QCD PREDICTIONS FOR LEPTON SPECTRA IN INCLUSIVE HEAVY FLAVOUR DECAYS

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Abstract

We derive the lepton spectrum in semileptonic beauty decays from a nonperturbative treatment of QCD; it is based on an expansion in $1/m_Q$ with $m_Q$ being the heavy flavour quark mass. The leading corrections arising on the $1/m_Q$ level are completely expressed in terms of the difference in the mass of the heavy hadron and the quark. Nontrivial effects appear in $1/m_Q^2$ terms affecting mainly the endpoint region; they are different for meson and baryon decays as well as for beauty and charm decays.

The weak decays of hadrons contain a wealth of information on the fundamental forces of nature. Yet the intervention of the strong interactions has prevented us from extracting this information in a reliable way. Heavy flavour decays promise to be more tractable since the mass of the heavy flavour quark $m_Q$ provides a powerful expansion parameter. Indeed significant successes have been scored by Effective Heavy Quark Theory (EHQT). Yet that approach has some intrinsic limitations: e.g., it deals with exclusive semileptonic modes only and it requires the presence of heavy quarks both in the initial and in the final state; thus it cannot be applied directly to $b \rightarrow u$ transitions. Its model-independent predictions so far include corrections through order $1/m_Q$ only. On the other hand the energy released in heavy flavour decays is much larger than ordinary hadronic energies. Our analysis will make use of this large energy release in treating $Q \rightarrow ql\nu$ transitions with $m_Q, m_Q - m_q \gg \Lambda_{QCD}$.

In previous papers we have shown how nonperturbative contributions to global quantities like lifetimes and semileptonic branching ratios can be obtained from a
model-independent treatment of QCD. The method was based on expanding the weak transition operator into a series of local operators of increasing dimension with coefficients that contain increasing powers of $1/m_Q$. The coefficients depend on the (inclusive) final state; the differences between the decay rates of different heavy flavour hadrons $H_Q$ – charged vs. neutral mesons vs. baryons – enter through the matrix elements of the local operators taken between $H_Q$. For example the total semileptonic $b \to u$ width through order $1/m_b^2$ is given by:

$$\Gamma(H_b \to l\nu X) \propto m_b^5 \cdot \frac{1}{2M_{H_b}} \langle H_b | \bar{b}b \rangle - \frac{1}{m_b^2} \bar{b}i\sigma G b | H_b \rangle$$ (1)

with $i\sigma G = i\gamma_\mu \gamma_\nu G_{\mu\nu}$, $G_{\mu\nu}$ being the gluonic field strength tensor.

In this note we will expand the general method to treat the lepton spectra in the semileptonic decays of heavy flavour hadrons. A novel feature is encountered: one is dealing with an expansion in powers of $1/(p_Q - p_l)^2$ with $p_Q$ and $p_l$ denoting the momenta of $Q$ and the lepton $l$, respectively. This series is singular at the endpoint of the lepton spectrum; thus some care has to be applied in interpreting the results. Two dimension five operators generate the leading nonperturbative corrections of order $1/m_b^2$: the colour magnetic operator $Q i\sigma G Q$ and the operator $Q(D^2 - (vD)^2)Q$ describing the kinetic energy of $Q$ in the gluon background field; $D_\mu$ denotes here the covariant derivative and $v_\mu$ is the 4-velocity vector of the hadron. Corrections actually arise only on the $1/m_Q$ level; it is crucial that in QCD those can be expressed completely in terms of the difference between the quark and the hadron mass.

We will phrase our discussion in terms of beauty decays with a few added comments on charm decays. It should be noted that the question of the inclusive lepton spectra in QCD was first addressed explicitly in ref. [3].

Ignoring gluon bremsstrahlung one obtains a lengthy expression for the lepton spectra in the semileptonic decays of a beauty hadron $H_b$:

$$\frac{d\Gamma}{dy}(H_b \to l\nu X_q) = \Gamma_0 \theta(1 - y - \rho) 2y^2\{(1 - f)^2 + 2f)(2 - y) + (1 - f)^3(1 - y) +$$

$$+(1 - f)\left[ (1 - f)(2 + \frac{5}{3}y - 2f + \frac{10}{3}f y) - \frac{f^2}{\rho}(2y + f(12 - 12y + 5y^2)) \right] G_b -$$

$$- \left[ \frac{5}{3}(1 - f)^2(1 + 2f)y + \frac{f^3}{\rho}(1 - f)(10y - 8y^2) + \frac{f^4}{\rho^2}(3 - 4f)(2y^2 - y^3) \right] K_b \}$$

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{qb}|^2, \quad f = \frac{\rho}{1 - y}, \quad \rho = \frac{m_q^2}{m_b^2}, \quad y = \frac{2E_b}{m_b},$$

$$K_b = 1/2M_{H_b} \cdot \langle H_b | \bar{b}(i\bar{D})^2 b | H_b \rangle / m_b^2, \quad G_b = 1/2M_{H_b} \cdot \langle H_b | \bar{b}i\sigma G b | H_b \rangle / 2m_b^2$$ (2)

with $m_q$ denoting the mass of the quark $q$ in the final state. Eq.(2) represents the master formula containing both relevant cases, namely $q = u, c$.

For $b \to ul\nu$ transitions with $m_u = 0$ this expression simplifies considerably:

$$\frac{d\Gamma}{dy} = \Gamma_0 2y^2\left[ 3 - 2y - (\frac{5}{3}y + \frac{1}{3}\delta(1 - y) - \frac{1}{6}(2y^2 - y^3) \delta'(1 - y)) K_b + (2 + \frac{5}{3}y - \frac{11}{6}\delta(1 - y)) G_b \right]$$ (3)
The $\delta$-functions and their derivative reflect the previously mentioned singular nature of the expansion at the endpoint; their emergence has a transparent meaning (see[4] for details). The spectrum is finite at the endpoint for $m_u = 0$ and thus contains a step function. The chromomagnetic interaction effectively ‘shifts’ the spectrum by changing the energy in either initial or final state; the shift in the argument of the step function thus yields a $\delta$-function. The singular structure in the $K_b$ term on the other hand reflects the motion of the $b$ quark inside the $H_b$ hadron which Doppler shifts the spectrum; in second order it generates $\delta'(1-y)$ for the step-like spectrum.

Due to these singular terms the expression given above can be identified with the observable spectrum only outside a finite neighbourhood of the endpoint region. (This distance remains constant in absolute units in the limit $m_b \to \infty$). Yet even this neighbourhood does not represent true ‘terra incognita’: for integrating our expression over this kinematical region yields a finite and trustworthy result that can be confronted with the data. This can be expressed through the function

$$\Gamma(E_l) = \int_{E_l}^{E_{max}} dE_l \frac{d\Gamma}{dE_l}, \quad E_l \leq E_{QCD} < E_{max}. \quad (4)$$

$E_{max}$ denotes the maximal kinematically allowed energy and $E_{QCD}$ the maximal energy for which one can still trust the QCD expansion in eqs.(2,3); its value depends on the size of $K_b$ and $G_b$. Clearly $\Gamma(0) = \Gamma_{SL}$ has to hold. This is not a trivial relation: for $\Gamma_{SL}$ is deduced from a completely regular expansion in $1/m_b$ (see eq.(1)) whereas $\Gamma(0)$ comes from integrating the expression in eq.(3) containing singularities; thus the singular terms are essential for recovering the correct decay width!

Similar considerations apply to $b \to cl\nu$ transitions. With $m_c$ as an infrared cutoff there arise no singular terms at the endpoint $y = 1 - \rho$; yet the expansion parameters $G_b/(1-y), K_b/(1-y)^2$ though finite become large there. The need for ‘smearing’ the spectrum over the endpoint region, eq.(4), thus still exists.

The contributions from higher dimensional operators that we are ignoring have terms of the schematic form $\sim [\mu_{hadr}/(m_b(1-y))]^n$; summing them all up would yield a well-behaved function. As long as these quantities are smaller than unity, we can trust the expressions given above for the unintegrated lepton spectrum.

The size of $G_b$ is easily determined: $G_b = \langle B|\bar{Q}i\sigma GQ|B\rangle/4m_b^3 \simeq 3(M^2(B^*) - M^2(B))/4m_b^3 \simeq 0.017$; for $\Lambda_b$ it vanishes. A recent QCD sum rules analysis yields $\langle B|\bar{b}(i\bar{D})^2 b|B\rangle \sim 0.4 \text{ GeV}^2 \cdot 2M_B$ in agreement with rather general expectations. For our subsequent discussion we will set $K_b = 0.02$. We use $m_b \simeq 4.8 \text{ GeV}$ as deduced from a QCD analysis of the Ypsilon system, $m_b - m_c = 3.35 \text{ GeV}$ as inferred from the beauty and charm meson masses which for $m_b = 4.8 \text{ GeV}$ yields $m_c = 1.45 \text{ GeV}$, and put $m_u = 0$. For $b \to u$ decays we estimate that $E_{QCD} \sim 0.9 \cdot m_b/2 \simeq 2.15 \text{ GeV}$; for $b \to c$ a somewhat smaller ‘smearing’ range near the endpoint seems to be required, namely $\Delta E_l \simeq 0.15 \text{ GeV}$ and thus $E_{QCD} \simeq 2.0 \text{ GeV}$. Numerically it implies that in the real world the $c$ quark is relatively heavy in $b$ decays: $m_c^2 > \mu_{hadr}m_b$; therefore the falling edge of the spectrum starts in the calculable region.

For a proper perspective we show the partially integrated spectrum $\Gamma(E_l)$ from three different prescriptions, namely (a) our QCD expansion; (b) the simple free quark picture with $G_b = K_b = 0, m_u = 0$; (c) the phenomenological treatment of
To summarize: We have shown here how an expansion in \( 1/m_b \) beyond the scope of our present analysis to treat lepton spectra \( b \to c \) incorporate nonperturbative corrections to the lepton spectra in inclusive semileptonic \( b \to u \) decays. Through order \( 1/m_b \) allows us meaningful statements about fully integrated quantities like \( \Gamma \) – their size can be extracted from the relationship to other observables. Improvements

Altarelli et al. [6] (hereafter referred to as ACM) where one attempts to incorporate some bound state effects. We have set here \( m_u = m_{spect} = 0.15 \text{ GeV} \), \( m_c = 1.67 \text{ GeV} \), \( p_F = 0.3 \text{ GeV} \) as suggested by a fit to CLEO data; \( p_F \) denotes the ‘Fermi momentum’. To be consistent we have ignored gluon bremsstrahlung both in ACM and in our QCD expressions (for fitting the data it should be added to eqs.(2-4)). In comparing the QCD formula with the ACM prediction one has to note a subtle distinction in the definition of the kinematical variables: in ACM energy is expressed in terms of the mass of the beauty hadron; yet in eqs.(2-3) \( y \) measures the energy in units of \( m_b \), which by itself introduces a shift of order \( 1/m_b \).

Comparing the results on \( \Gamma(E_l) \) from the three approaches we conclude: (i) The shape of the QCD, of the free quark model and of the ACM curves are very close over most of the range of \( E_l \leq E_{QCD} \) for the \( b \to u \) as well as the \( b \to c \) case. The main difference lies in the overall normalization. (ii) The QCD result for \( b \to u \) can be largely simulated by setting \( m_u \sim 0.3 \text{ GeV} \) in the free quark spectrum. The nonperturbative corrections thus effectively transform a current quark mass into a ‘constituent’ one of reasonable size. (iii) Once the ACM result is renormalized according to \( \Gamma(0)_{QCD}(b \to u) = 1.07 \times \Gamma(0)_{ACM}(b \to u) \) and \( \Gamma(0)_{QCD}(b \to c) = 1.13 \times \Gamma(0)_{ACM}(b \to c) \) the two curves are hardly distinguishable as functions of \( E_l \).

(iv) While the QCD results for \( \Gamma(E_l) \) are largely independent of the value of \( K_b \) for \( K_b \sim 0.02 \) in \( b \to u \), they are sensitive to it for \( b \to c \); that change can be easily understood \[4\]. (v) The QCD curves for semileptonic \( \Lambda_b \) decays are somewhat harder than for \( B \) meson decays. (vi) The largest differences between the models are found in the endpoint region, namely for \( E_l > 1.8 \text{ GeV} \) in \( b \to c \).

Some of these points are illustrated in the figures. In fig.1 we show \( \Gamma(E_l) \) for the endpoint region of \( b \to u \); in fig.2 we have plotted these partially integrated spectra for \( b \to c \). The unintegrated spectrum \( d\Gamma(E_L)/dE_l \) which can be calculated in the QCD treatment for \( E_l \leq 2 \text{ GeV} \) is shown in fig.3. The similarity in the shape of the two spectra is quite striking!

A few comments are in order about charm decays. The lepton spectra are quite different in beauty and in charm decays, especially in the endpoint region: for the charged lepton in beauty decays is an electron or muon whereas it is an antifermion \( l^+ \) in charm decays. Since the lepton spectrum vanishes at the end point in \( c \to ql\nu \) even for \( m_q = 0 \), the chromomagnetic operator can yield finite terms only while the kinetic energy operator produces a \( \delta(1 - y) \), but not a \( \delta'(1 - y) \) term. Yet since the nonperturbative corrections are much larger in charm than in beauty – \( G_c, K_c \sim 0.2 \) – the smearing required in \( y \) is much larger for charm, namely \( \Delta y \sim 0.5 \). While this allows us meaningful statements about fully integrated quantities like \( \Gamma_{SL} \), it appears beyond the scope of our present analysis to treat lepton spectra in charm decays.

To summarize: We have shown here how an expansion in \( 1/(p_b - p_l) \) allows us to incorporate nonperturbative corrections to the lepton spectra in inclusive semileptonic \( b \to c \) and \( b \to u \) decays. Due to the singular nature of this expansion at the endpoint the spectrum cannot be computed in detail in a close neighbourhood of the endpoint, yet smeared or partially integrated spectra can. Through order \( 1/m_b^2 \) these expressions are given in terms of the quantities \( m_b, m_c, G_b \) and \( K_b \). These are not free parameters – their size can be extracted from the relationship to other observables. Improvements
in the precision of their determination, in particular concerning $K_b$ and to a lesser
degree $m_b$, can be anticipated from future progress in theory.

We have compared our QCD results with the ACM description that so far has
allowed a decent fit to the data. Despite the obvious differences in the underlying
dynamics we have found the shape of the resulting lepton spectra remarkably similar,
even more so when one keeps the following in mind:
(a) Initially the scale for the kinematics is different in the two descriptions: for the
QCD expansion it is set by the quark mass $m_b$ whereas for ACM by the hadron mass
$M_B$ or $M_{\Lambda_b}$. The kinematical differences due to $m_b \neq M_{H_b}$ are actually of order $1/m_b$
and formally represent the leading corrections. This suggests an interesting observation:
an accurate measurement of the shape of the spectrum allows a determination
of the $b$ quark mass $m_b$ free of theoretical uncertainties. Unfortunately in practice one
would have to analyse $b \to u$; the shape of the $b \to c$ spectrum is basically determined
by $m_b - m_c$ with little sensitivity to $m_b$.
(b) The ACM ansatz contains three free parameters – $m_c$, $m_{sp}$ and $p_F$ – that are
to be fitted from the data. There are four quantities that set the scale in the QCD
expansion through order $1/m_b^2$, namely $m_b$, $m_c$, $G_b$ and $K_b$ but none of them is a free
parameter. That our QCD description containing therefore very little ‘wiggle room’
can nevertheless yield a description so similar to that of the ACM ansatz – after the
latter has been fitted to the data – has to be seen as quite remarkable!
(c) The ACM prescription can thus be viewed as a simple, though smart approxima-
tion to a more complete and complex QCD treatment. At the same time it would be
incorrect to interprete the fit parameters in ACM literally as real physical quantities;
it is thus not surprising that the numerical values for the former differ significantly
from what is known now about the corresponding quantities in QCD.

A more detailed analysis shows\[4\] that QCD provides a natural ‘home’ for the
Fermi motion originally introduced phenomenologically in ACM, and it is asymptot-
ically a dominant effect for the end point shape in $b$ decays. However it enters in a
somewhat different form. Its impact on the total widths is only quadratic in $1/m_b$
(in ACM it produces a linear shift). Nevertheless the shape of the lepton spectrum
gets $1/m_b$ corrections\[4\]; it will be discussed in detail elsewhere\[4].

Despite all similarities there arise relevant numerical differences as well, not only
in the overall normalization of the spectra, but also in the endpoint region: the
QCD spectra place a higher fraction of $b \to c$ events in the endpoint region. Such a
difference has important consequences for the extraction of $|V_{ub}/V_{cd}|$ from the data.
Furthermore our analysis shows that the lepton spectra in semileptonic $\Lambda_b$ decays
are distinct from those in $B$ decays. These issues will be discussed in a future paper\[4].

Another model ISGW\[7\] used for describing semileptonic spectra relies heavily on
model-dependent calculations of the formfactors for the exclusive final states. In the
limit of a heavy $c$ quark, $m_b - m_c \ll m_c$, EHFT yields the necessary formfactors to
some accuracy; therefore in that limit the prediction of this model coincides with the
model-independent QCD spectrum. On the other hand that is apparently not the
case for $b \to u$ decays where the difference is significant.

On the $1/m_Q^3$ level that was not treated here novel effects arise due to ‘Weak

\[1\]At this point we disagree with the conclusion of ref.\[3\]
Annihilation’ in the $b \rightarrow u$ channel; for details see [8].
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Figure Captions

Fig.1: The function $\Gamma(E_l)$ for $b \rightarrow u$ transitions calculated in the QCD expansion (solid line), the free spectator quark model (dotted line) and the ACM ansatz (dashed line); the ACM curve has been multiplied by a factor 1.07.

Fig.2: The function $\Gamma(E_l)$ for $b \rightarrow c$ transitions calculated in the QCD expansion (solid line), the free spectator quark model (dotted line) and the ACM ansatz (dashed line); the ACM curve has been multiplied by a factor 1.13.

Fig.3: The lepton spectrum $d\Gamma/dE_l$ for $b \rightarrow c$ transitions calculated in the QCD expansion (solid line), the free spectator quark model (dotted line) and the ACM ansatz (dashed line); the ACM curve has been multiplied by a factor 1.13.

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