Entropic Priors

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Abstract
The method of Maximum (relative) Entropy (ME) is used to translate the information contained in the known form of the likelihood into a prior distribution for Bayesian inference. The argument is guided by intuition gained from the successful use of ME methods in statistical mechanics. For experiments that cannot be repeated the resulting “entropic prior” is formally identical with the Einstein fluctuation formula. For repeatable experiments, however, the expected value of the entropy of the likelihood turns out to be relevant information that must be included in the analysis. As an example the entropic prior for a Gaussian likelihood is calculated.

1 Introduction
Among the methods used to update from a prior probability distribution to a posterior distribution when new information becomes available there are two that can claim the distinction of being systematic, objective, and of wide applicability: one is based on Bayes’ theorem (for applications to physics see [1]) and the other is based on the maximization of (relative) entropy [2]. The choice between the two methods is dictated by the nature of the information being processed.

Bayes’ theorem should be used when we want to update our beliefs about the values of quantities $\theta$ on the basis of observed values of data $y$ and of the known relation between them – the likelihood $p(y|\theta)$. The posterior distribution is $p(\theta|y) \propto \pi(\theta)p(y|\theta)$. The previous knowledge about $\theta$ is codified both in the prior distribution $\pi(\theta)$ and also in the likelihood $p(y|\theta)$.

The selection of the prior is a difficult problem [3] because it is not always clear how to translate our previous beliefs about $\theta$ into a distribution $\pi(\theta)$ in an objective way. One approach that seems to work, at least sometimes, is to rely

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on experience and physical intuition but this becomes unreliable in situations of increasing complexity. Attempts to achieve objectivity include arguments invoking symmetry – generalized forms of the principle of insufficient reason – and arguments that seek to identify that state of knowledge that reflects complete ignorance. The latter suggest connections with the notion of entropy \[4\] and have led to proposals for “entropic priors” \[5, 6\]. This brings us to the second method of processing information, the method of maximum entropy, which is designed for processing information given in the form of constraints on the family of posterior distributions \[2\].

In this paper, we use entropic arguments to translate information into a prior distribution \[7\]. Rather than seeking a totally non-informative prior, we translate information that we do in fact have: the knowledge of the likelihood function, \(p(y|\theta)\), already constitutes valuable prior information. The prior thus obtained is an “entropic prior.” The bare entropic priors discussed here apply to a situation where all we know about the quantities \(\theta\) is that they appear as parameters in the likelihood \(p(y|\theta)\). It is straightforward, however, to extend the method and incorporate additional relevant information beyond that contained in the likelihood.

The first proposal of priors of this form is due to Skilling \[5\] for the case of discrete distributions. The second proposal, due to Rodríguez \[6\], provided the generalization to the continuous case and further elaborations \[8, 9\]. In section 2, we give a derivation that is closer in spirit to applications of ME to statistical mechanics. A difficulty with the case of experiments that can be indefinitely repeated, which had been identified in \[10\], is diagnosed and resolved with the introduction of a hyper-parameter \(\alpha\) in section 3. The analogy to statistical mechanics is important: the interpretation of \(\alpha\) as a Lagrange multiplier affects how \(\alpha\) should be estimated and is an important difference between the entropic prior proposed here and those of Skilling and Rodríguez. The example of a Gaussian likelihood is given in section 4. In section 5, we collect our conclusions and some final comments.

2 The basic idea

We use the ME method \[2\] to derive a prior \(\pi(\theta)\) for use in Bayes’ theorem \(p(\theta|y) \propto p(y, \theta) = \pi(\theta)p(y|\theta)\). As discussed in \[10\], since Bayes’ theorem follows from the product rule, we must focus our attention on \(p(y, \theta)\) rather than \(\pi(\theta)\). Thus, the relevant universe of discourse is the product \(\Theta \times Y\) of \(\Theta\), the space of all \(\theta\)s, and the data space \(Y\). This important point was first made by Rodríguez \[8\] but both our derivation and final results differ from his \[8, 9\].

To rank distributions on the space \(\Theta \times Y\) we must first decide on a prior \(m(y, \theta)\). When nothing is known about the variables \(\theta\) – in particular, no relation between \(y\) and \(\theta\) is yet known – the prior must be a product \(m(y)\mu(\theta)\) of the separate priors in the spaces \(Y\) and \(\Theta\) because maximizing the relative entropy

\[
\sigma[p] = - \int dy d\theta p(y, \theta) \log \frac{p(y, \theta)}{m(y)\mu(\theta)},
\]  

(1)
yields \( p(y, \theta) \propto m(y) \mu(\theta) \). This distribution reflects our state of ignorance: the data about \( y \) tells us absolutely nothing about \( \theta \).

In what follows we assume that \( m(y) \) is known because it is an important part of understanding what data it is that has been collected. Furthermore, if the \( \theta \)s are parameters labeling some distributions \( p(y|\theta) \), then for each particular choice of the functional form of \( p(y|\theta) \) there is a natural distance in the space \( \Theta \) given by the Fisher-Rao metric \( ds^2 = g_{ij} d\theta^i d\theta^j \). [11]

\[
g_{ij} = \int dy \frac{\partial \log p(y|\theta)}{\partial \theta^i} \frac{\partial \log p(y|\theta)}{\partial \theta^j}. \tag{2}
\]

Therefore the prior on \( \theta \) is \( \mu(\theta) = g^{1/2}(\theta) \) where \( g(\theta) \) is the determinant of \( g_{ij} \).

Next we incorporate the crucial piece of information: of all joint distributions \( p(y, \theta) = \pi(\theta)p(y|\theta) \) we consider the subset where the likelihood \( p(y|\theta) \) has a fixed, known functional form. Notice that this is an unusual constraint: it is not an expectation value. Note also that the only information we are using about the quantities \( \theta \) is that they appear as parameters in the known likelihood \( p(y|\theta) \), nothing else. But, of course, should additional relevant information (i.e., an additional constraint) be known it should also be taken into account.

The preferred distribution \( p(y, \theta) \) is chosen by varying \( \pi(\theta) \) to maximize

\[
\sigma[\pi] = -\int dy d\theta \pi(\theta)p(y|\theta) \log \frac{\pi(\theta)p(y|\theta) g^{1/2}(\theta) m(y)}{g^{1/2}(\theta) m(y)}. \tag{3}
\]

Assuming that both \( \pi(\theta) \) and \( p(y|\theta) \) are normalized the result is

\[
\pi(\theta)d\theta = \frac{1}{\zeta} e^{S(\theta)} g^{1/2}(\theta) d\theta \quad \text{where} \quad \zeta = \int d\theta g^{1/2}(\theta) e^{S(\theta)}, \tag{4}
\]

and \( S(\theta) \) is the entropy of the likelihood,

\[
S(\theta) = -\int dy p(y|\theta) \log \frac{p(y|\theta)}{m(y)}. \tag{5}
\]

The entropic prior eq.\([4]\) is our first important result: it gives the probability that the value of \( \theta \) should lie within the small volume \( g^{1/2}(\theta)d\theta \). The preferred value of \( \theta \) is that which maximizes the entropy \( S(\theta) \) because this maximizes the scalar probability density \( \exp S(\theta) \). Note that eq.\([4]\) manifestly invariant under changes of the coordinates \( \theta \).

To summarize: for the special case of a fixed data space \( Y \), that is, for experiments that cannot be repeated, we have succeeded in translating the information contained in the model – the space \( Y \), its measure \( m(y) \), and the conditional distribution \( p(y|\theta) \) – into a prior \( \pi(\theta) \).

But for experiments that can be repeated indefinitely the prior \([4]\) yields nonsense and we have a problem. Indeed, let us assume that \( \theta \) is not a “random” variable, its value is fixed but unknown. For \( N \) independent repetitions of an experiment, the joint distribution in the space \( \Theta \times Y^N \) is

\[
p(y^{(N)}, \theta) = \pi^{(N)}(\theta) p(y^{(N)}|\theta) = \pi^{(N)}(\theta)p(y_1|\theta) \ldots p(y_N|\theta), \tag{6}
\]
and maximization of the appropriate $\sigma^{(N)}$ entropy gives [10]

$$\pi^{(N)}(\theta) = \frac{1}{Z^{(N)}} g^{1/2}(\theta) e^{NS(\theta)}, \quad (7)$$

which is clearly wrong. The dependence of $\pi^{(N)}$ on the amount $N$ of data would lead us to a perpetual revision of the prior as more data is collected. For large $N$ the data becomes irrelevant.

The problem, as we will see next, is not a failure of the ME method but a failure to include all the relevant information. Indeed, when an experiment can be repeated we actually know more than just $p(y^{(N)}|\theta) = p(y_1|\theta) \ldots p(y_N|\theta)$. We also know that discarding the values of say $y_2, \ldots y_N$, yields an experiment that is indistinguishable from the single, $N = 1$, experiment. This additional information, which is expressed by

$$\int dy_2 \ldots dy_N p(y^{(N)}, \theta) = p(y_1, \theta) \quad (9)$$

leads to $\pi^{(N)}(\theta) = \pi^{(1)}(\theta)$ for all $N$. Next we identify a constraint that codifies this information within each space $\Theta \times Y^N$.

3 More information: the Lagrange multiplier $\alpha$

For large $N$ the prior $\pi^{(N)}(\theta)$ in eq.(7) reflects an overwhelming preference for the value of $\theta$ that maximizes the entropy $S(\theta)$. Indeed, as $N \to \infty$ we have

$$\langle S \rangle = \int d\theta \pi^{(N)}(\theta) S(\theta) \xrightarrow{N \to \infty} S(\theta_{\text{max}}), \quad (8)$$

which is manifestly incorrect. This suggests that information about the actual numerical value $\bar{S}$ of the expected entropy $\langle S \rangle$ is very relevant (because if $\bar{S}$ were known the problem above would not arise) and that we should maximize $\sigma^{(N)}$ subject to an additional constraint on $\bar{S}$. Naturally, additional steps will be needed to estimate the unknown $\bar{S}$. A similar argument justifying the introduction of constraints in statistical physics is explored in [2].

We maximize the entropy

$$\sigma^{(N)}[\pi] = - \int d\theta dy^{(N)} \pi(\theta)p(y^{(N)}|\theta) \log \frac{\pi(\theta)p(y^{(N)}|\theta)}{g^{1/2}(\theta) m(y^{(N)})} \quad (9)$$

subject to constraints on $\langle S \rangle$ and that $\pi$ be normalized. (An unimportant factor of $N^{d/2}$ has been dropped from the Fisher-Rao measure $g^{(N)}(\theta)$.) The result is

$$\pi(\theta) = \frac{1}{\zeta} g^{1/2}(\theta) \exp \left[ (N + \lambda_N)S(\theta) \right] \quad (10)$$

The undesired dependence on $N$ is eliminated if the Lagrange multipliers $\lambda_N$ in each space $\Theta \times Y^N$ are chosen so that $N + \lambda_N = \alpha$ is a constant independent of $N$. The resulting entropic prior,

$$\pi(\theta|\alpha) = \frac{1}{\zeta(\alpha)} g^{1/2}(\theta) e^{\alpha S(\theta)} \quad (11)$$
is our second important result. The prior $\pi(\theta|\alpha)$ incorporates information contained in the likelihood plus information about

$$\langle S \rangle = \bar{S}(\alpha) = \frac{d}{d\alpha} \log \zeta(\alpha) \quad \text{where} \quad \zeta(\alpha) = \int d\theta \ g^{1/2}(\theta)e^{\alpha S(\theta)}. \quad (12)$$

The last step would be to estimate $\alpha$ and $\theta$ from Bayes’ theorem

$$p(\alpha, \theta|y^{(N)}) = \pi(\alpha)\pi(\theta|\alpha)p(y^{(N)}|\theta)/p(y^{(N)}), \quad (13)$$

where $\pi(\alpha)$ is a prior for $\alpha$. However, if we are only interested in $\theta$, we can just marginalize over $\alpha$ to get

$$p(\theta|y^{(N)}) = \int d\alpha \ p(\alpha, \theta|y^{(N)}) = \bar{\pi}(\theta)\frac{p(y^{(N)}|\theta)}{p(y^{(N)})} \quad (14)$$

where

$$\bar{\pi}(\theta) = \int d\alpha \ \pi(\alpha)\pi(\theta|\alpha). \quad (15)$$

The averaged $\bar{\pi}(\theta)$ is our final expression for the entropic prior. It is independent of the actual data $y^{(N)}$ as it should.

Next we assign an entropic prior to $\alpha$. We start by pointing out that $\alpha$ is not on the same footing and should not be treated like the other parameters $\theta$ because the relation between $\alpha$ and the data $y$ is indirect: $\alpha$ is related to $\theta$ through $\pi(\theta|\alpha)$, and $\theta$ is related to $y$ through $p(y|\theta)$. Once $\theta$ is given, the data $y$ contains no further information about $\alpha$. Since the whole significance of $\alpha$ is derived purely from $\pi(\theta|\alpha)$, eq.(11), the relevant universe of discourse is $A \times \Theta$ with $\alpha \in A$ and not $A \times \Theta \times Y^{N}$ as in [6] which requires the introduction of an endless chain of hyper-parameters.

We therefore consider the joint distribution $\pi(\alpha, \theta) = \pi(\alpha)\pi(\theta|\alpha)$ and obtain $\pi(\alpha)$ by maximizing the entropy

$$\Sigma[\pi] = - \int d\alpha \ d\theta \ \pi(\alpha, \theta) \log \frac{\pi(\alpha, \theta)}{\gamma^{1/2}(\alpha)g^{1/2}(\theta)} \quad (16)$$

where $\gamma^{1/2}(\alpha)$ is determined below. Since no reference is made to repeatable experiments in $Y^{N}$ there is no need for any further constraints – and no further hyper-parameters – except for normalization. The result is

$$\pi(\alpha) = \frac{1}{z} \gamma^{1/2}(\alpha)e^{\gamma(\alpha)}, \quad (17)$$

where using eqs.11 and 12, the Fisher-Rao measure $\gamma(\alpha)$ is

$$\gamma(\alpha) = \int d\theta \ \pi(\theta|\alpha) \left[ \frac{d}{d\alpha} \log \pi(\theta|\alpha) \right]^{2} = \frac{d^{2} \log \zeta(\alpha)}{d\alpha^{2}}, \quad (18)$$
and where $s(\alpha)$ is given by

$$s(\alpha) = -\int d\theta \pi(\theta|\alpha) \log \frac{\pi(\theta|\alpha)}{g^{1/2}(\theta)} = \log \zeta(\alpha) - \alpha \frac{d\log \zeta(\alpha)}{d\alpha}.$$  \hspace{1cm} (19)

This completes our derivation of the actual prior for $\theta$: the averaged $\bar{\pi}(\theta)$ in eq.(15) codifies information contained in the likelihood function, plus the insight that for repeatable experiments, information about the expected likelihood entropy, even if unavailable, is relevant.

4 Example: a Gaussian model

Consider data $y^{(N)} = \{y_1, \ldots, y_N\}$ that are scattered around an unknown value $\mu$,

$$y = \mu + \nu$$  \hspace{1cm} (20)

with $\langle \nu \rangle = 0$ and $\langle \nu^2 \rangle = \sigma^2$. The goal is to estimate $\theta = (\mu, \sigma)$ on the basis of $y^{(N)}$ and the information implicit in the data space $Y$, its measure $m(y)$ (discussed below), and the Gaussian likelihood,

$$p(y|\mu, \sigma) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp \left[ -\frac{(y - \mu)^2}{2\sigma^2} \right].$$  \hspace{1cm} (21)

We asserted earlier that knowing the measure $m(y)$ is part of knowing what data has been collected. In many physical situations where the data happen to be distributed according to eq.(21) the underlying space $Y$ is invariant under translations and we can assume $m(y) = m = \text{constant}$. Indeed, the Gaussian distribution can be obtained by maximizing an entropy with an underlying constant measure and constraints on the relevant information the mean $\mu$ and the variance $\sigma^2$.

From eqs. (5) and (21) the entropy of the likelihood is

$$S(\mu, \sigma) = \log \left[ \frac{\sigma}{\sigma_0} \right] \text{ where } \sigma_0 \overset{\text{def}}{=} \left( \frac{e}{2\pi} \right)^{1/2} \frac{1}{m},$$  \hspace{1cm} (22)

and the corresponding Fisher-Rao measure, from eq.(2) is

$$g(\mu, \sigma) = \det \begin{vmatrix} 1/\sigma^2 & 0 \\ 0 & 2/\sigma^2 \end{vmatrix} = \frac{2}{\sigma^4}.$$  \hspace{1cm} (23)

Note that both $S(\mu, \sigma)$ and $g(\mu, \sigma)$ are independent of $\mu$. This means that if we were concerned with the simpler problem of estimating $\mu$ in a situation where $\sigma$ happens to be known, the Bayesian estimate of $\mu$ using entropic priors coincides with the maximum likelihood estimate.

When $\sigma$ is unknown the $\alpha$-dependent entropic prior, eq.(11), is

$$\pi(\mu, \sigma|\alpha) = \frac{2^{1/2}}{\zeta(\alpha)} \frac{\sigma^{n-2}}{\sigma_0^n}. \hspace{1cm} (24)$$
Since \( \pi(\mu, \sigma|\alpha) \) is improper in both \( \mu \) and \( \sigma \) we must introduce high and low cutoffs for both \( \mu \) and \( \sigma \). The fact that without cutoffs the model is not well defined is interpreted as a request for additional relevant information, namely, the values of the cutoffs.

We write the range of \( \mu \) as \( \Delta \mu = \mu_H - \mu_L \) and introduce dimensionless quantities \( \varepsilon_L \) and \( \varepsilon_H \): \( \sigma \) extends from \( \sigma_L = \sigma_0 \varepsilon_L \) to \( \sigma_H = \sigma_0 / \varepsilon_H \). Then \( \zeta(\alpha) \) and \( \pi(\mu, \sigma|\alpha) \) are given by

\[
\zeta(\alpha) = \frac{2^{1/2} \Delta \mu \varepsilon_H^{1-\alpha} - \varepsilon_L^{\alpha-1}}{\sigma_0 \alpha - 1}.
\]  

(25)

and

\[
\pi(\mu, \sigma|\alpha) = \frac{1}{\Delta \mu \sigma_0 \varepsilon_H^{1-\alpha} - \varepsilon_L^{\alpha-1}} \left( \frac{\sigma}{\sigma_0} \right)^{\alpha-2}.
\]  

(26)

Note that \( \pi(\mu, \sigma|\alpha = 1) \) reduces to \( d\sigma/\sigma \) which is the Jeffreys prior usually introduced by imposing invariance under scale transformations, \( \sigma \rightarrow \lambda \sigma \).

Writing \( \varepsilon \equiv (\varepsilon_L \varepsilon_H)^{1/2} \), the prior for \( \alpha \), is obtained from eq.(18),

\[
\gamma(\alpha) = \frac{1}{(\alpha - 1)^2} - \left( \frac{2 \log \varepsilon}{\varepsilon^{1-\alpha} - \varepsilon^{\alpha-1}} \right)^2
\]  

(27)

and from eqs.(15) and (19),

\[
\pi(\alpha) = \frac{\gamma^{1/2}(\alpha) \varepsilon^{1-\alpha} - \varepsilon^{\alpha-1}}{\varepsilon^{1-\alpha} - \varepsilon^{\alpha-1}} \exp \left[ \frac{1}{\alpha - 1} + \alpha \varepsilon^{1-\alpha} + \varepsilon^{\alpha-1} \log \varepsilon \right],
\]  

(28)

where the normalization \( z \) has been suitably redefined.

Eqs.(27) and (28) simplify in the limit \( \varepsilon \rightarrow 0 \). Note that the same result is obtained irrespective of the order in which we let \( \varepsilon_H \rightarrow 0 \) and/or \( \varepsilon_L \rightarrow 0 \). The resulting \( \gamma(\alpha) \) and \( \pi(\alpha) \) are

\[
\gamma(\alpha) = \frac{1}{(\alpha - 1)^2},
\]  

(29)

and

\[
\pi(\alpha) = \begin{cases} 
\frac{1}{(1-\alpha)} \exp \left[ \frac{1}{\alpha - 1} \right] & \text{for } \alpha < 1 \\
0 & \text{for } \alpha \geq 1
\end{cases}
\]  

(30)

where \( \pi(\alpha) \) is normalized and is shown in Fig. 1.

\( \pi(\alpha) \) reaches its maximum value at \( \alpha = 1/2 \). Since \( \pi(\alpha) \sim \alpha^{-2} \) for \( \alpha \rightarrow -\infty \) the expected value of \( \alpha \) and all higher moments diverge. This suggests that replacing the unknown \( \alpha \) in the prior \( \pi(\theta|\alpha) \) by any given numerical value \( \hat{\alpha} \) is probably not a good approximation.

Since \( \alpha \) is unknown the effective prior for \( \theta = (\mu, \sigma) \) is obtained marginalizing \( \pi(\mu, \sigma, \alpha) = \pi(\mu, \sigma|\alpha) \pi(\alpha) \) over \( \alpha \), eq.(15). Since \( \pi(\alpha) = 0 \) for \( \alpha \geq 1 \) as \( \varepsilon \rightarrow 0 \) we can safely take the limit \( \varepsilon_H \rightarrow 0 \) or \( \sigma_H \rightarrow \infty \) while keeping \( \sigma_L \) fixed,

\[
\pi(\mu, \sigma, \alpha) = \begin{cases} 
\frac{1}{\Delta \mu \sigma_L} \exp \left[ \frac{1}{\alpha - 1} \right] \left( \frac{\sigma}{\sigma_L} \right)^{-2} & \text{for } \alpha < 1 \\
0 & \text{for } \alpha \geq 1
\end{cases}
\]  

(31)
Figure 1: The prior $\pi(\alpha)$ for various values of the cutoff parameter $\varepsilon$, as $\varepsilon \to 0$. (However we cannot take $\sigma_L \to 0$). The averaged prior for $\mu$ and $\sigma$ is

$$
\bar{\pi}(\mu, \sigma) = \frac{(\sigma_L/\sigma)^2}{\Delta\mu\sigma_L} \int_{-\infty}^{1} \frac{\exp\left[\frac{1}{\alpha-1}\right]}{1-\alpha} \left(\frac{\alpha}{\sigma_L}\right)^\alpha d\alpha = \frac{2}{\Delta\mu\sigma} K_0 \left(2\sqrt{\log \frac{\sigma}{\sigma_L}}\right),
$$

where $K_0$ is a modified Bessel function of the second kind. This is the entropic prior for the Gaussian model. The function

$$
P(x) = \frac{2}{x} K_0 \left(2\sqrt{\log x}\right)
$$

is shown in Fig. 2 as a function of $x = \sigma/\sigma_L$. The singularity as $x \to 1$ is integrable.

5 Final remarks

Using the method of maximum relative entropy we have translated the information contained in the known form of the likelihood into a prior distribution. The argument follows closely the analogous application of the ME method to statistical mechanics. For experiments that cannot be repeated the resulting “entropic prior” is formally identical with the Einstein fluctuation formula. For repeatable experiments, however, additional relevant information – represented in terms of a Lagrange multiplier $\alpha$ – must be included in the analysis. The important case of a Gaussian likelihood was treated in detail.

We have dealt with the simplest case where all we know about the quantities $\theta$ is that they appear as parameters in the likelihood $p(y|\theta)$. Our argument
The effective \( \bar{\pi}(\mu, \sigma) \) is shown as \( P(x) = \frac{2}{x} K_0 \left( 2\sqrt{\log(x)} \right) \) where \( x = \sigma / \sigma_L \).

Figure 2

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can, however, be generalized to situations where we know of additional relevant information beyond what is contained in the likelihood. Such information can be taken into account through additional constraints in the maximization of the entropy \( \sigma \).

To conclude we comment briefly on the entropic priors proposed by Skilling and by Rodríguez. Skilling’s prior, unlike ours, is not restricted to probability distributions but is intended for generic “positive additive distributions” \[5\]. Our argument, which consists in maximizing the entropy \( \sigma \) subject to a constraint \( p(y, \theta) = \pi(\theta) p(y|\theta) \), makes no sense for generic positive additive distributions for which there is no available product rule. Another important difference arises from the fact that Skilling’s entropy is not, in general, dimensionless and his hyper-parameter \( \alpha \) is interpreted some sort of cutoff carrying the appropriate corrective units. Difficulties with Skilling’s prior were identified in \[12\].

Rodríguez’s approach is, like ours, derived from a maximum entropy principle \[9\]. One (minor) difference is his treatment of the underlying measure \( m(y) \). For us knowing \( m(y) \) is part of knowing what data has been collected; for him \( m(y) \) is an initial guess and he suggests setting \( m(y) = p(y|\theta_0) \) for some value \( \theta_0 \). The more important difference, however, is that the number of observed data \( N \) is left unspecified. The space \( \Theta \times Y^N \) over which distributions are defined, and therefore the distributions themselves, also remain unspecified. It is not clear what the maximization of an entropy over such unspecified spaces could possibly mean but a hyper-parameter \( \alpha \) is eventually introduced and it is interpreted as a “virtual number of observations supporting the initial guess
θ_0." A different interpretation is given in [13]. Since α is treated on the same footing as the other parameters θ,- Rodríguez’s approach requires an endless chain of hyper-parameters.

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