Frequencies analysis in an infinite beams array

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Abstract.
This model consists of a periodic structure formed by solid beams equidistant from each other submerged in a fluid. The beams are clamped at both ends. The distance between the beams, the elastic properties of the solid and the fluid; and the geometric parameters of the beams determine a relationship between the frequencies of the mechanical waves that can propagate through the structure and the wave vector. Analysis within the first Brillouin zone with the Bloch periodicity condition gives rise to frequency bands in which there is the propagation of mechanical waves and bands in which no waves are propagated. Some propagation bands and forbidden regions were found in the examined frequency ranges for various geometric configurations.

1. Introduction
Solid-state physics studies the energy bands of electrons in crystalline structures, such as silicon, germanium, and other chemical compounds. In the last decades, the propagation of electromagnetic waves in periodic structures or photonic crystals has been studied, in which in some frequency bands there is wave propagation and frequency gaps where there is no propagation. These frequency bands are formed in dependence on the geometric parameters of the structure, but also on the optical and electromagnetic properties of the components of the structure.
Similar phenomena are found in many areas of science when there are structures that are regularly repeated in space in one, two, or three dimensions. In this case, the phenomenon of solid bars immersed in a fluid with certain geometric dimensions and periodicity patterns is studied. The bars are clamped at their ends and in this way a two-dimensional periodic structure is obtained. The frequencies that can propagate through this bar structure are studied.

2. Model of infinite beams array
The unit cell of this study is a prism with a square base of side $a$ and height $h$, which contains a solid beam with the same high immersed in a fluid as shown in figure 1.
This beam can oscillate in natural vibration modes according to the equation of motion given by Euler-Bernoulli theory, whose equation is

$$
\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial z^4} = 0 ,
$$

(1)

where $\rho$ is the density of the beam material, $A$ is the cross-section, $E$ is Young’s module of the beam material, and $I$ is the second moment of the cross-section of the beam about the neutral axis. When the beam is immersed in a fluid, external friction forces appear on the beam. Consequently, a term that is proportional to velocity is introduced into the Euler-Bernoulli equation. Then, the equation of the beam motion is

$$
\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial z^4} + \alpha E \frac{\partial y}{\partial t} = 0 ,
$$

(2)

where $\alpha E$ is the friction coefficient due to fluid action on the beam.

If the beam is fixed at both ends, the following boundary conditions apply

$$
y \big|_{z=0} = y' \big|_{z=0} = 0
$$

(3)

$$
y \big|_{z=h} = y' \big|_{z=h} = 0
$$

(4)

The beam eigenfrequencies in the fluid will be as following

$$
f_n = \frac{\omega_n}{2\pi} = \frac{\beta_n^2}{2\pi} \sqrt{\frac{EI}{\rho A}} \Gamma ,
$$

(5)

here, $\beta_n$ are given by

$$
\beta_n \approx (2n + 1) \frac{\pi}{2h}, \quad n = 1, 2, 3 \ldots
$$

(6)

and $\Gamma$ is a factor that depends on beam and fluid interactions.

This prism is surrounded by infinite similar prisms, forming an infinite plane $(x, y)$ as shown in figure 2, where mechanical waves can propagate with different frequencies depending on the periodicity of the structure and the natural oscillation frequencies of the beams and their interaction with the fluid.
Figure 2. Partial view of an infinite beams array.

Figure 3 illustrates the inferior view of the unit cell. The vectors \( \mathbf{a} \) and \( \mathbf{b} \) are the primitive vectors that form the lattice vector \( \mathbf{R} \), which satisfies the translational condition: \( \mathbf{R} = m\mathbf{a} + n\mathbf{b} \), with \( m \) and \( n \) integers. The operation of infinite translations of vector \( \mathbf{R} \) will fill the entire infinite plane with similar unit cells.

Figure 3. The unit cell for an infinite beams array.

In this way, mechanical waves can propagate across the plane where the prisms are located, the frequencies of waves vary as a function of the \( k_x \) and \( k_y \) wave vectors within the first Brillouin zone in each mode of vibration.

Figure 4. First Brillouin zone.

Bloch’s theorem establishes the conditions of spatial periodicity of the waves that propagate in a two-dimensional periodic structure

\[
\Phi_{nk}(\mathbf{r} + \mathbf{R}) = e^{i\mathbf{k} \cdot \mathbf{R}} \Phi_k(\mathbf{r}),
\]

(7)
where \( \mathbf{R} \) is the lattice vector and \( \mathbf{k} \) is the wave vector, defined by \( \mathbf{k} = k_x \mathbf{i} + k_y \mathbf{j} \). Therefore, the vectors \( \mathbf{R} \) and \( \mathbf{k} \) satisfy the following condition: \( \mathbf{k} \cdot \mathbf{R} = k_x m + k_y n \).

3. Implementation and results

The unit cell is made up of a silicon beam with width \( d \), depth \( w \), and height \( h \) surrounded by an air prism with a square base of side \( a \) and height \( h \). To evaluate the frequency response of this two-dimensional periodic structure simply requires unit cell analysis, with periodic Bloch boundary conditions spanning a certain range of wave vectors. It is enough to take a relatively small range of wave vectors covering the edges of the so-called irreducible Brillouin zone. In this case, it starts from point \( \Gamma \) to point \( X \), from \( X \) to \( M \), and finally returns to \( \Gamma \). In all cases, frequencies were studied up to the fifth eigenfrequency mode.

**Figure 5.** The frequency band diagram for unit cell parameters: \( d = 500 \ \mu m \), \( w = 400 \ \mu m \), \( h = 300 \ \mu m \) and \( a = 800 \ \mu m \).

**Figure 6.** The frequency band diagram for unit cell parameters: \( d = 500 \ \mu m \), \( w = 400 \ \mu m \), \( h = 600 \ \mu m \) and \( a = 1000 \ \mu m \).
Figure 5 shows the frequency intervals where the waves propagate through this periodic structure with dimensions $d = 500 \, \mu m$, $w = 400 \, \mu m$, $h = 300 \, \mu m$ and $a = 800 \, \mu m$. The figure also shows no wave propagation frequencies in the range of 434130 to 448240 Hz between the first and second modes, and 491720 to 500510 Hz between the third and fourth modes. Figure 6 also shows the frequency intervals where the waves propagate through this periodic structure. Here no wave propagation frequencies are in the range of 368450 to 385170 Hz between the fourth and fifth modes.

Figure 7. The frequency band diagram for unit cell parameters: $d = 500 \, \mu m$, $w = 400 \, \mu m$, $h = 500 \, \mu m$ and $a = 600 \, \mu m$.

Figure 7 shows the frequency ranges in which mechanical waves can be propagated for a periodic structure with dimensions $d = 500 \, \mu m$, $w = 400 \, \mu m$, $h = 500 \, \mu m$ and $a = 600 \, \mu m$. In this occasion, the figure shows no wave propagation frequencies in the range of 294740 to 321560 Hz between the first and second modes.

Figure 8. The frequency band diagram for unit cell parameters: $d = 400 \, \mu m$, $w = 400 \, \mu m$, $h = 500 \, \mu m$ and $a = 1000 \, \mu m$.

Finally, figure 8 shows that mechanical waves can propagate in all the frequency ranges
studied and there are no forbidden wave propagation regions. In this case the parameters of the primitive cell are $d = 400 \, \mu m$, $w = 400 \, \mu m$, $h = 500 \, \mu m$ and $a = 1000 \, \mu m$.

4. Conclusions
The propagation of mechanical waves through a periodic structure composed of an infinite beams array regularly separated by a fluid was studied. The propagation patterns were obtained within a frequency range below 600 kHz. In the selected cases, forbidden propagation regions were found except for the case when the beam has a square base of 400 $\mu m$ on each side.

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