Computed Tomography Parameter Calibration and Imaging Principle

Jiachen Jiang, Yifei Ma
Statistic Institute, Shanxi University of Finance and Economics, wucheng road, Taiyuan, China

Abstract. Since the successful development of Computed Tomography in 1972, both industrial and medical aspects have had a huge impact on our lives. In reality, Computed Tomography often has errors in its installation, which affects the quality of the image. However, the calibration of the parameters has a significant impact on the quality of the image of Computed Tomography. Therefore, this article takes a typical two-dimensional Computed Tomography as an example to perform parameter calibration on the installed Computed Tomography to find more accurate and stable parameters.

1. Introduction
Installation and commissioning are an essential task before a CT device is put into use. Since several key parameter indicators need to be accurately positioned during installation, if there is an error, the quality of the image will be affected. This paper discusses the parameter calibration method of Computed Tomography on this issue. According to the physical properties of Computed Tomography, we believe that three parameter indicators need to be considered when installing and debugging.

1 Rotation center of the entire transmitting-receiving system
2 Distance between detector units
3 Computed Tomography uses 180 directions of X-rays

Therefore, this paper studies and gives the calibration methods and values of the three parameter indicators. On the basis of the calibration parameters, the iterative method is used to reconstruct the image of irregular graphics.

2. Parameter calibration
According to the results of the calibration template data analysis, this paper believes that the center ray of the light must pass through the rotation center of Computed Tomography, and the center of rotation is inside the medium ellipse. Based on the knowledge of geometric principle, Radon transform theory and X-ray attenuation law, this paper obtains the calibration results of the distance between the detector unit of Computed Tomography, the 180 directions of X-ray and the position of the rotation center.

2.1. Unit distances
On the basis of data preprocessing, this paper uses the data of each column to make a line graph, and compares the shapes of these graphs. It is observed that the beam in the direction of the DE column is exactly the parallel light that is tangent to the ellipse and the circle. A light and the b light correspond to points a and b in the figure, respectively. Using the geometric relationship, the distance $\Delta L$ of the detector unit can be obtained.
As shown in the figure, the distance between a light and the b light is the diameter of the small circle, that is, \( L = 8 \text{ mm} \). It is calculated that the number of detectors between a light and the b light is \( N = 29 \), and \( \Delta L = \frac{L}{N} = 0.2759 \text{ mm} \) is obtained.

2.2. Direction of parallel light
In this paper, the origin of the ellipse is the origin of the Cartesian coordinate system, and the plane two-dimensional Cartesian coordinate system is established. Use knowledge of geometry, draw \( \theta = 45.217^\circ \)

Assume that the initial position is an angle towards the second quadrant, \( N=49 \), so \( \Delta \theta = \frac{\theta}{N} = 0.9228^\circ \).

Because a parallel light is the DE light beam, that is, the 109th direction, it can be obtained \( 100.5852^\circ \), so \( =55.3628^\circ \), so the first parallel light incident direction is \( 124.6138^\circ \) direction, and each rotation is \( 0.9228^\circ \), and the rotation is 180 times.

2.3. Rotation center positions
In the case where the initial ray direction and the detector unit distance are known, this paper uses the original plane two-dimensional Cartesian coordinate system X-O-Y to solve the position of the center of rotation.

Let the center of rotation to be \( (x, y) \), let N views per view, and a total of M views.

The projection when the rotation center has no offset is \( \rho_{\theta_{i}}(S_{i}), i = 1,2,3,..., N, i = 1,2,..., M \)
When the center of rotation has an offset, the projection is \( \rho_{\theta_j}(s_j) \). Assume \( S_j = i \). Then the coordinates of its mass point on the s-axis are

\[
\sum_{i=1}^{N} \frac{p_{\theta_j}(i)}{\sum_{i=1}^{N}} \rightarrow s_{\theta_j} = \frac{\sum_{i=1}^{N} p_{\theta_j}(i)}{\sum_{i=1}^{N}}
\]

For different angles, the calculated projection centroid coordinates can be obtained. Calculated using MATLAB, the coordinates of the center of rotation are (-9.2734, 5.5363).

![Figure 4. Coordinate map](image)

### 3. Filtered back projection reconstruction

**Fourier Center Slice Theorem:**

Assuming \( f(x, y) \) is the density function of the object to be reconstructed. \( p_{\phi}(x) \) is the parallel beam projection \( f(x, y) \) at angle \( \phi = \phi_0 \), the mathematical expression of the Fourier center slice theorem is:

\[
F_1(p_{\phi}(x)) = F(p_{\phi}(x))\phi = \phi_0
\]

Where: \( F_1(\cdot) \) represents the one-dimensional Fourier transform. \( F(p, \phi) \) is the polar coordinate representation of the two-dimensional Fourier transform.

The reconstruction of the filtered back projection (FBP) is used to reconstruct the CT image. The core of the filtered back projection reconstruction algorithm is the convolution operation and back projection algorithm. The back projection reconstruction algorithm is also called the accumulation method. The algorithm principle is that the density value of a point in the plane of the fault is regarded as the sum of all the ray projections passing through the point in the plane. The task of the back projection reconstruction algorithm is to project all the ray projections of a certain pixel point, uniformly distribute the projection values along the ray direction, and generate a reconstructed image according to the back projection result map.

The reconstruction formula derived from the Fourier Center Slices Theorem is:

\[
f(\gamma, \theta) = f(x, y) = \int_{0}^{2\pi} p(x, \phi) * h(x, \phi) d\phi = \int_{0}^{2\pi} g(x, \phi) d\phi
\]

\[
h(x, \phi) = F_1^{-1}(|\rho|), p(x, \phi) = F_1^{-1}(P(\rho, \phi))
\]
3.1. Image restoration of irregular graphics using iterative method

Phase recovery refers to extracting the phase information of an object from the intensity data only if the intensity can be measured. In order to obtain the shape of the phase restoration, the feasibility of the filtered back projection algorithm has been discussed above, but the filtered back projection algorithm is not suitable for image restoration of irregular patterns. After further analysis, we can segment these irregular unknown media, use a small amount of filtered back projection calculation, and then calculate the next small medium. This method is an iterative method.

Iterative method phase recovery mainly includes two main ideas: 1 error reduction algorithm, including Gerchberg-Saxton algorithm, steepest descent method, other gradient search method; 2 input-output algorithm (input output Algorithm), including basic input-output method, output-output method, and Hybrid input-output method.

The error reduction algorithm is stable and convergent, while the input and output algorithm is unstable and may diverge. So we use the error reduction method.

To determine the accuracy of the error and discuss the convergence of the iterative algorithm, define the relative root mean square error $E$ to evaluate the accuracy of the phase recovery:

$$E = \frac{\sum_{i,j} (\phi_{rec}^{i,j} - \phi_{true}^{i,j})^2}{\sum_{i,j} |\phi_{true}^{i,j}|^2}^{1/2}$$

The actual phase value $\phi_{rec}^{i,j}$ representing a pixel point $(i, j)$ of the phase object image, $\phi_{true}^{i,j}$ representing the phase value of the same pixel point $(i, j)$ of the phase restored image.

The figure above shows the relationship between the relative root mean square error of the recovered phase and the number of iterations. Therefore, the iterative phase recovery has a faster convergence when the number of iterations is small, and then the convergence is slower. Iteratively iterates in the air and frequency domain of the object, and extrapolates the bandwidth to finally realize the phase restoration of the object surface and the image plane.

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