M-Fivebranes Wrapped on Supersymmetric Cycles II

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ABSTRACT

We construct D=11 supergravity solutions dual to the twisted field theories arising when M-theory fivebranes wrap supersymmetric cycles. The cases considered are M-fivebranes wrapped on (i) a complex Lagrangian four-cycle in a D=8 hyper-Kähler manifold corresponding to a D=2 field theory with (2,1) supersymmetry (ii) a product of two holomorphic two-cycles in a product of two Calabi-Yau two-folds corresponding to a D=2 field theory with (2,2) supersymmetry and (iii) a product of a holomorphic two-cycle and a SLAG three-cycle in a product of a Calabi-Yau two-fold and a Calabi-Yau three-fold corresponding to a quantum mechanics with two supercharges. In each case we construct BPS equations and find IR superconformal fixed points corresponding to new examples of AdS/CFT duality arising from the twisted field theories.

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1 Introduction

An interesting way to generalise the AdS/CFT correspondence \[1\] is to construct supergravity duals of the twisted field theories arising \[2\] when branes wrap supersymmetric cycles. By exploiting the observation that the supergravity solutions can be first constructed in an appropriate gauged supergravity and then uplifted to D=10 or D=11, such solutions were presented in \[3\]. The examples considered in \[4\] involve M-fivebranes and D-3-branes wrapping two-cycles in Calabi-Yau two- or three-folds. The solutions describe a flow from a UV region, corresponding to the D=6 or D=4 twisted field theory on the brane wrapped on the cycle, to an IR region corresponding to a D=4 or D=2 dimensional field theory, where the energy scale is set by the inverse size of the cycle. In several cases AdS fixed points were found in the IR corresponding to new AdS/CFT examples.

In subsequent work, D=11 supergravity solutions corresponding to M-fivebranes wrapped on associative three-folds in manifolds of $G_2$ holonomy were constructed in \[4\] and many other examples were considered in \[5\]. Other cases involving D3-branes and M2-branes were considered in \[6\] and \[7\], respectively. In addition to these examples involving conformal branes, analogous supergravity solutions for other wrapped branes have been studied in \[8, 4, 9, 10, 11, 12, 13, 14, 15, 16\].

Here we would like to report on three outstanding cases involving M-fivebranes. As in \[5\], the solutions are constructed in maximal D=7 gauged supergravity \[17\] and then uplifted to D=11 using the results of \[18, 19\]. Indeed, we shall employ exactly the same techniques as \[5\] and we refer the reader to this paper for further background and details on notation and conventions. The first case we consider, which is probably the most interesting, is M-fivebranes wrapping a “complex Lagrangian” four-cycle in a D=8 hyper-Kähler manifolds. These are four-cycles that are complex (Kähler) with respect to one of the three complex structures and special Lagrangian with respect to another, which together imply that they are also special Lagrangian with respect to the third complex structure \[20\] (for a discussion in the physics literature, see \[21\]). A concrete example of such a supersymmetric four-cycle is $CP^2$ in the hyper-Kähler Calabi metric on $T^*(CP^2)$. At low-energies the wrapped fivebrane gives rise to a D=2 field theory preserving (2,1) supersymmetry, as we shall show. In the supergravity solutions we construct, the metric on the four-cycle $\Sigma_4$ is taken to be Kähler with constant holomorphic sectional curvature. In other words, $CP^2$ for positive curvature; the Bergmann metric on a unit open ball $D^2$ in $C^2$ for negative curvature; and flat space for zero curvature (see p. 170 of \[22\]). As in previous solutions we can also take
a quotient of these spaces by a discrete group of isometries and in particular we can obtain compact manifolds with negative curvature. We construct the BPS equations and demonstrate an $AdS_3 \times \Sigma_4$ IR fixed point when $\Sigma_4$ has negative curvature and we determine the central charge of the fixed point. We show that the numerical analyses of the BPS equations is essentially included in [5].

The second case to be considered is M-fivebranes wrapping a product of two-cycles, $\Sigma_1 \times \Sigma_2$, with each $\Sigma_i$ a Kähler (holomorphic) two-cycle in a Calabi-Yau two-fold. At low-energies this gives rise to a D=2 field theory with (2,2) supersymmetry. We find BPS equations when each $\Sigma_i$ has constant curvature and show that there is an IR $AdS_3 \times \Sigma_1 \times \Sigma_2$ fixed point in the special case that the four-cycle is Einstein with negative curvature. We again determine the central charge of the fixed point.

The final case we will examine is M-fivebranes wrapping a product of a three-cycle with a two-cycle, $\Sigma_1 \times \Sigma_2$, with $\Sigma_1$ a SLAG three-cycle in a Calabi-Yau three-fold and $\Sigma_2$ a Kähler two-cycle in a Calabi-Yau two-fold. At low-energies this gives rise to a quantum mechanics with 2 supercharges. We find BPS equations when each $\Sigma_i$ has constant curvature and show that there is an IR $AdS_2 \times \Sigma_1 \times \Sigma_2$ fixed point in the special case that the five-cycle is Einstein with negative curvature.

## 2 Four-Cycles in D=8 Hyper-Kähler Manifolds

In this section we consider fivebranes wrapping supersymmetric complex Lagrangian four-cycles in D=8 hyper-Kähler manifolds. Before turning to the construction of the supergravity solutions let us begin by describing the supersymmetry preserved by a probe fivebrane wrapping such a cycle. It will then be straightforward to impose the appropriate supersymmetry projections in the gauged supergravity context. Consider a D=11 background to be of the form $R^{1,2} \times M$ where $M$ is a hyper-Kähler eight-manifold. It will be convenient to introduce an orthonormal frame $e^a$, $a = 1, \ldots, 8$, with hyper-Kähler structure given by

\[
J^1 = e^1 \wedge e^2 - e^3 \wedge e^4 - e^5 \wedge e^6 + e^7 \wedge e^8 \\
J^2 = e^1 \wedge e^5 + e^2 \wedge e^6 + e^3 \wedge e^7 + e^4 \wedge e^8 \\
J^3 = e^1 \wedge e^6 - e^2 \wedge e^5 - e^3 \wedge e^8 + e^4 \wedge e^7 = J^1 J^2
\] (2.1)

Noting that $(1/2)J^1 \wedge J^1, -Re\Omega^{J^2} \equiv -Re(e^1 + ie^5)(e^2 + ie^6)(e^3 + ie^7)(e^4 + ie^8)$ and $-Re\Omega^{J^3} \equiv -Re(e^1 + ie^6)(e^2 - ie^5)(e^3 - ie^8)(e^4 + ie^7)$ can be expressed as $-e^{1234} + \ldots$ we conclude that a four-cycle whose volume form is the pull-back of $-e^{1234}$ is complex with respect to $J^1$ and special Lagrangian with respect to $J^2, J^3$. If we wrap a
fivebrane probe on this four-cycle the D=11 supersymmetry preserved satisfies

\begin{align}
\Gamma^{01234}\epsilon &= \epsilon \\
\Gamma^{12}\epsilon &= -\Gamma^{56}\epsilon \\
\Gamma^{34}\epsilon &= -\Gamma^{78}\epsilon \\
(\Gamma^{14} + \Gamma^{23} + \Gamma^{58} + \Gamma^{67})\epsilon &= 0
\end{align}

(2.2)

where the last three conditions are the projections imposed on the parallel spinors of the hyper-Kähler manifold (see, for example, [23]), and the first is due to the wrapped fivebrane. These conditions preserve 3/32 supersymmetry or more precisely (2,1) supersymmetry, where the chirality refers to the D=2 field theory living on the unwrapped directions of the fivebrane. To see this, first note that the first three projections preserve 1/8 supersymmetry. Next note that the last condition can be replaced with \((1-S_1-S_2-S_3)\epsilon = \epsilon\) where \(S_1 \equiv \Gamma^{1234}, S_2 \equiv \Gamma^{1458}\) and \(S_3 \equiv \Gamma^{2358}\). The \(S_i\) all have vanishing trace, commute with the other projectors, square to unity and satisfy \(S_1 S_2 S_3 = -1\). Working on the subspace where the other conditions in (2.2) are satisfied we can choose a basis where \(S_1 = diag(1,1,-1,-1), S_2 = diag(1,-1,1,-1)\) and \(S_3 = diag(-1,1,1,-1)\). It is then easy to see that the last condition preserves 3/4 of the supersymmetry corresponding to the spinors \(\epsilon_1 = (1,0,0,0), \epsilon_2 = (0,1,0,0), \epsilon_3 = (0,0,1,0)\). In addition, from the first condition in (2.2) we note that the D=2 chirality is specified by the action of \(\Gamma^{1234}\) and we see that \(\epsilon_1\) and \(\epsilon_2\) have positive helicity and \(\epsilon_3\) has negative helicity giving rise to (2,1) supersymmetry as claimed.

It is interesting to observe that the spinors \(\epsilon_1\) and \(\epsilon_2\) are annihilated by \(S_2 + S_3\) and hence for these spinors the last condition is simply \((1-S_1)\epsilon = 0\). Comparing with, e.g. [24], one now sees that this projection along with the first three in (2.2) are precisely those for a Kähler four cycle corresponding to \(J^1\). Similarly one finds that \(\epsilon_1\) and \(\epsilon_3\) are associated with the projections for a SLAG four-cycle with respect to the complex structure \(J^2\) and \(\epsilon_2\) and \(\epsilon_3\) are associated with the projections for a SLAG four-cycle with respect to \(J^3\).

Having finished this explicit discussion of the supersymmetry projections for the fivebrane probe we are ready to start with the construction of the corresponding gauged supergravity solutions. As noted, we shall first construct the solutions in maximal D=7 gauged supergravity [17] and we refer the reader to [5] for more details on notation. The ansatz for the D=7 metric is given by

\[ ds^2 = e^{2f}(-dt^2 + dx^2 + dr^2) + e^{2g}ds^2 \]

(2.3)

where \(t, x\) are coordinates of the unwrapped part of the fivebrane worldvolume, \(ds^2\)
is the metric on the four-cycle $\Sigma_4$ that the fivebrane wraps and $f, g$ are functions of $r$ only.

The ansatz for the $SO(5)$ supergravity gauge fields are directly determined by the “twisting” arising when an M-fivebrane probe wraps a supersymmetric cycle. This twisting is simply a consequence of the structure of the normal bundle of the supersymmetric cycle. It entails an identification of the structure group of the cycle with a subgroup of the $SO(5)$ R-symmetry and is required in order to preserve supersymmetry. We can thus determine the $SO(5)$ supergravity gauge field ansatz by consideration of the supersymmetry preserved by the M-fivebrane wrapping the four-cycle. In the language of gauged supergravity the appropriate supersymmetry projections discussed above are given by

\[
\begin{align*}
\gamma^r \epsilon &= \epsilon \\
(1 - \gamma^{12} \Gamma^{12}) \epsilon &= 0 \\
(1 - \gamma^{34} \Gamma^{34}) \epsilon &= 0 \\
(\gamma^{14} + \gamma^{23} + \Gamma^{14} + \Gamma^{23}) \epsilon &= 0
\end{align*}
\]

(2.4)

where $\gamma^\mu$ and $\Gamma^m$ are $SO(1,6)$ and $SO(5)$ gamma-matrices, respectively, the indices refer to an obvious orthonormal frame and the directions 1, 2, 3, 4 correspond to those of the cycle. By repeating a similar analysis to that above we conclude that these projections preserve (2,1) supersymmetry, where the chirality refers to the D=2 field theory living on the unwrapped directions of the fivebrane specified.

The “twisting condition” required by supersymmetry is given by [5]:

\[
(\bar{\omega}_{ab} \gamma^{ab} + 2mB_{mn} \Gamma^{mn}) \epsilon = 0
\]

(2.5)

where $\bar{\omega}$ is the spin connection of $\Sigma_4$ with $a, b = 1, \ldots, 4$ tangent space indices, and $B$ is the $SO(5)$ gauge-field with $m, n = 1, \ldots, 5$. Upon imposing the projections we see that this condition is satisfied if we demand that the metric on the cycle is Kähler (i.e. impose $\bar{\omega}_{31} = \bar{\omega}_{24}$ and $\bar{\omega}_{23} = \bar{\omega}_{14}$ corresponding to the Kähler form with non-vanishing entries given by $J_{12} = -J_{34} = 1$) and in addition we demand that the only non-vanishing gauge fields are in a $U(2)$ subgroup of $SO(5)$ and identified with the spin connection via $\bar{\omega} = 2mB$. In other words, we see that when a fivebrane wraps a complex Lagrangian four-cycle in a D=8 hyper-Kähler manifold, the appropriate twisted field theory is obtained by identifying the $U(2)$ spin connection of the cycle with a corresponding $U(2)$ subgroup of the $SO(5)$ R-symmetry.

As in [3], with the type of ansatz we consider, supersymmetry demands that the Kähler four-cycle is Einstein, and we take $\bar{R}_{ab} = l \bar{g}_{ab}$ with $l = \pm 1, 0$. To ensure
all equations of motion are satisfied we demand in addition that it has constant holomorphic sectional curvature\footnote{Another way to satisfy the equations of motion and preserve (2,2) supersymmetry, is to take the four-cycle to be a product of two constant curvature two-metrics. This will be discussed in the next section.}. Equivalently, we demand that the Riemann tensor of the four-cycle can be expressed as

\[\bar{R}_{ab}{}^{cd} = \frac{l}{3} \left[ J_{ab} J^{cd} + \delta_{ab}^{cd} + J_{[c} J_{d]}^a \right] \] (2.6)

For \( l = 1 \) we have \( CP^2 \), for \( l = 0 \) flat space and for \( l = -1 \) the Bergmann metric on the unit ball \( D^2 \) in \( C^2 \), or a quotient of these spaces by a discrete group of isometries.

We truncate the 15 scalar fields of maximal gauge supergravity to a single scalar field \( \lambda(r) \). As for the other cases of M-fivebranes wrapping four-cycles in eight dimensions considered in \[5\], and consistent with the twisting just discussed, we take

\[ \Pi_A^i = (e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \] (2.7)

Only one of the five three-forms, \( S_5 \), is non-zero and is given by

\[ S_5 = -\frac{e^{-8\lambda-4g+3f}}{3\sqrt{3}m^4} dt \wedge dx \wedge dr \] (2.8)

By setting the supersymmetry variations of the D=7 fermions to zero we find that the resulting BPS equations are given by

\[ e^{-f} f' = -\frac{m}{10} \left[ 4e^{-2\lambda} + e^{8\lambda} \right] + \frac{l}{5m} e^{2\lambda-2g} - \frac{l^2}{5m^3} e^{-4\lambda-4g} \]

\[ e^{-f} g' = -\frac{m}{10} \left[ 4e^{-2\lambda} + e^{8\lambda} \right] - \frac{3l}{10m} e^{2\lambda-2g} + \frac{2l^2}{15m^3} e^{-4\lambda-4g} \]

\[ e^{-f} \lambda' = \frac{m}{5} \left[ e^{8\lambda} - e^{-2\lambda} \right] + \frac{l}{10m} e^{2\lambda-2g} + \frac{l^2}{15m^3} e^{-4\lambda-4g} \] (2.9)

Any solution of these BPS equations, and others presented in the next sections, also solves the full equations of motion. We do not have a general solution to these BPS equations. However, the numerical analyses carried out for the BPS equations for other four-cycles in \[3\] is applicable here (set \( \alpha = l/m, \beta = 2/3m^2 \) in equation (6.14) of \[3\]). In particular figures 5 and 6 of \[3\] illustrate the corresponding behaviour of the flows from the UV to the IR for \( l = \pm 1 \).

Using the results of \[17, 18\] we can uplift solutions to the BPS equations to give supersymmetric solutions to \( D = 11 \) supergravity. The metric is given by

\[ ds^2_{11} = \Delta^{-\frac{2}{3}} ds^2_7 + \frac{1}{m^2} \Delta^\frac{2}{3} \left[ e^{2\lambda} DY^\alpha DY^\alpha + e^{-8\lambda} dY^5 dY^5 \right] \] (2.10)
where
\[
DY^a = dY^a + \bar{\omega}^{ab} Y^b
\]
\[
\Delta - \frac{\pi}{2} = e^{-2\lambda} Y^a Y^a + e^{8\lambda} Y^5 Y^5
\]  
(2.11)

and \((Y^a, Y^5)\) are constrained coordinates on the four-sphere satisfying \(Y^a Y^a + Y^i Y^i = 1\). The expression for the four-form can easily be read from the formulae in [17, 18] and we will not bother to write it explicitly here.

If we take the four-cycle to have constant negative holomorphic sectional curvature, \(l = -1\), we find that the BPS equations admit an \(AdS_3 \times \Sigma_4\) solution with:
\[
e^{10\lambda} = \frac{6}{5},
\]
\[
e^{2g} = \frac{e^{-6\lambda}}{m^2},
\]
\[
e^f = \frac{e^{2\lambda} 1}{m^r}.
\]  
(2.12)

The central charge of the corresponding D=2 superconformal field theory can be obtained from the radius of \(AdS_3\). Repeating the arguments in [5], we find, setting \(m = 2\),
\[
c = \frac{8N^3}{\pi^2} \frac{5}{192} Vol(\Sigma).
\]  
(2.13)

It is worth mentioning how this example interconnects with the more general class of solutions corresponding to fivebranes wrapping Kähler and SLAG four-cycles in Calabi-Yau four-folds discussed in [5]. In that paper, it was shown that if the four-cycle is Kähler-Einstein with the \(U(1) \subset U(2)\) part of the spin connection identified with the corresponding \(U(1)\) of \(U(2) \subset SO(5)\) then there are BPS equations preserving \((2,0)\) supersymmetry. The analysis in [4] only covered the case when the rest of the \(U(2) \subset SO(5)\) gauge fields vanished. Here we have shown that if they are switched on, for the special case when the full \(U(2)\) gauge fields are identified with the \(U(2)\) spin connection, and in addition the Kähler-Einstein metric is taken to have constant holomorphic sectional curvature, we get BPS equations preserving \((2,1)\) supersymmetry. The supersymmetric four-cycles we are considering in a hyper-Kähler manifold are also SLAG four-cycles (with respect to a different complex structure). The reason that the BPS equations presented here were not included in the SLAG four-cycles considered in [4] is that it was assumed there that the structure group of the four-cycle was in fact \(SO(4)\) and not a proper subgroup of it.

Another specialisation, to be discussed in the next section, is when the supersymmetric four-cycle is taken to be a product of two constant curvature metrics. In this
case the structure group of the spin connection of the four-cycle is $U(1) \times U(1)$ and this is identified with a corresponding $U(1) \times U(1) \subset SO(5)$. With these restrictions, $(2,2)$ supersymmetry is preserved. Again this can be viewed as a special case of a SLAG or Kähler four-cycles with supersymmetry enhanced to $(2,2)$.

3 A product of two Kähler two-cycles

Consider M-fivebranes wrapping a four-cycle consisting of a product of two-cycles, $\Sigma_1 \times \Sigma_2$, with each $\Sigma_i$ a Kähler two-cycle in a Calabi-Yau two-fold. This is another example of an M-fivebrane wrapped on a four-cycle in eight dimensions, but the product structure allows us to consider a slightly more general ansatz for the metric than in the previous example. Specifically, we now take

\[ ds_7^2 = e^{2f}(-dt^2 + dx^2 + dr^2) + e^{2g_1}ds_1^2 + e^{2g_2}ds_2^2 \]  

(3.1)

with $ds_i^2$ two-metrics on each of the two-cycles, and $f, g_1, g_2$ functions of $r$. Combined with the twisting to be discussed, supersymmetry forces these metrics to have constant curvature with $\bar{R}_{ab} = l_i\bar{g}_{ab}$ and $l_i = \pm 1, 0$. Each two-cycle must be $S^2$ for positive, flat space for zero curvature, $H^2$ for negative curvature, and again we can take quotients of these spaces by discrete isometry subgroups. Note that this ansatz allows for the four-cycle to be non-Einstein in general.

The appropriate supersymmetry projections are now given by

\[ \gamma^r \epsilon = \epsilon \]
\[ (1 - \gamma^{12}\Gamma^{12})\epsilon = 0 \]
\[ (1 - \gamma^{34}\Gamma^{34})\epsilon = 0 \]  

(3.2)

These preserve 1/8 of the supersymmetry, or more precisely, $(2,2)$ supersymmetry from the point of view of the unwrapped D=2 part of the M-fivebrane world-volume. These projections give rise to the appropriate ansatz for the $SO(5)$ gauge-fields via the twisting condition (2.5). We split $SO(5) \rightarrow U(1) \times U(1)$ and identify each $U(1)$ with a $U(1)$ factor of the $U(1) \times U(1)$ structure group of the four-cycle. In other words we set $\bar{\omega}_{12} = 2mB_{12}, \bar{\omega}_{34} = 2mB_{34}$ and all other gauge fields vanishing. Clearly this is just two copies of the twisting of a holomorphic two-cycle inside a Calabi-Yau two-fold considered in [3].

We choose a two-scalar ansatz consistent with $U(1) \times U(1)$ symmetry via

\[ \Pi_A = (e^{\lambda_1}, e^{\lambda_1}, e^{\lambda_2}, e^{\lambda_2}, e^{-2\lambda_1-2\lambda_2}) \]  

(3.3)
with $\lambda_i$ functions of $r$. Only one of the five three-forms, $S_5$, is non-vanishing and is given by

$$S_5 = -\frac{l_1 l_2 e^{-4\lambda_1 - 4\lambda_2 - 2g_1 - 2g_2 + 3f}}{2\sqrt{3m^4}} \, dt \wedge dx \wedge dr.$$  \hfill (3.4)

Setting the supersymmetry variations of the fermions to zero we obtain the following BPS equations

$$e^{-f} f' = -\frac{m}{10} \left[ 2e^{-2\lambda_1} + 2e^{-2\lambda_2} + e^{4\lambda_1 + 4\lambda_2} \right] + \frac{1}{10m} \left[ l_1 e^{2\lambda_1 - 2g_1} + l_2 e^{2\lambda_2 - 2g_2} \right] - \frac{3l_1 l_2}{10m^3} X$$

$$e^{-f} g_1' = -\frac{m}{10} \left[ 2e^{-2\lambda_1} + 2e^{-2\lambda_2} + e^{4\lambda_1 + 4\lambda_2} \right] - \frac{1}{10m} \left[ 4l_1 e^{2\lambda_1 - 2g_1} - l_2 e^{2\lambda_2 - 2g_2} \right] + \frac{l_1 l_2}{5m^3} X$$

$$e^{-f} g_2' = -\frac{m}{10} \left[ 2e^{-2\lambda_1} + 2e^{-2\lambda_2} + e^{4\lambda_1 + 4\lambda_2} \right] - \frac{1}{10m} \left[ 4l_2 e^{2\lambda_2 - 2g_2} - l_1 e^{2\lambda_1 - 2g_1} \right] + \frac{l_1 l_2}{5m^3} X$$

$$e^{-f} \lambda_1' = \frac{m}{5} \left[ e^{4\lambda_1 + 4\lambda_2} - 3e^{-2\lambda_1} + 2e^{-2\lambda_2} \right] + \frac{1}{10m} \left[ 3l_1 e^{2\lambda_1 - 2g_1} - 2l_2 e^{2\lambda_2 - 2g_2} \right] + \frac{l_1 l_2}{10m^3} X$$

$$e^{-f} \lambda_2' = \frac{m}{5} \left[ e^{4\lambda_1 + 4\lambda_2} - 3e^{-2\lambda_2} + 2e^{-2\lambda_1} \right] + \frac{1}{10m} \left[ 3l_2 e^{2\lambda_2 - 2g_2} - 2l_1 e^{2\lambda_1 - 2g_1} \right] + \frac{l_1 l_2}{10m^3} X$$  \hfill (3.5)

where $X \equiv e^{-2\lambda_1 - 2\lambda_2 - 2g_1 - 2g_2}$.

The metric of the corresponding D=11 supergravity solution now has the form

$$ds_{11}^2 = \Delta^{-\frac{2}{3}} ds_7^2 + \frac{1}{m^2} \Delta^{\frac{2}{3}} \left[ e^{2\lambda_1} D Y^a D Y^a + e^{2\lambda_2} D Y^\alpha D Y^\alpha + e^{-4\lambda_1 - 4\lambda_2} dY^5 dY^5 \right]$$  \hfill (3.6)

with $a, b = 1, 2, \alpha, \beta = 3, 4$ and

$$D Y^a = dY^a + \tilde{\omega}^{ab} Y^b$$

$$D Y^\alpha = dY^\alpha + \tilde{\omega}^{\alpha\beta} Y^\beta$$

$$\Delta^{-\frac{2}{3}} = e^{-2\lambda_1} Y^a Y^a + e^{-2\lambda_2} Y^\alpha Y^\alpha + e^{4\lambda_1 + 4\lambda_2} Y^5 Y^5$$  \hfill (3.7)

and $(Y^a, Y^\alpha, Y^5)$ are again constrained coordinates on the four-sphere. The expression for the four-form can be read off from the formulae in \[17, 18\].

It is straightforward to show that the only $AdS_3$ fixed point of the BPS equations has $l_1 = l_2 = -1$ and

$$\lambda_i = 0$$

$$e^{2g_i} = \frac{1}{m^2}$$

$$e^f = \frac{1}{m r}.$$  \hfill (3.8)

Note that the four-cycle is $H_2 \times H_2$ or a quotient thereof and is Einstein. In addition the warp factor in D=11 is trivial, so the D=11 metric is simply a twisted product...
of $AdS_3 \times H_2 \times H_2 \times S^4$. The central charge of the D=2 superconformal field theory can be obtained from the radius of $AdS_3$. Repeating the arguments in [5] we find, upon setting $m = 2$,

$$c = \frac{8N^3}{\pi^2} \frac{1}{32} Vol(\Sigma).$$  \hspace{1cm} (3.9)

We shall not attempt a numerical analysis of the general BPS equations here. However, if we restrict to the case $g_1 = g_2$, $\lambda_1 = \lambda_2$ and $l_1 = l_2$, the numerical analyses carried out for the BPS equations for other four-cycles in [5] is applicable here (set $\alpha = l_1/m, \beta = 1/m^3$ in equation (6.14) of [5]). In particular figures 5 and 6 of [5] illustrate the corresponding behaviour of the flows from the UV to the IR.

4 A product of a SLAG three-cycle with a Kähler two-cycle

Our final example concerns M-fivebranes wrapping a five-cycle consisting of a product of a SLAG three-cycle in a Calabi-Yau three-fold with a Kähler two-cycle in a Calabi-Yau two-fold. The metric ansatz is taken to be

$$ds^2_7 = e^{2f}(-dt^2 + dr^2) + e^{2g_1}d\bar{s}_1^2 + e^{2g_2}d\bar{s}_2^2$$  \hspace{1cm} (4.1)

with $d\bar{s}_1^2$ a three-metric of constant curvature with $\bar{R}_{ab} = l_1 g_{ab}$ and $d\bar{s}_2^2$ a two-metric of constant curvature with $\bar{R}_{\alpha\beta} = l_2 g_{\alpha\beta}$ with $l_i = \pm 1, 0$. As usual these restrictions on the metrics arise from supersymmetry and the equations of motion.

The supersymmetry projections for this case are given by

$$\gamma^r \epsilon = \epsilon$$
$$\gamma^{ab} \epsilon = -\Gamma^{ab} \epsilon$$
$$\gamma^{\alpha\beta} \epsilon = -\Gamma^{\alpha\beta} \epsilon$$  \hspace{1cm} (4.2)

where $a, b = 1, 2, 3$, $\alpha, \beta = 4, 5$ and preserve 1/16 of the supersymmetry i.e. the low-energy effective quantum mechanics arising from the wrapped fivebrane has two supercharges. These projections give rise to the following twisting. The spin connection has structure group $SO(3) \times SO(2)$ and we identify this with a corresponding subgroup of the $SO(5)$ R-symmetry. Concretely we set $\bar{\omega}_{ab} = 2mB_{ab}$, $\bar{\omega}_{\alpha\beta} = 2mB_{\alpha\beta}$ and set all other components of the gauge-fields to zero. This twisting is just the combination of the twisting associated with the SLAG three-cycle discussed in [5] and the Kähler two-cycle discussed in [3].
An ansatz for the scalar fields preserving the $SO(3) \times SO(2)$ symmetry is given by

$$\Pi_A^i = (e^{2\lambda}, e^{2\lambda}, e^{2\lambda}, e^{-3\lambda}, e^{-3\lambda}).$$  \hspace{1cm} (4.3)

Three of the five three-forms are active and we have

$$S_m = -l_1 l_2 e^{-2g_1 - 2g_2 + 4\lambda + 2f} \frac{dt \wedge dr \wedge e^m}{4\sqrt{3}m^4},$$  \hspace{1cm} (4.4)

for $m = 1, 2, 3$. With this ansatz the resulting BPS equations are given by

$$e^{-f} f' = -\frac{m}{10} \left[ 3e^{-4\lambda} + 2e^{6\lambda} \right] + \frac{3l_1}{20m} e^{4\lambda - 2g_1} + \frac{l_2}{10m} e^{-6\lambda - 2g_2} - \frac{9l_1 l_2}{20m^3} e^{2\lambda - 2g_1 - 2g_2}$$

$$e^{-f} g_1' = -\frac{m}{10} \left[ 3e^{-4\lambda} + 2e^{6\lambda} \right] - \frac{7l_1}{20m} e^{4\lambda - 2g_1} + \frac{l_2}{10m} e^{-6\lambda - 2g_2} + \frac{l_1 l_2}{20m^3} e^{2\lambda - 2g_1 - 2g_2}$$

$$e^{-f} g_2' = -\frac{m}{10} \left[ 3e^{-4\lambda} + 2e^{6\lambda} \right] + \frac{3l_1}{20m} e^{4\lambda - 2g_1} - \frac{2l_2}{5m} e^{-6\lambda - 2g_2} + \frac{3l_1 l_2}{10m^3} e^{2\lambda - 2g_1 - 2g_2}$$

$$e^{-f} \lambda' = \frac{m}{5} \left[ e^{6\lambda} - e^{-4\lambda} \right] + \frac{l_1}{10m} e^{4\lambda - 2g_1} - \frac{l_2}{10m} e^{-6\lambda - 2g_2} - \frac{l_1 l_2}{20m^3} e^{2\lambda - 2g_1 - 2g_2}. \hspace{1cm} (4.5)$$

The corresponding D=11 metric is given by

$$ds_{11}^2 = \Delta^{-\frac{2}{5}} ds_7^2 + \frac{1}{m^2} \Delta^{\frac{2}{5}} \left[ e^{4\lambda} DY^a DY^a + e^{-6\lambda} DY^a DY^a \right]$$

where

$$DY^a = dY^a + \bar{\omega}^{ab} Y^b$$

$$DY^a = dY^a + \bar{\omega}^{ab} Y^b$$

$$\Delta^{-\frac{6}{5}} = e^{-4\lambda} Y^a Y^a + e^{6\lambda} Y^a Y^a$$

and $(Y^a, Y^a)$ are constrained coordinates on the four-sphere. The expression for the four-form can again be easily found using the formulae in [17, 18].

We find that there is only an $AdS_2$ fixed point when $g_1 = g_2$ and $l_1 = l_2 = -1$.

$$e^{10\lambda} = 2$$

$$e^{2g_i} = \frac{e^{8\lambda}}{2m^2}$$

$$e^f = \frac{e^{4\lambda} 1}{2m r}.$$ \hspace{1cm} (4.8)

In particular the product manifold on the 5-cycle is $H^2 \times H^3$ or a quotient thereof and is Einstein.
5 Discussion

We have discussed new D=11 supergravity solutions describing M-fivebranes wrapped on supersymmetric cycles. We have demonstrated new AdS fixed points corresponding to the CFTs arising from the twisted M-fivebrane theory in the IR. It would be interesting if we can compare our supergravity solutions directly with the field theory arising on the M-fivebrane. For example it may be possible to check the central charges of the CFT fixed points that we derived from the supergravity point of view.

By simply enumerating cases it would seem that the solutions discussed here and those in [3, 5] cover all ways in which static M-fivebranes can wrap supersymmetric cycles in a non-trivial Riemannian manifold with parallel spinors. Of course our ansatz can be generalised in a number of ways and it would be interesting if more solutions can be found in closed form. It is worth noting that there are additional configurations of branes intersecting at angles in flat space that preserve supersymmetry. For example, fivebranes can lie along quaternionic planes in $R^8$ and preserve $(3, 0)$ supersymmetry. These are planes that are complex with respect to three complex structures (e.g., the plane $-e^{1256}$ with respect to the hyper-Kähler structure (2.1)). Supergravity solutions for these and other similar configurations were discussed in [25, 26, 27].

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