How Magnetic Field Enters Heat Current: Application to Fluctuation Nernst Effect

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A problem of the definition of the heat transported in thermomagnetic phenomena has been well realized in the late sixties, but not solved up to date. Ignoring this problem, numerous recent theories grossly overestimate the thermomagnetic coefficients in strongly interacting systems. Here we develop a gauge-invariant microscopic approach, which shows that the heat transfer should include the energy of the interaction between electrons and a magnetic field. We also demonstrate that the surface currents induced by the magnetic field transfer the charge in the Nernst effect, but do not transfer the heat to the Ettingshausen effect. Only with these two modifications of the theory, the physically measurable thermomagnetic coefficients satisfy the Onsager relation. We critically revised the Gaussian fluctuation model above the superconducting transition and show that the gauge invariance uniquely relates thermomagnetic phenomena in the Fermi liquid with the particle-hole asymmetry.

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I. INTRODUCTION

It is well known that in the electric field, \( \mathbf{E} = -\nabla \phi \), and magnetic field \( \mathbf{H} \), the energy of a system of charged particles with coordinates \( \mathbf{r}_\alpha \) and momentums \( \mathbf{p}_\alpha \) is

\[
E = \sum_\alpha \frac{p_\alpha^2}{2m} + \sum_\alpha e\phi(r_\alpha) + \sum_\alpha \frac{e}{2mc}(r_\alpha \times p_\alpha) \cdot H. \quad (1)
\]

Considering the heat transfer, it is important to realize which terms in the above equation should be associated with the thermal energy. The uniform electric field accelerates all particles in the same way, while the effect of the magnetic field depends on the particle state. Therefore, it is reasonable to assume that the electric term (the second term) contributes to the potential energy, while the magnetic term (the third term) provides contribution to the thermal energy. Of course, the above consideration cannot be considered as a proof. Moreover, there is no regular methods for identification of thermal energy. Correctness of our assumption will be verifed through the Onsager relation between the Ettingshausen and Nernst coefficients, which describe the heat and electric currents initiated by the electric field and temperature gradient, correspondingly. We will show that the magnetic term overlooked in previous works plays a crucial role and restores the gauge invariance. While in accordance with van Leeuwen’s theorem\(^1\), the magnetic contribution to the heat transfer does not allow a classical interpretation, it is critically important for consistent quantum description of thermomagnetic transport.

The definition of the heat current and the Onsager relation in the magnetic field are long standing problems, which were attracted significant attention in the late sixties in the context of thermomagnetic vortex transport in the type-II superconductors\(^2,3\). Maki\(^4\) has noted that the phenomenological\(^5\) and microscopic\(^6,7\) descriptions of thermomagnetic phenomena violate the third law of thermodynamic and the Onsager relation. All attempts to resolve this enigma were based on thermodynamic treatment. Various corrections suggested to the heat current are expressed in terms of the equilibrium magnetization currents\(^8\). As we will show, the magnetization currents do not transfer the heat and the thermodynamic approach cannot solve this problem.

Recently, much efforts have been dedicated to understanding of thermomagnetic effects in high-\(T_c\) superconductor\(^9\). Variety of phenomenological concepts were suggested to explain anomalously large Nernst signal above \(T_c\). In many papers\(^10,11,12,13\) the Nernst effect is attributed to the motion of vortices created by phase fluctuations. Microscopic models\(^14,15\) are based on the Gaussian-Fluctuation Theory (GFT), which includes both module and phase fluctuations, but is applicable only near \(T_c\)\(^16,17,18,19\) (for a review see\(^20\)). However, the theories related to vortices are based on the results of Refs\(^2\) and\(^3\), which inconsistency with basic principles was noted in\(^20\). GFT and other microscopic models also ignore corrections to the heat current.

While the large Nernst coefficient was obtained in all above models, there is still not much understanding of how superconducting fluctuations could lead to this effect. In the model of noninteracting electrons, which describes well ordinary metals, the Nernst coefficient is small, because it is proportional to the square of the particle-hole asymmetry (PHA)\(^21\). In this case, the
Nernst coefficient combines PHAs of the thermoelectric, \( \eta \), and Hall, \( \sigma_{xy} \), coefficients: \( N_a \simeq \eta \cdot \sigma_{xy} \). The inter-electron interaction cannot change PHA of \( \eta \) and \( \sigma_{xy} \). According to current points of view, the interaction in the Cooper channel leads to the Nernst coefficient in the zeroth order in PHA. This effect is \((\varepsilon_F/T)^2\) times larger than the corresponding correction to \( \eta \cdot \sigma_{xy} \) \((\varepsilon_F \text{ is the Fermi energy})\). However, experiments with ordinary superconductors, such as Nb, Al, Sn, do not support this prediction. This contradiction is a consequence of a number of misconceptions at microscopic level.

The microscopic formalism based on the Kubo method allows one to calculate the Ettingshausen coefficient \( \Upsilon \), which describes the heat flux induced by crossed electric and magnetic fields, \( j^h = -\Upsilon(\mathbf{E} \times \mathbf{H}) \). The temperature gradient cannot be directly introduced in the Lagrange formalism of the Kubo approach. Therefore, the Nernst coefficient, which relates the electric current with the temperature gradient, \( j^e = N [\nabla T \times \mathbf{H}] \), is found from the Onsager relation,

\[
N = \Upsilon/T.
\]

The coefficient \( \Upsilon \) is calculated for the infinite sample, while the Onsager relation is only valid for a finite sample\(^22\), where surface currents should be taken into account.

If magnetization depends on temperature, the temperature gradient generates surface magnetization currents, which provide important contribution to the charge transfer in the Nernst effect\(^23\). Several recent works\(^16,18,19,24\) state that surface magnetization currents also play an important role in the heat transfer and the Ettingshausen coefficient calculated for the infinite sample, \( \delta \Upsilon_{inf} \), should be corrected due to the surface heat currents: \( \delta \Upsilon = \delta \Upsilon_{inf} - \mu_0 \). According to\(^16,18,19,24\), GFT leads to the bulk coefficient, which is of the same order as the correction due to surface currents: \( \delta \Upsilon_{inf} = 3/2 \mu_0 \), where \( \mu_0 \) is the fluctuation magnetic susceptibility\(^20\),

\[
\mu = \frac{e^2 T}{6\pi c^2} \times \left\{ \frac{2\alpha/\eta}{\sqrt{\alpha/\eta}} \right\} \text{ for 2D},
\]

\[
\eta = (T - T_c)/T_c \text{ and } \alpha = (T - T_c)[\xi(T)]^2, \xi(T) \text{ is the coherence length}.
\]

In the current paper we reconsider basics of the microscopic description of the thermoelectric effects. This work will address the following questions: (1) Is the heat current operator modified in the presence of a magnetic field? (2) Do surface magnetization currents transfer the heat? (3) How do Lorenz coefficients satisfy the Onsager relation? (4) Finally, can the interelectron interaction change the PHA of the Nernst and Ettingshausen coefficients? While our results are quite general, we specify them for GFT, which will be critically revised. In particular, we show that \( \delta \Upsilon \sim T \delta N \sim (T_c/\varepsilon_F)^2\mu \).

II. ETTINGSHAUSEN COEFFICIENT: KUBO METHOD

In the gauge \( \mathbf{H} = i[\mathbf{k}_H \times \mathbf{A}_H] \) and \( \mathbf{E} = -i\mathbf{k}_E\phi \), contributions of static electric and magnetic fields to the energy of a charged particle are easy separated,

\[
\tilde{E} = \left( \mathbf{p} + e\mathbf{A}_H/c \right)^2 + e\phi = \xi_p + \frac{e}{c} (\mathbf{v} \cdot \mathbf{A}_H) + e\phi + \mu_0(4)
\]

where \( \xi_p = p^2/2m - \mu_0 \), \( \mu_0 \) is the chemical potential. Let us first show the importance of the second (magnetic) term for noninteracting electrons\(^22\). Thermal energy is counted from the electro-chemical potential, so only the first and the second terms contribute to the heat current. The corresponding diagrams for the Ettingshausen coefficient are shown in Fig. 1. The diagram (a) has a form of the Hall diagram\(^22\), where the electric current operator, \( e\mathbf{v} \), is replaced by the heat current, \( \xi_p \mathbf{v} \). The Hall effect is proportional to PHA, i.e. the corresponding integrant is an odd function of \( \xi_p \) and a nonzero Hall coefficient is obtained after expansion of all parameters in \( \xi_p/\varepsilon_F \) near the Fermi surface. With the heat current operator \( \xi_p \mathbf{v} \), the integrant becomes an even function and gives a nonzero Ettingshausen coefficient without expansion. However, this large contribution is canceled by the diagram (b). The well-known expression for \( \Upsilon \) in a system of noninteracting electrons is obtained in the second order in PHA, when both integrants (a) and (b) are expanded in \( (\xi_p/\varepsilon_F)^2 \) (for details see an Appendix in\(^22\)).

Now we consider electrons interacting in the Cooper channel. The energy current operator in this system\(^22\) is easily expanded to include external fields,

\[
\mathbf{J} = \sum_p v \xi_p a_p^+ a_p + \sum_p v e\mathbf{A}_H a_p^+ a_p + \sum_p v e\phi a_p^+ a_p + \sum_{p,p',p''} (v + v') a_p^+ a_{p'} - a_{p''} a_p a_{p'} a_{p''} + \sum_{p,p',R_i} (v + v') U_{imp}(R_i) \exp[i(p + p')R_i] a_p^+ a_{p'}^{-}, \text{ (5)}
\]

Here \( a_p^+ \) and \( a_p \) are the electron creation and annihilation operators, \( \lambda \) is the interaction constant in the Cooper channel, and \( U_{imp} \) is the impurity potential. The first and the forth terms in the Eq.\(^22\) describes the energy flux of noninteracting electrons without fields. The second and third terms are due to electron interaction with magnetic and electric fields. Two last terms are due to the electron-electron and electron-impurity interactions. These two terms generate diagram blocks (Aslamazov-Larkin blocks) proportional to PHA, but contributions of all blocks with PHA cancel each other\(^22\).

The thermal energy is defined as the electron energy counted from the electro-chemical potential \( e\phi + \mu \) and, therefore, the third and the forth terms in Eq.\(^22\) do not contribute to the heat current. Thus, calculating the heat current we should take into account only two heat current vertices \( \gamma_1^e \) and \( \gamma_2^e \) corresponding to the first (kinetic) and second (magnetic) terms in Eq.\(^22\).
Two leading terms in the heat current operator generate two diagrams, which describe $\Upsilon$ in the Aslamazov-Larkin (AL) approximation (see Fig. 2). The wavy lines correspond to the fluctuation propagator,

$$L_{R,A}^{R,A}(q,\omega) = (\lambda^{-1} - p_{R,A}^{R,A}(q,\omega))^{-1} ,$$

$$P_{R,A}^{R,A}(q,\omega) = -\nu \left( \frac{2}{\pi T} \frac{C_{\omega D}}{\pi T} - aq^2 \frac{i\pi \omega}{8T} + \gamma_{\omega} \right) ,$$

where $P(q,\omega)$ is the polarization operator, $\nu$ is the electron density of states, $\omega_{D}$ is the Debye frequency, and $C_{\gamma}$ is the Euler constant. The last term in Eq. 7 is proportional to PHA.

The AL blocks $B_{c,h,H}$ presented in Fig. 2 are built from electron Green functions and vertices $\gamma_{c,h,H}^{R}$ (see Tab. 1). The left blocks $B_{1}^{h}$ and $B_{2}^{h}$ in Figs. 2.a and 2.b are blocks with heat current vertices $\gamma_{1}^{h}$ (kinetic) and $\gamma_{2}^{h}$ (magnetic). The right block $B_{c}^{\nu}$ in both diagrams includes the electric current vertex $\gamma_{c}^{\nu} = e\nu \cdot e\nu$, and its contribution is $\delta L = e\nu \cdot e\nu = \frac{2}{\pi T} \frac{C_{\omega D}^{L_{c}}}{}$. Block $B_{H}^{R}$ includes the magnetic vertex $\gamma_{H}^{R} = (e/c) \nu \cdot A_{H}$. Results of calculation are summarized in Table I. AL blocks are obtained by inserting vertices $\gamma$ into the polarization operator and can be expressed through $\nabla_{q} P_{R}(q,0)$. Blocks $B_{c}^{\nu}$, $B_{H}^{R}$, and $B_{\nu}^{R}$ are well known. The block $B_{1}^{h}$ describing the heat current in the absence of magnetic field has been calculated in Eq. (8) (see also 18,19). Here we introduce $B_{2}^{h}$, which is based on the electron vertex $\gamma_{2}^{h}$ and describes the magnetic correction to the heat current.

The first AL diagram (Fig. 2.a) was investigated in Ref. 22 and its contribution is $\delta \Upsilon_{inf}^{(1)} = 3/2 \nu \omega$. The same result has been obtained in the time dependent Ginzburg-Landau formalism (TDGL).

Contribution of the second AL diagram (Fig. 2.b) is

$$\Upsilon_{2}^{inf} = 3 \int \frac{dq}{(2\pi)^{n}} \frac{d\omega}{2\Omega} \frac{B_{2}^{h}B_{c}^{\nu}}{2(\nu A_{c} + L_{A}^{L_{c}})}.$$

where $L_{c} = \text{coth}(\omega/2T)(L_{h} - L_{A})$, $L_{A}$ is used for $L_{q \pm k/2, \omega \pm 2\Omega/(2n)$, and $n$ is the system dimensionality with respect to the coherence length $\xi(T)$. Expanding the integral to the linear order in $\Omega$ and $k$ and calculating the integrals over $\omega$ and $q$, we find that the contribution of the second diagram, $\Upsilon_{2}^{inf}$ cancels completely the contribution of the first one.

Thus, without PHA the Ettingshausen effect is absent, $\delta \Upsilon = 0$. To get nonzero result, we should expand the fluctuation propagator (Eq. 6) up to the second order in PHA. Expanding the polarization operator (Eq. 7) to the second order in $\gamma$ we get

$$\delta \Upsilon = \frac{5e^{2}}{4\pi c} \left( \frac{8T}{\pi} \right) \left\{ \begin{array}{ll} 2a/\eta & \text{for } 2D; \\ \sqrt{a/\eta} & \text{for } 3D; \end{array} \right.$$
bracket should be replaced by the derivatives \( \partial P/\partial T \) in the Nernst coefficient.

To get the vector product \( \mathbf{H} \times \nabla T \), the \( \mathbf{H} \)-bracket should include the same polarization operator as the \( \nabla T \)-bracket. Thus, in the first order in \( \mathbf{H} \times \nabla T \) the nonequilibrium fluctuation propagator \( \delta L^C(q, \omega) \) is described by the diagrams shown in Fig. 3. Finally, calculating \( \delta L^C(q, \omega) \) and the corresponding Nernst current in the interior of the sample (Eq. \( \text{[11]} \)), we find

\[
\delta N_{\text{inf}} = \delta N + \frac{e^2}{3\pi c} \left( \frac{\alpha}{\eta^2} - \frac{\alpha}{\eta} - \frac{\partial \alpha}{\partial T} \right),
\tag{12}
\]

where \( \delta N \) is equal to the term, which was calculated in the previous section from the Onsager relation (see Eq. \( \text{[9]} \)). Thus, in the infinite sample or in the interior of the finite sample, the Nernst coefficient consists of two terms. The second term in Eq. \( \text{[12]} \) has the zeroth order in PHA and violates the Onsager relation. As it will be shown in the next section, in the finite sample this term is canceled by the contribution of the magnetization currents.

IV. ONSAGER RELATION IN MAGNETIC FIELD

The above results, Eqs. \( \text{[3]} \) and \( \text{[12]} \), have been calculated for the infinite sample. Referring to\(^{23,23,23,23}\), recent works\(^{18,19,24}\) state that for a finite sample both coefficients should be corrected due to charge and heat transfer by surface magnetization currents. Here we show that the magnetization currents contribute only to the charge transfer, and the results of \(^{23,23,23,23}\) have been misinterpreted.

The electric magnetization current \( \mathbf{j}_{\text{mag}} \) in the potential relief \( \phi(r) \) transfers the energy flux \( \mathbf{j}_{\text{mag}} = \phi \delta \mathbf{j}_{\text{mag}} \) (Eq. 37 in\(^{23} \)). Using \( \mathbf{j}_{\text{mag}} = c \mu k^2 \mathbf{A} \), we get

\[
\mathbf{j}_{\text{mag}} = c \mu [\mathbf{H} \times \mathbf{E}] .
\tag{13}
\]

This term is erroneously attributed to the heat flux\(^{18,19,20,24} \). As we discussed, the electric potential \( \phi \) and the corresponding vertex \( \gamma \phi = e v \phi \) do not contribute to the heat current, because the thermal energy should be counted from the electro-chemical potential.

In the interior of the sample the electric current consists of the transport and magnetization components, \( \mathbf{j}_{\text{intr}} = \mathbf{j}_e + \mathbf{j}_{\text{mag}} \). The magnetization component is\(^{23,23,23,23} \)

\[
\mathbf{j}_{\text{mag}} = c \frac{\partial \mu}{\partial T} (\nabla T \times \mathbf{H}) .
\tag{14}
\]

The magnetization currents are divergence-free. The total magnetization current through the sample cross-section must be zero, i.e. the bulk magnetization currents are canceled by the surface currents. Therefore, the Nernst coefficient measured in the finite sample is determined by the transport currents: \( N = \mathbf{j}_e / (\nabla T \times \mathbf{H}) \). The Nernst coefficient in the infinite sample (Eq. \( \text{[12]} \)) is associated with the bulk current in the finite sample, \( N_{\text{inf}} = \mathbf{j}_e / (\nabla T \times \mathbf{H}) \). Using Eq. \( \text{[14]} \) we get

\[
\delta N = \frac{\mathbf{j}_{\text{intr}} - \nabla \mathbf{j}_{\text{mag}}}{\nabla T \times \mathbf{H}} = \delta N_{\text{inf}} - c \frac{\partial \mu}{\partial T} .
\tag{15}
\]

Taking into account Eq. \( \text{[5]} \) we see that the second term in the last equation, \( c \partial \mu / \partial T \), cancels completely the second term in \( \delta N_{\text{inf}} \) (Eq. \( \text{[12]} \)). The rest is equal to \( \delta N \), which satisfies the Onsager relation \( \delta N = \delta T / T \).

Note that, if in contradiction to our results, the surface magnetization currents provide the heat transfer, this effect could be found in transport measurements. In the Gorbino disc geometry with the magnetic field perpendicular to the disc and the circular inductive electric field in the plane (Fig. 4), the heat current in the radial direction does not contain the surface components, which were predicted for the standard parallelepiped geometry in\(^{18,19,24} \). According to our results, both experiments will give the same results. The surface electric currents generated by \( \nabla T \) are very significant (see Eq. \( \text{[14]} \)). However, they cannot be experimentally separated from the interior currents, because contrary to the circular electric field, the circular temperature gradient does not exist. This difference between \( \mathbf{E} \) and \( \nabla T \) is reflected in the asymmetry of the Nernst and Ettingshausen coefficients calculated for the infinite sample.

V. PARTICLE-HOLE ASYMMETRY IN THERMOMAGNETIC EFFECTS

Now we show that in the general case, the interelectron interaction cannot change PHA requirements for \( N \) and \( \Upsilon \), i.e. the thermomagnetic coefficients are always proportional to the square of PHA.

Assuming that electron scattering from impurities is the main mechanism of the momentum relaxation, it is easy to see\(^{25,27,29} \) that the magnetic field and temperature gradient enter into the transport equation formalism through the distribution functions of noninteracting electrons and the Poisson brackets. In fact, the terms proportional to \( \nabla T \times \mathbf{H} \) can appear in three different ways\(^{25} \): (a) through the Nernst nonequilibrium distribution function of noninteracting electrons, \( (e^2 / cm) \mathbf{v} \cdot [\nabla T \times \mathbf{H}] (\partial S / \partial T) \), (b) through the \( \mathbf{H} \)-Poisson bracket that involves the nonequilibrium distribution function under the temperature gradient, \( -e \mathbf{v} \cdot (\mathbf{v} \times \mathbf{E}) / (\partial S / \partial T) \), and, finally, (c) due to double, \( \nabla T \) and \( \mathbf{H} \), Poisson brackets. It is evident that a and \( b \)-type terms have already includes the Hall PHA, which is proportional to \( (\partial \mu / \partial p) = 1/m \). The \( c \)-type terms in the form of the double Poisson brackets describe the AL process, which has been investigated above. As we have seen, the AL diagram gives the contribution in the zeroth order in PHA, however, this contribution is canceled by the contribution of the surface magnetization currents. Thus, we conclude that the interelectron interaction can provide many-body thermomagnetic effects only in the second order in PHA.
VI. CONCLUSIONS

We have shown that the magnetic term in the Hamiltonian of charged particles (Eq. 1) should be associated with the thermal energy. The corresponding term in the heat current operator (Eq. 3) restores the gauge invariance and gives important contribution to the Ettingshausen coefficient. We also found that the surface magnetization currents do not contribute to the heat current, but provide substantial contribution to the charge transfer in the Nernst effect (Eq. 15). Our gauge-invariant scheme gives the thermomagnetic coefficients that satisfy to the Onsager relation (see Eqs. 15 and 12). In the general case of the Fermi liquid with particle-hole excitations, we conclude that the measured thermomagnetic coefficients are always proportional to the square of PHA. Any interaction by itself, i.e. without changing the electron band structure or character of elementary excitations, cannot provide large thermomagnetic effects.

The developed approach has been applied to effects of superconducting fluctuations. We show that the gauge invariant form of the heat current operator of fluctuating pairs is $\gamma^h = \gamma^h_1 + \gamma^h_2 = \xi v + (q/c)(v \cdot A)_n v$. (A1)

Two vortices $\gamma^h_1$ and $\gamma^h_2$ create two diagrams shown in Fig. 1. Solid lines in diagrams represent the electron Green functions,

$$G^R_p = \{G_p^A\}^* = (\epsilon - \xi_p + i/2\tau)^{-1}. \quad \text{(A2)}$$

To get the Ettingshausen coefficient proportional to $A(k \cdot E)$, one should expand the Green function $G(p + k)$ in powers of $(k \cdot v)$. Then the contribution of the first diagram is given by

$$\gamma \sim \tau A_{\parallel}^2 (A3)$$

The production of the second diagram is

$$\gamma \sim \tau A_{\perp}^2 (A4)$$

Using GFT as an example, we have shown that the gauge-invariant form of the heat current is critical for description of the Ettingshausen effect and that the surface currents are important for the Nernst effect. All other models of thermomagnetic transport including vortex models should be reconsidered in accordance with the formalism developed above.

We would like to acknowledge useful discussions with I. Aleiner, A. Larkin, D. Livanov, A. Varlamov, and I. Ussishkin.

APPENDIX A: METHOD KUBO FOR NONINTERACTING ELECTRONS

Appendix A

In this appendix we present detailed calculations of the Ettingshausen coefficient for noninteracting electrons using the Kubo method.

According to Eq. 4, the heat current vertex for noninteracting electrons in the magnetic field is given by

$$\gamma^h = \gamma^h_1 + \gamma^h_2 = \xi v + (q/c)(v \cdot A)_n v. \quad \text{(A1)}$$

Two vortices $\gamma^h_1$ and $\gamma^h_2$ create two diagrams shown in Fig. 1. Solid lines in diagrams represent the electron Green functions,

$$G^R_p = \{G_p^A\}^* = (\epsilon - \xi_p + i/2\tau)^{-1}. \quad \text{(A2)}$$

To get the Ettingshausen coefficient proportional to $A(k \cdot E)$, one should expand the Green function $G(p + k)$ in powers of $(k \cdot v)$. Then the contribution of the first diagram is given by

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APPENDIX A: METHOD KUBO FOR NONINTERACTING ELECTRONS
Without taking into account PHA the total contribution of two diagrams goes to zero after integration over $\xi_p$, because
\[
\int d\xi_p \left( \xi_p (G_p^A)^2 + \text{Im} G_p^A \right) = 0. \quad (A9)
\]
Nonzero contribution arises from terms proportional to $\xi^2$, thus we should expand all electron parameters near the Fermi surface. For example for 3D conductor,
\[
v^A = v^q_0 \tau_0 \left[ 1 + \frac{5 \xi_p}{2 \epsilon_F} + \frac{15}{8} \left( \frac{\xi_p}{\epsilon_F} \right)^2 + \ldots \right], \quad (A10)
\]
\[
\tau^2 = \tau_0^2 \left[ 1 - \frac{\epsilon}{\epsilon_F} - \left( \frac{\epsilon}{\epsilon_F} \right)^2 + \ldots \right]. \quad (A11)
\]
Taking into account terms proportional to the square of PHA, e.g. $\xi^2/\epsilon_F^2$ or $\xi^4/\epsilon_F^4$, we get
\[
\int d\xi_p \, v^A = v^q_0 \tau_0 \frac{5 \xi^2}{4 \epsilon_F} = -\pi v^q_0 \tau_0^2 \nu_0 \frac{\xi^2}{4 \epsilon_F}. \quad (A12)
\]
Substituting this result into Eq. (A.8), and performing integration over $\epsilon$, we get the well-known result for the Ettingshausen coefficient of noninteracting electrons,
\[
\gamma_{3D} = -\frac{\pi^2 T^2}{6 \epsilon_F} (\Omega \nu_0) \sigma_{xx}/H, \quad (A13)
\]
where $\Omega = eH/mc$ is the cyclotron frequency and $\sigma_{xx}$ is the Drude conductivity. For 2D-conductors, the corresponding relation between $\gamma_{2D}$ and two-dimensional conductivity has an additional numeric factor of 2.

Thus, calculating the Ettingshausen coefficient of non-interacting electrons, we demonstrated that the magnetic field should be taken into account in the heat current vertex. The diagram with this magnetic vertex in the heat current cancels the basic diagrams in the zeroth order in PHA. The nonzero Ettingshausen coefficient arises only in the second order in PHA. As it has been shown in the main text, the above conclusions are also relevant to any many-body corrections to thermomagnetic coefficients.

**APPENDIX B: AL BLOCKS**

For an arbitrary electron momentum relaxation time $\tau$, the AL blocks $B^{c,h,H}_i$ built from electron Green functions $G^{H(A)}$ with electron vertices $\gamma$ ($\gamma^c$, $\gamma^H$, $\gamma^h$, and $\gamma^h$) are given by$^{20}$
\[
B^{c,h,H}_i = \text{Im} \int \frac{dp}{(2\pi)^n} \frac{de}{2\pi} \gamma^{c,h,H}_i S_0(e) \frac{(G_p^A)^2 G_q^R}{(1 - \zeta)^2}, \quad (B1)
\]
\[
\zeta = \frac{1}{\pi \nu \tau} \int \frac{dp}{(2\pi)^3} G_p^A G_q^R, \quad (B2)
\]
where the electron Green functions are given by Eq. A.2.

The block $B^c$ with the electric current vertex, $\gamma^c = e\mathbf{v} \cdot \mathbf{E}$, may be presented as$^{20}$
\[
B^c(q) = 2e \nabla_q P^R(q,0) \cdot \mathbf{E} = 2e\nu \alpha \mathbf{q} \cdot \mathbf{E}. \quad (B3)
\]

The block $B^H$ with the vertex $\gamma^H = (e/\epsilon)\mathbf{v} \cdot \mathbf{A}_H$ is given by$^{20}$
\[
B^H(q) = \frac{2e}{c} \nu \alpha \mathbf{q} \cdot \mathbf{A}_H. \quad (B4)
\]

The block $B^h$ with the kinetic heat current vertex, $\gamma^h = \mathbf{v} \cdot \mathbf{e}_j^h$ ($\mathbf{e}_j^h = \mathbf{j}^h/\mathbf{v} \parallel \mathbf{A}$), is given by$^{22}$ (see also$^{18,19,20}$)
\[
B^h(q, \omega) = \omega \nabla_q P^R(q,0) \cdot \mathbf{e}_j^h = \omega \nu \alpha \mathbf{q} \cdot \mathbf{e}_j^h. \quad (B5)
\]

Next, we calculate the block $B^h_\nu$ with the magnetic heat current vertex $\gamma^h_\nu = \mathbf{v} \cdot \mathbf{e}_j^h$. The integral over angles of the electron momentum involves only the vertex $\gamma^h_\nu$, because the heat current is in the direction of $\mathbf{A}_H$. To obtain an imaginary part in Eq. (B1), the integral
\[
\int \frac{d\xi}{(G_p^A)^2 G_{q-p}^R} = \frac{2\pi i}{(2\epsilon - \omega - \mathbf{q} \cdot \mathbf{v} - i/\tau)^2}, \quad (B6)
\]
should be expanded in $\omega$ (in calculations of $B^c$ it is expanded in $\mathbf{q} \cdot \mathbf{v}$). Finally, we get
\[
B^h_\nu(q, \omega) = 2\omega \nabla^2_q P^R(q,0) \mathbf{A}_H = 2(e/\epsilon)\omega \nu \alpha \mathbf{A}_H. \quad (B7)
\]

**APPENDIX C: GAUGE E = iΩA_E/c**

Here we will show how our results can be obtained in the gauge, where $\mathbf{E} = i\Omega \mathbf{A}_E/c$ and $\mathbf{H} = i[\mathbf{k} \times \mathbf{A}_H]$ (this was a question of one of our referees). In the electric and magnetic fields the kinetic energy has a form $K = (\mathbf{p} + e\mathbf{A}/c)^2/2m$, and the part of the Hamiltonian describing the interaction with external fields is given by
\[
H' = \frac{e}{mc} \mathbf{p} (\mathbf{A}_H + \mathbf{A}_E) + \frac{e^2}{2mc^2} (\mathbf{A}_H + \mathbf{A}_E)^2, \quad (C1)
\]
Calculating a response to $\mathbf{E} \times \mathbf{H} = (\Omega/c)[\mathbf{A}_H (\mathbf{k} \cdot \mathbf{A}_E) - \mathbf{k}(\mathbf{A}_E \cdot \mathbf{A}_H)]$, it is convenient to use the gauge conditions $\mathbf{A}_E = \mathbf{A}_H = 0$. In this gauge the second term in Eq. (C1) can be neglected. Including the interaction with the magnetic field we get the heat current operator,
\[
\mathbf{j}^h = \sum_p v \xi_p a_p^+ a_p + \sum_p \frac{ev}{c} (\mathbf{v} \cdot \mathbf{A}_H) a_p^+ a_p. \quad (C2)
\]
As it is expected, the term in the heat current describing the interaction with the magnetic field is independent on the presentation of the electric field (see Eqs. 5 and A1). Therefore, all further calculations of thermomagnetic coefficient are the same as in the gauge $\mathbf{E} = -\nabla \phi$. 
1. R.M. White, *Quantum Theory of Magnetism*, Springer-Velag (1983).
2. K. Maki, Phys. Rev. Lett. **21**, 1755 (1968).
3. C.-R. Hu, Phys. Rev. B **13**, 4780 (1976).
4. M.J. Stephen, Phys. Rev. Lett. **16**, 801 (1966).
5. C. Caroli and K. Maki, Phys. Rev. **164**, 591 (1967).
6. Z.A. Xu, N.P. Ong, Y. Wang et al., Nature **406**, 486 (2000).
7. Y. Wang, S. Ono, Y. Onose et al., Science **299**, 86 (2003).
8. Y. Wang, L. Li, and N.P. Ong, Phys. Rev. B. **73**, 024510 (2006).
9. P.A. Lee, N. Nagaosa, X.G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
10. V.J. Emery and S.A. Kivelson, Nature (London) **374**, 434 (1995).
11. O. Vafek and Z. Tesanovic, Phys. Rev. Lett. **91**, 237001 (2003).
12. P.W. Anderson, [cond-mat/0603726](http://arxiv.org/abs/cond-mat/0603726).
13. A.M. Tsvelik and A.V. Chubukov, [cond-mat/0610181](http://arxiv.org/abs/cond-mat/0610181).
14. H. Kontani, Phys. Rev. B. **69**, 064510 (2004).
15. S. Ullah and A.T. Dorsey, Phys. Rev. Lett. **65**, 2066 (1990); Phys. Rev. B **44**, 262 (1991).
16. A. A. Varlamov and D. V. Livanov, Sov. Phys. JETP 72, 1016 (1991).
17. I. Ussishkin, S.L. Sondhi, and D.A. Huse, Phys. Rev. Lett. **89**, 287001 (2002).
18. I. Ussishkin, Phys. Rev. B **68**, 024517 (2003).
19. A. Larkin and A. Varlamov, *Theory of Fluctuations in Superconductors*, Oxford Univ Press (2005).
20. T.C. Harman and J.M. Honig, *Thermoelectric and Thermomagnetic Effects and Applications*, McGraw-Hill Book Company, 1967.
21. J.J. Krempasky and A. Schmid, J. Low Temp. Phys. **34**, 197 (1979).
22. N.R. Cooper, B.I. Halperin, and I.M. Ruzin, Phys. Rev. B **55**, 02344 (1997).
23. S. Mukerjee and D.A. Huse, Phys. Rev. B **70**, 014506 (2004).
24. B. L. Altshuler, D. Khmel’nitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
25. M. Reizer and A. Sergeev, Phys. Rev. B **50**, 9344 (1994).
26. M. Reizer and A. Sergeev, Phys. Rev. B **50**, 9344 (1994).
27. H. Fukuyama, H. Ebisawa, and T. Tsuzuki, Prog. Theor. Phys. **46**, 1028 (1971).
28. A. Sergeev, M.Yu. Reizer, and V. Mitin, Phys. Rev. B. **66**, 104504 (2002).
29. B. L. Altshuler and A. G. Aronov, *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Polak (North-Holland, Amsterdam, 1985).
| $\gamma^e = e \cdot v \cdot E$ | $\gamma^H = (e/c) \cdot v \cdot A$ | $\gamma^h_1 = \xi \cdot e_j$ | $\gamma^h_2 = (v \cdot A_H)(v \cdot e_j)$ |
|-----------------------------|---------------------------------|---------------------------|---------------------------------|
| $B^e = 2e \nabla qP^R(q,0) \cdot E$ | $B^H = (2e/c) \nabla qP^R(q,0) \cdot A_H$ | $B^h_1 = \omega \nabla qP^R(q,0) \cdot e_j$ | $B^h_2 = 2\omega \nabla q^2P^R(q,0) A_H$ |
| $= 2e\nu\alpha q \cdot e$ | $= (2e/c)\nu\alpha q \cdot A_H$ | $= \omega\nu\alpha q \cdot e_j$ | $= (2e/c)\omega\nu\alpha A_H \cdot e_j$ |
Fig. 1. Diagrams for Ettingshausen coefficient of noninteracting electrons. The straight lines stand for the electron Green functions.

Fig. 2. Fluctuation AL diagrams describing the heat current in crossed electric and magnetic fields. Wavy lines stand for the fluctuation propagators and straight lines stand for the electron Green functions, which form the AL blocks.

Fig. 3. Nonequilibrium fluctuation propagator $\delta L^C$ due to $H$ and $\nabla T$ - Poisson brackets.

Fig. 4. The interior heat currents in the Ettingshausen effect can be measured in the Gorbino disc geometry with the circular inductive electric field. The interior electric currents in the Nernst effect cannot be measured, because the circular temperature gradient does not exist.