Improvement of the method of calculation of the parameters of the universal current collector with the increased motion speeds

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**Abstract.** The paper considers the main stages of development of a computer model of current collectors of an electric rolling stock. The original description of the kinematic scheme of a universal measuring current collector with the subsequent implementation of mathematical model in Matlab SimMechanics environment is offered. The model is based on the use of the method of homogeneous coordinates to represent the kinematic scheme of the current collector. It is shown that the computer model accurately and adequately describes the processes occurring with the current collector, is a universal instrument for simulating any types of current collectors, allows changing a wide range of parameters and characteristics incorporated in it.

1 Introduction

On the railways of the whole world, test trips using the latest methods of research and analysis of the current collection process are regularly carried out to assess the state of the contact suspensions and the process of their interaction with the current collectors of the electric rolling stock. The use of a special rolling stock for the testing of current collector devices is an economically costly measure, especially when using high-speed electric trains. Due to the large choice of high-speed current collectors with the required declared characteristics, the task of assessing the best available variants on the market is difficult. The production of a measuring current collector of an electric rolling stock requires a considerable amount of time and money. Thus, for test trips intended to determine the quality of current collection, a universal diagnostic tool with the capability to implement the characteristics of known prospective current collectors of electric rolling stock is necessary [1].

2 Materials and methods

At the current stage of development, most manufacturers of high-speed current collectors came to the conclusion that the scheme of asymmetric half-pantograph is optimal (Figure 1). It should be noted that there are significant differences in the mass-dimensional parameters

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of these current collectors and the values that are put into the design models (Table 1) [2, 3, 4, 5].

The greatest difficulty for modeling is the system of moving frames. For an adequate representation in the model of the arms’ movement, it is necessary to take into account the motion of all the masses in the system. When designing and developing a universal measuring current collector of electric rolling stock, it is important to choose the original model of a current collector, because the adjustment ranges, configuration and selection of parameters of the generated measuring current collector will depend on the original characteristics designed in the project. The universal measuring pantograph, the design of which allows varying the mass distribution by the elements in a wide range, was developed in the Omsk State Transport University at the department “Electric power supply of railway transport” [6].

Fig. 1. Examples of basic structures of high-speed pantographs and their design model: a) Faiveley CX, b) Contact ATR95, c) Stemman DSA380, d) three-mass model of current collector.

Table 1. Parameters of the design scheme of a pantograph of an electric rolling stock.

| Parameter | Unit of measurement | Faiveley CX | Contact ATR95 | Stemman DSA380 |
|-----------|---------------------|-------------|---------------|----------------|
| m1        | Kg                  | 5.58        | 10.14         | 7              |
| m2        | Kg                  | 8.78        | 13.05         | 11             |
| m3        | Kg                  | 7.75        | 9.45          | 10.5           |
| k1        | N/m                 | 178.45      | 7247.6        | 65.7           |
| k2        | N/m                 | 15487       | 30274         | 6700           |
| k3        | N/m                 | 7000        | 7978.7        | 20000          |
| c1        | N s/m               | 108.39      | 225.33        | 500            |
| c2        | N s/m               | 0.009       | 0.01          | 0              |
| c3        | N s/m               | 45.85       | 87.74         | 200            |

The three-mass model does not allow fully taking into account the features of the change in the weight characteristics of the pantograph from the height of its raise. For reliable simulation, the current collector is proposed to be represented as a spatially-rod finite element model [7]. In this case, the laws of motion of all the units and details of the model must be determined. It is proposed to use the generalized coordinates of the kinematic links included in the model.

To derive the equations of motion of the moving frame system, it is assumed that the asymmetric half-pantograph is a four-link. To determine the law of motion of point N (Figure 2), as well as the law of motion of links, it is necessary to choose a generalized coordinate. After determining the kinematic scheme to the four-link of the second-class Assur group [8], in which the OA-link corresponds to the lower arm, the AN-link - to the upper arm, the DB-link - to the lower pole, and the OD-link - to the base rigidly fixed with respect to the coordinate system. Within the model, it is necessary to know the dimensions of the links
The coordinates $X_A (\phi)$, $Y_A (\phi)$ and $X_D, Y_D$ of the arbitrarily chosen Cartesian coordinate system $XOY$ are given as a function of the angle $\phi$. The counter-clockwise direction is assumed as the positive direction of rotation of the link.

**Fig. 2.** Kinematic diagram of the moving frame system of a pantograph.

**Fig. 3.** Determination of the angle $\theta$ using the cosine theorem.
Taking into account the agreed notations for determining the coordinates of point A, we write the following expressions:

\[
X_A = X_O + l_{OA} \cdot \cos \phi \\
Y_A = Y_O + l_{OA} \cdot \sin \phi
\]  

(1)

(2)

To determine the coordinate of point B, it is necessary to find the angle \( \theta \). Using the variable length \( l_v \), which is determined by the cosine theorem with known values of the coordinates of points A and D, we determine the angle \( \beta \) in the triangle ADC (Figure 3).

\[
l_v = \sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2}
\]

(3)

Through the known coordinates of the points A and D, we define the angle \( \beta \)

\[
\beta = \arctg \frac{Y_D - Y_A}{X_D - X_A}
\]

(4)

For the second auxiliary angle \( \lambda \), we find the angle \( \lambda \) from the triangle ADB by the cosine theorem

\[
\lambda = \arccos \frac{\frac{l_{AB}^2 + (\sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2})^2 - l_{BD}^2}{2l_{AB} \cdot \sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2}}}{2}
\]

(5)

The value of the angle \( \theta \) can be determined through the additional angles \( \beta \) and \( \lambda \)

\[
\theta = \frac{\pi}{2} - \beta - \lambda
\]

(6)

Thus, the position of the point B is uniquely determined by the angle \( \varphi \):

\[
X_B = X_D + l_{BD} \cdot \cos \left( \frac{\pi}{2} - \arctg \frac{Y_D - Y_A}{X_D - X_A} + \arccos \frac{\frac{l_{AB}^2 + (\sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2})^2 - l_{BD}^2}{2l_{AB} \cdot \sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2}}}{2}
\]

(7)

\[
Y_B = Y_D + l_{BD} \cdot \sin \left( \frac{\pi}{2} - \arctg \frac{Y_D - Y_A}{X_D - X_A} + \arccos \frac{\frac{l_{AB}^2 + (\sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2})^2 - l_{BD}^2}{2l_{AB} \cdot \sqrt{(Y_D - Y_A)^2 + (X_D - X_A)^2}}}{2}
\]

(8)

Next, we need to find the coordinates of the point N. To do this, we construct the auxiliary triangle AFB (Figure 4)

\[\text{Fig. 4. Determination of the trajectory of the movement of the upper hinge of the moving frame system of a pantograph.}\]
Knowing the coordinates of the points A and B, we can find the coordinates of the point F

\[
XF =XA = XO + l_{OA} \cdot \cos\phi \tag{9}
\]
\[
YF =YB = YD + l_{DB} \cdot \sin\theta \tag{10}
\]

Next, we determine the lengths of the sides AF and FB

\[
AF =XA - YD + l_{BD} \cdot \sin\theta \tag{11}
\]
\[
FB =XD + l_{DB} \cdot \cos\theta \tag{12}
\]

We find the angle \(\mu\) by the cosine theorem

\[
\mu = \arccos \frac{l_{FB}}{l_{FA}} \tag{13}
\]

Now the coordinates of the point N can be written as follows

\[
X N =XA + l_{NA} \cdot \cos\mu \tag{14}
\]
\[
Y N =YA + l_{NA} \cdot \sin\mu \tag{15}
\]

Thus, the trajectory of the point N depends on the generalized coordinate of the angle \(\phi\). For numerical simulation of the reduced mass of the pantograph, the weight characteristic, the static pressing characteristic, it is proposed to implement the model of the current collector using the SimMechanics library, the Simulink package, the MATLAB environment [9].

Fig. 5. Graphical representation of a pantograph model: a) two-dimensional model, b) one-dimensional model, 1 – pantograph pan, 2 - upper hinge of moving frame system, 3 - pantograph base, Pyp - aerodynamic lift of pans, Pct - contact pressing, Pf - static pressing of the moving frame system.
This software complex uses the method of homogeneous coordinates as a computational mechanism, according to which the frame motion of the elements of the kinematic scheme can be represented by matrices of rotation and shift with respect to an arbitrary coordinate system [10]. This property allows calculating by the matrix method and converting the coordinates of the point N in different coordinate systems.

The homogeneous transformation matrix \( H_n^0 \) consists of the 3x3 rotation matrix \( R_n^0 \), the transport vector \( d_n^0 \), which consists of three lines, the central projection vector \( f \), and the scaling factor \( m \).

\[
H_n^0 = \begin{bmatrix} R_n^0 & d_n^0 & f \\ & & & m \end{bmatrix}
\]  

(16)

The application of the method of homogeneous coordinates with the help of matrix multiplication operations allows transforming the laws of motion known in local coordinate systems to global or any other. To do this, it is necessary to multiply several coordinate transformation matrices in sequence:

\[
H_3^0 = H_1^0 \cdot H_2^0 \cdot H_3^0
\]  

(17)

3 Results

The resulting matrix transforms the motion of point N relative to the base. To shorten the form of the record, it is suggested to use the conventions: \( \sin \) is denoted by the symbol \( s \), and \( \cos \) by the symbol \( c \). We write down the values of the transformation matrix \( H_3^0 \), corresponding to the coordinates \( x \) and \( y \) of the pans in the coordinate system of the pantograph base:

\[
XN_1 = \Delta H_k \cdot c \phi \cdot c(\pi - \phi - \mu) - s \phi \cdot s(\pi - \phi - \mu) \cdot -s\left(\frac{\pi}{2} + \mu\right) + c \phi \cdot -s \phi \cdot c(\pi - \phi - \mu) \cdot c\left(\frac{\pi}{2} + \mu\right) + c \phi \cdot c(\pi - \phi - \mu) - s \phi \cdot s(\pi - \phi - \mu) \cdot l_{NN_1} \\
\cdot s\left(\frac{\pi}{2} + \mu\right) + c \phi \cdot l_{AN} \cdot c(\pi - \phi - \mu) - s \phi \cdot l_{AN} \cdot s(\pi - \phi - \mu) + l_{OA} \cdot c \phi
\]  

(18)

\[
YN_1 = \Delta H_k \cdot s \phi \cdot c(\pi - \phi - \mu) + c \phi \cdot s(\pi - \phi - \mu) \cdot -s\left(\frac{\pi}{2} + \mu\right) + s \phi \cdot -s(\pi - \phi - \mu) + +c \phi \cdot c(\pi - \phi - \mu) + s \phi \cdot c(\pi - \phi - \mu) + c \phi \cdot s(\pi - \phi - \mu) \\
\cdot l_{NN_1} \\
\cdot c\left(\frac{\pi}{2} + \mu\right) + s \phi \cdot -s(\pi - \phi - \mu) + c \phi \cdot c(\pi - \phi - \mu) + s \phi \cdot l_{AN} \cdot c(\pi - \phi - \mu) \\
+ c \phi \cdot l_{AN} \cdot s(\pi - \phi - \mu) + l_{OA} \cdot s \phi
\]  

(19)

The model compiled according to the proposed method allows taking into account the base oscillations in space and the displacement of the parts of the pantograph under the action of contact pressing. Taking into account the law of motion of each described unit of the pantograph (Figure 6), the calculation of the weight characteristic of the current collector, its reduced masses, and the static pressing characteristics is performed (Figure 7).
Fig. 6. Coordinate systems of the elements of the spatial model of a pantograph.

Fig. 7. Calculation characteristics of the pantograph obtained with the help of the model: 1 – lowering characteristic, 2 - reduced mass, 3,4 - static pressing characteristics.

4 Conclusions

With the help of the proposed method, it was possible to remove a number of model assumptions that do not allow taking into account the laws of variation of the reduced mass, the characteristics of static pressing, and the lowering force of the pantograph. The use of the proposed model allows recreating more accurately the indicators of high-speed pantographs during the tests and increasing the quality and informativeness of the results.

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