Hurley, J. R., Tout, C. A. (1998). The binary second sequence in cluster colour-magnitude diagrams.

Originally published in Monthly Notices of the Royal Astronomical Society, 300(4), 977-980

Available from: http://dx.doi.org/10.1046/j.1365-8711.1998.01981.x

This version of the article copyright © 1998 The Authors.

This is the author’s version of the work. It is posted here with the permission of the publisher for your personal use. No further distribution is permitted. If your library has a subscription to this journal, you may also be able to access the published version via the library catalogue.

The definitive version is available at www.interscience.wiley.com.
The Binary Second Sequence in Cluster Colour–Magnitude Diagrams

Jarrod Hurley and Christopher A. Tout
Institute of Astronomy, The Observatories, Madingley Road, Cambridge CB3 0HA

ABSTRACT
We show how the second sequence seen lying above the main sequence in cluster colour magnitude diagrams results from binaries with a large range of mass ratios and not just from those with equal masses. We conclude that the presence of a densely populated second sequence, with only sparse filling in between it and the single star main sequence, does not necessarily imply that binary mass ratios are close to unity.

Key words: stars: clusters of – stars: binary

1 INTRODUCTION
With the recent surge of high quality colour–magnitude diagrams (hereinafter CMDs) of clusters obtained both from space (Rubenstein & Bailyn 1997, Richer et al. 1997 and Elson et al. 1998) and ground based (Ferraro et al. 1997) observations it is timely to reconsider the effect of binary stars on the observed stellar main sequence. It is well known that an unresolved binary system comprising two identical stars has the same colour but twice the luminosity of an equivalent single star and that such a system, comprising two equal-mass main-sequence stars, will appear in the cluster CMD displaced vertically by 0.753 magnitudes irrespective of the wavelength bands used (Haffner & Heckmann 1937). Because of this, the clear second sequences displaced by this amount above the main-sequence position in many cluster CMDs are taken to indicate a population of equal-mass binaries (Bergbusch et al. 1991, Kaluzny & Rucinski 1995 and Santiago et al. 1996). The fact that the region between the single-star main sequence and this second sequence is not very densely populated is then often taken to mean that the mass-ratio distribution in the binary systems is biased towards equal masses. However it turns out that this need not be the case. A system with two unequal main-sequence components has a combined colour that is redder than the colour of the brighter component as well as a luminosity greater than the single star but less than the corresponding equal-mass binary (Bolte 1991 and Romani & Weinberg 1991). Such a system is displaced both upwards and to the right relative to the main-sequence position of the brighter component. Thus if we consider the position of a system with a mass ratio slightly different from unity, relative to the equal-mass system it moves to fainter magnitude but also to redder colour. Now, because the intrinsic slope of the main sequence is such that less massive stars are fainter and redder, it is possible for systems with unequal components to follow the second sequence downwards and to the right in the CMD. We show (section 2) that, for certain mass ranges and choice of colours, systems with quite extreme mass ratios still lie close to the second sequence, 0.753 mag above the actual main sequence, appropriate to their combined colour. We then show (section 3) how an unbiased mass ratio distribution can lead to the clearly separated main and second sequences observed in cluster CMDs.

2 COMBINING THE COLOURS AND MAGNITUDES
For a given colour index X a main-sequence star of bolometric luminosity \( L \) has an absolute magnitude

\[
M_X = M_{X,\odot} - 2.5 \log_{10} \frac{L}{L_{\odot}} + \beta_X(T_{\text{eff}}),
\]

where \( \beta_X \) is the bolometric correction appropriate to a main-sequence star with effective temperature \( T_{\text{eff}} \) and \( M_{X,\odot} \) and \( L_{\odot} \) are the absolute magnitude and bolometric luminosity of the Sun. Given a second colour Y a corresponding absolute magnitude \( M_Y \) can be defined in a similar way and a colour by

\[
(X - Y) = M_X - M_Y.
\]

In the case of an unresolved binary the two components contribute differently to \( M_X \) and \( M_Y \) because of their different effective temperatures and consequently different bolometric corrections. Using the suffixes 1 and 2 for the components and 3 for the system we can write the combined magnitudes as

\[
M_{X,3} = M_{X,\odot} - 2.5 \log_{10} \frac{L_{X,1} + L_{X,2}}{L_{\odot}},
\]

where
The theoretical zero-age single star and equal-mass binary main sequences for a metallicity of $Z = 0.02$ covering a range of $M_V$. Also plotted for a range of primary masses, $M_1$, are binary points with $M_2 = q M_1$ where $q$ ranges from 1.0 to 0.0 in increments of 0.1. The point at $q = 0.5$ is an open square.

\[ L_{X,i} = L_i 10^{-0.4 \beta (T_{\text{eff},i})} \]  
\[ (X - Y)_3 = M_{X,3} - M_{Y,3} \]  

For a binary system of total mass $M = M_1 + M_2$ and mass ratio $q = M_2/M_1$, where we choose the primary mass $M_1 > M_2$ (corresponding to $L_1 > L_2$ on the main sequence) and hence $0 \leq q \leq 1$, we can calculate both $L_1$ and $T_{\text{eff},i}$ by means of the zero-age main-sequence formulae constructed by Haffner & Heckmann (1937) and later developed by Bettis (1975) and Dabrowski & Beardsley (1977) but neither of the latter two make any connection between magnitude difference and mass ratio. Our results, based on up to date stellar models and bolometric corrections, represent a thorough re-working.

3 CLUSTER COLOUR MAGNITUDE DIAGRAMS

To understand how these effects manifest themselves in observed CMDs we consider a distribution of 5000 stars of which 4000 are in unresolved binary systems. We select the masses of the single stars and of the binary primaries according to the initial mass function derived by Kroupa, Tout & Gilmore (1993) by means of the generating function

\[ \frac{M_1}{M_\odot} = 0.08 + \frac{0.19 X^{1.55} + 0.05 X^{0.6}}{(1.0 - X)^{0.58}}, \]  

where we choose $X$ uniformly distributed to give masses in the range $1 < M_1/M_\odot < 6$. We then choose the mass ratio in the binary systems so that $q$ is uniformly distributed between 0 and 1 to find the secondary masses

\[ M_2 = q M_1. \]  

For each single star and binary system we calculate the colours and magnitudes as described in the previous section and plot them in the CMD shown in Figure 2. We see that the densest population is close to the single star main sequence even though most stars are binary. This is because faint secondaries make no contribution. With this particular mass ratio distribution we see a clear second sequence for $(B - V) < 0.1$ corresponding to $M_1 > 2 M_\odot$. This is much clearer in the expanded region shown in Figure 2b than in the full CMD in Figure 2a and we note that this is due to the size of the symbols used to plot the stars relative to the space between them and care should be taken when interpreting observations by eye. This point CMD is in the form in which observations have been presented so far but we note that a density contour plot would be less open to false interpretation (see Figure 3). Between $0.1 < (B - V) < 0.3$ no clear second sequence is visible but for $(B - V) > 0.3$ it is again apparent in Figures 2c and 3c.

4 DISCUSSION

We have shown that not only equal mass main-sequence binary systems lie on the second sequence 0.75 mag above the single star sequence but also many with unequal masses. In particular for $B$ and $V$ colours and primary masses $M_1 > 2$ even systems with quite extreme mass ratios, as low as $q = 0.5$, lie on the second sequence. We have also shown how a cluster with uncorrelated binary masses can show a distinct second sequence in the CMD. However we must comment on what we mean by uncorrelated. For Figures 2 and 3 we have selected each primary mass $M_1$ from the IMF and then chosen $q < 1$ uniformly distributed. Note that choosing $q_{\text{max}} > q > 1$ uniformly would lead to a quite a different distribution and it can be argued that we really should be choosing $0 > \log q > \log q_{\text{max}}$ uniformly. Alternatively Eggleton, Fitchett & Tout (1989) defined uncorrelated to mean the two masses chosen independently from the same initial mass function. For a nonuniform initial mass function such a choice does not lead to a uniform distribution for $q$ overall nor for any particular primary mass. Indeed, because any star is most likely to have a low-mass companion, systems with massive primaries will tend to have extreme mass ratios while those with low-mass primaries will tend to have
mass ratios closer to unity. In this sense our uniform distribution in $0 < q < 1$ actually corresponds to a correlation in the masses for primaries with $M_1 > 2M_\odot$. A CMD with masses chosen independently from the initial mass function given above does not have a clear second sequence around $3M_\odot$ where extreme mass ratios are more likely but does at low masses where systems tend to have equal mass components. To distinguish between the possible mass ratio distributions we note that we can employ statistical methods such as those described by Kroupa and Tout (1992) to investigate the binary content of Praesepe. In that work the colours and magnitudes were implicitly combined according to the prescription of section 2.

**ACKNOWLEDGMENTS**

CAT is very grateful to PPARC for support from an advanced fellowship. JH thanks Trinity College and the Cambridge Commonwealth Trust for their kind support. We thank Sverre Aarseth, Steinn Sigurdsson, Melvyn Davies...
Figure 3. Number density plot corresponding to the same regions as the CMD of figure 2. 1,000,000 randomly generated binaries are considered with no single stars. The grid is $200 \times 200$ with 10 contour levels evenly spaced between 0.05 and 0.95 after normalisation of the data to the maximum density.

and Rebecca Elson for the stimulating questions that have led to this contribution.

REFERENCES

Bergbusch P. A., VandenBerg D. A., Infante L., 1991, AJ, 101, 2102
Bettis C., 1975, PASP, 87, 707
Bolte M., 1991, ApJ, 376, 514
Dabrowski J. P., Beardsley W. R., PASP, 89, 225
Eggleton P. P., Fitchett M. J., Tout C. A., 1989, ApJ, 347, 998
Elson R. A. W., Sigurdsson S., Davies M., Hurley J., Gilmore G., 1998, MNRAS, accepted
Ferraro F. R., Carretta E., Fusi Pecci F., Zamboni A., 1997, A&A, 327, 598
Haffner H., Heckmann O., 1937, Gött. Veröff., 55, 77
Kaaumen J., Rucinski S. M., 1995, A&AS, 114, 1
Kroupa P., Tout C. A., 1992, MNRAS, 259, 223
Kroupa P., Tout C. A., Gilmore G., 1993, MNRAS, 262, 545
Kurucz, R. L., 1992, in Barbuy, B., Renzini, A., eds., The Stellar Populations of Galaxies, IAU Symp. 149, Kluwer, Dordrecht,

© 1998 RAS, MNRAS
Richer H. B., et al. 1997, ApJ, 484, 741
Romani R. W., Weinberg M. D., 1991, 372, 487
Rubenstein E. P., Bailyn C. D., 1997, ApJ, 474, 701
Santiago B. X., Elson R. A. W., Gilmore G. F., 1996, MNRAS, 281, 1363
Tout C. A., Pols, O. R., Eggleton, P. P., Han, Z., 1996, MNRAS, 281, 257