Dynamic Soil-Structure Interaction Using ITM-FEM Approach

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Abstract. The effect of dynamic soil-structure interaction is really important to be examined. Moreover if the structure is nuclear power structure or multi-storey structure. In solving the dynamic problem caused by dynamic mass on the soil-structure and interaction, it will use wavelet transform. This study aims to examine the dynamic soil-structure interaction using ITM-FEM approach. In this study, soil is considered as a semi-infinite media in z-direction with unbounded domain in x-direction. The characteristic of material is considered as an isotropic, homogenous, and linear elastic material, and the material damping seems not dependent on the frequency. The dynamic soil-structure interaction using ITM-FEM approach is conducted by counting dynamic stiffness matrix \{ D^{FE} \}, space truss, hexahedron solid coupling element between FEM and ITM.

1. Introduction
The effect of dynamic soil-structure interaction is really important to be examined. Moreover if the structure is nuclear power structure or multi-storey structure. In solving the dynamic problem caused by dynamic mass on the soil-structure and interaction, it will use the Wavelet transform. The Wavelet transform is the development of Fourier transform where the signal is converted from time domain to Wavelet domain. They are translation and scale. Over the last two decades, the Wavelet transform keeps on improving. Many researchers uses it to complete either ordinary or partial differential equation. The Wavelet transform has more strengths than Fourier transform. By the Wavelet transform, it is possible to gain information about the frequency level and where it occurs, while by the Fourier transform it is only possible to indicate one dimension, namely frequency level, but it is incapable of knowing where it occurs. The other advantage of the Wavelet transform is dynamic mass which is able to be re-constructed from the result of response measurement gained. Also the Wavelet
method directly gives information from frequency domain which is really useful for some dynamic problem. Many studies related to the development of wavelet is still conducted till now. It cannot only be used for construction zone, but it can also be used for other fields, such as medicine, electronics, etc. Kim (2002) uses the wavelet transform to control the response of civil construction using low pass filter where the component of the highest frequency from external excitation will disappear because the response of civil construction is not determined by high frequency. Kalahasti (2011) uses Daubechies Wavelet to complete transient problem in dynamic structure by some finite element approach where the Nodal movement in time domain is gained by completing partial differential equation (PDE). Lenz (2003) research response of soil-structure interaction caused by the mass of train ran where its railway base is located on the layer half space and the problem is solved using Fourier and Wavelet transform. Soil is a semi infinite medium in z direction with unbounded domain in x and y direction. The material property from soil is regard as isotropic, homogenous, and linear elastic material, and the material damping does not depend on the frequency. Although the soil is considered as unbounded homogenous half space, the property is allowed to change with depth but it keeps constant in the individual layer. This configuration is called a layered half space. The structure is scattered by using Finite Element Method (FEM) and the soil modeled by using FEM ought to be coupled in a certain limit where the compatibility from strain and deformation is examined. Coupling method which is used in the soil-structure interaction is ITM-FEM coupling. The relationship between strain and displacement on the imaginary surface can be used to determine the element of matrix. This study aims to examine the dynamic soil-structure interaction using ITM-FEM approach. This study is expected to have useful contribution to develop civil engineering, especially in the context of Wavelet transform [1]-[13]

2. Experimental Details
In this study, soil is considered as semi infinite media in z direction with unbounded domain in x and y direction. The characteristic of material is considered as isotropic, homogenous, and linear elastic material, and the material damping is considered that it does not depend on the frequency. Although soil is considered as a half space homogenous unbounded medium, its property is probably various based on depth, but each layer keeps constant. This configuration is called a layer half space. The simplest Wavelet is Haar Wavelet which is invented by a Hungarian mathematics expert. His name is Alfred Haar. It is invented in 1910. The Haar Wavelet is used to complete differential equations in this study. Haar Wavelet can be divided into two big group, namely Father Wavelet or scaling function φ and Mother Wavelet or Wavelet Ψ. Multiresolution analysis is used to expand both vector space and scale space. The description of various signal resolution having length (soft and rough) is called discrete Wavelet transform (DWT). The Haar Wavelet is a constant function for an interval, orthonormal Wavelet which is simple and has pedestal compact [0,1] [14]-[16].

The procedures of this research is on the figure 1.
Figure 1. The procedures of research

3. Results and Discussion
3.1. Dynamic Stiffness Matrix \{ D^{FE} \}
The dynamic soil-structure interaction can be explained systematically on the figure 2.

Figure 2. The system of soil-structure interaction
- Node which is located in the structure
- Node which is located on the soil-structure limit
Subscript is used to explain nodal system which is separated.
Node located on the surface of soil-structure is symbolized with “h” and the other node is symbolized with “s”. The dynamic system consists of two substructures, namely FE structure and half space. The superscript is used to facilitate it. Structure is indicated with FE and half space is indicated with \( \infty \).
Motion equation of discrete system such as FEM model toward harmonious excitation

\[
[M] \{ \ddot{u} \} + [C] \{ \dot{u} \} + [K] \{ u \} = \{ P_0 \} e^{i\omega t}
\]  

(1)

Where:

- \([M]\) = Mass matrix in global coordinate
- \([C]\) = Attenuation matrix in global coordinate
- \([K]\) = Stiffness matrix in global coordinate

\[ \{ \ddot{u} \} = \text{Displacement in global coordinate} \]
\[ \{ \dot{u} \} = \text{Velocity in global coordinate} \]
\[ \{ u \} = \text{Acceleration in global coordinate} \]

To count the amount of each matrix in the global coordinate, it is needed to count the mass matrix \([M^e]\), stiffness matrix \([K^e]\), and attenuation matrix \([C^e]\) from each structural element in local coordinate with:

\[
[M^e] = \iiint \rho [N]^T [N] \, dV
\]

(2)

\[
[K^e] = \iiint [B]^T [D] [B] \, dV
\]

(3)

\[
[C^e] = \iiint [N]^T [N] \, dV
\]

(4)

Where:

- \([N]\) = Shape function of element
- \([D]\) = Constitutive matrix for isotropic material
- \([B]\) = Strain and displacement matrix of element
- \(\xi\) = Coefficient of attenuation

The amount of matrix \([\bar{D}]\) is:

\[
\bar{D} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix}
1-\mu & 1-\mu & 0 \\
1-\mu & 1-\mu & 0 \\
0 & 0 & \frac{1-2\mu}{2}
\end{bmatrix}
\]

(5)

Then mass matrix \([M^e]\), stiffness matrix \([K^e]\), and attenuation matrix \([C^e]\) from each element in global coordinate is counted by using transformation matrix \([T]\). Then mass matrix \([M^e]\), stiffness matrix \([K^e]\), and attenuation matrix \([C^e]\) are additions of each element.

\[
[M] = \sum_{e=1}^{n} [M^e]
\]

(6)

\[
[K] = \sum_{e=1}^{n} [K^e]
\]

(7)

\[
[C] = \sum_{e=1}^{n} [C^e]
\]

(8)

In this paper it only uses the hexahedron solid element to model soil from half space and the space truss element to model structure. For instance, a tower made of iron is sustained by four legs made of metal which size is on the figure 3 below. In order to facilitate it, the element of tower is just considered as a bar element, so that it only has 3 DOF on the node namely translation in \(x, y, \) and \(z\) direction. The concrete support belongs to stiffness, so that the translation is incapable of occurring. Thus the translation just occurs on the peak of tower caused by horizontal force. It is around 10.000 which works on the peak of tower in \(x\) direction. Figure 4 illustrates 3D form of Finite ElementMess.
The material is considered as an isotropic, homogenous, linear elastic material and the material attenuation does not depend on the frequency. These are the data of material below.

| Material | E (kg/m²) | µ | ρ (kg/m³) |
|----------|----------|---|-----------|
| Soil     | 5. e7    | 0.4 | 2.000    |
| Concrete | 2. 10e10 | 0.17| 2.400    |
| Iron     | 2. E11   | 0.005| 7.850   |

The tower is modeled with space truss element while the soil is modeled with hexahedron 3D solid element. Finite element mesh has 300 hexahedron solid elements, 4 space truss elements and 485 nodes where each node has 3 DOF like what is illustrates on the figure 5 and 6.

![Finite element mesh](image)

**Figure 5.** Sketch and piece of Finite element mesh

A = 0.005 m²

**Dynamic stiffness matrix of Finite Element**

\[
[D^{FE}] = \begin{bmatrix} [K] \cdot (1 + 2 \xi i) - \omega^2 [M] \end{bmatrix}
\]

\eqref{eq:9}

[K] is stiffness matrix of Finite Element Mesh, [M] is mass matrix of Finite Element Mesh, \(\xi\) is the ratio of attenuation taken = 5% and \(\omega\) is the frequency which is 50 rad/s.
3.2. Space truss

The space truss, where the local axis x is taken in the same direction with the axial displacement of element is illustrated on the figure 7.

\[ u(x) = u_1 + \left(u_2 - u_1\right) \frac{x}{L} \]

or \[ \{u(x)\} = [N] \{u^e\} \]

where

\[ [N] = \begin{bmatrix} 1 & \frac{x}{L} \\ \frac{x}{L} & \frac{x^2}{2L} \end{bmatrix} \]

\[ \{u^e\} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]

Then it can count the strain \( \epsilon_x \) below:

\[ \epsilon_x = \frac{\partial u(x)}{\partial x} = \frac{u_2 - u_1}{L} \]

\[ \{\epsilon_x\} = [B] \{u^e\} \]

Where

\[ [B] = \begin{bmatrix} -1 & 1 \\ \frac{1}{L} & \frac{1}{L} \end{bmatrix} \]

The relationship between strain and stretch is:

\[ \{\sigma_x\} = [D] \{\epsilon_x\} \]

The element of space truss which amount is [D] is the same as [E].

The element of stiffness matrix \([k^e]\) can be counted using equation 3.

\[ [k^e] = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

Where A is the width of element piece

To gain the stiffness matrix \([K^0]\) of element in global coordinate must use transformation matrix. The displacement of local element \(u_1\) and \(u_2\) on the figure (5.2) can be scattered as displacement component in global coordinate XYZ, namely \(U_{3i-2}, U_{3i-1}, U_{3i}, U_{3j-2}, U_{3j-1}\) and \(U_{3j}\). It has the same position as the global axis XYZ that it gain something below.

\[ \{u^e\} = (T) \{U^e\} \]

Where

\[ \{u^e\} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]
\[ \{ U^e \} = \begin{bmatrix} \begin{bmatrix} U_{3i-2} \\ U_{3i-1} \\ U_{3i} \\ U_{3j-2} \\ U_{3j-1} \end{bmatrix} \end{bmatrix} \]

\[ [T] = \begin{bmatrix} \cos \alpha & \cos \beta & \cos \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & \cos \beta & \cos \gamma \end{bmatrix} \] (18)

\( \alpha, \beta, \) and \( \gamma \) are angles between local element and global element XYZ

\[ \cos \alpha = \frac{x_j - x_i}{L}, \quad \cos \beta = \frac{y_j - y_i}{L}, \quad \cos \gamma = \frac{z_j - z_i}{L} \] from equation (18)

\( (x_i, y_i, z_i) \) and \( (x_j, y_j, z_j) \) are global coordinate of node i and j.

\[ L = ((x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2)^{1/2} \] (19)

Then the stiffness matrix element in global coordinate can be counted

\[ [K^e] = [T]^T [k^e] [T] \] (20)

by using matrix value \([T]\) from equation (17) and \([k^e]\) from equation on the formula 16 into formula 20 equation. It gains:

\[ [K^e] = \frac{AE}{L} \begin{bmatrix} T_1 & -T_1 \\ -T_1 & T_1 \end{bmatrix} \]

Where \[ [T_1] = \begin{bmatrix} \cos^2 \alpha & \cos \alpha \cos \beta & \cos \alpha \cos \gamma \\ \cos \alpha \cos \beta & \cos^2 \beta & \cos \beta \cos \gamma \\ \cos \alpha \cos \gamma & \cos \beta \cos \gamma & \cos^2 \gamma \end{bmatrix} \] (21)

consistent mass matrix \([m^e]\) of bar element can be counted by using equation on the formula 2. It gains:

\[ [m^e] = \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \] (22)

Besides consistent mass matrix, lumped mass matrix is also known, where the total mass of element is divided on the node that it gains:

\[ [m^e]_{lump} = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \] (23)

Bundle mass matrix is a diagonal matrix

Then the mass matrix is counted for truss space element like what is illustrated on the figure 8

**Figure 8.** The displacement of bar element in axis XYZ

\[ \{ u(x) \} = \begin{bmatrix} u(x) \\ v(x) \\ w(x) \end{bmatrix} = [N] \{ U \} \] (24)

Where

\[ [N] = \begin{bmatrix} 1 - \frac{x}{L} & 0 & 0 & \frac{x}{L} & 0 & 0 \\ 0 & 1 - \frac{x}{L} & 0 & 0 & \frac{x}{L} & 0 \\ 0 & 0 & 1 - \frac{x}{L} & 0 & 0 & \frac{x}{L} \end{bmatrix} \]
\{ \mathbf{U} \} = \{ \mathbf{U}_{3j-2} \ \mathbf{U}_{3j-1} \ \mathbf{U}_{3j} \ \mathbf{U}_{3j-2} \ \mathbf{U}_{3j-1} \ \mathbf{U}_{3j} \}^T \quad (25)

\mathbf{U}_{3j-2}, \mathbf{U}_{3j-1}, \mathbf{U}_{3j} \text{ is the displacement component of node } i \text{ dan } \mathbf{U}_{3j-2}, \mathbf{U}_{3j-1}, \mathbf{U}_{3j} \text{ is the displacement component of node } j \text{ in global coordinate XYZ. If the density } \rho \text{ and cross-sectional area } A \text{ of bar element is constant, the consistent mass matrix of bar space element can be counted by using equation on the formula 2.}

By inputting value of equation on the formula 25 to equation on the formula 2, it gains:

\[
[m^e] = \frac{\rho AL}{6} \begin{bmatrix}
2 & 0 & 0 & 1 & 0 & 0 \\
0 & 2 & 0 & 0 & 1 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 \\
1 & 0 & 0 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 & 2 & 0 \\
0 & 0 & 1 & 0 & 0 & 2
\end{bmatrix} \quad (26)
\]

For bundle mass matrix, the total element of each axis is divided on each node of element that for truss space element, it gains:

\[
[m^e] = \frac{\rho AL}{2} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \quad (27)
\]

For an element which degree of freedom which only consists of translation or without rotation the mass matrix element \([M^e]\) in global axis will be the same as the mass matrix element in local axis \([m^e]\), so that \([M^e] = [m^e]\).

In this paper the bundle mass matrix will be used to save the computer memory because \(M_{ij} = 0\) for \(i \neq j\), and the attenuation used is attenuation ratio \((\xi)\) which is constant.

For instance, an iron tower is modeled with bar space element. The data of iron tower are illustrated on the table 2.

| Element | \(X_i\) | \(Y_i\) | \(Z_i\) | \(X_j\) | \(Y_j\) | \(Z_j\) | \(L\) | cos \(\alpha\) | cos \(\beta\) | cos \(\gamma\) |
|---------|---------|---------|---------|---------|---------|---------|------|-------------|-------------|-------------|
| 1       | -4.80   | 4.80    | 0       | 0       | 0       | -20     | 21.121| 0.227       | -0.227      | -0.947      |
| 2       | -4.80   | -4.80   | 0       | 0       | 0       | -20     | 21.121| 0.227       | 0.227       | -0.947      |
| 3       | 4.80    | -4.80   | 0       | 0       | 0       | -20     | 21.121| 0.227       | 0.227       | -0.947      |
| 4       | 4.80    | 4.80    | 0       | 0       | 0       | -20     | 21.121| -0.227      | -0.227      | -0.947      |

Table 2. The data of iron tower

Element 1

Transformation Matrix \([T_1]\)

\[
[T_1] = \begin{bmatrix}
0.052 & -0.052 & -0.215 \\
-0.052 & 0.052 & 0.215 \\
-0.215 & 0.215 & 0.897
\end{bmatrix}
\]

\[
[K^{e1}]_{6x6} = \frac{AE}{L} \begin{bmatrix}
0.052 & -0.052 & -0.215 & -0.052 & 0.052 & 0.215 \\
-0.052 & 0.052 & 0.215 & 0.052 & -0.052 & -0.215 \\
-0.215 & 0.215 & 0.897 & 0.215 & -0.215 & -0.897 \\
-0.052 & 0.052 & 0.215 & 0.052 & -0.052 & -0.215 \\
0.052 & -0.052 & -0.215 & -0.052 & 0.052 & 0.215 \\
0.215 & -0.215 & -0.897 & -0.215 & 0.215 & 0.897
\end{bmatrix}
\]

\[
[M^{e1}] = \frac{\rho AL}{2} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
Element 2
Transformation Matrix \([T_1]\)

\[
[T_1] = \begin{bmatrix}
0.052 & 0.052 & -0.215 \\
0.052 & 0.052 & -0.215 \\
-0.215 & -0.215 & 0.897
\end{bmatrix}
\]

\[
[K^e2]_{6x6} = \frac{AE}{L} = \begin{bmatrix}
0.052 & 0.052 & -0.215 & -0.052 & -0.052 & 0.215 \\
0.052 & 0.052 & -0.215 & -0.052 & -0.052 & 0.215 \\
-0.215 & -0.215 & 0.897 & 0.215 & 0.215 & -0.897 \\
-0.052 & -0.052 & 0.215 & 0.052 & 0.052 & -0.215 \\
-0.052 & -0.052 & 0.215 & 0.052 & 0.052 & -0.215 \\
0.215 & 0.215 & -0.897 & -0.215 & -0.215 & 0.897
\end{bmatrix}
\]

\[
[M^e2] = \frac{\rho AL}{2}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Element 3
Transformation Matrix \([T_1]\)

\[
[T_1] = \begin{bmatrix}
0.052 & -0.052 & 0.215 \\
-0.052 & 0.052 & -0.215 \\
0.215 & -0.215 & 0.897
\end{bmatrix}
\]

\[
[K^e3]_{6x6} = \frac{AE}{L} = \begin{bmatrix}
0.052 & -0.052 & 0.215 & -0.052 & -0.052 & 0.215 \\
-0.052 & 0.052 & -0.215 & 0.052 & 0.052 & -0.215 \\
0.215 & -0.215 & 0.897 & -0.215 & -0.215 & 0.897 \\
-0.052 & -0.052 & 0.215 & 0.052 & 0.052 & -0.215 \\
-0.052 & -0.052 & 0.215 & 0.052 & 0.052 & -0.215 \\
-0.215 & 0.215 & -0.897 & 0.215 & 0.215 & -0.897
\end{bmatrix}
\]

\[
[M^e3] = \frac{\rho AL}{2}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Element 4
Transformation Matrix \([T_1]\)

\[
[T_1] = \begin{bmatrix}
0.052 & 0.052 & 0.215 \\
0.052 & 0.052 & 0.215 \\
0.215 & 0.215 & 0.897
\end{bmatrix}
\]

\[
[K^e4]_{6x6} = \frac{AE}{L} = \begin{bmatrix}
0.052 & 0.052 & 0.215 & -0.052 & -0.052 & -0.215 \\
0.052 & 0.052 & 0.215 & -0.052 & -0.052 & -0.215 \\
0.215 & 0.215 & 0.897 & -0.215 & -0.215 & -0.897 \\
-0.052 & -0.052 & -0.215 & 0.052 & 0.052 & 0.215 \\
-0.052 & -0.052 & -0.215 & 0.052 & 0.052 & 0.215 \\
-0.215 & -0.215 & -0.897 & 0.215 & 0.215 & 0.897
\end{bmatrix}
\]
3.3 The Element of Solid Hexahedron

To facilitate the calculation model of half space, it uses hexahedron solid element like what is illustrated on the figure 9.

a. Global coordinate system xyz  
b. Local coordinate system rst

Figure 9. The element of hexahedron with 8 nodes

The element of hexahedron has 8 nodes with 3 degrees of freedom per nodal namely displacement in x, y, and z direction. To facilitate it, the element of hexahedron is considered as an isoperimetric element where the shape function [N] is good either for displacement of node or coordinate of node. These are the procedures must be conducted for the element of hexahedron 3D solid.

a. Counting shape function [N] of each equation element:

\[ N_i(r,s,t) = \frac{1}{8} (1 + r r_i)(1 + s s_i)(1 + t t_i), \text{ where } i = 1, 2, \ldots, 8 \]  

(b) Counting stretch displacement matrix [B] by using equation below.

\[
[B_i]_{6x3} = \begin{bmatrix}
\frac{\partial N_i}{\partial x} & 0 & 0 \\
0 & \frac{\partial N_i}{\partial y} & 0 \\
0 & 0 & \frac{\partial N_i}{\partial z}
\end{bmatrix}, \quad i = 1, 2, \ldots, 8
\]  

(c) Counting Jacobian Matrix [J] by using equation below.

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\
\frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\
\frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{8} \frac{\partial N_i}{\partial r} x_i \\
\sum_{i=1}^{8} \frac{\partial N_i}{\partial s} y_i \\
\sum_{i=1}^{8} \frac{\partial N_i}{\partial t} z_i
\end{bmatrix}
\]  

(d) Counting differential of the shape function [N] toward r, s, and t by using equation below.

\[
\frac{\partial N_i}{\partial r} = \frac{1}{8} r_i (1 + s s_i)(1 + t t_i) \\
\frac{\partial N_i}{\partial s} = \frac{1}{8} s_i (1 + r r_i)(1 + t t_i) \\
\frac{\partial N_i}{\partial t} = \frac{1}{8} t_i (1 + r r_i)(1 + s s_i) \quad i = 1, 2, \ldots, 8
\]  

e. Counting differential of the shape function [N] toward x, y, and z by using equation below.
on the equation (Formula 1) is 1455 x 1455 and the displacement of node $M_0$ and $M_1$ is gained an equation (Rumus 1). The amount of matrix $M_e$ is 1455 x 1.

The stiffness matrix $\left[ K_e^i \right]_{24 \times 24}$ by using Gauss quadrature method below.

$\left[ K_e^i \right] = \sum_{i=1}^{n} \left[ B(r_i^e \cdot s_i^e \cdot t_i^e) \right]^T \left[ D \right] \left[ B(r_i^e \cdot s_i^e \cdot t_i^e) \right] \det \left[ J \left( r_i^e \cdot s_i^e \cdot t_i^e \right) \right] W_i W_k$ (33)

Table 3. Gauss element of hexahedron

| Point | $r_i$   | $s_i$   | $t_i$   | $W_{i,j,k}$ |
|-------|---------|---------|---------|-------------|
| 1     | -0.577735 | -0.577735 | -0.577735 | 1           |
| 2     | +0.577735  | -0.577735 | -0.577735 | 1           |
| 3     | +0.577735  | +0.577735 | -0.577735 | 1           |
| 4     | -0.577735  | +0.577735 | -0.577735 | 1           |
| 5     | -0.577735  | -0.577735 | +0.577735 | 1           |
| 6     | +0.577735  | -0.577735 | +0.577735 | 1           |
| 7     | +0.577735  | +0.577735 | +0.577735 | 1           |
| 8     | -0.577735  | +0.577735 | +0.577735 | 1           |

The amount of matrix $\left[ K_e^i \right]$ and $\left[ M_e^i \right]$ on the equation (Formula 1) is 1455 x 1455 and the amount of matrix $\left\{ U \right\}$ is 1455 x 1.

For instance, hexahedron 3D solid element can be divided into 3 elements like what is illustrated on the table 4, 5 and 6.
Table 6. Element 3

| Titik | Xi   | Yi   | Zi  |
|-------|------|------|-----|
| 9     | -7.00| -7.00| 1.00|
| 10    | -7.00| -5.60| 1.00|
| 11    | -5.60| -5.60| 1.00|
| 12    | -5.60| -7.00| 1.00|
| 13    | -8.00| -8.00| 0   |
| 14    | -8.00| -6.40| 0   |
| 15    | -6.40| -6.40| 0   |
| 16    | -6.40| -8.00| 0   |

### 3.4 Coupling between FEM and ITM

Think that the sector $\Gamma$ has no external force, the structure is like what is illustrated on the figure 10. It can be divided into 2 substructures namely $\Omega_{FE}$ dan $\Omega_{\infty}$

![Figure 10. The system of 2 substructures in balance](image)

The balance of sector limit is illustrated on the figure 10.

$$\sigma_{h}^{FE} - \sigma_{h}^{\infty} = 0$$  \hspace{1cm} (35)

or on the discrete system

$$\{p_{h}^{FE}\} - \{p_{h}^{\infty}\} = 0$$  \hspace{1cm} (36)

and

$$u_{h}^{FE} - u_{h}^{\infty} = 0$$  \hspace{1cm} (37)

From the equation (4.5), the relationship between $\{C_{mn}\}$ and $\{u_{h}^{\infty}\}$ is below.

$$\{u_{h}^{\infty}\} = [TR] \{C\}$$  \hspace{1cm} (38)

By using variation method for the potential internal one, it gains:

$$\delta U_{FE} = \delta \{u_{h}^{FE}\}^{T} \begin{bmatrix} [D_{FE}] & [D_{sh}] \end{bmatrix} \begin{bmatrix} [u_{h}^{FE}] \end{bmatrix}$$  \hspace{1cm} (39)

By substituting the equation between (5.64) and (5.65) into the equation (5.66), it gains:

$$\delta U_{FE} = [u_{h}^{FE}]^{T} \delta \{C\}^{T} \begin{bmatrix} [D_{FE}] & [D_{sh}] \end{bmatrix} [TR]^{T} [D_{hs}]^{T} [TR] \begin{bmatrix} [u_{h}^{FE}] \end{bmatrix}$$  \hspace{1cm} (40)

Now we have a new dynamic matrix from Finite element mesh $[D_{FE}]$ with new DOF.

$$\{u_{h}^{FE}\}^{T} = \{u_{h}^{FE}\}^{T} \{C\}$$  \hspace{1cm} (41)

Previously we have already counted dynamic matrix of half space.
\[ D^\infty = \int [U_{lmn}]^T [T_{lmn}] \, d\Gamma_s \]  

(42)

If there is no external force on the sector limit \( \Gamma \) we can combine both of substructures that it gains:

\[
\{ [p^F_s] \} = \begin{bmatrix} [D^F_{ss}] & [D^F_{sh}] & [D^F_{TR}] \end{bmatrix} \begin{bmatrix} [TR]^T [D^F_{hs}] & [TR]^T [D^F_{bh}] \end{bmatrix} \begin{bmatrix} [u^F_s] \end{bmatrix} + \{ [C] \}
\]

(43)

4. Conclusion

In this study, soil is considered as a semi-infinite media in z-direction with unbounded domain in x-direction. The characteristic of material is considered as an isotropic, homogenous, and linear elastic material, and the material damping seems not dependent on the frequency. The structure is scattered using Finite Element Method (FEM) and the soil modeled by FEM ought to be coupled in certain level where the compatibility of strain and deformation is observed. Coupling method which is used in soil-structure interaction is ITM-FEM Coupling. The relationship between strain and displacement on the imaginary surface can be used to determine the element of matrix. By Wavelet transform, it is possible to get information about frequency level, and where it occurs. Dynamic mass can be reconstructed from the result of response measurement which is gained and directly gives information from frequency domain which is really useful for some dynamic problem.

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