Application of Error Function in Dead-end/cross-flow Microfiltration Mode

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Abstract. In this study, error function was used to analyze the membrane fouling behavior (flux decay) for bovine serum album (BSA) solution in dead-end mode and for kaolin suspension in cross-flow mode under constant pressure. The results showed that the predictions of error function had good agreements with experimental data both in dead-end mode and cross-flow mode. Meanwhile, both higher concentration and higher TMP could accelerate the membrane fouling rate. In addition, the parameter $k$ in error function decreased with the increase of concentration or TMP. Thus, smaller parameter $k$ represented more serious membrane fouling.

1. Introduction

Membrane technology has been widely used in many fields like waste water treatment, desalination, purification [1]. However, membrane fouling which contents pore shrinking, pore blocking and cake formation is unavoidable during the filtration process [2]. Among all foulants, BSA as the representative foulant of protein was widely used to study the fouling behavior for microfiltration process[3–8]. For example, Ho and Zydney built up a model for complete pore blocking and cake filtration to describe the flux decay for BSA solution [3]. Hou et al. developed a precise model to predict the variation of the flux decay for BSA solution [4]. G. Bolton et al. established five different models and the model combined complete pore blocking and cake filtration was the best one for BSA solution[5]. The mathematical models above usually have many parameters. To find a simplified mathematical model to describe the BSA fouling behavior, an error function was used to predict the flux decay of BSA solution in constant pressure dead-end filtration process. Meanwhile, kaolin suspensions were used to verify the application of the error function in cross-flow filtration process.

2. Materials and Methods

2.1. Materials

Bovine serum album (BSA, MW = 67 kDa) was purchased from Fuchen (China). BSA-PBS solutions
were prepared by dissolving 1g BSA in 1L phosphate buffer saline (PBS) solution. And then dilute the concentrated solution to obtain the desired concentration solution. Kaolin was supplied by Datong Coal Jinyu kaolin chemical Co., Ltd (China). Kaolin suspensions were prepared by dissolving a certain mass of kaolin in mixed salt solution (c(NaHCO₃)=0.172 g/L and c(CaCl₂)=0.055 g/L). Flat sheet polyacrylonitrile (PAN) membrane with the average pore size of 0.1 μm (Beijing Ande Membrane Separation Technology and Engineering (Beijing) Co., Ltd.) was used. To remove glycerol from the membrane surface, the membrane should be soaked in DI-water at 4°C for at least 12h before each experiment.

2.2. Experimental Equipment
The experiments under constant pressure were carried out in a dead-end filtration cell (figure 1). BSA solutions and kaolin suspensions were used in dead-end experiments (figure 1 (a)) and cross-flow experiments (figure 1 (b)), respectively.

![Figure 1. Schematic diagram of filtration experimental setup for (a) dead-end mode and (b) cross-flow mode under constant pressure.](image)

2.3. Error Function Model
Error function is usually used in the description of heat transfer. While, the variation of the permeate flux in dead-end filtration process has the similar trend with temperature variation in cooling process. Thus, the permeate flux was analyzed by equation (1) [9]:

\[ \text{flux} = \frac{1}{\sqrt{\pi t}} \text{exp}(-t^2) \]
\[
\frac{J - J_s}{J_0 - J_s} = \text{erf} \left( \frac{k}{\sqrt{t}} \right)
\]  

(1)

3. Results & Discussion

3.1. The Application of Error Function in Dead-end Filtration for BSA Solution

The predictions of error function could fit the experimental data well both for different concentrations (0.3, 0.5 and 0.8 g/L) at a fixed TMP of 0.08 MPa and for different TMPs (0.06, 0.08 and 0.10 MPa) at a fixed BSA concentration of 0.5 g/L (figure 2 (a) and figure 2 (b)). The model parameters were shown in Table 1.

Table 1. The model parameters for dead-end filtration mode

| c(BSA) (g/L) | TMP (MPa) | \text{J/J}_0 | k |
|-------------|-----------|-------------|---|
| 0.3         | 0.08      | 2.29        | 0.27 |
| 0.5         | 0.08      | 1.28        | 0.26 |
| 0.8         | 0.06      | 0.94        | 0.24 |
| 0.5         | 0.08      | 1.28        | 0.26 |
|             | 0.10      | 0.87        | 0.22 |

Figure 2 (a) exhibited that the higher BSA concentration resulted in faster flux decline rate. This is because the higher BSA concentration contained more particles and led to faster mass transfer [10]. In addition, with the increasing of concentration, the parameter \(k\) decreased nonlinearly (figure 2 (c)).

Figure 2 (b) showed that higher TMP caused the more serious membrane fouling as the normalized flux decreased faster. This can be explained that higher TMP could provide much more driving force to make particles deposit on the membrane surface more quickly [4]. In addition, the parameter \(k\) decreased linearly with the increasing of TMP (figure 2 (c)).

![Figure 2(a) and 2(b)](image)

Figure 2. The experimental data and model prediction for (a) different BSA concentrations at a fixed TMP (0.08 MPa) and (b) different TMPs at a fixed BSA concentration (0.5 g/L). And (c) the model parameters for BSA solution.

![Figure 2(c)](image)
3.2. The Application of Error Function in Cross-flow Filtration for Kaolin Suspension

The predictions of error function could fit the experimental data well both for different concentrations (5, 10 and 15 g/L) at a fixed TMP of 0.09 MPa and for different TMPs (0.06, 0.09 and 0.12 MPa) at a fixed kaolin concentration of 10 g/L (figure 3 (a) and figure 3 (b)). The model parameters were shown in Table 2.

Table 2. The model parameters for cross-flow filtration mode

| c(kaolin) (g/L) | TMP (MPa) | \( k \)  | \( J/J_0 \) |
|---------------|-----------|--------|----------|
| 5             | 0.09      | 4.28   | 0.111    |
| 10            | 0.09      | 4.15   | 0.057    |
| 15            | 0.06      | 3.77   | 0.076    |
| 10            | 0.09      | 3.45   | 0.030    |
|               | 0.12      | 3.30   | 0.016    |

Figure 3 showed that higher concentration or higher TMP led to a more serious fouling process. Meanwhile, as shown in Table 2, the parameter \( k \) decreased with the increasing of concentration or TMP.

4. Conclusions

In this work, the fouling behaviors of BSA solutions and kaolin suspensions in dead-end filtration process and cross-flow filtration process under constant pressure were studied, respectively. And the predictions of error function showed excellent agreements with experimental data. Meanwhile, the parameter \( k \) decreased with the increasing of concentration or TMP, which means the smaller the parameter \( k \), the more serious membrane fouling could happen.

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References

[1] Kim M., Sankararo B., Lee S., Yoo C. (2013) Prediction and identification of membrane fouling mechanism in a membrane bioreactor using a combined mechanistic model, Ind. Eng. Chem.
[2] Wang Z., Chu J., Zhang X. (2007) Study of a cake model during stirred dead-end microfiltration, Desalination, 217: 127–138.

[3] Ho C.C., Zydney A.L. (2000) A combined pore blockage and cake filtration model for protein fouling during microfiltration, J. Colloid Interface Sci., 232: 389–399.

[4] Hou L., Wang Z., Song P. (2017) A precise combined complete blocking and cake filtration model for describing the flux variation in membrane filtration process with BSA solution, J. Memb. Sci. 542: 186–194.

[5] Bolton G., LaCasse D., Kuriyel R. (2006) Combined models of membrane fouling: Development and application to microfiltration and ultrafiltration of biological fluids, J. Memb. Sci. 277: 75–84.

[6] Kirschner A.Y., Cheng Y.H., Paul D.R., Field R.W., Freeman B.D. (2019) Fouling mechanisms in constant flux crossflow ultrafiltration, J. Memb. Sci., 574: 65–75.

[7] Iritani E., Katagiri N., Takenaka T., Yamashita Y. (2015) Membrane pore blocking during cake formation in constant pressure and constant flux dead-end microfiltration of very dilute colloids, Chem. Eng. Sci., 122: 465–473.

[8] Kelly S.T., Zydney A.L. (1995) Mechanisms for BSA fouling during microfiltration, J. Memb. Sci., 107: 115–127.

[9] Wang Z., Du X., Zhang H., Wang X., Song P., Natsagdorj K., Khan B., Khurram R., (2019) The application of error function for normalized flux prediction in dead-end microfiltration (MF) process, Sep. Sci. Technol., 1–12.

[10] Nataraj S., Schomäcker R., Kraume M. Mishra I.M., Drews A. (2008) Analyses of polysaccharide fouling mechanisms during crossflow membrane filtration, J. Memb. Sci. 308: 152–161.