Ultrasoft Quark Damping in Hot QCD

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Abstract

We determine the quark damping rates in the context of next-to-leading order hard-thermal-loop summed perturbation of high-temperature QCD where weak coupling is assumed. The quarks are ultrasoft. Three types of divergent behavior are encountered: infrared, light-cone and at specific points determined by the gluon energies. The infrared divergence persists and is logarithmic whereas the two others are circumvented.

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I. INTRODUCTION

RHIC results seem to indicate that in the temperature range between one to two $T_c \approx 200\text{MeV}$, the critical temperature of deconfinement from conventional hadronic states, there is an intermediary phase in which the deconfined quarks and gluons remain strongly interacting \[1\], a scenario introduced in \[2, 3, 4\]. Since in this temperature regime the quark binding energy is about 4 GeV, one order of magnitude higher than the temperature itself, perturbative treatment based on quasi-free quarks and gluons is not adequate. Rather, a hydrodynamic description of a near-perfect liquid seems to work and the picture emerging is that of non-conventional bound states of quarks and gluons: diquarks $qq$, baryons $qqq$, three-gluon bound states $ggg$ and polymeric chains $\bar{q}g \ldots gq$. A model of a strongly interacting classical chromo-electric plasma can account of much of the phenomenological results \[1\].

But the same quark binding energy decreases as the temperature increases away from $T_c \[1\]$, and here, starting from about $3T_c$, weak coupling between individual constituents may be expected. This is also supported by the behavior of particle susceptibilities: baryons and other bound states contribute in the intermediary temperature range, but only quarks survive in the high-temperature limit \[5\].

Therefore, as the temperature increases, the picture emerging is that of a hadronic phase with very short inter-quark binding lengths changing at $T_c$ into the so-called strongly coupled quark-gluon plasma (sQGP), a near-perfect liquid phase of quark and gluon bound states with longer inter-quasiparticle binding lengths. As the temperature increases further, the sQGP changes into a weakly coupled quark-gluon plasma (wQGP) of quasi quarks and gluons with chromo-neutralizing isotropic Debye clouds \[1\].

If perturbative QCD is to apply in the wQGP phase, the problem becomes: how to organize it? It is known for some time that the standard loop-expansion would break at some order, depending on the quantity of interest under consideration \[6, 7, 8\]. It has also proved inadequate when describing slow-moving particles since it does not reflect an expansion in powers of the coupling \[9, 10, 11, 12, 13\]. Regarding this last difficulty, an important improvement has been the dressing of the lowest-order propagators and vertices with the so-called hard thermal loops (HTL) \[14, 15, 16, 17, 18, 19, 20, 21\]. Since then, work has flourished and many phenomenological aspects of the presumed wQGP as well as
more theoretical aspects of high-temperature QCD have been addressed in the context of lowest and next-to-leading orders of the so-called HTL-perturbation theory. But HTL-dressed perturbative QCD is not itself safe from chronic problems, one important one being the non-screening of static chromo-magnetic fields at lowest order which plagues the theory with infrared divergences in next-to-leading order calculations. The determination of the chromo-magnetic correlation length may not even be accessible perturbatively.

The present work finishes a calculation started in. The context is next-to-leading order HTL-dressed perturbative QCD and weak coupling is assumed. The aim is to complete the determination of the quark damping rates to second order in the ultrasoft external momentum. The coefficients of zeroth order are the ones already found in; they are finite and positive. The coefficients of first order we find are also finite and free from any divergence, but the coefficients of second order are logarithmically divergent in the infrared. This work is meant to be an additional contribution investigating the analytic infrared behavior of QCD at high-temperature. It is certainly more pressing nowadays to have a better theoretical understanding of the sQGP phase of hadronic matter, but ultimately, two issues have to be addressed: (i) Must we look for a quantum field theory description of the different phases of quarks and gluons? (ii) Do we want to reproduce the eventual phase transitions in this context?

This article is organized as follows. After this introduction, section two recalls the essential results of to which the present work is a follow-up. The analytic expression of the damping rates is given in the form of integrals over functions involving products of spectral distributions and their first and second derivatives. Section three takes up from these expressions and perform the integrals. The steps of the calculations are detailed using a generic form. The occurrence of divergences is discussed. Three types occur: infrared, light-cone, and divergences at specific points determined by the gluon energies. The infrared divergences are extracted and will stay, whereas the two other kinds are dealt away with. Section four summarizes the work and finishes the article with concluding remarks.

II. QUARK DAMPING RATES IN HTL-SUMMED PERTURBATION

We consider a theory with $N_c$ colors and $N_f$ flavors. Imaginary-time formalism is used throughout. The euclidean momentum of the quark is $P^\mu = (p_0, \mathbf{p})$ such that $P^2 = p_0^2 + p^2$
with \( p_0 = (2n + 1)\pi T \), a fermionic Matsubara frequency; \( n \) is an integer. Once all intermediary steps are carried out, real-time amplitudes are obtained via the analytic continuation \( p_0 = -i\omega + 0^+ \) where \( \omega \) is the energy of the quark. The temperature \( T \) determines the hard scale, \( gT \) the soft scale where \( g \) is the (weak) coupling constant, and \( g^2T \) the ultrasoft scale. An infrared cutoff \( \eta \sim g^2T \) is introduced.

### A. Dressing the propagators and vertices

The HTL-dressed quark propagator can be written as:

\[
\*\Delta_F (P) = - \left[ \gamma_{+p} \Delta_+ (P) + \gamma_{-p} \Delta_- (P) \right].
\]  

(2.1)

\( \gamma^\mu \) are the euclidean Dirac matrices, \( \gamma_{\pm p} = (\gamma^0 \pm i \gamma \hat{\mathbf{p}}) / 2 \) and \( \Delta_{\pm} = (D_0 \mp D_s)^{-1} \) with:

\[
D_0 (P) = ip_0 - \frac{m_f^2}{p} Q_0 \left( \frac{ip_0}{p} \right); \quad D_s (P) = p + \frac{m_f^2}{p} \left[ 1 - \frac{ip_0}{p} Q_0 \left( \frac{ip_0}{p} \right) \right].
\]  

(2.2)

\( m_f = \sqrt{C_f / 8 \ gT} \) is the lowest-order quark thermal mass with \( C_f = (N_c^2 - 1) / 2N_c \), and \( Q_0 (x) = \frac{1}{2} \ln \frac{x + 1}{x - 1} \). The poles of \( \Delta_{\pm} (-i\omega, \mathbf{p}) \) determine the dispersion laws \( \omega_{\pm} (p) \) to lowest order in \( g \). The + sign is for real quarks and the − sign for the so-called ‘plasminos’ \( \oplus \), thermally excited quasiparticles. For soft quarks and using the notation \( \bar{\mathbf{p}} = \mathbf{p} / m_f \), one has:

\[
\omega_{\pm} (p) = m_f \left[ 1 \pm \frac{1}{3} \bar{\mathbf{p}} + \frac{1}{3} \bar{\mathbf{p}}^2 \pm \frac{16}{135} \bar{\mathbf{p}}^3 + \frac{1}{54} \bar{\mathbf{p}}^4 \pm \frac{32}{2835} \bar{\mathbf{p}}^5 - \frac{139}{12150} \bar{\mathbf{p}}^6 \pm \mathcal{O} (\bar{\mathbf{p}}^7) \right].
\]  

(2.3)

The quark damping rates \( \gamma_{\pm} (p) \) are obtained by including in the dispersion relations the HTL-dressed one-loop-order quark self-energy \( \*\Sigma (P) \). The inverse quark propagator becomes:

\[
\Delta_F^{-1} (P) = \*\Delta_F^{-1} (P) - \*\Sigma (P).
\]  

(2.4)

The decomposition \( \*\Sigma = \gamma^0 \*D_0 + i \gamma \*\hat{\mathbf{p}} \*D_s \) implies:

\[
\Delta_F^{-1} (P) = - \left[ \gamma^0 (D_0 + \*D_0) + i \gamma \*\hat{\mathbf{p}} (D_s + \*D_s) \right].
\]  

(2.5)

With the definitions \( \gamma_{\pm} (p) \equiv - \text{Im} \ \Omega_{\pm} (p) \) with \( \Omega_{\pm} \) the poles of \( \Delta_F (-i\Omega, \mathbf{p}) \), and since \( \*\Sigma \) is \( g \)-times smaller than \( \*\Delta_F^{-1} \), we have to order \( g^2T \):

\[
\gamma_{\pm} (p) = \frac{\text{Im} \ *f_\pm (-i\omega, p)}{\partial_\omega \ *f_\pm (-i\omega, p)} \bigg|_{\omega = \omega_{\pm} (p) + i0^+},
\]  

(2.6)
where $f_\pm = D_0 \mp D_s$, $*f_\pm = *D_0 \mp *D_s$ and $\partial_\omega$ stands for $\partial / \partial \omega$. Expanding the denominator in the above relation in powers of $\tilde{p}$ using the expressions in (2.2), one obtains:

$$
\gamma_\pm (p) = \frac{1}{2} \left[ 1 \pm \frac{2}{3} \tilde{p} - \frac{2}{9} \tilde{p}^2 \pm O (\tilde{p}^3) \right] \text{Im} *f_\pm (-i \omega, p) \Bigg|_{\omega=\omega_\pm (p)+i0^+}.
\tag{2.7}
$$

Therefore, determining $\gamma_\pm (p)$ amounts to calculating $\text{Im} *\Sigma (P)$.

In HTL-summed perturbation [14, 20, 21], the one-loop-order quark self-energy writes:

$$
*\Sigma (P) = *\Sigma_1 (P) + *\Sigma_2 (P).
\tag{2.8}
$$

The first contribution is:

$$
*\Sigma_1 (P) = -g^2 C_f \text{Tr}_{\text{soft}} [ *\Gamma^\mu (P, -Q; -K) *\Delta_F (Q) *\Gamma^\nu (-P, Q; K) *\Delta_{\mu \nu} (K)],
\tag{2.9}
$$

a QED-like loop formed by two quark-gluon vertices connected by one gluon propagator and one quark propagator. The second contribution is:

$$
*\Sigma_2 (P) = -\frac{i}{2} g^2 C_f \text{Tr}_{\text{soft}} [ *\tilde{\Gamma}^{\mu \nu} (P, -P; K, -K) *\Delta_{\mu \nu} (K)],
\tag{2.10}
$$

a purely hard-thermal-loop two-gluon-quark-antiquark vertex with one gluon propagator. In the above two expressions, $K$ is the soft loop momentum, $Q = P - K$ and $\text{Tr} \equiv T \sum_{k_0} \int \frac{d^3 k}{(2\pi)^3}$ with $k_0 = 2 n \pi T$ when bosonic or $k_0 = (2 n + 1) \pi T$ when fermionic. The subscript ‘soft’ means that only soft values of $K$ are allowed in the integrals; hard values have already dressed the propagators and vertices.

To be complete, we give the expressions of the gluon propagator and the vertices involved in (2.9) and (2.10). In the strict Coulomb gauge, $*\Delta_{00} (K) = *\Delta_l (K)$, $*\Delta_{0i} (K) = 0$ and $*\Delta_{ij} (K) = \left( \delta_{ij} - \hat{k}_i \hat{k}_j \right) *\Delta_l (K)$ with $*\Delta_l$ and $*\Delta_i$ given by:

$$
*\Delta_l (K) = \frac{1}{k^2 - \delta \Pi_l (K)}, \quad *\Delta_i (K) = \frac{1}{K^2 - \delta \Pi_i (K)},
\tag{2.11}
$$

where $\delta \Pi_l (K) = 3 m_g^2 Q_1 (\frac{i k_0}{k})$ and $\delta \Pi_i (K) = \frac{3}{5} m_g^2 \left[ Q_3 (\frac{i k_0}{k}) - Q_1 (\frac{i k_0}{k}) - \frac{5}{3} \right]$ are the gluonic hard thermal loops. Here $Q_1 (\frac{i k_0}{k})$ is a Legendre function of the second kind and $m_g = \sqrt{N_c + N_f / 2 g T} / 3$ is the gluon thermal mass. Finally, the HTL-dressed vertices $*\Gamma$ are as follows:

$$
*\Gamma^\mu (P, Q; R) = \gamma^\mu + m_f^2 \int \frac{d \Omega_s}{4 \pi} \frac{S^\mu S}{P S Q S};
\tag{2.12}
$$

$$
*\tilde{\Gamma}^{\mu \nu} (P, -P; K, -K) = -2 m_f^2 \int \frac{d \Omega_s}{4 \pi} \frac{S^\mu S^\nu S}{P S (P + K) S (P - K) S}.
\tag{2.13}
$$

In both (2.12) and (2.13), $S \equiv (i, \hat{s})$ and $\Omega_s$ is the solid angle of $\hat{s}$. 

}{
B. Analytic expressions

Let us recall the main results of [23]. Henceforth, the quark thermal mass \( m_f \) is set to one. Since there remains another soft mass in the problem, \( m_g \), we define the ratio
\[
\frac{m_g(N_c, N_f)}{m_f} = 4 \sqrt{\frac{N_c(N_c+N_f/2)}{N_f-1}}.
\]
It is easy to see that we always have \( m > 1 \). Since the gluon propagator is taken in the strict Coulomb gauge, there are uncoupled longitudinal and transverse contributions to \(^*\Sigma_1\) and \(^*\Sigma_2\). There is a further split in \(^*\Sigma_1\) due to positive and negative helicity quarks. Six contributions in all, each being treated separately.

After a preliminary manipulation of the gamma matrices, we perform the expansion
\[
\frac{1}{p^0} = \frac{1}{ip_0} \left[ 1 - \frac{p^2}{ip_0} + O(p^3) \right]
\]
in order to integrate analytically over the solid angle \( \Omega_s \). This expansion is valid in the region \( p < |ip_0| \), a condition always satisfied before analytic continuation because \( p_0 = (2n + 1) \pi T \) and \( p \sim g^2 T \), and after since for ultrasoft momenta, \( ip_0 = m_f + O(p) \sim gT \), see (2.3). A subsequent expansion in powers of \( p \) of functions of \( q = |p - k| \) is necessary in order to carry out analytically the integration over the internal three-momentum solid angle \( \Omega_k \). In the process of these angular integrations, special care must be taken when rotating the gamma matrices [23].

Next we perform the Matsubara sums. For this, the spectral decompositions of the dressed propagators and \( Q_0(K) \) with \( k_0 \) fermionic are used [21, 27, 28]:
\[
\Delta_\varepsilon(k_0, k) = \int_0^{1/T} d\tau e^{ik_0\tau} \int_{-\infty}^{+\infty} d\omega \rho_\varepsilon(\omega, k) (1 - \tilde{n}(\omega)) e^{-\omega\tau};
\]
\[
\Delta_l(k_0, k) = \int_0^{1/T} d\tau e^{ik_0\tau} \int_{-\infty}^{+\infty} d\omega \rho_l(\omega, k) (1 + n(\omega)) e^{-\omega\tau};
\]
\[
Q_0(ik_0/k) = \int_0^{1/T} d\tau e^{ik_0\tau} \int_{-\infty}^{+\infty} d\omega \rho_0(\omega, k) (1 - \tilde{n}(\omega)) e^{-\omega\tau}.
\]
(2.14)
\( \varepsilon \) stands for + or – and \( l \) for \( l \) or \( t \). The functions \( n(\omega) \) and \( \tilde{n}(\omega) \) are the Bose-Einstein and Fermi-Dirac distributions respectively, and the rho’s are the spectral densities, to be displayed in the next section. Before implementing the spectral decomposition, it is first necessary to rearrange terms in such a way that products of at most two functions necessitating a spectral decomposition occur to ensure the obtainment of only one energy denominator just before the extraction of the imaginary part. The steps of the calculation must also ensure the Matsubara frequency \( ik_0 \) appears only in the numerator of fractions. Hence, using (2.14), the sum over \( k_0 \) can be performed, yielding a delta function that automatically removes one integration over one imaginary time while the other integration produces an
energy denominator. At this stage, every $i p_0$ is to be replaced with $(2n + 1) \pi T$ except in the energy denominator.

Now the analytic continuation to real energies $i p_0 \rightarrow \omega_\pm(p) + i 0^+$ can be taken. The extraction of the imaginary part becomes straightforward using the relation $1/(x + i 0^+) = \text{Pr} \left(1/x\right) - i \pi \delta(x)$ where $\text{Pr}$ stands for the principal part. Further rearrangements are made and, according to the definitions in (2.7), the quark damping rates are given by the following expressions:

$$\gamma_\pm(p) = -\frac{g^2 C_f T}{8\pi} \left[ a_0 \pm a_1 \bar{p} + a_2 \frac{\bar{p}^2}{9} + O(\bar{p}^3) \right]. \quad (2.15)$$

Recall that we have set $m_f = 1$. The coefficients $a_i$ are given by the expressions:

$$a_0 = \int_\eta^\infty dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'} f_0 \delta;$$

$$a_1 = \int_\eta^\infty dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'} [f_1 - f_0 \partial_\omega] \delta;$$

$$a_2 = \int_\eta^\infty dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'} [f_2 - f_1 \partial_\omega - f_0 (3 \partial_\omega - \partial_\omega^2)] \delta, \quad (2.16)$$

with $\delta = \delta(1 - \omega - \omega')$. The three functions $f_i \equiv f_i(\omega, \omega'; k)$ are given by the following rather long expressions [23]:

$$f_0 = \sum_{\varepsilon = \pm} \left[ -k^2 (1 - \varepsilon k + \omega)^2 \rho_\varepsilon \rho'_t + \frac{1}{2} \left( 1 + 2 \varepsilon k + k^2 - \omega^2 \right)^2 \rho_\varepsilon \rho'_t \right] + \frac{1}{k} (k^2 - \omega^2) \rho_0 \rho'_t. \quad (2.17)$$

This expression is the one obtained in [24, 25] for the non-moving quark damping rates. In the above expression and the two subsequent ones, the notation is as follows: $\rho_{\varepsilon,0}$ stands for $\rho_{\varepsilon,0}(\omega, k)$ and $\rho'_{t,t}$ for $\rho_{t,t}(\omega', k)$. Remember also that $\partial_\omega$ stands for the partial derivative $\partial/\partial\omega$. The next function $f_1(\omega, \omega'; k)$ is given by:

$$f_1 = \sum_{\varepsilon = \pm} \left[ 2k^2 \left( -1 + k^2 - 2 \varepsilon k \omega + \omega^2 \right) \rho_\varepsilon \rho'_t + \left[ -\frac{2\varepsilon}{k} - 3 + 2 \varepsilon k + 4k^2 - k^4 \right. \right. $$

$$- (2 + 4 \varepsilon k + 2k^2) \omega + \left( \frac{4 \varepsilon}{k} + 2k + 2 \varepsilon k + 2k^2 \right) \omega^2 + 2 \omega^3 - \left( \frac{2 \varepsilon}{k} + 1 \right) \omega^4 \right] \rho_{t,t}$$

$$+ \varepsilon k^2 (1 - \varepsilon k + \omega)^2 \rho_{t,t} \partial_\omega \rho'_t + \left[ \frac{\varepsilon}{2} + 2k + 3 \varepsilon k^2 + 2k^3 + \frac{\varepsilon}{2} k^4 - (\varepsilon + 2k + \varepsilon k^2) \omega^2 + \frac{\varepsilon}{2} \omega^4 \right] \rho_{t,t} \partial_\omega \rho'_t$$

$$- \frac{2}{k} \left( k^2 - \omega^2 + 2 \frac{\omega^3}{k} \right) \rho_{t,t} \rho'_t - 2k^2 \delta (\omega^2 - k^2) \rho'_t + \frac{\omega}{k^2} (\omega^2 - k^2) \rho_{t,t} \partial_\omega \rho'_t + 2 \omega \rho_{0,0} \partial_\omega \rho'_t, \quad (2.18)$$

where $\partial_\omega$ is a short notation for the partial derivative $\partial/\partial\omega$ and $\varepsilon(\omega)$ is the sign function.
The last function \( f_2 (\omega, \omega'; k) \) to display is the longest of all three and is given by:

\[
\begin{align*}
f_2 &= \sum_{\varepsilon = \pm} \left[ \left( -\frac{9}{2} - k^2 - 6\varepsilon k^3 - \frac{1}{2} k^4 - (6\varepsilon k - 6k^2 + 2\varepsilon k^3) \omega + (9 + k^2) \omega^2 + 6\varepsilon k\omega^3 - \frac{9}{2} \omega^4 \right) \\
\times \rho_\varepsilon \rho'_l + \left( \frac{9}{2k^2} - \frac{14\varepsilon}{k} - \frac{8}{3} + 4\varepsilon k - \frac{19}{2} k^2 - 6\varepsilon k^3 + k^4 + \left( -\frac{3}{k^2} + \frac{25\varepsilon}{k} - 10 + 6\varepsilon k + 9k^2 - 3\varepsilon k^3 \right) \omega \\
+ \left( -\frac{6}{k^2} + \frac{2\varepsilon}{k} + 23 - 6\varepsilon k + k^2 \right) \omega^2 + \left( \frac{6}{k^2} - \frac{22\varepsilon}{k} - 6 + 6\varepsilon k \right) \omega^3 + \left( -\frac{3}{2k^2} + \frac{12\varepsilon}{k} - 5 \right) \omega^4 \\
- \left( \frac{3}{k^2} + \frac{3\varepsilon}{k} \right) \omega^5 + \frac{3}{k^2} \omega^6 \right) \rho_\varepsilon \rho'_l - k \left( 9 - 14\varepsilon k + 5k^2 + (12 - 2\varepsilon k - 6k^2) \omega - (3 - 12\varepsilon k) \omega^2 \\
- 6\omega^3 \right) \rho_\varepsilon \partial_k \rho'_l + \left( \frac{3}{2k} + 4\varepsilon + 7k + 8\varepsilon k^2 + \frac{7}{2} k^3 - \left( \frac{3}{k} + 6\varepsilon + 6k + 6\varepsilon k^2 + 3k^3 \right) \omega \\
- \left( \frac{3}{k} + 4\varepsilon + 5k \right) \omega^2 + \left( \frac{6}{k} + 6\varepsilon + 6k \right) \omega^3 + \frac{3}{2k} \omega^4 - \frac{3}{2} \omega^5 \right) \rho_\varepsilon \partial_k \rho'_l - \frac{3}{2} k^2 \left( 1 - \varepsilon k + \omega^2 \right) \rho_\varepsilon \partial^2_k \rho'_l \\
+ \left( \frac{3}{k} + 3\varepsilon k + \frac{9}{2} k^2 + 3\varepsilon k^3 + \frac{3}{4} k^4 - \frac{3}{2} \left( 1 + 2\varepsilon k + k^2 \right) \omega^2 + \frac{3}{4} \omega^4 \right) \rho_\varepsilon \partial^2_k \rho'_l - \frac{3}{k} \left( k^2 - \omega^2 \right) \rho_0 \rho'_l \\
+ \left( \frac{3}{k} + 2k + \frac{6}{k} \omega - \frac{15}{(k^3 + \frac{2}{k})} + \frac{18}{k^3} \omega^3 \right) \rho_0 \rho'_l + \left( 6 - 12\omega \right) \rho_0 \partial_k \rho'_l \\
+ \left( -3 + 6k\omega + \frac{3}{k^2} \omega^2 - \frac{6}{k^3} \omega^3 \right) \rho_0 \partial_k \rho'_l + 3k \rho_0 \partial^2_k \rho'_l - \frac{3}{2k} \left( k^2 - \omega^2 \right) \rho_0 \partial^2_k \rho'_l \\
+ 12k^2 \varepsilon (\omega - \omega') \rho'_l - 6\omega \varepsilon (\omega - \omega') \delta (\omega^2 - k^2) \rho'_l - 6k^2 \omega \varepsilon (\omega - \omega') \partial_\omega \delta (\omega^2 - k^2) \rho'_l. \right]
\end{align*}
\]

Note that, since only soft values of \( \omega \) and \( \omega' \) are to contribute, we have made use of the two approximations \( \tilde{n}(\omega) \simeq 1/2 \) and \( n(\omega) \simeq T/\omega \).

To complete the calculation of the damping rates, it remains to perform the integrals over the frequencies \( \omega \) and \( \omega' \) and then over the momentum \( k \). These integrations are not straightforward and necessitate numerical work. But before that, one has to hunt down divergences, particularly the infrared ones. Also, the dimensionless parameter \( m(N_c, N_f) \) is implicitly present in the spectral densities \( \rho_{l,t} \) and so, each case \((N_c, N_f)\) has to be treated separately. All this is discussed in the next section.

### III. INTEGRATION AND DIVERGENCES

In this work, we will consider three cases regarding the number of colors and flavors, namely, \((N_c, N_f) = (2, 1), (3, 2) \) and \((3, 3)\). All the terms in the expressions of the coefficients \( a_0, a_1, a_2 \) in (2.15) have the following generic structure:

\[
I_{rs} = \int \int \int \int \int \int g(\omega, k) \rho(\omega, k) \partial_k \rho'(\omega', k) \delta(1 - \omega - \omega').
\]
where \( g(\omega, k) \) is a polynomial in \( \omega \) with coefficients functions of \( k \). The density \( \rho(\omega, k) \) stands for the quark spectral functions given in \((3.6)\) below or for the spectral distributions given in \((3.9)\). The density \( \rho'(\omega', k) \) stands for the gluonic spectral functions \( \rho_{l,t}(\omega', k) \) given in \((3.2)\). The two indices \( r \) and \( s \) designate the \( r \)th and \( s \)th derivatives with respect to \( k \) and \( \omega \) respectively and are such that \( r, s = 0, 1, 2 \) with the condition \( r + s \leq 2 \).

First we give the expressions of the spectral functions. The gluonic spectral densities are as follows:

\[
\rho_{l,t}(\omega, k) = 3_{l,t}(k) [\delta(\omega - \omega_{l,t}(k)) - \delta(\omega + \omega_{l,t}(k))] + \beta_{l,t}(\omega, k) \theta(k - |\omega|). \tag{3.2}
\]

The gluon energies \( \omega_{l,t}(k) \) are the poles of the longitudinal and transverse gluon propagators \( *\Delta_{l,t}(K) \) given in \((2.11)\). The longitudinal and transverse residue functions are given by:

\[
3_l(k) = \frac{\omega_l(k)(k^2 - \omega_l^2(k))}{k^2(3m^2 - \omega_l^2(k) + k^2)}; \quad 3_t(k) = \frac{\omega_t(k)(\omega_t^2(k) - k^2)}{3m^2\omega_t^2(k) - (\omega_t^2(k) - k^2)^2}. \tag{3.3}
\]

The longitudinal cut function is given by:

\[
\beta_l(k, \omega) = \frac{-3m^2\omega}{2k\left[3m^2 + k^2 - 3m^2\omega \ln\left(k + \omega \over k - \omega\right) + (3\pi m^2\omega)^2\right]}, \tag{3.4}
\]

and the transverse one by:

\[
\beta_t(k, \omega) = \frac{3m^2\omega(k^2 - \omega^2)}{2k^3\left[\omega^2 - k^2 - \frac{3}{2}m^2\left(\omega^2 \over k^2 + \omega(k^2 - \omega^2) \ln\left(k + \omega \over k - \omega\right)\right)^2 + (3\pi m^2\omega)^2\right]}. \tag{3.5}
\]

The quark spectral functions are:

\[
\rho_\pm(\omega, k) = 3_\pm(k)\delta(\omega - \omega_\pm(k)) + 3_\mp(k)\delta(\omega + \omega_\pm(k)) + \beta_\pm(\omega, k) \theta(k - |\omega|), \tag{3.6}
\]

where the quark residue functions are:

\[
3_\pm(k) = -\frac{1}{2}(\omega_\pm^2(k) - k^2), \tag{3.7}
\]

and the quark cut functions given by:

\[
\beta_\pm(\omega, k) = \frac{-(k \mp \omega)}{2k^2\left[\omega \mp k + \frac{1}{2k^2}\left((k \mp \omega) \ln\left(k \mp \omega \over k + \omega\right) + 2k\right)^2 + (\pi(k \mp \omega))^2\right]}]. \tag{3.8}
\]

The other spectral distributions appearing in the functions \( f_0, f_1 \) and \( f_2 \) are as follows:

\[
\rho_0(\omega, k) = -\frac{1}{2} \theta(k^2 - \omega^2); \quad \rho_1(\omega, k) = \epsilon(\omega) \delta(k^2 - \omega^2); \quad \rho_2(\omega, k) = \epsilon(\omega) \partial_\omega \delta(k^2 - \omega^2). \tag{3.9}
\]
To show how to carry with the generic integral \( I_{rs} \), it is appropriate to chose the spectral function \( \rho(\omega, k) \) as a quark spectral density. This example is typical and general enough to encompasses all the major difficulties and subtleties we encounter throughout the work. Using the expressions in (3.6) and (3.2), we see that there are four kinds of contributions: pole-pole \((pp)\), pole-cut \((pc)\), cut-pole \((cp)\) and cut-cut \((cc)\). Let us start with the pole-pole contribution, which typically writes as:

\[
I^{pp}_{rs} = \int_{\eta}^{\infty} dk \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} \frac{d\omega'}{\omega'} g(\omega, k) \delta_{\pm}(k) \delta(\omega \mp \omega_{c}) \partial^{(r)}_{\omega} \delta_{k} \left[ \beta_{l,t}(k) \theta(k - |\omega'|) \right] \partial^{(s)}_{\omega} \delta(1 - \omega - \omega').
\]  

(3.10)

The integrand is nonzero only at the intersections of the supports of the delta functions, namely, when \( \omega \pm \omega_{c} = 0, \omega' \pm \omega_{l,t} = 0 \) and \( \omega + \omega' = 1 \). This is satisfied at the points \( k \) for which we have \( \pm \omega_{c}(k) = 1 \mp \omega_{l,t}(k) \). But for the three cases \((N_{c}, N_{f})\) considered in this work, these curves never intersect. Hence we can write:

\[
I^{pp}_{rs} = 0.
\]  

(3.11)

Next we look at the term pole-cut. It is of the form:

\[
I^{pc}_{rs} = \int_{\eta}^{\infty} dk \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} \frac{d\omega'}{\omega'} g(\omega, k) \delta(\omega \mp \omega_{c}) \partial^{(r)}_{\omega} \left[ \beta_{l,t}(\omega', k) \theta(k - |\omega'|) \right] \partial^{(s)}_{\omega} \delta(1 - \omega - \omega').
\]  

(3.12)

With the aim of making the illustration clear, let us consider first the specific case \( r = 1 \) and \( s = 0 \). The integration over \( \omega' \) becomes trivial. The derivation with respect to \( k \) yields two terms: 

\[
g(\omega, k) \delta(\omega \mp \omega_{c}) \partial_{\omega} \left[ \beta_{l,t}(1 - \omega, k) \theta(k - |1 - \omega|) \right] \text{ and } g(\omega, k) \delta(\omega \mp \omega_{c}) \partial_{\omega} \left[ \beta_{l,t}(1 - \omega, k) \theta(k - |1 - \omega|) \right].
\]

The second term is always zero because it imposes the conditions \( \omega_{c}(k) = \pm (1 \mp k) \), which are never satisfied kinematically. In the first term, the integration over \( \omega \) is straightforward and the theta function imposes the constraints \( -k \leq 1 \pm \omega_{c}(k) \leq k \) on the momentum integration. Only the minus sign can be satisfied, and for all values of \( k \). Hence we write for this contribution:

\[
I^{pc}_{10} = \int_{\eta}^{\infty} dk \frac{g(\omega_{c}, k)}{1 - \omega_{c}(k)} \delta(\omega_{c}) \partial_{\omega} \left[ \beta_{l,t}(1 - \omega, k) \theta(k - |1 - \omega|) \right] \bigg|_{\omega = 1 - \omega_{c}(k)}.
\]  

(3.13)

The case \( r = 2 \) and \( s = 0 \) is carried out in a similar way. Because of the second derivative in \( k \), there will be three contributions: 

(i) \( g(\omega, k) \delta(\omega \pm \omega_{c}) \partial_{\omega}^{2} \beta_{l,t}(1 - \omega, k) \theta(k - |1 - \omega|) \). The integration over \( \omega \) is trivial and the theta function imposes the minus sign with no constraints on \( k \). 

(ii) \( 2g(\omega, k) \delta(\omega \pm \omega_{c}) \partial_{\omega} \beta_{l,t}(1 - \omega, k) \delta(\omega \pm \omega_{c}) \delta(k - |1 - \omega|) \) and (iii)
$g(\omega,k)\delta_\varepsilon(k)\beta_{l,t}(1-\omega,k)\delta(\omega\pm\omega_z)\partial_k\delta(k-|1-\omega|)$. These two contributions cancel because of the same kinematics as in the previous case. Then we can write:

$$I_{20}^{pc} = \int_\eta^\infty \! dk \, \frac{g(\omega_\varepsilon,k)}{1-\omega_\varepsilon(k)} \delta_\varepsilon(k) \frac{\partial^2_s \beta_{l,t}(\omega,k)}{\omega} \bigg|_{\omega=1-\omega_z(k)}. \quad (3.14)$$

The other cases for $r$ and $s$ are worked out in similar steps. When a derivative with respect to $\omega$ intervenes over $\delta(1-\omega-\omega')$, it is first transformed into an integral over $\omega'$ and then brought back onto the other functions. The rest of the steps are straightforward. We can write generically:

$$I_{rs}^{pc} = (-)^s \int_\eta^\infty \! dk \, g(\omega_\varepsilon,k) \delta_\varepsilon(k) \frac{\partial^r_s \beta_{l,t}(\omega,k)}{\omega} \bigg|_{\omega=1-\omega_z(k)}. \quad (3.15)$$

The notation $\partial^r_{k,\omega}$ means derive $r$-times with respect to $k$ and $s$-times with respect to $\omega$.

### A. Infrared behavior

The generic integral $I_{rs}^{pc}$ is present in all three coefficients $a_0$, $a_1$ and $a_2$. From the explicit expression of $\delta_\varepsilon(k)$ given in (3.7) and those of $\beta_{l,t}(\omega,k)$ given in (3.4) and (3.5), one carries out the $k$-integration in (3.15) for the different specific functions $g(\omega,k)$ intervening in the different contributions to $f_0$, $f_1$ and $f_2$ given in relations (2.17), (2.18) and (2.19) respectively. Most of the work is numerical. For the coefficients $a_0$ and $a_1$, no particular difficulty arises and the limit $\eta \to 0$ is safe. However, the coefficient $a_2$ requires special attention since the infrared limit is sensitive. To see this concretely, take the explicit example of $\rho_+ \partial^2_k \rho_l$ in $f_2$ with $g(\omega,k) = -3k^2(1-k+\omega)^2$. Let us write the integral as $I_{20}^{pc} = \int_\eta^\infty \! dk F(k)$ with the integrand $F(k) = -3k^2(1-k+\omega_+)^2\delta_+/(1-\omega_+) \partial^2_k \beta_l(\omega,k)\big|_{\omega=1-\omega_+}$. With the explicit expressions of $\omega_+(k)$ given in (2.3), $\delta_+(k)$ in (3.7) and $\beta_l(\omega,k)$ in (3.4), we can perform a small-$k$ expansion of the integrand $F(k)$ to find for the different cases of $N_c$ and $N_f$:

$$\begin{align*}
(N_c; N_f) = (2,1) \longrightarrow F(k) &= -\frac{0.4486}{k} + 0.4486 + 0.2248k + O(k^2); \\
(N_c; N_f) = (3,2) \longrightarrow F(k) &= -\frac{0.4985}{k} + 0.5428 + 0.2466k + O(k^2); \\
(N_c; N_f) = (3,3) \longrightarrow F(k) &= -\frac{0.4431}{k} + 0.4825 + 0.2223k + O(k^2). \quad (3.16)
\end{align*}$$

The $1/k$ behavior indicates a logarithmic infrared divergence. This latter is extracted in the following manner. Writing $F(k) = \alpha/k + \text{finite}$, we have:

$$I_{20}^{pc} = -\alpha \ln \eta + \alpha \ln \lambda + \int_0^\lambda \! dk \Big( F(k) - \frac{\alpha}{k} \Big) + \int_\lambda^\infty \! dk F(k). \quad (3.17)$$

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In short, we have split the original integral into two: one from $\eta$ to an arbitrary number $\lambda$ and another one from $\lambda$ to $\infty$. The second integral is finite. From the integrand of the first one we have subtracted $\alpha/k$, which makes the integral safe in the infrared and the limit $\eta \to 0$ can be taken. The term $\alpha/k$ has to be integrated by itself from $\eta$ to $\lambda$. As said, the choice of $\lambda$ is arbitrary and we must (and do) check that the result is independent of it. For this specific example, we obtain the following results:

\[
(N_c; N_f) = (2, 1) \quad \rightarrow \quad I^{pc}_{20} = 0.4486 \ln \eta - 0.7641;
\]

\[
(N_c; N_f) = (3, 2) \quad \rightarrow \quad I^{pc}_{20} = 0.4985 \ln \eta - 0.8557;
\]

\[
(N_c; N_f) = (3, 3) \quad \rightarrow \quad I^{pc}_{20} = 0.4431 \ln \eta - 0.7539.
\]

All other infrared-sensitive contributions are worked out in a similar manner.

\[\text{(3.18)}\]

**B. Other circumvented singularities**

The upcoming cut-pole contribution is not sensitive to $\eta$ but has other difficulties we discuss now. Consider then:

\[
I^{cp}_{rs} = \int_{\eta}^{\infty} dk \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} \frac{d\omega'}{\omega'} g(\omega, k) \beta_\varepsilon(\omega, k) \theta(k - |\omega|) \partial_\omega [\delta(\omega' - \omega_i)] \partial_\omega \delta(1 - \omega - \omega'),
\]

(3.19)

where $i$ stands for $l$ or $t$. Let us be a little more specific and consider the case $r = 2$ and $s = 0$. The integration over $\omega'$ is trivial. We obtain:

\[
I^{cp}_{20} = \int_{\eta}^{\infty} dk \int_{-\infty}^{\infty} \frac{d\omega}{1 - \omega} \left[ g(\omega, k) \beta_\varepsilon(\omega, k) \theta(k - |\omega|) \left[ \delta''(\omega_i) + 2\delta'(\partial_k + \delta^2) \delta(1 - \omega - \omega_i) \right] \right].
\]

(3.20)

Here, $\delta''(\omega_i)$ is the first derivative and $\delta''(\omega_i)$ the second derivative of the residue function. The kinematics imposes $1 - k \leq \omega_i(k) \leq 1 + k$, conditions satisfied for all $k$ from the lower bound $k_i$ to $\infty$, where $k_i$ is the solution to the condition $\omega_i(k) = 1 + k$. In the last two contributions, the $k$-derivatives over the delta function are brought onto the other functions. With some algebra and using the fact that $\lim_{\omega \to \pm k} \beta_\varepsilon(\omega, k) = 0$ to eliminate some of the intermediary terms, we can write:

\[
I^{cp}_{20} = \int_{k_i}^{\infty} dk \left[ \delta''(\omega_i) + 2\delta'(\partial_k + \delta^2) \right] g(\omega, k) \beta_\varepsilon(\omega, k) / (1 - \omega) \bigg|_{\omega=1-\omega_i} + \frac{3\omega_i^2 g(1 - \omega_i, k)}{\omega_i |1 - \omega_i'|} \left[ 2 \partial_\omega \beta_\varepsilon(\omega, k) \bigg|_{\omega=1-\omega_i} - \frac{\beta_\varepsilon'(1 - \omega_i, k)}{|1 - \omega_i'|} \right]_{k=k_i}.
\]

(3.21)
In this expression, $\beta'(1 - \omega, k)$ indicates the total derivative with respect to $k$. Now we must be careful with this expression because the derivatives of the quark cut functions at $k_i$ are infinite. This is not related to the infrared limit. However, putting these infinities together cancels them. More specifically, if we call $y = k - k_i$, then the singular behavior of the integrand around $k_i$ coming from the derivatives of the quark cut functions has the form $A (\ln y, y) / y^2 + B (\ln y, y) / y$, where $A$ and $B$ are complicated but computable functions.

The task is then to systematically extract this singular behavior and put it together with a similar one coming from the terms not under the integral sign. It turns out that in each case and for every term, when put together, all the singularities cancel.

C. Light-cone behavior

Last we turn to the cut-cut contribution to $I_{rs}$. Here a different kind of potential singularities arises, light-cone, but circumvented too. The generic term has the following form:

$$I_{cc}^{rs} = \int_{\eta}^{\infty} dk \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' g(\omega, k) \beta_\varepsilon'(\omega, k) \theta(k - |\omega|)$$
$$\times \partial^2 \beta_i(1 - \omega, k) \partial(1 - |1 - \omega|).$$

Here too the case $r = 2$ and $s = 0$ is typical enough. Performing the integral over $\omega'$ trivially, we have:

$$I_{20}^{cc} = \int_{0}^{\infty} dk \int_{-\infty}^{\infty} d\omega \frac{g(\omega, k) \beta_\varepsilon(\omega, k)}{1 - \omega} \beta_\varepsilon(1 - \omega, k) \theta(k - |1 - \omega|)$$
$$\times \left[ \partial_k^2 \beta_i(1 - \omega, k) \theta(k - |1 - \omega|) + 2 \partial_k \beta_i(1 - \omega, k) \delta(k - |1 - \omega|) + \beta_i(1 - \omega, k) \partial k \delta(k - |1 - \omega|) \right].$$

The $\Theta\Theta$ term is constrained by $1 - k \leq \omega \leq k$ with $k \geq 0.5$. We obtain the contribution $\int_{0.5}^{\infty} dk \int_{1-k}^{k} d\omega g(\omega, k) \beta_\varepsilon(\omega, k) \partial_k^2 \beta_i(1 - \omega, k) / (1 - \omega)$. The $\Theta\partial$ contribution is left with one integral over $k$; it reads $2 \int_{0.5}^{\infty} dk g(1 - k, k) \beta_\varepsilon(1 - k, k) \partial_k \beta_i(1 - \omega, k) \delta(k - |1 - \omega|)$. The $\Theta\partial\delta$ contribution is treated similarly and yields $- \int_{0.5}^{\infty} dk \partial_k [g(\omega, k) \beta_\varepsilon(\omega, k) \beta_i(1 - \omega, k)]_{\omega = 1-k}$. However, it is easy to see that we can get rid of the second and third contributions by performing one integration by part in the first contribution. It will yield $\int_{0.5}^{\infty} dk \int_{1-k}^{k} d\omega \partial_k [g(\omega, k) \beta_\varepsilon(\omega, k)] \partial_k \beta_i(1 - \omega, k)$ plus the opposite of the second and third contributions. Hence, changing the integration from over $\omega$ to over $1 - \omega$, we can
\[ I_{20}^{cc} = - \int_{0.5}^{\infty} dk \int_{1-k}^{k} d\omega \frac{\partial_k [g (1 - \omega, k) \beta_\epsilon (1 - \omega, k)] \partial_k \beta_i (\omega, k)}{\omega} \partial_k \left[ g (1 - \omega, k) \beta(1 - \omega, k) \right] \partial_k \beta_i (\omega, k). \] (3.24)

But extra care must be taken because \( \lim_{\omega \rightarrow k} \partial_k \beta_i (\omega, k) \) in infinite. This situation is handled as follows. Given the explicit expressions (3.21) and (3.22) of the longitudinal and transverse cut functions respectively, one can see that the divergence in \( \partial_k \beta_i (\omega, k) \) comes from \( 1/\zeta \equiv 1/(k - \omega) \) and \( Y^{-1} \equiv \ln \zeta \). Then, writing \( I_{20}^{cc} = \int_{0.5}^{\infty} dk \int_{0}^{2k-1} d\zeta G (k, \zeta; Y) \), we expand the integrand \( G (k, \zeta; Y) \) in powers of \( \zeta \):

\[ G (k, \zeta; Y) = G_{-1} (k, Y) /\zeta + G_0 (k, Y) + G_1 (k, Y) \zeta + O (\zeta^2). \] (3.25)

The singular term \( G_{-1} (k, Y) /\zeta \) is singled out in the following manner:

\[ I_{20}^{cc} = \int_{0.5}^{+\infty} dk \int_{0}^{2k-1} d\zeta \left[ G (k, \zeta; 1/\ln \zeta) - G_{-1} (k, 1/\ln \zeta) /\zeta \right] + \int_{0.5}^{\infty} dk \int_{\ln^{-1}(2k-1)}^{0^-} \frac{dY}{Y^2} G_{-1} (k, Y). \] (3.26)

In this expression, the first integral is finite. In the second integral, knowing that \( G_{-1} (k, Y) = O (Y^3) \) close to zero, there is no divergence anymore. Hence the whole \( I_{20}^{cc} \) is finite. The same trick is used for all the cut-cut terms sensitive close to the light-cone and all singularities are circumvented.

IV. RESULTS AND CONCLUSION

All the terms in \( f_0, f_1 \) and \( f_2 \) given in (2.17), (2.18) and (2.19) respectively have to be calculated. There are no additional subtleties to mention. Assembling all the partial results together and reintroducing the quark thermal mass \( m_f \) such that \( \bar{\rho} = p/m_f \) and \( \bar{\eta} = \eta/m_f \), we find the following final results:

\[
\begin{align*}
(N_c; N_f) = (2, 1) \rightarrow & \quad \gamma_{\pm} (p) = \frac{3g^2 T}{64\pi} \left[ 5.6978 \mp 1.0452\bar{\rho} - (6.0427 \ln \bar{\eta} - 8.4684)\bar{p}^2 + O \left( \bar{p}^3 \right) \right]; \\
(N_c; N_f) = (3, 2) \rightarrow & \quad \gamma_{\pm} (p) = \frac{g^2 T}{12\pi} \left[ 5.6344 \mp 0.9492\bar{\rho} - (6.7141 \ln \bar{\eta} - 7.7539)\bar{p}^2 + O \left( \bar{p}^3 \right) \right]; \\
(N_c; N_f) = (3, 3) \rightarrow & \quad \gamma_{\pm} (p) = \frac{g^2 T}{12\pi} \left[ 5.7057 \mp 1.0568\bar{\rho} - (5.9681 \ln \bar{\eta} - 8.5536)\bar{p}^2 + O \left( \bar{p}^3 \right) \right].
\end{align*}
\] (4.1)

Remember that the present calculation of the quark damping rates is done in the context of next-to-leading order hard-thermal-loop summed perturbative QCD at high temperature.
where weak coupling is assumed. It is intended to be an additional contribution to probe the analytic properties of finite-temperature QCD, particularly in the infrared. As mentioned in the introductory remarks, it has no direct relevance to the physics of the strongly coupled quark-gluon plasma, but it may be useful if, eventually, quantum chromodynamics is still believed to be the bedrock of all theoretical hadronic physics.

One peculiarity of the results (4.1) is that the logarithmic sensitivity to the infrared is found only in the coefficients of \( p^2 \); the zeroth and first order coefficients are safe. The calculations have encountered other potential divergences: at the light-cone and at specific points determined by the gluon energies. But all these have been circumvented.

The persistence of the infrared divergence is most probably attributed to the non-screening of the static chromo-magnetic fields at lowest order. In a separate work [29], we have considered the context of scalar quantum electrodynamics, where calculations are more fluid, and have explicitly shown that an early momentum expansion like the one we perform in [23] and this work gives exactly the same results as a late expansion, after the Matsubara sum and analytic continuation to real energies are done. Remember that only soft values of the internal momentum have been included in the integration. It seems then that the ultrasoft region is important and needs to be investigated with probably non-perturbative means.

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