Current in normal and superconductors

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Abstract

In this paper the center of mass momentum operator is derived from its generating function. It is emphasized that this operator describes current in normal conductors. A state which is an eigenstate of this operator has a finite Drude weight and is also delocalized. An external potential is a necessary but not sufficient condition for insulation. Whether a model corresponds to an insulator or conductor depends on the ratio of the the external potential to the energy scale of the underlying interacting system. It is also shown that a normal conductor of finite size exhibits flux quantization, and it is only in the thermodynamic limit that the flux becomes continuous. The Hubbard model is analyzed, and it is demonstrated that it cannot exhibit insulating behaviour. Both the large $U$ and small $U$ states are perfect conducting states, the difference is that the former does not exhibit flux quantization whereas the latter does.

1 Introduction

A proper description of current is central to understanding conduction in quantum systems. In a perfect conductor charge carriers accelerate with the applied field. In a real conductor the motion of charge carriers is hindered by collisions among charge carriers and with the nuclei situated at lattice sites. To understand the behavior of normal conductors a description of the motion of the center of mass of all charge carriers (the drift current) is indispensable. In a speculative sense normal conduction was linked to a Berry phase which arises upon moving the center of mass across the periodic unit cell by Moulopoulos and Ashcroft [1]. Recently it was shown that it is possible to express the total current in this manner [2] and
from it it is possible to derive the Drude weight \[3\]. The formalism allows for an easy demonstration of Kohn’s original tenet which connects many-body localization with insulation \[4\].

The standard textbook expression for the current \[5\] consists of a sum over single particle momentum operators averaged over the ground state wavefunction. It turns out \[6\] that this definition hides a mathematical subtlety. The average over the sum of momenta is not equivalent to the sum of the average of one-body momenta. They may be equal in value, but they originate from different generating functions. The former from the generator of translations of the center of mass, the latter from single particle translations. In fact it is also possible to break the sum over momenta into pairs of particles as well \[6\]. In this paper we derive these different current expressions using the appropriate generating functions. We stress that normal conduction is described by the center of mass momentum operator. It is also emphasized, and shown through examples, that many of the consequences of this subtlety have been overlooked until now.

It is shown that a normal conductor which is finite exhibits flux quantization, the magnetic flux becomes continuous in the thermodynamic limit. Applying these ideas to the Hubbard model \[7, 8, 9, 10\] we find that it is always conducting. The Hamiltonian commutes with both the total current and the total position shift operators, thus the eigenstates of the Hubbard Hamiltonian must be eigenstates of the total current. This means that these states give rise to a finite Drude weight \[3\], as well as a diverging localization length \[11, 12\]. The phase transition which occurs at zero interaction strength in one dimension \[13\] separates a perfect conductor with flux quantization \((U = 0)\) and one with continuous flux. The Gutzwiller wavefunction \[7, 8\] provides a qualitatively correct description of the Hubbard model. These results contradict many previous studies \[13, 14, 15\], but the point is that these analyses lacked a proper expression for current corresponding to a normal conductor, and the associated transport coefficient, the Drude weight. Note that some of the formalism used in this paper was published recently \[6\]. There the connections of different current expressions with off-diagonal long-range order were derived. Here the connection between conduction and flux quantization and its consequences are investigated.

Finally the Ginzburg-Landau theory is placed under scrutiny. In the original Ginzburg-Landau theory the quantity \(|\Psi(X)|^2\) is assumed to coincide with the superfluid weight. It is shown that to truly obtain a finite superfluid weight the function \(\Psi(X)\) must be a plane wave.
2 Phenomenological considerations: perfect conductors vs. normal conductors

In this motivational section we will consider conduction in the case of a perfect conductor and a normal conductor from a phenomenological point of view. In textbooks on superconductivity [16, 17, 18, 19] and in relevant chapters in books on solid state physics [20, 21] Faraday’s law is used to derive the London equations, and also equations which describe the phenomenological aspects of the Meissner effect (London penetration depth). Here we point out that starting with the London equations, which are valid for a perfect conductor, one can arrive at Faraday’s law. The connection between the two is made by invoking the Drude model for conduction.

Let our starting point be the London equations,

\[ j = -\frac{n e^2}{mc} A, \]

\[ \nabla \cdot A = 0. \]

where \( n \) denotes the density of superconducting charge carriers. The second equation fixes the gauge (London gauge).

Taking the time derivative of Eq. (1) results in

\[ \frac{dj}{dt} = \frac{n e^2}{m} E, \]

which is the defining equation of a perfect conductor. The simplest way to account for collisions is through the Drude model, in which a particle accelerates until it experiences a collision. The collision zeroes the particle velocity. If the average time between collisions is \( \tau \) Ohm’s law results,

\[ j = \frac{n e^2 \tau}{m} E. \]

Note that in this case \( j = nev_d \), where \( v_d \) indicates the drift velocity, which corresponds to the average velocity of the charge carriers. In a normal conductor the velocity of individual charge carriers can differ by many orders of magnitude from the drift velocity.

Starting again from Eq. (1) one can also derive the characteristic features of the Meissner effect, and obtain the London penetration depth. Here we would like to point out that from Eq. (1) Faraday’s law can be obtained via the application of the Drude model. Let us take the curl and the time derivative of both sides of Eq. (1), resulting in

\[ \nabla \times \frac{dj}{dt} = -\frac{n e^2}{m} \frac{1}{c} \frac{dB}{dt}. \]
Now, assuming that the conductor is not perfect, we can replace the derivative of the current with respect to time with the ratio of the current and the collision time as was done in the Drude model. This results in

$$\nabla \times \frac{j}{\tau} = -\frac{ne^2}{mc} \frac{dB}{dt}. \tag{6}$$

Combining with Ohm’s law we obtain Faraday’s law

$$\nabla \times E = -\frac{1}{c} \frac{dB}{dt}. \tag{7}$$

Recently Hirsch [22, 23, 24, 25] has argued that the Meissner effect and the effects associated with Faraday’s law can be described by the similar physics (same physics with minor modifications). The above reasoning also suggests this.

3 Constructing the current operator

We now construct the current operator, using the appropriate generating function. Our starting point is the Hamiltonian with a vector potential

$$H(A) = \sum_{i=1}^{N} \left( \frac{\hat{p}_i - eA}{2m} \right)^2 + \hat{V}. \tag{8}$$

For simplicity a one-dimensional system is considered here, but the generalization to larger dimensions is trivial. The ground state energy is given by

$$E(A) = \langle \Psi(A)|H(A)|\Psi(A) \rangle. \tag{9}$$

The current is proportional to the derivative of $E(A)$ with respect to $A$, which is, using the Hellman-Feynman theorem,

$$J_{N}(A) = \frac{\partial E(A)}{\partial A} = -\frac{Ne}{mc} A + \frac{e}{mc} \langle \Psi(A)| \left[ \sum_{i=1}^{N} \hat{p}_i \right] |\Psi(A) \rangle. \tag{10}$$

The first term in $J(A)$ is usually referred to as the paramagnetic current, and corresponds to the contribution due to perfect conduction. The second term is known as the diamagnetic current, and accounts for the effect of collisions. Indeed, neglecting the second order term leads to the first London equation (Eq. 1).

The diamagnetic contribution consists of a sum over average momenta

$$J_{D}(A) = \frac{e}{mc} \langle \Psi(A)| \left[ \sum_{i=1}^{N} \hat{p}_i \right] |\Psi(A) \rangle. \tag{11}$$
It turns out that in this case the sum over average momenta or the average of the sum are not necessarily equivalent. In other words one cannot be replaced by the other. This may seem counterintuitive, but one has to consider the following. A many-body wavefunction of identical particles if it is an eigenstate of $p_i$ it is also an eigenstate of $\left[\sum_{i=1}^{N} p_i\right]$, but the converse of this statement is not necessarily the case. And this fact turns out to be the key to interpreting conduction.

The above statement can be made more explicit by considering that momentum operators are intimately linked with generators of translation. The one-body momentum operator is constructed by

$$|\Psi(x + \delta x)\rangle = \exp(i\hat{p}\delta x)|\Psi(x)\rangle. \quad (12)$$

Taylor expansion leads to

$$\hat{p} = -i\frac{\partial}{\partial x}. \quad (13)$$

For comparison one can also consider shifts in the center of mass of a many-body system:

$$|\Psi(x_1 + x_{cm} + \delta x, \ldots, x_N + x_{cm} + \delta x)\rangle = \exp(i\hat{P}_{cm}\delta x)|\Psi(x_1 + x_{cm}, \ldots, x_N + x_{cm})\rangle, \quad (14)$$

which leads to

$$\hat{P}_{cm} = -i\frac{\partial}{\partial x_{cm}}. \quad (15)$$

But the operator $\hat{P}_{cm}$ can also be written

$$\hat{P}_{cm} = \sum_{i=1}^{N} \hat{p}_i. \quad (16)$$

The diamagnetic current corresponds to the average of the momentum of the center of mass of the system, and we have

$$J_N(A) = -\frac{Ne}{mc}A + \frac{e}{mc}\langle \Psi(A)|\hat{P}_{cm}||\Psi(A)\rangle. \quad (17)$$

From the current one can also obtain the Drude weight which is the criterion of dc conduction

$$D_N = \left[\frac{\partial J(A)}{\partial A}\right]_{A=0} = -\frac{Ne}{mc} + \frac{e}{mc}(\partial_A \Psi(A)||\hat{P}_{cm}||\Psi(A)) + \frac{e}{mc}\langle \Psi(A)||\hat{P}_{cm}||\partial_A \Psi(A)\rangle. \quad (18)$$
The derivative in $A$ can be written in terms of the total momentum shift operator, resulting in

$$\lim_{\Delta A \to 0} \frac{|\Psi(A + \Delta A)\rangle - |\Psi(A)\rangle}{\Delta A}.$$  

(19)

Since $A$ is a momentum shift, we can write $|\Psi(A + \Delta A)\rangle$ in terms of the total momentum shift operator,

$$|\Psi(A + \Delta A)\rangle = \exp \left( iN \sum_{i=1}^{N} \hat{x}_i \right) |\Psi(A)\rangle.$$

(20)

Substituting Eqs. (19) and (20) into (18) results in

$$D = -\frac{Ne}{mc} A + \frac{Ne}{mc} \langle \Psi | [\hat{X}_{cm}, \hat{P}_{cm}] |\Psi\rangle.$$  

(21)

From evaluating the commutator it can be verified [3] that $D = 0$, unless $|\Psi\rangle$ is an eigenfunction of $\hat{P}_{cm}$ in which case $D = -\frac{Ne}{mc}$. Thus metallicity corresponds to being an eigenfunction of $\hat{P}_{cm}$. Whether a wavefunction is an eigenfunction of $\hat{P}_{cm}$ can be decided by considering the spread of $\hat{P}_{cm}$ given by

$$\sigma_{\hat{P}_{cm}}^2 = \lim_{\Delta x \to 0} \frac{2}{\Delta x^2} \text{Re} \log \langle \Psi | \exp(i\hat{P}_{cm} \Delta x) |\Psi\rangle.$$  

(22)

If this spread is zero, the wavefunction $|\Psi\rangle$ is an eigenfunction of $\hat{P}_{cm}$ and the system is conducting. Equivalently one can consider the spread in total position [11, 12] given by

$$\sigma_{\hat{X}_{cm}}^2 = \lim_{\Delta x \to 0} \frac{2}{\Delta x^2} \text{Re} \log \langle \Psi | \exp(i\hat{X}_{cm} \Delta x) |\Psi\rangle.$$  

(23)

In this case a system is conducting if $\sigma_{\hat{X}_{cm}}^2$ diverges. Indeed, if $|\Psi\rangle$ is an eigenfunction of $\hat{P}_{cm}$, then $\langle \Psi |$ and $\exp(i\hat{X}_{cm} \Delta k) |\Psi\rangle$ will be orthogonal, since $\exp(i\hat{X}_{cm} \Delta k)$ is the total momentum shift operator. In summary, the quantities $D$, $\sigma_{\hat{P}_{cm}}^2$, and $\sigma_{\hat{X}_{cm}}^2$ contain the same information. The Drude weight $D$ was first derived by Kohn [4] as the criterion to distinguish metals from insulators. Kohn also initiated the idea that many-body localization is a criterion is also a criterion for insulation.

For future reference we also define the $q$-body currents

$$J_q(A) = -\frac{Ne}{mc} A + \frac{Ne}{qmc} \langle \Psi(A) | \sum_{i=1}^{q} \hat{P}_i |\Psi(A)\rangle,$$

(24)

their associated transport coefficients,

$$D_q = -\frac{Ne}{mc} + \frac{Nie}{qmc} \sum_{i=1}^{q} \langle \Psi | [\hat{x}_i, \hat{P}_i] |\Psi\rangle.$$  

(25)
and the corresponding momentum spreads

\[ \sigma^2 = \lim_{\Delta x \to 0} \frac{2}{\Delta x^2} \text{Re} \log \langle \Psi | \exp(i \sum_{i=1}^{q} \hat{p}_i \Delta x) | \Psi \rangle. \]  

We have derived the current associated with the motion of the center of mass and the associated transport susceptibility, the Drude weight. We emphasize that the mathematical subtlety indicated in the beginning of this section has heretofore been, to the best of the knowledge of the author, overlooked, which means that in the theory of conduction an essential element, the appropriate representation of the current has been lacking. The important point is that \( D_q \)'s are distinct. In particular, since in a normal conductor it is the collective motion of the charge carriers which is of interest, normal conductivity corresponds to \( D_N \). Below, it is shown that \( D_q \) can be made to correspond to flux quantization rules depending on the number \( q \).

### 4 General criteria for normal conduction

In general the electronic Hamiltonian consists of kinetic energy, interaction and external potentials. The center of mass momentum operator commutes with the first two. To see this for the interaction, consider that the shift of the center of mass does not alter the interaction potential. Thus, the behavior of the external potential will determine whether a given Hamiltonian is conducting or insulating. Let us write the total Hamiltonian as

\[ \hat{H} = \hat{H}_0 + \hat{V}_{\text{ext}}, \]

where \( \hat{H}_0 \) indicates the kinetic energy plus the interaction potential and \( \hat{V}_{\text{ext}} \) denotes the external potential.

For a second-order metal-insulator transition an approximate quantitative criterion can be obtained from second-order perturbation theory. In this case the spread in current will be given by

\[ \sigma_{P,cm}^2 = \lim_{\Delta x \to 0} \frac{2}{\Delta x^2} \text{Re} \log \left( 1 + \sum_{j \neq 0} \exp(i P_j \Delta x) \frac{|V_{j0}|^2}{(\epsilon_j - \epsilon_0)^2} \right), \]

where \( V_{j0} \) denotes the matrix elements of \( \hat{V}_{\text{ext}} \) in the basis of \( \hat{H}_0 \) and \( \epsilon_j \) are the energy eigenvalues of \( \hat{H}_0 \). While this criterion is very approximate, one sees that
for a system with an external potential to become metallic the term
\[ \sum_{j \neq 0} \exp(iP_j \Delta x) \frac{|V_{j0}|^2}{(\epsilon_j - \epsilon_0)^2} \to 0 \] (29)
must go to zero in the thermodynamic limit. Thus conduction results from the
competition between the energy scale of the interacting system and the external
potential.

5 Off-diagonal long-range order and flux quantization

By expressing the transport coefficients in terms of reduced density matrices it can
be shown that they are sensitive to off-diagonal long-range order [6]. In particular,
\( D_1 \) is sensitive to ODLRO in the first order reduced density matrix, \( D_2 \) in the
second, and so on. Yang [26] has shown that if ODLRO is found in RDM of
order \( q \) then also RDMs of higher orders will exhibit it. Thus we can define the
following criteria to interpret conduction: let \( q_n \) denote the smallest number for
which ODLRO is found in the RDM of order \( q_n \). If \( q_n \) is a number of microscopic
size, then superconduction (or superflow occurs). The only known examples are
\( q_n = 1 \), superflow in bosonic helium, and \( q_n = 2 \), superconductivity with BCS
pairing. If \( q_n \) is thermodynamically large, then normal conduction occurs. It
needs to be emphasized though, that even if \( q_n \) is thermodynamically large, it is
possible to have perfect conduction. A macroscopic cluster of interacting particles
in the absence of an external potential responds to electric fields by accelerating
with the applied field, hence in that sense is a perfect conductor. In fact \( D_{q_n} \)
just counts the number of particles in “mobile units” responsible for particle flow
(conduction in the case of charged system). The number of particles in a mobile
unit \( (q) \) is one in bosonic helium, two in paired systems (BCS superconductors),
and a thermodynamically large number in conductors. We now show that this
picture is paralleled in flux quantization.

We consider an \( N \) particle system and write the expectation value of the current
over some ground state wavefunction \( |\Psi(A)\rangle \) as
\[ J(A) = \langle \Psi(A) \left| \sum_{i=1}^{N} \frac{\hat{p}_i - \vec{z}A}{m} \right| \Psi(A) \rangle. \] (30)
We can also express this current in terms of the reduced density matrix of order \( q \)
associated with the wavefunction \( |\Psi(A)\rangle \) as
\[ J(A) = Tr \left[ \hat{\rho}_q \sum_{i=1}^{q} \frac{\hat{p}_i - \vec{z}A}{m} \right], \] (31)
or in terms of the eigenstates of the reduced density matrix as

\[
J(A) = \frac{N}{q} \sum_{I=1}^{N/q} \left\langle \chi_I(A) \right| \sum_{i=1}^{q} \frac{\hat{p}_i - \frac{e}{c} A}{m} \left| \chi_I(A) \right\rangle.
\] (32)

Suppose that one of the states \( |\chi_I(A)\rangle \) is an eigenstate of the \( q \)-body current. For this state it will hold that

\[
\left\langle \chi_I(A) \right| \sum_{i=1}^{q} \frac{\hat{p}_i - \frac{e}{c} A}{m} \left| \chi_I(A) \right\rangle = \left\langle \chi_I(A) \right| \frac{(\hat{p}_q - \frac{e}{c} A)}{m} \left| \chi_I(A) \right\rangle,
\] (33)

the state \( \chi_I(A) \) will be of the form

\[
\chi_I(A) = \exp(i\theta(\hat{X}_q))
\] (34)

and the flux quantization rule that follows will be

\[
\oint dX_q \cdot A = \frac{nhc}{qe}.
\] (35)

For a superconductor \( q = 2 \) due to electron pairing, and this flux quantization rule is the usual one. For a normal conductor \( q = N \) or at least a thermodynamically large number, meaning that the flux can take on continuous values. One can summarize these results together with the results of Ref. [6] as follows. If for some system the number \( q_n \) denotes the lowest order density matrix in which ODLRO is exhibited, then the flux quantization rule for that system is

\[
\oint dX_q \cdot A = \frac{nhc}{q_n e}.
\] (36)

If \( q_n \) is a thermodynamically large number the system is a normal conductor and the flux is continuous. If the system does not exhibit ODLRO in any of its reduced density matrices, then it is localized and therefore an insulator.

6 Strongly correlated lattice models

In this section we analyze conductivity in lattice models. We will make specific statements about the Hubbard model. For simplicity we consider one dimension. In this case the Hubbard model is given by

\[
H = -t \sum_{i,\sigma} \{ c_{i\sigma}^\dagger c_{i+1\sigma} + c_{i+1\sigma}^\dagger c_{i\sigma} \} + U \sum_i n_{i\uparrow} n_{i\downarrow}.
\] (37)
Let us already remark that this model has no external potential, only a distance dependent pair potential. The external field in this case is usually represented by a phase applied to the hopping parameter $t$ as

$$H(\Phi) = -t \sum_{i \sigma} \{ e^{i \Phi} c_{i \sigma}^\dagger c_{i+1 \sigma} + e^{-i \Phi} c_{i+1 \sigma}^\dagger c_{i \sigma} \} + U \sum_{i} n_{i \uparrow} n_{i \downarrow}. \quad (38)$$

The Drude weight in this case reads as

$$\left[ \frac{\partial^2 E(\Phi)}{\partial \Phi^2} \right]_{\Phi=0} = -\langle \hat{T} \rangle + i \langle \Psi | [\hat{J}, \hat{X}] | \Psi \rangle, \quad (39)$$

where $\hat{J}$ indicates the total current, and is defined as

$$\hat{J} = -i t \sum_{i \sigma} \{ c_{i \sigma}^\dagger c_{i+1 \sigma} - c_{i+1 \sigma}^\dagger c_{i \sigma} \} \quad (40)$$

and $\hat{X}$ denotes the total position defined as

$$\hat{X} = \sum_{i \sigma} i n_{i \sigma}. \quad (41)$$

Again the commutator in Eq. (39) can be shown to be zero, unless the state $|\Psi\rangle$ is an eigenstate of the total current $\hat{J}$. The total current in reciprocal space can be written as

$$\hat{J} = 2 t \sum_{k \sigma} \sin(k) n_{k \sigma}. \quad (42)$$

This operator is closely related to the total position shift operator

$$\hat{\Pi} = \sum_{k \sigma} k n_{k \sigma}, \quad (43)$$

which evidently commutes with the Hubbard Hamiltonian, since shifting all the positions does not change the number of double occupations. This is also proven in Ref. [27]. The ground state of the Hubbard model is an eigenstate of the total position shift with zero eigenvalue, which means the $k$ values associated are symmetrically distributed around the origin. It follows that the ground state is also an eigenstate of $\hat{J}$ with eigenvalue zero. Also, the spread in position (Eq. 23) for this state diverges. This implies that the ground state of the Hubbard model is a delocalized state. To see this consider that the spread in position is given by Eq. (23), which is an overlap between the ground state $|\Psi\rangle$ and the state $\langle \Psi | \exp(i \frac{2\pi}{L} \hat{X})$. The operator $\exp(i \frac{2\pi}{L} \hat{X})$ is the total momentum shift operator. Thus the overlap

$$\langle \Psi | \exp(i \frac{2\pi}{L} \hat{X}) | \Psi \rangle \quad (44)$$
for an eigenstate of the operator $\hat{\Pi}$ has to vanish, meaning that the spread in total position is infinite. $|\Psi\rangle$ is a delocalized state.

Since a delocalized state is conducting, it follows that Hubbard model can not, as previously thought, produce an insulating state even at half-filling. The celebrated Lieb and Wu solution does not correspond to a metal-insulator transition at $U = 0$, but to a transition of a different type. On both sides of the transition the system is metallic. The $t = 0$ state is a Fermi sea, which has all $D_1, \ldots, D_N$ finite and exhibits flux quantization according to $\Phi_B = n\frac{hc}{e}$, whereas the finite $U$ state corresponds to a conducting state with only $D_N$ finite and flux quantization according to $\Phi_B = n\frac{hc}{Ne}$, in other words the flux in the thermodynamic limit is continuous.

Let us emphasize that this conclusion regarding the Hubbard model is a simple consequence of the fact that the ground state is an eigenstate of the total position shift, proven in Ref. [27], and that when this is the case, then the state itself must be delocalized, proven in Ref. [3], and that a delocalized state must be conducting, which is suggested in Ref. [4], and proven in Ref. [28] and also in Ref. [3]. Actually, one can also view it as a consequence of the fact that the Hubbard model only has a kinetic energy and a distance dependent interaction potential, but no external potential.

The reason this has been overlooked is due to the lack of the proper definition of the drift current which is essential in describing normal conduction. In fact, if $D_1$ is used to gauge the Hubbard model, then there is a jump from a finite value at $U = 0$ to zero for finite $U$.

In fact the behaviour of the Hubbard model is qualitatively well-described by the Gutzwiller wavefunction, which is of the form

$$|\Psi(\gamma)\rangle = e^{-\gamma \sum_i n_i n_i} |FS\rangle.$$  \tag{45}$$

The total position shift commutes with the Gutzwiller projector, hence a conducting state with only $D_N$ finite results, and the flux quantization rule is Eq. \eqref{eq:flux} with $q_n = N$. On the other hand $D_p$ for $p \neq N$ are all zero unless $\gamma = 0$, the Fermi sea.

\section{Ginzburg-Landau theory}

Before concluding let us also investigate how the formalism developed in this paper relates to Ginzburg-Landau theory. In particular we will investigate the relation between the function $\Phi_0(X)$ and the superfluid order parameter which we take to be proportional to the second derivative of the relevant free energy. Usually the function $|\Phi_0(X)|^2$ is assumed \cite{29} to correspond to the superfluid density.
We use a one-dimensional theory, but the results are general. The Ginzburg-Landau free-energy functional used to describe superconductors is of the form

\[ F(A) = \int dX - \frac{1}{2} \Phi_A^*(X) \left( i \partial_X - \frac{e}{c} A \right) \Phi_A(X) + \frac{r_0}{2} |\Phi_A(X)|^2 + \frac{\mu_0}{4!} |\Phi_A(X)|^4. \]  

The function \( \Phi_A(X) \) represents the macroscopic wavefunction associated with the superconductor. The function \( \Phi_A(X) \) is found by optimizing\( \delta F(A) / \delta \Phi_A(X) = 0. \)

One can write the function \( \Phi_A(X) \) in terms of the unperturbed function as

\[ \Phi_A(X) = \exp \left( i \frac{e}{c} X \right) \Phi_0(X). \]

We take the transport coefficient (which for a superconductor is the Meissner weight) to be

\[ D \equiv \left[ \frac{\partial F(A)}{\partial A^2} \right]_{A=0}. \]

This transport coefficient can be shown to be

\[ D = \frac{e}{c} \int dX |\Phi_A(X)|^2 - i \int dX \Phi_A^*(X) [\partial_A, \partial_X] \Phi_A(X). \]

To derive this equation one must use the analog of the Hellman-Feynman theorem in this case, which holds due to the Eq. (48). Using Eq. (49) one finds

\[ D = \frac{e}{c} \int dX |\Phi_0(X)|^2 - \Phi_0^*(X) \{X, \partial_X\} \Phi_0(X), \]

which is zero due to the commutator, unless the function \( \Phi_0(X) \) is an eigenfunction of \( \partial_X \) which in this case means a plane-wave. In that case the commutator is zero and we have

\[ D = \frac{e}{c} \int dX |\Phi_0(X)|^2. \]

If the system is conducting, then only the kinetic energy term contributes (the \( |\Phi_0(X)|^2 \) and \( |\Phi_0(X)|^4 \) are irrelevant constants). Note that to have a finite \( D \) must be an eigenstate of \( \partial_X \). Thus the transport coefficient is sensitive to the form of the function \( \Phi_0(X) \) rather than its magnitude.
8 Conclusion

In this paper the center of mass current for a quantum mechanical system was derived from the generator of center of mass translations. It was emphasized that normal conduction can only be described by use of the center of mass momentum operator. It was shown that finite normal conducting systems exhibit flux quantization, and the flux inside the cavity of a conductor becomes continuous in the thermodynamic limit. Subsequently we applied the theory of conduction based on the center of mass momentum operator to the Hubbard model and showed that this model is always insulating. What was interpreted as a metal-insulator transition before is a transition between a flux-quantized to a continuous flux state. We also find that a qualitatively correct description of the Hubbard model results from the Gutzwiller wavefunction, which reproduces these properties. We also refine the Ginzburg-Landau theory, in showing that the form of the function which defines the free energy is sensitive to conduction rather than the magnitude.

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