Laminar-turbulent transition in Taylor-Dean flow

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Abstract. An experimental study is reported of flows produced in a moderate Taylor-Couette system, $\Gamma \approx 15$, closed or azimuthally opened. In the last case, the flow is bounded by $\theta = 0$ and $(2\pi - \theta_1)$ where $\theta_1 = 30^\circ$ represents the region cut-off by a diaphragm. The basic flow is a combination of a flow caused by the rotation of the inner cylinder and a flow provided azimuthally in the gap by external pumping. Our observations of the laminar-turbulent transition for a wide range of $\tau$, the ratio of pumping and rotation flow rates, reveals competition between the Taylor-Dean flow due to rotation and the Dean flow due to pumping leading to new flow regimes which have not counterpart in the closed Taylor-Couette flow.

1. Introduction

Taylor-Couette flow refers to the viscous flow in the fully filled annulus of concentric rotating cylinders. Its stability is of both academic and practical applications interest. The flow occurs in many engineering fields and has important applications in bearing lubrication and viscometry [1]. Taylor-Couette apparatus have been also used as a reaction system to quantify the relationship between shear and aggregation by coagulation or flocculation, cooling of rotating electrical machinery, electrolytic applications and catalytic chemical reactors. When the annulus is only partially filled, the flow has also applications in electrogalvanizing line in the steel-making-industry [2], in textile industries or paper-making. It has also been shown that wall curvature can have a significant effect on the performance of film cooling over turbine blades [3]. Fundamentally, the study of the secondary motions induced by centrifugal forces in curved channel flows is an important area of theoretical, numerical or experimental investigations. The closed Taylor-Couette system [4] was the most used configuration to study hydrodynamic instability, laminar-turbulent transition or deterministic chaos [5]. Dean [6] studied the case where the motion of viscous fluid in the fully filled annulus of concentric rotating cylinders was driven by an azimuthal pressure gradient as a model for curved channel flow. The curved channel flow was then called Dean flow and received much attention in the past few years. Excepted for Finlay et al.[7] who tried to find, numerically, analogy with Taylor-Couette flow in its evolution towards turbulence, most of these studies were concerned only with the primary instability.

Some authors considered more general flows between coaxial cylinders when, in addition to rotation, a constant transverse pressure gradient is present. The flow obtained is called the Taylor-Dean flow. Theoretical investigations were carried out by Di Prima [8], Hughes and Reid [9] and Raney and chang[10]. In practice, a fully developed Taylor-Dean flow is hardly realizable because
providing external pressure gradient in the azimuthal direction needs a breakdown of the symmetry of the annulus. The only experimental work prior to the present study is that by Brewster et al. [11].

Elsewhere, over the last twenty years, new ideas have been formulated for the stability of open flows and shear flows [12,13]. In open flows, the instabilities can be distinguished according to either they are convective or absolute. These new ideas are applied to the case of an axially open Taylor-Couette system where convective instabilities are studied experimentally [14]. On the other hand, at the end of the last century, experiments have been performed in the Taylor-Couette system when the cylinders are horizontal and the gap not completely filled. The existence of a back flow gives as a basic flow a combination of a Couette flow and a Poiseuille flow. The stability of this flow, named Taylor-Dean flow, was studied by many authors [15-19]. Because it exhibits a large variety of patterns, this flow needs much theoretical, numerical and experimental investigations.

Our contribution is experimental. The device we realized is a generalized version of the Taylor-Couette system.

As shown in Fig. 1, three different flows may be produced in this new system:
1. A flow can be obtained by pumping a fluid around the annulus while the cylinders are at rest: the Dean flow.
2. A flow can be produced by the inner cylinder rotating clockwise or counter-clockwise. The flow induced is forced to reverse by a diaphragm: the “closed” Taylor-Dean flow.
3. A flow can be obtained by the simultaneous action of pumping and of the rotation of the inner cylinder: the open Taylor-Dean flow.

As control parameter of the open Taylor-Dean flow, we define \( \tau = \frac{2Vq}{Vc} \), the ratio of pumping and rotation flow rates, where \( Vq \) is the mean velocity of the Poiseuille flow and \( Vc \) the linear velocity of the rotating cylinder. According to Rayleigh’s stability criterion for curved flows, the basic flow velocity profile shows potentially unstable zones and potentially stable zones, depending on \( \tau \) (Fig. 2).

The classical closed Taylor-Couette flow (Fig. 3a) was investigated separately. The main sequence of its transition towards turbulence is now well known. In contrast to the closed Taylor-Couette system, the open Taylor-Couette system (Fig. 3b) does not have rotational symmetry around the
cylinder axis. Each flow produced in the open system undergoes noticeable changes with spatial location along the stream. So, our study is focused on the Dean and Taylor-Dean flows which need more information.

2. Experimental set-up and methods
The devices used, which have nearly the same aspect ratio, are shown in Fig.3. One for the closed flow (Fig.3a), an other for the open flow (Fig.3b). For this last, the curved section consists of two concentric cylinders with an inner rotating cylinder of radius \( R_1 = 3.85 \text{cm} \), a gap \( d = R_2 - R_1 = 0.6 \text{cm} \), a radius ratio \( \eta = 0.865 \) and the cylinders length \( L = 10 \text{cm} \), giving an aspect ratio \( \Gamma = L/d = 16.6 \). We make our observations in a cell delimited axially between \( 0 < z < 10 \text{cm} \) and azimuthally between \( 0^\circ < \theta < 330^\circ \). The inner cylinder can rotate in the range \(-17 \text{ rd/s} < \Omega < 17 \text{ rd/s} \). The flow rate, varying from 0 to 1000 \( \text{dm}^3/\text{h} \), is provided by a pump controlled by an electromagnetic flow meter. The fluid, recycled from a tank, is a mixture of water and Emkarox with added kalliroscope for the visualization. Experiments are down for different viscosities to obtain the first instability and the high turbulence in the range of rotation allowing easy observation. The viscosity is varied in the range \( 10^{-6} < \eta < 5 \times 10^{-6} \text{m}^2/\text{s} \).

Information about flow velocity are obtained with Laser Doppler Anemometry. We measured the azimuthal component. The signal is collected at \( \theta = 280^\circ \) in the mid-axis of the gap for different radial locations. We used the Burst Signal Analyzer Flow Software to analyze it. The BSA gives the azimuthal mean velocity and the RMS; power spectra density is calculated by the FFT techniques.

![Figure 3a. Closed Taylor-Couette system](image1)

![Figure 3b. Open Taylor-Dean system](image2)

3. Experimental results
The evolution of the flow is described in a two parameter space \((Ta, \tau)\) or \((De, \tau)\) where \( Ta \) is the Taylor number, \( De \) the Dean number and \( \tau \) the ratio of pumping and rotation flow rates.

3.1. Taylor-Couette flow
We consider here the classical closed Taylor-Couette flow in the case where only the inner cylinder rotates. The flow is then parameterised by the Taylor number \( Ta = \frac{\Omega_1 R_1 d}{\nu} \sqrt{\frac{d}{R_1}} \) where \( \Omega_1 \) is the inner cylinder rotation velocity.
3.1.1. Visualisation. The natural transition observed from the basic Couette flow to turbulent flow is as reported on the photographic sequence of Fig.4: Taylor vortex flow → Wavy vortex flow → Modulated wavy vortex flow → Weak turbulence.

![Figure 4](image-url)

Figure 4. The main sequence transition of the closed Taylor-Couette flow

3.1.2 Routes to turbulence. The characteristic spectrum of the modulated wavy vortex flow (Fig.5) are generally described by two or three fundamental frequencies. As in literature, we found the three scenarios leading to the chaos: quasi periodicity with frequency locking, period doubling and intermittency.

![Figure 5](image-url)

Figure 5. a) Period doubling b) Frequency locking \( \frac{f_2}{f_1} = \frac{1}{2} \)

3.2. Dean flow

Now, the two cylinders are maintained at rest. The outer one is opened and we pump fluid into the gap azimuthally. The flow obtained, the Dean flow, is parameterized by \( D_e = \frac{V_a d}{\nu \sqrt{R_l}} \), the Dean number, where \( V_a \) is the mean velocity of the flow.

Initial experiments determined the critical values of the flow rate at the appearance of the Dean cells for different viscosities of fluid. We obtained a linear curve.

3.2.1. Visualisation. The photographic sequence of Fig.6 shows the different flow regimes observed during the transition laminar-turbulent.

The cells appear first between \( 280^\circ < \theta < 330^\circ \) and \( 7 < z < 10 \text{cm} \). They are not stationary but move slightly in the axial direction: they are alternately inclined towards the right and the left and seem disappear temporarily. Ligrani et al. [20] described similar phenomena in a curved channel as a “rocking motion”. They observed the first signs of cells at \( \theta = 85^\circ \) for \( De = 64 \). The critical value obtained theoretically by Dean for the appearance of the cells in a closed azimuthal flow between
static concentric infinite cylinders with a small gap is equal to 35.92. The alone experimental verification of this value, known till now, is obtained by [11] in a gap with aspect ratio $\Gamma=35$, almost entirely filled. In our case, open system with a moderate aspect ratio, $\Gamma=16.6$, and a gap filled on 330° the instability is convective: the cells, non present in the core of the flow till $De=52$, occupy almost entirely the gap at $De=58$. We consider this last value as critical.

Beyond $De=58$, the cells split and merge alternately. This mixed cellular regime is characterized by two sets of cells which wavelengths are approximately in a ratio 5/3.

At $De=1.3 \, De_c$, the cells undulate slowly close the exit of the flow. Beyond $De=1.7 \, De_c$, the cells split into wavy rolls which merge into flat cells and so on. The undulating vortices of this mixed wavy-cellular regime are similar to the wavy Taylor vortices obtained in the closed Taylor-Couette flow even if the subsequent regime is not so well established. They seem to be modulated with increasing $De$. At nearly $De=1.9 \, De_c$, the split-merging phenomena disappears and we note the presence of the first turbulent bursts in the flow.

Beyond $De=2.2\, De_c$, short oblique waves make their appearance on the cells. They correspond to the "twisting vortices" obtained numerically by [7]. Then, until $De=17 \, De_c$, the highest Dean number reached by our system, the flow seems like the "soft" turbulence regime which arises in the closed Taylor-Couette flow after the disappearance of the travelling waves.

3.2.2. Local velocity measurement. As shown in Fig.7, the velocity distribution for different Dean numbers have a parabolic profile like, with the location of maximum velocity shifted towards the outer wall. RMS values are greater near the walls, particularly near the inner wall.

![Dean cells](image1)
!["Undulating waves"](image2)
!["Twisting waves"](image3)

**Figure 6.** Photographic sequence showing the characteristic regimes of the Dean flow.

**Figure 7.** Dean velocity profiles for different values of $De$. Measurements of $V_{mean}$ are done nearly at 1, 2, 3 and 4mm of the inner wall.
For any position sufficiently far from the walls, the evolution of the mean velocity $V_{\text{mean}}$ and the RMS versus Dean number is quasi linear. Fig.8 shows an example at approximately 2.5mm of the wall. Near the wall, measurements are perturbed.

![Mean velocity and RMS](image)

**Figure 8.** Evolution of $V_{\text{mean}}$ and RMS versus Dean number. Spectral analysis reveals three distinct steps in the spatio-temporal evolution of the flow: $De < 2De_c$ (Fig.9), $2De_c < De < 11De_c$ and $11De_c < De < 17De_c$.

![Dean(P2): Spectral evolution](image)

**Figure 9.** Spectral evolution for $De < 2De_c$. The fundamental frequency of the flow increases with $De$ and vanishes at $De = 99$.

### 3.3 “Closed” Taylor-Dean flow

Since the flow induced only by the inner cylinder rotation is completely reversed, we consider it as “closed”. The “entry” and the “exit” of the flow are defined following the direction of the rotating cylinder. The flow is parameterized by the Taylor number $Ta$.

**3.3.1 Visualisation.** Without pumping, for $\tau = 0$, we increase the inner cylinder rotation velocity from rest. At first, for low $Ta$, we observe axial longitudinal thin vortices at $\theta = 0^\circ$ and $\theta = 330^\circ$(Fig.10a). The fluid layers dragged by the inner cylinder make a 180° turn at the exit and flow back towards the entry where the flow is reversed again. On the longitudinal thin vortices appear some distortions like bulbs. They give rise to corner vortex and cells around them (Fig.11.1 and 10e) at the entry and “necking” cell with inclined travelling rolls at the exit (Fig.10d). With increasing $Ta$, the inclined rolls become
wavy (Fig.11.3) while other patterns as split-merging cellular and then diagonally propagating rolls (Fig.11.2) develop themselves in the core. The visualisation shows the stochastic flow (Fig.11.4) spreading from the exit to the entry. Pattern formation at the exit are well documented by [15-19] contrary to those observed at the entry which did not receive much attention. To our knowledge, the corner vortex are reported here for the first time in the Taylor-Dean flow. Notice that a similar flow sequence has been observed in the convection in binary fluid mixtures by [21] who described the motion of rolls generated in a corner and moving continuously to the opposite corner.

In the core, the primary flow is purely azimuthal. With increasing Ta, the flow undergoes a series of transitions leading to the turbulence: Taylor cells, Dean cells, split-merging cells, travelling waves, modulated waves, vortex path, diagonally propagating rolls and finally stochastic flow where all structures crumble.

![Diagram of Flow Patterns](image)

**Figure 10.** Rise and growth of the entry and exit instabilities for τ=0

![Images of Flow Patterns](image)

**Figure 11.** Structures observed at: 1) the entry at Ta=380; 2) the core at Ta=380; 3) the exit at Ta=180; 4) stochastic flow in the core at Ta=450.

### 3.4 Open Taylor-Dean flow

This case constitutes a synthesis of the Dean and the “closed” Taylor-Dean flow.

**3.4.1 Visualisation.** We distinguish two cases:

When τ → ±∞, the reversed flow is not noticeable. Structures observed are like those described for pure pumping. Dean flow dominates.

For −2/3<τ<2/3, we observe, qualitatively, the same phenomena as in the case τ=0. The flow is partially reversed and gives rise, at the entry and exit zones, to the structures described before, ie
corner vortex in the entry and “necking” cell with inclined axially propagating rolls in the exit. The
flow regimes observed in the core are presented in Fig.12. The basic flow, combination of the Couette
and Poiseuille flows, is purely azimuthal. At the instability onset, we can observe either axially
regularly spaced cells for $\tau<0$ or spirals for $\tau>0$. With increasing $Ta$ for a fixed $De$, a succession of
transitions lead the flow to turbulence: split-merging of the cells, travelling waves with two trains in
the same or opposite direction, turbulent bursts, vortex paths in the same or opposite direction,
diagonally traveling rolls, chaos where the different structures coexist and then the stochastic flow
where all the structures crumble as in fig.11.4. The chaos occurs earlier than in closed Taylor-Couette
flow and the stochastic flow, which is not reached in closed Taylor-Couette flow, propagates from
the exit to the entry. The structures around the corner vortex at the entry are the last to be destroyed.

Figure 12. Open Taylor-Dean flow regimes (A.T.: axially translation motion; Spi.: Spirals)

3.4.2 Discussion Assuming that in the core of the flow velocity distribution is the same than
for a completely filled gap we can compare our experimental results with theory, at least for
the primary instability. The curve ($Ta_c$, $\tau$) we have obtained has a similar behavior than that
predicted theoretically by Chandrasekar [22] who offered a highly plausible explanation of
the very peculiar dependence of $Ta_c$ on $\tau$. His analysis is based on Rayleigh’s criterion.

Near $\tau=-0.222$, point of maximum stability which focalized numerous studies, the curve of neutral
stability shows an unusual behavior. [9] found that the curve exhibits two loops, due to a discontinuity
in the critical neutral wave number. They demonstrate that the two lowest stationary modes do not
exist. [10] showed after them that the stationary modes are replaced by axially nonsymmetrical
oscillatory modes. In our experiments, we observed that from $\tau=-1$, large stationary cells appear in
the outside layer while finest cells are present in the inside layer. Then the two sets of cells seem to
be in competition. The flow does not choose its cellular mode. It changes from a state to another by
the split merging phenomenon.
3.4.3 Local measurements. Measurements made at nearly \(d/3\) of the rotating cylinder show that in absolute value, the mean velocity and its RMS (Fig.13) have the same evolution and become quasi superposed when \(\tau\) is varied toward zero, i.e when the unstable regions are of equal extend.

![Open Taylor-Dean flow De=16; P2](image)

![Open Taylor-Dean flow De=66; P2](image)

**Figure 13.** Evolution of the mean velocity and its RMS vs \(\tau\) at De=16 and De=66 for the same \(\theta = 280^\circ\)-section and a measurement point situated at nearly 2.5 mm of the inner cylinder.

Measured frequency spectra displays some features similar to that observed in the Taylor-Couette transition. We observed quasi-periodic flow with three independent frequencies and the chaos is signaled by the emergence of a broadband spectrum.

Fig.14 represents two spectrum obtained at the same spatial location for nearly opposite values of \(\tau\) in the region of Fig.12 where the Kelvin-Helmoltz instability induces a periodical azimuthally propagating structure, the vortex path.

![Open Taylor-Dean](image)

![Open Taylor-Dean](image)

**Figure 14.** Spectrum at \((\tau = -0.115 ; \text{Ta}=286)\) and \((\tau = 0.1 ; \text{Ta}=320)\) for De=16. Energy is in arbitrary units.

4. Conclusions
The study we presented has revealed a great number of physical phenomena. We described qualitatively the instabilities and the subsequent regimes observed during the laminar-turbulent transition of the Taylor-Couette, Dean and Taylor-Dean flows. The description is completed with some local measurements. Many additional experiments remain to be done, particularly concerning the velocity field and the spatio-temporal organization of the Dean and Taylor-Dean flows during their evolution towards turbulence. Further theoretical and numerical studies are also needed.

Fundamentally, it is expected that an understanding of the origin of the observed instabilities and their role in the crumble process of the structures of the flow could help to throw some light on the rise of turbulence. A general relation between closed and open systems is to sought.
The system studied can also have many practical applications as in textile industry or paper fabrication. So it will be interesting, for example, to investigate the effect of the observed patterns as the vortices split-merging or the vortex path on the mass transfer enhancement. This is being done with an electrochemical method.

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