Learning Engagement as a Moderator between Self-Efficacy, Math Anxiety, Problem-Solving Strategy, and Vector Problem-Solving Performance

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Abstract: Vector problem-solving abilities are fundamental to everyday life and higher education; thus, improving them is important in education and research. However, the role of cognitive and affective factors and learning engagement in vector problem-solving performance is still unclear. This study examines the processes associated with vector problem-solving performance, focusing on the problem-solving strategy as a cognitive factor and math anxiety and task-specific self-efficacy as affective factors. In addition, this study examines the impact of learning engagement as a moderator in this process. A total of 245 Japanese 11th-grade high school students completed questionnaires. A multiple-group structural equation modelling revealed that (1) task-specific self-efficacy, math anxiety, and problem-solving strategies contribute to vector problem-solving performance when learning engagement is above average; (2) task-specific self-efficacy contributes to math anxiety, whereas task-specific self-efficacy and math anxiety contribute to problem-solving strategies when learning engagement is above average and stable; (3) task-specific self-efficacy is a positive predictor of vector problem-solving performance regardless of learning engagement. The results suggest that learning engagement moderates the association between math anxiety, task-specific self-efficacy, problem-solving strategy, and vector problem-solving performance. In addition, task-specific self-efficacy is a strong predictor of vector problem-solving performance.

Keywords: math anxiety; self-efficacy; problem-solving strategy; problem-solving; learning engagement; vector

1. Introduction

Mathematical problem-solving is central to the way people understand and represent events in everyday life and in many areas of science. Therefore, developing mathematical problem-solving skills has long been one of the main goals of mathematics education in many countries [1–3]. In this context, the development of vector problem-solving skills is the basis for considering the properties of shapes and motion in everyday life, as well as for the study of motion in physics, linear algebra, vector spaces, and data science in higher education [1,4].

Previous studies have investigated vector problem-solving situations and their cognitive and affective factors in students [5–11]. Students have difficulties solving vector problems, particularly in the geometric interpretation of inner products, unit vectors, and outer products, and in the first-order independence of unit and space vectors [5–7]. Furthermore, the limited understanding of the connection between the algebraic (i.e., matrix notation and operations) and geometric (i.e., arrow vectors) aspects of vectors [8,9] might explain students’ difficulties in solving vector problems. Shimizu [10] examined the relationship between vector problem-solving and affective factors and showed that mastery and performance-approach goals in the achievement goal theory mediate behavioural and emotional engagement and positively influence performance on textbook-level standardized vector problems.
However, three important questions remain unresolved in the research to date. First, to my knowledge, there are no studies that simultaneously address cognitive and affective factors and examine their relationship to vector problem-solving. In research on mathematical problem-solving, both cognitive factors (i.e., knowledge, skills, and problem-solving strategies) and affective factors (i.e., self-efficacy and math anxiety) play an important role in mathematical problem-solving [12–14]. Therefore, the simultaneous consideration of cognitive and affective factors makes it possible to identify factors and processes that explain the problem-solving performance of vectors in a detailed and meaningful way. Second, the variables that focus on cognitive and affective factors of vector problem-solving are limited to knowledge, skills, and performance goals. It has long been established that problem-solving strategies, knowledge, and skills contribute significantly to the main cognitive factors of mathematical problem-solving [12,14,15]. Moreover, the results of the OECD PISA survey and meta-analysis have attracted attention as they show a significant contribution of self-efficacy and math anxiety as affective factors in mathematical problem-solving [16–18]. Therefore, engagement with these variables could provide a stronger explanation for performance in vector problem-solving, and there is room for consideration. Third, the impact of engagement in vector learning on vector problem-solving is not yet fully understood. As far as I know, Shimizu [10] is the only study that investigates the relationship between vector problem-solving and engagement in vector learning. It is of great academic and pedagogical importance to find out how students’ engagement in vector learning improves their performance in vector problem-solving and related processes. It provides scientifically sound guidance not only for research but also for pedagogical practice. Therefore, it is also important to consider engagement in vector learning when examining the relationship between vector problem-solving and cognitive and affective factors.

With this in mind, this study examines the processes associated with performance in vector problem-solving, focusing on problem-solving strategy as a cognitive factor and math anxiety and self-efficacy as affective factors, in order to make suggestions for improving vector problem-solving among struggling students. In addition, this study also examines how students’ engagement in learning affects this process. The following section describes the variables discussed in this study and the assumed associated processes.

1.1. Math Anxiety

Math anxiety refers to tension, anxiety, and other feelings that affect number operations and mathematical problem-solving in a wide variety of ordinary life and academic situations [19]. Math anxiety is distinct from anxieties such as test anxiety and general anxiety [20] and is, therefore, a mathematics-specific emotion. Meta-analyses of math anxiety have shown that it inhibits mathematics-related activities, situations, and mathematical problem-solving [16,20–22]. For example, in a previous study, math anxiety was found to be low to moderately, negatively, and statistically significantly correlated with the amount of high school mathematics ($r = -0.31$) [20], intention to enrol in university ($r = -0.31$) [20], and mathematics achievement ($r = -0.28$, Barroso et al. [16]; $r = -0.27$, Ma [21]; $r = -0.32$, Zhang et al. [22]).

The negative relationship between math anxiety and mathematical problem-solving may increase as the students advance through the educational levels. Hill et al. [23] found a significant negative correlation between math anxiety and secondary students’ mathematics achievement, but no significant correlation between math anxiety and primary students’ mathematics achievement. Zhang et al. [22] found that the negative correlation was stronger from primary to secondary school ($r = -0.27$ for elementary school, $r = -0.39$ for junior high school, and $r = -0.44$ for high school). Therefore, math anxiety may play a significant role as an affective factor in vector problem-solving.

The disruption account is a popular theoretical explanation that math anxiety inhibits mathematical problem-solving [24]. This explanation states that math anxiety temporarily inhibits cognitive resources such as working memory, thus reducing mathematical performance. Disruption theory draws on attentional control theory [25], which states that
general anxiety reduces task performance by inhibiting executive functions. In addition, empirical studies supporting the disruption account include meta-analyses that showed a significant negative indirect effect of math anxiety on mathematics performance mediated by working memory [26,27].

1.2. Self-Efficacy

Self-efficacy is a judgement about one’s ability to plan and execute actions to achieve a particular performance [28]. Bandura [29] has shown that self-efficacy in relation to personal achievement is universal and utilitarian. Self-efficacy tends to be lower in East Asian countries such as Japan and Korea, but in all cultures, self-efficacy and mathematical literacy are positively correlated [17]. Furthermore, self-efficacy is considered to be a strong predictor of mathematical problem-solving, as it is significantly positively associated with mathematical problem-solving performance, even when self-concept, engagement, and prior academic performance are statistically controlled [30,31].

Self-efficacy can be divided into general, domain-specific, and task-specific levels [32]. General self-efficacy is an assessment of one’s ability to deal with a variety of situations and is measured by items such as “Thanks to my resourcefulness, I can handle unforeseen situations” [33]. Domain-specific self-efficacy is a judgement about one’s competence in a domain such as mathematics or writing and is measured by items such as “I feel confident enough to ask questions in my mathematics class” [32]. Task-specific self-efficacy is an assessment of competence in specific tasks and problems, such as algebra and geometry. It is measured using questions such as “On a certain map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map is 3 1/2 inches?” [34]. Empirical studies have shown that task-specific self-efficacy explains mathematical achievement better than general and domain-specific self-efficacy [31,35]. Therefore, the present study focuses on task-specific self-efficacy.

Self-efficacy has also been shown to be indirectly related to mathematical problem-solving via affective factors such as math anxiety [36,37] and cognitive factors such as the deep-learning approach [38] and metacognitive and cognitive learning strategies [39]. This indirectly related process between self-efficacy and mathematical problem-solving is consistent with control value theory [40], according to which cognitive evaluation with elements such as self-efficacy leads to performance via mediating performance-related emotions such as math anxiety and cognitive resources such as attention and memory. Furthermore, the relationship between self-efficacy and math anxiety is in line with the “interpretation account”, according to which the cognitive evaluation of past experiences and outcomes related to mathematics is responsible for the development and benefits of math anxiety [24].

1.3. Problem-Solving Strategy

Problem-solving strategy is considered one of the most important cognitive factors in solving mathematical problems. The problem-solving strategy does not guarantee the solution of a problem but is a technique that guides problem-solving [41]. Problem-solving strategies can be broadly divided into domain-specific and general problem-solving strategies [41]. Domain-specific problem-solving strategies are task-specific, such as addition, linear equations, and proof problems for congruent figures. Typical examples of domain-specific strategies are counting, decomposing, and retrieving, which are used to find solutions to additive problems such as 5 + 5 and 13 + 5 [42]. On the other hand, general problem-solving strategies can be applied to various tasks and domains, such as addition, linear equations, and proof problems for congruent figures. Polya’s [43] heuristics, such as “draw a picture” and “guess and check”, are typical examples.

Many studies have looked at a general problem-solving strategy in mathematics education and psychology for suggestions on how to improve general mathematical problem-solving. In one of the leading studies, Schoenfeld [14] used an experimental method to investigate the effects of teaching general problem-solving strategies to university stu-
The study showed that students in the experimental group who were taught general problem-solving strategies performed significantly better in solving mathematical problems and used these strategies more frequently. Ishida [44] used an experimental method to examine the effects of teaching general problem-solving strategies to Japanese sixth-grade students and found similar results to Schoenfeld [14]. Hembree [15] showed that general problem-solving strategies are significantly positively correlated with solving mathematical problems ($r = 0.31$ for drawing a diagram, $r = 0.20$ for using equations, and $r = 0.42$ for guessing and testing). The present study focuses on a general problem-solving strategy based on the above.

It is also hypothesized that self-efficacy and math anxiety are significant predictors of the general problem-solving strategy. Self-efficacy is expected to be positively related to general problem-solving strategy, as self-efficacy positively influences mathematical problem-solving via cognitive factors [38,39]. It is assumed that math anxiety inhibits cognitive resources and thus prevents problem-solving strategy since problem-solving strategy depends on cognitive resources such as working memory [45,46].

1.4. Learning Engagement

There are many conceptual and operational definitions of learning engagement [47]. However, it generally refers to active participation and involvement in learning activities [48] and is a motivational variable that captures the quality of learning [49]. Learning engagement is often divided into three broad categories: behavioural, emotional, and cognitive [47]. Behavioural engagement is behavioural involvement in learning, such as initiating action, effort, exertion, and hard work. Emotional engagement is a positive emotional response to learning, such as enthusiasm, interest, and pleasure. Cognitive engagement is cognitive involvement in learning, such as striving for goals and seeking strategies. Behavioural, emotional, and cognitive engagement in learning mathematics has been shown to positively influence academic performance in mathematics and performance in solving mathematical problems [10,50,51].

In addition, previous studies have examined the effects of learning engagement on affective factors and academic achievement [48,52–54]. Reeve and Lee [53] argued that high-quality engagement plays an essential role in understanding, predicting, and promoting motivation, as engagement is a significant positive predictor of psychological need satisfaction and self-efficacy. Umemoto and Ito [54] conducted a three-item longitudinal study among Japanese university students and found that emotional engagement is a significant positive predictor of self-efficacy and intrinsic value. These findings suggest that learning engagement is a necessary condition for affective factors such as self-efficacy and academic achievement.

Learning engagement could have a moderating effect on the relationship between cognitive and affective factors and vector problem-solving because learning engagement is necessary for academic achievement and affective factors. In other words, the relationship between cognitive and affective factors and mathematical problem-solving might be more pronounced when the level of learning engagement is high but only slight when it is low. In an empirical study, learning strategies, an aspect of cognitive engagement, have been shown to be a significant moderator of motivation and mathematical competence [55]. Focusing on learning engagement as a moderator of the relationship between cognitive and affective factors and vector problem-solving is important for academic research and educational practice. It could identify the desirable mathematical learning engagement to improve vectorial problem-solving.

Furthermore, learning engagement can vary by date, time, and situation, as it arises from people’s interaction with particular situations [47]. However, previous studies have investigated whether the level of learning engagement contributes to academic achievement and affective factors but have ignored aspects of instability. More recently, several studies have shown that the lower the instability of motivation, the more adaptive learning is. Okada et al. [56] showed that instability of motivation was a significant negative pre-
dictor of introjected regulation, identified regulation, and intrinsic regulation. Umemoto and Inagaki [57] found that introjected regulation was significantly positively associated with the cognitive and metacognitive strategy only when the instability of motivation was low. Research on the variability of self-esteem, a supporting source of motivational activation, has also shown that the combination of the level and variability of self-esteem has differential effects on depression [58,59]. Given the above, it is possible that learning engagement level and instability are not independent. Their combination could act as a moderator for the relationship between cognitive and affective factors and vector problem-solving performance.

While previous studies have examined combinations of level and instability as high and low, combinations of level and instability can also be other than high and low, respectively. Therefore, in the present study, combinations of level and variability of learning engagement are investigated by hierarchical clustering, and the obtained combinations are used as adjustment variables.

1.5. The Current Study

The current study investigates the processes associated with performance in vector problem-solving, focusing on problem-solving strategy as a cognitive factor and math anxiety and self-efficacy as affective factors. In addition, the current study also investigates the influence of learning engagement as a moderator on this process. Based on the above-mentioned previous studies, the hypotheses Hypothesis 1 to Hypothesis 4 are set up in the present study.

- Hypothesis 1: Self-efficacy is negatively related to math anxiety and positively related to problem-solving strategy and vector problem-solving performance.
- Hypothesis 2: Math anxiety is negatively related to problem-solving strategy and vector problem-solving performance.
- Hypothesis 3: Problem-solving strategy is positively related to vector problem-solving performance.
- Hypothesis 4: The combination of the level of learning engagement and instability has a significant moderate effect on the association between Hypothesis 1 to Hypothesis 3.

The hypothesized model (Figure 1) constructed based on the above hypotheses is examined in this study.

![Figure 1. Hypothesized model of the current study. Solid lines assume a positive association, dashed lines a negative association, and dotted lines a moderating effect.](image-url)

2. Methods

2.1. Participants and Procedure

The participants in this study were 245 11th-grade high school students (51% females, 49% males) enrolled in private high school A in the Tokyo area. All participants were ethnic Japanese. Eleventh-grade high school students in typical Japanese are 16–17 years old.

The survey consisted of nine surveys conducted weekly in school A's plane vector class from May to July 2020. Learning engagement in-plane vectors were measured in the
first to eighth surveys. In that order, self-efficacy, math anxiety, vector problem-solving performance, and problem-solving strategy were measured in the ninth survey. The surveys were administered without the students’ names, but subjects were asked to provide their student numbers to link the data and give feedback on the results. Before the survey, the author explained the study’s purpose to school A’s teacher, responsible for managing the math department, and requested his cooperation. After obtaining the teacher’s consent, the author conducted the survey. At the time of the survey, the author explained to the students both orally and through a clear statement on the questionnaire that (1) the responses to the survey were voluntary and not related to class math grades, (2) the survey contents were statistically processed, so the privacy of the subjects was protected, (3) the survey form for vector problem-solving performance was returned to the subjects through the teacher in charge, and (4) the author was responsible for the disposal of the survey forms other than the vector’s problem-solving performance.

2.2. Measures

The summary of measures used in this study is shown in Table 1. In the following sections, I provide details of each measure.

### Table 1. Summary of measures used in this study.

| Variable                          | Sub-Scales                                    | Number of Items | Example Item                                                                 |
|-----------------------------------|-----------------------------------------------|-----------------|------------------------------------------------------------------------------|
| Math anxiety                      | Math learning anxiety                          | 4               | When I take a math class, I am . . .                                        |
|                                  | Math evaluation anxiety                        | 4               | When I think about tomorrow’s math, I am . . .                               |
| Self-efficacy                     |                                               | 6               | I solve Q1                                                                   |
| Problem-solving strategy          |                                               | 4               | I would use formulae and theorems                                            |
| Level of learning engagement and  | Behavioural engagement                         | 3               | I work as hard as I can on mathematics learning                             |
| instability                       | Emotional engagement                           | 3               | I enjoy learning mathematics                                                 |
|                                  | Cognitive engagement                           | 3               | I try to connect what I am learning with my knowledge                        |
| Vector problem-solving performance|                                               | 6               | If magnitudes of $\vec{a}$ and $\vec{b}$ are equal, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.|

2.2.1. Math Anxiety

Eight items measuring math anxiety were developed based on items with high factor loadings in the Japanese math anxiety scale [60] based on Richardson and Suinn [19]. The measure assessed students’ math learning anxiety ($n = 4$; e.g., when I take a math class) and math evaluation anxiety ($n = 4$; e.g., when I think about tomorrow’s math). The students rated the items on a 6-point Likert scale ranging from “not very anxious” (1) to “very anxious” (6).

2.2.2. Self-Efficacy

Six items were used to measure task-specific self-efficacy. Like Pajares and Miller [61], students were asked about their confidence in answering each correctly before solving the vector problems (Table 2). The students rated the items on a 6-point Likert scale ranging from “not confident at all” (1) to “completely confident” (6).
Table 2. Vector problems in this study.

| Q1  | If magnitudes of $\vec{a}$ and $\vec{b}$ are equal, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular. |
| Q2  | Show that the diagonals of a rhombus are orthogonal using the concept of vector. |
| Q3  | In $\triangle ABC$, if P, Q, and R are the midpoints of AB, BC, and CA, respectively, then show that the centres of gravity of $\triangle ABC$ and $\triangle PQR$ are congruent. |
| Q4  | If $3\vec{AP} + 4\vec{BP} + 5\vec{CP} = \vec{0}$ is true for a point P inside $\triangle ABC$, then show and illustrate where P is located. |
| Q5  | If $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, then represent $\vec{OR}$ by $\vec{a}$ and $\vec{b}$. |
|     | Q5.1: If $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, then represent $\vec{OR}$ by $\vec{a}$ and $\vec{b}$. |
|     | Q5.2: If point R is the intersection of the straight line OR and the side A, then answer AD: DB. |

2.2.3. Problem-Solving Strategy

Four items were used to measure general problem-solving strategy in mathematics. An example of an item was “I would use formulae and theorems”. These items were based on Seo’s [62] and Ichikawa et al.’s [63] general problem-solving strategy in math and asked whether students attempted to use diagrams, formulae, theorems and prior knowledge when solving vector problems (Table 2). The scale used in this study is also consistent with Polya’s [43] and Shoenfeld’s [14] problem-solving strategy. After completing the vector problems, the students rated the items on a 6-point Likert scale ranging from “not at all” (1) to “very much” (6), with the instruction “How much did you do the following when solving Q1 to Q5.1?”.

2.2.4. Level of Learning Engagement and Instability

Nine items measuring learning engagement in-plane vectors were developed based on Skinner et al.’s [49] and Reeve and Tseng’s [64] engagement scale. The measure assessed students’ behavioural engagement ($n = 3$; e.g., “I work as hard as I can on mathematics learning”), emotional engagement ($n = 3$; e.g., “I enjoy learning mathematics”), and cognitive engagement ($n = 3$; e.g., “I try to connect what I am learning with my knowledge”). The students rated the items on a 6-point Likert scale ranging from “completely disagree” (1) to “completely agree” (6), with the instruction “Please answer the following questions about your work in mathematics class this week”.

In this study, based on Okada et al. [56], the mean of the scores at eight time points was used as the level of learning engagement and the intra-individual standard deviation as the instability of learning engagement.

2.2.5. Vector Problem-Solving Performance

Six items were used to measure performance in solving vector problems (Table 2). These tasks were modified problems treated as representative and standard examples and exercises for understanding the content of plane vectors in authorized textbooks of Japanese high school mathematics [65–67]. The tasks included Q1 and Q2 as proof problems using the perpendicular condition of vectors, Q3 as a proof problem using the position vectors of the quantum and the centroid, Q4 as a problem to prove and illustrate the position of a point, and Q5.1 and Q5.2 as problems to find the position vector of the intersection point. These questions were based not only on authorized Japanese textbooks [65–67] but also on the course of study for mathematics in senior high school [1], so they were considered appropriate as a scale for measuring performance in solving vector problems. For each problem in this study, correct answers were coded as 1 and incorrect answers as 0.
2.3. Data Analysis

The data analysis comprised five main steps. First, a preliminary analysis was conducted to confirm the validity of the scale. Confirmatory factor analysis (restricted maximum likelihood method) was used to check the structure of the scale. The \( \omega \) coefficient with the number of group factors set to 1 and composite reliability (CR) were used to examine the scale’s reliability. The average variance extracted (AVE) was determined to verify the convergent validity. It is generally recommended that the AVE value is 0.50 or over [68]. However, even if the AVE value is 0.50 or less, the scale’s validity is sufficient if the CR value is 0.60 or over [68]. Second, descriptive statistics and correlation coefficients were calculated for the level and instability of learning engagement, math anxiety, self-efficacy, problem-solving strategy and vector problem-solving performance to confirm the underlying information of the measures used in this study. Third, a hierarchical cluster analysis was conducted to examine the combinations of the level of learning engagement and instability. The reason why hierarchical cluster analysis was conducted in this study rather than grouping with central values as cut-off values is that combinations of the level of learning engagement and instability can also be other than high and low, respectively. The study produced a scree plot of the sum of squares within clusters (WSS), and the number of clusters was defined as the point at which the decay status of WSS changes from rapid to gradual. Fourth, descriptive statistics and correlation coefficients were calculated for each measure for each combination of the level of learning engagement and instability obtained. Fifth, structural equation modelling was conducted with multiple groups to test the hypothesized model (Figure 1), using the obtained combinations of learning engagement and instability as group variables. The study selected the best model among the following four models in multiple-group structural equation modelling.

- Model 1: a model that does not impose equal constraints.
- Model 2: a model that imposes equal constraints on the intercept.
- Model 3: a model that imposes equal constraints on intercept and variance.
- Model 4: a model that imposes equal constraints on intercept, variance, and path coefficient.

3. Results

3.1. Preliminary Analysis

The results of a confirmatory factor analysis of the assumed scale structures for math anxiety, self-efficacy, problem-solving strategy, learning engagement and vector problem-solving performance are shown in Table 3. All goodness-of-fit indicators were good for problem-solving strategy, learning engagement and vector problem-solving performance. Math anxiety and self-efficacy had a poor RMSEA, but the other goodness-of-fit indicators were good. Therefore, the assumed scale structure of math anxiety, self-efficacy, problem-solving strategy, learning engagement and vector problem-solving performance was valid.

Table 3. Results of confirmatory factor analysis.

| Variable                                | CFI   | TLI   | RMSEA  | SRMR  |
|-----------------------------------------|-------|-------|--------|-------|
| 1. Self-efficacy                        | 0.96  | 0.93  | 0.11   | 0.04  |
| 2. Math anxiety                         | 0.95  | 0.92  | 0.12   | 0.05  |
| 3. Problem-solving strategy             | 0.99  | 0.96  | 0.09   | 0.02  |
| 4. Learning engagement                  | 0.96-1.00 | 0.92-0.99 | 0.03-0.09 | 0.02-0.09 |
| 5. Vector problem-solving performance   | 1.00  | 1.00  | 0.02   | 0.06  |

The goodness-of-fit indicators in learning engagement show the range from the first to eighth surveys.

The \( \omega \) coefficients, CR and AVE are shown in Table 4. The reliability of each scale was confirmed to be good since the value of the \( \omega \) coefficient and CR was greater than 0.69. Regarding the convergent validity, the AVE values of cognitive engagement and vector problem-solving were less than 0.50, while the CR values of these scales were greater than
Moreover, the AVE values for these other scales were greater than 0.50. Thus, all scales in this study had adequate convergent validity.

Table 4. Results of $\omega$ coefficients, CR and AVE.

| Variable                          | $\omega$ | CR     | AVE    |
|----------------------------------|----------|--------|--------|
| 1. Math learning anxiety         | 0.88     | 0.88   | 0.64   |
| 2. Math evaluation anxiety       | 0.92     | 0.92   | 0.74   |
| 3. Self-efficacy                 | 0.92     | 0.89   | 0.66   |
| 4. Problem-solving strategy      | 0.89     | 0.89   | 0.66   |
| 5. Behavioural engagement        | 0.83–0.87| 0.82–0.86| 0.60–0.68|
| 6. Emotional engagement          | 0.88–0.93| 0.88–0.93| 0.72–0.81|
| 7. Cognitive engagement          | 0.69–0.87| 0.69–0.87| 0.42–0.69|
| 8. Vector problem-solving performance | 0.77     | 0.77   | 0.38   |

The $\omega$ coefficients, CR and AVE in learning engagement show the range from the first to eighth surveys.

Therefore, the following analysis used the arithmetic mean of each item as the score for math anxiety, self-efficacy and problem-solving strategy sub-scales. The total number of vector problems answered correctly was the vector problem-solving performance score. As mentioned in Section 2.2.4, the level of learning engagement was the mean of the scores at eight-time points, and the instability of learning engagement was the intra-individual standard deviation.

3.2. Descriptive Statistics

The descriptive statistics and correlation coefficients are shown in Table 5. The means of level of learning engagement, math evaluation anxiety and problem-solving strategy scores were above the six-point Likert scale semantic median of 3.50, while the means of self-efficacy and math learning anxiety scores were below the six-point Likert scale semantic median of 3.50. In addition, the vector problem-solving performance score was less than half the number of problems.

Table 5. Descriptive statistics and correlation coefficients.

| Variable                          | N   | M     | SD    | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|-----------------------------------|-----|-------|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. Behavioural engagement level   | 240 | 5.12  | 0.65  | —   | —   | —   | —   | —   | —   | —   | —   | —   | —   |
| 2. Behavioural engagement instability | 238 | 0.38  | 0.22  | −0.48| —   | —   | —   | —   | —   | —   | —   | —   | —   |
| 3. Emotional engagement level     | 240 | 4.30  | 0.92  | 0.59 | −0.31| —   | —   | —   | —   | —   | —   | —   | —   |
| 4. Emotional engagement instability | 238 | 0.57  | 0.32  | −0.13| 0.47 | −0.30| —   | —   | —   | —   | —   | —   | —   |
| 5. Cognitive engagement level     | 240 | 4.86  | 0.68  | 0.77 | −0.57| 0.70 | −0.23| —   | —   | —   | —   | —   | —   |
| 6. Cognitive engagement instability | 238 | 0.44  | 0.24  | −0.23| 0.49 | −0.30| 0.56 | −0.40| —   | —   | —   | —   | —   |
| 7. Self-efficacy                  | 217 | 2.22  | 1.07  | 0.16 | −0.32| 0.30 | −0.06| 0.26 | −0.15| —   | —   | —   | —   |
| 8. Math learning anxiety          | 218 | 3.19  | 1.26  | −0.19| 0.07 | −0.35| 0.15 | −0.26| 0.20 | −0.30| —   | —   | —   |
| 9. Math evaluation anxiety        | 218 | 4.77  | 1.34  | −0.09| 0.14 | −0.27| 0.21 | −0.16| 0.22 | −0.31| 0.66| —   | —   |
| 10. Problem-solving strategy      | 229 | 4.53  | 0.95  | 0.39 | −0.22| 0.34 | −0.06| 0.41 | −0.14| 0.41 | −0.07| −0.03| —   |
| 11. Vector problem-solving performance | 241 | 1.61  | 1.76  | 0.28 | −0.20| 0.29 | −0.11| 0.33 | −0.16| 0.55 | −0.26| −0.30| 0.36|

3.3. Combination of Level and Instability of Learning Engagement

The WSS of level and instability of learning engagement is shown in Figure 2. The number of clusters in this study was three, as the change from a rapid to a gradual attenuation status occurred in a number of three clusters. Table 6 shows the results of the mean and standard deviation of the level and instability of learning engagement for each cluster, the analysis of variance and the Holm multiple comparisons. Cluster 1 was named “Medium and Unstable” because the level of learning engagement was between clusters 2 and 3, and the instability of learning engagement was greater than in Cluster 3. Cluster 2 was named “High and Stable” because it had the highest level of learning engagement and the lowest instability of learning engagement. Cluster 3 was named “Low and Unstable” because it had the lowest level of learning engagement and the highest instability of learning engagement.
The results of the mean, standard deviation, analysis of variance, and multiple comparisons using the Holm method for math anxiety, self-efficacy, general problem-solving strategy, and vector problem-solving performance for each cluster are in Table 7. High and Stable (Cluster 2) had the highest self-efficacy, problem-solving strategy, and vector problem-solving performance and the lowest math anxiety. On the other hand, Low and Unstable (Cluster 3) had the lowest self-efficacy, problem-solving strategy, and vector problem-solving performance, and the highest math anxiety. Medium and Unstable (Cluster 1) had lower self-efficacy and problem-solving strategies and vector problem-solving performance than High and Stable (Cluster 2) and lower mathematics learning anxiety than Low and Unstable (Cluster 3).

Table 7. The differences in self-efficacy, math anxiety, and vector problem-solving between different clusters and the results of multiple comparisons by using the Holm test.

| Variable                      | Cluster 1 (n = 110) | Cluster 2 (n = 73) | Cluster 3 (n = 51) | F     | η²   | Multiple Comparison |
|-------------------------------|---------------------|--------------------|--------------------|-------|------|---------------------|
|                               | M      | SD    | M      | SD    | M      | SD    |       |       |       |       |       |       |
| 1. Behavioural engagement level| 5.15   | 0.43  | 5.66   | 0.31  | 4.30   | 0.54  | 154.89 | *    | 0.57  | 2 > 1 > 3 |
| 2. Behavioural engagement instability | 0.41   | 0.21  | 0.29   | 0.22  | 0.44   | 0.22  | 9.89   | *    | 0.08  | 1 = 3 > 2 |
| 3. Emotional engagement level  | 4.12   | 0.50  | 5.29   | 0.42  | 3.26   | 0.75  | 221.24 | *    | 0.66  | 2 > 1 > 3 |
| 4. Emotional engagement instability | 0.65   | 0.33  | 0.45   | 0.26  | 0.58   | 0.34  | 9.15   | *    | 0.07  | 1 > 2    |
| 5. Cognitive engagement level  | 4.86   | 0.36  | 5.50   | 0.34  | 3.94   | 0.48  | 251.87 | *    | 0.69  | 2 > 1 > 3 |
| 6. Cognitive engagement instability | 0.50   | 0.26  | 0.33   | 0.18  | 0.47   | 0.22  | 12.33  | *    | 0.10  | 1 = 3 > 2 |

* p < 0.001.

** p < 0.001, * p < 0.05.

3.4. Multiple-Group Structural Equation Modelling

A multiple-group structural equation modelling was conducted based on Figure 1 using the three obtained combinations of the level of learning engagement and instability as grouping variables. The analysis used the scale scores described in Section 3.1. As basic statistical information, the correlation coefficients for math anxiety, self-efficacy, general problem-solving strategy and vector problem-solving performance in each group are given in Table 8. In the study, insignificant paths were eliminated at the 5% level for all three groups in models 1 to 4. The information criterion and goodness-of-fit indices for models 1
to 4 are shown in Table 9. The results of the likelihood ratio test for models 3 and 4 were significant at the 1% level, the AIC had the lowest value and the goodness-of-fit indices (CFI, TLI, RMSEA and SRMR) were sufficient, so model 2 was adopted in this study.

**Table 8.** Correlation matrices between different clusters.

| Variable                                    | Medium and Unstable | High and Stable | Low and Unstable |
|---------------------------------------------|---------------------|-----------------|-----------------|
| 1. Self-efficacy                            | —                   | —               | —               |
| 2. Math learning anxiety                    | −0.21               | −0.40           | −0.03           |
| 3. Math evaluation anxiety                  | −0.27               | 0.68            | 0.12            |
| 4. Problem-solving strategy                 | 0.29                | 0.14            | 0.33            |
| 5. Vector problem-solving performance       | 0.43                | −0.23           | −0.32           |

| Variable                                    | Medium and Unstable | High and Stable | Low and Unstable |
|---------------------------------------------|---------------------|-----------------|-----------------|
| 1. Self-efficacy                            | —                   | —               | —               |
| 2. Math learning anxiety                    | −0.21               | −0.40           | −0.03           |
| 3. Math evaluation anxiety                  | −0.27               | 0.68            | 0.12            |
| 4. Problem-solving strategy                 | 0.29                | 0.14            | 0.33            |
| 5. Vector problem-solving performance       | 0.43                | −0.23           | −0.32           |

Table 9. Comparison of the hypothesized model.

| Model    | AIC       | BIC       | χ²  | df | Δχ² | CFI | TLI | RMSEA | SRMR |
|----------|-----------|-----------|-----|----|-----|-----|-----|-------|------|
| Model 1  | 2529.80   | 2691.40   | 4.29| 6  | -   | 1.00| 1.00| 0.00  | 0.02 |
| Model 2  | 2384.00   | 2514.50   | 14.29| 14 | 14.00| 1.00| 1.00| 0.02  | 0.05 |
| Model 3  | 2392.10   | 2496.50   | 38.34| 22 | 24.06| 0.92| 0.89| 0.11  | 0.12 |
| Model 4  | 2398.20   | 2457.00   | 72.51| 36 | 34.17| 0.83| 0.86| 0.13  | 0.17 |

* p < 0.01.

The standardised path coefficients and coefficient of determination (R²) resulting from model 2 are shown in Figure 3. Self-efficacy was positively associated with problem-solving strategy and vector problem-solving performance in all groups. Self-efficacy was also significantly negatively associated with math learning anxiety and evaluation anxiety in High and Stable and Medium and Unstable. Math evaluation anxiety showed a significant negative association with vector problem-solving performance in Medium and Unstable and Low and Unstable, but a significant positive association with problem-solving strategy in High and Stable. The problem-solving strategy was positively associated with vector problem-solving performance in Medium and Unstable.

![Figure 3](image-url)
The results of the difference in path coefficients test underlined those that showed significant differences (Figure 3). The results showed that the association between self-efficacy and vector problem-solving performance was significantly greater for High and Stable than for Medium and Unstable. The association between problem-solving strategy and vector problem-solving performance was significantly greater for Medium and Unstable than for High and Stable.

The model of Figure 3 explained 5–15% of the variance in math learning anxiety (5% for the Low and Unstable, 5% for the Medium and Unstable, and 15% for the High and Stable), 5–13% for the variance in math evaluation anxiety (5% for the Low and Unstable, 6% for the Medium and Unstable, and 13% for the High and Stable), 5–19% of the variance in problem-solving strategy (5% for the Low and Unstable, 7% for the Medium and Unstable, and 19% for the High and Stable), and 5–39% of the variance in vector problem-solving performance (5% for the Low and Unstable, 30% for the Medium and Unstable, and 39% for the High and Stable).

4. Discussion

The current study examined the processes associated with vector problem-solving performance, focusing on problem-solving strategy as a cognitive factor and math anxiety and self-efficacy as affective factors. In addition, the current study also investigated the influence of learning engagement as a moderator on this process. A structural equation modelling for several groups with the level and instability of learning engagement as grouping variables showed the following results. Self-efficacy was a positive predictor of problem-solving strategy and vector problem-solving performance regardless of learning engagement and a negative predictor of math anxiety when learning engagement was above the mean level, supporting Hypothesis 1. Math evaluation anxiety was a negative predictor of vector problem-solving performance when learning engagement was medium or below, and a positive predictor of problem-solving strategy when learning engagement was high and stable. The first result partially supports Hypothesis 2, while the second contradicts it. The problem-solving strategy was a positive predictor of vector problem-solving performance when learning engagement was medium and unstable, which partially supports Hypothesis 3. Furthermore, variance explained that the rate of vector problem-solving performance was relatively low only when learning engagement was low and unstable. On the other hand, the variance explanation rates for math anxiety and problem-solving strategy were relatively high only when learning engagement was high and stable. Therefore, Hypothesis 4 is supported, as learning engagement has a moderate influence on the relationship between math anxiety, self-efficacy, problem-solving strategy and vector problem-solving performance.

A notable finding of this study is that the level and instability of learning engagement have a moderating effect on the relationship between math anxiety, self-efficacy, problem-solving strategy, and vector problem-solving performance. Wu et al. [55] showed that the level of learning strategies, an aspect of cognitive engagement, had a moderating effect on the relationship between motivation and mathematical competence. The present study shows the following two functions of learning engagement as a moderator in solving vector problems, in which many students have difficulties.

First, this study suggests that an intermediate or higher level of learning engagement is necessary for task-specific self-efficacy, math evaluation anxiety or problem-solving strategy to explain strongly vector problem-solving performance. Students with a medium or higher level of learning engagement in learning plane vectors might have acquired knowledge and skills in plane vectors through learning plane vectors and could assess their abilities in plane vector problem-solving appropriately. As a result, vector problem-solving performance may be more explained to these students. On the other hand, students with low learning engagement may have acquired little knowledge and skills in plane vectors and could not adequately assess their competence in solving plane vector problems. Therefore, their performance in solving vector problems may be largely unexplained.
Second, this study suggests that high and stable learning engagement is one of the necessary conditions for task-specific self-efficacy to contribute to math anxiety and for task-specific self-efficacy and math evaluation anxiety to contribute to problem-solving strategy. It is likely that task-specific self-efficacy serves as a frame of reference for the cognitive processing of math anxiety. Students with high and stable levels of learning engagement could confidently and appropriately assess their competence in solving vector problems in the plane. As a result, their task-specific self-efficacy might have contributed to math anxiety and problem-solving strategy, while their math evaluation anxiety might have contributed to problem-solving strategy.

Another important finding is that task-specific self-efficacy strongly predicted performance in vector problem-solving. Specifically, the positive relationship between task-specific self-efficacy and performance in solving vector problems was greatest for students whose learning engagement was stable at a high level. This fits previous findings [31,35] that task-specific self-efficacy strongly predicts mathematical problem-solving performance, especially when learning engagement is stable at a high level.

The significant positive relationship between math evaluation anxiety and problem-solving strategy among students whose learning engagement was stable at a high level is interesting because this was contrary to Hypothesis 2. Tsui and Mazzocco [69] found that math anxiety, as facilitating anxiety, positively predicted mathematical problem-solving performance among mathematically talented sixth-grade students. In the present study, students with high and stable levels of learning engagement showed higher performance in solving vector problems (Table 7). Therefore, it is likely that for these students, anxiety about math evaluation functioned as eustress for the problem-solving strategy.

It is also surprising that no significant positive associations between problem-solving strategy and vector problem-solving performance were found except for Medium and Unstable. It is assumed that students whose learning engagement was Medium and Unstable had insufficient knowledge and skills specific to the typical and standardized problems of plane vectors. Therefore, it is likely that the general problem-solving strategy of using formulae, theorems, diagrams and previously learned content contributed in a complementary way to a performance in solving vector problems in Medium and Unstable. On the other hand, students with high and stable learning engagement may have relatively more knowledge and skills specific to the typical and standard plane vector problems. As a result, problem-specific knowledge and skills, rather than general problem-solving strategy, contributed to vector problem-solving performance in High and Stable. Students with low and unstable levels of learning engagement may have little or no problem-specific knowledge and skills. Therefore, the general problem-solving strategy would not have contributed to the vector’s problem-solving performance in Low and Unstable.

4.1. Implications for Education

The results suggest that stabilizing learning engagement at a high level could strengthen the positive association between task-specific self-efficacy and improvement in representative and standardized problem-solving performance on plane vectors. Reeve and Tseng [64] have shown that the satisfaction of three basic psychological needs (autonomy, competence, and the desire for relationships) is positively associated with high school students’ engagement in learning mathematics. Therefore, it is likely that engagement in vector learning can be enhanced by providing opportunities for involvement (i.e., educator shows his/her students affection), structure (i.e., educator respects his/her students’ space and privacy) and autonomy support (i.e., educator encourages his/her students to decide things for their self) [70] to satisfy the basic psychological needs. Furthermore, sources of self-efficacy include mastery experience, vicarious experience, social persuasion, and physiological arousal [71]. In particular, as mastery experience is a significant antecedent of self-efficacy [72], more opportunities to solve vector problems may be effective in increasing task-specific self-efficacy.
4.2. Limitations

Finally, I would like to ask four questions about this study. First, since this study was based on a limited sample of Japanese high school students, it is not certain to what extent the results are generalizable to other cultures, such as the Western world. Furthermore, the vector tasks in this study are based on typical and standard examples and exercises for understanding the content of plane vectors. Therefore, it is not certain whether similar trends can be observed in more advanced or applied problems. Future research should include studies with students from other cultures and on other vector problems. Second, because the results of the present study are from a one-time cross-sectional survey, there is no guarantee that the causality condition of the independent variable preceding the dependent variable in time is satisfied. Therefore, it is not possible to speak of strict causality. Future research should investigate task-specific self-efficacy, math anxiety, problem-solving strategy and vector problem-solving performance at multiple time points and examine them with cross-lagged effects models. Third, this study did not include “check the work” and “guess and test” [15], which are significantly related to mathematical problem-solving as a general problem-solving strategy. Therefore, different findings could be obtained if these were included as a general problem-solving strategy. Future research should include “check the work” and “guess and test” as a general problem-solving strategy. Fourth, this study did not examine antecedents of learning engagement and self-efficacy. This is an important issue for future research to clarify better the processes associated with mathematical and vector problem-solving performance. Previous studies in achievement goal theory and control value theory showed that achievement goals, particularly mastery and mastery approach goals, were strong predictors of learning engagement and self-efficacy [40,48,51,73]. Therefore, future research should examine achievement goals as antecedents of learning engagement and self-efficacy.

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Institutional Review Board Statement: The study was conducted according to the guidelines of the Declaration of Helsinki. Ethical review and approval were waived for this study because the survey was conducted as part of the improvement of math education practice in School A, informed consent was obtained from all subjects in advance, an anonymous survey in which individuals were not identified, and no research invasion occurred.

Informed Consent Statement: Informed consent was obtained from all students involved in the study.

Data Availability Statement: The datasets of the current study are not publicly available. However, data from the current study will be available from the corresponding author upon reasonable request with permission.

Conflicts of Interest: The author declares no conflict of interest.

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