Local Daily Temperature Averaging Reveals Semiannual Climate Cycles

Yunxiang Song\textsuperscript{1}, Kyle Lawlor\textsuperscript{1}, Thomas A. Witten\textsuperscript{1}

\textsuperscript{1}James Franck Institute, University of Chicago, Chicago, Illinois, USA

Key Points:

- Analysis emphasizing long historical local temperature records yields precise measurement of annual and semiannual temperature cycles.
- Annual temperature phase lags are confined to a narrow ten day window.
- Semiannual to annual temperature amplitude ratios suggest potential non-linear response in the semiannual cycle.

Corresponding author: Yunxiang Song, songy1@uchicago.edu
Abstract

The annual temperature cycle of the earth closely follows the annual cycle of solar flux. At temperate latitudes, both driving and response cycles are well described by a strong annual component and a non-vanishing semiannual component. We report features of these cycles revealed by historic data taken from 64 weather stations in the United States. The use of daily temperatures yields well-resolved determination of annual and semiannual temperature amplitudes. We compare these temperature amplitudes with the known amplitudes of the primary solar flux. Annual temperature phase lags mostly fell within a narrow ten-day band, well separated from an in-phase response. Semiannual temperature cycles were much stronger than expected based on the semiannual solar driving. Instead, these cycles were consistent with combined effects of two annual cycles. Thus, our methods provide a quantitative window into the climate’s nonlinear response to solar driving, which is of potential value in testing climate models.

Plain Language Summary

Underlying the random day to day variations of the surface air temperature is a regular part that has period a year. The variability in the solar flux has an annual cycle, and it is the primary contributor to this periodic behavior in the earth’s thermal response. For the purpose of revealing robust seasonal variations in the local temperature response to the sun’s periodic driving, one can average the temperature at a given day of the year over many years. Averaging over historical records preserves global factors affecting local temperature responses that are important for understanding climate. Here we show that daily local weather station records provide a remarkably precise measure of this temperature response without regional or monthly averaging. Harmonic analysis of these records provides annual responses that consistent with the literature. It also provides well-resolved semiannual responses for many stations, which can serve as a window onto studying nonlinear temperature responses.

1 Introduction

Global climate features are often studied through terrestrial averaging of monthly mean temperatures over long periods of time. Several authoritative studies (Stine et al., 2009) (Stine & Huybers, 2012) (McKinnon et al., 2013) (Legates & Willmott, 1990) (Eliseev & Mokhov, 2003) have revealed important global factors and secular trends. Comprehensive weather maps constructed with this data reveal accurate proportionality between the solar flux and the annual variation of temperature, especially at temperate latitudes. Thus, describing the temperature response amounts to stating the relationship between the input and response sine waves. This relationship can be characterized by two quantities: the gain (the magnitude of response, defined as the ratio between temperature and driving amplitudes); and the phase lag (the time delay of response, defined as the difference between temperature and driving cycles) (Stine et al., 2009) (McKinnon et al., 2013). Our analysis of 50 to 130 years of temperature data for 64 weather stations builds strongly on this previously established method. Applying such gain-phase analysis to the semiannual part for mid-latitude stations shows anomalously large semiannual gains, which are not well-explained by linear response theory. Below, we interpret these semiannual amplitudes in terms of nonlinear response.

The gains and phase lags reflect universal factors that are largely consistent throughout the northern hemisphere. It is known that the seasonal albedo, the local absorption of short wave radiation, the re-radiated long wave radiation, and the energy transported elsewhere along prevailing winds and temperature gradients can modulate the net solar flux at any given location (Pierrehumbert, 2010). A significant fraction of the 20 GJ/m² annual variation in solar energy must be absorbed and re-emitted every year. The only accessible reservoir of this magnitude is the oceans, and these have their own well-studied
annual temperature cycles (Levitus, 1984). The physical mechanism for this global heat transport is time invariant and can thus be probed by a historical analysis of the annual cycles.

2 Methods

We measured gains and phase lags for 64 weather stations spread over the United States, as shown in Fig. 3. To improve precision, the stations were selected for consistency over long historic records of 50 to 130 years; to avoid urban effects, they were also selected for low population densities. To minimize short-term weather effects, we used the daily low temperature as our sampled quantity. In the interest of uniform data-taking, all stations were taken in a single country with institutional continuity. To ensure consistencies in gains and phase lags, we cross-checked our findings with an additional 9 weather stations scattered across the globe, selected on the basis of similar criteria. All annual temperature gains were similar to the well-established, uniform annual gains (20 °C/m²/W) across the mid-latitude oceans (Stine et al., 2009). We express all annual gains henceforth as dimensionless quantities by normalizing to this mid-ocean gain.

2.1 Analysis of data

Our source of data was the National Oceanic and Atmospheric Association (NOAA) Global Historical Climatology Network dataset, using temperature data through 2010 (Menne, Durre, Vose, et al., 2012) (Menne, Durre, Korzeniewski, et al., 2012). We converted each calendar day in the records to fraction of an astronomical year, \( t \), measured from the winter solstice. We averaged all the temperatures for a given \( t \), for all years in each station record.

The data for each station were then subject to a least-squares fitting to the function \( T(t) \) given by

\[
T(t) = a \cos(2\pi t) + b \sin(2\pi t) + c \cos(4\pi t) + d \sin(4\pi t) + e. \tag{1}
\]

We determined the coefficients \( a \) to \( e \) using Mathematica's `LinearModelFit` (Wolfram Research, Inc., 2019). We verified that the covariance matrix was very nearly diagonal, reflecting the orthogonality of the components in the fitting function. Given the values of coefficients \( a \) to \( e \), we could readily determine the amplitude and phase \( B_1 \) and \( \phi_1 \) of the annual cycle and their counterparts \( B_2 \) and \( \phi_2 \) for the semiannual cycle. The temperature variation is thus given by

\[
T(t) = T_0 + B_1 \cos(2\pi t - \phi_1) + B_2 \cos(4\pi t - \phi_2). \tag{2}
\]

We compare this temperature variation at each station with the daily average incident solar flux \( S(\alpha, t) \) at that station’s latitude \( \alpha \) (Pierrehumbert, 2010). We may express the known \( S(\alpha, t) \) to good accuracy in the form

\[
S(\alpha, t) = S_0 + A_1 \cos(2\pi t) + A_2 \cos(4\pi t). \tag{3}
\]

From the \( B_n \) and \( A_n \) we could compute the gains, \( G_n \), for each harmonic. This gain is the temperature amplitude \( B_n \) divided by the corresponding amplitude \( A_n \) of the primary solar flux \( (G_n = B_n/(A_n)) \). The \( A_n \) are given explicitly as Fourier series coefficients in Appendix A. The solar flux amplitudes all have a phase shift of 0. Thus, the angles \( \phi_n \) are the phase lags between the solar power \( A_n \cos(2\pi nt) \) and the temperature \( B_n \cos(2\pi nt - \phi_n) \). We report these gains and phase lags for representative stations in Table 1, and full results in the Supporting Information. Their quoted uncertainties derive entirely from the standard errors in the fitted parameters \( a \) to \( e \) (Wolfram Research, Inc., 2019). These uncertainties do not reflect the consistency of gains or phase lags over the historical record.
2.2 Consistency of gains and phase lags

To assess the variability of the annual phase lags over time, we performed a separate year-by-year analysis of the temperature records. For each station, we performed the five parameter fit (1) on the individual year data and obtained phase lags and annual and semiannual gains for each year. As a measure of consistency over time, we report the standard deviation in these individual year annual gains and phase lags as an additional uncertainty, presented in boldface in Table 1. The semiannual quantities for individual years were ill-defined and are not included.

Figure 1. (a) Historically averaged temperature vs. day of year for two local weather stations showing quality of fit to sinusoidal dependence. Left plot is the station with maximal annual gain, marked L in Fig. 3; right plot is the station with minimal gain. Horizontal axis is the fraction of year measured from the winter solstice, marked R in Fig. 3. Red curve is the five-parameter fit (1) including the average temperature and annual and semiannual frequencies. Histograms show individual year phase lags to show consistency over time. (b) Semiannual variation vs. day of year for the same stations. Red curve is the seasonal cycle with the average and the annual component used in (a) subtracted. Orange band gives the error range arising from statistical uncertainties in the fitted parameters, as given in Table 1.

3 Results

The annual gains for 50 out of 64 stations were determined to within half a percent; the annual phase lags for all stations were determined to within half a day. Two such examples of the seasonal temperature cycle are given in Fig. 1(a). A complete depiction of the annual gain-phase distribution with location is given in Fig. 2. For most stations in the United States, the annual gains, normalized by the mid-ocean gain, varied across a range from 1 to 4, following a well-established geographic pattern (Stine et
Table 1: Gain phase information for best and worst determined stations

| Location | Normalized Temperature Amplitude | Annual Phase Shift |
|----------|----------------------------------|-------------------|
|          | Annual Gain | Semiannual to annual ratio | (Degrees) |
| 1        | 3.8 ± 0.24% | 0.08 ± 2.8% | 25.5 ± 0.14 |
|          | 4.0 ± 5%     |                   | 27.7 ± 2.2 |
| Best 5-parameter fits | 2   | 3.9 ± 0.26% | 0.05 ± 6% | 26.0 ± 0.15 |
|          | 4.2 ± 7%     |                   | 28.3 ± 3.1 |
|          | 3            | 3.7 ± 0.27% | 0.08 ± 3% | 27.4 ± 0.13 |
|          | 4.0 ± 6%     |                   | 28.9 ± 3.0 |
| Best determined semiannual to annual ratio | 4   | 3.2 ± 0.3% | 0.15 ± 2.0% | 28.4 ± 0.17 |
|          | 3.5 ± 8%     |                   | 30.1 ± 5.0 |
|          | 5            | 3.2 ± 0.3% | 0.17 ± 2.0% | 30.5 ± 0.19 |
|          | 3.6 ± 10%    |                   | 32.1 ± 3.8 |
|          | 6            | 4.0 ± 0.3% | 0.14 ± 2.3% | 21.9 ± 0.19 |
|          | 4.3 ± 5%     |                   | 23.8 ± 3.3 |
| Worst determined semiannual to annual ratio | 7   | 3.3 ± 0.5% | 0.01 ± 53% | 28.4 ± 0.27 |
|          | 3.5 ± 9%     |                   | 30.1 ± 3.7 |
|          | 8            | 3.2 ± 0.4% | 0.01 ± 42% | 30.5 ± 0.26 |
|          | 3.5 ± 10%    |                   | 31.9 ± 4.1 |
|          | 9            | 3.3 ± 0.3% | 0.01 ± 24% | 25.6 ± 0.17 |
|          | 3.7 ± 9%     |                   | 33.1 ± 3.0 |

Table 1. Amplitude and phase information for best and worst determined stations, as identified in the first column. Locations in the second column are indicated on Fig. 3. Annual temperature gains are normalized by the mid-ocean annual gain magnitude defined in the text. The semiannual to annual amplitude ratio is given in the fourth column. Historically averaged quantities with standard fit parameter errors as uncertainty ranges, are given in the upper row for each station. Individual year annual gains and phase shifts, with standard deviations as uncertainty ranges, are given in the bottom row and set in bold. The spread for individual year semiannual quantities are omitted as discussed in Methods. Full information on all stations analyzed is reported in the Supporting Information.

The annual phase lags mostly converged within a narrow range of 20.7 to 30.6 degrees (each degree is approximately a day), with no obvious geographic trends, as shown by the marker orientations in Fig. 2. The individual differences in annual gains and phase lags were typically of order ten times greater than the uncertainties arising from best-fit parameters of the seasonal cycles.

To investigate the higher harmonics, we plot temperature profiles with the annual component and average removed in Fig. 1(b). The remaining semiannual signals are well resolved outside the noise. The error range of the pure semiannual fit is small, as represented by the orange band. Of all stations, 97% had relative errors less than 25% uncertainty in the semiannual gain.

The semiannual temperature amplitude relative to the corresponding annual amplitude is mapped in Fig. 3. The amplitude ratios are systematically smaller than 20 percent, with no clear geographic trends. One possible driver of the semiannual amplitude is the semiannual solar amplitude $A_2$. To investigate the effect of $A_2$, we plotted the temperature amplitude ratio $B_2/B_1$ vs. the solar amplitude ratio $A_2/A_1$ in Fig. 4. A $B_2$ driven...
entirely by $A_2$ would appear as a straight line through the origin on this plot. On the actual plot, no such straight-line correlation is apparent. Moreover, many gain ratios, given by $(B_2/B_1)/(A_2/A_1)$, are as much as several hundred, implying large disparities between annual and semiannual gains (see Supporting Information).

4 Discussion

Our nonstandard selection of data requires some justification. The daily low temperatures averaged over historical records at individual stations produce sinusoidal temperature cycles that agree well with the broad surveys of references (Stine et al., 2009) (Stine & Huybers, 2012) (McKinnon et al., 2013). We acknowledge that our selection of daily low temperatures is not an optimal measure of mean temperature (Baker, 1975) (Schaal & Dale, 1977). Nevertheless, our data reveal annual and semiannual variation with high precision.

Our annual variations reveal some features not clear in the comprehensive findings of Stine et al. (2009). One of these is the well-resolved phase lags of individual stations. Stine et al. (2009) noted a correlation between regional phase lags, gains, and geographic position. These correlations are not apparent in our local data. Instead, our data show a clustering of phase lags within a narrow band of 20.7 to 30.6 degrees, suggesting some time-invariant, lag mechanism beyond the regional scale. Our analysis suggests that these lags are constant over time to within a few days, consistent with the observed station-to-station variability. The relative constancy of these lags over wide geographical regions.
Figure 3. Heat map of the ratio of semiannual to annual temperature amplitude for all stations of this study. Grey circles are locations of the stations in our selected sample. Stations enumerated in the data table are numbered 1 to 9. Station on the left in Fig. 1 is labeled L, and the right, labeled R.

Figure 4. Scatter plot of the ratio of semiannual to annual solar driving amplitudes $A_2/A_1$ vs. relative semiannual temperature amplitude $B_2/B_1$ for all weather stations. The lack of weak semiannual responses below the solid line suggests a floor set by the direct semiannual solar driving.
was not noted in prior studies, though a similar narrow spread is visible in the supplementary data of Stine et al. (2009).

Our annual temperature amplitudes, in agreement with prior studies, are well characterized by the concept of gain, giving amplitudes proportional to the solar driving amplitude. Likewise, the semiannual temperature cycle is certainly driven by the semiannual solar driving. Yet, as Fig. 4 suggests, an explanation of the semiannual temperatures based on semiannual gain is incomplete. These data suggest an apparent floor in semiannual relative temperature, of roughly 1.1 times the relative solar driving. This may indicate the contribution of semiannual gain.

The bulk of the semiannual amplitude must come from another source. One natural source is nonlinear driving; that is, two annual harmonics combining to produce a semi-annual harmonic. To show how two annual periodicitics in climate features can lead to a semiannual temperature response, we focus on the relation between the incident astronomical flux $S(\alpha, t)$ and the absorbed solar flux that is converted to heat, denoted $J(t)$. At any given station this $J(t)$ is smaller than $S(\alpha, t)$ because some of the incident radiation is re-radiated and not converted to heat. Thus $J(t)$ has the form $\beta(t)S(\alpha, t)$, where $\beta(t)$ is the absorbed fraction. The fraction $\beta$ depends on climatic factors such as cloud cover. This $\beta$ can certainly be affected by the solar flux itself. Thus over historic time, $\beta$ may be expressed as $\beta(t) = \beta_0(1 + \beta_1(S(\alpha, t) - \bar{S}) + \mathcal{O}((S(\alpha, t) - \bar{S})^2))$. Here we demonstrate how this variation of $\beta$ can lead to semiannual periodicity in the temperature. To illustrate our point, we suppose that the induced variation is weak. Accordingly, we neglect any contribution beyond the linear term proportional to $\beta_1$. Further, we keep only the annual sinusoidal modes of $S(\alpha, t)$ so that, $\beta(t) \approx \beta_1(1 + \beta_C \cos(2\pi t) + \beta_S \sin(2\pi t))$, where $\beta_C$ and $\beta_S$ are unknown coefficients.

Using this form, we may readily find the time dependence of the absorbed flux $J(t)$, taking $S(\alpha, t)$ from (3)

$$J(t) = \bar{\beta} \bar{S} \left( 1 + \beta_C \cos(2\pi t) + \beta_S \sin(2\pi t) \right) \left( 1 + S_1 \cos(2\pi t) + S_2 \cos(4\pi t) \right),$$

where $\bar{S} \equiv S_0$, $S_1 \equiv A_1/\bar{S}$, and $S_2 \equiv A_2/\bar{S}$. Expanding this expression using trigonometric identities, we obtain

$$J(t) = \bar{\beta} \bar{S} \left[ \left( 1 + \frac{1}{2}(\beta CS_1) \right) + \left( (\beta C + S_1) \cos(2\pi t) + \beta_S \sin(2\pi t) \right) \right. \\
\left. + \left[ \frac{1}{2} \beta_C S_1 + S_2 \right] \cos(4\pi t) + \frac{1}{2} \beta_S S_1 \sin(4\pi t) \right] + \mathcal{O}(\beta_C S_2, \beta_S S_2),$$

where the first square bracket term is constant in time; the second has an annual period; and the third has a semiannual period.

The semiannual component has the expected part proportional to $S_2$ from the semiannual part of the solar flux $S$. In addition, it has a contribution proportional to $S_1$ and to $\beta_C$ or $\beta_S$. Combining these annual and semiannual modes, $J(t)$ can be written

$$J(t) = \bar{J} \left( 1 + J_1 \cos(2\pi t + \delta_1) + J_2 \cos(4\pi t + \delta_2) \right),$$

where $J_1, \delta_1, J_2,$ and $\delta_2$ can be expressed in terms of $S_1, S_2, \beta_S,$ and $\beta_C$.

We now consider the effect of the $\beta_C$ and $\beta_S$ on the temperature $T(t)$ from (2). If the temperature response is proportional to the $J(t)$ with a frequency-dependent gain $G$, we then have $B_1 = G_1 J_1$ and $B_2 = G_2 J_2$, where $G_1$ is the gain at the annual frequency and $G_2$ is the gain at the semiannual frequency. Thus $B_2/B_1 = (G_2/G_1)(J_2/J_1).$ The gain $G$ gives the response of the steady-state earth system to an arbitrarily weak periodic driving. In general, such gains depend smoothly on frequency (Van Trees, 2004), so that $G_2/G_1$ would be a number of order unity. Any $G_2/G_1$ much larger or smaller than unity calls for an explanation.
Without the $\beta_C$ and $\beta_S$ factors, the temperature ratio $B_2/B_1$ is given by

$$\frac{B_2}{B_1} = \frac{G_2 S_2}{G_1 S_1}.$$

(7)

Thus, this formula predicts a simple proportionality between the $S_2/S_1$ and $B_2/B_1$, which are the quantities plotted in Fig. 4. Instead, the figure shows a broad scatter of points, implying $G_2/G_1$ values as high as several hundred. (The largest implied gains arise from latitudes near 44 degrees where the $S_2$ goes to 0.) The semiannual temperature variations are thus too large to be plausibly explained in terms of a linear gain treatment.

When we add an annual cyclic variation to the absorption factor $\beta$, the absorbed solar flux $J(t)$ acquires a new semiannual contribution, which adds to the conventional $S_2$. From (5), this new contribution dominates when $\beta_C \gg S_2 S_1$. At latitudes where $S_2$ vanishes, any nonzero $\beta_C$ part must dominate. The term in $\beta_S$ causes similar effects.

To estimate the magnitude $\beta_C$ and $\beta_S$ needed to explain the semiannual temperature cycle, we neglect the $S_2$ contribution to $J(t)$, so that the entire semiannual contribution comes from $\beta_C$ and $\beta_S$. For example, if only $\beta_C$ were present, (7) becomes

$$\frac{B_2}{B_1} = \frac{G_2 \beta_C S_2 / 2}{G_1 (S_1 + \beta_C)} = \frac{G_2}{G_1} \frac{\beta_C/2}{1 + \beta_C/S_1}.$$

(8)

At latitudes near 44 degrees, the measured $B_2/B_1$ ratios from the full data table in the supporting information lie in the range of 5-10 percent. One readily verifies that this range is compatible with (8) for $G_2/G_1 \approx 2$ and $0.1 \lesssim \beta_C \lesssim 0.2$. Other latitudes are also consistent with this range.

This amount of periodic annual variation is consistent with other evidence. Kukla and Robinson (1980) reported measurements of annual variation in the reflectivity of relative magnitude $0.05 - 0.15$, similar to our inferred $\beta_C$.

This example shows that the primary solar flux cycle, combined with the induced annual variation in some other quantity, such as the absorbed fraction $\beta$ can plausibly account for our observed semiannual cycles in temperature. Many other induced variations would have the same effect. Examples include the induced thermal radiation into space via the Stefan-Boltzmann law or the atmosphere’s effective thermal conductivity (Pierrehumbert, 2010). Our observations are thus compatible with many potential causes. Without further study, the specific nonlinear effect producing $B_2$ is not clear. However, the existence of this semiannual variation sets a lower bound on the importance of these anharmonic effects. Thus it provides an alternate route to proving the sensitivity of properties like the albedo to the solar flux. Further, our study gives strong evidence against the simple picture wherein the semiannual amplitude $B_2$ is simply caused by the corresponding solar amplitude $S_2$.

5 Conclusion

The potential consequences of global warming depend strongly on ill-understood, non-linear responses to solar heating. This study shows a novel source of information about these responses in historic weather station data. Extensions of such studies beyond the limited geographic scope presented here show promise in distinguishing global from regional effects and in distinguishing baseline behavior from human impacts.
Acknowledgments
The authors thank professors Noboru Nakamura, Mary Silber, Sidney Nagel, Dorian Abbot, and Robert Rosner for constructive discussions and valuable comments on the manuscript; professors Alexander Stine and Peter Huybers provided useful insights during the early stages of the work. The data used in this paper can be found by accessing the National Oceanic and Atmospheric Association (NOAA) Global Historical Climatology Network dataset. Yunxiang Song acknowledges support from a James Franck Institute summer fellowship.

References
Baker, D. G. (1975). Effect of observation time on mean temperature estimation. Journal of Applied Meteorology, 14(4), 471–476.
Eliseev, A. V., & Mokhov, I. I. (2003). Amplitude-phase characteristics of the annual cycle of surface air temperature in the northern hemisphere. Advances in atmospheric sciences, 20(1), 1–16.
Kukla, G., & Robinson, D. (1980). Annual cycle of surface albedo. Monthly Weather Review, 108(1), 56–68.
Legates, D. R., & Willmott, C. J. (1990). Mean seasonal and spatial variability in global surface air temperature. Theoretical and applied climatology, 41(1-2), 11–21.
Levitus, S. (1984). Annual cycle of temperature and heat storage in the world ocean. Journal of Physical Oceanography, 14(4), 727–746.
McKinnon, K. A., Stine, A. R., & Huybers, P. (2013). The spatial structure of the annual cycle in surface temperature: Amplitude, phase, and lagrangian history. Journal of Climate, 26(20), 7852–7862.
Menne, M. J., Durre, I., Korzeniewski, B., McNeal, S., Thomas, K., Yin, X., … others (2012). Global historical climatology network-daily (ghcn-daily), version 3. NOAA National Climatic Data Center, 10, V5D21VHZ.
Menne, M. J., Durre, I., Vose, R. S., Gleason, B. E., & Houston, T. G. (2012). An overview of the global historical climatology network-daily database. Journal of Atmospheric and Oceanic Technology, 29(7), 897–910.
Pierrehumbert, R. T. (2010). Principles of planetary climate. Cambridge University Press.
Schaal, L. A., & Dale, R. F. (1977). Time of observation temperature bias and climatic change. Journal of Applied Meteorology, 16(3), 215–222.
Stine, A. R., & Huybers, P. (2012). Changes in the seasonal cycle of temperature and atmospheric circulation. Journal of Climate, 25(21), 7362–7380.
Stine, A. R., Huybers, P., & Fung, I. Y. (2009). Changes in the phase of the annual cycle of surface temperature. Nature, 457(7228), 435.
Van Trees, H. L. (2004). Detection, estimation, and modulation theory, part i: detection, estimation, and linear modulation theory. John Wiley & Sons.
Wolfram Research, Inc. (2019). Mathematica, Version 12.0. Retrieved from https://reference.wolfram.com/language/ref/LinearModelFit.html