Massive torsion modes, chiral gravity, and the Adler-Bell-Jackiw anomaly

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Regularization of quantum field theories introduces a mass scale which breaks axial rotational and scaling invariances. We demonstrate from first principles that axial torsion and torsion trace modes have non-transverse vacuum polarization tensors, and become massive as a result. The underlying reasons are similar to those responsible for the Adler-Bell-Jackiw (ABJ) and scaling anomalies. Since these are the only torsion components that can couple minimally to spin $\frac{1}{2}$ particles, the anomalous generation of masses for these modes, naturally of the order of the regulator scale, may help to explain why torsion and its associated effects, including CPT violation in chiral gravity, have so far escaped detection. As a simpler manifestation of the reasons underpinning the ABJ anomaly than triangle diagrams, the vacuum polarization demonstration is also pedagogically useful. In addition it is shown that the teleparallel limit of a Weyl fermion theory coupled only to the left-handed spin connection leads to a counter term which is the Samuel-Jacobson-Smolin action of chiral gravity in four dimensions.

PACS numbers: 11.15.-q, 11.40.Ha, 04.62.+v, (Journal-ref: Class. Quantum Grav. 20 (2003) 1379-1387)

I. INTRODUCTION AND OVERVIEW

Torsion ($T_A = de_A + A_{AB} \wedge e^B$) arises naturally in Riemann-Cartan spacetimes when the vierbein, $e_{\mu A}$, and spin connection, $A_{\mu AB}$, are assumed to be independent. In minimal coupling schemes only spinors couple to torsion, and even then only the axial and trace modes of torsion couple to spin $\frac{1}{2}$ particles. Actually, in a hermitian theory, only the axial torsion mode, $\tilde{A}_\mu \equiv \frac{1}{2} g_{\mu \lambda} \epsilon^{\lambda \gamma \alpha \beta} e_{\lambda \alpha} T^A_{\alpha \beta}$, interacts minimally with spin $\frac{1}{2}$ matter. Recent works have however revealed that even the well-studied Adler-Bell-Jackiw (ABJ) anomaly [1] receives further contributions from torsional invariants [2-4]. Moreover, as we shall demonstrate, the vacuum polarization diagram with two external axial torsion vertices is not transverse, and its divergence is controlled by the Nieh-Yan term [5]. This breakdown in transversality occurs in addition to that manifested by the ABJ triangle diagrams that give rise to a term quadratic in the curvatures. A striking consequence of this non-transversality is the generation of mass. We may think of the axial torsion mode as an “axial torsion photon” [4], coupled to a current whose conservations has been compromised because of anomaly considerations. The associated “gauge” invariance, the $\gamma^5$ rotational symmetry, is therefore broken, and a mass results. In this paper, we describe explicitly how this phenomenon takes place. It should be noted that the breakdown in current conservation poses no consistency problems; since axial torsion modes are not gauge field modes, and are not responsible for any local symmetries.

The standard model incorporates maximal parity and charge conjugation non-conservation by assigning left- and right-handed fermions to different representations of the internal gauge group. It behooves us to pursue the extent to which chirality can be used as a defining characteristic of particle interactions, including gravity. That context leads us to use only Weyl spinors and the Ashtekar formulation of gravity [6,7]. The result in using only left-handed fermion fields is a minimal coupling $J^\mu_L(iB_{\mu} + A_{\mu}) \equiv J^\mu_L C_{\mu}$ of both the axial torsion and torsion trace fields to the total singlet current $J^\mu_L$ traced over all left-handed fermion fields. Here, $A_{\mu} \equiv \frac{1}{2} \tilde{A}_\mu$, while $B_{\mu} \equiv \frac{1}{2} \epsilon^{\mu A} T_{\mu \nu A}$ is the trace of the torsion field. The first term in $C_{\mu}$ is anti-hermitian relative to the second, and hermitizing the action would eliminate it completely. But doing so requires bringing in right-handed conjugate Weyl fields, which in turn are coupled to the self-dual or right-handed spin connection fields. These fields are distinct components of the spin connection, and it has already been shown in Ref. [6] that general relativity field equations can be reproduced without reference to them. The system can be defined by a holomorphic action that is dependent only upon left-handed fields, thereby extending this characteristic feature of the standard model to cover gravity interactions as well [7,8]. In this scheme of things, the appearance of $iB_{\mu}$ is entirely natural. In the regularization scheme adopted in this paper [9], this property is also maintained, therefore both torsional modes must have the same mass. The generation of mass for the $B$ field
is not self-evident, since it has a vector coupling to the fermion fields rather than an axial vector coupling. Based upon our experience with Quantum electrodynamics (QED), we might have argued that the associated quanta remain massless. The difference arises because of the anti-hermitian coupling. The operator appearing in the regularized integrals involves the positive definite form in Euclidean space \( i\hat{D} \hat{D} \). Now \( (i\hat{D})^\dagger \) differs from \( i\hat{D} \), because of the anti-hermitian nature of the coupling. For conventional internal gauge fields, the two forms of the covariant derivatives are the same, leading to hermitian Dirac operators. We shall show how this difference causes the vacuum polarization tensor for \( B_\mu \) to be non-transverse as well.

**A. Vacuum polarization and the ABJ Current**

Consider first the action of a bispinor theory in teleparallel spacetimes with flat vierbein \( \epsilon^A = \delta^A_\mu \) but with nontrivial axial torsion coupling

\[
S = -\frac{1}{2} \int d^4x \overline{\Psi} \gamma^\mu (i\partial_\mu + A_\mu \gamma^5) \Psi + H.c. \tag{1.1}
\]

The ABJ current \( J^5 \) has naive expectation value

\[
\langle J^5(x) \rangle = -\lim_{x\to y} \text{Tr} \{ \gamma^5 \gamma^5 \frac{1}{(i\hat{D} + A\gamma^5)} \delta(x - y) \}, \tag{1.2}
\]

and the corresponding vacuum polarization tensor, \( \Pi^{\mu\nu} \), defined by the Fourier transform

\[
\frac{\delta \langle J^5(x) \rangle}{\delta A_\nu(y)} \mid_{A_\mu = 0} = \int \frac{d^4k}{(2\pi)^4} \Pi^{\mu\nu}(k) e^{ik(x-y)}, \tag{1.3}
\]

leads to

\[
\Pi^{\mu\nu}(k) \propto \int d^4p \text{Tr} \{ \gamma^\mu \gamma^5 \frac{1}{\not{p} + \not{k}} \gamma^\nu \gamma^5 \frac{1}{\not{p}} \}. \tag{1.4}
\]

Had this expression been well defined, it would have been no more than

\[
\Pi^{\mu\nu}(k) \propto \int d^4p \text{Tr} \{ \gamma^\mu \frac{1}{\not{p} + \not{k}} \gamma^\nu \frac{1}{\not{p}} \}, \tag{1.5}
\]

since the two \( \gamma^5 \)'s cancel out in the trace, and we would have obtained the usual vacuum polarization amplitude for which we do not expect a longitudinal component. But it is not, for the integration over fermion loop momentum diverges. We will need a regularization scheme, Pauli-Villars for instance, to tame this divergence before performing any Dirac algebra. Any gauge-invariant scheme however will compromise symmetry generated by the axial current. Summing over the propagators for all the fields, including the massive regulators, results in

\[
\Pi^{\mu\nu}(k) \propto \sum_n C_n \int d^4p \text{Tr} \{ \gamma^\mu \gamma^5 \frac{1}{\not{p} + \not{k} + im_n} \gamma^\nu \gamma^5 \frac{1}{\not{p} + im_n} \}, \tag{1.6}
\]

with \( C_n = \pm 1 \), depending on whether the regulators are anti-commuting or commuting. (For the original fermion multiplet, \( C_0 = 1 \) and \( m_0 = 0 \). We assume analytic continuation to Euclidean Green functions.) By moving the second \( \gamma^5 \) to the left to cancel out the first, we observe that \( m_n \) changes its relative sign with respect to \( (\not{p} + \not{k}) \) in the denominator. Consequently,

\[
\Pi^{\mu\nu}(k) \propto \sum_n C_n \int d^4p \text{Tr} \{ \gamma^\mu \frac{1}{(\not{p} + \not{k}) - im_n} \gamma^\nu \frac{1}{\not{p} + im_n} \}. \tag{1.7}
\]

The integrals over the loop momentum will be well defined for a suitable set \( \{ C_n, m_n \} \). An explicit set will be presented for the Weyl theory. It is also applicable to the bispinor theory here, consistently yielding a polarization magnitude which is twice that of the Weyl theory. The important point is that if we had started with a vector (instead of the axial vector) coupling, the result for Eq. (1.7) would have been

\[
\Pi^{\mu\nu}(k) \propto \sum_n C_n \int d^4p \text{Tr} \{ \gamma^\mu \frac{1}{(\not{p} + \not{k}) + im_n} \gamma^\nu \frac{1}{\not{p} + im_n} \}. \tag{1.8}
\]
Instead. This integral would have produced a transverse polarization tensor. As it is, by rewriting one of the propagators in Eq. (1.7) as
\[
\frac{1}{(p + k) - im_n} = \frac{1}{(p + k) + im_n} - \frac{2im_n}{(p + k)^2 + m_n^2}.
\] (1.9)
we obtain two terms, the first of which is identical to the integral in Eq. (1.8). However, there is now an additional anomalous term
\[
\sum_n C_n \int d^4p \text{Tr}\{\gamma^{\mu} \frac{2im_n}{(p + k)^2 + m_n^2} \gamma^{\nu} (p - im_n)\} = 8g^{\mu\nu} \sum_n C_n \int d^4p \frac{m_n^2}{((p + k)^2 + m_n^2)p^2 + m_n^2}.
\] (1.10)
Clearly the anomalous component arises from both the axial vector coupling and the nontrivial regulator masses. From
\[
\Pi^{\mu\nu} = (k^{\mu}k^{\nu} - g^{\mu\nu}k^2)\Pi + g^{\mu\nu}\Pi',
\] (1.11)
we deduce
\[
k_\mu\Pi^{\mu\nu} \propto k^{\nu} \quad \text{and} \quad (\partial_\mu J^{5\mu})_{\text{Reg}} \propto \partial_\mu \tilde{A}^\nu \neq 0
\] (1.12)
at the level of vacuum polarization diagrams. The anomalous \(g^{\mu\nu}\Pi'\) contribution in the vacuum polarization tensor also implies that besides the usual \(F_{\mu\nu}F^{\mu\nu}\) piece required for the logarithmic divergence of transverse polarization, a mass counter term of the form \(A_\mu \tilde{A}^\nu\) is also needed for the non-zero longitudinal component in the effective action. \(\tilde{A}_\mu\) becomes massive as a result.

In vector QED, curbing divergences by naive momentum truncation also results in a non-transverse photon polarization tensor [10]. But this apparent breakdown of gauge invariance is an artifact of symmetry breaking “regularization” which can be removed altogether by proper gauge-invariant regularization schemes. In contradistinction, the non-transverse torsion polarization exhibited here is not the consequence of a “fake anomaly” resulting from “improper regularization” which breaks the symmetry of axial rotations. The complete ABJ anomaly assures us there are no regularization schemes that preserve singlet axial rotations, and at the same time respect all of the local symmetries, such as Lorentz and other gauge symmetries, which are present. The non-transverse polarization for axial torsion can thus be regarded as another manifestation of this phenomenon. \(\tilde{A}_\mu\) however transforms covariantly under diffeomorphisms, and is Lorentz invariant. The counter term necessary to compensate for a non-transverse polarization tensor, \(A_\mu \tilde{A}^\nu\), is therefore completely consistent with these local invariances. A similar term in QED would have violated local gauge invariance.

It is also instructive to exhibit and confirm the same effect using an alternative regularization method. For bispinors, we may write the effective action in curved space as
\[
\Gamma_{\text{eff.}} = -i \text{Tr} \ln[ e^{\frac{\gamma^5}{2} i\Delta} e^{-\frac{\gamma^5}{2}}],
\]
\[
i\Delta = \gamma^{\mu}(i\partial_\mu + \frac{i}{2} \omega_{\mu AB} \sigma^{AB} + A_\mu \gamma^5).
\] (1.13)
With heat kernel regularization and the Schwinger-DeWitt expansion [11], the ABJ anomaly with Euclidean signature has been demonstrated to take the form [2]
\[
\langle \partial_\mu J^{5\mu}\rangle_{\text{Reg.}} = 2i \lim_{t \to 0} \lim_{x \to x'} \frac{1}{(4\pi t)^2} \text{Tr}(x' \gamma^5 \exp[-t(i\Delta)^2]x')
\]
\[
= 2i \lim_{t \to 0} \frac{1}{(4\pi t)^2} \text{Tr}[\sum_{n=0}^{\infty} e^{\gamma^5 a_n t^n}].
\] (1.14)
Furthermore, it is known that the traces of the coefficients \(a_0, a_1\) and \(a_2\) contribute to the divergent part of the effective action. For instance, \(a_0 = \beta\) leads to the renormalization of the cosmological constant. We focus on the relevant coefficient \(a_1\) which, in our notation, is \(a_1 = -\frac{1}{24}R - 2A_\mu A^\mu - \gamma^5 e^{-i\partial_\mu (eA^\mu)}\). Clearly \(\text{Tr}(ea_1)\) (and thus the effective action) contains both the familiar Einstein-Hilbert term, \(eR\), as well as the \(eA_\mu A^\mu\) mass term which we also found to be required by the vacuum polarization computations above. It follows from Eq.(1.14) that the ABJ anomaly is
\[
\langle \partial_\mu J^5_{\mu} \rangle_{\text{Reg.}} = -\frac{2i}{(4\pi)^2 t} \partial_\mu A^\mu + \frac{2i}{(4\pi)^2} \text{Tr}(e\gamma^5 a_2). \tag{1.15}
\]

\(\text{Tr}(e\gamma^5 a_2)\) is the more familiar regulator scale independent part of the ABJ anomaly. But there is also the \(t\)-dependent first term \((t\) has the physical dimension of inverse regulator mass squared\). This is precisely the Nieh-Yan contribution to the ABJ anomaly. The linear dependence on \(A_\mu\) shows that the vacuum polarization is indeed the correct Feynman diagram process to consider for this purpose \([4]\). Therefore the non-transversality of the polarization tensor is not feasible to compute the polarization of torsion fields while preserving explicit left-handedness as well as Lorentz and determinant line bundle, and the effective action is not a straightforward functional determinant. Nevertheless, it is to the ABJ anomaly. The linear dependence on \(L\) and the original fermion multiplet is projected as \(\Psi^t\) from \(\Psi\) introduced by Frolov and Slavnov \([12]\). Specifically, to form invariant regulator masses, the internal space is doubled in the internal space (please see Ref. \([9]\) for further details.) This generalizes the invariant scheme first introduced by Frolov and Slavnov \([12]\). Specifically, to form invariant regulator masses, the internal space is doubled from \(T^a\) to

\[
T^a = \left( \begin{array}{cc} (-T^a)^\dagger & 0 \\ 0 & T^a \end{array} \right); \tag{2.2}
\]

and the original fermion multiplet is projected as \(\Psi_L = \frac{1}{2}(1 - \sigma^3)\Psi_L\), where

\[
\sigma^3 = \begin{pmatrix} 1_d & 0 \\ 0 & -1_d \end{pmatrix}. \tag{2.3}
\]

The chirality of the regularized theory with respect to the gravitational interaction is thus preserved even in the teleparallel limit. It can be shown \([9]\) the net effect of the regularization is to replace the \(\frac{1}{2}(1 - \sigma^3)\) projection of the bare currents by \(\frac{1}{2}(f(\frac{\beta\hat{h}_i}{\Lambda}) - \sigma^3)\) where \(f\) is the regulator function,

\[
f(\hat{\mathcal{D}}\hat{\mathcal{D}}^\dagger/\Lambda^2) = \sum_n C_n \left\{ \frac{i\hat{\mathcal{D}}(i\hat{\mathcal{D}})^\dagger}{[i\hat{\mathcal{D}}(i\hat{\mathcal{D}})^\dagger + m^2_n]} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n i\hat{\mathcal{D}}(i\hat{\mathcal{D}})^\dagger}{[i\hat{\mathcal{D}}(i\hat{\mathcal{D}})^\dagger + n^2\Lambda^2]} \right\}. \tag{2.4}
\]

To concentrate on the vacuum polarization of the torsion fields, we specialize to \(W_\mu^a = 0\), and flat vierbein \(e_{\mu A} = \eta_{\mu A}\) but retain nontrivial torsion couplings in the teleparallel limit. To wit, the Weyl action reduces to

\[
S_L = \int d^4 x [-e\overline{\Psi}_L \gamma^\mu i\partial_\mu \Psi_L + C_\mu \overline{\Psi}_L e_{\mu A} \gamma^A \Psi_L]. \tag{2.5}
\]

The bare current

\[
\frac{\delta S_L}{\delta C_\nu(x)} = (J^L_\nu)_{\text{bare}} = \lim_{x \to y} \text{Tr}(\gamma^\mu \frac{1}{2}(1 - \gamma^5) \frac{1}{i\hat{\mathcal{D}}} \frac{1}{2}(1 - \sigma^3)\delta(x - y)) \tag{2.6}
\]

is modified by the Pauli-Villars regulators to become
As demonstrated in Ref. [9], the $\sigma^3$ part vanishes automatically for fermion loops with four or less external vertices, and hence does not contribute to vacuum polarization diagrams.

With respect to the Euclidean\(^1\) inner product \((X|Y) = \int d^4x e^{x\mathcal{X}} Y\), the full curved space Dirac operator obeys \((i\mathcal{D})^\dagger = i\mathcal{D} + 2i\mathcal{B}\), and the positive-definite operator which appears in the regulator function \(f\) is \(i\mathcal{D}(i\mathcal{D})^\dagger\). In computing the vacuum polarization \(\Pi_{\mu\nu}\) defined as

\[
\Pi_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \Pi_{\mu\nu} e^{ik.(x-y)} \equiv \delta^{(2)}(J^\mu_L(x)) \bigg| \frac{\delta C_{\nu}(y)}{\delta C_{\nu}(y)} \bigg|_{C_{\nu}=0},
\]

we need retain only terms linear in $\tilde{A}_\mu$ and $B_\mu$ in the regularized current. So displaying just the relevant terms,

\[
\langle J^\mu_L(x) \rangle_{\text{Reg}} = \lim_{x \to y} \text{Tr} \left\{ \gamma^\nu \frac{i}{2}(1 - \gamma^5) \frac{1}{i\mathcal{D}} \frac{1}{2}(f - \sigma^3) \delta(x-y) \right\}
\]

\[
= \lim_{x \to y} \text{Tr} \left\{ \gamma^\nu \frac{i}{2}(1 - \gamma^5) \left( \frac{1}{2} \sum_n \frac{(-1)^n (i\mathcal{D})^\dagger}{i\mathcal{D}(i\mathcal{D})^\dagger + m_n^2} \right) - \frac{i}{2} \frac{1}{2}(f - \sigma^3) \delta(x-y) \right\},
\]

(2.7)

where $\sigma^3$ part vanishes automatically for fermion loops with four or less external vertices, and hence does not contribute to vacuum polarization diagrams.

In continuing to Euclidean signature $(+,-,+,+)$ our Dirac matrices satisfy $\gamma^{\mu\dagger} = \gamma^\mu$.\(^2\)
In the effective propagator with vacuum polarization insertions, it is known that a non-trivial longitudinal polarization causes a shift to a physically massive pole, even if the bare propagator is massless in the beginning (see, for instance, Ref. [10]). Since $\Pi^{\mu\nu}$ is the Fourier transform of $\delta^3_{\mathrm{R}}(k,\alpha)\left[\frac{1}{C_{\alpha}}\right]_{C=0}$, it follows that in addition to the more familiar curvature squared counter term $g^{\mu\nu}g^{\rho\beta}(\partial_\mu C_\nu - \partial_\nu C_\mu)(\partial_\beta C_\rho - \partial_\rho C_\beta)$ required by the logarithmic divergence of the transverse part of $\Pi^{\mu\nu}$, a counter term proportional to $g^{\mu\nu}C_\mu C_\nu$ for the longitudinal component of the polarization is also needed in the Lagrangian when the $\Lambda \to \infty$ limit is taken. The presence of these terms in the effective action implies that as a result $C_\mu$ becomes massive and obeys the Proca equation.

### III. CONCLUDING REMARKS

The Weyl action of Eq. (2.5) would be gauge invariant under local $\gamma^5$ and scaling transformations

$$\Psi_L \to \exp(i\alpha(x)\gamma^5 - \frac{3}{2}\beta(x))\Psi_L = T\Psi_L, \quad e\overline{\Psi}_L\gamma^5 \to e\overline{\Psi}_L\gamma^5T^{-1},$$

with $T(x) = \exp[-(i\alpha + \frac{3}{2}\beta)]$ if we pretend that $C_\mu = (ib_\mu + A_\mu)$ is a complex Abelian gauge connection which transforms as

$$C_\mu \to TC_\mu T^{-1} - Ti\partial_\mu T^{-1} \quad \text{i.e.} \quad A_\mu \to A_\mu + \partial_\mu \alpha, \quad B_\mu \to B_\mu - \frac{3}{2}\partial_\mu \beta. \quad (3.1)$$

Note that $B_\mu$ comes with an $i$ in the complex combination $C_\mu$ because, unlike $\gamma^5$ rotations, the group parametrized by $\exp(-\frac{3}{2}\beta)$ is noncompact rather than $U(1)$. However massive regulators break both of these symmetries, and the current $J_\mu^F$ coupled to $C_\mu$ is not conserved. The full Weyl theory however exhibits no inconsistencies because these invariances are really not gauged as local symmetries. The theory is on the other hand diffeomorphism, and local internal gauge and Lorentz invariant, with internal symmetries gauged by $W_{\mu a}$ and local Lorentz invariance by the spin connection $A_{\mu AB}$. $C_\mu$ is in fact a composite which transforms covariantly under general coordinate transformations, and is invariant under local Lorentz and gauge transformations.

The appearance of a mass term for $C^\mu$ does have an important consequence. The combination $C_\mu$ is complex, therefore the counter terms of the Lorentzian signature Lagrangian required by vacuum polarization diagrams include anti-hermitian cross terms which are Lorentz invariant, but CPT-odd. In a related work [8] it was pointed out that a consequence is that both axial torsion as well as vector torsion trace are needed, but with a relative phase which ruins CPT invariance because of the chiral nature of the fields. Massive modes appear as consequences of the anomalous non-conservation of the current to which these torsion modes are coupled. At low energies compared to the torsion mass, the fermion-torsion interaction reduces to a four-fermion coupling. Present high energy experimental data on four-fermion vertices sets the lower bound for torsion mass at above the 200GeV scale [15].

\[ \text{Eq. (3.1)} \]
The question of mass generation via anomalies has had a storied past. In the Schwinger model in 2D, the physical degree of freedom of the photon is equivalent to a free massive boson [16,17] since the interaction term can be transformed away by an axial rotation. The mass of the boson stems from the ABJ anomaly, which gives rise to an infra-red pole in the polarization tensor. On the other hand, the chiral Schwinger model in 2D is anomalous albeit exactly solvable. The resultant photon mass, while still finite, carries an ambiguity as the previous condition of gauge invariance is now absent. It can be made to vanish, while preserving the (V-A) form for the coupling, but we then lose unitarity [18]. This paper discusses the corresponding chiral situation in 4D, but without loss of any local gauge invariance. By retaining explicitly all local gauge symmetries and the holomorphic dependence on the left-handed spin connection in the regularization, we end up with a vacuum polarization tensor that is non-transverse, and gives a mass to $C_\mu = iB_\mu + A_\mu$. These complex torsion modes are massive because of ABJ and scaling anomalies, with generated masses of the order of the regulator scale. Since these are the only modes that can couple to spin 1/2 fermions, large torsion masses, or high cut-off scales in the context of effective field theories, naturally explain why torsion and its associated effects, including CPT violations from $B_\mu$ couplings, have so far escaped detection.

ACKNOWLEDGMENTS

The research for this work has been supported in part by funds from the U.S. Department of Energy under Grant No. DE-FG05-92ER40709, and the National Science Council of Taiwan under Grant Nos. NSC 90-2112-M-006-012 and NSC 91-2112-M-006-018.

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