Improving cosmological parameter estimation with the future gravitational-wave standard siren observation from the Einstein Telescope

Xuan-Neng Zhang,1 Ling-Feng Wang,1 Jing-Fei Zhang,1 and Xin Zhang*1,2,3,4,†

1Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China
2Center for High Energy Physics, Peking University, Beijing 100080, China
3CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
4Center for Gravitation and Cosmology, Yangzhou University, Yangzhou 225009, China

Detection of gravitational waves produced by merger of binary compact objects could provide an independent way for measuring the luminosity distance to the gravitational-wave burst source, indicating that gravitational-wave observation, combined with observation of electromagnetic counterparts, can provide “standard sirens” for investigating the expansion history of the universe in cosmology. In this work, we wish to investigate how the future gravitational-wave standard siren observations would break the parameter degeneracies existing in the conventional optical observations and how they help improve the parameter estimation in cosmology. We take the third-generation ground-based gravitational-wave detector, the Einstein Telescope, as an example to make an analysis. By simulating 1000 events data in the redshift range between 0 and 5 based on the ten-year observation of the Einstein Telescope, we find that the gravitational-wave data could largely break the degeneracy between the matter density and the Hubble constant, thus significantly improving the cosmological constraints. We further show that the constraint on the equation-of-state parameter of dark energy could also be significantly improved by including the gravitational-wave data in the cosmological fit.

The observations from the Planck satellite mission strongly favor a 6-parameter base Λ cold dark matter (ΛCDM) cosmology [1]. That is to say, in the current stage, one can use only 6 parameters to reproduce the evolution of the universe, including the expansion history and the large-scale structure formation, based on the ΛCDM model, which is in good agreement with the current various cosmological observations. However, it is hard to believe that the eventual model of cosmology indeed consists of only 6 parameters. Actually, one believes that this is because the current observations are not precise enough to tightly constrain other possible parameters beyond the base ΛCDM model, and in the future the base model must be extended in several aspects with the help of future highly accurate observational data.

In fact, it seems that some cracks appear in the base ΛCDM cosmology, which is hinted by the fact that some tensions exist between different astrophysical observations based on the base ΛCDM model, e.g., the well-known issues concerning H0 and σ8Ωmα (with α taken to be 0.3–0.5) measurements [1–3]. A possible way to solve the tensions is to consider some extensions to the base ΛCDM model, but this leads to the fact that the extra parameters are rather difficult to be tightly constrained by the current observational data, in particular, the parameter degeneracies might be strong for these extra parameters. All these facts actually are making requests for the current cosmology: (i) cosmological probes should be further developed; and (ii) cosmological model should also be further extended.

Currently, the major cosmological probes mainly include: cosmic microwave background anisotropies, type Ia supernovae, baryon acoustic oscillations, direct determination of the Hubble constant, weak gravitational lensing, clusters of galaxies, and redshift-space distortions. The combinations of these cosmological data have provided precise measurements for some cosmological parameters, but for several important extra parameters, e.g., the equation-of-state parameter of dark energy w(z) (with z being redshift), the total mass of neutrinos mν, the effective number of relativistic species Neff, and so forth, the current observations still cannot provide tight constraints [1]. The future major dark energy experiments (e.g., DESI, LSST, Euclid, WFIRST, etc.) will definitely play a crucial role in determining these parameters, but all these experiments are optical (or near-infrared imaging or spectroscopy) observations, which implies that any new observational means would be helpful in avoiding the systematic errors in these optical observations. The promising new cosmological probes mainly include the radio observations (i.e., 21 cm neutral hydrogen survey) and the gravitational-wave observations.

It is well known from Schutz’s work in the mid-1980s [4] that the gravitational waves carry information of cosmic distance of the source. Actually, from the observation of the waveform of gravitational waves released by binary compact objects merger, one can independently measure the luminosity distance to the source of gravitational-wave burst. Furthermore, if the redshift of the source can also be observed by identifying the electromagnetic (EM) counterpart of the source (like the cases of the merger of two neutron stars and the merger of black hole and
neutron star), then one can establish a true distance-redshift relation based on the observation of large amount gravitational-wave events from which the expansion history of the universe can be inferred [5–12]. Compared with the observation of type Ia supernovae (SN) that can also measure the luminosity distance in some sense, the gravitational-wave (GW) observation as a cosmological probe has the following advantages: (i) The SN observation actually cannot measure the absolute luminosity distance, but can only measure the ratio of luminosity distances at different redshifts, due to the fact that the intrinsic luminosity of type Ia SN is not precisely known to us. But the GW observation, definitely, can provide measurement for the absolute luminosity distance to the source. (ii) The SN observation can only provide measurements for events with redshifts less than 1.4, but the GW observation can provide measurements for events with much higher redshifts. These advantages ensure that the future GW observations could play a significant role in breaking the parameter degeneracies and help improve the cosmological parameter estimation.

Some forecast studies on cosmology by using the future GW observations (with short γ-ray bursts or other EM counterparts, as standard sirens) have been performed in the literature (see, e.g., Refs. [13–22]). For example, in Ref. [13], it is shown that the observation from a network of advanced LIGO detectors can constrain the Hubble constant to a 5% accuracy. In Ref. [14], it is demonstrated that the observation of 1000 GW events from the next-generation ground-based GW detector, the Einstein Telescope (ET), is possible to constrain the cosmological parameters up to $\sigma(h) \sim 5 \times 10^{-3}$ and $\sigma(\Omega_m) \sim 0.02$ using a Fisher matrix approach (see also Ref. [16] in which similar results are found using a MCMC method). In Ref. [16], it is found that using the Gaussian Process method the equation-of-state parameter of dark energy can be constrained to be $\sigma(w) \sim 0.03$ with the future GW observations. Also, in Ref. [22], it is shown that with the help of 1000 GW events observed from the ET the constraints on the neutrino mass can be improved by about 10%. Furthermore, based on the space-based detector LISA, the expansion of the universe and the interacting dark energy have also been investigated in Refs. [17, 18].

In this work, we wish to investigate how the future GW standard siren observation would break the cosmological parameter degeneracies (existing in the conventional observations) and thus help improve the parameter estimation for cosmology. We will take the third-generation ground-based detector ET as an example to make an analysis. In this analysis, to be in accordance with the previous studies [14, 16, 22], we will simulate 1000 GW events data in the redshift range of $z \in [0, 5]$ based on the ET’s ten-year observation. The GW data simulation method used in this paper is in exact accordance with the prescription given in Refs. [16, 22], and thus we do not repeat it here; we refer the reader to Refs. [16, 22] for details. There are some parameter degeneracies in cosmological models constrained by the current conventional observations, such as cosmic microwave background (CMB), baryon acoustic oscillations (BAO), and type Ia supernovae (SN). Using the GW data to make parameter estimation, there will also be some degeneracies, but in this case the orientation of the degeneracy in some parameter plane would be different from the above case. Thus, the GW standard siren observation will play an important role in improving the parameter estimation because it could break parameter degeneracies in the conventional observations. In this paper, we will take the ΛCDM model and the $w$CDM model (where the equation-of-state parameter of dark energy $w$ is taken to be a constant) as examples to see how this happens.

First, we will use the current observations of CMB, BAO, and SN to constrain the two cosmological models, and see how the parameters degenerate with each other, leading to the result that they cannot be well constrained. Then, we will use the simulated GW data from the ET to constrain the models, and we will observe different degeneracy cases, which leads to the fact that the previous degeneracies are broken when the GW data are included in the data combination.

We now briefly describe the current observations used in this paper. For CMB data, we use the Planck temperature and polarization power spectra (Planck TT, TE, EE+lowP) [1]. For BAO data, we use the measurements from 6dFGS ($z_{\text{eff}} = 0.106$) [23], SDSS-MGS ($z_{\text{eff}} = 0.15$) [24], and BOSS LOWZ ($z_{\text{eff}} = 0.32$) and CMASS ($z_{\text{eff}} = 0.57$) [25]. For SN data, we use the JLA compilation [26]. We use the data combination of CMB+BAO+SN to constrain the ΛCDM and $w$CDM models (by employing the Markov-chain Monte Carlo package CosmoMC [27]), and then take the best-fitted models as the fiducial models to generate the simulated GW data. In Fig. 1, we show the simulated GW data consisting of $d_L^{\text{meas}}$ and $\sigma_{d_L}$ in the redshift range of $z \in [0, 5]$ for the two cases in which the ΛCDM model and the $w$CDM model are taken as fiducial models, respectively.

We then use the simulated GW data to constrain the models. The constraints from the data combination of CMB+BAO+SN+GW will show how the GW data help improve the parameter estimation in the considered two cases. We summarize the fitting results in Tables I and II. In Table I we show the fitting values of cosmological parameters, and in Table II we show the constraint errors and constraint accuracies for the concerned parameters (i.e., $\Omega_m$, $H_0$, and $w$). Here, the error $\sigma$ is taken to be the average of $\sigma_+$ and $\sigma_-$, and $\varepsilon(P)$ for a parameter $P$ is defined as $\varepsilon(P) = \sigma(P)/P$.

In Fig. 2, we show the one-dimensional posterior distributions of $H_0$ in the ΛCDM model (left) and the $w$CDM model (right) using the CMB+BAO+SN, GW, and CMB+BAO+SN+GW data combinations. Note that, for convenience, hereafter we use the abbreviation “CBS” to denote the combination CMB+BAO+SN. Here we can clearly see that the GW observation solely can tightly constrain the Hubble constant. In the case of ΛCDM, we have $\sigma(h) = 4.6 \times 10^{-3}$ and $\varepsilon(h) = 0.68\%$.
We find that the accuracy of data are included in the fit. In the case of CBS+GW, we find that the accuracy of CMB+BAO+SN.

TABLE II: Constraint errors and accuracies for parameters of ΛCDM and wCDM (right) models, respectively, constrained by the current CMB+BAO+SN data.

| Model | ACMD | wCDM |
|-------|------|------|
| | CBS | GW | CBS+GW | CBS | GW | CBS+GW |
| $\Omega_m h^2$ | 0.02233 ± 0.00014 | 0.0501^{+0.00099}_{-0.0014} | 0.02229 ± 0.00010 | 0.02230 ± 0.00015 | 0.051^{+0.00097}_{-0.0014} | 0.02231 ± 0.00013 |
| $\Omega_c h^2$ | 0.11853 ± 0.00101 | 1.019^{+0.00050}_{-0.00051} | 0.11808^{+0.00050}_{-0.00051} | 0.11809^{+0.000124}_{-0.000125} | 0.081^{+0.00046}_{-0.00046} | 0.11879^{+0.000105}_{-0.000104} |
| $100\theta_{MC}$ | 1.04092 ± 0.00030 | 0.96^{+0.012}_{-0.011} | 1.04086^{+0.00027}_{-0.00026} | 1.04087 ± 0.00030 | 0.95 ± 0.11 | 1.04088 ± 0.00028 |
| $\tau$ | 0.086 ± 0.016 | 0.083 ± 0.016 | 0.084 ± 0.017 | – | – | 0.084 ± 0.017 |
| $\ln(10^{10}A_s)$ | 3.103 ± 0.032 | – | 3.098 ± 0.032 | 3.101 ± 0.033 | – | 3.101 ± 0.033 |
| $\Omega_m$ | 0.3075^{+0.0060}_{-0.0061} | 0.3119^{+0.0071}_{-0.0073} | 0.3103 ± 0.0024 | 0.3093^{+0.0094}_{-0.0093} | 0.2890 ± 0.0150 | 0.3061 ± 0.0023 |
| $H_0$ [km/s/Mpc] | 67.84 ± 0.46 | 67.57 ± 0.22 | 67.62 ± 0.16 | 68.3^{+1.9}_{-1.3} | 67.79^{+0.32}_{-0.33} | 68.04^{+0.24}_{-0.22} |

from CBS, $\sigma(h) = 2.2 \times 10^{-3}$ and $\varepsilon(h) = 0.33\%$ from GW, and $\sigma(h) = 1.6 \times 10^{-3}$ and $\varepsilon(h) = 0.24\%$ from CBS+GW. We find that the accuracy of $H_0$ is improved from 0.68% to 0.24% in the ΛCDM case when the GW data are included in the fit. In the case of wCDM, we have $\sigma(h) = 1.05 \times 10^{-2}$ and $\varepsilon(h) = 1.53\%$ from CBS, $\sigma(h) = 3.3 \times 10^{-3}$ and $\varepsilon(h) = 0.48\%$ from GW, and $\sigma(h) = 2.4 \times 10^{-3}$ and $\varepsilon(h) = 0.35\%$ from CBS+GW. We find that the accuracy of $H_0$ is improved from 1.53% to 0.35% in the wCDM case when the GW data are included in the fit. Obviously, it is shown from this analysis that the GW observation is capable of significantly improve the constraint accuracy of $H_0$, in particular in dynamical dark energy models.

Figure 3 shows constraints on the ΛCDM model (left) and the wCDM model (right) in the $H_0-\Omega_m$ plane. In this figure, we can clearly see that, from the CBS constraint, in both the ΛCDM and wCDM models, $\Omega_m$ and $H_0$ are in strong anti-correlation. In the case of ΛCDM, the GW constraint still gives an anti-correlation for $\Omega_m$ and $H_0$, but its degeneracy orientation in the parameter plane is evidently different from the former, resulting in
FIG. 2: One-dimensional posterior distributions of $H_0$ in the $\Lambda$CDM model (left) and the $w$CDM model (right) using the CBS, GW, and CBS+GW data combinations. Here, CBS stands for CMB+BAO+SN.

FIG. 3: Constraints (68.3% and 95.4% CL) on the $\Lambda$CDM model (left) and the $w$CDM model (right) in the $H_0$–$\Omega_m$ plane using the CBS, GW, and CBS+GW data combinations. Here, CBS stands for CMB+BAO+SN.

FIG. 4: Constraints (68.3% and 95.4% CL) on the $w$CDM model in the $H_0$–$w$ plane using the CBS, GW, and CBS+GW data combinations. Here, CBS stands for CMB+BAO+SN.

The breaking of the degeneracy. In the case of $w$CDM, we find that the GW constraint leads to a weak positive correlation for $\Omega_m$ and $H_0$, and thus the orthogonality of the two degeneracy orientations results in a complete breaking of the parameter degeneracy. Thus, although CBS and GW give similar constraints on $\Omega_m$, the combination of the two could tremendously improve the constraint on $\Omega_m$. In the $\Lambda$CDM case the constraint on $\Omega_m$ is improved from 1.97% to 0.77%, and in the $w$CDM case the constraint on $\Omega_m$ is from 3.08% to 0.75%, when the GW data are included in the fit.

In Fig. 4, we show the constraints on the $w$CDM model in the $H_0$–$w$ plane. From this figure, we can also clearly see that the parameter degeneracy could be largely broken by including the GW observation in the cosmological fit. The CBS data give $\sigma(w) = 0.042$ and the GW data give $\sigma(w) = 0.058$, indicating that their constraints on $w$ are similar. But the combination of the two gives $\sigma(w) = 0.020$, showing that the error is largely decreased. By considering the GW observation, the constraint accuracy of $w$ is improved from 4.1% to 2.0%. Therefore, in this analysis, we have shown that the GW observation could help greatly improve the constraint accuracy of $w$. 

The constraint accuracy of $w$. Therefore, in this analysis, we have shown that the GW observation could help greatly improve the constraint accuracy of $w$. 

...
In summary, it is shown in this work that the future GW standard siren observation is capable of breaking the parameter degeneracies existing in the conventional optical observations and thus could help improve the parameter estimation for cosmology. We have simulated 1000 GW events data based on the ET’s ten-year observation. In order to show how the GW data break the parameter degeneracies, we employ the current CMB+BAO+SN data to make comparison and combination. We take the $\Lambda$CDM and $w$CDM models as examples to make an analysis. We find that the degeneracy between $\Omega_m$ and $H_0$ can be greatly broken by including the GW data, particularly for the case of $w$CDM. Thus, for both $\Omega_m$ and $H_0$, the constraint accuracies are tremendously improved by considering the GW data from ET. Although $w$ is hard to be tightly constrained, the GW observation from ET could help improve its constraint accuracy from 4% to 2%, according to our analysis. In this work, we only make a preliminary analysis, because we only consider the improvement for the parameter estimation based on the current CMB+BAO+SN data, but not the future optical observations. Moreover, we only consider the simplest dynamical dark energy model, i.e., the $w$CDM model, in this work. We will leave a comprehensive analysis to a future work.

Acknowledgments

We would like to thank Zhou-Jian Cao, Tao Yang, and Wen Zhao for helpful discussions. This work was supported by the National Natural Science Foundation of China (Grants No. 11690021 and No. 11522540) and the National Program for Support of Top-Notch Young Professionals.