Space anisotropy search at colliders

I. S. Karpikov\textsuperscript{1}, D. A. Tlisov\textsuperscript{2}, D. V. Kirpichnikov\textsuperscript{3}

Institute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary prospect 7a, Moscow 117312, Russia

Abstract

In the framework of model with Lorentz violation (LV) we discuss a physical observables for $q\bar{q}$ pair production at lepton-lepton colliders and describe the experimental signal to be detected. We obtain a conservative limits on Lorentz-violating dimensionless coupling for quark sector from LEP data. We also make a phenomenological prediction for LV model at the future lepton collider.

PACS: 12.60.Cn, 13.60.Fz, 14.65.Fy Keywords: Lorentz violation, Standard Model Extension (SME), lepton colliders.

1 Introduction

The problem of space-time anisotropy is a great challenge of high energy physics. The attempts to measure the space anisotropy for a relatively low energy scale are widely performed by an astrophysical experimental searches. Nevertheless, the ability to search fundamental properties of space-time on a high energy scales appears with LHC launching. A violation of Lorentz invariance is one of the possible reason of space anisotropy. There are various self-consistent setups of quantum gravity which admit the violation of Lorentz invariance: a models of quantum loop gravity \cite{1,2}, string model setups \cite{3,4}, a models of Horava-Lifshitz with extra spatial derivatives \cite{5,6,7}, the models of the analogue gravity \cite{8}. The most general Lorentz-violating Lagrangian with gauge invariant renormalizable terms was performed by \cite{9,10} for the particles of standard model (SM). The former framework is known as Standard-Model Extension (SME) of Alan Kosteletsky. The current constraints on SME parameters are presented in \cite{11}. In particular, the limits for leptons have been set at the level of $10^{-6} - 10^{-20}$.

A very recent result \cite{12} claims that \textit{CPT}-even coefficients for LV in the quark sector (e.g. for $u$ and $d$ quarks) can be bounded at the level about $10^{-5} - 10^{-6}$ from HERA experimental data on deep inelastic scattering (DIS) of $e^-p$. A framework of SME has been explored carefully also in the context of Tevatron collider phenomenology. In particular Ref. \cite{13} provides the bounds on \textit{CPT}-even LV couplings of top quark from dependence of the $t\bar{t}$ production cross-section on sidereal time as the orientation of the D0 detector changes with the rotation of the Earth. Test of \textit{CPT}-odd symmetry violation for $B$-mesons was performed in Refs. \cite{14,15}.

However the limits on \textit{CPT}-even LV coupling $(c_{Q(U,D)})_{ZZAB}$ for quarks hasn’t been obtained yet. In the present paper we discuss a possible implication of the SME phenomenology for quark

\textsuperscript{1}e-mail: karpikov@inr.ru
\textsuperscript{2}e-mail: dtlisov@cern.ch
\textsuperscript{3}e-mail: kirpich@ms2.inr.ac.ru
sector at a lepton-lepton colliders. Namely, we calculate the production cross-section of $q\bar{q}$ pair via $\gamma$ and $Z^0$-boson for the SME couplings that affect the quark field.

The paper is organized as follows. In Sec. 2 we consider general Lagrangian of the SME and perform LV couplings of quarks to be constrained by collider experiment. In Sec. 3 we derive the matrix element squared for the process $e^+e^- \rightarrow q\bar{q}$ in SME. In Sec. 4 we consider the spatial transformations from a Sun-centered reference frame to the Earth-based laboratory frame. In Sec. 5 we obtain a very conservative limits for LV coupling of $u,d,s,c$ and $b$ quarks from ALEPH and OPAL data. In Sec. 6 we derive time-dependent cross-section for the process $e^+e^- \rightarrow q\bar{q}$ and make SME prediction for the future lepton-lepton collider.

2 SME Lagrangian

We begin with a general SME Lagrangian, which can be expressed in the following form

$$\mathcal{L}_{SME} = \mathcal{L}_{SM} + \mathcal{L}_{LV},$$

(1)

where $\mathcal{L}_{SM}$ is the standard model (SM) Lagrangian and $\mathcal{L}_{LV}$ contains renormalizable Lorentz-violating terms for SM fields. Now let us consider CPT even Lagrangian for the quark sector

$$\mathcal{L}_{LV} \supset \mathcal{L}_{\text{quarks}}^{\text{LV}} = i (c_{Q})_{\mu\nu AB} \bar{Q}_A \gamma^\mu D^\nu Q_B + i (c_{U})_{\mu\nu AB} \bar{U}_A \gamma^\mu D^\nu U_B + i (c_{D})_{\mu\nu AB} \bar{D}_A \gamma^\mu D^\nu D_B,$$

(2)

where index $A$ labels the quark flavor, $A = 1,2,3$, here $u_A = (u,c,t)$ and $d_A = (d,s,b)$. We denote left- and right-handed quarks in (2) by $Q_A = (u_A,d_A)_L$, $U_A = (u_A)_R$ and $D_A = (d_A)_R$. The dimensionless LV coefficients $(c_{Q})_{\mu\nu AB}$, $(c_{U})_{\mu\nu AB}$ and $(c_{D})_{\mu\nu AB}$ can be assumed symmetric in flavor indices, $A,B$ and traceless in space-time indices, $\mu,\nu$. For definiteness in the present paper we consider a very specific case of Lorentz violation instead of treating full SME Lagrangian (2) for quarks, when $A = B$. Namely, the subjects of our interest are the Lagrangians for quarks in the $SU(2) \times U(1)$ breaking sector, $\mathcal{L}_{LV} = \sum_q (\mathcal{L}_{\gamma q}^{\text{quarks}} + \mathcal{L}_{Z q}^{\text{quarks}})$. As an illustration we perform below the lagrangian for $b$ quark

$$\mathcal{L}_{\gamma b}^{\text{quarks}} = Q_b \bar{e} \left( c_{Q \mu\nu} \left( \frac{1 - \gamma_5}{2} \right) + c_{D \mu\nu} \left( \frac{1 + \gamma_5}{2} \right) \right) \gamma^\mu b A^\nu,$$

(3)

$$\mathcal{L}_{Z b}^{\text{quarks}} = \frac{e}{\sin 2\theta_W} \bar{b} \left( c_{Q \mu\nu} C_{L}^f \left( \frac{1 - \gamma_5}{2} \right) + c_{D \mu\nu} C_{R}^f \left( \frac{1 + \gamma_5}{2} \right) \right) \gamma^\mu b Z^\nu,$$

(4)

here we denote for simplicity $c_{Q \mu\nu} \equiv c_{Q \mu\nu 33}$ and $c_{D \mu\nu} \equiv c_{D \mu\nu 33}$. For other flavors only diagonal elements have been left, say, for $c$-quark we have $c_{Q \mu\nu} \equiv c_{Q \mu\nu 22}$ and $c_{U \mu\nu} \equiv c_{U \mu\nu 22}$. These coefficients to be constrained by the collider experiment. All remaining LV coefficients for quarks in (2) can be set to zero without loss of generality. We also use a convenient SM notations $C_{L}^f = 2T_3^f - 2Q_f \sin^2 \theta_W$ and $C_{R}^f = -2Q_f \sin^2 \theta_W$ in (3) and (4).
In this section we calculate the matrix element squared for the signal process $e^+e^- \rightarrow q\bar{q}$ at lepton-lepton collider for the case of Lorentz violation \([3]\) and \([4]\). The amplitude squared, which corresponds to $q\bar{q}$ pair production via $\gamma$ and $Z^0$ boson can be written as sum of SM term and SM-SME interference terms in the leading order of LV couplings $c^Q_{\mu\nu}, c^U_{\mu\nu}$ and $c^D_{\mu\nu}$

$$\sum_{s.c.} |M(e^+e^- \rightarrow q\bar{q})|^2 \approx \sum_{s.c.} |M_\gamma + M_Z|^2 + \sum_{s.c.} (2M^\dagger_\gamma \delta M_\gamma + 4M^\dagger_\gamma \delta M_Z + 2M^\dagger_Z \delta M_Z), \quad (5)$$

in the expression above we average the amplitude squared over the initial state of lepton polarization and sum over the quark colors. For the sake of simplicity we now set $c^Q_{\mu\nu} = c^U_{\mu\nu} = c^D_{\mu\nu} \equiv c^V_{\mu\nu}$, then the partial amplitudes take the following forms

$$\sum_{s.c.} 2M^\dagger_\gamma \delta M_Z = \frac{2N_c e^4}{\sin^4 2\theta_W} \frac{c^V_{\mu\nu}}{(s-M_Z^2)^2} (C^q_{\mu}+C^q_{\nu})(C^Q_{\mu}+C^Q_{\nu})L^\mu\nu + (C^q_{\mu}-C^q_{\nu})(C^Q_{\mu}-C^Q_{\nu})L_A^\mu\nu, \quad (6)$$

$$\sum_{s.c.} 4M^\dagger_\gamma \delta M_Z = \frac{2N_c e^4 Q_q Q_l}{\sin^2 2\theta_W} \frac{c^V_{\mu\nu}}{s(s-M_Z^2)} ((C^q_{\mu}+C^q_{\nu})(C^Q_{\mu}+C^Q_{\nu})L^\mu\nu + (C^q_{\mu}-C^q_{\nu})(C^Q_{\mu}-C^Q_{\nu})L_A^\mu\nu), \quad (7)$$

$$\sum_{s.c.} 2M^\dagger_\gamma \delta M_Z = 2N_c 2e^4 Q_q^2 Q_l^2 \frac{1}{s^2} 2c^V_{\mu\nu} L^\mu\nu, \quad (8)$$

where $L^\nu_{\mu\nu}$ and $L^A_{\mu\nu}$ are the vector and the axial Lorentz violating tensors respectively, which depend on the 4-momenta of the incoming and produced particles $e^-(p_1)e^+(p_2) \rightarrow q(k_1)\bar{q}(k_2)$:

$$L^A_{\mu\nu} = ((p_2k_1)^2 - (p_2k_2)^2)g_{\mu\nu} - (p_2k_1)(k_2p_1 + k_1p_2) + (p_2k_2)(k_1p_1 + k_2p_2), \quad (9)$$

$$L^V_{\mu\nu} = (p_2k_1)(k_1\nu p_2\mu + k_2\nu p_1\mu) + (p_2k_2)(k_1\nu p_1\mu + k_2\nu p_2\mu) - (p_1p_2)(k_2\mu k_1\nu + k_1\mu k_2\nu + p_2\nu p_1\mu + p_1\nu p_2\mu) + g_{\mu\nu}(p_1p_2)^2. \quad (10)$$

In Sec. \([5]\) and Sec. \([6]\) we compare the matrix element \([5]\) to the SM expectation in order to estimate the contribution of LV coefficients $c^V_{\mu\nu}$ to the production rate of $q\bar{q}$ pair at the lepton-lepton colliders.
Figure 2: Left panel: orientation of the beam direction for ALEPH and OPAL detectors. Right panel: schematic illustration of the Sun-centered and Earth-based reference frames.

|               | ALEPH | OPAL | L3    | DELPHI |
|---------------|-------|------|-------|--------|
| Beam orientation ($\alpha$) | 33.92° | 54.50° | 55.60° | 34.87° |
| Colatitude ($\chi$)      | 43.77° | 43.77° | 43.77° | 43.77° |

Table 1: The location of the LEP detectors at Earth-based reference frame.

4 The reference frame transformation

If one takes into account the Earth’s rotation effect, then we should replace $c_{ij}^q \rightarrow c_{ij}^q(t) = c_{ij}^q R_i^I(t) R_j^J(t)$ in Eqs. (6-8), the indices $I$ and $J$ numerate the coordinates of the Sun-centered frame, $I, J = (X, Y, Z)$; the indices $i$ and $j$ are associated with Earth-based reference frame (see e.g. Fig. 2 for details). For the sake of simplicity we set also $c_{TT}^q = c_{TI}^q = c_{IT}^q = 0$ throughout the paper. We assume that the relative velocity of Sun-centered and Earth-based reference frames is negligible, so the transformation operation involves only rotations. The explicit form of the rotation matrix $\hat{R}(t) = R_i^I(t)$ is given by the following partial transformations

$$\hat{R}(t) = R_z(\omega t) R_y(\chi) R_x(\pi/2) R_y(\alpha).$$

The corresponding matrices, $R_x(\phi)$, $R_y(\theta)$ and $R_z(\psi)$ are defined by the following way

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix}, R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}, R_z(\psi) = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

here $\omega = 2\pi/T_{sid}$ is related to the sidereal period, $T_{sid} = (23\ h, 56\ m, 4.091\ s)$, $\chi$ is the colatitude of the detector, $\chi = (90^\circ - \text{Latitude})$, and $\alpha$ is the angle between the lepton beam and detector’s longitude. One can see from Eqs. (6-8) that SME amplitudes squared have the terms which are proportional to the vector part, $c_{IJ}^q R_i^I(t) R_j^J(t) L_{ij}^V$, and to the axial part, $c_{IJ}^q R_i^I(t) R_j^J(t) L_{ij}^A$. Thus the effect of Earth’s rotation will introduce a time dependence in the SME contribution, $\delta |M|_{SME}^2(t)$, to the production rate of $q{\bar{q}}$ pair.
5  \( q \bar{q} \) pair production at LEP

The differential cross-section for \( q \bar{q} \) pair production at LEP including Lorentz-violating contribution from SME can be written in the following form

\[
\frac{d\sigma}{d\Omega}(e^+e^- \rightarrow q\bar{q}) = \frac{1}{64\pi^2 s} \sum_{s.c.} (|M|_{SM}^2 + \delta|M|_{SME}^2(t)).
\]

(12)

In this section we restrict our analysis to the case

\[
c_{IJ}^q = \begin{pmatrix}
  c_{qXX}^q & c_{qXY}^q & c_{qXZ}^q \\
  c_{qYX}^q & c_{qYY}^q & c_{qYZ}^q \\
  c_{qZX}^q & c_{qZY}^q & c_{qZZ}^q
\end{pmatrix}.
\]

(13)

The traceless condition for \( c_{IJ}^q \) requires that \( c_{qXX}^q + c_{qYY}^q = -c_{qZZ}^q \). In order to estimate the collider sensitivity to SME coefficient we average \( \delta|M|_{SME}^2(t) \) over the sideral period, \( T_{sid} \). The explicit calculation revealed that time-averaged SME amplitude is proportional to SM matrix element squared

\[
\langle \delta|M|_{SME}^2(t) \rangle_t = \frac{1}{T_{sid}} \int_0^{T_{sid}} \delta|M|_{SME}^2(t) dt = C_{SME} \cdot |M|_{SM}^2
\]

(14)

where

\[
C_{SME} = \frac{c_{qZZ}^q}{8} (1 + 3(\cos 2\alpha + \cos 2\chi - \cos 2\alpha \cos 2\chi)).
\]

(15)

Which means that SME coefficients contribute to the signal cross-section up to the multiplicative factor in the following way

\[
\sigma_{e^+e^- \rightarrow q\bar{q}}^{SME} = \sigma_{e^+e^- \rightarrow q\bar{q}}^{SM} \cdot (1 + C_{SME}).
\]

(16)

It must be point out that after time-averaging (14) our analysis is not sensitive to \( XY, XZ \) or \( ZY \) elements of (13). So we can constrain only \( ZZ \) component of SME coupling. Since ALEPH and OPAL detectors at LEP measured directly the production rate of \( q\bar{q} \) events, from experimental uncertainties on \( \sigma_{e^+e^- \rightarrow q\bar{q}}^{SM} \) we can derive the limits on \( |c_{qZZ}^q| \) under assumption \( |c_{ZZ}^q| = |c_{ZZ}^q| = |c_{ZZ}^q| = |c_{ZZ}| = |c_{ZZ}| \), Tab. 2 shows relevant constraints. Beyond this assumption, namely for \( |c_{ZZ}^q| \neq |c_{ZZ}^q| \neq |c_{ZZ}^q| \neq |c_{ZZ}^q| \neq |c_{ZZ}^q| \), the systematic uncertainty of \( b\bar{b} \) - and \( c\bar{c} \) - pairs fraction in total \( q\bar{q} \) production needs to be taken into account. It follows from \( \sigma_{b\bar{b}(c\bar{c})} = R_{b(c)} \cdot \sigma_{q\bar{q}} \) that the relative uncertainties on \( b\bar{b}(c\bar{c}) \) cross-section can be expressed in the following way \( \Delta\sigma_{b\bar{b}(c\bar{c})}/\langle \sigma_{b\bar{b}(c\bar{c})} \rangle = \Delta\sigma_{q\bar{q}}/\langle \sigma_{q\bar{q}} \rangle + \Delta R_{b(c)}/\langle R_{b(c)} \rangle \). In this case conservative bounds can be found in Tab. 3 for ALEPH and OPAL detectors.

6  The prospects of SME probes in quark sector

In this section we briefly discuss a possible implications of the SME phenomenology for collider experiments and for low energy searches of LV. In contrast to the Sec. 5 now we consider the
Table 2: Conservative bounds on LV coupling of all quarks assuming $|c_{ZZ}^u| = |c_{ZZ}^d| = |c_{ZZ}^s| = |c_{ZZ}^c| = |c_{ZZ}^b| = |c_{ZZ}^c|$.

|                | ALEPH          | OPAL          |
|----------------|----------------|---------------|
| $\Delta \sigma_{q\bar{q}}/\langle \sigma_{q\bar{q}} \rangle$ | 0.78%, see Tab. 4 of Ref. [16] | 1.21%, see Tab. 5 of Ref. [18] |
| $|c_{ZZ}|$     | < 0.027        | < 0.036       |

Table 3: Conservative bounds on LV coupling of $c$- and $b$-quarks.

|                | ALEPH          | OPAL          |
|----------------|----------------|---------------|
| $\Delta \sigma_{q\bar{q}}/\langle \sigma_{q\bar{q}} \rangle$ | 0.78%, see Tab. 4 of Ref. [16] | 2.2%, see Tab. 2 of Ref. [19] |
| $\Delta R_b/\langle R_b \rangle$ | 9.2%, see Sec. 7.1 of Ref. [16] | 13.5%, see Sec. 2.2 of Ref. [19] |
| $\Delta R_c/\langle R_c \rangle$ | 10.8%, see Sec. 7.2 of Ref. [16] | -            |
| $|c_{ZZ}^c|$   | < 0.35         | < 0.46        |
| $|c_{ZZ}^b|$   | < 0.4          | -             |

The analyses of Refs. [12, 13] are very sophisticated and comprehensive test of SME. However, in the light of prospect study, it is instructive to probe SME for the low energy observables [20] as well as for the phenomenological quantities at the high-energy scales [21]. Indeed, Lorentz-violation in quark sector [3] affects the photon polarization operator. Moreover, one can show that Lorentz-violating kinetic term of quark Lagrangian modifies the dispersion relation of quarks at the tree level as well as the photon’s dispersion relation at the one-loop level [20]. This effectively means that the velocity of the photon acquires the additional contribution from the terms which ”run” via renormalization-group. Therefore, one can constrain the LV coupling of quarks with a high accuracy from laser experiments by measuring the speed of light. This sophisticated analysis is a subject of our study in the nearest future.
7 Acknowledgments

This work was supported by RFBR grant 16-32-00803. We thank A. Kostelecky, P. Satunin, D. Gorbunov, S. Gninenko, N. Krasnikov and V. Matveev for fruitful discussions.

References

[1] R. Gambini and J. Pullin, Phys. Rev. D 59, 124021 (1999).
[2] J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, Phys. Rev. D 65, 103509 (2002).
[3] V. A. Kostelecky and S. Samuel, Phys. Rev. D 39, 683 (1989).
[4] V. A. Kostelecky and R. Potting, Nucl. Phys. B 359, 545 (1991).
[5] P. Horava, Phys. Rev. D 79, 084008 (2009).
[6] D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. 104, 181302 (2010).
[7] S. Liberati and L. Maccione, Ann. Rev. Nucl. Part. Sci. 59, 245 (2009).
[8] S. Fagnocchi, S. Finazzi, S. Liberati, M. Kormos and A. Trombettoni, New J. Phys. 12, 095012 (2010).
[9] D. Colladay and V. A. Kostelecky, Phys. Rev. D 58, 116002 (1998).
[10] V. A. Kostelecky, Phys. Rev. D 69, 105009 (2004).
[11] V. A. Kostelecky and N. Russell, Rev. Mod. Phys. 83, 11 (2011).
[12] A. Kostelecky, E. Lunghi and A. R. Vieira, arXiv:1610.08755 [hep-ph].
[13] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 108, 261603 (2012).
[14] K. R. Schubert, arXiv:1607.05882 [hep-ph].
[15] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 116, no. 24, 241601 (2016) arXiv:1603.04804 [hep-ex].
[16] S. Schael et al. [ALEPH Collaboration], Eur. Phys. J. C 49 (2007) 411.
[17] R. Barate et al. [ALEPH Collaboration], Eur. Phys. J. C 12, 183 (2000) doi:10.1007/s10052990223 [hep-ex/9904011].
[18] G. Abbiendi et al. [OPAL Collaboration], Eur. Phys. J. C 33 (2004) 173.
[19] G. Abbiendi et al. [OPAL Collaboration], Eur. Phys. J. C 6, 1 (1999) doi:10.1007/s100520050318, 10.1007/s100529801027 [hep-ex/9808023].
[20] P. Satunin, arXiv:1705.07796 [hep-th].
[21] M. S. Berger, V. A. Kostelecky and Z. Liu, Phys. Rev. D 93, no. 3, 036005 (2016)