Mechanism for the Suppression of Intermediate-Mass Black Holes

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A model for the formation of supermassive primordial black holes in galactic nuclei with the simultaneous suppression of the formation of intermediate-mass black holes is presented. A bimodal mass function for black holes formed through phase transitions in a model with a “Mexican hat” potential has been found. The classical motion of the phase of a complex scalar field during inflation has been taken into account. Possible observational manifestations of primordial black holes in galaxies and constraints on their number are discussed.

INTRODUCTION

There is almost no doubt that the nuclei of most structured galaxies host supermassive black holes (SMBHs) with masses $\sim 10^6 - 10^{10} M_\odot$ (Antonucci 1993; Falcke and Hehl 2003; Ferrarese and Ford 2004). There is also observational evidence for the presence of intermediate-mass ($\sim 10^3 - 10^5 M_\odot$) black holes (IMBHs) in galactic disks and globular star clusters (Gebhardt et al. 2002; Lanzoni et al. 2007; Noyola et al. 2008; Strohmayer and Mushotzky 2009). In contrast to stellar-mass black holes produced by the explosions of massive supernovae, the origin of SMBHs and IMBHs remains enigmatic. Several scenarios for the formation and subsequent growth of SMBHs have been proposed (for reviews and references, see Rees 1984, Begelman et al. 1984, Dokuchaev 1991, and Dokuchaev et al. 2007), which can arbitrarily divided into two groups: astrophysical and cosmological. In astrophysical scenarios, IMBHs and SMBHs result from the dynamical evolution and gravitational collapses of gas clouds or of central star clusters in galactic nuclei or from multiple mergers of stellar black holes. Cosmological scenarios suggest a very early formation of primordial black holes (PBHs) at the radiation-dominated stage from adiabatic density perturbations or through phase transitions in the early Universe. A cosmological scenario for the formation of massive PBHs through the collapse of isothermal baryonic charge fluctuations that arise during phase transitions at the inflationary stage has also been proposed (Dolgov and Silk 1993; Dolgov et al. 2009).

The currently most popular models are those of hierarchical merging where SMBHs are formed by multiple mergers of less massive IMBHs (Miller and Colbert 2004; Kawakatu et al. 2005). The latter could be formed, for example, during the collapses of gas clouds in small dark matter halos at pregalactic epochs, during hierarchical merging of stellar-mass black holes that are the remnants of the first stars (Islam et al. 2003), or during accretional swallowing of the first massive stars by PBHs in their interiors (Bambi et al. 2008).
One of the scenarios for the formation of PBHs through phase transitions was developed by Rubin et al. (2000, 2001a, 2001b), Khlopov and Rubin (2004), Khlopov et al. (2005), and Dokuchaev et al. (2005, 2008). Apart from the presence of SMBHs in galactic nuclei, this scenario also predicts the existence of IMBHs in galactic halos at a considerable distance from their centers. The distinctive features of the scenario are the formation of black holes already at the radiation-dominated stage and a characteristic spatial distribution of black holes in clusters. Whereas “astrophysical” black holes are formed in potential wells at the centers of dark matter halos (Dokuchaev et al. 2007), PBHs are capable of producing an induced dark matter halo around themselves that appears as a dwarf galaxy (Dokuchaev and Eroshenko 2001, 2003; Dokuchaev et al. 2008) with a sharp increase in density toward the center. The mergers of IMBHs formed through the mechanism proposed by Rubin et al. (2000, 2001a, 2001b), Khlopov and Rubin (2004), and Khlopov (2005) and the formation of a peculiar class of galaxies and quasars around primordial SMBHs were considered by Dokuchaev et al. (2005, 2008).

It is hard to imagine that absolutely all IMBHs will merge and produce central SMBHs in presentday galaxies. It is more likely that only some fraction of them, \( \sim 1 \), will merge, while the remaining IMBHs form a population of IMBHs in galactic halos. A similar situation arises in many models with primordial SMBHs. Indeed, if some mechanism predicts the formation of one primordial SMBH in the bulk of one galaxy, then it is hard to avoid the presence of a large number of less massive IMBHs in the same galaxy. In addition, even if SMBHs in galactic nuclei are not related by their origin to IMBHs, the IMBH mass functions tend to increase toward low masses.

Thus, a common feature of the above SMBH formation mechanisms is the predicted abundance of IMBHs. Nevertheless, observational data constrain significantly the possible number of IMBHs in galaxies. Only a small number of objects can be attributed to IMBHs with some probability, for example, IMBHs in some globular star clusters. However, as yet there is no unequivocal proof of the presence of IMBHs at their centers, because the available observational evidence for their existence is questioned and comes under justified criticism (Illingworth and King 1977; McNamara et al. 2003; Baumgardt et al. 2003; van der Marel and Anderson 2009). Besides, the formation of IMBHs in globular clusters is severely hampered by the heating of their central parts by binary stars (hard pairs) formed during dissipative two-body encounters of single stars (Ozernoy and Dokuchaev 1982; Dokuchaev and Ozernoy 1982).

IMBHs could accrete gas and manifest themselves as ultraluminous X-ray sources (ULXs), those with a luminosity \( L_X \geq 10^{39} \text{ erg s}^{-1} \). However, various models have been proposed to explain ULXs observed in the disks of some galaxies even without invoking IMBHs (Roberts 2007; King 2008). We have to say that the currently available observational data give no unambiguous answer to the question about the existence and abundance of IMBHs in the Universe. There are only specific examples and some evidence and theoretical arguments for their presence. In this paper, we propose a model in which the IMBH formation is suppressed under effective SMBH formation in galactic nuclei. This scenario can become topical if future observations will constrain significantly the number
of IMBHs. We propose a modification of the mechanism initially suggested by Rubin et al. (2001b) that allows not only the presence of SMBHs at galactic centers but also the suppression of IMBH formation to be explained.

**THE FORMATION OF PRIMORDIAL BLACK HOLES WITH ALLOWANCE MADE FOR THE CLASSICAL MOTION OF THE PHASE OF A COMPLEX SCALAR FIELD**

The subsequent discussion is based on the primordial SMBH formation model developed by Rubin et al. (2000, 2001a, 2001b), Khlopov and Rubin (2004), Khlopov et al. (2005), and Dokuchaev et al. (2005, 2008), who showed that if the inflaton potential has a local maximum, then the SMBH formation probability at the post-inflationary epoch is high. These authors described in detail the formation of a black hole through quantum field fluctuations near this maximum. The consideration is based on the shape of a potential known as a “tilted Mexican hat”, where the local maximum or, more precisely, the saddle point is located at points $\theta = \pi, 2\pi, \ldots$ ($\theta$ is the phase of a complex scalar field). This potential, which is used, for example, to describe inflation on a pseudo-Nambu-Goldstone field (Dolgov and Freese 1995), is

$$V(|\Phi|) = \lambda[\Phi^*\Phi - f^2/2]^2,$$

with the minimum at $\langle \Phi \rangle = f e^{i\phi}/\sqrt{2}$. However, we will emphasize that the field $\Phi$ under consideration is additional to the inflaton one.

The problem is slightly simplified on energy scales smaller than $f$, because the massive radial mode of $\Phi$ ($m_{rad} = \lambda^{1/2} f$) can be discarded. The remaining light degree of freedom is defined by the angular variable $\theta = \phi/f$ that acts as a pseudo-Nambu-Goldstone boson (PNGB). Through spontaneous $U(1)$ symmetry breaking on energy scales $\sim \Lambda \ll f$, the PNGB potential acquires an additional term,

$$V(\theta) = \Lambda^4[1 - \cos \theta],$$

In the equation of motion for the field $\theta$ with potential (1), (2),

$$\ddot{\theta} + 3H\dot{\theta} + \frac{\Lambda^4}{f^2} \sin \theta = 0$$

where the Hubble parameter $H$ remains constant during inflation, because the field $\theta$ is not the inflaton one. During inflation, the quantity $\theta$ changes slowly and only after the end of the inflationary stage does $\theta$ execute rapid oscillations near the minima necessary for efficient production of matter particles and heating of the Universe. Depending on initial conditions, the field $\theta$ can roll down to one of the minima, $\theta_{\text{min}} = 0$ or $\theta_{\text{min}} = 2\pi$.

During the inflationary stage, when the field $\theta$ is in the valley of the “Mexican hat” $|\Phi| = f/\sqrt{2}$, the space is divided into many causally disconnected regions. The values of $\theta$ in these regions slightly differ due to quantum fluctuations. In some of them, the field $\theta$ may turn out to be on the other side of the maximum $\theta_{\text{max}} = \pi$ and after the end...
of inflation may roll down to a value of the minimum different from that to which most other regions tend. Thus, a Universe that consists of chaotically distributed domains with fields of 0 or $2\pi$ inside emerges after inflation. The neighboring domains are separated by the field walls whose subsequent evolution leads to the formation of PBHs. The number of formed PBHs and their mass depend strongly on the tilt of the potential $\Lambda$ and the symmetry breaking scale $f$ at the onset of inflation. Dokuchaev et al. (2005) chose model parameters at which clusters consisting of initially massive PBHs, the largest of which reached a mass $\sim 4 \times 10^7 M_\odot$, were formed. Such massive PBHs can subsequently serve as the nuclei of protogalaxies, increasing their mass to $\sim 10^9 M_\odot$ through accretion. However, in addition to massive PBHs, a large number of intermediate-mass ($\sim 10^4 M_\odot - 10^6 M_\odot$) PBHs are formed. As was discussed in the Introduction, their number may turn out to be too large, which will be in conflict with observational data. The mechanism for the suppression of IMBH formation considered here is based on the following almost obvious fact. The farther the initial phase at which the present-day Universe was formed from the maximum (in our case, $\theta = \pi$), the smaller the volume of space will be filled with a phase $\theta > \pi$. Indeed, after each e-fold, spatial regions with phases advanced toward the maximum of the potential by some small $\delta \theta$ appear due to field fluctuations. The time, the number of e-folds needed for the maximum of the potential to be reached, increases sharply with increasing difference between the initial phase and $\theta = \pi$. Therefore, the total number of closed walls and, hence, black holes formed from them also decreases sharply. In other words, the dispersion of the phase distribution increases with time and the farther the mean field from $\theta = \pi$, the fewer the black holes will appear subsequently. If the mean phase recedes from $\theta = \pi$ with time, then the formation of IMBHs will be suppressed. This is possible if the classical motion of the phase is taken into account.

So far the classical motion of the phase $\theta$ of a complex scalar field at the stage of inflation has been neglected. This is true if the tilt of the potential $V(\theta)$ defined by the parameter $\Lambda$ is negligible. If, however, this assumption does not hold, then, as a result of the classical motion, the mean field recedes from the maximum and the IMBH formation probability turns out to be suppressed. Since the main idea of our paper is to take this effect into account, let us consider it in more detail. We will measure the time in e-folds $N$ before the end of inflation. Let there be some region of space appeared $N_r$ e-folds before the end of inflation and filled with a field with phase $\theta_r(N_r)$.

We will be interested in the phase change with time through both quantum fluctuations and classical motion. An appropriate mathematical apparatus was developed by Rey (1987), who proposed representing the field as the sum of the classical term $\Theta$ and fluctuations $\vartheta$ about it,

$$\theta = \Theta(t) + \vartheta, \quad \vartheta \ll \Theta$$

The probability density $P_f(\vartheta)$ for detecting the fluctuational part of the phase $\vartheta$ satisfies the Fokker-Planck equation, whose solution has the form of a Gaussian distribution (Starobinsky 1982). Obviously, the phase distribution of interest to us can be obtained by the simple substitution $P(\theta) = P_f(\Theta - \theta)$.

The tilt of potential (1), (2) is small at the chosen parameters $\Lambda = 1.75 H$ and $f =$
10H and we will take it into account only when calculating the main, classical contribution \( \Theta(t) \) in (4). The small corrections due to the fluctuations \( \theta \) in (4) will be taken into account by neglecting the tilt of the potential lest the accuracy of the calculations be exceeded. In addition, analytical results are known for the spatial phase distribution (see, e.g., Starobinsky, 1982; Rey 1987).

In terms of e-folds, the probability of finding the phase in the interval \( (\theta, \delta\theta) \) at a certain point of space is (Khlopov and Rubin 2004; Rubin et al. 2001b)

\[
\delta P(\theta) = \frac{1}{\sqrt{2\pi (N_r - N)}} \exp \left[ -\frac{(\theta_r(N) - \theta)^2}{2\delta\theta^2 (N_r - N)} \right]; \quad \delta\theta = \frac{H}{2\pi f},
\]

Here, as was said above, we neglected the tilt of the potential. The classical part of the phase \( \Theta(t) \) is represented as \( \theta_r(N) \), where the number of e-folds is \( N = Ht \).

The condition for the appearance of one, and only one, SMBH at time \( N_0 \)

\[
\delta P(\pi) = \frac{1}{e^{3(N_r-N_0)}}
\]

indicates that black holes are formed when the extremum of the potential \( \theta = \pi \) is crossed. Now, from the equation

\[
e^{3(N_r-N_0)} = \sqrt{2\pi (N_r - N_0)} \exp \left[ \frac{(\theta_r(N_0) - \pi)^2}{2\delta\theta^2 (N_r - N_0)} \right],
\]

we will find the phase \( \theta_r(N_0) \) from which the spatial region must be produced for the SMBH formation conditions to emerge \( N_0 \) e-folds before the end of inflation,

\[
\theta_r(N_0) = \pi - \delta\theta \sqrt{(N_r - N_0) \left[ 6 (N_r - N_0) - \ln 2\pi (N_r - N_0) \right]}.
\]

Let us now determine the formation probability of lower-mass black holes produced at an e fold \( N \), so that \( N < N_0 < N_r \). Figure 1 show the dependence of the phase \( \theta \) on time expressed in e-folds obtained by numerically solving Eq. (3) and in the slow-rolling approximation. The excellent agreement indicates that the system is in the regime of slow rolling. In the subsequent calculation of the probability of quantum fluctuations in field \( \theta \), it is appropriate to use not the numerical solution of Eq. (3) but its linear fit. Since intermediate-mass PBHs are formed from large domains emerging in a small number of e-folds, the solution of (3) is fitted in the initial segment \( (N < 20) \). It is convenient to fit the dependence by a linear function,

\[
\theta_r(N) \simeq \theta_r(N_0) - \alpha \cdot (N - N_0)
\]

The probability for the emergence of IMBH formation conditions during an e-fold number \( N \) is then

\[
\delta P(\pi) = \frac{1}{\sqrt{2\pi (N_r - N)}} \exp \left[ -\frac{(\theta_r(N_0) - \alpha \cdot (N - N_0) - \pi)^2}{2\delta\theta^2 (N_r - N)} \right].
\]
Given the number of causally disconnected regions \( \exp[3(N - N_r)] \) formed since the formation of the spatial region \( N_r \), we can find the total number of IMBHs in this region. A detailed description was given by Khlopov and Rubin (2004), who, in contrast to our case, disregarded the classical motion of the field, i.e., \( \alpha = 0 \).

Figure 2 presents the PBH mass distribution for \( \Lambda = 1.75H \) and \( f = 10H \). As we see, the IMBH formation actually turns out to be suppressed. The mass function is bimodal in shape: the region of low-mass black holes is separated by a dip from SMBHs.

**ASTROPHYSICAL IMPLICATIONS**

In the preceding section, we showed that the PBH mass spectrum could have two isolated regions. The maximum at the highest masses gives SMBHs that can become SMBHs in galactic nuclei, possibly, by additionally increasing their mass through gas accretion. Less massive IMBHs form a population of black holes in galactic halos. Since their formation is suppressed, these IMBHs are very rare and they can be virtually unobservable. Let us discuss in more detail the paths of black-hole evolution from these two sets.

Under the influence of dynamical friction, a black hole that was initially in the halo loses its angular momentum and approaches the galactic center. However, the dynamical friction mechanism is efficient only for fairly massive objects, more specifically, objects with a mass of more than \( \leq 10^7 M_\odot \) sink to the galactic center in the Hubble time. An important factor that contributes to the motion of black holes to the galactic center is the formation of an induced dark matter halo around the black hole (Dokuchaev and Eroshenko 2001a, 2003; Mack et al. 2007). The mass of the induced halo exceeds the black-hole mass approximately by two orders of magnitude. Thus, black holes with a mass \( \geq 10^5 M_\odot \) surrounded by the induced halo accumulate and merge at the galactic center. If the internal structure of the induced halo depends only on the PBH mass, then the boundary of the induced halo is determined by the environment in which the halo is formed. The neighboring density fluctuations at a pregalactic stage will be an alternative center of attraction for dark matter starting from some distance from the black hole (Dokuchaev and Eroshenko 2001a, 2003).

If a pair of IMBHs was formed at the galactic center and if a third IMBH approached the pair under the action of dynamical friction, then the slingshot effect is possible: the pair becomes even closer as energy is exchanged during the gravitational interaction, while the third IMBHacquires a high velocity and is ejected away from the galactic center (Haehnelt and Kauffmann 2002). The detection of such ejected IMBHs would serve as evidence for the formation of SMBHs from merging lower-mass IMBHs. Another method for testing various SMBH formation models is to search for gravitational wave bursts from black-hole mergers. For example, in the model from Dokuchaev et al. (2008), IMBHs in central clusters in galactic nuclei collide with one another with the generation of gravitational wave bursts (Dokuchaev et al. 2009), with the burst redshift distribution differing from that provided by “astrophysical” scenarios for the origin of SMBHs. Therefore, observations with the LISA telescope planned to be launched can provide a choice between various scenarios for the origin of SMBHs (Dokuchaev et al. 2009).
In an alternative scenario, for example, that considered here, the central black hole initially had a high mass. In the hierarchical picture of galaxy formation, a SMBH fell into a large galaxy together with one of the small protogalaxies and rapidly sank to its dynamical center through dynamical friction. The advantage of the model with massive primordial SMBHs is that it can successfully explain the presence of quasars at high redshifts (Dokuchaev et al. 2005).

Lower-mass ($\leq 10^5 M_\odot$) black holes together with the surrounding induced dark matter halos have no time to sink to the center and remain in the galactic halo. They can be observable as ultraluminous X-ray sources when passing through dense molecular clouds in the disk. Since the gas density in the galactic halo is low, accretion directly from the halo will not give a significant X-ray flux. However, the case where an IMBH moving in the halo carries a small induced dark matter halo and, most importantly, a cloud of baryons and an accretion disk is possible. In this case, the X-ray emission from the IMBH will be determined by the accretion of baryons from the induced halo. This scenario appears problematic, because no cooling of baryons, in particular, no star formation, takes place in low-mass halos. For this reason, there is no flow of gas onto the central IMBH. This also implies that the observed ultraluminous X-ray sources, candidates for the role of IMBHs, are present only in galactic disks.

Let us discuss the growth of IMBH masses through accretion in the time of their passages through the galactic disks. For Bondi-Hoyle accretion, a black hole has a luminosity $L_X = 4\pi\eta c^2 G^2 M^2 \rho v^{-3}$ (Mii and Totani 2005; Mapelli et al. 2006), where $v$ is the velocity with which the black hole moves through a gas with density $\rho$ and $\eta$ is the mass-to-energy conversion efficiency. This parameter is $\eta \approx 0.06$ for disk accretion onto a Schwarzschild black hole and reaches $\eta \approx 0.42$ accretion onto an extreme Kerr black hole. Typically, $\eta \approx 0.1$. In this case, the black hole mass increases with the characteristic time $t_{BH} = (4\pi G^2 M \rho v^{-3})^{-1}$. If the luminosity is assumed to correspond to the luminosity of observed ultraluminous X-ray sources, then

$$t_{BH} \approx 6 \times 10^{10} \left(\frac{M}{10^4 M_\odot}\right) \left(\frac{L_X}{10^{39} \text{ erg s}^{-1}}\right)^{-1} \text{ yr.}$$

Thus, an IMBH has no time to increase appreciably its mass in the lifetime of the Universe even if it always moves in the disk. A larger mass growth could be expected for the Eddington luminosity of a black hole at rest in a molecular cloud. However, the probability of an IMBH being captured into the galactic disk is very low. The probability of an IMBH being directly in a cloud of cold gas is even lower, because molecular clouds occupy only a disk volume fraction $f_{MC} \approx 0.017$ (Mapelli et al. 2006). Consequently, no black hole mass redistribution occurs and the region of the IMBH mass function remains suppressed.

**CONCLUSIONS**

Here, we presented a new SMBH formation mechanism in which the formation of IMBHs is suppressed. Whereas the existence of SMBHs at galactic centers is already
beyond doubt, the currently available observational data provide only circumstantial evidence for the existence of IMBHs. Since in some models for the formation of black holes their mass spectrum is close to a power law, the formation of SMBHs is inevitably accompanied by the formation of a large number of IMBHs. In the model we developed, this does not happen. On the contrary, the formation of IMBHs is suppressed through the classical motion of the phase of a scalar field additional to the inflaton. If detailed astronomical observations will establish that the number of IMBHs in the Universe is very small, then this fact can be explained in the presented model.

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Figure 1: Dependence of $\theta$ on the number of e-folds at $\Lambda = 1.75H$ and $f = 10H$: the solid line and the dots represent the numerical calculation and the analytical calculation in the slow-rolling approximation, respectively; the fitting straight line is indicated by dashes, its slope is $\alpha = 0.028$. 
Figure 2: PBH mass distribution: the solid line indicates the distribution for $\alpha = 0.028$ and $\Lambda \approx 1.75H$; the dashed line indicates the distribution for $\alpha = 0$. 