RPA-Approach to the Excitations of the Nucleon

Part I: Theory

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Abstract

In this paper we develop a theoretical framework which allows us to study excitations of the nucleon. Assuming an effective two-body interaction as a model for low-energy QCD, we derive a relativistic TDHF equation for a many-body system of quarks. To render the Dirac-sea contribution to the mean field finite, we introduce a symmetry conserving regularization scheme. In the small amplitude limit we derive an RPA equation. The structure of the ph interaction and modifications due to the regularization scheme are discussed. We give a prescription to obtain a nucleon state with good angular momentum \((J)\) and isospin \((T)\) quantum numbers on mean-field level. To study excitations, we develop a tensor-RPA approach, which is an extension of the conventional RPA techniques to systems with a nonscalar ground state. This allows us to construct excited states with good \((J/T)\) quantum numbers. We discuss a method to reduce the overcomplete \(ph\)-space and compute the tensor-RPA interaction matrix elements. Finally we extend our scheme to include \((\frac{3}{2}^+, \frac{3}{2})\)-states.
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I. INTRODUCTION

While lattice QCD is on the verge of making definite predictions for certain quantities like the static $q\bar{q}$ potential or the order of the chiral phase transition, hadron spectroscopy still remains a difficult task due to finite size effects and uncertainties connected with the valence approximation \[1\]. Therefore the need for effective models of low energy QCD persists, especially in the hadronic sector. Over the years, such approaches have led to a deeper insight into various aspects of the relation between the hadronic world and the underlying field theory, which is formulated with quark and gluon degrees of freedom. We want to focus our attention on models with quark degrees of freedom only, thus incorporating effects originating from instantons or monopole condensation at best in an indirect way. Such fermionic models have been quite successful from a phenomenological point of view \[2\].

For the baryonic spectrum the nonrelativistic quark model, based on the work of Isgur and Karl \[3\], and related models provide a satisfactory description of the data. Recently interest in the nonrelativistic quark model was renewed when it was discovered that a spin-flavor interaction allows for the correct ordering of the negative parity states with respect to the positive parity ones \[4\].

The mesonic part of the spectrum, at least in the pseudoscalar and, with some restriction, in the vector channel, was increasingly well understood due to work based on the Nambu-Jona-Lasinio (NJL) model \[5,6\]. In that regard the role of the pion as the Goldstone boson of the spontaneously broken chiral symmetry was clarified. The NJL model possesses solitonic solutions in the $B=1$ sector which can be viewed as a mean-field description of the ground-state baryons. On this basis the phenomenology of ground-state baryons could be reproduced quite well \[7,8\].
The aim of the present work is to develop a general framework for a field theoretical quark model which describes the nucleon ground state within relativistic Hartree-Fock theory and allows to study excited states within an RPA approach. A subsequent paper is devoted to the application of our method.

Whereas in the nonrelativistic quark model the baryon wavefunctions are built from three-quark configurations exclusively, we also take into account quark-antiquark admixtures to the wavefunctions. The quark-antiquark components are included as a coherent superposition to the excitations of the valence shell.

In contrast to the bozonization techniques commonly used in the NJL model, we derive a Dirac-RPA scheme in close analogy to the approximation schemes known from nonrelativistic many-body theory. The essence of such an approach is a particle-hole expansion around a mean-field ground state. We emphasise that the methods we develop in part I of this paper are to a large extent model independent. Only in part II we make a specific choice for the interaction.

Several versions of a Dirac-RPA approach have been developed in the context of nuclear-structure physics. In such a relativistic RPA approach it is difficult to achieve a restoration of symmetries that are broken by the mean-field configuration. We solve this problem on a very general basis by introducing a symmetry-conserving regularization scheme.

The aspect of symmetry is crucial for our approach also in another regard. Rotational symmetry of the interaction in coordinate and isospin space leads to the requirement that the physical states carry good angular momentum ($J$) and isospin ($T$) quantum numbers. In most attempts to construct a self-consistent mean-field state for the nucleon the rotational symmetries are treated in a rather approximate way. Examples are the Skyrme model and the NJL soliton model which are based on a hedgehog symmetry. In such a description the full symmetry group is truncated to an invariance under simul-
taneous rotations in coordinate and isospin space, leading to conservation of the grand spin $G = J + T$.

In contrast to that we construct our states as tensors of the full symmetry group $SU(2)_J \otimes SU(2)_T$. On mean-field level this is achieved by a simple projection technique. On RPA level the situation is more complicated. The constraints set by the invariance properties of the underlying quark model force us to introduce an extended RPA scheme, the so called tensor RPA. The tensor-RPA method has been developed in the context of nuclear structure physics to study excitations of nuclei with an odd number of protons or neutrons [13]. We generalize this method and adjust it to the Dirac-RPA theory for the nucleon. As a result we obtain a powerful scheme which allows us to compute wavefunctions and excitation energies for states in any $(J/T)$-channel.

The tensor RPA is also a quite efficient method to build the correct representations of the symmetric group, since it is based on second quantization. In the nonrelativistic quark model a complicated classification scheme has to be introduced to construct representations of the various symmetries (see, e. g. [14]). Compared to that the construction of states via coupled particle-hole operators constitutes an alternative and more straightforward scheme for building the space of $(J/T)$-projected baryon wavefunctions.

Thus the focus of this theoretical part is on two novel techniques we were forced to introduce in order to maintain the symmetry structure of a quark model for the nucleon: on the one hand a special regularization scheme which guarantees the restoration of broken symmetries on RPA level, on the other hand a tensor RPA scheme which allows for the construction of states with good angular momentum and isospin quantum numbers.

The paper is divided into the following sections: In section 2 we derive an equation of motion for the one-body density matrix of the system, starting from the Martin-Schwinger hierarchy for time-ordered Green’s functions. In section 3 we discuss the issue of consistent implementation of a regularization scheme. We focus on the Dirac Hartree-Fock
problem for the nucleon in section 4. Section 5 is devoted to the discussion of excited states belonging to multiplets of the group $SU(2)_J \times SU(2)_T$. The paper closes with a discussion and a summary of the results.

II. EQUATIONS OF MOTION FOR A RELATIVISTIC SYSTEM OF FERMIONS

In the framework of field theoretical quark models the common way to derive many-body equations of motion is via bosonization techniques [15]. There one considers an Euclidean path integral and derives the equivalent of the Hartree-Fock approximation by restricting the functional integration to the stationary phase configuration. The equivalent of RPA can be derived by allowing for a time dependence of the mean field. We follow a different path, because we want to stay as close as possible to the usual formulation of many-body physics in terms of multi-particle multi-hole operators acting on a mean-field ground state.

Since we do not perform a Wick rotation to Euclidean space-time we avoid ambiguities with the treatment of vector potentials [7]. We start from a model state given by the Dirac sea with three valence quarks on top. Dynamic correlations are treated on the level of small-amplitude oscillations of the one-body density matrix. In that spirit we derive many-body equations of motion using a relativistic version of the Martin-Schwinger hierarchy [16,17].

The class of models we consider is defined by a Lagrangian of the form

$$L = \int d1 \Psi(1)(i\gamma^\mu \partial_\mu - m)\Psi(1) - \frac{1}{2} \int d1d2d1'd2' \langle 12|u|1'2'\rangle \bar{\Psi}(1)\bar{\Psi}(2)\Psi(2')\Psi(1'),$$

where $1, 2, 1', 2'$ denote the quantum numbers necessary to label the single particle states, including the time label. It is assumed that the quarks interact via an effective two-body potential $u$. We define time ordered Green’s functions by
\[ G(1 \ldots n, 1' \ldots n') = \left( \frac{1}{i} \right)^n \langle T[\Psi(1) \cdots \Psi(n)\bar{\Psi}(n') \cdots \bar{\Psi}(1')] \rangle. \] (2)

With that we obtain an equation of motion for the two-point function

\[ D(1)G(1, 1') = \delta(1 - 1') - i \int d2d1''d2'' \langle 12|u|1''2'' \rangle G(1''2'', 1'2') \] (3)

and the adjoint

\[ D^*(1')G(1, 1') = \delta(1 - 1') - i \int d2d1''d2'' \langle 1''2''|u|1'2 \rangle G(12-, 1''2''). \] (4)

The Dirac operator \( D \) is given by

\[ D = i\gamma^\mu \partial_\mu - m \] (5)

and the notation \( D(1)G(1, 1') \) denotes the action of \( D \) on the first argument of the Green’s function. The superscripts + (−) indicate that the argument has to be evaluated at an infinitesimally later (earlier) time compared to the integration variable.

Following closely the derivation of time dependent Hartree-Fock theory given in [18], we approximate the 4-point function by an antisymmetrized product of 2-point functions

\[ G(12, 1'2') = G(1, 1')G(2, 2') - G(1, 2')G(2, 1'). \] (6)

Furthermore we introduce a one-body density matrix by

\[ \rho(1, 1') = iG(1, 1')\gamma_0(1'), \] (7)

where \( \gamma_0 \) is the Dirac matrix times a unit matrix with respect to space-time, color and flavor indices.

If \( u \) parametrizes a one-boson exchange interaction with a local vertex we encounter a divergence in the equations of motion, because in this case one of the density operators appearing in the integral is evaluated at the same points in space-time. Thus, the theory requires regularization. In section 3 we will discuss a regularization scheme which strictly
preserves the symmetries of the equation of motion. For the time being we perform all algebraic manipulations as they can be applied to finite quantities.

We introduce new time variables by

$$\tau = t_1 - t_1', \quad t = (t_1 + t_1')/2.$$  \hfill (8)

Multiplying Eq. (3) and (4) by $\gamma_0$ and taking the difference leads to an equation which only contains a time-derivative with respect to $t$

$$[i\partial_t - (\alpha \cdot p + \beta m)(1) + (\alpha \cdot p + \beta m)(1')] \rho(1, 1')$$

$$= - \int d2d1''d2''\langle 12|\bar{v}|1''2''\rangle \rho(1'', 1')\rho(2'', 2^+)$$

$$+ \int d2d1''d2''\langle 12|\bar{v}|1''2''\rangle \rho(1, 1'')\rho(2^-, 2'').$$  \hfill (9)

In this expression

$$v \equiv \gamma_0 \otimes \gamma_0 \ u$$  \hfill (10)

plays the role of a two-body potential as defined in a Hamiltonian formulation. The shorthand $\bar{v}$ indicates antisymmetrization with respect to particles 1 and 2. The tensor product of the two Dirac matrices defines a two-body operator. In the above derivation we have assumed that $v$ is hermitian.

If the interaction is local in time, we obtain a closed equation for the evolution in $t$

$$i\partial_t \rho(1, 1', t) = [H_{MF}, \rho](1, 1', t),$$  \hfill (11)

where the limit $\tau = t_1 - t_1' \to 0^-$ was taken. The mean-field Hamiltonian is defined by

$$H_{MF}(1, 1', t) = [\alpha \cdot p + \beta m](1, 1') + \int d2d2'(12|\bar{v}|1'2')\rho(2', 2, t).$$  \hfill (12)

We thus recover the same structure as in the TDHF equation known from nonrelativistic physics [19]. Although not written in a manifestly covariant manner, Eq. (11) is Poincaré-invariant if the same is true for the two-body interaction $v$. This can be traced back to
the approximation introduced in Eq. (6), which respects the covariance properties of the 4-point function.

In most cases Eq. (11) is of little use for practical purposes unless we specify a regularization scheme. The two-point function will be infinite due to contributions from the Dirac sea, so we need to filter the spectral density of negative energy states to obtain finite results.

In the next chapter we will discuss a method that allows us to regularize the TDHF-equation in a consistent way. This method will lead to the derivation of a self-consistent Dirac Hartree-Fock + RPA approach.

III. REGULARIZATION OF THE TDHF EQUATION

A covariant theory as the one developed in the previous chapter confronts us with single particle spectra which are unbound from below. In that case the standard RPA approach for the excitations of the system proves to be inappropriate. Even when one considers a mean field that is made up by positive energy states only, one has to take special care to develop a consistent RPA scheme, because the response function picks up contributions from negative energy states [3].

We use the term ‘consistent’ in the following way: Given a Hartree-Fock ground state a method to compute the low-lying excitations of system is consistent if it respects the symmetries of the underlying theory. If a symmetry is broken in the ground state, a mode that corresponds to a symmetry transformation (e. g. a translation in space) has to appear at zero excitation energy. Then the spurious modes separate from the intrinsic excitations. A scheme that does not preserve the symmetries will lead to unphysical admixtures of spurious modes to the excited states.

There are, however, a few exceptional cases where no regularization is needed (see, e. g. [20]).
In the nonrelativistic case the consistency problem has been discussed in much detail \cite{21,22}. The most important result is that RPA modes corresponding to symmetry transformations appear at zero excitation energy. However, a straightforward extension of the nonrelativistic RPA scheme does not suffice to describe the linear response of the Dirac sea. Such a scheme does not allow for the decoupling of spurious modes in the relativistic case.

If the mean field is exclusively made up from positive energy states the simplest way to achieve consistency is to shift the negative energy poles of the Feynman propagator into the lower half plane \cite{4}. This corresponds to a drastic change of the structure of Dirac-hole states from backward to forward motion in time. Effectively this simulates an unoccupied Dirac sea. As a consequence the RPA equations include contributions from configurations where a particle is scattered into a Dirac-hole state.

We follow a different path for the regularization of our time dependent mean-field theory. On the one hand we include explicitly the effects of an occupied Dirac sea. On the other hand we avoid the appearance of unphysical states in the $ph$ basis. On the RPA level our regularization scheme amounts to using a modified (with respect to the original theory) interaction. The eigenstates are defined as a superposition of the physical (positive energy) $ph$ states and their time-reversed analogues.

The basic idea of our regularization scheme is to replace the divergent mean-field Hamiltonian, which is a functional of the one-body density, by an effective Hamiltonian of the form

$$H'_{MF}(\rho) = H_{MF}(R[H'_{MF}\rho R[H'_{MF}]].$$ \hspace{1cm} (13)

Thus only contributions to the mean field from the regularized density

$$\rho' = R[H'_{MF}\rho R[H'_{MF}]]$$ \hspace{1cm} (14)

are taken into account. $R[x]$ is a cut-off function with the property
\[ R[x] \to 0, \quad (x \to -\infty) \] (15)

for a real argument \( x \). The falloff for large negative arguments should be fast enough to guarantee that the effective particle number of the ground state \( \sum_i R^2[\epsilon_i] \) remains finite. The sum is to be taken over the occupied single-particle states with energies \( \epsilon_i \). We assume that the cut-off function can be expanded into a power series, which provides the definition of \( R[x] \) for an operator-valued argument. Written down in a specific model space Eq. (13) is an implicit relation between the matrix elements of the effective Hamiltonian \( H'_{MF} \), which appear on both sides of the equation. The modified TDHF equation reads

\[ i\partial_t \rho = [H'_{MF}(\rho), \rho]. \] (16)

The new description allows to solve for a set of states that simultaneously diagonalize \( \rho \) and \( H'_{MF} \), corresponding to the stationary limit of Eq. (16). When we write down \( H'_{MF} \) in a basis of such eigenstates, contributions from states arbitrarily deep in the Dirac sea are excluded by the cut-off function \( R \). However, the density operator itself will still diverge in a basis of eigenstates of the position operator, corresponding to a completely filled Dirac sea.

The regularization scheme we propose preserves the symmetries of the underlying interaction \( v \). To show this, consider a unitary transformation

\[ \rho(\theta) = U^\dagger \rho(0) U, \] (17)

with

\[ U = \exp(i\theta_a S_a). \] (18)

\( S_a \) are the generators of any group of transformations that leave the interaction invariant. Let us assume that \( \rho(0) \) is a solution of Eq. (16). If we can show that \( H'_{MF} \) transforms as
we have clearly proven the invariance property of the regularized equation of motion. In the above expression the abbreviation

\[ H'_{MF}(\theta) \equiv H'_{MF}(\rho(\theta)) \]  
(20)

is used.

From the defining equation for \( H'_{MF} \) we have

\[ H'_{MF}(\theta) = H_{MF}(R[H'_{MF}(\theta)]\rho(\theta)R[H'_{MF}(\theta)]). \]  
(21)

The proof of the transformation law Eq. (19) is based on the fact that Eq. (21) is to be considered as an implicit definition of \( H'_{MF}(\theta) \). We assume that we are given a solution \( H'_{MF}(0) \) of this equation for \( \theta = 0 \). To prove the validity of the transformation law, we insert the right hand side of Eq. (19) for \( H'_{MF}(\theta) \)

\[ U^\dagger H'_{MF}(0)U = H_{MF}(R[U^\dagger H'_{MF}(0)U] U^\dagger \rho(0)UR[U^\dagger H'_{MF}(0)U]). \]  
(22)

The cut-off function transforms as

\[ R[U^\dagger H'_{MF}(0)U] = U^\dagger R[H'_{MF}(0)]U, \]  
(23)

as one finds from a power-series expansion for \( R \). The transformation law for the original mean-field Hamiltonian is

\[ H_{MF}(U^\dagger \rho(0)U) = U^\dagger H_{MF}(\rho(0))U. \]  
(24)

With that Eq. (22) becomes

\[ U^\dagger H'_{MF}(0)U = U^\dagger H_{MF}(R[H'_{MF}(0)]\rho(0)R[H'_{MF}(0)])U, \]  
(25)

which is the defining equation for the regularized mean-field Hamiltonian at \( \theta = 0 \). Thus the expression given in Eq. (19) is a solution of the defining equation whenever the same
is true for $H'_{MF}(0)$. Note that a proof of the transformation law can only be given in such an indirect manner, since Eq. (21) can in general not be solved for $H'_{MF}(\theta)$.

Our regularization scheme relies on the fact that the cut-off is provided by a description which is self-contained, i.e. with no reference to an external set of basis states. Had we done an expansion with respect to any complete set of states, e.g. momentum eigenstates, we would have violated the symmetry requirements if we had cut off the high momentum tails of the density matrix.

In fact we have introduced a whole set of regularization schemes, because so far the form of the cut-off function $R$ has not been specified. Clearly the many-body-theory defined above is not unique and in general we expect the results to depend on the cut-off function. Any parametrization which includes the effects of the most important states close to the fermi surface should, however, lead to similar results for physical observables. This can only be checked by applying the relativistic TDHF theory to a specific model. A similar situation is encountered in the NJL-soliton model, where different schemes are used to regularize the diverging contributions from the Dirac sea.

We now proceed to develop an RPA equation by considering the small amplitude limit of Eq. (16). For this purpose we write the density operator as

$$\rho = \rho_0 + \delta \rho. \tag{26}$$

$\rho_0$ is time independent and fulfills

$$[H'_{MF}(\rho_0), \rho_0] = 0, \tag{27}$$

$\delta \rho$ is a small perturbation. Inserting this ansatz into the TDHF equation and keeping terms linear in $\delta \rho$ we obtain

$$i \partial_t \delta \rho = \left[ \frac{\partial H'_{MF}}{\partial \rho} |_{\rho_0} \delta \rho, \rho_0 \right] + [H'_{MF}(\rho_0), \delta \rho] \tag{28}$$

In the unregularized RPA we would have obtained an expression where $H'_{MF}$ is simply replaced by $H_{MF}$. The second term of Eq. (28) contains the information about single-
particle energies, since we can make use of the fact that $H'_{MF}$ and $\rho_0$ are simultaneously diagonal. The derivative in the first term defines the $ph$ interaction which enters the RPA equation

$$\langle ij | \bar{v}_{eff} | i'j' \rangle \equiv \frac{\partial (H'_{MF})_{ii'}}{\partial \rho_{jj'}}. \quad (29)$$

A similar definition of an effective interaction enters the Landau-Migdal theory of Fermi liquids [23].

In the standard derivation of the RPA equation one uses the fact that all particle-particle and hole-hole matrix elements of $\delta \rho$ vanish [21]. In case that the mean-field ground state can be written as the vacuum of a set of single-particle annihilation operators the same statement will hold for the Dirac RPA.

The nucleon, which is a system built from a filled Dirac sea with three valence quarks on top, requires some special care. If we model the nucleon by single-particle wavefunctions taken from a mean-field calculation, we have to introduce explicit correlations to construct a $(J=1/2, T=1/2)$ ground state. In a strict sense this does not allow us anymore to define the terms ‘particle’ and ‘hole’. However, the old concepts will remain unaltered for all states outside of the valence shell. Since such ground-state correlations are restricted to the valence shell, we do not expect to make a big mistake if for the derivation of the RPA equation we ignore them.

A similar situation is encountered in nuclei away from magic numbers. Already by statistical reasons such systems are characterized by ground-state occupation probabilities less than unity in the valence shells. Superimposed are dynamical correlations due to residual interactions. In the theory of Fermi-systems the various contributions to such ground-state correlations are being subsumed into the quasiparticle concept [24]. This picture takes into account that the mean-field model states, e. g. from Hartree-Fock theory, are distributed over a range of many-body eigenstates.

We now focus our attention on the effective $ph$ interaction which originates from the reg-
ularization of our theory. In order to compute the interaction matrix elements we have
to examine the derivative of the regularized mean-field Hamiltonian with respect to the
density. The derivative has to be evaluated at $\rho_0$, the stationary solution of the TDHF
equation. In the following all single particle labels refer to states of the self-consistent
basis, i.e. the set of states which simultaneously diagonalize the regularized mean-field
Hamiltonian and the density matrix. As shown in Appendix A, the implicit equation for
the effective interaction is obtained as

$$\frac{\partial}{\partial \rho_{kk'}} H'_{MF} \bigg|_{\rho_0} = \frac{\partial H_{MF}}{\partial \rho_{kk'}} R[\epsilon_k] R[\epsilon_{k'}]$$

$$+ \frac{\partial H_{MF}}{\partial \rho_{ll'}} \bigg|_{\rho_0} \frac{\partial (H_{MF})_{ll'}}{\partial \rho_{kk'}} R[\epsilon_{l'}] R[\epsilon_l] \left( \theta(k_F - l) R[\epsilon_l] + \theta(k_F - l') R[\epsilon_{l'}] \right).$$

(30)

As an example which reveals the structure of the above equation let us consider a cut-off
function of the form

$$R_l[\epsilon_i] = e^{-e^{-l(\epsilon_i + \Lambda)}},$$

(31)

where $l$ has dimension $[MeV^{-1}]$. Clearly $R_l$ has a power series representation and

$$\lim_{l \to \infty} R_l[\epsilon_i] = \theta(\epsilon_i + \Lambda).$$

(32)

In the following let us restrict ourselves to very large $l$, where the step function is repro-
duced with high accuracy. We want to emphasize that the above choice for the cut-off
function is made in order to provide an interpretation of Eq. (30) in terms of $ph$ states
and to compare our approach to the previously mentioned Dirac-RPA method of Dawson
and Furnstahl [9]. Their approach allows for particle states in the Dirac sea. At the
moment we do not touch the question if the special choice for the cut-off function in
Eq. (31) is suited for practical purposes, because our aim is to illuminate the structure
of the equations rather than going into details of a numerical solution. In fact, as has
been shown in Ref. [23], a sharp cut-off might lead to problems when attempting to solve
self-consistent mean-field equations by numerical methods.

We write the matrix elements of the effective interaction as

$$V_{kk',ll'} \equiv \frac{\partial (H_{MF}^{'})_{kk'}}{\partial \rho_{ll'}}.$$  \hspace{1cm} (33)

When two-body states $\alpha = (kk')$, $\beta = (ll')$ are introduced the defining equation for the effective interaction can be written as a matrix equation of the form

$$V_{\alpha\beta} = M_{\alpha\mu} V_{\mu\beta} + N_{\alpha\beta},$$  \hspace{1cm} (34)

where the matrices $M$ and $N$ can be read off from Eq. (30). We introduce model spaces $\mathcal{E}$ and $\mathcal{D}$ of two-body states defined by

$$\mathcal{E}: \text{ph states } (kk') \text{ with } \begin{cases} \epsilon_k > \epsilon_F \\ \epsilon_{k'} < \epsilon_F \end{cases} \text{ or } (k \leftrightarrow k')$$

$$\mathcal{D}: \text{states } (kk') \text{ with } \begin{cases} -\Lambda < \epsilon_k < \epsilon_F \\ \epsilon_{k'} < -\Lambda \end{cases} \text{ or } (k \leftrightarrow k').$$

\(\epsilon_F\) denotes the Fermi energy. Clearly the spaces $\mathcal{E}$ and $\mathcal{D}$ are orthogonal.

Let $P$ and $Q$ be the projectors onto $\mathcal{E}$ and $\mathcal{D}$. For an arbitrary operator $A$ we introduce the notation $A_{PP} \equiv PAP$, $A_{QQ} \equiv QAQ$, similarly for other combinations of $P$ and $Q$.

Note that $P$ and $Q$ are operator-valued functionals of the self-consistent density $\rho_0$.

The structure of $M$ in Eq. (34) is such that the sum over $\mu$ is restricted to states in $\mathcal{D}$. This fact is due to the special choice for the cut-off function Eq. (31). With that we obtain the two relations

$$V_{PP} = M_{PQ}V_{QP} + N_{PP}$$

$$V_{QP} = M_{QQ}V_{QP} + N_{QP}.$$  \hspace{1cm} (35)

When we solve the second equation for $V_{QP}$ and insert the solution into the first equation, we find
\[ V_{PP} = M_{PQ}(1 - M_{QQ})^{-1}N_{QP} + N_{PP}. \] (36)

Similar structures involving projection operators are encountered in the theory of valence forces in nuclei [26]. There one considers the modification of the interaction due to the truncation of the Hilbert space to a smaller model space.

For the TDHF equation we exclusively need \( ph \) matrix elements of \( V \), i.e. matrix elements in \( \mathcal{E} \). The model space \( \mathcal{D} \) contains the de-excitation states of Dawson and Furnstahl [9], which can be viewed as negative energy \( ph \) states with respect to a ‘Fermi surface’ at cut-off energy. In contrast to their prescription the de-excitation states only enter as intermediate states in our RPA scheme and do not contribute to the collective \( ph \) amplitudes describing the excited states of the system.

When the RPA eigenvalue equation is solved with the effective \( ph \) interaction defined by Eq. (36), the spurious modes decouple from the excitation spectrum.

IV. MEAN-FIELD THEORY OF THE NUCLEON

For the ground state of the nucleon we have to solve the stationary limit of the regularized TDHF equation, which amounts to finding the eigenvalues of the regularized mean-field Hamiltonian. The fact that \( H_{MF} \) is replaced by \( H'_{MF} \) formally leads to a highly nonlinear problem. The eigenvalues are, however, determined iteratively.

In order to exemplify the procedure without the quite complicated valence-shell contributions to the mean field, let us consider the analogous problem for the vacuum. Initially we choose a set of states which should not be too far away from the self-consistent set in order to guarantee convergence. We then compute the regularized mean-field Hamiltonian assuming that the states we have chosen are eigenstates. The Hartree-Fock potential of the vacuum acquires the form

\[ V_{MF} = \sum_{pqi} \langle pi|\bar{v}|qi\rangle a_p^i a_q \epsilon_i \theta(-\epsilon_i) \theta(\epsilon_i), \] (37)
where in the argument of the regularization function $H'_{MF}$ has been replaced by an approximation for its eigenvalues. After that we solve the eigenvalue problem for the new $H'_{MF}$ and repeat the procedure until convergence is achieved. By construction a configuration obtained in such a way represents a solution of Eq. (27).

A well known model for vacuum structure and spontaneous chiral symmetry breaking is the Nambu-Jona-Lasinio (NJL) model (for a review, see e.g. [27]). Due to the simple momentum structure of the NJL model it is easy to derive a nonlinear equation which determines the chiral condensate $\langle \bar{\Psi} \Psi \rangle$. With the assumption of a translationally invariant vacuum state this so called gap equation can also be derived within our Dirac-Hartree-Fock scheme applied to the NJL model. One finds

$$\langle \bar{\Psi} \Psi \rangle \propto \int_{m}^{\infty} dE \sqrt{E^2 - m^2} m R^2[E],$$

(38)

where $m$ is the constituent quark mass which includes contributions from the scalar part of the mean-field potential. Eq. (38) is identical to the gap equation with an $O(3)$-invariant cut-off discussed in [27] when a step-function parametrization is chosen for $R^2[E]$. From this discussion it should become clear that the Dirac-Hartree-Fock method set forth in the previous chapter contains previously developed methods for the determination of the mean-field structure of fermionic systems. The special form of the regularization scheme based on an implicit definition of the mean-field Hamiltonian mainly has an effect on the RPA sector of the theory where it leads to a modification of the $ph$ interaction.

Compared to the vacuum sector additional complications are introduced when one includes valence quarks and studies a nucleonic system. As mentioned in the introduction, our aim is to construct states that carry good $J/T$ quantum numbers. However, the $J/T$ symmetries are not automatically self-consistent symmetries of the nucleon mean-field Hamiltonian. This means that the mean-field of a system described by a many-body state with good $J/T$ quantum numbers is no longer invariant under rotations in coordinate and isospin space. Subsequently it will be only be possible to fulfill the symmetry
requirements by acting with a projection operator on the eigenstates of this symmetry-broken Hamiltonian.

A possible approach would be to define the nucleon as a state with grand spin $G = 0$, where the grand spin is defined as

$$G = J + T.$$  \hspace{1cm} (39)

This so called ‘hedgehog’ ansatz is used in the chiral soliton models, because the grand spin symmetry is a self-consistent symmetry of the mean-field Hamiltonian. After solving the mean-field equation states with good $J/T$ quantum numbers are projected out, usually by a cranking procedure. In the language of nonrelativistic nuclear physics this approach would be called a ‘variation before projection’ method \[21\].

A superior but technically more involved approach is provided by the ‘variation after projection’ techniques. There the mean-field approximation is applied to a projection of the full hamiltonian onto a subspace of $J/T$ eigenstates (see \[28,29\] and references therein). The projected ground-state wave function can in general only be written as a superposition of several slater determinants.

In order to keep the theoretical discussion more transparent and numerical investigations feasible we restrict ourselves to a somewhat simplified ‘variation after projection’ technique in the following. We assume that the valence shell is a $\kappa = 1$ or $\kappa = -1$ state, thus carrying an angular momentum of $J = 1/2$. Although the notation we use suggests a valence level of positive energy, the positivity of the valence energy is not mandatory for our discussion.

The vacuum $|0\rangle$ which we refer to as ‘Dirac sea’ is defined by the single-particle states which are lower in energy than the valence level.

A wavefunction with the quantum numbers of the nucleon ($(J^p, T) = (1^+, 1/2)$) is given by \[2\]

$$|\Gamma_N, \Gamma_z\rangle = \frac{1}{\sqrt{18}} \sum_{\Gamma_{z1}, \Gamma_{z2}, \Gamma_{z3}} T^{(\Gamma_z)}_{\Gamma_{z1}\Gamma_{z2}\Gamma_{z3}} a_{\Gamma_{z1}}^{1\dagger} a_{\Gamma_{z2}}^{2\dagger} a_{\Gamma_{z3}}^{3\dagger} |0\rangle.$$  \hspace{1cm} (40)
The quark states are labelled by their color quantum number (superscripts) and angular-momentum/isospin quantum number (subscripts). The \((J,T)\) quantum numbers of the nucleon are denoted with \(\Gamma_N \equiv (1/2, 1/2)\), the projection quantum numbers with \(\Gamma_z\).

In the following we take the convention that doublets of \((J/T)\) quantum numbers are subsumed in one single symbol

\[
\Gamma_i \equiv (J_i, T_i). \tag{41}
\]

Projection quantum numbers are abbreviated in a similar way

\[
\Gamma_{zi} \equiv ((J_z)_i, (T_z)_i). \tag{42}
\]

Unless stated otherwise, we define for any function of the \((J/T)\) doublets

\[
f(\Gamma_1, \Gamma_2, \ldots, \Gamma_n) \equiv f(J_1, J_2, \ldots, J_n)f(T_1, T_2, \ldots, T_n). \tag{43}
\]

State vectors and tensor operators which are labelled by \(\Gamma\)-symbols are an exception to this convention. To express the nucleon wavefunction in terms of creation operators we have introduced coupling coefficients \(T_{\Gamma_1 \Gamma_2 \Gamma_3}^{(\Gamma_z)}\) of the group \(SU(2)_J \otimes SU(2)_T\). The \(T\)-coefficients do not split up according to Eq. (43).

For the further discussion it is appropriate to introduce a creation operator for a 3-quark state with nucleonic quantum numbers

\[
A^\dagger_N(\Gamma_z) \equiv \frac{1}{\sqrt{18}} \sum_{\Gamma_{z1}, \Gamma_{z2}, \Gamma_{z3}} T_{\Gamma_{z1} \Gamma_{z2} \Gamma_{z3}}^{(\Gamma_z)} a_{\Gamma_{z1}}^{\dagger} a_{\Gamma_{z2}}^{\dagger} a_{\Gamma_{z3}}^{\dagger}. \tag{44}
\]

By definition this operator fulfills

\[
A^\dagger_N(\Gamma_z)|0\rangle = |\Gamma_N, \Gamma_z\rangle
\]

\[
A_N(\Gamma_z)|0\rangle = 0. \tag{45}
\]

The diagonal matrix elements of the density operator defined with respect to a nucleon state with projection quantum numbers \(\Gamma_z\) then become
\[
\rho^{(\Gamma_z)}(m) = \langle 0 | A_N(\Gamma_z) \Psi^\dagger(m) \Psi(m) A_N^\dagger(\Gamma_z) | 0 \rangle \\
= \langle 0 | \Psi^\dagger(m) \Psi(m) | 0 \rangle + \langle 0 | [[A_N(\Gamma_z), \Psi^\dagger(m) \Psi(m)], A_N^\dagger(\Gamma_z)] | 0 \rangle.
\]

From this expression it becomes obvious that the density operator contains contributions from the Dirac sea and from the valence quark configuration, respectively. It is easy to verify Eq. (46) by writing out the double commutator and making use of Eq. (45). The different contributions are evaluated as \[30\]

\[
\rho^{(\Gamma_z)}(m) = g^{(\Gamma_z)}(m)/18 + \theta(-m),
\]

where the symbolic \(\theta\) function is defined as

\[
\theta(-m) = \begin{cases} 
1 & : m \epsilon \text{ Dirac sea} \\
0 & : \text{otherwise}
\end{cases}
\]

The valence-occupation factors \(g\) are color independent and given by

\[
g^{(\Gamma_z)}(m) = \begin{cases} 
\sum_{\Gamma_{z1},\Gamma_{z2}} \langle T_{\Gamma_{zm}\Gamma_{z1}\Gamma_{z2}}(\Gamma_z) \rangle^2 & : m \epsilon \text{ valence shell} \\
0 & : \text{otherwise}
\end{cases}
\]

From these relations it is apparent that the valence part of the full wavefunctions leads to a dependence of \(\rho^{(\Gamma_z)}(m)\) on the \(J/T\) projection quantum numbers of \(m\). In a basis of eigenstates of \(H'_{MF}\) the mean-field potential for a nucleon with projection quantum numbers \(\Gamma_z\) is defined as

\[
V_{MF} = \sum_{pq} \langle p|\bar{v}|q\rangle a_p^\dagger a_q R^2[\epsilon_i] \rho^{(\Gamma_z)}(i).
\]

Due to the noninvariant structure of the density matrix the mean-field potential is no longer invariant under rotations in coordinate and isospin space. Such complications are well known from nuclear structure physics and have been treated on mean-field level allowing for angular momentum and isospin mixing in the wavefunctions \[31\].

In the present context the approximation scheme we use to describe the excited states
requires a single particle basis of $J/T$ eigenstates. In order to enforce that the mean-field potential is invariant with respect to rotations in coordinate and isospin space we use occupation probabilities averaged over projection quantum numbers

$$\rho^{(\Gamma_z)} \rightarrow \bar{\rho} \equiv \frac{1}{4} \sum_{\Gamma_z} \rho^{(\Gamma_z)}.$$  \hfill (51)

This approximation neglects the nonscalar parts of the density matrix. As a result, the averaged density becomes independent of the projection quantum numbers and is given by

$$\bar{\rho}(m) = \frac{1}{4}$$  \hfill (52)

for any state $m$ of the valence shell.

Comparing this prescription to the usual hedgehog ansatz we find that $\bar{\rho}$ contains contributions from configurations with $G \neq 0$. Due to the unconstraint summation over angular momentum and isospin projection quantum numbers $G_z$ can take the values $0, \pm 1$.

Another difference with respect to the hedgehog ansatz is related to the Lorentz-structure of the mean field. Due to the averaging procedure only the Lorentz-scalar and the time-like component of the Lorentz-vector part contributes. Pionic contributions to the stationary mean field which are a major building block of the hedgehog soliton are thus excluded.

Note, however, that before the averaging the expectation value $\langle \Gamma_N, \Gamma_z | \bar{\Psi} \gamma_5 \tau \Psi | \Gamma_N, \Gamma_z \rangle$ can take a nonzero value. For the same reason the pion field will contribute in the more elaborate projection scheme of Refs. [28,29]. With such a description the tensor-RPA matrixelements to be discussed in the following section will exhibit a quite complicated structure. Thus in the present work we restrict ourselves to the averaging prescription of Eq. (51).

Within this simplified scheme the pionic contributions are recovered on the level of ground-state correlations. Such correlations are inherent in an RPA description, where the information on improvements of the ground state beyond the static mean-field level enters
via the $Y$-amplitudes. In [32] a simple RPA scheme for the nucleon was proposed where the tensor structure of the excited states is ignored. In such a scheme the single particle occupation probabilities for the RPA ground state are given by

$$n_h = 1 - (2\Gamma_h + 1)^{-1} \frac{1}{2} \sum_{\Gamma,\Gamma_z,\rho,\nu} (2\Gamma + 1)|Y^\nu_{ph}(\Gamma, \Gamma_z)|^2(\bar{\rho}(h) - \bar{\rho}(p))^2$$

$$n_p = (2\Gamma_h + 1)^{-1} \frac{1}{2} \sum_{\Gamma,\Gamma_z,h,\nu} (2\Gamma + 1)|Y^\nu_{ph}(\Gamma, \Gamma_z)|^2(\bar{\rho}(h) - \bar{\rho}(p))^2,$$

(53)

where $n_h$ and $n_p$ are the occupation probabilities for hole and particle states, respectively. The backward amplitudes $Y$ are defined in the usual way (see, e.g. [21]). The depletion of hole states and occupation of particle states is due to $2p$-$2h$-admixtures to the ground state. These admixtures contain contributions from $ph$ excitations of the vacuum, where the pionic modes are expected to be most important. In [32] it has been shown that the vacuum contributions are in fact sizeable, leading to a depletion of the Dirac sea of about 10%.

With such an improved description of the ground state not only the coupling to pionic modes is recovered, but it is also possible to study the spin structure of the nucleon. In the nucleon wavefunction of Eq. (40) the spin is carried by the valence quarks exclusively. When ground-state correlations are taken into account, a part of the angular momentum is transferred to $ph$ admixtures.

The ground state density of Eq. (41) or Eq. (51) does not allow an unambiguous assignment of particles and holes, since the valence shell is only partially filled and the vacancy can be populated in excitations. The structure of the $ph$ operators to be included in the RPA basis will be discussed in the next section.

Applying Eq. (51) to a valence quark configuration coupled to $\Gamma_\Delta \equiv (3/2, 3/2)$ the same averaged occupation probabilities as in Eq. (52) are found. Thus, the $(1/2^+, 1/2)$ and the $(3/2^+, 3/2)$ valence quark configuration are indistinguishable on the level of the averaged density matrix. However, as discussed in the next section, this degeneracy will be lifted.
from configuration mixing from residual interactions. When we take the expectation value of the full Hamiltonian with an explicit representation of the ground state like Eq. (40) we in general find different values for nucleon and \( \Delta \). This means that the expectation value of the residual interaction is nonzero for both states due to ground-state correlations.

For the derivation of the RPA equation in the previous chapter we have assumed that these correlations are negligible, i.e. we have derived the equation in the same manner as for a closed-shell system. A measure for the importance of correlations is the difference

\[
|\langle \Gamma_N | H | \Gamma_N \rangle - \langle \Gamma_\Delta | H | \Gamma_\Delta \rangle|.
\]

If this difference is not small compared to the lowest single particle excitation energies a RPA scheme assuming an uncorrelated ground state must be taken with precaution.

The important point of our description is the fact that we can extract a single-particle basis with good \( J/T \) quantum numbers from the mean-field Hamiltonian defined with the averaged density. This will be a crucial prerequisite for the tensor-RPA method discussed in the next section.

**V. TENSOR RPA**

The tensor-RPA approach is a special case of the tensor equations of motion method developed more than 20 years ago by Rowe et al. [13], which, however, never seems to have been used in realistic calculations. It allows to compute excited states of a system with a nonscalar (with respect to a \( SU(2) \)-symmetry) ground state. Usually RPA calculations are restricted to closed-shell systems with a spin/isospin zero ground state. In this case the excited states carry the quantum numbers of the \( ph \) configuration admixed to the ground-state wavefunction.

For a nonscalar system the situation is more complicated. We assume that the Hamiltonian which governs the dynamics is invariant under spatial rotations (\( SO(3) \)) and isorotations (\( SU(2) \)). Since \( SO(3) \) and \( SU(2) \) are isomorphic, the invariance group is
\[ SU(2)_J \otimes SU(2)_T. \] The eigenstates of the Hamiltonian should transform as tensors with respect to this symmetry. An excited state of given \( J/T \) quantum numbers can in general be defined by coupling \( ph \) pairs of different tensor ranks on the ground state, according to the rules for outer products of \( SU(2) \) representations.

For the nucleon we have to set up such an extended RPA formalism in order to compute excitations. For the time being we want to stay in the \( SU(2) \) sector of the flavor group. This seems to be a reasonable approximation, because hints for strangeness in the excitation spectrum of the nucleon are restricted to the \( N(1535) \) resonance, which decays to a sizeable amount into a nucleon and an \( \eta \) meson. A \( SU(3) \) tensor RPA could be formulated as a straightforward extension of the method discussed here, but in order to apply it to baryon phenomenology one would have to think of a mechanism which splits the multiplets into several \( SU(2) \) representations and which can be traced back to a symmetry breaking mass matrix [4].

The formal derivation of the tensor-RPA equation for the nucleon is independent of a specific representation of the ground state. The general formalism is therefore applicable also in the context of a more sophisticated description than the one used in the previous chapter. Details of the method as the number of linearly independent basis states or the structure of the interaction matrix elements, however, depend on the ground-state structure.

**A. The Tensor-RPA Equation**

We assume that an excited state of the nucleon is defined by the action of an excitation operator on the ground state

\[ |x\Gamma_\Delta\rangle = (Q_\Delta^\dagger \times |\Gamma_N\rangle)^\Gamma_\Delta. \quad (54) \]
ΓΔ(ΓN) denotes the SU(2)J ⊗ SU(2)T tensor rank of the excited state (ground state) and x includes all additional quantum numbers. Since the adjoint of a tensor state is found to transform as a tensor only when supplied with appropriate phase factors [13], we use the definition

\[ |\Gamma, \Gamma_z\rangle \equiv |\Gamma, \Gamma_z\rangle \]
\[ \langle\langle \Gamma, \Gamma_z| \equiv (-1)^{\Gamma_z} \langle \Gamma, -\Gamma_z|, \]

where Γz is the projection quantum number. A similar definition is used for operators

\[ O^\dagger(\Gamma, \Gamma_z) \equiv O^\dagger(\Gamma, \Gamma_z) \]
\[ O(\Gamma, \Gamma_z) \equiv (-1)^{\Gamma^+ \Gamma_z} O(\Gamma, -\Gamma_z). \]

The notation for the tensor coupling we use in Eq. (54), applied to arbitrary tensors R and S, stands for

\[ \left( R^{\Gamma_1} \times S^{\Gamma_2} \right)^\Gamma_{\Gamma_z} \equiv \sum_{\Gamma_{z1}\Gamma_{z2}} \langle \Gamma_1 \Gamma_2 \Gamma_{z1} \Gamma_{z2} | \Gamma \Gamma_z \rangle R^{\Gamma_1} R^{\Gamma_2} S^{\Gamma_{z1}} S^{\Gamma_{z2}}, \]

with Clebsch-Gordan coefficients that split up according to Eq. (43). The excitation operator Q†Δ contains a sum over several tensor ranks Γi

\[ Q^\dagger_\Delta = \sum_i O^\dagger_{x\Gamma_i}, \]

according to angular-momentum/isospin selection rules. We demand

\[ Q_\Delta |\Gamma_N\rangle \rangle = 0 \]

to ensure that the excited states are orthogonal to the ground state. Furthermore we assume that the excitation operators can be written as a superposition of certain operators η† yet to be specified

\[ O^\dagger_{x\Gamma_i} = \sum_\beta Z_{\beta\Gamma}(x) \eta^\dagger_\beta(\Gamma_i). \]
Based on these assumptions a general equation of motion for the excited states of the nucleon can be derived, which in structure resembles the results of Ref. [13]

\[ MZ(x) = \omega_\Delta NZ(x), \]

where \( \omega_\Delta \) are the excitation energies. The hermitian matrices \( M \) and \( N \) are defined as

\[
M_{\alpha i, \beta j} = \sum_{\Gamma} \hat{\Gamma} (-1)^{\Gamma_N - \Gamma_\Delta - \Gamma_i} \langle \Gamma_N | [\eta_\alpha (\hat{\Gamma}_i), H, \eta_\beta (\Gamma_j)]^\Gamma | \Gamma_N \rangle \\
\cdot W(\Gamma_i \Gamma_j \Gamma_N \Gamma_N, \Gamma \Gamma_\Delta) \\
N_{\alpha i, \beta j} = \sum_{\Gamma} \hat{\Gamma} (-1)^{\Gamma_N - \Gamma_\Delta - \Gamma_i} \langle \Gamma_N | [\eta_\alpha (\hat{\Gamma}_i), \eta_\beta \dagger (\Gamma_j)]^\Gamma | \Gamma_N \rangle \\
\cdot W(\Gamma_i \Gamma_j \Gamma_N \Gamma_N, \Gamma \Gamma_\Delta).
\]

In this expression \( H \) is the full Hamiltonian of the system and \( W \) are the \( SU(2)_J \otimes SU(2)_T \) Wigner 6j-symbols [33]. We have introduced a frequently needed symbol for the dimension of multiplets by

\[ \hat{\Gamma} \equiv (2\Gamma + 1)^{\frac{1}{2}}. \]

A coupled commutator of two operators \( P \) and \( Q \) is defined as

\[
[P_{\Gamma_1}^{\Gamma}, Q_{\Gamma_2}^{\Gamma}]_{\Gamma_z}^{\Gamma} \equiv \sum_{\Gamma_{z1} \Gamma_{z2}} \langle \Gamma_1 \Gamma_2 \Gamma_{z1} \Gamma_{z2} | \Gamma \Gamma_z \rangle \left[ P_{\Gamma_{z1}}^{\Gamma_1}, Q_{\Gamma_{z2}}^{\Gamma_2} \right].
\]

A symmetrized double commutator will be denoted by

\[
2 [A, B, C] \equiv [A, [B, C]] + [[A, B], C].
\]

When one of the three operators is a scalar, as in Eq. (62), we use the notation of Eq. (64) for the coupling of the remaining two operators.

Depending on the choice for the operators \( \eta \) one can derive several approximation schemes from this equation, e. g. Tamm Dancoff, RPA or quasiparticle RPA. The states are normalized according to
with a metric tensor $N$ which will in general be different from the identity matrix. For deriving an RPA equation, we make the specific choice

$$ O_{x_{\Gamma k}} = \sum_{m,i} \left[ X_{(mi)k}(x) A_{mi}^{\dagger}(\Gamma_k) - Y_{(mi)k}(x) A_{mi}(\bar{\Gamma}_k) \right] . $$

(67)

An operator $A^{\dagger}_{mi}(\Gamma_k)$ creates a $ph$ pair $(mi)$ with $J/T$ quantum numbers $\Gamma_k$ and is defined as

$$ A^{\dagger}_{mi}(\Gamma_k \Gamma_{zh}) = \sum_{\Gamma z, m, i} \langle \Gamma_m \Gamma_i \Gamma_{zm} \Gamma_{zi} | \Gamma_k \Gamma_{zk} \rangle a^{\dagger}_{\Gamma_m \Gamma_{zm}} a_{\Gamma_i \Gamma_{zi}}. $$

(68)

We adopt the convention that particle states are denoted by $m, n$ and hole states by $i, j$. A state above the valence shell is always a particle, a state in the Dirac sea always a hole. Within the valence shell there can be both types of states. This opens up the possibility of having $ph$ states where both particle and hole are in the valence shell. Such states would induce a recoupling of the ground state and can be used to express the wavefunction of the $\Delta(1232)$, which is degenerate with the nucleon on mean-field level. However, introducing such recoupling states into the set of excitation operators requires special care. This issue will be discussed later.

With Eq. (67) the tensor equation of motion takes the form

$$ \left( \begin{array}{cc} M^{(1)} & -M^{(2)} \\ -M^{(3)} & M^{(4)} \end{array} \right) \begin{pmatrix} X \\ Y \end{pmatrix} = \omega_{\Delta} \left( \begin{array}{cc} N^{(1)} & -N^{(2)} \\ -N^{(3)} & N^{(4)} \end{array} \right) \begin{pmatrix} X \\ Y \end{pmatrix}. $$

(69)

With the definition

$$ C^{\Gamma}_{k,l} \equiv (-1)^{\Gamma_N - \Gamma_\Delta - \Gamma_k} \hat{\Gamma} \hat{W}(\Gamma_k \Gamma_i \Gamma_N \Gamma_N, \Gamma \Gamma_\Delta) $$

(70)

the submatrices are given by

$$ M_{(mi)k,(nj)l}^{(1)} = \sum_{\Gamma} C^{\Gamma}_{k,l}(\Gamma_N || [A_{mi}(\bar{\Gamma}_k), H, A^{\dagger}_{nj}(\Gamma_i)]^\Gamma || \Gamma_N) $$
We obtain the matrices \( N \) when we replace the double commutators by ordinary commutators and leave away the Hamiltonian \( H \). As a consequence we have

\[
N^{(2)} = N^{(3)} = 0.
\] (72)

It is understood that we use the nucleon ground state as defined in Eq. (40) in these expressions. Note, however, that with a suitably generalized definition of particle and hole states the description remains valid for any ground state with the correct tensor structure.

So far we have suppressed the color degree of freedom. According to the confinement property of QCD we demand that the excited states and the ground state are color singlets. Then the excitation operator is restricted to color singlet \( ph \) pairs. We can easily incorporate the color degree of freedom in Eq. (71) by making the replacement

\[
a_i^\dagger a_m \rightarrow \frac{1}{\sqrt{3}} \sum_c a_i^\dagger c a_m c.
\] (73)

A similar expression holds for the adjoint \( ph \) operators. We will display the color quantum numbers only when they are explicitly needed.

In case of a scalar ground state with respect to the group \( SU(2)_J \otimes SU(2)_T \) the tensor-RPA equation (69) can be shown to reduce to an ordinary RPA equation [30].

Given a real interaction, we find that \( M^{(1)} \) is equal to \( M^{(4)} \) and that \( M^{(2)} \) is symmetric in case of a time-reversal invariant (in terms of \( SU(2)_J \)) system, i.e., a scalar ground state.

For a nonscalar ground state these relations are found not to be fulfilled anymore. However, restricting the summation in Eq. (71) to \( \Gamma = (0, 0) \) we recover symmetric expressions for the RPA matrices. The symmetry is violated because we need to superimpose terms.
with different $\Gamma$.

The implication of time-reversal invariance and the resulting structure of the RPA matrix is a symmetry in the spectrum of eigenvalues. We find that in this case the eigenvalues occur in pairs such that for every $\omega_\Delta$ there is an adjoint $-\omega_\Delta^*$. This way we are guaranteed to obtain as many positive as negative eigenvalues, the negative ones are then discarded as unphysical.

On a nonscalar ground state this symmetry is broken. In part II of this paper [34] we will find that for the cases under investigation the spectrum is still approximately symmetric as long as the eigenvalues are real.

### B. The Basis States

A genuine feature of the tensor-RPA approach is the overcompleteness of the space of basis states defined by Eq. (67). A basis state can be labelled by its $ph$ quantum numbers $(m_i, \Gamma_k)$, where $m$ and $i$ denote the quantum numbers of the shells in which particle and hole are created and $\Gamma_k$ defines the $J/T$ quantum numbers the $ph$ pair is coupled to. A novel feature of the tensor RPA is the fact that the $ph$ space for fixed quantum numbers $\Gamma_\Delta$ of the excited state contains states which are degenerate in energy but carry different quantum numbers $\Gamma_k$. We will distinguish four different classes of basis states:

A. States with a hole in the Dirac sea and a particle in an empty shell.

B. States with a hole in the Dirac sea and a particle in the valence shell.

C. States with a hole in the valence shell and a particle in an empty shell.

D. States with a hole and a particle in the valence shell.
For the moment we do not consider the states of class D, since they require a modified
RPA scheme. We will come back to this issue later.

The metric tensor \( N \) contains the mutual scalar products of the basis states, so in principle
we can answer all questions concerning linear dependence and orthogonality by computing
the relevant matrix elements of \( N \).

One can easily show that the states of class A are mutually orthogonal and normalized,
just like in the usual RPA. The states of class B and C do not share that feature, but we
can derive some general statements about the number of linearly independent states in
each case.

We define the degeneracy \( d \) of a \( ph \) pair as the number of allowed \( J/T \) values it can
be coupled to in order to excite the ground state to a state with quantum numbers
\( \Gamma_\Delta = (J_\Delta, T_\Delta) \).

If we restrict ourselves to class B(C) we will always have a particle(hole) in the valence
shell, the corresponding hole(particle), denoted by \( r \), being in the Dirac sea (in an empty
shell). In case of the nucleon we obtain for the states of class B and C

\[
\begin{align*}
1 : & \quad j_r = J_\Delta \pm 1, \quad t_r = T_\Delta \pm 1 \\
2 : & \quad \begin{cases} j_r = J_\Delta, \quad t_r = T_\Delta \pm 1 \\ j_r = J_\Delta \pm 1, \quad t_r = T_\Delta \end{cases} \\
4 : & \quad j_r = J_\Delta, \quad t_r = T_\Delta.
\end{align*}
\]

The number of linearly independent basis states for given quantum numbers \((m_i)\), which
we denote by \( d_r \), will be less than or equal to \( d \). In general the basis states are of the form

\[
| (\Gamma_k)_{m_i} \rangle = \left( (a_{m_i}^i \times a_{t_i})^{\Gamma_k} \times | \Gamma_N \rangle \right)^\Gamma_\Delta.
\]

The ansatz for the RPA operator also contains time reversed \( ph \) operators, but the metric
tensor \( N \) can be written in terms of mutual scalar products of the states in Eq. \((73)\) only.

We rewrite this expression by making use of the Racah-recoupling theorem, but we have
to make a distinction between the states B and C.

For the B-states (i.e. states of class B) we obtain

\[
\begin{align*}
&\left((a_m^i \times a_i^j)^{\Gamma_k} \times |\Gamma_N\rangle\right)^{\Gamma_\Delta} \\
= &\ (-1)^{\Gamma_m + \Gamma_i - \Gamma_k + 1} \sum_{\Gamma} \Gamma_k \Gamma \ W(\Gamma_m, \Gamma_i, \Gamma, \Gamma_k, \Gamma) \left(a_i^j \times (a_m^i \times |\Gamma_N\rangle)^{\Gamma}\right)^{\Gamma_\Delta}.
\end{align*}
\]  

(76)

Effectively this is a unitary transformation from states \(|(\Gamma_k)_{mi}\rangle\) with a coupled \(ph\) pair to states \(|\Gamma_{4q}\rangle\) with 4 coupled valence quarks.

In a similar way the C-states are reexpressed in terms of states with two coupled valence quarks. Here the situation is slightly more complicated, because we need an explicit representation of the nucleon wavefunction. It is not difficult to verify that

\[
|\Gamma_N\Gamma_z\rangle = \left(\frac{\sqrt{2}}{3} a^1_{(\Gamma_N, \Gamma_z)} (a^2_{\Gamma_N} \times a^3_{\Gamma_N})^{(0,0)} - a^2_{(\Gamma_N, \Gamma_z)} (a^1_{\Gamma_N} \times a^3_{\Gamma_N})^{(0,0)} + a^3_{(\Gamma_N, \Gamma_z)} (a^1_{\Gamma_N} \times a^2_{\Gamma_N})^{(0,0)}\right)|0\rangle.
\]  

(77)

Only the \(J/T\) and color quantum numbers have been displayed. When we insert this expression into Eq. (75) we get a contribution from each term of the sum. After recoupling, the contribution from, e.g. the third term, reads

\[
|(\Gamma_k)_{mi}, 3\rangle \equiv \sum_{\Gamma} \Gamma_k \Gamma \ W(\Gamma_m, \Gamma_i, \Gamma, \Gamma_k, \Gamma) \\
\cdot \left(a_m^i \times \left(a_i^j \times (a^1_{\Gamma_N} \times a^3_{\Gamma_N})^{(0,0)}\right)^{\Gamma_\Delta}\right)|0\rangle.
\]  

(78)

Corresponding relations are found for the other terms. Bearing in mind that \(i\) is a state in the valence shell, we can simplify this expression by commuting \(a_i^j\) to the right. We obtain

\[
|(\Gamma_k)_{mi}, 3\rangle = \sum_{\Gamma} \Gamma_k \Gamma \ W(\Gamma_m, \Gamma_i, \Gamma, \Gamma_k, \Gamma) \\
\cdot \left(\frac{1}{2} \left(a_m^i \times (a^2_{\Gamma_N})^{\Gamma_\Delta}\right) - \frac{1}{2} \left(a_m^i \times (a^3_{\Gamma_N})^{\Gamma_\Delta}\right) + \delta_{\Gamma,(0,0)} \left(a_m^i \times (a^2_{\Gamma_N})^{\Gamma_\Delta}\right)|0\rangle.
\]  

(79)
The first and second term in Eq. (77) give similar expressions. This shows that we can express the C-states as linear combinations of states $|\Gamma_2q\rangle$ with two coupled valence quarks and a third quark in a higher shell.

As mentioned before, the number of linearly independent B- and C-states is smaller than the degeneracy $d$. The reason for this is the fact that the states $|\Gamma_4q\rangle$ and $|\Gamma_2q\rangle$ do not all comply with the Pauli principle for any given $\Gamma_4q, \Gamma_2q$. By applying Young-tableaux techniques it is found that the value $(J=0,T=0)$ is not allowed for $\Gamma_4q$. For $\Gamma_2q$ the values $(0,1)$ and $(1,0)$ are excluded. The derivation of these results is presented in Appendix B.

For given $ph$ quantum numbers $(mi)$ the forbidden states do not in any case contribute to the right hand side of Eq. (76) or (79). Due to the triangle selection rules some of the $6j$-symbols vanish for certain values $\Gamma = \Gamma'$. For this reason the states with the corresponding quantum numbers do not appear in the sum over $\Gamma$. If, however, a $6j$-symbol multiplying a forbidden state is nonzero the number of linearly independent states $d_r$ is lowered by one, since the forbidden state has zero norm. An analysis of the triangle selection rules for the $6j$-symbols reveals a simple relation between $d_r$ and the degeneracy $d$. For the B-states we obtain

$$d_r = \begin{cases} 
1 : d = 1 \\
2 : d = 2 \\
3 : d = 4 
\end{cases} \tag{80}$$

whereas for the C-states

$$d_r = \begin{cases} 
1 : d = 1 \\
1 : d = 2 \\
2 : d = 4 
\end{cases} \tag{81}$$

In a practical calculation one would have to reduce the dimension of the naive $ph$ basis to $d_r$ in each degenerate subspace. The reduced basis need not to be orthogonal, because nonorthogonal states are taken care of by the metric tensor $N$, which contains the mutual
scalar products. Alternatively, a Schmidt-orthogonalization procedure might be used. Bearing in mind this reduction scheme the basis states of Eq. (75) completely span the space of physically allowed $ph$ excitations on the nucleon. They are color singlet states which belong to $SU(2)_J \otimes SU(2)_T$ multiplets and allow us to treat excitations of the valence quarks and excitations of the Dirac sea on the same footing.

In the nonrelativistic limit the $C$-states span the space of spin-flavor wavefunctions one encounters in the nonrelativistic quark model. There one discards certain spin-flavor representations from the very beginning, because they constitute spurious center of mass excitations (e.g. the symmetric spin-flavor representation for the lowest $\left(\frac{1}{2}^-, \frac{1}{2}^+\right)$ state). In contrast to that we include the spurious states in our basis and rely on the fact that they decouple from the excitation spectrum in a consistent RPA calculation. In a relativistic framework this is the method of choice, especially when there are broken symmetries other than the translational symmetry, like in the case of mesonic modes related to chiral symmetry.

C. The Interaction Matrix Elements

So far we have only discussed issues which do not depend on the structure of the Hamiltonian $H$. We now focus our attention on the interaction matrix elements and give two different prescriptions to compute these.

We have to compute reduced matrix elements of the double commutators appearing in Eq. (71). To do so, we insert the definition of the reduced matrix elements according to the Wigner-Eckart theorem. In case of, e.g. $M^{(1)}$ we get

$$
\langle \Gamma_N || [A_{mi}(\Gamma_k), H, A_{nj}^\dagger(\Gamma_l)] || \Gamma_N \rangle = \hat{\Gamma}_N \frac{\langle \Gamma_N \Gamma_z || [A_{mi}(\bar{\Gamma}_k), H, A_{nj}^\dagger(\Gamma_l)] || \Gamma_N \Gamma_z \rangle}{\langle \Gamma_N \Gamma_z 0 || \Gamma_N \Gamma_z \rangle}. \quad (82)
$$
This expression is obviously independent of the projection quantum numbers $\Gamma_z$, but we have to make a choice for $\Gamma_z$ in order to perform the calculation. In the following we will not always display the dependence of all quantities on the projection quantum numbers and assume a fixed value of $\Gamma_z$ in case of doubt. We write the interaction in the general form

$$H_{\text{int}} = \frac{1}{2} \sum_{P,Q,R,S} \langle PQ|v|RS\rangle a_P^{\dagger} a_Q^{\dagger} a_S a_R.$$  (83)

The sum is taken over the self-consistent single particle states. We use the convention that upper case letters denote labels that include the color quantum number (e.g. $P = (p,c_p)$), whereas lower case letters label all quantum numbers besides color.

Usually one splits the interaction into a mean field and a residual part by making use of Wick’s theorem. The residual part is defined via a normal ordered product with respect to the ground state

$$N(a_P^{\dagger} a_Q^{\dagger} a_S a_R)_{(\Gamma_N,\Gamma_z)}.$$  (84)

With the theorems for normal ordered products [35] it is then easy to compute the RPA interaction matrix elements.

However, the application of Wick’s theorem is only possible when we can define a set of quasiparticle operators that annihilate the ground state. This is no longer true for the nucleon state Eq. (40). Even the most general ground state we can write down in the framework of our model

$$|N'\rangle = \sum_{\Gamma_z} C_{\Gamma_z} |\Gamma_N,\Gamma_z\rangle$$  (85)

contains correlations beyond the level of Bogolyubov quasiparticles [21] for any choice of the coefficients $C_{\Gamma_z}$.

To compute the interaction matrix elements one can still assume normal ordering, thus defining an approximation that neglects the explicit ground-state correlations which are
due to the coupling of the valence quarks. This approximation is solely based on algebraic relations for the normal ordering operator and allows us to express the RPA interaction matrix elements by the one-body density of the ground state.

By Eq. (83) the residual interaction part of the reduced matrix element becomes

$$D_R(m_i, n_j) \equiv \frac{1}{2} \sum_{PQRS} \langle PQ|v|RS \rangle \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z \Gamma_{\bar{z}} | \Gamma_N \Gamma_z \rangle} \cdot \frac{1}{3} \langle \Gamma_N, \Gamma_z | [a_{\mu}^i a_{mc}, N(a_{\mu}^i a_{mc} a_{SR}(\Gamma_N, \Gamma_z), a_{nc}^j a_{nc'}^j)] | \Gamma_N, \Gamma_z \rangle. \quad (86)$$

In this expression all sums needed for the tensor coupling have been suppressed and only the term entering $M^{(1)}$ has been displayed. We obtain the analogous expressions for $M^{(2)}, M^{(3)}$ and $M^{(4)}$ if we make suitable interchanges of the quantum numbers $m, i, n$ and $j$.

For the evaluation of the normal ordered product we use the theorem \[ \begin{aligned} N(\hat{O}_1 \ldots \hat{O}_{n-1} \hat{O}_n)\hat{O}_{n+1} &= N(\hat{O}_1 \ldots \hat{O}_n \hat{O}_{n+1}) + \ldots + N(\hat{O}_1 \ldots \hat{O}_n \hat{O}_{n+1}) + N(\hat{O}_1 \ldots \hat{O}_n \hat{O}_{n+1}), \quad (87) \end{aligned} \]

where $\hat{O}_i \hat{O}_j = \langle \hat{O}_i \hat{O}_j \rangle$ denotes a contraction with respect to $|\Gamma_N, \Gamma_z\rangle$.

The $\hat{O}_i$ can be any of the creation or annihilation operators appearing in Eq. (86). By successive use of the above relation we can express the double commutator in Eq. (86) by contractions of two operators and arrive at

$$D_R(m_i, n_j) = \frac{1}{3} \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z \Gamma_{\bar{z}} | \Gamma_N \Gamma_z \rangle} \langle (mc)(jc')|\bar{v}|(ic)(nc') \rangle \cdot [\rho(i) - \rho(m)] [\rho(j) - \rho(n)], \quad (88)$$

where $\rho$ is the density matrix as defined in Eq. (47). In this approximation the modifications with respect to the usual RPA expression are basically the weight factors that multiply the $ph$ matrix element. These weight factors account for the fact that the valence shell is not fully occupied.

To compute the mean-field part of the RPA matrix elements, we assume that the one-body part of the Hamiltonian, which is the sum of the relativistic kinetic energy and the mean-field potential Eq. (50), has been diagonalized.
\[ H_{MF} = \sum_K \epsilon_K a_K^\dagger a_K. \] (89)

At this stage an average over the projection quantum numbers of the nucleon is assumed to identify the mean-field Hamiltonian with the expression discussed in the previous chapter. The mean-field contribution to the reduced matrix elements is then defined as

\[ D_{MF}(m_i, n_j) \equiv \sum_K \epsilon_K \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z | 0 | \Gamma_N \Gamma_z \rangle} \cdot \frac{1}{3} \langle \Gamma_N, \Gamma_z | \left[ a_{i_c}^\dagger a_{m_c}, a_K^\dagger a_K, a_{n_c}^\dagger a_{j_c} \right] | \Gamma_N, \Gamma_z \rangle, \] (90)

which can be simplified to give

\[ D_{MF}(m_i, n_j) = \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z | 0 | \Gamma_N \Gamma_z \rangle} \delta_{mn} \delta_{ij} (\epsilon_m - \epsilon_i) \cdot [\rho(i) - \rho(m)]. \] (91)

Again we find an expression well known from the usual formulation of the RPA multiplied by weight factors.

In order to correctly incorporate the effects that are related to details of the ground-state structure, we have to go beyond the normal ordering approximation. Among the basis states with lowest energy there will be many B- and C-states, and we expect the lowest lying excitations to have a sizeable admixture of these. The B- and C-states are those which are sensitive to the structure of the valence shell, as can be seen from the occupation factors in Eq. (88). Therefore it is worthwhile to compute the exact interaction matrix elements and compare them to Eq. (88).

For this purpose we do not a priori split the interaction in two parts, but simply compute the reduced matrix element of the full interaction

\[ D_{int}(m_i, n_j) \equiv \frac{1}{2} \sum_{PQRS} \langle PQ | v | RS \rangle \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z | 0 | \Gamma_N \Gamma_z \rangle} \]

\[ \cdot \frac{1}{3} \langle \Gamma_N, \Gamma_z | \left[ a_{i_c}^\dagger a_{m_c}, a_{P}^\dagger a_{Q}^\dagger a_S a_R, a_{n_c}^\dagger a_{j_c} \right] | \Gamma_N, \Gamma_z \rangle. \] (92)

From this expression we subtract the contributions from the mean-field potential Eq. (50) and define the remaining terms to be the residual interaction matrix elements \( D_R \). By definition, the mean-field part of the RPA matrix elements is then the same as in the
normal ordering approximation.

For the evaluation of the expectation value in Eq. (92) we express the ground state by Eq. (40) and commute the annihilation operators in $\langle \Gamma_N, \Gamma_z \rvert$ to the right until they act on the vacuum state $\lvert 0 \rangle$. We find that there are three different kinds of contributions to $D_R$

$$D_R = D_R^{(1)} + D_R^{(2)} + D_R^{(3)}. \quad (93)$$

The first one consists of all terms of Eq. (88) except those weighted with a product of two occupation factors $g$

$$D_R^{(1)}(m_i, n_j) = \frac{1}{54} \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z \lvert 0 \rvert \Gamma_N \Gamma_z \rangle} \langle (mc)(jc')|\bar{v}|(ic')(nc') \rangle$$

$$\cdot [g(j)\theta(-i) + g(i)\theta(-j) - g(m)\theta(-j) - g(n)\theta(-i)]$$

$$- g(j)\theta(-m) - g(i)\theta(-n) + g(m)\theta(-n) + g(n)\theta(-m)$$

$$+ 18(\theta(-j)\theta(-i) - \theta(-j)\theta(-m) - \theta(-i)\theta(-n) + \theta(-n)\theta(-m))] . \quad (94)$$

The second term only gives a contribution for two $ph$ states with either the same particle or the same hole quantum numbers

$$D_R^{(2)}(m_i, n_j) = \frac{1}{648} \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z \lvert 0 \rvert \Gamma_N \Gamma_z \rangle} (g(i) - g(m) + g(j) - g(n))$$

$$\cdot \sum_k g(k) \left[ \delta_{mn} \langle (jc')(kc')|\bar{v}|(ic)(nc') \rangle - \delta_{ij} \langle (mc)(kc')|\bar{v}|(nc)(kc') \rangle \right] . \quad (95)$$

The third term cannot be expressed in terms of the occupation factors $g$, but contains the coupling coefficients $T$ explicitly

$$D_R^{(3)}(m_i, n_j) = \frac{1}{216} \frac{\hat{\Gamma}_N}{\langle \Gamma_N \Gamma_z \lvert 0 \rvert \Gamma_N \Gamma_z \rangle}$$

$$\cdot \sum_{P,Q,R,S} C_{pq,rs} \langle PQ|\bar{v}|RS \rangle \left[ \delta_{ij} \delta_{rn} G_{pq,ms} + \delta_{mn} \delta_{ij} G_{rs,ip} \right.$$ 

$$+ \delta_{ij} \delta_{qm} G_{rs,np} + \delta_{mn} \delta_{ri} G_{pq,js} - 4\delta_{sn} \delta_{qm} G_{ip,jr} - 4\delta_{pj} \delta_{ri} G_{nq,ms} \right] . \quad (96)$$

The projection on color-singlet configurations is ensured by
\[ C_{pq,rs} = \sum_{c,c' = 1}^{3} \delta_{cp} \delta_{cq} (\delta_{cr} \delta_{cs} - \delta_{c'r} \delta_{c's}) . \]  

(97)

The color operator \( C_{pq,rs} \) represents those contributions from the two-body interaction \( a_P a_Q a_S a_R \) where a creation and an annihilation operator are coupled to an intermediate color-octet. Note the + sign in front of the exchange term in Eq. (96), which is due to the antisymmetry of \( C_{pq,rs} \) under exchange of \( r \) and \( s \).

The structure of the valence shell is reflected in a matrix \( G \), which is defined as

\[
G_{pq,rs} = \begin{cases} 
\sum_{\Gamma} T_{\Gamma P} T_{\Gamma Q} T_{\Gamma S} T_{\Gamma R} : p, q, r, s \in \text{valence shell} \\
0 : \text{else}
\end{cases} .
\]

(98)

Through terms \( D^{(2)}_R \) and \( D^{(3)}_R \) new types of matrix elements are introduced to the RPA theory. Eq. (95) describes the scattering of particles and holes by the mean field of the valence quarks. In addition to scattering processes (i.e. processes in which particles or holes keep their identity) Eq. (96) also describes the transition of particles and holes into valence states. Similar structures are found in QRPA, where \( ph \) and \( pp/hh \) matrix elements contribute.

**D. Description of \( (\frac{3}{2}^+, \frac{3}{2}^-) \) states**

With the formalism presented so far it is not possible to study excitations of the nucleon in the \( (\frac{3}{2}^+, \frac{3}{2}^-) \) channel. The three valence quarks can as well be coupled to the quantum numbers of the \( \Delta(1232) \), which can be achieved by acting with a \( ph \) operator of special structure on the nucleon ground state

\[ |\Gamma_{\Delta}\rangle = (A^\dagger_{m_1}(\Gamma) \times |\Gamma_N\rangle)^{\Gamma_{\Delta}} , \]

(99)

where \( \Gamma = (1, 1) \). In this expression both the particle \( m \) and the hole \( i \) are in the valence shell. If the mean-field level of the theory is defined in the sense of the previous section
(with an averaged ground-state density) such a state is clearly energetically degenerate with the nucleon. For the \( ph \) operator in Eq. (99) we have

\[
A_{m_i}^\dagger(\Gamma) = A_{m_i}(\Gamma).
\] (100)

The adjoint of an RPA excitation operator including contributions from the recoupling operator Eq. (99) will in general not fulfill the relation

\[
O_{x\Gamma_k}|\Gamma_N\rangle \approx 0,
\] (101)

since the \( ph \) annihilation operators taken from Eq. (99) are of the same structure as the creation operators. Hence, there is no reason to believe that the \( Y \)-Amplitudes multiplying the recoupling contribution are small. However, in the derivation of the RPA equation we have assumed that Eq. (101) is fulfilled.

From investigations of hyperfine splitting in the MIT-bag model it is strongly suggested that the wavefunction of the \( \Delta(1232) \) has a dominant contribution from the recoupling configuration \[36\]. Thus an RPA approach in a \( ph \) space without the state Eq. (99) can at best account for the higher lying resonances in the \((\frac{3}{2}^+, \frac{3}{2}^-)\) channel.

In the following derivation we neglect the tensor coupling of operators and states in order to point out more clearly the special features of the equations for the \( \Delta \)-channel. A straightforward way to account for admixtures of both types of configurations is to write down the full Hamiltonian in a model space containing the recoupled ground state as well as the nondegenerate \( ph \) excitations. For this purpose we write an excited state as

\[
|x\Gamma\Delta\rangle = |\Delta_v\rangle + |\Delta_k\rangle,
\] (102)

where \( x \) denote additional quantum numbers. The projection on the recoupling configuration is given by

\[
|\Delta_v\rangle = Z_v|v\rangle,
\] (103)
where $|v\rangle$ is the properly normalized state of Eq. (99). The projection on the space of $ph$ states (of nonzero energy) is given by

$$|\Delta_k\rangle = \sum_i Z_{k_i} |k_i\rangle,$$

where

$$|k_i\rangle = \eta_{k_i}^\dagger |\Gamma_N\rangle$$

and $\eta_{k_i}^\dagger$ are $ph$ operators including the states of class A, B, and C. Again we assume proper normalization of the basis states. All quantum numbers, including angular momentum and isospin, have been absorbed in the label $k_i$. Clearly, the two subspaces are orthogonal, i.e. $\langle k_i|v\rangle = 0 \forall i$. Configuration mixing is contained in the amplitudes $Z_v, Z_{k_i}$, where

$$|Z_v|^2 + \sum_i |Z_{k_i}|^2 = 1.$$

To proceed, we split off a residual interaction $V$ from the Hamiltonian, in the same way as this was done in the previous section. We assume that the mean-field part of the Hamiltonian is diagonal in the basis we have chosen. The projection of the eigenvalue equation on the different subspaces is given by

$$(H_{vv} - E)Z_v + \sum_i V_{vk_i} Z_{k_i} = 0$$

$$\sum_j (H_{k_i k_j} - \delta_{k_i k_j} E)Z_{k_j} + V_{k_i v} Z_v = 0.$$

Clearly $H_{k_i k_j}$ and $V_{vk_i}$ are matrices according to the dimensionality of the $ph$ space. The matrix elements are in general divergent due to contributions from the Dirac sea. A prescription to regularize these matrix elements in agreement with the cut-off scheme for the mean-field Hamiltonian has been given in [37]. When we solve the first equation for $Z_v$ and insert the result into the second, we obtain

$$\sum_j (H_{k_i k_j} + \Sigma_{k_i k_j} (E) - \delta_{k_i k_j} E)Z_{k_j} = 0,$$
where a self-energy
\[ \Sigma_{k_i k_j} = -V_{k_i v} G_v V_{v k_j} \] (109)
accounts for the coupling to the valence configuration. \( G_v \) is the projection of the many-body propagator and is given by
\[ G_v = \frac{1}{H_{vv} - E}. \] (110)
Thus in the \( ph \) channel we obtain an equation where the residual interaction is supplemented by a self-energy term, compared to the conventional formulation without coupling to the valence configuration. We immediately find that the self energy will give a considerable contribution to Eq. (108) for energies close to the pole value \( H_{\alpha \alpha} \). Since Eq. (107) is a diagonalization problem for a hermitian operator, the lowest eigenvalue will be located below \( H_{\alpha \alpha} \).

Denoting the energy-expectation value of the nucleon ground state Eq. (40) by \( E_0 \), we obtain for the \( N-\Delta \) mass splitting
\[ \Delta E_{N\Delta} < H_{\alpha \alpha} - E_0. \] (111)
This quantity must be compared to the phenomenological value for the mass splitting of about 300\( MeV \). In the last section it was argued that \( H_{\alpha \alpha} - E_0 \) should already be a rather small quantity for reasons of consistency with the assumptions entering the derivation of the RPA equation. Thus it might be difficult to reproduce the observed value for the mass splitting on the level of Eq. (111).

A solution to this problem could be provided when we treat the nucleon ground state on the same basis as the recoupling configuration. Due to the averaging of the ground-state density over projection quantum numbers in the course which information about the detailed structure of the nucleon is lost we can lower the nucleon energy \( E_0 \) by infinitesimal \( 1p - 1h \) admixtures. In the usual Hartree-Fock description such admixtures are excluded.
by the self-consistency condition. The derivation presented in this section effectively takes into account $1p - 1h$ admixtures in the $(\frac{3}{2}^+, \frac{3}{2})$ channel to the recoupling configuration $|\alpha\rangle$. In a similar way we have to account for $1p - 1h$ admixtures to the nucleon in the $(\frac{1}{2}^+, \frac{1}{2})$ channel. This is required by reasons of consistency when comparing the $(\frac{1}{2}^+, \frac{1}{2})$ and the $(\frac{3}{2}^+, \frac{3}{2})$ channel. By such admixtures the energy of the nucleon will be lowered, thus the mass splitting is increased with respect to the value obtained from configuration mixing in the $\Delta$-channel.

In addition to that we have to take into account another important point. So far the spectrum of $(\frac{3}{2}^+, \frac{3}{2})$ states was derived from a diagonalization problem of the full Hamiltonian in a model space. It is important to note that the usual RPA scheme is not equivalent to such a diagonalization problem. However, even when we choose excitation operators $\eta^\dagger_{ki}$ of RPA form in Eq. (67) we still obtain a diagonalization problem, since the contribution of the time reversed operators vanishes. In the subspace of $ph$ states we recover the well known RPA structure when we replace the operator products in Eq. (107) by double commutators

$$\langle \Gamma_N | \eta_{ki} H \eta^\dagger_{kj} | \Gamma_N \rangle = \langle \Gamma_N | [\eta_{ki}, H, \eta^\dagger_{kj}] | \Gamma_N \rangle + E_0 \langle \Gamma_N | [\eta_{ki}, \eta^\dagger_{kj}] | \Gamma_N \rangle.$$  

(112)

Note that within the space of $ph$ operators we have

$$\eta_{ki} | \Gamma_N \rangle = 0,$$  

(113)

which is an approximate identity when RPA operators are acting on a HF ground state. This justifies the replacement in Eq. (112). Note that we are not allowed to introduce double commutators in the full space of excitation operators, since the recoupling operator does not annihilate the ground state. Abbreviating the recoupling operator of Eq. (99) with $\eta^\dagger_v$, we are, however, allowed to subtract the ground state energy

$$\langle \Gamma_N | \eta_v H \eta^\dagger_v | \Gamma_N \rangle = \langle \Gamma_N | \eta_v [H, \eta^\dagger_v] | \Gamma_N \rangle + E_0 \langle \Gamma_N | \eta_v \eta^\dagger_v | \Gamma_N \rangle.$$  

(114)
Together with Eq. (112) we thus obtain an eigenvalue equation for the excitation energy \( \omega = E_\Delta - E_0 \). It is easy to show that the replacements of Eq. (112) and (114) preserve the symmetry of the eigenvalue problem (i.e. hermiticity) if Eq. (113) is fulfilled exactly, which is the case on Tamm-Dancoff level. For excitation operators of RPA type we obtain, as usual, a diagonalization problem for a nonhermitian matrix. Whereas in case of a scalar system such an RPA problem can in general be reformulated as an eigenvalue equation for a hermitian matrix \[21\], the tensor RPA scheme does not allow for such a reformulation. This fact is connected to the asymmetry of the spectrum under inversion of the energy axis.

In such a more general formulation Eq. (111) looses its validity. The upper bound for the \( N-\Delta \) mass splitting is based on the Rayleigh-Ritz variational principle for hermitian matrices. Thus for the description of \( (3^+, \frac{3}{2}) \) states it seems to be crucial to go beyond the Tamm-Dancoff level and introduce commutators instead of operator products.

On RPA level there is no a priori estimate for the mass of the \( \Delta(1232) \) which stands in contradiction to the consistency requirements set by the Hartree-Fock theory. Only in a numerical calculation for a specific model it can be decided if such a RPA scheme allows for a realistic description of the \( (\frac{3}{2}^+, \frac{3}{2}) \) spectrum.

### VI. SUMMARY, DISCUSSION AND CONCLUSION

The theory presented in this paper is intended to provide a consistent approach to many-body effects in the ground state and excited states of the nucleon. By using a field theoretical formulation essential features of low-energy hadron physics are accounted for. The theory is based on an effective two-body quark-quark interaction which must be determined empirically. The approach describes baryon spectroscopy by an expansion in terms of many-body correlation functions, where at the present stage RPA contributions were investigated.
The nucleon is considered as a system of three valence quarks which interact with the vacuum. The ground state of the full system, including valence and vacuum parts, was described with Hartree–Fock methods which clearly represents a nonperturbative approach. By introducing a symmetry conserving regularization of the mean-field potential a self-consistent RPA scheme including excitations from the Dirac sea was defined.

A central issue of the model is that states of good angular momentum $J$ and isospin $T$ are used. This we consider as an important advantage over other approaches which are based on a grand spin description, as e. g. the hedgehog soliton [7,8]. However, it was discussed that this leads to mean–field self–energies violating rotational and isospin invariance. Although many–body theory provides in principle methods to treat such a problem, e. g. in nuclear structure physics [21], we have chosen a somewhat simplified approach by averaging over projection quantum numbers. This led to a one–body density matrix which is the same for the $(\frac{1}{2}^+, \frac{1}{2})$ nucleon and the $(\frac{3}{2}^+, \frac{3}{2})$ Δ-like configuration. A mean-field Hamiltonian defined with the averaged density allows to extract single particle states with good $J/T$. From these a nucleon state can be built which contains important details of the many-body structure related to angular momentum and isospin coupling.

Excited states of the nucleon were described by RPA–methods. Since we are dealing with a nonscalar ground state the theory must account for the mixing of $ph$ configurations with different $J/T$ quantum numbers in an excited state. Appropriate methods are provided by the tensor RPA which originally was introduced in nuclear structure physics by Rowe et al. [13]. Although technically rather involved, the tensor RPA method relies on a similar representation of excited states as collective $1p – 1h$ configurations on a mean-field ground state as the standard RPA scheme. In the present case, this includes transitions from the valence sector and the Dirac–sea into positive energy states. The application of RPA theory to the vacuum sector is well established for the NJL-model [3]. As an extension of such techniques we have made a first attempt to describe a baryon with good
J/T quantum numbers in the ground state and in the excited states.
The description of excited states provides additional information on the residual interaction beyond those properties which contribute to the ground state. It was pointed out that the regularization of the Hamiltonian led to a modification of the residual interaction. This reflects the fact that a projection onto a model space is used which includes vacuum states only up to a certain cut-off energy. For practical purposes it might be preferable to use a parametrization of the residual interaction and account for the modifications implied by the cut-off scheme by a readjustment of the parameters.
It was also pointed out that the RPA theory allows to obtain information on the ground-state structure beyond mean-field level. On the one hand there will be $1p-1h$ admixtures which are due to the averaging prescription for the ground-state density. On the other hand an RPA ansatz includes implicit $2p-2h$ correlations. An improved ground-state wavefunction including such contributions will allow to recover the coupling to pion degrees of freedom which are excluded on mean-field level for reasons of symmetry. Furthermore ground-state correlations will lead to a depolarization of the valence-quark core and allow to study the spin transfer to, e. g. excitations of the Dirac sea. Such admixtures have already been investigated in \cite{B2} using a somewhat simplified RPA scheme.

APPENDIX A

In this appendix we derive an operator equation for the effective $ph$ interaction which has to be used in the Dirac-RPA equation.

By definition, the mean-field Hamiltonian depends on $\rho$ only via the the regularized density $\rho'$

$$
\frac{\partial}{\partial \rho_{kk'}} H'_{MF} = \frac{\partial}{\partial \rho_{kk'}} H_{MF}(\rho') = \left( \frac{\partial}{\partial \rho_{ll'}} H_{MF} \right) \frac{\partial \rho_{ll'}}{\partial \rho_{kk'}}.
$$

\[\text{(115)}\]
We have adopted the convention to sum over repeated indices. The regularized density depends on $\rho$ also via the regularization function $R$
\[
\frac{\partial \rho_{li}'}{\partial \rho_{kk'}} = \frac{\partial R_{ii}}{\partial \rho_{kk'}} \rho_{ii}' R_{kk'}' + R_{ii} \frac{\partial \rho_{ii}'}{\partial \rho_{kk'}} R_{kk'}' + R_{ii} \rho_{ii}' \frac{\partial R_{kk'}}{\partial \rho_{kk'}}.
\]
(116)

We arrive at
\[
\frac{\partial}{\partial \rho_{kk'}} H_{MF}' = \left( \frac{\partial}{\partial \rho_{ll'}} H_{MF} \right) R_{lk} R_{lk'}' + \left( \frac{\partial}{\partial \rho_{ll'}} H_{MF} \right) \rho_{ii}' \frac{\partial}{\partial (H_{MF}')_{mm'}} \left[ \frac{\partial R_{ii}'}{\partial (H_{MF}')_{mm'}} R_{kk'}' + R_{ii} \frac{\partial R_{kk'}}{\partial (H_{MF}')_{mm'}} \right].
\]
(117)

The effective interaction $\partial H_{MF}' / \partial \rho$ appears on both sides of the equation. Expression (117) has to be evaluated at $\rho_0$, the stationary solution of the TDHF equation. For the remaining parts of this section we adopt the convention that single particle labels refer to the basis in which $\rho_0$ is diagonal.

Denoting the eigenvalues of $H_{MF}'$ by $\epsilon_i$, we find that the first term on the right hand side of Eq. (117) is given by
\[
\left( \frac{\partial}{\partial \rho_{kk'}} H_{MF} \right) \bigg|_{\rho_0} R[\epsilon_k] R[\epsilon_{k'}]
\]
(118)

where all density-dependent quantities have been evaluated at $\rho_0$. In the self-consistent basis the density matrix acquires the form
\[
(\rho_0)_{kk'} = \delta_{kk'} \theta(k_F - k),
\]
(119)

for states normalized to unity. The symbolic $\theta$-function indicates that $k$ has to be below the Fermi surface which is defined by $k_F$. In an infinite system $k_F$ denotes the Fermi momentum.

To evaluate the second term in Eq. (117), we write the regularization function as
\[
R[x] = \sum_{n=0}^{\infty} a_n x^n.
\]
(120)

For an arbitrary operator $A$ we have
\[
\frac{\partial R_{i_l}}{\partial A_{m_{m'}}} = \sum_{n=0}^{\infty} a_n \left( \frac{\partial A_{i_l}}{\partial A_{m_{m'}}} A_{i_{l2}} \ldots A_{i_{n-1}l} + A_{i_l} \frac{\partial A_{i_{l2}}}{\partial A_{m_{m'}}} \ldots A_{i_{n-1}l} + \ldots + A_{i_l} A_{i_{l2}} \ldots \frac{\partial A_{i_{n-1}l}}{\partial A_{m_{m'}}} \right),
\]
where the Einstein convention for repeated indices is used. Evaluating this expression for 
\( A = H'_{MF} \) gives
\[
\frac{\partial R_{i_l}}{\partial (H'_{MF})_{m_{m'}}} \bigg|_{\rho_0} = \delta_{i_l m} \frac{\partial A_{i_l}}{\partial \rho_{k_{k'}}} \sum_{n=0}^{\infty} a_n (\epsilon_{m'}^{n-1} + \epsilon_m \epsilon_{m'}^{n-2} + \ldots + \epsilon_m^{n-2} \epsilon_{m'} + \epsilon_m^{n-1}).
\]
(122)
The expression in brackets can be cast into the form of a finite geometrical series. Then the derivative of the cut-off function simplifies to
\[
\frac{\partial R_{i_l}}{\partial (H'_{MF})_{m_{m'}}} \bigg|_{\rho_0} = \delta_{i_l m} \frac{\partial A_{i_l}}{\partial \rho_{k_{k'}}} \left( \sum_{n=0}^{\infty} a_n \epsilon_{m'}^{n} - \sum_{n=0}^{\infty} a_n \epsilon_m^{n} \right).
\]
(123)
On the right hand side we recover the defining power series for the cut-off function \( R \) and obtain
\[
\frac{\partial R_{i_l}}{\partial (H'_{MF})_{m_{m'}}} \bigg|_{\rho_0} = \delta_{i_l m} \frac{\partial A_{i_l}}{\partial \rho_{k_{k'}}} \left( \frac{R[\epsilon_{m'}] - R[\epsilon_m]}{\epsilon_{m'} - \epsilon_m} \right).
\]
(124)
With that we finally obtain an operator equation from which the effective interaction can be determined
\[
\frac{\partial}{\partial \rho_{k_{k'}}} H'_{MF} \bigg|_{\rho_0} = \frac{\partial H_{MF}}{\partial \rho_{k_{k'}}} R[\epsilon_{k}] R[\epsilon_{k'}]
\]
(125)
+ \left( \frac{\partial (H'_{MF})_{l_{l'}}}{\partial \rho_{k_{k'}}} \bigg|_{\rho_0} \frac{R[\epsilon_{l'}] - R[\epsilon_{l}]}{\epsilon_{l'} - \epsilon_{l}} \right) \theta(k_F - l) R[\epsilon_{l}] + \theta(k_f - l') R[\epsilon_{l'}].
\]

**APPENDIX B**

The structure of 3-quark wavefunctions has been discussed in detail in the literature, e. g. in the book by Close [38]. In this appendix we investigate the group representations which can build from the 2- and 4-quark wavefunctions we encounter when we recouple the RPA basis states.
First we discuss the B-states, which contain a hole in the Dirac sea and four quarks in the valence shell.

A single particle state is characterized by its quantum numbers in Dirac, isospin and color space. We assume that the valence shell is a $|\kappa| = 1$ state, so that the Dirac-space part of the wavefunction is uniquely determined by specifying the angular-momentum projection quantum number. We can simply imagine having spin $\frac{1}{2}$ particles with additional isospin and color degrees of freedom, so we will label the Dirac-space representations with a subscript $J$, denoting the total angular momentum. The isospin and color representations carry subscripts $T$ and $C$, respectively.

We have to construct irreducible representations of the group $SU(2)_J \otimes SU(2)_T \otimes SU(3)_C$. In order to be allowed by the Pauli principle, these states have to form antisymmetric representations of the symmetric group. The space of allowed 4-quark wavefunctions is further restricted to those combinations which can be reached by coupling one quark to a $(J=\frac{1}{2}, T=\frac{1}{2})$ color singlet 3-quark state.

According to the rules for outer products of Young tableaux [39,40] the only relevant $SU(2)$ diagrams are $(3,1)$ and $(2,2)$ with dimensionalities

$$d_{SU(2)} = \begin{cases} 
3 & : (3,1) \\
1 & : (2^2)
\end{cases} \quad (126)$$

When we take the inner product of a $J$ with a $T$ representation we can classify the resulting representation by a $SU(4)_{JT}$ Young tableaux. We could now deduce the $SU(2)_J \otimes SU(2)_T$ content of the $SU(4)$ representations by using the Clebsch-Gordan coefficients of the symmetric group [40]. In many cases this is not necessary, since one often can conclude on which product representations of lower dimensional unitary groups are contained in a Young tableaux by considering the symmetry of the diagrams and matching the dimensionalities of the representations.

One finds that the product representations of the relevant $J$ and $T$ diagrams are contained
in $SU(4)_{JT}$ diagrams, according to

| $SU(2)_J \times SU(2)_T$ | $SU(4)_{JT}$ |
|--------------------------|-------------|
| $(3, 1) \times (3, 1)$   | $(4), (3, 1), (2^2), (2, 1^2)$ |
| $(3, 1) \times (2^2)$    | $(3, 1), (2, 1^2)$ |
| $(2^2) \times (3, 1)$    | $(3, 1), (2, 1^2)$ |
| $(2^2) \times (2^2)$    | $(4), (2^2), (1^4)$ |

(127)

The $SU(3)_C$ representation is fixed to be $(2, 1^2)$, since the three valence quarks of the nucleon form a color singlet. Coupling this $C$-representation on the $SU(4)$ diagrams one can embed the resulting product representations in multiplets of the group $SU(12)_{JTC}$, according to

| $SU(4)_{JT} \times SU(3)_C$ | $SU(12)_{JTC}$ |
|-----------------------------|----------------|
| $(4) \times (2, 1^2)$       | $(2, 1^2)$ |
| $(3, 1) \times (2, 1^2)$   | $(1^4), (2, 1^2), (2^2), (3, 1)$ |
| $(2^2) \times (2, 1^2)$    | $(3, 1), (2, 1^2)$ |
| $(2, 1^2) \times (2, 1^2)$ | $(4), (3, 1), (2^2), (2, 1^2)$ |
| $(1^4) \times (2, 1^2)$    | $(3, 1)$ |

(128)

The $SU(12)$ representation is restricted to the completely antisymmetric $(1^4)$. Therefore the only allowed $SU(4)$ diagram is $(3, 1)$. From the spin-singlet isospin-singlet diagrams we cannot construct this $SU(4)$ representation, so a configuration of four valence quarks with $(J=0,T=0)$ is excluded by the Pauli principle.

For the states of class $C$ we can apply quite similar arguments based on a configuration of two valence quarks. To obtain the possible 2-quark Young tableaux, we again label the Dirac space diagrams with a subscript $J$, since all other quantum numbers are the same. Then the dimensionalities of the Dirac and isospin diagrams are

$$d_{SU(2)} = \begin{cases} 
3 : (2) \\
1 : (1^2) 
\end{cases}$$

(129)
These representations can be coupled according to

\[
SU(2)_J \times SU(2)_T | SU(4)_{JT}
\]

| (2) × (2)  | (2)   |
| (2) × (1^2) | (1^2) |
| (1^2) × (2) | (1^2) |
| (1^2) × (1^2) | (2)   |

(130)

When we take the outer product of the 2-quark \( SU(4) \) diagrams with a third particle in a higher shell, we obtain

\[
(2) \otimes (1) = (3) + (2, 1)
\]

\[
(1^2) \otimes (1) = (2, 1) + (1^3).
\]

(131)

In color space the three quarks form a completely antisymmetric representation, so the residual wavefunction has to be symmetric under exchange of particles. Only the symmetric \( SU(4) \) representation for two quarks can be coupled to a symmetric 3-quark representation, so the \((J=0, T=1)\) and the \((J=1, T=0)\) 2-quark configurations are forbidden by the Pauli principle.
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