Viscous cosmologies in scalar-tensor theories for Kasner type metrics.

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Abstract: In a viscous Bianchi type I metric of the Kasner form, it is well known that it is not possible to describe an anisotropic physical model of the universe, which satisfies the second law of thermodynamics and the dominant energy condition (DEC) in Einstein’s theory of gravity.

We examine this problem in scalar-tensor theories of gravity. In this theory we show that it is possible to describe the growth of entropy, keeping the thermodynamics and the dominant energy condition.

PACS number(s): 98.80.Hw, 98.80.Bp

I. INTRODUCTION

The standard Friedmann-Robertson-Walker (FRW) model has attracted considerable attention in the relativistic cosmology literature, since the FRW model is in accordance with the large scale spatial homogeneity and isotropy of the observable universe. Perhaps, one of the most important characteristics of this model is, as predicted by inflation [1], the flatness, which agrees with the observed cosmic microwave background radiation.

One can consider as source for FRW models not only a perfect fluid, without dissipative viscous processes, but also a bulk viscosity, which is compatible with the isotropy of the universe [2,3]. Therefore, the imperfect fluid also can be considered as a source for FRW models. It is shown in ref. [4], for a “tilting” four velocity with a spacelike component, that FRW cosmologies, in particular flat models, do not necessarily represent perfect fluid solutions. They may correspond to exact solutions of the vector field equations of viscous fluid in which bulk and shear solutions. They may correspond to exact solutions of the

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where the set of given anisotropic Kasner coefficients (1) are different. The expansion factors \( t^{p_i} \), \( t^{p_2} \) and \( t^{p_3} \) will be determined via Einstein’s field equations. The space is anisotropic if at least two of the three \( p_i \) (i = 1, 2, 3) are different.

In the next section we give the definitions of the corresponding expressions that we shall use throughout this paper. Also, in the same section, we show that, in standard Einstein relativity, a viscous cosmological fluid does not permit the Kasner metric to be anisotropic [5,6]. We work out the general scalar-tensor theory in Sect.III, and go on to analyze specific models in Sect.IV. Whereas in most cases conflicts between anisotropy/viscosity and the dominant energy condition still turn out to be the case, there are a few exceptions, as exemplified in Sect.IV.C, where the set of given anisotropic Kasner coefficients leads to a thermodynamically consistent description of

\[ ds^2 = dt^2 - t^{2p_1} dx^2 - t^{2p_2} dy^2 - t^{2p_3} dz^2, \]

where \( p_1, p_2 \) and \( p_3 \) are three parameters that we shall require to be constants. The expansion factors \( t^{p_1}, t^{p_2} \) and \( t^{p_3} \) will be determined via Einstein’s field equations. The space is anisotropic if at least two of the three \( p_i \) (i = 1, 2, 3) are different.

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the viscous universe. In section V we calculate the generation of entropy for a thermodynamically consistent cosmological model. Finally, in section VI we conclude our work.

II. VISCOUS KASNER TYPE UNIVERSE IN EINSTEIN THEORY

The Kasner universe, in Einstein’s theory (with cosmological constant $\Lambda = 0$), refers to a vacuum cosmological model given by (1)}, where the numbers $p_1$, $p_2$ and $p_3$ satisfy the constraints

$$p_1 + p_2 + p_3 = 1, \quad p_1^2 + p_2^2 + p_3^2 = 1.$$  \hfill (2)

An anisotropic Kasner type universe can be considered to be filled with an ideal (nonviscous) fluid which has an equation of state $p = \rho$ (stiff matter - the velocity of sound coincides with the speed of light), where $\rho$ is the energy density and $p$ is the isotropic pressure. In this case only the constraint $p_1 + p_2 + p_3 = 1$ is satisfied.

When one considers a fluid with bulk and shear viscosities, i.e. a viscous fluid, the mass density and the isotropic pressure energy density and $p$ are proportional to $1/t$, and the constraints (2) are not satisfied any more \(\boxed{}\). In general the energy-momentum tensor for a viscous fluid is given by

$$T_{\alpha\beta} = \left[ \rho + (p - \xi \theta) \right] u_\alpha u_\beta - (p - \xi \theta) g_{\alpha\beta} + 2\eta \sigma_{\alpha\beta},$$  \hfill (3)

where $u_\alpha$, $\rho$, $p$, $\xi$ and $\eta$ are the fluid’s four velocity, the energy density, the isotropic pressure, the bulk and shear (or coefficient of dynamic viscosity) viscosities, respectively. The scalar expansion and the traceless shear tensor are defined by $\theta = u^\alpha_{\alpha}$ and

$$\sigma_{\alpha\beta} = h^\gamma_{\alpha} u_\gamma (\xi) h^\delta_{\beta} - \frac{1}{3} \theta h_{\alpha\beta},$$  \hfill (4)

respectively. Here $h_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta$ is the projection tensor.

From the metric (1) and the Einstein equations we get for the shear viscosity the following expression

$$\eta = \frac{1}{16\pi G t} (1 - p_1 - p_2 - p_3).$$  \hfill (5)

It could be shown from thermodynamic principles that we should impose the conditions $\xi \geq 0$ and $\eta \geq 0$ in order to have a positive entropy generation \(\boxed{}\). When these conditions are applied in an anisotropic Kasner type model, the parameters entering into the metric have to satisfy the bound $p_1 + p_2 + p_3 \leq 1$ in order to deal with an appropriate physical model. But, as was shown in refs. \(\boxed{}\), it is not possible to describe the growth of entropy in an anisotropic model of the Kasner form while keeping the 2nd law of thermodynamics together with the DEC.

On the other hand, we require the model to satisfy the DEC. This becomes specified by the range $-\rho \leq P_j \leq P_j$, where $\rho$ is the energy density and $P_j$ (with $j = x, y, z$) are the effective momenta related to the corresponding coordinate axes. Note that both $\rho$ and $P_j$ scale as $t^{-2}$. Thus, the DEC will give some specific relations among the Kasner parameters $p_i$ ($i = 1, 2, 3$). In the following, we introduce the symbols $S \equiv p_1 + p_2 + p_3$ and $Q \equiv p_1^2 + p_2^2 + p_3^2$, following ref. \(\boxed{}\).

The conditions $P_j \leq \rho$, with $j = x, y, z$, yield three inequalities, which, by adding them, reduce to just one inequality given by

$$2S(S-1) \geq 0.$$  \hfill (6)

In a similar way, from $P_j \geq -\rho$ we get the inequality

$$S \geq \frac{AS^2}{2},$$  \hfill (7)

where $A = 3Q/S^2 - 1$ \(\boxed{}\).

From expression (6), we see that $S \geq 0$, since $A \geq 0$ \(\boxed{}\). With this condition on $S$, we obtain from expression (4) that necessarily $S$ should be greater than one. This conclusion yields to a negative shear viscosity, as can be seen from expression (3). Therefore, from this result, we observe that the entropy in this sort of universe decreases instead of increasing.

We should note here that the DEC, specified by the range for $P_i$ ($i = x, y, z$), $-\rho \leq P_i \leq \rho$, is equivalent to the range $-\rho \leq P_i \leq \rho$, where $\rho$ is the isotropic pressure. Effectively, for a fluid with energy density $\rho(t)$ and principal pressures $P_z(t)$, $P_y(t)$ and $P_x(t)$, the energy-momentum tensor becomes $T^\mu_\nu = \text{diag}(\rho, -P_z, -P_y, -P_x, -P_z)$ \(\boxed{}\). Thus, from eqs. (1) and (3), and using $u_\alpha = \delta^\alpha_\alpha$, we obtain

$$T^t_t = \rho,$$  \hfill (8)

$$T^i_i = -p_i = -\rho + 2\eta \sigma^i_i, \quad (i = x, y, z),$$  \hfill (9)

where there is no sum over $i$, and $\xi = 0$. After adding the inequalities $P_i \leq \rho$ ($i = x, y, z$), one obtains $p \leq \rho$. On the other hand, after adding $-\rho \leq P_z$ ($i = x, y, z$) we get $-\rho \leq \rho$. In both cases we have used the result $\sigma^1_1 + \sigma^2_2 + \sigma^3_3 = 0$. This result is easy generalized to the case where the bulk viscosity is taken into account. Here, we obtain $-\rho + \theta \xi \leq p \leq \rho + \theta \xi$.

III. SCALAR-TENSOR THEORIES AND THE FIELD EQUATIONS

We start with the Lagrangian density for the scalar tensor theory of gravity
\[ L = \sqrt{-g} \left[ \phi(R - 2\Lambda(\phi)) + \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] , \] (10)

where \( g = \text{det}(g_{\alpha\beta}) \). The arbitrary functions \( \omega = \omega(\phi) \) and \( \Lambda = \Lambda(\phi) \) distinguish the different scalar-tensor gravitational theories: \( \omega(\phi) \) is the coupling function and \( \Lambda(\phi) \) is a potential function and plays the role of a cosmological constant.

The Euler-Lagrange equations of motion for \( g_{\alpha\beta} \) and \( \phi \) obtained from the action \( S = \int (L + L_m) d^4x/16\pi \) are

\[ R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{8\pi}{\phi} T_{\alpha\beta} - \Lambda(\phi) g_{\alpha\beta} - \frac{\omega(\phi)}{\phi^2} \left[ \phi,_{\alpha}\phi,_{\beta} - \frac{1}{2} g_{\alpha\beta} \phi,_{\gamma}\phi,_{\gamma} \right] - \frac{1}{\phi} \left[ \phi,_{\alpha\beta} - g_{\alpha\beta} \phi,_{\gamma}\phi,_{\gamma} \right] , \] (11)

and

\[ \phi,_{\gamma} = \frac{2\phi^2 \delta^2 (\phi)}{2\omega(\phi) + 3} = \frac{1}{2\omega(\phi) + 3} \times \left[ 8\pi T_{\gamma\gamma} - \frac{d\omega(\phi)}{d\phi} \phi,_{\gamma}\phi,_{\gamma} \right] \] (12)

respectively, where the partial derivatives are denoted by a prime and covariant derivatives are denoted by a semicolon. \( T_{\alpha\beta} \) is the stress energy-tensor which be calculated from \( L_m \) through the definition \( T_{\alpha\beta} = \frac{\delta S}{\delta g_{\alpha\beta}} \). General relativity is recovered in the limit \( \omega \to -\infty \) and \( \phi = \text{const.} \equiv 1/G \). We can conveniently write (11) as

\[ R_{\alpha\beta} = \left[ \Lambda(\phi) - \frac{1}{2\phi} \phi,_{\gamma} \right] g_{\alpha\beta} - \frac{8\pi}{\phi} \left[ T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right] - \frac{1}{\phi} \left[ \phi,_{\alpha\beta} + \frac{\omega(\phi)}{\phi} \phi,_{\alpha}\phi,_{\beta} \right] . \] (13)

We now apply this theory to a homogeneous and anisotropic cosmology of the Kasner type, expressed by the metric (1), together with the energy-momentum tensor specified by eq. (3). The signature here is \((+---)\), and the Ricci tensor is given by \( R_{\alpha\beta} = \mathcal{R}_{\alpha\beta} \). The scalar expansion and the traceless shear tensor are

\[ \theta = S t^{-1} , \]

\[ \sigma_{00} = 0 , \quad \sigma_{ij} = t^{2p_i - 1} \left[ \frac{S}{3} - \frac{p_i}{t} \right] , \]

with

\[ \sigma^2 = \frac{1}{2t^2} \left[ Q - \frac{S^2}{3} \right] , \]

respectively. Then, eq. (12) reduces to the following field equations:

\[ \frac{\dot{\phi}}{\phi} + \frac{S}{t} = \frac{2\phi^2 \Lambda(\phi) - 2\phi \Lambda(\phi)}{2\omega(\phi) + 3} \]

\[ = \frac{8\pi}{2\omega(\phi) + 3} \left( \rho - \frac{S}{t} \xi \right) - \frac{\omega'}{2\omega(\phi) + 3} \phi'' . \] (14)

Eq. (13) gives rise to the following set of eqs.:

\[ - \frac{S - Q}{t^2} = \Lambda(\phi) \quad \frac{\dot{\phi}}{\phi} - \frac{1}{\phi} \left[ \frac{S}{t} \dot{\phi} + \frac{\omega(\phi)}{\phi} \phi'' \right] ^2 , \] (15)

and

\[ p_i \left[ 1 - S - \frac{16\pi}{\phi} \eta t - \frac{\phi}{\phi} \right] t^{-2} = -\Lambda(\phi) - \frac{4\pi}{\phi} \left[ \rho - \left( -\frac{S}{t} \xi \right) + \frac{4S}{3t} \right] + \frac{1}{2\phi} \left[ \phi + \frac{S}{t} \phi \right] , \] (16)

where the dots specify derivatives with respect to the cosmological time.

Since the \( p_i \) \((i = 1, 2, 3)\) are all linearly independent we get from eq. (16) that

\[ \eta = \frac{\phi}{16\pi t} \left( 1 - S - \frac{\phi}{\phi} t \right) > 0 . \] (17)

From the same eqn., we get a condition that, after substituting eq. (17) into this expression we get

\[ 8\pi \rho = \left( S^2 - Q \right) \frac{\phi}{2t^2} + \frac{S \dot{\phi}}{t} - \frac{\omega(\phi) \phi''}{2\phi} - \phi \Lambda(\phi) \] (19)

and

\[ 8\pi p = \left( S - Q \right) \frac{\phi}{2t^2} - \frac{\phi}{3t} + \frac{S(1 - S)}{6t^2} \phi - \frac{\omega(\phi) \phi''}{2\phi} + \phi \Lambda(\phi) + 8\pi \frac{S}{t} \xi . \] (20)

These two expressions can be substituted into the equation of motion for the scalar field \( \phi \), eq. (14), and obtain

\[ \frac{\dot{\phi}}{\phi} + \frac{S}{t} \phi + \frac{2\phi^2 \Lambda(\phi) - 2\phi \Lambda(\phi)}{2\omega(\phi) + 3} = \frac{1}{(2\omega(\phi) + 3)} \times \left[ \left( \frac{S^2 + Q - 2S}{t^2} \right) \phi + \frac{3S}{t} \phi + \frac{\omega(\phi)}{\phi} \phi'' - 4\Lambda(\phi) \right] \] (21)

\[ - \frac{\omega'}{2\omega(\phi) + 3} \phi'' . \]

We should note that, when \( \phi = \text{const.} = 1/G \) and \( \Lambda(\phi) = 0 \), this set of equations coincides with that considered by the authors of ref [8]. In the following we shall consider some particular cases.
IV. SOME SPECIFIC MODELS

The set of equations described above can be solved for the particular situation $\omega(\phi) = \omega_0$ and $\Lambda(\phi) = \Lambda_0\phi^{-\frac{4}{n}}$, where $\omega_0$, $\Lambda_0$ and $n$ are arbitrary real constants. This choice allows us to write the following particular solution for the scalar field: $\phi = \phi_0 \left( \frac{t}{t_0} \right)^{2/n}$ as can see from (21).

Here $\phi_0$ and $t_0$ are two constants that would be associated with the present values of $\phi$ and $t$, respectively. Replacing these expressions into eq. (21) we obtain the following expression for the $\Lambda_0$ parameter:

$$\Lambda_0 = \frac{n \phi_0^{2/n}}{2n + 4} \left[ n \omega_0 (2 - 2S - n) - 2S + S^2 + Q \right]. \quad (22)$$

These conditions give for $\rho$, $p$ and $\eta$ the following expressions:

$$\rho = \frac{\phi_0 t^{n-2}}{8\pi(2-n)} \left[ S^2 - Q + nQ - n^2S + nS - n^2\omega_0S \right], \quad (23)$$

$$p = \frac{\phi_0 t^{n-2}}{24\pi(2-n)} \left[ -3Sn - nS^2 + 4S - S^2 - 3Q + 3n^3 - 9n^2 + 2Sn^2 + 6n + 3\omega_0n^3 + 3\omega_0n^2S - 6\omega_0n^2 \right] + \frac{S}{t} \xi, \quad (24)$$

and

$$\eta = \frac{\phi_0}{16\pi} \left[ 1 - S - n \right] t^{n-1}. \quad (25)$$

We should note that the set of values $(\omega_0, n, \Lambda_0)$ will define interesting cases, which we consider in the following.

A. Einstein Kasner type universe

Einstein theory is obtained for $n = 0$, $\omega_0 \rightarrow \infty$ and $\phi_0 = \frac{1}{2}$. In this case the solutions take the form

$$\rho = \frac{t^{-2}}{16\pi G} \left[ S^2 - Q \right], \quad (26)$$

$$p = \frac{t^{-2}}{48\pi G} \left[ 4S - S^2 - 3Q \right] + \frac{S}{t} \xi, \quad (27)$$

and

$$\eta = \frac{t^{-1}}{16\pi G} \left[ 1 - S \right]. \quad (28)$$

The corresponding particular cases are studied in detail in ref. [9]. We have shown above that the shear coefficient of viscosity is always negative. As we saw earlier, the DEC imply that $-\rho + \theta \xi \leq p \leq \rho + \theta \xi$. Then, from (24), (27) and (28) for $p \leq \rho + \theta \xi$ we have that $-S^2 \leq 0$ and from $-\rho + \theta \xi \leq p$ we obtain that $-S^2 - 2S + 3Q \leq 0$. Thus, from these expressions it is easy to see that one always gets that $\eta \leq 0$ and $\rho \geq 0$.

B. Brans-Dicke in a Kasner-type universe

In this subsection we consider the Brans-Dicke theory, i.e. we take $\Lambda_0 = 0$ with $n$ and $\omega_0$ for the moment arbitrary parameters. Thus, from (22), we have that the shear viscosity is still given by (23) and the energy density and pressure are given by

$$\kappa \rho = \frac{\phi_0 t^{n-2}}{2} \left[ S^2 + 2Sn - Q - \omega_0 n^2 \right] \quad (29)$$

and

$$\kappa \rho = \frac{\phi_0 t^{n-2}}{6} \left[ 4S - S^2 - 3Q - n(6n - 6 + 4S) - 3\omega_0 n^2 \right], \quad (30)$$

respectively.

From eqs. (24) and (30) we see that a barotropic state equation is satisfied ($p = \gamma \rho$), and then

$$\gamma = \frac{4S - S^2 - 3Q - n(6n - 6 + 4S) - 3\omega_0 n^2}{3(S^2 + 2Sn - Q - \omega_0 n^2)}, \quad (31)$$

and the constraint for the constants is

$$2n\omega_0 (1 - S - n/2) = 2S - S^2 - Q, \quad (32)$$

which is obtained from eq. (24). This solution must satisfy the second law of thermodynamics ($\eta \geq 0$) and the DEC ($\rho \geq 0$ and $-1 \leq \gamma \leq 1$), i.e. there are four conditions to be satisfied. It is easy to show that the three inequalities of the DEC are linearly dependent. Then we get from $\rho \geq 0$

$$S^2 + 2Sn - Q - \omega_0 n^2 \geq 0 \quad (33)$$

and from $p \leq \rho$

$$4S^2 - 4S + 6n^2 - 6n + 10nS \geq 0. \quad (34)$$

Now, from the conditions $\eta \geq 0$ and (34) we obtain

$$S(1 + n) \geq Q + \omega_0 n^2. \quad (35)$$

Thus, a physically consistent model must satisfy the conditions (32), (34), and (35).

It can be shown that the condition (32) is compatible with the inequality (33), but not with the condition (34). Therefore, in this particular model based on Brans-Dicke theory, there are no physically reasonable solutions for a viscous Kasner type model. This is similar to what is found in Einstein’s theory.

C. Scalar-tensor Kasner type universe

In the following we do not consider the bulk viscosity, since as is well known, the shear viscosity at early time is much greater than the bulk viscosity [11]; therefore, we could take $\xi = 0$. In this case we can write a barotropic
equation of state $p = \gamma \rho$, where, in agreement with the DEC, $\gamma$ lies in the interval $-1 \leq \gamma \leq 1$.

It is easy to check that in this case there is a family of solutions which satisfy the second law of thermodynamics together with the DEC, simultaneously. For instance, we have found the following particular solutions. When the parameters $\omega$ and $n$ take the values $\omega_0 = 0.1$ and $n = -0.1$, it is found that the parameters $p_1 = 0.1$, $p_2 = 0.12$ and $p_3 = 0.8$ represent a solution of the field equations. Here, all the relevant quantities, such that the effective pressure $p$, the shear viscosity $\eta$, the energy density $\rho$, etc. all these quantities are physically acceptable. Thus, there are particular situations in scalar-tensor theories of gravity where are not found the problems appearing in Einstein theory.

Note that if $\eta = 0$, i.e. $n = 1 - S$, one has the solution
\[
\rho = S \frac{\phi_0 t^{-3(1+S)}}{8 \pi (1+S)} \times (2 \omega_0 S - \omega_0 - \omega_0 S^2 + 2 S - S^2 - Q),
\]
from which we get the barotropic equation of state, $p = \frac{1}{3} \rho$, with $-1 \leq 1/S \leq 1$. A dust dominated universe is obtained when $S \rightarrow \infty$. If $S = 1$ ($n = 0$), then the model becomes stiff matter dominated. In the latter case we obtain
\[
\rho = p = \frac{\phi_0}{16 \pi t^2} (1 - Q),
\]
which coincides with equation (21) of ref. [8].

V. GENERATION OF ENTROPY

In the following we shall determine the generation of entropy in a scalar-tensor theory. Here we include the shear coefficient of viscosity only, since $\xi \ll \eta$.

It is well known that the production of entropy could be related to the anisotropy of the universe [12]. In order to see this we introduce the entropy current four-vector $S^\mu$ as
\[
S^\mu = n_b k_b \sigma u^\mu,
\]
where, as before, $u^\mu$ represents the four-velocity, $n_b$ the baryon number density, $k_b$ is the Boltzmann constant and $\sigma$ the nondimensional entropy per baryon. Since
\[
S^\mu_{\nu} = \frac{2}{T} \eta \sigma^{\mu \nu} \sigma_{\mu \nu},
\]
($\xi = 0$), it is obtained that [12]
\[
\dot{\sigma} = \frac{2 \sigma^2}{n_b k_B T t^2} \eta A,
\]
where $A = \frac{3Q}{S^2} - 1$.

We note that from (25) one has $\eta t^{1-n} = \text{const}$. This fact permits us to write all the quantities if we know the value of $\eta$ at a given moment of time. We use here the values given in ref [3] for some parameters at $t = 1000$ s (the universe is characterized by ionized H and He in approximate equilibrium with radiation). At that time, the number densities of electrons and protons, $n_{e1000} = n_{p1000} \approx 10^{10} cm^{-3}$, the temperature $T_{1000} \approx 4 \times 10^8 K$ and $n_{1000} \approx 2.8 \times 10^{14} g cm^{-1}s^{-1}$. Then we get at any time
\[
\eta(t) = n_{1000} \left( \frac{1000}{t} \right)^{1-n}.
\]
Note that from (41) we have $\frac{n_b T t^2 \dot{\sigma}}{\eta} = \text{const}$. Thus the generation of entropy at any time can be written as
\[
\dot{\sigma}(t) = \dot{\sigma}_{1000} \frac{n_{e1000} T_{1000}}{n_{e}(t) T(t)} \left( \frac{1000}{t} \right)^{3-n}.
\]

In order to determine how the generation of entropy decreases as time goes on, we determine this generation of entropy for two different instants in the evolution of the universe. We consider the plasma era (radiation era), where the temperature ranges $10^{10} K \geq T \geq 4000 K$ [11]. For the recombination of hydrogen we have $t_{rec} \approx 0.126 \times 10^{14}$, $n_{e rec} = n_{p rec} \approx 4 \times 10^{13} cm^{-3}$, the temperature $T_{rec} \approx 4000 K$. Then, for the ratio between values of generation of entropy at time of recombination $t_{rec}$ and the time $t = 1000$ we get for the model studied above ($n = -0.1$), that $\frac{\dot{\sigma}_0}{\dot{\sigma}_{1000}} \approx 10^{-11}$, which implies a drastic reduction in the rate of entropy production during the plasma era.

VI. CONCLUSIONS

We have studied the scalar-tensor theories for Kasner type metrics in the presence of a viscous fluid. We have explored the possibility of describing reasonable physical models for a Kasner type universe when we keep the thermodynamics ($\eta \geq 0$ and $\xi \geq 0$) together with the dominant energy condition in theories more general than Einstein’s gravity. We concluded, for the specific model treated here, that in the Brans-Dicke theory, like in the Einstein theory, it is not possible to describe the growth of entropy.

In the case of the scalar-tensor theory we have found, for our particular choice of the scalar field $\phi$, the coupling function $\omega(\phi) = \omega_0$ and the potential function $\Lambda(\phi) = \Lambda_0 \phi^{-2/n}$, that it is possible to keep the thermodynamic conditions ($\eta \geq 0$ and $\xi \geq 0$) together with the DEC, as was described in Sect.IV.C. For these models we have found that, during the plasma era, i.e. from $T \sim 10^{10}$ K to $T_{rec} \sim 4.000$ K, a drastic reduction in the rate of entropy occurred.
VII. ACKNOWLEDGEMENTS

We thank Paul Minning for carefully reading the manuscript. This work was supported by COMISION NACIONAL DE CIENCIAS Y TECNOLOGIA through Grants FONDECYT N° 1990601 (MC and PS) and N° 1000305 (SdC). Also MC was supported by Dirección de Promoción y Desarrollo de la Universidad del Bio-Bío, SdC was supported from UCV-DGIP 123.752/2000 and PS was supported in part by Dirección de Investigación de la Universidad de Concepción through Grant N 98.011.023-1.0.

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