A cooperative covering problem under disruption considering backup coverage

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Abstract: In this paper, we study the location of emergency centres considering cooperative and backup coverage while natural disasters occur which can result in facility disruption. In this regard, a reliable version of cooperative covering problem is presented considering two types of candidate sites, i.e., reliable and unreliable. To achieve a fortified system against disaster, reliable candidate sites are selected from areas which are far away from the disaster harms. Furthermore, backup coverage is considered to compensate unsatisfied coverage of the demand zones due to facility disruption. The performance of the model is investigated solving numerical examples with different approaches utilising commercial software. The results confirm accurate performance of the model. They also show that both facility failure and backup coverage considerations lead to a more efficient network by incurring some additional cost.

Keywords: backup coverage; cooperative covering; disruption; natural disaster; reliability.

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1 Introduction

One of the basic discrete facility location problems is the un-capacitated fixed-charge location problem (UFLP). This problem aims to locate a number of new facilities whose number is not predefined. The classic objective is to minimise the total fixed costs of establishing new facilities as well as transportation costs between demand nodes and new facilities. One of the major assumptions of this classical problem is that no disruption occurs in facilities. Conversely, in real world situations, there is a probability for disruption in service facilities which can be occurred intentionally or unintentionally. Natural disasters such as earthquake, hurricane, and flood are some instances of unintentional occurrences. In such situations, emergency servicing to victims is of great importance, especially when the facility becomes unavailable due to the disruption.

One of the most important activities during natural disasters is the emergency evacuation of injured people. This stage plays a substantial role in the emergency response phase and can considerably decrease the loss of human lives. In case of a severe earthquake, there is a possibility that a subset or most of the servicing facilities, such as hospitals and/or medical centres in stricken areas, are destroyed, and therefore some demands cannot be met; this can clearly lead to critical circumstances for rescue missions during natural disasters.

Most researches in establishing emergency centres are based on location covering models. In one of the first researches in this field, Toregas et al. (1971) have proposed a covering model to locate emergency service facilities. Some applications of covering problems have been presented in locating health care centres, ambulances, emergency medical services (EMS), and medical facilities (Boffey and Narula, 1998; Drezner et al., 2004). Also, developed version of classical covering models are utilised in emergency facility location problems (Sorensen and Church, 2010). Therefore, some characteristics of covering models provide them as an appropriate candidate to apply in emergency facility location problems. However, gradual and cooperative covering models may be more effective in establishing emergency centres, as they are more flexible than the
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classical covering models (Li et al., 2011). In the classical covering models, a demand zone is considered to be covered, if it is exactly placed within the coverage radius of a facility, otherwise it is uncovered. This rigid assumption is eliminated in gradual covering in which coverage is expressed by a non-increasing function of distance between the facility and the demand zone. Another assumption of classical covering models is that, if at least one facility covers a demand zone, then the coverage is fully done while in cooperative covering models the full coverage of a demand zone is done if all the cooperated facilities provide a predefined coverage threshold (Berman et al., 2010).

In this paper, considering the importance of servicing time to injured people in real world conditions, a cooperative version of covering problems is investigated. Furthermore, due to the existence of natural disasters, the established emergency centres may also be disrupted, which could lead to disorder in a number of emergency centres and facing system-side risk. Consequently, lack of coverage of the demand areas that were formerly covered can be resulted another type of risk which is called demand-side risk. The resulted unmet demands can provide a sever situation for the injured during natural disasters. In this regard, backup coverage concept is used to compensate the unmet demands. Therefore, the purpose of this paper is to decrease both system and demand side risks simultaneously in locating emergency centres.

The rest of the paper is organised as follows: related literature review is presented in Section 2. Problem formulation is given in Section 3. Numerical analysis of the given model is argued in Section 4, and finally Section 5 concludes the paper and presents the future research directions.

2 Literature review

Based on characteristics of the proposed model, concepts of failure occurrence in facilities, cooperative covering problem, and backup coverage are briefly reviewed in this section.

In one of the initial researches based on failure occurrence in facilities, Drezner (1987) has discussed facility failures in classical location models of p-median problem (PMP) and p-centre problem. Snyder and Daskin (2005) have considered this concept for both PMP and fixed-charge location problems (FCLP) and have proposed multi-layer backup facilities. In another research by Snyder et al. (2006), this issue has been studied on FCLP using a different approach in which facilities are divided into two categories of reliable and unreliable, and failure probabilities are considered only for unreliable facilities. One of the drawbacks of the given models is consideration of identical failure probabilities for all the facilities; this makes the model far from the real world conditions. As an example, in case of natural disasters such as earthquake, facilities closer to the earthquake occurrence point are more likely to be disrupted compared with the further facilities. Berman et al. (2007) and Shen et al. (2011) have relaxed this assumption for PMP and FCLP, and have proposed a scenario based approach to formulate their problems. Due to natural disaster situation which results in location-based failure in facilities, proposed model in this paper also considers diverse and location-based failure probabilities for candidate sites. When failure probabilities are assumed to be different, dependency or independency of failures should be defined. Dependency means that, neighbouring facilities of the disrupted facility tend to simultaneously fail by the disaster.
Li and Ouyang (2010) have presented a model based on UFLP considering dependent failure probabilities. Peng et al. (2011) have studied this issue for capacitated facility location problem (CFLP) in which failures are defined in the form of scenario based structure. In that sense, each scenario encompasses all facilities that fail simultaneously. Shishebori and Babadi (2015) have addressed a scenario-based facility location network design problem (NDP) for locating medical service centres considering both facility and transfer links disruptions in defined scenarios. On the other side, some researches have considered independent failure probabilities. Shishebori et al. (2014) have proposed a model involving aspects of both UFLP and PMP in which failure cost that occurs for any disruption is calculated regardless of disruption probability. They have also applied budget constraints for maximum allowable disruption cost as well as investment cost. In another study on UFLP by Lim et al. (2010), two types of reliable and unreliable facilities are considered in which reliable ones are those which have been fortified using more investments. Li et al. (2012) have applied fortification concept for both PMP and UFLP. In their models, a limited budget is considered for fortification and it is assumed that if a facility is selected to be fortified, it will be fortified completely and marked as a reliable facility which never fails. Li and Savachkin (2013) have proposed a model in which a facility can reach various levels of reliability through fortification. In some cases as disaster situations, a drawback arises that even a facility at a high level of fortification may be disrupted due to the fact that its location is close to the disaster occurrence points. In the proposed model of this study, it is attempted to apply fortification concept as considering candidate sites for locating reliable centres to be away from the disaster harms. Using this strategy, it is acceptable to overlook failure possibilities in centres which have been located in reliable candidate sites.

Berman et al. (2009) have introduced a new concept of coverage considering cooperation of facilities in coverage of demand nodes and named it as a cooperative covering problem. They have considered the planar version of this model based on two types of discrete and continuously distributed demands. Berman et al. (2011) have proposed three discrete cooperative covering models based on set covering, maximal covering, and p-centre models. In its set covering version, the minimum number of required source points is defined, while no information is provided on the cooperated source points in coverage of each demand point due to their modelling approach. Averbakh et al. (2014) have considered maximal covering version of cooperative covering problem on network in which facilities are allowed to be located at both nodes and along the edges. Regarding other studies on the backup coverage, we can address the research by Hogan and Revelle (1986) in which this concept is considered as a decision criterion in locating emergency service facilities in order to achieve a consistent service level in the zones with high density of population. Pirkul and Schilling (1989) have considered the location of emergency facilities in which both primal and backup coverage are possible for each customer. In another research, Araz et al. (2007) have assumed that a part of population can receive backup coverage, while the population who have received this type of coverage is maximised. Erdemir et al. (2010) have applied the concept of backup coverage in maximal covering problem in order to locate emergency medical centres for road accidents. Table 1 summarises the features of previous works comparing to the characteristics of the presented model in this paper.

To the best of authors’ knowledge there is no research in the literature considering facility failure in the cooperative covering problem with backup coverage to risky demand points when natural disasters occur. In this paper, failure probability depends on
the geographical aspect of the corresponding candidate site. As a result, two types of reliable and unreliable candidate sites are considered. By selecting reliable candidate sites under some constraints, system fortification is applied to minimise the system side risk occurred as a result of disruption in unreliable centres. On the other hand, with regard to backup coverage, we try to minimise demand side risk in which uncovered demand zones are covered by the second type coverage.

Table 1  Summary of previous works with focus on characteristics of the presented model

| Paper | Modelling approach | Failure event | Failure probability | Layers of backup | System risk approach |
|-------|--------------------|---------------|---------------------|------------------|----------------------|
| Snyder and Daskin (2005) | PMP, UFLP | Independent | Equal | Multiple | – |
| Snyder et al (2006) | UFLP | Independent | Equal | Single | – |
| Shen et al. (2011) | UFLP | Independent | Location specific | Multiple | – |
| Li and Ouyang (2010) | UFLP | Dependent | Location specific | Multiple | – |
| Peng et al. (2011) | CFLP | Dependent | – | – | Defining desired level of robustness |
| Shishebori and Babadi (2015) | NDP | Dependent | – | – | Defining desired level of robustness |
| Shishebori et al. (2014) | Combination of UFLP and PMP | Independent | – | Single | Defining maximum allowable disruption cost |
| Lim et al (2010) | UFLP | Independent | Location specific | Single | Facility fortification |
| Li et al. (2012); Li and Savachkin (2013) | PMP, UFLP | Independent | Location specific | Single | Facility fortification |
| Berman et al. (2011) | Cooperative covering problem | – | – | – | – |
| This paper | Cooperative covering problem | Independent | Location specific | Single | Selection of reliable candidate sites |

3  Problem formulation

3.1  Problem description and notation

We consider a number of demand zones which may be affected by natural disasters such as earthquake. A number of candidate sites are also considered that divided into two sets of reliable and unreliable sites. Other assumptions of the model are as follows:
1 reliable candidate sites are located away from the disaster harms; therefore, the chance to become out of reach for mentioned sites is negligible
2 unreliable candidate sites are located nearly close to the disaster harms; therefore, they can be affected by the disaster and may become out of reach
3 a reliable (unreliable) emergency centre is called a centre established in reliable (unreliable) candidate site
4 failure in an unreliable emergency centre depends on the corresponding candidate site’s characteristics and is independent from other centres
5 emergency centres are assumed to be uncapacitated.

The proposed model in this paper aims to minimise the total costs of establishing reliable and unreliable emergency centres as well as transportation costs. Coverage approach is considered for locating emergency centres where a threshold is considered for covering each demand zone; i.e., if the demand zone is located somewhere whose overall provided coverage by the centres is equal or less than the given threshold, then it can be covered. This assumption is valid for both primary and backup coverages. Primary coverage can be provided by both the reliable and unreliable centres, while backup coverage is only provided by the reliable centres. The role of backup coverage is that if an unreliable centre is disrupted, then the backup is activated and a set of backup emergency centres established in reliable sites are responsible for supporting an unreliable emergency centre.

The notations are given as sets, parameters and decision variables.

| Sets          | Description                                                                 |
|---------------|-----------------------------------------------------------------------------|
| \( I \)       | Number of reliable candidate sites, \((i = 1, 2, \ldots, I)\)               |
| \( K \)       | Number of unreliable candidate sites, \((k = 1, 2, \ldots, K)\)             |
| \( D \)       | Number of demand zones, \((d = 1, 2, \ldots, D)\)                          |

| Parameters    | Description                                                                 |
|---------------|-----------------------------------------------------------------------------|
| \( FR_i \)   | Fixed cost of establishing an emergency centre in reliable candidate site \(i\) |
| \( FU_k \)   | Fixed cost of establishing an emergency centre in unreliable candidate site \(k\) |
| \( dem_d \)  | Demand of customers at zone \(d\)                                           |
| \( \beta_d \) | Minimal coverage level by which demand zone \(d\) can be covered             |
| \( q_k \)    | Failure probability associated with unreliable candidate site \(k\)         |
| \( d_{i,k} (d'_{i,k}) \) | Distance between reliable (unreliable) emergency centre \(i(k)\) and demand zone \(d\) |
| \( a_{i,d} (a'_i,d) \) | Unit transportation cost per distance unit between reliable (unreliable) emergency centre \(i(k)\) and demand zone \(d\) |
| \( l_i (l'_i) \) | Minimal coverage radius of reliable (unreliable) emergency centre \(i(k)\) |
| \( u_i (u'_i) \) | Maximal coverage radius of reliable (unreliable) emergency centre \(i(k)\) |
| \( c_{d,i} (c'_{d,i}) \) | Percentage of coverage provided for demand zone \(d\) by reliable (unreliable) emergency centre \(i(k)\) |
| \( M \)      | A large positive number                                                     |
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### Decision variables

- **XR<sub>i</sub>** Binary variable which equals 1 if an emergency centre is established at reliable candidate site <i>i</i>; otherwise, 0.

- **XU<sub>k</sub>** Binary variable which equals 1 if an emergency centre is established at unreliable candidate site <i>k</i>; otherwise, 0.

- **YP<sub>d,i</sub>** Binary variable which equals 1 if demand zone <i>d</i> is covered by reliable emergency centre <i>i</i> in primal coverage condition; otherwise, 0.

- **YYP<sub>d,k</sub>** Binary variable which equals 1 if demand zone <i>d</i> is covered by unreliable emergency centre <i>k</i> in primal coverage condition; otherwise, 0.

- **YB<sub>d,i,k</sub>** Binary variable which equals 1 if reliable emergency centre <i>i</i> cooperates to cover demand zone <i>d</i> in backup coverage condition, in order to compensate the provided primal coverage by unreliable emergency centre <i>k</i> which was disrupted; otherwise, 0.

### 3.2 Mathematical model

With respect to the given notations, the mathematical model is proposed. In order to determine the coverage level for a demand zone, a non-increasing linear function as given in Berman et al. (2010) is utilised.

\[
c_{d,i,j} = \begin{cases} 
1 & d_{d,i,j} < l_i \\
\frac{u_i - d_{d,i,j}}{u_i - l_i} & l_i < d_{d,i,j} < u_i \\
0 & d_{d,i,j} > u_i
\end{cases}
\]  

The model can be stated as (2) – (11).

\[
\text{Min } f = \sum_{i=1}^{I} FR_i XR_i + \sum_{k=1}^{K} FU_k XU_k + \sum_{d=1}^{D} \sum_{i=1}^{I} YP_{d,i,d} \text{dem}_d \text{a}_{d,i} \\
+ \sum_{d=1}^{D} \sum_{k=1}^{K} (1 - q_k) YYP_{d,k,d} \text{dem}_d \text{a}'_{d,k} + \sum_{d=1}^{D} \sum_{k=1}^{K} \sum_{i=1}^{I} q_k YB_{d,i,k,d} \text{dem}_d \text{a}_{d,i}
\]

Subject to:

\[
\sum_{j=1}^{J} c_{d,i,j} YP_{d,i,j} + \sum_{k=1}^{K} c_{d,k,i} YYP_{d,k} \geq \beta_d \quad \forall d
\]

\[
c'd,k (1 - q_k) YYP_{d,k} + \sum_{i=1}^{I} c_{d,i} q_k YB_{d,i,k} \geq c'_{d,k} YYP_{d,k} \quad \forall d, k
\]

\[
\sum_{j=1}^{J} YB_{d,i} \geq q_k YYP_{d,k} \quad \forall d, k
\]

\[
\sum_{j=1}^{J} YB_{d,i,k} \leq M YYP_{d,k} \quad \forall d, k
\]
The objective function (2) minimises total costs including fixed costs of establishing reliable and unreliable emergency centres, and transportation costs. Transportation costs are composed of three parts: the first part is the transportation costs from reliable emergency centres in primal coverage condition to the covered demand zones; the second part is the transportation costs from unreliable emergency centres in primal coverage condition to the covered demand zones; the third part is the transportation costs from reliable emergency centres in backup coverage condition to the covered demand zones.

Constraint (3) guarantees that the sum of primal coverage provided by the reliable and unreliable emergency centres for a given demand zone to be equal or greater than the corresponding coverage level. Constraint (4) ensures that the provided coverage level by each unreliable centre for a demand zone in primal coverage is supported by the expected cooperative coverage resulted from that centre (if not failed) and a set of reliable centres (if failed). Constraint (5) represents that if an unreliable centre contributes in covering a demand zone, then at least a reliable centre should be considered as backup facility. Constraint (6) states that backup coverage is not considered for a demand zone, if no unreliable centre takes part in primal coverage of that zone. Constraint (7) prevents a demand zone to be covered by an unreliable centre which failure probability is equal to 1. Constraint (8) addresses the fact that contribution of an unreliable centre for coverage is not possible, unless that centre is established in one of the unreliable candidate sites. Constraint (9) states the same fact as constraint (8) but for reliable centres. Furthermore, it enforces that each reliable centre can contribute either in primal or backup coverages. Constraints (10) and (11) are standard integrality constraints for the binary variables.

In order to show the performance of the proposed model, it should be compared with a similar basic model. As mentioned before, based on existing literature, the presented cooperative location set covering problem by Berman et al. (2011) has some similarities to our model if failure possibility of source points is not considered. In their model, the minimum number of required source points is predefined, while no information is provided on the cooperated source points in coverage of each demand point due to their modelling approach. It is also important to mention that their objective function is number-based, while our model is cost-based. To compare their model with our model some changes should be done. Two new models called basic models 1 and 2 are presented regardless of failure possibility in service facilities. Consideration of these two basic models is due to no existence of assignment variables in the model proposed by Berman et al. (2011). To make it more clear, the features of the proposed model in comparison with the basic models 1 and 2 are presented in Table 2.
According to Table 2, the difference between two basic models is based on this fact that the establishment and coverage decisions are taken hierarchically in basic model 1 and simultaneously in basic model 2. So it is expected that the basic model 2 will present a better solution due to separation of the feasible region in basic model 1. Furthermore, the purpose for presenting basic model 2 is to investigate the performance of the proposed model under zero failure probabilities, because it can make location and allocation decisions simultaneously same as the presented model and also it does not consider reliability issues as backup coverage in the model which is relaxed in the proposed model for this comparison.

3.2.1 Basic model 1

This model is divided into two parts. In the first part, the selected candidate sites for establishment of centres are determined by the classical cooperative covering model which is based on set covering problem and has been presented in Berman et al. (2011). It should be noted that in this comparison due to no consideration of failure probabilities for facilities, all of facilities are regarded as reliable ones and have the same fixed costs. In the second part, using the presented assignment model and the determined centres in part one, the allocation of the demand zones to the established centres are determined. First part of model is presented as follows.

\[ \begin{align*}
\text{Min } f_1 &= \sum_{i=1}^{I} FR_i X_{R_i} \\
\sum_{i=1}^{I} c_{d,i} X_{R_i} &\geq \beta_d Y_d \quad \forall d \\
\sum_{d=1}^{D} Dem_d Y_d &\geq \sum_{d=1}^{D} \beta_d Dem_d \\
X_{R_i}, Y_d &\in \{0,1\} \quad \forall d, i
\end{align*} \]  

(12)  

(13)  

(14)  

(15)

\(Y_d\) is a binary variable and has a value of 1 if the demand zone \(d\) is covered. Other variables were presented previously. The result of the model contains selected candidate sites for locating emergency centres. The second part of the model which is used for determination of assignments is presented below.
\[
\text{Min } f_2 = \sum_{d=1}^{D} \sum_{i=1}^{I} Z_{d,i} \text{dem}_d d_{d,i} a_{d,i} 
\]

\[
\sum_{i=1}^{I} c_{d,i} Z_{d,i} \geq \beta_d \quad \forall d 
\]

\[
Z_{d,i} \in \{0, 1\} \quad \forall d, i 
\]

\(Z_{d,i}\) is also a binary variable and will be 1 if the demand zone \(d\) is covered by emergency centre \(i\).

The overall objective function of the model is as follows.

\[
f = \text{Min } f_1 + \text{Min } f_2 
\]

### 3.2.2 Basic model 2

This model is also based on set covering version of classical cooperative covering model which has been proposed in Berman et al. (2011). Unlike basic model 1, this model takes location and allocation decisions simultaneously. In this regard, \(Z_{d,i}\) which has been used in the second part of basic model 1 is applied. This variable replaces \(XR_i\) in constraint (13). On the other hand, calculation of allocation costs in addition to the fixed costs is provided by using this variable.

\[
\text{Min } f = \sum_{i=1}^{I} FR_i XR_i + \sum_{d=1}^{D} \sum_{i=1}^{I} Z_{d,i} \text{dem}_d d_{d,i} a_{d,i} 
\]

\[
\sum_{i=1}^{I} c_{d,i} Z_{d,i} \geq \beta_d Y_d 
\]

\[
\sum_{d=1}^{D} \text{Dem}_d Y_d \geq \sum_{d=1}^{D} \beta_d \text{Dem}_d 
\]

\[
Y_d, Z_{d,i} \in \{0, 1\} \quad \forall d, i 
\]

To make the differences of the relaxed proposed model, basic model 1, and basic model 2 more clear their characteristics are as follows. In the proposed model, there is a possibility of failure in source points and backup coverage is considered to compensate the unmet coverage by failure occurrence in each emergency centre. In the relaxed version of the proposed model, there is no possibility of failure, so all the emergency centres are the same and are not distinguished by the concept of reliability; therefore, there is not any need for backup coverage as no centre is disrupted. These two concepts of source point failures and backup coverage are relaxed to have a proper comparison with the basic models in which none of these two concepts are considered. Also, the difference between two basic models is based on this fact that the decision variables relevant to establishment and coverage are taken hierarchically in basic model 1 and
simultaneously in basic model 2. So this is expected that the second model will present a better solution than the first one due to separation of the feasible region in basic model 1.

In the next section, above mentioned models are used to analyse the accurate performance of the proposed model.

4 Numerical analysis

In this section, the proposed model is investigated using numerical analysis. Initially, a numerical example is designed to indicate the necessity of considering reliability for the cooperative location set covering problem presented by Berman et al. (2011); then, relaxed version of the proposed model and the basic models 1 and 2 (presented in Section 3) are compared using the given numerical example in order to investigate the accuracy of the proposed model. At the end, the effects of the proposed model parameters are discussed.

4.1 Consideration of reliability for the cooperative location set covering problem

To begin, a numerical example with 14 demand zones and seven candidate sites for location of emergency centres is considered. This example is extracted from the dataset presented in Snyder and Daskin (2005), by changing the values of some parameters in order to adapt with the proposed model. Parameter values generation scheme are presented in Table 3.

| Parameter | Value |
|-----------|-------|
| FRi       | Uniform (1,800,000, 2,000,000) |
| FUk       | Uniform (900,000, 1,000,000) |
| demd      | Uniform (9,000, 10,000) |
| βd        | Uniform (0.8, 0.99) |
| dk        | Uniform (100, 500) |
| qk        | Uniform (7, 240) |
| qk        | Uniform (0.01, 0.09) |
| a, i     | Uniform (0.3, 0.5) |
| l, i     | Uniform (150, 160) |
| u, i     | Uniform (350, 400) |

Optimal objective function value of the proposed model without taking into account the possibility of failures in facilities (qk = 0) is equivalent to 6,874,500 and candidate sites 1, 2 and 5 are selected for locating emergency centres. In Table 4, the percentage of covered demands by each established centre is presented. Then, it is assumed that each of these established centres becomes out of reach as disaster occurs. In each case, two remaining centres are responsible for meeting the lost amount of demands, so the system will face
new kind of cost which is presented in the failure cost column and generated by failure in each established centre.

### Table 4 Results for the relaxed version of the proposed model

| Established centre | Covered demand (%) | Failure cost | Increase (%) |
|--------------------|--------------------|--------------|--------------|
| 1                  | 21                 | 7818500      | 14           |
| 2                  | 36                 | 8223500      | 20           |
| 5                  | 43                 | –            | –            |

As can be seen in Table 4, failure in centre 2 (among centres 1 and 2) leads to higher failure cost since it provides higher demand coverage and also the geographical locations of the addressed demands are far from centres 1 and 5. Furthermore, the problem does not reach a feasible solution when failure occurs in centre 5; in other words, centres 1 and 2 are not able to cover demand zones which were covered by centre 5. Thus, it can be seen that 43% of the total demand remain un-served, if failure possibility is not considered for the centres, which leads to critical situation in relief activities when a natural disaster occurs. Then, reliable versions of the problem are solved providing that in each version, failure probability for one of the established centres in classical problem is equal to 0.01. The solved problem again determines centres 1, 2 and 5 to be established. By comparing failure cost columns in Tables 4 and 5, it can be observed that system costs will be decreased, if failure probabilities are taken into account. Moreover, application of backup coverage chooses centre 6 as a backup centre for centre 5, thus system will not face uncovered demands which were inevitable in the relaxed version of the proposed model.

### Table 5 Results of the proposed model under disruption and backup coverage consideration

| Established centre | Covered demand (%) | Backup centres | Failure cost | Increase (%) |
|--------------------|--------------------|----------------|--------------|--------------|
| 1                  | 21                 | 2, 5           | 6902445      | 0.4          |
| 2                  | 36                 | 1, 5           | 6897990      | 0.3          |
| 5                  | 43                 | 3, 6           | 6928500      | 0.8          |

By comparing increase (%) columns in Tables 4 and 5, it is observed that consideration of failure probabilities as well as backup coverage in the cooperative location set covering problem presented by Berman et al. (2011) leads to fewer cost increasing while the reliability has considerable growth. Also consideration of backup coverage along with disruption is of great importance as it prevents the system to face uncovered demands when failures occur in service facilities.

### 4.2 Performance analysis of the proposed model

For the numerical analysis of the model, 14 demand zones and seven reliable and seven unreliable candidate sites are considered. Other parameter values are based on Table 3. The proposed model under zero failure probability for unreliable centres is compared with two basic models 1 and 2 in terms of objective function values and selected candidate sites. The comparison is based on same examples and parameter values which are presented in Table 3.
Results have been obtained using GAMS optimisation software, CPLEX solver in a personal computer (Intel Core i3 and 4.00 GB RAM). Performance of the proposed model (with no failure probability for facilities) is compared with basic models 1 and 2, in Table 6.

| Models            | Established centres | Covered demand zones | Objective value |
|-------------------|---------------------|----------------------|-----------------|
| Basic model 1     | 2                   | 3, 4, 6–11, 13       | 8,156,000       |
|                   | 4                   | 1, 2, 5, 8, 9, 11, 12, 14 |               |
| Basic model 2     | 3                   | 2–5, 8, 9            | 4,644,000       |
|                   | 5                   | 1, 6, 7, 10–14       |                 |
| Proposed model    | 3                   | 2–5, 8, 9            | 4,644,000       |
|                   | 5                   | 1, 6, 7, 10–14       |                 |

According to Table 6, basic model 1 has greater objective value in comparison with the proposed model, because basic model 1 does not make location and allocation decisions simultaneously. On the other hand, same objective function values for both basic model 2 and the proposed model guarantees proper performance of the proposed model. So it can be validated the accuracy of the proposed model.

### 4.3 Parameter analysis of the proposed model

To investigate the impact of defined failure probability values (related to unreliable facilities) on the number of selected emergency centres, in the primal covering, equal values for failure probability of all unreliable facilities are considered in the range of (0.1, 0.9). Furthermore, fixed values are applied for the other parameters of the model during solving each example.

![Figure 1](image)

As can be seen in Figure 1, increasing of failure probability values decreases the number of established unreliable centres and increases that of established reliable ones which verify the reasonable performance of the model; in a way that model tends to select more
reliable centres than unreliable ones in the primal covering which leads to less backup costs.

To evaluate the effect of fixed cost values on the number of selected reliable and unreliable emergency centres in the primal covering, unreliable to reliable fixed cost ratios are varied from 20% to 600%.

**Figure 2** Effect of various unreliable to reliable fixed cost ratios on the number of established emergency centres in primal covering

As illustrated in Figure 2, when unreliable fixed cost is less than reliable one, model tends to choose more unreliable centres. On the contrary, when unreliable fixed cost is greater than reliable one, model chooses more reliable centres. When reliable and unreliable fixed cost valued the same, model selects more reliable centres than unreliable ones in the primal covering in order to reduce backup coverage cost. Mentioned behaviours prove the validity of the proposed model.

In the further analysis, the same approach is applied for total number of established centres in both primal and backup coverage stages.

**Figure 3** Effect of various reliable to unreliable fixed cost ratios on the total number of established emergency centres (in both primal and backup coverage)

As demonstrated in Figure 3, either increase or decrease in reliable fixed costs to unreliable ones leads to decreasing the total number of established reliable centres in both primal and backup coverage. In that sense, model tends to select fewer reliable centres and incurs more transportation cost. Also, mentioned increase in reliable fixed cost leads
to increase in the number of established unreliable centres in the primal covering. These behaviours also prove the validity of the proposed model.

5 Conclusions

In this paper, a location covering model is presented in which both concepts of cooperative and backup coverage are considered. Due to the devastation of natural disasters, disruption in candidate sites at which emergency centres are established can occur. Therefore, geographic features of some locations can result in unavailability of related established centres which are named as unreliable centres in our model. The proposed model attempts to reduce both system risk (which resulted from facility failure) and facing uncovered demand zones (by considering backup covering) in disaster situation. The presented results show that if there is a possibility of failure in facilities, it should be considered when the system is modelled; otherwise by failure occurrence the system will face significant increase in cost. On the other hand, application of backup coverage leads to compensating uncovered demands which are resulted from failure in operating facilities. Such an issue is of great importance during natural disasters as well as relief missions. Investigation of numerical examples show that both facility failure and backup coverage considerations can incur additional cost and result in significant increase in the system reliability. Also, the results of proposed model were compared with that of two basic models 1 and 2 to show its accurate performance. Furthermore, different approaches have been used during sensitivity analysis of the model in order to show its validity. As future studies and more adaption of the model to the real world situation, uncertain demands or probabilistic covering radius as well as servicing capacity for the centres can be investigated.

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