This paper studies the control problem of an uncertain nuclear refueling machine (RM) system in the presence of uncertainty, external disturbance, input backlash, and asymmetric output constraint. Taking into account the effects of external disturbance and input backlash, the RM system with fuel rod is modeled as a coupling partial differential equation-ordinary differential equation (PDE-ODE). Using the backstepping method, an adaptive neural network control scheme is designed to drive the RM and bridge to the desired positions and simultaneously reduce the vibration of the fuel rod. A novel asymmetric tangent-type barrier Lyapunov function (Tan-BLF) is constructed to restrict the position tracking error into the given range. The formulation of backlash nonlinearity is transformed into the expected input and nonlinear error. Then the combination of input backlash error and boundary disturbance is defined as a disturbance-like item. Applying the robust control strategy and adaptive technique, two auxiliary input signals are proposed to offset the impact of the disturbance-like item. The adaptive neural network (NN) is employed to compensate for the system uncertainty. The practical stability of the RM system with the proposed control is demonstrated via the theoretical Lyapunov analysis. Simulation verifies the performance of the designed control scheme.

**INDEX TERMS** Adaptive control, nuclear refueling machine, neural network control, input backlash, output constraint.

**I. INTRODUCTION**

Nuclear energy is widely applied in power generation to satisfy the growing energy requirement. The fuel rod is installed in the reactor core of the nuclear plant to produce the energy. RM transports the fuel rod to the given position along the bridge to complete the maintenance and installation tasks. However, both external disturbance and movement of RM would induce the vibration of the fuel rod. These undesired vibrations could result in fatigue damage to fuel rods and even lead to serious nuclear safety accidents. Therefore, the vibration and position controls of the RM with the fuel rod are fundamental to ensure the safe operation of the fuel assembly.

Many researchers have developed different techniques to achieve vibration reduction and position tracking for the RM with fuel rod. In [1], a theoretical method is proposed to analyze the vibration characteristics of the fuel rod subjected to axial flow. In [2], the authors provide a solution to compute the flow-induced vibration of the fuel rod by analyzing the operational modal. In [3], the simulation experiments are conducted to show the nonlinear vibration responses of the nuclear fuel assembly, where the system boundaries are clamped. Nevertheless, for convenience, the fuel rod is simplified as the finite-dimensional ODE in previous works, which only describes the finite system modes. These results would bring inaccurate system response, and the un-modeled system modes could destabilize the system. Mathematically, the fuel rod is a typical distributed parameter system governing PDE. Control design of the PDE system is more difficult than the classical ODE system [4], [5], [6], [7], [8]. In this paper, considering the coupling between the motion of RM and the vibration of the fuel rod, the nuclear RM system is modeled as a coupling PDE-ODE.
Numerous control methods have been employed to stabilize distributed parameter systems [9], [10], [11], [12], [13], [14], [15]. In [16], the authors propose a distributed control to track the given signal for a constrained flexible structural system. The mixed control scheme consisting of boundary control and distributed control is employed to stabilize the spacecraft in the presence of external disturbance and actuator fault in [17]. For a flexible gantry crane system with nonuniform tension, the authors design two boundary control laws to regulate the deformation of cable and simultaneously move the tip payload to the desired position in [18]. In [19], by installed boundary control, the elastic deformation of the rigid-flexible wing is suppressed. Among these control methods, distributed control and boundary control are the two chief control methods. From the view of practical engineering, compared with distributed control, boundary control possesses better economy and realizability since its implementation only installs sensors and actuators at the system boundary [20], [21], [22], [23]. Therefore, we design two boundary control laws to offset the transverse and lateral vibration displacements of the fuel rod and simultaneously drive RM and bridge to the desired positions.

In [24], the authors propose a PD control scheme to control the RM system without any constraints. To guarantee the efficient and safe operation of the nuclear plant, the position tracking errors of RM and bridge are generally restricted in the desired ranges. Hence, the proposed control scheme in [24] is not suitable for the RM system with asymmetric output constraint. In this paper, by embedding a Tan-BLF into the process of control design, the proposed control scheme can ensure that the asymmetric constraint of position tracking error is never violated. Nonlinear input backlash generally occurs in actuators. The unknown backlash nonlinearity would produce the input error and even affect the stability of the control system [25], [26], [27]. For designing control, the expression of backlash nonlinearity is decomposed into the desired input and the bounded input error. Additionally, the disturbance rejection should be considered in control design. The sum of input backlash error and boundary disturbance is defined as a disturbance-like item. Using the control system [25], [26], [27]. For designing control, the system structural parameters are listed as follows: $EI$, $l$, $d$ and $\rho$ are the bending stiffness, length, diameter, and mass per unit length of fuel rod respectively; $m_b$ and $m_r$ are the masses of bridge and RM respectively.

For convenience, note that $(\cdot)' = \frac{\partial (\cdot)}{\partial t}$, $(\cdot)'' = \frac{\partial^2 (\cdot)}{\partial t^2}$, $(\cdot)''' = \frac{\partial^3 (\cdot)}{\partial t^3}$ and $(\cdot)'''' = \frac{\partial^4 (\cdot)}{\partial t^4}$.

In order to obtain the dynamical equations of the nuclear RM system, the fuel rod with a large aspect ratio can be regarded as an Euler-Bernoulli beam. The system kinetic energy, potential energy, and virtual work are given by

$$K(t) = \frac{\rho}{2} \int_0^l \left[ \ddot{s}(x,t) + \ddot{y}(t) \right]^2 dx + \frac{m_b + m_r}{2} \ddot{y}^2(t)$$

$$+ \frac{\rho}{2} \int_0^l \left[ \ddot{\omega}(x,t) + \ddot{z}(t) \right]^2 dx + \frac{m_b}{2} \ddot{z}^2(t) \quad (1)$$

$$P(t) = \frac{EI}{2} \int_0^l \left[ \dddot{s}(x,t) \right]^2 dx + \frac{EI}{2} \int_0^l \left[ \dddot{w}(x,t) \right]^2 dx \quad (2)$$

$$\delta W(t) = -c \int_0^l \left[ \dddot{s}(x,t) + \dddot{y}(t) \right] \delta s(x,t) + c \dddot{y}(t) \delta y(t) dx$$

$$-c \int_0^l \left[ \dddot{\omega}(x,t) + \dddot{z}(t) \right] \delta w(x,t) + c \dddot{z}(t) \delta z(t) dx$$

$$+ \int_0^l F_3(x,t) \delta s(x,t) + y(t) dx$$

$$+ \int_0^l F_3(x,t) \delta w(x,t) + z(t) dx$$

$$+ u_r(t) \delta y(t) + u_b(t) \delta z(t)$$

$$+ d_3(t) \delta y(t) + d_3(t) \delta z(t) \quad (3)$$
Applying the Hamilton’s principle: $\int_{t_1}^{t_2} \delta[K(t) - P(t) + W(t)]dt = 0$ derives the system governing equations as

$$\rho\dddot{x}(t) + E\dddot{x}(t) + c\dot{x}(t) = f_1(t)$$
$$\rho\dddot{\theta}(t) + E\dddot{\theta}(t) + c\dot{\theta}(t) = f_2(t)$$

and the corresponding boundary conditions as

$$w(0, t) = s(0, t) = w'(0, t) = s'(0, t) = 0$$

$$w''(l, t) = s''(l, t) = w'''(l, t) = s'''(l, t) = 0$$

In this paper, the following nonlinear backlash models are investigated

$$u_y(t) = B(v_y(t)) = \begin{cases} \eta_y(v_y(t) - B_{ry}) & \text{if } \dot{v}_y(t) > 0 \\ \eta_y(v_y(t) - B_{ry}) & \text{if } \dot{v}_y(t) < 0 \\ u_y(t') & \text{otherwise} \end{cases}$$
$$u_z(t) = B(v_z(t)) = \begin{cases} \eta_z(v_z(t) - B_{rz}) & \text{if } \dot{v}_z(t) > 0 \\ \eta_z(v_z(t) - B_{rz}) & \text{if } \dot{v}_z(t) < 0 \\ u_z(t') & \text{otherwise} \end{cases}$$

where $v_y(t)$ and $v_z(t)$ are the expected control commands, $\eta_y, \eta_z > 0$ are the slope, $B_{ry}, B_{rz} > 0$ and $B_{ly}, B_{lz} < 0$ are the constant parameters, $u_y(t')$ and $u_z(t')$ imply that $u_y(t)$ and $u_z(t)$ have not changed.

To facilitate control design, (10) and (11) are transformed as

$$u_y(t) = \eta_y v_y(t) + d(v_y)$$
$$u_z(t) = \eta_z v_z(t) + d(v_z)$$

where $d(v_y)$ and $d(v_z)$ denote the nonlinear backlash errors and can be expressed as

$$d(v_y) = \begin{cases} -\eta_y B_{ry} & \text{if } \dot{v}_y(t) > 0 \\ -\eta_y B_{ry} & \text{if } \dot{v}_y(t) < 0 \\ u_y(t') - \eta_y v_y(t') & \text{otherwise} \end{cases}$$
$$d(v_z) = \begin{cases} -\eta_z B_{rz} & \text{if } \dot{v}_z(t) > 0 \\ -\eta_z B_{rz} & \text{if } \dot{v}_z(t) < 0 \\ u_z(t') - \eta_z v_z(t') & \text{otherwise} \end{cases}$$

To proceed, we employ the following lemmas and assumption.

**Lemma 1** [28]: Let $y(x, t) : [0, l] \times \mathbb{R}^+ \rightarrow \mathbb{R}$ be a continuously differentiable function, we have

$$y^2(x, t) \leq y^2(0, t) + l \int_0^l [y'(x, t)]^2 dx, \quad \forall t \in \mathbb{R}^+$$

**Lemma 2** [28]: For any $|\xi| < 1$, the following inequality holds

$$\tan(\pi \xi^2) \leq \pi \xi^2 \sec^2(\pi \xi^2)$$

**Assumption 1**: In this paper, we suppose that all external disturbances $F_1(x, t), F_2(x, t), d_1(t), d_2(t)$ are bounded, i.e., there exist four constants $\bar{F}_1, \bar{F}_2, \bar{d}_1, \bar{d}_2 > 0$ such that $|F_1(x, t)| < \bar{F}_1, |F_2(x, t)| < \bar{F}_2, |d_1(t)| < \bar{d}_1$, $|d_2(t)| < \bar{d}_2$, $\forall (x, t) \in [0, l] \times [0, +\infty)$.

**Remark 1**: To handle the effects of unknown backlash and external disturbance, we define two disturbance-like terms as

$$d_1(t) = d(v_y) + d(v_z)$$
$$d_2(t) = d(v_y) + d(v_z)$$

According to (14) and (15), it is obviously observed that $|d(v_y)| \leq D_y = \max(\eta_y B_{ry}, -\eta_y B_{ry})$ and $d(v_z) \leq D_z = \max(\eta_z B_{rz}, -\eta_z B_{rz})$. Thus, the disturbance-like terms $d_1(t)$ and $d_2(t)$ are also bounded, and $|d_1(t)| \leq \bar{d}_1 = \bar{d}_y + D_y$ and $|d_2(t)| \leq \bar{d}_2 = \bar{d}_z + D_z$.

From (12), (13) and (16), the boundary conditions (6) and (7) can be rewritten as

$$m_y \dddot{y}(t) + E\dddot{y}(t) - d_1(t) = \eta_y v_y(t)$$
$$m_z \dddot{z}(t) + E\dddot{z}(t) - d_2(t) = \eta_z v_z(t)$$

**III. CONTROL DESIGN**

In this part, two adaptive neural network boundary control laws are designed to achieve the following targets:

1. suppress the transverse and lateral vibrations of the fuel rod and simultaneously drive RM and bridge to the desired positions;
2. regulate the position tracking errors into the given ranges;
3. offset the impacts of uncertainty, external disturbance, and input backlash.

For the desired positions $y_d$ and $z_d$, we define the tracking errors as $e_y(t) = y(t) - y_d$ and $e_z(t) = z(t) - z_d$. In order to use the backstepping method to design control, the following transformation is developed

$$\begin{cases} y_1 = x_1 \equiv e_y(t) \\ y_2 = x_2 - \tau_y = \dot{y}(t) - \tau_y \\ z_1 = x_3 \equiv e_z(t) \\ z_2 = x_4 - \tau_z = \dot{z}(t) - \tau_z \end{cases}$$

where $\tau_y$ and $\tau_z$ are virtual controls.

**Step 1**: To avoid the violation of asymmetric constraint, construct an asymmetric Tan-BLF as

$$V_1(t) = \frac{\alpha}{2} y_1^2 + \frac{\alpha}{2} y_2^2 \tan(\pi y_1^2 / 2S_1^2) + \frac{\alpha}{2} z_1^2 + \frac{\alpha}{2} z_2^2 \tan(\pi z_1^2 / 2S_2^2)$$

where $\alpha > 0$, and $S_1$ and $S_2$ are defined as

$$S_1 = \text{sign}(y_1) a_1 + (1 - \text{sign}(y_1)) a_2$$
$$S_2 = \text{sign}(z_1) a_3 + (1 - \text{sign}(z_1)) a_4$$
with $\text{sign}(\cdot) = 1$, if $\cdot > 0$, otherwise $\text{sign}(\cdot) = 0$. $a_i(i = 1, \ldots, 4) > 0$ are the limited values of tracking error.

Differentiating $V_1(t)$ leads to

$$
\dot{V}_1(t) = \alpha y_1(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2}))(y_2 + \tau_y) + \alpha z_1(1 + \sec^2(\frac{\pi z_1^2}{2S_2^2}))(z_2 + \tau_z)
$$

(22)

From the backstepping method, we chose the virtual control as

$$
\begin{align*}
\tau_y &= -k_1 y_1 - \frac{s''(0, t)}{1 + \sec^2(\frac{\pi y_1^2}{2S_1^2})} \\
\tau_z &= -k_2 z_1 - \frac{w''(0, t)}{1 + \sec^2(\frac{\pi z_1^2}{2S_2^2})}
\end{align*}
$$

(23)

where $k_1$ and $k_2$ are positive control gains.

Substituting (23) into (22) yields

$$
\begin{align*}
\dot{V}_1(t) &= -\alpha k_1 y_1^2 \sec^2(\frac{\pi y_1^2}{2S_1^2}) - \alpha k_2 z_1 \sec^2(\frac{\pi z_1^2}{2S_2^2}) - \alpha k_1 y_1^2 - \alpha k_2 z_1 - \alpha y_1 s''(0, t) - \alpha z_1 w''(0, t) - \alpha k_1 y_1 y_2(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2})) \\
&= -\alpha k_1 y_1 y_2(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2}))
\end{align*}
$$

(24)

The first four terms of (24) are negative definite, the other terms will be handled in the next step.

**Step 2**: To stabilize the unmanned nuclear RM system, the lyapunov function is adjusted as

$$
V_2(t) = V_1(t) + \frac{a(m_r + m_s)}{2} y_2^2 + \frac{a m_s}{2} z_2^2
$$

(25)

The derivative of $V_2(t)$ is computed as

$$
\dot{V}_2(t) = \dot{V}_1(t) + \alpha(m_r + m_s) y_2 \dot{z}_2 + a m_s \dot{z}_2
$$

(26)

Substituting (17) and (18) into the above equation leads to

$$
\begin{align*}
\dot{V}_2(t) &= -\alpha k_1 y_1^2 \sec^2(\frac{\pi y_1^2}{2S_1^2}) - \alpha k_2 z_1 \sec^2(\frac{\pi z_1^2}{2S_2^2}) - \alpha k_1 y_1^2 - \alpha k_2 z_1 - \alpha y_1 s''(0, t) - \alpha z_1 w''(0, t) + (m_r + m_s) \dot{y}_1 + y_1(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2})) \\
&- \alpha k_1 y_1 y_2(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2})) - \alpha k_2 z_1 z_2(1 + \sec^2(\frac{\pi z_1^2}{2S_2^2})))
\end{align*}
$$

(27)

From the backstepping method, we propose the expected boundary control laws as

$$
\begin{align*}
v_y(t) &= \frac{1}{\eta_y}[E s''(0, t) - (m_b + m_r) \dot{y}_1 - d_1(t)] \\
&- y_1(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2})) - k_3 y_2 \\
&- k_4 y_2(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2}))
\end{align*}
$$

(28)

$$
\begin{align*}
v_z(t) &= \frac{1}{\eta_z}[E w''(0, t) - z_1(1 + \sec^2(\frac{\pi z_1^2}{2S_2^2}))) \\
&- m_r \dot{z}_2 - k_5 z_2 - k_6 z_2(1 + \sec^2(\frac{\pi z_1^2}{2S_2^2}))) - d_2(t)]
\end{align*}
$$

(29)

Substituting the above control laws, $\dot{V}_2(t)$ becomes

$$
\begin{align*}
\dot{V}_2(t) &= -\alpha k_1 y_1^2 - \alpha k_1 y_1 y_2(1 + \sec^2(\frac{\pi y_1^2}{2S_1^2})) - \alpha k_2 z_1^2 \\
&- \alpha k_2 z_1 z_2(1 + \sec^2(\frac{\pi z_1^2}{2S_2^2})) - \alpha y_1 s''(0, t) - \alpha z_1 w''(0, t)
\end{align*}
$$

(30)

Nevertheless, in practice, both of the system parameters $E$, $m_b$, $m_r$ and the disturbance-like terms $d_1(t)$, $d_2(t)$ are unknown. The unknown terms of the proposed control laws will be handled in the next step.

**Step 3**: To deal with the impact of the disturbance-like term, the following auxiliary input signals are developed by robust control strategy

$$
\begin{align*}
u_{d1}(t) &= \frac{\tilde{d}_1^2(t)}{d_1(t) z_1 + \tau_1 y_2} \\
u_{d2}(t) &= \frac{\tilde{d}_2^2(t)}{d_2(t) z_2 + \tau_2}
\end{align*}
$$

(31)

(32)

where $\tau_1, \tau_2 > 0$, and $\tilde{d}_1(t)$ and $\tilde{d}_2(t)$ are the estimates of $\tilde{d}_1$ and $\tilde{d}_2$ respectively.

The adaptive disturbance laws are defined as

$$
\begin{align*}
\dot{\tilde{d}}_1(t) &= -\theta_\psi \tilde{d}_1(t) + \alpha y_2 \\
\dot{\tilde{d}}_2(t) &= -\theta_\psi \tilde{d}_2(t) + \alpha z_2
\end{align*}
$$

(33)

(34)

where $\theta_\psi, \theta_\zeta > 0$ are the accommodation coefficients.

In this step, we employ the radial basis function neural network (RBFNN) to approximate the system uncertainty. According to the property of RBFNN, the following equations hold

$$
\begin{align*}
E s''(0, t) - (m_b + m_r) \dot{y}_1 &= \Phi_y^T h_1(Y) + \Delta_y \\
E w''(0, t) - m_r \dot{z}_2 &= \Phi_z^T h_2(Z) + \Delta_z
\end{align*}
$$

(35)

(36)
where \( \Phi^*_y \) and \( \Phi^*_z \) are the ideal weight vectors, \( Y = [y''(0, t), \hat{y}_1, \hat{y}_2]^T \) and \( Z = [w''(0, t), \hat{w}_1, \hat{w}_2]^T \) are the input vectors, the approximation errors \( \Delta_y \) and \( \Delta_z \) satisfy \( |\Delta_y| < \Delta_1 \) and \( \Delta_z < \Delta_2 \), and the Gaussian functions \( h_i(Y) = [h_1(Y), \ldots, h_m(Y)]^T \) and \( h_j(Z) = [h_1(Z), \ldots, h_n(Z)]^T \) are expressed as

\[
\begin{align*}
    h_i(Y) &= \exp\left(-\frac{\|Y-N_i\|^2}{\omega_{yi}}\right), \quad i = 1, \ldots, m \\
    h_j(Z) &= \exp\left(-\frac{\|Z-N_j\|^2}{\omega_{yj}}\right), \quad j = 1, \ldots, n
\end{align*}
\]

\( N_i \) and \( N_j \) are the center vectors, \( \omega_{yi} \) and \( \omega_{yj} \) are the widths of Gaussian function.

Now, the actual adaptive neural network boundary control laws are proposed as

\[
\begin{align*}
    v_y(t) &= \frac{1}{\eta_y} \left[ \Phi^T_y h_i(Y) - y_1(1 + \sec^2(\pi y_1^2/2\Sigma_1^2)) \right] \\
    &\quad - k_3 y_2 - k_4 y_2(1 + \sec^2(\pi y_2^2/2\Sigma_2^2)) - u_d(t) \\
    v_z(t) &= \frac{1}{\eta_z} \left[ \Phi^T_z h_j(Z) - z_1(1 + \sec^2(\pi z_1^2/2\Sigma_2^2)) \right] \\
    &\quad - k_5 z_2 - k_6 z_2(1 + \sec^2(\pi z_2^2/2\Sigma_2^2)) - u_d(t)
\end{align*}
\]

where \( \hat{\Phi}_y \) and \( \hat{\Phi}_z \) are the estimates of \( \Phi^*_y \) and \( \Phi^*_z \) respectively.

The adaptive weight laws of NNs are defined as

\[
\begin{align*}
    \dot{\hat{\Phi}}_y &= -\Lambda^{-1}_1(\gamma_1 \hat{\Phi}_y + \alpha y_2 h_1(Y)) \\
    \dot{\hat{\Phi}}_z &= -\Lambda^{-1}_2(\gamma_2 \hat{\Phi}_z + \alpha z_2 h_2(Z))
\end{align*}
\]

where \( \gamma_1, \gamma_2 > 0 \) are the modification coefficients which can improve the system robustness, \( \Lambda_1 \) and \( \Lambda_2 \) are the positive diagonal matrices.

The augmented Lyapunov function is further modified as

\[
V_3(t) = V_2(t) + \frac{1}{2} \hat{d}_1^2(t) + \frac{1}{2} \hat{d}_2^2(t) + \frac{1}{2} \hat{\Phi}^T_y \Lambda_1 \hat{\Phi}_y + \frac{1}{2} \hat{\Phi}^T_z \Lambda_2 \hat{\Phi}_z
\]

where \( \hat{d}_1 = \hat{d}_1(t) - \bar{d}_1 \), \( \hat{d}_2 = \hat{d}_2(t) - \bar{d}_2 \), \( \Phi_y = \hat{\Phi}_y - \Phi^*_y \) and \( \Phi_z = \hat{\Phi}_z - \Phi^*_z \) are the error variables.

Taking the derivative of \( V_3(t) \) and then substituting the adaptive control given by (39) and (40), we can conclude that

\[
\begin{align*}
    \dot{V}_3(t) &\leq -ak_1 y_1^2 - ak_2 y_2^2 \sec^2(\pi y_1^2/2\Sigma_1^2) - ak_3 z_1^2 \\
    &\quad - ak_4 z_2^2 \sec^2(\pi z_2^2/2\Sigma_2^2) - \alpha(k_3 - 1/2) y_2^2 \\
    &\quad - ak_5 y_2(1 + \sec^2(\pi y_2^2/2\Sigma_2^2)) - \alpha(k_4 - 1/2) z_2^2 \\
    &\quad - ak_6 y_2(1 + \sec^2(\pi z_2^2/2\Sigma_2^2)) - \theta_1 \hat{d}_1^2(t) - \theta_2 \hat{d}_2^2(t) \\
    &\quad - \frac{\gamma_1}{2\lambda_{\max}(\Lambda_1)} \hat{\Phi}^T_y \Lambda_1 \hat{\Phi}_y - \frac{\gamma_2}{2\lambda_{\max}(\Lambda_2)} \hat{\Phi}^T_z \Lambda_2 \hat{\Phi}_z
\end{align*}
\]

\[
\begin{align*}
    &- \alpha y_1 s''(0, t) - \alpha z_1 w''(0, t) + \frac{\alpha}{2} \Delta_1^2 + \frac{\alpha}{2} \Delta_2^2 \\
    &+ \theta_1 \hat{d}_1^2 + \theta_2 \hat{d}_2^2 + \frac{\gamma_1}{2} \hat{\Phi}^T_y \hat{\Phi}_y + \frac{\gamma_2}{2} \hat{\Phi}^T_z \hat{\Phi}_z + v_1 + v_2
\end{align*}
\]

where \( \lambda_{\max}(\Lambda_1) \) and \( \lambda_{\max}(\Lambda_2) \) are the maximum eigenvalues of \( \Lambda_1 \) and \( \Lambda_2 \) respectively.

IV. STABILITY ANALYSIS

Under the aforementioned analysis and the Lyapunov stability theory, the final Lyapunov function is given as

\[
V(t) = V_3(t) + V_w(t) + V_e(t)
\]

where

\[
\begin{align*}
    V_w(t) &= \frac{\beta \rho}{2} \int_0^t [\hat{s}(x, t) + \hat{y}(t)]^2 dx + \frac{\beta \rho}{2} \int_0^t [\hat{w}(x, t) + \hat{z}(t)]^2 dx \\
    &\quad + \frac{\beta EI}{2} \int_0^t [s''(x, t)]^2 dx \\
    V_e(t) &= \int_0^t [s(x, t) + y_1][\hat{s}(x, t) + \hat{y}(t)] dx \\
    &\quad + \frac{\gamma \rho}{2} \int_0^t [w(x, t) + z_1][\hat{w}(x, t) + \hat{z}(t)] dx
\end{align*}
\]

in which \( \beta, \gamma > 0 \).

Combined Lemma 1 with the definition of \( V_w(t) \) yields

\[
|V_w(t)| \leq \frac{\gamma \rho}{2} \int_0^t [s''(x, t)]^2 dx + \frac{\gamma \rho}{2} \int_0^t [w''(x, t)]^2 dx
\]

\[
+ \frac{\gamma \rho}{2} \int_0^t [\hat{s}(x, t)]^2 dx + \frac{\gamma \rho}{2} \int_0^t [\hat{w}(x, t)]^2 dx
\]

According to the above inequality with \( 0 < \gamma \leq \min\{\frac{\beta}{\rho}, \frac{\beta EI}{\rho^2}, \frac{\alpha}{\rho}\} \), we have

\[
|V(t)| \leq \lambda_0[V_w(t) + V_3(t)]
\]

where \( \lambda_0 = \max\{\frac{\gamma \rho}{2}, \frac{\gamma \rho}{2}, \frac{\gamma \rho}{2}\} \).

Invoking (48) and (49) results in

\[
0 \leq \lambda_1[V_w(t) + V_3(t)] \leq V(t) \leq \lambda_2[V_w(t) + V_3(t)]
\]

where \( \lambda_1 = \min\{1 - \frac{\gamma}{\beta}, 1 - \frac{\gamma \rho}{\beta EI}, 1 - \frac{\gamma \rho}{\alpha}\} > 0 \)

and \( \lambda_2 = \max\{1 + \frac{\gamma}{\beta}, 1 + \frac{\gamma \rho}{\beta EI}, 1 + \frac{\gamma \rho}{\alpha}\} > 0 \).

Lemma 3: \( \dot{V}(t) \) satisfies that \( \dot{V} \leq -\lambda V(t) + \varepsilon \), where \( \lambda, \varepsilon > 0 \).

Proof: The derivative of \( V(t) \) is expressed as

\[
\dot{V}(t) = \dot{V}_3(t) + \dot{V}_w(t) + \dot{V}_e(t)
\]
Differentiating $V_c(t)$ leads to
\[
\dot{V}_c(t) = \beta \rho \int_0^t [\dot{s}(x, t) + \dot{\gamma}(t)][\ddot{s}(x, t) + \ddot{\gamma}(t)]dx \\
+ \beta EI \int_0^t [\ddot{s}''(x, t) + \ddot{e}(x, t)]dx \\
+ \beta \rho \int_0^t [\dot{w}(x, t) + \dot{\gamma}(t)][\ddot{w}(x, t) + \ddot{\gamma}(t)]dx \\
+ \beta EI \int_0^t [\ddot{w}''(x, t) + \ddot{e}(x, t)]dx
\] (52)

Substituting the governing equations into (52) and then utilizing the integration by parts derive
\[
\dot{V}_c(t) = -\beta c \int_0^t [\dddot{s}(x, t) + \dddot{\gamma}(t)]^2 dx \\
-\beta c \int_0^t [\dddot{w}(x, t) + \dddot{\gamma}(t)]^2 dx \\
+ \beta \int_0^t [\dddot{s}(x, t) + \dddot{\gamma}(t)]F_s(x, t)dx \\
+ \beta \int_0^t [\dddot{w}(x, t) + \dddot{\gamma}(t)]F_c(x, t)dx \\
+ \beta EI \dddot{e}(t) + \beta EI \dddot{\gamma}(t)''(0, t) + \beta EI \dddot{\gamma}(t)'''(0, t)
\] (53)

Differentiating $V_c(t)$ and substituting (4) and (5), we have
\[
\dot{V}_c(t) = yEIy_1s''''(0, t) + yEIz_1w''''(0, t) \\
- yEI \int_0^t [s''''(x, t)]^2 dx - yEI \int_0^t [w''''(x, t)]^2 dx \\
- y \int_0^t [s(x, t) + y_1][F_s(x, t) - c(\dot{s}(x, t) + \dot{\gamma}(t))]dx \\
- y \int_0^t [w(x, t) + z_1][F_c(x, t) - c(\dot{w}(x, t) + \dot{\gamma}(t))]dx
\] (54)

Using Young inequality for (55) one can get
\[
\dot{V}_c(t) \leq -(yEI - 2\gamma \delta s l^3 - 2\gamma \delta_6 c l^3) \int_0^t [s''''(x, t)]^2 dx \\
-(yEI - 2\gamma \delta_7 l^3 - 2\gamma \delta_8 c l^3) \int_0^t [w''''(x, t)]^2 dx \\
+ \frac{\gamma c}{\delta_8} \int_0^t [\dot{s}(x, t) + \dot{\gamma}(t)]^2 dx + \frac{\gamma c}{\delta_8} \int_0^t [\dot{w}(x, t) + \dot{\gamma}(t)]^2 dx \\
+(2\gamma \delta_1 l + 2\gamma \delta_6 cl)\gamma_1^2 + (2\gamma \delta_7 l + 2\gamma \delta_8 cl)\gamma_2^2 \\
+ yEIFY_1s''''(0, t) + yEIz_1w''''(0, t) + \frac{\gamma_1^2 F_1^2}{\delta_5} + \frac{\gamma_1^2 F_2^2}{\delta_7}
\] (55)

Invoking (44), (53) and (55) and Lemma 2 and using Young inequality, \(\dot{V}(t)\) satisfies the following inequality
\[
\dot{V}(t) \leq -\lambda_3 [V_c(t) + V_3(t)] + \varepsilon \\
\leq -\lambda \nu(t) + \varepsilon
\] (58)

where \(\lambda = \frac{\lambda_3}{\lambda_5} > 0\), and \(\lambda_3\) is given by
\[
\lambda_3 = 2 \min \left\{ \frac{\mu_1}{\rho^2}, \frac{\mu_2}{\rho^2}, \frac{\mu_3}{\rho\delta_1}, \frac{\mu_4}{\rho\delta_2}, \frac{\mu_5}{\rho\delta_1}, \frac{\mu_6}{\rho\delta_2}, \frac{k_1}{m}, \frac{\delta_3}{\delta_5}, \frac{k_3}{m}, \frac{k_4}{m}, \frac{\delta_2}{\delta_5}, \frac{\gamma_1}{\lambda_{\max}(A_1)}, \frac{\gamma_2}{\lambda_{\max}(A_2)} \right\}
\] (59)

Theorem 1: For a nuclear RM system subject to external disturbance, input backlash, and asymmetric output constraint, under Assumption 1, the proposed control laws (39) and (40) can ensure that: (1) the lateral and transverse vibration dissipations \(x(t)\) and \(w(x, t)\) of fuel rod and the position tracking errors \(y_1\) and \(z_1\) are uniformly bounded and
eventually converge to the compact sets; (2) the asymmetric constraints of tracking errors are never violated.

Proof: Multiplying (58) by $e^{\lambda t}$ and then integrating for the computed result, we can obtain

$$V(t) \leq [V(0) - \frac{E}{\lambda} e^{-\lambda t} + \frac{E}{\lambda}] V(t) e^{-\lambda t} + \frac{E}{\lambda} \quad (60)$$

Form (60), it can get that $V(t)$ is bounded. Applying Lemma 1, (20) and (46) yields

$$\begin{cases}
\beta EI \sum^2(x, t) \leq \frac{\beta EI}{2} \int^1_0 \sum''(x, t)^2 dx \leq \frac{V(t)}{\lambda_1} \\
\beta EI \sum^2(x, t) \leq \frac{\beta EI}{2} \int^1_0 \sum''(x, t)^2 dx \leq \frac{V(t)}{\lambda_1} \\
\alpha \sum^2 \leq V_3(t) \leq \frac{V(t)}{\lambda_1} \\
\alpha \sum^2 \leq V_3(t) \leq \frac{V(t)}{\lambda_1}
\end{cases} \quad (61)$$

Substituting (60) into (61) derives

$$\begin{cases}
|s(x, t)| \leq \sqrt{\frac{2l^2}{\beta EI \lambda_1}} \left[V(0) e^{-\lambda t} + \frac{E}{\lambda}\right] \\
|w(x, t)| \leq \sqrt{\frac{2l^2}{\beta EI \lambda_1}} \left[V(0) e^{-\lambda t} + \frac{E}{\lambda}\right] \\
|y_1| \leq \frac{2}{\alpha \lambda_1} \left[V(0) e^{-\lambda t} + \frac{E}{\lambda}\right] \\
|z_1| \leq \frac{2}{\alpha \lambda_1} \left[V(0) e^{-\lambda t} + \frac{E}{\lambda}\right]
\end{cases} \quad (62)$$

Furthermore, taking the limit of (62) leads to

$$\begin{cases}
lm_{t \to \infty} |s(x, t)| \leq \sqrt{\frac{2l^2 E}{\beta EI \lambda_1}} \\
lm_{t \to \infty} |w(x, t)| \leq \sqrt{\frac{2l^2 E}{\beta EI \lambda_1}} \\
lm_{t \to \infty} |y_1| \leq \frac{2E}{\alpha \lambda_1} \\
lm_{t \to \infty} |z_1| \leq \frac{2E}{\alpha \lambda_1}
\end{cases} \quad (63)$$

According to (45) and (60), we can obtain that $V_1(t)$ is bounded. From the expression of $V_1(t)$, it can conclude that $V_1(t) \to \infty$ as $y_1 \to a_1$ or $-a_2$, $z_1 \to a_3$ or $-a_4$. Using Lemma 1 of [29] implies that the tracking errors $y_1$ and $z_1$ satisfy that $-a_2 < y_1 < a_1$ and $-a_4 < z_1 < a_3$.

V. SIMULATIONS

From the above analysis, the practical stability of the studied refueling machine system is demonstrated theoretically. To verify the performance of the proposed control scheme further, simulations are carried out by using the finite difference method. The physical parameters of the studied refueling machine system are given as $EI = 10$ Nm², $\rho = 0.038$ kg/m, $m_y = 3.2$ kg, $m_x = 6.8$ kg, $l = 1$ m. The system initial states are represented as $s(x, 0) = w(x, 0) = \hat{s}(x, 0) = \hat{w}(x, 0) = 0$, and $y(0) = z(0) = \hat{y}(0) = \hat{z}(0) = 0$. The desired positions are $y_d = 0.6$ and $z_d = 0.2$. The constrained values of tracking errors are $a_1 = 0.05$, $a_2 = 0.65$, $a_3 = 0.1$ and $a_4 = 0.25$. The external disturbances are given by $d_0(t) = 0.3\sin(t)$, $d_4(t) = 0.2\sin(t)$, $F_3(x, t) = 0.5(1 + \sin(0.5t)) + \cos(2\pi t))$ and $F_4(x, t) = 0.5(2 + \sin(0.5t) + \cos(2\pi t))$. 

FIGURE 1. Nuclear refueling machine with a fuel rod.

FIGURE 2. Lateral vibration of the fuel rod without control.

FIGURE 3. Transverse vibration of the fuel rod without control.

FIGURE 4. Lateral vibration of the fuel rod with the designed control.
FIGURE 5. Transverse vibration of the fuel rod with the designed control.

Figs. 2 and 3 show the lateral and transverse vibration displacements of the fuel rod without control. From Figs. 2 and 3, we can see that there are large and continuous vibrations in the fuel rod. Choosing the control parameters $k_1 = 20$, $k_2 = 50$, $k_3 = 100$, $k_4 = 5$, $k_5 = 80$, $k_6 = 10$, $\theta_y = \theta_z = 3$, $\gamma_1 = \gamma_2 = 0.5$, $\eta_y = \eta_z = 1$, $B_{ry} = B_{rz} = 10$, $B_{rl} = B_{lz} = 15$, the developed control laws (39) and (40) are implemented to realize the expected control targets. Under the designed cooperative control scheme, the lateral and transverse vibration displacements of the fuel rod, the positions of RM and bridge, and the tracking errors are displayed in Figs. 4-8, respectively. It is easily seen from Figs. 4 and 5 that the lateral and transverse vibration displacements of the fuel rod are reduced in the small neighborhood of zero. Figs. 6 and 7 imply that the designed cooperative control scheme can move RM and bridge to the desired positions. According to Fig. 8, it is clear that the tracking errors satisfy the given constrained conditions, i.e., $-a_2 < y_1 < a_1$ and $-a_4 < z_1 < a_3$. By analyzing these results, the designed adaptive control scheme is efficient.

VI. CONCLUSION

This paper has designed two adaptive neural network boundary control laws to stabilize a nuclear RM system with uncertainty, external disturbance, input backlash, and asymmetric output constraint. The system dynamic equation has been given by a coupling PDE-ODE. Under the proposed control scheme, the transverse and lateral vibrations of the fuel rod have been completely reduced, whereas the RM and bridge positions have been moved to the expected positions. The position tracking errors have been restricted in the desired ranges by using an asymmetric Tan-BLF. RBFNN has been employed to compensate for the uncertainty of control system. Both the effects of input backlash and boundary disturbance have been handled by constructing two auxiliary input signals. The system practical stability has been proven. Simulation has verified that the developed control scheme can realize all control targets.

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