Towards a Unified Approach to Electromagnetic Analysis by Multilayer Embedded Objects

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Abstract—In this paper, an efficient and accurate unified approach is proposed to solve transverse magnetic scattering problems by multilayer embedded objects. In the proposed approach, an equivalent current density is derived when the equivalent theorem is recursively applied on each boundary from inner to exterior regions. Then, the objects are replaced by the background medium and the equivalent electric current density enforced on the outermost boundary remains fields in the exterior region unchanged. Therefore, the scattering problems by the multilayer embedded objects can be solved with the electric field integral equation (EFIE). Compared with other two-region formulations (TRFs), like the PMCHWT, CTF, the proposed approach shows significant benefits; only the surface electric current density instead of both the electric and magnetic current densities is required to model the complex objects. Furthermore, the single electric current density is only enforced on the outermost boundary of objects. Therefore, the overall count of unknowns can be significantly reduced, especially when the number of boundaries between different homogenous media is large. At last, several numerical experiments are performed to validate the accuracy and efficiency of the proposed approach.

Index Terms—Multilayer, single-source formulation, surface equivalent theorem, two-region formulation

I. INTRODUCTION

The multilayer embedded objects are widely used in the practical engineering applications, such as multilayer planar integrated circuits [1], thin layer coated fibers [2], power cables [3], cloaking coated aircrafts [4], and so on. There are various numerical methods proposed to model such objects. One of the powerful numerical tool is the method of moment (MOM) [5] based on the surface integral equation (SIE), which is widely used to model electrically large, multiscale or multilayer structures due to its unknowns residing on the interfaces of different homogenous media. Therefore, the overall count of unknowns is significantly less than those of partial differential equation (PDE) based methods, like the finite element method (FEM) [6] and the finite-difference time-domain (FDTD) method [7], which requires the volumetric meshes.

There are various SIEs, like the Poggio-Miller-Chan-Harrington-Wu-Tsai (PMCHWT) [8], CTF [9], are proposed to model multilayer embedded objects. In these formulations, both the surface equivalent electric and magnetic current densities are introduced and several two-region problems are required to be solved simultaneously. Therefore, as the layer number increases, the overall count of unknowns will greatly increase.

To mitigate the problem above, a number of efforts are made to introduce single-source formulations. In [10], a novel single-source surface-volume-surface formulation is proposed to model penetrable objects. In this formulation, the volumetric integral operator is mapped from surface to volume operator and then back from volume to surface operator. Therefore, significant efficiency improvement can be obtained. However, compared with the SIEs, it still requires to evaluate the volume integral operator. Various other single-source formulations based on the surface equivalent theorem are proposed in [11]–[13]. Only single source current source is required in those formulations. Therefore, more efficient formulations can be obtained compared with their dual source counterparts. However, in those formulations, those two-regions problems are required to be solved simultaneously. Therefore, it still suffers from the problem stated above. In [14], a single-source formulation based on the differential surface admittance operator (DSAO) to model high-speed interconnects is proposed. The DSAO is derived through the surface equivalent theorem and electric fields in the equivalent problem are enforced to equal to those in the original problem. Its capability is further enhanced to model arbitrary shaped interconnects [15], [16], circular solid and hollow cables [17]. In [18], a single electric current density based on the DSAO and magnetic vector potential is introduced to model circular hole cables. However, the authors confine their models to canonical objects. In [19], it has been extended to solve three-dimensional scattering problems and antenna arrays [20]. Especially, in [21], each element in a large antenna array is replaced by a fictitious enclosed surface and an equivalent surface current density is introduced to ensure the fields unchanged, which is similar to the equivalence principle algorithm (EPA) [21]. This formulation shows better condition in the coefficient matrix, therefore, better convergence properties compared with the direct MOM and great performance improvement.

In this paper, we proposed a unified single-source SIE based on the DSAO to solve two dimensional transverse magnetic (TM) scattering problems by multilayer embedded objects to address the problem above. The equivalent theorem is applied recursively from the innermost to outermost boundaries. An equivalent object is derived in the formulation, which is filled with the background medium, and an electric current density
enforced on the original outermost boundary. Compared with the traditional TRFs, like the PMCHWT formulation [8], CTF [9], the overall count of unknowns can be significantly reduced, especially when the layer number is large. The formulation shows three obvious merits: (1) Only the single-source electric current is required rather than both the electric and magnetic current densities in the other two-region formulations (TRFs). (2) The surface equivalent electric current density is only enforced on the outermost boundary rather than all the boundaries between different homogenous media. (3) If the multilayered cylinder objects are involved, the proposed method can avoid the troublesome multilayer Green function evaluation, the proposed approach only requires to evaluate the free space Green function. The approach proposed in this paper can significantly simplify the analysis and provide a unified approach to analyze the multilayer embedded objects and reduce the overall number of unknowns for the complex objects with large number of multilayer enclosed media. In [24], we reported some preliminary results upon this idea. In this paper, we greatly extended technique contents and presented comprehensive investigations upon the proposed approach.

The main contributions in this paper are mainly into three aspects as follows:

1) A novel and general SIE is proposed to model the multilayer embedded objects. In the proposed formulation, which is derived based on the DSAO, only an electric current density is required to be enforced on the outermost boundary of objects, which retain the fields the same as those in the original problem. Unknowns involved only the electric current density in the proposed SIE only reside on the outermost boundary. Therefore, significant efficiency improvement can be obtained.

2) Two extensions based on the proposed approach are derived. One is that perfectly electric conductor (PEC) scatters embedded in the multilayer embedded media. The other is that a fictitious boundary resides in the same medium, which is similar to the EPA [21] and the macromodeling approach [20]. However, the approach proposed in this paper is much more general and applicable for various objects. Therefore, the approach in [20] can be considered as a special case of the proposed approach in this paper.

3) The proposed approach is comprehensively investigated through several numerical examples. Results demonstrate that the proposed approach can significantly reduce the overall count of unknowns compared with TRFs when the number of interfaces is large.

This paper is organized as follows. In Section II, detailed formulations for the proposed approach are presented. In Section III, two special scenarios are derived based on the proposed approach. In Section IV, its accuracy and efficiency are comprehensively investigated through several numerical examples. At last, we draw some conclusions in Section V.

Fig. 1. (a) Original model and (b) the equivalent model with inner media replaced by its background medium and enforcing the surface electric current density on the outermost boundary

II. METHODOLOGY

A. The Problem Configuration

In this paper, a two-dimensional TM scattering problem by objects embedded by multilayer media are considered as shown in Fig. 1(a). To derive a unified approach to solve it, we resort to the equivalence theorem to derive an equivalent model, in which all the media are replaced by the background one and the exterior fields are exactly the same as those of the original model. Our goal is to introduce only one single surface equivalent electric or magnetic current density on the outermost boundary \( \gamma_n \) as shown in Fig. 1(b) in the equivalent model to keep the fields in outermost region unchanged. We keep this goal in mind and will derive an equivalent current density enforced on the outermost boundary \( \gamma_n \) from inner to exterior layer by layer. The original multilayer embedded objects are shown in Fig. 1(a), where \( \varepsilon_i, \mu_i \) denote the permittivity and the permeability of the \( i \)th layer medium, which is bounded by two adjacent boundaries, \( \gamma_{i-1} \) and \( \gamma_i \), respectively.

Before we derive the detailed formulations, let’s make more remarks on the single source formulations. Our goal is to derive an equivalent model as shown in the Fig. 1(b), in which the equivalent electric current density \( \vec{J}_{s_i} \) and magnetic current \( \vec{M}_{s_i} \) are introduced on the outermost fictitious boundary, \( \gamma_i \). According to the equivalent theorem [22], the equivalent electric and magnetic currents can be expressed as

\[
\vec{J}_{s_i}(\vec{r}) = \vec{H}_{i}(\vec{r}) - \hat{\vec{H}}_{i}(\vec{r}), \\
\vec{M}_{s_i}(\vec{r}) = \vec{E}_{i}(\vec{r}) - \hat{\vec{E}}_{i}(\vec{r}),
\]

where \( \vec{r} \in \gamma_i \), \( \vec{E}_{i}(\vec{r}), \vec{H}_{i}(\vec{r}), \hat{\vec{E}}_{i}(\vec{r}), \hat{\vec{H}}_{i}(\vec{r}) \) are the surface tangential electric and magnetic fields in the original and equivalent model, respectively. All quantities with a hat denote their values in the equivalent model.

Since the electric and magnetic fields in the equivalent model can be arbitrary, we can obtain the single electric current source by enforcing that \( \vec{E}_{i}(\vec{r}) = \hat{\vec{E}}_{i}(\vec{r}) \) and \( \vec{M}_{s_i}(\vec{r}) = 0 \) are expressed as

\[
\vec{J}_{s_i}(\vec{r}) \neq 0, \vec{M}_{s_i}(\vec{r}) = 0.
\]
On the other hand, we can have the single magnetic current source by enforcing that \( \hat{\mathbf{M}}_m(\vec{r}) = \hat{\mathbf{H}}_i(\vec{r}) \) and (1) and (2) are expressed as
\[
\hat{\mathbf{M}}_m(\vec{r}) \neq 0, \hat{\mathbf{J}}_m(\vec{r}) = 0.
\]
(4)

Both of them can give us a single-source formulation. In this paper, we select the first option to obtain the single source integral formulation and derive the surface equivalent electric current using the contour integral method for the multilayer embedded objects. The key point in the proposed approach is to derive the differential surface admittance operator \( \mathbf{Y}_{m} \), according to the equivalence theorem, which relates the outermost equivalent electric and magnetic current density of the object on the outermost boundary.

**B. The Single Source SIE for A Penetrable Object**

Let us first consider the simplest scenario, in which a single penetrable object with the homogeneous medium of the permittivity \( \varepsilon_1 \) and the permeability \( \mu_1 \), respectively, as shown in Fig. 2(a). Its boundary is denoted as \( \gamma_1 \). The permittivity and the permeability of the background medium are \( \varepsilon_0 \) and \( \mu_0 \), respectively. According to the equivalence theorem \[22\], an equivalent model, in which the object is replaced by its surrounding medium and a surface equivalent electric and magnetic current is introduced at \( \gamma_1 \) to ensure the fields in the exterior region unchanged as shown in the Fig. 2(b). Here, we only summarize the key points of the MOM idea. Interested readers are referred to \[16\] for more details.

Let’s consider that the penetrable object does not include any sources, electric fields must satisfy the following scalar Helmholtz equation inside \( \gamma_1 \)
\[
\nabla^2 E_1 + k_1^2 E_1 = 0,
\]
subject to the boundary condition
\[
E_1(\vec{r})|_{\vec{r} \in \gamma_1} = \hat{E}_1(\vec{r})|_{\vec{r} \in \gamma_1},
\]
where \( E_1 \) and \( \hat{E}_1 \) denote the electric field inside \( \gamma_1 \) for the original and equivalent models, respectively, and \( E_1(\vec{r})|_{\vec{r} \in \gamma_1} \) and \( \hat{E}_1(\vec{r})|_{\vec{r} \in \gamma_1} \) denote their values on the inner side of \( \gamma_1 \).

(5) can be solved through the second scalar Green theorem \[23\]. Then, the electric field \( E_1 \) inside \( \gamma_1 \) can be expressed in terms of \( E_1 \) and its normal derivative value on \( \gamma_1 \) as
\[
TE_1(\vec{r}) = \int_{\gamma_1} \left[ G_1(\vec{r}, \vec{r}^\prime) \frac{\partial E_1(\vec{r}^\prime)}{\partial n^\prime} - \frac{\partial G_1(\vec{r}, \vec{r}^\prime)}{\partial n^\prime} E_1(\vec{r}^\prime) \right] d\vec{r}^\prime,
\]
(7)

where the constant \( T = 1/2 \) when the source and observation points are located on the same boundary, otherwise, \( T = 1 \) and \( G_1(\vec{r}, \vec{r}^\prime) \) is the Green function expressed as \( G_1(\vec{r}, \vec{r}^\prime) = -jH_0^{(1)}(k_1 r)/4 \), where \( j = \sqrt{-1} \) and \( k_1 \) is the wavenumber in the penetrable object and \( H_0^{(1)}(\cdot) \) is the zero-th order Hankel function of the second kind. In addition, the tangential magnetic field relates to the electric field on \( \gamma_1 \) through the Poincare-Steklov operator \[15\]
\[
H_1(\vec{r}) = \frac{1}{j \omega \mu_1} \frac{\partial E_1(\vec{r})}{\partial n} \bigg|_{\vec{r} \in \gamma_1},
\]
(8)

where \( \mu_1 \) is the permeability of the object. We discretize \( \gamma_1 \) into \( m_1 \) segments and use the pulse basis function to expand \( E_1 \) and \( H_1 \) as
\[
E_1(\vec{r}) = \sum_{n=1}^{m_1} e_n f_n(\vec{r}),
\]
\[
H_1(\vec{r}) = \sum_{n=1}^{m_1} h_n f_n(\vec{r}),
\]
(9)
(10)

where \( f_n(\vec{r}) \) denotes the \( n \)th basis function. We use the Galerkin scheme to test (7) and \[3\] at each segment of \( \gamma_1 \) and collect all \( E_1 \) and \( H_1 \) expansion coefficients into two column vectors \( \mathbf{E}_1 \) and \( \mathbf{H}_1 \) as
\[
\mathbf{E}_1 = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1m_1} \end{bmatrix}^T,
\]
\[
\mathbf{H}_1 = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m_1} \end{bmatrix}^T.
\]
(11)
(12)

Therefore, (7) can be rewritten into the matrix form as
\[
\mathbf{P}_1^{(1)} \mathbf{H}_1 = \mathbf{U}_1^{(1)} \mathbf{E}_1,
\]
(13)

where the subscript 1 denotes that the testing procedure is applied on \( \gamma_1 \), and the superscript (1) denotes the equivalence theorem applied on \( \gamma_1 \), and elements of \( \mathbf{P}_1^{(1)} \) and \( \mathbf{U}_1^{(1)} \) are expressed as
\[
\begin{bmatrix} \mathbf{U}_1^{(1)} \end{bmatrix}_{m,n} = -2k_1 \int_{\gamma_1} \frac{d^2 m \cdot \hat{n}'}{d m} G_1(\vec{r}_m, \vec{r}) \, d\vec{r}',
\]
\[
\begin{bmatrix} \mathbf{P}_1^{(1)} \end{bmatrix}_{m,n} = 2\omega \mu j \int_{\gamma_1} G_1(\vec{r}_m, \vec{r}') \, d\vec{r}'.
\]
(14)
(15)

Then, through inverting the square matrix \( \mathbf{P}_1^{(1)} \), we obtain the surface admittance operator \( \mathbf{Y}_1 \) \[16\] as
\[
\mathbf{H}_1 = \mathbf{P}_1^{(1)} \mathbf{Y}_1 \mathbf{U}_1^{(1)} \mathbf{E}_1.
\]
(16)

When all the parameters are replaced by those of its surrounding medium, the equivalent model is obtained. The \( \mathbf{H}_1(\vec{r}) \) is expanded through the pulse basis function as
\[
\mathbf{H}_1(\vec{r}) = \sum_{n=1}^{M_1} \hat{h}_n f_n(\vec{r}).
\]
(17)

With the similar procedure in the original model, for the equivalent model we can obtain
\[
\mathbf{H}_1 = \mathbf{P}_1^{(1)} \mathbf{Y}_1 \mathbf{U}_1^{(1)} \mathbf{E}_1.
\]
(18)
be careful to handle the current density. In it, the Helmholtz equation is again used to calculate electric fields in the inner region, and note that the inhomogeneous Helmholtz equation is required due to the existence of the equivalent surface current density introduced in Section II-B to represent the penetrable object. Therefore, the inhomogeneous Helmholtz equation can be expressed as

$$\nabla^2 E_2 + k_2^2 E_2 = j\omega\mu_2 J_1, \quad (23)$$

where $E_2$ is the electric field inside the boundary $\gamma_2$, and $J_1$ is the equivalent current density on $\gamma_1$ after the first equivalent theorem applied in Section II-B. Through solving $\text{(23)}$ with the second scalar Green function theorem $\text{(22)}$, we obtain

$$\int_{\gamma_1} \int_{\gamma_2} G_2(\bar{r}, \bar{r}') \frac{\partial E_2(\bar{r})}{\partial n'} - \frac{\partial G_2(\bar{r}, \bar{r}')}{\partial n'} E_2(\bar{r}') \, d\bar{r}' - \int_{\gamma_1} G_2(\bar{r}, \bar{r}) \cdot (j\omega\mu_2 J_1) \, ds. \quad (24)$$

It should be noted that $T = 1/2$ when the source and observation points are located on $\gamma_2$, otherwise, $T = 1$.

We expand the electric field $E_2$ and the magnetic field $H_2$ in Equ. $\text{(24)}$ using the pulse basis functions and then test it using the Galerkin scheme on $\gamma_1$. Then, the electric field on the left side of Equ. $\text{(24)}$ is the electric field on $\gamma_1$. We collect the expansion coefficient into column vectors and write it into compact form as follows

$$E_1 = U_1^{(2)} E_2 + P_1^{(2)} H_2 + G_1^{(2)} J_1. \quad (25)$$

Then, we further test Equ. $\text{(24)}$ on $\gamma_2$, and obtain the following compact form as

$$E_2 = U_2^{(2)} E_2 + P_2^{(2)} H_2 + G_2^{(2)} J_1, \quad (26)$$

where $E_1$ and $E_2$ denote the electric field expansion coefficients on $\gamma_1$ and $\gamma_2$ respectively, expressed as $E_1 = [ e_{11} e_{12} \ldots e_{1m_1} ]^T$, $E_2 = [ e_{21} e_{22} \ldots e_{2m_2} ]^T$, and $H_2$ is the magnetic field on $\gamma_2$ expressed as $H_2 = [ h_{21} h_{22} \ldots h_{2m_2} ]^T$. $J_1$ is the surface equivalent current density expansion vector, defined in Equ. $\text{(21)}$, on the first layer boundary $\gamma_1$ obtained in section II-B. The elements of $U_2^{(2)}$, $P_2^{(2)}$ and $G_2^{(2)}$ are expressed as

$$[U_2^{(2)}]_{m,n} = 2k \int_{\gamma_2} \frac{d_m}{d_m} G_2(\bar{r}, \bar{r}') \, d\bar{r}', \quad (27)$$

$$[P_2^{(2)}]_{m,n} = 2\omega\mu_j \int_{\gamma_2} G_2(\bar{r}, \bar{r}') \, d\bar{r}', \quad (28)$$

$$[G_2^{(2)}]_{m,n} = -2\omega\mu_j \int_{\gamma_1} G_2(\bar{r}, \bar{r}') \, d\bar{r}', \quad (29)$$

where $\tilde{d}_m = \bar{r}' - \bar{r}_m$, $d_m = |\bar{r}' - \bar{r}_m|$. After substituting Equ. $\text{(21)}$ into Equ. $\text{(25)}$ and making some mathematical manipulations, $E_1$ is expressed in terms of $E_2$ and $H_2$ as

$$E_1 = \left( I_1 - G_1^{(2)} Y_{s_1} \right)^{-1} \left( U_1^{(2)} E_2 + P_1^{(2)} H_2 \right). \quad (30)$$

C. The Single Source SIE for Two Layered Embedded Objects

1) The original model: We now consider a slightly more complex scenario in which a layered medium with the permittivity $\varepsilon_2$ and the permeability $\mu_2$ encloses a penetrable object, which is considered in section II-B, as shown in Fig. 3(a). For this problem, we derive an equivalent model similar to that in Section II-B, in which objects are replaced by its surrounding medium with the permittivity $\varepsilon_2$ and the permeability $\mu_2$, and an equivalent current density $\hat{J}_1$ on $\gamma_1$ as shown in Fig. 3(b) is enforced to ensure the fields in the outer region unchanged. By comparing Fig. 2 (a) and Fig. 3 (b), it is easy to find that this problem is similar to each other. The only difference is that there is an equivalent current density, which is derived in the previous subsection, inside the new object. Therefore, it should

where

$$\hat{H}_1 = \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} & \ldots & \hat{h}_{1m_1} \end{bmatrix}^T \quad (19)$$

and $\hat{H}_1$ denotes the discretized magnetic fields in the equivalent model.

The surface equivalent current density on $\gamma_1$ is expanded through pulse functions, and the coefficients are extracted into the column vector $J_1$,

$$J_1 = \begin{bmatrix} j_{11} & j_{12} & \ldots & j_{1m_1} \end{bmatrix}^T. \quad (20)$$

Since $\text{(19)}$ is enforced, only single equivalent electric current density $\hat{J}_1$ is required. By substituting $\text{(16)}$ and $\text{(18)}$ into $\text{(1)}$, $J_1$ is obtained as

$$\hat{J}_1 = Y_{s_1} E_1, \quad (21)$$

where $\hat{J}_1 = J_1$, $Y_{s_1}$ is the differential surface admittance operator $\text{(16)}$ and can be expressed as

$$Y_{s_1} = Y - \hat{Y} = [P_1^{(1)}]^{-1} U_1 - [\hat{P}_1^{(1)}]^{-1} \hat{U}_1. \quad (22)$$

Fig. 3. (a) Original two layered object, (b) the equivalent model with the innermost medium replaced by its surrounding (the second) medium and enforcing the surface current density on $\gamma_2$, (c) the equivalent model with the inner medium replaced by its background medium and enforcing the surface current density on $\gamma_2$. Therefore, it should
Then, after substituting (30) into Equ. (26) and making some mathematical manipulations, the relationship of $E_2$ and $H_2$ on $\gamma_2$ is obtained as

$$H_2 = Y_{s_2}E_2.$$  \hspace{1cm} (31)

where

$$Y_{s_2} = \left[ P_2^{(2)} + G_2^{(2)}Y_{s_1}V_1^{(2)}P_1^{(2)} \right]^{-1}$$

$$U_2^{(2)}(I_2 - [U_2^{(2)}]^{-1}G_2^{(2)}V_2^{(2)}U_1^{(2)}).$$  \hspace{1cm} (32)

2) The equivalent model for two layered objects: According to the equivalence theorem, the equivalent model is shown in Fig. 3(c), where the inner medium is replaced by its surrounding medium with $(\varepsilon_0, \mu_0)$. In the equivalent model, there is no current sources existing inside the equivalent object. However, another surface equivalent current density, $\hat{J}_2$, on the boundary $\gamma_2$ is introduced to enforce the fields exactly the same as those in the original model.

With the similar procedure in Section II-B for the equivalent model of a penetrable object, the relationship between the electric field $E_2$ and the magnetic field $H_2$ on $\gamma_2$ in equivalent model is obtained as

$$\hat{H}_2 = [\hat{P}_2^{(2)}]^{-1}\hat{U}_2^{(2)}E_2.$$  \hspace{1cm} (33)

After substituting Equ. (32) and Equ. (33) into Equ. (3), we can obtain the equivalent current $\hat{J}_{s_2}$ on the boundary $\gamma_2$ by the surface admittance operator $Y_{s_2}$

$$\hat{J}_{s_2} = H_2 - \hat{H}_2 = Y_{s_2}E_2.$$  \hspace{1cm} (34)

As shown in Fig. 3(c), the two layered embedded objects are replaced by its background medium and only a single surface electric current density is introduced on $\gamma_2$. The traditional TRFs have unknowns on both $\gamma_1$ and $\gamma_2$. However, the proposed approach only has unknowns residing on the outermost boundary $\gamma_2$, which will greatly reduce the overall number of unknowns. To derive a unified approach for any multilayer embedded objects, we further generalize the proposed approach in the next subsection.

**D. Generalization of the Proposed Approach to Arbitrary Multilayer Embedded Objects**

Once we have presented the approach to model the two layered embedded objects as shown in the previous two subsections, it is straightforward to extend the approach to model any multilayer embedded objects.

Similarly, for objects embedded a $n$ layered medium, we can apply the proposed approach recursively and obtain the surface equivalent current on the outmost boundary. The first equivalent procedure is similar to that presented in the Section II-B, and the subsequent equivalent procedure is similar to that in the Section II-C. The surface equivalent current density induced from the $(i-1)$th medium is expressed as

$$\hat{J}_{i-1} = Y_{i-1}E_{i-1}. \hspace{1cm} (35)$$

For the original object with the $l$th ($l \geq 2$) layer media, it is required to test Equ. (24) on the $i$th layer as

$$E_i = U_i^{(i)}E_i + P_i^{(i)}H_i + G_i^{(i)}J_{i-1}, \hspace{1cm} (36)$$

and on the boundary of the $(i-1)$th layer as

$$E_{i-1} = U_{i-1}^{(i)}E_i + P_{i-1}^{(i)}H_i + G_{i-1}^{(i)}J_{i-1}, \hspace{1cm} (37)$$

where $E_i, H_i$ denote the electric field on the boundary of the $i$th layered medium and the entities of other three matrices are denoted as follows

$$U_{i/i-1}^{(i)}_{m,n} = 2k_{i/i-1} \int \frac{\tilde{d}_{m'}\cdot\tilde{n}'}{d_m} G_i(\tilde{r}_{m',\tilde{r}}')d\tilde{r}' \hspace{1cm} (38)$$

$$P_{i/i-1}^{(i)}_{m,n} = 2\omega\mu_{i/i-1} \int G_i(\tilde{r}_{m',\tilde{r}}')d\tilde{r}' \hspace{1cm} (39)$$

$$G_{i/i-1}^{(i)}_{m,n} = -2\omega\mu_{i/i-1} \int G_i(\tilde{r}_{m',\tilde{r}}')d\tilde{r}' \hspace{1cm} (40)$$

Through some mathematical manipulations using Equ. (35), Equ. (36) and Equ. (37), we obtain

$$H_i = \left[ P_i^{(i)} + G_i^{(i)}Y_{s_i}V_{i-1}^{(i)}P_{i-1}^{(i)} \right]^{-1}$$

$$U_i^{(i)}(I_i - U_i^{-1}G_i^{(i)}V_{i-1}^{(i)}U_i^{(i)})E_i, \hspace{1cm} (41)$$

then, it can be rewritten into a more compact form as

$$H_i = Y_iE_i,$$  \hspace{1cm} (42)

where

$$Y_i = \left[ P_i^{(i)} + G_i^{(i)}Y_{s_i}V_{i-1}^{(i)}P_{i-1}^{(i)} \right]^{-1}$$

$$U_i^{(i)}(I_i - U_i^{-1}G_i^{(i)}V_{i-1}^{(i)}U_i^{(i)})E_i.$$  \hspace{1cm} (43)

For the equivalent model, the procedure is similar to that in the equivalent model as shown in Section II-C. The surface discretized magnetic and electric fields can be expanded as

$$\hat{H}_i = \hat{P}_i^{-1}\hat{U}_iE_i.$$  \hspace{1cm} (44)

Therefore, the equivalent current density $J_i$ on $\gamma_i$ is expressed as

$$J_i = H_i - \hat{H}_i = \left[ Y_i - \hat{Y}_i \right]E_i.$$  \hspace{1cm} (45)

Then, the equivalent model obtained is shown in Fig. 4(b), and the single surface equivalent current $J_n = Y_{s_n}E_n$ on the outermost layer boundary $\gamma_n$ is obtained. The original object is replaced by its surrounding medium along with an alternate equivalent current $J_n$ on $\gamma_n$ as shown in Fig. 4(b).
E. Multiple Scattering Objects

When there are multiple scattering objects involved on $\gamma_i$, we first compute the equivalent current density for each object according to the proposed approach in section II-D as

$$\mathbf{J}_i^{(p)} = \mathbf{Y}_s^{(p)} \mathbf{E}_i^{(p)}, \quad (46)$$

where the superscript $(p)$ represents the $p$th object, $\mathbf{J}_i^{(p)}$, $\mathbf{E}_i^{(p)}$ denote the electric current density and electric field of the $p$th object, respectively.

Then, we collect all the surface equivalent current together and obtain

$$\mathbf{J}_i = \mathbf{Y}_s \mathbf{E}_i, \quad (47)$$

where the surface admittance operator $\mathbf{Y}_s$ of the $i$th layer is a diagonal block matrix assembling from the surface equivalent operator $\mathbf{Y}_s^{(p)}$ of each object and can be expressed as

$$\mathbf{Y}_s = \begin{bmatrix} \mathbf{Y}_s^{(1)}_{s_i} & \mathbf{Y}_s^{(2)}_{s_i} & \cdots & \mathbf{Y}_s^{(p)}_{s_i} \end{bmatrix}. \quad (48)$$

The electric field $\mathbf{E}_s$ is a column vector assembling all electric field coefficients of each object and is expressed as

$$\mathbf{E}_s = \begin{bmatrix} E_1^1 & \cdots & E_m^1 & \cdots & E_m^p & \cdots & E_m^p \end{bmatrix}^T, \quad (49)$$

and the equivalent current is expressed as

$$\mathbf{J}_i = \begin{bmatrix} J_1^1 & \cdots & J_m^1 & \cdots & J_m^p & \cdots & J_m^p \end{bmatrix}^T. \quad (50)$$

F. Scattering Modeling of the Exterior Problem

The electric field $\mathbf{E}$ outside the object is the superposition of the incident field $\mathbf{E}^i$ and the scattered field $\mathbf{E}^s$. Since the surface equivalent current $\mathbf{J}_s$ exists on the outermost layer of the equivalent object, the induced electric field and magnetic field by $\mathbf{J}_n$ are expressed as

$$\mathbf{E}^s(\mathbf{r}) = -j\omega\mu \int_{\gamma_n} \mathbf{J}_n(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') ds', \quad (51)$$

where $\mathbf{r}'$ on the outermost boundary $\gamma_n$ of the object, $G_0$ is the Green function expressed as $G_0 = -jH_0^{(2)}(k_0r)/4$, where $k_0$ is the wavenumber in the background medium. Obviously, $\mathbf{J}_n$ is the equivalent current on $\gamma_n$.

Therefore, the electric and magnetic fields can be simplified as

$$\mathbf{E}_n = \hat{L}(\mathbf{J}_n) + \mathbf{E}^i \quad (52)$$

We substitute Equ. (51) into Equ. (52) and obtain the electric field $\mathbf{E}_n$ on the outermost boundary $\gamma_n$ as

$$\mathbf{E}_n = (\mathbf{I} + \hat{L}\mathbf{Y}_s)^{-1} \mathbf{E}^i, \quad (53)$$

where $\mathbf{I}$ is identity matrix. Once the electric field $\mathbf{E}_n$ on the boundary $\gamma_n$ is obtained, we can easily calculate the surface equivalent current $\mathbf{J}_n$ through the surface admittance operator $\mathbf{Y}_s$ and other interested parameters.

III. Two Special Cases

A. PEC Embedded Objects

In this section, we derive the formulation in the proposed approach for the PEC with multilayer embedded media.

1) A single PEC object: For a single PEC object, there is a current density $\mathbf{J}_1$ on its boundary $\gamma_1$ as shown in Fig. 5 and the internal fields vanish. This scenario is different from that present in the Section II-B. Therefore, for a single PEC object, we do not need to apply the equivalence theorem to obtain the surface equivalent current on its boundary. The current $\mathbf{J}_1$ is the true surface current density flowing on $\gamma_1$.

2) A PEC object with a layer of medium: The PEC object with a layer of medium is shown in Fig. 5(a). It can be seen from section III-A that the object can be equivalent to Fig. 5(b). The permittivity and the permeability of medium inside $\gamma_2$ are $\varepsilon_1$ and $\mu_1$, and the current $\mathbf{J}_1$ exists on $\gamma_1$.

According to the inhomogeneous Helmholtz equation, we can solve the electric field on $\gamma_2$ as

$$TE_2(\mathbf{r}) = \int_{\gamma_2} \left[ G_2(\mathbf{r}, \mathbf{r}') \frac{\partial E_2(\mathbf{r}')}{\partial n'} - \frac{\partial G_2(\mathbf{r}, \mathbf{r}')}{\partial n'} E_2(\mathbf{r}') \right] ds' - \int_{\gamma_1} G_2 \cdot (j\omega\mu_2 \mathbf{J}_1) ds. \quad (54)$$

Then, we test the above equation on $\gamma_1$. Since the electric field vanishes on $\gamma_1$, we obtain the following formulation

$$0 = U_1^{(2)} \mathbf{E}_2 + P_1^{(2)} \mathbf{H}_2 + G_1^{(2)} \mathbf{J}_1. \quad (55)$$

Next, test (54) on $\gamma_2$ and obtain

$$\mathbf{E}_2 = U_2^{(2)} \mathbf{E}_2 + P_2^{(2)} \mathbf{H}_2 + G_2^{(2)} \mathbf{J}_1. \quad (56)$$

The entries of each matrix are the same as those in Equ. (27)-(29).

According to the section II-C, by using Equ. (55) and Equ. (56) along with some mathematical manipulations, we can obtain the relationship between the equivalent current $\mathbf{J}_2$ on $\gamma_2$ and $\mathbf{E}_2$, which can be expressed as

$$\mathbf{J}_2 = \mathbf{Y}_{s_2} \mathbf{E}_2. \quad (57)$$

where

$$\mathbf{Y}_{s_2} = \left[ \begin{pmatrix} P_2^{(2)} - G_2^{(2)} & + G_1^{(2)} \left[ P_1^{(2)} \right]^{-1} U_1^{(2)} \end{pmatrix} \right]^{-1} \left[ I_2 - U_2^{(2)} + G_2^{(2)} \left[ G_1^{(2)} \right]^{-1} U_1^{(2)} \right] - G_2^{(2)} \left[ P_1^{(2)} \right]^{-1} U_1^{(2)}. \quad (58)$$
3) A PEC object embedded into multilayer media: When the PEC object has multiple layered media enclosed, we can obtain the equivalent current \( J_2 \) on the boundary \( \gamma_2 \) through section II-D. Then, according to the equivalence theorem and the method proposed in section II-D, we can derive the outermost boundary equivalent current \( J_{n+1} \) layer by layer and then obtain the final equivalent model.

B. Surface Extension in the Same Medium

We further extend the proposed approach to model the fictitious extension boundary in the same medium, like the EPA [20], [21]. According to the method proposed in the section II-D, we have obtained the equivalent current at the outermost boundary of a multilayered enclosed object as

\[
J_n = Y_{s_n}E_n, \quad (59)
\]

and the relationship between electric and magnetic fields on the outermost boundary can be expressed as

\[
H_n = Y_s E_n. \quad (60)
\]

We can make a fictitious boundary \( \gamma_{n+1} \) in the background medium as shown in Fig. 5(a). Then, an equivalent model is obtained through enforcing an equivalent surface current at \( \gamma_{n+1} \). If we treat the medium between the target boundary \( \gamma_{n+1} \) and the outermost boundary \( \gamma_n \) of the object as a new one with the same constitutive parameters as those of the background medium, the proposed approach can be further extended to handle this objects.

The method in the section II-D is used to conduct the equivalence of a new layer of medium, as shown in the Fig. 5(b). We can obtain the relationship between tangential magnetic field and normal electric field on \( \gamma_{n+1} \) of the original model as

\[
H_{n+1} = Y_{s_{n+1}}E_{n+1}, \quad (61)
\]

where

\[
Y_{n+1} = \left[ P_{n+1}^{-1} + G_{n+1}^{(n+1)} V_n V_{n+1} P_{n+1}^{-1} \right]^{-1} U_{n+1} \quad (62)
\]

Then, in the equivalent model, an surface equivalent current is enforced at \( \gamma_{n+1} \) as shown in Fig. 5(c), where the relationship between tangential magnetic field and normal electric field on boundary \( \gamma_{n+1} \) can expressed as

\[
H_{n+1} = \hat{P}_{n+1}^{-1} \hat{U}_{n+1} \hat{E}_{n+1}. \quad (63)
\]

For the \( (n + 1) \)th layered medium, since the outermost medium is equal to the background medium, we get the following equation

\[
P^{(n+1)}_{n+1} = \hat{P}^{(n+1)}_{n+1}, \quad (64)
\]

\[
U^{(n+1)}_{n+1} = \hat{U}^{(n+1)}_{n+1}. \quad (65)
\]

Other procedures for this scenario is exactly the same as those in the previous proposed method. Finally, we can get the equivalent current \( J_{n+1} \) of the target surface \( \gamma_{n+1} \) as

\[
\tilde{J}_{n+1} = H_{n+1} - \tilde{H}_{n+1} = Y_{s_{n+1}}E_{n+1}. \quad (66)
\]

IV. NUMERICAL RESULTS AND DISCUSSION

A. A Single Layer Coated Cylindrical Conductor

An infinitely long single dielectric layer coated cylindrical conductor is first considered to demonstrate the accuracy of the proposed approach. As shown in Fig. 7, the radius of the copper with the conductivity of \( 5.6 \times 10^7 \) S/m and the coated layer dielectric medium with \( \varepsilon_r = 2.3 \) are 10 mm and 4 mm, respectively. In all the simulations, the background medium is air. A plane wave incidents from the \( x \)-axis with the frequency of 30 GHz. The radar cross section (RCS) is compared with the proposed approach, the TRF, and the Comsol.

Two scenarios are considered for the proposed approach: the single layer object with/without a fictitious extension boundary in the background medium as shown by the dashed line in Fig. 7. This boundary is selected as 6 mm away from the outermost boundary as shown in Fig. 7. However, this boundary can be selected arbitrarily. There is only one implication that the should be enclosed.

As shown in Fig. 8, all the results obtained from the proposed approach with/without fictitious extension boundary, the TRF, the Comsol, show excellent agreement. It demonstrates that the proposed approach can accurately model the multilayer embedded objects. In addition, the fictitious extension boundary can model objects accurately. As shown in [20], it can be used to model complex structures with significant performance improvement in terms of time cost, condition number of the coefficient matrix.

B. A Three-layered Dielectric Cylinder

A three-layered dielectric concentric cylinder is considered as shown in Fig. 9(a). The radii of each cylinder are 1 m, 2 m, and 3 m. The relative permittivity of each layer are 2, 3, and 1.5. A plane wave with the frequency of 300 MHZ incidents from the \( x \)-axis. The RCS is obtained from the Comsol, the TRF, and the proposed approach.

As shown in Fig. 2(b), results obtained from three approaches agree well with each other. It shows that the proposed approach can accurately model the multilayer embedded dielectric media.
Fig. 7. A cylindrical conductor coated with one layer of dielectric medium and an extended fictitious boundary is introduced in the exterior region. Dash line denotes the fictitious boundary.

Fig. 8. RCS obtained from the Comsol and the proposed approach applied to boundary $\gamma_2$ and $\gamma_3$.

Fig. 9. Configurations of a three layered dielectric cylinder.

Table I shows the overall count of unknowns for the three approaches. The count of degree of free (DOF) for the Comsol is 505,477, which is significantly more than the other two approaches. This is because the Comsol is a software package based on the FEM and volumetric meshes are required. For the TRF approach, 3784 unknowns are required to solve this problem. However, only 632 unknowns are required in the proposed method, which only has 16% and 43% of time cost of unknowns of the TRF. Since both the electric and magnetic currents are required on all the interfaces of different media in the TRF, the overall count of unknowns is significantly larger than that of the proposed approach. However, since a single surface electric current is only enforced on the outermost boundary, it requires much less unknowns without compromising accuracy.

Fig. 10. RCS obtained from the Comsol and the proposed method

Fig. 11. Configurations of the multilayer embedded objects.
In this paper, we proposed a novel and unified single-source SIE to model multilayer embedded objects. The proposed approach only needs a single electric current source on the outermost boundary of objects to solve this complex problem compared with 529,123 unknowns for the FEM, 6268 for the TRF, which only has 12% and 48% of time cost of unknowns of the TRF. Similar to the previous two examples, the proposed approach uses significantly less unknowns compared with other two approaches. This is because only a single electric current source is enforced on the outermost boundary of objects in the proposed approach.

V. CONCLUSION

In this paper, we proposed a novel and unified single-source SIE to model multilayer embedded objects. The proposed approach only needs a single electric current density on the outermost boundary of objects, which can be derived by recursively applying the equivalent theorem on each boundary from inner to exterior regions. The surface electric field is related to the magnetic field on the outermost boundary through a differential surface admittance operator. Then, with combining the EFIE with the equivalent current density, we can accurately solve the scattering problems by multilayer embedded objects. Numerical results demonstrate that the proposed approach can significantly improve the performance in terms of the count of unknowns and time costs, which implies great potential for electromagnetic analysis of complex multilayer embedded objects. Currently, development of the proposed approach into three-dimensional general scenarios is in progress. We will report more results on this topic.

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