Non-standard neutrinos interactions in a 331 model with minimum Higgs sector

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We present a detailed analysis of a class of extensions to the SM Gauge chiral symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ (331 model), where the neutrino electroweak interaction with matter via charged and neutral current is modified through new gauge bosons of the model. We found the connections between the non-standard contributions on 331 model with non-standard interactions. Through limits of such interactions in cross section experiments we constrained the parameters of the model, obtaining that the new energy scale of this theory should obey $V > 1.3$ TeV and the new bosons of the model must have masses greater than 610 GeV.

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I. INTRODUCTION

The confirmation of flavor neutrino oscillation \cite{1, 2} by the combination of a variety of data from solar \cite{3-5}, atmospheric \cite{8-12} and reactor \cite{13-17} neutrino experiments established the incompleteness of the Standard Model of electroweak interactions, leaving room for other non-standard neutrino properties.

One convenient way to describe neutrino new interactions with matter in the electro-weak (EW) broken phase are the so-called non-standard neutrino interactions (NSI), that is a very widespread and convenient way of parameterizing the effects of new physics in neutrino oscillations \cite{18-23}. NSI with first generation of leptons and quarks for four of electroweak interactions, leaving room for other non-standard neutrino properties.

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\( \bar{g}_2^a = g_2^a + \varepsilon_{\alpha \alpha} R \), leading to the following differential scattering cross section:\[27\] 28

\[
\frac{d\sigma_{\alpha}}{dT} = \frac{2G_F m_e}{\pi} \left\{ (g_1^\alpha + \varepsilon_{\alpha L})^2 + (g_2^\alpha + \varepsilon_{\alpha R})^2 \left(1 - \frac{T^e}{E}\right)^2 
- (g_1^\alpha + \varepsilon_{\alpha L}) (g_2^\alpha + \varepsilon_{\alpha R}) m_T^e \frac{T^e}{E}\right\}.
\]

(4)

Our goal is to investigate how NSI with matter can be induced by new physics generated by 331 models and based in constraint on this NSI parameters constraint the model some values expected for 331 model. In section II

II. 331 MODEL

The success of the standard model (SM) implies that any new theory should contain the symmetry \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) (\( G_{321} \)) in a low energy limit. Then, it is natural that one possible modification of SM involves extensions of the representation content in matter and Higgs sector, leading to extension of symmetry group \( G_{321} \) to groups \( SU(N)_C \otimes SU(m)_L \otimes U(1)_X \) with \( SU(N)_C \otimes SU(m)_L \otimes U(1)_X \supset G_{321} \).

In early 90’s, F. Pisano and V. Pleitez [29,30] and latter P. H. Frampton [31] suggested an extension of the symmetry group \( SU(2)_L \otimes U(1)_Y \) of electroweak sector to a group \( SU(3)_L \otimes U(1)_X \), i.e. with \( N_C = m = 3 \). The 331 models present some interesting features, as for instance, they associate the number of families to internal consistence of the theory, preserving asymptotic freedom.

In these models, the SM doublets are part of triplets. In quark sector three new quarks are included to build the triplets, while in lepton sector we can use the right-handed neutrino to such role[29,31]. Another option is to invoke three new heavy leptons, charged or not, depending on the choice of charge operator[32,33]. In SM the electric charge operator is constructed as a combination of diagonal generators of symmetry group \( SU(2)_L \otimes U(1)_Y \).

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To have local gauge invariance we have the following covariant derivative:

\[
D_\mu = \partial_\mu - i \frac{e}{2} \lambda_\alpha W^\alpha_\mu - ig_x X B_\mu
\]

where we have two free parameters to obtain the charge of fermions, \( a \) and \( b \) (\( X \) can be determined by anomalies cancellation). However, \( a = 1 \) is necessary to obtain doublets of isospins \( SU(2)_L \otimes U(1)_Y \) correctly incorporated in the model \( SU(3)_L \otimes U(1)_X [32,33] \). Then we can vary \( b \) to create different models in 331 context, being a signature which differentiate such models. For \( b = -3/2 \), we have the original 331 model[29,30].

To have local gauge invariance we have the following covariant derivative: \( D_\mu = \partial_\mu - i \frac{e}{2} \lambda_\alpha W^\alpha_\mu - ig_x X B_\mu \) and a total of 17 mediator bosons: one field \( B_\mu \) associated with \( U(1)_X \), eight fields associated with \( SU(3)_C \), and another eight fields associated with \( SU(3)_L \), written in the form:

\[
W_\mu = W^\alpha_\mu \lambda_\alpha = \left(\begin{array}{ccc}
W^8_\mu & \sqrt{2}W^+_\mu & \sqrt{2}K^Q_\mu \\
\sqrt{2}W^-_\mu & W^3_\mu + \frac{1}{2}W^8_\mu & \sqrt{2}K^-_\mu \\
\sqrt{2}K^Q_\mu & \sqrt{2}K^-_\mu & W^3_\mu - \frac{1}{2}W^8_\mu
\end{array}\right),
\]

(7)

where

\[
W^\pm_\mu = \frac{1}{\sqrt{2}} (W_{1\mu} \mp i W_{2\mu}),
\]

(8)

\[
K^{\pm Q_1}_\mu = \frac{1}{\sqrt{2}} (W_{4\mu} \mp i W_{5\mu}),
\]

(9)

\[
K^{\pm Q_2}_\mu = \frac{1}{\sqrt{2}} (W_{6\mu} \mp i W_{7\mu}),
\]

(10)
Therefore, charge operator in eq. (11) applied over eq. (7) leads to:

\[ Q_W \rightarrow \begin{pmatrix} 0 & +1 & \frac{1}{2} + b \\ -1 & 0 & -\frac{1}{2} + b \\ \frac{1}{2} + b & -\frac{1}{2} + b & 0 \end{pmatrix} \]  

(11)

The mediator bosons will have integer electric charge only if \( b = \pm 1/2, \pm 3/2, \pm 5/2, \ldots, \pm (2n+1)/2, n = 0, 1, 2, 3, \ldots \). A detailed analysis shows that if \( b \) is associated with the fundamental representation \( 3 \) then \( -b \) will be associated with antisymmetric representation \( 3^* \).

A. The representation content

There are many representations for the matter content, for instance \( b = 3/2 \). But we note that if we accommodate the doublets of SU(2)_L in the superior components of triplets and anti-triplets of SU(3)_L, and if we forbid exotic charges for the new fermions, we obtain from eq. (14) the constrain \( b = \pm 1/2 \) (assuming \( a = 1 \)). Since a negative value of \( b \) can be associated to the anti-triplet, we obtain that \( b = 1/2 \) is a necessary and sufficient condition to exclude exotic electric charges in fermion and boson sector.

The fields left and right-handed components transform under SU(3)_L as triplets and singlets, respectively. Therefore the theory is quiral and can present anomalies of Alder-Bell-Jackiw. In a non-abelian theory, in the fermionic sector without exotic electric charges for quarks and with three new leptons without charged singlets under SU(3)_L.

\[ \sum_{a} \{T^a_L(R), T^b_L(R)\}T^c_L(R) = \{T^a_R(R), T^b_R(R)\}T^c_R(R) \],

(12)

where \( T^a(R) \) are symmetry group matricial representation. The indexes \( R \) and \( L \) relate to the quiral property of the fields. Therefore, to eliminate the pure anomaly \( [SU(3)_L]^3 \) we should have that \( A^{abc} \propto \sum_{R} Tr \{\{T^a_L(R), T^b_L(R)\}T^c_L(R)\} = 0 \). We use the fact that \( SU(3)_L \) has two fundamental representations, 3 and \( 3^* \), where \( T^a = -T^a \), which is equivalent to say that \( T^{a*}_L(R^*) = -T^a_L(R) \). Then:

\[ \sum_{R} Tr \{\{T^a_L(R'), T^b_L(R')\}T^c_L(R)\} = \sum_{R} Tr \{\{T^a_L(R), T^b_L(R)\}T^c_L(R)\} + \sum_{R^*} Tr \{\{T^{a*}_L(R'), T^{b*}_L(R')\}T^{c*}_L(R^*)\} \]

(13)

\[ = \sum_{R} Tr \{\{T^a_L(R), T^b_L(R)\}T^c_L(R)\} - \sum_{R} Tr \{\{T^c_L(R), T^a_L(R)\}T^c_L(R)\} \].

(14)

So, we can see that for the anomalies to be canceled, the number of fields that transform as triplets (first term in eq. (14)) and anti-triplets under \( SU(3)_L \) has to be the same. This implies that two families of quarks should transform different then the third family, as will be discussed in next section.

Usually the third quark family is closed to transform in a different way that the first two families. But we will assume that the first family transform differently, to address the fact that \( m_u < m_d, m_u < m_\ell \) while \( m_e >> m_s \) and \( m_\ell >> m_b \). To state in a more clear way, we remember that in SM the \( SU(2)_L \) doublets are: \((u, \ell)\), \((u, \ell)^T\), \((e, s)^T\), \((t, b)^T\), with \( \ell = e, \mu, \tau \). We can see that the first component of leptons doublets and first quark family is lighter that the second component. But for the second and third quark families is the opposite. Then we use this idea to justify that first quark family transform as leptons.

B. Minimal 331 model on scalar sector

Among the different possibilities of 331 models, we will present a detailed study on a Minimal Model on scalar sector without exotic electric charges for quarks and with three new leptons without charged singlets \( b = 1/2 \), where the fermions present the following transformation structure under \( SU(3)_C \otimes SU(3)_L \otimes U(1)_X \):

\[ \psi_{LL} = (\ell^-, \nu_\ell, N^0_L) \sim (1, 3^*, -1/3), \]

\[ \nu_{LR} \sim (1, 1, 0), \]

\[ \ell^0_R \sim (1, 1, -1), \]

\[ N^0_{LR} \sim (1, 1, 0), \]
\[ Q_{1L} = (d, u, U_1)^T_L \sim (3, 3^*, 1/3), \]
\[ u_{iR} \sim (3, 1, 2/3), \]
\[ d_{iR} \sim (3, 1, -1/3), \]
\[ U_{1R} \sim (3, 1, 2/3), \]
\[ Q_{aL} = (u_a, d_a, D_a)^T_L \sim (3, 3, 0), \]
\[ D_{aR} \sim (3, 1, -1/3), \]

where \( i = 1, 2, 3 \), \( \ell = e, \mu, \tau \), \( a = 2, 3 \). We note that the quarks multiplets \( \psi_{1L} \) consist of three fields \( \ell = \{ e, \mu, \tau \} \), the corresponding neutrinos \( \nu = \{ \nu_e, \nu_\mu, \nu_\tau \} \) and new neutral leptons \( N^0_\ell = \{ N^0_e, N^0_\mu, N^0_\tau \} \). We can also see that the multiplet associated with the first quark family \( Q_{1L} \) consists of quarks down, up and a new quark with the same electric charge of quark up (named \( U_1 \)), while the multiplet associated with second (third) family \( Q_{aL} \) consist of SM quarks of second (third) family and a new quark with the same electric charge of quark down (named \( D_2 \) (\( D_3 \))). The numbers on parenthesis refer to the transformation properties under \( SU(3)_C, SU(3)_L \) and \( U(1)_X \) respectively. With this choice the anomalies cancel in a non-trivial way \[40\], and asymptotic freedom is guaranteed \[41\] \[42\].

1. **Scalar sector and Yukawa couplings**

The scalar fields have to be coupled to fermions by Yukawa terms, invariants under \( SU(3)_L \otimes U(1)_X \). In lepton sector, these couplings can be written as:

\[ \bar{\psi}_{1L} \ell_R \sim (1, 3, 1/3) \otimes (1, 1, -1) = (1, 3, -2/3), \]
\[ \bar{\psi}_{1L} \nu_{1R} \sim (1, 3, 1/3) \otimes (1, 1, 0) = (1, 3, 1/3), \]
\[ \bar{\psi}_{1L} N_{1R}^0 \sim (1, 3, 1/3) \otimes (1, 1, 0) = (1, 3, 1/3), \]

and in quarks sector:

\[ \bar{Q}_{1L} u_{1R} \sim (3^*, 3, -1/3) \otimes (3, 1, 2/3) = (1, 3, 1/3) \oplus (8, 3, 1/3), \]
\[ \bar{Q}_{1L} d_{1R} \sim (3^*, 3, -1/3) \otimes (3, 1, -1/3) = (1, 3, -2/3) \oplus \ldots, \]
\[ \bar{Q}_{1L} U_{1R} \sim (3^*, 3, -1/3) \otimes (3, 1, 2/3) = (1, 3, 1/3) \oplus \ldots, \]
\[ \bar{Q}_{aL} u_{1R} \sim (3^*, 3^*, 0) \otimes (3, 1, 2/3) = (1, 3^*, 2/3) \oplus \ldots, \]
\[ \bar{Q}_{aL} d_{1R} \sim (3^*, 3^*, 0) \otimes (3, 1, -1/3) = (1, 3^*, -1/3) \oplus \ldots, \]
\[ \bar{Q}_{aL} D_{aR} \sim (3^*, 3^*, 0) \otimes (3, 1, -1/3) = (1, 3^*, -1/3) \oplus \ldots, \]

As usual in these class of models, we impose non-colored Higgs, selecting only the multiplets that transform as singlets under \( SU(3)_C \). We note that we need only three Higgs multiplets \( \rho, \chi \) and \( \eta \), to couple the different fermionic fields and generate mass through spontaneous symmetry breaking. In eqs. \[16\] and \[17\] we note that quantum numbers of triplets \( \chi \) and \( \eta \) are the same, which leads us to consider models with two or three Higgs triplets. We will adopt the first option, two Higgs triplets, due to the simpler scalar sector in comparison with the scenario with three triplets \[32\] \[33\].
C. Model with two Higgs triplets

For the models with two Higgs triplets, we obtain

\[ \Phi_1 = (\phi_1^-, \phi_1^0, \phi_1^0)^T \sim (1, 3^*, -1/3), \]
\[ \Phi_2 = (\phi_2^0, \phi_2^+, \phi_2^-)^T \sim (1, 3^*, 2/3). \]  \hspace{1cm} (18)

Assuming the following choice to the Higgs triplets vacuum expectation value (VEV) \[ \langle \Phi_1 \rangle_0 = (0, \vartheta_1, V)^T \] and \[ \langle \Phi_2 \rangle_0 = (\vartheta_2, 0, 0)^T \] we associate \( V \) with the mass of the new fermions, which lead us to assume \( V >> \vartheta_1, \vartheta_2 \). We expand the scalar VEV’s in the following way:

\[ \varphi_0 = V + \frac{H_0^0 + iA_0^0}{\sqrt{2}}, \quad \varphi_1^0 = \vartheta_1 + \frac{H_1^0 + iA_1^0}{\sqrt{2}}, \quad \varphi_2^0 = \vartheta_2 + \frac{H_2^0 + iA_2^0}{\sqrt{2}}. \] \hspace{1cm} (19)

The real (imaginary) part \( H_{\varphi_i} \) \( (A_{\varphi_i}) \) is usually called CP-even (CP-odd) scalar field. The most general potential can be written as:

\[ V(\Phi_1, \Phi_2) = \mu^2 \Phi_1^\dagger \Phi_1 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_1^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_1^\dagger \Phi_1). \] \hspace{1cm} (20)

Demanding that in the displaced potential \( V(\Phi_1, \Phi_2) \) the linear terms on the field should be absent, we have, in tree level approximation, the following constraints:

\[ \mu^2 + 2\lambda_1 (\vartheta_1^2 + V^2) + \lambda_3 \vartheta_2^2 = 0, \]
\[ \mu^2 + 2\lambda_3 (\vartheta_1^2 + V^2) + 2\lambda_2 \vartheta_2^2 = 0. \] \hspace{1cm} (21)

The analysis of such equations shows that they are related to a minimum in scalar potential with the value:

\[ V_{\text{min}} = -\vartheta_1^2 \lambda_2 - (\vartheta_1^2 + V^2) \left[ (\vartheta_1^2 + V^2) \lambda_1 + \vartheta_2^2 \lambda_3 \right] = V(\vartheta_1, \vartheta_2, V). \] \hspace{1cm} (22)

Replacing eq. \[ \text{(19)} \] and \[ \text{(21)} \] in eq. \[ \text{(20)} \] we can calculate the mass matrix in \( (H_{\varphi_1}^0, H_{\varphi_2}^0, H_{\varphi_1}^0) \) basis through the relation \( M_{ij}^2 = \frac{\partial^2 V(\Phi_1, \Phi_2)}{\partial H_{\varphi_i}^0 \partial H_{\varphi_j}^0} \), obtaining:

\[ M_H^2 = 2 \begin{pmatrix} 2\lambda_1 \vartheta_1^2 V & \lambda_3 \vartheta_2^2 V & 2\lambda_1 \vartheta_1 V \\ \lambda_3 \vartheta_2^2 V & 2\lambda_3 \vartheta_1 \vartheta_2 & \lambda_3 \vartheta_1 V \\ 2\lambda_1 \vartheta_1 V & \lambda_3 \vartheta_1 \vartheta_2 & 2\lambda_1 \vartheta_1^2 \end{pmatrix}. \] \hspace{1cm} (23)

Since eq. \[ \text{(23)} \] has vanishing determinant, we have one Goldstone boson \( G_1 \) and two massive neutral scalar fields \( H_1 \) and \( H_2 \) with masses

\[ M^2_{H_1, H_2} = 2\lambda_1 (\vartheta_1^2 + V^2) + 2\lambda_2 \vartheta_2^2 \pm 2\sqrt{[\lambda_1 (\vartheta_1^2 + V^2) + \lambda_2 \vartheta_2^2]^2 + \vartheta_2^2 (\vartheta_1^2 + V^2) \left( \lambda_3^2 - 4\lambda_1 \lambda_2 \right)}, \] \hspace{1cm} (24)

where real values for \( \lambda \)'s produce positive mass to neutral scalar fields only if \( \lambda_1 > 0 \) and \( 4\lambda_1 \lambda_2 > \lambda_3^2 \), which implies that \( \lambda_2 > 0 \). A detailed analysis shows that when \( V(\Phi_1, \Phi_2) \) in eq. \[ \text{(20)} \] is expanded around the most general vacuum, given by eq. \[ \text{(19)} \] and using constrains in eq. \[ \text{(21)} \], we don’t obtain pseudo-scalar fields \( A_{\varphi_i}^0 \). This allows us do identify

\[ ^1 \text{ Note that in this model we assumed } \Phi_1 = \chi, \eta \in \Phi_2 = \rho \]
\[ ^2 \text{ Note that if } \lambda_3^2 = 4\lambda_1 \lambda_2 \text{ we obtain two Goldstone bosons, } G_1 \text{ and } H_2, \text{ and a massive scalar field } H_1 \text{ with mass } M^2_{H_1} = 4 \left[ \lambda_1 (\vartheta_1^2 + V^2) + \lambda_2 \vartheta_2^2 \right] \text{ where } \lambda_1 \lambda_2 > 0, \text{ then imposing } M^2_{H_1} > 0 \text{ leads to } \lambda_1 > 0 \text{ and } \lambda_2 > 0. \]
three more Goldstone bosons $G_2 = A^0_{\Phi_1}$, $G_3 = A^0_{\Phi_2}$ and $G_4 = A^0_{\Phi_3}$. For the mass spectrum in charged scalar sector on $(\phi_1^+, \phi_2^+, \phi_2^0)$ basis the mass matrix will be given by:

\[
M^2_\phi = 2\lambda_4 \begin{pmatrix}
\partial_1^2 & \partial_1 \partial_2 & \partial_2 \partial_2 \\
\partial_1 \partial_2 & \partial_2^2 & \partial_2 V \\
\partial_2 V & \partial_2 V & V^2
\end{pmatrix},
\]

(25)

with two eigenvalues equal to zero, equivalent to four Goldstone bosons $G_5^\pm$, $G_6^\pm$ and two physical charged scalar fields with large masses given by $\lambda_4 (\partial_1^2 + \partial_2^2 + V^2)$, which leads to the constrain $\lambda_4 > 0$.

This analysis shows that, after symmetry breaking, the original twelve degrees of freedom in scalar sector leads to eight Goldstone bosons (four electrically neutral and four electrically charged), four physical scalar fields, two neutral (one of which being the SM Higgs scalar) and two charged. Eight Goldstone bosons should be absorbed by eight gauge fields as we will see in next section.

1. Gauge Sector with two Higgs triplets

The gauge bosons interaction with matter in electroweak sector appears with the covariant derivative for a matter field $\varphi$ as:

\[
D^\mu_\varphi = \partial^\mu - \frac{i}{2} g W^a_\mu \lambda^a L - i g x X_\varphi B^\mu = \partial^\mu - \frac{i}{2} g M^\mu_\varphi,
\]

(26)

where $\lambda^a_L$, $a = 1, \ldots, 8$ are Gell-Mann matrices of $SU(3)_L$ algebra, and $X_\varphi$ is the charge of abelian factor $U(1)_X$ of the multiplet $\varphi$ in which $D^\mu_\varphi$ acts. The matrix $M^\mu_\varphi$ contain the gauge bosons with electric charges $q_i$ defined by the generic charge operator in eq. (3) and eq. (10). For $b = 1/2$ the matrix $M^\mu_\varphi$ will have the form:

\[
M^\mu_\varphi = \begin{pmatrix}
W_{3\mu} + \frac{W_{2\mu}}{\sqrt{2}} + 2t X_\varphi B^\mu & \frac{\sqrt{2} W^+}{\sqrt{2} K^-} & \frac{\sqrt{2} K^+}{\sqrt{2} K^-} \\
-\frac{W_{3\mu}}{\sqrt{2}} + \frac{W_{2\mu}}{\sqrt{2}} + 2t X_\varphi B^\mu & \sqrt{2} K^- & \sqrt{2} K^0 + \frac{2W_{3\mu}}{\sqrt{3}} + 2t X_\varphi B^\mu
\end{pmatrix},
\]

(27)

where $t = g_s/g$ and non physical gauge bosons on non-diagonal entries, $W_\mu^\pm$ and $K_\mu^\pm$, are defined in eq. (8) and (9) with $Q_1 = 1$ respectively, and:

\[
K^0_\mu = \frac{1}{\sqrt{2}} (A_{6\mu} - i A_{7\mu}), \quad (28)
\]

\[
\tilde{K}^0_\mu = \frac{1}{\sqrt{2}} (A_{6\mu} + i A_{7\mu}). \quad (29)
\]

Then for the 331 model we are considering ($b = 1/2$) we have two neutral gauge bosons, $K^0_\mu$ and $\tilde{K}^0_\mu$, and four charged gauge bosons, $W^\pm_\mu$ and $K^\pm_\mu$. The three physical neutral eigenstates will be a linear combination of $W_{3\mu}$, $W_{5\mu}$ and $B_\mu$. After breaking the symmetry with $\langle \Phi_i \rangle$, $i = 1, 2$, and using covariant derivative $D_\mu = \partial_\mu - \frac{i}{2} g M^\mu_\varphi$ for the triplets $\Phi_i$ we obtain the following masses for the charged physical fields:

\[
M^2_{W_\mu} = \frac{1}{2} g^2 \partial_1^2, \quad M^2_{K_\mu} = \frac{1}{2} g^2 (\partial_1^2 + \partial_2^2 + V^2),
\]

(30)

and the following physical eigenstates:

\[
W^\pm_\mu = \frac{1}{\sqrt{\partial_1^2 + V^2}} (-\partial_1 K^\pm_\mu + V W^\pm_\mu), \quad K^\pm_\mu = \frac{1}{\sqrt{\partial_1^2 + V^2}} (V K^\pm_\mu + \partial_1 W^\pm_\mu).
\]

(31)

The neutral sector in approximation $(\partial_1^2)^n \approx 0$ for $n > 2$ leads to the following masses for the neutral physical fields:

\[
M^2_{\Phi_{\text{non}}} = 0, \quad (32)
\]

\[
M^2_{\Phi_{\text{charged}}} = \frac{1}{2} g^2 (V^2 + \partial_1^2),
\]

(33)
We can see from eqs. (30) and (32) that we have one non-massive boson, which we associate with the photon, and four massive neutral fields, where the mass of one of them is proportional to $\vartheta_2$ while the other three have masses proportional to $V$ (new energy scale). Therefore we can associate the field $Z$ with SM $Z_\mu$, and the fields $Z'$, $K'_0$ and $K''_R$, with three new neutral bosons. We also have four massive charged fields, where two of them have masses proportional to $\vartheta_2$. Therefore we can associate the fields $W'^\pm$ to the SM fields $W^\pm$, while the fields $K'^\pm$ are new bosons. The eigenstates $B_\mu$, $W_3\mu$, $W_8\mu$ and $K_{R\mu}$ can be related to the physical eigenstates $A_\mu$, $Z'^0_\mu$, $Z^0_\mu$ and $K'^0_\mu$ by:

$$M^{-1} = \begin{pmatrix}
-\frac{1}{2} T_W^2 C_W + \beta_1 & \frac{1}{\sqrt{3}} T_W + \beta_2 \\
S_W & -\frac{1}{2} \frac{\vartheta_1}{\vartheta_2} C_W + \beta_3 \\
\frac{1}{\sqrt{3} S_W} & -\frac{1}{\sqrt{3}} T_W S_W + \beta_5 \\
0 & 1 - \beta_7
\end{pmatrix},$$

where, again, $t = g_x/g$ and:

$$S_W = \frac{\sqrt{3} g_x}{\sqrt{2} g^2 + 4 g_x^2}, \quad C_W = \sqrt{1 - S_W^2}, \quad T_W = \frac{S_W}{C_W},$$

$$\beta_1 = -\frac{\vartheta_1}{4 V^2} T_W^2 C_W^{-3}, \quad \beta_2 = -\frac{3 \vartheta_2}{4 V^2} T_W^3 C_W^{-2},$$

$$\beta_3 = -\frac{\vartheta_1^2}{2 V^2} C_W^{-1}, \quad \beta_4 = -\sqrt{3} \frac{(2 C_W^2 \vartheta_1^2 + \vartheta_2^2)}{4 V^2} T_W C_W^{-2},$$

$$\beta_5 = \frac{6 C_W^4 \vartheta_1^4 + 3 \vartheta_2^4}{4 \sqrt{3} V^2 C_W^5} T_W, \quad \beta_6 = \frac{(6 C_W^4 \vartheta_1^4 + S_W^2 \vartheta_2^4)}{4 V^2 C_W^5} T_W,$$

$$\beta_7 = \frac{2 \vartheta_2^2}{V^2}.$$

We note that all $\beta_i$ are of order $O\left(\left(\frac{\vartheta_1}{\vartheta_2}\right)^2\right)$. So, assuming $\vartheta_1 \sim O\left(10^{-1}\right)$ TeV, for a new energy scale of order $V \sim 10$ TeV all the $\beta_i$s are negligible.

2. Charged and Neutral Currents

The interaction between gauge bosons and fermions in flavor basis is given by the following Lagrangian density:

$$\mathcal{L}_f = \bar{R} i \gamma^\mu (\partial_\mu + i g_x B_\mu X_R) R + \bar{L} i \gamma^\mu (\partial_\mu + i g_x B_\mu X_L + \frac{i g}{2} \lambda_3 W'^0_\mu) L,$$

where $R$ represents any right-handed singlet and $L$ any left-handed triplet. We can write $\mathcal{L}_f = \mathcal{L}_{lep} + \mathcal{L}_{Q_3} + \mathcal{L}_{Q_9}$ and in lepton sector we obtain:

$$\mathcal{L}_{lep} = \mathcal{L}_{lep}^{kin} + \mathcal{L}_{lep}^{CC} + \mathcal{L}_{lep}^{NC},$$

where $\mathcal{L}_{lep}^{kin}$ is the kinetic term, $\mathcal{L}_{lep}^{CC}$ is the charged-current term, and $\mathcal{L}_{lep}^{NC}$ is the neutral-current term.
where

\[ \mathcal{L}^{\text{kin}}_{\text{lep}} = \tilde{R}i \gamma^\mu \partial_\mu R + \tilde{L}i \gamma^\mu \partial_\mu L, \quad (38) \]

\[ \mathcal{L}^{\text{CC}}_{\text{lep}} = -\frac{g}{\sqrt{2}} \tilde{\ell}_L \gamma^\mu \nu_L W^+_\mu - \frac{g}{\sqrt{2}} \tilde{\ell}_L \gamma^\mu N^0_{L\ell} K^+_\mu + \text{h.c.}, \quad (39) \]

\[ \mathcal{L}^{\text{NC}}_{\text{lep}} = \frac{g_s}{3} \left[ \tilde{\ell}_L \gamma^\mu \ell_L + \tilde{\nu}_L \gamma^\mu \nu_L + N^0_{L\ell} \gamma^\mu N^0_{L\ell} \right] B_\mu + g_x \tilde{\ell}_R \gamma^\mu \ell_R B_\mu - \frac{g}{2\sqrt{3}} \left[ \tilde{\ell}_L \gamma^\mu \ell_L + \tilde{\nu}_L \gamma^\mu \nu_L - 2N^0_{L\ell} \gamma^\mu N^0_{L\ell} \right] W_{8\mu} - \frac{g}{2\sqrt{3}} \tilde{\nu}_L \gamma^\mu N^0_{L\ell} K^0_{\mu} - \frac{g}{2} \left[ \tilde{\ell}_L \gamma^\mu \ell_L - \tilde{\nu}_L \gamma^\mu \nu_L \right] W_{3\mu} - \frac{g}{2\sqrt{3}} N^0_{L\ell} \gamma^\mu \nu_L K^0_{\mu}. \quad (40) \]

In quark sector we have that for the first family triplet \( X = 1/3 \), and for the singlets \( d, u \), and \( U_1 \) we have \( X = -1/3, 2/3 \) and \( 2/3 \), respectively. Then we have

\[ \mathcal{L}^{\text{kin}}_{Q_1} = \tilde{Q}_1 R i \gamma^\mu \partial_\mu Q_1 R + \tilde{Q}_1 L i \gamma^\mu \partial_\mu Q_1 L, \quad (41) \]

\[ \mathcal{L}^{\text{CC}}_{Q_1} = -\frac{g}{\sqrt{2}} \tilde{d}_L \gamma^\mu u_L W^+_\mu - \frac{g}{\sqrt{2}} \tilde{d}_L \gamma^\mu U_1 L K^+_\mu + \text{h.c.}, \quad (42) \]

\[ \mathcal{L}^{\text{NC}}_{Q_1} = \frac{g_s}{3} \left[ \tilde{d}_L \gamma^\mu d_R - 2\tilde{u}_R \gamma^\mu u_R - 2\tilde{u}_R \gamma^\mu U_1 R \right] B_\mu + \frac{g}{2\sqrt{3}} \tilde{d}_L \gamma^\mu u_L W_{3\mu} - \frac{g}{2\sqrt{3}} \tilde{u}_R \gamma^\mu U_1 L K^0_{\mu} \quad (43) \]

For second and third families we know that \( X = 0 \) for the triplets and \( X = 2/3, -1/3 \) and \(-1/3\) for the singlets \( u_{2,3}, d_{2,3}, D_{2,3} \), respectively, where \( u_2 = c, u_3 = t, d_2 = s, d_3 = b \). Then we obtain for \( a = 2, 3 \):

\[ \mathcal{L}^{\text{kin}}_{Q_a} = \tilde{Q}_a R i \gamma^\mu \partial_\mu Q_a R + \tilde{Q}_a L i \gamma^\mu \partial_\mu Q_a L, \quad (44) \]

\[ \mathcal{L}^{\text{CC}}_{Q_a} = -\frac{g}{\sqrt{2}} \tilde{u}_a R \gamma^\mu u_a L W^+_\mu - \frac{g}{\sqrt{2}} \tilde{u}_a R \gamma^\mu D_a L K^+_\mu + \text{h.c.}, \quad (45) \]

\[ \mathcal{L}^{\text{NC}}_{Q_a} = \frac{g_s}{3} \left[ -2\tilde{u}_a R \gamma^\mu u_a R + \tilde{d}_a R \gamma^\mu d_a R + \tilde{d}_a R \gamma^\mu D_a R \right] B_\mu - \frac{g}{2\sqrt{3}} \left[ \tilde{u}_a L \gamma^\mu u_a L + \tilde{d}_a L \gamma^\mu d_a L - 4D_a L \gamma^\mu D_a L \right] W_{8\mu} - \frac{g}{2\sqrt{3}} \tilde{d}_a L \gamma^\mu D_a L K^0_{\mu} - \frac{g}{2} \left[ \tilde{u}_a L \gamma^\mu u_a L - \tilde{d}_a L \gamma^\mu d_a L \right] W_{3\mu} - \frac{g}{2\sqrt{3}} \tilde{d}_a L \gamma^\mu D_a L K^0_{\mu}. \quad (46) \]

### III. Neutrinos Interactions with Matter in 331 Model

It is well known that neutrino oscillation phenomenon in a material medium, as the sun, earth or in a supernova, can be quite different from the oscillation that occurs in vacuum, since the interactions in the medium modify the dispersion relations of the particles traveling through it \[ 19 \]. From the macroscopic point of view, the modifications of neutrino dispersion relations can be represented in terms of a refractive index or an effective potential. And according to \[ 43, 44 \], the effective potential can be calculated from the amplitudes of coherent elastic scattering in relativistic limit.

In the present 331 model, the coherent scattering will be induced by neutral currents, NC, mediated by bosons \( Z_\mu^0, Z'_\mu, K_{R\mu}^0 \), and by charged currents, CC, mediated by bosons \( W_\mu^\pm \) and \( K_\mu^\pm \). Following \[ 44 \], we calculate in next sections the neutrino effective potentials in coherent scattering.

#### A. Charged Currents

The first term of eq. \[ 39 \] shows that the interaction of charged leptons with neutrinos occurs only through the gauge bosons \( W_\mu^\pm \), then by eq. \[ 41 \] we obtain that the interaction through charged bosons is given by:

\[ \frac{g}{\sqrt{2}} \tilde{\ell}_L \gamma^\mu \nu_L W^+_\mu = \frac{Vg}{\sqrt{2} \sqrt{1 + \frac{1}{2} \Sigma^2}} \tilde{\ell}_L \gamma^\mu \nu_L W^+_\mu - \frac{g\theta_1}{\sqrt{2} \sqrt{1 + \frac{1}{2} \Sigma^2}} \tilde{\ell}_L \gamma^\mu \nu_L K^+_{\mu}. \quad (47) \]
The amplitude for the neutrino elastic scattering with charged leptons in tree level through CC is given by  

$$
\mathcal{L}_{\text{int}}^{CC} = - \left( \frac{g \ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} - p_{1} \right)^{2}}{2 \left( \sqrt{\omega_{1}^{2} + V^{2}} \right)^{2}} \right) \frac{\bar{\nu}_{L} \gamma_{\mu} \nu_{L} \left( p_{2} \right)}{\left( p_{2} - p_{1} \right)^{2} - M_{W}^{2}} \nu_{L} \gamma_{\omega} \nu_{L} \left( p_{4} \right)
$$

For low energies $M_{W}^{2}, M_{K}^{2}, >> \left( p_{2} - p_{1} \right)^{2}$, the effective Lagrangian is given by:

$$
\mathcal{L}_{\text{eff}}^{CC} \approx - \left( \frac{g^{2}}{2 \left( \sqrt{\omega_{1}^{2} + V^{2}} \right)^{2}} \right) \frac{\ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} \right)}{\left( p_{2} - p_{1} \right)^{2} - M_{W}^{2}} \nu_{L} \gamma_{\omega} \nu_{L} \left( p_{4} \right),
$$

where we used Fierz transformation\cite{[55]} to go from eq. (48) to eq. (49). Replacing eq. (30) in eq. (49) we obtain:

$$
- \mathcal{L}_{\text{eff}}^{CC} \approx \left[ \frac{1}{2 \sqrt{2}} - \frac{\ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} \right)}{2 \sqrt{2} V^{2}} \right] + O \left( \frac{1}{V^{4}} \right) \nu_{e \ell} \gamma_{\omega} \nu_{e \ell},
$$

The modifications on electronic neutrino dispersion relations can be represented by the following effective potential:

$$
V_{\text{CC}}^{CC} \approx \frac{1}{2 \sqrt{2}} - \frac{\ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} \right)}{2 \sqrt{2} V^{2}} + O \left( \frac{1}{V^{4}} \right).
$$

Disregarding the term $(\ell_{i} \nu \ell_{i} \nu)$, since we are assuming $V \gg \theta_{i}$, and remembering that in subsection II C we associated boson $W'$ with SM boson $W$, we can easily associate:

$$
\sqrt{2}G_{F} \approx \frac{1}{2 \sqrt{2}} - \frac{\ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} \right)}{2 \sqrt{2} V^{2}}.
$$

We note that eq. (53) gives limits for the VEV of one of Higgs triplets. Under assumption $\theta_{1} \sim \theta_{2} \ll V$, we can write $G_{F} \approx \frac{1}{2 \sqrt{2} V^{2}} \left( 1 - \frac{\ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} \right)}{2 \sqrt{2} V^{2}} \right)$, from which we can see that the maximum value of $\theta_{2}^{2}$ is achieved when we consider $\ell_{i}^{\nu} \nu_{i \ell} \left( p_{2} \right) = 0$, in which replacing $G_{F} = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$ leads to

$$
\theta_{2} \lesssim 174.105 \text{ GeV} 
$$

**B. Neutral Current**

The lagrangian for neutrino elastic-scattering with fermions $f = e, u, d$ through NC is given by:

$$
- \mathcal{L}_{\text{int}}^{NC} = \bar{f}(p_{1}) \gamma^{\mu} \left( g_{\nu \ell L} + g_{\nu \ell R} \right) f(p_{2}) \frac{-ig_{\mu \lambda} \nu_{L} \left( p_{3} \right) \gamma_{\lambda} \nu_{L} \left( p_{4} \right)}{\left( p_{2} - p_{1} \right)^{2} - M_{W}^{2}} \nu_{L} \gamma_{\nu} \nu_{L} \left( p_{4} \right)
$$

$$
- \bar{f}(p_{1}) \gamma^{\mu} \left( g_{\nu \ell L} + g_{\nu \ell R} \right) f(p_{2}) \frac{-ig_{\mu \lambda} \nu_{L} \left( p_{3} \right) \gamma_{\lambda} \nu_{L} \left( p_{4} \right)}{\left( p_{2} - p_{1} \right)^{2} - M_{K}^{2}} \nu_{L} \gamma_{\nu} \nu_{L} \left( p_{4} \right).
$$

\[\text{Note from eq. 47 that only left-handed leptons interact with neutrinos, as in SM.}\]
For low energies, we have that $M^2_L, M^2_L, M^2_L \gg (p_2 - p_1)^2$ with $p_3 = p_4 = p$ and eq. (55) can be written as:

$$-\mathcal{L}^{NC}_{eff} \approx \frac{G_{\nu e}}{M^2_L} \left( \langle \bar{f}(p_1)\gamma^\mu \left( g^a_{\mu \nu} + g^a_{\mu \nu} \right) f(p_2) \rangle \bar{\nu}_L \gamma_\mu \nu_L \right. $$

$$+ \frac{G_{\nu e}}{M^2_L} \left. \left( \bar{f}(p_1)\gamma^\mu \left( g^a_{\mu \nu} + g^a_{\mu \nu} \right) f(p_2) \right) \bar{\nu}_L \gamma_\mu \nu_L \right.$$ 

$$+ \frac{G_{\nu e}}{M^2_L} \left. \left( \bar{f}(p_1)\gamma^\mu \left( g^a_{\mu \nu} + g^a_{\mu \nu} \right) f(p_2) \right) \bar{\nu}_L \gamma_\mu \nu_L \right).$$  \hspace{1cm} (56)

Following the same procedure of section 11A we obtain:

$$-\mathcal{L}^{NC}_{eff} \approx \sum_{\mu, \nu, \rho} \left( g^a_{\mu \nu} \frac{G_{\nu e}}{M^2_L} + g^a_{\mu \nu} \frac{G_{\nu e}}{M^2_L} + g^a_{\mu \nu} \frac{G_{\nu e}}{M^2_L} \right) \frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L \right).$$  \hspace{1cm} (57)

1. Leptons sector

From eqs. 60 and 33, we obtain that for the known neutral leptons:

$$\frac{g}{3} \bar{\nu}_L \gamma^\mu \nu_L B_{\mu} = \bar{\nu}_L \gamma^\mu \nu_L \left[ -\frac{g}{3} S_W A_\mu + \left( \frac{1}{2} T^a_{W} C_W + \frac{g}{3} \beta_1 \right) Z^a_\mu \right]$$

$$\frac{g}{3} \bar{\nu}_L \gamma^\mu \nu_L W^a_\mu = \bar{\nu}_L \gamma^\mu \nu_L \left[ g \frac{1}{2} S_W A_\mu - \frac{g}{2} \frac{1}{2} Z^a_\mu + \left( \frac{g}{2} C_W + \beta_2 \right) Z^a_\mu + \frac{g}{2} \right] \right],$$  \hspace{1cm} (58)

$$\frac{-g}{2 \sqrt{3}} \bar{\nu}_L \gamma^\mu \nu_L W^a_\mu = \bar{\nu}_L \gamma^\mu \nu_L \left[ g \frac{1}{2} S_W A_\mu - \frac{g}{2} \frac{1}{2} Z^a_\mu + \left( \frac{g}{2} S_W - \frac{g}{2} C_W - \beta_3 \right) Z^a_\mu \right]$$

$$\frac{-g}{2 \sqrt{3}} \bar{\nu}_L \gamma^\mu \nu_L K^0_{R^\mu} = \bar{\nu}_L \gamma^\mu \nu_L \left( \frac{g}{2} - \frac{g}{2} C^{-1} \right) T_W + \eta_2 \equiv G_{\nu K}, \hspace{1cm} (60)

By eqs. 68, 59 and 69 we obtain that vertex interactions with neutrinos can be written as:

$$\bar{\nu}_L \gamma^\mu \nu_L A_\mu \propto 0,$$  \hspace{1cm} (61)

$$\bar{\nu}_L \gamma^\mu \nu_L Z^0_\mu \propto -\frac{g_1}{V} = G_{\nu Z}, \hspace{1cm} (62)

$$\bar{\nu}_L \gamma^\mu \nu_L K^0_{R^\mu} \propto \left( 3g_1 - \frac{g_1}{2} \frac{1}{6} T_W + \eta_2 \equiv G_{\nu K}, \hspace{1cm} (64)

where

$$\eta_1 = \frac{-4 g C^2_W \eta_1 + g \left( 1 - 2 S_W^2 \right) \eta_2}{8 V^2 C^2_W},$$

$$\eta_2 = \frac{g t \left( 1 - 4 C^2_W \right) \eta_2 - \left( -g t^3 - \frac{8 g t^3 C^2_W + 8 g t^3 C^4_W + 6 g t S_W^4}{24 \sqrt{3} V^2 C_W S_W} \right) \eta_2}{2 V^2 C_W S_W}. $$

We note from eq. 61 that neutrinos does not interact electrically, as expected. For charged leptons, from eqs. 41 and 33 we obtain:

$$\frac{g}{3} \ell_L \gamma^\mu \ell_L B_{\mu} = \ell_L \gamma^\mu \ell_L \left[ -\frac{g}{3} S_W A_\mu + \left( \frac{1}{2} T^a_{W} C_W + \frac{g}{3} \beta_1 \right) Z^a_\mu \right]$$

$$\frac{g}{3} \ell_L \gamma^\mu \ell_L W^a_\mu = \ell_L \gamma^\mu \ell_L \left[ g \frac{1}{2} S_W A_\mu - \frac{g}{2} \frac{1}{2} Z^a_\mu + \left( \frac{g}{2} C_W + \beta_2 \right) Z^a_\mu + \frac{g}{2} \right] \right],$$  \hspace{1cm} (65)

$$\frac{-g}{2} \ell_L \gamma^\mu \ell_L W^a_\mu = \ell_L \gamma^\mu \ell_L \left[ g \frac{1}{2} S_W A_\mu + \frac{g}{2} \frac{1}{2} Z^a_\mu - g \left( C_W + \beta_3 \right) Z^a_\mu - \frac{g}{2} \frac{1}{2} \right] \right],$$  \hspace{1cm} (66)
\[ -\frac{g}{2\sqrt{3}} \ell_L^\gamma \ell_L W_\mu^\mu = \ell_L^\gamma \ell_L \left[ -\frac{g}{6} S_W A_\mu - \frac{g \beta_1}{2 \sqrt{3}} Z_\mu^0 + \left( \frac{g S_W^2}{6 C_W} - \frac{g \beta_5}{2 \sqrt{3}} \right) Z_\mu^0 \right] + \frac{g}{2\sqrt{3}} \left( \frac{1}{t} T_W - \beta_0 \right) K_{R\mu}^0, \]

\[ g_x \ell_R^\gamma \ell_R B_\mu = \ell_R^\gamma \ell_R \left[ -g S_W A_\mu + (g T_W^0 C_W + g_x \beta_1) Z_\mu^0 - g_x \left( \frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}^0 \right], \]

and therefore:

\[ \ell_L^\gamma \ell_L A_\mu \propto -g S_W, \]

\[ \ell_L^\gamma \ell_L Z_\mu^0 \propto 0 \equiv g_{\ell_L}^e = g_{\ell_L}^\mu, \]

\[ \ell_R^\gamma \ell_R Z_\mu^0 \propto \frac{1}{2} g \left( 1 + T_W^2 \right) C_W + \eta_4 \equiv g_{\ell_R}^e, \]

\[ \ell_L^\gamma \ell_L K_{R\mu}^0 \propto \frac{1}{6 \sqrt{3} t} (3g - 2tg_x) T_W + \eta_4 \equiv g_{\ell_R}^\mu, \]

\[ \ell_R^\gamma \ell_R K_{R\mu}^0 \propto -g_{\ell_R}^e T_W + \eta_6 \equiv g_{\ell_R}^\mu, \]

where

\[ \eta_3 = \frac{\left( -1 + 2 C_W^2 \right) g_x \theta_2^2}{8 t V^2 C_W^5}, \]

\[ \eta_4 = \frac{\left( g t^3 \left( 1 + 2 C_W^2 \right)^2 - 12 g t^3 S_W^2 C_W^2 - 6 g_x S_W^4 \right)}{24 \sqrt{3} t V^2 C_W^5 S_W}, \]

\[ \eta_5 = -\frac{g_x \theta_2^2}{4 t V^2 C_W^5} T_W^3, \]

\[ \eta_6 = -\frac{\sqrt{3} g_x \theta_2^2}{2 t V^2 C_W^5} T_W^3, \]

and, again, \( t = g_x / g \). We note that by eq. \((69)\) we can make the association \( g S_W = |e| \). Then for \( f = e \), eqs. \((62)-(64)\) and \((70)-(74)\) lead to:

\[ \mathcal{L}_{\text{effNC}} \approx -\sum_{P=L,R} \frac{1}{2} \left( g_{\ell_L}^e G_{\nu_L}^e M_2^2 + g_{\ell_L}^\mu G_{\nu_L}^\mu M_2^2 + g_{\ell_R}^e G_{\nu_L}^e M_2^2 \right) n_e \bar{\nu}_{L\gamma} \gamma_0 \nu_{L\gamma}, \]

\[ \approx -\left\{ \left[ \frac{T_W^4}{144 t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8 V^2} \left( 1 - T_W^2 \right) \right] + \frac{1}{2} \left( \frac{1}{2 \theta_2^2} - \frac{\theta_2^2}{2 V^2 \theta_2^2} \right) \left( 1 - 2 C_W^2 \right) \right\}_{L} \]

\[ + \left[ \frac{T_W^4}{24 g_x^2 V^2} (2tg_x - 3g) - \frac{T_W^2}{4 V^2} + \left( \frac{1}{2 \theta_2^2} - \frac{\theta_2^2}{2 V^2 \theta_2^2} \right) S_W^2 \right]_{R} \right\} n_e \bar{\nu}_{L\gamma} \gamma_0 \nu_{L\gamma}. \]

Since intermediate neutral bosons in eq. \((55)\) does not distinguish between different lepton flavors, the interaction through NC with electron is described by the following effective potential.

\[ V_{NC}^e = V_{NC}^\mu = V_{NC}^\nu = V_{NC}^\tau, \]

\[ V_{NC}^e = V_{NC}^{LL} + V_{NC}^{LR}, \]

where

\[ V_{NC}^{LL} = \left[ \frac{T_W^4}{144 t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8 V^2} \left( 1 - T_W^2 \right) \right] \]
We note that on limit $V \to \infty$ we recover SM. The NSI are a sub-leading interaction, as expected. By eq. (81) and eq. (80) we obtain: $\varepsilon_{ee} \approx -2S_{W}^{2}\varepsilon_{\ell \ell} - \frac{\vartheta_{2}^{2}}{8V^{2}}T_{W}^{4}$.

### 2. Quarks sector

For the quarks of first family, Lagrangian density in eq. (13) describes the interactions with gauge bosons $W_{3\mu}$, $W_{8\mu}$ and $B_{\mu}$, then by eqs. (81) and (80) we obtain the following interactions for quarks up:

$$
\begin{align*}
-\frac{g}{3}u_{L}^{\gamma \mu}u_{L}B_{\mu} &= \frac{g}{3}u_{L}^{\gamma \mu}u_{L}\left[\frac{g}{3}S_{W}A_{\mu} - \frac{g}{3}\left(\frac{1}{t}T_{W}^{2}C_{W} + \beta_{1}\right)Z_{\mu}^{0} + \frac{g}{3}\left(\frac{1}{t}T_{W}^{2}C_{W} + \beta_{2}\right)K_{R_{\mu}}^{0}\right], \\
\frac{g}{2}\sqrt{3}u_{L}^{\gamma \mu}u_{L}W_{3\mu}^{\mu} &= \frac{g}{2}\sqrt{3}u_{L}^{\gamma \mu}u_{L}\left[\frac{g}{2}S_{W}A_{\mu} - \frac{g\theta_{1}}{2V}Z_{\mu}^{0} + \frac{g}{2}C_{W} + \frac{g\beta_{3}}{2}Z_{\mu}^{0} + \frac{g\beta_{4}}{2}K_{R_{\mu}}^{0}\right], \\
-\frac{g}{2}\sqrt{3}u_{L}^{\gamma \mu}u_{L}W_{8\mu}^{\mu} &= \frac{g}{2}\sqrt{3}u_{L}^{\gamma \mu}u_{L}\left[\frac{g}{6}S_{W}A_{\mu} - \frac{g\theta_{1}}{2V}Z_{\mu}^{0} + \frac{g}{2}\left(\frac{1}{t}T_{W}^{2}C_{W} - \beta_{5}\right)Z_{\mu}^{0} + \frac{g}{2}\left(\frac{1}{t}T_{W}^{2}C_{W} - \beta_{6}\right)K_{R_{\mu}}^{0}\right], \\
-\frac{2g}{3}u_{R}^{\gamma \mu}u_{R}B_{\mu} &= \frac{2g}{3}u_{R}^{\gamma \mu}u_{R}\left[\frac{2g}{3}S_{W}A_{\mu} - \frac{2g}{3}\left(\frac{1}{t}T_{W}^{2}C_{W} + \beta_{1}\right)Z_{\mu}^{0} + \frac{2g}{3}\left(\frac{1}{t}T_{W}^{2}C_{W} + \beta_{2}\right)K_{R_{\mu}}^{0}\right].
\end{align*}
$$
The couplings quark-quark-boson for the first family are given by:

\[ \mathcal{L}^u_{\text{NC}} \approx \frac{1}{2} \sum_{P=L,R} \left( g_{uP}^u \frac{G_{\nu z}^u}{M_2} + g_{zP}^u \frac{G_{\nu z}^u}{M_2} + g_{kP}^u \frac{G_{\nu k}^u}{M_2} \right) n_u \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \]

where

\[ \zeta_1 = \frac{g_x \left( -12C_W \partial_1^2 + (1 + 2C_W^2) \partial_2^2 \right)}{24tV^2C_W^5} \]
\[ \zeta_2 = \frac{12g_x^2C_W^2 \left( 1 - 4C_W^2 \right) \partial_1^2 + \left( gt^3 \left( 1 - 2C_W^2 - 8C_W^4 \right) + 6g_x S_W^5 \right) \partial_2^2}{24\sqrt{3}t^2V^2C_W^5} \]
\[ \zeta_3 = \frac{g S_W^2 \partial_2^2}{6C_W^2 V^2} \]
\[ \zeta_4 = \frac{g_x S_W^3 \partial_2^2}{2\sqrt{3}t^2V^2C_W^5} \]

We note that eqs. (91) and (92) reflect the fact that quarks interact electrically through photons with coupling constant \( Q_f \sin \theta_W \), as in SM. The effective lagrangian at low energies for neutrino interaction with quarks up through neutral currents are given by eq. (97) with \( f = u \):

\[ \mathcal{L}^u_{\text{NC}} \approx - \frac{1}{2} \sum_{P=L,R} \left( g_{uP}^u \frac{G_{\nu z}^u}{M_2} + g_{zP}^u \frac{G_{\nu z}^u}{M_2} + g_{kP}^u \frac{G_{\nu k}^u}{M_2} \right) n_u \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \]

where \( n_u \) is the up-quarks average density.

SM predictions, using result of eq. (53), can be written as:

\[ \mathcal{V}^u_{NC} = \mathcal{V}^{uL}_{NC} + \mathcal{V}^{uR}_{NC} = \left( \frac{1}{2\partial_2^2} - \frac{\partial_1^2}{2V^2\partial_2^2} \right) \left( \frac{1}{2} - \frac{4}{3} S_W^2 \right) n_u \]

where

\[ \mathcal{V}^{uL}_{NC} = \left( \frac{1}{2\partial_2^2} - \frac{\partial_1^2}{2V^2\partial_2^2} \right) \left( \frac{1}{2} - \frac{2}{3} S_W^2 \right) n_u \]
\[ \mathcal{V}^{uR}_{NC} = - \frac{2}{3} \left( \frac{1}{2\partial_2^2} - \frac{\partial_1^2}{2V^2\partial_2^2} \right) S_W^2 n_u. \]
By comparison, we obtain:

\[
V_{NC}^{uL} \approx V_{NC}^{uR} + \left[ \frac{1}{24 V^2} (3 + T_W^4) + \frac{T_W^4}{1444V^2} (9 - 4t^4) - \frac{\vartheta_1^2}{4V^2 \vartheta_2^2} \right] n_u,
\]

(104)

\[
V_{NC}^{uR} \approx V_{NC}^{uL} + \left[ \frac{T_W^4}{6V^2} + \frac{T_W^4}{36tg_xV^2} \right] n_u.
\]

(105)

Then we can say that \( \epsilon^{uL}_{\ell \ell} = \epsilon^{uL}_{\ell \ell} + \epsilon^{uR}_{\ell \ell} \) where

\[
\epsilon^{uL}_{\ell \ell} \approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2 C_W} (9 - 8S_W^2),
\]

(106)

\[
\epsilon^{uR}_{\ell \ell} \approx \frac{\vartheta_2^2}{6V^2 C_W}.
\]

(107)

again, we obtain universal NSI, as for the electrons. We note that \( \epsilon^{uL}_{\ell \ell} = -\frac{\vartheta_1^2}{2V^2} + \frac{3\vartheta_2^2}{8V^2 C_W} - 2\epsilon^{uR}_{\ell \ell} \) and in the limit \( V \to \infty \) we recover SM.

For quarks down by eq. (113) and (114) we obtain that:

\[
-\frac{g_2}{3} \overline{d}_L \gamma^\mu d_L B_\mu = \overline{d}_L \gamma^\mu d_L \left[ \frac{g S_W}{3} A_\mu - \frac{g_x}{3} \left( \frac{1}{t} T_W C_W + \beta_1 \right) Z_\mu^0 \right]
+ \frac{g_x}{3} \left( \frac{1}{\sqrt{3}} T_W - \beta_2 \right) K_{R\mu}^0,
\]

(108)

\[
-\frac{g}{2\sqrt{3}} \overline{d}_L \gamma^\mu d_L W_3^\mu = \overline{d}_L \gamma^\mu d_L \left[ -\frac{g S_W}{2} A_\mu + \frac{g \vartheta_1}{2V} Z_\mu^0 - \frac{g (C_W + \beta_3)}{2} Z_\mu^0 - \frac{g \beta_4}{2} K_{R\mu}^0 \right],
\]

(109)

\[
-\frac{g}{2\sqrt{3}} \overline{d}_L \gamma^\mu d_L W_8^\mu = \overline{d}_L \gamma^\mu d_L \left[ -\frac{g S_W}{6} A_\mu + \frac{g \vartheta_1}{2V} \left( \frac{1}{t} T_W S_W - \beta_5 \right) Z_\mu^0 \right]
+ \frac{g \vartheta_1}{2V} Z_\mu^0 + \frac{g}{2\sqrt{3}} \left( \frac{1}{t} T_W - \beta_6 \right) K_{R\mu}^0,
\]

(110)

\[
\frac{g_2}{3} \overline{d}_R \gamma^\mu d_R B_\mu = \overline{d}_L \gamma^\mu d_L \left[ \frac{g S_W}{3} A_\mu + \frac{g_x}{3} \left( \frac{1}{t} T_W C_W + \beta_1 \right) Z_\mu^0 \right]
+ \frac{g_x}{3} \left( \frac{1}{\sqrt{3}} T_W + \beta_2 \right) K_{R\mu}^0.
\]

(111)

Then the couplings quark-quark-boson for the first family are given by:

\[
\overline{d}_L \gamma^\mu d_L A_\mu \propto -\frac{1}{3} g S_W,
\]

(112)

\[
\overline{d}_R \gamma^\mu d_R A_\mu \propto -\frac{1}{3} g S_W,
\]

(113)

\[
\overline{d}_L \gamma^\mu d_L Z_\mu^0 \propto 0 \equiv g_{zz}^d,
\]

(114)

\[
\overline{d}_R \gamma^\mu d_R Z_\mu^0 \propto 0 \equiv g_{zz}^d,
\]

(115)

\[
\overline{d}_L \gamma^\mu d_L Z_\mu^0 \propto -\frac{1}{6} g (3 + T_W^2) C_W + \zeta_5 \equiv g_{zz}^d,
\]

(116)

\[
\overline{d}_R \gamma^\mu d_R Z_\mu^0 \propto \frac{g T_W^2}{3} C_W + \zeta_7 \equiv g_{zz}^d,
\]

(117)

\[
\overline{d}_L \gamma^\mu d_L K_{R\mu}^0 \propto \frac{1}{6\sqrt{3}t} (3g + 2tg_x) T_W + \zeta_6 \equiv g_{zz}^d,
\]

(118)

\[
\overline{u}_R \gamma^\mu u_R K_{R\mu}^0 \propto -\frac{1}{3\sqrt{3}} g_x T_W + \zeta_8 \equiv g_{zz}^d,
\]

(119)

where

\[
\zeta_5 = \frac{g \vartheta_2^2}{24V^2 C_W} (3 - 2S_W^2),
\]

(120)
\[ \zeta_6 = \frac{-1 + 3C_W^2 + 6C_W^4 - 8C_W^6}{24\sqrt{3}V^2C_W^5S_W^3}, \]
\[ \zeta_7 = -\frac{gS_W^2\vartheta_2^2}{12V^2C_W^2}, \]
\[ \zeta_8 = -\frac{g_sS_W^2\vartheta_2^2}{4\sqrt{3}V^2C_W^4}. \]

(120)

then by eq. (57) for \( f = d \) we obtain the following effective lagrangian for NC:

\[ \mathcal{L}_{\text{quark, } d}^{\text{NC}} \approx -\left( g_d^d \frac{G_{\nu\nu}^d}{M_z^2} + g_d^e \frac{G_{\nu\nu}^e}{M_z^2} + g_d^e \frac{G_{\nu\nu}^e}{M_w^2} \right) n_d \bar{\nu}_L \gamma_5 \nu_L. \]

and the effective potential felt by neutrinos when crossing a medium composed by a density \( n_d \) of down quarks is \( V_{NC} = V_{NC}^{dL} + V_{NC}^{dR} \) where

\[ V_{NC}^{dL} \approx \left[ \frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9 - 4t^4)}{144t^4V^2} T_W^4 \right] n_d, \]
\[ V_{NC}^{dR} \approx \left[ -\frac{S_W^2}{24V^2C_W^4} + \frac{1}{3} \left( \frac{1}{2} - \frac{3}{2} S_W^2 \right) S_W^2 \right] n_d. \]

(121)

(122)

Then we can easily see that in SM the NC effective potential for neutrinos in a d-quark medium, using result of eq. (53) will be given by:

\[ V_{NC}^d = V_{NC}^{dL} + V_{NC}^{dR} \approx -\left( \frac{1}{2} - \frac{3}{2} S_W^2 \right) n_d, \]
\[ V_{NC}^{dL} = \left( \frac{1}{2} - \frac{3}{2} S_W^2 \right) n_d, \]
\[ V_{NC}^{dR} = \left( \frac{1}{3} \left( \frac{1}{2} - \frac{3}{2} S_W^2 \right) \right) S_W^2 n_d. \]

(123)

(124)

(125)

(126)

Then from eq. (120)- (126), we obtain:

\[ V_{CN}^{dL} \approx V_{NC}^{dL} + \left[ \frac{(3S_W^2 - 2S_W^4)}{24V^2C_W^4} + \frac{(9 - 4t^4)}{144t^4V^2} T_W^4 \right] n_d, \]
\[ V_{CN}^{dR} \approx V_{NC}^{dR} - \frac{S_W^2}{24V^2C_W^4} n_d, \]

(127)

(128)

and neglecting terms of order \( (\lambda^d)^n \), for \( n > 2 \) we obtain that \( \epsilon_{\ell\ell}^d = \epsilon_{\ell\ell}^{dL} + \epsilon_{\ell\ell}^{dR} \)

\[ \epsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2C_W^2} \left( 3 - 2S_W^2 \right), \]
\[ \epsilon_{\ell\ell}^{dR} \approx \frac{S_W^2\vartheta_2^2}{12V^2C_W^4}. \]

(129)

(130)

Then we obtain \( \epsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{8V^2C_W^2} + \epsilon_{\ell\ell}^{dR} \). Note that again in limit \( V \to \infty \) we recover the SM.
and the constrains in $\epsilon$ on that results, we obtain the following inferior limits for the new Gauge bosons masses:

$$V > \vartheta \text{GeV}$$

we obtain $142\text{GeV}$ qualitatively address the mass hierarchy problem in standard model. Finally we obtained limits for the triplets VEV's will assume $\sin^2 \theta$ in the model, leading to new constraints on NSI. beyond SM. We restrained our work to a simple scenario, but flavor-changing interactions can be naturally introduced in the model, leading to new constraints on NSI.

We presented in this work a procedure to show that models with extended Gauge symmetries $SU(3)_C \times SU(3)_L \times U(1)_X$ can lead to neutrino non-standard interactions, respecting the Standard Model Gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, without spoiling the available experimental data and reproducing the known phenomenology at low energies. We also have shown that with an assumption about a mass hierarchy for the Higgs triplets VEV's we could qualitatively address the mass hierarchy problem in standard model. Finally we obtained limits for the triplets VEV's based on limits for NSI in cross-section experiments.

We believe that the class of model presented here is an interesting theoretical possibility to look for new physics beyond SM. We restrained our work to a simple scenario, but flavor-changing interactions can be naturally introduced in the model, leading to new constraints on NSI.

### Table I: Values for NSI in 331 model and experimental limits

| Model331                  | Exp. 90% C.L. [46] |
|---------------------------|---------------------|
| $\epsilon_{\ell\ell}^L \approx (1-2\sin^2 \theta_W) \frac{\theta_l^2}{8V^2c_W}$ | $0.114 \left( \frac{\theta_l^2}{V^2} \right)$ |
| $-0.07 < \epsilon_{\ell\ell}^L < 0.11$ |
| $-0.025 < \epsilon_{\mu\mu}^L < 0.03$ |
| $-0.6 < \epsilon_{\tau\tau}^L < 0.4$ |

| $\epsilon_{\ell\ell}^R \approx -2S_W^2 \epsilon_{\ell\ell}^L - \frac{\theta_l^2}{V^2} T_W^4$ | $0.143 \left( \frac{\theta_l^2}{V^2} \right)$ |
| $-1 < \epsilon_{\ell\ell}^R < 0.5$ |
| $-0.027 < \epsilon_{\mu\mu}^R < 0.03$ |
| $-0.4 < \epsilon_{\tau\tau}^R < 0.6$ |

| $\epsilon_{\ell\ell}^U \approx - \frac{\theta_l^2}{V^2} + \frac{\theta_l^2}{24V^2c_W} (9 - 8S_W^2)$ | $0.50 \left( \frac{\theta_l^2}{V^2} \right)$ |
| $-1 < \epsilon_{\ell\ell}^U < 0.3$ |
| $\left| \epsilon_{\mu\mu}^U \right| < 0.003$ |
| $\left| \epsilon_{\tau\tau}^U \right| < 1.4$ |

| $\epsilon_{\ell\ell}^U \approx \frac{\theta_l^2}{V^2} \frac{S_W^2}{c_W}$ | $0.065 \left( \frac{\theta_l^2}{V^2} \right)$ |
| $-0.4 < \epsilon_{\ell\ell}^U < 0.7$ |
| $-0.008 < \epsilon_{\mu\mu}^U < 0.003$ |
| $\left| \epsilon_{\tau\tau}^U \right| < 3$ |

| $\epsilon_{\ell\ell}^D \approx - \frac{\theta_l^2}{24V^2c_W} (3 - 2S_W^2)$ | $0.179 \left( \frac{\theta_l^2}{V^2} \right)$ |
| $-0.3 < \epsilon_{\ell\ell}^D < 0.3$ |
| $\left| \epsilon_{\mu\mu}^D \right| < 0.003$ |
| $\left| \epsilon_{\tau\tau}^D \right| < 1.1$ |

| $\epsilon_{\ell\ell}^D \approx - \frac{S_W^2 \theta_l^2}{12V^2c_W}$ | $-0.033 \left( \frac{\theta_l^2}{V^2} \right)$ |
| $-0.6 < \epsilon_{\ell\ell}^D < 0.5$ |
| $-0.008 < \epsilon_{\mu\mu}^D < 0.015$ |
| $\left| \epsilon_{\tau\tau}^D \right| < 6$ |

### IV. RESULTS

In last sections we saw that in 331 model we choses, all NSI parameters are universal and diagonal, and will not affect oscillation experiments. However, measurements of cross-section will be sensitive to such parameters, through modifications on $\theta_3^3$ [28]. We will now compare our results with those obtained in cross-section measurements. We will assume $\sin^2 \theta_W = 0.23149(13)$.

In Table II we can see that constrains in $\epsilon_{\ell\ell}^P$ lead to $V^2 > 5.3\theta_3^2$, while the constrains in $\epsilon_{\ell\ell}^U$ lead to $V^2 > 21.7\theta_3^2$, and the constrains in $\epsilon_{\ell\ell}^D$ ($\left| \epsilon_{\mu\mu}^D \right| < 0.003$) lead to $V^2 > 60\theta_3^2$. If $\theta_2$ has its maximum value of 174.105 GeV then $V \gtrsim 1.3$ TeV. We note also that by $\left| \epsilon_{\mu\mu}^D \right| < 0.003$ we obtain $|\theta_3^2 - \theta_2^2| < 0.006V^2$, then for $V \sim 1.3$TeV and $\theta_2 = 174$ GeV we obtain $142\text{GeV} < \vartheta_1 < 201\text{GeV}$. We therefore can not predict any hierarchy to the VEV’s $\vartheta_1$ and $\vartheta_2$. Based on that results, we obtain the following inferior limits for the new Gauge bosons masses:

$$M_{K_1} = M_{Z'} > 610\text{GeV},$$
$$M_{K'} > 613\text{GeV},$$
$$M_{K_R} > 740\text{GeV}.$$
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