Effect of Measurement Errors on the Multivariate CUSUM CoDa Control Chart for the Manufacturing Process

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Abstract

Control charts, one of the main tools in Statistical Process Control (SPC), have been widely adopted in manufacturing sectors as an effective strategy for malfunction detection throughout the previous decades. Measurement errors (M.E’s) are involved in the quality characteristic of interest. The authors explored the impact of a linear covariate error model on the multivariate cumulative sum (CUSUM) control charts for a specific kind of data known as compositional data (CoDa). The average run length (ARL) is used to assess the performance of the proposed chart. The results indicate that M.E’s significantly affects the multivariate CUSUM-CoDa control charts. The authors have used the Markov chain method to study the impact of different involved parameters using four different cases for the variance-covariance matrix (i.e. uncorrelated with equal variances, negatively correlated with equal variances, uncorrelated with unequal variances, positively correlated with unequal variances). The authors concluded that the ARL of the multivariate CUSUM-CoDa chart increase with an increase in the value of error variance-covariance matrix, while the ARL decreases with an increase in the subgroup size $m$ or the constant powering $b$. For the implementation of the proposal, two illustrated examples have been reported for multivariate CUSUM-CoDa control charts in the presence of M.E’s. One deals with the manufacturing process of uncoated aspirin tablets, and the other is based on monitoring machines in the muesli manufacturing process.

Keywords: MCUSUM Chart; CoDa; Measurement Error; Manufacturing process; Performance;

1 Introduction

To understand and monitor a process, statistical process control (SPC) is a method for regularly collecting and analysing data on quality characteristics and taking appropriate action whenever the actual quality differs from the specifications or standard. SPC is a decision-making tool used in almost all industrial manufacturing processes to achieve process stability and sustainable product quality enhancement. Walter A. Shewhart discovered a method to differentiate between common and special variability in a process during the 1920s. The goal of a control chart is to find assignable causes of shifts to minimize the production of a large number of defective items. In practice, determining which control chart is appropriate for specific data can be difficult. It’s possible to figure out by looking at the distribution of the underlying data process. Wang et al. (2021) suggested a control system that provides strategies for controlling and monitoring the welding width during the manufacturing of wire arc accurately and adaptively, and control of layer width in wire-arc additive manufacturing by model predictive control is also studied by Xia et al. (2020). Research work examined by Patil et al. (2021) focused on developing a deposition geometry extraction method that can automatically determine the parameters for components manufacturing by an additive manufacturing process.

Compositional data (CoDa) vectors are positive components presented as percentages, ratios, proportions, or parts of a whole. CoDa has applications in various fields, including chemical research surveys, engineering sciences, and econometric data analysis. Some of the most recent articles dealing with statistical methods and CoDa processing are discussed here. Morais et al. (2018) examined an automobile market application in which the authors model brand share of the market as a function of media investments while taking account of the brand’s cost. At the turn
of the millennium, Egozcue and Pawlowsky-Glahn (2019) examined the sample space’s algebraic-geometric structure to adhere to the principles of sample space and the structure of CoDa. J. Aitchison’s principles and statistical tools from the 1980s Aitchison (1982) have been used to solve many practical problems, and for more information about CoDa, readers are referred to Aitchison (2011) and Pawlowsky-Glahn et al. (2015). Because the CoDa variable aggregates are limited to constant values, they can’t be used the same way as standard multivariate data studied by Carreras-Simó and Coenders (2020). Recently, Zaidi et al. (2019) investigated the impact of M.E’s on Hotelling $T^2$ control chart Average run length (ARL) performance. After that, Zaidi et al. (2020) examine the impact of M.E’s on multivariate exponentially moving average (MEWMA) $CoDa$ control chart’s ARL performance.

Hawkins and Olwell (1998) showed the CUSUM charts to be well-known for detecting smaller and more frequent changes than Shewhart charts and found them among the most common and widely used in practice. It is essential to optimize several multiple objective functions at the same time when performing multi-objective optimization, and the multivariate control chart method is unaffected by small and moderate variable shifts discussed by Hotelling (1947). Such multivariate charts are commonly used to detect changes in process parameters around their corresponding axes, which must be illustrated in a specific direction. MCUSUM charts for detecting shifts in means and covariance matrix have been discussed by Healy (1987). Hawkins and Olwell (1998) examine the optimal ARL performance technique’s anticipated transition is provided to identify the shift if the mechanism has changed unexpectedly or without warning. Genetic algorithms optimize the parameters for a new control chart to match the desired in-control (IC) ARL and decrease the out-of-control (OOC) ARL for a given mean shift while also optimizing the gauge dimensions proposed by Hu et al. (2016).

Control charts are often created with the hypothesis that the factor of the quality feature is assessed accurately. However, M.E’s can occur due to various circumstances. Imprecise measurement instruments or human errors are two causes that contribute to M.E’s. The presence of M.E’s must be taken into account while implementing control charts because it has an impact on their performance studied by Nguyen et al. (2021). Bellotti et al. (2020) proposed cumulative sum (CUSUM) control charts for the drilling process to monitor the variations of discharge pulses for sensing the actual condition of the whole drilling process. For a finite horizon production process, Tran et al. (2021) suggest one-sided Shewhart-type charts to handle the ratio of two normal random variables. For the $X$ and $S^2$ charts, Liina and Woodall (2001) presented a linear covariate error model $X = A + BY + e$, where $e$ is a random error (RE) due to measurement imprecision. Many researchers examined the impact of M.E’s on control charts. Bennett (1954) proposed the model $X = Y + e$ to investigate the effects of M.E’s on the Shewhart $X$ chart, where $Y$ represents the actual value of the quality characteristic, $X$ represents the analyzed value provided by the measurement equipment. Typically, practitioners monitor processes without considering the possibility of M.E’s. Despite sophisticated measurement techniques and methodologies, M.E’s is commonly caused by operational and environmental factors. Like Montgomery et al. (1994) outlined, Gage capacity tests commonly quantify M.E’s variability. Maleki et al. (2017) studied the impacts of M.E’s on the functionality of a multivariate control chart for the combined monitoring of a normal process’s mean vector and variance-covariance matrix. Maravelakis et al. (2004) conducted a thorough examination of the impact of M.E’s in SPC. Hu et al. (2016) examined the performance testing of univariate adaptive control charts in the existence of M.E’s. They investigated the effect of M.E’s on the Variable Sampling Intervals (VSI) and Variable Sample Sizes (VSS) $X$ control charts, respectively. The findings in both studies revealed that those univariate charts’ performance is negatively influenced when M.E’s is taken into account, examined by Sabalino et al. (2019). The effect of M.E’s on the performance of the Variable Sample Sizes and Sampling Intervals VSSI $X$ control chart was explored by Sabalino and Amiri (2017). Maravelakis et al. (2004) conducted a thorough examination of the impact of M.E’s in SPC. Although many studies on the assessment of control charts in the presence of M.E’s have been conducted, few have been focused on multivariate data, and few researchers have contributed to CoDa so, the purpose of this paper is to close this gap and, as a result, to investigate the impact of M.E’s on the MCUSUM – CoDa control chart.

The remainder of this paper is organized as follows: In Section 1, the modelisation and transformation of CoDa are briefly explained. Then, in Section 2, the linearly covariate M.E’s model for CoDa is introduced. Section 3 details the MCUSUM – CoDa control chart in the presence of M.E’s, and Section 4 investigates the performance of this control chart. Finally, two very detailed illustrative examples will be provided in Section 5.2, and conclusions and future research directions will be presented in Section 6.
Compositional Data (CoDa)

Compositional data is defined as a \( p \)-part composition consisting of a row vector \( y = (y_1, \ldots, y_p) \) defined on the simplex space \( \mathcal{S}^p \). Where \( \mathcal{S}^p \) can be defined as

\[
\mathcal{S}^p = \left\{ y = (y_1, y_2, \ldots, y_p) | y_i > 0, i = 1, 2, \ldots, p \quad \& \quad \sum_{i=1}^{p} y_i = \kappa \right\}.
\]

Where \( \kappa \) is a constant and is always greater than zero. \( \kappa \) can take different values, such as \( \kappa = 100 \) if we deal with proportions and \( \kappa = 1 \) if the composition components are in probabilities or proportions. In this paper, all the compositional vectors are supposed to be row vectors. If two vectors carry the same relative information, they are compositionally equivalent. For example, \( y = (0.75, 0.1, 0.15) \) and \( z = (75, 10, 13) \) are not equal numerically but they convey same information. So, in this case, a closure function can be used that is defined as

\[
\mathcal{C}(y) = \left( \frac{\kappa y_1}{\sum_{i=1}^{p} y_i}, \frac{\kappa y_2}{\sum_{i=1}^{p} y_i}, \ldots, \frac{\kappa y_p}{\sum_{i=1}^{p} y_i} \right).
\]

Using the closure mentioned above function, we can say that \( \mathcal{C}(y) = \mathcal{C}(z) \).

Because of the constant sum, the standard Euclidean geometry used for real space (i.e., \( \mathbb{R}^p \)) cannot be used. For example, if we have two compositional vectors, \( y = (0.1, 0.65, 0.35) \in \mathcal{S}^p \) and \( z = (0.25, 0.5, 0.25) \in \mathcal{S}^p \) then their sum using Euclidean geometry will be \( y + z \neq (0.35, 1.15, 0.6) \notin \mathcal{S}^p \) and similarly if we multiple a compositional vector with a scalar such that \( 5 \times y \neq (0.5, 3.25, 0.0875) \notin \mathcal{S}^p \). So, the Euclidean geometry operators are not suitable in the case of CoDa. [Aitchison (2011)] proposed a specific geometry known as Aitchison’s geometry with new operators to overcome this problem. These operators are defined as

- the perturbation operator \( \oplus \) of \( y \in \mathcal{S}^p \) by \( z \in \mathcal{S}^p \) as (substitute of “+”)

\[
y \oplus z = \frac{y_1 z_1 y_2 z_2 \cdots y_p z_p}{y_1 + z_1, y_2 + z_2, \ldots, y_p + z_p},
\]

- the powering operator \( \odot \) of \( y \in \mathcal{S}^p \) by a constant \( c \in \mathbb{R} \) (substitute of multiplication with a scalar) is defined as

\[
c \odot y = \mathcal{C}(y_1^c, y_2^c, \ldots, y_p^c).
\]

CoDa can be dealt with in two ways; one way is to use the original data. But in that case, we have to deal with the constraint of a constant sum. The other option is to transform the data into real data using the pre-defined log-ratio transformations. One of them for compositional vector \( y \in \mathcal{S}^p \) is the centered log-ratio transformation that is defined as

\[
\text{clr}(y) = \left( \ln \frac{y_1}{\bar{y}_G}, \ln \frac{y_2}{\bar{y}_G}, \ldots, \ln \frac{y_p}{\bar{y}_G} \right),
\]

where \( \bar{y}_G \) is the component-wise geometric mean of \( y \), i.e.

\[
\bar{y}_G = \left( \prod_{i=1}^{p} y_i \right)^{\frac{1}{p}} = \exp \left( \frac{1}{p} \sum_{i=1}^{p} \ln y_i \right).
\]

Another log-ratio transformation for a compositional vector \( y \in \mathcal{S}^p \) is the isometric log-ratio transformation defined as

\[
\iilr(y) = y^* = \text{clr}(y) \mathbf{B}^T,
\]

where there are many possible options for \( \mathbf{B} \) [Pawlowsky-Glahn et al. (2015)]. Where \( \mathbf{B} \) is a matrix of size \( (p-1, p) \). The one used in this paper is given below:

\[
B_{i,j} = \begin{cases} \sqrt{\frac{1}{(p-j)(p-j+1)}} & j \leq p - i \\ -\sqrt{\frac{1}{p-j+1}} & j = p - i + 1 \\ 0 & j > p - i + 1 \end{cases}.
\]

Conversely, the inverse isometric log-ratio can be used to transform the ilr-coordinates \( y^* \) into the composition coordinates \( y \). The ilr transformation is defined as

\[
\iilr^{-1}(y^*) = y = \mathcal{C}(\exp(y^* \mathbf{B})).
\]

In this paper, the compositional coordinates \( y \in \mathcal{S}^p \) are denoted as vectors or matrices without a "*" sign. At the same time, the ilr transformed coordinates \( y^* \in \mathbb{R}^{p-1} \) are denoted as vectors or matrices with a "*".
2 Linearly Covariate Measurement Errors Model for CoDa

It is necessary to explain how to model and transform such data to continue working with CoDa before implementing any SPC techniques. As a result, the authors included a brief introduction to CoDa in the Appendix and a table of the major notations used in the paper to make the model understandable.

Assume that \( y_i = (y_{i,1}, \ldots, y_{i,p}) \in S^p \) are \( p \)-part compositions at time \( t = 1, 2, \ldots \) and \( y_1, y_2, \ldots \) are independent multivariate normal random compositions with mean \( \mu^* \) and variance \( \Sigma^* \), i.e. \( y_i \sim \text{MNOR}_{S^p}(\mu^*, \Sigma^*) \). Assume that each \( y_i \) cannot be directly observed but it can be nevertheless measured through observable quality characteristics \( x_{i,1}, \ldots, x_{i,m} \), where each \( x_{i,j} \), \( j = 1, \ldots, m \), satisfies the following linearly covariate M.E’s model:

\[
x_{i,j} = a \oplus (b \odot y_i) \oplus \varepsilon_{i,j},
\]

where \( a \in S^p \) is known as the perturbation constant, \( b \in \mathbb{R} \) is known as the powering constant and \( \varepsilon_{i,j} \) is a random error term (independent of \( y_i \)) that is assumed to follow a multivariate normal distribution \( \text{MNOR}_{S^p}(0, \Sigma^*_a) \), where \( \Sigma^*_a \) is the known M.E’s variance-covariance matrix. For "classical" univariate and multivariate data, Linna and Woodall (2001) introduced this type of M.E’s model. The researchers in both of these papers advocated averaging the multiple (\( m \)) measurements to minimise the impact of M.E’s and keep the M.E’s component’s variance to a minimum. In this paper, our study corresponds to a similar approach of Linna and Woodall (2001), and the authors define the composition mean \( \bar{x}_i \) at the time \( t = 1, 2, \ldots \) as (see Pawlowsky-Glahn et al. [2015], page 132):

\[
\bar{x}_i = \frac{1}{m} \odot (x_{i,1} \odot \cdots \odot x_{i,m})
\]

\[
= a \oplus (b \odot y_i) \oplus \left( \frac{1}{m} \odot (\varepsilon_{i,1} \odot \cdots \odot \varepsilon_{i,m}) \right).
\]

Let us define \( a^* = \text{ilr}(a) \). Using theorem 6.20 in Pawlowsky-Glahn et al. [2015], page 117, we have \( \bar{x}_i \sim \text{MNOR}_{S^p}(\mu^*_x, \Sigma^*_x) \) with

\[
\mu^*_x = a^* + b\mu^*,
\]

\[
\Sigma^*_x = \psi^2 \Sigma^* + \frac{1}{m} \Sigma^*_m.
\]

3 Multivariate CUSUM Control Chart for Compositional Data

Assume, we have \( y_i \sim \text{MNOR}_{S^p}(\mu_0^*, \Sigma^*) \) and \( y_t \sim \text{MNOR}_{S^p}(\mu_1^*, \Sigma^*) \), where the IC and OOC process mean is defined by \( \mu_0^* \) and \( \mu_1^* \), respectively. The \( \text{MCUSUM} - \text{CoDa} \) control chart corresponding to the linearly covariate M.E’s model monitors the following statistic

\[
C_t = \left[ (s_t^T \left( \frac{1}{m} s_{t-1} \right)^{-1} s_{t-1} ) \right]^{1/2}, t = 1, 2, \ldots
\]

where \( s_t \) is the vector in \( \mathbb{R}^{p-1} \) defined as

\[
s_t = 0 \quad \text{if} \quad Q_t \leq k
\]

\[
s_t = (s_{t-1} + \bar{x}_t^* - a^* - b\mu_0^*)/(1 - k/Q_k) \quad \text{if} \quad Q_t > k,
\]

with \( s_0 = 0 \) and \( k > 0 \). Let

\[
Q_t = \left[ (s_{t-1} + \bar{x}_t^* - a^* - b\mu_0^*)^T \left( \frac{1}{m} s_{t-1} \right)^{-1} (s_{t-1} + \bar{x}_t^* - a^* - b\mu_0^*) \right]^{1/2}.
\]

The \( \text{MCUSUM} - \text{CoDa} \) control chart displays a signal when \( C_t > h \), where \( h \) is the upper control limit(UCL). A pre-defined IC ARL_0 is used to select the suitable value of \( h \).

To assess the \( \text{MCUSUM} - \text{CoDa} \) control chart’s run-length efficiency, the authors implement a Markov chain approximation originally suggested by Crosier (1986). Lowry et al. [1992] proposed ARL of MCUSUM charts depends on the parameter \( \delta \) for non-centrality. Where \( \delta \) can be written as,

\[
\delta = (\mu_1 - \mu_0)(\Sigma)^{-1}(\mu_1 - \mu_0)^T.
\]
When M.E’s is included in the process, the non-centrality parameter is denoted by $\delta_M$ and is equal to

$$
\delta_M = b^2(\mu_1^* - \mu_0^*) \left( b^2 \Sigma^* + \frac{1}{m} \Sigma_M^* \right)^{-1} (\mu_1^* - \mu_0^*)' .
$$

\text{(13)}

Linna and Woodall (2001) examined the effectiveness of the shift detection of multivariate charts and found them to be more effective in detecting a shift in one direction and ineffective in detecting a shift in the other direction. For this reason, Linna and Woodall (2001) proposed using the minimum and maximum values of $\delta_M$, i.e. $\delta_B$ (minimum value of $\delta_M$) and $\delta_W$ (maximum value of $\delta_M$). Using the results in Zaidi et al. (2019), we get

$$
\delta_B = \delta \lambda_1, \\
\delta_W = \delta \lambda_{p-1},
$$

where $\lambda_1$ and $\lambda_{p-1}$ are the smallest and the largest eigenvalues of the $(p-1, p-1)$ matrix $b^2 \Sigma^*(b^2 \Sigma^* + \frac{1}{m} \Sigma_M^*)^{-1}$, respectively.

The ARL of the MCUSUM – CoDa chart can be obtained using the Markov chain technique suggested by Lowry et al. (1992).

The authors assumed the unit sample size $n = 1$, and in that case, we can only detect the mean shift in the MCUSUM – CoDa chart. But if we use $n > p$, then it is also possible to monitor variability using the statistic defined in Gnanadesikan and Gupta (1970). (see, for instance, Zaidi et al. (2020)).

4 Performance of the MCUSUM-CoDa Control Chart in the Presence of M.E’s

This section aims to investigate the performance of the MCUSUM – CoDa control chart in the presence of M.E’s. The M.E’s variance-covariance matrix equals $\Sigma^*_M = \sigma_M^2 I$, where $\sigma_M$ is the standard-deviation M.E’s (common for all dimensions), and $I$ is the (2,2) identity matrix. The optimal couple $(k, H)$ that minimizes the OOC ARL for a fixed value of the shift $\delta_M$ subject to a constrained value for the IC ARL is chosen first in the design of the MCUSUM – CoDa control chart. The procedure for determining the optimal couple consists of three steps:

1. Select the most appropriate value for the IC ARL0. In this case, we choose ARL0 = 370.
2. Select a particular value of k. In this case, following Montgomery (2007) we choose $k = 0.5$.
3. Obtain the value of H such that ARL = ARL0 when no shift occurs, i.e., when $\delta = \delta_M = 0$.
4. Select the optimal value of $H^*$ from all the $H$ values such that the OOC ARL value is minimum for a given value of the shift $\delta$ for $\mu^*$ (to attain the best statistical performance).

Similar to Zaidi et al. (2020), the following four cases for the CoDa variance-covariance matrix $\Sigma^*$ are considered,

Case #I uncorrelated with equal variances

$$
\Sigma^* = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix},
$$

Case #II negatively correlated with equal variances

$$
\Sigma^* = \begin{pmatrix} 3 & -1/2 \\ -1/2 & 3 \end{pmatrix},
$$

Case #III uncorrelated with unequal variances

$$
\Sigma^* = \begin{pmatrix} 1.5 & 0 \\ 0 & 3 \end{pmatrix},
$$

Case #IV positively correlated with unequal variances

$$
\Sigma^* = \begin{pmatrix} 1.5 & 1/2 \\ 1/2 & 3 \end{pmatrix}.
$$

We will now separately investigate the influence of parameters $\sigma_M$, $b$, $p$ and $m$ on the performance of the MCUSUM – CoDa control chart in the presence of M.E’s for the above-defined cases.
When we increase the values of $\sigma_M$, the ARL values are (ARL$_B = 9.71$, ARL$_W = 9.66$). But, when we increase the value of $\delta$, the OOC ARL also indicates an increase. For example, when we select $\sigma_M = 0.1$ and $\delta = 1.5$ the ARL values are (ARL$_B = 9.69$, ARL$_W = 9.69$). But, when $\sigma_M = 0.6$ (i.e. $\sigma_M$ increased), the ARL values are (ARL$_B = 7.51$, ARL$_W = 7.64$).

### Table 1: Case I.

| $\delta$ | $H$ | $P$ | $ARL_B$ | $ARL_W$ | $H_B$ | $ARL_B$ | $ARL_W$ | $H_P$ | $ARL_B$ | $ARL_W$ | $H_P$ | $ARL_B$ | $ARL_W$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 7.14 | 5 | 7.12 | 5.62 | 4.17 | 5.62 | 5.17 | 5.62 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 0.25 | 7.14 | 5 | 7.12 | 6.12 | 4.17 | 6.12 | 5.17 | 6.12 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 0.5 | 7.14 | 5 | 7.12 | 6.63 | 4.17 | 6.63 | 5.17 | 6.63 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 0.75 | 7.14 | 5 | 7.12 | 7.14 | 4.17 | 7.14 | 5.17 | 7.14 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 1 | 7.14 | 5 | 7.12 | 7.65 | 4.17 | 7.65 | 5.17 | 7.65 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 1.25 | 7.14 | 5 | 7.12 | 8.16 | 4.17 | 8.16 | 5.17 | 8.16 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 1.5 | 7.14 | 5 | 7.12 | 8.67 | 4.17 | 8.67 | 5.17 | 8.67 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 1.75 | 7.14 | 5 | 7.12 | 9.18 | 4.17 | 9.18 | 5.17 | 9.18 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |
| 2 | 7.14 | 5 | 7.12 | 9.69 | 4.17 | 9.69 | 5.17 | 9.69 | 7.14 | 9.71 | 9.71 | 9.71 | 9.71 |

4.1 Case I

This subsection investigates the influence of the involved parameters $\sigma_M$, b, and p corresponding to Case #1 (i.e. uncorrelated case with equal variances). For shifts $\delta \in [0, 2]$, different ARL values are given in Table 1 for the $MCUSUM - CoDa$ control chart with and in the presence of M.E’s. From Table 1 we can draw the following conclusions:

- When we increase the values of $\sigma_M$, the OOC ARL also indicates an increase. For example, when we select $\sigma_M = 0.1$ and $\delta = 1.5$ the ARL values are (ARL$_B = 6.97$, ARL$_W = 6.99$). But, when $\sigma_M = 0.6$ (i.e. $\sigma_M$ increased), the ARL values are (ARL$_B = 7.51$, ARL$_W = 7.64$).

- An increase in the value of $m$ imposes a slight decrease in the ARL values. For example, for the fixed values of $\sigma_M = 0.3$, b = 1 and p = 3, when $m = 1$ and $\delta = 1.5$ , (ARL$_B = 8.04$, ARL$_W = 7.84$). But, when we increase $m = 6$, (ARL$_B = 7.02$, ARL$_W = 7.05$).

- An increase in the value of $b$ imposes a significant decrease in the ARL values. For example, for the fixed values of $\sigma_M = 0.3$, $m = 3$ and p = 3, when $b = 1$ and $\delta = 1.5$ , (ARL$_B = 11$, ARL$_W = 13.88$). But, when we increase $m = 6$, (ARL$_B = 8.07$).

- An increase in the values of $p$ imposes a significant increase in the ARL. For example,for the fixed values of $\sigma_M = 0.3$, b = 1 and $m = 3$, when $p = 10$ and $\delta = 1.5$ the (ARL$_B = 7.18$, ARL$_W = 7.25$). But, when $p = 10$ (i.e. p increased), the (ARL$_B = ARL$_W = 9.36$).

6
4.2 Case II

This subsection investigates the influence of the involved parameters $\sigma_M$, $b$, and $m$ corresponding to Case #2 (i.e., negatively correlated case with equal variances). For shifts $\delta \in [0, 2]$, different ARL values are given in Table 2 for the MCUSUM – CoDa control chart without and in the presence of M.E.’s. From Table 2, we can draw the following conclusions:

- When we increase the values of $\sigma_M$, the OOC ARL also indicates an increase. For example, when we select $\sigma_M = 0.1$ and $\delta = 1.5$ the ARL values are ($ARL_B = 6.96, ARL_W = 7.01$). But, when $\sigma_M = 0.6$ (i.e. $\sigma_M$ increased), the ARL values are ($ARL_B = 7.45, ARL_W = 7.75$).
- An increase in the value of $m$ imposes a slight decrease in the ARL values. For example, for the fixed values of $\sigma_M = 0.3$, $b = 1$ and $p = 3$, when $m = 1$ and $\delta = 1.5$ , ($ARL_B = 8.21, ARL_W = 7.75$). But, when we increase $m = 6$, ($ARL_B = 7.08, ARL_W = 7.01$).
- An increase in the value of $b$ imposes a significant decrease in the ARL values. For example, for the fixed values of $\sigma_M = 0.3$, $m = 3$ and $p = 3$, when $b = 1$ and $\delta = 1.5$ , ($ARL_B = 15, ARL_W = 10.9$). But, when we increase $m = 6$, ($ARL_B = 6.87$).

4.3 Case III

This subsection investigates the influence of the involved parameters $\sigma_M$, $b$, and $m$ corresponding to Case #1 (i.e., uncorrelated case with unequal variances). For shifts $\delta \in [0, 2]$, different ARL values are given in Table 3 for the MCUSUM – CoDa control chart without and in the presence of M.E.’s. From Table 3, we can draw the following conclusions:

- When we increase the values of $\sigma_M$, the OOC ARL also indicates an increase. For example, when we select $\sigma_M = 0.1$ and $\delta = 1.5$ the ARL values are ($ARL_B = 7.07, ARL_W = 6.97$). But, when $\sigma_M = 0.6$ (i.e. $\sigma_M$ increased), the ARL values are ($ARL_B = 8.17, ARL_W = 7.5$).
- An increase in the value of $m$ imposes a slight decrease in the ARL values. For example, for the fixed values of $\sigma_M = 0.3$, $b = 1$ and $p = 3$, when $m = 1$ and $\delta = 1.5$ , ($ARL_B = 10.5, ARL_W = 11.85$). But, when we increase $m = 6$, ($ARL_B = 9.43, ARL_W = 9.64$).
Table 3: Case III.

| wot M.E's | $H$ | ARL | $D_{\sigma}$ | $H_{\sigma}$ | $W_{\sigma}$ | $L_{\sigma}$ | $D_{\sigma}$ | $H_{\sigma}$ | $W_{\sigma}$ | $L_{\sigma}$ | $D_{\sigma}$ | $H_{\sigma}$ | $W_{\sigma}$ | $L_{\sigma}$ | $D_{\sigma}$ | $H_{\sigma}$ | $W_{\sigma}$ | $L_{\sigma}$ |
|----------|-----|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.25     | 7.14 | 158.88 | 0.26 | 7.14 | 122.12 | 0.26 | 7.14 | 102.66 | 0.25 | 7.14 | 127.95 | 0.25 | 7.14 | 123.57 | 0.24 | 7.14 | 136.69 | 0.24 | 7.14 | 127.95 |
| 0.50     | 7.14 | 62.60 | 0.51 | 7.14 | 44.90 | 0.51 | 7.14 | 44.37 | 0.49 | 7.14 | 47.04 | 0.49 | 7.14 | 45.43 | 0.46 | 7.14 | 50.36 | 0.46 | 7.14 | 47.04 |
| 0.75     | 7.14 | 23.75 | 0.75 | 7.14 | 22.38 | 0.75 | 7.14 | 22.09 | 0.72 | 7.14 | 23.58 | 0.72 | 7.14 | 22.68 | 0.68 | 7.14 | 25.41 | 0.68 | 7.14 | 23.58 |
| 1.00     | 7.14 | 12.74 | 0.99 | 7.14 | 13.17 | 0.99 | 7.14 | 8.64 | 12.97 | 0.95 | 8.52 | 13.97 | 0.95 | 8.58 | 13.37 | 0.90 | 8.39 | 15.19 | 0.89 | 8.52 | 13.97 |
| 1.25     | 7.14 | 9.87 | 1.24 | 7.14 | 9.05 | 1.24 | 7.14 | 9.37 | 1.19 | 7.14 | 9.67 | 1.19 | 7.14 | 9.67 | 1.12 | 7.14 | 9.86 | 1.12 | 7.14 | 9.67 |
| 1.50     | 7.14 | 7.24 | 1.48 | 7.14 | 7.07 | 1.48 | 7.14 | 7.09 | 1.42 | 7.14 | 7.55 | 1.42 | 7.14 | 7.55 | 1.37 | 7.14 | 7.81 | 1.37 | 7.14 | 7.55 |
| 1.75     | 7.14 | 5.98 | 1.73 | 7.14 | 5.97 | 1.73 | 7.14 | 6.15 | 1.66 | 7.14 | 6.51 | 1.66 | 7.14 | 6.51 | 1.59 | 7.14 | 6.93 | 1.59 | 7.14 | 6.51 |
| 2.00     | 7.14 | 4.79 | 1.97 | 7.14 | 4.85 | 1.97 | 7.14 | 6.38 | 1.89 | 7.14 | 6.70 | 1.89 | 7.14 | 6.70 | 1.81 | 7.14 | 7.06 | 1.81 | 7.14 | 6.70 |

- An increase in the value of $b$ imposes a significant decrease in the ARL values. For example, for the fixed values of $\sigma = 0.3$, $m = 3$ and $p = 3$, when $b = 1$ and $\delta = 1.5$, (ARL$_B = 12.58$, ARL$_W = 11.13$). But, when we increase $m = 6$, (ARL$_B = ARL_W = 6.87$).

4.4 Case IV

This subsection investigates the influence of the involved parameters $\sigma$, $b$, and $m$ corresponding to Case #1 (i.e., positively correlated case with unequal variances). For shifts $\delta \in [0, 2]$, different ARL values are given in Table 4 for the $MCUSUM\rightarrow CoDa$ control chart without and in the presence of M.E's. From Table 4, we can draw the following conclusions:

- When we increase the values of $\sigma$, the OOC ARL also indicates an increase. For example, when we select $\sigma = 0.1$ and $\delta = 1.5$ the ARL values are (ARL$_B = 7.09$, ARL$_W = 6.96$). But, when $\sigma = 0.6$ (i.e. $\sigma$ increased), the ARL values are (ARL$_B = 8.32$, ARL$_W = 7.47$).
- An increase in the value of $m$ imposes a slight decrease in the ARL values. For example, for the fixed values of $\sigma = 0.3$, $b = 1$ and $p = 3$, when $m = 1$ and $\delta = 1.5$, (ARL$_B = 10.44$, ARL$_W = 12.16$). But, when we increase $m = 6$, (ARL$_B = 9.42$, ARL$_W = 9.69$).
- An increase in the value of $b$ imposes a significant decrease in the ARL values. For example, for the fixed values of $\sigma = 0.3$, $m = 3$ and $p = 3$, when $b = 1$ and $\delta = 1.5$, (ARL$_B = 12.27$, ARL$_W = 11.16$). But, when we increase $m = 6$, (ARL$_B = ARL_W = 6.87$).

5 Practical Implementation of the Proposal in the Manufacturing Scenario

For the proposal implementation, two illustrated examples have been incorporated in this study: one involving manufacturing uncoated aspirin tablets with the composition aspirin, micro-crystalline cellulose, talcum powder and magnesium stearate and the other is the monitoring of machines that are responsible for the percentages of three components (whole-grain cereal, dried fruits, and nuts) in museli production.
5.1 Proposal Implementation Related to the Aspirin Manufacturing Process

Pharmaceutical manufacturing is the process of synthesising pharmaceutical medications on a large scale as part of the pharmaceutical business. The medication production process may be divided into several unit activities, such as milling, granulation, coating, tablet pressing, and so on. Here the authors simulated an example of a company is producing uncoated aspirin tablets. The base parameters have been taken from Tiwari et al. (2018). According to Tiwari et al. (2018), the composition of the tablet consists of 150 mg of Aspirin, 60 mg of micro crystalline cellulose, 20 mg of talcum powder and magnesium stearate.

In the start, the company wants to check the measuring unit in charge of measuring raw material (i.e. estimate \( \hat{a} \), \( \hat{b} \) and \( \hat{\Sigma}_M \)). For this purpose, the firm cautiously prepared \( k = 4 \) samples of aspirin and measured them \( n = 10 \) times each. The simulation results are in Table 6 and Figure 6. Using the data in Table 6, one can easily obtain

\[
\hat{\mu}_{\bar{x}} = (1.376, 0.719)
\]

\[
\hat{\Sigma}_{\bar{x}} = \begin{pmatrix}
0.0009 & 0.0024 \\
0.0024 & 0.00219
\end{pmatrix}
\]

Using the above values, we can obtain \( \hat{\mu}_0 \) and \( \hat{\Sigma} \).

\[
\hat{\mu}_0 = (1.284, 0.646)
\]

\[
\hat{\Sigma} = \begin{pmatrix}
0.0005 & 0.0018 \\
0.0018 & 0.00164
\end{pmatrix}
\]
Table 5: Phase-0 data for the Asprin Manufacturing.

| i | j | y_i | y_i^* | x_{i,j} | x_{i,j}^* |
|---|---|-----|-------|---------|-----------|
| 1 | 1 | 0.33 | 0.33 | 0.33 | 0.33 |
| 2 | 0.37 | 0.32 | 0.32 | 0.06 | 0.11 |
| 3 | 0.32 | 0.33 | 0.35 | -0.06 | -0.04 |
| 4 | 0.34 | 0.33 | 0.33 | 0.01 | 0.01 |
| 5 | 0.37 | 0.30 | 0.34 | -0.02 | 0.15 |
| 6 | 0.37 | 0.30 | 0.33 | 0.02 | 0.14 |
| 7 | 0.34 | 0.32 | 0.34 | -0.01 | 0.04 |
| 8 | 0.38 | 0.28 | 0.34 | -0.03 | 0.23 |
| 9 | 0.32 | 0.34 | 0.34 | -0.01 | -0.05 |
| 10 | 0.34 | 0.32 | 0.33 | 0.00 | 0.04 |

2 1 0.25 0.25 0.50 -0.57 0.00 0.24 0.25 0.51 -0.61 -0.05
2 0.26 0.24 0.50 -0.58 0.05
3 0.23 0.26 0.52 -0.62 -0.08
4 0.24 0.25 0.51 -0.61 -0.03
5 0.26 0.25 0.50 -0.56 0.03
6 0.25 0.25 0.50 -0.57 0.01
7 0.23 0.26 0.51 -0.61 -0.09
8 0.24 0.23 0.53 -0.66 0.03
9 0.26 0.23 0.52 -0.62 0.08
10 0.24 0.26 0.50 -0.56 -0.04

3 1 0.25 0.50 0.25 0.28 -0.49 0.26 0.50 0.24 0.33 -0.46
2 0.27 0.49 0.24 0.33 -0.41
3 0.26 0.50 0.24 0.33 -0.47
4 0.21 0.56 0.23 0.34 -0.68
5 0.20 0.56 0.24 0.26 -0.75
6 0.26 0.48 0.26 0.26 -0.44
7 0.26 0.50 0.24 0.33 -0.47
8 0.27 0.50 0.24 0.36 -0.43
9 0.25 0.51 0.24 0.30 -0.52
10 0.28 0.47 0.25 0.30 -0.37

4 1 0.50 0.25 0.25 0.28 0.49 0.56 0.21 0.23 0.32 0.69
2 0.52 0.23 0.25 0.25 0.58
3 0.53 0.23 0.23 0.34 0.58
4 0.49 0.26 0.25 0.29 0.44
5 0.52 0.23 0.25 0.25 0.57
6 0.50 0.26 0.24 0.32 0.46
7 0.49 0.26 0.25 0.29 0.44
8 0.51 0.23 0.25 0.26 0.55
9 0.54 0.21 0.25 0.26 0.67
10 0.55 0.21 0.23 0.31 0.68

Figure 1: Data used for calibrating the measurement system: $y_i$ and $x_{i,j}$ in $S^p$ (left side), $y_i^*$ and $x_{i,j}^*$ in $\mathbb{R}^2$ (right side).
| i | j | $x_{i,j}$ | $R_i$ | $R_i^*$ | $C_i$ |
|---|---|---------|-------|-------|------|
| 1 | 1 | 0.72    | 0.711 | 0.369 | 0.841 |
| 2 | 0.73  | 0.216   | 0.620 | 0.841 | 1.486 |
| 3 | 0.68  | 0.073   | 1.486 | 0.073 | 1.486 |
| 2 | 1 | 0.69    | 0.674 | 0.711 | 0.369 |
| 2 | 0.67  | 0.253   | 0.674 | 0.692 | 1.246 |
| 3 | 0.67  | 0.073   | 1.246 | 0.073 | 1.246 |
| 3 | 1 | 0.68    | 0.687 | 0.711 | 0.369 |
| 2 | 0.71  | 0.237   | 0.711 | 0.753 | 1.581 |
| 3 | 0.7   | 0.073   | 1.581 | 0.073 | 1.581 |
| 4 | 1 | 0.68    | 0.677 | 0.711 | 0.369 |
| 2 | 0.7   | 0.250   | 0.711 | 0.705 | 1.547 |
| 3 | 0.65  | 0.073   | 1.547 | 0.073 | 1.547 |
| 5 | 1 | 0.75    | 0.708 | 0.711 | 0.369 |
| 2 | 0.71  | 0.216   | 0.711 | 0.841 | 2.338 |
| 3 | 0.66  | 0.077   | 2.338 | 0.077 | 2.338 |
| 6 | 1 | 0.7    | 0.714 | 0.711 | 0.369 |
| 2 | 0.73  | 0.216   | 0.711 | 0.844 | 2.405 |
| 3 | 0.71  | 0.073   | 2.405 | 0.073 | 2.405 |
| 7 | 1 | 0.71   | 0.711 | 0.711 | 0.369 |
| 2 | 0.74  | 0.219   | 0.711 | 0.833 | 3.244 |
| 3 | 0.68  | 0.073   | 3.244 | 0.073 | 3.244 |
| 8 | 1 | 0.66   | 0.660 | 0.711 | 0.369 |
| 2 | 0.64  | 0.260   | 0.711 | 0.660 | 2.216 |
| 3 | 0.68  | 0.080   | 2.216 | 0.080 | 2.216 |
| 9 | 1 | 0.52   | 0.523 | 0.711 | 0.369 |
| 2 | 0.53  | 0.387   | 0.711 | 0.614 | 3.566 |
| 3 | 0.52  | 0.080   | 3.566 | 0.080 | 3.566 |
| 10 | 1 | 0.69  | 0.687 | 0.711 | 0.369 |
| 2 | 0.69  | 0.240   | 0.711 | 0.743 | 2.652 |
| 3 | 0.68  | 0.073   | 2.652 | 0.073 | 2.652 |
| 11 | 1 | 0.65  | 0.660 | 0.711 | 0.369 |
| 2 | 0.68  | 0.263   | 0.711 | 0.650 | 3.163 |
| 3 | 0.65  | 0.077   | 3.163 | 0.077 | 3.163 |
| 12 | 1 | 0.64  | 0.640 | 0.711 | 0.369 |
| 2 | 0.64  | 0.280   | 0.711 | 0.585 | 4.174 |
| 3 | 0.64  | 0.080   | 4.174 | 0.080 | 4.174 |
| 13 | 1 | 0.72  | 0.710 | 0.711 | 0.369 |
| 2 | 0.72  | 0.220   | 0.710 | 0.830 | 3.218 |
| 3 | 0.69  | 0.077   | 3.218 | 0.077 | 3.218 |
| 14 | 1 | 0.64  | 0.674 | 0.711 | 0.369 |
| 2 | 0.67  | 0.253   | 0.711 | 0.694 | 4.834 |
| 3 | 0.71  | 0.073   | 4.834 | 0.073 | 4.834 |
| 15 | 1 | 0.64  | 0.660 | 0.711 | 0.369 |
| 2 | 0.66  | 0.260   | 0.711 | 0.660 | 3.411 |
| 3 | 0.68  | 0.080   | 3.411 | 0.080 | 3.411 |
| 16 | 1 | 0.74  | 0.724 | 0.711 | 0.369 |
| 2 | 0.7   | 0.206   | 0.724 | 0.888 | 5.566 |
| 3 | 0.73  | 0.073   | 5.566 | 0.073 | 5.566 |
| 17 | 1 | 0.64  | 0.664 | 0.711 | 0.369 |
| 2 | 0.67  | 0.256   | 0.711 | 0.673 | 4.842 |
| 3 | 0.68  | 0.080   | 4.842 | 0.080 | 4.842 |
| 18 | 1 | 0.72  | 0.730 | 0.711 | 0.369 |
| 2 | 0.73  | 0.200   | 0.730 | 0.916 | 5.573 |
| 3 | 0.74  | 0.073   | 5.573 | 0.073 | 5.573 |
| 19 | 1 | 0.65  | 0.650 | 0.711 | 0.369 |
| 2 | 0.65  | 0.270   | 0.711 | 0.621 | 3.397 |
| 3 | 0.65  | 0.080   | 3.397 | 0.080 | 3.397 |
| 20 | 1 | 0.7   | 0.688 | 0.711 | 0.369 |
| 2 | 0.64  | 0.239   | 0.711 | 0.748 | 0.004 |
| 3 | 0.72  | 0.073   | 0.004 | 0.073 | 0.004 |
Figure 2: Phase I data for the aspirin example: $\bar{x}_i$ (left side) and $\bar{x}^*_i$ (right side).

Figure 3: MCUSUM-CoDa chart for Asprin Phase-I data.
The results of \textit{MCUSUM – CoDa} charts are also presented in Table \ref{tab:MCUSUM-CoDa}. Figure \ref{fig:MCUSUM-CoDa} also shows the values of the \textit{MCUSUM – CoDa} chart along with UCL. In Phase-II of the production process, again 20 batches of Asprin have been produced and sampled three times each. These values are shown in Table \ref{tab:MCUSUM-CoDa-II} and Figure \ref{fig:MCUSUM-CoDa-II} also shows \textit{MCUSUM – CoDa} chart and UCL values. It can be seen from Figure \ref{fig:MCUSUM-CoDa-II} that the process was IC till sample \#3, but sample \#4 went OOC because of an error. Hence the process stops at sample \#4 to study the reason for the OOC point and restart again after fixing the error, and it is shown in Figure \ref{fig:MCUSUM-CoDa-II} that after fixing the error, the process remains IC throughout all the samples.

### 5.2 Proposal Implementation Related to the Muesli Manufacturing Process

For the second example, the authors are using the same example used in Zaidi et al. (2019) and Zaidi et al. (2020) of a company that manufactures breakfast muesli with (A) 66 percent of cereals, (B) 24 percent dried fruits, and (C) 10 percent nuts in every 100 grams. The company has decided to begin by calibrating the measurement device used to determine the percentage of each component in the manufactured muesli (i.e. estimate $\mathbf{a}$, $\mathbf{b}$ and $\Sigma^*_\mathbf{M}$). The company meticulously prepared $k = 4$ samples with perfectly-known percentages for each component and measured them $m = 7$ times. Table \ref{tab:muesli-cereal} shows the results obtained along with the ilr transformed values to perform this calibration.

The following estimates are obtained using Table \ref{tab:muesli-cereal} with $d = 2$, $\mathbf{y}_i = \mathbf{y}_i^*$ and $\mathbf{u}_{i,j} = \mathbf{x}_{i,j}^*$ for $i = 1, \ldots, 4$ and $j = 1, \ldots, 7$, we get the following estimators $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\Sigma}_M^*$ for $\mathbf{a}$, $\mathbf{b}$ and $\Sigma_M^*$:

- $\hat{\mathbf{a}} = (0.3354, 0.3357, 0.3289)$ or, equivalently, $\hat{\mathbf{a}}^* = (0.0162972, -0.0006318)$,
- $\hat{\mathbf{b}} = 1.1070$,
- $\hat{\Sigma}_M^* = \begin{pmatrix} 0.0014346 & 0.0007812 \\ 0.0007812 & 0.0102893 \end{pmatrix}$.

To estimate the IC composition parameters (Phase I) $\hat{\mu}_0^*$ and $\hat{\Sigma}_0^*$, 20 batches of muesli have been measured $m = 3$ times. Table \ref{tab:muesli-cereal-II} shows the results with the values of $\mathbf{x}_{i,j}$, $\bar{\mathbf{x}}_i$ and $\bar{\mathbf{x}}_i^*$, $i = 1, \ldots, 20$. The results for $\bar{\mathbf{x}}_i$ and $\bar{\mathbf{x}}_i^*$ are plotted in Figure \ref{fig:muesli-composition}. From the values $\bar{\mathbf{x}}_i^*$, it is easy to obtain

- $\hat{\mu}_0^* = (1.2766, 0.7657)$,
- $\hat{\Sigma}_0^* = \begin{pmatrix} 0.0146362 & 0.0105839 \\ 0.0105839 & 0.0510887 \end{pmatrix}$.
Table 7: Phase-II data for the Asprin Manufacturing.

| i | j | x_{i,j} | R_i | R_j | D_i | C_j |
|---|---|---------|-----|-----|-----|-----|
| 1 | 1 | 0.65 0.27 0.08 | 0.6535 0.2566 0.08 | 1.3483 0.6343 0.3577 |
| 2 | 0.64 0.28 0.08 |
| 3 | 0.67 0.25 0.08 |
| 2 | 1 | 0.69 0.24 0.07 | 0.6834 0.2434 0.0732 | 1.4024 0.7362 1.9677 |
| 2 | 0.69 0.24 0.07 |
| 3 | 0.67 0.25 0.08 |
| 3 | 1 | 0.69 0.24 0.07 | 0.6822 0.2462 0.0716 | 1.4243 0.7266 6.8861 |
| 2 | 0.68 0.25 0.07 |
| 3 | 0.68 0.25 0.07 |
| 4 | 1 | 0.64 0.28 0.08 | 0.6580 0.2723 0.0697 | 1.4723 0.624 11.5462 |
| 2 | 0.69 0.24 0.07 |
| 3 | 0.65 0.20 0.07 |
| 5 | 1 | 0.64 0.28 0.08 | 0.6605 0.2562 0.0833 | 1.3038 0.6698 7.6805 |
| 2 | 0.69 0.23 0.08 |
| 3 | 0.65 0.26 0.09 |
| 6 | 1 | 0.64 0.28 0.08 | 0.6592 0.2698 0.08 | 1.3512 0.6221 7.5254 |
| 2 | 0.67 0.25 0.08 |
| 3 | 0.64 0.28 0.08 |
| 7 | 1 | 0.65 0.27 0.08 | 0.657 0.263 0.0801 | 1.3448 0.6474 6.8381 |
| 2 | 0.68 0.24 0.08 |
| 3 | 0.64 0.28 0.08 |
| 8 | 1 | 0.73 0.2 0.07 | 0.7139 0.2128 0.0733 | 1.3646 0.8558 4.8713 |
| 2 | 0.73 0.2 0.07 |
| 3 | 0.68 0.24 0.08 |
| 9 | 1 | 0.66 0.26 0.08 | 0.6668 0.2567 0.0765 | 1.3777 0.675 5.8349 |
| 2 | 0.68 0.25 0.07 |
| 3 | 0.66 0.26 0.08 |
| 10 | 1 | 0.65 0.27 0.08 | 0.625 0.295 0.080 | 1.368 0.531 7.907 |
| 2 | 0.56 0.36 0.08 |
| 3 | 0.66 0.26 0.08 |
| 11 | 1 | 0.68 0.24 0.08 | 0.664 0.257 0.080 | 1.339 0.672 6.744 |
| 2 | 0.66 0.26 0.08 |
| 3 | 0.65 0.27 0.08 |
| 12 | 1 | 0.66 0.26 0.08 | 0.653 0.263 0.083 | 1.312 0.643 5.217 |
| 2 | 0.66 0.26 0.08 |
| 3 | 0.64 0.27 0.09 |
| 13 | 1 | 0.68 0.24 0.08 | 0.687 0.237 0.077 | 1.357 0.753 4.127 |
| 2 | 0.7 0.23 0.07 |
| 3 | 0.68 0.24 0.08 |
| 14 | 1 | 0.63 0.28 0.09 | 0.643 0.273 0.083 | 1.321 0.605 4.753 |
| 2 | 0.65 0.27 0.08 |
| 3 | 0.65 0.27 0.08 |
| 15 | 1 | 0.65 0.27 0.08 | 0.653 0.267 0.080 | 1.349 0.634 5.299 |
| 2 | 0.66 0.26 0.08 |
| 3 | 0.65 0.27 0.08 |
| 16 | 1 | 0.68 0.24 0.08 | 0.677 0.250 0.073 | 1.409 0.704 5.545 |
| 2 | 0.68 0.25 0.07 |
| 3 | 0.67 0.26 0.07 |
| 17 | 1 | 0.71 0.22 0.07 | 0.730 0.200 0.070 | 1.385 0.917 3.106 |
| 2 | 0.74 0.19 0.07 |
| 3 | 0.74 0.19 0.07 |
| 18 | 1 | 0.72 0.21 0.07 | 0.697 0.229 0.073 | 1.385 0.786 2.949 |
| 2 | 0.71 0.22 0.07 |
| 3 | 0.66 0.26 0.08 |
| 19 | 1 | 0.71 0.22 0.07 | 0.710 0.220 0.070 | 1.413 0.829 4.684 |
| 2 | 0.71 0.22 0.07 |
| 3 | 0.71 0.22 0.07 |
| 20 | 1 | 0.68 0.24 0.08 | 0.667 0.253 0.080 | 1.336 0.685 2.709 |
| 2 | 0.65 0.27 0.08 |
| 3 | 0.67 0.25 0.08 |
Figure 5: MCUSUM-CoDa chart for Asprin Phase-II data.

Table 8: Data used for calibrating the measurement system.

| i | j | \( y_i \) | \( x_{i,j} \) | \( y_i^* \) | \( x_{i,j}^* \) |
|---|---|---|---|---|---|
| 1 | 1 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.0000 | 0.0000 | 0.0122 | 0.0211 |
| 2 |   | 0.32 | 0.35 | 0.33 | 0.0115 | -0.0634 |
| 3 |   | 0.33 | 0.35 | 0.32 | 0.0491 | -0.0416 |
| 4 |   | 0.31 | 0.35 | 0.34 | -0.0259 | -0.0858 |
| 5 |   | 0.36 | 0.34 | 0.30 | 0.1255 | 0.0404 |
| 6 |   | 0.34 | 0.32 | 0.34 | -0.0247 | 0.0429 |
| 7 |   | 0.31 | 0.36 | 0.33 | 0.0100 | -0.1057 |
| 2 | 1 | 0.60 | 0.20 | 0.20 | 0.4485 | 0.7768 | 0.4828 | 0.8363 |
|   | 2 | 0.65 | 0.17 | 0.18 | 0.5009 | 0.9484 |
|   | 3 | 0.59 | 0.22 | 0.19 | 0.5224 | 0.6976 |
|   | 4 | 0.61 | 0.20 | 0.19 | 0.4971 | 0.7885 |
|   | 5 | 0.67 | 0.15 | 0.18 | 0.4621 | 1.0583 |
|   | 6 | 0.65 | 0.17 | 0.18 | 0.5009 | 0.9484 |
|   | 7 | 0.61 | 0.19 | 0.20 | 0.4343 | 0.8248 |
| 3 | 1 | 0.20 | 0.60 | 0.20 | 0.17 | 0.65 | 0.18 | 0.4485 | -0.7768 | 0.5009 | -0.9484 |
|   | 2 | 0.14 | 0.69 | 0.17 | 0.4926 | -1.1279 |
|   | 3 | 0.21 | 0.60 | 0.19 | 0.5103 | -0.7423 |
|   | 4 | 0.19 | 0.62 | 0.19 | 0.4828 | -0.8363 |
|   | 5 | 0.20 | 0.62 | 0.18 | 0.5479 | -0.8000 |
|   | 6 | 0.16 | 0.66 | 0.18 | 0.4823 | -1.0020 |
|   | 7 | 0.20 | 0.62 | 0.18 | 0.5479 | -0.8000 |
| 4 | 1 | 0.20 | 0.20 | 0.60 | 0.18 | 0.20 | 0.62 | -0.8970 | 0.0000 | -0.9668 | -0.0745 |
|   | 2 | 0.19 | 0.19 | 0.62 | 0.9657 | 0.0000 |
|   | 3 | 0.21 | 0.19 | 0.60 | -0.8980 | 0.0708 |
|   | 4 | 0.21 | 0.18 | 0.61 | -0.9336 | 0.1090 |
|   | 5 | 0.20 | 0.18 | 0.62 | -0.9668 | 0.0745 |
|   | 6 | 0.19 | 0.19 | 0.62 | -0.9657 | 0.0000 |
|   | 7 | 0.21 | 0.17 | 0.62 | -0.9702 | 0.1494 |
Then, using (10) and (11), we have

- $\hat{\mu}_0^* = \frac{1}{b} (\bar{x}_N - \hat{\alpha}^*) = (1.1385, 0.6922)$,

- $\hat{\Sigma}^* = \frac{1}{b} \left( \bar{\Sigma}_N - \frac{1}{m} \bar{\Sigma}_M \right) = \left( \begin{array}{cc} 0.0115533 & 0.0084242 \\ 0.0084242 & 0.038891 \end{array} \right)$.

The authors have computed the $MCUSUM - CoDa$ statistics $C_i^*$ for $i = 1, \ldots, 20$ using Equation (12). The UCL for $\delta = 1.5$ is $H = 9.715$. The $MCUSUM - CoDa$ statistics $C_i^*$ values are listed in Table 9 and plotted in Figure 7 with the UCL $= H = 9.715$. One can see that the process is IC, as all the values in Figure 7 are smaller than the UCL.

Concerning Phase II, $i = 1, \ldots, 20$ batches of muesli have also been measured $m = 3$ times. Table 10 shows the results with the values $x_{i,j}$, $\bar{x}_i$ and $\bar{x}_i^*$, $i = 1, \ldots, 20$. The results $\bar{x}_i$ and $\bar{x}_i^*$ are also plotted in Figure 8. The $MCUSUM - CoDa$ statistics $C_i^*$ values in the presence of M.E’s are listed in Table 10 and plotted in Figure 9 with the UCL $= H = 9.715$ obtained in Phase I.

The process seems to be IC up to sample #14, but sample #15 and sample #16 are clearly OOC. Then, in samples #15, #16 and #17, it was discovered that a hatch malfunction had caused a sudden drop in the amount of whole-grain cereals, causing a shift from $\mu_0^* = (1.1385, 0.6922)$ to $\mu_N^* = (1.2766, 0.7657)$. Using Equation (13), we found...
Table 9: Phase I data for the muesli example.

| i  | j  | x_{i,j} | x_i | x_j | x_{i,j}^* | C_i | C_j |
|----|----|---------|-----|-----|----------|-----|-----|
| 1  | 1  | 0.77    | 0.16| 0.07| 0.709    | 0.8678 | 0.5965 |
| 2  | 0.67| 0.24    | 0.09|     |          |       |      |
| 3  | 0.68| 0.23    | 0.09|     |          |       |      |
| 2  | 1  | 0.64    | 0.27| 0.09| 0.6301   | 0.2767 | 0.5819 | 0.6735 |
| 2  | 0.63| 0.28    | 0.09|     |          |       |      |
| 3  | 0.62| 0.28    | 0.1 |     |          |       |      |
| 3  | 1  | 0.76    | 0.16| 0.08| 0.6958   | 0.2108 | 1.1523 | 0.8445 | 2.0531 |
| 2  | 0.65| 0.25    | 0.1 |     |          |       |      |
| 3  | 0.67| 0.23    | 0.1 |     |          |       |      |
| 4  | 1  | 0.65    | 0.27| 0.08| 0.6434   | 0.2766 | 0.8175 | 0.7871 |
| 2  | 0.64| 0.28    | 0.08|     |          |       |      |
| 3  | 0.64| 0.28    | 0.08|     |          |       |      |
| 5  | 1  | 0.5     | 0.38| 0.12| 0.5268   | 0.3634 | 0.1098 | 1.2829 | 0.2625 | 4.2017 |
| 2  | 0.54| 0.36    | 0.1 |     |          |       |      |
| 3  | 0.54| 0.35    | 0.11|    |          |       |      |
| 6  | 1  | 0.8     | 0.14| 0.06| 0.7677   | 0.1657 | 0.6666 | 1.3699 | 1.0843 | 1.2242 |
| 2  | 0.74| 0.19    | 0.07|     |          |       |      |
| 3  | 0.76| 0.17    | 0.07|     |          |       |      |
| 7  | 1  | 0.75    | 0.18| 0.07| 0.7304   | 0.1995 | 0.0701 | 1.3843 | 0.9177 | 0.4566 |
| 2  | 0.71| 0.22    | 0.07|     |          |       |      |
| 3  | 0.73| 0.2     | 0.07|     |          |       |      |
| 8  | 1  | 0.65    | 0.26| 0.09| 0.6534   | 0.2533 | 0.0932 | 1.203   | 0.67    | 0.3416 |
| 2  | 0.65| 0.25    | 0.1 |     |          |       |      |
| 3  | 0.66| 0.25    | 0.09|     |          |       |      |
| 9  | 1  | 0.65    | 0.27| 0.08| 0.6635   | 0.2565 | 0.08    | 1.3389 | 0.6721 | 1.0116 |
| 2  | 0.66| 0.26    | 0.08|     |          |       |      |
| 3  | 0.68| 0.24    | 0.08|     |          |       |      |
| 10 | 1  | 0.76    | 0.17| 0.07| 0.7534   | 0.1766 | 0.07    | 1.3478 | 1.0257 | 1.1556 |
| 2  | 0.75| 0.18    | 0.07|     |          |       |      |
| 3  | 0.75| 0.18    | 0.07|     |          |       |      |
| 11 | 1  | 0.67    | 0.24| 0.09| 0.6601   | 0.2499 | 0.09    | 1.2302 | 0.6868 | 0       |
| 2  | 0.65| 0.26    | 0.09|     |          |       |      |
| 3  | 0.66| 0.25    | 0.09|     |          |       |      |
| 12 | 1  | 0.53    | 0.36| 0.11| 0.5367   | 0.3533 | 0.11    | 1.1234 | 0.2956 | 3.2053 |
| 2  | 0.54| 0.35    | 0.11|     |          |       |      |
| 3  | 0.54| 0.35    | 0.11|     |          |       |      |
| 13 | 1  | 0.75    | 0.16| 0.09| 0.7251   | 0.1813 | 0.0936  | 1.1058 | 0.9803 | 3.8491 |
| 2  | 0.67| 0.23    | 0.1 |     |          |       |      |
| 3  | 0.75| 0.16    | 0.09|     |          |       |      |
| 14 | 1  | 0.67    | 0.24| 0.09| 0.6701   | 0.2333 | 0.0966  | 1.151   | 0.746  | 5.1974 |
| 2  | 0.67| 0.23    | 0.1 |     |          |       |      |
| 3  | 0.67| 0.23    | 0.1 |     |          |       |      |
| 15 | 1  | 0.64    | 0.27| 0.09| 0.6468   | 0.2632 | 0.09    | 1.2431 | 0.6357 | 5.1296 |
| 2  | 0.64| 0.27    | 0.09|     |          |       |      |
| 3  | 0.66| 0.25    | 0.09|     |          |       |      |
| 16 | 1  | 0.72    | 0.21| 0.07| 0.744    | 0.1928 | 0.0632  | 1.4614 | 0.9548 | 2.0026 |
| 2  | 0.74| 0.2     | 0.06|     |          |       |      |
| 3  | 0.77| 0.17    | 0.06|     |          |       |      |
| 17 | 1  | 0.73    | 0.2  | 0.07 | 0.7436  | 0.1899 | 0.0665  | 1.4137 | 0.9652 | 0.59    |
| 2  | 0.75| 0.18    | 0.07|     |          |       |      |
| 3  | 0.75| 0.19    | 0.06|     |          |       |      |
| 18 | 1  | 0.65    | 0.28| 0.07 | 0.6568  | 0.2767 | 0.0665  | 1.5169 | 0.6112 | 3.5386 |
| 2  | 0.65| 0.28    | 0.07|     |          |       |      |
| 3  | 0.67| 0.27    | 0.06|     |          |       |      |
| 19 | 1  | 0.76    | 0.17| 0.07 | 0.7604  | 0.173  | 0.0665  | 1.3846 | 1.0468 | 4.0517 |
| 2  | 0.74| 0.19    | 0.07|     |          |       |      |
| 3  | 0.78| 0.16    | 0.06|     |          |       |      |
| 20 | 1  | 0.67    | 0.23| 0.1  | 0.7013  | 0.2055 | 0.0931  | 1.1475 | 0.8678 | 2.1401 |
| 2  | 0.68| 0.22    | 0.1 |     |          |       |      |
| 3  | 0.75| 0.17    | 0.08|     |          |       |      |
that $\delta = 1.62$. Here, a shift of size $\delta = 1.5$ in $\hat{\mu}_x$ has been interpreted. For $m = 3$ and $\delta = 1.5$, the UCL of $MCUSUM - CoDa$ chart is $H = 9.715$. The process can be restarted without incident after the hatch has been repaired.

6 Conclusions

Control charts are an effective statistical process monitoring tool that are frequently used to assess the stability of manufacturing processes. They are used to monitor the industrial process by giving practitioners an early signal about an OOC process to keep the process IC, but M.E’s negatively impact the performance of control charts in quality control applications. In this paper, the authors investigate the impact of M.E’s on the $MCUSUM - CoDa$ control chart’s performance. The identity matrix assumption has been incorporated into the diagonal matrix in the linear covariate error model. This makes it easier to see how this parameter affects the performance of the proposed charts. Four cases have been considered for the variance-covariance matrix of CoDa. Various values of each parameter were investigated to inspect the influence of the parameters on the control chart. The following are some key findings of the study from this research. For all four cases, (i). when we increase the values of $\sigma_M$, the OOC ARL also indicates an increase, (ii). an increase in the value of $m$ imposes a slight decrease in the ARL values, (iii). an increase in the value of $b$ imposes a significant decrease in the ARL values (iv). an increase in the values of $p$
Table 10: Phase II data for the muesli example.

| i | j | $x_{i,j}$ | $\bar{x}_i$ | $\bar{x}_j$ | $\bar{x}_i^*$ | $\bar{c}_j$ |
|---|---|-----------|-------------|-------------|--------------|-----------|
| 1 | 1 | 0.68      | 0.23        | 0.09        | 0.6504       | 0.0688    |
|   | 2 | 0.63      | 0.27        | 0.1         | 0.6463       | 0.0933    |
|   | 3 | 0.64      | 0.27        | 0.09        | 0.6504       | 0.0688    |
| 2 | 1 | 0.68      | 0.24        | 0.08        | 0.6731       | 0.0933    |
|   | 2 | 0.66      | 0.25        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.68      | 0.24        | 0.09        | 0.6504       | 0.0688    |
| 3 | 1 | 0.62      | 0.31        | 0.07        | 0.6547       | 0.0933    |
|   | 2 | 0.67      | 0.26        | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.67      | 0.26        | 0.07        | 0.6504       | 0.0688    |
| 4 | 1 | 0.67      | 0.22        | 0.11        | 0.6637       | 0.0933    |
|   | 2 | 0.65      | 0.24        | 0.11        | 0.6504       | 0.0688    |
|   | 3 | 0.67      | 0.22        | 0.11        | 0.6504       | 0.0688    |
| 5 | 1 | 0.63      | 0.27        | 0.1         | 0.650        | 0.0933    |
|   | 2 | 0.66      | 0.25        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.66      | 0.25        | 0.09        | 0.6504       | 0.0688    |
| 6 | 1 | 0.77      | 0.15        | 0.08        | 0.770        | 0.0933    |
|   | 2 | 0.76      | 0.16        | 0.08        | 0.6504       | 0.0688    |
|   | 3 | 0.78      | 0.14        | 0.08        | 0.6504       | 0.0688    |
| 7 | 1 | 0.67      | 0.24        | 0.09        | 0.6571       | 0.0933    |
|   | 2 | 0.66      | 0.24        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.64      | 0.27        | 0.09        | 0.6504       | 0.0688    |
| 8 | 1 | 0.64      | 0.28        | 0.08        | 0.678        | 0.0933    |
|   | 2 | 0.72      | 0.21        | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.67      | 0.25        | 0.08        | 0.6504       | 0.0688    |
| 9 | 1 | 0.72      | 0.21        | 0.07        | 0.700        | 0.0933    |
|   | 2 | 0.68      | 0.24        | 0.08        | 0.6504       | 0.0688    |
|   | 3 | 0.7       | 0.23        | 0.07        | 0.6504       | 0.0688    |
| 10| 1 | 0.64      | 0.26        | 0.1         | 0.650        | 0.0933    |
|   | 2 | 0.66      | 0.25        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.65      | 0.26        | 0.09        | 0.6504       | 0.0688    |
| 11| 1 | 0.73      | 0.2         | 0.07        | 0.744        | 0.0933    |
|   | 2 | 0.74      | 0.19        | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.76      | 0.17        | 0.07        | 0.6504       | 0.0688    |
| 12| 1 | 0.81      | 0.12        | 0.07        | 0.803        | 0.0933    |
|   | 2 | 0.8       | 0.13        | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.8       | 0.13        | 0.07        | 0.6504       | 0.0688    |
| 13| 1 | 0.64      | 0.28        | 0.08        | 0.657        | 0.0933    |
|   | 2 | 0.68      | 0.24        | 0.08        | 0.6504       | 0.0688    |
|   | 3 | 0.65      | 0.27        | 0.08        | 0.6504       | 0.0688    |
| 14| 1 | 0.82      | 0.11        | 0.07        | 0.779        | 0.0933    |
|   | 2 | 0.74      | 0.17        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.77      | 0.15        | 0.08        | 0.6504       | 0.0688    |
| 15| 1 | 0.61      | 0.25        | 0.14        | 0.641        | 0.140     |
|   | 2 | 0.66      | 0.2         | 0.14        | 0.6504       | 0.0688    |
|   | 3 | 0.65      | 0.21        | 0.14        | 0.6504       | 0.0688    |
| 16| 1 | 0.7       | 0.23        | 0.07        | 0.694        | 0.0933    |
|   | 2 | 0.71      | 0.22        | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.67      | 0.25        | 0.08        | 0.6504       | 0.0688    |
| 17| 1 | 0.74      | 0.19        | 0.07        | 0.740        | 0.0933    |
|   | 2 | 0.73      | 0.2         | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.75      | 0.18        | 0.07        | 0.6504       | 0.0688    |
| 18| 1 | 0.56      | 0.35        | 0.09        | 0.534        | 0.0933    |
|   | 2 | 0.51      | 0.35        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.53      | 0.38        | 0.09        | 0.6504       | 0.0688    |
| 19| 1 | 0.55      | 0.35        | 0.1         | 0.547        | 0.0933    |
|   | 2 | 0.55      | 0.36        | 0.09        | 0.6504       | 0.0688    |
|   | 3 | 0.54      | 0.37        | 0.09        | 0.6504       | 0.0688    |
| 20| 1 | 0.77      | 0.16        | 0.07        | 0.754        | 0.0933    |
|   | 2 | 0.76      | 0.17        | 0.07        | 0.6504       | 0.0688    |
|   | 3 | 0.73      | 0.2         | 0.07        | 0.6504       | 0.0688    |
imposes a significant increase in the ARL. All these findings are relevant to the existing studies in the literature. In the end, two illustrated examples are given to demonstrate for implementation of the proposed control charts in the presence of M.E.’s. One is based on the manufacturing process of uncoated aspirin tablets, and the other is based on monitoring machines in the muesli manufacturing process.

**Contribution Authorship Statement**

Muhammad Imran: Conceptualization, Resources, Methodology, Investigation, Validation, Writing Original Draft, Software.

Jinsheng Sun: Investigation, Resources, Methodology, Validation, Writing Original Draft, Supervision, Project administration.

Fatima Sehar Zaidi: Investigation, Methodology, Resources, Validation, Writing Original Draft, Software.

Zameer Abbas: Investigation, Validation, Review & Editing.

Hafiz Zafar Nazir: Investigation, Validation, Review & Editing.

**Consent to Participate and Publish**

Consent was obtained from all authors for their participation in completing this manuscript and their consent to publish it.

**Conflicts of Interest**

The authors declare no competing interests.

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