To the problem on the Universe evolution and the condensed description in the field theory

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On the basis of hypotheses, that a density of weakly interacting particles in the Universe has an order of nuclear matter density or more the Lagrangian is offered, through which one can be obtained a propagator of a vector boson with a non-zero rest-mass. The dependence of vector bosons masses on the time allows to explain the availability of the hot stage of the Universe evolution, not using to the hypothesis of the Universe expansion.

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From last astronomical observations (see for example [1]) it follows that not more than 5% of the Universe matter has the baryon nature. Thereof it is convenient to divide the Universe matter on rapid and slow subsystems. We shall consider that all known particles (it is possible excluding only neutrinos) belong to the rapid subsystem. Thus we ought to consider fundamental particles as coherent frames in open systems characterized by quasi–group structure. It results in necessity of usage inhomogeneous (quasi–homogeneous) space-time endowed by a nontrivial geometrical structure.

Let’s consider the Boltzmann hypothesis of the Universe birth owing to a gigantic fluctuation not in an empty space but in a medium which consists of weakly interacting particles characterized by zero temperature and forming the Bose condensate. Certainly, if the particles are fermions they should be in the coupled state. For the description of such state of the Universe matter (this state we shall consider pure one) it is necessary to introduce an amplitude of probability $B$ with components $B^b_a(\omega)$ ($a, b, c, d, e, f, g, h = 1, 2, ..., r$) dependent from points $\omega$ of a manifold $M_r$ (not excepting a limiting case $r \to \infty$). In this case we can not define the metric but for its definition we need a density matrix $\rho(B)$ (the rank of which equals to 1 for a pure state), determining its standard mode $BB^+ = \rho \text{tr}(BB^+)$ ($\text{tr} \rho = 1$, $\rho^+ = \rho$, the top index “+” is the symbol of the Hermitian conjugation).

Let as a result of a fluctuation the disintegration of the Bose condensate will begin with formation of fermions (for their description we shall introduce an amplitude of probability $\Psi$) and with an increase of pressure in some local area of Universe (in addition some time the temperature of the Universe particles could remain equal or close to zero point — so–called the inflation period). As a result the rank of the density matrix $\rho$ will begin to grow that characterizes the appearance of mixed states. An inverse process of relaxation (characterized by the formation of the Bose condensate and by the pressure decline) should go with an energy release which will go on heating of the Fermi liquid with formation of excited states — of known charged fermions (quarks and leptons). From this moment it is possible to introduce the metric and use the results obtained for the hot model of Universe (with those by inflationary modifications which have appeared recently [2]) interpreting the Universe evolution as the process characterized by the increase of the entropy $S = - \text{tr}(\rho \ln \rho)$. Now Universe is at that stage of an evolution when the dominating number of particles has returned to the Bose condensate state. They are also displaed only at a weak interaction with particles of a visible matter.

For the description of a matter it is convenient to use differentiable fields given in a differentiable manifold $M_n$ which one we shall call as a space-time and the points $x$ of which one will have coordinates $x^i$ ($i, j, k, l... = 1, 2, ..., n$). Probably the rank of the density matrix $\rho$ equals $n$, but it is impossible to eliminate that the generally given equality is satisfied only approximately when some components of a density matrix can be neglected. In any case we shall consider that among fields $B$ the mixtures $\Pi^i_a$ were formed with non-zero vacuum means $h^i_a$ which determine differentiable vector fields $\xi^i_a(x)$ as:

$$\Pi^i_a = B^b_a \xi^i_b$$  \hspace{1cm} (1)
for considered area \( \Omega_n \subset M_n \) (field \( \xi'_a(x) \) determine a differential of a projection \( d\pi \) from \( \Omega_n \subset M_n \in \Omega_n \)). It allows to define a space–time \( M_n \) as a Riemannian manifold, the basic tensor \( g_{ij}(x) \) of which we shall introduce through a reduced density matrix \( \rho'(x) \).

So let components \( \rho'_i \) of a reduced density matrix \( \rho'(x) \) are determined by the way:

\[
\rho'_i = \xi'^a_i \rho_a \xi'_b / (\xi'^c_{ki} \rho'^c_{kd} \xi^d_k).
\]

and let fields

\[
g^{ij} = \eta^{k(i} \rho'^{j)}_k (g^{lm} \eta_{lm})
\]

are components of a tensor of a converse to the basic tensor of the space–time \( M_n \). By this components \( g_{ij}(x) \) of the basic tensor must be the solutions of following equations:

\[
g^{ij} \delta_k = \delta_i^j. \] (Hereinafter \( \eta_{ij} \) are metric tensor components of a tangent space to \( M_n \) and \( \eta^{ik} \) are determined as the solution of equations: \( \eta^{ij} \eta_{ik} = \delta_i^j. \)

The influencing of the macroscopic observer will display in an approximation of the transition operator \( T \) by the differential operators \( \partial_i \). First of all we shall require that for fermion fields deviations \( X_a(\Psi) = T_a(\Psi) - \xi'_a \partial_i \Psi \) were minimum in the “mean” [3]. For this purpose we shall consider a following integral

\[
\mathcal{A} = \int_{\Omega_n} \mathcal{L}(\Psi) d_{n}V = \int_{\Omega_n} \mathcal{X}^i(\Psi) \rho^j_a(x) X_b(\Psi) d_{n}V
\]

(\( \kappa \) is a constant, \( \mathcal{L}(\Psi) \) is a Lagrangian; the bar means the Dirac conjugation that is to be the superposition of Hermitian conjugation and the spatial inversion) which is the action. Let the action \( \mathcal{A} \) is quasi-invariant at infinitesimal substitutions \( x^i \to x^i + \delta x^i = x^i + \delta \omega_a \xi'_a(x), \) \( \Psi \to \Psi + \delta \Psi = \Psi + \delta \omega_a T_a(\Psi) \) of Lie local loop \( G_r \), the structural tensor components \( C^{ab}_c \) of which satisfy to Jacobi generalized identity [3]. The given requirement causes to introduce the full Lagrangian (instead of a Lagrangian \( \mathcal{L}(\Psi) \)) recording it as

\[
\mathcal{L}_1 = \mathcal{L}(\Psi) + \kappa_0 \mathcal{F}_c^{ab} \mathcal{F}_d^{ef} [t^{ad}(s^e_c s_f^b - v s_c^b s_f^e) + t^{be}(s^a_d s_c^d - v s_a^d s_c^e) + u_{ce}(t^{ad} s_c^e - v t^{ab} t^{de})]/4
\]

(\( \kappa_0, v \) are constants). In addition intensities \( \mathcal{F}_c^{ab} (B) \) of the boson (gauge) fields \( B^c_a(x) \) will look like

\[
\mathcal{F}_c^{ab} = \Theta^{c}_{a} (\Pi^{c}_{a} \partial_{b} B_{d} - \Pi^{c}_{b} \partial_{d} B_{a} + \Xi^{c}_{ad}),
\]

where

\[
\Theta^{c}_{b} = \delta_{b}^{c} - \xi_{b}^{c} \Pi^{d}_{i} (B^{c}_{d} - \beta^{c}_{d}), \quad \Xi^{b}_{ad} = (B^{c}_{a} T_{ca}^{d} - B^{c}_{d} T^{c}_{ca}) B^{b}_{e} - B^{c}_{a} B^{e}_{d} C^{ce}_{bd}.
\]

Hereinafter a selection of fields \( \Pi^{c}_{i}() \) and \( \beta^{a}_{c} \) are limited by the relations:

\[
\Pi^{c}_{j} \Pi^{i}_{a} = \delta^{i}_{j}, \quad \beta^{a}_{c} s^{i}_{a} = h^{i}_{c}
\]

(\( \delta^{i}_{j} \) are Kronecker deltas). If

\[
s^{b}_{a} = \delta^{b}_{a}, \quad t^{ab} = \eta^{ab}, \quad u_{ab} = \eta_{ab}
\]
(\eta_{ab} \text{ are metric tensor components of the flat space and } \eta^{ab} \text{ are tensor components of a converse to basic one}) then the given Lagrangian is most suitable one at the description of the hot stage of the Universe evolution because it is most symmetrical one concerning intensities of the gauge fields \( \mathcal{F}_{ab}^c \). What is more we shall require the realization of the correlations: \( T^b_a \eta^{ab} + T^d_a \eta^{cb} = 0 \), that the transition operators \( T^b_a \) generate the symmetry, which follows from the made assumptions. In absence of fields \( \Pi^i_a(x) \) and \( \Psi(x) \) at earlier stage of the Universe evolution the Lagrangian (5) becomes even more symmetrical \( (\mathcal{L}_t \propto \mathcal{B}^4) \), so that the formation of fermions (the appearance of fields \( \Psi \) in a full Lagrangian \( \mathcal{L}_t \)) from primary bosons is a necessary condition (though not a sufficient one) of the transition of Universe to the modern stage of its development with a spontaneous symmetry breaking. Only the formation of the Bose condensate from pairs of some class of fermions (the neutrinos of different flavors) has resulted in a noticeable growth of rest–masses of those vector bosons \( (W^+, W^-, Z^0) \), which interact with this class of fermions. In parallel there could be a growth of rest–masses and other fundamental particles, though and not all (photon, directly with a neutrino not interacting, has not a rest-mass).

Let’s connect non-zero vacuum means \( \beta^b_a \) of gauge fields \( B^b_a \) with a spontaneous violation of a symmetry, which has taken place in the early Universe and which is a phase transition with a formation of Bose condensate from fermion pairs. The transition to the modern stage of the Universe evolution for which it is possible to suspect the presence of cluster states of weakly interacting particles will be expressed in following formula for tensors \( s^b_a \), \( t^{ab} \), \( u_{ab} \) and \( h^i_a \):

\[
s^b_a = s^i_a h^b_i + \xi^c_a e^b_c, \quad t^{ab} = t^a_i e^b_j \eta^{ij} + \xi^a_c e^b_c \eta^{cd},
\]

\[
u_{ab} = u^i_a \xi^j_b h^d_j \eta_{cd} + \xi^c_a \xi^d_b \eta_{cd}, \quad h^i_a = h^i_j e^j_a \] (10)

((i), (j), (k), (l), ... = 1, 2, ..., n; \( \alpha, \beta, \gamma, \delta \) = \( n + 1, n + 2, ..., n + \nu, \nu/r \ll 1 \)), where fields \( h^i_a(x) \), taking into account the relations (10), are uniquely determined from equations: \( h^i_b h^b_a = \delta^i_k \). Similarly tensors \( \eta^{(i)(j)} \), \( \eta^{ab} \) are determined from equations: \( \eta^{(i)(k)} \eta^{(j)(k)} = \delta^{(i)(j)} \), \( \eta^{ab} \eta_{cb} = \delta^{ac} \), while tensors \( \eta^{ij} \), \( \eta_{ab} \) are determined as follows \( \eta^{ij} = \eta^{ab} \), \( \eta_{ab} = \eta_{cd} \). We shall connect constants \( \epsilon^a_i \), \( \epsilon^c_a \) with a selection of the gauge fields \( \Pi^i_a(x) \) recording them in the form

\[
\Pi^i_a = \Phi^i_a \epsilon^b_j + \Phi^c_a \epsilon^a_j \] (11)

and let \( \epsilon^a_\alpha = 0 \). Besides we shall apply the decomposition of fields \( B^b_a(x) \) in the form

\[
B^b_a = \zeta^a_b \Pi^c + \zeta^a_\alpha A^b_\alpha \] (12)

where \( A^b_\alpha = B^b_\alpha \). In addition components of intermediate tensor fields \( \zeta^a_b(x) \), \( \zeta^b_\alpha(x) \), \( \zeta^a_\alpha(x) \) should be connected by the relations: \( \zeta^a_b \zeta^b_\alpha = \delta^a_\alpha \), \( \zeta^a_b \zeta^b_a = 0 \), \( \zeta^a_\alpha \zeta^a_\alpha = 0 \), \( \zeta^c_b \zeta^b_\alpha = \delta^c_\alpha \).

Let \( n = 4, \ v = 2, \ tu = s^2 \), \( T^a_{(k)} = T^a_{cb} \epsilon^b_{(k)} = T^a(i) \epsilon^b_{(i)} \), \( T^a_{(k)} = \zeta^a_i T^a_{(i)} \), \( T^a_{(k)} = \zeta^a_\alpha T^a_{(\alpha)} \)

and

\[
T^a_{(i)} \eta^{(k)(i)} + T^a_{(k)} \eta^{(k)(i)} = 0, \quad \Phi^a_{ab}(x) \] (13)
so that the full Lagrangian (5) will be rewritten as follows

\[ L_t = L(\Psi, D\Psi) + \eta^{(j)(m)} [\kappa_o E_{(i)(j)}^a E_{(k)(m)}^b \eta^{(i)(k)} \eta_{ab} + \]

\[ \kappa_1 (F_{(i)(j)}^k F_{(l)(m)}^n \eta^{(i)(l)} \eta_{(k)(n)} + 2 F_{(i)(j)}^i F_{(k)(m)}^k - 4 F_{(i)(j)}^i F_{(k)(m)}^k)] / 4, \]  

(14)

where

\[ \kappa_o = \kappa_o' t^2, \quad \kappa_1 = \kappa_1' t^2. \]  

(15)

\[ F_{(i)(j)}^k = F_{ab} \epsilon_{(i)}^a \epsilon_{(j)}^b h_{(k)}^i h_{(k)}^j = \Phi_{(i)}^k F_{mn} \Phi_{(i)}^m \Phi_{(j)}^n = \]

\[ (\Phi_{(i)}^l \nabla_l \Phi_{(j)}^m) - \Phi_{(j)}^l \nabla_l \Phi_{(i)}^m) \Phi_{(i)}^m + \Phi_{(i)}^l T_{(j)}^{(k)} - \Phi_{(j)}^l T_{(i)}^{(k)} + \Lambda_{(i)}^a T_{(j)}^{(k)} - \Lambda_{(j)}^a T_{(i)}^{(k)}, \]

(16)

\[ E_{(i)}^a \Phi_{(j)}^k = E_{(k)(l)}^a \Phi_{(i)}^k \Phi_{(j)}^l = F_{bc} \epsilon_{(k)}^c \epsilon_{(k)}^b \epsilon_{(l)}^i \epsilon_{(l)}^j \Phi_{(i)}^k \Phi_{(j)}^l = \]

\[ \nabla_i A_{(k)}^a - \nabla_j A_{(a)}^i + A_{(b)}^j A_{(c)}^j + C_{(a)bc} A_{(b)}^j + C_{(a)}^j + C_{(a)ij}, \]

(17)

\[ \Phi_{(i)}^k = \Pi_{(i)}^k \epsilon_{(k)}, \quad A_{(j)}^b = A_{(j)}^b \Phi_{(j)}^k, \]

(18)

\[ C_{(a)bc} = \zeta_{(a)bc} \epsilon_{(d)} \epsilon_{(e)} \epsilon_{(f)} \epsilon_{(g)} \epsilon_{(h)} \epsilon_{(i)} \epsilon_{(j)} \epsilon_{(k)} \epsilon_{(l)} \epsilon_{(m)} \epsilon_{(n)} \epsilon_{(o)} \epsilon_{(p)} \epsilon_{(q)} \epsilon_{(r)} \epsilon_{(s)} \epsilon_{(t)} \epsilon_{(u)} \epsilon_{(v)} \epsilon_{(w)} \epsilon_{(x)} \epsilon_{(y)} \epsilon_{(z)}, \]

(19)

As a result of an equation of fields \( \Phi_{(i)}^k(x) \) may be received in a standard manner [4] as the Einstein gravitational equations

\[ D_i \Psi \frac{\partial L}{\partial D_j \Psi} g_{jk} - g_{ik} L(\Psi, D_m \Psi) + \kappa_o \eta_{ab} g^{ij} (E_{ij}^a E_{jk}^b - \frac{1}{4} g_{ik} g^{mn} E_{jm}^a E_{ln}^b) = \]

\[ \kappa_1 (2 R_{ij}^k j - g_{ik} g^{lm} R_{jlm}^n), \]  

(20)

\( R_{ij}^k \) is the curvature tensor of the connection \( \Gamma_{ij}^k \) of Riemannian space–time \( M_n \), \( \kappa_o = 1/(4\pi) \), \( \kappa_1 = 1/(4\pi G_N) \), \( G_N \) is the gravitational constant; hereinafter the system of units are used \( h/(2\pi) = c = 1 \), where \( h \) is the Planck constant, \( c \) is the speed of light). Naturally, that the Einstein equations express the modern physical state of the Universe matter. All this confirms a capability for interpretations of fields \( \Phi_{(i)}^k(x) \) or fields \( \Phi_{(j)}^k(x) \) as gravity potentials, but taking into account their dependence from properties of medium (vacuum), and also the opinion to consider components \( g_{ij}(x) \) of a metric tensor of the space–time by potentials of a gravitational field, it is meaningful to call \( \Phi_{(i)}^k(x) \) and \( \Phi_{(j)}^k(x) \) by polarization fields. It is necessary to note that the condensed description of a medium, consisting of weakly interacting particles, by polarization (gravitational) fields \( \Phi_{(i)}^k(x) \) allows to “hide” the Bose condensate with the help of a nontrivial geometrical structure, in particular, using a Riemannian space–time of General Relativity.

Let’s study an approaching, in which the space–time is possible to consider as a Minkowski space, the fields \( \Phi_{(k)}^k \), \( \Phi_{(k)}^i \) are constants and let \( \omega = 1 \), that assumes \( C_{ab}^c = 0 \). For obtaining
equations of fields $A^a_\perp(x)$ in Feynman perturbation theory the calibration should be fixed. For this we shall add the following addend:

$$\mathcal{L}_q = \kappa_o \eta_{\bar{a}a} g^{ij} g^{kl} (\partial_i A^b_j - \eta_{o C_i A^b_j})(\partial_k A^b_l - \eta_{o C_k A^b_l})/2$$

(21)

to a Lagrangian (14), where $q_o = \eta_{\bar{a}a}/\eta_{\bar{b}b}$, $C_i = C^b_{ia}$. Besides let

$$T^{(i)}_{a(k)} \eta^{(j)(k)} + T^{(j)}_{a(k)} \eta^{(i)(k)} = \frac{\eta_{\bar{a}}}{\eta_{\bar{b}}} \eta^{(i)(j)}.$$  

(22)

As a result of this equations of a vector field $A^b_\perp(x)$ will be written as:

$$g^{jk}[\partial_j \partial_k A^a_\perp - (1 - 1/q_o) \partial_j A^a_\perp + (1 - q_o)C_i C_j A^a_\perp] + m^2 A^a_\perp = I^a_\perp/\kappa_o ,$$

(23)

where $I^a_\perp = \frac{g_{\bar{a}a}}{\eta_{\bar{a}a}} \frac{\partial \mathcal{L}(\Phi)}{\partial A^a_\perp}$ and

$$m^2 = (n - 1)(n - 2)\kappa_t \eta_{\bar{a}a}^2/(2\kappa_o \eta_{\bar{a}a}) - g^{jk} C_j C_k .$$

(24)

Notice that owing to the vacuum polarization ($C_i \neq 0$) the propagator of a vector boson has the rather cumbersome view [5]

$$D_{ij}(p) = \frac{[1 - q_o] (p_ip_j - C_i C_j)(p^k p_k - q_o m^2) + (1 - q_o) p^k C_k (p_i C_j + C_i p_j)}{(p^i p_i - q_o m^2)^2 + (1 - q_o)^2 (p^i C_i)^2}$$

$$- g_{ij}/(p^m p_m - m^2),$$

(25)

which is simplified and receives the familiar form $(-g_{ij}/(p^k p_k - m^2), p^k$ is the 4-momentum, and $m$ is the mass of the vector boson) only in the Feynman calibration ($q_o = 1$).

So, the transition to the hot state of Universe was connected with the destruction of the Bose condensate and the increase of the Fermi gas pressure accordingly. In addition some time a temperature of background particles of Universe could remain equal or close to zero (the stage of the inflation). As a result the rest--mass of $W^+, W^-, Z^\alpha$ bosons have decreased so, that the weak interaction has stopped to be weak and all (or nearly so all) particles from a ground (vacuum) state started to participate in an installation of a thermodynamic equilibrium. The given phenomenon also has become the cause of an apparent increase of a density of particles in the early Universe. Suggesting, that mean density $n_o$ of particles in the Universe did not vary at the same time and the script of the hot model in general is correct, we come to its following estimation $n_o \sim m_\pi^3 \sim 10^{-3} GeV^3$ ($m_\pi$ is a mass of a $\pi$ meson). This result allows to give explanation to a known ratio [6] $H_o/G_N \approx m_\pi^3$, if to consider, that the Hubble constant $H_o$ gives an estimation $1/H_o$ to the length $l \sim 1/(n_o \sigma_\nu)$ of a free run of particle in “vacuum” on the modern stages of the Universe evolution ($\sigma_\nu$ is a scattering cross--section of neutrinos on a charged particle) and to take into account the estimation given earlier [7] for the gravitational constant $G_N$ ($G_N \sim \sigma_\nu$). Thus the gravitational constant $G_N$ is inversely proportional to the time of a free run of a charged particle in the neutrinos medium characterizing a kinetic phase of a relaxation process in the Universe.
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