SCALAR BLACK BRANES WITH NON-ADS ASYMPTOTICS

Mariano Cadoni
Dipartimento di Fisica, Università di Cagliari and INFN, Sezione di Cagliari,
Cittadella Universitaria, 09142 Monserrato, Italy

Black brane solutions with scalar hair and with non-AdS asymptotics have several features that make them very interesting for holographic applications: (1) They allow to circumvent usual no-hair theorems; (2) They may have a non singular extremal limit in the form of a scalar soliton that interpolates between AdS and non-AdS vacua; (3) They allow for phase transitions between the Schwarzschild-AdS (SADS) and scalar dressed black branes; (4) They give an holographic realization of hyperscaling violation in critical systems. In this note I illustrate these features using an exact integrable Einstein-scalar gravity model.

I. INTRODUCTION

In the weak field approximation (i.e. far away from the sources) the gravitational field generated by localized sources is described by Minkowski spacetime $ds^2 = -dt^2 + dr^2 + dx_i^2$. In the presence of a negative cosmological constant the spacetime becomes Anti de Sitter (AdS) $ds^2 = r^2(-dt^2 + dx_i^2) + r^{-2}dr^2$, which is particularly relevant for holographic applications, in particular for the AdS/CFT correspondence.

One is therefore naturally led to consider spacetimes that interpolate between Minkowskian and AdS behaviour

$$ds^2 = r^\eta(-dt^2 + dx_i^2) + r^{-\eta}dr^2, 0 \leq \eta \leq 2.$$  

(1.1)

This spacetime is obviously not maximally symmetric and describes a brane, which must be sourced by a non trivial scalar field behaving as $\ln r$.

Nevertheless, spacetimes of the form (1.1) have recently attract much attention because they play an important role in holographic applications, in particular for the AdS/condensed matter (CM) correspondence.

Simply putting in the AdS background a black brane (BB) with a non trivial scalar field and eventually a finite Electromagnetic charge gives a rich phenomenology in the dual QFT: phase transitions triggered by scalar condensates, $U(1)$ symmetry breaking (superconductivity), non-trivial transport properties of the dual QFT.

In a wide class of models the near-horizon behavior, corresponding to the infrared (IR) regime of the dual QFT, is described by the metric (1.1) or by its Lifshitz generalization $ds^2 = r^{\theta-2}(-r^{-2(z-1)}dt^2 + dx_i^2 + dr^2)$.

(1.2)

where $\theta$ is the hyperscaling violation parameter and $z$ the dynamic critical exponent.

Although most of the conformal isometries of the AdS spacetime are broken whenever $\eta \neq 2$, the backgrounds (1.1) and (1.2) still preserve some scaling symmetries, namely they are scale covariant. In the dual QFT this corresponds to hyperscaling violation.

Most of the models with hyperscaling violation in the IR investigated in the literature are low-energy effective models, which flow in the ultraviolet (UV) to an AdS spacetime. However, we will show that an UV completion of the models is not strictly necessary. In fact all the thermodynamical parameters and properties are well defined also for models with hyperscaling violation in the UV. Moreover, we will show that a wide class of models exist which have an AdS phase in the IR and flow to an hyperscaling violating phase in the UV.

This note is mainly based on the papers [10, 11]. Its structure is the following. In sect. II we will describe the BB solutions of Einstein-scalar gravity with non-AdS asymptotics. In Sect. III we will investigate their thermodynamics and phase transitions. In Sect. IV we will be concerned with the symmetries of the solutions.

II. BLACK BRANES OF ADS EINSTEIN-SCALAR GRAVITY

We consider a model of minimally coupled Einstein-scalar gravity in 4D [10]:

$$S = \int d^4x \sqrt{-g} \left[ R - 2(\partial \phi)^2 + \frac{6}{\gamma L^2} \left( e^{2\sqrt{3}\beta \phi} - \beta^2 e^{2\sqrt{3}L^2} \right) \right],$$

(2.1)

* Talk given at the Thirteenth Marcel Grossmann Meeting, Stockholm, 1-7 July 2012.
where \( L \) is the AdS length, \( \beta \) is a free-parameter and \( \gamma = 1 - \beta^2 \). Note that the action is invariant under duality transformation \( \beta \rightarrow 1/\beta \).

The model has several interesting features \([10]\). It is a fake supergravity (SUGRA) model \([12, 13]\). For the asymptotically AdS solutions holds a positivity energy theorem and related no-hair theorem forbidding hairy BB solutions. The field equations for static, planar, radially symmetric solutions are exactly integrable (reduce to those of a Toda molecule). The model allows for BB solutions with scale-covariant asymptotics and it emerges as Kaluza-Klein compactifications of \( p \)-brane solutions of SUGRA theories.

The field equations stemming from Eq. (2.1) can be reduced to that of a Toda Molecule and admit a two-parameters \((\nu_1, \nu_2)\) family of scalar BB solutions \([14]\), which asymptotically approach to a scale-covariant metric \([14]\) with exponents: \( z = 1, \theta = \frac{6\beta^2}{1 + 3\beta^2} \). The extremal limit is a completely regular scalar soliton interpolating between AdS in the near-horizon (IR) region and a scale covariant metric asymptotically (UV) region. In the IR limit conformal invariance is restored. This corresponds to an IR fixed point of the dual QFT. In the UV the metric takes the form \([14]\). This corresponds to hyperscaling violation in the dual QFT. Apart from the scalar BB the model obviously admits the SADS solution with \( \phi = 0 \).

III. THERMODYNAMICS AND PHASE TRANSITION

Thermodynamics of the two-parameter family of BB solutions of Ref. \([10]\) is not straightforward. Temperature and entropy density are given by standard formulae \([11]\)

\[
T = \frac{1}{4\pi} \frac{3\gamma}{1 + 3\beta^2 (\nu_1 + \nu_2)^{-\frac{3\beta^2}{2}\nu_1^{1/3}}}, \quad S = \frac{\Omega R^2}{4G^2} = 4\pi \Omega (\nu_1 + \nu_2)^{\frac{3\beta^2}{2}\nu_1^{2/3}},
\]

(3.1)

where \( \Omega \) is the volume of the transverse 2-dimensional space. On the other hand the energy density (BB mass density) of the solution diverges. This is due to the explicit dependence of the scalar from \( \nu_1 \) (the temperature). The problem can be solved imposing the constraint \( \nu_2 (\nu_2 + \nu_1) = 1 \). Using an Euclidean action formalism we get the BB mass and free energy \([11]\): \( M = \frac{20}{1+3\beta^2} \left( \frac{1}{\nu_2} + (2\beta^2 - 1)\nu_2 \right) \), \( F_{SB}(T) = \frac{4}{1+3\beta^2} \left( \frac{3\beta^2 - 1}{\nu_2^2} + (\beta^2 + 1)\nu_2 \right) \). One can easily check that the first principle \( dM = T dS \) is satisfied. Comparing the free energy of the scalar BB with that of the SADS brane one finds that exists a critical temperature \( T_c \) above which the system undergoes a first order phase transition between SADS and the scalar BB \([11]\).

IV. HYPERSCALING VIOLATION

The features discussed in the previous sections have simple interpretation in terms of symmetries of the solution in the UV and IR region. In the IR phase we have AdS\(_4\), hence full conformal symmetry and critical parameters \( z = 1, \theta = 0 \). Conversely in the UV phase we have the scalar BB, hyperscaling violation and critical parameters \( z = 1, \theta = 6\beta^2/(3\beta^2 - 1) \). The problem is that for \( \beta^2 < 1/3 \) the hyperscaling violation parameter \( \theta \) is negative, whereas for \( \beta^2 > 1/3 \) we have \( \theta > 2 \). It has been shown that both regions \( \theta < 0 \) and \( \theta > 2 \) are compatible with the null energy conditions for the bulk stress-energy tensor \([8, 11]\). However, in the region \( \theta > 2 \) the scalar BB phase is unstable. This instability is confirmed by our results for the free energy and for specific heat. On the other hand negative values of \( \theta \) are not common in CM physics. In fact they correspond to a rising of the effective dimensions of the dual QFT \([11]\). This fact is probably related to an other main difference between the two cases. In CM physics the hyperscaling-violating phase is stable at small \( T \) where long wavelength fluctuations (for instance induced by a random field) dominate over thermal fluctuations. In our case the opposite is true: the hyperscaling violating phase is stable at large \( T \). This interchange of UV and IR physics is a rather puzzling point.

In this note we have considered the four-dimensional case, but our results can be also extended to arbitrary dimensions \([14]\).

---

[1] M. Cadoni, S. Mignemi and M. Serra, Phys. Rev. D 84 (2011) 084046 [arXiv:1107.5979 [gr-qc]].
[2] M. Cadoni, G. D’Appollonio and P. Pani, JHEP 1003 (2010) 100 [arXiv:0912.3520 [hep-th]].
[3] M. Cadoni and P. Pani, JHEP 1104 (2011) 049 [arXiv:1102.3820 [hep-th]].
[4] M. Cadoni, P. Pani and M. Serra, arXiv:1304.3279 [hep-th].
[5] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, Phys. Rev. Lett. 101 (2008) 031601 [arXiv:0803.3295 [hep-th]].
[6] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, JHEP 0812 (2008) 015 [arXiv:0810.1563 [hep-th]].
[7] C. Charmousis, B. Gouteraux and J. Soda, Phys. Rev. D 80 (2009) 024028 [arXiv:0905.3337 [gr-qc]].
[8] K. Goldstein, S. Kachru, S. Prakash and S. P. Trivedi, JHEP 1008 (2010) 078 [arXiv:0911.3586 [hep-th]].
[9] X. Dong, S. Harrison, S. Kachru, G. Torroba and H. Wang, JHEP 1206 (2012) 041 [arXiv:1201.1905 [hep-th]].
[10] M. Cadoni, S. Mignemi and M. Serra, Phys. Rev. D 85 (2012) 086001 [arXiv:1111.6581 [hep-th]].
[11] M. Cadoni and S. Mignemi, JHEP 1206 (2012) 056 [arXiv:1205.0412 [hep-th]].
[12] D. Z. Freedman, C. Nunez, M. Schnabl and K. Skenderis, Phys. Rev. D 69 (2004) 104027 [hep-th/0312055].
[13] P. K. Townsend, Phys. Lett. B 148 (1984) 55.
[14] M. Cadoni and M. Serra, JHEP 1211 (2012) 136 [arXiv:1209.4484 [hep-th]].