A DIMENSIONAL STUDY OF DISK GALAXIES
(AN EXCUSE TO TALK ABOUT THE HUBBLE SEQUENCE)

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RESUMEN
El resumen será traducido al español por los editores. We present a highly simplified model of the
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momentum. We show through simple physical scalings that once the mass has been fixed, the angular momen-
tum parameter λ is expected to regulate such critical galactic disk properties as colour, thickness of the disk
and disk to bulge ratio. It is hence expected to be the determinant physical ingredient resulting in a given
Hubble type. A simple analytic estimate of λ for an observed system is provided. An explicit comparison of the
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ABSTRACT
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IES:STRUCTURE

1. INTRODUCTION

The first thing an astronomer wants to know
about a galaxy is its morphological type, typically
expressed through the Hubble classification scheme,
introduced about 80 years ago.

The continued usage of the Hubble classification
owes its success to the fact that on hearing the Hub-
ble entry of a galaxy, one immediately forms an im-
age of the type of galaxy that is being talked about.
A wide variety of physical features of galaxies show
good monotonic correlations with Hubble type, de-
spite the presence of significant overlap and disper-
sion. To mention only a few, total magnitudes de-
crease towards later types and colours become bluer
while gas fractions diminish from early to late types
e.g. Roberts & Haynes 1994). Bulge magnitudes
and bulge to disk ratios decrease (e.g. Pahre et al.
2004) and disks become thinner (e.g. Kregel et al.
2005) in going to later types.

However, galactic classification schemes suffer
from several shortcomings. First, the diagnostics
which enter into the assignment of type, do so in a
fundamentally subjective manner. One has to take
the images to an expert who will somehow integrate
various aspects of the galaxy in question to arrive
at the type parameter. That different authors gen-
erally agree, together with the correlations between
type and various galactic parameters is indicative
of a solid physical substrate to galaxy classification
schemes, the particular nature of which however, has
remained elusive. Second, type parameters are es-
sentially qualitative which make it difficult to relate
type to definitive quantitative aspects of a galac-
tic system, and bring into question the validity of
any statistical mathematical analysis performed on
galactic populations based on type. Finally, the type
given to a galaxy is highly dependent on the infor-
mation one has regarding the system. This stems
from the inputs that determine the galactic type,
the relevance of the bulge, typically redder than the
disk, means that when observing at longer rest frame
bands galaxies shift to earlier types, an effect com-
pounded with the high wavelength sensitivity of the

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bright HII regions which define the structure and morphology of the spiral arms. Similarly, projection effects become important, as the structure of the arms completely disappears in edge-on disks, eliminating one of the diagnostics relevant to the assignment of type.

Monotonic trends with Hubble type suggest that after having fixed the mass, there might exist one other parameter whose variation gives rise to the Hubble sequence. Many approaches to galaxy formation and evolution have appeared over the years, generally reaching the conclusion that it is the angular momentum of a galactic system what determines its main characteristics, e.g. Fall & Efstathiou (1980), Firmani et al. (1996).

2. THEORETICAL FRAMEWORK

Having chosen the angular momentum as our second parameter we now construct a simple physical model for a galaxy which allows to estimate this parameter directly from observed galactic properties.

In the interest of capturing the most essential physics of the problem, the model we shall develop will be the simplest one could possibly propose, little more than a dimensional analysis of the problem. We shall model only two galactic components, the first one a disk having a surface mass density $\Sigma(r)$ satisfying:

$$\Sigma(r) = \Sigma_0 e^{-r/R_d}, \quad (1)$$

Where $r$ is a radial coordinate and $\Sigma_0$ and $R_d$ are two constants which are allowed to vary from galaxy to galaxy. The second component will be a fixed dark matter halo having an isothermal $\rho(r)$ density profile, and responsible for establishing a rigorously flat rotation curve $V_d$ throughout the entire galaxy, such that

$$\rho(r) = \frac{1}{4\pi G} \left( \frac{V_d}{r} \right)^2, \quad (2)$$

and a halo mass profile $M(r) \propto r$. We shall refer not to the total angular momentum $L$, but to the dimensionless angular momentum parameter

$$\lambda = \frac{L}{GM^{3/2}}, \quad (3)$$

where $E$ is the total energy of the configuration. We now assume $|E|$ is given by $1/2$ the potential energy of the halo, which dominates through a halo to disk mass ratio $M_H/M_d = F \approx 25$. If the specific angular momenta of disk and halo are equal, $I_d = I_H$, as would be the case for an initially well mixed protogalactic state, we can introduce a disk Tully-Fisher relation: $M_d = A_{TF} V_d^{3.5}$ into to yield:

$$\lambda = 21.8 \frac{R_d/kpc}{(V_d/kms^{-1})^{3/2}}. \quad (4)$$

Where $A_{TF}$ and $F$ were calibrated from Galactic parameters. The above equation allows a direct estimate of $\lambda$, a dimensionless numerical parameter with a clear physical interpretation for any observed galaxy, with no degree of subjectivity and little sensitivity to orientation effects. Also, given the Tully-Fisher relation, it is clear that $V_d$ in eq (4) can be replaced for total magnitude on a given band, for cases where the rotation velocity is not available.

We now study the expected scaling between $\lambda$ and other observables of a disk galaxy, for example colour, gas fraction, star formation activity, and degree of flatness of the disk. The main ingredient will be the Toomre parameter,

$$Q(r) = \frac{\kappa(r)\sigma(r)}{\pi G \Sigma(r)}. \quad (5)$$

This parameter measures the degree of internal gravitational stability of the disk, with $\kappa(r)$ accounting for the disruptive shears induced by the differential rotation, $\sigma(r)$ modeling the thermal pressure of a component, both in competition with $\Sigma(r)G$, the self gravity of the disk. Values of $Q \leq 1$ are interpreted as leading to gravitational instability in a 'cold' disk. We replace $\kappa(r)$ in eq (5) with $\Omega(r) = V_d/r$, the angular velocity of the disk.

The ratio of disk scale height to disk scale length, $h/R_d$, is a measurable characteristic of galaxies which it is easy to show, will scale with $\lambda$. Assuming a thin disk, virial equilibrium in the vertical direction yields a relation between $h$ and $\Sigma$,

$$h = \frac{\sigma_g^2}{2\pi G \Sigma}. \quad (6)$$

We use this relation for $h$ to replace the gas velocity dispersion appearing in equation (5) for a combination of $h$ and the surface density. The dependence on $\Sigma$ is replaced by a dependence on $M_d$, $M_H$ and $\lambda$ to get a new expression for the Toomre’s stability criterion, which evaluating radial dependences at $r = R_d$ yields,

$$Q^2 = e25/2 \left( \frac{M}{M_d} \right) \left( \frac{h}{R_d} \right) \lambda \quad (7)$$

With $F = 1/25$, evaluating at $Q = 1$, the stability threshold suggested by self-regulated star formation cycles, the ratio $h/R_d$ is obtained as:
Thinking of star formation as determined by a Schmidt law, $\dot{\Sigma} = \pi G \Sigma^2 g$, we can use a characteristic time for the star formation process in the disk $\tau_{sf} = \Sigma / \dot{\Sigma}$, that gives an idea of the time a galaxy can support a star formation rate of constant magnitude. From the above relation $\tau_{sf}$ becomes

$$\tau_{sf} = \frac{\pi G}{\Sigma g} = \frac{2\pi^2 GR_d^2}{M_d} \propto \lambda^2,$$ (9)

at fixed disk mass. Alternatively, one could start from an empirical Schmidt law of power $n$, a scaling $\tau_{sf} \propto \lambda^n$ will always result, which does not alter our conclusions provided $n > 1$, a general trait of Schmidt laws found in the literature.

Large values of $\lambda$ will correspond to long star formation periods; for this case we expect to have galaxies with young populations, looking relatively blue and having the high gas fractions of late type disks. Galaxies with low $\lambda$ will have short star formation periods; for them we will hardly see young stars and they will look red and gas poor, as is the case for early types. A scatter on this trend will be introduced by variations in total mass, together with a systematic reduction in $\tau_{sf}$ in going to larger masses (c.f. equation 10), in accordance with larger average galactic masses found at earlier types.

3. DATA AND MODEL COMPARISONS

As mentioned in the preceding section, we expect a correlation between the $\lambda$ parameter and the Hubble type, since the analysis of section 2 leads one to suspect that it is this what primarily determines the properties of the galaxy which are taken into account for its classification. We now test these expectations by direct comparison against two galaxy samples: the atlas of Courteau (1996) and the sample from Kregel et. al (2005), henceforth ASSSG and dGKK, respectively.

In figure 1 we present the distribution of the ASSSG sample as a function of $\lambda$, estimated through $\Lambda$. For the entire sample, the mean value is $< \lambda > = 0.0645$. In general, the bulk of a galactic population moves to larger values of $\lambda$ for later types, the mean value for each group is: $< \lambda >_{Sb} = 0.0574, < \lambda >_{Sbc} = 0.0649, < \lambda >_{Sc} = 0.0689$.

We see in figure 1 an approximately Gaussian shape, with a considerable extension towards the larger values of $\lambda$. We do not explore the details of this distribution further, as the ASSSG sample is not intended to be complete or statistically unbiased in any sense. However, use of eq(4) in one such sample should prove useful in obtaining an empirical distribution of $\lambda$ parameters, to compare for example, against predictions from cosmological models.

The analysis of the distribution of $h/R_d$ ratios was made with the dGKK sample. It contains galaxies with more morphological types; from Sa to Scd galaxies. Figure 2 shows early galaxies presenting varied values for the $h/R_d$ ratio, although the means show a clear diminishing trend in going to latter types. Late type galaxies at the Sc end clearly show on average much thinner disks than the earlier types, as confirmed by several authors over the last few years.

The complementary plot to figure 2 is given in figure 3, where instead of presenting $h/R_d$ vs $\lambda$, we use a logarithmic plot to better detect the presence of a relation between these two parameters of the type the dimensional analysis of section 2 leads us to expect, equation(5). The line drawn in figure 3 is the best fit straight line for the sample, having a slope of $-0.4 \pm 0.9$, not at odds with value of $-1$ predicted by the simple model. Trends for colour distributions

![Fig. 1. Distribution of values of $\lambda$ through eq(4) for the complete ASSSG sample, with dots showing the binning.](image1)

![Fig. 2. $h/R_d$ ratios for galaxies according to their Hubble type, for galaxies in the dGKK sample.](image2)
Fig. 3. Logarithmic plot of the relation between $\lambda$ and $h/R_d$, for galaxies in the dGKK sample, the solid line shows a linear fit to the data, yielding a slope of $-0.4 \pm 0.9$.

and bulge to disk ratios (not shown) are also qualitatively very similar when plotting against Hubble type or $\lambda$, Hernandez & Cervantes-Sodi (2006).

We must now check our estimate of $\lambda$ in cases where this parameter is in fact known, galaxy formation simulations where $\lambda$ acts as an input parameter. We use results from two groups, models calculated in connection with a study of star formation in disk galaxies applied to the Milky Way (Hernandez et al. 2001), and the models published by van den Bosch (2000).

Both models simulate the cosmological dark matter and baryonic mass accretion through a primordial fluctuation spectrum, and the subsequent, self-consistent dynamical and star formation evolution of a galaxy. The details of both models vary in numerical approaches, resolution, time step issues and the level of approximation and inclusion of the many different physical aspects of the complicated problem. Figure 4 shows the $\lambda_{\text{model}}$ values given as input by the above authors to their detailed formation codes, with the $y$-axis showing $\lambda$ from equation (4) applied to the final results of the code evolution which followed the model for $\sim 13$ Gyr.

It is interesting that the particular features of a galactic formation scenario which we are interested in treating here, $\lambda$, final resulting $R_d$ and flat rotation region $V_d$, are evidently very well modeled by the trivial physics that went into eq(4), across the more than 2.5 orders of magnitude covered by the masses in the modeled galaxies. This can be understood by considering that eq(4) is the result of two fundamental hypothesis:

To conclude, we have presented an easily applicable way of estimating the $\lambda$ parameter, a theorist’s favourite descriptor of a galaxy type, for any real observed disk galaxy, which is therefore proposed as a physically motivated, objective classification parameter for disk galaxies. An extensive development of these ideas can be found in Hernandez & Cervantes-Sodi (2006).

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