; Y^n_1|W_2) + H(W_1|Y^n_1, W_2) \]
\[ = I(W_1; Y^n_1|W_2) + n\epsilon_1 \]
\[ \leq I(W_1; Y^n_1 Y^n_2|W_2) + n\epsilon_1 \]
\[ = \sum_{i=1}^{n} I(W_1; Y_1 Y_2|Y_1^{i-1} Y_2^{i-1} W_2) + n\epsilon_1 \]
\[ = \sum_{i=1}^{n} I(W_1; Y_1 Y_2|Y_1^{i-1} Y_2^{i-1} W_2 X_2^i) + n\epsilon_1 \]
\[ = \sum_{i=1}^{n} I(X_1 i; Y_1 Y_2|U_i X_2^i) + n\epsilon_1 \] (190) (191) (192) (193) (194) (195) (196)

(192) is due to Fano’s inequality; (195) is because the codeword $X_2$ is a function of $W_2$ and $Y_2^{i-1}$ as stated in (135); (196) is due to the memoryless channel model.

\[ nR_2 = H(W_2) = I(W_2; Y^n_3) + H(W_2|Y^n_3) \]
\[ \leq I(W_2; Y^n_3) + n\epsilon_2 \]
\[ = \sum_{i=1}^{n} I(W_2; Y_3|Y_3^{i-1}) + n\epsilon_2 \]
\[ = \sum_{i=1}^{n} \{ h(Y_3|Y_3^{i-1}) - h(Y_3|Y_3^{i-1} W_2 X_2^i) \} + n\epsilon_2 \]
\[ \leq \sum_{i=1}^{n} \{ h(Y_3) - h(Y_3|Y_3^{i-1} W_2 X_2^i) \} + n\epsilon_2 \]
\[ \leq \sum_{i=1}^{n} \{ h(Y_3) - h(Y_3|Y_3^{i-1} W_2 X_2^i) \} + n\epsilon_2 \]
\[ \leq \sum_{i=1}^{n} \{ h(Y_3) - h(Y_3|U_i X_2^i) \} + n\epsilon_2 \]
\[ = \sum_{i=1}^{n} I(U_i X_2^i; Y_3) + n\epsilon_2 \] (197) (198) (199) (200) (201) (202) (203) (204)

(202) follows from Lemma [8] When $b \leq 1$, the capacity region of the cognitive ZIC is equivalent to the cognitive ZIC such that given $X_2$,

\[ X_1 \rightarrow Y_1 \rightarrow Y_3. \] (205)

Therefore it suffices to establish the outer bound for the cognitive ZIC satisfying (205). Due to (205), conditioning on $(X_2^{i-1}, Y_1^{i-1})$, the random vector $Y_3^{i-1}$ is independent of all other variables, including $Y_3$. Thus, given $X_2^i, W_2$,

\[ Y_3 \rightarrow Y_1^{i-1} \rightarrow Y_3^{i-1}. \] (206)
Hence,

\[ h(Y_3|Y_3^{i-1}W_2X_2^i) \geq h(Y_3|Y_1^{i-1}W_2X_2^i). \] (207)

Thus, (202) follows.

Next, we derive another outer bound for the specific case where \( bZ \leq 1 \) and \( Z \) is independent of \( Y \). Define \( U = (U_l, I) \), \( X_1 = X_{1l} \), \( X_2 = X_{2l} \), \( Y_1 = Y_{1l} \), \( Y_2 = Y_{2l} \) and \( Y_3 = Y_{3l} \). Then, we have

\[
\begin{align*}
R_1 & \leq \sum_{i=1}^{n} I(X_1; Y_3) + n\epsilon_1 \\
& = nI(X_1; Y_3) + n\epsilon_1 \\
& \leq nI(U_lX_{2l}; Y_{3l}) + n\epsilon_2 \\
& = nI(U_lX_{2l}; Y_{3l}) + n\epsilon_2 \\
& \leq nI(U_lX_{2l}; Y_{3l}) + n\epsilon_2
\end{align*}
\] (208)

This establishes (187)-(188) but it requires \( Y_3 \) satisfy Markov chain (205), i.e., the noise \( Z_3 = bZ_1 + \tilde{Z}_3 \) as in Lemma 3. However, due to the special form of (187)-(188), we now show that this outer bound can be equivalently evaluated by assuming \( Z_3 \) to be independent of \( Z_1 \) as in the original channel model. According to the channel model and the Markov chain (189), \((U, X_1, X_2)\) are independent of all the noises \( Z_1, Z_2 \) and \( Z_3 \). Therefore, to compute (188),

\[
I(UX_2; Y_3) = h(bX_1 + X_2 + Z_3) - h(bX_1 + Z_3)UX_2)
\] (215)

Thus, any value that \( I(UX_2; Y_3) \) can achieve with \( Z_3 = bZ_1 + \tilde{Z}_3 \) can also be achieved with \( Z_3 \) independent of \( Z_1 \). Therefore, the relaxation of \( Z_3 \) to its original model will yield the same outer bound. This proves Theorem 8.

To recap, the proof of Theorem 8 is accomplished in two steps. The first step utilizes Lemma 3 and establishes the outer bound for the equivalent channel satisfying the Markov chain conditions (205). In the second step, we prove that the outer bound can be equivalently evaluated using the original channel model. Theorem 8 is valid for all cognitive ZIC as long as \( b \leq 1 \). This outer bound is analogous to the capacity region of a degraded broadcast channel where receiver \( Y_3 \) sees a degraded channel compared with that of \((Y_1, Y_2)\).

Next, we derive another outer bound for the specific case where \( b \leq 1 \) and \( K \geq 1 \). Before that, we introduce another lemma.

**Lemma 4**: If \( K \geq 1 \), the capacity region of the cognitive ZIC (131)-(133) is the same as the cognitive ZIC given below:

\[
\begin{align*}
\tilde{Y}_1 &= \frac{1}{K} \tilde{Y}_2 + \tilde{Z}_1 \\
\tilde{Y}_2 &= KX_1 + Z_2 \\
\tilde{Y}_3 &= bX_1 + X_2 + Z_3
\end{align*}
\] (217) (218) (219)
where \( \tilde{Z}_1 \sim N(0, 1 - \frac{1}{K^2}) \) is independent of all the other random variables. Thus,

\[
X_1 \implies \tilde{Y}_2 \implies \tilde{Y}_1. \tag{220}
\]

Further, for all random variable \( U \) such that \( U \implies (X_1, X_2) \implies (\tilde{Y}_1, \tilde{Y}_2, \tilde{Y}_3) \),

\[
X_1 \implies (U, X_2, \tilde{Y}_2) \implies \tilde{Y}_1. \tag{221}
\]

**Proof:** The argument is similar to that of Lemma 3. Since \( Y_1 \) does not cooperate with \( Y_2 \) or \( Y_3 \), the capacity region only depends on \( p(y_1|x_1x_2) \) and \( p(y_2, y_3|x_1, x_2) \). Therefore, making the noise at \( Y_1 \) to be correlated with that of \( Y_2 \) will have no effect on the capacity region. Thus, (220) holds. Furthermore, since \( \tilde{Z}_1 \) is independent of all other variables, (221) also holds. ■

When \( K \geq 1 \), according to Lemma 4, we only need to consider the cognitive ZIC such that (221) is satisfied. Since \( b \leq 1 \) and due to the fact that Theorem 8 holds for the original cognitive ZIC channel model, the outer bound (187)-(188) is still an outer bound for the system defined in Lemma 4. For all \( U \) under condition (189), due to (221),

\[
I(X_1; Y_1 | UX_2 Y_2) = 0. \tag{222}
\]

Thus, we can rewrite the outer bound \( R_o \) as

\[
R_1 \leq I(X_1; Y_2 | UX_2) \tag{223}
\]

\[
R_2 \leq I(UX_2; Y_3). \tag{224}
\]

Next, we give the second outer bound in the theorem below.

**Theorem 9:** If \( b \leq 1 \) and \( K \geq 1 \), an outer bound to the capacity region of the cognitive ZIC is the union of all nonnegative rate pairs \((R_1, R_2)\) satisfying

\[
R_1 \leq C_1 \tag{225}
\]

\[
R_2 \leq C_2 \tag{226}
\]

\[
R_1 \leq \gamma(K^2 \alpha P_1) \tag{227}
\]

\[
R_2 \leq \gamma \left( \frac{b^2 \alpha P_1 + P_2 + 2b\sqrt{\alpha P_1 P_2}}{1 + b^2 \alpha P_1} \right) \tag{228}
\]

for \( 0 \leq \alpha \leq 1 \) and \( \alpha + \bar{\alpha} = 1 \).

**Proof:** (225) and (226) are trivial outer bounds. (227) and (228) are derived from (223) and (224), respectively. Consider (223).
where \( \alpha \in [0, 1] \). Thus, bound (223) becomes
\[
R_1 \leq \frac{1}{2} \log(2\pi e (1 + K^2 \alpha P_1)).
\] (235)

By Lemma 1 of [23],
\[
h(Y_2 | U X_2) \leq h(K X_1 + Z_2 | X_2)
\leq h(K X_1^* + Z_2 | X_2^*)
= \frac{1}{2} \log(2\pi e (1 + K^2 \text{Var}(X_1^*|X_2^*)�))
\] (238)

where \( X_1^* \) and \( X_2^* \) are Gaussian distributed variables with the same covariance matrix with that of \( X_1 \) and \( X_2 \). Combining (234) and (238), we obtain
\[
\text{Var}(X_1^*|X_2^*) \geq \alpha P_1
\] (239)

Since,
\[
\text{Var}(X_1^*|X_2^*) = E[(X_1^*)^2] - E[(E[X_1^*|X_2^*])^2],
\] (240)

Combined with (239), we obtain
\[
E[(E[X_1^*|X_2^*])^2] \leq \bar{\alpha} P_1.
\] (241)

Thus,
\[
E(X_1 X_2) = E(X_1^* X_2^*)
\leq (E[(E[X_1^*|X_2^*])^2]E[(X_2^*)^2])^{\frac{1}{2}}
\leq \sqrt{\bar{\alpha} P_1 P_2}.
\] (244)

Next we consider (224). Since \( K \geq 1 \) and \( b \leq 1 \), given \( X_2, Y_3 \) is a degraded version of \( Y_2 \). By the entropy power inequality,
\[
2^{2h(Y_3|U X_2)} \geq 2^{2h(Y_2|U X_2)} + 2^{2h(Z')}
\]
\[
= \frac{b^2}{K^2} 2^{2h(Y_2|U X_2)} + 2\pi e(1 - \frac{b^2}{K^2})
\]
\[
= \frac{b^2}{K^2} 2\pi e(1 + K^2 \alpha P_1) + 2\pi e(1 - \frac{b^2}{K^2})
\]
\[
= 2\pi e(1 + b^2 \alpha P_1)
\] (248)

Therefore,
\[
h(Y_3|U X_2) \geq \frac{1}{2} \log(2\pi e (1 + b^2 \alpha P_1))
\] (249)

Thus, (224) becomes
\[
I(U X_2; Y_3) = h(Y_3) - h(Y_3|U X_2)
\leq \frac{1}{2} \log \left( \frac{b^2 P_1 + P_2 + 2b \sqrt{\bar{\alpha} P_1 P_2} + 1}{1 + b^2 \alpha P_1} \right)
\] (251)
This completes the proof of Theorem 9.

**Corollary 1:** If $K \leq 1$ and $b \leq 1$, then $(C_1, \gamma(\frac{P_2}{1+bP_1^2}))$ is the corner point of the capacity region for the cognitive Z channel.

**Proof:** The achievability of this point is trivial, as this is indeed the sum-rate capacity of the classical ZIC, i.e., one can achieve this point in the absence of the cooperating link.

For the converse part, if $b \leq 1$, when $K = 1$ and $R_1 = C_1$, according to Theorem 9, $R_2 \leq \gamma(\frac{P_2}{1+bP_1^2})$. Thus, this corner point is on the boundary of the capacity region of the cognitive ZIC when $K = 1$. Since the capacity outer bound for the case with $K = 1$ is also an outer bound for the case with $K < 1$ if all other parameters remain the same, this is also a corner point of the capacity region for all $K \leq 1$.

That is to say, when the interference is weak ($b \leq 1$) and the cooperating link is weak ($K \leq 1$), the cooperating link becomes useless when user 1 is transmitting at its maximum rate. While this does not imply that the entire capacity region will not be affected by the cooperating link, this corner point is particularly important in practice: it is often desirable in a cognitive radio system with primary-secondary user pairs that the primary user’s rate is not affected by the interference of the secondary user. We note that in the absence of the cooperating link, this is also the sum-rate capacity corner point. However, no such statement can be made for the cognitive Z channel as the slope of the outer bound at this corner point is not necessarily smaller than 45°. The comparison of the outer bound $R_o$ and the inner bounds are given in Fig. 9. Note that $R_4$ and $R_5$ coincide with each other in (a) when $K = 1$ while $R_4$ outperforms $R_5$ in (b) when $K > 1$. Also, when $K = 1$, the optimal corner point $(C_1, \gamma(\frac{P_2}{1+bP_1^2}))$ is shown in (a).

![Fig. 9. Comparison of $R_5$, $R_4$ and $R_o$.](image)

(a) $P_1 = P_2 = 6, K = 1, b = 0.6$. (b) $P_1 = P_2 = 6, K = 1.2, b = 0.6$.

**IV. CONCLUSION AND DISCUSSION**

This paper studied the interference channel with one cognitive transmitter (ICOCT), from both the noncausal and causal perspectives. For the noncausal ICOCT (or interference channel with degraded message sets), we proposed a new achievable rate region which generalizes existing results reported in the literature. The proposed coding scheme leads to a rate region that reduces to...
Marton’s achievable rate region for the general broadcast channels in the absence of the primary transmitter. This proposed achievable region, together with a new outer bound, establishes the capacity region of a class of semi-deterministic IC-DMS.

The causal ICOT, which is a special case of interference channels with generalized feedback, imposes causality constraint on the way the cognitive transmitter can cooperate with the primary user. Motivated by practical constraint on the so-called interference temperature in the spectrum sharing cognitive radio system, we focus on the Gaussian Z interference channel (ZIC) in which the interference link from the cognitive transmitter and the primary receiver is negligible. Both capacity inner bounds and outer bounds are proposed for the cognitive Gaussian ZIC. The capacity bounds established some intuitive results with regard to the usefulness of the cognitive capability. In general, when the feedback link is weak, it is generally not useful to utilize the cognitive capability, as far as the capacity region is concerned.

APPENDIX

A. Proof of Theorem 2

Proof: Codebook generation: Generate $2^{nR_{10}}$ independent and identically distributed (i.i.d.) codewords $u_{10}(j_{10}), j_{10} = 1, \ldots, 2^{nR_{10}}$, according to $\prod_{i=1}^{n} p(u_{10,i})$. For each codeword $u_{10}(j_{10})$, generate $2^{nR_{11}}$ i.i.d. codewords $u_{11}(j_{11}, j_{10}), j_{11} = 1, \ldots, 2^{nR_{11}}$, according to $\prod_{i=1}^{n} p(u_{11,i}|u_{10,i})$. For each pair of $(u_{11}, u_{10})$, generate one codeword $x_{1}(j_{11}, j_{10})$ according to $\prod_{i=1}^{n} p(x_{1,i}|u_{11,i}, u_{10,i})$.

For each codeword $u_{10}(j_{10})$, also generate $2^{nL_{20}}$ i.i.d. codewords $v_{20}(l_{20}, j_{10}), l_{20} = 1, \ldots, 2^{nL_{20}}$, according to $\prod_{i=1}^{n} p(v_{20,i}|u_{10,i})$ and randomly place them into $2^{nR_{20}}$ bins. For each codeword pair $(v_{20}(l_{20}, j_{10}), u_{10}(j_{10}))$, generate $2^{nL_{21}}$ i.i.d. codewords $v_{11}(l_{11}, l_{20}, j_{10}), l_{11} = 1, \ldots, 2^{nL_{21}}$, according to $\prod_{i=1}^{n} p(v_{11,i}|v_{20,i}, u_{10,i})$ and randomly place them into $2^{nR_{21}}$ bins. For each codeword pair $(v_{20}(l_{20}, j_{10}), u_{10}(j_{10}))$, generate $2^{nL_{22}}$ i.i.d. codewords $v_{22}(l_{22}, l_{20}, j_{10}), l_{22} = 1, \ldots, 2^{nL_{22}}$, according to $\prod_{i=1}^{n} p(v_{22,i}|v_{20,i}, u_{10,i})$ and randomly place them into $2^{nR_{22}}$ bins. For each set $(v_{22}, v_{20}, v_{11}, u_{11}, u_{10})$, generate one codeword $x_{2}(l_{22}, l_{20}, l_{11}, j_{11}, j_{10})$ according to $\prod_{i=1}^{n} p(x_{2,i}|v_{22,i}, v_{20,i}, v_{11,i}, u_{11,i}, u_{10,i})$.

Encoding: For user 1 to send message $(j_{11}, j_{10})$, it simply transmits the codeword $x_{1}(j_{11}, j_{10})$.

Suppose user 2 wants to send message $(j_{22}, j_{20})$, encoder 2 first looks into bin $j_{20}$ for codeword $v_{20}(l_{20}, j_{10})$ such that

\[(v_{20}(l_{20}, j_{10}), u_{11}(j_{11}, j_{10}), u_{10}(j_{10})) \in T_{c}(V_{20}, U_{11}, U_{10})\]  \hspace{1cm} (252)

where $T_{c}(\cdot)$ denotes jointly typical set. After it finds $v_{20}(l_{20}, j_{10})$, it looks for a codeword $v_{22}(l_{22}, l_{20}, j_{10})$ in bin $j_{22}$ and a codeword $v_{11}(l_{11}, l_{20}, j_{10})$ in bin $j_{11}$ such that

\[(v_{22}(l_{22}, l_{20}, j_{10}), v_{11}(l_{11}, l_{20}, j_{10}), u_{11}(j_{11}, j_{10}), v_{20}(l_{20}, j_{10}), u_{10}(j_{10})) \in T_{c}(V_{22}V_{11}U_{11}V_{20}U_{10})\]  \hspace{1cm} (253)

If there are more than one such codewords, pick the one with the smallest index; if there is no such codeword, declare an error. Then, user 2 sends $x_{2}(l_{22}, l_{20}, l_{11}, j_{11}, j_{10})$. The diagram of the encoding scheme is illustrated in Fig. 2.

Decoding: Given $y_{1}$, receiver 1 looks for all sequences $v_{11}, u_{11}, u_{10}, v_{20}$ such that

\[(v_{11}, u_{11}, u_{10}, v_{20}, y_{1}) \in T_{c}(V_{11}U_{11}U_{10}V_{20}Y_{1})\]  \hspace{1cm} (254)

If all the codewords $v_{11}$ have the same bin indices $j_{11}$, all $u_{11}$ and $u_{10}$ have the same message indices $\hat{j}_{11}$ and $\hat{j}_{10}$ respectively, and if $\hat{j}_{11} = \hat{j}_{11}, \hat{j}_{10} = \hat{j}_{10}$; otherwise, declare an error.

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Given \( y_2 \), receiver 2 looks for all sequences \( v_{22}, v_{20}, u_{10} \) such that

\[
(v_{22}, v_{20}, u_{10}, y_2) \in T_i(V_{22}V_{20}U_{10}Y_2)
\]

(255)

If all the codewords \( v_{22} \) and \( v_{20} \) have the same bin indices \( j_{22} \) and \( j_{20} \) respectively, we declare \( j_{22} = \hat{j}_{22}, j_{20} = \hat{j}_{20} \); otherwise, declare an error.

**Error analysis:** By the symmetry of random code generation, without loss of generality, we assume the messages \( (j_{11}, j_{10}, j_{22}, j_{20}) = (1, 1, 1, 1) \) are sent. Let \( P_{e,enc2}, P_{e,dec1}, P_{e,dec2} \) denote the error probabilities at encoder 2, decoder 1 and decoder 2 respectively. Denote the codeword \( v_{20}(l_{20}, j_{10}) \) as \( v_{20}(j_{20}, k_{20}, j_{10}), \) where \( j_{20} \) is its bin index and \( k_{20} \) is its index within the bin. Similarly, we denote \( v_{11}(l_{11}, l_{20}, j_{10}) \) and \( v_{22}(l_{22}, l_{20}, j_{10}) \) as \( v_{11}(j_{11}, j_{20}, k_{20}, j_{10}) \) and \( v_{22}(j_{22}, k_{20}, j_{20}, j_{10}) \) respectively.

1) Error occurs at encoder 2 when one or both of the following events occur.

\[
E_1: \text{there is no } v_{20} \text{ such that (252) holds}.
E_2: \text{there is no pair } (v_{22}, v_{11}) \text{ such that (253) holds}.
\]

(256)

where event \( E_1 \) can be further divided into two sub-events:

\[
E_{11}: u_{11}(j_{11}, j_{10}) \text{ and } u_{10}(j_{10}) \text{ are not jointly typical.}
E_{12}: \text{With } u_{11}(j_{11}, j_{10}) \text{ and } u_{10}(j_{10}) \text{ jointly typical, there is no } v_{20} \text{ that satisfies (252).}
\]

(257)

According to the code book generation, it is obvious that \( P(E_{11}) \leq \epsilon \). For \( P(E_{12}) \), we have

\[
P(E_{12}) = (1 - P[(v_{20}(1, k_{20}, 1), u_{11}(1, 1), u_{10}(1)) \in T_i(V_{20}U_{11}U_{10})])^{2^n(L_{20} - R_{20})}
\]

\[
\leq (1 - 2^{-n(I(V_{20};U_{11}|U_{10}) + \epsilon)})^{2^n(L_{20} - R_{20})}
\]

\[
\leq \exp(-2^n(L_{20} - R_{20} - I(V_{20};U_{11}|U_{10}) - \epsilon))
\]

(258)

(259)

(260)

So, (43) guarantees \( P(E_1) \to 0 \) as \( n \to \infty \).

Event \( E_2 \) can be divided into the following three sub-events:

\[
E_{21}: \text{There is no codeword } v_{11} \text{ such that } (v_{11}, u_{11}, v_{20}, u_{10}) \in T_i(V_{11}V_{20}U_{10}).
E_{22}: \text{There is no codeword } v_{22} \text{ such that } (v_{22}, u_{11}, v_{20}, u_{10}) \in T_i(V_{22}V_{10}U_{10}).
E_{23}: \text{Provided } E_{21} \text{ and } E_{22} \text{ occur, there is no pair } (v_{22}, v_{11}) \text{ such that (253) holds.}
\]

(261)

Following similar derivations as for event \( E_{12} \), the error probability for events \( E_{21} \) and \( E_{22} \) will be arbitrarily small as \( n \to \infty \) if (44) and (45) hold respectively. By using the second moment method as in [14], one can also show that \( P(E_{23}) \to 0 \) as \( n \to \infty \) if (46) holds. Since

\[
P_{e,enc2} = P\{E_1 \cup E_2\} \leq P(E_1) + P(E_2)
\]

(262)

(43) and (46) ensure \( P_{e,enc2} \to 0 \) as \( n \to \infty \).

2) Error occurs at decoder 1 if

\[
E_3: \text{The transmitted } (u_{10}(1), v_{20}(1, k_{20}, 1), v_{11}(1, k_{11}, 1, k_{20}, 1), u_{11}(1, 1)) \text{ do not satisfy (254).}
E_4: \text{If } (u_{10}(j_{10}), v_{20}(j_{20}, k_{20}', j_{10}), v_{11}(j_{11}, k_{11}, j_{20}, k_{20}', j_{10}), u_{11}(j_{11}, j_{10})) \text{ satisfy (254)}
\]

where \( (j_{11}, j_{10}) \neq (1, 1) \).

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Let $A(j_{10}, j_{20}, k'_{20}, j_{11}, k'_{11})$ denote the event $E_4$. We have
\[
P_{e,dec1} \leq P\{E_3 \cup E_4\} \leq P(E_3) + P(E_4)
\]
(263)
\[
\leq \epsilon + \sum_{(j_{10}, j_{20}, j_{11}) \neq (111), k'_{20}, k'_{11}} P(A(j_{10}, j_{20}, k'_{20}, j_{11}, k'_{11}))
\]
(264)
\[
\leq \epsilon + \sum_{j_{10} \neq 1, j_{20}, k'_{20}, j_{11}, k'_{11}} P(A(j_{10}, j_{20}, k'_{20}, j_{11}, k'_{11}))
\]
(265)
\[
+ \sum_{j_{20} \neq 1, k'_{20}, j_{11} \neq 1, k'_{11}} P(A(1, j_{20}, k'_{20}, j_{11}, k'_{11})) + \sum_{j_{11} \neq 1, k'_{11}} P(A(1, 1, k'_{20}, j_{11}, k'_{11}))
\]
(266)
Take $P(A(1, 1, k'_{20}, j_{11}, k'_{11}))$ for example,
\[
P(A(1, 1, k'_{20}, j_{11}, k'_{11})) = \sum_{(u_{10}v_{20}, u_{11}v_{10}, Y_1) \in T_e} p(u_{10}v_{20})p(u_{11}u_{10})p(v_{11}|v_{20}u_{10})p(y_1|u_{10}v_{20})
\]
(267)
\[
\leq 2^{-n[H(U_{10}V_{20})+H(U_{11}|U_{10})+H(V_{11}|V_{20}U_{10})+H(Y_1|V_{20}U_{10})-H(U_{10}V_{20}V_{11}Y_1) - \epsilon]}
\]
(268)
\[
\leq 2^{-n[H(U_{11}Y_1V_{20}|U_{10})+I(V_{11};Y_1U_{11}|V_{20}U_{10})]}
\]
(269)
So,
\[
P_{e,dec1} \leq \epsilon + 2^{-n[I(V_{11}U_{11}V_{20}U_{10};Y_1) - (L_{11}+L_{20}+R_{10}) - \epsilon]}
\]
(270)
\[
+ 2^{-n[I(V_{11}U_{11}V_{20};Y_1) - (L_{11}+L_{20}) - \epsilon] + 2^{-n[I(U_{11};V_{11}V_{20}|U_{10}) + I(V_{11};Y_1U_{11}|U_{10}V_{20}) - L_{11} - \epsilon]}
\]
(271)
\[
\leq \epsilon + 2^{-n[I(V_{11}U_{11}V_{20}U_{10};Y_1) - (L_{11}+L_{20}+R_{10}) - \epsilon]}
\]
(272)
\[
+ 2^{-n[I(V_{11}U_{11}V_{20};Y_1) - (L_{11}+L_{20}) - \epsilon] + 2^{-n[I(U_{11};V_{11}V_{20}|U_{10}) + I(V_{11};Y_1U_{11}|U_{10}V_{20}) - L_{11} - \epsilon]}
\]
\[
\text{\textit{\textbf{REFERENCES}}}
\]
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