Boundary String Field Theory of the \( D\overline{D} \) System

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We develop the boundary string field theory approach to tachyon condensation on the \( D\overline{D} \) system. Particular attention is paid to the gauge fields, which combine with the tachyons in a natural way. We derive the RR-couplings of the system and express the result in terms of Quillen’s superconnection. The result is related to an index theorem, and is thus shown to be exact.
1. Introduction

Tachyon condensation is the conceptually simple process of fields rolling down a potential towards a minimum. The technical challenge of describing this phenomenon in string theory is that such a shift of the vacuum cannot be studied in the standard first quantized formalism; the physics is necessarily off-shell and so in the domain of string field theory (SFT). The purpose of this article is to develop the boundary SFT for the basic unstable system of Type II string theory, the \( \mathcal{D\overline{D}} \)-system. The physics of both BPS D-branes and “wrong \( p \)” non-BPS D-branes can be recovered from this system by appropriate projections. We will particularly emphasize the role of gauge fields and their interplay with the tachyons. We also derive the RR-couplings of the branes, expressing the results in a compact form using Quillen’s superconnection.

The best developed approach to SFT is the cubic SFT for the open bosonic string [1]. This version gives a well defined theory with interactions that are closely tied to worldsheet geometry. Additionally, the truncation to the first few string levels provides a practical method for studying tachyon condensation, yielding quantitative results [2,3]. Much intuition about the subject has been developed this way. The difficulty in using the cubic SFT is that an infinite number of component fields acquire expectation values as the tachyon condenses. This makes it difficult to obtain an analytical description of tachyon condensation.

In recent months the background independent version of open SFT [4,5,6] has been established as a viable alternative to the cubic SFT [7,8]. Classical solutions in string theory are conformal field theories (CFTs) so it is natural to interpret SFT as a theory on the space of all two dimensional field theories, conformal or not. Background independent SFT is an attempt to make this concrete. It was originally derived through the Batalin-Vilkovisky formalism, but it is also possible to take a more intuitive approach, generalizing ordinary sigma models [9,10,11,12,13]. This is the strategy we pursue.

The idea is the following. We are interested in open string field theory, so the closed strings are treated as an on-shell background. For conformally invariant theories it is well known that the classical spacetime action of the open string theory is given by the partition function on the disc. The new ingredient in string field theory is that we allow boundary interactions which break conformal invariance — this is precisely what we mean by taking open strings off-shell. The working assumption is that the spacetime action can be identified with the partition function also in this more general setting. All of our
computations thus boil down to computing partition functions on the disk of theories which are conformal in the bulk but not necessarily on the boundary.

This approach to SFT can also be motivated from a different point of view \([14]\). The renormalization group flow in the space of 2D quantum field theories, from one 2D CFT to another, is characterized by the \(c\)-functional. On surfaces with boundaries the flow between theories with identical conformal bulk but different boundary theories is similarly characterized by the \(g\)-functional \([15]\), also known as the boundary entropy. The boundary entropy is a measure of the size of phase-space, as the name suggests, but it also measures D-brane tension \([16]\). The tension is thus a function of the boundary interaction, suggesting an interpretation as an action on the space of theories, whether conformal or not. Since the \(g\)-functional is in fact nothing other than the partition sum we return to the identification between the disc partition sum and the spacetime action.

For the bosonic string, it is not quite correct to equate the spacetime action with the disk partition sum, but the needed modifications can be obtained either from the BV formalism or from general considerations \([6]\). However, for the superstring we are not aware of any problems with this identification (for discussion see \([7]\)), and the fact that we obtain reasonable results serves \textit{a posteriori} as further justification.

Let us consider the strategy a little more concretely. The starting point is an on-shell closed string background described by one of the standard methods. It is convenient to describe it in the Schr"odinger representation as a closed string wave functional \(\Psi_{\text{bulk}}\). We consider very simple bulk states, corresponding to either the NS-NS or RR vacua, but generalizations are possible. The remaining ingredient is the boundary interaction, described by the wave functional \(\Psi_{\text{bndy}}\). The boundary wave functional is defined for general boundary interactions, but those giving rise to free field theories are singled out as being particularly simple. The final step is to combine the two ingredients by projecting the boundary wave functional on to the bulk wave functional. The result is the disk partition sum, which we then identify with the spacetime action for open string fields, as explained above. In view of the significance of the boundary interaction, this form of SFT is sometimes referred to as boundary SFT rather than background independent SFT; either way it is BSFT.

The contributions of the present paper fall in the following categories:

(1) We compute the effective action for the tachyons of the \(D\overline{D}\) system, and show that lower dimensional D-branes arise as solitons in the expected fashion. All other D-
branes – non-BPS and BPS – appear as special cases of the $D\overline{D}$. Our organization of
the computation is somewhat different from some recent presentations, and offers –
in our opinion – conceptual and computational advantages. As discussed above, our
method is similar to the sigma model approach [8,10,11,12,13,17].

(2) We include gauge fields on the branes. The introduction of gauge fields via boundary
fermions is explained in some detail. It becomes apparent that in the $D\overline{D}$-system it
is economical to consider gauge fields and tachyons simultaneously, as the problem
thus acquires its natural generality. As a concrete result we derive various terms in
the combined tachyon-gauge field action.

(3) We consider the system in the background of a constant RR potential. The role of D-
branes as sources of RR-charges makes this a natural problem. In the presence of the
RR-background the modings of fermions and bosons are identical, effectively reducing
the problem to the fermion zero-modes. The couplings we derive are summarized in
the action

$$S = T_{D9} \int C \wedge \text{Str} \ e^{2 \pi \alpha' iF}, \quad (1.1)$$

where $F$ is the curvature of the superconnection [18,19]

$$iA = \begin{pmatrix} iA^+ & T \\ T & iA^- \end{pmatrix}. \quad (1.2)$$

This action generalizes the well-known RR-couplings of BPS D-branes to the $D\overline{D}$
system, and was conjectured by Kennedy and Wilkins [20]. Additionally, we find the
corresponding couplings for the non-BPS D-branes. From the result (1.1) we verify
that lower dimensional D-branes described as solitons carry the correct RR-charge.
We also discuss some of the connections to index theorems.

The organization of this paper is as follows. In section 2 we introduce the basic
concepts of bulk and boundary wave functionals and the tachyon effective action. In
section 3 we add non-abelian degrees of freedom to the problem by discussing boundary
fermions in some detail. We also define the partition function for the $D\overline{D}$ system. In
section 4 we discuss lower dimensional branes as solutions in the resulting theory, and
derive terms in the combined tachyon gauge field action. In section 5 we consider the
couplings to background RR potentials and discuss their implications. We conclude with
a discussion in section 6.
2. Boundary String Field Theory

The purpose of this section is to state the general procedure defining boundary string field theory. We also work out simple examples, yielding results needed later.

As explained in the introduction, we wish to compute the path integral over all fields on the unit disk in the presence of specified boundary interactions. It is convenient to perform the analysis in several steps:

1. Integrate over fields in the bulk. The result is a closed string wave functional, a functional of the fields restricted to the boundary of the disk.

2. Include boundary interactions, as described by a boundary wave functional.

3. Project the boundary wave functional on to the bulk wave functional. The result is the disc partition function, a functional of the spacetime fields appearing as boundary couplings.

4. The partition function thus computed is divergent and must be regularized and renormalized. We do so by zeta function methods. In superstring theory, the resulting renormalized partition function is identified with the spacetime action.

In the following we make this procedure explicit through several important examples.

2.1. The Bosonic String Partition Function

We begin by computing the disc partition function of the bosonic string, formally defined as

\[ Z = \int \mathcal{D}X e^{-\left( S_{\text{bulk}} + S_{\text{bndy}} \right)} , \]  

with

\[ S_{\text{bulk}} = \frac{1}{4\pi\alpha'} \int d^2x \sqrt{\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu . \]  

As explained above, we first perform the integral over bulk field configurations with fixed boundary conditions. We write the metric on the disc as

\[ ds^2 = d\rho^2 + \rho^2 d\tau^2 , \]  

with boundary at \( \rho = 1 \), and specify the field on the boundary as

\[ X^\mu|_{\rho=1} = X_0^\mu + \sqrt{\frac{\alpha'}{2}} \sum_{n=1}^{\infty} \left( X_n^\mu e^{in\tau} + X_{-n}^\mu e^{-in\tau} \right) . \]
The unique regular solution to the bulk equation of motion, \( \nabla^2 X^\mu = 0 \), is

\[
X^\mu = X^\mu_0 + \sqrt{\frac{\alpha'}{2}} \sum_{n=1}^{\infty} \rho^n (X^\mu_n e^{in\tau} + X^\mu_{-n} e^{-in\tau}) ,
\]

and the bulk action evaluated on this solution is

\[
S_{\text{bulk}} = \frac{1}{4\pi \alpha'} \int d\tau X^\mu \partial_\rho X^\mu |_{\rho=1} = \frac{1}{2} \sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_n .
\]

The corresponding bulk wave functional

\[
\Psi_{\text{bulk}} = e^{-S_{\text{bulk}}} = \exp \left( -\frac{1}{2} \sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_n \right) ,
\]

characterizes the conformally invariant closed string vacuum. The computation only took the saddle point contribution into account. This is justified because the overall factor from the fluctuations around the classical field is independent of the boundary field \((2.4)\), and therefore irrelevant for our further considerations.

The result obtained so far is the starting point irrespective of the boundary interactions. We now consider the quadratic boundary interaction \([5,7]\)

\[
S_{\text{bndy}} = \int \frac{d\tau}{2\pi} u X^2 = uX^2_0 + u\alpha' \sum_{n=1}^{\infty} X_{-n} X_n ,
\]

where the index on \( X \) is omitted because we focus on a specific \( X \). This boundary interaction breaks conformal invariance and therefore takes the theory off-shell. It is a relevant interaction, inducing an RG flow between two CFTs. This flow describes tachyon condensation: the boundary interaction \((2.8)\) is interpreted in spacetime as a tachyon profile of the form \( T(X) = uX^2 \), as explained in \([7]\).

The above evaluation of the bulk integral amounts to summing over all field configurations at \( \rho < 1 \); what remains then is to integrate over the fields at \( \rho = 1 \). The result is

\[
Z(u) = \int \frac{dX_0}{\sqrt{2\pi \alpha'}} \prod_{n=1}^{\infty} \frac{dX_n dX_{-n}}{4\pi} e^{-(S_{\text{bulk}} + S_{\text{bndy}})} = \frac{1}{\sqrt{2\alpha' u}} \prod_{n=1}^{\infty} \left( \frac{1}{n + 2\alpha' u} \right) .
\]

We chose a convenient measure; the factors in the denominator affect only the \( u \) independent normalization factor, which we are not keeping track of anyway. The infinite product
is divergent (more accurately, $1/Z(u)$ diverges) and needs to be regularized. We use zeta function regularization:

$$
\prod_{n=1}^{\infty} \left( \frac{1}{n + 2\alpha'u} \right) = \exp \left\{ \frac{d}{ds} \sum_{n=1}^{\infty} (n + 2\alpha'u)^{-s} \right\}_{s=0} = \exp \left\{ \frac{d}{ds} \left[ \zeta(s, 2\alpha'u) - (2\alpha'u)^{-s} \right] \right\}_{s=0}
$$

$$
= \exp \left\{ \ln \Gamma(2\alpha'u) - \frac{1}{2} \ln 2\pi + \ln 2\alpha'u \right\} = \frac{2\alpha'u \Gamma(2\alpha'u)}{\sqrt{2\pi}},
$$

(2.10)

where we used the zeta function

$$
\zeta(z, q) = \sum_{n=0}^{\infty} \frac{1}{(q + n)^z}.
$$

(2.11)

Our final result for the partition function is thus

$$
Z(u) = \frac{\sqrt{2\alpha'u} \Gamma(2\alpha'u)}{\sqrt{2\pi}}.
$$

(2.12)

This expression differs by a factor of $e^{2\alpha'w_{\gamma}/\sqrt{2\pi}}$ from that of [5] due to a different choice of regularization scheme. This difference does not affect any physical quantities such as D-brane tensions.

2.2. Including Gauge Fields

Another important example is that of an abelian gauge field, coupled to the worldsheet via the boundary term

$$
S_A = -i \int d\tau A_\mu (X^\mu) \dot{X}^\mu,
$$

(2.13)

where $\dot{X} = dX/d\tau$. This coupling exhibits the gauge invariance

$$
\delta A_\mu = \partial_\mu \alpha.
$$

(2.14)

In the case of a constant field strength we can write

$$
A_\mu = -\frac{1}{2} F_{\mu\nu} X^\nu,
$$

(2.15)

and the boundary coupling is

$$
S_A = \frac{i}{2} \int d\tau F_{\mu\nu} \dot{X}^\mu X^\nu.
$$

(2.16)
Using mode expansions of the form (2.5) gives

\[ S_A = \pi \alpha' F_{\mu\nu} \sum_{n=1}^{\infty} n X_{-n}^\mu X_n^\nu. \]  

(2.17)

So for constant field strengths and tachyon profile \( T = \sum_{\mu} u_{\mu}(X^\mu)^2 \), and using the results (2.8) and (2.9), the total action is

\[ S_{\text{bulk}} + S_{\text{bndy}} = u_{\mu}(X^\mu_0)^2 + \frac{1}{2} \sum_{n=1}^{\infty} X_{-n}^\mu \{(n + 2\alpha'u_{\mu})\delta_{\mu\nu} + 2\pi\alpha' n F_{\mu\nu}\} X_n^\nu. \]  

(2.18)

(sum over \( \mu, \nu \) implied). The partition function is obtained from the obvious higher dimension generalization of (2.9).

To proceed we skew-diagonalize \( F_{\mu\nu} \), with eigenvalues \( f_\beta \equiv F_{2\beta,2\beta+1} \). The integration gives

\[ Z(u, F) = \int \prod_{\mu} \frac{dX^\mu_0}{\sqrt{2\pi \alpha'}} e^{-u_{\mu}(X^\mu_0)^2} \prod_{\beta=0}^{(d-1)/2} \prod_{n=1}^{\infty} \frac{1}{(n + 2\alpha'u_{2\beta})(n + 2\alpha'u_{2\beta+1}) + (2\pi\alpha' f_\beta n)^2}. \]  

(2.19)

For vanishing tachyon, \( u_{\mu} = 0 \), the zeta function prescription is

\[ \prod_{n=1}^{\infty} \frac{1}{n^2 + a^2 n^2} = e^{2\zeta'(0)(1 + a^2) - \zeta(0)} = \frac{\sqrt{1 + a^2}}{2\pi}, \]  

(2.20)

which yields

\[ Z(0, F) = C \int d^{d+1}X_0 \sqrt{\det [\delta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]} , \]  

(2.21)

with the overall normalization constant working out to be \( C = (4\pi^2\alpha')^{-(d+1)/2} \). This is the standard Born-Infeld action, as expected.

For tachyon profiles that are identical in pairs, \( u_{2\beta} = u_{2\beta+1} \), (2.19) reduces to (12)

\[ Z(u, F) = \prod_{\beta=0}^{(d-1)/2} \sqrt{1 + (2\pi\alpha' f_\beta)^2} |Z\left(\frac{u_{2\beta}}{1 + 2\pi\alpha' f_\beta}\right)|^2 , \]  

(2.22)

where \( Z \) is the partition function (2.12) when a single \( u_{\mu} \) is turned on.

To include non-abelian tachyons and gauge fields on multiple D-branes replace \( T \) and \( A_{\mu} \) by Hermitian matrices and introduce a path ordered exponential,

\[ e^{-S_{\text{bndy}}} = \text{Tr} \ e^{-\int d\tau (T(X) + iA_{\mu}X^\mu)}, \]  

(2.23)
Path ordering is necessary in order that the partition function be invariant under the gauge transformation
\[ \delta A_\mu = \partial_\mu \alpha + i[\alpha, A_\mu] \]
\[ \delta T = i[\alpha, T]. \] (2.24)

Due to the nontrivial matrix orderings, no closed form expression is known for the general non-abelian partition function. Another approach to including non-abelian degrees of freedom is to introduce auxiliary boundary fermions instead of explicit Chan-Paton factors. We will have more to say about this approach when we come to the superstring.

2.3. Preliminaries for the Superstring

We would like to carry out the corresponding computations for the superstring. The starting point is the bulk action for the NSR string
\[ S_{\text{bulk}} = \frac{1}{4\pi} \int d^2z \left( \frac{2}{\alpha'} \partial X^\mu \overline{\partial X}_\mu + \psi^\mu \overline{\partial \psi}_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu \right). \] (2.25)

We work in the NS sector so the fermions are anti-periodic on the disk. The mode expansion of \( \psi^\mu \) at the boundary of the disc is
\[ \psi^\mu(\tau) = \sum_{r = \frac{1}{2}}^{\infty} \left( \psi^\mu_{r+} e^{i r \tau} + \psi^\mu_{r-} e^{-i r \tau} \right). \] (2.26)

Fermions are simpler in rectangular coordinates than in polar so it is convenient to extend into the bulk by going to the upper half-plane. We therefore temporarily complexify the coordinate \( \tau \), imposing regularity as \( \text{Im} \tau \to \infty \). This prescription gives
\[ \psi^\mu(\tau) = \sum_{r = \frac{1}{2}}^{\infty} \left( \psi^\mu_{r+} e^{i r \tau} + \psi^\mu_{r-} e^{-i r \tau} \right), \] (2.27)

and the bulk action works out to be
\[ S_{\text{bulk}} = \frac{1}{2} \sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_n + i \sum_{r = \frac{1}{2}}^{\infty} \psi^\mu_{r+} \psi^\mu_{r}. \] (2.28)

The right-moving fermions are \( \tilde{\psi}^\mu(\tau) = \psi^\mu(\tau) \), so they simply contributed a factor of 2.

\footnote{1}{The left-moving classical field \( \psi^\mu \) necessarily depends on both holomorphic and anti-holomorphic coordinates; indeed, otherwise the on-shell action would vanish because it is proportional to \( \overline{\partial} \psi^\mu \). The treatment of fermion wave functionals is discussed in \textit{e.g.} [21].}
The next step is to introduce boundary interactions. An important principle that we must respect is worldsheet supersymmetry. We do so by working in boundary superspace with coordinates $\hat{\tau} = (\tau, \theta)$ and

$$X^\mu = X^\mu + \sqrt{\alpha'} \theta \psi^\mu, \quad D = \partial_\theta + \theta \partial_\tau.$$  \hspace{1cm} (2.29)

The simplest example of a boundary action corresponds to the gauge field. For an abelian gauge field we use the supersymmetric generalization of (2.13),

$$S_A = -i \int d\tau d\theta A_\mu(X) D X^\mu = -i \int d\tau \left[ A_\mu(X) \dot{X}^\mu + \frac{1}{2} \alpha' F_{\mu\nu} \psi^\mu \psi^\nu \right].$$  \hspace{1cm} (2.30)

It is a simple matter to expand this boundary action in modes and compute the partition function. The unsurprising result \cite{22} is that the fermion determinant is independent of $F_{\mu\nu}$ so that we recover the Born-Infeld action (2.21).

3. Boundary Fermions

In our approach to the $D\overline{D}$ system boundary interactions will be introduced using auxiliary boundary fermions. In order to motivate and explain the construction we first consider the related question of describing non-abelian gauge fields. It will turn out that tachyons are similarly described.

3.1. Non-abelian Gauge Fields

Boundary interactions for non-abelian gauge fields in superstring theory must simultaneously preserve spacetime gauge invariance and worldsheet supersymmetry. There are several ways to achieve this.

One option is to use supersymmetric path ordering and consider

$$e^{-S_A} = \text{Tr} \hat{P} e^{\int d\tau d\theta A_\mu(X) D X^\mu}.$$  \hspace{1cm} (3.1)

The $\hat{P}$ symbol is

$$\hat{P} e^{\int \hat{M}(\hat{\tau})} = \sum_{N=0}^{\infty} \int d\hat{\tau}_1 \ldots d\hat{\tau}_N \Theta(\hat{\tau}_{12}) \Theta(\hat{\tau}_{23}) \ldots \Theta(\hat{\tau}_{N-1,N}) M(\hat{\tau}_1) \ldots M(\hat{\tau}_N),$$  \hspace{1cm} (3.2)

where $\hat{\tau}_{12} = \tau_1 - \tau_2 - \theta_1 \theta_2$, and $\Theta$ is the step function. The delta function term in the expansion $\Theta(\hat{\tau}_{12}) = \Theta(\tau_1 - \tau_2) - \theta_1 \theta_2 \delta(\tau_1 - \tau_2)$ gives contact terms which are essential for
worldsheet supersymmetry. An interesting feature is that these same contact terms are crucial for gauge invariance \cite{11}, as they contribute the $[A_\mu, A_\nu]$ in the non-abelian field strength $F_{\mu\nu}$. Indeed, performing the $d\theta$ integrals in (3.1) gives

$$e^{-S_A} = \text{Tr} \ P e^{i \int d\tau [A_\mu(X) \dot{X}^\mu + \frac{\alpha'}{4} F_{\mu\nu} \psi^\mu \psi^\nu]} , \quad (3.3)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$.

The drawback of describing non-abelian interactions this way is that path ordering is awkward, whether supersymmetric or not. An alternative that is sometimes convenient is to trade the explicit path ordering in (3.1) for a path integral over boundary fermions $\eta^a$ in the fundamental representation of the gauge group \cite{11}. Introducing the boundary superfield $\hat{\eta}^a = \eta^a + \theta \zeta^a$, (3.1) can be rewritten as

$$e^{-S_A} = \int D\hat{\eta} \ D\hat{\eta}^* e^{-\int d\tau d\theta \left[ \frac{1}{4} D\hat{\eta}^a D\hat{\eta}^a - \hat{\eta}^a A^a_{\mu}(X) DX^\mu \hat{\eta}^b \right]} . \quad (3.4)$$

Rather than (3.1), (3.3), or (3.4), we will use yet another description \cite{23,24} which is particularly well adapted to the problem of tachyon condensation \cite{24,14,7}. Consider $2^m$ branes with the corresponding gauge group $U(2^m)$ generated by $2^m \times 2^m$ matrices. These matrices can be expanded in terms of $SO(2m)$ gamma matrices,

$$A_{\mu}^{ab} = \sum_{k=0}^{2m} \frac{1}{2k!} A_{\mu}^{I_1 \cdots I_k} \gamma_{ab}^{I_1 \cdots I_k} , \quad (3.5)$$

where $\gamma^{I_1 \cdots I_k}$ denote anti-symmetrized products of gamma matrices with unit weight (e.g. $\gamma^{12} = \gamma^1 \gamma^2$). All that is needed for the expansion (3.5) is a representation of the Clifford algebra. So instead of gamma matrices, introduce $2m$ boundary fermion superfields $\Gamma^I = \eta^I + \theta F^I$ with action $S = -\int d\tau d\theta \frac{1}{4} \Gamma^I D\Gamma^I$. Canonically quantizing, one arrives at the anti-commutation relations $\{\eta^I, \eta^J\} = 2\delta^{IJ}$, so $\eta^I$ represent the Clifford algebra as desired. So we write the boundary action for the non-abelian gauge field as

$$S_A = -\int d\tau d\theta \left[ \frac{1}{4} \Gamma^I D\Gamma^I + i \sum_{k=0}^{2m} \frac{1}{2k!} A_{\mu}^{I_1 \cdots I_k} DX^\mu \Gamma^{I_1} \cdots \Gamma^{I_k} \right] . \quad (3.6)$$

In this action the correct ordering is enforced by the boundary fermions. Indeed, integrating out $\Gamma^I$ using the standard formula for a transition amplitude,

$$\int D\Phi e^{-S} = \text{Tr} \ P e^{-\int d\tau H(\tau)} , \quad (3.7)$$
one recovers a path ordered expression.

The action (3.6) is manifestly invariant under the \( U(1) \otimes SO(2m) \) gauge transformations

\[
\delta A_\mu = \partial_\mu \alpha , \\
\delta A_\mu^{IJ} = \partial_\mu \alpha^{IJ} + i(\alpha^{IK} A^K_J - A^K_I \alpha^K_J) ,
\]

and with \( \Gamma^I, A^I_\mu, A^{IJK}_\mu, \ldots \), transforming in anti-symmetric tensor representations of \( SO(2m) \). The remainder of the \( U(2^m) \) gauge symmetry is realized in a more involved fashion, mixing fields with different number of indices.

Actually there is a problem with (3.6) as it stands: terms in the action with \( k \) odd are fermionic rather than bosonic, leading to an incorrect algebra. This must be remedied by the introduction of anti-commuting cocycle factors. Happily, we will see that this complication is absent in the \( D\overline{D} \) system. We therefore disregard cocycles in the following.

Then the boundary fermion representation has an important effect on the rules for matrix multiplication. Consider a general \( 2^m \times 2^m \) matrix,

\[
M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \sum_{k=0}^{2m} M^{I_1 \cdots I_k} \gamma^{I_1 \cdots I_k}.
\]

We allow each submatrix \( A \cdot \cdots \cdot D \) to be either bosonic or fermionic. We keep track of this by defining, e.g., \((-)^a\) to be +1 if \( A \) is bosonic or −1 if \( A \) is fermionic. To this matrix we associate the quantity \( M = \sum_{k=0}^{2m} M^{I_1 \cdots I_k} \Gamma^{I_1 \cdots I_k} \). Now consider the procedure of multiplying together two matrices expressed in terms of fermions. We need to enforce the rule that all \( \Gamma \)’s are taken to the right of \( M^{I_1 \cdots I_k} \’s. \) We then have

\[
MM' = \sum_{k,k'=0}^{2m} M^{I_1 \cdots I_k} \Gamma^{I_1 \cdots I_k} M'^{J_1 \cdots J_{k'}} \Gamma^{J_1 \cdots J_{k'}}
\]

\[
= \sum_{k,k'=0}^{2m} (-)^{km'} M^{I_1 \cdots I_k} M'^{J_1 \cdots J_{k'}} \Gamma^{I_1 \cdots I_k} \Gamma^{J_1 \cdots J_{k'}}.
\]

Here \( m' \) is 0 or 1 depending on whether \( M'^{J_1 \cdots J_{k'}} \) is bosonic or fermionic. So we pick up an extra minus sign whenever \( M \) is associated with an odd number of gamma’s and \( M' \) is fermionic. Choosing an off-diagonal basis for our gamma matrices and translating back to matrix notation, (3.10) tells us that the correct rule for matrix multiplication is

\[
MM' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'} BC' & AB' + (-)^{d'} BD' \\
DC' + (-)^{a'} CA' & DD' + (-)^{b'} CB' \end{pmatrix}.
\]

This rule is standard in the theory of superconnections [18, 19]. It will be crucial for us later in obtaining the correct tachyon covariant derivative.
3.2. BSFT for the $D\overline{D}$ system

There are two unstable D-brane systems in type II string theory: the “wrong p” non-BPS D-branes and the $D\overline{D}$ system. It is sufficient to consider the string field theory of the $D\overline{D}$ system, since the theory of the non-BPS D-brane can then be obtained by restricting to couplings which are invariant under $(-)^{F_L}$ [20]. Hence we focus on $D\overline{D}$.

Consider for definiteness $N D9−\overline{D9}$ pairs in IIB. This system has a $U(N) \otimes U(N)$ gauge group with a tachyon transforming in the $(N, N)$ representation. The gauge fields and tachyons are naturally packaged as $2N \times 2N$ matrices indicating from which open string sector the fields arise,

$$\begin{pmatrix} A_\mu^+ & 0 \\ 0 & A_\mu^- \end{pmatrix}, \quad \begin{pmatrix} 0 & T \\ T & 0 \end{pmatrix}.$$

To write their boundary couplings it is convenient to combine them with the boundary superfield $D\mathbf{X}^\mu$ and write

$$M(\mathbf{X}) = \begin{pmatrix} iA_\mu^+(\mathbf{X})D\mathbf{X}^\mu & \sqrt{\alpha'} T(\mathbf{X}) \\ \sqrt{\alpha'} T(\mathbf{X}) & iA_\mu^-(\mathbf{X})D\mathbf{X}^\mu \end{pmatrix}.$$  \hfill (3.13)

Now take $N = 2^{m-1}$ so $M$ is a $2^m \times 2^m$ matrix, naturally expanded in $SO(2m)$ gamma matrices as in (3.5),

$$M^{ab} = \sum_{k=0}^{2m} \frac{1}{2k!} M_{I_1 \cdots I_k} \gamma_{ab}^{I_1 \cdots I_k}.$$  \hfill (3.14)

Introducing $2m$ boundary fermion superfields $\mathbf{F}^I$ as before, the boundary interaction is

$$S_{bndy} = -\int d\tau d\theta \left[ \frac{1}{4} \mathbf{F}^I D\mathbf{F}^I + \sum_{k=0}^{2m} \frac{1}{2k!} M_{I_1 \cdots I_k} \mathbf{F}^{I_1} \cdots \mathbf{F}^{I_k} \right].$$  \hfill (3.15)

The theory has $U(2^{m-1}) \otimes U(2^{m-1})$ gauge invariance, of which a $U(1) \otimes SO(2m)$ subgroup is manifest.

The lowest component of the superfield $M$ is fermionic and, as for gauge fields, the components $M^{I_1 \cdots I_k}$ with odd $k$ have opposite statistics, i.e. the lowest component is bosonic. In the present context the bosonic components are tachyons so the formalism automatically assigns them the correct statistics. Thus we do not need to introduce any cocycle factors in the $D\overline{D}$-system. In fact, it may be useful to realize a non-abelian gauge field on a BPS D-brane in terms of a $D\overline{D}$-system with $T = \overline{T} = A^- = 0$. This construction
has two extra fermions compared to the discussion of the pure gauge field around (3.6), and these additional degrees of freedom provide the cocycle factors which were rather awkwardly needed in the pure gauge system.

The upper component of the superfield \( \Gamma^I = \eta^I + \theta F^I \) is an auxiliary field, as it has no kinetic term. It can be eliminated as follows. We write the matrix \( M \) as \( M = M_0 + \theta M_1 \) and carry out the \( \theta \)-integral in the action (3.15)

\[
S_{\text{bndy}} = - \int d\tau \left[ \frac{1}{4} \dot{\eta}^I \eta^I + \frac{1}{4} F^I F^I + \sum_{k=0}^{2m} \frac{1}{2k!} \left( M_1^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k} - (-1)^k k M_0^{I_1 \cdots I_k} F^{I_1} \eta^{I_2} \cdots \eta^{I_k} \right) \right].
\]  

(3.16)

The Gaussian integral over the \( F^I \) can now be carried out. The result is a term \(- \frac{1}{4} F^I F^I\) under the integral, where

\[
F^I = \sum_{k=1}^{2m} \frac{(-1)^k}{(k-1)!} M_0^{I_2 \cdots I_k} \eta^{I_2} \cdots \eta^{I_k}.
\]  

(3.17)

To write the result in a more compact form, consider a general \( 2m \times 2m \) matrix, represented in terms of SO\((2m)\) matrices as in (3.3). Simplifying the products of gamma matrices using the Clifford algebra yields an identity of the form

\[
\sum_{k,k'=0}^{2m} \frac{1}{2k!2k'} \Phi^{I_1 \cdots I_k} \gamma^{I_1} \cdots \gamma^{I_k} \Phi^{J_1 \cdots J_{k'}} \gamma^{J_1} \cdots \gamma^{J_{k'}} = \sum_{k,k'=0}^{2m} \frac{(-1)^{k+k'}}{4(k-1)!(k'-1)!} \Phi^{I_2 \cdots I_k} \gamma^{I_2} \cdots \gamma^{I_k} \Phi^{J_2 \cdots J_{k'}} \gamma^{J_2} \cdots \gamma^{J_{k'}} + \text{higher contractions}.
\]  

(3.18)

Using this identity for \( \Phi = M_0 \) and temporarily ignoring the higher contractions we find the action

\[
S_{\text{bndy}} = - \int d\tau \left[ \frac{1}{4} \dot{\eta}^I \eta^I + \sum_{k=0}^{2m} \frac{1}{2k!} (M_1 - M_0^2)^{I_1 \cdots I_k} \eta^{I_1} \cdots \eta^{I_k} \right].
\]  

(3.19)

Here \( M_0^2 \) is defined using the matrix multiplication rule of (3.11) with \( a' = d' = 1 \) and \( b' = c' = 0 \) (because off-diagonal entries of \( M_0 \) are bosonic while diagonal entries are fermionic).

In the Gaussian computation yielding \(- \frac{1}{4} F^I F^I\) the interacting fermions were treated naïvely, omitting the presence of divergences when two fermions coincide. In more precise
computations there are additional contractions between the fermions in the two $F^I$'s. In [24] it is shown that these contractions exactly match the higher contractions in (3.18), rendering (3.19) the complete result. The $M_0^2$ terms in (3.19) have a nice interpretation: they give the commutator terms in the non-abelian field strengths of $A^+$ and $A^-$, as well as in the gauge covariant derivatives of the tachyon field. These results were guaranteed by gauge invariance; they are nevertheless nontrivial to recover because not all of the gauge invariance is manifest in the present formalism.

4. Explicit Computations

In this section we make these general considerations explicit for $m = 1$, corresponding to a single $D9 - D9$ pair. In this case gamma matrices reduce to Pauli matrices, and the expansion (3.14) takes the form

$$M = \frac{i}{2} A_\mu DX^\mu 1 + \frac{1}{2} \sqrt{\alpha'} T^I \sigma_I + \frac{i}{4} A^{IJ}_\mu DX^\mu \sigma_{IJ}, \quad I, J = 1, 2,$$

with $\sigma_{IJ} = [\sigma_I, \sigma_J]/2$ and

$$A^{I\pm}_\mu = \frac{1}{2} (A_\mu \pm i A^{12}_\mu),$$

$$T = \frac{1}{2} (T^1 + iT^2).$$

Therefore the boundary interaction is

$$S_{\text{bndy}} = -\int d\tau d\theta \left[ \frac{1}{4} \Gamma^I D^I \Gamma^I + \frac{i}{2} A_\mu DX^\mu + \frac{1}{2} \sqrt{\alpha'} T^I \Gamma^I + \frac{i}{4} A^{IJ}_\mu DX^\mu \Gamma^I \Gamma^J \right].$$

This action exhibits the full $U(1) \otimes U(1)$ gauge symmetry. Expanding (4.3) in components and integrating out the auxiliary fields $F^I$ (recall $\Gamma^I = \eta^I + \theta F^I$) yields

$$S_{\text{bndy}} = -\int d\tau \left[ -\frac{\alpha'}{4} T^I T^I + \frac{1}{4} \eta^I \eta^I + \frac{\alpha'}{2} D_\mu T^I \psi^\mu \eta^I + \frac{i}{2} (\dot{X}^\mu A_\mu + \frac{1}{2} \alpha' F_{\mu\nu}\psi^\mu\psi^\nu) \right. $$

$$+ \left. \frac{i}{4} (\dot{X}^\mu A^{IJ}_\mu + \frac{1}{2} \alpha' F_{\mu\nu}\psi^\mu\psi^\nu) \eta^I \eta^J \right] ,$$

where the derivative of the tachyon comes out correctly covariantized,

$$D_\mu T^I = \partial_\mu T^I - i A^I_{\mu j} T^j.$$

(4.4) is in agreement with the general results in the previous section.

The interactions in (4.4) are nontrivial so the corresponding partition function cannot be computed explicitly in its entirety. In the following we consider various special cases.
4.1. Tachyon Condensation on $D\overline{D}$.

To begin, we set the gauge fields to zero, $A_\mu^+ = A_\mu^- = 0$. Then it is simple to integrate out $\eta^I$, with the result

$$S_{\text{bndy}} = \frac{\alpha'}{4} \int d\tau \left[ T^I T^I + \alpha'(\psi^\mu \partial_\mu T^I) \frac{1}{\partial_\tau} (\psi^\nu \partial_\nu T^I) \right].$$  \hspace{1cm} (4.6)

The operator $1/\partial_\tau$ is defined by

$$\frac{1}{\partial_\tau} f(\tau) = \frac{1}{2} \int d\tau' \epsilon(\tau - \tau') f(\tau'),$$  \hspace{1cm} (4.7)

where $\epsilon(\tau)$ is +1 or -1 for positive or negative $\tau$.

For constant tachyon we have simply

$$\Psi_{\text{bndy}} = e^{-S_{\text{bndy}}} = e^{-2\pi \alpha'T\overline{T}}.$$  \hspace{1cm} (4.8)

The disc partition function is obtained by projecting this onto the bulk wave functional corresponding to (2.28), and integrating over all fields. No further tachyon dependence is introduced in this process so we learn that the tachyon potential for the $D\overline{D}$ system is

$$V(T, \overline{T}) = 2T_{D9} e^{-2\pi \alpha'T\overline{T}},$$  \hspace{1cm} (4.9)

where we fixed the overall normalization by hand, though this presumably can be verified independently as in [27].

Next we turn to spatially dependent tachyon configurations. Linear tachyon profiles are singled out as leading to free worldsheet theories. By a combination of spacetime and gauge rotations we can bring $T^I$ to the form

$$\sqrt{\alpha'} T^I = u^I X^I.$$  \hspace{1cm} (4.10)

Substituting the mode expansions (2.4) and (2.26) into (4.6) and combining with (2.28) gives the action

$$S_{\text{bulk}} + S_{\text{bndy}} = 2\pi \alpha'T\overline{T} + \sum_{l=1}^{2} \left[ \frac{1}{2} \sum_{n=1}^{\infty} (n + \pi \alpha'y^I) X_{-n}^I X_n^I + i \sum_{r=1}^{\infty} (1 + \pi \alpha' y^I_r^I) \psi^I_{-r} \psi^I_r \right].$$  \hspace{1cm} (4.11)

The first term is simply the zero-mode part of (4.10) and we defined

$$y^I = (u^I)^2.$$  \hspace{1cm} (4.12)
The partition function is
\[
Z(y^I) = \int \frac{d^{10}X_0}{(2\pi\alpha')^5} \prod_{I=1}^{2} \left( \prod_{n=1}^{\infty} \frac{dX_n^I dX_{-n}^I}{4\pi} \prod_{r=\frac{1}{2}}^{\infty} d\psi_r^I d\psi_{-r}^I \right) e^{-(S_{\text{bulk}} + S_{\text{bndy}})}
\]
(4.13)

\[
= \int \frac{d^{10}X_0}{(2\pi\alpha')^5} \prod_{I=1}^{2} \prod_{r=\frac{1}{2}}^{\infty} \frac{(1 + \pi\alpha'y_r^I)}{\prod_{n=1}^{\infty} (n + \pi\alpha'y_r^I)}.
\]

The bosonic product in the denominator was computed in (2.10) and the fermionic product is similarly
\[
\prod_{r=\frac{1}{2}}^{\infty} (r + \pi\alpha'y_r^I) = \prod_{n=1}^{\infty} (n + \pi\alpha'y_r^I - \frac{1}{2}) = \frac{\sqrt{2\pi}}{\Gamma(\pi\alpha'y_r^I + \frac{1}{2})} = \frac{4\pi\alpha'y_r^I}{\sqrt{2}} \Gamma(\pi\alpha'y_r^I).
\]
(4.14)

Defining the function
\[
F(x) = \sqrt{2\pi} \prod_{r=\frac{1}{2}}^{\infty} (1 + \frac{x}{r}) \prod_{n=1}^{\infty} (n + x) = \frac{4x \Gamma(x)^2}{2\Gamma(2x)},
\]
(4.15)

our result for the partition function becomes
\[
Z(y^I) = 2T_{D9} \int d^{10}X_0 \ e^{-2\pi\alpha'T\bar{T}} \prod_{I=1}^{2} F(\pi\alpha'y^I).
\]
(4.16)

The overall normalization was fixed by comparison with (4.9).

The partition function (4.16) gives the spacetime action evaluated on linear tachyon profiles. (4.10) shows that it can be written in terms of the tachyon by the substitution \(y^I \rightarrow \alpha'(\partial_\mu T^I)^2\). This result gives an expression for the action to all orders in derivatives. However, a significant ambiguity remains: any term with at least second derivatives acting on \(T\) can be added. At quadratic order in derivatives the result is unambiguous:
\[
S(T, \bar{T}) \simeq 2T_{D9} \int d^{10}x \ e^{-2\pi\alpha'T\bar{T}} \left[ 1 + 8\pi(\alpha')^2 \ln(2) \partial^\mu \bar{T} \partial_\mu T + \cdots \right].
\]
(4.17)

We used the expansion
\[
F(x) \simeq 1 + 2\ln(2)x + O(x^2), \quad x \rightarrow 0.
\]
(4.18)

Next we turn to the description of lower dimensional D-branes as solitons on the \(D9 - \bar{D9}\) system. According to the conjectures of Sen (for a review see [28]), a kink
represents a non-BPS $D8$-brane and a vortex represents a BPS $D7$-brane. There are three fixed points of the RG flow depending on whether zero, one, or both of the $y^I$ are taken to infinity, and these represent the $D9 - D9$, the non-BPS $D8$-brane, and the BPS $D7$-brane, respectively. A single nonzero $y^I$ gives a tachyon profile $T \sim x^1$, which indeed describes a kink; and two nonzero $y^I$'s gives $T \sim x^1 + i x^2$ which describes a vortex. To compute the tension of these solitons we simply evaluate (4.16) at the endpoint of the RG flow using the limiting behavior

$$F(x) \simeq \sqrt{\pi x} + O(x^{-\frac{1}{2}}), \quad x \to \infty.$$  

(4.19)

We find:

1. A non-BPS D8-branes corresponds to $y^1 = \infty$ and $y^2 = 0$. This gives

$$Z(y^1, 0) = 2T_{D9} \int d^{10} X_0 e^{-\frac{\pi}{2} y^1 (X_0^1)^2} F(\pi \alpha' y^1)$$

$$= 2T_{D9} \int d^9 X_0 \sqrt{\frac{2}{y^1}} F(\pi \alpha' y^1)$$

$$\to 2\pi \sqrt{2\alpha'} T_{D9} \int d^9 X_0 ,$$  

(4.20)

which correctly identifies the tension as $T_8 = \sqrt{2(2\pi \sqrt{\alpha'})} T_{D9} = \sqrt{2} T_{D8}$.

2. The BPS D7-brane corresponds to $y^1 = y^2 = \infty$. This gives

$$Z(y^1, y^2) = 2T_{D9} \int d^{10} X_0 e^{-\frac{\pi}{2} [y^1 (X_0^1)^2 + y^2 (X_0^2)^2]} F(\pi \alpha' y^1) F(\pi \alpha' y^2)$$

$$= 2T_{D9} \int d^9 X_0 \sqrt{\frac{2}{y^1}} F(\pi \alpha' y^1) \sqrt{\frac{2}{y^2}} F(\pi \alpha' y^2)$$

$$\to 4\pi^2 \alpha' T_{D9} \int d^9 X_0 ,$$  

(4.21)

which correctly gives the tension as $T_{D7} = (2\pi \sqrt{\alpha'})^2 T_{D9}$.

Higher codimension branes can be described similarly. The details of this generalized construction is discussed in sec 5.4.

### 4.2. Gauge fields on the $D\bar{D}$ system

We now consider simple examples with both tachyons and gauge fields on the $D\bar{D}$ system. To do so, we return to the boundary action (4.4).

Setting $A^I_\mu = 0, F_{\mu\nu} = \text{constant}, T^I = \text{constant}$ leads to the action

$$S = 2T_{D9} \int d^{10} x e^{-\frac{\alpha'}{2} T^I T^I} \mathcal{L}_{B1}(F/2)$$

$$= \int d^{10} x V(T, \bar{T}) \mathcal{L}_{B1}(F^+) ,$$  

(4.22)
where $V$ is the tachyon potential (4.9). When the two gauge fields are identical, $F^+ = F^-$, we thus find a Born-Infeld action times an overall factor equal to the tachyon potential $2g$.

Next, consider $F_{\mu\nu}^I$ and $F_{IJ}^\mu$ constant and $T^I = 0$. First integrate out $\eta^I$ using

$$\int D\eta e^{-\int d\tau \left[ \frac{1}{4} \dot{\eta}^I \eta^I + \frac{i}{2} N(\tau) e^{IJ} \eta^I \eta^J \right]} = e^{\int d\tau N(\tau)} + e^{-\int d\tau N(\tau)} .$$

(4.23)

The partition function then becomes

$$Z(A^+, A^-) = \int D\mathcal{X} D\mathcal{Y} e^{-S_{\text{bulk}}} \left[ e^{-S_{\text{bndy}}^+} + e^{-S_{\text{bndy}}^-} \right] ,$$

(4.24)

with

$$S_{\text{bndy}}^\pm = -i \int d\tau \left[ \dot{X}^\mu A_\mu^\pm + \frac{1}{2} \alpha' F^\pm_{\mu\nu} \psi^\mu \psi^\nu \right] .$$

(4.25)

Therefore, the partition function for this background is a sum of two Born-Infeld actions,

$$Z(A^+, A^-) = T_{D9} \int d^{10} x \left[ \mathcal{L}_{\text{BI}}(F^+) + \mathcal{L}_{\text{BI}}(F^-) \right] .$$

(4.26)

This is correct, since for vanishing tachyon the gauge fields on the two D-branes are decoupled from one another.

**4.3. Mixing of Gauge Fields and Tachyons**

Finally, we turn to the more nontrivial case of constant and nonzero $F_{\mu\nu}^I$, $F_{IJ}^\mu$, and $T^I$. In this case we will work out the partition function perturbatively in $A_{\mu}^I$, which corresponds to expanding in $D_{\mu} T^I$ and $F_{IJ}^\mu$. From (4.4) it follows that each such term has the tachyon dependence $e^{-\frac{\alpha'}{2} T^I T^J}$ times a polynomial in $T^I T^J$; in particular, all the terms vanish in the closed string vacuum $T^I T^J \rightarrow \infty$. Now let’s work out the explicit terms quadratic in field strengths. Integrating out $\eta^I$ at this order yields the partition function

$$Z(T, A^+, A^-) = \int D\mathcal{X} D\mathcal{Y} e^{-S_{\text{bulk}}} \left[ e^{-S_{\text{bndy}}^+} + e^{-S_{\text{bndy}}^-} \right] ,$$

(4.27)

with

$$S_{\text{bndy}}^\pm = \int d\tau \left[ \frac{\alpha'}{4} T^I T^I + \frac{(\alpha')^2}{4} \left( D_{\mu} T^I \psi^\mu \right) \frac{1}{\partial \tau} \left( D_{\nu} T^I \psi^\nu \right) - i \left( \dot{X}^\mu A_\mu^\pm + \frac{\alpha'}{2} F^\pm_{\mu\nu} \psi^\mu \psi^\nu \right) \right] .$$

(4.28)

We stress that (4.28) is only correct to order $A^2$. In terms of $A^\pm$ we have

$$D_{\mu} T^I = \partial_{\mu} T^I - (A^+_\mu - A^-_\mu) \epsilon^{IJ} T^J .$$

(4.29)
We write the background as

\[ A^\pm_\mu = -\frac{1}{2} F^\pm_{\mu\nu} X^\nu, \quad F^\pm_{\mu\nu}, T^I = \text{constant}, \quad (4.30) \]

hence

\[ D_\mu T^I = \frac{1}{2} \epsilon^{IJ} T^J (F^+_{\mu\nu} - F^-_{\mu\nu}) X^\nu. \quad (4.31) \]

At order \( A^2 \) we can separate (4.27) into two terms, \( Z = Z^{(0)} + Z^{(1)} \), corresponding to expanding (4.27) to zeroth and first order in \( (D_\mu T^I \psi^\mu) \frac{1}{\partial_\tau} (D_\nu T^I \psi^\nu) \). The zeroth order term is (4.26) and the first order term is

\[ Z^{(1)} = \frac{(\alpha')^2}{16} e^{-\frac{\pi \alpha'}{2} T^I T^J (F^+_{\mu\alpha} - F^-_{\mu\alpha}) (F^+_{\nu\beta} - F^-_{\nu\beta})} \int d\tau d\tau' \epsilon(\tau - \tau') \langle X^\alpha(\tau) X^\beta(\tau') \psi^\mu(\tau) \psi^\nu(\tau') \rangle, \quad (4.32) \]

where we used (4.7), and where

\[ \langle X^\alpha(\tau) X^\beta(\tau') \psi^\mu(\tau) \psi^\nu(\tau') \rangle = \int \mathcal{D}X \mathcal{D}\psi e^{-S_{\text{bulk}}} X^\alpha(\tau) X^\beta(0) \psi^\mu(\tau) \psi^\nu(0). \quad (4.33) \]

Separating out the \( X \) zero mode, we write

\[ \langle X^\alpha(\tau) X^\beta(\tau') \psi^\mu(\tau) \psi^\nu(\tau') \rangle = \int \mathcal{D}X \mathcal{D}\psi e^{-S_{\text{bulk}}} X^\alpha(\tau) X^\beta(0) \psi^\mu(\tau) \psi^\nu(\tau'), \quad (4.34) \]

where the correlators

\[ G^{-1}(\tau, \tau') = \frac{1}{4\pi^2 \alpha'} \sum_{n=1}^{\infty} n \cos n(\tau - \tau'), \]
\[ K^{-1}(\tau, \tau') = -\frac{1}{4\pi^2} \sum_{r=\frac{1}{2}}^{\infty} \sin r(\tau - \tau'), \quad (4.35) \]

are defined so that the bulk action (2.28) reads

\[ S_{\text{bulk}} = \int d\tau d\tau' \left[ X^\mu(\tau) G^{-1}(\tau, \tau') X_\mu(\tau') + \psi^\mu(\tau) K^{-1}(\tau, \tau') \psi^\nu(\tau') \right]. \quad (4.36) \]

Now insert (4.34) into (4.32). The contribution with an explicit dependence on the zero modes \( X_0 \) combine with the earlier result (4.17) to provide the gauge covariant tachyon kinetic term \( (D_\mu T^I)^2 \). The remainder contributes to the gauge kinetic terms as

\[ \frac{\beta}{4} (\alpha')^2 e^{-2\pi \alpha' T^I T^J (F^+_{\mu\nu} - F^-_{\mu\nu})^2}, \quad (4.37) \]
where we defined

\[ \beta = \int d\tau d\tau' \epsilon(\tau - \tau')G(\tau, \tau')K(\tau, \tau') . \]  

(4.38)

Combining this result with the order \( F^2 \) expansion of (4.26) gives our result for the partition function at this order

\[
Z(T, \overline{T}, A^+, A^-) = 2T_{D9} \int d^{10}x e^{-2\pi \alpha' T \overline{T}} \left[ 1 + 8\pi \alpha' \ln(2) D^\mu T D_\mu T \right. \\
\left. + \frac{(2\pi \alpha')^2}{8} (F^{+}_\mu)^2 + \frac{(2\pi \alpha')^2}{8} (F^{-}_\mu)^2 + \frac{\beta}{8} (\alpha')^2 T \overline{T} (F^{+}_{\mu\nu} - F^{-}_{\mu\nu})^2 \right] .
\]  

(4.39)

The actual numerical value of \( \beta \) could be computed from (4.38), after regularization and renormalization.

### 4.4. Non-BPS Branes

Before ending this section let us comment on non-BPS D-branes. They are defined as projections by \((-1)^{F_L}\) of the \( D\overline{D} \) system [26], so are included as special cases of our formalism. Results for a single non-BPS brane can be obtained from the boundary interaction (4.4) after the following substitutions:

1. The tachyon is taken to be real. So take \( T_2 = 0 \) and thus \( T = \overline{T} = \frac{1}{2} T_1 \).
2. The two gauge fields are identified, \( F^+ = F^- = F_{\text{non-BPS} D} \).
3. After the above restrictions, the fermion \( \eta^2 \) decouples. One should not perform the path integral over this fermion. This corresponds to changing the overall normalization from \( 2T_{D9} \) to \( \sqrt{2}T_{D9} \), which is the correct tension of a non-BPS D9-brane.

For example, the tachyon action (4.16) translates to the non-BPS action

\[
S(T) = \sqrt{2}T_{D9} \int d^{10}x e^{-2\pi \alpha' T^2} F(2\pi (\alpha')^2 \partial^\mu T \partial_\mu T) .
\]  

(4.40)

This is the same result found in [7], after the identification \( T^2_{\text{there}} = 8\pi \alpha' T^2_{\text{here}} \). In sec 5.5 we use the same identifications to compute the coupling to RR fields.

### 5. Couplings to RR-fields

In this section we derive the Chern-Simons couplings between unstable brane systems and background RR-fields. The computations reduce to integrals over fermion zero-modes. The results are presented in (5.34) and (5.48).
5.1. Wave Functionals Revisited

So far we have studied the action for a D-brane in the closed string NS-NS vacuum. To include closed string excitations we should compute the path integral on the disk with insertions of bulk vertex operators. By first performing the path integral over bulk fields we obtain a bulk wave functional representing the closed string background. Projecting against the boundary wave functional then yields their coupling to open strings. In this way we arrive at a prescription for coupling off-shell open strings to on-shell closed strings.

The interest in this section is to study couplings to RR-fields. As the starting point we need the bulk wave functional of the RR-vacuum. We begin by developing the wave functional formalism a little further before taking couplings to RR-fields into account.

The bulk wave functionals are the state vectors of the closed string, and operators acting on them form representations of the closed string operator algebra. An explicit construction of the bosonic operators in terms of the modes $X^\mu_n$ is

$$\alpha^\mu_n = -\frac{i}{2} X^\mu_n - i \frac{\partial}{\partial X^\mu_n}, \quad \tilde{\alpha}^\mu_n = -\frac{i}{2} X^\mu_n - i \frac{\partial}{\partial X^\mu_n}. \quad (5.1)$$

These operators indeed satisfy the standard commutation relations

$$[\alpha^\mu_n, \alpha^\nu_m] = n \delta^{\mu\nu} \delta_{n+m}, \quad [\tilde{\alpha}^\mu_n, \tilde{\alpha}^\nu_m] = n \delta^{\mu\nu} \delta_{n+m}, \quad [\alpha^\mu_n, \tilde{\alpha}^\nu_m] = 0. \quad (5.2)$$

The zero-mode operators are identical, $\alpha^\mu_0 = \tilde{\alpha}^\mu_0$, as they should be.

These considerations are independent of the specific wave functional being considered. In vacuum, the bosonic part of the wave functional is \((2.7)\)

$$\Psi^\text{bos}_{\text{bulk}} = N_{\text{bos}} \exp \left[-\frac{1}{2} \sum_{n=1}^\infty n X^{-n}_n X^\mu_n \right]. \quad (5.3)$$

In this particular state

$$\alpha^\mu_n \Psi^\text{bos}_{\text{bulk}} = \tilde{\alpha}^\mu_n \Psi^\text{bos}_{\text{bulk}} = 0, \quad (5.4)$$

for $n \geq 0$. This confirms that the state is the bosonic vacuum.

The next step is to include fermions. In the NS sector the fermion field $\psi$ has modes $\psi_r, r \in \mathbb{Z} + \frac{1}{2}$. Functionals of such fields are acted on by the closed string fermion operators

$$\beta^\mu_r = \frac{1}{\sqrt{2i}} (\psi^\mu_r + i \frac{\partial}{\partial \psi^\mu_r}), \quad \tilde{\beta}^\mu_r = \sqrt{\frac{i}{2}} (\psi_r - i \frac{\partial}{\partial \psi^{-\mu}_r}), \quad (5.5)$$
satisfying the standard anti-commutation relations
\[ \{ \beta^\mu_r, \beta^\nu_s \} = \delta^{\mu\nu} \delta_{r+s} , \quad \{ \tilde{\beta}^\mu_r, \tilde{\beta}^\nu_s \} = \delta^{\mu\nu} \delta_{r+s} , \quad \{ \beta^\mu_r, \tilde{\beta}^\nu_s \} = 0 . \tag{5.6} \]

Again, consider as a definite example the vacuum wave functional
\[ \Psi^{NS-NS}_{\text{bulk}} = \mathcal{N}_{NS-NS} \exp \left[ -\frac{1}{2} \sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_{n} - i \sum_{r=\frac{1}{2}}^{\infty} \psi^\mu_{-r} \psi^\mu_{r} \right] . \tag{5.7} \]

In this state
\[ \beta^\mu_r \Psi^{NS-NS}_{\text{bulk}} = \tilde{\beta}^\mu_r \Psi^{NS-NS}_{\text{bulk}} = 0 , \tag{5.8} \]
for \( r > 0 \) and (5.4) remains satisfied. This confirms that the state is the NS-NS vacuum.

Our convention for fermionic derivatives is that they act from the left.

We are now ready to determine the RR vacuum. The fermions now have integer modes \( \psi^\mu_n , n \in \mathbb{Z} \) and realize the closed string algebra
\[ \{ \beta^\mu_n, \beta^\nu_m \} = \delta^{\mu\nu} \delta_{n+m} , \quad \{ \tilde{\beta}^\mu_n, \tilde{\beta}^\nu_m \} = \delta^{\mu\nu} \delta_{n+m} , \quad \{ \beta^\mu_n, \tilde{\beta}^\nu_m \} = 0 , \tag{5.9} \]
through
\[ \beta^\mu_n = \frac{1}{\sqrt{2i}} (\psi^\mu_n + i \frac{\partial}{\partial \psi^\mu_n}) , \quad \tilde{\beta}^\mu_n = \frac{\sqrt{i}}{2} (\psi^\mu_n - i \frac{\partial}{\partial \psi^\mu_n}) . \tag{5.10} \]

A fundamental aspect of the RR-sector is the role played by the algebra of fermion zero-modes. Spinorial representations of the Lorentz group are realized by the identification \( \Gamma^\mu = \sqrt{2} \beta^\mu_0 \) and similarly for the right movers, \( \tilde{\Gamma}^\mu = \sqrt{2} \tilde{\beta}^\mu_0 \). These operators act on wave functionals of the zero-modes \( \psi^\mu_0 \). In the RR-vacuum the wave functional is thus
\[ \Psi^{RR}_{\text{bulk}} = \mathcal{N}_{RR} \exp \left[ -\frac{1}{2} \sum_{n=1}^{\infty} n X^\mu_{-n} X^\mu_{n} - i \sum_{n=1}^{\infty} \psi^\mu_{-n} \psi^\mu_{n} \right] \sum_{p \text{ odd}} \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_0 \cdots \mu_p} \psi^{\mu_0}_0 \cdots \psi^{\mu_p}_0 , \tag{5.11} \]
where the \( C_{\mu_0 \cdots \mu_p} \) are numerical coefficients that will be identified with RR-potentials momentarily. We chose for definiteness the type IIB GSO projection, acting on the zero-mode vacua by an even number of \( \Gamma^\mu \) as well as an even number of \( \tilde{\Gamma}^\mu \). In type IIA the result is the same except that \( p \) is restricted to even integers.

It is instructive to compare these considerations with standard worldsheet technology. By the state-operator correspondence we can associate each wave functional on the boundary of the disk to a vertex operator inserted at the center of the disk. The RR-vacuum wave functional (5.11) corresponds to an insertion of
\[ \nu^{RR}_{\frac{-\frac{9}{2}}{2}} = S^a C_{ab} \tilde{S}^b e^{-\frac{1}{2} \phi(0)} e^{-\frac{2}{3} \phi(0)} . \tag{5.12} \]
The RR-potential is written here in a spinorial form with the component expansion²

\[ C_{ab} = \sum_{p \text{ odd}} \frac{1}{(p+1)!} C_{\mu_0 \cdots \mu_p} (\Gamma_{\mu_0 \cdots \mu_p})_{ab}. \] (5.13)

After the vertex insertion the fermion field \( \psi \) becomes integer moded. Additionally, there is an overall factor related to the RR-potential. The conclusion is that the coefficients \( C_{\mu_0 \cdots \mu_p} \) in (5.11) can be identified with the RR-potential. In the construction of (5.11) normalizations were determined using the correspondence \( \Gamma^\mu \sim \sqrt{2} \beta_0^\mu \sim i^{1/2} \psi_0^\mu \) when acting on the zero-mode vacuum (and similarly for right movers). The coefficients \( C_{\mu_0 \cdots \mu_p} \) in (5.11) are therefore correctly normalized. In other words, the overall factor \( N_{RR} \) is independent of \( p \). It is possible to determine the numerical value of \( N_{RR} \) explicitly by computations familiar from the boundary state formalism (see e.g. [30]). In the following it will be fixed by requiring the correct D9-brane charge.

The operator (5.12) is written in the \((-\frac{1}{2}, -\frac{3}{2})\) picture. The total number of superconformal ghosts is thus \(-2\), saturating the superconformal Killing symmetries on the disc. As usual, on the disc the ordinary ghosts are taken care of by fixing the bulk vertex operator at the origin. This leaves one CKV, corresponding to the azimuthal symmetry of the disc boundary. In principle this means one boundary operator must be fixed, but integrating instead over all boundary operators, as we will find convenient, simply results in an overall numerical factor that can be ignored. The partition sums computed here are thus interpretable as generating functionals of string amplitudes. In the previous sections ghosts and superghosts were simply ignored, as usual in sigma-model constructions. That is also correct, because it amounts to ignoring an overall volume of the super-Möbius group, which is in fact finite [11]. In the RR-vacuum considered in this section the ghosts must be considered; fortunately, we see that they introduce no significant complications.

5.2. RR-couplings of the D-brane

We now have a wavefunctional representation of the RR-vacuum. The next step is to project the result (5.11) onto a boundary wave functional. As an example we begin by considering a single BPS D-brane, including its world-volume gauge field. The boundary action is simply (2.30) and the mode expansions are (2.4) for the bosons and

\[ \psi^\mu = \sum_{n=-\infty}^{\infty} \psi_\mu^n e^{i n \tau} \] (5.14)

² This is for type IIB. For type IIA the corresponding spinors have opposite chirality; the potential is \( C_{ab} \).
for the periodic fermions. For constant field strength the boundary wave functional is therefore

$$\Psi_{\text{bndy}}^{BPS} = e^{-S_{\text{bndy}}} = \exp \left[ -2\pi\alpha' F_{\mu\nu} \left( \sum_{n=1}^{\infty} \left( \frac{1}{2} nX_{-n}X_{n} - i\psi_{-n}^{\mu} \psi_{n}^{\nu} - \frac{i}{2} \psi_{0}^{\mu} \psi_{0}^{\nu} \right) \right) \right]. \tag{5.15}$$

Except for the integer moding, the derivation of this result is identical to the NS-NS sector computation leading to the Born-Infeld action.

The projection of the bulk wave functional (5.11) onto (5.15) proceeds by integration over all field components. The integrals over non-zero modes are trivial in the present situation, the contributions from bosons and the fermions cancelling by supersymmetry. We are thus left with the zero-mode integrals

$$Z_{\text{RR}}^{BPS} = T_{D9} \int D\psi_{0} e^{2\pi\alpha' F_{\mu\nu} \psi_{0}^{\mu} \psi_{0}^{\nu}} \sum_{p \text{ odd}} \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_{0}...\mu_{p}} \psi_{0}^{\mu_{0}} \cdots \psi_{0}^{\mu_{p}}. \tag{5.16}$$

The physically significant combination of the normalization $N_{\text{RR}}$ and various measure factors was determined by comparison with the known result for a single $D9$-brane without a gauge field. The bosonic zero-mode integral gives an overall volume integral and the remaining fermionic zero-mode integral is readily evaluated with the result

$$Z_{\text{RR}}^{BPS} = T_{D9} \int C \wedge e^{2\pi\alpha' iF}, \tag{5.17}$$

in the familiar representation as a formal sum of differential forms, i.e

$$C = \sum \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_{0}...\mu_{p}} dx_{0}^{\mu_{0}} \wedge \cdots \wedge dx_{0}^{\mu_{p}}. \tag{5.18}$$

This is the correct result, including coefficients. It was previously derived using the boundary state formalism [31], a close relative to the present set-up, and independently by anomaly inflow [32].

Suppose we allow $F_{\mu\nu}$ to be nonconstant and try to derive a generalization of (5.17). The new feature is that in (5.15) we replace $F_{\mu\nu}$ by $F_{\mu\nu}(X)$. Bosons and fermions no longer cancel (because a generic zero-mode background $(X_{0}, \psi_{0})$ breaks supersymmetry) and the integrals are non-Gaussian. Nevertheless there is a precise sense in which the formula (5.17) remains correct, but we defer discussion of this point to section 5.6.
5.3. RR-couplings of the $\mathcal{D}\overline{\mathcal{D}}$-system

We now compute the RR-couplings of the $\mathcal{D}\overline{\mathcal{D}}$ system. The wave functional describing the bulk by definition does not depend on the boundary interactions so it is still \((5.11)\). The boundary action describing the $\mathcal{D}\overline{\mathcal{D}}$ system is given in \((3.19)\):

$$S_{\text{bndy}} = -\int d\tau \left[ \frac{1}{4} \dot{\eta}^{I} \eta^{I} + \sum_{k=0}^{2m} \frac{1}{2k!} (M_{1} - M_{0}^{2})^{I_{1}}...I_{k} \eta^{I_{1}}...\eta^{I_{k}} \right]. \quad (5.19)$$

Our task is to compute

$$Z_{\mathcal{D}\overline{\mathcal{D}}}^{RR} = N_{RR} \int \mathcal{D}X \mathcal{D}\psi \mathcal{D}\eta e^{-S_{\text{bndy}}} \Psi_{\text{bulk}}^{RR}, \quad (5.20)$$

with fields obeying periodic boundary conditions.

What we now wish to argue is that it is justified to set all nonzero modes of $X^{\mu}$ and $\psi^{\mu}$ to zero. To proceed, allow a general $\tau$ periodicity, $\tau \sim \tau + \beta$, and write \((5.20)\) in the canonical formalism as

$$Z_{\mathcal{D}\overline{\mathcal{D}}}^{RR} = \text{Tr}(-)^{F} e^{-\beta H}, \quad (5.21)$$

for some supersymmetric Hamiltonian $H$. \((5.21)\) is a Witten index \([33]\). Because states of nonzero energy cancel between bosons and fermions, $Z_{\mathcal{D}\overline{\mathcal{D}}}^{RR}$ is independent of $\beta$, and is also constant with respect to smooth deformations of $H$ which preserve supersymmetry. Using our freedom to smoothly deform the theory, we will introduce into \((5.20)\) a conventional looking 0 + 1 dimensional kinetic term,

$$Z_{\mathcal{D}\overline{\mathcal{D}}}^{RR} = N_{RR} \int \mathcal{D}X \mathcal{D}\psi \mathcal{D}\eta e^{-S_{0} - S_{\text{bndy}}} \Psi_{\text{bulk}}^{RR}, \quad (5.22)$$

with

$$S_{0} = \frac{1}{4} \int d\tau \left( (\dot{X}^{\mu})^{2} + \dot{\psi}^{\mu} \psi^{\mu} \right) \sim \sum_{n=1}^{\infty} \left( \frac{n^{2}}{\beta} X_{-n}^{\mu} X_{n}^{\mu} + in\psi_{-n}^{\mu} \psi_{n}^{\mu} \right). \quad (5.23)$$

Following a standard line of attack \([34]\), we consider the path integral in the limit $\beta \to 0$. To avoid introducing spurious $\beta$ dependence, the spacetime fields should be rescaled as one takes the limit. Considering the path integral for constant fields (in which case the path integral is Gaussian) and demanding $\beta$ independence, one finds the rescalings: $T^{I} \to \beta^{-\frac{1}{2}} T^{I}$, $C_{\mu_{0}...\mu_{p}} \to \beta^{\frac{p+1}{2}} C_{\mu_{0}...\mu_{p}}$. Now, think of $S_{0}$ as supplying the propagators of the theory, and the remainder as interaction terms. Performing the $X^{\mu}$ and $\psi^{\mu}$ nonzero mode path integrals with no interaction insertions gives $\beta^{-\frac{d}{2}}$, where $d$ is the spacetime
dimension. Next, note that we need to saturate \(d\) zero modes integrals of \(\psi^\mu_0\). From the form of the action and from the rescaling of \(C_{\mu_0\ldots\mu_p}\) one sees that each fermion zero mode is accompanied by a factor of \(\beta^{\frac{d}{2}}\), so saturating the fermion zero modes precisely cancels the earlier factor of \(\beta^{-\frac{d}{2}}\). Now it is easy to see that \(X^\mu\) and \(\psi^\mu\) nonzero modes can be dropped from the interaction terms, since any of the associated diagrams carry positive powers of \(\beta\). So we have arrived at the desired result: in computing (5.20) it is valid to set all nonzero modes of \(X^\mu\) and \(\psi^\mu\) to zero.

We have now reduced the computation to

\[
Z_{RR} = N_{RR} \int D X_0 D\psi_0 D\eta e^{-S_{\text{bndy}}} \sum_{p \text{ odd}} \frac{(-i)^{\frac{p+\mu}{2}}}{(p+1)!} C_{\mu_0\ldots\mu_p} \psi_0^{\mu_0} \cdots \psi_0^{\mu_p},
\]

(5.24)

with \(S_{\text{bndy}}\) given by restricting (5.19) to zero-modes of \(X^\mu\) and \(\psi^\mu\). Recalling the definition of \(M\) in (3.13) we find that its components, \(M = M_0 + \theta M_1\), are represented by the matrices

\[
M_0 = \sqrt{\alpha'} \begin{pmatrix} iA^+_\mu \psi_0^\mu & T \\ T & iA^-_\mu \psi_0^\mu \end{pmatrix} \equiv i\sqrt{\alpha'} A,
\]

(5.25)

\[
M_1 = \alpha' \begin{pmatrix} \frac{i}{2} (\partial_\mu A^+_\nu - \partial_\nu A^+_\mu) \psi_0^\mu \psi_0^\nu & \partial_\mu T \psi_0^\mu \\ \partial_\mu T \psi_0^\mu & \frac{i}{2} (\partial_\mu A^-_\nu - \partial_\nu A^-_\mu) \psi_0^\mu \psi_0^\nu \end{pmatrix} \equiv i\alpha' dA.
\]

(5.26)

The matrix \(A\) is the superconnection \([18,19]\). We will adopt the notation of differential forms; e.g. for a \(k\)-form,

\[
B^{(k)} = \frac{1}{k!} B_{\mu_1\ldots\mu_k} \psi_0^{\mu_1} \cdots \psi_0^{\mu_k}.
\]

(5.27)

Similarly, in (5.26) \(d\) denotes the exterior derivative. Now note that the combination appearing in (5.19) is

\[
M_1 - M_0^2 = i\alpha' (dA - iA \wedge A) \equiv i\alpha' F.
\]

(5.28)

\(F\) is the curvature of the superconnection, given explicitly by

\[
iF = \begin{pmatrix} iF^+ - \frac{T T}{DT} \\ \frac{DT}{iF^+} \end{pmatrix}.
\]

(5.29)

Here \(F^\pm\) are the full non-abelian field strengths,

\[
F^\pm = dA^\pm - iA^\pm \wedge A^\pm,
\]

(5.30)
and the covariant derivatives are

\[ DT = dT + iTA^+ - iA^-T, \]
\[ D\overline{T} = d\overline{T} - iA^+\overline{T} + i\overline{T}A^- . \]  (5.31)

Here \( TA^+ \) and \( \overline{T}A^- \) appear with an extra minus sign because, as emphasized after (3.19), we must multiply matrices using (3.11). Now, upon performing the \( \eta \) path integral, (5.24) becomes

\[ Z_{\overline{D}D} = \mathcal{N}_{RR} \int D\psi_0 \mathcal{D}X_0 \text{Str} 2\pi i\alpha'F \sum_{p \text{ odd}} \frac{(-i)^{\frac{p}{2}}}{(p + 1)!} C_{\mu_0 \cdots \mu_p} \psi_0^{\mu_0} \cdots \psi_0^{\mu_p} . \]  (5.32)

The supertrace arises because \( \eta' \) are periodic, and is defined by

\[ \text{Str} M = \text{Tr}(-)^F M = \text{Tr} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M. \]  (5.33)

The remaining step is to do the integral over fermion zero-modes, which simply picks out the 10-form part of the integrand. Hence our final result is

\[ Z_{\overline{D}D} = T_{D9} \int C \wedge \text{Str} 2\pi i\alpha'F , \]  (5.34)

where factors of \( i \) are included in the definition of \( C \), as in (5.18). We fixed the overall normalization from the BPS computation.

The RR-couplings of the \( D\overline{D} \)-system were conjectured to be of the form (5.34) by Kennedy and Wilkins [20]. The evidence for the conjecture came from an S-matrix computation of the \( d\overline{T}dT \) term. Such an approach of course carries with it some ambiguity when one attempts to write down an action which is valid off-shell. It is a welcome surprise that the full formula can be derived unambiguously rather simply from open string field theory. A feature of the computation worth repeating is that we did not limit ourselves to a linear tachyon profile nor to constant gauge field strength. This was possible because it was sufficient to consider the \( X^\mu \) and \( \psi^\mu \) zero-modes. This in turn is closely connected to the fact that what we are computing is the index of a certain operator, as we discuss in section 5.6.

The curvature of the superconnection satisfies some important properties which make it suitable for appearing in the Chern-Simons term. In particular we note the Bianchi identity,

\[ \mathcal{D}\mathcal{F} = d\mathcal{F} - i\mathcal{A} \wedge \mathcal{F} + i\mathcal{F} \wedge \mathcal{A} = 0 , \]  (5.35)
and the transgression formula
\[ \text{Str} F^{n+1} = d\omega_{2n+1}(A, F) . \] (5.36)

As examples we consider some special cases of the RR-couplings. If the tachyon vanishes the couplings become
\[ Z^{DD}_{RR}(T = 0) = T_{D9} \int C \wedge (e^{2\pi\alpha'F^+} - e^{2\pi\alpha'F^-}) . \] (5.37)
This is clearly recognized as the RR-couplings (5.17) of two BPS D-branes, one of each sign.

Next, take vanishing gauge fields and consider linear tachyon fields \( \sqrt{\alpha'} T_{1,2} \). In section 4.1 we identified this configuration with a BPS D7-brane and we would like to check that is has the correct charge. After expansion the partition sum becomes
\[ Z^{DD}_{RR} = T_{D9} \int C \wedge e^{-2\pi\alpha' T T} (2\pi\alpha')^2 dT \wedge dT = T_{D7} \int C_8 , \] (5.38)
where \( T_{D7} = (2\pi\sqrt{\alpha'})^2 T_{D9} \) is the correct tension of a D7-brane. Note that the result is independent of the parameters \( u_{1,2} \); in particular, the result is valid before taking the fixed point limit \( u_{1,2} \to \infty \). This is a reflection of the fact that the D-brane charge is a topological invariant.

5.4. Example: the ABS Construction

Consider the ABS construction of a BPS \( D(9-2m) \)-brane on \( 2^{m-1} D9 - D9 \) pairs. The gauge fields vanish and the tachyons are
\[ \sqrt{\alpha'} \begin{pmatrix} 0 \\ T \\ 0 \end{pmatrix} = u \gamma^i x^i , \] (5.39)
where \( i = 1, \ldots, 2m \) is a vector index over the directions transverse to the \( D(9-2m) \)-brane. The gamma matrices \( \gamma^i \) represent the Clifford algebra of the \( SO(2m) \) transverse rotation group as \( 2^m \times 2^m \) matrices. They can be chosen in the form
\[ \gamma_{i=1,\ldots,2m-1} = \begin{pmatrix} 0 & \tilde{\gamma}^i \\ \tilde{\gamma}^i & 0 \end{pmatrix}, \quad \gamma^{2m} = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix} , \] (5.40)
where the \( \tilde{\gamma}^i \) represent the \( SO(2m-1) \) Clifford algebra. The chirality matrix \( \gamma^5 \) is
\[ \gamma^{2m+1} = \gamma^1 \cdots \gamma^{2m} = i^m \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} . \] (5.41)
The nonvanishing RR-couplings become
\[
S = T_{D9} \int C \wedge \left( \frac{1}{(2m)!} e^{-2\pi u^2 x^2} \text{Str} \left( 2\pi \sqrt{\alpha'} u \gamma^i dx^i \right)^{2m} \right)
\]
\[
= T_{D(9-2m)} \int C_{10-2m}(u \int e^{-2\pi u^2 x^2} dx) 2m_i m^{i-m} \text{Str} \gamma^{2m+1} = T_{D(9-2m)} \int C_{10-2m}
\]
(5.42)
corresponding to a single $D(9-2m)$ brane, as expected. For $m = 1$ the ABS construction reduces to the vortex solution considered around (5.38).

5.5. Non-BPS Branes

The non-BPS D-branes are defined as projections by $(-1)^{F_L}$ of the $D\overline{D}$ system. Their RR-couplings are computed as above, with the following substitutions:

(1) The tachyon is Hermitian, $T = \overline{T}$.

(2) The two gauge fields are identified, $A^+ = A^- = A$.

(3) After the above restrictions, the fermion $\eta^{2m}$ decouples. One should not perform the path integral over this fermion.

(4) The type IIA forms are defined as
\[
C = \sum_{p \text{ even}} \frac{(-i)^{\frac{s-p}{2}}}{(p+1)!} C_{\mu_0 \ldots \mu_p} dx^\mu_0 \wedge \ldots \wedge dx^\mu_p.
\]
(5.43)

Repeating the same steps as in the $D\overline{D}$ case, we find the coupling
\[
S = \frac{T_{D9}}{\sqrt{2}} \int C \wedge \text{Str} \exp \left[ 2\pi \alpha' \begin{pmatrix} iF & T^2 \\ DT & iF - T^2 \end{pmatrix} \right]
\]
(5.44)
with the convention that the super-trace operation is defined with $\sigma_1$-type weight, i.e.
\[
\text{Str} M = \text{tr} M \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},
\]
(5.45)
and the covariant derivative of the tachyon is now
\[
D_\mu T = \partial_\mu T - i[A_\mu, T].
\]
(5.46)

It is clear that the result can be rewritten in terms of the curvature of a superconnection of the form (5.26), with the identifications detailed above.

In these considerations the origin of the non-BPS D-brane as a projection of $D\overline{D}$ is emphasized. When interpreted as an object in its own right it is awkward to write the
coupling of \( N = 2^m \) branes in terms of \( 2N \times 2N \) matrices. An alternative representation is obtained by writing the gamma matrices as

\[
\gamma_{i=1,\ldots,2m-1} = \begin{pmatrix} \tilde{\gamma}_i & 0 \\ 0 & \tilde{\gamma}_i \end{pmatrix}.
\] (5.47)

Then the result becomes

\[
S = \sqrt{2T_{D9}} \int C \wedge \text{tr} \, e^{2\pi\alpha'(iF + T^2 + DT)} ,
\] (5.48)

where the trace is an ordinary trace over group indices. Only odd forms are retained in the expansion of the exponential because \( C \) contains only odd forms in the type IIA theory. The linear tachyon terms in (5.48) were discussed in [36,37,7].

We end this section with a simple example, the \( D8 \)-brane as a domain wall on \( D9-D9 \). Taking the tachyon \( \sqrt{\alpha'} T = ux \) and integrating over the linear profile we find

\[
S = \sqrt{2T_{D9}} \int C_9 \wedge (e^{-2\pi\alpha'T^2} 2\pi\alpha'dT) = T_{D8} \int C_9 ,
\] (5.49)

where \( T_{D8} = 2\pi\sqrt{\alpha'T_{D9}} \). The example verifies that the overall factors of \( \sqrt{2} \) were incorporated correctly.

5.6. **RR-charge as an Index**

Our computation of the RR-couplings boiled down to a path integral in supersymmetric quantum mechanics with periodic boundary conditions. It is a famous result [34] that various index theorems can be derived in precisely this way. In particular

\[
Z = \int \mathcal{D}\Phi \, e^{-S} = \text{Tr} \, (-)^F \, e^{-\beta H} = \text{ind}(Q) ,
\] (5.50)

where the index of the supercharge \( Q \) counts the number of bosonic zero eigenvalues minus the number of fermionic zero eigenvalues. To prove an index theorem, on the one hand one writes \( Q \) as an operator in the canonical formalism, and on the other hand evaluates the path integral expression for \( Z \) in the \( \beta \rightarrow 0 \) limit.

In section 5.3 we essentially computed the path integral in (5.50). The result was proportional to an infinite volume integral over \( C^{(p+1)} \) which is factored out in the index computation. The remainder was the transverse integral over the generalized Chern character \( \exp(2\pi\alpha'iF) \). Previously we didn’t pay close attention to the normalization of the zero-mode integral since this was absorbed into \( Z^{\mathcal{D}\mathcal{D}}_{RR} \); it is

\[
\left( \frac{-i}{4\pi^2\alpha'} \right)^n \int \mathcal{D}X_0 \mathcal{D}\psi_0 ,
\] (5.51)
where we integrate over \(2n = 9 - p\) transverse dimensions, \(\mu = p + 1 \cdots 9\).

All that remains is to determine the operator form of the supercharge \(Q\). The boundary action \(S_0 + S_{\text{bndy}}\) in (5.22) has the detailed form

\[
S = \frac{1}{4\alpha'} \int d\tau d\theta D X^\mu D^2 X_\mu - \int d\tau \left[ \frac{1}{4} \dot{\eta}^I \dot{\eta}^I + \sum_{k=0}^{2m} \frac{1}{2k!} (M_1^I - M_0^I) \eta^I \cdots \eta^I \right],
\]

with \(\mu\) running over the transverse directions. It is invariant under the supersymmetry transformations:

\[
\delta X^\mu = \epsilon \sqrt{\alpha'} \psi^\mu, \\
\delta \psi^\mu = \epsilon \frac{1}{\sqrt{\alpha'}} \dot{X}^\mu, \\
\delta \eta^I = \epsilon F^I,
\]

where \(F^I\) is expressed in terms of the \(\eta^I\) as in (3.17). Canonically quantizing (5.52) we find the commutations relations

\[
[X^\mu, P_\nu] = i\delta^\mu_\nu, \\
\{\psi^\mu, \psi^\nu\} = -2\delta^\mu\nu, \\
\{\eta^I, \eta^J\} = 2\delta^{IJ},
\]

with the (Euclidean) momentum \(P_\mu\),

\[
-iP_\mu = \frac{1}{2\alpha'} \dot{X}_\mu - i \sum_{k \text{ even}} \frac{1}{2k!} A^I_{\mu} \eta^I \cdots \eta^I.
\]

Now we want to show that the supercharge is

\[
Q = i\sqrt{\alpha'} \psi^\mu P_\mu - \sum_{k \text{ even}} \frac{1}{2k!} M_0^I \eta^I \cdots \eta^I = i\sqrt{\alpha'} \psi^\mu P_\mu - iA.
\]

To show this we need to show that commuting fields with \(\epsilon Q\) reproduces the variations in (5.53). The first and third variations are easily derived. For the second recall that

\[
\sum_{k \text{ even}} \frac{1}{2k!} M_0^I \eta^I \cdots \eta^I = i\sqrt{\alpha'} \sum_{k \text{ even}} \frac{1}{2k!} \psi^\mu A^I_{\mu} \eta^I \cdots \eta^I
\]

\[+ \sqrt{\alpha'} \sum_{k \text{ odd}} \frac{1}{2k!} T^I \cdots \eta^I \cdots \eta^I.
\]

Then we see that

\[
[\epsilon Q, \psi^\mu] = -2i\epsilon \sqrt{\alpha'} P_\mu + 2i\epsilon \frac{\sqrt{\alpha'}}{\sqrt{\alpha'}} \sum_{k \text{ even}} \frac{1}{2k!} A^I_{\mu} \eta^I \cdots \eta^I = \epsilon \frac{1}{\sqrt{\alpha'}} \dot{X}^\mu,
\]

\[
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\]
as desired. To write $Q$ as a differential operator we use

$$P_\mu \rightarrow - i \partial_\mu, \quad \psi^\mu \rightarrow i \gamma^\mu, \quad \eta^I \rightarrow \gamma^I,$$  

(5.59)

where $\gamma^\mu$ are spacetime gamma matrices, and $\gamma^I$ are “internal” gamma matrices. Therefore, we can write $Q$ as

$$Q = \begin{pmatrix} i \bar{\theta} + A^+ & \bar{T} \\ T & i \bar{\theta} + A^- \end{pmatrix}.\quad (5.60)$$

The index of $Q$ counts zero eigenvalues weighted by $(-)^F$. In contrast with (5.33) the $(-)^F$ here anti-commutes with all fermions and is given by $(-)^F = \prod_\mu \psi^\mu \prod_I \eta^I$. Choosing gamma matrices of the form (5.40) and using the rules (5.59) we have

$$(-)^F = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.\quad (5.61)$$

We finally combine the ingredients. (5.50) was evaluated first as a path integral with normalization (5.51), and then as the index of the expression (5.60) for $Q$. We arrive at the index theorem

$$\text{ind}\left( \begin{array}{cc} i \bar{\theta} + A^+ \\ T \\ i \bar{\theta} + A^- \end{array} \right) = \left( \frac{-i}{4\pi^2 \alpha'} \right)^n \int \text{Str} \ e^{2\pi i \alpha' F}.\quad (5.62)$$

Applying this result to tachyon condensation, we find that the net D-brane charge of a solitonic configuration is equal to the index written in (5.62).

Finally, we comment on the nature of corrections to Chern-Simons couplings. We nowhere assumed constant field strengths, so our formula $\int C \wedge \text{Str} \ e^{2\pi i \alpha' F}$ incorporates the contribution of derivatives of open string fields. However, it is important to emphasize that this holds only for the integrated value of the coupling. In other words, upon explicitly computing derivative corrections one would arrive at a coupling (see for instance [38])

$$\int C \wedge \left\{ \text{Str} \ e^{2\pi i \alpha' F} + dV \right\}.\quad (5.63)$$

This correction does not contribute to D-brane charges, nor to open string correlation functions as long as $C$ is constant. So our results are exact for constant $C$ in this sense. On the other hand, if one were to make $C$ nonconstant then the correction would have physical consequences.
6. Discussion

We end this paper with a brief discussion of field redefinitions. This issue is important if one wants to compare our results with those obtained using different methods. There are now three approaches to constructing D-branes as solitons in string field theory: BSFT \cite{7}, level truncation \cite{39}, and noncommutative geometry \cite{40,41} — all presumably related by field redefinitions.

For example, the tachyon potential has been computed in level truncated superstring field theory with the leading order result \cite{42}

\[
V(T, \overline{T}) = 2TD_9 \frac{1}{2}(\alpha' T \overline{T} - 1)^2 + \cdots .
\] (6.1)

Several of the corrections to this potential are known \cite{43} and it is believed that the result to all orders is qualitatively similar. An obvious difference is that in (6.1) the minima lie at finite values of $T$, whereas the minima in BSFT are at $|T| = \infty$. A more serious discrepancy is that level truncation violates gauge invariance since the gauge transformations act on the entire string field, in contrast to BSFT and the effective field theories employed in the noncommutative approach. So it is clear that the field redefinition relating the theories will have to involve all components of the string field \cite{44}.

The BSFT and noncommutative geometry approaches also realize gauge invariance differently from each other, being related by some generalization of the Seiberg-Witten map \cite{45}. In BSFT the soliton solutions have vanishing gauge fields so the RR-charge of solitons arises entirely from the tachyon in this setup. On the other hand, a crucial element of soliton solutions in the noncommutative approach at finite $B$ is the presence of a gauge field which sets all gauge covariant derivatives to zero \cite{46}. In this approach the gauge field strength contributes to RR-charge. Apparently, the map between BSFT and noncomutative variables can take a solution with vanishing gauge fields to one with non-vanishing gauge fields. Formulating BSFT with a background $B$-field in terms of noncommutative variables is a step in finding this field redefinition \cite{47}.

The last issue we would like to address concerns tachyon derivative corrections to the action \cite{29}

\[
S = \int V(T, \overline{T}) \sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})} .
\] (6.2)

It has been proposed that these can be accounted for by the substitution $F_{\mu\nu} \rightarrow F_{\mu\nu} + \partial_{\mu} T \partial_{\nu} T$ \cite{48}. This simple prescription does not occur in BSFT, but to settle the issue one must determine whether a field redefinition exists that factorizes the action this way.
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