GOSSIP-BASED INFORMATION SPREADING IN MOBILE NETWORKS

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Abstract

In this paper, we analyze the effect of mobility on information spreading in geometric networks through natural random walks. Specifically, our focus is on epidemic propagation via mobile gossip, a variation from its static counterpart. Our contributions are twofold. Firstly, we propose a new performance metric, mobile conductance, which allows us to separate the details of mobility models from the study of mobile spreading time. Secondly, we utilize geometrical properties to explore this metric for several popular mobility models, and offer insights on the corresponding results. Large scale network simulation is conducted to verify our analysis.

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Index Terms

Conductance, Gossip, Information Spreading, Mobile Networks, Mobility Models.
I. INTRODUCTION

A. Motivation

Mobile networks receive increasing research interest recently; mobile ad hoc networks (MANET) and vehicular ad hoc networks (VANET) are two prominent examples. In many real world networks, an interesting application is to broadcast the information from some source node to the whole network. For wireless ad hoc and sensor networks, a node triggered by the event of interest may want to inform the whole network about the situation as quickly as possible. For social networks, rumors and stories are forwarded by people via different communication media. In these and many other applications, how fast a message can be spread to the whole network is of particular interest as opposed to the general network throughput.

Mobility introduces challenges as well as opportunities. It is known to improve the network throughput as shown in [1]. However, its effect on information spreading is still not very well understood. In mobile networks, will the information spreading speed up or slow down? How may the different mobility patterns affect the information spreading? These problems are of major importance and deserve further study.

B. Related Works

Information spreading in static networks has already been well studied in literature [2], [3]. Gossip algorithm, dated back to [4], is a simple but effective fully distributed information spreading strategy, in which every node randomly selects only one of its neighbors for message exchange during the information spreading process. In contrast to the more aggressive flooding algorithms [5]–[10] (and the references therein), the gossip algorithms can achieve near-optimal performance for a class of static network graphs including random geometric graphs [2]. It is also found that the spreading time is closely related to the geometry of a network, named “conductance” [11], [12], which essentially represents the bottleneck for information exchange within a network.

Mobile networks have drawn significant research interest in recent years. Traditionally, mobility is viewed as a negative feature as it adds additional uncertainty to wireless networks, and incurs more challenges in channel estimation. Recently, mobility has been revisited for its potential to improve network performance. In the seminal works [1], mobility is shown
to significantly increase the sum-throughput of the network under the fully random mobility model; later the study is extended to the one-dimensional mobility model \[13\]. Subsequently, the throughput-delay tradeoff is further investigated in the context of mobile ad-hoc networks \[14\]–\[19\]. Various mobility models have been investigated, including fully random mobility \[1\], velocity constrained mobility \[20\], virtual mobility \[21\], one-dimensional \[13\], \[22\] and two-dimensional \[23\], \[24\] area constrained mobility.

There has been extensive study on both information spreading and mobility of networks, separately. Recently, some interesting analytical results for information spreading in dynamic wireless networks have emerged. The scaling properties of information propagation between a pair of nodes in large mobile wireless networks are explored in \[23\], for the constrained i.i.d. mobility and discrete-time Brownian motion model. A stationary Markovian evolving graph model is introduced in \[25\], and the completion time of a flooding mechanism is analyzed. Some other recent progress along this line can be found in \[26\] and \[27\]. For delay tolerant networks \[5\], both inter-contact time \[6\] \[7\] and message delay \[8\]–\[10\] have been studied. An upper bound of the information propagation speed is derived in \[28\] for the random walk mobility model. Mobile spreading time is also studied for a velocity-limited mobility model in \[20\], and for a random mobility model with a few virtually mobile agents in \[21\]. In \[29\], node mobility is considered for the average consensus problem, where the focus is on the communication complexity (energy efficiency). The effect of mobility on the consensus time of pairwise gossip algorithms is studied in \[30\]. Gossip information dissemination in mobile networks under extreme resource constraints is studied in \[31\], where the stopping criteria is considered explicitly. The applications of gossip algorithms in different distributed tasks, including information dissemination, can be found in \[32\] and \[33\], and the references therein.

C. Summary of Contributions

In this work, we intend to develop a more general framework, especially for studying information spreading under mobile gossip. The main contributions of this paper are summarized below.

1) Based on a “move-and-gossip” information spreading model, we propose a new metric, mobile conductance, which represents the capability of a mobile network to conduct information flows. Mobile conductance is dependent not only on the network geometry structure,
but also on the mobility patterns. Facilitated by the definition of mobile conductance, a general result on the mobile spreading time is derived for a class of mobile networks modeled as a stationary geometric Markovian evolving graph.

2) We evaluate the mobile conductances for various mobility models, including fully random mobility, partially random mobility, velocity constrained mobility, one-dimensional and two-dimensional area constrained mobility, and offer insights on the results. The results are summarized in Table. I. In particular, the study on the fully random mobility model reveals that the potential improvement in information spreading time due to mobility is dramatic: from $\Theta (\sqrt{n})$ to $\Theta (\log n)$. We have also carried out large scale simulations to verify our analysis.

The rest of this paper is organized as follows. System, mobility, and information spreading models are presented in Section II. Mobile conductance is defined in Section III and a general result on the mobile spreading time is derived. In Section IV mobile conductances of several popular mobility models are evaluated, leading to some interesting insights. Finally Section V concludes the work.

| TABLE I | CONDUCTANCES OF DIFFERENT MOBILITY MODELS |
|----------------|------------------------------------------|
| Static Conductance | $\Phi_s = \Theta \left( \sqrt{\frac{\log n}{n}} \right)$ |
| Mobility Model | Mobile Conductance $\Phi_m$ |
| Fully Random | $\Theta (1)$ |
| Partially Random | $\left( \frac{n-k}{n} \right)^2 \Phi_s + \frac{k(2n-k)}{2n^2}$ |
| Velocity Constrained | $\Theta (r), \quad v_{\max} << r$ |
| | $\Theta (r) \text{ or } \Theta (v_{\max}), \quad v_{\max} = \Theta (r)$ |
| | $\Theta (v_{\max}), \quad v_{\max} >> r$ |
| Area Constrained: One-Dim | $\frac{n^2 + n^2}{n^2} \Phi_s + \frac{n v_{\max}}{n^2}$ |
| Area Constrained: Two-Dim | $\Theta (r), \quad r_c << r$ |
| | $\Theta (r) \text{ or } \Theta (r_c), \quad r_c = \Theta (r)$ |
| | $\Theta (r_c), \quad r_c >> r$ |
II. Problem Formulation

A. System Model

We consider an \( n \)-node mobile network on a unit square \( \Omega \), modeled as a time-varying graph \( G_t \triangleq (V, E_t) \) evolving over discrete time steps. The set of nodes are represented as the vertex set \( V \) and identified by the first \( n \) positive integers. One key difference between a mobile network and its static counterpart is that, the locations of nodes change over time according to certain mobility models, and so do the connections between the nodes represented by the edge set \( E_t \). Denote the position of node \( i \) at time \( t \) as \( X_i(t) \), and the transmission range as \( r \). If \(|X_i(t) - X_j(t)| < r\), \((i, j)\) belongs to the edge set \( E_t \) and \( j \) belongs to \( N_i(t) \), the neighboring set of \( i \) at the beginning of time slot \( t \).

It is assumed that the moving process of all the nodes \( \{X_i(t), t \in \mathbb{N}\}, i \in [n] \), are independent and identically distributed (i.i.d.) stationary Markov chains with the transition distribution \( q \), and collectively denoted by \( \{X(t), t \in \mathbb{N}\} \) with the transition distribution \( Q = q^n \). While not necessary, we assume the celebrated random geometric graph (RGG) model \([34]\) for the initial node distributions for concreteness (particularly in Section IV), i.e., \( G_0 = G(n, r) \), where \( r \) is the common transmission range. Under most existing random mobility models \([1]\), \([20]\), \([23]\), \([25]\), \([29]\), nodes will maintain the uniform distribution on the state space \( \Omega \) over the time. We also assume that the network graph remains connected under mobility; for RGG this implies \( v_{\text{max}} + r = \Omega \left( \sqrt{\frac{\log n}{n}} \right) \), where \( v_{\text{max}} \) is the maximal node speed\(^{1}\).

B. Mobility Model

For notation convenience, the unit square is discretized into a grid with a sufficiently high resolution \( \delta \): \( \Omega = \{(i\delta, j\delta) | 0 \leq i, j \leq \lfloor 1/\delta \rfloor \} \). Denote \( Q_{xy} = \Pr(X_i(t+1) = y | X_i(t) = x) \), \( x, y \in \Omega \), \( \forall i \), as a generic element of the transition matrix \( Q \). The following mobility models are considered in this study:

Fully Random Mobility \([1]\): \( X_i(t) \) is uniformly distributed on \( \Omega \) and i.i.d. over time. In this case, \( Q_{xy} = 1/|\Omega|, \forall x, y \in \Omega \). This idealistic model is often adopted to explore the largest possible improvement brought about by mobility.

\(^{1}\)This requirement is already a relaxation as compared to \( r = \Omega \left( \sqrt{\frac{\log n}{n}} \right) \) demanded for static networks. Actually our result only requires \( E_{Q}[N_S(t+1)] > 0 \); see \([5]\)
Partially Random Mobility: \( k \) randomly chosen nodes are mobile, following the fully random mobility model, while the rest \( n - k \) nodes stay static. This is one generalization of the fully random mobility model.

Velocity Constrained Mobility \[20\], \[25\]: This is another generalization of the fully random mobility model, with the node speed bounded as \(|X_i(t + 1) - X_i(t)| \leq v_{\text{max}}\). In this case, \( Q_{xy} = 1/|\mathcal{C}(x)|, \forall y \in \mathcal{C}(x) \), where \( \mathcal{C}(x) = \{y \in \Omega||y - x| \leq v_{\text{max}}\} \); and \( Q_{xy} = 0 \), otherwise.

One-dimensional Area Constrained Mobility \[13\], \[22\]: In this model, the mobile nodes move either vertically (named V-nodes) or horizontally (named H-nodes), reminiscent of trains or automobiles moving on the railways or city streets. It is assumed that both V-nodes and H-nodes are uniformly and randomly distributed on \( \Omega \), and the mobility pattern of each node is “fully random” on the corresponding one-dimensional path. Let \( x \triangleq (x_i, x_j) \in \Omega \) and \( y \triangleq (y_i, y_j) \in \Omega \). For a V-node, \( Q_{xy} = 1/([1/\delta] + 1), \forall y \in \mathcal{V}(x) \), where \( \mathcal{V}(x) = \{y \in \Omega|y_i = x_i\} \); and \( Q_{xy} = 0 \), otherwise. The transition probability for an H-node is similarly defined.

Two-dimensional Area Constrained Mobility \[23\], \[24\]: In this model, each node \( i \) has a unique home point \( i_h \), and moves around the home point within a disk of radius \( r_c \) uniformly and randomly. The home points are fixed, independently and uniformly distributed on \( \Omega \). For node \( i \), \( Q_{xy} = 1/K, \forall x, y \in \Omega_i \), where \( \Omega_i = \{y \in \Omega||y - i_h| \leq r_c\} \) while \( K \) is the number of grid points in a circle of radius \( r_c \); and \( Q_{xy} = 0 \), otherwise. \( r_c \) is also called mobility capacity. This model may simulate the patrol scenarios by police or automatic mobile agents.

C. Spreading Model: Move-and-Gossip

We consider the problem of single-piece information dissemination through a natural randomized gossip algorithm in \[2\]. The extension to the multi-piece dissemination problem is straightforward. In this study, we adopt a move-and-gossip model to facilitate our analysis. Specifically, information spreading in a mobile network is decomposed into two steps in each time slot. First, all nodes move according to some mobility model as discussed above, and then each node gossips with one of its new neighbors. During the gossip step, it is assumed that each node independently contacts one of its neighbors, and during each meaningful contact (where at least one node has the piece of information), the message is successfully delivered in either direction (through the “push” or “pull” operation). Denote \( S(t) \subset V \) as the set of nodes that have the message, at the beginning of time slot \( t \). Initially only the source node \( s \) has the message,
i.e. $S(0) = \{s\}$.

![Diagram of move-and-gossip spreading strategy]

**Fig. 1. Move-and-Gossip Spreading Strategy**

The move-and-gossip strategy is illustrated in Fig. 1 where we adopt the following conventions for our following discussion: $X_i(t)$ changes at the middle of each time slot (after the move step), while $S(t)$ is not updated till the end (after the gossip step). We use $P_{ij}(t+1)$ to denote the probability that node $i$ contacts one of its new neighbors $j \in \mathcal{N}_i(t+1)$ in the gossip step of slot $t$. Without loss of generality, $P_{ij}(t+1)$ is set as $1/|\mathcal{N}_i(t+1)|$ for $j \in \mathcal{N}_i(t+1)$, and 0 otherwise. In the static case, $P_{ij} = \Theta \left(\frac{1}{n^{1/2}}\right) = \Theta \left(\frac{1}{\log n}\right)$ when $j \in \mathcal{N}_i$ [2], [34]. In the mobile case, the stochastic matrix $P(t) = [P_{ij}(t)]_{i,j=1}^n$ changes over time (in terms of connections) governed by the transition matrix $Q$ of the homogeneous Markov chain $\{X_i(t)\}$, $\forall i$, but the values of non-zero $P_{ij}(t)$’s remain on the order of $\Theta \left(\frac{1}{\log n}\right)$.

Our performance metric is the $\varepsilon$-dissemination time, defined as (where * stands for static or mobile):

$$T_* (\varepsilon) \triangleq \sup_{s \in \mathcal{V}} \inf \{t : \Pr (|S(t)| \neq n | S(0) = \{s\}) \leq \varepsilon\}.$$  \hspace{1cm} (1)

**III. MOBILE CONDUCTANCE**

A. Preliminaries on Static Networks

We first recall some relevant results in static networks. According to [2], the static spreading time is

$$T_{static} (\varepsilon) = O \left(\frac{\log n + \log \varepsilon^{-1}}{\Phi_s}\right),$$  \hspace{1cm} (2)
where $\Phi_s$ is the static conductance defined as

$$\Phi_s = \min_{S \subseteq V, |S| \leq n/2} \left( \frac{\sum_{i \in S, j \in S} P_{ij}}{|S|} \right) = \min_{S \subseteq V, |S| \leq n/2} \left( \frac{P(r) N_S}{|S|} \right), \quad (3)$$

where $P_{ij} = P(r) = \Theta \left( \frac{1}{\log n} \right)$ when $j \in N_i$, and $N_S$ is the number of connecting edges between set $S$ and its complement $\bar{S}$. Note that $N_S$ is a constant for a given set $S$ in the static case, but becomes a random variable in the mobile case when the nodes in $S(t)$ and $\bar{S}(t)$ move at each time step.

It has been shown that the conductance for a static random geometric graph scales as $\Theta \left( r \sqrt{\log n/n} \right)$, i.e. $\Phi_s = \Theta \left( \sqrt{\log n/n} \right)$, and the static spreading time scales as

$$T_{\text{static}} = O \left( \frac{\log n}{\sqrt{\log n/n}} \right) \approx O \left( \sqrt{n} \right).$$

It is worth mentioning that the above result is actually tight in the order sense. The network radius is on the order of $\Theta \left( 1 \right)$, and the distance of one-hop transmission is $\Theta \left( \sqrt{\log n/n} \right)$. Thus, the minimal number of hops is on the order of $\Theta \left( \sqrt{n \log n/n} \right) \approx \Theta \left( \sqrt{n} \right)$. This indicates that the spreading time in the static network scales as $\Theta \left( \sqrt{n} \right)$.

**B. Mobile Conductance and Mobile Spreading Time**

Conductance essentially determines the network bottleneck in information spreading. Intuitively, node movement introduces dynamics into the network structure, thus can facilitate the information flows. In this work we define a new metric, mobile conductance, to measure and quantify such improvement.

Given an arbitrary node set $S'(t)$ at the beginning of time slot $t$ (cf. Fig. 11), we denote the number of contact pairs between $S'(t)$ and $\bar{S}'(t)$ after the move as $N_{S'}(t + 1) = \sum_{i \in S'(t), j \in \bar{S}'(t)} I_{ij}(t + 1)$, where $I_{ij}(t + 1)$ is the indicator function for the event that node $i \in S'(t)$ and $j \in \bar{S}'(t)$ are neighbors after the move and before the gossip step in slot $t$, i.e., $j \in N_i(t + 1)$. As we mentioned, this quantity is a random variable because of node motions. Nonetheless, under the framework of stationary Markovian evolving graphs considered in this work, its expected value is well defined with respect to the transition matrix $Q$. This motivates us to define the counterpart of static conductance as follows.
**Definition:** The mobile conductance of a stationary Markovian evolving graph with transition distribution \(Q\) is defined as:

\[
\Phi_m(Q) \triangleq \min_{|S'(t)| \leq n/2} \left\{ \mathbb{E}_Q \left( \sum_{i \in S'(t), j \in S'(t)} \frac{P_{ij}(t+1)}{|S'(t)|} \right) \right\} \quad (4)
\]

\[
\min_{|S'(t)| \leq n/2} \left\{ \frac{P(n,r)}{|S'(t)|} \mathbb{E}_Q [N_{S'}(t+1)] \right\}. \quad (5)
\]

**Remarks:**

1) Some explanations for this concept are in order. Similar to its static counterpart, we examine the cut-volume ratio for an arbitrary node set \(S'(t)\) at the beginning of time slot \(t\). Different from the static case, due to the node motion \((X_i(t) \rightarrow X_i(t+1))\) in Fig. 1, the cut structure (and the corresponding contact probabilities \(\{P_{ij}(t)\}\)) changes. Thanks to the stationary Markovian assumption, its expected value (conditioned on \(S'(t)\)) is well defined with respect to the transition distribution \(Q\). Minimization over the choice of \(S'(t)\) essentially determines the bottleneck of information flow in the mobile setting.

2) For a RGG \(G(n,r)\), the stochastic matrix \(P(t) = [P_{ij}(t)]_{i,j=1}^n\) changes over time (in terms of connections) governed by the transition distribution \(Q\) of the stationary Markovian moving process, but the values of non-zero \(P_{ij}(t+1)\)'s remain on the order of \(P(n,r) = \Theta\left(\frac{1}{n \pi r^2}\right)\) given that nodes are (almost) uniformly distributed. This allows us to focus on evaluating the expected connecting edges (contact pairs) between \(S'(t)\) and \(S'(t)\) after the move: \(N_{S'}(t+1) = \sum_{i \in S'(t), j \in S'(t)} I_{ij}(t+1)\). Therefore for network graphs where nodes keep (almost) uniform distribution over the time, mobile conductance admits a simpler expression (5).

3) This definition may naturally be extended to the counterpart of \(k\)-conductance in [2], with the set size constraint of \(n/2\) in (4) replaced by \(k\), to facilitate the study of multi-piece information spreading in mobile networks.

Based on the above definition, we have the mobile counterpart of (2) as shown below.

**Theorem 1:** For a mobile network with mobile conductance \(\Phi_m(Q)\), the mobile spreading time scales as

\[
T_{\text{mobile}}(\varepsilon, Q) = O\left(\frac{\log n + \log \varepsilon^{-1}}{\Phi_m(Q)}\right). \quad (6)
\]

**Proof:** We follow the proof of (2) in [2], with suitable modifications to account for the difference between static and mobile networks. Starting with \(|S(0)| = 1\), the message set \(S(t)\)
monotonically grows through the information spreading process, till the time $|S(t)| = n$ which we want to determine. The main idea is to find a good lower bound on the expected increment $|S(t+1)| - |S(t)|$ at each slot. It turns out that such a lower bound is well determined by the conductance of the network. Since the conductance is defined with respect to sets of size no larger than $n/2$, a two-phase strategy is adopted, where the first phase stops at $|S(t)| \leq n/2$. In the first phase, only the “push” operation is considered (thus the upper bound on the spreading time is safe); while in the second phase, the emphasis is on the “pull” aspect of the nodes in $S(t)$ (whose size is no larger than $n/2$). Since the two phases are symmetric, we will only focus on the first one.

In the first phase, for each node $j \in S(t)$, define a random variable $\Delta_j(t)$. If at least one node with the message moves to the $j$’s neighboring area in slot $t$ and “pushes” the message to $j$ in the gossip step, one new member is added to the message set. We let $\Delta_j(t+1) = 1$ in this case, and 0 otherwise. In the following, we will evaluate the expected increase $|S(t+1)| - |S(t)|$ conditioned on $S(t)$. The key difference between the static and mobile case is that, there is an additional move step in each slot; therefore, the expectation is evaluated with respect to both the moving and gossiping process. This is where our newly defined metric, mobile conductance, enters the scene and takes place of the static conductance. Specifically, due to the independent actions of nodes in $S(t)$ after the move, we have

$$E[\Delta_j(t+1) | S(t)] = E_Q \left[ 1 - \prod_{i \in S(t)} (1 - P_{ij}(t+1)) \right]$$

$$\geq E_Q \left[ 1 - \prod_{i \in S(t)} \exp(-P_{ij}(t+1)) \right]$$

$$\geq \frac{P(r)}{2} E_Q \left[ \sum_{i \in S(t)} I_{ij}(t+1) \right],$$

where the first and the second inequalities are due to the facts of $1 - x < \exp(-x)$ for $x \geq 0$ and $1 - \exp(-x) \geq \frac{x}{2}$ for $0 \leq x \leq 1$, respectively. Then

$$E[|S(t+1)| - |S(t)| | S(t)] = \sum_{j \in S(t)} E[\Delta_j(t+1) | S(t)]$$

$$\geq \frac{P(r)}{2} E_Q \left[ \sum_{i \in S(t), j \in S(t)} I_{ij}(t+1) \right]$$
\[ \frac{|S(t)|}{2} \frac{P(r)}{|S(t)|} E_Q [N_S (t + 1)] \leq \frac{|S(t)|}{2} \min_{S'(t) \subseteq V} \left\{ \frac{P(r)}{|S'(t)|} E_Q [N_{S'(t)}^+ (t + 1)] \right\} \]

\[ \frac{|S(t)|}{2} \Phi_m (Q). \quad (7) \]

The form of (7) is consistent with the counterpart in static networks [2], [3]. Therefore, we can follow the same lines of [2] in the rest part of the proof.

\[ \text{IV. APPLICATION} \]

The general definition of mobile conductance allows us to separate the details of mobility models from the study of mobile spreading time. In this section, we will evaluate the mobile conductances of several popular mobility models. The main efforts lie in finding the bottleneck segmentation, and then determining the expected number of contact pairs.

A. Evaluation of Several Mobility Models

1) Fully Random Mobility:

Theorem 2: In fully random mobile networks, the mobile conductance scales as \( \Theta (1) \), and the corresponding mobile spreading time scales as \( O (\log n) \).

Proof: Since this mobility model is memoryless, for an arbitrary \( S'(t) \), the nodes in both \( S'(t) \) and \( \overline{S'}(t) \) are uniformly distributed after the move, with density \( |S'(t)| \) and \( |\overline{S'}(t)| \) respectively. For each node in \( S'(t) \), the size of its neighborhood area is \( \pi r^2 \), therefore, the expected number of contact pairs

\[ E_Q [N_{S'} (t + 1)] = |S'(t)| \left| \overline{S'}(t) \right| \pi r^2. \quad (8) \]

Noting that

\[ \frac{P(r)}{|S'(t)|} E_Q [N_{S'} (t + 1)] = \Theta (1), \]

regardless the choice of \( S'(t) \) (with size no larger than \( n/2 \)), we have \( \Phi_m = \Theta(1) \). There is no bottleneck segmentation in this mobility model.

\[^2\text{In the following calculation, the resolution parameter } \delta \text{ in Section II-B goes to 0.}\]
Remark 1: In the gossip algorithms, only the nodes with the message can contribute to the increment of $|S(t)|$. Consider the ideal case that each node with the message contacts a node without message in each step, which represents the fastest possible information spreading. We have the following straightforward arguments:

$$
|S(t+1)| - |S(t)| \leq |S(t)|
$$

$$
\Rightarrow |S(t+1)| \leq 2 |S(t)|
$$

$$
\Rightarrow |S(t)| \leq 2^t = O\left(\epsilon^t\right).
$$

When $|S(T)|$ reaches $(1 - \epsilon)n$, the message has largely been spread to the whole network. Therefore, $T_{mobile}(\epsilon) = \Omega\left(\log n\right)$ for arbitrary constant $\epsilon$, and the optimal performance in information spreading is achieved in the fully random model.

Remark 2: While this model is not practical, it reveals that the potential improvement on information spreading time due to mobility is dramatic: from $\Theta\left(\sqrt{n}\right)$ to $\Theta\left(\log n\right)$.

2) Partially Random Mobility:

Theorem 3: For the partially random mobility model, where $k$ out of $n$ nodes are fully mobile, and the rest $n-k$ nodes stay static, the mobile conductance $\Phi_m = \left(\frac{n-k}{n}\right)^2 \Phi_s + \frac{k(2n-k)}{2n^2}$.

Proof: For each node that already has the message, say $i$, among all its neighbors, there are on average $(n-k)\pi r^2$ static nodes and $k\pi r^2$ mobile nodes. We denote the set of $k$ mobile (dynamic) nodes at time $t$ as $D(t)$, the set of $n-k$ static nodes at time $t$ as $\overline{D(t)}$ and calculate the number of contacted pairs separately as follows.

$$
E_Q \left[N_{S'}(t+1)\right] = E_Q \left[\sum_{i \in S'(t) \cap D(t), j \in S'(t) \cap \overline{D(t)}} I_{ij}(t+1)\right. 
$$

$$
+ \sum_{i \in S'(t) \cap \overline{D(t)}, j \in S'(t) \cap \overline{D(t)}} I_{ij}(t+1) 
$$

$$
+ \sum_{i \in S'(t) \cap D(t), j \in S'(t) \cap \overline{D(t)}} I_{ij}(t+1) 
$$

$$
+ \sum_{i \in S'(t) \cap \overline{D(t)}, j \in S'(t) \cap \overline{D(t)}} I_{ij}(t+1) \right],
$$

(9)

where the former two terms are the number of contact pairs within static nodes and mobile nodes, respectively, while the latter two terms are those between static and mobile nodes.

The links within the static nodes remain unchanged after the move, therefore $I_{ij}(t+1) = I_{ij}$ for $i \in S'(t) \cap \overline{D(t)}, j \in S'(t) \cap \overline{D(t)}$. Since the $k$ mobile nodes are fully random, the links
involving the mobile nodes (the last three terms) can be estimated similarly as in the fully random model. Putting together (with some reorganization), we have

\[
E_Q[N_{S'}(t + 1)] = E_Q \left[ \sum_{i \in S'(t) \cap D(t)} \sum_{j \in S'(t) \cap D(t)} I_{ij} \right] \\
+ \left( \frac{n-k}{n} \right) |S'(t)| \left( n \pi r^2 \frac{|S'(t)|}{n} \right) \\
+ \left( \frac{k}{n} \right) |S'(t)| \left( n \pi r^2 \frac{|S'(t)|}{n} \right),
\]

(10)

According to the definition of mobile conductance,

\[
\Phi_m = \min_{S'(t) \subseteq V, |S'(t)| \leq n/2} \left\{ \frac{P(r)}{|S'(t)|} \sum_{i \in S'(t) \cap D(t), j \in S'(t) \cap D(t)} I_{ij} \right\} \\
+ \min_{S'(t) \subseteq V, |S'(t)| \leq n/2} \left\{ \frac{2n-k}{n} \frac{|S'(t)|}{n} + \frac{k}{n} \frac{|S'(t)|}{n} \right\} \\
= \left( \frac{n-k}{n} \right)^2 \Phi_s + \frac{k(2n-k)}{2n^2}.
\]

(11)

Note that the two minima are achieved simultaneously when \(|S'(t)| = |S'(t)| = \frac{n}{2}\).

Remarks: Since \(\Phi_s = \Theta\left( \sqrt{\log n/n} \right)\) and \(\frac{k}{2n} < \frac{k(2n-k)}{2n^2} < \frac{k}{n}\), the number of mobile nodes needs to achieve \(\omega\left( \sqrt{n \log n} \right)\) in order to bring significant benefit over the static one. Partially mobility model is a mixture of the static network and fully random mobile network. It can be seen that as \(k\) grows, the mobile conductance increases: as \(k \to \Theta(n)\), \(\Phi_m \to \Theta(1)\).

3) Velocity Constrained Mobility:

Theorem 4: For the mobility model with velocity constraint \(v_{max}\), when \(v_{max} = w(r)\), the mobile conductance scales as \(\Theta(v_{max})\), and when \(v_{max} = o(r)\), the mobile conductance scales as \(\Theta(r)\). If \(v_{max} = \Theta(r)\), then mobile conductance scales both as \(\Theta(v_{max})\) and \(\Theta(r)\).

Proof: Given arbitrary \(S'(t)\) satisfying \(|S'(t)| = n_0 < n/2\), it is necessary to minimize \(E_Q[N_{S'}(t + 1)]\) in order to achieve the minimum of mobile conductance defined in (4).

The expected number of contact pairs after the move can be represented as

\[
E_Q[N_{S'}(t + 1)]
\]
The first term equals to $n_0 n \pi r^2$ and can be treated as a constant. Therefore minimizing $E_Q[N_{S'}(t + 1)]$ is equivalent to maximizing the second term. To be specific, the second term $E_Q[I_{ij}(t + 1)] > 0$ only if $i$ and $j$ can possibly move to positions within a distance of $r$, i.e., $|X_i(t) - X_j(t)| < 2v_{\text{max}} + r$, and the maximum is reached when the number of such node pairs in $S'(t)$ is maximized. Therefore, the nodes in $S'(t)$ should be placed as close as possible, until forming a continuous block. Then there will be a border between $S'(t)$ and $\overline{S'(t)}$, and the nodes at least $2v_{\text{max}} + r$ away from the border cannot have meaningful contact after the move.

According to Lemma 7 (See Appendix A), the bottleneck segmentation between $S'(t)$ and $\overline{S'(t)}$ is the straight line bisection and the density of nodes before and after move is illustrated in Fig. 2. The darkness of the color represents the density of nodes bearing the message (i.e. in $S'(t)$). We can see that before the move, the nodes in $S'(t)$ and $\overline{S'(t)}$ are strictly separated by a straight line border. After the move, with some nodes in both $S'(t)$ and $\overline{S'(t)}$ crossing the border to enter the other half, a mixture strip as wide as $2 \times v_{\text{max}}$ emerges in the middle of the graph.

We take the center of the graph as the origin. Denote $\rho_{S'(t)}(l)$ and $\rho_{\overline{S'(t)}}(l)$ as the density of nodes with and without message before moving, and $\rho'_{S'(t)}(l)$ and $\rho'_{\overline{S'(t)}}(l)$ as the density of nodes with and without message after moving, with $l$ the horizontal coordinate. As shown in the upper subfigure of Fig. 2 at time $t$, the nodes in the circle of radius $v_{\text{max}}$ have equal probabilities to move to the center point at time slot $t + 1$. Therefore, $\rho'_{S'(t)}(l)$ is given by the proportion of dark area in the circle over the total area of the circle (thus is uniform over the vertical line $x = l$). $\rho'_{\overline{S'(t)}}(l)$ can be obtained similarly.

Therefore, for $-v_{\text{max}} < l < v_{\text{max}}$:

$$\rho'_{S'}(l) = \frac{\text{sizeof (dark area in circle)}}{\pi v^2_{\text{max}}} \times n,$$

$$\rho'_{\overline{S'}}(l) = \frac{\text{sizeof (white area in circle)}}{\pi v^2_{\text{max}}} \times n.$$
After some derivation, we have

$$\rho'_{S'}(l) \frac{n}{n} = \begin{cases} 
1, & l < -v_{\text{max}}, \\
\arccos \left( \frac{l}{v_{\text{max}}} \right), & -v_{\text{max}} < l < v_{\text{max}}, \\
-\frac{l}{v_{\text{max}}} \sin \left( \arccos \frac{l}{v_{\text{max}}} \right), & l > v_{\text{max}}, \\
0, & l > v_{\text{max}},
\end{cases}$$

and

$$\rho'_{S'}(l) \frac{n}{n} = 1 - \rho'_{S'}(l) \frac{n}{n}.$$

Fig. 2. Velocity Constrained Mobility
Fig. 3. Calculating the Number of Contact Pairs in Velocity Constrained Mobility

The contact region with the above bottleneck segmentation is the $2 \times (v_{\text{max}} + r)$ wide vertical strip in the center. All nodes outside this region will not contribute to $N_{S'}(t + 1)$.

The number of contact pairs after the move can be calculated according to Fig. 3. The center of the circle with radius $r$ is $x$ away from the middle line. For node $i$ located at the center, the number of nodes that it can push message to is equal to the number of nodes without message in the circle. Since the density of nodes without message at positions $l$ away from the middle line is $\rho'_{S'}(l)$, the number of nodes that $i$ can ‘push’ information to is

$$\int_{x - r}^{x + r} \rho'_{S'}(l) 2\sqrt{r^2 - (l - x)^2} \, dl.$$  

Taking all nodes with message in the contact region into consideration, the expected number of contact pairs after the move is

$$E_Q[N_{S'}(t + 1)] = \int_{-v_{\text{max}} - r}^{v_{\text{max}} + r} \int_{x - r}^{x + r} \rho'_{S'}(l) 2\sqrt{r^2 - (l - x)^2} \, dl \, dx.$$  

(12)

Since $S'(t)$ and $\overline{S'}(t)$ here is the bottleneck segmentation that minimize the conductance, the mobile conductance is $\Phi_m(Q) = \frac{2}{n \pi r^2} E_Q[N_{S'}(t + 1)]$. According to the calculation in Appendix B we can obtain the results in Theorem 4.

Remarks: Theorem 4 indicates that, when $v_{\text{max}} = O(r)$, $\Phi_m = \Theta(r)$, and the spreading time scales as $O(\log n/r)$, which degrades to the static case; when $v_{\text{max}} = \omega(r)$, $\Phi_m = \Theta(v_{\text{max}})$, and the spreading time scales as $O(\log n/v_{\text{max}})$, which improves over the static case and approaches...
the optimum when \( v_{\text{max}} \) approaches \( O(1) \). These observations are further verified through the simulation results below.

4) One-dimensional Mobility:

**Theorem 5:** For the one-dimensional area constrained mobility model, where among the \( n \) nodes, \( n_V \) nodes only move vertically and \( n_H \) nodes only move horizontally, The mobile conductance \( \Phi_m = \frac{n_V^2 + n_H^2}{n^2} \Phi_s + \frac{n_V n_H}{n^2} \).

**Proof:** Denote the subset of V-nodes as \( S_V \), and the subset of H-nodes as \( S_H \). Similar to the partially random mobility case, the calculation of the expected number of contact pairs is decomposed into four groups as follows.

\[
E_Q \left[ N_{S'} (t + 1) \right] = \sum_{i \in S_V \cap S'(t), j \in S_V \cap S'(t)} I_{ij} (t + 1) + \sum_{i \in S_H \cap S'(t), j \in S_H \cap S'(t)} I_{ij} (t + 1) + \sum_{i \in S_V \cap S'(t), j \in S_H \cap S'(t)} I_{ij} (t + 1) + \sum_{i \in S_H \cap S'(t), j \in S_V \cap S'(t)} I_{ij} (t + 1).
\]

Consider the first term, the expected number of contact pairs within V-nodes. Because all nodes in this case follow a one-dimensional “fully random” mobility model on their corresponding vertical paths, this number remains unchanged after the move. Therefore, the bottleneck segmentation is the same as in the static case, i.e. letting all V-nodes on the left half belong to \( S' (t) \) and those on the right half belong to \( S'(t) \). However, the density of the V-nodes on both halves is \( n_V \), instead of \( n \). With respect to this bottleneck segmentation, the first term of (13), translated into (4), gives \( \left( \frac{n_V}{n} \right)^2 \Phi_s \). Analogously, the bottleneck segmentation for the second term is formed by letting all H-nodes on the upper half belong to \( S' (t) \) and those on the bottom half belong to \( S'(t) \), which contributes \( \left( \frac{n_H}{n} \right)^2 \Phi_s \) to the mobile conductance. These two bottleneck segmentations can be combined.

Now we move on to the latter two terms of (13). One key observation is that the contact probability between one V-node and one H-node is independent of the positions of their paths in the unit square. To see this, let us check Fig. 4 for any \( i \in S_H \cap S'(t) \) located \( x \) away from the vertical path of \( j \in S_V \cap S'(t) \), the probability that \((i, j)\) is a contact pair is the proportion of the
Fig. 4. Contact Probability of V-node and H-node in One-dimensional Area Constrained Mobility

chord length $|AB|$ over the unit side length. Taking the integral over all $i$’s possible positions on the horizontal path, their contact probability $p_{H-V}$ is

$$p_{H-V} = \int_{-r}^{r} 2\sqrt{r^2 - x^2} dx = \pi r^2.$$  

Similarly, the contact probability between any $i \in S_V \cap S'(t)$ and $j \in S_H \cap \overline{S'}(t)$, $p_{V-H}$ is also $\pi r^2$. Thus, the latter two terms can be evaluated as $p_{V-H} |S_V \cap S'(t)| |S_H \cap \overline{S'}(t)|$ and $p_{H-V} |S_H \cap S'(t)| |S_V \cap \overline{S'}(t)|$, respectively, which are both independent of the segmentation before the move.

To sum up, the mobile conductance for the one-dimensional mobility model is:

$$\Phi_m = \left( \frac{n_V}{n} \right)^2 \Phi_s + \left( \frac{n_H}{n} \right)^2 \Phi_s + \min_{\substack{S'(t) \subset V \\ |S'(t)| < n/2}} \left\{ \frac{P(r)}{|S'(t)|} \left( \frac{p_{H-V} |S'(t)| \frac{n_H}{n}}{p_{V-H} |S'(t)| \frac{n_V}{n}} \right) \right\}$$

$$= \frac{n_V^2 + n_H^2}{n^2} \Phi_s + \min_{\substack{S'(t) \subset V \\ |S'(t)| < n/2}} \left\{ \frac{2n_V n_H |S'(t)|}{n^3} \right\}$$
\[ n^2 V + n^2 H + \frac{n^2 V n_H}{n^2}. \]  \hspace{1cm} (14)

**Remarks:** We can see that, when all nodes move in one direction, the mobile conductance is the same as the static case. On the contrary, when half (or a constant proportion) of the nodes are V-nodes and the other half are H-nodes, the mobile conductance achieves its maximum of \( \Theta(1) \), the same order as in the fully random mobility model. The implication is that multidirectional movement spreads information faster than unidirectional movement.

5) **Two-dimensional Mobility**:

**Theorem 6:** For the two-dimensional area-constrained mobility model with mobility capacity \( r_c \), the mobile conductance scales as \( \Theta(\max(r_c, r)) \).

**Proof:** Denote by \( H_S \triangleq \{i_h\}, i \in S'(t) \) the set of home points for \( S'(t) \), and \( H_\overline{S} \triangleq \{i_h\}, i \in S'(t) \) the set of home points for \( \overline{S}'(t) \). Let \( X_{ih} \) and \( X_{jh} \) denote the positions of home points \( i_h \) and \( j_h \), then \( i \) and \( j \) can possibly move to positions within a distance of \( r \) only if their home points are within a distance of \( 2r_c + r \), i.e., \( E_Q[I_{ij}(t + 1)] > 0 \) only if \( |X_{ih} - X_{jh}| < 2r_c + r \). This is similar to the velocity constrained mobility model, except that the node’s position before the move \( X_i(t) \) is replaced by the position of its home point \( X_{ih} \), and \( v_{\text{max}} \) replaced by \( r_c \).

We now show that the two-dimensional area-constrained mobile conductance can be obtained similarly to the velocity constrained mobile conductance.

1) Here the node positions of \( S'(t) \) and \( \overline{S}'(t) \) after the move are not conditioned on their positions before the move, but determined by the positions of their home points. When calculating the expected number of contact pairs, \( H_S \) and \( H_\overline{S} \) play the same roles as \( S'(t) \) and \( \overline{S}'(t) \) before the move, respectively.

2) The mobility capacity \( r_c \) has the same effect on information spreading as the maximal velocity \( v_{\text{max}} \) in the velocity constrained model, both of which set a limit on the nodes’ moving ability.

Instead of finding the bottleneck segmentation between \( S'(t) \) and \( \overline{S}'(t) \) before the move as in the velocity constrained model, we need to find the bottleneck segmentation between the home points: \( H_S \) and \( H_\overline{S} \). Since the home points also form a random geometric graph, it can be shown that the bottleneck segmentation is formed by dividing the home points into two halves using a
straight vertical line bisecting the unit square, as illustrated in Fig. 5.

Therefore, we may follow the same line of evaluating the velocity constrained mobile conductance to obtain the two-dimensional area-constrained mobile conductance. The only difference is that $v_{\text{max}}$ in (15) is replaced with the mobility capacity $r_c$, which leads to the final results in Theorem 6.

Remarks: When the mobility capacity is much greater than the transmission radius, the mobile conductance is dominated by the mobility capacity, i.e. $\Phi_m = \Theta (r_c)$. When the mobility capacity is much smaller than the transmission radius, the mobile conductance is dominated by the transmission radius, i.e. $\Phi_m = \Theta (r)$, as in static networks. The similarity between this model and the velocity constrained mobility model is worth noting.

B. Simulation Results

We have conducted large-scale simulations to verify the correctness and accuracy of the derived theoretical results. In our simulation, $n$ nodes are randomly deployed on a unit square and move
according to certain mobility models, as described in Section II. The transmission radius $r(n)$ is set as $\sqrt{\frac{C_0 \log n}{n}}$ with $C_0 = \frac{8}{\pi}$ [35]. The spreading time is measured by the number of time slots. For each curve, we simulate one thousand Monte-Carlo rounds and present the average.

The spreading time results for static networks and fully random mobile networks are shown in Fig. 6 to Fig. 9 as the upper and lower bounds. We observe that the spreading time in mobile networks is significantly reduced, and as network size $n$ grows, the static spreading time increases much faster than the mobile counterpart. The bottommost curve (fully random mobility) grows in a trend of $\log n$ (note that the x-axis is on the log-scale), which confirms Theorem 2.

Fig. 6 further confirms our remarks on Theorem 3. When the proportion of mobile nodes is a constant (0.1), the corresponding curve exhibits a slope almost identical to that for the fully random model. We also observe that $k = \Theta \left( \sqrt{n \log n} \right)$ is a breaking point, below which ($k = \Theta \left( \sqrt{\log n} \right)$) the performance degrades to the static case.

Fig. 7 confirms our remarks on Theorem 4. When $v_{\text{max}} = 0.1$, the corresponding curve exhibits a slope almost identical to that for the fully random model. We also observe that $v_{\text{max}} = \Theta (r)$ is a breaking point: velocity that is lower ($v_{\text{max}} = o(r) = \Theta \left( \sqrt{\frac{1}{n}} \right)$) leads to a performance similar to the static case.

The spreading time results for the one-dimensional area constrained mobility model is shown in Fig. 8, which exhibit slopes almost identical to that for the fully random model. It is also shown that when half of the nodes are V-nodes and the other half are H-nodes, the best performance is achieved.

Fig. 9 confirms our remarks on Theorem 6. When $r_c = 0.1$, the corresponding curve exhibits a slope almost identical to that for the fully random model. We also observe that $r_c = \Theta (r)$ is a breaking point, a mobility capacity ($r_c = \Theta \left( \sqrt{\frac{1}{n}} \right)$) below which leads to a performance similar to the static case. Also note the similarity between this figure and Fig. 7.

V. CONCLUSION

In this paper, we analyze information spreading in mobile networks, based on the proposed move-and-gossip information spreading model. We have derived the spreading time of single message in mobile networks by gossip algorithms with respect to the newly defined metric mobile conductance, and shown that mobility can significantly speed up information spreading. We have considered five types of mobility models (fully random mobility, partially random...
mobility, velocity constrained mobility, one-dimensional and two-dimensional area constrained mobility), and utilized geometrical properties to analyze the effects of various mobility patterns on mobile conductance and information spreading. Large-scale simulation results have been provided to support our theoretical analysis. Our proposed metric and methodologies may be extended to other network/mobility models and applications.

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Fig. 7. Average Spreading Time under the Velocity-Constrained Mobility Model

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APPENDIX A

BOTTLENECK SEGMENTATION FOR VELOCITY CONSTRAINED MOBILITY MODEL

Lemma 1: The minimum in mobile conductance under velocity constrained mobility is achieved through the bottleneck segmentation using a vertical straight line bisecting the unit square.

Proof: Nodes at most \(2v_{\text{max}} + r\) away from the border are able to move and contact the nodes on the other side of the border. Denote by \(B(t)\) the length of border and partition the area near the border into bins of area (approximately) \(S_b = (2v_{\text{max}} + r)^2\) as in Fig. 10. The nodes in a bin can contact nodes in three bins on the opposite side. For example, the nodes in \(S_0\) can reach \(S_1, S_2,\) and \(S_3\) after the move. Given \(v_{\text{max}}, r\) and \(n\), the number of possible contact pairs for \(S_0\) is on the order of \(\Theta(n^2 S_b^2)\). The total number of contact pairs after the move is

\[
N_{S'}(t+1) = \frac{B(t)}{2v_{\text{max}} + r} \Theta(n^2 S_b^2).
\]

Therefore, the number of contact pairs after the move is proportional to the length of the border before the move, i.e. \(N_{S'}(t+1) \propto B(t)\), and the mobile conductance is \(\Phi_m \propto \frac{B(t)}{|S'(t)|}\).

Following the same argument in static conductance [11], the ratio of \(\frac{B(t)}{|S(t)|}\) is minimized by a vertical straight line bisecting the unit square. Therefore, the mobile conductance under velocity constrained mobility is also minimized through this bottleneck segmentation.

APPENDIX B

EVALUATION OF VELOCITY CONSTRAINED MOBILE CONDUCTANCE

The accurate evaluation in [12] over the circle is rather involved, therefore we loosen the requirement by only calculating over the small dashed square in the circle, as illustrated in Fig. 10.
Fig. 3 Specifically, we replace (12) with

\[ E_Q [N_{S'} (t + 1)] \]

\[ \cong \int_{-v_{\text{max}} - r}^{v_{\text{max}} + r} \rho_{S'} (x) \int_{x - \frac{r}{\sqrt{2}}}^{x + \frac{r}{\sqrt{2}}} \rho_{S'} (l) \sqrt{2}r dl dx. \]

This will result in a smaller mobile conductance, but the scaling law will not be affected in the order sense. After the integral, the mobile conductance is approximated by

\[ \Phi_m (P_X) \]

\[ \mathbb{R} \left\{ \begin{array}{ll}
\frac{1}{2}r + \frac{v_{\text{max}}^2}{3r}, & \text{for } v_{\text{max}} \leq \frac{1}{2}r, \\
-\frac{r^3}{48v_{\text{max}}^3} + \frac{r^2}{6v_{\text{max}}} + \frac{2}{3}v_{\text{max}}, & \text{for } v_{\text{max}} > \frac{1}{2}r.
\end{array} \right. \]