Disruption of co-orbital (1:1) planetary resonances during gas-driven orbital migration

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ABSTRACT
Planets close to their stars are thought to form farther out and migrate inwards due to angular momentum exchange with gaseous protoplanetary discs. This process can produce systems of planets in co-orbital (Trojan or 1:1) resonance, in which two planets share the same orbit, usually separated by 60 deg. Co-orbital systems are detectable among the planetary systems found by the \textit{Kepler} mission either directly or by transit timing variations. However, no co-orbital systems have been found within the thousands of \textit{Kepler} planets and candidates. Here, we study the orbital evolution of co-orbital planets embedded in a protoplanetary disc using a grid-based hydrodynamics code. We show that pairs of similar-mass planets in co-orbital resonance are disrupted during large-scale orbital migration. Destabilization occurs when one or both planets are near the critical mass needed to open a gap in the gaseous disc. A confined gap is opened that spans the 60 deg azimuthal separation between planets. This alters the torques imparted by the disc on each planet – pushing the leading planet outwards and the trailing planet inwards – and disrupts the resonance. The mechanism applies to systems in which the two planets’ masses differ by a factor of 2 or less. In a simple flared disc model, the critical mass for gap opening varies from a few Earth masses at the inner edge of the disc to 1 Saturn mass at 5 au. A pair of co-orbital planets with masses in this range that migrates will enter a region where the planets are at the gap-opening limit. At that point, the resonance is disrupted. We therefore predict an absence of planets on co-orbital configurations with masses in the super-Earth to Saturn mass range with similar masses.

Key words: accretion, accretion discs – hydrodynamics – methods: numerical – planets and satellites: formation – planet–disc interactions.

1 INTRODUCTION
A striking feature of the current population of exoplanets is the broad diversity in system architectures that have been discovered. Among the known multiplanetary systems, a significant number contains bodies in resonant or near-resonant configurations. For example, the two giant planets in the Kepler-9 system are trapped in 2:1 resonance (Holman et al. 2010). The formation of mean-motion resonances is generally explained by the convergent migration of two planets in the highly damped environment of protoplanetary discs (e.g. Snellgrove, Papaloizou & Nelson 2001; Lee & Peale 2002; but see Raymond et al. 2008b).

The origin of close-in systems of low-mass planets is debated (see Raymond, Barnes & Mandell 2008a; Raymond et al. 2013). Many of these systems’ characteristics can be reproduced by either \textit{in situ} accretion within high-mass discs (Raymond et al. 2008a; Hansen & Murray 2012, 2013; Chiang & Laughlin 2013) or by accretion during inward migration of planetary embryos (Terquem & Papaloizou 2007; Ogihara & Ida 2009; Cossou et al. 2014). The \textit{in situ} accretion model requires extremely massive inner discs with a broad range of surface density slopes, which is inconsistent with any known disc theory (Raymond & Cossou 2014). The inward migration model is viable but remains to be tested quantitatively. There exist other plausible models that invoke large-scale inward migration of small bodies to produce a surplus in mass in the inner parts of the disc (Boley & Ford 2013; Chatterjee & Tan 2014), essentially producing the initial conditions needed for the \textit{in situ} migration model. Systems of low-mass planets are rarely in resonance. Rather, pairs of adjacent planets are often found just wide of resonance (Lissauer et al. 2011). It is unclear whether this supports the \textit{in situ} (Petrovich, Malhotra & Tremaine 2013) or migration (Baruteau & Papaloizou 2008) model.
The inward migration model proposes that populations of embryo
migrants (Ward 1997; Tanaka, Takeuchi & Ward 2002) inwards in so-called type I migration from large orbital radii. During
this migration, they interact gravitationally and occasionally
merge (Terquem & Papaloizou 2007; Ogihara & Ida 2009; Cos-
sou et al. 2014). Using hydrodynamical simulations, Cresswell &
Nelson (2006, 2008) found that the formation of co-orbital planets
engulfed in a 1:1 resonance is a natural outcome of close encoun-
ters between embryos during this process. Trojan planets are formed in
~30 per cent of their simulations. Although they noted an increase
in the amplitude libration about the $L_3/L_4$ point during large-scale
migration, Cresswell & Nelson (2009) confirmed that Trojan planet
systems embedded in protoplanetary discs are stable. These studies
concluded that co-orbital planets on short-period orbits should be
common.

Trojan planets could be discovered as two planets transiting the
same star but separated by 60° in phase. They can also be detected
by comparing the time of central transit with the time of zero stellar
reflex velocity (Ford & Gaudi 2006) or by transit timing variations
(Kepler data (Janson 2013). One reason for that may be related to
the fact that only Trojans with coplanar orbits can be detected with
Kepler. It is also possible that existing Trojans evolve on horseshoe
or tadpole orbits with large libration, and thus are easily missed
from planet-search algorithms since they induce large TTV (Janson
2008). We use hydrodynamical simulations to show that the gap region located between the
planets’ orbits is far more gas depleted than the rest of the disc.
This essentially produces a gap that is confined in azimuth between
the planets. This makes the trailing planet experience a negative
torque whereas the leading component feels a positive torque from
the disc. The planets experience divergent migration and migrate
divergently, resulting in the disruption of the 1:1 resonance. In an
evolved, flared disc this mechanism destabilizes co-orbital systems
with two Saturn-mass planets at several au. Trojan configurations
with two 5–10 $M_\oplus$ planets become unstable in the inner disc due
to the ability of such planets to open a partial gap in the disc
when the disc aspect ratio is small. Co-orbital systems with more
massive trailing planets are less stable than systems with more mas-
sec leading planets. This mechanism relies on non-axisymmetric
perturbations to the disc structure and requires that the two plan-
net systems be within roughly a factor of 2. Trojan systems with
disparate masses or in the Jupiter-mass range are stable during
migration.

The paper is organized as follows. We describe the numerical set-
up in Section 2. In Section 3, we present the physical mechanism
responsible for the unstable evolution of co-orbital systems and
examine the orbital migration of Trojan planets for a variety of
planet masses and disc models. Finally, we summarize and conclude
in Section 4.

## 2 Numerical Set-up

Simulations were performed using the GENESIS (De Val-Borro et al.
2006) numerical code which solves the equations governing the disc
evolution on a polar grid ($R$, $\phi$) using an advection scheme based
on the monotonic transport algorithm (Van Leer 1977). It includes a
fifth-order Runge–Kutta integrator (Press et al. 1992) that computes
the planet orbits.

The computational units we adopt are such that the mass of the
central star $M_*$ = 1 corresponds to one solar mass, the gravitational
constant is $G = 1$ and the radius $R = 1$ in the computational domain
corresponds to 5 au. In the following, time is measured in units of
the orbital period at $R = 1$.

We use $N_R = 768$ radial grid cells uniformly distributed between
0.25 and 5, and $N_\phi = 1200$ azimuthal grid cells. For a planet-to-star mass ratio $q = 3 \times 10^{-5}$, which corresponds to a 10-
Earth-mass planet and a disc aspect ratio $h = 0.05$, the dimension-
less half-width of the horseshoe region is $x_h \sim 1.2 \sqrt{q/h} \sim 0.027$ (e.g. Paardekooper et al. 2010). This means that the half-width of
the horseshoe region is resolved by about 10 cells in the radial
direction. To avoid wave reflections at the disc edge, we employ
wave-killing boundary conditions (Pierens & Nelson 2008), where
the radial velocity in the ghost zones is set to $v_R = \beta \sqrt{R_h}$, where $v_R(R_h) = -3 \sqrt{2} R_h$ is the gas drift velocity due to viscous evolution and $\beta$ is a free parameter which was set to $\beta = 5$.

For most of the simulations presented here, we adopt a locally
isothermal equation of state such that both the disc temperature and
the aspect ratio profiles are constant in time. Our fiducial disc model
adopts a constant aspect ratio with $h = 0.05$ or 0.02 but we also considered a flared disc model with $h = h_0(R/R_c)^{\alpha}$, where $\alpha = 0.3$ is the flaring index and $h_0$ the disc aspect ratio at $R_c = 5$ au. We have also performed an additional set of simulations using a radiative
disc model, in order to test the dependence of our results upon the
choice of the equation of state. The disc viscosity is $\nu = 10^{-5}$ in
code units which corresponds to an alpha viscous stress parameter
$\alpha = 4 \times 10^{-3}$ for $h_0 = 0.05$. The initial surface density profile is
chosen to be $\Sigma = \Sigma_0(R/R_c)^{-3/2}$ with $\Sigma_0 = 2 \times 10^{-5}$.

The trailing and leading components of the co-orbital system are
both initiated on circular orbits at $R_c = 1$ and are initially located at
their mutual $L_3/L_2$ Lagrange points. In the following, we denote by
$m_t$ and $m_l$ the masses of the trailing and leading planets, respective-
ly, for which we consider values between 1.6 $M_\oplus$ and 1 $M_\odot$, where $M_\odot$ is the mass of Jupiter.

## 3 Results

### 3.1 Disc torques exerted on the trailing and leading components

We first examine the torques experienced by co-orbital planets held
on fixed orbits. We use a simple disc model with constant aspect ratio
$h = 0.05$. Fig. 1 (upper panel) shows the evolution of the torques $\Gamma$
normalized by $\Gamma_0 = (q/h)^2 \Sigma_0 R_\odot^2 \Omega_\odot^2$, where $\Sigma_0$ is the planet angular
velocity, experienced by the trailing and leading components of a co-
orbital system with $m_t, m_l = 1 M_\oplus$, where $M_\odot$ is the mass of Saturn.
The disc torques exerted on the two co-orbitals are different, even
though the planets share the same orbit. The trailing planet feels a
positive torque and the leading planet feels a negative torque. This
should lead to divergent orbital migration of the co-orbital pair. To
explain the differences in the magnitudes of the torques, we show the
torque density distribution acting on the trailing and leading
components in the lower panel of Fig. 1. Significant differences arise from inside the horseshoe region of the planets, whose half-width for a Saturn-mass planet is estimated to be $x_s = 2.45R_H$ (Masset, D’Angelo & Kley 2006), where $R_H = R_p(M_S/3M_p)^{1/3}$ is the Hill radius of the planet. Compared with the torque exerted on a single planet (the dashed line in the lower panel of Fig. 1), the torque exerted on the leading planet takes more positive values over the whole horseshoe region whereas the torque acting on the trailing planet takes more negative values. Slight differences between the torques exerted on the two co-orbitals can also be observed outside of the horseshoe region. In the outer disc, the torque felt by the leading planet is in good agreement with the torque exerted on a single planet. In the inner disc, it is the torque felt by the trailing planet that matches the single-planet torque. This suggests that there is an additional positive contribution from the outer disc region bounded by $1.2 < R < 1.3$ to the torque exerted on the trailing planet, as well as an additional negative contribution from the inner disc region bounded by $0.8 < R < 0.9$ to the torque acting on the leading planet. As shown below, the differences in the torque density distribution outside of the horseshoe region result from the interaction of the trailing (resp. leading) planet with the wake of the leading (resp. trailing) planet. However, the fact that the total torque exerted on the leading planet is positive suggests that these additional contributions to the torque density distribution are smaller than the differences in the torques arising from inside the horseshoe region.

Fig. 2 shows contours of the perturbed surface density $(\Sigma - \Sigma_0)/\Sigma_0$, where $\Sigma_0$ is the initial surface density profile, for simulations with two equal-mass co-orbital planets with, from left to right, $m_c, m_i = 10 \text{M}_J, 100 \text{M}_J \approx M_5$, and $300 \text{M}_J \approx M_5$. In the figure, the trailing planet is located at $\phi = \pi$ rad and the leading one at $\phi = 4\pi/3$ rad. Overplotted are a few streamlines that delimit two different horseshoe regions: (i) one of azimuthal extension $\Delta \phi = \pi/3$ rad and located ahead the trailing planet and behind the leading component and (ii) one of larger azimuthal extension $\Delta \phi = 5\pi/3$ and bounded by $\phi = \phi_i$ and $\phi = 2\pi + \phi_i$.

As expected, the density structure is not significantly perturbed for low-mass planets ($m_c, m_i = 10 \text{M}_J$). However, Trojan planets with $m_c, m_i > M_5$ generate gaps in the disc. The azimuthal density structure of the gap is strongly asymmetric in the simulation with $m_c, m_i = M_5$, with a much more gas-depleted region located ahead the trailing planet and behind the leading one. The implication is that in this case, the trailing (resp. leading) planet tends to feel a strong negative (resp. positive) corotation torque due to the lower surface density ahead (resp. behind) of the planet. This explains the torque density distribution in the lower panel of Fig. 1. This figure also shows that the differences in the torque distribution outside of the horseshoe region are likely to result from the interaction between one component of the co-orbital system and the wake of the second component. In the outer disc, the wake of the leading planet and shearing past the trailing planet tends to exert a positive torque on the latter, clearly seen in the torque density distribution of the trailing planet from $R = 1.2$ to 1.3. Alternatively, the wake of the trailing planet and shearing past the leading component in the inner disc exerts a negative torque on it, in agreement with the torque distribution of the leading planet for $0.8 < R < 0.9$. The asymmetry in the azimuthal gap structure in the middle panel of Fig. 2 should occur for co-orbital planets that are able to open a partial gap in the disc. For a single planet on a circular orbit with semi-major axis $a_p$, the first condition for gap clearance is $R_H > H$ (Ward 1997), where $H$ is the disc scaleheight, so that the wake of the planet forms a shock and the flux of angular momentum carried by the wake is deposited locally. Gap formation also requires that the planetary tidal torque exceeds the viscous torque $J_{\text{visc}}$, which is given by (Lynden-Bell & Pringle 1974)

$$J_{\text{visc}} = 3\pi a_p^3 \rho_0^2 \Omega_p.$$  

This condition leads to the so-called viscous criterion for gap opening $q > 40 a_p^3 \rho_0^2$ (Lin & Papaloizou 1993). Crida et al. (2006) provided a single criterion for gap opening which combines the two aforementioned conditions and reads $P < 1$, where the gap-opening parameter $P$ is given by

$$P = 1.1 \left( \frac{q}{H^3} \right)^{-1/3} + \frac{50 \nu}{q a_p^3 \Omega_p}. $$  

We now turn to the issues of the conditions for gap clearance for two co-orbital planets located at their mutual $L_5/L_4$ Lagrange points. The main difference with the single planet case is that the integrated viscous torque now depends on which part of the horseshoe region is considered. For the horseshoe region of azimuthal extension $\Delta \phi = \pi/3$ located ahead the trailing planet and behind the leading body, the integrated viscous torque over the region is $J_{\text{visc}}(\Delta \phi = \pi/3) = J_{\text{visc}}/6$, whereas for the part of the horseshoe region with azimuthal extension $\Delta \phi = 5\pi/3$ and bounded by $\phi_i < \phi < 2\pi + \phi_i$, the integrated viscous torque is $J_{\text{visc}}(\Delta \phi = 5\pi/3) = 5 J_{\text{visc}}/6$, where $J_{\text{visc}}(\Delta \phi = 2\pi) = J_{\text{visc}}$. Consequently, it is straightforward to show that the gap-opening criterion of Crida et al. (2006) can be
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Figure 2. Contours of the perturbed surface density \((\Sigma - \Sigma_0) / \Sigma_0\), where \(\Sigma_0\) is the initial surface density profile, for simulations with equal-mass co-orbitals of masses \(m_t, m_l = 10 \text{M}_\odot\) (left-hand panel), \(1 \text{M}_\oplus\) (middle panel), \(1 \text{M}_J\) (right-hand panel). A few streamlines are overplotted as white lines.

written for the two regions

\[
P_1 = 1.1 \left( \frac{q}{h} \right)^{-1/3} + \frac{1}{6} \frac{50v}{qa_p^2 \Omega_p} \quad \text{for} \quad \phi_t < \phi < \phi_l, \tag{3}
\]

\[
P_2 = 1.1 \left( \frac{q}{h} \right)^{-1/3} + \frac{5}{6} \frac{50v}{qa_p^2 \Omega_p} < 1 \quad \text{for} \quad \phi_l < \phi < 2\pi + \phi_t, \tag{4}
\]

For \(h = 0.05\) and \(v = 10^{-5}\), equation (3) predicts that a gap is formed in this region for \(m_t, m_l = \text{M}_S\), whereas from equation (4), we find that a mass of \(m_t, m_l = \text{M}_J\) is required to clear a gap in the region bounded by \(\phi_t\) and \(2\pi + \phi_l\). In that case, and as illustrated by the right-hand panel of Fig. 2, the azimuthal structure of the gap becomes axisymmetric such that the difference in the torques acting on the trailing and leading planets is weakened in comparison with the case with \(m_t, m_l = \text{M}_S\).

The gap tends to be cleared more quickly in the region bounded by \(\phi_t < \phi < \phi_l\) than in the rest of the disc. Inspection of surface density maps at different times suggests that it takes approximately \(\sim 10\) orbits to form the gap in the region in between the two planets while the gap is carved in \(\sim 100\) orbits in the rest of the disc. Consequently, we expect that the asymmetry in the gap structure, and therefore the differences in the torques felt by the trailing and leading planets, is maximal after \(\sim 10\) orbits. The gap structure then becomes slightly more symmetric due to the formation of a shallow gap in the rest of the disc and the differences in torques weaken, consistent with the time evolution of the torques in Fig. 1.

3.2 Stability of migrating co-orbital systems

3.2.1 Disc with constant aspect ratio

Fig. 3 (top panel) shows the evolution of three co-orbital systems in a disc with aspect ratio \(h = 0.05\). Each system has two equal-mass planets with masses of \(10 \text{M}_\oplus\) (black curves), \(100 \text{M}_\oplus\) (blue curves), and \(300 \text{M}_\oplus\) (red curves). A locally isothermal equation of state is adopted in these simulations such that all systems migrate inwards. For \(m_t, m_l = 10 \text{M}_\oplus\), the orbital evolution of the trailing and leading components are almost indistinguishable, indicating that the libration amplitude about the \(L_4/L_5\) Lagrange points is small and that the system is likely to be stable. Using a more realistic equation of state may lead to a different mode of evolution since it has been shown that in non-isothermal discs, the non-barotropic part of the corotation torque can make type I migration slow down or even reverse (Paardekooper & Mellema 2006; Baruteau & Masset 2008). The issue of the equation of state is probably not important for the simulations with \(m = \text{M}_S\) or \(\text{M}_J\) in which the planets open a gap in the disc and deplete their horseshoe region, weakening thereby the effect of the corotation torque (Kley & Crida 2008).

Figure 3. Upper panel: time evolution of the orbital distance of co-orbital systems with \(m_t, m_l = 10 \text{M}_\oplus\) (black), \(\text{M}_S\) (blue), \(\text{M}_J\) (red) for a disc model with constant aspect ratio \(h = 0.05\). Lower panel: same but for \(h = 0.02\) and \(m_t, m_l = 1.6 \text{M}_\oplus\) (green), \(10 \text{M}_\oplus\) (black) and \(\text{M}_S\) (blue).
We will discuss in more details the sensitivity of our results on the equation of state in Section 4.1. Different evolution occurs for more massive co-orbitals. Trojan planets with $m_0 = M_j$ open a deep gap in the disc and undergo stable type II migration. However, the co-orbital configuration with $m_0 = M_j$ is disrupted at $t \sim 1000$ orbits. This occurs because the structure of the gap cleared by both planets is such that the leading planet feels a positive torque from the disc. The trailing planet feels a negative torque, resulting in divergent migration. In the case with $m_0 = M_j$, the surface density in the gap is only very slightly non-axisymmetric so the torques exerted on both planets are similar and the system is stable.

We now demonstrate that the mechanism presented in Section 3.1 is responsible for breaking the co-orbital resonance for the case $m_0 = M_j$. We performed similar hydrodynamical simulations but for a disc model with $h = 0.02$. The co-orbitals that were destabilized should have masses in the partial gap-opening regime for which $q/h^3 \sim 1$ (e.g. Baruteau & Papaloizou 2013). We therefore expect Trojan planets with masses typical of Super-Earths to become unstable when $h = 0.02$. Fig. 3 (bottom panel) shows the orbital evolution for simulations with $m_0 = 1.6 M_\oplus$, $10 M_\oplus$ and $M_j$. As expected, the co-orbital system with $m_0 = 10 M_\oplus$ is indeed unstable and the one with $m_0 = M_j$ is stable. The perturbed density has an asymmetric gap structure to that in the middle panel of Fig. 2. In this thin ($h = 0.02$) disc, the system with Saturn-mass co-orbitals carves a deep, axisymmetric gap and is stable.

3.2.2 The case of a flared disc

So far, we have considered a simplified disc with constant aspect ratio. In a more realistic model, the aspect ratio is a sensitive function of the opacity and energy flux due to both viscous heating and stellar irradiation (Bitsch et al. 2013). For an evolved disc, we expect stellar heating to dominate over viscous heating and the disc to become flared with an aspect ratio $h \propto R^{3/7}$ (Chiang & Goldreich 1997; D’Angelo & Marzari 2012). What is particularly interesting about a flared disc is that the partial gap-opening mass is a function of orbital radius.

To investigate the evolution of co-orbital systems embedded in a flared disc, we performed an additional suite of simulations with $m_0 = 10 M_\oplus$, $33 M_\oplus$ and $M_j$. The disc model’s aspect ratio is $h = h_0 (R/R_p)^{0.3}$ where $h_0 = 0.05$ and $R_p$ is the initial position of the planets, set to be 5 au. Due to the computational expense of the simulations, we study the large-scale migration of these systems using three different sets of simulations covering different radial zones: $1 < R < 5$ au, $0.5 < R < 1$ au and $0.15 < R < 0.5$ au.

Fig. 4 (left-hand panel) shows the orbital evolution of co-orbital systems for $1 < R < 5$ au. As for a constant aspect ratio with $h = 0.05$, the co-orbital configuration with $m_0 = M_j$ is unstable once the planets reach $R_j \sim 4.5$ au. At this location, the planets are in the partial gap-opening regime, with $q/h_j^3 \sim 2$. The planets open a non-axisymmetric gap since $P_1 \sim 1$ and $P_2 \sim 2$. For $m_0 = 10 M_\oplus$ ($m_0 = 33 M_\oplus$), $q/h_j^3 \sim 0.2$ ($q/h_j^3 \sim 0.8$) so that the planets do not open a gap and their evolution is stable.

As the planets migrate inwards, the disc’s local aspect ratio continuously decreases and we expect these systems to carve a gap in the disc and to become unstable once $q/h_j^3 \sim 1$. This is indeed what happens. The co-orbital system with $m_0 = 10 M_\oplus$ is unstable at $\sim 0.9$au (middle panel of Fig. 4) where the aspect ratio is $h_0 \sim 0.03$ ($q/h_j^3 \sim 3.5$). At this point, $P_1 \sim 1.5$ and $P_2 \sim 5$ which confirms that the gap is strongly non-axisymmetric in that case. The system with $m_0 = 10 M_\oplus$ is unstable at $\sim 0.23$au (right-hand panel of Fig. 4), where $h \sim 0.02$ ($q/h_j^3 \sim 3.7$). At this location, we note that $P_1 \sim 3$ and $P_2 \sim 15$.

Our results suggest that in a flared disc, equal-mass co-orbital systems become unstable wherever the planets can clear a partial gap. This corresponds to Saturn-mass planets located in the giant planet formation region outside the snow-line $R \sim 2.7$ au (Lecar et al. 2006) or co-orbital planets of a few Earth masses in the inner regions of the disc $R \lesssim 1$ au. A co-orbital system can remain stable during migration if the two components are approximately of Jupiter-mass and carve deep gaps in the disc. Lower mass co-orbitals could remain stable in a dead-zone where the viscosity is small enough for low-mass planets to open deep gaps and undergo type II migration (e.g. Matsumura, Pudritz & Thommes 2007).

4 DISCUSSION

Here, we discuss how our results depend on certain physical parameters. We focus on the effect of the equation of state of the disc, the disc’s viscosity and the mass ratio between the trailing and leading planets. We also examine the possible fates of planet pairs after disruption of the co-orbital resonance.

4.1 Effect of the equation of state

The simulations presented so far used a locally isothermal equation of state. The disc’s aspect ratio was constant. Although this
is reasonable when modelling the outer, optically thin regions of protoplanetary discs (see, e.g., fig. 20 in Pierens & Raymond 2011), this approximation breaks down in the optically thick inner parts where the equation of state is more likely to be adiabatic. To test the effect of the equation of state on our results, we performed an additional set of simulations using a non-isothermal disc model. The energy equation that we use includes the contribution from viscous heating plus a radiative cooling term $Q_{\text{rad}} = 2\sigma_{\text{eff}}T^4$, where $\sigma_{\text{eff}}$ is the Stefan–Boltzmann constant and $T$ the effective temperature which is computed using the opacity law of Bell & Lin (1994). For these calculations, the viscosity is $\nu = 10^{-5}$ and the disc surface density at $R = 1$ was chosen such that the disc aspect ratio is $h_R \sim 0.05$ at this location.

The results of these simulations are presented in Fig. 5. The figure shows the evolution of the orbital distance for $m_1, m_1 = 10 \text{M}_\oplus$, $1 \text{M}_\oplus, 1 \text{M}_J$. We note that for the disc parameters employed here, a simulation performed with a single $10 \text{M}_\oplus$ planet resulted in outward migration, which is consistent with the presence of a negative entropy gradient in the disc (Baruteau & Masset 2008). The fact that a pair of $10 \text{M}_\oplus$ co-orbital planets is observed to migrate inwards is possibly related to the saturation of the corotation torque in the region located in between the planets, where the libration period is very short. In that case, only half of the corotation torque remains, which is clearly not enough to counterbalance the effect of the differential Lindblad torque. Comparing Fig. 5 and the upper panel of Fig 3 – which corresponds to a locally isothermal equation of state with $h = 0.05$ – we see that very similar evolution outcomes are obtained. In both cases, we indeed find that the co-orbital resonance for $m_1, m_2 = 1 \text{M}_\oplus$ is broken whereas the 1:1 resonance for $m_1, m_1 = 10 \text{M}_\oplus$ or $1 \text{M}_J$ remains stable. The 1:1 resonance is disrupted more quickly for the radiative disc model simply because the disc mass is higher in that case. This suggests that our results are fairly robust against the choice of the equation of state.

4.2 Effect of the viscosity

As discussed earlier, the mechanism responsible for the destabilization of the 1:1 resonance occurs for planets in the partial gap-opening regime. This typically requires $q/h^3 \sim 1$. According to equations (3) and 5, we expect our results to depend on the value of the viscosity. For an inviscid disc, the gap structure tends to be axisymmetric and therefore gap-opening co-orbitals are stable, whereas for high values of the disc viscosity, the planets do not open a gap which prevents the mechanism to operate. In order to examine the effect of varying the disc viscosity, we performed a series of simulations for an isothermal disc model with $h = 0.02$ and $\nu = 10^{-4}$. We remind the reader that in the case where $\nu = 10^{-5}$, we found that $10 \text{M}_\oplus$ co-orbitals become unstable due to the non-axisymmetric depletion of the horseshoe region. Considering the case with $\nu = 10^{-4}$, we have $(p_1, p_2) \sim (30, 140)$ for $m_1, m_1 = 10 \text{M}_\oplus$ whereas $(p_1, p_2) \sim (3, 14)$ for $m_1, m_1 = 1 \text{M}_S$, so that the gap tends to be strongly non-axisymmetric in the latter case while $10 \text{M}_\oplus$ co-orbitals are not expected to open a gap for $\nu = 10^{-5}$. For this disc model, the orbital distance versus time for co-orbital planets with $m_1, m_1 = 10 \text{M}_\oplus, 10 \text{M}_\oplus, 1 \text{M}_S$ is plotted in Fig. 6. In agreement with the previous expectation, we indeed find that $10 \text{M}_\oplus$ co-orbitals are now stable whereas the 1:1 resonance between planets with $m_1, m_1 = 1 \text{M}_S$ is much more chaotic, with the planets’ eccentricity reaching values of $e_\text{p} \sim 0.4$. Over longer time-scales, the co-orbital pair is found to migrate outwards due to the high value reached by the eccentricity, which is still growing. Although the evolution outcome for this run remains uncertain over the time-scale covered by the simulation, it seems likely that this system will ultimately become unstable.

4.3 Evolution of co-orbital of different mass

We now discuss how the stability of the 1:1 resonance depends on the mass ratio $q_c = m_c/m_1$ between the trailing and leading planets. To investigate this issue, we have performed a suite of runs in a disc with constant aspect ratio $h = 0.05$ in which the trailing or the leading planet is a Saturn-mass planet while the mass of the second component is varied in the range $[10 \text{M}_\oplus, 1 \text{M}_J]$.

Fig. 7 shows the outcomes of the simulations. In the simulations in the upper panel, $q_c < 1$ (the leading planet is more massive), and in the lower panel all simulations have $q_c > 1$ (the trailing planet is more massive). There is a clear tendency for the 1:1 resonance to be more stable with a more massive leading planet ($q_c < 1$). For $q_c < 1$, only one system is unstable, with $m_1 = M_5$ and $m_1 = 1.7 \text{M}_S$. But for $q_c > 1$, the simulation with corresponding masses ($m_1 = 1.7 \text{M}_S$ and $m_1 = M_5$) was unstable on a much shorter time-scale and two additional simulations were unstable, with $m_1 = M_5$, $m_1 = 66 \text{M}_\oplus$ ($q_c = 1.5$) and $m_1 = 0.7 \text{M}_S$, $m_1 = 1 \text{M}_S$ ($q_c = 2.3$). Co-orbital systems with higher values of $q_c$ tend to be stable due to the ability of the more massive component to create a deep gap in the disc.

Co-orbital systems with more massive leading planets are more stable because when the two planets open a partial gap in the disc,
the positive (resp. negative) contribution to the torque exerted on the leading (resp. trailing) component due to the more gas-depleted disc region located behind (resp. ahead of) this planet counterbalances the effect of a stronger (resp. smaller) negative differential Lindblad torque. In the case where the trailing planet is more massive, the contribution from the gap region to the torque exerted on the trailing planet and the differential Lindblad torque add, resulting in amplified differences in torques felt by the leading and trailing planets.

4.4 Possible fates of planet pairs after disruption of the 1:1 resonance

For the isothermal simulations in which the 1:1 resonance is unstable, the long-term evolution outcome remains uncertain but it principle, systems in which the most massive planet is ejected to a larger radius may be trapped in a wider, mean-motion resonance, since the planets tend to undergo convergent migration after disruption of the co-orbital resonance in that case. For example, we checked that the period ratio is continuously decreasing after disruption of the co-orbital resonance in the runs of Fig. 7 with $q_c = 1.5$ and 2.3, which suggests that the planets may eventually become trapped in an MMR. An alternative possibility is that the planets become locked in a co-orbital resonance again. It has been indeed shown that co-orbital planets can be formed in isothermal discs during the relaxation of a swarm of low-mass planets migrating inwards (Cresswell & Nelson 2006).

In the case of radiative disc models, capture in a new 1:1 resonance may also occur for unstable systems with equal-mass planets, for example during the convergent migration of protoplanets towards a zero-torque radius. Formation of co-orbital planets was indeed observed to arise in the radiative simulations of Pierens, Cosou & Raymond (2013), in which 3 M$_\odot$ embryos migrate towards a convergence line created by a change in the opacity regime.

5 CONCLUSION

We have used hydrodynamical simulations to study the orbital evolution of co-orbital planets located at their mutual $L_1/L_2$ Lagrange points embedded in a protoplanetary disc. Co-orbital (also called Trojan or 1:1 resonant) configurations are disrupted when the planets open a partial gap around their orbit. This occurs because the gap that opens between the two planets is far more depleted than the rest of the co-orbital region (Fig. 2). The trailing planet feels a negative torque and the leading planet feels a positive torque, resulting in divergent migration of the two planets’ orbits. For a constant disc aspect ratio of $h = 0.05$ and a viscosity typical to that in the active regions of protoplanetary discs, this mechanism destabilizes co-orbital systems in the Saturn-mass range (Fig. 3, top panel). For a thinner disc with $h = 0.02$, the 1:1 resonance is destabilized for planets of a few Earth masses (Fig. 3, bottom panel). Evolved protoplanetary discs are expected to be flared due to heating from the central star such that the partial gap-opening mass is a function of the orbital radius. As they migrate inwards, co-orbital configurations with different masses are therefore disrupted at different orbital radii. Saturn-mass co-orbitals are disrupted beyond the snow line, 30 M$_\oplus$ co-orbitals are disrupted at $\sim 1$ au and 10 M$_\oplus$ co-orbitals are disrupted at a few tenths of an au (Fig. 4).

Although most of our simulations were performed with an isothermal equation of state, a series of runs using radiative disc models indicated that our results are very robust regarding the choice of the equation of state. This occurs because the mechanism presented here is at work for planets that partially deplete their horseshoe region, and for which the effect of the corotation torque is substantially weakened.

In the partial gap-opening regime, co-orbital configurations are more stable in systems with more massive leading planets. This is because the positive contribution to the torque from the gap region exerted on the leading component is balanced by a stronger (negative) differential Lindblad torque. For systems with a more massive trailing planet, the co-orbital system becomes unstable if $q_c = m_l/m_t \lesssim 2.5$. Systems with higher mass ratios are more stable due to the ability of the more massive component to create a deep gap in the disc.

As they migrate inwards, co-orbital systems can remain stable under certain conditions. First, if both planets are in the full gap-opening regime (e.g. Jupiter-mass planets at 5 au), then the gap is axisymmetric and too deep for this mechanism to operate. Secondly, if the planets have a mass ratio larger than roughly a factor of 2, then the perturbed surface density in the co-orbital region is nearly axisymmetric and the resonance remains stable.

We therefore predict that no close-in co-orbital systems will be discovered with near-equal-mass planets with masses between a few Earth masses and a Saturn mass.

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