

Hill-climbing Higgs inflation

Ryusuke Jinno,1 Kunio Kaneta,1 and Kin-ya Oda2

1Center for Theoretical Physics of the Universe, Institute for Basic Science (IBS), Daejeon 34051, Korea
2Department of Physics, Osaka University, Osaka 560-0043, Japan

(Received 18 May 2017; published 24 January 2018)

We propose a realization of cosmic inflation with the Higgs field when the Higgs potential has degenerate vacua by employing the recently proposed idea of hill-climbing inflation. The resultant inflationary predictions exhibit a sizable deviation from those of the ordinary Higgs inflation.

DOI: 10.1103/PhysRevD.97.023523

I. INTRODUCTION

Inflation plays an essential role in modern cosmology [1–3], not only by addressing the horizon and flatness problems [3], but also by giving primordial seeds for late-time structures [4]. The properties of primordial perturbations have been strongly constrained by precision cosmology, especially by the cosmic microwave background (CMB) observations [5], and such observations are expected to explore the inflationary physics much further in the forthcoming decade. Nevertheless, the identity of the inflaton, the scalar field causing inflation, is still veiled in mystery.

The Higgs particle—the quantum fluctuation of the Higgs field around its potential minimum—had long been the last missing element of the standard model (SM), and was finally discovered in 2012 [6,7]. Ever since, the Higgs field has been the only (possibly) elementary scalar field observed by human beings. The possibility of realizing inflation with this Higgs field has been studied extensively, and it has turned out that the Higgs field can indeed be identified as the inflaton with the help of a large nonminimal coupling $\xi \sim 10^5$ to the Ricci scalar [8]. This scenario, now called the Higgs inflation, has been found to fit in the most favored region by CMB observations [5].

Regarding the mass of the Higgs particle $m_H = 125.09 \pm 0.24$ GeV [19], there was an interesting prediction based on the multiple point principle (MPP) [20]. The MPP requires that there exist another vacuum in the Higgs potential around the Planck scale, in addition to the electroweak one. This means that the Higgs quartic coupling and its beta function both vanish there, $\lambda \sim \beta_\lambda \sim 0$. The observed Higgs mass has turned out to be almost within 1σ from the value $m_H = 135 \pm 9$ GeV predicted in this way [20].

Even though these two scenarios, the Higgs inflation and the MPP, seem attractive, difficulties arise when it comes to combining them. The MPP requires a degenerate vacuum around the Planck scale, which spoils the monotonicity of the Higgs potential which is necessary for a successful inflation [25]. On this regard, an interesting proposal has recently been made by two of the present authors: the hill-climbing inflation [26]. This is a general framework which enables a successful inflation with an inflaton potential with multiple vacua. This idea opens up a new possibility of identifying the Higgs field as the inflaton while having degenerate vacua in the Higgs potential. The aim of this paper is to pursue this possibility.

This paper is organized as follows. In Sec. II we briefly summarize inflationary behavior and predictions in the hill-climbing inflation. Then in Sec. III we propose an inflation model using the Higgs field as the inflaton. We conclude in Sec. IV.

1In earlier Ref. [9], the Higgs inflation with essentially the same parameters $\xi \sim 10^4$ and $\lambda \sim (\xi/10^5)^2 \sim 10^{-2}$ has also been sketched; see also Refs. [10–14]. It is noted that we may also cope with a smaller $\xi \sim 10–10^2$ under the SM criticality [15–17]. See also Ref. [18] for the explosive production of longitudinal gauge bosons and possible strong coupling issues under the presence of the large nonminimal coupling.

2See Appendix D in Ref. [21] for a review, and Ref. [22] for possible generalizations.

3See, e.g., Refs. [23,24] for more recent analyses. Especially, it is intriguing that the bare Higgs mass can also vanish around the Planck scale, and hence there can be a triple coincidence [23].

4The gauge-Higgs unification models fit in the periodic potential case in the general consideration of the hill-climbing inflation [26]. Such a possibility will be pursued in a separate publication.
II. HILL-CLIMBING INFLATION AND ITS PREDICTIONS

In this section, we briefly summarize the inflaton behavior and inflationary predictions in the general hill-climbing inflation. We start from the Jordan-frame action that has a nonminimal coupling between the inflaton and gravity:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_J \right],$$

where (and throughout the paper) we work in the Planck units $M_p = 1/\sqrt{8\pi G} = 1$ unless otherwise stated; the subscript J indicates that the quantity is given in the Jordan frame; $\phi$, $R_J$ and $V_J(\phi)$ are the inflaton, the Ricci scalar and the inflaton potential, respectively; and we assume that the conformal factor $\Omega(\phi)$ is positive for the inflaton field values we consider. Under the Weyl rescaling $g_{\mu\nu} = \Omega g_{\mu\nu}$, the Ricci scalar transforms as

$$R_J = \Omega \left[ R + 3 \Box \ln \Omega - \frac{3}{2} (\partial \ln \Omega)^2 \right],$$

and we obtain the Einstein-frame action that has a canonically normalized Ricci scalar:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - K (\partial \phi)^2 - V \right],$$

where

$$K = \frac{1}{\Omega} + \frac{3}{2} \left( \frac{d \ln \Omega}{d \phi} \right)^2,$$  

$$(\partial \phi)^2 = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,$$ and the Einstein-frame potential reads $V = V_J/\Omega^2$. If the second term dominates in Eq. (4), the kinetic term in the action (3) reduces to [27]

$$- \frac{K}{2} (\partial \phi)^2 \approx - \frac{3}{4} (\partial \ln \Omega)^2.$$  

This means that $\phi = \sqrt{3/2} \ln \Omega$ works as a canonically normalized inflaton. If the potential can be expanded as a series of $\Omega$, it will become exponentially flat in terms of $\ln \Omega$ as we will see below.

In Ref. [26] it has been proposed to use $\Omega \ll 1$ for the inflationary dynamics, instead of $\Omega \gg 1$. In this class of models, called the hill-climbing inflation, it is assumed that $V_J$ and $\Omega$ vanish at the same point $\phi_J$, in the field space, and the Taylor expansion of the potential starts from $\Omega^2$ around $\phi_1 = \phi_J$:

$$V_J = \sum_{k=2}^{\infty} V_{J,k} \Omega^k,$$  

with $V_{J,k}$ being constants. One of the examples is the case where the potential has a quadratic minimum with vanishing cosmological constant at $\phi_1 = \phi_J$, and $\Omega$ vanishes linearly in $\phi_1 - \phi_J$, there. In Sec. III we seek for a realization of this inflationary setup with the Higgs field. With the Jordan-frame potential (6), the corresponding Einstein-frame potential $V = V_J/\Omega^2$ reads

$$V = V_0 \left( 1 - \sum_{k=1}^{\infty} \eta_k \Omega^k \right) = V_0 \left( 1 - \sum_{k=1}^{\infty} \eta_k e^{-k/\ln \Omega} \right)$$

at $\Omega < 1$, where we have written $V_0 = V_{J,2}$ and $\eta_k = -V_{J,k+2}/V_{J,2}$ and the leading exponent $n \geq 1$ dominantly determines the inflationary predictions. The last expression in Eq. (7) tells that the potential is exponentially flat for the canonical inflaton field. In Sec. III we will see that the leading power depends on the explicit form of the conformal factor we take.

It is remarkable that the Einstein-frame potential $V = V_J/\Omega^2$ has been lifted up by the small $\Omega$ and made monotonic, even around a local minimum of $V_J$. As is pointed out in Ref. [26], inflation at $\Omega \ll 1$ means that the Jordan-frame potential $V_J = \Omega^2 V$ increases in time, that is, the inflaton climbs up the Jordan-frame potential hill. This observation is crucial in making successful inflation with inflaton potentials having multiple vacua, as stressed in that paper. In Sec. III we propose taking the SM Higgs field as the inflaton.

For the inflationary predictions, this class of models show attractor behavior called $\eta$-attractor. Following the standard procedure, the slow-roll parameters with the potential (7) are obtained as

$$\frac{dV_J}{d\Omega} = \left( \frac{dV_J}{d\phi_J} \right) \left( d\Omega/d\phi_J \right) = 0$$

at the minimum.

5In fact, we have $dV_J/d\Omega = (dV_J/d\phi_J)/(d\Omega/d\phi_J) = 0$ at the minimum in this case. However, it should be noted that this is just one of the possible realizations and that the expansion (6) is possible even if $d\Omega/d\phi_J = 0$ at the vanishing point $\phi_J = \phi_J$. Also, if we allow $\Omega$ to be proportional to a fractional power of $\phi_J - \phi_J$, around the vanishing point, it is even possible to remove the assumption that $V_J$ at $\phi_J = \phi_J$, is a local minimum.

6In the context of MPP, the existence of another local minimum of $V_J$ is an essential requirement, as explained in Sec. I. The vanishing $\Omega$ at the coincident point can be regarded as a realization of the generalized MPP discussed in Ref. [22].

7Having $n \geq 2$ means that we assume $V_{J,k} = 0$ for $k = 3, \ldots, n + 1$.

8It can be shown that these models share the inflationary predictions with some branch of $\alpha$-attractor [28,29] at the leading order in the $e$-folding [26]. However, there are several reasons to distinguish $\xi$- and $\eta$-attractors from $\alpha$-attractor: First of all, their actions generically differ even after the Weyl transformation. Second, the reheating and preheating processes depend on the preferred frame in which the canonically normalized matter fields are introduced; see Ref. [18] for example. Finally, such a distinction is important in constructing inflation models with particle-Physics motivated potentials, as stressed in Ref. [26].
\[\epsilon_V = \frac{1}{2} \left(\frac{V''}{V}\right)^2 = \frac{1}{4} \frac{1}{n^2 N^2}, \quad \eta_V = \frac{V''}{V} = -\frac{1}{N}, \]

where we used the following expression for the \(e\)-folding number \(N\):

\[N = \frac{3}{2} \frac{1}{n^2 \eta_\phi \Omega^a}.
\]

The inflationary predictions at the leading order in \(N\) become

\[n_s = 1 - \frac{2}{N}, \quad r = \frac{12}{n^2 N^2},
\]

where \(n_s\) and \(r\) are the spectral index and the tensor-to-scalar ratio, respectively.

### III. HILL-CLIMBING HIGGS INFLATION

Now let us take the Higgs field as the inflaton. We write its effective potential as

\[V_1(\phi_1) = \frac{1}{4} \lambda_{\text{eff}}(\phi_1)\phi_1^4.
\]

Around \(\phi_1 = M \sim 10^{17-18}\) GeV, the effective coupling can be approximated by [17]

\[\lambda_{\text{eff}}(\phi_1) = \lambda_{\text{min}} + \beta_2 \left(\ln \frac{\phi_1}{M}\right)^2 + \beta_3 \left(\ln \frac{\phi_1}{M}\right)^3 + \cdots,
\]

where \(\beta_2 \approx 2 \times 10^{-5} = \beta_2^{\text{SM}}\) in the SM [25]. The cubic and higher order terms are loop-suppressed, \(\beta_3, \cdots \ll \beta_2\), and will be neglected hereafter.

In the following we set \(\lambda_{\text{min}} = 0\) so that the potential becomes zero at \(\phi_1 = M\) by assuming the MPP. In the SM, this is realized with the top quark mass \(m_t = 171.4\) GeV for the strong coupling \(\alpha_s = 0.1185\), leading to \(M \approx 4 \times 10^{18}\) GeV [17]. However, the precise values of the \(\beta_2\) and \(M\) that realize \(\lambda_{\text{min}} = 0\) are altered by extra particles such as the heavy right-handed neutrinos and the Higgs-portal dark matter; see, e.g., Refs. [31–36]. Therefore we take them as free parameters hereafter.

Also, we consider the following forms for the conformal factor in this paper:\(^{10}\)

\(^{10}\)The precisely-measured top mass \(m_t^{\text{MC}} = 173.1 \pm 0.6\) GeV [30] is the Monte-Carlo mass, which is just a parameter in the Monte-Carlo code. The mass relevant to our discussion is the pole mass. A theorists’ naive average gives a combined result \(m_t^{\text{pole}} = 173.5 \pm 1.1\) GeV [30], and the MPP value of the top pole mass 171.4 GeV is within 1.9\(\sigma\) of this bound.

\(^{11}\)We take \(\Omega = 0\) at \(\phi_1 = M\) as discussed in footnote 6. Then Eq. (13) is the two simplest possibilities that lead to \(\Omega = 1\) at \(\phi_1 = 0\) at the same time.

\[\Omega = \begin{cases} 1 - \left(\frac{\phi_1}{M}\right)^2 & (\text{Model 1}), \\ 1 - \left(\frac{\phi_1}{M}\right)^4 & (\text{Model 2}). \end{cases}
\]

We summarize the setup in Fig. 1. Given this setup, the Einstein-frame potential is expanded as

\[V = \begin{cases} \frac{\beta_2 M^4}{16} \left(1 - \Omega - \cdots\right) & (\text{Model 1}), \\ \frac{\beta_2 M^4}{64} \left(1 - \frac{1}{12} \Omega^2 - \cdots\right) & (\text{Model 2}). \end{cases}
\]

Therefore, the leading exponent is given by \(n = 1\) and 2 for Model 1 and 2, respectively, and the potential height in the Einstein frame is given by \(V_0 \approx \beta_2 M^4\). Taking Eq. (8) and the curvature perturbation \(A_s \sim V_0/\epsilon_V\) into account, one sees that the observed value \(A_s \approx 2.2 \times 10^{-9}\) constrains the model parameters along \(M \propto \beta_2^{1/4}\). Figure 2 shows such a constraint for each of Model 1 and 2. The two bands correspond to Model 1 and 2, and the upper and lower lines for each band correspond to \(N = 50\) and 60, respectively. In making this figure we numerically solved for the \(e\)-folding \(N\) under the slow-roll assumption, defining the end of inflation by \(\max(\epsilon_V, \eta_V) = 1\). It should be mentioned that while we have investigated only two simple models, there are various possible choices of \(\Omega\) which gives different viable parameter spaces. In addition, as mentioned
above, the values of $\beta_2$ and $M$ may easily change in models beyond the SM by the existence of additional particles and associated intermediate scales; see e.g., Refs. [31–35].

Figure 3 shows the inflationary predictions in the hill-climbing Higgs inflation. It is seen that the prediction of the tensor-to-scalar ratio differs between Model 1 and 2 because of the difference in the leading exponent. See also Table I. These predictions fall within the planned sensitivities by near-future experiments such as the POLARBEAR-2 [37], LiteBIRD [38] and CORE [39], and in Model 1 there is a good possibility of distinguishing from the ordinary Higgs inflation by these experiments.

Note that the prediction for $r$ differs from the rough estimate (10) by $\mathcal{O}(10)\%$. This is because Eq. (10) is derived by taking only the leading term in Eq. (14) into account, while higher order terms can contribute to the inflaton dynamics as the conformal factor grows toward the end of inflation. Such a contribution is larger if the coefficient of the leading term is smaller, and this is why Model 2 shows a larger deviation from Eq. (10) compared to Model 1.

In Table I we summarize the allowed value for $M$ and corresponding inflationary predictions for $\beta_2 = 2 \times 10^{-5}$. One sees that $M \sim 0.1 M_P$ is favored for this value of $\beta_2$ and also that $\phi_1$ at the CMB scale corresponds to $\sim 0.01 M$ away from the potential minimum at $\phi_1 = M$.

IV. CONCLUSION

In this paper we have proposed a realization of cosmic inflation using the Higgs field with degenerate vacua. This realization utilizes the recently proposed idea of hill-climbing inflation [26], which is a general framework to enable a successful inflation using an inflaton potential with multiple vacua. It has been shown that a successful inflation occurs while the inflaton is climbing up the potential hill from the high-scale vacuum around the Planck scale to the electroweak vacuum, and that the resulting inflationary predictions come well within the region favored by the CMB observations, while showing a sizable deviation from those of the ordinary Higgs inflation.

Though in this paper we have considered only the case where the Higgs field has degenerate vacua, the original proposal in Ref. [26] can work also when the Higgs potential becomes negative at some scale. Such a study will be presented in a separate publication.

ACKNOWLEDGMENTS

The authors are grateful to Y. Hamada, H. Kawai, Y. Nakanishi, T. Onogi, and S. Rusak for useful discussions. The work of R. J. and K. K. is supported by Institute for Basic Science IBS under the Project No. IBS-R018-D1. The work of K. O. is supported in part by JSPS KAKENHI Grants No. 16J06151, 23104009 and 15K05053.
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