SENSING STRONG INTERACTION EFFECTS IN SPECTROSCOPY OF HADRONIC ATOMS

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Abstract. The theoretical studying the strong interaction shifts and widths from X-ray spectroscopy of kaonic atoms is fulfilled. Sensing the strong interaction effects and theoretical estimating spectra of kaonic atomic systems can be considered as a new tool for studying nuclear structure and strong K-nucleus interaction.

Keywords: strong interaction effects, spectroscopy, kaonic atoms
of intensive theoretical and experimental interest (e.g., radiative corrections, first-order vacuum polarization effects, kaon-nuclear strong interaction effects as QED problems connected with accurate accounting for nuclear structure and strong kaon-nucleus interaction). The standard way is based on solution of the Klein-Gordon equation. The relativistic dynamic of a spinless particle can be described by the Klein-Gordon equation. The electromagnetic interaction between a negatively charged spin-0 particle with a charge equal to $q = -e$ and the nucleus can be taken into account introducing the nuclear potential $A_n$ in the KG equation via the minimal coupling $p_{\nu} \rightarrow p_{\nu} - qA_n$. The wave functions of the zeroth approximation for kaonic atoms are found from the Klein-Gordon equation [5]:

$$m^2c^2\Psi(x) = \frac{1}{c^2} [i\hbar\partial_t + eV_0(r)]^2 + h^2\nabla^2 \Psi(x)$$

(1)

where $h$ is the Planck constant, $c$ the velocity of the light and the scalar wavefunction $\Psi(x)$ depends on the space-time coordinate $x = (ct, r)$. Here it is considered a case of a central Coulomb potential $V_0(r)$.

2. New quantum Klein-Gordon equation approach in the kaonic atoms theory

Let us describe the key moments of our new approach to quantum calculation of the spectra for multi-electron kaonic (pionic) atoms with an account of nuclear and radiative effects (more details applying to the multi-electron heavy atoms can be found in refs. [16-23]). It is well known that the relativistic dynamic of a spinless particle can be described by the Klein-Gordon equation. The electromagnetic interaction between a negatively charged spin-0 particle with a charge equal to $q = -e$ and the nucleus can be taken into account introducing the nuclear potential $A_n$ in the KG equation via the minimal coupling $p_{\nu} \rightarrow p_{\nu} - qA_n$. The wave functions of the zeroth approximation for kaonic atoms are found from the Klein-Gordon equation [5]:

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A. Usually, we consider the stationary solution of Eq. (1). In this case, we can write:

$$\Psi(x) = \exp(-iEt/\hbar)\phi(x)$$

(2)

and Eq. (1) becomes:

$$\frac{1}{c^2} [E + eV_0(r)]^2 + h^2\nabla^2 - m^2c^2\phi(x) = 0$$

(3)

where $E$ is the total energy of the system (sum of the mass energy $mc^2$ and binding energy $e_0$). In prin-
principle, the central potential $V_0$ should include the central Coulomb potential, the vacuum-polarization potential as well as the kaon-nucleus strong interaction potential (optical model potential). Earlier we have calculated some characteristics of hydrogen-like and other multi-electron ions with using the nuclear charge distribution in the form of a uniformly charged sphere and Gaussian form (c.f. [19–21]). The advantage of the Gaussian form nuclear charge distribution is provided by using the smooth function instead of the discontinuous one as in the model of a uniformly charged sphere [22]. It is obvious that it simplifies the calculation procedure and permits to perform a flexible simulation of the real distribution of the charge in a nucleus. In last years to define the nuclear potential it is usually used the Fermi model for the charge distribution in the nucleus $\rho(r)$ (c.f.[21]):

$$c(r) = c_0 / [1 + \exp((r-c)/a)]$$

(4)

where the parameter $a=0.523$ fm, the parameter $c$ is chosen by such a way that it is true the following condition for average-squared radius: $<r^2>1/2 = (0.836/A^{1/3}+0.5700)$fm. Further let us present the formulas for the finite size nuclear potential and its derivatives on the nuclear radius. If the point-like nucleus has the central potential $W(R)$, then a transition to the finite size nuclear potential is realized by exchanging $W(r)$ by the potential [19]:

$$W(r|R) = W(r) dr r^2 \rho(r|R) +$$

$$+ \int_{r}^{\infty} dr r^2 W(r) \rho(r|R).$$

(5)

We assume it as some zeroth approximation. Further the derivatives of various characteristics on $R$ are calculated. They describe the interaction of the nucleus with outer electron; this permits calculation of results, when $R$ varies within reasonable limits. The Coulomb potential for the spherically symmetric density $\rho(r|R)$ is:

$$V_{nucl}(r|R) = -\left(1/r\right) \int_{0}^{\infty} dr r^2 \rho(r|R) +$$

$$+ \int_{r}^{\infty} dr r^2 \rho(r|R).$$

(6)

It is determined by the following system of differential equations [19]:

$$V_{nucl}(r,R) = \left(1/r^2\right) \int_{0}^{\infty} dr r^2 \rho(r|R) = \left(1/r^2\right) y(r,R)$$

$$y'(r,R) = r^2 \rho(r,R)$$

(7)

$$c'(r) = (c_0 / a) \exp((r-c)/a) [1 + \exp((r-c)/a)]^{2}$$

with the boundary conditions:

$$V_{nucl}(0,R) = -4/(\pi r)$$

$$y(0,R) = 0,$$

(8)

$$c(0) = c_0 / [1 + \exp(-c/a)]$$

The new important topic is connected with a correct accounting the radiation QED corrections and, first of all, the vacuum polarization correction. Procedure for an account of the radiative QED corrections in a theory of the multi-electron atoms is given in detail in refs. [17–22]. Regarding the vacuum polarization effect let us note that this effect is usually taken into account in the first PT order by means of the Uehling potential:

$$U(r) = -2a \alpha \mu Z \exp(-2rt/\alpha Z) \times$$

$$\times (1+1/2\pi) \sqrt{\frac{r^2-1}{\pi r^2}} = -\frac{2a}{3\pi r} \tilde{C}(g),$$

(9)

where $g = \frac{r}{\alpha Z}$. In our calculation we usually use more exact approach. The Uehling potential, determined as a quadrature (9), may be approximated with high precision by a simple analytical function. The use of new approximation of the Uehling potential [21] permits one to decrease the calculation errors for this term down to $0.5-1\%$. Besides, using such a simple analytical function form for approximating the Uehling potential allows its easy inclusion into the general system of differential equations.

As it is well known, the nuclear absorption is defined by the strength of the strong interaction and overlapping the kaonic atomic wave function with the nuclear ones. The widespread approach to treating the strong interaction between the nucleus and orbiting kaon is in using the phenomenological optical potential of the following form [1,5,10]:

$$V_N = -\frac{2\pi}{\mu} [1 + \frac{M_k}{M_N}] [A_{kp}\rho_p(r) + A_{kn}\rho_n(r)],$$

(10)

where $\mu$ is the kaon-nucleus reduced mass, $M_k$ and $M_N$ are the kaon and nucleon masses, $\rho_p(r), \rho_n(r)$ are the proton and neutron densities in the nucleus and $A_{kp}, A_{kn}$ are the corresponding complex effective $Kp$ and $Kn$ scattering lengths. It si well known the Batty simplifying assumption of the following kind [4]:

$$\rho_p(r) = \rho_n(r) = \rho(r).$$
\[ V_N = \frac{2\pi}{\mu} [1 + \frac{M_K}{M_N}] [\alpha_p(r)] , \]

where \( \alpha \) is the effective averaged K-nucleon scattering length. Batty et al. had analyzed the previous kaon data and found the acceptable value for the length as follows [4]:

\[ a = [(0.34 \pm 0.03) + i(0.84 \pm 0.03)] \text{ (fm)}. \]

The presented value of the length \( a \) has been indeed chosen to describe the low and middle Z nuclei [4]. The disadvantage of the usually used approach is connected with approximate definition of the proton and neutron densities and using the effective averaged K-nucleon scattering length. More correct approach is in the relativistic mean-field (RMF) model for the ground-state calculation of the nucleus. Though we have no guaranty that these wave-functions yield a close approximation to nature, the success of the RMF approach supports our choice [24]. These wave functions do not suffer from known deficiencies of other approaches, e.g., the wrong asymptotics of wave functions obtained in a harmonic oscillator potential. The RMF model has been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei [24]. The realization of nonlinear self-interactions of the scalar meson led to a quantitative description of nuclear ground states. As a self-consistent mean-field model (for a comprehensive review see ref. [22-24]), its ansatz is a Lagrangian or Hamiltonian that incorporates the effective, in-medium nucleon-nucleon interaction. Recently [22] the self-consistent models have undergone a reinterpretation, which explains their quantitative success in view of the facts that nucleons are composite objects and that the mesons employed in RMF have only a loose correspondence to the physical meson spectrum. They are seen as covariant Kohn-Sham schemes and as approximations to the true functional of the nuclear ground state. As a Kohn-Sham scheme, the RMF model can incorporate certain ground-state correlations and yields a ground-state description beyond the literal mean-field picture. RMF models are effective field theories for nuclei below an energy scale of 1 GeV, separating the long- and intermediate-range nuclear physics from short-distance physics, involving, i.e., short-range correlations, nucleon form factors, vacuum polarization etc, which is absorbed into the various terms and coupling constants. As it is indicated in refs. [24] the strong attractive scalar (\( S: -400 \text{ MeV} \)) and repulsive vector (\( V: +350 \text{ MeV} \)) fields provide both the binding mechanism (\( S + V: -50 \text{ MeV} \)) and the strong spin-orbit force (\( S - V: -750 \text{ MeV} \)) of both right sign and magnitude. In our calculation we have used so called NL3-NLC (see details in refs. [24]), which is among the most successful parameterizations available.

### 3. Results and conclusions

In ref. [5] we have presented some calculations for a selection of kaonic atom transitions. Such calculations are obtained solving numerically the Klein-Gordon equation using the effective Dirac Superatom code developed by Ivanov et al [16-21] that has been modified to include spin-0 particles case, even in the presence of electrons [1]. The kaon mass was assumed to be 493.677 \pm 0.013 \text{ MeV} [11].

In table 1 we present the calculated electromagnetic (EM) X-ray energies of kaonic atoms for transitions between circular levels. The transitions are identified by the initial (\( n_i \)) and final (\( n_f \)) quantum numbers. The calculated values of transition energies are compared with available measured (\( E_m \)) and other calculated (\( E_c \)) values [1-7]. In a case of the close agreement between theoretical and experimental data, the corresponding levels are less sensitive to strong nuclear interaction. In the opposite case one could point to a strong-interaction effect in the exception cited above.

| Nucl. | Transition | \( E_{c,our \ theor} \) | \( E_{c, [4]} \) | \( E_{c, [6]} \) | \( E_{c, [7]} \) | \( E_m \) |
|-------|------------|----------------|----------------|----------------|----------------|---------|
| W     | 8-7        | 346.572        | 346.54         | -              | -              | 346.624(25) |
| W     | 7-6        | 535.136        | 535.24         | -              | -              | 534.866(92) |
| Pb    | 8-7        | 426.174        | 426.15         | -              | -              | 426.221(57) |
| U     | 8-7        | 538.528        | 538.72         | 538.013        | 537.44         | 538.315(100) |

In table 2 we present the calculated (\( C \)) and measured (\( M \)) strong interaction shifts \( \Delta E \) and widths \( G \) (in keV) for the kaonic atoms X-ray transitions. The subscripts M and C stands for measured and calculated values correspondingly. The width \( G \) is the strong width of the lower level which was obtained by
subtracting the electromagnetic widths of the upper and lower level from the measured value. The shift \( \Delta E \) is defined as difference between the measured \( E_M \) and calculated \( E_{EM} \) (electromagnetic) values of transition energies; the calculated value is obtained by direct solving the equation (1) with kaon-nuclear potential. Besides, the measured values by Miller et al and Cheng et al (from refs. [1,4]) are listed in table 2 too. It should be noted that Cheng et al did not make any energy calibration above the 511 e\(^+\) annihilation and Batty et al [4] indicated that the corresponding difference between the energy values is not serious.

| Nucl | \( \Delta E_c \) (d) | \( G_c \) (d) | \( \Delta E_c \) (c) | \( G_c \) (c) | \( \Delta E_{EM} \) | \( G_{EM} \) |
|------|----------------|-------------|----------------|-------------|-----------------|-------------|
| W, 8-7 | 0.038 | 0.072 | -0.003 | 0.065 | 0.079 | 0.052 | 0.070 (15) |
| W, 7-6 | -0.294 | 3.85 | -0.967 | 4.187 | -0.353 | -0.250 | 3.72 (35) |
| Pb, 8-7 | 0.035 | 0.281 | -0.023 | 0.271 | 0.072 | 0.047 | 0.284 (14) |
| Pb, 8-7 | 0.035 | 0.281 | -0.023 | 0.271 | 0.072 | 0.047 | 0.284 (14) |
| U, 8-7 | -0.205 | 2.620 | -0.189 | 2.531 | 0.120 | 0.032 | 1.50 (75)+ |

From the other side, more correct definition of proton and neutron densities is of a great importance for physically reasonable agreement between the measured and calculated (this work) shifts and widths. In whole we can conclude that the measured strong interaction parameters are reasonably well reproduced by present theory. To understand further information on the low-energy kaon-nuclear interaction, new experiments to determine the shift and width of kaonic atoms are now in preparation in J-Parc and in LNF, respectively (look, for example, refs. [1,8]).

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