Doorway states, the super-radiant mechanism and nuclear reactions

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Abstract. In the 1950s the possibility of forming a ‘‘super-radiant’’ (SR) state in a gas of atoms confined to a volume of a size smaller than the wave length of radiation was suggested by Dicke. The atoms, with two levels, are coupled through their common radiation field. This indirect coupling leads to a redistribution of lifetimes among unstable intrinsic states. A strongly decaying SR state is created at the expense of the rest of the states of the system. Recently the connection of this mechanism to the notion of doorway states in low-energy nuclear reactions, was pointed out. The conditions for appearance of such doorways in nuclear physics processes are discussed.

1. Introduction

In the early 1950s the possibility of forming a ‘‘super-radiant’’ (SR) state in a gas of atoms confined to a volume of a size smaller than the wave length of radiation was suggested by Dicke [1]. In the absence of a direct interaction, the atoms are coupled through their common radiation field. This indirect interaction through the continuum leads to a redistribution of lifetimes among unstable intrinsic states. A rapidly decaying SR state is created at the expense of the rest of the states of the system that are ‘‘robbed’’ of their decay probability and become narrow. This mechanism has a general origin and analogous phenomena should appear in many quantum systems when quasi-bound states are strongly coupled through common decay channels. The SR approach has been used in many different fields. For example, it was applied in chemistry [2], atomic physics [3], condensed matter physics [4, 5], intermediate energy nuclear physics [6, 7], in particle physics [8], in the theory of nuclear reactions [5, 9, 10]. In the next section we will develop the SR formalism for some simple situations in order to arrive in a heuristic fashion to the idea of a super-radiant state and its connection to the concept of a doorway.

2. The effective complex Hamiltonian.

Following the projection formalism [6,11], let us divide the Hilbert space of nuclear states into two parts, the \( Q \)-subspace involving complicated many-body states \( |q\rangle \), and the subspace \( P \) of open channels \( |c\rangle \). We use the notations \( Q \) and \( P \) for the corresponding projection operators onto the above subspaces. The wave function of the total system,

\[
|\Psi\rangle = Q|\Psi\rangle + P|\Psi\rangle
\]

satisfies the Schrödinger equation

\[
H|\Psi\rangle = E|\Psi\rangle
\]
that can be written as a set of coupled equations,
\[
(E - H_{QQ})|\Psi\rangle = H_{QP} |\Psi\rangle
\]
and
\[
(E - H_{PP})|\Psi\rangle = H_{QQP} |\Psi\rangle
\]
where we use the notations \( H_{ab} = AHB \). Eliminating the part \( P|\Psi\rangle \), we obtain an equation in the \( Q \)-space,
\[
(E - \tilde{H}_{QQ})Q|\Psi\rangle = 0
\]
with the effective Hamiltonian
\[
\tilde{H}_{QQ} = H_{QQ} + \frac{1}{E^{(+)} - H_{PP}} H_{QP}
\]
Here \( E^{(+)} = E + i0 \) contains the infinitesimal imaginary term \( +i0 \) ensuring correct asymptotic conditions for the continuum wave functions. The second term of the effective Hamiltonian (6) contains a real and imaginary part of the propagator
\[
G^{(+)}(E) = \frac{1}{E^{(+)} - H_{PP}}
\]
resulting from the principal value and the delta-function \( \delta(E - H_{PP}) \) (on-shell contributions from channels \( c \) open at energy \( E \), respectively. The imaginary part of the effective Hamiltonian is \( \tilde{\Gamma} = \frac{E - (i/2)W}{W} \), with
\[
W = 2\pi \sum_c H_{QP} |c\rangle \langle c| H_{QP}
\]
Thus, the effective Hamiltonian (7) in \( Q \)-space is anti-Hermitian,
\[
\tilde{H}_{QQ} = \tilde{\Gamma}_{QQ} - \frac{i}{2}W
\]
where \( \tilde{\Gamma}_{QQ} \) is a symmetric and real matrix that includes, apart from the original Hamiltonian in the \( Q \)-space \( H_{QQ} \), the principal value contribution of the \( PQ \)-coupling. The second part is anti-Hermitian. The eigenvalues of \( \tilde{H} \), \( \tilde{\Gamma} = E - (i/2)\Gamma \) are complex poles of the scattering matrix corresponding to the resonances in the cross sections.
To demonstrate in a simple way the role of the anti-Hermitian term we assume that only one channel is open. Then the matrix \( W \), Eq. (8), has a completely separable form,
\[
\langle q|W|q'\rangle = 2\pi A^*_q A_{q'}
\]
where:
\[
A^*_q = \langle q|H_{QP}|c\rangle
\]
The rank of the matrix \( W \) is 1, so that all the eigenvalues of this matrix are zero except one that has the value equal to the trace of the matrix:
\[
\Gamma_s = \sum_q \langle q|W|q\rangle = 2\pi \sum_q |A^*_q|^2 \equiv \sum_q \Gamma_q^s
\]
with \( \Gamma_q^s \) denoting the escape width of the individual levels before the \( W \)-matrix is diagonalized. The special unstable state with width \( \Gamma_s \) is often referred to as the super-
radiant (SR), in analogy to the Dicke coherent state \([1]\) of a set of two-level atoms coupled through the common radiation field. Here the coherence is generated by the common decay channel. The stable states are trapped and decoupled from the continuum.

In the more general case of \(N\) intrinsic states and \(N_c\) open channels with \(N_c \ll N\) the super-radiant mechanism survives if the mean level spacing \(D\) of internal states and their decay widths \(\Gamma\) obey:

\[
\kappa = \frac{\Gamma}{D} < 1
\]

In this case we have \(N_c\) broad states, while the rest of states \(N - N_c\) become very narrow.

3. Doorways

It is often the case that only a subset of intrinsic states \(\{Q\}\) connects directly to the \(\{P\}\) space of channels. The rest of states in \(\{Q\}\) will connect to \(\{P\}\) only when they obtain admixtures of these selected states of the first type. The special states directly coupled to continuum are the doorways, \(\{d\}\). They form the doorway subspace \(\{D\}\) within \(\{Q\}\), and the corresponding projection operator will be denoted here as \(D\). The remaining states in \(\{Q\}\) will be denoted as \(\{\tilde{Q}\}\) and the subspace as \(\{\tilde{Q}\}\).

The full Hamiltonian can be decomposed the following way:

\[
H = \left( H_{\tilde{Q}Q} + H_{DD} + H_{\tilde{Q}D} + H_{DQ} \right) + \left( H_{PP} + H_{DP} + H_{PD} \right)
\]

Note that the terms \(H_{PQ}\) and \(H_{QP}\) are missing because in accordance with the doorway hypothesis they are very small. Also note that diagonalizing the operator in the upper line of (14) would give back the states \(\{q\}\) with the components of \(\{d\}\) mixed with \(\{\tilde{q}\}\) states. The two last items in the above equation couple the doorway states, and therefore all the \(\{\tilde{q}\}\) states to the open channels.

3.1 The case of a single doorway

We start our discussion with the case when there is only one important doorway state \(\{d\}\).

The matrix elements of the effective operator \(W\) in the intrinsic space are now given by:

\[
\langle q | W | q' \rangle = 2\pi \sum_{c=1}^{N_c} \langle q | H_{DP} | c \rangle \langle c | H_{PD} | q' \rangle
\]

with the doorway assumption:

\[
\langle q | H_{DP} | c \rangle = \langle q | d \rangle \langle d | H_{DP} | c \rangle
\]

where \(\langle q | d \rangle\) is the amplitude of the admixture of the doorway into the \(\{q\}\) state. Eq. (16) becomes:

\[
\langle q | W | q' \rangle = 2\pi \sum_{c=1}^{N_c} \langle q | H_{DP} | c \rangle \langle c | H_{PD} | q' \rangle
\]

with the doorway assumption:
where $\langle q | d' \rangle$ is the amplitude of the admixture of the doorway into the $|q\rangle$ state. Eq. (16) becomes:

$$\langle q | W | q' \rangle = 2\pi \langle q | d' \rangle \langle d | q' \rangle \sum |d | H_{DP} | c\rangle|^2$$

again we have separable matrix elements of the matrix $W$, this time irrespective of the number of open channels $N_c$.

As discussed above, the criterion of validity of the SR mechanism is that the average spacing between the levels in $\{Q\}$ is smaller than the decay width of such a state “before” the SR mechanism takes effect. This can be expressed, in the case of doorways, the following way.

Consider the spreading width $\Gamma_d$, of the doorway state representing the fragmentation of $|d\rangle$ into compound states $|q\rangle$. If $N_q$ is the number of compound states in the interval covered by the spreading width, their average energy spacing is:

$$\bar{D} \approx \frac{\Gamma_d}{N_q}$$

Before the SR mechanism is turned on, the average decay width of a typical $|q\rangle$ state is:

$$\Gamma_q^\uparrow = 2\pi \sum_c \langle q | H_{QP} | c\rangle |^2$$

that can be estimated as:

$$\Gamma_q^\uparrow = \frac{\Gamma_q}{N_q}$$

Thus:

$$\frac{\Gamma_q^\uparrow}{D_q} \approx \frac{\Gamma_q}{\Gamma_d} \approx \frac{\Gamma_d}{\Gamma_d}$$

We conclude that the requirement for the SR doorway mechanism to be valid can be formulated as:

$$\frac{\Gamma_q^\uparrow}{\Gamma_d} > 1$$

4. Examples

4.1 Isobaric Analog States

The isobaric analog state (IAS), $|A\rangle$, is obtained from the parent state with certain isospin $T$ by acting with the isospin lowering operator $T_-$ that changes a neutron into a proton:

$$|A\rangle = \text{const} \cdot T_- |T_\pi\rangle$$

In a compound nucleus, the IAS is surrounded by many compound states $|q\rangle$ of lower isospin $T_q = T - 1$. The Coulomb interaction violates the isospin symmetry fragmenting the strength of the IAS over many states $|q\rangle$ giving rise to the spreading width $\Gamma_d^\uparrow$ of the IAS. If located above thresholds, the IAS can also decay into several continuum channels that gives rise to
The decay width $\Gamma_A^\uparrow$. In medium and heavy mass nuclei the condition the in Eq. (24) is satisfied. For example in $^{208}\text{Pb}$ the spreading width is about 80 keV while the escape width is about 160 keV, [12], thus $\Gamma_A^\uparrow / \Gamma_A^\downarrow \approx 2$.

The SR mechanism is therefore relevant to this case providing a straightforward explanation why the IAS appears as a single resonance with the decay width given by that of $A$:

$$\Gamma_A^\uparrow = 2\pi \langle A | H_{Q\beta} | P \rangle^2$$

(26)

### 4.2 Giant resonances

Giant resonances in nuclei (or atomic clusters) present another example of similar physics [13, 14]. Usually, the giant resonances are discussed in terms of $p$–$h$ configurations with identical spin-parity quantum numbers. The coherent residual interactions form a correlated state that carries much of the transition strength of a corresponding multipole operator. The giant resonances are mostly located in the particle continuum decaying via particle emission to the ground and excited states in the daughter nucleus.

Since the $p$–$h$ giant resonance $|G\rangle$ is surrounded usually by a dense spectrum of $2p$–$2h$ and more complex configurations, the residual strong interaction will mix $|G\rangle$ with this background. Each of the resulting states denoted as $|b\rangle$ will contain the admixture $\langle G | b \rangle$ of the giant resonance. The mixed states $|b\rangle$ couple to the continuum. If we assume that the dominant coupling is that through the admixture of the giant state, then $|G\rangle$ serves as a doorway, and the matrix $W$ is separable, given by:

$$\langle b | W | b' \rangle = \langle b | G \rangle \langle G | b' \rangle \sum_{c=1}^{N_G} \langle G | c \rangle \langle c | b \rangle$$

(27)

The matrix $W$ is again of rank one and the non-zero eigenvalue is:

$$\Gamma_G^\uparrow = 2\pi \sum_c |\langle G | c \rangle|^2$$

(28)

This holds under the assumption that the energy spread of the background states $|b\rangle$ is small compared to their decay width:

$$\Gamma_b^\uparrow = 2\pi \sum_c |\langle b | c \rangle|^2$$

(29)

As before, this condition can be expressed in this case as:

$$\Gamma_G^\uparrow / \Gamma_b^\uparrow > 1$$

(30)

the spreading width of the giant resonance is smaller than its total decay width. When this condition is not satisfied, so that the energy intervals between the $2p$–$2h$ states are larger than their decay widths $\Gamma_b^\uparrow$, the situation of a single decay peak for the giant resonance might not hold. Still one could expect some bunching of background states into groups and have spacing within the group smaller than their decay widths $\Gamma_b^\uparrow$. Each such group then can be treated separately using the SR mechanism and appear as a single peak in the decay (or excitation) curve. Then one will observe intermediate structure resonances in the GR energy domain.

In some continuum RPA calculations it was found that high lying giant resonances have substantial decay widths [14] larger than the spreading widths. For example the isovector monopole resonances calculated in heavy nuclei had decay widths of the order of 8-10 MeV, which is larger than the spreading width which is of the order of several MeV. In these cases the condition Eq. [29] for super-radiance is satisfied.
5. Conclusions
We have considered few examples of well known nuclear physics phenomena and described the formation of doorways in the framework of the SR mechanism in these phenomena. Because of the generic nature of this mechanism it would be interesting to extend this approach to other mesoscopic systems. In the physics of cold atoms [15] in a trap, the coupling of the internal states to the continuum via the Feshbach resonance influences profoundly the system affecting not only the imaginary part of the effective Hamiltonian but also the real part. This enables one to manipulate the properties of the system by changing the sign and/or the strength of the two-body interaction. The Feshbach resonance in this case serves as a doorway. The traditional theory of intermediate structure [16, 17] considers usually statistical sequences in time of multi-step particle interactions with possible pre-compound (or pre-equilibrium) decay to the continuum. The present formulation emphasizes complementary aspects of underlying physics, namely those that follow from the strict quantum-mechanical description of complicated many-body dynamics generated by the mean field and residual interactions. The approach based on the effective non-Hermitian Hamiltonian predicts the existence of different scales in the energy dependence of observed cross sections. These scales are defined by the intrinsic dynamics that combines collective (coherent) and chaotic (incoherent) features. The new collectivity responsible for the emergence of the doorways, under certain physical conditions, is due to the factorized nature of corresponding terms in the effective Hamiltonian. This separability is related to the properties of unitarity of the scattering matrix. In this way we obtain a supplementary description of many-body dynamics.

We should emphasize that the SR approach with respect to doorways is not in any way contradicting the more conventional approach to the notion of doorways [11, 12]. It presents a different view of the same physical phenomenon, stressing the collective nature of the decay width. The SR formalism in the present application is analogous to the calculation of the collective states in nuclear structure. However in the SR mechanism one deals with the imaginary part of the effective Hamiltonian and not with the real part. This analogy can bridge nuclear structure and nuclear reactions, especially when dealing with weakly bound exotic nuclei. In certain situations the SR mechanism may provide an understanding of the appearance of unexpectedly narrow resonances. As a consequence of the creation of a SR resonance the widths of the other resonances coupled to the same channel, become narrow.

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