Consistency of Calculation Results of Two Typical Vehicle Collision Models

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Abstract

In order to study consistency of calculation results of typical vehicle collision models based on the law of momentum conservation, the difference of coordinate systems of two typical models is analyzed. Consistency of two typical models is studied by using empirical formulas based on vehicle collision tests. From the above study, a vehicle collision accident is analyzed by using two typical models. Results show that calculation results of two typical models are identical under reasonable selection of coefficients in these models, thus two typical models are consistent. Study results provide the basis for consistent analysis of similar models.

Keywords: typical vehicle collision model; consistency; pre-impact speed; empirical formula

1. Introduction

Vehicle collision models based on the law of momentum conservation have advantages of simple structure and easy programming. They are widely used in calculating the pre-impact speed in vehicle collision accidents. Their formulas are \( A_0 v_0 = A v \), which \( A_0 \) is a pre-impact parameter matrix; \( v_0 \) is a pre-impact speed matrix; \( A \) is a post-impact parameter matrix; \( v \) is a post-impact speed matrix. The pre-impact speed can be calculated easily by solving \( A_0 v_0 = A v \). Two typical models among them are given on related literature [1,2,4]. Because two typical models are established by using different coordinate systems, their parameter matrices are different. Are calculation results of two typical models identical for a traffic accident? Thus, it is necessary to study consistency of calculation results of two typical models.

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From advantages of two typical models, some researches have been completed by scholars. Hereinafter two typical models are referred to as model 1 and model 2. The optimization program of model 1 has been compiled so that pre-impact speeds of accident vehicles can be accurately calculated by using the program [1,2,4]. With strong practicability of two typical models, the method based on dent superposition has been used in analyzing vehicle collision accidents [3]. Simple algorithm of model 1 is studied so that the pre-impact speed can be calculated quickly [1,2,4,10]. Based on model 2, Zhou [5,6] established a three-dimensional mechanical model of vehicle collision accidents. According to the principle of model 2, Neades J and Smith R [7] studied a method for calculating pre-impact speeds of accident vehicles. However, the study on consistency of calculation results of two typical models had few related reports. Therefore, the problem is studied by using the matrix theory.

2. Two typical models

Two typical models based on the law of momentum conservation are shown in formula (1) ~ formula (2).

$$A_{01}v_{01} = A_{1}v_{1}$$

$$A_{02}v_{02} = A_{2}v_{2}$$ (1) (2)

where $A_{01}$ and $A_{02}$ are pre-impact parameter matrixes, $v_{01}$ and $v_{02}$ are pre-impact speed matrixes, $A_{1}$ and $A_{2}$ are post-impact parameter matrixes, $v_{1}$ and $v_{2}$ are post-impact speed matrixes.

$$v_{01} = (v_{01n}, v_{011}, v_{02n}, v_{021}, \omega_{01}, \omega_{02})^T$$

$$v_{i} = (v_{in}, v_{i1}, v_{in}, v_{i2}, \omega_{i}, \omega_{i})^T$$ (3) (4)

where $i=1, 2$ represents model 1 and model 2 in this paper, $j=1, 2$ represents vehicle 1 and vehicle 2 in this paper, $v_{01n}$ and $v_{011}$ are the normal pre-impact speed and the tangential pre-impact speed (m/s), $\omega_{01}$ is the pre-impact angular velocity around each vehicle mass centre (rad/s), $v_{jn}$ and $v_{j1}$ are the normal post-impact speed and the tangential post-impact speed (m/s), $\omega_{j}$ is the post-impact angular velocity around each vehicle mass centre (rad/s).

$$A_{01} = \begin{bmatrix} m_1 & 0 & m_2 & 0 & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 & 0 \\ 0.5m_1x_{1n} & -0.5m_1x_{11} & -0.5m_2x_{11} & 0.5m_2x_{1n} & J_1 & 0 \\ -0.5m_1x_{21} & 0.5m_1x_{2n} & 0.5m_2x_{21} & -0.5m_2x_{2n} & 0 & J_2 \\ \mu & -1 & \mu & -1 & 0 & 0 \\ -\varepsilon_{1n} & 0 & \varepsilon_{1} & 0 & \varepsilon_{1n}x_{11} & -\varepsilon_{1n}x_{21} \end{bmatrix}$$

$$A_{02} = \begin{bmatrix} m_1 & 0 & m_2 & 0 & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 & 0 \\ 0 & 0 & a_1m_1 & 0 & 0 & J_1 \\ 0 & 0 & a_2m_2 & -b_1m_2 & 0 & J_2 \\ \varepsilon_{1n} & 0 & -\varepsilon_{1} & 0 & -a_1\varepsilon_{1} & a_2\varepsilon_{1} \\ 0 & \varepsilon_{1} & 0 & -\varepsilon_{1} & b_1\varepsilon_{1} & -b_2\varepsilon_{1} \end{bmatrix}$$ (5) (6)
where \( m_j \) is each vehicle mass (kg), \( J_j \) is the rotational inertia around each vehicle mass centre (kg·m\(^2\)), \( x_{jT} \) and \( x_{jn} \) are the tangential coordinate and the normal coordinate with mass centre origins (m), \( a_j \) and \( b_j \) are the tangential coordinate and the normal coordinate with impact point origins (m), \( \mu \) is the tangential friction coefficient at impact points, \( \varepsilon_n \) is the normal elasticity coefficient at impact points, \( \varepsilon_t \) is the tangential elasticity coefficient at impact points. \( A_1 \) can be obtained when line 6 in \( A_{01} \) is substituted by 1, 0, -1, 0, \( -x_{1T}, x_{2n} \). \( A_2 \) can be obtained when line 5 and line 6 in \( A_{02} \) are substituted by -1, 0, 1, 0, \( a_1, -a_2 \) and 0, -1, 0, 1, \( -b_1, b_2 \).

Coordinate systems of two typical models are illustrated in Fig. 1.

The following can be observed from Fig. 1.

\( O \) and \( c_j \) are impact points and each vehicle mass centre. In model 1, \( c_j \) is taken as the coordinate origin. In model 2, \( O \) is taken as the coordinate origin.

In model 1, \( n \) is the normal axis whose direction is same and from \( c_1 \) to its front, \( \tau \) is the tangential axis formed by rotating \( n \) axis anticlockwise 90°. In model 2, \( \tau \) is the tangential axis whose direction is from \( O \) to its right, \( n \) is the normal axis formed by rotating \( \tau \) axis anticlockwise 90°. In two coordinate systems, the anticlockwise direction is positive and the clockwise direction is negative.

\( A_{0i} \) and \( A_i \) can be calculated by formula (5) ~ formula (6). \( v_i \) can be calculated by formula (7) ~ formula (9).

\[
v_{jn} = (2g\varphi_j s_j)^{1/2} \cos \theta_j \tag{7}
\]

\[
v_{jT} = (2g\varphi_j s_j)^{1/2} \sin \theta_j \tag{8}
\]

\[
\omega_j = \pi (\alpha_j - \alpha_{j0}) (2g\varphi_j s_j)^{1/2} / 180 \tag{9}
\]

where \( g \) is the acceleration of gravity, \( \varphi_j \) is the adhesion coefficient between wheels and road, \( s_j \) is the sliding distance of vehicles, \( \theta_j \) is the sliding angle of vehicles, \( \alpha_j \) is the parking angle of vehicles, \( \alpha_{j0} \) is the pre-impact angle of vehicles. \( v_{0j} \) can be calculated by formula (1) ~ formula (2).

3. Assumption of consistency

The introduction of a vehicle collision accident: At the intersection of a city in China, vehicle 1 travelling from west to east collided with vehicle 2 turning left from south to west. The right front of vehicle 2 bumped into the right side of vehicle 1, two vehicles were seriously damaged. Accident data on the case are shown in Table 1. Calculate pre-impact speeds of two vehicles.
Table 1. Data measured on site

| Vehicles | m/kg | \( J \)/\((\text{kg} \cdot \text{m}^2)\) | \( l(a) \)/m | \( l(b) \)/m | \( a_0 \)/\( \ell \) | \( s \)/m | \( \theta \)/\( \ell \) | \( \alpha \)/\( \ell \) | \( \phi \) |
|----------|------|------------------|-------------|-------------|----------------|-------|-------------|-------------|--------|
| Vehicle 1 | 2035 | 4800             | 0.891(-0.891)| 0.860(0.860)| 0              | 12    | 20.0        | 50.0        | 0.5    |
| Vehicle 2 | 1200 | 1300             | -0.835(0.835)| -1.790(-1.790)| 137           | 4.4   | 156.0       | 170.0       | 0.5    |

According to damage conditions of two vehicles, \( \varepsilon_1=0.30, \mu=0.40 \) in model 1. From accident data in Table 1, by formula (7) ~ formula (9), the post-impact speed in model 1 is \( v_1=(-3.709, 10.190, -2.671, -5.999, 0.788, 0.859)^T \). \( A_{01} \) and \( A_1 \) can be obtained by formula (5). By formula (1), the pre-impact speed in model 1 is \( v_{01}=(-0.226, 11.584, -8.577, -8.361, -0.020, 0.210)^T \).

The pre-impact speed of vehicle 1 is \( v_{10}=3.6\sqrt{(v_{10}^n)^2+(v_{10}^I)^2}=3.6\sqrt{(-0.226)^2+(11.584)^2}=41.710 \) km/h.

The pre-impact speed of vehicle 2 is \( v_{20}=3.6\sqrt{(v_{20}^n)^2+(v_{20}^I)^2}=3.6\sqrt{(-8.577)^2+(-8.361)^2}=43.121 \) km/h.

The pre-impact angle of vehicle 1 is \( \theta_{10}=\arctan\left(\frac{v_{10}^n}{v_{10}^I}\right)=\arctan\left(-\frac{0.226}{11.584}\right)=-1.118^\circ \).

The pre-impact angle of vehicle 2 is \( \theta_{20}=\arctan\left(\frac{v_{20}^n}{v_{20}^I}\right)=\arctan\left(-\frac{8.577}{-8.361}\right)=45.731^\circ \).

4. Verification of consistency

The empirical formula of determining \( \varepsilon_1 \) has been established based on a large number of vehicle to vehicle crash tests [8,9]. Reliable test data are given by a large number of vehicle crash tests [10]. The consistency of calculation results of two typical models can be studied by combining the empirical formula of determining \( \varepsilon_1 \) with reliable test data.

(1) The empirical formula

\[
\varepsilon_1 = 0.0396G_{IR}^2 - 0.4501G_{IR} + 0.3066
\]  (10)

where \( G_{IR} \) is the equivalent friction coefficient between vehicle 1 and vehicle 2.

(2) The test data

The corresponding range between \( G_{IR} \) and \( \varepsilon_1 \) is shown in Table 2.

| Collision form | \( G_{IR} \) | \( \varepsilon_1 \) |
|----------------|-------------|-------------------|
| Side collision | \( 0<G_{IR} <4.5 \) | \( 0.5 \geq \varepsilon_1 \geq -0.9 \) |
|                | \( G_{IR} >5 \) | \( -0.9 >\varepsilon_1 >1.0 \) |

\( \varepsilon_1=-0.907 \) is substituted into formula (10) and its solution is

\[
G_{IR1}=6.973, G_{IR2}=4.393
\]
According to Table 2, the analysis of GIR1 and GIR2 is as follows.

(1) Although GIR2=4.393 is in 0<\(GIR\)<4.5, \(\varepsilon_1=-0.907\) is not in -0.5 < \(\varepsilon_1\) < -0.9 so that GIR2=4.393 is not the solution of formula (10).

(2) Because GIR1=6.973 is content to GIR>5 and \(\varepsilon_1=-0.907\) is in -0.9 > \(\varepsilon_1\) > -1.0, GIR1=6.973 is the unique solution of formula (10).

According to Table 2, study results show that GIR1=6.973 can ensure \(\varepsilon_1=-0.907\) to be in a reasonable range and to be the unique solution of formula (10). Study results prove the above assumption to be true.

5. Conclusions

Through study on the consistency of calculation results of two typical models, the following conclusions can be obtained.

Analysis method of combining empirical formulas with reliable test data can be applied to study the consistency of calculation results of two typical models based on the law of momentum conservation. The reasonable selection of \(\varepsilon_1\) can ensure that two typical models have identical calculation results. The calculation results of a vehicle collision accident by using two typical models can be verified each other.

Compared with the literature [10], the innovation of this paper is that the consistency of calculation results of two typical models can be studied by combining the empirical formula with reliable test data. Study results provide a valuable experience for solving similar problems. The contribution of the paper is the foundation of a simple and practical method of studying the consistency of calculation results of two typical models based on the law of momentum conservation. Future study will shed more light on this.

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