The problem of "anomalous" ion transport in high-current vacuum discharges

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Abstract. This paper gives an exhaustive theoretical description of the so-called "anomalous" ion transport phenomenon that exists in vacuum diodes with cathode plasma emission. The phenomenon is represented by the cathode plasma directed movement towards anode. In general terms, the "anomaly" means that metal ions are accelerated towards an electrode with a higher electrostatic potential (anode). The theoretical description is given within the framework of a one-dimensional kinetic model, which is based on the Vlasov-Poisson system of equations for plasma and electrostatic field components. Time-dependent accurate numerical solution of this system describes the process evolution leading to "anomalous" ion transport. It was shown that the “anomalous” transport has fast electrodynamic nature.

1. Introduction

Among the fundamental research subjects concerning vacuum electronics, a special place is devoted to the problems connected with studying of various dynamic effects associated with plasma motion that can be a medium for the electric current transfer [1, 2]. An important case here is the formation of an electron beam in a vacuum diode with plasma cathode. Various experimental papers (e.g. [3-5]) point out that the near-cathode plasma expansion rates are much higher than thermal ones, as well as to the fact that an “anomalous” transfer of ions from cathode to anode occurs, for example, in a vacuum discharge. “Over-thermal” ion motion corresponds to kinetic energy at the level of tens and even hundreds of electron volts [6]. In vacuum arcs, ion fluxes with such energies are actually observed, and their physical parameters are reliably measured [7-9]. Since the kinetic energies of ions (> 100 eV) can manifold exceed a vacuum arc discharge combustion voltage (< 80 V), they are commonly called ions with "anomously" high energies. As a percentage, the anode of the vacuum arc receives ions that carry up to 8–12% of the total electric charge in arc discharge [10].

2. Theory

In this paper, we do not use simplified approaches to modelling current flow in vacuum diodes, such as the PIC method or hydrodynamic (liquid) description, which are traditionally widely used in problems of vacuum electronics [11, 12]. Based on the fundamental principles of physical kinetics, we will characterize the state of the plasma by non-stationary classical distribution functions of individual plasma components: electrons $f_e$ and ions $f_i$. For simplicity, we will consider in the problem only one type of metal singly charged ions. In the specified geometry, the distribution functions will be
parameterized with one spatial variable x, one component of the momentum \( p_x \), and time \( t \). For the simplicity reasons in this paper we consider a nonrelativistic plasma transport in the absence of external magnetic fields. The distribution functions obey the collisionless Boltzmann (Vlasov) equations [13]:

\[
\frac{\partial f_{e,i}}{\partial t} + \frac{p_x}{m_{e,i}} \frac{\partial f_{e,i}}{\partial x} - qE_x \frac{\partial f_{e,i}}{\partial p_x} = 0,
\]

\[
\frac{\partial f_{e,i}}{\partial t} + \frac{p_x}{m_{e,i}} \frac{\partial f_{e,i}}{\partial x} + qE_x \frac{\partial f_{e,i}}{\partial p_x} = 0,
\]

(1)

where \( m_e \) and \( m_i \) are the masses of the electron and ion, respectively, \( q \) is elementary charge, \( E_x \) is the electric field strength. The concentrations of electrons and ions \( n_e \) and \( n_i \), as well as their current densities \( j_e \) and \( j_i \), can be expressed in the form of the corresponding moments of the distribution functions

\[
n_{e,i}(x,t) = \int_{-\infty}^{\infty} f_{e,i}(x,p_x,t) dp_x,
\]

\[
j_{e,i}(x,t) = \frac{q}{m_{e,i}} \int_{-\infty}^{\infty} f_{e,i}(x,p_x,t) p_x dp_x.
\]

(2)

In vacuum discharge gaps an external strong field is applied, so the global plasma “quasineutrality” condition is not met due to space-charge effects are dominant. Therefore, there is no possibility to calculate the electric field strength from "plasma approximation", i.e. \( n_e = n_i \). To account the electric field self-consistently, we supplement system (1) with the Poisson’s equation

\[
\frac{\partial^2 \varphi}{\partial x^2} = -\frac{q}{\varepsilon_0} (n_i - n_e), \quad E_x = -\frac{\partial \varphi}{\partial x},
\]

(3)

where \( \varphi \) is the electrostatic potential.

As will be shown below, we will consider the cathode plasma dynamics in a vacuum diode at the initial stages. Therefore, in this problem formulation, we can neglect the external circuit for the diode connection and assume diode to be connected to an ideal voltage source, i.e. cathode is grounded - \( \varphi(x = 0) \) while anode voltage is equal to voltage of the source \( \varphi(x = D) = U(t) \). Then the equation (3) has a simple solution in quadratures

\[
E_x(x,t) = -\frac{U(t)}{D} - \frac{q}{\varepsilon_0} \int_{-\infty}^{x} [n_i(x',t) - n_e(x',t)] dx' + \frac{q}{\varepsilon_0} \int_{-\infty}^{x} \int_{-\infty}^{x'} [n_e(x'',t) - n_i(x'',t)] dx'' dx'.
\]

(3)

The charged particle currents through the anode can be calculated straightforwardly using the expression (2). To obtain a total current density \( j(t) \) of the diode the current conservation law can be used

\[
\varepsilon_0 \frac{\partial E_x(x,t)}{\partial t} + j_e(x,t) - j_i(x,t) = j(t).
\]

(4)

By integrating (5) over all \( x \) for this simple circuit we obtain total current density \( j(t) \),

\[
j(t) = \frac{1}{D} \int_{-\infty}^{x} [j_e(x,t) - j_i(x,t)] dx - \frac{\varepsilon_0}{D} \frac{dU(t)}{dt}.
\]

(5)

It is assumed that at staring time point \( t = 0 \), anode voltage with amplitude of \( U_0 \) is launched with a short front edge of \( t_{\text{vac}} \). Initially, the vacuum diode is empty, i.e. \( f_e = f_i = 0 \). The vacuum discharge development occurs due to the anode voltage increase. Cathode plasma inflow is ensured by
continuous electrons and ions emission from the cathode surface. It is assumed to be quasi-neutral having Maxwellian velocity distribution with slightly different temperatures. In mathematical model (1)-(6), the continuous emission condition is implemented by the following boundary conditions at cathode

\[ f_{e,i}(x=0,p_x,t) = \frac{n_0}{\sqrt{2\pi m_{e,i}T_{e,i}}} \exp\left(-\frac{p_x^2}{2m_{e,i}T_{e,i}}\right), \]

where \( T_e \) and \( T_i \) are ion and electron thermodynamic temperatures, correspondingly, and \( n_0 \) is the cathode plasma number density. The condition (7) ensures the emission of a quasi-neutral cathode plasma, which components of have different temperatures. Typical values of ion and electron temperatures are set at several electron volts, e.g. \( T_e \sim 1 \text{ eV} \) and \( T_i \sim 3 \text{ eV} \). In calculations it is assumed that cathode plasma number density varies within a wide range of several orders of magnitude. The lower \( n_0 \) threshold value is determined by the emission current magnitude, i.e. at least an order of magnitude higher than the Child-Langmuir currents for a flat vacuum diode.

3. Numerical results
The system of equations (1), (2) and (4) together with boundary conditions (7) forms a closed mathematical model of a one-dimensional vacuum diode with plasma emission from the cathode. The specified system of equations is solved numerically. The solution of the Vlasov equations (1) has been obtained using semi-Lagrangian methods. In order to perform the numerical solution accurately, the PIC4 high-precision non-oscillatory numerical scheme was used [14]. It was implemented on uniform 1000 by 4000 elements phase space grid both for ions and electrons. The Vlasov equations for electrons and ions were solved with a constant time step. High precision calculations require the correct choice of the time step. The time stepping for solving the electrons Vlasov equation has been set to 1-2 ps, while for ions it needs to be about 20-50 times larger, so the intermittent solution scheme has been used. To calculate the time-derivatives of voltage pulse waveforms in expression (6), the numerical differentiation schemes with smoothing were used. All necessary integration was carried out using the Simpson method on uniform grids of the phase space. The numerical scheme has been implemented in Mathworks MATLAB enforced with built-in GPU high-performance computational features.

Figure 1. Electron distribution function density plot a), ion distribution function density plot b) and spatial electric potential distribution c) at \( t = 5 \text{ ns} \).

The calculation was carried out within the framework of the proposed model where copper \((m_i = 63.5 \text{ a.m.})\) was chosen as the cathode material (ion plasma component). The question was studied whether the movement of ions from the cathode to the anode is possible if continuous emission of the cathode plasma is provided from the cathode. Therefore, there is no need to cover large computation
end times $t_{\text{end}}$. For a $D = 1$ cm vacuum gap, provided that a voltage of $U_0 = 2$ kV is applied to it, it is sufficient to observe the dynamics of processes in the diode for $t_{\text{end}} = 100$ ns.

In figures 1-3 the step-by-step dynamics of the electron and ion distribution functions is presented in details. Additionally, the description completeness is provided by the instantaneous electric potential plots and time profiles of the total diode current density and the anode current density.

During the first hundreds of picoseconds (figure 1), the motion of the plasma in the diode insufficiently differs from those in vacuum diode with predominant electron emission (see [13]). Namely, a rapid movement of electrons occurs towards anode, as well as damped (relaxation) oscillations of the anode current arise due to the space charge temporal inhomogeneity. After the relaxation oscillations decay the anode current establishes near the Child’s current values. These results confirm the observations of paper [13].

![Figure 2](image1.png)

**Figure 2.** Electron distribution function density plot a), ion distribution function density plot b) and spatial electric potential distribution c) at $t = 50$ ns.

![Figure 3](image2.png)

**Figure 3.** Electron distribution function density plot a), ion distribution function density plot b) and spatial electric potential distribution c) at $t = 100$ ns.

Already at the initial stage of vacuum breakdown development (figure 1), certain prerequisites for the directed ions movement towards anode are formed (figure 1 (c)). Namely, a negative voltage drop region known as a “virtual cathode” is formed near the cathode. The appearance of this region is caused by the high electron density emission from cathode plasma. Taking into account the continuous cathode plasma supply, the inhomogeneous electron density cloud slowly moves towards anode. This, in turn, causes the virtual cathode region displacement directed from physical cathode to anode region. The dynamic region behind the virtual cathode is filled with a quasi-neutral plasma, the average field inside which is approximately zero. The strong difference in the emissivity of the cathode and the
throughput of the diode gives rise to some features of the process. Its details are already noticeable at the initial stage of the process (figure 1). It looks like a small peak of the electric potential near the cathode of ~ 20 V and the corresponding decrease in the plasma number density of about an order of given boundary $n_0 = 10^{20} \text{ m}^{-3}$ value. This peak is maintained during ~ 1 ns while the “starting” electron Child-Langmuir current [13] is established.

The relatively low electric field strength in plasma accelerates ions to the anode, contributing to the formation of an even deeper (than the initial drop) potential well in the gap. The ion distribution function phase portraits shown in figure 2 and figure 3 clearly show how the starting and subsequent acceleration of ions to high velocities is formed. Further cathode to anode plasma dynamics is accompanied by the "virtual cathode” region displacement and further average ion energy increase. The described process represents a purely electrodynamic ions acceleration phenomenon at the plasma channel tip, which come from the quasi-neutral plasma region between the physical cathode and the "virtual cathode”.

Profiles of a full diode current density (6) and the electron current density through the anode (2) represent non-trivial time-dependencies. The anode electron current density at the beginning of the electrons arrival at anode process depicts classic relaxation oscillations (figure 4) [13]. As during the initial breakdown stage, the anode voltage is supplied with a short leading edge $t_{\text{rise}} = 100 \text{ ps}$; therefore, this process involves nontrivial contribution to the total diode current density due to the second term existence in (6). Once the anode voltage is set equal to the peak value of $U_{0}$, the displacement current density associated with $dU/dt$ completely disappears. As the cross section of the virtual cathode approaches the anode, the electron current in the gap increases, and the amplitude of the total current density also increases (figure 4). These points out to the increase of diode gap electrical conductivity caused by “anomalous” ion transport and to the increase of quasi-neutral plasma number density in the interelectrode gap.

![Figure 4. Time profiles of the full diode current density (red) and the anode electron current (blue).](image)

4. Conclusions
On the basis of a strict physical kinetic description of the electron and ion ensembles of particles behaviour in a self-consistent electric field, the initial stages of a non-stationary electron beam formation process have been simulated for a vacuum diode with the cathode plasma emitter.

We unambiguously showed for the first time that the cathode plasma velocity towards anode is determined by a self-consistent electric field dynamics accelerating ions to energies of tens of electron-volts, and not by the fact that the initial distribution of the cathode plasma has nonzero thermal velocities. The above mentioned ion energies are observed in experiments with vacuum arcs in a large number of experimental papers (for example, [9] and [10]). Given theoretical explanation
was obtained without introducing various artificial processes (e.g. so-called "electron wind" acceleration), which consideration always requires the formulation of artificial interactions between charged particles having number of arbitrary speculative parameters.

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References
[1] Beilis I I 1995 Vacuum Arc Science and Technology ed R Boxman, D Sanders and P Martin (Ridge Park: Noyes Publishing)
[2] Brown I G, Galvin J E and MacGill R A 1985 Appl. Phys. Lett. 47 358
[3] Anders A 2008 Cathodic Arcs: From Fractal Spots to Energetic Condensation (New York: Springer)
[4] Beilis I I 2001 IEEE T. Plasma Sci. 29 657
[5] Chapelle P, Bellot J P, Duval H, Jardy A and Ablitzer D 2001 J. Phys. D: Appl. Phys. 35 137
[6] Hantzsche E 2003 IEEE T. Plasma Sci. 31 799
[7] Davis W D and Miller H C 1969 J. Appl. Phys. 40 2212
[8] Brown I and Oks E 2005 IEEE T. Plasma Sci. 33 1931
[9] Anders A 1997 Phys. Rev. E 55 969
[10] Oks E M, Savkin K P, Yushkov G Y, Nikolaev A G, Anders A and Brown I G 2006 Rev. Sci. Instrum. 77 03B504
[11] Yang W, Sun Q and Zhou Q 2020 J. Appl. Phys. 128 060905
[12] Shmelev D L and Uimanov I V 2015 IEEE T. Plasma Sci. 43 2261
[13] Kozhevnikov V Y, Kozyrev A V and Semeniuk N S 2017 IEEE T. Plasma Sci. 45 2762
[14] Umeda T, Nariyuki Y and Kariya D 2012 Comput. Phys. Commun. 183 1094