Interaction Graphs for Reliability Analysis of Power Grids: A Survey

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Abstract—Reliability analysis of power grids has been the focus of many researchers for years. However, the complex interactions among the large number of components in these systems and their contributions to the reliability of the system are not yet completely understood. Therefore, various techniques have been developed and used to model and analyze the underlying interactions among the components of the power grid with respect to reliability challenges, such as cascading failures. Such methods are important to reveal the essential information that may not be readily available from power system physical models and topologies. Based on these models, the influences and interactions among the components of the system may occur both locally and at distance due to the physics of electricity governing the power flow dynamics as well as other functional and cyber dependencies among the components of the system. These methods use data-driven approaches or techniques based on the physics of power and develop a graph to model the interactions among the components of the power grid. In this survey, we review various methods of developing interaction graphs as well as studies on reliability analysis of power grids using these graphs.

Index Terms—reliability, interaction graphs, cascading failures, data-driven, power grids, system modeling

I. INTRODUCTION

Reliability analysis of critical infrastructures such as power grids has been the focus of many researchers and practitioners for years. However, understanding and addressing certain reliability issues in power systems remain a challenge due to the large size of these systems and complex and sometimes hidden interactions among the components of these systems during various events. An example of such complex reliability challenge in power grids is cascading failures. Cascading failures are successive failures of components in the system in a relatively short period of time that can lead to large blackouts, such as the case of US Northeast Blackout in 2003 [1]. Various studies and models have been developed to understand and control these complex phenomena including methods based on power system simulation [2], [3], deterministic analytical models [4], probabilistic models [5], and graph-based models [6]–[48]. For a survey of various methods for studying cascading failures see [49], [50]. Each of these approaches shed light on different aspects of these phenomena.

Among these categories of approaches, graph-based methods have attracted a lot of attention due to the simplicity of the models and ability to describe the propagation behavior of the failures on the graph of the system [52], [53]. Many initial graph-based models were developed based on the physical topology of the power system, where the connections among the nodes represent the actual physical connections among the components of the system [54], [55]. However, a study in [56] showed lack of strong connection between the physical topology of the system and failure propagation in cascading failures in power grids. In general, influences and interactions among the components of the system during cascade process may occur both locally and at distance due to the physics of electricity governing the power flow dynamics as well as other functional and cyber dependencies among the components of the system. For instance, historical as well as simulation data verify that failure of a critical transmission line in the power grid may cause overload/failure of another transmission line that may or may not be topologically close. Therefore, graph models based on the physical topology of the system are not adequate in describing the propagation behavior of failures in power grids. Hence, new methods are emerging to reveal the complex and hidden interactions that may not be readily available from power system physical models and topologies. These new approaches are focused on extracting and modeling the underlying graph of interactions among the components of the system. In this survey, we will review various techniques in building such interaction graphs. While the main focus of this survey is on methods for constructing interaction graphs, we will also briefly review studies and analysis performed using such graphs. The benefit of interaction graphs is that the interactions among the components will be topologically local, which will simplify the study and analysis of propagation behavior of failures and properties and roles of various components in the system during the cascade process or other reliability challenges. Note that as cascading failures are attributes of transmission networks in power grids, the focus of such studies are mainly on the transmission network; however, similar methods can be used for modeling interaction graphs in distribution layer of power grids. While the majority of studies based on interaction graphs are focused on reliability challenges in power grids, the interaction graphs can be used for other applications in power grids or other networked systems with component interactions such as transportation networks [57], [58].

We categorize various methods for building interactions graphs into two main classes: data-driven approaches and power physics-based approaches as shown in Table I. As the name implies, the data-driven approaches for building interaction graphs rely on data collected from the system

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(historical and real data or simulation data) for extracting and characterizing interactions among the components of the system. We further define five sub-categories for data-driven interaction graphs based on the method used for analyzing the data. These include (1) methods based on outage sequence statistics [6]–[18], (2) influence-based methods [19]–[22], (3) markovian methods [23], (4) risk-graph methods [24]–[27], and (5) correlation-based methods [28], [59]. On the other hand, power physics-based approaches exploit properties based on physics of power and electricity governed by Kirchoff’s laws to define interactions among the components. Thus, instead of interactions between components being represented by topological distances, interactions are represented by electrical distances, which illustrate the properties of the electrical interactions based on power flows between components. We define two sub-categories for power physics-based interaction graphs based on the electrical properties utilized for creating the graphs. These include (1) impedance-based methods, which define interactions by replacing multiple paths between components by a single distinct equivalent path using impedance values [29]–[35], effective resistances [36]–[38], or weighted impedance [39], and (2) methods that define the interactions based on changes in power flow due to outage in transmission lines [40]–[44] or sensitivities in components’ states due to changes in voltage magnitudes [45]–[48], [51]. We will discuss these methods in detail in Section III.

In addition to review of various methods of building interaction graphs, we will briefly review various reliability analysis studies performed using these graphs. Some studies of interaction graphs are focused on identifying critical components in the cascade process of power grids [7]–[17], [19]–[26], [29], [36]–[39]. These studies can have different purposes such as (1) identifying the vulnerable or most influential components of the system in the cascade process by utilizing standard centrality metrics [7]–[13], [31], [35] or defining new centrality metrics [14]–[17] and (2) identifying the set of components that their upgrade (for instance, by increasing the power flow capacity of transmission lines) or protection can help in mitigating the risk of cascading failures and large blackouts [19], [20], [23], [37], or quantifying the performance of the grids after addition of new transmission lines [37], [38]. To characterize the latter, some works [9], [24]–[26], [38] focus on response of power grids to attacks and failure scenarios using metrics that quantify the efficiency of the grid before and after attacks.

Furthermore, to characterize the role of components in the cascade process, some efforts are focused on characterizing the patterns and structures in interaction graphs using community detection approaches [21], [22] or tree structures [41], [40]. Moreover, the work in [41] uses interaction graphs to identify transmission lines that if switched off creates partitions that limit the propagation of failures in the power grid. Similarly, structures in interaction graphs have been used for reliability analysis of zonal patterns [32] and partitioning into voltage control regions [51]. We discuss these analysis in Section IV.

II. GRAPH OF PHYSICAL TOPOLOGY

As mentioned in Section I, initial graph-based studies of power grids, such as [52]–[55], were based on the physical topology of the power grid. In general, a power grid can simply be represented by graph, $G = (V, E)$, where $V$ represents the set of generator, transmission, and substations buses and $E$ represents the set of power lines. Such graph shows the physical connectivity among the components of the system. Various studies have been performed on the physical topology of the power grids by analyzing their structural properties [54], [55], such as average path length, clustering coefficient, and degree distribution, for analyzing power grids with respect to standard complex networks such as small world, random, and scale-free graphs. Some studies performed on the physical topology also focus on properties of the electrical connections [60] identified using centrality measures such as degree, eigenvector, closeness, and betweenness (for a review and definition of centrality measures refer to [61]). However, it has also been discussed that physical graphs may be inadequate in representing and capturing the interactions among the components of the power grid [47], [56] specifically for analyzing cascading failures.

III. GRAPH OF INTERACTIONS

In this section, we review the methods of representing power systems by graphs of interactions in two categories: data-driven methods and power physics-based methods. The outcome of these methods is a graph of interactions, denoted by $G = (V, E)$ in which the set of vertices $V$ can be
the set of buses or transmission lines depending on the interactions of interest for analysis. The set of $E_i$ in this case represents the set of interactions/influences among the components, which may be directed, undirected, weighted (representing the strength of interactions or influences) or unweighted depending on the analysis of interest.

A. Data-driven Methods for Interaction Graphs

Various data-driven approaches have been proposed for revealing and modeling interactions among the components of the power grid. These approaches rely on data from simulation or historical and real outage datasets. As the historical datasets are limited, majority of the studies use simulation data. In this paper, we have identified and reviewed five classes of data-driven methods for modeling interaction graphs for studying cascading failures in power grids, as discussed next.

1) Interaction Graphs based on Outage Sequences in Cascading Failures: This class of methods rely on cascading failure data in the form of sequence of failures in each cascade. For instance, the sequence $l_5 \rightarrow l_7 \rightarrow l_3 \rightarrow l_6$ represents an example of sequence of transmission line failures in a cascade scenario. These methods are based on analysis of sequence of failures for extracting interactions and focus on the cause and effect interactions among failure of components. Methods in this category use various techniques and statistics to analyze such data as discussed next.

a) Interaction Graph based on Consecutive Failures: In this category of outage sequence analysis, only direct consecutive failures in a sequence are used for deriving the interaction links among the components. In other words, two components in the system have a directed interaction link, $e_{ij} \in E_i$, if they appear as successive outages in the order $l_i \rightarrow l_j$ in a cascade scenario in the dataset. For instance, if the sequence $l_5 \rightarrow l_7 \rightarrow l_3 \rightarrow l_6$ represents an example sequence of transmission line failures in a cascade scenario, the following directed interaction edges will belong to $G_i$: $\{e_{57}, e_{73}, \text{ and } e_{36}\} \in E_i$. The strength of interactions among the components in this case can be characterized using the statistics of occurrences of pairs of successive outages in cascade scenarios in the dataset. For instance, the work in [6] assigns weights to the interaction edges by statistical analysis of number of times that a pair of successive line outages occur in the cascade dataset (i.e., $|l_a \rightarrow l_b|/|\text{total number of successive pairs}|$, where $|l_a \rightarrow l_b|$ is the number of times failures $l_a$ and $l_b$ occur successively in the cascade dataset). These weights can be interpreted as the probability of occurrence of each pair of successive line outages. Examples of studies using this method to develop the power grid’s graph of interactions include [6]–[13], where they consider transmission lines in the system as the vertices $V_i$ of the interaction graph $G_i$.

In the works presented in [7]–[9], [11]–[13], the sequences of consecutive failures are called fault chains. The dataset of fault chains are created by tripping a single transmission line in the simulation and identifying consecutive failures based on power flow re-distributions. In these works, for a power grid with $n$ transmission lines, $n$ fault chains are created and the edges among consecutive failures in each chain is weighted based on power flow changes in a line after the failure. In the work in [10], fault chains are created by considering multiple initial failures such that, a system with $n$ transmission lines may have more than $n$ fault chains. In addition to power flow re-distributions, the work in [12] also considers the temperature evolution process of transmission lines during cascades while constructing fault chains. Thus, the fault chain graph is capable of reflecting thermo-physical effects of transmission lines during cascades. Finally, a fault chain graph is developed by combining all fault chains together into a single graph where the vertices are all the components that have failed in the fault chains and the edges between the vertices exist if the outages have successively occurred in the fault chains. For pairs of outages $(i \rightarrow j)$ that have reoccurred in multiple fault chains, their combined edge weight in the fault chain graph is averaged.

b) Interaction Graph using Generation-based Failures: The method based on the consecutive failures discussed in Section III-A1a focuses on one to one impact that the outage of a line has on the outage of another line. However, in cascading failures, instead of pair-wise interactions among successive failures, a group of failures may contribute to failures of other components. Therefore, it is important to consider the effects of groups of failures and characterize interactions among the components based on the effects among groups of components. The works presented in [14]–[16] define such groups as generation of failures within a cascade process, which are failures that occur within short temporal distance of each other. In these works, the sequence of failures in the cascade are divided into sequence of generations and the failure induced cause and effect relationships are considered between consecutive generations. Specifically, outages occurring in generation $m+1$ are assumed to be caused by outages in generation $m$.

The interactions based on successive generations are defined in different ways in the literature. For instance, the authors in [19] assume that all components in generation $m$ have interactions with all components in generation $m+1$, i.e., if generation $m$ has $n_1$ number of components and generation $m+1$ has $n_2$ number of components, then the number of interactions between generation $m$ and $m+1$ will be $n_1 \times n_2$. But some studies argue that considering all possible pairs of interactions among components of two consecutive generations overestimate the interactions among components [14]–[18]. Specifically, all line outages in one generation may not be the cause of a line outage in the next generation. Therefore, the works presented in [14]–[18] define the line interactions between consecutive generations selectively as following. In [14], [16], the cause of failure of a line $k$ in generation $m+1$ is considered to be due to the failure of a line in generation $m$ with the maximum influence value on the line $k$. The influence value for component $j$ in generation $m$ is defined as the number of times that the component $j$ has failed in a generation $m$ before the failure of line $k$ in the successive generation $m+1$ in the cascade dataset. For cases where two or more lines in generation $m$ have the same maximum influence values on line $k$ in generation $m+1$, all such components are assumed to interact with line $k$. In the works discussed so far in this section, the interaction among component $j$ in generation $m$ and component $k$ in generation...
$m + 1$ will be represented by a directed link $e_{jk}$. The weight of the link can be defined as the ratio of number of times that the pair of components appeared in two successive generations over the total number of times that component $k$ has appeared in the dataset. This weight can be interpreted as the probability of failure of component $k$ in the next generation given the failure of component $j$ in the current generation.

While the work in [14], [16] ignores many interactions by only considering maximum influence values in current generation as probable cause of component failures in the next generation, the work in [15] gives an estimate of the interactions between successive generations using the expectation maximization (EM) algorithm. Initially, all failed components in generation $m$ are assumed to be causes of failure of all components in generation $m + 1$. However, the actual components in generation $m$ (hidden variables) that cause failure of components in generation $m + 1$, are found using the iterative process of updating the probabilities of failures. Thus, after the iterative update process is completed, some probabilities of failures between components may be zero, which removes the overestimated interactions of the initial assumption. The work in [17] also generates the graph of interactions based on the maximum influences among generations in cascades, similar to the method used in [14], [16]. However, instead of statistically assigning the weights of the interaction link, the weights of the links are assigned based on the amount of load shed that has occurred after the failures in generation $m$. Therefore, the dataset requires additional information about the amount of load shed during the cascade process. The study in [18] considers both statistical properties as well as the amount of load shed that has occurred between successive generations to assign interaction link weights. However, the study in [18] identified islands formed in the power grids during outages and then, selectively assigned links between components of successive generations only if the generations were located in islands that were direct consequences of one another.

c) Influence-based Interaction Graph: In this method, the interactions among the components are derived based on successive generations in cascades; however, the weights of the interactions are characterized based on the influence model and the branching process probabilistic framework. The influence model is a networked Markov chain framework, originally introduced in [62] and was first applied to cascade dataset to derive the weights of interactions of a randomly generated network [63].

In this survey, we review studies that use influence model in the context of power grids to develop graph of interactions, where they consider transmission lines in the system as the vertices $V_i$ of the interaction graph $G_i$ and influences/interactions between the lines as the edges $E_i$. For instance, in [19], authors consider interaction links among all pairs of components of two successive generations in a cascade. Then, the weights of the directed links are derived in two steps. In the first step, a branching process approach is used in which each component can produce a random number of outages in the next generation. The number of induced outages by each component is assumed to have a Poisson distribution based on the branching process model. Parameter $\lambda_i$ specifies the propagation rate (mean number of outages) in generation $m+1$ for the outage of component $i$ in generation $m$. In other words, this step defines the impact of components on the process of cascade by describing how many failures their failure can generate.

In the second step, it is assumed that given that component $i$ causes $k$ outages in the next generation, some components are more likely to outage than others. Therefore, they calculate the conditional probability $g(j|i)$, which is the probability of component $j$ failing in generation $m + 1$, given the failure of component $i$ in generation $m$. If only $g(j|i)$ values based on the statistical analysis of data are considered, then the probability of failure of component $j$ given component $i$ failure will be known; however, the expected number of failures from failure of component $i$ is not known. Hence, both steps are important in characterizing the influences among components.

The final step consists of combining the information from the first and second steps into a single influence matrix $H$ (representing the links of graph of interactions and their weights). The elements of the matrix are defined based on the conditional probability that a particular component $j$ fails in the next generation $m + 1$, given that component $i$ has failed in generation $m$ and that generation $m + 1$ includes exactly $k$ failures. This probability can be defined as $P(j|i, k) = 1 - (1 - g(j|i))^k$. Then, the conditional probability $h(i, j, m)$ that component $j$ fails in generation $m + 1$, given that component $i$ failed in generation $m$, over all possible values of $k$ represents the actual elements of $H$, and is found by multiplying $P(j|i, k)$ with the probability of $k$ failures occurring as follows:

$$h_{i,j,m} = \sum_{k=0}^{\infty} (1 - (1 - g(j|i))^k) \frac{\lambda_i^k m^k}{k!} e^{-\lambda_i m}.$$  

Based on the influence graph, cascading failures can start with a line outage at a node of the graph and propagate probabilistically along the directed links in the graph. Examples of other works, which have used the influence-based approach to derive the graph of interactions for power grids include [20]–[22].

2) Markovian Interaction Graph: This method, which can be considered as the generalized influence graph, is presented in [23]. The main goal of this method is to address the problem of the effect of multiple simultaneous outages within generations on the characterization of the interactions among the components of the successive generations in a cascade. In this case, the developed graph is a Markov chain, where the nodes represent the states of the Markov chain defined as the set of line outages in a generation of the cascade and the links represent the transition among the states (i.e., interactions between successive generations of outages). Hence, each node in the graph may represent the outage of a single line or multiple lines. Markovian interaction graphs differ from generation-based and influence-based interaction graphs as edges are the interactions between successive generations of sets of line outages instead of the individual interactions between line outages in successive generations. Markovian interaction graphs also consider a node with a null state, which
represents the state where the cascade stops. This state occurs at the end of all cascade scenarios. The transition probabilities among the states (i.e., the weight of the links) from state \( i \) to state \( j \) can be estimated by counting the number of consecutive states in which state \( i \) and state \( j \) occur in all the cascades and dividing by the number of occurrences of state \( i \).

3) Risk Graphs for Interaction Graph: The work presented in [24] introduces the risk-based interaction graphs, which describes the interactions or relationships among the nodes (i.e., buses/substations) of the power grid based on effects of their simultaneous failures in causing damage in the system. This graph is not solely focused on analysis of interactions among components during cascading failures. Instead, it is focused on the vulnerability analysis of the power grid and the effect of failures is assessed using metrics such as net-ability, which measures the effectiveness of a power grid subjected to failures, based on power system attributes including power injection limitation and impedance among the components.

Construction of risk graphs are done in two steps. The first step includes generating and tracking the sets of strongest node combinations whose simultaneous failures have significant effects on the power grid. Identification of such sets of strong node combinations can be done by reducing the search space or exhaustive search methods [24], [25]. Reduced search space strategy is the preferred method for computational purposes. For instance, in [25] the search works as following: given \( m \)-node combination, which causes damage in the network, \( m+k \)-node combination (where \( k \) represents additional components) should cause an even greater damage.

In the second step, these sets of strong node combinations are used to form the risk graphs. If a node appears at least once in the sets of strong node combinations, then the node becomes a vertex of the risk graph. Links among nodes in the risk graph exist if they appear in the same set of strong node combination. Both nodes and links in the risk graph are weighted based on the frequency of their appearance in the sets of strong node combinations. This approach results in a weighted but undirected node risk graph, where higher weight values on the links suggest stronger node combinations. Node risk graphs are dependent on the system parameters such as ratio of capacity to the initial load of the nodes in the system. To remove dependencies on system parameters, node risk graphs can be constructed for multiple parameter values and combined together to form the node integrated risk graph using the risk graph additivity property [24], [25]. The aforementioned risk graph can also be extended to a directed risk graph, where the removal of components in a specific order in strong node combinations are considered. The study in [26] constructs the directed node risk graphs and the directed node integrated risk graph with the same concept as its undirected counterpart in the studies in [24] and [25].

Another similar concept to risk graph is the double contingency graph introduced in [27]. While \( m \) contingency combinations of attack scenarios for the power grid was studied in the risk graphs, many methods focus on N-2 contingency analysis as the power grid is considered to be N-1 protected [64]. In the double contingency graph, the vertices of the graph are the transmission lines and the links between vertices show pairs of transmission lines whose simultaneous failure as initial triggers can affect the reliability of the system by, for instance, violating the thermal constraint rules in the power grid. Similar to the risk graph, double contingency graph only considers combinations of initial triggers and lacks information about the components, which will be affected due to the outage of the initial triggers. Therefore, the work in [27] uses a combination of the double contingency graph with influence graph for reliability analysis of the power grid.

4) Correlation-based Interaction Graph: The work in [28] presents a graph of interactions for power grids based on correlation among the failures of the components. In the correlation-based interaction graph in [28], vertices represent the transmission lines and the edges represent the pairwise correlation between line failures in the cascade dataset. The correlation dependence between failures are captured in the correlation matrix, whose \( ij \)th elements are positive Pearson correlation coefficient between the failure statuses of components \( i \) and \( j \) in the cascade dataset. The resulting correlation matrix is symmetric and can be interpreted as an undirected and weighted interaction graph, where the nodes are the failed lines, the edges are the interactions between the lines and the weights are the correlation values among the components. Similarly, the studies in [21] and [22] also construct correlation-based interactions graphs from simulated cascade dataset consisting of sequences of transmission line failures.

B. Power Physics-based Interaction Graphs

While there is abundant literature focused on data-driven interaction graphs, various power physics-based approaches that consider the electrical properties of the power system for modeling interactions among the components have also been proposed. In this paper, one main class of power physics-based method i.e. electric distance-based method for constructing interaction graphs is discussed.

1) Electric Distance-based Interaction Graph: In a power grid, electricity does not flow through the shortest path between two nodes \( i \) and \( j \). Instead, it can flow through parallel paths between nodes \( i \) and \( j \) based on the physics of electricity (i.e., Ohm’s law). The concept of electric distance was first introduced by Lagonotte et al. [51] in 1989 as a measure of coupling between buses in the power system and was based on sensitivities in the power system due to changes in voltage magnitudes. It can be broadly classified into two categories: sensitivity-based and impedance-based. We introduce some preliminaries and discuss both of these categories in detail. For a power system with \( n \) buses, we have \( I = YV \), where \( I \) is an \( n \times 1 \) nodal current injection vector, \( V \) is an \( n \times 1 \) nodal bus voltage vector and \( Y \) is an \( n \times n \) admittance matrix. The admittance matrix \( Y \) is symmetric and sparse as it contains non-zero elements only if there is a direct physical connection between the buses. Matrix \( Y \) can be viewed as an undirected but weighted graph of the physical topology of the system. Next, we discuss the electric distance-based interaction graphs of the power grid that capture connections between components that extend beyond the physical topology.
a) Interaction Graphs using Impedance-based Electric Distance: Impedance-based electric distance interaction graphs $G_i$ consist of vertices $V_i$ that represent buses and edges $E_i$ that represent electrical connections between pairs of buses weighted by their corresponding electrical distances. Electrical distances between pairs of buses are commutative in nature such that $e_{ij} = e_{ji}$. Thus, impedance-based interaction graphs are undirected and can broadly be divided into three categories: inverse admittance, effective resistance and weighted impedance, as discussed next.

1) Inverse Admittance Interaction Graph:
Inverse admittance matrix, more commonly known as the impedance matrix $Z$, is found by inverting the system admittance matrix $Y$, i.e., $Z = Y^{-1}$. Similar to the $Y$ matrix, matrix $Z$ is symmetric and shows the relationship between the nodal bus voltage vector and the nodal current injection vector. However, impedance matrix $Z$ is non-sparse as it represents the changes in nodal voltage throughout the system due to a single nodal current injection between a pair of nodes in the system. The impedance matrix $Z$ shows electrical connections between pairs of buses in the system. Therefore, edges in the inverse admittance interaction graph are the connections between the elements in the $Z$ matrix with weights between buses $i$ and $j$ corresponding to their absolute value of the impedance $|Z_{ij}|$ [29]–[33]. Smaller values of impedance represent shorter electric distance between buses. The studies in [34], [35] also adopt the concept of representing electrical distances between buses by the equivalent impedance between buses but apply the condition that power only flows from generator buses to load buses such that impedance values between generator buses and load buses suggest edges in the interaction graph.

2) Effective Resistance Interaction Graph:
Effective resistance, $R_{ij}$, between nodes $i$ and $j$, also known as Klein resistance distance [65] is the equivalent resistance of all parallel paths between the nodes. It was initially introduced in the study in [65] and shows the potential difference between nodes $i$ and $j$ due to unit current injection at node $i$ and withdrawal at node $j$. While impedance between nodes account for non-linear approximations of power flow, effective resistances only consider linear approximations of power flows in the grid. In the studies in [36]–[38], effective resistance between nodes $i$ and $j$ is found as $R_{ij} = Q_{ij}^\dagger - 2Q_{ij}^\dagger Q_{jj}^\dagger$ where, $Q_{ij}$ is the row $i$ and column $j$ element of $Q^\dagger$, which is the penrose pseudo-inverse of the Laplacian matrix $Q$. Matrix $Q$ is defined as the difference between the weighted diagonal degree matrix and the weighted adjacency matrix derived from the physical topology, and shows the relationship between the buses and transmission lines in the grid. In the studies in [36]–[38], the weights of the edges in the physical topology required for finding the weighted diagonal degree and adjacency matrix are the susceptance values between the nodes. Thus, edges $E_i$ in the effective resistance interaction graph reflect the electrical connections between the buses with weights between nodes $i$ and $j$ being the corresponding $R_{ij}$ values.

3) Weighted Impedance Interaction Graph:
Weighted impedance interaction graphs are similar to inverse admittance interaction graphs as they consider impedance between pairs of nodes in the system as the electrical distances between the nodes. However, they also consider the power flows through the lines along the path between the pairs of nodes [39]. Therefore, edges $E_i$ in the weighted impedance interaction graph, which correspond to the electrical connections between buses $i$ and $j$ along path $k$, are weighted by the impedance between buses $i$ and $j$ as well as the power transfer distribution factor of the lines along the path $k$ of power flows between the buses.

b) Interaction Graphs using Sensitivity-based Electric Distance: Power system sensitivities to various conditions such as changes in power flows due to transmission line outages and their impacts (e.g., measured using line outage distribution factor (LODF) [66]) can be utilized for forming interaction graphs. Sensitivities showing changes in voltage magnitudes and voltage phase angles during normal conditions can also be used to form interaction graphs. Thus, sensitivity-based electric distance interaction graphs can broadly be classified into two categories: outage-induced and non-outage induced. Both categories are discussed next.

1) Outage Induced Interaction Graphs:
As the name suggests, this class of methods for forming electric distance-based interaction graphs are focused on interactions among the components of the power grid during outage conditions. For instance, in the study in [41], interactions among the components as well as their weights are derived using the changes in power flows in transmission lines during outage conditions. Thus, the outage induced interaction graph $G_i$ consists of vertices $V_i$ that represent transmission lines and edges $E_i$ that represent the impact of the outage among the lines. This impact is characterized using LODF [66], which is derived from the spectral graph properties of the physical topology of the power grid. Specifically, in the study in [40], [41], LODF for line $e_{ij} \in E_i$ is the ratio of two variables: the numerator variable measures the impact of outage of line $i$ on line $j$ using the reactance of all possible spanning tree paths between the lines and the denominator variable measures the impact of outage of line $i$ on line $j$ using the reactance of all alternative spanning tree paths that the power can flow (i.e., excluding the spanning tree path of line $i$). Thus, the direction and weight of the edges are specified by the corresponding LODF between the lines.

However, the impact of an outaged line on the remaining lines is not limited to changes in power flows. In a power grid, if two or more lines share a bus, outage of one line may expose the remaining lines (connected through the same bus) to incorrect tripping due to malfunctioning of the protection system such as relays. The exposed lines are prone to failure and increase in power flow in the exposed lines exacerbates their tripping probability causing further outages. Such failures are known as hidden failures. In the studies in [42]–[44], vertices $V_i$ represent transmission lines as well as hidden failures’ condition state and edges $E_i$ represent inter-line interactions as well as interactions between lines and hidden failures. Thus, the interaction graph $G_i$ will have $n + 1$ nodes where $n$ is the number of transmission lines and the extra one node represents the effect of hidden failures. The hidden failure node has bidirectional links from itself to every other
node in the power grid. However, the hidden failure node does not have influence on itself. The inter-line interaction \( e_{ij} \in E_i \) shows the increase of power flow in line \( j \) due to outage of line \( i \). The interaction from the hidden failure node to a line reflects the tripping probability of the line caused due to the increase in power flow exceeding the line flow limits. Interaction between a line to the hidden failure node reflects the influence that the outage of the line has on the hidden failure through other lines.

2) Non-outage Induced Interaction Graphs:

Electric distance-based interaction graphs can also be constructed using the sensitivity matrix of the power grid during normal conditions. In such interaction graphs \( G_i \), vertices \( V_i \) represent the buses and the edges \( E_i \) represent the electrical interactions in terms of sensitivities between the buses. These sensitivities can be found using the Jacobian matrix, which is obtained during Newton Raphson-based load flow computation. Jacobian sensitivity matrix \( J \), shows the effect of complex power injection at a bus on the voltage magnitude and voltage phase angles of other buses. It consists of four sub-matrices: matrix \( J_{P0} \), which shows the relationship between nodal active power injections and voltage phase angle changes, matrix \( J_{PV} \), which shows the relationship between nodal active power injections and voltage magnitude changes, matrix \( J_{Q0} \), which shows the relationship between nodal reactive power injections and voltage phase angle changes, and matrix \( J_{QV} \), which shows the relationship between nodal reactive power injections and voltage magnitude changes. The inverse of any of these Jacobian sub-matrices, denoted as \( J^{-1} \), can be used to find the sensitivity matrix by using the Klein Resistance distance [65], whose individual element is calculated as \( x_{ii} + x_{jj} - x_{ij} - x_{ji} \), where \( x_{ij} \) represents the element in row \( i \) and column \( j \) of the inverted Jacobian sub-matrix \( J^{-1} \) in consideration. In the study in [33], all of the four Jacobian sub-matrices are applied to Klein Resistance distance to form sensitivity matrices, which in turn can be used to form interaction graphs whose edges \( E_i \) represent the electrical interactions between the components of the sensitivity matrices weighted by their corresponding elements.

However, literature has revealed that most studies are focused on any one of the four Jacobian sub-matrices. The seminal work of electrical distances by Lagonotte et al. in [51] focuses on using the Jacobian sub-matrix \( J_{QV} \), also known as the voltage sensitivity matrix \( \partial V / \partial Q \), to find the electric distance between buses. Similarly, the study in [45] also uses the voltage sensitivity matrix. In both of these studies, the matrix of maximum attenuations is found, which consists of columns of voltage sensitivity matrix divided by the diagonal values. Finally, electrical interactions \( e_{ij} \in E_i \) between buses \( i \) and \( j \) weighted by their electric distance is derived as the logarithm of the individual elements of the attenuation matrix. The study in [46] also uses the voltage sensitivity matrix to find electric distance between buses but instead of finding matrix of attenuations, the study applies the sensitivity matrix to Klein Resistance distance formulation. Similarly, the studies in [47] and [48], apply the Jacobian sub-matrix \( J_{P0} \), or the sensitivity matrix \( \partial P / \partial \theta \), to Klein Resistance distance and finds electrical distances between buses.

IV. RELIABILITY ANALYSIS USING INTERACTION GRAPHS OF POWER GRIDS

Interaction graphs constructed in Section III can be used for various analysis; specifically related to the reliability of the power grid; including analyzing role of components and finding critical ones that contribute heavily in a cascade process, predicting distribution of cascades sizes, and studying patterns and structures that reveal connections and properties of the components in the power grid that extend beyond physical topology-based graphs. Thus, we divide these reliability studies performed using the interaction graphs into various categories as discussed below.

A. Critical Component Analysis

We classify studies that identify and analyze role of critical components in power grid’s reliability into three broad categories that include 1) using pre-existing as well as novel measures to find critical buses/transmission lines, 2) evaluating attack strategies that cause significant damage in the power grid, and 3) employing mitigation measures such as upgrading transmission lines or adding new components to protect the identified critical components.

1) Critical Component Identification: This class of reliability analyses focuses on finding critical buses/transmission lines by analyzing structural properties of interaction graphs using standard centrality measures such as degree, betweenness etc. (for a review of standard centrality measures refer to [61]) or by defining novel interaction graph based metrics.

a) Critical Component Analysis using Standard Centrality Measures: In the studies presented in [7]–[9], [11]–[13], fault chain-based interaction graphs are found to be scale free graphs, indicating that most nodes possess low degrees but a limited number of nodes possess high in and out degrees. Thus, in the fault chain-based interaction graphs, vertices with higher degrees are assumed to be the critical components of the system. Similar conclusions are obtained by authors in the studies in [31], [35], where the inverse admittance interaction graph is observed to be scale-free and consisting of limited number of nodes with high degrees, which are considered as the critical components of the system. These are examples of works that consider the degree centrality measure to identify critical components of the system. Other centrality measures such as betweenness, eigenvector, and pagerank have also been considered on interaction graph-based representations of power grids including [42], [44], [46] to find critical components of the system.

b) Critical Component Analysis using New Centrality Measures: In addition to studies that rely on standard centrality measures; some works develop new centrality measures in the context of power grids and the developed interaction graphs to analyze criticality of the components. For instance, in the generation-based interaction graphs [14]–[18], out-strength measure, which is the sum of the weights of the interaction links originating from a node, is used to find critical transmission lines. Such lines are the ones whose failure at any stage of the cascade including the initial stage or propagation stage induces failure in significant number of other transmission lines. Outages in the initial stages are caused by external
factors such as bad weather conditions, improper vegetation management, and exogenous events, whereas outages in the propagation stage is caused due to power flow re-distributions, hidden failures, and other interactions between components. Influence-based [19], [20] and markovian [23] interaction graphs are also used to find critical transmission lines but they explicitly focus on lines whose failure during the propagation stage of cascading failures cause large cascades. Particularly, the studies in [19], [20] use a cascade probability vector derived using the influence-based interaction graph to quantify the probability of failure of lines during propagation stage of cascades and defines critical lines as the ones whose corresponding entries in the probability vector have higher values. Similarly, the study in [23] finds the probability distribution of states of the markovian interaction graph and defines critical lines as the ones that belong to states with higher probability of occurrence. Influence-based and correlation-based interaction graphs constructed in the studies in [21], [22] are also used to find the critical transmission lines during cascade processes by using a community-centrality measure. As the name suggests, the measure quantifies the criticality of transmission lines based on their community membership, where critical lines are the ones that belong to multiple communities or act as bridges between communities. Note that communities are defined as groups of vertices with strong connections among themselves and few connections outside (for definition of communities and a review of community detection methods on graphs refer to [67]).

Identification of critical lines is not limited to data-driven interaction graphs. Multiple studies use electric distance-based interaction graphs for such analysis as well. Effective resistance between components in the effective resistance-based interaction graph can be summed for all node pairs in the graph to find the effective graph resistance metric of the power grid. Effective graph resistance metric was initially defined in the study in [65] as Kirchhoff index and used in the study in [68] as a robustness metric. Lower values of this metric suggests that the power grid is robust to cascading failures. Effective graph resistance can also be found using the eigenvalues and eigenvectors of the Laplacian matrix of the grid [69]. In the study in [36], critical transmission lines are found by measuring the changes in effective graph resistance before and after the removal of the line. In a similar manner, weighted impedance interaction graph constructed in the study in [39] and inverse admittance interaction graph constructed in the study in [34] are also used to find critical transmission lines by measuring the changes in net-ability metric before and after removal of a line. Net-ability reflects the performance of a network by quantifying the ability of a generator to transfer power to a load within the power flow limits.

2) Studying the Effect of Line Upgrades and Line Additions on Reliability of Power Grids: While identification of critical components in the power grid is necessary, assessing the impact of modifications and protection of such critical components in the overall power grid is the next step in the study. In the studies presented in [19] and [20], an influence interaction graph-based metric is used to quantify the impact of upgrading the critical lines (for example, by improving vegetation management around the lines or by improving protection systems) on cascade propagation. The work in [23] uses the markovian interaction graph to do a similar study. In both interaction graphs, the authors conclude that upgrading lines that take part in propagation of cascades reduces the risk of large cascades compared to upgrade of lines that initiate cascades. While the studies in [19], [20] investigate the performance of the power grid networks after line upgrades, the studies in [37] and [38] use effective graph resistance metric to study the impact of adding transmission lines in optimal locations of the power grid. However, in [38] the authors warn that placing an additional line between a pair of nodes does not necessarily imply increased robustness of the grid. Infact, grid robustness may decrease after adding additional lines (due to Braess’s paradox [70]) if the additions are done haphazardly.

3) Analyzing Response to Attack/Failure Scenarios: In addition to identifying critical components and characterizing the impact of their modifications in the reliability of power grids, the study of the response of power grids to attacks and failures is also necessary. Such studies can be used to find critical components and attack strategies that threaten the reliability of the overall power grid. In the studies in [24]–[26], node integrated risk graphs are used to find groups of transmission lines whose removal from the graph causes the largest drop in net-ability of the power grid, as discussed in Section III-A3. These groups can be found in real-time independent of system parameters. The study in [38] also studies the robustness of power grids to deliberate attacks using the effective graph resistance metric, as discussed in Section IV-A1b.

B. Prediction of Cascade Sizes

Forecasting cascade sizes is another challenge in the reliability analysis of power grids. In the study in [28], a correlation graph-based statistical model, known as the co-susceptibility model, is used to predict cascade size distributions in transmission network of power grids using individual failure probabilities of transmission lines as well as failure correlations between transmission lines found from the correlation matrix. The study exploits the idea that groups of components that have higher correlations are likely to fail together and uses the correlation matrix to find such co-susceptible groups which is an approximate estimate of the cascade size given an initial trigger failure. Similar idea is used in the studies in [21], [22], where components within the same community are assumed to be likely to fail together and the size of communities gives an approximation of cascade sizes. The study in [23] also characterizes size of cascades by using the states of the Markov chain to find the probability distribution of the number of generations in a cascade.

C. Studying Patterns and Structures in Interactions

Structures and patterns in networks are important in describing the spread of various processes such as infectious diseases, behaviours, rumors etc. [71]–[73]. For instance, in the studies in [21], [22], [40], [41], structures present in the interaction graphs of power grids are used to study the impact of cascading failures in the transmission network and...
utilize the graph structure to mitigate large cascades. The graph structure considered in the studies in [21], [22] are communities whereas the graph structure considered in the studies in [40], [41] are tree partitions.

In the study in [40], tree structures present in the outage induced interaction graph showed that transmission line failures could not propagate across common areas of tree partitions. Further, the extended works of [40] in [41] found the critical components of the tree partitions, known as bridges. The failure of bridge lines plays a crucial role in the propagation of cascading failures. However, failure of non-bridge components do not propagate failures and the impact is more likely to be contained inside smaller regions/cells. This important property of bridge lines is used in mitigation of cascading failures by switching off transmission lines that cause negligible network congestion as well as improve the robustness of the system. Similarly, in the studies in [21], [22], influence-based and correlation-based interaction graphs were used to limit propagation of cascading failures inside community structures. Particularly, the authors used the idea that failures can be trapped within communities by protecting the bridge/overlap nodes, which connect multiple communities together.

Purpose of analyzing structures present in the interaction graphs are not limited to mitigation of cascading failures. For instance, in the study in [30], network structures were used for contingency analysis. The inverse admittance interaction graph in [30] was pruned by removing edges above an operator defined threshold. Then, the common structure between the pruned inverse admittance interaction graph and the topological graph of the power grid were analyzed and verified to be the contingencies that violate transmission line limits and cause overloads. Similarly, the study in [32] identified zonal patterns in the inverse admittance interaction graph for reliability assessment of zones for load deliverability analysis.

V. Summary

In this survey paper, we reviewed existing work on various techniques deployed for constructing interaction graphs beyond the physical structure of the power grid. We also briefly reviewed studies performed using such interaction graphs for the reliability analysis of power grids. We started by reviewing the physical topology of the power grid as a graph and its limitations. Then, we discussed the need for constructing interaction graphs that capture non-local influences/interactions between components of the power grid. Next, we developed a novel taxonomy for classification of existing research studies on various techniques for constructing interaction graphs and discussed each construction technique in detail. And finally, we discussed the types of reliability analysis performed using these interaction graphs.

The key findings from this survey are:

1) A comprehensive study of the techniques used in the construction of interaction graphs.
2) Classification of previous research studies into two broad categories: data-driven and power physics-based approaches. We found that the earliest study of finding interactions between components was done in 1989. However, we observed a hiatus in research studies that adopted graph models to represent interaction between components and most research studies in this survey paper are fairly recent. Lately, studies in both areas of data-driven and power physics-based approaches are ongoing.

3) Recurring theme in most data-driven interactions graphs is the analysis of components during cascading failures. Though, risk-based interaction graphs are one class of data-driven interaction graphs that are more focused towards the vulnerability analysis of power grids. Power physics-based approaches are also focused towards analyzing vulnerabilities in the power grids.

4) While there is no consensus on which approach, data-driven or power physics-based, is better suited for modeling power grids as interaction graphs, we observed that recent research studies using both methods are moving towards identifying structures and patterns in the interaction graphs to study the vulnerabilities in power grids.

In order to create resilient and reliable power grids, there is a need to understand the role of components in causing failures and vulnerabilities in power grids. Hence, we observed that most studies based on interaction graphs are focused on identification of the critical components of the power grid. However, there is a greater need to analyze mitigation strategies that can be applied effectively to create reliable power grids. The research studies discussed in this survey, to some extent, suggest such mitigation strategies using the interaction graphs. In the future, practical as well as economically feasible mitigation strategies need to be studied. Future research studies should also focus on the study of real-time implementation of mitigation strategies using interaction graphs. We observed that computational power required for the construction of interaction graphs increase with the increase in size and scale of power grids and as such, most data-driven as well as power physics-based methods discussed in this survey performed offline reliability analysis of the constructed interaction graphs. However, on-line mitigation strategies for power grids of any scale is a research area that needs to be explored.

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