Nonmesonic Weak Decay of Light Hypernuclei with Coherent $\Sigma$ Mixing

K. Sasaki$^{a,b}$, T. Inoue$^c$, and M. Oka$^b$

$^a$ Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan

$^b$ Department of Physics, Tokyo Institute of Technology
Meguro, Tokyo 152-8551 Japan

$^c$ Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC
Institutos de Investigación de Paterna, Apdo. correos 22085, 46071, Valencia, Spain

March 30, 2022

Abstract

Nonmesonic weak decays of the $A = 4$, and 5 hypernuclei are studied. The short range parts of the hyperon-nucleon weak interactions are described by the direct quark (DQ) weak transition potential, while the longer range interactions are given by the $\pi$ and $K$ meson exchange processes. Virtual $\Sigma$ mixings of the coherent type are found to give significant effects on the decay rates of $^4_A\text{He}$. A large violation of the $\Delta I = 1/2$ rule is predicted in the $J = 0$ transition amplitudes.

1 Introduction

Recent progress in experimental studies of hypernuclei has opened a new era of hypernuclear physics that challenges high precision understanding of hypernuclear structures and interactions. Weak decays of hyperons especially provide us with rich phenomena in which nuclear
many-body dynamics and nonperturbative QCD effects on hadronic weak interaction are important.

Nonmesonic weak decay of hypernucleus is unique as a new type of weak interaction process, which involves only baryons. In weak decays of heavy hypernuclei, pion emissions via \( \Lambda \rightarrow p\pi^-, n\pi^0 \) are hindered due to the Pauli blocking on the final state nucleon. Instead, the decay is mainly induced by a two-body transition, \( \Lambda N \rightarrow NN \), and does not emit pions. This nonmesonic weak baryon-baryon interaction is analogous to the parity violating part of the nuclear force \( |\Delta S| = 1 \) sector.

The mechanism of nonmesonic decay of hypernuclei is a long standing problem due to some disagreements between theory and experiment. One of them is the \( \Gamma_{nn}/\Gamma_{pn} \) ratio, i.e., the ratio of \( \Gamma_{nn} = \Gamma(\Lambda n \rightarrow nn) \) and \( \Gamma_{pn} = \Gamma(\Lambda p \rightarrow pn) \) transitions. Another, to the validity of \( \Delta I = 1/2 \) rule. Many studies have been done. The one-pion-exchange (OPE) is the simplest to describe the \( \Lambda N \rightarrow NN \) transition. It has a similar structure as the OPE in nuclear force and in both of them the tensor transition is strong. As it enhances \( \Gamma_{pn} \), the \( \Gamma_{nn}/\Gamma_{pn} \) ratio is suppressed. Typical prediction for OPE is \( \Gamma_{nn}/\Gamma_{pn} \sim 0.1 \), while most experimental data indicate \( \Gamma_{nn}/\Gamma_{pn} \sim 1 \) or larger [1].

In the nonmesonic decay, the \( \Lambda-N \) mass difference is reflected in a high momentum transfer between the baryons, and therefore, short range interaction effects must be important. The lifetime measurements of heavy hypernuclei show saturation at large \( A \) and therefore suggest importance of short range interactions [2].

An attempt was made in refs. [3, 4, 5] by considering the effect of the polarization of the pion propagator, which seems to enhance \( \Gamma_{nn}/\Gamma_{pn} \) slightly. The \( \rho \) meson contribution was calculated in refs. [3, 4], but it could not reproduce the experimental data. The two-pion exchange and the effective \( \sigma \) meson exchanges [6, 7, 8] may play an important role because it increases the central attraction which enhances the \( \Gamma_{nn}/\Gamma_{pn} \) ratio through the enhancement of the \( J = 0 \) channels. The pseudoscalar and vector mesons (\( \pi, K, \eta, \rho, \omega, \) and \( K^* \)) are considered in [11, 12]. The contribution of each meson comes out large, especially for kaon exchange, and significant cancellations between the different contributions are found [13]. The final result is 4-6 times larger than the OPE one [14].

Apart from the meson exchange model, several authors pointed out the importance of quark degrees of freedom in baryons [15, 16, 17, 18, 19]. Recently we proposed to treat the short range part using the valence quark picture of the baryon and the effective four quark weak hamiltonian. We found that the weak quark transition, called the direct quark (DQ) process, gives significantly large contribution and shows qualitatively different features from the meson exchange mechanism, especially in its isospin structure [16]. In this process the \( \Delta I = 3/2 \) contribution is naturally involved, which is found to be important in the \( J = 0 \) decay channels. We think that DQ is a key process to solve the puzzle of \( \Gamma_{nn}/\Gamma_{pn} \) ratio and to reveal
the mechanisms of the $\Delta I = 1/2$ rule for the weak $|\Delta S| = 1$ decay.

In a previous paper [18], we proposed a quark-meson hybrid model, which includes the DQ transition potential supplemented by the long-range part that comes from one-pion (OPE) and one-kaon (OKE) exchanges. It was shown that the model predicts fairly large $\Gamma_{nn}/\Gamma_{pn}$ for the decay of $\Lambda$ in nuclear matter. In this report, we apply this model to the weak decay of $^4\Lambda$He, $^4\Lambda$H, and $^5\Lambda$He. We especially concentrate on the effect of virtual $\Sigma$ mixing in $A = 4$ system.

In Sect.2, we discuss the basic ingredients of the calculation, and the weak transition potential is explained in Sect.3. The $\Sigma$ mixing contributions are considered in Sect.4. We show our results in Sect.5, and give the conclusions in Sect.6.

2 Light hypernuclei

Light hypernuclei have advantages in order to extract pure information of the elementary process, as the emitted nucleons are less distorted than in medium and heavy hypernuclei, as $^{12}_{\Lambda}$C, where final state deformations seem to be significant according to the recent data from KEK [20]. Observables of the weak decay of light hypernuclei give us a clue to clear up some puzzles concerning the nonmesonic decay, the $\Gamma_{nn}/\Gamma_{pn}$ ratio and the $\Delta I = 1/2$ dominance. Block and Dalitz [1] performed an analysis based on the lifetime data of light hypernuclei, which were updated by some other authors [21, 22]. They tried to confirm the $\Delta I = 1/2$ dominance in medium, but they were unsuccessful so far due to large uncertainties in the data. Another advantage of the light hypernuclei is that the decay observables may give us evidence of virtual $\Sigma$ excitation in $\Lambda$ hypernuclei. This is the main interest of this paper.

For s-shell hypernuclei, the initial $YN$ system can be assumed to be in the relative $s$-wave state, and we consider the $YN \rightarrow NN$ transition with the six $^{2S+1}L_J$ combinations listed in Table 1. Note that $^4\Lambda$He may be mixed with $^4\Sigma_+\Lambda$He, in which the $\Sigma^+p$ pair induces a new decay channel with the $I_f^I = +1$ final states, i.e., $\Sigma^+p \rightarrow pp$. Thus in Table 1 we have extra amplitudes $a_{pp}$, $b_{pp}$, and $f_{pp}$, which are absent in the previous approaches. As the two proton final state does not appear without the virtual $\Sigma$ state, it should give a good signature of the $\Sigma$ mixing in $^4\Lambda$He.

The main observables are the decay rates. The total decay rates are the sum of the proton induced $\Gamma_{pn}$ and the neutron induced $\Gamma_{nn}$ decay rates. They are given by summing up the squared amplitudes of the relevant channels in Table 1. The decay rates from $J = 0$, $\Gamma_{J=0}$, and $J = 1$, $\Gamma_{J=1}$, channels are often useful, though they are not directly measurable. The ratio of the parity violating, $\Gamma_{PV}$, and the parity conserving, $\Gamma_{PC}$, decay rates, $PV/PC$, is also an interesting quantity. Among the six channels given in Table 1, $a$, $c$, and $d$ are PC, while $b$, $e$, and $f$ are PV. Although this ratio is not directly observable, the asymmetry of the
Table 1: Possible $^2S^1L_J$ combination and amplitudes for the $YN \rightarrow NN$ transitions.

| $^2S^1L_J$ Comb. | Final Isospin $I^f$ | Amplitudes $I_z^f = 0$, $I_z^f = -1$, $I_z^f = +1$ |
|------------------|-------------------|------------------------------------------------|
| $^1S_0 \rightarrow ^1S_0$ | $I^f = 1$ | $a_{pn}$ $a_{nn}$ $a_{pp}$ |
| $^3P_0$ | $I^f = 1$ | $b_{pn}$ $b_{nn}$ $b_{pp}$ |
| $^3S_1 \rightarrow ^3S_1$ | $I^f = 0$ | $c_{pn}$ |
| $^3D_1$ | $I^f = 0$ | $d_{pn}$ |
| $^1P_1$ | $I^f = 0$ | $e_{pn}$ |
| $^3P_1$ | $I^f = 1$ | $f_{pn}$ $f_{nn}$ $f_{pp}$ |

Proton emitted from the spin polarized hypernucleus is sensitive to $PV/PC$. The asymmetry parameter is obtained by

$$\alpha = \frac{2(\sqrt{3}|ae| - |bc| + \sqrt{2}|bd| + \sqrt{6}|cf| + \sqrt{3}|df|)}{|a|^2 + |b|^2 + 3(|c|^2 + |d|^2 + |e|^2 + |f|^2)}$$

(1)

where we define $[ae] \equiv \text{Re}(a_{pn}^*e_{pn})$, etc. Note that there appear interference terms between the $J = 0$ and $J = 1$ amplitudes, such as $[ae]$ and $[bc]$, in eq. (1). The previous calculations often neglected these interference terms, but they are important because their magnitudes are similar to the other terms.

The wave functions of the $s$-shell hypernuclei are rather simple. We assume that the nucleons reside in the lowest energy state of the harmonic oscillator shell model, i.e., given by $(0s)^n$ configuration. The size parameter is chosen so as to reproduce the size of the nucleus without $\Lambda$. Recent theoretical and experimental studies suggest that the size of the nucleus shrinks due to the intruded $\Lambda$ [24], but here we take a conservative approach. Calling the $(0s)^n$ nucleons as the core, the $\Lambda$-core relative motion is described by the solution of the Schrödinger equation with a $\Lambda$-core potential obtained by the convolution of the realistic $\Lambda - N$ interaction.

It was shown that the short-range repulsion between $\Lambda$ and $N$ results in a repulsion at the center of the core and thus the $\Lambda$ is pushed out from the core region. Such a wave function was shown to explain the mesonic decay rates of the light hypernuclei, which are sensitive to the overlap of the $\Lambda$ and $N$ wave functions [23]. When we consider the virtual $\Sigma$ mixing, we assume that the $\Sigma$ single particle wave function is given by a Gaussian, whose $b_\Sigma$ parameter is adjusted according to the $\Sigma$ mass,

$$b_\Sigma = \sqrt{\frac{M_\Sigma + M_N}{M_\Sigma}} b_N$$

(2)
Thus \( \Sigma \) resides more in the central region than \( \Lambda \) and its effect on the weak decay is enhanced. This enhancement is about 10\% in magnitude, as can be shown by comparing with the calculation assuming that \( \Sigma \) wave function is identical to \( \Lambda \).

In computing the two-body decay matrix elements, it is necessary to take into account the short-range correlation. Thus we take the wave function of the \( Y \)-\( N \) two body systems in the form,

\[
\phi_Y(\vec{r}_Y)\phi_N(\vec{r}_N) \left[ (1 - e^{-r^2/a^2})^n - br^2e^{-r^2/c^2} \right]
\]  

with \( r = |\vec{r}_Y - \vec{r}_N| \) and determine the parameters for the SRC so that it reproduces the realistic \( \Lambda N \) correlation [26], which gives \( a = 0.5, b = 0.25, c = 1.28 \) and \( n = 2 \).

The wave function of the final two nucleons emitted in the two-body weak process is assumed to be the plane wave with SRC:

\[
e^{i\vec{K} \cdot \vec{R}_I} e^{i\vec{k} \cdot \vec{r}_I} [1 - j_0(q_c r')] (4)
\]

where \( \vec{r}' = \vec{r}_{N_2} - \vec{r}_{N_1}, \vec{R}' = (\vec{r}_{N_2} + \vec{r}_{N_1})/2 \) and \( q_c = 3.93 \text{ [fm}^{-1}] \). This approximation may be justified for light nuclei as the momenta of the emitted nucleons are relatively high (\( \sim 400 \text{ MeV/c} \)).

### 3 Transition potential

We employ a hybrid model to describe the weak \( YN \leftrightarrow NN \) transition potential [16, 17, 18]. At long and medium distances, the transition is induced mainly by the one pion exchange (OPE) and one kaon exchange (OKE) mechanisms. For example the \( \Lambda p \rightarrow np \) transition potential induced by \( \pi^0 \) exchange is given by

\[
V_{\Lambda p \rightarrow np}(\vec{q}) = G_Fm^2_\pi[\bar{u}_n(A^\Lambda_\pi + B^\Lambda_\pi \gamma_5)u_p] \frac{I_1 \times I_2}{\vec{q}^2 + \vec{m}_2^2} \left( \frac{\Lambda^2 - \vec{m}_2^2}{\Lambda^2 + \vec{q}^2} \right)^2 g_{\pi NN}[\bar{u}_p \gamma_5 u_p] (5)
\]

where \( I_1 \) and \( I_2 \) are the isospin factors and, in this case, given as

\[
I_1 = \langle n|\tau_1^3|\Lambda \rangle = -1, \quad I_2 = \langle p|\tau_2^3|p \rangle = 1. \quad (6)
\]

Here \( \Lambda \) is regarded as a \( \{I, I_3\} = \{1/2, -1/2\} \) state, called spurion. This form guarantees that this transition is purely given by \( \Delta I = 1/2 \) amplitude. The coupling constants \( g_{\pi NN}, G_F, A_\pi \)
and \( B_\pi \) are determined phenomenologically so that the free \( \Lambda \) decay rate is reproduced. A double pole form factor with the cutoff parameter \( \Lambda_\pi = 800 \) MeV is employed. As the energy transfer is significantly large we introduce the effective pion mass

\[
\tilde{m}_\pi = \sqrt{m_\pi^2 - (m_\Lambda - m_N)^2}/4 \simeq 110 \text{MeV}. \tag{7}
\]

The Dirac spinors and the \( \gamma \) matrices in eq. (5) are reduced into the nonrelativistic form in the standard way. After the spin and angular momentum projections, we obtain a local potential for each transition channel in Table 1. The cut off \( \Lambda_\pi = 800 \) MeV for OPE is rather soft compared to a harder cut off \( \Lambda_\pi = 1300 \) MeV employed by the one-boson exchange (OBE) potential model of nuclear force \[27\]. On the other hand, the soft form factor is preferred in the study of \( NN \to NN\pi \) process \[28\]. A reason for this discrepancy is that the OBE potential model requires reasonably strong repulsion and spin-dependent forces induced by vector meson exchange. As the vector mesons are heavy, they need a hard form factor. In our approach, however, the short distance part is described in terms of quark substructures of baryons and only the pion and kaon are employed for the meson exchange potential. We therefore believe that the soft form factor is more consistent for our calculation.

The kaon exchange potential (OKE) can be constructed similarly. Both the strong and weak coupling constants are evaluated under the assumption of the flavor SU(3) symmetry. The cut off for the kaon vertex is taken as \( \Lambda_K = 1200 \) MeV, according to \[12\]. All the couplings used in our calculation are listed in Table 2. Note that, for OKE, it involves the strangeness transfer and thus the strong and weak vertices are exchanged.

For shorter distances, we employ the direct quark (DQ) transition potential based on the constituent quark picture of baryons. In this mechanism, a strangeness changing weak interaction between two constituent quarks induces the transition of two baryons. Here we sketch the derivation, while the details are given in \[16\].

The DQ transition potential is derived by evaluating the matrix elements,

\[
V_{DQ}(k, k')_{L_i, S_i, J_i}^{L_f, S_f, J_f} \equiv \langle NN(k', L_f, S_f, J_f)|H_{\Delta S=1}^{\text{eff}}|YN(k, L_i, S_i, J_i)\rangle. \tag{8}
\]

Here the two-baryon states, \(|BB(k, L, S, J)\rangle\), are expressed by six-quark wave functions, constructed in the quark cluster formalism. As we use nonrelativistic quark model, we employ the transition potential given by nonrelativistic reduction of the low-energy effective weak Hamiltonian for \(|\Delta S| = 1\), consisting of 4-quark weak vertices:

\[
H_{\text{eff}}^{\Delta S=1} = -\frac{G_f}{\sqrt{2}} \sum_{r=1, r\neq 4}^{6} K_r O_r \tag{9}
\]
Table 2: The strong and weak meson baryon coupling constants. The weak coupling constants are given in units of $G_F m_\pi^2 = 2.21 \times 10^{-7}$.

| Meson | Strong c.c. | Weak c.c. | Cut off $\Lambda$ [MeV] |
|-------|-------------|-----------|-------------------------|
|       | $g_{NN\pi} = 13.3$ | $B_{\pi}^A = -7.15$ | $A_{\pi}^A = 1.05$ | 800 |
|       | $g_{\Lambda\Sigma\pi} = 12.0$ | $B_{\pi}^{\Sigma^+} = -18.3$ | $A_{\pi}^{\Sigma^+} = 0.04$ |          |
|       |                  | $B_{\pi}^{\Sigma^0} = -12.2$ | $A_{\pi}^{\Sigma^0} = -1.39$ |          |
|       |                  | $B_{\pi}^{\Sigma^-} = -8.78$ | $A_{\pi}^{\Sigma^-} = 0.95$ |          |
|       |                  | $B_{\pi}^{\Sigma^0} = 12.2$ | $A_{\pi}^{\Sigma^0} = 1.39$ |          |
|       |                  | $B_{\pi}^{\Sigma^-} = 0.74$ | $A_{\pi}^{\Sigma^-} = 1.87$ |          |
| $K$   | $g_{\Lambda NK} = -14.1$ | $C_{\pi K}^{PC} = -18.9$ | $C_{K}^{PV} = 0.76$ | 1200 |
|       | $g_{\Sigma NK} = 4.28$ | $D_{\pi K}^{PC} = 6.63$ | $D_{K}^{PV} = 2.09$ |          |

where $O_r$'s are the 4-quark operators whose explicit forms are given below.

This Hamiltonian is derived by the renormalization-group-improved perturbation theory of QCD from the standard electro-weak vertex,

$$\begin{align*}
\begin{cases}
s \rightarrow u + W^- \\
u + W^- \rightarrow d
\end{cases}
\end{align*}$$

or

$$s + u \rightarrow u + d.$$  \(11\)

Note that eq.\((11)\) contains both $\Delta I = 1/2$ and $3/2$ contributions with similar strengths. It is known that the QCD correction enhances the $\Delta I = 1/2$ part, while it suppresses the $\Delta I = 3/2$ part at the same time \[29, 30\]. The mechanism of $\Delta I = 1/2$ enhancement can be intuitively understood by considering one gluon exchange between quarks in the initial or final two quark states. The spin dependent part of the one gluon exchange interaction lowers the energy of color antisymmetric $J = 0$ pair, which restricts the final state to $I = 0$. Therefore the $\Delta I = 1/2$ transition from $I_i = 1/2$ to $I_f = 0$ is enhanced. Further enhancement comes from the so-called penguin diagrams, which induce the $O_5$ and $O_6$ operators given below \[31\].

We employ the values of the coefficients $K_r$ evaluated by solving the renormalization-group equations \[30, 37\],

| $K_1$ | $K_2$ | $K_3$ | $K_5$ | $K_6$ |
|-------|-------|-------|-------|-------|
| $-0.284$ | $0.009$ | $0.026$ | $0.004$ | $-0.021$ |

7
each of which correspond to the four-quark operators,
\[
\begin{align*}
O_1 &= (\bar{d}_\alpha s_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A} - (\bar{u}_\alpha s_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A} \\
O_2 &= (\bar{d}_\alpha s_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A} + (\bar{u}_\alpha s_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A} \\
&\quad + 2(\bar{d}_\alpha s_\alpha)_{V-A}(\bar{d}_\beta d_\beta)_{V-A} - 2(\bar{d}_\alpha s_\alpha)_{V-A}(\bar{s}_\beta s_\beta)_{V-A} \\
O_3 &= 2(\bar{d}_\alpha s_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A} + 2(\bar{u}_\alpha s_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A} \\
&\quad - (\bar{d}_\alpha s_\alpha)_{V-A}(\bar{d}_\beta d_\beta)_{V-A} - (\bar{d}_\alpha s_\alpha)_{V-A}(\bar{s}_\beta s_\beta)_{V-A} \\
O_5 &= (\bar{d}_\alpha s_\alpha)_{V-A}(\bar{u}_\beta u_\beta + \bar{d}_\beta d_\beta + \bar{s}_\beta s_\beta)_{V+A} \\
O_6 &= (\bar{d}_\alpha s_\beta)_{V-A}(\bar{u}_\beta u_\alpha + \bar{d}_\beta d_\alpha + \bar{s}_\beta s_\alpha)_{V+A} \\
\end{align*}
\]

Among the above four-quark operators, only \(O_3\) induces \(\Delta I = 3/2\) transitions. The large value of \(K_1\), and the appearance of \(K_5\) and \(K_6\) (from the penguin diagrams) show the enhancement of \(\Delta I = 1/2\) transition. It is, however, realized that the above perturbative enhancement is not enough to explain the observed \(\Delta I = 1/2\) dominance in the decays of the kaon and the hyperons. Further enhancement, most probably due to nonperturbative QCD corrections, is required. Several possibilities have been suggested, which include effects of isospin dependent final state interactions for the \(K \to \pi\pi\) decays \[32, 33\], and for the hyperon decays, the suppression of the \(\Delta I = 3/2\) by the color antisymmetrization of the valence quarks \[34, 35\]. Neither of them seem to be effective in the two-baryon transitions in the present analysis. Thus it is possible that the nonmesonic weak decays show significant deviation from the \(\Delta I = 1/2\) dominance.

4 \(\Lambda-\Sigma\) mixing contribution

Virtual \(\Sigma\) can be mixed in \(\Lambda\) hypernuclei via the strong \(\Lambda N \to \Sigma N\) transition. Effects of \(\Sigma\) mixing has been considered by many authors in the context of both the hypernuclear structure and its transitions.

Recently, it is advocated that the \(\Sigma\)-mixing is crucial in solving the overbinding problem of the \(s\)-shell hypernuclei \[10\]. Namely, the coherent \(\Sigma\) mixing, which is important in \(A = 4\) hypernuclei, gives enough attraction for the \(A = 4\) binding energy even if we take weaker central attraction that is preferable for the smaller binding of \(^{5}\Lambda\text{He}\). A sophisticated four-body calculation of \(A = 4\) hypernuclear structure also indicates significant mixing of virtual \(\Sigma\) of 1-2\% level and thus supports the above idea \[24\].

If the mixing probability of the virtual \(\Sigma\) is 1\%, the mixing amplitude \(\beta\) is \(|\beta| \sim 0.1\). Although its effects on the binding energy are proportional to \(|\beta|^2\) perturbatively, those on the
transition amplitude are of the order of $|\beta|$. The latter is also sensitive to the phase of the mixing and therefore to the mixing mechanism. Thus we consider the coherent $\Sigma$ mixing in nonmesonic decays of the $A = 4$ hypernuclei.

The diagrams shown in fig. 1 are two types of nonmesonic weak decays of the virtual $\Sigma$ in nuclei. A previous study [38] considered two-pion exchange process between $\Lambda$ and $N$, one of which induces weak transition (fig. 1(a)). The intermediate $\Sigma$-$N$ state is restricted to $I = 1/2$. It is, however, possible that the virtual $\Sigma$ decays with the assistance of a second nucleon, (fig. 1(b)). This “three body” type process is taken into account by considering the coherent $\Sigma$ mixing. They are important for two reasons.

(1) It involves the weak interaction of the $\Sigma$-$N$ ($I = 3/2$) states, which does not contribute in fig. 1(a).

(2) The coherent mixing of $\Sigma$ hypernuclear states is prohibited in a hypernucleus with $I = 0$, such as $\Lambda^4$He, due to the isospin conservation. In contrast, for $I \neq 0$ hypernuclei, the coherent $\Sigma$ mixing allows the virtual $\Sigma$ to interact with all the nucleons equally and therefore the 3-body weak process fig. 1(b) becomes important.

Let us write down the $\Sigma$ mixing effect explicitly for $\Lambda^4$He. We suppose that the $\Sigma + 3N$ state with the same quantum numbers mixes to the $|\Lambda + ^3\text{He}\rangle$ state,

$$|\Lambda^4\text{He}\rangle^0_{1/2} = \sqrt{1 - \beta^2}|\Lambda + ^3\text{He}\rangle^0_{1/2} + \beta|\Sigma + 3N\rangle^0_{1/2}$$  \hspace{1cm} (13)$$

where the superscripts and subscripts are the total angular momentum ($J$) and isospin ($I$)
respectively and
\[
|\Sigma + 3N\rangle_{1/2}^0 = \sqrt{\frac{2}{9}}(|\Sigma^+p\rangle^0(\Sigma^0n)^0)_{1/2}^0 + \sqrt{\frac{1}{9}}(|\Sigma^0n\rangle^0(pp)^0)_{1/2}^0
\]
\[
- \sqrt{\frac{1}{6}}[\sqrt{\frac{2}{3}}(|\Sigma^+n\rangle^0 + \sqrt{\frac{1}{3}}(|\Sigma^0p\rangle^0)(\Sigma^0n)^1)_{1/2}^1 + \sqrt{\frac{1}{2}}[\sqrt{\frac{2}{3}}(|\Sigma^+n\rangle^1 - \sqrt{\frac{1}{3}}(|\Sigma^0p\rangle^1)(\Sigma^0n)^1)_{1/2}^1].
\]  
(14)

One sees that \(\Sigma + 3N\) state contains the \(\Sigma^+p\) \((J = 0)\) component with 22% probability for each pair of baryons. The \(\Sigma\) mixing strength is denoted by \(\beta\). According to the recent calculation by Hiyama et al. \cite{24}, \(|\beta|^2\) is about 1% in the ground state of \(4^4\Lambda\)He. But we need not only the magnitude but also the relative phase between the \(\Lambda + 3N\) and \(\Sigma + 3N\) states. We therefore attempt to estimate \(\beta\) roughly in the first order perturbation as follows
\[
\beta = -\frac{\langle \Sigma^+3\mathrm{He}|V_{\Sigma N \rightarrow \Lambda N}|\Lambda^+3\mathrm{He}\rangle}{M_\Sigma - M_\Lambda}.
\]  
(15)

For the transition potential, we employ the D2 potential of the paper \cite{40}. Evaluating eq.\((15)\) by using the Gaussian wave function, we obtain
\[
\beta = -0.05
\]  
(16)

We should note that the magnitude of \(\beta\) is rather sensitive to the Gaussian \(b\) parameter, which is chosen here as \(b = 2.24\) \([\text{fm}]\). Although this estimate of \(\beta\) may be too crude to be quantitative, we can at least determine the sign of the mixing. In the next section, we assume the coherent \(\Sigma\) mixing of 1% for \(4^4\Lambda\)He and evaluate its effects on the nonmesonic decay rates. We consider the similar \(\Sigma\) mixing to the isospin partner \(\Lambda\)H.

The transition potential for the \(\Sigma N \rightarrow NN\) is derived similarly to the \(\Lambda N \rightarrow NN\) transition. For example the OPE induced \(\Sigma^0p \rightarrow np\) transition potential is
\[
V_{\Sigma^0p \rightarrow np}(\vec{q}) = G_Fm_\pi^2[\bar{u}_n(A_{\pi0}^{\Sigma^0} + B_{\pi0}^{\Sigma^0}\gamma_5)u_{\Sigma^0}] \frac{I_2}{\vec{q}^2 + \bar{m}_\pi^2} \left(\frac{\Lambda_\pi^2 - \bar{m}_\pi^2}{\Lambda_\pi^2 + \vec{q}^2}\right)^2 g_{\pi NN}[\bar{u}_p\gamma_5u_p]
\]  
(17)

where \(I_2 = 1\). The potential for the OKE induced \(\Sigma N \rightarrow NN\) transition is also written similarly.
5 Results

5.1 $A=5$ system

Table 3 shows the results for $^5\Lambda$He. The $\pi$ exchange process has a large proton-induced decay rate $\Gamma_{pn}$ because of the large tensor transition amplitude ($d_{pn}$-channel). The $K$ exchange reduces the $\pi$ contribution of $\Gamma_{pn}$ by about 30%, while it enhances the $J = 1$ part of $\Gamma_{nn}$ ($f_{nn}$-channel) and the $\Gamma_{nn}/\Gamma_{pn}$ ratio.

The DQ process gives significantly large contribution and large $\Gamma_{nn}/\Gamma_{pn}$ ratio, which is the major difference from the meson exchange. It contributes constructively to the meson exchange amplitudes. As a result, we have $\Gamma_{nn}/\Gamma_{pn}$ ratio largely enhanced to 0.720. Considering the large error bars in the experimental data, the agreement with experiment is fairly good.

Exchanges of the heavy mesons i.e. $\eta$, $\rho$, $\omega$, and $K^*$, are the other possibility for the short range weak interaction. It was pointed out that the nonmesonic decay rates are significantly enhanced by the heavy mesons, while the change of $\Gamma_{nn}/\Gamma_{pn}$ ratio is not large \cite{14}. In contrast, the DQ process induces a large $\Gamma_{nn}$ and it gives $\Gamma_{nn}/\Gamma_{pn} \simeq 1.216$ by itself. From the $\pi + K$ exchange contribution the $\Gamma_{nn}/\Gamma_{pn}$ ratio is enhanced by more than 50% due to the DQ transition. Ref.\cite{14} examined the realistic final state interactions and found a stronger effect that reduces the total nonmesonic decay rates by more than 50% and the $\Gamma_{nn}/\Gamma_{pn}$ ratio by more than 20%. As the recent measurement of the nucleon spectrum \cite{41} suggests significantly large final state interaction, further quantitative studies are important and urgent.

The last column of Table 3 shows the asymmetry parameter of the emitted proton against the polarization of $\Lambda$. This quantity is sensitive to the ratio of parity violating amplitude and parity conserving amplitude. Our result shows that the asymmetry parameter is negative and large as it is enhanced by the DQ contribution compared to the value given by OPE. This again is a reflection of strong $f_{pn}$ amplitude. The only available experimental data taken at KEK indicate positive value in contrast to the theoretical predictions \cite{13}. This requires further study as the sign of the asymmetry seems robust in theoretical calculations as far as the meson exchange and direct quark processes are concerned.

5.2 $A=4$ system

An advantage of the $A = 4$ and 5 hypernuclei regarding the nonmesonic weak decay is their selectivity of the two-body channels. To the leading order, the ground state of $^4\Lambda$He contains two protons forming spin 0 and a neutron and a $\Lambda$ forming spin 0. Thus there is no $\Lambda-n$ pair with spin 1, which forbids the $f_{nn}$-channel in Table 4. As a result, the neutron induced decay rate, $\Gamma_{nn}$, is strongly suppressed, which is consistent with what current experimental data indicate (}
Table 3: Nonmesonic decay rates of $^5\Lambda$He in units of $\Gamma_{\Lambda}$

| $^5\Lambda$He | total | $\Gamma_{pn}$ | $\Gamma_{nn}$ | $\Gamma_{nn}/\Gamma_{pn}$ | PV/PC | $\alpha$ |
|----------------|-------|--------------|--------------|--------------------------|--------|---------|
| $\pi$          | 0.372 | 0.328        | 0.044        | 0.133                    | $0.481$| $-0.441$|
| $\pi+K$        | 0.304 | 0.207        | 0.097        | 0.466                    | $1.336$| $-0.362$|
| DQ             | 0.066 | 0.030        | 0.036        | 1.216                    | $4.403$| $-0.398$|
| $\pi+K+DQ$     | 0.523 | 0.304        | 0.219        | 0.720                    | $4.845$| $-0.678$|

| EXP [38]       | 0.41±0.14 | 0.21±0.07 | 0.20±0.11 | 0.93±0.55 | —— | —— |
| EXP [42]       | 0.50±0.07 | 0.17±0.04 | 0.33±0.04 | 1.97±0.67 | —— | —— |
| EXP [43]       | ——        | ——        | ——        | ——        | —— | 0.24±0.22 |

Table 4.

It is interesting to see how large the effect of the $\Sigma$ mixing is. As we discussed in the previous section, we consider virtual $\Sigma$-hypernucleus component, whose probability is given by $|\beta|^2$. In Table 4, we show the decay rates of $\Lambda$=4 hypernuclei for two values of $\beta$, such that $\beta^2 \sim 1\%$. One sees that the mixing changes the total decay rate by about 20% for $\beta^2 \sim 1\%$. The sign of $\beta$ determined above is negative. One sees that the results for $\beta = -0.1$ give better agreement with the current experimental data. It has been also found that the negative $\beta$ is consistent with the study of the magnetic moments due to the $\Sigma$ mixing in hypernuclei [44]. The negative $\beta$ is found to reduce the proton induced decay rate, $\Gamma_{pn}$, while a positive $\beta$ will enhance the rates. We find that the mixing does not change the qualitative behaviors of the $\Gamma_{nn}/\Gamma_{pn}$ ratio and the proton asymmetry.

A new interesting decay channel is available when we consider the $\Sigma^+$ mixing in $^4\Lambda$He. The system consists of a virtual $\Sigma^+$, $p$ and two $n$. When $\Sigma^+$ meets the proton, it decays into two $p$, i.e., $\Sigma^+ p \rightarrow pp$ decay. The calculated decay rate for $\Sigma^+ p \rightarrow pp$ is

$$\Gamma_{pp} = 0.0003 \Gamma_{\Lambda} \quad (18)$$

at $\beta = 0.1$. Although the branching ratio is tiny, it gives a clean signal as a back to back $p-p$ in the final state. This will be a direct evidence for virtual $\Sigma$ mixing in $\Lambda$ hypernuclei.

The selectivity is reversed in $^4\Lambda$H, that is, the $J = 1$ part of $\Gamma_{pn}$ is absent. As the $\Gamma_{pn}(J = 1)$ is the largest contribution of OPE due mainly to the strong tensor force, the OPE predicts very small total nonmesonic decay rate. The kaon exchange and DQ enhances $\Gamma_{nn}$, and the total decay rate reaches up to 0.19. Thus the total nonmesonic decay rate of $^4\Lambda$H is a good indicator of the non-pion contributions to the decay process.
Table 4: Nonmesonic decay rates of $^{4}_{\Lambda}$He in units of $\Gamma_{\Lambda}$

| $^{4}_{\Lambda}$He | total | $\Gamma_{pn}$ | $\Gamma_{nn}$ | $\Gamma_{nn}/\Gamma_{pn}$ | PV/PC | $\alpha$ |
|-------------------|-------|---------------|---------------|--------------------------|--------|--------|
| $\pi$             | 0.272 | 0.250         | 0.022         | 0.089                    | 0.353  | -0.417 |
| $\pi + K$         | 0.155 | 0.145         | 0.009         | 0.064                    | 0.146  | -0.357 |
| DQ                | 0.032 | 0.021         | 0.011         | 0.516                    | 2.093  | -0.373 |
| $\pi + K + DQ$    | 0.218 | 0.214         | 0.004         | 0.019                    | 2.321  | -0.679 |

$\beta = +0.1$ 0.276 0.270 0.006 0.021 —— -0.678
$\beta = -0.1$ 0.168 0.165 0.003 0.017 —— -0.645

Table 5: Nonmesonic decay rates of $^{4}_{\Lambda}$H in unit of $\Gamma_{\Lambda}$

| $^{4}_{\Lambda}$H | total | $\Gamma_{pn}$ | $\Gamma_{nn}$ | $\Gamma_{nn}/\Gamma_{pn}$ | PV/PC |
|-------------------|-------|---------------|---------------|--------------------------|--------|
| $\pi$             | 0.040 | 0.011         | 0.029         | 2.597                    | 7.953  |
| $\pi + K$         | 0.071 | 0.005         | 0.067         | 14.225                   | 6.882  |
| DQ                | 0.040 | 0.013         | 0.027         | 2.028                    | 4.238  |
| $\pi + K + DQ$    | 0.187 | 0.030         | 0.157         | 5.318                    | 8.622  |

$\beta = +0.1$ 0.205 0.025 0.181 7.171 ——
$\beta = -0.1$ 0.168 0.034 0.134 3.938 ——

Table 5: Nonmesonic decay rates of $^{4}_{\Lambda}$H in unit of $\Gamma_{\Lambda}$

| $^{4}_{\Lambda}$H | total | $\Gamma_{pn}$ | $\Gamma_{nn}$ | $\Gamma_{nn}/\Gamma_{pn}$ | PV/PC |
|-------------------|-------|---------------|---------------|--------------------------|--------|
| $\pi$             | 0.040 | 0.011         | 0.029         | 2.597                    | 7.953  |
| $\pi + K$         | 0.071 | 0.005         | 0.067         | 14.225                   | 6.882  |
| DQ                | 0.040 | 0.013         | 0.027         | 2.028                    | 4.238  |
| $\pi + K + DQ$    | 0.187 | 0.030         | 0.157         | 5.318                    | 8.622  |

$\beta = +0.1$ 0.205 0.025 0.181 7.171 ——
$\beta = -0.1$ 0.168 0.034 0.134 3.938 ——

5.3 Breaking of the $\Delta I = 1/2$ rule

It is worthy to note that our model does not predict $\Delta I = 1/2$ dominance. In the present nonmesonic weak decays, the $\Delta I = 1/2$ rule requires

$$
\begin{bmatrix}
    a_{nn} \\
    b_{nn} \\
    f_{nn}
\end{bmatrix}
= \sqrt{2} \begin{bmatrix}
    a_{pn} \\
    b_{pn} \\
    f_{pn}
\end{bmatrix}
$$

(19)

between the amplitudes in Table 4. It is, however, strongly violated in the $J = 0$ amplitudes, i.e., $a$ and $b$. The amplitudes calculated in our $\pi + K + DQ$ model are given in Table 5. Since we use the $\Delta I = 1/2$ rule at the $\Lambda \to N\pi$ and $\Sigma \to N\pi$ vertices, the relations are satisfied in
Table 6: The $a$, $b$, and $f$ amplitudes for $A = 5$ system in arbitrary units.

| $J = 0$ | $J = 1$ |  |  |
| --- | --- | --- | --- |
| $a$ | $b$ | $f$ | 
| $\pi + K$ | $\pi + K + DQ$ | $\pi + K$ | $\pi + K + DQ$ |
| $pn$ | $nn$ | $nn/pn$ | $pn$ | $nn$ | $nn/pn$ |
| 0.061 | 0.086 | $\sqrt{2}$ | 0.122 | 0.023 | 0.189 |
| 0.015 | 0.021 | $\sqrt{2}$ | 0.088 | 0.042 | 0.477 |
| 0.211 | 0.298 | $\sqrt{2}$ | 0.332 | 0.465 | 1.401 |

the $\pi + K$ model.

However, in the $\pi + K + DQ$ model, one sees that the $\Delta I = 1/2$ relations are largely violated in $J = 0$ amplitudes, while they are almost maintained in the $J = 1$ channels. Of course this violation comes from the DQ contribution. Because the $J = 0$ amplitudes are relatively small in magnitude, their $\Delta I = 1/2$ breaking does not show up in the decay rate of $^{5}\Lambda$He. The ratio of the average $\Gamma_{nn}$ and $\Gamma_{pn}$ is not affected much by the $\Delta I = 3/2$ components. Therefore the large $\Gamma_{nn}/\Gamma_{pn}$ ratio predicted in the DQ model is independent of the breaking of the $\Delta I = 1/2$ dominance.

In order to check whether the $\Delta I = 1/2$ dominance is realized in the nonmesonic decays, the selectivity in the $A = 4$ system is again useful. In fact, we can write

$$\kappa \equiv \frac{\Gamma_{nn}(^{4}\Lambda{\text{He}})}{\Gamma_{pn}(^{4}\Lambda{\text{H}})} \sim \frac{|a_{nn}|^2 + |b_{nn}|^2}{|a_{pn}|^2 + |b_{pn}|^2}.$$  \hspace{1cm} (20)

to the leading order. This ratio should be 2 if the $\Delta I = 1/2$ transition is dominant. We can test the $\Delta I = 1/2$ dominance in the $J = 0$ amplitudes $a$ and $b$ by seeing the deviation of $\kappa$ from 2. Our $\pi + K + DQ$ model predicts $\kappa = 0.133$ reflecting the large $\Delta I = 3/2$ contribution. Unfortunately, we cannot confirm our result because no experimental data is available for $\Gamma_{pn}(^{4}\Lambda{\text{H}})$ so far. Further experimental studies are highly desirable.

## 6 Conclusions

We calculate the nonmesonic decay rates of $\Lambda$ hypernuclei by using the quark-meson hybrid model. We have found that the $J = 1$ part of $\Gamma_{pn}$ is reduced by the $K$ exchange contribution, and the $J = 1$ part of $\Gamma_{nn}$ is enhanced both by the $K$ exchange and the DQ contribution. Thus the $\Gamma_{nn}/\Gamma_{pn}$ ratio becomes large, and the result is consistent with the current experimental data.
We also estimate the effect of the virtual $\Sigma$ excitation which is known to be important when we consider the property of $\Lambda$ in nuclei. In this paper we add the $\Sigma N \rightarrow NN$ amplitude to the $\Lambda N \rightarrow NN$ amplitude by using the mixing parameter $\beta$. We find that it changes the magnitudes of the nonmesonic decay rate significantly and our estimates of the decay rates agree with the experimental data fairly well for a negative $\beta \simeq -0.1$. We have also pointed out that observation of final $p-p$ state from the decay of $^{4}_\Lambda$He gives us a chance to show a clear evidence of the $\Sigma$ component in $\Lambda$ hypernucleus.

Our model predicts that the “$\Delta I = 1/2$ rule” is largely violated in the $J = 0$ transitions. In order to confirm the prediction, a careful experiment of the $^{4}_\Lambda$H decay is indispensable, which is now underway. The origin of the $\Delta I = 1/2$ dominance in nonleptonic $|\Delta S| = 1$ weak transitions is still under heavy discussion. The vertex corrections plus the Penguin diagrams were shown to enhance $\Delta I = 1/2$. They alone, however, are not enough to reproduce the large ratio of the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes. There must be significant effects from the nonperturbative QCD corrections. The $\Lambda N \rightarrow NN$ transition will be a new tool to determine the nonperturbative mechanism of the $\Delta I = 1/2$ enhancement.

Acknowledgments

The authors thank Prof. A. Gal, Prof. Y. Akaishi, Dr. E. Hiyama, and Prof. M. Iwasaki for discussions. This work is supported in part by the Grant-in-Aid for Scientific Research (C) (2) 11640261 of Japan Society for the Promotion of Science.

References

[1] M. M. Block and R. H. Dalitz, Phys. Rev. Lett. 11 (1963) 96.
[2] H. Bhang et al., Phys. Rev. Lett. 81 (1998) 4321. H. Park et al., Phys. Rev. C61 (2000) 054004.
[3] E. Oset and L. L. Salcedo, Nucl. Phys. A443 (1985) 704.
[4] A. Ramos, E. Oset, and L. L. Salcedo, Phys. Rev. C50 (1994) 2314.
[5] W. M. Alberico, A. De Pace, G. Garbarino, and R. Cenni, Nucl. Phys. A668 (2000) 113.
[6] B. H. J. McKellar and B. F. Gibson, Phys. Rev. C30 (1984) 322.
[7] K. Takeuchi, H. Takaki, and H. Bando, Prog. Theor. Phys. 73 (1985) 841.
[8] M. Shmatikov, Phys. Lett. B322 (1994) 311; Nucl. Phys. A580 (1994) 538.

[9] K. Itonaga, T. Ueda, and T. Motoba, Nucl. Phys. A585 (1994) 331c; ibid A639 (1998) 329c; Phys. Rev. C65 (2002) 034617.

[10] D. Jido, E. Oset, and J. E. Palomar, Nucl. Phys. A694 (2001) 525.

[11] J. F. Dubach, G. B. Feldman, B. R. Holstein, and L. de la Torre, Ann. Phys. 249 (1996) 146.

[12] A. Parreño, A. Ramos, and C. Bennhold. Phys. Rev. C56 (1997) 339.

[13] J. F. Dubach, Nucl. Phys. A450 (1986) 71c.

[14] A. Parreño and A. Ramos, Phys. Rev. C65 (2002) 015204.

[15] C. Y. Cheung, D. P. Heddle, and L. S. Kisslinger, Phys. Rev. C27 (1983) 335.

[16] T. Inoue, S. Takeuchi, and M. Oka, Nucl. Phys. A577 (1994) 281c; ibid A597 (1996) 563.

[17] T. Inoue, M. Oka, T. Motoba, and K. Itonaga, Nucl. Phys. A633 (1998) 312.

[18] K. Sasaki, T. Inoue, and M. Oka, Nucl. Phys. A669 (2000) 331; ibid A678 (2000) 455 (E).

[19] K. Maltman and M. Shmatikov, Phys. Lett. B331 (1994) 1.

[20] O. Hashimoto et al., Phys. Rev. Lett. 88 (2002) 042503.

[21] C. B. Dover, Few-Body Systems Suppl. 2 (1987) 77.

[22] R. A. Schumacher, Nucl. Phys. A547 (1992) 143c.

[23] H. Nabetani, T. Ogaito, T. Sato, and T. Kishimoto Phys. Rev. C60 (1999) 017001.

[24] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, and Y. Yamamoto, Nucl. Phys. A670 (2000) 273.

[25] T. Motoba, Nucl. Phys. A547 (1992) 115c.

[26] A. Parreño, A. Ramos, and E. Oset, Phys. Rev. C51 (1995) 2477.

[27] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. 149 (1987) 1.
[28] T. S. H. Lee and A. Matsuyama, Phys. Rev. C36 (1987) 1459.
[29] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33 (1974) 108.
[30] G. Altarelli and L. Maiani, Phys. Lett. B52 (1974) 351.
[31] A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Sov. Phys. JETP 45 (1977) 670.
[32] T. Morozumi, C. S. Lim, and A. I. Sanda, Phys. Rev. Lett. 65 (1990) 404.
[33] M. Takizawa, T. Inoue, and M. Oka, Prog. Theor. Phys. Suppl. 120 (1995) 335.
[34] K. Miura and T. Minamikawa, Prog. Theor. Phys. 38 (1967) 954.
[35] J. C. Pati and C. H. Woo, Phys. Rev. D3 (1971) 2920.
[36] F. J. Gillman and M. B. Wise, Phys. Rev. D20 (1979) 2392.
[37] E. A. Paschos, T. Schneider, and Y. L. Wu, Nucl. Phys. B332 (1990) 285.
[38] H. Bando, Y. Shono, and H. Takaki, Int. J. Mod. Phys. A3 (1988) 1581.
[39] J. J. Szymanski et al., Phys. Rev. C43 (1991) 849.
[40] Y. Akaishi, T. Harada, S. Shinmura, and Khin Swe Myint, Phys. Rev. Lett. 84 (2000) 3539.
[41] S. Okada, Master Thesis, Tokyo Institute of Technology (2001).
[42] H. Noumi et al., in proceedings of the IV International Symposium on Weak and Electromagnetic Interactions in Nuclei, edited by H. Ejiri, T. Kishimoto and T. Sato (World Scientific, 1995) p.550.
[43] S. Ajimura et al., Phys. Rev. Lett. 84 (2000) 4052.
[44] M. Oka, in proceedings of Physics with GeV Electrons and Gamma-Rays, edited by T. Tamae et al. (Universal Academic Press, 2001).
[45] V. J. Zeps et al., Nucl. Phys. A639 (1998) 261c.
[46] H. Outa et al., Nucl. Phys. A639 (1998) 251c.