SIGNALS FROM SUPERSYMMETRY IN ELECTROWEAK PRECISION DATA?

WOLFGANG HOLLIK
Institut für Theoretische Physik, Universität Karlsruhe
D-76128 Karlsruhe, Germany

ABSTRACT

The predictions of the Standard Model and the minimal supersymmetric standard model (MSSM) for the electroweak precision parameters are discussed in the light of the recent precision data. The results from global fits yield lower $\chi^2$ values in the MSSM than in the Standard model. The fits prefer regions in the MSSM parameter space with $M_2 \simeq |\mu|$ and allow chargino masses higher than the present exclusion limits of LEP 1.5.

1. Introduction

The present generation of high precision experiments imposes stringent tests on the Standard Model and its possible extensions. Besides the impressive achievements in the determination of the $Z$ boson parameters and the $W$ mass, the most important step has been the confirmation of the top quark at the Tevatron with the mass average value $m_t = 180 \pm 12$ GeV.

The lack of direct signals from “New Physics” makes the high precision experiments also a unique tool in the search for indirect effects: through deviations of the experimental results from the theoretical predictions of the minimal Standard Model. We discuss the minimal supersymmetric standard model as a special example of particular theoretical interest.

2. Electroweak precision observables

The $Z$ boson observables and the $W$ mass are conveniently calculated in terms of effective couplings at the $Z$ peak and the quantity $\Delta r$ in the correlation between $M_{W,Z}$ and the Fermi constant $G_\mu$. The formal relations are identical for the Standard Model and the MSSM. For details on the Standard Model calculations see, and for calculations in the MSSM we refer to ref.

Effective $Z$ boson couplings: The effective couplings follow from the set of 1-loop diagrams without virtual photons, the non-QED or weak corrections. These weak corrections can conveniently be written in terms of fermion-dependent overall normalizations $\rho_f$ and effective mixing angles $s_f^2$ in the NC vertices, which contain the details of the models:

$$J_\nu^{NC} = \left( \sqrt{2} G_\mu M_Z^2 \right)^{1/2} (g_\nu^f \gamma_\nu - g_\nu^f \gamma_\nu \gamma_5)$$

(1)
\[
(\sqrt{2}G_\mu M_Z^2 \rho_f)^{1/2} \left((I_3^f - 2Q_fs_f^2)\gamma_\nu - I_3^f \gamma_\nu \gamma_5\right).
\]

**Asymmetries and mixing angles:** The effective mixing angles are of particular interest since they determine the on-resonance asymmetries via the combinations

\[
A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}.
\]  

(2)

Measurements of the asymmetries hence are measurements of the ratios

\[
g_V^f/g_A^f = 1 - 2Q Fs_f^2
\]  

(3)

or the effective mixing angles, respectively.

**Z widths:** The fermionic partial widths of the Z boson, when expressed in terms of the effective coupling constants read up to 2nd order in the (light) fermion masses:

\[
\Gamma_f = \Gamma_0 \left((g_V^f)^2 + (g_A^f)^2 \left(1 - \frac{6m_f^2}{M_Z^2}\right)\right) \cdot \left(1 + Q_f^3 \frac{3\alpha}{4\pi}\right) + \Delta\Gamma^f_{QCD}
\]

with

\[
\Gamma_0 = N_C^f \frac{\sqrt{2}G_\mu M_Z^3}{12\pi}, \quad N_C^f = 1 \text{ (leptons), } = 3 \text{ (quarks)}.
\]

**The W mass:** The correlation between the masses \(M_W, M_Z\) of the vector bosons in terms of the Fermi constant \(G_\mu\) is given by:

\[
\frac{G_\mu}{\sqrt{8}} = \frac{\pi\alpha}{2s_W^2 M_W^2} \frac{1}{1 - \Delta r}
\]  

(4)

with the higher order quantity \(\Delta r\), containing the details of the models.

3. **Standard model predictions versus data**

In table 1 the Standard Model predictions for Z pole observables and the W mass are put together. The first error corresponds to the variation of \(m_t\) in the observed range (1) and \(60 < M_H < 1000\) GeV. The second error is the hadronic uncertainty from \(\alpha_s = 0.123 \pm 0.006\), as measured by QCD observables at the Z. The recent combined LEP results on the Z resonance parameters, under the assumption of lepton universality, are also shown in table 1, together with \(s_e^2\) from the left-right asymmetry at the SLC.
Table 1: Precision observables: experimental results (from refs. 1,2,3) and standard model predictions.

| observable | exp. (1995) | Standard Model prediction |
|------------|-------------|---------------------------|
| $M_Z$ (GeV) | $91.1884 \pm 0.0022$ | input |
| $\Gamma_Z$ (GeV) | $2.4963 \pm 0.0032$ | $2.4976 \pm 0.0077 \pm 0.0033$ |
| $\sigma_0^\text{had}$ (nb) | $41.4882 \pm 0.078$ | $41.457 \pm 0.011 \pm 0.076$ |
| $\Gamma_\text{had}/\Gamma_c$ | $20.788 \pm 0.032$ | $20.771 \pm 0.019 \pm 0.038$ |
| $\Gamma_\text{inv}$ (MeV) | $499.9 \pm 2.5$ | $501.6 \pm 1.1$ |
| $\Gamma_b/\Gamma_\text{had} = R_b$ | $0.2219 \pm 0.0017$ | $0.2155 \pm 0.0004$ |
| $\Gamma_c/\Gamma_\text{had} = R_c$ | $0.1540 \pm 0.0074$ | $0.1723 \pm 0.0002$ |
| $A_b$ | $0.841 \pm 0.053$ | $0.9346 \pm 0.0006$ |
| $\rho_\ell$ | $1.0044 \pm 0.0016$ | $1.0050 \pm 0.0023$ |
| $s_\ell^2$ (LEP) | $0.23186 \pm 0.00034$ | $0.2317 \pm 0.0012$ |
| $s_\ell^2(\Lambda_{LR})$ | $0.23049 \pm 0.00050$ | $0.2317 \pm 0.0012$ |
| LEP+SLC | $0.23143 \pm 0.00028$ |
| $M_W$ (GeV) | $80.26 \pm 0.16$ | $80.36 \pm 0.18$ |

Significant deviations from the Standard Model predictions are observed in the ratios $R_b = \Gamma_b/\Gamma_\text{had}$ and $R_c = \Gamma_c/\Gamma_\text{had}$. The experimental values, together with the top mass (1) from the Tevatron, are compatible with the Standard Model at a confidence level of less than 1% (see (1)), enough to claim a deviation from the Standard Model. The other precision observables are in perfect agreement with the Standard Model. Note that the experimental value for $\rho_\ell$ exhibits the presence of genuine electroweak corrections by nearly 3 standard deviations.

Assuming the validity of the Standard Model a global fit to all electroweak results from LEP, SLD, $p\bar{p}$ and $\nu N$ constrains the parameters $m_t, \alpha_s$ as follows: 

$$m_t = 178 \pm 8^{+17}_{-20} \text{GeV}, \quad \alpha_s = 0.123 \pm 0.004 \pm 0.002$$

with $M_H = 300 \text{ GeV}$ for the central value. The second error is from the variation of $M_H$ between 60 GeV and 1 TeV. The fit results include the uncertainties of the Standard Model calculations.

4. The minimal supersymmetric standard model (MSSM):

The MSSM deserves a special discussion as the most predictive framework beyond the minimal model. Its structure allows a similarly complete calculation of the electroweak precision observables as in the standard model in terms of one Higgs mass (usually taken as $M_A$) and $\tan \beta = v_2/v_1$, together with the set of SUSY soft breaking parameters fixing the chargino/neutralino and scalar fermion sectors. It
has been known since quite some time that light non-standard Higgs bosons as well as light stop and charginos predict larger values for the ratio $R_b$ and thus diminish the observed difference $\Delta r$. Complete 1-loop calculations are meanwhile available for $\Delta r$ and for the $Z$ boson observables.

Figure 1 displays the range of predictions for $M_W$ in the minimal model and in the MSSM. Thereby it is assumed that no direct discovery has been at LEP2. As one can see, precise determinations of $M_W$ and $m_t$ can become decisive for the separation between the models.

The range of predictions for $\Delta r$ and the $Z$ boson observables in the MSSM is visualized in Figure 2 (between the solid lines) together with the standard model predictions (between the dashed lines) and with the present experimental data (dark area). $\tan \beta$ is thereby varied between 1 and 70, the other parameters are restricted according to the mass bounds from the direct search for non-standard particles at LEP I and the Tevatron. From a superficial inspection, one might get the impression that the MSSM, due to its extended set of parameters, is more flexible to accommodate also the critical observable $R_b$. A more detailed analysis shows, however, that those parameter values yielding a "good" $R_b$ are incompatible with other data points. An example is given in Figure 3: a light $A$ boson together with a large $\tan \beta$ can cure $R_b$, but violates the other hadronic quantities and the effective leptonic mixing angle. Whereas the hadronic quantities can be repaired (at least partially) by lowering the value of $\alpha_s$, the mixing angle and $A_{FB}^b$ remain off for small Higgs masses. Thus, even in the MSSM it is not possible to simultaneously find agreement with all the individual precision data.

The main results can be summarized as follows:
- $R_c$ can hardly be moved towards the measured range.
- $R_b$ can come closer to the measured value, in particular for light $\tilde{t}_R$ and light charginos.
- $\alpha_s$ turns out to be smaller than in the minimal model because of the reasons explained in the beginning of this section.
- There are strong constraints from the other precision observables which forbid parameter configurations shifting $R_b$ into the observed 1$\sigma$ range.

| $\tan \beta$ | $\chi^2$ | $M_{\chi^+_1}/M_{\chi^+_2}$ (GeV) | $\alpha_s$ | $m_t$ | $M_{h^0}$ (GeV) |
|--------------|----------|---------------------------------|-----------|------|-----------------|
| 1            | 14.7     | 88/93                           | 0.110     | 175  | 105             |
| 1.2          | 15.6     | 83/99                           | 0.112     | 176  | 107             |
| 1.6          | 16.8     | 72/110                          | 0.115     | 176  | 114             |
| 50           | 17.3     | 64/299                          | 0.114     | 165  | 50              |

Table 2: Variables for the best fit results
Figure 1: The $W$ mass range in the standard model (---) and the MSSM (- - -). Bounds correspond to the possible situation that no Higgs bosons and SUSY particles are found at LEP2.
Figure 2: Range of precision observables in the standard model (dash-dotted) and in the MSSM (solid), and present experimental data (dark area). The MSSM parameters are restricted by the mass bounds from direct searches at LEP I and Tevatron, the dotted lines indicate the bounds to be expected from LEP II.
Figure 3: Precision observables as function of the pseudoscalar Higgs mass $M_A$ for $\tan \beta = 0.7(\cdots), 1.5(\cdots), 8(\cdots\cdots), 20(\cdots\cdots), 70(\cdots\cdots)$. $m_t = 174$ GeV, $\alpha_s = 0.123$. $m_{\tilde{t}} = 800$ GeV, $m_{\tilde{q}} = 500$ GeV, $\mu = 100$ GeV, $M_2 = 300$ GeV.
For obtaining the optimized SUSY parameter set, therefore, a global fit to all the electroweak precision data (including the top mass measurements) has to be performed, as done in refs. 12, 13, 14. As an example, Figure 4 displays the experimental data normalized to the best fit results in the SM and MSSM (for \( \tan \beta = 1 \)), with the data from the 1995 summer conferences 13. For the SM, \( \alpha_s \) identified with the experimental number, therefore the corresponding result in Figure 6 is centered at 1. The most relevant conclusions are:

(i) The difference between the experimental and theoretical value of \( R_b \) is diminished by a factor \( \simeq 1/2 \),
(ii) the central value for the strong coupling is close to the value obtained from deep inelastic scattering,
(iii) the other observables are practically unchanged,
(iv) the \( \chi^2 \) of the fit is slightly better than in the minimal model.

\[ \chi^2/d.o.f = 14.7/10 \]
\[ \chi^2/d.o.f = 25.3/12 \]

Figure 4: Experimental data normalized to the best fit results in the SM and MSSM.

In table 2 we put together the variables for the best fits (minimum \( \chi^2 \)) for a few values of low \( \tan \beta \) and for \( \tan \beta = 50 \). The mass of the nearly righthanded scalar top is taken at 48 GeV, the other sfermions and Higgs bosons are heavy. Only for
the large tan $\beta$ scenario, we have also a pair of light Higgs bosons $M_{h^0} \simeq M_{A^0} = 50$ GeV. The mixing in the scalar top sector is small, but not zero. A non-diagonal $\tilde{t}$ mass matrix is required to make the $h^0$ sufficiently massive.

In the fits, the SUSY mass parameters $\mu$ and $M_2$ are varied independently. In the low tan $\beta$ regime, the values in table 2 correspond to the situation $|\mu| \simeq M_2$. The charginos in this case do not have large mass splittings. They appear as a mixture of wino and Higgsino. In all cases, the masses for the charginos are not yet excluded by the searches at LEP 1.5.

5. Conclusions

The experimental data for testing the electroweak theory have achieved an impressive accuracy. The observed deviations of several $\sigma$’s in $R_b, R_c, A_{LR}$ reduce the quality of the Standard Model fits significantly, but the indirect determination of $m_t$ is remarkably stable. Still impressive is the perfect agreement between theory and experiment for the whole set of the other precision observables. Supersymmetry can improve the situation due to an enhancement of $R_b$ by new particles in the range below 100 GeV, but it is not possible to accommodate $R_c$. Within the MSSM analysis, the value for $\alpha_s$ is close to the one from deep-inelastic scattering.

***

The MSSM fits to the precision data were done in a collaboration with A. Dabelstein, W. de Boer, W. Möslle, U. Schwickerath. The results were presented partially at the EPS Conference on High Energy Physics, Brussels 1995.
$e^+e^-$ Collisions”, Ringberg Castle, February 1995, Ed. B.A. Kniehl, World Scientific 1995 (p. 345)

6. S. Bethke, in: Proceedings of the Tennessee International Symposium on Radiative Corrections, Gatlinburg 1994, Ed. B.F.L. Ward, World Scientific 1995

7. SLD Collaboration, K. Abe et al., Phys. Rev. Lett. 73 (1994) 25; M. Woods (SLD Collaboration), talk at the International Europhysics Conference on High Energy Physics, Brussels 1995 (to appear in the Proceedings)

8. A. Denner, R. Guth, W. Hollik, J.H. Kühn, Z. Phys. C51 (1991) 695;
J. Rosiek, Phys. Lett. B252 (1990) 135;
F. Cornet, W. Hollik, W. Möslle, Nucl. Phys. B428 (1994) 61;
M. Boulware, D. Finnell, Phys. Rev. D44 (1991) 2054

9. G. Altarelli, R. Barbieri, F. Caravaglios, CERN-TH.7536/94 (1994);
C.S. Lee, B.Q. Hu, J.H. Yang, Z.Y. Fang, J. Phys. G19 (1993) 13;
Q. Hu, J.M. Yang, C.S. Li, Comm. Theor. Phys. 20 (1993) 213;
J.D. Wells, C. Kolda, G.L. Kane, Phys. Lett. B338 (1994) 219;
G.L. Kane, R.G. Stuart, J.D. Wells, Phys. Lett. B354 (1995) 350

10. P. Chankowski, A. Dabelstein, W. Hollik, W. Möslle, S. Pokorski, J. Rosiek,
Nucl. Phys. B417 (1994) 101;
D. García, J. Solà, Mod. Phys. Lett. A9 (1994) 211

11. D. García, R. Jiménez, J. Solà, Phys. Lett. B347 (1995) 309; B347 (1995) 321;
D. García, J. Solà, Phys. Lett. B357 (1995) 349

12. P. Chankowski, S. Pokorski, preprint IFT-UW-95/5 (1995);
P. Chankowski, talk at the International Europhysics Conference on High Energy Physics, Brussels 1995 (to appear in the Proceedings)

13. W. De Boer, S. Meyer, A. Dabelstein, W. Hollik, W. Möslle, U. Schwickerath (to be published)

14. W. Hollik, plenary talk at the International Europhysics Conference on High Energy Physics, Brussels 1995 (to appear in the Proceedings), KA-TP-15-1995, hep-ph/9512232

15. The LEP Collaborations, Joint Seminar on the First Results from LEP 1.5, CERN, 12 December 1995
Signals from supersymmetry in electroweak precision data?

W. Hollik
Institut für Theoretische Physik
Universität Karlsruhe
D-76128 Karlsruhe, Germany

Invited talk at the XIXth International Conference on Theoretical Physics
“Particle Physics and Astrophysics in the Standard Model and Beyond”
19 - 26 September 1995
Bystra, Poland