Real photons from nonequilibrium QGP

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Abstract

We calculate the rate of the emission of the photons from the QGP. We base on the real-time kinetic approach [1] without an explicit assumption about a complete thermal equilibrium in the emitting system.

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1. The subject of this note is as follows. In recent papers devoted to the rate of emission of the real photons from the quark-gluon plasma one may find very different final answers. Most of them agree as for the main logarithmic terms which appropriately estimate rate of emission only at unphysically large energies. Next terms which come to be significant at the typical value of $E/T \sim 10$ different authors write in manifestly different way [2,3]. It looks extremely surprising even the different functional dependence from the basic parameters, though all authors start from the same initial formulas. I have seen no papers were the procedure of analytic approximation would have been discussed in detail except paper [4] were expression for the rate of emission was given in the form of one-dimensional integrals which allow the reliable further analysis by very simple means.

2. Expression for the rate of emission, more accurate then the main logarithm, is needed in connection with some modern scenario of the QGP formation which predict suppression of the quark component against background of the hot glue rather than complete thermalization of the QGP [5].

For sake of completeness we shall start with reminding of the definitions and the main framework of calculations. For photon spectrum the rate of emission is given by [1,4]

$$k^0 \frac{dN_{\gamma}}{dkd^3x} = \frac{ig_{\mu\nu}}{2(2\pi)^3} \pi^{\mu\nu}_{10}(-k),$$

(1)

with the usual definition of two currents correlation function

$$\pi^{\mu\nu}_{10}(x,y) = i\langle T^+(j^\mu(x)S^+)T(j^\mu(x)S)\rangle,$$

(2)

In what follows we use the same way of derivation of the two loop expansion of $\pi^{\mu\nu}_{10}$ as in [1,4] and denote $g_{\mu\nu}\pi^{\mu\nu}_{10}(-k)$ as $\pi(k)$.

3. As the threshold of the first Born’s term and the one-loop radiative corrections is $k^2 > 4m^2$ they are absent for real photons. Physically, this mean that the two fermion states with the time-like momenta can not produce the massless photon. In the vertex which emits the photon one of the fermion legs should carry the time-like momenta and another - a space-like one. The fermion field excitations with space-like momentum can not belong to normal set of the normalizable states and therefore to contribute the density matrix. They can be only virtual. So only the real processes of the next perturbation order can contribute the rate of emission. The desired virtuality of the fermion line in the electromagnetic vertex comes from the intermediate state of the annihilation or the Compton process.

It would have been inconsistent to include the Born’s term with the fermion lines dressed by the one-loop self-energy in the whole scheme where the two-loop and the next graphs are considered perturbatively. It will lead to an over-count in the subset of graphs of the self-energy type and to the under-count in the group of graphs of the vertex type.
In fact, an account for some elements of a collective behavior in the ensemble of the partons is needed only in a very narrow region of a collinear geometry of the real processes which leads to the logarithmic divergence of the emission rate. A simple restriction of the size of the domain of the coherence of the interaction between the partons reflects the physics of the partonic ensemble and resolve the problem of a divergence.

This allows us to consider only the real processes, annihilation and Compton, where the necessary virtuality is provided by the gluons. In this case the trace of the electromagnetic polarization tensor reads as [6]

\[ \pi_{\text{real}} = -\frac{ie^2 g^2 N_c C_F}{2\pi^5} \int d^4p d^4q \delta(q^2) \delta((p - k)^2 - m^2) \delta((p + q)^2)(SW_a + SW_c) \times \]

\[ \times \left\{ 1 + \frac{A}{(p^2 - m^2)^2} + \frac{B}{p^2 - m^2} + \frac{C}{p^2 - m^2 + 2kq} \right\} \]

where we denoted:

\[ A = 4m^2, \quad B = 3m^2 + 2(kq) - \frac{2m^4}{(kq)}, \quad C = m^2 + \frac{2m^4}{(kq)}. \]

The expression in the curly brackets is a sum of the squared moduli of the matrix elements of the annihilation process, \( q\bar{q} \rightarrow g\gamma \), or Compton process, \( qg \rightarrow q\gamma \) and \( \bar{q}g \rightarrow \bar{q}\gamma \). Just for the purpose of the following calculations it is written not in terms of habitual Mandelstam variables \( (s, t, u) \). Specification of the process as well as their kinematic boundaries are due to statistical weights.

In the formal approximation when distributions of initial particles are taken to be Boltzmann ones the statistical weight of the annihilation process looks as

\[ SW_a = \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0) n_F(k_0 - p_0) n_F(q_0 + p_0) n_B(q_0) \approx \]

\[ \approx \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0) \frac{e^{-(ku)/T}}{e^{-(qu)/T} - 1} \]

For the Compton rates the statistical weight is equal to

\[ SW_c = -\theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0) n_F(p_0 + q_0) [1 - n_F(p_0 - k_0)] n_B(-q_0) - \]

\[ -\theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0) n_F(k_0 - p_0) [1 - n_F(-p_0 - q_0)] n_B(-q_0) \approx \]

\[ \approx -e^{-(ku)/T} \left[ \theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0) \exp((pu - ku)/T) + 1 \right]^{-1} - \]

\[ -\theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0) \exp((qu)/T) + 1 \right]^{-1} \]

This approximation was proved to be quantitatively effective for the case of the true thermal equilibrium of the emitting plasma.
We may also assume that quarks and gluons are not in thermal equilibrium. Computer simulation of the parton cascade usually produces something similar to the Boltzmann distributions damaged by the specific parameters $\zeta_Q$ and $\zeta_G$. We adopt for quarks and gluons

$$n_F(p) = \zeta_Q e^{-pu/T}, \quad n_B(p) = \zeta_G e^{-pu/T},$$

where $u^\mu$, the 4-velocity of the nonequilibrium partonic media, fugacities $\zeta$ and temperature $T$ are very smooth functions of space-time coordinates. Scenario of the hot glue leads to $\zeta_Q < \zeta_G < 1$ and all these quantities have no habitual thermodynamical meaning.

In this parametrization of the partons distributions which we shall name the partons cascade, the statistical weight of the annihilation process with emission of a gluon looks as

$$SW^{\text{casc}}_a \approx \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0)\zeta_Q^2 e^{-(ku)/T} e^{-(qu)/T} + \zeta_G e^{-(2qu)/T}$$

For the Compton rate of the photon emission the statistical weight equals to

$$SW^{\text{casc}}_c \approx -\zeta_Q\zeta_G \{\theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0) e^{-pu/T} [1 - \zeta_Q e^{-(pu-ku)/T}] + \theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0) e^{-ku/T} e^{(qu+pu)/T} [1 - \zeta_Q e^{(qu+pu)/T}]\}$$

For any kind of these distributions we can perform an exact integration over $p$ using the Breit reference system where $k + q = 0$, c.m.s. of the reaction $q\bar{q} \rightarrow g\gamma$.

For the equilibrium distributions of the partons this integration leads to

$$\pi_{\text{ann}} = -\frac{i e^2 g^2 N_c C_F}{4 \pi^4} e^{-ku/T} \int d^4q \delta(q^2) \theta(q_0) \theta[(k + q)^2 - 4m^2] \frac{\mathcal{F}_a(kq)}{e^{qu/T} - 1}$$

where

$$\mathcal{F}_a(x) = (1 + \frac{2m^2}{x} - \frac{2m^4}{x^2}) \ln \frac{1 - \sqrt{1 - 2m^2/x}}{1 + \sqrt{1 - 2m^2/x}} + (1 + \frac{2m^2}{x}) \sqrt{1 - \frac{2m^2}{x}}$$

For the Compton process it is reasonable to start with a chain of changes of variables, $p \rightarrow -p + k - q$, in the first term and $q \rightarrow -q - p$ in both terms. After that we may also easily perform the integration over $p$ using the same Breit reference system. It gives

$$\pi_{\text{compt}} = -\frac{i e^2 g^2 N_c C_F}{4 \pi^4} e^{-ku/T} \int d^4q \delta(q^2 - m^2) \theta(q_0) \frac{\mathcal{F}_c(kq)}{e^{qu/T} + 1}$$
where $q$ is the momentum of the (anti)quark in the final state and
\[ F_c(x) = (1 - \frac{2m^2}{x} - \frac{2m^4}{x^2}) \ln \frac{m^2 + 2x}{m^2} + \frac{4m^2}{x} + \frac{2x(m^2 + x)}{(m^2 + 2x)^2} \tag{13} \]

In course of these calculations we could easily learn that the energy $p_0$ of a virtual quark in the annihilation process equals to zero in the Breit system. In the Compton process it equals to the ratio $m^2/2(k_0 + q_0)$ which is small for the hard photon also. At least in both cases the momenta of virtual quark are space-like. Because of the high energy of the photon the 4-momentum $p^\mu$ of a virtual quark is very close to the light cone in the rest frame of the media.

4. Further integration is somewhat tricky. Indeed, for the photons $\pi_{\text{compt}}$ depends only on one time-like 4-vector $u$, $u^2 = 1$. So to continue calculations in a reasonable way we are to chose reference frame $\bar{u} = 0$. In this frame the integral of Eq.(9) takes form,
\[- \pi T \int_0^\infty d[\ln(1 + e^{-\sqrt{q^2 + m^2}})]q \int_{-1}^1 dz F_c(k_0 \sqrt{q^2 + m^2} + |k|qz) \tag{14}\]

where $k_0 = |k|$ stands for $ku$. The trick which allows one to reduce this double integral to a simple quadrature is as follows [4]:

i) change the variable $z$ for the new one, $|k|qz = \eta$;

ii) integration with respect to $q$ by parts;

iii) successive changes of variables: $q = m \sinh \phi$, then $me^\phi/T = 2y$.

The result reads as
\[ \pi_{\text{compt}} = -\frac{ie^2g^2N_cCF}{2\pi^3}e^{-ku/T}T^2 \int_0^\infty dy \ln(1 + e^{-y - \frac{x^2}{4y}}) \times \tag{15} \]
\[ \times [(1 - \frac{\xi}{y} - \frac{\xi^2}{2y^2}) \ln(1 + \frac{4y}{\xi}) + \frac{2\xi}{y} + \frac{4y(\xi + 2y)}{(\xi + 4y)^2}] \]

where $\xi = m^2/(kuT)$, $\lambda = m/T$ and singular point $y = 0$ corresponds to the infinite rapidity $\phi$ of the massless quark.

For the $q\bar{q}$ annihilation into gluon and real photon, $k^2 = 0$, the angular integration is also performed in the rest frame of the media and may be reduced to
\[ \pi T \int_0^\infty d[\ln(1 - e^{-q/T})]q \int_{-1}^1 d\theta(q(k_0 + |k|z) - 2m^2)F_a((k_0 + |k|z)q) \tag{16} \]

After changing variable $z$ for $\eta = q(k_0 + |k|z)$ integration over $q$ by parts and changing $q$ for $y = (m^2/ku)$ we arrive at
\[ \pi_{\text{ann}} = -\frac{ie^2g^2N_cCF}{2\pi^3}T^2e^{-ku/T} \int_\xi^\infty dy \ln(1 - e^{-y}) \times \tag{17} \]
\[ \times [(1 + \frac{\xi}{y} - \frac{\xi^2}{2y^2})2\cosh^{-1}\sqrt{\frac{y}{\xi}} + (1 + \frac{\xi}{y})\sqrt{1 - \frac{\xi}{y}]} \]
It is easy to see that the spectrum of gluons in the annihilation process is effectively cut from the above at gluon momenta $\sim T$. So from a simple kinematics it follows that at high photon energy, $E >> T$, the energies of the initial quarks should be large, time-like and slightly above the light cone.

The Compton process can be more precisely identified as the bremsstrahlung of a photon due to the hard scattering of a quark on a gluon. Maximum rate of the emission is produced by the infinite rapidity of the massless quark in the final state.

5. Both annihilation and Compton rates as they are given above logarithmically diverge when quark mass is going to zero and the geometry of collision is collinear. In this case all the participants of the reaction interact infinitely long. This can not be true in the media and all the momenta should be cut off from the below.

The external cut-off should come from the smallest of the three lengths. The first one is the Compton wave-length $l_c \sim 1/m$. The second one is the mean free path defined via the ordinary or transport cross-section, $l_{mfp} \sim 1/g^2 T$ (or $\sim 1/g^4 T$). The third one is connected with the amplitude of the forward scattering, $l_{fs} \sim 1/g T \sim 1/m_{therm}$. As long as we keep quarks massless and consider coupling as the small parameter we must chose the length $l_{fs}$ as the smallest one. So whenever we meet the collinear singularity the momenta of both real and virtual quarks should be cut off from the below at $\sim g T$.

For the production of photons with the energy $E$ much exceeding any of these scales the particular choice is not so significant within the reasonable accuracy. The cut-off mass appears only under the logarithm and its variation is not so valuable.

6. Having reached our goal to present the expression for the rates of emission from the equilibrium plasma in the form of simple quadratures we can easily find them numerically. We can analyze the reliability of different analytic approximations and compare them with those known in the literature.

The way of calculations chosen in the papers [2] and [3] had followed paper [7] where at the very initial stage it was performed change of variables from original momenta to the invariant variables ($s, t, u$). This trick produces a set of delta-functions which are naturally used to get rid of angular integrals. This results in a cumbersome four-dimensional integral which can be simplified only by assuming that $s >> t$. This assumption (as it was used in [7]) do not touch the main logarithmic term which is contributed basically by the low-angle scattering (small $t$). But it violates the constant term which is mainly due to large-angle scattering, not to tell about the next terms. Though these are parametrically small, they are not small numerically in the physically actual region.

I failed to trace out the way of analytic approximation in Ref.[3] but the final result disagree even with the well established functional form of the leading logarithmic terms.

Let us start with the annihilation mechanism of the real photons production.
Notice that rate of photon emission depend upon a single parameter $\xi = m^2/(ku)T$ and contains the logarithmic (collinear) divergence when $m \to 0$. Assuming the regularization by means of the mass cut-off at $m \sim gT$ we see that $\xi << 1$ at least because of $(ku) >> m \sim T$.

My statement is that the approximation which picks up only the main logarithm of the small parameter $\xi$ fails at reasonable values of this parameter (actually, $\xi \sim 0.1$). I will show that terms like $\xi \ln \xi, \xi \ln^2 \xi, \ldots$ appear as well and that at reasonable values of $\xi$ they essentially change the rate of emission.

Let us consider the leading integral of $\pi_{ann}$,

$$- \int_\xi^\infty dy \ln(1 - e^{-y}) \ln \frac{4y}{\xi}$$

Its main part which is easily obtained by taking zero for the low limit equals to

$$\frac{\pi^2}{6} (\ln \frac{4}{\xi} + 1) + \{ \int_0^\infty \frac{y \ln y \, dy}{e^y - 1} \approx -0.24 \}$$

while estimation of the residue at small $\xi$ adds a term like

$$\frac{1}{2}(\xi \ln \frac{1}{\xi} - \xi)$$

Estimation of the next terms is somewhat more complicated but it is very easy to understand the type of corrections: as the integrand has logarithm in the nominator so the extra powers of $y$ in the denominator will produce the powers of $\ln \xi$.

Just similar analysis can be performed for the Compton rate of emission. The integral (15) is effectively cut from the below by $y_{min} \sim \lambda^2/4$. The main term may be approximated as

$$\int_{\lambda^2/4}^\infty \ln(e^{-y} + 1)(\ln \frac{4y}{\xi} + \frac{1}{2})$$

Proceeding as above we get the leading part,

$$\frac{\pi^2}{12} (\ln \frac{4}{\xi} + \frac{1}{2}) + \{ \int_0^\infty \ln(1 + e^{-y}) \ln y \, dy = -0.372 \}$$

and the corrections,

$$-\frac{\lambda^2}{4} (\ln \frac{\lambda^2}{4} - 1) \ln 2 + \frac{\pi^2}{12} \frac{3\lambda^2}{8} \ln 2$$

The "reasonably small" value of the parameter $\lambda = m/T$ is $\mathcal{O}(1)$ and the corrections are comparable with the main term at the previous value of $\xi \sim 0.1$. The rest terms in the integral (15) will produce corrections with the next powers of $\ln \lambda$, just as in the case of the annihilation.
This kind of analysis helps us to understand why it may be dangerous to use the limit of the high photon energy in the many-dimensional integrals. In this form it is hard to figure out the true structure of corrections.

At this circumstances it seems more reliable to use the quadratures (15) and (17) as the definitions without further approximations. The result of numerical calculations is plotted at Fig.1 apart from the common constant factor and the Boltzmann factor \( \exp(-E/T) \). The difference between exact function and the approximate ones is easily seen to be almost constant at the photon energies exceeding 2 Gev. An approximation given in Ref.[2] approaches the exact result only at \( E \sim 100 \text{Gev} \). At \( E < 4 \text{Gev} \) this approximation leads even to the wrong relation between the annihilation and Compton rates.

7. The partons distributions (7) of the nonequilibrium regime lead to another expressions for the rate of emission. All technique of calculations remains the same and the final results immediately follow from the next very simple observation. Inspecting statistical weights given by Eqs. (5) and (8) we see that the following changes are required in the annihilation channel:

\[
[e^{qu/T} - 1]^{-1} \rightarrow e^{-qu/T} \rightarrow \zeta_Q^2 [e^{-qu/T} + \zeta_G e^{-2qu/T}]
\]

The middle of this string corresponds to the straightforward Boltzmann approximation which results in the replacement of the \( \ln(1 - e^{-y}) \) in Eq.(17) by the \( -e^{-y} \). Now the prescription is evident,

\[
\pi_{\text{ann}}^{casc} = \frac{ie^2 g^2 N_c C_F}{2\pi^3} T^2 e^{-ku/T} \zeta_Q^2 \int_0^\infty dy [e^{-y} + \frac{\zeta_G}{2} e^{-2y}] \times
\]
\[
\times [(1 + \frac{\xi}{y} - \frac{\xi^2}{2y^2})^2 \cosh^{-1} \left( \frac{y}{\xi} + (1 + \frac{\xi}{y}) \sqrt{1 - \frac{\xi}{y}} \right)]
\]

The first exponent in the integrand corresponds to spontaneous emission of a gluon, the second exponent with the extra factor \( \zeta_G \) corresponds to the induced emission.

The rate of photons production in the Compton channel modifies in a similar way. A string of changes in the Eq.(12),

\[
[e^{qu/T} + 1]^{-1} \rightarrow e^{-qu/T} \rightarrow \zeta_Q \zeta_G [e^{-qu/T} - \zeta_Q e^{-2qu/T}]
\]

results in

\[
\pi_{\text{compt}}^{casc} = -\frac{ie^2 g^2 N_c C_F}{2\pi^3} e^{-ku/T} T^2 \zeta_Q \zeta_G \int_0^\infty dy [e^{-\left(y + \frac{\xi^2}{4y} \right)} - \frac{\zeta_G}{2} e^{-2\left(y + \frac{\xi^2}{4y} \right)}] \times
\]
\[
\times [(1 - \frac{\xi}{y} - \frac{\xi^2}{2y^2}) \ln(1 + \frac{4y}{\xi}) + \frac{2\xi}{y} + \frac{4y(\xi + 2y)}{(\xi + 4y)^2}]
\]
where the second exponent is due to the Pauli suppression of a quark in the final state.

The actual values of the fugacities which are the measure of the chemical equilibrium are about $\zeta_Q \sim 0.5$ and $\zeta_G \sim 0.75$ \cite{5}. In order to find out the influence of the kinetic nonequilibrium produced by the Boltzmann distributions we plot the rates given by (25) and (27) using $\zeta_Q = \zeta_G = 1$ (also apart from the common Boltzmann factor $\exp(-E/T)$).

We consciously do not integrate the rates over the history of the system which can be easily done for a simple form of the hydrodynamic background. The very approach of nonequilibrium field kinetics \cite{1,6} was designed in order to use the nonequilibrium partons distributions generated in a "realistic" cascade.

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