Randomised Variable Neighbourhood Search for Multi Objective Optimisation

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Abstract

Various local search approaches have recently been applied to machine scheduling problems under multiple objectives. Their foremost consideration is the identification of the set of Pareto optimal alternatives. An important aspect of successfully solving these problems lies in the definition of an appropriate neighbourhood structure. Unclear in this context remains, how interdependencies within the fitness landscape affect the resolution of the problem.

The paper presents a study of neighbourhood search operators for multiple objective flow shop scheduling. Experiments have been carried out with twelve different combinations of criteria. To derive exact conclusions, small problem instances, for which the optimal solutions are known, have been chosen. Statistical tests show that no single neighbourhood operator is able to equally identify all Pareto optimal alternatives. Significant improvements however have been obtained by hybridising the solution algorithm using a randomised variable neighbourhood search technique.

Keywords: Hybrid Local Search, Multi Objective Optimisation, Flow Shop Scheduling.

1 Introduction

Machine scheduling considers in general the assignment of a set of resources (machines) \( M = \{M_1, \ldots, M_m\} \) to a set of jobs \( J = \{J_1, \ldots, J_n\} \), each of which consists of a set of operations \( J_j = \{O_{j1}, \ldots, O_{jo}\} \) [5]. The operations \( O_{jk} \) typically may be processed on a single machine \( M_i \in M \) involving a nonnegative processing time \( t_{jk} \). Usually, precedence constraints are defined among the operations of a job, reflecting its technical nature. In the specific case of the here considered permutation flow shop scheduling problem, the machine sequences are identical for all jobs [21]. Furthermore, the sequence of the jobs on the machines is assumed to be the same, therefore leading to a single job permutation as a possible schedule representation. A solution of the problem, a schedule, defines starting times for the tasks respecting the chosen job sequence and the defined constraints of the problem.

Optimality of schedules can be judged with respect to a single or multiple objective functions, expressing the quality of the solution in a quantitative way. While the multi criteria nature of scheduling in manufacturing environments has been mentioned quite early [24], the problem is treated only recently as a multi objective optimisation problem [30]. Here, the goal is to identify all efficient alternatives, the set of Pareto optimal solutions \( P \) [31]. Most important optimality criteria are based on the completion times \( C_j \) of the jobs \( J_j \) in the schedule:

- The minimisation of the maximum completion time (makespan) \( C_{max} = \max\{C_1, \ldots, C_n\} \).
- The minimisation of the sum of the completion times \( C_{sum} = \sum_{j=1}^{n} C_j \).

In the case of existing due dates \( d_j \) for each job \( J_j \), it is possible to compute due date violations in the form of tardiness values \( T_j = \max\{C_j - d_j, 0\} \). Possible optimality criteria are:

- The minimisation of the maximum tardiness \( T_{max} = \max\{T_1, \ldots, T_j\} \).
- The minimisation of the total tardiness \( T_{sum} = \sum_{j=1}^{n} T_j \).
• The minimisation of the number of tardy jobs $U = \sum_{j=1}^{n} U_j$ where $U_j = \begin{cases} 1 & : T_j > 0 \\ 0 & : T_j = 0 \end{cases}$

In terms of machine efficiency, idle times $I_i$ of the machines $M_i$ may be considered up to the completion of the last job. Optimality criteria are therefore:

• The minimisation of the maximum machine idleness $I_{\text{max}} = \max\{I_1, \ldots, I_m\}$.

• The minimisation of the total machine idleness $I_{\text{sum}} = \sum_{i=1}^{m} I_i$.

An important factor for the resolution of the permutation flow shop scheduling problem is the circumstance that the functions are regular [6]. It is therefore possible to map a given sequence of jobs $\pi = \{\pi_1, \ldots, \pi_n\}$ into an active schedule, one of which is optimal [8]. Consequently, many existing local search approaches are based on this idea of a schedule representation, and neighbourhood definitions manipulate the job permutations $\pi$ by changing the positions of jobs [23]:

• Exchange neighbourhood (EX), exchanging the position of two jobs $\pi_j$ and $\pi_k$, $j \neq k$.

• Forward shift neighbourhood (FSH), removing a job $\pi_j$ and reinserting it at position $\pi_k$ with $k > j$.

• Backward shift neighbourhood (BSH), removing a job $\pi_j$ and reinserting it at position $\pi_k$ with $k < j$.

• Inversion neighbourhood (INV), inverting the positions of a chosen subset of $\pi$.

Important applications of local search algorithms for the permutation flow shop scheduling problem comprise local search descent [11, 23, 32], simulated annealing [10, 11, 13, 19, 20, 25], tabu search [22, 27, 33], and evolutionary algorithms [1–4, 14, 15, 17, 18, 26]. The majority of the previously conducted studies consider the problem as a single objective optimisation problem with respect to the makespan objective $C_{\text{max}}$. Here, shift neighbourhood operators lead in most cases to superior results [27]. A generalisation of the conclusions with respect to multi objective problem settings seems however questionable, and corresponding studies have not been conducted yet.

2 An investigation of neighbourhood operators

2.1 Local search framework and experimental setup

The effectiveness of local search operators for multi objective flow shop scheduling problems has been investigated using the framework described in algorithm 2.1 [29].

Algorithm 2.1 Multi Objective Local Search Descent

1: Generate initial solution $x$
2: $P^{\text{approx}} = \{x\}$
3: repeat
4: Select $x \in P^{\text{approx}}$ for which $nh(x)$ has not been investigated yet
5: Generate $nh(x)$
6: Update $P^{\text{approx}}$ with all $x' \in nh(x)$
7: if $x \in P^{\text{approx}}$ then
8: Mark $nh(x)$ as 'investigated'
9: end if
10: until $\nexists x \in P^{\text{approx}}$ with $nh(x)$ still to be investigated

Starting from a random initial solution, neighbouring solutions are generated until no further improvement is possible. With respect to the goal of approximating a Pareto set $P$ of alternatives, an approximation set $P^{\text{approx}}$ is maintained during search. The update of $P^{\text{approx}}$ is done according to the the Pareto dominance relation, given in definition 2.1 for the chosen objective vector $G(x) = (g_1(x), \ldots, g_k(x))$. $P^{\text{approx}}$ therefore only contains nondominated alternatives. Solutions $x' \in nh(x)$ that dominate $x$ lead to the removal of $x$ from $P^{\text{approx}}$, making step 7 after the update of $P^{\text{approx}}$ necessary.
Definition 2.1 (Pareto dominance): A vector of objective functions $G(x)$ is said to dominate an vector $G(x')$ if and only if $\forall i \ g_i(x) \leq g_i(x') \land \exists i \ | g_i(x) < g_i(x')$.

The neighbourhood operators applied within the framework have been $k$-FSH, $k$-BSH, and $k$-EX. A control parameter $k$ determines the number of succeeding jobs in the sequence being involved in the shift or exchange operation, resulting in a block shift or block exchange neighborhood for $k > 1$. Settings of $k = 1$, $k = 2$, and $k = 3$ have been tested. 100 Test instances have been generated with $n = m = 10$ following the proposal of TAILLARD [28], and the due dates of the jobs were generated adopting the methodology of DEMIRKOL ET AL. [9]. The size of the instances allows the determination of the true Pareto set by enumerating all possible alternatives within reasonable time.

| Abbreviation | No objectives | Classification $\alpha \mid \beta \mid \gamma$ |
|--------------|---------------|-----------------------------------------------|
| $\gamma_1$  | 2             | $F \mid prmu, d_j \mid C_{max}, T_{max}$          |
| $\gamma_2$  | 2             | $F \mid prmu, d_j \mid C_{max}, T_{sum}$          |
| $\gamma_3$  | 2             | $F \mid prmu, d_j \mid C_{max}, T_{sum}$          |
| $\gamma_4$  | 2             | $F \mid prmu, d_j \mid T_{max}, T_{sum}$          |
| $\gamma_5$  | 2             | $F \mid prmu, d_j \mid C_{sum}, T_{max}$          |
| $\gamma_6$  | 2             | $F \mid prmu, d_j \mid C_{sum}, T_{sum}$          |
| $\gamma_7$  | 3             | $F \mid prmu, d_j \mid C_{max}, T_{max}, T_{sum}$ |
| $\gamma_8$  | 3             | $F \mid prmu, d_j \mid C_{max}, C_{sum}, T_{max}$ |
| $\gamma_9$  | 3             | $F \mid prmu, d_j \mid C_{max}, C_{sum}, T_{sum}$ |
| $\gamma_{10}$ | 3           | $F \mid prmu, d_j \mid C_{sum}, T_{max}, T_{sum}$ |
| $\gamma_{11}$ | 4            | $F \mid prmu, d_j \mid C_{max}, C_{sum}, T_{max}, T_{sum}$ |
| $\gamma_{12}$ | 6            | $F \mid prmu, d_j \mid C_{max}, C_{sum}, T_{max}, T_{sum}, I_{sum}, U$ |

Each neighbourhood operator has been tested on each problem instance in 100 test runs regarding twelve different combinations of optimality criteria as given in table 1, leading to a total of 1,080,000 local search runs. The approximation quality of the obtained set $P_{approx}$ to the Pareto set $P$ has been measured using the $D_1$ (average deviation of $P_{approx}$ to $P$) and $D_2$ (maximum deviation of $P_{approx}$ to $P$) metrics of Czyżak and Jaszkiewicz [7].

2.2 Results

The average results obtained by the local search operators have been investigated for statistical significance using a t-test at a given level of significance of 0.01. Table 2 and 3 give the number of test instances, in which a neighbourhood definition led to a significantly best approximation for $D_1$ and $D_2$.

It is possible to observe that neighbourhoods with a control parameter of $k > 1$ are only superior for very few problem instances. Comparing 1-FSH, 1-BSH and 1-EX, no operator turns out to be the most appropriate in all cases. While forward shift tends to lead more frequently to significantly best results, there are still numerous instances for which other operators are more favourable. It can also be noticed that this result is not primarily depending on the optimality criteria involved.

On a more detailed level it is possible to observe that the identification probability of a single Pareto optimal alternative depends on the choice of the neighbourhood operator. To illustrate this circumstance, figure 1 plots for each $x \in P$ of a problem instance, involving the optimality criteria combination $\gamma_{11}$, the frequency of its identification. No correlation between the identification frequencies can be seen, and the analysis reveals the existence of Pareto optimal alternatives which may be identified rather easily using e.g. 1-FSH while not being found by the 1-EX neighbourhood and vice versa.
3 Multi Objective Variable Neighbourhood Search

3.1 Description of the approach

Based on the investigation presented in section 2, an improved local search approach has been proposed, described in algorithm 3.1. Its main idea is the randomised application of a set of neighbourhood operators to increase the overall identification probability of every Pareto optimal alternative. Compared with existing concepts of variable neighbourhood search [16], the applied operator is randomly chosen at every step of the algorithm from a predefined set of neighbourhoods. Also, only a single neighbourhood is generated, even if it is not possible to achieve further improvements with the selected operator. The computational complexity of this approach is therefore identical with the multi objective local search framework in section 2.

The MOVNS heuristic has been applied to the 100 test instances regarding the twelve optimality criteria definitions in 100 test runs each. Two different configurations have been considered:

1. MOVNS/3, applying a neighbourhood from the set \{1-BSH, 1-FSH, 1-EX\}.
2. MOVNS/9, selecting in each step from all nine neighbourhood definitions as given in section 2.

### Tab. 2: Significance test results for $D_1$ over all 100 test instances.

| $\gamma$ | 1-BSH | 2-BSH | 3-BSH | 1-FSH | 2-FSH | 3-FSH | 1-EX | 2-EX | 3-EX |
|----------|-------|-------|-------|-------|-------|-------|------|------|------|
| $\gamma_1$ | 0 | 0 | 0 | 37 | 0 | 0 | 15 | 0 | 0 |
| $\gamma_2$ | 20 | 0 | 0 | 9 | 0 | 0 | 10 | 0 | 0 |
| $\gamma_3$ | 11 | 0 | 0 | 9 | 0 | 0 | 16 | 0 | 0 |
| $\gamma_4$ | 2 | 0 | 0 | 30 | 0 | 0 | 13 | 0 | 0 |
| $\gamma_5$ | 1 | 0 | 0 | 39 | 0 | 0 | 15 | 0 | 0 |
| $\gamma_6$ | 32 | 0 | 0 | 1 | 0 | 0 | 15 | 0 | 0 |
| $\gamma_7$ | 2 | 0 | 0 | 44 | 0 | 0 | 19 | 0 | 0 |
| $\gamma_8$ | 3 | 0 | 0 | 48 | 0 | 0 | 18 | 0 | 0 |
| $\gamma_9$ | 20 | 0 | 0 | 17 | 0 | 0 | 17 | 0 | 0 |
| $\gamma_{10}$ | 6 | 0 | 0 | 29 | 0 | 0 | 15 | 0 | 0 |
| $\gamma_{11}$ | 8 | 0 | 0 | 47 | 0 | 0 | 18 | 0 | 0 |
| $\gamma_{12}$ | 5 | 0 | 0 | 42 | 0 | 0 | 37 | 0 | 0 |

### Tab. 3: Significance test results for $D_2$ over all 100 test instances.

| $\gamma$ | 1-BSH | 2-BSH | 3-BSH | 1-FSH | 2-FSH | 3-FSH | 1-EX | 2-EX | 3-EX |
|----------|-------|-------|-------|-------|-------|-------|------|------|------|
| $\gamma_1$ | 0 | 0 | 0 | 53 | 1 | 0 | 23 | 0 | 0 |
| $\gamma_2$ | 22 | 0 | 0 | 14 | 0 | 0 | 12 | 0 | 0 |
| $\gamma_3$ | 13 | 0 | 0 | 15 | 0 | 0 | 24 | 0 | 0 |
| $\gamma_4$ | 3 | 0 | 0 | 41 | 0 | 0 | 20 | 0 | 0 |
| $\gamma_5$ | 4 | 0 | 0 | 49 | 1 | 0 | 24 | 0 | 0 |
| $\gamma_6$ | 30 | 0 | 0 | 2 | 0 | 0 | 19 | 0 | 0 |
| $\gamma_7$ | 2 | 0 | 0 | 49 | 2 | 0 | 24 | 0 | 0 |
| $\gamma_8$ | 5 | 1 | 0 | 56 | 0 | 0 | 19 | 0 | 0 |
| $\gamma_9$ | 20 | 0 | 0 | 23 | 0 | 0 | 23 | 0 | 0 |
| $\gamma_{10}$ | 14 | 1 | 0 | 34 | 0 | 0 | 27 | 0 | 0 |
| $\gamma_{11}$ | 10 | 2 | 0 | 51 | 1 | 0 | 21 | 0 | 0 |
| $\gamma_{12}$ | 10 | 1 | 0 | 32 | 0 | 0 | 53 | 0 | 0 |
Algorithm 3.1 Multi Objective Variable Neighbourhood Search (MOVNS)

1: Generate initial solution $x$
2: $P_{\text{approx}} = \{x\}$
3: repeat
4: Select $x \in P_{\text{approx}}$ for which $nh(x)$ has not been investigated yet
5: Select a neighbourhood $nh$
6: Generate $nh(x)$
7: Update $P_{\text{approx}}$ with all $x' \in nh(x)$
8: if $x \in P_{\text{approx}}$ then
9: Mark $nh(x)$ as 'investigated'
10: end if
11: until $\exists x \in P_{\text{approx}}$ with $nh(x)$ still to be investigated

3.2 Results

Statistical tests of significance show that MOVNS/3 is able to outperform the multi objective local search approach involving a single operator in most test instances. Table 4 and 5 give the number of problem instances in which the approach led to significantly best results for $D_1$ and $D_2$. This behaviour does not seem to depend on the defined optimality criteria. For a few problem instances however, and especially for the maximum deviation $D_2$, no improvements are possible.

The configuration of MOVNS with all presented neighbourhood definitions MOVNS/9 does not lead to satisfying results. Integrating relatively weaker neighbourhood operators can consequently not be regarded as a promising concept of hybridising local search heuristics.

The running times for the different algorithms are almost identical as the approaches differ only in terms of the additional choice of the neighbourhood in MOVNS. On a Intel Pentium 4 PC running at 1.8 GHz and 256 MB RAM, neighbouring solutions may be computed within 0.0003 milliseconds, while the evaluation of a solution takes 0.0402 milliseconds. The comparison of the resolution behaviour of different neighbourhood operators shows no significant difference in terms of running time.

4 Conclusions

A study of local search neighbourhoods for multi objective flow shop scheduling has been presented. First experiments involving twelve different combinations of optimality criteria revealed that no single operator is superior for
all considered problem instance. Furthermore, it has been possible to show that the identification probability of a specific Pareto optimal alternative depends on the choice of the neighbourhood in the local search procedure.

Improvements have been obtained in a significantly large number of test cases by applying a concept of randomised variable neighbourhood search. While the in average superior behaviour of the MOVNS approach is independent from the choice of the optimality criteria, a restriction to favourable neighbourhood definitions is necessary as weaker operators do not contribute to the quality of the obtained results.

The study demonstrates the advantages of hybridising traditional local search techniques with multi neighbourhood strategies. This is especially of importance as in the proposed setting of alternatively selecting operators the computational complexity of the approach is not increased while superior results are obtained.

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