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Polarized spots in anisotropic open universes

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Abstract

We calculate the temperature and polarization patterns generated in anisotropic cosmological models drawn from the Bianchi classification. We show that localized features in the temperature pattern, perhaps similar to the cold spot observed in the Wilkinson Microwave Anisotropy Probe (WMAP) data, can be generated in models with negative spatial curvature, i.e. Bianchi types V and VII$_h$. Both these models also generate coherent polarization patterns. In Bianchi VII$_h$, however, rotation of the polarization angle as light propagates along geodesics can convert E modes into B modes but in Bianchi V this is not necessarily the case. It is therefore possible, at least in principle, to generate localized temperature features without violating existing observational constraints on the odd-parity component of the cosmic microwave background polarization.

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1. Introduction

Observations of the temperature anisotropies of the cosmic microwave background, particularly those from the Wilkinson Microwave Anisotropy Probe (WMAP) [1, 2], form the foundations of the current (‘concordance’) cosmological model [3]. However, WMAP has also uncovered tantalizing evidence of departures from the standard framework. In particular, detailed analysis of the pattern of temperature fluctuations has led to the identification of a Cold Spot [4–10]. The level of departure from isotropy is relatively small but it is highly significant from a statistical point of view. The presence of this feature seems to be inconsistent with the assumption of global isotropy upon which the concordance cosmology is based and could be evidence that we live in a globally anisotropic universe, i.e. one not described by a Friedmann–Robertson–Walker (FRW) model.

The Bianchi classification arranges all possible spatially homogeneous but anisotropic relativistic cosmological models into types depending on the symmetry properties of their spatial hypersurfaces [11, 12]. It has been known for some time that localized features in the
radiation background can occur in Bianchi models with negative spatial curvature [13–17]. The physical origin of such features lies in the focussing effect of spatial curvature on the geodesics that squeezes the pattern of the anisotropic radiation field into a small region of the sky. Only a few of the Bianchi types contain the FRW model as a limiting case and, from this subset the model which appears to best able to reproduce the anomalous cold spot is the Bianchi VII$_h$ case [19–23]. However, as well as forming distinctive features in the temperature pattern, anisotropic cosmological models also generate characteristic signatures in the polarized component of the background radiation. Thomson scattering generates polarization as long as there is a quadrupole anisotropy in the temperature field of the radiation incident upon the scattering particle. In the concordance cosmology the temperature and polarization patterns are (correlated) stochastic fields arising from their common source in scalar and tensor perturbations arising from inflation. In a Bianchi cosmology, however, the patterns are coherent and have a deterministic relationship with one another owing to their common geometric origin. It has recently been shown [24–26] that the properties of the polarization field produced in Bianchi VII$_h$ are inconsistent with the latest available WMAP polarization data [27] because they inevitably involve a large odd-parity (B-mode) contribution that exceeds the experimental upper limit.

In a forthcoming paper we present an exhaustive study of the temperature and polarization anisotropies produced by those Bianchi types that possess an FRW limit [26]. The purpose of studying these models is to characterize as fully as possible the radiation fields they can produce in order to separate them as clearly as possible from residual systematics and thus provide the strongest possible constraints on exotic cosmologies. In the present paper, however, we focus on the very specific question of whether localized spots necessarily involve a large B-mode polarization.

2. Bianchi cosmologies

The models we consider are based on Einstein’s general theory of relativity and we use the field equations in the form

$$G_{ab} \equiv R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} - \Lambda g_{ab},$$

with $R_{ab}$ being the Ricci tensor, $R$ the Ricci scalar, $T_{ab}$ the energy–momentum tensor and $\Lambda$ the cosmological constant. Indices $a$ and $b$ run from 0 to 3. We use units where $8\pi G = c = 1$.

In terms of a coordinate system $x^a$, the metric $g_{ab}$ is written as

$$ds^2 = g_{ab} dx^a dx^b = (h_{ab} - u_a u_b) dx^a dx^b,$$

where $u^a$ is the fluid velocity; the signature of $g_{ab}$ is $(−+++)$.

Starting from a local coordinate system $x^i$ we construct a tetrad basis [12]

$$e_a = e^i_a \frac{\partial}{\partial x^i}$$

such that

$$g_{ab} = e^i_a e^j_b \delta_{ij} = e^i_a e^j_b = \text{diag}(-1, +1, +1, +1)$$

meaning that the tetrad basis $e_a$ is orthonormal. The Ricci rotation coefficients,

$$\Gamma^c_{abc} = e^c_i e_{vi,j} e^j_b,$$

are the tetrad components of the Christoffel symbols; semicolons denote covariant derivatives.

In general, the operators defined by equation (3) do not commute: they generate a set of relations of the form

$$[e_a, e_b] = \gamma^c_{ab} e_c.$$
The Ricci rotation coefficients are just
\[ \Gamma_{abc} = \frac{1}{2} (\gamma_{abc} + \gamma_{cab} - \gamma_{bca}). \]
(7)
The matter flow is described in terms of the expansion \( \vartheta_{ab} \) and shear \( \sigma_{ab} \):
\[ u_{a;b} = \omega_{ab} + \vartheta_{ab} - \dot{u}_a u_b, \]
\[ \sigma_{ab} = \vartheta_{ab} - \frac{1}{3} \vartheta \sigma_{ab} \equiv \vartheta_{ab} - H_{ab}, \]
(8)
where \( \vartheta = \text{Tr}(\vartheta_{ab}) = \vartheta_{aa} \) and the magnitude of the shear is
\[ \sigma^2 = \sigma_{ab} \sigma_{ab}/2. \]
We now take the time-like vector in our basis to be the fluid flow velocity so that
\[ u_a = \delta_a^0 \text{ and } u_a = -\delta^a_0. \]
The remaining space-like vectors form an orthonormal triad, with a set of commutation relations like that shown in equation (6) but with an explicit time dependence in the ‘structure constants’ describing the spatial sections:
\[ [e_i, e_j] = \gamma_{ik}^{(t)} e_k. \]
(9)
Without loss of generality we can write
\[ \gamma_{ij}^k = \epsilon_{ijl} n^{lk} + \delta_{kj} a_i - \delta_{ki} a_j, \]
for some tensor \( n_{ij} \) and some vector \( a_i \). The Jacobi identities require that
\[ n_{ij} a^j = 0 \] so we choose \( a^j = (a, 0, 0) \) and \( n_{ij} = \text{diag}(n_1, n_2, n_3) \). The remaining four free parameters are used to construct the Bianchi classification described in detail elsewhere [11, 12].

We are interested in cosmological models that are close to the homogeneous and isotropic FRW case, but not all the Bianchi types contain this solution. Those that do are types I, V, VII\(_h\), VII\(_v\) and IX. Bianchi I and Bianchi VII\(_h\) are spatially flat, Bianchi IX is positively curved and Bianchi types V and VII\(_v\) have negative spatial curvature. The open cases are of particular interest in this paper as they permit the focussing of anisotropic patterns into small regions of the sky. The scalar curvature \( R \) of the spatial sections is given in terms of the Bianchi parameters as
\[ R = \frac{1}{2} \left( 2n_1 n_2 + 2n_1 n_3 + 2n_2 n_3 - n_1^2 - n_2^2 - n_3^2 \right) - 6a^2. \]
(11)
For Bianchi V we have \( n_1 = n_2 = n_3 = 0 \) so that \( R = -6a^2 \). In Bianchi VII\(_h\) we have \( n_1 = 0 \) but \( n_2 \neq 0 \) and \( n_3 \neq 0 \); the parameter \( h \) is defined by \( h = a^2/n_2 n_3 \). If we take \( n_2 = n_3 = n \), the canonical form of the model, we also have \( R = -6a^2 = -6n^2 h \).

The models we consider have a single preferred axis of symmetry. The alignment of the shear eigenvectors relative to this preferred axis determines not only the dynamical evolution of the model through the field equations, but also the temperature and polarization pattern that gets imprinted into the cosmic background radiation.

3. Temperature and polarization anisotropies

The transfer equation for polarized radiation propagating through spacetime can be described by a combination of a propagation vector and a polarization vector. In the tetrad frame these can be written as
\[ \hat{N} = \left( \begin{array}{c} N^0 \\ N^2 \end{array} \right) = \frac{1}{e^2 v^3} \left( \begin{array}{c} I + iV \\ Q - iU \end{array} \right), \]
(12)
in terms of the standard Stokes parameters, \( I, Q, U \) and \( V \) where the superscripts indicate the parts of the (complex) photon distribution involving spin weights 0 and 2. The direction of a light ray in three-dimensional space is defined by \( k^i = (\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi) \) and we have also used the complex unit vector
\[ m^i = -\frac{1}{\sqrt{2}} \partial k^i \equiv -\frac{1}{\sqrt{2}} \left( \frac{\partial}{\partial \theta} + \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right) k^i. \]
(13)
Neglecting shear-dependent terms in the geodesic equation, the following equation can be obtained [14, 15] that describes the evolution of the direction of propagation:

$$\frac{1}{\varepsilon} \frac{d\kappa_i}{dt} = \epsilon_{i,jk} n^j k^k + a_i k_i - a_1,$$

(14)

where $\varepsilon$ is the photon energy. This leads to

$$\frac{1}{\varepsilon} \frac{d\theta}{dt} = (a + (n_3 - n_2) \cos \phi \sin \theta) \sin \theta,$$

$$\frac{1}{\varepsilon} \frac{d\phi}{dt} = [n_1 - n_3 + (n_3 - n_2) \cos^2 \phi] \cos \theta.$$

(15)

The radiation distribution described by $N^A$ is affected by the properties of the geodesics along which photons propagate but is also altered by scattering. If only elastic scattering is included, this is governed by a Boltzmann equation

$$\mathcal{D} N^A = \tau (-N^A + J^A),$$

(16)

involving the operator

$$\mathcal{D} = \frac{\partial}{\partial t} - \varepsilon \gamma^0 \frac{\partial}{\partial \varepsilon} + \sqrt{2} \left( i \delta_{ik} k^j k^m - \delta_{ik} k^j m^k \right) \epsilon_{ikm},$$

(17)

where $\tau$ is the optical depth; the amplitude of the polarization depends strongly on $\tau$ but we shall not explore its dependence in detail in this paper. The horizontal bar denotes complex conjugation. We also have $\gamma^a = \Gamma^a_{\bar{i}0} k^j + \Gamma^a_{ij} k^j$, so the non-zero connection terms are given by

$$\Gamma^0_{ij} = \Gamma^i_{j0} = \delta_{ij},$$

$$\Gamma^i_{jk} = \frac{1}{2} \left( \epsilon_{ijk} n^l + \epsilon_{ji} n^l_k + \epsilon_{kj} n^l_i + (\delta^l_{ij} a_l - \delta^l_{jk} a_i) \right),$$

(18)

if we assume no vorticity ($\omega^k = 0$) and no acceleration ($\Gamma^0_{\bar{i}i} = \bar{u}_i$). The Boltzmann operator (17) involves terms that describe the propagation of the radiation field along the geodesics, the effect of the shear on the photon energy distribution and the rotation of the polarization angle $\chi$ defined by $2\chi = \arctan(U/Q)$. The emission term $J^A$ (which involves terms describing the fluid flow) contains only harmonics up to $l = 2$, since the radiation modes with $l \leq 2$ are damped as well as re-radiated by Thomson scattering, while higher-order modes $l > 2$ are only damped. It has become conventional to decompose the polarized component of the cosmic microwave background into modes classified according to their parity. The even modes are called the E-modes and the odd modes are the B-modes. The latter are of particular interest in the context of inflationary cosmology as they cannot be sourced by scalar perturbations and are therefore generally supposed to be a signature of the presence of primordial tensor perturbations, i.e. gravitational waves [28, 29],

$$N^2 \equiv N^2_{ij} m^i m^j = (Q - iU) (\hat{n}) = \sum_{lm} a_{-2,lm} \bar{Y}_{lm} (\hat{n}),$$

$$\bar{N}^2 \equiv \bar{N}^2_{ij} m^i m^j = (Q + iU) (\bar{n}) = \sum_{lm} a_{2,lm} Y_{lm} (\bar{n})$$

$$a_{E,lm} = -\frac{(a_{2,2} + a_{-2,2})}{2}, \quad a_{B,lm} = i \frac{(a_{2,2} - a_{-2,2})}{2}.$$

(19)

The overall level of polarization (in both E and B modes) increases strongly with the optical depth to Thomson scattering $\tau$ through (16).

However, the equations of the previous section for the evolution of the polarized radiation distribution allow us to establish some firm implications for the relative size E and B modes just by considering their symmetry. First, the scattering term $J^A$ produces a pure E-mode
quadrupole anisotropy. However, depending on the initial conditions, the redshifting effect of shear can produce either E or B modes (just as a gravitational wave perturbation of FRW can). The physical processes described by equations (14)–(17) can, in principle, convert E modes into B modes and vice-versa. In Bianchi VII$_h$ these effects are unavoidable so, even if there is no initial B mode, one is inevitably generated as the universe evolves. In Bianchi V, however, the last term in equation (17) does not contribute to the mixing of E and B modes, at least at this level of perturbation theory. Since initial conditions exist in which the polarization is purely E mode, there are therefore models of this type that can produce spots without any B modes. Figure 1, which was plotted using the Healpix software [30], shows illustrative examples of Bianchi VII$_h$ and Bianchi V that show localized patterns with and without B modes respectively.

4. Discussion and conclusions

We have shown that it is possible, in a Bianchi V cosmological model, to generate a localized temperature anomaly qualitatively similar to the known Cold Spot without necessarily producing a large B-mode polarization. Observational limits on the B mode alone are therefore not sufficient to exclude global anisotropy as a possible explanation of the famous Cold Spot. Although the set of Bianchi models that can evade the limits set by B mode polarization is small, a more rigorous analysis requires a fuller parametrization in which the Bianchi parameters are constrained with other cosmological parameters (such as the optical depth) using a complete description of the polarized radiation field.

We stress that there are severe difficulties with anisotropic universes as explanations for the overall pattern of observed CMB anomalies. Most important among these is that a significant (negative) spatial curvature seems to be at odds with measurements that clearly prefer a flat universe [1, 2]. Nevertheless, these models may provide important clues that can lead to more effective and efficient use of observations to test exotic cosmologies. For example, there are
other ways in which the polarization angle for CMB photons can be rotated [31]; the lack of any observed B mode allows present observations to place strong constraints on such models also [32].

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