Transport Coefficients of Quark Gluon Plasma for Pure Gauge Models *

A.Nakamura,* S.Sakai and K.Amemiya

Faculty of Education, Yamagata University, Yamagata

Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka

The transport coefficients of quark gluon plasma are calculated on a lattice $16^3 \times 8$, with the pure gauge models. Matsubara Green’s functions of energy momentum tensors have very large fluctuations and about a few million MC sweeps are needed to reduce the errors reasonably small in the case of the standard action. They are much suppressed if Iwasaki’s improved action is employed. Preliminary results show that the transport coefficients roughly depend on the coupling constant as $a^{-3}(g)$.

1. Introduction and Formulation

Quark gluon plasma (QGP) is expected to be realized in high energy heavy ion collisions in near future and thought to play an important roll at the very early stage of the universe. Phenomenologically it will be treated as a fluid and it will be very useful to calculate the transport coefficients from the fundamental theory of QCD.

The calculation of the transport coefficients of QGP has been done within the Kubo’s linear response theory. According to this theory, the transport coefficients are calculated by the space time integral of retarded Green’s function at finite temperature. It is generally difficult to calculate the retarded Green’s function at finite temperature directly. Then the shortcut is that the Matsubara Green’s function is calculated and by analytic continuation retarded Green’s function is obtained. For the analytic continuation the spectral representation of the both Green’s functions is used, for which the following ansatz is assumed,

$$\rho(p',\omega) = \frac{A}{\pi} \frac{\gamma}{(m-\omega)^2 + \gamma^2} - \frac{\gamma}{(m+\omega)^2 + \gamma^2},$$

where $\gamma$ is related to the imaginary part of self energy. From these parameters, the transport coefficients are calculated as,

$$\alpha(A, \gamma, m) = 2A \frac{2\gamma m}{(\gamma^2 + m^2)^2},$$

where $\alpha$ represents heat conductivity, bulk and shear viscosities.

We notice that if $m = 0$ or $\gamma = 0$, transport coefficient becomes zero. In order to fix these parameters, at least three independent data points in temperature direction are required for the Matsubara Green’s function.

In the pioneering work of Karsch and Wyld, they performed lattice QCD calculations on $8^3 \times 4$ lattice and the resolution was not enough to determine these three parameters independently.

2. Matsubara Green’s Function

We carry out the simulation on $16^3 \times 8$ lattice for U(1),SU(2) and SU(3) pure gauge theories. In the gauge theory, the energy momentum tensor is expressed as follows,

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma}),$$

where $F_{\mu\nu}$ are field strength tensor defined by plaquette variables as $U_{x,\mu\nu} = \exp(i\alpha^2 g F_{\mu\nu})$. From the plaquette variable $F_{\mu\nu}$ is obtained either by taking log or by expanding with respect to $ga^2$. We call the former method as ‘diagonal method’ and a latter one as ‘perturbative method’.

We have started the simulation from compact U(1) case, because it has two phase, the confinement and the Coulomb phase which are separated at $\beta \sim 1.0$, and is a good place to test the formulation. In the following we denote the
Matsubara Green’s function of energy momentum tensor $T_{\mu\nu}$ as $G_{\mu\nu}(T)$.

In Fig. 1, we show how the error decreases with number of MC sweeps in confined and deconfined phase for U(1) case. In this work the measurements are done with every sweep. The difference in the fluctuations of $G_{\mu\nu}(T)$ between the confined and deconfined phase are observed. In the confined phase, we could not reduce the ratio $Errors/ < G(T) >$ less than unity within more than million data even at $T = 2$. Similar situation is observed in SU(2) case. We interpret this as follows; the energy momentum tensor in the confined phase should be written by the hadron fields. Namely $T_{\mu\nu}$ used in this calculations are not good operators in the confined phase. Therefore in the following we calculate the transport coefficients only in the deconfined phase.

The fluctuations of $G_{\mu\nu}(T)$ for U(1),SU(2) and SU(3) cases in the deconfined phase are shown in Fig.2, where transition point is $\beta \sim 6.05$ for SU(3) and cross over region is $\beta \sim 2.4$ for SU(2) for $N_t = 8$. The fluctuation becomes larger as we proceed to SU(2) and SU(3). And as the fluctuation increases as $T$ becomes large, we need about million data for SU(2) $\beta = 3.0$ case to make the ratio $Error(T)/ < G(T) > \leq 1$ for $T = 4$, and more data will be needed for SU(3) case.

In order to decrease the fluctuation, we try to use an improved action proposed by Iwasaki for the SU(3) case. The phase transition region for the Iwasaki’s improved action on $16^3 \times 8$ lattice is $\beta \sim 2.7 - 2.9$ and we started our simulation at $\beta = 3.3$. In Fig.2 we have also shown how errors decrease with number of MC sweeps for the improved action. The fluctuation is much reduced by the use of the improved action. From Fig.2 we expect that within a few hundred thousand data, we could make the ratio $Error(T)/ < G(T) > \leq 1$ and get the transport coefficients of QGP with reasonable accuracy.

Figure 1. Error as a function of number of MC sweeps.

Figure 2. Error as a function of number of MC sweeps at $T = 2$ for U(1) $\beta = 1.2$, SU(2) $\beta = 3.0$, SU(3) $\beta = 6.25$ and improved action for SU(3) $\beta = 3.3$.

3. Transport Coefficients

We determine the parameters in the spectral function by making the fit of Matsubara Green’s function. The fit of $G_{11}(T)$ is shown in Fig.3 for SU(2) at $\beta = 3.0$, where the parameters and their errors are determined by the jackknife method using the least square package SALS. In this work we excludes the points where $Error/ < G(T) >$ larger than unity from the fit range, and the data at $T = 0$ is not included.

We could not find the difference between the definitions of field strength tensor from the pla-
quette variables (‘diagonal’ and ‘perturbative’ definitions) in the present accuracy of data. Then at this $\beta$ region, the higher order effects in $g a^2$ may be small for SU(2) gauge theory.

Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$.

In the region $T = 3 - 4$, $G_{11}(T)$ becomes very flat, which is also observed for U(1) and SU(3). This behavior could not be found if we do the simulation in the smaller lattice of $N_t = 4$.

The shear viscosity $\eta$ and bulk viscosity $\xi$ are separated by taking the following combination,

$$\eta = \alpha(A, \gamma, m)_{12},$$

$$\xi = \alpha(A, \gamma, m)_{12} - \frac{4}{3} \alpha(A, \gamma, m)_{11},$$

where the subscript means that the transport coefficients determined from $G_{\mu\nu}$. The shear and bulk viscosities are shown in Fig.4 for SU(2) case. The errors are estimated by jackknife method.

The calculation for the SU(3) case is now under simulation at $\beta = 3.3$ and 3.2. The result for the transport coefficients at $\beta = 3.3$ are $\eta \times a^3 = 0.93 \times 10^{-3} \pm 0.107 \times 10^{-2}$, and $\xi \times a^3 = 0.10 \times 10^{-2} \pm 0.13 \times 10^{-2}$. The results have still large errors and are very preliminary, but we think that within a few month the result will be improved and the data at $\beta = 3.2$ will also be presented.

Figure 4. The sheer and bulk viscosity of SU(2) case as a function of $\beta$.

We have found that, in the confined phase, it is very difficult to reduce the error of $G_{\mu\nu}$. We interpret it because the energy momentum tensor in the confined phase should be written by the hadron fields. In the deconfined phase the fluctuation for $G_{\mu\nu}$ is still large, but by the use of Iwasaki’s improved action, it is much suppressed and the transport coefficients for SU(3) gauge theory is calculated on $16^3 \times 8$ lattice. However there are many thing to do before we obtain the result for the transport coefficients which are used for the quantitative phenomenology of quark gluon plasma.

ACKNOWLEDGEMENT Calculations of the SU(2) and SU(3) part have been done on VPP/500 at KEK and by AP1000 at Institute for Nuclear Study. We would like to express our thanks to the members of KEK and INS for their warm hospitality.

REFERENCES

1. A.Hosoya, M.Sakagami and M Takao, Annals of Phys.154(1984) 229.
2. F.Karsch and H.W.Wyld, Phys.Rev.D35(1987) 2518.
3. R.Horsley and W.Schoenmaker, Nuclear Phys. B280[FS18](1987),716, ibid.,735.
4. QCDTARO collaboration, private communications.

5. Y. Iwasaki preprint UTHEP-118, 
   S.Itoh, Y.Iwasaki, Y.Oyanagi and T. Yoshié, 
   Nuclear Phys. B274(1986), 33. 
   Y.Iwasaki, K.Kanaya, S.Sakai and T. Yoshié, 
   Nuclear Phys B(Proc. Suppl.) 42(1995), 502.
Fig. 1:
Error as a function of number of MC sweeps in the case of U(1) for confined and deconfined phase.
Error/\langle G_{11} \rangle_{all} \quad T=2

**Fig. 2:**
Error as a function of number of MC sweeps at $T=2$ for $U(1) \beta=1.2$, $SU(2) \beta=3.0$, $SU(3) \beta=6.25$ and improved action for $SU(3) \beta=3.3$. 
Fig. 3:
Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta=3.0$. 
Fig. 4:
The Shear and Bulk Viscosity of SU(2) gauge theory as a function of $\beta$. 