Evidence for resonant structures in $e^+e^- \rightarrow \pi^+\pi^- h_c$

Chang-Zheng Yuan

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

(Dated: November 19, 2014)

The cross sections of $e^+e^- \rightarrow \pi^+\pi^- h_c$ at center-of-mass energies from 3.90 to 4.42 GeV were measured by the BESIII and the CLEO-c experiments. Resonant structures are evident in the $e^+e^- \rightarrow \pi^+\pi^- h_c$ line shape, the fit to the line shape results in a narrow structure at a mass of $(4216 \pm 18)$ MeV/$c^2$ and a width of $(39 \pm 32)$ MeV, and a possible wide structure of mass $(4293 \pm 9)$ MeV/$c^2$ and width $(222 \pm 67)$ MeV. Here the errors are combined statistical and systematic errors. This may indicate that the $Y(4260)$ state observed in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ has fine structure in it.

PACS numbers: 14.40.Rt, 14.40.Pq, 13.66.Bc

The observation of the $Y$-states in the exclusive production of $\pi^+\pi^- J/\psi$ and $\pi^+\pi^- \psi(3686)$ from the B-factories is a great puzzle in understanding the vector charmonium states. According to the potential models, there are 5 vector states above the well-known 1D states $\psi(3770)$ and below around 4.7 GeV/$c^2$, namely, the 3S, 2D, 4S, 3D, and 5S states. However, experimentally, besides the three well known structures observed in inclusive hadronic cross section, i.e., the $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$, there are four $Y$-states, i.e., the $Y(4008)$, $Y(4260)$, $Y(4360)$, and $Y(4660)$. This suggests that at least some of these structures are not charmonium states, and thus has arisen various scenarios in interpreting one or more of them.

The BESIII experiment running near the open charm threshold supplies further information to understand the properties of these vector states. Amongst these information, the most relevant measurement is the study of $e^+e^- \rightarrow \pi^+\pi^- h_c$ [11]. Besides the observation of a charged charmoniumlike state $Z_c(4020)$, BESIII reported the cross section measurement of $e^+e^- \rightarrow \pi^+\pi^- h_c$ at 13 center-of-mass (CM) energies from 3.90 to 4.420 GeV [11]. The measurements are listed in Table I. In the studies, the $h_c$ is reconstructed via its electric-dipole (E1) transition $h_c \rightarrow \gamma\eta_c$ with $\eta_c$ to 16 exclusive hadronic final states: $pp, 2(p^+p^-), 2(K^+K^-), K^+K^-\pi^+\pi^-, pp\pi^+\pi^-, 3(p^+p^-), K^+K^-2(p^+p^-), K^0K^+\pi^-, K^0K^+\pi^+\pi^+, K^+K^-\pi^0, pp\pi^0, \pi^+\pi^-\eta, K^+K^-\eta, 2(p^+p^-)\eta, \pi^+p^-\pi^0\pi^0, \pi\pi\pi\eta, K^+K^-\eta, 2(p^+p^-)\eta, \pi^+\pi^-\pi^0\pi^0$.

The CLEO-c experiment did a similar analysis, but with significant signal only at CM energy 4.17 GeV [13], the result is $\sigma = (15.6 \pm 2.3 \pm 1.9 \pm 3.0) \text{ pb}$, where the third error is from the uncertainty in $B[\psi(3686) \rightarrow \pi^0 h_c]$. The cross sections are of the same order of magnitude as those of the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ measured by BESIII [14] and other experiments [3, 4], but with a different line shape (see Fig. II). There is a broad structure at high energy with a possible local maximum at around 4.23 GeV. We try to use the BESIII and the CLEO-c measurements to extract the resonant structures in $e^+e^- \rightarrow \pi^+\pi^- h_c$.

As the systematic error ($\pm 18.1\%$) of the BESIII experiment is common for all the data points, we only use the statistical errors in the fits below. The CLEO-c measurement is completely independent from the BESIII experiment, and all the errors added in quadrature ($\pm 4.2 \text{ pb}$) is taken as...
TABLE I: $e^+e^- \rightarrow \pi^+\pi^-h_c$ cross sections measured from the BESIII experiment. For the first three energy points, besides the upper limits, the central values and the statistical errors which will be used in the fits below are also listed. The second errors are systematic errors and the third ones are from the uncertainty in $B(h_c \rightarrow \gamma \eta_c)$ [12].

| $\sqrt{s}$ (GeV) | $\sigma(e^+e^- \rightarrow \pi^+\pi^-h_c)$ (pb) |
|------------------|-----------------------------------------------|
| 3.900            | 0.0 ± 6.0 or < 8.3                            |
| 4.009            | 1.9 ± 1.9 or < 5.0                            |
| 4.090            | 0.0 ± 7.4 or < 13                             |
| 4.190            | 17.7 ± 9.8 ± 1.6 ± 2.8                       |
| 4.210            | 34.8 ± 9.5 ± 3.2 ± 5.5                       |
| 4.220            | 41.9 ± 10.7 ± 3.8 ± 6.6                      |
| 4.230            | 50.2 ± 2.7 ± 4.6 ± 7.9                       |
| 4.245            | 32.7 ± 10.3 ± 3.0 ± 5.1                      |
| 4.260            | 41.0 ± 2.8 ± 3.7 ± 6.4                       |
| 4.310            | 61.9 ± 12.9 ± 5.6 ± 9.7                      |
| 4.360            | 52.3 ± 3.7 ± 4.8 ± 8.2                       |
| 4.390            | 41.8 ± 10.8 ± 3.8 ± 6.6                      |
| 4.420            | 49.4 ± 12.4 ± 4.5 ± 7.6                      |

the total error and is used in the fits. We use a least $\chi^2$ method with

$$
\chi^2 = \sum_{i=1}^{14} \frac{(\sigma_i^{\text{meas}} - \sigma_i^{\text{fit}(m_i)})^2}{(\Delta \sigma_i^{\text{meas}})^2},
$$

where $\sigma_i^{\text{meas}} \pm \Delta \sigma_i^{\text{meas}}$ is the experimental measurement, and $\sigma_i^{\text{fit}(m_i)}$ is the cross section value calculated from the model below with the parameters from the fit. Here $m_i$ is the energy corresponds

FIG. 1: The comparison between the cross sections of $e^+e^- \rightarrow \pi^+\pi^-h_c$ from BESIII (dots with error bars) [11] and those of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ from Belle (open circles with error bars) [4]. The errors are statistical only.
to the \( i \)th energy point.

As the line shape above 4.42 GeV is unknown, it is not clear whether the large cross section at high energy will decrease or not. We try to fit the data with two different scenarios.

Assuming the cross section follows the three-body phase space and there is a narrow resonance at around 4.2 GeV, we fit the cross sections with the coherent sum of two amplitudes, a constant and a constant width relativistic Breit-Wigner (BW) function, i.e.,

\[
\sigma(m) = |c \cdot \sqrt{PS(m)} + e^{i\phi} BW(m)\sqrt{PS(m)/PS(M)}|^2,
\]

where \( PS(m) \) is the 3-body phase space factor, \( BW(m) = \frac{\sqrt{12\pi\Gamma_{e^+e^-} B(\pi^+\pi^- h_c)\Gamma_{\text{tot}}}}{(m^2-M^2+iM\Gamma_{\text{tot}})} \), is the Breit-Wigner (BW) function for a vector state, with mass \( M \), total width \( \Gamma_{\text{tot}} \), electron partial width \( \Gamma_{e^+e^-} \), and the branching fraction to \( \pi^+\pi^- h_c \), \( B(\pi^+\pi^- h_c) \), keep in mind that from the fit we can only extract the product \( \Gamma_{e^+e^-} B(\pi^+\pi^- h_c) \). The constant term \( c \) and the relative phase, \( \phi \), between the two amplitudes are also free parameters in the fit together with the resonant parameters of the BW function.

The fit indicates the existence of a resonance (called \( Y(4220) \) hereafter) with a mass of \((4216 \pm 7)\) MeV/\( c^2 \) and width of \((39 \pm 17)\) MeV, and the goodness-of-the-fit is \( \chi^2/\text{ndf} = 11.04/9 \), corresponding to a confidence level of 27%. There are two solutions for the \( \Gamma_{e^+e^-} B(Y(4220) \rightarrow \pi^+\pi^- h_c) \) which are \((0.32\pm0.15)\) eV and \((6.0\pm2.4)\) eV. Here all the errors are from fit only. Fitting the cross sections without the \( Y(4220) \) results in a very bad fit, \( \chi^2/\text{ndf} = 72.75/13 \), corresponding to a confidence level of \( 2.5 \times 10^{-10} \). The statistical significance of the \( Y(4220) \) is calculated to be \( 7.1\sigma \) comparing the two \( \chi^2 \)s obtained above and taking into account the change of the number-of-degree-of-freedom. Figure 2 (left panel) shows the final fit with the \( Y(4220) \).

FIG. 2: The fit to the cross sections of \( e^+e^- \rightarrow \pi^+\pi^- h_c \) from BESIII and CLEO-c (dots with error bars). Solid curves show the best fits, and the dashed ones are individual component. Left panel is the fit with the coherent sum of a phase space amplitude and a BW function, and the right panel is the coherent sum of two BW functions.

Assuming the cross section decreases at high energy, we fit the cross sections with the coherent sum of two constant width relativistic BW functions, i.e.,

\[
\sigma(m) = |BW_1(m) \cdot \sqrt{PS(m)/PS(M_1)} + e^{i\phi} BW_2(m) \cdot \sqrt{PS(m)/PS(M_2)}|^2,
\]

where both \( BW_1 \) and \( BW_2 \) take the same form as \( BW(m) \) above but with different resonant parameters.
The fit indicates the existence of the $Y(4220)$ with a mass of $(4230 \pm 10)$ MeV/$c^2$ and width of $(12 \pm 36)$ MeV, as well as a broad resonance, the $Y(4290)$, with a mass of $(4293 \pm 9)$ MeV/$c^2$ and width of $(222 \pm 67)$ MeV. The goodness-of-the-fit is $\chi^2/\text{ndf} = 1.81/7$, corresponding to a confidence level of 97%, an almost perfect fit. There are two solutions for the $\Gamma_{e^+e^-} \times B[Y(4220)/Y(4290) \rightarrow \pi^+\pi^- h_c]$ which are $(0.07 \pm 0.07)$ eV/(16.1 $\pm$ 2.2) eV and $(2.7 \pm 4.9)$ eV/(19.0 $\pm$ 5.9) eV. Again, here the errors are from fit only. Fitting the cross sections without the $Y(4220)$ results in a much worse fit, $\chi^2/\text{ndf} = 30.65/11$, corresponding to a confidence level of $1.3 \times 10^{-3}$. The statistical significance of the $Y(4220)$ is calculated to be $4.5\sigma$ comparing the two $\chi^2$s obtained above and taking into account the change of the number-of-degree-of-freedom. Figure 2(right panel) shows the final fit with the $Y(4220)$ and $Y(4290)$.

From the two fits showed above, we conclude that very likely there is a narrow structure at around 4.22 GeV/$c^2$, although we are not sure if there is a broad resonance at 4.29 GeV/$c^2$. We try to average the results from the fits to give the best estimation of the resonant parameters. For the $Y(4220)$, we obtain

$$M(Y(4220)) = (4216 \pm 18) \text{ MeV}/c^2,$$

$$\Gamma_{\text{tot}}(Y(4220)) = (39 \pm 32) \text{ MeV},$$

$$\Gamma_{e^+e^-} \times B[Y(4220) \rightarrow \pi^+\pi^- h_c] = (4.6 \pm 4.6) \text{ eV}.$$ 

While for the $Y(4290)$, we obtain

$$M(Y(4290)) = (4293 \pm 9) \text{ MeV}/c^2,$$

$$\Gamma_{\text{tot}}(Y(4290)) = (222 \pm 67) \text{ MeV},$$

$$\Gamma_{e^+e^-} \times B[Y(4290) \rightarrow \pi^+\pi^- h_c] = (18 \pm 8) \text{ eV}.$$ 

Here the errors include both statistical and systematic errors. The results from the two solutions and the two fit scenarios are covered by enlarged errors, the common systematic error in the cross section measurement is included in the error of the $\Gamma_{e^+e^-}$.

It is noticed that the uncertainties of the resonant parameters of the $Y(4220)$ are large, this is due to two important facts: one is the lack of data at CM energies above 4.42 GeV which may discriminate which of the two above scenarios is correct, the other is the lack of high precision measurements around the $Y(4220)$ peak, especially between 4.23 and 4.26 GeV. The two-fold ambiguity in the fits is a nature consequence of the coherent sum of two amplitudes [16], although high precision data will not resolve the problem, they will reduce the errors in $\Gamma_{e^+e^-}$ from the above fits. As the fit with a phase space amplitude predicts rapidly increasing cross section at high energy, it is very unlikely to be true, so the results from the fit with two resonances is more likely to be true. More measurements from the BESIII experiments at CM energies above 4.42 GeV and more precise data at around the $Y(4220)$ peak will also be crucial to settle down all these problems.

There are thresholds of $\bar{D}D_1$ [17], $\omega_{\chi_{cJ}}$ [18, 19], $D_s^+D_s^{-}$ [9] at the $Y(4220)$ mass region, these make the identification of the nature of this structure very complicated. The fits described in this paper supply only one possibility of interpreting the data. In Ref. [20], the BESIII measurements [11] were described with the presence of one relative S-wave $\bar{D}D_1 + c.c.$ molecular state $Y(4260)$ and a non-resonant background term; while in Ref. [21], the BESIII data [11] were fitted with a model where the $Y(4260)$ and $Y(4360)$ are interpreted as the mixture of two hadron-charmonium states. It is worth to point out that various QCD calculations indicate that the charmonium-hybrid lies in the mass region of these two $Y$ states [22] and the $c\bar{c}$ tend to be in a spin-singlet.
state. Such a state may couple to a spin-singlet charmonium state such as \( h_c \) strongly, this makes the \( Y(4220) \) and/or \( Y(4290) \) good candidates for the charmonium-hybrid states.

In summary, we fit \( e^+e^- \rightarrow \pi^+\pi^-h_c \) cross sections measured by BESIII and CLEO-c experiments, evidence for a narrow structure at around 4.22 GeV, as well as a wide one at 4.29 GeV, is observed. More high precision measurements at above 4.42 GeV and around 4.22 GeV are desired to better understand these structures.

This work was supported in part by the Ministry of Science and Technology of China under Contract No. 2009CB825203, and National Natural Science Foundation of China (NSFC) under Contracts Nos. 10825524, 10935008, and 11235011.

---

[1] B. Aubert et al. (BaBar Collaboration). Phys. Rev. Lett. 95, 142001 (2005).
[2] C. Z. Yuan et al. (Belle Collaboration). Phys. Rev. Lett. 99, 182004 (2007).
[3] J. P. Lees et al. (BaBar Collaboration), Phys. Rev. D 86, 051102(R) (2012).
[4] Z. Q. Liu et al. (Belle Collaboration), Phys. Rev. Lett. 110, 252002 (2013).
[5] B. Aubert et al. (BaBar Collaboration). Phys. Rev. Lett. 98, 212001 (2007).
[6] X. L. Wang et al. (Belle Collaboration). Phys. Rev. Lett. 99, 142002 (2007).
[7] B. Aubert et al. (BaBar Collaboration). [arXiv:1211.6271](http://arxiv.org/abs/1211.6271).
[8] For a recent review, see N. Brambilla et al., Eur. Phys. J. C 71, 1534 (2011).
[9] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
[10] M. Ablikim et al. (BESIII Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 614, 345 (2010).
[11] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 111, 242001 (2013).
[12] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 104, 132002 (2010).
[13] T. K. Pedlar et al. (CLEO Collaboration), Phys. Rev. Lett. 107, 041803 (2011).
[14] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 110, 252001 (2013).
[15] For the three low statistics energy points, the \( \chi^2 \) is not well defined. We take the central values listed in Table 1 as nominal values, and vary the central values and statistical errors in a wide range to estimate the possible bias in this assumption. The bias is found to be small and is considered as systematic error of the results.
[16] K. Zhu, X. H. Mo, C. Z. Yuan and P. Wang, Int. J. Mod. Phys. A 26, 4511 (2011).
[17] Q. Wang, C. Hanhart and Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013).
[18] L. Y. Dai, M. Shi, G. -Y. Tang and H. Q. Zheng, [arXiv:1206.6911](http://arxiv.org/abs/1206.6911) [hep-ph].
[19] C. Z. Yuan, P. Wang and X. H. Mo, Phys. Lett. B 634, 399 (2006).
[20] M. Cleven, Q. Wang, F. -K. Guo, C. Hanhart, U. -G. Meißner and Q. Zhao, [arXiv:1310.2190](http://arxiv.org/abs/1310.2190) [hep-ph].
[21] X. Li and M. B. Voloshin, [arXiv:1309.1681](http://arxiv.org/abs/1309.1681) [hep-ph].
[22] T. Barnes, F. E. Close and E. S. Swanson, Phys. Rev. D 52, 5242 (1995); P. Guo, A. P. Szczepaniak, G. Galata, A. Vassallo and E. Santopinto, Phys. Rev. D 78, 056003 (2008); J. J. Dudek and E. Rrapaj, Phys. Rev. D 78, 094504 (2008).