Shortcuts to adiabatic passage for multiparticles in distant cavities: applications to fast and noise-resistant quantum population transfer, entangled states’ preparation and transition

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Abstract
In this letter, we study the fast and noise-resistant population transfer, quantum entangled states’ preparation and quantum entangled states’ transition by constructing shortcuts to adiabatic passage (STAP) for multiparticles based on the approach of “Lewis–Riesenfeld invariants” in a distant cavity quantum electronic dynamics (QED) system. Numerical simulation demonstrates that all of the schemes are fast and robust against the decoherence caused by atomic spontaneous emission and photon leakage. Moreover, it is not only the total operation time but also the robustness in each scheme against decoherence that is irrelevant to the number of qubits. This might lead to a useful step toward realizing fast and noise-resistant quantum information processing in current technology.

Keywords: shortcuts to adiabatic passage, population transfer, multiparticle entangled state generation, multiparticle entangled state transition

(Some figures may appear in colour only in the online journal)
with increasing interaction time. There are many instances for which we would like, or need, to quicken the operations in an experiment. An idea or method to generate and manipulate various entangled states should be fast, robust and easy realized with current technology. Combining the best aspects of the resonant pulses route and adiabatic method, accelerating the dynamics of adiabatic passage towards a final outcome is a perfect way to realize fast and noise-resistant population transfer, quantum entangled states’ preparation and quantum entangled states’ transition under the current technology.

‘Shortcuts to adiabatic passage’ (STAP) [9–11] which is a recent and timely introduction to describe schemes that usually speed up a quantum adiabatic process, although not necessarily, through a non-adiabatic route, has attracted a great deal of attention and promises to overcome the harmful effects caused by decoherence, noise or losses during a long operation time. In recent years, various reliable, fast and robust schemes have been proposed to find shortcuts to a slow adiabatic passage in theory [9, 11–16] and in experiment [17–22]. For example, Chen et al [9] have put forward a scheme to speed up the adiabatic passage techniques in two- and three-level atoms. Later, they also proposed a scheme [14] to perform a fast population transfer (FPT) in three-level systems with the help of invariant-based inverse engineering and resonant laser pulses. However, it is worth noticing that although many methods for STAP have been realized in the internal states of a single atom in different systems, it is very hard to directly generalize these ideas to two- and multi-particle cases. In view of that we ask if it is possible to construct STAP for multiparticle systems. In this scenario, Lu et al [23] have proposed a scheme to implement the FPT and a fast maximum entanglement preparation between two atoms in a cavity QED based on the transitionless quantum driving proposed by Berry [16]. Then, Lu et al [24] also put forward another scheme to realize the fast quantum state transfer between two three-level atoms using the invariant-based inverse engineering in a cavity QED. In 2014, motivated by quantum Zeno dynamics, Chen et al [25] constructed shortcuts for performing the FPTs of ground states in multiparticle systems with invariant-based inverse engineering [14]. This scheme is not only implemented without requiring extra complex conditions, but is also insensitive to variations of the parameters.

We note that [23–26] have successfully introduced STAP into cavity QED systems which concern the interaction of atoms and photons within a cavity and are very promising and highly inventive for QIP [27–30]. However, they [23–26] all assume that all the operations are implemented in only one place, for example, a cavity QED. In view of the requirements for long-distance quantum computation and quantum information processing, it is desirable to extend the approach to distant cavities systems. But, when it comes to more complex systems, for example, a multi-cavity-fibre-atom combined system, the above schemes [23–26] are useless. That is, new designs are required in a complex situation. In fact, constructing STAP for multiparticles in cavity-fibre-atom combined systems is very complicated since it is difficult to look for a Hamiltonian operator $\hat{H}(t)$ (i.e. $\hat{H}(t) = \{H(t), \hat{I}(t)\}$) that is related to the original Hamiltonian $H(t)$ but drives the eigenstates $\{|\Psi_n(t)\rangle\}$ exactly. Therefore, until now, this problem has not been addressed.

In this letter, we use the approach of ‘Lewis–Riesenfeld (LR) invariants’ to construct shortcuts to speed up the rate of the quantum population transfer, the quantum entangled states’ generation and the quantum entangled states’ transition for multiparticles in a multi-cavity-fibre-atom combined system. That is, we first study how to construct STAP by inverse engineering for $N(N \in \{1, 2, 3, \ldots, +\infty\})$ atoms which are trapped in $N$ distant optical cavities, respectively. Then we use STAP to realize the fast and noise-resistant quantum population transfer, Bell states [31], Greenberger–Horne–Zeilinger (GHZ) states [2] and $W$ states’ [32] preparation and entangled states’ transition [33–37] in spatially separated cavities which are connected by fibres. Compared with previous works, the present schemes have the following advantages. First, the FPT, the fast quantum entangled states’ generation and the fast quantum entangled states’ transition for multiparticles in spatially separated atoms can be achieved in one step. Secondly, our schemes are not only fast, but also robust against variations in the experimental parameters and decoherence caused by atomic spontaneous emission and photon leakage. In fact, further research shows that, both the total operation time and the robustness against decoherence in each of the schemes is irrelevant to the number of qubits.

The letter is structured as follows. In section 2, we briefly describe the LR phases. In section 3, we construct STAP for FPT in a system with two spatially separated atoms trapped in different cavities which are connected by a fibre by designing resonant time-dependent laser pulses by invariant-based inverse engineering. In section 4, we use STAP to quickly prepare the Bell state, GHZ state and $W$ state in two distant cavities which are connected by a fibre. In section 5 we generalise the schemes in section 4 to implement the fast entangled state transition. In section 6, we provide the numerical simulation and discussion for our schemes. The conclusion appears in section 7.

2. Lewis–Riesenfeld phases

We would like to give a brief description of the LR theory [38, 39]. We consider a time-dependent quantum system whose Hamiltonian is $H(t)$. Associated with the Hamiltonian there are time-dependent Hermitian invariants of motion $I(t)$ that satisfy

\[
\frac{i\hbar}{\partial t} I(t) = \{H(t), I(t)\} = 0.
\]

For any solution $|\Psi(t)\rangle$ of the time-dependent Schrödinger equation $i\hbar\partial_t |\Psi(t)\rangle = H(t)|\Psi(t)\rangle \left(\frac{\partial}{\partial t}, I(t)\right)$, $|\Psi(t)\rangle$ is also a solution and $|\Psi(t)\rangle$ can be expressed as a linear combination of invariant modes

\[
|\Psi(t)\rangle = \sum_n C_n e^{i\lambda_n t} |\phi_n(t)\rangle,
\]

where $C_n$ is the $n$th constant, $|\phi_n(t)\rangle$ is the $n$th eigenvector of $I(t)$ and the corresponding real eigenvalue is $\lambda_n$. The LR phases $\phi_n$ fulfill

\[
\hbar \frac{d\phi_n}{dt} = \langle \phi_n(t) | i\hbar \frac{\partial}{\partial t} - H(t) | \phi_n(t)\rangle.
\]
3. Fast population transfer in two spatially separated atoms

For the sake of clarity, as shown in figures 1(a) and (b), we first assume that two (N = 2) Λ-type atoms are trapped in two distant optical cavities c₁ and c₂ which are connected by a fibre, respectively. Each atom has an excited state and two ground states. The atomic level configuration for the two atoms is shown in Figure 1(d). The set-up diagram is shown in Figure 1(e). The atomic level configuration for the atoms and the auxiliary ground state |ψ₁⟩ in the schemes of the Bell-state and GHZ-state generations.

The initial state is |ψ₀⟩ = |f⟩₁ |g⟩₁ |0⟩₀ |0⟩₁, the whole system evolves in the subspace spanned by

\[ |ψ₁⟩ = |f⟩₁ |g⟩₁ |0⟩₀ |1⟩₁, \]
\[ |ψ₂⟩ = |g⟩₁ |f⟩₁ |0⟩₀ |1⟩₁, \]
\[ |ψ₃⟩ = |g⟩₁ |g⟩₁ |0⟩₀ |0⟩₁, \]
\[ |ψ₄⟩ = |g⟩₁ |f⟩₁ |0⟩₀ |0⟩₁, \]
\[ |ψ₅⟩ = |f⟩₁ |f⟩₁ |0⟩₀ |0⟩₁, \]
\[ |ψ₆⟩ = |g⟩₁ |g⟩₁ |0⟩₀ |0⟩₁. \]

The states |ψ₂⟩ − |ψ₆⟩ can be regarded as ‘intermediate’ states when we transfer the population from the state |ψ₁⟩ to |ψ₇⟩. Hence, we regard the Hamiltonian \( H_{\text{im}} = H_{\text{ac}} + H_{\text{cf}} \) as an ‘intermediate’ Hamiltonian. Then we rewrite the above Hamiltonian in equation (1) with a set of vectors \{ |ψ₁⟩, |ψ₂⟩, |ψ₃⟩, |ψ₄⟩, |ψ₅⟩, |ψ₆⟩ \} as

\[ H_{\text{im}} = \frac{\lambda}{2 \sqrt{2 \alpha^2 + 2 \xi^2}} \left( |ψ₁⟩ + \sqrt{2 \alpha^2 + 2 \xi^2} |ψ₃⟩ + \frac{2 \xi}{\alpha} |ψ₅⟩ \right), \]

\[ H_{\text{cf}} = \frac{\lambda}{2 \sqrt{2 \alpha^2 + 2 \xi^2}} \left( |ψ₁⟩ - \sqrt{2 \alpha^2 + 2 \xi^2} |ψ₃⟩ + \frac{2 \xi}{\alpha} |ψ₅⟩ \right). \]

are the eigenvectors of \( H_{\text{im}} \) corresponding eigenvalues \( E₀ = 0, E₁ = \lambda, E₂ = -\lambda, E₃ = \chi = \sqrt{2 \alpha^2 + 2 \xi^2} \) and \( E₄ = -\chi = -\sqrt{2 \alpha^2 + 2 \xi^2} \). And we obtain

\[ H_{\text{ac}} = \frac{\lambda}{2 \sqrt{2 \alpha^2 + 2 \xi^2}} \left( |ψ₁⟩ - \sqrt{2 \alpha^2 + 2 \xi^2} |ψ₃⟩ + \frac{2 \xi}{\alpha} |ψ₅⟩ \right) \]

\[ + \frac{\lambda}{2 \sqrt{2 \alpha^2 + 2 \xi^2}} \left( |ψ₁⟩ + \sqrt{2 \alpha^2 + 2 \xi^2} |ψ₃⟩ + \frac{2 \xi}{\alpha} |ψ₅⟩ \right). \]

Through solving the eigenvalue equation of \( H_{\text{ac}} \), the instantaneous dark state is given by

\[ |D⟩ = \frac{1}{N₀} |Ω⟩ |ψ₁⟩ - \frac{\sqrt{2 \alpha^2 + 2 \xi^2}}{\lambda} |ψ₃⟩. \]

In light of an adiabatic process, we know that when the adiabatic condition \( \langle θ₀ | 0 | θ_{m} \rangle \approx ξ_{m} \) is fulfilled, where |θ₀⟩ and ξ(l = 0, 1, … 6) denote the instantaneous eigenstates and eigenvalues of \( H_{\text{ac}} \), respectively, the initial state undergoes
the evolution decided by equation (8). However, the simplest way of speeding up the evolution is to construct non-adiabatic processes for the system. The states $|\theta_m\rangle$ are no longer completely negligible but are required for the STAP process. Actually, most of the eigenstates are still completely negligible ($\langle \theta_0 | \hat{a} | \theta_m \rangle \ll \xi_m$ is fulfilled), only $|\theta_1\rangle$ and $|\theta_2\rangle$ whose eigenvalues $\xi_1$ and $\xi_2$ are closest to zero have a chance to participate in the evolution, otherwise, the dynamics is so different from adiabatic passage that we can no longer name it as ‘shortcuts to adiabatic passage’. To speed up the transfer by using the dynamics of invariant based inverse engineering, we need to introduce an invariant Hamiltonian operator $I(t)$ which satisfies $i\partial_t I(t) = \{H_{\text{re}}, I\}$ [9, 13, 25, 38], where it is obvious that directly designing such an operator for $H_{\text{re}}$ is very challenging. Therefore, some simplifications should be made first.

With the help of Zeno space division [42], we partition the ‘intermediate’ states (the terms governed by $H_{\text{re}}$) into three parts which are independent of each other:

$$S_0 = \{ |\phi_0\rangle \}, \quad S_1 = \{ |\mu_1\rangle, |\mu_2\rangle \}, \quad S_2 = \{ |\mu_3\rangle, |\mu_4\rangle \}.$$  

One can find from equation (7) that the states $|\mu_2\rangle$ and $|\mu_4\rangle$ can only be transformed from the states $|\mu_1\rangle$ and $|\mu_3\rangle$ respectively. If we set limiting conditions to make the states $|\mu_1\rangle$ and $|\mu_3\rangle$ negligible during the evolution, the states $|\mu_2\rangle$ and $|\mu_4\rangle$ become independent from the system. To neglect the state $|\mu_4\rangle$, we regard the Hamiltonian $H_{\text{re}}$ as $H_{\text{re}} = H_1 + H_0$, where $H_0 = \chi_0 |\phi_0\rangle \langle \phi_0| - \chi |\phi_2\rangle \langle \phi_2| + |\phi_1\rangle \langle \phi_1|$ and we set $\chi \gg \Omega_1(t), \Omega_2(t)$, $\lambda$ so that we can perform the unitary transformation $U = e^{i\theta(t)}$. By discarding the terms with a high oscillating frequency $\lambda$, an effective Hamiltonian is obtained

$$H_{\text{eff}} = \frac{\chi}{2\lambda} |\phi_0\rangle \langle \phi_0| (\Omega_1 |\psi_1\rangle + \Omega_2 |\psi_2\rangle) + \frac{1}{\sqrt{2}} |\mu_1\rangle \langle -\Omega_1 |\psi_1\rangle + \Omega_2 |\psi_2\rangle| + \chi |\mu_1\rangle \langle |\mu_2\rangle + \text{H.c.}.$$  

In fact, setting $\chi \gg \Omega_1(t), \Omega_2(t)$, $\lambda$ helps to shorten the operation time because when $\chi$ is too small, traversing of photons between the cavities and the fibre will be very difficult which increases the interaction time. Through analysing the proportions of the base vectors in equation (10) in the eigenstates $|\theta_1\rangle$ and $|\theta_2\rangle$, where $|\theta_1\rangle$ and $|\theta_2\rangle$ are the eigenstates whose eigenvalues are closest to zero of $H_{\text{eff}}$, the relation between the proportions of the states $|\phi_0\rangle$ and $|\mu_1\rangle$ is given by:

$$r = \frac{|P_{\phi_0}|}{P_{\phi_1}} = \sqrt{\frac{(5\Omega_1^2 - 5\Omega_2^2)^2 + 4\Omega_1^2\Omega_2^2 + 12(\Omega_1^2 + \Omega_2^2) + 36 + 7}{12(\Omega_1^2 - \Omega_2^2)}}.$$  

We assume $v = g = 1$ as a typical example here in order to simplify the analysis. In general, to transfer the population from the initial state $|\psi(t)\rangle$ to the target state $|\psi_{\text{st}}\rangle$ via adiabatic passage, the form of Rabi frequencies can be chosen as $\Omega_1(t) = \Omega_0 \sin \beta t$ and $\Omega_2(t) = \Omega_0 \cos \beta t$, where $\Omega_0$ denotes the amplitude of the laser pulse and $\beta$ is a time-related parameter. In this case,

$$r = \sqrt{\frac{25\Omega_0^2 \sin^2 \beta t + 12\Omega_0^2 + 36 + 7}{12\Omega_0^2}}.$$  

It is obvious that when $r^2 \gg 1$, the population of the state $|\mu_1\rangle$ is far less than that of the state $|\phi_0\rangle$. That means when $r^2 \gg 1$, the state $|\mu_1\rangle$ is considered as negligible since the state $|\phi_0\rangle$ is only populated in a limited manner during the evolution [25]. Then the whole system can be divided into two parts which are independent of each other: the main subsystem $S_m = \{ |\mu_1\rangle, |\psi_1\rangle, |\psi_2\rangle \}$ and the assistant subsystem $S_a = \{ |\mu_2\rangle \}$. And the Hamiltonian for the main subsystem is

$$H_m = \frac{\chi}{2} |\phi_0\rangle \langle \phi_0| (\Omega_1 |\psi_1\rangle + \Omega_2 |\psi_2\rangle) + \chi |\mu_1\rangle \langle |\mu_2\rangle + \text{H.c.}.$$  

Then, as $H_m$ possesses SU(2) dynamical symmetry [43], the invariant Hamiltonian operator $I(t)$ which satisfies $i\partial_t I(t) = \{H_m, I\}$ can be easily given [13, 25]

$$I(t) = \chi_0 (\cos \gamma \sin \beta |\phi_0\rangle \langle \psi_1| + \cos \gamma \cos \beta |\phi_0\rangle \langle \psi_2| + i \sin \gamma |\psi_2\rangle \langle \psi_1| + \text{H.c.}),$$  

where $\chi_0$ is an arbitrary constant with units of frequency to keep $I(t)$ with dimensions of energy, $\gamma$ and $\beta$ are both time-dependent auxiliary parameters. Through solving the relation $i\partial_t I(t) = \{H_m, I\}$, $\Omega_1(t)$ and $\Omega_2(t)$ are obtained,

$$\Omega_1(t) = \frac{\gamma}{\lambda} \left( \hat{\beta} \cot \gamma \sin \beta + \hat{\gamma} \cos \beta \right),$$  

$$\Omega_2(t) = \frac{\lambda}{\tilde{\beta}} \left( \hat{\lambda} \cot \gamma \cos \beta - \hat{\gamma} \sin \beta \right),$$  

where the dot represents a time derivative. The general solution of the Schrödinger equation $i\partial_t |\psi(t)\rangle = H_m |\psi(t)\rangle$ with respect to the instantaneous eigenstates of $I(t)$ is written as

$$|\psi(t)\rangle = \sum_{n=0,2} C_n e^{i\omega_n t} |\tilde{\theta}_n(t)\rangle,$$  

where $C_n$ are the LR phases mentioned in section 2 and $|\tilde{\theta}_n\rangle$ are the instantaneous eigenstates of $I(t)

$$|\tilde{\theta}_0\rangle = \cos \gamma \cos \beta |\psi_1\rangle - i \sin \gamma |\phi_0\rangle - \cos \gamma \sin \beta |\psi_2\rangle,$$

$$|\tilde{\theta}_2\rangle = \frac{1}{\sqrt{2}} \left[ (\sin \gamma \sin \beta \pm i \cos \beta) |\psi_1\rangle + i \cos \gamma |\phi_0\rangle - (\sin \gamma \sin \beta \mp i \cos \beta) |\psi_2\rangle \right].$$

To transfer population from the initial state $|\psi(t)\rangle$ to the target state $-|\psi_{\text{st}}\rangle$, a simple choice for the parameters is

$$\gamma = \epsilon, \beta = \pi t / 2t,$$
where \( \epsilon \) is a time-independent small value and \( \tau_f \) is the interaction time. And we obtain

\[
\begin{align*}
\Omega_1(t) &= \frac{\chi \pi \cot \epsilon}{2v_f} \sin \frac{\pi t}{2f}, \\
\Omega_2(t) &= \frac{\chi \pi \cot \epsilon}{2v_f} \cos \frac{\pi t}{2f}.
\end{align*}
\]

(19)

In the present case, when \( t = \tau_f \),

\[
|\psi(\tau_f)\rangle = \left( -i \sin \epsilon \cos \alpha + i \sin \epsilon \cos \alpha e^{i \tau_f} \right) |\psi_0\rangle
\]

(20)

where \( \alpha = \pi/(2 \sin \epsilon) = |\alpha_2| \). When, however, we choose \( \alpha = 2N\pi(N = \pm 1, \pm 2 \ldots) \), \( |\psi(t_f)\rangle = [0, 0, -1]' \). Meanwhile, in the assistant subsystem \( S_2 \), the time-dependence population of the state \( |\alpha_2| \) is mainly dominated by the dark state evolution \([25]\) and when \( t = \tau_f \), its population becomes zero and the dark state also evolves into the state \(-|\psi_f\rangle\). That is, with joint efforts of both the subsystems, the whole system quickly evolves from the initial state \( |\psi_0\rangle \) to the final state \(-|\psi_f\rangle\).

### 4. Fast preparation of entangled states in two separated cavities which are connected by a fibre

#### 4.1. Bell states

In this section, we will put forward two methods to generate Bell states via the STAP proposed in section 3. The first method is simple and easy. We only have to introduce an auxiliary ground state \( |\alpha| \) not interacting with other states in the atom 1 as shown in figure 1(c) and set the initial state as follows

\[
|\psi_0\rangle = \frac{1}{\sqrt{2}}(|f\rangle + |\alpha\rangle) |g\rangle_2 |0\rangle_3 |0\rangle_4 |0\rangle_f
\]

(21)

where \( |\psi_0\rangle = |\alpha_1\rangle |g\rangle_2 |0\rangle_3 |0\rangle_4 |0\rangle_f \). Similar to the FPT in section 3, the term \( |f\rangle |g\rangle_2 |0\rangle_3 |0\rangle_4 |0\rangle_f \) will evolve along the STAP constructed above while the other term \( |\alpha_1\rangle |g\rangle_2 |0\rangle_3 |0\rangle_4 |0\rangle_f \) will remain the same. The evolution of the system is governed by the Hamiltonian in equation (4). With the parameters we have chosen, when \( t = \tau_f \), the final state of the system becomes

\[
|\psi(\tau_f)\rangle = \frac{1}{\sqrt{2}} \left[ (i \sin \epsilon \sin \alpha) |\psi_0\rangle + (i \sin \epsilon \sin \alpha |\phi_0\rangle - (\cos \epsilon \sin \alpha) |\psi_0\rangle + |\psi_0\rangle \right] \]

(22)

When we choose \( \epsilon = \arcsin 1/(4N) \), equation (22) becomes the maximally entangled state \( |\psi(\tau_f)\rangle = \frac{1}{\sqrt{2}} (-|\psi_1\rangle + |\psi_2\rangle) = |Bell\rangle \).

In the second method for generating a two-atom maximally entangled state, we trap two atoms in the cavity \( c_3 \) while the others are unchanged (the set-up diagram and the atomic level configuration for each atom are similar to that in section 3).

In this case, the Hamiltonian in the interaction picture for the whole system is

\[
H_i = H_{id} + H_{ac} + H_{cf},
\]

(23)

where

\[
H_{id} = \sum_{n=1}^{2} \Omega_n(t) |\epsilon_n\rangle \langle \epsilon_n| + H. c.,
\]

\[
H_{ac} = \lambda |\alpha| |\epsilon(\alpha)| + \sum_{n=2}^{3} \lambda |\alpha| |\epsilon(\alpha)| |\epsilon_n| + H. c.,
\]

\[
H_{cf} = \lambda^3 (a_1 + a_2) + H. c.,
\]

and the closed subspace is spanned by

\[
|\psi_0\rangle = \frac{v}{\sqrt{2} \alpha + 2 \lambda} \left( |\psi_2\rangle - \frac{\lambda}{v} |\psi_4\rangle + \frac{1}{\sqrt{2}} |\psi_6\rangle + \frac{1}{\sqrt{2}} |\psi_8\rangle \right) \]

(25)

Introducing a vector \( \sigma = (|\psi_0\rangle + |\psi_6\rangle) / \sqrt{2} \) and eliminating the states which are irrelevant to the evolution, eigenstates of the intermediate Hamiltonian which have the same form as those in equation (6) are obtained. Afterwards, setting \( \Omega_2(t) = \Omega_3(t) \), we can similarly rewrite the Hamiltonian in equation (21) as

\[
H'_{ac} = \frac{v}{\lambda} |\psi_0\rangle \left( \Omega_1(\psi) + \Omega_2(\sigma) \right) + \frac{1}{\sqrt{2}} \lambda |\mu_1\rangle (-\Omega_2(\psi) + \Omega_2(\sigma))
\]

(26)

\[
+ \frac{\lambda}{\sqrt{2} \alpha} |\mu_1\rangle (\Omega_1(\psi) + \Omega_2(\sigma)) + \lambda |\mu_1\rangle \langle \mu_1| + H. c.,
\]

where

\[
|\psi_1\rangle = \frac{1}{\sqrt{2}} (|\psi_0\rangle + |\psi_6\rangle),
\]

\[
|\mu_1\rangle = \frac{1}{\sqrt{2}} (-|\psi_0\rangle + |\sigma\rangle),
\]

\[
|\mu_2\rangle = \frac{1}{\sqrt{2}} (-|\psi_0\rangle + |\psi_4\rangle),
\]

(27)

\[
|\mu_3\rangle = \frac{\lambda}{\sqrt{4 \alpha^2 + 2 \lambda^2}} (|\psi_2\rangle + \frac{2v}{\lambda} |\psi_4\rangle + |\sigma\rangle),
\]

\[
|\mu_4\rangle = \frac{1}{\sqrt{2}} \left( |\psi_0\rangle + |\psi_6\rangle \right).
\]
It is obvious that equation (26) is equal to equation (7). The method used to construct the STAP in section 3 also applies to the present three-atom system. Therefore, with the Rabi frequencies in equation (19), when \( t = t_f \), the system evolves from the initial state |\( \Psi_0 \rangle \) to the final state |\( \Psi_f \rangle \). Meanwhile, the two atoms in the cavity \( c_2 \) are turned into a maximally entangled state:

\[
[\text{Bell}] = \frac{1}{\sqrt{2}} (|f\rangle_2 |g\rangle_3 + |g\rangle_2 |f\rangle_3). \tag{28}
\]

### 4.2. \( M \)-atom Greenberger–Horne–Zeilinger states

The present STAP method in section 4.1 can actually be generalized to build an \( M \)-atom (\( M \in \{1, 2, 3, \ldots, \infty\} \)) GHZ state. We trap atom 1 and atom \( M \) in cavities \( c_1 \) and \( c_2 \), respectively. The other \( M - 2 \) atoms are discretionarily trapped in the two cavities. The atomic level configuration is the same as that mentioned in the first STAP method for Bell-state generation. An assistant ground state |\( \alpha \rangle \) is also required for each atom. We assume atom \( M \) is initially in the state |\( g \rangle \) and the rest of the atoms are initially in an \((M - 1)\)-atom GHZ state

\[
\frac{1}{\sqrt{2}} (|a, a, g, \ldots, g\rangle + | f, a, a, \ldots, a\rangle)_{1,2,3,\ldots,M-1}. \tag{29}
\]

In this case, the term \(|a, g, \ldots, g\rangle_{1,2,3,\ldots,M-1}|g\rangle_M\) will remain the same during the evolution. And since the state |\( \alpha \rangle \) is only an assistant ground state, atoms 2, 3, ..., \( M - 1 \) are actually irrelevant to the evolution, only \(|f, g\rangle_{1,2}\) participates in the evolution. Therefore, the whole system can be regarded as an effective two-atom system which is similar to the first method for Bell-state generation. The shortcut is constructed easily by setting the Rabi frequencies \( \Omega_\ell(t) \) and \( \Omega_\beta(t) \) as the form of equation (19). Similarly, when \( t = t_f \), the final state is obtained:

\[
|\psi(t_f)\rangle = \frac{1}{\sqrt{2}} \left( (i \sin \epsilon \sin \alpha) |\Psi_0\rangle + (\cos^2 \epsilon - \sin^2 \epsilon \cos \alpha) |\Psi_0^\mp\rangle + |\Psi_0^\dagger\rangle \right). \tag{30}
\]

where

\[
|\Psi_0\rangle = |a, a, \ldots, a\rangle_{1,2,3,\ldots,M-1},
\]

\[
|\Psi_0^\mp\rangle = |f, g\rangle_{1,2} |\Psi_0\rangle_{0,0,0_{c_1c_2}},
\]

\[
|\Psi_0^\dagger\rangle = |a, g, g, \ldots, g\rangle_{1,2,M-1} |\Psi_0\rangle_{0,0,0_{c_1c_2}},
\]

\[
|\Phi_0\rangle = \frac{\nu}{\sqrt{2} \nu^2 + \lambda^2} \left( |f, g\rangle_{1,2} |\Psi_0\rangle_{0,0,0_{c_1c_2}} + |g, e\rangle_{1,2} |\Psi_0\rangle_{0,0,0_{c_1c_2}} + \frac{\lambda}{\nu} |g, g\rangle_{1,2} |\Psi_0\rangle_{0,0,0_{c_1c_2}} \right). \tag{31}
\]

There is no doubt that by choosing \( \epsilon = \arcsin(1/4N) \) \((N = \pm 1, \pm 2, \ldots)\), equation (29) becomes an \( M \)-atom GHZ state:

\[
|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|a, g, g, \ldots, g\rangle + |g, a, a, \ldots, a\rangle)_{1,2,3,\ldots,M-1} |M\rangle. \tag{32}
\]

If we switch off the classical field \( \Omega_M \), the state |\( \Psi_f \rangle \) can also be regarded as an assistant ground state |\( \alpha \rangle_M \). An \( M \)-atom GHZ state is generated by using an \((M - 1)\)-atom GHZ state. And the transition \(|f\rangle + |g\rangle\rangle \) of the atom 1 can be driven without question by a classical laser field. Therefore, when we trap one more \( A \)-type atom whose initial state is |\( g \rangle \) in the cavity \( c_2 \), the present \( M \)-atom GHZ state can be converted into an \((M + 1)\)-atom GHZ state.

### 4.3. \( M \)-atom W states

We will demonstrate how to generate an \( M \)-atom \( W \) entangled state via STAP in this section. The model we used is nearly the same as that in section 3 except that we trap \( M \) identical \( A \)-type atoms in the cavity \( c_2 \). In a typical setup with neutral atoms in the cavity at least several microns apart, direct interaction between the atoms in the cavity \( c_2 \) is negligible. The Hamiltonian of the whole system in the interaction picture can be written as

\[
H_{\text{int}} = H_{\text{int}} + H_{\text{ac}} + H_{\text{rf}}.
\]

\[
H_{\text{int}} = \Omega_p(t) |e\rangle_p \langle f| + \sum_{n=1}^{M} \Omega_n(t) |e\rangle_n \langle f| + H. c. .
\]

\[
H_{\text{ac}} = \lambda_p a |e\rangle_p \langle g| + \sum_{n=1}^{M} \lambda_n a_n |e\rangle_n \langle g| + H. c. .
\]

\[
H_{\text{rf}} = v b^\dagger (a_1 + a_2) + H. c. , \tag{33}
\]

where subscript \( p \) denotes the atom which is trapped in the cavity \( c_1 \) and \( n(n \in \{1, M\}) \) denotes the \( n \)th atom which is trapped in the cavity \( c_2 \). For simplicity, we assume \( \Omega_p(t) = \Omega_\beta(t) \) and \( \lambda_n = \lambda_p \). Since the atoms in the cavity \( c_2 \) are indistinguishable, each atom could be excited by the photon which is emitted from the atom trapped in the cavity \( c_1 \) with the same probability; we can describe the excited state of the atoms in the cavity \( c_2 \) as

\[
|\Phi_g = \frac{1}{\sqrt{M}} (|e, g, \ldots, g\rangle + |g, e, \ldots, g\rangle + \ldots + |g, g, \ldots, e\rangle)_{1,2,\ldots,M}. \tag{34}
\]

The transition \(|\Phi_g\rangle \leftrightarrow |\Phi_f\rangle\) is coupled to the cavity mode with an effective coupling strength \( \sqrt{M} \lambda_p \), which \(|\Phi_f\rangle = |g, g, \ldots, g\rangle_{1,2,\ldots,M} \). In the case of \( \Omega_n = \Omega_\beta \), \(|\Phi_f\rangle\) might be driven to

\[
|\Phi_w = \frac{1}{\sqrt{M}} (|f, g, \ldots, g\rangle + |g, f, \ldots, g\rangle + \ldots + |g, g, \ldots, f\rangle)_{1,2,\ldots,M}. \tag{35}
\]

Then we assume the initial state is \( |\Psi_0\rangle = |f\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j \), the whole system evolves in the subspace spanned by

\[
|\Psi_0\rangle = |f\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_1} = |e\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_2} = |e\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_3} = |g\rangle_p |\Phi_f\rangle_{1,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_4} = |g\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_5} = |g\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_6} = |g\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_7} = |g\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j,
\]

\[
|\Phi_{\phi_8} = |g\rangle_p |\Phi_f\rangle_{0,0,0_{c_1c_2}} |f\rangle_j.
\]
In this subspace, the Hamiltonians $H_{ad}$ and $H_{ac}$ can be written as
\begin{equation}
H_{ad} = \Omega_p (t) \langle \Phi_2 | (\Phi_1 + \lambda (t) | \Phi_0 \rangle + \text{H.c.}, \quad H_{ac} = \lambda_p (t) | \Phi_2 \rangle \langle \Phi_1 | + \sqrt{M} \lambda (t) | \Phi_0 \rangle \langle \Phi_3 | + \text{H.c.}, \quad (36)
\end{equation}
When we choose $\lambda_p = \sqrt{M} \lambda = \lambda$ and
\begin{equation}
\Omega_p = \Omega_1 = \frac{\gamma \pi \cot e}{2vtf} \sin \frac{\pi t}{2t_f}, \quad \Omega_2 = \frac{\gamma \pi \cot e}{2vtf} \cos \frac{\pi t}{2t_f}, \quad (37)
\end{equation}
the total Hamiltonian in the present system equals the Hamiltonian in equation (4). The dark state for this system is given by:
\begin{equation}
|D_W \rangle = \frac{1}{N_W} \left( \Omega_p | \Phi_1 \rangle - \Omega_1 | \Phi_1 \rangle + \frac{\sqrt{M} \lambda_p}{\lambda} | \mu_2 \rangle \right), \quad (38)
\end{equation}
where $| \mu_2 \rangle = (| \Phi_2 \rangle + | \Phi_3 \rangle) / \sqrt{2}$ is the intermediate state. And we have to make it independent from the whole system by setting a limiting condition which is similar to equation (11). Then the shortcuts are easily constructed in the same way as in section 3 and the evolution of the system is approximatively described as
\begin{equation}
| \Phi (t) \rangle = \sum_{n=0, \pm} C_n e^{i n t} | \tilde{\Phi}_n (t) \rangle. \quad (39)
\end{equation}
When $t = t_f$, the final state becomes $| \Phi (t_f) \rangle = - | \Phi_1 \rangle$, meanwhile, the $M$ atoms in the cavity $c_2$ collapse to the $M$-qubit $W$ state
\begin{equation}
|W \rangle = \frac{1}{\sqrt{M}} \left( | f, f, g, \cdots, g \rangle + | g, f, g, \cdots, g \rangle + | g, g, f, \cdots, g \rangle \right. \left. + \cdots + | g, g, g, \cdots, f \rangle \right)_{1,2,3, \ldots, M}. \quad (40)
\end{equation}

5. The fast transition of two-atom entangled states via shortcuts to the adiabatic passage

In the following, we discuss the fast transition of the entangled state from the cavity $c_1$ to $c_2$ via STAP. In this section, we assume that there are four identical atoms, two of which are trapped in the cavity $c_1$ while the other two are trapped in the cavity $c_2$. The atomic level configuration is the same as that in section 3. Hence, the Hamiltonian of the present system in the interaction picture is
\begin{equation}
H_1 = H_{ad} + H_{ac} + H_{if},
\end{equation}
\begin{equation}
H_{ad} = \sum_{k=1}^{4} \Omega_{a_k} e^{i \epsilon_k} \langle f | + \text{H.c.}, 
H_{ac} = \sum_{k=1,2} \lambda_{a_k} e^{i \epsilon_k} \langle g | + \sum_{k=3,4} \lambda_{a_k} e^{i \epsilon_k} \langle g | + \text{H.c.}, 
H_{if} = vb' (a_1 + a_2), \quad (41)
\end{equation}
We assume that the initial state is
\begin{equation}
| \Psi_0 \rangle = \frac{1}{\sqrt{2}} (| f g \rangle + | g f \rangle)_{1,2} | g g \rangle_{3,4} | 0 \rangle_{f}, \quad \text{which is a two-atom entangled state. The system evolves into the subspace spanned by}
| \Psi_1 \rangle = | f g \rangle_{1,2} | g g \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f},
| \Psi_2 \rangle = | g f \rangle_{1,2} | g g \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f},
| \Psi_3 \rangle = | g g \rangle_{1,2} | g g \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f},
| \Psi_4 \rangle = | g f \rangle_{1,2} | g g \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f},
| \Psi_5 \rangle = | g f \rangle_{1,2} | g f \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f},
| \Psi_6 \rangle = | f g \rangle_{1,2} | g f \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f},
| \Psi_7 \rangle = | f g \rangle_{1,2} | g f \rangle_{3,4} | 0 \rangle_{c_2} | 0 \rangle_{f}, \quad (42)
\end{equation}
We regard the atoms in the same cavity as an integral whole as they are indistinguishable. Similar to section 4, we describe the excited states of the atoms in the cavities as
\begin{equation}
| \Psi_2 \rangle = \frac{1}{\sqrt{2}} (| e g \rangle + | e g \rangle)_{1,2} | g g \rangle_{3,4}, \quad (43)
| \Psi_4 \rangle = \frac{1}{\sqrt{2}} (| e g \rangle + | g e \rangle)_{3,4} | g g \rangle_{1,2}.
\end{equation}
The atomic transitions $| \Psi_0 \rangle \leftrightarrow | \Psi_4 \rangle$ are coupled to the cavity mode with an effective coupling strength $\lambda$ and so is $| \Psi_4 \rangle \leftrightarrow | \Psi_6 \rangle$, where $| \Psi_6 \rangle = | g g g g \rangle_{1,2,3,4}$ and $\lambda_k = \lambda / \sqrt{2}$. By choosing $\Omega_{a_1, a_2} = \Omega_p$ and $\Omega_{a_3, a_4} = \Omega_p$, we rewrite the Hamiltonian in equation (41) with the basis vectors in equations (42) and (43) as
\begin{equation}
H_1 = H_{ad} + H_{ac} + H_{if}, \quad H_{ad} = \Omega_p | \Psi_2 \rangle \langle c_0 | + \Omega_1 | \Psi_2 \rangle \langle c_0 | + \text{H.c.}, \quad (44)
H_{ac} = \lambda | \Psi_2 \rangle \langle c_0 | + \lambda | \Psi_2 \rangle \langle c_0 | + \text{H.c.}, \quad H_{if} = vb' (a_1 + a_2), \quad (45)
\end{equation}
where $| \Psi_2 \rangle = \frac{1}{\sqrt{2}} (| g g \rangle_{1,2} | e g \rangle + | e g \rangle)_{3,4} | 00 \rangle_{c_2} | 0 \rangle_f$ and $| c_0 \rangle = | 00 \rangle_{c_2} | 0 \rangle_f$. It is obvious that the present Hamiltonian has the same form as that in equation (4). When we choose
\begin{equation}
\Omega_p = \Omega_1 = \frac{\gamma \pi \cot e}{2vtf} \sin \frac{\pi t}{2t_f}, \quad \Omega_2 = \frac{\gamma \pi \cot e}{2vtf} \cos \frac{\pi t}{2t_f}, \quad (46)
\end{equation}
the shortcut is obtained. The whole system is approximately described as
\begin{equation}
| \Phi (t) \rangle = \sum_{n=0, \pm} C_n e^{i n t} | \tilde{\Phi}_n (t) \rangle. \quad (46)
\end{equation}
When \( t = t_f \), the final state is

\[
|\Psi(t_f)\rangle = |\Psi_f\rangle = \frac{1}{\sqrt{2}}|\text{gg}\rangle_{1,2}|(\text{eg})\rangle_{3,4}|00\rangle_{5,6}|0\rangle_f.
\] (47)

In fact, the present scheme can be generalised to implement the transition of a \( M \)-atom \( W \) entangled state via trapping \( M \) atoms in respective cavities \( c_1 \) and \( c_2 \) and setting \( \lambda_k = \lambda / \sqrt{M} \).

6. Numerical simulation and discussion

We will first analyse the relation between the cavity-fibre coupling \( v \) and the interaction time \( t_f \) as \( v \) plays a very important role in the evolution. The fidelity \( F_f \) of the target state \( |\psi_f\rangle \) versus \( v \) and \( \lambda \) \( t_f \) is shown in figure 2 when \( \epsilon = \arcsin0.25 \), where the fidelity of a state is given through the relation \( F = |\langle \psi | \rho (t_f) | \psi \rangle| \). Figure 2 shows that the increasing value of \( v \) does not help to shorten the interaction time, which is different to the findings we mentioned above in section 3. The reason is that the relation between the coupling \( v \) and the amplitude of the laser pulses \( \Omega_0 \) at that time was not taken into consideration. We know from \([9, 12–14, 23, 25]\), that shortening the time requires increasing the amplitude of the laser pulses. The amplitude of the laser pulses in equation (19) inverses the proportion to the coupling \( v \) and the smaller the amplitude is, the longer the interaction time. Consequently, in our method it is wise to choose \( v = g \). Furthermore, figure 2 shows that in the present case, the shortest interaction time required for an ideal population transfer from \( |\psi_i\rangle \) to \( |\psi_f\rangle \) is only about \( 8/\lambda \). The time dependences of \( \Omega_1(t)|\lambda \rangle \) and \( \Omega_2(t)|\lambda \rangle \) are shown in figure 3(a) versus \( \lambda \) \( t \) when \( \epsilon = \arcsin0.25 \), \( t_f = 10/\lambda \) and \( v = \lambda \). The amplitude of the laser pulse \( \Omega_0 \) is 1.05\( \lambda \) which meets the conditions mentioned above. And such an intensity is safe to assume linear optic models. A similar laser amplitude has been used and discussed in \([25, 44–47]\). In \([46]\) proposed by Song et al, even high drive intensity \( \Omega_0 = \sqrt{5/2} \lambda \approx 2.6591\lambda \) was used. Figure 3(b) shows the time evolution of the populations in states \( |\psi_i\rangle \) and \( |\psi_f\rangle \). We contrast the interaction time required for achieving the target state via an adiabatic process with the present STAP method in figure 3(c). The result obviously shows that the present STAP method effectively shortens the interaction time of the adiabatic method even with the same laser intensity (we choose \( \Omega_1 = \lambda \sin \beta \) and \( \Omega_2 = \lambda \cos \beta \) in the adiabatic method). A similar process in an adiabatic scheme in \([45]\) shows that the interaction time required to achieve the target state is almost \( t_f = 250/\lambda \) when all the populations of the excited states should be restrained to reduce the influence of dissipation. To demonstrate that the conditions for the STAP process are fulfilled, especially, the condition that the state \( |\mu_i\rangle \) is negligible during the evolution, we plot figure 3(d) in the case of \( |\epsilon = \arcsin0.25, t_f = 10/\lambda \rangle \). We find that the states \( |\mu_1\rangle \) and \( |\mu_4\rangle \) remain negligible all the time and the state \( |\mu_1\rangle \) is very slightly populated but still considered as negligible since the maximum value of its population is only 4.36\( \% \). Besides, from equation (8), it is evident that the population of the state \( |\mu_2\rangle \) has a special relationship with \( \lambda t_f \).

\[
P = \frac{\sqrt{2}\Omega_1\Omega_2}{N_p\lambda} = \frac{2\epsilon^2}{(\lambda t_f)^2 + 2\epsilon^2},
\] (48)

where \( C = (\pi\sqrt{1 - \epsilon^2 - \epsilon^2 \cos \beta})/2\). Although the STAP process is constructed based on nonadiabatic processes, the dark state \( |D\rangle \) can still approximatively describe the evolution of the system because the adiabatic condition has not been absolutely destroyed. That is also why equation (48) is useful to describe the population of the state \( |\mu_2\rangle \). The result of equation (48) shows that there is an inverse relationship between \( P \) and \( \lambda t_f \), which means, increasing the population of the state \( |\mu_2\rangle \) is a simple and effective way to shorten the interaction time. Moreover, we have mentioned above that the state \( |\mu_2\rangle \) can only be transformed from the state \( |\mu_1\rangle \). Therefore, under the premise of the negligible state \( |\mu_1\rangle \) during the evolution, properly increasing the population of the state \( |\mu_1\rangle \) helps to shorten the interaction time (this result also applies to most of the adiabatic schemes when the decoherence is not taken into consideration). In figure 4, we analyse the efficiency of the STAP for the generation of Bell entangled states. Figure 4(a) displays the time evolution of the populations in states \( |\psi_i\rangle \), \( |\psi_f\rangle \) and \( |\psi_a\rangle \) in the first STAP for Bell-state generation and we give a comparison between the performances of the fidelity of the two-atom Bell state based on the first STAP method for Bell-state generation and an adiabatic passage method governed by a Hamiltonian whose form is the same as that in equation (4) with parameters \( \Omega_1 = \lambda \sin \beta, \Omega_2 = \lambda \cos \beta, t_f = 60/\lambda \) satisfying the adiabatic condition. Figure 4(c) shows the populations in states \( |\zeta_i\rangle \), \( |\zeta_7\rangle \) and \( |\zeta_9\rangle \) in the second STAP for Bell-state generation and the second STAP method is contrasted with adiabatic passage method for the two-atom Bell state, demonstrating that the STAP method is much faster when one more atom is used. The green dashed curve in figure 4(d) is plotted based on a model with two atoms trapped in a cavity whose interaction Hamiltonian is \( H = \sum_k \lambda_k a_k^\dagger a_k^\dagger (I + \lambda a_k a_k^\dagger) + H.c. \) and the parameters \( \Omega_1 = 0.5\lambda (\sin \beta)^{1/2}, \Omega_2 = \lambda (\cos \beta)^{1/2}, t_f = 60/\lambda \) are set to satisfy the adiabatic condition. These models and parameters are chosen according to the principle of fair comparison. Both figures 4(a) and (b), beyond doubt, give a result where the STAP methods can generate Bell-entangled states.
much faster than the adiabatic methods without requiring extra complex operations and harsh conditions.

For simplicity, in the following, when a comparison is made between the adiabatic and the STAP methods, the form of the Hamiltonian for an adiabatic method is chosen as the same as that for a STAP method. Figure 5(a) shows the time-dependent populations for states $|\Psi_1\rangle$, $|\Psi_2\rangle$ and $|\Psi_3\rangle$ in the scheme of a fast GHZ state generation when $|\epsilon\rangle = \text{arsin}0.25$, $t_f = 10/\lambda$ and figure 5(b) gives the time comparison between the adiabatic method and the STAP method. The dashed curve in figure 5(b) is plotted with parameters $|\epsilon\rangle = \text{arsin}0.02$, $t_f = 60/\lambda$ satisfying the adiabatic condition via adiabatic passage. It is obvious that figures 5(a) and (b) are almost the same as figures 4(a) and (b), respectively. A similar phenomenon is shown in the generation of W state when we contrast figures 5(c) and (d) with figures 4(c) and (d). Here, figure 5(c) displays the populations for the states $|\Phi_1\rangle$, $|\Phi_2\rangle$ and $|\Phi_3\rangle$ when $|\epsilon\rangle = \text{arsin}0.25$, $t_f = 10/\lambda$, where $|\Phi_1\rangle = |g\rangle_p|f, g, g\rangle_{1,2,3}|0, 0, 0\rangle_{c1, c2, f}$, $|\Phi_2\rangle = |g\rangle_p|g, f, g\rangle_{1,2,3}|0, 0, 0\rangle_{c1, c2, f}$ and $|\Phi_3\rangle = |g\rangle_p|g, g, f\rangle_{1,2,3}|0, 0, 0\rangle_{c1, c2, f}$ and figure 5(d) gives the comparison between the STAP method and the adiabatic method for W state generation. We choose parameters $|\epsilon\rangle = \text{arsin}0.02$, $t_f = 100/\lambda$ to satisfy the adiabatic condition in the adiabatic method. As a matter of fact, the STAP methods for GHZ state and W state generations are equivalent to the first and second STAP methods for Bell-state generations, respectively. When $M = 2$, the STAP methods for $M$-particle-entangled-state generations are exactly the STAP methods for the Bell-state generations. From the result of the analysis, a significant inference is obtained: the interaction time required for each of the generations of the GHZ state and the W state is irrelevant to the number of qubits. Whereas, when there are too many atoms trapped in the cavity $c_2$, the system is possibly not safe to assume linear optic models since the coupling $\lambda_s$ is proportionally reduced while $V_\theta(t)$ remains unchanged in section 4.3 if we do not change the parameters. Therefore, in fact, the shortest interaction time required for the W state generation is limited by the number of qubits. But that only has a small effect on the STAP method since, even when $M = 100$, we can still choose the interaction time as $t_f = 50/\lambda$ which is much shorter than that in an adiabatic method to make sure the system is safe to assume linear optic models (in this case, $\Omega_\theta = 2\lambda_s$). Figure 5(e) shows the time evolution of the populations for the initial entangled state $|\Phi_0\rangle$ and target entangled state $|\Psi\rangle$ in the scheme of the transition of the two-atom entangled state when $|\epsilon\rangle = \text{arsin}0.25$, $t_f = 100/\lambda$ and $v = \lambda$. It is merited that figure 5(e) is the same as figure 3(b), the reason for this phenomenon is also that the Hamiltonian in equation (44) is precisely the same as the Hamiltonian in equation (4). And figure 5(f) is the two-atom entanglement transfer via adiabatic passage in the case of $|\epsilon\rangle = \text{arsin}0.02$.

It is well known that a scheme’s potential application for quantum information processing and quantum computing depends on its robustness against possible mechanisms of decoherence. The dissipation has not been taken into account in the above discussion. And once the dissipation is considered, the evolution of the system can be modelled by a master equation in Lindblad form,

$$\dot{\rho} = i[\rho, H] + \sum_k \left[ L_k \rho L_k^\dagger - \frac{1}{2} (L_k^\dagger L_k \rho + \rho L_k^\dagger L_k) \right].$$

(49)
where the \( L_\ell \)'s are the so-called Lindblad operators. For the first scheme, there are seven Lindblad operators:

\[
\begin{align*}
L_1^\ell &= \sqrt{\kappa_1} a_1, \\
L_2^\ell &= \sqrt{\kappa_2} a_2, \\
L_3^\ell &= \sqrt{\kappa_3} b, \\
L_4^\ell &= \sqrt{\Gamma_1} |f\rangle_1 \langle e|, \\
L_5^\ell &= \sqrt{\Gamma_2} |g\rangle_1 \langle e|, \\
L_6^\ell &= \sqrt{\Gamma_3} |f\rangle_2 \langle e|, \\
L_7^\ell &= \sqrt{\Gamma_4} |g\rangle_2 \langle e|,
\end{align*}
\]  

(50)

where \( \kappa_m (m = 1, 2) \) are the decays of the cavities, \( \kappa_j \) is the decay of the fibre and \( \Gamma_n (n = 1, 2, 3, 4) \) are the spontaneous emissions of atoms. For simplicity, we assume \( \kappa_\alpha = \kappa_\beta = \kappa_\gamma = 0.02 \) and \( \Gamma_\alpha = 100/\lambda \). The advantage of the adiabatic passage technique is that the fidelity of the target state is independent of the fibre decay and atomic spontaneous emission because the intermediate states including the excited state of the fibres and atoms are always adiabatically eliminated and the decoherence is usually caused by the cavity decay since the dark state always includes the terms including the cavity-excited states. We prove this by numerical simulation shown in figure 6(a). Figure 6(a) displays the fidelity \( F_7 \) versus each of the three noise resources when the other two are zero via adiabatic process (we choose parameters \( \epsilon = 0.02, t_f = 100/\lambda \) to satisfy the adiabatic condition). Figure 6(b) shows the relationship between \( F_7 \) and the three noise resources via STAP. Contrast these two figures; the benefits of the STAP method are shown obviously; though the STAP method is a little more sensitive to fibre decay and atomic spontaneous emission, it is far more robust against cavity decay than the adiabatic method. In a word, the STAP method is not only fast but also robust.

The decoherence, as a general rule, is dominated by the populations of the excited states and the total operation time.

In previous articles, the decoherence is always prevented by decreasing the populations of the excited states. However, the excited states are required to speed up the transfer in the present STAP method. Therefore, we plot figure 7(a) to demonstrate that in the present case, the decoherence is mainly dominated by the total operation time rather than the populations of the excited states. The result obviously shows that the fidelity is highest when the operation time for achieving an ideal target state is the shortest i.e. \( t_f = 8.9/\lambda \). And the fidelity decreases gradually as \( \lambda t_f \) increases while oscillating. When \( t_f < 8.9/\lambda \), as the conditions for constructing STAP are no longer faultlessly satisfied, the fidelity is lower than that when \( t_f = 8.9/\lambda \). The maximum population of the intermediate states \( |f_0\rangle \) and \( |g_2\rangle \) which practically cause the decoherence versus \( \lambda t_f \) are shown in figure 7(b). The maximum population clearly decreases with the increasing of \( \lambda t_f \). When we join up these two results, an inference is drawn, that is, at least when \( 9/\lambda \leq t_f \leq 20/\lambda \), the decoherence is mainly dominated by the total operation time since decreasing the population of the excited states does not obviously prevent the decoherence. Figure 7(a) is plotted with parameters \( \epsilon = \arcsin0.25, \mu = g \) and \( \kappa_\alpha = \kappa_\beta = \kappa_\gamma = \Gamma = 0.054 \) and figure 7(b) is plotted with parameters \( \epsilon = \arcsin0.25, \nu = g \) and \( \kappa_\alpha = \kappa_\beta = \kappa_\gamma = \Gamma = 0 \).

The robustness of the STAP for the Bell-state generations is shown in figure 8(a) with \( \Gamma/\lambda = \kappa_\ell/\lambda = \kappa_\ell/\lambda = \lambda = Y \). And when we trap one more atom in the cavities, two more Lindblad operators \( L_\ell^\rho = \sqrt{\Gamma_\ell} |S\rangle \langle e| \) (\( S = f, g \)) governing spontaneous emissions should be considered in the master equation. The result proves that the first STAP method for Bell-state generation is much more robust against decoherence than the second.

Figure 4. (a) Time evolution of the populations for the states \( |\phi_1\rangle, |\psi_\ell\rangle \) and \( |\psi_\ell\rangle \) in the first scheme of the Bell-state generation. (b) Time evolution of the fidelities of Bell states via STAP and adiabatic methods. (c) Time evolution of the populations for the states \( |\psi_\ell\rangle, |\psi_\ell\rangle \) and \( \langle \zeta_\ell \rangle \) in the second scheme of Bell-state generation. (d) Time evolution of the fidelities of Bell states via STAP and adiabatic methods.
Figure 5. Time-dependent populations in STAP schemes and the comparison between STAP methods and adiabatic methods: (a) time evolution of the populations for the states $|\Psi_1\rangle$, $|\Psi_f\rangle$ and $|\Psi_a\rangle$ of the three-atom GHZ-state via STAP. (b) The comparison of total interaction times required between the STAP method and the adiabatic method for GHZ-state generation. (c) Time evolution of the populations for the states $|\Phi_1\rangle$, $|w_1\rangle$, $|w_2\rangle$ and $|w_3\rangle$ of the three-atom $W$ via STAP. (d) The comparison of total interaction times required between the STAP method and the adiabatic method for $W$-state generation. (e) Time evolution of the entanglement transfer form the initial entangled state $|\Psi_0\rangle$ to target entangled state $|\Psi_f\rangle$ via STAP. (f) Time evolution of the entanglement transfer form the initial entangled state $|\Psi_0\rangle$ to the target entangled state $|\Psi_f\rangle$ via adiabatic passage.

Figure 6. (a) The influence of decays on the fidelity $F_7$ of the target state via the adiabatic method: each of the three curves denotes $F_7$ versus each of the three noise resources when the other two are zero, for example the blue curve denotes the relationship between the fibre decay $\kappa_f/\lambda$ and the $F_7$ when $\kappa_c = \Gamma = 0$. (b) The influence of decays on the fidelity $F_7$ of the target state via the STAP method.
one. That is, because only the term \( \frac{1}{\sqrt{2}} |\psi_i\rangle \) in equation (21) participates in the evolution. From such an initial state to the target state \( \frac{1}{\sqrt{2}} |\psi_f\rangle \) along the STAP, the evolution is very closely analogous to the evolution of the FPT scheme in section 3 except that all of the coefficients of the states should be multiplied by \( \frac{1}{\sqrt{2}} \). However, the second scheme of the Bell-state generation is actually equivalent to the scheme for FPT. That means the population for each of the effective excited states in the second scheme is twice as large as that in the first scheme, at any time during the evolution. Figure 8(b) displays the fidelities of the three-atom GHZ and W states versus the decay when \( \Gamma\lambda = \kappa\lambda = \kappa, \lambda = \Upsilon \). It is apparent that figure 8(b) is almost identical with figure 8(a). Further research shows that, when it comes to the generation of more qubits GHZ and W states, the robustness of the schemes against decoherence still remains the same as that of three atoms. It is implied that not only the interaction time required for the generation of entangled states but also the robustness of the scheme against decoherence is irrelevant to the number of qubits. The reason for this phenomenon is not hard to understand: however many atoms are trapped in the cavities, the photon emitted from the atom 1 in the cavity \( c_1 \) can only excite one of the atoms in cavity \( c_2 \). In other words, the holistic spontaneous emissions of \( M \) atoms is equivalent to that of one atom in the cavity \( c_2 \). The result also applies to the scheme of the transition of entangled states.

Finally, we present a brief discussion about the basic elements in the real experiment. Cesium atoms can be used to implement the schemes, the state \( |g\rangle \) corresponds to \( F = 3, m = 2 \) the hyperfine state of \( ^6\text {Sr}_{1/2} \) electronic ground state, \( |a\rangle \) corresponds to \( F = 3, m = 4 \) the hyperfine state of \( ^6\text {Sr}_{1/2} \) electronic ground state, \( |f\rangle \) corresponds to \( F = 4, m = 3 \) hyperfine state of \( ^6\text {Sr}_{1/2} \) electronic ground state and \( |e\rangle \) corresponds to \( F = 4, m = 3 \) the hyperfine state of \( ^6\text {P}_{1/2} \) electronic state. An almost perfect fibre-cavity coupling with an efficiency larger than 99.9% can be realized using fibre-taper coupling to high-Q silica microspheres [48]. The fibre loss at 852 nm wavelength is only about 2.2 dB km\(^{-1} \) [49], in this case, the fibre decay rate is only 0.152 MHz. While in recent experimental conditions [50, 51], it was predicted to achieve a strong atom-cavity coupling \( \lambda = 2\pi \times 750 \text{ MHz} \). That means the fibre decay can actually be neglected in a real experiment with these parameters. And by choosing another set of parameters (\( \lambda, \Gamma, \kappa \)) = (2500, 10, 10) MHz in the microsphere cavity QED experiment reported in [51, 52], the fidelities of all of the STAP schemes in this letter are higher than 99%.

Until now, all of the above descriptions and discussion are based on atoms that are trapped in two distant cavities which are connected by a fibre, in fact, the present STAP can also be generalized to the system with \( \mathcal{N}(\mathcal{N} \geq 2) \) atoms respectively trapped in \( \mathcal{N} \) cavities which are connected by \( \mathcal{N} - 1 \) fibres [53]. In such a system, there exists only one eigenvector whose eigenvalue is zero for the intermediate Hamiltonian and
one dark state. Similar to equations (6)–(10) in section 3, in light of quantum Zeno subspace division, we can rewrite the Hamiltonian interaction with some special vectors and partition the intermediate states into parts which are independent of each other. After discarding most of the parts except $S_0$ and $S_1$ whose eigenvalues are Zero and closest to zero, respectively, the effective Hamiltonian which has the same form as that in equation (10) is easily obtained. Then the STAP can also be constructed to realize the fast and noise-resistant population transfer, quantum entangled states' transition in a multi-cavity-fibre-atom combined system.

7. Conclusion

‘Shortcuts to the adiabatic process’ provide alternative fast and robust processes which reproduce the same final populations, or even the same final state as the adiabatic process in a finite and shorter time. In general, the shorter the operation time the better for any method, otherwise, the method may be useless. Many experiments of quantum information science also desire fast and robust theoretical methods since high repetition rates contribute to the achievement of better signal-to-noise ratios and better accuracy. Therefore, in this letter, by dividing the whole system into parts, we construct shortcuts in different parts in the same time and implement the fast quantum information processing via the STAP. Different schemes are proposed to perform fast population transfer, entanglement state generations and entangled state transition. By using the STAP, we have successfully overcome the problem of proposing effective schemes which are not only fast, but also robust, to perform population transfer, prepare entangled states and implement entangled state transition in multi-cavity-fibre-atom combined systems. Numerical simulation demonstrates that all of the schemes are fast and robust versus variations in the experimental parameters and decoherence. Moreover, similar operations can also be applied to other cavity QED systems, for example, a multi-cavity-cavity coupled system, to realize fast and noise-resistant quantum information processing. We believe that the present work is promising and will make a great contribution to the experimental research in quantum information science.

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