Rate of photon production from hot hadronic matter

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Thermal photon emission rates from hot hadronic matter are studied to order $e^2 g^4$, where $g$ indicates a strong-interaction coupling constant. Radiative decay of mesons, Compton and annihilation processes for hadrons, and bremsstrahlung reactions are all considered. Compared to the standard rates from the literature, one finds two orders of magnitude increase for low photon energies stemming mainly from bremsstrahlung and then a modest increase (factor of 2) for intermediate and high energy photons owing to radiative decays for a variety of mesons and from other reactions involving strangeness. These results could have important consequences for electromagnetic radiation studies at RHIC.

Since production rates are increasing functions of temperature, the inverse slope on a transverse momentum spectrum, for instance, is a good measure of the kinetic properties of the early (thermal) hadronic source. And yet, photons are produced during the entire reaction. Pre-equilibrium dynamics contribute, parton dynamics contribute, and then, hadron dynamics might be considered as a background to the more sought-after quark gluon plasma contributions. For quantitative interpretation of photon measurements at the Relativistic Heavy Ion Collider (RHIC), a complete determination of all components, including the hadronic background, is clearly a prerequisite.

The early comparison of photon production from quark gluon plasma versus hadronic matter by Kapusta et al. revealed that the two phases have equal production rates at fixed temperature. Technically, the advancements here were made possible through an application of hard-thermal-loop resummation methods and effective hadronic field theory. This physically appealing result is however, somewhat discouraging in terms of allowing distinction between the two phases using photons. The equal-luminous property of the two phases has survived even after next-to-leading-order (NLO) QCD processes have been considered. NNLO parton processes have even been analyzed, and the new rate is still consistent with the hadron rates. The inevitable conclusion is that photon production as a QGP diagnostic is not as definitive as was originally hoped. However, with the tremendous recent advancements in the determination of QCD processes—to three-loop order, including annihilation with scattering and also bremsstrahlung with multiple scattering interferences, it behooves theory to put the hadronic formalism on a more equal footing. The purpose of this paper is to report on photon production from hadronic matter computed at NNLO in the strong-interaction coupling constant, namely, to order $e^2 g^4$.

The photon production rate to lowest order in the electromagnetic coupling and to all orders in the strong coupling can be compactly written as

$$E_\gamma \frac{dR}{d^3 p_\gamma} = -\frac{2g^{\mu\nu}}{(2\pi)^2}\text{Im}\Pi^{\mu\nu}_\gamma(p_\gamma) \frac{1}{e^{\beta E_\gamma} - 1},$$

where the retarded photon self-energy includes all possible hadron topologies—that is, to arbitrary loop order. The imaginary part corresponds to emission and absorption, of which the former is relevant here. Absorption was studied in Ref. and found to be negligible. At the one-loop level, the imaginary part corresponds to such lowest-order reactions as $\omega \rightarrow \pi\gamma$ and $\rho \rightarrow \pi\gamma$. There is however, a course a long list of other resonance decays to be considered, as discussed below. Typically, in order to quantify production rates, a model for the strong interaction (e.g. an effective Lagrangian) is necessary. However, the one-loop reactions have the desirable feature that the rate can be computed in a model independent way, requiring only the measured radiative decay rates. If such measurements are not available, then vector meson dominance plus a chiral Lagrangian is an appropriate approach.

For the process $a \rightarrow 1 + \gamma$, where $a$ and 1 are mesons with allowed quantum numbers and masses, the thermal rate can be written as

$$E_\gamma \frac{dR}{d^3 p_\gamma} = \frac{N m_a^2 \Gamma(a \rightarrow 1\gamma)}{16\pi^3 E_\gamma E_0} \int_{E_{\text{min}}}^{\infty} dE_a f_{BE}(E_a).$$
The first channel, which is the Compton type, will be considered here while the second and third will be discussed later in the higher orders, since no unique process, will be appropriate and a minus sign in the second factor of the integrand would suppress phase-space occupation for species 1. The above integral is typically reported in the literature, and then a numerical exercise ensues. However, there is no need to leave it here since the integral has a closed-form expression with the result

$$\Gamma = \frac{N m^2 \Gamma(a \to 1\gamma)}{16\pi^3 E_p E_0} f_{BE}(E_{\gamma}) \times [1 + f_{BE}(E_a - E_{\gamma})]$$

(2)

where $N$ is the spin and isospin degeneracy for the parent, $E_0$ is the photon energy in the rest frame of the parent, $E_{\gamma,\text{min}} = m_a (E_p^2 + E_{\gamma,\text{0}}^2) / 2E_p E_0$, and $f_{BE}$ is the Bose-Einstein distribution to account for quantum statistics and medium-enhancement for species 1. If baryons were studied, e.g. $\Delta \to N\gamma$, then Fermi-Dirac distributions would be appropriate and a minus sign in the second factor of the integrand would suppress phase-space occupation for species 1. The above integral is typically reported in the literature, and then a numerical exercise ensues. However, there is no need to leave it here since the integral has a closed-form expression with the result

$$E_\gamma \frac{d\Gamma}{dp_{\gamma}} = \frac{N m^2 \Gamma(a \to 1\gamma)}{16\pi^3 E_p E_0} f_{BE}(E_{\gamma}) \times T \left[ \ln \left( \frac{f_{BE}(E_{\gamma,\text{min}} - E_{\gamma})}{f_{BE}(E_{\gamma,\text{min}})} \right) - E_\gamma / T \right]$$

(3)

Empirical radiative decay widths for several low-lying resonances are available\(11\) and included in Table I. The resulting thermal decay rates are displayed in Fig. 1.

Next-to-leading-order contributions enter at order $\epsilon^2 m^2$ and come from two-loop self-energy structures. Coupling strength and phase space arguments suggest that $\pi\rho \to \pi\gamma$, $\pi\pi \to \rho\gamma$ and $\rho \to \pi\pi\gamma$ will be important contributors. The first channel, which is the Compton type process, will be considered here while the second and third will be discussed later in the higher orders, since they are essentially the on-shell components of the full, higher order contributions. For the Compton process, a particular interaction Lagrangian was assumed in Ref. 4\(a\) describing $\pi\rho$ dynamics, and then calibrated to $\rho \to \pi\pi$ decay. In terms of practical kinetic theory formalism, the rate for the general reaction $a \to b + 1 + \gamma$ is

$$E_\gamma \frac{d\Gamma}{dp_{\gamma}} = \frac{N}{4E_p} \int \frac{d^3 p_a}{(2\pi)^3} \frac{d^3 p_b}{(2\pi)^3} \frac{d^3 p_l}{(2\pi)^3} \frac{d^3 p_1}{(2\pi)^3} \sum_{1}^{M} |\bar{M}|^2 (2\pi)^4 \delta^4 (p_a + p_b - p_l - p_1) (4)$$

where $\bar{M}$ is the amplitude for the Compton process $\pi\rho \to \pi\gamma$. In Fig. 1 the $a_1$ radiative decay has been included while in Fig. 2 only the $\pi$-exchange result for the Compton process $\pi\rho \to \pi\gamma$ is shown. This has the slight incomplete feature of ignoring the $u$ channel...
FIG. 2: Thermal photon production rate from important channels at order $e^2 g^2$.

FIG. 3: Bremsstrahlung contributions (order $e^2 g^4$) to thermal photon production rate at $T = 200$ MeV.

A treatment of the bremsstrahlung mechanisms in terms of complete phase space is a bit tedious, and so to first assess their relative importance, a soft-photon approximation is adopted here. The rate is then

$$E_\gamma \frac{dR}{d^3 p_\gamma} = \mathcal{N} T^2 \frac{\lambda}{16\pi^4} \int_{z_{\text{min}}}^{\infty} dz (z^2 T^2, m_a^2, m_b^2) K_1(z) \times \left[ E_\gamma \frac{d\sigma}{d^3 p_\gamma} \frac{R_2(s_2, m_a^2, m_b^2)}{R_2(s, m_a^2, m_b^2)} \right],$$

where $z_{\text{min}}T = E_\gamma + \sqrt{M^2 + E_\gamma^2}$, with $M = \max(m_a + m_b, m_1 + m_2)$, where the full squared amplitude is approximated as $|M|^2 = |M_0|^2 (-J^2)$, where $M_0$ is the elastic scattering amplitude, $J^\mu$ is the hadronic current including the fundamental charge $e$, $K_1$ is the modified Bessel function of order 1, and finally, where the ratio of the two-body phase space for $s_2 = s - 2\sqrt{s E_\gamma}$ and $s$ respectively, is a phase-space correction factor to minimize error at finite $E_\gamma$.

Bremsstrahlung reactions are shown in Fig. 3 for pions, rho mesons, $K$ and $K^*$ mesons. Interactions are once again modeled with an SU(3) chiral Lagrangian with coupling constants fitted by respecting hadronic phenomenology. As expected, the partial rates are very strong at low photon energy. But the striking feature is the strength surviving even for intermediate photon energies.

Two cautionary remarks must however be made. First, Eq. (5) does not include multiple scattering interference effects of Landau-Pomeranchuk-Migdal, which will reduce the rates when the photon energy is less than the inverse of the mean time between strong interactions and second, the soft photon approximation becomes less reliable with increasing photon energy. Still, one does not
FIG. 4: Comparison of conventional hadronic rate to order $e^2 g^2$ ("old rate" as parameterized by Nadeau et al.\textsuperscript{[26]}) with an updated rate including additional radiative decays, strangeness reactions, and bremsstrahlung ("new rate") which goes to order $e^2 g^4$. The old rate includes $\pi\pi \rightarrow \rho\gamma$, $\rho \rightarrow \pi\pi\gamma$, $\pi\pi \rightarrow \gamma\gamma$ and $\omega \rightarrow \pi\gamma$.

expect more than a factor of 2–4 suppression for $E_\gamma \sim 0.5$ GeV.

Finally, the most useful comparison one can make regarding the relative importance of the higher-order contributions is to add up all the partial rates from radiative decays (order $e^2$), Compton processes (order $e^2 g^2$), and from bremsstrahlung (order $e^2 g^4$) and then compare to the conventional hadronic rate from the literature (which includes $\omega \rightarrow \pi^0 \gamma$, $\pi\pi \rightarrow \rho\gamma$, $\rho \rightarrow \pi\pi\gamma$ and $\pi\pi \rightarrow \gamma\gamma$). This comparison is reported in Fig. 4. A factor of 2 increase is seen over most of the intermediate and high energy region up to 3 GeV. Below 1 GeV photon energy, the increase is rather large, approaching two orders of magnitude around 200 MeV photon energy.

One might summarize this study as follows. Photon production rates in hot hadronic matter have been investigated up to next-to-next-to-leading order in the strong coupling. This means contributions to order $e^2 g^4$ have been included, most notably, elastic scattering of mesons with accompanying bremsstrahlung. In addition, strangeness reactions were included and found to be somewhat important. Quantitatively, the new photon rate with strangeness and with NNLO hadronic processes is two orders of magnitude greater at low photon energies and a factor of approximately 2 greater at intermediate and high energies as compared with the standard photon production rate from the literature\textsuperscript{[26].}

Since multiple scattering interferences have not yet been included, the precise increase at low photon ener-

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