Ward and Nielsen Identities for ABJM Theory in $\mathcal{N} = 1$ Superspace

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The structures and the associated gauge algebra of ABJM theory in $\mathcal{N} = 1$ superspace are reviewed. We derive the Ward identities of the theory in the class of Lorentz-type gauges at quantum level to justify the renormalizability of the model. We compute the Nielsen identities for the two-point functions of the theory with the help of enlarged BRST transformation. The identities are derived in ABJM theory to ensure the gauge independence of the physical poles of the Green’s functions.

Keywords: ABJM theory; Ward identities; Nielsen identity; BRST symmetry.

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I. INTRODUCTION

Aharony, Bergman, Jafferis and Maldacena (ABJM) [1] have been found a breakthrough in understanding M2-branes in M-theory that the worldvolume theory of $N$ multiple M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$ is described by the $\mathcal{N} = 6$ Chern-Simons-matter theory which celebrates the gauge group $U(N) \times U(N)$ and levels $k$ and $-k$. Before this important discovery, the search of worldvolume theory of multiple M2-branes by supersymmetrizing the three-dimensional Chern-Simons theory begins to the pioneering studies in [2]. The study of three dimensional conformal field theories is relevant in condensed matter systems also as they could describe interesting conformal fixed points. More recently, the study on perturbative part in the ABJM theory resulting a novel instanton contribution in the orbifold theory has also been made in [3]. The ABJM theory follows less supersymmetries than the three-dimensional Chern-Simons theory constructed by Bagger, Lambert and Gustavsson [4–8] which follows $\mathcal{N} = 8$ supersymmetry. The BLG theory was conjectured to be related to a specific theory of M2-branes for $k = 1, 2$ [9, 10]. The $\mathcal{N} = 1$ supersymmetric higher-order terms that follow from the BLG theory in its expansion with respect to the inverse squared gauge coupling constant is analysed in [11]. The BLG theory follows a Lie three-algebra. The underlying gauge symmetry of the theory is an ordinary gauge theory based on Lie algebras [12].

As it is well known, whatever the scheme employed to quantize a gauge theory, a gauge-fixing is required in order to keep on the quantization program. The gauge-fixing can be implemented to the theory by adding a non-invariant term, so called gauge-breaking term, to the classical action. Consequently, the resulting (effective) action becomes a gauge parameter dependent functional on the field configuration manifold. However, the gauge invariance of the quantum theory is desired because the expectation values of physical quantities become independent of the choice of a gauge condition. The best way to realize the gauge independence is to observe the on-shell quantum effective action, evaluated at those configurations that extremize it, when estimating S-matrix elements (or expectation values) of the gauge independent quantities. According to the Nielsen identities [13], the variation of the quantum effective action due to changes in the functions that fix the gauge is linear in the quantum corrected equations of motion for the mean fields which follows that the on-shell quantum effective action does not depend on the choice of the gauge breaking term. Even though the mean fields do depend on the gauge-fixing, but this dependence

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gets canceled by the explicit gauge-fixing dependence of the quantum effective action [14-16].

Apart from such investigations, the BRST quantization [17] of both the BLG and the ABJM theory have been explored in recent past [18-23]. For instance, the spontaneous breaking of BRST symmetry in ABJM theory in both non-linear as well as in maximally Abelian gauge and the occurrence of the ghost condensation have been studied [19]. The connection of different gauges through generalized BRST transformation is investigated in [20]. However the Batalin-Vilkovisky quatzization of ABJM theory is analysed in [24]. The Ward identities and gauge flow for M-theory in \( N=3 \) superspace has been studied recently [25] where the gauge dependence of one-particle irreducible amplitudes in such superconformal Chern-Simons theory is shown to be generated by a canonical flow with respect to the extended Slavnov-Taylor identity. The \( N=2 \) supersymmetric Chern-Simons theory coupled to matter fields is studied in the large \( N \) limit and the two-loop anomalous dimensions of certain operators are also computed [26]. Although these progresses have been made towards the complete understanding of the ABJM theory, Ward identities as well as the Nielsen identities for the two-point functions of the ABJM theory which guarantees the physical observables to be gauge independent have not been studied yet. This provides a motivation to us for the present investigations.

The aim of this paper is to investigate explicitly the Ward identities at quantum level to show the renormalizability of the model algebraically and to compute Nielsen identities for \( N=1 \) ABJM theory which will be helpful in the investigations of the gauge dependence of the effective potential. First of all, we review the ABJM theory in \( N=1 \) superspace with their gauge structure. The BRST quantization as well as Ward identities for the model are analysed in the covariant gauge. The Slavnov-Taylor identities, gauge condition, antighost equation, ghost number and spinor number are also computed. Subsequently, we discuss the renormalizability of the theory with the help of gauge conditions as well as antighost equation at all order. We show that these ward identities hold at quantum level. We derive the Nielsen identities Green’s function for ABJM theory following the method discussed by Piguet and Sibold in [27]. It is evident in what follows that the Nielsen identities are very helpful in investigations of on-mass shell Green’s functions and on-shell renormalization constants. So, the present investigations may be helpful to the investigations of the gauge dependence of the effective potential in \( N=1 \) ABJM theory as well as in the gauge independence of the physical poles of the propagator.

The rest of the paper are assembled as following. In section II, we present the preliminaries of the ABJM theory in \( N=1 \) superspace and show how it leads to a gauge symmetry. In section III, we quantize the model utilizing Faddeev-Popov method and compute the BRST symmetry and consequently Slavnov-Taylor identities of the effective action. In section IV, we derive the Nielsen identities for the two-point functions of \( N=1 \) the ABJM theory which gives relation between various Green’s functions.

II. THE ABJM THEORY IN \( N=1 \) SUPERSPACE

In this section, we recapitulate the Lagrangian construction of \( N=1 \) ABJM model [28, 30]. The generators of \( N=1 \) supersymmetry is given by \( Q_{a} = \partial_{a} - (\gamma^{\mu} \partial_{\mu})^{b} \theta_{b} \), where \( \theta^{a} \) is a two component anti-commutating parameter used to specify the three dimensional \( N=1 \) superspace together with the three spacetime coordinates. To describe the ABJM theory in \( N=1 \) superspace, we first define the Chern-Simons Lagrangian \( \mathcal{L}_{CS} \) as follows

\[
\mathcal{L}_{CS} = \frac{k}{2\pi} \int d^{2}\theta \ Tr \left[ \Gamma^{a} \Upsilon_{a} - \bar{\Gamma}^{a} \bar{\Upsilon}_{a} \right],
\]
where $k$ is an integer and
\[
\mathcal{L}_M = \frac{1}{2} D^b D_a \Gamma_b - \frac{i}{2} [\Gamma^b, D_b \Gamma_a] - \frac{1}{6} [\Gamma^b, \{\Gamma_b, \Gamma_a\}] - \frac{1}{6} [\Gamma^b, \Gamma_{ab}],
\]
where the gauge superfields $\Gamma^a_k$ where $a$ is an integer and $k$ is an integer and $\Gamma^{A}_{ab} = D^{A}_{ab} \Lambda$, $\tilde{\Gamma}^{a}_{b} = D^{a}_{b} \tilde{\Lambda} - i\{\tilde{\Gamma}^{a}_{b}, \Gamma^{b}\}$.

Here the gauge superfields $\Gamma^a$ and $\tilde{\Gamma}^a$ are matrix valued spinor superfields suitably contracted with generator $T^a$ of Lie algebra as follows: $\Gamma^a = \Gamma^a_{A} T^A$ and $\tilde{\Gamma}^a = \tilde{\Gamma}^a_{A} T^A$, respectively. The generators $Q^a$ commute with the superspace derivative, $D_a = \partial_a + (\gamma^a \partial_a)^b \theta_b$, which plays an important role in construction of the Lagrangian for ABJM theory in $N = 1$ superspace. In component form, these superfields are expressed by
\[
\Gamma^a = \chi^a + B \theta^a + \frac{1}{2} (\gamma^a)_d A^d + i \theta^2 \left[ \lambda_a - \frac{1}{2} (\gamma^a \partial_a) \chi^a \right],
\]
\[
\tilde{\Gamma}^a = \tilde{\chi}^a + \tilde{B} \theta^a + \frac{1}{2} (\gamma^a)_d \tilde{A}^d + i \tilde{\theta}^2 \left[ \tilde{\lambda}_a - \frac{1}{2} (\gamma^a \partial_a) \tilde{\chi}^a \right].
\]

The Lagrangian for the matter sector $\mathcal{L}_M$ is given by
\[
\mathcal{L}_M = \frac{1}{4} \int d^2 \theta \, Tr \left[ \nabla^a X^I \nabla_a X^J + \mathcal{V} \right],
\]
with the super-covariant derivatives of matrix valued complex scalar superfields $X^I$ and $X^I$,\[
\nabla_a X^I = D_a X^I + i \Gamma^a_a X^I - i X^I \tilde{\Gamma}^a_a, \\
\nabla_a X^I = D_a X^I - i X^I \Gamma^a_a + i \tilde{\Gamma}^a_a X^I,
\]
and the potential term $\mathcal{V}$,
\[
\mathcal{V} \propto [X_I X^I, X^I X^J X^K].
\]

With the help of Chern-Simons and matter Lagrangian, the Lagrangian for ABJM theory having gauge group $U(N)_k \times U(N)_{-k}$ is written by
\[
\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS}.
\]

The gauge symmetry of ABJM follows $U(N) \times U(N)$ group. As the Lagrangian for each gauge superfield consists a Chern-Simons term, the gauge invariance requires the coupling constant to be integer valued [31, 32]. Under a gauge transformation the scalars and gauge superfields transform as
\[
\delta \Gamma^a = \nabla^a \Lambda, \quad \delta \tilde{\Gamma}^a = \tilde{\nabla}^a \tilde{\Lambda}, \\
\delta X^I = i (AX^I - X^I \Lambda), \quad \delta X^I = i (\tilde{A}X^I - X^I \tilde{\Lambda}),
\]
where $\Lambda = \Lambda^A T^A$ and $\tilde{\Lambda} = \tilde{\Lambda}^A \tilde{T}^A$ are the global transformation parameters. The above transformations leave the classical Lagrangian of the model [9] invariant.

III. ABJM THEORY: SLAVNOV-TAYLOR IDENTITY

In order to give quantum description the ABJM theory, one must add the gauge-fixing term and the corresponding Faddeev-Popov term to the invariant Lagrangian [9] [24, 32]. By doing so, the gauge
fixing term breaks the gauge invariance and, thus, removes the divergence of the functional integral. However, the Faddeev-Popov term improves the integration measure to provide correct predictions for gauge invariant observables. Here, the gauge superfields satisfy the following gauge conditions: $G_1 = D^a \Gamma_a = 0$, $G_1 = D^a \tilde{\Gamma}_a = 0$. The corresponding gauge-fixing term with gauge parameter $\alpha$ is constructed by

$$L_{gf} = \int d^2 \theta \, Tr \left[ b(D^a \Gamma_a) + \frac{\alpha}{2} bb - \tilde{b}(D^a \tilde{\Gamma}_a) - \frac{\alpha}{2} \tilde{b} \tilde{b} \right],$$

where $b$ and $\tilde{b}$ are Nakanishi-Lautrup type auxiliary fields. With the help of of ghost fields $c, \tilde{c}$ and corresponding anti-ghost fields $\bar{c}, \bar{\tilde{c}}$, the Faddeev-Popov ghost term is written explicitly by

$$L_{gh} = - \int d^2 \theta \, Tr[\bar{c}D^a \nabla_a c - \tilde{\bar{c}}D^a \tilde{\nabla}_a \tilde{c}].$$

The sum of gauge fixing and ghost terms is defined by

$$L_g = L_{gf} + L_{gh},$$

which is BRST exact and hence justifies its own BRST invariance due to the nilpotency property. For the present ABJM model, the nilpotent BRST transformations (i.e. $\delta^2 = 0$) are

$$\delta_b \Gamma_a = \lambda \nabla_a c, \quad \delta_b \tilde{\Gamma}_a = \tilde{\lambda} \tilde{\nabla}_a \tilde{c},$$

$$\delta_b c = -\frac{1}{2} \lambda [c, \tilde{c}], \quad \delta_b \tilde{c} = -\frac{1}{2} [\tilde{\lambda}, \bar{c}],$$

$$\delta_b \bar{c} = \lambda b, \quad \delta_b \bar{\tilde{c}} = \tilde{\lambda} \bar{b},$$

$$\delta_b b = 0, \quad \delta_b \tilde{b} = 0,$$

$$\delta_b X^I = i \lambda c X^I - i X^I \tilde{\lambda},$$

$$\delta_b X^{I\dagger} = i \tilde{\lambda} \tilde{c} X^{I\dagger} - i X^{I\dagger} \lambda,$$

where $\lambda$ and $\tilde{\lambda}$ are Grassmannian parameters. The effective ABJM Lagrangian, defined by the sum of classical and BRST-exact parts ($L_{ABJM} = L_c + L_g$), is invariant under the above BRST transformations. In terms of gauge-fixing fermion, the gauge-fixing and ghost parts of the effective Lagrangian can also be expressed as

$$L_g = i s_b \int d^2 \theta \, Tr \left[ \hat{\bar{c}}D^a \hat{\Gamma}_a + \frac{\alpha}{2} \hat{b} - \hat{\bar{c}}D^a \Gamma_a - \frac{\alpha}{2} \hat{\bar{b}} \right].$$

Since, the original action is modified by a BRST-exact piece only, which cannot alter the BRST cohomology, and thereby, in turn, cannot alter the notion of physical states.

The Ward identities are obtained by exploiting this invariance by adding sources corresponding to the non-linear transformations. For this, we first couple a source to each non-linear variation of the fields, i.e. to $s_b \Gamma_a, s_b \tilde{\Gamma}_a, s_b c, s_b \bar{c}, s_b X^I$ and $s_b X^{I\dagger}$. Then, we add them together to write the auxiliary part as following:

$$L_{ext} = \int d^2 \theta \, Tr \left[ K^a (\nabla_a c) - \tilde{K}^a (\tilde{\nabla}_a \tilde{c}) - \frac{1}{2} K_c [c, \tilde{c}] + \frac{1}{2} \tilde{K}_c [\tilde{c}, \bar{c}] + K_I (i c X^I - i X^I \tilde{c}) + (i \tilde{c} X^{I\dagger} - i X^{I\dagger} \lambda) K_I \right],$$

whereby $K^a, \tilde{K}^a$ are the Grassmann supersources, $K_c, \tilde{K}_c$ are the bosonic supersources and $K_I$ and $\tilde{K}_I$ are the matrix valued supersources. Now, the effective action

$$\Sigma = \int d^3 x (L_c + L_g + L_{ext}),$$

leads to the following identities.
• The Slavnov-Taylor identity is given by

\[
S(\Sigma) = \int d^3 x d^2 \theta \left[ \frac{\delta \Sigma}{\delta K^a} \frac{\delta \Sigma}{\delta \bar{Y}^a} + \frac{\delta \Sigma}{\delta K^a} \frac{\delta \Sigma}{\delta \bar{K}^a} + \frac{\delta \Sigma}{\delta K^c} \frac{\delta \Sigma}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta K^c} \frac{\delta \Sigma}{\delta \bar{\bar{c}}} + \frac{\delta \Sigma}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta \bar{\bar{c}}} + \frac{\delta \Sigma}{\delta \bar{\bar{c}}} \right] = 0.
\] (18)

• The gauge conditions are given by

\[
\frac{\delta \Sigma}{\delta b} = D_a \Gamma_a + \alpha b,
\]

\[
\frac{\delta \Sigma}{\delta \tilde{b}} = D_a \tilde{\Gamma}_a + \tilde{\alpha} \tilde{b},
\] (19)

Though these symmetries are linearly broken, these are allowed due to the Quantum Action Principle (QAP) [17, 34–36].

• The action enjoys anti-ghost equations

\[
\left( \frac{\delta}{\delta \bar{c}} - D_a \frac{\delta}{\delta K^a} \right) \Sigma = 0,
\]

\[
\left( \frac{\delta}{\delta \bar{\bar{c}}} - D_a \frac{\delta}{\delta \bar{K}^a} \right) \Sigma = 0.
\] (20)

• We also notice that the action preserves the ghost number

\[
G_n(\Sigma) = \int d^3 x d^2 \theta \left[ \frac{\delta}{\delta c} - \frac{\delta}{\delta \bar{c}} - \tilde{c} \frac{\delta}{\delta \bar{c}} - \bar{\bar{c}} \frac{\delta}{\delta \bar{c}} - K^a \frac{\delta}{\delta K^a} - \bar{K}^a \frac{\delta}{\delta \bar{K}^a} - 2 \bar{K}^a \frac{\delta}{\delta \bar{K}^a}
\right. \left. - 2 \bar{\bar{K}}^a \frac{\delta}{\delta \bar{\bar{K}}^a} - \bar{\bar{K}}^a \frac{\delta}{\delta \bar{\bar{K}}^a} \right] \Sigma = 0.
\] (21)

• The action also preserves the spinor number

\[
S_n(\Sigma) = \int d^3 x d^2 \theta \left[ X_I \frac{\delta}{\delta X_I} - X_I \frac{\delta}{\delta \bar{X}_I} + \bar{K}_I \frac{\delta}{\delta K_I} - \bar{\bar{K}}_I \frac{\delta}{\delta \bar{\bar{K}}_I} \right] \Sigma = 0.
\] (22)

These identities will be very helpful to show the algebraic renormalizability of the $\mathcal{N} = 1$ ABJM theory. However, we should notice that the spinor number is not a necessary Ward identity to prove the renormalizability. The counter terms can also be derived by following these identities.

IV. WARD IDENTITIES AT THE QUANTUM LEVEL

Now, by considering gauge conditions and antighost equations, we try to prove that all the Ward identities can be transformed to the quantum level.

1. The gauge conditions

Let us start with the gauge conditions (19), we would like to prove that identities (19) are not affected by the radiative corrections.
The QAP [17] translates the symmetry to the quantum level as
\[
\frac{\delta \Sigma}{\delta b} = D^a \Gamma_a + \alpha b + \bar{\Delta} \cdot \Sigma, \\
\frac{\delta \Sigma}{\delta \tilde{b}} = D^a \tilde{\Gamma}_a + \alpha \tilde{b} + \tilde{\Delta} \cdot \Sigma.
\] (23)

As we know that \(\bar{\Delta}\) and \(\tilde{\Delta}\) can only start from order \(\hbar\), so let us assume that these start with order \(\hbar^n\), \(n \geq 1\)
\[
\frac{\delta \Sigma}{\delta b} = D^a \Gamma_a + \alpha b + \hbar^n \bar{\Delta} + O(\hbar^{n+1}), \\
\frac{\delta \Sigma}{\delta \tilde{b}} = D^a \tilde{\Gamma}_a + \alpha \tilde{b} + \hbar^n \tilde{\Delta} + O(\hbar^{n+1}),
\] (24)

where \(\bar{\Delta}\) and \(\tilde{\Delta}\) are the local polynomials of the sources and superfields of dimensions 3/2 with ghost number zero and therefore given by
\[
\bar{\Delta}(x) = F(\Gamma^a, c, \bar{c})(x) + \omega b(x), \\
\tilde{\Delta}(x) = \tilde{F}(\tilde{\Gamma}^a, \tilde{c}, \tilde{\bar{c}})(x) + \tilde{\omega} \tilde{b}(x),
\] (25)

written in terms of the local polynomials \(F, \tilde{F}\) and constants \(\omega\) and \(\tilde{\omega}\). Here, we assume that identities [19] hold below the order \(n\) in \(\hbar\) and, therefore, the most general breaking are compatible with the power-counting constraints above. Now, the consistency conditions
\[
\frac{\delta}{\delta b(x)} \bar{\Delta}(y) - \frac{\delta}{\delta b(y)} \bar{\Delta}(x) = 0, \\
\frac{\delta}{\delta \tilde{b}(x)} \tilde{\Delta}(y) - \frac{\delta}{\delta \tilde{b}(y)} \tilde{\Delta}(x) = 0,
\] (26)

follow from the facts that \([\delta/\delta b(x), \delta/\delta b(y)] = 0\), \([\delta/\delta \tilde{b}(x), \delta/\delta \tilde{b}(y)] = 0\), respectively. Utilizing (25) and (26), it is easy to write
\[
\bar{\Delta}(x) = \frac{\delta}{\delta b(x)} \int d^3y \left[ F(\Gamma^a, c, \bar{c})(y) + \frac{1}{2} \omega b(y)b(y) \right], \\
\tilde{\Delta}(x) = \frac{\delta}{\delta \tilde{b}(x)} \int d^3y \left[ \tilde{F}(\tilde{\Gamma}^a, \tilde{c}, \tilde{\bar{c}})(y) + \frac{1}{2} \tilde{\omega} \tilde{b}(y)\tilde{b}(y) \right].
\] (27)

Now, we redefine the effective action as
\[
\bar{\Sigma} = \Sigma - \hbar^n \int d^3y \left[ F(\Gamma^a, c, \bar{c})(y) + \frac{1}{2} \omega b(y)b(y) \right] + \hbar^n \int d^3y \left[ \tilde{F}(\tilde{\Gamma}^a, \tilde{c}, \tilde{\bar{c}})(y) + \frac{1}{2} \tilde{\omega} \tilde{b}(y)\tilde{b}(y) \right],
\] (28)

which follow,
\[
\frac{\delta \bar{\Sigma}}{\delta b} = D^a \Gamma_a + \alpha b + O(\hbar^{n+1}), \\
\frac{\delta \bar{\Sigma}}{\delta \tilde{b}} = D^a \tilde{\Gamma}_a + \alpha \tilde{b} + O(\hbar^{n+1}).
\] (29)

We repeat this argument at each consecutive order. Consequently, this ends the recursive proof of the renormalizability of the gauge conditions.
2. Antighost equations

Let us now investigate the antighost equations (20) to prove that

\[
\begin{align*}
\left( \frac{\delta}{\delta \bar{c}} - D_a \frac{\delta}{\delta \bar{K}_a} \right) \bar{\Sigma} &= 0, \\
\left( \frac{\delta}{\delta \bar{c}} - D_a \frac{\delta}{\delta \bar{K}_a} \right) \tilde{\bar{\Sigma}} &= 0,
\end{align*}
\]

where \( \bar{\Sigma} \) has already been defined in (28). To write these equations into a more simple form, we redefine the superfields to yield

\[
\begin{align*}
\frac{\delta}{\delta \bar{K}_a} &= \frac{\delta}{\delta \hat{\bar{K}}_a}, \\
\frac{\delta}{\delta \bar{c}} &= \frac{\delta}{\delta \hat{\bar{c}}} - D_a \frac{\delta}{\delta \hat{\bar{K}}_a}, \\
\frac{\delta}{\delta \tilde{\bar{K}}_a} &= \frac{\delta}{\delta \tilde{\hat{\bar{K}}}_a}, \\
\frac{\delta}{\delta \tilde{\bar{c}}} &= \frac{\delta}{\delta \tilde{\hat{\bar{c}}}} - D_a \frac{\delta}{\delta \tilde{\hat{\bar{K}}}_a}.
\end{align*}
\]

Thus, the antighost equations become

\[
\begin{align*}
\frac{\delta}{\delta \hat{\bar{c}}} \hat{\bar{\Sigma}} &= 0, \\
\frac{\delta}{\delta \tilde{\hat{\bar{c}}}} \tilde{\bar{\Sigma}} &= 0,
\end{align*}
\]

where \( \hat{\Sigma} \) is the effective action written for new variables \( \hat{\bar{K}}_a, \hat{\bar{c}}, \tilde{\hat{\bar{c}}} \). Now, we apply QAP and thus obtain

\[
\begin{align*}
\frac{\delta}{\delta \hat{\bar{c}}} \hat{\bar{\Sigma}} &= \bar{\Delta} \hat{\bar{\Sigma}}, \\
\frac{\delta}{\delta \tilde{\hat{\bar{c}}}} \tilde{\bar{\Sigma}} &= \tilde{\bar{\Delta}} \tilde{\bar{\Sigma}}.
\end{align*}
\]

Here, we assume again that the breaking starts at order \( \hbar^n \), with \( n \geq 1 \),

\[
\frac{\delta}{\delta \hat{\bar{c}}} \hat{\bar{\Sigma}} = h^n \bar{\Delta} + O(h^{n+1}), \\
\frac{\delta}{\delta \tilde{\hat{\bar{c}}}} \tilde{\bar{\Sigma}} = h^n \tilde{\bar{\Delta}} + O(h^{n+1}),
\]

with local polynomials of the sources and superfields of dimensions 3/2 with ghost number 3/2, \( \bar{\Delta} \) and \( \tilde{\bar{\Delta}} \).

\[
\begin{align*}
\bar{\Delta}(x) &= G(\Gamma^a, \bar{c})(x) + \bar{v}(\bar{c})\hat{\bar{c}}(x), \\
\tilde{\bar{\Delta}}(x) &= \tilde{G}(\tilde{\Gamma}^a, \tilde{\bar{c}})(x) + \tilde{v}(\tilde{\bar{c}})\tilde{\hat{\bar{c}}}(x).
\end{align*}
\]

Here, we found that

\[
\frac{\delta}{\delta \hat{\bar{c}}(x)} \bar{\Delta}(y) - \frac{\delta}{\delta \hat{\bar{c}}(y)} \bar{\Delta}(x) = 0, \\
\frac{\delta}{\delta \tilde{\hat{\bar{c}}}(x)} \tilde{\bar{\Delta}}(y) - \frac{\delta}{\delta \tilde{\hat{\bar{c}}}(y)} \tilde{\bar{\Delta}}(x) = 0,
\]

which have the following solutions:

\[
\begin{align*}
\bar{\Delta}(x) &= \frac{\delta}{\delta \hat{\bar{c}}(x)} \int d^3y \left( \hat{\bar{c}}G(\Gamma^a, \bar{c})(y) + \frac{1}{2} \bar{v}(\bar{c})\hat{\bar{c}}(y) \right), \\
\tilde{\bar{\Delta}}(x) &= \frac{\delta}{\delta \tilde{\hat{\bar{c}}}(x)} \int d^3y \left( \tilde{\hat{\bar{c}}}\tilde{G}(\tilde{\Gamma}^a, \tilde{\bar{c}})(y) + \frac{1}{2} \tilde{v}(\tilde{\bar{c}})\tilde{\hat{\bar{c}}}(y) \right).
\end{align*}
\]

We can redefine the action analogously as in equation (28), so the antighosts as well as the gauge conditions hold to order \( h^n \). Similarly, we are able to prove that all these identities hold to all orders.
V. Nielsen identity for ABJM theory

In this section, we analyse the Nielsen identity for the ABJM theory in $\mathcal{N} = 1$ superspace following [21, 28]. To do so, we first perform a shift in the Lagrangian density as follows:

$$\mathcal{L}_{ABJM} \rightarrow \mathcal{L}'_{ABJM} = \mathcal{L}_{ABJM} + \int d^2 \theta \text{ Tr} \left( \frac{\chi b}{2} c \right),$$

(38)

where $\chi$ is a global Grassmannian variable, i.e., $\chi^2 = 0$. It is clear upon a little reflection that this extra term does not change the dynamics of the theory. The resulting Lagrangian (38) remains invariant under the following extended set of BRST transformations:

$$\delta^+_b \Gamma_a = \lambda \nabla_a c, \quad \delta^+_b \tilde{\Gamma}_a = \tilde{\lambda} \tilde{\nabla}_a \tilde{c},$$
$$\delta^+_b c = -\frac{1}{2} \lambda [c, c], \quad \delta^+_b \tilde{c} = -\frac{1}{2} \tilde{\lambda} [\tilde{c}, \tilde{c}],$$
$$\delta^+_b \tilde{c} = \lambda b, \quad \delta^+_b \tilde{b} = \lambda \tilde{b},$$
$$\delta^+_b b = 0, \quad \delta^+_c \tilde{b} = 0,$$
$$\delta^+_b \alpha = \lambda \chi, \quad \delta^+_b \chi = 0,$$
$$\delta^+_b X^I = i\lambda c X^I - iX^I \tilde{c} \tilde{\lambda},$$
$$\delta^+_b X^{I\dagger} = i\tilde{\lambda} \tilde{c} X^{I\dagger} - iX^{I\dagger} c \lambda,$$

(39)

where $\lambda$ and $\tilde{\lambda}$ are the Grassmann parameters. The interesting point noted here is that the gauge parameter also changes under the transformation. To exploit this invariance to derive the Nielsen identities, we construct the following generating functional:

$$Z = \int [D\phi] \exp \left[ i \int d^3 x \left( \mathcal{L}'_{ABJM} + \int d^2 \theta \text{ Tr} \left\{ J^a \Gamma_a - \tilde{J}^a \tilde{\Gamma}_a + \tilde{J}^I X_I + X^I J^I + b J_b - \tilde{b} J_{\tilde{b}} \right\} \right] \right].$$

(40)

The various sources denoted by $J$ with a different subscript are the obvious ones, however, the purpose of the additional, rather exotic looking, sources denoted by $K$'s will become apparent in a moment. The terms with such additional sources of the Lagrangian may be rewritten as

$$K_I \left( \delta^+_b X^I \frac{\delta \Delta}{\delta \lambda} + \delta^+_b X^{I\dagger} \frac{\delta \Delta}{\delta \lambda} \right) + \left( \delta^+_b X^{I\dagger} \frac{\delta \Delta}{\delta \lambda} + \delta^+_b X^I \frac{\delta \Delta}{\delta \lambda} \right) K_I.$$  

(41)

To study the gauge dependence of the gauge and matter propagators, we now introduce the generating functional of proper Green functions

$$\Delta(\Gamma_a, \tilde{\Gamma}_a, X_I, X_I^\dagger, c, \tilde{c}, \tilde{\Gamma}_{\tilde{c}}, \tilde{\Gamma}_{\tilde{c}}^\dagger, \beta, \alpha, \chi, K_a, \tilde{K}_a, K_I, \tilde{K}_I) = W(J^a, \tilde{J}^a, J^I, \tilde{J}^I, J_b, \tilde{J}_b, J_\beta, \tilde{J}_\tilde{b}, J_\alpha, \tilde{J}_\tilde{c}, K_a, \tilde{K}_a, K_I, \tilde{K}_I) - \int d^3 x \int d^2 \theta \text{ Tr} \left[ J^a \Gamma_a - \tilde{J}^a \tilde{\Gamma}_a + \tilde{J}^I X_I + X^I J^I + b J_b - \tilde{b} J_{\tilde{b}} + b J_\beta - \tilde{b} J_{\tilde{b}} + \beta J_\beta - \tilde{\beta} J_{\tilde{c}} + \tilde{\beta} J_{\tilde{c}} - \tilde{\beta} J_{\tilde{c}} \right].$$

(42)

The invariance of above functional under (39) leads to

$$\delta^+_b \Delta = 0 \Rightarrow \delta^+_b \Gamma_a \frac{\delta \Delta}{\delta \Gamma_a} + \delta^+_b \tilde{\Gamma}_a \frac{\delta \Delta}{\delta \Gamma_a} + \delta^+_b \tilde{c} \frac{\delta \Delta}{\delta \tilde{c}} + \delta^+_b \tilde{\tilde{c}} \frac{\delta \Delta}{\delta \tilde{c}} + \delta^+_b c \frac{\delta \Delta}{\delta c} + \delta^+_b \tilde{c} \frac{\delta \Delta}{\delta \tilde{c}} + \delta^+_b c \frac{\delta \Delta}{\delta \tilde{c}} + \delta^+_b \tilde{c} \frac{\delta \Delta}{\delta \tilde{c}} + \delta^+_b X^I \frac{\delta \Delta}{\delta X^I} + \delta^+_b X^{I\dagger} \frac{\delta \Delta}{\delta X^{I\dagger}}.$$  

(43)
Here, the terms corresponding to the fields which vanish under the transformation (39) would not be appear. Utilizing (41) together with (42), we rewrite (43) as follows:

\[
\frac{\delta \Delta}{\delta K_a} \frac{\delta \Delta}{\delta \Gamma_a} - \frac{\delta \Delta}{\delta K^a} \frac{\delta \Delta}{\delta \Gamma_a} + b \frac{\delta \Delta}{\delta \bar{c}} + b \frac{\delta \Delta}{\delta \bar{c}} + \frac{\delta \Delta}{\delta \bar{c}} \frac{\delta \Delta}{K_c} \frac{\delta \Delta}{\delta K_c} - \delta \Delta \frac{\delta \Delta}{\delta K_c} \frac{\delta \Delta}{\delta K_c} + \frac{\delta \Delta}{\delta \Gamma_c} \frac{\delta \Delta}{\delta \Gamma_c} + b \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} + b \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} - \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} - \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} + \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} + \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} + \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} + \frac{\delta \Delta}{\delta \chi} \frac{\delta \Delta}{\delta \chi} = 0. \tag{44}
\]

Now, we are able to have the Nielsen identities for the \( \mathcal{N} = 1 \) ABJM theory. For this, we differentiate equation (44) with respect to \( \chi \) and then set \( \chi = 0 \). This yields

\[
\frac{\delta \Delta}{\delta \alpha} + \frac{\delta^2 \Delta}{\delta \chi K_a} \frac{\delta \Delta}{\delta \Gamma_a} - \frac{\delta \Delta}{\delta K^a} \frac{\delta^2 \Delta}{\delta \chi \Gamma_a} - \frac{\delta^2 \Delta}{\delta K^a} \frac{\delta^2 \Delta}{\delta \Gamma_a} \frac{\delta \Delta}{\delta K_a} + b \frac{\delta^2 \Delta}{\delta \chi \bar{c}} + b \frac{\delta^2 \Delta}{\delta \chi \bar{c}} + b \frac{\delta^2 \Delta}{\delta \chi \bar{c}} \frac{\delta \Delta}{\delta K_c} \frac{\delta \Delta}{\delta K_c} + \frac{\delta^2 \Delta}{\delta \chi \Gamma_c} \frac{\delta \Delta}{\delta \Gamma_c} + b \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} + \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} + \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} + \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} + \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} + \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} + \frac{\delta^2 \Delta}{\delta \chi \chi} \frac{\delta \Delta}{\delta \chi \chi} = 0. \tag{45}
\]

From these results, we can generate the Nielsen identities for the two-point functions of ABJM theory. The investigations of the gauge dependence of the effective potential in the ABJM theory as well as the gauge independence of the physical poles of the propagator can schematically be derived from the above Nielsen identities Green’s function.

VI. CONCLUSION

Since M2-branes are three-dimensional objects embedded in an eleven-dimensional manifold, so the world-volume theory of such branes must be a three-dimensional gauge theory. However, in the low-energy limit, the theory must flow to a non-trivial fixed point. The promising candidate for the theories fulfilling all these requirements was constructed by Aharony, Bergman, Jafferis, and Maldacena (ABJM). The ABJM model is a three-dimensional superconformal Chern-Simons-matter theory with gauge group \( U(N) \times U(N) \).

In this paper, we have reviewed the gauge symmetry of ABJM theory in \( \mathcal{N} = 1 \) superspace. According to the standard quantization method, a theory having gauge symmetry can be quantizing only after breaking the gauge invariance by adding a gauge variant term which induces a ghost term to the action. The resulting action follows the BRST symmetry. With the help of BRST symmetry, we have computed the Slavnov-Taylor identities, gauge condition, anti-ghost equation, ghost number and spinor number. With the help of Ward identities, namely, gauge condition and anti-ghost equation at quantum level, we established the renormalizability of the ABJM theory in \( \mathcal{N} = 1 \) superspace at all order. Further, we have investigated the Nielsen identities for the two-point functions of ABJM theory in \( \mathcal{N} = 1 \) superspace in the covariant formalism. The Nielsen identities offer possibilities to check one’s calculations, however, they also allow us to see where physical meaning may be found in apparently gauge dependent Green’s functions. The present investigations will be very helpful to show the gauge dependence of the effective potential in a gauge theory (with scalar fields) as well as the gauge independence of the physical poles of the propagator and on-shell renormalization constants. Since the on-shell renormalization scheme is not commonly used in ABJM theory, so it will be subject of future investigation.

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