Ant Colony Optimization for Optimal Low-Pass State Variable Filter Sizing

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ABSTRACT

In analog filter design, discrete components values such as resistors (R) and capacitors (C) are selected from the series following constant values chosen. Exhaustive search on all possible combinations for an optimized design is not feasible. In this paper, we present an application of the Ant Colony Optimization technique (ACO) in order to selected optimal values of resistors and capacitors from different manufactured series to satisfy the filter design criteria. Three variants of the Ant Colony Optimization are applied, namely, the AS (Ant System), the MMAS (Min-Max AS) and the ACS (Ant Colony System), for the optimal sizing of the Low-Pass State Variable Filter. SPICE simulations are used to validate the obtained results/performances which are compared with already published works.

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1. INTRODUCTION

The optimal sizing of analog circuits is one of the most complicated activities, due to the number of variables involved, to the number of required objectives to be optimized and to the constraint functions restrictions. The aim is to automate this task in order to accelerate the circuits design and sizing. Recently, the used of the metaheuristics have proved a capacity to treat these problem efficiently, such as Tabu Search (TS) [1], Genetic Algorithms (GA) [2], Local search (LS) [3], Simulated Annealing (SA) [4], Ant Colony Optimization (ACO) [5-7] and Particle Swarm Optimization (PSO) [8].

Active analog filters are constituted of amplifying elements, resistors and capacitors; therefore, the filter design depends strongly of passive component values. However, the manufacturing constraints makes difficult an optimal selection of passive component values.

Indeed, the search on all possible combinations in preferred values for capacitors and resistors is an exhaustive process, because discrete components are produced according to a series of values constants such as the series: E12, E24, E48, E96 or E192.

Consequently, an intelligent search method requires short computation time with high accuracy, must be used. The ACO technique has been applied successfully to solve a variety of optimization problems, such as the prediction of the consumption of electricity [9], the traveling salesman problem (TSP) [10], the vehicle routing problem [11], the optimization of power flow [12], the learning problem [13] and the field of analog circuits design [5-7].

In this work, we propose to apply three variants of the ACO technique such as, the AS (Ant System), the MMAS (Max-Min Ant System) and the ACS (Ant System), for the optimal sizing of the Low-
Pass State Variable Filter considering two objectives functions, the cutoff frequency and the selectivity factor.

The remainder of the paper is structured as follows: The second section presents an overview of the ACO technique and highlights its three most important variants. The third section deals with the application example. The fourth section presents the simulation and the ACO variants comparison. The fifth section gives some comparisons with published works. The last section summarizes the main results of the work.

2. ANT COLONY OPTIMIZATION: ACO TECHNIQUE: AN OVERVIEW

ACO has been inspired by the foraging behavior of real ant colonies. Figure 1 shows an illustration of the ability of ants to find the shortest path between food and their nest [14], [15]. It is illustrated through the example of the appearance of an obstacle on their path. Every ant initially chooses path randomly to move and leaves a chemical substance, called pheromone in the path. The quantity of pheromone deposited will guide other ants to the food source. The indirect communication between the ants via the pheromone trail allows them to find shortest paths from their nest to the food source.

Figure 1. Self -adaptive behavior of a real ant colony, (a) Ants go in search of food; (b) Ants follow a path between nest and food source. They; choose, with equal probability, whether to shortest or longest path; (c) The majority of ants have chosen the shortest path.

2.1. Ant System

The first variant of the ACO is « Ant System » (AS) which is used to solve combinatorial optimization problems such as the traveling salesman problem (TSP), vehicle routing problem. For solving such problems, ants randomly select the vertex to be visited. When ant $k$ is in vertex $i$, the probability of going to vertex $j$ is given by (1):

$$P_{ij}^k = \begin{cases} 
\frac{(\tau_{ij})^{\alpha}(\eta_{ij})^{\beta}}{\sum_{l \neq i} (\tau_{il})^{\alpha}(\eta_{il})^{\beta}} & \text{if } i \in J^k_j \\
0 & \text{if } i \notin J^k_j 
\end{cases} \quad (1)$$

Where $J^k_i$ is the set of neighbors of vertex $i$ of the $k$th ant, $\tau_{ij}$ is the amount of pheromone trail on edge $(i, j)$, $\alpha$ and $\beta$ are weightings that control the pheromone trail and the visibility value, i.e. $\eta_{ij}$, which expression is given by (2):

$$\eta_{ij} = \frac{1}{d_{ij}} \quad (2)$$

The $d_{ij}$ is the distance between vertices $i$ and $j$.

Once all ants have completed a tour, the pheromone trails are updated. The update follows this rule:

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta \tau_{ij}^k \quad (3)$$

Where $\rho$ is the evaporation rate, $m$ is the number of ants, and $\Delta \tau_{ij}^k(t)$ is the quantity of pheromone laid on edge $(i, j)$ by ant $k$. 

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\[
\Delta t_{ij}^k = \begin{cases} 
\frac{Q}{L_k} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour} \\
0 & \text{otherwise}
\end{cases}
\]

\( Q \) is a constant and \( L_k \) is the length of the tour constructed by ant \( k \).

2.2. Max-min Ant System

The Max-Min Ant System is another variant of ACO, which was developed by Stützle & Hoos [15], [16] to improve convergence of AS.

Max-Min ant system has always been to achieve the optimal path searching by allowing only the best solution to increase the information and use a simple mechanism to limit the pheromone, which effectively avoid the premature stagnation. MMAS which based on the ant system does the following areas of improvement:

a. During the operation of the algorithm, only a single ant was allowed to increase the pheromone. The ant may be the one which found the best solution in the current iteration or the one which found the best solution from the beginning of the trial.

b. In order to avoid stagnation of the search, the range of the pheromone trails is limit to an interval \([\tau_{\text{min}}, \tau_{\text{max}}]\).

c. The pheromone is initialized to \( \tau_{\text{max}} \) in each edge.

2.3. Ant Colony System:

The ACS algorithm represents an improvement with respect to the AS. The ACS incorporates three main differences with respect to the AS algorithm:

a. ACS introduced a transition rule depending on a parameter \( q_0 \), which provides a direct way to balance between diversification and intensification. In the ACS algorithm, an ant positioned on node \( i \) chooses the city \( j \) to move to by applying the rule given by:

\[
j = \left\{ \begin{array}{ll}
\arg \max_{u \in J^i} \left[ \tau_{uw}(t) \cdot (\eta_{ij})^\beta \right] & \text{if } q \leq q_0 \\
1 & \text{if } q > q_0
\end{array} \right.
\]

Where \( q \) is a random number uniformly distributed in \([0, 1]\), \( q_0 \) is a parameter \((0 \leq q_0 \leq 1)\).

b. The global updating rule is applied only to edges which belong to the best ant tour. The pheromone level is updated as follows:

\[
\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \rho \cdot \Delta \tau_{ij}
\]

Where

\[
\tau_{ij} = \begin{cases} 
\frac{1}{L} & \text{if } (i, j) \in \text{best} - \text{global} - \text{tour} \\
0 & \text{otherwise}
\end{cases}
\]

c. While ants construct a solution a local pheromone updating rule is applied:

\[
\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \rho \cdot \tau_{\text{int}}
\]

3. APPLICATION TO THE OPTIMAL DESIGN OF LOW PASS STATE VARIABLE FILTER

The three proposed variants of ACO algorithm were used to optimize the analog circuit, namely State Variable Filter. Analog active Filters are important building blocks in signal processing circuits. They are widely used in the separation and demodulation of signals, frequency selection decoding, and estimation of a signal from noise [17]. Analog active filters are characterized by four basic properties: the filter type (low-pass, high-pass, bandpass, and others), the passband gain (generally all the filters have unity gain in the passband), the cutoff frequency (the point where the output level has fallen by 3 dB from the maximum level within the passband), and the quality factor \( Q \) (determines the sharpness of the amplitude response curve) [18]. A state variable
filter (SVF) realizes the state-space model directly. The instantaneous output voltage of one of the integrators corresponds to one of the state-space model’s state variables.

SVF can generate three simultaneous outputs: low-pass, high-pass, and bandpass. This unique characteristic comes from the filter’s implementation using only integrators and gain blocks. A second order SVF is illustrated in Figure 2. In this paper, the low pass output is supposed to be the desired output.

The response of a second order low-pass circuit is specified by the passband gain (H), the cutoff frequency (ω=2πf), and the selectivity factor (Q). These quantities are given in terms of passive component values as follows:

\[ H = \frac{R_2}{R_1 + R_2} \]  
\[ \omega = \frac{1}{R_3 \left( \frac{C_1 C_2 R_5 R_6}{C_1 R_5 R_6} \right) \frac{1}{R_7}} \]  
\[ Q = \frac{R_3 (R_1 + R_2)}{R_1 (R_2 + R_3) \sqrt{C_2 R_3 R_6}} \]  

The specification chosen here is \( \omega_0 = 10 \text{ k rad/s} \) (f= 10 000/ (2π) = 1591.55Hz) and \( Q_0 = 0.707 \) for reduced peak on low-pass response.

In order to generate \( \omega \) and \( Q \) approaching the specified values; the values of the resistors and capacitors to choose should be able to satisfy desired constraints. For this, we define the Total Error (TE) which expresses the offset values, of the cut-off frequency and the selectivity factor, compared to the desired values, by:

\[ Total \text{- Error} = 0.5 \Delta \omega + 0.5 \Delta Q \]  

Where:

\[ \Delta \omega = \frac{|\omega_{SVF} - \omega_0|}{\omega_0}, \quad \Delta Q = \frac{|Q - 0.707|}{0.707} \]  

The objective function considered is the Total Error. The decision variables are the resistors and capacitors forming the circuit. Each component must have a value of the standard series (E12, E24, E48, E96, and E192). The resistors have values in the range of \( 10^3 \) to \( 10^6 \Omega \). Similarly, each capacitor must have a value in the range of \( 10^{-9} \) to \( 10^{-6} \text{F} \). The aim is to obtain the exact values of design parameters (R1...6; C1, 2) which equate the Total-Error to a very close value to 0.
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3. RESULTS AND ACO VARIANTS COMPARISON

In this section we applied ACO algorithms to perform optimization of a Second order State Variable Low-Pass Filter. The studied algorithms parameters are given in Table 1 with a generation algorithm of 1000. The optimization techniques work on C codes and are able to link SPICE to measure performances.

Table 1. The ACO Algorithmus Parameters

| Parameter             | Value      |
|-----------------------|------------|
| Number of Ants        | 200        |
| Evaporation rate (\( \rho \)) | 0         |
| Quantity of deposit pheromone (Q) | 0.4   |
| Pheromone Factor (\( \alpha \)) | 1        |
| Heuristics Factor (\( \beta \)) | 1        |
| \( \tau_{\text{min}} \) | 0.5        |
| \( \tau_{\text{max}} \) | 1.5        |

The optimal linear values of resistors and capacitors forming the low-pass state variable Filter and the performance associated with these values using the three variants of the ACO technique: The MMAS, the AS and the ACS are shown in Table 2.

Table 2. Linear values of component and related filter performance for the AS, The MMAS and the ACS

| Component | \( \text{ACO 'MMAS'} \) | \( \text{ACO 'AS'} \) | \( \text{ACO 'ACS'} \) |
|-----------|-------------------------|---------------------|-----------------------|
| R1 (KΩ)   | 65.4                    | 80.2                | 63.2                  |
| R2 (KΩ)   | 19.4                    | 58.1                | 88.8                  |
| R3 (KΩ)   | 28.9                    | 24.0                | 29.9                  |
| R4 (KΩ)   | 84.3                    | 74.9                | 39.3                  |
| R5 (KΩ)   | 56.2                    | 70.4                | 12.6                  |
| R6 (KΩ)   | 15.7                    | 22.8                | 48.3                  |
| C1 (nF)   | 0.038                   | 0.024               | 0.054                 |
| C2 (nF)   | 0.087                   | 0.081               | 0.040                 |
| \( \Delta \omega \) | 0.000001 | 0.000008 | 0.000005 |
| \( \Delta Q \) | 0.000008 | 0.000014 | 0.000002 |
| TE        | 0.000004                | 0.000011            | 0.000004              |

The optimal values of resistors and capacitors forming the low-pass state variable Filter and the performance associated with these values for the different series using the three variants of the ACO technique: The MMAS, the AS and the ACS are shown in Table 3, Table 4 and Table 5 respectively.

Table 3. Values of component and related filter performance for the MMAS

| Component | E12 | E24 | E48 | E96 | E192 |
|-----------|-----|-----|-----|-----|------|
| R1(KΩ)    | 68.0 | 68.0 | 64.9 | 64.9 | 65.7 |
| R2(KΩ)    | 18.0 | 20.0 | 19.6 | 19.6 | 19.3 |
| R3(KΩ)    | 27.0 | 30.0 | 28.7 | 28.7 | 29.1 |
| R4(KΩ)    | 82.0 | 82.0 | 82.5 | 84.5 | 84.5 |
| R5(KΩ)    | 56.0 | 56.0 | 56.2 | 56.2 | 56.2 |
| R6(KΩ)    | 15.0 | 16.0 | 15.4 | 15.8 | 15.8 |
| C1(nF)    | 3.90 | 3.90 | 3.83 | 3.83 | 3.79 |
| C2(nF)    | 8.20 | 9.10 | 8.66 | 8.66 | 8.66 |
| \( \Delta \omega \) | 0.06328 | 0.07287 | 0.000069 | 0.00015 | 0.00182 |
| \( \Delta Q \) | 0.002898 | 0.00723 | 0.02377 | 0.00483 | 0.00338 |
| TE        | 0.04613 | 0.04005 | 0.01231 | 0.00250 | 0.00260 |
Table 4. Values of component and related filter performance for the AS

|     | E12   | E24   | E48   | E96   | E192  |
|-----|-------|-------|-------|-------|-------|
| R1(KΩ) | 82.0  | 82.0  | 78.7  | 80.6  | 80.6  |
| R2(KΩ) | 56.0  | 56.0  | 59.0  | 57.6  | 58.3  |
| R3(KΩ) | 22.0  | 24.0  | 23.7  | 24.3  | 24.0  |
| R4(KΩ) | 68.0  | 75.0  | 75.0  | 75.0  | 75.0  |
| R5(KΩ) | 68.0  | 68.0  | 71.5  | 69.8  | 70.6  |
| R6(KΩ) | 22.0  | 22.0  | 22.6  | 22.6  | 22.9  |
| C1(nF) | 2.20  | 2.40  | 2.37  | 2.43  | 2.4   |
| C2(nF) | 8.20  | 8.20  | 8.20  | 8.06  | 8.06  |
| Δω   | 0.07018 | 0.03026 | 0.02467 | 0.0052 | 0.00039 |
| ΔQ   | 0.06842 | 0.02974 | 0.03184 | 0.00611 | 0.00085 |
| TE   | 0.06930 | 0.03000 | 0.02826 | 0.003316 | 0.00062 |

Table 5. Values of component and related filter performance for the ACS

|     | E12   | E24   | E48   | E96   | E192  |
|-----|-------|-------|-------|-------|-------|
| R1(KΩ) | 68.0  | 62.0  | 61.9  | 63.4  | 63.4  |
| R2(KΩ) | 82.0  | 91.0  | 90.9  | 88.7  | 88.7  |
| R3(KΩ) | 27.0  | 30.0  | 30.1  | 30.1  | 29.8  |
| R4(KΩ) | 39.0  | 39.0  | 40.2  | 39.2  | 39.2  |
| R5(KΩ) | 12.0  | 13.0  | 12.7  | 12.7  | 12.6  |
| R6(KΩ) | 47.0  | 47.0  | 48.7  | 48.7  | 48.1  |
| C1(nF) | 5.60  | 5.60  | 5.36  | 5.36  | 5.42  |
| C2(nF) | 3.90  | 3.90  | 4.02  | 4.02  | 4.02  |
| Δω   | 0.08289 | 0.01298 | 0.00108 | 0.01145 | 0.00192 |
| ΔQ   | 0.07117 | 0.09045 | 0.01873 | 0.00821 | 0.00110 |
| TE   | 0.07702 | 0.05172 | 0.00990 | 0.00983 | 0.00151 |

From the results, we notice that the AS achieved a smaller design error.

Figure 3 to 5 show the PSPICE simulation of the filter gain for the optimal values. The practical cuts off frequency using the three variants of the ACO technique: The MMAS, the AS and the ACS are equal to 1.610 KHz, 1.601 KHz and 1.595 KHz respectively.

Figure 3. Frequency responses of low-pass State variable filter using the MMAS

Figure 4. Frequency responses of low-pass State variable filter using the AS

Figure 5. Frequency responses of low-pass State variable filter using the ACS
Table 6. Comparisons between the theoretical and practices for the error on the cut-off frequency

|       | \( \Delta \omega \) theoretical | \( \Delta \omega \) Practical |
|-------|----------------------------------|-----------------------------|
| MMAS  | 0.00015                          | 0.01194                     |
| AS    | 0.00039                          | 0.00628                     |
| ACS   | 0.00192                          | 0.00251                     |

Table 6 shows the comparison between the theoretical values and those practices for the error on the cut-off frequency for the optimal results. From Table 6, we notice that there is a slight difference between the simulation results and the theoretical results which is mainly due to imperfections of the op-amp which are considered perfect in the theoretical calculations.

4. COMPARISON AND DISCUSSIONS

4.1. Accuracy and Time Computing

The optimal component selection of the Low-Pass State Variable Filter has been elaborated by other metaheuristics. Table 7 presents the ACO results for series E96 and E192, compared to those of the GA, ABC and PSO techniques. One can notice that the ACO techniques provide acceptable results than those achieved by the GA, PSO, and ABC algorithms. The AS technique has a deviation of about 0.06% from what is expected, which presents a highly accurate in the field of analog circuit design.

Table 8 shows the run time of the three variants of the ant colony optimization compared to those of other metaheuristics. The comparison shows that the ABC algorithm achieved the shortest execution time, followed by the ACO techniques, in particular the ACS, which presents an execution time less than the half of those of the GA algorithm and PSO algorithm.

4.2. Convergence Rate and Optimum Rapidity

In order to check the convergence rate of the proposed algorithms, a robustness test was performed. i.e. the three algorithms are applied a hundred times for optimizing the Total Error (TE) objective. In Figure 6 we present obtained results for the algorithms: AS, MMAS and ACS.

The good convergence ratio can be easily noticed, despite the probabilistic aspect of the ACO algorithms. We can, also, notice that the robustness of the MMAS algorithm is better than the robustness of the AS and ACS algorithms; in fact the convergence rates to the same optimal value are 26%, 49% and 19% respectively for AS, MMAS and ACS.

The Total Error (TE) values versus iteration number are plotted in Figure 7 and Figure 8 for the AS, the MMAS and the ACS algorithms for linear values and E192 series, respectively. From these figures, it can be seen that the number of iterations required to achieve the quality requirements are slightly different for each algorithm. In fact for the AS, the optimal value of the TE is reached on the 42nd iteration, for the MMAS it is reached on the 253rd iteration and for the ACS it is reached on the 14th iteration.
We notice that the ACO methods are faster in term of the number of iterations to achieve the optimal values of the TE, in particular the ACS, compared to the GA algorithm and ABC algorithm, with 4441 iterations and 175 iterations, to reach the optimal design respectively [18].

![Box plot for the convergence rate for TE](image1)

**Figure 6.** Box plot for the convergence rate for TE

![TE values versus iteration number for the AS, ACS, and MMAS algorithms (linear values)](image2)

**Figure 7.** TE values versus iteration number for the AS, ACS, and MMAS algorithms (linear values)

![TE values versus iteration number for the AS, ACS, and MMAS algorithms (E192 series)](image3)

**Figure 8.** TE values versus iteration number for the AS, ACS, and MMAS algorithms (E192 series)

5. **CONCLUSION**

We presented in this paper an application of the three important variants of the Ant Colony Optimization technique for optimal sizing of a state variable filter. SPICE simulation confirms the validity of the proposed methods. The AS performs the smaller Total Error, the AS and ACS give the rapid convergence to the optimal values and the MMAS provide a better convergence rate. The comparison, with already published works, showed that the ACO techniques present alternative and competitive methods for the analog filter design automation and optimization.

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