$B_d \to \pi^- K^{(*)+}$ and $B_s \to \pi^+(\rho^+)K^-$ decays with QCD factorization and flavor symmetry

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Abstract

The QCD factorization (QCDF) method usually contains infrared divergences which introduce large model dependence to its predictions on charmless B decays. The amplitudes of charmless B decays can be decomposed into “tree” and “penguin” parts which are conventionally defined, not from the topology of the dominant diagrams, but through their associated CKM factors $V_{ub}^*V_{ud}$ and $V_{tb}^*V_{td}$, respectively, with $q = d, s$. We find that for $B_{d,s} \to \pi^\pm K^\mp$ decays, the “tree” amplitude can be well estimated in QCDF with small errors, as the endpoint singularities have been canceled to a large extent. With this as the only input from QCDF and combined with flavor symmetry, the branching ratio of $B_s \to \pi^+ K^-$ is estimated to be significantly larger than the CDF measurement. This contradiction could be solved if the form factor $F_{B s K}$ is smaller than the light cone sum rules estimation or the “tree” amplitude has been over estimated in QCDF. The latter possibility could happen if charming penguins are nonperturbative and not small, as argued in soft collinear effective theory. To differentiate between these two possibilities, we examine the similar $B_s \to \rho^+ K^-$ decay with the same technique. It is found that a large part of the uncertainties are canceled in the ratio $\mathcal{B}(B_s \to \rho^+ K^-)/\mathcal{B}(B_s \to \pi^+ K^-)$. In QCDF, it is predicted to be $2.5 \pm 0.2$ which is independent on the form factor. However if charming penguins are important, this ratio could be very different from the QCDF prediction. Therefore the ratio of these two branching ratios could be an interesting indicator of the role of charming penguins in charmless B decays.

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I. INTRODUCTION

Charmless hadronic B decays provide various possibilities for the determination of the CKM angles, but often the efforts are hindered by the low energy QCD dynamics which may lead to large, sometimes even uncontrolled, systematic uncertainties. To proceed further, one may resort to either factorization methods or flavor symmetries. However it is known that here factorization only holds to the lowest order of power expansion in $1/m_b$, which means there is no (known) model-independent way to estimate power corrections in $1/m_b$ systematically. For flavor symmetries, in general SU(3) flavor symmetry breaking effects may also introduce large uncertainties, though isospin is a good symmetry at the few percent level. Recently, Descotes-Genon et al. [1] observed that, by the combined use of QCD factorization (QCDF) and flavor symmetries, it is possible to obtain new sum rules for $B_{d,s} \rightarrow KK$ decays with better control on systematic uncertainties. This, together with the first observation of $B_s \rightarrow \pi^+K^-$ by CDF Collaboration [2], stimulates us to investigate $B_d \rightarrow \pi^-K^{(*)+}$ and $B_s \rightarrow \pi^+(\rho^+)K^-$ decays in the spirit of Ref. [1].

Factorization aims to separate the calculable short distance effects from the nonperturbative long distance part. In addition, the nonperturbative structure can be simplified as form factors, decay constants and light-cone distribution amplitudes (LCDAs) in hadronic B decays at leading power expansion in $1/m_b$. For charmless B decays, there are three factorization approaches available in the market: QCDF [3–5], the perturbative QCD method (PQCD) [6–8] and soft collinear effective theory (SCET) [9–11]. We shall not discuss PQCD in this paper. For QCDF, phenomenologically power suppressed corrections such as chirally enhanced power corrections and annihilation topology have to be included to fit the experimental data. Unfortunately these terms contain endpoint singularities which break factorization and can only be estimated in a model-dependent way. In SCET, the chirally enhanced power corrections and weak annihilation diagrams are calculable [13, 14], while the diagrams with internal charm quark loops are claimed to be nonperturbative. Consequently these charming penguin amplitudes\footnote{It was first proposed in [15–17] that the charming penguins may play an important role in charmless B decays.} can only be determined by fitting to the data. Generally as weak annihilation and charming penguins have the same topology, it is hard in practice to tell which one is really important. In this paper we find that the ratio
\[ \mathcal{B}(B_s \to \rho^+ K^-)/\mathcal{B}(B_s \to \pi^+ K^-) \] may provide some insight into this issue.

This paper is organized as follows. In the next section, we first discuss \( B_{d,s} \to \pi^\pm K^\mp \) decays with a minimal use of QCDF combined with flavor symmetry. We then extend the discussions to \( B_d \to \pi^- K^{*+} \) and \( B_s \to \rho^+ K^- \) decays in the third section. Finally we conclude with a summary in section IV.

II. \( B_{d,s} \to \pi^\pm K^\pm \) DECAYS

Recently the CDF Collaboration reported new results on the branching ratios and direct CP violation for \( B^0_{d,s} \) decay channels \([2]\), among them is the first observation of \( B_s \to \pi^+ K^- \) decay:

\[ \mathcal{B}(B_s \to \pi^+ K^-) = (5.0 \pm 0.7 \pm 0.8) \times 10^{-6}, \] \[ A_{CP}(B_s \to \pi^+ K^-) = 0.39 \pm 0.15 \pm 0.08, \] where the first errors are statistical and the second systematic. It was first proposed in \([12]\) to determine the CKM angle \( \gamma \) from \( B_{d,s} \to \pi^\pm K^\pm \) decays using the U-spin symmetry. It was further noticed in \([18]\) that here the flavor symmetry breaking effects should be unusually small since the strong phases from final state interactions are exactly the same due to the charge conjugation symmetry of the final states. Therefore it could be a robust test of the Standard Model vs New Physics to check the equality of the direct CP asymmetries of these two decay channels. In this paper we will instead focus on their branching ratios, together with \( (B_d \to \pi^- K^{*+}, B_s \to \rho^+ K^-) \) decays, with a minimal use of QCDF combined with flavor symmetry\(^2\).

For \( \pi K \) channels, the decay amplitudes can usually be expressed as

\[ A(B_d \to \pi^- K^+) = A^d_{\pi K}(V_{ub}^* V_{us} T^d + V_{cb}^* V_{cs} P^d), \] \[ A(B_s \to \pi^+ K^-) = A^s_{\pi K}(V_{ub}^* V_{ud} T^s + V_{cb}^* V_{cd} P^s), \] with

\[ A^d_{\pi K} = \frac{G_F}{\sqrt{2}} (m_{B_d}^2 - m_{\pi}^2) f_K F^{B\pi}, \quad A^s_{\pi K} = \frac{G_F}{\sqrt{2}} (m_{B_s}^2 - m_{K}^2) f_\pi F^{B_s K}. \]

\(^2\)A recent analysis of \( B_{d,s} \to K \pi \) decays using flavor symmetry can be found in \([19]\).
$T_{d,s}$ and $P_{d,s}$, multiplied by a common factor $A_{πK}^{d,s}$, denote the “tree” and “penguin” amplitudes accompanied by the CKM factors $V_{ub}^*V_{us}$ and $V_{cb}^*V_{cd}$, respectively, with $q = d, s$. In QCDF, 

$$T = \alpha_1 + \alpha_4^u + \alpha_{4,ew}^u + \beta_3^u - \frac{1}{2}\beta_{3,ew}^u,$$

$$P = \alpha_4^c + \alpha_{4,ew}^c + \beta_3^c - \frac{1}{2}\beta_{3,ew}^c,$$  

(5)

where the weak annihilation contributions are contained in $\beta$ terms, while all the other parts are grouped into $\alpha$ terms, including the vertex corrections and the hard spectator scattering contributions. The explicit expressions of $\alpha$’s and $\beta$’s can be found in [5]. In QCDF, the endpoint singularities appear in annihilation topology and hard spectator scattering diagrams, which introduce significant uncertainties to $\alpha$’s and $\beta$’s. Therefore neither $T$ nor $P$ can be predicted reliably in QCDF with small errors. However, by implementing the unitarity condition of the CKM matrix, the above decay amplitudes can be reexpressed as

$$A(B_d \to π^- K^+) = A_{πK}^{d}(V_{ub}^*V_{us}\tilde{T}_d^d - V_{tb}^*V_{ts}P_d^d),$$  

(6)

$$A(B_s \to π^+ K^-) = A_{πK}^{s}(V_{ub}^*V_{ud}\tilde{T}_s^s - V_{tb}^*V_{td}P_s^s),$$  

(7)

with $\tilde{T} = T - P$. It was first observed in [1] that for $B_{d,s} \to K^0\bar{K}^0$ decays, although both $T$ and $P$ contain endpoint singularities, $\tilde{T}$ is infrared safe as the endpoint singularities in $T$ and $P$ cancel completely. Therefore $\tilde{T}$ can be estimated reliably in QCDF. However such case is rare in $B$ decays. For $B \to K\pi$ decays, using Eq. (5), one obtains

$$\tilde{T} = \alpha_1 + (\alpha_4^u - \alpha_4^c) + (\alpha_{4,ew}^u - \alpha_{4,ew}^c) + (\beta_3^u - \beta_3^c) - \frac{1}{2}(\beta_{3,ew}^u - \beta_{3,ew}^c).$$  

(8)

Interestingly, for every bracket in the above equation, the endpoint singularities are canceled exactly, so the only residual endpoint singularity lies in $\alpha_1$ which comes from the hard spectator scattering diagrams. Therefore, although $\tilde{T}$ in $B \to K\pi$ decays is not totally free of infrared divergence, in practice it could be estimated in QCDF with much smaller uncertainty.

Numerically, the errors of $\tilde{T}$ are dominated by the residual endpoint divergence from hard spectator scattering diagrams which is usually modeled as

$$X_H = \ln \left(\frac{m_B}{\Lambda_h}\right) \left(1 + \rho_H e^{i\phi_H}\right),$$  

(9)
and the parameter $\lambda_B$, which parameterizes the integral of the B meson wave function as

$$
\int_0^1 \frac{d\Phi_B^B(\xi)}{1-\xi} \equiv \frac{m_B}{\lambda_B}.
$$

To estimate the uncertainties, we will take $\lambda_B = 300 \pm 100$ MeV, $0 \leq \rho_H \leq 1$, $0 \leq \phi_H \leq 2\pi$ and adopt the default values of $[5]$ for all the other parameters. We then find

$$
|\tilde{T}^s| = 0.99^{+0.03}_{-0.05}, \quad |\tilde{T}^d| = 1.00^{+0.03}_{-0.04},
$$

which respects the U-spin symmetry $\tilde{T}^s = \tilde{T}^d$ at one percent level. Thus in the following we will not differentiate between $\tilde{T}^s$ and $\tilde{T}^d$ and simply take in the flavor symmetry limit

$$
|\tilde{T}^d| \simeq |\tilde{T}^s| \equiv \tilde{T} = 0.99^{+0.03}_{-0.05}.
$$

It is worth reminding that charming penguins, accompanied by the CKM factors $V_{cb}^* V_{cq}$, will contribute to both $\tilde{T}$ and $P$. So if charming penguins are nonperturbative and not small as argued in SCET, the above estimation of $\tilde{T}$ could change.

As mentioned earlier, in QCDF the “penguin” amplitude $P$ is significantly affected by the endpoint divergences and therefore hard to be estimated in a reliable way. So the above equation of $\tilde{T}$ is the only result that we will adopt from QCDF. In this sense $B_{d,s} \to K\pi$ decays are analyzed with “a minimal use of QCDF”. For all the rest we will resort to flavor symmetry. As observed in $[18]$, the flavor symmetry breaking effects in $B_{d,s} \to \pi^+K^\pm$ should be unusually small due to charge conjugation which is a strict symmetry for the strong phases from final state interactions. So by taking the factors $A_{\pi K}^{d,s}$ out which have included explicitly part of the U-spin symmetry breaking effects, it should be reasonable to take the approximation of the U-spin symmetry limit $P^d \simeq P^s \equiv P$. This amounts to neglect hard spectator scattering contributions sensitive to the difference between $B_d$ and $B_s$ distributions, and also annihilation contributions when the gluon is emitted from the spectator quark. Then the decay amplitudes can be rewritten as

$$
A(B_d \to \pi^- K^+) = A_{\pi K}^d(V_u e^{i\gamma}|\tilde{T}| - V_t |P|e^{i\delta}) ,
$$

$$
A(B_s \to \pi^+ K^-) = A_{\pi K}^s(V_u e^{i\gamma}|\tilde{T}| + V_t' e^{-i\beta} |P|e^{i\delta}) ,
$$

where $V_q \equiv |V_{qb}^* V_{qs}|$ and $V_q' \equiv |V_{qb'}^* V_{qd}|$ for $q = u, t$. $\delta$ is the relative strong phase between $P$ and $\tilde{T}$:

$$
P/\tilde{T} = -|P/\tilde{T}|e^{i\delta},$$

5
where the convention is chosen such that $\delta = 0$ in the naive factorization limit. It is then straightforward to get a well-known relation between the direct CP violations of $B_{d,s} \to \pi K$ decays

$$
A_{CP}(B_s \to \pi^+ K^-) = -\frac{\mathcal{B}(B^0 \to \pi^- K^+) \tau(B_s)(A_{sK}^d)^2}{\mathcal{B}(B_s \to \pi^+ K^-) \tau(B^0)(A_{sK}^s)^2}.
$$

(15)

Concerning the currently large experimental uncertainty of $A_{CP}(B_s \to \pi^+ K^-)$, this relation is well consistent within errors with the current data of $B_s \to \pi K$ in Eqs.(1,2) and of $B_d \to \pi K$.

![Image](image.png)

In this sense, there are three independent experimental observables: two branching ratios and one direct CP violation. If the related form factors are taken as input, there are also three unknowns: CKM angle $\gamma$, the "penguin" amplitude $|P|$, and the strong phase $\delta$. Then in principle, it should be possible to extract $\gamma$ from $B_{d,s} \to \pi^\pm K^\mp$ decays, if the QCDF knowledge of $\tilde{T}$ is reliable. However in practice, it is hard to obtain a strong constraint on the CKM angle $\gamma$ through this way, especially considering the uncertainties of the input parameters and also the mild dependence of the branching ratios on the angle $\gamma$. As currently no clear evidence of new physics beyond SM exists in rare B decays, we will instead take the global fit determination of $\gamma = (67.9^{+4.3}_{-3.8})^\circ$ from CKMfitter [21] as input. Then our focus is what information one may obtain about the soft QCD dynamics in $B \to K \pi$ decays.

Now only the "penguin" amplitude $|P|$ and the strong phase $\delta$ are unknown, which can be extracted solely from the observables of $B_d \to \pi^- K^+$ decay in Eq. (16). With the CKM parameters taken from CKMfitter [21]:

$$
\lambda = 0.2251, \quad |V_{ts}| = 0.04042^{+0.00037}_{-0.00118}, \quad |V_{ub}| = 0.00351^{+0.00015}_{-0.00016},
$$

(17)

, the determination of $|V_{ub}|F^{B\pi} = (9.1 \pm 0.7) \times 10^{-4}$ [22] from the semi-leptonic $B \to \pi \ell \bar{\nu}$ decay and $\tau_{B_d} = 1.525 \pm 0.009$ ps [20], we obtain

$$
|P| = 0.129 \pm 0.012, \quad \delta = -(18.9 \pm 2.9)^\circ
$$

(18)

with the uncertainties coming mainly from the variation of $|V_{ub}|F^{B\pi}$. Notice that the determination of $|P|$ is rather insensitive to the change of $\tilde{T}$. For instance, about 20% change of $|\tilde{T}|$ from 0.99 to 0.80, with all the other parameters unchanged, will lead to only 1% variation of the central value of $|P|$ from 0.129 to 0.128. This is because, as a penguin dominant
TABLE I: Input parameters in numerical calculations of $B_s \to \pi^+ K^-$ decay. The CKM parameters are taken from CKMfitter \cite{21}, The $B$-meson lifetimes and the branching ratio are taken from \cite{20} and the form factors are given in \cite{22,23}.

decay, the branching ratio of $B_d \to \pi^- K^+$ decay largely determines the magnitude of $P$, irrespective of how much the charming penguins may contribute to $P$.

It is also known that $B^+ \to \pi^+ K^0$ decay is almost pure penguin process, in addition its penguin amplitude is equal to that of $B_d \to \pi^- K^+$ decay except for the negligible color-suppressed electroweak penguin contributions. As a consistency check, we do find that the penguin amplitude of $B^+ \to \pi^+ K^0$ is $|P| = 0.130 \pm 0.012$, which is in well agreement with Eq. (18). Therefore the above estimation of the “penguin” amplitude $|P|$ in Eq. (18) should be, up to few percent, independent on whether the charming penguins are important or not.

Taking Eq. (18) as input, we can now estimate the branching ratio of $B_s \to \pi^+ K^-$ decay. The CP-averaged amplitude square can be expressed as

$$|A(B_s \to \pi^+ K^-)|^2 = (A_{\pi K}^e)^2 [|V_u^\prime T|^2 \left(1 + \frac{|V_u^\prime P|^2}{|V_u^\prime T|^2} + 2 \frac{|V_u^\prime P|}{|V_u^\prime T|} \cos \delta \cos (\beta + \gamma) \right) + \frac{|V_d^\prime T|^2}{|V_u^\prime T|^2}].$$

Due to the accidental fact that $\beta + \gamma \sim 90^\circ$, the “tree”-“penguin” interference term in $B_s \to \pi^+ K^-$ decay is negligibly small. Thus approximately the strong phase $\delta$ does not affect the branching ratio and one has

$$|A(B_s \to \pi^+ K^-)|^2 \approx (A_{\pi K}^e)^2 \left(|V_u^\prime T|^2 + |V_d^\prime T|^2 \right).$$

If one observes that the $|P|^2$ term can be directly related to the pure penguin process $B^+ \to \pi^+ K^0$ as

$$\left(\frac{V_d^\prime}{V_t} A_{\pi K}^e\right)^2 |A(B^+ \to \pi^+ K^0)|^2,$$

numerically one can then obtain

$$\mathcal{B}(B_s \to \pi^+ K^-) = \left(\frac{F_{B_s K}}{F_{B\pi}}\right)^2 \left[(6.2 \pm 1.0)|T|^2 + 0.6\right] \times 10^{-6},$$

\[7\]
with the input parameter listed in Table I. Here the uncertainty of $|P|$ term, which can be estimated by using Eq. (21), is much smaller than that of $\tilde{T}$ term and has been therefore neglected. We want to stress that Eq. (22) is not just valid in QCDF, but rather be a model independent result to a large extent.

The form factors ratio has been estimated in light-cone sum rules [23], as shown in Table I. As discussed earlier, $\tilde{T}$ can also be estimated in a relatively reliable way in QCDF in Eq. (11), then we get

$$B^{th}(B_s \rightarrow \pi^+ K^-) = 8.8^{+1.4+2.8}_{-1.4-1.3} \times 10^{-6},$$

where the first error comes mainly from the uncertainties of $|V_{ub}|F_{B\pi}$ and $\tilde{T}$, while the second error comes from the form factors ratio. This estimation is significantly larger than the CDF measurement $(5.0 \pm 1.1) \times 10^{-6}$.

Notice that $|V_{ub}|F_{B\pi}$ is extracted from $B \rightarrow \pi \ell \nu$ decays in a model independent way. Thus if the updated experimental measurement does not change greatly, it must be that either the form factors ratio is much smaller than the light-cone sum rules prediction or $\tilde{T}$ is overestimated in QCDF, or both. The possibility of a smaller ratio of the form factors has been discussed in a flavor symmetry analysis [19] and also in a recent QCDF analysis [24, 25] where the central values of $F^{B_s K} = 0.24$ and $F^{B\pi} = 0.25$ are taken. The latter possibility of a smaller $\tilde{T}$ can not be easily realized in QCDF, though it could happen in SCET if charming penguins are not small. In practice, however, it is difficult to tell experimentally which possibility is chosen by nature. This is because $\tilde{T}$ is not directly observable in charmless B decays. For the form factor $F^{B_s K}$, in principle it could be determined via semileptonic $B_s \rightarrow K\ell\nu$ decays. But this measurement is not easy at hadron colliders, such as the Tevatron and the LHC. For the future super B factories, the challenge is to accumulate enough $B_s$ mesons for precise measurement of the kaon momentum spectrum of this semileptonic decay. $F^{B_s K}$ may also be predicted by lattice QCD computations at small recoil, while unfortunately charmless B decays happen at large recoil.

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3 A direct QCDF calculation in [5] gives $10.2^{+1.5+3.8+0.7+0.8}_{-3.9-3.2-1.2-0.7} \times 10^{-6}$ as their “default results”. Here the first error corresponds to the variations of the CKM factors and the second error comes mainly from the variation of the form factor. The readers are referred to [5] for more details of the analysis.
III. $B_d \to \pi^- K^{*+}$ AND $B_s \to \rho^+ K^-$ DECAYS

To find a way out of this problem, we may examine similar decay channels $B_d \to \pi^- K^{*+}$, $B_s \to \rho^+ K^-$ and $B^+ \to \pi^+ K^{*0}$. At first sight, it may seem better to discuss $B_{d,s} \to \rho^\pm K^{\mp}$ decays as the final states are related by charge conjugation symmetry. However these two decay channels are actually not related by U-spin symmetry. Again we may express the relevant decay amplitudes in the following form:

$$A(B_d \to \pi^- K^{*+}) = A_{\pi K^*} (V_u e^{i\gamma} |\tilde{T}_{K^*}| - V_t |P_{K^*}| e^{i\delta_{K^*}}) ,$$

(24)

$$A(B_s \to \rho^+ K^-) = A_{\rho K} (V'_u e^{i\gamma} |\tilde{T}_{\rho}| + V'_t e^{-i\beta} |P_{\rho}| e^{i\delta_{\rho}}) ,$$

(25)

with

$$A_{\pi K^*} = \sqrt{2} G_F m_{B_d} p_c f_{K^*} F_{B\pi} , \quad A_{\rho K} = \sqrt{2} G_F m_{B_s} p_c f_{\rho} F_{B_s K} .$$

Here $p_c$ is the c.m. momentum of the final state mesons. In QCDF, the expression of $\tilde{T}_{\rho}$ is formally the same as Eq. [8] of $\tilde{T}$, though the explicit formulae of the coefficients $\alpha$’s and $\beta$’s are different which can also be found in [5]. Since now the final states $\pi K^*$ and $\rho K$ are not related to each other under charge conjugation, there is no reason to expect that here the U-spin symmetry breaking effects would be particularly small. This means the difference between $(\tilde{T}_{\rho}, P_{\rho}, \delta_{\rho})$ and $(\tilde{T}_{K^*}, P_{K^*}, \delta_{K^*})$ could reach the level of 20% and can not be simply ignored. But if we focus only on the $B_s \to \rho^+ K^-$ decay, the strong phase $\delta_{\rho}$ again does not affect the CP-averaged branching ratio

$$|A(B_s \to \rho^+ K^-)|^2 \simeq (A_{\rho K}^s)^2 \left(|V'_u|^{2} + |V'_t P_{\rho}|^2\right)$$

(26)

in good approximation due to the accidental fact that $\beta + \gamma \simeq 90^\circ$, just like the case of $B_s \to \pi^+ K^-$ decays. Similarly we find $|\tilde{T}_{\rho}| = 0.95^{+0.03}_{-0.05}$ with the uncertainties mainly from $\lambda_B$ and the hard spectator parameters $\rho_{PV}^H$ and $\phi_{PV}^H$. We will soon find the ratio $\tilde{T}_{\rho}/\tilde{T}$ to be useful. As they share the common parameter $\lambda_B$, their errors are partly correlated and the uncertainty related to $\lambda_B$ will be canceled in the ratio. But the hard spectator parameters are independent for $B_s \to PP$ and $B_s \to PV$ decays. Concerning this, we find the ratio to be

$$\left|\frac{\tilde{T}_{\rho}}{\tilde{T}}\right| = 1.00 \pm 0.04$$

with the uncertainty coming dominantly from the variation of hard spectator parameters.
Then the last missing piece required is the “penguin” amplitude $|P_\rho|$. Notice that $B_s \to \rho^+ K^-$ decays are dominated by the “tree” amplitude $\tilde{T}_\rho$. Furthermore it is known that generally the penguin amplitudes of $B \to PV$ decays are smaller than those of $B \to PP$ decays. As the $|P|$ term contribute about 10% of the final branching ratio of $B_s \to \pi^+ K^-$ decay, it is expected that the contribution of $|P_\rho|$ term should be even smaller. Therefore for such a small term, an estimation based on SU(3) flavor symmetry should be enough which will not introduce large uncertainty to the final branching ratio result.

In the flavor symmetry limit, one has $|P_\rho| = |P_K^*|$. Then one may estimate $|P_K^*|$ from either $B_d \to \pi^- K^{*+}$ decay or the almost pure penguin $B^+ \to \pi^+ K^{*0}$ decay, just like the case of $B \to \pi K$ decays. Taking $f_\rho = 216$ MeV, $f_{K^*} = 220$ MeV and the parameters listed in Table I, we find $|P_K^*| = 0.0663^{+0.0068}_{-0.006}$ using [20].

$$B(B_d \to \pi^- K^{*+}) = (8.6 \pm 0.9) \times 10^{-6}, \quad A_{CP}(B_d \to \pi^- K^{*+}) = (-18 \pm 7)\% , \quad (27)$$

which is consistent with the determination of $|P_K^*| = 0.0643^{+0.007}_{-0.006}$ from $B(B^+ \to \pi^+ K^{*0}) = (9.9 \pm 0.8) \times 10^{-6}$ [20]. Then we obtain

$$|A(B_s \to \rho^+ K^-)|^2 \simeq (A_{\rho K}^s)^2 \left( |V_u^\dagger \tilde{T}_\rho| \right)^2 + \left( V^t V^\prime \frac{|A(B^+ \to \pi^+ K^{*0})|}{A_{\pi K^*}} \right)^2 . \quad (28)$$

Numerically the corresponding branching ratio can then be estimated

$$B(B_s \to \rho^+ K^-) = \left( \frac{F_{B_s K}^{B_{s K}}}{F_{B_s \pi}} \right)^2 \left[ (15.7 \pm 2.4)|\tilde{T}_\rho|^2 + 0.4 \right] \times 10^{-6} . \quad (29)$$

where the uncertainty from $|P_\rho|$ term is much smaller than that from $\tilde{T}_\rho$ term and has therefore been ignored. We want to stress again that Eq. (29), just like Eq. (22), is a model independent result to a large extent.

Notice that $B_s \to \pi^+(\rho^+)K^-$ decays depend upon many common parameters such as $F_{B_s K}^{B_s K}$, $|V_{ub}| F_{B\pi}$ etc., their uncertainties are highly correlated. It is then clear that the ratio of the branching ratios should have much smaller error. Actually the error of the ratio depends almost solely on $\tilde{T}_\rho/\tilde{T}$

$$\frac{B(B_s \to \rho^+ K^-)}{B(B_s \to \pi^+ K^-)} \approx \frac{f_\rho^2}{f_{K^*}^2} \left( \frac{|\tilde{T}_\rho|^2}{|\tilde{T}|^2} \right) \frac{1 + 0.4/(15.7|\tilde{T}_\rho|^2)}{1 + 0.6/(6.2|\tilde{T}|^2)} \simeq 2.5 \left( \frac{|\tilde{T}_\rho|}{|\tilde{T}|} \right)^2 . \quad (30)$$

In the above estimation, we have adopted flavor symmetries to relate the “penguin” amplitude of $B_s \to \pi^+(\rho^+)K^-$ to that of $B_d \to \pi^- K^{(*)+}$, which may introduce 20% level of uncertainty. But fortunately as tree-dominant decays, the ratio $B(B_s \to \rho^+ K^-)/B(B_s \to \pi^+ K^-)$
is not sensitive to the variation of the relevant “penguin” amplitudes. As an illustration, we assign conservatively 30% uncertainty to the “penguin” amplitudes to take account of the flavor symmetry breaking effects, and adopt the QCDF estimation $|\tilde{T}_p/\tilde{T}| = 1.00 \pm 0.04$, it is found that

$$\frac{\mathcal{B}(B_s \to \rho^+ K^-)}{\mathcal{B}(B_s \to \pi^+ K^-)} \bigg|_{QCDF} \simeq 2.5 \pm 0.1 \pm 0.2$$

(31)

with the first error comes from the flavor symmetry breaking effects and the second error comes from the variation of $|\tilde{T}_p/\tilde{T}|$. While in SCET, if the charming penguins are not small, it could alter $\tilde{T}$ and $\tilde{T}_p$. It is worth reminding that there is no known reason, at least to our knowledge, to expect that the charming penguins of $B_s \to \pi^+ K^-$ and $\rho^+ K^-$ channels should be the same. This means the ratio $\mathcal{B}(B_s \to \rho^+ K^-)/\mathcal{B}(B_s \to \pi^+ K^-)$ in SCET could be quite different from Eq. (31). For example, a recent global fit analysis of charmless B decays using SCET gives $[26]$ (in units of $10^{-6}$)

$$\mathcal{B}(B_s \to \pi^+ K^-) = 5.7^{+0.6+0.5}_{-0.6-0.5}, \quad \mathcal{B}(B_s \to \rho^+ K^-) = 7.6^{+0.3+0.8}_{-0.1-0.8} \quad \text{(Solution I)}$$

(32)

$$\mathcal{B}(B_s \to \pi^+ K^-) = 5.5^{+0.6+0.5}_{-0.6-0.4}, \quad \mathcal{B}(B_s \to \rho^+ K^-) = 10.2^{+0.4+0.9}_{-0.5-0.9} \quad \text{(Solution II)}$$

(33)

In these two solutions, the ratio $\mathcal{B}(B_s \to \rho^+ K^-)/\mathcal{B}(B_s \to \pi^+ K^-)$ equals to 1.3 or 1.9, significantly different from the QCDF prediction $2.5 \pm 0.2$. Therefore a precise measurement of this ratio in the near future could be an interesting test of the importance of the charming penguins in charmless B decays.

IV. SUMMARY

In this paper we first studied $B_{d,s} \to \pi^\pm K^\mp$ decays using QCDF combined with flavor symmetry. In general, the QCDF calculations contain infrared divergence when weak annihilation topology and hard spectator scattering diagrams are included. But if we express the decay amplitude as

$$A(B_s \to \pi^+ K^-) = A_{\pi K}^s (V_{us}^* V_{ud} \tilde{T} \tilde{V}_{tb} V_{td} P),$$

The “tree” amplitude $\tilde{T}$ can be (relatively) reliably estimated in QCDF to be $0.99^{+0.03}_{-0.05}$. This is because the endpoint singularities inside $\tilde{T}$ has been canceled to a large extent. In

\footnote{The results of $\pi^+ K^-$ were provided by Wei Wang, one author of [26], in a private communication.}
addition, the CP-averaged branching ratio is insensitive to the relative strong phase between \( \tilde{T} \) and \( P \) because the interference term is proportional to \( \cos(\beta + \gamma) \) which is (accidently) very close to zero. Noticed that the final states of \( B_{d,s} \to \pi^\pm K^\pm \) are invariant under charge conjugation, implying that the flavor symmetry breaking effect should be unusually small as pointed out in [18]. So the penguin amplitude \(|P|\) can be determined from \( B_d \to \pi^- K^+ \) decay with small error. Then we obtain

\[
B(B_s \to \pi^+ K^-) = \left( \frac{F_{B_s K}}{F_{B^0\pi}} \right)^2 \left( 6.2 \pm 1.0 \right) \left| \tilde{T} \right|^2 + 0.6 \times 10^{-6} .
\]  

With the light cone sum rules estimation [23] \( F_{B_s K}/F_{B^0\pi} = 1.15^{+0.17}_{-0.09} \) and the QCDF prediction of \( \tilde{T} \), the expected branching ratio would be \( 8.8^{+3.1}_{-1.9} \times 10^{-6} \), much larger than the CDF observation \( (5.0 \pm 1.1) \times 10^{-6} \). One possibility is that the form factors ratio \( F_{B_s K}/F_{B^0\pi} \) has been overestimated by the light cone sum rules calculation. But it is also possible that the QCDF prediction of \( \tilde{T} \) is too large. This could happen, for example, if the charming penguins play an important role in charmless B decays.

To differentiate between these two possibilities, we examined the similar decay channels \( B_s \to \rho^+ K^- \), \( B_d \to \pi^- K^{*+} \) and \( B^+ \to \pi^+ K^{*0} \) with the same method. We find a large part of uncertainties has been canceled in the ratio \( B(B_s \to \rho^+ K^-)/B(B_s \to \pi^+ K^-) \) which equals approximately to be \( 2.5 \left| \tilde{T}_\rho/\tilde{T} \right|^2 \). In QCDF, \( \left| \tilde{T}_\rho/\tilde{T} \right| \) is also well predicted to be about one, so we get an interesting prediction \( B(B_s \to \rho^+ K^-)/B(B_s \to \pi^+ K^-) = 2.5 \pm 0.1 \pm 0.2 \) in QCDF, with the first error reflects the flavor symmetry breaking effects and the second error reflects the uncertainty of \( \left| \tilde{T}_\rho/\tilde{T} \right| \). However if charming penguins are not small, this ratio could vary significantly from 2.5, which means the ratio of these two branching ratios could be useful to clarify the role of charming penguins in charmless B decays.

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