Excitation of magnon spin photocurrents in antiferromagnetic insulators

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In this Letter, we propose a new mechanism for generating spin currents in antiferromagnetic insulators, which involves circularly polarized light and is motivated by the photogalvanic effect in metals and the direction of the current depends on the polarization. We suggest that an analogous nonlinear effect exists for antiferromagnetic insulators wherein the total spin of light and spin waves is conserved. In consequence, a spin angular momentum is expected to be transferred from photons to magnons so that a circularly polarized electromagnetic field will generate a direct magnon spin current. The direction of the current is determined by the helicity of the light. We show that this resonant effect appears as a second order light-matter interaction. We find also a geometric contribution to the spin photocurrent, which appears for materials with complex lattice structures and Dzyaloshinskii-Moriya interactions.

Antiferromagnetic insulators are promising candidates with which to address the problem of creating and transmitting spin currents—one of the key issues facing spintronics. Spin currents in these materials are carried by magnons and possess nontrivial topological properties including chirality and its conservation. A number of possible applications have been proposed that exploit unique properties of antiferromagnetic magnons. Magnon spin currents in antiferromagnets can be created in several ways. There is a spin pumping mechanism, injection from metallic layer across the interface, and generation using a temperature gradient via a magnonic spin Seebeck effect. Additionally, a magnonic spin Nernst effect was proposed for quasi-two-dimensional hexagonal antiferromagnets with Dzyaloshinskii-Moriya interactions (DMI) and found later experimentally in MnPS3. Ultrafast optical excitation of coherent magnon dynamics is another prominent area, which is in the basic concept of antiferromagnetic optospintronics—a direction targeting optical control of spin states in antiferromagnetic insulators.

In this Letter, we propose a new mechanism for generating spin currents in antiferromagnetic insulators by optical excitation of spin dynamics. The mechanism involves polarized light and is motivated by the photogalvanic effect. In metals, the circular photogalvanic effect is a nonlinear optical response to the electric field \( \mathbf{E}(\omega) \) of a circularly polarized electromagnetic wave that generates an electron photocurrent proportional to \( \mathbf{E}(\omega) \times \mathbf{E}^*(\omega) \). The photocurrent reverses its direction when the polarization of light is switched (see Fig. 1a).

Similar to metallic systems, a symmetry argument suggests that the magnetic component of a light wave, \( \mathbf{B}(\omega) \), propagating in an insulating antiferromagnet is able to create a magnonic spin current. The current will be proportional to \( \mathbf{B}(\omega) \times \mathbf{B}^*(\omega) \), as schematically shown in Fig. 1b.

The argument goes as follows. Electromagnetic chirality is a quantity useful in optics and plasmonics for characterizing the asymmetry in light-matter interactions. Exactly like spin current in an antiferromagnet, the chirality of an electromagnetic field is odd under spatial inversion and even under time-reversal operations. The magnetic field part of the optical chirality is \( \chi^{(m)}_k \sim i k \cdot (\mathbf{B}_k \times \mathbf{B}^*_k) \), where \( k \) is the wave vector of the electromagnetic field, and corresponds to unequal numbers of left- and right-polarized photons. Excitation of antiferromagnetic magnons by this field will likewise create unequal populations of left and right polarized magnon states. The unequal magnon populations can be interpreted as the transfer of spin angular momentum from the light to the antiferromagnet, and results in

![FIG. 1. Schematic picture of the circular photogalvanic effect in metals (a), and the proposed optical excitation of the spin current in antiferromagnetic insulators (b). The circularly polarized light is propagating along the magnetic ordering direction with the wave vector \( k \) exciting magnon spin current \( J_s \). The precession of the sublattice magnetizations \( M_1 \) and \( M_2 \) is shown above.](attachment:image-url)
the generation of a spin current. This effect should be most effective when the light resonantly couples to the spin system.

The optical generation of spin currents can be illustrated semiclassically. For a cubic antiferromagnet with two sublattices \( M_1 \) and \( M_2 \), the energy density can be written as \( w = \delta M_1 \cdot M_2 + \alpha \nabla M_1 \cdot \nabla M_2 - \beta \sum_{i=1,2} (M_i^2)^2 \), where \( \delta \) and \( \alpha \) are the intersublattice exchange parameters and \( \beta \) is the easy-axis anisotropy along the \( z \)-direction. The linearized Landau-Lifshitz equations of motion have the form, \( \partial_t \mathbf{m}_k = -\varepsilon^{(l)}_{k} \hat{z} \times \mathbf{l}_k \) and \( \partial_t \mathbf{l}_k = -\varepsilon^{(m)}_{k} \hat{z} \times \mathbf{m}_k \), where \( \varepsilon^{(m)}_k = \gamma M_s (\delta + 2\beta + \alpha k^2) \), and \( \varepsilon^{(l)}_k = \gamma M_s (2\beta - \alpha k^2) \). \( \hat{z} \) is a unit vector along the \( z \)-direction, \( M_s \) is the saturation magnetization, and \( \gamma \) is the gyromagnetic ratio. The fluctuating components of the sublattice magnetizations are written as \( \mathbf{m}_k = m_{1k} \hat{z} + m_{2k} \), where \( m_{1k} \) and \( m_{2k} \) are the components of the sublattice magnetizations along the \( z \)-direction and \( \hat{z} \) is the easy-axis anisotropy.

We now transform the Hamiltonian assuming symmetric exchange: \( H = \sum_{(ij)} \frac{J_{ij}}{2} \left( S_i^{(+)S_j^{(-)} + S_i^{(-)}S_j^{(+)}} \right) + \sum_{(ij)} J_{ij} S_i^z S_j^z - K \sum_i (S_i^z)^2 \), (2)

which includes symmetric exchange interactions \( J_{ij} \) between the first and the second nearest neighboring sites, the single ion anisotropy constant \( K \sim \beta a^{-3} \) (in what follows we take the lattice constant \( a = 1 \)). Later we will comment on effects that can arise when a Dzyaloshinski-Moriya interaction (DMI) appears as an asymmetric contribution to the exchange interaction.

We now transform the Hamiltonian using a Hostein-Primakoff representation with \( S_i^{(+)} = \sqrt{2S_a} a_i \), \( S_i^{(-)} = \sqrt{2S_a} a_i^\dagger \), \( S_i^z = S - a_i^\dagger a_i \), and \( S_i^z = -S + b_i^\dagger b_i \), where \( A \) and \( B \) are sublattice indexes. Terms to the second order in \( a_i \) and \( b_i \) are kept. Using spin wave variables defined as \( a_i = N^{-1/2} \sum_k e^{i k \cdot r} a_k \) and \( b_i = N^{-1/2} \sum_k e^{i k \cdot r} b_k \), we rewrite Eq. (2) as

\[
\mathcal{H} = \sum_k \left[ A_k \left( a_k^\dagger a_k + b_k^\dagger b_k \right) + B_k a_k b_k + B_k a_k^\dagger b_k^\dagger \right].
\]

(3)

Here \( A_k = 2K \mathcal{S} + z J_1 S - 2S J_2 G \) includes exchange interactions between the first (\( J_1 \)) and the second (\( J_2 \)) nearest neighbors with the lattice form factor \( G_k = \sum_{\delta} \sin^2(\mathbf{k} \cdot \delta) \), and the summation is over next nearest neighbor sites. The parameter \( B_k = |B_k| \exp(-i \varphi_k) \) contains information about the lattice configuration and intersublattice DMI interactions. Without DMI terms, \( B_k = J_1 SCK_k \), where the structure factor is given by \( C_k = \sum_{\delta} \exp(-i \mathbf{k} \cdot \delta) \). The vector \( \delta \) connects nearest neighboring sites.

We consider optical excitation of spin dynamics. Interaction with the electromagnetic field is represented by a Zeeman coupling as \( H_I = -\mu B \sum_i \mathbf{B}(r_i) \cdot \mathbf{S}_i \), where \( \mu B \) denotes the Bohr magneton and \( g \) is the Landé factor, as used in cavity electrodynamics for the magnon-photon interaction [11][12]. The interaction term in Eq. (4) can be written as

\[
H_I = -\mu B \sum_k \left[ \mathcal{B}_k^{(-)}(t) \left( a_k + b_k^\dagger \right) + \text{h.c.} \right],
\]

(4)

where \( \mathcal{B}_k^{(\pm)} = \mathcal{B}_k^\pm \mp i \mathcal{B}_k^0 \) is the circular Fourier component of the magnetic field defined as \( \mathcal{B}_k(r) = N^{-1/2} \sum_t \exp(-i \mathbf{k} \cdot \mathbf{r} \cdot \phi_k \mathcal{B}_k \). Note that this satisfies the identity \( (\mathcal{B}_k^{(-)})^* = \mathcal{B}_k^{(+) \dagger} \). We do not consider coupling between \( S^2 \) and \( \mathcal{B}^2 \), since we consider only electromagnetic waves traveling along the \( z \)-direction.

To diagonalize Eq. (3), we apply a Bogolyubov transformation using two parameters

\[
\begin{pmatrix}
    a_k \\
    b_k^\dagger \end{pmatrix} = \begin{pmatrix}
    \cosh \theta_k e^{i\phi_k} & -\sinh \theta_k \\
    -\sin \theta_k & \cosh \theta_k e^{-i\phi_k} \\
\end{pmatrix} \begin{pmatrix}
    \alpha_k \\
    \beta_k^\dagger \\
\end{pmatrix},
\]

(5)

where \( \alpha_k \) and \( \beta_k \) are operators in the transformed frame. The parameters of the transformation are given by \( \tan \theta_k = |B_k|/A_k \), and \( \phi_k = \varphi_k \). After the transformation, the Hamiltonian in Eq. (4) becomes

\[
H = \sum_k \varepsilon_k \left( \alpha_k \alpha_k^\dagger + \beta_k^\dagger \beta_k \right),
\]

(6)

where the energy dispersion relation is given by \( \varepsilon_k = \sqrt{\alpha_k^2 - |B_k|^2} \).

We now define the magnon spin current. Because the \( z \)-component of the total spin is a conserved quantity, the local magnon density \( n(r_i) = z^{-1} \sum_\delta b_{i+\delta}^\dagger b_{i-\delta} - a_i^\dagger a_i \)
is proportional to the magnon group velocity describing wave packet propagation $v_{\text{pk}} = \nabla v_{\text{pk}}$:

$$\langle J_s^{(1)} \rangle = \frac{1}{4} \sum_k \frac{|M_k|^2 (\epsilon^2 + \omega^2) v_{\text{pk}} h_k^-(\omega) h_k^+(\omega)}{(\epsilon^2 - \omega^2)^2}.$$  

The second term is related to the fast oscillating intersublattice dynamics in Eq. (7):

$$\langle J_s^{(2)} \rangle = \frac{1}{4} \sum_k \Re (K_k M_k^2) h_k^-(\omega) h_k^+(\omega),$$

where $\omega$ denotes the frequency of the electromagnetic wave. In the most common situation when $\varepsilon_k = -\varepsilon_{-k}$, both $v_{\text{pk}}$ and $K_k$ are odd functions of $k$, and the only nonzero contribution in Eqs. (12) and (13) comes from the asymmetric part of the field intensity. As mentioned above, this quantity is proportional to difference between number of left and right polarized photons and determines the chirality of optical field. For $\varphi_k = 0$, we can combine Eqs. (12) and (13) in the following form

$$\langle J_s \rangle = \frac{i}{2} \sum_k \frac{\gamma_k^2 \nabla q_k + \omega^2 \nabla q_k^h}{(\varepsilon_k^2 - \omega^2)^2} [h_k^*(\omega) \times h_k(\omega)]_z,$$

where $\gamma_k = A_k + |B_k|$ and $q_k = A_k - |B_k|$. This is in agreement with the semiclassical picture [40].

The most interesting region for experiment occurs near the antiferromagnetic resonance, $\omega \approx \varepsilon_k$. In this frequency region, dissipation plays a crucial role, and can be included in our formalism phenomenologically by the replacement $\varepsilon_k - \omega \approx \pm i \Gamma$ and $\varepsilon_k + \omega \approx 2 \omega_{\text{rs}} \pm i \Gamma$ in the equations above, where $\omega_{\text{rs}}$ is the resonant frequency and $\Gamma$ denotes the spin-wave damping. Ballistic transport will occur in materials with small damping and large resonant frequencies, $\omega_{\text{rs}} \gg \Gamma$ where the dominant contribution comes from the term proportional to group velocity. Near the resonance, the spin current is given by

$$\langle J_s \rangle \approx \frac{i q_k}{4 \hbar \omega_{\text{rs}}^2 v_{\text{pk}}^2} [h_k^* \times h_k]_z.$$  

We note that experiments have demonstrated [44] that a thin NiO or CoO layer ($\sim 1 \text{ nm}$) provides a considerable enhancement of spin current transmission in a multilayer system [45, 46]. This motivates making an estimate of the magnitude of currents that may be expected for the present mechanism for NiO. The magnitude of the resonant spin current can be estimated as

$$\langle J_s \rangle \approx \chi g \mu_B J_1 S^2 c_s I_B / (2 \omega^2 \eta^2 \omega_{\text{rs}}^2),$$

where $\chi = \pm 1$ is the polarization helicity, $c_s$ is the velocity of spin waves, $c$ denotes the speed of light, $I_B = (25 \omega)^2$ is the intensity of magnetic field, and we take $\Gamma = \eta \hbar \omega_{\text{rs}}$, where $\eta$ is the Gilbert damping. For a typical antiferromagnetic insulator such as NiO, we assume $c_s = 3 \times 10^7 \text{ m/s}$, $J_1 = 200 \text{ K}$, $\omega_{\text{rs}} = 30 \text{ THz}$, $\eta = 10^{-4}$, $a = 0.5 \text{ nm},$
which gives $\langle J_z \rangle \approx 1.5 \times 10^4 \text{ A/m}^2$ (in electric units $e/\hbar$) for magnetic field $B \approx 10 \text{ mT}$. Such magnetic field corresponds to electric field strength of the laser beam $\approx 30 \text{ kV/cm}$, which is below the maximum field strength achieved in THz laser pulses [47, 48]. For a focused spot size about 100 µm the total spin current through the spot area will be $\approx 0.1 \text{ mA}$, which is the same order as the current estimated for the magnon Nernst effect [27, 28].

Lastly, we discuss the effects of DMI and magnon geometrical phase. Generally, with DMI we can excite the spin current even with linearly polarized light. For illustration, we consider a two dimensional antiferromagnet where electromagnetic wave polarized in $x$-direction is traveling along the $y$-direction with a wave number $k$. There are many possibilities of DMI configurations in a two-dimensional system with some being summarized in Ref. [49]. We describe these configurations by modifying the first term in the Hamiltonian such that a phase term and effective interaction appear, $J_{ij} \rightarrow J_{ij} \exp(i\varphi_{ij})$. We choose $D_{ij}$ to point along the $z$ direction, and include phase factors with $\tan \varphi_{ij} = D_{ij}/J_{ij}$, with an effective exchange parameter $\tilde{J}_{ij} = (J_{ij}^2 + D_{ij}^2)^{1/2}$.

Let us first consider the case of uniform DMI on a square lattice, $\sum_{i,j} D_{ij}(S_i \times S_j)_z$, where $D_{ij} = D_1$ for nearest neighboring $i$ and $j$ along the $x$-direction, see Fig. 2 (a). Such a configuration does not give a complex phase in Eq. (3), but it does shift the origin of $\varepsilon_k$ by $Q \sim D_1/J_1$, which leads to a finite group velocity $v_g \sim D_1$ at $k_x = 0$. Similar to equation [15], linearly polarized light in the resonant region induces a spin current in the $x$-direction proportional to the intensity of magnetic field $\langle J_{s,x}^{(c)} \rangle \approx q_k v_g / (4h \omega_c \Gamma^2) |\tilde{h}_k^{(c)}(\omega_c)|^2$. The magnitude of this effect will be typically $D_1/J_1 \approx 10^{-3}$ times smaller than estimated above for circularly polarized field. We note that a similar result is reported in Ref. [50].

Equation [12] shows that there is also a geometrical contribution to the spin current from the phase gradient term in Eq. [9], which is given by

$$\langle J_{s,x}^{(c)} \rangle = \frac{1}{2} \sum_k |B_k| \sin \varphi_k \nabla_{k} \varphi_k \tilde{h}_{k}^{(-)}(\omega)\tilde{h}_{k}^{(+)}(-\omega).$$

This phase $\varphi_k$ is an offset between the dynamics of the $A$ and $B$ magnetic sublattices, $a_k(t) \sim \exp(i\varepsilon_k t)$ and $b_k^{(+)}(t) \sim \exp(i\varepsilon_k t - i\varphi_k)$, owing to the effect of DMI [19]. Alternatively, the magnon phase can be generated by the electric field, through the the Aharonov-Casher effect, which was proposed in Refs. [51, 52] to realize topological magnonic states. This mechanism opens a possible way to manipulate the spin current in Eq. [16] by the electric field.

To demonstrate phase effects, let us take the example of a two-dimensional antiferromagnet on a honeycomb lattice. Even without DMI, this model is characterized by finite phase [27, 28], which satisfies the symmetry condition $\varphi_k = -\varphi_{-k}$. We break this symmetry by a constant phase originating from the nearest neighboring staggered DMI, $D_{ij} = D$ for $ij = \{BA\}$ and $-D$ for $ij = \{AB\}$ (see Fig. 2 (b)). In this case, we have $B_k = JSC_k \exp(i\varphi_0)$, where $J = (J_1^2 + D^2)^{1/2}$, $\tan \varphi_0 = 1 + 2i\sin(k_x/2)\left[\cos(k_x/2) - \cos(\sqrt{3}k_y/2)\right]$, $C_k = 2C \cos(k_x/2)\cos(\sqrt{3}k_y/2) - 1 - 2i\sin(k_x/2)\left[\cos(k_x/2) - \cos(\sqrt{3}k_y/2)\right]$ is the structure factor for the honeycomb lattice. In the long wavelength limit we can approximate $B_k = JS \exp(i\varphi_0 + i\varphi_k)$, where $\varphi_k \approx k_x(3k_y^2 - k_x^2)/2$. Taking the phase gradient $\nabla_{k_x} \varphi_k$ in the $k_x \rightarrow 0$ limit, we can generate a spin current perpendicular to the direction of wave propagation with a direction controlled by the sign of $\phi_0$, i.e. $\langle J_{s,x}^{(c)} \rangle = 3g_r^2 \mu_B^2 J_S/(8\hbar^2 c^2) \sin \phi_0 I_B \omega^2 / (\omega^2 - \varepsilon_k^2)$. Away from the resonance, this expression will be independent of $\omega$.

The phase contributions may be relevant in such quasi-two-dimensional honeycomb materials as MnPS$_3$ [53] and BiMn$_4$O$_{12}$(NO$_3$)$_2$ [54], although these materials have different configurations of DMI terms than assumed here.

In summary, we have described a resonant induced spin photocurrent for insulating antiferromagnets with a magnitude that may be possible to measure in THz optical experiments. The direction of spin current is determined by the polarization of the optic beam, which is similar to the photogalvanic effect in metals. In our analysis, we assumed a Zeeman magnon coupling with the electromagnetic field, as used in cavity spintronics [42, 50]. We also demonstrated that in the presence of

FIG. 2. (a) Schematic picture of 2D antiferromagnet on a square lattice with uniform DMI, $D_{ij}(S_i \times S_j)_z$. The sign of $D_{ij}$ is positive for $i \rightarrow j$ in the direction of green arrows. Spin dynamics is excited by a linearly polarized wave traveling along the $y$-direction, which generates $J_{s,x}$; (b) The same setup for 2D antiferromagnet on a honeycomb lattice with staggered DMI. The sign of $D_{ij}$ is positive for $i \rightarrow j$ pointing from $A$ to $B$ sites (marked by the arrows).
asymmetric Dzyaloshinskii-Moriya interactions it is possible to induce spin photocurrents using linearly polarized light. The geometric contribution to the spin current from the magnonic Aharonov–Casher phase has been demonstrated for an example honeycomb lattice with staggered Dzyaloshinskii-Moriya interaction.

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Supplemental material: “Excitation of magnon spin photocurrents in antiferromagnetic insulators”

Model formulation

In order to derive the magnon Hamiltonian, we consider the following spin Hamiltonian for an antiferromagnetic insulator with two magnetic sublattices \( S_A \) and \( S_B \)

\[
\mathcal{H} = \sum_{(ij)} J_{ij} S_i \cdot S_j + \sum_{(ij)} D_{ij} (S_i \times S_j)_z - K \sum_i (S_i^z)^2
\]

\[
= \sum_{(ij)} \frac{\tilde{J}_{ij}}{2} \left( e^{i \varphi_{ij}} S_i^{(+)(-)} S_j^{(-)} + e^{-i \varphi_{ij}} S_i^{(-)} S_j^{(+)(-)} \right) + \sum_{(ij)} J_{ij} S_i^z S_j^z - K \sum_i (S_i^z)^2, \tag{S1}
\]

where \( J_{ij} \) is the exchange interaction between lattice sites \( i \) and \( j \), \( D_{ij} \) is the \( z \)-component of the Dzyaloshinskii-Moriya (DM) vector between \( i \) and \( j \), and \( K \) is the single ion magnetic anisotropy, which stabilizes a uniform antiferromagnetic ordering along the \( z \)-axis \([39]\). The Dzyaloshinskii-Moriya interaction (DMI) determines the phase factors \( \varphi_{ij} = \tan^{-1}(D_{ij}/J_{ij}) \) and the effective exchange constant \( \tilde{J}_{ij} = \sqrt{J_{ij}^2 + D_{ij}^2} \) in the second equation, where \( S_i^{(+)} = S_i^z \pm i S_i^y \) are the circular components of the spin operators. In general, the spin ordering may become canted in the presence of DMI. However, in what follows, we imply that the uniform order is stabilized by the presence of anisotropy term. We do not specify explicitly the lattice configuration at this point.

The spin wave part of the Hamiltonian in Eq. (S1) can be separated by using the Holstein-Primakoff representation of spin operators

\[
\begin{align*}
S_{iA}^{(+)z} &= \sqrt{2S} a_i, & S_{iB}^{(+)z} &= \sqrt{2S} b_i, \\
S_{iA}^{(-)z} &= \sqrt{2S} a_i^\dagger, & S_{iB}^{(-)z} &= \sqrt{2S} b_i^\dagger, \\
S_{iA}^z &= S - a_i a_i^\dagger, & S_{iB}^z &= -S + b_i b_i^\dagger,
\end{align*}
\tag{S2}
\]

where \( a_i \) and \( b_j \) satisfy boson commutation rules, and we keep only second order terms in \( a_i \) and \( b_j \). We will further use the Fourier transform of these operators defined as follows

\[
\begin{align*}
a_i &= \frac{1}{\sqrt{N}} \sum_k e^{i k \vec{r}_i} a_k, & a_i^\dagger &= \frac{1}{\sqrt{N}} \sum_k e^{-i k \vec{r}_i} a_k^\dagger, \\
b_j &= \frac{1}{\sqrt{N}} \sum_k e^{i k \vec{r}_j} b_k, & b_j^\dagger &= \frac{1}{\sqrt{N}} \sum_k e^{-i k \vec{r}_j} b_k^\dagger.
\end{align*}
\tag{S3}
\]

The magnon Hamiltonian

We apply the Holstein-Primakoff transformation to the spin Hamiltonian in Eq. (S1). The anisotropy energy transforms as follows

\[
- K \sum_i (S_i^z)^2 = - K \sum_{i \in A} (S_i^z_{iA})^2 - K \sum_{j \in B} (S_j^z_{jB})^2 = 2KS \sum_k \left( a_k^\dagger a_k + b_k^\dagger b_k \right). \tag{S4}
\]

For the nearest neighboring exchange interaction, \( J_{ij} = J_1 \), we obtain

\[
J_1 \sum_{(ij)} S_i \cdot S_j = J_1 \sum_{i \in A} \sum_\delta S_{iA} \cdot S_{i+\delta B} = J_1 S \sum_{i \in A} \sum_\delta \left( a_i b_{i+\delta} + a_i^\dagger b_{i+\delta}^\dagger + a_{i+\delta} a_i + b_{i+\delta}^\dagger b_i \right) + O(a^4)
\]

\[
= zJ_1 S \sum_k \left( a_k^\dagger a_k + b_k^\dagger b_k + C_k a_k b_{-k} + C_k^* a_k^\dagger b_{-k}^\dagger \right). \tag{S5}
\]

where \( \delta \) connects the \( i \)-th site on \( A \) sublattice with \( z \) nearest neighboring sites on \( B \) sublattice. The lattice form factors in the last line are given by \( C_k = \frac{1}{z} \sum_\delta e^{-i k \delta} \).
Similarly, for the next-nearest-neighboring exchange interaction, \( J_{ij} = J_2 \), we have

\[
J_2 \sum_{\langle \langle ij \rangle \rangle} S_i \cdot S_j = \frac{J_2}{2} \sum_{i \in A} \sum_{\delta'} S_i A S_i + S_i A' + \frac{J_2}{2} \sum_{j \in B} \sum_{\delta'} S_j B S_j + S_j B' = \frac{S J_2}{2} \sum_{i \in A} \sum_{\delta'} \left( a_i a_{i+\delta'}^\dagger + a_i^\dagger a_{i+\delta'} - a_i a_{i+\delta'} - a_i^\dagger a_{i+\delta'} \right)
\]

\[+ \frac{S J_2}{2} \sum_{j \in B} \sum_{\delta'} \left( b_j b_{j+\delta'} + b_j^\dagger b_{j+\delta'} - b_j^\dagger b_j - b_{j+\delta'}^\dagger b_{j+\delta'} \right) = -2 z' J_2 S \sum_{k} G_k \left( a_k^\dagger a_k + b_k^\dagger b_k \right), \quad \text{(S6)}
\]

where \( \delta' \) connect the next-nearest-neighboring sites, \( G_k = \frac{1}{2} \sum_{\delta'} \sin^2 (k \cdot \delta') \), and \( z' \) is the total number of the next-nearest-neighboring sites.

There can be various possible configurations of DMI depending on how the direction on the lattice is set (see e.g. Ref. [49]). Here, we consider two situations relevant for our discussion.

- **Uniform nearest neighboring DMI along the \( d \)-direction.** In this case, the DM vector \( D_{ij} = D_d \hat{z} \) is nonzero only for the neighboring lattice sites connected along \( d \)-direction, which gives

\[
\sum_{\langle \langle ij \rangle \rangle} D_1 (S_i \times S_j)_z = i S D_1 \sum_k \left( D_k a_k b_{-k} - D_k^* a_k^\dagger b_{-k} \right), \quad \text{(S7)}
\]

where \( D_k = \sum_\delta T_\delta e^{-i k \cdot \delta} \), \( \delta \) connects the \( i \)th site on \( A \) sublattice with all the neighboring sites, and the ordering factor \( T_\delta \) defined as follows

\[
T_\delta = \begin{cases} +1, & \text{if } \delta \uparrow \uparrow \text{ with } d, \\ -1, & \text{if } \delta \uparrow \downarrow \text{ with } d, \end{cases} \quad \text{(S8)}
\]

and 0 otherwise. For example, for a square lattice with DMI along the \( x \)-direction, it gives \( D_k = \frac{1}{2} \left( e^{ikx} - e^{-ikx} \right) = \frac{i}{2} \sin k_x \). In the longwavelength limit, this type of DMI is responsible for the appearance of \( k \)-linear terms.

- **Staggered nearest neighboring DMI.** The sign convention for this type of DMI is defined as follows

\[
D_{ij} = \begin{cases} +D, & \text{if } i \in A \text{ and } j \in B, \\ -D, & \text{if } i \in B \text{ and } j \in A, \end{cases} \quad \text{(S9)}
\]

leading to the following expression

\[
\sum_{\langle \langle ij \rangle \rangle} D_{ij} (S_i \times S_j)_z = D \sum_{i \in A} \sum_\delta \left( S_{i A}^x S_{i+\delta B}^y - S_{i A}^y S_{i+\delta B}^x \right)
\]

\[= i S D \sum_{i \in A} \sum_\delta \left( a_i b_{i+\delta} - a_i^\dagger b_{i+\delta}^\dagger \right) = i S D \sum_k \left( C_k a_k b_{-k} - C_k^* a_k^\dagger b_{-k} \right), \quad \text{(S10)}
\]

where the form factors are the same as in Eq. (S5).

Combining together Eqs. (S5), (S6), (S7), and (S10), we obtain the final expression for the magnon Hamiltonian

\[
\mathcal{H} = \sum_k \left[ A_k \left( a_k^\dagger a_k + b_{-k}^\dagger b_{-k} \right) + B_k a_k b_{-k} + B_k^* a_k^\dagger b_{-k} \right], \quad \text{(S11)}
\]

where \( A_k = 2 KS + z J_1 S - 2 z' S J_2 G_k \), and \( B_k = z J_1 S C_k e^{i \varphi_0} + i D_1 SD_k \), where \( \tan \varphi_0 = D/J_1 \), and \( \tilde{J} = \sqrt{J_1^2 + D^2} \).

**Interaction with the electromagnetic field**

We take the interaction term with the electromagnetic field in the form of the Zeeman coupling between spin operators and the magnetic vector of the electromagnetic wave

\[
\mathcal{H}_I = -g_B \sum_i \mathbf{B}(t, \mathbf{r}_i) \cdot \mathbf{S}_i = -g_B \sqrt{\frac{S}{2}} \sum_{i \in A} \left[ \mathbf{B}(-)(t, \mathbf{r}_i) a_i + \mathbf{B}(+)(t, \mathbf{r}_i) a_i^\dagger \right]
\]

\[\quad - g_B \sqrt{\frac{S}{2}} \sum_{j \in B} \left[ \mathbf{B}(-)(t, \mathbf{r}_j) b_j^\dagger + \mathbf{B}(+)(t, \mathbf{r}_j) b_j \right] + g_B S \sum_{i \in A} \mathbf{B}^z(t, \mathbf{r}_i) a_i a_i - g_B S \sum_{j \in B} \mathbf{B}^z(t, \mathbf{r}_j) b_j^\dagger b_j. \quad \text{(S12)}
\]
\[ \mathcal{W}^{(\pm)} = \mathcal{W}^z \pm i \mathcal{W}^y. \] We will ignore the last term in this expression, which describes coupling of \( \mathcal{W}^z \) with local magnon density, since in the geometry of our model \( \mathcal{W}^z \) can be neglected. In the Fourier space, the expression above can be rewritten as

\[ H_I = -\frac{1}{2} \sum_k \left[ h_k^{(-)}(t) \left( a_k + b_{-k}^\dagger \right) + h_k^{(+)}(t) \left( a_{-k}^\dagger + b_{-k} \right) \right], \]  

where we use a shorthand notation \( h_k^{(\pm)}(t) = g \mu_B \sqrt{2s} \mathcal{W}_k^{(\pm)}(t) \), and the Fourier transforms of the field is defined as \( \mathcal{W}_k^{(\pm)}(t, r) = \frac{1}{\sqrt{V}} \sum_k e^{-i k \cdot r} \mathcal{W}_k^{(\pm)}(t) \), which satisfies the identity \( (\mathcal{W}_k^{(-)})^* = \mathcal{W}_{-k}^{(+)} \).

**Semiclassical dynamics in the continuous limit**

Let us consider semiclassical dynamics of a three-dimensional cubic antiferromagnet without DMI in the continuous approximation. In this case, the spin Hamiltonian in Eq. (S1) can be rewritten in the form of classical magnetic energy density

\[ w(r, t) = \frac{\alpha}{2} \left[ (\nabla M_A)^2 + (\nabla M_B)^2 \right] + \alpha' \nabla M_A \cdot \nabla M_B + \delta M_A \cdot M_B - \frac{\beta}{2} \left[ (M_A \cdot n)^2 + (M_B \cdot n)^2 \right], \]  

where \( M_A \) (\( M_B \)) is the magnetization of \( A \) (\( B \)) magnetic sublattice.

Semiclassical precession dynamics of sublattice magnetization can be described by Landau-Lifshitz equations of motion

\[ \frac{\partial M_A}{\partial t} = \gamma (M_A \times H_A^{\text{eff}}), \quad \frac{\partial M_B}{\partial t} = \gamma (M_B \times H_B^{\text{eff}}), \]  

where the effective fields are determined by the magnetic energy functional, \( W = \int d^3r w(r, t) \), as follows \( H_i^{\text{eff}} = -\delta W/\delta M_i \) (\( i = A, B \)).

In the linear approximation \( M_A(t, r) = (M_s - m_A^2/(2M_s)) n + m_A(t, r) \), \( M_B(t, r) = -(M_s - m_B^2/(2M_s)) n + m_B(t, r) \), where \( m_A \cdot n = m_B \cdot n = 0 \) (\( n \) unit vector along the magnetic ordering direction, \( M_s \) denotes the saturation magnetization), the energy density becomes

\[ w = \frac{\alpha}{2} \left[ (\nabla m_A)^2 + (\nabla m_B)^2 \right] + \alpha' \nabla m_A \cdot \nabla m_B + \frac{\delta}{2} (m_A + m_B)^2 + \frac{\beta}{2} (m_A^2 + m_B^2), \]  

We introduce \( m = m_A + m_B \) and \( l = m_A - m_B \). With the help of these notations, the total energy of the spin waves can be written as

\[ W = \frac{1}{4} \sum_k \left[ \varepsilon^{(m)}_k m_k \cdot m_k + \varepsilon^{(l)}_k l_k \cdot l_k \right], \]  

where \( \varepsilon^{(l)}_k = \gamma M_s(\beta + (\alpha - \alpha')k^2) \) and \( \varepsilon^{(m)}_k = \gamma M_s(2\delta + (\alpha + \alpha')k^2) \). The Fourier components of \( m \) and \( l \) are defined as

\[ m(t, r) = \frac{1}{\sqrt{V}} \sum_k e^{i k \cdot r} m_k(t), \quad l(t, r) = \frac{1}{\sqrt{V}} \sum_k e^{i k \cdot r} l_k(t). \]  

For the Fourier components \( m_k(t) \) and \( l_k(t) \), the linearized Landau-Lifshitz equations (S15) have the following form

\[ \partial_t m_k = -\varepsilon^{(l)}_k (n \times l_k), \]
\[ \partial_t l_k = -\varepsilon^{(m)}_k (n \times m_k). \]

**Conservation law for the spin current**

Let us consider the equation of motion for the z-component of the magnetization

\[ M^z(t, r) = \frac{1}{2M_s} \left[ m_B^2(t, r) - m_A^2(t, r) \right] = \frac{1}{2M_s V} \sum_{kk'} e^{i (k+k') \cdot r} \left[ m_{Bk} \cdot m_{Bk'} - m_{Ak} \cdot m_{Bk'} \right]. \]  

\[ \]
The Fourier transform of $M^z(t, r)$, $M^z_q = \int d^3 r e^{-i q \cdot r} M^z(r)$, is given by

$$M^z_q = \frac{1}{2M_s} \sum_k \left[ m^*_{Bk} \cdot m_{Bk} - m^*_{Ak} \cdot m_{Ak} \right] = - \frac{1}{2M_s} \sum_k m^*_{k-q} \cdot l_k. \quad (S22)$$

The equation of motion for $M^z_q$ has the following form

$$\frac{\partial M^z_q}{\partial t} = - \frac{1}{2M_s} \sum_k \left( \frac{\partial m^*_{k-q}}{\partial t} \cdot l_k + m^*_{k-q} \cdot \frac{\partial l_k}{\partial t} \right) = \frac{1}{2M_s} \sum_k \left\{ \varepsilon^{(l)}_{k-q} n \cdot [l^*_{k-q} \times l_k] - \varepsilon^{(m)}_{k} n \cdot [m^*_{k-q} \times m_k] \right\}, \quad (S23)$$

where we used Eqs. (S19) and (S20). With the help of the following identities

$$\sum_k \varepsilon^{(l)}_{k-q} [l^*_{k-q} \times l_k] = - \sum_k \varepsilon^{(l)}_{-k} [l^*_{k-q} \times l_k],$$

$$\sum_k \varepsilon^{(m)}_{k-q} [m^*_{k-q} \times m_k] = - \sum_k \varepsilon^{(m)}_{k-q} [m^*_{k-q} \times m_k],$$

we rewrite Eq. (S23) as follows

$$\frac{\partial M^z_q}{\partial t} = \frac{1}{4M_s} \sum_k \left\{ \left( \varepsilon^{(l)}_{k-q} - \varepsilon^{(l)}_{-k} \right) n \cdot [l^*_{k-q} \times l_k] + \left( \varepsilon^{(m)}_{k-q} - \varepsilon^{(m)}_{k} \right) n \cdot [m^*_{k-q} \times m_k] \right\}. \quad (S24)$$

Taking into account that $\varepsilon^{(m)}_{-k} = \varepsilon^{(m)}_{k}$ and $\varepsilon^{(l)}_{-k} = \varepsilon^{(l)}_{k}$, we can set $q \to 0$ and rewrite the expression above in the form of continuity equation

$$\frac{\partial M^z_q}{\partial t} + \frac{1}{4M_s} q \cdot \sum_k \left\{ \frac{\partial \varepsilon^{(m)}_{k}}{\partial k} (n \cdot [m^*_{k} \times m_k]) + \frac{\partial \varepsilon^{(l)}_{k}}{\partial k} (n \cdot [l^*_{k} \times l_k]) \right\} = 0, \quad (S25)$$

which allows us to identify the spatially uniform spin current associated with the magnetization dynamics

$$J_s = -\frac{i\hbar}{4M_s} \sum_k \left\{ \frac{\partial \varepsilon^{(m)}_{k}}{\partial k} (n \cdot [m^*_{k} \times m_k]) + \frac{\partial \varepsilon^{(l)}_{k}}{\partial k} (n \cdot [l^*_{k} \times l_k]) \right\}. \quad (S26)$$

where the factor $\hbar/M_s$ has been added to convert from the magnetization current to spin current. Note that the identities $m_k = m^*_{-k}$ and $l_k = l^*_{-k}$ warrant that $J_s$ is real.

**Spin current in circularly polarized magnetic field**

In order to study field-induced magnetization dynamics, we add the interaction term $-\mathbf{h}(t) \cdot (M_A + M_B)$ in Eq. (S13), where $\mathbf{h}(t)$ denotes the magnetic vector of the electromagnetic wave. The equations of motion for $m_k(t)$ and $l_k(t)$ in this case become

$$\partial_t m_k = -\varepsilon^{(l)}_{k} (n \times l_k), \quad (S27)$$

$$\partial_t l_k = -\varepsilon^{(m)}_{k} (n \times m_k) + 2\gamma M_s n \times h_k, \quad (S28)$$

where $h_k$ denotes the Fourier component of the magnetic field with the wave vector $k$. For $h_k \sim \exp(i\omega t)$, we immediately obtain the solutions, $m_k(\omega) = 2\gamma M_s \chi_m(k, \omega) h_k(\omega)$ and $l_k(\omega) = 2\gamma M_s \chi_l(k, \omega) [n \times h_k(\omega)]_n$, where the susceptibilities are given by $\chi_m(k, \omega) = \varepsilon^{(l)}_{k} / (\varepsilon^2_{k} - \omega^2)$ and $\chi_l(k, \omega) = \omega / (\varepsilon^2_{k} - \omega^2)$ with $\varepsilon_{k} = \sqrt{\varepsilon^{(m)}_{k} \varepsilon^{(l)}_{k}}$ being the dispersion relation for spin waves. With the help of these expressions, the field-induces spin current takes the following form

$$J_s = -i\hbar\gamma^2 \sum_k \left( \chi^2_{m}(k, \omega) \nabla_k \varepsilon^{(m)}_{k} + \chi^2_{l}(k, \omega) \nabla_k \varepsilon^{(l)}_{k} \right) [h^*_k(\omega) \times h_k(\omega)]_n. \quad (S29)$$

Note that since $\nabla_k \varepsilon^{(m)}_{k}$ and $\nabla_k \varepsilon^{(l)}_{k}$ are proportional to $k$, the asymmetric combination $k_n [h^*_k(\omega) \times h_k(\omega)]_n$ will appear on the right hand side for the wave propagating along $n$. 
**Bogolyubov transformation**

Since, in general, the parameter $B_k$ in Eq. (S11) can be complex, we need to use the Bogolyubov transformation with two parameters in order to diagonalize the Hamiltonian in Eq. (S11) [27, 28]. Below, we outline the essential steps of this transformation for the sake of completeness.

Two-parametric Bogolyubov rotation from $(a_k, b_{-k}^\dagger)^T$ to the new field operators $(\alpha_k, \beta_{-k}^\dagger)^T$, which preserves boson commutation relations $[\alpha_k, \alpha_k^\dagger] = [\beta_k, \beta_k^\dagger] = 1$, has the following form

$$
\begin{pmatrix}
\alpha_k \\
\beta_{-k}^\dagger
\end{pmatrix}
= 
\begin{pmatrix}
\cosh \theta_k e^{i\phi_k} & -\sinh \theta_k \\
-\sinh \theta_k & \cosh \theta_k e^{-i\phi_k}
\end{pmatrix}
\begin{pmatrix}
\alpha_k \\
\beta_{-k}^\dagger
\end{pmatrix},
$$

(S30)

where $\theta_k$ and $\phi_k$ are real parameters. Applying this transformation to the magnon Hamiltonian in Eq. (S11), we obtain

$$
U_k^\dagger \begin{pmatrix} A_k & B_k^* \\ B_k & A_k^* \end{pmatrix} U_k = 
\begin{pmatrix}
A_k \cosh 2\theta_k - \text{Re}[B_k e^{i\phi_k}] \sinh 2\theta_k \\
e^{i\phi_k} \left\{ \text{Re}[B_k e^{i\phi_k}] \cosh 2\theta_k - A_k \sinh 2\theta_k + i \text{Im}[B_k e^{i\phi_k}] \right\}
\end{pmatrix},
$$

(S31)

where $U_k$ denotes the transformation matrix

$$
U_k = 
\begin{pmatrix}
\cosh \theta_k e^{i\phi_k} & -\sinh \theta_k \\
-\sinh \theta_k & \cosh \theta_k e^{-i\phi_k}
\end{pmatrix}.
$$

(S32)

We require that off-diagonal terms should disappear in Eq. (S31), which can be achieved if we choose $\tan \phi_k = -\text{Im} B_k / \text{Re} B_k$ and $\tanh 2\theta_k = |B_k|/A_k$. After that, we obtain diagonal magnon Hamiltonian

$$
\mathcal{H} = \sum_k \left( \varepsilon_k \alpha_k^\dagger \alpha_k + \varepsilon_{-k} \beta_{-k}^\dagger \beta_{-k} \right),
$$

(S33)

where $\varepsilon_k = \sqrt{A_k^2 - |B_k|^2}$ denotes the magnon energy dispersion relation.

The interaction term in Eq. (S13) transforms as follows

$$
\mathcal{H}_i = -\frac{1}{2} \sum_k \left[ h_k^\dagger(t) \left( M_k^\star \alpha_k + M_k \beta_{-k}^\dagger \right) + h_k(t) \left( M_k \alpha_k^\dagger + M_k^\star \beta_{-k} \right) \right],
$$

(S34)

where $M_k = \cosh \theta_k e^{-i\phi_k} - \sinh \theta_k$.

**Magnon spin current**

Our approach for defining the magnon spin current in this section is similar to the derivation of the magnetization current in Sec. . Since $z$-component of the total spin is a conserving quantity, we can find the expression for the spin current from the continuity equation. With the help of the expressions $a_i^\dagger a_i = N^{-1/2} \sum_{kq} e^{-i\mathbf{q} \cdot \mathbf{r}_i} a_{k+q}^\dagger a_{k}$, and $b_j^\dagger b_j = N^{-1/2} \sum_{kq} e^{-i\mathbf{q} \cdot \mathbf{r}_j} b_{k+q}^\dagger b_k$, the local magnon density is defined as follows

$$
n(\mathbf{r}_i) = \sum_\delta b_{i+\delta}^\dagger b_{i+\delta} - a_i^\dagger a_i = \frac{1}{N} \sum_{kq} e^{-i\mathbf{q} \cdot \mathbf{r}_i} \left[ e^{-i\mathbf{q} \cdot \mathbf{r}_i} \delta_{k+q}^\dagger b_k - a_{k+q}^\dagger a_k \right],
$$

(S35)

where $\delta$ connects the $i$th site on $A$ sublattice neighboring sites on $B$ sublattice.

Let us obtain the equation of motion for $n(\mathbf{r}_i)$. For this purpose, we use Eq. (S11) to obtain Heisenberg equations of motions for $a_k$ and $b_k$

$$
\dot{a}_k = \frac{i}{\hbar} [\mathcal{H}, a_k] = -\frac{i}{\hbar} \left( A_k a_k + B_k^\star b_{-k}^\dagger \right),
$$

(S36)

$$
\dot{b}_{-k} = \frac{i}{\hbar} [\mathcal{H}, b_{-k}] = -\frac{i}{\hbar} \left( A_k b_{-k} + B_k a_k^\dagger \right),
$$

(S37)
which gives us (after we change $k \rightarrow -k$ and $-k + q \rightarrow -k$ in the first term of Eq. (S35))
\[
\frac{\partial n(r_i)}{\partial t} = -\frac{i}{\hbar N} \sum_{kq\delta} e^{-iqr_i} \left\{ (A_{k+q} - A_k) \left[ e^{-i\delta} b_k^+ b_{-k-q} + a_{k+q}^+ a_k \right] + (B_{k+q} - B_k e^{-i\delta}) a_k b_{-k-q} + (e^{-i\delta} B_{k+q}^* - B_k^*) a_{k+q}^+ b_{-k}^\dagger \right\}. \tag{S38}
\]

Now we consider the long-wave-length limit, $q \rightarrow 0$. In the case when $\sum \delta = 0$, the lattice form factors do not contribute at $q \rightarrow 0$, and we obtain $\partial n_q/\partial t + i q \cdot J_s = 0$, where the magnon spin current is defined as
\[
J_s = \sum_k \left[ \frac{\partial A_k}{\partial k} (a_k^+ a_k + b_k^+ b_{-k}) + \frac{\partial B_k}{\partial k} a_k b_{-k} + \frac{\partial B_k^*}{\partial k} a_{k+q}^+ b_{-k}^\dagger \right]. \tag{S39}
\]

By applying the Bogolyubov transformation in Eq. (S30) to this expression, we obtain
\[
J_s = \sum_k \left( \alpha_k \right) \left( \nabla_k \varepsilon_k \right) \left( K_k \varepsilon_k \right) \left( \beta_{-k}^\dagger \right)
\]
where the off-diagonal elements are determined by
\[
K_k = e^{i\phi_k} \left( \frac{A_k \nabla_k |B_k| - |B_k| \nabla_k A_k}{\sqrt{A_k^2 - |B_k|^2}} - i |B_k| \nabla_k \phi_k \right). \tag{S41}
\]

**Second-order response**

In order to calculate the spin current generated by the electromagnetic wave, we use the response theory (see e.g. [43]). It is easy to show that both the equilibrium magnon spin current and the first order response vanish. The second-order response term to $H_f$ is expressed as
\[
\langle J_s(t) \rangle = -\sum_{\omega_1 \omega_2} \int_{-\infty}^{t_1} dt_1 \int_{-\infty}^{t_2} dt_2 e^{i(t_1 + t_2 - t) \omega} \langle \left[ [\hat{J}_s(t), \hat{H}_f^{(\omega_1)}(t_1)], \hat{H}_f^{(\omega_2)}(t_2) \right] \rangle, \tag{S42}
\]
where the operators are in the Heisenberg picture
\[
\hat{J}_s(t) = e^{i\hat{H}_f} J_s e^{-i\hat{H}_f}, \quad \hat{H}_f(t) = e^{i\hat{H}_f(t)} \hat{H}_f e^{-i\hat{H}_f(t)}, \tag{S43}
\]
and $\hat{H}_f$ is defined by
\[
\hat{H}_f = -\frac{1}{2} \sum_k \left[ h_k^{(-)}(\omega) \left( M_k \alpha_k + M_k^* \beta_{-k}^\dagger \right) + h_k^{(+)}(\omega) \left( M_k \alpha_k^+ + M_k^* \beta_{-k} \right) \right], \tag{S44}
\]
where $h_k^{(\pm)}(\omega) = \sum_\omega e^{i\omega t} h_k^{(\pm)}(\omega)$ are the Fourier components of the magnetic field, which satisfy the identity $h_k^{(-)}(-\omega)^\dagger = h_k^{(+)}(\omega)$. Using the Heisenberg equations of motion for $\alpha_k$ and $\beta_{-k}$, we find
\[
\hat{H}_f = -\frac{1}{2} \sum_k \left[ h_k^{(-)}(\omega) \left( M_k e^{-i\epsilon_{k t} \alpha_k} + M_k e^{i\epsilon_{k t} \beta_{-k}^\dagger} \right) + h_k^{(+)}(\omega) \left( M_k e^{i\epsilon_{k t} \alpha_k^+} + M_k^* e^{-i\epsilon_{k t} \beta_{-k}} \right) \right]. \tag{S45}
\]
and
\[
J_s = \sum_k \left[ \nabla_k \varepsilon_k \left( \alpha_k^+ \alpha_k + \beta_{-k}^\dagger \beta_{-k} \right) + K_k e^{-2i\epsilon_{k t}} \alpha_k \beta_{-k} + K_k^* e^{2i\epsilon_{k t}} \alpha_k^+ \beta_{-k}^\dagger \right]. \tag{S46}
\]
The commutators in Eq. (S42) can be calculated straightforwardly
\[
\left[ \hat{J}(t), \hat{H}_f^{(\omega_1)}(t_1) \right] = -\frac{1}{2} \sum_k \left\{ -\nabla_k \varepsilon_k h_k^{(-)}(\omega_1) \left[ M_k e^{-i\epsilon_{k t} t_1} \alpha_k - M_k e^{i\epsilon_{k t} t_1} \beta_{-k}^\dagger \right] + \nabla_k \varepsilon_k h_k^{(+)}(\omega_1) \left[ M_k e^{i\epsilon_{k t} t_1} \alpha_k - M_k e^{-i\epsilon_{k t} t_1} \beta_{-k} \right] \right\}
\]
and
\[
\left[ \hat{J}(t), \hat{H}_{\omega_1}(t_1), \hat{H}_{\omega_2}(t_2) \right] = \frac{1}{4} \sum_k \left\{ -\nabla_k \varepsilon k |M_k|^2 \left( e^{-i\varepsilon k(t_1-t_2)} + e^{i\varepsilon k(t_1-t_2)} \right) + K_k M_k^2 e^{i\varepsilon k(t_1+t_2-2t)} + K^*_k M_k^* e^{-i\varepsilon k(t_1+t_2-2t)} \right\} \times \left( h_k^{(-)}(\omega_1) h_k^{(+)}(\omega_2) + h_k^{(+)}(\omega_1) h_k^{(-)}(\omega_2) \right). \tag{S48}
\]

The integration over \( t_1 \) and \( t_2 \) in Eq. \( \text{(S42)} \) is performed as follows
\[
\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 e^{i(\omega_1 t_1 + \omega_2 t_2)} e^{-i\varepsilon k(t_1-t_2)} = \frac{e^{i(\omega_1+\omega_2)t+et}}{(\varepsilon k + \omega_2 - i\epsilon)(\omega_1 + \omega_2 - 2i\epsilon)}, \tag{S49}
\]
\[
\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 e^{i(\omega_1 t_1 + \omega_2 t_2)} e^{i\varepsilon k(t_1-t_2)} = \frac{e^{i(\omega_1+\omega_2)t+et}}{(\varepsilon k - \omega_2 + i\epsilon)(\omega_1 + \omega_2 - 2i\epsilon)}, \tag{S50}
\]
\[
\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 e^{i(\omega_1 t_1 + \omega_2 t_2)} e^{i\varepsilon k(t_1+t_2-2t)} = \frac{e^{i(\omega_1+\omega_2)t+et}}{(\varepsilon k + \omega_2 - i\epsilon)(\omega_1 + \omega_2 + 2i\epsilon)}, \tag{S51}
\]
\[
\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 e^{i(\omega_1 t_1 + \omega_2 t_2)} e^{-i\varepsilon k(t_1+t_2-2t)} = \frac{e^{i(\omega_1+\omega_2)t+et}}{(\varepsilon k - \omega_2 + i\epsilon)(\omega_1 - \omega_2 + 2i\epsilon)}. \tag{S52}
\]

Combining together Eqs \( \text{(S47)} \) - \( \text{(S52)} \), we obtain the following expression for the magnon spin current
\[
\langle J_s(t) \rangle = \frac{1}{4} \sum_{\omega_1, \omega_2, k} e^{i(\omega_1+\omega_2)t+et} \left\{ |M_k|^2 \nabla_k \varepsilon k \left( \frac{1}{\omega_1 - \omega_2 - 2i\epsilon} - \frac{1}{\varepsilon k - \omega_2 - i\epsilon} \right) + K_k M_k^2 \left( \frac{1}{\varepsilon k + \omega_2 + i\epsilon} - \frac{1}{\varepsilon k - \omega_2 - i\epsilon} \right) \right\} \times \left( h_k^{(-)}(\omega_1) h_k^{(+)}(\omega_2) + h_k^{(+)}(\omega_1) h_k^{(-)}(\omega_2) \right). \tag{S53}
\]

By changing, \( \omega_2 \rightarrow -\omega_2 \) \( (\omega_1 \rightarrow -\omega_1) \) in the term proportional to \( h_k^{(-)}(\omega_1) h_k^{(+)}(\omega_2) \) \( (h_k^{(+)}(\omega_1) h_k^{(-)}(\omega_2)) \), we can rewrite the expression above in the manifestly real form
\[
\langle J_s(t) \rangle = \frac{1}{4} \sum_{\omega_1, \omega_2, k} e^{i(\omega_1-\omega_2)t+et} \left\{ |M_k|^2 \nabla_k \varepsilon k \left( \frac{1}{\omega_1 - \omega_2 - 2i\epsilon} - \frac{1}{\varepsilon k + \omega_2 + i\epsilon} \right) + K_k M_k^2 \left( \frac{1}{\varepsilon k - \omega_2 - i\epsilon} - \frac{1}{\varepsilon k - \omega_2 + i\epsilon} \right) \right\} \times h_k^{(-)}(\omega_1) h_k^{(-)}(\omega_2) \tag{S54}
\]
\[
+ \frac{1}{4} \sum_{\omega_1, \omega_2, k} e^{i(\omega_1-\omega_2)t+et} \left\{ |M_k|^2 \nabla_k \varepsilon k \left( \frac{1}{\omega_1 - \omega_2 + 2i\epsilon} - \frac{1}{\varepsilon k + \omega_2 - i\epsilon} \right) + K_k M_k^2 \left( \frac{1}{\varepsilon k - \omega_2 + i\epsilon} - \frac{1}{\varepsilon k - \omega_2 - i\epsilon} \right) \right\} \times h_k^{(+)}(\omega_1) h_k^{(-)}(\omega_2). \tag{S55}
\]

If we take the diagonal part of this expression at \( \omega_1 = \omega_2 = \omega_k \), we obtain time-independent component of the spin current
\[
\langle J_s \rangle = \frac{1}{4} \sum_k \left\{ |M_k|^2 \nabla_k \varepsilon k \left[ \frac{1}{(\varepsilon k + \omega_k)^2 + 4\epsilon^2} + \frac{1}{(\varepsilon k - \omega_k)^2 + 4\epsilon^2} \right] \right\} \times h_k^{(-)}(\omega_k) h_k^{(-)}(\omega_k). \tag{S56}
\]

Small damping case can be included phenomenologically by treating \( \epsilon \) as the inverse magnon lifetime \( \Gamma \).
Clean limit expression

In the limit $\epsilon \to 0$, we can rewrite Eq. (S55) in the following form

$$\langle J \rangle = \frac{1}{2} \sum_k \left[ \frac{|M_k|^2 (\epsilon_k^2 + \omega_k^2) \nabla_k \epsilon_k}{\epsilon_k^2 - \omega_k^2} + \frac{\text{Re} \left[ K_k M_k^2 \right]}{\epsilon_k - \omega_k^2} \right] h_{k}^{(-)*}(\omega_k) h_{k}^{(-)}(\omega_k),$$  \hspace{1cm} (S56)

where the coefficients can be calculated using the identities

$$|M_k|^2 = \frac{A_k - |B_k| \cos \phi_k}{\sqrt{A_k^2 - |B_k|^2}}, \quad M_k^2 = e^{-i\phi_k} \left( \frac{A_k \cos \phi_k - |B_k|}{\sqrt{A_k^2 - |B_k|^2}} - i \sin \phi_k \right),$$  \hspace{1cm} (S57)

In the special case, when $\phi_k = 0$, we can obtain a compact expression

$$\langle J \rangle = \frac{1}{2} \sum_k \left[ \frac{(A_k - |B_k|)^2 \nabla_k (A_k + B_k)}{(\epsilon_k^2 - \omega_k^2)^2} + \frac{\omega_k^2 \nabla_k (A_k - B_k)}{(\epsilon_k^2 - \omega_k^2)^2} \right] h_{k}^{(-)*}(\omega_k) h_{k}^{(-)}(\omega_k),$$  \hspace{1cm} (S58)

which coincides with the expression in Eq. (S29) obtained in the semiclassical Landau-Lifshitz analysis if we identify $\epsilon_k^{(l)} = A_k - |B_k|$ and $\epsilon_k^{(m)} = A_k + |B_k|$. Without DMI, $\epsilon_k = \epsilon_{-k}$, and the coefficient in brackets is an odd function of $k$, which selects the antisymmetric part of the total intensity of the magnetic field

$$h_{k}^{(-)*}(\omega) h_{k}^{(-)}(\omega) \to -i[h_k^{(\omega) \times k_k^{(\omega)}}].$$

In addition, for lattices with $\nabla_k \phi_k \neq 0$, we find a geometrical contribution to the spin current in Eq. (S56)

$$\langle J_s \rangle = -\frac{1}{2} \sum_k |M_k|^2 \left[ \text{Im} \phi_k \nabla_k \phi_k \right] h_{k}^{(-)*}(\omega_k) h_{k}^{(-)}(\omega_k).$$  \hspace{1cm} (S59)

Phase factor for a honeycomb lattice

As an example of a lattice with nonzero phase gradient, let us consider a honeycomb lattice. The structure factor for the honeycomb lattice in Eq. (S5) is given by

$$C_k = \frac{1}{3} \left( e^{-i\delta_1 k} + e^{-i\delta_2 k} + e^{-i\delta_3 k} \right) = \frac{1}{3} \left[ 2 \cos \frac{k_x}{2} \left( \cos \frac{k_x}{2} + \cos \frac{3k_y}{2} \right) - 1 + 2i \sin \frac{k_x}{2} \left( \cos \frac{k_x}{2} - \cos \frac{3k_y}{2} \right) \right],$$  \hspace{1cm} (S60)

where $\delta_1 = (-1,0,0)$, $\delta_2 = (1/2, \sqrt{3}/2, 0)$, and $\delta_3 = (1/2, -\sqrt{3}/2, 0)$ (see Fig. S1). In the long wavelength limit, this expression reduces to

$$C_k = 1 - \frac{k_x^2 + k_y^2}{4} - \frac{ik_x}{8} (3k_y^2 - k_x^2),$$  \hspace{1cm} (S61)

which gives $\phi_k \approx ik_x(3k_y^2 - k_x^2)/8$. Note that $\partial \phi_k/\partial k_x = 3k_y^2/8$ at $k_x = 0$. 

FIG. S1. Schematic picture of a honeycomb lattice with $A$ and $B$ magnetic sublattices.