Analytic Amplitudes for Hadronic Forward Scattering: COMPETE update

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We consider several classes of analytic parametrizations of hadronic scattering amplitudes, and compare their predictions to all available forward data ($pp$, $\bar{p}p$, $\pi p$, $Kp$, $\gamma p$, $\gamma\gamma$, $\Sigma p$). Although these parametrizations are very close for $\sqrt{s} \geq 9$ GeV, it turns out that they differ markedly at low energy, where a universal Pomeron term $\sim \ln^2 s$ enables one to extend the fit down to $\sqrt{s} = 4$ GeV. We present predictions on the total cross sections and on the ratio of the real part to the imaginary part of the elastic amplitude ($\rho$ parameter) for present and future $pp$ and $\bar{p}p$ colliders, and on total cross sections for $\gamma p \rightarrow \text{hadrons}$ at cosmic-ray energies and for $\gamma\gamma \rightarrow \text{hadrons}$ up to $\sqrt{s} = 1$ TeV.

Analytic parametrizations of forward ($t = 0$) hadron scattering amplitudes is a well-established domain in strong interactions.

However, in the past, the phenomenology of forward scattering had quite a high degree of arbitrariness: i) Excessive attention was paid to $pp$ and $\bar{p}p$ scattering; ii) Important physical constraints were often mixed with less general or even ad-hoc properties; iii) The cut-off in energy, defining the region of applicability of the high-energy models, differed from one author to the other; iv) The set of data considered by different authors was often different; v) No rigorous connection was made between the number of parameters and the number of data points; vi) No attention was paid to the necessity of the stability of parameter values; vii) The experiments were performed in the past in quite a chaotic way: huge gaps are sometimes present between low-energy and high-energy domains or inside the high-energy domain itself.

The COMPETE (COMputerized Models and Parameter Evaluation for Theory and Experiment) collaboration has cured as much as possible the above discussed arbitrariness.

The $\chi^2/dof$ criterium is not able, by itself, to cure the above difficulties: new indicators have to be defined. Once these indicators are defined\cite{1}, it is possible to estimate the overall performance of each model, and to establish a ranking: the highest the numerical value of the rank the better the model under consideration.

The final aim of the COMPETE project is to provide our community with a periodic cross assessments of data and models via computer-readable files on the Web\cite{2}.

We consider the following exemplar cases of the imaginary part of the scattering amplitudes:

$$Im F^{ab} = s \sigma_{ab} = P_1^{ab} + P_2^{ab} + R_+^{ab} \pm R_-^{ab}$$

where:
- the $\pm$ sign in formula\cite{2} corresponds to antipar-
article (resp. particle) - particle scattering amplitudes.
- \( R_{\pm} \) signify the effective secondary-Reggeon \(((f, a_2), (\rho, \omega))\) contributions to the even (odd)-under-crossing amplitude

\[
R_{\pm}(s) = Y_{\pm} \left( \frac{s}{s_1} \right)^{\alpha_{\pm}},
\]

where \( Y \) is a constant residue, \( \alpha \) - the reggeon intercept and \( s_1 \) - a scale factor fixed at 1 GeV\(^2\);
- \( P_1(s) \) is the contribution of the Pomeron Regge pole

\[
P_{1}^{ab}(s) = C_{1}^{ab} \left( \frac{s}{s_1} \right)^{\alpha_{P_1}},
\]

\( \alpha_{P_1} \) is the Pomeron intercept \( \alpha_{P_1} = 1 \), and \( C_{1}^{ab} \) are constant residues.
- \( P_{2}^{ab}(s) \) is the second component of the Pomeron corresponding to three different \( J \)-plane singularities:

a) a Regge simple - pole contribution

\[
P_{2}^{ab}(s) = C_{2}^{ab} \left( \frac{s}{s_1} \right)^{\alpha_{P_2}},
\]

\( \alpha_{P_2} = 1 + \epsilon, \) \( \epsilon > 0 \), and \( C_{2}^{ab} \) constant;
b) a Regge double-pole contribution

\[
P_{2}^{ab}(s) = s \left[ A^{ab} + B^{ab} \ln \left( \frac{s}{s_1} \right) \right],
\]

with \( A^{ab} \) and \( B^{ab} \) constant;
c) a Regge triple-pole contribution

\[
P_{2}^{ab}(s) = s \left[ A^{ab} + B^{ab} \ln^2 \left( \frac{s}{s_0} \right) \right],
\]

where \( A^{ab} \) and \( B^{ab} \) are constants and \( s_0 \) is an arbitrary scale factor.

We consider all the existing forward data for \( pp, \bar{p}p, pp, Kp, \gamma \gamma \) and \( \Sigma p \) scatterings. The number of data points is: 904, 742, 648, 569, 498, 453, 397, 329 when the cut-off in energy is 3, 4, 5, 6, 7, 8, 9, 10 GeV respectively. A large number of variants were studied. All definitions and numerical details can be found in Ref. 1.

The 2-component Pomeron classes of models are RRPE, RRPL and RRPL2, where by RR we denote the two effective secondary-reggeon contributions, by P - the contribution of the Pomeron Regge-pole located at \( J = 1 \), by E - the contribution of the Pomeron Regge-pole located at \( J = 1 + \epsilon \), by L - the contribution of the component of the Pomeron, located at \( J = 1 \) (double pole), and by L2 - the contribution of the component of the Pomeron located at \( J = 1 \) (triple pole). We also studied the 1-component Pomeron classes of models RRE, RRL and RRL2.

The highest rank are get by the RRPL2\(_u\) models (see Table 1), corresponding to the \( \ln^2 s \) beh-

| Model Code         | Rank |
|--------------------|------|
| RRPL2\(_u\)(19)    | 230  |
| RRP\(_nf\)L2\(_u\)(21) | 222  |
| RRL\(_nf\)(19)     | 222  |
| (RR\(_c\))\(_PL2\(_u\)(15) | 204  |
| (RR\(_a\))\(_P\)L2\(_u\)(19) | 194  |
| (RR\(_c\))\(_P\)L2\(_u\)(15) | 180  |
| (RR\(_c\))\(_P\)L2\(_u\)(14) | 155  |
| (RR\(_c\))\(_P\)L2\(_u\)(16) | 133  |
| (RR\(_c\))\(_P\)L2\(_u\)(20) | 170  |
| (RR\(_c\))\(_P\)L2\(_u\)(17) | 174  |
| RRPL(21)           | 173  |
| RR\(_c\)L\(_P\)(15) | 172  |
| RRL2\(_P\)(17)     | 170  |
| RR\(_c\)\(_P\)L2\(_P\)R\(_c\)(12) | 170  |
| RR\(_c\)\(_P\)L(17) | 162  |
| RRPE\(_a\)(19)     | 158  |
| RR\(_c\)\(_P\)L(14) | 155  |
| RRL2(18)           | 152  |
| RR\(_c\)PL(19)     | 142  |
| RRL(18)            | 133  |

Table 1
Ranking of the the 21 models having nonzero area of applicability. The number between parenthesis, in the Model Code column, denotes the number of free parameters.

The behaviour of total cross sections first proposed by Heisenberg 50 years ago [3]. The \( u \) index denotes
Table 2
Predictions for $\sigma_{\text{tot}}$ and $\rho$, for $\bar{p}p$ (at $\sqrt{s} = 1960$ GeV) and for $pp$ (all other energies). The central values and statistical errors correspond to the preferred model RRPL2$_u$.

| $\sqrt{s}$ (GeV) | $\sigma$ (mb) | $\rho$ |
|------------------|--------------|--------|
| 100              | 46.37 ± 0.06 | 0.1058 ± 0.0012 |
| 200              | 51.76 ± 0.12 | 0.1275 ± 0.0015 |
| 300              | 55.50 ± 0.17 | 0.1352 ± 0.0016 |
| 400              | 58.41 ± 0.21 | 0.1391 ± 0.0017 |
| 500              | 60.82 ± 0.25 | 0.1413 ± 0.0017 |
| 600              | 62.87 ± 0.28 | 0.1416 ± 0.0018 |
| 1960             | 78.27 ± 0.55 | 0.1450 ± 0.0018 |
| 10000            | 105.1 ± 1.1  | 0.1382 ± 0.0016 |
| 12000            | 108.5 ± 1.2  | 0.1371 ± 0.0015 |
| 14000            | 111.5 ± 1.2  | 0.1361 ± 0.0015 |

Table 3
Predictions for $\sigma_{\text{tot}}$ for $\gamma p \rightarrow \text{hadrons}$ for cosmic-ray photons. The central values and the statistical errors are as in Table 2.

| $p_{\text{lab}}$ (GeV) | $\sigma$ (mb) |
|------------------------|--------------|
| 0.5 · 10^6             | 0.243 ± 0.009 |
| 1.0 · 10^6             | 0.262 ± 0.010 |
| 0.5 · 10^7             | 0.311 ± 0.014 |
| 1.0 · 10^7             | 0.333 ± 0.016 |
| 1.0 · 10^8             | 0.418 ± 0.022 |
| 1.0 · 10^9             | 0.516 ± 0.029 |

the universality property: the coupling $B$ of the $\ln^2 s$ term is the same in all hadron-hadron scatterings and $s_0$ is the same in all reactions.

We note that the familiar RRE Donnachie-Landshoff model is rejected at the 98% C.L. when models which achieve a $\chi^2/dof$ less than 1 for $\sqrt{s} \geq 5$ GeV are considered.

The predictions of the best RRPL2$_u$ model, adjusted for $\sqrt{s} \geq 5$ GeV, are given in Tables 2-4.

The uncertainties on total cross sections, including the systematic errors due to contradictory data points from FNAL (the CDF and E710/E811 experiments, respectively), can reach 1.9% at RHIC, 3.1% at the Tevatron, and 4.8% at the LHC, whereas those on the $\rho$ parameter are respectively 5.4%, 5.2%, and 5.4%. The global picture emerging from fits to all data on forward observables supports the CDF data and disfavors the E710/E811 data at $\sqrt{s} = 1.8$ TeV.

Any significant deviation from the predictions based on model RRPL2$_u$ will lead to a re-evaluation of the hierarchy of models and presumably change the preferred parametrisation to another one. A deviation from the “allowed region” would be an indication that strong interactions demand a generalization of the analytic models discussed so far, e.g. by adding Odderon terms, or new Pomeron terms, as suggested by QCD.

REFERENCES
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2. See the preliminary version of a Web interface at the adress [http://sirius.ihep.su/~kuyanov/OK/eng/intro.html](http://sirius.ihep.su/~kuyanov/OK/eng/intro.html).
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