The Third Frequency Range of the Sound Insulation Plot of the Single-Layer Partitions

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Abstract. The main concepts of sound insulation theory of the massive plates were developed in the mid-20th-century. There were obtained the frequency dependence of sound insulation from the plate mass and the plate stiffness [1, 2]. The multiple researches on dependence of insulation from sound incidence angle [1, 2, 3, 4] and coefficients of internal friction for plates materials were carried out [1, 2, 3, 4]. The comparison of theoretical and experimental results showed the generally statistical matching, but in the same time, there were frequently observed their considerable differences [1, 2]. There was a necessity in additional studies in that period. Due to the electronics development and perfect appliances on its base, the observational researches begun to prevail, such as in issues [5, 6, 7, 8, 9, 10]. Thereafter, the reference curves with the three frequency ranges for sound isolation calculation were obtained, due to the large mass of experimental data processing. The first two ranges are divided by the wave coincidence frequency, and the third one limits the maximum possible insulation irrespective of frequency. The articles [11, 12, 13] show, that the existing formulas of sound propagation through the interface of two media and the flat layers, which are developed from the continuity conditions, mismatch to these conditions on all incident angles, except of the normal. So the present insulation theories, which are depending from the sound incident angle, make no sense. The issue [14] shows, that the calculation sound insulation method, which is based on discrete (concentrated) models of sound propagation with the application of momentum conservation law and the kinetic energy for the plates with the mass more than 50kg/m², gives the more exact results of comparison with the experimental data [15, 16], than with the calculation data by the standard curve. The contributions [15, 16] contains the evidences about the sound insulation values mainly in first two frequency ranges. The third frequency range is become beyond the limits of measurements. This article contains the results of publication materials, involving the sound insulation data at the third frequency range.

1. Introduction

Accordingly the Russian normative documents, an airborne sound insulation capacity of single-layer partitions is normalized in frequency diapason from 100 till 3200 Hz [17].
Figure 1. The sound insulation graphs for reinforced concrete single-layer partition of 300 mm thickness: 1 - are the experimental data [15]; 2 - is the sound insulation line by [17]; 3 - is the graph by the proposed method; I, II, III - are the frequency plots of considering spectrum.

The figure 1 depicts the outline (curve 2) for reference calculated curve of the airborne isolation spectrum by the massive one-layer wall, that consists from three linear regions. The first linear region denotes the insulation of massive plate without elasticity, that is incompressible by its thickness. Here the sound insulation is defined by the "Mass Action Law" and by the natural shear oscillation frequencies of the plate.

The second spectrum region reveals the diapason of so called "wave coincidence", where the plate is treated as possessing mass and elasticity, but being incompressible by its thickness. This is the area of flexural wave existence in the plate, at which the waves have the phase matching with longitudinal air oscillations and by this way they determine the sound insulation level. The first and the second graph regions of the normative spectrum have their elaborated theories and algorithms of the calculation [1, 2, 17]. The insulation regularities at the third range have not yet the complete proof and clear calculation technique. Its value is taken equal to 65 dB at the normative documents due to the experimental data base generalization.

Therefore, the scientific rational of acoustical process and the creation of simple calculation algorithm for this range, in case of one-layer massive walls with the surface weight (density) from 100 till 800 kg/m², appear to be of a great interest. These problems can be successfully solved under the using of the sound insulation calculation method on the base of a physical model with the concentrated (lumped) parameters, that model is not contradictory to the continuity conditions [11, 12, 13, 14, 18, 19, 20].

2. Materials and methods

The third frequency range of the sound insulation curve begins with the frequencies, where the flexural waves cease to exist and the shear waves start to occur. These frequencies lay nearby to the limit frequencies of dilatational waves appearance. The dilatational waves propagate along the plate (partition) thickness. The dilatational and shear waves have the constant velocities values, which do not depend on oscillation frequency, unlike the flexural ones. So the third diapason segment is flat-lying at the sound insulation spectrogram.

As it was pointed out in annotation, the continuity conditions in currently accepted model of the sound transmittance are fulfilled only under the normal sound incidence on the plate or on the interface media. In case of the oblique incidence, the continuity conditions are satisfied with an allowance for the Snell law, accordingly which, the angles of incidence and of refraction for sound beams belong to one plane, and besides, the thickness of the media layer is constant at this plane, while the changing of cross section areas of the incident and the refracted beam takes place due to their width variation [11, 12, 13]. The incident and refracted oblique beam changing compensates the changing of oscillation velocity vectors insomuch, that the transmission and reflection coefficients at any angles are equal to their values in case of the normal incidence of the sound on the interface of two media or on the separating plate.

The standard sound insulation curve always represents the minimum value of insulation from all of every possible variants. So that, the purpose of this research is a determination of waves type, which provides the lowest insulation value.
The notion of "reduced mass", μ, is introduced by the applied method in this article:

\[ \mu = \frac{p\lambda}{2\pi} = \frac{\rho c}{2a} [\text{kg} \cdot \text{m}^{-2}], \]

where \( \rho \) - is the medium density, [kg \cdot m\(^{-3}\)]; \( c \) and \( \lambda \) - are the propagation velocity, [m \cdot s\(^{-1}\)], and the wave length of given form (dilatational, flexural or shear), [m]; \( f \) - is the oscillation frequency, [Hz].

In other words, in this treated model, the reduced mass is represented by the concentrated mass, which is equivalent buy its magnitude to the mass of the medium fragment, which is contained in the sound beam with the lateral section in 1 m\(^2\), on the segment of \( 1/2\pi \) wave length at the current frequency. Such mass can be taken into consideration beginning from the frequencies, lying higher than the "ultimate" oscillation frequency, \( f_{ult.} \):

\[ f_{ult.} = \frac{c}{2aL} \ [\text{Hz}], \]

where \( L \) - is the dimension of an object, along which the sound wave propagates, [m]; \( c \) - is the propagation velocity of the definite wave type in the object, [m \cdot s\(^{-1}\)].

In this case, the sound transmittance through the interface of two media can be represented as the reduced masses interaction (collision), that is described by the equation of the momentum conservation:

\[ \mu_1 \cdot v = \mu_2 \cdot v \cdot \beta + \mu_2 \cdot v \cdot \alpha; \]

and by the equation of kinetic energy conservation:

\[ \frac{\mu_1 v^2}{2} = \frac{\mu_1 (\beta v)^2}{2} + \frac{\mu_2 (\alpha v)^2}{2}, \]

where \( v \) - is the unite velocity of the media fragment movement, [m \cdot s\(^{-1}\)]; \( \mu_1 \) - is the reduced mass of the first medium, [kg \cdot m\(^{-2}\)]; \( \mu_2 \) - is the reduced mass of the second medium, [kg \cdot m\(^{-2}\)]; \( \alpha \) and \( \beta \) - are the transmittance and the reflection coefficients of the oscillation speed respectively.

Simultaneous solution of the equations (3) and (4) gives the following formula for the transmission coefficient \( \alpha \): \[ \alpha = 2\mu_1 \frac{v}{\mu_1 + \mu_2}; \]

Let us consider the transmittance of the dilatation wave through the plate thickness. The frequency, after which such oscillation movement is possible, is determined by the formula (2), where \( c \) - is the propagation velocity of the dilatational wave in a plate and \( L \) - is a plate thickness.

In case of longitudinal waves appearance in the plate (partition), it is necessary to take into consideration the summary insulation at the two interfaces of the two media: air-plate and plate-air. It can be written mathematically in the form (6):

\[ R_{dil.} = 10\log \frac{1}{\alpha_1^2} + 10\log \frac{1}{\alpha_2^2} = 10\log \left( \frac{\mu_a + \mu_{pl}}{2\mu_a} \right)^2 + 10\log \left( \frac{\mu_{pl} + \mu_a}{2\mu_{pl}} \right)^2 = 2 \cdot 10\log \left( \frac{\rho_{pl} + \rho_a}{4\mu_a \mu_{pl}} \right)^2 \ [\text{dB}], \]

where \( R_{dil.} \) - is the sound insulation for dilatational waves, [dB]; \( \alpha_1 \) and \( \alpha_2 \) - are the transmittance coefficients for oscillation velocities at the first and the second interfaces of two media respectively; \( \mu_a \) - is the reduced mass of an air, [kg \cdot m\(^{-2}\)]; \( \mu_{pl} \) - the reduced mass of the plate (partition) material, [kg \cdot m\(^{-2}\)].

Under the condition \( \mu_{pl} \gg \mu_a \):

\[ R_{dil.} = 20\log \frac{\rho_{pl}}{4\mu_a} = 20\log \frac{\rho c_{dil.}}{2\rho_0 \phi_0} \ [\text{dB}], \]

where \( \rho \) - is a partition material density, [kg \cdot m\(^{-3}\)]; \( \rho_0 \) - is the air specific weight, [kg \cdot m\(^{-3}\)]; \( c_0 \) - is the sound speed in the air, [m \cdot s\(^{-1}\)]; \( c_{dil.} \) - is the dilatational wave propagation velocity in partition material, [m \cdot s\(^{-1}\)], by formula (8):

\[ c_{dil.} = \sqrt{\frac{E}{\rho(1-\nu^2)}} \ [\text{m} \cdot \text{s}^{-1}]; \]

where \( E \) - is an elasticity modulus, [N \cdot m\(^{-2}\)]; \( \nu \) - is the Poisson ratio of material.

As was mentioned above, the shear oscillations occur along with the dilatational waves at the third range of the acoustical spectrum. The frequency of the third range beginning for the shear waves is
also found by the formula (2). These waves velocity, \(c_{sh.}\), by formula (9) will not depend on the frequency, and their wave coincidence angle will be the same for every frequency too.

\[
c_{sh.} = \frac{E}{2\rho(1+\nu)} \text{[m/s]}, \tag{9}
\]

Then, the sound isolation for a such type of the waves can be written as the equation (10):

\[
R_{sh.} = 10\log\left(1 + \left(\frac{\pi \mu_{sh.}}{\rho \omega c_0}\right)^2\right) \text{[dB]}, \tag{10}
\]

where \(\mu_{sh.}\) - is the reduced mass of the partition for shear waves, that is defined by the formula (1).

It should be noted, that the experimental sound insulation values at the third range are lower, than calculated by formulas (7) and (10) for both dilatational and shear waves. That's due to the sound insulation decrease because of the resonance phenomena in the plate, which arise in case of dilatational and shear waves reflections from the connection joints of the partition between the adjacent walls and floor slabs, equation (11):

\[
R_{tot.,(sh.)} = R_{dil.,(sh.)} + \Delta R_{dil.,(sh.)} \text{[dB]}, \tag{11}
\]

where \(R_{tot.,(sh.)}\) - is the total partition isolation under considering the dilatational or shear waves, [dB]; \(R_{dil.,(sh.)}\) - is the partition isolation derived from formulas (7) and (10), [dB]; \(\Delta R_{dil.,(sh.)}\) - is the correction on sound insulation decrease from resonances in the process of dilatational or shear waves propagation, [dB].

In accordance with the [12], the formula for the determination of resonance corrections has a form (12):

\[
\Delta R_{dil.,(sh.)} = -20\log\left(1 - \beta_{dil.,(sh.)}\right) \text{[dB]}, \tag{12}
\]

where \(\beta_{dil.,(sh.)}\) - is the reflection coefficient of the oscillation speed from the fixed ends of the partition during the dilatational or shear waves action.

The dilatational waves in a partition (wall) material cause the shear waves in material of the adjacent constructions in case of cross (perpendicular) joints. In the first instance, the horizontal floor slabs refers to such constructions. Thereat, the reflection coefficients can be defined by the formula (13):

\[
\beta_{dil.1} = \beta_{dil.2} = \frac{\mu_{pl.,dil.} - \mu_{dil.,sh.}}{\rho_{pl.,dil.} - \rho_{dil.,sh.}} \tag{13}
\]

where \(\mu_{pl.,dil.}\) - is the reduced mass of the partition under consideration of the dilatational waves and its thickness, [kg\cdot m\(^{-1}\)]; \(\mu_{dil.,sh.}\) - is the reduced mass of the floor slab under consideration of the shear waves and its thickness, [kg\cdot m\(^{-1}\)].

Thereof, if the shear oscillations appears in the partition (wall), the dilatational waves will occur in the perpendicularly adjacent constructions, first of all, in the floor slabs. The reflection coefficients from the upper and the lower fixing points of the partition, \(\beta_{sh.1}\) и \(\beta_{sh.2}\) can be found by the equation (14):

\[
\beta_{sh.1} = \beta_{sh.2} = \frac{\mu_{pl.,sh.} - \mu_{pl.,dil.}}{\rho_{pl.,sh.} + \rho_{pl.,dil.}} \tag{14}
\]

where \(\mu_{pl.,sh.}\) - is the reduced mass of the partition under consideration of the shear waves and its thickness, [kg\cdot m\(^{-1}\)]; \(\mu_{sl.,dil.}\) - is the reduced mass of the floor slab under consideration of the dilatational waves and its thickness, [kg\cdot m\(^{-1}\)].

3. Results and discussion

In such a way, either of the two considering wave types will have a predominant impact on eventual (minimal) sound isolation values, as example, figure 1. The values of reflection coefficients will depend heavily from the slabs and walls (partitions) connection variants.
As a rule, the accurate information on this point is not reported in the specialized literature, that makes rather problematic the reflection coefficients search. Nonetheless, within the framework of this article, there were performed the computations for the six indoor partitions variants with the available experimental insulation plots. The floor slab was taken into consideration with a thickness of 200 mm.

**Figure 1.** The sound insulation graphs for reinforced concrete single-layer partition of 300 mm thickness at the third spectrum range: 1 - are the experimental data [15]; 2 - is the sound insulation line by [17]; 3 - is the graph by the proposed method for dilatational waves; 4 - is the same for shear waves.

| Material of a partition | Partition thickness, h, (m) | Density, ρ, (kg/m³) | Elasticity modulus, E, ×10¹₀ (N/m²) | Sound insulation of dilatational waves, R_{dil.} (dB) | Sound insulation of shear waves, R_{sh.} (dB) | Measured sound insulation, R_{exp.}, (dB) |
|------------------------|--------------------------|---------------------|-----------------------------------|---------------------------------------------|---------------------------------------------|-------------------------------------------|
| Reinforced concrete    | 0,3                      | 2300                | 2,98                              | 69,09                                       | 75,87                                       | 68,47                                     |
| Reinforced concrete    | 0,206                    | 2300                | 2,98                              | 71,25                                       | 77,32                                       | 66,45                                     |
| Lightweight concrete   | 0,161                    | 1500                | 1,5                               | 69,14                                       | 73,89                                       | 65,58                                     |
| Brickwork              | 0,52                     | 1800                | 1,6                               | 64,64                                       | 71,48                                       | 70,0                                      |
| Brickwork              | 0,38                     | 1800                | 1,6                               | 66,54                                       | 72,77                                       | 65,40                                     |
| Brickwork              | 0,25                     | 1800                | 1,6                               | 68,77                                       | 74,16                                       | 64,95                                     |

Accordingly performed calculations, the arithmetic average deviation from the experimental data for all partitions variants by proposed method is 1,43 dB in case of dilatational waves and 7,44 dB for the shear oscillations. It should be noted, that the details of the experiments undertaking are not covered in the source of field measurements, so why the results of comparison between the obtained figures and the experimental data need in further consideration and refinement.

**4. Conclusions**

The considered in article sound wave propagation model in third frequency range, which is based on the discrete parameters method, allows to obtain and to explain theoretically the insulation value of 65 dB, that is given in [17].

The formulas for the determination of the lower frequency bound for the sound insulation graph in the third diapason are represented.

The general technique for engineering sound insulation calculation in the third range is given, with an allowance for the appearance of shear and dilatational oscillations in the material of a partition.

The comparison between the computation and the experimental results is carried out. There were found out that the lager results matching occurred in case of dilatational waves against the case of shear oscillations.
Thus, the proposed sound insulation calculation method in third frequency spectrum range is globally proven by the actual measurements results, however, it needs the following elaborations and researches, including further experiments.

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