Rate based call gapping with priorities and fairness between traffic classes

Benedek Kovács\textsuperscript{a,b},
\textsuperscript{a} Ericsson Research Hungary
\textsuperscript{b} Department of Mathematical Analysis, Budapest University of Technology and Economics H-1111, Egry József u. 1, Hungary
E-mail: benedek.kovacs.mr@gmail.com

Abstract—This paper presents a new rate based call gapping method. The main advantage is that it provides maximal throughput, priority handling and fairness for traffic classes without queues, unlike Token Bucket which provides only the first two or Weighted Fair Queuing that uses queues.

The Token Bucket is used for call gapping because it has good throughput characteristics. For this reason we present a mixture of the two methods keeping the good properties of both.

A mathematical model has been developed to support our proposal. It defines the three requirements and proves theorems about if they are satisfied with the different call gapping mechanisms. Simulation, numerical results and statistical discussion are also presented to underpin the findings.

I. INTRODUCTION

There are many overload and load sharing problems to be solved in telecommunication networks of various kind e.g. in the Internet Multimedia Subsystem. Considering any type of network and signalling protocol a protocol operation flow consists of messages. The network nodes are entities receiving these messages and they process them using their resources such as CPU capacity or memory. In case they lack the resource to process the message we say that the node is overloaded.

To avoid such situations the node itself can deny to serve the request and reject (send a negative reply message) or drop (ignore) it. Another solution can be that the sender (source) does not send out the message if for some reason it knows that the target will not be able to serve it. In both cases there is a decision logic deciding upon admission of the request or sending out the request (see Figure 1). This entity is called the throttle which is in the center of our interest. From now on we use the following terminology and model (see Figure 2).

- The throttle decision function is a function mapping from the offer load point process to the set \{admission, rejection\}. (Each throttle is uniquely assigned a function $\gamma$ that transforms the intensity process $\rho(t)$ of the income process to an intensity process of the admissions $\gamma(\rho(t)) = a(t)$.)
- An offer is the event for which the throttle has to decide on admission or rejection. If an offer is admitted it cannot be rejected (dropped) and vice versa, and there is no third possibility. An offer has properties: arrival time, priority level and class which can be measured.
- The traffic class and the priority level sets have finite elements.
- The offered traffic (or offer load) is the flow of offers modeled with a progressively measurable not necessarily stationary point process marked with the marks from the mark space that is the direct product of the set of priorities and classes. (This implies that the probability of two offer events occurring at the same time is zero.)
- The admitted traffic (or throughput) is the flow (i.e. the point process) of admitted offers (offers for which the throttle yields admission). The flow of admitted offers can be conditioned upon the whole history (past) of the offer load flow and upon the throttle parameters and of course on the decision strategy.

The above assumptions and definitions are natural and obvious and also necessary to make the discussion clear.

The throttle entity discussed here is one very important and well defined part of overload control systems of any type as it has the role to reject (or drop) an offer or to let it go through: admit it. The throttle realizes a call gapping mechanism if it makes the decision based only on previous offers i.e. no offers in the future are examined. This also means that in our case the non-anticipative throttle is not allowed to delay an
offer and only one offer arrives at a time i.e. the call gapping mechanism cannot buffer the offer and admit it later than it has arrived. This makes a fundamental difference from Weighted Fair Queuing and mechanisms like those in [4], [3].

Many call gapping mechanisms have been developed for different purposes with different characteristics. One of the most important call gapping algorithms is the Crawford algorithm [5]. It does not differentiate between incoming offers. One of the most common solutions that handles priority levels and some kind of traffic classes is the Token Bucket call gapping mechanism [11]. This one is also popular because it is also used to characterize telecommunication traffic [3].

The aim here is to present a traffic estimation based call gapping mechanism that can provide traffic share Service Level Agreement, like weighted fair queuing mechanisms but without queuing the traffic. We discuss the following requirements for a call gapping rate limiting throttle mechanism. The typical verbal definitions given here preliminary, are not precise and many contradict and can have multiple exact (i.e. mathematical) definitions with different results.

- **Requirement-A Maximal throughput with bound:** No offer should be rejected if there is enough available capacity in the system to serve it, but no offer should be admitted if there is not enough available capacity to serve it in the system.
- **Requirement-B Priority levels:** Each offer may be assigned a priority level and the offer with higher priority shall be admitted in favor of the one with the lower priority level.
- **Requirement-C Throughput share for traffic classes:** The offers can be classified and for the traffic class \( i \) the \( s_i \) portion of the capacity of the target shall be provided.

In this paper we give exact definitions of these requirements.

In Section [II] we show how the Token Bucket mechanism meets Requirement-A and then for Requirement-B. Token Bucket does not fulfill Requirement-C. In Section [III] our new method is presented. We give mathematical definitions of all the requirements and prove that our new method meets Requirement-A and Requirement-C. Then we present a call gapping method that is a mixture of the latter two and also show that it meets the requirements. In Section [IV] we present our simulations and some figures about the offer and admission traffic flows with the three mechanisms. Using statistics we show how each mechanism meets Requirement-B.

## II. Token Bucket Throttle

We do not want to go into details discussing the throughput regulation properties of a Token Bucket algorithm (defined e.g. in patent [7] and used e.g. in standard [11]), but it is necessary to give a brief description to underpin the assumptions of our model. At first we present the concept of Token Bucket then show how it was extended to meet Requirement-B.

### A. The Token Bucket with parameters \((r, W)\)

The Token Bucket call gapping mechanism is the following: there is a bucket of available tokens representing available resources (free capacity) of the system. Requests are offered to the system and each of them is assigned a number of tokens needed i.e. the amount of resources it requires to be served. Once there are enough tokens in the bucket the request is admitted and dropped otherwise. (Thus no queues are applied and no delay is present in the system because of the Token Bucket call gapping algorithm.)

By the definition of the original Token Bucket the tokens are generated into the bucket with exponential distribution and the offers arrive with a Poisson process in most models that means that the time interval between the arrivals is also exponentially distributed. We analyze and describe a variant of this.

At first we mention that decision about serving a request are often implemented differently. The most important difference is in the interpretation: rather than consuming the tokens the bucket fill \( b \) is increased when a request arrives. The token generation is then realized with decreasing the bucket fill. The maximum fill is the watermark \( W \) that cannot be exceeded and also the bucket fill can not be lower than 0. This concept is equivalent to the original algorithm.

Secondly we consider deterministic token generation instead of the exponential one that is used in most cases (e.g. [11]), because it is much easier to implement and sometimes to analyze, as well.

Then the Token Bucket mechanism we discuss works as follows: When a new request arrives at \( t_n \) than the needed bucket fill is calculated: \( b(t_n) \) as if the request was served. This is done with calculating the expected number of tokens that would have been generated from the time the former service was served \( (t_{n-1}) \) then multiply it with the throughput capacity of the bucket i.e. the Token Bucket rate at \( t_n \): \( r(t_n) \) and subtract it from the former bucket size at \( b(t_{n-1}) \). Then it tests it against the preset constant watermark: \( W \).

### Definition 1 (Token Bucket call gapping strategy \( \gamma(r,W) \))

\[
b(t_n) = \max\{\chi(t), b(t_{n-1})-r(t_{n-1})(t_n-t_{n-1})+\chi(t)\},
\]

where \( \chi(t) = 1 \) iff there is an offer. Admit if \( b(t_n) \leq W \). If the offer is admitted, the above definition is used for the next value of the bucket fill \( b \). If the offer is rejected, then \( b(t_n) \) is recalculated with \( \chi(t) = 0 \).

(In many solutions the offers for the bucket can be of different types with different resource needs and thus \( S_i \chi_i(t) \) is used for update, where \( S_i \) is the so-called “splash amount” i.e. the expected number of tokens needed to serve the request of type \( i \). From now on we suppose that \( S_i = 1 \), since the calculations would be much more difficult without any qualitatively different result with respect to the requirements we consider now.)

### B. Priority handling with Token Bucket

Once the offered traffic is modeled with a point process and the throttle meets Requirement-A we cannot provide priority between the offers. Why? Suppose that we have an offer in the system and we have to decide if we should admit it or not. Requirement-A tells us to admit the offer if we have
the capacity to serve it. Suppose that this is the case and see that if the throttle would not admit the current offer to reserve this capacity for offers of higher priority then it might happen that there will be no higher priority offer in the future and the throttle would suffer a loss of workload.

However, giving up the maximal throughput requirement some priority handling naturally can be done. In the Token Bucket concept different watermarks are assigned to each priority level. The offers of lower priority are checked with a lower watermark. This is kind of reserving a set of tokens (system resources) to the higher priority traffic. This method violates Requirement-A whenever \( \hat{b}(t) \) declines to 0 before rejecting an offer. Whenever this event has a low probability, using different watermarks for different priority levels is a good solution to meet Requirement-B with a Token Bucket throttle.

### III. Call Gapping With Rate Estimation

In this Section our new method, the proposed rate based call gapping throttle is presented. At first we introduce the complete proposed procedure clearly. Then we discuss and prove how it provides all the requirements and what possible extensions, modifications or other solutions might result a similar good algorithm. At the end of the discussion we present relationship between the new method and the original Token Bucket algorithm.

#### A. The new call gapping algorithm \( \gamma_g(c, T, g, s) \)

Suppose that the consecutive offers arrive to the throttle at \( ... < t_{n-1} < t_n < t_{n+1} < ... \) time instants respectively. Each offer has a well defined priority level \( j \), \( j \in 1..J \) and traffic class \( i \), \( i \in 1..I \). Each priority level \( j \) has a constant priority parameter \( T_j \) assigned (\( T_j \geq T_k \)) if the offer with priority \( k \) has the higher priority) and each traffic class \( i \) has a pre-configured weight \( \sum s_i = 1 \). For each \( i \) the algorithm maintains an estimation of the incoming offer rate \( \hat{\rho}_i(t) \), a provisional admission rate \( \hat{\alpha}_i(t) \) from which it calculates a bounding rate \( g_i(t) \) and then according to the decision it estimates an admission rate \( \hat{\alpha}_i(t) \).

We suppose that the rate of the throttle varies with the following function: \( c(t) \). (This value is determined and given for the algorithm and represents the capacity of the throttle and might be different from \( v(t) \)).

**Definition 2** (The rate based call gapping \( \gamma_g(c, T, g, s) \)). Define the proposed throttle decision strategy \( \gamma_g \) in the following way. Suppose that at \( t_n \) an offer arrives and the system is in state \( \{t_{n-1}, \hat{\rho}_i(t_{n-1}), \hat{\alpha}_i(t_{n-1})\} \) and \( c(t_n) \):

1. Determine priority constants, i.e. calculate \( T_j \);
2. Update the incoming rates estimate for all \( i \): \( \hat{\rho}_i(t_n) \) with \( \chi_k(t_n) = 1 \) iff \( i = k \), 0 otherwise;
3. Calculate a provisional admission rate for all \( i \): \( \hat{\alpha}_i(t_n) \) with \( \chi_k(t_n) = 1 \) iff \( i = k \), 0 otherwise;
4. Calculate the bounding rate for class \( i \) only: \( g_i(t_n) \);
5. If \( \hat{\alpha}_i \leq g_i \) then admit the offer and \( a(t_n) := \alpha(t_n) \) else reject the offer and update \( \hat{\alpha}_i(t_n) \) with \( \chi_k(t_n) = 0, \forall k(t_n) \);
6. (Continue with 1. for the next event).

We propose to update \( \hat{\rho}_i, \hat{\alpha}_i \) according to the following equation:

\[
\hat{\lambda}(t_n) := \frac{\chi(t_n)}{T_j} + \max\{0, \frac{T_j \hat{\lambda}(t_{n-1}) - (t_n - t_{n-1})\hat{\lambda}(t_{n-1})}{T_j}\},
\]

where \( \hat{\lambda} \) is an estimator asymptotically unbiased for the \( \lambda(t) \) real intensity of a point process thus to be replaced by \( \hat{\rho}_i, \hat{\alpha}_i \) and indicator \( \chi_i(t_{n-1}) = 1 \) iff the offer is of type \( i \) and 0 otherwise (or further specified like in step 5). Note that the time parameter \( T_j \) changes in time too according to the priority level and the former one always has to be remembered.

To calculate the bound rate at first we introduce \( u(t) \) the provisional used capacity according to Requirement-B:

\[
u(t) := \sum_{\forall i} \min\{s_i c(t), \hat{\rho}_i(t)\}
\]

\[
= \sum_{\hat{\rho}_i(t) \leq s_i c(t)} \hat{\rho}_i(t) + \sum_{s_i c(t) < \hat{\rho}_i(t)} s_i c(t)
\]

Then the remaining (unused) capacity in the system is \( c(t) - u(t) \). This has to be split between traffic classes with higher incoming rate then the agreed share \( \hat{\rho}_i(t) > s_i c(t) \). Then

\[
g_i(t) := \min\{\hat{\rho}_i(t), s_i c(t) + (\hat{\rho}_i(t) - s_i c(t)) \} c(t) - u(t) \]

\[
\rho - u(t) \quad \text{(4)}
\]

It is important to see that our method is capable to handle other class-wise throughput criteria than fair sharing and maximal throughput. Giving upper or lower bounds for \( g \) one can implement fairly simple throttle mechanisms.

As one can see the new method is more complex than the original token bucket mechanism. However, the processing cost of updating the few variables introduced is significantly smaller than processing the offers thus does not count even in case of overload.

#### B. \( \gamma_g \) Meets All the Requirements

Now that the strategy is introduced we prove that it meets all the requirements. At first we define each requirement mathematically then we show how they are satisfied. We introduce some notation to make the discussion clear.

- \( c(t) \) represents the true capacity of the system expressed in rate, i.e. some deterministic value coming from an external input source.
- \( \rho(t) \) is the real intensity of the offered traffic and \( \hat{\rho}(t) \) its estimate with \( \hat{\rho}(t) \).
- \( a(t) \) is the real intensity of the admitted traffic and \( \hat{\alpha}(t) \) is the estimation of the rate intensity with \( \hat{\rho}(t) \).
- \( \alpha(t) \) is the preliminary admitted traffic intensity for which the following stands: \( \alpha(t) = a(t), \forall t < t_n \) and \( \alpha(t_n) \) is the intensity \( \alpha(t_n) \) would have if the offer was admitted at time \( t_n \), and its estimate is \( \hat{\alpha}(t) \) accordingly.
1) **Requirement-A**: This requirement consists of two parts. At first it says that there exists an upper bound for the system that should not be exceeded, i.e. it limits the admission rate to avoid overload. Secondly, it tells us that once the limit is not exceeded then all the offers should be admitted to maximize the utilization. However, in theory the words capacity and bound can have many different definitions depending on the model we use for the target node.

The target node is often modeled with an inverse Token Bucket, i.e. server with deterministic serving rate \( s \) and a queue of maximal length \( Q \). It is very easy to see that the Token Bucket throttle \( \gamma(s, Q) \) can perfectly meet the requirement in this case. (Note that this is true supposed that there is no delay in the system between the throttle and the protected entity while \( s(t) = r(t) \) is satisfied.)

Another approach is to assume that the target can handle requests on a maximal call rate \( c \) that is used as the bound at the throttle.

Both models have benefits and drawbacks while a mixture of them is used in practice. Speaking about the capacity of a node in Next Generation Networks engineers often refer the call rate value in industrial contracts and Service Level Agreements. It is very important to note that the feedback driven overload control mechanisms work with call rate information too (see \[\Pi\]). On the other hand a server with queue is a common model in the academic literature for the CPU capacity and Token Bucket (or versions of it) is proposed in many standards (e.g. \[\Pi\] again) and implemented into nodes.

As a consequence we say that although it is rather difficult to give exact definition for **Requirement-A** we can give some definition grabbing a few properties depending on the method we use.

**Definition 3. Call rate bound.** **Requirement-A** is met if
\[
\sum E[a_i(t_n)] \leq c(t_n) \quad \text{(the throughput rate is bounded in expected value)}.
\]

**Theorem 1.** The throttle with strategy \( \gamma_g \) meets the call rate bound requirement.

**Proof:** The proof relies on the fact that the estimator is asymptotically unbiased i.e. \( \lim_{T \to + \infty} E[\hat{\alpha}; T] = E[\alpha] \) with negative bias if \( T \geq 1/a_i \) (thus \( E[\hat{a}_i] < E[a_i] \)). The proposed strategy \( \gamma_g \) limits \( a_i \) so that \( a_i \leq g_i \) thus we are ready if we show that \( g(t) := \sum g_i(t) = c(t) \).

Define \( u_1(t) := \sum_{i:j_i(t)<\hat{\rho}_i(t)} s_i c(t) \) and \( u_2(t) := \sum_{i:j_i(t)<\hat{\rho}_i(t)} s_i c(t) \) thus \( u = u_1 + u_2 \) and then \( g_i = \min(\hat{\rho}_i, s_i c + (\hat{\rho}_i - s_i c) \frac{c - u}{\rho - u}) \). Although the system is non-stationary it is homogenous in time so \( f(t) = \text{const.} \) for all functions. Now calculate \( g(t) \):

\[
g = \sum g_i = \sum \min\{\hat{\rho}_i, s_i c + (\hat{\rho}_i - s_i c) \frac{c - u}{\rho - u}\} = \\
= \sum_{i:j_i(t)<\hat{\rho}_i} \hat{\rho}_i + \sum_{i:j_i(t)<\hat{\rho}_i} s_i c + (\hat{\rho}_i - s_i c) \frac{c - u}{\rho - u} = \\
g = u_1 + u_2 + (\rho - u_1 - u_2) \frac{c - u_1 - u_2}{\rho - u_1 - u_2} = c. \tag{5}
\]

**Corollary 1.** The following calculation of \( g \) can also be used:

\[
g_i(t) = \min\{\hat{\rho}_i, s_i c(t) + (\hat{\rho}_i(t) - s_i c(t)) \frac{c(t) - u(t)}{\rho - u(t)}\}, \tag{6}
\]

where \( u(t) = \sum_{i:t_i(t)<s_i c(t)} \hat{\rho}_i(t) = \alpha(t) \). Then \( g \) becomes:

\[
g' = u_1 + u_2 + (\rho - u_1 - u_2) \frac{c - u_1 - u_2}{\rho - u_1 - u_2} = c. \tag{7}
\]

The difference between the two strategies is that in case of \( g \) the remaining capacity is split between the classes with higher offer rates proportionally to their weights while using \( g' \) the remaining capacity is split proportionally the remaining offer rates. Both satisfies **Requirement-A** and as we will see **Requirement-C**. From now on \( g \) means either \( g \) or \( g' \) and the results will be the same obviously.

2) **Requirement-B**: As pointed out before, the priority requirement for call gapping is the most complex in a way since in the gapping algorithms it is supposed that we make decisions using measures on the past and the present offer. No future events can be used thus **Requirement-B** is always satisfied. There is always one offer in the system and the throttle can admit or reject it according to **Requirement-A** and **Requirement-B**.

In case of the Token Bucket call gapping different watermarks \( W_j \) are introduced for each priority level \( j \). One interpretation is that the bucket allows larger peaks for traffics with higher priority thus \( W_j < W_k \) whenever \( k \) represents the higher priority level. Doing this, the bucket implicitly reduces the throughput for lower priority traffics (the extra peak in the bucket has to be refilled with tokens i.e. \( b(t) \) has to decline below the low watermarks to admit low priority traffic). Note that the different watermark levels has no effect if the offer rate is low with small peaks thus the rejection probability is small i.e. if there is no overload. Supposed that the true bound is \( W = \max\{W_j\} \) this system preserves capacity for high priority traffic.

We give a similar solution for the problem through the timer parameter of the estimators: \( T \). As it was defined we introduce a function of \( T : j \mapsto T_j \) where \( T_k \leq T_j \) if \( k \) represents the higher priority. (Note that it is the other way around for \( W_j \)s.) The interpretation is that the estimator forgets the high offer rates faster for the traffic of the higher priority. Let \( T_m = \min\{T_j\} \), the true bound on the throttle using different \( T_j \)s, means that for low priority traffic it remembers the high peaks for a longer period thus reserves capacity for the higher priorities similarly to the Token Bucket.

The two methods have different characteristics, but one thing is common. Both reserve capacity for higher priority traffic. Now we say that to meet **Requirement-B** the system has to have this ability and define it in the following way.

**Definition 4** (Requirement on priorities.). Suppose that the throttle has rejected an offer at time \( t_{n-1} \). Let \( t_{n,j} \) be the closest time the throttle is able to admit an offer of priority
level j. **Requirement-B** is met iff \( \forall k, l(t_{n,k} \leq t_{n,l}) \Leftrightarrow (k \geq l) \) (\( k \) represents a higher priority).

The exact proof of this statement is not ready yet. Simulation result shows that the proposed strategy satisfies **Requirement-B**. We discuss the statement in the Numerical Results Section.

3) **Requirement-C**: This is referred to as the throughput share requirement and tells us that there should be at least an \( s_i \) portion of the capacity dedicated to traffic class \( i \).

**Definition 5** (Requirement-C). The Minimum share requirement is met if \( \forall i : (\rho_i(t_n) \leq s_i c(t)) \Rightarrow E[a_i(t_n)]=\rho_i(t_n) \) i.e. if the offer rate of a traffic class is less than the agreed share it should be fully admitted.

**Theorem 2.** The throttle with strategy \( \gamma \) meets Requirement-C in expected value.

Proof: At first we have the asymptotical unbiasedness for our estimators thus \( \lim_{t \rightarrow +\infty} E[a_i(t_n)] = E[a_i] \) the proof is true for the expected value of \( a_i \).

Statement \( \hat{\alpha}_i(t_n) = \hat{\rho}_i(t_n) \) whenever \( \forall i \hat{\rho}_i(t_n) \leq s_i c(t) \) is equivalent to the statement \( g_i(t_n) \geq \hat{\alpha}_i(t_n) \) thus \( g_i(t_n) \geq \hat{\alpha}_i(t_n) \) whenever \( \hat{\rho}_i(t_n) \leq s_i c(t) \). According to strategy \( \gamma \): \( g_i(t_n) = \hat{\rho}_i(t_n) \) whenever \( \hat{\rho}_i(t_n) \leq s_i c(t) \) and since \( \hat{\alpha}_i(t_n) \leq \hat{\rho}_i(t_n) \) it is true that \( \hat{\alpha}_i(t_n) \leq g_i(t_n) \) thus the offer is admitted (and also \( \hat{\alpha}_i(t_n) \leq g_i(t_n) \)).

**C. Rate model for Token Bucket and a joint algorithm merging the methods**

In this section we introduce a model for Token Bucket that is equivalent to the definition in Section 3 but makes calculations easier.

**Definition 6.** Token Bucket Rate Model Strategy: \( \gamma \) Let us define \( T(t) = W/r(t) \) and use the following equation for updating the bucket rate variable:

\[
\hat{\alpha}(t_n) = \frac{\chi(t_n)}{T} + \max\{0, \frac{T\hat{\alpha}(t_{n-1}) - (t_n - t_{n-1})r(t_n)}{T} \}
\]

where \( \chi(t) = 1 \) iff there is an offer at time \( t \). Admit the offer iff \( \hat{\alpha}(t_n) \leq r(t_n) \). If the offer is admitted then the above definition is the used for the next value of the bucket rate variable \( \hat{\alpha}(t) \). If the offer is rejected then \( \hat{\alpha}(t_n) \) is recalculated with \( \chi(t) = 0 \).

**Theorem 3.** The Token Bucket and the Token Bucket Rate Model Strategy are the same: \( \gamma \) = \( \gamma \).

Proof: It is easy to show that \( b(t_{n-1}) = \hat{\alpha}(t_{n-1})T \Rightarrow b(t_n) = \hat{\alpha}(t_n)T \) and the decision is \( b = T\hat{\alpha}(t) \leq Tr(t) = W \) also trivial.

If one extends the Token Bucket for traffic class handling with some role like in the proposed mechanism it will not provide traffic class fairness. The reason is hidden in the fact that unlike \( \hat{\rho}, \hat{\alpha}, \hat{\beta}\) and all such estimators is not asymptotically unbiased i.e. \( E[\hat{\gamma}] = \gamma \) as \( t \rightarrow +\infty \) is not true for the estimators defined with:

\[
\hat{\lambda}(t_n) = \frac{\chi(t_n)}{T} + \max\{0, \frac{T\hat{\lambda}(t_{n-1}) - (t_n - t_{n-1})r(t_n)}{T} \}
\]

The bucket fill does not represent at all the used capacity in the system it only measures the peakedness of the traffic but these peaks can happen on low offer rates too.

On the other hand, the proposed method does not allow such big transient peaks in the traffic. Now we aim to make the proposed new call gapping to behave like Token Bucket. We define the following strategy that is a mixed architecture.

**Definition 7.** Rate Based Call Gapping with Bucket-type Aggregates Characteristics: \( \gamma \) Take all the definition from the new call gapping mechanism \( \gamma \) for \( \hat{\rho}, \hat{\alpha}, \hat{\beta}, \hat{\eta}, \hat{\gamma} \) and define \( T_j(t) = W_j/r(t) \). Take \( W_j \) and the bucket fill change definition \( b \) from the original token bucket \( \gamma \). Perform all the steps like in \( \gamma \) but decide using the following constraint equation:

\[
\frac{\hat{b}(t_n)}{W_j} \hat{\alpha}(t_n) \leq \hat{g}_i(t_n).
\]

We will show numerically that the mixed algorithm behaves like Token Bucket on aggregate level and meets all the requirements. The source of the idea comes from the fact that \( \hat{\alpha}(t) \) places a strict bound on the rate thus \( \hat{\alpha}(t) \leq r(t) \) is always true as required. However we decrease the value of \( \hat{\alpha} \) and thus allow peaks in the traffic like Token Bucket does. (See that Token Bucket \( \gamma \) allows temporary bounding violation rate-wise unlike \( \gamma \) but like \( \gamma \). The bucket size related to the whole bucket is a kind of measure of this violation.)

1) \( \gamma \) and \( \gamma \) and Requirement-A : Here we discuss how the different algorithms meet the maximal throughput requirement. It is obvious that Token Bucket cannot meet Requirement-A in the way it was defined before since that definition assumed that the target has an infinite queue.

We do not aim to give an exact definition to Requirement-A but we derive relations between the bucket and the estimator based throughput characteristics. The number of admitted offers i.e. the probability of admission is in the center of our interest.

The probability of admission for token bucket depends on the offer rate with the following formula: \( 1 - \text{Erlang}[^{\rho}][r] \). Thus the probability of losing calls is only defined at given values of \( \rho \).

For rate based call gapping, since the estimator always overestimates the rate \( \rho < \lambda \) and cuts the traffic strictly with \( c \) the admission rate is always below the target. But for the same reason it is possible that the offer is rejected although it could have been accepted according to the bound. The probability of this is the probability of estimating higher rate than \( c \) while the true offer rate is lower: \( P[\hat{\alpha} > c | \alpha < c] = 1 - \frac{P[\alpha < c - B[T]]}{P[\alpha < c]} \), where \( B[T] = 1/(T(1 - F[T]) + E[\Delta(t < T)]) - \alpha \) is the bias. (Knowing the exact bias if constant intensity is supposed for the offer rate, the bound can be modified to have maximal throughput and strict bound at the same time.)

The two methods can only be compared at a given value of the intensity. For all those values when the
intensity is not between $c - B[T]$ and $c$ the $\gamma_g$ strategy works perfectly. The Token Bucket drops a call with positive probability for any value of the offer rate and also might admit when the intensity is higher than allowed. This means that we cannot tell which method is better or has the higher throughput since it depends very much on the offer rate.

Theorem 4. The mixed strategy $\gamma_x$ meets Requirement-A with appropriate watermark settings.

Proof: It is shown in Theorem 1 that $\sum g_i(t) = g(t) = c(t)$ and since the definition of $g$ was not changed we should only examine what means to compare $g_i$ to $\frac{a}{W_j}$ rather than to $a$.

When we admit a request then $1 \leq b(t_n) \leq W_j \leq W_{\text{max}}$ thus $\frac{1}{W_{\text{max}}} \hat{\alpha}_i \leq \frac{1}{W_j} \hat{\alpha}_i \leq \frac{b(t_n)}{W_j} \hat{\alpha}_i \leq \hat{\alpha}_i$. This tells us that $\gamma_x$ lets through more messages than $\gamma_g$ since $E[b(t_n) \hat{\alpha}_i] \leq E[\hat{\alpha}_i]$. Fortunately the maximal watermark limits this overflow error $\frac{1}{W_{\text{max}}} E[\hat{\alpha}_i] \leq E[\frac{b(t_n)}{W_j} \hat{\alpha}_i]$. It tells us that there is a setting of watermarks that guarantees bounding. (It is obvious that if $W_{\text{max}} \to +\infty$ then $\frac{1}{W_{\text{max}}} \hat{\alpha}$ becomes very small and we always admit the request thus the theorem cannot be proved for any watermark settings.)

2) $\gamma_x$ and Requirement-B: Some simple theorems are proved to show that the mixed strategy meets the priority and the throughput share requirements.

Theorem 5. Token Bucket strategy $\gamma_t$ meets Requirement-B.

Proof: Obviously, the time to accept the next offer of priority level $j$ is the time when the bucket level declines sufficiently to $b(t) \leq W_j$. For all levels $k > j$, $W_k > W_j$ i.e. $b(t)$ declines under the lower threshold later in time and the requirement is met.

Again it is rather hard to show that the mixed strategy $\gamma_x$ meets Requirement-B. However, it seems to be trivial that $\gamma_x$ satisfies Requirement-B more drastically than $\gamma_t$ does. We have interesting simulation results presented about this property. We can see numerical results about this in Section IV.

3) $\gamma_x$ and Requirement-C:

Theorem 6. The mixed strategy $\gamma_x$ meets Requirement-C.

Proof: As pointed out $\gamma_x$ admits at least all the offers $\gamma_g$ does since $\forall i, \frac{b}{W_j} \hat{\alpha}_i \leq \hat{\alpha}_i$ is compared to $s_i \hat{\alpha}$ while a comparison of $\hat{\alpha}$ would be enough. This means that the mixed strategy provides minimum throughput share and fulfills Requirement-C.

IV. Numerical results and analysis

Although we have nice proofs on the good behavior of the proposed rate based call gapping mechanism the complete mathematical discussion about the differences and similarities with Token Bucket is not ready yet. It is also true that the requirements can be interpreted with definitions slightly different from those we gave. Therefore we would like to present some simulation results and show that the findings are valid.

The simulation is written in Mathematica [13] and a notebook is available at http://www.math.bme.hu/kovacsbl/rcbg/BENEDEK-KOVACS-rate-based-call-gapping-PRELIMINARY-VERSION.nb as an electronic appendix.

A. Requirement-A

The figures shows that all the mechanisms limit the admitted offer rate while try to keep the highest throughput. In this scenario we examine the traffic on aggregate level i.e. there is only one traffic class for which the capacity of the throttle should be maximized and limited. The capacity is 1 offer/sec for the simple simulation case while the average number of offers per sec raises from 0.8 to 2 meaning that there is a 200% load on the node.

![Fig. 3. The new algorithm ($\gamma_g$) on aggregate level](image)

As it can be seen in Figure 3 all three mechanisms limit the admitted traffic although Token Bucket allows considerable peak at the beginning. (The size of the peak depends on the parameters we set. Here the 1 offer/sec capacity is very small compared to the watermark what is set to 10.) On the other hand, rate based call gapping seems to under-utilize the system while the joint mechanism seems to have the smoothest and also maximal throughput.

After a total 600 offers from each traffic with the same exact trajectory the results shows that $\gamma_t, \gamma_g, \gamma_x$ has admitted 415, 386, 404 number of calls respectively.

The problem with the mathematical discussion of maximal throughput is that the results depend very much on the value of the offer rate and capacity. It is only possible to compare the mechanisms at given rates what is not available in the world.

B. Requirement-B

To discuss Requirement-B we provide the reader with some statistical results. The sample is generated with our simulation program. Generally there are two priority levels: normal and emergency calls. Each call is one of the two types with 1/2 probability. The means and the standard deviation are presented of 100 samples with 10 000 offers handled in each
The further setups for the simulation can be seen on Table IV-B. It can be seen that all three methods reject less offer from those of higher priority but Token Bucket ($\gamma_t$) and the mixed mechanisms ($\gamma_x$) enforce a more strict priority handling than the simple proposal. Note that in case of sustained overload (row 2) almost all dropped offers are the lower priority ones.

### C. Requirement-C

The results tell explicitly that unlike the new rate based call gapping proposal the original Token Bucket algorithm does not meet Requirement-C. We consider a scenario when there are two traffic classes Class A and Class B. The agreed share for Class A is the 20% of the total capacity of the node while the share for Class B is the remaining 80%. The offer rates set for the simulator are exactly the inverse of this for the two type of traffic.

The aggregate offer rate increases from 0.7 offers/sec to 2 offers/sec and reaches the scenario of 100% overload (the capacity of the node is mean 1 offer/sec while the offered rate is a mean 2 offers/sec). The offer rate of traffic Class B is 0.4 i.e. it is still under its provided share thus all such calls are admitted. On the other hand the whole remaining capacity should be granted to traffic Class A and it should be admitted on a higher level than the agreed share and only those exceeding the capacity limit are to be rejected.

Both methods are able to reach the accepted share for Class A but Token Bucket (γt) is not able to get it since it never offers on a higher rate than the agreed share. With the proposed method there is no rejected message of Class B since it never offers on a higher rate than the agreed share. The throughput of the throttle is limited but also maximized since Class A is granted all remaining capacity.

### V. Conclusions

We have presented the “rate based call gapping” mechanism and its extension with the original Token Bucket mechanism. These unique mechanisms meet the maximal throughput with bound requirement, handle priorities and give minimum share for different traffic classes without using message buffers or queues.

Examing the properties of the mechanisms we gave mathematical definitions of the three requirements and accompanied the mathematical model with several theorems. Still the proof of priority handling is missing for the new methods, rather we have statistical analysis with the simulation we have coded to underpin our proposal and findings.

Our rate based call gapping strategy can use different traffic intensity estimators. It is still an open question to find the optimal estimator or the optimal parameter setting of the estimators considering Poisson input traffic with variable intensity or even non-Poisson (e.g. general renewal or Hawkes type) input process.

### VI. Appendix

#### Notations:

| No | H | $\gamma_{H}$ | 0.1 | 0.38 | 0.1 | 0.01 |
|---|---|---|---|---|---|---|
| 90 | 100 | W_H = 10 | [0.1] | [0.38,0.62] | [0.01,0.99] |
| 150 | 100 | W_H = 10 | [0.2,0.98] | [0.4,0.6] | [0.05,0.95] |
| 10 | 10 | W_L = 10 | [0.1] | [0.31,0.69] | [0.1] |
| 10 | 10 | W_L = 10 | [0.5,0.5] | [0.5,0.5] | [0.5,0.5] |

**Fig. 4.** The new algorithm ($\gamma_g$) with two traffic classes.

**Fig. 5.** The special bucket algorithm that is not able to do meet any criteria because the bucket size has nothing to do with the offer and admission rates.
| Symbol | Definition |
|--------|------------|
| \( a, \alpha(t) \) | Real admission rate |
| \( \hat{a}, \hat{\alpha}(t) \) | Estimated admission rate |
| \( b, b(t) \) | Actual bucket fill |
| \( c(t) \) | Maximal capacity of the target (rate) |
| \( g_i, g_i(t) \) | Goal rate for traffic class \( i \) |
| \( g, g(t) \) | Sum of goal rates of all traffic classes |
| \( r, r(t) \) | Token Bucket token generation rate |
| \( T \) | Parameter of the estimator |
| \( T_j \) | Parameter of the estimator for priority level \( j \) |
| \( u, u(t) \) | Used capacity according to Requirement-B |
| \( W \) | Watermark for Token Bucket |
| \( W_j \) | Watermark for offers of priority level \( j \) |
| \( \hat{\alpha}, \hat{\alpha}(t) \) | Estimated preliminary admission rate |
| \( \beta, \beta(t) \) | Preliminary bucket size |
| \( \gamma_t \) | The token bucket throttle function |
| \( \gamma_g \) | The rate based call gapping throttle function |
| \( \gamma_g' \) | The variant of the rate based call gapping throttle function |
| \( \gamma_x \) | The rate based call gapping throttle function with Token Bucket extension |
| \( \lambda, \lambda(t) \) | Intensity (rate) of a Poisson process |
| \( \hat{\lambda}, \hat{\lambda}(t) \) | Estimated rate (intensity) |
| \( \rho, \rho(t) \) | Real offer rate |
| \( \hat{\rho}, \hat{\rho}(t) \) | Estimated offer rate |

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