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An extended TODIM approach for group emergency decision making based on bidirectional projection with hesitant triangular fuzzy sets

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ABSTRACT

With the recent Covid-19 outbreak, group emergency decision-making (GEDM), as a new management model to pursue both social stability and decrease the negative impact of emergencies, has become highly popular. Evaluating and choosing the best emergency response is the core of the GEDM and selecting the choices can be regarded as a multi-attribute group decision-making (MAGDM) problem. Due to the increasing complexity and fuzziness of emergency decision-making environment, decision-makers (DMs) often cannot express completely rational preference information in many real EDM situations. At the same time, the existing methods seldom consider the DM’s psychological mindset at the point of decision making. In this paper, an extended TODIM (an acronym for interactive and multi-criteria decision-making in Portuguese) method based on bidirectional projection is proposed to solve the GEDM problem in the context of hesitant triangular fuzzy sets (HTFSs) and the novel method is applied to a case study and compared with other existing methods. The validity and applicability of the proposed method are discussed.

1. Introduction

With the growing frequency of emergencies, governments and policymakers increasingly face the need to mitigate the effect of such emergencies on society and the economy such as the recent Covid-19 outbreak. Decisions are made on a collective level, weighing the advantages and disadvantages of each solution alternative such as city lockdowns versus home quarantines for workers for two weeks. Indeed, choosing the solution alternative has become an important issue in emergency decision-making. When a catastrophe occurs, Group Emergency Making (GEDM) can help to mitigate the effect on the loss of properties and other potentially serious consequences caused by emergency outbreaks (Gao, Xu, Liao, 2019; Wang, Wang, & Martinez, 2017; Xu, Wang, Chen, & Liu, 2019). Selecting the best solution alternative is an important part of the GEDM, which can directly influence the quality of decision making (Xu, Zhang, & Chen, 2019). It can be regarded as a MAGDM problem involving many evaluation attributes throughout all levels of society and individuals. In practice, because of the fluidity of the emergency, most of the detailed information about the available crisis solution options are based on a contingency approach and the solution alternative assessment is not always clean and clear. Therefore, precise information may not be available to fully describe the crisis situation and the corresponding solution afforded. In such cases, fuzzy sets could be considered as a viable option for handling the ambiguity of the situation and the uncertainty. Examples of such fuzzy sets include the interval fuzzy sets (Gorzalczy, 1987; Wang et al., 2017), hesitant fuzzy sets (Alcantud & Giarlotta, 2019; Garg & Kaur, 2020; Torra, 2010), intuitionistic fuzzy sets (Alcantud, Khameneh, & Kilicman, 2020; Atanassov, 1986; Garg & Kumar, 2019), and type-2 fuzzy sets (Wang, Pan, Yan, Yao, & He, 2020; Zhang & Hu, 2020). This paper focuses on the case when the attribute values of the emergency response alternatives are expressed as Hesitant Triangular Fuzzy Sets (HTFS).

Many studies focus on solving the MAGDM problem (Liu, Diao, Zou, & Deng, 2020; Sun et al., 2020; Xiong, Chen, Chang, & Chin, 2019) and have made important inroads into large scale crisis management, but the decision maker’s psychological behavior has not been well studied.

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From the extant literature, the Decision Makers (DMs) are assumed to be bounded rational rather than completely rational under when operating in a uncertain environment (Li & Cao, 2019; Wang et al., 2017; Xu et al., 2019). GEDM problems often have risks and uncertainties, thus, the DM’s psychological behavior should be considered in the EDM process. In this paper, an extended TODIM approach based on bidirectional projection is proposed to deal with the EDM problems, in which DM’s psychological behavior is taken into account. The classical TODIM (an acronym in Portuguese of interactive and multi-criteria decision-making) method was proposed in 1992 (Gomes & Lima, 1992), which is a MADM method derived from prospect theory (Kahneman & Tversky, 1979). Though some methods based on prospect theory have discussed the psychological behavior of DMs (Liu & Li, 2019; Sun, Hu, & Chen, 2019; Zhou, Wang, Liao, & Lev, 2019), it is necessary to note that prospect theory has a drawback: it requires the aspiration levels of the attributes to be predetermined (Li & Cao, 2019), but it is difficult for DMs to determine it. The TODIM method does not require the aspiration levels of attributes to be known in advance. Besides, in incomplete rationality decision-making, the DM pursues maximum utility, while in the TODIM method; the DM aims to maximum value function. Therefore, the TODIM method is a useful tool to deal with MADM/MAGDM problems considering DM’s psychological behavior (Liang, Tu, Ju, & Shen, 2019; Liang, Wang, Xu, & Liu, 2020; Wang, Wang, & Martínez, 2020). Although the TODIM method is a good tool in solving MADM/MAGDM problems, there is also a shortcoming, such as Zhang and Xu (Zhang & Xu, 2014) proposed novel measured functions and combine it with the TODIM method to deal with service quality evaluation and ranking problems on domestic airlines under hesitant fuzzy environment. However, this method only considers the distance and ignores the angle of approach. As the bidirectional projection method has advantages not only in considering the distance and angle but also in the bidirectional projection between each alternative and the ideal solution, therefore, this paper intends to combine the bidirectional projection model with the TODIM method for handling GEDM problems.

Due to the complexity and uncertainty of the real world, there are some decision-making problems may lead DMs to use HTFSs to express their preference information (Wei, Wang, Zhao, & Lin, 2014; Zhao, Lin, & Wei, 2014). The membership degree of HTFS is presented by several possible triangular fuzzy numbers, which are more suitable and adequate to solve real-life decision-making problems than real numbers (Fahmi, Abdullah, Amin, Ali, & Khan, 2018). However, up till now, many fuzzy sets theory has been extended to EDM problem, but little attention has been paid to the HTFS environment to handle the GEDM problems (see Table 1). Therefore, this paper proposes an extended TODIM approach for GEDM based on bidirectional projection with HTFS. This not only improves the model’s ability to handle fuzzy situations and uncertainty but also solves the problem of crisis alternative selection with imprecise and fuzzy information.

Based on the analysis aforementioned, the main advantages of this paper are as follow:

1. The TODIM-based bidirectional projection model is defined. Compared to existing related methods, the TODIM-based bidirectional projection model, in which DM’s psychological behavior is taken into account.
2. Compared to the projection model, which only considers the distance and projection information. The proposal in this paper can avoid the situation where two projection values are consistent due to the projection model has only one reference point. Therefore, this paper intends to combine the bidirectional projection model with the TODIM method for handling group emergency decision making problems.
3. The TODIM-based bidirectional projection model is a novel and comprehensive way paid attention to handle EDM problems under hesitant triangular fuzzy information.

The rest of this paper is set as follows. The classical TODIM is introduced in Section 2, Section 3 describes a bidirectional projection method based on the HTFS. An extended TODIM method is proposed for GEDM based on bidirectional projection in Section 4. Section 5 provides a case study and compares with the other existing methods and some discussions are presented. Finally, Section 6 concludes the paper.

2. HTFS and classical TODIM method

As a preliminary to the rest of the paper, this section reviews the relevant knowledge of the HTFS and the classical TODIM method that will be used in later sections.

2.1. HTFS

In many practical situations, there is often incomplete and uncertain information, and decision makers cannot easily and accurately express the judgment of the alternatives. Therefore, triangular fuzzy sets (Wei et al., 2014) are generally more suitable to simulate real-life decision problems than crisp values. However, triangular fuzzy set cannot fully describe a comprehensive cognition of the decision maker, because the decision makers may hesitate among several triangular fuzzy sets due to a lack of relevant knowledge on the decision issues or time limitation to indicate that their assessment is more flexible and complex. To deal with this issue and to simulate this vagueness caused by the hesitation, we propose to use a hesitant triangular fuzzy set (Zhao et al., 2014), in which the membership degrees of an element to a given set are expressed as triangular fuzzy numbers.

**Definition 1.** (Zhao et al., 2014) Let X be a fixed set, the hesitant triangular fuzzy set (HTFS) on X can be written as:

\[ E = \left\{ \left( x, \tilde{h}_{E}(x) \right) \mid x \in X \right\} \]  \hspace{0.5cm} (1)

where \( \tilde{h}_{E}(x) \) is a set of triangular fuzzy numbers in the interval \([0, 1] \). These numbers represent the possible membership degree of an element \( x \in X \) in the HTFS \( E \). For convenience, \( \tilde{h} = \tilde{h}_{E}(x) = \left\{ \left( \mu^x, \mu^y, \mu^z \right) \mid \mu \in \tilde{h}_{E}(x) \right\} \) is taken as a symbol of a hesitant triangular fuzzy element.

**Example 1.** Let HTFS \( E \) on a reference set \( X = \{ x_1, x_2 \} \), then the hesitant triangular fuzzy element \( \tilde{h}_{E}(x) \) can be expressed as follows:

\[ \tilde{h}_{E}(x) = \left\{ \left( (0.6, 0.7, 0.8), (0.7, 0.8, 0.9) \right), \left( (0.4, 0.5, 0.7), (0.5, 0.6, 0.7), (0.7, 0.8, 0.9) \right) \right\} \]

### Table 1
Review of research on crisis solution alternative selection.

| Category                        | Method                     | Source                       |
|---------------------------------|----------------------------|------------------------------|
| Fuzzy rough set                 | Probabilistic approaches   | (Sun, Ma, & Chen, 2015)     |
| Hesitant fuzzy set              | Prospect theory            | (Zhou et al., 2019)         |
| Fuzzy linguistic set            |                             | (Zhang, Wang, Rodríguez, Wang, & Martínez, 2017) |
| Interval-valued fuzzy set       | New information measure    | (Peng & Garg, 2018)         |
| Intuitionistic fuzzy set        | Entropy Weight Method      | (Narayanamoorthy, Geetha, Rakkipappan, & Joo, 2019) |
| Type-2 fuzzy set                | Choquet method             | (Cheng, Chen, & Liu, 2013)  |
|                                | Extended TODIM method      | (Qin, Liu, & Pedrycz, 2017) |

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2.2. Classical TODIM method

The TODIM method (Gomes & Lima, 1992), which takes into account the DM’s psychological behavior, can be used to solve the GEDM problem. In this section, we introduce the classical TODIM method.

Consider an MADM problem with \( m \) attributes and \( n \) alternatives. Let \( C_j (j = 1, ..., m) \) be a set of attributes, whose weight vector is \( \omega = (\omega_1, ..., \omega_m) \), with \( \omega_j \in [0, 1] \) and \( \sum \omega_j = 1 \). Suppose \( A_j (i = 1, ..., n) \) is a set of alternatives, and the decision matrix \( X \) can be denoted as:

\[
X = \begin{bmatrix}
    x_{11}, x_{12}, ..., x_{1n} \\
    x_{21}, x_{22}, ..., x_{2n} \\
    \vdots & \ddots & \vdots \\
    x_{m1}, x_{m2}, ..., x_{mn}
\end{bmatrix}_{m \times n}
\]

where \( x_{ij} \) is hesitant triangular fuzzy number, which is the value of alternative \( A_i \) under attribute \( C_j \).

The algorithm for the TODIM method is introduced as follows:

Step 1: Normalize the decision matrix \( X = [x_{ij}]_{m \times n} \) into \( X = [\bar{x}_{ij}]_{m \times n} \),

\[
\text{proj}_B(A) = |A| \cos(A, B)
= \sqrt{\frac{\sum_{j=1}^{n} \left( \frac{1}{ln} \sum_{i=1}^{m} \bar{x}_{ij} (\bar{x}_{ij}) + \left( \frac{1}{ln} \sum_{i=1}^{m} (1 - \bar{x}_{ij}) \right)^2 \right)^2}{\left( \frac{1}{ln} \sum_{i=1}^{m} (1 - f_{Bx} (\bar{x}_{ij}) \right)^2}}
\]

where \( x_{ij} (i = 1, ..., n; j = 1, ..., m) \) is a crisp number.

Step 2: Obtain the relative weight \( \omega_j \) of attribute \( C_j \) to reference \( C_j \), where \( \omega_j = \omega_j / \sum_{j=1}^{m} \omega_j \), \( j = 1, ..., m \), and \( \omega_j = \max \{ \omega_j | j = 1, ..., m \} \).

Step 3: Compute the dominance degree of alternative \( A_i \) over alternative \( A_q \) with respect to \( C_j \) using

\[
\phi_{ij} = \begin{cases} 
\frac{(\tau_j - \tau_i) \omega_j}{\sum_{j=1}^{m} \omega_j}, & \tau_j - \tau_i > 0 \\
0, & \tau_j - \tau_i = 0 \\
\frac{1}{\theta} \frac{(\tau_j - \tau_i) \omega_j}{\sum_{j=1}^{m} \omega_j}, & \tau_j - \tau_i < 0
\end{cases}
\]

where \( \theta > 0 \) is the attenuation factor of the losses. A larger \( \theta \) means less aversion to loss. If \( 0 < \theta < 1 \), then the influence of the loss will increase. If \( \theta > 1 \), then the influence of the loss will decrease.

Step 4: Calculate the comprehensive dominance degree of alternative \( A_i \) over alternative \( A_q \).

\[
\phi_{i} = \sum_{j=1}^{m} \phi_{ij}
\]

Step 5: Calculate the perceived dominance degree of alternative \( A_i \),

\[
\xi(A_i) = \frac{\sum_{j=1}^{m} \phi_{ij} - \max \left\{ \sum_{j=1}^{m} \phi_{ij} \right\}}{\max \left\{ \sum_{j=1}^{m} \phi_{ij} \right\} - \min \left\{ \sum_{j=1}^{m} \phi_{ij} \right\}}
\]

Step 6: The alternatives are sorted according to the perceived dominance degree. The larger the value of \( \xi(A_i) \) is, the is better alternative \( A_i \).

3. Research method based on HTFS

So far, little attention has been paid to the projection model and bidirectional projection model based on the HTFS environment to handle the GEDM problem. In this section, we first give the concept of the existing projection method and bidirectional projection method, which are based on fuzzy numbers. In Section 3.1, the existing projection model and bidirectional projection model are introduced and the limitations of the methods are listed. To overcome the limitations of the methods mentioned, Section 3.1 proposes a bidirectional projection method based on the HTFS.

3.1. Existing projection model and bidirectional projection model

**Definition 2.** (Ye, 2017) Let \( A = \left\{ x_1, \left( \bar{t}_{Ax} (x_1), \bar{l}_{Ax} (x_1), \bar{f}_{Ax} (x_1) \right) \right\} \) and \( B = \left\{ x_2, \left( \bar{t}_{Bx} (x_2), \bar{l}_{Bx} (x_2), \bar{f}_{Bx} (x_2) \right) \right\} \) be two multi-valued neutrosophic sets. The projection of \( A \) on \( B \) as follows:

\[
B_{proj}(A, B) = \frac{1}{1 + \left| \frac{\|A\| \|B\|}{\|A\| \|B\|} - 1 \right|} = \frac{\|A\| \|B\|}{\|A\| \|B\| - \|A\| - \|B\| + \|A\| \|B\|}
\]

where \( \|A\| = \sqrt{\sum_{j=1}^{n} (d_{Aj} + u_{Aj} \inf t)^2 + (d_{Aj} + u_{Aj} \sup t)^2} \) is the modulus of \( A \), and the modulus of \( B \) is \( \|B\| = \sqrt{\sum_{j=1}^{n} (d_{Bj} + u_{Bj} \inf t)^2 + (d_{Bj} + u_{Bj} \sup t)^2} \). The inner product between \( A \) and \( B \) is \( A \cdot B = \sum_{j=1}^{n} \left[ (d_{Aj} + u_{Aj} \inf t) (d_{Bj} + u_{Bj} \inf t) + (d_{Aj} + u_{Aj} \sup t) (d_{Bj} + u_{Bj} \sup t) \right] \).

There are some limitations to the above methods. First, projection methods have been widely used in many disciplines; it can provide a measure of the tightness between two vectors. However, the projection method cannot guarantee that every calculation is significant. The projection method determines the order of the alternatives utilizing the projection of the alternative on the reference alternative. As the projection method only has one reference point, so when two projection
values are equal, the decision maker does not know how to choose the better alternative. Second, the bidirectional projection model mentioned above overcomes the drawback of the projection model and simultaneously considers the distance and angle, but the above-mentioned bidirectional projection model neglects the psychological behavior of the decision maker that is important in decision making. As such, this paper proposes a new model as follows:

**Definition 4.** ([Xu, 2005, 2015]) Let \( a = (a_1, a_2, \ldots, a_n) \) and \( \beta = (\beta_1, \beta_2, \ldots, \beta_n) \) be two n-dimensional vectors, then

\[
a \beta = \sum_{j=1}^{n} a_j \beta_j
\]

(7)

called is the inner product between \( a \) and \( \beta \). The cosine \( \cos(a, \beta) = \frac{a \beta}{|a||\beta|} \) between \( a \) and \( \beta \) is used to compute the bidirectional projection value, where \( |a| = \sqrt{\sum_{j=1}^{n} a_j^2} \) is the modulus of vector \( a \).

**Definition 5.** Let \( b = (b_1, b_2, \ldots, b_m) \), \( b_i = (b_{i1}, b_{i2}, b_{i3}, \ldots, b_{in}) \), \( i = 1, 2, \ldots, m \) is any m-dimensional triangular fuzzy vector column, \( E = \left\{ x, \bar{h}_{g(x)} \right\} \mid x \in X \) be an HTFS, and then the modulus of an m-dimensional triangular fuzzy vector \( b = (b_1, b_2, \ldots, b_m) \) and a hesitant triangular fuzzy element \( \bar{h}_{g(x)} = \left\{ (\gamma^L_i, \gamma^M_i, \gamma^U_i) \mid x \in g(x) \right\} \) are defined as

\[
|b| = \sqrt{\sum_{i=1}^{n} b_i^2} = \sqrt{\sum_{i=1}^{n} \left| b_i^e \right|^2 + \left( b_i^m \right)^2 + \left( b_i^b \right)^2}
\]

(8)

\[
|E| = \sqrt{\sum_{i=1}^{k} \frac{1}{\# \bar{h}} \left( \frac{1}{\prod_{j=1}^{q} \left| \gamma_j \right|} \right)^2}
\]

(9)

where \( \# \bar{h} = \max\left\{ \# \bar{h}_{g(x)} \right\} \), \( \# \bar{h}_{g(x)} \) represents the length of the HTFE.

**Definition 6.** Let \( \bar{h}_{g(x)} = \left\{ (\gamma^L_1, \gamma^M_1, \gamma^U_1) \mid x \in g(x) \right\} \) and \( \bar{h}_{g(x)} = \left\{ (\gamma^L_2, \gamma^M_2, \gamma^U_2) \mid x \in g(x) \right\} \) are two HTFEs, then the cosine of the included angle between \( \bar{h}_{g(x)} \) and \( \bar{h}_{g(x)} \) is defined as

\[
\cos(\bar{h}_{g(x)}, \bar{h}_{g(x)}) = \frac{\bar{h}_{g(x)} \cdot \bar{h}_{g(x)}}{|\bar{h}_{g(x)}||\bar{h}_{g(x)}|} = \frac{\gamma^L_1 \gamma^L_2 + \gamma^M_1 \gamma^M_2 + \gamma^U_1 \gamma^U_2}{\sum_{i=1}^{n} \frac{1}{\# \bar{h}_{\gamma_i}} \left( \prod_{j=1}^{q} \left| \gamma_j \right| \right)^2}
\]

(10)

where \( 0 \leq \cos(\bar{h}_{g(x)}, \bar{h}_{g(x)}) \leq 1 \). \( \cos(\bar{h}_{g(x)}, \bar{h}_{g(x)}) \) reflects the changing direction between \( \bar{h}_{g(x)} \) and \( \bar{h}_{g(x)} \), and can reflect whether the directions of \( \bar{h}_{g(x)} \) and \( \bar{h}_{g(x)} \) are consistent.

**Definition 7.** Let \( E_1 = \left\{ (x, \bar{h}_{g(x)}) \mid x \in X \right\} \) and \( E_2 = \left\{ (x, \bar{h}_{g(x)}) \mid x \in X \right\} \) be two HTFSs, \( \bar{h}_{g(x)} = \left\{ (\gamma^L_1, \gamma^M_1, \gamma^U_1) \mid x \in g(x) \right\} \), then the vector formed by \( E_1 \) and \( E_2 \) is defined as

\[
E_1E_2 = \left\{ (\gamma^L_1 \gamma^M_1 + \gamma^U_1, \gamma^L_2 \gamma^M_2 + \gamma^U_2, \gamma^L_1 \gamma^M_2 + \gamma^U_1 \gamma^U_2, \gamma^L_2 \gamma^M_1 + \gamma^U_1 \gamma^U_2, \gamma^L_1 \gamma^M_2 + \gamma^U_1 \gamma^U_2) \right\}
\]

(11)

where \( \# \bar{h} \) represents the max length of the two HTFSs and \( \{ (\gamma^L_1 - \gamma^L_2, \gamma^M_1 - \gamma^M_2, \gamma^U_1 - \gamma^U_2), \ldots, (\gamma^L_1 - \gamma^L_2, \gamma^M_1 - \gamma^M_2, \gamma^U_1 - \gamma^U_2) \} \) represents the projection of \( \bar{h}_{g(x)} \) on \( \bar{h}_{g(x)} \), \( \bar{h}_{g(x)} \) is the further the distance between \( \bar{h}_{g(x)} \) and \( \bar{h}_{g(x)} \), \( \bar{h}_{g(x)} \) is the projection of \( \bar{h}_{g(x)} \) on \( \bar{h}_{g(x)} \), \( \bar{h}_{g(x)} \) is the projection of \( \bar{h}_{g(x)} \) on \( \bar{h}_{g(x)} \), \( \bar{h}_{g(x)} \) is the larger the projection value of \( \bar{h}_{g(x)} \), \( \bar{h}_{g(x)} \), \( \bar{h}_{g(x)} \).
and $0 < \text{Proj}_{E} (\tilde{h}_i(x)) \leq 1$ for any two sets $\tilde{h}_i(x_1)$ and $\tilde{h}_i(x_2)$, which is a normalized measure.

**Definition 10.** Let $E^+$, $E^-$, and $E^r$ be the PIS, NIS, and $ith$ alternative, respectively, then the bidirectional projection model defined on the based on two formulas as follows

$$\text{Proj}_{E^-} (E^E) = |E^E \cos(E^E, E^-) |$$

$$\text{Proj}_{E^r} (E^E) = |E^E \cos(E^E, E^-) |$$

where $ \text{Proj}_{E^-} (E^E)$ indicates the projection of the vector, which formed by the NIS and $ith$ alternative, on the vector, which formed by the NIS and PIS and $ \text{Proj}_{E^r} (E^E)$ represents the projection of the vector formed by PIS and NIS on the vector formed by $ith$ alternative and PIS. The measure of the projection can be illustrated in Fig. 1.

**Theorem 2.** The larger the value of $\text{Proj}_{E^-} (E^E)$, the closer is $E^E$ to $E^-$. Similarly, the smaller the value of $\text{Proj}_{E^r} (E^E)$, the farther is distance between $E^r$ and $E^-$. The larger the value of $\text{Proj}_{E^r} (E^E)$, the closer is $E^E$ to $E^-$, the smaller the value of $\text{Proj}_{E^r} (E^E)$, the farther distance between $E^r$ and $E^-$. In other words, alternative $A_i$ is closer from the PIS. A similar argument holds for $\text{Proj}_{E^-} (E^E)$.

4. **Extended TODIM method based on bidirectional projection**

Although many scholars have studied how the TODIM method solves the MADM problems, studies combine the bidirectional projection model of HTFS with TODIM in dealing with EDM problems are scant. This paper aims at developing a new method to deal with emergency model of HTFS with TODIM in dealing with EDM problems are scant.

4.1. **Normalize the decision matrix**

For the convenience of analysis and computation, we need to normalize the decision matrix $X = [x_{ij}]_{m \times n}$ into $\hat{X} = [\hat{x}_{ij}]_{m \times n}$, $x_{ij}(i = 1, ..., n; j = 1, ..., m)$. The normalized matrix can be calculated by

$$\hat{x}_{ij} = \begin{cases} \tilde{h}_{j(x_i)}, & \tilde{g}(x_i) \in G_1 \\ \tilde{h}_{j(x_i)}, & \tilde{g}(x_i) \in G_2 \end{cases}$$

where $G_1$ represents the benefit index, $G_2$ the cost index, and $\tilde{h} = (\hat{p}^1, \hat{p}^2, \hat{p}^3)$, $\tilde{g} = (\hat{q}^1, \hat{q}^2, \hat{q}^3)$.

4.2. **Determine the attribute weight**

In this paper, we use the method of combining the weighted total deviation minimum method of all alternatives with the entropy weight method to solve the attribute weights.

According to **Theorem 2**, the deviation between the evaluation value of the alternative $E^r$ and the PIS $E^+$ is $1 - \text{Proj}_{E^-} (E^E)$. In order to remove the influence of the symbolic factor, the square of the deviation sum is taken as the target value, and then the sum of weighted deviations of all attributes $E^r$ and PIS $E^+$ is modeled as follows:

$$\begin{aligned}
\min G(o) &= \sum_{j=1}^{m} \sum_{i=1}^{n} \omega_i (1 - \text{Proj}_{E^-} (E^E)) \\
\text{s.t.} \quad \sum_{j=1}^{m} \omega_i &= 1 \\
0 \leq \omega_i &\leq 1.
\end{aligned}$$

Due to the uncertainty of the value of the attribute weight, the idea based on the maximum entropy principle is to select one with the largest entropy among all the feasible solutions, the maximum entropy means the least amount of information obtained and the amount of information added during the solution process is least. Therefore, it is reasonable in the case of incomplete data, and the comprehensive model for the attribute weight is:

$$\begin{aligned}
\min H(o) &= C_1 \sum_{j=1}^{m} \sum_{i=1}^{n} \omega_i (1 - \text{Proj}_{E^-} (E^E)) + C_2 \sum_{j=1}^{m} \omega_j \ln \omega_j \\
\text{s.t.} \quad \sum_{j=1}^{m} \omega_i &= 1 \\
0 \leq \omega_i &\leq 1.
\end{aligned}$$

where $C = (C_1, C_2)^T$ is the balance factor matrix; the balance factor value will be given before the decision-making, Eq. (23) can be determined using LINGO.

4.3. **Calculation of gains and losses**

As computed, the dominance degree based on the gain and loss is the basic content of the proposed extended TODIM method, this subsection proposes some relations for finding the gain and loss between two HTFSs. Let $E^+$, $E^-$ and $E^r$ be the hesitant triangular fuzzy positive ideal solution (PIS), hesitant triangular fuzzy negative ideal solution (NIS), and another alternative, respectively, then the difference measure based on the projection value $\text{Proj}_{E^-} (E^E)$ and $\text{Proj}_{E^r} (E^E)$ can be written as

$$d_{\text{proj}}(\hat{x}_i, \hat{x}_g) = \frac{C(E_g) - C(E_{\hat{x}_i})}{|E|} > 0,$$

and
C(E_i) = \left[ \frac{Proj_{E'}(E-E')}{\sum_{j=1}^{n} \frac{r_{ij}}{C(E_i) - C(E_j)}} \right]_{j=1}^{n} + \left[ \frac{Proj_{E}(E-E)}{\sum_{j=1}^{n} \frac{r_{ij}}{C(E_i) - C(E_j)}} \right]_{j=1}^{n} (25)

C(E_j) = \left[ \frac{Proj_{E'}(E-E')}{\sum_{j=1}^{n} \frac{r_{ij}}{C(E_i) - C(E_j)}} \right]_{j=1}^{n} + \left[ \frac{Proj_{E}(E-E)}{\sum_{j=1}^{n} \frac{r_{ij}}{C(E_i) - C(E_j)}} \right]_{j=1}^{n} (26)

4.4. Compute dominance degree

Based on the classical TODIM method (see Eq. (2)), the dominance degree of alternative A_i over alternative A_j concerning attribute C_j can be calculated.

\[
\phi_{ij}^{(1)} = \sqrt{d_{\max}(\bar{x}_i, \bar{x}_j) \omega_r C(E_i) - C(E_j)} > 0
\]

\[
\phi_{ij}^{(2)} = \sqrt{d_{\max}(\bar{x}_i, \bar{x}_j) \omega_r C(E_i) - C(E_j)} = 0
\]

\[
\phi_{ij}^{(3)} = \sqrt{d_{\max}(\bar{x}_i, \bar{x}_j) \omega_r C(E_i) - C(E_j)} < 0
\]

where \(\omega_r = a_{ij}/a_{ij} \neq 0\), and \(a_{ij} = \max\{a_{ij} \neq 1, ..., m\}\), and \(\theta\) is the attenuation factor as stated earlier. Obviously, \(0 \leq \phi_{ij}^{(1)}, \phi_{ij}^{(2)}, \phi_{ij}^{(3)} \leq 0\).

4.5. Ranking of alternatives

From Eqs. (3) and (4), the value of a comprehensive dominance degree and perceived dominance degree can be calculated and we obtain the ranking of the alternatives. Clearly, \(0 \leq \xi(A_i) \leq 1\), and the bigger the value of \(\xi(A_i)\) the better alternative \(A_i\). Therefore, based on the descending order of the perceived values of all the alternatives, we can rank order all the alternatives to choose the ideal alternative(s).

In summary, the procedures for the proposed method are presented below (see Fig. 2):

Step 1. Add its largest (or smallest) element to each HTFS, which is determined by the risk attitude of the decision maker, so that the length of each HTFS is \#\(\hat{h} = \max\{\#\hat{h}_{(x_i)}\}\), \#\(\hat{h}_{(x_i)}\) represents the length of the HTFS.

Step 2. Utilize Eq. (21) to normalize the decision matrix \(X = [x_{ij}]_{m \times n}\) into \(\tilde{X} = [\tilde{x}_{ij}]_{m \times n}\).

Step 3. According to the hesitant triangular fuzzy weighted geometric operator, preference information from different DMs are aggregated (Zhao et al., 2014).

Step 4. Obtain the attribute weights according to Eq. (23).

Step 5. Construct the dominance degree matrix \(\phi' = [\phi'_{ij}]_{m \times n}\) using Eq. (27).

Step 6. Obtain the comprehensive dominance degree matrix \(\xi = [\xi(A_i)]_{m \times n}\) using Eq. (3).

Step 7. Calculate the perceived dominance degree of \(A_i\) using Eq. (4).

Step 8. The rank order of the alternatives is determined based on the obtained perceived dominance degree.

5. Case study and comparison analysis

To better demonstrate the method proposed in this paper, this method presents a new bidirectional projection model that avoids the situation where two projection values are consistent and extend the TODIM method based on bidirectional projection model. To demonstrate how the proposal not only considers the psychological behavior of decision makers but also solves the GEDM problems, this section presents an example based on the background of the recent COVID-19 outbreak caused by the coronavirus and compares against another related method. To acquire the evaluation information of this crisis management, expert survey and questionnaire methods are used to obtain the attribute values in this paper. The expert panel consists of three experts. DM_1 is a scholar, who is a senior professor studying infectious disease and crisis management, possesses rich research experience and has published many senior SCI papers in this field. DM_2 is the head of the province as he is familiar with details of the Covid-19 outbreak in the city, therefore, his support is needed to deal with this crisis. DM_3 is the leader of the city’s medical emergency management department and has the quality and ability to handle infectious diseases professionally. In this paper, the role of the expert panel is to evaluate the qualitative evaluation attributes of each crisis solution alternative and provide the evaluation values in form of an HTFS for a better selection of the optimal choices to solve the medical crisis. Table 2 contains the evaluation values given by three experts.

5.1. Description of the Covid-19 crisis

The Covid-19 outbreak has threatened the lives of thousands of people in different countries and slowed down the economic livelihoods of many industries. In China, the Department of Emergency Management has already activated an emergency team of experts to deal with the challenging medical issues by building hospitals to house the infected and contain the infection locally, put several cities into lockdown mode. To determine the best course of action, the emergency group, which consists of three DMs, quickly developed the following three crisis management modes through an analysis of the situation and previous experience gained from the H1N1 virus avian flu outbreak. Without any loss of generality, we assume equal weights \(\omega_i = [1/3, 1/3, 1/3]\) for each DM.

A_1: Airborne transporters are used to transport the relevant medical professionals and small infectious disease monitoring equipment to the lockdown areas, moving the quarantined people to safer zones. Doing so allows the medical professionals to directly treat the infected and reduce the spread of disease at the scene of the containment, but the cost is relatively high.

A_2: Rescue buses are allowed to transport the relevant medical workers and small rescue equipment to the disaster site, moving people who are not yet affected to safe zones outside of the city. The cost of this alternative is relatively low.

A_3: Building special infectious disease hospitals to house the infected and contain the spread of the disease. Doing so will take time and the cost of bringing the construction equipment and people is high, without a sense of how many people will be affected and the spread of the infection.

The following indicators need to be considered when DMs make decisions: prevention of secondary infectious disease spread \((C_1)\), the safety of medical personnel brought in to the lockdown area \((C_2)\), the best evacuation process and route to take \((C_3)\), and the state of economic growth \((C_4)\). After reviewing and discussing the relevant information by three experts, the weight range of attributes was given \([0.26 \leq \omega_1 \leq 0.25, 0.25 \leq \omega_2 \leq 0.26, 0.25 \leq \omega_3 \leq 0.27, 0.24 \leq \omega_4 \leq 0.26]\). After that, Eq. (23) is transformed into Eq. (28), as shown below.
be calculated as obtained, as shown in Table 3.

6.2. Comparison analysis

To illustrate the novelty, effectiveness, and feasibility of the proposed method, the following two comparisons are made in this subsection.

(1) The method proposed in this paper is compared with the HTF information aggregation method of (Zhao et al., 2014), which ignores the psychological behavior of DMs in the decision making process.

(2) The proposed method is compared with the gray projection method (Zhang, Jin, & Liu, 2013), which ignores the relationship with the positive ideal solution and the situation that the two alternatives cannot be distinguished when the two projection values are equal.

The comparison results are shown in Table 4. Because the HTF information aggregation method (Zhao et al., 2014) puts forward two aggregation operators, to make a fair comparison, the proposed method is compared with two different aggregation operators to determine the decision results. From Table 4, it is obvious that the rank results for different aggregation operators differ. Therefore, the use of the extended TODIM method to reflect the DM’s psychological behavior makes the decision different, which indicates that the DM’s psychological behavior is considered in GEDM. This will enable decision making to better reflect the way humans think in reality. Thus, the proposed method is better than the HTF information aggregation method, and better able to mimic human response and behavioural uncertainty.

Then, to demonstrate that the proposed extended TODIM method

\[
\begin{align*}
\xi(A_i) = & \frac{\sum_{l=1}^{m} \varphi_{l}}{\max \{\sum_{l=1}^{m} \varphi_{l}\}} - \min \{\sum_{l=1}^{m} \varphi_{l}\} \\
\varphi_{l} = & \left[ \begin{array}{cccc}
\omega_{1} & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 \\
-4.124 & 0.000 & -3.714 & -1.540 \\
0.000 & 0.000 & -3.646 & -4.809 \\
0.000 & -2.207 & 0.000 & -4.556 \\
\end{array} \right]
\end{align*}
\]

Here, \( \theta \) is set to one (Gomes, 2009).

Further, using Eqs. (3) and (4), the perceived dominance value can be calculated as \( \xi(A_1) = 0 \), \( \xi(A_2) = 0.3688 \), \( \xi(A_3) = 1 \). Therefore, the rank of the alternatives is \( A_3 > A_2 > A_1 \) and the best one is \( A_3 \). This choice is fully in line with the actual emergency solution undertaken by the provincial government in Hubei Province of China and proves the validity and feasibility of the method to some extent.

...
### Table 2
Original decision matrix of three DMs.

|     | $C_1$                      | $C_2$                      | $C_3$                      | $C_4$                      |
|-----|---------------------------|---------------------------|---------------------------|---------------------------|
| $DM_1$ | $A_1$                     | (0.103.05)                | (0.307.08),(0.104.08),(0.508.09) | (0.203.05)                | (0.203.04),(0.304.05)      |
|      | $A_2$                     | (0.607.08)                | (0.204.05)                | (0.304.07),(0.304.08)     | (0.203.05),(0.405.06),(0.506.07) |
|      | $A_3$                     | (0.103.05)                | (0.508.09),(0.609.10)     | (0.203.04),(0.405.06)     | (0.405.07),(0.506.07)      |
| $DM_2$ | $A_1$                     | (0.408.09)                | (0.307.08),(0.203.08),(0.308.09) | (0.102.03)                | (0.306.07),(0.407.08)      |
|      | $A_2$                     | (0.607.08)                | (0.203.04),(0.506.07)     | (0.405.06)                | (0.203.04)                |
|      | $A_3$                     | (0.607.08),(0.708.09)     | (0.506.07)                | (0.405.07),(0.506.07),(0.708.09) | (0.103.05)                |
| $DM_3$ | $A_1$                     | (0.306.07),(0.407.08)     | (0.305.07),(0.406.08),(0.208.09) | (0.304.07),(0.304.08)     | (0.204.05)                |
|      | $A_2$                     | (0.203.04),(0.103.05)     | (0.102.03)                | (0.207.08),(0.708.09)     | (0.405.07),(0.506.07),(0.708.09) |
|      | $A_3$                     | (0.203.05),(0.405.06),(0.506.07) | (0.203.05)                | (0.506.07)                | (0.306.07),(0.407.08)      |

### Table 3
Aggregated values from decision makers.

|     | $C_1$                      | $C_2$                      | $C_3$                      | $C_4$                      |
|-----|---------------------------|---------------------------|---------------------------|---------------------------|
| $A_1$ | (0.71.055.25),(0.71.055.25),(0.68.052.23) | (0.31.080.90),(0.31.066.08),(0.18.039.77) | (0.18.029.49),(0.18.029.49),(0.18.029.47) | (0.58.048.29),(0.58.048.29),(0.52.042.23) |
| $A_2$ | (0.68.053.33),(0.68.053.33),(0.63.053.42) | (0.22.036.47),(0.22.036.47),(0.16.029.39) | (0.44.054.76),(0.44.054.76),(0.29.052.70) | (0.63.052.41),(0.55.045.34),(0.52.036.25) |
| $A_3$ | (0.68.052.33),(0.66.049.30),(0.58.040.23) | (0.39.055.07),(0.39.055.07),(0.37.052.68) | (0.52.062.72),(0.46.066.06),(0.34.045.058) | (0.65.050.27),(0.56.050.27),(0.63.045.23) |
based on bidirectional projection is closer to reality than the gray projection method (Zhang et al., 2013), the comparison results are as shown in the first column and the last column of Table 4. From Table 4, we can see that alternative A3 is the best one, which is consistent with reality. Using the proposed method to calculate the decision results can avoid the deficiency of the projection method, which cannot choose the better alternative when the projection values of the two alternatives are equal. When using a projection method to compute decision results, it is unavoidable that two projection values are equal. The combination of the extended TODIM method and bidirectional projection method can make the decision results more reasonable and reliable. This is another advantage of group decision making and an indirect way to demonstrate the superiority and effectiveness of the proposed method.

5.3. Discussion

Through the above comparative analysis with the existing relevant methods, the main novelty and advantage of the proposed method can be summarized as follows.

(1) The extended TODIM approach for GEDM based on bidirectional projection in this paper not only considers the psychological behavior of the DMs but also avoids the situation where the projection values are equal and a better alternative cannot be selected objectively. These are the two significant differences between the proposed method and existing related methods.

(2) This paper focuses on the case that the attribute value of emergency solution alternatives expressed as HTFSs, because of their useful and simple technique to represent situational uncertainty.

(3) A new approach to determining the attribute weights is established by using a nonlinear programming model based on the minimum total deviation of the alternatives projection and the maximum entropy principle.

(4) The proposed method in this paper is easy to understand and the result of the decision is closer to the actual situation. Moreover, the computation process is simpler.

6. Conclusion

This paper presents an extended TODIM approach for group emergency decision making based on bidirectional projection. This method takes into account the psychological behavior of the DMs during the decision-making process, which is currently under studied. We combine the TODIM method with a bidirectional projection method to overcome the limitation of the traditional projection method, which cannot accurately rank the solution alternatives. Therefore, the proposed method is quite different from the existing GEDM method and provides a better decision making outcome. In the proposed method, because of the incomplete information and complexity of the emergencies, the DMs use the HTFS to express their personal preference information, which is better than crisp numbers. A new approach to determining the attribute weights is established by using a nonlinear programming model based on the minimum total deviation of the alternatives projection and maximum entropy principle. The weight of attributes is used to make a fair comparison with the existing related method in the case study to demonstrate the novelty, feasibility, and effectiveness of the proposed method. In future, to accommodate complex decision environments, the individual preference value of DMs may be expressed by a variety of different fuzzy types, based on a hybrid fuzzy environment.

CRediT authorship contribution statement

Quanyu Ding: Writing - original draft. Ying-Ming Wang: Supervision, Project administration, Funding acquisition. Mark Goh: Validation.

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