Optimization Design of Halbach Permanent Magnet Motor Based on Multi-objective Sensitivity

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Abstract—The halbach permanent magnet synchronous motor (HPMSM) combines the advantages of permanent magnet motors and halbach arrays, which make it very suitable to act as a robot joint motor, and it can also be used in other fields, such as electric vehicles, wind power generation, etc. At first, the sizing equation is derived and the initial design dimensions are calculated for the HPMSM with the rated power of 275W, based on which the finite element parametric model of the motor is built up and the key structural parameters that affect the total harmonic distortion of air-gap flux density and output torque are determined by analyzing multi-objective sensitivity. Then the structure parameters are optimized by using the cuckoo search algorithm. Last, in view of the problem of local overheating of the motor, an improved stator slot structure is proposed and researched. Under the condition of the same outer dimensions, the electromagnetic performance of the HPMSM before and after the improvement are analyzed and compared by the finite element method. It is found that the improved HPMSM can obtain better performances.

Index Terms—Halbach permanent magnet synchronous motor, multi-objective sensitivity, cuckoo search algorithm, electromagnetic characteristics, finite element analysis.

I. INTRODUCTION

JOINT robots not only can replace manpower to complete work in harsh environments such as high temperature, radiation, and corrosion, but also can be used to improve production efficiency, shorten production cycles, and reduce labor intensity in product production [1]. It has an irreplaceable position in the transformation and upgrading of traditional manufacturing industries [2]. The motor is one of the most important parts of the robot because it is essential for the smooth, accurate, and reliable operation of the robot. The main joint motors currently used in robots include stepping motor, direct current motor, and permanent magnet motor [3]-[7].

The permanent magnet motor has become a research hot spot in joint motors because of its small size, wide speed range, and large starting torque. In order to further satisfy the requirements for low-mass, high-torque joint motors of biological robots, the different topologies have been researched in recent years. Axial flux permanent magnet machine with high torque density was proposed and compared with radial flux BLDC motor in [8] and [9]. It was found that the axial field permanent magnet machine could provide larger output torque than radial flux BLDC motor. However, the torque ripple was not able to be ignored due to the large cogging torque. W. Zhang studied a permanent magnet motor with fractional slot concentrated winding [10]. Although the torque was largely improved, it also brought the problems of low efficiency and large loss. In 1980, Dr. Klaus Halbach first proposed halbach permanent magnet array [11]. The halbach array is made by inserting permanent magnets with different magnetization directions according to a certain law. It forms a self-shielding effect [12]. Applying the halbach array into the permanent magnet synchronous motor and optimizing the structural parameters of the motor, the output torque can be increased [13]. In addition, the torque ripple and losses can be reduced. These are beneficial to the application of joint motors.

At present, there are many optimization methods were used for permanent magnet motors, such as hybrid genetic-simulated annealing algorithm, fuzzy-based Taguchi method, hybrid multi-objective optimization algorithm and so on. In [14], the fuzzy-based Taguchi method was applied to the multi-objective optimization of direct-driven permanent magnet synchronous motor, by which solved the problem that the fuzzy algorithm was limited by the number of objective functions. However, it had a slow convergence rate. A hybrid multi-objective optimization algorithm had a fast convergence rate, but the global search ability was poor [15]. In [16] and [17], the multi-objective analysis was carried out to optimize the motor by applying a hybrid genetic-simulated annealing algorithm and conformal mapping method, respectively. By which the performance of the machine could be improved, but both methods were prone to premature stagnation. In order to improve the magnetic density of halbach array and reduce harmonic contents, the genetic algorithm was used to optimize halbach array with unequal thickness [18]. It was found if there
were many parameters for multi-objective optimization, the ability of this algorithm to quickly optimize would be weakened. The Non-dominated Sorting Genetic Algorithm technique was used to optimize the efficiency, size and mass of a permanent-magnet generator in which could arbitrarily select the number of optimization goals by non-dominated set ordering [19]. However, it increased the complexity and amount of calculation. Based on a Tabu search algorithm and finite element method, permanent magnet motor with halbach magnets was optimized [20]. Although this algorithm overcame the shortcomings of genetic and simulated annealing algorithm, which were prone to mature prematurely, it relied heavily on the initial design values. Poor initial values may result in the bad optimization results.

According to the above mentioned, in some extent, the optimization results of permanent magnet motors can be improved by the intelligent algorithms. However, there are still empirical limitations in the choice of initial design variables. It can lead to time-consuming and inefficient optimization design. Therefore, the key structural parameters that have the greatest impact on optimal goals are quickly chosen for halbach permanent magnet synchronous motors (HPMSM) based on multi-objective sensitivity in this paper.

The experiments are done for the improved HPMSM in Section IV, and initial design parameters of HPMSM are determined. In Section III, the HPMSM is optimized and improved by multi-objective sensitivity and cuckoo search algorithm (CSA). The experiments are done for the improved HPMSM in Section VI. The conclusion is given in Section V.

II. BASIC PRINCIPLE AND INITIAL DESIGN OF HPMSM

A. Basic Structure of HPMSM

Fig. 1 shows the 3D topology of three-phase HPMSM, which composed of stator and rotor iron cores, armature windings and 20 groups of halbach arrays. The materials of the stator and rotor iron core are silicon steel sheets, 100 blocks of NdFeB permanent magnets with different magnetization directions labeled as A, B1, B2, C1, and C2, respectively, and those magnetization angles are α, β1, β2, γ1, and γ2, respectively, and defined as (1).

\[ \alpha = \pi + k \cdot 18 \]  \( (k = 0, 1, 2 \ldots 19) \]
\[ \beta_1 = \pi - \beta_2 \]
\[ \gamma_1 = \pi - \gamma_2 \]

where k is the rotation offset coefficient.

B. Original Design of HPMSM

The output power \( P_2 \), the induced electromotive force \( E_m \) and the permanent magnetic flux \( \Phi_m \) are shown in (2) ~ (4), respectively.

\[ P_2 = \eta P_1 = \eta m E_m I_m \]  \( (2) \)
\[ E_m = N_{ph} \Omega I_m \]  \( (3) \)
\[ \Phi_m = k_i k_p B_{g\text{max}} C_s \frac{\pi}{P_s} (D_{io}^2 - D_{si}^2) \]  \( (4) \)

where \( m \) is the number of phases, \( I_m \) is the magnitude of the stator phase winding, \( \eta \) is the efficiency, \( N_{ph} \) is the turn number of the stator phase windings, \( \omega_r \) is the angular velocity of the rotor, \( k_i \) is the winding factor, \( k_F \) is the waveform factor of the air-gap flux, \( B_{g\text{max}} \) is the maximum value of the air-gap flux density, \( C_s \) is the calculating pole-arc coefficient, \( P_s \) is the number of stator slots, \( D_{io} \) and \( D_{so} \) are the inner and outer diameter of the stator, respectively.

Substituting (4) into (3), (5) can be derived.

\[ E_m = N_{ph} \omega_r 2 P k_i k_p B_{g\text{max}} C_s \frac{\pi}{P_s} (D_{io}^2 - D_{si}^2) \]  \( (5) \)

The armature current amplitude \( I_a \) is given by (6).

\[ I_a = \sqrt{2} I_m = \sqrt{2} \frac{A_s \pi D_{si}}{m N_{ph}} \]  \( (6) \)

where \( A_s \) is the line load of the winding inner diameter, \( I_m \) the effective value of the armature winding phase current.

Substituting (5) and (6) into (1), (7) is derived.

\[ P_2 = \sqrt{2} \frac{\pi}{60} k_i k_p k_{io} D_{si} (D_{io}^2 - D_{si}^2) A_s B_{g\text{max}} C_s \eta \]  \( (7) \)

(8) is derived from (7).

\[ D_{io}^2 = \frac{P_2}{\sqrt{2} \frac{\pi}{60} P_s k_i k_p k_{io} (1 - k_{io}^2) A_s B_{g\text{max}} C_s \eta} \]  \( (8) \)

where \( k_{io} \) is split ratio of HPMSM, \( k_{io} = D_{si} / D_{so} \).

The initial design parameters are calculated by (8) and as shown in Table I, according to the performance indexes...
demands of the rated power $P_r = 275$ W, the rated speed $n_r = 2100$ r/min, the rated voltage $U_N = 48$ V, and $\eta = 89\%$. The geometric parameters of HPMSM are depicted in Fig.3.

III. OPTIMIZATION AND STRUCTURAL IMPROVEMENT OF HPMSM

The optimal design flow chart of HPMSM is shown in Fig. 4. In this paper, the initial parameters are designed for HPMSM with the rated power of 275 W by (2) ~ (8). Then a parametric model of HPMSM is established to evaluate the electromagnetic performance of the motor. Due to the huge workload of the traditional single parameter optimization method, the multi-objective sensitivity method is used to obtain the key structural parameters that have the greater impact on the optimization targets of HPMSM can be determined. According to the constraints of actual manufacture condition, among the multiple structural parameters in Table I, the eight structural parameters of the HPMSM are selected as design variables to analyze multi-objective sensitivity. The variables and their optimization ranges are shown in Table II.

According to (9), the sensitivity of different main structural parameters $S(X)$ are calculated, and the analysis results are shown in Fig.5.

$$S(X) = \frac{1}{n} \sum_{i=1}^{n} \frac{F(X_0 + \Delta X_i) - F(X_0)}{F(X_0)}$$

where $X$ is the design parameter variable, $X_0$ is the initial value of $X$, and $X_i$ is defined as 10%, 15%, and 20% of its initial value, respectively. $S(X)$ is the sensitivity index of $X_i$, $AvgS(X)$ is the average value of $S(X)$, $F(X)$ is the value of optimization objective under the condition of $X_0$. 

![Fig. 4. Flow chart of optimal design of HPMSM.](image)

![Fig. 3. The geometric parameters of HPMSM.](image)
Fig. 5 demonstrates the sensitivity analysis of the THD_{Br} and output torque under the condition of the different design variables, in which the red dot represents positive data of sensitivity, and the green dot is negative data. If the sensitivity is far away from the center of the circle, the parameter is the more sensitive.

The sensitivity less than 0.4 is defined as low sensitive parameters. The sensitivity greater than 0.6 is defined as high sensitive parameters. The sensitivity between 0.4 and 0.6 is defined as medium sensitive parameters.

It can be seen from Fig. 5(a) that w_{st}, h_{pm} are high sensitivity parameters for THD_{Br}. In addition, γ, h_{ss}, w_{os} and h_{sy} are low sensitive parameters, β is medium sensitive parameters for THD_{Br}.

Similarly, h_{pm} is high sensitive parameters for the output torque that can be quickly found from Fig. 5(b). Meanwhile, β, h_{ss}, w_{os} and h_{sy} are low sensitive parameters, w_{st} and γ are medium sensitive parameters for the output torque.

Therefore, through parameter sensitivity analysis, the crucial variables that have greater impacts on the optimization targets are determined intuitively and quickly, which is beneficial to reduce time consumption and improve efficiency of the multi-objective optimization.

**B. Optimization Based on CAS**

According to Fig. 5, the w_{st}, h_{pm} are chosen as key sensitivity parameters and they are optimized by CSA. First, the two optimal targets are fitted by the surface response method, and the function expressions of the output torque and the THD_{Br} are obtained, as shown in (10) and (11), respectively.

\[
T_{st} = 0.028w_{st}^2 - 0.017h_{pm}^2 + 0.012w_{os}h_{pm} + 0.061w_{st} + 0.223h_{pm} - 0.041
\]

\[
f_{THD_{Br}} = 1.32w_{st}^2 + 1.68h_{pm}^2 + 2.99w_{st}h_{pm} - 13.14w_{st} - 13.97h_{pm} + 47.93
\]

In addition, the Pareto optimal solution sets of the objective functions are obtained by the CSA, as shown in Fig. 6.

Because the output torque and the THD_{Br} are of equal importance to the performance of HPMSM, the optimal solution is chosen and labeled in Fig. 6 and the optimal values are shown in Table III. It is found from TABLE III that the proposed optimal method can obtain the optimal parameters quickly and efficiently.

**C. Air-gap flux Density Cloud Map of HPMSM**

Fig. 7 is the air-gap flux density cloud map of the after optimization of HPMSM. It can be found that the stator teeth have local magnetic saturation, which could cause the problem of motor overheating, so that the risk of motor burnout and permanent magnet demagnetization may be both increased.

**D. Improved structure of HPMSM**

In view of the above problems, the stator slot of the HPMSM is improved without changing the other structures of HPMSM. The improved stator slot structure is shown in Fig. 8. For the convenience of description, the HPMSM before improvement is indicated by HPMSM_{1}, and the improved HPMSM is indicated by HPMSM_{2}.

As shown Fig. 8, the stator slot is divided into an upper slot and a lower slot by a slot wedge in the radial direction. The upper slot has a fan-shaped structure in the radial section, and the lower slot has a circular wine glass-shaped structure in the radial section.
E. Air-gap flux density cloud map of HPMSM2

Fig. 9 is the air-gap flux density cloud map of HPMSM2. By comparing Fig. 7 and Fig. 9, it can be seen that the local teeth magnetic density of the HPMSM1 is as high as 2.163 T while the local teeth magnetic density of HPMSM2 is reduced by about 9.1%, the over saturation point is significantly reduced and the local overheating problem of the motor is effectively solved.

F. Air-gap Flux Density

Fig. 10 and Fig. 11 show the flux density distribution and the harmonic analysis of air-gap flux density of HPMSM1 and HPMSM2, respectively. It can be seen from Fig. 10 that the flux density amplitude of HPMSM2 is larger than that of HPMSM1, and the THD of HPMSM1 is 19.5%, while the THD of HPMSM2 is reduced by 6.1%. Thus, the fundamental amplitude of the air gap flux density of the improved HPMSM is increased. Those are beneficial to reduce torque ripple and losses, and improve output torque.

G. No-load Back EMF

Fig. 12 shows the comparison of no-load back EMF of HPMSM1 and HPMSM2. When the two machines operate at the speed of 2100 r/min, the total harmonic distortion rates of the no-load back EMF waveforms are both less than 5%. In Fig. 12, the effective value of no-load back EMF of HPMSM2 is 16.5 V, and it is increased by 17.9% than that of HPMSM1.

H. Torque Characteristics

The 48V DC power supply is as input source to drive the both motors respectively. As shown in Fig. 13, the output torque of the HPMSM2 is 1.25 Nm, which is 25% higher than HPMSM1. And the torque ripple of HPMSM2 is 4.8%, which is 1.7% less than that of HPMSM1. It can be seen from Fig. 13 that the torque characteristic of HPMSM2 is significantly improved in contrast with HPMSM1. The torque ripple is calculated by (12).

\[ k_{\text{ripple}} = \frac{T_{\text{out(min)}} - T_{\text{out(avg)}}}{T_{\text{out(avg)}}} \times 100\% \] (12)

IV. EXPERIMENTAL VALIDATION

The allocation diagram of the prototype is shown in Fig. 14.
In order to verify the proposed design method, the experimental platform is built, as shown in Fig. 15. The experimental platform consists of oscilloscope, performance tester, PC, DC power supply, driver, hysteresis dynamometer and HPMSM.

![Fig. 14. Allocation diagram of HPMSM prototype.](image)

**Fig. 14. Allocation diagram of HPMSM prototype.**

When the HPMSM operates at 2100 r/min, the waveform of the measured no-load back-EMF is shown in Fig. 16. It can be seen that the back-EMF waveform is sinusoidal and symmetrical, and the amplitude of voltage reaches 22.9V. It is found that the amplitude value of the back-EMF in Fig. 12 is 23.3V, which is similar to the experiment value.

![Fig. 16. No-load back-EMF waveform.](image)

**Fig. 16. No-load back-EMF waveform.**

In Fig. 17, the measured output torque of the HPMSM is got by the hysteresis dynamometer. It can be observed that the motor torque reaches 1.247 Nm at rated load. Meanwhile, the torque ripple is about 8.66% which is larger than that of the simulation result in Fig. 13, because of the accuracy of manufacture and measure error.

![Fig. 17. Output torque at rated speed.](image)

**Fig. 17. Output torque at rated speed.**

### V. CONCLUSION

Among the many structural parameters of HPMSM, the sensitivity parameters that have the greatest impact on HPMSM air-gap flux density and torque are selected by multi-objective sensitivity analysis. And combined with the CSA, the optimal structure parameters of the HPMSM are quickly and efficiently determined. Then, in order to reduce local over saturation of HPMSM, the stator slot is improved under the condition of the same outer dimension. The simulation and experiment results are done to verify the design and optimization method. It is found that the improved HPMSM can increase the output torque largely and reduce the torque ripple effectively.

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