THERMAL INSTABILITY BEHIND A SHOCK WAVE IN H I AND MOLECULAR CLOUDS

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Received 2012 December 26; accepted 2013 July 23; published 2013 August 30

ABSTRACT

We performed one-dimensional hydrodynamic simulations with detailed cooling, heating, and chemical processes to examine the thermal stability of shocked gas in cold neutral medium (CNM) and molecular clouds. We find that both CNM and molecular clouds can be thermally unstable in the cooling layer behind the shock wave. The characteristic wavelength of the thermal instability ranges from $10^{-5}$ pc to 0.1 pc in the CNM, and from $10^{-7}$ pc to 0.1 pc in the molecular clouds. This coincides with the size of observed tiny scale structures in the CNM and molecular clouds, indicating that the thermal instability in the shocked gas could be a formation mechanism of these tiny structures in the interstellar medium. We have also calculated the $e$-folding number of the thermal instability to estimate the amplification of the density fluctuation in the shocked gas. Density perturbations in the CNM grow by a factor of $\exp (5) \simeq 150$, whereas the perturbations in the molecular clouds grow only by a factor of a few behind a high Mach number shock. The amplification factor is larger at lower densities and higher velocities. Formation of very small scale structures by thermal instability in shocked gas is more effective in lower densities.

Key words: ISM: clouds – ISM: molecules – ISM: structure – shock waves

1. INTRODUCTION

Thermal instability is an important physical process to determine the structure in the interstellar medium (ISM; Field 1965). It is well established that the neutral gas in the ISM consists of two distinct phases: cold neutral medium (CNM) and warm neutral medium (WNM; e.g., Field et al. 1969; Wolfire et al. 1995; Heiles & Troland 2003). Field et al. (1969) calculated the thermal equilibrium state in ISM considering the cosmic-ray heating and line cooling by H, O, and C [\textit{ii}]. They showed that there are three physical states under the pressure equilibrium: two stable states and one unstable state. The stable states correspond to the CNM and the WNM. When the WNM transforms to the CNM in converging flows and/or shocks, the gas goes through a thermally unstable state (e.g., Hennebelle & Pérault 1999; Koyama & Inutsuka 2000; Heitsch et al. 2005; Vázquez-Semadeni et al. 2006).

Many authors have studied the dynamical condensation and fragmentation processes of the ISM driven by the thermal instability in the shock in WNM. Koyama & Inutsuka (2000, 2002) showed that small-scale clumps of CNM are formed behind the shock front in WNM (see also Hennebelle & Pérault 1999; Hennebelle & Audit 2007; Inoue & Inutsuka 2008, 2009; Heitsch et al. 2008; Vázquez-Semadeni et al. 2007). Koyama & Inutsuka (2000) also investigated the shock propagation within the CNM and found that the shocked layer in the CNM is thermally unstable as well. More recently, Inoue & Inutsuka (2012) has succeeded in forming very turbulent molecular clouds by accretion of CNM mixed with WNM.

The thermal instability works as follows. Consider an isobaric gas with small density perturbation. If the gas in small density enhancement (i.e., temperature decline) has a larger cooling rate than the surrounding gas, the density enhancement grows. In other words, the condition is determined by how the cooling rate depends on the density and temperature. Thermal instability thus could occur in denser regions like molecular clouds as well, and has actually been studied. de Jong et al. (1980) investigated the thermal-chemical instability (e.g., Glassgold & Langer 1975) and showed that the thermal-chemical instability is not active. Gilden (1984) investigated thermal instability in molecular clouds with density $n \sim 10^{3}$ cm$^{-3}$ and temperature $T \simeq 35$–75 K and found that such a cloud is thermally unstable. Nejad-Asghar (2007, 2011) investigated thermal instability in molecular cloud cores ($n \sim 10^{4}$–$10^{6}$ cm$^{-3}$) with the effect of the ambipolar diffusion and showed that the thermal instability can grow in quasi-magnetohydrostatic, self-gravitating slab (Nejad-Asghar 2007) and axisymmetric cylindrical core (Nejad-Asghar 2011).

On the other hand, to the best of our knowledge, the role of thermal instability in shock-heated molecular clouds has not yet been studied. Molecular clouds are characterized by supersonic velocity dispersions, which are most probably due to turbulence (Larson 1981; Solomon et al. 1987). Dissipation of the supersonic turbulence would be accompanied by shocks. Shock waves are also driven by collisions of protostellar outflows with ambient gas.

Molecular clouds are known to have clumpy structures. While many of the clumps are gravitationally bound, there are very small scale structures as well: a size of $\sim 1000$ AU and density $n \sim 10^4$ cm$^{-3}$ (Langer et al. 1995; Heithausen 2002; Sakamoto & Sunada 2003; Tachihara et al. 2012). Since they are gravitationally unbound ($M \lesssim 0.05 M_{\odot}$), they cannot be formed by gravitational instability. Although shock compression by turbulence often makes a gravitationally unbound structure, it would not be easy to make a very small scale structure (<0.1 pc) by shock compression alone. First, the shock compresses the gas as a whole, and thus cannot make fragments. Since turbulence has eddies of various spatial scales, one may imagine that the shock compression at small scale eddied can make very small scale structures or fragments. However, it should be noted that the turbulence (i.e., velocity dispersion in molecular clouds) becomes subsonic at <0.1 pc according to Larson’s law (see, e.g., Heyer & Brunt 2004). Structures smaller than this scale thus cannot be formed by shock compression by turbulence. If the shocked molecular gas is thermally unstable, it could generate very small scale fragments. The effect of thermal instability on the shocked molecular gas must be explored.
Table 1
Heating and Cooling Processes

| Cooling and Heating Process | Reference |
|-----------------------------|-----------|
| Lyα cooling                 | Spitzer (1978) |
| C\(^+\) fine-structure cooling(158 \(\mu\)m) | de Jong et al. (1980), Hollenbach & McKee (1989) |
| O fine-structure cooling(63 \(\mu\)m) | de Jong et al. (1980), Wolfire et al. (2003) |
| CO rotational cooling      | Hollenbach & McKee (1979), Hosokawa & Inutsuka (2006) |
| CO vibrational cooling     | Hollenbach & McKee (1989) |
| Cooling due to recombination on grains | Bakes & Tielens (1994) |
| Cooling by the collision with dust | Hollenbach & McKee (1989) |
| Photoelectric heating by PAH | Bakes & Tielens (1994), Wolfire et al. (2003) |
| Cosmic-ray heating         | Goldsmith & Langer (1978) |
| H\(_2\) photodissociation heating | Black & Dalgarno (1977) |

Figure 1. Schematic view of our 1D shock model.

In this paper, we examine the thermal stability of shocked gas in molecular clouds using one-dimensional (1D) hydrodynamic simulations including detailed cooling, heating, and chemical processes. Although our main target is molecular clouds, we also calculate one model of CNM, in order to compare our results with Koyama & Inutsuka (2000), and to compare the e-folding numbers in CNM and molecular clouds. This paper is organized as follows. In Section 2, we describe our physical and chemical models. Then, we explain the condition for the thermal instability and how we evaluate the growth of perturbation in Section 3. In Section 4, we show the results of our simulations of shock propagation in CNM and molecular clouds. Finally, we summarize our results in Section 5.

2. THE MODEL

We investigate the evolution of ISM swept by a shock wave in a plane-parallel gas. Figure 1 schematically shows the configuration of our model; we consider a collision of oppositely oriented gas flows which have the same density, temperature, and chemical composition. The external radiation irradiates both ends of the numerical domain. We calculate the temporal variation of temperature, density, and chemical composition in the shocked region.

2.1. Basic Equations

We solve the following equations:

\[ \frac{\partial U(t, x)}{\partial t} + \frac{\partial F_x}{\partial x} = S \]

\[ U = (\rho, \rho v_x, E) \]

where \(\rho\), \(v_x\), \(p\), \(\gamma\), \(\Gamma\), and \(\Lambda\) are the gas mass density, velocity, thermal pressure, ratio of specific heat, and heating and cooling rate per unit mass, respectively. We use an operator-splitting technique to solve these equations, which are split into three parts: (1) ideal hydrodynamics, (2) cooling and heating, and (3) chemical reactions (e.g., Inoue & Inutsuka 2008). The first part, ideal hydrodynamics, is calculated by employing a second-order Godunov method with Lagrangian coordinates (Van Leer 1979). We solve the exact Riemann problem iteratively at each grid cell interface to calculate numerical fluxes, and determine the position of the grid cell interface in the next time step. Thus, we can appropriately calculate small-scale compressed dense regions and large-scale pre-shock regions at once. The energy equation

\[ \frac{\partial E}{\partial t} = \rho (\Gamma - \Lambda) \]

is solved by the second-order explicit method. Temporal variation of the number density of chemical species is determined by the rate equations

\[ \frac{dn_i}{dt} = \sum_j k_{ij} n_j + \sum_{j,l} k_{ijl} n_j n_l, \]

where \(k\) is the rate coefficient of the chemical reactions. The rate equations are calculated by a first-order implicit method (Hersant et al. 2009).

The time step of the integration is set to be small enough to satisfy the Courant-Friedrichs-Lewy (CFL) condition, and to be much smaller (\(\lesssim 4\)%) than the cooling timescale of the gas. The details of the physical and chemical processes considered in our model are described in the following:

2.2. Heating and Cooling Processes

A full list of the thermal processes included in our model is listed in Table 1. Our model includes the cooling by the line emission of H, C\(^{[\text{ii}]}\), O\(^{[\text{i}]}\), and CO with the effect of radiative trapping, recombination on grains, and dust–gas collision. We adopt the formula of escape probability by de
Heating processes include photoelectric heating by polycyclic aromatic hydrocarbons (PAH; Bakes & Tielens 1994), cosmic-ray (Goldsmith & Langer 1978), and H$_2$ photodissociation (Black & Dalgarno 1977). In the calculation of photoelectric heating, we consider the attenuation of external radiation. Visual extinction, $A_V$, is calculated by

$$A_V = \frac{\int n_H dx}{1.89 \times 10^{21} \text{cm}^{-2}} \text{mag},$$

where the numerator is the column density of hydrogen nuclei integrated from the edge of the numerical domain to the center of each grid cell (e.g., Mathis et al. 1983).

### 2.3. Chemical Reactions

We calculate the chemical reaction network in the gas phase, which consists of 462 species and 9578 reactions. Chemical reactions and rate coefficients are adopted mainly from the OSU network (http://www.physics.ohio-state.edu/~eric), which is developed for interstellar chemistry. Namely, we use the network of Garrod & Herbst (2006) at $T \lesssim 100 \text{K}$ and Harada et al. (2010) (see also Harada et al. 2012, the errata of Harada et al. 2010) at $T > 100 \text{K}$. These networks contain cosmic-ray ionization, ion–molecule reactions, neutral–neutral reactions, recombinations of ion, photoreactions, and grain surface reactions. We also include the collisional dissociations in Table A1 of Willacy et al. (1998), but some rate coefficients are modified (see the appendix of Furuya et al. 2012). We do not consider grain-surface reactions except for H$_2$ formation. Cosmic-ray ionization rate is set to be $1.3 \times 10^{-17} \text{ s}^{-1}$. In the model of CNM gas, the total column density of hydrogen nuclei is only $N_H = 1.0 \times 10^{20} \text{ cm}^{-2}$, and the molecules are destroyed by photoreactions. We take into account the self-shielding effects of H$_2$, CO, and C atom referring to Lee et al. (1996) and Tielens & Hollenbach (1985); the shielding factors are given as a function of $A_V$.

### Table 2

Element Abundance in the Gas Phase Relative to Hydrogen

| Element | Abundance |
|---------|-----------|
| He      | $9.75 \times 10^{-2}$ |
| O       | $4.5 \times 10^{-4}$ |
| C       | $3.02 \times 10^{-4}$ |
| N       | $2.47 \times 10^{-5}$ |
| S       | $1.94 \times 10^{-8}$ |
| Si      | $2.47 \times 10^{-9}$ |

Notes.

- Relative abundance of chemical species to hydrogen nuclei ($n_H$). $a \sim b$ means $a = 10^{-b}$.

### 2.4. Initial and Boundary Conditions

The initial temperature and chemical composition of the colliding gases are determined by calculating the thermal and chemical equilibrium. In the model of CNM, these initial conditions vary spatially depending on the visual extinction $A_V$ at each position in the numerical domain. In the model of molecular clouds, $A_V$ is set to be 5 mag for all grid cells; our numerical domain is an embedded small portion of the molecular cloud. In total, we calculate one CNM model and 45 molecular cloud models. The model parameters are summarized in Table 3.

Our initial condition of the CNM is basically the same as that of Koyama & Inutsuka (2000), see their Section 3.4); we assume the number density to be $n_H = 10 \text{ cm}^{-3}$. Thermal equilibrium determines the gas temperature, which is about 110 K at any position. The velocity of the colliding gas $V_{\text{fluid}}$ (see Figure 1) is 10 km s$^{-1}$, which corresponds to the velocity of gas flow associated with old supernova remnants. Total column density of hydrogen nuclei is set to be $1.0 \times 10^{20} \text{ cm}^{-2}$, i.e., $A_V = 0.05 \text{mag}$.

For the molecular cloud models, we explore the parameter space of the number density $n_H$: $1.0 \times 10^{2}, 3.0 \times 10^{2}, 1.0 \times 10^{3}, 3.0 \times 10^{3}$, and $1.0 \times 10^{4} \text{ cm}^{-3}$. The velocity $V_{\text{fluid}}$ ranges from 0.5 to 4.5 km s$^{-1}$, referring to the turbulent velocities in molecular clouds ( Larson 1981; Fukui et al. 2008). We also investigate a model with $V_{\text{fluid}} = 10 \text{ km s}^{-1}$, which is a typical velocity of protostellar outflows (e.g., Moriarty-Schieven & Snell 1988).

In the model of the CNM, the size of our numerical domain $L_{\text{cal}}$ is 3.24 pc, which covers the whole region of the CNM. In order to calculate the post-shock region with a high spatial resolution, we set 225 grid cells at $x < L_{\text{in}}$ and 35 grid cells at...

### Table 3

Initial State

| $n_H$ (cm$^{-3}$) | $T$ (K) | $G_0$ | H atom | H$_2$ | C atom | CO | C$^+$ | $L_{\text{cal}}$ (pc) |
|------------------|--------|-------|--------|-------|--------|----|------|----------------------|
| 10               | 110    | 1.7   | 1.0    | 2.5(−5) | 3.8(−8) | 1.8(−12) | 3.0(−4) | 3.24 |
| 30               | 15     | 1.0   | 3.0(−3) | 0.5 | 3.8(−7) | 3.0(−4) | 1.1(−7) | 0.05 |
| 1 $\times$ 10$^3$ | 13     | 1.0   | 1.0(−3) | 0.5 | 9.7(−8) | 3.0(−4) | 4.3(−8) | 0.02 |
| 3 $\times$ 10$^3$ | 10.5   | 1.0   | 4.2(−4) | 0.5 | 2.3(−8) | 3.0(−4) | 1.4(−8) | 0.01 |
| 1 $\times$ 10$^4$ | 9.3    | 1.0   | 1.3(−4) | 0.5 | 2.3(−8) | 3.0(−4) | 1.4(−8) | 0.01 |

Notes.

- External far-UV radiation normalized to the local interstellar radiation field ($1.6 \times 10^{-3} \text{ erg cm}^{-2} \text{ s}^{-1}$) of Habing (1968).
- Relative abundance of chemical species to hydrogen nuclei ($n_H$). $a \sim b$ means $a = 10^{-b}$.
- Initial numerical domain (see Figure 1).
\( x > L_{\text{in}} \) (see Figure 1). Initially, \( L_{\text{in}} \) is 0.972 pc. Both \( L_{\text{gal}} \) and \( L_{\text{n}} \) change with time, since we are using Lagrangian coordinates. In the models of molecular clouds, our numerical domain varies from 0.01 pc to 0.28 pc depending on the model parameters (see Table 3). The numerical domain is divided into 300 grid cells with equal intervals.

We adopt the free–boundary condition:

\[
\frac{\partial \rho}{\partial x} = 0, \quad \frac{\partial \rho}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial x}{\partial t} = 0,
\]

i.e., we assume that the physical values of the left and right sides of the boundary are the same when we solve the exact Riemann problem at the boundary. Basically, the boundary condition is not important, because we stop the calculation before the shock wave reaches the boundary.

### 3. AMPLITUDE OF THERMAL INSTABILITY

Balbus (1986) showed that for any unperturbed state, the gas is thermally unstable if

\[
\left[ \frac{\partial}{\partial s} \left( \frac{\Lambda - \Gamma}{T} \right) \right]_s < 0,
\]

where \( s \) is specific entropy, \( T \) is temperature, and \( \Lambda \) is a thermodynamic variable kept constant in the perturbation. Since the ISM is mostly in pressure equilibrium, the isobaric condition, \( A = \rho \), is satisfied, which leads to

\[
\left[ \frac{\partial}{\partial T} \left( \frac{\Lambda - \Gamma}{T} \right) \right]_T < 0.
\]

Schwarz et al. (1972) and Koyama & Inutsuka (2000) performed a linear analysis of thermal instability in isochorically cooling gas and isobarically contracting gas, respectively. When the gas is thermally unstable, the density perturbation grows as \( \rho = \rho_0 \exp(\sigma t) \). The growth rate in isobarically contracting gas is

\[
\sigma = -\frac{m T (\gamma - 1)}{\gamma k_B} \left[ \frac{\partial}{\partial T} \left( \frac{\Lambda - \Gamma}{T} \right) \right]_T
= \frac{1}{\gamma} \left[ 1 + s_{\rho} - s_T \right] \frac{s_{\rho}}{\tau_{\text{cool}}} - \frac{1 + r_{\rho} - r_T}{\tau_{\text{heat}}},
\]

where \( m \) is the mean molecular mass and \( \tau_{\text{cool}} \equiv k_B T / (\gamma - 1) / (m \Lambda) \) and \( \tau_{\text{heat}} \equiv k_B T / (\gamma - 1) / (m \Gamma) \) are the cooling and heating timescales, respectively. \( \sigma \) is a function of density and temperature that changes with time as the gas goes through the shock front and enters the cooling region. When the gas is thermally unstable, \( \sigma \) takes a positive value.

In this paper, we evaluate the integrated e-folding number \( \int \sigma \, dt \) as an indicator of the amplification of the perturbation, where the e-folding number is calculated for each fluid element in Lagrangian coordinates, and the integral is executed only when the Balbus criterion (Equation (1)) is satisfied. Then, using the e-folding number, we obtain the amplification of the thermal instability \( \exp(\int \sigma \, dt) \).

In the linear analysis, the growth rate is a function of the wavelength of the perturbation, and Equation (3) is the rate for the most unstable mode at

\[
\lambda_{\text{max}} = \sqrt{\pi l_a}.
\]

The Field length \( l_F \) is

\[
l_F = \left\{ \frac{\kappa T}{\rho (\Gamma - \Lambda)} \right\}^{1/2},
\]

where the thermal conductivity \( \kappa \) is \( 2.5 \times 10^3 T^{0.5} \text{ cm}^{-1} \text{ K}^{-1} \text{ s}^{-1} \) (Parker 1953). The acoustic length \( l_a \) is

\[
l_a = c_s \left( \frac{e}{\Gamma - \Lambda} \right).
\]

where \( c_s \) is the sound speed, \( e \) is the specific internal energy, and \( e / (\Gamma - \Lambda) \) is the net cooling timescale (Field 1965). It should be noted, however, that the dependence of the growth rate on perturbation wavelength is considerably weak around the most unstable wavelength. The growth rate is comparable to Equation (3) in the wavelength range of

\[
l_F < \lambda < l_a
\]

(Field 1965; see also Appendix B of Koyama & Inutsuka 2000).

Ideally, the growth of density perturbation should be measured in hydrodynamic simulations starting from an initial condition with small-amplitude perturbations, but it is not easy in practice. First of all, the thickness of the unstable layer is comparable to or smaller than the most unstable wavelength (see Figure 3). In 1D flow, the perturbation grows as long as Equation (1) is satisfied, but when the gas is compressed to be in a thermally stable state, the perturbation is dispersed, which is an artifact. In two-dimensional (2D) and/or three-dimensional (3D) simulations, large-scale perturbations up to the acoustic length \( l_a \) can grow in directions parallel to the shock front, and these clump structures remain in the post-shock regions (e.g., Inoue & Inutsuka 2008). The 2D/3D simulation is, however, very time consuming, especially if we are to resolve the perturbations with very small wavelengths. It is well established that the growth rate derived from the linear analysis is applicable to the nonlinear regime of thermal instability. For instance, we can see in Figure 1 of Inoue et al. (2007) that the isobaric condition, which is required for the thermal instability to grow with the linear growth rate, is met throughout the evolution without interceptive feedback effects. Therefore, it is reasonable to estimate the amplification of the density perturbation by integrating the e-folding number along the 1D flow. It should at least be done before investing a large computational time on 2D hydrodynamic simulation with perturbation and very high spatial resolution. We also note here that our present work is analogous to Koyama & Inutsuka (2000); they performed 1D shock calculations to find that the gas becomes thermally unstable in the shock-compressed layer and predicted that small clumps would be formed. Later, Koyama & Inutsuka (2002) indeed showed that such clumps are formed in the 2D simulation.

### 4. RESULTS

We have calculated the generation and propagation of shock waves in CNM and molecular clouds by solving the basic equations in Section 2.1. In the following, we show the spatial distribution of physical parameters and the e-folding number when the shock wave reaches a steady state.
4.1. Shock Propagation in CNM

Figure 2(a) shows the distribution of temperature, number density of hydrogen nuclei \(n(\text{H})\), thermal pressure, and integrated e-folding number \(\sigma t\) at \(t = 88,000\) yr in the CNM model. A similar simulation was performed by Koyama & Inutsuka (2000). Temperature and density distributions in Figure 2 are indeed similar to Figure 2 of Koyama & Inutsuka (2000), who found that the shocked CNM evolves through a thermally unstable state. The gray shade in Figure 2 depicts the thermally unstable region, in which condition (1) is satisfied. CNM becomes thermally unstable immediately behind the shock and then evolves to a thermally stable dense gas with basically isobaric conditions (see Figure 7(a) of Koyama & Inutsuka 2000). The main coolants are \(\text{CII}(158\ \mu\text{m})\) and \(\text{OI}(63\ \mu\text{m})\). Now, we go one step further from Koyama & Inutsuka (2000) and calculate the e-folding number. The integrated e-folding number \(\int \sigma dt\) is approximately 5, which means that the density fluctuation grows by a factor of \(\exp(5) \sim 150\). Note that the density profile shown in Figure 2 is the unperturbed value. For example, if the pre-shock gas has a density fluctuation of 10%, i.e., \(11\ \text{cm}^{-3}\) in the pre-shock gas of \(10^{-3}\) cm\(^{-3}\), the fluctuation grows by a factor of \(150, 1.6 \times 10^4\ \text{cm}^{-3}\) in the post-shock gas of \(1.0 \times 10^3\ \text{cm}^{-3}\).

We also show the distribution of assorted chemical species (Figure 2(b)). Molecular hydrogen in the pre-shock region in our model is more abundant than that of Koyama & Inutsuka (2000) due to the difference in the self-shielding model; Koyama & Inutsuka (2000) used the formulation by Tielens & Hollenbach (1985), while we use Table 10 in Lee et al. (1996). In the dense stable region \(x \lesssim 10^{-3}\ \text{pc}\), on the other hand, the \(\text{H}_2\) abundance in our model is almost the same as that in Koyama & Inutsuka (2000). \(\text{H}_2\) is formed by the association of H atoms on grain surfaces and destroyed by photodissociation. Figure 2(b) also shows the abundances of \(\text{C}^+\), \(\text{C}\), and \(\text{CO}\). Although Koyama & Inutsuka (2000) did not show the spatial distribution of \(\text{CO}\), they reported that 0.02% of the carbon is in \(\text{CO}\) in the dense stable gas in the post-shock region. In our model 0.03% of carbon is in \(\text{CO}\). It should be noted that we solve the detailed chemical network, whereas Koyama & Inutsuka (2000) adopted a simplified chemical model that assumed a direct conversion of \(\text{C}^+\) to \(\text{CO}\) without accounting explicitly for the intermediate reactions. Consistency of our model results with Koyama & Inutsuka (2000) validates their simplified model.

Figure 2(c) shows the Field length, acoustic length, and the most unstable wavelength of perturbation \(\sqrt{lt}L\) in the unstable region. The Field length is \(10^{-5}–10^{-3}\ \text{pc}\), and the acoustic length is \(10^{-3}–10^{-1}\ \text{pc}\). The most unstable wavelength ranges from \(10^{-4}\) to \(10^{-2}\ \text{pc}\), which coincides with the size of the tiny scale structures observed in CNM, \(\sim 10^{-3}\ \text{pc}\) (see Table 1 of Heiles 1997). Such tiny structures can be formed by thermal instability (Koyama & Inutsuka 2000).
4.2. Shock Propagation in Molecular Clouds

Figure 3(a) shows the spatial distribution of temperature, number density of hydrogen nuclei \((n_H)\), thermal pressure, and integrated e-folding number \((\int \sigma dt)\) in the model with \(n_H = 10^2 \text{ cm}^{-3}\) at \(t = 53,000 \text{ yr}\). The gas becomes thermally unstable immediately behind the shock, where the gas temperature reaches near 1000 K. The gas evolves in the post-shock region with the isobaric condition. The main coolant is CO in this warm gas. The integrated e-folding number is 1.25; if we calculate the hydrodynamic (HD) simulation with a small perturbation, it grows only by a factor of 3.5.

Figure 3(b), on the other hand, shows a model with a higher initial density \(n_H = 1 \times 10^4 \text{ cm}^{-3}\) at \(t = 1500 \text{ yr}\). In this model, the post-shock gas becomes thermally unstable right behind the shock front, but then becomes stable, although the temperature (several hundreds of K) in this region is similar to that in the unstable post-shock gas in Figure 3(a). Once the number density \(n_H\) reaches several times \(10^5 \text{ cm}^{-3}\), the dust–gas collisional cooling becomes dominant and the gas becomes thermally unstable again. The integrated e-folding number is even smaller than in the model of Figure 3(a).

Figures 3(c) and (d) show the spatial distribution of the Field length, acoustic length, and the most unstable wavelength \(\sqrt{\ell_F \ell_a}\) in the two models of molecular gas. In the model with higher gas density, the cooling rate is higher (see below), and thus these scale lengths become shorter (see Equations (5) and (6)).

We summarize the integrated e-folding number in our molecular cloud models in Table 4. We can see that the shocked molecular gas is unstable when \(V_{\text{fluid}}\) is larger than \(\sim 1.5 \log(n_H \text{[cm}^{-3}\text{])} - 2 \text{ km s}^{-1}\), and that the integrated e-folding number increases with increasing \(V_{\text{fluid}}\) and decreasing initial gas density. These dependences can be understood as follows. When the net cooling rate per unit mass \(L\) is proportional to \(n^{6} T^{4}\), the criterion for thermal instability by Balbus (1986)
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The power-law index of temperature, $\gamma = \frac{\partial \ln(\Lambda)}{\partial \ln(T)}$, is calculated at a given density $n_H = 10^3$ cm$^{-3}$, CO abundance of $1.6 \times 10^{-4}$, and CO column density of $N_{\text{CO}} = 7.0 \times 10^{17}$. The power-law index of density, $\beta = \frac{\partial \ln(\Lambda)}{\partial \ln(n)}$, is calculated at $T = 100$ K, CO abundance of $1.6 \times 10^{-4}$, and $N_{\text{CO}} = 7.0 \times 10^{17}$. (c) The power-law index $\gamma$ of the dust–gas collisional cooling is calculated at $T_{\text{dust}} = 10$ K.

In Table 4, asterisks indicate that the dust–gas collisional cooling dominates over the CO cooling in the model. The cooling rate by dust–gas collision per unit mass is given as

$$1.2 \times 10^{33} n_H \left(\frac{T_{\text{gas}}}{1000}\right)^{0.5} \left(\frac{100 \text{Å}}{a_{\text{min}}}\right) \left(\frac{T_{\text{gas}} - T_{\text{dust}}}{m}\right) \text{erg g}^{-1} \text{s}^{-1},$$

where $m$ is the mean molecular mass and we set $a_{\text{min}} = 100$ Å and $T_{\text{dust}} = 10$ K (Hollenbach & McKee 1989). It is obvious that $\beta$ is always 1 in any density region, and that dust–gas collisional cooling is important at high densities. Figure 4(c) shows the power index $\gamma$. The cooling rate is proportional to $T_{\text{gas}}^{1.5}$ when $T_{\text{gas}} \gg T_{\text{dust}}$, but $\gamma$ becomes larger than 1.5 when $T_{\text{gas}} \approx T_{\text{dust}}$ (see Appendix B). Since the dust temperature is 10 K in our model, the gas is more unstable at higher temperatures.

5. SUMMARY AND DISCUSSION

We performed the 1D hydrodynamic simulations with the effects of heating, cooling, and chemical reactions in order to study the thermal stability of shocked gas in CNM and molecular clouds. Taking advantage of the fact that the growth rate derived from the linear analysis (Schwarz et al. 1972; Koyama &
Inutsuka (2000) is applicable to the non-linear regime in thermal instability, we calculate the e-folding number along the flow to evaluate the amplification of density perturbation behind the shock wave. Our findings are as follows.

1. Both CNM and molecular cloud can be thermally unstable behind a shock wave.

2. A molecular cloud becomes thermally unstable behind a shock when \( V_{\text{fluid}} \gtrsim 1.5 \log(n \text{ cm}^{-3}) - 2 \text{ km s}^{-1} \).

3. The integrated e-folding number in the shocked molecular cloud increases with increasing \( V_{\text{fluid}} \) and decreasing pre-shock density.

4. The wavelength, \( l_{p} \lesssim \lambda \lesssim l_{a} \), the perturbation of which can grow within the cooling timescale, ranges from \( 10^{-5} \text{ pc} \) to 0.1 pc in the CNM, and from \( 10^{-7} \text{ pc} \) to 0.1 in molecular clouds. The unstable wavelength is a decreasing function of pre-shock density and fluid velocity, since both the Field length \( l_{p} \) and acoustic length scale \( l_{a} \) decrease with gas density.

The unstable wavelength of the thermal instability coincides with the size of the tiny scale structures observed in the CNM (Heiles 1997; Stanimirović & Heiles 2005) and molecular clouds (Langer et al. 1995; Heithausen 2002; Sakamoto & Sunada 2003; Tachihara et al. 2012). Thermal instability could thus explain the formation of such small gravitationally unbound clumps in the ISM. In the CNM, the initial perturbation is amplified by a factor of \( 10^{2} \) in the thermally unstable region behind a shock. In molecular clouds, on the other hand, the initial perturbation is amplified only by a factor of a few. It should be noted, however, that the supersonic velocity dispersion is ubiquitous in molecular clouds. Small clumps would be formed if the molecular cloud is swept by multiple shocks.

Finally, we discuss the fate of the structure formed by the thermal instability. In the H1 medium, the CNM can coexist with the WNM thanks to the thermally bistable nature (Field et al. 1969). If the molecular clouds are isothermal uni-phase medium, the density fluctuations enhanced by the thermal instability could exist only in the very narrow shock transition layer, because the post-shock gas eventually returns to the same temperature as the pre-shock gas. However, recent numerical simulations of molecular cloud formation (e.g., Banerjee et al. 2009; Inoue & Inutsuka 2012) have shown that molecular clouds are composed of the cold molecular gas \((T \sim 10 \text{ K} \text{ and } n > 100 \text{ cm}^{-3})\) and the non-equilibrium diffuse warm gas \((T > 1000 \text{ K} \text{ and } n \sim 1 \text{ cm}^{-3})\). The diffuse warm component, which is generated by the cloud-forming shocks at the envelope of the molecular cloud, is in high pressure and thermally unstable. The implication of this result is twofold. First, it shows that a bistability of thermal equilibrium gas is not needed for density fluctuations to survive in post-shock gas. Second, in such a “non-equilibrium two-phase medium,” the structure formed by the thermal instability behind the shock within molecular clouds would be more likely to survive in the non-equilibrium diffuse gas. If the perturbed gas can fragment and coexist with the diffuse gas, the very small scale structure could survive until at least the diffuse gas cools down \((\sim 0.1-1 \text{ Myr})\). However, it is still an open question, and multi-dimensional simulations are necessary to confirm our expectation.

We are grateful to the anonymous referee for helpful comments, which have improved the manuscript. This work is supported by Grant-in-aids from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan, No. 21740146 and No. 23740154 (T.I.), and No. 21244021, No. 23540266, and No. 23103004 (Y.A.). Numerical computations were in part carried out on Cray XT4 at the Center for Computational Astrophysics, CfCA, of the National Astronomical Observatory of Japan.

APPENDIX A

MODIFICATION OF BALBUS CRITERION

We assume a gas with the cooling rate \( An^{\gamma}T^{\gamma} \) (see Elmegreen 1991). The number density and temperature of an unperturbed state are \( n_{0} \) and \( T_{0} \). Then the gas is compressed isobarically; the perturbed gas density and temperatures are \( n_{p} = \alpha n_{0} (\alpha > 1) \) and \( T_{p} = T_{0}/\alpha \). The Balbus criterion becomes

\[
\frac{\partial}{\partial T} \left( \frac{\Lambda - \Gamma}{T} \right)_{p} \simeq \frac{A T_{p}^{\alpha} n_{p}^{\beta}/T_{p} - A T_{0}^{\alpha} n_{0}^{\beta}/T_{0}}{T_{p} - T_{0}} < 0. \tag{A1}
\]

Considering \( T_{p} - T_{0} < 0, n_{p} = \alpha n_{0} \), and \( T_{p} = T_{0}/\alpha \), it is straightforward to derive

\[
A T_{p}^{\alpha} n_{p}^{\beta}/T_{p} - A T_{0}^{\alpha} n_{0}^{\beta}/T_{0} = A T_{0}^{\gamma-1} n_{0}^{\beta}(\alpha^{\beta-\gamma+1} - 1) > 0. \tag{A2}
\]

We can see that the Balbus criterion is satisfied if \( \beta - \gamma + 1 > 0 \).

APPENDIX B

TEMPERATURE DEPENDENCE OF DUST–GAS COLLISIONAL COOLING

The cooling function of dust–gas collisional cooling is

\[
f[T] = AT^{0.5}(T - T_{\text{dust}}), \tag{B1}
\]

where \( A \) is constant and \( T \) and \( T_{\text{dust}} \) are temperatures of gas and dust, respectively. The power index \( \gamma = \partial \ln(f[T])/\partial \ln(T) \) becomes

\[
\gamma \simeq \frac{\ln(f[(1 + d\delta)T]) - \ln(f[T])}{\ln(1 + d\delta T) - \ln(T)}, \tag{B2}
\]

where \( d\delta \simeq 0. \) We set \( T = \alpha T_{\text{dust}}, \) and it is straightforward to derive

\[
\gamma \simeq 0.5 + \frac{\ln(1 + d\delta(1 - 1/\alpha^{-1}))}{\ln(1 + d\delta)}. \tag{B3}
\]

If \( \alpha \gg 1 \) \((T \gg T_{\text{dust}}), \ gamma \simeq 1.5 \), and if \( \alpha \approx 1 \) \((T \approx T_{\text{dust}}), \ gamma \) becomes larger than 1.5.

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