Two-stage Reactive Power Optimization Strategy for Active Distribution Network Based on Extreme Scenarios

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Abstract. Uncertainties from distributed generations (DGs) make conventional reactive power optimization (RPO) methods difficult to be applied in the distribution network (DN). To settle this issue, a novel two-stage RPO strategy for active distribution network (ADN) based on extreme scenarios is proposed in this paper. Random variables are handled through extreme scenarios, then flexible and inflexible variables are decided in two different stages to adapt to DG’s randomness. Firstly, based on second order cone relaxation (SOCR), power flow equations are convexly relaxed to transform original model into mixed integer second order cone programming (MISOCP). Then, extreme scenario method (ESM) is employed to determine DG’s deployment. Finally, numerical tests are performed on a modified IEEE 33-bus DN to demonstrate the effectiveness and feasibility of the proposed strategy.

1. Introduction

In recent years, DGs have been continuously integrated into DN, imposing great challenges on original voltage regulation mode [1-3]. DG has strong volatility, while some reactive power adjustable devices such as on-load tap changers (OLTCs) fail to respond rapidly [4]. To cope with it, other devices need to act frequently, largely changing power flow, and some nodal voltages may exceed limit if slow-motion devices like capacitor banks (CBs) aren’t set properly. Hence, it’s necessary to investigate how to set slow-motion devices to make flexible devices respond quickly and satisfy operation constraints. Scholars have started relevant research on the RPO problem of ADN. Reference [5] proposed a solution method based on heuristic search and variable correction, and validated its feasibility through IEEE test systems. For nonlinear power flow equations of RPO, reference [6] put forward a novel continuous linear approximation method, which improved solution speed and also guaranteed the accuracy. A three-phase RPO model based on branch flow model (BFM) was established in [7], and SOCR was adopted for model transformation. All the above studies employ deterministic models to describe DGs, but these models have limitations when applied in practice. Moreover, the problem of coordinated reactive power dispatch between DGs and conventional devices isn’t considered in these studies.

To deal with uncertain output of DGs, stochastic programming methods have been used. Reference [8] considered output characteristics of different reactive power sources and DG’s forecast error, and put forward a multi-stage multi-source coordinated RPO method based on chance-constrained programming (CCP). In [9], the authors modelled uncertainties in the form of interval numbers and used interval programming to solve it. A two-stage robust RPO model based on BFM was built in [10], and SOCR was used to convexly relax the model. ESM is also a method handling random variables.
[11], and extreme scenario sets can cover all the value space, which avoids massive scenarios generated by Monte Carlo sampling in scenario method-based stochastic programming.

In this paper, a two-stage RPO strategy for ADN is investigated. Via SOCR, power flow equations are convexly relaxed and ADN’s model is built based on MISOCP. This model is processed by ESM and solved by commercial software. The remainder of this paper is organized as follows. In Section 2, a RPO model is presented. Then in Section 3, this model is transformed using SOCR. In Section 4, a two-stage optimization algorithm based on extreme scenarios is clarified. Computational results of a modified IEEE 33-bus DN are given in Section 5, and conclusions are drawn at last in Section 6.

2. Model of RPO

2.1. Objective Function

For RPO in this paper, the objective is to minimize network’s operation cost, which is given by

$$F = \min \sum_{i=1}^{n} \sum_{j=1}^{n} G_{ij} \left( V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij} \right)$$

(1)

Where $G_{ij}$ is the conductance of branch $i$-$j$, and $V_i$, $V_j$ denotes voltage amplitude of node $i$, $j$ respectively. $\theta_{ij}$ is the phase angle difference between node $i$ and $j$, and $n$ represents the total number of nodes in ADN.

2.2. Form of Branch Power Flow in Radial ADN

Figure 1 gives an overview of the radial ADN, and its branch power flow [12-14] is described as follows.

For node $j$ in the grid, there are

$$\sum_{(\text{parent}(j))} \left[ P_j - r_j \left( \frac{(P_j)^2 + (Q_j)^2}{(V_j)^2} \right) \right] = \sum_{k=(j)} P_k + P_j$$

$$\sum_{(\text{child}(j))} \left[ Q_j - x_j \left( \frac{(P_j)^2 + (Q_j)^2}{(V_j)^2} \right) \right] = \sum_{k=(j)} Q_k + Q_j$$

(2)

and for branch $i$-$j$, there is

$$(V_j)^2 = (V_i)^2 - 2(r_j P_i + x_j Q_i) + [(r_j)^2 + (x_j)^2] \frac{(P_i)^2 + (Q_i)^2}{(V_i)^2}$$

(3)

Where $u(j)$ is set of all parents of bus $j$, and $v(j)$ is set of all children. $r_j$, $x_j$ denotes resistance, reactance of branch $i$-$j$ respectively. $P_j$ and $Q_j$ are power in the head of branch $i$-$j$. $P_j$ and $Q_j$ are active and reactive power injected to node $j$. $P_{j,PV}$ and $Q_{j,PV}$ are active and reactive power of photovoltaics (PV) connected to node $j$. $Q_{j,\text{CB}}$ is CB’s discrete compensation power at node $j$. $P_{j,d}$ and $Q_{j,d}$ are load demand at node $j$. 
2.3. Constraints
Power flow constraints are given by (2) and (3). For system’s secure operation, there are
\[
\begin{align*}
V_i^\text{min} & \leq V_i \leq V_i^\text{max} \\
I_j & \leq I_j^\text{max}
\end{align*}
\] (4)

Where \( I_j \) is current amplitude of branch \( i-j \), and \( I_j^\text{max} \) is upper bound. \( V_i^\text{max}, V_i^\text{min} \) are maximum and minimum allowable nodal voltage levels, respectively.

Specific operation constraints of CBs are given by
\[
\begin{align*}
Q_{i,CB}^\text{op} & = N_{i,CB} \cdot Q_{i,CB}^\text{op} \\
0 & \leq N_{i,CB} \leq N_{i,CB}^\text{max} \\
N_{i,CB} & \in \text{int}
\end{align*}
\] (5)

Where \( Q_{i,CB}^\text{op} \) is compensation power of a batch of CBs at node \( i \), and \( Q_{i,CB} \) is actual compensation power. \( N_{i,CB} \) and \( N_{i,CB}^\text{max} \) are actual and maximum batches, respectively.

\[
(V_{i,\text{min}})^2 \leq (V_{i,h})^2 \leq (V_{i,\text{max}})^2
\]
\[
k_{i,\text{min}} \leq k_i \leq k_{i,\text{max}}
\] (6)

(6) are constraints for OLTC, where \( V_{i,h} \) is high-side voltage and \( V_{i,\text{min}}, V_{i,\text{max}} \) are lower and upper bounds. \( k_i \) is the square of ratio, and \( k_{i,\text{min}}, k_{i,\text{max}} \) confine its range. \( k_i \) is discrete and can be processed [15] as

\[
k_i = k_{i,\text{min}} + \sum_s k_{i,s} \cdot \gamma_{i,s}^\text{OLTC}
\] (7)

Where \( k_{i,s} \) is the difference between \( s \) and \( s-1 \) tap position. \( \gamma_{i,s}^\text{OLTC} \) is a 0-1 variable.

\[
|Q_{PV}|_{\text{max}} = \sqrt{S_{PV,\text{max}}^2 - P_{PV}^2}
\] (8)

The maximum reactive power of a grid-connected PV inverter is determined by (8), where \( P_{PV} \) and \( Q_{PV} \) are its active and reactive output. \( S_{PV,\text{max}} \) is inverter’s rated capacity. With a reasonable control strategy, the inverter can maximize its active output and realize reactive power compensation simultaneously [16].

3. Cone Transformation of Optimization Model
Based on characteristics of SOCP [17-18], branch power flow equations are relaxed. Firstly, we define

\[
(I_{i,j})^2 = \left( \frac{P_{i,j}}{V_j} \right)^2 + \left( \frac{Q_{i,j}}{V_j} \right)^2
\] (9)

Here, let \( I_{i,j} = (I_{i,j})^2 \), \( V_{i,j} = (V_{i,j})^2 \), then (2) and (3) are transformed as

\[
\begin{align*}
\sum_{i \in \text{net}(j)} [P_{i,j} - r_j \bar{I}_{i,j}] & = \sum_{k \in \text{net}(j)} P_{j,k} + P_j \\
\sum_{i \in \text{net}(j)} [Q_{i,j} - x_j \bar{I}_{i,j}] & = \sum_{k \in \text{net}(j)} Q_{j,k} + Q_j \\
P_j & = P_{j,PV} + P_{j,\text{dis}} - P_{j,\text{ch}} - P_{j,d} \\
Q_j & = Q_{j,PV} + Q_{j,CB} - P_{j,d} \\
\bar{V}_j & = \bar{V}_i - 2(r_j \bar{P}_j + x_j \bar{Q}_j) + \left[ (r_j)^2 + (x_j)^2 \right] \bar{I}_{i,j}
\end{align*}
\]

(10)

(11)

\[
I_{i,j} = \left( \frac{P_{i,j}}{V_j} \right)^2 + \left( \frac{Q_{i,j}}{V_j} \right)^2
\] (12)

After the above deformation, the original power flow equations have changed into linear equations (10), (11) and a simple quadratic equation (12). Then (12) is relaxed as
\[ I_{ij} \geq \left( \frac{P_{ij}}{V_{ij}} \right)^2 + \left( \frac{Q_{ij}}{V_{ij}} \right)^2 \]  

(13)

Furthermore, (13) can be transformed into the standard form of SOC as given by
\[ \begin{bmatrix} 2P_{ij} \\ 2Q_{ij} \\ I_{ij} - V_{ij} \end{bmatrix} \leq \begin{bmatrix} \bar{V}_{ij} \\ \bar{V}_{ij} \end{bmatrix} \]  

(14)

4. Two-stage Optimization Algorithm Based on Extreme Scenarios

As is known from Section 2, the problem of RPO in ADN with DGs can be formulated as
\[ \begin{align*}
& \text{min } q(\xi, x, y) \\
& \text{s.t. } f(\xi, x, y) = 0 \\
& \quad g(\xi, x, y) \leq 0
\end{align*} \]  

(15)

Where \( \xi \) is the parameter of DG’s output. \( x \) is flexible variables for those flexible sources and \( y \) is inflexible variables for slow-motion adjustable devices. \( q(\xi, x, y) \) and \( f(\xi, x, y) \) are linear, and \( g(\xi, x, y) \) is downward convex. The objective function and constraints both satisfy determination conditions that convex programming requires, and (14) meets SOC’s definition, so the solution of transformed model belongs to SOCP. It has been demonstrated in [13-15] that SOCP is a convex programming. At this point, if \( y \) is to be decided to ensure that there exists \( x \) that can minimize \( q(\xi, x, y) \) for any \( \xi \), all the potential values of \( \xi \) can be taken as scenario constraints, then (15) is transformed as
\[ \begin{align*}
& \text{min } \sum_{i} q(\xi_i, x_i, y) \\
& \text{s.t. } f(\xi_i, x_i, y) = 0, g(\xi_i, x_i, y) \leq 0 \\
& \quad \vdots \\
& f(\xi_n, x_n, y) = 0, g(\xi_n, x_n, y) \leq 0
\end{align*} \]  

(16)

Where \( \xi_i \) is the \( i \)-th potential value of \( \xi \) and \( x_i \) is the corresponding value of \( x \). Decision variables \( x \) and \( y \) are solved in two stages, and the detailed process is as follows.

1) When DG’s output varies, \( y \) doesn’t change accordingly and it’s the first-stage decision variables.

2) After \( y \) is decided, \( x \) changes with \( \xi \) and it’s the second-stage decision variables.

However, if the value space of \( \xi \) is not a set with finite discrete values, (16) will have infinite constraints, making the solution infeasible. Thus, it’s needed to find finite values instead of entire space. Here, extreme scenario is introduced. Extreme scenario is a scenario where all the random variables are at their confidence limit. If there are \( n \) \((n \geq 3)\) random power generations, the output value space is then an \( n \)-dimensional convex polyhedron with \( 2^n \) vertices, and the number of extreme scenarios is \( 2^n \). Take three random variables for example, and its diagram of extreme scenarios is depicted in figure 2.

\[ \begin{align*}
\bar{\xi}_1 & \quad \text{value space of} \\
\bar{\xi}_2 & \quad \text{random variables} \\
\bar{\xi}_3 & \quad \text{extreme scenario} \\
& \quad \text{forecasted scenario}
\end{align*} \]

Figure 2. Diagram of the extreme scenarios.
It has proved that for inflexible variables \( y \) decided, as long as flexible variables \( x \) adapt to the extreme scenarios, \( x \) can adapt to all scenarios of \( \xi \)'s value space. Then (16) is transformed and ultimate two-stage model based on ESM is given by

\[
\min \sum_i q(\xi_i, x_i, y) \\
\text{s.t. } f(\xi_i, x_i, y) = 0, g(\xi_i, x_i, y) \leq 0
\]

(17)

Where \( \xi_1, \ldots, \xi_i \) are values of random variables in the extreme scenarios.

As is stressed above, \( \xi \) must have finite values. Moreover, extreme scenarios increase exponentially as random power generations increases, so it's inevitable to use scenario reduction technique to ease the computation burden. It's necessary to mention that as \( n \) increases, the representativeness of scenario set generated by Monte Carlo weakens rapidly and conflict between computation time and computation accuracy will be irreconcilable. Compared with Monte Carlo, however, the solution based on ESM has more adaptability to DG’s randomness when scenario’s number is the same.

5. Simulation Analysis

5.1. Development Environment

Based on Yalmip in Matlab, the RPO problem is modelled considering DG’s reactive power adjustable ability, and large-scale mathematical programming optimizer Gurobi is used to solve the model.

5.2. Case Description

In view of PV as the main form of DG’s integration, this paper uses a modified IEEE 33-bus DN as the test system, which is shown in figure 3, and PV, OLTC and CB are considered in it. OLTC’s adjustable range is between 0.95 and 1.05, divided into 5 positions. Each CB’s rated capacity is 0.4 MVA and it has 20 batches. The rated capacity of PV is 0.5 MVA and its active output ranges between \((1-\alpha)P_{PV} \) and \((1+\alpha)P_{PV} \), where \( \alpha \leq 1 \). \( \alpha \) is the parameter measuring fluctuation level of PV’s output.

*Figure 3. Diagram of a modified IEEE 33-bus DN.*

5.3. Simulation Results

To analyse the robustness and economy of the method proposed in this paper, deterministic optimization method (DOM) and ESM are adopted to solve the problem, respectively. Based on forecasted scenario, the optimization results using DOM are shown in table 1.

| Operation cost (MW) | First-stage decision variables | Second-stage decision variables |
|--------------------|-------------------------------|---------------------------------|
|                    | OLTC Location                 | CB Location                    |
|                    |                               | Switched batches               |
|                    |                               | PV Location                    |
|                    |                               | Reactive output (Mvar)         |
| 0.1526             | 1.025                         | 2                              |
|                    | 13                            | 19                             |
|                    |                               | 0.2126                         |
In the test system, there are four random power generations (at location 19, 24, 27 and 30), so the number of extreme scenarios is 16. To ease the computation burden, scenario reduction technique is used to merge similar scenarios and extreme scenarios are then reduced to 10. Table 2 shows the results of first-stage decision variables in extreme scenarios considering different $\alpha$.

**Table 2.** Results of first-stage decision variables in extreme scenarios considering different $\alpha$.

| $\alpha$ | OLTC | Switched batches |
|---------|------|------------------|
|         |      | CB2 | CB9 |
| 0.2     | 1.025| 13  | 16  |
| 0.4     | 1.025| 12  | 16  |
| 0.6     | 1.025| 12  | 16  |
| 0.8     | 1    | 13  | 17  |
| 1       | 1    | 13  | 17  |

In extreme scenarios, the grid is mostly affected and its operation cost is large. In this paper, extreme scenario with maximum cost is defined as the “worst scenario”. To verify the optimization effect of the proposed two-stage algorithm, operation cost of stochastic optimization and deterministic optimization in the worst scenario are analysed, and results are shown in table 3.

**Table 3.** Comparison of the operation cost between DOM and ESM in the worst scenario.

| $\alpha$ | Operation cost of DOM (MW) | Operation cost of ESM (MW) |
|---------|-----------------------------|-----------------------------|
| 0.2     | 0.1632                      | 0.1630                      |
| 0.4     | 0.1761                      | 0.1758                      |
| 0.6     | 0.1911                      | 0.1906                      |
| 0.8     | voltage violation           | 0.2076                      |
| 1       | voltage violation           | 0.2267                      |

Table 4 shows the results of second-stage decision variables in three different scenarios. Scenario 1 is a forecasted scenario with ESM utilized. Scenario 2 and scenario 3 are both the worst scenario, but ESM is used in the former while DOM is adopted in the latter.

**Table 4.** Results of second-stage decision variables in different scenarios when $\alpha = 0.4$.

| Scenarios | Reactive output (Mvar) |
|-----------|------------------------|
|           | PV19 | PV24 | PV27 | PV30 |
| scenario 1| 0.1927 | 0.3662 | 0.2931 | 0.4051 |
| scenario 2| 0.2056 | 0.3787 | 0.2472 | 0.4211 |
| scenario 3| 0.1816 | 0.3582 | 0.3992 | 0.3992 |

As is shown in table 3, when $\alpha$ is small, DOM can also ensure ADN’s reliable operation, but the results are inferior to that obtained by ESM. When $\alpha$ is getting larger, however, DOM’s deficiency starts to appear, i.e. voltage exceeds specified limits. On the contrary, ESM can still guarantee the secure and stable operation of the system at this time. Therefore, ESM employed in this work is more robust.

From table 3 and table 4, it’s indicated that although the operation cost between DOM and ESM in the worst scenario are similar ($\alpha = 0.4$), power flow distribution differs a lot from each other.

**6. Conclusion**

A novel two-stage RPO strategy is proposed in this paper, and simulation analysis shows that:
1) This strategy based on ESM can better adapt to DG’s uncertain output, and optimizes network’s cost under the condition of ADN’s robust operation and DG’s safe consumption;
2) When PV’s output fluctuates in a small scale, i.e. $\alpha$ is small, DOM can still apply to the worst scenario, but is economically inferior to ESM. When $\alpha$ is large, only ESM can cope with it;
3) The method proposed can theoretically obtain the problem’s optimal solution and can be realized by commercial software directly and conveniently, which is applicable to large-scale network.

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