Mathematical Modelling of Low Temperature Solder Direct Extrusion

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Abstract. Defining of process technological parameters is very important for low temperature solder extrusion. A mathematical model of low temperature solder wire and rods direct extrusion, which allows determining the energy-power process parameters, has been developed. The validation of the mathematical model for adequacy was checked during a laboratory experiment on a hydraulic press. In laboratory the force and speed of pressing were measured and recorded using the data collection system installed on it. The validation of the model for adequacy was carried out by pressing Ø8.00 and Ø15 mm rods and Ø2.00 mm wire and showed fine precision of calculations with experimental measurements. The error did not exceed 10%. The resulting mathematical model was used for analytical studies of Sn-In alloy bars and wires direct pressing technological modes. Calculations have shown that a decrease in the finished rod (wire) diameter from 16.00 to 2.00 mm while maintaining the workpiece size Ø30.00 mm leads to an increase in the pressing force from 86 kN to 131 kN at the initial moment of pressing. It increases the elongation ratio from 4 to 256. The pressing force of Ø8.00 mm rod increases from 25 to 171 kN after an increase in the diameter of the workpiece from 12.0 to 40.0 mm. Analytical studies on the model showed that it is possible not only to study the pressing process in order to understand the mechanisms of forming the mechanical and operational properties of the finished rod (wire), but also to design resource-saving pressing modes for various product gauges, to carry out substantiated selection of the required equipment and accessories.

1. Introduction
The works devoted to the methods and technologies of soldering using lead-free solders have been actively developed recently [1–6]. The reason for this is that the European Union's RoHS (Restriction of use of Certain Hazardous Substances) directive limited the use of lead in new electrical and electronic equipment to no more than 0.01% in 2006. These limits are established not on the weight of a product, assembly or component, but on each individual homogeneous material [7].

A number of lead-free solders of the Sn–Cu, Sn–Ag, Sn–Ag–Cu systems have been developed in recent years to solve this problem. Lead-free technologies are distinguished by a significant increase in soldering temperatures from 230 °C for Sn63Pb37 (melting point 183 °C) to 265 ... 270 °C for the most low-melting, generally recognized lead-free solder Sn3Ag0.7Cu (melting temperature 217 °C). Such a temperature load requires not only a change of solders, but also a change of fluxes, base materials of printed-circuit boards, renewal of technological equipment, and revision of specific energy consumption. This leads to the need for investments in the renovation of production, the use of more expensive heat-resistant materials, and the restructuring of technologies. However, low
melting point solders have also been developed and applied in recent years. These are alloys of the Sn–In, Sn–Bi systems.

Low-melting Sn–In solder consist of 51–53% indium and the rest of tin. This solder is one of the most commonly used in 2016 (after the introduction of restrictions on the use of cadmium and lead) in semiconductor technology.

The Processes and Machines of Metal Forming Department at SUSU has been developing and mastering the technology for the production of solder Sn-In alloy wire and rod under the conditions of low-rate production since 2019 [8, 9].

The purpose of this study is to develop a mathematical model for direct pressing of solder from low-melting materials, which makes it possible to determine the energy-power modes of pressing wires and bars of various diameters.

2. Mathematical Description of the Extrusion Power Mode

Figure 1 shows a diagram of the deformation zone during pressing. The energy for pressing is supplied to the extrusion ram by force $F$, while the total work of process $A$ is the sum of the plastic deformation work $A_{def}$ and the friction forces work $A_f$.

$$A = A_{def} + A_f.$$  \hspace{1cm} (1)

![Figure 1. Scheme of the deformation zone during extrusion.](image)

The extrusion energy consumption (while neglecting the sliding friction forces on the contact surface of the ram and the container) is proposed to be estimated as a sum of forces in [10, 11]

$$F = F_{def} + F_f,$$  \hspace{1cm} (2)

where $F_{def}$ – forming (plastic deformation) force; $F_f$ – efforts to overcome sliding friction forces on the processed metal with the container and with the die contact surface. The derivation of the equation for determining the plastic deformation required force can be based on the Fink dependence. This dependence determines the work of metal shaping, spent on the implementation of plastic deformations,
determined by the initial and final dimensions of the processed metal (without taking into account the peculiarities of the plastic deformation conditions)

\[ A_p = \theta \cdot \sigma_{av} \cdot \ln \mu \]  

(3)

where \( \theta \) – volume of processed metal; \( \mu \) – extrusion ratio

\[ \mu = \frac{D_{ent}^2}{D_{exit}^2} \]  

(4)

\( D_{ent}=D_{con} \) – billet diameter at the entrance to the die equal to the container diameter; \( D_{con} \) – diameter of the wire (rod) at the exit from the die; \( \sigma_{av} \) – the average value of the processed metal deformation resistance, which can be determined as (5), under the assumption that the specific resistivity of the billet metal deformation is equal to the average value of the yield stress

\[ \sigma_{av} = \sqrt{\sigma \cdot \sigma_{exit}}, \]  

(5)

\( \sigma \) – billet material yield strength; \( \sigma_{exit} \) – metal yield stress at the exit from the die.

A distinctive feature of the considered extrusion process is the value of the deformation zone shape parameter \( \Delta_B = \frac{l_{arc}}{L_{cont}} = \ll 1 \), where \( l_{arc} \) is the length of the arc perpendicular to the tool contact line in the middle of the deformation zone; \( L_{cont} \) – the length of the tool contact line (Figure 1).

The value \( \Delta_B \) is calculated by the equation [12]:

\[ \Delta_B = \frac{\alpha}{\mu - 1 - \frac{:\mu - 1}{\mu}} \]  

(6)

where \( \alpha \) – half-angle of the die.

The shape parameter \( \Delta_B \ll 1 \) leads to an increase in power conditions similarly to how it is accepted in sheet rolling [13, 14]. It should be taken into account when calculating:

\[ A_{def} = \frac{1}{\Delta_B} \cdot A_p = \frac{1}{\Delta_B} \cdot \theta \cdot \sigma_{av} \cdot \ln \mu. \]  

(7)

Taking into account that the deformation power on one side is

\[ N_{def} = F_{def} \cdot V_{pr}, \]  

(8)

where \( V_{pr} \) – pressing speed (movement speed of the ram), on the other hand, can be found as

\[ N_{def} = \frac{dA_{def}}{dt} = \frac{1}{\Delta_B} \cdot \sigma_{av} \cdot \ln \mu \cdot \frac{d\theta}{dt} = \frac{1}{\Delta_B} \cdot \sigma_{av} \cdot \ln \mu \cdot S_{rod} \cdot V_{pr}, \]  

(9)

then provided that the area of the round billet

\[ S_{rod} = \frac{\pi \cdot D_{ent}^2}{4} \]  

(10)

the equation for calculating the deformation force is:

\[ F_{def} = \frac{\pi \cdot D_{ent}^2}{4} \cdot \frac{1}{\Delta_B} \cdot \sigma_{av} \cdot \ln \mu. \]  

(11)
The friction force $F_{frI}$ is determined by the geometry of the deformation zone (Figure 1) and consists of three components – the effort aimed at overcoming the processed metal sliding friction against the container $F_{frI}$, the surface of the die $F_{frII}$ and the parallel land of the die $F_{frIII}$ [15]

$$F_{fr} = F_{frI} + F_{frII} + F_{frIII}$$ (12)

The formation of friction forces according to Sybel’s law is

$$\tau = f \cdot \sigma$$ (13)

Components of friction forces can be found:

$$F_{frI} = S_{con} \cdot \tau = \pi \cdot D_{con} \cdot L_{bil} \cdot f \cdot \sigma;$$ (14)

$$F_{frII} = S_{die} \cdot \tau = \frac{\pi}{4 \cdot \sin \alpha} \cdot (D_{con}^2 - D_{exit}^2) \cdot f \cdot \sigma ;$$ (15)

$$F_{frIII} = S_{cal} \cdot \tau = \pi \cdot D_{exit} \cdot L_{cal} \cdot f \cdot \sigma_{cal},$$ (16)

where $f$ – friction coefficient; $S_{con}, S_{die}, S_{cal}$ – lateral surface area of the container inner liner, die and its parallel land; $D_{con}$ – container diameter; $\sigma_t$ – plastic shear strength

$$\sigma_t = \sigma_{as} = \frac{\sqrt{\sigma \cdot \sigma_{cal}}}{2},$$ (17)

$L_{bil}$ – length of pressed billet; $L_{cal}$ – length of die parallel land.

The mathematical model of direct extrusion is implemented in the Excel mathematical table. An example of calculating the power conditions for pressing rods $\odot$ 2.00; 8.00 and 15.00 mm from Sn-In alloy are given in table 1.

**Table 1. Results of mathematical model calculation.**

| Parameter                        | Unit | Value  | Exper.1 | Exper.2 | Exper.3 |
|----------------------------------|------|--------|---------|---------|---------|
| Diameter of billet, $D_{out}$    | mm   | 30     | 30      | 20      |         |
| Diameter of the wire (rod), $D_{exit}$ | mm   | 15     | 8       | 2       |         |
| Billet length, $L$              | mm   | 120    | 120     | 120     |         |
| Container diameter, $D_{con}$   | mm   | 31     | 32      | 21.8    |         |
| Pressing speed, $V_{pr}$        | mm/s | 12     | 3.5     | 0.5     |         |
| Billet material yield strength, $\sigma$ | MPa | 10.5   | 10.5    | 10.5    |         |
| Half-angle of the die, $\alpha$ | °    | 33.5   | 40      | 18.4    |         |
| Length of die parallel land, $L_{cal}$ | mm | 5      | 5       | 5       |         |
| Friction coefficient, $f$       | -    | 0.5    | 0.5     | 0.5     |         |

### Calculation results

|                         |      |        |        |        |
|-------------------------|------|--------|--------|--------|
| Extrusion ratio, $\mu$  | -    | 4.3    | 16.0   | 118.8  |
| Extrusion speed, $V_{ext}$ | mm/s | 51     | 56     | 59     |
| Force at the start of pressing, $F_{start}$ | kN | 92.8   | 106    | 105    |
| Force at the end of pressing, $F_{end}$ | kN | 41.0   | 55.6   | 71.9   |
3. Experimental Studies

Validation of a model for adequacy was carried out in the laboratory of the Processes and Machines of Metal Forming Department experiment by Pressure on a hydraulic press model D2428 equipped with a data collection and storage system (Figure 2). The press allows us to determine the speed and pressing force after modernization, which included replacing the oil pump (8) and equipping it with pressure sensors and the stroke (2) with displacement sensors. The press can work both with the included data collection system (10), displaying the pressing diagram (Figure 3) on the monitor (9) and recording the measurement results into the database, or without it. The press is controlled from the control panel (5) with the data collection system turned off, and the change in pressure can be observed on the pressure gauge (6).

![Figure 2](image)

Figure 2. Scheme of laboratory hydraulic press: 1 – main cylinder; 2 – stroke; 3 – housing; 4 – bench; 5 – control desk; 6 – manometer; 7 – slide; 8 – oil pump; 9 – monitor; 10 – block of control and data collecting.

Wire (rods) were obtained from cast sections during experiment in accordance with the conditions presented in Table 2.

| Parameter                          | Unit | Value  | Value  | Value  |
|------------------------------------|------|--------|--------|--------|
|                                    |      | Exper.1| Exper.2| Exper.3|
| Force at the start of pressing, $F_{\text{start}}$ | kN   | 85     | 116    | 100    |
| Force at the end of pressing, $F_{\text{end}}$    | kN   | 42     | 61.5  | 70     |

Table 2. Experimental results.

Figure 3 shows the results of measuring the pressing force for a billet $\varnothing$ 30.00 mm from alloy into Sn-In a bar $\varnothing$ 8.00 mm. The pressing force decreases with the billet length decreasing in the container. Reduction of losses for overcoming friction forces is a consequence of a decrease in the surface between the billet and the container.

The fine precision of the calculated energy-power parameters (Table 1) and experimental results (Table 2) is quite high, the error does not exceed 10%. This allows us to conclude about the adequacy of the compiled mathematical model and the possibility of its further use for the rods and wires direct extrusion process analytical studies.
4. Analytical Study

The developed mathematical model can be used for operational calculation and extrusion modes selection for wires and rods of various assortments.

As an example, let us consider how the energy-power indexes change when extruding a various diameter rods from a billet Ø30.00 mm at a pressing speed $V_{pr} = 3.5$ mm/s. The simulation results are shown in Table 3.

**Table 3.** Results of power extruding parameters mathematical modeling of various diameters rod (wire) from billet Ø30.00 mm.

| Diameter of the wire (rod) $D_{exit}$, mm | Extrusion ratio, $\mu$ | Extrusion speed, $V_{exit}$, mm/s | Force at the start of pressing $F_{start}$, kN | Force at the end of pressing $F_{end}$, kN |
|-----------------------------------------|------------------------|----------------------------------|-----------------------------------------------|----------------------------------|
| 16.00                                   | 4.0                    | 0.14                             | 86                                            | 36                              |
| 14.00                                   | 5.22                   | 0.18                             | 91                                            | 41                              |
| 12.00                                   | 7.11                   | 0.25                             | 96                                            | 46                              |
| 10.00                                   | 10.24                  | 0.36                             | 101                                           | 51                              |
| 8.00                                    | 16                     | 0.56                             | 106                                           | 56                              |
| 6.00                                    | 28.44                  | 1.00                             | 112                                           | 61                              |
| 4.00                                    | 64                     | 2.24                             | 119                                           | 68                              |
| 2.00                                    | 256                    | 8.96                             | 131                                           | 80                              |
A decrease in the finished rod (wire) diameter from 16.00 to 2.00 mm while maintaining the billet size leads to an increase in the pressing force from 86 kN to 131 kN at the initial moment of pressing, as follows from the table. The reason for this is that the stretch ratio increases significantly (from 4 to 256).

The calculations of the \( \emptyset 8.00 \) bar pressing force with a change in the billet diameter \( V_{pr} = 3.5 \) mm/s are presented in Table 4.

**Table 4.** Results of energy-power parameters mathematical modeling of rod (wire) a \( \emptyset 8.00 \) mm extrusion from a different diameters billet.

| Diameter of billet \( D_{ent} \), mm | Extrusion ratio, \( \mu \) | Extrusion speed, \( V_{extr} \), mm/s | Force at the start of pressing \( F_{start} \), kN | Force at the end of pressing \( F_{end} \), kN |
|-------------------------------------|-----------------|-------------------------------|-------------------|-------------------|
| 40.00                               | 27.6            | 0.97                          | 171               | 103               |
| 36.00                               | 22.6            | 0.79                          | 143               | 82                |
| 32.00                               | 18.1            | 0.63                          | 118               | 64                |
| 28.00                               | 14.1            | 0.49                          | 95                | 48                |
| 24.00                               | 10.6            | 0.37                          | 74                | 35                |
| 20.00                               | 7.6             | 0.27                          | 56                | 23                |
| 16.00                               | 5.1             | 0.18                          | 39                | 14                |
| 12.00                               | 3.1             | 0.11                          | 25                | 7                 |

The pressing force with an increase in the diameter of the billet from 12.0 to 40.0 mm increases as follows from the calculation results. It is quite predictable even without the use of a model, but a mathematical model is needed in order to estimate the quantitative value. The presence of an adequate mathematical model allows not only to reasonably select equipment for production, depending on the planned assortment, but also to optimize the tooling design for the extrusion process. So, we see that even a 10-tonnes press can be used to produce a bar of \( \emptyset 8.00 \) mm. However, this is only possible with billet with a less than 28.00 mm diameter.

### 5. Conclusions

A mathematical model of low temperature solder direct extrusion has been developed. It allows determining the energy-power parameters of the process. Testing the mathematical model adequacy in the laboratory of the Processes and Machines of Metal Forming Department of SUSU on a hydraulic press equipped with a data collection and storage system showed fine precision. The error in both the pressing force and the temperature of the metal at the exit from the die did not exceed 10%, which is quite acceptable for analytical studies of the process and the extrusion technological modes choice.

Analytical studies on the model showed that it is possible not only to study the extrusion process in order to understand the mechanisms of finished rod (wire) mechanical and operational properties formation, but also to select resource-saving pressing modes for various product mixes, to make a reasonable choice of the required equipment and tooling.

### References

[1] Medvedev A M 2004 *Electronic components* (Moscow: ID Electronics)

[2] Medvedev A M 2006 *Components and Technologies* (Moscow: Finestreet)

[3] Grigoriev V 2001 *Electronic components* (Moscow: ID Electronics)

[4] Zenin V V, Belyaev V N, Segal Yu E, Kolbenkov A A 2003 *Microelectronics* (Moscow: Nauka Publishes)

[5] Bleifreiliten 2000 *Silber und Kupfer statt Blei. Krempelsauer. Elector* (BRD 5)

[6] Shapiro L 2007 *Bulletin of Electronics* 2

[7] Shapiro L 2006 *Electronic components* (Moscow: ID Electronics)
Acknowledgments
The work was carried out with the financial support of the Ministry of Science and Higher Education of the Russian Federation within the framework of a subsidy for financial support for the fulfillment of a state task (fundamental scientific research), contract № FENU-2020-0020 (2020071GZ).