Kharita: Robust Map Inference using Graph Spanners

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ABSTRACT
The widespread availability of GPS information in everyday devices such as cars, smartphones and smart watches make it possible to collect large amount of geospatial trajectory information. A particularly important, yet technically challenging, application of this data is to identify the underlying road network and keep it updated under various changes. In this paper, we propose efficient algorithms that can generate accurate maps in both batch and online settings. Our algorithms utilize techniques from graph spanners so that they produce maps can effectively handle a wide variety of road and intersection shapes. We conduct a rigorous evaluation of our algorithms over two real-world datasets and under a wide variety of performance metrics. Our experiments show a significant improvement over prior work. In particular, we observe an increase in Biagioni $f$-score of up to 20\% when compared to the state of the art while reducing the execution time by an order of magnitude. We also make our source code open source for reproducibility and enable other researchers to build on our work.

KEYWORDS
Map Inference, Graph Spanners

1 INTRODUCTION
Map Construction from Crowd-sourced GPS Data: The widespread adoption of smartphones with GPS and sensors like accelerometers makes it possible to collect large amounts of geospatial trajectory data. This has given rise to an important data science research question: Is it possible to accurately infer the underlying road network solely from GPS trajectories? An affirmative answer to this question will have a profound impact on map making. It will reduce the cost and democratize map construction and reduce the lag between changes in the underlying road network (e.g., due to construction) and its reflection on the map. The latter is particularly useful if autonomous vehicles are to become part of the mainstream traffic profile. For example, Uber has announced last July that it will invest $500 million into a global mapping project\textsuperscript{1} and Toyota has showcased in CES 2016 its ambitious project for creating maps using GPS devices\textsuperscript{2}. Not surprisingly, there has been extensive prior work by the data mining community \cite{3, 11, 13, 14} for map creation. However, most of them exhibit a large gap in quality between inferred and actual road maps.

Challenges for Accurate Map Inference: There are a number of technical challenges that a map construction algorithm using crowdsourced data needs to overcome including: (i) GPS sensors in smartphones, while reliable in general, can have non-trivial GPS errors. The errors are often acute in “urban canyons” with dense buildings or other structures; (ii) There is a substantial disparity in data due to the opportunistic data collection from smartphones. For example, while popular highways might have large number of data points, residential areas might only have handful of points; (iii) There exist a wide variety of sampling rates in which the GPS information is collected, often due to power concerns; (iv) Existing algorithms tend to overfit on the data sets on which they were tested. For example, many existing algorithms make some implicit assumptions that are specific to the road structures common to the United States and Europe. For instance, many countries in the Middle East have circular intersections (roundabouts) that are notoriously hard for existing algorithms to capture; (v) Finally, real-world roads often undergo various changes such as closure, new construction, shifting, temporary blocking due to accidents etc. Despite this, most prior work often treat the map construction as a static problem, resulting in maps that eventually become outdated. For example, in the city of Doha, with a dynamic road infrastructure, both Google maps and OpenStreetMap take over a year to update several major changes to the road network causing frustration to many drivers.

Crossing the Quality Chasm: In this paper, we propose concrete steps towards reducing the gap between the inferred and the true map by deploying a diverse collection of tools and techniques:

Principled Approach through Graph Spanners: We propose Kharita\textsuperscript{3} – a two-phase approach – that combines clustering with a map construction algorithm based on graph spanners. Graph

\begin{footnotesize}
\begin{enumerate}
\item \url{https://www.ft.com/content/e0df4a5e-5522-11e6-bfed-2fe9c26b3c60}
\item \url{https://www.cnet.com/au/news/toyota-wants-to-make-google-mapping-tech-obsolete/}
\item Kharita means map in Arabic.
\end{enumerate}
\end{footnotesize}
spanners are subgraphs that approximate the shortest path distance of the original graph. We show that these provide us a parameterized way to obtain sparse maps that retain the connectivity information effectively. The spanner formulation is particularly useful in handling different types of road and intersection shapes.

Online Map Construction: We propose Kharita as an online version of the map construction algorithm which combines the clustering and graph spanner phases.

Rigorous Evaluation: We conduct extensive evaluation of our approaches using standard performance metrics that include comparing the true and the inferred maps using geometric and topological metrics.

Reproducibility: In addition to providing high level details about our algorithms, we also make our source code open so that other researchers can readily build on top our work. The code can be found at this url: https://github.com/vipyoung/kharita

Solution in a nutshell:

We briefly describe our offline solution. We abstract the map inference task as that of constructing a directed graph $G = (V, E)$ from a collection of GPS trajectories. We proceed as follows:

1. We begin by first treating trajectories as a collection of points, by ignoring the implicit ordering in the trajectories.
2. The next step is clustering these points into those that belong to the same road segments using the location and heading information.
3. We then bring back the trajectory information and assign each point in the trajectory to its cluster center. If two consecutive points in the trajectory $tr$, say $tr : x_i$ and $tr : x_{i+1}$ are assigned to cluster centers $u$ and $v$ respectively, then we create an edge from $u$ to $v$. We now have a graph $G = (V, E)$ whose nodes are cluster centers derived in the previous step.
4. In the final step, the edge set $E_C$ is pruned to $E$ by invoking a spanner algorithm which sparsifies the graph while maintaining the connectivity and approximate shortest path distances.

Figure 1 provides an illustration of the major steps of the offline algorithm. While our approach is conceptually simple it significantly outperforms the state-of-the-art by improving the Biagioni TOPO (the de-facto standard metric for measuring map quality, see Sec. 5) $f$-score of up to 20%. The uplift in performance comes mainly from the efficient use of angle and speed information and elegant handling of geographic information (via clustering) and topological structure (via spanner).

2 DEFINITIONS AND PROBLEM STATEMENT

Definition 2.1. A GPS point is a five-tuple $(lat, lon, t, s, a)$, where lat is the latitude, lon the longitude, $t$ the timestamp, $s$ the speed and $a$ the angle.

Definition 2.2. A trajectory $tr$ is a chronologically ordered collection of GPS points. We represent a trajectory $tr$ as $x_1, x_2, \ldots, x_{|tr|}$ where each $x_i$ is a GPS point and $|tr|$ is number of GPS measurements in trajectory $tr$.

Definition 2.3. A geometric, weighted and directed graph $G = (V, E)$ consists of vertex set $V$ where each $v \in V$ is a $(lat, lon, angle)$ triplet.

Definition 2.4. The length of the shortest path between two nodes $u$ and $v$ in $G$ is denoted as $d_G(u, v)$.

Definition 2.5. A graph $H = (V, E')$ is an $(\alpha > 1)$ spanner of $G = (V, E)$ if $E' \subseteq E$ and for any pair of nodes $u$ and $v$, $d_G(u, v) \leq d_H(u, v) \leq \alpha * d_G(u, v)$.

2.1 Problem Definition

Given: A collection $T$ of GPS trajectories.

Objective: Infer a geometric, directed and weighted $G$ which represents the underlying road network.

3 KHARITA: OFFLINE ALGORITHM

In this section, we describe Kharita, a two-phase algorithm that processes a collection of GPS trajectories to produce a routable map.

3.1 Distance Metric

A key novelty of our paper is the extensive use of the angle information that is widely available as part of GPS output. One
can measure the distance between two (latitude, longitude) pairs \((L_1, L_2)\) using Vincenty distance formula. We denote it by \(v(L_1, L_2)\) and it provides the distance in meters. The distance between two angles can be computed using the unit circle metric defined as \(d_\theta(a_1, a_2) = \min(|a_1 - a_2|, 360^\circ - |a_1 - a_2|)\). For example, the distance between 350° and 10° is 20°. In order to compute the distance between two GPS points, we should consider both the location and the heading (angle of movement). Given two GPS points \((L_1, a_1)\) and \((L_2, a_2)\), we combine the aforementioned metrics as

\[
d_\theta((L_1, a_1), (L_2, a_2)) = \sqrt{v(L_1, L_2)^2 + \left(\frac{d_\theta(a_1, a_2)}{180^\circ}\right)^2},
\]

We denote the heading penalty parameter through \(\theta\). Intuitively, we want to penalize points that are very close to each other based on location but has diametrically opposite direction. This is often the case for parallel roads where each lane corresponds to the traffic in opposite directions.

**Lemma 3.1.** The distance metric \(d_\theta(., .)\) is a metric.

**Proof.** This follows directly from Cauchy-Schwarz inequality and the fact that both \(v\) and \(d_\theta\) are metrics:

\[
\begin{align*}
(d_\theta((L_1, a_1), (L_2, a_2)) + d_\theta((L_2, a_2), (L_3, a_3)))^2 &= v(L_1, L_2)^2 + (\frac{d_\theta(a_1, a_2)}{180^\circ})^2 + v(L_2, L_3)^2 + (\frac{d_\theta(a_2, a_3)}{180^\circ})^2 + \left(\frac{d_\theta(a_1, a_2)}{180^\circ}\right)\left(\frac{d_\theta(a_2, a_3)}{180^\circ}\right) \\
2v(L_1, L_2)v(L_2, L_3) + 2\frac{d_\theta(a_1, a_2)}{180^\circ} + 2\frac{d_\theta(a_2, a_3)}{180^\circ} &= 2v(L_1, L_2)v(L_2, L_3) + 2\frac{d_\theta(a_1, a_2)}{180^\circ} + 2\frac{d_\theta(a_2, a_3)}{180^\circ} \\
&\geq v(L_1, L_3)^2 + \left(\frac{d_\theta(a_1, a_3)}{180^\circ}\right)^2 = d_\theta((L_1, a_1), (L_3, a_3))^2.
\end{align*}
\]

### 3.2 Densification

An important pre-processing step, before inferring the nodes of the graph, is data densification that addresses two challenges: (a) different sampling rates and (b) data disparity. Due to power concerns, GPS measurements might not be reported at an adequate and constant rate. Similarly, based on the speed limits, two GPS points can be far away from each other. At a sampling rate of one GPS point every 10 seconds, for a car driving at 60 kmph, two consecutive points could be as far as 170 meters even without GPS noise. This issue is especially pronounced when there is data disparity. Specifically, in low-volume road segments (e.g., residential areas), the amount of data available is limited, leading to potentially missing those road segments.

The intuition behind densification is to introduce artificial GPS points along the trajectory of the moving vehicle in the following manner. If the two consecutive GPS points \(x_1 = (lon_1, lat_1, a_1)\), \(x_2 = (lon_2, lat_2, a_2)\) on the trajectory have headings with angular difference of less than a threshold (say 5°), we add \(s_r\) GPS points between them in an equidistant manner. \(s_r\) can be an application and dataset dependent. For our paper, we set it as \(s = \lfloor v(L_1, L_2)/20\text{m} \rfloor\). By doing this we effectively create a point every (approximately) 20m along trajectories on the straight roads.

We would like to note that densification is an optional step that can be skipped if the data is dense enough, or the map accuracy in low-frequency roads is not important for the end-user of the map. Since, our design objective is to achieve high levels of map accuracy for both major and minor roads, we densify the original data before clustering.

#### 3.3 Seed Selection and Cluster Assignment

**Seed Selection:** After the optional densification step, we cluster the GPS data points in \(T\) while ignoring the trajectory information. The cluster centers obtained will form the nodes \(V\) of the inferred road network \(G\) and are then connected using the trajectory information. In this paper, we use k-Means algorithm for clustering that has been shown to work well in prior work even without usage of the angle information [1, 13]. We use the following process for selecting a good set of seed nodes.

We start with an empty seed list and go through the GPS points sequentially. A GPS point \((l, a)\) is added to the seed list if there are no other seeds within a specified radius \(seed-radius\) using the distance function \(d_\theta(.)\). This process ensures that every point is within a distance \(seed-radius\) to some seed point. Additionally, this also results in the seeds being uniformly spread throughout the space of the map by making sure there are no two seeds within a distance \(seed-radius\) of each other. The parameter \(seed-radius\) determines the density of the clusters. For example using a small \(seed-radius\), say 3m can result in inferring each lane in a highway as an independent road segment, while choosing large \(seed-radius\) may result into merging points from close (parallel) streets into the same cluster.

**k-Means Clustering:** After the initial cluster centers are selected, we run the standard k-Means algorithm (where \(k\) is the number of seeds selected). In the assignment step, a GPS point is assigned to the closest centroid using the distance metric \(d_\theta\) defined in Section 3.1. In the update step, the cluster centroid is updated using standard mean along the \(lat/lon\) coordinates and the mean of circular quantities for the heading coordinate. Formally, the centroid for a cluster of points \(S_i\) is:

\[
m_i = \left(\bar{lat}, \bar{lon}, \bar{a}\right)
\]

where:

\[
\bar{lat} = \frac{1}{|S_i|} \sum_{x \in S_i} x.lat, \quad \bar{lon} = \frac{1}{|S_i|} \sum_{x \in S_i} x.lon
\]

and

\[
\bar{a} = atan2\left(\frac{1}{|S_i|} \sum_{x \in S_i} \sin x.\alpha, \frac{1}{|S_i|} \sum_{x \in S_i} \cos x.\alpha\right).
\]

It is well known that \(\bar{a}\) is a maximum likelihood estimator of the Von-Mises distribution (the spherical Gaussian) [16].

**Cluster Splitting:** After the clustering is done, we perform a post-processing routine to ensure that GPS points in each cluster are homogeneous. Specifically, if we observe that the heading angles in a cluster have a large variance, we proceed by splitting the cluster into smaller ones that are individually more homogeneous. This additional step is necessary to handle some complex intersections.
such as roundabouts that present a wide range of angles. Splitting clusters ensures that our map can naturally handle complex intersection shapes.

For a cluster \( v_i \) with mean heading \( \overline{\alpha} \), we define heading variability \( h_{\alpha} \) as the average of \( d_\alpha (x, \alpha, \overline{\alpha}) \), across all points \( x \) within the cluster\(^4\). If the heading variability \( h_{\alpha} \) is greater than some threshold (say, \( 10^3 \)) we split \( v_i \) into two clusters by partitioning \( v_i \) based on the angle information of its points. We then recalculate the centroids of the new clusters accordingly.

### 3.4 Edge Inference

In the previous subsection we used the clustering process to infer the nodes \( V \) of the output graph. In this subsection, we seek to infer the candidate set of edges \( E_C \) of \( G \) by using the trajectory information. We will then prune \( E_C \) by constructing a spanner of the graph \( G = (V, E) \).

The output of the clustering stage is a set of cluster centroids \( V = \{v_1, v_2, \ldots, v_k\} \) where each centroid is a triple of \((\text{lat}, \text{lon}, a)\). We now construct a cluster connectivity graph \( G_C = (V, E_C) \) that integrates the clustering information and trajectory information.

To create the edges we use the set of densified trajectories \( \{\text{tr}_1, \text{tr}_2, \ldots, \text{tr}_n\} \). We transform the \( \text{tr} \) sequence of GPS points into a sequence of cluster centroids \( \text{tr}_v = \{v_{i_1}, v_{i_2}, \ldots, v_{i_{|\text{tr}_v|}}\} \) where \( v_{i_j} \) is the closest centroid to point \( x_i \) from \( l \). For each pair of consecutive centroids \( (v_{i_j}, v_{i_{j+1}}) \in \text{tr}_v \), we add an edge between them if the following conditions are satisfied:

**Self Loop Avoidance**: \( v_{i_j} \neq v_{i_{j+1}} \). If two consecutive data points fall into the same cluster, then we do not add a self edge as it does not provide any additional information.

**Spurious Edge Avoidance**:

\[
f_e(v_{i_j}, v_{i_{j+1}}) = \max(1, \ln(\min(f(v_{i_j}), f(v_{i_{j+1}})))) - 1
\]

where \( f(u) \) is the number of datapoints assigned to cluster \( u \) and \( f_e(u, v) \) number of trajectories which draw the edge \((u, v)\) in the cluster connectivity graph \( G_C \). This condition eliminates spurious edges which may be created due to noise or abnormal vehicle behaviour (e.g. going in the wrong direction in a one-way street).

### 3.5 Graph Sparsification through Spanners

The inferred graph \( G = (V, E_C) \) from the previous section has potentially several redundant edges which makes \( G \) unusable for routing. Recall from Section 3.2 that for a car driving at 60 kmph (around 40 mph), two consecutive points obtained ten seconds apart could be as far as 170 meters (560 feet). However, due to the densification step, the cluster centroids could be as near as 20m (65 feet). So these two points could create a very long edge which may not follow the underlying road geometry.

Our objective is to construct a sparsified graph that retains the same connectivity information but also connects clusters that are near each other maintaining the road shapes. Consider a trajectory based edge \((v_i, v_j) \in E_C \). This implies that \( v_j \) is reachable from \( v_i \) through the underlying road network. However, due to the sampling rate, they might not necessarily be adjacent to each other. Hence, we would like to identify a smooth path between \( v_i \) and \( v_j \) that connects them through clusters that are near each other. Let us now formalize this problem. Given candidate graph \( G = (V, E_C) \), our objective is to obtain a sparsified graph \( H = (V, E) \) where:

\[
E \subseteq E_C, \text{ i.e. } H \text{ is a subgraph of } G
\]

\[
\forall e = (v_i, v_j) \in E_C, d_H(v_i, v_j) \leq \alpha \times d_{G}(v_i, v_j)
\]

This constraint ensures that the sparsification preserves approximate distance between each pair of vertices in \( G \). In other words, for any pair of vertices \((v_i, v_j) \in V\), their distance \( d_H(v_i, v_j) \) in \( H \) is at most \( \alpha \) times their distance \( d_G(v_i, v_j) \) in \( G \). Setting larger values of \( \alpha \) generates sparser graphs. Throughout the paper we use \( \alpha = \sqrt{2} \approx 1.41 \) which removes the cross edges in the orthogonal streets.

This problem can be abstracted as a well studied combinatorial problem of graph spanners [19]. Given a graph \( G = (V, E_C) \), a subgraph \( H = (V, E) \) is called a \( \alpha \)-spanner of \( G \), if for every \( u, v \in V \), the distance from \( u \) to \( v \) in \( G \) is at most \( \alpha \) times the corresponding distance in \( G \). There has been extensive prior work on developing efficient algorithms for spanner construction. For the purpose of our paper, we use a simple greedy spanner algorithm depicted in Algorithm 2. A naive implementation has a time complexity of \( O(n^3 \log n) \) while it can be improved to \( O(n^2 \log^2 n) \) using advanced data structures [19]. However, we empirically observe that by not recomputing the paths we can obtain a sub-quadratic runtime complexity while retaining full connectivity.

The final step in Kharita is ‘duplexification’ of slow road segments. Namely, certain low-tier streets receive very few data points, and often in only one direction which may confuse the Kharita to consider it as a one-way street. We make sure that these capillary roads are two way by adding an edge \((u, v)\) to the final graph \( H \) if:

\[
(u, v) \in H \text{ and maximum observed speed of all data points assigned to } u \text{ and } v \text{ is } \leq 60 \text{ kmph.}
\]

#### Algorithm 1 Two-Phase Kharita Algorithm

1. **Input**: A collection \( T \) of trajectories, \( \alpha \)
2. **Output**: A directed planar graph \( G = (V, E) \).
3. **Phase 1**:
   4. *(Optional)* \( T_D \leftarrow \) Densification\((T)\)
   5. \( G_D \leftarrow \) DistinctPoints\((T_D)\)
   6. Let \( S \leftarrow \) SelectSeeds\((G_D)\)
   7. \( V \leftarrow \) K-Means\((|S|, G_D)\)
4. **Phase 2**:
   9. Let \( E_C \leftarrow \) EdgeAssignment\((V, T_D)\)
10. \( G(V, E') \leftarrow \) Spanner\((V, E_C, \alpha)\)
11. \( G(V, E) \leftarrow \) Duplexification\((G(V, E'))\)

\(^4\)Note that angular arithmetics do not allow us to use standard measures of variability such as standard variation or variance.

#### Algorithm 2 Greedy Spanner Algorithm

1. **Input**: \( G = (V, E_C, \alpha) \)
2. \( E \leftarrow \emptyset, H = (V, E) \)
3. for each edge \((v_i, v_j) \in E_C \) in the order of decreasing weight do
   4. if \( d_G(v_i, v_j) > \alpha d_H(v_i, v_j) \) then
   5. \( E \leftarrow E \cup (v_i, v_j) \)
6. end if
7. end for
8. return \( H \)
4 KARITA*: ONLINE ALGORITHM

In this section, we present Karita*, an online algorithm that can update the map as new trajectories arrive. In this algorithm, the two phases of the offline algorithm - clustering and sparsification - are combined into one phase. This enables us to process trajectories that arrive in streaming fashion.

**Streaming Setting:** We consider the following streaming setting: our algorithm is provided a pair of GPS data points, $x_i, x_{i+1}$, that are taken consecutively.

Our algorithm is generic enough to handle various GPS streaming models that have been previously studied. As an example, it generalizes trajectory streaming model where the algorithm is provided trajectory at a time. Given a trajectory $tr$, one can convert it to our streaming model by considering every consecutive pair of points. Similarly, our model is equivalent to the sliding window streaming model with the widow size of 2. Our model cannot directly handle the GPS navigational stream format where points arrive one at a time. However, with a simple modification, where we store the previous data point from every unique source, we can still apply our online algorithm.

**Challenges:** Recall that our offline algorithm has two phases: (i) clustering and (ii) graph sparsification. Both of them required access to the entire data. The clustering phase consists of seed selection and actual k-Means. The graph sparsification required to order the edges of the graph in the decreasing order of weight. However, in the streaming setting, we have to process each pair of GPS points immediately and it is not feasible to conduct expensive operations. Our solution combines online adaptation of k-Means clustering algorithm and online spanner algorithm.

**Online Algorithm for k-Means and Spanners:** We begin by providing a brief intuition behind prior work on online k-Means algorithm [17] that was published last year. In this problem, the data points arrive one at a time and the objective is to provide a clustering that is competitive with the offline variant of k-Means that has access to all the data points. The algorithm assigns the first $k+1$ as cluster centroids. When a new data point arrives, it is assigned to the nearest centroid if the distance is less than some threshold $d$. If not, the data point becomes a new cluster on its own. As the number of clusters becomes larger, the threshold is periodically doubled thereby reducing the likelihood that a new cluster is created for new points. The online version of spanner algorithm is adapted from [18]. Given an $\alpha$-spanner graph $G$ and a new edge $e = (u, v)$, we add the edge to $G$ if $\alpha \times \omega(e)$ is less than the distance $dq(u, v)$.

**Online Map Construction Algorithm – Karita*: We begin with an empty graph $G = (V, E)$. When a pair of GPS points $(x_i, x_{i+1})$ arrives, we first densify them by creating a set of equidistant points $P = \{p_1, \ldots, p_l\}$ where $p_1 = x_i$ and $p_l = x_{i+1}$ and each consecutive pair of points differ by a fixed densification threshold of $sr$ meters. Each point $p_l$ is assigned to the closest node (cluster) $v^* \in V$ if it is within a radius distance $cr$ and the difference in angle is less than some heading angle tolerance $ha$. If not, we create a new node and assign $p_l$ to it. Next, the algorithm checks whether an edge should be created from node $v^*_{\text{prev}}$ (which correspond to the node to which the point $pt_{i-1}$ was assigned) and nodes $v^*$. This edge is created if and only if the angle difference between $v^*_{\text{prev}}$ and $v^*$ as well as that between $v^*_{\text{prev}}$ and the vector $(pt_i, pt_{i+1})$ are both lower or equal to $ha$. In addition, we also ensure that the graph remains an $\alpha$-spanner. Given the online and streaming nature of Karita*, we had to give up on the angular distance and both lower or equal to $ha$. In addition, we also ensure that the graph remains an $\alpha$-spanner. Given the online and streaming nature of Karita*, we had to give up on the angular distance.

**Challenges:** Recall that our offline algorithm has two phases: (i) clustering and (ii) graph sparsification. Both of them required access to the entire data. The clustering phase consists of seed selection and actual k-Means. The graph sparsification required to order the edges of the graph in the decreasing order of weight. However, in the streaming setting, we have to process each pair of GPS points immediately and it is not feasible to conduct expensive operations. Our solution combines online adaptation of k-Means clustering algorithm and online spanner algorithm.

**Algorithm 3 Karita* for online map inference**

1. **Input:** Trajectory $tr = \{x_1, x_2, \ldots, x_{|tr|}\}$, Road graph $G = (V, E)$, that can be empty
2. **Parameters:** clustering radius ($cr$, in meters), sampling rate ($sr$, in meters), heading angle tolerance ($ha$, in degrees)
3. for each consecutive pair of points $e(x_i, x_{i+1}) \in tr$ do
4. $tr_{pre} \leftarrow$ Densification$(x_i, x_{i+1})$, $sr$
5. for each point $p_l \in tr_{pre}$ do
6. $v^* = \text{argmin}(d(p_l, v_i) | v_i \in V)$
7. if $v^*$ is within the distance tolerance $cr$ and angle tolerance $ha$ then
8. Assign $p_l$ to cluster $v^*$
9. else
10. Create a new node $v^*$ for $p_l$
11. $V \leftarrow V \cup \{v^*\}$
12. end if
13. if $d_G(v^*, v^*) > \alpha \times d(edge(v^*_{\text{prev}}, v^*))$ then
14. $E \leftarrow E \cup \{(v^*_{\text{prev}}, v^*)\}$
15. end if
16. $v^*_{\text{prev}} \leftarrow v^*$
17. end for
18. end for
19. **return** $G$

**Figure 2:** Example of streaming densification and merging of trajectories. The first trajectory $tr_1$ is densified and all its points are considered as new clusters, whereas the densification of the second trajectory $tr_2$ results in the assignment of some of its points to the clusters generated for $tr_1$

**Incremental Map Maintenance:** A key advantage of the online algorithm Karita* is that it can be easily modified so that both the
geometry and connectivity is updated based on the traffic patterns. Let us consider the major scenarios.

1) **New Road:** No special processing is needed. As we observe new trajectories from new roads, the map will be automatically updated.

2) **Shifting of an Existing Road:** There are two cases. If there is only a slight shift (of less than \( sr \)), then our algorithm will not detect it. However, \( sr \) is often set to a low value and hence the shift might not have a major impact in routing. If the shift is major, we will create a parallel road based on the traffic observed on the new road. A time-stamp indicating the last time a cluster was visited is used to decide whether the cluster should be disabled. We can use a time decaying function to periodically eliminate those nodes and edges that have not seen any new traffic recently. Of course, this threshold has to be based on the prior traffic over the nodes and edges. In other words, major roads must be marked inactive while we provide some leeway for roads with low traffic.

3) **Road closure and blocking:** Our method can handle both temporary and permanent closure of the roads using the same idea as above. We store with each node and edge the time-stamp of the last GPS point that got assigned to it and the "typical" travel pattern. If the delay is large, we mark the edge as "inactive". If the closure is temporary, a road is marked as active.

4) **Shortest Distance Maintenance:** Due to the online spanner algorithm, there might eventually be multiple paths between two nodes \((u,v)\). We periodically invoke the offline spanner algorithm to ensure that the graph is as sparse as possible and remove the non-shortest paths between them.

5 **EVALUATION**

We discuss in this section the results of comparing our proposed solution Kharita to state of the art map construction algorithms. The baseline algorithms are selected in a way to cover all major approaches for map inference, namely Edelkamp [9] for \( k \)-means based techniques, Cao [6] for trace merging based techniques, Biagioni [4] for KDE based techniques, and Chen [11] which is the closest to our work in that it uses data with similar features to ours (e.g., angle, speed). In the following, we first introduce the datasets and evaluation metrics used. Next, we report the comparison of Kharita to state of the art algorithms. Finally, we assess the robustness of our solution to different parameter initialization settings.

### 5.1 Data and methodology

| Dataset | # Days | # GPS points | # Vehicles | Covered roads (km) |
|---------|--------|--------------|------------|--------------------|
| Doha    | 30     | 5.3M         | 432        | 300                |
| UIC     | 29     | 200K         | 13         | 60                 |

As we discussed earlier, our map inference process uses data generated by a fleet of vehicles with GPS-enabled devices. In this paper we utilize two datasets from Doha (Qatar) and UIC (Chicago) with basic statistics reported in Table 2.

The two datasets are different in that Doha dataset reports the speed and the heading of the moving direction in addition to the location \((lon, lat)\) of the vehicle, while UIC datasets reports only the location. Since Kharita utilizes both speed and heading information, we infer them from the consecutive data points. Heading is measured in angles against the North axis in degrees reporting values from 0 to 360°.

**Doha** dataset covers a rectangle (in \( lat, lon \) coordinates) of 6km x 8km in an urban region in the city of Doha with a mixture of highways, high and medium volume roads, capillary streets, and roundabouts. The UIC dataset covers an area of approximately 2km x 3km in downtown Chicago and is generated by a fleet of University buses with relatively regular routes.

We preprocessed both datasets to eliminate those datapoints with speed \( \leq 5 \text{kmph} \) which are known to have non-trivial noise when reporting location.

Kharita has two main parameters. In Doha dataset we set \( seed \_radius = 20\text{m} \) to cover 3-lane roads with minimal noise, while in UIC dataset we set \( seed \_radius = 80\text{m} \) as we observe much higher noise levels, which go as high as 60m for a single road segment. In both cases we use \( \theta = 2 \cdot seed \_radius \) to ensure that two datapoints from the same street going in opposite directions are not assigned to the same cluster. We also experimented with other choice of parameters and observe low sensitivity of final maps to the parameter choice. However, a more detailed examination of parameter space is left for future work.

Evaluating the quality of an automatically generated map is a challenging question. The de-facto standard for measuring the quality of the map is the holes-and-marbles method introduced by Biagioni and Eriksson [4] which we describe now.

**Holes-and-marbles methods**. Biagioni and Eriksson [4] proposed the following two methods for measuring the accuracy of the inferred map.

**GEO** method evaluates how well the given map geometrically matches a ground truth map. Throughout the paper we use the OpenStreetMap (OSM) snapshot of the same region as the ground truth map\(^5\). GEO method samples points every 3 meters from both maps and marks a marble in each sampled point of the tested map and puts a hole in each sampled point of the ground truth map. We say that marble (hole) is matched if there is a hole (marble) within \( matching \_threshold \). We vary this threshold in the range of \( 5 \text{m} \) to \( 30 \text{m} \) and evaluate \( precision \), \( recall \) and \( f \_score \) as follows:

\[
\text{precision} = \frac{\# \text{matched}_\text{marbles}}{\# \text{all}_\text{marbles}}, \quad \text{recall} = \frac{\# \text{matched}_\text{holes}}{\# \text{all}_\text{holes}}
\]

\[
\text{f}_\text{score} = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}
\]

**TOPO** method evaluates the topological characteristics of the map. As it was done in other map-evaluation studies we eliminate the edges of the ground-truth map without a single trajectory passing through it. This way TOPO method measures the topological quality of the map: how accurately the inferred map assesses the connectivity structure of the inferred roads. Namely from a randomly sampled marble (hole) we first find all the marbles (holes) which could be reached from the starting point within a \( radius \);
throughout the paper we set the radius = 2000m. For the neighborhoods of each sampled starting point we calculate $f$-score as above and report the mean $f$-score over a sample of 200 randomly selected starting points. Note, that starting marble and starting hole belong to two different maps and virtually never match exactly; hence we enforce them to be within 1m distance and belong to the roads with the same direction (within a small angle difference).

5.2 Comparison with state-of-the-art

In this section we compare Kharita with state of the art (SOTA) map-inference algorithms: Cao [6], Edelkamp [9], Biagioni[4], and Chen [11]. For Cao, Biagioni and Edelkamp we use the source code developed and provided by the authors, while for Chen we use our implementation of the algorithm as the original implementation was not made available to us.

An important characteristics of some of the existing solutions is poor scalability. Namely it takes 3,558 and 6,621 seconds to run Cao/Edelkamp on a single day worth of data ($\approx$ 200K data points), with only one iteration of Cao’s clarification step. Hence we report the comparison of the different algorithms using one day worth of data from Doha. Furthermore, trying to scale these algorithms to one week of data has resulted in “out of memory” problems in the machines we used for the evaluation.

5.2.1 GEO Comparison. In Figure 3 we report the GEO $f$-score for the four map inference solutions, measured against the underlying OSM map in the rectangles of interest for both Doha and UIC datasets. The GEO $f$-score is predominately determined by the amount of the data and their coverage of the underlying road network, but also depends on the edge-creation process. Namely some algorithms are more conservative in the way they infer a road segment and hence have lower $f$-score; Cao and Biagioni belong to this category. Other algorithms require a small number of trajectories to infer a particular road segment and consequently have larger $f$-score. Note that difference in $f$-score among those (Chen, Edelkamp, Kharita, Kharita*) is relatively small which is a result of the fact that geometrically each one of them closely covers the street segments which have at least one trajectory passing by. However, GEO method is oblivious to the connectivity structure of the road network which is captured by TOPO method.

5.2.2 TOPO Comparison. The true test of the map quality is the TOPO method. It measures how accurately intersection are inferred with the corresponding connectivity among different road segments. In Figure 4 we report the TOPO $f$-score for the same six maps and both datasets. We observe that Kharita and Kharita* manage to achieve largest $f$-score among the examined algorithms. This improvement comes from the improved accuracy of inferring connections (at the intersections) between the road segments; a task at which most of the previous methods fail, especially when oblivious of incoming trajectory information.

Kharita achieves TOPO $f$-score (with matching radius of 30m) of 0.91 on the UIC dataset and 0.8 on the Doha dataset which improves state-of-the-art by 20% and 10%, respectively. In general, TOPO $f$-score are greater on the UIC dataset compared to Doha. This is the result of the fact that data is generated by buses following (mostly) regular routes, hence the network structure is fairly regular with many trajectories covering most inferred road-segments.
In contrast, Doha dataset is much more diverse both in terms of routes and intersection types, and hence more difficult to accurately infer. Another interesting observation is that despite the fact that Kharita\textsuperscript{a} is presented with a very limited future information regarding the incoming trajectories, due to the online streaming constraints, it was able to achieve higher $f_{score}$ (up to 0.75) than all other algorithms except Kharita which acts offline. Finally, note that some solutions perform well on one of the datasets and not so well on the other suggesting that their approach is data-dependent, while Kharita effectively handles both datasets.

5.2.3 Visual Comparison. We depict the output of the 6 mapping algorithms on the same roundabout in Figure 5. Note that each map has its own way of creating edges which determines the final quality of the map. Both Kharita and Kharita\textsuperscript{a} faithfully map the underlying road geometry and connectivity, with very few redundant edges. A visual inspection of the roundabout generated by Biagioni algorithm explains its poor GEO $f_{score}$ performance as it misses too many road segments. Likewise, the fact that Chen’s algorithm generates too many undesirable edges increases its GEO $f_{score}$. However, because the edges are not correctly connected, its TOPO $f_{score}$ performance is not very good. It is important to notice that for navigation purposes, the most important aspect of the map to be captured is the connectivity, which translates in the case of complex intersections and roundabouts to being able to correctly infer all in/out links and turns.

5.2.4 Time performance of SOTA algorithms. Another important aspect to explore is the time performance of the different solutions. Table 3 reports the execution time of the six algorithms to process 1 days of data from Doha and one month of data from UIC. Note that the two datasets are very similar in terms of number of GPS points ($\approx 200K$). However, the rate at which points are generated is different, which is reflected on the times achieved. Doha data being denser, it requires generally more time to process compared to the sparse UIC data.

As previously reported in [4][11], Edelkamp, Biogioni and Cao are expensive in terms of execution time. For Edelkamp, the algorithm runs 10 iterations in the case of Doha data and 19 iterations in the case of UIC data before it converges. The time we report in Table 3 as well as the $f_{score}$ reported previously are all the results of these iterations. Cao needs almost one hour in the case of Doha to make one iteration of its clarification step. Thus, it was unrealistic to aim for a larger number of iterations. Due to its myriad steps, Biagioni algorithm required 01h20min to digest one day of Doha data. Surprisingly, our two solutions Kharita and Kharita\textsuperscript{a} outperformed all existing algorithms. The reason for which Kharita runs in less time than its online counterpart Kharita\textsuperscript{a} is due to the use of the efficient “bash” ball-search in the KD-tree index to find nearest neighbors, not possible in the online setting as the data comes one point at a time.

Next, we will explore how GEO and TOPO $f_{score}$ scale with additional data for Kharita (such study is not possible for other algorithms, due to scalability issues mentioned above).

5.3 Kharita at scale

In previous paragraphs we compared the Kharita against other map inference solutions. In this section we will examine how the quality of the maps changes when more data is available and look into computational scalability of our algorithms.

We run both Kharita and Kharita\textsuperscript{a} on slices of 1 day, 7 days, and 30 days of Doha data. We report for each slice the amount of GPS points processed, the execution time, and both GEO and TOPO $f_{scores}$.

We empirically observe that the execution times scale close to linearly with the size of the input dataset making the proposed algorithms highly scalable.

In terms of $f_{score}$, with more data GEO $f_{score}$ improves for both Kharita and Kharita\textsuperscript{a} as simply more data implies better coverage of the map. More interestingly, we observe that Kharita TOPO $f_{score}$ also improves implying Kharita’s ability to improve the topological accuracy when additional data is available. Oddly, Kharita\textsuperscript{a} TOPO performance suffers with more data due to the creation of spurious edges. Since Kharita\textsuperscript{a} is online in nature, it is designed to work over short time windows and poor scalability is not a major concern of Kharita\textsuperscript{a}.

6 RELATED WORK

Due to the widespread availability of smartphones and the advent of autonomous cars, the problem of map construction from opportunistically collected GPS traces has been extensively study by various communities including data mining, geo spatial computing, transportation and computational geometry [2–4, 6, 8, 9, 11]. In
In this paper, we propose two efficient algorithms $Kharita$ and $Kharita^*$ for constructing maps from opportunistically collected GPS information. The first is a two-phase algorithm that clusters GPS points followed by a sparse graph construction using spanners. The second is an online algorithm that can create and update the map when the GPS data points arrive in a streaming fashion. While our approach is conceptually simple it significantly outperforms the state-of-the-art due to efficient exploitation of angle and speed information and elegant handling of geographic information (via clustering) and topological structure (via graph spanner).

7 CONCLUSION

In this paper, we proposed two efficient algorithms $Kharita$ and $Kharita^*$ for constructing maps from opportunistically collected GPS information. The first is a two-phase algorithm that clusters GPS points followed by a sparse graph construction using spanners. The second is an online algorithm that can create and update the map when the GPS data points arrive in a streaming fashion. While our approach is conceptually simple it significantly outperforms the state-of-the-art due to efficient exploitation of angle and speed information and elegant handling of geographic information (via clustering) and topological structure (via graph spanner).

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