Doing spin physics with unpolarized particles

Igor P. Ivanov
CFTP, Instituto Superior Tecnico, Universidade de Lisboa, Lisbon 1049-001, Portugal

Nikolai Korchagin
Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

Alexandr Pimikov
Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China and Research Institute of Physics, Southern Federal University, Rostov-na-Donu 344090, Russia

Pengming Zhang
School of Physics and Astronomy, Sun Yat-sen University, Zhuhai 519082, China

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Twisted, or vortex, particles refer to freely propagating non-plane-wave states with helicoidal wave fronts. In this state, the particle possesses a non-zero orbital angular momentum with respect to its average propagation direction. Twisted photons and electrons have been experimentally demonstrated, and creation of other particles in twisted states can be anticipated. If brought in collisions, twisted states offer a new degree of freedom to particle physics, and it is timely to analyze what new insights may follow. Here, we theoretically investigate resonance production in twisted photon collisions and twisted $e^+e^-$ annihilation and show that these processes emerge as a completely novel probe of spin and parity-sensitive observables in fully inclusive cross sections with unpolarized initial particles. This is possible because the initial state with a non-zero angular momentum explicitly breaks the left-right symmetry even when averaging over helicities. In particular, we show how one can produce almost 100% polarized vector mesons in unpolarized twisted $e^+e^-$ annihilation and how to control its polarization state.

Introduction. Spin-parity properties of hadrons are a fascinating chapter in modern particle phenomenology. The rich hadron spectrum exhibits a variety of spin-parity quantum numbers possible in the $q\bar{q}$ and $qqq$ quark combinations and in multiquark states [1]. Deep inelastic scattering (DIS) with polarized lepton or proton allows one to investigate how spin of the ultrarelativistic proton emerges from spins and orbital angular momenta of its constituents [2,3]. Reconstructing the proton spin structure in 3D brings in new spin-sensitive variables, which can be encoded via transverse momentum distributions and explored experimentally in semi-inclusive DIS with transversely polarized protons [4].

Spin-parity structure of hadrons is explored in experiment via two approaches: either one collides polarized initial particles and investigates the response of the cross section to flipping the polarization sign or direction, or one studies exclusive or semi-inclusive reactions and reconstructs the spin properties of the target hadron or intermediate resonances from the final state angular distributions. There seems to be no other way to access spin-dependent observables.

Here, we propose a novel tool for spin physics in particle collisions, complementary to all existing approaches. We demonstrate that parity- and spin-sensitive observables can be accessed in fully inclusive processes with unpolarized initial particles — provided they are prepared in the so-called twisted states. This initial state explicitly breaks the left-right symmetry even when averaging over the initial particle helicities. We calculate resonance production in unpolarized twisted $\gamma\gamma$ collision or in twisted $e^+e^-$ annihilation and demonstrate that such processes offer unprecedented control over polarization of the produced resonance.

Twisted photons and electrons. A twisted particle, be it a photon, an electron, or a hadron, is a wave packet with helicoidal wave fronts. It propagates, as a whole, in a certain direction and carries a non-zero angular momentum (OAM) projection with respect to that direction, which can be adjusted experimentally. Twisted photons are known since long ago [5-8]; following suggestions of [9], twisted electrons [10,11] were also recently created [12,13]. Relying on ideas of how to bring them into the GeV energy range [15,16] and steer them in accelerators [17], and on future experimental progress, one can imagine collider-type experiments with GeV-range twisted photons or twisted $e^+e^-$ annihilation.

Description of twisted particles adapted to calculation of their high energy collisions was presented in [15,16] and further developed in [18,20], see also recent reviews [8,10] and the Supplementary information. For each sort of fields (photons, electrons, etc), one begins with
Bessel twisted states $|E, \kappa, m\rangle$, which represent solutions of the free wave equation with definite energy $E$, longitudinal momentum $k_z$, modulus of the transverse momentum $|k_\perp| = \kappa$, the $z$-projection of the total angular momentum (AM) $m$, and definite helicity. (We use natural units $\hbar = c = 1$ and denote three-dimensional vectors by bold symbols, labeling the transverse momenta with $\perp$.) A Bessel twisted photon is defined, in the Coulomb gauge, as

$$A_{\kappa m k, \lambda}(r) = e^{ik_z z} \int a_{\kappa m}(k_\perp) e^{ik_\perp \cdot r} \frac{d^2k_\perp}{(2\pi)^2},$$

where the Fourier amplitude $a_{\kappa m}(k_\perp)$ is given by

$$a_{\kappa m}(k_\perp) = i^{-m} e^{im\kappa} \sqrt{2\pi} \frac{\delta(|k_\perp| - \kappa)}{\kappa}.$$

It is a superposition of plane wave (PW) photons with equal $E$, $k_z$, $|k_\perp| = k \sin \theta$, and helicity $\lambda = \pm 1$, but arriving from different azimuthal angles $\varphi_k$. Each PW component of a twisted photon contains its polarization vector $e_k$, orthogonal to its momentum: $e_k \cdot k = 0$. Notice that a twisted photon with given total AM $m$ and helicity $\lambda$ is not an eigenstate neither of the OAM $\zeta$-component operator $\hat{L}_z = -i\partial/\partial \varphi_k$ nor of the spin $\zeta$-component operator $\hat{\sigma}_z$. This is a manifestation of the spin-orbital interaction of light, which gives rise to a variety of remarkable optical phenomena \[21\]. Nevertheless, in the paraxial approximation $\kappa/k_z = |\tan \theta| \ll 1$, the spin-orbital coupling is suppressed and one can deal with approximately conserved $s_z \approx \lambda$ and $L_z \approx \ell \approx m - \lambda$.

Similarly, a Bessel twisted electron \[10, 20, 22\] as a monochromatic solution of the Dirac equation with definite $E$, $k_z$, $\kappa$, half-integer total AM $m$, and helicity $\lambda = \pm 1/2$:

$$\Psi_{\kappa m k, \zeta}(r) = e^{ik_z z} \int a_{\kappa m}(k_\perp) \frac{\psi_{\zeta}(k)}{\sqrt{2E}} e^{ik_\perp \cdot r} \frac{d^2k_\perp}{(2\pi)^2},$$

with the standard expressions for the plane wave bispinor $\psi_{\zeta}(k)$ and with the same Fourier amplitude $a_{\kappa m}(k_\perp)$ as in Eq. (2). Similar expression holds for the negative frequency solutions of the Dirac equation $\psi_{\zeta}(k)$ used to describe positrons. Again, spin and OAM $\zeta$-projections are not separately conserved in twisted electron due to the intrinsic spin-orbital interaction \[22, 23\], but in the paraxial approximation both $s_z \approx \zeta$ and $\ell = m - \zeta$ are approximately conserved in the focal spot.

A pure Bessel state $|E, \kappa, m\rangle$ is not normalizable in the transverse plane. We construct a realistic normalizable monochromatic twisted beam as a superposition of Bessel states with equal $E$, $m$, and helicity but with a distribution over $\kappa$,

$$|E, \kappa, \sigma, m\rangle = \int d\kappa f(\kappa)|E, \kappa, m\rangle,$$  

with a weight function $f(\kappa)$ peaked at $\kappa$ and having a width $\sigma$.

**Unpolarized twisted photons or electrons.** For PW initial particles, the polarization and coordinate degrees of freedom factorize, and the definition of the unpolarized cross section is straightforward. For twisted photons and electrons, where they are coupled, this notion requires clarification. When averaging cross section over initial helicities, should one keep their $m$ unchanged or not?

In fact, there is no unique definition of unpolarized twisted photon or electron beam, because it will eventually depend on the details of experimental realization. If an experimental device is capable of selecting electrons in a single $m$ state irrespective of $\zeta$, then one calculates the cross sections $\sigma_+$ with the twisted state $|m, \zeta = +1/2\rangle$ and $\sigma_-$ with the twisted state $|m, \zeta = -1/2\rangle$ and obtains the unpolarized cross section as $\sigma_0 = (\sigma_+ + \sigma_-)/2$. These $\sigma_+$ and $\sigma_-$ are not expected to be always equal. Indeed, although $|m, \zeta = +1/2\rangle$ is not an OAM eigenstate, it is dominated by the OAM component $\ell = m - 1/2$. Similarly, the state $|m, \zeta = -1/2\rangle$ is dominated by $\ell' = m + 1/2 = \ell + 1$. These states have different current and spin densities \[22\]. Therefore, they may lead to non-equal collision cross sections even if the fundamental interactions are $P$-invariant.

Another possible definition of an unpolarized twisted beam is to keep $\ell = m - \zeta$ fixed. However since a twisted beam is not an OAM eigenstate, its downstream evolution leads to spin-to-orbital conversion, which was experimentally verified for twisted light  \[25\]. In real experiments, the sensitivity of the cross section to the helicity flip will depend on experimental set-up and cannot be predicted unambiguously. However, the effect will be there for any setting. To elucidate its particle physics consequences, we use below the fixed-$m$ definition and demonstrate that twisted particle annihilation is a remarkably rich tool for spin physics.

**Resonance production in twisted particle collisions.** We apply the formalism developed in \[19\] to a generic $2 \rightarrow 1$ process of resonance production in annihilation of two counter-propagating twisted initial particles defined with respect to the same axis $z$ with energies $E_1$, $E_2$, moduli of the transverse momenta $s_1$, $s_2$, and AM $m_1$ and $m_2$, respectively. For simplicity, the two initial particles are assumed to be massless. The final particle with mass $M$ is described in terms of plane waves with momentum $K$ and energy $E_K$.

The S-matrix amplitude of twisted particle annihilation can be written as a superposition of PW S-amplitudes $S_{PW}$ with different transverse momenta of the initial particles:

$$S = \int \frac{d^2k_{1,\perp}}{(2\pi)^2} \frac{d^2k_{2,\perp}}{(2\pi)^2} a_{s_{1,\perp}, m_1}(k_{1,\perp}) a_{s_{2,\perp}, -m_2}(k_{2,\perp}) S_{PW}$$

$$= i(2\pi)^4 \delta(S_\Sigma) \delta(S_{\Sigma_0}) \frac{(-i)^{m_1 - m_2}}{\sqrt{8E_1 E_2 E_K}} \frac{1}{(2\pi)^6} \sqrt{s_1 s_2} \cdot J,$$  

(5)
where \( \delta(\Sigma E) \equiv \delta(E_1 + E_2 - E_K) \), \( \delta(\Sigma k_z) \equiv \delta(k_{z1} + k_{z2} - K_z) \). The twisted amplitude \( \mathcal{J} \) is defined as
\[
\mathcal{J} = \lambda \kappa \delta^{(2)}(k_{z1} + k_{z2} - K_z) \mathcal{M},
\]
where \( \mathcal{M} \) is the usual PW invariant amplitude. The integral (6) receives contributions from exactly two
\[
E \kappa \equiv |K_\perp| \text{ satisfy the triangle inequalities}
\]
and form a triangle with the area \( \Delta \). Out of the many PW components “stored” in the initial twisted particles, the integral \([8]\) receives contributions from exactly two plane wave combinations shown in Fig. 1 (a) \( \phi_1 = \varphi_K + \delta_1 \), \( \phi_2 = \varphi_K - \delta_2 \), (b) \( \phi_1 = \varphi_K + \delta_1 \), \( \phi_2 = \varphi_K - \delta_2 \), where \( \delta_1 \) and \( \delta_2 \) are the inner angles of the triangle. As a result, \( \mathcal{J} \) can be calculated exactly \([8]\):
\[
\mathcal{J} \propto \frac{\varphi_1 \varphi_2}{2\Delta} \left[ M_a e^{i(m_1 \delta_1 + m_2 \delta_2)} + M_b e^{-i(m_1 \delta_1 + m_2 \delta_2)} \right].
\]
One observes the hallmark feature of twisted particle collisions: interference between two PW amplitudes \( M_a \) and \( M_b \) calculated for the two distinct initial PW pairs shown in Fig. 1 but the same final momentum \( K_\perp \).

The cross section can be written as
\[
\frac{d\sigma}{dE} \propto |\mathcal{J}|^2 \delta(E_1 + E_2 - E_K) d^2K_\perp.
\]
The prefactor here is inessential since the new effects come not from the overall magnitude of the cross section but from its kinematic and helicity dependence. Integration with respect to \( K \) can be used to eliminate the energy delta-function in \([9]\). For fixed \( E_1 \), \( \varphi_1 \), and \( M \), the energy-momentum conservation fixes \( K_z = k_{z1} + k_{z2} \) and, therefore, \( K = \sqrt{E_k^2 - M^2 - K_z^2} \). Thus, the polar angle of the produced resonance is defined by
\[
\cos \theta_K = \frac{K_z}{\sqrt{(E_1 + E_2)^2 - M^2}}.
\]
The experiment can be repeated at different total energy \( E_K = E_1 + E_2 \), but, as long as \( E_K \) satisfies
\[
E_- \leq E_K \leq E_+, \quad E_\pm = \sqrt{(\varphi_1 \pm \varphi_2)^2 + K_z^2 + M^2},
\]
the resonance with mass \( M \) can be produced with a non-zero cross section. The polar angle \( \theta_K \) can also be adjusted by varying the ratio between \( E_1 \) and \( E_2 \).

Notice how different this situation is with respect to the PW collisions. In the PW case, the (narrow) resonance production cross section is \( \propto \delta(E_1 + E_2 - E_K) \), and the production occurs only at the resonance. In twisted particle annihilation, there is a finite range of energies to produce a resonance with mass \( M \). The cross section \( \sigma(E_K) \) varies with \( E_K \) in a periodic fashion, revealing the interference fringes induced by the varying coefficients in front of \( M_a \) and \( M_b \) in \([3]\). Moreover, a twisted annihilation experiment running at fixed energy can simultaneously produce two or more resonances with close but different masses, provided they satisfy \([11]\). According to \([10]\), these resonances will be emitted at different polar angles. Thus, twisted annihilation has a built-in mass spectrometric feature. These kinematic peculiarities will be explored in detail in the follow-up paper \([20]\).

**Twisted annihilation as parity analyzer.** To illustrate physics opportunities offered by twisted annihilation, consider production of a hypothetical spin-0 resonance in collision of two twisted photons \([27]\). It can be either a pure scalar \( S \), a pure pseudoscalar \( P \), or their mixture. For a pure scalar, whose effective interaction Lagrangian with photons is described by \( \mathcal{L}_S = g F_{\mu\nu} F^{\mu\nu} S \), one finds that the PW helicity amplitude is non-zero only for \( \lambda_1 = \lambda_2 = \lambda : M_S = -2g E_1 E_2 \delta_{\lambda_1 \lambda_2} e_{1 \lambda} e_{2 \lambda} \). For Bessel photons, one obtains the twisted amplitude \( \mathcal{J}_S \propto \mathcal{J}_1 + \mathcal{J}_2 \) with
\[
\mathcal{J}_1 = \cos(m_1 \delta_1 + m_2 \delta_2) \left[ \cos(\delta_1 + \delta_2) (1 - c_1 c_2) - s_1 s_2 \right], \\
\mathcal{J}_2 = \sin(m_1 \delta_1 + m_2 \delta_2) \sin(\delta_1 + \delta_2) (c_1 - c_2),
\]
where we used the short notation \( c_1 \equiv \cos \theta_1 \), \( s_1 \equiv \sin \theta_1 \). The cross section explicitly depends on the photon helicities: \( \sigma_\lambda = \sigma_0 + \lambda \sigma_s \), where \( \sigma_0 \propto J_1^2 + J_2^2 \) is the unpolarized cross section and \( \sigma_\lambda \propto 2 \mathcal{J}_1 \mathcal{J}_2 \) is the spin asymmetry.

This dependence on \( \lambda \) may look surprising since the fundamental interaction is parity-invariant. However, unlike in the PW case, here we explicitly break the left-right symmetry of the initial state. If the AM values \( m_i \) are fixed, then the helicity choices \( \lambda_1 = \pm 1 \) are not equivalent because the corresponding OAM contributions differ. The process would be invariant under the simultaneous sign flips \( m_i \rightarrow -m_i \) and \( \lambda_i \rightarrow -\lambda_i \); but not with respect to \( \lambda_i \rightarrow -\lambda_i \) alone.

For a pure pseudoscalar with the interaction Lagrangian \( \mathcal{L}_P = ig \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} P \), one gets \( M_P = \lambda M_S \). If the spin-0 particle does not possess definite parity, one can write its PW production amplitude as \( \mathcal{M} = a M_S + b M_P = (a + b\lambda) M_S \), where the (complex) coefficients \( a \) and \( b \) describe the scalar/pseudoscalar contributions to the amplitude. In the usual PW collision with circularly polarized photons, the cross section...
is \( \sigma_\lambda \propto |a|^2 + |b|^2 + 2\lambda \text{Re}(a^*b) \). Averaging it over the initial photon polarizations yields \( \sigma_0 \propto |a|^2 + |b|^2 \); it reveals the overall production intensity but cannot measure the amount of scalar-pseudoscalar mixing. Twisted photons offer access to this mixing even in the unpolarized cross section. Using the fixed-\( m \) convention for unpolarized twisted photon beams, we obtain the twisted amplitude

\[
\mathcal{J} = (aJ_1 + bJ_2) + \lambda (bJ_1 + aJ_2),
\]

with \( J_1, J_2 \) given in [12]. Averaging \(|\mathcal{J}|^2\) over the initial photon helicities, we get the unpolarized cross section

\[
\sigma_0 \propto (J_1^2 + J_2^2)(|a|^2 + |b|^2) + 4J_1J_2\text{Re}(a^*b).
\]

Now both the total intensity \(|a|^2 + |b|^2\) and the scalar-pseudoscalar mixing \( \text{Re}(a^*b) \) can be extracted experimentally from the energy dependence of \( \sigma_0(E_K) \) and, specifically, from the exact location and heights of the interference fringes. Since \( J_1 \) and \( J_2 \) have different dependence on \( \delta_1 \) and \( \delta_2 \), their combinations \( J_1J_2 \) and \( J_1^2 + J_2^2 \) produce different interference patterns and can be distinguished. A detailed numerical investigation of this effect with realistic twisted beams will be given elsewhere [28].

**Twisted annihilation as spin polarizer.** Our second example is production of a spin-1 resonance \( V \) of mass \( M \) in twisted \( e^+e^- \) annihilation. We take the PW helicity amplitude in the form

\[
\mathcal{M}_{\zeta_1\zeta_2\lambda_V} = g\bar{u}_\zeta_1(k_2)\gamma_\mu u_\zeta_2(k_1)V_{\lambda_V}^\mu(K),
\]

where \( \zeta_1, \zeta_2 \), and \( \lambda_V \) are the helicities of the electron, positron, and the produced vector resonance, respectively, and \( V_{\lambda_V}^\mu \) is the polarization vector of the spin-1 resonance. To illustrate the main effect, we evaluate this PW helicity amplitude in the paraxial limit \( \theta_1 \to 0 \), \( \theta_2 \to \pi \) but with generic \( \theta_K \), and observe that it is non-zero only for \( \zeta_1 = -\zeta_2 \equiv \zeta = \pm 1/2 \):

\[
\mathcal{M}_{\zeta,-\zeta\lambda_V} \propto e^{-i\zeta(\varphi_1 + \varphi_2 - 2\varphi_K)} (\lambda_V \cos \theta_K + 2\zeta).
\]

For twisted \( e^+e^- \) annihilation, we get

\[
\mathcal{J}_{\zeta,-\zeta\lambda_V} \propto (\lambda_V \cos \theta_K + 2\zeta) \cos[m_1\delta_1 + m_2\delta_2 - \zeta(\delta_1 - \delta_2)] ,
\]

where \( m_1, m_2 \) are half-integer. Using the fixed-\( m \) definition of the unpolarized electron and positron beams, we obtain the unpolarized twisted \( e^+e^- \) cross section:

\[
\sigma_{\lambda_V = \pm 1} \propto (1 + \cos^2 \theta_K) \\
\times [1 + \cos(2m_1\delta_1 + 2m_2\delta_2)\cos(\delta_1 - \delta_2)] + 2\lambda_V \cos \theta_K \sin(2m_1\delta_1 + 2m_2\delta_2)\sin(\delta_1 - \delta_2) .
\]

This result shows another remarkable phenomenon, which was impossible in the PW case. Even with unpolarized electrons and positrons, one produces vector meson with \( \lambda_V = +1 \) and \( \lambda_V = -1 \) with unequal intensities. The imbalance is enhanced at \( \theta_K \) far from \( \pi/2 \) and is sensitive to the exact position at the interference fringes. Thus, the produced vector meson is polarized on average and its polarization can be controlled.

To give a numerical example, we consider production of the \( J/\psi \) meson with \( M = 3.1 \) GeV and a finite width of \( \Gamma = 93 \) keV in unpolarized twisted \( e^+e^- \) annihilation with the following parameters:

\[
E_1 = 1.8 \text{ GeV}, \ E_2 = 1.338 \text{ GeV}, \ (m_1, m_2) = (5/2, 1/2), \ \bar{\zeta}_1 = 0.2 \text{ GeV}, \ \bar{\zeta}_2 = 0.1 \text{ GeV}, \ \sigma_1 = \bar{\zeta}_1/5 . \quad (19)
\]

Although the energies of both incoming particles are fixed, smearing over \( \zeta_i \) produces a distribution in \( \theta_K \). In Fig. 2 we show the resulting differential cross section \( d\sigma_{\lambda_V}/d\cos \theta_K \) computed beyond the paraxial approximation for all three polarization states \( \lambda_V = \pm 1, 0 \). With the parameter choice (19), the cross section is strongly dominated by the polarization state \( \lambda_V = +1 \), with a \( \approx 10\% \) admixture of the \( \lambda_V = 0 \) state and even smaller contribution from \( \lambda_V = -1 \).

**Conclusions.** In summary, we proposed a completely novel, complementary tool for doing spin physics in particle collisions. We demonstrated that by preparing initial particles in twisted states and adjusting their angular momenta and kinematics, one can access parity- and spin-dependent observables even in unpolarized inclusive cross section. Fundamentally, it is possible because the initial twisted states explicitly break the left-right symmetry, which leads to non-vanishing spin effects even for unpolarized cross section. We gave two illustrations of this remarkable effect: accessing scalar-pseudoscalar mixing of a spin-0 particle produced in unpolarized twisted photon collisions, and production of polarized vector mesons in unpolarized \( e^+e^- \) annihilation. None of these effects is possible with the usual plane-wave collisions.

Experimental exploration of these phenomena requires significant development of accelerator instrumentation to make high-energy physics with twisted particles possi-
ble. We believe that the novel opportunities in hadronic physics offered by twisted particles present a compelling scientific case to justify this dedicated work.

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† igor.ivanov@tecnico.ulisboa.pt
§ korchagin@impcas.ac.cn
‡ pimikov@mail.ru
† korchagin@impcas.ac.cn

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SUPPLEMENTARY INFORMATION

Description of twisted photons and electrons

We describe twisted photons with the formalism developed in [8, 15, 16]. For definiteness, we work in the Coulomb gauge. A monochromatic plane-wave electromagnetic field with helicity \( \lambda = \pm 1 \) is described by

\[
A_{k\lambda}(r) = e_{k\lambda} e^{i k r}.
\]

The polarization vector is orthogonal to the wave vector: \( e_{k\lambda} k = 0 \). To construct a Bessel twisted photon, we fix a reference frame, select an axis \( z \), and write it as a superposition of PW photons with fixed longitudinal momentum \( k_z = |k| \cos \theta \), fixed modulus of the transverse momentum \( \kappa = |k_{\perp}| = k \sin \theta \), but arriving from different azimuthal angles \( \varphi_k \). A twisted photon with a definite \( z \)-projection of the total AM \( m \) and definite helicity \( \lambda = \pm 1 \) is written in Eqs. (1) and (2). The usual dispersion relation holds for every plane wave component: \( k_z^2 + \kappa^2 = E^2 \).

The Fourier amplitude [2] is an eigenstate not only of the \( z \)-component of the total AM operator \( \hat{J}_z \) but also the OAM operator \( \hat{L}_z = -i \partial / \partial \varphi_k \). For a scalar twisted particle, it would imply that such a state possesses a well-defined OAM. However, this property is not shared by \( e_{k\lambda} \), which is an eigenstate of \( \hat{J}_z \) with a zero eigenvalue but not of \( \hat{L}_z \) or \( \hat{s}_z \) separately. This is a manifestation of the spin-orbital interaction of light, which gives rise to a variety of remarkable optical phenomena [21]. Nevertheless, in the paraxial approximation \( \kappa / |k_z| = \tan \theta \ll 1 \), the spin-orbital coupling is suppressed and one can deal with approximately conserved \( s_z \approx \lambda \) and \( \ell = m - \lambda \).

Each PW component of a twisted photon contains its polarization vector \( e_{k\lambda} \), which is orthogonal to the momentum of that particular PW component: \( e_{k\lambda} k = 0 \). To describe it in the chosen coordinate frame, we first define the eigenvectors \( \chi_\sigma, \sigma = \pm 1 \), of the helicity operator \( \hat{s}_z \) defined with respect to the fixed axis \( z \): \( \hat{s}_z \chi_\sigma = \sigma \chi_\sigma \).

Their explicit form is

\[
\chi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \chi_{\pm1} = \mp \sqrt{1 \over 2} \begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}, \quad \chi_\sigma^{\dagger} \chi_{\sigma'} = \delta_{\sigma \sigma'}.
\]

(21)

The polarization vector can be expanded in the basis of \( \chi_\sigma \):

\[
e_{k\lambda} = \sum_{\sigma = 0, \pm 1} e^{-i \sigma \varphi_k} d^1_{\sigma \lambda}(\theta) \chi_\sigma.
\]

(22)

The explicit expressions for Wigner’s \( d \)-functions [29] are:

\[
d^1_{\sigma \lambda} = \begin{pmatrix} \cos^2 \theta / 2 & -1 \sqrt{2} \sin \theta & \sin^2 \theta / 2 \\ -1 \sqrt{2} \sin \theta & \cos \theta & -1 \sqrt{2} \sin \theta \\ \sin^2 \theta / 2 & 1 \sqrt{2} \sin \theta & \cos^2 \theta / 2 \end{pmatrix}
\]

(23)

The first, second, and third rows and columns of this matrix correspond to the indices \( +1, 0, -1 \). Performing the summation in Eq. (22), one gets explicit expressions for the polarization vectors:

\[
e_{k\lambda} = \lambda \sqrt{m \over 2} \begin{pmatrix} -\cos \theta \cos \varphi_k + i \lambda \sin \varphi_k \\ -\cos \theta \sin \varphi_k - i \lambda \cos \varphi_k \\ \sin \theta \end{pmatrix}, \quad \lambda = \pm 1.
\]

(24)

Finally, when describing a counter-propagating twisted photon defined in the same reference frame with respect to the same axis \( z \), one can use the above expressions assuming that \( k_z < 0 \) and replacing \( m \rightarrow -m \) in the Fourier amplitude [2]. The expression for the polarization vector (24) stays unchanged, but \( \cos \theta < 0 \).

Twisted states have been experimentally demonstrated not only for photons but also for electrons [12,14]. To describe them in a fully relativistic manner, we use the definitions of [10, 30]. Other works, such as [20, 22], use slightly different conventions. The PW electron with energy \( E \), momentum \( k \), and helicity \( \zeta = \pm 1/2 \) is described by \( e^{i k r} u_{\zeta}(k) / \sqrt{2E} \) with the bispinor

\[
u_{\zeta}(k) = \begin{pmatrix} \sqrt{E + m_e} w^{(\zeta)} \\ 2 \zeta \sqrt{E - m_e} w^{(\zeta)} \end{pmatrix},
\]

(25)

where

\[
w^{(+1/2)} = \begin{pmatrix} \cos \theta / 2 e^{-i \varphi_k} \\ \sin \theta / 2 e^{i \varphi_k} \end{pmatrix}, \quad w^{(-1/2)} = \begin{pmatrix} -\sin \theta / 2 e^{-i \varphi_k} \\ \cos \theta / 2 e^{i \varphi_k} \end{pmatrix}.
\]

(26)

The bispinors are normalized as \( \bar{u}_{\zeta}(k) u^{\dagger}_{\zeta'}(k) = 2m_e \delta_{\zeta, \zeta'} \). The negative-frequency solutions of the Dirac equation are constructed as

\[
u_{\zeta}(k) = \begin{pmatrix} -\sqrt{E - m_e} w^{(-\zeta)} \\ 2 \zeta \sqrt{E + m_e} w^{(-\zeta)} \end{pmatrix},
\]

(27)

with the same spinors \( w \) as in (26). We use this basis of plane-wave solutions of the Dirac equation to construct the Bessel twisted state of the electron in Eq. (3). The similar expression holds for the negative-frequency solutions. For the sake of simplicity, we performed calculations of the twisted \( e^+e^- \) annihilation in the massless limit \( m_e \rightarrow 0 \). Restoring the finite mass of the electron does not change the results in any noticeable way.

Calculating twisted particle annihilation

In the PW case, the \( S \)-matrix amplitude of the \( 2 \rightarrow 1 \) process has the form

\[
S_{PW} = i(2 \pi)^4 \delta^{(4)}(k_1 + k_2 - K) M(k_1, k_2; K) \sqrt{8E_1 E_2 E_K}.
\]

(28)

Here, \( M(k_1, k_2; K) \) is the plane-wave invariant amplitude calculated according to the standard Feynman rules.
Squaring this amplitude, regularizing the squares of delta-functions, and diving by flux, as described, for instance, in [31], one gets the cross section

$$
\sigma = \frac{\pi \delta(\Sigma E)}{4E_1E_2E_Kv} |\mathcal{M}|^2 \delta(3)(k_1 + k_2 - K) \, d^3K,
$$

where \( \delta(\Sigma E) \equiv \delta(E_1 + E_2 - E_K) \). As should be expected in the PW case, the final momentum is fixed at \( K = k_1 + k_2 \) and the dependence on the total energy of the colliding particles is proportional to \( \delta(E_1 + E_2 - E_K) \). The production process occurs only when the initial particles are directly “at the resonance”.

Let us now consider collision of two Bessel states or arbitrary particles \( |\chi_1, m_1\rangle \) and \( |\chi_2, m_2\rangle \) which are defined in the same reference frame, and with respect to the same axis \( z \). The final particle with mass \( M \) is still described in the basis of plane waves, and its momentum \( K \) and energy \( E_K \) satisfy \( E_K^2 = M^2 + K^2 \). We follow the procedure of [13] [18] [32], which adapts the general theory of scattering of non-monochromatic, arbitrarily shaped, partially coherent beams developed in [33] to the collisions of Bessel twisted states. The S-matrix element of this process is

$$
S = \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{d^2k_{2\perp}}{(2\pi)^2} a_{\chi_1, m_1}(k_{1\perp})a_{\chi_2, -m_2}(k_{2\perp}) S_{PW}.
$$

The negative sign in front of \( m_2 \) reflects the fact that the second particle propagates on average in the \(-z\) direction. Substituting in [30] the Fourier amplitudes of the Bessel states, we get

$$
S = i(2\pi)^{-1} \frac{\delta(\Sigma E)\delta(\Sigma k_\perp)}{\sqrt{8E_1E_2E_K}} \frac{(-i)^{m_1-m_2}}{(2\pi)^4\sqrt{\chi_1\chi_2}} \cdot J,
$$

where \( \delta(\Sigma k_\perp) \equiv \delta(k_1z + k_2z - K_z) \). The twisted amplitude \( J \) is defined in [6]. Since it contains the equal number of integrations and delta-functions, it can be calculated exactly [18]. It is non-zero only if the moduli of the transverse momenta \( \chi_\perp = |k_{1\perp}| \) and \( K_{\perp} = |K_{\perp}| \) satisfy the triangle inequalities [7] and form a triangle with the area

$$
\Delta = \frac{1}{4} \sqrt{2K^2\chi_1^2 + 2K^2\chi_2^2 + 2\chi_1^2\chi_2^2 - K^4 - \chi_1^2 - \chi_2^2}.
$$

Out of many plane wave components “stored” in the initial twisted particles, the integral [6] receives contributions from exactly two plane wave combinations shown in Fig. 1 with the following azimuthal angles:

configuration a: \( \varphi_1 = \varphi_K + \delta_1 \), \( \varphi_2 = \varphi_K - \delta_2 \),
configuration b: \( \varphi_1 = \varphi_K - \delta_1 \), \( \varphi_2 = \varphi_K + \delta_2 \).

The inner angles of the triangle \( \delta_1 \), \( \delta_2 \) are determined by

$$
\cos \delta_1 = \frac{\chi_1^2 + K^2 - \chi_2^2}{2\chi_1 K}, \quad \cos \delta_2 = \frac{\chi_2^2 + K^2 - \chi_1^2}{2\chi_2 K}.
$$

As a result, \( J \) can be calculated exactly [18]:

$$
J = e^{i(m_1-m_2)\varphi_K} \frac{\chi_1\chi_2}{2\Delta} \left[ M_a e^{i(m_1\delta_1 + m_2\delta_2)} + M_b e^{-i(m_1\delta_1 + m_2\delta_2)} \right].
$$

This expression displays the hallmark feature of the twisted particle collision processes: the interference between two PW amplitudes \( M_a \) and \( M_b \) calculated for the two distinct initial PW pairs shown in Fig. 1, but the same final momentum \( K_{\perp} \). They represent two distinct paths, in momentum space, to arrive at the same final state from the initial twisted states [32].

When presenting numerical results, we use not the pure Bessel states, which are not normalizable in the transverse plane, but the monochromatic \( \kappa \)-smeared wave packets given in Eq. [4]. Instead of repeating the derivation of the cross section, one just applies this smearing procedure with the functions \( f_1(\chi_1) \) and \( f_2(\chi_2) \) to S-matrix amplitude [31]. Therefore, \( \mathcal{J} = \delta(k_{1z} + k_{2z} - K_z) \) for the pure Bessel states turns now into

$$
\langle J \rangle = \int_{0}^{E_1} dx_1 \int_{0}^{E_2} dx_2 f_1(x_1) f_2(x_2) \delta(k_{1z} + k_{2z} - K_z) \frac{J}{\sqrt{x_1 x_2}}.
$$

In numerical calculations, we use the Gaussian smearing functions of the following form:

$$
f_i(x_i) = n_i \sqrt{\pi} \exp\left[-\frac{(x_i - \bar{x}_i)^2}{2\sigma_i^2}\right].
$$

The normalization condition \( \int_{0}^{E_i} dx_i |f_i(x_i)|^2 = 1 \) fixes the normalization constants \( n_i \). The differential cross section [6] turns into

$$
d\sigma \propto |\langle J \rangle|^2 \delta(E_1 + E_2 - E_K) \, d^3K.
$$

Removing the energy delta-function, one obtains a non-trivial angular distribution over a finite range of polar angles:

$$
d\sigma \propto E_K^2 \beta_K |\langle J \rangle|^2 \, d\Omega_K.
$$