LIGHT AS CAUSED NEITHER BY BOUND STATES NOR BY NEUTRINOS

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Participants of this workshop pursue the old Neutrino Theory of Light vigorously. Other physicists have long ago abandoned it, because it lacks gauge invariance. In the recent Quantum Induction (QI), all basic Bose fields $B^P$ are local limits of quantum fields composed of Dirac’s $\Psi$ (for leptons and quarks). The induced field equations of QI even determine all the interactions of those $B^P$. Thus a precise gauge invariance and other physical consequences are unavoidable. They include the absence of divergencies, the exclusion of Pauli terms, a prediction of the Higgs mass and a ‘minimal’ Quantum Gravity. As we find in this paper, however, photons can’t be bound states while Maxwell’s potential $A_\mu$ contains all basic Dirac fields except those of neutrinos.

1 The Desired Gauge Symmetry

A topic of this workshop has been the Neutrino Theory of Light; its history and present problems have been reviewed by Perkins thoroughly. Others such as Bjorken and Wightman have abandoned it, mainly because they could not attain gauge invariance. This important symmetry poses a serious problem as follows. In the usual theory, Dirac’s field $\Psi$ and Maxwell’s potential $A_\mu$ are
basically independent; hence their gauge transformations can be postulated separately. However, when photons are expected as bound states of basic fermions, should one not derive the gauge transformation of $A_\mu$ from that of $\Psi$? In the Neutrino Theory, this question is not even mentioned.

The problem has been solved as a by-product of QI, a version of Quantum Field Theory where we prevent the familiar divergencies. Since such an unconventional theory cannot be explained briefly, either of the following simplifications must here be adopted:

(a) We can treat a mathematical theory similar to QI. Then such restrictions may be imposed that we can prove simple results which formally resemble those of QI.

(b) We can explain some results of QI itself completely, but then most proofs must be omitted for brevity.

Having shown an example of (a) at another place, we find (b) more suitable for comparing QI with the Neutrino Theory.

In Section 2 we review the postulates of QI and the basic fields entering there. No further postulate is needed, however, for deriving the perfect gauge invariance in Section 3. The composite Bose fields $B^P$ of QI and their gauge covariant derivatives are in Section 4 distinguished from any bound state. Further extensions mentioned in Section 5 go beyond the expectations from the Neutrino Theory of Light, but fail to satisfy its old desires. Thus we are reminded of Quantum Mechanics, which has explained much more than Bohr’s theory of ‘planetary’ atoms did, but still cannot yield the ‘electron orbits’ envisioned there.

2 Fields of Quantum Induction

With a basically simple, but unorthodox action, we could start from Feynman’s path integral. For a faster introduction to QI, we (with the hermitian conjugate $\Psi^\dagger$ and the transpose $\Psi^T$ of the Dirac spinor $\Psi$) postulate the anticommutators

$$\begin{align*}
[\Psi(x), \Psi(0)]_+ \delta(x_0) &= \delta(x), \\
[\Psi(x), \Psi(0)^T]_+ \delta(x_0) &= 0
\end{align*}$$

(1)

and Dirac’s equation

$$(i \partial - B)\Psi = 0.$$
mathematical models. The Bose field \( B \) in (1) is a member of Dirac’s Clifford algebra \( \mathcal{C}_D \), but also a (non-canonical) quantum field. Hence it can with a suitable basis of \( \mathcal{C}_D \) be written

\[
B = S^+ + i\gamma_5 S^- + \gamma^\mu V_\mu^+ + \gamma^\mu \gamma_5 V_\mu^- + \sigma^{\mu\nu} T_{\mu\nu},
\]

(2)

where \( S^\pm, V_\mu^\pm \) and \( T_{\mu\nu} \) act only on the flavors and colors of \( \Psi \).

In (2) the ‘tensor potential’ \( T_{\mu\nu} = -T_{\nu\mu} \) must strictly be absent. Hence we must impose

\[
T_{\mu\nu} = 0 \quad \text{so that} \quad B = S + \gamma^\mu V_\mu
\]

(3)

(where \( S, V_\mu \) are obvious combinations of \( S^\pm, V_\mu^\pm \) and \( \gamma_5 \)). In other words, \( B \) is not spanned by all 16 basis elements of Dirac’s Clifford algebra \( \mathcal{C}_D \), but only by those ten that couple Dirac’s \( \Psi \) with \( S \) and \( V_\mu \) (which contain the observed Higgs and Yang-Mills fields).

This total absence of Pauli terms \( \sigma^{\mu\nu} T_{\nu\mu} \) from (2) is necessary whenever postulates similar to (1) are maintained. Hence (3) holds as well in the mathematical theory of heat kernels, but not necessarily in the usual ‘effective’ theory, where the infinite renormalizations make (1) invalid. The connections of tensor potentials \( T_{\mu\nu} \neq 0 \) with themselves have been investigated extensively. In the literature we cannot find, however, any hint about their interactions with Dirac’s \( \Psi \). If a tensor potential directly couples only to itself (and perhaps to other Bose fields), we evidently cannot reach the conclusion (3); but how could that hypothetical \( T_{\mu\nu} \neq 0 \) be observed?

3 Implied Gauge Invariance

Whenever Dirac’s \( \Psi \) satisfies (1) with any Bose field \( B \) (even if (3) is violated), this \( B \) can be recovered from a time ordered, bilocal field as the local limit

\[
B(x) = 8\pi^2 \gamma^\mu \lim_{z \to 0} T \Psi(x - z) \frac{\partial}{\partial z^n} \overline{\Psi}(x + z) \varphi^3.
\]

(4)

Under the restriction to any non-quantized Bose field, its representation (4) can be derived easily. For clarity, however, we totally exclude those ‘purely classical’ fields \( B \) from QI. Hence the proof of (4) requires precautions not explainable briefly; but its technical aspects remain the same as for non-quantized \( B \). Another local limit, which follows more directly than (4), but also breaks down under infinite renormalizations, is

\[
\lim_{z \to 0} \varphi^3 b(x, z) = i \quad \text{with} \quad b(x, z) := (4\pi)^2 T \Psi(x + z) \overline{\Psi}(x - z).
\]

(5)
Here and in (4), $z = 0$ can be approached on any path which does not touch the cone $z^2 = 0$.

For any member of Dirac’s Clifford algebra $\mathcal{C} \ell_D$, we must now define the adjoint

$$\Gamma := \gamma_0 \Gamma^\dagger \gamma_0$$

for every $\Gamma \in \mathcal{C} \ell_D$. (6)

From (4) and (5), one easily concludes then that the gauge transformation

$$\Psi \rightarrow e^{-i\omega} \Psi$$

(hence $\bar{\Psi} \rightarrow \bar{\Psi} e^{i\omega}$) (7)

implies

$$B \rightarrow e^{-i\omega}(B - i \partial) e^{i\omega}.$$ (8)

Here $\omega$ is a non-quantized matrix, which contains $\gamma_5$ but no other Dirac matrix (hence $\omega$ acts on flavors, colors and chiralities, but not on spins or helicities). We admit $\gamma_5 := i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ in order to include chiral transformations; but we make $\omega$ hermitian in order to exclude conformal mappings (hence the $\omega$ obeying $\gamma^\mu \omega = \omega \gamma^\mu$ simply equals $\omega$ with $\gamma_5 \rightarrow -\gamma_5$). From (8) it is also evident that the gauge transformations (7) are in general non-abelian and depend on time and space. For some readers, (8) will be more familiar when by (3) it is split into

$$S \rightarrow e^{-i\omega} S e^{i\omega}$$

and

$$V_\mu \rightarrow e^{-i\omega}(V_\mu - i\partial_\mu) e^{i\omega}.$$ (9)

or even linearized in an ‘infinitesimal’ $\omega$.

We have presented (8) for two reasons:

(a) Its derivation from (7) by means of (4) and (5) is easier than any other proof. Hence it can be left to the reader (just insert (7) in (4) and apply the product rule of differentiations; the result (8) follows when (4) and (5) are used).

(b) That proof of (8) not only demonstrates the implied gauge invariance of (1), but also why this symmetry has not been achieved with composite photons. In fact, (4) does not involve any forces or form factors and not even adjustable constants.

4 Composite Fields versus States

The strongest contrast between (4) and any physical ‘composite’ lies in the following distinction. For particles to form bound states, they must first exist themselves. Hence their own states must be created by operators such as

$$\Psi_f := \int f(x) dx \Psi(x) \quad \text{with} \quad f \in \mathcal{D},$$ (10)
obtained by smearing an operator valued distribution with an \( f \) from a space \( D \) of smooth test functions. The sharp limit (4), however, produces first another local field \( B(x) \), only \textit{afterwards} to be smeared into operators \( \int f(x) dx B(x) \) which create bosons.

Further consequences of (1) are Dirac induced equations for the Higgs and Yang-Mills fields contained in (9). They show that \( QI \) prevents the familiar divergencies very \textit{differently} for Dirac’s \( \Psi \) and the basic Bose fields in \( B \), namely as follows. Due to (5), the ‘coincident’ product \( \Psi(x) \overline{\Psi}(x) \) does not exist; but it is never needed. The components of \( B \), however, must form products \( \mathcal{B}^p(x) \mathcal{B}^q(y) \) which exist (as operator valued distributions) even for \( x \equiv y \). Therefore, the quantum fields in \( B \) must be non-canonical, just in order to keep \( \Psi \) forever canonical. Even without proofs, these complete results cannot be explained briefly.

Splitting (4) as in (3), we also get \( S \) and \( V_\mu \) as such local limits. They can be inserted into

\[
S_\mu := S_{\mu} + i \left( \nabla_\mu S - SV_\mu \right) \quad \text{and} \quad V_{\mu\nu} := V_{\mu,\nu} - V_{\nu,\mu} + i \left[ V_{\mu}, V_{\nu} \right].
\]

These are called gauge covariant derivatives because they transform under (9) homogeneously as \( S \) (in contrast to \( V_\mu \)) does. Then highly complicated expressions by \( \Psi \) are obtained, involving second derivatives of \( \Psi, \overline{\Psi} \) and terms with \( \Psi \overline{\Psi} \Psi \overline{\Psi} \).

Very surprisingly, however, there are alternative representations of (11) in which not even \( \Psi_\mu \) occurs. Instead, every component of (11) can (with a specific constant \( c^F_{\mu} \)) be written

\[
F(x) = \lim_{z \to 0} \left( z^\mu T \overline{\Psi}(x - z) c^F_{\mu} \Psi(x + z) \right)_{z^2}.
\]

Here \( \langle \ldots \rangle_{z^2} \) denotes the Lorentz invariant average over the direction of \( z \), while the matrix \( c^F_{\mu} \) acts on the flavors, colors, chiralities and helicities of \( \Psi \). Thus (12) evidently is not more complicated than (4), but even much simpler \textit{logically}.

5 Observable Boson States

Let \( \psi_f \) describe a single lepton or quark of the electric charge \( e_f \). Then the Maxwell component of (12) is

\[
F_{\nu\rho}(x) = \text{const} \cdot \sum_f e_f \lim_{z \to 0} \left( z^\mu T \overline{\psi}_f(x - z) \gamma_{\mu\nu\rho} \psi_f(x + z) \right)_{z^2}.
\]
with the ‘axial’ $\gamma_{\mu\nu\rho} := \gamma_{[\mu} \gamma_{\nu] \gamma_{\rho]} \in C\ell_D$. Hence the state of a single photon, created with the test function $t^{\mu\rho} = - t^{\rho\mu} \in \mathcal{D}$, becomes

$$|t\rangle = \int t^{\nu\rho}(x) dx \sum_{f} e_{f} \lim_{z \to 0} (z^{\mu} T_{f}(x - z) \gamma_{\mu\nu\rho} \psi_{f}(x + z))_{z^2} \rangle. \quad (14)$$

Here we have used the fact that Maxwell’s potential $A_{\mu}$ is not needed for observable photons, although it remains indispensible for electromagnetic forces and Coulomb clouds. Every pure state with at least one photon follows from (14), when the Poincare invariant vacuum $|\rangle$ is replaced by the most general state in the Hilbert space.

In order to generalize (14) to any state with at least one observable boson, we need only replace (13) by other components (12) of (11). Not only all Bose fields and their states are thus recovered, but the Dirac induced field equations of QI also determine all their interactions (which for hypothetically composite photons are never mentioned). Thus QI has achieved much more than one expected from the Neutrino Theory (even a minimal extension to gravity), but not what there had been desired originally. For instance, the gauge bosons are clearly not bound states, because neither forces nor form factors occur in (14) or in its extensions to other bosons. Whenever a state contains an additional photon, however, this involves due to (14) all basic Dirac components $\psi_{f}$ except those of neutrinos.

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