Modelling the movement of log trucks with a hydro-mechanical transmission

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Abstract. The paper proposes a mathematical model of the movement of log trucks with hydro-mechanical transmission, which represents the analytical functional dependence of the speed of the log truck on the covered path. On the basis of real data it is shown that the traction characteristic of hydro-mechanical transmission is best approximated by hyperbolic regression. In this case, the solution of heterogeneous nonlinear differential equation of motion can be obtained by the approximate analytical method through decomposition of a logarithmic component in series. The calculation example showed that the obtained approximate analytical solution deviates from the numerical solution by no more than 0.002%. The obtained model allows one to determine the analytical dependencies for speed and time, which can be used to calculate the modes of movement of log trucks with the hydro-mechanical transmission.

1. Introduction

Logging enterprises in transport operations use log trucks equipped with various types of transmission: mechanical, electrical, hydromechanical, hydrostatic, etc. At the same time, as practice has shown, log trucks are operated not only on public roads and forestry roads of permanent validity, but also on temporary roads, as well as in difficult operating conditions, where the use of log trucks with the hydromechanical transmission is preferable. [1, 2].

There are three hydraulic hybrid drive train architectures, namely: parallel, series and power-split. The parallel architecture assists the conventional drive train by storing and releasing excessive kinetic energy as needed. The series architecture is a hydrostatic transmission with high- and low-pressure hydraulic accumulators placed between two pump/motors as a means to store energy. The power-split architecture splits the input power into two power paths, allowing for both full engine control and efficient power transfer. The power-split architecture is also known as the Hydro-mechanical Transmission (HMT) when hydraulic components are used. Hydro-mechanical transmission are widely used in off-road and agricultural vehicles due to their reliability, smoothness, and high-efficiency power transfer [3–5].

The problems of optimization and control of a hydro-mechanical power split transmission are discussed in [5–9]. In paper [10] summarizes developments, key techniques and trends, analyses problems and difficulties in design and application of hydro-mechanical power split transmission and discusses the methods for solving these problems. At the same time, despite the existence of a lot of work on research of hydro-mechanical transmissions [11–17], the issue of truck traffic modeling with this transmission remains relevant. To resolve this issue, you need information about the dynamic characteristics and modes of movement of log trucks.
2. Materials and methods
The mathematical model of a log truck movement can be represented as a nonlinear heterogeneous differential equation (1) [18]:

\[ \frac{G \delta}{g} \frac{dv}{ds} = T(v), \]  

(1)

where \( v \) – speed of the log truck [m/s];
\( s \) – distance covered [m];
\( G \) – log truck weight [kg];
\( \delta \) – rotational mass inertia ratio, [];
\( g = 9.81 \) – free fall acceleration [m/s\(^2\)];
\( T(v) \) – traction characteristic of a log truck [kg].

In [18] it was shown that for approximation of the traction characteristic of the hydromechanical transmission hyperbolic dependence of the species is best suited:

\[ T(v) = \frac{A + B}{v}, \]  

(2)

where \( A, B \) – hyperbolic regression coefficients.

Taking into account the inhomogeneous part (2), the differential equation (1) after integration in quadrature has an analytic general solution of the form:

\[ s - \frac{1}{2} g \delta v(s)^2 + \frac{G \delta v(s)B}{gA^2} - \frac{G \delta B^2 \ln(Av(s) + B)}{gA^3} + _C1 = 0, \]  

(3)

where \(_C1\) – constant integration.

Since the Cauchy problem is being solved, to obtain the private solution of equation (1) it is necessary to set the initial conditions:

\[ v(s_0) = v_0 \]  

(4)

where \( v_0 \) – initial speed of the log truck;
\( s_0 = 0 \) – initial value of the path covered - this assumption is adopted to facilitate obtaining a convenient type of decision (1).

The value of the constant \(_C1\) is obtained from equation (3) after substitution of initial conditions (4) in it:

\[ _C1 = \frac{1}{2} \frac{G \delta}{gA^3} \left( v_0^2 A^2 - 2 v_0 B A + 2 B^2 \ln(A v_0 + B) \right). \]  

(5)

Now the required private solution of the log truck movement is represented by the system of equations (3) and (5):

\[ \begin{cases} 
  s - \frac{1}{2} g \delta v(s)^2 + \frac{G \delta v(s)B}{gA^2} - \frac{G \delta B^2 \ln(Av(s) + B)}{gA^3} + _C1 = 0 \\
  _C1 = \frac{1}{2} \frac{G \delta}{gA^3} \left( v_0^2 A^2 - 2 v_0 B A + 2 B^2 \ln(A v_0 + B) \right) 
\end{cases} \]  

(6)

Unfortunately, the presence of the speed function under the logarithm makes it impossible to obtain an accurate analytical solution to algebraic equation (6).

Due to the need to obtain speed characteristics of the road train in the form of functional dependencies for further research, as an alternative, it is proposed to find an approximate analytical solution. For this, the logarithmic term in (6) is proposed to be expanded in a Taylor series.

Such an expansion is well known for a function of the form \( \ln(x + 1) \) in a vicinity of the point \( x = a \)

\[ \ln(x + 1) = \ln(a + 1) + \frac{x - a}{a + 1} - \frac{(x - a)^2}{2(a + 1)^2} + \frac{(x - a)^3}{3(a + 1)^3} + \cdots + \frac{(-1)^n(x - a)^{n+1}}{(n + 1)(a + 1)^{n+1}} \pm \cdots. \]  

(7)

For our case, under the logarithm there is a function from (3) \( Av(s) + B \). The point in the vicinity
of which it is necessary to expand the logarithm in a series is determined from the initial conditions
\[ A v_0 + B. \]
As a result we get a substitution:
\[
\begin{cases}
x + 1 = A v(s) + B \\
a = A v_0 + B
\end{cases}
\]  
(8)

We use only the first three terms for the series expansion so that the degree of the approximating polynomial is no higher than the second:
\[
\ln(x + 1)_{x=a} = \ln(a + 1) + \frac{x - a}{a + 1} - \frac{(x - a)^2}{2(a + 1)^2}.
\]  
(9)

Then, using the replacement of variables (8) in the expansion (9), we obtain the final expression for the logarithmic component of the solution:
\[
\ln(A v(s) + B) = \ln(A v_0 + B + 1) + \frac{A v(s) - 1 - A v_0}{A v_0 + B + 1} - \frac{1}{2} \frac{(A v(s) - 1 - A v_0)^2}{(A v_0 + B + 1)^2}.
\]  
(10)

Finally, when expression (10) is added to system (6), we obtain a system of algebraic equations, from which it is possible to express the required speed of the log truck in the form of an analytical functional dependence:
\[
\left\{
\begin{array}{l}
\frac{s}{2} - \frac{1}{2} g \delta v(s)^2 \frac{G \delta v(s) B}{g A} + \frac{G \delta B^2 \ln(A v(s) + B)}{g A^3} + c_1 = 0 \\
\end{array}
\right.
\]  
(11)

Substituting the second and third expressions of system (11) into the first expression, and grouping this expression by powers of \( v(s) \), we obtain a quadratic equation for \( v(s) \) of the form:
\[
k_2 v(s)^2 + k_1 v(s) + s + k_0 = 0,
\]  
(12)

where \( k_2, k_1, k_0 \) – coefficients at the corresponding powers of \( v(s) \), the values of which are determined by the following formulas:
\[
k_2 = -\frac{G \delta}{2 g A} + \frac{1}{2} \frac{G \delta B^2}{g A (A v_0 + B + 1)^2}.
\]  
(13)

\[
k_1 = -\frac{G \delta B^2 (A v_0 + B + 1) - \frac{(-1 - A v_0) A}{(A v_0 + B + 1)^2}}{g A^3} + \frac{G \delta B}{g A^3}.
\]  
(14)

\[
k_0 = -\frac{G \delta B^2 \left( \ln(A v_0 + B + 1) + \frac{-1 - A v_0}{A v_0 + B + 1} - \frac{1}{2} \frac{(-1 - A v_0)^2}{(A v_0 + B + 1)^2} \right)}{g A^3} + \frac{1}{2} \frac{G \delta (v_0^2 A^2 - 2 v_0 B A + B^2 \ln(A v_0 + B))}{g A^3}.
\]  
(15)

As a result, one of the two solutions of equation (12), which gives a positive value of the speed of movement, has the form:
\[
v(s) = -\frac{1}{2} k_1 + \sqrt{k_1^2 - 4 k_2 s - 4 k_2 k_0}
\]  
(16)
Thus, analytical expressions (13)–(16) represent a mathematical model of the speed of a log truck with a hydromechanical transmission. It is obvious that from (16) it is also easy to obtain an expression for estimating the time of movement:

\[ t(s) = \frac{s}{v(s)}. \]  

(17)

3. Results and Discussion
We consider an example of calculating the characteristics of a log truck movement according to the obtained model (13)–(16).

Let us consider the ways of approximation of the real traction characteristic of the log truck. In Figure 1 you can see the initial data of the traction force dependence on the speed of the log truck, as well as their approximation curves.

![Figure 1. Approximation of traction characteristic of hydromechanical transmission.](image)

For approximation, five types of functions were used, the form of which, together with the results of the approximation quality, is presented in Table 1.

| Type of approximation function | Mean Absolute Percentage Error (MAPE), % | Determination coefficient $R^2$ |
|-------------------------------|------------------------------------------|-------------------------------|
| $F_1(v) = A - B\sqrt{v}$     | 23.3%                                     | 0.981                         |
| $F_2(v) = A - Bv^2$          | 43.6%                                     | 0.933                         |
| $F_3(v) = A + B/v$           | 2.0%                                      | 0.999                         |
| $F_4(v) = A - Bv$            | 30.5%                                     | 0.968                         |
| $F_5(v) = A + Bv + Cv^2$     | 4.7%                                      | 0.998                         |

From Table 1 it follows that the best approximation results are obtained using the hyperbolic regression function of the form:

\[ F_3(v) = A + B/v, \]  

(18)

where the corresponding coefficients take the values $A = 580$ and $B = 3119$.

Thus, for model (13)–(16), we set the following calculated parameters:
Taking into account (19), the equation for the speed of the road train (16) takes the form:

\[ v(s) = 2.120 + 0.223\sqrt{478.510 + 8.966s}. \]  

Figure 2 shows a graph of changes in the speed of the log truck at a distance of 0 to 100 meters. The circles also mark the numerical solution of equation (1). The maximum MAPE between the approximate analytical solution (16) and the numerical solution of equation (1) did not exceed 0.002%. This is a good result for a model that can be used for engineering calculations of log truck driving modes.

\[ G = 30000 \text{ kg}; \delta = 1.048; g = 9.81 \text{ m/s}^2; v_0 = 7 \text{ m/s}; A = 580; B = 3119. \]  

4. Conclusion
As a result of the research, an approximate analytical solution of the differential equation of motion of a log truck with a traction characteristic of a hyperbolic type was obtained. Decomposition in series of logarithmic components of analytical solution allowed one to obtain an approximate analytical solution of differential equations of motion. High accuracy of the obtained solution (mean absolute percentage error of no more than 0.002% from the numerical solution) will allow one to use this solution for scientific research and calculation of motion modes of a log truck depending on changes of model parameters.

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