Far-field super-resolution imaging with a planar hyperbolic metamaterial lens

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Abstract – We demonstrate that achieving the far-field super-resolution imaging can be realized by using a planar hyperbolic metamaterial lens (PHML), beyond the Fabry-Perot resonance condition. Although the thickness of the PHML is much larger than the wavelength, the PHML not only can transmit radiative waves and evanescent waves with high transmission, but also can collect all the waves in the image region having amplitudes of the same order of magnitude. We present a design for a PHML to realize the far-field super-resolution imaging, with a distance between the sources and the images 10 times larger than the wavelength. We show that the super-resolution of our PHML is robust against losses, and the PHML can be fabricated by periodic stacking of metal and dielectric layers.

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Introduction. – Remarkable progress has been made over the past decade in achieving an optical super-resolution imaging [1] that breaks the traditional resolution limit, offering substantial advances for the imaging and lithography systems that are the cornerstones of modern biology and electronics. Metamaterial-based super-lenses have shown the super-resolution imaging in the near field [2,3], but magnification of subwavelength features in the far field has not been possible. Recently, super-oscillatory lenses [4] and hyperlenses [5,6] have been demonstrated in the far-field super-resolution imaging. However, both the super-oscillatory lenses and hyperlenses suffer from some limitations: super-oscillatory lenses come at a price of losing most of the optical energy into diffuse sidebands; and hyperlenses need complex concentrically curved-layer structures, which not only the bring more challenges in fabrication, but also fundamentally lack the Fourier transform function due to their inability to focus plane waves [7]. To avoid these limitations, planar hyperbolic metamaterial lenses (PHMLs) were reported [8,9] and have proved capable of providing not only the Fourier transform function in the near field [8,9] (with the distances between source and image being much smaller than the unit wavelength). Recently, Belov and co-authors proposed an alternative regime based on the Fabry-Perot resonance effect, which is then capable of transmitting images to the far field [10]. Nevertheless, the regime itself assumes that the thickness of the PHML should be equal to an integer number of half-wavelengths [10]. In this paper, we demonstrate that...
the far-field super-resolution imaging also arises in well-designed PHMLs when the Fabry-Perot resonance condition does not hold.

Generally, achieving the far-field super-resolution imaging through a planar lens is very challenging. For the sub-wavelength imaging, both evanescent waves and radiative waves are required, and their amplitudes should be comparable with each other in the image. If the amplitude of radiative waves were much larger than that of evanescent waves, the subwavelength information taken by evanescent waves would be concealed by radiative waves, and then the imaging system loses the super-resolution capability. Therefore, the amplitude of radiative waves and evanescent waves should be of the same order of magnitude in the image region for super-resolution. But satisfying this condition is generally very difficult for the far-field imaging, because of the inherent nature of evanescent waves with amplitudes exponential decaying in the air [11,12]. One of the most striking properties of hyperbolic metamaterials is their strong ability to carry evanescent waves to the far field [5], which enables PHMLs to be good candidates for far-field super-resolution imaging. To realize the far-field super-resolution, however, a PHML should overcome at least four difficulties: First, beyond the Fabry-Perot condition, the PHML should be able to transmit radiative waves and evanescent waves from a source with high transmission and low loss. Second, it should be able to collect the radiative waves and evanescent waves in the image, with amplitudes being of the same order of magnitude. Third, the distance between source and image should be much larger than the unit wavelength. Fourth, the resolution of the PHML must be robust against losses to a certain degree, including radiative losses and material losses.

In this work, we present our design for a PHML that can be used to realize the far-field super-resolution imaging beyond the Fabry-Perot condition, with resolution less than 0.1 wavelengths and simultaneously with a distance beyond the Fabry-Perot condition, with resolution less than 0.1 wavelengths and simultaneously with a distance 

\[ \Delta Z \]

The distance between the two sources is \( \Delta Z \). The hyperbolic metamaterial has a matrix form permittivity \( \varepsilon = \text{diag}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \) with \( \varepsilon_x = \varepsilon_y \), and \( \varepsilon_x \varepsilon_z < 0 \), and so its dispersion relation \( k_x^2 \varepsilon_x + k_z^2 \varepsilon_z = k_0^2 \)

is hyperbolic [9,13], where \( k_0 = 2\pi/\lambda \) is the wave vector, with \( \lambda \) the wavelength in air. The equi-frequency contours (EFCs) of hyperbolic metamaterials are hyperbolic. In the case of \( \varepsilon_x < 0 \) and \( \varepsilon_z > 0 \), two typical examples are shown in fig. 1(b). In this work, only the transverse electric (TE) mode (i.e., \( H = H_y \neq 0 \)) is considered.

**Choice of hyperbolic metamaterials.** – For a PHML, there are many choices of the transverse axis’ direction of the hyperbolic EFC. In this work we choose the one that is parallel to the \( z \)-axis (corresponding to \( \varepsilon_z < 0 \) and \( \varepsilon_x > 0 \) as shown in fig. 1(b)), we find it is much suitable for the far-field imaging. If the transverse axis is perpendicular to the \( z \)-axis (in this case we have \( \varepsilon_z > 0 \) and \( \varepsilon_x < 0 \), the radiative waves in air with wave vector \( k_z < \sqrt{\varepsilon_z} k_0 \) would exponentially decay in the lens, so this case is unsuitable for far-field imaging. If the transverse axis is tilted by an angle with respect to the \( z \)-axis [14], this kind of PHML can be directly used as a Fourier transform lens, but it is also unsuitable for far-field super-resolution because of its large lateral deviation of the image through a long distance.

After fixing the transverse axis, i.e., \( \varepsilon_z < 0 \) and \( \varepsilon_x > 0 \), the value \( |\varepsilon_z/\varepsilon_x| \) determining the eccentricity of the hyperbola is the core parameter of our system. We choose a large \( |\varepsilon_z/\varepsilon_x| \) for our PHML. There are two reasons for our choice. First, if \( |\varepsilon_z/\varepsilon_x| \) is not large enough, the image will be formed at a long distance \( \Delta Z \) from the interface of the lens [15]. In this case the evanescent waves would be too small to carry subwavelength information for the long-distance imaging. Second, for a PHML, a larger \( |\varepsilon_z/\varepsilon_x| \) corresponds to a better self-collimation of light, which is very significant for the super-resolution imaging. Hyperbolic metamaterials with a larger \( |\varepsilon_z/\varepsilon_x| \) have a flatter EFCs, and so the light diffraction can be lower when light waves propagate in the hyperbolic metamaterials with a larger \( |\varepsilon_z/\varepsilon_x| \). To show this, we present two typical examples: one case is \( |\varepsilon_z/\varepsilon_x| = 5 \) (with \( \varepsilon_z = 1.5, \varepsilon_x = -7.5 \)), and the other case is \( |\varepsilon_z/\varepsilon_x| = 140 \) (with \( \varepsilon_z = 1.5, \varepsilon_x = 5 \)).

![Fig. 1: (Colour on-line) (a) The schematic diagram of our model. (b) The EFCs of hyperbolic metamaterials with \( \varepsilon_x/\varepsilon_z = 140 \) (blue solid line) and \( \varepsilon_x/\varepsilon_z = 5 \) (green dashed line), and the EFC of air (red dash-dotted line).](image)
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Fig. 2: (Colour on-line) (a) The distribution of $|\vec{H}|^2$ of a single source placed at the interface of PHMLs with $|\varepsilon_x/\varepsilon_z| = 5$ (left panel), and $|\varepsilon_x/\varepsilon_z| = 140$ (right panel). Both lenses’ width are $d = 10\lambda$. (b) The distribution of $|\vec{H}|^2$ at the interface (along the white dash line in (a)) for the PHMLs with $|\varepsilon_x/\varepsilon_z| = 140$ (blue solid line) and $|\varepsilon_x/\varepsilon_z| = 5$ (red dashed line), respectively.

$-210$, whose EFCs are shown in fig. 1(b). From this figure, we can see the latter has much flatter EFC than that of the former. As a comparison, the two hyperbolic metamaterials are used for imaging, according to the model shown in fig. 1(a), with $\Delta x = 0, d = 10\lambda$. Their magnetic field intensity distributions calculated by Green’s function method [11,16,17] are shown in fig. 2, in which (a) shows the distributions in the $x-z$ plane, and (b) shows the distributions in an imaging region along the upper interface of the hyperbolic metamaterials. Comparing the left panel with the right panel in fig. 2(a), we can see the right one (corresponding to the latter with $|\varepsilon_x/\varepsilon_z| = 140$) has much better capability of self-collimation. Also, from fig. 2(b), we can see the intensity of the image of the latter is more than 35 times larger than that of the former, which indicates that the latter can focus more waves in the image. Therefore, hyperbolic metamaterials with larger $|\varepsilon_x/\varepsilon_z|$ are more suitable for super-resolution imaging.

Far-field super-resolution. In fig. 2(b), the full width at half-maximum (FWHM) of the intensity distribution for the case of $|\varepsilon_x/\varepsilon_z| = 140$ (blue solid line) is about $0.2\lambda$ that is much smaller than half of the wavelength. In addition, the distance between the source and the image are $10\lambda$ that is much larger than the unit wavelength, suggesting our system has the ability for the far-field subwavelength imaging.

To show it more clearly, we use such a PHML for imaging. As shown in fig. 1(a), the two sources are spaced at a distance $\Delta x = 0.3\lambda$, the thickness of the PHML is $d = 10\lambda$. For the hyperbolic metamaterial, the real part of the relative permittivity is $\text{Re}[\varepsilon_x] = 1.5$ and $\text{Re}[\varepsilon_z] = -210$ (i.e., $\text{Re}[\varepsilon_x]/\text{Re}[\varepsilon_z] = 140$), and the imaginary part is much smaller than the real part, which is phenomenologically introduced by

$$\text{Im}[\varepsilon_x(z)] = \gamma|\text{Re}[\varepsilon_x(z)]|,$$

where $\gamma$ is the loss coefficient. Based on the Green’s function method, we calculated the field ($H_{\text{rad}}, H_{\text{eva}}$ and $H_{\text{tot}}$) distributions and the intensity distributions in the image region, as shown in fig. 3(a)–(e). Here $H_{\text{rad}}, H_{\text{eva}}$ and $H_{\text{tot}} = H_{\text{rad}} + H_{\text{eva}}$ represent the radiative waves, the evanescent waves, and the total field, respectively. Note that $H_{\text{rad}}, H_{\text{eva}}$ and $H_{\text{tot}}$ are all complex. Figure 3(c) shows our PHML’s ability for the far-field super-resolution imaging. In this figure, we show the intensity distributions of $|H_{\text{rad}}|^2$, $|H_{\text{eva}}|^2$, and $|H_{\text{tot}}|^2$ in the image region that is very close to the upper interface of the PHML ($\Delta z \sim 0$). In fig. 3(c), for the intensity distribution of the total field (corresponding to the blue solid line), it is very clear that the two images (corresponding to the two main peaks) can be well resolved, although their distance $\Delta x = 0.3\lambda$ is smaller than $0.5\lambda$. Furthermore, we stress that the distance between the sources and the images is $d + \Delta z \sim 10\lambda$, which is much larger than $\lambda$. So our PHML does have the ability for the far-field super-resolution imaging.

Physically, the reason why our PHML exhibits the far-field super-resolution capability can be explained as follows. First, as shown in fig. 3(a) and fig. 3(b), both radiative waves and evanescent waves from the sources can be transmitted through our PHML with high transmission (although the thickness of the PHML is very large), and they can be collected in the image region. Second, also from fig. 3(a)–(c), we can see the amplitudes of the
radiative waves and the evanescent waves are comparable with each other, and they are constructive at the image peaks but simultaneously destructive at the side peaks.

Evanescent waves play a very important role on the super-resolution. Without evanescent waves, our PHML would lose its super-resolution capability. In fig. 3(c), the images formed only by the radiative waves become a large spot without any subwavelength information that totally cannot be resolved. This fact can also be found in fig. 3(d)–(f). In fig. 3(d), the images formed only by the radiative waves cannot be resolved at all in the image region; while in fig. 3(f), the two images can be resolved well in the image region within a certain range of Δz with the help of the evanescent waves. As Δz increases, the amplitudes of the evanescent waves exponentially decay, which lessens the resolution of the PHML. As shown in fig. 3(f), when Δz > 0.05λ, the two images cannot be resolved at all. To show the increase of Δz can impair the resolution of the PHML (i.e., the minimal distinguishable distance Δxmin would become larger as Δz increases), Δxmin vs. Δz is plotted in fig. 4(d) (blue line). When Δz = 0, our PHML can achieve its highest resolution, with the minimal distinguishable distance Δxmin = 0.24λ. In fig. 4(d), whether the images can be resolved or not follows the Rayleigh criterion [18]. In this work, we choose the criterion that two distinguishable images should be smaller than the critical contrast Istd/Imax = 75%, where Istd and Imax represent the saddle point intensity and the maximum intensity, respectively. Our criterion is a little bit stricter than that used in ref. [4].

Far-field super-resolution beyond the Fabry-Perot resonance condition. – Our PHML can be capable of far-field super-resolution beyond the Fabry-Perot resonance condition. To prove that, here we would like to show the far-field super-resolution ability of several of our PHMLs with different thicknesses (i.e., d = 9.5, 9.6, ..., 10.0λ) as shown in fig. 4, in which the Fabry-Perot resonance condition is satisfied only when d = 9.8λ, while other thickness values correspond to the nonresonant case. From fig. 4(b), by comparing the |Htot|2 distributions in the resonant case with those in the nonresonant case, we can see that their |Htot|2 distributions are very similar and all the PHMLs are able to perform super-resolution. Also, from fig. 4(a), we can see that the transmissions (here we only plot the radiative waves) of the PHMLs with different thicknesses are similar, too. These similarities of super-resolution and transmission in the PHMLs with different thicknesses indicate that the far-field super-resolution ability of our PHMLs is not linked to the Fabry-Perot resonance effect, but related to the nonresonant impedance matching.

We find in our case that a better (nonresonant) impedance matching corresponds to a better super-resolution, as shown in fig. 4(c) and (d). For the PHMLs in fig. 4(c) and (d), the eccentricity |εz/εx| = 140 is fixed, but the values of εz are different, i.e., 1.0, 1.5, and 2.0, respectively. Other parameters are the same as in fig. 3. Clearly, when εz becomes closer to unity, the nonresonant impedance matching will become better. From fig. 4(d) we can find that, when the impedance is matching better, the minimal distinguishable distance Δxmin becomes smaller, and the largest Δz becomes larger, which corresponds to a better super-resolution. In the following, we will show that to realize a PHML with quasi-perfect nonresonant impedance matching is not very difficult.

Robustness against losses. – The super-resolution of our PHML has robustness against losses, which is also important for the imaging system. To show this, three PHMLs with three different losses are, respectively, used for imaging. The losses are phenomenologically introduced by eq. (1), with the loss coefficient γ = 0.01, 0.05 and 0.1, respectively. The intensity distributions of the three PHMLs’ image region at Δz = 0 are shown in fig. 5(a). In fig. 5(a), the parameters are the same as in fig. 3, except for the loss coefficient γ. In this figure, the maximum intensity value of each intensity distribution has been normalized to unity. From fig. 5(a), we can see that, even for a big loss (γ = 0.1), our PHML still has the far-field super-resolution capability. For our PHMLs with different |εz/εx|, the robustness against losses is also valid, as shown in fig. 5(b). From fig. 5(b), we can also see that the larger |εz/εx|, the higher the resolution of our PHML, which agrees with our previous discussion.

Fig. 4: (Colour on-line) (a) The transmission rate of the PHML with different thicknesses. (b) The |Htot|2 distribution at the interface of the PHMLs with different thicknesses. Other parameters in (a) and (b) are the same as in fig. 3. (c) The transmission of the PHML with εz = 1.0 (red dash line), εz = 1.5 (blue solid line) and εz = 2.0 (black dash-dotted line). (d) Δxmin vs. Δz with εz = 1.0 (red dash line), εz = 1.5 (blue solid line) and εz = 2.0 (black dash-dotted line). In (c) and (d), the thickness of the PHML d = 10λ, and eccentricity |εz/εx| = 140 are fixed.
The fact that the super-resolution ability of the PHML is robust against losses does not mean that the losses has nothing to do with the transmitted light waves. Actually, the losses play a very important role in the amplitude of the transmitted light waves. For such a thick PHML (e.g. $d \sim 10\lambda$), the intensities of the transmitted light waves with different losses can differ by orders of magnitude in the case with $\gamma = 0.01$ (blue solid line), $\gamma = 0.05$ (red dashed line) and $\gamma = 0.1$ (black dash-dotted line). Other parameters are the same as in fig. 3. (b) The minimal distinguishable distance $\Delta x_{\min \mathbf{e}_x}$, $|\varepsilon_x/\varepsilon_z|$ with $\gamma = 0.01$ (blue solid line), $\gamma = 0.05$ (red dashed line) and $\gamma = 0.1$ (black dash-dotted line).

Anisotropic layered structure for PHMLs. – Our PHMLs can be realized by an anisotropic structure comprising alternately layered metal and dielectric films, as shown in fig. 6(a). The thickness of the cell (a cell means a combination of one metal layer and one dielectric layer) is much smaller than the wavelength. Using the transfer-matrix method and imposing the Bloch theorem [14], the effective permittivity of such a layered structure can be obtained as $\varepsilon_x = \varepsilon_m f_m + \varepsilon_d f_d$ and $\varepsilon_x^{-1} = f_m/\varepsilon_m + f_d/\varepsilon_d$. Here $\varepsilon_m$ and $\varepsilon_d$ are the permittivities of the metal and dielectric, respectively. $f_m$ and $f_d$ are the filling factors for the metal and dielectric layer, respectively. In the realization, the number of cells of the layered structure is finite. For a real PHML, the number of cells has much influence on the super-resolution imaging. To see it, here we present a comparison of the $|\tilde{\mathbf{H}}_{\text{tot}}|^2$ distributions at the upper interface of an ideal PHML with $\varepsilon_x = 1.5$, $\varepsilon_x = -210$ and the real PHMLs with different cell number $N$ in fig. 6(b). All the PHMLs in fig. 6(b) have a fixed thicknesses $d = 10\lambda$. For the real PHMLs with different cell number $N$, the thicknesses of each cell $d/N$ are different, but in each cell the filling factors $f_d = 0.52$, $f_m = 0.48$ and the permittivities $\varepsilon_d = 12.25$, $\varepsilon_m = -10.16 + 0.03i$ are fixed. In order to show the resolution ability of each PHML, the maximum intensity value of each intensity distribution in fig. 6(b) is also normalized to unity. From fig. 6(b), we can see that the super-resolution can be reached with the cell number $N = 300$, and when $N$ increases, the distribution of $|\tilde{\mathbf{H}}_{\text{tot}}|^2$ of the layered structure becomes more similar to that of the ideal one, which is expected.

Next, we will discuss the feasibility of the fact that the real PHML achieves higher resolution. According to fig. 5(b), we can see that the resolution can be 0.1$\lambda$ or even smaller when $|\varepsilon_z/\varepsilon_x| > 1200$. This can be realized by choosing the suitable parameters $(f_m, f_d, \varepsilon_m, \varepsilon_d)$ for the multilayered structure as shown in fig. 6(a). For example, for the 1000-cell multilayered structure shown in fig. 6(b), with the parameters remaining unchanged except for the filling factors that become $f_m = 0.545$ and $f_d = 1 - f_m = 0.455$, the effective permittivities $\varepsilon_x$ and $\varepsilon_z$ become about 0.037 and −60.31, respectively, and the eccentricity $|\varepsilon_z/\varepsilon_x|$ is about 1638. Using such 1000-cell multilayered structure as the PHML, we find the resolution is about 0.098$\lambda$.

Conclusion. – In conclusion, we present an approach to achieve super-resolution at a long-distance ($10\lambda$) through a PHML beyond the Fabry-Perot resonance condition, which can distinguish two sources with distance 0.1$\lambda$. The resolution of our imaging system is robust against losses. The PHMLs can be structured by periodic stacking of metal and dielectric layers.

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