Nucleon-Nucleon t-matrix Effective Interaction for (p, 2p) Reactions.

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Abstract. In (p, 2p) reactions the p-p interaction is the main knockout interaction which acts in its full glory. Recent finite range (p, 2p) calculations used a parametrized spin-isospin dependent finite range $t_{NN}(r)$ - effective interaction operator which is a function of the distance between the nucleons alone. In the present work we have evaluated the nucleon-nucleon (N-N) t-matrix effective interaction using the Reid soft core N-N interaction potential. In this we find that the proper t-matrix effective interaction is not only a function of spin and isospin of the interacting nucleons but is a strong function of the relative orbital angular momentum, $L$ also. The radial behaviour of the $t_L(r)$ is also found to be very unconventional in that it is found to go to zero at $r = 0$ while in the conventional usage the $t(r)$ is taken to peak at $r = 0$ because of its representation in terms of Yukawa functions. This very different behaviour of our calculated N-N t-matrix effective interaction operator is expected to make large changes in the Finite Range (FR-DWIA) predictions for (p, 2p) reaction cross sections. Besides this it is hoped that the marked differences one obtains in the behaviour of these $t_L(r)$'s from different realistic N-N interactions through their resulting fits obtained for the (p, 2p) reactions will provide us a tool to select the proper N-N interaction out of the many available in literature.

1. Introduction

For a long time the medium energy (p, 2p) reactions on light medium mass nuclei were considered to be very good tools to obtain the absolute proton spectroscopic factors in those target nuclei. These expectations grew further with the recent advancements made in this field due to the incorporation of the sophisticated relativistic finite range methodology and the development of related computer programs [1, 2, 3, 4, 5, 6]. For performing these advanced calculations one requires not only the scalar and vector components of the optical potentials which in the Dirac phenomenology fit the proton elastic scattering data but also requires the proper p-p t-matrix effective interaction, $t(\vec{r})$. The conventional finite range distorted wave impulse approximation FR-DWIA calculations employ the meson exchange model t-matrix effective interaction by Love and Franey or Horowitz [7, 8, 9] or others in the same category [10, 11] where the direct and exchange contributions to the amplitude are parametrized separately in terms of a number of Yukawa type functions in the first-order Born approximation. Thus far, the use of the Dirac phenomenology in the present relativistic finite range-distorted wave impulse approximation (RFR-DWIA) has not resulted much improvement in agreements with the data in comparison to the non-relativistic FR-DWIA formalisms. The existing studies of the sensitivities of the analyzing power to both the zero range (ZR) and finite range (FR) approximations on $^{40}$Ca...
and \(^{208}\text{Pb}\) targets have not resulted in any definite conclusion as to which approximation gives a consistently better description of the data \([3, 6]\). The RFR-DWIA calculations using all the inputs mentioned above sometimes describe the shape of the energy sharing spectra reasonably well but often the analyzing power data fits show significant disagreement. Even the inclusion of the assumed 10\% to 20\% medium modification of the interaction parameters could not explain the inconsistencies completely indicating thereby that something more basic is missing in the assumed \(p-p\ t\)-matrix effective interaction.

2. Knockout Reactions

The knockout reaction \(A(a, a'b)B\) of Fig.1 is akin to a free \(a-b\) scattering \([12, 13]\). Here the incident proton, \(a\) of momentum \(k_0\) and kinetic energy \(E_0\) interacts with the weakly bound proton, \(b\) with its momentum, \(-k_3\) and binding energy, \(-Q\). After scattering, the proton, \(a'\) (just another name for \(a\) after the scattering) goes out with momentum \(k_1\) and kinetic energy \(E_1\) and the ejected proton, \(b\) now goes out with momentum \(k_2\) and kinetic energy \(E_2\). The residual nucleus, \(B\) behaving as a spectator moves with momentum \(k_3\) and kinetic energy \(E_3\). Now as the incident energy, \(E_0\) is much larger than the binding energy, \(-Q\) one can consider it to be negligible in comparison to \(E_0\) thus the \((a, ab)\) reaction can be regarded to be close to free \(a-b\) scattering (\(k_3 = 0\)). The energy momentum conservation equations for the free \(a-b\) scattering are:

\[
E_o = E_1' + E_2' \quad \text{and} \quad k_0 = k_1' \cos \theta_1' + k_2' \cos \theta_2'.
\]

However if the particle \(b\) is bound to \(B\) by \(-Q\) (a few MeV) in the target nucleus, \(A\), and as the momentum \(k_3\) of the recoiling nucleus is small, taking it to be zero one gets:

\[
E_o = E_1 + E_2 - Q.
\]

As the \(Q\) here is negative we have

\[
E_1 < E_1' \quad \text{and} \quad E_2 < E_2' \quad \text{and} \quad (k_0 = k_1 \cos \theta_1 + k_2 \cos \theta_2).
\]

because,

\[
k_1 = \sqrt{2m_1E_1}, \quad k_2 = \sqrt{2m_2E_2}
\]

Figure 1. Knockout reaction schematics.
Hence $k'_1$ and $k'_2$ are more than $k_1$ and $k_2$ respectively. Similarly the angles $\theta_1$ and $\theta_2$ of $\vec{k}_1$ and $\vec{k}_2$ will be slightly less than $\theta'_1$ and $\theta'_2$ of the free scattering.

Now for the $A(a,a')B$ reaction the transition amplitude $T^{bL,\wedge}_{f_1}$ is written as [12]:

$$T^{bL,\wedge}_{f_1}(\vec{k}_f,\vec{k}_i) = \int \chi^{(-)*}_{1}(\vec{k}_{aB},\vec{r}_{aB})\chi^{(-)*}_{2}(\vec{k}_{bB},\vec{R}_{bB})t_{ab}(\vec{r}_{ab})$$

$$\chi^{(+)}_{0}(\vec{k}_{1-A},\vec{r}_{1-A})\varphi_{b\wedge}(\vec{R}_{bB})d\vec{r}_{aB}d\vec{R}_{bB}$$

(1)

The scattering state distorted waves $\chi_0$, $\chi_1$ and $\chi_2$ of this expression are evaluated using the optical potentials for the $p_o-A$, $p_1-B$ and $p_2-B$ systems with the relative coordinates $\vec{r}_{aB}$, $\vec{r}_{aB}$ and $\vec{R}_{bB}$ respectively [14]. The bound state of $b$ and $B$ is represented by $\varphi_{b\wedge}(\vec{R}_{bB})$ where $\wedge$ represents all the quantum numbers collectively. Finally all these relative coordinates are expressed in terms of $\vec{r}_{ab}(\equiv \vec{r})$ and $\vec{R}_{bB}(\equiv \vec{R})$. While using the zero range approximation the transition matrix element, $T_{f_1}$ of this expression is factorized into a half off-shell $p-p$ scattering amplitude and a distorted momentum distribution of the struck proton in the target nucleus. On the other hand when one uses the full finite range $t_{ab}(\vec{r}_{ab})$ the factorization is not possible due to the presence of optical distortions. In the FR-DWIA formalism the chosen relative coordinates $\vec{r}$ and $\vec{R}$ get coupled through the distorted waves $\chi^{(+)}_{0}(\vec{k}_{aA},\vec{r}_{aA})$ and $\chi^{(-)*}_{1}(\vec{k}_{bB},\vec{r}_{bB})$ leading to no-factorization of the scattering amplitude of particles $a$ and $b$ [12, 13]. Usually one treats the $t_{ab}(\vec{r}_{ab})$ as a series expansion in terms of Yukawa functions of various range parameters [4] and that too only a function of $|r_{ab}|$. The parameters in this expansion are chosen to fit the $N-N$ amplitudes rather than the data directly.

As indicated earlier the quasi-free scattering ($p, 2p$) reaction kinematic is only slightly modified because of the binding of the proton in the target nucleus ($-Q$, a small value). Therefore the assumption made that one can replace $t_{pp}(\vec{r}_{pp})$ in ($p, 2p$) reaction by free $p-p$ $t$ - matrix effective interaction.

We can obtain the $p-p$ $t$-matrix effective interaction, $t_{ab}(\vec{r}_{ab})$ through another approach where the $N-N$ interaction potentials obtained directly by fitting $N-N$ scattering data are employed in solving the Schrödinger equation. Our method uses solution of the Schrödinger equation in configuration space and is much different from the conventional method (where this equation is solved in momentum space by matrix inversion only to first order).

To start with we use the Reid soft core realistic $N-N$ potential [15], seen in Fig.2 which is spin and isospin dependent and contains tensor and spin-orbit interaction in triplet states (where $L \neq J$, $L$ and $J$ also $\neq 0$). In such states $L$ is not a good quantum number and $L = J-1$ and $L = J+1$ are present in the same $J^+$ state. Here one faces the difficulty that there are coupled equations for these triplet states such as $(^3S_1-^3D_1$ and $^3P_2-^3F_2$ etc.).
In order to resolve this coupling of different L’s which lead to coupled second order differential equations a semi-classical treatment has been developed for the N-N scattering problem. This method is similar to the deuteron problem solved earlier with tensor interaction [16]. The coupled wave functions, \( \Psi^{(+)}(k, r) \) and \( \Psi^{(-)}(k, r) \) are obtained as,

\[
\begin{align*}
\Psi^{(+)}(k, r) &= u(k, r) \cos \frac{\theta(k, r)}{2} - w(k, r) \sin \frac{\theta(k, r)}{2} \\
\Psi^{(-)}(k, r) &= u(k, r) \sin \frac{\theta(k, r)}{2} + w(k, r) \cos \frac{\theta(k, r)}{2},
\end{align*}
\]

(2)

where \( \theta(k, r) \) is function of tensor potential, \( V_t(r) \), spin-orbit potential \( V_{LS}(r) \) and the difference between the centrifugal repulsions between the two \( \ell \) - states, \( u(k, r) \) (for the lower \( \ell \)-value) and \( w(k, r) \) (for the larger \( \ell \)-value). As \( \theta(k, r) \rightarrow 0 \) for \( r \) beyond the range of \( V_t(r) \) these \( u(k, r) \) and \( w(k, r) \)’s are used to get the phase shifts in these \( J \ell \) states.

For the singlet states as well as for the triplet states (with \( L \neq J-1 \) and \( L \neq J+1 \) cases) there exist no \( L \)-coupling hence the radial Schrödinger equations are easily solvable. These solutions are then matched with the known forms of the wave functions in the external region to obtain the phase shifts.

The phase shifts for the singlet as well as triplet states for the \( p-p \) and \( n-p \) scattering at various energies have been found to be matching exceedingly well with those obtained by other techniques. We are now assured that the radial distorted wave functions found by this method are correct. We now use these distorted wave functions in the \( t \)-matrix equation below for various spin, \( S \) and isospin, \( T \) values.

3. Evaluation of the \( t \)-matrix
The \( N-N \) \( t \)-matrix effective interaction, \( t_{12}(\vec{r}_{01}) \) evaluated at the relative energy \( E \), is given by [17, 18]:

\[
t_{12}^{ST}(S, T, E, \vec{r}) = e^{-ikz}V^{ST}(\vec{r})\Psi_{12}^{+}(\vec{r}) = \sum_{L=0,1,2,3...}t_{L}^{ST}(E, \vec{r})P_{L}(\hat{r})
\]

(3)

where,

\[
\Psi_{12}^{+}(\vec{r}) = \sum_{\ell=0,1,2,3...}i^{\ell}(2\ell + 1)\frac{u_{\ell}(kr)}{kr}e^{i\sigma_{\ell}}P_{\ell}(\hat{r}).
\]

(4)

\[
t_{L}^{ST}(E, r) = \frac{(2L + 1)}{2} \sum_{\ell, n}V_{\ell}^{ST}(r)i^{\ell}(2\ell + 1)\frac{u_{\ell}(kr)}{kr}J_{n}(kr)(-i)^{n}
\]

\[
(2n + 1)e^{i\sigma_{n}}\int_{-1}^{1}P_{L}^{*}(\cos \theta)P_{l}(\cos \theta)P_{n}(\cos \theta)d(\cos \theta).
\]

(5)

where \( j_{\ell}(kr) \)’s are the spherical Bessel functions and \( P_{l}(x) \) are the Legendre polynomials, \( \sigma_{\ell} \)’s are the Coulomb phase shifts and \( u_{\ell}(kr) \)’s are the radial wave functions in the \( \ell \) channel.

These \( t_{L}^{ST}(E, r) \)’s form our \( t \)-matrix effective interactions. For \( p-p \) case the total isospin \( T = 1 \), therefore it will be only \((S = 0, \text{ - even})\) and \((S = 1, \text{ - odd})\) states present in the realistic interactions \( V_{t}^{ST}(r) \). Each of these states will provide \( t_{L}^{ST}(E, r) \)’s which will be complex in general. It is easily seen from Eqs.(3) to (5) that for one \( \ell \)-value of \( V_{t}^{ST}(r) \), there will be several possible \( L \)-values in the effective interaction \( t_{L}^{ST}(E, r) \), i.e. \( L \) here will vary from 0 to \( \infty \). It is
to be emphasized at this stage that even though the interactions $V_L^{ST}(r)$’s are real, the $t$-matrix effective interactions $t_L^{ST}(E, r)$’s could, in general be complex. Moreover for each orbital angular momentum $\ell$ of the realistic interactions $V_L^{ST}(r)$, the $L$ of the effective interaction, $t_L^{ST}(E, r)$ will vary and peak at some other value of $L$. The observation of different $L$’s for the same $\ell$ - value indicate that the symmetry character (decided by the $\ell$ - value) of the state will be modified in the effective interaction, $t_L^{ST}(E, r)$. In fact the derived $t$- matrix effective interaction will have mixed symmetry which should not be disturbed by any re-symmetrization. This is because all these $t_L$’s are intricately interwoven so as to preserve the symmetry in the overall expansion in terms of the series. It is easy to observe in Eq.(3) that the symmetrization of $t^+_{\ell}(\vec{r})$ due to symmetry of $\Psi^{ST}_\ell(r)$ is modified due to the multiplication $e^{ikz}$ or $e^{i\vec{k}_o\cdot\vec{r}}$ (which is not symmetrized) in the expression of $t_L^{ST}(E, r)$. One must also realize that as long as one uses the matrix elements of these $t^{ST}(E, \vec{r})$ between the initial plane wave state and the final plane wave state the (effectively taking the Fourier transform of $t^{ST}(E, \vec{r})$) the symmetry is restored. Therefore, after taking the Fourier transform, one can perform any operation what so ever one wants with these matrix element because the symmetry is already restored in this procedure of taking the Fourier transform.

Figure 3. $t_L^{ST}(E, r)$ $N$-$N$ effective interaction at $E_{inc.} = 200$ MeV for various $L$ values for central, tensor and spin-orbit potentials. Real $t_L^{ST}(E, r)$ (Solid Line), Imaginary $t_L^{ST}(E, r)$ (Dotted Line).
4. Results and Discussion

The $t^{ST}_{L}(E, r)$ for 200 MeV incident energy using the Reid soft core $N-N$ potential and the results are shown in Figs.3 and 4. The results are calculated for $(2S+1)(2L+1)$ states as $^{13}S_0$, $^{13}P_1$, $^{13}D_2$, $^{33}P_0$, $^{31}S_1$, $^{35}P_1$, $^{35}P_2$, $^{31}D_1$, $^{31}D_2$ and $^{35}F_2$ for various $L$ - values of $t^{ST}_{L}(E, r)$. As there are too many $L$ - values contributing to all these states, only certain representative and significant results are presented and discussed here.

It is observed that whatever be the state $^{13}S_0$, $^{13}P_1$, $^{13}D_2$, $^{33}P_0$, $^{31}S_1$, $^{35}P_1$, $^{35}P_2$, $^{31}D_1$, $^{31}D_2$ or $^{35}F_2$ and whatever the $L$ - value, the $t^{ST}_{L}(E, r)$ vanish at $r = 0$ which can be understood in terms of vanishing of $\Psi_{LJ}(r)$ at $r = 0$. In the case of a hard core, there will be a $\delta(r-r_c)$ at $r = r_c$ as seen in Eq. 53 of [19]. In the case of a soft repulsive core however, this will result in a $t^{ST}_{L}(E, r = 0) = 0$ while there will be a rise in $t^{ST}_{L}(E, r)$ below $r = r_c$ and then it will fall back to zero as $r$ goes to zero (in the case where there is no mixing due to the tensor interaction), this behaviour is clearly seen in our results in Fig.3. Besides, the real and imaginary components are observed to have large variations in shape and strengths for this energy. Except for the triplet states with $\ell \neq J$ (as well as for $L$ or $S$ or $J \neq 0$) and for the tensor interaction, there is a node at the value of $r$ where the $V^{ST}_{L}(r)$ goes to zero. It is seen that in the case of triplet states with $\ell \neq J$ this node does not appear necessarily. The range of $t^{ST}_{L}(E, r)$ is not shifted outwards for larger $L$’s as is normally expected from the centrifugal barrier considerations. The general impression one gets is that $t^{ST}_{L}(E, r)$’s behave much differently in comparison to the corresponding $V^{ST}_{L}(r)$’s. From these one can easily conclude that the $N-N$ $t$ - matrix effective interaction from Love and Franey [7, 8], Satchler and Love [11], Bertsch et al [10] and that of Horowitz [9] which behave like sums of Yukawa functions are not properly behaved as they all peak at $r = 0$. We have seen that there are many more multipole $(L)$-components of the $t^{ST}_{L}(E, r)$’s instead of just the $L = 0$ component. The symmetry character is built-in if large number of $t^{ST}_{L}(E, r)$’s are incorporated. Moreover the imaginary part of these $t^{ST}_{L}(E, r)$’s, in the present situation, cannot be interpreted as absorptive potentials because they are present even when the phase shifts are real.

In conclusion one can say that the $t$ - matrix effective interactions have many more multipole components in its structure and their individual components may have symmetry characteristics while the overall symmetry is taken care of in their interwoven behaviour. Because of this as well as because of wide variations in their values and shapes one can not have the $L$ - averaging as it will change their symmetry character as also their rotation characteristics. The tensor interaction makes an order of magnitude large contributions in the $t^{ST}_{L}(E, r)$’s as compared to the uncoupled multipole components; therefore, this interaction should be treated with caution, without the use of Born approximation. The repulsive core character is also affected by the couplings caused by the tensor interaction, although the $t$ - matrix effective interactions vanishes at $r = 0$ the vanishing of $t_L(r)$ where $V_{Central}(r)$ goes to zero may be altered due to the hard core for coupled states.
5. References

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