Effective design features of rotor shafts

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Abstract. An important technological progress factor in mechanical engineering is improving structural and process solutions to production problems. The paper suggests a rational method of designs for hard shafts operating in a subcritical region, and for flexible shafts operating in a supercritical region.

A high rate of development of all mechanical engineering industries requires to improve the production process at industrial enterprises while solving a number of technical and organizational problems [1-18].

A large number of parts, such as shafts or rotors, have center-mounted unbalanced discs, e.g., in machines such as mills and sieves (Figure 1). During their operation, they perform not only rotational movement. The specificity is that, during rotation, a weight-bearing shaft axis bends and performs a precession movement [19].

The shaft center bending $y$ is determined by the following expression [20, 21]:

$$y = \frac{e}{\omega_{kp}^2} - 1,$$

where $e$ is the mass center eccentricity of the disc; $\omega$ is the rotational angular velocity of the disc; $\omega_{kp}$ is the critical angular velocity.

Figure 1. Diagram of a rotor with a center-mounted unbalanced disc.
The dependency of the shaft’s bending at the disc mounting point on the angular velocity is plotted in Figure 2 [21].

![Figure 2](image)

**Figure 2.** Shaft’s bending vs angular velocity.

Depending on the ratio of the critical $\omega_{kp}$ and working $\omega_p$ angular velocities, shafts are subdivided into flexible ones ($\omega_p > \omega_{kp}$), when balancing is not possible, and hard ones ($\omega_p < \omega_{kp}$). As $\omega_p > \omega_{kp}$ increases, the disc mass center approaches the rotational axis followed by the rotor’s self-centering.

If we do not take into account the gyroscopic effect, the shaft’s critical rotational velocity $\omega_{kp}$ is equal to the natural frequency of transverse (bending) oscillations of the system $p$ [19]:

$$ p = \omega_{kp} = \frac{k}{l} \sqrt{\frac{E \cdot J}{M \cdot l}}, $$

where $E \cdot J$ is the shaft’s bending stiffness;

- $l$ is the shaft length;
- $M$ is the disc mass;
- $k$ is a factor depending on the shaft support conditions ($k = 6.93$ – with hinged shaft ends support; $k = 13.85$ – with the shaft supports restraint).

The above dependencies were obtained without taking into account the shaft mass $m$. With comparable values of $m$ and $M$, the shaft mass should be taken into account. In a first approximation, we can assume that $m$ is concentrated in the shaft center. In this case, the value $M$ in expression (1) is the sum of the disc mass and the shaft mass, equaling to $(M + m)$.

Then, the shaft’s critical velocity value $\omega_{kp,m}$, taking into account its mass $m$, is determined using the following expression:

$$ \omega_{kp,m} = \frac{k}{l} \left( \frac{E \cdot J}{M \cdot l \left( 1 + \frac{m}{M} \right)} \right)^{1/2}. $$

Figure 3 shows a graph of the dependency of the critical velocity ratios $\omega_{kp,m}/\omega_{kp}$ on the ratio of the shaft mass to the disc mass $m/M$. 

The graph shows that taking into account the shaft mass reduces the design value of the rotor’s critical rotational velocity $\omega_{кр}$. Given the above, we can draw the following conclusions:

- For rotors with a hard shaft, the critical velocity $\omega_{кр}$ should be, where possible, higher than the working value $\omega_{кр,р}$ (see the graph in Figure 2, where $\omega_{кр,р} < 0.7\omega_{кр}$). It follows from the graph in Figure 3 that, in this case, the shaft mass $m$ should be as low as possible. With the set rotor parameters (diameter $D$, length $l$), its mass can be reduced by using a hollow shaft with a bore diameter $d$.

- For rotors with a flexible shaft, the critical rotational velocity $\omega_{кр}$ should be lower than the working value $\omega_{кр,р}$ ($\omega_{кр,р} < 1.3\omega_{кр}$). For this purpose, the shaft mass should be as low as possible, and in the system with preset parameters, a solid shaft should be used.

The quantitative analysis for choosing rational shaft parameters for a simplified design is confirmed by the analysis of an accurate solution of critical velocities for shafts with a distributed mass.

The critical rotational velocity for a shaft with a distributed mass, that is equal to the first natural bending vibration frequency, is determined by the following formula [22]:

$$p_p = \omega_{кр,р} = \frac{\lambda}{l} \sqrt{\frac{E \cdot J}{m}},$$

where $J = \frac{\pi D^4}{64}$ is the axial moment of the shaft section inertia;
$m$ is the mass of the shaft unit mass;
$\lambda$ is a factor depending on the boundary conditions ($\lambda = \pi^2 = 9.8$ for a hinged support; $\lambda = 22.4$ for the fixed ends of the shaft).

The shaft mass intensity is:

$$m = \rho \cdot A,$$

where $\rho$ is the shaft material density;
$A$ is the shaft’s cross-sectional area.
For a shaft with a solid round cross-section with a diameter $D$, the cross-sectional area is:

$$A = \frac{\pi D^2}{4}.$$

Then, using Formula (3), the critical rotational velocity of a shaft with a solid cross section is:
\[
\omega_{kp,c} = \frac{\lambda}{l^2} \left( \frac{E \cdot D^2}{16 \rho} \right)^{1/2}.
\]  

(6)

For a hollow shaft with an outer diameter \(D\) and an inner diameter \(d\), the cross-sectional area is:

\[
A = \frac{\pi D^2}{4} (1 - c^2),
\]

(7)

where \(c = \frac{d}{D}\) is the ratio of the shaft inner diameter \(d\) to the outer diameter \(D\).

The axial inertia moment for a hollow shaft is:

\[
J = \frac{\pi D^4}{64} (1 - c^2).
\]

(8)

By substituting (7) and (8) in Formula (3), we will have, after simple transformations, a formula for determining the critical rotational velocity for a hollow shaft:

\[
\omega_{kp,n} = \frac{\lambda}{l^2} \left( \frac{E \cdot D^2}{4 \rho} \left(1 + c^2\right) \right)^{1/2}.
\]

(9)

By comparing expressions (6) and (9), we can conclude that the critical rotational velocity for a hollow shaft is higher than that for a shaft of the same diameter with a solid cross section, that is:

\[
\omega_{kp,n} = \omega_{kp,c} \cdot \sqrt{1 + c^2}.
\]

Conclusion

For hard shafts operating in a subcritical region, it is rational to use a design with an inner bore, whereas it is advisable to make flexible shafts operating in a supercritical region with a solid cross section [23]. Due to the need for targeted use of the above shafts, it is desirable to mark them without disturbing the properties of the surface layer [24, 25].

These results can be illustrated by a plot of the bending dependencies \(y/e\) on the shaft’s rotational velocity \(\omega\) (Figure 4).

![Figure 4. Bending V/e vs rotational velocity \(\omega\).](image-url)
It follows from the graph that, for a hollow hard shaft, the operating region $\omega$ increases ($\omega < 0.7\omega_{kp} = 0.7\omega_{kp,n}$) with the increase in the value $\omega_{kp} = \omega_{kp,n}$. For a solid flexible shaft, the operating region $\omega$ increases ($\omega > 1.3\omega_{kp} = 1.3\omega_{kp,c}$) as the value $\omega_{kp} = \omega_{kp,c}$ decreases.

References

[1] Bratan, S., Roshchupkin, S. Synthesis of lunberger stochastic observer for estimation of the grinding operation state (2018) MATEC Web of Conferences, 224, paper No. 01133.
[2] Gorbatyuk, S.M., Gerasimova, A.A., Radyuk, A.G. Using the coating for the diffusion layer OBTAINING on the walls of the mold (CCM) (2015) Metallurgical and Mining Industry, 7 (9), pp. 1085-1088.
[3] Roshchupkin, S., Kharchenko, A. Method of building dynamic relations, estimating product and grinding circle shape deviations (2018) MATEC Web of Conferences, 224, paper No. 01001.
[4] Gerasimova, A.A., Radyuk, A.G., Tityyanov, A.E. Wear-resistant aluminum and chromonickel coatings at the narrow mold walls in continuous-casting machines 2016 Steel in Translation 46(7), p. 456-462
[5] Bratan, S., Roshchupkin, S., Revenko, D. Probabilistic Approach for Modeling Electroerosion Removal of Grinding Wheel Bond (2017) Procedia Engineering, 206, pp. 1426-1431. DOI: 10.1016/j.proeng.2017.10.656
[6] Gerasimova, A., Mishedchenko, O., Devyatiarova, V. Determination of temperature conditions for steel plate rolling at Vyksa Steel Works (AO VMZ) (2020) IOP Conference Series: Materials Science and Engineering, 709 (2), No. 022016. DOI: 10.1088/1757-899X/709/2/022016
[7] Keropyan, A., Gorbatyuk, S., Gerasimova, A. Tribotechnical Aspects of Wheel-Rail System Interaction (2017) Procedia Engineering, 206, pp. 564-569. DOI: 10.1016/j.proeng.2017.10.517
[8] Gorbatyuk, S.M., Morozova, I.G., Naumova, M.G. Reindustrialization principles in the heat treatment of die steels (2017) Steel in Translation 46(5), pp. 308-312. DOI: 10.3103/S0967091217050047
[9] Chichenev, N.A. Import-replacing re-engineering of the drive of the rollers in the intermediate roller table of a continuous bloom caster (2015) Metallurgist, 58 (9-10), pp. 892-895. DOI: 10.1007/s11015-015-0013-9
[10] Gorbatyuk, S., Kondratenko, V., Sedykh, L. Investigation of the Deep Hole Drill Stability When Using a Steady Rest (2019) MATERIALS TODAY-PROCEEDINGS DOI: 10.1016/j.matpr.2018.12.140
[11] Jordan, D. W., and Smith, P., 2007, Nonlinear Ordinary Differential Equations: An Intro-duction for Scientists and Engineers,4th edition, Oxford University Press, Oxford, UK
[12] Gorbatyuk, S., Pashkov, A., Chichenev, N. Improved Copper-Molybdenum Composite Material Production Technology (2019) Materials Today: Proceeding, 11, pp. 31-35. DOI: 10.1016/j.matpr.2018.12.102
[13] Kobelev OA, Albul SV, Kirillova NL (2020) Research and development of broaching methods on mandrel of large-sized pipe forgings. IOP Conference Series: Materials Science and Engineering 709(3):044104
[14] Osadchiy VA, Albul SV, Kuprienko NS, Kirillova NL (2020) Future developments of a roll forming mill design algorithm. IOP Conference Series: Materials Science and Engineering 709(3):044079
[15] Glukhov, L. M., Gorbatyuk, S. M., Morozova, I. G. and Naumova, M. G. (2016). Effective laser technology for making metal products and tools. Metallurgist, 60(3-4), pp. 306-312. doi:10.1007/s11015-016-0291-x.
[16] Slobodyanik, T.M., Balakhnina, E.E. Dynamics of elementary differential composed of elastic bodies (2019) Mining Informational and Analytical Bulletin, 2019 (9), pp. 204-210. DOI: 10.25018/0236-1493-2019-09-0-204-210
[17] Radyuk, A.G., Androsov, N.V., Kopylov, A.F., Glebovskij, A.E., Mazurov, V.M., Bokarev,
S. P. Mold reconditioning by gas-thermal coating (1998) Stal', (7), pp. 22-26.

[18] Bardovskii, A. D., Gerasimova, A. A., Keropyan, A. M., Bibikow, P. Y. Influence of the mechanical characteristics of harp screen material on screening process (2018) Izvestiya Ferrous Metallurgy, 61 (9), pp. 678-682. DOI: 10.17073/0368-0797-2018-9-678-682

[19] Gorbatyuk S, Kondratenko V, Sedykh L, Influence of critical speed when working shafts with symmetrically located monolithic weighting on the accuracy of work surfaces, Materials Today, Article MATPR10005, 13-AUG-2019, DOI: 10.1016/j.matpr.2019.07.695

[20] Birger I. A. i dr. Raschet na prochnost' detalei mashin: Spravochnik/ I. A. Birger, B. F. Shorr, G. B. Iosilevich. – 4-e izd., pererab. i dop. – M.: Mashinostroenie, 1993. – 640 s

[21] Busygin A.M « Power calculation of a crawler excavator mechanism with three degrees of freedom that is used in mining operations» // Mining information and analytical Bulletin (scientific and technical journal) – 2018. – №1. p.133–142.

[22] Biderman V.L. Teoriya mekhanicheskikh kolebanii: Uchebnik dlya vuzov. – URSS Fizikomatematicheskoe nasledie: fizika (mekhanika), 2017. – 416 s.

[23] Efremov, D. B., Gerasimova, A. A., Gorbatyuk, S. M., Chichenev, N. A. Study of kinematics of elastic-plastic deformation for hollow steel shapes used in energy absorption devices (2019) CIS Iron and Steel Review, 18, pp. 30-34. DOI: 10.17580/cisisr.2019.02.06

[24] Naumova, M. G., Morozova, I. G., Zaraapin, A. Y. and Borisov, P. V. (2018). Copper alloy marking by altering its surface topology using laser heat treatment. Metallurgist, 62(5-6), pp. 464-469. doi:10.1007/s11015-018-0682-2.

[25] A.A. Gerasimova, A.M. Keropyan, A.M. Girya. Study of the Wheel-Rail System of Open-Pit Locomotives in Traction Mode.// ISSN 1052-6188, Journal of Machinery Manufacture and Reliability, 2018. Vol. 47, № 1, pp. 35–38.