Nonlinearly Realized Extended Supergravity

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Abstract

We provide nonlinear realization of supergravity with an arbitrary number of supersymmetries by means of coset construction. The number of gravitino degrees of freedom counts the number of supersymmetries, which will be possibly probed in future experiments. We also consider goldstino embedding in the construction to discuss the relation to nonlinear realization with rigid supersymmetries.
1 Introduction

Supersymmetry (SUSY) is expected to be a basic structure of Nature within the description in terms of effective relativistic field theory with fundamental bosons and fermions. It is even a leading candidate as low-energy physics just beyond the standard model of elementary particles at the electroweak scale.

Furthermore, quantum theory with the maximal SUSY, as a hidden ingredient broken in some way, seems unique enough as a unified theory of elementary particles. As is well known, it is investigated under the name of string/M theory, which has no arbitrary parameter to be defined. In such a perspective, SUSY may be a defining property of the basic laws in Nature.

The above point of view implies that a possible experimental discovery of extended SUSY would constitute a rather direct evidence to elucidate the string/M theory as a fundamental theory of Nature. Although several phenomenological motivations are advocated for simple SUSY, it is uncertain whether SUSY is relevant at low energy. In fact, we have no direct experimental evidence of SUSY yet, which means that extended SUSY is on an equal footing as simple SUSY to be discovered in future experiments.

One of the characteristic features of extended SUSY is the existence of multiple superpartners. For definiteness, let us consider $\mathcal{N} = 2$ SUSY in four space-time dimensions. Possible extra superpartners to the standard model consist of “mirror” partners of gauginos, higgsinos, quarks and leptons. In particular, multiple gravitinos seem most compelling as an evidence of extended SUSY, since the presence of the other mirror partners might look like mere presence of extra matter multiplets observationally.

In this paper, we construct nonlinear realization of extended supergravity in four dimensions. Although it is unclear how SUSY is broken at a fundamental level, nonlinear realization possibly allows us to investigate a manifestation of the hidden SUSY structure behind interactions of elementary particles. The gravitino masses turn out to determine universal interactions of the gravitinos with matter fields. As an example of concrete

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1 As an analogy, we may think that a possible discovery of extra dimensions would come up with five-dimensional space-time or yet higher-dimensional one on an equal footing. We also note that SUSY in higher dimensions implies extended SUSY in four dimensions.

2 Various investigations have been pursued already to consider $\mathcal{N} = 2$ particle physics. For a (surely incomplete) list of related literature, see Ref. [1].
implications, we note an exotic possibility that the mirror gravitino may be discovered prior to the usual gravitino in the minimal SUSY standard model.

Nonlinear realization describes low-energy effective field theory of light degrees of freedom in a way independent of concrete symmetry-breaking models. The form of the effective action is determined solely by the low-energy symmetries of the theory. The nonlinear realization of $\mathcal{N} = 1$ global SUSY was given by Akulov and Volkov [2] (for recent investigations, see Ref.[3]). Their action reads

$$\Gamma_{AV} = -M_s^4 \int d^4x \det \left( \delta^\nu_\mu - \frac{i}{M_s^4} \lambda^{\nu \sigma} \bar{\lambda}^{\sigma \mu} \right),$$  

with

$$x'^\mu = x^\mu + \frac{i}{\sqrt{2}M_s^2} [\xi^{\mu \sigma} \bar{\lambda}(x) - \lambda(x) \bar{\sigma}^{\mu} \xi],$$  

where $\lambda_\alpha(x), \bar{\lambda}_{\dot{\alpha}}(x)$ denote the goldstino field and $M_s$ is the SUSY-breaking scale. The SUSY transformation law is given by

$$\lambda'_\alpha(x') = \lambda_\alpha(x) + \sqrt{2}M_s^2 \xi_\alpha, \quad \bar{\lambda}'_{\dot{\alpha}}(x') = \bar{\lambda}_{\dot{\alpha}}(x) + \sqrt{2}M_s^2 \bar{\xi}_{\dot{\alpha}},$$  

with $M_{pl}$ the reduced Planck mass scale. We henceforth adopt the Planck unit $M_{pl} = 1$. In a similar manner, the case of extended SUSY was considered by Ferrara, Maiani, and West [5]. They discussed the extended local SUSY nonlinear realization up to the second order just like the $\mathcal{N} = 1$ case by Deser and Zumino.

In contrast, coset construction of nonlinear realization developed by Callan, Coleman, Wess, and Zumino [6] does not involve such an approximation and is valid to all orders.
with regard to the fields contained in the action. Therefore, we employ this technique to construct nonlinearly realized extended supergravity, which contains gravitinos in addition to ordinary matters without the need for their superpartners. The case with $\mathcal{N} = 1$ local SUSY is considered by Clark, Love, Nitta, and ter Veldhuis [7]. Our construction is a generalization of theirs to the case with extended SUSY.

The rest of the paper is organized as follows. In the next section, we introduce nonlinearly realized extended supergravity by means of coset construction technique. In section 3, we discuss goldstino embedding in our locally SUSY theories. The final section concludes the paper with some comments on future directions.

2 Coset Construction

In this section, we perform the coset construction of nonlinearly realized supergravity with an arbitrary number of SUSY. We first recapitulate coset construction for a general case of global symmetries.

Here some definitions follow. $G$ is the symmetry group of internal and space-time symmetries of a theory. For example, in our case, $G$ is given by the extended SUSY algebra,

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\sigma}),$$

$$[M_{\mu\nu}, P_\lambda] = i(P_\mu\eta_{\lambda\nu} - P_\nu\eta_{\mu\lambda}),$$

$$[M_{\mu\nu}, Q_{A\alpha}] = -\frac{1}{2}(\sigma^{\mu\nu})_{\alpha}{}^{\beta}Q_{A\beta},$$

$$[M_{\mu\nu}, \bar{Q}_{A\dot{\alpha}}] = \frac{1}{2}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}}\bar{Q}_{A\dot{\beta}},$$

$\{Q_{A\alpha}, \bar{Q}_{B\dot{\alpha}}\} = 2\sigma_{\alpha\dot{\alpha}}^{\mu} \delta_{AB} P_\mu$,

$\{Q_{A\alpha}, Q_{B\beta}\} = \epsilon_{\alpha\beta} X_{AB}$,

$\{\bar{Q}_{A\dot{\alpha}}, \bar{Q}_{B\dot{\beta}}\} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{X}_{AB}$,

with the other commutators vanishing and $\eta_{\mu\nu} = \text{diag}[+1, -1, -1, -1]$.

The generators of the group $G$ can be divided into the following three sets:

- $P_\mu$ : the generators of space-time translations,
- $\Gamma_a$ : the generators of spontaneously broken internal and space-time symmetries,
• $\Gamma_i$: the generators of unbroken internal symmetries and space-time rotations, which form a subgroup $H$ of $G$.

Then, we define the coset space $G/H$ with the equivalence relation $\Omega \sim \Omega h$ for $\Omega \in G$ and $h \in H$. A representative element of the coset space $G/H$ is written as

$$\Omega = \exp[i x_\mu P^\mu] \exp[i N^a(x) \Gamma_a],$$

(6)

where $x_\mu$ denote space-time coordinates and $N^a(x)$ are generalized Nambu-Goldstone fields which contain goldstinos associated with spontaneously broken global SUSY.

The action of a group element $g$ of $G$ on a coset space element as $\Omega \to \Omega'$ is defined by

$$g\Omega = \Omega' h,$$

(7)

where

$$\Omega' = \exp[i x'_\mu P^\mu] \exp[i N'^a(x') \Gamma_a],$$

(8)

and $h = \exp[i \alpha^i(g, x, N) \Gamma_i] \in H$ with $\alpha^i(g, x, N)$ as a function of $g$, $x$, and $N$. That is, the induced transformations of $x_\mu$ and $N^a(x)$ as

$$x_\mu \to x'_\mu, \quad N^a(x) \to N'^a(x')$$

(9)

are given through the group action.

In order to construct invariant actions under the symmetry group $G$, we now proceed to introduce the Maurer-Cartan 1-form, which is a Lie algebra valued 1-form defined by

$$\Omega^{-1} d\Omega = i(\omega^\mu P_\mu + \omega^a \Gamma_a + \omega^i \Gamma_i),$$

(10)

where $d$ denotes the space-time exterior derivative. The transformation of the Maurer-Cartan 1-form under $G$ turns out to be

$$\Omega^{-1} d\Omega \to h(\Omega^{-1} d\Omega) h^{-1} + h dh^{-1}.$$  

(11)

Notice that only the $h$ appears in the above expression. This property is crucial to construct invariant actions of nonlinearily realized extended supergravity below.
2.1 The transformation laws

Now we further discuss basic building blocks and their transformation properties to construct invariant actions of nonlinearly realized extended supergravity, taking into account the local nature of SUSY transformations in supergravity.

We adopt the extended SUSY algebra given in the above Eq. (5) with $H$ as the Lorentz group. Then, an element of the coset space $G/H$ is written as

$$\Omega(x) = e^{i x^\mu P_\mu} e^{i \lambda_A^a(x) Q_{Aa} + i \lambda_{Aa}(x) Q_A^a + C_{AB}(x) X_{AB} + \bar{C}_{AB}(x) X_{AB}]} ,$$

where the indices $A$ and $B$ run from 1 to $\mathcal{N}$, that is, the number of SUSY in four space-time dimensions, and $\lambda_A^a(x)$ are the goldstino fields associated with $\mathcal{N}$ SUSY. The $C_{AB}(x)$ and $\bar{C}_{AB}(x)$, which are antisymmetric in $A$ and $B$, are the Nambu-Goldstone fields associated with $\mathcal{N} (\mathcal{N} - 1)$ central charges.

The action of an element $g$ of the group $G$ to $G/H$ is given in the same way as for the global case by

$$g\Omega = \Omega' h,$$

though, in the local case with $x$-dependent $g$, we have

$$g(x) = e^{i x^\mu P_\mu} e^{i \lambda_A^a(x) Q_{Aa} + i \lambda_{Aa}(x) Q_A^a + C_{AB}(x) X_{AB} + \bar{C}_{AB}(x) X_{AB}]} e^{i \gamma_{\alpha}(x) M_{\alpha\beta}},$$

$$\Omega'(x') = e^{i x'^\mu P_\mu} e^{i \lambda_A'^a(x') Q_{Aa} + i \lambda_{Aa}'(x') Q_A^a + C_{AB}'(x') X_{AB} + \bar{C}_{AB}'(x') X_{AB}]} e^{i \gamma_{\alpha}(x') M_{\alpha\beta}},$$

where $\gamma^\mu(x), \xi_A^a(x), \zeta_{AB}(x)$, and $\alpha_{\mu\nu}(x)$ denote local infinitesimal parameters.

The Baker-Campbell-Hausdorff formula,

$$e^X e^Y = e^{X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] + [Y,[Y,X]] + \cdots},$$

yields the nonlinear extended local SUSY transformation laws of the generalized Nambu-Goldstone fields, which are given by

$$\lambda'_{Aa}(x') = \lambda_{Aa}(x) + \xi_{Aa}(x) + \frac{i}{4} \alpha_{\mu\nu}(x) (\sigma^{\mu\nu})_{\alpha}^\beta \lambda_{A\beta}(x),$$

$$\bar{\lambda}'_{\dot{A}\dot{a}}(x') = \bar{\lambda}_{\dot{A}\dot{a}}(x) + \bar{\xi}_{\dot{A}\dot{a}}(x) - \frac{i}{4} \alpha_{\mu\nu}(x) (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} \lambda_{\dot{A}\dot{\beta}}(x),$$

$$C'_{AB}(x') = C_{AB}(x) + \zeta_{AB}(x) - \frac{i}{2} \xi_{[A\lambda_B]},$$

$$\bar{C}'_{\dot{A}\dot{B}}(x') = \bar{C}_{\dot{A}\dot{B}}(x) + \bar{\zeta}_{\dot{A}\dot{B}}(x) - \frac{i}{2} \bar{\xi}_{[\dot{A}\bar{\lambda}_{\dot{B}}]},$$

where $\alpha_{\mu\nu}(x)$ are the local infinitesimal parameters.
with
\[ x'^\mu = x^\mu + \Delta x^\mu \equiv x^\mu + \epsilon^\mu(x) + i[\xi_A(x)\sigma^\mu \lambda_A(x) - \lambda_A(x)\sigma^\mu \xi_A(x)] - \alpha^\mu(x)x_\nu, \] (17)
where the square bracket \([AB]\) for the indices indicates their antisymmetrization. These constitute extended local SUSY generalization of the nonlinear transformation law Eq. (2) proposed by Akulov and Volkov.

How about the Maurer-Cartan form? We would like to maintain the transformation property of the Maurer-Cartan 1-form in the global case, though we now consider the local transformation \(g(x)\), which results in a deviated transformation property of \(\Omega^{-1}d\Omega\). Therefore, we define our locally covariant Maurer-Cartan 1-form through replacing the exterior derivative by a covariant derivative, that is,
\[ \omega = \Omega^{-1}D\Omega \equiv \Omega^{-1}(d + iE)\Omega \]
\[ = i\left[ \omega^m P_m + \frac{1}{2}\omega^\alpha Q_{A\alpha} + \frac{1}{2}\tilde{\psi}_{\dot{A}\dot{a}} Q^\dot{A}_{\dot{a}} + \omega_{XAB} X_{AB} + \tilde{\omega}_{\bar{X}AB} \bar{X}_{AB} + \frac{1}{2}\gamma^mn M_{mn} \right], \] (18)
where the indices \(m, n = 0, 1, 2, 3\) are hereafter used for the tangent space local Lorentz transformation, and
\[ E = \hat{E}^m P_m + \frac{1}{2}\psi^\alpha Q_{A\alpha} + \frac{1}{2}\tilde{\psi}_{\dot{A}\dot{a}} Q^\dot{A}_{\dot{a}} + A_{AB} X_{AB} + \bar{A}_{AB} \bar{X}_{AB} + \frac{1}{2}\gamma^mn M_{mn} \] (19)
is the 1-form gravitational field that is also a Lie algebra valued 1-form. As we see later, \(\hat{E}^m\) represents the graviton, \(\psi^\alpha, \tilde{\psi}_{\dot{A}\dot{a}}\) are the gravitinos, \(A_{AB}, \bar{A}_{AB}\) are the gauge fields associated with the central charges, and \(\gamma^mn\) is the spin connection. Here, the 1-form gravitational field transforms as a gauge field:
\[ E'(x') = g(x)E(x)g^{-1}(x) - ig(x)dg^{-1}(x). \] (20)
Then, the form of the transformation law for this modified Maurer-Cartan 1-form is the same as that for the original one, namely,
\[ \omega'(x') = h(x)\omega(x)h^{-1}(x) + h(x)dh^{-1}(x). \] (21)

We can extract the transformation laws of the components of Maurer-Cartan 1-form by expanding the above transformation in terms of the charges of the symmetry group...
The transformation laws of the component 1-form fields are given by

\[
\begin{align*}
\omega'(x') &= \omega(x)\Lambda^m_n(\alpha(x)), \\
\omega'_{QA}\alpha(x') &= D_{\alpha}^{(\frac{1}{2},0)}(\alpha(x))\omega_{QA}, \\
\bar{\omega}'_{QA}\dot{\alpha}(x') &= D^{(0,\frac{1}{2})}_{\dot{\beta}}(\alpha(x))\bar{\omega}'_{QA}, \\
\omega'_{XAB}(x') &= \omega_{XAB}(x), \\
\bar{\omega}'_{XAB}(x') &= \bar{\omega}_{XAB}(x), \\
\omega'_{M}^{mn}(x') &= \omega_{M}^{rs}(\alpha(x))\Lambda^m_r(\alpha(x))\Lambda^n_s(\alpha(x)) - d\alpha^{mn}(x),
\end{align*}
\]

where the infinitesimal local Lorentz transformation and the corresponding spinor transformation matrices are denoted by

\[
\begin{align*}
\Lambda^m_n(\alpha(x)) &= \delta^m_n - \alpha^m_n(x), \\
D_{\alpha}^{(\frac{1}{2},0)}(\alpha(x)) &= \delta_{\alpha} + i\frac{1}{4}\alpha_{mn}(x)(\sigma^{mn})_{\alpha}, \\
D^{(0,\frac{1}{2})}_{\dot{\beta}}(\alpha(x)) &= \delta_{\dot{\beta}} + i\frac{1}{4}\alpha_{mn}(x)(\bar{\sigma}^{mn})_{\dot{\beta}}.
\end{align*}
\]

The above expressions show that the components of locally covariant Maurer-Cartan 1-form are subject to the appropriate local Lorentz transformations for their indices.

The gauge transformation of the gravitational 1-form can be also expanded by the charges of the symmetry group $G$. The transformation laws of the components are given by

\[
\begin{align*}
\hat{E}'^m &= \hat{E}^m + \gamma^{mn}\epsilon_n + i(\xi_A\sigma^m\bar{\psi}_A - \psi_A\sigma^m\bar{\xi}_A) - \alpha^{mn}\hat{E}_n - d\epsilon^m, \\
\psi'_A &= \psi_A - i\frac{1}{4}\alpha_{mn}(\bar{\psi}_A\sigma^{mn})\alpha + i\frac{1}{2}\gamma_{mn}(\bar{\xi}_A\sigma^{mn})\alpha - 2d\xi_A, \\
\bar{\psi}'_{A\dot{a}} &= \bar{\psi}_{A\dot{a}} - i\frac{1}{4}\alpha_{mn}(\bar{\psi}_{A\dot{a}}\sigma^{mn})\dot{\alpha} + i\frac{1}{2}\gamma_{mn}(\bar{\xi}_{A\dot{a}}\sigma^{mn})\dot{\alpha} - 2d\xi_{A\dot{a}}, \\
\Lambda'_{AB} &= \Lambda_{AB} - i\frac{1}{2}\xi_{[A}\psi_{B]} - d\zeta_{AB}, \\
\bar{\Lambda}'_{AB} &= \bar{\Lambda}_{AB} - i\frac{1}{2}\xi_{[A}\bar{\psi}_{B]} - d\bar{\zeta}_{AB}, \\
\gamma^{mn} &= \gamma^{mn} + (\alpha^{mr}\xi_r^n - \alpha^{mr}\xi_r^m) - d\alpha^{mn},
\end{align*}
\]

which amount to the usual transformation laws of the vierbein, gravitino, and spin connection in supergravity.
We finally expand the locally covariant Maurer-Cartan 1-form $\omega$ by the charges of $G$ to obtain

$$
\omega^m = dx^m - i[\lambda_A \sigma^m(d\bar{\lambda}_A + \bar{\psi}_A) - (d\lambda_A + \psi_A)\sigma^m\bar{\lambda}_A] + E^m \\
+ \frac{1}{4} \gamma_{rs} \lambda_A (\sigma^m \bar{\sigma}^s + \sigma^s \bar{\sigma}^m) \bar{\lambda}_A,
$$

$$
\omega_{QA} = 2d\lambda^A + \psi^A - i \frac{1}{2} \gamma_{mn} (\lambda_A \sigma^{mn})^A,
$$

$$
\bar{\omega}_{QA} = 2d\bar{\lambda}_{A\dot{A}} + \bar{\psi}_{A\dot{A}} - i \frac{1}{2} \gamma_{mn} (\bar{\lambda}_A \bar{\sigma}^{mn})_{\dot{A}},
$$

$$
\omega_{XAB} = i \frac{1}{2} \lambda_A [d\lambda_B] + dC_{AB} + i \frac{1}{2} \lambda_A \bar{\psi}_B + A_{AB} + \frac{1}{8} \gamma_{mn} \lambda_A \sigma^{mn} \lambda_B,
$$

$$
\bar{\omega}_{XAB} = i \frac{1}{2} \bar{\lambda}_A [d\bar{\lambda}_B] + d\bar{C}_{AB} + i \frac{1}{2} \bar{\lambda}_A \bar{\psi}_B + \bar{A}_{AB} + \frac{1}{8} \gamma_{mn} \bar{\lambda}_A \bar{\sigma}^{mn} \bar{\lambda}_B,
$$

$$
\omega_{mn} = \gamma^{mn}.
$$

Where $E^m = \hat{E}^m - \gamma^{mn} x_n$. In the next subsection, we construct invariant actions under $G$ by using the locally covariant Maurer-Cartan 1-form and its transformation law obtained above.

### 2.2 The invariant actions

We now proceed to construct gauge-invariant actions under the symmetry group $G$ and express them in terms of the component fields. Let us first describe the components $\omega^m$ of locally covariant Maurer-Cartan 1-form by the space-time coordinate differential as $\omega^m = dx^\mu e^m_\mu$, where $e^m_\mu$ is the "vierbein" of extended local SUSY given by

$$
e^m_\mu = \delta^m_\mu - 2i \lambda_A \bar{\psi}_A \sigma^m \bar{\lambda}_A + E^m_\mu - i(\lambda_A \sigma^m \bar{\psi}_A - \psi_A \sigma^m \bar{\lambda}_A) + \frac{1}{4} \gamma_{rs} \lambda_A (\sigma^m \bar{\sigma}^s + \sigma^s \bar{\sigma}^m) \bar{\lambda}_A.
$$

As is presupposed in Eq. (17), the coordinate transformation induced by the $G$ transformation is given by $x'^\mu = x^\mu + \Delta x^\mu$. This leads to the following transformation law of the space-time coordinate differential:

$$
d x'^\mu = d x^\nu G^\mu_\nu(x), \quad G^\mu_\nu(x) = \frac{\partial x'^\mu}{\partial x^\nu}.
$$

This expression, together with the transformation Eq. (22), yields the transformation law of the "vierbein" as

$$
e^m_\mu (x') = G^{-1}_\nu(x) e^m_\nu(x) \Lambda^m_n(\alpha(x)).
$$
We can also define the "metric tensor" of extended local SUSY by means of the "vierbein" as

\[ g'_{\mu\nu}(x') = G_{\mu}^{-1\rho}(x)g_{\rho\sigma}(x)G_{\nu}^{-1\sigma}(x). \]  

(29)

The above preparation reveals that the construction of invariant actions under the local \( G \) transformation can be performed, with the aid of locally covariant Maurer-Cartan 1-form, in the same way as the construction of invariant actions under the general coordinate transformation. Namely, we can write such invariant actions as

\[ \Gamma = \int d^4x \, \det e(x) \mathcal{L}(x), \]  

(30)

with \( \mathcal{L}'(x') = \mathcal{L}(x) \), since \( d^4x' = d^4x \, \det G \) and thus \( d^4x' \, \det e'(x') = d^4x \, \det e(x) \).

We can construct invariant actions by using covariant derivatives of locally covariant Maurer-Cartan 1-form. The covariant derivatives of the components, \( \omega_{QA\alpha} = dx^\nu \omega_{QA\nu\alpha} \) and \( \bar{\omega}^\alpha_{\dot{Q}A} = dx^\nu \bar{\omega}^\alpha_{\dot{Q}A\nu} \), are given by

\[
\nabla_\mu \omega_{QA\alpha} = \partial_\mu \omega_{QA\alpha} + \frac{i}{4} \gamma_{mn}^{\alpha} (\sigma_{mn})^\beta_{\alpha} \omega_{QA\nu\beta} - \Gamma^\rho_{\mu\nu} \omega_{QA\alpha},
\]

\[
\nabla_\mu \bar{\omega}^\alpha_{\dot{Q}A} = \partial_\mu \bar{\omega}^\alpha_{\dot{Q}A} + \frac{i}{4} \gamma_{mn}^{\alpha} (\bar{\sigma}_{mn})^\dot{\beta}_{\alpha} \bar{\omega}^\dot{\beta}_{\dot{Q}A\nu} - \Gamma^\rho_{\mu\nu} \bar{\omega}^\alpha_{\dot{Q}A\rho},
\]

(31)

where

\[
\Gamma^\nu_{\sigma\rho} = \epsilon^\nu_n \partial_\rho e^m_\sigma - \epsilon^\nu_n \gamma^m_{\sigma\rho} \epsilon^s_\sigma \eta_{rs}
\]

(32)

is the "affine connection" in the extended supergravity.

Moreover, we are led to define the "Riemann curvature tensor" in the same way as that in the case of general relativity:

\[
R^\rho_{\sigma\mu\nu} = \partial_\nu \Gamma^\rho_{\sigma\mu} - \partial_\mu \Gamma^\rho_{\sigma\nu} + \Gamma^\lambda_{\sigma\mu} \Gamma^\rho_{\lambda\nu} - \Gamma^\lambda_{\sigma\nu} \Gamma^\rho_{\lambda\mu}.
\]

(33)

We can contract indices of the "Riemann curvature tensor" to obtain the "Ricci tensor," \( R_{\mu\nu} = R^\rho_{\mu\nu\rho} \), and the "scalar curvature," \( R = g^{\mu\nu} R_{\mu\nu} \).

With the aid of all these ingredients, we finally arrive at invariant actions of nonlinearly realized extended supergravity. In particular, the minimal action without the fields for
central charges that has only quadratic terms of the components in the locally covariant Maurer-Cartan 1-form is given by

\[ \Gamma = \int d^4x \det e \left\{ \Lambda - \frac{1}{2} R + \epsilon^{\mu \nu \rho \sigma} \omega_{Q A \mu} \sigma_{\sigma} \nabla_{\rho} \bar{\omega}_{Q A \nu} \\
+ \frac{i}{2} m^{\frac{3}{2}}_{AB} \{ \omega_{Q A \mu} \sigma^\alpha_{\alpha} \omega_{Q B \nu} + \bar{\omega}_{Q A \mu} \bar{\sigma}^\alpha_{\alpha} \bar{\omega}_{Q B \nu} \} \right\}, \tag{34} \]

where \( \Lambda \) is the cosmological constant. Note that the normalizations of kinetic terms can be so chosen without loss of generality by field rescalings.

When we adopt the unitary gauge, \( \lambda_{A \alpha} = \bar{\lambda}_{A \dot{\alpha}} = 0 \), the components of locally covariant Maurer-Cartan 1-form become

\[ \omega^m = dx^m + E^m = dx^\mu e^m_\mu; \]

\[ \omega^\alpha_{Q A} = \psi^\alpha_A, \quad \bar{\omega}_{Q A \dot{\alpha}} = \bar{\psi}_{A \dot{\alpha}}, \quad \omega^{mn} = \gamma^{mn}, \tag{35} \]

and the minimal action reduces to

\[ \Gamma = \int d^4x \det e \left\{ \Lambda - \frac{1}{2} R + \epsilon^{\mu \nu \rho \sigma} \psi_{A \mu} \sigma_{\sigma} \nabla_{\rho} \bar{\psi}_{A \nu} \\
+ \frac{i}{2} m^{3/2}_{AB} \{ \psi_{A \mu} \sigma^\mu \psi_{B \nu} + \bar{\psi}_{A \mu} \bar{\sigma}^\mu \bar{\psi}_{B \nu} \} \right\}, \tag{36} \]

where \( m^{3/2}_{AB} \) are gravitino masses. As has been anticipated, we now see that \( E^m \) is none other than the graviton, \( \psi^\alpha_A \) are the gravitinos, and \( \gamma^{mn} \) is the spin connection.

We can also consider minimal actions with the fields for central charges. For example, an action for the \( N = 2 \) case is given by

\[ \Gamma = \int d^4x \det e \left[ \Lambda - \frac{1}{2} R + \epsilon^{\mu \nu \rho \sigma} \omega_{Q A \mu} \sigma_{\sigma} \nabla_{\rho} \bar{\omega}_{Q A \nu} \\
+ \frac{i}{2} m^{3/2}_{AB} \{ \omega_{Q A \mu} \sigma^\mu \omega_{Q B \nu} + \bar{\omega}_{Q A \mu} \bar{\sigma}^\mu \bar{\omega}_{Q B \nu} \} \\
- \frac{M^2_X}{4} \left\{ \nabla_\mu (\omega_{X \nu} + \bar{\omega}_{X \nu}) - \nabla_\nu (\omega_{X \mu} + \bar{\omega}_{X \mu}) \right\}^2 \tag{37} \\
+ \frac{M^2_X}{2} m^2_X (\omega_{X \mu} + \bar{\omega}_{X \mu})^2 \\
+ f_{AB} M_X \epsilon^{\mu \nu \rho \sigma} \omega_{Q A \mu} \sigma_{\sigma} (\omega_{X \rho} + \bar{\omega}_{X \rho}) \bar{\omega}_{Q B \nu} \right], \]
where the components associated with the central charges of locally covariant Maurer-Cartan 1-form are denoted by

\[
\omega_{X\mu} = \frac{i}{2M_{s1}^2 M_{s2}^2} \lambda_1 \partial_\mu \lambda_2 + \frac{1}{M_X^2} \partial_\mu C + \frac{i}{2 \sqrt{2}} \left( \frac{1}{M_{s1}^2} \lambda_1 \psi_{2\mu} - \frac{1}{M_{s2}^2} \lambda_2 \bar{\psi}_{1\mu} \right) + \frac{1}{M_X} A_\mu + \frac{1}{16 M_{s1}^2 M_{s2}^2} \gamma_{m\mu} \lambda_1 \sigma^{mn} \lambda_2,
\]

\[
\bar{\omega}_{\bar{X}\mu} = \frac{i}{2M_{s1}^2 M_{s2}^2} \bar{\lambda}_1 \partial_\mu \bar{\lambda}_2 + \frac{1}{M_X^2} \partial_\mu \bar{C} + \frac{i}{2 \sqrt{2}} \left( \frac{1}{M_{s1}^2} \bar{\lambda}_1 \bar{\psi}_{2\mu} - \frac{1}{M_{s2}^2} \bar{\lambda}_2 \bar{\psi}_{1\mu} \right) + \frac{1}{M_X} \bar{A}_\mu + \frac{1}{16 M_{s1}^2 M_{s2}^2} \gamma_{m\mu} \bar{\lambda}_1 \bar{\sigma}^{mn} \bar{\lambda}_2,
\]

and the corresponding covariant derivative is given by \( \nabla_\mu \omega_{X\nu} = \partial_\mu \omega_{X\nu} - \Gamma^\rho_{\mu\nu} \omega_{X\rho} \). The constants \( M_X \) and \( M_{sA} \) are introduced here for normalization.

When we adopt the unitary gauge, \( \lambda_A = \bar{\lambda}_A = C = \bar{C} = 0 \), the action reduces to

\[
\Gamma = \int d^4x \det e \left[ \Lambda - \frac{1}{2} R + e^{\mu\nu\rho\sigma} \psi_{A\mu} \sigma_\rho \psi_{A\nu} - \bar{\psi}_{A\mu} \sigma_\rho \bar{\psi}_{A\nu} \right] + \frac{i}{2} m_{AB} \{ \psi_{A\mu} \sigma^{\mu\nu} \psi_{B\nu} + \bar{\psi}_{A\mu} \sigma^{\mu\nu} \bar{\psi}_{B\nu} \} - \frac{1}{4} (\nabla_\mu B_\nu - \nabla_\nu B_\mu)^2 + \frac{m_{X}^2}{2} B_\mu^2 + f_{AB} e^{\mu\nu\rho\sigma} \psi_{A\mu} \sigma_\rho \bar{\psi}_{B\nu},
\]

where \( B_\mu \equiv A_\mu + \bar{A}_\mu \). This action is equivalent to the one given in Ref.[5], where a nonlinearly realized local \( \mathcal{N} = 2 \) action was considered by reconciling the nonlinear realization of \( \mathcal{N} = 2 \) global SUSY with the \( \mathcal{N} = 2 \) supergravity multiplet so that the full theory has \( \mathcal{N} = 2 \) local SUSY up to the second order in the fields. Note that the \( \mathcal{N} = 2 \) supergravity multiplet has one spin-1 gauge boson, while we can introduce two spin-1 gauge fields associated with two central charges in general, although we have omitted the \( (\omega_{X\mu} - \bar{\omega}_{\bar{X}\mu}) \) dependence in the above example.

### 2.3 The interactions with matters

Let us introduce interactions of gravitinos with matters. We define the \( G \) transformation of a matter field \( M(x) \) by

\[
M'(x') = \tilde{h} M(x), \quad \tilde{h} = e^{\frac{i}{2} \alpha_{mn}(x) \bar{M}^{mn}},
\]
where $\tilde{M}^{mn} = 0$ for a scalar field,

$$(\tilde{M}^{mn})_\alpha^\beta = \frac{1}{2} (\sigma^{mn})_\alpha^\beta, \quad (\tilde{M}^{mn})^\beta_\alpha = \frac{1}{2} (\sigma^{mn})^\beta_\alpha,$$

for a fermion field $\psi_\alpha(x)$, $\bar{\psi}_\alpha(x)$, and so forth. Then the covariant derivative of the matter field is given by

$$\nabla_\mu M = \left( \partial_\mu + \frac{i}{2} \gamma^{mn} \tilde{M}^{mn} \right) M,$$

and its transformation law reads

$$(\nabla_\mu M)'(x') = \tilde{h} G^{-1\nu}_\mu \nabla_\nu M(x).$$

In terms of such matter fields, we obtain invariant actions under the local symmetry. Namely, the matter action is given by

$$\Gamma_{\text{matter}} = \int d^4 x \ det e \mathcal{L}_{\text{matter}},$$

where

$$\mathcal{L}_{\text{matter}} = \mathcal{L}_{\text{matter}}(M, \nabla_\mu M, \omega_{QA}, \bar{\omega}_{Q\bar{A}}, \nabla_\mu \omega_{QA}, \nabla_\mu \bar{\omega}_{Q\bar{A}}, e^m_\mu; R_{\mu\nu\rho\sigma}),$$

with $\mathcal{L}'_{\text{matter}}(x') = \mathcal{L}_{\text{matter}}(x)$.

When we may identify the above matter fields as the Standard Model particles, we obtain a theory which contains gravitino degrees of freedom as many as the number of SUSY interacting with the Standard Model fields. This serves as a starting point to experimental predictions of extended SUSY structure, which may be realized in Nature.

### 3 Goldstino Embedding

In the previous section, we have constructed our actions of nonlinearly realized extended supergravity, which enables us to obtain experimental predictions of these theories.

As a first step, let us restrict ourselves to high-energy processes where we can ignore the gravitino masses. In these situations, we naively expect that nonlinear realization of extended global SUSY might provide a good approximation to the original theory with local SUSY.
In the case of simple SUSY, this guess goes through as expected [4, 8]. However, as we will see below, in the case of extended SUSY, such simplification generically does not give a good approximation to the full supergravity theory even for high-energy processes. Consequently, we have to extract experimental predictions from our actions for extended local SUSY, instead of the global theories from the beginning.

### 3.1 The nonlinear realization of extended global SUSY

We first review nonlinear realization of extended global SUSY based on recent works by Nishino [9] and Clark and Love [10]. The goldstino fields are denoted by $\lambda_{A\alpha}(x)$, $\bar{\lambda}_{\dot{A}\dot{\alpha}}(x)$, and their extended SUSY transformation laws read

$$
\lambda'_{A\alpha}(x') = \lambda_{A\alpha}(x) + \sqrt{2}M_{sA}^2 \xi_{A\alpha},
$$

$$
\bar{\lambda}'_{\dot{A}\dot{\alpha}}(x') = \bar{\lambda}_{\dot{A}\dot{\alpha}}(x) + \sqrt{2}M_{sA}^2 \bar{\xi}_{\dot{A}\dot{\alpha}},
$$

with

$$
x'^\mu = x^\mu + \frac{i}{\sqrt{2}M_{sA}^2} [\xi_A \sigma^\mu \bar{\lambda}_A(x) - \lambda_A(x) \bar{\sigma}^\mu \bar{\xi}_A],
$$

where $M_{sA}$ are superficial SUSY breaking scales. The invariant action under the above transformations is a generalization of the Akulov-Volkov effective action, which is given by

$$
\Gamma_{AV} = \int d^4x \mathcal{L}_{AV}, \quad \mathcal{L}_{AV} = -M_s^4 \det \left( \delta^\nu_\mu - \frac{i}{M_{sA}^2} \lambda_A \bar{\sigma}^\nu \bar{\xi}_A \right).
$$

Although we can introduce such superficial SUSY-breaking scales as many as the number of SUSY in the action, the goldstino field redefinitions,

$$
\lambda_A \rightarrow \frac{M_{sA}^2}{M_s^2} \lambda_A,
$$

render the above Lagrangian into

$$
\mathcal{L}_{AV} = -M_s^4 \det \left( \delta^\nu_\mu - \frac{i}{M_s^4} \lambda_A \bar{\sigma}^\nu \bar{\lambda}_A \right),
$$

where $M_s$ is the common SUSY-breaking scale. This shows that physically only one SUSY-breaking scale is present in the action of nonlinear realization with extended global SUSY.
3.2 Comparison with the Deser-Zumino construction

We here turn to identify goldstino degrees of freedom in the Deser-Zumino construction \[^{[4]}\] with those in our construction and compare ours to the nonlinear realization for the global case.

The minimal action with nonlinearly realized extended supergravity is approximately given by

\[
\Gamma \simeq \int d^4x \det e \left[ \Lambda - \frac{1}{2}R + \epsilon^{\mu\nu\rho\sigma} \omega_{Q\mu\nu} \sigma_{\sigma} \partial_{\rho} \bar{\omega}_{Q\lambda} + \frac{i}{2} m_{3/2} A \left\{ \omega_{Q\mu}^\alpha \sigma_{\alpha}^\mu \bar{\omega}_{Q\nu}^\beta + \bar{\omega}_{Q\mu}^\alpha \bar{\sigma}_{\alpha}^\mu \bar{\omega}_{Q\nu}^\beta \right\} \right],
\]

(52)

where

\[
\omega_{Q\mu}^\alpha = \frac{\sqrt{2}}{M_{sA}^2} \partial_{\mu} \lambda_A^\alpha + \psi_A^\mu,
\]

\[
\bar{\omega}_{Q\mu\dot{\alpha}} = \frac{\sqrt{2}}{M_{sA}^2} \partial_{\mu} \bar{\lambda}_{A\dot{\alpha}} + \bar{\psi}_{A\dot{\alpha}},
\]

(53)

\[
e_{\mu} = \delta_{\mu}^{\nu} + E_{\mu}^{\nu} - \frac{i}{M_{sA}^4} \lambda_A \sigma^{m\nu} \partial_{\mu} \lambda_A - \frac{i}{\sqrt{2} M_{sA}^2} \left( \lambda_A \sigma^{m} \bar{\psi}_{A\mu} - \psi_{A\mu} \sigma^{m} \bar{\lambda}_A \right).
\]

Here, we have reduced the action up to the second order in the fields and introduced normalization factors \(M_{sA}\). We further set the cosmological constant and the non-diagonal components of gravitino masses\[^{[3]}\] to be vanishing, that is, \(\Lambda = 0\) and \(m_{3/2} AB = 0\) for \(A \neq B\), which results in

\[
\Gamma \simeq \int d^4x \left[ -\frac{1}{2}R + \epsilon^{\mu\nu\rho\sigma} \psi_{A\mu} \sigma_{\sigma} \partial_{\rho} \bar{\psi}_{A\nu} + \frac{i}{2} m_{3/2} A \psi_{A\mu}^\alpha \sigma_{\alpha}^\mu \bar{\psi}_{A\nu} + \frac{i}{2} m_{3/2} A \bar{\psi}_{A\mu\dot{\alpha}} \bar{\sigma}_{\dot{\alpha}}^\mu \bar{\psi}_{A\nu} + \frac{\sqrt{2} m_{3/2} A}{M_{sA}^2} i \partial_{\mu} \lambda_A \sigma_{\alpha}^\mu \psi_{A\nu} + \frac{\sqrt{2} m_{3/2} A}{M_{sA}^2} i \partial_{\mu} \bar{\lambda}_{A\dot{\alpha}} \bar{\sigma}_{\dot{\alpha}}^\mu \bar{\psi}_{A\nu} \right].
\]

(54)

This form has the local SUSY within the present approximation.

The above expression seems to have no goldstino kinetic terms, though it has mixed kinetic terms of goldstinos and gravitinos. Thus, we are led to redefine the gravitino fields\[^{[4]}\] they imply a possibility of gravitino oscillation.
so that the action has the canonical goldstino kinetic terms without the mixed terms. The redefinition of the gravitino fields is given by

\[
\psi_A^\mu \rightarrow \psi_A^\mu + i C_A \sigma^\mu \lambda_A,
\]

\[
\bar{\psi}^\mu_A \rightarrow \bar{\psi}^\mu_A - i C^*_A \bar{\sigma}^\mu \lambda_A,
\]

where \( C_A \) are constants to be determined below. This makes the action to be

\[
\Gamma \simeq \int d^4x \left[ -\frac{1}{2} R + \epsilon^{\mu\nu\rho\sigma} \psi_A^\mu \sigma^\rho \partial_\sigma \bar{\psi}_A^\nu + i \lambda_A \sigma^\mu \partial_\mu \bar{\lambda}_A 
\right.
\]

\[+ \left( i \frac{3 \sqrt{2} C_A m_{\frac{3}{2} A}}{M_{s A}^2} - i \frac{3 \sqrt{2} C_A^* m_{\frac{3}{2} A}}{M_{s A}^2} + 6 |C_A|^2 \right) i \lambda_A \sigma^\mu \partial_\mu \bar{\lambda}_A
\]

\[+ \left( 2 C_A + \frac{\sqrt{2} m_{\frac{3}{2} A}}{M_{s A}^2} \right) i \partial_\mu \lambda_A \sigma^\mu \psi_A^\nu + \left( 2 C_A^* + \frac{\sqrt{2} m_{\frac{3}{2} A}}{M_{s A}^2} \right) i \partial_\mu \bar{\lambda}_A \bar{\sigma}^\mu \bar{\psi}_A^\nu
\]

\[- \frac{3}{2} i C_A m_{\frac{3}{2} A} \psi_A^\mu \sigma^\mu \bar{\lambda}_A + \frac{3}{2} i C_A^* m_{\frac{3}{2} A} \bar{\psi}_A^\mu \bar{\sigma}^\mu \lambda_A - 6 |C_A|^2 m_{\frac{3}{2} A} \left( \lambda_A \lambda_A + \bar{\lambda}_A \bar{\lambda}_A \right) \left( \lambda_A \lambda_A + \bar{\lambda}_A \bar{\lambda}_A \right) \right].
\]

(55)

We now choose \( C_A \) so that the mixed kinetic terms of goldstinos and gravitinos disappear, which is achieved by

\[
C_A = -\frac{m_{\frac{3}{2} A}}{\sqrt{2} M_{s A}^2}.
\]

(57)

We also impose the condition that the action has the canonical goldstino kinetic terms, which requires

\[
6 C_A^2 = 1.
\]

(58)

Together with Eq. (57), this determines the normalization factors \( M_{s A} \) as physical scales:

\[
m_{\frac{3}{2} A} = \frac{M_{s A}^2}{\sqrt{3}},
\]

(59)

which is a generalization of the usual relation Eq. (4). Hence we have finally obtained a desired form

\[
\Gamma \simeq \int d^4x \left[ -\frac{1}{2} R + \epsilon^{\mu\nu\rho\sigma} \psi_A^\mu \sigma^\rho \partial_\sigma \bar{\psi}_A^\nu + i \lambda_A \sigma^\mu \partial_\mu \bar{\lambda}_A
\right.
\]

\[+ \left( i \frac{3 \sqrt{2} C_A m_{\frac{3}{2} A}}{M_{s A}^2} - i \frac{3 \sqrt{2} C_A^* m_{\frac{3}{2} A}}{M_{s A}^2} + 6 |C_A|^2 \right) i \lambda_A \sigma^\mu \partial_\mu \bar{\lambda}_A
\]

\[+ \left( 2 C_A + \frac{\sqrt{2} m_{\frac{3}{2} A}}{M_{s A}^2} \right) i \partial_\mu \lambda_A \sigma^\mu \psi_A^\nu + \left( 2 C_A^* + \frac{\sqrt{2} m_{\frac{3}{2} A}}{M_{s A}^2} \right) i \partial_\mu \bar{\lambda}_A \bar{\sigma}^\mu \bar{\psi}_A^\nu
\]

\[- \frac{3}{2} i C_A m_{\frac{3}{2} A} \psi_A^\mu \sigma^\mu \bar{\lambda}_A + \frac{3}{2} i C_A^* m_{\frac{3}{2} A} \bar{\psi}_A^\mu \bar{\sigma}^\mu \lambda_A - 6 |C_A|^2 m_{\frac{3}{2} A} \left( \lambda_A \lambda_A + \bar{\lambda}_A \bar{\lambda}_A \right) \left( \lambda_A \lambda_A + \bar{\lambda}_A \bar{\lambda}_A \right) \right].
\]

(60)
which coincides with the Deser-Zumino construction for $\mathcal{N} = 1$. Note that the matter interaction Eq. (44) contains the physical SUSY-breaking scales $M_{sA}$ in the factor $\text{det} e$, which determine the universal interactions of the goldstinos with matter fields.

In the case of $\mathcal{N} = 1$ SUSY, the Akulov-Volkov global action may be regarded as the first approximation to the full supergravity action since it is the starting point of the Deser-Zumino construction. However, when we consider the case of extended SUSY, the story is different from the $\mathcal{N} = 1$ case. As we have seen in the previous subsection, one common SUSY-breaking scale essentially appears in the action for nonlinear realization of extended global SUSY. In contrast, the above goldstino embedding in supergravity generically yields physically independent SUSY-breaking scales as many as the number of SUSY. Hence we conclude that the nonlinear realization of extended global SUSY does not properly approximate the nonlinearly realized extended supergravity theory even for high-energy processes.

4 Conclusion

In this paper, we have provided nonlinear realization of extended supergravity by using coset construction. One of the motivations to construct such a theory is to serve for investigating the number of SUSY by counting the number of gravitinos at the scale within experimental reach.

We also consider goldstino embedding in the locally invariant theory. In the case of $\mathcal{N} = 1$ SUSY, we may use the nonlinear realization of global SUSY to extract approximate experimental predictions of the local theory. However, in the case of extended SUSY, we cannot use the nonlinear realization of extended global SUSY, or rather, we generically have to use the nonlinearly realized extended supergravity action even for high-energy processes.

It seems intriguing to investigate characteristic experimental signatures of theories with more than one kind of gravitinos. Along the way, inclusion of linear SUSY multiplets into the present formalism may be useful for further research in connection with

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4One obvious exception is the case that there is only one SUSY-breaking scale so that all the gravitino masses take the same value in the original action from the start.
phenomenology under the setup of SUSY Standard Model. Mirror superpartners other than multiple gravitinos such as mirror gauginos, higgsinos, quarks and leptons might play crucial roles in future particle phenomenology. As such, low-energy extended SUSY structure is a candidate window into basic laws in Nature.

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