Nonlinear evaluation of the long-term critical force

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Abstract. An evolution of a normal sections stiffness generated by a creep and corrosion of materials, changing its mechanical characteristics are taken into account. The aim of the paper is to refine the known estimates based on the concept of a statistical distribution of the disabilities links a combination of which constitutes an element. This concept allows us to derive non-linear rheological equations. The proposed generalization of the Boltzmann creep strains superposition principle is used. The destruction of a part of the links results in a redistribution of the forces to the links capable to resist. This generates a non-linear dependence of deformations on the calculated stresses obtained under the assumption of equality for all links durability. By solving it we obtain a structural stress. The product of its limit per an area of the normal element section is a long structural critical force. The significant decline of the critical force value is due to the degradation of the mechanical characteristics. Accounting for non-linearity assessment of the long-term critical force gives the values less than the well-known ones from the literature values.

1. Introduction

The forecast of a long-term constructive safety of structures takes into account the stability of compressed by a longitudinal force elements. This gives rise to the need to estimate the critical force that causes the compressed elements equilibrium loss at the end of a large period of time. The force estimates are the various modifications of the classical Euler formula. They take into account the evolution of the normal sections stiffness generated by the creep and corrosion of the material, changing its mechanical characteristics. The aim of the paper is to refine the known estimates based on the concept of the statistical distribution of the disabilities links, the combination of which constitutes an element. Traditionally, the stability of the structural element is understood as its ability to maintain the equilibrium form given to it during the manufacturing process [1, 2]. In these monographs and in a number of other papers, the study of the compressed bars stability is based on the classical Euler formula. The creep and corrosion of structural materials leads to the need to clarify the critical strength in the structures calculation for a long safe period of operation. In our opinion, it is necessary to take into account the rods stiffness degradation due to the creep and corrosion and structural material damage. A structural damage is the destruction of parts of the links the combination of which forms the structural element. In contrast to the traditional approach, considering the material isotropically strong, we assume some statistical distribution of the strengths of the links. This circumstance leads to the fact that the relationship between stresses and deformations becomes essentially nonlinear, given by a nonlinear rheological equation. The establishment of this equation is based on the modification of
the creep strains Boltzmann principle of superposition proposed in this paper. According to Euler, the articulated rod loses its rectilinear shape as a result of the action of the longitudinal force

\[
P_E = \left(\frac{\pi}{l}\right)^2 D
\]

where \(D = lE\) is the flexural rod stiffness; \(I\) is the moment of the normal section inertia; \(E\) is the elasticity modulus; \(l\) is the length of the rod. The longitudinal force, that changes the equilibrium positions of the rod, is called critical. In the Euler formula the rigidity is assumed to be independent on the current time \(\tau\) and force \(P_\sigma\).

2. Rheological equations of state

The physicochemical processes in the rod material gives rise to a change in its mechanical parameters \(E(\tau)\) and \(C(t, \tau)\) that are the creep measures at the moment of time \(t\), under the action of the normal stress \(\sigma(\tau)\). The phenomenon of strains increase generated by \(\sigma(\tau)\) when \(\tau > t_0\) is called creep. When \(\sigma(\tau) = \sigma(t_0)\) for a period of time \([t_0, t]\) the creep strain has the form

\[
\varepsilon_c(t, t_0) = C(t, t_0)\sigma(t_0),
\]

then as the total strain is determined by formula

\[
\varepsilon(t, t_0) = \varepsilon_{\text{inst}}(t) + C(t, t_0)\sigma(t_0),
\]

where \(\varepsilon_{\text{inst}}(t) = \sigma(t)/E(t)\) is the instantaneous strain.

It is assumed in a linear formulation that with the increasing stress \(\sigma(t)\) the creep strain increment \(d\varepsilon_c(t, \tau)\) is associated with the increment of the stress \(d\sigma(\tau)\) by the relation

\[
d\varepsilon_c(t, \tau) = C(t, \tau)d\sigma(\tau).
\]

According to the well-known Boltzmann superposition principle of dependence \(\Delta\varepsilon_c(t, \tau)\) only on \(\Delta\sigma(\tau)\) we obtain the equality

\[
\varepsilon_c(t, t_0) = \int_{t_0}^t C(t, \tau)d\sigma(\tau)
\]

Integrating by parts this equation we obtain

\[
\varepsilon_c(t, t_0) = C(t, t)\sigma(t) - \int_{t_0}^t \sigma(\tau)\frac{\partial C(t, \tau)}{\partial \tau}d\tau
\]

where value \(C(t, t)\) is usually called short-term creep [4]. Adding to \(\varepsilon_c(t, t_0)\) the instantaneous strain \(\varepsilon_{\text{inst}}(t) = \sigma(t)/E(t)\) we obtain the linear equation of the mechanical state under the uniaxial loading [5]

\[
\varepsilon(t, t_0) = \int_{t_0}^t \sigma(\tau)\frac{\partial C(t, \tau)}{\partial \tau}d\tau
\]

Now we establish the form of a nonlinear rheological equation. We will assume that the material of structural elements consists of a combination of links with same statistically distributed strengths \(R(\tau)\) and the increasing stress \(\sigma(\tau)\) destroys the links with \(R(\tau) < \sigma(\tau)\). This entails redistribution \(\sigma(\tau)\) to the other units capable to a force resistance. If \(A\) and \(A(\tau)\) are the areas of the normal cross-sections of the rod, whole at the moment \(\tau\) its links, then the values

\[
\sigma(\tau) = \frac{P}{A}, \quad \sigma(\tau) = \frac{P}{A(\tau)}
\]
represent respectively the calculated and structural (true) normal stresses. In accordance with the expressions (8) and (9), we have relation

\[ \sigma_{\text{st}}(t) = S^0(t)\sigma(t). \]  

(10)

The function \( S^0(t) = A/A(t) \) describes the process of the stresses redistribution to the whole links and determines the nonlinear dependence of the instantaneous strains on the stresses \( \sigma(t) \)

\[ e_{\text{st}}(t) = S^0(t)\sigma(t)/E(t). \]  

(11)

According to the Boltzmann principle the each increment \( \Delta e_i(t, \tau) = C(t, \tau) \Delta \sigma(\tau) \) depends only on the magnitude and duration \( \Delta \sigma(\tau_i) \), what is realized at the same strength \( R_i(\tau) \) for all links of the material. The strength distribution \( R(\tau) \) entails an increase \( \Delta e_i(t, \tau) \) from the effect of the each subsequent increment \( \Delta \sigma(\tau_i) \) [6, 7].

Consider, for example, the stress increment given by the expression

\[ \Delta \sigma(\tau) = \begin{cases} 
\Delta \sigma(\tau_1); & \tau_1 \leq \tau < \tau_2; \quad \tau_1 = t_0 \\
\Delta \sigma(\tau_3) + \Delta \sigma(\tau_2); & \tau_2 \leq \tau < t 
\end{cases} \]

Because \( S^0(\tau_2, t_0) > S^0(\tau_1, t_0) \), the strain due to the action \( \Delta \sigma(\tau_1) \) depend on \( \Delta \sigma(\tau_2) \) as well. This circumstance does not allow the application of the superposition principle of the creep strains together with the stress increments \( \sigma(t) \). This makes it necessary to modify the Boltzmann superposition principle. Now we show that this superposition is realized relatively of the fraction increments of the structural stress \( \sigma_i(t) \) [8]. The interdependence on \( e_i(t, \tau_i) \) take place for an ideal element when all the fractions have the same strengths \( R(\tau) \) and consequently the retribution of \( N(\tau) \) is excluded. Therefore parallel with the given element we consider the geometrically identical to it ideal element with the same parameters \( E(t) \) and \( C(t, \tau) \). The ideal element under the axial loading \( P(\tau) = A P(\tau)/A(\tau) = S^0(\tau) P(\tau) \) has the stress \( \sigma_i(\tau) \) and the increment \( \Delta \sigma_i(\tau_i) \) generate \( \Delta e_i(t, \tau) \).

Thus we have

\[ \Delta e_i(t, \tau) = C(t, \tau_i) \Delta \sigma_i(\tau_i), \]

(12)

\[ \Delta e_i(t, t_0) = \sum_{i=1}^{n} C(t, \tau_i) \Delta \sigma_i(\tau_i) \]

(13)

\[ \Delta e_i(t, t_0) = \int_{t_0}^{t} C(t, \tau) d \sigma_i(\tau) \]

(14)

\[ \Delta e_i(t, t_0) = C(t, t_0) \sigma_i(t) - C(t, t_0) \sigma_i(t) - \int_{t_0}^{t} \sigma_i(\tau) \frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} + C(t, \tau) \right] d \tau \]

(15)

\[ e_i(t, t_0) = C(t, t_0) \sigma_i(t) - \int_{t_0}^{t} \sigma_i(\tau) \frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} + C(t, \tau) \right] d \tau \]

(16)

According to the equation (16) and \( \sigma_i(\tau) = S^0(\tau) \sigma(\tau) \) we obtain the nonlinear rheological state equation

\[ e(t, t_0) = S^0(\tau) \sigma(t) \left[ \frac{1}{E(t)} + C(t, t) - \int_{t_0}^{t} S^0(\tau) \sigma(\tau) \frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} + C(t, \tau) \right] d \tau \right] \]

(17)

We can present an alternate vision of the deduction for nonlinear rheological equation (17).
Consider to this end the part \( V \) of the constructive element consisting from the union of all the remaining entire in the interval \([t_0, t]\) fractions. The loading \( P \) generates on its cross-section (with the area \( A(t) \)) the same structural stress \( \sigma(t) \) that is on the cross-section of the considered element (with the area \( A(t) \)). The element and its part \( V \) have the same fraction creep deformations and the absence of the one in the interval \([t_0, t]\) of the redistribution of the loading \( P(t) = A(t)/A(t) \) implies their inter-indency. Then, for \( V \), we have the relations (14)-(18) and consequently again obtain the rheological equation (17). Evidently the linear equation (7) contradicts to the well known non-linear diagrams "\( \sigma - \varepsilon \)". We note that according to A.A. Gvozdev a non-linear part of "\( \sigma - \varepsilon \)" is generated by quick creep at the moment of loading [9]. In addition he assumes the linearity of the dependence \( \varepsilon(t) \) on \( A(t) \). By A.A. Gvozdev the creep deformation is the composition of the linear component \( \varepsilon(t,t_0) = \int_0^t C(t,\tau)d\sigma(\tau) \) and the non-linear component \( \varepsilon_{nl}(t,t_0) = \int_0^t L(t,\tau,\sigma)d\sigma(\tau) \) due to the structural damages. This conception implies a necessity of the choice of the functions \( C^* \) and \( L \) corresponding to the diagram "\( \sigma - \varepsilon \)". We note that this case reduces to the considerable inconvenience for the applications [10]. According to the relation \( \sigma_{\varepsilon}(\tau) = S^0(\tau)\sigma(\tau) \) the equation (17) is represented in the form

\[
\varepsilon(t,t_0) = \sigma_{\varepsilon}(\tau) \left[ \frac{1}{E(t)} + C(t,t_0) \right] - \int_0^t \sigma_{\varepsilon}(\tau) \frac{\partial C(t,\tau)}{\partial \tau} d\tau
\]

(18)

Thus with respect to the structural stress \( \sigma_{\varepsilon}(\tau) \) we have the linear integral equation. This circumstance makes it possible in particular to fulfill the solution of the relaxation problems [10] with the standard methods [8, 11]. According to the equation (3) the quantity \( L(t,t_0) = 1/E(t) + C(t,t_0) \) is elastic-plastic compliance, whereas \( E(t,t_0) = E(t)[1 + E(t)C(t,t_0)] \) is elastic-plastic module при \( \sigma(\tau) = \sigma(t_0) \) on the interval \([t_0, t]\).

Now we will estimate the critical strength. According to the formula (1) the critical force \( P_{cr}(t) \) when loading at the moment \( t \) is given by

\[
P_{cr}(t) = (\pi/l)^2 D(t).
\]

(19)

In a linear setting, the stiffness of a rod can be determined by the formula

\[
D(t) = IE(t,t_0).
\]

(20)

Taking into account the distribution of the strengths of its links, the rigidity will be written so

\[
D(t) = \frac{IE(t)}{S^0(t)[1 + E(t)C(t,t_0)]}
\]

(21)

The value \( P_{cr}(\infty) = (\pi/l)^2 D(\infty) \) is a long-term critical force in the nonlinear formulation as well

\[
P_{cr}(\infty) = \frac{(\pi/l)^2 IE(\infty)}{S^0(\infty)[1 + E(\infty)C(\infty,t_0)]}.
\]

(23)

Assuming the isotropy of the material for strength, we obtain

\[
P_{cr}(\infty) = \frac{(\pi/l)^2 IE(\infty)}{1 + E(\infty)C(\infty,t_0)}.
\]

(24)
Because the $S^0(\infty)>1$ we obtain in accordance with these two formulas the inequality

$$P_e(\infty) < P_{cr \text{lin}}(\infty)$$  \hspace{1cm} (25)

Thus the long-term nonlinear critical force is always less than the long-term linear critical one. For example, for concrete brands B50 ($R(t) = 36\text{MPA}$, $V = 0.8$, $m = 4$) and with $P_{cr}(t)/AR(t) = 3/4$ we obtain $S^0(t) = 5/4$. Since $P_{cr \text{lin}}(t)/P_{cr} = S^0(t)$ the critical force is reduced by 20\% compared with the linear case. The value $\varphi = E(\infty)C(\infty, t_0)$ is the limiting value of the creep characteristic $\varphi(t, \tau)$ at constant loading. According to the expression (1), (23) and (24) we can write

$$P_{cr}(\infty) = \frac{P_e}{S^0(\infty)(1 + \varphi)}$$  \hspace{1cm} (26)

$$P_{cr \text{lin}}(\infty) = -\frac{P_e}{1 + \varphi}$$  \hspace{1cm} (27)

Here $P_e$ is the critical force of Euler.

In applications the stress nonlinearity function $S^0(t) = S^0[\sigma(t)]$ is given in the form

$$S^0[\sigma(t)] = 1 + V[\sigma(t)/R(t)]^m$$ \hspace{1cm} or \hspace{1cm} $$S^0[\sigma(t)] = a[\sigma(t)/R(t)]^b$$

where $V, m, a$ and $b$ are the empirical parameters; $R(t)$ is strength of material. We estimate the long-term critical force by solving the equation (18) with $S^0(\infty) = 1 + V[P_{cr}(\infty)/AR(\infty)]^m$ or $S^0(\infty) = a[P_{cr}(\infty)/AR(\infty)]^b$. For the concrete it is usually assumed $m = 4$ [4].

We write the equation (17) in the form $\varepsilon(t, t_0) = S^0(t)\ddot{\varepsilon}(t, t_0)$ by introducing the notation

$$\ddot{\varepsilon}(t, t_0) = \dot{\sigma}(t)\left[\frac{1}{E(t)} + C(t, t)\right] - \int_0^t S^0(\tau)\dot{\sigma}(\tau) \frac{\partial C(t, \tau)}{\partial \tau} d\tau.$$  \hspace{1cm} (28)

Assuming $\dot{\sigma}(t) = S^0(\tau)\dot{\sigma}(\tau)/S^0(t)$ and taking into account the equality $\ddot{\sigma}(t) = \sigma(t)$ we obtain

$$\ddot{\varepsilon}(t, t_0) = \dot{\sigma}(t)\left[\frac{1}{E(t)} + C(t, t)\right] - \int_0^t \dot{\sigma}(\tau) \frac{\partial C(t, \tau)}{\partial \tau} d\tau.$$  \hspace{1cm} (29)

The value $\ddot{\varepsilon}(t, t_0)$ is the deformation of the element under the equality condition of the strengths of all its links and therefore it is an invertible part of the strain $\varepsilon(t, t_0)$ [7]. The strain $\varepsilon(t) = S^0(t)\ddot{\varepsilon}(t, t_0)$ is linear with respect to $\sigma(t)$ part of $\varepsilon(t, t_0)$. Therefore the relation $\varepsilon(t, t_0) = S^0(t)\ddot{\varepsilon}(t, t_0)$ is the quasilinear representation of the strain $\varepsilon(t, t_0)$. In various applications the level function of strain $S^0(t) = \exp(\varepsilon(t)/\varepsilon_n(t))$ is used [8, 12]. When $\sigma(\tau) \leq \sigma(t_0)$ for all $\tau \geq t_0$ we have equality $S^0(\tau) = S^0(t_0)$. The value $\ddot{\varepsilon}(t, t_0)$ is defined by the formula (7). Since $S^0(\infty) = S^0(t_0)$ then according to the formula (25)

$$P_{cr}(\infty) = \frac{P_e}{S^0(t_0)(1 + \varphi)}.$$  \hspace{1cm} (30)

In the work [13] instead of the incorrectly applied creep characteristic $\bar{\varphi}(t, \tau) = \varepsilon(t, \tau)/\varepsilon_{\text{inst \text{lin}}}(t_0)$ (here $\varepsilon_{\text{inst \text{lin}}}(t_0) = \sigma(t_0)/E(t_0)$) the nonlinear characteristic $\bar{\varphi}_{\text{nonlin}}(t, \tau) = \varepsilon(t, \tau)/\varepsilon_{\text{inst \text{lin}}}(t_0)$ is proposed. The subscript $\text{lin}$ indicates the nonlinearity of the creep characteristics.

Since $\varepsilon_{\text{inst \text{lin}}}(t_0) = S^0(t_0)\varepsilon_{\text{inst \text{lin}}}(t_0)$ then
\[ \tilde{\varphi}_{\text{lin}}(t, \tau) = \tilde{\varphi}(t, \tau)/S^0(t). \] (31)

When \( \sigma(\tau) \neq \text{const} \) we have
\[ \varphi_{\text{lin}}(t, \tau) = -\int_{t_0}^{t} \sigma(\tau) \frac{\partial C(t, \tau)}{\partial \tau} d\tau / E(t) ; \quad \varepsilon_{\text{inst}}(t) = \frac{\sigma_{\text{lin}}(t)}{E(t)}, \] (32)
\[ \varepsilon_c(t, t_0) = -\int_{t_0}^{t} \varepsilon_{\text{inst}}(t) \frac{\partial \varphi(t, \tau)}{\partial \tau} d\tau. \] (33)

In the linear formulation we have \( \varphi_{\text{lin}}(t, \tau) = -\int_{t_0}^{t} \sigma(\tau) \frac{\partial C(t, \tau)}{\partial \tau} d\tau / E_{\text{lin}}(t) \). In accordance with the expressions (31)-(33) there is the relation \( \tilde{\varphi}_{\text{lin}}(t, \tau) = \varphi_{\text{lin}}(t, \tau) \) when \( \tilde{\varphi}(t, \tau) = S^0(t) \varphi_{\text{lin}}(t, \tau) \) and the characteristic \( \tilde{\varphi}(t, \tau) \) meets the creep measure \( \tilde{C}(t, \tau) = S^0(t)C(\tau, \tau) \). In the case \( \tilde{C}(t, \tau) = S^0(t)C_\infty[1 - \beta e^{-\gamma(\tau - t)}] ; E(\tau) = E \) the equation (7) reduces to the differential equation
\[ E \frac{dE}{dt} + \gamma E_E = \frac{d\sigma}{dt} + \gamma(1 + C_\infty) \sigma \] (34)

In turn, this equation reduces to the equation [14]
\[ \frac{\partial M}{\partial t} + \gamma(1 + C_\infty)M = D \left( \frac{\partial \kappa}{\partial t} + \frac{\kappa}{1 + \beta C_\infty} \right), \] (35)

where \( M \) is the bending moment, \( \kappa = -\frac{\partial^2 w}{\partial \tau^2} \) is the curvature of the rod (\( \nu \) is the axis coordinate of the rod), \( w = y(\nu) y(t) \) is the transverse displacement of the normal section, \( 0 \leq \nu \leq l \).

According to the equation (34) the differential equation relatively \( y(t) \) is deduced. Its solution when the force \( P_c(t_0) = (\pi/l)^2 D \left[ 1 + \beta C_\infty / S^0(t) \right] \) for all \( \tau \geq t_0 \) has the form \( y(\tau) = y(t_0) \) [13]. As \( \tilde{C}(t, \tau) = S^0(t)C_\infty \), then \( P_c(t_0) = \frac{P_c}{1 + \beta \varphi_{\text{lin}}} \). Thus it is obvious that proposed in [13] the approach for estimating the long-term critical force is in the linear formulation, since the proposed here the nonlinear formulation when \( \tilde{\varphi}(t, \tau) = \tilde{\varphi}(t, \tau)/S^0(t) \) and \( \tilde{\varphi}_{\text{lin}}(t, \tau) = \varphi(t, \tau) \) reduces to a linear one.

**Remark 1.** In the works [9, 14, 15] is believed \( \beta = 1 \). This implies the relation \( C(t, t_0) = 0 \) and excludes from the equation of mechanical state the term \( C(t, t_0) \sigma_{\text{lin}}(t) \), called a short-term creep [8].

The nonlinear formulation with the help of the energy method [2] estimates the dissipation of the rod energy and its corrosion, and the compliance of the normal cross sections to the strains of compression and shear [16]. The method given in [16] can also be used to estimate the long-term critical force. The critical Euler force and the estimates given above assume that the rod is not subject to the compression and shear. To take into account their influence, we use the energy method. First we consider the linear problem taking into account the compression. Due to the decrease in length by an amount \( \Delta \tilde{L}(t) = \frac{P_c(t)}{AE(t, t_0)} \) the displacement \( \Delta \tilde{L}(t) = \left[ 1 - \frac{P_c(t)}{D(t, t_0)} \right] \Delta l(t) \) correspond to the curved length rod of length \( \tilde{L}(t) = \left[ 1 + \frac{P_c(t)}{AE(t, t_0)} \right] \Delta l(t) \). (Here \( \Delta l(t) \) is the end rod displacement when \( l/D_1(t, t_0) = 0 \) and \( l/D_2(t, t_0) = 0 \).

According to the energy balance

\[ P_c(t) \left[ 1 - \frac{P_c(t)}{D_1(t, t_0)} \right] \Delta l(t) = P_E(t) \Delta l(t) \] (36)

we obtain the quadratic equation. Its smaller root is

\[ P_c(t) = \frac{D_1(t, t_0) - \sqrt{D_1^2(t, t_0) - 4P_E(t)D_1(t, t_0)}}{2} \] (37)
and represents a critical force in the case under consideration. It is seen \( P_{cr}(t) > P_E(t) \).

In the process of the bifurcation there are the shifts in the normal cross sections that generate a decrease in the projection of the axis of the rod onto the axis \( x \) and consequently an increase of \( \Delta l(t) \).

As a result of taking into account the shifts, we obtain an increment \( \Delta l(t) \) by value \( \Delta l(t) = P_{cr}(t)/D_2(t, t_0) \). When \( 1/D_1(t, t_0) = 0 \) we obtain the relation
\[
P_{cr}(t) \left[ 1 + P_{cr}(t)/D_2(t, t_0) \right] \Delta l(t) = P_{cr}(t) \Delta l(t),
\]
from which the formula follows
\[
P_{cr}(t) = \frac{D_2(t, t_0)}{2} \left[ \sqrt{1 + 4P_{cr}(t)/D_2(t, t_0)} - 1 \right].
\]
When taking into account the compression and shift compliances \( D_1(t, t_0) \) and \( D_2(t, t_0) \) from the energy balance relation
\[
P_{cr}(t) \left[ 1 + P_{cr}(t) \left( \frac{1}{D_2(t, t_0)} - \frac{1}{D_1(t_0)} \right) \right] \Delta l(t) = P_{cr}(t) \Delta l(t)
\]
we obtain the critical force
\[
P_{cr}(t) = \frac{1}{2\alpha} \left( \sqrt{1 + 4P_{cr}(t)\alpha} - 1 \right);
\]
\[
\alpha = \frac{1}{D_2(t, t_0)} - \frac{1}{D_1(t_0)}.
\]

**Remark 2.** The formulas of a look (39) and (41) on the basis of the corresponding equations in variations are received in [17]. The given energy approach is more evident and simple one. As it is noted in [17] the known Engesser formula \( P_{cr}(t) = P_E/(1 + N_E/D_2) \) isn’t correct. Really, according to our approach it is equivalent to same impossible power balance. The assessment of the value \( P_{cr}(t) \) is connected with the inevitable dissipation of the energy of a bar resistance together with an influence of the environment generating the degradation of the compliances. When accounting the corrosive attacks and the set nonlinear functions of damages are given we receive a similar formula. It is essential that the statistical distribution of the value \( R(t) \) along with the nonlinear dependence between the strain and stress generates the dependence of these parameters on \( P_{cr}(t) \).

3. Conclusion
The formula (41) takes into account all the significant factors affecting the magnitude of the longitudinal critical force. It determines the long-term critical force when \( t \to \infty \) providing a long-term stability of the rod under the condition \( P(t) < P_{cr}(t) \). The basis of our constructions is the concept of the strengths statistical distribution of the links that make up a structural element. The transformation of a rectilinear form of balance is followed by destruction of a part of the whole by the time of bifurcation core links.

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