Effect of the r-value change on the forming limit analysis for a ultra-low carbon steel sheet

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Abstract. The effect of the r-value change on the accuracy of the forming limits predicted using the Marciniak-Kuczyński-type (M-K) forming limit analysis for a cold rolled interstitial-free (IF) steel sheet is investigated. Uniaxial tensile tests with a digital image correlation system are used to measure the r-value change. A tube subjected to tension-expansion loading under linear paths in the first quadrant of the stress space are performed to measure the multiaxial plastic deformation behavior and the forming limits of the test material. The observed differential hardening (DH) behavior is approximated by changing the material parameters of the Yld2000-2d yield function (Barlat et al, 2003) as functions of the reference plastic strain. The M-K analyses are performed using the r-value change and r-value constant DH model. It is concluded that the DH model considering the r-value change leads to the more accurate predicted forming limits.

1. Introduction

A forming limit curve (FLC) is widely used in industry. It is well known that material models have a significant effect on the prediction accuracy of FLCs [1, 2]. Therefore, yield functions that are capable of accurately reproducing the plastic deformation behavior of a test material are crucial.

One of the most popular constitutive models is isotropic hardening. However, the yield surface shape can change even in monotonic loading because of the change in texture with plastic deformation. Hill and co-authors [3, 4] proposed a new constitutive analysis method by focusing on the contours of equal plastic work (henceforth referred to as work contour) in the stress space; the change in shape of successive work contours with increasing deformation was formulated. One of the present authors developed biaxial tensile testing methods for sheet metals using cruciform specimens [5] and for tubular specimens [6, 7] to observe the differential hardening (DH) behavior of cold-rolled ultralow-carbon steel sheets [5, 7], high-strength steel sheet [8], aluminum alloy sheet [9], pure titanium sheet [6], and a magnesium alloy sheet [10].

Several authors developed constitutive models that enable us to reproduce the DH behavior in sheet forming simulations [8, 9, 11-15]. However most of them ignored the r-value change. Xu and Weinmann [11] calculated the FLCs of AK steel using the r-value change DH model based on the Marciniak-Kuczyński-type (M-K) approach [16]; however, the validity of the material models used in
these analyses was not fully discussed.
Present authors investigated the effect of the DH behavior on the predictive accuracy of the forming limit for a cold-rolled IF steel sheet and concluded that the effect of the DH behavior was negligible [17]. However, the r-value change was not considered. The objective of the present study is to clarify the effect of the constitutive model considering the r-value change on the predictive accuracy of the FLC and forming limit stress curve (FLSC) based on the M-K analysis for the sample used in [7, 17]. The calculated FLC and FLSC were compared with the measured ones.

2. Material modeling

2.1. Uniaxial tensile test procedures and results
The test material used was a 0.7 mm thick cold-rolled IF steel sheet. The work hardening characteristics and r-values measured at 0, 45 and 90° to the rolling direction (RD) are listed in table 1. Hereafter, the RD, transverse (TD) and thickness (ND) directions of the material are defined as the x-, y- and z-axes, respectively.

The digital image correlation system (ARAMIS®, Gom) was used to measure the development of the strain field for the uniaxial tensile tests. Standard uniaxial tensile specimens (JIS 13 B-type) were used. Figure 1 shows the variations in the instantaneous r-values, \( r' \), in the RD and TD with increasing the longitudinal plastic strain \( \varepsilon^p \). The values of \( r' \) are almost constant for \( \varepsilon^p \leq 0.1 \) and \( 0.3 \leq \varepsilon^p \leq 0.3 \).

The variation in \( r' \) was approximated as a function of \( \varepsilon^p \) using the following equation:

\[
\begin{align*}
\frac{A_1 - A_2}{1 + \exp\left(\frac{(\varepsilon^p - A_3)}{A_4}\right) + A_2}
\end{align*}
\]

where \( A_i \ (i=1-4) \) are parameters. The approximated results were also depicted in figure 1.

| Tensile direction (°) | \( \sigma_{0.2} \) (MPa) | \( \varepsilon^s \) (MPa) | \( n^p \) | \( \alpha \) | r-value\(^b\) |
|----------------------|--------------------------|--------------------------|---------|---------|----------------|
| 0 (RD)              | 164                      | 574                      | 0.273   | 0.008   | 2.27           |
| 45                  | 173                      | 574                      | 0.272   | 0.008   | 1.77           |
| 90 (TD)             | 170                      | 564                      | 0.273   | 0.009   | 2.65           |

\(^a\) Approximated using \( \sigma = c(\alpha + \varepsilon^p) \) for \( 0.002 \leq \varepsilon^p \leq 0.093 \).
\(^b\) Measured at nominal strain \( \varepsilon_n = 0.1 \).

![Figure 1. Variation in the instantaneous r-values, \( r'_0 = d\varepsilon_y / d\varepsilon_z \) and \( r'_{90} = d\varepsilon_y / d\varepsilon_z \), with increasing the logarithmic plastic strain \( \varepsilon^p \).](image-url)
2.2. **Multiaxial tube tension-expansion forming test procedures**

Multiaxial tube tension-expansion forming tests (MTET) were performed to measure the multiaxial plastic deformation behavior of the test material from initial yield to fracture using the servo-controlled multiaxial tube tension-expansion machine [7]. Tubular specimens with an inner diameter of 44.6 mm and a length of 200 mm were fabricated by bending a sheet sample into a cylindrical shape and 

\[ \text{CO}_2 \text{laser-welding the sheet edges together.} \]

Axial loads \( T \) and an internal pressure \( P \) were applied to the specimen and caused multiaxial stresses at the center of the specimen. Linear stress paths were applied to tubular specimens; the true stress ratios \( \sigma_x : \sigma_y \) were chosen to be 4:1, 2:1, 4:3, 1:1, 3:4, 1:2 and 1:4. All the MTET data considered in this study are those obtained in [7].

2.3. **Material modeling**

The concept of the work contour in the stress space [3, 4] was introduced to evaluate the work hardening behavior of the test material under biaxial tension. The true stress-logarithmic plastic strain curve obtained from a uniaxial tension test along the RD was selected as a reference datum for work hardening; the uniaxial true stresses \( \sigma_0 \) and the plastic work per unit volume \( W_0 \) performed during the test up to a specified value of the uniaxial logarithmic plastic strain \( \varepsilon_0^p \). The uniaxial true stress \( \sigma_{90} \) in the TD and the biaxial true stress components \( (\sigma_x, \sigma_y) \) were then determined for the same plastic work as \( W_0 \). The stress points plotted in the stress space form a work contour associated with \( \varepsilon_0^p \).

Figure 2 shows the measured stress points forming work contours. Also depicted in the figure are the theoretical yield loci calculated using two DH models; one is based on constant \( r \)-values (DH-I) and the other one is based on instantaneous \( r \)-values (DH-II). The DH-I and II models were approximated by changing the anisotropic parameters \( \alpha_i (i=1-8) \) and exponent \( M \) of the Yld2000-2d yield function [18] as functions of \( \varepsilon_0^p \) using the following equations:

\[
M, \alpha_i = B_1 - B_2 \exp(-B_3\varepsilon_0^p) + B_4/(B_5 + \varepsilon_0^p)
\]  

(2)

where \( B_i (i=1-5) \) are the parameters. The \( r \)-values calculated using the DH-II model are also depicted in figure 1.

3. **Forming limit analyses**

Figure 3 shows the forming limit strains and stresses measured using the uniaxial tensile tests, MTETs, and hydraulic bulge tests. Also depicted in the figure are the FLCs and FLSCs calculated using the M-K approach [16]. The material parameters assumed in the M-K analyses are as follows: the elastic modulus is 200 GPa, the Poisson’s ratio 0.3, and the strain rate sensitivity exponent (\( m \)-value) 0.02. The strain hardening function was described as \( \sigma_0 = 670(e_0^p + 0.01)^{0.33} - 147e_0^p \) (MPa) [17]. The magnitude of the initial imperfection was set to 0.997. Additional details on the calculation

![Figure 2. Measured stress points forming contours of plastic work compared with those calculated using the DH-I and II models based on the Yld2000-2d yield function.](image-url)
procedures and constitutive equations for the M@K approach can be found in [1, 8]. The DH@II model has a better agreement with the measured FLC and FLSC than the DH@I model for the stress paths with $\sigma_x > \sigma_y$.

4. Conclusions
The DH-II model (considering the $r$-value change) had a better agreement with the measured FLC and FLSC than the DH-I model (assuming constant $r$-values).

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