The future of the local large scale structure: the roles of dark matter and dark energy

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Received 5 June 2007
Accepted 5 October 2007
Published 29 October 2007

Online at stacks.iop.org/JCAP/2007/i=10/a=016
doi:10.1088/1475-7516/2007/10/016

Abstract. We study the distinct effects of dark matter and dark energy on the future evolution of nearby large scale structures using constrained N-body simulations. We contrast a model of cold dark matter and a cosmological constant (ΛCDM) with an open CDM (OCDM) model with the same matter density $\Omega_m = 0.3$ and the same Hubble constant $h = 0.7$. Already by the time the scale factor has increased by a factor of 6 (29 Gyr from now in ΛCDM; 78 Gyr from now in OCDM) the comoving position of the Local Group is frozen. Well before that epoch the two most massive members of the Local Group, the Milky Way and Andromeda, will merge. However, as the expansion rates of the scale factor in the two models are different, the Local Group will be receding in physical coordinates from Virgo exponentially in a ΛCDM model and at a roughly constant velocity in an OCDM model. More generally, in comoving coordinates the future large scale structure will look like a sharpened image of the present structure: the skeleton of the cosmic web will remain the same, but clusters will be more ‘isolated’ and the filaments will become thinner. This implies that the long-term fate of large scale structure as seen in comoving coordinates is determined primarily by the matter density. We conclude that although the ΛCDM model is accelerating at present due to its dark energy component while the OCDM
model is non-accelerating, their large scale structures in the future will look very similar in comoving coordinates.

**Keywords:** dark matter, dark energy theory, superclusters and voids, cosmological simulations

**ArXiv ePrint:** 0705.4477

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### 1. Introduction

Cosmologists usually study the past evolution of the universe. Indeed, the Local Universe has been used before as a laboratory for studying dynamics and cosmological parameters, for example the timing argument [26,10], or N-body methods [24]. Similarly, we can gain insight into the dynamics and the role of the dark matter and dark energy components of the universe by considering the future evolution of nearby large scale structure. The background cosmology is now reasonably well established based on the measurements of the cosmic microwave background, the large redshift surveys, Supernovae Ia data and other probes (e.g. [28] and references therein). These measurements support a ‘concordance’ model in which the universe is flat and contains approximately 4% baryons, 21% cold dark matter and 75% dark energy, with a possible small contribution of massive neutrinos. The nature of the dark matter and the dark energy are still to be understood, in particular the possibility of a more general equation of state of the dark energy component, \( w = P/\rho \), which might also be evolving with cosmic epoch. The case \( w = -1 \) corresponds to Einstein’s cosmological constant \( \Lambda \), which we shall consider in this paper, as it is consistent with the current observations. We shall refer to this model as \( \Lambda \) cold dark matter (\( \Lambda \)CDM).
The fate of a $\Lambda$CDM universe as a whole has been discussed before (e.g. [17, 12]). The goal of the present study is to contrast the $\Lambda$CDM model with an OCDM model with the same matter density parameter, to investigate the roles of the dark matter and the dark energy in the evolution of clustering. When the universe will be completely dominated by the $\Lambda$ term the scale factor will expand exponentially, $a(t) \propto \exp(\sqrt{\Lambda / 3} t)$ (a ‘de Sitter phase’). It is less clear what will happen to the growth of structure and to individual objects in the universe. e.g. will they manage to survive as bound objects in an exponentially expanding universe? Papers by Nagamine and Loeb [22] and Busha et al [2] have already considered some of these questions.

In particular Nagamine and Loeb [22] used a constrained $N$-body simulation for a $\Lambda$CDM universe to predict the future evolution of nearby large scale structure. They found that structures will freeze in comoving coordinates. In particular they found that the Local Group will get somewhat closer to the Virgo cluster in comoving coordinates, but will be pulled away from Virgo in physical coordinates due to the accelerated expansion of the universe. However, one should recall that the freeze out of the growth of structure is not the signature of the cosmological constant but rather of the fact that the density of matter is less than the critical. Namely, in a universe dominated either by the negative curvature or by the $\Lambda$ term gravitational instability stalls. The linear theory growth of density perturbations in a universe with and without a cosmological constant is illustrated in numerous papers (e.g. figure 2 of [15]). This has motivated us to explore the fate of the universe in the general case where the mean matter density is subcritical and compare the cases of a $\Lambda$ dominated universe with an open one.

Here we extend the study of [22] to explore to what extent this fate of the local structure depends on the dark matter and dark energy contents of the universe. We use the tool of constrained realizations of Gaussian random fields [9] to constrain the initial conditions for the $N$-body simulations by observational data. The resulting simulations reproduce quite faithfully the observed LSS out to a few tens of Mpc from the LG. We apply the simulations to both $\Lambda$CDM model and an open cold dark matter (OCDM) model with the same $\Omega_m = 0.3$ and the same Hubble constant $h = 0.7$. We look in particular at the universe at present epoch $a = 1$ (where the age of the universe is 13.5 and 11.3 Gyr in $\Lambda$CDM and OCDM, respectively) and at $a = 6$ (where the universe will be 42.4 Gyr old for $\Lambda$CDM and 89.2 Gyr old in OCDM). We find that in comoving coordinates the evolution of large scale structure is quite similar. The main difference is in physical coordinates, as the evolution of the scale factor is obviously different for the two models.

The outline of the paper is as follows. In section 2 we summarize briefly the properties of the Local Group and the Local Supercluster. In section 3 and in the appendix we present analytic considerations for the evolution of structure and in section 4 we give details of the constrained simulations. The results and plots from the simulations are shown in section 5 and we discuss the results in section 6.

2. Nearby structures in the local universe

The Local Universe has been mapped in great detail since the 1980s, with the aid of whole sky galaxy surveys like IRAS, 2MASS and peculiar velocity surveys (e.g. [30] for a review). Roughly speaking by Local Universe we mean the volume of a sphere of radius $\approx 200h^{-1}$ Mpc centred at the Milky Way (MW). Here we focus on the Local Group (LG) and the Local Supercluster (LSC). For definitions of the local structure see e.g. [34] and [33].
Here we only summarize briefly the terminology of local structures relevant for our study. The Milky Way, Andromeda (M31) and other 30 other small galaxies within a few Mpc form the LG. At the present epoch the two major galaxies of the LG are approaching one another at an infall velocity of about 120 km s$^{-1}$ and thus they constitute a bound dynamical system. The distance between the Milky Way and M31 is $740 \pm 40$ kpc. The total mass of the LG is estimated to be $2.3 \pm 0.6 \times 10^{12} M_\odot$ [34].

Until the early 1980s the Virgo cluster was regarded as the centre of an overdense region called the LSC (e.g., [4,16]) and it was assumed to be the major supercluster in the local universe. However, in the late 1980s it was recognized in whole sky surveys that much larger superclusters, such as the Great Attractor and Perseus–Pisces, dominate the local universe. These and other superclusters and voids generate tidal forces which affect the motion of the LG towards the Virgo cluster. Despite this complexity of the local structure we find it useful below to consider the time evolution of the distance between the LG and the centre of Virgo. Obviously in principle we can define other distances or statistics to quantify the local structure. We also note the supergalactic coordinate system, defined based on a planar structure in the galaxy distribution [5], which we shall use for convenience in some of our plots.

The Virgo–LG system is much less dynamically evolved than the LG itself. The observed mean overdensity in the number count of galaxies within the Virgocentric sphere is $\approx 2$ [4] and the (line of sight) peculiar velocity of the Virgo cluster relative to the LG is 932 km s$^{-1}$ at a distance of $16 \pm 2$ Mpc [34].

Both the LG and the LSC constitute departure from the homogeneous and isotropic expanding universe. An unperturbed open universe or a flat $\Lambda$ dominated universe expand forever. Bound objects that have collapsed and virialized by the present epoch will remain so in spite of the future expansion of the universe. The question is what is the fate of the objects in the nearby universe when the universe will be freely or exponentially expanding. In particular we shall follow the evolution of the two objects that dominate local dynamics, namely the LG and LSC.

3. Theoretical expectations

A rough estimate on the dynamical evolution is provided by the spherical top-hat model [8]. The evolution of spherical density perturbations in a general Friedmann universe dominated by non-relativistic matter and a cosmological constant was considered e.g. by [14,35,18,19] and [25]. A brief summary of the results of [18] and an extension to include velocity perturbations as well are presented in the appendix.

Using the formalism presented in the appendix the following values for the critical overdensity to future collapse are obtained. For the case of a vanishing velocity perturbation the critical overdensity is 17.6 ($\Lambda$CDM; in agreement with [22]) and 2.33 (OCDM). Adopting a Virgocentric infall velocity of $16H_0 - 932 = 188$ km s$^{-1}$ (for $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$) the critical value is 14.6 ($\Lambda$CDM) and 1.3 (OCDM) (cf appendix). Given the observed Virgocentric overdensity (in galaxy count) of $\approx 2$ one can safely assume that in the $\Lambda$CDM model the Virgocentric infall is expected not to proceed to a collapse and virialization but rather to reach a freeze out. The case of the OCDM is not clear as the current estimation of the Virgocentric infall and overdensity are only marginally consistent with a freeze out of the infall. Numerical simulations are needed here to resolve the issue of the future of the LSC in the OCDM model.
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The LG has already passed its turn-around phase and the MW and M31 are heading at a (line of sight) velocity of 120 km s\(^{-1}\) towards a merger to become one object. The mean overdensity in the LG is \(\approx 260\) (by averaging the mass of the MW and M31 over a sphere with a radius which is half the distance MW-M31) and it is much larger than \(\Delta_{\text{crit}}\) for both the \(\Lambda\)CDM and OCDM models. It follows that we expect the LG to collapse and merge into one object in both cosmologies.

4. \(N\)-body simulations: methodology

The goal of the present paper is to study the future evolution of the nearby LSS in both OCDM and a flat \(\Lambda\)CDM cosmologies. The nearby universe seems to constitute a very typical realization of the flat-\(\Lambda\)CDM and the OCDM models. The LG, in particular, is not a unique or an unusual object in the universe, yet not being a relaxed object in virial equilibrium it has its own characteristics that affect the outcome of any dynamical test that would be applied to it. The key to a successful numerical study of the LG and the nearby universe is the ability to reproduce the characteristic dynamical properties of the LG and its surrounding. Namely a quasi-linear object located at about \((10–15)h^{-1}\) Mpc from a Virgo-like cluster, within a supercluster that extends as filament connecting two major structures, the Perseus–Pisces supercluster and the Great Attractor that are located some \(90h^{-1}\) Mpc apart. The LG is caught in a ‘tug of war’ in between these two major structures, which affects the local dynamics. The key for a successful numerical study of the local universe is the ability to design numerical simulations which reproduce the main dynamical features of the local universe. The optimal way of achieving that goal is by the use of constrained simulations (CSs; [13,21,11]), namely simulations based on initial conditions set by means of constrained realizations of Gaussian fields [9].

The data used to constrain the initial conditions of the simulations are made of two kinds. The first data set is made of radial velocities of galaxies drawn from the MARK III [38], SBF [32] and the Karachentsev [10] catalogues. Peculiar velocities are less affected by non-linear effects and are used as constraints as if they were linear quantities [39]. This follows the CSs performed by Kravtsov et al [13] and Klypin et al [11]. The other constraints are obtained from the catalogue of nearby x-ray selected clusters of galaxies [27]. Given the virial parameters of a cluster and assuming the spherical top-hat model one can derive the linear overdensity of the cluster. The estimated linear overdensity is imposed on the mass scale of the cluster as a constraint. It should be noted that neither the MW and M31 nor the LG have been imposed directly on the simulations by the constraints. The constraints used here constrain quite closely the structure on scale larger than \(\approx 5h^{-1}\) Mpc [11]. Different CSs with different random realizations have been calculated and they all exhibit a clear and unambiguous LSC-like structure that dominates the entire simulation, much in the same way as in the actual universe in which the LSC dominates the nearby LSS. The lack of explicit LG constraints causes the simulations to vary with respect to the particular details of the LG-like object. Yet, the fact that the large scale structure of the local universe is faithfully reproduced implies that the CSs have a high probability of producing quite reasonable LG-like objects. This is definitely confirmed by the present simulations.

We assume a flat \(\Lambda\)CDM model with \(\Omega_m = 0.3\), \(\sigma_8 = 0.9\) and \(h = 0.7\) (where \(\sigma_8\) is the power spectrum normalization factor). A somewhat refined set of cosmological parameters
Table 1. The main dynamical parameters that characterize the simulated local universe: mass of Milky Way (\(M_{\text{MW}}\)), mass of M31 (\(M_{\text{M31}}\)), mass of LG (\(M_{\text{LG}}\), for the present epoch it is the sum of \(M_{\text{MW}}\) and \(M_{\text{M31}}\)), mass of the Virgo cluster (\(M_{\text{Virgo}}\)) and the LG–Virgo distance (\(R_{\text{LSC}}\)). Here, MW, M31 and the Virgo cluster refer to the simulated objects.

| Model/epoch | \(\Lambda\)CDM \(a = 1\) | OCDM \(a = 1\) | \(\Lambda\)CDM \(a = 6\) | OCDM \(a = 6\) |
|-------------|----------------|----------------|----------------|----------------|
| \(M_{\text{MW}}\) \(\left(\h M\right)\) | \(7.9 \times 10^{11}\) | \(5.5 \times 10^{11}\) | | |
| \(M_{\text{M31}}\) \(\left(\h M\right)\) | \(1.0 \times 10^{12}\) | \(1.5 \times 10^{12}\) | | |
| \(M_{\text{LG}}\) \(\left(\h M\right)\) | \(1.8 \times 10^{12}\) | \(2.0 \times 10^{12}\) | \(5.7 \times 10^{12}\) | \(2.4 \times 10^{12}\) |
| \(R_{\text{LG}}\) \(\left(\h \text{Mpc}\right)\) | 1.1 | 0.44 | | |
| \(M_{\text{Virgo}}\) \(\left(\h M\right)\) | \(1.7 \times 10^{14}\) | \(6.8 \times 10^{13}\) | \(3.4 \times 10^{14}\) | \(5.6 \times 10^{14}\) |
| \(R_{\text{LSC}}\) \(\left(\h \text{Mpc}\right)\) | 13.0 | 11.6 | 10.6 | 15.6 |

has been adopted after WMAP third year data release [28], but the change in the values of the cosmological parameters would not affect the results presented here in any significant way. The other model used here is the OCDM, which is obviously inconsistent with recent observations of the CMB [28]. The model is considered here so as to show that the main effect that drives the freeze out of structure is the fact that \(\Omega_m\) is less than unity. For the sake of concreteness we assume that the cosmological parameters of the OCDM model coincide with that of the \(\Lambda\)CDM model apart from setting \(\Omega_\Lambda = 0\), and thereby implying an open universe. The simulations correspond to a periodic cubic box of \(64\h^3\) Mpc on a side, spanned by a \(256^3\) grid. This translates into a mass per particle of \(1.3 \times 10^9\h M\). The softening of the gravitational force corresponds to an equivalent Plummer softening of comoving 5 kpc \(\h^{-1}\), which reproduce the exact Newtonian force at 2.8 times this scale, implying a force resolution of 15 kpc \(\h^{-1}\). The mass resolution is adequate for resolving a LG-like object made of two \(\approx 10^{12}\h M\) objects and following its internal dynamics. Yet, the internal dynamics of the main halos cannot be resolved. This has to be compared with the mass resolution of \(3.6 \times 10^{11}\h M\) of [22], which does not resolve the LG. The current simulations are among the ones used by Martinez-Vaquero et al [20]. The parallel TREEPM N-body code GADGET2 [29] has been used to run the simulation. A more detailed description of the simulation is presented in [20]. The halos are found by the AMIGA [7] halo finder, and the mass \(M\) corresponds to the mass enclosed within the virial radius.

5. Constrained simulation of the local universe

The CS clearly manifests the main characteristics of the local universe. Figure 1 shows the projected dark matter density of a \(12\h^{-1}\) Mpc thick slice centred on the supergalactic plane in both the \(\Lambda\)CDM and OCDM models. The local structure is dominated by the LSC filaments which crosses horizontally the supergalactic plane at roughly \(SGY \approx 15\h^{-1}\) Mpc. The simulated LG is located in a filaments that run perpendicularly to the LSC at \(SGX \approx -7\h^{-1}\) Mpc. Apart from a general shift by a few Mpc in the \(-SGX\) direction of the of the whole cosmic web, the simulated structure recovers the observed one. It should be noted at the outset that the present (and future) LSS exhibited by the two models look very similar. The main dynamical parameters that characterize the simulated local
universe at the present and future epochs for both the ΛCDM and OCDM models are presented in table 1. Both models reproduce the structure of the LSC at the present epoch with a simulated LG located 13.0 (ΛCDM) and 11.6 Mpc h⁻¹ (OCDM) away from the LG, compared with the actual value of 11.2 Mpc h⁻¹. The mass of the simulated Virgo is $1.7 \times 10^{14} h^{-1} M_\odot$ (ΛCDM) and $0.7 \times 10^{14} h^{-1} M_\odot$ (OCDM), compared with the observed $\approx 10^{14} h^{-1} M_\odot$. The mass of the simulated LG is very close to the observationally inferred value of $1.6 \times 10^{12} h^{-1} M_\odot$ [34]. The OCDM simulated MW-M31 distance is close to the observed value of 0.7 Mpc, but the ΛCDM simulated distance is more than twice larger. That last discrepancy does not affect in any way the conclusions drawn here on the future evolution of the local universe.
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Figure 2. The evolution of the distance of the LG from the Virgo cluster in the ΛCDM (blue curves) and OCDM (red curves) models is presented in comoving (solid line) and physical (dashed lines) coordinates. The upper panel presents the distance as a function of the scale factor \( a \) and the lower one as a function of time. In the ΛCDM case the Virgocentric infall extends to about twice the present Hubble time before it converges to its asymptotic value, where the inflow freezes. In physical coordinates the distance increases at all time and eventually it grows exponentially. In the OCDM the time evolution of the distance does not show the freezing of the flow. The position of the LG indeed becomes frozen in comoving coordinates but the Virgo cluster moves away from it because of the internal dynamics within the LSC.

5.1. The evolution of the LSC

The ΛCDM simulated local universe is presented at the present epoch of \( a = 1 \) and future epoch of \( a = 6 \) (figure 1), where \( a \) is the expansion factor. The projected density maps are presented in term of co-moving supergalactic coordinates. An inspection of figure 1 reveals that the cosmic web at the two epochs has hardly changed. Yet, the late epoch web is more ‘skinny’ and is more dominated by massive halos and less by the smaller mass halos.

It should be noted here that the similarity between the present and future structure exists only upon using co-moving coordinates. In physical coordinates the cosmic web gets more empty and the typical length scale of the web expands exponentially. This is clearly seen in figures 2 which shows the evolution of the Virgo–LG distance (in physical and co-moving coordinates). The first 30 Gyr reflect the Virgocentric infall of the LG towards the Virgo. The infall is manifested in the co-moving sense only and it levels off
in about 35 Gyr. In physical coordinates the LG is receding away from the Virgo cluster almost exponentially. Note that after $t_0 + t_H$ the comoving distance remains constant with time, where $t_H = H^{-1} = \sqrt{\Lambda/3} \sim 1.7H_0^{-1}$, in agreement with [22].

The present ($a = 1$) and future ($a = 6$) simulated local structure in the OCDM model are displayed in figure 1. The LSS, viewed in comoving coordinates exhibited by the $\Lambda$CDM and OCDM models at the two epochs shows a great resemblance. The general appearance of the cosmic web of the two models is almost indistinguishable. Yet, a close inspection of the LSC shows some differences. The LSC almost freezes out in the $\Lambda$CDM model. In the OCDM case the centre of the LSC keeps on evolving. In particular the Virgo cluster moves along the filament that constitutes the LSC, from right to left in the supergalactic projection, and merges with other clusters. The LG, on the other hand, is essentially frozen in comoving coordinates. As a result the Virgocentric distance of the LG keeps on growing in comoving coordinates.

5.2. The evolution of the LG

As the observed distance between the Milky Way and Andromeda is 740 kpc and they are approaching one another at 120 km s$^{-1}$, we can estimate naively that they will collide in about 6 Gyr if relative transverse velocity is ignored [3]. The simulated $\Lambda$CDM LG is shown at the present epoch in figure 3 exhibits two DM halos with masses of about $10^{12}h^{-1}M_\odot$, at a distance 1.1$h^{-1}$ Mpc and located in a slightly overdense filament that connects the LG group with the Virgo cluster of a mass of $10^{14}h^{-1}M_\odot$, located 10$h^{-1}$ Mpc away. The relative infall velocity of the two halos is about 140 km s$^{-1}$ and the transverse relative velocity is 50 km s$^{-1}$. It follows that the CS has reproduced a LG-like object that resembles the actual LG apart from the MW-M31 distance which is larger than the true distance of 0.74 Mpc. From the $\Lambda$CDM simulation we find that the simulated MW and M31 merge into one object, at the epoch of $a = 3$, corresponding for $h = 0.7$ to 17.3 Gyr from now (we note this is longer than that expected from just the two-body radial motion based on the simulation values: with infall of 140 km s$^{-1}$ starting at relative distance of 1570 kpc, we expect 11.0 Gyr).

Our large simulated present epoch MW-M31 distance obviously implies that the estimated time till the LG merger is overestimated. Indeed Cox and Loeb [3] have recently estimated that the merger will take place in 5 Gyr.

The simulated OCDM LG is very similar to the $\Lambda$CDM one, both in structure and evolution (figure 3). The present epoch MW-M31 distance of the OCDM LG is somewhat smaller than in the $\Lambda$CDM case, $\approx 0.5h^{-1}$ Mpc and therefore the MW-M31 merger is taking place earlier, at $a \sim 1.1$, corresponding to (12.7–11.3) Gyr = 1.4 Gyr from now. Figure 3 shows the final merged object much later (at $a = 6$) in both simulations.

It is clear that in the present CSs the internal structure of the LG is very weakly constrained and different realizations exhibit a large scatter with respect to the structure of the LG. This leads to a considerable scatter in the collapse time of the LG. Yet, all simulated LG-like objects will collapse much before $a = 6$.

5.3. General remarks on the future evolution

The cumulative mass function $n(M)$ of DM halos serves as a good indicator of the growth of structure of a cosmological model, where $n(M)$ is the number of DM halos
Figure 3. The mass distribution around the LG in the supergalactic plane in the \( \Lambda \)CDM (left column) and OCDM (right column) models presented at the present epoch \((a = 1; \text{upper row})\) and the future \((a = 6; \text{lower row})\). The plot shows the projection of the particles in a slab of thickness \(3h^{-1}\) Mpc. The colour coding of the particles indicates the density field. At the present epoch the LG is dominated by two massive halos of mass \(\approx 10^{12}h^{-1} M_{\odot}\) at a distance of \(\approx 1.1h^{-1}\) Mpc and \(0.5h^{-1}\) Mpc in \(\Lambda \)CDM and OCDM respectively. There is very little difference between the dynamics of the simulated LG in the \(\Lambda \)CDM and OCDM models.

more massive than \(M\) per unit co-moving volume. We have calculated (see figure 4) the mass function of both the \(\Lambda \)CDM and OCDM models in the present \(a = 1\) and future \(a = 6\) epochs. Generally speaking the mass function of the two models are very similar at both epochs. This similarity follows from the very similar dynamical nature of the two models, when analysed in co-moving coordinates. The cumulative mass function hardly evolves in both models, yet at \(a = 6\) there are somewhat fewer small, \(\approx 10^{12}h^{-1} M_{\odot}\), DM halos and roughly twice as many rich cluster-like halos. A detailed analysis shows that the depletion in the number of small halos and the appearance of more massive clusters is somewhat more pronounced in the OCDM model compared with the \(\Lambda \)CDM one.
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Figure 4. The cumulative mass function $n > M$ is plotted for both models and epochs, ΛCDM $a = 1$ (solid line, black) and $a = 6$ (dotted, black) and for OCDM $a = 1$ (dot–dashed, red) and $a = 1$ (dashed, red). DM halos are defined by the AMIGA halo finder and $M$ corresponds to the mass within the virial radius. The cumulative mass function $n(M)$ gives the number of DM halos per unit volumes with mass greater than $M$.

A close inspection of figure 2 shows that the LG–Virgo distance is already frozen by $a = 6$ in the ΛCDM model, while it is still slowly increasing in the OCDM case at that epoch. This is consistent with the evolution displayed by the mass function, namely by $a = 6$ the OCDM model is somewhat more evolved than the ΛCDM model. The slight differences between the models is clearly understood.

A simple analytical reasoning leads to the understanding that once the dynamics of the universe is no longer dominated by the cold matter density, gravitational instability comes to an end and the growth of structure freezes out. This property is shared by both the ΛCDM and OCDM models. Yet, the rate of transition to the asymptotic state is different. In the ΛCDM case the asymptotic state is achieved when the (constant for $w = -1$) dark energy term dominates the cold matter term (which decays like $a^{-3}$) in the Friedmann equation. In the OCDM case, on the other hand, the freeze out is achieved when the curvature term (which decays as $a^{-2}$) dominates. It follows that the ΛCDM model converged much faster, in time and scale factor as well, to the freeze out state than the OCDM model.

6. Discussion

Exploring the future of evolution of the large scale structure of the local universe is more than just a curiosity. Busha et al [2] have demonstrated that by running simulations into
the future it is possible to identify more efficiently which objects will form bound systems. It also allows us to get further insight into the effect of dark matter and dark energy in the evolution of large scale structure. In particular we extended here the study of Nagamine and Loeb [22] who analysed with the aid of constrained $N$-body simulations the evolution of nearby large scale structure only in a flat $\Lambda$CDM model. By contrasting the $\Lambda$CDM simulation with an OCDM with the same matter density $\Omega_m = 0.3$ and the same Hubble constant $h = 0.7$ we can learn what is the role of the dark matter compared with the role of the dark energy.

Our main conclusion is that the long-term fate of large scale structure as seen in comoving coordinates is determined primarily by the matter density. This generalizes the result of Nagamine and Loeb [22], and clarifies the distinct role of the matter density in defining the freezing out of structure, and the combined effects of both the dark matter and dark energy in the evolution of the scale factor and hence in the physical coordinates.

In more detail our main results are:

(i) The Milky Way and Andromeda will merge already at $a = 3$ and at $a = 1.1$ according to our $\Lambda$CDM and OCDM simulations (where the present epoch distances between the two galaxies in our simulations are somewhat different from the true observed value of 740 kpc).

(ii) Already by $a = 6$ the comoving position of the LG becomes frozen at its asymptotic value. In the $\Lambda$CDM case the comoving LG–Virgo distance reaches its asymptotic value. In the OCDM case the Virgo cluster still moves within the LSC and hence that distance is still slightly growing. However, in physical coordinates the LG will be receding from Virgo exponentially in a $\Lambda$CDM model and at an almost constant velocity in an OCDM model. This is just a manifestation of the different evolution of the scale factor $a(t)$ in the two models.

(iii) When we consider the overall cosmic web we find that qualitatively the future large scale structure will look like a sharpened image of the present structure: the skeleton of the cosmic web will remain the same, but clusters will be more ‘isolated’ and the filaments will become thinner.

The discussion has focused so far on the dynamical evolution of our cosmological neighbourhood, which hardly distinguishes between the comoving structure of the $\Lambda$CDM and OCDM models. Yet, the two models appear fundamentally very different when viewed in physical coordinates and in particular when the notion of the horizon is considered. The comoving event horizon in the $\Lambda$CDM model of the present epoch at the present time is $3.4h^{-1}$ Gpc and at $a = 6$ it is $0.6h^{-1}$ Gpc. As was pointed by Loeb [17] the future LG observer in the $\Lambda$CDM model will be living in an Island Universe. This will happen at $a \approx 350$ (namely, at $t \approx 8H_0^{-1}$), when the Virgo cluster will be at the event horizon of the LG observer. The cosmic web will not be accessible to these observers and the whole notion of comoving coordinates will be meaningless [12].

The work presented here can be extended further in a number of ways. The evolution of the skeleton structure with time in different models could be quantified by advanced statistics (e.g. the probability distribution function and Minkowski functionals). The models could be extended to study the much discussed equation of state parameter $w$, and to allow it to be epoch dependent, e.g. $w(a) \approx w_0 + w_a(1 - a)$. Major surveys of dark energy are underway to quantify $w(a)$ (e.g. the DETF report [1] and ESO/ESA
The future of the local large scale structure report [23]), and it would be interesting to understand the implications for the growth of structure in the universe if $w(a)$ turns out to be different than the $w = -1$ assumed in this paper. In addition, one may explore the impact of changes in the value of $\Omega_m$, the effect of massive neutrinos (which suppress structure on small scales) and the variations of the normalization of power spectrum $\sigma_8$.

Finally, we remark on the connection between our simulation results and the anthropic principle. The basic anthropic argument is that if the matter density was too high the universe would have collapsed by the present epoch without providing enough time for life to emerge and evolve. On the other hand, if the matter density was too small, structure would have not collapsed and again life would not have evolved. It has been argued further that the observed value of the cosmological constant can also be supported by anthropic arguments, (e.g. [36,6,31,37] and references therein). In a nutshell, Weinberg’s original argument was that the cosmological constant (in the form of a vacuum energy) cannot be too large and positive, because then galaxies would not form. The vacuum density should be less than the matter density of the universe at the epoch when galaxies formed. The probability of a random observer seeing the value of the vacuum energy as small as observed is about 15%, under certain assumptions [37]. We note in particular the sensitivity of the argument to the assumed amplitude of density fluctuations [31]. More recently, the possibility of a $\Lambda$CDM universe has been considered in the context of a ‘string landscape’, where the universe we live in is only one of many in the ‘multiverse’ ([37] and references therein). It has been argued in the above references that a $\Lambda$CDM universe is suitable for the emergence of life, while other universes with other extreme values of matter and vacuum densities would be hostile to life.

While the observed value of the cosmological constant is consistent with this argument, we argue that as the large scale structures expected in $\Lambda$CDM and the OCDM scenarios look so similar at the present epoch ($a = 1$), there is no preference from anthropic arguments alone for one or the other. The dominant effect of the formation of structure (as seen in comoving coordinates) is the matter density rather than the vacuum energy.

Acknowledgments

Fruitful discussions with Avi Loeb are gratefully acknowledged. We thank CIEMAT (Spain) for the use of the SGI-ALTIX supercomputer and to NIC Jülich (Germany) for the access to the IBM-Regatta p690+JUMP supercomputer. GY would like to thank also MEC for financial support under project numbers FPA2006-01105 and AYA2006-15492-C03. YH acknowledges the support of ISF-143/02, the Sheinborn Foundation and a PPARC Visiting Fellowship at UCL. OL acknowledges a PPARC Research Senior Fellowship.

Appendix

The fate of an individual object in an expanding universe can be readily calculated under the assumption of spherical symmetry. Here we follow the analysis of [18] for the evolution of density perturbations in a universe made of (non-relativistic) matter and gravitational constant of an arbitrary curvature. The analysis of [18] is repeated here for the sake of completeness and it is simply extended to include velocity as well as density perturbations.
The dynamical analysis of spherical perturbations often starts with the Tolman–Bondi energy-like term of a spherical shell of radius \( r \),
\[
\epsilon = \frac{v^2}{2} - \frac{GM(r)}{r} - \frac{4\pi}{3}G\rho_{\Lambda}r^2,
\]
where \( M(r) \) is the mass enclosed within a radius \( r \) and \( \rho_{\Lambda} = \Lambda c^2/8\pi G \). In the absence of shell crossing \( \epsilon \) is a constant of motion. This can be rewritten as:
\[
\epsilon = \left( \frac{v}{v_H} \right)^2 - \Omega_m(1 + \Delta) - \Omega_{\Lambda} \frac{H_0^2r^2}{2},
\]
where \( \Delta \) is the cumulative overdensity within the radius \( r \), \( v_H = Hr \) and \( H \) is Hubble’s constant. It follows that the integral of motion can be evaluated at any time, denoted here as \( t_i \). Thus the RHS of equation (A.2) is evaluated at \( t_i \) when the cosmological parameters equal \( \Omega_{m,i}, \Omega_{\Lambda,i} \) and \( H_i \) and the density perturbation is \( \Delta_i \). For convenience \( t_i \) is assumed to precede the time of turn-around of the given shell.

The equation of motion of the radius of the shell is readily given by
\[
\frac{ds}{dt} = H_i \left[ 1 + \Omega_{m,i}(\Delta_i + 1) \left( \frac{1}{s} - 1 \right) + \Omega_{\Lambda,i}(s^2 - 1) \right]^{1/2},
\]
where \( s \) is the radius scaled by its value at \( t_i \), \( s = r/r_i \). The turn-around radius is found by setting the RHS of (A.3) to zero. The condition for a turn-around radius to exist is that \( \Delta_i \) should be larger than a critical density given by:
\[
\Delta_{cr,i} = \frac{1}{\Omega_i} p(\Omega_{\Lambda,i}) - 1,
\]
where
\[
p(\Omega_{\Lambda,i}) = 1 + \frac{5\Omega_{\Lambda,i}}{4} + \frac{3\Omega_{\Lambda,i}(8 + \Omega_{\Lambda,i})}{4q(\Omega_{\Lambda,i})} + \frac{3q(\Omega_{\Lambda,i})}{4}
\]
and
\[
q(\Omega_{\Lambda,i}) = \{\Omega_{\Lambda,i}[8 - \Omega_{\Lambda,i}^2 + 20\Omega_{\Lambda,i} + 8(1 - \Omega_{\Lambda,i})^{3/2}]\}^{1/3}.
\]
The solution of Lokas and Hoffman [18] (equations (A.4)–(A.6)) is obtained for the case of no velocity perturbation, namely \( v/v_H = 1 \).

The analysis of [18] can be easily extended to accommodate also velocity perturbations. This is done by expressing the velocity perturbation as a change of the global Hubble constant into a local one. Consider a spherical shell at the fiducial time \( t_i \) with a peculiar velocity \( v_{p,i} \). The Tolman–Bondi energy equation (equation (A.2)) is rewritten as
\[
\epsilon = \left( 1 + \frac{v_p}{v_H} \right)^2 - \Omega_m(1 + \Delta) - \Omega_{\Lambda} \frac{H_0^2r^2}{2},
\]
and here a positive \( v_p \) indicates an outflow. An effective local Hubble constant is defined by
\[
h_l = H \left( 1 + \frac{v_p}{v_H} \right)
\]
and effective local density parameters by
\[
\omega_x = \Omega_x \left(1 + \frac{v_p}{v_H}\right)^{-2},
\] (A.9)

where \(x\) stands for \(m\) or \(\Lambda\).

Equation (A.3) can now be easily solved by replacing the global cosmological parameters \(H, \Omega_m\) and \(\Omega_{\Lambda}\) by the local values of \(h_l, \omega_m\) and \(\omega_{\Lambda}\) evaluated at \(t_i\).

Equation (A.4) implies that in the case of no velocity perturbation the critical overdensity is 17.6 (\(\Lambda\)CDM) and 2.33 (OCDM). In the case of velocity perturbations equation (A.4) is still valid, but with a modified local Hubble constant and density parameter (equations (A.8) and (A.9)). Adopting for the Virgo cluster a distance of 16 Mpc, a Hubble constant of \(H_0 = 70\) km s\(^{-1}\) Mpc\(^{-1}\) and a radial recession velocity of 932 km s\(^{-1}\) (cf section 3), one finds \((1 + v_p/v_H) = 0.83\). It follows that the critical overdensity for the Virgocentric infall is 14.6 (\(\Lambda\)CDM) and 1.3 (OCDM).

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