Multi-Pair D2D Communications Aided by an Active RIS Over Spatially Correlated Channels With Phase Noise

Zhangjie Peng, Xue Liu, Cunhua Pan, Member, IEEE, Li Li, and Jiangzhou Wang, Fellow, IEEE

Abstract—This letter investigates a multi-pair device-to-device (D2D) communication system aided by an active reconfigurable intelligent surface (RIS) with phase noise and direct link. The approximate closed-form expression of the ergodic sum rate is derived over spatially correlated Rician fading channels with statistical channel state information (CSI). When the Rician factors go to infinity, the asymptotic expressions of the ergodic sum rates are presented to give insights in poor scattering environment. The power scaling law for the special case of a single D2D pair is presented without phase noise under uncorrelated Rician fading condition. Then, to solve the ergodic sum rate maximization problem, a method based on genetic algorithm (GA) is proposed for joint power control and discrete phase shifts optimization. Simulation results verify the accuracy of our derivations, and also show that the active RIS outperforms the passive RIS.

Index Terms—Reconfigurable intelligent surface (RIS), active RIS, device-to-device (D2D) communication, ergodic sum rate, phase noise, spatial correlation, power control.

I. INTRODUCTION

RECENTLY, reconfigurable intelligent surface (RIS) has emerged as a brand new communication paradigm to reconfigure the radio propagation environment in a desired manner [1]. To be specific, RIS is an array of reflecting elements, each of which can independently induce a phase shift on the incident signals. By carefully tuning the phase shifts, RIS can be utilized for reducing the sum power [2] and enhancing the system sum rate [3]. The RIS also possesses the advantages of small size, light weight, convenient deployment and low cost.

On the other hand, device-to-device (D2D) technology is also regarded as a promising solution to alleviate the capacity demand of local transmission. The integration of RIS in D2D communications has sparked extensive research efforts [4], [5]. In [4], the power allocation scheme was presented and an RIS was deployed to improve the system sum rate by eliminating the interference of D2D communications. In [5], the energy efficiency was maximized through jointly optimizing the power control and the phase shifts in D2D communication network.

However, the RIS considered in [1]–[5] is passive, which can only reflect the incident signals without any signal processing operations. The performance gain brought by the passive RIS is limited due to the “multiplicative fading” effect with strong direct links between the base station and the users [6]. To address this issue, active RIS has been proposed, which can amplify the incident signals in the electromagnetic level and adjust the phase shifts, concurrently [7]. Different from amplify-and-forward relays, the active RIS operates in full-duplex mode and is equipped with low-power reflection-type amplifiers instead of power-hungry radio frequency chains [8]. The active RIS inherits the advantages of the passive RIS, while its superiorities have been demonstrated in [9]. However, the contributions in [6]–[9] were based on the assumption of the availability of instantaneous channel state information (CSI), which results in high channel estimation overhead.

To tackle this issue, the authors in [10]–[13] designed the RIS phase shifts exploiting statistical CSI, which varies much slower and can relax the necessity for configuring the RIS frequently. Besides, the impact of spatial correlation is non-negligible since the reflecting elements are located close to each other due to the small size of the RIS. The performance of RIS-aided cell-free massive multiple-input multiple-output (MIMO) system was analyzed under the presence of spatial correlation [10]. The ergodic sum rate was studied and maximized in the RIS-aided MIMO multiple access channel system over spatially correlated Rician fading with statistical CSI [11]. The outage probability in RIS-aided communication system was investigated in [12] and the impact of the Von Mises phase errors of the reflecting elements was further studied in [13].

In this letter, we investigate a multi-pair D2D communication system aided by an active RIS over spatially correlated Rician fading channels, where only statistical CSI is available to reduce the channel estimation overhead. It is noted that our work differs from [14] where the RIS was used as a receiver. The contributions of this letter are summarized as follows: 1) The approximate closed-form expression of the ergodic sum rate is derived; 2) We propose a method based on genetic algorithm (GA) to maximize the ergodic sum rate through joint power control and discrete phase shifts optimization; 3) Simulation results validate the accuracy of our derivations and the superiority of the active RIS over the passive RIS.

II. SYSTEM MODEL

Consider a D2D overlaying communication system as depicted in Fig. 1, which serves K pairs of single antenna users. The base station acts as a controller, which not only allocates the power and the spectrum resources for the D2D user-pairs, collects all the statistical CSI estimated by
the devices but also performs the power control and the optimization of RIS phase shifts. The active RIS with $N = N_H N_V$ reflecting elements is deployed to further enhance the communication performance. The phase shift matrix is expressed as $\Theta = \text{diag}(e^{j\theta_1}, \ldots, e^{j\theta_n}, \ldots, e^{j\theta_N}) \in \mathbb{C}^{N \times N}$, where $n = 1, \ldots, N$. The phase noise is considered due to the hardware limit of the active RIS. The phase noise matrix is denoted as $\Phi = \text{diag}(e^{j\phi_1}, \ldots, e^{j\phi_k}, \ldots, e^{j\phi_N})$ and $\theta_n$ follows the circular normal distribution with zero mean and concentration parameter $\kappa \tilde{g}$ [15]. The amplification factor matrix is $\Lambda = \text{diag}(\kappa_1, \kappa_2, \ldots, \kappa_{\tilde{N}})$. The amplification noise is $\eta_F \sim \mathcal{CN}(0, \sigma_F^2 I)$ and $\eta_F \in \mathbb{C}^{N \times 1}$.

We denote the $i$-th D2D user-pair as $U_{A,i}$ and $U_{B,i}$ for $i = 1, \ldots, K$. The power of $U_{A,i}$ is $P_i$, and $s_{A,i}$ stands for the information symbol with unit power.

The direct channel link between $U_{A,i}$ and $U_{B,j}$ is denoted as $h_{i,j}$, which is assumed to follow the Rician fading in the short communication distance, i.e., $h_{i,j} \sim \mathcal{CN}(h_{i,j}, \sigma_{i,j}^2/(1 + \gamma_{i,j}))$ where $h_{i,j} = \sigma_{i,j}\sqrt{\gamma_{i,j}}/(1 + \gamma_{i,j})$. $h_{i,j}$ is the line-of-sight (LoS) component, $\sigma_{i,j}^2$ is the large-scale fading coefficient and $\gamma_{i,j}$ is the Rician factor. The reflecting channels $U_{A,i} \rightarrow$ RIS and RIS $\rightarrow$ $U_{B,j}$ are assumed to follow the spatially correlated Rician fading since the active RIS is usually deployed in high altitude with LoS components [16]:

$$g_{A,i} = \sqrt{\alpha_i \gamma_{A,i}}/(1 + \gamma_{A,i}) \tilde{g}_{A,i} + \sqrt{\alpha_i/(1 + \gamma_{A,i})} \tilde{g}_{A,i},$$

$$g_{B,i} = \sqrt{\beta_i \gamma_{B,i}}/(1 + \gamma_{B,i}) \tilde{g}_{B,i} + \sqrt{\beta_i/(1 + \gamma_{B,i})} \tilde{g}_{B,i},$$

where $g_{A,i}$, $g_{B,i} \in \mathbb{C}^{N \times 1}$. $\alpha_i$ and $\beta_i$ denote the large-scale fading coefficients. $\gamma_{A,i}$ and $\gamma_{B,i}$ are the Rician factors. $g_{A,i}$ and $\tilde{g}_{A,i}$ are non-line-of-sight (NLoS) components, which are distributed as $\tilde{g}_{A,i} \sim \mathcal{CN}(0, \mathbf{R})$ and $\tilde{g}_{B,i} \sim \mathcal{CN}(0, \mathbf{R})$, respectively. $\mathbf{R}$ stands for the normalized spatial correlation matrix under the isotropic scattering model, whose $(p, q)$-th element $[\mathbf{R}]_{p,q}$ is [16, eq. (10)].

$$r_{p,q} = \sin(c) \left(2\sqrt{(h(p) - h(q))^2 + (v(p) - v(q))^2}\right)/\lambda,$$

where $h(p) = \text{mod}(p - 1, N_H)$ and $v(p) = \lfloor(p - 1)/N_H \rfloor$. $d_H$ and $d_V$ are the horizontal and vertical element-spacing, $\lambda$ is the wavelength and we set $d_H = d_V = 1/2 \lambda$. $g_{A,i}$ and $\tilde{g}_{B,i}$ represent the LoS components under the uniform planar array (UPA) model, which are respectively written as

$$g_{A,i} = \left[1, \ldots, e^{j\pi^2 \sin(\theta_1) \sin(\phi_1) \cos(\phi_1^0 + \phi_2^0) \sin(\phi_1^0) + \cdots + \sin(\phi_N) \sin(\phi_N^0) + \cdots}\right]^T,$$

$$\sum_{i=1}^K P_i \mathbb{E}\left\{\|\Lambda \Theta \Phi A_{i} \|^2\right\} + \mathbb{E}\left\{\|\Lambda \Theta \Phi F \|^2\right\} = P_R,$$

where $\alpha_i$ and $\sigma_F^2$ denote the azimuth and elevation angles of arrival (AoA), $\phi_1$ and $\phi_2$ denote the azimuth and elevation angles of departure (AoD). The amplification power of the active RIS is given by

$$g_{B,i} = \left[1, \ldots, e^{j\pi^2 \sin(\theta_1) \sin(\phi_1) \cos(\phi_1^0 + \phi_2^0) \sin(\phi_1^0) + \cdots + \sin(\phi_N) \sin(\phi_N^0) + \cdots}\right]^T,$$

$$\sum_{i=1}^K P_i \mathbb{E}\left\{\|\Lambda \Theta \Phi A_{i} \|^2\right\} + \mathbb{E}\left\{\|\Lambda \Theta \Phi F \|^2\right\} = P_R,$$

where $\varepsilon$ is the amplifier efficiency, $P_{DC}$ and $P_{SW}$ represent the direct current biasing power consumption and the power consumption of the switch and control circuit at each reflecting element [7]. In addition, the signal received at the $j$-th D2D receiver is given by

$$\gamma_j = \frac{P_j |g_{j,i}|^2}{\sum_{i \neq j} P_i |g_{i,j}|^2 + \|g_{B,j} \Lambda \Theta \Phi A_{i} \|^2 \sigma_F^2 + \sigma_j^2}.$$

Then, the ergodic rate is expressed as

$$R_j = \mathbb{E}\left\{\log_2(1 + \gamma_j)\right\},$$

and the ergodic sum rate is $R = \sum_{j=1}^K R_j$.

### III. Ergodic Sum Rate Analysis

In this section, the approximate closed-form expression of the ergodic sum rate is derived with statistical CSI, which contains the spatial correlation, the location and angle information of the transceivers. First, we assume that the amplification factor of each element is the same, i.e., $\eta_h = \eta$, which is derived below.

**Lemma 1:** The amplification factor for each element on the active RIS is given by

$$\eta = \sqrt{P_j} / \sqrt{N \left(\sum_{i=1}^K P_i \alpha_i + \sigma_F^2\right)}.$$

**Proof:** With $\mathbb{E}\{\Phi H \Theta H \Lambda H \Theta \Phi\} = \eta^2 I_{N \times N}$, we can derive $\mathbb{E}\{\|\Lambda \Theta \Phi A_{i} \|^2\} = \alpha_i \mathbb{E}\{\Phi H \Theta H \Lambda H \Theta \Phi\} + \mathbb{E}\{\|\Lambda \Theta \Phi F \|^2\} = \eta^2 N \alpha_i$. Similarly, we have $\mathbb{E}\{\|\Lambda \Theta \Phi F \|^2\} = \eta^2 N \sigma_F^2$. Substituting these results into (6) yields $\gamma_j = \eta^2 N \alpha_i$. Based on Lemma 1, the approximation of the ergodic rate is given in the following theorem.

**Theorem 1:** For the considered multi-pair D2D communication system assisted by an active RIS over spatially
correlated channels with phase noise, the closed-form expression of the approximate ergodic rate of $U_{B,j}$ is given by $\bar{R}_j \approx \hat{R}_j$, where

$$\hat{R}_j = \log_2 \left( 1 + \frac{P_j \left( \eta^2 \Omega_{i,j} + 2\alpha \gamma_{ij} + \sigma_f^2 \right)}{\sum_{i:j} P_i \left( \eta^2 \Omega_{i,j} + 2\alpha \gamma_{ij} + \sigma_f^2 \right) + \eta^2 N \beta^2 \sigma_f^2 + \sigma_f^2} \right),$$

(12)

in which $\Omega_{i,j} = \alpha \beta_j N + \tau_i, \gamma_{ij} = \gamma_{A,i} \gamma_{B,j} A_j, \gamma_{B,j} L_{\phi,i}^\alpha \phi_i + \gamma_{A,i} L_{\phi,i}^\alpha \phi_i + L_0$, $1 - \bar{T}_j = \frac{(1+\alpha_{i,j} \gamma_{ij}) N}{1+\alpha_{i,j} \gamma_{ij}}$, $\tau_i, \gamma_{ij}$, $L_{\phi,i}, L_{\phi,i}$, and $Y_{i,j}$ are given in (23), (25), (27) and (30) in Appendix A.

Proof: See Appendix A.

Remark 1: 1) By setting the values of $\eta$ and $\sigma_f^2$ in (12) with $\eta = 1$ and $\sigma_f^2 = 0$, the ergodic rate in the passive RIS case is written as

$$\hat{R}_j^{\text{noRIS}} = \log_2 \left( 1 + \frac{P_j^i \bar{T}_j \sigma_f^2}{\sum_{i:j} P_i \bar{T}_j \sigma_f^2 + \sigma_f^2} \right).$$

(13)

2) Furthermore, when $\Omega_{i,j} = \Omega_{j,i} = 0$ and $Y_{i,j} = Y_{j,i} = 0$ for any $i$ and $j$, we derive the ergodic rate in the case without RIS, which is given by

$$\hat{R}_j^{\text{noRIS}} = \log_2 \left( 1 + \frac{P_j^i \bar{T}_j \sigma_f^2}{\sum_{i:j} P_i \bar{T}_j \sigma_f^2 + \sigma_f^2} \right).$$

(14)

The total power consumption in the passive RIS case and the case without RIS are $P_{\text{pass}} = \sum_{i=1}^{K} P_i^i + N P_{\text{SW}}$ and $P_{\text{noRIS}} = \sum_{i=1}^{K} P_i^i$, respectively. For fairness, we assume $P_T = P_{\text{pass}} = P_{\text{noRIS}} = P$. Besides, when the RIS hardware is ideal, i.e., $\Phi = Y_{i,j} N$, the ergodic rates without phase noise are rewritten by substituting $\kappa = 1$ into (12) and (13).

Corollary 1: When the Rician factors go to infinity, i.e., $\gamma_{A,i} \gamma_{B,j} \gamma_{A,i} \gamma_{B,j} = \gamma_{ij} \rightarrow \infty$ for any $i$ and $j$, the ergodic rates $\hat{R}_j$ in (12) and $\hat{R}_j^{\text{pass}}$ in (13) converge to

$$\hat{R}_j \rightarrow \hat{R}_j, \hat{R}_j^{\text{pass}} \rightarrow \hat{R}_j^{\text{pass}},$$

(15)

where $\hat{R}_j$ and $\hat{R}_j^{\text{pass}}$ are given by (16) and (17), shown at the bottom of the page, respectively.

Corollary 1 means that when the communication environment has limited scatters, the ergodic rates converge to fixed values depending on the phase shifts.

Corollary 2: If the total power consumption $P \rightarrow \infty$ (i.e., $P_R \rightarrow \infty$, $P_i \approx P_i^i \approx P_j \approx P_j^j \rightarrow \infty$ for any $i$ and $j$) and the direct links are non-existent due to the blockage, we can derive that $\hat{R}_j \rightarrow \tilde{R}_j$ and $\hat{R}_j^{\text{pass}} \rightarrow \tilde{R}_j$, where

$$\tilde{R}_j = \log_2 \left( 1 + \frac{\Omega_{i,j}}{\sum_{i:j} \Omega_{i,j}} \right).$$

(18)

Corollary 2 shows that the ergodic rate of the active RIS aided system with high transmit power converges to the same value as the passive RIS counterpart. This is because the dynamic noise caused by active RIS and the static noise are negligible, and the power of both the desired signal and the multi-pair interferences increase by the same factor of $\eta^2$. In other words, the ergodic sum rate in passive RIS case is close to the active counterpart at high power consumption.

Remark 2: If $R = Y_{i,j} N$, the reflecting links are simplified to uncorrelated Rician fading channel. Then, we consider a special case that there is only one user-pair ($U_{A,i}$ and $U_{B,i}$) and the direct link is blocked, the ergodic rate becomes $\bar{R}_i = \log_2 \left( 1 + \frac{P_i P_B^i \gamma_{A,i} \gamma_{B,i}}{P_{\text{pass}} \gamma_{A,i} + \gamma_{B,i}} \right)$.

Proof: See Appendix B.

IV. POWER CONTROL AND PHASE SHIFTS OPTIMIZATION

In this section, we propose a GA-based method to jointly optimize the transmit power and the phase shifts to maximize the ergodic sum rate. Due to the hardware limit, the discrete phase shift design is considered and the optimization problem is formulated as follows:

$$\max_{p,\theta} \tilde{R} = \sum_{i=1}^{K} \tilde{R}_i,$$

(19a)

s.t. $0 < p_i \leq p_{\text{max}}, \forall i = 1, \ldots, K,$

$$\theta_n \in \{0, 2\pi / 2^B, \ldots, (2\pi - 1) 2\pi / 2^B\}, \forall n = 1, \ldots, N,$$

(19b)

where $p = [p_1, p_2, \ldots, p_K], \theta = [\theta_1, \theta_2, \ldots, \theta_N]$ and $B$ is the number of bits for controlling the phase shifts.

Note that GA is a globally search method inspired by imitating the biological evolution, which can automatically accumulate knowledge about the search space, and adaptively control the search process to obtain the optimal solution. The details of our proposed GA-based method are given in Algorithm 1 on the next page.

V. NUMERICAL RESULTS

In this section, we provide simulation results to demonstrate the correctness of our derivations and evaluate the performance of the proposed algorithm. The RIS is deployed at (30 m, 0 m, 8 m) with $N = 32$ elements. The number of D2D user-pairs are set to $K = 6$, which are distributed in a rectangular plane with four points (0 m, 0 m, 1.6 m), (60 m, 0 m, 1.6 m), (0 m, 25 m, 1.6 m) and (60 m, 25 m, 1.6 m). The large-scale fading coefficients are $PL = -30 - 10 \log_{10}(d)$ dB, where $\chi$ is the path-loss exponent, and $d$ is the distance between the D2D users (or the D2D user and the RIS) in meters. We set $\chi_d = 3.8$ for the direct links and $\chi_r = 2.2$ for the reflecting links. The AoA and AoD of all channels are randomly generated from $[0, 2\pi]$. The Rician factors are set to 10 dB. Unless stated otherwise, we set $P = 30$ dBm, $B = 3$, $\kappa = 4$, $\sigma_f^2 = -70$ dBm, $\sigma_f^2 = -80$ dBm [8], $P_{DC} = -5$ dBm, $P_{\text{SW}} = -10$ dBm, $\varepsilon = 0.8$ [7]. Note that the curves marked “PCPSO”
Algorithm 1 GA-Based Method

Input: the population size $N_{po}$, the number of parents $N_{pa}$, the number of mutated chromosomes $N_{mu}$, the maximum iteration number $N_i$ and the termination fitness value $f$.

Output: the elite $e_i = \{p_i, \theta_i\}$ of each generation.

Initialize: the first generation $e_s$ for $s = 1, \ldots, N_{po}$.

1: for $t = 1$ to $N_i$ do
2: Calculate the fitness value $f(e_s) = \frac{1}{\mathcal{R}(p_s, \theta_s)}$.
3: Note down $e_t$ with the minimum fitness value;
4: if $f(e_t) > f$ then
5: Applying roulette wheel selection on the individuals (except $e_t$) to get $N_{po}$ parent chromosomes;
6: Cut off the parent chromosomes at a random place $c$ for $c = 1, \ldots, K + N$;
7: Hybridize them to produce children chromosomes;
8: Chose $N_{mu}$ chromosome and mutated places $n'_1$, $n'_2$ at random for $n'_1 = 1, \ldots, K$ and $n'_2 = 1, \ldots, N$;
9: Update $p_{s_{n'_1}}$ with a random value subject to (19b);
10: Update $\theta_{n'_2}$ with a random value subject to (19c);
11: end if
12: end for

This letter investigated the system performance for a multi-pair D2D communication system aided by an active RIS over spatially correlated channels with phase noise and direct link. The approximate closed-form expression of the ergodic sum rate was derived and analyzed under different channel conditions. The impact of the phase noise on the ergodic sum rate was also discussed. The proposed GA-based method was shown to be an effective way on performance improvement compared with equal power allocation. Therefore, deploying an active RIS with a small number of reflecting elements is sufficient to achieve significant performance gains.

VI. CONCLUSION

This letter investigated the system performance for a multi-pair D2D communication system aided by an active RIS over spatially correlated channels with phase noise and direct link. The approximate closed-form expression of the ergodic sum rate was derived and analyzed under different channel conditions. The impact of the phase noise on the ergodic sum rate was also discussed. The proposed GA-based method was effective for ergodic sum rate maximization. The simulation results showed that the active RIS outperforms the passive RIS.

APPENDIX A

By using [18, Lemma 1], the ergodic rate can be approximated by

$$R_j \approx \log_2 \left( 1 + \frac{P_j \mathbb{E}\{|g_{i,j}|^2\}}{\sum_{i \neq j} P_i \mathbb{E}\{|g_{i,j}|^2\} + \mathbb{E}\left\{ \left| \mathbf{h}_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \right|^2 \right\} \sigma_k^2 + \sigma_j^2} \right).$$

Since $\mathbf{g}_{A,i}$, $\mathbf{g}_{B,i}$ and $\mathbf{h}_{i,j}$ are independent of each other, we have $\mathbb{E}\{|g_{i,j}|^2\} = \mathbb{E}\{|g_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i}|^2\} + \sigma_{i,j}^2 + 2\mathbb{E}\{\text{Re}\{\mathbf{g}_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \mathbf{h}_{i,j}\}\}$. First, we derive that

$$\mathbb{E}\{|\mathbf{g}_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i}|^2\} = \mathbb{E}\{|\mathbf{g}_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{\Gamma}_{B,A}^H \mathbf{\Phi}^H \mathbf{\Gamma}_{A,B} \mathbf{g}_{B,j}|^2\}$$

$$= \gamma_{A,i} \gamma_{B,j} \mathbb{E}\{|\mathbf{g}_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{\Gamma}_{B,A} \mathbf{\Phi}^H \mathbf{\Gamma}_{A,B} \mathbf{g}_{B,j}|^2\} + \gamma_{B,j} \mathbb{E}\{|\mathbf{g}_{B,j}^T \mathbf{A} \mathbf{\Theta} \mathbf{\Gamma}_{B,A} \mathbf{\Phi}^H \mathbf{\Gamma}_{A,B} \mathbf{g}_{B,j}|^2\}.$$
where \((\alpha)\) is derived by removing zero terms. Then, we can obtain the following result according to the Euler’s formula.

\[
\mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \right\} = \mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \right\} = \mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \right\} + \mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \right\} \tag{20}
\]

We can substitute (11), (28)-(30) into (20) to complete the proof.

\[\begin{align*}
\gamma_A, & \mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \mathbf{H} \mathbf{g}_{B,j}^* \right\} \\
+ & \mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \mathbf{H} \mathbf{g}_{B,j}^* \right\} \\
+ & \mathbb{E}\left\{ \mathbf{g}_{B,j}^\text{T} \mathbf{A} \mathbf{\Theta} \mathbf{g}_{A,i} \mathbf{H} \mathbf{g}_{B,j}^* \right\},
\end{align*}\]

**APPENDIX B**

From (22), we can infer that
\[N + \bar{\Gamma}_{i,i} \leq \left( \sum_{n=1}^{N} \left| e^{j\theta_n} \left[ \mathbf{g}_{B,i}^\text{T} \mathbf{g}_{A,i} \right]_n \right| \right)^2 = N^2, \tag{31}\]

which indicates that the optimal phase shift of each element on the active RIS with ideal hardware is \(\theta_n^{\text{opt}} = -\frac{\pi}{2} \left( h(n) d_H (\sin \phi^e_i \cos \phi^e_i + \sin \phi^c_i \cos \phi^c_i) + v(n) d_V (\sin \phi^c_i + \sin \phi^c_i) + C \right)\), where \(C\) is an arbitrary constant. Recalling \(\Omega_{i,i}^{\text{opt}}\) in Remark 2, we have \(\Omega_{i,i}^{\text{opt}} = \tau_{i,i} (\gamma_A, \gamma_B) N^2 + (\gamma_A, \gamma_B) N + N\). Substituting \(\Omega_{i,i}^{\text{opt}}\) with \(\Omega_{i,i}^{\text{opt}}\) into (21) yields
\[\bar{R}_1^{\text{opt}} = \log_2 \left( 1 + \frac{E_s \mathbb{P}_E \mathbb{S}_i (\gamma_A, \gamma_B)^2 + \frac{1}{4} (\gamma_A, \gamma_B) + 1}{\rho_0 \beta \sigma^2 + \rho_0 \alpha \sigma^2 + 2 \sigma^2} \right).\]

When \(N \to \infty\), we have \(\frac{1}{N} \to 0\) and \(\bar{R}_1 \to \bar{R}_1^{\text{opt}}\). We arrive at the final result after removing the zero terms.

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