THE RAMSAUER-TOWNSEND EFFECT AND THE de BROGLIE-BOHM QUANTUM MECHANICS

J. M. F. Bassalo\textsuperscript{1}, P. T. S. Alencar\textsuperscript{2}, A. Nassar\textsuperscript{3} and M. Cattani\textsuperscript{4}

\textsuperscript{1} Fundação Minerva, R. Serzedelo Correa 347, 1601 - CEP 66035-400, Belém, Pará, Brasil

E-mail: bassalo@amazon.com.br

\textsuperscript{2} Universidade Federal do Pará - CEP 66075-900, Guamá, Belém, Pará, Brasil

E-mail: tarso@ufpa.br

\textsuperscript{3} Extension Program-Department of Sciences, University of California, Los Angeles, California 90024, USA

E-mail: nassar@ucla.edu

\textsuperscript{4} Instituto de Física da Universidade de São Paulo. C. P. 66318, CEP 05315-970, São Paulo, SP, Brasil

E-mail: mcattani@if.usp.br

ABSTRACT - In this work we study the Ramsauer-Townsend effect. First, we use the Quantum Mechanical Formalism of Schrödinger. After, it is calculated with the Quantum Mechanical Formalism of de Broglie-Bohm. In this case, we use the Kostin equation, taking into account the energy dissipation of the electrons scattered by sharp edged potential wells.
1) INTRODUCTION

In 1921,[1] the German physicist Carl Wilhelm Ramsauer (1879-1955) studied the scattering of low-energy electrons (0.75-1.1 eV) in inert gases argon (A), krypton (Kr) and xenon (Xe). For A, for instance, he observed that the effective cross-section was much smaller than that calculated by the Kinetic Theory of Gases. In 1922[2], for higher energies it was observed a surprising modification of the cross section.

In the same year, 1922[3], the English physicist Sir John Sealy Edward Townsend (1868-1957) and V. A. Bailey analyzed electronic scattering, for electrons with energy between 0.2-0.8 eV. Using a different method from the adopted by Ramsauer, have found that the maximum mean free path of the electrons occurs around 0.39 eV. This result, that is known as the Ramsauer-Townsend effect, was confirmed by Ramsauer and R. Kollath in 1929[4]. This would imply that the noble gases were transparent for a critical energy value[5,6].

2) THE RAMSAUER-TOWNSEND EFFECT AND THE SCHRÖDINGER’S EQUATION

As is well known, the linear Schrödinger’s equation is defined by

\[
\frac{i}{\hbar} \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + V(x, t) \psi(x, t) . \quad (2.1)
\]

Let us consider an stationary electronic flux with incident energy E, colliding with a potential well with height V and width L:

\[
V(x, t) = \begin{cases} 
0 , & x \neq 0, L, \\
-V , & 0 < x < L ,
\end{cases} \quad (2.2a)
\]

which define the following regions:

Incidence Region (1): \( x < 0 \), \quad (2.3a)

Scattering Region (2): \( 0 < x < L \), \quad (2.3b)

Transmission Region (3): \( x > L \). \quad (2.3c)
Since \( E > 0 \), the solution of the Schrödinger’s equation (2.1), for these three mentioned above regions is given by[7]

\[
\psi_1(x, t) = (e^{i k_1 x} + A e^{-i k_1 x}) e^{-i \omega t}, \quad (2.4)
\]

\[
\psi_2(x, t) = (C e^{i k_2 x} + D e^{-i k_2 x}) e^{-i \omega t}, \quad (2.5)
\]

\[
\psi_3(x, t) = (B e^{i k_1 x}) e^{-i \omega t}, \quad (2.6)
\]

where

\[
k_1^2 = \frac{2 m E}{\hbar^2}, \quad k_2^2 = \frac{2 m (E + V)}{\hbar^2}. \quad (2.7a-b)
\]

So, the reflection \( |R|^2 = A A^* \) and the transmission \( |T|^2 = B B^* \) coefficients will be given by[8]

\[
|R|^2 = \frac{\left( \frac{k_2^2 - k_1^2}{2 k_1 k_2} \right)^2 \text{sen}^2 (k_2 L)}{1 + \left( \frac{k_2^2 - k_1^2}{2 k_1 k_2} \right)^2 \text{sen}^2 (k_2 L)}, \quad (2.8)
\]

\[
|T|^2 = \frac{1}{1 + \left( \frac{k_2^2 - k_1^2}{2 k_1 k_2} \right)^2 \text{sen}^2 (k_2 L)}. \quad (2.9)
\]

Using Eqs. (2.8)-(2.9) we will analyze a particular case assuming that[9]

\[
L = \frac{\lambda_2}{2}, \quad (2.10)
\]

and considering also the de Broglie “pilot wave”, that is \( k = \frac{p}{\hbar} = \frac{2 \pi p}{\hbar}, \) we get:

\[
\lambda_2 = \frac{\hbar}{p_2} = \frac{2 \pi}{k_2}, \quad k_2 L = \pi. \quad (2.11-12)
\]

Substituting Eqs. (2.8)-(2.9) into Eq. (2.12), results:

\[
|T|^2 = 1, \quad |R|^2 = 0. \quad (2.13a-b)
\]
These equations show that when the incident electron wavelength is two times larger than the well width L, there is no reflection and the transmission is complete. This is the way how Schrödinger quantum mechanics explains the Ramsauer-Townsend effect.

3) THE RAMSAUER-TOWNSEND EFFECT AND THE DE BROGLIE-BOHM FORMALISM

In this section we study the Ramsauer-Townsend effect when in the collision process there is an energy dissipation governed by Kostion Equation that is defined by

\[
i \hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + [V(x, t) + \frac{\hbar \nu}{2i} \ln \frac{\psi(x, t)}{\psi^*(x, t)}] \psi(x, t), \quad (3.1)
\]

where \(\psi(x, t), V(x, t)\) and \(\nu\) represent, respectively, the wavefunction, potential and dissipation constant of our system. This Eq. (3.1) will be studied within the formalism of the de Broglie-Bohm.

Putting \(\psi(x, t)\) as

\[
\psi(x, t) = \Phi(x) \exp \left[ -\frac{iE}{\hbar} (1 - e^{-\nu t}) \right], \quad (3.2)
\]

we can write,

\[
i \hbar \frac{\partial \psi(x, t)}{\partial t} = E e^{-\nu t} \psi(x, t), \quad (3.3a)
\]

\[
-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} = -\frac{\hbar^2}{2m} \frac{\Phi''(x)}{\Phi(x)} \psi(x, t), \quad (3.3b)
\]

\[
\frac{\hbar \nu}{2i} \ln \frac{\psi(x, t)}{\psi^*(x, t)} = \frac{\hbar \nu}{2i} \ln \frac{\Phi(x)}{\Phi^*(x)} - E (1 - e^{-\nu t}). \quad (3.3c)
\]

Inserting Eqs.(3.3a-c) into Eq.(3.1) and using Eq.(2.2b) we obtain,

\[
\Phi''(x) + \left[ q^2 - \frac{m \nu}{\hbar} \ln \frac{\Phi(x)}{\Phi^*(x)} \right] \Phi(x) = 0, \quad q^2 = \frac{2m}{\hbar^2} (E + V). \quad (3.4a-b)
\]

Now, considering \(\phi(x, t)\) given by the Madelung-Bohm transformation[8]:
\[ \Phi(x) = \phi(x) e^{iS(x)}, \quad (3.5) \]

we see that Eq.(3.4a) becomes

\[
\frac{d\Phi(x)}{dx} \equiv \Phi'(x) = \phi'(x) e^{iS(x)} + i \phi(x) e^{iS(x)} S'(x)
\]

\[
\frac{d^2\Phi(x)}{dx^2} \equiv \Phi''(x) = \frac{d}{dx} \frac{d\Phi(x)}{dx} =
\]

\[
\Phi'' = e^{iS} \left[ \phi'' + 2i \phi' S' - \phi(x) (S')^2 + i \phi S'' \right],
\]

\[
\phi'' + 2i \phi' S' - \phi (S')^2 + i \phi S'' + [q^2 - \frac{2m\nu}{\hbar} S] \phi = 0. \quad (3.6)
\]

Separating real and imaginary parts of the above expression we have,

\[
\phi'' + (q^2 - \frac{2m\nu}{\hbar} S) \phi = (S')^2 \phi, \quad 2 \phi' S' + \phi S'' = 0. \quad (3.7a-b)
\]

Defining,

\[
\rho(x) = \phi^2(x), \quad (3.8)
\]

and integrating Eq.(3.7b) results:

\[
\frac{(S')'}{S'} = -\frac{2\phi'}{\phi} \rightarrow \int \frac{(S')'}{S'} = -\int \frac{2\phi'}{\phi} \rightarrow
\]

\[
\ln S' = -2 \ln \phi + \ln C = -\ln \phi^2 + \ln C = \ln C \phi^2 \rightarrow
\]

\[
S'(x) = \frac{C}{\rho(x)}, \quad S(x) = S(0) + C \int_o^x \frac{dx'}{\rho}. \quad (3.9a-b)
\]

Multiplying Eq.(3.7a) by \(\phi'\) and using Eqs.(3.8,9a) we have,

\[
\phi'' \phi' + [q^2 - (S')^2] \phi \phi' = \frac{2m\nu}{\hbar} S \phi \phi' \rightarrow
\]

\[
\frac{d}{dx} \left[ \frac{1}{2} (\phi')^2 + \frac{1}{2} q^2 \phi^2 + \frac{1}{2} \frac{C^2}{\phi^2} \right] = \frac{2m\nu}{\hbar} S \phi \phi'. \quad (3.10)
\]
Since Eq.(3.8) can be written as,
\[ \rho' = 2 \phi \phi' , \quad \text{(3.11)} \]
Eq.(3.10) can be rewritten, using also Eq.(3.8), as:
\[ I'(x) = \frac{2m \nu}{h} S(x) \rho'(x) , \quad I(x) = \frac{[\rho(x)]^2}{4 \rho(x)} + q^2 \rho(x) + \frac{C^2}{\rho(x)} . \quad \text{(3.12a-b)} \]

One can easily see that, taking Eq.(3.9a), we obtain,
\[ \frac{d}{dx} (S \rho) = S' \rho + S \rho' = S \rho' + C = S \rho' + \frac{4}{dx} (C x) \to \]
\[ S \rho' = \frac{d}{dx} (S \rho - C x) . \quad \text{(3.13)} \]

Substituting Eq.(3.13) into Eq.(3.12a) and considering Eqs.(3.9b,12b) results,
\[ \frac{dI}{dx} = \frac{2m \nu}{h} \frac{d}{dx} (S \rho - C x) \to \frac{d}{dx} \left[ I - \frac{2m \nu}{h} (S \rho - C x) \right] = 0 \to \]
\[ I - \frac{2m \nu}{h} (S \rho - C x) = \text{constante} = I_o \to \]
\[ I(x) = I_o + \frac{2m \nu}{h} [S(x) \rho(x) - C x] , \quad \text{(3.14a)} \]
\[ I_o = \frac{[\rho(x)]^2}{4 \rho(x)} + q^2 \rho(x) + \frac{C^2}{\rho(x)} - \]
\[ - \frac{2m \nu}{h} \left[ \rho(x) \left( S(0) + C \int_0^x \frac{dx'}{\rho(x')} \right) - C x \right] . \quad \text{(3.14b)} \]

Now, let us solve the differential equation (3.12b) using the Variational Parameters technique[10] putting
\[ \rho(x) = \frac{1}{2q^2} \left[ I(x) + \sqrt{I^2(x) - 4q^2C^2 \cos \left( 2q [x - \beta(x)] \right)} \right] , \quad \text{(3.15)} \]
where \( \beta(x) \) is the variational unknown function. To determine \( \beta(x) \) we derive Eq.(3.15), that is,
\[ \rho'(x) = \frac{1}{2 \, q^2} \left( I'(x) + \frac{I(x) \, I'(x) \cos \left( 2 \, q \, [x - \beta(x)] \right)}{\sqrt{I^2(x) - 4 \, q^2 \, C^2}} \right) - \sqrt{I^2(x) - 4 \, q^2 \, C^2} \, \text{sen} \left( 2 \, q \, [x - \beta(x)] \right) \times 2 \, q \, [1 - \rho'(x)] \right) , \quad (3.16) \]

where the following conditions must be obeyed:

\[
I'(x) + \frac{I(x) \, I'(x) \cos \left( 2 \, \theta(x) \right)}{\sqrt{I^2(x) - 4 \, q^2 \, C^2}} + \sqrt{I^2(x) - 4 \, q^2 \, C^2} \times \\
\times 2 \, q \, \beta'(x) \, \text{sen} \left( 2 \, \theta(x) \right) = 0 , \quad \theta(x) = q \left[ x - \beta(x) \right] . \quad (3.17a-b)
\]

This implies that Eq.(3.16) is written as,

\[ \rho'(x) = - \sqrt{I^2(x) - 4 \, q^2 \, C^2} \, q \, \text{sen} \left( 2 \, \theta(x) \right) . \quad (3.18) \]

From Eqs.(3.12a) and (3.18) we get,

\[ \beta'(x) = \frac{m \, \nu \, S(x)}{\hbar \, q^2} \left( 1 + \frac{I(x) \, \cos \left( 2 \, \theta(x) \right)}{\sqrt{I^2(x) - 4 \, q^2 \, C^2}} \right) . \quad (3.19) \]

We study now the scattering of a stationary flux of particles with energy \( E \, e \, k^2 = \frac{2 \, m \, E}{\hbar^2} \) [see Eq. (2.5a)] by a potential well defined by Eqs.(2.2a-b).

The particles flux, incident \((x < 0)\) and transmitted \((x > L)\), will be given by

\[
\psi_I(x) = e^{i \, k \, x} + A \, e^{-i \, k \, x} = \phi(x) \, e^{i \, S(x)} , \quad (3.20a)
\]

\[
\psi_T(x) = B \, e^{i \, k \, x} = \phi(x) \, e^{i \, S(x)} . \quad (3.20b)
\]

Since \( \psi \) and \( \frac{\partial \psi}{\partial x} \) are assumed to be continuous at the boundaries of the potential well, we obtain:

a) \( x = 0 \)

Using Eq.(3.20a) results:
\( \psi(x = 0) \rightarrow 1 + A = \phi(0) e^{i S(0)} , \quad (3.21a) \)

\[
\frac{\partial \psi}{\partial x} \bigg|_{x = 0} \rightarrow 1 - A = \frac{e^{i S(0)}}{k} \left[ - i \phi'(0) + \phi(0) S'(0) \right] . \quad (3.21b)
\]

Adding Eqs.(3.21a-b), we get the following expression:

\[
2k = \left[ \cos S(0) + i \sin S(0) \right] \left( \phi(0) \left[ k + S'(0) \right] - i \phi'(0) \right) ,
\]

that, can be divided in two parts, real and imaginary:

\[
2k = \cos S(0) \phi(0) \left[ k + S'(0) \right] + \phi'(0) \sin S(0) , \quad (3.22a)
\]

\[
0 = \sin S(0) \phi(0) \left[ k + S'(0) \right] - \phi'(0) \cos S(0) . \quad (3.22b)
\]

Multiplying Eq.(3.22a) by \( \sin S(0) \) and Eq.(3.22b) by \( \cos S(0) \) and subtracting the expressions, results:

\[
2k \sin S(0) = \phi'(0) . \quad (3.23a)
\]

On the other hand, multiplying Eq.(3.22a) by \( \cos S(0) \) and Eq.(3.22b) by \( \sin S(0) \) and adding the expressions, we obtain:

\[
2k \cos S(0) = \phi(0) \left[ k + S'(0) \right] . \quad (3.23b)
\]

Squaring and adding Eqs.(3.23a-b) the following expression is found:

\[
4k^2 = \left[ \phi'(0) \right]^2 + \phi^2(0) \left[ k + S'(0) \right]^2 . \quad (3.23c)
\]

b) \( x = L \)

Using Eq.(3.20b), we find:

\[
\psi(x = L) \rightarrow B e^{i k L} = \phi(L) e^{i S(L)} , \quad (3.24a)
\]

\[
\frac{\partial \psi_T}{\partial x} \bigg|_{x = L} \rightarrow
\]
\[
B e^{i k L} = \frac{1}{k} e^{i S(L)} \times \left[ -i \phi'(L) + S'(L) \phi(L) \right]. \tag{3.24b}
\]

Taking the real and imaginary parts of Eqs. (3.24a-b) and using Eqs. (3.9a,11):

\[
\phi(L) = \frac{1}{k} \left[ -i \phi'(L) + S'(L) \phi(L) \right] \rightarrow \phi(L) = \frac{1}{k} S'(L) \phi(L) \rightarrow
\]

\[
S'(L) = k, \quad \rho(L) = \frac{C}{k}, \tag{3.25a-b}
\]

\[
\phi'(L) = 0, \quad \rho'(L) = 0. \tag{3.26a-b}
\]

Subtracting Eqs. (3.21a-b) and taking into account Eqs. (3.8,9a,11,23a-b), we will find:

\[
2 A = e^{i S(0)} \left( \phi(0) \left[ 1 - \frac{S'(0)}{k} \right] + \frac{i}{k} \phi'(0) \right) \rightarrow
\]

\[
A = \frac{2i}{k} \frac{\left| k \rho(0) - C \right| - \rho'(0)}{\left| k \rho(0) + C \right| + \rho'(0)}. \tag{3.27}
\]

Taking the above expression let us calculate the reflection (|R|^2) and transmission (|T|^2) coefficients:

\[
|R|^2 = A A^* = \frac{4}{4} \frac{|k \rho(0) - C|^2 + |\rho'(0)|^2}{|k \rho(0) + C|^2 + |\rho'(0)|^2}, \tag{3.28a}
\]

\[
|T|^2 = 1 - |R|^2 = \frac{4kC}{4 \rho'(0)^2 + C^2 + k^2 \rho(0) + 2kC}. \tag{3.28b}
\]

Putting \(x = 0\) into Eq. (3.14), results,

\[
I_o = \frac{[\rho'(0)]^2}{4 \rho(0)} + q^2 \rho(0) + \frac{C^2}{\rho(0)} - \frac{2m \nu}{h} \rho(0) S(0). \tag{3.29}
\]

Now, substituting Eq. (3.29) into Eq. (3.28b) the transmission coefficient is written as,
\[ |T|^2 = \frac{4 \frac{k}{C} + 2 k C}{l_o + [k^2 - q^2 + \frac{2 m \nu}{k} S(0)] \rho(0) + 2 k C}. \quad (3.30) \]

The above expression can be written in a different form. Indeed, considering Eqs. (3.8) and (3.25b), and using Eq. (3.24a) we can write:

\[ |T|^2 = B B^* = \phi^2(L) = \rho(L) = \frac{C}{k}. \quad (3.31) \]

To obtain the final form for \( |T|^2 \) we need to determine the constant \( C \). To do this, it is necessary to accomplish some intermediate steps. Thus, taking Eqs. (3.12b,25b,26b) we have,

\[ I(L) = k C (1 + n^2), \quad n = \frac{q}{k}. \quad (3.32a-b) \]

Starting from Eqs. (3.14a,25b,32a) we will find

\[ I_o = C \left( k (1 + n^2) + \frac{2 m \nu}{k} [k L - S(L)] \right). \quad (3.33) \]

On the other hand, from Eq. (3.14a),

\[ I(0) = I_o + \frac{2 m \nu}{k} S(0) \rho(0). \quad (3.34a) \]

Now, using Eq. (3.23c) and Eqs. (3.8,9a,11,29,32b,34a) the function \( I(0) \) becomes,

\[ I(0) = 4 k^2 - 2 k C - k^2 (1 - n^2) \rho(0). \quad (3.34b) \]

From Eqs. (3.8,9a,11,17b,18,23a-b,26b,33,34a-b) we can also verify that

\[ S(0) = arctg \left( \frac{\rho'(0)}{2 \sqrt{[k \rho(0) + C]} \right), \quad \beta(L) = L, \quad (3.35-36) \]

\[ I(0) = C \left( k (1 + n^2) + \frac{2 m \nu}{k} [k L - S(L)] \right) + \frac{2 m \nu}{k} S(0) \rho(0), \quad (3.37a) \]

\[ C \left( k (3 + n^2) + \frac{2 m \nu}{k} [k L - S(L)] \right) = 4 k^2 - \]


\[ - \rho(0) \left[ k^2 (1 - n^2) + \frac{2 m \nu}{\hbar} S(0) \right]. \quad (3.37b) \]

Substituting Eq. (3.37) into Eq. (3.15) results

\[ 2 q^2 \rho(0) = I(0) + \sqrt{I^2(0) - 4 q^2 C^2} \cos [2 q \beta(0)] = \]

\[ = C \left( k (1 + n^2) + \frac{2 m \nu}{\hbar k} [k L - S(L)] \right) + \frac{2 m \nu}{\hbar} S(0) \rho(0) + \]

\[ + \sqrt{D_\nu} \cos [2 q \beta(0)], \quad (3.38a) \]

where [taking only the first order terms in \( \nu \) and using Eq. (3.32b)]:

\[ D_\nu = \left( C^2 k^2 (1 + n^2)^2 + \frac{4 m \nu C^2}{\hbar} (1 + n^2) [k L - S(L)] \right) + \]

\[ + \frac{4 m \nu}{\hbar} C k (1 + n^2) \rho(0) S(0) - 4 k^2 n^2 C^2. \quad (3.38b) \]

Analyzing the above equation, we note that its first term is given by \( \rho(0) \nu \). As only first terms in \( \nu \) are being taken into account, no terms involving \( \nu \), as a factor, in the \( \rho(0) \) expression will be considered. This term will be represented by \( \rho_\nu \). Thus, using Eq. (3.15), we can write:

\[ \rho_\nu(0) = \frac{1}{2 q^2} \left( \sqrt{I^2(0) - 4 q^2 C^2} \cos [2 q \beta(0)] + I(0) \right). \quad (3.38c) \]

Now, taking Eq. (3.37a) without \( \nu \) factors, that is, \( I(0) \sim C k (1 + n^2) \), inserting it in Eq. (3.38c) and also using Eq. (3.32b) results,

\[ \rho_\nu(0) = \frac{C}{n^2 k} \left[ (n^2 - 1) \times \cos^2 [q \beta(0)] + 1 \right]. \quad (3.39) \]

Substituting Eq. (3.39) into Eq. (3.38b), putting \( \sqrt{1 + x} \sim 1 + x/2 \) for \( x \ll 1 \) and that \( q \sim k \rightarrow n > 1 \), according to Eqs. (2.5a) and (3.4b), we get:
\[
\sqrt{D_\nu} = C k (n^2 - 1) \left(1 + \frac{2 m \nu (1 + n^2)}{\hbar k^2 (n^2 - 1)} \right) \left[k L - S(L) + \right.
\]
\[
+ \frac{S(0)}{n^2} \left( (n^2 - 1) \cos^2 [\theta(0)] + 1 \right) \left(k L - S(L) \right) + \left.
\right]
\]

Considering the Eq. (3.38a) and inserting into the above expression, the following expression is obtained for \( \rho(0) \),

\[
\rho(0) = \frac{C k}{2 |q^2 - \frac{m \nu \beta(0)}{\hbar}|} E , \quad (3.40a)
\]

where

\[
E = (n^2 - 1) \left(1 + \frac{2 m \nu (1 + n^2)}{\hbar k^2 (n^2 - 1)} \right) \left[k L - S(L) + \right.
\]
\[
+ \frac{S(0)}{n^2} \left( 1 + (n^2 - 1) \cos^2 [\theta(0)] \right) \left \right] \cos \left[ 2 \theta(0) \right] + \left.
\right]
\]
\[
+ (1 + n^2) \left( 1 + \frac{2 m \nu}{\hbar k^2 (1 + n^2)} [k L - S(L)] \right) \right) . \quad (3.40b)
\]

Now, substituting Eq. (3.40a) into Eq. (3.37b), we see that

\[
\frac{C}{\ell} = \frac{1}{F} , \quad (3.41a)
\]

with:

\[
F = 3 + n^2 + \frac{2 m \nu}{\hbar k^2} [k L - S(L)] + \frac{k^2 (1 - n^2) + \frac{2 m \nu S(0)}{2 |q^2 - \frac{m \nu \beta(0)}{\hbar}|} E . \quad (3.41b)
\]

In this way, the transmission coefficient [see Eqs. (2.2a-b)] will written in terms of the Eqs. (3.31,41a), that is,

\[
| T |^2 = \frac{4}{F} . \quad (3.42)
\]
Finally, the Ramsauer-Townsend effect will be studied considering the above expression for dissipative regions having small $\nu$ values. First, let us use the Taylor expansion of $\beta(x)$ and $S(x)$ for $x \sim L$:

$$
\beta(x) = \beta(L) + \beta'(L)(x - L) + \beta''(L)\frac{(x - L)^2}{2!} + \ldots, \quad (3.43a)
$$

$$
\beta(0) = L - \beta'(L)L + \beta''(L)\frac{L^2}{2!} + \ldots, \quad (3.43b)
$$

$$
S(x) = S(L) + S'(L)(x - L) + S''(L)\frac{(x - L)^2}{2!} + \ldots, \quad (3.44a)
$$

$$
S(0) = S(L) - S'(L)L + S''(L)\frac{L^2}{2!} + \ldots. \quad (3.44b)
$$

From Eqs. (3.17b) and (3.43b), results,

$$
\cos [q \beta(0)] \sim 1 \rightarrow \frac{S(0)}{n^2} \left(1 + (n^2 - 1) \cos^2 [q \beta(0)] \right) = \frac{S(0)}{n^2} (1 + n^2 - 1) = S(0).
$$

Substituting the above relations into Eqs. (3.40b), the function $E$, in the first order $\nu$, will be given by:

$$
E_\nu = (n^2 - 1) \left(1 + \frac{2m \nu S(0)}{\hbar k^2 (n^2 - 1)^2} \left[kL - S(L) + S(0)\right]\right) \times \cos [2q \beta(0)] + (1 + n^2) \left(1 + \frac{2m \nu}{\hbar k^2 (1 + n^2)} [kL - S(L)]\right). \quad (3.45)
$$

Now, taking into account only first order $\nu$ terms into Eq. (3.32b), remembering that $(1 + x)^m \sim 1 + m x$, for $x \ll 1$, we find,

$$
\frac{k^2 (1 - n^2) + \frac{2m \nu S(0)}{\hbar q^2 [1 - \frac{m \nu S(0)}{\hbar q^2}]}}{2q^2 [1 - \frac{m \nu S(0)}{\hbar q^2}]} = \frac{(1 - n^2)}{2n^2} \left[1 + \frac{m \nu S(0)}{\hbar q^2} \frac{n^2 + 1}{1 - n^2}\right].
$$

Putting the above relation into Eq. (3.41b) and using also Eqs. (3.42,45), the function $F$ in the first order $\nu$ approximation becomes:
\[ F_\nu = 3 + n^2 + \frac{2 m \nu}{\hbar k^2} [k L - S(L)] + \]
\[ + \left( \frac{1 - n^2}{2 n^2} \right) \left[ 1 + \frac{m \nu S(0)}{\hbar q^2} \left( \frac{n^2 + 1}{1 - n^2} \right) \right] E_\nu , \quad (3.46) \]

and, consequently,
\[ |T|^2 = \frac{4}{F_\nu} . \quad (3.47) \]

Eqs. (3.45-46) show that the transmission coefficient \( T^2 \) in a dissipative potential well [see Eq. (3.47)] depends on \( S(0) \) and \( S(L) \). To calculate these functions we take Eq. (3.19) and Eqs. (3.25a,32a-b), remembering that \( \cos [2 \theta(L)] \sim 1 \) and that \( n > 1 \), obtaining:
\[ \beta'(L) = \frac{2 m \nu S(L)}{\hbar (q^2 - k^2)} , \quad \beta''(L) = \frac{2 m \nu k}{\hbar (q^2 - k^2)} . \quad (3.48a-b) \]

Substituting the above Eqs. (3.48a-b) into Eq. (3.43b), \( \beta(0) \) will given by
\[ \beta(0) \sim L \left( 1 + \frac{2 m \nu}{\hbar (q^2 - k^2)} \left[ \frac{k L}{2} - S(L) \right] \right) . \quad (3.49) \]

Since \( k^2 = \frac{2 m E}{\hbar^2} \), Eqs. (3.4b,9a,26b) can be written as follows,
\[ q^2 - k^2 = \frac{2 m}{\hbar^2} (E + V) - \frac{2 m E}{\hbar^2} = \frac{2 m V}{\hbar^2} , \quad S''(L) = 0 . \quad (3.50a-b) \]

This permit us to write Eqs. (3.25a,44b,49,50b) as,
\[ S(0) \sim S(L) - k L , \quad (3.51a) \]
\[ \beta(0) \sim L \left( 1 - \frac{\nu \hbar}{V} \left[ \frac{k L}{2} + S(0) \right] \right) , \quad (3.51b) \]
\[ \beta^2(0) \sim L^2 \left( 1 - \frac{\nu \hbar}{V} \left[ \frac{k L}{2} + S(0) \right] \right) . \quad (3.51c) \]

Now, substituting Eq. (3.51a) into Eqs. (3.45-47) and remembering that only first order \( \nu \) terms are being considered, we obtain
\[ E_{\nu}[S(0)] = (n^2 - 1) \cos [2 \, q \, \beta(0)] + 1 + n^2 - \frac{2 \, m \, \nu \, S(0)}{\hbar \, k^2} , \quad (3.52a) \]

\[ F_{\nu}[S(0)] = 4 \left( 1 + \sin^2 [q \, \beta(0)] \left( \frac{1 - n^2}{2 \, n} \right)^2 \right. \times \]

\[ \times \left[ 1 - \frac{m \, \nu \, S(0)}{\hbar \, q^4} \left( \frac{n^2 + 1}{n^2 - 1} \right) \right] , \quad (3.52b) \]

\[ | T |^{-2} = 1 + \sin^2 [q \, \beta(0)] \left( \frac{1 - n^2}{2 \, n} \right)^2 \times \]

\[ \times \left[ 1 - \frac{m \, \nu \, S(0)}{\hbar \, q^4} \left( \frac{n^2 + 1}{n^2 - 1} \right) \right] . \quad (3.52c) \]

Note that Eq. (2.8) is obtained, putting \( \nu = 0 \) into Eq. (3.52c) and using Eqs. (2.6a-b) and (3.4b,49).

After these tedious calculations we are almost in conditions to write the final expression to explain the Ramsauer-Townsend effect. Indeed, as was shown in Section 2, this effect is characterized [see Eq. (2.12)] by:

\[ q \, L = \pi . \quad (3.53a) \]

In this way, with Eq. (3.53a), Eq. (3.51b) can be written as,

\[ q \, \beta(0) \sim \pi \rightarrow 2 \, q \, \beta(0) \sim 2 \, \pi . \quad (3.53b-c) \]

The validity of Eqs. (3.17b,18,53b-c) permit us to put,

\[ \rho'(0) = 0 , \quad \sin [q \, \beta(0)] = 0 . \quad (3.54a-b) \]

\[ 2 \, k \, \sin S(0) = \phi'(0) = \frac{\rho'(0)}{2 \, \rho(0)} = 0 \rightarrow S(0) = 0 . \quad (3.55) \]

Finally, Eqs. (3.52c,54b) show that the transmission coefficient is given by:
\[ | T |^2 = 1 , \quad (3.56) \]

in agreement with Eq. (2.13a), which characterizes the Ramsauer-Townsend effect.

**NOTES AND REFERENCES**

1. RAMSAUER, C. W. 1921. *Annalen der Physik* 64, p. 513.
2. RAMSAUER, C. W. 1921. *Annalen der Physik* 66, p. 545.
3. TOWNSEND, J. S. E. and BAILEY, V. A. 1922. *Philosophical Magazine* 43; 44, p. 593; 1033.
4. RAMSAUER, C. W. und KOLLATH, R. 1929. *Annalen der Physik* 3, p. 536.
5. Stephen G. Kukolich, in 1968, *American Journal of Physics* 36, p. 701), has shown this effect for the Xenonian (Xe).
6. For more details about this effect, see MOTT, N. F. and MASSEY, H. S. W. 1971. *The Theory of Atomic Collisions*, Clarendon Press, Oxford; BRODE, R. B. 1933. *Reviews of Modern Physics* 5 (p. 257).
7. The scattering of particles by potential wells is studied in many textbooks. See, for instance:
   - BOHM, D. 1951. *Quantum Theory*. Prentice-Hall, Inc.
   - DAVYDOV, A. S. 1968. *Quantum Mechanics*. Addison-Wesley Publishing Company, Inc.
   - MESSIAH, A. 1961. *Quantum Mechanics I*. North-Holland Publications Company.
   - LEITE LOPES, J. 1992. *A Estrutura Quântica da Matéria*. Editora UFRJ/ERCA Editora e Gráfica.
   - SCHIFF, L. I. 1955. *Quantum Mechanics*. MacGraw-Hill Book Company, Inc.
   - SPROULL, R. L. and PHILLIPS, W. A. 1980. *Modern Physics*. John Wiley and Sons.
8. BASSALO, ALENCAR, P. T. S., CATTANI, M. S. D. e NASSAR, A. B. 2003. Tópicos da Mecânica Quântica de de Broglie-Bohmo. EDUFPA.

9. SPROULL and PHILLIPS, op. cit.

10. NASSAR, A. B. 1998. Effect of dissipation on scattering and tunneling through sharp-edged potential barriers (DFUFPA, preprint).