Faithful state transfer between two-level systems via an actively cooled finite-temperature cavity

Lőrinc Sárkány,1 József Fortágh,1 and David Petrosyan1,2

1Physikalisches Institut, Eberhard Karls Universität Tübingen, D-72076 Tübingen, Germany
2Institute of Electronic Structure and Laser, FORTH, GR-71110 Heraklion, Crete, Greece

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We consider state transfer between two qubits – effective two-level systems represented by Rydberg atoms – via a common mode of a microwave cavity at finite temperature. We find that when both qubits have the same coupling strength to the cavity field, at large enough detuning from the cavity mode frequency, quantum interference between the transition paths makes the SWAP of the excitation between the qubits largely insensitive to the number of thermal photons in the cavity. When, however, the coupling strengths are different, the photon number-dependent differential Stark shift of the transition frequencies precludes efficient transfer. Nevertheless, using an auxiliary cooling system to continuously extract the cavity photons, we can still achieve a high-fidelity state transfer between the qubits.

I. INTRODUCTION

Quantum information protocols and quantum simulations with cold atomic systems extensively utilize strong dipole-dipole interaction between the laser-excited Rydberg states of atoms [1–8]. Atoms are routinely trapped and manipulated near the surfaces of superconducting atom chips [9, 10], and can couple to on-chip microwave planar resonators [11, 12]. The cavity field can then serve as a quantum bus to mediate Rydberg states of atoms [1–8]. Atoms are routinely trapped and manipulated near the surfaces of superconducting atom chips [9, 10], and can couple to on-chip microwave planar resonators [11, 12]. The cavity field can then serve as a quantum bus to mediate Rydberg states of atoms [1–8]. Atoms are routinely trapped

In this paper we consider a compound system consisting of two two-level (Rydberg) atoms \( i = 1, 2 \) coupled to a common mode of a microwave resonator, as sketched in Fig. 1(a). The system is thus equivalent to the Jaynes–Cummings model for two atoms interacting with a cavity field [27, 28]. In the frame rotating with the frequency of the cavity mode \( \omega \), the Hamiltonian is given by

\[
\mathcal{H}/\hbar = \sum_{i=1,2} \left[ \hbar \Delta_i (\hat{\sigma}_{bb}^{(i)} - \hat{\sigma}_{aa}^{(i)}) + g_i (\hat{c} \hat{\sigma}_{ba}^{(i)} + \hat{\sigma}_{ab}^{(i)} \hat{c}) \right]. \tag{1}
\]

Here \( \Delta_i \equiv \omega_{ba}^{(i)} - \omega \) is the detuning of atom \( i \) on the transition \( |a\rangle \leftrightarrow |b\rangle \) from the cavity mode frequency, \( \hat{\sigma}_{mn}^{(i)} \equiv |\mu\rangle_i \langle \nu| \) (\( \mu, \nu = a, b \)) are atomic operators, and \( g_i \) is the coupling strength of atom \( i \) to the cavity mode described by the photon creation \( \hat{c} \) and annihilation \( \hat{c}^\dagger \) operators. The coupling strength \( g_i = (\varphi_{ab}/\hbar)\varepsilon_c u_i(r_i) \) is proportional to the dipole matrix element \( \varphi_{ab} \) of the atom on the transition \( |a\rangle \rightarrow |b\rangle \), field per photon \( \varepsilon_c \) within the cavity volume, and the cavity mode function.
We assume that both Rydberg states Stark shift of one of the Rydberg states [29]. Inhomogeneous electric (dc Stark) or magnetic (Zeeman) fields or by a focused non-resonant laser inducing an ac Stark shift of one of the Rydberg states [29].

Several relaxation processes are affecting the system. We assume that both Rydberg states $|a\rangle$ and $|b\rangle$ decay with approximately the same rate $\Gamma$ to some lower state(s) $|s\rangle$ which are decoupled from the cavity field. These decay processes are described by the Liouvillians [27, 28]

$$\mathcal{L}_{a,\hat{\rho}} = \frac{1}{2} \Gamma (2\hat{\sigma}_{aa}^{(i)} \hat{\rho} - \hat{\rho} \hat{\sigma}_{aa}^{(i)}) + \frac{1}{2} \Gamma (2\hat{\sigma}_{bb}^{(i)} \hat{\rho} - \hat{\rho} \hat{\sigma}_{bb}^{(i)})$$

acting onto the density operator $\hat{\rho}$ of the total system. The relaxation of the cavity field toward the thermal equilibrium with rate $\kappa$ is described by

$$\mathcal{L}_{c,\hat{\rho}} = \frac{1}{2} \kappa (1 + \tilde{n}_{\text{th}}) (2\hat{c} \hat{\rho} \hat{c}^\dagger - \hat{c}^\dagger \hat{c} \hat{\rho} - \hat{\rho} \hat{c} \hat{c}^\dagger \hat{\rho}) + \frac{1}{2} \kappa \tilde{n}_{\text{th}} (2\hat{c}^\dagger \hat{\rho} \hat{c} - \hat{c} \hat{\rho} \hat{c}^\dagger - \hat{\rho} \hat{c} \hat{c}^\dagger)$$

where $\tilde{n}_{\text{th}} = (e^{\hbar \omega/\kappa n T} - 1)^{-1}$ is the mean number of thermal photons in the cavity mode at temperature $T$ [27, 28].

The density operator of the total system obeys the master equation

$$\partial_t \hat{\rho} = -\frac{i}{\hbar} [\hat{\mathcal{H}}, \hat{\rho}] + \mathcal{L}_{a,\hat{\rho}} + \mathcal{L}_{b,\hat{\rho}} + \mathcal{L}_{c,\hat{\rho}}.$$

We solve numerically the equations for the density matrix of the system whose Hilbert space is a tensor product space of two three-state atoms, $\{|a\rangle, |b\rangle, |s\rangle\}$, and the cavity field with the photon number states $\{|n\rangle\}$ truncated at sufficiently large $n \leq 100$.

**B. Adiabatic elimination of the cavity mode**

We are interested in the state or excitation transfer between the atoms 1 and 2 using the cavity mode as a quantum bus. Consider the states $|ba, n\rangle$ and $|ab, n\rangle$ with either one or the other atom excited, while the cavity mode contains $n$ photons, see Fig. 1(b). There are two transition paths between these states via the intermediate states $|aa, n + 1\rangle$ and $|bb, n - 1\rangle$ involving a photon addition or subtraction from the cavity mode. In order to minimize the effects of relaxation and thermalization of the cavity mode during the transfer, the atoms should exchange virtual cavity photons. We therefore choose the atomic detunings to be similar and large enough, $\Delta_{1,2} \gg |\Delta_1 - \Delta_2|$, $g_{1,2}/n_{\text{max}}$, where $n_{\text{max}}$ is the maximal number of photons that can be in the cavity with appreciable probability; typically, $n_{\text{max}}$ can be taken as $10 \times n_{\text{th}}$, and we recall that at thermal equilibrium the probability distribution of the cavity photon number is

$$P_n = \frac{\tilde{n}^n_{\text{th}}}{(1 + \tilde{n}_{\text{th}})^{n+1}}.$$  (5)

Using the perturbation theory, we adiabatically eliminate the nonresonant intermediate states $|aa, n + 1\rangle$ and $|bb, n - 1\rangle$. We then obtain that the energies of states $|ba, n\rangle$ and $|ab, n\rangle$ are Stark shifted by the cavity field as

$$E_{ba,n} \simeq \frac{1}{2} \delta + \frac{g_{12}^2 (n + 1)}{\Delta_1 + S(n + 1)} - \frac{g_{12}^2 n}{\Delta_2 + S n},$$  (6a)

$$E_{ab,n} \simeq \frac{1}{2} \delta + \frac{g_{12}^2 (n + 1)}{\Delta_2 + S(n + 1)} - \frac{g_{12}^2 n}{\Delta_1 + S n}.$$  (6b)

where $\delta = \Delta_1 - \Delta_2$ and $S \equiv g_{12}^2 + g_{12}^2$. Simultaneously, the transition amplitude between the states $|ba, n\rangle$ and $|ab, n\rangle$ is given by

$$G(n) \simeq \frac{g_{12} n (n + 1)}{2} \left[ \frac{1}{\Delta_1 + S(n + 1)} + \frac{1}{\Delta_2 + S(n + 1)} \right] - \frac{g_{12} n}{2} \left[ \frac{1}{\Delta_1 + S n} + \frac{1}{\Delta_2 + S n} \right],$$  (7)

where the first and the second terms correspond to the amplitudes of transitions via the states $|aa, n + 1\rangle$ and $|bb, n - 1\rangle$, respectively.

**C. Transfer probability**

States $|ba, n\rangle$ and $|ab, n\rangle$ have the energy difference $\delta E(n) = E_{ba,n} - E_{ab,n}$ and are coupled with rate $G(n)$. Since we assumed $\Delta_1 \approx \Delta_2 = \Delta \gg \delta$, we can cast the energy difference and transition rate as

$$\delta E(n) \simeq \delta + \frac{g_{12}^2 - g_{12}^2}{\Delta} (2n + 1),$$  (8)

$$G(n) \simeq \frac{g_{12} g_{21}}{\Delta} \left( 1 - \frac{\delta}{2 \Delta} \right) \left( \frac{g_{12}^2 + g_{12}^2}{\Delta^2} (2n + 1) \right),$$  (9)

where we neglected terms of the order of $\delta^2/\Delta^2$, $\delta^4/\Delta^4$ and higher. We thus see that both $\delta E(n)$ and $G(n)$ depend on the cavity photon number $n$. If the coupling strengths

**FIG. 1. Schematics of the system. (a) Two atoms with levels $|a\rangle$, $|b\rangle$ are coupled to the cavity field $\hat{c}$ with the coupling strengths $g_1$ and $g_2$ and detunings $\Delta_1$ and $\Delta_2$, respectively. (b) Transitions between states $|ba, n\rangle$ and $|ab, n\rangle$ containing $n$ cavity photons can occur via two paths involving intermediate nonresonant states $|bb, n - 1\rangle$ and $|aa, n + 1\rangle$.**
Without relaxations, the probability for the system to be in state $|b, n\rangle$ becomes $n$ independent. Then, by choosing $\delta = 0$, we obtain that both atoms have the same transition frequency, $\delta E(n) = 0 \forall n$. Simultaneously, due to interference between the two transition paths, $G(n)$ only weakly depends on the cavity photon number $n$, leading to resonant exchange of excitation between the atoms. In Fig. 2 upper panel, we show the oscillations between the states $|ba, n\rangle$ and $|ab, n\rangle$ of the two atoms with equal couplings $g_1 = g_2$ to the cavity which has a large mean thermal photon number $\bar{n}_{th}$, corresponding to a broad photon number distribution $P_n$.

When, however, the coupling strengths $g_1$ and $g_2$ are different, the energy difference $\delta E(n)$ is photon number dependent. Then, in a thermal cavity, $\bar{n}_{th} > 0$, with a broad distribution $P_n$ of photon numbers, $\delta E(n)$ cannot be made to vanish for all $n$. In fact the difference between $\delta E(n)$ for various $n$ is of the order of the transition rate $G$, which suppresses the amplitude of oscillations between the states $|ba\rangle$ and $|ab\rangle$, as shown in Fig. 2 lower panel.

We can derive an approximate expression for the transfer probability between the states $|ba\rangle$ and $|ab\rangle$ as follows. Assume that we start in state $|ba, n\rangle$ at time $t = 0$. Without relaxations, the probability for the system to be in state $|ab, n\rangle$ would be given by

$$p_{ab,n}(t) = \frac{|G(n)|^2}{\bar{G}(n)} \sin^2 \left[ \bar{G}(n)t \right],$$

where $\bar{G}(n) = \sqrt{|G(n)|^2 + \frac{1}{4} |\delta E(n)|^2}$ is the generalized Rabi frequency for the oscillations between $|ba, n\rangle$ and $|ab, n\rangle$. Relaxations will result in exponential damping of the transfer probability. The decay of states $|a\rangle$ and $|b\rangle$ of each atom with rate $\Gamma$ leads to multiplication by $e^{-\Gamma t}$. We neglect for now the cavity relaxation and will consider its effect later. Thus the transfer probability from state $|ba, n\rangle$ to state $|ab, n\rangle$ is given by

$$p_{ab,n}(t) = e^{-2\Gamma t} \frac{|G(n)|^2}{\bar{G}(n)} \sin^2 \left[ \bar{G}(n)t \right].$$

If initially there is an equilibrium photon number distribution $P_n$ in the cavity, the total probability of transfer between $|ba\rangle$ and $|ab\rangle$ is given by

$$p_{ab}(t) = \sum_n P_n p_{ab,n}(t),$$

This expression approximates well the exact dynamics of the system for small $\kappa \ll G$ as verified in Fig. 2.

III. REDUCING THE NUMBER OF THERMAL PHOTONS

The thermal photons in the cavity thus preclude efficient state transfer between the atoms when their couplings to the cavity mode have, in general, different strength, $g_1 \neq g_2$. We now outline a method to reduce the number of photons in the cavity, which will significantly increase the transfer probability. We will use an ensemble of stationary trapped atoms extracting photons from the cavity by continuous optical pumping, attaining thereby an equilibrium with a smaller mean number of photons. We note a conceptually similar approach [30–32] to extract thermal photons from a microwave cavity by sending across it a sequence of atoms prepared in the lower Rydberg state, which is typically done in the transient regime to achieve a nearly empty cavity until it equilibrates with the thermal environment.

A. Cavity cooling by photon extraction

Our strategy to cool the cavity is to continuously extract the photons from it with a rate $\gamma_c \gg \kappa$. This process is described by the Liouvillian

$$\mathcal{L}'_c \hat{\rho} = \frac{i}{2} \gamma_c (\hat{c} \hat{c}^\dagger - \hat{c}^\dagger \hat{c} - \hat{\rho} \hat{c} \hat{c}^\dagger),$$

which should be added to $\mathcal{L}_c \hat{\rho}$ in Eq. (3). The photon extraction thus breaks the balance between the usual photon decay to and addition from the thermal reservoir. It follows from the Master equation for the cavity field that the photon number probabilities obey the equation

$$\partial_t P_n = d(n + 1)P_{n+1} + aP_{n-1} - dP_n - a(n + 1)P_n,$$

where $a = \kappa \bar{n}_{th}$ and $d = \kappa (\bar{n}_{th} + 1) + \gamma_c$. In the steady state, $\partial_t P_n = 0$, we have the detailed balance $d(n + 1)P_{n+1} = a(n + 1)P_n$ and $dP_n = aP_{n-1}$ for any transition $n \leftrightarrow n \pm 1$. This leads to $P_n = (a/d)^n P_0.$
volves absorption of a laser photon and cavity photon. and frequency $\Omega$ and large detuning $\Delta$ the two-photon transition $|1\rangle \rightarrow |g, n\rangle$ from state $|g\rangle$ to lower excited state $|r\rangle$ with large $\bar{n}$, we obtain the incoherent transition rate $\gamma_{inco}$, which is highly peaked around $n = 0$.

**B. Photon extraction by optical pumping**

To extract the photons from the thermal cavity, we envision a system depicted in Fig. 4. An ensemble of $N_c$ “cooling” atoms in the ground state $|g\rangle$ are trapped near the cavity antinode. A laser field acts on the transition from state $|g\rangle$ to a Rydberg state $|i\rangle$ with the Rabi frequency $\Omega$ and large detuning $\Delta_c \gg \Omega$. Each atom is coupled to the cavity field $\hat{e}$ on the Rydberg transition $|i\rangle \rightarrow |r\rangle$ with strength $g_e \ll \Delta_c$. Upon adiabatic elimination of the nonresonant intermediate state $|i\rangle$, we obtain an effective Rabi frequency $\Omega_e^{(2)} = \Omega g_e \sqrt{\bar{n}}/\Delta_c$ for the two-photon transition $|g, n\rangle \rightarrow |r, n - 1\rangle$ which involves absorption of a laser photon and a cavity photon. We can write the equations for the amplitudes $A_{g,n}$ and $A_{r,n-1}$ of states $|g, n\rangle$ and $|r, n - 1\rangle$ as

$$\partial_t A_{g,n} = -i \Omega_e^{(2)} A_{r,n-1},$$
$$\partial_t A_{r,n-1} = -\frac{i}{2} \Gamma_r A_{r,n-1} - i \Omega_e^{(2)} A_{g,n},$$

where $\Gamma_r$ is the population decay rate of the Rydberg state $|r\rangle$ and we assume the two-photon resonance. Assuming $\Gamma_r \gg \Omega_e^{(2)}$ (see below), we can set $\partial_t A_{r,n-1} = 0$ and obtain the incoherent transition rate $\frac{1}{2} \Gamma_r = \frac{\Omega_e^{(2)}^2}{\Delta_c^2} \frac{1}{\Gamma_r}$ from $|g, n\rangle$ to $|r, n - 1\rangle$. With $N_c$ cooling atoms, we can then identify the photon extraction rate via $\gamma_{c} = N_c \Gamma_r$, leading to

$$\gamma_{c} = N_c \frac{4 \Omega_e^{(2)}^2 g_e^2}{\Delta_c^2 \Gamma_r}.$$  

Note that contributions of individual atoms add incoherently to the total extraction rate $\gamma_{c}$ and the possible variation of the coupling strength $g_e$ for different atoms can be absorbed into redefinition of the atom number $N_c$. In the above analysis, we have also neglected the (Rydberg blockade) interactions between the atoms in state $|r\rangle$. This approximation is justified for moderate number of photons $n$ and large enough $\Gamma_r \gg \Omega_e^{(2)}$, such that the probability of having simultaneously two or more atoms in state $|r\rangle$ is small.

Typically, Rydberg states have slow population decay rates. Larger decay rate $\Gamma_r$ can be achieved by laser-induced ionization of state $|r\rangle$, which, however, will result in continuous depletion of the number $N_c$ of cooling atoms. A better alternative is to use an auxiliary laser of Rabi frequency $\Omega_a$ to resonantly couple the Rydberg state $|r\rangle$ to a lower excited state $|e\rangle$ having large decay rate $\Gamma_a \gg \Omega_a$, back to the ground state $|g\rangle$, as shown in Fig. 4. This will induce a cascade from $|r\rangle$ to $|g\rangle$ with sufficiently rapid rate $\Gamma_r \approx \frac{\Omega_a^2}{\Gamma_a}$. Thus, each cooling cycle $|g, n\rangle \rightarrow |r, n - 1\rangle \rightarrow |g, n - 1\rangle$ will extract with rate $\gamma_{c}$ a cavity photon, while the number of cooling atoms will be preserved. For $\gamma_{c} \gg \kappa \bar{n}_{th}$, we can then approach a cavity vacuum by optically pumping out thermal photons.
IV. STATE TRANSFER VIA COOLED CAVITY

In Fig. 5 we demonstrate significant enhancement of the amplitude of oscillations between the initial $|ba\rangle$ and the target $|ab\rangle$ states in the presence of rapid extraction of thermal photons from the cavity. Since the probability distribution of the cavity photons is now highly peaked at $n=0$, we set the frequency mismatch $\delta = \frac{g^2}{\kappa}$ to satisfy the resonant condition $\delta E(0) = 0$ in Eq. (8).

A. Optimizing the transfer fidelity

From Eq. (9) we have for the transition rate $G(0) \simeq \frac{g_1 g_2}{\Delta} \left(1 - \frac{\delta^2}{\Delta^2} - \frac{\Delta^2 + g_2^4}{\Delta^4}\right)$. Our aim is to transfer the population of state $|ba\rangle$ to state $|ab\rangle$. Using Eq. (11), we can estimate the lower bound for the transfer probability as being determined mainly by the $n=0$ term,

$$p_{ab}(t) > p_{0ab,0}(t) = \frac{1}{1 + n_{\text{eff}}} e^{-\Gamma t} \times \frac{1}{\Gamma} \left(1 - \cos(2G(0)t)e^{-\kappa_{\text{eff}}t}\right),$$

where we included the effective damping rate $\kappa_{\text{eff}} = (\kappa + \gamma_c)\frac{g^2}{\Delta} + \frac{\kappa_1^2}{\Delta^2}$ of the oscillation amplitude, which can be intuitively understood as follows: During the transfer, states $|ba,0\rangle$ and $|ab,0\rangle$ have small admixture, $\sim \frac{\gamma_c}{\Delta}$, of state $|aa,1\rangle$ containing the additional exchange photon which is damped with rate $(\kappa + \gamma_c)$. The transfer probability is peaked at time $t_{\text{tr}} = \frac{\pi}{2G(0)}$ when $\cos(2G(0)t_{\text{tr}}) = -1$. Assuming $n_{\text{eff}} \ll 1$ and $(\kappa_{\text{eff}}, \Gamma)t_{\text{tr}} \ll 1$, the transfer fidelity is then

$$F \equiv P_{0ab,0}(t_{\text{tr}}) \geq \left[1 - \frac{\kappa n_{\text{th}}}{\kappa + \gamma_c}\right] \left[1 - \frac{\pi \Delta}{2\kappa g_1 g_2}\right] \left[1 - \frac{\pi \kappa + \gamma_c - g^2}{4\Delta g_1 g_2}\right],$$

where $g^2 = \frac{1}{3}(g_1^2 + g_2^2)$. Although the right-hand side of this expression underestimates the maximal fidelity, we can still use it to optimize the parameters of the system. Thus, the transfer fidelity is reduced by three factors, and we therefore require that each of them be small:

(i) $\gamma_c + \kappa \gg \kappa n_{\text{th}}$, i.e. the cooling rate $\gamma_c$ should be sufficiently large to have the mean photon number small, $n_{\text{eff}} \ll 1$;

(ii) $\kappa g_2 \gg \Gamma$, i.e., the transition rate $G(0)$ should be large enough to have small probability of the atomic decay during the transfer.

(iii) $\Delta \gg \gamma_c + \kappa$, i.e., the cavity field should be strongly detuned to have small photon population and therefore decay during the transfer.

With $\gamma_c \gg \kappa$, the total reduction of the fidelity, or infidelity, can be estimated as

$$1 - F \simeq \frac{\kappa n_{\text{th}}}{\gamma_c} + \frac{\pi \Gamma \Delta}{g_1 g_2} + \frac{\gamma_c g^2}{4\Delta g_1 g_2}.$$  \hspace{1cm} (17)

We can minimize this expression with respect to $\Delta$, with the other parameters fixed, obtaining

$$1 - F \simeq \frac{\kappa n_{\text{th}}}{\gamma_c} + \frac{\pi g}{g_1 g_2} \sqrt{\gamma_c},$$  \hspace{1cm} (18)

for $\Delta = g\sqrt{\bar{n}_c}$. Next, we minimize the resulting infidelity with respect to $\gamma_c$, finally obtaining

$$\min(1 - F) \leq 3(\kappa n_{\text{th}})^{1/3} \left(\frac{\pi g \sqrt{\gamma_c}}{g_1 g_2}\right)^{2/3}$$  \hspace{1cm} (19)

for $\gamma_c = \left(\frac{2\kappa n_{\text{th}} g_2^2}{\pi g \sqrt{\gamma_c}}\right)^{2/3}$.

B. Experimental considerations

We assume the following realistic parameters of the system: The cavity mode resonant frequency is $\omega = 2\pi \times 5$ GHz, and its quality factor is $Q = 10^8$ leading to the decay rate $\kappa \simeq 300$ kHz. The Rydberg–cavity coupling rates are $g_1 = 2\pi \times 5$ MHz and similar for $g_2$. The decay rates of the atoms are $\Gamma = 10$ kHz. With $n_{\text{th}} \sim 10$ at cryogenic environment, we need to choose the cooling rate $\gamma_c \gtrsim 15$ MHz, which can be achieved with $N_t \sim 1000$ cooling atoms, with $\Omega \simeq g_\gamma = 2\pi \times 0.1$ MHz, $\Delta_c = 100\Omega$ and $\Gamma \simeq 1$ MHz. We then choose the detunings $\Delta_1 \simeq 30g_1$ and $\Delta_2 = \Delta_1 + \frac{g_1^2 - g_2^2}{\Delta}$ for the resonant
transfer of the excitation between the atoms. The resulting fidelity is \( F \gtrsim 0.95 \) which we verified numerically. To turn off the transfer, one of the atoms can be strongly detuned by \( |\Delta_1 - \Delta_2| \gg G(0) \approx \frac{2a_2}{\Delta} \), which can be achieved by, e.g., Stark shifting the resonance with a focused laser beam [29].

V. CONCLUSIONS

We have elaborated the conditions for coherent state transfer between two two-level systems through a thermal microwave cavity. We have demonstrated that by actively cooling a cavity mode by continuously removing photons with a laser-driven ensemble of atoms, high-fidelity swap operation between pairs of spatially separated Rydberg-atom qubits is possible in state-of-the-art experimental systems [12, 15, 16, 20].

The SWAP is a universal entangling quantum gate [27], which can also be realized by the present scheme.

Trapped ground-state atoms have good coherence properties and can serve as reliable qubits. The atoms can be excited on demand by focused lasers to the Rydberg states for realizing short distance quantum communication and quantum logic gates. Our results will thus pave the way for the realization of scalable quantum information processing with cold atoms trapped on the integrated superconducting atom chips.

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