FIXING MATCH-FIXING∗

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Abstract. In the last round of the World Cup group stage, games whose outcomes do not affect the selection of the qualified teams are played with little enthusiasm. Furthermore, teams that have already qualified may take into account other factors, such as the opponents it will face in the next stage of the competition. Thus, depending on the results in the other groups and the scheduling of the next stage, winning the game may not be in its best interest. Even more critically, there may be situations in which a simple draw will qualify both teams for the next stage of the competition. Any situation in which the two opposing teams do not play competitively is detrimental to the sport, and, above all, can lead to collusion and match-fixing opportunities. We here develop a method of evaluating competitiveness and apply it to the current format of the World Cup group stage. We then propose changes to the current format in order to increase the stakes in the last round of games of the group stage, making games more exciting to watch and reducing any collusion opportunities. We appeal to the same method to evaluate the “groups of 3” format which may be introduced in the 2026 World Cup edition, as well as a “groups of 5” format.

Key words. tournament structure, fairness, soccer, FIFA World Cup, modeling match outcomes, Monte Carlo simulations

AMS subject classifications. 91A99, 90B90, 65C05

1. Introduction. The soccer World Cup is a global sport event that attracts one of the highest audiences: according to the organizing entity, FIFA, over one billion television viewers watched the final between France and Croatia. To further improve the worldwide World Cup audience, FIFA is currently trying to include more countries in the final stages of the tournament and increase the attractiveness of all of the games played during this one-month competition.

In its current format, the World Cup consists of 32 qualified teams (via continental qualifying tournaments) that are distributed into 8 groups of 4 teams each. In the group stage, teams in the same group play against each other once (for a total of 6 matches per group) with the group ranking based on the current football points system: 3 points for a win, 1 for a draw and 0 for a loss. The first two teams in each group qualify for the knockout stage.

This schedule can lead to games being played in an unethical and unattractive way, with a good example being the infamous match between Austria and West Germany (Germany being at that time still divided) during the 1982 World Cup, in Gijon (Spain). Sometimes called the “disgrace of Gijon”, the game is known in German as the “Gijon non-aggression pact”. West Germany and Austria both played in group 2, with Algeria and Chile. To everyone’s surprise, the German Mannschaft stumbled against Algeria, losing its first match (1-2) (the first time that a European team had lost to an African team in a World Cup), and Austria beat Chile (1-0). In the second round of games, West Germany beat Chile (4-1) while Algeria lost to Austria (0-2).
At this point, Chile was already eliminated. The last two games in Group 2 were thus decisive for Germany, Austria and Algeria. On June 24, Algeria beat Chile 3-2. From then on, an arrangement became possible between the West Germans and Austrians. A calculation shows that, with a simple 1-0 victory for the Mannschaft, both the Germans and Austrians would qualify while (oddly) either a large German victory, an Austrian victory or a draw would lead to Algeria qualifying. After 10 minutes of the game, the Germans scored, following which both teams almost stopped for the remaining, long, 80 minutes, under the booing of infuriated Spanish spectators. After the scandal of Gijon (and similar games, the last two games in each group are now played simultaneously, but this has not completely eliminated collusion opportunities.

The groups are formed using a draw procedure that changes slightly from year to year depending on the origin of the qualified teams (FIFA tries to spread out teams from the same continent in an even manner across all of the groups). Nevertheless, the main structure of the draw is the following:

- Teams are divided into pots, each of which is supposed to contain teams with similar levels of performance: the 8 best teams in pot A, the second best 8 teams in pot B, and so on. Team performance is based on a ranking that has changed over time, and has been criticized by some football experts. The country hosting the tournament (which qualifies automatically) is included in pot A in order to maximize its chances of proceeding to the knockout stage of the tournament.

- Groups are formed by picking one team from each pot such that all groups have a "top-level" pot-A team, a "second-level" pot-B team, a pot-C team and a "weaker" pot-D team.

- Finally, the schedule of the games, i.e. the order in which the teams play against each other is drawn randomly. As such, in some groups the last round of games will consist of "pot A" vs "pot B", and "pot C" vs "pot D", while in other groups the last matches consists of "pot A" vs "pot C", and "pot B" vs "pot D", or "pot A" vs "pot D" and "pot B" vs "pot C".

We here develop a method to evaluate the competitiveness and fairness of the last-round games, choose a model to simulate the group-stage outcomes and look for the optimal setting that maximizes our competitiveness metric. We also apply this method to real World-Cup data (starting from the 1998 competition, in which the new format was adopted) and conclude that games were sub-optimally scheduled. We find out that the points-attribution scheme does not affect the quality of games in that a victory produces more points than a draw, which produces more points than a loss. However, the order in which games are played, and specifically the schedule for the last round of games, is critical and can substantially improve the competitiveness of the last round if well-designed.

Many models have been developed to predict or simulate the outcome of football games. For example, Lee [1] and Dyte and Clarke [2] treat the goals scored by each team as conditionally-independent Poisson variables whose parameters depend on team attributes and the match venue. Maher [3] found that introducing a correlation between the number of goals scored via a bivariate Poisson distribution improved predictive power in data from the English League. Reep, Pollard and Benjamin [4] construct a model based on the negative binomial distribution, while Karlis and Ntzoufras [5] use Skellam’s distribution to model the difference in the number of goals (the margin of victory). In a following article, the latter develop a robust fitting
method to account for abnormal large scores [6]. Most of this previous work has looked at national championships, but only few have considered the FIFA World Cup. The 1998 World Cup is covered in [2]. Suzuki et al. [7] use a Bayesian approach to predict the result of the 2006 World Cup, while Groll et al. [8] apply their model to the 2014 World Cup. Among the articles looking at international competitions, the focus has been on the predictive power of the model, rather than considerations of fairness and competitiveness. An exception is the new penalty shoot-out approach proposed by Brams and Ismail [10], which improves fairness at the knock-out stage of the World Cup. The aim of the current article is to propose a method to assess and improve the competitiveness and fairness of games in the group stage of the competition.

We first benchmark and calibrate the different models, based on the results in previous World Cups and team rankings. We next develop a classification method that allows us to quantify the attractiveness of the last round of games. Based on the chosen model and our original method, we then use Monte Carlo simulations to determine the key factors that affect the quality of the last round of games in the current World Cup format. We also use our metric to assess the quality of games in previous World Cups (with 8 groups of 4 teams) and propose a remedy to improve the competitiveness of the last round of the group stage. The last section applies our method to the new enlarged version of the World Cup. We here propose two options: 16 groups of 3 teams or 8 groups of 5 teams.

2. Group-stage model. We here benchmark the different models that will be used to simulate the game outcomes. For the sake of simplicity, a team’s strength is completely described by one single variable. Instead of using FIFA rankings, we choose the ELO index as a proxy for team performances. The advantage of this index is that it is more transparent and is a more accurate reflection of a team’s real level than the FIFA ranking [9]. The calculation method has not changed over time, and it is thus better-suited for analyses over long time periods (we will here cover all the World Cups starting from that in 1998). Nevertheless, the ELO and FIFA point systems produce very similar country rankings.

Each team has an ELO index of the form \([a, b]\), with \(a\) and \(b\) positive real numbers. The higher the ELO index, the better the team’s performance. As in the official draw procedure, we form groups of four teams with different ELO indices as follows: team A’s ELO is uniformly drawn from the interval \([b - \frac{b-a}{4}, b]\), team B from \([b - \frac{b-a}{2}, b - \frac{b-a}{4}]\), team C from \([a + \frac{(b-a)}{4}, b - \frac{b-a}{2}]\) and team D from \([a, a + \frac{b-a}{4}]\). The bounds \(a\) and \(b\) are parameters in our model and will be calibrated in the next section. Intuitively, the greater the \(b - a\) gap, the larger the performance gap between teams within the group. Based on the ELO indices of the teams in the group, we simulate the outcomes of their matches.

2.1. Simulating match outcomes. As we only consider a team’s ELO index, we analyze the following relatively simple parametric models:

1. Simple Poisson model: Each time a team has the ball it can attack and score a goal. With \(n\) attack opportunities and a probability \(p\) of scoring per attack, the number of goals scored follows a binomial distribution \(\mathcal{B}(n, p)\). On average, \(\lambda = np\) goals will be scored by the team per game. The binomial distribution limits the number of goals scored per game to the total amount

\[\text{All historical and current ELO ratings, as well as the details on how they are calculated, can be found at https://www.eloratings.net}\]
of attacks $n$. If instead of considering discrete attacks, we look at ball possession and introduce the probability of scoring per unit of ball possession, $\lambda$, the number of goals scored is distributed Poisson with parameter $\lambda$. This distribution is the limit case of the binomial distribution as $n \to \infty$ (every ball possession signifies an attack) and $p = \frac{\lambda}{n} \to 0$ (the strict probability becomes a probability density of scoring per unit of time possession). The probability that team $i$ score $k$ goals against team $j$ is:

$$P(\text{goals} = k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

(2.1)

where $\lambda$ is

$$\lambda = \alpha \cdot \frac{e_i}{e_i + e_j}$$

(2.2)

Here $e_i$ is the ELO index of team $i$, $e_j$ the ELO index of its opponent $j$, and $\alpha$ a parameter of the model to be calibrated. The stronger the scoring team (the higher is $e_i$) and the weaker its opponent (the lower is $e_j$), the easier it will be for team $i$ to score goals. The parameter $\alpha$ reflects how prolific games are in terms of goals scored (a higher $\alpha$ produces games with higher scores). Consequently, the result $(k_i, k_j)$ of a game between team $i$ with ELO index $e_i$ and team $j$ with ELO index $e_j$ is distributed:

$$(k_i, k_j) \sim (X, Y)$$

(2.3)

where $X$ and $Y$ are independent, and $X \sim \mathcal{P}(\alpha \cdot \frac{e_i}{e_i + e_j})$, $Y \sim \mathcal{P}(\alpha \cdot \frac{e_j}{e_i + e_j})$.

2. **Bivariate Poisson model**: The bivariate Poisson model is very similar to that above. The only difference is that it accounts for correlations between the number of goals scored by the two teams. The underlying idea here is that if one team scores, the other will attempt to equalize and put more effort into scoring. This leads to open games with a greater number of goals on both sides. On the contrary, if neither team scores the game will remain “closed” with few goals. The final score of the game $(k_i, k_j)$ is:

$$k_i = X + Z \quad k_j = Y + Z$$

(2.4)

where $X$, $Y$ and $Z$ are independent, and $X \sim \mathcal{P}(\alpha \cdot \frac{e_i}{e_i + e_j})$, $Y \sim \mathcal{P}(\alpha \cdot \frac{e_j}{e_i + e_j})$, $Z \sim \mathcal{P}(\beta)$. The correlation between $k_i$ and $k_j$ comes from the term $Z$, and the greater is $\beta$ the higher this correlation. This model has one more parameter than that above (namely $\beta$).

3. **Negative binomial model**: In this model, when one team scores a goal, it becomes more motivated and has a greater probability of scoring a second goal. The scoring model starts as a Poisson distribution, and each time a goal is scored the probability of scoring the next goal rises by a given constant. The probability distribution of goals can be calculated and has a so-called negative binomial distribution. The probability that team $i$ score $k_i$ goals against team $j$ is:

$$P(k_i) = \left( \frac{k_i + r - 1}{k_i} \right) \cdot (1 - \alpha \cdot \frac{e_i}{e_i + e_j})^r \cdot (\alpha \cdot \frac{e_i}{e_i + e_j})^{k_i}$$

(2.5)

where $r \in \mathbb{N}$ and $\alpha > 0$ are two parameters to be calibrated.
2.2. Rescaling the ELO distribution. The ELO indices usually fluctuate between 1500 and 2200 for the teams that qualify for the World Cup. As such, \( \frac{e_i}{e_i + e_j} \) varies between \( \frac{1500}{1500 + 2200} = 0.4054 \) and \( \frac{2200}{1500 + 2200} = 0.5946 \). Consequently the “raw” ELO indices will barely affect the number of goals scored by teams. We amplify the performance difference between teams via a linear transformation of the original ELO indices:

\[
e'_i = 1 + e^{gap} \cdot \frac{e_i - \min_j(e_j)}{\max_j(e_j) - \min_j(e_j)}
\]

After this transformation, the weakest team will have an index of 1 and the strongest an index of \( 1 + e^{gap} \), where \( gap \) is a parameter to be calibrated. The higher is \( gap \), the larger the performance gap between teams and the greater the impact of the ELO indices on a game’s outcome.

2.3. Model selection and calibration. We now carry out maximum-likelihood estimation of the three models presented above, in order to decide which will be used to carry out our group simulations. The data used for these models is the results of the first two rounds of the World Cup group stage from 1998 up to 2018. This covers 192 games: the last round of games is not included as factors other than team performance may play a role here (teams may prefer to lose or draw in the last game, and finish second in the group in order to have easier knock-out games). Table 1 shows the results from this maximum-likelihood estimation.

First, since the optimal value of \( \beta \) is \( 2.7518 \cdot 10^{-10} \), we conclude that the introduction of a correlation term between the goals does not improve the accuracy of the simple Poisson model (there is no change in the log-likelihood between the simple and the bivariate Poisson models either). According to the AIC and BIC criteria, which penalize the last two models as compared to the simple Poisson model, we decide to adopt the latter for our simulation procedure. This has the advantage of being simple to implement, tractable and having very similar performance to the Negative-Binomial distribution. In our data, the bivariate Poisson distribution performs no better than the univariate distribution.

Figure (1) shows the likelihood in the simple Poisson and Negative-Binomial distributions as a function of the model parameters \( (r = 13 \) for the Negative-Binomial

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3All of which, historical and current, are online at https://www.eloratings.net
We can clearly see that the optimal value of $gap$ is strictly positive, so that the ELO indices do have predictive power for game outcomes.

### 2.4. Additional performance checks for the Poisson model

We now compare a number of statistics from our chosen Poisson model to those in our sample data at three different levels of aggregation:

- **Detailed level**: Each result is considered as a separate event. As our data set is composed of 192 games, we use the Poisson model to carry out 15,000 simulations of each of the 192 games. We then count the number of occurrences of each event in each simulation run, and average the results over the 15,000 simulations and calculate the standard deviations for each event. The results appear in Table 3, to be compared to the actual outcomes in Table 2. The games in which at least one team scores more than four goals are rare, and do not appear in the tables. The Poisson model provides a good approximation to the actual data.

- **Compact level**: Here an event is characterized by the difference in the scores, so that games finishing 3-1 or 4-2 would be similarly categorized as "2-goal differences" events. Figure 2 shows the results of our simulations (in red), compared to the actual data (in blue). As above, the Poisson model performs well in reproducing the actual scores.

- **Draw frequency**: This is extreme case where we group all non-zero goal difference outcomes into a single event, with draws being the complement (a goal difference of zero). This allows us to see if the ratio of draws to total number of games in our model matches the sample data. The frequency of draws in the Poisson model is 0.2133 with a standard deviation of 0.0297, while the sample frequency is $0.2552$. As the data only includes 192 games, we cannot say whether the fit could be improved (by reducing the $a$ parameter, for example) or if our data is not truly representative of the underlying generating process.

Overall, the previous results suggest that the Poisson model fits the score-generating process well.
Table 2: Number of goal outcomes in our World Cup data sample of 192 games.

| Final score | 0   | 1   | 2   | 3   | 4   |
|-------------|-----|-----|-----|-----|-----|
| 0           | 14  | 44  | 22  | 11  | 8   |
| 1           | 23  | 29  | 13  | 1   |     |
| 2           | 11  | 3   | 2   |     |     |
| 3           | 1   | 0   |     |     |     |
| 4           |     |     |     |     | 0   |

Table 3: Average number of occurrences of scores ± standard deviation based on 15000 Monte Carlo simulations.

| Final score | 0   | 1   | 2   | 3   | 4   |
|-------------|-----|-----|-----|-----|-----|
| 0           | 15.5 ± 3.8 | 39 ± 5.6 | 30.6 ± 5.1 | 17.9 ± 4.0 | 8.5 ± 2.9 |
| 1           | 18.4 ± 4.1 | 23.2 ± 4.5 | 11.4 ± 3.2 | 4.7 ± 2.1  |     |
| 2           | 6.0 ± 2.4 | 5.0 ± 2.2 | 1.8 ± 1.3  |     |     |
| 3           | 0.91 ± 0.95 | 0.59 ± 0.77 |     |     |     |
| 4           |     |     |     |     | 0.08 ± 0.3 |

Fig. 2: Score difference: Poisson model vs. actual data

3. The group-stage classification method. We now assess the attractiveness of the group format using the previous model to simulate all the rounds of games in a group, except for the last. For example, when considering the current format of the World Cup with groups of four countries, we simulate the first two rounds of games (for a total of four games). Then, given the points system, we calculate the ranking in the group. It is worth noting that the group ranking is based only on points, and does
not include other criteria such as the number of goals scored, goal difference etc. We consequently cannot distinguish between two teams with the same number of points.

During the last round of games, the qualification of team $i$ will likely not only depend on the outcome of its own game against team $j$ but also on that in the other game, between teams $k$ and $l$. As the last-round games are played simultaneously, we assume that team $i$ does not know the outcome of the other game when playing against $j$: all three scenarios are possible ($k$ beats $l$, $l$ beats $k$ and $k$ draws with $l$). For each outcome of the $i$ vs $j$ game, we check under which scenarios team $i$ qualifies for the next phase (i.e. ends up in the top two group teams). In the case that the team ends up second position with the same number of points as the other teams, we count the number of teams in the second position. Occupying the second position alone (and so qualifying) is better than sharing the second position with other teams (other criteria that are not included in our model are then used to rank the teams, and lead to team $i$ not progressing). Team $i$ will choose the lowest-effort outcome that maximizes the chances of qualification: a win (winning is better than drawing in at least one scenario), a draw (winning=drawer in all scenarios is and better than losing in at least one scenario) or indifference (win=draw=losing in all scenarios). Note that the last situation refers to the case where team $i$ is already qualified or cannot qualify regardless the result of game $k$ vs. $l$.

For example, suppose that any scenario other than $k$ beating $l$ automatically qualifies $i$. If team $k$ wins, a draw between $i$ and $j$ will qualify $i$, while a loss for $i$ will not lead to qualification (if $i$ wins, it will obviously progress as wins gains more points than draws). In this case, team $i$ will play for a draw. Even though the team may qualify if it loses against $j$, a draw will increase its chances of qualification (all scenarios are possible, including a victory for $k$). A victory will also qualify $i$, although it does not improve the probability of qualification as compared to a draw. If, in a given scenario, a draw leads to a shared second place while a win leads to a "clean" second place, or even first place, then team $i$ will play to win.

Note that, in our model, we do not distinguish between first and second place in the group, in the sense that teams only care about qualification to the next round. In the actual World Cup, teams do not always want to finish first in their groups. There have been many occasions where teams seem to have intentionally lost in order to finish second in their group and play against weaker opponents in the knock-out stage. This may well have occurred in the 2018 World Cup in the group with England and Belgium, in which the winner faced more-difficult opponents (Brazil and France) in the knock-out stage.

After having determined what team $i$ would prefer, we carry out an analogous analysis for its opponent $j$, yielding the following classification for the game $i$ vs $j$:

- **Stake-less games**: At least one of the teams is indifferent between winning, drawing or losing. In these games, the indifferent team has, in general, nothing to gain and may field second-team players. This is unfair for the other teams, $k$ and $l$, as they played against a stronger opponent $i$ in the previous rounds. In addition, teams that are already qualified may take into account other factors such as the opponents it will face in the next stage of the competition. Thus, depending on the results in other groups and the scheduling of the next stage, winning the game may not be in its best interest. These games are not competitive and therefore should be avoided.

- **Collusive games**: Both teams here are looking to draw, as this will put
both teams in the best possible situation. These games should be avoided.

- **Competitive games:** Neither team is indifferent here, and at least one hopes to win. As one team wants to win and the other, at least, hopes to draw, their aims are incompatible: if one team reaches its objective the other will not. The two teams will thus give their maximum in this competitive game. Our aim here is to ensure that this type of game occurs as often as possible.

4. **Assessing the current World Cup format.** We use the above to assess the last round of the current World Cup format, with groups of four teams of which the top two qualify for the next round. We test both different point-attribution systems as well as changes to the scheduling of the last round of games:

- **Setting 1:** pot A vs. pot D and pot B vs. pot C
- **Setting 2:** pot A vs. pot C and pot B vs. pot D
- **Setting 3:** pot A vs. pot B and pot C vs. pot D

We carry out 15000 simulations for each setting and point-attribution system: the results appear in Table 4. Our conclusions are as follows:

- The points system has no impact on the quality of games (systems with 4-points for a win produce no visible changes in the results);
- Collusive games are relatively rare, but stake-less games are not; and
- The setting has a considerable impact on the quality of games, with setting 1 being the best and setting 3 the worst.

The last of the above results is intuitive: in setting 3, A and B have already played against the weakest teams in the previous rounds. Before the the last game, they are likely to a good number of points, while teams C and D have few or no points. The last round matches the best two teams (who are already or almost qualified) against the weakest two teams (who are already or almost eliminated): the outcome of both games has very little impact on the final group ranking.

We use our real data set to assess the impact of the last round of games as a function of their setting. The new format has applied to six World Cups with 8 groups each. The game schedule is drawn randomly, so that each setting is equally likely. Table 5 shows the frequency of each setting in the previous World Cups. We use our method to calculate the frequencies of each game type as a function of the setting. Table 6 shows the results: setting 3 produces the least exciting last round of games, in line with our predictions. Nevertheless, the sample size is only small to check whether the sample estimates fit our model predictions.
Fig. 3: The cumulative frequencies in Monte Carlo simulations for a win giving 3 points and a draw 1 point (setting 1).

| Setting type : | 1 | 2 | 3 | 1 | 2 | 3 |
|----------------|---|---|---|---|---|---|
| Points for :   |   |   |   |   |   |   |
| Draw = 1       | 63.05% | 59.50% | 42.95% | 63.14% | 59.49% | 42.69% |
|                | 1.02% | 1.15% | 1.68% | 0.94% | 1.32% | 1.76% |
|                | 35.93% | 39.35% | 55.37% | 35.92% | 39.19% | 55.55% |
| Draw = 2       | - | - | - | 63.13% | 59.76% | 43.54% |
|                | - | - | - | 0.88% | 1.30% | 1.60% |
|                | - | - | - | 35.99% | 38.94% | 54.87% |

Table 4: Results of the Monte Carlo simulations with 15000 iterations per run (competitive, collusive and stakeless games).

| Setting | 1 | 2 | 3 |
|---------|---|---|---|
| Occurrences | 15 | 19 | 14 |
| Frequencies  | 31.25% | 39.58% | 29.17% |

Table 5: Number of occurrences and frequencies of the different group settings in our sample data.

| Type of game | Competitive | Stake-less | Collusion opportunity |
|--------------|-------------|------------|-----------------------|
| Setting 1    | 50%         | 50%        | 0%                    |
| Setting 2    | 57.89%      | 36.84%     | 5.26%                 |
| Setting 3    | 32.14%      | 67.86%     | 0%                    |

Table 6: Frequencies of types of games as a function of the group setting.
5. The 2026 World Cup. In 2026, FIFA plans to have 48 qualified teams, distributed into 16 groups of 3 teams. The first two teams in each group (2/3 of teams) then qualify to the knockout stage. The transition from 1/2 to 2/3 teams qualifying has already been tested in the second biggest soccer competition: the UEFA Euro. Between 1996 and 2012, the proportion of teams qualifying for the knockout stage was 1/2 in a 16-team tournament (4 groups of 4 teams), growing to 2/3 for the 2016 UEFA Euro (24 teams divided into 6 groups of 4). As a result, the number of goals scored fell (from an average of over 2.5 goals per match in the group phases between 1996 and 2012 to a figure of 1.92 goals per match during the 2016 group phase), with the games becoming unattractive. This natural experiment then suggests that tournaments with a high proportion of teams qualifying lead to less attractive games.

As such, alternative propositions have been made for the enlargement of the number of qualified teams for the next FIFA World Cups: for instance a 40-team tournament (8 groups of 5 teams) in which only 2 teams per group (2/5) qualify for the knockout stage. In addition, the group-match schedule (the order in which the teams play against each other) appears to have a considerable impact on the quality of the games in this phase, and in particular during the last round of games.

In this last section, taking into consideration FIFA’s intention to increase the number of teams participating, we evaluate two potential new World-Cup formats.

5.1. First option: 48 groups of 3. Some FIFA officials are currently proposing a “48-team, groups of 3” format, in which the best two teams in each group qualify for the next knock-out stage. As the groups contain an odd number of teams, one team per group will not play in the last round of games. There are therefore only three possibilities in the last round:

- Setting 1: The weakest team is the passive team in the last round
- Setting 2: The middle team is the passive team in the last round
- Setting 3: The strongest team is the passive team in the last round

We carry out 15000 Monte-Carlo simulations to assess the quality of the last round: the results appear in Table 7. We have also checked that the number of points attributed for wins and draws do not affect the results and that “collusive” games, as defined previously, cannot occur here.

The conclusion is clear: it is key that the passive team in the last round be the strong team. In our model, when the strongest team plays the first two rounds of games, there is a 92% chance of a last game in which both teams will give their best. The only two situations in which the last game is stakeless or even conducive to unfair behavior are as follows:

- **2 losses for the passive team**: Both of the other teams have already qualified, and the last game is completely stakeless. In some particular situations, depending on the results in other groups and the playoff schedule, some qualified teams may want to lose in order to finish second in the group and thus meet a less-strong team in the next round. This leads to uninteresting games that should be avoided.

- **1 loss, 1 draw for the passive team**: Of the two other teams, one has already qualified and the other only needs a draw to do so. The latter will face a team that has already qualified and may end up with a much-easier game than that which the passive team had against the already qualified team (which had not yet qualified at that time). This is detrimental to the fairness of the game. In our model, this type of game is qualified as
Table 7: Results of the Monte Carlo simulations with 15000 iterations.

| Setting type     | 1     | 2     | 3     |
|------------------|-------|-------|-------|
| Interesting games| 23.10%| 79.12%| 92.23%|
| Stakeless games  | 76.90%| 20.88%| 7.77% |
| Collusion games  | 0%    | 0%    | 0%    |

“stakeless” as one team is indifferent while the other is aiming for at least a draw. Nevertheless, a “stakeless” game in this World Cup format can produce much more unfair behavior as the passive team no longer has any impact on the group’s outcome. In the 4-team per group format, there is “pressure” from the unknown result in the other last-round game (which is played at the same time). This no longer applies in the 3-team per group format.

When the strongest team plays the first two rounds, it is unlikely that they will be in such situations. However, when the weakest team plays the first two games, these outcomes become more probable (they actually happen 77% of the time!). In case this World Cup format would be carried forward. In order to preserve the fairness and beauty of the game, FIFA should drop its group randomization draw and implement a predefined schedule in which the pot A team will be the passive team in the last round.

5.2. Second alternative: 8 groups of 5. A second alternative is to consider a World Cup with 40 teams divided into 8 groups of 5 teams. The first two teams in each group qualify for the knock-out stage. In this new format, each team will have to play one more game in the group stage, and the last round of games will also include a “passive” team due to the odd number of teams per group. This passive team will have played all its games previously and will have to watch the other four teams play during the last round.

As in the previous cases, we carry out a Monte Carlo simulation to determine which combination of games should be played in the last round. The results appear in Table 8, where the attribute “game quality” is that defined previously, “passive team” is the team that has already played four games and “setting” corresponds to the matching combination of the four teams left. The results are quite intuitive:

- Setting 3 produces far worse results than settings 1 and 2. This is the one in which the strongest teams left play against each other, and the weakest two play each other.
- The choice of the passive team affects the quality of results via two mechanisms:
  1. The stronger the passive team, the more chances it has to win its games and be qualified for the next round. There will consequently only be one spot left for the other four teams playing in the last round. This increases the chances of there being some teams that cannot qualify among the four left, leading to uninteresting last-round games. Based on this mechanism, the weaker the passive team the better the quality of last-round games.
  2. The second effect is related to the position of the four teams left relative to the passive team. If the passive team is the weakest “pot E” (or the strongest “pot A”), then it is likely that the four teams left had positive (negative) results against the passive team. What matters here
Table 8: Results of the Monte Carlo simulations with 5000 iterations.

| Setting: | 1          | 2          | 3          |
|----------|------------|------------|------------|
|          | Game quality: |            |            |
|          | Comp | Col | Stkless | Comp | Col | Stkless | Comp | Col | Stkless |
| Passive team |                  |            |            |
| Pot A    | 48.31% | 0.10% | 51.59% | 47.85% | 0.39% | 51.76% | 45.49% | 1.09% | 53.42% |
| Pot B    | 48.61% | 0.21% | 51.18% | 46.83% | 0.81% | 52.36% | 43.42% | 1.63% | 54.95% |
| Pot C    | 49.87% | 0.33% | 49.80% | 48.28% | 0.67% | 51.05% | 37.70% | 3.22% | 59.08% |
| Pot D    | 52.33% | 0.83% | 46.84% | 49.45% | 1.31% | 49.24% | 41.29% | 1.91% | 56.80% |
| Pot E    | 63.81% | 1.12% | 35.07% | 61.47% | 1.46% | 37.07% | 53.05% | 1.94% | 45.01% |

is not whether they won or lost, but that these four teams had the same outcome against the passive team. They thus start the last-round games with the same head start, leading to competitive last-round games. On the contrary, if the passive team is from pot C, the teams coming from pot A and B were likely to have won against it, while the teams from pot D and E were likely to have lost. As such, the last round starts with a considerable gap between the teams in terms of points, so that some teams may be already qualified (or not be able to qualify). This phenomenon stands out clearly in setting 3, where the pot A and B teams have played against the weakest teams and play each other in the last round. It is likely that they have both already qualified before the last round, making this game stakeless. Consequently, the more extreme is the passive team (pots A or E), the better is the last round of games.

Based on the previous results, we conclude that setting 3 should be avoided (setting 1 is the optimal one), while the pot-E team should be the passive team in the last round.

6. Conclusion. This article has developed an assessment method of the competitiveness and attractiveness of the last round of games in the FIFA World Cup group stages. Using this new method, we note that the scheduling of games, in particular the choice of teams playing each other in the last round, is crucial for obtaining exciting and fair last-round games. Furthermore, our results underline that the point-attribution system has no impact in small groups. The optimal game schedule depends on the format of the World Cup, but our clear recommendation is that FIFA should drop its current schedule-randomization process in the draw for the group matches. In the current World Cup format, we recommend that the last group games should be pot-A teams against pot-D teams, and pot-B teams against pot-C teams. Scheduling these games in advance has no negative impact on any aspect of the competition (logistics, fairness etc.), but increases the attractiveness and competitiveness of the last round. In the forthcoming 48 teams in groups of 3 format, the “pot-A” team should be the passive team in the last round.

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