Study on genetic algorithm (GA) approaches for solving Flow Shop Scheduling Problem (FSSP)

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Abstract. The scheduling problem is known as one of the well-known optimization problems. It occurs in many situations of our daily-life applications, especially in industrial fields. One type of scheduling problem is called Flow Shop Scheduling Problem (FSSP). It belongs to the class of NP-complete problem. During the last decades, researches on exploring more accurate and efficient heuristic methods to solve hard optimization problems have taken considerable attention from researchers. Among them, GA has been one of the powerful and widely used algorithms. In this paper, we present two GA approaches to solve FSSP. The main objective is to investigate the effectiveness and the efficiency of GA based on different variations of the chromosome representation, referred to as the job-based GA (jb-GA) and machine-based GA (mb-GA). We conducted numerical experiments using standard test problems (Benchmark test problems). We also present the comparison of results with those given by another heuristic algorithm (NBH Algorithm) and the optimal solutions reported in the literature. Those demonstrate the jb-GA is more effective and efficient almost all of the time. The current limitation of this approach, like many other heuristic methods, is that it still sometimes gives the near-optimal solutions.

Keyword: genetic algorithm, scheduling, benchmark, flow shop

1. Introduction
The scheduling problem concerns the allocation of limited sources over time to perform the task to satisfy specific criteria. This problem exists everywhere in our daily life, especially in industrial applications. However, it is known as one of the hard combinatorial optimization problems. It has highly complex constraints and belongs to the class of NP-hard problems. Despite its NP-Hardness and its importance, during the last decades, many solution methods have also been proposed to solve it [1]. There have been many variations of scheduling problems for different applications [2,3]. In general, there are two types of scheduling problems discussed in the literature. Those are Flowshop Scheduling Problem (FSSP) and Jobshop Scheduling Problem (JSSP).
FSSP occurs when \( m \) machine process \( n \) jobs in the same sequence. A different series usually will differ in terms of processing time. An example of FSSP occurs in manufacturing facilities where jobs moved from machine-to-machine. It is widely known as one of the NP-complete optimization problems with \( n! \). Possible schedule. Recently, there have been many variations of FSSPs intensively studied in the literature for various applications. There have many variants of solution methods to solve FSSP; most of them are heuristic methods [4].

Nowadays, as computers rapidly increased, researchers have more attention on applying heuristic methods such as Genetic Algorithm (GA), Tabu Search (TS), and Simulated Annealing (SA) for solving various NP-hard/NP-complete optimization problems including Scheduling Problem. Most of the objective is to develop both accurate and efficient heuristic methods. Among them, GA is the most powerful and widely used [5]. Several researchers reported the successful implementation of GA to solve a wide variety of real-world applications, including engineering, economics, finance, manufacturing, agriculture, business, etcetera. In our previous works, we also have reported the success of GA for various combinatorial optimization problems [6], [7], [8], and [9]. Though GA has been a versatile approach for searching the global optimality, it also has a disheartening weakness in gaining too many time to reach optimal solutions. The success of GA depends on several factors, including an efficient design of the chromosome representation, method of crossover and mutation, selection methods, and the value of GA parameters. Thus, research on determining an efficient design of the GA approach for a specific problem becomes very crucial.

In this research work, we present two GA approaches called job-based GA (jb-GA) and machine-based GA (mb-GA) to solve FSSP. These approaches differ in the way to represent the chromosome. Our primary intention is to investigate the effectiveness and efficiency of GAs to solve FSSP. We carried out some numerical experiments using Benchmark test problems to see the performances of the algorithms [10]. The results are comparable with those given by another algorithm called the NBH algorithm [11].

We organize the remainder of this paper as follows: In Section 2, we give a brief overview of the FSSP. Section 3 describes the design strategies of the proposed GA approaches, including the design of chromosome representations, genetic operations, and selection strategy. We present the numerical experiment results and the comparison with other heuristic methods in Section 4. Finally, Section 5 describes the conclusion showing the remarkable effectiveness of the approaches.

2. Flow Shop Scheduling Problem

Flow shop scheduling is one of the problems that is follows: There is a set of \( m \) number machines and \( n \) number of jobs. Each job consists of \( m \) operation(s) that must be processed with a different device. The sequence for processing all jobs in the \( m \) machine(s) is the same. \( t_{ij} (i = 1, \ldots, n; j = 1, \ldots, m) \) denote the processing time of job \( i \) by using machine \( j \). For FSSP, we have the following assumption:
- Every job has to be processed on all machines in the order \( j = 1, 2, \ldots, m \).
- Each machine processes just one job at a time.
- Operations are not preemptive.
- The processing times include Set-up times for the operations.
- Operation sequences of the jobs are the same on every machine.

The usual objective function is to determine the schedule (the processing job sequence on the machine or machine sequence to process jobs) with minimum Makespan. There are also some different objective functions used, i.e., total tardiness, mean flowtime, and so on. We can find the mathematical formulation of FSSP in [12].

3. Design of The GAs

In this section, we describe the GA, which is one of the accurate and efficient heuristic methods to solve hard optimization problems. It was first introduced by Holland [13] and popularised by several researchers, including [14], [15], and [4].

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3.1. Initialization
When implementing GA for a specific application, the first step is to find a way to represent the possible problem solution. Here, we applied the permutation-based representation called job-based GA (jb-GA) and machine-based representation (mb-GA). For jb-GA, a list of $n$ jobs represents the chromosome. Each job appears once in the list. Thus, this representation will always yield a feasible schedule. The order represents the sequence of the job processed in each machine. We construct the schedule following the order in the list.

Similarly, for mb-GA, the chromosome represents the order of the machine to process each job. It will be a list of $m$ machines. We generate the value of each gene in the chromosome randomly. As an example, Figure 1 illustrates a chromosome for the problem ta001 (20 jobs and five machines).

![Figure 1. An example of chromosome representation](image)

3.2. Crossover and Mutation
The purpose of crossover operation is to make replication of the chromosome. It has a vital role in the success of GA. For permutation-based representation, we cannot use simple two-point crossover operations. There are many variants of crossover operations usually used for permutation representation, such as Partially Matched Crossover (PMX), Position-based crossover (PX), and Weight mapping crossover (WMX) [4]. Here, we adopt the PMX crossover as follows:

**Procedure: PMX**
- **Step 1**: Select a section of chromosome randomly
- **Step 2**: Exchanged each substring
- **Step 3**: Determine the mapping of genes in each substring
- **Step 4**: Update the chromosome with information on Step 3

Another essential feature of GA is the mutation operation. It is usually done by exchanging the data within a chromosome to prevent premature loss of information. In this paper, we adopt the inversion mutation that selects two positions within a chromosome at random and then inverts the sub-string between these two positions. We illustrate the inversion mutation operation as follows:

![Figure 2. Example of inversion mutation](image)

3.3. Evaluation and Selection
When using GA, we have to assess each chromosome on how well it fits with the problem requirements. Here, we use the makespan as the fitness value. The selection process is also known as an essential step in applying GA. The main objective is to guide in determining the chromosome for the next population. The selection process is done based on the fitness value. There have been many selection strategies reported for various GA applications [5]. We adopt the elitist selection strategy by selecting the best $pop_size$ chromosome to the next generation.

4. Experimental Design and Results
There are two purposes of this section: First is to explain the design of the numerical experiments, including the design of the test problems and parameter setting. The second is to evaluate the effectiveness and efficiency of GAs to solve FSSP.
4.1. Design of Test Problems

Several numerical experiments have been done to show the effectiveness and efficiency of the proposed approaches. In our experiments, we used a total of 18 different size Benchmark instances provided by literature [10]. Those problems have the number of jobs 20-100 and the number of machines 5-20. Those approaches were implemented in C++ and run on PC with processor Intel Core i5. We set the crossover and mutation probabilities as 0.4 and 0.2, respectively.

The population size is varied based on the size of the problems. Table 1 summarizes the overall results of the experiments.

| No. | Test problems | Number of jobs | Number of machines | max_gen | Optimal mb-GA | jb-GA | NEH* | time** |
|-----|---------------|----------------|--------------------|--------|----------------|-------|-------|--------|
| 1   | ta001         | 20             | 5                  | 600    | 1278           | 1376  | 1297  | 1286   | 25.18  |
| 2   | ta006         | 20             | 5                  | 600    | 1195           | 1373  | 1195  | 1228   | 25.18  |
| 3   | ta011         | 20             | 10                 | 600    | 1582           | 1716  | 1592  | 1680   | 33.88  |
| 4   | ta016         | 20             | 10                 | 600    | 1397           | 1515  | 1412  | 1453   | 33.88  |
| 5   | ta021         | 20             | 20                 | 600    | 2297           | 2346  | 2316  | 2410   | 36.83  |
| 6   | ta026         | 20             | 20                 | 600    | 2226           | 2302  | 2230  | 2349   | 36.83  |
| 7   | ta031         | 50             | 5                  | 600    | 2724           | 2899  | 2729  | 2733   | 34.36  |
| 8   | ta036         | 50             | 5                  | 600    | 2829           | 3068  | 2832  | 2850   | 34.36  |
| 9   | ta041         | 50             | 10                 | 700    | 3025           | 3459  | 3098  | 3146   | 40.91  |
| 10  | ta046         | 50             | 10                 | 700    | 3006           | 3470  | 3116  | 3178   | 40.91  |
| 11  | ta051         | 50             | 20                 | 700    | 3875           | 4192  | 3995  | 4038   | 49.56  |
| 12  | ta056         | 50             | 20                 | 700    | 3698           | 4080  | 3829  | 3918   | 49.56  |
| 13  | ta061         | 100            | 5                  | 700    | 5493           | 5646  | 5495  | 5567   | 47.82  |
| 14  | ta066         | 100            | 5                  | 700    | 5135           | 5553  | 5144  | 5139   | 48.8   |
| 15  | ta071         | 100            | 10                 | 800    | 5770           | 6366  | 5842  | 5848   | 63.23  |
| 16  | ta076         | 100            | 10                 | 800    | 5303           | 5965  | 5344  | 5373   | 63.23  |
| 17  | ta081         | 100            | 20                 | 2000   | 6286           | 6974  | 6456  | 6661   | 187    |
| 18  | ta086         | 100            | 20                 | 2000   | 6437           | 7074  | 6661  | 6761   | 187    |

*Nawaz, Encore and Ham (NEH) [11]

**Computational Time of jb-GA (in the second)

The above results that jb-GA outperforms mb-GA on the solution quality, all of the time. It can reach the optimal/near-optimal solutions to the problem. The comparison with the results of the NEH algorithm shows an impressive performance of the jb-GA on the quality of solutions (95 percent). The above results also indicate the reasonable computational time of jb-GA.

In these experiments, we define the error as the percentage of (Obtained Solution – Optimal Solution)/Optimal Solution. The next Figure 3 illustrates the comparison of the errors for each instance.
Figure 3. The comparative error of the methods

Like many other heuristic methods, the above results show that jb-GA still has a limitation on the number of instances solved (NIS) optimally. For some significant size problems, it even often reaches the near-optimal solutions. So there is always a place for improvement in the quality of solution or computational cost. Finally, in the next Figure 4, we illustrate the obtained schedule and the convergence of objective value in the generation for test problem ta061.

Figure 4. Makespan of the schedule for test problem ta061

5. Conclusions
We have presented two GA approaches called job-based GA (jb-GA) and machine-based GA (mb-GA) to solve FSSP. To demonstrate the effectiveness of the methods, we conducted several numerical experiments. We use the Benchmark scheduling test problems given in the literature. We compare the results with those provided by another heuristic algorithm (NEH algorithm). The results demonstrate that jb-GA has higher accuracy and achieves the optimal solution with reasonable computational time. This finding confirms the usefulness of GA to solve FSSP. The current limitation of this approach, like many other heuristic methods, is that jb-GA still sometimes gives the near-optimal solution.
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