Self-adaptive improved PSO algorithm to solve optimal operation of cascade reservoir systems

Y Q Xiang$^{1,2}$ and L Mo$^{1,2,3}$

$^1$School of Hydropower and Information Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China
$^2$Hubei Key Laboratory of Digital Valley Science and Technology, Wuhan 430074, China

E-mail: moli@hust.edu.cn

Abstract. The optimal scheduling problem of cascade reservoirs is an optimal control problem for a dynamic complex nonlinear system with a large number of constraints, which has not been satisfactorily solved so far. This paper presents a new improved particle swarm optimization (PSO) algorithm, which based on self-adaptive improvement of relative progress, to solve the problem of optimal scheduling of cascade reservoirs. The results obtained by the improved PSO algorithm are found to be encouraging when compared with PSO algorithm under similar test condition.

1. Introduction

The joint optimization of river basin cascade reservoirs is the main means to improve the operational management benefits of cascaded reservoirs in the basin. It needs to coordinate the head, flow and output relationship between reservoirs at all levels under the premise of meeting market and grid load demand, hydropower system constraints, upstream and downstream flood control safety, ecological security, and shipping safety [1].

At present, the methods used to solve the optimal scheduling model of cascade reservoirs are roughly divided into two categories. The first category is traditional linear and nonlinear methods [2,3], dynamic programming method [4] and other methods, but such methods have some problems such as dimensionality disaster, complex algorithm and unstable convergence. The second type of method is the swarm intelligence optimization algorithm (SIA), such as particle swarm optimization (PSO) algorithm [5], artificial fish swarm algorithm (AFSA) algorithm [6], ant colony algorithm (ACO) algorithm [7], the group spider optimization (SSO) algorithm [8], cuckoo search (CS) algorithm [9] and others. Because of its self-learning, self-organizing, self-adaptive features and simple, easy-to-understand, robust and parallel processing, it has been widely used in the optimal scheduling of cascade reservoirs and has achieved certain practical effects. However, as the search space becomes more and more complex and the search scale becomes larger and larger, the traditional intelligent optimization algorithms tend to be weak. Therefore, the improvement and fusion of various algorithms have become the hot research content for solving the reservoir optimization scheduling problem.

In this paper, based on the mathematical model of the reservoir group with the largest total power generation in the scheduling period as the optimal scheduling target, an improved PSO algorithm is applied to solve the optimal scheduling of a cascade reservoir group. It is characterized by adaptively...
adjusting the particle swarm parameters based on the relative progress of the particles, and then replacing
the worst individual optimal values in the particle swarm with the average of the individual optimal
values of all the particles. In this way, while improving the search efficiency and convergence
speed of the particle swarm, the diversity of the particle population can be increased and the local
optimality can be avoided, and the good information of other particles can be guided to the search, and
the probability of searching for the global optimal value is increased. Finally, through the calculation
in the simulation example, the solution results are compared with the results of the PSO algorithm to
verify the feasibility and effectiveness of the improved PSO algorithm in the optimization of the
cascade reservoir model.

2. Mathematical model for optimal scheduling of cascade reservoirs

2.1. Objective function
The mathematical model of the maximum power generation $E$ in the dispatching cycle is the optimal
scheduling target. The mathematical model can be expressed as:

$$\text{Max} E = \text{Max} \sum_{i=1}^{R} \sum_{t=1}^{T} A_i q_i H_i \Delta T$$  \hspace{1cm} (1)

where $A_i$ is the $i^{\text{th}}$ power station output coefficient; $q_i$ is the average power generation flow rate of the $i^{\text{th}}$ power station during the $t^{\text{th}}$ period, $\text{m}^3/\text{s}$; $H_i$ is the average power head of the $i^{\text{th}}$ power station in the $t^{\text{th}}$ period, which is calculated by subtracting average downstream water level from average upstream water level during the $t^{\text{th}}$ period; $R$ is the total number of hydropower stations; $T$ is the total number of hours; $\Delta T$ is the length of the period.

2.2. Constraints

2.2.1. Water balance constraint

$$V_{i,t+1} = V_{i,t} + (Q_{i,t} - q_{i,t} - S_{i,t}) \Delta T$$  \hspace{1cm} (2)

Where $V_i$ is the initial storage capacity of the $i^{\text{th}}$ reservoir and $V_{i,t+1}$ is the terminal storage capacity of the $i^{\text{th}}$ reservoir; $Q_{i,t}$ is the inflow of the $i^{\text{th}}$ reservoir in the $t^{\text{th}}$ period; $q_{i,t}$ is the power flow of the $i^{\text{th}}$ reservoir in the $t^{\text{th}}$ period; $S_{i,t}$ is the surplus water of the $i^{\text{th}}$ reservoir in the $t^{\text{th}}$ period.

2.2.2. Output constraints

$$N_{i,t,\text{min}} \leq N_{i,t} \leq N_{i,t,\text{max}}$$  \hspace{1cm} (3)

$$N_{i,t} = A_i q_i H_i$$  \hspace{1cm} (4)

Where $N_{i,t,\text{min}}$ and $N_{i,t,\text{max}}$ are, respectively, the minimum and maximum limit power outputs of the $i^{\text{th}}$ hydropower station during the $t^{\text{th}}$ period. $N_{i,t}$ is the power outputs of the $i^{\text{th}}$ hydropower station during the $t^{\text{th}}$ period.

2.2.3. Discharge flow constraint

$$q_{i,t,\text{min}} \leq q_{i,t} + S_{i,t} \leq q_{i,t,\text{max}}$$  \hspace{1cm} (5)
In which \( q_{it,\text{min}} \) and \( q_{it,\text{max}} \) are, respectively, the minimum and maximum allowable discharges of the \( i^{th} \) hydropower station during the \( t^{th} \) period.

2.2.4. Water level constraint

\[
Z_{it,\text{min}} \leq Z_{it} \leq Z_{it,\text{max}}
\]  

(6)

Where \( Z_{it,\text{min}} \) and \( Z_{it,\text{max}} \) are, respectively, the minimum and maximum allowable water levels of the \( i^{th} \) hydropower station during the \( t^{th} \) period.

3. Improved PSO algorithm

3.1. PSO algorithm

The Particle Swarm Optimization Algorithm [10] (PSO) is an evolutionary algorithm developed by Dr. Eberhart and Dr. Kennedy in 1995 from the behavioral study of bird predation. The algorithm finds the optimal solution by simulating the mutual cooperation mechanism in the foraging behavior of the group of animals such as birds and fish. On the basis of observing the activities of fauna, the PSO algorithm utilizes the information sharing of individuals in the group, so that the movement of the whole group produces an evolutionary process from disorder to order in the process of obtaining the optimal solution.

The update of the velocity and position of the particle is calculated by:

\[
V_{i}^{k+1} = \omega V_{i}^{k} + c_{1} \text{rand}_1 \left( pbest_{i} - X_{i}^{k} \right) + c_{2} \text{rand}_2 \left( gbest_{i} - X_{i}^{k} \right)
\]

(7)

\[
X_{i}^{k+1} = X_{i}^{k} + V_{i}^{k+1}
\]

(8)

Where \( k \) is the number of iterations; \( V_{i}^{k} \) and \( X_{i}^{k} \) are the velocity and position of the \( i^{th} \) particle at the \( k^{th} \) iteration; \( \text{rand}_1( ) \) and \( \text{rand}_2( ) \) are random numbers between 0 and 1; \( pbest_{i} \) is the best position for the \( i^{th} \) particle. \( gbest_{i} \) is the best position for the entire population at the \( k^{th} \) iteration; \( c_1 \) and \( c_2 \) are constants, Representing the weight of each particle to \( pbest_{i} \) and \( gbest_{i} \) and \( \omega \) is the inertia weight.

The steps of PSO algorithm are as follow:

Step 1: Initializing a group of particles randomly. Including the position, velocity and population size of the particle \( D \), and specifying the upper and lower limits of velocity \( V_{\text{max}} \) and \( V_{\text{min}} \), setting the inertia weight \( \omega \), learning factors \( c_1 \) and \( c_2 \), the maximum number of iterations \( K \) (In this calculation, \( D=20, \omega=0.8, c_1=1.5, c_2=1.5, V_{\text{max}}=2, V_{\text{min}}=-2, K=100 \));

Step 2: For the first time, the fitness value is calculated to select the individual extreme value and the global extreme value. According to the fitness function (related to the specific problem to be solved), the current fitness of each particle can be calculated, which can measure the position of the particle, and select the individual extreme value and the global extreme value.

Step 3: Iterating according to the formula to update the position and velocity of particles. If the velocity is greater than the maximum velocity, set it as the maximum velocity, and if it is less than the minimum velocity, set it as the minimum velocity;

Step 4: Continue calculating fitness values to evaluate particles and update individual extremes and global extremes.

Step 5: Check whether it meets the end conditions. If the number of iterations reaches the maximum number of times set in advance, stop iterating and output the optimal solution, otherwise go to Step 3.
3.2. Improved PSO algorithm
The difference between the fitness of two times of particle optimization is called particle progress. The ratio of the progress of the particle itself to the average progress of all the particles of the group is called the relative progress [11]. The greater the relative progress, the better the optimization effect of the particles in all particles. The influence of the "inertia" part of the particle should be increased to enhance the exploration ability of the particle, its own experience and the influence of group experience. The specific calculation is as follows:

\[
\Delta f_i^k = \begin{cases} 
  f_i^k - f_i^{k-1}, & f_i^k - f_i^{k-1} > 0 \\
  0, & f_i^k - f_i^{k-1} \leq 0 
\end{cases}
\]  

\[
\Delta h_i^k = \frac{\Delta f_i^k \cdot D}{\sum_{i=1}^D \Delta f_i^k}
\]

\[
o_i^k = o_{\text{min}} + (o_{\text{max}} - o_{\text{min}}) \frac{1}{1 + e^{-\Delta h_i^k}}
\]

\[
c1_i^k = c1 + \frac{1}{1 + e^{-\Delta h_i^k}}
\]

\[
c2_i^k = c2 - \frac{1}{1 + e^{-\Delta h_i^k}}
\]

Where \(\Delta f_i^k\) is the difference in fitness of the particle i between the \(k^{th}\) generation and the \((k-1)^{th}\) generation, \(\Delta h_i^k\) is the relative progress of particle i in the \(k^{th}\) generation, \(o_i^k\), \(c1_i^k\), and \(c2_i^k\) are the inertia weights and acceleration coefficients of particle i in the \(k^{th}\) generation.

The particle speed update formula is improved as follows:

\[
V_i^{k+1} = \omega V_i^k + c1 \cdot rand(0,1) \left( \sum_{i=1}^{D} \frac{pbest_i^k - X_i^k}{D} \right) + c2 \cdot rand(0,1) \left( \text{gbest} - X_i^k \right)
\]

The particle position update formula is unchanged.
Specific steps are as follows:

Step 1: Random initialization of a group of particles. Including the position, velocity and population size of the particle D, and specifying the upper and lower limits of velocity \(V_{\text{max}}\) and \(V_{\text{min}}\), setting upper and lower limits of inertia weight \(o_{\text{min}}\) and \(o_{\text{max}}\), learning factors \(c1\) and \(c2\), the maximum number of K=100;

Step 2: If the iteration k=0, then \(o_0\), \(c1^0\) and \(c2^0\) are \(o_{\text{max}}, c1\) and \(c2\) respectively; if k\geq1, each \(o_i^k\), \(c1_i^k\) and \(c2_i^k\) are calculated according to equations (9)~(13).

Step 3: According to the fitness function (related to the specific problem to be solved), the current fitness of each particle can be calculated.

Step 4: Update the speed and position of the particle i, update the individual optimal value, and update the global optimal value.
Step 5: Check whether it meets the end conditions. If the number of iterations reaches the maximum number of times set in advance, stop iterating and output the optimal solution, otherwise go to Step 2.

The difference between PSO algorithm and improved PSO algorithm can be seen from the comparison of figures 1 and 2 below.

![Figure 1. PSO algorithm flow chart.](image1)

![Figure 2. Improved PSO algorithm flow chart.](image2)

4. Simulation example
A cascade reservoir consists of two annual regulation reservoirs, A and B. Reservoir A is located upstream of reservoir B. The Capacity-elevation relation and the tail water level flow curve of reservoir A and B are shown in figure 3. The actual scheduling period is one year, and the year is divided into 12 periods, and every month is a period of time. Reservoir A is mainly for power generation, providing power supply for the power grid around the basin, and has other benefits such as flood control, water supply, shipping, tourism and fish farming. The normal water level of the power station is 1140 m, the total storage capacity is 4.497 billion m$^3$, and the regulating storage is 3.361 billion m$^3$. The total installed capacity is 600 MW, the guaranteed output is 157.1 MW, and the average annual power generation is 1.56 billion kW·h. Reservoir B is the second-stage power station of the main stream, 65 km from the upstream reservoir A. The reservoir is mainly for power generation. The normal storage level of the reservoir is 970 m. The storage capacity below the normal storage level is 0.864 billion m$^3$, the total storage capacity is 1.025 billion m$^3$, and the regulating storage is 0.491 billion m$^3$. The total installed capacity is 695 MW, and the guaranteed output is 236.1 MW. The annual average annual power generation is 2.24 billion kW·h. The inflow of reservoir A and reservoir AB are known.
Figure 3. (a) Capacity-elevation relation of reservoir A, (b) Tail water level flow curve of reservoir A, (c) Capacity-elevation relation of reservoir B and (d) Tail water level flow curve of reservoir B.

4.1. Algorithm Implementation Steps As described in Section 3.2, the optimal scheduling results for each program run are shown in the following tables 1-3:

| Time | Reservoir inflow (m³/s) | Reservoir A (initial water level 1076m) | Reservoir B (initial water level 936m) | PSO algorithm | Improved PSO algorithm |
|------|-------------------------|------------------------------------------|------------------------------------------|----------------|------------------------|
|      |                         | Water level at the end of the month (m) | Water level at the end of the month (m) | Generated Energy (Billion kW·h) | Generated Energy (Billion kW·h) |
| 6    | 194.0                   | 1082.93                                  | 1082.77                                  | 0.068           | 85.84                  | 0.069                  |
| 7    | 254.0                   | 1093.24                                  | 1092.98                                  | 0.074           | 90.66                  | 0.075                  |
| 8    | 245.0                   | 1101.58                                  | 1101.25                                  | 0.080           | 94.66                  | 0.081                  |
| 9    | 103.0                   | 1101.30                                  | 1100.85                                  | 0.083           | 94.95                  | 0.084                  |
| 10   | 82.6                    | 1099.86                                  | 1099.25                                  | 0.082           | 94.31                  | 0.083                  |
| 11   | 95.0                    | 1099.07                                  | 1098.34                                  | 0.082           | 93.87                  | 0.083                  |
| 12   | 58.2                    | 1096.04                                  | 1095.19                                  | 0.080           | 92.59                  | 0.081                  |
| 1    | 66.9                    | 1093.42                                  | 1092.36                                  | 0.078           | 91.28                  | 0.079                  |
| 2    | 68.4                    | 1090.81                                  | 1089.59                                  | 0.077           | 89.98                  | 0.077                  |
| 3    | 43.5                    | 1086.27                                  | 1084.83                                  | 0.074           | 87.87                  | 0.075                  |
| 4    | 63.7                    | 1082.95                                  | 1081.25                                  | 0.072           | 86.01                  | 0.072                  |
| 5    | 93.0                    | 1076.00                                  | 1076.00                                  | 0.112           | 157.30                 | 0.098                  |
|   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|   |     |     |     |     |     |     |     |     |     |     |
|   | 191.7 | 232.3 | 239.0 | 215.0 | 165.4 | 141.8 | 60.8 | 84.0 | 68.6 | 85.5 | 97.5 | 129.8 |
|   | 951.88 | 970  | 970  | 970  | 970  | 970  | 961.56 | 956.05 | 946.88 | 940.07 | 936  | 936  |
|   | 207.2 | 228.54 | 340  | 323  | 273.4 | 249.8 | 225  | 212.7 | 204.11 | 196.88 | 191.62 | 306.25 |
|   |     |     |     |     |     |     |     |     |     |     |     |     |
|   | 0.144 | 0.170 | 0.271 | 0.258 | 0.218 | 0.199 | 0.174 | 0.164 | 0.154 | 0.143 | 0.135 | 0.179 |
|   |     |     |     |     |     |     |     |     |     |     |     |     |
|   | 954.05 | 970  | 970  | 970  | 970  | 970  | 963.37 | 960.21 | 953.73 | 950.26 | 948.79 | 936  |
|   | 200.04 | 242.44 | 349  | 325  | 275.4 | 251.8 | 209.63 | 206.84 | 201.21 | 197.8 | 196.24 | 345.06 |
|   | 0.139 | 0.182 |     |     |     |     |     |     |     |     |     | 0.216 |

Annual power generation of cascade hydropower stations 3.170 billion kW·h 3.226 billion kW·h

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
|   | 16 | 17 | 18 | 19 | 20 | 21 |
|   |     |     |     |     |     |     |
|   | 232.3 | 239.0 | 215.0 | 165.4 | 141.8 | 60.8 |
|   | 970  | 970  | 970  | 970  | 970  | 961.56 |
|   | 228.54 | 340  | 323  | 273.4 | 249.8 | 225  |
|   | 0.170 | 0.271 | 0.258 | 0.218 | 0.199 | 0.174 |
|   | 970  | 970  | 970  | 970  | 970  | 963.37 |
|   | 242.44 | 349  | 325  | 275.4 | 251.8 | 209.63 |
|   | 0.182 |     |     |     |     | 0.167 |

Table 2. Algorithm to optimize average power head statistics.

|   | PSO algorithm | Improved PSO algorithm |
|---|---------------|------------------------|
|   | 134.00        | 134.95                 |
|   | 124.09        | 124.29                 |

Table 3. The algorithm to solve the results of the statistical table.

|   | Annual power generation of cascade hydropower stations (billion kW·h) | Average computing time(s) |
|---|-------------------------------------------------------------------------|---------------------------|
|   | PSO algorithm 3.170          | 471                       |
|   | Improved PSO algorithm 3.226 | 450                       |

4.2. Result analysis

Table 1 compare of the monthly power generation and the water level at the end of each month between the two algorithms, it shows the annual power generation of cascade hydropower stations for the improved PSO algorithm is better optimized. Table 2 lists the average power head of each power station under the two algorithms. The improved PSO algorithm improves the average power head of the power station compared with the PSO algorithm. From the table 3, the total annual power generation of cascade reservoirs calculated by improved PSO algorithm is about 3.226 billion kW·h, and the calculation time is 450 s. The result of PSO algorithm is 3.17 billion kW·h and the calculation time is 471 s. Compared with the PSO algorithm, the total annual power generation of the improved PSO algorithm is increased by 1.77%, and the calculation time is reduced. To conclude, the optimized results of the improved PSO algorithm for model simulation are obviously better than that of the PSO algorithm, and it has better optimization accuracy and global optimization ability.

5. Conclusion

In this paper, the water level of each period of the reservoir dispatch period is used as the decision variable, and the maximization power generation is taken as the objective function. Taking the cascaded reservoir system of a basin as the research object, the improved PSO algorithm and PSO algorithm are used to optimize the calculation, and the total annual power generation of each group is obtained. It can be seen from the optimization results of each algorithm that the improved PSO algorithm has better performance than the PSO algorithm. From the above, the superiority of the improved PSO algorithm in solving the problem of medium and long-term power generation optimization scheduling in cascade reservoirs is observed.
Acknowledgments
This work was supported by the National Key R&D Program of China (No. 2016YFC0402210) and the National Natural Science Foundation of China (No. 51479075), and the Fundamental Research Funds for the Central Universities (No. 2017KFYXJJ199).

References
[1] Ma L and Zhi Y T 2015 Review of joint optimal dispatching algorithms for cascade reservoir groups Yellow River 37 126-32
[2] Shawwash Z K, Siu T K and Russell S O D 2000 The BC hydro short term hydro scheduling optimization model IEEE T Power Syst 15 1125-31
[3] Guan X, Luh P B and Zhang L 2000 Nonlinear approximation in Lagrangian relaxation-based for hydrothermal scheduling problems IEEE T Power Syst 10 772-8
[4] Yang J S and Chen N M 1989 Short term hydrothermal coordination using multi-pass dynamic programming IEEE T Power Syst 4 1050-6
[5] Yang D H, Ma G W and Guo X M 2006 Application of particle swarm optimization in optimal dispatching of hydropower stations J Hydroelectr Eng 25 5-7
[6] Peng Y, Tang G L and Xue Z C 2011 Optimal dispatching of cascade reservoirs based on improved artificial fish swarm algorithms Syst Eng-Theory P 31 1118-25
[7] Xu G, Ma G W and Liang W H 2005 Application of ant colony algorithms in reservoir optimal dispatching Adv Water Sci 16 397-400
[8] Wang W C, Lei G J and Qiu L 2015 Application and efficiency analysis of swarm spider optimization algorithm in optimal dispatch of hydropower station J Hydroelectr Eng 34 80-7
[9] Ming B, Huang Q and Wang Y M 2015 Study on optimal operation of cascade reservoirs based on improved cuckoo algorithm J Hydroelectr Eng 46 341-9
[10] Kennedy J and Eberhart R C 1995 Particle swarm optimization Proceedings of IEEE International Conference on Neural Networks (ICNN’95) (Perth Australia) IV 1942-8
[11] Deng J B, Ma R and Hu Z W 2018 Optimal scheduling of micro grid with CCHP systems based on improved particle swarm optimization algorithm J Electr Power Sci Tec 33 35-42