Application of Grey Theory in Prediction of Ground Settlement in Foundation Pit

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Abstract. The grey system is a mathematical method to solve the incomplete information system; this model is often combined with BP neural network in foundation pit engineering to produce a new model. Among them, GM (1, 1) model is widely used in foundation pit engineering. In the process of subway construction, the foundation pit is often the most important one of them, therefore, this paper takes a subway station as an example for surface subsidence analysis, and selects GM (1, 1) model and kalman filter as the basic prediction model, programs with MATLAB, and predicts the results respectively, compares the results with the actual values to analyse the good results of the prediction. Through comparative analysis, it indicates that the prediction results of GM (1, 1) in surface settlement prediction are ideal and more suitable for practical engineering.

1. Introduction
There are two commonly used types of prediction methods for foundation pit deformation: empirical statistics and mechanical model. The empirical statistics are divided into linear regression and nonlinear regression analysis, which is the traditional method. Through the actual observation of the foundation pit monitoring points, the mathematical statistics method is used to establish the model of the prediction of foundation pit deformation; But the mechanical model is based on the change of the deformation of the enclosure of the foundation pit and the foundation pit itself, and the prediction model is established to realize the deformation prediction of the foundation pit. At present, the prediction theory of foundation pit deformation mainly includes the following models, such as regression analysis, time series analysis, artificial neural network model, grey system model, etc.

Regression analysis is a mathematical model that establishes a function expression between a dependent variable and an independent variable by mathematical statistics, and in the premise of mastering a large amount of observation data, we model and reveal the deformation law and the dynamic characteristic of the variable body. The time series analysis is the model to predict the mathematical model of deformation that uses the sequences in chronological order and deal with the problem in mathematical statistics. Its characteristic is: successive observations are usually not independent, and the analysis must consider the observation data of time sequence, when successive observations related, the future value can be predicted by the past observations, it can describe the dynamic characteristics of objective phenomena by using the correlation between the observed datas. Artificial neural network model is a mathematical model that simulating the structure and function of human brain cells. At present, BP neural network model is widely applied in the prediction of the settlement of deep foundation pit and the horizontal displacement of the top and pile of the pile. BP network is a multi -
layer feed forward network training by error - back propagation algorithm. The learning process consists of forward propagation and reverse propagation. At present, many scholars at home and abroad have conducted extensive research on BP neural network and applied it to prediction of foundation pit deformation.

In the 1980s, professor Deng Julong of China proposed a grey system model to solve the mathematical problem of incomplete information system. Employing the behaviour of the system as a random process, we explore the deformation law of observation objects from the huge data through the probability of statistical method. Gray system is widely used in the field of deformation monitoring, however, there are still great defects in some pit deformation monitoring, thus Yuan Jinrong and Zhao Fuyong combined previous research results and the measured analysis, putting forward that the grey system is not suitable for predicting horizontal displacement of underground continuous wall.

2. GM (1, 1) model modelling principle and process

Based on the analysis of the station’s actual engineering situation we choose the GM (1, 1) model as the data processing model. In this model, it describes the system state equation, and provide a description method for the uncertainty relation between the system's main behavior and other behavioral factors.

2.1. General situation of the engineering

The subway station adopts the structure of two - storey double columns. The length of the station’s main body is 202 meters, the standard section is 22.20 meters wide, the total height is 14.87 meters, the structure floor is buried about 18.22m deep, and the top cover of the central range is about 3.2m thick, the structure floor is buried about 17.170m deep, and the roof covering is about 3.200m thick. The total construction area of the station is 15609.27m², the main building area is 13853.29m², there are three entrance and two wind pavilions in this station, and the entrance and exit of the station are set along the two sides of xijin east road, and the two groups of wind pavilions are located on the south and north side of xijin east road. The station is constructed with a clear cut (half-paved) method.

The principal palisade structure uses bored piles, the pile’s body adopts C30 concrete, the diameter is 800 mm, the thickness of concrete cover is 70 mm, and width on both ends of the interval shield diameter is 1500 @ 1800, using glass fiber reinforcement. The pile was levelled with 100 thick net shotcrete. The first support is supported by 800*800mm reinforced concrete support on the crown beam. The second, third support using Φ 600 steel support, in double I45b steel purlin around. At the same time adopt dewatering well precipitation.

The structure of the railway station is mainly bored with bored piles, and the construction drilling pile is constructed by mechanical construction. It is necessary to use mechanical construction to locate the bored piles. When the construction deviation occurs, it is difficult to find and rectify deviation. Bored pile construction technology is using a special rotating drill which use the function of machine itself spin dig dug in the ground to set the diameter of the deep well, making the reinforcing cage well on the ground and putting them in the slot, pouring the concrete underwater by the catheter method, and finally completing every concrete pile spacing.

Table 1 is the surface settlement monitoring table of a subway station , it can be seen from the table, that the ground surface’s subsidence of the subway station settlement are all in a normal range, the largest settlement of the dot is DB02 and DB05-05-02, the variation is 1.6 mm, the change rate is -0.53 mm/d, the settlement of the minimum dot is DB05-02, the variation is 0.2 mm, the rate of change is 0.07 mm/d, from a subway station monitoring schedules and figure 2.5 we can see that on May 13, the dot DB06-04 appeared a peak, its cumulative settlement reached -3.4 mm, but it still in reasonable settlement range.
Table 1. Surface settlement monitoring table of a subway station

| Number of monitoring points | Initial value (m) | last cumulative change (mm) | Cumulative variation (mm) | Variation of this time (mm) | Rate of change (mm/d) | Monitoring control value monitoring (mm) / (mm/d) | monitoring conclusion |
|-----------------------------|-------------------|-----------------------------|---------------------------|----------------------------|----------------------|------------------------------------------------|----------------------|
| DB01-01                     | 1524.4419         | 0.30                        | 1                         | 0.60                       | 0.23                 | ±30/±2                                        | normal               |
| DB01-02                     | 1524.3691         | 0.40                        | 1.2                       | 0.80                       | 0.27                 | ±30/±2                                        | normal               |
| DB02-01                     | 1524.4151         | 0.50                        | 1.1                       | 0.60                       | 0.20                 | ±30/±2                                        | normal               |
| DB02-02                     | 1524.3217         | 0.10                        | -1.5                      | -1.60                      | -0.53                | ±30/±2                                        | normal               |
| DB03-01                     | 1524.3536         | 0.30                        | 1.3                       | 1.00                       | 0.33                 | ±30/±2                                        | normal               |
| DB03-02                     | 1524.2918         | 0.10                        | 1                         | 0.90                       | 0.30                 | ±30/±2                                        | normal               |
| DB04-01                     | 1524.2921         | 0.50                        | 1.5                       | 1.00                       | 0.33                 | ±30/±2                                        | normal               |
| DB04-02                     | 1524.2833         | 0.60                        | 1.3                       | 0.70                       | 0.23                 | ±30/±2                                        | normal               |
| DB05-01                     | 1524.4135         | 1.00                        | 1.4                       | 0.40                       | 0.13                 | ±30/±2                                        | normal               |
| DB05-02                     | 1524.2435         | 1.00                        | 1.2                       | 0.20                       | 0.07                 | ±30/±2                                        | normal               |
| DB05-03                     | 1524.1916         | 1.00                        | 1.3                       | 0.30                       | 0.10                 | ±30/±2                                        | normal               |
| DB05-04                     | 1524.1835         | 0.00                        | 1.3                       | 1.30                       | 0.43                 | ±30/±2                                        | normal               |
| DB05-05                     | 1524.8936         | 0.80                        | -0.8                      | -1.60                      | -0.53                | ±30/±2                                        | normal               |

2.2. Modeling principles and processes of GM (1, 1)

In the GM (1, 1) model, GM represents the differential equation model of grey theory. GM (1, 1) model is a dynamic prediction model, and its modeling principle is as follows:

Let's set the non-negative discrete number:

\[ x(0) = \{x(0)(1), x(0)(2), ..., x(0)(n)\} \]  

(1)

The above sequence is referred to as the original sequence, in which, \( n \) is the length of the original sequence, it accumulates the original sequence, and then generates the new sequence:

\[ x(1) = \{x(1)(1), x(1)(2), ..., x(01)(n)\} \]  

(2)

Among them:

\[ x(1) = \sum_{i=1}^{k} x(0)(i) \]  

(3)

Then do the first order of \( x(1) \), get the following:

\[ x = x(2), x(3), ... x(n) \]  

(4)

Among them: \( x(1)(k) = -1/2(x(1)(k-1) + x(1)(k)) \) \( (k=2, 3, ..., n) \) it forms a grey module. Grey model can be established, the general formula of the GM (1, 1) model is:

\[ \frac{dx^{(1)}}{dt} + ax = \mu \]  

(5)

Solve this differential equation, get the following:

\[ x^{(1)}(k+1) = \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-ax} + \frac{u}{a} \]  

(k=1, 2, ...)

In the formula, “a, u” can be obtained by least square method:

\[ \hat{\theta} = [a, u]^T = (B^TB)^{-1}B^Ty_N \]

Among them:
Through the accumulative reduction, we can get the prediction model of $x^{(0)}$:

$$
\begin{bmatrix}
-1/2 \left(x^{(1)}(2) + x^{(1)}(1)\right) & 1 \\
-1/2 \left(x^{(1)}(3) + x^{(1)}(2)\right) & 1 \\
\vdots & \vdots \\
-1/2 \left(x^{(1)}(n + 1) + x^{(1)}(n)\right) & 1
\end{bmatrix}
\begin{bmatrix}
y^{(0)}(2) \\
y^{(0)}(3) \\
\vdots \\
y^{(0)}(n)
\end{bmatrix}
$$

The main purpose of modeling is to predict the deformation of foundation pit deformation. In order to improve the accuracy and effect of prediction, the accuracy of filtering is guaranteed. At present, there are three methods to evaluate the accuracy of the model, namely the residuals, the correlation degree and the post-test. Generally, the grey system is usually tested by the posterior difference method, and the posterior difference is the statistical characteristic test of the residual distribution, which is jointly described by the posterior difference ratio $C$ and the small error probability $P$.

3. Prediction results and analysis
After the model was established, MATLAB was used for programming to analyze the observed data. The observation data is from ground subsidence, and the observation lasted 12 days, respectively, the settlement amount May 4 to May 26, it’s $x^{(0)}$ = {0.7, 0.8, 0.6, -1.6, 1.0, 0.9, 1.0, 0.7, 0.4, 0.2, 0.3, 1.3}, the unit is mm. The parameters of the calculation $a$=-0.0953, $u$=0.2214. The prediction results of surface settlement are shown in table 2.

| Serial number | Predictive value | Measured value | Residual |
|---------------|-----------------|----------------|---------|
| 1             | 0.72            | 0.7            | 0.02    |
| 2             | 0.83            | 0.8            | 0.03    |
| 3             | 0.49            | 0.6            | -0.11   |
| 4             | -1.48           | -1.6           | 0.12    |
| 5             | 1.31            | 1              | 0.31    |
| 6             | 0.85            | 0.9            | -0.05   |
| 7             | 0.87            | 1              | -0.13   |
| 8             | 0.71            | 0.7            | 0.01    |
| 9             | 0.38            | 0.4            | -0.02   |
| 10            | 0.17            | 0.2            | -0.03   |
| 11            | 0.26            | 0.3            | -0.04   |
| 12            | 1.46            | 1.3            | 0.16    |
As can be seen from figure 1, although the settlement volume fluctuates greatly, it is still within safe range. The predicted value is relatively close to the measured value. Only on the fourth day, the gap is larger at the fifth day, which is -1.45mm and 1.35mm respectively. From the overall feedback, GM (1, 1) model can well predict the settlement of foundation pit surface, but still need to pay attention to, the GM (1, 1) model has its limitations, deformation analysis does not apply to any situation, for example, in the foundation pit engineering, have to be cautious with the deformation of underground continuous wall, should be particular case is particular analysis, choose a reasonable model.

Figure 2. Kalman filter surface settlement prediction chart.
At the same time we use kalman filtering model, and use the MATLAB programming to deal with the deformation data analysis, the analysis result is shown in figure 2, the forecast trend is basically in line with the actual situation, but overall, the effect is not very ideal, compared with other models, such as BP neural network, polynomial regression, the finite element analysis, gray system, in the aspect of data processing model for prediction of foundation pit, the optimal model is still a neural network model and grey system, the two more in line with the actual engineering situation.

As you can see from the predicted results graph, as shown in figure 2, using the Calman filter model, it is concluded that the surface settlement of a subway station is consistent with the results obtained by GM (1, 1) model, indicating that GM (1, 1) model has a better prediction effect in predicting the surface settlement of foundation pit.

4. Conclusion
Through analyzing and processing the monitoring data of a subway station on rail transit line 1, the grey system GM (1, 1) and Calman filter can reflect the deformation trend, the predicted values are in a reasonable range, but compared to the two models, the grey system GM (1, 1) model is more consistent with the actual project, and the prediction results is better than the Calman filter model.

Because the foundation pit engineering involves a lot of variables, all kinds of predictable and unpredictable existence, the soil body is complex and changeable. At present, the research on deformation mechanism of foundation pit engineering is still in progress, and any model can only be approximated by infinite approximation or trend.

Acknowledgements
School youth fund of Lanzhou Jiao Tong University, Application of dynamic inversion in deformation early warning of deep foundation pit, No. 2017001.

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