AUV Tunnel Tracking Method Based on Adaptive Model Predictive Control

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Abstract. In this paper, a model predictive path tracking controller is designed for the path tracking of the autonomous underwater vehicle (AUV). Firstly, Serret-Frenet coordinate system is introduced, and the path tracking error system model is established in this coordinate system. Based on Lyapunov theory and Backstepping method, the forward heading of AUV is converged to the target value. Then, a nonlinear model predictive control design constraint path following control law of the AUV based on rolling to solve constrained optimization problems, satisfy the constraint of extension control input, to hit the target heading control. In the end, AUV-T, an underwater robot developed by Harbin engineering university, was used to test the proposed control law. The results showed that the controller showed good tracking effect.

1. Introduction

With the application of autonomous underwater vehicle (AUV) in ocean research and development, path tracking control has become one of the important technologies of AUV motion control. Path tracking is an expectation trajectory that controls AUV to track a course requirement and performance constraint, and is independent of time. Is the research background of this paper is tunnel detection, AUV in the tunnel and tunnel side wall fixed distance within the same horizontal plane along the tunnel track, and through the AUV stem of the front camera and midship section to install 5 groups of lights and cameras to see inside the tunnel environment, thus achieve the purpose of the inspection on tunnel inside detection.

The serret-frenet coordinate system is one of the more classical path tracking control methods, which is widely used in the research of the path tracking control of robots and unmanned ships. There are some literatures\cite{1-7} have studied the path tracking control problem of the motion control system, but it is not comprehensive enough to consider the constraint of AUV physical actuator during the design of the controller. While the model predictive control (MPC) is an advanced intelligent feedback control method, it through to the definition of some time domain in the future on the prediction and forecast input is presented to solve constrained optimization problems to get the control input, the mechanism itself make it has a good ability to deal with constraints explicitly\cite{8-12}.

Based on the above research results, this article will first introduce Serret - Frenet frame, and path tracking error system model in this system, and based on the Lyapunov theory and Backstepping method will advance of the AUV heading converge to the target. Then, a nonlinear model predictive control design constraint path following control law of the AUV based on rolling to solve constrained
optimization problems, satisfy the constraint of extension control input, thus to control target heading, achieve the result of path tracking.

2. Problem description
In this paper, the AUV is fully driven due to the motion of only the horizontal direction, and the force of x direction and y direction and the control moment of the heading direction are respectively present. The horizontal kinematics equation is:

\[
\begin{align*}
\dot{x} &= u \cos \psi - v \sin \psi \\
\dot{y} &= u \sin \psi + v \cos \psi \\
\psi &= r
\end{align*}
\]

(1) (2) (3)

The rotation Angle of \{B\} around the Z axis is obtained by the new coordinate system \{W\}. Under \{W\}, the AUV motion equation can be expressed as:

\[
\begin{align*}
\dot{x} &= v_t \cos \psi_W \\
\dot{y} &= v_t \sin \psi_W \\
\psi_W &= r + \beta
\end{align*}
\]

(4) (5) (6)

For the dynamic equation of AUV, based on the literature [13], the AUV horizontal motion and vertical plane motion decouple, so it can ignore the vertical swing and pitch motion. In addition, the AUV-T cross section of the tunnel underwater robot developed by Harbin engineering university is an ellipse, which can be neglected. The above assumptions are combined. The dynamic equation of AUV is as follows:

\[
\begin{align*}
m(\dot{u} - vr) &= X_{rr}r^2 + X_{uu}\dot{u} + X_{uv}uv + X_{uu}u^2 + X_{vv}v^2 + T_x \\
m(\dot{v} - ur) &= Y_{rr}\dot{r} + Y_{uu}\dot{u} + Y_{uv}\dot{v} + Y_{uu}u^2 + Y_{uv}uv + Y_{vv}v^2 + T_y \\
I_z\ddot{\psi} &= N_r \dot{r} + N_{r|v|}r\dot{r} + N_{\psi|v|}\dot{v} + N_{\psi|v|}\psi + N_{\psi|v|}v|v| + M_{Tz}
\end{align*}
\]

(7) (8) (9)

Among them, \(m\) is the quality of AUV, \(X_{ij}, Y_{ij}, Z_{ij}\) is the hydrodynamic coefficients, \(I_z\) is the moment of inertia, \(T_x\) and \(T_y\) is control, \(M_{Tz}\) is to control the torque.

3. AUV path tracking control
We need to study the kinematics model of AUV in frame \{F\}.

\[
\begin{align*}
\dot{s}_1 &= -s(1 - c_e s_1) + v_t \cos \psi_e \\
\dot{s}_2 &= -c_e s_1 + v_t \sin \psi_e \\
\psi_e &= r + \beta - c_e \dot{s}
\end{align*}
\]

(10) (11) (12)

Where \(c_e(s)\) is the curvature of the expected path, and \(s\) is the parameter variable of the expected path.

The AUV kinematics model is formula (1) - (3), and the path tracking error model in serret-frenet coordinate system is formula (10) - (12). Based on literature [14], the Angle of sight navigation method, for a given expected path \(S\) and the expected surge speed \(u_d\) of AUV, design control as follows, for any initial \(Q\) position, make the AUV tracking heading error converges to converge to zero.

First of all, choose the Lyapunov function as \(V_1 = \frac{1}{2}(\psi_e - \delta)^2\), and design the control as follows:

\[
r = \delta - \beta - k_1(\psi_e - \delta) + c_e \dot{s}
\]

(13)

Where \(k_1\) is the controller gain, then there is:

\[
\dot{V}_1 = -k_1(\psi_e - \delta)^2 \leq 0
\]

(14)

Under the guidance of the navigation Angle \(\delta\), AUV, the trajectory convergence:

\[
\{O_1|(x,y) \in R^2, \psi_e = \delta\}
\]

(15)

After obtaining the AUV target heading, the design path tracking prediction controller is designed to control the heading of the target. Basic principle of path tracking controller is shown in figure 1.
Figure 1. A schematic diagram of AUV path tracking control based on MPC.

Based on literature [15], the cost function of defining the constraint optimization problem at each moment $t_k$ is:

$$f\left( x(t_k), \bar{e}(\cdot), \bar{\theta}(\cdot), \bar{u}(\cdot), \bar{v}(\cdot) \right) = \int_{t_k}^{t_k+T_p} F\left( \bar{e}(\tau), \bar{\theta}(\tau), \bar{u}(\tau), \bar{v}(\tau) \right) d\tau$$

(16)

Where $F(\bar{e}, \bar{\theta}, \bar{u}, \bar{v}) = [e^T \theta] Q [e \theta]^T + [u \ \ v]^T R [u \ v]^T$ and $E(x, z) = [x^T \ z^T] P [x^T \ z^T]^T$ are called the state cost function and the terminal cost function. $Q \in \mathbb{R}^3$ and $R \in \mathbb{R}^2$ are respectively the state cost weight matrix and the input cost weight matrix. $T_p$ is prediction horizon, $\bar{e}(\cdot)$ and $\bar{u}(\cdot)$ is the error and input of prediction horizon. $e(\cdot)$, $u(\cdot)$ is the error and input of the actual system, $\bar{v}(\cdot)$ is the virtual input of the control parameters. The constraint optimization problem solved by $t_k$ at each moment is:

$$\min_{\forall \tau \in [t_k, t_k + T_p]} f\left( x(t_k), \bar{e}(\cdot), \bar{\theta}(\cdot), \bar{u}(\cdot), \bar{v}(\cdot) \right)$$

(17)

$$\dot{x}(\tau) = f(\bar{x}(\tau), \bar{u}(\tau)), \bar{x}(t_k) = x(t_k)$$

(18)

$$\dot{\bar{e}}(\tau) = \eta(\tau) - p(\bar{z}(\tau))$$

(19)

$$\dot{\bar{\theta}}(\tau) = \bar{z}_1, \bar{u}(\tau) \in U$$

(20)

$$\dot{\bar{z}}(\tau) \in z, \bar{v}(\tau) \in V$$

(21)

(22)

(23)

For constrained optimization problems (17-23), the starting position of AUV can be used as the initial value of AUV prediction model. In order to ensure that AUV can converge to the reference path as soon as possible, the path parameters corresponding to the nearest path reference point can be selected as the initial point of the path prediction model, namely:

$$z_0 = [\theta_{min}, p(\theta_{min})]^T$$

(24)

$$\theta_{min} = \arg\min \left\{ \| \theta - [\theta, p(\theta)]^T \| \right\}$$

(25)

At the same time, the path parameter state constraint domain can be set to ensure that $\dot{\bar{\theta}} \geq 0$, that is, the forward orientation of the guaranteed path. The optimal input of constraint optimization problem (16) at time $t_k$ is:

$$(u_k^*(\cdot), v_k^*(\cdot)) = \arg\min_{\bar{u}(\cdot), \bar{v}(\cdot)} f\left( x(t_k), \bar{e}(\cdot), \bar{\theta}(\cdot), \bar{u}(\cdot), \bar{v}(\cdot) \right)$$

(26)

If the sampling period of AUV path tracking system is $\delta$, namely $t_k = k \delta$, through the system at time $t$ model to calculate the system input and the state and to solve the constrained optimization problem (17-23) optimized input solution is $u_k^*(\cdot)$, when applied to the system input is:

$$(u_k, v_k) = (u_k^*(t - t_k; x(t_k)), v_k^*(t - t_k; x(t_k))) \quad \forall \tau \in [t_k, t_{k+1})$$

(27)

The optimal input process (26) for each time $t$ is applied to the control system only (27). Meanwhile, the time domain is predicted from $[t_k, t_k + T_p]$ to $[t_{k+1}, t_{k+1} + T_p]$, and the constraint path planning problem of $t_{k+1}$ time is obtained.

To sum up, the constraint path tracking controller algorithm is:
1) obtain the initial value of path parameters $z_0$ by solving formula (24-25);
2) at the sampling time, $t_k$, measure system state $x_{tk}$, solve the optimization problem equation (17-23), and obtain the optimized solution formula (26);
3) apply equation (27) to the system and roll the predicted time domain to the next moment, when the sampling time is $t_{k+1}$;
4) let $k=k+1$, back to 2).

The MPC algorithm can guarantee the stability of the algorithm by means of zero terminal constraint or terminal state constraint and terminal cost function. Therefore, when designing the MPC controller in the engineering practice, the terminal state constraint is often dropped, and the control system is restrained by the longer prediction time domain.

4. Experimental analysis.
In order to prove the effectiveness of the tracking controller proposed, the auv-t robot developed by Harbin engineering university was used in this section to conduct experiments in a canal in Hangzhou.

About test, has provided about a quarter of voltage, ensure AUV in section 1 of the lateral speed all the way, the robot began to trace the initial heading Angle $\psi = -60^\circ$, tracking distance $S = 3$ m. The experimental site and the schematic diagram are shown in figure 2 and figure 3.

![Figure 2. Actual scene of the test site.](image)

![Figure 3. AUV path tracking schematic.](image)

Figure 4, Figure 5 and Figure 6 are respectively the heading change, distance change and voltage change of the AUV path tracking.

![Figure 4. AUV path follows the forward change.](image)

![Figure 5. AUV path tracking range changes.](image)
5. Conclusion
In this paper, the path tracking of the plane motion AUV is studied. Firstly, the adaptive control method based on the Lyapunov theory and Backstepping technique is proposed in the Serret-Frenet coordinate system, and the target heading of AUV is determined. Then, a nonlinear MPC is used to track the reference path of AUV under constraint conditions. During the design of the controller, the freedom of control process is added, and the reference model is introduced to predict the controller. The last test in the outfield lake, a simulated tunnel environment on the wall is distance tracking, finally the analysis of experimental data proved that the design of the underactuated AUV path tracking predictive controller has good tracking performance.

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