Mathematical modeling of the transmission pipeline under the influence of earthquake seismic waves

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Abstract. The stress-strain state of an underground transmission pipeline under the action of earthquake seismic waves is considered. The main objective of this study is the construction of a mathematical model with the analytical solution for lower estimation of the influence of earthquake seismic waves on the transmission pipeline. The spectra of the design seismic actions of the earthquake are determined on the basis of parameters of real seismograms of past earthquakes. The generalized functions of seismic impulses are constructed. The transmission pipeline is modeled by a thin bar. Equations of pipeline motion using the Laplace transformation in time are solved, stresses are determined. The numerical example is considered. The results of calculations on the theories of Kirchhoff and Timoshenko are compared. Graphs of stress changes depending on the earthquake’s strength in points are given. The limiting value of the earthquake magnitude resulting in large stresses and deformations is determined. As a result, the transmission pipeline is destroyed.

1. Introduction

Past data shows that earthquake damage to underground transmission pipeline can lead to extended outage and unpredictably high socio-economic losses in society, not to mention the environmental consequences that will be manifested in the future. In a relatively recent study [1], an extensive review of the literature on this issue was carried out. The analysis of existing models for calculating the stress-strain state of the transmission pipeline under the influence of earthquake seismic waves. This work makes a disappointing conclusion that modern seismic standards and guidance documents do not reflect the important results of the latest research. There are also no general principles with detailed step-by-step methodology for designing aseismic underground pipeline systems. On the contrary, there is basically a set of empirical recommendations that are sometimes incomplete or outdated.

The nature of deformation and destruction of constructions under the influence of earthquake seismic waves indicates that underground structures are less damaged than above-ground structures. There are several reasons for this. First, underground objects are constrained by the surrounding soil, which significantly reduces the oscillations amplitude. Second, seismic wave energy is dissipated into the surrounding soil. Third, the most dangerous surface waves have no greater effect on underground objects than on above-ground structures. However, in the book of Poliakov S.V. [2], it is noted that some underground structures have experienced significant deformation and damage during major earthquakes of the 20th century, including the San Francisco USA in 1906, Tokyo Japan in 1923, Chile in 1943, etc. The book [3] is devoted to the review of experimental studies of soil movement in surface and deep layers.
It should be noted that in soils with significantly different mechanical characteristics, an increase in the intensity of the influence of earthquake seismic waves. They cause deformation or destruction of long underground structures, which leads to their failure or reduce performance. It is accepted to allocate two factors of influence of ground conditions on the seismic stability of structures. The first factor is due to the dynamic characteristics of soils in the surface layers. The second is determined by the decrease of dispersed soil bearing capacity arising from oscillations. As a result, this leads to a significant change in the stress-strain state of the underground structure. At certain combinations of characteristics, in particular, the frequencies of natural oscillations of surface layers with the frequencies of forced oscillations of bedrock, resonance phenomenon in surface layers may occur [2].

In the work [4], the regularities of propagation of the incident quasicompressional and transverse seismic waves in the intermediate monoclinic layer between strongly aeolotropic soil layers were analyzed. It is established that the critical angles for incident quasicompressional and transverse waves are different. It is shown that the angles of incidence and the width of the soil layer have a significant effect on the resultant displacement, velocity and reflectance in the considered soil layer.

Work [5] explores the influence of seismic waves on an underground structure – power station. It is established that the deformation of the underground structure is obviously affected by the angles of incidence of the transverse SV-wave and the longitudinal P-wave. The deformation reaches a maximum at an angle of incidence of 30° for SV- and P-waves. Weak parts of the underground structure of the power station under seismic load are places with a large free surface, such as arches, roofs and floors. It is shown that the state of the contact surface between the surrounding rock and the underground structure plays a key role in the strength of the underground structure. The surrounding soil has a damping effect on the underground structure. It is also a favorable factor for the underground structural competence under seismic load.

The purpose of this work is the lower estimate of the influence of earthquake seismic waves on the transmission pipeline. In the opinion of the authors of this work, it is extremely relevant to determine the limit earthquake magnitude, at which the destruction of the pipeline may occur, leading to environmental disasters.

A seismic wave, or a seismic impulse, as can be seen from the seismograms, is a short-term oscillation with a duration of 3-4 dominant oscillation periods. In the first and terminal phases, oscillations with a high frequency and low amplitude are registered. The wave coming from great depths from the seismic center to the earth surface, is refracted, reflected from layers with curved interface boundaries with different wave propagation velocities, and wave interference occurs. Therefore, as the distance from the seismic center increases, the wave pattern on the earth’s surface changes and becomes more complex. The impulse shape also changes depending on the soil properties at the observation point, attenuation constant and oscillation frequency. The most intense oscillations are recorded in the second – main phase. Oscillations occur with high amplitude and relatively low frequency (fig. 1). In work [6], 200 seismograms recorded during earthquakes in different regions of the Russian Federation and neighboring countries are presented. To perform mathematical modeling for the seismogram shown in figure 1, an approximation of the seismogram was constructed (fig. 2).

Oscillations with high amplitude and low frequency carry the maximum fraction of seismic impulse energy and affect structures more strongly than the oscillations of the first and last phases [7]. The main phase of seismic impulses is usually approximated by expressions [8]:

\[ v_1(t) = \tilde{V}te^{-\eta t} \sin \Omega t, \quad v_2(t) = Ve^{-(\eta t)^2} \sin \Omega t, \]  

(1)

\[ v_1(t) \] and \( v_2(t) \) – soil particles displacement; \( \tilde{V} \) – velocity and displacement amplitudes; \( \eta \) – attenuation constant; \( \Omega \) – oscillation frequency; \( t \) – time. The choice of expressions (1) is based on the superficial similarity with the main phase of seismograms (fig. 2). The velocities and accelerations of soil particles can be found by differentiating expressions (1):

\[ \ddot{v}_1(t) = -\ddot{V}e^{-\eta t} \left\{ \left[ 1 - \left( \frac{\eta^2}{\Omega^2} \right) \right] \Omega t + 2 \frac{\eta}{\Omega} \right\} \sin \Omega t - 2(1 - \eta t) \cos \Omega t, \]
However, engineering seismology manuals refer to earthquake manifestations as successive shocks and aftershocks that follow one after another after a certain period of time [9]. The theory of seismic stability, based on the assumption that the soil motions are performed according to harmonic law, ignores the phenomenon of "shocks" and "aftershocks". Eyewitness accounts show that even with a relatively weak earthquake, there are facts of displacement or overturning of individual solid objects [9].

Below, the harmonic functions (1) are replaced by generalized functions [10] and the action of the seismic impulse on the transmission pipeline is considered as a sequence of shocks.

2. Generalized seismic wave functions

We place the origin of the rectangular coordinate system \(x, z\) on the pipeline axis at the point before the seismic impulse, directing the \(x\)-axis along its axis, the \(z\)-axis perpendicular to the earth surface. We compatible the main phase initiation of seismic waves with the origin of the coordinate axes. At time:

\[
t_i = \left(\frac{1}{4} + n\right) \frac{T_i}{\Omega} = \frac{2\pi}{\Omega \sqrt{1 - \left(\frac{n}{\Omega}\right)^2}} \left(\frac{1}{4} + n \right) = \frac{2\pi}{\Omega} \left(\frac{1}{4} + n\right), \quad n = 0, 1, 2...
\]
The function \( \sin \Omega t \) takes the largest positive values, but \( \cos \Omega t \) equals zero. Therefore, in expressions (1) and their derivatives, the terms containing \( \cos \Omega t \) are assumed to be zero. We also neglect the value of \( \eta \Omega \), as small in comparison with the unit. As a result we get:

\[
\dot{v}_1(t) = -V \Omega^2 e^{-\eta \Omega t} \sin \Omega t, \quad \dot{v}_2(t) = -V \Omega^2 e^{-\eta \Omega t} \sin \Omega t. \tag{2}
\]

Then, in expressions (1) and (2), the function \( \sin \Omega t \) is replaced by its values at times \( t \), and time \( t \) is replaced by time \( t \). Thus, the following generalized functions of the seismic impulse are obtained:

\[
v_1(t) = \frac{2\pi \bar{V}}{\Omega} \sum_{n=0}^{j-1} \left( \frac{1}{4} + n \right) \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right], \tag{3}
\]

\[
v_2(t) = \bar{V} \sum_{n=0}^{j-1} \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right], \tag{3}
\]

\[
\dot{v}_1(t) = -2\pi \bar{V} \Omega \sum_{n=0}^{j-1} \left( \frac{1}{4} + n \right) \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right],
\]

\[
\dot{v}_2(t) = -\Omega^2 \sum_{n=0}^{j-1} \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right]^2,
\]

where \( n = j - 1, \ j \) – number of shocks during seismic impulse action. Impulsive forces are displaced along the pipeline with the compressional velocity \( a \), and act at intervals \( T_0 = T = 2\pi/\Omega \).

3. Pipeline movement equations according to Timoshenko model and their solution

The transmission pipeline is modeled by a thin bar. Using dimensionless quantities and notation in the form:

\[
\xi = \frac{x}{r}, \quad \tau = \frac{c_1 t}{r}, \quad w = \frac{W}{r}, \quad c_1 = \frac{E}{\rho}, \quad c_2 = \frac{kG}{\rho}, \quad r^2 = \frac{J}{F}, \quad \gamma = \frac{\gamma}{\left(1 + \frac{m_2}{\rho F}\right)}, \quad \gamma_r = \frac{c_1}{c_2}, \tag{4}
\]

Bar motion equations in displacements, taking into account the shear deformation and rotary inertia, are written as follows:

\[
\frac{\partial^2 w}{\partial \xi^2} - \frac{\partial \theta}{\partial \xi} - \gamma \frac{\partial \theta}{\partial \tau} = \frac{r p(\xi, \tau)}{pc_2^2}, \quad \frac{\partial^2 w}{\partial \xi^2} - \theta + \gamma_r \left( \frac{\partial^2 \theta}{\partial \xi^2} - \frac{\partial^2 \theta}{\partial \tau^2} \right) = 0. \tag{5}
\]

In expressions (4) and (5): \( r \) – radius of inertia; \( E, G, \rho \) – elastic, shear modulus and density of pipe material; \( k' = 1.1 \) – form of section factor; \( F, J \) – cross-sectional area and axial moment of inertia; \( w, \theta \) – deflection and angle of rotation due to the bending moment; \( \tau \) – time; \( m_2 = \rho_2 \left[D(h-0.39D) + h^2 \tan(0.7\varphi)\right] \) – soil mass above the pipe; \( h \) – distance from the top of the pipeline burial to its pitch line. Pressure on the pipeline by seismic wave has the form:

\[
p(\xi, \tau) = \rho_s(\xi, \tau) \delta(\xi - \tau/\beta) = \bar{\beta} p_s(\xi, \tau) \delta(\tau - \beta \xi),
\]

where \( \delta \) – Dirac delta function, \( \xi_0 = x_0/r, \beta = c_1/\delta, \)

Equations (5) are solved using the Laplace transformation in time:

\[
\frac{d^2 \bar{w}}{d \xi^2} - \frac{d \bar{\theta}}{d \xi} - \gamma^2 = k \gamma e^{-\rho \xi},
\]

\[
\frac{d \bar{w}}{d x} + \gamma_r \frac{d \bar{\theta}}{d \xi} + \left( \gamma, s^2 + 1 \right) \bar{\theta} = 0. \tag{6}
\]
where \( k = -r[p][\xi]/EF \); \( \vec{w} \), \( \vec{\theta} \) – images of deflection \( w \) and angle of rotation \( \theta \). From the first equation (6) is excluded \( \theta \):

\[
\frac{d^2 \vec{w}}{d\xi^2} - (\gamma + 1)s^2 \frac{d\vec{w}}{d\xi} + \gamma s^2 \left(s^2 + \frac{1}{\gamma}\right) \vec{w} = kf_2 e^{-\beta \xi},
\]

\[
\vec{\theta} = \left(\frac{\gamma}{(\gamma, s^2 + 1)}\right) \left[ \frac{d^2 \vec{w}}{d\xi^2} - \left(\gamma s^2 - \frac{1}{\gamma}\right) \frac{d\vec{w}}{d\xi} + k\beta s e^{-\beta \xi} \right],
\]

\( f_2 = (\beta^2 - 1)\gamma, s^2 - 1. \)

The system solution in images (7) is:

\[
\vec{w} = A_1 e^{-\lambda_1 \xi} + A_2 e^{-\lambda_2 \xi} + \frac{k f_2 e^{-\beta \xi}}{h_1 s^2 (s^2 + a_1^2)},
\]

\[
\vec{\theta} = \left(\lambda_1 - \gamma s^2\right) \frac{A_1}{\lambda_1} e^{-\lambda_1 \xi} + \left(\lambda_2 - \gamma s^2\right) \frac{A_2}{-\lambda_2} e^{-\lambda_2 \xi} + \frac{k \beta s e^{-\beta \xi}}{h_1 s (s^2 + a_1^2)},
\]

where \( A_1, A_2 \) – integration constants; \( h_1 = (\beta^2 - \gamma)(\beta^2 - 1), a_1^2 = \gamma/(\gamma, h_1); \lambda_{1,2} \) – two (of four) roots of the characteristic equation:

\[
\lambda^2 - (\gamma + 1)s^2 \lambda^2 + \gamma s^2 \left(s^2 + \frac{1}{\gamma}\right) = 0,
\]

satisfying the condition of attenuation \( \vec{w} \) and \( \vec{\theta} \) to infinity.

Bar cross-section \( \xi = 0 \) is a sliding restraint, the angles of rotation and shear of the cross-section are equal to zero:

\[
\xi = 0, \quad \vec{\theta} = 0, \quad \frac{d\vec{w}}{d\xi} - \vec{\theta} = 0.
\]

The integration constants are determined by boundary conditions (8). The image of the dimensionless bending moment is found by the formula:

\[
\vec{m}(s, \tau) = \frac{M r}{EJ} \frac{d\vec{\theta}}{d\xi},
\]

where \( M \) – dimensional bending moment.

The originals of deflection and bending moment are determined using the inversion formula:

\[
\frac{1}{2\pi i} \int_{\tau} F(s)e^{i\tau s} ds = \left\{ \begin{array}{ll} f(\tau), & \tau > 0, \\
0, & \tau < 0. \end{array} \right.
\]

The integrands have simple poles and branching points. Calculations were made by formula:

\[
I = \sum \text{res}(s) - \sum \int_t,
\]

where \( \text{res}(s) \) – residue, \( \gamma \) – integration paths along the banks of the cut and arcs of a circle of infinitesimal radius. In the case of the action of a unit mobile force for a bending moment the expression is obtained:

\[
m(s, \tau) = \sum_{i=1}^{s} \text{res}(s_i), \quad 0 \leq \xi_i \leq \xi_s = \tau/\beta.
\]

The expanded expression (9) can be written in the form:

\[
m(s, \tau) = \frac{r[p)(\xi_s)\beta}{EF} \sum_{i=1}^{s} R_i,
\]

\[
R_i = \frac{\beta}{4\sqrt{2a_i h_i}} \left[ a_1 (\sin \xi_1 - \sin \xi_2) + 2a_2 e^{-a_2 \xi_1} \cos a_1 \tau_2 \right],
\]

\( a_i \) – integration paths along the banks of the cut and arcs of a circle of infinitesimal radius.
\[ R_z = \frac{\beta (2\beta^2 - \gamma - 1)}{4\sqrt{2}(\gamma - 1)h \sqrt{a^2 + a_i^2}} \left[ a_i \left( \sin \varepsilon - \sin \varepsilon_i \right) - 2a_i \cos \alpha \cos \alpha_i \right] \]

\[ R_i = \frac{\beta^2}{a_i h} \sin \left[ a_i (\tau_2 - \beta \xi) \right]. \]

\[ \varepsilon_{i,2} = a_i \left( \tau_2 \pm \frac{\alpha_i \xi}{2} \right), \quad a_{i,2} = \left[ (\gamma - 1) \sqrt{1 + \frac{a_i^2}{a_i^2} \pm (\gamma + 1)} \right]^2, \]

\[ a^2 = \frac{4\gamma}{\gamma (\gamma - 1)}, \quad \tau_2 = \tau - \tau, \quad \tau_i = \frac{\pi a_i}{2\Omega}. \]

In the case of the action of several forces following each other, using the principle of composition of forces, the result is the following expressions for bending moments:

\[ m_i(\xi, \tau) = \frac{2\pi \rho \bar{V} \Delta}{\Omega EF} \sum_{n=0}^{j-1} \left( \frac{1}{4} + n \right) \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right] \sum_{i=1}^{3} R_i, \]

\[ m_i(\xi, \tau) = \frac{r \bar{V} \Delta}{EF} \sum_{n=0}^{j-1} \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right]^2 \sum_{i=1}^{3} R_i. \]  \hspace{1cm} (10)

4. Solution of the equation of the bar bending technical theory

Using dimensionless quantities (4), we write the equation of motion according to Kirchhoff’s theory:

\[ \frac{\partial^4 w}{\partial \xi^4} + \frac{\partial^2 w}{\partial \tau^2} = \frac{r \rho p(\xi, \tau)}{EF}, \]

where \( \gamma = 1 + (m_i/\rho, F) \). After the Laplace transformation in time we get:

\[ \frac{\partial^4 \bar{w}}{\partial \xi^4} + 4\lambda \frac{\partial \bar{w}}{\partial \xi^2} = k e^{-\beta \xi}. \]

The dimensionless bending moment is found by the formula:

\[ \bar{m} = \frac{\partial^2 \bar{w}}{\partial \xi^2}. \] \hspace{1cm} (11)

The inverse transformation (11) leads the expression of the bending moment to the form:

\[ m(\xi, \tau) = \frac{2\beta \rho p(\xi, \tau)}{\sqrt{\gamma EF}} \sin \left[ \frac{\sqrt{\gamma}}{\beta} (\tau_2 - \beta \xi) \right]. \] \hspace{1cm} (12)

**Table 1.** Calculation of maximum bending moment values

| Line | No of seismogram for [1] | \( V \cdot 10^{-2}, \text{m/s} \) | \( v \cdot 10^{3}, \text{mm/s} \) | Parameters \( \Omega, j, \eta/\Omega \) | \( j \) | \( m_i \) | \( m_\sigma \) |
|------|---------------------------|-------------------------------|--------------------------------|---------------------------------|-----------|-------------|-------------|
| 1    | 156(III)                  | 9                            | 12                            | 9                               | 67       | 0.04        | 0.021       | 0.062      |
| 2    | 163(IV)                   | 8                            | 19                            | 9.6                             | 39       | 0.038       | 0.059       | 0.091      |
| 3    | 119(IV)                   | 8.5                          | 21                            | 9.75                            | 38       | 0.039       | 0.067       | 0.099      |
| 4    | 95(I)                     | 8.3                          | 20                            | 9.7                             | 30       | 0.05        | 0.061       | 0.079      |
| 5    | type II, p. 86            | 0.9                          | 5                             | 21                               | 0.043    | 3           | 0.00034     | 0.0027     |
| 6    | type II, p. 86            | 11                           | 8.75                          | 27                               | 0.056    | 3           | 0.0101      | 0.045      |
| 7    | 100(9)                    | 0.31                         | 1.4                           | 44                               | 0.034    | 7           | 0.00087     | 0.0046     |
| 8    | 8(3)                      | 0.5                          | 1.4                           | 4.1                             | 0.018    | 11          | 0.0056      | 0.0069     |
| 9    | 20(4)                     | 3.8                          | 7.25                          | 4                                | 0.038    | 6           | 0.0162      | 0.043      |
| 10   | 141(5)                    | 3.75                         | 7.2                           | 32.5                             | 0.034    | 7           | 0.035       | 0.054      |
| 11   | 31(1)                     | 3.2                          | 7                             | 31.4                             | 0        | 1           | 0.0014      |            |
In the case of the action of the seismic impulse shown in Figure 1, the maximum moment is determined by formulas:

\[
m_{ii} = \frac{4\pi \beta V \Delta}{\sqrt{\gamma \Omega E F}} \sum_{n=0}^{1} \left( \frac{1}{4} + n \right) \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right].
\]

\[
m_{ii} = \frac{2\pi \beta V \Delta}{\sqrt{\gamma \Omega E F}} \sum_{n=0}^{1} \exp \left[ -\frac{2\pi \eta}{\Omega} \left( \frac{1}{4} + n \right) \right].
\]

It should be noted that the solution on Kirchhoff model is much easier than on Timoshenko model.

5. Model results

We will determine the stresses in the transmission pipeline during earthquakes, the seismograms of which are given in [6]. Table 1 contains the number of the seismogram record, the peak amplitudes of the velocity and displacement, the oscillation frequency of the main phase and the earthquake intensity on the scale [11], depending on the displacement value of the seismometer pendulu. In the work [11] it is also noted that seismic oscillation of the soil have a number of features. In particular, particles on the soil surface make spatial oscillation, and the components of the displacement vector along the coordinate axes are comparable with each other.

The amplitude ratios of the different maximum impulse phases depend on: \( \eta \): the smaller \( \eta \), the greater the amplitude ratio of the second maximum to the amplitude of the first maximum. By plotting graphs \( v_i(t) \) \( (i=1,2) \) using the formulas (3) for different values of the ratio \( \eta/\Omega \) and visual comparison of these graphs with seismograms, Ratio value \( \eta/\Omega \) is determined. The parameters of the seismogram in fig. 1 are given in line 7 of table 1.

Initial data: the pipeline with a diameter \( D = 1 \) m and a wall thickness of \( h_w = 0.01 \) m is laid in sandy soil at the depth of \( h = 1.5 \) m. Soil consistency \( \rho_s = 1.53 \) t/m\(^3\), accept \( a_s = 660 \) m/s, \( b = 310 \) m/s, angle \( \phi = 30^\circ \), coefficient \( K_2 = 2.4 \cdot 10^4 \) MPa/m. Elastic modulus, shear modulus and density of pipe material are equal \( E = 2 \cdot 10^4 \) MPa, \( G = 0.8 \cdot 10^4 \) MPa, \( \rho = 8 \) t/m\(^3\), \( c_1 = 5 \cdot 10^3 \) m/s, \( c_2 = 3.3 \cdot 10^3 \) m/s, \( k' = 1.1 \). Bending moment calculations were performed according to formulas (10), (13) and they are given in Table 1, stresses are determined according to formula \( \sigma = \sqrt{2Em} \).

![Figure 3: Models: Timoshenko (o-\( \sigma_i \)), Kirchhoff (x-\( \sigma_i \)).](image)

Figure 3 shows the stresses in the pipeline depending on the earthquake intensity. The solid and dotted lines show the maximum allowable stress values for the above initial data for the Kirchhoff and Timoshenko models respectively. As can be seen, the technical theory of beam bending (Kirchhoff model) gives overestimated stress values compared to the Timoshenko model. It should be noted that the stress level in the pipeline along with the earthquake magnitude is significantly affected by the
number of shocks and aftershocks. With an increase in the number of shocks, stresses in the pipeline increase, see, for example, lines 5, 6, and 11 and lines 9, 10, and 11 of table 1.

From the obtained results of calculations, it follows that for this pipeline, an earthquake of 4 points is the limit.

6. Conclusion
Mathematical calculations of the influence of seismic waves of an earthquake on the transmission pipeline were carried out using two analytical models. It is shown that the technical theory of beam bending gives overestimated stress values compared to the Timoshenko model. From the obtained results of calculations, it follows that for this pipeline, an earthquake of 4 points is the limit. Of course, the obtained research results are approximate, but having a fairly simple analytical model, a lower estimate can be made. It should be noted that depending on various terrain conditions, the presence of voids near the pipeline, as well as the water cut of the surrounding soil, there may be an increase in the effect of earthquake seismic waves on the transmission pipeline, due to the occurrence of resonance phenomenon as previously noted [2]. In addition, these simple mathematical models can be complicated by introducing various methods and ways of protecting structures from the effects of seismic waves of an earthquake. This will certainly lead to nonlinearities in the differential equations of the transmission pipeline movement, which can be solved only numerically or with the use of complex software systems such as ANSYS.

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