Hypothetical learning trajectory design in development of mathematics learning didactic design in madrasah

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Abstract. Mathematics learning for most student is still problem. Meanwhile, mathematics is very important in everyday life. The problem is that students experience difficulties in understanding the concept. These difficulties are called epistemological obstacles. It can be reduced by designing teaching materials in accordance with the students learning obstacle. This research was conducted to identify the students’ learning obstacle and create a hypothetical learning trajectory. The research subjects were 30 mathematics teachers at Madrasah Tsanawiyah and Aliyah in South Tangerang. The method used is Participatory Action Research (PAR). In this PAR, the participants involved were madrasah mathematics teachers, Head of Madrasah Education Section of South Tangerang, Head of Development Aliyah Madrasah, and lecturers from Mathematics Education Department. There are two Focused Group Discussion activities of the PAR. The results showed that students in madrasah still experienced many obstacles in understanding mathematical concepts, especially on the topic of functions, circles, sequences and series, as well as comparison due to limitations in conceptual explanations in the learning process. Based on students’ learning obstacle, material hierarchy, and analysis of previous teaching materials, a didactic design was created in the form of hypothetical learning trajectory (HLT). Henceforth, HLT will be developed into teaching materials.

1. Introduction
The quality of madrasah is very important to improve. Why not, madrasah society has a comparison with public schools. Meanwhile, some madrasahs are only a second choice for the community. Although several other madrasahs have a high and superior reputation, there are still many madrasahs that need intensive guidance and empowerment related to the quality of madrasah. Several academic competitions such as Mathematics and Science Olympiad at local, national and international levels are still dominated by public schools.

The achievements of Indonesian students generally at the international level such as PISA and TIMSS are at the bottom position. In PISA 2015 [1], overall (science, reading, mathematics) Indonesian students only reached 42.3% at levels 1-4 and 0.8% at levels 5 and 6. Indonesian students’ mathematics achievement was the lowest. Mathematics reached a score of 386, while science reached a score of 403 and reading reached a score of 397. In the 2018 PISA results [2], the achievements of Indonesian students in the mathematics category rank of 72 out of 78 countries. In the 2018 PISA which was released in 2019, Indonesian students only achieved an average math score of 379 with a standard deviation of 79, far below the overall average of 489 with a standard deviation of 91. This result is at level 1 out of 6 levels on the scale of PISA. Even the mathematics achievement on PISA in
2019 decreased from the previous achievement at PISA in 2015 of 7 points from an average of 386 to 379.

The students who were sent to take PISA were students in the high or smart category among their peers. This means that if only students are smart and their choice of international achievement is still below it, what more would other students who are in the normal category and generally students. This shows that students have obstacle in learning. One of the duties of the teacher in overcoming these student obstacles is the need for the teacher to develop an appropriate learning trajectory. One way of making teaching materials can be through design research which is equipped with anticipation of student responses during learning where the anticipation is based on the previously identified obstacles of students.

Learning obstacles can also be caused by teachers' teaching habits that do not pay attention to the needs of students' cognitive situations. The teacher dominates the style he is accustomed to rather than involving students in the learning process. There is the shift in emphasis from teaching to learning from both a theoretical and practice perspective. According to Bergström [3], the shift in emphasis from teaching to learning is based upon a shift in symbolic power and control for teachers. Currently, teachers should pay more attention to the potential and abilities of students and follow the cognitive needs of students in learning so that students can follow learning easily. Therefore it is important for teachers to review the learning materials developed to be revised according to understanding needs.

Didactic design is a learning design in the form of teaching materials developed by Freudental based on the hypothetical learning trajectory [4] of the identification of learning obstacles in the mathematics learning process that has appeared previously. During the learning process in the classroom, didactic design teaching materials were made through a series of didactic situations along with predictions of responses and anticipations. Ruthven [5] states that didactic design is a learning sequence design in a learning environment to study a particular topic so that it is effective in achieving learning objectives with the specified method. The didactic design was designed with the aim of overcoming or reducing the learning obstacle that appeared in previous learning, so that students were able to fully understand the concept of a material in mathematics. By using the didactic design it is expected that the learning obstacle experienced by students can be reduced so that the objectives of learning mathematics can be achieved properly.

The initial didactic design was prepared based on the problems of students' obstacles found in the results of measuring the abilities of the previous respondents with the aim of developing the abilities of students to be developed. The didactic design also contains predictions of student responses that appear in the implementation of the initial didactic design and is complemented by the teacher's anticipatory action on the responses of students that appear. Suryadi [6] adds that in the didactic triangle the teacher-material relationship is called the didactic-pedagogical anticipation relationship (ADP), didactical relationship (DR), and pedagogical relationship (PR). The Figure 1 is a didactic triangle modification scheme according to Suryadi.

The target of community service activities in the form of Participatory Action Research (PAR) assistance is a community of high school level mathematics teachers within the Ministry of Religion. PAR is a collective, self-reflective inquiry conducted by researchers and participants, so that they can understand and improve the practices in which they participate and the situations in which they find themselves [7,8]. This community is a community of mathematics teachers through the Mathematics Teacher Meeting of Madrasah Aliyah and Tsanawiyah in the City of South Tangerang, totalling 30 people. Mathematics teachers need to be provided with assistance and empowerment in overcoming learning problems in their madrasah respectively.

Some of the empowerment carried out on teachers is usually related to the development of learning methods. In this study, empowerment is more focused on how to identify students learning obstacles and develop student learning trajectories as a first step in improving the quality of student learning.
2. Method
The method used is Participatory Action Research (PAR). This method is empowering teachers through the participation given in identifying and solving problems experienced. The strategy is carried out by conducting Focus Group Discussions (FGD) and mentoring. There are two FGD activities conducted in the PAR. FGD 1 designed to identifying the students learning obstacles, and FGD 2 designing to develop the hypothetical learning trajectory. In this PAR, the number of teacher participants who took part was 30 mathematics teachers (Tsanawiyah and Aliyah). The 30 teachers were from the Mathematics Subject Teacher community who were invited through the Head of the Madrasah Education Section of the Ministry of Religion, South Tangerang City. The stages in data collection are as follows:
1) review and determine the material that will be used as mentoring material. These materials may differ according to the level of the mentoring participant madrasah, both madrasah tsanawiyah and aliyah;
2) analyse the selected material (repersonalization). The selected material is limited to only one basic competency or at least for two face-to-face learning meetings;
3) analyse learning obstacles of students based on interviews and teacher experiences;
4) develop a hypothetical learning trajectory that is tailored to the needs of students to overcome the learning obstacle, conceptual hierarchy and analysis of learning resources that have been used so far.

3. Results and Discussion
3.1. Preparation
3.1.1. Coordination with the Head of the Madrasah Education Section of the Ministry of Religion, South Tangerang City
The preliminary study activity was carried out for the first time in coordination with the Head of the Madrasah Education Section at the Ministry of Religion Office, South Tangerang City. This coordination was carried out as a follow-up to the Cooperation Agreement between the Faculty of Educational Sciences of Syarif Hidayatullah State Islamic University Jakarta and the Office of the Ministry of Religion of South Tangerang City which was carried out on July 25, 2019 at the South Tangerang Ministry of Religion Office.
Based on the results of discussions with the Madrasah Education Section, the following were approved:
1) due to the large number of madrasah, both tsanawiyah and aliyah in South Tangerang City, the number of the first mentoring activities was limited. The original plan was that the number was limited to only 30 mathematics teacher participants representing madrasah tsanawiyah and aliyah;
2) the head of the Madrasah Education Section is responsible for inviting mentoring participants from math teachers in the madrasah;
3) the Head of the Madrasah Education Section provides direction and policies related to solving problems in education and learning mathematics in madrasah;
4) the community service assistance team from Faculty of Educational Sciences Syarif Hidayatullah State Islamic University is responsible for facilitating / accommodating and assisting the mentoring participants to make the transformation for identification and formulation of solving mathematics learning problems in madrasah.

3.1.2. Interview with teachers
Interview activities with madrasah mathematics teachers were held on Monday, October 14, 2019. Interviews with mathematics subject teachers were conducted in order to identify problems experienced by teachers in the process of developing mathematics education in madrasah. The results of interviews with several madrasah teachers regarding mathematics learning and the results can be summarized as follows:
1) mathematical literacy problems that are planned to be developed in mathematics learning in madrasah are still too high to be directly developed in learning because it turns out that the fundamental problem lies in mastering the basic concepts of mathematics. Most of the students' mastery of basic mathematical concepts is still inadequate. Therefore, in the end, this assisting activity is more for the development of didactic designs for learning fundamental concepts first before developing mathematical literacy in learning;
2) there needs to be a method or strategy or a relevant learning approach that is developed in order to overcome the learning problems experienced by students;
3) teachers need refreshment related to methodologies that can be applied in mathematics learning, learning material content, and research methodologies that can be developed in learning;
4) teachers need support and motivation to be able to develop mathematics learning professionally.

3.2. Implementation
3.2.1. Focus Group Discussion (FGD) and Teacher Empowerment
FGD 1 and Mentoring 1 activities were held on Saturday, 26 October 2019. This activity is focused on identifying learning problems that have so far been constrained. As the first activity, mentoring participants in FGD 1 were grouped based on madrasah and class levels. This was done so that the discussion was focused. Initially, the number of mathematics teachers invited at Tsanawiyah and Aliyah madrasah was 30 teachers from 30 madrasahs in the City of South Tangerang. There are 12 mathematics teachers aliyah from 12 madrasah aliyah, 9 mathematics teachers from 9 madrasah tsanawiyah, and 9 mathematics teachers from 9 Islamic junior high schools.

In this FGD 1 activity, sources will be presented from the Office of the Ministry of Religion of South Tangerang City as well as opening and inaugurating the Madrasah Teacher Assistance activity. It is hoped that the Madrasah Education Section of the Ministry of Religion of South Tangerang City as the government can provide direction for Madrasah Education policies for the quality of madrasah mathematics education in South Tangerang City. In addition, it is hoped that it can facilitate the development and improvement of the quality of mathematics learning in madrasah.

In the first mentoring activity, discussions were held about problems in learning mathematics. Before proceeding to discussion and brainstorming, first the mentoring participants were divided into 7 groups. The grouping is based on the madrasah level and the class in which the accompanying participants teach.

The problem map identified in the FGD 1 activities covers the problem of conceptual mastery and development of mathematical literacy. However, the development of mathematical literacy cannot be discussed during this mentoring because it turns out that the problem of mastering basic concepts alone still needs attention first.
The mentoring technique is carried out by pairing the mentoring team with the FGD groups of the mentoring participants, in which one mentor from the lecturer to facilitate one group intensively. Each group produces an identification of learning problems for which the solution will be developed in FGD 2 and Mentoring activities 2. The facilitator with the related group discusses all problems related to learning obstacle that occur in students. Brousseau [9] states that there are three factors that cause learning obstacles, namely ontogenic, didactic, and epistemological obstacles. The learning obstacle in question is an epistemological obstacle, not an ontogenic or didactic obstacle. Epistemological obstacles are obstacles that occur because of the limited scope of the explanation of conceptual material. Students have obstacle learning concepts because the illustrations or problem situations provided are not relevant to the cognitive development conditions of the students themselves. Students are less able to reach learning material with the didactic situation given.

**Table 1. Results of identification of students' learning obstacles.**

| No | Concept   | Madrasah Level | Learning Obstacles experienced by Students |
|----|-----------|----------------|--------------------------------------------|
| 1  | Circle    | Junior High School | Students have obstacle understanding and connecting the concept of arcs and circles  |
|    |           |                 | Students have obstacle in finding the circumference and area of the potsherd |
| 2  | Ratio     | Junior High School | Students have obstacle in determining the variables contained in the problem situation |
|    |           |                 | Students have obstacle in modelling the information contained on the graph into an equation |
| 3  | Sequence and | Senior High     | Students have obstacle finding patterns    |
| No | Concept | Madrasah Level | Learning Obstacles experienced by Students |
|----|---------|----------------|------------------------------------------|
| 6  | Series  | School         | that are in the sequence of numbers       |
|    |         |                | Students have obstacle applying the formula to find the number of n terms using the n<sup>th</sup> term formula |
| 4  | Function| Senior High School | Students have obstacle making mathematical models based on story problems |
|    |         |                | Students have obstacle determining the inverse rational form function |
|    |         |                | Students have obstacle determining the inverse of the function composition |

3.2.2. Focus Group Discussion (FGD) and Teacher Empowerment II

FGD 2 and Mentoring 2 activities are a continuation of the first FGD and Mentoring. This activity was carried out on Saturday, November 2, 2019. From the identification of the problems produced in FGD 1, in this second FGD and Mentoring, a possible didactic situation was developed, assignments that were in accordance with the level of obstacle experienced by students, as well as predictions of responses and anticipation. This concept is known as the hypothetical learning trajectory (HLT). This is the product or output of the first cycle from didactic design development assistance. The second cycle and so on will be carried out at the next time according to the time and opportunity available for the teachers through the Mathematics Teachers Meeting activities.

In FGD 2, speakers were presented related to the problems of developing didactic designs in improving students' conceptual understanding skills in mathematics learning. Resource persons are presented to provide a theoretical description of didactic design research (DDR) as a method to anticipate obstacles to learning mathematics in understanding mathematical concepts. After describing the DDR concept, the mentoring participants proceeded to focus discussions (FGD).

In this second mentoring a didactic situation was developed in accordance with the learning obstacles that exist among students. The teachers together with their companions formulate a didactic situation that is sufficient to develop learning according to the concepts identified in FGD 1. From this didactic situation, assignments are made to accommodate basic competencies related to indicators developed according to the obstacle level of their students. The teachers compile the didactic design in the form of a hypothetical learning trajectory which is called a hypothetical learning trajectory (HLT).

In further development, of course the HLT (blueprint for didactic design) will be developed teaching materials. This HLT will be a reference for teachers to develop teaching materials that are in accordance with the level of obstacles or learning obstacles of students. HLT and teaching materials developed with DDR will imply the better the teaching materials that are made and are expected to make it easier for students to learn them.

The second mentoring activity was carried out after a conceptual discussion regarding the possibility of implementing didactic design research was completed. In this second mentoring activity, the mentoring participants together with the companion team discussed the didactic design formula for learning mathematics according to the topic/material they chose which was an obstacle for students in learning it.

The output of mentoring activity 2 is a didactic design in the form of a Hypothetical Learning Trajectory (HLT). The developed HLT consists of didactic situations which are developed based on the students' learning obstacle, the material hierarchy, and the previously used learning materials. For each set didactic situation (learning trajectory), assignments related to the basic competencies...
developed and the didactic situation were made. For each assignment a predictive response may be made in learning, it is possible to make several predictions of the response for each didactic situation. Here is an example of HLT on the circle concept in Table 2.

| Epistemological Obstacle | Activities | Response Prediction | Anticipation |
|--------------------------|------------|----------------------|--------------|
| Student have difficulty in distinguishing between radius and diameter | The teacher explains the definition of a circle in front of the class | With different cases student find in difficult to distinguish radius and diameter | The teacher provides different illustrations to develop competences for distinguishing radius and diameter |
|                         | The teacher illustrates the relationship of radius and diameter by presenting two circles of different sizes |                         | The teacher illustrates the different circles |
|                         | Students are asked to explain the relationship of radius and diameter |                         |                         |
| Students find it difficult to identify the circumference of the circle. | The teacher gives an overview of the shape around the circle. | Students still have difficulty identifying the circumference of the circle. | The teacher describes and explains that the diameter is the bowstring that passes through the center point. |
| Student have difficulty distinguishing radius and arcs | Students make a circular arc illustration with the instructions given. | Students find it difficult to compare bowstrings with diameters | The teacher illustrates the definition of the perpendicular in several cases. |
|                         | Students are asked to explain the relationship between the arc and the circumference of the circle. | Students illustrate the bowstring and apothem according to the instructions. |                         |
|                         | Students illustrate the bowstring and apothem according to the instructions. | Students are asked to explain the definitions of the bowstring and circle apothem. |                         |
|                         | Students are asked to explain the definitions of the bowstring and circle apothem. | Students make a circle illustration according to the instructions given. |                         |

### 3.3. Analysis of results and follow-up plans

Based on the results of the identification of students’ learning barriers, in general, students experiencing learning obstacles indicate that:

1) students’ understanding of basic concepts is still weak;
2) students’ prior knowledge is inadequate for ongoing learning;
3) teaching materials previously made do not make use of relevant learning theories and do not accommodate the potential abilities of students;
4) the learning indicators developed do not refer to higher order thinking skills (hots) so that students are less familiar with developing their mathematical thinking skills as a soft skill;
5) learning that is developed is still around mechanistic skills or just counting.

Based on the results obtained from the mentoring activities of madrasah mathematics teachers, both madrasah tsanawiyah and madrasah aliyah, in general the process of learning mathematics in the classroom is not adequate. The results of identification and discussion on mentoring that have been carried out are already at the didactic design stage in the form of a hypothesis learning trajectory (HLT). This HLT cannot be used in the implementation of learning because the form is not a teaching
material but a didactic design blueprint. Therefore, as a follow-up to this mentoring activity, the following assistance will be continued in the form of didactic design development in the form of teaching materials, didactic design implementation, metapedaddidactic analysis, and retrospective analysis. The output of this retrospective analysis is the revised learning trajectory and revised learning materials.

However, the results of the teacher's first analysis of the learning trajectories found will form the basis for the development of subsequent learning. Self-awareness of teachers in paying attention to and improving student learning trajectories will make changes to better learning development. Giménez, Font, and Vanegas [10] explains the complexity of analysis that the teacher should realize to value his/her own practice to go beyond from narratives and descriptions.

The development of student learning trajectories can help build social constructivism in the learning process. In the research conducted by Uygun [11], it was found that the learning trajectory developed could support middle school mathematics teachers in developing mathematics learning practices in a social context.

4. Conclusion
Based on the results of the didactic design development assistance activities for mathematics learning in madrasahs, the following conclusions were obtained:

1) the epistemological obstacles of students identified related to mathematical concepts include the concept of functions, circles, comparisons, and sequences and series;
2) the hypothetical learning trajectory for the initial didactic design was developed based on the results of the learning obstacle analysis by taking into account the potential level of students into didactic situations, assignments, predictive responses, and pedagogical didactic anticipations that match the characteristics of students' learning obstacles;
3) the resulting didactic design was developed by utilizing learning theory that is relevant to the potential and level of cognitive development of students.

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