Parity nonconservation in dielectronic recombination of multiply charged ions

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Abstract. We discuss a parity nonconserving (PNC) asymmetry in the cross section of dielectronic recombination of polarized electrons on multiply charged ions with $Z \gtrsim 40$. This effect is strongly enhanced for close doubly-excited states of opposite parity in the intermediate compound ion. Such states are known for He-like ions. However, these levels have large energy and large radiative widths which hampers observation of the PNC asymmetry. We argue that accidentally degenerate states of the more complex ions may be more suitable for the corresponding experiment.

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1. Introduction

First discussion of parity nonconservation (PNC) effects in multiply charged ions (MCI) appeared soon after formulation of electroweak theory (Gorshkov & Labzovskii 1974). Since then many authors attempted to suggest realistic experiments (Schafer et al. 1989, Karasiov et al. 1992, Zolotorev & Budker 1997). Recently PNC asymmetry in electron scattering was calculated by Milstein & Sushkov (2005). Here we focus on the PNC asymmetry in dielectronic recombination (DR) on MCI. A particular case of DR on H-like ions has been discussed in Gribakin et al. (2005). PNC asymmetry there is strongly enhanced because of the near-degeneracy of doubly-excited $2\ell/2\ell'$ states of opposite parity in He-like ions. Similar enhancement can take place for other ions if there are close levels of opposite parity. In this paper we use expressions for the PNC asymmetry derived by Gribakin et al. (2005) to analyze the optimal conditions for observation of the PNC effects in DR and compare them with the H-like ions. We show that if suitable levels in non-hydrogenic ions are found, the statistical sensitivity of the PNC experiment can be improved by several orders of magnitude.
2. Feasibility of PNC experiment

In this section we briefly outline the main results from Gribakin et al. (2005). The feasibility of measuring the PNC effect in DR depends on the sensitivity requirements for an experimental apparatus to observe the PNC asymmetry

\[ A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}, \]  

where \( \sigma^\pm \) are the recombination cross sections for electrons with positive and negative helicity. One can consider two limiting cases: the monoenergetic beam and the beam with wide energy distribution in comparison to the energy difference between the DR resonances of opposite parity \( |E_1 - E_2| \). For the most interesting case of the very close resonances the typical experimental conditions are likely to be closer to the latter case. However, it is easier to start with the former, simpler case.

The number of counts in an ideal experiment with a fully polarized electron beam with positive helicity is given by:

\[ N_+ = j_e N_i t \sigma^+ \equiv I \sigma^+, \]  

where \( j_e \) is the electron flux, \( N_i \) is the number of target ions, \( t \) is the acquisition time. The number of counts for negative helicity is \( N_- = \sqrt{N_+ + N_-} \).

For a beam with polarization \( P \), to detect the PNC asymmetry, the difference between the counts, \( P |N_+ - N_-| \) should be greater than statistical error, \( \sqrt{N_+ + N_-} \), which gives:

\[ I > \frac{\sigma^+ + \sigma^- + 2\sigma_b}{P^2(\sigma^+ - \sigma^-)^2}, \]  

where \( \sigma_b \) is the magnitude of any background occurring through direct radiative recombination or as an experimental artifact (e.g. detector dark counts). This is the effective cross section to which the apparatus background would correspond. For the rest of this analysis we consider the ideal limit, \( P = 1, \sigma_b = 0 \).

If the electron energy spread in the beam is greater than the resonance spacing and widths, then the flux \( j_e \) in (2) should be replaced by the flux density \( dj_e/d\varepsilon \). The counts \( N_\pm \) are obtained by integrating over the electron energy and the effect can be detected if

\[ \mathcal{I}_{av} > \int (\sigma^+ + \sigma^-) d\varepsilon \left( \int (\sigma^+ - \sigma^-) d\varepsilon \right)^{-2}. \]  

The first integral above is equal to \( 2(S_1 + S_2) \), where

\[ S_i = \frac{\pi^2}{2p^2} \frac{\Gamma_i^{(r)} \Gamma_i^{(a)}}{\Gamma_i}, \]  

is the strength of resonance \( i \) (we use atomic units). In this expression \( p \) is the momentum of the incident electron, \( \Gamma_i^{(a)} \) and \( \Gamma_i^{(r)} \) are autoionizing and radiative widths of the resonances of opposite parity, \( \Gamma_i = \Gamma_i^{(a)} + \Gamma_i^{(r)} \) being their total widths.
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The integral \( \int (\sigma^+ - \sigma^-) d\varepsilon \) in (4) can be written as \( 2S_{1,2}^{\text{PNC}} \), where

\[
S_{1,2}^{\text{PNC}} \equiv \int \sigma_{\text{PNC}}^{\text{av}} d\varepsilon \approx -\frac{\pi^2}{p^2} \sqrt{\Gamma_1^{(a)} \Gamma_2^{(a)} h_{\text{PNC}}^{(a)}} \frac{(\Gamma_1^{(r)}/\Gamma_1 + \Gamma_2^{(r)}/\Gamma_2)}{(E_1 - E_2)^2 + \frac{1}{2}(\Gamma_1 + \Gamma_2)^2} \times [(E_1 - E_2) \cos \delta_{1,2} + \frac{1}{2}(\Gamma_1 + \Gamma_2) \sin \delta_{1,2}],
\]

is the PNC strength of the two resonances. Here \( h_{\text{PNC}}^{(a)} \) is the PNC matrix element which mixes the resonances, \( E_i \) are their energies, and \( \delta_{1,2} \) is the relative Coulomb phase between two channels (see Gribakin et al. (2005) for details). Equation (4) now reads:

\[
I_{\text{av}} > \frac{1}{2}(S_1 + S_2)/(S_{1,2}^{\text{PNC}})^2.
\]

We see that for the limiting case of wide energy distribution in the beam the feasibility of the PNC experiment depends on the function:

\[
F_{\text{av}} = \int (\sigma^+ + \sigma^-) d\varepsilon \left[ \int (\sigma^+ - \sigma^-) d\varepsilon \right]^{-2}.
\]

Optimal experimental conditions correspond to the minimum of the function \( F_{\text{av}} \). In the next section we examine how this function depends on the parameters of the resonances.

3. Optimization

Substituting equations (5) and (6) in (8) we obtain an explicit expression for \( F_{\text{av}} \):

\[
F_{\text{av}} = \frac{p^2}{4\pi^2} \frac{\Gamma_1^{(r)} \Gamma_1^{(a)} / \Gamma_1 + \Gamma_2^{(r)} \Gamma_2^{(a)} / \Gamma_2}{(h_{\text{PNC}}^{(a)})^2 \left( \Gamma_1^{(r)}/\Gamma_1 + \Gamma_2^{(r)}/\Gamma_2 \right)^2} \left( \frac{\Delta^2 + \Gamma^2}{\Delta \cos \delta_{1,2} + \Gamma \sin \delta_{1,2}} \right)^2,
\]

where \( \Delta \equiv E_1 - E_2 \) and \( \Gamma \equiv \frac{1}{2}(\Gamma_1 + \Gamma_2) \). Function \( F_{\text{av}} \) has dimension area\(^{-1} \times \)energy\(^{-1} \). It is convenient to write it as a product, \( F_{\text{av}} = F_1 \times F_2 \times F_3 \), where \( F_1 \) is a dimensional factor, and \( F_3 \) depends only on \( \Delta \) and \( \Gamma \), but not on \( \Gamma_1^{(a)} \) or \( \Gamma_1^{(r)} \):

\[
F_1 = p^2 \frac{\Gamma^2 (2\pi h_{\text{PNC}})^{-2}}{\Delta^2 + \Gamma^2},
\]

\[
F_2 = \frac{\Gamma \left( \Gamma_1^{(r)} \Gamma_1^{(a)} / \Gamma_1 + \Gamma_2^{(r)} \Gamma_2^{(a)} / \Gamma_2 \right)}{(\Gamma_1^{(a)} \Gamma_2^{(a)})^2 \left( \Gamma_1^{(r)}/\Gamma_1 + \Gamma_2^{(r)}/\Gamma_2 \right)^2},
\]

\[
F_3 = \left( \Delta^2 + \Gamma^2 \right)^2 \Delta^{-2} (\Delta \cos \delta_{1,2} + \Gamma \sin \delta_{1,2})^{-2}.
\]

Since the three factors above depend on different parameters, it is convenient to analyze them separately. The dimensional factor (10a) depends on the PNC matrix element, the average DR width \( \Gamma \) and the electron momentum \( p \), which is defined by the energy difference between the ground state \( E_0 \) of the target ion and the autoionizing resonances, \( p^2/2 = E_{1,2} - E_0 \). Thus, the optimal conditions correspond not only to the close levels of opposite parity, but also to the lowest possible energy of the beam. For H-like ions
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$p^2 \approx Z^2/2$ and $|\hbar^{\text{PNC}}| \approx 1.1 \times 10^{-18} Z^5$ (Gribakin et al. 2005). Thus, the overall scaling of the first factor is:

$$F_1(\text{H-like}) \approx 1.0 \times 10^{34} Z^{-8} \Gamma.$$  \hspace{1cm} (11)

This expression shows that it would be much easier to observe PNC effects in heavy ions. Below we will focus on $Z > 30$, where $\Gamma$ is dominated by radiative widths $\Gamma_{1,2}^{(r)}$ and grows rapidly with $Z$:

$$\Gamma \approx 0.76 \times 10^{-8} Z^4,$$  \hspace{1cm} (12)

which yields the following estimate:

$$F_1(\text{H-like}) \approx 0.76 \times 10^{34} Z^{-4}.$$  \hspace{1cm} (13)

We see that because of the growth of $p^2$ and $\Gamma$ with $Z$, the overall scaling of $F_1$ is much weaker than one might expect from the scaling of the matrix element $\hbar^{\text{PNC}}$ alone. For non-hydrogenic MCI this matrix element is of the same order of magnitude. Therefore, if one could find low-lying autoionizing levels of opposite parity, it would be possible to reduce both $p$ and $\Gamma$ in Equation (10a).

The two remaining factors, $F_2$ and $F_3$, are dimensionless. Factor $F_3$ depends on the relation between $\Gamma$ and $\Delta$: $F_3 \approx \sin^{-2} \delta_{1,2}$ for $\Gamma \gg \Delta$, and $F_3 \approx (\Delta/\Gamma)^2 \cos^{-2} \delta_{1,2}$ for $\Gamma \ll \Delta$. Optimization of this factor requires that $\Gamma > \Delta$. For H-like ions this condition is met for $Z \gtrsim 40$, where $F_3$ is practically optimal and of the order of unity.

Finally, we turn to the factor $F_2$ which depends on the relative size of the autoionizing and radiative widths. Consider first the “symmetric” case $\Gamma_i^{(r)} = \Gamma_i^{(a)}$. Then $F_2 = 2 \Gamma^2 / (\Gamma_1 \Gamma_2)$ and the minimum is reached for $\Gamma_1 = \Gamma_2$:

$$\min F_2|_{\Gamma_i^{(r)} = \Gamma_i^{(a)}} = 2.$$  \hspace{1cm} (14)

In the “asymmetric” case $\Gamma_i^{(r)} \neq \Gamma_i^{(a)}$ the denominator becomes small if either both radiative widths, or both autoionizing widths become small. Therefore, optimal conditions correspond to the case when one radiative widths is large and another is small. Let us assume that $\Gamma_1^{(r)} \ll \Gamma_1^{(a)}$ and $\Gamma_2^{(r)} \gg \Gamma_2^{(a)}$. That gives

$$F_2 = \frac{\Gamma_1^{(r)} + \Gamma_2^{(a)}}{\Gamma_1^{(a)} + \Gamma_2^{(r)}}.$$  \hspace{1cm} (15)

Now the minimum is reached for $\Gamma_1 \gg \Gamma_2$ and $\Gamma_1^{(r)} \ll \Gamma_2^{(a)}$:

$$\min F_2|_{\Gamma_1^{(r)} \ll \Gamma_1^{(a)}} = \frac{1}{2}.$$  \hspace{1cm} (16)

We see that it is beneficial if the width of the narrower resonance is dominated by the radiative decay, while that of the broader one — by autoionization. However, the gain in comparison to the symmetric case is only a factor of 4. It is important though to have at least one radiative and one autoionizing width of the order of the total width $\Gamma$. In H-like ions the autoionizing widths of both resonances are smaller than $\Gamma_i^{(r)}$ for $Z > 30$. Using expressions for the widths from Gribakin et al. (2005) we can estimate the $F_2$ factor for H-like ions:

$$F_2(\text{H-like})|_{Z>30} \approx 2.3 \times 10^{-6} Z^4.$$  \hspace{1cm} (17)
For H-like ions with $40 \lesssim Z \lesssim 60$ the function $F_2$ is 1 – 2 orders of magnitude larger than minimal value (16). Taking into account equation (13) and the fact that for the H-like ions with $Z \gtrsim 40$ the factor $F_3$ is of the order of unity we get the estimate:

$$F_{av}(\text{H-like})|_{Z \gtrsim 40} \approx 1.7 \times 10^{20}.$$  \hfill (18)

Thus, in spite of the strong dependence of the PNC interaction on $Z$, the feasibility function for the H-like ions is practically independent of $Z$. The main reason for this is the rapid growth of the radiative widths of the resonances, $\Gamma^{(r)} \propto \omega^3|E1|^2$. For H-like ions the radiative transition frequency is $\omega \approx \frac{3}{8}Z^2$, and the transition amplitude $E1$ decreases as $Z^{-1}$, so that $\Gamma^{(r)} \propto Z^4$.

For non-hydrogenic ions the transition frequencies are typically much smaller, while the $E1$ amplitudes are of the same order of magnitude. Therefore we can expect smaller values of the functions $F_1$ and $F_2$. In addition, for non-hydrogenic ions the autoionizing resonances typically lie at lower energies. This means that the electron momentum $p$ in equation (10a) is smaller. Of course, it is still necessary to find closely-spaced resonances of opposite parity with $\Delta \sim \Gamma$, to keep the function $F_3$ small. If such resonances are found for ions with $Z \gtrsim 40$, one can expect a noticeable improvement of the experimental sensitivity to the PNC asymmetry in comparison with similar H-like ions.

4. Conclusions

We see that the feasibility of observing PNC asymmetry in DR on H-like ions is increased by the proximity of the autoionizing resonances of opposite parity and decreased by the large energy and radiation widths of these resonances. For non-hydrogenic ions the autoionizing resonances usually lie at lower energies and have smaller radiative widths, but are further apart. On the other hand, the number of such ions is huge and there can be accidental degeneracies between levels of opposite parity. Level widths grow rapidly with $Z$, increasing the probability to find pairs of levels with $\Delta \sim \Gamma$ for heavier ions where PNC effects are larger. Finding such close levels for ions with $Z \gtrsim 40$ can significantly simplify observation of the PNC asymmetry in comparison to H-like ions.

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