Neutrino mixing

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Abstract

In the first part of these lectures the neutrino mixing hypothesis will be considered in detail. We will discuss the possible schemes of neutrino mixing and present the data of the recent experiments searching for effects due to nonvanishing neutrino masses and mixing angles. In the second part of these lectures the physics of solar neutrinos will be considered. We will discuss the MSW resonance solution of the equation of evolution of a neutrino in matter and present data of solar neutrino experiments.
1 Introduction

One of the most important question in neutrino physics is the problem of neutrino masses and mixing. From the point of view of many models beyond the standard one it is very natural for neutrinos to be massive. In the minimal standard model with only left-handed neutrino fields, neutrinos are massless particles. However, the standard model can be easily generalized if we assume that singlet right-handed neutrino fields are presented in the Lagrangian. In this enlarged standard model neutrinos, like leptons and quarks, are massive particles.

Leptons and quarks are electrically charged Dirac particles (particle ≠ antiparticle). For massive neutrinos there are two possibilities. Neutrinos with definite masses can possess some conserved lepton number and be Dirac particles or, in theories where there is no conserved lepton numbers, massive neutrinos are truly neutral Majorana particles (particle ≡ antiparticle). Theories with massive Majorana neutrinos are beyond the standard theory.

If neutrinos are particles with Dirac or Majorana masses their fields can appear in the weak currents in mixed form. This is the so-called neutrino mixing hypothesis \[1, 2\]. Mixing of fermion fields is a characteristic feature of modern gauge theories with spontaneous violation of symmetry. The Cabibbo-Kobayashi-Maskawa mixing of quarks is a well-known phenomenon. Does neutrino mixing take place too? In more than 50 experiments the problem of neutrino mixing is being investigated by different methods. Up to now no indications in favour of nonzero neutrino masses and mixing were obtained in experiments with terrestrial neutrinos. Experiments are continuing, however, and in the nearest future new level of accuracy will be achieved in experiments searching for neutrinoless double β-decay and other experiments.

The solar neutrino experiments play a special role in the test of the neutrino mixing hypothesis. These experiments are sensitive to values of neutrino masses and neutrino mixing angles so small that could not be reached in experiments with reactor and accelerator neutrinos.

In the first part of these lectures we will consider possible schemes of neutrino mixing and possible experiments to search for effects of neutrino masses and mixing. We will present also data of some of the most recent experiments. In the second part we will discuss the solar neutrino experiments. Resonance transitions of neutrinos in matter (MSW-mechanism) \[8\] will be considered in detail. As an introduction, it seems appropriate to recall the standard Higgs mechanism for the generation of quark masses.

2 Higgs mechanism of quark mass generation.

Let us introduce the doublet of scalar Higgs fields
\[ \phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \]  

(1)

and assume that the \( SU(2) \times U(1) \) invariant Lagrangian of interaction of quarks and Higgs boson has the Yukawa form

\[ \mathcal{L} = -\sqrt{2} \frac{v}{2} \sum_{i=1,2,3} \bar{q}_i M_{iq} q'_R \phi + h.c. \]  

(2)

Here \( \psi_{1L} = (u'_L, d'_L), \psi_{2L} = (c'_L, s'_L), \psi_{3L} = (t'_L, b'_L) \) are doublets of quark fields, \( q'_R \) are right-handed singlets, \( M \) is a complex nondiagonal \( 3 \times 3 \) matrix, \( v \) is a parameter. After spontaneous violation of symmetry we can put (unitary gauge)

\[ \phi(x) = \begin{pmatrix} 0 \\ \frac{v + \chi(x)}{\sqrt{2}} \end{pmatrix}, \]  

(3)

where \( \chi(x) \) is the field of the scalar, neutral Higgs particles. An arbitrary complex matrix \( M \) can be diagonalized with the help of a biunitary transformation:

\[ M = V_L m V_R^+, \]  

(4)

where \( V_L \) and \( V_R \) are \( 3 \times 3 \) unitary matrices; \( m_{ik} = m_i \delta_{ik}, \ m_i > 0 \). From Eq.(2), Eq.(3) and Eq.(4) we have

\[ \mathcal{L} = - \sum_{q=d,s,b} m_q \bar{q}q - \sum \left( \frac{m_q}{v} \right) \bar{q}q \chi, \]  

(5)

where

\[ \begin{pmatrix} d'_{L,R} \\ s'_{L,R} \\ b'_{L,R} \end{pmatrix} = V_{L,R} \begin{pmatrix} d_{L,R} \\ s_{L,R} \\ b_{L,R} \end{pmatrix}, \]  

(6)

The first term in expression Eq.(5) is the mass term of ”down” quarks. As it is seen from Eq.(6), the L(R) components of the fields that enter into the initial multiplets are connected with the L(R) components of quarks fields with definite masses by unitary transformations. Analogously, for the fields of ”up” quark we have

\[ \begin{pmatrix} u'_{L,R} \\ c'_{L,R} \\ t'_{L,R} \end{pmatrix} = U_{L,R} \begin{pmatrix} u_{L,R} \\ c_{L,R} \\ t_{L,R} \end{pmatrix}, \]  

(7)

where \( U_{L,R} \) are unitary matrices.

Further, the standard charged current is given by
\[ j^W_\alpha = 2[\bar{u}_L^e \gamma_\alpha d_L^e + \bar{e}_L^\gamma \gamma_\alpha s_L^e + \bar{t}_L^\gamma \gamma_\alpha b_L^e] \] (8)

Using Eq.(6) and Eq.(7) we can rewrite current \( j^W_\alpha \) in terms of physical quark fields as follows

\[ j^W_\alpha = 2[\bar{u}_L^e \gamma_\alpha d_L^e + \bar{e}_L^\gamma \gamma_\alpha s_L^e + \bar{t}_L^\gamma \gamma_\alpha b_L^e] \] (9)

Here

\[ d_L^e = \sum_{q=d,s,b} V_{qq}^L q_L, \quad s_L^e = \sum_{q=d,s,b} V_{cq}^L q_L, \quad b_L^e = \sum_{q=d,s,b} V_{tq}^L q_L \] (10)

and \( V = V_U^L U_L \) is the unitary mixing matrix.

Thus, in the general case of spontaneous violation of symmetry quark fields appear in the charged current in mixed form.\[^1\]

Now let us turn to the subject our lectures, namely the lepton sector. The standard lepton charged current has the form

\[ j_\alpha = 2 \sum_{l=e,\mu,t} \bar{\nu}_{lL} \gamma_\alpha l_L \] (11)

What are neutrino fields \( \nu_{lL} \)? Are they real flavour neutrino fields or mixture of fields of neutrinos with definite masses:

\[ \nu_{lL} = \sum_i U_{li} \nu_i L \] (12)

(\( \nu_i \) is the field of neutrinos with mass \( m_i \), \( U \) is unitary matrix). This last assumption is the so called neutrino mixing hypothesis. In the next chapter we will consider different possibilities of neutrino mixing.

### 3 Schemes of neutrino mixing.

#### 3.1 Dirac mass term.

Schemes of neutrino mixing are usually characterized by the type of the relevant mass terms. From Lorentz invariance it follows that in general three possible neutrino mass terms can be built \[^2\]. All of them may appear in different gauge models. In this section we will consider the mixing scheme, that corresponds to Dirac mass term.

\[ \mathcal{L}^D = -\sum_{\nu_j=e,\mu,\tau} \bar{\nu}_{jR} M_{\nu_j} \nu_{jL} + h.c., \] (13)

\[^1\]Let us notice that due to the unitarity of the matrices \( V_{L,R} \) and \( U_{L,R} \) the standard neutral current is diagonal in the quark fields.
that is analogous to the quark mass term (see Eq.(5)) and could be generated by the
standard Higgs mechanism we discussed in the previous paragraph. In the expression
Eq.(13) $M$ is a $3 \times 3$ complex, nondiagonal matrix, $\nu_{L}$ are neutrino fields that appear
in the standard charged and neutral currents (current fields). We will assume that
right-handed fields $\nu_{R}$ enter only into mass term $\mathcal{L}^{D}$. After the standard procedure
of diagonalization (see Eq.(4)) we have

$$\mathcal{L}^{D} = -\sum_{i=1}^{3} m_{i} \bar{\nu}_{i} \nu_{i}, \quad (14)$$

$$\nu_{L} = \sum_{i=1}^{3} U_{li} \nu_{iL}, \quad (15)$$

where $U^{+}U = 1$. It follows from Eq.(14) that $\nu_{i}$ is the field of neutrinos with mass $m_{i}$. So, if the neutrino mass term $\mathcal{L}^{D}$ is present in the Lagrangian, neutrinos are particles
with nonzero masses and the current fields $\nu_{L}$ are linear,unitary combinations of
left-handed components of the fields of the massive neutrinos.

It follows from Eq.(13) that in the case under consideration the lepton numbers
$L_{e}, L_{\mu}, L_{\tau}$ are not conserved separately. However, it is easy to see that invariance under
the global gauge transformation $\nu_{i} \to e^{i\alpha} \nu_{i}, \, l \to e^{i\alpha} l$ holds to the total lepton number

$$L = L_{e} + L_{\mu} + L_{\tau}$$

is conserved, and neutrinos with masses $m_{i}$ are Dirac particles ($\nu_{i}$ differs from $\bar{\nu}_{i}$ by the value of $L$).

Let us stress in conclusion that in the case of Dirac mass term there is a full
analogy between the quark and lepton sectors of the theory.

### 3.2 Majorana mass term.

In the Dirac mass term both left-handed and right-handed neutrino fields enter. Neutrino mass term can be built, however, using left-handed fields only, if we
assume that there are no conserved lepton numbers $\mathbb{I}$. Indeed, let us assume that in
the Lagrangian of the system the following mass term appear

$$\mathcal{L}^{M} = -\frac{1}{2} \sum_{\nu_{i}} (\nu_{iL})^{c} M_{\nu_{iL}} \nu_{iL} \quad + \quad h.c. \quad (16)$$

Here $(\nu_{iL})^{c} = C \bar{\nu}_{iL}^{T}$ is a right handed component ($C$ is the matrix of charge conjugation,
$C_{\gamma_{\alpha}}^{*} C^{-1} = -\gamma_{\alpha}, \, C^{T} = -C$) and $M$ is a complex, nondiagonal matrix. From the Pauli principle it follows that $M$ is a symmetric matrix. For any symmetric matrix $M$ we have

$$M = (U^{+})^{T} m U^{+} \quad (17)$$
where \( U^+ U = 1, \ m_{ik} = m_i \delta_{ik}, \ m_i > 0. \) From Eq.(16) and Eq.(17) it follows that

\[
\mathcal{L}^M = -\frac{1}{2} \sum_{i=3}^{3} m_i \bar{\chi}_i \chi_i, \quad (18)
\]

where

\[
\chi = U^+ \nu_L + (U^+ \nu_L) = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} \quad (19)
\]

and

\[
\nu_L = \begin{pmatrix} \nu_e L \\ \nu_{\mu} L \\ \nu_{\tau} L \end{pmatrix}.
\]

It is clear from Eq.(19) that the field \( \chi_i \) satisfies the condition

\[
\chi_i^c = \chi_i \quad (20)
\]

which is called Majorana condition. For any fermion field \( \chi(x) \) we have

\[
\chi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3 p}{\sqrt{2p^0}} \left( u^r(p) e^{-ipx} c_r(p) + u^r(-p) e^{ipx} d^+_r(p) \right) d^3 p \quad (21)
\]

where \( c_r(p) \) (\( d^+_r(p) \)) is the operator of annihilation of a particle (creation of antiparticle) with momentum \( p \) and helicity \( r \). If a fermion field \( \chi \) satisfies the Majorana condition Eq.(20), then we have

\[
c_r(p) = d_r(p)
\]

So the field that satisfies the Majorana condition is a field of truly neutral (Majorana) particles with spin 1/2 (particle \( \equiv \) antiparticle). It is clear from equations Eq.(18)-Eq.(20) that \( \mathcal{L}^M \) is the mass term of Majorana particles. This term is not invariant under any global gauge invariance. Further from Eq.(19) we will find easily that

\[
\nu_L = U \chi_L \quad \text{or} \quad \nu_{iL} = \sum_{i=1}^{3} U_{iL} \chi_i L \quad (22)
\]

Thus, if a neutrino mass term is of the form Eq.(16), the current neutrino fields \( \nu_{iL} \) are unitary combinations of the left-handed components of fields of Majorana neutrinos with definite masses. In this case there are no conserved lepton numbers.
3.3 Dirac and Majorana mass term

The scheme that corresponds to a Majorana mass term is the most economic mixing scheme— only left-handed current neutrino fields enter both the interaction Lagrangian and the neutrino mass term. The most general neutrino mixing scheme corresponds to Dirac and Majorana mass term, which is built with the help of left-handed and right-handed fields under the assumption that there are no conserved lepton numbers. Thus, let us assume that in the Lagrangian of the system the following neutrino mass term enters [2,3]

$$\mathcal{L}_{D-M} = -\frac{1}{2} \sum_{i,l} (\bar{\nu}_l^c M^c_{l'i} \nu_{iL} - \bar{\nu}_{l' R} M^D_{l'i} \nu_{iL} - \bar{\nu}_{l' R} M^R_{l'i} (\nu_{l R})^c + h.c.,)$$

(23)

where $M^L$, $M^D$ and $M^R$ are $3 \times 3$ complex matrices.

It is clear that, as in the case of Majorana mass term, neutrinos with definite masses in the case under consideration are Majorana particles. However, the number of massive particles in this case is twice of the number of lepton flavours. From Eq.(23), after standard procedure of the diagonalization of a $6 \times 6$ matrix $M$ we have

$$\mathcal{L}_{D-M} = -\frac{1}{2} \sum_{i=1}^{6} m_i \bar{\chi}_i \chi_i,$$

(24)

where $\chi_i = \chi_i^c$ is the field of Majorana neutrinos with mass $m_i$. The current fields $\nu_{iL}$ and fields $(\nu_{iR})^c = C \bar{\nu}_{i R}^T$ (left-handed components) are connected with left-handed components of massive Majorana fields $\chi_{iL}$ by a unitary transformation

$$\nu_{iL} = \sum_{i=1}^{6} U_{ii} \chi_{iL}$$

$$\quad \quad (\nu_{iR})^c = \sum_{i=1}^{6} U_{ii} \chi_{iL}$$

(25)

where $U$ is a unitary $6 \times 6$ matrix.

If all masses $m_i$ are small enough, the quanta of the fields $\nu_{iL}$ are usual flavor left-handed neutrinos and right-handed antineutrinos, while the quanta of the fields $\nu_{iR}$ are "sterile" right-handed neutrinos and left-handed antineutrinos. These last particles are sterile in the sense that they do not take part in the standard weak interaction (fields $\nu_{iR}$ do not enter the standard Lagrangian of interaction).

The masses $m_i$ and the mixing matrix $U$ are determined by the complex matrices $M^L$, $M^D$ and $M^R$. One of the most popular mechanism of neutrino mass generation is based on the assumption that $M^L = 0$ and elements of $M^D$ are much smaller than the
nonzero elements of $M_R$ (see-saw mechanism [3]). If the see-saw mechanism is realized, then the particles with definite masses are three very light Majorana neutrinos and three very heavy Majorana particles. In the next section we will consider the see-saw mechanism in some detail.

### 3.4 See-saw mechanism of neutrino mass generation.

From existing experimental data it follows that the mass of the neutrino in each generation (if any) is much smaller than the mass of the fermion in the same generation. The see-saw mechanism of neutrino mass generation [3] naturally incorporates this experimental fact. Let us consider the D-M mass term in the simplest case of one generation. We have

$$\mathcal{L}_{D-M} = -\frac{1}{2} m_L (\nu_L)^c \nu_L - m_D \bar{\nu}_R \nu_L - \frac{1}{2} m_R \bar{\nu}_R (\nu_R)^c + \text{h.c.}$$

$$= -\frac{1}{2} (\nu_L)^c M (\nu_L) + \text{h.c.}$$

(26)

Here

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix},$$

(27)

$m_L, m_D, m_R$ are parameters (for simplicity, real). For a symmetrical matrix $M$ we have

$$M = O \, m \, O^T,$$

(28)

where $O^T O = 1$, $m_{ik} = m_i \delta_{ik}$. From Eq.(26) and Eq.(28) we have

$$\mathcal{L}_{D-M} = -\frac{1}{2} \sum_{i=1}^2 m_i \bar{\chi}_i \chi_i,$$

(29)

where

$$\nu_L = \cos \theta \chi_1 - \sin \theta \chi_2$$

$$(\nu_R)^c = -\sin \theta \chi_1 + \cos \theta \chi_2.$$

(30)

Here $\chi_1$ and $\chi_2$ are fields of Majorana neutrinos with masses $m_1, m_2$. The masses $m_1, m_2$ and the mixing angle $\theta$ are connected to the parameters $m_L, m_D$ and $m_R$ by the relations
\[ m_{1,2} = \frac{1}{2} |m_R + m_L \mp a| \]
\[ \sin 2\theta = \frac{2m_D}{a}, \quad \cos 2\theta = \frac{m_R - m_L}{a} \]

(31)

where
\[ a = \sqrt{(m_R - m_L)^2 + 4m_D^2} \]

(32)

Relations Eq.(31) are exact. Let us assume now that
\[ m_L = 0, \ m_D \simeq m_F, \ m_R \gg m_F, \]

(33)

where \( m_F \) is the mass of the lepton or quark of the corresponding generation. From Eq.(31) we have
\[ m_1 \simeq \frac{m_F^2}{m_R}, \ m_2 \simeq m_R, \ \theta \simeq \frac{m_D}{m_R} \]

(34)

Thus, if the conditions Eq.(33) are satisfied, the particles with definite masses are a very light Majorana neutrino with mass \( m_1 \ll m_F \) and a very heavy Majorana particle with mass \( m_2 \simeq m_R \). The current neutrino field \( \nu_L \) practically coincides with \( \chi_1L \) and \( \chi_2 \simeq \nu_R + (\nu_R)\zeta \)

Usually it is assumed that \( m_R = M_{GUT} \); \( M_{GUT} \) is grand unification scale. The value of \( M_{GUT} \) depends on the model. Different possibilities were considered: from \( m_R \simeq 10^{10} \) GeV (some intermediate scale) up to \( m_R \simeq 10^{19} \) GeV (Planck mass). If \( m_R \) lies in this interval, for the heaviest neutrino \( \nu_\tau \), for example, we have
\[ 3.10^{-10} \text{eV} \leq m_{\nu_\tau} \leq 3.10^{-1} \text{eV} \]

or
\[ 2.10^{-6} \text{eV} \leq m_{\nu_\tau} \leq 2.10^{3} \text{eV} \]

depending on which see-saw formula we use: \( m_{\nu_\tau} \simeq \frac{m^2}{m_R} \) or \( m_{\nu_\tau} \simeq \frac{m^2}{m_R} \). Let us stress in conclusion that the idea of a see-saw mechanism is the following. Assume that in D-M mass term Dirac masses are of order of usual fermion masses, the right-handed Majorana masses, responsible for lepton numbers violation, are of order of a GUT mass and the left-handed Majorana masses are equal zero. In such a scheme neutrinos are Majorana particles with masses much smaller than masses of the other fermions. Concrete predictions of neutrino masses depend on the value of the GUT mass.
4 Physical consequences of neutrino mixing hypothesis. Experimental data.

4.1 “Direct” method of the measurement of neutrino mass ($\beta$-spectrum of $^3H$).

If neutrino masses are different from zero, the hard part of a $\beta$-spectrum that correspond to emission of soft neutrinos will be modified. The classical method is the investigation of the spectrum of the decay

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$

(35)

The electron spectrum in this superallowed decay is determined by the phase factor. Assuming that the mass of $\nu_e$ is not equal to zero, for the electron spectrum in the tritium decay we have

$$\frac{dN}{dT} = CpE(Q - T)\sqrt{(Q - T)^2 - m_\nu^2} \, F(E)$$

(36)

Here $p$ and $E = m_e + T$ are momentum and energy of the electron, $Q$ is the energy release ($Q \approx 18.6$ kev), $F(E)$ is the Fermi function, $m_\nu$ is the neutrino mass, $C$ is a constant.

The experiments on the measurement of neutrino mass by the tritium method are very complicated. Spectrometer energy resolution, molecular and other effects must be correctly taken into account. The measured spectrum is usually fitted with the help of $m_\nu^2$ and $Q$, and additional parameters that take into account energy resolution, background and normalization. The data that have been obtained from recent experiments are presented in Table 1.

Table 1. Upper bounds for the mass $m_\nu$ obtained by the tritium method [4].

| Group   | Upper bound for $m_\nu$ |
|---------|-------------------------|
| Zurich  | $< 11eV$                |
| Tokyo   | $< 13eV$                |
| LANL    | $< 9.3eV$               |
| Mainz   | $< 7.2eV$               |
| Livermore | $< 8eV$               |

As it is seen from Table 1, experiments on the precise investigation of the hard part of the tritium $\beta$-spectrum give only upper bounds on the mass of neutrino (about 10 $eV$). Notice that modern upper bounds of masses of $\nu_\mu$ and $\nu_\tau$ are

$$m_{\nu_\mu} < 270 \, keV$$
\[ m_{\nu_e} < 31 \text{ MeV} \]

### 4.2 Neutrinoless double $\beta$-decay.

The process

\[(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \]  \hspace{1cm} (37)

is allowed only if the lepton number L is not conserved (M or D-M mass terms). The neutrino masses enter into the neutrino propagator. In the case of neutrino mixing

\[ \nu_{eL} = \sum_i U_{ei} \chi_i L \]

with \( \chi_i = C \bar{\chi}_i^T \) we have

\[ \nu_{eL}(x_1) \nu_{eL}(x_2) = -\sum U_{ei}^2 \frac{1+\gamma_5}{2} \chi_i(x_1) \chi_i(x_2) \frac{1+\gamma_5}{2} \]

\[ = \frac{-i}{(2\pi)^4} \sum U_{ei}^2 m_i \int \frac{e^{ip(x_1-x_2)}}{p^2 + m_i^2} dp \frac{1+\gamma_5}{2} C \] \hspace{1cm} (38)

If the neutrino masses \( m_i \) are small enough (\( \leq \text{MeV} \)), \( m_i^2 \) in the integral Eq.(38) can be neglected and all the dependence of the matrix element on neutrino masses and mixing matrix elements is in the factor

\[ < m > = \sum U_{ei}^2 m_i \] \hspace{1cm} (39)

Notice that if there is a hierarchy in the lepton sector, similar to the hierarchy in the quark sector, the mixing matrix will be almost diagonal and the main contribution to \( < m > \) will come from the lightest neutrino mass.

More than 30 experiments searching for neutrinoless double $\beta$-decay (($\beta\beta$)$_{0\nu}$-decay) of different nuclei are going on at present. Up to now there are no positive indications in favour of the existence of ($\beta\beta$)$_{0\nu}$-decay. In Table 2 some latest lower bounds on the lifetime of this process are presented.

At present new generation of experiments on the search for ($\beta\beta$)$_{0\nu}$-decay with enriched $^{76}Ga$, $^{100}Mo$ and $^{136}Xe$ are going on and prepared. It is expected [7] that these new experiments will be sensitive to \( |< m >| \approx 0.2-0.3 \text{ eV} \).

### 4.3 Neutrino oscillations.

If there are neutrino mixing, then neutrino oscillations, that are analogous to the well known $K^0 \leftrightarrow \bar{K}^0$ oscillations, become possible [8]. Due to the fact that
oscillations are interference phenomena, their search is the most sensitive method of investigation of neutrino mixing. We will consider here briefly neutrino oscillations. The vector of state of flavour neutrino with momentum \( \vec{p} \) in the case of any type of neutrino mixing is given by

\[
| \nu_l > = \sum_i U_{li}^* | i >
\]

(40)

Here \( | i > \) is eigenstate of the free Hamiltonian

\[
H_0 | i > = E_i | i >,
\]

(41)

where

\[
E_i = \sqrt{m_i^2 + p^2} \simeq p + \frac{m_i^2}{2p}.
\]

(42)

Thus in the case of mixing, the states of flavour neutrinos are coherent superpositions of the states of neutrinos with definite energies. Notice that this is correct if neutrino mass differences are small enough.

Assume that at the time \( t=0 \) flavour neutrinos \( \nu_l \) with momentum \( \vec{p} \) are produced. At \( t > 0 \) the state vectors of neutrinos are given by

\[
| \nu_l >_t = \sum | \nu_{l'} > A_{\nu_{l'},\nu_l}(t)
\]

(43)

where

\[
A_{\nu_{l'},\nu_l}(t) = \sum_i U_{l'i} e^{-iE_i t} U_{li}^*
\]

(44)

is the amplitude of transition \( \nu_l \rightarrow \nu_{l'} \) during the time \( t \). Let us notice that in the case of D-M mixing in the right-handed side of Eq.(43) the sum over the states of sterile antineutrinos could enter. So, in the case of neutrino mixing, at some distance from the place where \( \nu_l \) were produced, neutrinos \( \nu_{l'} \) different from \( \nu_l (\nu_{l'} \neq \nu_l) \) could be observed. For the probability of the transition \( \nu_l \rightarrow \nu_{l'} \) during time \( t \simeq R \) from Eq.(44) we have
\[ P(\nu \rightarrow \nu') = \delta_{ll'} + 2\text{Re} \sum_{i<k} U_{li}^* U_{l'i} U_{l'k} U_{ki}^* (1 - e^{-i\Delta m^2_{ik} R / p}) \] (45)

where \( \Delta m^2_{ik} = m_i^2 - m_k^2 \). If \( \Delta m^2_{ik} R / p \ll 1 \) at all \( i \neq k \) in this case \( P(\nu \rightarrow \nu') \simeq \delta_{ik} \).

For neutrino oscillations to be observed, it is necessary that at least one neutrino mass squared difference satisfies the condition

\[ \Delta m^2 \geq \frac{p}{R} \] (46)

Typical values of the parameter \( \frac{p}{R} \) for reactors, meson factories, accelerators and the sun are equal respectively to \( 10^{-2}eV^2, 10^{-1}eV^2, 1eV^2, 10^{-11}eV^2 \).

Experimental data are usually analyzed under the simplest assumptions of oscillations between two neutrino types \( \nu_i \leftrightarrow \nu_{i'} \) \( (i' \neq i) \). In this case the mixing matrix \( U \) has the form

\[
U = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\]

where \( \theta \) is the mixing angle. From Eq.(45) we have

\[
P(\nu_i \rightarrow \nu_{i'}) = \frac{1}{2} \sin^2 2\theta (1 - \cos \frac{\Delta m^2 R}{2p})
\]

\[
P(\nu_i \rightarrow \nu_i) = P(\nu_{i'} \rightarrow \nu_{i'}) = 1 - P(\nu_i \rightarrow \nu_{i'})
\] (47)

There are no indications in favour of neutrino oscillations in experiments with terrestrial neutrinos. At present new experiments searching for the transition \( \nu_\mu \rightarrow \nu_\tau \) are going on [9] and are planned [10]. We will discuss these important experiments later on.

5 Solar neutrinos.

5.1 Experimental data. Comparison with the standard solar model.

Experiments on the detection of neutrinos from the sun are of utmost importance from the point of view of investigation of the sun as well as of neutrino properties (neutrino masses and mixing, neutrino magnetic moment and so on). Detection of neutrinos from the sun began in 1970. During many years the only solar neutrino experiment was Davis experiment [11]. Beginning from 1985 solar neutrinos are detected also by the Kamiokande II collaboration [12]. In both these experiments only high energy solar neutrinos, the flux of which is \( \simeq 10^{-4} \) of the total flux, are detected. A very important step in the investigation of neutrinos from the sun began at present.
First results of new experiments SAGE \cite{13} and Gallex\cite{14} have appeared. In these experiments low energy neutrinos, whose flux constitute most of the solar neutrino flux, are also detected.

In Table 3 the main neutrino producing reactions of the solar \textit{pp} and CNO cycles are presented. In the second column of Table 3 we give the neutrino energies and in the third the predicted fluxes (in units $10^{10} cm^{-2} sec^{-1}$).

Table 3. Reaction of \textit{pp} and CNO cycles in which neutrinos are produced.

| Reaction | Neutrino energies (MeV) | Predicted by the SSM flux ($10^{10} cm^{-2} sec^{-1}$) |
|----------|-------------------------|------------------------------------------------------|
| $pp \rightarrow de^+ \nu_e$ | 0 - 0.42 | 6.0 |
| $pep \rightarrow dv_e$ | 1.44 | $1.4 \times 10^{-2}$ |
| $^7Be \ e^- \rightarrow ^7Li \ \nu_e$ | 0.86(90\%) | $4.9 \times 10^{-1}$ |
| $0.38(10\%)$ |  |  |
| $^8B \rightarrow ^8Be \ e^+ \nu_e$ | 0 - 14 | $5.7 \times 10^{-4}$ |
| $^{13}N \rightarrow ^{13}C \ e^+ \nu_e$ | 0 - 1.2 | $4.9 \times 10^{-2}$ |
| $^{15}O \rightarrow ^{15}Ne \ e^+ \nu_e$ | 0 - 1.73 | $4.3 \times 10^{-2}$ |

In the Davis et.al. experiment neutrino are detected by the observation of $^{37}Ar$ production in the reaction

$$\nu_e + ^{37}Cl \rightarrow e^- + ^{37}Ar$$ \hspace{1cm} (48)

About 10 atoms of $^{37}Ar$ produced during one month are extracted from 615 t of $C_2Cl_4$, and in a small proportional counter K-capture is detected. The average rate of $^{37}Ar$ production in Davis et al. experiment is $[11]$

$$2.10 \pm 0.30 \ SNU$$

where $1SNU = 10^{-36} \text{ capture atoms sec}$. The threshold of the reaction Eq.(48) is equal to $E_{th} = 0.814 MeV$. Thus in the Davis experiment one detects neutrinos mainly from $^8B$-decay ($\simeq 77\%$) and from $^7Be$ K-capture ($\simeq 15\%$). From standard solar model it follows that $^{37}Ar$ production rate is

$$8.0 \pm 1.0 \ SNU \quad \text{(Bahcall)} \ [14]$$
$$5.8 \pm 1.0 \ SNU \quad \text{(Turck-Chieze)} \ [15]$$

Thus, the rate of $^{37}Ar$ production measured in Davis et al. experiment is much less than the predicted rate. This inconsistency was called the solar neutrino problem. In the Kamiokanda II experiment $[12]$ solar neutrinos are detected by the observation of the process
\[ \nu + e \rightarrow \nu + e \]

The threshold in this experiment is rather high \((E_{th} \simeq 7.5\text{MeV})\). Thus only \(^8\text{Be}\) neutrinos are detected in K II experiment. For the ratio of the detected number of events to the predicted by SSM number it was found

\[
\frac{\text{data}}{\text{SSM}} = 0.46 \pm 0.05 \pm 0.06 \quad \text{(Bahcall)}
\]

\[
\frac{\text{data}}{\text{SSM}} = 0.70 \pm 0.08 \pm 0.09 \quad \text{(Turck-Chieze)}
\]

Recently the result of two new radiochemical solar neutrino experiments GALLEX and SAGE were published \([13, 14]\). In these experiments solar neutrinos were detected by the observation of \(^{71}\text{Ge}\) production in the reaction

\[ \nu_e + ^{71}\text{Ga} \rightarrow e^- + ^{71}\text{Ge} \quad (49) \]

The threshold of this reaction is 0.233 MeV. Thus, the \(Ga - Ge\) method allows us to detect solar neutrinos from all sources, including the main \(pp\) source. In the GALLEX experiment the target is a water solution of gallium chloride (30.3 tons of \(Ga\)). The result published is based on 14 runs of exposition (about 1 year). For the average rate of \(^{71}\text{Ge}\) production the value \([14]\)

\[(83 \pm 19(\text{stat}) \pm 8(\text{syst})) \text{ SNU}\]

was found, which is only about two standard deviations below the predicted values:

\[
131.5 \pm 7 \text{ SNU} \quad \text{(Bahcall)}
\]

\[
124.0 \pm 5 \text{ SNU} \quad \text{(Turck-Chieze)}
\]

From the thermodynamical point of view solar energy is produced by

\[ 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e + 27 \text{ MeV} \]

So the total flux of solar neutrinos \(I\) is connected to the sun luminosity \(L\) by the relation

\[ I \simeq \frac{2L}{4\pi R^227\text{MeV}} \quad (50) \]

where \(R\) is Sun-Earth distance. For the rate \(Q\) of \(^{71}\text{Ge}\) production we have
\[ Q = \sum_i \bar{\sigma}_i I_i \]  

(51)

where a sum of over-all neutrino sources \( i \) is assumed and \( \bar{\sigma}_i \) is average cross section of the process Eq.(49). From Eq.(50) and Eq.(51) it follows

\[ Q \geq \bar{\sigma}_{pp} I \simeq 80 \pm 2 \text{ SNU} \]

The observed rate satisfies this bound. From our point of view this means that new experiments are needed to solve the solar neutrino problem if this problem exists.

In the other \( Ga - Ge \) solar neutrino experiment SAGE metallic \( Ga \) is used (30 tons of \( Ga \) in the first runs and 57 tons now). The latest data of this experiment \[ 13 \]

\[ (58 \pm 17 - 24 \pm 14) \text{ SNU} \]

are in agreement with the GALLEX result.

### 5.2 Neutrino mixing and solar neutrinos

#### 5.2.1 Introduction

In this section we will discuss solar neutrino experiments from the point of view of the neutrino mixing hypothesis. If there are neutrino oscillations the beam of solar neutrinos that initially was a \( \nu_e \) beam at the Earth will be described by a mixture of neutrinos of different types. Radiochemical methods allow us to detect only \( \nu_e \). Thus, from the point of view of neutrino oscillations it is naturally to expect that the detected \( \nu_e \) flux \( I_{\nu_e} \) is lower than the initial \( \nu_e \) flux \( I_{\nu_e} \). We have

\[ I_{\nu_e} = P(\nu_e \rightarrow \nu_e) I_{\nu_e}^0 \]  

(52)

where \( P(\nu_e \rightarrow \nu_e) \) is the probability for \( \nu_e \) to survive. This last quantity depends on the neutrino mass squared difference, \( m_{ik}^2 = |m_i^2 - m_k^2| \) on the elements of the mixing matrix and on the parameter \( \frac{R}{p} \) (\( R \) is the Sun-Earth distance, \( p \approx E \) is the neutrino energy). If for each \( \Delta m_{ik}^2 \), the condition

\[ \Delta m_{ik}^2 \geq \frac{E}{R} \]

is satisfied, from Eq.(45) for the averaged survival probability we have

\[ P(\nu_e \rightarrow \nu_e) = \sum_i |U_{ei}|^4 \]  

(53)

In the simplest case of the mixing of two neutrino types from Eq.(53) it follows
\[
P(\nu_e \rightarrow \nu_e) = \frac{1}{2}(1 + \cos^2 2\theta)
\]  
(54)

We see from Eq.(54) that the detected $\nu_e$ flux could be as small as $\frac{1}{2}$ of the initial flux. This maximal suppression of $\nu_e$ flux takes place at maximum mixing $\theta \simeq \frac{\pi}{4}$. In the general case of oscillations between $n$ neutrino types we have from Eq.(53)

\[
P_{\text{min}}(\nu_e \rightarrow \nu_e) = \frac{1}{n}
\]  
(55)

This minimum is reached at $|U_{ei}|^2 = \frac{1}{n}$ (maximum mixing). Thus, in the case of D or M mixing

\[
P_{\text{min}} = \frac{1}{3}
\]

and in the case of D-M mixing

\[
P_{\text{min}} = \frac{1}{6}
\]

Up to now we have considered only oscillations of solar neutrinos in vacuum. It was shown, however, that matter effects could be important. Now we will discuss these effects.

### 5.2.2 Equation of evolution of neutrinos in matter.

Let us consider a beam of neutrinos with momentum $p$ and helicity 1. The vector of state of the beam has the form

\[
|\psi(t)\rangle = \sum_i |\nu_l\rangle a_{\nu_l}(t)
\]  
(56)

where $a_{\nu_l}(t)$ is the amplitude of probability to find $\nu_l$ at the time $t$. In vacuum we have

\[
i \frac{\partial a(t)}{\partial t} = H_0 a(t)
\]  
(57)

where

\[
<\nu_{l'}|H_0|\nu_l> = \sum_i <\nu_{l'}|i> E_i <i|\nu_l>
\]  
(58)

Here $|i\rangle$ is eigenstate of $H_0$

\[
H_0|i\rangle = E_i|i\rangle
\]  
(59)

where
\[ E_i = \sqrt{m_i^2 + \vec{p}^2} \simeq p + \frac{m_i^2}{2p} \quad (60) \]

Further, we have

\[ |\nu_i> = \sum_i |i><i|\nu_i> = \sum U_{ii}|i> \quad (61) \]

where \( U \) is the neutrino mixing matrix in vacuum. From this relation it follows

\[ <\nu_i|i> = U_{ii} \quad (62) \]

From Eq.(58)-Eq.(62) for the free Hamiltonian in the flavour representation we have

\[ H_0 = UEU^+ \simeq p + \frac{m^2}{2p}U^+ \quad (63) \]

Now let us discuss the Hamiltonian of interaction of neutrino with matter. First, let us notice that the neutral current contribution to the Hamiltonian of interaction is proportional to the unit matrix (\( \nu_e - \nu_\mu - \nu_\tau \) symmetry) and can be dropped. The Hamiltonian of charged current interaction of \( \nu_e \)'s with electrons can be written in the form

\[ \mathcal{H} = \frac{G}{\sqrt{2}} \overline{\nu}_e \gamma_\alpha(1 + \gamma_5)\nu_e \gamma_\alpha(1 + \gamma_5)e \quad (64) \]

Taking into account that electrons of matter are nonrelativistic particles for the effective Hamiltonian we have

\[ (H_I(x))_{\nu_e;\nu_e} = \frac{G}{\sqrt{2}} <\overline{\nu}_e(x)\gamma_\alpha(1 + \gamma_5)\nu_e(x)|\vec{p}> <\text{mat}\overline{e}(x)\gamma_\alpha(1 + \gamma_5)e(x)|\text{mat}> = 2\frac{G}{\sqrt{2}}\rho(x) \quad (65) \]

where \( \rho(x) \) is electron density at the point \( x \). Thus, finally we have the following equation of the evolution of the neutrino beam in matter \[ 3 \]

\[ i\frac{\partial a(x)}{\partial x} = (U\frac{m^2}{2p}U^+ + \sqrt{2}G\rho(x)\beta)a(x) \quad (66) \]

Here \( x \simeq t \) is the distance from the point where neutrinos where born; \( (\beta)_{\nu_e;\nu_e} = 1 \), other elements of the matrix \( \beta \) are equal to zero.
5.2.3 Solutions of the equation of evolution of neutrinos in matter.

In this section we will discuss the solutions of Eq.(66). For simplicity let us limit ourselves to the simplest case of two neutrino flavours ($\nu_e$ and, say, $\nu_\mu$). For the mixing matrix in vacuum we have in this case

$$O = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$ (67)

Let us present the Hamiltonian in the form

$$H_0 + H_I = H + \frac{1}{2} Tr(H_0 + H_I)$$

The unit matrix $\frac{1}{2} Tr(H_0 + H_I)$ can be omitted. For the total Hamiltonian from Eq.(66) and Eq.(67) we have

$$H(x) = \frac{1}{4p} \begin{pmatrix} 2\sqrt{2}G\rho(x)p - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & -2\sqrt{2}G\rho(x)p + \Delta m^2 \cos 2\theta \end{pmatrix}$$ (68)

where $\Delta m^2 = m^2_2 - m^2_1$.

Let us transform now the matrix $H(x)$ to diagonal form. We have

$$H(x) = O(x)E(x)O^T(x)$$ (69)

where

$$O(x) = \begin{pmatrix} \cos \theta(x) & \sin \theta(x) \\ -\sin \theta(x) & \cos \theta(x) \end{pmatrix}$$

and $E_{ik}(x) = E_i(x)\delta_{ik}$. Here $E_{1,2}(x)$ are eigenvalues of $H(x)$, and $\theta(x)$ is the mixing angle in matter. From Eq.(68) and Eq.(69) we have

$$\sin 2\theta(x) = \frac{\Delta m^2 \sin 2\theta}{\sqrt{X^2 + \Delta m^4 \sin^2 2\theta}}$$

$$\cos 2\theta(x) = \frac{X}{\sqrt{X^2 + \Delta m^4 \sin^2 2\theta}}$$

$$E_{1,2} = \mp \frac{1}{4p} \sqrt{X^2 + \Delta m^4 \sin^2 2\theta}$$ (70)

where

$$X(x) = \Delta m^2 \cos 2\theta - 2\sqrt{2}G\rho(x)E$$ (71)
As it is seen from Eq.(70) the mixing angle in matrix depends on $x$ through the density $\rho(x)$. In the sun the density is maximal at the center and decreases in the direction of periphery. Let us assume that at some point $x = x_R$ we have

$$\triangle m^2 \cos 2\theta = 2\sqrt{2}G\rho(x_R)E$$

(we will assume that $\triangle m^2 > 0$). From Eq.(70) and Eq.(71) it follows, that at this point the mixing in matter is maximal $\theta(x_R) = \frac{\pi}{4}$ for any values of $\theta$ different from zero. The condition Eq.(72) is called resonance condition. Let us notice that at the point $x = x_R$ the distance between levels is minimal

$$E_2(x_R) - E_1(x_R) = \frac{\triangle m^2 \sin 2\theta}{2E}$$

The resonance condition can be rewritten in the form

$$\triangle m^2 \cos 2\theta \simeq 0.7 \times 10^{-7} \frac{\rho_m}{g/cm^3} \frac{E}{MeV} eV^2$$

In the center of the sun $\rho_m \simeq 10^2 g/cm^3$ and for solar neutrinos $E \simeq MeV$. So for solar neutrinos the resonance condition is realized at $\triangle m \simeq 10^{-5} eV^2$.

Let us return now to the evolution equation. From Eq.(66) and Eq.(69) we have

$$i\frac{\partial a(x)}{\partial x} = O(x)E(x)O^T(x)a(x)$$

(75)

Now determine the function

$$a'(x) = O^T(x)a(x)$$

(76)

For this function we have the following equation

$$i\frac{\partial a'(x)}{\partial x} = (E(x) - iO^T(x)\frac{\partial O(x)}{\partial x})a'(x)$$

(77)

it is easy to show that

$$-iO^T(x)\frac{\partial O(x)}{\partial x} = \begin{pmatrix} 0 & i\frac{d\theta(x)}{dx} \\ i\frac{d\theta(x)}{dx} & 0 \end{pmatrix}$$

(78)

Further, with the help of Eq.(70) we have

$$\frac{d\theta}{dx} = \frac{\sqrt{2}G\triangle m^2 \sin 2\theta E\frac{d\rho}{dx}}{X^2 + \triangle m^4 \sin^2 2\theta}$$

(79)

So we derivative $\frac{d\theta}{dx}$ is determined by $\frac{d\rho}{dx}$. Non let us assume that the density varies slowly enough and
\[ \left| \frac{d\theta(x)}{dx} \right| \ll \frac{1}{2}(E_2 - E_1) \]  

(80)

In this case we have

\[ i \frac{\partial a'(x)}{\partial x} = E(x)a'(x) \]  

(81)

The solution of this equation has the form

\[ a'(x) = e^{-i \int_{x_0}^{x} E_i(x) dx} a_i(x_0) \]  

(82)

The condition Eq.(80) is called adiabatic condition. If this condition is satisfied, neutrinos remain at the same energy level. From Eq.(76) and Eq.(82) we obtain the following solution of the evolution equation in the adiabatic approximation

\[ a(x) = O(x)e^{-i \int_{x_0}^{x} E(x) dx} O^T a(x_0) \]  

(83)

For the averaged survival probability from Eq.(83) we get

\[ P(\nu_e \to \nu_e) = \sum O^2_{\nu_e i}(x) O^2_{\nu_k i}(x_0) = \frac{1}{2}(1 + \cos 2\theta(x) \cos 2\theta(x_0)) \]  

(84)

Let us consider neutrinos that were produced in the region of the sun where \( \rho > \rho(x_R) \). Assume also that \( \theta \) is small. From Eq.(70) for the initial mixing angle we have \( \theta(x_0) \simeq \frac{\pi}{2} \). From Eq.(84) it follows that survival probability is equal in this case

\[ P(\nu_e \to \nu_e) \simeq \frac{1}{2}(1 - \cos 2\theta) \simeq 0 \]  

(85)

Thus, if the adiabatic condition Eq.(80) is satisfied, all the electron neutrinos that are produced in the region with \( \rho > \rho(x_R) \) are transformed into muon neutrinos that cannot be registered by the radiochemical detectors.

In the general case for the probability of transition \( \nu_e \to \nu_e \) we have

\[ P(\nu_e \to \nu_e) = \sum O^2_{\nu_e k}(x) P_{ki} O^2_{\nu_k i}(x_0) \]  

(86)

where \( P_{ik} \) is the probability of transition between levels with energies \( E_i \) and \( E_k, x_0 \) is the initial point and \( x \) is the final point. We have from unitarity

\[ P_{12} = P_{21}, \quad P_{11} = P_{22} = 1 - P_{12} \]  

(87)

From Eq.(86) and Eq.(87), for the survival probability we obtain the following expression
\[ P(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \frac{1}{2} \cos 2\theta(x) \cos 2\theta(x_0)(1 - 2P_{12}) \]  \hspace{1cm} (88)

To determine \( P_{12} \) it is necessary to solve the evolution equation. A rather good approximation is the Landau-Zenner approximation that is based on the assumption of linear behavior of density near the resonance. In this approximation\[17\]

\[ P_{12} = e^{-\gamma(x_R)} \]  \hspace{1cm} (89)

where

\[
\gamma(x_R) = \frac{1}{2} \left( E_2(x_R) - E_1(x_R) \right) \left| \frac{d\theta}{dx} \right|_{x=x_R} = \frac{\Delta m^2 \sin^2 2\theta}{2E} \cos 2\theta \left| \frac{d \ln \rho}{dx} \right|_{x=x_R} \]  \hspace{1cm} (90)

is the adiabaticity parameter. If the adiabatic approximation is valid, then \( \gamma(x_R) \gg 1 \) and \( P_{12} = 0 \). Let us assume now that \( \gamma(x_R) \ll 1 \) and the resonance condition is fulfilled. In this case for survival probability from Eq.\( (88) \) we will get

\[ P(\nu_e \rightarrow \nu_e) \simeq \frac{1}{2} \left( 1 + \cos 2\theta \right) \simeq 1 \]  \hspace{1cm} (91)

Thus, for resonance transition of \( \nu_e \) into \( \nu_\mu \) to take place, two conditions must be satisfied:

1. Resonance condition

\[ \frac{\Delta m^2}{E} \leq \frac{2\sqrt{2}G\rho_0}{\cos 2\theta} \]  \hspace{1cm} (92)

where \( \rho_0 \) is electron density at the center of the sun,

2. Condition \( \gamma(x_R) \geq 1 \)

\[ \frac{\Delta m^2}{E} \geq \frac{2 \cos \theta \left| \frac{d \ln \rho}{dx} \right|_{x=x_R}}{\sin^2 2\theta} \]  \hspace{1cm} (93)

In conclusion we would like to remark that all existing solar neutrino experimental data could be described by the MSW mechanism if we assume that the standard solar model is valid. For the allowed values of the parameter \( \Delta m^2 \) and \( \sin^2 \theta \) the following three regions were found\[18,19\]

\[ 3.2 \times 10^{-6} eV^2 \leq \Delta m^2 \leq 1.2 \times 10^{-5} eV^2 \]
\[ 5.0 \times 10^{-3} \leq \sin^2 2\theta \leq 1.6 \times 10^{-2} \]
5.4 \times 10^{-6} eV^2 \leq \Delta m^2 \leq 1.1 \times 10^{-4} eV^2 \\
0.18 \leq \sin^2 2\theta \leq 0.86 \\
10^{-7} eV^2 \leq \Delta m^2 \leq 1.8 \times 10^{-6} eV^2 \\
0.74 \leq \sin^2 2\theta \leq 0.93 \quad (94)

6 Some remarks about future experiments on the search for neutrino oscillations.

As we have discussed before, no indications in favour of neutrino oscillations were obtained in experiments with terrestrial neutrinos. The only indications that neutrino masses are different from zero come from solar neutrino data. All data available give some clue in favour of a possible scenario \[20\] that we would like to discuss briefly in conclusion. Let us assume that there is a mass hierarchy

\[ m_1 \ll m_2 \ll m_3 \quad (95) \]

and that \( \Delta m^2_{21} = m_2^2 - m_1^2 \) is so small that

\[ \frac{\Delta m^2_{21} R}{p} \ll 1 \]

where R is a terrestrial distance (say \( 10^{-7} eV^2 < \Delta m^2_{21} < 10^{-4} eV^2 \)). For the average transition probability we have

\[ P(\nu_l \rightarrow \nu_{l'}) = 2|U_{l'3}|^2|U_{l3}|^2, \quad l' \neq l. \quad (96) \]

So only the matrix elements that connect lepton flavours with the heaviest neutrino \( \nu_3 \) determine the transition probabilities in this case.

Now, let us assume also that there is in the lepton sector a hierarchy of couplings between generations

\[ |U_{e3}|^2 \ll |U_{\mu 3}|^2 \ll |U_{\tau 3}|^2, \quad (97) \]

that is analogous to the hierarchy in the quark sector. From the unitarity of the mixing matrix we have \( |U_{\tau 3}|^2 \simeq 1 \). That means that preferred transitions are those into \( \nu_\tau \). Further, from Eq.(96) and Eq.(97) we have

\[ P(\nu_e \rightarrow \nu_\tau) \ll P(\nu_\mu \rightarrow \nu_\tau) \]

\[ P(\nu_\mu \rightarrow \nu_e) = \frac{1}{2} P(\nu_e \rightarrow \nu_\tau) P(\nu_\mu \rightarrow \nu_\tau) \ll P(\nu_e \rightarrow \nu_\tau) \quad (98) \]

Thus, if there is a hierarchy Eq.(95) and Eq.(97), \( P(\nu_\mu \rightarrow \nu_\tau) \) is the largest transition probability.
New experiments in search for $\nu_\mu \rightarrow \nu_\tau$ oscillations are prepared and planned at present. These experiments will be much more sensitive to $\nu_\mu \rightarrow \nu_\tau$ transitions than the previous one. For example, if the mass $m_3$ is $\simeq 1 eV (\simeq 10 eV)$ neutrino oscillations could be seen in these experiments if $\sin^2 2\theta \geq 10^{-2} (\sin^2 2\theta \geq 10^{-3})$.

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