KINETIC THEORY AND MESOSCOPIC NOISE *

M. P. Das

Department of Theoretical Physics
Research School of Physical Sciences and Engineering
The Australian National University
Canberra ACT 0200, Australia

F. Green

GaAs IC Prototyping Facility
CSIRO Telecommunications and Industrial Physics
PO Box 76, Epping NSW 1710, Australia

1. INTRODUCTION

There are two truisms in the theory of transport. One states that two-particle
correlations carry far more information about microscopic charge dynamics, than do
single-particle processes such as direct-current response. The other asserts that, for
the first insight to bear fruit, one must look beyond the near-equilibrium limit.

Nowhere are these notions more apt than in testing the relation between hot-
electron noise and shot noise, the leading effects of nonequilibrium mesoscopic fluc-
tuations. Hot-electron noise is generated by spontaneous energy exchanges between
a driven conductor and its thermal bath. Shot noise is generated by random entry
and exit of the discrete carriers. Neither species is detectable unless a current flows.

Our thesis is that the relationship between shot noise and hot-electron noise is
absolutely fundamental to understanding mesoscopic fluctuations. At least from the
vantage point of orthodox microscopics and kinetics, their relation is a long way from
being settled. Its resolution calls for the tools of many-body theory.

In Section 2 we motivate the many-body approach to noise. Sec. 3 surveys
key mesoscopic experiments; we review the analysis of conductance and noise within
linear diffusive theories, and the physical transition, or smooth crossover, linking
thermal noise and shot noise [1,2]. In Sec. 4 we outline a kinetic theory of nonequi-
librium fluctuations [3] and discuss how this conventional formulation directly negates
diffusive explanations of the smooth crossover. We sum up in Sec. 5.

2. BACKGROUND

At sub-micrometer scales, device sizes approach the mean free path for scat-
tering and, often, the phase-breaking length for coherent propagation; they are
“mesoscopic”[4-6], no longer fitting the usual picture of bulk transport. Certain struc-
tures, such as quantum dots, are tinier still. Multi-particle correlations are clearly important for devices supporting only a few carriers at most (strongly quantized dots, say), but they remain relevant even in a semiclassical setting. That prompts two questions: What are the experimental signatures of two-particle correlations at mesoscopic scales? In which ranges of the driving potential should they be probed?

In dealing with themes similar to the above, many-body physicists know the value of the van Hove formula. For solid-state plasma excitations, it connects the dynamic polarizability of the electrons directly with inelastic momentum-energy loss, whenever the system is probed from outside. It forms the basis of much experimental analysis. For carrier motion in a conductor, there is a recipe comparable to van Hove’s: the Johnson-Nyquist formula [2]. This connects thermal fluctuations of the current directly with energy dissipation, both ultimately induced by the same processes for microscopic scattering.

The connecting principle of the Johnson-Nyquist formula provides a major consistency criterion for transport models. Like the van Hove relation, it is an example of the fluctuation-dissipation theorem [2]. In the electron gas, both share a common basis since fluctuations, and hence current noise, are microscopically related to the dynamic polarizability. Each of the two effects, in its way, reflects the form and action of the underlying electron-hole excitations. This drives home a vital, if obvious, message: *a true theory of current noise cannot avoid being a many-body theory.*

Despite these interconnections, many-body methods are under-represented in mainstream noise research [7]. With few exceptions [8], the field is served by special developments of weak-field, single-particle formalisms. In both quantum coherent and semiclassical stochastic versions [1,2], the formalisms rest on novel mesoscopic re-interpretations of drift-diffusion phenomenology [2,4]. Since noise is intrinsically a multi-particle effect, the internal logic of single-particle diffusive approaches (coherent and stochastic) bears closer inspection [9].

In extending many-body ideas to driven fluctuations, there are two linked issues:

- Real mesoscopic devices, in real operation, cannot be characterized by low-field response alone. This is easy to see in a typical structure 100 nm long, subject to a potential of 0.1 V. The mean applied field is $10^4$ V cm$^{-1}$; hardly weak.
- Given the need for a high-field kinetic description, one must still preserve all the definitive low-field properties of the electron gas.

While the leading rule of linear transport is the fluctuation-dissipation theorem (FDT), it is by no means the sole guiding principle in degenerate Coulomb systems. The FDT applies within the context of a nonequilibrium ensemble’s adiabatic connection to the global equilibrium state, whose nature thus exerts a governing effect on noise. An electron gas in equilibrium is anything but a collection of independent carriers. It is a correlated plasma, best known for the dominance of both degeneracy and quasi-neutrality, which persists down to distances not far above the Fermi wavelength and certainly well below mesoscopic [10].

Heuristically, much has been made of the Johnson-Nyquist formula [2] and the Einstein relation [4], which ties diffusion quantitatively to conduction in a restricted sense. However, a model built on these precepts alone is inadequate to characterize electronic fluctuations [11]. The sum rules must be respected, notably compressibility and perfect screening [10]. These ensure quasi-neutrality throughout the degenerate
plasma. Sum rules cannot emerge from an independent-particle analysis because they refer explicitly to electron-hole correlations.

Theories of fluctuations have less claim to be reliable if they make many-body predictions by inductive extrapolation from noninteracting single-particle physics [1,2]. Consider a case in point: the first aim of all diffusive models is to compute the (one-body) conductance. To that end, diffusive phenomenology must simply assume that Einstein’s relation and its parent, the microscopic FDT, are valid [4]. The need to take these theorems on faith removes the logical possibility of proving them. Without such proof, no diffusive model can demonstrate its control over the microscopic multi-particle structure. Without such control, the inner consistency of any subsequent noise prediction is uncertain.

It is the inability to describe multi-particle correlations \textit{ab initio} that bars access to proof of the fluctuation-dissipation theorem and the sum rules [9]. All of these constraints will follow naturally in a canonical description of fluctuations. Conversely, within a given model of noise in a degenerate conductor, logical derivation of the constraints is first-hand evidence of tight control, reliability, and predictive strength.

\section{3. LOW-FIELD MESOSCOPIC NOISE}

There is now a large collection of mesoscopic conductance and noise measurements. Sample sizes range from a few nanometers, to hundreds. The experiments generally cover three aspects: (a) behavior of current noise at low fields, the only domain in which diffusive theory is valid; (b) relation of noise to conductance, forming the tangible link between transport and fluctuations; and (c) crossover from thermally induced noise to shot noise, providing a unique signature of the underlying microscopics. We cite References [1] and [2] for reviews of noise, and [4], [5], and [6] for mesoscopic transport.

Two simple classes of metallic conductive structures have been tested: point-contact junctions and diffusive wires [12]. We address each case separately, and then examine issues of interpretation common to both.

\textit{Point contacts}

A point contact is a small conducting region between external leads. Its aperture approaches the scale of the Fermi wavelength, narrow enough for transport through it to be ballistic and strongly quantized [1,4,5]. The contact forms an “electron waveguide” with discrete modes, that is, subbands of quasi-one-dimensional states propagating through the constriction. If the junction is fully transmissive, its conductance is quantized in steps of the universal value $G_0 \equiv 2e^2/h$. This is explained as follows. Each step signals the opening of a new channel as, with increasing carrier density, the Fermi surface successively and discretely intersects higher subbands (in analogy with the integer quantum-Hall effect [5]). If the junction has nonideal transmission, there will be a forward-scattering probability $T_n < 1$ for the $n$th mode at the Fermi energy $\mu$. Each crossing then augments the total intrinsic conductance in Landauer’s formula.
where $\varepsilon_n$ is the $n$th subband threshold. There have been many verifications of this result. Two of the earliest are by van Wees et al. [13] and Wharam et al. [14].

One may ask why, if transport through a point contact is ballistic (collisionless), its conductance should be finite. The answer is that the contact is not a closed circuit on its own. It is open to a larger electrical environment where scattering effects are strong. The influence of the leads (Landauer’s massive banks), supplying and receiving the current, is paramount [6]. Through dissipative collisions or by geometrical mismatch at the interfaces, the leads couple the modes in the contact to an arbitrarily large set of asymptotic degrees of freedom. This introduces irreversibility and stabilizes the transport. The details of asymptotic relaxation should not affect the response; relaxation serves only to ensure boundedness of the current and the electromotive potential. In every other way, the relation between them is an irreducible property of the mesoscopic channel, albeit in contact with the macroscopic environment [15].

Mesoscopic current-noise measurements are more challenging, owing to very low signal levels. Good representative data for point contacts are in Reznikov et al. [16] and Kumar et al. [17]. In the zero-temperature limit, there are no thermal fluctuations; shot noise is the only active form of carrier correlations. The noise-power spectrum at low frequencies is [18,19]

$$S = 2eVG_0 \sum_n \theta(\varepsilon_n - \mu)T_n(1 - T_n)$$

for voltage $V$ across the contact boundaries. This theoretical expression is well confirmed by experiment.

To see how $S$ relates to the current, take the single-channel case, for which $G = G_0 T_1$. Since we are limited to weak voltages, the current response is linear: $I = GV$. We then have

$$\frac{S}{2eI} = \frac{2eVG_0 T_1(1 - T_1)}{2eGV} = 1 - T_1.$$  

We have normalized to Schottky’s expression [2] for classical shot noise, $2eI$, associated with current $I$. Equation (3) shows that fluctuations in the point contact do in fact behave as shot noise, suppressed below the classical value depending on transmission. In such a small, quasi-ballistic device, suppression can only be a quantum effect. If the contact is ideal, then $T_1 = 1$; quantum shot noise vanishes completely, because the incoming and outgoing scattering wave functions overlap fully with an eigenstate of the system. The asymptotic occupancies are totally anti-correlated by Pauli exclusion [18,19]. If $T_1 < 1$, the state of the system is no longer asymptotically pure, but mixed. The occupancies are partly decorrelated, allowing scope for the appearance of fluctuations. Evidently, the fluctuations and their associated shot noise have a nonlocal character.
For finite temperatures, with $k_B T \geq eV$, the current noise displays an appreciable thermal component. In place of Eq. (2), experimental data [17] follow the expression (again we keep one channel for simplicity)

$$S(V) = 4k_B T G \left[ T_1 + (1 - T_1) \frac{eV}{2k_B T} \coth \left( \frac{eV}{2k_B T} \right) \right].$$  \hspace{1cm} (4)

This is the prototypical smooth crossover, melding thermal noise and shot noise into a continuum [19-21].

At equilibrium, Eq. (4) for point-contact noise gives the classic Johnson-Nyquist form: $S(0) = 4k_B T G$. For $eV \gg k_B T$ the second term dominates and yields quantum shot noise with suppression, just as in Eq. (3). At intermediate potentials $eV \sim k_B T$, Eq. (4) takes on a hybrid character, more than thermal but less than shot.

From Eq. (4) one sees that the suprathermal contribution $S(V) - S(0)$ has a quite complex nonlinear dependence on $T$ and $V$. Eq. (4) is certainly well supported empirically. In our view, however, the cause of its nonlinearity is a puzzle in the light of models which depend (by design) on a strictly linear drift-diffusion approach to transport. We revisit this issue shortly. The smooth crossover also dominates noise in larger conductors, as we now discuss.

**Diffusive wires**

Transport in a diffusive wire is not ballistic, but may still be quantum-coherent. This is especially so at low temperatures and weak fields, where scattering is almost perfectly elastic. If collisions preserve the quantum phase of the carrier wave function, its total phase shift in transmission depends only on the total length of the randomized path; this is quantum diffusion. Samples are too cold, and still too short, for local dissipative heating. Instead, carriers thermalize in the access leads [22].

Quantum-mechanically one can think of a diffusive wire as the extreme limit of a point contact. The subband mode distribution becomes complicated and quasicontinuous, but Eqs. (1) and (4) still apply. With a statistical estimate of $\mathcal{T}_n(E)$ at the Fermi energy $E = \mu$, one can do an ensemble average [1,20] to get $\sum_n T_n^2 \rightarrow \frac{2}{3} \sum_n T_n$. In the context of multiple modes, Eq. (4) generalizes to

$$S(V) = 4k_B T G_0 \sum_n \left[ T_n^2 + T_n(1 - T_n) \frac{eV}{2k_B T} \coth \left( \frac{eV}{2k_B T} \right) \right]$$

$$\rightarrow 4k_B T G \left[ \frac{2}{3} + \frac{1}{3} \frac{eV}{2k_B T} \coth \left( \frac{eV}{2k_B T} \right) \right],$$  \hspace{1cm} (5)

presenting the smooth-crossover formula in its best-known guise [1,2,19-21,23,24]. Once more, the wire at zero voltage exhibits plain Johnson-Nyquist thermal noise. For $eV \gg k_B T$ there is shot-noise behavior with the famous threefold suppression; even in conductors physically much bigger than a point contact, quantum suppression of shot noise is a robust effect.

As with Eq. (4), there is solid corroboration of Eq. (5) by experiments [22,25-27]. The fit to measurements is not invariably good. Besides the survey of boundary-heating effects by Henny et al. [22], we note the very early, interesting test of Eq.
(5) by Liefrink et al. [25] in a two-dimensional electron gas. That experiment shows clear and systematic departures from the expected $\frac{1}{3}$ suppression. Although tentative explanations have been offered for those deviations, we consider that the work of Liefrink et al. in two-dimensional wires has ongoing importance, and we suggest that it be repeated with better control over carrier uniformity in the structure [9].

So far, we have reviewed the quantum-coherent interpretation of diffusive noise theory. Diffusive wires are at the large end of the mesoscopic range, and elastic scattering need not necessarily be phase-preserving. A semiclassical Boltzmann analysis might be justified if one accepts that many sequential, locally incoherent collisions should give much the same diffusive transport as the superposition of many coherent, but randomly determined, quantum paths. That is the basis of diffusive adaptations of Boltzmann-Langevin theory [2]. Such a basis lacks the clarity of the quantum-coherent descriptions of pure (zero-temperature) shot noise.

This presents an interesting juxtaposition of alternatives: pure quantum mechanics alongside semiclassical stochastics, each offering a quite different computational strategy. We do not retrace the semiclassical derivation here; theoretical details can be found in the literature [1,2,23,24]. Most important is the fact that these disparate approaches both converge on Eq. (5). Their agreement, which may seem surprising, suggests that it is the common assumptions about linear diffusive transport above all, which matter for the crossover. If the theoretical crossover were to be disconfirmed, by whatever means, both derivations would be equally suspect.

A Theoretical Issue: Nonlinearity

Having already noted the nonlinearity of the crossover formula, we now examine it more closely. Eq. (5), derived either quantum-coherently or semiclassically, describes all of the fluctuations about a mean current which is understood to be rigidly linear [1,2,4]. Linearity of the $I-V$ relation means that the resistive power dissipation in the conductor is strictly quadratic in $V$.

Mesoscopic systems are quite amenable to linear-response analysis at the microscopic level [3]. If one followed a normal plan for linear response (such as Kubo’s), one would compute a coefficient (the conductivity) for the local, quadratic, power density. The calculation, actually a microscopic proof of the FDT, would furnish the coefficient as a current autocorrelation proportional to the current-noise spectral density within the conductor. As an ensemble average at equilibrium, the coefficient could not depend on the external field, that is, on $V$. After integrating it over the sample, the local quantity would finally lead to $S(0)$: Johnson-Nyquist noise, and nothing more.

In arriving at $S(V)$ rather than just $S(0)$, diffusive theories cannot have followed a normal plan for linear response. Let us run this in reverse. The crossover formula shows marked dependence of the noise on voltage. On the other hand, it is derived in a model whose $I-V$ response is perfectly linear. Its power dissipation $IV = G V^2$ is perfectly quadratic; the coefficient $G$ must, and does, scale with Johnson-Nyquist noise as required by the FDT (naturally so, since the models at hand invoke some form of Einstein relation, or drift-diffusion FDT, to secure linearity between $I$ and $V$). Assuming that the FDT is applicable to any diffusive model (without benefit of its proof within the same model), it follows that the excess noise $S(V) - S(0)$
has no coupling to the equilibrium coefficient fixing the (strictly quadratic) resistive dissipation. Thus the excess noise is **nondissipative**.

It is evident that the smooth-crossover formula does not fit the accepted linear-response canon, even though its associated transport model is in the linear-response regime. This shows how diffusively based accounts of the crossover fall short of consistency. However, it does not touch upon the established experimental validity of Eq. (5). Indeed, the experiments bring out one of our themes: the importance of nonequilibrium, nondissipative noise as a sensitive marker of physical effects on a fine scale [3].

In terms of theory, two situations arise. Diffusive models of mesoscopic noise either (i) violate the microscopic fluctuation-dissipation theorem despite their need to invoke its offshoot, the Einstein relation, or (ii) they are somehow covertly nonlinear, despite their manifestly linear construction. One way or the other, there appear to be problems with diffusive accounts of the crossover. Eq. (5) requires a new explanation.

### 4. KINETIC APPROACH

We begin by asserting our formalism’s most striking conclusion: *the nonequilibrium thermal noise of a degenerate conductor always scales with bath temperature $T$*. Since shot noise does not scale with $T$, there is an immediate corollary. Within kinetic theory, thermal noise and shot noise cannot be subsumed under a unified formula.

The focus of this section is on the conceptual structure of the formalism, with only a brief mathematical overview. Ref. [3] has more detail. The kinetic approach to nonequilibrium transport in a metallic conductor works with a set of assumptions and boundary conditions identical to those of every other model of current and noise in metals, including every version of diffusive theory [1,2,4-6]. They are:

- an ideal thermal bath regulating the size of energy exchanges with the conductor, while itself always remaining in the equilibrium state;
- ideal macroscopic carrier reservoirs (leads) in open contact with the conductor, without themselves being driven out of their local equilibrium;
- local charge neutrality of the leads, and overall neutrality of the intervening conductor.

This standard scheme, consistently applied within a standard semiclassical Boltzmann framework, puts tight and explicit constraints on the behavior of nonequilibrium current noise [3], constraints that are less transparent in a purely diffusive framework [9].

The assumption of ideal leads implies that, regardless of the voltage across the active region, the electron distributions “far away” from the conductor remain quiescent and never depart from their proper equilibrium, characterized by $T$ and by a uniform density $n$. In practice, these extended populations need not be further away than a few Thomas-Fermi screening lengths. The associated interfacial screening zones will buffer any charge redistribution; these boundary zones should be included in the kinetic description of the system.
The electron gas in each asymptotic lead is unconditionally neutral, and satisfies the canonical sum rules [10]. Gauss’ theorem then implies that the central region must be overall neutral. Global neutrality and asymptotic equilibrium together condition the form of the nonequilibrium fluctuations in the mesoscopic conductor.

Our goal is to show that nonequilibrium correlations are linear functionals of the equilibrium ones. In the degenerate electron gas, the immediate consequence of this is that all thermally induced noise must scale with ambient temperature $T$. Therefore it is impossible for shot noise to couple to the thermal bath. Otherwise, shot noise too would be seen to scale with $T$, which is not the case.

The kinetic approach to fluctuations, sketched out below, takes as its input the electron-hole pair excitations in the equilibrium state. Fermi-liquid theory shows that these pair correlations form an essential unit, always with an internal kinematic coupling. Generally, they cannot be factorized into two stochastic components autonomously located, so to speak, on the single-electron energy shell. In that respect we do not follow Boltzmann-Langevin analysis for degenerate electrons [2,28].

It is straightforward to specify the distribution of free electron-hole fluctuations, $\Delta f_{eq}^k(r)$, for wavevector $k$ at position $r$:

$$\Delta f_{eq}^k(r) \equiv k_BT \frac{\partial f_{eq}^k}{\partial \mu} = f_{eq}^k(r)[1 - f_{eq}^k(r)].$$

(6)

The one-electron equilibrium distribution is

$$f_{eq}^k(r) = \left[1 + \exp\left(\frac{\varepsilon_k + U_0(r) - \mu}{k_BT}\right)\right]^{-1},$$

(7)

where the conduction-band energy $\varepsilon_k$ can vary (implicitly) with $r$ if the local band structure varies, as in a heterojunction. The electronic potential $U_0(r)$ vanishes asymptotically in the leads, and satisfies the self-consistent Poisson equation ($\epsilon$ is the background-lattice dielectric constant)

$$\nabla^2 U_0 = e \frac{\partial}{\partial r} \cdot \mathbf{E}_0 = -\frac{4\pi e^2}{\epsilon} \left(\langle f_{eq}(r)\rangle - n^+(r)\right)$$

(8)

in which, for later use, $\mathbf{E}_0(r)$ is the internal field at equilibrium and $\langle \rangle$ denotes the trace over spin and wave vector $k$. The (nonuniform) neutralizing background density $n^+(r)$ goes to $n$ in the (uniform) leads.

The semiclassical Boltzmann equation, subject to the total internal field $\mathbf{E}(r,t)$, can be written as

$$\left(\frac{\partial}{\partial t} + D_{k,r}[\mathbf{E}(r,t)]\right)f_k(r,t) = -C_{k,r}[f].$$

(9)

Here $D_{k,r}[\mathbf{E}] \equiv v_k \cdot \partial/\partial r - (e\mathbf{E}/\hbar) \cdot \partial/\partial k$ is the convective operator and $C_{k,r}[f]$ is the collision operator, whose kernel (local in real space) is assumed to satisfy detailed balance, as usual [1-3]. Even for single-particle impurity scattering, of immediate concern, Pauli blocking of the outgoing scattering states still means that $C$ is nonlinear in the nonequilibrium solution $f_k(r,t)$. 

8
Since we follow the standard Boltzmann formalism, all of our results will comply with the conservation laws. The nonlinear properties of these results will extend as far as the inbuilt limits of the Boltzmann framework; much further than if they were restricted to the weak-field domain, as demanded by the drift-diffusion Ansatz [4]. Moreover, since we rely directly on the whole fluctuation structure provided by Fermi-liquid theory [10], the sum rules are incorporated.

Our prescription starts by developing the steady-state nonequilibrium distribution \( f_k(r) \) as a mapping of the equilibrium distribution, which satisfies

\[
D_{k,r}[E_0(r)]f^\text{eq}_k(r) = 0 = -C_{k,r}[f^\text{eq}],
\]

the last equality following by detailed balance. Subtracting the corresponding sides of Eq. (10) from both sides of the time-independent version of Eq. (9), and introducing the difference \( g_k(r) \equiv f_k(r) - f^\text{eq}_k(r) \), we obtain

\[
\int dr' \int \frac{2dk'}{(2\pi)^d} \left( I_{kk',rr'} D_{k',r'}[E(r')] + C'_{kk',kr'}[f] \right) g_k(r') = \frac{e}[E(r) - E_0(r)] \cdot \frac{\partial f^\text{eq}_k(r)}{\partial k} - C''_{k,r}[g].
\]

The unit operator in \( d \) dimensions is \( I_{kk',rr'} \equiv (2\pi)^d \delta(k - k') \delta(r - r') \), and the linearized operator \( C'[f] \) is the variational derivative \( C'_{kk',kr'}[f] \equiv \delta C_{k,r}[f]/\delta f_k(r) \). Last, \( C''[g] \equiv C[f] - C'[f] \cdot g \) carries the residual nonlinear contributions. Global neutrality enforces the important constraint \( \int dr \langle g(r) \rangle = 0 \).

The leading right-hand term in Eq. (11) is responsible for the functional dependence of \( g \) on the equilibrium distribution. This is important because dependence on equilibrium-state properties carries through to the derived steady-state fluctuations. The electric-field factor can be written as \( E - E_0 \equiv E_{\text{ext}} + E_{\text{ind}} \) where \( E_{\text{ext}}(r) \) is the external driving field, and the induced field \( E_{\text{ind}}(r) \) obeys

\[
\frac{\partial}{\partial r} E_{\text{ind}} = -\frac{4\pi e}{\epsilon} \langle g(r) \rangle.
\]

Now we consider the nonequilibrium fluctuation \( \Delta f_k(r, t) \). It satisfies the linearized Boltzmann equation [29, 30]

\[
\int dr' \int \frac{2dk'}{(2\pi)^d} \left[ I_{kk',rr'} \left( \frac{\partial}{\partial t} + D_{k',r'}[E(r')] \right) + C'_{kk',kr'}[f] \right] \Delta f_{k'}(r', t) = 0.
\]

Given the temporal and spatial boundary constraints for this equation (causality and global neutrality), all of the relevant dynamical properties of the fluctuating electron gas, notably its current noise, can be obtained. Its adiabatic \( t \to \infty \) limit, \( \Delta f_k(r) \), represents the average strength of the spontaneous background fluctuations, induced in steady state by the ideal thermal bath. It is one of two essential components that determine the dynamical fluctuations (the other is the Green function for the
inhomogeneous form of Eq. (13)). In particular, $\Delta f_k(r)$ dictates the explicit $T$-scaling of all thermal effects through its functional dependence on the equilibrium distribution $\Delta f_{k}^{eq}(r)$. We now show how this comes about.

Define the variational derivative $G_{kk'}(r, r') \equiv \delta g_k(r)/\delta f_{k'}^{eq}(r')$. This is a Green-function-like operator obeying a steady-state equation obtained from Eq. (11) by taking variations on both sides [3]. The explicit form of $G$ can be derived from knowledge of the Green function for Eq. (13). One can verify that

$$\Delta f_k(r) = \Delta f_k^{eq}(r) + \int dr' \int \frac{2d k'}{(2\pi)^d} G_{kk'}(r, r') \Delta f_{k'}^{eq}(r')$$  \hspace{1cm} (14)$$
satisfies the steady-state form of Eq. (13) identically. This establishes the linear relationship between nonequilibrium and equilibrium thermal fluctuations, and the need for the former to be proportional to $T$ in a degenerate conductor, since then $\Delta f_k^{eq}(r) \rightarrow k_B T \delta(\epsilon_k + U_0(r) - \mu)$.

Again, charge neutrality enforces upon Eq. (14) the constraint

$$\int dr \int \frac{2d k}{(2\pi)^d} G_{kk'}(r, r') = 0$$

for all $k'$ and $r'$. Over volume $\Omega$ of the whole conductor, including its buffer zones, this leads to the normalization

$$\int_{\Omega} dr \langle \Delta f(r) \rangle = \int_{\Omega} dr \langle \Delta f^{eq}(r) \rangle.$$  \hspace{1cm} (15)$$

One can compare the strict equality in Eq. (15) with the analogous situation in any of the diffusive noise formulations [1,2,18-21,23,24]; diffusive fluctuations do not fulfill this most basic of physical constraints. They do not fulfill it because local equilibrium and neutrality are not guaranteed, in one lead or more (depending on where a given model chooses to locate its “absolute” chemical potential $\mu$). Although those asymptotic conditions are implicitly respected at the level of one-body transport, they are no longer respected by fluctuations produced in the diffusive theories’ passage to the two-body level. Such inconsistency could never arise if the sum rules for the electron gas [10] were in place and operative.

For semiclassical diffusive models, Eq. (15) restores conformity of the local $\Delta f_k(r)$ with the FDT, at the price of suppression. If a source of semiclassical suppression does exist, it is genuinely nonequilibrium and it accords with global neutrality. Furthermore, there is no compelling reason to expect that any semiclassical description – including ours – must recover a priori the quantum-coherent result for elastic diffusive wires.

As far as we can see, only a quantum treatment can capture genuinely nonlocal physics in mesoscopic systems. However, we still differ on the separate conceptual issue of a smooth quantum crossover (as such), based on drift-diffusion ideas. We speculate that the S-matrix formalism can work without recourse to drift-diffusion phenomenology. Its coherent nonlocal nature gives it a certain numerical robustness against violations of neutrality, a feature not shared by local theories.
Finally, using the tools that we have outlined, it is possible not only to display the linear functional dependence of hot-electron thermal noise on $\Delta f^{eq}$, but also to prove the mesoscopic FDT explicitly for semiclassical noise in the weak-field limit \cite{3}. Beyond this limit there is a systematic, nonperturbative way of classifying the appreciable hot-electron contribution. This type of excess noise has two features: it is not dissipative, and it still scales with temperature. It is just about impossible for it to “cross over” into shot noise, which is indisputably non-thermal \cite{3,9}.

5. SUMMARY

Transport and fluctuations at mesoscopic scales reveal new, intriguing physics requiring theoretical models beyond the normal methods for extended, uniform systems. In the mesoscopic regime, even strongly metallic conductors may become nonuniform and sharply quantized. In addition, mesoscopic devices are very likely to operate at high fields. To date, however, most experimental and theoretical work has engaged only the low-field linear limit.

There are two theoretical responses to these challenges: make greater efforts within standard microscopics and kinetics, or revisit simpler phenomenologies and try to stretch those. For one-body mesoscopics, notably low-field conductance, the success of diffusively inspired phenomenologies is impressive. For many-body effects such as current fluctuations, diffusive theories have also had considerable success, as witness the prediction of suppressed mesoscopic shot noise, whose quantum origin is not in debate.

Regardless of their triumphs and their intuitive appeal, diffusive phenomenologies have not adduced a microscopic rationale for the apparent attempt to extrapolate the drift-diffusion Ansatz upward into the hierarchy of multi-particle correlations. In particular, diffusively based noise models fail to address the sum rules, and hence to secure them. The sum rules set fundamental constraints, whose satisfaction is crucial to the correct representation of many-body phenomena in the electron gas \cite{7}. Current noise is one such phenomenon. Therefore, noise predictions based on diffusive arguments are less sure to be well controlled.

One of diffusive noise theory’s key results is the smooth crossover of equilibrium thermal noise (scaling with ambient temperature $T$) into nonequilibrium shot noise (independent of $T$). We have pointed out that the accepted explanation for the crossover is incompatible with conventional kinetics and the theory of charged Fermi liquids. This is shown by the mandatory $T$-scaling of nonequilibrium thermal noise in degenerate mesoscopic conductors. Such scaling clearly precludes any continuous crossover between shot noise and noise that is generated purely thermally.

In sum, the account of the smooth crossover given by diffusive analysis is not supported theoretically by orthodox kinetic theory. We believe that its empirical truth still stands in need of a more rigorous, and certainly quite different, description. New experiments would be needed to test any alternative. We end with a question: since diffusive analysis itself is widely advertised as a serious first-principles procedure, could some of its subsidiary assumptions be defective?
REFERENCES

[1] M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, edited by L. P. Kouwenhoven, G. Schön, and L. L. Sohn, NATO ASI Series E (Kluwer Academic, Dordrecht, 1997).

[2] Sh. M. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, 1996).

[3] F. Green and M. P. Das, [cond-mat/9809339](http://arxiv.org/abs/cond-mat/9809339) (Report RPP3911, CSIRO, unpublished, 1998).

[4] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995).

[5] D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, Cambridge, 1997).

[6] Y. Imry and R. Landauer, *Rev. Mod. Phys.* **71**, S306 (1999).

[7] Many-body issues in mesoscopic noise tend to be presented as secondary, rather than central, in mainstream thinking. See for example R. Landauer in *Proceedings of New Phenomena in Mesoscopic Structures, Kauai, 1998* (submitted to *Microelectronic Engineering*).

[8] A noteworthy Monte-Carlo study is P. Tadyszak, F. Danneville, A. Cappy, L. Reggiani, L. Varani, and L. Rota *Appl. Phys. Lett.* **69**, 1450 (1996).

[9] F. Green and M. P. Das, in *Proceedings of the Second International Conference on Unsolved Problems of Noise, Adelaide, 1999* edited by D. Abbott and L. B. Kiss (AIP, in preparation). See also [cond-mat/9905080](http://arxiv.org/abs/cond-mat/9905080).

[10] D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966).

[11] Indeed the Johnson-Nyquist formula in itself shows no sensitivity to carrier degeneracy, and neither does the Einstein relation. See C. Kittel, *Elementary Statistical Physics* (Wiley, New York, 1958), pp 143-5. Their forms are the same whether the conductor is classical or degenerate. Therefore, additional microscopic input is essential to any systematic treatment of the fluctuations.

[12] We do not discuss noise in a third important, but more complex, class: tunnel-junction devices [5]. See for example H. Birk, M. J. M. de Jong, and C. Schönenberger, *Phys. Rev. Lett.* **75**, 1610 (1995). There are also remarkable results for tunneling shot noise in the fractional-quantum-Hall regime. Refer to R. de Picciotto *et al.*, *Nature* **389**, 162 (1997); L. Saminadayar *et al.*, *Phys. Rev. Lett.* **79**, 2526 (1997); and M. Reznikov *et al.*, *Nature* **399**, 238 (1999).

[13] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, *Phys. Rev. Lett.* **60**, 848 (1988).

[14] D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, *J. Phys. C* **21**, L209 (1988).

[15] Landauer’s conception extends further, to a perfect duality between voltage and current by which either quantity can equally well induce the other [6]. In con-
ventional kinetic theory, the electromotive force is always distinguished as the prime cause of the current. Duality is problematic beyond the weak-field linear limit; recall the non-monotonic response of a resonant-tunneling diode [5].

[16] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. 75 3340 (1995).

[17] A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 76 2778 (1996).

[18] V. A. Khlus, Sov. Phys. JETP 66, 1243 (1987); G. B. Lesovik, JETP Lett. 49 592 (1989).

[19] M. Büttiker, Phys. Rev. Lett. 65 2901 (1992); Phys. Rev. B 46 12485 (1992).

[20] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B 46 1889 (1992).

[21] Th. Martin and R. Landauer, Phys. Rev. B 45 1742 (1992).

[22] M. Henny, S. Oberholzer, C. Strunk, and C. Schönenberger, Phys. Rev. B 59, 2871 (1999).

[23] K. E. Nagaev, Phys. Lett. A 169 103 (1992); Phys. Rev. B 52 4740 (1994).

[24] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B 51 16867 (1995).

[25] F. Liefrink, J. I. Dijkstra, M. J. M. de Jong, L. W. Molenkamp, and H. van Houten, Phys. Rev. B 49 14066 (1994).

[26] A. H. Steinbach, J. M. Martinis, and M. H. Devoret, Phys. Rev. Lett. 76 3806 (1996).

[27] R. J. Schoelkopf, P. J. Burke, A. A. Kozhevnikov, D. E. Prober, and M. J. Rooks, Phys. Rev. Lett. 78 3370 (1997).

[28] For a critique of Langevin methods in correlated-particle kinetics, see N. G. van Kampen, Stochastic Processes in Physics and Chemistry (North-Holland, Amsterdam, 1981), pp 246-52.

[29] S. V. Gantsevich, V. L. Gurevich, and R. Katilius, Nuovo Cimento 2 1 (1979).

[30] Here we freeze the response of the self-consistent fields. This is equivalent to probing the nonequilibrium analog of the long-wavelength, “screened” Lindhard function [10] prior to including internal Coulomb screening correlations. Screening effects are especially important for inhomogeneous systems [3]. They can be treated systematically in Eq. (13) in the spirit of a Landau-Silin approach [10].