Extended Gravity Description for the GW190814 Supermassive Neutron Star

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Very recently a compact object with a mass in the range $2.50 \div 2.67 M_{\odot}$ has been discovered via gravitational waves detection of a compact binary coalescence. The mass of this object makes it among the heaviest neutron star never detected or the lightest black hole ever observed. Here we show that a neutron star with this observed mass, can be consistently explained with the mass-radius relation obtained by Extended Theories of Gravity. Furthermore, equations of state, consistent with LIGO observational constraints, are adopted. We consider also the influence of rotation and show that masses of rotating neutron stars can exceed $2.6M_{\odot}$ for some equations of state compatible with LIGO data.

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I. INTRODUCTION

Compact astrophysical objects, such as Neutron Stars (NSs), can be described by General Relativity (GR) in the strong field regime. The structure of a NS is strictly correlated with the equation of state (EoS), i.e. the relation between pressure and density in its interior \cite{1}. Given an EoS, a mass-radius ($M-R$) relation and a corresponding maximal mass can be derived, in principle, for any NS. Furthermore, the knowledge of these parameters provides significant information on the mechanism responsible for formation, stability and possible effects on the evolutionary history of NSs. For a detailed introduction to the theory of relativistic stars, see for example \cite{2}.

On the other hand, NSs are natural laboratories to test strong gravitational field regimes that can hardly be reached in any other part of the Universe and, so, their internal structure cannot be easily reproduced because of the extreme conditions in which it operates. Thus, theoretical models can be formulated where a very large number of EoS candidates can be taken into account. Starting from microscopic information, the task is to reproduce consistently the observed macroscopic parameters, and, vice versa, from these parameters, to select and constrain reliable classes of EoS. In this perspective, astrophysical measurements of mass, radius and rotation, besides selecting realistic EoS, give also insight on the behavior of matter in extreme gravity regimes.

It is important to stress that GR gives very strict limits for the stability of compact objects made up of degenerate matter, like NSs or white dwarfs. In particular, Chandrasekhar fixed a theoretical upper bound of $M \sim 1.44 M_{\odot}$ \cite{3} for non-rotating compact objects. For masses around this limit, gravity and degenerate matter achieve stable configurations around a radius of $R \sim 10$ Km. Beyond this limit, considering also secondary effects which can improve it, gravity cannot be stopped by degenerate matter pressure and black holes originate.

From an observational point of view, the mass determination can accurately be achieved only for NSs in binary systems. Observations of these systems have found some NSs that could violate this limit \cite{4,9}. In particular, a very recent observation \cite{10} detected a compact object in the mass range $2.50 \div 2.67 M_{\odot}$ via gravitational waves. It could represent the most massive NS ever observed. This result is well beyond the Chandrasekhar limit so it cannot be in agreement with the standard theory also stretching parameters and processes in the GR context. The way out to these shortcomings could be finding some exotic EoS capable of stabilizing the stellar structure by some form of degenerate, strange matter, or considering alternative gravity where Chandrasekhar limit can be relaxed or improved. On the other hand, the massive object could be a very light black hole, but also in this case there are difficulties in explaining it by the standard theory.

In the research line of modified gravity alternative description, Extended Theories of Gravity (ETGs) \cite{11} could play a prominent role in explaining consistently the problem at hand. Such theories are straightforward extensions of GR. Specifically, GR is a particular case of a large class of theories which proved to be particularly useful in the IR limit of cosmology (see \cite{12} for a recent review). Detailed studies of anomalous astrophysical compact objects in the framework of ETGs have been already performed in some previous works under the hypothesis that very massive NSs could be materialized by gravitational (geometric) effects \cite{13,16}. In particular $f(R)$ gravity, i.e. a class of Lagrangians considering a generic function of the Ricci curvature scalar, has been investigated. Clearly, for $f(R) \to R$ the standard GR is restored.
$f(R)$ gravity has been used to solve and explain theoretically a large number of astrophysical and cosmological issues, i.e. the cosmic acceleration [17,22], the inflationary paradigm, the dark matter, the dark energy, and some stellar structures [23,30].

For the astrophysical GW190814 event, the primary objective is to obtain the $M - R$ relation of NSs allowing to derive the maximum mass value. This should be achieved considering realistic EoS.

In this paper, we want to demonstrate that, measurements reported by the LIGO collaboration [10,31] for the GW190814 event, can be theoretically framed in the context of EIGs, if the $M - R$ relation is obtained by a system of modified Tolman-Oppenheimer-Volkoff (TOV) equations [32,33]. Clearly, in the limit $f(R) \to R$, the standard TOV system is recovered [34].

The paper is organized as follows. In Sect. II we derive the TOV equations for $f(R)$ gravity. Specific $f(R)$ models are also discussed. Then, in Sect. III, we consider rotating stars in the framework of $f(R)$ gravity. Sect. IV is devoted to the numerical results for static and rotating cases. Conclusions are drawn in Sec. V.

II. THE TOLMAN-OPPENHEIMER-VOLKOFF EQUATIONS IN $f(R)$ GRAVITY

Let us start from the $f(R)$ action given by

$$\mathcal{A} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[ f(R) + L_{\text{matter}} \right], \quad (1)$$

where $g$ is the determinant of the metric $g_{\mu\nu}$ and $L_{\text{matter}}$ is the standard perfect fluid matter Lagrangian. The variation of $\mathcal{A}$ with respect to $g_{\mu\nu}$ gives the field equations [11,28,29,35]:

$$\frac{df(R)}{dR} R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \left[ \nabla_\mu \nabla_\nu - g_{\mu\nu} \Box \right] \frac{df(R)}{dR} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (2)$$

where $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} L_m)}{\delta g^{\mu\nu}}$ is the energy momentum tensor of matter. Here we adopt the signature $(+, -, -, -)$. The metric for systems with spherical symmetry has the usual form

$$ds^2 = c^2dt^2 - e^{2\lambda}dr^2 - e^{2\lambda}d\theta^2 - r^2(d\phi^2 + \sin^2 \theta d\phi^2), \quad (3)$$

where $\psi$ and $\lambda$ are functions depending only on the radial coordinate $r$. Within the stellar structure, matter is described as a perfect fluid, whose energy-momentum tensor is $T_{\mu\nu} = \text{diag}(\epsilon \rho c^2, \epsilon \rho c^2, \rho, \rho, \rho)$. Here $\rho$ is the matter density and $p$ is the pressure [36]. The equations for the stellar configuration are obtained adding the condition of hydrostatic equilibrium which can be derived from the contracted Bianchi identities

$$\nabla^\mu T_{\mu\nu} = 0, \quad (4)$$

that give the Euler conservation equation

$$\frac{dp}{dr} = -(\rho + p) \frac{d\psi}{dr}. \quad (5)$$

From the metric (3) and the field equations (2), it is possible to derive the equations for the functions $\lambda$ and $\psi$ in the form [13]

$$\frac{d\lambda}{dr} = \frac{e^{2\lambda} \left[ r^2 (16\rho + f(R)) - f'(R) (r^2 R + 2) \right] + 2R^2 f''(R) r^2 + 2rf''(R) \left[ rR_{rr} + 2R_r + 2f'(R) \right]}{2r \left[ 2f'(R) + rR_f + f''(R) \right]}, \quad (6)$$

$$\frac{d\psi}{dr} = \frac{e^{2\lambda} \left[ r^2 (16\rho - f(R)) + f'(R) (r^2 R + 2) \right] - 2(2rf''(R) R_r + f'(R))}{2r \left[ 2f'(R) + rR_f + f''(R) \right]}, \quad (7)$$

respectively. In both Eqs. (5) and (7), the prime denotes a derivative with respect to the Ricci scalar $R(r)$.

The above equations are the modified TOV equations that, for $f(R) = R$, reduce to the standard TOV equations of GR [38,39]. It is important to stress that, in $f(R)$ gravity, the Ricci scalar is a dynamical variable and then we need a further equation to solve the system of Eqs. (5), (6) and (7). The corresponding equation takes the form

$$\frac{d^2R}{dr^2} = R_r \left( \lambda_r + \frac{1}{r} \right) + \frac{f'(R)}{f''(R)} \left[ \frac{1}{r} \left( 3\lambda_r - \lambda_r + \frac{2}{r} \right) - e^{2\lambda} \left( \frac{R^2}{2} + \frac{2}{r^2} \right) \right] - R^2 f'''(R) \frac{f''(R)}{f''(R)} . \quad (8)$$
which can be derived from the trace of Eqs. (2) inserting the metric (3).

Let us now consider two physically motivated functional forms of the \( f(R) \) function and derive the TOV equations for these cases. The aim is to demonstrate that minimal modifications with respect to GR can give relevant results capable of explaining consistently the problem of having supermassive NSs, without the need for a stiff EoS. In fact, our description can incorporate even more massive NSs, which may eventually will not be able to be described by standard GR, even with stiff EoS being used.

A. The \( f(R) = R + \alpha R^2 \) model

We consider here the specific form of \( f(R) \):

\[
f(R) = R + \alpha R^2, \tag{9}
\]

where \( \alpha \) is the coupling parameter of the quadratic curvature correction. This model is specially suitable to account for cosmological inflation, where higher-order curvature terms naturally lead to cosmic accelerated expansion. The quadratic term emerges in strong gravity regimes and as an effective contribution in quantum field theory on curved spacetime [37]. However, at Solar System scales and, more in general, in the weak field regime, the linear term predominates.

It is worth noticing that in the interior of a NS, the physical conditions quantified by the energy and pressure, could be analogous to those during the early Universe [15]. Due to this feature, the model (9) is particularly suitable for our considerations. Specifically, Eqs. (6), (7) and (8) take the explicit form:

\[
d\lambda = \frac{e^{2\lambda}[16\pi r^2\rho - 2 - \alpha R(r^2 R + 4)] + 4\alpha(r^2 R_\epsilon + 2 r R_\epsilon + R) + 2}{4 r [1 + \alpha(2 R + r R_\epsilon)]}, \tag{10}
\]

\[
d\psi = \frac{e^{2\lambda}[16\pi r^2 p + 2 + \alpha R(r^2 R + 4)] - 4\alpha(2 r R_\epsilon + R) - 2}{4 r [1 + \alpha(2 R + r R_\epsilon)]}, \tag{11}
\]

\[
d^2 R = R_\epsilon \left( \lambda_\epsilon + \frac{1}{r} \right) + \frac{1 + 2\alpha R}{2\alpha} \left[ 1 + \frac{3\psi - \lambda_\epsilon + 2}{r} - e^{2\lambda} \left( \frac{R}{2} + \frac{2}{r^2} \right) \right]. \tag{12}
\]

 Clearly, GR is restored for \( \alpha = 0 \).

B. The \( f(R) = R^{1+\varepsilon} \) model

Another interesting class of models are the power law models \( f(R) \sim R^n \) with \( n \in \mathbb{R} \). As shown in [40], these models are related to the existence of Noether symmetries. For \( n = 1 \), the Noether symmetry gives the standard Schwarzschild radius as a conserved quantity. We can assume the form

\[
f(R) = R^{1+\varepsilon}, \tag{13}
\]

where \( n = 1 + \varepsilon \), to study small deviation with respect to GR for \( |\varepsilon| \ll 1 \). In this limit, it is possible to write a first-order Taylor expansion as

\[
R^{1+\varepsilon} \approx R + \varepsilon R \log R + O(\varepsilon^2), \tag{14}
\]

which is interesting in order to define the correct physical dimensions of the coupling constant and to control the magnitude of the corrections with respect to the standard Einstein gravity [50].

A term in the Lagrangian of the form (14) has been widely tested starting from Solar System up to cosmological scales. Indeed, the value of the parameter \( \varepsilon \) can straightforwardly relate a weak field curvature regime (\( \varepsilon \approx 0 \)) to a regime where strong curvature effects start to become relevant (\( \varepsilon \neq 0 \)). In this perspective, \( \varepsilon \) could be different from zero in NSs and then probe deviations with respect to GR. The explicit forms of Eq. (6) and (7) for the action (14) are:

\[
d\lambda = \frac{8\pi G\varepsilon^{2\lambda} R_\rho}{c^2[2 R(1 + \varepsilon + s \log R) + r e R'] + e^{2\lambda} R^2 [r^2 R e - 2(1 + \varepsilon + s \log R)]} + \frac{e^{2\lambda} R^2 [r^2 R e - 2(1 + \varepsilon + s \log R)]} {2 r R [2 R(1 + \varepsilon + s \log R) + r e R']} \tag{15}
\]
while the equation for R is

\[
\frac{d^2 R}{dr^2} = \frac{R^2}{R} + R' \left( \lambda - 2 \frac{r}{R} - w' \right) - \frac{e^{21} R[c^4(1-e) + 8G\pi(3P-c^2\rho) + c^4 R e \log R]}{3c^4e}.
\]

Also in this case, GR is restored for \( \epsilon \to 0 \). The final aim of this mathematical apparatus is to investigate if physical relations of supermassive NSs, like the \( M - R \) diagram, can be realized by modified TOV systems according to the values of parameters \( \alpha \) and \( \epsilon \). Before tackling this task, let us discuss also rotating NSs in the framework of \( f(R) \).

### III. ROTATING NEUTRON STARS IN \( f(R) \) GRAVITY

Studying spinning NSs is very important from a theoretical point of view because realistic stellar structures are always rotating objects. NSs in binary systems, after merging, can produce black holes or supermassive fast rotating NSs which then collapse into black holes \[41\]. Parameters of post-merging gravitational wave signals are strongly depending on angular momenta, masses and other secondary parameters of NSs so then a multimessenger characterization of relativistic stellar objects could help also in selecting the theory of gravity working in these systems.

Let us consider now a star rotating along the polar axis with angular frequency \( \Omega \). It is convenient to use metric in quasi-isotropic coordinates namely

\[
ds^2 = e^{2\psi} c^2 dr^2 - e^{2\lambda}(dr^2 + r^2 d\theta^2) - e^{2\mu} r^2 \sin^2 \theta (d\phi - \omega dt)^2,
\]

where metric functions \( \psi, \lambda, \mu \) and \( \omega \) depend only on coordinates \( r \) and \( \theta \). It is worth noticing that \( \lambda \), in this metric, does not reduce immediately to \( \lambda \) in the previous section also in the limit \( \omega \to 0 \).

In GR, a \((3 + 1)\) formalism is usually adopted for rotating stars (see for details \[42\]-\[44\]). In the case of \( f(R) \) gravity, being this theory a straightforward extension of GR, the same formalism can be used without significant changes. Dropping technical details, let us give the system of field equations

\[
f'(R)\Delta_{(3)}\psi + \frac{1}{2} \Delta_{(3)} f'(R) = 4\pi e^{2\lambda}(\epsilon + \sigma) - \frac{1}{2} e^{2\lambda} (f'(R)R - f(R)) - f'(R)\partial \psi \partial (\psi + \mu) - \partial \psi \partial f'(R) - \frac{1}{2} \partial \ln(\psi + \mu) \partial f'(R) + f'(R) \frac{1}{2} e^{2(\mu - \psi)} r^2 \sin^2 \theta (\partial \omega)^2,
\]

\[
f'(R)\Delta_{(4)}(\psi + \mu) + \Delta_{(4)} f'(R) = 8\pi e^{2\lambda}(\sigma_\phi + \sigma_\theta^2) - e^{2\lambda} (f'(R)R - f(R)) - f'(R) (\partial \psi + \mu) (\partial \psi + \mu) - 2 \partial (\psi + \mu) \partial f_R,
\]

\[
f'(R)\Delta_{(2)}(\psi + \lambda) + \Delta_{(2)} f'(R) = 8\pi e^{2\lambda}\sigma_\phi - \frac{1}{2} e^{2\lambda} (f'(R)R - f) - f'(R) (\partial \psi)^2 - \partial \psi \partial f'(R) + \frac{3}{8} f'(R) e^{2(\mu - \psi)} r^2 \sin^2 \theta (\partial \omega)^2,
\]

\[
f'(R)\Delta_{(5)} \omega = - \frac{16\pi e^{\psi + 2(\lambda - \mu)}}{r^2 \sin^2 \theta} p_\phi + \frac{4\omega}{r} \partial R \partial \omega + 4 \omega \partial \mu \partial R + \frac{4\omega}{r} \left( \frac{\partial R}{\partial r} + \frac{1}{r \tan \theta} \frac{\partial R}{\partial \theta} \right) - 3 f'(R) \partial \mu \partial \omega + f'(R) \partial \psi \partial \omega.
\]
For any two given quantities $g_1$ and $g_2$, we define, for brevity, the notation
\[
\partial g_1 \partial g_2 \equiv \left( \frac{\partial g_1}{\partial r} \frac{\partial g_2}{\partial r} + \frac{1}{r^2} \frac{\partial g_1}{\partial \theta} \frac{\partial g_2}{\partial \theta} \right).
\]
$
\Delta_{(\alpha)}$
defines the Laplace operators in Euclidean space including derivatives of radial and polar coordinates:
\[
\Delta_{(\alpha)} = \frac{1}{r^{n-1}} \frac{\partial}{\partial r} \left( r^{n-1} \frac{\partial}{\partial r} \right) + \frac{1}{r \sin^{n-2} \theta} \frac{\partial}{\partial \theta} \left( \sin^{n-2} \theta \frac{\partial}{\partial \theta} \right)
\]
Source terms $\epsilon, \sigma^\phi, \sigma^\rho, \sigma^r$ are defined, according to the standard notations, as
\[
\epsilon = \Gamma^2 \left( \rho + \frac{p}{c^2} \right) - \frac{p}{c^2}, \quad (23)
\]
\[
\sigma^r = \sigma^\rho = \frac{p}{c^2}, \quad \sigma^\phi = \frac{p}{c^2} + \left( \epsilon + \frac{p}{c^2} \right) \frac{U^2}{c^2}, \quad (24)
\]
\[
p_\phi = \epsilon^\prime \left( \epsilon + \frac{p}{c^2} \right) \frac{U}{c} r \sin \theta, \quad (25)
\]
where $\Gamma$ is the Lorentz factor
\[
\Gamma = \left( 1 - \frac{U^2}{c^2} \right)^{-1/2}, \quad U = \epsilon^\prime \phi (\Omega - \omega) r \sin \theta.
\]
and $U$ is the linear velocity of rotation. The equation for the scalar curvature in quasi-isotropic coordinates has the following form:
\[
\Delta_{(3) f'}(R) = \frac{8 \pi}{3} \epsilon^\prime \left( \frac{3p}{c^2} - \rho \right) - \frac{2 \epsilon}{3} \left( f'(R) - 2 f(R) \right) - \partial (\phi + \mu) \partial f'(R). \quad (26)
\]
It is straightforward to plug models $f(R) = R + \alpha R^2$ and $f(R) = R^{1+r}$ into the system (19), (26) and then to develop analysis for rotating case in analogy to non-rotating one.

IV. NUMERICAL RESULTS

Considering the previous rotating and non-rotating cases, let us report now results relevant to the conclusion we are looking for. For a complete analysis of static stellar configurations see [32] and [33].

For the $f(R) = R + \alpha R^2$ model, results are reported in Fig. 1. Here we note that the larger the value $\alpha$ is, the larger the NS mass becomes. It is immediate to see that, for appropriate values of $\alpha$, we can reproduce the values reported in [10]. Considering the MPA1 as EoS, reported in [32], the mass value of $2.6M_\odot$ is easily achieved. This is more difficult considering the case SLy for EoS. See Fig. 1.

It is interesting to consider both the influence of rotation with high frequency and deviation from GR on NS parameters. From the observational data, it follows that the highest measured rotation frequency is 716 Hz for the pulsar PSR J1748-2446ad [45]. For various EoS, such a frequency leads to an increasing of maximal mass of the order $\sim 0.07 \div 0.1 M_\odot$, in the GR context, which is not sufficient to explain the data reported by LIGO [10].

Let us consider, as an illustrative example, the EoS GM1 without hyperons [46] and, for frequency, let us assume the value $f = 700$ Hz. Results of our calculations show that maximal mass for non-rotating stars in GR is $2.39M_\odot$. For stars rotating with $f = 700$ Hz, the value increases up to $2.49M_\odot$. For $f(R) = R + \alpha R^2$ gravity, the maximal mass of static star is $2.50M_\odot$ assuming $\alpha = 2.5$. In the case of rotation with $f = 700$ Hz, the maximal mass is $2.63M_\odot$ showing that the LIGO limit can easily be achieved (see Fig. 2).

According to the data in [47], the maximal mass in the case of uniform rotation for GM1 as EoS is $2.84M_\odot$ assuming GR. However, this mass-shedding limit is reached for a Keplerian frequency of 1.49 kHz and the existence of so fast rotating stars seems unrealistic. On the other hand, it seems possible that, in the context of $R + \alpha R^2$ gravity, supermassive NSs, with masses close to $3M_\odot$, can appear for observed rotation frequencies.

Furthermore, it is worth mentioning that some stiff EoS were proposed with the maximal mass limit for non-rotating stars in the range $\sim 2.75 \div 2.8 M_\odot$ (see for example MS1 [48], NL3 [49]). For frequencies $\sim 700$ Hz, in $R + \alpha R^2$ gravity with large values of $\alpha$, the maximal NS mass can also be close to $3M_\odot$. 

Source terms $\epsilon, \sigma^\phi, \sigma^\rho, \sigma^r$ are defined, according to the standard notations, as
\[
\epsilon = \Gamma^2 \left( \rho + \frac{p}{c^2} \right) - \frac{p}{c^2}, \quad (23)
\]
\[
\sigma^r = \sigma^\rho = \frac{p}{c^2}, \quad \sigma^\phi = \frac{p}{c^2} + \left( \epsilon + \frac{p}{c^2} \right) \frac{U^2}{c^2}, \quad (24)
\]
\[
p_\phi = \epsilon^\prime \left( \epsilon + \frac{p}{c^2} \right) \frac{U}{c} r \sin \theta, \quad (25)
\]
where $\Gamma$ is the Lorentz factor
\[
\Gamma = \left( 1 - \frac{U^2}{c^2} \right)^{-1/2}, \quad U = \epsilon^\prime \phi (\Omega - \omega) r \sin \theta.
\]
FIG. 1: \( M - R \) diagram for NSs in the \( f(R) = R + \alpha R^2 \) compared with GR considering the SLy and MPA1 as EoS. Parameter \( \alpha \) is given in units of \( r_g^2 = G^2 M_{\odot} / c^4 \). Here \( r_g \) is the gravitational radius of the Sun.

FIG. 2: \( M - R \) diagram of NSs for \( f(R) = R + \alpha R^2 \) compared with GR. We are considering the GM1 as EoS without hyperons in cases with and without rotation.

In the case of \( f(R) = R + \varepsilon R^2 \), following [32, 33], we adopt SLy as EoS and the results of our numerical analysis are shown in Fig. 3. Here we can notice that the value of \( \varepsilon \) influences greatly the \( M - R \) relation. In particular the larger \( \varepsilon \) is, the larger the NS mass becomes. In Fig. 3 we reported the \( M - R \) relation for \( \varepsilon \) between 0.005 and 0.008, which are consistent with the mass in the range \( 2.50 \div 2.67 \, M_{\odot} \) reported by [10].
V. CONCLUSIONS

In this paper, we presented a way to theoretically explain the anomalous mass of compact object recently detected by [10] with the hypothesis that it is a supermassive NS. Specifically, for $f(R) = R + \alpha R^2$ gravity with maximal observed rotation and for $f(R) = R^{1+\varepsilon}$ gravity without rotation, it is straightforward to obtain results consistent with LIGO detection without invoking exotic EoS. The fact that ETGs are consistent with observations which cannot be explained by standard GR is fundamental not only because we can shed new light on the extreme gravity regimes that are realized in compact objects like NSs, but also because these observations could validate more and more the theoretical grounding of ETGs. It is worth noticing that it could be not only an alternative explanation of the reported results, but a sort of experimentum crucis for these theories, if such a kind of (present or future) observations cannot be explained in the framework of GR.

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