SUSY SU(6) GIFT for Doublet-Triplet Splitting and Fermion Masses

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Abstract

The supersymmetric SU(6) model equipped by the flavour-blind discrete gauge symmetry $Z_3$ is considered. It provides simultaneous solution to the doublet-triplet splitting problem, $\mu$-problem and leads to natural understanding of fermion flavour. The Higgs doublets arise as Goldstone modes of the spontaneously broken accidental global $SU(6) \times U(6)$ symmetry of the Higgs superpotential. Their couplings to fermions have peculiarities leading to the consistent picture of the quark and lepton masses and mixing, without invoking the horizontal symmetry or zero texture concepts. In particular, the only particle that has direct $O(1)$ Yukawa coupling with the Higgs doublet is top quark. Other fermion masses arise from the higher order operators, with natural mass hierarchy described in terms of small ratios $\varepsilon_\Sigma = V_\Sigma/V_H$ and $\varepsilon_H = V_H/M$, where $V_H$ and $V_\Sigma$ correspondingly are the $SU(6)$ and $SU(5)$ symmetry breaking scales, and $M$ is a large (Planck or string) scale. The model automatically implies almost precise $b-\tau$ Yukawa unification. Specific mass formulas are also obtained, relating the down quark and charged lepton masses. Neutrinos get small ($\sim 10^{-5}$ eV) masses which can be relevant for solving the solar neutrino problem via long wavelength vacuum oscillation.
1. Introduction

The evidence of the gauge coupling unification [1] in the minimal supersymmetric standard model (MSSM) suggests the following paradigm: at the Planck or string scale \( M \sim 10^{18-19} \text{ GeV} \) the ultimate “Theory of Everything” reduces to a field theory given by the SUSY GUT, which then is broken at the scale \( M_X \approx 10^{16} \text{ GeV} \) down to the \( SU(3) \times SU(2) \times U(1) \) MSSM, with minimal content of chiral superfields including the standard fermion families and the Higgs doublets \( h_{1,2} \).

The central question dubbed a gauge hierarchy problem concerns the origin of scales: why the electroweak scale \( M_W \) is so small as compared to the GUT scale \( M_X \), which in itself is not far from the Planck scale? It is well known [2] that supersymmetry can stabilize the Higgs mass (\( \sim M_W \)) against radiative corrections, provided that the soft SUSY breaking scale \( m \) (typically given by the gaugino and sfermion masses) does not exceed few TeV. Most likely, the electroweak scale \( M_W \) emerges from the SUSY scale \( m \) itself. In particular, it is suggestive to think that the MSSM Higgs doublets \( h_{1,2} \) would stay massless in the exact SUSY limit, and the only source of their non-zero masses is related to the soft SUSY breaking terms. However, in the context of grand unification the gauge hierarchy problem has the following puzzling aspects:

A. The problem of the doublet-triplet (DT) splitting [2]: the Higgs doublets should stay light, while their colour triplet partners in GUT supermultiplet should have \( O(M_X) \) mass. Otherwise the latter would cause unacceptably fast proton decay (mainly via the Higgsino mediated \( d = 5 \) operators [3]), and also spoil the gauge coupling unification.

B. The \( \mu \)-problem [4]: the resulting low energy MSSM should contain the supersymmetric \( \mu h_1 h_2 \) term defining the higgsino masses, with \( \mu \sim M_W \). It is questionable why the supersymmetric mass \( \mu \) should be of the order of soft SUSY breaking mass \( m \).

Another theoretical weakness of SUSY GUTs is a lack in the understanding of flavour. Although GUTs can potentially unify the Yukawa couplings within each fermion family, the origin of inter-family hierarchy and weak mixing pattern remains open. Moreover, for the light families the Yukawa unification simply contradicts to the observed mass pattern, though the \( b - \tau \) Yukawa unification may constitute a case of partial but significant success. In order to deal with the flavour problem in GUT frameworks, some additional ideas (horizontal symmetry, specific textures) have to be invoked [5, 6].

An attractive possibility towards the solution of these problems is suggested by the GIFT (Goldstones Instead of Fine Tuning) mechanism in SUSY \( SU(6) \) model [7, 8, 9], which is a minimal extension of \( SU(5) \): the Higgs sector contains supermultiplets \( \Sigma \) and \( H + \bar{H} \) respectively in adjoint 35 and fundamental \( 6 + \bar{6} \) representations, in analogy to 24 and 5 + \( \bar{5} \) of \( SU(5) \). However, this model drastically differs from the other GUT approaches. Usually in GUTs the Higgs sector consists of two different sets: one is for the GUT symmetry breaking (e.g. 24-plet in \( SU(5) \)), while another containing the Higgs doublets (like \( 5 + \bar{5} \) in \( SU(5) \)) is just for the electroweak symmetry breaking and fermion mass generation. In contrast, the \( SU(6) \) theory has no special superfields for the second function: 35 and \( 6 + \bar{6} \) constitute a minimal Higgs content needed for the local \( SU(6) \)

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1) The Goldstone boson mechanism for the DT splitting was first suggested in the context of SUSY \( SU(5) \) [10, 11] (in [11] it was elegantly named as GIFT), by assuming an \textit{ad hoc} \( SU(6) \) global symmetry of the Higgs superpotential. Our results, however, are specific of the gauged \( SU(6) \) theory.
symmetry breaking down to MSSM. As for the light Higgs doublets $h_{1,2}$, they arise from the doublet fragments in $\Sigma$ and $H, \bar{H}$, as Goldstone modes of the accidental global symmetry $SU(6)_{\Sigma} \times U(6)_H$. This global symmetry arises if mixing terms of the form $\bar{H} \Sigma H$ are suppressed in the Higgs superpotential. Thus $h_{1,2}$ being strictly massless in the exact SUSY limit, acquire non-zero mass terms (including the $\mu$-term) only due to the spontaneous SUSY breaking and subsequent radiative corrections.

On the other hand, in the GIFT picture the Yukawa couplings have peculiarities leading to new possibilities towards the understanding of flavour. Indeed, if the Yukawa terms also respect the $SU(6)_{\Sigma} \times U(6)_H$ global symmetry, then $h_1$ and $h_2$ being the Goldstone modes should have vanishing Yukawa couplings with the fermions that remain massless after the $SU(6)$ symmetry breaking down to MSSM, that are ordinary quarks and leptons. Thus, the couplings relevant for fermion masses should explicitly violate $SU(6)_{\Sigma} \times U(6)_H$. This constraint leads to striking predictions for the fermion mass and mixing pattern even in completely ‘democratic’ approach, without invoking the horizontal symmetry arguments. In particular, it was shown in [9] that only the top quark can get $\sim 100$ GeV mass through renormalizable $SU(6)$ invariant Yukawa coupling. For the other fermion masses one has to appeal to the higher order operators, scaled by inverse powers of the Planck scale. In order to achieve a proper operator structure, additional discrete symmetry was invoked. The model suggested in [9] succeeded in appealing description of the third and second fermion families, but the first family was rendered massless.

In order to built a consistent GIFT model, one has to find some valid symmetry reasons to forbid the mixing terms like $\bar{H} \Sigma H$: otherwise the theory has no accidental global symmetry. It is natural to use for this purpose the discrete gauge symmetries, which can naturally emerge in the string theory context. In the present paper we suggest a consistent SUSY $SU(6)$ model equipped with the flavour-blind discrete $Z_3$ symmetry. The role of the latter is important: it forbids the mixing terms in the Higgs superpotential thus ensuring the accidental $SU(6)_{\Sigma} \times U(6)_H$ symmetry, and provides the proper higher order operators for generating a realistic mass and mixing pattern of all fermions.

2. $SU(6) \times Z_3$ model

Let us assume that below the Planck or string scale $M$ the theory is given by SUSY GUT with the $SU(6)$ gauge symmetry, containing the following chiral superfields – ‘Higgs’ sector: vectorlike supermultiplets $\Sigma_1(35), \Sigma_2(35), H(6), \bar{H}(\bar{6})$ and an auxiliary singlet $Y$; ‘fermion’ sector: chiral, anomaly free supermultiplets $(\bar{6} + 6')_i, 15_i$ ($i = 1, 2, 3$ is a family index) and 20; and some heavy vector-like matter multiplets like $15_F + \bar{15}_F$, etc., which we recall later on as $F$-fermions. According to survival hypothesis [12], these should have $SU(6)$ invariant large ($\sim M$) mass terms and thus decouple from the lighter sector. However, they can play a crucial role in the light fermion mass generation [13]. In Sect. 4

2) In order to maintain the gauge coupling unification, $SU(6)$ must be first broken to $SU(5)$ by $H, \bar{H}$ at some scale $V_H$. At this stage the fermion sector is also reduced to the minimal $SU(5)$ content. Then at the scale $V_\Sigma \simeq 10^{16}$ GeV, $\Sigma$ breaks the intermediate $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$.

3) The survival hypothesis does not apply to 20, since it is a pseudo-real representation and the mass term $M_{20}$ is vanishing (the singlet is contained only in antisymmetric tensor product $20 \times 20$). More generally, if in the original theory 20-plets present in odd number then one of them inevitably ‘survives’ to be massless.
we use the $F$-fermion exchanges for inducing the masses of all light fermions, except the top which gets mass from the direct Yukawa coupling.

We introduce also two flavour-blind discrete symmetries. One is usual matter parity $Z_2$, under which the fermion superfields change the sign while the Higgs ones stay invariant. Such a matter parity, equivalent to R parity, ensures the proton stability. Another discrete symmetry is $Z_3$ acting in the following way \( \omega = e^{\frac{2\pi i}{3}} \):

\[
20 \rightarrow \omega 20, \quad 15_i \rightarrow \omega 15_i, \quad \bar{6}_i, \bar{6}_i' \rightarrow \omega \bar{6}_i, \bar{6}_i', \quad \Sigma_1 \rightarrow \omega \Sigma_1, \quad \Sigma_2 \rightarrow \bar{\omega} \Sigma_2, \quad (1)
\]

while $H, \bar{H}$ and $Y$ are invariant. One can easily check that this $Z_3$ symmetry satisfies the anomaly cancellation constraints \[14\] so that it can be regarded as the gauge discrete symmetry. The matter parity $Z_2$ is also known to be free of discrete anomalies \[14\].

Let us consider first the Higgs sector. The most general renormalizable superpotential compatible with the $SU(6) \times Z_3$ symmetry is\[7\]

\[
W = M_\Sigma \Sigma_1 \Sigma_2 + \lambda_1 \Sigma^3_1 + \lambda_2 \Sigma^3_2 + \lambda S \Sigma_1 \Sigma_2 + M_H \bar{H} H + \rho Y(\bar{H} H - \Lambda^2) + M_Y Y^2 + \xi Y^3 \quad (2)
\]

This superpotential automatically has the global symmetry $SU(6)_\Sigma \times U(6)_H$, related to independent transformations of $\Sigma$ and $H$.\[8\] In the exact SUSY limit the condition of vanishing $F$ and $D$ terms allows, among the other degenerated vacua, the following VEVs,$^7$\[\]

\[
\langle \Sigma_{1,2} \rangle = V_{1,2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ -2 & -2 \end{pmatrix}, \quad \langle H \rangle = \langle \bar{H} \rangle = V_H \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle Y \rangle = V_Y \quad (3)
\]

where, provided that $\Lambda \gg V_\Sigma = (V_{1,2}^2 + V_{2,2}^2)^{\frac{1}{2}}$, we have:

\[
V_Y = \frac{M_H}{\rho}, \quad V_{1,2} = \frac{M_\Sigma + \lambda V_Y}{(\lambda_1 \lambda_2)^{\frac{1}{2}}}, \quad V_H = \Lambda + O \left( \frac{V_\Sigma^2}{\Lambda} \right) \quad (4)
\]

These VEVs lead to needed pattern of the gauge symmetry breaking: $H, \bar{H}$ break $SU(6)$ down to $SU(5)$, while $\Sigma_{1,2}$ break $SU(6)$ down to $SU(4) \times SU(2) \times U(1)$. Both channels

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4) we assume that all coupling constants are of the order of 1, say within factor of 3–4. For comparison, we remind that the gauge coupling constant at the GUT scale is $g_X \approx 0.7$

5) In fact, $SU(6)_\Sigma \times U(6)_H$ is not a global symmetry of a whole Lagrangian, but only of the Higgs superpotential. In particular, the Yukawa as well as the gauge couplings (D-terms) do not respect it. However, in the exact supersymmetry limit (i) it is effective for the field configurations on the vacuum valley, where $D = 0$, (ii) owing to non-renormalization theorem, it cannot be spoiled by the radiative corrections from the Yukawa interactions.

6) Discrete degeneration of the $\langle \Sigma \rangle$ is not essential and will be immediately removed for the proper range of the soft SUSY breaking parameters $A, B$ (see below, eq.\[6\]). However, for $\langle H \rangle, \langle \bar{H} \rangle$ fixed as in eq.\[6\] there is also continuous degeneration related to independent rotation of $\langle \Sigma \rangle$: any configuration obtained by the unitary transformation $U \langle \Sigma_{1,2} \rangle U$ is a vacuum state as well. Actually this flat direction gives rise to Goldstone mode which can be identified to the Higgs doublets provided that true vacuum is given by $U = 1$, i.e. the relative orientation of the VEVs is fixed as in eq.\[6\]. For a proper parameter range, this configuration can indeed appear as a true vacuum state after lifting the vacuum degeneracy by the effects of SUSY breaking and subsequent radiative corrections\[6\].
together break the local symmetry down to $SU(3) \times SU(2) \times U(1)$. At the same time, the
global symmetry $SU(6)_\Sigma \times U(6)_H$ is broken down to $[SU(4) \times SU(2) \times U(1)]_\Sigma \times U(5)_H$.
The Goldstone degrees which survive from being eaten by the $SU(6)$ gauge superfields via the Higgs mechanism, constitute a couple of the MSSM Higgs doublets $h_1 + h_2$ which in terms of the doublet (anti-doublet) fragments in $\Sigma_{1,2}$ and $H, \tilde{H}$ are given as
\begin{equation}
\begin{aligned}
h_2 &= c_\eta (c_\sigma h_{\Sigma_1} + s_\sigma h_{\Sigma_2}) - s_\eta h_H, \\
h_1 &= c_\eta (c_\sigma \bar{h}_{\Sigma_1} + s_\sigma \bar{h}_{\Sigma_2}) - s_\eta \bar{h}_H
\end{aligned}
\end{equation}
(here and in the following we use notations $c_\sigma = \cos \sigma$, $s_\sigma = \sin \sigma$, etc.), where $\tan \eta = 3V_2/V_H$ and $\tan \sigma = V_2/V_1 = (\lambda_1/\lambda_2)^{\frac{1}{2}}$. In the natural range of constants $\lambda_{1,2}$ allowed to deviate from 1 no more than a factor of 4, $\tan \sigma \simeq 1$ within a factor of 2.

After the SUSY breaking enters the game (presumably through the hidden supergravity sector), the Higgs potential, in addition to the (supersymmetric) squared $F$ and $D$ terms, includes also the soft SUSY breaking terms $[15]$. These are
\begin{equation}
V_{SB} = AmW_3 + BmW_2 + m^2 \sum_k |\phi_k|^2,
\end{equation}
where $\phi_k$ imply all scalar fields involved, $W_{3,2}$ are terms in superpotential respectively trilinear and bilinear in $\phi_k$, and $A, B, m$ are soft breaking parameters. Due to these terms the VEVs $V_{1,2}$ are shifted by an amount of $\sim m$ as compared to the ones in eq. $[8]$ being calculated in the exact SUSY limit. Via the $\Sigma^3$ terms in the superpotential, this shift gives rise to term $\mu h_1 h_2$ contributing the higgsino masses. Thus, the GIFT scenario automatically solves the $\mu$-problem: the (supersymmetric) $\mu$-term for the resulting MSSM in fact arises in consequence of SUSY breaking, with $\mu \sim m$.

The scalar components of $h_{1,2}$ acquire the soft SUSY breaking mass terms, but not all of them immediately. Clearly, $V_{SB}$ also respects the larger global symmetry $SU(6)_\Sigma \times U(6)_H$, so that only the combination $h = h_1 - h_2^*$ of scalars gets a $\sim m$ mass, while the orthogonal state $\tilde{h} = h_1 + h_2^*$ remains massless as a truly Goldstone boson. Taking into the account also the structure of $D$-term, we see that there is a vacuum valley with $v_2/v_1 = 1$, where $v_{1,2}$ are the VEVs of $h_{1,2}$ while the value of the $v_1 = v_2$ remains arbitrary.

However, SUSY breaking relaxes radiative corrections (mainly due to the large top Yukawa coupling) which lift the vacuum degeneracy and provide non-zero mass to $\tilde{h}$, fixing thereby the VEVs $v_1$ and $v_2$. It is natural to expect that renormalization effects will not deviate these VEVs very strongly from the valley given by $v_1 = v_2$, so that the magnitude of $\tan \beta = v_2/v_1$ will be very moderate. The effects of radiative corrections leading to the electroweak symmetry breaking were studied in ref. $[8]$. It was shown that in spite of earlier claims $[11, 12]$ the GIFT scenario does not imply any upper bound on the top mass, and it can go up to its infrared fixed limit $M_t = (190 - 210) \sin \beta \text{ GeV} [17]$.

Thus, our model naturally solves both the DT splitting and the $\mu$ problems. The Higgs doublets $h_{1,2}$ remain light, while their triplet partners are superheavy. Indeed, the triplet fragments from $\Sigma_{1,2}$ have masses $\sim V_\Sigma$, and the triplets from $H, \tilde{H}$ are the Goldstone modes eaten up by the $SU(6)$ gauge superfields. In the following we assume that $V_H \gg V_\Sigma$, as it is suggested by the gauge coupling unification, and show how the observed hierarchy of fermion masses can be naturally explained in terms of small ratios $\varepsilon_\Sigma = V_\Sigma/V_H$ and $\varepsilon_H = V_H/M$. In this case the Higgs doublets dominantly come from $\Sigma_{1,2}$ while in $H, \tilde{H}$ they are contained with small weight $\sim 3\varepsilon_H$. 

4
3. Fermion masses: general operator analysis

The most general Yukawa superpotential allowed by the $SU(6) \times Z_3$ symmetry is

$$W_{Yuk} = G 20\Sigma_1 20 + \Gamma 20 H 15_3 + \Gamma_{ij} 15_i \bar{H} \bar{6}_j', \quad i, j = 1, 2, 3$$

(7)

where all Yukawa coupling constants are assumed to be $O(1)$. Without loss of generality, one can always redefine the basis of $15$-plets so that only the $15_3$ state couples $20$-plet in (7). Also, among six $6$-plets one can always choose three of them (denoted in eq. (4) as $\bar{6}_{1,2,3}$) which couple $15_{1,2,3}$, while the other three states $\bar{6}_{1,2,3}$ have no Yukawa couplings.

Already at the scale $V_H$ of the gauge symmetry breaking $SU(6) \rightarrow SU(5)$ the fermion content of our theory reduces to the one of minimal $SU(5)$. Indeed, the $SU(5) \supset SU(3) \times SU(2) \times U(1)$ decomposition of the fermion multiplets under consideration reads

$$20 = 10 + \overline{10} = (q + u^c + e^c)_{10} + (Q^c + U + E)_{\overline{10}}$$
$$15_i = (10 + 5)_i = (q_i + u_i^c + e_i^c)_{10} + (D_i + L_i)_{5}$$
$$\bar{6}_i = (5 + 1)_i = (d_i^c + l_i)_{\overline{5}} + n_i$$
$$\bar{6}'_i = (5 + 1)'_i = (D'_i + L_i)_{\overline{5}'} + n'_i, \quad i = 1, 2, 3$$

(8)

According to eq. (6), the extra fermion pieces with non-standard $SU(5)$ content, namely $\overline{10}$ and $\bar{5}_{1,2,3}$, form massive particles being coupled to $10_3$ and $\bar{5}_{1,2,3}$:

$$\Gamma V_H \overline{10} 10_3 + \Gamma_{ij} V_H \bar{5}_i \bar{5}'_j + G V_1 (U u^c - 2 E e^c),$$

(9)

and thereby decouple from the light states which remain as $\bar{5}_{1,2,3}$, $10_{1,2}$ and $10$ (we neglect the small ($\sim \varepsilon_{1,2}$) mixing between the $u^c - u_3^c$ and $e^c - e_3^c$ states) and singlets $n_i, n'_i$.

The couplings of $20$-plet in (7) explicitly violate the global $SU(6)_\Sigma \times U(6)_H$ symmetry. Hence, the up-type quark from $20$ (to be identified as top) has non-vanishing coupling with the Higgs doublet $h_2$. As far as $V_H \gg V_\Sigma$, it essentially emerges from $G 20 \Sigma_1 20 \rightarrow G qu^c h_2$. Thus, in our scheme only the top quark can have $\sim 100$ GeV mass due to the large Yukawa constant $\lambda_t = G \sim 1$. Other fermions would stay massless unless we invoke the higher order operators scaled by inverse powers of the large mass $M$. Such operators could appear due to quantum gravity effects, with $M \sim M_{Pl}$. Alternatively, they can arise by integrating out heavy fermions with masses $M \gg V_H$ (see Sect. 4).

Nevertheless, before addressing the concrete scheme with heavy fermion superfields, let us start with the general operator analysis. Obviously, $Z_3$ symmetry forbids the $d = 5$ ‘Yukawa’ terms in the superpotential (7). However, the $d = 6$ operators are allowed and they are the following:

$$\mathcal{B} = \frac{B}{M^2} 20 \bar{H} (\Sigma_1 H) \bar{6}_3, \quad \mathcal{C} = \frac{C_{i}}{M^2} 15_i H (\Sigma_2 H) 15_j$$

(10)

7) Operators involving an odd number of fermion superfields are forbidden by $Z_2$ matter parity.

8) The way of the $SU(6)$ indices convolution in these operators is indicated by the parentheses so that the combinations inside transform as effective $\bar{6}$ or $6$. We remind that operators which are relevant for the light fermion masses should explicitly violate the global $SU(6)_\Sigma \times U(6)_H$ symmetry. The possible terms $15 \bar{H} (\Sigma_1 \Sigma_2 \bar{6})$ and $15 \bar{H} \bar{6} \cdot \text{Tr}(\Sigma_1 \Sigma_2)$ actually do not violate it and therefore are irrelevant. We also omit the operators obtained by trivial replacing $\Sigma_1 \rightarrow \Sigma_2$ in $\mathcal{S}$. 

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\[
S = \frac{S_{ik}^{(1)}}{M^2} 15_i (\Sigma_1 \Sigma_2 H) 6_k + \frac{S_{ik}^{(2)}}{M^2} 15_i (\Sigma_1 H) (\Sigma_2 6_k)
\]
(11)

\[
N = \frac{N_{kl}}{M^2} 6_k H (\Sigma_1 H) 6_l
\]
(12)

(clearly, matrices \( C_{ij} \) and \( N_{kl} \) are symmetric) where \( B, \ldots N_{kl} \) are the \( O(1) \) constants.

First we focus on the operators \( B, C \) and \( S \) generating the charged fermion masses. \((N) \) is relevant only for the neutrino masses, and we consider it later in this section). Similar operators involving heavy \( \bar{6}' \) states are irrelevant, since the charged fragments of the latter are already massive. According to eq. (9), the state \( 10 \subset \bar{6}' \) is also heavy and it is decoupled from the light particle spectrum. Therefore, these operators are relevant only for \( 10 \subset 20, 10_i \subset 15 \) \((i = 1, 2)\) and \( \bar{6}_k \subset \bar{6}_i \) \((k = 1, 2, 3)\) states. Without loss of generality, we redefine the basis of \( \bar{6}-\)plets so that only the \( \bar{6}_3 \) state couples 20 in eq. (11).

Obviously, the operator \( B \) is responsible for the \( b \) quark and \( \tau \) lepton masses, and at the MSSM level it reduces to the Yukawa couplings \( \varepsilon_H^2 c_\sigma B \left( qd_3 + e^c l_3 \right) h_1 \). Hence, though \( b \) and \( \tau \) belong to the same family as \( t \) (namely, to 20-plet), their Yukawa constants are substantially (by factor \( \sim \varepsilon_H^2 \)) smaller than \( \lambda_t \). Moreover, we automatically have almost precise \( b - \tau \) Yukawa unification at the GUT scale:

\[
\lambda_b = \varepsilon_H^2 c_\sigma B, \quad \lambda_\tau = \varepsilon_H^2 c_\sigma B \left[ 1 - \varepsilon_H^2 (c_\sigma G/T)^2 \right] \cong \lambda_b
\]
(13)

where the \( \sim \varepsilon_H^2 \) correction is due to the mixing of \( e^c \) and \( e^c \) states in eq. (11).

As far as the third family fermions are already defined as the states belonging to 20 and \( \bar{6}_3 \), operators \( C \) and \( S \) induce mass terms for the fermions of the first two families, which in general would appear unsplit. Indeed, for the Yukawa matrices of the corresponding upper and down quarks and charged leptons we obtain:

\[
\lambda_{ij}^{\text{up}} = \varepsilon_H^2 s_\sigma C_{ij}, \quad \lambda_{ik}^{\text{down}} = \varepsilon_\Sigma \varepsilon_H^2 s_\sigma c_\sigma (S_{ik}^{(1)} - S_{ik}^{(2)}) , \quad \lambda_{ik}^{\text{lept}} = \varepsilon_\Sigma \varepsilon_H^2 s_\sigma c_\sigma (S_{ik}^{(1)} + 2S_{ik}^{(2)})
\]
(14)

Thus, for \( \varepsilon_H, \varepsilon_\Sigma \sim 0.1 \) a feasible description of the third and second family masses can be achieved: we naturally (without appealing to any flavour symmetry) obtain \( \lambda_t \gg \lambda_{\tau(b)}, \lambda_\tau \gg \lambda_{\mu,S} \). The charm quark Yukawa constant \( \lambda_c \sim \varepsilon_H^2 \), as well as the bottom-tau constant \( \lambda_{b,\tau} \), whereas the \( \lambda_{\mu,s} \) are smaller by factor of \( \sim \varepsilon_\Sigma \) in addition, the Yukawa couplings \( \lambda_\mu \) and \( \lambda_\mu \) are split due to different contribution of the second term in (14). Finally, the operator \( S \) involving the \( \bar{6}_3 \) state gives rise to the \( O(\lambda_\mu/\lambda_b) \) CKM mixing angle between the second and third families.

However, a completely general operator analysis implies that \( \lambda_\mu \sim \lambda_c \) and \( \lambda_{d,e} \sim \lambda_{s,\mu} \).

In order to explain the observed mass hierarchy between the first and the second families, some additional ideas are needed. For example, one can assume that the ‘Yukawa’ matrices \( C_{ij} \) and \( S_{ik}^{(1,2)} \) are rank-1 matrices, and in addition \( S_{ik}^{(1,2)} \) are alligned, so that these operators provide only one non-zero mass eigenvalue for each type of charged fermions. Then, without loss of generality, we can redefine the basis of \( 15_{1,2} \) and \( \bar{6}_{1,2} \) states so that

\[
C_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & C \end{pmatrix}, \quad S_{ik}^{(1,2)} \propto \begin{pmatrix} 0 & s_\theta S_2 & s_\theta S_3 \\ 0 & c_\theta S_2 & c_\theta S_3 \end{pmatrix}
\]
(15)

\(^9\) As we have commented earlier, the natural value of \( \tan \sigma \) is of about 1. The fact that the physical masses of \( b, \tau \) and \( c \) are all in the GeV range hints that \( \tan \beta \) should be close to 1, in agreement with our earlier remark that the natural value of \( \tan \beta \) in the GUT scenario should be very moderate.
Hence, in this basis only $C_{22} = C$ component of the matrix $C_{ij}$ is nonzero, and $c$ quark should be identified as an up-quark state from $q_2, u_2 \subset 15_2$. Then $s$ and $\mu$ are the down quark and charged lepton states contained in $q_2' \subset 15_2'$ and $d_2' \subset 6_2$, where $15_2' = s_05_1 + \epsilon_0 15_2$ is an effective combination which couples $3_2$ and $6_3$ states (it is not difficult to recognize that in fact $\theta$ is the Cabibbo angle). In this way operators $\mathcal{C}$ and $\mathcal{S}$ provide masses of $c, s$ and $\mu$, rendering the $u, d$ and $e$ states massless. Then for the latter one can appeal to the $d = 7$ operators ($15_1'$ is defined as a state orthogonal to $15_2'$):

$$
\mathcal{D} = \frac{D_{ik}^{(1)}}{M^3} 15_i' (\Sigma_1^1 H) \bar{6}_k + \frac{D_{ik}^{(2)}}{M^3} 15_i' (\Sigma_1^2 H) (\Sigma_3^1 \bar{6}_k) + \frac{D_{ik}^{(3)}}{M^3} 15_i' (\Sigma_1^2 \bar{6}_k) + \frac{D_{ik}^{(4)}}{M^3} 15_i' (\Sigma_1^2 H) \bar{6}_k Tr (\Sigma_1^2)
$$

$$
\mathcal{U} = \frac{U_{ij}^{(1)}}{M^3} 15_i H (\Sigma_1^1 H) 15_j + \frac{U_{ij}^{(2)}}{M^3} 15_i H (\Sigma_1^1 H) (\Sigma_1^1 15_j)
$$

(16)

Operator $\mathcal{D}$ induces the following Yukawa couplings at the GUT scale:

$$
\tilde{\lambda}_{ik}^{\text{down}} = \varepsilon^2 \varepsilon_H^3 \varepsilon \bar{H}_{\sigma}^3 (3D_{ik}^{(1)} - D_{ik}^{(2)} + D_{ik}^{(3)} + 12 D_{ik}^{(4)})
$$

$$
\tilde{\lambda}_{ik}^{\text{lept}} = \varepsilon^2 \varepsilon_H^3 \varepsilon \bar{H}_{\sigma}^3 (3D_{ik}^{(1)} + 2D_{ik}^{(2)} + 4D_{ik}^{(3)} + 12 D_{ik}^{(4)})
$$

(18)

which provide $\lambda_{d,e}$ in the proper range when $\varepsilon, \varepsilon_H \sim 0.1$. As for the operator $\mathcal{U}$, for $U_{11} \sim 1$ it would lead to $\lambda_u \sim \varepsilon \varepsilon_H^3 c_\sigma^2$, which is parametrically one order of magnitude larger then $\lambda_d$. It is more suggestive to assume that the matrices $U_{ij}^{(1,2)}$ have a Fritzsch-like structure [3], with $U_{11}^{(1,2)} = 0$. Then the above estimate holds rather for $(\lambda_u, \lambda_c)^{1/2}$, and we obtain the appealing estimate $\lambda_u \sim \varepsilon^2 \varepsilon_H^3 c_\sigma^2 s^{-1}$. As we show in sect. 4, this pattern of the Yukawa couplings can be indeed obtained in the heavy fermion exchange scheme.

Let us conclude this section by considering the neutrino mass pattern. After the GUT symmetry breaking the operator (12) reduces to the following terms:

$$
\frac{N_{kl}}{M^2} c_\sigma [V^2_H V^2_\Sigma n_k n_l + V^2_H (l_k n_l + l_m n_k) h_2 - 3 V_\Sigma l_k l_l h_2^2]
$$

(19)

It is not difficult to recognize in this pattern the well-known ‘seesaw’ picture for the neutrino mass generation. Indeed, the ‘right-handed’ neutrinos $n_k$ acquire large ($\sim \varepsilon^2 H \varepsilon_\Sigma$) Majorana masses, while the second term in eq. (19) is nothing but Dirac mass terms $\sim \varepsilon^2 H v_2$ obtained after substituting the VEV $\langle h_2 \rangle$. As a result of the seesaw mixing, small Majorana masses are induced for the ordinary neutrino states $\nu_k \subset l_k$:

$$
m^\nu_{kl} = \frac{\varepsilon_H}{\varepsilon_\Sigma} \frac{N_{kl}}{M} c_\sigma v_2^2
$$

(20)

Thus, for $\varepsilon, \varepsilon_H \sim 0.1$ the neutrino masses are in the range $m_\nu \sim M^2_H / M \sim 10^{-5}$ eV. (Notice, that the same estimate follows in Standard Model or $SU(5)$ with possible gravity induced non-renormalizable operators $\frac{1}{M} l h h$ [13]). It is well-known that this mass range together with large neutrino mixing angles, also naturally implied in our ‘democratic’ approach with $N_{kl} \sim 1$, can provide a feasible solution to the solar neutrino problem through the long wavelength ‘just-so’ neutrino oscillations (for recent discussions of the experimental status of this scenario see [13]).
4. Yukawa couplings from heavy particle exchanges

From the previous section, we are left with the problem how to split the masses of the first two families (eq. (13) for the coupling constants in $C$ and $S$ was assumed ad hoc). Now we show that this problem can be solved, still without appealing to any flavour symmetry, by assuming that all higher order operators are generated by the exchanges of heavy superfields with $\sim M$ masses [13]. As we will see shortly, it is possible to find a proper set of the heavy fermions, which after their decoupling lead to the needed rank-1 pattern of the higher order operators fulfilling eq. (15), and thus providing the following Yukawa matrices at the GUT scale:

$$
\begin{align*}
&u^c_1 \left( \begin{array}{ccc}
0 & \epsilon U & u^c \\
\epsilon S \epsilon H c^2 & \epsilon^2 & 0 \\
\epsilon S & 0 & G \\
\end{array} \right) \cdot h_2, \\
&q \left( \begin{array}{ccc}
0 & \epsilon S \epsilon^2 H c^2 & d^c_1 \\
0 & 0 & d^c_2 \\
0 & 0 & d^c_3 \\
\end{array} \right) \cdot h_1, \\
&e^c \left( \begin{array}{ccc}
\epsilon S \epsilon^2 H c^2 & \epsilon S \epsilon^2 H c^2 & \epsilon S \epsilon^2 H c^2 \\
0 & \epsilon S \epsilon^2 H c^2 & \epsilon S \epsilon^2 H c^2 \\
0 & 0 & \epsilon S \epsilon^2 H c^2 \\
\end{array} \right) \cdot h_1 \\
\end{align*}
$$

(15)

(notice, that the basis of down quarks in 15'$_{1,2}$ is already ‘Cabibbo’ rotated with respect to the one of the upper quarks 15'$_{1,2}$ by the angle $\theta$), where $J$ and $K$ are some Clebsch factors. As we see below, the heavy fermion mechanism leads also to the specific predictions for the coefficients $J$ and $K$ distinguishing the down quark and charged lepton masses.

Let us introduce the set of heavy vectorlike superfields (in the following referred as $F$-fermions) with $\sim M$ masses and transformation properties under $SU(6) \times Z_3$ given in Table 1. Certainly, we prescribe negative matter parity to all of them.

Then the operators $B$, $C$ and $S$ are uniquely generated by $F$-fermion exchanges shown in Fig. 1, with the rank-1 coupling matrices [13] in $C$ and $S$. Indeed, operator $B$ defines the 6$_3$ state. On the other hand, the coupling with 20$_F$ defines the 15$_2$ state, so that the operator $C$ induces only the $c$ quark mass. The coupling $(G_1 15_1 + G_2 15_2) \Sigma_1 \Sigma_3$ defines the state 15'$_2 = c_9 15_1 + s_9 15_2$ with tan$\theta = G_1 / G_2$, and the couplings of 15'$_F$ define the 6$_2$ state. Thus, the operator $S$ induces only the $s$ and $\mu$ masses, and in general leads

| $Z_3$: | Higgs | fermions | $F$-fermions |
|-------|-------|----------|--------------|
| $\omega$ | $\Sigma_1$ | 6$_1$, 6$_2$, 20 | 15'$_F$, 15'$_F$, 20$_F$, 35$_F$, 70$_F$, 84$_F$ |
| $\bar{\omega}$ | $\Sigma_2$ | 15$_i$ | 15'$_F$, 15'$_F$, 20$_F$, 35$_F$, 70$_F$, 84$_F$ |
| inv. | $H$, $\bar{H}$, $Y$ | – | 15$_F$, 15$_F$, 20$_F$, 105$_F$, 105$_F$, 210$_F$, 210$_F$ |

Table 1: $Z_3$-transformations of various supermultiplets.
to the large Cabibbo mixing. It acts as $S \propto S_1 + 2S_2$, where $S_{1,2}$ are the two possible combinations in (11), so that $S^{(2)}_{ik} = 2S^{(1)}_{ik}$. Then eq. (14) leads to $K = -1/5$.

The exchange of $35_F$ and $35_F$ induces the operator $N$ relevant for the neutrino mass (see Fig. 2). Clearly, only one combination of neutrino states gets small Majorana mass in this way, since $N_{kl}$ in eq. (12) appears to be rank-1 matrix. Then neutrino oscillations are described by one large mixing angle.

Finally, operators $D, U$ are generated from the $F$-fermion exchanges shown in Fig. 3. The operator $D$ built in this way acts as $D \propto D_1 + D_3 - D_4$ with $D_{1,2,3,4}$ being the possible convolutions in eq. (10). According to eq. (13) this leads to $J = 8/5$. On the other hand, the operator $U$ built as in Fig. 3, can only mix $15_1$ state containing $u$ quark, with $15_2$ state containing $c$ quark, but cannot provide direct mass term for the former. As a result, the higher order operators obtained by the exchange of $F$-fermions given in Table 1, unambiguously reproduce the ansatz given in eqs. (21), with $J = 8/5$ and $K = -1/5$.

Before addressing the obtained fermion mass and mixing pattern, let us remark that actually our choice of the $F$-fermion content is a result of a rather general analysis. In constructing the higher order operators we have taken into account the following constraints:

(A) In order to ensure the rank-1 form (15) of the coupling matrices, each of the $d = 6$ operators $C, S$ should be induced by the unique exchange chain.

(B) Once the exchanges generating $C$ and $S$ are selected, the $d = 7$ operators $D$ and $U$ should be constructed by the exchange chains which are irreducible to $d = 6$ operators: otherwise the mass hierarchy between the first and second families would be spoiled. In other words, the exchange chains should not allow to replace $\Sigma_1 \times \Sigma_1$ by $\Sigma_2$, so that the (symmetric) tensor product $\Sigma_1 \times \Sigma_1$ should effectively act as the 189 or 405 representations of $SU(6)$. This condition requires the large representations like 105, 210, etc. to be involved into the game.

In fact, one can classify all possible exchanges satisfying the conditions (A) and (B). In particular, besides the exchange in Fig. 3, operator $D$ can be induced only by few irreducible chains involving even larger representations. These are:

$$
\begin{align*}
15_1 & \Sigma \left[ 21_F(384_F) + 21_F(384_F) \right] \Sigma \left[ 315_F + 315_F \right] H [120_F + 120_F] \Sigma 6_k \\
15_1 & \Sigma \left[ 384_F + 384_F \right] H \left[ 840_F(1260_F) + 840_F(1260_F) \right] \Sigma \left[ 84_F(120_F) + 84_F(120_F) \right] \Sigma 6_k \\
15_i & \Sigma \left[ 384_F + 384_F \right] H \left[ 840_F + 840_F \right] \Sigma \left[ 120_F + 120_F \right] \Sigma 6_k
\end{align*}
$$

(22)

where $\Sigma$ can be either $\Sigma_1$ or $\Sigma_2$. These exchanges induce $D$ respectively in the combinations $D_1 - D_2 + D_3 + D_4$: $J = 1$, $D_1 \mp D_4$: $J = 1$, $D_1 - 2D_2 - D_4$: $J = 11/17$, and thus they all lead to unacceptable situation $\lambda_d \leq \lambda_c$. Hence, $J = 8/5$ is selected as the only one feasible choice.

One can also classify the exchanges inducing the operator $S$. By scanning the relevant representations for the $F$-fermions, we have obtained that $S$ can appear only in the combinations $S_1$: $K = 1$, $S_2$: $K = -1/2$, $S_1 \pm S_2$: $K = 0, -2$ respectively, $S_1 - 2S_2$: $K = -1$, and $S_1 + 2S_2$: $K = -1/5$. We have chosen the latter case uniquely selected by the exchange in Fig. 1. All other cases are unacceptable: $K = 0$ ($|K| \geq 1$) leads to

---

10) In fact, by removing the $F$-fermions $20^2_F$, one could leave the $u$ quark massless. Though this possibility is somewhat dubious, it would naturally solve the strong CP-problem without invoking an axion.
massless (or too heavy) s quark, while $K = -1/2$ [9] in combination with $J = 8/5$ leads to unacceptably small $m_d/m_s (\approx 1/70)$.

Thus, among all possible exchanges only the selected ones lead to acceptable pattern for $D$ and $S$. As for the operators $C$ and $U$, the only possible exchanges obeying conditions (A) and (B) are the ones given in Figs. 1, 3.

Let us now analyse the obtained pattern of the Yukawa matrices [21]. The Yukawa coupling eigenvalues and CKM weak mixing matrix at the GUT scale are the following:

$$
\begin{align*}
3^{rd} \text{ family:} & \quad \lambda_t = G \sim 1, \quad \lambda_\tau = \lambda_b = \varepsilon_H^2 c_\sigma B \\
2^{nd} \text{ family:} & \quad \lambda_c = \varepsilon_H^2 s_\sigma C, \quad \lambda_\mu = -5 \lambda_s = \varepsilon_\Sigma \varepsilon_H^2 c_\sigma s_\sigma S_2 \\
1^{st} \text{ family:} & \quad \lambda_u = \varepsilon_\Sigma^2 \varepsilon_H^4 c_\sigma^4 s_\sigma^{-1} (U^\prime U/C), \quad \lambda_e = (5/8) \lambda_d = \varepsilon_\Sigma^2 \varepsilon_H^3 c_\sigma^3 D_1
\end{align*}
$$

(where the small corrections due to the mixing terms are neglected) and

$$
V_{\text{CKM}} \approx \begin{pmatrix}
1 & s_{12} & s_{12} s_{23} - s_{13} e^{-i\delta} \\
-s_{12} & 1 & s_{23} + s_{12} s_{23} e^{-i\delta} \\
s_{13} e^{i\delta} & -s_{23} & 1
\end{pmatrix}, \quad s_{12} \approx \frac{G_1}{G_2}, \quad s_{23} \approx \frac{S_3 \lambda_s}{S_2 \lambda_b}, \quad s_{13} \approx \frac{D_3 \lambda_d}{D_1 \lambda_b}
$$

where $\delta$ is the CP-phase. In order to confront these Yukawa constants to the masses of the quarks and leptons, we have to account for the renormalization group running. For the heavy quarks $f = t, b, c$ we take the values of their running masses at $\mu = m_f$, while for the light quarks $f = s, d, u$ at $\mu = 1$ GeV. Then we have [8, 20]:

$$
\begin{align*}
m_t &= 165 \pm 15 \text{ GeV} = A_u \eta_t y^6 \lambda_t v \sin \beta \\
m_b &= 4.25 \pm 0.10 \text{ GeV} = A_d \eta_b y \lambda_r v \cos \beta, \quad m_\tau = 1.784 \text{ GeV} = A_e \eta_\tau \lambda_r v \cos \beta \\
m_c &= 1.27 \pm 0.05 \text{ GeV} = A_u \eta_c y^3 \lambda_c v \sin \beta \\
m_s &= 100 - 250 \text{ MeV} = A_d \eta_s K \lambda_u v \cos \beta, \quad m_\mu = 105.6 \text{ MeV} = A_e \eta_\mu \lambda_u v \cos \beta \\
m_u &= (0.4 \pm 0.4) m_d = A_u \eta_u y^3 \lambda_u v \sin \beta \\
m_d &= (0.05 \pm 0.01) m_s = A_d \eta_d J \lambda_c v \cos \beta, \quad m_e = 0.51 \text{ MeV} = A_e \eta_e \lambda_c v \cos \beta
\end{align*}
$$

where $v = 174$ GeV,

$$
y = \exp \left[ -\frac{1}{16 \pi^2} \int_{\ln m_t}^{\ln M_X} \lambda^2_t(\mu) d(\ln \mu) \right]
$$

and, for $\alpha_s(M_Z) = 0.11 - 0.125$

$$
\eta_b = 1.5 - 1.6, \quad \eta_c = 1.8 - 2.3, \quad \eta_s, d, u = 2.1 - 2.8, \quad \eta_\tau, \mu, e = 0.99 \\
A_u = 3.3 - 3.8, \quad A_d = 3.2 - 3.7, \quad A_e = 1.5
$$

It is well-known that the $b - \tau$ Yukawa unification and moderate $\tan \beta$, both implied in our scheme, require rather large $\lambda_t$ ($\lambda_t \geq 2$, so that $y = 0.75 - 0.6$). Then the top ‘pole’ mass is given by its infrared fixed limit [17]

$$
M_t = m_t \left[ 1 + \frac{4}{3\pi} \alpha_3(m_t) \right] = (190 - 210) \sin \beta \text{ GeV} = 140 - 210 \text{ GeV}
$$

in agreement with the CDF result $M_t = 174 \pm 10 \pm 13 \text{ GeV}$ [21]. Clearly, in our model $\tan \beta$ should be rather moderate: $\tan \beta = 1.2 - 2$. Interestingly, this range is also favoured.
by the electroweak symmetry radiative breaking picture in the presence of \( b - \tau \) Yukawa unification. It is worth to mention the striking correlation between \( M_t \) and the mass of lightest Higgs boson \( M_h \). As far as \( M_t \) appears to be in the infrared fixed regime, this correlation is essentially determined by the value of \( \tan \beta \), providing strong upper limit on \( M_h \) for the low values of the latter (see [3] and refs. therein).

Then the experimental values of \( m_\tau \) and \( m_c/m_\tau \) respectively imply that \( \varepsilon_H^2 c_s B \approx 10^{-2} \) and \((C/B) \tan \sigma \tan \beta \approx 0.4 - 0.6\). From \( m_\mu/m_\tau \) and \( m_c/m_\mu \) we obtain \( \varepsilon_H s_a S_2 / B \approx 0.06 \) and \( \varepsilon H c_s^2 S_2^{-1} (D_1 / S_2) \approx 5 \cdot 10^{-3} \). The CKM mixing pattern \( |V_{us}| = 0.22, |V_{cb}| = 0.04 \pm 0.01 \) and \( |V_{ub} / V_{cb}| = 0.1 \pm 0.05 \) implies respectively \( G_2 / G_1 \approx 4, S_3 / S_2 \approx 3 \) and \( D_3 / D_1 \approx 3 - 4 \). Taking all these into the account, we see that our scheme gives an elegant understanding of all fermion masses and their mixing in terms of small ratios \( \varepsilon_H, \varepsilon_\Sigma \approx 0.1 \) and of the \( O(1) \) parameters \( G, B \ldots \) and \( \tan \sigma \).

Moreover, we obtain the relations \( \lambda_d = \frac{1}{5} \lambda_\mu \) and \( \lambda_d = \frac{8}{5} \lambda_e \), with possible \( \sim \varepsilon_\Sigma \) corrections that can arise due to mixing terms in (21). Thus, we have

\[
\frac{m_d}{m_s} \approx 8 \frac{m_e}{m_\mu} \approx \frac{1}{20} \left[ 1 + O(\varepsilon_\Sigma) \right]
\]  (29)

while for the quark running masses at \( \mu = 1 \) GeV we obtain

\[
m_s = \frac{1}{5} \frac{A_d \eta_s}{A_e \eta_\mu} m_\mu = 100 - 150 \text{ MeV}, \quad m_d = \frac{8}{5} \frac{A_d \eta_d}{A_e \eta_e} m_e = 4 - 7 \text{ MeV}
\]  (30)

5. Discussion

As we have seen above, the fermion mass and mixing pattern can be naturally explained in our scheme without appealing to any horizontal symmetry, provided that the scales \( M, V_\Sigma \) and \( V_H \) are related as \( V_\Sigma / V_H \sim V_H / M \sim 0.1 \). As far as the scale \( V_\Sigma \approx 10^{16} \) GeV is fixed by the \( SU(5) \) unification of gauge couplings, these relations in turn imply that \( V_H \approx 10^{17} \) GeV and \( M \approx 10^{18} \) GeV, so that \( M \) is indeed close to the string or Planck scale. On the other hand, the superpotential (2) includes mass parameters \( M_\Sigma \) and \( M_H \) which are not related to the large scale \( M \) and thus the origin of this hierarchy remains unclear. However, bearing in mind the possibility that our \( SU(6) \) theory could be a stringy SUSY GUT, one can assume that the superfields \( H, \bar{H} \) and \( \Sigma_{1,2} \) are zero modes, and the Higgs superpotential has the form not containing their mass terms:

\[
W = \lambda Y (\bar{H} H - \Lambda^2) + \lambda_1 \Sigma_1^3 + \lambda_2 \Sigma_2^3 + \frac{(\bar{H} H)}{M} (\Sigma_1 \Sigma_2)
\]  (31)

The last term can be effectively obtained by exchange of the singlet superfield \( Z \) with a large mass \( M \), as shown in Fig. 4. More explicitly, the relevant superpotential has the form

\[
\lambda_1 \Sigma_1^3 + \lambda_2 \Sigma_2^3 + \lambda Y \Sigma_1 \Sigma_2 + \lambda' Z \Sigma_1 \Sigma_2 + \rho Y (\bar{H} H - \Lambda^2) + \rho' Z \bar{H} H + M Z^2 + M' Y^2 + \ldots
\]  (32)

(obviously, the basis of two singlets always can be redefined so that only one of them, namely \( Y \) has a linear term). Then the relation \( V_\Sigma / V_H \sim V_H / M = \varepsilon_H \) follows naturally.
Certainly, the origin of small linear term ($\Lambda = \varepsilon_H M$) in (31) remains unclear. It may arise due to some hidden sector outside the GUT.

Let us conclude with the following remark. In our scheme all the higher order operators are induced by exchanges of the heavy particles with masses $\sim M$. In doing so, all higher order operators are under control and the unwanted higher order operators can be always suppressed by the proper choice of the heavy particle content. However, the higher order operators scaled by inverse powers of the Planck mass could appear also due to non-perturbative effects, in an uncontrollable way. If all such operators unavoidably occur, this would spoil the GIFT picture. For example, already the operator $\frac{1}{M_{Pl}}(\bar{H}\Sigma_1)(\Sigma_2H)$ would provide an unacceptably large ($\sim V^2_H/M_{Pl}$) mass to the Higgs doublets. One may hope, however, that not all possible structures appear in higher order terms. Alternatively, one could try to suppress dangerous high order operators by symmetry reasons, in order to achieve a consistent ‘all order’ solution. Some possibilities based on additional discrete (or R-type discrete) symmetries are suggested in [22].

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\[ B: \hspace{1cm} \begin{array}{c|c|c|c} 20 & 70_F & 15^3_F \times 15^3_F & \bar{6}_3 \\ \hline \Sigma_1 & \bar{H} & \bar{H} \end{array} \]

\[ C: \hspace{1cm} \begin{array}{c|c|c|c|c} 15 & 20_F & 20_F & 20_F & 20_F & 15 & 12 \\ \hline H & \Sigma_2 & H \end{array} \]

\[ S: \hspace{1cm} \begin{array}{c|c|c|c} 15' & 15^1_F \times 15^1_F & 15^2_F \times 15^2_F & \bar{6}_{2,3} \\ \hline \Sigma_1 & \Sigma_2 & \bar{H} \end{array} \]

Figure 1: diagrams giving rise to the operators \( B, C, S \) respectively.

\[ N: \hspace{1cm} \begin{array}{c|c|c|c} \bar{6}_k & 35_F & 35_F & \bar{6}_l \\ \hline H & \Sigma_1 & H \end{array} \]

Figure 2: the diagram giving rise to the operator \( N \) for neutrino mass.

\[ D: \hspace{1cm} \begin{array}{c|c|c|c|c} 15_i & 105_F \times 105_F & 210_F \times 210_F & 84_F \times 84_F & \bar{6}_k \\ \hline \Sigma_1 & \bar{H} & \Sigma_1 & \Sigma_1 \end{array} \]

\[ U: \hspace{1cm} \begin{array}{c|c|c|c} 15_i & 105_F \times 105_F & 20^1_F \times 20^2_F & 20_F \times 20_F & 15 & 12 \\ \hline \Sigma_1 & H & \Sigma_1 & H \end{array} \]

Figure 3: diagrams giving rise to the operators \( D \) and \( U \) respectively.
Figure 4: Diagram generating the operator \( \frac{1}{M}(\bar{H}H)(\Sigma_1 \Sigma_2) \).