Influence of Initial Disturbance on Flight Trajectory of Uncontrolled Projectiles on the Plateau

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Abstract. Initial disturbance will cause nutation motion of projectile and affect aerodynamic force, thus affecting the altitude flight trajectory of uncontrolled projectile. Through the analysis and simulation of angular motion caused by initial disturbance under plateau conditions and the simulation calculation of the variation of ballistic parameters with position altitude under the same initial disturbance conditions, it was found that the influence of initial disturbance on plateau ballistics decreases under plateau conditions. As for the change of ballistic parameters, it has great influence on range, lateral deviation and maximum ballistic height, but little influence on full flight time and falling speed.

1. Introduction

Because of the eccentricity of projectile mass, firing vibration, clearance between projectiles and guns, impact of aftereffect period and so on, the initial disturbance of projectiles at muzzle is inevitable. Initial disturbance will cause nutation motion of projectile and affect aerodynamic force, thus affecting flight trajectory of uncontrolled projectile[1]. For uncontrolled projectiles, this effect can not be eliminated by the change of aerodynamic force as for controlled projectiles. In the plateau condition, because of the decrease of air density and air pressure, the influence of initial disturbance on the flight trajectory of uncontrolled projectile is different from that in the plain condition. Based on the analysis of the attack angle equation, the influence of initial disturbance on the flight trajectory of uncontrolled projectiles at high altitude was studied by using a six-degree-of-freedom rigid body motion model. The variation of trajectory data with altitude of position under the same initial disturbance was simulated. The research results will help us to understand the trajectory changes caused by the initial disturbance of uncontrolled projectiles at high altitude.

2. Angular motion model generated by initial disturbance

2.1. Angle of attack equation

According to the external ballistics theory[2][3], the complex attack angle $\Delta$ can be expressed as:

$$\Delta = \delta_1 + i\delta_2$$

For the symmetrical spinning projectile, according to the theory of exterior ballistics, the variation rule of attack angle $\Delta$ can be described by the following models:

$$\Delta'' + (H - iP)\Delta' - (M + iP)\Delta = 0$$

(1)
In the formula, \( H = k_zz + b_y - b_x - g \sin \theta / v^2 \), \( P = \Gamma / n \), \( M = k_z \), \( T = b_y - nk_y \), speed ratio \( \Gamma = \gamma / v \), dimensionless quantity \( n = A/C \), and about \( b_x, b_y, k_y, k_z, k_{2D} \), there are:

\[
b_x = \frac{\rho S_u}{2m} C_D, \quad b_y = \frac{\rho S_u}{2m} C_{la}, \quad k_y = \frac{\rho S_l d}{2A} C_{mua}, \quad k_z = \frac{\rho S_l}{2A} C_{ma}, \quad k_{zz} = \frac{\rho S_l^2}{2A} C_{mua}.
\]

And:

\[
H = \frac{\pi \rho d^2 l^2}{8A} C_{mua} + \frac{\rho \pi d^2}{8m} (C_{la} - C_D) - g \sin \theta / v^2, \quad P = \frac{C \gamma}{Av}, \quad M = \frac{\pi \rho d^2 l}{8A} C_{ma},
\]

\[
T = \frac{\pi \rho d^2}{8m} C_{la} - \frac{\pi \rho d^2}{8C} C_{mua}.
\]

In the formula, \( \Delta' \) is the derivation of the attack angle \( \Delta \) to the dimensionless distance \( \Delta' = S = x/d \); \( x \) is distance; \( \Delta'' \) is the corresponding two derivative; \( d \) is the diameter of the projectile; \( m \) is the mass of the projectile; \( \rho \) is the air density; \( C \) is the polar moment of inertia; \( A \) is the equatorial moment of inertia; \( C_D \) is the drag coefficient; \( \gamma \) is the rotational speed.

According to the theory of differential equations, the solution of formula (1) is:

\[
\Delta = k_1 e^{i \phi} + k_2 e^{j \phi} \tag{2}
\]

In the formula, \( k_j = k_{j0} e^{i \phi_j}, \quad \phi_j = \phi_{j0} + \phi_{j} S, \quad j = 1, 2 \)

The solution is the attack angle equation.

2.2. Angular motion model

According to the vector representation of the complex, \( e^{i \phi} \) represents a vector whose modulus is 1 and whose amplitude angle is \( \phi_j \). When the frequency of the angle of motion is \( \phi_j \), the motion trajectory of the complex vector endpoint complex plane is a unit circle[4].

\[
\phi_j = \frac{1}{2} \left( P \pm \sqrt{P^2 - 4M} \right) \quad \lambda_j = -\frac{1}{2} \left( H \pm \sqrt{P(H - 2T)} \right) \sqrt{P^2 - 4M} \tag{3}
\]

As mentioned earlier, there are initial disturbances caused by eccentricity of projectile mass, firing vibration, clearance between projectiles and artillery, and impact of aftereffect period in the process of artillery launching. These factors are not analyzed one by one, but are unified into two angular parameters \( \Delta \) and \( \Delta_0 \) of projectile initial motion at muzzle. \( \Delta_0 \) is the derivative of \( \Delta_0 \) to time. Since the origin of the initial disturbance is basically the same in plains and plateaus, it can be considered that the initial disturbance does not change in plateau and plain conditions. However, due to the change of air density in Plateau conditions, the influence of initial disturbance on ballistic trajectory is bound to change.

Under the plateau conditions, the angular motion generated by the initial disturbance \( \Delta_0 \) can be described as:

\[
k_{10} e^{i \phi_{10}} = -\frac{i \Delta_0}{\sqrt{P^2 - 4M}} \quad k_{20} e^{i \phi_{20}} = \frac{i \Delta_0}{\sqrt{P^2 - 4M}} \tag{4}
\]

Under the plateau conditions, the angular motion generated by the initial disturbance \( \Delta_0 \) can be described as:

\[
k_{10} e^{i \phi_{10}} = -\frac{\phi' \Delta_0}{\sqrt{P^2 - 4M}} \quad k_{20} e^{i \phi_{20}} = \frac{\phi' \Delta_0}{\sqrt{P^2 - 4M}} \tag{5}
\]

According to equation (2) and the above two formulas, the angular motion caused by the initial disturbance is a two-circle motion composed of fast and slow circular motion, in which \( k_{10} \) and \( k_{20} \)
are the initial radius of the two circles, $\varphi_{10}$ and $\varphi_{20}$ are the initial angles of the two circles, $\varphi'_1$ and $\varphi'_2$ are the angular frequencies of the two-circle motion respectively, and also represent the frequency of the two-circle motion. With the flight of projectile, the radius of fast and slow circular motion should be attenuated. Otherwise, the impact of initial disturbance can not be overcome and the stable flight state can not be maintained. Similarly, the ability of the projectile to overcome the influence of initial disturbance also reflects the ability of the projectile to overcome random disturbance at any point in the trajectory, that is, the dynamic stability of the projectile.

3. Influence of initial disturbance on flight of uncontrolled projectiles at high altitude

3.1. Variation of angular motion caused by initial disturbance with elevation of position

Taking a certain grenade as an example, the angular motion caused by initial disturbance was calculated by using 6D rigid body ballistic model. In the case of initial disturbance $\dot{\delta}_{10} = 10 \text{ rad/s}$, $\dot{\delta}_{20} = 10 \text{ rad/s}$, $\delta_{10} = 0$, $\delta_{20} = 0$, the angular trajectory caused by different position elevations is shown in figures 1 to 3.

![Figure 1](image1.png)  Angular motion with a height of 1000m.
![Figure 2](image2.png)  Angular motion with a height of 3000m.
![Figure 3](image3.png)  Angular motion with a height of 5000m.

From the angular motion trajectory in the figure, it can be seen that, no matter how high the position is, the radius of the fast circle and the slow circle both attenuates, and the radius of the fast circle decreases faster. The two circle motion is finally reflected as one attenuated circular motion.

1. Under the plateau condition, the frequency of fast circular motion increases obviously, while the slow circular motion frequency decreases.

2. Under the plateau condition, the fast circular motion radius and the slow circular motion attenuation speed all slow down.

3. Under the plateau conditions, the radius of the two-circle motion decreases, which indicates that the angular motion attenuates faster under the plateau conditions, that is to say, the influence of the initial disturbance on trajectory decreases under the plateau conditions.

Under plateau conditions, the angular motion of the initial disturbance $\Delta_0$ is similar to that of $\Delta_0'$, because $\Delta_0'$ is the driving force in essence and the direct result is the change of the angle of attack $\Delta_0$. As long as the angular motion law produced by $\Delta_0'$ is analyzed clearly, the angular motion law produced by the initial disturbance $\Delta_0$ is similar.

3.2. Variation of ballistic parameters caused by initial disturbance with elevation of position

Initial disturbances also affect ballistic parameters. In the case of initial disturbance $\dot{\delta}_{10} = 10 \text{ rad/s}$, $\dot{\delta}_{20} = 10 \text{ rad/s}$, $\delta_{10} = 0$, $\delta_{20} = 0$, the variation of ballistic parameters with the change of altitude is shown in Table 1.
Table 1. Influence of initial disturbance $\Delta_0$ on trajectory with the change of altitude.

| H(M) | T/ (s) | V/ (m/s) | X/ (m) | Z/ (m) | Y_d/ (m) |
|------|--------|----------|--------|--------|----------|
| 0    | Undisturbed | 92.5 | 352.7 | 29209 | 819.6 | 10340.9 |
|      | Disturbance  | 91   | 349.9 | 28448 | 706.6 | 10017.3 |
|      | Percentage deviation | -1.6% | -0.8% | -2.6% | -13.8% | -3.1% |
| 1000 | Undisturbed | 95.5 | 375.8 | 32116.4 | 912 | 12004.2 |
|      | Disturbance  | 94.3 | 372.4 | 31382.6 | 786.7 | 11730.7 |
|      | Percentage deviation | -1.3% | -0.9% | -2.3% | -13.7% | -2.3% |
| 2000 | Undisturbed | 98.5 | 402.4 | 35396.5 | 1034.4 | 13699.5 |
|      | Disturbance  | 97.5 | 399.1 | 34716.1 | 892.7 | 13451 |
|      | Percentage deviation | -1.0% | -0.8% | -1.9% | -13.7% | -1.8% |
| 3000 | Undisturbed | 101.6 | 431.6 | 38904.4 | 1197 | 15420.9 |
|      | Disturbance  | 100.6 | 428.4 | 38252.2 | 1036.8 | 15185.3 |
|      | Percentage deviation | -1.0% | -0.7% | -1.7% | -13.4% | -1.5% |
| 4000 | Undisturbed | 104.5 | 463.3 | 42560.9 | 1377.6 | 17158.3 |
|      | Disturbance  | 103.6 | 459.9 | 41923.9 | 1201 | 16929.3 |
|      | Percentage deviation | -0.9% | -0.7% | -1.5% | -12.8% | -1.3% |
| 5000 | Undisturbed | 107.4 | 496.8 | 46283.4 | 1549.4 | 18900.2 |
|      | Disturbance  | 106.5 | 493.4 | 45655.2 | 1358 | 18673.9 |
|      | Percentage deviation | -0.8% | -0.7% | -1.4% | -12.4% | -1.2% |

From the data in the table, it can be seen that the initial perturbation will affect the variation of ballistic elements in the two cases of perturbation and non-perturbation. With the increase of the elevation of position, the variation of the trajectory deviation caused by the initial disturbance is as follows:

1. The initial disturbance has little effect on the total flight time, which will make the total flight time decrease and the influence degree decreases with the increase of altitude.
2. The initial disturbance has little effect on the velocity of the landing point, and the influence degree has little change, which makes the velocity of the landing point decrease about 8%.
3. The influence of initial disturbance on the range is relatively obvious, which makes the range decrease, and the degree of reduction decreases with the increase of altitude.
4. The initial disturbance has a relatively large influence on the sideslip of the landing point, which makes the sideslip distance of the landing point decrease, and the degree of decrease decreases with the increase of altitude.
5. The influence of initial disturbance on the maximum ballistic height is obvious, which makes the maximum ballistic height decrease, and the degree of decrease decreases with the increase of altitude.

4. Conclusion
Initial disturbance is inevitable in the process of artillery launching, and changes its aerodynamic force during the flight of uncontrolled projectiles, eventually affecting the flight trajectory, and this effect has new characteristics under plateau conditions. Through the analysis and simulation of angular motion caused by initial disturbance in plateau, and the simulation calculation of the variation of
ballistic parameters with elevation of position under the same initial disturbance, it is found that the influence of initial disturbance on plateau ballistics decreases under the plateau conditions. As for the change of ballistic parameters, it has great influence on range, lateral deviation and maximum ballistic height, but little influence on full flight time and falling speed.

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