On anti-gravitational phenomenon of excited states in quantum systems

Lijia Jiang and Jun-Hui Zheng
Institute of Modern Physics, Northwest University, 710127 Xi’an, China

It is common belief that gravity is an attractive interaction between all things with mass or energy, affecting the motion of objects at the macroscopic scale and determining the large-scale structure of the universe. Contrary to the conventional cognition, here we reveal that gravitational repulsion is also ubiquitous in quantum systems — the anomalous response of the position of the particles at (topologically) excited states to the gravitational field in confined systems. We prove that this anti-gravitational phenomenon results from a principle called ‘quantum-state exclusion’ inherited from the orthogonality of quantum states. We further predict that, in an inflating space, this gravitational anomaly may cause quantum matter in excited states to expand even faster than space, leading to an observable accumulation of quantum matter near the boundary of the space. These unique phenomena can be simulated in ultracold atom experiments by using Bose-Einstein condensates with solitons. The accelerating expansion phenomenon in quantum systems also sheds new light on understanding the evolution of the universe, where the vacuum state may also be an excitation with topological defects.

I. INTRODUCTION

Newton’s law of universal gravitation is the first systemic description of gravitational attraction between objects, unifying projectile motion and stellar motion. Einstein further developed general relativity, depicting gravity using curved spacetime [1], after taking into account the equivalence between mass and energy and the indistinguishability between gravitational mass and inertial mass. The theory remains a classical theory, unlike the other three known fundamental interactions that are jointly described by the standard model in quantum field theory. Although the quantization of the gravitational field itself faces enormous theoretical difficulties [2], gravity encounters quantum mechanics in many other practical situations and their meetings bring exotic phenomena and novel physical insights, including but not limited to the Hawking radiation, entropy and information of black holes [3–5], the Unruh effect of the vacuum viewed from an accelerating observer [6], the new interpretation of the equivalence principle [7–13], gravity-induced redshift of matter waves [14–17], precise measurements [18, 19], and the essence of the vacuum and cosmic evolution [20].

In this Letter, we reveal another exotic phenomenon — the quantum-effect-induced gravitational repulsion — the expectation value of the position (i.e., the center of mass) of the particles at (topological) quantum excited states ascends when the gravitational field is enhanced. This anti-gravitational phenomenon also implies anti-inertial behavior — in the weightless environment, the average velocity of the particles confined in a microscopic box well can be greater than the box’s velocity when the acceleration of the box is increased. We further prove that this gravitational anomaly obeys the energy-conservation law: as increasing the gravitational field strength, the particle at excited states rises its height by releasing the kinetic energy. We also demonstrate that this anti-gravitational phenomenon is a consequence of a principle we name as ‘quantum-state exclusion’: the particle at the ground state descents to a lower position under stronger gravity and, correspondingly, the excited states are forced to ascend to a higher position due to the orthogonality between quantum states. Moreover, we analyze the impact of antigravity on the evolution of quantum matter in expanding space. In the context of topologically excited Bose-Einstein condensate (BEC) with solitons, we show that quantum matter could expand even faster than space owing to gravitational repulsion. We predict the peripheral accumulation effect induced by antigravity and accelerating expansion, which are capable to be observed in the current ultracold atom experiments.

II. GRAVITATIONAL RESPONSE IN QUANTUM SYSTEMS

We start with the simplest case that a single particle is confined in a stationary microscopic elevator under a gravitational field \( \mathbf{G} = -g \mathbf{e}_x \) (see Fig. 1a). For simplicity, we employ a one-dimensional infinite square well to model the elevator. The evolution of the state of the trapped particle follows the Schrödinger equation,

\[
\frac{i \hbar}{\partial t} \phi = \frac{-\hbar^2}{2m} \frac{\partial^2 \phi}{\partial x^2} + [V(x) + mgx] \phi, \tag{1}
\]

where the box well \( V(x) \) vanishes for \( |x| < \frac{L}{2} \) and is infinite elsewhere. \( L \) is the width of the well.

In Fig. 1b, we plot the probability density distribution \( |\phi_n|^2 \) for the steady-state solutions of Eq. (1) in real space, exhibiting substantial changes when gravity is present. Specifically, the density distribution of the ground state is pulled down to reduce the gravitational potential energy. For excitations, the wavefunction nodes where \( \phi_n = 0 \), move downwards, indicating that the average wavelength of the matter wave above the node becomes longer than that below. This reflects the gravitational redshift effect of matter waves [14–17], which originates from the fact that the kinetic energy converts...
into the gravitational potential energy as the particle inside the elevator moves upwards. Moreover, the peaks of the probability density of the excitations are enhanced at the upper part but suppressed at the lower part due to the existence of the gravitational field, indicating that the particle favors moving upwards when gravity is present.

To confirm this conjecture, in Fig. 1c, we plot the center of mass $x_c$ as a function of the gravitational field strength $g$ for the first several steady states. If we further assume that the field strength slowly increases over time so that the system adiabatically remains at the same energy level, then the curve $x_c(g(t))$ also represents the trajectory of the center of mass of the particle in the elevator. Moreover, we define the gravitic susceptibility for the adiabatic state, $\chi = \delta x_c / \delta g$. According to the sign of $\chi$, the quantum states are classified to be para-gravitic and anti-gravitic, corresponding to different responses that the center of mass sinks ($\chi < 0$) and rises ($\chi > 0$) when slightly increasing $g$, respectively. From the figure, we find that in contrast to the ground state which is para-gravitic, the excitations are all anti-gravitic in the relative weak gravity regime and turn to be para-gravitic in sequence as the increase of the gravitational field strength (see the curves $n = 1, 2$ for the tendency).

Note that it was proved that a stationary system in a gravitational field $\mathbf{G} = -g\mathbf{e}_z$ is equivalent to a moving quantum system with acceleration $g\mathbf{e}_x$ [13]. Therefore, the phenomenon of antigravity implies the emergence of anti-inertia, i.e., the excitations (in a weightless environment) will rise even faster than the elevator, when the elevator starts increasing its upward-acceleration slowly.

The anti-gravitational behaviors are counter-intuitive, so they are doubted frequently whether the law of energy conservation is kept [21]. Thus, before explaining the origination of antigravity, we illustrate the rationality of antigravity in quantum systems. In the differential form, the work on the particle contributed by gravity due to the change of the field strength is $\delta W = -mg\delta x_c$. By denoting the corresponding change of kinetic energy as $\delta E_k$ and the change of the wavefunction as $\delta \phi_n$, we have

$$\delta E_k - \delta W = \left[ \langle \delta \phi_n | \hat{p}^2/2m | \phi_n \rangle + mg \langle \delta \phi_n | \hat{x} | \phi_n \rangle \right] + \text{c.c}$$

$$= \varepsilon_n \left[ \langle \delta \phi_n | \phi_n \rangle + \text{c.c} \right] = 0,$$

where $\varepsilon_n$ is the eigenenergy for the state $\phi_n$. The work done by gravity totally transfers into the change of kinetic energy, obeying the law of energy conservation.

When increasing $g$ slowly from zero, the total work done by gravity is $W = -m \int gdx_c$, where the integral is exactly the area left covered by the curve $x_c(g)$ (see the shadows in Fig. 1c for cases $n = 0, 1$). The work is positive for the ground state, but negative for excitations for relatively small $g$. It is worth stressing that, due to the non-monotonicity of $x_c(g)$ for excitations, the work can be different even when the center of mass is shifted by the same distance, and a positive $x_c$ does not always mean that the work done by gravity is negative (for the example shown in Fig. 1c, the work is negative from the origin to point A but positive from the origin to B).

To see how the kinetic energy of the particle is changed by gravity, in Fig. 1d, we plot the particle’s probability density distribution in momentum space for different $g$. For the excitation $n = 1$, the probability density in the low-momentum regime is enhanced by gravity and that in the high-momentum regime is suppressed, which is opposite to the gravity effect on the ground state. If we interpret kinetic energy as the system’s internal energy (like as in thermodynamic systems), then the excitation lifts its gravitational potential energy by releasing its internal energy, inducing the anti-gravitational phenomenon. Or we can say, a quantum particle realizes antigravity by ‘cooling’ itself. This is unique for quantum systems, since due to the uncertainty principle, the quantum state always has finite kinetic energy and thus has a chance to ‘cool down’.
III. WHY ANTIGRAVITY IS POSSIBLE?

Now we explain the origination of antigravity in quantum systems. For quantum systems with arbitrary finite size, the arithmetic mean of the center of mass over all eigenstates of the Hamiltonian, \( \langle x_c \rangle \equiv \text{Tr}[\hat{x}] / \text{Tr}[1] = \int x dx / \int dx, \) is well defined and is invariant against any disturbance. Therefore, the average shift of the center of mass by gravity over all eigenstates remains zero. As a result, it is not allowed that states are all para-gravitic or all anti-gravitic. We name this principle as quantum-state exclusion. This exclusion stems from the orthogonality and completeness of quantum states. It is an intrinsic property of quantum states, no matter whether the states are occupied and whether the particle is boson or fermion. This is quite different from the Pauli exclusion principle that describes the rule of occupying the states are occupied and whether the particle is boson or fermion. This is quite different from the Pauli exclusion principle which is inspired by the cosmic evolution, we are curious how quantum systems with arbitrary finite size are suppressed by the repulsive interaction, the main properties of these steady solitonic solutions \( \Phi_n \) match with those of single particle eigenstates of Eq.(1), including the para/anti-gravitonic effects and the gravitational redshift effect (see Appendix). As a result, the center of mass of the solitonic BEC \( (n \geq 1) \) will move upwards (due to antigravity) and the solitons will move downwards (due to redshift) when the gravitational field strength starts increasing slowly.

\[
\chi \equiv \frac{\delta \langle \hat{\Phi} \hat{x} \rangle}{\delta g} = 2m \sum_{n \geq 1} \frac{\langle \Phi_n | x | \phi_n \rangle^2}{\varepsilon_0 - \varepsilon_n},
\]

is negative since \( \varepsilon_0 < \varepsilon_n \) for all \( n \geq 1 \). Together with the quantum-state exclusion, it generally implies that some excitations must be anti-gravitic for finite-size systems.

IV. OBSERVING ANTIGRAVITY IN THE BEC SYSTEM

The anti-gravitational phenomena are rarely observed in conventional physical systems, since antigravity cannot occur in thermal equilibrium systems as shown in the Appendix. To observe the phenomenon of antigravity in experiments, it is necessary to excite the quantum system and measure the probability density distribution of the particle in position space. Topologically excited BEC in ultracold atom systems are favorable for this purpose due to the following advantages. First, topological excitations can be experimentally realized by using the phase imprinting technique [22–24] or produced as defects in quench dynamics of phase transition according to the Kibble-Zurek mechanism [25–27]. Second, the excitations are topologically (sometimes also symmetrically) protected and thus they have extremely long lifetime [23]. Third, in BEC systems, all particles occupy a same state, so the probability density distribution of the particle in space is identical to the particle density distribution of the macroscopic BEC, which is detectable by absorption imaging technique in cold atom experiments [28, 29].

The BEC can be described by a single macroscopic wavefunction \( \Phi(x, t) \), and its dynamics follows the Gross-Pitaevskii (GP) equation,

\[
\frac{i \hbar}{\partial t} \Phi = -\frac{\hbar^2}{2m} \frac{\partial^2 \Phi}{\partial x^2} + [V(x) + mgx] \Phi + \lambda |\Phi|^2 \Phi,
\]

where the last term refers to the two-body repulsive interaction \( (\lambda > 0) \), and \( \Phi \) is normalized to one. Both the square well \( V(x) \) and the linear potential \( mgx \) can be realized by using optical beams [30]. The GP equation is a many-body extension of the Schrödinger equation. Numerically, we obtain steady and self-consistent solitonic solution \( \Phi_n(x, t) = \Phi_n(x) e^{-i \lambda t} \) of this nonlinear equation by cyclic iteration, where \( n \) is the node number of wavefunctions. In the many-body case, each node corresponds to a dark soliton. Even though the density peaks are suppressed by the repulsive interaction, the main properties of these steady solitonic solutions \( \Phi_n \) match with those of single particle eigenstates of Eq.(1), including the para/anti-gravitonic effects and the gravitational redshift effect (see Appendix). As a result, the center of mass of the solitonic BEC \( (n \geq 1) \) will move upwards (due to antigravity) and the solitons will move downwards (due to redshift) when the gravitational field strength starts increasing slowly.

V. A TOY MODEL OF AN EXPANDING QUANTUM WORLD UNDER GRAVITY

As the responses of quantum matter and classical matter to gravitational fields are significantly different, inspired by the cosmic evolution, we are curious how quantum matter behaves in an expanding space. The controllable BEC system in the ultracold atom experiments provides an ideal simulation platform for us to explore the related dynamics.

Imagine a quantum world that is full of BEC and we make the following assumptions. First, the size of the world is finite and thus the world can be modeled as a box well. Second, the position space expands extremely slowly over time, i.e., the width of the well changes from the initial width \( L \) to \( a_S(t) L \) with a slowly varying scale factor \( a_S(t) \) [20]. Third, the BEC is in an effective central gravitational field (to simulate the gravitation between matter in macroscopic world). For simplicity, we will naively suppose that the gravitational potential keeps the form \( mg x \), and neglect the changes of the field strength \( g \) during expansion.

In experiments, by loading BEC in a ring trap and tuning the ring’s radius, it is possible to simulate an expanding space, as is done in the study of the inflation of BEC in topological geometric spaces [31–34]. An additional potential is needed to confine the BEC in a sector of the ring trap, as the region surrounded by the brown lines shown in Fig.2a. Using the phase imprinting technique, the initial steady solitonic state can be prepared. With these assumptions, we are able to numerically solve the GP equation (4) with the inflation of space \( x \rightarrow a_S(t)x \). When the spatial expansion is sufficiently slow \( (a_S \rightarrow 0) \), the adiabatic approximation can be applied to further simplify the evolution. In this case, the evolved BEC is at the instantaneous steady state of the GP equation.

Without loss of generality, we select \( n = 11 \) to visual-
FIG. 2. a) Left: Density plot of the BEC’s particle number density $|\Phi_n|^2$ at different scales with $a_S = 1, 2, \cdots, 6$ for the zero two-body interaction limit ($\lambda = 0$). The initial BEC is supposed to be loaded in a sector of a ring trap (confined by the brown solid lines) with $a_S = 1$ and evolves as the enlargement of the radius of the ring. Along with the expansion of space, the solitonic BEC expands and accumulates at the boundary of the arc. At large $a_S$, the solitonic BEC ceases expansion (see $|\phi_n|^2$ at $a_S = 4, 5, 6$). Right: The comparison of density profile of the BEC at $a_S = 1, 4, 6$ for different interaction strength $\lambda = 0$ (red-solid lines) and for $\lambda = 100$ (blue-dotted lines). b) The matter expansion factor $a_M$ with respect to the spatial scale factor $a_S$, the ratio between the matter expansion rate and spatial expansion rate $\dot{a}_M/\dot{a}_S$, and the acceleration of matter expansion $\ddot{a}_M$. All results are for $n = 11$ at the adiabatic limit ($\dot{a}_s \rightarrow 0$), and the unit for $\lambda$ is $h^2/mL$. See Appendix for the results of non-adiabatic evolution.

We have discovered the phenomena of antigravity in quantum systems, which is a consequence of quantum-state exclusion that forbids quantum states in finite-size systems to be all para-gravitic or all anti-gravitic. This anomaly obeys the law of energy conservation — quantum systems realize antigravity by releasing the kinetic energy of particles. The impact of gravity on the motion of quantum matter and classical matter are significantly different. Especially, the expansion of quantum matter can be accelerated by the antigravity effect during the spatial expansion. Both the anti-gravitic response of the center of mass of quantum matter to the gravitational field and the antigravity-induced accelerating expansion effect can be simulated and visualized in ultracold atom experiments, by using the soliton-excited BEC. Regarding the present progress in the study of the inflation of BEC [31–34], our study also provides new and unique observable phenomena in topological quantum systems.
further speculate about the evolution of the universe. The astronomical observation shows that the expansion of the universe is accelerating [35–40], which contradicts the slowing effects caused by the inter-matter attractive gravitation. In modern cosmology, this observed result is attributed to accelerating expansion of space — the cosmological constant (which represents the vacuum) is introduced to overwhelm the gravity effects from radiation and matter [20]. Since the early universe underwent spontaneous symmetry breaking in theories, suggesting the vacuum may possess topological defects [41, 42], our finding indicates that the antigravity effect on topological vacuum state may also influence the universe’s expansion.

ACKNOWLEDGMENTS

We thank the funding supports from the NSFC under grant no. 12105223 and no. 12175180. Zheng acknowledges the support from the research start-up funding from Northwest University. Both authors performed calculations, interpreted the results, and cowrote the manuscript. Correspondence should be addressed to Zheng, junhui.zheng@nwu.edu.cn.

Appendix A: A proof of the claim that the system in thermal equilibrium is para-gravitic

In thermal equilibrium, the system’s mass center is,

\[
\bar{x} = \frac{1}{N_p} \sum_n \langle \phi_n | x | \phi_n \rangle f(\varepsilon_n - \mu), \tag{A1}
\]

where \( f(\varepsilon_n - \mu) = 1/[\exp[\beta(\varepsilon_n - \mu)] \pm 1] \) is the Fermi-Dirac or Bose-Einstein distribution. The chemical potential is determined by the total particle number \( N_p = \sum_n f(\varepsilon_n - \mu) \). When increasing \( g \to g + \delta g \), using the perturbation theory, the changed position becomes,

\[
\delta \bar{x} = \frac{N_p m \delta g}{N_p} \sum_{n \neq q} \frac{1}{e^{\beta(\varepsilon_n - \mu)} \pm 1} - \frac{1}{e^{\beta(\varepsilon_q - \mu)} \pm 1} \left| x_{nq} \right|^2 e^{-\frac{\beta(\varepsilon_n - \mu)}{2}} \left( \delta \varepsilon_n - \delta \mu \right)^2, \tag{A2}
\]

where \( x_{nq} = \langle \phi_n | x | \phi_q \rangle \), \( \delta \varepsilon_n = mgx_{nn} \), and \( \delta \mu \) is the change of chemical potential. It is not difficult to prove that \( \delta \bar{x} \) is always negative for \( \delta g > 0 \). Thus, the thermal equilibrium system is para-gravitic.

Appendix B: The steady states \( \Phi_n \) with \( n \) solitons in a linear potential

The properties of these steady solitonic excitations qualitatively match with that of excitations in the single particle case. As shown in Fig. 3, the solitonic BECs are anti-gravitic and the solitons (nodes) move downwards when the gravity is present. However, it is worth to mention that the quantum-state exclusion is weakened by the many-body effects comparing to the one-body system (see Fig. 3b): the self-consistent solutions are not necessary to be orthogonal, i.e., \( \langle \Phi_m | \Phi_n \rangle \neq 0 \) for \( m \neq n \), even through the overlap between the many-body wavefunction is arbitrary small (orthogonality), i.e., \( |\langle \Phi_m | \Phi_n \rangle| \rightarrow 0 \), where \( N_p \) is the total number of particles.

FIG. 3. a) The wavefunction \( \Phi_n \) for the steady states with \( n \) solitons. b) The center of mass of each steady state. The interaction strength is \( \lambda = 100 \).

FIG. 4. Upper: The matter expansion factor with respect to the spatial scale factor. Bottom: The ratio of the matter expansion rate and spatial expansion rate, \( \dot{a}_M/\dot{a}_S \). The dashed lines are for adiabatic limit (\( \dot{a}_S \to 0 \)). The solid lines are the results of the dynamical real-time evolution for \( \dot{a}_S(t) = 4\hbar^2/mL^2 \), which qualitatively agree with the adiabatic result except the oscillations.
Appendix C: Non-adiabatic effect in the dynamical evolution

For the case that $\dot{\lambda}(t)$ is sufficient large, the particles can not stay at the instantaneous steady state of the GP equation during dynamical evolution. The non-adiabatic effect will introduce tunneling between different states and thus induce oscillation to the matter expansion during the evolution. We present the result in Fig.4.
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