State-space Coil Modelling in Plasma Magnetic Confinement Devices

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Abstract. The need of robust and optimal control schemes is a key factor for the development of future fusion reactors. This paper has dealt with the state-space modelling of the Ultra-Low Iota Super Elongated Stellarator of the UPV/EHU, using a physical lumped parameter equivalent circuit approach. The model obtained has been validated by means of experimental output data showing an excellent matching with the real system. Besides, it has been designed a MPC scheme that has been successfully implemented both in simulation and experimentally using a real-time control platform.
The rest of the paper is organized as follows: The model development is stated in Section 2. Section 2.1 describes the physical state-space model of the system considered, obtaining some relevant unknown parameter values by means of parameter identification over the real system. Then, the numerical solution achieved is benchmarking with experimental output data in Section 2.2, so as to validate the model. Section 3 is devoted to present the Model Predictive Controller (MPC) scheme used, both parameter description -Section 3.1- and design and implementation of the real time control system -Section 3.2- employed to test the controller in the next section. Section 4 presents the corresponding result, showing the correspondence between the responses of the real system and the numerical model obtained. Finally, some concluding remarks are stated in the last section.

2 Model statement

In this section, the analytical development of the plant model is provided. For this purpose, the circuit formed by the coils and resistors composing the system plant is analysed so as to derive an adequate state-space representation. Afterwards, the model is validated by comparing the numerical plant output with the real system for different shots. It is shown that the output of the numerical model presents excellent matching with that of the real plant for all them.

2.1 System modelling

Several studies on the behaviour of the device have been carried out in order to achieve an adequate equivalent system model [6]. It has been determined that the internal circuit of the stellarator can be represented by the following lumped parameter electrical system (Figure 2). This lumped parameter approach is widely employed when in fusion devices [7-9]:

In this figure, \( R_1 \) represents the total resistance of the first large coil and the first small inner coil. Analogously, \( R_2 \) is the total resistance for the other two coils, so \( R \) represents the total resistance of the circuit. Note also that the inner small coils are coupled, so that total inductance of the circuit may be expressed as:

\[
K = L_{s1} + L_{s2} + L_{12} + L_{s1} + M,
\]  

with \( M \) being the mutual inductance of the two coupled coils and \( K \) the total inductance. The voltage input \( (V_{IN}) \) represents the DC voltage supply and the voltage output is due to the drop in the second part of coils, so that:

\[
V_{OUT} = V_{R2} + V_{Ls2} + V_{12}.
\]

Thus, the two basic equations which govern the circuit can be written as:

\[
V_{IN} = I(R_1 + R_2) + K \frac{dl}{dt} \quad (3)
\]

\[
V_{OUT} = I \cdot R_2 + (L_{s2} + L_{12}) \frac{dl}{dt} \quad (4)
\]

where \( I \) represents the current in the circuit. In order to obtain a state-space representation, the following state variables are chosen:

\[
\begin{align*}
 x_1 &= I \\
 x_2 &= \frac{dl}{dt}
\end{align*}
\]

Then, from equations (3) and (5), the following expression is achieved:

\[
\begin{align*}
 x_1 &= \frac{dl}{dt} = -\left(\frac{R_1 + R_2}{K}\right) I + \frac{1}{K} V_{IN} \\
 x_2 &= \frac{d^2l}{dt^2} = -\left(\frac{R_1 + R_2}{K}\right) \frac{dl}{dt} = -\left(\frac{R_1 + R_2}{K}\right) x_2
\end{align*}
\]

Besides, deriving the equation (7) the result is:

\[
\begin{align*}
 x_2 &= \frac{d^2l}{dt^2} = -\left(\frac{R_1 + R_2}{K}\right) \frac{dl}{dt} = -\left(\frac{R_1 + R_2}{K}\right) x_2
\end{align*}
\]

Therefore, substituting equations (5) in equations (7) and then (8), the state-space model of the system can
be established in a convenient form with non-coupled state variables as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
\frac{-\left( R_1 + R_2 \right)}{K} & 0 \\
0 & -\frac{\left( R_1 + R_2 \right)}{K}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
1 \\
0
\end{bmatrix} V_{IN}
\] (9)

\[
\begin{bmatrix}
I_{OUT} \\
V_{OUT}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{R_2} & 0 \\
L_{s2} + L_{l2} & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\] (10)

Now, is necessary to determine the real values of the parameters of the real system. For this purpose, some experimental tests are performed, collecting the results through the data acquisition system. In order to identify the elements of the matrices it is necessary to manage the differential equations of the system, so that using equations (7) and (8) it is possible to obtain an analytical expression for \(x_1(t)\):

\[
x_1(t) = I(t) = \frac{V_{IN}}{R} \left( 1 - e^{-\frac{R}{K}t} \right)
\] (11)

Analogously, from (6) one has:

\[
x_2(t) = \frac{df}{dt} = \frac{V_{IN}}{K} e^{-\frac{R}{K}t}
\] (12)

Taking into account these results, it is possible to use a set of real data from appropriate experiments to calculate the parameters of the analytical solutions of \(x_1(t)\) and \(x_2(t)\) obtained. In this case, a typical step voltage signal with a 6V magnitude has been used as input to the system. Considering a fixed instant during the transient response -0.05s from the enforcement of the step voltage it is obtained a voltage output value of 2.47V and a current value of 9.4A. Besides, the steady-state current value is 25.73A, so that the total resistance of the system can be directly stablished from Ohm’s law providing value of 0.233Ω.

On the one hand, it is now possible from these data to obtain the total inductance of the circuit from equation (11), resulting in a value \(K=0.021\). In the same way, from the solution (12) and the value of \(K\) obtained, it is possible to calculate the value of the current derivative of current at this specific point of the transient regime, resulting in 163.98A/s. On the other hand, due to the symmetry of the stellarator, both large and small inner coil pairs present the same resistance, so that the value of is also known -\(R_1=R_2=0.1166\ Ω\)-. Finally, from these results and equation (4) it can also be calculated the value of \(L_{s2} + L_{l2} = 6.32\ mH\)-.

According to these results, the state-space model at this point may be expressed as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
-11.1 & 0 \\
0 & -11.1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
47.62 \\
0
\end{bmatrix} V_{IN}
\] (13)

\[
\begin{bmatrix}
I_{OUT} \\
V_{OUT}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0.1166 & 0.00728
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\] (14)

It has to be taken into account that the previous process has been performed for a particular state of the system during its transient response. In order to obtain a more accurate space-state model it has been used a grey-box modelling procedure, where the information acquired from the theoretical model determines the form of the identified system and the estimation of the elements of the state-space matrices is based on real input-output experimental data using the values previously obtained as initial values for the system identification algorithms [10]. In this way, it is possible to stablish a model-based grey-box state-space representing the system:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
-15.5 & 0 \\
0 & -15.5
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
69.3 \\
0
\end{bmatrix} V_{IN}
\] (15)

\[
\begin{bmatrix}
I_{OUT} \\
V_{OUT}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0.111 & 0.00632
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\] (16)

As a result, the values of the Lumped parameter equivalent circuit physical circuit are: \(K=0.0144\ Ω\), \(R=0.2236\ Ω\), \(R_2=0.111\ Ω\) and \(L_{s2} + L_{l2} = 6.32\ mH\).

### 2.2 Model validation

In this section the obtained model is compared with the real system output. For this purpose it has been used a set of validation data obtained for the same open-loop high demanding step-wise voltage input response. Results are shown in the following figures 3 and 4 both for voltage and current output, respectively. As it may be observed, the response of the grey-box model matches that of the real system, with a fit percentage of 74.2% for voltage output and 84.3% for current output case, being in part the error due to the noisy unfiltered experimental output.
3 Model predictive control law

3.1 Controller description

The control objective is to regulate the voltage input supply to the stellarator to successfully solve the trajectory tracking problem for the voltage reference to be achieved in the chamber coils. In turn, current is controlled indirectly through voltage, as these signals are dependent each other. There exist different control approaches for this purpose, being the most used traditional PID-based schemes, robust (sliding-mode) schemes and MPC schemes [11-23]. In particular, when dealing with fusion devices where the available energy is limited, MPC is especially interesting due to its ability to ensure that both the control action and the solution error are minimal [9].

The working principle of MPC (Model Predictive Control) is to predict the future evolution of the outputs or manipulated variables [24-26]. For this purpose it is necessary to define a prediction horizon $N$ representing the time interval in which the output predictions evolve, so that the following sequence of output predictions are calculated according with system model considered and the information of evolution of the process until time $t$:

$$\hat{y}(t+\frac{1}{2}), \hat{y}(t+\frac{3}{2}), \ldots, \hat{y}(t+\frac{N}{2})$$ (17)

Besides, a control horizon $N_u$ representing the sequence of future control signals within $N$ is employed:

$$u\left(\frac{1}{2}\right), u\left(t+\frac{1}{2}\right), \ldots, u(t+N-\frac{1}{2})$$ (18)

Usually, it is considered that the control signal remains constant for future time exceeding the control horizon ($N > N_u$).

A further feature is the sliding horizon representing the number of components of the sequence of control signals vector $u$ to be considered, neglecting the rest of the sequence at each iteration. The MPC scheme used is shown in Figure 5.

The model used for the design of the MPC implementation is based on the previously obtained state-space (15-16) [27,28]. In order to develop an optimal controller, and given that the state variables are not coupled, the state-space model may be expressed as follows:

$$x_1 = -15.5x_1 + 69.3V_{IN}$$ (19)

$$V_{OUT} = 0.111x_1$$ (20)

The control interval of the controller is established in accordance with the experimental sample time of the real system (0.00025s). The prediction horizon considered is set to 25 and the control horizon used is set to 13 (See Table 1). With these MPC parameters, the weight rate was 0.0225 and the weight of the output variable was established at 4 [29,30].

| Table 1. MPC parameters. |
|--------------------------|
| Control Interval          | 0.00025s |
| Prediction Horizon (Intervals) | 25       |
| Control Horizon (Intervals) | 13       |

3.2 Real-time control system

In order to demonstrate the feasibility and performance of the control scheme previously proposed, it has been implemented a real-time control design for the UPV/EHU Stellarator. The system has been designed using the real-time environment shown in Figure 6. The real-time applications are programmed and modified on an engineering PC and then launched to run on a dedicated real-time target computer connected to the physical system [31-32]. In particular, the control strategy is aimed to control a 15 kW EA-PSI 8080-510 3U power supply. A NI PCIe-6323 data acquisition device with two terminal blocks NI SCB-68A has been used for the communication between the real-time control computer and the power source, the external sensors and the stellarator. The external sensors include a Hall effect inductive current sensor able to measure up to 600A with an 1% error for measuring the current in the four-coil system of the stellarator and a scalable voltage sensor for measuring the output voltage.
This structure allows to use the kernel of the real-time target computer only to run the real-time process, with an effective sample time below 250μs, updating the data in the engineering PC at a lower rate via a RJ-45 Ethernet connection.

4 Results: real system vs proposed model

As it has been stated in the introduction the final aim is to develop different controllers for the UPV/EHU Stellarator. As a first step towards achieving this goal a grey-box model has been developed. This model has been initially validated against the real system. In a second step, a model predictive control (MPC) has been designed and implemented, both in simulation with the proposed model and experimentally. In order to validate the real-time MPC over the system, experimental voltage and current output results for a demanding shot may be seen, respectively, in Figure 7 and Figure 8.

As it may be observed, the controlled response of the real system is very similar to the desired one for both voltage and current, which shows that the real time MPC of the system provides an adequate performance over time, since the only deviation is a non-significant peak.

Besides, taking into account that the voltage is sufficiently controlled with the MPC and that the current depends basically on the voltage and on the resistance of the system, it can be assumed that this error is a result of a variation of the resistance of the system as the experiment evolves. This variation of the resistance may be due to the change of the temperature in the coils of the stellarator. Likewise, other factor that affects to the error is the environmental noise inherent to the experimental output, and in particular to the inductive current sensor of the coils.

Therefore, the real-time MPC has been proven to be an excellent candidate as a control scheme since it is able to optimize the future behaviour of the states while intrinsically minimizing the control action. In particular, Figure 7 and Figure 8 show how this scheme evidences the optimal prediction effect at each step in the reference value, where the overshoot is minimized, due to the control ability to predict the future flat-top regime.

5 Conclusions

This paper deals with the grey-box modelling of the magnetic confinement coil system of the UPV/EHU Stellarator. The state-space model obtained has been successfully validated by comparing the results of the model with those of the real system. Besides, a Model Predictive Control law has been experimentally implemented over the real-time control system using the proposed state-space model showing an excellent reference trajectory tracking behaviour. A benchmarking of the proposed MPC with a traditionally used PID controller has been also performed. It has been shown that the MPC presents a superior performance due to its ability to predict the state of the plant during the dynamic operation, while PID controller needs a fine tuning when dealing with time-delay or changing dynamics during operation. In particular, the proposed MPC scheme allows ensuring both minimal control action and solution error, which
is especially relevant in the case of fusion devices where the available energy is limited.

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