WFIRST Exoplanet Mass Measurement Method Finds a Planetary Mass of $39 \pm 8 M_\oplus$ for OGLE-2012-BLG-0950Lb

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ABSTRACT

We present the analysis of the simultaneous high resolution images from the Hubble Space Telescope and Keck Adaptive Optics system of the planetary event OGLE-2012-BLG-0950 that determine that the system consists of a $0.58 \pm 0.04 M_\odot$ host star orbited by a $39 \pm 8 M_\oplus$ planet at projected separation of $2.54 \pm 0.23$ AU. The planetary system is located at a distance of $2.19 \pm 0.23$ kpc from Earth. This is the second microlens planet beyond the snow line with a mass measured to be in the mass range $20–80 M_\oplus$. The runaway gas accretion process of the core accretion model predicts few planets in this mass range, because giant planets are thought to be growing rapidly at these masses and they rarely complete growth at this mass. So, this result suggests that the...

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core accretion theory may need revision. This analysis also demonstrates the techniques
that will be used to measure the masses of planets and their host stars by the WFIRST
exoplanet microlensing survey: one-dimensional microlensing parallax combined with
the separation and brightness measurement of the unresolved source and host stars
to yield multiple redundant constraints on the masses and distance of the planetary
system.

Subject headings: gravitational lensing: micro, planetary systems

1. Introduction

Gravitational microlensing is currently the only technique to detect planets just outside the
snow line (Gould & Loeb 1992) with masses as low as that of the Earth (Bennett & Rhie 1996).
This method has so far discovered about 70 planets. For most events, the light curve modeling
of these microlensing exoplanets provides the planet-star mass ratio, but it does not provide the
masses for either the planet or the host star. Bennett et al. (2007) showed theoretically that we
can detect the foreground lens (host star and the planet) and the background source separately
with high angular resolution follow-up observations taken a few years after the peak magnification,
and these observations can determine the host star mass and distance. The planet mass is then
determined from the microlensing light curve determination of the planet-star mass ratio. High
angular resolution follow-up observations of the planetary microlensing event OGLE-2005-BLG-
169 using the Hubble Space Telescope (HST) (Bennett et al. 2015) and Keck adaptive optics (AO)
system (Batista et al. 2015) demonstrated this method, and measured the masses and distance
of this planetary system. These observations also confirmed the planetary interpretation of the
microlensing light curve because the lens-source relative proper motion predicted from the planetary
signal was consistent with the one measured by the follow-up observations.

In addition to this event, there are a number of planetary microlensing events that have
had excess starlight detected at the position of the source in the high angular resolution follow-
up observations, however very few managed to measure the lens-source separation. Under the
assumption that this excess flux is due to the planetary host star, a number of papers have claimed
to determine the host star mass (Janczak et al. 2010; Kubas et al. 2012; Batista et al. 2014), but
further follow-up observations by Bhattacharya et al. (2017) have indicated that the excess flux for
one of these events was not due to the host star. A detailed analysis by Koshimoto et al. (2017b)
showed that excess star light that is unresolved from the source can often be due to stars other than
the lens, such as companions to the source or lens, or unrelated stars. In cases where we were able
to measure the microlensing parallax, we do not require confirmation of the predicted lens-source
relative motion. Instead, high angular resolution follow-up observations that don’t resolve the lens
and source stars can confirm and refine the microlensing parallax mass measurement (Gaudi et
al. 2008; Bennett et al. 2010; Beaulieu et al. 2016) or distinguish between degenerate light curve
models (Bennett et al. 2016; Sumi et al. 2016).

This paper presents the first result from the NASA Keck Key Strategic Mission Support (KSMS) program, in support of WFIRST, entitled “Development of the WFIRST Exoplanet Mass Measurement Method,” with follow-up observations of planetary microlensing event OGLE-2012-BLG-0950. This analysis demonstrates all of the methods that are expected to be the major host star and planet measurement methods for the WFIRST exoplanet microlensing survey (Bennett et al. 2007). The HST observations are part of a pilot program to use near simultaneous observations with Keck and HST to measure the separation of the source and planet host stars using the color dependent centroid shift method (Bennett et al. 2006) in the optical and infrared. The Keck and HST images can be used separately to measure the source-host star separation using the image elongation method (Bennett et al. 2015). With the image elongation method, the lens and the source are partially resolved such that their total point spread function (PSF) is substantially elongated. So, in order to detect the lens we need to fit multiple stellar profiles to the target. However, if the PSF is not substantially elongated, we can still detect the lens from the color-dependent centroid shift method. In this case, when the lens and the source have very different colors, their relative brightness is very different in different passbands. As a result, the combination of the source+lens flux will have different centroids in different passbands at the same time. Hence, observing the target nearly simultaneously in three different passbands will give three different centroids for the same target. The shift of these centroids between different passbands can yield the separation and hence the detection of the lens. To demonstrate this method, we took a near simultaneous observation of the event OGLE-2012-BLG-0950 with HST I, V and Keck K passbands.

The microlensing event OGLE-2012-BLG-0950 (Koshimoto et al. 2017a) was observed by the microlensing survey telescopes of the Optical Gravitational Lensing Experiment (OGLE) and the Microlensing Observations in Astrophysics (MOA) collaborations. The anomaly at HJD$'$ =6149 was observed primarily in the MOA data, in addition to a single OGLE observation during the anomaly. No significant finite source effect from the background source was detected in the light curve modeling, but a significant microlensing parallax signal was seen. However, as is often the case (Muraki et al. 2011), only a single component of the parallax vector ($\pi_{E,E}$) was well measured with any precision. The second component ($\pi_{E,N}$) is constrained only with an upper limit on $|\pi_{E,N}|$. Koshimoto et al. (2017a) attempted to determine the mass of the lens system with the microlensing parallax measurement and the excess flux seen at the position of the source in 2013 AO follow-up imaging with the Keck NIRC2 camera, but the uncertainty in the $\pi_{E,N}$ and the possibility that some or all of the excess flux could be due to a star other than the planetary host star, the masses of the host star and its planet remained uncertain.

In this paper, we present the first planetary microlensing event in which the lens-source separation is measured, which allows us to convert the one-dimensional microlensing parallax measurement to a complete microlensing parallax measurement (Ghosh et al. 2004). We combine this with the lens-source separation, measured in three passbands, to obtain a direct measurement of the lens system mass (Gould et al. 2004). The paper is organized as follows: Section 2 discusses the light curve
modeling done to extract the light curve constraints to be combined with the followup observations to determine the properties of the lens system. Section 3 describes the details of our high resolution followup observations and their photometry calibrations. Section 4 focuses on the HST astrometric and photometric analyses with single star and dual star PSF fits of the blended source plus lens target. Section 5 explores the astrometric and photometric analyses of Keck AO images. Section 6 describes the color dependent centroid shifts between the positions of the blended source plus lens target in different passbands. In sections 7 and 8, we determine the geocentric relative lens-source proper motions and show that the identification of the lens constrains the parallax vector. Finally in sections 9 and 10, we discuss the lens exoplanet system properties and its implications.

2. Light Curve Models

Our analysis includes constraints on the lens system from both the microlensing light curve and the high angular resolution follow-up observations, and we have found it most convenient to redo the light curve analysis of this event that was previously presented by Koshimoto et al. (2017a). Our re-analysis uses the same data set used by Koshimoto et al. (2017a) except that the MOA-II survey light curve data have been re-reduced using the procedure described in Bond et al. (2017). This re-reduction procedure includes a light curve detrending procedure that is designed to remove systematic photometry errors due to the differential refraction of neighboring stars (Bennett et al. 2012), as well as other seeing and air mass effects.

The modeling was done with the image centered ray shooting method (Bennett & Rhie 1996; Bennett 2010), and the results are summarized in Table 1. The parameters of these models can be separated into several categories. There are three parameters that are required for single lens light curves: the Einstein radius crossing time, $t_E$, the time of closest lens-source alignment, $t_0$, and the lens source separation at the time of closest alignment, $u_0$, which is dimensionless because it is given in units of the Einstein radius. For a binary lens system, $t_0$ and $u_0$ refer to the time of closest alignment between the source and the lens system center-of-mass. Four additional parameter are generally included to describe binary lens systems. These are: the mass ratio, $q$, between the two lens masses, the angle, $\alpha$, between the source trajectory and the lens axis, the separation, $s$, between the two lens masses, in units of the Einstein radius, and the source radius crossing time, $t_*$, which is needed for most planetary events because the sharp planetary light curve features often resolve the finite angular size of the source star. Finally, there are the two components of the microlensing parallax vector, $\pi_{E,N}$ and $\pi_{E,E}$, which describe the effect of the orbital motion of the observers on the Earth around the Sun.

As explained in the discovery paper, (Koshimoto et al. 2017a), there are 4 degenerate solutions, due to two well-known degeneracies. The first is the usual close-wide degeneracy that occurs for events have planetary signals associated with the central caustic. Normally, the close-wide degeneracy relates models that differ mainly in the $s \leftrightarrow 1/s$ substitution, but in this case with $s \sim 1$, the planetary caustics have merged with the central caustic, which ruins the usual $s \leftrightarrow 1/s$
Table 1. Best Fit Unconstrained Model Parameters

| Parameter | $u_0 < 0$ | $u_0 > 0$ | $s < 1$ | $s \approx 1$ | $s < 1$ | $s \approx 1$ | MCMC averages |
|-----------|-----------|-----------|---------|--------------|---------|--------------|----------------|
| $t_E$ (days) | 70.823 | 71.059 | 70.093 | 70.062 | 69.8 ± 2.0 |
| $t_0$ (HJD') | 6151.4951 | 6151.4988 | 6151.4805 | 6151.4785 | 6151.492 ± 0.028 |
| $u_0$ | -0.09764 | -0.09734 | 0.09634 | 0.09648 | 0.004 ± 0.098 |
| $s$ | 0.89915 | 1.00082 | 0.89937 | 1.00243 | 0.942 ± 0.056 |
| $\alpha$ (rad) | -1.94169 | -1.94301 | 1.94842 | 1.94873 | -0.07 ± 1.95 |
| $q \times 10^4$ | 1.6469 | 1.5668 | 1.6669 | 1.6295 | 1.95 ± 0.38 |
| $t_*$ (days) | 0.03932 | 0.01433 | 0.01943 | 0.01324 | 0.057 ± 0.030 |
| $\pi_{E,N}$ | -0.2364 | -0.2470 | -0.2040 | -0.2020 | -0.09 ± 0.22 |
| $\pi_{E,E}$ | -0.1190 | -0.1179 | -0.0988 | -0.1023 | -0.122 ± 0.028 |
| $I_s$ | 19.299 | 19.303 | 19.315 | 19.314 | 19.253 ± 0.038 |
| $V_s$ | 20.807 | 20.810 | 20.823 | 20.821 | 20.762 ± 0.038 |
| fit $\chi^2$ | 6488.84 | 6490.48 | 6488.99 | 6490.60 | 6489.60 |

Table 2. Best Fit Model Parameters with $\mu_{rel}$ and Magnitude Constraints

| Parameter | $u_0 < 0$ | $u_0 > 0$ | $s < 1$ | $s \approx 1$ | $s < 1$ | $s \approx 1$ | MCMC averages |
|-----------|-----------|-----------|---------|--------------|---------|--------------|----------------|
| $t_E$ (days) | 68.007 | 67.628 | 68.919 | 68.995 | 68.1 ± 1.2 |
| $t_0$ (HJD') | 6151.4702 | 6151.4749 | 6151.4978 | 6151.4999 | 6151.484 ± 0.027 |
| $u_0$ | -0.09968 | -0.09734 | 0.10088 | 0.10073 | 0.043 ± 0.091 |
| $s$ | 0.89783 | 1.00136 | 0.89791 | 0.99000 | 0.928 ± 0.052 |
| $\alpha$ (rad) | -1.94592 | -1.94719 | 1.93987 | 1.94146 | -0.84 ± 1.75 |
| $q \times 10^4$ | 1.7255 | 1.7442 | 1.6726 | 1.6165 | 2.01 ± 0.39 |
| $t_*$ (days) | 0.03634 | 0.03633 | 0.03609 | 0.01324 | 0.0366 ± 0.0013 |
| $\pi_{E,N}$ | 0.2107 | 0.2103 | 0.2192 | 0.2178 | 0.213 ± 0.017 |
| $\pi_{E,E}$ | -0.1536 | -0.1536 | -0.1685 | -0.1672 | -0.157 ± 0.016 |
| $I_s$ | 19.274 | 19.303 | 19.315 | 19.314 | 19.265 ± 0.023 |
| $V_s$ | 20.783 | 20.773 | 20.768 | 20.769 | 20.734 ± 0.023 |
| fit $\chi^2$ | 6490.25 | 6491.87 | 6491.88 | 6493.45 | 6492.45 |
relation. So, the two solutions have \( s \approx 0.9 \) and \( s \approx 1.0 \), instead. The second degeneracy is the well known microlensing parallax degeneracy that involves a flipping the orientation of the lens plane with respect to the orbit of the Earth. This is indicated by sign changes of the \( \alpha \) and \( u_0 \) parameters.

One unexpected feature of this new analysis is that the best fit \( \pi_{E,N} \) value has changed sign from the discovery paper (Koshimoto et al. 2017a) from \( \sim 0.12 \) to \( \sim -0.22 \), but in both cases, the uncertainty is quite large, as is often the case for ground-based microlensing parallax measurements (Muraki et al. 2011; Gould 2014). This change is due to an improvement in the detrending algorithm that we have applied to the MOA data, and it does not make a significant difference in our final conclusions.

In Section 8, we discuss constrained light curve models that employ constraints on the lens and source star relative proper motion, \( \mu_{rel} \) and magnitudes that have been derived in Sections 4, 5, and 7. The best fit models from this analysis are shown in Table 2. As we discuss in Section 8, the \( \mu_{rel} \), the \( \mu_{rel} \) constraints from our high angular resolution measurements greatly improve the precision of our \( \pi_{E,N} \) measurements, which, in turn, enables precise determinations of the masses of the host star and planet.

The final column in Tables 1 and 2 is the average of each parameter over Markov Chain Monte Carlo calculations for all the different models in a weighted sum. In Table 1, the weighting is based only on the \( \chi^2 \) difference (\( \Delta \chi^2 \)) between the different models, but in Table 2, we also include weightings from a Galactic model. Note that the probability distributions the parameters, \( u_0 \), \( s \), and \( \alpha \), that take significantly different values in the different degenerate models have double peaks.

3. Follow up observations

The event OGLE-2012-BLG-0950 was observed with HST on May 22, 2018 with Wide Field Camera 3 - Ultraviolet Visible (WFC3-UVIS) instrument as part of the program GO 15455. Seven dithered images, each with 62 seconds exposure time, and eight dithered images, each with 111 seconds exposure time, were taken in F814W and F555W passbands (which are HST equivalent of \( I \) and \( V \) bands) respectively. The pixel scale for WFC3-UVIS instrument is \( \sim 40 \) mas on a side. The images, corrected for CTE (Charge Transfer Efficiency) losses (Anderson & Bedin 2010), were obtained from Mikulski Archive for Space Telescopes (MAST) and were reduced and stacked following the methods described in Anderson & King (2000, 2004). The stars from the HST stack images were matched and calibrated to the OGLE III catalog (Szymański et al. 2011), which is already calibrated to Cousins \( I \) (Cousins 1976) and Johnson \( V \) (Johnson 1966). Nine bright, isolated calibration stars with magnitude \( I_{\text{OGLEIII}} \leq 17.5 \) and color \( 1.0 \leq (V - I)_{\text{OGLEIII}} \leq 2.0 \) were matched in both frames. We obtained the following calibration relations:

\[
I_{\text{OGLEIII}} = 28.764 + I_{\text{HST}} + 0.0467(V - I)_{\text{HST}} \pm 0.02
\]
\[
V_{\text{OGLEIII}} = 30.602 + V_{\text{HST}} - 0.0641(V - I)_{\text{HST}} \pm 0.03
\]
These uncertainties are the RMS scatter of the fit, divided by the square root of the number of stars used for the transformation.

The same event was observed nearly simultaneously with the Keck AO (Adaptive Optics) NIRC2 instrument during the early morning of May 23, 2018 as part of our Keck NASA KSMS program. Five dithered exposures, each of 30 seconds, were taken in $K_S$ short passband with the wide camera. In this paper, from now on we refer to the $K_S$ band as the $K$ band. Each wide camera image covers a 1048 × 1048 square pixel area, and each pixel size is about $40 \times 40$ mas$^2$. These images were flat field and dark current corrected using standard methods, and then stacked using the SWarp Astrometrics package (Bertin et al. 2002). The details of our methods are described in Batista et al. (2014). We used aperture photometry method on these wide images with SExtractor software (Bertin & Arnouts 1996). These wide images were used to detect and match as many bright isolated stars as possible to the VVV catalog for the calibration purposes. Twenty seven isolated bright stars in $K$ band were calibrated to VVV with a 0.02 magnitude dispersion.

This event was observed on the same date with Keck NIRC2 narrow camera in the $K$-band using laser guide star adaptive optics (LGSAO). The main purpose of these images is to derive a proper PSF for the astrometric and photometric analysis of the lens and source stars (section 5). Thirty-nine dithered observations were taken with 60 second exposures. The images were taken with a small dither of 0.7" at a position angle (P.A.) of 0° with each frame consisting of 2 co-added 30 seconds integrations. The observations were taken in 8 dither positions with atleast 4 images in each dither position. The seeing of these narrow camera images was $\sim 0.06-0.08"$. There are 1048×1048 pixels in each image with each pixel subtending 10 mas on each side. The stars from the narrow camera image were cross matched to the wide camera image for calibration. The photometry used for narrow camera images are from DAOPHOT analysis (section 5).

4. HST ePSF Fitting

Like a many other space telescope cameras, the HST-WFC3-UVIS pixel scale undersamples the point spread function (PSF) in a compromise between field-of-view and angular resolution. Fortunately, accurate photometry (Lauer 1999) and astrometry (Anderson & King 2000) can still be obtained if the image pointings are dithered to recover the spatial sampling lost to undersampling. To overcome this problem, we adopt the method of Anderson & King (2006) to construct an Effective Point Spread Function (ePSF) from the dithered images. This method has proved effective at measuring the separation of stars separated by < 1 Full-Width-Half-Maximum (FWHM) (Bennett et al. 2015; Bhattacharya et al. 2017). Eight main sequence stars, within 120 pixel radius of the target and similar brightness as the target were chosen to build the ePSF. The ePSFs of all these eight stars were computed in each image frame and then the average ePSF over all the frames was obtained. It is this averaging of the ePSFs from all dithered images that helps to overcome the undersampling problem. The procedure was iterated until the average ePSFs converged. We used this final ePSF model to fit single and dual stellar profiles for our target object as described
in next paragraphs. Our methodology for fitting single and multiple stellar profiles with ePSFs is described in detail in Bhattacharya et al. (2017).

The first step of ePSF modeling is to do a single star fit of our target object. There are three model parameters for such a fit: the two dimensional position of the star and the stellar flux. We selected a region centered on the target that included 151 pixels in our 7 I-band images and 200 pixels in our 8 V-band images. The calibrated magnitudes of the target object from the best fit solution were 18.64 ± 0.02 and 20.41 ± 0.03 in the I and V bands, respectively. The magnitude uncertainties are the combination of the ePSF fitting and calibration uncertainties. As Figure 1 shows, the residual image from the fit indicates that the best single star fit was a poor fit. It was also clear that the I and V magnitudes of the target were significantly brighter than the source magnitude determined from the microlensing light curve analysis: $I_S = 19.26 ± 0.05$, and $V_S = 20.65 ± 0.07$. This indicates the presence of at least one additional star blended with the source star. This leads to the dual star ePSF fitting analyses described below.

The dual star ePSF fitting method requires six model parameters: the two dimensional positions of each of the stars $(x_1,y_1,x_2,y_2)$, the total flux $(Z)$ and the flux fraction $(f_1)$ of star # 1. The first stage or our dual star fitting process is a grid search. The positions of each star were allowed to explore the full grid of 151 and 200 pixels for I and V bands with a step size of 0.02 pixel. For each combination of positions of the two stars, the flux fraction, $f_1$, was varied between (0.0 -1.0) with a step size of 0.02. At each step, the $\chi^2$ value was computed and the minimum $\chi^2$ solution was stored. This intense grid search yielded our first best fit result on the dual star ePSF fit. The grid search was performed to cover the full parameter space. The best fit result was used as an initial condition for a Markov Chain Monte Carlo (MCMC) (Verde et al. 2003) ePSF fitting in order to obtain a more precise measurement with error bars. Our detailed methodology of MCMC ePSF fitting for multiple stars is described in detail in Bhattacharya et al. (2017). The best fit photometry from the dual star MCMC PSF models is shown in Table 3, and the best fit relative astrometry is shown in Figure 4. The uncertainties are determined from the distribution of 29727 and 37132 accepted MCMC links in the I and V-bands, respectively. The residual of the dual star shown in Figure 1 shows that it is a good model.

The dual star ePSF model results (Table 4) show that the target consists of two stars with a separation of $\sim 34$ mas, with consistent separations measured in all three passbands. The magnitudes of star # 1 (the southernmost star) are $I_1 = 19.24 ± 0.06$ and $V_1 = 20.65 ± 0.09$, and the magnitudes of the second star are $I_2 = 19.57 ± 0.09$ and $V_2 = 22.27 ± 0.21$. The magnitudes for star # 1 match the approximately calibrated source magnitudes (Koshimoto et al. 2017a), and they are within 1$\sigma$ of the source brightness predicted from our light curve modeling based on improved MOA photometry. However, star # 2 does not match with the predicted source flux from light curve of $I_S = 19.24 ± 0.03$ and $V_S = 20.65 ± 0.07$. Thus, we identify star # 1 as the source star. Since both stars have similar brightnesses in the I band and the lens is located nearer than the source in the bulge, the lens should be redder than the main sequence source. So, the star we identify as the source is the bluest and significantly brighter in the V band. The second star is
Fig. 1.— *Top left:* The stack image in the HST $I$ (F814W) passband with the target indicated by the yellow circle. *Top right:* The $100\times$ supersampled summed image of the target object. The source and lens positions obtained from the best fit dual star PSF model to the individual images. *Bottom left:* The residual image from the best fit single star PSF model. The wings are under-subtracted and the core is over-subtracted indicating that the best fit single star PSF model is not consistent with the flux distribution of the target object. *Bottom right:* The residual image from the best fit dual star PSF model. This indicates a much better fit consistent with Poisson noise (which is larger at the location of the subtracted stars), so the best fit dual star model is compatible with the flux distribution of the target. Both the bottom images are demonstrated using the same photometry scale.
Table 3. Photometry from Dual Star PSF Fits

| Star   | Passband | Mag   |
|--------|----------|-------|
| Lens   | HST $I$  | 19.57 ± 0.09 |
|        | HST $V$  | 22.27 ± 0.21 |
|        | Keck $K$ | 17.27 ± 0.04 |
| Source | HST $I$  | 19.24 ± 0.06 |
|        | HST $V$  | 20.65 ± 0.09 |
|        | Keck $K$ | 17.68 ± 0.05 |

*Magnitudes are calibrated Cousins $I$-band, Johnson $V$-band and 2MASS $K$-band magnitudes.

Table 4. Measured Lens-Source Separation and Relative Proper Motion

| Passband | Separation(mas) | $\mu_{\text{rel,H}}$(mas/yr) |
|-----------|-----------------|-------------------------------|
|           | East            | North                        | East         | North         |
| HST $I$   | $-15.5 \pm 0.4$ | $30.5 \pm 0.7$               | $-2.66 \pm 0.05$ | $5.23 \pm 0.13$ |
| HST $V$   | $-15.6 \pm 1.4$ | $30.8 \pm 2.6$               | $-2.68 \pm 0.27$ | $5.28 \pm 0.47$ |
| Keck $K$  | $-15.9 \pm 2.9$ | $29.3 \pm 3.0$               | $-2.7 \pm 0.49$  | $5.03 \pm 0.52$ |
a candidate for the planetary host and lens star. The lens-source separations in both East and North are consistent in independent analysis of the $I$ and the $V$ passbands. The separation of these two stars is also measured to high precision. The one dimensional separation, as measured in the $I$-band is $34.2 \pm 0.06$ mas, which is a precision of 2%, even though the separation is $< 0.5$ FWHM of the PSF. This is consistent with our previous analysis of the event MOA-2008-BLG-310, where we were able to measure the separation between the source and blend star (which was not the lens) to a precision of 27%, when the separation was 12 mas (Bhattacharya et al. 2017).

5. Keck AO PSF Fitting

Thirty-nine images taken with the Keck NIRC2 narrow camera were reduced. These images taken with the narrow camera are not undersampled, so we did not need to adopt the ePSF method for the analysis of these images. For the reduction of these images, we used $K$-band dome flats taken with narrow camera on the same day as the science images. There were 5 dome flat images with the lamp on and 5 more images with the lamp off, each with 65 seconds exposure time. Also at the end of the night, we took 20 sky images using a clear patch of sky at a (RA, Dec) of (18:08:04.62, -29:43:53.7) with an exposure time of 30 seconds each. All these images were used to flat field, bias subtract and remove bad pixels and cosmic rays from the thirty-nine raw science images. Finally these clean raw images were stacked into one image and that we used for the final photometry and astrometry analysis.

Because the source and candidate lens stars are separated by $< 1$ FWHM, we must analyze the Keck data with a PSF fitting code to measure the astrometry and photometry of this 2-star system. However, PSF fitting photometry codes that employ analytic PSF models may have problems with AO images that often have highly non-Gaussian PSFs. We have previously found that DAOPHOT (Stetson 1987) does quite well with such images (Bennett et al. 2010), so we use DAOPHOT for the analysis of our OGLE-2012-BLG-0950 Keck data.

To start our analysis, we needed to construct a proper PSF. We built the PSF in three stages. In the first stage, we run the FIND and PHOTOMETRY commands of DAOPHOT to find all the possible stars in the image. Then we used the PICK command to find twenty-three bright ($K < 19.24$) isolated stars to be used for constructing our PSF. Our target object was excluded from this list of PSF stars because it is expected to consist of two stars that are not in the same position. We build a PSF from these stars and fit all the stars in the field with this PSF. In the second stage, we carefully check the residual image that has all the identified stars subtracted. We noticed several stars which had large residuals. These were either very bright stars that were near saturation (where the detector becomes non-linear) or elongated PSFs due to multiple blended stellar images. At this stage, we carefully checked and found that two of our PSF contributing stars had a significant residual from the PSF subtraction. A close look showed that both of these had slight elongations that are probably due to binary companions. So, these two stars were removed from our PSF star list, and the PSF was constructed again. Next, we ran the PSF fitting again
on the field with the new PSF and moved on to the third stage. In the third stage, we removed all the neighbor stars of the PSF stars and computed a clean final PSF from these 21 stars. We then did a final round of PSF fitting for all the stars in the image with this clean PSF.

After finding a good PSF model, we started our analysis with a single star PSF fit to the target object. The residual of this fit is shown in Figure 2. This residual shows a clear pattern that indicates that it is elongated compared to the PSFs of single stars. From our reanalysis of this event (with improved MOA photometry), we find an extinction corrected magnitude and color of the source star $I_{S,0} = 18.40 \pm 0.07$ and $(V - I)_{S,0} = 0.74 \pm 0.07$, which is consistent with the Koshimoto et al. (2017a) analysis. From the color-color relation in Kenyon & Hartmann (1995), $(V - I)_{S,0} = 0.74$ corresponds to the dereddened color $(I - K)_{S,0} = 0.75$. From Cardelli et al. (1989), the extinction in $K$ band is $A_K = 0.12$ at 8.2 kpc (see section 9). Hence, the source $K$ band calibrated magnitude is $(18.40 - 0.75 + 0.12) \pm 0.07 = 17.77 \pm 0.07$.

The total measured $K$-band flux from our single star fit was $16.71 \pm 0.03$ which indicated an excess flux of $K = 17.22$ on top of the source. This excess flux, combined with the residual shown in the lower left panel of Figure 2 implies that we should proceed with dual star PSF fitting for this target. The dual star fit with DAOPHOT limited the stars to move up to a 30 mas or 3 pixel radius from their initial positions. In the HST astrometry analysis (section 1), we found that the separation between the two stars is $\sim 34$ mas. So, the movement of star positions by a limit of 30 pixels should not impede our dual star modeling. The best dual star fit yielded two stars with calibrated $K$ magnitude $17.27 \pm 0.04$ and $17.68 \pm 0.05$, as listed in Table 3, which matches the predicted source brightness from the discovery paper. The separation of these two stars, given in Table 4 is consistent with their separations in the HST $I$ and $V$ bands. Both the single star and dual star fits were done using Newton-Raphson method following Stetson (1987). The uncertainties were calculated following King (1983):

$$\sigma_x = 0.65238 \times \text{FWHM} \times \sqrt{\frac{1}{3} \times \frac{\sigma_F}{F}} \quad (3)$$

where $F$ is the flux of the respective star. The uncertainty in $y$ direction can be presented by the same equation. The FWHM of the $K$ band narrow image measured from DAOPHOT is 84 mas and 87 mas in $x$ and $y$ directions.

6. Color Dependent Centroid Shift with Simultaneous HST and Keck Observations

The color dependent centroid shift is a method that can be used to confirm the identification of excess blended flux on top of the source with the lens star. This is possible because of the constraints on the properties of the blended image of the lens and source are known from the microlensing light curve. These known properties always include the source star brightness, and they usually include the lens-source relative proper motion, $\mu_{rel}$ or the microlensing parallax, $\pi_E$.

The source stars are biased toward the brightest stars in the bulge, since a brighter source
Fig. 2.— *Top left:* The stack image of 39 Keck K-band images, taken with the narrow camera. The target is indicated by the yellow circle. *Top right:* A closer look at the target object. The source and lens positions are obtained from the best fit dual star PSF model. *Bottom left:* The residual image after subtracting the best fit single star PSF model. The under-subtracted wings and the over-subtracted core indicates that the best fit single star PSF model is not consistent with the data. *Bottom right:* The residual image after subtracting the best fit dual star PSF model. This shows only noise, which implies that the best fit dual star model can account for the flux distribution of the target. Both the bottom images are demonstrated using the same photometry scale.
Fig. 3.— The blended image of the source plus lens stars is shown in three different passbands: Keck $K$ (left), HST $I$ (middle) and HST $V$ (right). The source and lens positions, determined from the best fit dual star PSF models, are shown in red and green dots, respectively. The centroids are calculated according to the flux of the lens and the source in the respective passbands (from Table 3). The size of the source and lens dots are proportional to the flux fraction for the lens and source stars. In the Keck $K$-band the lens is brighter than the source hence the green dot is bigger than the red dot and the centroid is shifted toward the lens. In the $I$-band, the source is slightly brighter than the lens, so the centroid is slightly closer to the source. In the $V$-band, the centroid is moved toward the source, since the source is much brighter than the lens.
provides a stronger microlensing signal. However, the lens stars are detected with a probability that scales as lens mass, $M_L^n$ where $0.5 \leq n \leq 1$, which is a much shallower dependence. Thus, the lens stars tend to have a lower mass with a redder color than the source stars as the cases of OGLE-2005-BLG-169 (Bennett et al. 2015; Batista et al. 2015) and OGLE-2012-BLG-0950, presented in this paper, demonstrate.

When the source and lens stars have different colors, their blended image will have different centroids in follow-up images in different passbands due to the separation between the lens and source. This is the color dependent centroid shift, which was first demonstrated for the first planet found by microlensing (Bennett et al. 2006) using HST data. The main advantage of this method over the measurement of image elongation is that the S/N of color dependent centroid shift scales as time interval since the magnification peak, $\Delta t$, whereas the image elongation signal grows as $\Delta t^2$ if the source brightness is known, or as $\Delta t^3$ when the source brightness is not constrained (Bennett et al. 2007). Thus, the color dependent centroid method might be the most effective method for determining the lens-source separation shortly after an event.

The color dependent centroid shift is largest with the maximum color difference between passbands, and this was the justification for the near simultaneous HST and Keck AO imaging. Previous studies (Lu et al. 2014) have argued that it is possible to obtain astrometry with an accuracy of 150 mas from Keck NIRC2 images, but this requires a large number of careful corrections. Since our primary science result comes from the dual star fits, we do not require this very high precision astrometry for our science results. So, we present a preliminary test of the color dependent centroid method here. We plan to present more a more refined analysis in a future paper.

In order to measure the color dependent centroid shift between each pair of passbands, we must perform a coordinate transformation between the images in the different passbands. We have corrected for achromatic differential refraction (Yelda et al. 2010) and geometric distortion (Service et al. 2016). Based on Gubler & Tytler (1998), the chromatic differential refraction effect would be \( \pm 0.2 \) mas. Hence this effect is ignored for our preliminary analysis. For our preliminary analysis, we

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
Passbands & Single-Star & & Dual-Star & & Single-Star – Dual-Star & \\
& $\Delta E$(mas) & $\Delta N$(mas) & $\Delta E$(mas) & $\Delta N$(mas) & $\Delta E$(mas) & $\Delta N$(mas) \\
\hline
$V - I$ & $-3.764$ & $7.249$ & $-3.460$ & $5.950$ & $-0.304$ & $1.299$ \\
$V - K_S$ & $-6.400$ & $12.324$ & $-6.815$ & $13.358$ & $0.415$ & $-1.034$ \\
$I - K_S$ & $-2.635$ & $5.074$ & $-5.709$ & $7.863$ & $3.074$ & $-2.789$ \\
\hline
\end{tabular}
\caption{Blended Lens + Source Centroid Shifts}
\end{table}

*The positions are relative to the $K$-band position of the source at RA 18:08:04.61228 and Dec -29:43:53.3565 (J2000).
have performed linear coordinate transformations with a relatively small number of stars around the target. For the HST-\(I\) to Keck-\(K\) transformation, we used 10 stars, which resulted in a one-dimensional RMS scatter of 0.9 mas. For the HST-V to Keck-\(K\) transformation, we used 8 stars with a one-dimensional RMS scatter of 1.3 mas. Finally, for the HST-\(I\) to HST-V transformation, we used 19 stars with an RMS scatter of 0.2 mas, which is consistent with the astrometric precision obtained in a previous attempt to measure the color dependent centroid shift (Bennett et al. 2006). These coordinate transformations were used to compare the centroids of the blended lens plus source image from the single PSF fits, and these results are reported in the second and third columns of Table 5.

We can also use our dual star fits from Sections 4 and 5 to calculate the expected color dependent centroid shifts. From section 4, the flux fraction of the source (or star \# 1), is \(f_1\), which implies that the flux fraction of the excess flux is \((1 − f_1)\). For the remainder of this section we will refer to this excess flux as being due to the lens star. (see Section 9 and Figure 5). We denote the source position vector as \(x_1\) and the lens position as \(x_2\). The lens-source separation is given by \(Δx_{L−S}\). For a passband “\(i\)” the centroid of the combined source and the lens flux, \(x_{c,i}\), is given by:

\[
x_{c,i} = f_{1,i}x_1 + (1 − f_{1,i})x_2
\]

\[
x_2 = x_1 + Δx_{L−S}
\]

\[
x_{c,i} = f_{1,i}x_1 + (1 − f_{1,i})(x_1 + Δx_{L−S})
\]

For a different passband \(j \neq i\), the centroid of the blended source plus lens image is given by:

\[
x_{c,j} = f_{1,j}x_1 + (1 − f_{1,j})(x_1 + Δx_{L−S})
\]

Subtracting Equation (6) from Equation (7) we obtain the centroid shift between passbands \(j\) and \(i\) as shown in Equation (8). Rearranging the terms, we can derive the lens-source separation from this centroid shift and the flux ratios in two different passbands.

\[
Δx_{c,(j,i)} = x_1(f_{1,j} − f_{1,i}) + (f_{1,i} − f_{1,j})(x_1 + Δx_{L−S})
\]

\[
Δx_{c,(j,i)} = (f_{1,i} − f_{1,j})Δx_{L−S}
\]

\[
Δx_{L−S} = \frac{Δx_{c,(j,i)}}{(f_{1,i} − f_{1,j})}
\]

We can now use Equation (9) to predict the color dependent centroid shifts from the dual star fits. The results are shown in the fourth and fifth columns of Table 5. The sixth and seventh columns of this table show the difference between these two predictions. The centroid shift estimate from the dual star fits seems to be a rough match to the centroid measurements from the single star fits, but the difference between the single star fit measurements and the dual star fit estimates are much larger than we would like if we are to use the color dependent centroid shift measurement to confirm lens-source separation predictions at small separations. We expect that an astrometric precision in each of the passbands of \(\sim 0.3\) mas to be achievable (Bennett et al. 2006; Lu et al. 2014).
so that we should be able to achieve relative astrometric precision of \( \lesssim 0.5 \) mas between pairs of passbands. However, based on the scatter in our transformations between passbands, it is clear that this is only possible between the HST \( V \) and \( I \)-bands. Our transformations between the HST passbands and the Keck \( K \)-band have a one-dimensional scatter of \( \gtrsim 1 \) mas, so sub-mas precision is not possible until these transformation are improved. Fortunately, there are several ways in which these transformation can be improved. First, we need to double-check the differential refraction and optical distortion corrections that we have made to the \( K \)-band data. (Lu et al. 2014) have handled this with higher order coordinate transformations. This could be a problem with only 8-10 comparison stars, however. We will try increasing the number of comparison stars by selecting stars closer to the edge of the Keck narrow frames, but we will also consider changes in our observing program for future simultaneous Keck plus HST observing campaigns. For example, we might take deeper NIRC2-narrow camera images to allow the use of fainter comparison stars. Or we might try the NIRC2-medium camera to increase the field-of-view by a factor of 4, but with the pixel size increased from 10 to 20 mas.

We should note that the lens-source separation of 34 mas could be too large for Equations 4-10 to apply, particularly in the HST \( V \)-band with a PSF FWHM of \( \sim 48 \) mas. Thus, the comparison shown in Table 5 might not be completely fair. From the Table 3, we can conclude that the positions of the lens derived from the single star fit would be different by \( \sim 10-20\% \) from the measured positions of the lens using dual fits. Hence, the large elongation could be a big reason why this centroid shift method is not yielding a result close to the dual fits.

7. Determination of Relative Lens-Source Proper Motion

Our high resolution observations were taken 5.83 years after the microlensing event magnification peak. If these images were taken 6 years after the microlensing magnification, then the HST frame would have coincided with the heliocentric frame. However, this time difference of 0.17 years between the heliocentric frame and our current HST frame, with a lens system at 2.1 kpc (see section 9) produces less than 1\( \sigma \) difference in the lens-source separation. Hence we approximate the HST frame to the heliocentric frame. At the time of peak magnification, the separation between lens and source was \( \sim u_0 \theta_E \sim 0.1 \) mas. Hence, by dividing the measured separation by the time interval 5.83 years, we obtained the heliocentric lens-source relative proper motion, \( \mu_{rel,H} \). A comparison of these values from our independent dual star fits is shown in Table 4. In the galactic coordinates, the \( \mu_{rel,H} \) value from the \( I \)-band measurement is \( \mu_{rel,H,I} = 3.23 \) mas/yr and \( \mu_{rel,H,b} = 4.84 \) mas/yr, with an amplitude of \( \mu_{rel,H} = 5.87 \pm 0.12 \) at an angle of \( \sim 57^\circ \) from the direction of Galactic rotation. The dispersion in the motion of stars in the local Galactic disk is \( \sim 30 \) km/sec, which corresponds to a proper motion dispersion of \( \sim 3 \) mas/yr (in both directions) at the lens distance of \( D_L \approx 2.1 \) kpc as presented in section 9. The source is in the bulge, with a proper motion dispersion of \( \sim 2.5 \) mas/yr in each direction. Thus, the measured \( \mu_{rel,H} \) is entirely consistent with the combination of the mean relative proper motion of 6 mas/yr in the direction of...
Galactic rotation combined with the proper motion dispersion of bulge source and disk lens stars.

Our light curve models were done in a geocentric reference frame that differs from the heliocentric frame by the instantaneous velocity of the Earth at the time of peak magnification, because the light curve parameters can be determined most precisely in this frame. However, this also means that the lens-source relative proper motion that we measure with follow-up observations is not in the same reference frame as the light curve parameters. This is an important issue because, as we show below, the measured relative proper motion can be combined with the microlensing parallax light curve parameter to determine the mass of the lens system. The relation between the relative proper motions in the heliocentric and geocentric coordinate systems are given by (Dong et al. 2009b):

\[ \mu_{\text{rel},H} = \mu_{\text{rel},G} + \frac{v_\oplus \pi_{\text{rel}}}{\text{AU}}, \]

where \( v_\oplus \) is the projected velocity of the earth relative to the sun (perpendicular to the line-of-sight) at the time of peak magnification. The projected velocity for OGLE-2012-BLG-0950 is \( v_{\oplus,E,N} = (4.096, -0.448) \text{ AU/yr} \) at the peak of the microlensing, HJD' = 6151.48. The relative parallax is defined as \( \pi_{\text{rel}} \equiv 1/D_L - 1/D_S \), where \( D_L \) and \( D_S \) are lens and source distances. The lens distance for this event can be determined in two different ways. At each possible lens distance, we can use the \( \mu_{\text{rel},G} \) value from equation 11 to determine the angular Einstein radius, \( \theta_E = \mu_{\text{rel},G} t_E \). As we explain below, the \( \mu_{\text{rel},G} \) can also be used to convert a one-dimensional microlensing parallax measurement into a full measurement of the microlensing parallax vector. The angular Einstein radius, microlensing parallax, and the three lens flux measurements in the \( V, I, \) and \( K \)-bands all constrain the mass and distance of the lens, as we explain in the next section.

8. \( \mu_{\text{rel},H} \) and Lens Flux Constraints on \( \pi_E \) and Light Curve Models

The OGLE-2012-BLG-0950 light curve shows a significant improvement of \( \Delta \chi^2 = 85.9 \) due to the measurement of the microlensing parallax effect. But as it is often the case (Muraki et al. 2011; Gould 2014), only the \( \pi_{E,E} \) component of the microlensing parallax vector is measured precisely. As shown in the left panel of Figure 4, the 2-σ range for \( \pi_{E,N} \) is \(-0.39 < \pi_{E,N} < 0.43 \). However, the microlensing parallax vector, \( \pi_E \) is parallel to the \( \mu_{\text{rel},G} \) vector, and the two quantities are related by

\[ \pi_E = \frac{\pi_{\text{rel}}}{t_E} \frac{\mu_{\text{rel},G}}{|\mu_{\text{rel},G}|^2}, \]

so with measurements of \( \pi_{E,E} \) and \( \mu_{\text{rel},H} \), we can use equations 11 and 12 to solve for \( \pi_{E,N} \) (Ghosh et al. 2004; Bennett et al. 2007). Gould (2014) has shown that equations 11 and 12 can be converted to a quadratic equation in \( \pi_{E,N} \). A quadratic equation means two solutions, and Gould argued that this presented an important degeneracy that could lead to an ambiguous interpretation, but we find that this is generally not the case. For our measured value of \( \mu_{\text{rel},H} \), the degenerate solutions require that one solution had \( \pi_{\text{rel}} < 0 \), which would imply that the lens is (unphysically) more distant than the source. So, there is a unique solution in the case of OGLE-2012-BLG-0950. If the
Fig. 4.—Left panel: The $\pi_E$ distribution from light curve modeling without any constraint from follow-up observations. Right panel: The $\pi_E$ distribution resulting from the addition of the high resolution follow-up imaging constraints. The following color scheme is used to denote the $\chi^2$ differences from the best fit light curve model: black represents $\Delta \chi^2 < 1$, red represents $\Delta \chi^2 < 4$, green represents $\Delta \chi^2 < 16$, cyan represents $\Delta \chi^2 < 25$, and magenta represents $\Delta \chi^2 \geq 25$. The right panel clearly shows that the relative proper motion measurements from HST and Keck constrain $\pi_{E,N}$, which is the North component of $\pi_E$, that was largely unconstrained by the light curve. Without the $\mu_{\text{rel},H}$ measurement, in the left panel, the light curve slightly favors solutions with $\pi_{E,N} < 0$, but the constraint forces $\pi_{E,N} > 0$. Note that this figure combines both the degenerate $u_0 > 0$ and $u_0 < 0$ models.
sign of $\mu_{\text{rel}, H}$ was reversed, then there would be some degeneracy in the $\pi_{E,E}$ values at large $|\pi_{E,E}|$, but these would only be important for $D_L < 0.08 \text{kpc}$. In general, this degeneracy is not important when $|\mu_{\text{rel}, H}| \gg |v_0 \pi_{\text{rel}}/\text{AU}|$, as is the case for virtually all microlensing events observed towards the Galactic bulge.

In order to obtain good sampling of light curves that are consistent with our constraints, we apply constraints inside our modeling code to ensure that the Heliocentric proper motion and lens magnitudes are consistent with the Keck and HST observations. These constraints are $\mu_{\text{rel}, H,N} = 5.33 \pm 0.26$, $\mu_{\text{rel}, H,E} = -2.46 \pm 0.26$, $V_L = 22.18 \pm 0.26$, $I_L = 19.51 \pm 0.09$, and $K_L = 17.21 \pm 0.14$. They are implemented by calculating a $\chi^2$ contribution from each of the constraints and adding it to the light curve fit $\chi^2$ inside the modeling code (Bennett 2010). This requires the use of a mass-luminosity relation. As argued in Bennett et al. (2018), an empirical mass luminosity relation is preferred for lens masses $\lesssim 0.7 M_\odot$. Following Bennett et al. (2018), we use a combination of mass-luminosity relations for different masses. For $M_L \geq 0.66 M_\odot$, $0.54 M_\odot \geq M_L \geq 0.12 M_\odot$, and $0.10 M_\odot \geq M_L \geq 0.07 M_\odot$, we use the relations of Henry & McCarthy (1993), Delfosse et al. (2000), and Henry et al. (1999), respectively. In between these mass ranges, we linearly interpolate between the two relations used on the boundaries. That is, we interpolate between the Henry & McCarthy (1993) and the Delfosse et al. (2000) relations for $0.66 M_\odot > M_L > 0.54 M_\odot$, and we interpolate between the Delfosse et al. (2000) and Henry et al. (1999) relations for $0.12 M_\odot > M_L > 0.10 M_\odot$.

For the mass-luminosity relations, we must also consider the foreground extinction. At a Galactic latitude of $b = -4.634^\circ$, and a lens distance of $\sim 2 \text{kpc}$, the lens system is likely to be behind some, but not all, of the dust that is in the foreground of the source. We assume a dust scale height of $h_{\text{dust}} = 0.10 \pm 0.02 \text{kpc}$, so that the extinction in the foreground of the lens is given by

$$A_{i,L} = \frac{1 - e^{-|D_L (\sin b)/h_{\text{dust}}|}}{1 - e^{-|D_S (\sin b)/h_{\text{dust}}|}} A_{i,S} \ , \quad (13)$$

where the index $i$ refers to the passband: $I$, $V$, or $K$. In the Markov Chain calculations themselves, we fix $D_S = 8.0 \text{kpc}$ for our source star at a Galactic longitude of $l = 1.7647$, and we fix the dust scale height at $h_{\text{dust}} = 0.10 \text{kpc}$. But, we remove these restrictions by reweighting the links in the Markov Chain when we sum them for our final results.

These five constraints have a very small effect on the overall $\chi^2$. The addition of these constraints increases $\chi^2$ by $\Delta \chi^2 = 1.41$, so it is clear that the light curve is quite consistent with these constraints.

While these constraints have almost no impact on the best fit model $\chi^2$, they have a dramatic effect on the allowed range of microlensing parallax parameters, as Figure 4 indicates. The 2-$\sigma$ range for $\pi_{E,N}$ is reduced from $-0.39 < \pi_{E,N} < 0.43$ to $0.18 < \pi_{E,N} < 0.25$, a reduction of a factor of 12 in uncertainty. This yields a microlensing parallax amplitude of $\pi_E = 0.265 \pm 0.21$, which will be used in Section 9 to determine the lens mass.
9. Lens Properties

For most planetary microlensing events, finite source effects provide a measurement of the source radius crossing time, \( t^* \). This allows the angular Einstein radius, \( \theta_E \), to be determined with the equation

\[
\theta_E = \theta^* t_E / t^*,
\]

where \( \theta^* \) is the angular source radius, which can be determined by the source brightness and color (Kervella et al. 2004; Boyajian et al. 2014). However, \( t^* \), was not measured for OGLE-2012-BLG-0950, because this event did not reveal any finite source effects. Fortunately, it is also possible to determine \( \theta_E \) from \( \mu_{rel,G} \), which can be determined from the measured values of \( \mu_{rel,H} \) and \( \pi_{E,E} \) using equations 11 and 12. The relation between the length of the \( \mu_{rel,G} \) vector and \( \theta_E \) is

\[
\theta_E = \mu_{rel,G} \cdot \theta_E.
\]

The measurement of the either the angular Einstein radius, \( \theta_E \), or the microlensing parallax amplitude, \( \pi_E \), will provide a mass-distance relation, if we assume that the source distance, \( D_S \), is known (Bennett 2008; Gaudi 2012),

\[
M_L = \frac{c^2 \theta^2_E}{4G} \frac{D_S D_L}{D_S - D_L} = \frac{c^2 \text{AU} D_S - D_L}{4G \pi^2 E}.
\]  \hspace{1cm} (14)

When both \( \theta_E \) and \( \pi_E \) are known, the two mass-distance relations in equation 14 can be multiplied together, yielding

\[
M_L = \frac{c^2 \theta_E \text{AU}}{4G \pi_E} \frac{\theta_E}{(8.1439 \text{mas})} M_\odot,
\]  \hspace{1cm} (15)

which is a direct mass measurement with no dependence on \( D_L \) or \( D_S \).

To solve for the planetary system parameters, we sum over our MCMC results using the Galactic model employed by Bennett et al. (2014) as a prior, weighted by the microlensing rate and the measured \( \mu_{rel,H} \) value. The lens magnitude measurements were applied as constraints in the light curve modeling, so we do not apply them again in the sum over the MCMC results. We do constrain the source distances to follow the microlensing rate weighted distribution according to our Galactic model, and we evaluate the extinction in the foreground of the lens using equation 13 with the assumed error bar for \( h_{dust} \). The Galactic model is used to consider the uncertainty of the source distance. However, using a fixed source distance of \( D_S = 8.2 \text{ kpc} \) does not alter the results.

Figure 5 provides a graphical summary of the constraints on the host star in the mass-distance plane. The constraint from the one-dimensional microlensing parallax only is the magenta shaded region, while the red, black and blue curves give the \( K, I \) and \( V \)-band constraints from the Keck and HST follow-up observations, with 1-\( \sigma \) error bars as dashed lines. Note that a single passband flux measurement combined with the one-dimensional parallax constraint does not yield a unique host star mass. The combination of lens flux constraints in 3 passbands does somewhat better, but it is the \( \mu_{rel,H} \) measurement that gives the full \( \pi_E \) determination indicated by the green shaded region. This is the critical feature that provides the precise determination of the host star mass, planet mass and lens distance following the method described in section 8. The host mass is measured to be \( M_* = 0.58 \pm 0.04 M_\odot \), an early M or late K dwarf star, orbited by a twice Neptune-mass planet, \( m_p = 39 \pm 8 M_\oplus \) at a projected separation of \( a_\perp = 2.54 \pm 0.23 \text{AU} \). This also implies a lens system
Fig. 5.— The mass-distance relation obtained from the microlensing parallax parameter determined by the light curve models with the $\mu_{\text{rel,H}}$ constraint from the HST and Keck follow-up observations is plotted in green. The mass-distance relation from the light curve model only, with no additional constraint on $\pi_{E,N}$ is shaded in magenta. The mass-distance relations obtained from the $K$, $I$, and $V$-band mass luminosity relations with the lens flux constraints from Table 3 are plotted in red (Keck $K$), black (HST $I$) and blue (HST $V$). The solid lines are best fit values as a function of mass and distance. The dashed lines show the $1\sigma$ error bars. All three independent flux measurements in three different passbands result in the same solution. This confirms our identification of the lens star.
distance of $D_L = 2.19 \pm 0.23$ kpc. The fact that all three excess flux measurements give the same mass and distance indicates that there is no contamination of measurements from additional flux from another star (Bhattacharya et al. 2017; Koshimoto et al. 2017b).

We also use the lens flux and relative lens-source proper motion measurements to constrain the light curve models. The results of our final sum over the Markov Chain light curve models are given in Table 6 and Figure 6. This table gives the mean and RMS uncertainty plus the central 95.4% confidence interval range for each parameter except the 3D separation, $a_{3D}$, where we give the median the central 68.3% confidence interval. The lens flux and the parallax measurements exclude most of the masses and distances for this planetary system that were compatible with Bayesian analysis a MCMC of light curve model without any $\mu_{\text{rel},H}$ or lens brightness constraints. This constrains the parameters: host star mass, planet mass, their separation and their distance from earth as shown in Table 4. They are consistent with the parameters measured using the empirical mass-luminosity relations described in section 8 as described in previous paragraph. Assuming a random orientation, this implies 3-dimensional separation of $a_{3d} = 3.0^{+1.7}_{-0.5}$ AU. The uncertainties are calculated as RMS over the MCMC links.

10. Discussion and Conclusions

With near simultaneous high angular resolution follow-up observations from Keck and HST, we have measured the angular separation of the source and planetary host star to be 34 mas in three different passbands, $K$, $I$, and $V$. This separation measurement allows us to convert the partial measurement of the microlensing parallax from the light curve into a complete measurement of the two dimensional $\pi_E$ vector. The combination of this microlensing parallax measurement and the magnitude of the lens, determined independently in three different passbands. This rules out alternative explanations of this event involving additional stars such as a companion to the lens or source. The one, highly unlikely possibility that is not yet excluded is that the detected excess flux could come from a $\sim 0.6 M_\odot$ companion to a white dwarf planetary host star that is also $\sim 0.6 M_\odot$. According a statistical study by Holberg et al. (2013), the probability of a white dwarf hosting a

| parameter | units | values & RMS | 2-$\sigma$ range |
|-----------|-------|--------------|-----------------|
| Host star mass, $M_*$ | $M_\odot$ | $0.58 \pm 0.04$ | 0.51–0.67 |
| Planet mass, $m_p$ | $M_\oplus$ | $39 \pm 8$ | 26–59 |
| Host star - Planet 2D separation, $a_\perp$ | AU | $2.54 \pm 0.23$ | 2.13–3.03 |
| Host star - Planet 3D separation, $a_{3D}$ | AU | $3.0^{+1.7}_{-0.5}$ | 2.2–10.8 |
| Lens distance, $D_L$ | kpc | $2.19 \pm 0.23$ | 1.77–2.68 |
Fig. 6.— The Bayesian posterior probability distributions for the planetary companion mass, host mass, their separation and the distance to the lens system are shown with only light curve constraints in blue and with the additional constraints from our Keck and HST follow-up observations in red. The central 68.3% of the distributions are shaded in darker colors (dark red and dark blue) and the remaining central 95.4% of the distributions are shaded in lighter colors. The vertical black line marks the median of the probability distribution of the respective parameters.
main sequence companion is $\sim 8\%$, but the vast majority of these white dwarf-main sequence star binaries have separations that are much too large for the main sequence star to be confused with the lens. So, the fraction of white dwarfs with binary companions that could be confused with the lens star is about 1%. However, it is not guaranteed that a binary companion to a white dwarf lens would have $V$, $I$, and $K$-band magnitudes compatible with the lens mass inferred from the $\pi_{E,E}$ measurement and the apparent $\mu_{rel,H}$ measurement, so the probability that a white dwarf host with a binary companion could produce the light curve and followup data for this event is $\lesssim 0.1\%$. To avoid a light curve signal from this binary companion, this companion would need to be separated by $\gtrsim 10\theta_E \approx 13$ mas from the lens at the time of the event. This means that this unlikely possibility of a white dwarf host star with a main sequence companion could be tested with an additional epoch of followup observations to confirm that the relative proper motion of the main sequence star does extrapolate back to the position of the source at the time of the event, as we have shown for planetary microlensing event OGLE-2005-BLG-169 (Bennett et al. 2015; Batista et al. 2015).

The measured planetary mass of $m_p = 39 \pm 8 M_\oplus$ is of particular interest because the core accretion theory predicts that such planets should be rare. The core accretion theory includes a runaway gas accretion phase (Pollack et al. 1996; Lissauer et al. 2009) that is thought to imply that planets in the mass range 20-80$M_\oplus$ are rare. According to this theory, beyond the snow line, a planetary core rapidly grows by the accumulation of planetesimals until it reaches a mass of $\sim 10 M_\oplus$ (Pollack et al. 1996; Rafikov 2011). Further growth is dominated by gas accretion that starts slowly, but when the gas mass grows to equal the core mass, but growth is thought to become a runaway exponential growth process. This process is thought to continue very rapidly until it is terminated by a lack of gas at a mass similar to that of Jupiter ($318 M_\oplus$) or possibly Saturn ($95 M_\oplus$).

The cold exoplanet mass ratio function measured by Suzuki et al. (2016) finds no evidence for a dearth of planets at these intermediate 20-80$M_\oplus$ masses. The measured mass ratio function increases smoothly from a mass ratio of $q = 0.03$ down to a mass ratio of $q \approx 10^{-4}$, where it reaches a peak (Udalski et al. 2018). There is no evidence of a mass ratio gap at $1-4 \times 10^{-4}$, where we would expect to see this expected low occurrence rate of 20-80$M_\oplus$ mass planets. However, since the Suzuki et al. (2016) considers only mass ratios and not masses, it remains possible that a gap in the exoplanet mass distribution is smoothed out by the combination of a range of host stars from very low mass stars up to solar type stars. Perhaps planet formation is different for very low mass stars in a way that smooths out the gap in the exoplanet mass distribution around solar type stars.

Our follow-up high angular resolution imaging program addresses this issue directly by determining the masses of the microlens host stars and their planets. The planet OGLE-2012-BLG-0950Lb is the first planet from the Suzuki et al. (2016) sample to have a mass measured to be in the 20-80$M_\oplus$ range, but our ongoing follow-up observing program will measure masses of more host stars and planets from the Suzuki et al. (2016) sample to provide a more definitive answer to this question. Our program has also identified a similar mass planet OGLE-2012-BLG-0026Lb
(Beaulieu et al. 2016) with a mass of $46.0 \pm 2.5 M_\odot$ orbiting a solar type star. This planet is one of two planets detected in this microlensing event that is, unfortunately, not part of the Suzuki et al. (2016) sample.

This work is also an important step development in the development of the exoplanet mass measurement methods (Bennett et al. 2007) for the WFIRST microlensing exoplanet survey (Spergel et al. 2015). This requires accurate relative astrometry of the blended source plus lens stars, and in this analysis we have measured the lens-source relative proper motion to a precision (in the $I$-band) of 2%, despite the fact that the lens and source were separated by $\lesssim 0.5$ FWHM. The Bhattacharya et al. (2017) measurement of the separation of a source and blend star 0.17 FWHM to a precision of 27% might be considered even more impressive, but this case of OGLE-2012-BLG-0950 involves a lens star, instead of an unrelated blend, and it has been independently confirmed in three passbands (instead of only 2).

This analysis has also been the first example of a full microlensing parallax measurement being obtained from light curve measurement of one component of the $\pi_E$ vector and a follow-up measurement of the Heliocentric lens-source relative proper motion, $\mu_{\text{rel,H}}$. This is also an important part of the WFIRST exoplanet mass measurement tool set (Bennett et al. 2007), and in most cases, it will allow for mass measurements that are independent of the flux detected from the host star. The flux of the lens is also measured in three passbands giving rise to additional redundant consistent constraints on the host star and planet masses and distance. The lens and source are not separately resolved in any of the images, but still we measure the separation at a high significance. This combination of the unresolved lens-source separation measurement and flux measurement in multiple passbands, plus 1-d parallax measurement combines all the major WFIRST mass measurement methods, as discussed in Bennett et al. (2007).

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