The Fourier-T method to solve the heat transfer problem in combustion chambers of various configurations

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Annotation. With the use of laws of conservation of different substances, the transfer equation in the combustion chambers is formulated as a partial differential equation of the second order. The solution of the obtained equation by the Fourier-T method is proposed, which helps to evaluate the influence of various factors on the space-time temperature field of the combustion chamber quickly; it is shown that the generalized Nusselt number mainly affects the solution. The developed technique allows us to estimate the temperature field of the furnace at a certain flow rate of the combustion products at the inlet of the combustion chamber as well as their composition and temperature using the analogy with heat conductivity processes. The conditions for the reliable application of the developed model are determined. Applying the theorem on multiplication of solutions, we may estimate the temperature field in the combustion chambers of various structural configurations which is an extension of a zoned method.

Introduction

Industry and science pay constant attention to the improvement of characteristics of the heat engineering equipment, since this issue is closely connected with economic and environmental performance of the units. This trend is reflected in the international program THERMY-II and in the works of leading companies producing heat power equipment. With the extensive production experience of the service of combustors of various purposes and design [1] it is not always possible to solve the problem of the efficient and safe use of various fuels quickly, as well as to change the modes of equipment operation [2-5]. The available norms for thermal calculation of combustors and modern numerical methods for their calculation require considerable time and expenditures. In addition, sometimes there is a contradiction in some methods of solving thermophysical problems including a variable separation method which has become classical in solving equations of mathematical physics. The urgency of this problem is unquestionable, and in the authors’ opinion, it is necessary to solve it within fundamental laws of conservation of substances with the use of modern methods of thermophysical research of processes.

Geometric and physical conditions must correspond to the rules of the theory of similarity and modeling, the initial and boundary conditions are related to the correctness of the solution of the problem. In accordance with the statement of academician A.N. Tikhonov [6], the formulation and solution of the problem are considered to be correct, provided there is a solution which is unique and stable to small changes in the parameters of the problem. It should be noted, that the correctness corresponds to more rigid criteria than the reliability, therefore the results of solving the correct problem can be considered reliable.
When presenting the fundamentals of the method of separation of variables (the Fourier method) all publications admit “equality” of the functions, obtained as a result of separation of the main variable into a time and space function. Therefore, as a result of integrating the separated functions, there are two constants, each must include the initial temperature, and eventually the main function must include the square of the initial temperature which violates the rules of dimensional analysis. This inaccuracy is then manifested in the theorem on multiplication of solutions.

The way out of this situation is possible, if we assume immediately that the main information about the temperature field of the bodies is carried by the second spatial derivative, and the time function is a factor reflecting the dynamics of changes in spatial temperature field. This time function can be chosen any, however, as it is shown by the analysis, spontaneous processes of heat and mass transfer (dissipative processes) are best described by an exponential with a factor of 1,0; and this factor corresponds to an exponential with power 0, that is, to the beginning of the process. In this case the index of power has a separation constant, which guarantees the reliability of mathematical description, since the initial condition is a condition for the existence and uniqueness of the solution.

Formulation of the problem and derivation of differential equation of the temperature field in the furnace

For the case under consideration when \( \rho, c, \lambda = \text{const} \); \( q_i = 0 \), \( \partial T/\partial z = 0 \) for isotropic media with temperature-independent thermophysical properties the energy transfer equation for a stationary regime in the Cartesian reference system, the Fourier-Kirchhoff equation has the form

\[
\rho c \left( \frac{w_x}{\partial x} \frac{\partial T}{\partial x} + \frac{w_y}{\partial y} \frac{\partial T}{\partial y} \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right),
\]  

(1)

The law of conservation of energy in the form of a balance of heat flows for the volume of the combustion medium \( \partial V \) in the channel of the furnace of the boiler plant can be written in the form \( \sum_i \text{div} q_i = 0 \), where \( i \) is the heat transfer vectors for the enthalpy flow, the thermal conductivity mechanisms for the turbulent and molecular and beam mechanism respectively: \( q_c = w_x \rho c T_x \); \( q_r = -\lambda_r \nabla T_x \); \( q_a = -\lambda_a \nabla T_x \); \( q_v = -\lambda_v \nabla T_v \), W/m². This helps to transform equation (1) to the form

\[
\rho c w_x \nabla T_x - \text{div} (\lambda_c \nabla T_x + \rho c D \nabla T_x) = 0
\]  

(2)

and then, with the introduction of the relative temperature \( \theta = T_x - T_0 \) we go over to the canonical form of the differential equation

\[
\frac{\partial^2 \theta}{\partial x^2} - \frac{w_x \partial \theta}{a_x \partial x} + \frac{\partial^2 \theta}{\partial y^2} = 0.
\]  

(3)

In equation (3) the generalized temperature conductivity coefficient \( a_x = \lambda_x/\rho c \), the generalized thermal conductivity coefficient \( \lambda_x \) is equal to the sum of the coefficients of molecular \( \lambda_{mol} \), turbulent \( \lambda_t \), mass \( \lambda_m \) and radiation \( \lambda_r \) mechanisms of the substance transfer. The coefficient of radiation mechanism is \( \lambda_r = 16 \sigma_0 T_x^3/3k_a \), W/m²K, where \( \sigma_0 = 5.67 \cdot 10^{-8} \), \( k_a \) is the average attenuation coefficient over the combustion medium cross section, 1/m. Heat transfer is also possible together with mass transfer under conditions of temperature gradient \( T_x \) with mass diffusion coefficient \( D \), which is of the order \( (0,1…1,0) \cdot 10^{-4} \), m²/s.
Equation (3) solution and adaptation

The separation of the variables $\theta = \theta_x, \theta_y$ and the introduction of the separation constant $-k^2$ allows us to replace (3) by two ordinary differential equations and write down their solutions as

$$\theta_x'' - \theta_x \frac{w_x}{a_z} + k^2 \theta_x = 0; \quad \theta_x = C_1 \exp(\gamma_1 x) + C_2 \exp(\gamma_2 x); \quad (4)$$

$$\theta_y'' - k^2 \theta_y = 0; \quad \theta_y = C_3 \sin(k y) + C_4 \cos(k y). \quad (5)$$

In the formulas (4), (5) the constants $C_{1,2,3,4}$ are determined by the single-valued conditions, and the parameters $\gamma_{1,2}$ are calculated by the formula

$$\gamma_{1,2} = \frac{w_x}{2a_z} \pm \sqrt{\frac{w_x^2}{4a_z} - k^2}. \quad (6)$$

analysis of the first level has shown that because of the small value of $\gamma_1$ for $w_x/a_z >> 1$ the constant $C_1$ can be ignored, and $C_2 = 1$.

With the assumption of the symmetry of the medium flow relative to the $x$ axis in the furnace channel, the uniqueness condition is considered

$$\frac{\partial \theta_y}{\partial y} \bigg|_{y=0} = 0, \quad (7)$$

which leads to a derivation with respect to the constant $C_3 = 0$, and the general solution of the energy equation (3) will have the form

$$\theta = \theta_x \cdot \theta_y = C_4 \exp(\gamma_2 x) \cos(k y). \quad (8)$$

A boundary condition of the third kind for the heat flow on the heat reception surface $y = \delta$

$$\frac{\partial \theta_y}{\partial y} \bigg|_{y=\delta} = -\frac{\alpha}{\lambda_z} \theta_y \bigg|_{y=\delta}, \quad \text{from which} \quad \frac{k \delta}{\text{Nu}_z} = \text{ctg}(k \delta), \quad (9)$$

where the Nusselt number $\text{Nu}_z = \alpha \delta / \lambda_z$ reflects the ratio of the thermal resistances of the generalized thermal conductivity of the medium and the heat transfer on the heat reception surface: $\text{Nu}_z = (\delta / \lambda_z)/(1/\alpha)$. In formula (8) a refined solution is given in accordance with the Fourier-T method, which slightly differs from the solution in reference [7].

The only parameter affecting the solution of equation (8) is the number $\text{Nu}_z$, in the problems of thermal conductivity and diffusion such parameter are the Biot numbers $Bi$. Replacing the Biot number by the Nusselt number we can use the extensive reference material [8, 9] to define $C_4 = D/\theta_0$, where $\theta_0$ is the initial temperature of the surface. In the form of the ratio of two thermal resistances to heat transfer along the $y$ axis the Nusselt number establishes a relationship between the distributed resistance $\delta / \lambda_z$ and resistance to heat transfer concentrated on the heat-sensing wall $1/\alpha$. Therefore, to determine $\delta / \lambda_z$ it is rational to apply the dependence

$$\delta / \lambda_z = \delta / \left(\left\lfloor \frac{16 \sigma_0 T_{\text{m}}^3}{3 k_g} \right\rfloor \right), \quad (10)$$
and to determine \( 1/\alpha \) we apply the dependence for the resulting flow on the wall

\[
1/\alpha = \frac{(1/A_w - 1/2)(T_S - T_w)}{\sigma_0(T_S^4 - T_w^4)}.
\]

Thus, the Nusselt number

\[
Nu_S = \frac{3}{16} \frac{Bu}{(1/A_w - 1/2)} \frac{T_S^4 - T_w^4}{T_S^3(T_S - T_w)} = \frac{3}{16} \frac{Bu}{(1/A_w - 1/2)} \left[ 1 + \frac{T_w}{T_S} + \frac{T_w^2}{T_S^2} + \frac{T_w^3}{T_S^3} \right]^{-1}
\]

where the temperature factor is transformed to a polynomial of the third degree by algebraic transformations, with \( T_w/T_S < 1 \). The Bueger number \( Bu = k_ps; k_p \) is the coefficient of attenuation of the radiation flow by the gases of the combustor, \( 1/m \) MPa, \( p \) is the pressure in the combustor, MPa, \( s \) is the effective thickness of the emitting layer of gases, m.

In the solution of equation (8) the exponential factor characterizes the change in temperature along the channel length, so the definition of \( \gamma_{1,2} = \omega_s/2a_S \pm (\omega_s/4a_S^2 + k_S^2)^{0.5} \) is crucial for calculating the temperature along the channel length.

According to the estimates \( \gamma_2 = \omega_s/2a_S[1-(1+4k_S^2/\omega_s^2)]^{0.5} \), using the dependence

\[
(1+\bar{c})^{0.5} = 1 + 0.5\bar{c} - 0.0625\bar{c}^2
\]

the parameter of the solution \( \gamma_2 \) is reduced to the form \( \gamma_2 = 0.0625\omega_s \bar{c}(\bar{c}^2 - 2)/a_S \), where \( \bar{c} = 4k_S^2/\omega_s^2 \). The last expression makes it possible to determine the condition for the applicability of the parameter \( \gamma_2; \bar{c} \geq 2 \).

Taking into account the periodicity of the function \( \cotg(k\delta) \) the solution of equation (3) in the dimensionless form is determined as the sum of particular solutions

\[
\Theta = \frac{\theta}{\theta_0} = \sum\delta_i \exp(\mu_i X) \cos(\mu_i Y),
\]

for \( \mu = k\delta, X = x/\delta, Y = y/\delta \).

The boundary condition (9) makes it possible to determine the location of the convergence point of the tangents to the temperature curves \( \Theta_{x=1} \) on the heat reception surface \( x = \delta \), and the ordinate of this point \( Y_0 \) is also counted from the surface \( x = \delta \); \( Y_0 = 1/Nu_S \) [10]. The analogy with the processes of thermal conductivity in solids helps to use the Fourier-T method in application to combustors of various shapes and flow-through heat-mass transfer units with the replacement of the Biot number by the Nusselt number, it must be taken into account that the temperature scale is directed towards the flow of the medium [11-13].

Since the ordinate \( Y_0 \) defines a figure in space, that is equidistant to the heat reception surface, any change in the latter is easily taken into account in calculating the volume temperature field of the furnace, including the change in the Nusselt number using the zone method. In this case, the uniform distribution of the flow velocity of the medium in the channel becomes unnecessary. By setting the condition for the absence of slagging in a certain section we may define the coordinate of this section by substituting in (13) \( \Theta = \Theta_{di} \).

**Conclusion**

On the basis of the laws of conservation of substances the energy transfer equation in the channel of the combustor is formulated in the form of a partial differential equation, which is solved by the Fourier-T method. The obtained equation of temperature variation along and across the combustor...
makes it possible to estimate the influence of different factors on temperature changes in combustors of various configurations and thermal stress.

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