Feasibility and Uncertainty Analysis of Constitutive Parameters Identification for Composite Materials Using Displacement Field Data

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Abstract. Due to the normalized average strain error is remarkably higher than the displacement error, a new cost-effective method based on the Levenberg-Marquardt method, which is a gradient-based optimization method, was presented in this paper. The unknown constitutive properties are determined through minimization of the square difference between the reconstructed DIC-measured and the FEM-calculated displacements. The feasibility analysis of this method is presented by synthetic displacement data. The covariance matrix of the extracted material constitutive parameters has been given explicitly. It has been proved that this method can significantly reduce the uncertainty of the extracted material constitutive parameter induced by the approximation error due to reconstruction algorithm and the measurement noise. The material constants show strong robustness to the measurement noise. And the optimization procedure is not sensitive to the formulation and parameters in the initial approximation of the constitutive model. A sensitive matrix was derivation and evaluation iterative in this method to improve the convergence of the optimization procedure.

1. Introduction

It is a typical inverse problem to obtain the constitutive relationship of composite materials by mechanical experiments. In solving this inverse problem, a lot of related research has been done and lots of important results have been achieved [1].

Grédiac and Pierron [2] proposed the virtual field method (VFM), and first carried out a series of theoretical and experimental studies. The most significant advantage of this method is that the explicit relationship between measured data and unknown material parameters can be established, which avoids a large number of numerical iterations and correction processes and greatly improves the computational efficiency[3]. However, the VEM method depends strongly on the initial assumptions of the constitutive parameters. If the initial assumptions deviate greatly, the correct recognition results may not be obtained.

The finite element model updating technique identifies multiple parameters in the constitutive relationship iteratively by minimizing the objective function, which is established by the variance between experimental results and numerical results. Molimard[4] presents a first step towards a global approach of mechanical identification for composite materials by open-hole tensile test using
Levenberg-Marquardt (L-M method) algorithm. Makeev [5] applies the non-standard short beam shear test combined with the full-field deformation data to identify the mechanical properties of composite firstly. Comparing with the V-notched shear test, the results validate the superiority and reliability of SBS test in obtaining the non-linear in-plane shear constitutive relation of materials. He [6] presents an uncertainly analysis about the measurement noise and reconstruction parameters on extracted material properties. It is evidenced that there resist an appropriate ROI and the number of images, from which reliable material parameters can be identified.

Various types of constitutive models have been used in the identification process, such as Johnson-Cook model[7-9], power-law model[10], Zerilli-Armstrong model[11]. In previous study, Ramberg-Osgood expression has been used to characterize nonlinear shear relationship[5,6].

Based on the above research results, this paper mainly studies the feasibility analysis of identification constitutive parameters by constructing the objective function with measurement and numerical displacement data. The robustness of the identification method is carried out from the aspects of the composition of the objective function, the identification method and the initial value of the identification process, respectively.

2. Experiment

In a short beam shear test, which following the ASTM D2344 standard [12], a customized specimen is loaded under three point bending at room temperature (figure 1 (a)). The specimen is 6.4mm thick and 6.4 mm width, the diameter of loading nose is 50.8mm. The support diameter is 3.2mm. The specimen length is 44.5mm and the support length is 30.5mm. The specimen were loaded at 1.27mm/min crosshead displacement rate in the transverse (1-3 plane) material direction. The material direction is shown in figure 1 (b).

![Figure 1. (a) SBS test setup combined with DIC measurement system; (b)Direction of composite.](image)

In order to quantify the magnification of random errors, a loading free test is performed, and then the reconstructed strain field is obtained by full-field finite element approximation reconstruction method[6,13]. The ratio of the average normalized errors of displacement data and strain data under various reconstruction parameters with three different random error levels is shown in figure 2. It is illustrated that the error level of strain field is much higher than that of displacement field. Therefore, it can be inferred that using displacement data instead of strain data to identify constitutive parameter can significantly reduce the impact of errors on identification results.

![Figure 2. Error amplification scale of normalized average error.](image)
3. Synthetic displacement data of SBS test
A 1/4 three-dimensional finite element model with symmetry boundary condition along width direction and length direction is established. The loading nose and supports are assumed as analytical rigid body. There are totally 58140 elements in the model. The element type is C3D8I. The model and the location of ROI is shown in figure 3. ROI is a 2mm width rectangular in the middle of loading nose and support. There are totally 3072 nodes on the surface of ROI. The maximum load is P=421N after 10 continuous loading steps. The constitutive parameters are provided by material suppliers. The random error with mean value of 0 and standard deviation of $\mu=2.79 \times 10^{-5}$mm are superimposed on the displacement data to generate the synthetic data.

![Figure 3. Finite element model and location of ROI.](image)

4. Finite element model updating
The flowchart in figure 4 shows the identification process. The objective function is formed by the variance of synthetic displacement and numerical displacement. The convergence criteria to stop the iterative procedure is the relative value of identified parameters updates is less than 0.5% or the variation of the objective function value is less than 1%. The process of calculation and formula derivation can be referred to previous study[6,14].

![Figure 4. Schematic flowchart for the material constant identification procedure](image)

5. Results
The parameters of the identification process are shown in table 1.

|                  | $E_T$(GPa) | $E_C$(GPa) | $G_{13}$(MPa) | $\nu_{13}$ |
|------------------|------------|------------|---------------|------------|
| Truth value      | 187.93     | 142.35     | 4368.79       | 0.36       |
| Initial approximation | 239.11 | 102.24     | 5240.62       | 0.27       |
| Convergence results | 187.6916 | 142.2857   | 4370.518      | 0.359      |
| Standard deviation | 0.104    | 0.0283     | 0.532         | 0.00068    |
| COV              | 0.13%      | 0.05%      | 0.04%         | 0.28%      |

![Figure 5(a) shows the variation of the objective function value with iterations. The value decreases after one iteration and convergence after 36 iterations. Figure 5(b) shows iteration results for the Young’s module in tension and compression direction, the shear module and Poisson’s ratio with the iteration number. The Young’s module in tension and compression direction and the shear module convergence after 12 iterations.](image)
6. Uncertainly analysis of the extracted constitutive parameters

6.1. Influence of the cost function

![Figure 5](image-url)

Figure 5. Variation of values with the iteration numbers (a) Objective function value; (b) Normalized identified results.

The identification results of displacement data and strain data are compared in Table 2. It can be seen that the standard deviation of the identification results based on reconstructed displacement data is much smaller than that of reconstructed strain data.

| Parameter | Identification Results | Standard Deviation | Relative Deviation |
|-----------|------------------------|--------------------|--------------------|
| Displacement | | | |
| Mean | 187.69 | 142.29 | 4371.24 | 0.36 |
| Standard deviation | 0.00104 | 0.000283 | 0.000532 | 0.0068 |
| Relative deviation | 0.13% | 0.04% | 0.06% | 0.28% |
| Strain | | | |
| Mean | 197.41 | 142.13 | 4273.01 | 0.34 |
| Standard deviation | 0.1026 | 0.0632 | 0.0897 | 0.082 |
| Relative deviation | 5.04% | 0.15% | 2.19% | 5.56% |

6.2. Influence of the identification method

In Table 3, the CPU time of L-M method, which is 36 iterations and 370 minutes, are compared with those of simplex search method without sensitivity matrix which is 44 iterations and 1800 minutes. The L-M method with sensitivity matrix saves 4/5 of the computation time since the introduction of sensitivity matrix greatly improves the optimization efficiency.

Although the iterative process costs 370 minutes, it has advantages than the analytical method such as the accuracy aspect[15,16]. Besides, it is required to update the properties in the identification process[17].

| Algorithm | Convergence results | Relative deviation | Iterations | CPU time (minutes) |
|-----------|---------------------|--------------------|------------|--------------------|
| Simple search method | | | | |
| Convergence results | 187.61 | 142.27 | 4371.07 | 0.362 |
| Relative deviation | 0.17% | 0.06% | 0.05% | 0.67% |
| L-M method | | | | |
| Convergence results | 187.69 | 142.29 | 4370.24 | 0.359 |
| Relative deviation | 0.12% | 0.04% | 0.03% | 0.27% |
6.3. Robustness to the initial approximation and random error

In order to analyze the sensitivity of the method to initial approximation, various initial parameters in ±30% range of the true values are used as the initial values to identify the constitutive parameters. The convergence results, standard deviation, relative deviation are shown in figure 6. It is shown that the identification results show strong robustness to the initial approximation. The relative deviation of Poisson's ratio is slightly larger, but also less than 10%.

Figure 7 shows the mean and standard deviation from synthetic displacement data with 1 to 10 times of Gauss distribution random errors. It can be seen from figure 7 that the mean values of $E_T$, $E_C$ and $G_{13}$ are less sensitive to the random error. Even if the random error is increased to 10 times, the mean values of the constitutive parameters obtained by inversion are basically consistent with the true values of the constitutive parameters, and the standard deviation is less than 0.5%.

Figure 6. Parameters of various initial values (a) Normalized mean value (b) Standard deviation

Figure 7. Identified results with various random error. (a) Mean value; (b) Standard deviation.

On the contrary, the mean value of identification results of Young's modulus, which identified by strain field data, is changing with the level of random noise due to the lower positive strain level involved in identification of Young's modulus. When the noise level increases to three times, it is impossible to identify the theoretical true value while the theoretical true value of the parameters can still be obtained when the noise level reaches 10 times. This shows that the identification with displacement as the objective function has strong robustness to random errors in the data, especially for the identification of constitutive parameters under low strain conditions.

On the other side, the standard deviation of identification results increases linearly with the increase of the level of random error. As for the Poisson's ratio, which is more difficult to invert, it can be seen from the figure 7 that the normalized mean of Poisson's ratio identification results fluctuates obviously, and the standard deviation from the real value is close to 10%, which is much larger than the standard deviation of the normalized mean of Young's modulus and shear modulus. This is because Poisson's ratio is determined by the ratio of displacement along the thickness direction to displacement along the fiber direction. Compared with other constitutive parameters, the identification uncertainty is higher, which results in a larger standard deviation between the inversion Poisson's ratio and the real value.

7. Conclusion
In this paper, the constitutive parameters of carbon fiber reinforcement composite (IM7/8552) unidirectional composite laminates are identified by using finite element model updating method combined with L-M method with synthetic data of random error of $\mu=2.79\times10^{-3}$mm superimposed in the 2 mm width area between loading nose and support point. Different from the previous identification process, the objective function is constructed by using the variance between the numerical displacement data of the finite element model and the reconstructed displacement data. In the identification process, the non-linear L-M optimization method is adopted, and the explicit expression of sensitivity matrix in the optimization process is given by numerical difference. Four elastic constants including material Poisson's ratio are identified by this method.

The study shows that this method has strong robustness to the random error and the initial approximation, can reduces the influence of errors on the identification process significantly comparing with the identification method which used the strain field data and the explicit expression of sensitivity matrix can greatly speeds up the convergence speed of the identification process. Since it is not necessary to obtain strain field data by differential calculation from displacement field data, directly constructing objective function from displacement data can save calculation cost and improve efficiency.

This identification method will be improved by induced user subroutine UMAT to FEM model established by ABAQUS to identify shear nonlinear stress-strain relationship in the future. As the identification method shows strong robustness to the random error and the initial approximation, it can solve the problem of constitutive parameters identification in low strain or high noise level experiments, such as ceramic matrix composites.

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