Based on Polarized Light Navigation: According to The Analysis of Bias of Polarized Angle Measurement

Xiaofang SHEN, Jun XU, Zhanrong ZHOU, Yang GAO, Jiating XU
Science college, The rockets army engineering university, Xi’an, China

Abstract. Polarized light navigation is a interesting research hotspot in the world today. People study the mechanism of polarized light navigation for a long time. However, less progress has been done on the navigation precision. This article discussed on azimuth calculation and polarized angle measurement. Polarized angle measuring system is designed to detect polarized light information. The bias of polarized angle measurement is analyzed based on the system, from qualitative and quantitative point of view. The quantitative influence on polarized angle measurement and polarized navigation are achieved.

1. Introduction

There exists a kind of insect named desert ant in Sahara. They must have some special talents so that they can survive in such abominable environment. It is inscrutable that they can hunt food far away from their nest and return accurately. The desert is extremely dry and landform changes quickly. Besides, their cerebra are only 0.1 milligram. So, desert ant does not have complex memory and judgment. The question is how they complete the long journey? Numerous evidences proved that life evolutions endow desert ants a mystical ability that can be used for orientation. The ability is making use of polarized light to determine direction.

With the development of science and technology, people already had a deep understanding on desert ant’s navigation gift. Now, fundamental principles have been discovered.

So far, many navigation ways are mature. Such as Geomagnetism navigation, Inertial navigation, satellite navigation and so on. However, new ways are constantly appear. Satellite navigation is the most advanced way to orientate and navigate right now. Nevertheless, considering part-conflict conditions in present-days, the satellite become the first attack target. So, depending on satellite only cannot fit in with the needs of future wars. New and better ways for orientation and navigation are what we need to study. Using the talent of desert ants to manufacture navigating instruments gives us a new choice.

Precision of every navigating ways are always given more attentions. Polarized orientation is especially so. It produces effects on if this way can be used in practical situation. The importance of studying the precision of polarized orientation is obvious. This article puts emphasis on the influence of the bias of polarized angle measurement to the precision of polarized navigation..

2. Polarized Navigation

2.1. Fundamental Principles of Polarized Navigation
As everyone knows, the position of the sun itself represents a kind of direction information. Polarized light, which produced while sun light passing through atmosphere, also takes direction information. It can be gained useful direction information if handling polarized light in right way. Two variables are included in polarized navigation. They are azimuth of sun and angle of navigation. Azimuth of sun $\theta_s$ is the angle between solar meridian and the longitude on the point of the observer. The angle of navigation $\mu$ is the angle between the trend of the observer and the solar meridian. The following is a sketch map of polarized navigation:

![Sketch map of polarized navigation](image)

The precision of $\theta_s$ and $\mu$ directly influence the accuracy of $\psi$. The position of the sun and the position of observer decide the azimuth of sun. If the position of the sun and the position of observer are known, the azimuth of sun is sure to some degree. We can conclude that the bias of polarized navigation mainly comes from the bias of the angle of navigation. Detailed contents are followed.

2.2. Measurement of Polarized Angle

The light in the sky is a mix of polarized light and natural light. So, polarized slice is needed when measuring polarized light. We just suppose that natural light intensity is $I_1$ and Lined polarized light intensity is $I_2$ and the angle between Lined polarized light and the polarized direction of polarized slice is $\theta$. We can measure the light intensity when the polarized direction of polarized slice is in accordance with the set direction. We can use the formula to express it.

$$I(\theta) = \frac{I_1}{2} + I_2 \cos^2 \theta$$

Revolving the polarized slice a circle, it can appear two maxima and two minima. The maximum and minimum can be written as $I_{\text{max}}$ and $I_{\text{min}}$. After analyzing:

$$I_{\text{max}} = \frac{I_1}{2} + I_2, \quad I_{\text{min}} = \frac{I_1}{2}, \quad \text{So, } \cos \theta = \sqrt{\frac{I(\theta) - I_{\text{min}}}{I_{\text{max}} - I_{\text{min}}}}.$$  At last, $\theta$ is known.

Since the light in the sky is partial-polarized light. If wanting to measure polarized angle, the separation of natural light and polarize light is the key. The polarized slice is the best choice. Only the direction of vibration is same to the polarized direction of polarized slice can pass through. Light filter can get rid of chaotic light. Light-electricity transformation equipment records light intensity. The polarized slice revolves by step-by-step motor. In this way, measuring maximum and minimum can be realized.

2.3. Angle of Navigation Analysis

Building an coordinate system for polarized navigation information calculation by supposing the direction of observer as Y axis and the top of sky as Z axis. Two polarized angle measurement direction are 1 and 2. The two directions are all in OYZ plane and separately have an angle $\sigma$ and $\tau$.
with Z axis. We can use θ1 and θ2 to replace polarized angles provisionally. In order to calculate conveniently, we make σ and τ all 45°. μ is the angle between the trend of the observer and the solar meridian. ν is the angle between the top of sky and the direction of the sun. Then, polarized vectors of two directions are:

\[
P_1 = (\sin \theta_1, -\frac{\sqrt{2}}{2} \cos \theta_1, \frac{\sqrt{2}}{2} \cos \theta_1), \quad Q_2 = (\cos \theta_2, \frac{\sqrt{2}}{2} \sin \theta_2, \frac{\sqrt{2}}{2} \sin \theta_2)
\]

Vector of the sun: \( S = (\sin \nu \sin \mu, \sin \nu \cos \mu, \cos \nu) \)

Due to OS is perpendicular to \( P_1 \) and \( Q_2 \). So:

\[
\tan \theta_1 = \frac{\sqrt{2} \sin \nu \cos \mu - \frac{\sqrt{2}}{2} \cos \nu}{\sin \nu \sin \mu}; \quad \tan \theta_2 = -\frac{\sin \nu \sin \mu}{\frac{\sqrt{2}}{2} \cos \nu + \frac{\sqrt{2}}{2} \sin \nu \cos \mu}
\]

Unite two formulas: \( \tan \mu = \frac{\sqrt{2} \cos \theta_1 \sin \theta_2}{\sin \theta_1 \sin \theta_2 - \cos \theta_1 \cos \theta_2} \)

In the end, μ is figured out.

Figure 3. Angle of Navigation calculation
3. Analysis on Precision of Polarized Navigation

Polarized angle measurement equipment is driven by stepper motor. Every time the motor takes an action, the equipment revolves an angle as $\varepsilon$. But the position maybe not the right place. So, the light intensity perhaps is not the real maximum or minimum. If the angle between ideal position and present position is $\omega$ when measure maximum. The light intensity:

$$I_{\text{max}} = \frac{1}{2} + I_2 \cos^2 \omega,$$

While measuring minimum, the angle is $\rho$. The light intensity:

$$I_{\text{min}} = \frac{1}{2} + I_2 \cos^2 \left( \frac{\pi}{2} - \rho \right),$$

It is no bias when measure light intensity which is the result of the set direction:

$$I(\theta) = \frac{1}{2} + I_2 \cos^2 \theta = b$$

In order to calculate conveniently, we choose the supreme bias $\frac{\varepsilon^2}{2}$.

$$I_{\text{max}} = \frac{1}{2} + I_2 \cos^2 \frac{\varepsilon}{2} = a, \quad I_{\text{min}} = \frac{1}{2} + I_2 \cos^2 \left( \frac{\pi}{2} - \frac{\varepsilon}{2} \right) = c,$$

However, from calculating:

$$\cos \theta = \sqrt{\frac{I(\theta) - I_{\text{min}}}{I_{\text{max}} - I_{\text{min}}}} = \sqrt{\frac{b - c}{a - c}}$$

In theory:

$$\cos \theta = \sqrt{\frac{b - c \cos^2 \frac{\varepsilon}{2} + a \sin^2 \frac{\varepsilon}{2}}{a - c}}, \quad \cos \varepsilon = 1 - \frac{\varepsilon^2}{2} \quad \text{choose the two terms in front},$$

$$\cos^2 \theta \cos \theta - \cos \theta = \frac{\varepsilon^2 (a + c)}{4(a - c)}.$$

After simplifying:

$$\cos 2 \theta - \cos 2 \theta' = \frac{\varepsilon^2 (a + c)}{2(a - c)}.$$

Figure 4. Curve of $\cos 2 \theta$

Because of $\cos 2 \theta - \cos 2 \theta'$ is constant, we choose smooth curve to calculate supreme bias.

Make $\theta'$ equal to $90^\circ$. So:

$$\theta = \frac{1}{2} ar \cos \left[ \frac{\varepsilon^2 (a + c)}{2(a - c)} - 1 \right].$$

Nowadays, it can be $0.009^\circ$ of the angle every time the motor acts. It is the most advanced stepping motor. In this situation, $a + c$ and $a - c$ are the same rate. $\frac{a + c}{a - c} \approx 1$. We can know: $\theta = 89.7421^\circ$, $\theta' - \theta = 0.2579^\circ$. The bias is already prominent. According to the foregoing formula:
\[
\tan(\mu + \Delta \mu) = -\frac{\sqrt{2} \cos \left( \theta_1 + \Delta \theta \right) \sin \left( \theta_2 + \Delta \theta \right)}{\cos \left( \theta_1 + \theta_2 + 2\Delta \theta \right)}
\]

\[
\tan (\mu + \Delta \mu) - \tan \mu = \frac{\sqrt{2} \cos \theta_1 \sin \theta_2}{\cos (\theta_1 + \theta_2)} - \frac{\sqrt{2} \cos (\theta_1 + \Delta \theta) \sin (\theta_2 + \Delta \theta)}{\cos (\theta_1 + \theta_2 + 2\Delta \theta)}
\]

After simplifying and reducing: \( \tan (\mu + \Delta \mu) - \tan \mu \approx \Delta \theta - \frac{1}{2} \Delta \theta^3 \).

From aforesaid information, \( \Delta \theta \) is 0.2579°. So: \( \Delta \theta - \frac{1}{2} \Delta \theta^3 = 0.2493 \). We limit that \( \mu \) is from 0 to 90°. In order to gain the supreme bias, we make \( \mu = 0 \). So, \( \Delta \mu = 13.9985^\circ \). We can conclude that the bias of polarized angle measuring will bring about the supreme bias of 10° to polarized navigation.

4. Analysis and Conclusion
The article has been researching the influences that the angle of navigation _ one of the two important factors has on the precision of polarized navigation. Due to the measuring instruments itself have imperfections that the calculation of angle of navigation has bias. It has been concluded that the bias of polarized angle measuring will bring about the supreme bias of 10° to polarized navigation. Amending parameter is the aim. The work will make polarized navigation more precise theoretically. Of course, the article only considers one factor of all. It is a reference for researchers.

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