Long distance modifications of gravity in four dimensions.

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We discuss some general characteristics of modifications of the 4D Einstein-Hilbert action that become important for low space-time curvatures. In particular we focus on the chameleon-like behaviour of the massive gravitational degrees of freedom. Generically there is at least one extra scalar that is light on cosmic scales, but for certain models it becomes heavy close to any mass source.

1 The models

In this talk we will look at some aspects of modifications of the four dimensional Einstein-Hilbert action that are of the following general form:

\[ S = \int d^4 x \sqrt{-g} \frac{1}{16\pi G_N} \left[ R + F(R, P, Q) \right], \]

with \( P \equiv R_{\mu\nu} R^{\mu\nu} \) and \( Q \equiv R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} \). We want to modify gravity at long distances or low curvatures, so we introduce some crossover scale \( \mu \) where the modification kicks in, such that we have conventional general relativity (GR) for large curvatures and something different for low curvatures:

\[ R \gg F(R, P, Q) \quad \text{for} \quad R^2, P, Q \gg \mu^4 \quad \text{and} \quad R \ll F(R, P, Q) \quad \text{for} \quad R^2, P, Q \ll \mu^4. \]

Models of this kind have been proposed as a way to generate late time acceleration without a cosmological constant. For this to work the crossover scale has to be of the order of today’s Hubble constant: \( \mu \sim H_0 \). In the Friedmann equation obtained for these theories, the modification then only becomes important at the present stage of the expansion of the Universe. Obviously the detailed predictions will depend on the specific model at hand. So far the only model that has been put to test is the \( n = 1 \) case of:

\[ F(R, P, Q) = -\frac{\mu^{2+4n}}{(aR^2 + bP + cQ)^n}, \]

and it was found that it can fit the SN data for a certain range of parameters \( a, b, c \) (but see for stability constraints).

2 The excitations

To understand the physics of the models a good starting point is to examine their excitations. In the case of GR, the two only degrees of freedom are contained in the massless spin 2 graviton.
For a massless spin 2 particle the weak field limit is unique and this mediates the gravitational force in a very specific way. For the models (\text{I}), one will have six more degrees of freedom in addition to the massless spin 2 graviton. On vacuum, one of them is a massive scalar and the other five are contained in a massive spin 2 ghost, whose negative kinetic energy would arguably lead to the decay of a homogeneous background into a complete inhomogeneous state full of negative and positive energy excitations. To check the validity of these type of models one could try to calculate the decay time of the vacuum and check if the result is compatible with observations, but it is probably safer not to have ghosts at all.

Fortunately one can show that the massive spin 2 ghost disappears altogether for modifications of the form $F(R, Q - 4P)^5$, and we will assume this form from now on. The modification is then characterized by one extra scalar degree of freedom in addition to the massless spin 2 graviton of GR. Remember that by the conditions (2) we can neglect the corrections due to the modification for large curvatures. This will translate itself to a large mass of the extra scalar on backgrounds that have a large curvature. Indeed, since the interaction range of an excitation is typically of the order of the inverse mass (we use units such that $c = \hbar = 1$), the scalar will effectively decouple on those backgrounds. For the models (3) for instance, the running of the mass $m_s$ with the background curvature $R$ is given by an expression like:

$$m_s^2 \sim R \left( \frac{R}{\mu^2} \right)^{2n+1},$$

(4)

where $R$ stands for a certain combination of components of the background Riemann tensor. Also the effective Newton’s constant, describing the coupling of the gravitational excitations to matter, will run with the background curvature:

$$G_N^{\text{eff}} \sim \frac{G_N}{1 + (\frac{\mu^2}{R})^{2n+1}},$$

(5)

and again we see that we recover GR for large curvatures ($G_N^{\text{eff}} \rightarrow G_N$).

This background dependence of the mass of the scalar and of the effective Newton’s constant is a manifestation of the violation of the strong equivalence principle for this type of theories: the properties of the local gravitational excitations depend intrinsically on the background, in that sense they behave like a gravitational chameleon. In general, one can have a complete breakdown of the local Lorentz symmetry for the short distance excitations. To assess the stability of a certain background under short distance fluctuations, one will have to look at the propagation of the degrees of freedom on that background. On general FRW backgrounds for instance, the propagation of the spin 2 graviton can be sub- or superluminal and its kinetic energy can be positive or negative.

3 The Schwarzschild solution

Gravity, like electromagnetism, has infinite range. This means that GR influences a huge variety of phenomena on a vast range of distance scales. So if you fiddle with it to get some interesting modification for the expansion of the Universe, you should check the effect on all the other gravitational phenomena. Now, if we think about the modification in terms of extra degrees of freedom there seems to be a problem. On one hand, if we want a modification at cosmic scales, we need the mass of these extra degrees of freedom to be at most of the order of today’s Hubble constant, giving them an effectively infinite interaction range. But on the other hand we know that GR has been tested in our Solar System to very high accuracy which seems to exclude any extra light degrees of freedom. The way out of this problem is to have some mechanism that can decouple the extra degrees of freedom in the Solar System. As explained in the previous
section, for the type of models the mass of the extra scalar grows for large curvatures which suggests that such a mechanism could take place. It is instructive to see how this happens and what the modification will be for a spherically symmetric solution corresponding to a central mass source $M$ on a cosmological background, which we will take to be de Sitter for simplicity. (Notice that flat Minkowski space typically won’t be solution.) From (4) we see that on the cosmic background the extra scalar is indeed very light: $m_s \sim H_0 (\sim \mu)$. And at first order in the weak field expansion the Schwarzschild solution reads (for distances $r \ll H_0^{-1}$):

$$ds^2 \sim -\left(1 - \frac{2G eff M}{r} - \frac{2G eff M}{3r}\right) dt^2 + \left(1 + \frac{2G eff M}{r} - \frac{2G eff M}{3r}\right) dr^2 + r^2 d\Omega_2^2, \quad (6)$$

where we have written separately the contributions from the spin 2 graviton and the scalar. If this was the solution in the Solar system, the theory would be clearly ruled out. Indeed, the contribution from the scalar in (6) would make it impossible to fit both the orbits of the planets and the observed light bending with the same value for Newton’s constant. However, the weak field expansion breaks down at a huge distance $r_V = (G N M/H_0^3)^{1/4}$. For the Sun this distance is of the order of 10 kpc. For shorter distances, inside the Solar system for instance, one can not trust the perturbative solution anymore. This is in stark contrast with GR, where the weak field expansion can be trusted throughout the whole Solar System.

In fact we could have guessed the perturbative expansion to break down when approaching the mass source, simply because we know that we should recover something very close to the GR solution for short distances. This happens because the curvature of the GR Schwarzschild solution ($Q = 48(G_N M)^2/r^6$) blows up close to the mass source, thereby killing the modification. We can then estimate the distance $r_c$ where the modification becomes important by looking at the extra scalar. This scalar will produce an order one modification for distances $r$, smaller than its inverse mass. But we know that the mass depends on the background curvature (Eq. 4).

For the GR Schwarzschild solution this gives a mass that runs with the distance as:

$$m_s(r) \sim \mu \left(\frac{G_N M}{\mu^2 r^3}\right)^{n+1}. \quad (7)$$

So we can expect the scalar to really modify things, only for distances for which:

$$r < \frac{1}{m_s(r)} \implies r > r_c \equiv \left(\frac{(G_N M)^{n+1}}{\mu^{2n+1}}\right)^{\frac{1}{3n+2}}. \quad (8)$$

For the Sun, $r_c$ is at least of the order of 10 pc (for $n \geq 1$), so the background dependence of the scalar mass indeed provides a mechanism that decouples the scalar in the Solar System. We recover therefore GR, up to small corrections. We have calculated these corrections in an expansion on the GR solution. For the GR Schwarzschild solution this gives a mass that runs with the distance as:

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So the situation for a static mass source on a cosmic background can be summarized as follows. At ultra large distances we get the perturbative solution corresponding to an extra massless scalar with, in addition, a rescaled Newton’s constant $G_{eff}^N$. For distances smaller than $r_V$ this perturbative picture breaks down and we enter a non-perturbative regime. So far we can not say that much about this regime, since the expansions that we have used break down. But then, at distances smaller than $r_c(< r_V)$, the scalar decouples and we can expand on GR, finding a solution that is very close to the GR one and where the corrections can be quantified.

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$^a$ A more realistic set up for the Sun would be to treat it as a probe on the background of the Milky Way, one then finds $r_V \sim 10$ pc.

$^b$ We are assuming a modification that contains the Kretschmann scalar $Q$. This observation does not apply to $F(R)$ modifications.
Let us now put the centre of a galaxy as the mass source. One could impose as a condition in these theories that they produce no modification in the dynamics of the galaxy. This will be the case for large values of $n$ in (3), that give $r_c \sim 1\text{Mpc}$. A more ambitious alternative is to look for models that give a modification that could simulate the effects of dark matter at the galactic level. First of all, one can take a model for which $G_{\text{eff}}^N > G_N$. Measuring rotation curves at ultra large distances, one would then infer the mass of the galaxy to be larger than what it actually is. In that sense the non-perturbative region indeed seems to have the characteristics of a dark matter halo. However, from the success of MOND, we know that the distance ($r_c$) where the would-be dark matter halo begins should correspond to a universal acceleration $a_0 \sim H_0$. This is precisely the case for logarithmic actions. But due to lack of space we will have to refer to the proceedings of another talk, given by one of us at a different session of this year’s Rencontres de Moriond, for a report on the interesting phenomenology of this class of models.

4 Conclusions

Models of the type (1) clearly have potential as a possible alternative for $\Lambda$CDM. Obviously, for them to become a real contender there are still a lot of questions that need to be answered. What is for instance the effect of the chameleon-like behaviour of the gravitational excitations on the CMB? Another concern, derived from the non-perturbative behaviour of these models, is that it is not yet clear how to estimate the effects of quantum corrections for this type of actions. This would be necessary, for instance, to have an idea of the amount of fine tuning required in these effective actions. Still, we believe that it is worthwhile to explore the bottom-up approach of building a generally covariant classical action to model long distance modifications of gravity; the advantage being that one can make clear contact with experiment at many levels.

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