ON TESTING THE EQUIVALENCE PRINCIPLE WITH EXTRAGALACTIC BURSTS

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ABSTRACT
An interesting test of Einstein’s equivalence principle (EEP) relies on the observed lag in the arrival times of photons emitted from extragalactic transient sources. Attributing the lag between photons of different energies to the gravitational potential of the Milky Way (MW), several authors derive new constraints on deviations from EEP. It is shown here that potential fluctuations from the large-scale structure are at least two orders of magnitude larger than the gravitational potential of the MW. Combined with the larger distances, for sources at redshift \( z \gtrsim 0.5 \) the rms of the contribution from these fluctuations exceeds the MW by more than four orders of magnitude. We provide actual constraints for several objects based on a statistical calculation of the large-scale fluctuations in the standard ΛCDM cosmological model.

Key words: gravitation – large-scale structure of universe

1. INTRODUCTION
Any deviation from Einstein’s equivalence principle (EEP) will have far-reaching consequences on all fundamental theories of physics (Will 2006). The inability to distinguish between properties of motion in non-inertial frames of reference and certain gravitational fields in inertial frames (Landau & Lifshitz 1975) implies that the world line of a massless particle is independent of its energy. Delays between the arrival times of different types of radiation from astronomical burst events have been proposed (Krauss & Tremaine 1988; Sivaram 1999) to constrain deviations from the EEP through the effect of the Shapiro (gravitational) time delay (Shapiro 1964). Recently, Gao et al and Wei et al (Gao et al. 2015; Wei et al. 2015) have applied this test to gamma-ray bursts (GRBs) and fast radio bursts (FRBs; Lorimer et al. 2007). Their strongest constraints are based on FRBs. Photons with different frequencies, \( \nu \), from these millisecond transients are observed as arriving at different times. The observed time delay (with respect to a reference frequency) follows \( \Delta t_{\text{obs}} \sim \nu^{-2} \), as expected from the dispersion of radio waves propagating in an ionized medium. The dispersion measure (DM) is large, indicating sources of cosmological origin. Constraints on deviations from EEP are obtained by taking \( \Delta t_{\text{obs}} \) as an upper limit on the difference between the Shapiro time delay for photons at two distinct frequencies.

Adopting the parameterized post-Newtonian approximation (PPN), deviations from the EEP are described in terms of the parameter \( \gamma \) (Will 2006), where \( \gamma = 1 \) in general relativity. The Shapiro time delay is then

\[
\Delta t_{\text{gra}} = -\frac{1}{c^3} \int_{r_e}^{r_o} \frac{U(r(t), t) dr}{c^2},
\]

(1)

where the integration is along the path of the photon emitted at \( r_e \) and received at \( r_o \). Gao et al and Wei et al focus on the contribution from the gravitational potential of the MW. Assuming a Keplerian potential for the MW they use a corresponding shift

\[
\Delta t_{\text{graw}} = \Delta \gamma \frac{GM_{\text{MW}}}{c^3} \ln \left( \frac{d}{b} \right),
\]

(2)

where \( \Delta \gamma \) is the difference between the \( \gamma \) value for the two photons, \( M_{\text{MW}} \) is the mass of the MW, \( d \) is the distance to the source, and \( b \) is the impact parameter of the light path with respect to the Galactic center. For \( M_{\text{MW}} = 6 \times 10^{11} M_{\odot} \), \( d = 1500 \text{ Mpc} \) and \( b = 5 \text{ kpc} \), this equation gives \( \Delta t_{\text{graw}} / \Delta \gamma = 3.5 \times 10^7 \text{ s} \). One of the objects Wei et al use is FRB 110220 (Thornton et al. 2013). Since the observed time delay depends on frequency as \( \nu^{-2} \), most of the lag is due to the dispersion of photons. Using the observed DM, the inferred redshift for this object is \( z \sim 0.81 \) (corresponding to \( d = 1500 \text{ Mpc} \)). Taking the 1 s observed shift between arrival times of 1.5 and 1.2 GHz photons as an upper limit on \( \Delta t_{\text{graw}} \), they obtain \( \Delta \gamma < 2.5 \times 10^{-8} \). Wei et al point out that this is a conservative upper limit since the 1 s time delay should mostly be due to the dispersion of radio waves.

Wei et al argue that incorporating the gravitational potential from the large-scale (\( \gtrsim 10 \text{ Mpc} \)) structure (hereafter, LSS) tightens the constraint, but they do not estimate this effect. The current paper assesses the contribution of the LSS potential field and shows that it should greatly exceed the local MW contribution. In generalizing Equation (1) to cosmology we assume (i) distances well within the horizon, (ii) assume the mechanism for the EEP breaking is decoupled from the cosmological background and is induced solely by spatial fluctuations of the gravitational potential, \( U_{\text{LSS}} \), resulting from the LSS distribution of matter, and (iii) assume a PPN for the cosmological metric (Futamase 1988; Hwang et al. 2008) with \( \gamma \) appearing in the time and spatial components of the metric as \( g_{00} \approx -(1 - 2\gamma U_{\text{LSS}}/c^2) \) and \( g_{0i} = a(t)(1 + 2\gamma U_{\text{LSS}}/c^2) \), where \( U_{\text{LSS}} \ll c^2 \) and \( a(t) \) is the scale factor of the Universe. We write the shift in the arrival times of photons of two different frequencies due to the Shapiro effect as

\[
\Delta t_{\text{gra}} (\hat{r}) = \frac{\Delta \gamma}{c^3} \int_{r_e}^{r_o} U_{\text{LSS}} (\hat{r}, z) a(z) dr
\]

(3)

where \( r_e \) and \( r_o \) are now comoving distances and \( a = (1 + z)^{-1} \) corresponds to a comoving distance \( r(z) \), at a cosmological redshift \( z \). This cosmological Shapiro shift may acquire negative as well as positive values since \( U_{\text{LSS}} \) fluctuates around zero.
2. ORDER OF MAGNITUDE BASED ON THE OBSERVED LSS MOTIONS

The expected amplitude of LSS potential fluctuations can be found from the observed peculiar velocities (deviations from a pure Hubble flow), $v_p$, of galaxies. For a nearly homogeneous matter distribution at early cosmic times, linear theory provides the intuitive relation $v_p \sim t_g$, where $g$ is the gravitational force field generated by mass density fluctuations and $t \sim H_0^{-1}$ is the age of the universe as the only possible timescale. Peculiar velocity data yield a bulk peculiar velocity of $v_p \sim 300 \text{ km s}^{-1}$ for the sphere of radius $R \sim 100 \text{ Mpc}$ around us. This corresponds to a gravitational potential of $U_{100} \sim v_p R H_0 \approx (5 \times 10^{-3}c)^2 \approx 500 U_{\text{MW}}$, where $U_{\text{gw}}$ is the gravitational potential depth associated with the MW. Note that $U_{\text{gw}} = \mathcal{O}(10^{-5})c^2$ is of the same order of magnitude as the potential fluctuations inferred from the temperature fluctuations in the cosmic microwave background (Bennett et al. 1994). Since the gravitational time lag is proportional to the line-of-sight integral over the potential, the contribution from an LSS should greatly dominate the MW.

3. THEORETICAL ESTIMATE BASED ON ΛCDM

We provide a statistical estimate of the shift in the gravitational time lag, $\Delta\tau_{\text{gra}}$, due to a LSS in the framework of the ΛCDM model (Planck Collaboration et al. 2015). We are interested in the rms value, $\sigma = \langle (\Delta\tau_{\text{gra}})^2 \rangle^{1/2} = \Delta\tau_{\text{gra}}$ where the averaging is over all directions. We express $\Delta\tau_{\text{gra}} = \sum_j \frac{2j+1}{4\pi} C_j$ in terms of the angular power spectra, $C_l$, and write (Nusser et al. 2013)

$$C_l = \frac{2}{\pi c^2} \int dk k^2 P_V(k) \int_{r_1}^{r_2} dr D(t_1) j_1(kr) kr^2, \quad (4)$$

where $P_V$ is the power spectrum of the gravitational potential at redshift $z = 0$. Furthermore, we have used the linear theory result in which the gravitational potential $U(r, t) = (D/a) U_0(r, t_0)$, where $D(t)$ is the linear growth factor (Peebles 1980). Adopting the ΛCDM cosmology, these expressions are computed numerically as a function of the redshift of the burst. The lower curve in Figure 1 is the upper limit $\Delta\tau_{\text{gra}} < 1 \text{ s}$ for $\Delta\tau_{\text{gra}} < 1 \text{ s}$ between photons emitted by a burst at redshift $z$. For comparison, the upper curve represents the limit obtained by considering the MW alone according to Equation (2). The curve is obtained for $M_{\text{MW}} = 2 \times 10^{12} M_\odot$ in accordance with the mass determination from the dynamics of the Local Group (Phelps et al. 2013).

4. ACTUAL CONSTRAINTS

The expected time shift has been estimated in a statistical way. The rms LSS contribution is overwhelmingly greater than the MW, and therefore we could derive strange constraints even without an actual measurement of the LSS gravitational potential to the extragalactic burst. According to the figure, the probability that the LSS contribution at $z \sim 1$ acquires values smaller than the MW’s is $10^{-4.8}$, i.e., a 4.3σ rejection level. Values 200 times smaller than the MW’s are rules out at the 3σ level. We now derive constraints from several bursts that have already been considered in the literature.

1. **FRB 110220**: This is the object used in Wei et al. (2015). Based on the figure, for FRB 110220 at $z \sim 0.8$ (estimated from the DM) we derive the limit

$$\gamma_{1.2 \text{ GHz}} - \gamma_{1.5 \text{ GHz}} < 4.5 \times 10^{-11} \quad (3\sigma)$$

$$< 2.8 \times 10^{-12} \quad (2\sigma). \quad (5)$$

for $\Delta\tau_{\text{gra}} < 1 \text{ s}$. Since the observed arrival times of photons depend on frequency, as expected from the propagation of radio waves in an ionized medium, we infer that deviations from the EEP make a sub-dominant contribution to the observed lag. This gives us confidence in the DM-based redshift estimate and in adopting the observed lag of 1 s as an upper limit on gravitational delays.

2. **FRB 150418**: Keane et al. (2016) associate this FRB with a subsequent fading radio source at the position of a galaxy at $z = 0.492 \pm 0.008$ (but see Williams & Berger 2016 for a different point of view). For this redshift, Tingay & Kaplan (2016) estimate $\Delta\tau_{\text{gra}}$ to be less than 5%–10% of the total time delay of 0.8 s. Following Tingay & Kaplan (2016), we take $z = 0.49$ and $\Delta\tau_{\text{gra}} < 0.04 \text{ s}$ to obtain the constraint

$$\gamma_{1.2 \text{ GHz}} - \gamma_{1.5 \text{ GHz}} < 2.4 \times 10^{-12} \quad (3\sigma)$$

$$< 1.4 \times 10^{-13} \quad (2\sigma). \quad (6)$$

compared to their hard constraint $\Delta\tau < 10^{-9}$.

3. **GRB 090510**: The firmest constraint obtained in Gao et al. (2015) is for GRB 090510 at $z = 0.903 \pm 0.003$ (Rau et al. 2009) and a time delay of 0.83 s between GeV and MeV photons. For this object we obtain the limit

$$\gamma_{\text{GeV}} - \gamma_{\text{MeV}} < 4 \times 10^{-11} \quad (3\sigma)$$

$$< 2.3 \times 10^{-12} \quad (2\sigma). \quad (7)$$

4. **GRB 080119B**: This GRB is at $z = 0.937$ (Vreeswijk et al. 2008), with an upper limit on the time delay of 5 s between eV and MeV photons. The corresponding limit
that we derive here is
\[
\gamma_{eV} - \gamma_{\text{MeV}} < 2.3 \times 10^{-10} \quad (3\sigma)
\]
\[
< 1.3 \times 10^{-11} \quad (2\sigma).
\]

These numbers are smaller by a factor of a few 100s than the corresponding constraints obtained in Gao et al. (2015). Future deep galaxy redshift surveys, e.g., Euclid (Laureijs et al. 2011), and peculiar velocity data, will allow an actual estimation of the gravitational potential along the line of sight to some relevant transient events. This should yield robust constraints on the accuracy of the EEP from events with cosmological origins. Obtaining the redshifts of FRBs is the focus of intense observational activity. Thus, measured redshifts, especially of repeating events, are expected be available in the very near future. This would be very rewarding since FRBs offer important constraints on several aspects of deviations from standard physics, such as the photon mass (Bonetti et al. 2016; Wu et al. 2016), in addition to the EEP.

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