Parity Odd Domain Structure with Generalized $\theta$ Vacuum

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Abstract

Recent experiments in heavy ion collisions have shown the possibility of creating parity-odd domains resulting from the $\theta$ term in strong interaction Lagrangian. The $\theta$ term originates from the nontrivial solution of QCD vacuum known as the $\theta$ vacuum, and the value of $\theta$ is taken to be a function of spacetime coordinates in the parity-odd domains. This means that we have to consider different theories at each point so that we need to devise a new approach to define the QCD vacuum. In this Letter, we suggest a method to generalize the $\theta$ vacuum by exploiting the dimension 2 condensates and to calculate the parity-odd domain structure as the union of gauge slices defined by the constant value of dimension 2 condensate.

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In heavy ion collisions it has been reported that metastable domains leading to P and CP violations are observed as a realization of an excited vacuum domain [1]. These metastable domains are described as “P-odd bubbles” where the parameter $\theta$ introduced as a conjugate variable to the integral of the topological charge density becomes non-zero. In contrast to the stringent limit $\theta < 3 \times 10^{-10}$ obtained from the measurement of neutron electric dipole moment [2], the parity violating parameter $\theta$ measured in heavy ion collisions turns out to be of order $10^{-2}$. The large difference between these measurements cannot be easily accounted for without introducing new idea to the definition of quantum chromodynamic (QCD) vacuum.

The variation of the value of $\theta$ up to the order of $10^8$ can be assigned to the existence of different $\theta$-worlds [3] generated by instantons which induce tunnelling from one vacuum to a gauge-rotated vacuum. One possible explanation of the large variation of $\theta$ between the hadronic phase and the quark-gluon plasma phase could be the formation of instanton liquid in the hot and dense matter created in heavy ion collisions. Since the average size of an instanton is taken to be about $\frac{1}{3}$ fermi [4], the formation of instanton liquid needs at least 2 or 3 fermi size domain which is rare in hadronic phase. The formation of large size domain is induced by the fusion process of the colliding hadrons and we need to devise new method to describe these changes of vacuum domain. In this Letter, we will give a general idea on the construction of topological spaces of gluonic vacuum domain and introduce appropriate measure for the description of domain structure. The characteristics of gluonic vacuum domain can be represented by the value of $\theta$ or equivalently by the value of dimension 2 condensate $\langle A^2_{\mu\nu} \rangle$ [5].

The relation between dimension 2 condensate and instanton contribution has been confirmed by lattice calculations. The existence of dimension 2 condensate can be checked by considering the two-point correlation function compared with the lattice gluon propagator. Quantitative estimation of instanton contribution to dimension 2 condensate can be carried out through the instanton shape recognition procedure [6] in which the topological charge density

$$Q = \frac{g^2}{32\pi^2} \int d^4x F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}$$

(1)

is compared with the lattice one

$$Q_{\text{latt}}(x) = \frac{1}{2g^2\pi^2} \sum \bar{\epsilon}_{\mu\nu\rho\sigma} \text{Tr}[\Pi_{\mu\nu}(x)\Pi_{\rho\sigma}(x)],$$

(2)
where $\tilde{\epsilon}_{\mu \nu \rho \sigma}$ is the antisymmetric tensor and $\Pi_{\mu \nu}(x)$ is the field tensor defined on the lattice. In this way we can measure the radius of the identified instanton and count the numbers $n_I$ of instantons and $n_A$ of anti-instantons. Then we get

$$\langle A_{\text{inst}}^2 \rangle = \frac{n_I + n_A}{V} \int d^4 x \sum_{\mu, a} A_{\mu}^{(I)a}(x) A_{\mu}^{(I)a}(x),$$

assuming that the QCD vacuum is approximated by the ensemble of non-interacting instantons. The estimated result is consistent with the one obtained by operator product expansion so that we can conclude that the instanton liquid picture is useful in deducing the value of dimension 2 condensate for the long range region with nonperturbative interactions.

The ordinary $\theta$ vacuum is defined by the eigenstate of the gauge transformation performed between different vacuum states fixed by the gauge field components with different topological charges. The topological charge density can be added to the Lagrangian and then the strong interaction Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu}^a F_{\mu \nu}^a - \theta \frac{g^2}{32\pi^2} F_{\mu \nu}^a F_{\mu \nu}^a + \sum_f \bar{\psi}_f \left[ i \gamma^\mu (\partial_\mu - ig A_\mu) - m_f \right] \psi_f,$$

where $\theta$ is the parameter characterizing the vacuum state. Effectively the $\theta$-term can be transferred into the mass term of up quark by using axial anomaly and the results are

$$\mathcal{L}_\theta = -m \cos \theta (\bar{u}_L u_R + \bar{u}_R u_L) - im \sin \theta (\bar{u}_L u_R - \bar{u}_R u_L)$$

representing the flip of handedness in the quark field. There existed a stringent limit $\theta < 3 \times 10^{-10}$ from the measurement of neutron electric dipole moment, however, the observations of parity-odd domains in relativistic heavy ion collisions by STAR Collaboration and by ALICE Collaboration give strong support for the metastable state with the value of $\theta$ in the order of $10^{-2}$. These large differences in the value of $\theta$ imply that the metastable state has to be localized in space and time and the vacuum domain has to be characterized by $\theta = \theta(x, t)$. This spacetime dependence of $\theta$ can be viewed from a different point of view when we classify the points of the vacuum domain by the conditions $\theta(x, t) = \theta_i$ with fixed value of $\theta_i$. The classified points form a set of surfaces in the vacuum domain and the time evolution of the surfaces generates the unstability of the domain. For a fixed $\theta_i$, the instanton contributions can be estimated by the value of dimension 2 condensate $\langle A_{\mu}^2 \rangle$, and for another $\theta'_i$ we can assign another value of $\langle A_{\mu}^2 \rangle$. This situation can be represented by

$$\langle A_{\mu}^2 \rangle_\theta = C_\theta$$

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with different $C_\theta$ for each $\theta$. Then the spacetime dependence of $\theta$ is naturally transferred into that of $\langle A_\mu^2 \rangle$, that is

$$\langle A_\mu^2 \rangle = C(x, t),$$

(7)

and for a given $C_\theta$ the points satisfying the condition $C(x, t) = C_\theta$ form a set corresponding to the given $\theta$ vacuum. The whole metastable domain can be described by the collections of these sets representing different $\theta$ vacua.

The change of viewpoints from collections of different $\theta$ vacua into the sets of gauge slices defined by the value of dimension 2 condensate $\langle A_\mu^2 \rangle$ gives a good chance to construct a model for gluonic domains. The gluonic domains appearing in the metastable state formed by heavy ion collisions are mainly controlled by the positions of quarks and antiquarks which behave as sources and sinks of the gluons mediating the strong interactions. Since the gluonic domains can be combined or divided according to the movements of quarks and antiquarks, we can introduce the union and the intersection operations on the open sets assigned to the gluonic domains. These assignments can be summarized as

- Open sets are the gluonic domains.
- The union of gluonic domains is a gluonic domain.
- The intersection between a connected gluonic domain and disconnected gluonic domains is the reverse operation of the union.

Now we can construct the topological spaces of gluonic domains classified by the numbers of quarks and antiquarks existing inside the given gluonic domain. If we represent the gluonic domain with $a$ quarks and $b$ antiquarks as $D_{a,b}$, then the topological space encompassing $i$ baryons and $j$ antibaryons becomes

$$T_{i,j} = \{ \phi, D_{3,0}^i D_{0,3}^j, D_{3,0}^{i-1} D_{0,3}^{-1} D_{2,2}, D_{3,0}^{i-2} D_{0,3}^{-2} D_{2,2}^2, \cdots \}. \quad (8)$$

For example, the space with 3 baryons and 1 antibaryon is given by

$$T_{3,1} = \{ \phi, D_{3,0}^3 D_{0,3}^0, D_{3,0}^2 D_{2,2}, D_{3,0} D_{4,1}, D_{6,0} \}. \quad (9)$$

The last domain $D_{6,0}$ represents the case of 6 quarks and we can find that this domain is divided into 3 baryon domains and 1 antibaryon domain through fragmentation processes. During the fragmentation processes, any number of meson domains $D_{1,1}$ can be created and these domains can be added to the classified domains shown in Eq. (9) [10]. The creation
process is affected by the non-zero value of \( \theta \) assigned to the original metastable domain formed by the strong collision of heavy nuclei.

In order to calculate the structures of metastable domain related to the spacetime dependent \( \theta \), we need to introduce a systematic measure defined between the positions of quarks existing in the domain. One of the most general measure can be assigned to the amplitude defined by the nonlocal condensate \([11]\)

\[
\langle \bar{q}(x)U(x,0)q(0) : \rangle \equiv \langle \bar{q}(0)q(0) : \rangle Q(x^2),
\]

where \( U(x,0) \) represents the connection through the gluonic domain. Since the gluonic domain is characterized by the value of \( \langle A^2_{\mu} \rangle \) at each point, we can relate this value to the function of \( Q(x^2) \) by assuming

\[
\langle \bar{q}(x)U(x,y)A^a_{\mu}(y)A^\mu_a(y)U(y,0)q(0) : \rangle \propto \langle \bar{q}(x)U(x,y)q(y)\bar{q}(y)U(y,0)q(0) : \rangle,
\]

which implies the proportionality of the value of \( \langle A^2_{\mu} \rangle \) to the probability amplitude to have a quark pair at that point. The functional form of \( Q(x^2) \) can be deduced by introducing a measure \( \mathcal{M}(Q) \) with the condition

\[
\mathcal{M}(Q) \text{ decreases as } Q \text{ increases.}
\]

The second condition can be stated for two independent \( Q_1 \) and \( Q_2 \) as

\[
\mathcal{M}(Q_1) + \mathcal{M}(Q_2) = \mathcal{M}(Q_1Q_2).
\]

Then we get the solution

\[
\mathcal{M}(Q) = -k \ln \frac{Q}{Q_0},
\]

where \( Q_0 \) is a normalization constant and \( k \) is an appropriate parameter. If we try to represent the measure \( \mathcal{M}(Q) \) as a metric function of the distance between the quark pair, it is possible to write the form of \( Q \) as \([10]\)

\[
Q = \frac{Q_0}{r^\beta} \exp \left\{ -\frac{1}{k} \frac{r^2 - r}{\ln r} \right\},
\]

where \( r = \frac{1}{r} |x - y| \) with \( \ell \) being a scale parameter, and \( \beta \) represents the singular behavior near the quark. In the case of 6 quark domain, the value of \( \langle A^2_{\mu} \rangle \) at the point \( x \) becomes
FIG. 1: (Color online) Profiles of gauge slices for 6 quark domain represented by $S_1$ and $S_2$. The quarks are at $(-0.15, 0.3, 0.0)$, $(-0.25, 0.15, 0.0)$, $(-0.25, -0.3, 0.0)$, $(-0.1, -0.4, 0.0)$, $(0.3, -0.1, 0.0)$, and $(0.25, 0.05, 0.0)$ with $\beta = 1.0$ and $k = 1.0$. The collision axis is the $z$-axis.

\[
\langle A_\mu^2 \rangle = A_0^2 \prod_{i=1}^{6} |\mathbf{x} - \mathbf{r}_i|^{-\beta} \exp\left\{-\frac{1}{k} \frac{|\mathbf{x} - \mathbf{r}_i|^2 - |\mathbf{x} - \mathbf{r}_i|}{\ln |\mathbf{x} - \mathbf{r}_i|}\right\}
\]

\[
\cdot \sum_{i=1}^{6} \prod_{r_j, r_k \neq r_i} |\mathbf{r}_j - \mathbf{r}_k|^{-\beta} \exp\left\{-\frac{1}{k} \frac{|\mathbf{r}_j - \mathbf{r}_k|^2 - |\mathbf{r}_j - \mathbf{r}_k|}{\ln |\mathbf{r}_j - \mathbf{r}_k|}\right\}
\]

\[
+ \sum_{r_j, r_k} |\mathbf{r}_j - \mathbf{r}_k|^{-\beta} \exp\left\{-\frac{1}{k} \frac{|\mathbf{r}_j - \mathbf{r}_k|^2 - |\mathbf{r}_j - \mathbf{r}_k|}{\ln |\mathbf{r}_j - \mathbf{r}_k|}\right\}
\]

\[
\cdot \prod_{r_\alpha, r_\gamma \neq r_j, r_k} |\mathbf{r}_\alpha - \mathbf{r}_\gamma|^{-\beta} \exp\left\{-\frac{1}{k} \frac{|\mathbf{r}_\alpha - \mathbf{r}_\gamma|^2 - |\mathbf{r}_\alpha - \mathbf{r}_\gamma|}{\ln |\mathbf{r}_\alpha - \mathbf{r}_\gamma|}\right\},
\]

where $\mathbf{r}_i$ are the positions of the 6 quarks and $A_0^2$ is a normalization factor. The first term in the square bracket corresponds to the amplitude to have a meson and a changed 6 quark structure after quark pair creation at $\mathbf{x}$, and the second term represents the amplitude to have a baryon and a pentaquark structure [12]. The calculated results are shown in Fig. 1.

In Fig. 2 we have given the two baryon domains when they are colliding in the direction of
FIG. 2: (Color online) Two baryon domains merging with displaced quarks.

During the collision the domains are expected to change according to the movements of quarks. The final formation of 6 quark domain [13] could be processed via one quark pair creation forming a baryon and a pentaquark domains and then the union of domains with quark pair annihilation leads to the larger structure.

The main difference between the hadronic phase and the quark-gluon plasma phase is the range of strong interactions. Since the strong interactions are mediated by the propagation of gluons, it is important to check the quantization process leading to the definition of gluon propagator. In path-integral quantization, the functional integration cannot be carried out without the gauge fixing procedure. The well known approach proposed by Faddeev and Popov [14] is to factorize the integration space into the volume of the orbit traced by gauge group and some surface that intersects the gauge orbit only once. The equation which fixes the integration surface is called the gauge condition and in perturbation theory this condition is usually given by constant $\langle A_\mu^2 \rangle$. However, in nonperturbative region, the value of $\langle A_\mu^2 \rangle$ has to be position-dependent because the condensate values are taken to be non-zero only within the region where the quarks and gluons interact [15]. Then we can define the
surfaces $S_1$ and $S_2$ by $\langle A_\mu^2(x) \rangle = C_1$ and $\langle A_\mu^2(x) \rangle = C_2$ as in Fig. 1. The gauge field $A_\mu$ can be quantized on these gauge slices and we can find that the gluons can propagate long distance in the nonperturbative region. The relation between $S_1$ and $S_2$ can be deduced from the equation

$$A_i^{g'} = \left( \delta_{ij} - \nabla_i \frac{1}{\sqrt{2}} \nabla_j \right) A_j + O(A_\mu^2),$$

where the gauge transformed field $A_i^{g'}$ is represented in terms of original $A_j$. In traditional approaches we neglect the $O(A_\mu^2)$ terms, but the $O(A_\mu^2)$ terms cannot be neglected when the dimension 2 condensate $\langle A_\mu^2 \rangle$ exists and the volume of the gauge orbit becomes dependent on the value of $A_\mu^2$. This situation can be interpreted such that the surfaces $S_1$ and $S_2$ are the gauge slices fixed by constant values of $\langle A_\mu^2 \rangle$ and these gauge slices are related by field-dependent gauge transformations inducing new picture of nonperturbative QCD vacuum.

In summary, we have tried to generalize the $\theta$ vacuum by exploiting the dimension 2 condensates and to introduce gluonic vacuum domains as the sets of gauge slices defined by the constant value of $\langle A_\mu^2 \rangle$. We can construct the topological spaces of gluonic domains and the functional form of $\langle A_\mu^2 \rangle$ has been deduced from the measure assigned to the amplitude of nonlocal quark condensate. The calculated 6 quark domain is large enough to encompass instanton liquid so that the observed value of order $10^{-2}$ for the parity violating parameter $\theta$ can be explained in contrast to the limit $\theta < 3 \times 10^{-10}$ obtained from neutron data. The effects of gluon propagation on the gauge slices extended over the whole gluonic domain need further study.

Acknowledgments

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