Experimental Observation of Higher-Order Topological Anderson Insulators

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Recently, a new family of symmetry-protected higher-order topological insulators has been proposed, and was shown to host lower-dimensional boundary states. However, with the existence of the strong disorder in the bulk, the crystal symmetry is broken, and the associated corner states are disappeared. It is well known that the emergence of robust edge states and quantized transport can be induced by adding sufficient disorders into a topologically trivial insulator, that is the so-called topological Anderson insulator. The question is whether disorders can also cause the higher-order topological phase. This is not known so far, because the interaction between disorders and the higher-order topological phases is completely different from that with the first-order topological system. Here, we demonstrate theoretically that the disorder-induced higher-order topological corner state can appear in a modified Haldane model. In experiments, we construct the classical analog of such higher-order topological Anderson insulators using electric circuits, and observe the disorder-induced corner state through the voltage measurement. Our work defies the conventional view that the disorder is detrimental to the higher-order topological phase, and offers a feasible platform to investigate the interaction between disorders and the higher-order topological phases.
Topological physics has become one of the most fascinating reach areas in recent years\textsuperscript{1-6}. Based on the bulk-boundary correspondence principle, the conventional topological phase is always featured by the boundary states with one-dimensional lower than the bulk that hosts them, e.g., the robust one-dimensional edge state exists in the two-dimensional quantum Hall system. Recently, a novel class of the symmetry-protected higher-order topological insulator that possesses lower-dimensional boundary states have been proposed\textsuperscript{7-10} and realized in many systems, including solid materials\textsuperscript{11,12}, mechanics\textsuperscript{13}, acoustics\textsuperscript{14,15}, microwaves\textsuperscript{16}, photonics\textsuperscript{17-19}, and electrical circuits\textsuperscript{20, 21}. The key property of both first-order and higher-order topological insulators is their robustness of boundary states against weak disorders in the bulk. However, under the strong disorder, the non-trivial topological-bandgap gets closed, and the system goes into the trivial phase associated with the Anderson localization\textsuperscript{22}.

Contrary to this conventional wisdom, many previous theoretical results indicate that the emergence of protected edge states and quantized transport can be induced, rather than inhibited, by adding sufficiently strong disorder to the bulk of a topologically trivial insulator\textsuperscript{23-28}. These disorder-induced topological phases, called topological Anderson insulators, have been experimentally demonstrated in one-dimensional (1D) synthetic atomic wires\textsuperscript{29}, 1D sonic crystals\textsuperscript{30}, and 2D helical waveguides\textsuperscript{31}. So far, the study of topological Anderson insulators is all focused on the first-order topological phase, while, the in-depth investigation on the disorder-induced higher-order topological insulators remains lacking. A direct idea is extending the method used for the first-order topological Anderson insulator to the higher-order cases. However, compared to the first-order topological phase, the functionality of the disorder is drastically altered when it interacts with the higher-order topological system. For example, it has been pointed out that the on-site disorder, which is able to transform the trivial system to the first-order topological insulator, is detrimental to the higher-order topology\textsuperscript{32}. Hence, the current method for realizing the first-order topological Anderson insulators cannot be directly used to fulfill the higher-order topological Anderson insulator (HOTAI). Thus, it is straightforward to ask whether the HOTAI exists, and how to realize it in experiments? These open questions are yet to be clarified.

Here, we provide a solution to realize the HOTAI. Our scheme is based on a modified Haldane model with different values of the intra-cell and inter-cell couplings. It is found that
the topological phase transition from the anomalous quantum Hall phase to the second-order topological crystalline insulator appears when the sufficiently strong disorder of next-nearest-neighbor couplings is added into the bulk. Furthermore, the classical analog of the proposed HOTAI is realized experimentally using the electric circuit. The disorder-induced zero-dimensional (0D) corner state in the 2D circuit, which is verified through the circuit simulation and voltage measurement, manifests the appearance of HOTAI s. Our work offers a feasible platform to investigate the relationship between the higher-order topological phases and disorders, giving rise to the possibility for catching energy at a corner through the import of disorders.

The lattice model for the higher-order topological Anderson insulator. We start with considering the modified Haldane model with different values of intra-cell and inter-cell couplings, as depicted in Fig. 1a. It is found that there are six sites in the unit cell, and four types of hopping named nearest-neighbor (NN) intra-cell coupling ($\gamma_1$, black solid lines), NN inter-cell coupling ($\gamma_2$, red solid lines), next-nearest-neighbor (NNN) intra-cell coupling ($\lambda_1 e^{\pm i \phi}$, green dash lines), and NNN inter-cell coupling ($\lambda_2 e^{\pm i \phi}$, yellow dash lines), respectively. This lattice model can be effectively described by a tight-binding Hamiltonian, which can be expressed as:

$$H = \sum_i U a_i^\dagger a_i + \sum_{i,j} \gamma_i a_i^\dagger a_j + \sum_{i,j,k} \lambda_k e^{i\phi} a_i^\dagger a_j^\dagger a_k + \text{h.c.}$$

(1)

with $a_i^\dagger (a_i)$ being the creation (annihilation) operator at the site $i$. $U$ is the onsite potential. $\langle \ldots \rangle$ ($\ll \ldots \gg$) indicates that the summation is restricted within the NN (NNN) sites. $\phi$ is the geometrical phase of the NNN coupling. Additionally, we have $\gamma_{i,j} = \gamma_1$ ($\gamma_{i,j} = \gamma_2$) and $\lambda_{i,j} = \lambda_1$ ($\lambda_{i,j} = \lambda_2$), when the sites $i$ and $j$ belong to the same unit (adjacent units).

At first, we investigate the system without disorders, where the corresponding parameters are set as $U=0$, $\gamma_1=1$, $\gamma_2=5$, $\lambda_1=0.5$, $\lambda_2=3$, and $\phi=\pi/2$, respectively. Figures 1b and 1c show the calculated eigen-spectrum of the open lattice, which contains a total of nineteen units with $C_6$ symmetry, and the distribution of the associated zero-energy mode, respectively. It is clearly seen that the gapless edge state exists, ensuring the system is topologically equivalent to the anomalous quantum Hall phase (AQHP) of the standard Haldane model\textsuperscript{13}. Then, we introduce
NNN coupling disorders, where the NNN intra-cell (inter-cell) hopping strengths at different lattice sites are set as $w_i \lambda_1 (w_i \lambda_2)$ with $w_i$ being the independent random number in the range of $[W, 1]$ ($W \leq 1$). Here, $W$ weighs the disorder strength. As shown in Fig. 1d, upon introducing the NNN coupling disorder and increasing its strength (decrease $W$), the initially gapless edge state (shown in the red region) gradually gets opened and the midgap modes are generated (shown in the green region). The configuration average (100 times) is performed to eliminate the accidental result. To further illustrate the property of this disorder-induced midgap mode, Figs. 1e and 1f display the eigen-spectrum and the distribution of the midgap mode with $W=-1$. In this case, the NNN coupling coefficients at different sites are assigned as the random number in the range of [-1, 1]. It is clearly shown that the NNN coupling disorders can open a spectral-gap of the original edge state and generate 0D corner states near the zero energy (the slightly deviation from the zero-energy is mainly because of the finite size effect and the influence of NNN couplings around the corner). It is worthy to note that the disorder should break the required symmetries for the non-trivial secondary topological index$^{34}$, which can induce the Wannier-type 0D corner states in this 2D system. And, the definition of the disordered higher-order topological insulator, relying on either physical quantities or topological indices, is still unclear. In such a situation, one appropriate way to define the higher-order topological phase in the disordered system is to adopt the existence of the corner state as a working definition$^{32}$. In such a case, the presence of disorder-induced zero-energy corner modes is precisely a strong evidence of HOTAIs. By further increasing the disorder strength, the band-gap gets closed (shown in the purple region in Fig. 1d) and the system eventually goes into the topologically trivial phase (TTP) associated with the Anderson localization. The physical origin for the disorder-induced corner state can be clarified by considering the competition of two kinds of Dirac masses, where the one is induced by the directional-dependent NNN coupling and the other is caused by the lattice deformation through modulating the intra-cell and inter-cell couplings (see S1 in the Supporting Information for details).
Figure 1 | The lattice model and numerical results of the higher-order topological Andersen insulator. (a) The schematic diagram of the modified Haldane model with different values of intra-cell and inter-cell couplings. There are six sites in the unit cell, and four types of hopping named NN intra-cell coupling ($\gamma_1$, black solid lines), NN inter-cell coupling ($\gamma_2$, red solid lines), NNN intra-cell coupling ($\lambda_1 e^{i\phi}$, green dash lines), and NNN inter-cell coupling ($\lambda_2 e^{i\phi}$, yellow dash lines), respectively. (b) The calculated eigen-spectrum of the open lattice without disorders. The lattice parameters are set as $U=0$, $\gamma_1=1$, $\gamma_2=5$, $\lambda_1=0.5$, $\lambda_2=3$, and $\phi=\pi/2$. (c) The distribution of the zero-energy edge mode without disorders. (d) The eigen-spectrum of the open lattice as the function of the NNN coupling disorders. The eigen-spectrum (e) and distribution of the zero-energy mode (f) of the open lattice with the disorder strength being $W=-1$.

The theoretical demonstration for realizing the classical analog of the higher-order topological Anderson insulator in electric circuits. Due to the complex site couplings, which are required to fulfill the HOTAI, constructing the above lattice model is not an easy task no matter using solid materials or classical wave systems. Recently, based on the similarity between circuit Laplacian and lattice Hamiltonian, simulating topological states with electric circuits has attracted lots of interests\cite{35-44}. Compared with other classical platforms, circuit networks possess remarkable advantages of being versatile and reconfigurable. Consequently, many extremely complex topological states, which have never been observed in any other systems, are also experimentally fulfilled in circuit networks. In the following, we discuss the experimentally feasible scheme for the realization of the above proposed HOATI based on electric circuits.
The key to map the above modified Haldane model into the circuit network is to construct various types of site-couplings. Four charts in Fig. 2a illustrate the schematic diagram for the construction of four types of site-couplings, respectively. The triangle, which contains three nodes (blue dots) connected by the capacitance $C$ and grounded via a capacitance $C_g$, is considered to form an effective lattice site. The voltages at these three nodes are marked by $V_{i,1}$, $V_{i,2}$ and $V_{i,3}$, which are suitably manipulated to form a pair of pseudospins for implementing the above modified Haldane model. To simulate the NN intra-cell (inter-cell) coupling, three inductors with inductances being $L_{1\text{NN}}$ ($L_{2\text{NN}}$) are used to directly link two triangles labeled by $i$ and $j$ with $V_{i,a}$ connecting to $V_{j,a}$ ($a=1, 2, 3$), as shown in the up-left (up-right) chart. As for the realization of NNN intra-cell (inter-cell) couplings, the two triangles are cross-connected via three inductors $L_{1\text{NNN}}$ ($L_{2\text{NNN}}$), where $[V_{i,1}, V_{i,2}, V_{i,3}]$ is connected to $[V_{j,2}, V_{j,3}, V_{j,1}]$, as presented in the bottom-left (bottom-right) chart. Such a type of cross-connection can introduce a $U(1)$ hopping term accompanied by a geometrical phase$^{35,36}$. 
Figure 2 | The circuit design for the realization of the HOTAI and numerical results for the HOTAI in the electric circuit. (a) The schematic diagrams for the construction of the NN intra-cell, NN inter-cell, NNN intra-cell, and NNN inter-cell couplings in the lattice model using electric circuits. (b)-(d) The voltage distributions at the resonance frequency (2.77MHz) with the disorder strength being 0, 0.2 and 0.5, respectively. The value of $L_{1\text{NN}}$ ($C_{1\text{NN}}$), $L_{2\text{NN}}$ ($C_{2\text{NN}}$), $L_{1\text{NNN}}$ ($C_{1\text{NNN}}$) and $L_{2\text{NNN}}$ ($C_{2\text{NNN}}$) are taken as 5uH (0.66nF), 1uH (3.3nF), 10uH (0.33nF) and 3.3uH (1nF), respectively.

To present the validity of our designed electric circuit, we derive the eigen-equation of the periodic circuit based on the Kirchhoff’s law. By inducing a pair of pseudospins, that are $V_{i\uparrow}=V_{i\downarrow}+e^{i2\pi/3}V_{i\uparrow}+e^{i2\pi/3}V_{i\downarrow}$ and $V_{i\downarrow}=V_{i\uparrow}+e^{i2\pi/3}V_{i\uparrow}+e^{i2\pi/3}V_{i\downarrow}$, the two independent equations related to each pseudospin can be written as:

$$EV_{i\uparrow}=UV_{i\uparrow} + \frac{1}{L_{1\text{NN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\uparrow} + \frac{1}{L_{2\text{NN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\downarrow} + \frac{1}{L_{1\text{NNN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\uparrow} + \frac{1}{L_{2\text{NNN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\downarrow}$$

(2)

$$EV_{i\downarrow}=UV_{i\downarrow} + \frac{1}{L_{1\text{NN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\downarrow} + \frac{1}{L_{2\text{NN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\uparrow} + \frac{1}{L_{1\text{NNN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\downarrow} + \frac{1}{L_{2\text{NNN}}} \sum_{j\neq i} e^{i\Omega_j} V_{j\uparrow}$$

(3)

where $E=3\omega^2 C$ and $U=\omega^2 C_g-2/L_{1\text{NN}}-1/L_{2\text{NN}}-2/L_{1\text{NNN}}-4/L_{2\text{NNN}}$. The detailed derivation can be found in the S2 of the Supporting Information. For convenience, the grounding capacitance is set as: $C_g=2C_{1\text{NNN}}+C_{2\text{NNN}}+2C_{1\text{NNN}}+4C_{2\text{NNN}}$, where each pair of the LC circuit has the same resonance frequency $1/(L_{1\text{NN}}C_{1\text{NN}})_{1/2}=1/(L_{2\text{NN}}C_{2\text{NN}})_{1/2}=1/(L_{1\text{NNN}}C_{1\text{NNN}})_{1/2}=1/(L_{2\text{NNN}}C_{2\text{NNN}})_{1/2}$. Actually, the Eq. (2) and Eq. (3) are the eigenfunction of the modified Haldane model (Eq. (1)) with $U=0$, $\gamma_1=1/L_{1\text{NN}}$, $\gamma_2=1/L_{2\text{NN}}$, $\lambda_1=1/L_{1\text{NNN}}$, $\lambda_2=1/L_{2\text{NNN}}$ and $\varphi=2\pi/3$ at the resonance frequency (see S2 in the supporting information for details). Hence, it is straightforward to infer that our designed electric circuit can implement the HOTAI by suitably introducing the NNN coupling disorders, which can be achieved by randomly changing the value and connection pattern of the linking inductors on different nodes. It is worthy to note that for the purpose of ensuring the onsite potential at each circuit-site remain constant, the grounding capacitances $C_{g,i}$ should also be suitably adjusted at each site associated with the NNN coupling disorder. This makes the experimental implementation become extremely difficult due to the need of large amounts of different values of grounding capacitors and connected inductors. Hence, to alleviate this problem, we chose a simplified disorder scheme by fixing the values of $L_{1\text{NNN}}$ and $L_{2\text{NNN}}$ and only randomly setting the connection pattern of the NNN coupling on different sites. In this case,
even the NNN coupling disorder is introduced, the on-site potential $U_i$ can remain unchanged in the circuit network. As a general demonstration of disorders induced higher-order topological phases, in the following, we prove that this type of the connection-pattern disorder can bring the system into the higher-order topological phase.

Now, we turn to the designed electric circuit with open boundaries. Here, a corner-modified (deleting the NNN intra-cell coupling at the corner unit) rhombus lattice is used. It is worthy to note that the grounding of this finite circuit, which is different from the periodic case because of lacking neighbors for the edge and corner sites, should be suitably designed to ensure the same on-site energy of each node at the resonance frequency. To validate our scheme, we perform steady-state simulations of the designed open circuit using the LTSpice software. We firstly consider the circuit without disorders. The value of $L_{1\infty}$ ($C_{1\infty}$), $L_{2\infty}$ ($C_{2\infty}$), $L_{1\infty}$ ($C_{1\infty}$) and $L_{2\infty}$ ($C_{2\infty}$) are taken as 5uH (0.66nF), 1uH (3.3nF), 10uH (0.33nF) and 3.3uH (1nF), respectively. In this case, the resonance frequency of the circuit is $f_0=\omega_0/2\pi=2.77$MHz. To obtain the mode response of the circuit, we excite the ‘corner site’ (marked by the star) with $[V_{1,1}, V_{1,2}, V_{1,3}]=[V_0, V_0e^{2\pi i/3}, V_0e^{-2\pi i/3}]$ ($V_0=1$V) and calculate the voltage distribution at the resonance frequency (at $f=2.77$MHz), as shown in Fig. 2b. In this case, one of the pseudospins $V_{1,\downarrow}$ is effectively excited. It is clearly shown that the dominant voltage signals exist at the edge sites and no bulk penetration, manifesting the appearance of edge state belonging to the AQHP.

Then, we gradually increase the disorder strength of the NNN coupling by increasing the number of nodes with disordered NNN connections ($N_{\text{disorder}}$). Figures 2c and 2d present the voltage distribution at the resonance frequency with the disorder strength ($r_{\text{dis}}=N_{\text{disorder}}/N_{\text{total}}$) being 0.2 and 0.5, respectively. Here, ten types of disorder patterns are averaged to eliminate the accidental result. We find that there is still a little voltage signal on the edge, but the voltage on the corner node is significantly increased when the disorder strength of the NNN coupling is $r_{\text{dis}}=0.2$. And, by further increasing the disorder strength to $r_{\text{dis}}=0.5$, the original edge-focused voltage is completely concentrated on the corner, manifesting the edge state is gapped and only the midgap corner state is excited. In this case, we can see that, similar to the tight-binding lattice model, the disorder-induced corner state has been achieved in the electric circuit network.

The experimental observation of 0D corner states in the higher-order topological
Anderson insulator with electric circuits. To experimentally observe the disorder-induced 0D corner state, we fabricate a series of electric circuits with different disorder strengths. The photograph image of the fabricated sample (without disorders) is shown in Fig. 3a. We can see that the sample contains 6×6 units, which are fabricated on a total of four printed circuit boards (PCBs) with each PCB containing 3×3 units. The sub-PCB is suitably connected to each other by external circuit elements (corresponds to white wires). The inset of Fig. 3a presents the enlarged view of the unit cell, where the LC elements to achieve intra- and inter-cell couplings are marked in the photo. It is worthy to note that the tolerance of the circuit elements is only 1% to avoid the detuning of corner resonance. To ensure the effective excitation of the circuit with a required pseudospin, NI PXle-8840 Quad-Core Embedded Controller is used to tune the excitation phase and amplitude on the three nodes of the corner site. Details of the sample fabrication and experimental measurements are provided in the Methods section.

Figure 3 | The photograph of fabricated sample and experimental results for the HOTAIIs in the electric circuit. (a) The photograph of the fabricated electric circuit without disorders. The inset presents the enlarged view of the unit cell. (b)-(d) The measured voltage distribution at the resonance frequency (2.77MHz) with the disorder strength being 0, 0.2 and 0.5, respectively.
At first, we measure the voltage distribution of the circuit without NNN disorders at 2.77MHz, as shown in Fig. 3b (the star marked the excitation position). It is noted that, similar to the simulation results shown in Fig. 2b, the dominant voltage signals exist at edge sites, manifesting the system is belonging to the AQHP. Then, we turn to the fabricated sample, where the ratio of the lattice sites with random NNN connections ($r_{\text{dis}}=N_{\text{disorder}}/N_{\text{total}}$) equals to 0.2 and 0.5. Figures 3c and 3d display the measured voltage distributions at the resonance frequency with $r_{\text{dis}}=0.2$ and 0.5, respectively. Ten types of disorder patterns are averaged to eliminate the accidental result. It is clearly shown that the original edge-dominated voltage signal is gradually concentrated on the corner with the increase of the disorder strength of NNN couplings. At $r_{\text{dis}}=0.5$, the original edge-focused voltage is completely concentrated on the corner, which corresponds very well to the simulation result shown in Fig. 2d. In addition, we also measure the voltage signal at corner, edge and bulk sites at different frequencies for both clean and disorder samples. And, the measured voltage signal in the frequency domain also demonstrates the disorder-induced corner mode. The detailed results can be found in the S3 of the Supporting Information. In general, both theoretical and experimental results show that the corner states can be induced by adding sufficiently strong disorder to the bulk, and the HOTAI can be realized by using the electric circuit.

**Discussion.**

In conclusion, we have not only demonstrated theoretically that the disorder can induce higher-order topological insulators from a topologically trivial phase based on the modified Haldane model, but also realized experimentally the classical analog of HOTAIIs carried out in the electric circuits. Through the direct circuit simulation and voltage measurement, the 0D corner states induced by the bulk disorder in the two-dimensional system are verified. Such disorder-induced 0D corner states are expected to implement the analog circuit filter that relies on disorders in the bulk. Our work offers a feasible platform to investigate the relationship between disorders and higher-order topological phases, giving rise to the possibility for catching energy at a corner through the import of disorders.

**References**
1. Hasan, M. Z. & Kane, C. L. Colloquium: Topological insulators. Rev. Mod. Phys. 82, 3045-3067 (2010).
2. Lu, L., Joannopoulos, J. D. & Soljačić, M. Topological photonics, Nat. Photonics 8, 821-829 (2014).
3. Ozawa, T., Price, H. M., Amo, A., Goldman, N., Hafezi, M., Lu, L., Rechtsman, M. C., Schuster, D., Zilberberg, O. & Carusotto, L. Topological photonics, Rev. Mod. Phys. 91, 015006 (2019).
4. Ma, G., Meng, X. & Chan, C. T. Topological phases in acoustic and mechanical systems, Nat. Rev. Phys. 1, 281-294 (2019).
5. Harari, G. et al. Topological insulator laser: theory. Science 359, eaar4003 (2018).
6. Bandres, M. A. et al. Topological insulator laser: experiments. Science 359, eaar4005 (2018)
7. Benalcazar, W. A., Bernevig, B. A. & Hughes, T. L. Quantized electric multipole insulators, Science 357, 61 (2017).
8. Song, Z., Fang, Z. & Fang, C. (d-2)-Dimensional Edge States of Rotation Symmetry Protected Topological States, Phys. Rev. Lett. 119, 246402 (2017).
9. Langbehn, J., Peng, Y., Trifunovic, L., Oppen, F. & Brouwer, P. W. Reflection-Symmetric Second-Order Topological Insulators and Superconductors, Phys. Rev. Lett. 119, 246401 (2017).
10. Ezawa, M. Higher-Order Topological Insulators and Semimetals on the Breathing Kagome and Pyrochlore Lattices, Phys. Rev. Lett. 120, 026801 (2018).
11. Schindler, F., Wang, Z., Vergniory, M. G., Cook, A. M., Murani, A., Sengupta, S., Kasumov, A. Yu., Deblock, R., Jeon, S., Drozdov, I., Bouchiat, H., Guéron, S., Yazdani, A., Bernevig, B. A. & Neupert, T. Higher-order topology in bismuth, Nat. Phys. 14, 918 (2018).
12. Schindler, F. et al. Higher-order topological insulators. Sci. Adv. 4, eaat0346 (2018).
13. Serra-Garcia, M., Peri, V., Süsstrunk, R., Bilal, O. R., Larsen, T., Villanueva, L. G. & Huber, S. D. Observation of a phononic quadrupole topological insulator, Nature 555, 342 (2018).
14. Xue, H., Yang, Y., Gao, F., Chong, Y. & Zhang, B. Acoustic higher-order topological insulator on a kagome lattice. Nat. Mater. 18, 108–112 (2019).
15. Ni, X., Weiner, M., Alù, A. & Khanikaev, A. B. Observation of higher-order topological acoustic states protected by generalized chiral symmetry. Nat. Mater. 18, 113–120 (2019).
16. Peterson, C. W., Benalcazar, W. A., Hughes, T. L. & Bahl, G. A. quantized microwave quadrupole insulator with topologically protected corner states, Nature 555, 346 (2018).
17. Mittal, S., Orre, V. V., Zhu, G., Gorlach, M. A., Poddubny, A. & Hafezi, M. Photonic quadrupole topological phases, Nat. photonics 13, 692-696 (2019).

18. Zhang, W., Xie, X. & Hao, H. et al. Low-threshold topological nanolasers based on the second-order corner state. Light: Science & Application 9, 109 (2020).

19. Noh, J., Benalcazar, W. A., Huang, S., Collins, M. J., Chen, K. P., Hughes, T. L. & Rechtsman, M. C. Topological protection of photonic mid-gap defect modes, Nat. Photonics 12, 408 (2018).

20. Imhof, S. Berger, C., Bayer, F., Brehm, J., Molenkamp, L. W., Kiessling, T., Schindler, F., Lee, C. H., Greiter, M., Neupert, T. & Thomale, R. Topolectrical-circuit realization of topological corner modes, Nat. Phys. 14, 925 (2018).

21. Bao, J., Zou, D., Zhang, W., He, W., Sun, H. & Zhang, X. Topolectrical-circuit octupole insulator with topologically protected corner states, Phys. Rev. B 100, 201406(R) (2019).

22. Anderson, P. W. Absence of diffusion in certain random lattices. Phys. Rev. 109, 1492–1505 (1958).

23. Li, J., Chu, R.-L., Jain, J. K. & Shen, S.-Q. Topological Anderson insulator. Phys. Rev. Lett. 102, 136806 (2009).

24. Jiang, H. Wang, L., Sun, Q.-F. & Xie, X. C. Numerical study of the topological Anderson insulator in HgTe/CdTe quantum wells, Phys. Rev. B 80, 165316 (2009).

25. Groth, C. W., Wimmer, M., Akhmerov, A. R., Tworzydlo, J. & Beenakker, C. W. J. Theory of the Topological Anderson Insulator, Phys. Rev. Lett. 103, 196805, (2009).

26. Guo, H.-M., Rosenberg, G., Refael, G. & Franz. M. Topological Anderson Insulator in Three Dimensions, Phys. Rev. Lett. 105, 216601, (2010).

27. Altland, A., Bagrets, D., Fritz, L., Kamenev, A. & Schmiedt, H. Quantum criticality of quasi-one-dimensional topological Anderson insulators, Phys. Rev. Lett. 112, 206602, (2014).

28. Titum, P., Lindner, N. H., Rechtsman, M. C. & Refael, G. Disorder-induced Floquet topological insulators, Phys. Rev. Lett. 114, 056801, (2015).

29. Meier, E. J. et al. Observation of the topological Anderson insulator in disordered atomic wires, Science 362, 929 (2018).

30. Zangeneh-Nejad, F. & Fleury, R., Disorder-induced signal filtering with topological metamaterials, Advanced Materials, 32(28), 2001034, (2020).

31. Stützer, S., Plotnik, Y. & Lumer, Y. et. al. Photonic topological Anderson insulators, Nature 560,
32. Araki, H., Mizoguchi, T. & Hatsugai, Y. Phase diagram of a disordered higher-order topological insulator: A machine learning study, Phys. Rev. B 99, 085406 (2019).
33. Haldane, F. D. M. Model for a quantum Hall effect without Landau levels: Condensed-matter realization of the parity anomaly, Phys. Rev. Lett. 61, 2015 (1988).
34. Benalcazar, W. A., Li, T. & Hughes, T. L. Quantization of fractional corner charge in Cn-symmetric higher-order topological crystalline insulators. Phys. Rev. B. 99, 245151 (2019).
35. Albert, V. V., Glazman, L. I. & Jiang, L. Topological properties of linear circuit lattices. Phys. Rev. Lett. 114, 173902 (2015).
36. Ning, J., Owens, C., Sommer, A., Schuster, D. & Simon, J. Time and site resolved dynamics in a topological circuit. Phys. Rev. X 5, 021031 (2015).
37. Olekhno, N., Kretov, E., Stepanenko, A., Filonov, D., Yaroshenko, V., Cappello, B., Matekovits, L. & Gorlach, M. A. Topological edge states of interacting photon pairs realized in a topolectrical circuit, Nat. Commun. 11, 1436 (2020).
38. Hofmann, T., Helbig, T., Lee, C. H. & Thomale, R. Chiral voltage propagation and calibration in a topolectrical Chern circuit. Phys. Rev. Lett. 122, 247702 (2019).
39. Lee, C., Imhof, S., Berger, C. et al. Topolectrical circuits, Communications Physics, 1, 39 (2018).
40. Ezawa, M. Electric circuit simulations of nth-Chern-number insulators in 2n-dimensional space and their non-Hermitian generalizations for arbitrary, Phys. Rev. B 100, 075423 (2019).
41. Yu, R., Zhao, Y. & Schnuder, A. P. 4D spinless topological insulator in a periodic electric circuit. National Science Review, nwaa065, (2020).
42. Li, L., Lee, C. & Gong, J. Emergence and full 3D-imaging of nodal boundary Seifert surfaces in 4D topological matter, Communications physics 2, 135 (2019).
43. Wang, Y., Price, H. M., Zhang, B. & Chong, Y. D. Circuit Realization of a Four-Dimensional Topological Insulator, Nat. Commun., 11, 2356, (2020).
44. Zhang, W., Zou, D., He, W., Bao, J., Pei, Q., Sun, H. & Zhang, X. arXiv:2001.07931, (2020).

Methods.

Sample fabrications and circuit measurements. We exploit the electric circuits by using PADs program software, where the PCB composition, stackup layout, internal layer and
grounding design are suitably engineered. Here, each well-designed PCB possesses totally eight layers to arrange the NN/NNN intra-cell and inter-cell site-couplings. It is worthy to note that the ground layer should be placed in the gap between any two layers to avoid their coupling. Moreover, all PCB traces have a relatively large width (0.5mm) to reduce the parasitic inductance and the spacing between electronic devices is also large enough (1.0mm) to avert spurious inductive coupling. On the other hand, due to the size limit of the PCB fabrication, we cut the whole sample into four parts. The inter-cell couplings between each sub-PCB is realized by external circuit elements. To ensure the same grounding condition, we link the copper pillar of each sub-PCB together. As for the circuit excitation, we use NI PXIe-8840 Quad-Core Embedded Controller to input three signals simultaneously with controlled phases and amplitudes, which are required to excite the pseudospin of the circuit. Three SMP connectors are welded on corner, edge and bulk of the PCB for the signal input. In addition, the frequency spectrograph is used to measure the voltage on different sites of the circuit.

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