Towards an effective description for the $J/\psi$ decays

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Abstract. In this work we propose a phenomenological Resonance Chiral Theory inspired vertex $J/\psi \rho \pi$, which is a basic ingredient for a description of the $J/\psi$ phenomenology. This vertex is characterized by only one coupling constant which is computed through the analysis of the $J/\psi \rightarrow \gamma \pi^0$ decay. Assuming $\rho$ meson dominance we compute the $\text{BR}(J/\psi \rightarrow \pi^0 \pi^+ \pi^-)$, finding a good agreement with the experimental measurement.

1. Introduction
It is well known that Chiral Perturbation Theory ($\chi$PT) [1–3] is a rigorous effective field theory framework for a description of Quantum Chromodynamics (QCD) at low energies, well below the $\rho(770)$ meson mass. At energies $m_\rho \lesssim E \lesssim 2$ GeV the Resonance Chiral Theory ($R\chi T$) [4,5] emerges as a framework to describe the interactions between the light quark resonances and the chiral pseudo-goldstone bosons. The $R\chi T$ lagrangian fulfills the discrete symmetries of QCD and chiral symmetry and contains an even sector [4,6] and an odd intrinsic-parity sector [7,8]. It also models a large-$N_C$ expansion [9,10]. This framework has been applied for instance to the study hadronic tau decays [11–16] at leading order in the $N_C \rightarrow \infty$ limit and to a description [17] of the pion vector form factor up to the next to leading order in the large number of colours expansion.

However the low-energy couplings of both $\chi$PT and $R\chi T$ cannot be determined from chiral symmetry alone, they are free parameters of the lagrangian. Nonetheless demanding a short distance behaviour in accordance with QCD and its Operator Product Expansion (OPE) [18,19], one is able to extract constraints between several couplings. This procedure has been applied in Refs. [7, 20] in order to impose restrictions on the couplings of the odd intrinsic-parity sector, obtaining a set of relations between them. Still this method does not provide full information on the couplings and some of them remain unknown. Alternatively one can rely on the phenomenology for their determination or at least to establish some relations between them.

One of the couplings of the odd-intrinsic-parity $R\chi T$ Lagrangian (OIPL) that remains unknown after the first procedure above is the corresponding to an operator of the four that contribute to the vertex $VVP$ (being $V$ short for vector resonance and $P$ for pseudoscalar meson, all of them of the lightest $SU(3)$ multiplets), namely $d_4$ in of Ref. [7] (we take this notation in the following) or $\kappa_{4VV}$ in Ref. [8]. It appears, for instance, in the leading order determination of the odd-intrinsic-parity $\mathcal{O}(p^6)$ chiral perturbation theory couplings $C_{17}^W$, $C_{19}^W$, $C_{20}^W$ and $C_{21}^W$ [8,21] (though with a factor that suppresses its contribution) and, more importantly, in processes like the radiative decay of the tau lepton into two pions [22], relevant for the determination of the muon anomalous magnetic moment. One way to determine the $d_4$ coupling...
is to study the $J/\psi(1S)$ ($J/\psi$ for short) resonance phenomenology, in particular we can focus on the $J/\psi \rightarrow \omega(782)\pi^+\pi^-$ decay.

The phenomenology of this process extracted from the experimental analyses is still a matter of discussion. The BES Collaboration [23] has performed a fit to the decay data by considering a wide set of intermediate resonance states and as a result they reported a strong signal of resonances $f_0(500)$, $f_2(1270)$, $b_1(1235)$ in this particular $J/\psi$ decay. However they did not include the virtual $\rho(770)$ transition, namely, $J/\psi \rightarrow \rho(770)\pi \rightarrow \omega(782)\pi^+\pi^-$ channel in their analysis. The inclusion of this contribution within the chiral unitary approach was performed in Ref. [24], where some additional final state interactions where also taken into account. The $\rho(770)\pi$ intermediate state was later considered in Ref. [25] where it was found that, depending on the choice of the parameters of the form factor, the contribution of the $\rho(770)$ to the total decay width of $J/\psi \rightarrow \omega(782)\pi^+\pi^-$ extends from 3.1% to 21%. The inclusion of the $\rho(770)$ was also found to give a solution to the problem put forward in Ref. [23] of the disagreement between the value obtained for the width of the $b_1(1235)$ and the PDG value [26].

In order to disentangle the $\rho(770)$ contribution in the $J/\psi \rightarrow \omega(782)\pi^+\pi^-$ decay one has to consider two vertices: $\rho - \omega - \pi$ and $J/\psi - \rho - \pi$. Here we focus in the latter, by investigating its participation in the $J/\psi \rightarrow \gamma\pi^0$ decay, assuming that $\chi_T$ is reliable until $E \sim 2$ GeV and considering a phenomenological Lagrangian that describes the $J/\psi - \rho - \pi$ vertex. Subsequently with our result we give a prediction for the $B(J/\psi \rightarrow \pi^+\pi^-\pi^0)$. Our result will be useful for the determination of the $d_4$ coupling, where the knowledge of both vertices is mandatory [27].

Another point of discussion is the coupling of the $J/\psi$ vector resonance to the lightest hadrons, both vectors and pseudoscalar mesons. We will take the realization of the three-gluon-annihilation OZI rule put forward in Ref. [28] in the frame of the OIPL and where the $J/\psi$ is considered as a $SU(3)$ singlet in the Proca representation and the lightest mesons obey the constraints of chiral $SU(3)$ symmetry. This complements nicely our framework.

In Section 2 we present the basis of $\chi_T$ and our proposal for the $J/\psi VP$ vertex; in Section 3 we analyse the $J/\psi \rightarrow \gamma\pi^0$ decay and we estimate the $B(J/\psi \rightarrow \pi^0\pi^+\pi^-)$ decay with our assumptions; finally we give our conclusions in Section 4.

2. Resonance Chiral Theory

Resonance Chiral Theory [4, 5] provides an appropriate setting for the study of resonance contributions to low-energy ($E \lesssim 2$ GeV) hadrodynamics. Essentially it involves the construction of a phenomenological Lagrangian that describes the interaction of the lightest resonances ($N_f = 2$, 3, being $N_f$ the number of flavours) with the pseudoscalar stable mesons, namely pions and kaons. A detailed account of the method is given in Ref. [29].

On the other hand, it is customary to use the antisymmetric formulation to describe the light quark spin-1 resonances. This is because in the even-intrinsic-parity sector the $\mathcal{O}(p^4)$ $\chi$PT low-energy constants, consistent with higher energy constraints, are obtained directly upon integration of the resonances if one uses that formalism. If instead we resort to the Proca formalism, for instance, the same goal would need to add local contributions to the pure resonance Lagrangian [5]. However we will use the latter representation to describe the $J/\psi(1S)$ field.

In the antisymmetric tensor field formalism, spin-1 resonances are described by a field that satisfies:

$$
(0|V_{\mu\nu}|V,p) = \frac{i}{m_V}[p_\mu\eta_\nu(p) - p_\nu\eta_\mu(p)]
$$

where $m_V$ is the mass of the resonance, $p$ its momentum and $\eta_\nu$ its polarization vector. Its
propagator in this formalism reads:
\[
\Omega^{\alpha\gamma\tau\delta}(p) = 2i \left[ \frac{\Omega^{\alpha\gamma\tau\delta}(p)}{m_V^2 - p^2} + \frac{\Omega^{\alpha\gamma\tau\delta}(p)}{m_V^2} \right].
\] (2)

The transverse (T) and longitudinal (L) part of the propagator are given by:
\[
\Omega_T^{\alpha\gamma\tau\delta}(p) = \frac{1}{2p^2} \left[ p^2 g^{\alpha\tau} g^{\delta\gamma} \right. - g^{\alpha\tau} p^\gamma p^\delta - g^{\gamma\tau} p^\alpha p^\delta - \left( \tau \leftrightarrow \delta \right) \right],
\]
\[
\Omega_L^{\alpha\gamma\tau\delta}(p) = \frac{1}{2p^2} \left[ g^{\alpha\tau} p^\gamma p^\delta \right. - g^{\gamma\tau} p^\alpha p^\delta - \left( \tau \leftrightarrow \delta \right) \right].
\] (3)

The leading and next-to-leading R\(\chi\)T phenomenological Lagrangian in the even-intrinsic-parity sector has already been worked out \[4,6\]. Here we collect the pieces relevant for our purposes:
\[
\mathcal{L}_{\text{RT}} = \mathcal{L}_2 + \mathcal{L}_{\text{kin}} + \mathcal{L}_V + \mathcal{L}_{J/\psi VP}.
\] (4)

Here \(\mathcal{L}_2\) is the \(\mathcal{O}(p^2)\) \(\chi\)PT Lagrangian:
\[
\mathcal{L}_2 = \frac{F^2}{4} \left( u_{\mu} u^{\mu} + \chi_+ \right),
\] (5)

where \(F \sim 92.2\) MeV \[26\] is the pseudoscalar meson decay constant in the chiral limit. The brackets indicate a trace in the flavour space. The notation here and in the following is the one in Ref. \[6\]. The next piece involves kinetic term of the vector resonances \(\mathcal{L}_{\text{kin}}\), including also the one of the \(J/\psi\) state. The leading interacting operators of the even-intrinsic-parity sector, linear in the light quark vector resonance fields, are given by:
\[
\mathcal{L}_V = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_{+}^{\mu\nu} \rangle + i \frac{G_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle,
\] (6)

where \(V_{\mu\nu}\) indicates the \(U(3)\) nonet of the lightest vector meson resonances, namely \(\rho(770)\), \(K^*(892)\), \(\omega(782)\), \(\phi(1020)\), and \(F_V\) and \(G_V\) coupling constants not determined by the symmetries. As commented above the enforced symmetries do not give information on the couplings of the operators: \(F\), \(F_V\), \(G_V\). Within the \(\chi\)PT scheme some information on them can be extracted by requiring that relevant Green functions, determined with our phenomenological Lagrangian in Eq. (4) satisfy, asymptotically (i.e. at large energies or transfers of momenta), the leading behaviour described by QCD in the large-\(N_C\) limit. To proceed one would need to determine those Green functions using our \(\chi\)PT Lagrangian in a model of the large-\(N_C\) limit in which we consider only one multiplet of light resonances. Usually the constraints translate into relations between the coupling constants. Though reducing the infinite tower of resonances (of the large-\(N_C\) limit) to only one could look, admittedly, a strong assumption, several works along this line indicate that this is a good approximation when spin-1 resonances are involved \[7,20,30,31\]. See Ref. \[32\] for limitations on this approach. For instance, it was established \[20\] that \(F_V^2 = 3F^2\), which is in good agreement with phenomenological determinations of the \(F_V\) coupling constant \[12\]. In the following we will assume this condition.

Hadron decays of the \(J/\psi\) involve a rich dynamics mainly driven by light-flavoured meson resonances. Accordingly those decays are a very important source of information on the effective couplings of the \(\chi\)PT framework. The study of the processes we are interested in requires to add the \(\mathcal{L}_{J/\psi VP}\) piece in Eq. (4) that corresponds to the vertex of a \(J/\psi\) vector resonance with one light \((N_f = 3)\) vector nonet \((V)\) and pseudoscalar \((P)\) mesons. As the \(J/\psi\) is a pure \(\overline{c}c\) state, the decay \(J/\psi \to VP\) is OZI suppressed. The operators ruling the vertex will obey the
chiral and unitary symmetry in the part that corresponds to the light quark hadrons. For the description of the \( J/\psi \) state we will take a Proca realization of the field and it will be a singlet under the light quark symmetries. With this proviso we have just one leading operator:

\[
L_{J/\psi VP} = m_\psi H_1 \varepsilon_{\mu\nu\rho\sigma} \Psi^\mu \langle u^\nu V^{\rho\sigma} \rangle ,
\]

neglecting \( SU(3) \) breaking and double OZI suppressed contributions [28]. Here \( H_1 \) is an unknown coupling constant and \( \Psi_\mu \) is the field of the \( J/\psi \) vector meson; the fact that we have taken a Proca realization for the latter warrants that the number of operators, describing the vertex, is smaller than in the case of the antisymmetric realization, in fact only one in this case.

3. The \( J/\psi \to \gamma \pi^0 \) decay and the \( H_1 \) coupling .

As our main goal is to obtain information on the \( J/\psi VP \) vertex given in Eq. (7), here we proceed to do a phenomenological determination of \( H_1 \) from the study of \( J/\psi \to \pi^0 \gamma \) and confirm its consistency in the \( J/\psi \to \pi^+\pi^-\pi^0 \) decay.

Within our framework the \( J/\psi \to \pi^0 \gamma \) decay is driven, at leading order, by the vertex in Eq. (7). Hence we can use the experimentally determined \( B(J/\psi \to \pi^0 \gamma) \) [33] to determine the \( H_1 \) coupling. The only diagram that contributes is the one in Figure 1.

![Figure 1](image1)

Figure 1. The \( J/\psi \to \pi^0 \gamma \) decay encloses the \( H_1 \) coupling in Eq. (7). Only the \( \rho(770) \) contributes to this process and its coupling to the photon is given by Eq. (6).

The amplitude for this diagram, contributing to \( J/\psi(\eta, P) \to \pi^0(\epsilon, k) \gamma \) reads:

\[
\mathcal{M}_{\pi^0\gamma} = -i 2\sqrt{2} e H_1 \frac{e_p m_\psi}{2 m_\rho} \varepsilon_{\mu\nu\rho\sigma} \eta^\mu(\epsilon) k^\nu q^\rho .
\]

Taking \( F_V = \sqrt{3} F \) and the central value of the experimental result \( B(J/\psi \to \pi^0 \gamma) = (3.49^{+0.33}_{-0.30}) \times 10^{-5} \) [26], the absolute value of the coupling is found to be,

\[
|H_1| \approx 2.36 \times 10^{-5} .
\]

On the other hand, the \( J/\psi \to \pi^+\pi^-\pi^0 \) decay has recently been measured at BESIII [34], showing a \( \rho(770) \)-dominance in the \( \pi\pi \) invariant mass distribution. Indeed the known \( \rho\pi \) puzzle that has challenged for long many theoretical determinations is precisely to understand why other higher-mass intermediate states are suppressed and the dynamics is \( \rho\pi \) dominated [35–39]. The role of the \( J/\psi \rho\pi \) vertex has widely been studied in different frameworks [24,35,40–43]. Here, in order to obtain information on the Lagrangian given in Eq. (7) we use this system.

The \( J/\psi \rho\pi \) vertex is given by Eq. (7) that, for the fields of our interest, reads:

\[
L_{\psi\rho\pi} = -\sqrt{2} \frac{m_\psi}{F} H_1 \varepsilon_{\mu\nu\rho\sigma} \Psi^\mu \left[ \partial^\nu \pi^+ \rho^-_\sigma + \partial^\nu \pi^- \rho^+_\sigma + \partial^\nu \pi^0 \rho^0_\sigma \right] ,
\]
and preserves, by construction, the chiral symmetry.

Now we can compute the amplitude for the process \( J/\psi \to \pi^0\pi^-\pi^+ \) that proceeds through the diagrams in Figure 2. The description of the \( \rho\pi\pi \) vertex is given by Eq. (6). The amplitude for the \( J/\psi(\varepsilon) \to \pi^0(p_0)\pi^-(p_-)\pi^+(p_+) \) decay is:

\[
\mathcal{M}_{3\pi} = -i\frac{\sqrt{2}}{m_\rho} \frac{G_V H_1}{2s m_\rho} \varepsilon_{\sigma\rho\mu} p^\sigma_+ p^-_0 p^0_\rho e^{i\theta} \left[ \frac{m_\psi^2}{m_\rho^2(s) - s} + \frac{m_\pi^2}{m_\rho^2(u) - u} + \frac{m_\rho^2}{m_\rho^2(t) - t} \right],
\]

where the Mandelstam variables are defined as \( s = (p_+ + p_-)^2, t = (p_0 + p_+)^2 \) and \( u = (p_0 + p_-)^2 \). Here the finite width of the \( \rho(770) \) resonance is taken into account by the mass term, \( \hat{m}_\rho^2(s) = m_\rho^2 - im_\rho \Gamma_\rho(s) \), and where [44]:

\[
\Gamma_\rho(s) = \frac{m_\rho s}{96\pi F^2} \left[ \left( 1 - \frac{4m_\pi^2}{s} \right)^{3/2} \left( \theta(s - 4m_\pi^2) + \frac{1}{2} \left( 1 - \frac{4m_\pi^2}{s} \right)^{3/2} \theta(s - 4m_K^2) \right) \right].
\]

The decay width depends on two Mandelstam variables, we choose the pair \( (s, u) \), whose minimum and maximum values are given as follows,

\[
4m_\pi^2 \leq s \leq (m_\psi - m_\pi)^2,
\]

\[
u_- \leq u \leq u_+(s),
\]

where

\[
u_+(s) = \frac{1}{4s} \left[ (m_\psi^2 - m_\pi^2)^2 - \left( \lambda^{1/2}(s, m_\pi^2) \right)^2 \right].
\]

Using our finding given in Eq. (9), our prediction is found to be,

\[
B(J/\psi \to \pi^+\pi^-\pi^0) = 1.7097 \times 10^{-2},
\]

which is not far away from the value \( (2.11 \pm 0.07) \times 10^{-2} \) reported in Ref. [26].

4. Conclusions

Summarizing the work done, we have determined the value of \( H_1 \) except by a sign, and we have proved the reliability of our finding by estimating the \( BR(J/\psi \to \pi^+\pi^-\pi^0) \) which competes very well with the measurements. With this result we have a trustworthy model for an effective description of the \( J/\psi VP \) vertex. More analysis in others \( J/\psi \) decays could shed light on the determination of the correct sign, the central value of this coupling and the truthful impact of the possible direct term \( J/\psi\pi\pi\pi \) which is not considered here.

Acknowledgments

This research has been supported in part by Conacyt-SNI and by UANL through the grant PAICYT-2015. Flores-Baez thanks the hospitality and support from IFIC-Valencia where part of this work was done and also thanks to J Portoles for his valuable reading of this manuscript.

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