Contagious Sets in Dense Graphs

Abstract

We study the activation process in undirected graphs known as bootstrap percolation: A vertex is active either if it belongs to a set of initially activated vertices or if at some point it had at least \( r \) active neighbors. The threshold \( r \) is identical for all vertices. A contagious set is a subset of vertices whose activation results with the entire graph being active.

In the first part of the talk we prove that \( G \) has a contagious set of size 2 if \( r = 2 \) and \( G \) is a Dirac graph, meaning it has minimum degree \( n/2 \).

In the second part of the talk, we investigate \( M(n, k, r) \), the maximum number of edges an \( n \)-vertex graph can have without having a contagious set of size \( k \). Noticing that any disconnected graph cannot have a contagious set of size \( k \) if all thresholds are \( k \), we find that \( M(n, k, k) \geq \binom{n-1}{2} \).

In the second part of the talk we then prove that for \( n \geq 2k - 2 \) this lower bound is in fact tight.

Joint work with Matthias Poloczek and Daniel Reichman