Observed Asymmetry in $\bar{p}p \rightarrow \pi^+ K^- K^0 / \pi^- K^+ \bar{K}^0$
and Relation to Reciprocity

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**ABSTRACT**

The charge-asymmetry observed in a recent CPLEAR experiment was interpreted by the authors as a direct observation of T-noninvariance. While this is the simplest and most natural inference, and the observed effect agrees in sign and magnitude with theoretical expectation, adherents of T-invariance may argue that other interpretations are also possible. If $K^0$ and $\bar{K}^0$ are produced equally in $\bar{p}p$ annihilation, and T-invariance is assumed to hold, the asymmetry observed in CPLEAR must be attributed to TCP-noninvariance of kaon beta-decays. If that were the case, the charge-asymmetry in $K^0_S \rightarrow \pi l\nu$ decays should be three times larger than the one observed for $K^0_L$ decays.

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1 Introduction

The CPLEAR collaboration has measured [1] a hitherto unreported C- and CP-asymmetry in $\bar{p}p$ annihilation which, under reasonable assumptions, can be identified with a previously predicted [2] T-asymmetry. Until now, there has been no credible evidence of any departure from reciprocity in any reaction; also, questions have been raised [3] about the significance of the test proposed in Ref. [2]. Therefore, it may be useful to critically examine the circumstances under which the C- and CP-asymmetry reported by CPLEAR can be interpreted as a demonstration of deviation from T-invariance. We indicate further tests, and the conditions which must be satisfied for the CP-asymmetry found by CPLEAR to be consistent with T-invariance.

2 Expectation of T-Asymmetry

The departure from CP-invariance in neutral kaon decays has been reliably established [4] by a number of independent measurements, including a predicted asymmetry [5] in $K_L \to \pi^+\pi^-e^+e^-$ decays which has been observed recently [6]. However, despite many searches, there has been no clear evidence of CP-noninvariance in any phenomenon other than neutral kaon decays. There, the observed effects can be attributed entirely to $K^0 - \bar{K}^0$ mixing, which could arise from CP-noninvariant interactions much weaker [7] than the weak interactions responsible for the decay of kaons. This may explain the failure to see measurable CP-asymmetric effects in other phenomena.

Invariance of physical laws under inversion [8] of 4-dimensional space-time — which is not required by Lorentz-invariance but obtains in most Lorentz-invariant theories with further minimal analytic properties, e.g. field theories described by local Lagrangians — can be given a consistent interpretation only if space-time inversion PT is accompanied by particle-antiparticle conjugation C. Within the class of such TCP-invariant theories, lack of symmetry with respect to any of the constituent operations, e.g. particle-antiparticle exchange C or “combined inversion” CP, in which space-coordinates are inverted simultaneously with particle-antiparticle interchange, must be compensated by a corresponding asymmetry with respect to one or more of the other constituent operations, to preserve the overall TCP-symmetry. On this basis, Lee, Oehme and Yang [9] showed that the possible noninvariance with respect to space-inversion proposed [10] to explain the “tau-theta puzzle” necessarily required another presumed symmetry to be broken; they showed that observation of the suggested P-noninvariant effects would require C-invariance also to be broken. An elegant way to preserve the symmetry of space, even if P is abandoned,
suggested by several authors [11], is to require exact CP-symmetry, in which case TCP-invariance would automatically assure exact T-invariance as well. The subsequent discovery [4] that CP is not a valid symmetry in K-meson decays, requires T-invariance also to fail if TCP-invariance is to survive. Following the discovery [12] of parity-nonconservation, searches [13] for T-noninvariance were based largely on philosophical grounds: if physical laws are not indifferent to space-inversion, perhaps they might not be symmetric with respect to t-inversion either. After the discovery of CP-nonconservation, the search for T-noninvariance became a logical imperative. Either T-invariance would also fail, as TCP-invariance requires, or one would face the even greater challenge of TCP-noninvariance.

As long as deviations from CP-symmetry are confined to neutral kaon decays and associated effects, the only place where one has a definite expectation of seeing T-noninvariance must be in the same phenomena. Furthermore, if TCP-invariance is valid, the observed CP-noninvariance manifested in neutral kaon decays must be accompanied by corresponding deviations from T-invariance, which is more precisely described as symmetry with respect to motion-reversal. TCP-invariance requires

\[ (\tilde{a}_T|S|\tilde{b}_T) = (b|S|a) \]  

(1)

where \( \tilde{c} \) represents the CP-transform of the channel \( c \) and \( c_T \) represents its time-reverse, viz. the channel \( c \) with all particle momenta and spins reversed. The requirement of CP-invariance:

\[ (\tilde{b}|S|\tilde{a}) = (b|S|a) \]  

(2)

taken together with Eq. (1), would require that

\[ (\tilde{a}_T|S|\tilde{b}_T) = (\tilde{b}|S|\tilde{a}) \]  

(3)

i.e. CP-invariance requires reciprocity if TCP-invariance is valid. Conversely, if the requirement, Eq. (2), of CP-invariance fails for a related pair of transition matrix-elements, there must be a corresponding failure of reciprocity in the same case [14].

We already mentioned that a very feeble CP-noninvariant interaction contributing to \( K^0 - \bar{K}^0 \) mixing suffices to account for all observed CP-asymmetric effects. Therefore, the departure from T-invariance expected on the basis of TCP-invariance must also appear in \( K^0 - \bar{K}^0 \) mixing. Departure from reciprocity would appear in a difference between the rates of \( \bar{K}^0 \to K^0 \) and \( K^0 \to \bar{K}^0 \) transitions, expressed by a T-asymmetry parameter[2,15]:

\[ A_T = \frac{P_{\bar{K}K}(\tau) - P_{KK}(\tau)}{P_{\bar{K}K}(\tau) + P_{KK}(\tau)} \]  

(4)
which is found to be a constant in the generalized Weisskopf-Wigner approximation. Its value is given by

$$A_T^{1b} = 2\text{Re}(\epsilon_S + \epsilon_L) = 2\text{Re}\langle K_L|K_S \rangle$$  \hspace{1cm} (5)

to lowest order in the CP-nonconserving parameters $\epsilon_{S,L}$, defined by

$$K_{S,L} \propto [1 + \epsilon_{S,L}]K^0 \pm [1 - \epsilon_{S,L}]\bar{K}^0.$$  \hspace{1cm} (6)

TCP-invariance requires $[9] \epsilon_S$ and $\epsilon_L$ to be equal; on that basis, the value of $A_T$ could be predicted to be $4\text{Re}\epsilon = (6.4 \pm 1.2) \times 10^{-3} [17]$. Even without assuming any symmetry, the last quantity on the right-hand side of Eq. (5) can be deduced by appeal to unitarity [18]. On the basis of reasonable assumptions, the most relevant of which were subsequently verified [17], about upper limits on minor modes of neutral kaon decay, it was shown [2] that the expected T-asymmetry should have substantially the value predicted for the TCP-invariant case, whether that symmetry is assumed or not.

3 CP-Asymmetry Measured by CPLEAR

$\bar{p}p$ annihilations into $[19] \pi^+ K^- “K^0n”$ and $\pi^- K^+ “\bar{K}^0n”$, which are expected to occur equally frequently by CP-invariance, were selected by kinematic analysis, and the frequencies of beta-decay of the neutral kaons were compared for the two cases. If we accept the $\Delta S = \Delta Q$ rule [20] which requires that $\pi^- e^+ \nu$ and $\pi^+ e^- \bar{\nu}$ arise only from $K^0$ and $\bar{K}^0$, respectively, and assume that the two decay rates are equal, as required by TCP-invariance, then the observed $\pi^- e^+ \nu$ and $\pi^+ e^- \bar{\nu}$ rates at any time $\tau$ measure the $K^0$ and $\bar{K}^0$ populations at that time. Assuming initial equality[22] of $K^0$ and $\bar{K}^0$ populations and survival probabilities, any inequality between the observed annihilation rates into:

$$\bar{p}p \rightarrow \pi^+ K^- \{\pi^+ e^- \bar{\nu}\} \text{ and } \pi^- K^+ \{\pi^- e^+ \nu\}$$  \hspace{1cm} (7)

must arise from a difference between $\bar{K}^0 \rightarrow K^0$ and $K^0 \rightarrow \bar{K}^0$ transition rates. This is the conclusion drawn by CPLEAR.

The CP-asymmetry which they measure is:

$$A_l = \frac{R[\pi^+ K^- \{\pi^+ e^- \bar{\nu}\}] - R[\pi^- K^+ \{\pi^- e^+ \nu\}]}{R[\pi^+ K^- \{\pi^+ e^- \bar{\nu}\}] + R[\pi^- K^+ \{\pi^- e^+ \nu\}]}$$  \hspace{1cm} (8)

In Eqs. (7) and (8), the $\pi ev\nu$ configurations in braces are observed as (delayed) end-products deduced kinematically to arise from beta-decays of neutral kaons. Assuming the validity of the $\Delta S = \Delta Q$ rule, this asymmetry can be written as:

$$A_l = \frac{P_{KK}(\tau)R[K^0 \rightarrow \pi^- e^+ \nu] - P_{KK}(\tau)R[\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}]}{P_{KK}(\tau)R[K^0 \rightarrow \pi^- e^+ \nu] + P_{KK}(\tau)R[\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}]}.$$  \hspace{1cm} (9)
TCP-invariance requires that

\[ R[K^0 \rightarrow \pi^- e^+ \nu] = R[\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}] , \]

therefore, under the assumption of TCP-invariance, the CPLEAR asymmetry becomes

\[ A^\text{TCP}_l = \frac{P_{K\bar{K}}(\tau) - P_{\bar{K}K}(\tau)}{P_{K\bar{K}}(\tau) + P_{\bar{K}K}(\tau)} = A_T \]

which is a measure of T-asymmetry at the same time as CP-asymmetry. Over a time-interval \( \tau_S < \tau < 20\tau_S \), the observed asymmetry is consistent with being a constant, with a value reported as[1]

\[ A_T^{\text{exp}} = (6.6 \pm 1.3) \times 10^{-3} \] (11)

which agrees with the theoretical prediction. On the other hand, if we insist on exact reciprocity,

\[ P_{K\bar{K}}(\tau) = P_{\bar{K}K}(\tau) , \]

then Eq. (9) reduces, for the case of exact T-invariance, to

\[ A^T_l = \frac{R[K^0 \rightarrow \pi^- e^+ \nu] - R[\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}]}{R[K^0 \rightarrow \pi^- e^+ \nu] + R[\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}]} (13) \]

and represents a (CP- and) CPT-violating effect. The observed asymmetry \( A_l \), Eq. (8), requires the beta-decay rate for \( K^0 \rightarrow \pi^- e^+ \nu \) to exceed that for \( \bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu} \) by about 1.3%, if exact T-invariance is imposed. If we parametrize the deviation from TCP-invariance of kaon beta-decay amplitudes by setting [23,24]:

\[ \langle \pi^+ e^- \bar{\nu} | T | K^0 \rangle = (1 + y) \langle \pi^- e^+ \nu | T | K^0 \rangle \] (14)

where \( y \) can be taken to be real without loss of generality, the CP-asymmetry, Eq. (13), is given, to lowest order in \( y \), by \(-y\); \( y \) is therefore required to have the value:

\[ y = -(6.6 \pm 1.3) \times 10^{-3} \] (15)

if exact T-invariance is demanded.

The charge-asymmetry in \( K^0_L \rightarrow \pi e\nu \) decays was accurately measured in several concordant experiments, whose combined result is quoted as [17]:

\[ \delta_l = (3.27 \pm 0.12) \times 10^{-3} \] (16)

The phenomenological analysis without assumption of any symmetry, but assuming the validity of \( \Delta Q = \Delta S \), yields [23]

\[ \delta_{l,L} = 2 \text{Re} \epsilon_L - y . \] (17)
The corresponding quantity for \( K_S^0 \) decays is

\[
\delta_{l,S} = 2 \text{Re} \epsilon_S - y. \tag{18}
\]

T-invariance requires \( \epsilon_S = -\epsilon_L \), therefore Eqs. (17) and (18) would constrain the leptonic charge-asymmetry from \( K^0_S \) decays to have the value:

\[
\delta_{l,S}^T = -\delta_{l,L} - 2y = (9.9 \pm 1.3) \times 10^{-3}, \tag{19}
\]

viz. three times the value, Eq. (16), for \( K_L^0 \to \pi\nu \) decays, if T-invariance is to be sustained. The CPLEAR data probably contain the information required to confirm or refute this expectation [26]. If not, \( \Phi \)-decays from DAΦNE, which provide a certified \( K_S^0 \) in association with each \( K_L^0 \) decay, should provide a clean \( K_S^0 \) sample to test the unambiguous prediction (19) required by the hypothesis of T-invariance.

4 Conclusions

The simplest interpretation of the CPLEAR asymmetry, reported in Eq. (11), is that it exhibits the T-asymmetry predicted previously, and confirms the sign and magnitude of the expected effect. To this, the logical objection may be raised that the CP-asymmetry measured by CPLEAR translates into the T-asymmetry factor \( A_T \) defined in Eq. (4) only if the \( \bar{p}p \) annihilation rates into \( \pi^+K^-K^0 \) and \( \pi^-K^+\bar{K}^0 \) and the beta-decay rates for \( K^0 \to \pi^-e^+\nu \) and for \( \bar{K}^0 \to \pi^+e^-\bar{\nu} \) are assumed to be equal. The latter is required by TCP-invariance; but if one is prepared to accept TCP as an article of faith, then T-noninvariance follows as soon as CP-invariance fails and no further demonstration is required. Analysis of the CPLEAR asymmetry, without assuming equality of \( K^0 \) and \( \bar{K}^0 \) beta-decay rates, shows that, subject to the \( \Delta Q = \Delta S \) rule, the leptonic charge-asymmetry for \( K_S^0 \to \pi\nu \) decays should be three times larger than the measured asymmetry for \( K_L^0 \) decays, if T-invariance is valid. Thus, it should not be too difficult to distinguish between the simple interpretation of the CPLEAR charge-asymmetry as a direct demonstration of T-noninvariance, and the desperate and radical resort to TCP-noninvariance required to preserve T-invariance; these are the only two alternatives unless one is willing to countenance unequal \( production \) of \( K^0 \) and \( \bar{K}^0 \) in \( \bar{p}p \) annihilations.

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References

[1] CPLEAR Collaboration. A. Angelopoulos et al., Phys. Lett. B444, 43 (1998).

[2] P. K. Kabir, Phys. Rev. D2, 540 (1970).

[3] L. Wolfenstein, private communication.

[4] V. L. Fitch. Rev. Mod. Phys. 53, 387 (1981).

[5] L.M. Sehgal and M. Wanninger, Phys. Rev. D46, 1035 (1992); 5209 (1992) (E); P. Heiliger and L.M. Sehgal, Phys. Rev. D48, 4146 (1993).

[6] KTeV Collaboration. M. Arenton, Heavy Quark Workshop, FermiLab, October 1998.

[7] L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

[8] J. Schwinger, Phys. Rev. 82, 914 (1951).

[9] T.D. Lee, R. Oehme and C.N. Yang, Phys. Rev. 106, 340 (1957).

[10] T.D. Lee and C.N. Yang, Phys. Rev. 104, 254 (1956).

[11] G.C. Wick, A.S. Wightman, and E.P. Wigner, Phys. Rev. 88, 101 (1952); L.D. Landau, Nucl. Phys. 3, 127 (1957).

[12] C.S. Wu, E. Ambler, R. Hayward, D. Hoppes, and R. Hudson, Phys. Rev. 105, 1413 (1957); ibid. 106, 1361 (1957).

[13] For a review of such searches, see E. M. Henley, Ann. Rev. Nucl. Sc. 19, 367 (1969).

[14] I thank W. Blum, H.-P. Duerr, and L. Stodolsky for a spirited debate on this point.

[15] Doubts (e.g. in Refs. [3] and [16]) about the applicability of reciprocity to unstable particles are unwarranted. For a proper discussion, see P.K. Kabir, to be published.

[16] R.G. Sachs, The Physics of Time-Reversal, Univ. of Chicago Press, Chicago, 1987.

[17] Particle Data Group. Eur. Phys. J. C3, 1 (1998).

[18] J.S. Bell, in High Energy Physics, Les Houches 1965, C. DeWitt and M. Jacob, eds. Gordon & Breach, New York, 1965.

[19] Quotation marks indicate the strangeness assigned on the basis of the assumption that strangeness is conserved.
There are strong theoretical arguments in favour of the $\Delta S = \Delta Q$ rule [21]. In the standard model, the strangeness-changing weak current ($\bar{u}s$) necessarily obeys this rule. For analysis without this assumption, see Ref. [23].

[21] R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[22] G.V. Dass and A. Pilaftsis reminded me that another (radical) interpretation of the CPLEAR asymmetry, Eq.(8), could be through unequal [C- and CP-noninvariant] annihilation rates into $K_0^0$ and $\bar{K}_0^0$.

[23] G.V. Dass and P.K. Kabir, Proc. Roy. Soc. (Lond) A330, 331 (1972).

[24] Non-vanishing $y$ would introduce corresponding differences between $K^+ \rightarrow \pi^0 e^+ \nu$ and $K^- \rightarrow \pi^0 e^- \nu$ decay amplitudes, but available measurements [17] have not attained the precision for a significant test. Comparison of $K^\pm$ lifetimes with a relative accuracy better than $10^{-4}$ could also provide useful limits on the value of $y$.

[25] T.D. Lee and C.S. Wu, Ann. Rev. Nucl. Sc. 16,511 (1966).

[26] Formulas for the complete time-distributions of leptonic decays, including effects of possible $\Delta Q = -\Delta S$ admixtures, are given in Ref. [23].