From LHC physics to Dirac-Weyl materials

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Abstract. The quantum field theoretical description of particle physics under extreme
conditions, namely, at finite temperature, density and in the presence of external magnetic
fields, can naturally be extended to describe phenomenology in other branches of physics. In
this contribution, I review some aspects of particle physics in the realm of condensed matter
physics, particularly graphene and other Dirac-Weyl materials carried out in Mexico. I explore
several features of the dynamics of fermions in (2+1)-dimensions which are relevant to heavy
ion experiments, but that can be tested in tabletop experiments.

1. Introduction
Particle physics under extreme conditions –finite temperature and density, as well as in the
presence of background magnetic fields– offers a natural opportunity to link ideas that have
been developed in other fields, particularly condensed matter physics, and vice versa (see, for
instance, Ref.[1]). These scenarios are realized either in heavy ion collision experiments in RHIC
and ALICE and other natural sources, e.g., compact stars, where several aspects of different
phase transitions that hadron matter has experienced since the early instants after the Big-Bang
can be tested from a quantum field theoretical point of view. Lattice QCD, Schwinger-Dyson
equations and effective field theories are examples of frameworks that have been developed to
describe the QCD phase diagram. Inclusion of temperature, density and/or magnetic fields
results in a dimensional reduction of the corresponding quantum field theory under study.

On the other side of the bridge, a new era of materials science has emerged since the isolation
of graphene membranes back in 2004 [2]. Although, theoretically speaking, graphene has been
known since 1947 [3], the one-atom thickness of the membranes brings two-dimensional (2D)
crystals to a natural platform for the development of nano-devices. Apart from the potential
technological applications of graphene due to its high thermo-electric conductivity, stiffness
yet flexibility and almost transparent for visible light, it is the nature of charge carriers in
this material that makes it outstanding; at low energies, these quasi-particles behave as ultra-
relativistic Dirac fermions. The difference is that these fermions move at a maximum, constant,
energy independent Fermi velocity $v_F$ which is some 300 times smaller than $c$. Therefore,
graphene offers an opportunity to test aspects of quantum electrodynamics where relativistic
effects are enhanced 300 times. After the isolation of graphene, a number of new materials were
found which also exhibit relativistic charge carriers (Dirac-Weyl) [4].

In view of all the above, the study of fermions in (2+1)-dimensions naturally links the
studies of particle physics under extreme conditions and condensed matter systems. A group
of researchers, postdocs fellows, graduate and undergraduate students from UMSNH, UNAM, UNISON and UCOL have joined efforts to understand several aspects of these systems. In this contribution, I present the main results of our group in the past years to celebrate the XXX anniversary of the Division of Particles and Fields of the Mexican Physical Society.

2. Fermions in (2+1)-dimensions

Planar dynamics offers a number of interesting features, including generalized parity and time-reversal, fractional (anyonic) statistics and more (see, for instance, Ref. [5]). Particularly, for Dirac fermions, the Clifford Algebra

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}
\]  

(1)

can be realized in terms of 2 \times 2 matrices, the Pauli \(\sigma\) matrices. There are, in fact, two irreducible representations, as can be seen from the product of all matrices

\[
\Gamma = \gamma^0\gamma^1\gamma^2 = \pm iI_{2\times2},
\]  

(2)

where \(I_{2\times2}\) is the 2 \times 2 identity matrix. A possible realization of these representations is

\[
\gamma^0 = \sigma_z, \quad \gamma^1 = i\sigma_x, \quad \gamma^2 = \pm i\sigma_y.
\]  

(3)

Thus, at first glance, a chiral transformation cannot be defined. On top of that, the mass term \(m\bar{\psi}\psi\) is odd under parity and time reversal transformations, and thus the spectrum for the massive Dirac equation,

\[
(i\gamma^\mu \partial_\mu - m)\psi = 0 ,
\]  

(4)

is not parity invariant. Notice, nevertheless, that the fermion propagator \(S(p)\) is still of the form

\[
iS(p) = \frac{i}{\gamma^\mu p_\mu - m + i\epsilon},
\]  

(5)

regardless the representation we select to work with.

A fermion Lagrangian which is invariant under discrete transformations can be constructed by considering both irreducible representations simultaneously in the form

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \bar{\phi}(i\gamma^\mu \partial_\mu + m)\phi ,
\]  

(6)

where \(\psi\) and \(\phi\) are the fermion fields associated to each representation, but the \(\gamma\)-matrices are taken in a single irreducible one. We observe that the difference between the two fermion families can be realized in the sign of its mass term. This Lagrangian allows two chiral-like transformations, as has been discussed in [6]. Moreover the two irreducible representations can be merged into a single 4 \times 4 reducible representation of the Dirac matrices (a detailed discussion can be found in Ref. [7], for instance) and hence we are led to the usual Dirac Lagrangian with the third spatial component set to zero, namely

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi , \quad \mu = 0, 1, 2.
\]  

(7)

\(\mathcal{L}\) above (7) is invariant under discrete symmetry transformations, describes fermions with two spin orientations and in its massless form, is invariant under the chiral transformation \(\psi \rightarrow e^{i\alpha_5}\psi\) where \(\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3\) as usual. The fermion propagator is still of the form (5), although the gamma matrices are now the usual 4 \times 4. Furthermore, since \(\gamma^3\) does not enter
in (7), the massless reducible Lagrangian is invariant under two types of chiral transformations, namely

$$\psi \to e^{i\alpha \gamma^5} \psi, \quad \text{and} \quad \psi \to e^{i\beta \gamma^3} \psi,$$

and thus two mass terms can be considered in the Lagrangian,

$$m\bar{\psi}\psi \quad \text{and} \quad m_o \bar{\psi}\tau \psi, \quad \tau = \frac{1}{2}[\gamma^3, \gamma^5].$$

The former, which is the usual Dirac mass term, is invariant under discrete transformations, but breaks the chiral symmetries derived from the transformations in Eq. (8), while the latter, referred to in condensed matter physics literature as Haldane mass term [8], preserves both chiral symmetries, but is odd under parity and time-reversal transformations. Thus, the general form of the massive Dirac Lagrangian becomes

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m - m_o \tau)\psi.$$  \hspace{1cm} (10)

Neither $m$ nor $m_o$ can be associated with poles of the corresponding propagator. By introducing the chiral projectors $\chi^\pm = (1 \pm \tau)/2$, along with the left- and right-fields $\psi^\pm = \chi^\pm \psi$, the above Lagrangian (10) can be cast in a more familiar form

$$L = \bar{\psi}^+(i\gamma^\mu \partial_\mu - m_+)\psi^+ + \bar{\psi}^-(i\gamma^\mu \partial_\mu - m_-)\psi^-,$$  \hspace{1cm} (11)

where $m^\pm = m \pm m_o$. This Lagrangian (11) describes two fermion species with well defined poles at $p^2 = m^2_\pm$, which are non-degenerate in mass. The corresponding (Euclidean) propagator is

$$S(p) = -S^+(p)\chi^+ - S^-(p)\chi^- = -\frac{\gamma^\mu p_\mu + m_+}{p^2 + m^2_+}\chi^+ - \frac{\gamma^\mu p_\mu + m_-}{p^2 + m^2_-}\chi^-.$$  \hspace{1cm} (12)

We describe the electromagnetic interactions from this Lagrangian in different contexts below.

3. Quantum Electrodynamics

Quantum electrodynamics in (2+1)-dimensions (QED$_3$) is an interesting theory in several aspects. On one hand, it is a toy model for QCD under extreme conditions in the following sense: Any given quantum field theory at finite temperature is dimensionally reduced as the temperature $T \to \infty$. In the particular case of QCD, it is known that the non-Abelian corrections are suppressed as the inverse of the number of fermion families involved. Therefore, QED$_3$ can be considered as an effective description of QCD in these circumstances. The parallelism goes further by recalling that this theory exhibits dynamical chiral symmetry breaking and confinement. Thus, QED$_3$ resembles more to QCD than to ordinary QED.

Within the context of Schwinger-Dyson equations (SDE), QED$_3$ has been widely explored in Mexico [9, 10, 11, 12, 13, 14, 15, 16]. Early efforts were made to explore the possible gauge dependence of physical observables associated to the non-perturbative truncation of SDEs in quenched [9, 10, 11, 12] and unquenched [13, 14, 15] QED$_3$. The mathematical structure of the gap equation was deeply studied as well [16], establishing the criteria for which this equation could support multiple solutions. In these studies, the role of gauge invariance constraints like the Ward-Takahashi identities (WTI) and the Landau-Khalatnikov-Fradkin transformation (LKFT) of the fermion propagator played a crucial role. Proposals for the fermion-boson vertex satisfying these constraints were presented in [15, 17].
Special mention deserves the Mexican contribution to studies of LKFT for the fermion propagator and its implications to the perturbative and non-perturbative form of this two-point function. LKFT relates the fermion propagator in any covariant gauge, labeled by $\xi$, to the corresponding propagator in Landau gauge, $\xi = 0$, through the relation

$$S(x; \xi) = S(x; 0)e^{-\alpha \xi x/2}, \quad (13)$$

where $\alpha = e^2/(4\pi)$ as usual. Perturbative constraints arising from LKFT (13) were explored in parity preserving [18] and parity non-preserving [19] QED$_3$. Because the transformation (13) is written in coordinate space, its usefulness in non-perturbative studies was not previously considered. After exploring the asymptotic form of the solution to the SDE in the light of LKFT [10], a significant contribution from Mexican participants was to establish the form of the said transformations in momentum space. Writing the propagator in the form

$$S(p; \xi) = \sigma_v(p; \xi)\gamma^\mu p_\mu + \sigma_s(p; \xi), \quad (14)$$

the transformation (13) in momentum space is equivalent to the relations [11]

$$\sigma_v(p; \xi) = \frac{\alpha \xi}{2\pi p^2} \int_0^{\infty} dk \ k^2 \sigma_v(k; 0) \left[ \frac{1}{\lambda^-} - \frac{1}{\lambda^+} + \frac{1}{2kp} \ln \left| \frac{\lambda^-}{\lambda^+} \right| \right],$$

$$\sigma_s(p; \xi) = \frac{\alpha \xi}{2\pi p} \int_0^{\infty} dk \ k \sigma_s(k; 0) \left[ \frac{1}{\lambda^-} - \frac{1}{\lambda^+} \right], \quad (15)$$

with $\lambda^\pm = (\alpha \xi/2)^2 + (k \pm p)^2$. Out of these relations, it is possible to establish the gauge invariance of physical observables associated to a non-perturbative fermion propagator, like the chiral condensate [11] $-\langle \bar{\psi} \psi \rangle = Tr[S(x = 0; \xi)]$, which is the order parameter for chiral symmetry breaking, the critical number of fermion families above which chiral symmetry is restored [13], the parameters associated to confinement [11, 15], and the position of the nodes of multiple solutions to the SDE [16]. To conclude this section, the one-loop corrections to the vacuum polarization tensor, fermion propagator and fermion-boson vertex were considered in [20] including the influence of a Chern-Simons term.

4. Fermions in External Magnetic Fields
The dynamics of fermions in a background magnetic field is also characterized by a dimensional reduction of the dynamics. Therefore, planar fermions are also participants in such scenarios found, for instance, during peripheral heavy-ion collisions. Solutions to the planar Dirac equation in a magnetic field perpendicularly aligned to the plane of motion of fermions were discussed in Ref. [21] in connection with the magnetic catalysis for the formation of the chiral condensate. Parity preserving and parity breaking condensates were studied. The corresponding fermion propagator was diagonalized in momentum space through the Ritus method [22] which was extended also to the case where the background magnetic field was not uniform, but varies with an exponential decay along one spatial direction [22, 23]. In this context, the condensate and electric current were found to follow the profile of the external field [23].

The combined effect of a magnetic field and a thermal bath on the dynamics of fermions induces interesting phase transitions. Whilst the magnetic field promotes the formation of the chiral condensate, a thermal bath at temperature $T$ tries to melt the condensate. The impact of these two ingredients on parity preserving and parity breaking condensates for weak, intermediate and strong magnetic fields was explored in [24]. Magnetization and pair production rate through Schwinger mechanism were also explored in this scenario.

More recently, in an attempt to mimic the dynamics of quarks under extreme conditions, particularly under the influence of a magnetic field, a connection between the chiral magnetic
subject to the conditions

\[ d \text{propagating in } \psi \text{ includes matter fields } \]

\[ \text{magnetic field parallel effect (CME) in a condensed matter environment was established in Ref. [25]. The pseudo chiral magnetic effect is realized as the generation of a non-dissipative current by effects of a magnetic field parallel to the plane of motion of planar fermions in a medium characterized by a finite density in a thermal bath at temperature } T. \]

\[ \text{The effect of the external fields is to provide an imbalance between left and right reducible fermions with Dirac and Haldane mass gaps, resembling the imbalance of chiralities in the CME. Under these circumstances, it was established that an electric current is generated along the magnetic field which is proportional to the chiral number, in complete analogy to the CME.} \]

5. Graphene

Low-energy dynamics of graphene can be effectively described by a version of massless QED with photons living in 4 space-time dimensions. Inspired by brane-world scenarios, such a dynamics can be modeled by the Reduced QED action \[ [26, 27] \]

\[ I_{d_x, d_e}[A_{\mu}, \psi(x)] = \int d^d x \mathcal{L}_{d_x, d_e}, \]

where the Lagrangian

\[ \mathcal{L}_{d_x, d_e} = \bar{\psi}(x) i \gamma ^{\mu} \gamma^5 D_\mu \psi(x) \delta^{(d_x-d_e)} - \frac{1}{4} F^{\mu_1 \nu_1 \rho_1 \gamma_1} F_{\mu_1 \nu_1} - \frac{1}{2} \xi_1 (\partial_\mu A^{\mu} \gamma^5)^2 \]

includes matter fields \[ \psi(x) \] restricted to a \[ d_e \text{-dimensional brane and gauge fields } A_\mu \], propagating in \[ d_x \text{-bulk dimensions, with } d_\gamma > d_e \]. Here \[ D_\mu \] represents the covariant derivative and \[ F_{\mu \nu} \] the field strength tensor. Gauge covariance of the associated fermion propagator was studied in Ref. [28] through LKFT.

The problem of light absorption in this material has been addressed by our group. From the action in Eq. (16), we can describe the propagation of electromagnetic waves through a graphene sample according to the modified Maxwell’s equations

\[ \partial_\mu F_{\mu \nu} + \delta(z) \Pi^{\nu \rho} A_\rho = 0, \]

subject to the conditions

\[ A_\mu \bigg|_{z=0^+} - A_\mu \bigg|_{z=0^-} = 0, \]

\[ (\partial_\mu A_\mu) \bigg|_{z=0^+} - (\partial_\mu A_\mu) \bigg|_{z=0^-} = \Pi^{\nu \rho} A_\rho \bigg|_{z=0}, \]

where \[ A_\mu \] represents the incoming monochromatic plane wave and \[ \Pi^{\nu \rho} \] represents the vacuum polarization tensor. Considering that the incident wave is linearly polarized, with frequency \( \omega \), the reflected and transmitted waves can be described as

\[ A = e^{-i\omega t} \left\{ \hat{e}_x e^{ikxz} + (r_{xx} \hat{e}_x + r_{xy} \hat{e}_y) e^{-ikxz}, \quad z < 0, \right. \]

\[ (t_{xx} \hat{e}_x + t_{xy} \hat{e}_y) e^{ikxz}, \quad z > 0, \]

where \( \hat{e}_x, \hat{e}_y \) are the unit vectors along the directions \( x \) and \( y \) on the membrane. Then, the intensity of transmitted light \( I \) and the angle of polarization rotation \( \theta_F \) (Faraday effect) can, respectively, be expressed as

\[ I = |t_{xx}|^2, \quad \theta_F = \frac{1}{2} \arg \left( \frac{t_{xx} - it_{xy}}{t_{xx} + it_{xy}} \right). \]

A calculation of \( \Pi^{\nu \rho} \) for massless fermions in a weak magnetic field \( B \) perpendicular to the membrane [29] reveals that the 2.3% opacity of the membrane measured [30] gets corrected by a factor \( -\alpha \pi (eB)^2 / \omega^4 \) and \( \theta_F = 0 \). Furthermore, it was observed that the Haldane mass term in absence of any magnetic field is capable of generating a finite value for \( \theta_F = \alpha \) [31].
6. Final remarks
In this contribution we have reviewed several aspects of planar Dirac fermions as a bridge between the physics of LHC and the Dirac-Weyl materials carried out by a group of Mexican researchers, postdocs fellows and students. We would like to mention the independent work of another group regarding the influence of Chern-Simons terms in the dynamics of topological insulators [32] which are relevant for the studies carried out by ours. We hope this effort continues to shed light for fundamental physics in condensed matter environments.

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