Extraction of main elements from a TEM composite laser pattern

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Abstract We examined the atypical formation and superposition of transverse electromagnetic modes (TEM) of high order in a commercial He-Ne laser (@543.5 nm). The competence of the dominant modes exhibits a periodic and stable dynamic, uncommon in industrialized systems. Also, the intensity shows a smooth transition. The origin of the phenomenon is only a product of intra-cavity diffractive components. We found the promotors are a stable optical cavity but quasi-degenerated, with large transversal section (the Fresnel number is irregular), and a breaking the axial symmetry (producing astigmatism). The analysis of the average intensity patterns, and a correlation algorithm—via bi-orthogonal decomposition based on Hermite-Gauss polynomials—expose the five principal TEM elements. These results could be meaningful and inspiring both in courses of optics, laser engineering as in image processing.

1. Introduction

The patterns defined by the high-orders of Hermite-Gaussian (HG) and Laguerre-Gaussian (LG) polynomials are highlighted because of its feature of carrying optical angular momentum [1-3]. This extraordinary characteristic can promote many novel optical technologies; such as super-resolution microscopy, quantum information processing, and optical tweezers for microparticles manipulation. In general, the production of such laser beams has two experimental approaches: 1) extra-cavity methods and 2) direct production within the laser cavity. Some variations of extra-cavity configuration include diffractive optics, computer-generated holograms, and mode converters composed of cylindrical lenses [1, 2]. The imperfections and misalignment of the optical elements lead to unstable and degraded beams. However, direct generation intra-cavity improves the beam quality because of an inherent feedback effect. Usually, this configuration implies the alteration of one cavity mirror. For instance: using a spiral phase plate, a circular absorber, or a spot defect [1-3]. In any case, the production of stable high-order-TEM beams is a laborious and delicate experimental task; for that reason, most of the engineering and physics students have their first
experience with these subjects in advanced laboratories. Fortunately, we acquired a malfunctioning He-Ne laser system: emits periodically mixed and high order TEM patterns; working for over 3000 hrs. (in the last 15 years), this laser has been appropriate in lab demonstrations about optical physics and laser engineering.

In the literature, the reports of diode lasers with undesirable emissions of TEM-high orders are regulars; even some CO2 laser systems possess stable TEM patterns different from the Gaussian TEM00 distribution. However, as far as we know, this is the first report of a market He-Ne laser which emits repeatedly stable high orders patterns for a long time. Here, we demonstrate the systematic observation and numerical simulations of the periodic evolution of transversal electromagnetic modes (TEM) in a particular gas laser. We show the singularities in intensity and polarization of a defective low-intensity laser at room temperature (~20°C) and around -3°C. Using a base of normalized HG images; we identified four particular laser TEM-patterns and their ordered numerical evolution. These results can illustrate a lesson; besides, they contribute with a different point of view in the production and control of structured and complex laser beams.

2. Fundamental theory of transversal laser modes in Cartesian coordinates

All lasers systems possess a finite transverse size perpendicular to the optical-cavity axis. Most of the gas lasers have one or both concave mirrors at the tube extremes to get better alignment and stability in the laser emission, in a process equivalent to the basic-principles of waveguides [1]. Then, cavities show both longitudinal and transversal modes; each mode represents a particular electromagnetic field pattern, which can replicate itself after one round trip within the optical cavity. In other words, incorporating Maxwell’s equations and solving the paraxial form of the Helmholtz equation, with the boundary conditions of the cavity, it can determine the allowed electromagnetic wave patterns. So, it is necessary to incorporate optical gain into these calculations. Electromagnetic fields normal to the cavity axis represent all these modes, named transverse TEM modes. Each tolerable mode corresponds to a definite spatial field distribution at a reflector. The TEM-configurations depend on the optical cavity dimensions, reflector, and aperture sizes inside the cavity. In this way, the modes either have Cartesian or polar symmetry about the cavity axis, Cartesian symmetry arises whenever a feature of the optical cavity imposes a more favorable field direction. For example, setting a Brewster window at one extreme of the cavity. Otherwise, the patterns exhibit circular symmetry. The Hermite–Gaussian polynomials describe Cartesian solutions. Two indices identify a particular mode \((n, m)\), they are the number of nodes in the \(x\) and \(y\) directions, respectively. A mathematical expression for \(HG_{nm}\) modes is [1-2]:

\[
HG_{nm}(x, y) \propto e^{\frac{(x^2+y^2)}{\omega^2}} H_n\left(\frac{\sqrt{2}x}{\omega}\right) H_m\left(\frac{\sqrt{2}y}{\omega}\right) e^{-\frac{i\zeta}{2R}} e^{-\frac{jk(x^2+y^2)}{2R}} \tag{1}
\]

where \(R\) is the wavefront radius of curvature, \(\omega\) is the radius for which the Gaussian term falls to 1/e of its on-axis value, \(\zeta\) is the Gouy phase; and \(H_n\) and \(H_m\) are Hermite polynomials with the non-negative integer indexes \(n\) and \(m\), respectively. The lowest order mode TEM00 exhibits symmetric radial intensity distribution about the cavity axis, in fact, a Gaussian intensity profile in the beam cross-section, and a minimum divergence angle. All these properties render \((0, 0)\) mode desirable in many applications. For these reasons, most of the laser designs optimize only \((0, 0)\) while suppressing higher modes, as usual restricting the transverse size of the cavity. However, when the transversal size of the optical cavity is broad enough, the laser light can display multiple transversal modes. The composite TEM patterns are a linear superposition of higher modes as a function of time:
\[ F(x, y, t) = \sum_{j=0}^{M} \sum_{k=0}^{N} L_{jk} \rho_{jk}(x, y) \psi_{jk}(t) \]  
\hspace{1cm} (2)

Here, \( \psi \) represents the temporal existence of the mode, \( \rho \) the spatial mode (which is described by the HG-polynomials), \( L \) is a weight coefficient; and the subscripts \( j \) and \( k \) represents the order of the laser mode, e.g. \( \rho_{00} \) is the TEM00-laser mode. Then, the laser pattern \( F \) can be static or dynamic.

3. Experimental conditions, numerical simulations, and matching process

3.1. Laser system characteristics

We analyze a He-Ne laser (Melles-Griot, model 05-LHB-670) with an optical cavity length of 0.351 m, provided with two mirrors: the total reflector has a nominal radius of curvature \( R_1 = 0.6 \text{ m} \), and the output mirror an \( R_2 = 0.45 \text{ m} \) with a transmission coefficient of 1.4\%. A Brewster window is after the exit. The fabricator does not inform the radius of the capillary tube. Even so, very close to the output, this radius of the exit aperture is around 1.25 mm.

3.2. Laser system emission spectrum

Figure 1 shows UV-Vis spectra from the implemented laser system, the radiation of a standard He-Ne device, and the light background in the laboratory. The transmitted laser beam was captured by a multi-modal quartz optical fiber (600\( \mu \text{m} \) core aperture), which was connected to an optical spectrometer (Ocean Optics, HR4000CG UV-NIR). It is clear, the implemented laser transmits many lines of radiation, a typical feature in defected gas laser-systems [3]. Still, the local maximums show low intensities (lesser of the 5\%) compared to the largest laser emission at \( \lambda = 543.7 \text{ nm} \); this is the wavelength of the studied TEM-patterns.

3.3. Video-recorded laser patterns

The gas laser system directly lighted —at 3 m of distance— a thin and semitransparent paper screen. Behind the spotless and free-of-wrinkles screen, over a tripod, a digital camera was arranged (Canon EOS-Rebel-T1i). The video-records cover from just a moment before of turn-on the gas laser until 30 minutes of continuous emission. Afterward, a linear polarizer between the screen and the laser was mounted, with the optical axis in the vertical position. The videos show a period of \( 0.67 \pm 0.049 \text{ s} \) (15 full cycles and a percent relative error = 7.3\%) between each TEM-pattern. Besides, the prominent morphological variations in the patterns allowed the identification of three prime shapes in the transversal laser radiation. We separated these photo-frames to get their components.

3.4. Amendments in images

We separated the green channel from the rest of the RGB-photo because this channel possesses information about the laser radiation. By a circular filter (radius =10 pixels), we scaled down the level of random noise (a speckle effect) in the original images, as Fig. 2 shows. 1200x900 pixels conform to the raw images. Yet some zones are full dark and data-less. For this reason, we auto-selected a square of 300x300 pixels around the laser patterns to get a cropped image, where most of the pixels are significant and possess information. After, in the match stage, this cropped window was vertically and horizontally displaced (till 50 pixels of distance) and rotated (from 0 to 90\(^\circ\)) to reduce any error in translation and rotation, respectively. Besides, the small illumination variations, the scale changes, and geometric distortions between the laser patterns were insignificant; we assume that all the observed modes cover symmetrically the same area since the translucent screen was perpendicularly mounted referring to the laser beam. All these precautions enhanced the greatest correlation value, letting to get a more reliable matching between the test and reference images. Finally, we rescaled the images to the same size.
3.5. Algorithm of correlation in pattern recognition

Correlation is a popular and accurate method for pattern recognition and is the core-function used in different emerging matching applications of image processing, such as automatic target recognition in biomedical, forensic science, and products [4]. The method is based on selecting or programming a reference image $R[m,n]$ (with a size of $m \times n$ pixels) and selecting a threshold statistic to a test image $T[m,n]$ matches with the reference. We get in mathematical notation:

$$C[m,n] = \sum_{k=1}^{m} \sum_{l=1}^{n} T[k,l] \cdot R[k + m, l + m]$$  \hspace{1cm} (3)

The correlation output $C[m,n]$ achieves the maximum global only if the test image matches the reference image for each pixel. By comparing the complete image against a reference, the correlation is less sensitive to tiny obstructions and mismatches [4]. Yet, the correlation is prone to decrease when the tested sample shows illumination variations, scale changes, image rotation, translation, and noise [4].

Then, a factual image is contrasted with the HG-database, from (0, 0) to (10, 10). After, via the correlation between the experimental photo and the best first match, we calculated the next higher coefficient from the numerical set. Afterward, the empirical frame is compared again with the previous modulated pattern plus the images in the database. If there is an improvement in the correlation, we determined the new coefficient to get two elemental modes and their respective weights. This accumulative process was repeated five times. Additional operations significantly do not increase the match. Figure 3 shows the algorithm in a flowchart.
3.6. Training test of the proposed algorithm and principal results

Before applying our algorithm with the recorded laser patterns, we probed the method in a training test. In literature, we found adequate experimental and theoretical images about the superposition of TEM-patterns [3]. But, the authors of this previous work mentioned neither the coefficients nor the basic polynomials HG-components of this particular model. Also, this experimental image includes a non-random noise, some horizontal strips crossing the laser pattern, and slight asymmetry respect to the central vertical axis. We unknown if these features were considered in the previous model. Figure 4 illustrates the matching analysis between the experimental image, reference pattern, and our proposed HG-model. In the referential and trivial case, the autocorrelation of the factual image displays 1, as the top value. Hence, the referential model and our proposed model reached values of 0.83 and 0.87, respectively. Thus, our proposed model is a more accurate statistical approximation of the experimental image. Hence, this matching algorithm is proper for the recorded laser patterns. Figure 5 displays the principal result of this work. The sum of five TEM modes and the respective coefficients construct each of the three theoretical patterns. The via to compare the models and the original raw photos was by a correlation matrix. As we expected, the best values are over the diagonal, without confusion among different modes and photo frames. In short, the composite patterns show relatively low matches in the TEMs as a result that higher values require more energy and are less stable. Future works have to study the transitions of the evolution temporal.

4. Conclusions

We recorded the evolution of TEM modes of a He-Ne laser system and extracted the three key patterns of the video. By reducing noise and selecting the region of interest, we compared the experimental images with 2D-models, which were constructed with Hermite-Gauss polynomials. Through a cyclical correlation process, we found the five principal components and their coefficients. The results show higher correlations than 0.926 and unambiguous in pattern identification. This analysis can inspire and illustrate concepts in digital imaging and laser engineering courses and could be a significant demonstration in optics laboratories.
Figure 5. The sum of five components and the respective coefficients results in the numerical pattern. These three proposed models and the experimental images conform to the correlation matrix, where the diagonal shows the highest values, as we expected.

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References
[1] H Joachim, E Jürgen and E O Lux 2018 Lasers, Basics, Advances and Applications, Springer.
[2] K F Renk 2017 Basics of Laser Physics, For Students of Science and Engineering, Springer.
[3] G Machavariani, et al 2007 Effect of the spiral phase element on the radial-polarization (0, 1)* LG beam, Opt. Comm. 271, 190-19.
[4] B V K Vijaya Kumar, A Mahalanobis and R D Juday 2010 Correlation Pattern Recognition. Cambridge.