Further evidence for three-nucleon spin-orbit interaction in isotope shifts of $Z =$ magic nuclei

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(Dated: April 29, 2015)

We pointed out [Phys. Rev. C 91, 021302(R)] that the isotope shifts of the Pb nuclei, the kink at $N = 126$ in particular, can be well described by the Hartree-Fock-Bogolyubov calculations if a density-dependent LS interaction derived from the $3N$ interaction is incorporated. Effects of the density-dependence in the LS channel on the isotope shifts are extensively investigated for the Ca, Ni and Sn isotopes, using the semi-realistic M3Y-P6 interaction and its LS modified variant M3Y-P6a, as in the Pb case. It is found that almost equal charge radii between $^{40}$Ca and $^{48}$Ca are reproduced, as well as the isotope shifts in a long chain of the Sn nuclei, owing to the density-dependence in the LS channel. A kink is predicted at $N = 82$ for the isotope shifts of the Sn nuclei, in clear contrast to the interactions without the density-dependence.

PACS numbers: 21.10.Ft, 21.30.Fe, 21.60.Jz

I. INTRODUCTION

In the nuclear shell structure, which is formed by a series of the nucleonic single-particle (s.p.) orbits under the nuclear mean field (MF), the spin-orbit ($\ell$s) splitting plays an essential role. However, though known from the data, size of the $\ell$s splitting has been difficult to be accounted for only by the two-nucleon ($2N$) interaction [1]. Recent development of the chiral effective-field theory ($\chi$EFT) [2] indicates that the three-nucleon ($3N$) interaction gives significant density-dependence in an LS channel when it is converted to an effective 2N interaction [3, 4], and that this may account for the missing part of the $\ell$s splitting [4, 5].

There have been precise measurements on isotope shifts [6, 7]. As well as the $\ell$s splitting, they have supplied problems in nuclear structure theory that have not been solved for decades. As an important example, the isotope shifts of the Pb nuclei show a conspicuous kink at $N = 126$ in particular, can be well described by the Hartree-Fock-Bogolyubov calculations if a density-dependent LS interaction derived from the $3N$ interaction is incorporated. Effects of the density-dependence in the LS channel on the isotope shifts are extensively investigated for the Ca, Ni and Sn isotopes, using the semi-realistic M3Y-P6 interaction and its LS modified variant M3Y-P6a, as in the Pb case. It is found that almost equal charge radii between $^{40}$Ca and $^{48}$Ca are reproduced, as well as the isotope shifts in a long chain of the Sn nuclei, owing to the density-dependence in the LS channel. A kink is predicted at $N = 82$ for the isotope shifts of the Sn nuclei, in clear contrast to the interactions without the density-dependence.

The isotope shifts of the Pb nuclei, without degeneracy between the $1g_{9/2}$ and $0i_{11/2}$ orbitals [12]. This effect on the isotope shifts provides us with evidence for the 3N LS interaction that is practically independent of the $\ell$s splitting. Thus the two problems, origin of the $\ell$s splitting and the isotope shifts in Pb, may simultaneously be solved by the 3N LS interaction. In this paper we extensively examine effects of the density-dependent LS channel on the isotope shifts of other $Z =$ magic nuclei.

Although a density-dependent LS interaction was argued in Ref. [13] motivated to account for the model-dependence of the isotope shifts in the Pb nuclei, primary origin of the difference between the relativistic and the Skyrme approaches turns out to be the isospin-dependence of the LS channels. Since then the density-dependence has been considered in the LS channel only in a limited number of studies [14], and its influence on physical quantities other than energies has not been explored sufficiently.

II. FRAMEWORK

We implement the spherical Hartree-Fock-Bogolyubov (HFB) calculations for the Ca, Ni and Sn nuclei, as for Pb in Ref. [12]. The computational method is an extensive application of the Gaussian expansion method [15], and has been summarized in Ref. [16]. The effective Hamiltonian is comprised of the nuclear, Coulomb and center-of-mass (c.m.) parts: $H = H_N + V_C - H_{c.m.}$, The nuclear part $H_N$ is taken to be non-relativistic and isoscalar,

$$H_N = K + V_N; \quad K = \sum_i \frac{p_i^2}{2M}, \quad V_N = \sum_{i \neq j} v_{ij}, \quad (1)$$

with $i$ and $j$ representing the indices of individual nucleons. We set the nucleon mass to be $M = (M_p + M_n)/2$.
throughout this paper, where \( M_p \) (\( M_n \)) is the measured mass of a proton (a neutron) \[^{17}\]. In consistency with \( K \), the c.m. part is \( H_{\text{c.m.}} = P^2/2AM \) and contains one- and two-body terms. \( P = \sum_i p_i \) is the total momentum and \( A (= Z + N) \) is the mass number. Effective interactions considered in this paper have the following form:

\begin{align*}
v_{ij} &= v_{ij}^{(C)} + v_{ij}^{(LS)} + v_{ij}^{(TN)} + v_{ij}^{(CP)} + v_{ij}^{(LSP)}; \\
v_{ij}^{(C)} &= \sum_n (t_n^{(SE)} P_{SE} + t_n^{(TE)} P_{TE} + t_n^{(SO)} P_{SO} + t_n^{(TO)} P_{TO}) f_n^{(C)}(r_{ij}) |L_{ij}, \cdot (s_i + s_j), \\
v_{ij}^{(LS)} &= \sum_n (t_n^{(LS)} P_{TE} + t_n^{(LSO)} P_{TO}) f_n^{(LS)}(r_{ij}) L_{ij} \cdot (s_i + s_j), \\
v_{ij}^{(TN)} &= \sum_n (t_n^{(TNE)} P_{TE} + t_n^{(TNO)} P_{TO}) f_n^{(TN)}(r_{ij}) r_{ij}^2 S_{ij}, \\
v_{ij}^{(CP)} &= C^{(SE)}(\rho_{ij}) P_{SE} + C^{(TE)}(\rho_{ij}) P_{TE} \delta(r_{ij}), \\
v_{ij}^{(LSP)} &= 2i D[\rho_{ij}] \rho_{ij} \times \delta(r_{ij}) p_{ij} \cdot (s_i + s_j).
\end{align*}

Here \( s_i \) denotes the spin operator (for the \( i \)-th nucleon), \( r_{ij} = r_i - r_j, r_{ij} = |r_{ij}|, p_{ij} = (p_i - p_j)/2, L_{ij} = r_{ij} \times p_{ij}, \) \( R_{ij} = (r_i + r_j)/2, S_{ij} = 4 [s_i \cdot r_{ij} (s_j \cdot r_{ij}) - s_i \cdot s_j] \) with \( r_{ij} = r_i/r_{ij}, \) and \( \rho(r) \) is the nucleon density. \( P_y \) stands for the projection operator on the channel \( Y \) (\( Y = \text{SE}, \text{TE}, \text{SO} \) or \( \text{TO} \)) of the two-nucleon states,

\begin{align*}
P_{SE} &= \frac{1 - P_\sigma}{2} + \frac{1 - P_\tau}{2}, \\
P_{TE} &= \frac{1 + P_\sigma}{2} - \frac{1 + P_\tau}{2}, \\
P_{SO} &= \frac{1 + P_\sigma}{2} - \frac{1 + P_\tau}{2}, \quad P_{TO} = \frac{1 + P_\sigma}{2} + \frac{1 + P_\tau}{2},
\end{align*}

where \( P_\sigma \) (\( P_\tau \)) stands for the spin (isospin) exchange operator. In the former papers \[^{18}^{21}\] \( v^{(C)} \) was denoted by \( v^{(DD)} \). The density-dependent functional in \( v^{(C)} \) is assumed to be \( C^{(Y)}(\rho) = t^{(Y)}_\rho [\rho(\rho)]^{\alpha^{(Y)}}, \) where \( t^{(Y)}_\rho \) and \( \alpha^{(Y)} \) are parameters (\( Y = \text{SE} \) or \( \text{TE} \)).

The Michigan-three-range-Yukawa (M3Y) interactions were obtained from the \( G \)-matrix \[^{21}^{22}\], and have been widely applied to nuclear reactions. By modifying the M3Y-Paris interaction \[^{22}\] phenomenologically, the semi-realistic M3Y-Pn interactions have been developed \[^{20}^{23}\] and applied to nuclear structure studies in the mean-field (MF) \[^{20}^{24}^{26}\] and random-phase (RPA) \[^{27}^{28}\] approximations. In the M3Y-Pn interactions, the Yukawa function \( f_n^{(X)}(r) = e^{-\mu_n r}/\mu_n r \) is adopted for density-independent terms \( v^{(X)} \) (\( X = \text{C}, \text{LS} \) and \( \text{TN} \)) in Eq. (2). The modification has been made so as to obtain the saturation properties and the \( \ell s \) splitting reasonably. The \( v^{(CP)} \) term is relevant to the former, representing a part of the \( 3N \) force effects. Responsible for the shell structure, the \( \ell s \) splitting is significant to describing nuclear structure. Except M3Y-P6a whose results we shall mainly discuss in this paper, we have enhanced \( v^{(LS)} \) of the original M3Y-Paris interaction, in the absence of \( v^{(LSP)} \), to reproduce s.p. level sequence. Keeping the tensor channels \( v^{(TN)} \) of the M3Y-Paris interaction, we obtain \( Z \)- or \( N \)-dependence of the shell structure reasonably well, as exemplified by the level inversion of \( 20 \delta_{3/2} \) and \( 1 \delta_{1/2} \) from \( ^{40}\text{Ca} \) to \( ^{48}\text{Ca} \[^{27}\]. Among several parameter-sets, we have found that M3Y-P6 gives prediction of magic numbers compatible with most available experimental data, in wide range of the nuclear chart including unstable nuclei \[^{26}\]. We take M3Y-P6 as a yardstick for investigating effects of the \( 3N \) interaction. The values of the parameters in M3Y-P6 have been given in Ref. \[^{26}\].

The \( \chi \) EFT derives density-dependence in the LS channel as an effect of the \( 3N \) interaction \[^{2}^{2} \], which may complement the \( 2N \) LS interaction with respect to the \( \ell s \) splitting. So as to be consistent with the \( \chi \) EFT, we have introduced \( v^{(LSP)} \) in Ref. \[^{12}\], instead of enhancing \( v^{(LS)} \), which yields a variant of the M3Y-P6 interaction called M3Y-P6a. However, since the currently available \( \chi \) EFT is not yet convergent at \( \rho \approx \rho_0 \), where \( \rho_0 \) denotes the saturation density, we do not completely follow the \( \chi \) EFT results in quantitative respect. The functional \( D[\rho] \) is taken to be

\begin{equation}
D[\rho(r)] = -w_1 \frac{\rho(r)}{1 + d_1 \rho(r)}.
\end{equation}

The \( d_1 \) term of the denominator works to avoid instability for increasing density, whereas the results are not sensitive to \( d_1 \) and we fix it to be \( d_1 = 1.0 \text{fm}^3 \). Since the effective interactions have been more or less adjusted to the observed \( \ell s \) splitting, size of the \( \ell s \) splitting should not change much even when the density-dependence is taken into account. For this reason the remaining parameter \( w_1 \) has been fitted to the splitting of the \( n0i \) orbits obtained with M3Y-P6 at \( ^{208}\text{Pb} \[^{12}\]. Note that all the parameters except \( v^{(LS)} \) and \( v^{(LSP)} \) are identical to those of M3Y-P6. We shall apply this M3Y-P6a interaction to other \( Z = \text{magic nuclei} \).

Since \( v^{(LSP)} \) in Eq. (2) has zero-range, its contribution to the total energy can be represented by a functional of local currents. The particle-hole terms, which appear in the Hartree-Fock (HF) regime, are summarized as

\begin{equation}
E^{(LSP)}_{\text{PH}} = \frac{1}{4} \int d^3r D[\rho(r)] \left\{ \rho(r) \nabla \cdot J(r) + \sum_{\tau = p,n} \rho_{\tau}(r) \nabla \cdot J_{\tau}(r) \right. \\
+ i J(r) \cdot i J(r) + i \sum_{\tau = p,n} J_{\tau}(r) \cdot J_{\tau}(r) \\
- i J_{\tau}^{*}(r) \cdot J(r) - i \sum_{\tau = p,n} J_{\tau}^{*}(r) \cdot J_{\tau}(r) \\
- Q(r) \cdot \sigma(r) - \sum_{\tau = p,n} Q_{\tau}(r) \cdot \sigma_{\tau}(r) \right\}
\end{equation}

where the density \( \rho(r) \) and the other local currents are defined by

\begin{align*}
\rho(r) &= \sum_{\tau = p,n} \rho_{\tau}(r), \\
\rho_{\tau}(r) &= \sum_{\alpha, \beta \in \tau} \rho_{\alpha \beta} \phi_{\beta}(r) \phi_{\alpha}(r), \\
J(r) &= \sum_{\tau = p,n} J_{\tau}(r), \\
J_{\tau}(r) &= -i \sum_{\alpha, \beta \in \tau} \rho_{\alpha \beta} \phi_{\beta}(r) \nabla \phi_{\alpha}(r), \\
Q(r) &= \sum_{\tau = p,n} Q_{\tau}(r), \\
Q_{\tau}(r) &= i \sum_{\alpha, \beta \in \tau} \rho_{\alpha \beta} \nabla \phi_{\beta}(r) \times \nabla \phi_{\alpha}(r),
\end{align*}
\[ \sigma(r) = \sum_{\tau=p,n} s_{\tau}(r), \quad \sigma_r = 2 \sum_{\alpha,\beta \in \tau} g_{\alpha\beta} \phi_{\beta}^\dagger(r) s \phi_{\alpha}(r), \]

\[ J(r) = \sum_{\tau=p,n} J_{\tau}(r), \quad J_{\tau}(r) = 2i \sum_{\alpha,\beta \in \tau} g_{\alpha\beta} \phi_{\beta}^\dagger(r) s \nabla \phi_{\alpha}(r) \]

with the s.p. basis function \( \phi_{\alpha}(r) \) and the one-body density matrix \( g_{\alpha\beta} = \langle \Phi | a_{\beta}^\dagger a_{\alpha} | \Phi \rangle \) for the HF or HFB state \( | \Phi \rangle \). It should be noticed that, because of the presence of \( D[\rho] \), integration by parts does not simplify the expression of \( E_{ph}^{(LS)} \).

Under the spherical symmetry, \( Q_{\tau}(r) = \sigma_{\tau}(r) = 0 \), \( i\hat{J}_{\tau}(r) = \nabla \rho_{\tau}(r) \) and \( J_{\tau}(r) = J_{\tau}^s(r) = \langle r/r^2 \rangle \sum_{\alpha,\beta \in \tau} g_{\alpha\beta} \phi_{\beta}^\dagger(r) (2\ell \cdot s) \phi_{\alpha}(r) \) with \( \ell = r \times p \) (\( \phi_{\alpha} \), \( \phi_{\beta} \) are postulated to be spherical bases). \( E_{ph}^{(LS)} \) yields the \( \ell s \) potential as follows:

\[ -\frac{1}{2r} \left[ D[\rho(r)] \frac{d}{dr}(\rho(r)+\rho_{\tau}(r)) + \frac{1}{2} \frac{\delta D[\rho(r)]}{\delta \rho} (\rho(r)+\rho_{\tau}(r)) \right] \]

\[ \text{We apply the same Hamiltonian to the pairing channels in the HFB calculations. Contribution of } \psi^{(LS)} \text{ to the pair energy can be expressed as} \]

\[ E_{pp}^{(LS)} = \frac{i}{2} \int d^3r D[\rho(r)] \sum_{\tau=p,n} \hat{J}_{\tau}(r) \cdot [s \times \hat{J}_{\tau}(r)], \]

\[ \text{where} \]

\[ \hat{J}_{\tau}(r) = -i \sum_{\alpha,\beta \in \tau} \kappa_{\alpha\beta} \left\{ \phi_{\beta}(r) [\nabla \phi_{\alpha}(r)] - [\nabla \phi_{\beta}(r)] \phi_{\alpha}(r) \right\} \]

\[ \text{with the pairing tensor } \kappa_{\alpha\beta} = \langle \Phi | a_{\beta}^\dagger a_{\alpha} | \Phi \rangle \text{ and the nucleon spins coupled to } 1 \text{ as indicated by the subscript. The scalar } (\cdot) \text{ and the vector } (\times) \text{ products in Eq. 8 apply both to the ordinary space and the spin space.} \]

We primarily apply the semi-realistic M3Y-P6a interaction, and compare its results with those of M3Y-P6 to exhibit effects of the density-dependence in the LS channel. As mentioned above, the strength parameter \( \omega_{21} \) in M3Y-P6a has been determined so as to equate the \( n\bar{n} \) splitting at \( ^{208}\text{Pb} \) to its counterpart in M3Y-P6. This makes influence on other \( \ell s \) splitting insignificant as well. For instance, the \( n\bar{n} \) splitting at \( ^{40}\text{Ca} \) decreases merely by 4% if replacing M3Y-P6 by M3Y-P6a. The equal-filling approximation is applied to odd-\( N \) nuclei, whose ground states have one quasiparticle. The Gogny-D1M interaction is employed to show that qualitative features of M3Y-P6 are shared with the other interactions that lack density-dependence in the LS channels.

### III. RESULTS AND DISCUSSIONS

The frequency difference of corresponding deexcitations among isotopes is converted to the difference in mean-square (m.s.) charge radii of nuclei. It is hence

![FIG. 1.](image.png)

FIG. 1. (Color online) Isotope shifts of the Ca nuclei \( \Delta(r^2)^p(A_{\text{Z}}) \), obtained from the HFB calculations with M3Y-P6a (red solid line), in comparison to those with M3Y-P6 (green dashed line) and D1M (thin brown dot-dashed line). Experimental data are quoted from Ref. 31 (crosses).

Because \( \langle r^2 \rangle_c(p) \) is canceled out, we denote the isotope shift by \( \Delta(r^2)^p(A_{\text{Z}}) = \langle r^2 \rangle_c(A_{\text{Z}}) - \langle r^2 \rangle_c(A_{\text{Z}}), \) in this paper, with \( ^{40}\text{Z} \) taken as a reference.

#### A. Ca isotopes

We take \( ^{40}\text{Ca} \) as a reference nuclide for the Ca (i.e. \( Z = 20 \)) isotopes, defining isotope shifts as \( \Delta(r^2)^p(\text{A}_{\text{Ca}}) = \langle r^2 \rangle_c(\text{A}_{\text{Ca}}) - \langle r^2 \rangle_c(^{40}\text{Ca}) \). The HFB results are depicted and compared with the data in Fig. 1. It is commented that the weak instabilities against the octupole and the pairing correlations in \( ^{40}\text{Ca} \) are lifted with M3Y-P6a. The inversion of the \( p0d_{3/2} \) and \( p1s_{1/2} \) levels from \( ^{40}\text{Ca} \) to \( ^{48}\text{Ca} \) is reproduced with M3Y-P6a as well as with the previous interactions [25], in which \( \psi^{(TN)} \) plays a significant role.

As doubly magic nuclei, both \( ^{40}\text{Ca} \) and \( ^{48}\text{Ca} \) are expected to be well described within the HF regime. Although it has been known that their charge radii are close to each other, this property has been difficult to be reproduced by self-consistent nuclear structure calculations so far. It is interesting to see \( \Delta(r^2)^p(^{48}\text{Ca}) \approx 0 \), i.e. \( \langle r^2 \rangle_c(^{48}\text{Ca}) \approx \langle r^2 \rangle_c(^{40}\text{Ca}) \), in the M3Y-P6a result. Attraction from the neutrons occupying \( 0f_{7/2} \) determines
\(\Delta (r^2)_p(48\text{Ca})\). As illustrated for \(n\ell i\) orbits in Fig. 1 of Ref. \([12]\), the density-dependence in the LS channel tends to shrink the \(j = \ell + 1/2\) orbits while extends the \(j = \ell - 1/2\) orbits. Therefore the m.s. radius of neutrons occupying the \(0f_{7/2}\) orbit comes smaller; 0.23 fm\(^2\) decrease at \(^{40}\text{Ca}\) when we switch from M3Y-P6 to M3Y-P6a. Moreover, the m.s. radius of \(n0f_{7/2}\) increases from \(^{40}\text{Ca}\) to \(^{48}\text{Ca}\) only by 0.035 fm\(^2\) with M3Y-P6a, in contrast to 0.365 fm\(^2\) with M3Y-P6. This is interpreted as the neutrons on \(0f_{7/2}\) feel mutual attraction and the density-dependent LS channel blocks them to distribute more broadly, which further suppresses \(\langle r^2 \rangle_p\). These effects lead to the result \(\Delta (r^2)_p(48\text{Ca}) \approx 0\). Thus the very small difference in the charge radii between \(^{40}\text{Ca}\) and \(^{48}\text{Ca}\) can be regarded as evidence of the density-dependence of the LS channel originated from the 3N interaction.

The isotope shifts of \(^{42-46}\text{Ca}\) show sizable deviation from those of \(^{40,48}\text{Ca}\). Since \(\langle r^2 \rangle_p(\text{Ca})\) varies almost linearly from \(^{40}\text{Ca}\) to \(^{48}\text{Ca}\) in the spherical HFB calculations, none of the interactions including M3Y-P6a reproduce the observed \(N\)-dependence of \(\Delta (r^2)_p(\text{Ca})\) in \(N = 22 - 26\). This discrepancy may be ascribed to effects beyond MF, including influence of the \(\alpha\)-clustering. On the contrary, the rising of \(\Delta (r^2)_p(\text{Ca})\) from \(^{48}\text{Ca}\) to \(^{50}\text{Ca}\) in the M3Y-P6a result is in good agreement with the data, while not so good in the M3Y-P6 and D1M results. This also supports presence of the density-dependent LS interaction.

### B. Ni isotopes

For the Ni (\(i.e.\) \(Z = 28\)) isotopes we use \(^{60}\text{Ni}\) as a reference, defining isotope shifts as \(\Delta (r^2)_p(\text{Ni}) = \langle r^2 \rangle_p(\text{Ni}) - \langle r^2 \rangle_p(\text{Ni})\). The results are depicted in Fig. 2.

We obtain steeper slope of \(\Delta (r^2)_p(\text{Ni})\) with M3Y-P6a than with M3Y-P6 in \(28 \leq N \leq 40\). This is attributed to contribution of neutrons occupying \(n0f_{5/2}\), which distribute more broadly when the density-dependence is incorporated in the LS channel. Although the HFB results with M3Y-P6a are in good agreement with the existing data, it is not obvious to tell whether the density-dependent LS channel improves the theoretical results on \(\Delta (r^2)_p(\text{Ni})\) or not. It is desired to measure neutron-rich or neutron-deficient isotopes, \(e.g.\) \(^{56}\text{Ni}\).

### C. Sn isotopes

For the Sn (\(i.e.\) \(Z = 50\)) isotopes \(^{120}\text{Sn}\) is adopted as a reference nuclide, by which isotope shifts are defined as \(\Delta (r^2)_p(\text{Sn}) = \langle r^2 \rangle_p(\text{Sn}) - \langle r^2 \rangle_p(\text{Sn})\). The results are presented in Fig. 3.

We find that \(\Delta (r^2)_p(\text{Sn})\) is well described with M3Y-P6a in a long chain of the Sn isotopes, owing to the density-dependence in the LS channel. The density-dependence gives narrower distribution of neutrons occupying the \(0h_{11/2}\) orbit, whose attraction reduces \(\Delta (r^2)_p(\text{Sn})\) in \(N > 70\).

It is remarked that a kink is predicted at \(N = 82\) with the M3Y-P6a interaction. Origin of this kink is accounted for analogously to the Pb case. Attraction from neutrons occupying \(0h_{11/2}\) contributes to the broader proton distribution. While occupation probability on \(n0h_{9/2}\) is negligibly small in \(N \leq 82\), \(n0h_{9/2}\) is partially occupied in \(N > 82\) owing to the pairing correlation, although the lowest s.p. level above \(N = 82\) is \(n1f_{7/2}\). The relatively large m.s. radius of \(n0h_{9/2}\), for which the density-dependence in the LS channel is responsible, produces the kink at \(N = 82\). The reduction of the radii in \(N < 82\) as an effect of the density-dependent LS channel on \(n0h_{11/2}\) makes the kink more conspicuous. This kink in \(\Delta (r^2)_p(\text{Sn})\) at \(N = 82\) is in clear contrast to the results of M3Y-P6 and D1M which have no density-dependent LS channel. Thus the density-dependence in the LS channel is essential to the kink. Future measurements with respect to this kink (\(i.e.\) \(\Delta (r^2)_p(\text{Sn})\) beyond \(N = 82\)) will be intriguing, which could be a touchstone of the 3N LS effects.
TABLE I. Root-m.s. charge radii of the reference nuclei. Spherical HFB results with M3Y-P6 and M3Y-P6a are compared with experimental data \cite{31}. \((\mu p)\) and \((ep)\) corresponds to the data on \((r^2)_{c}(p)\).

| nuclide   | M3Y-P6 | M3Y-P6a | Exp. |
|-----------|--------|---------|------|
| \(^{16}\mathrm{O}\) \((\mu p)\) | 2.732  | 2.725   | \(2.6991 \pm 0.0052\) |
| \((ep)\)   | 2.743  | 2.737   | \(2.6991 \pm 0.0052\) |
| \(^{40}\mathrm{Ca}\) \((\mu p)\) | 3.491  | 3.487   | \(3.4776 \pm 0.0019\) |
| \((ep)\)   | 3.500  | 3.496   | \(3.500 \pm 0.001\)   |
| \(^{60}\mathrm{Ni}\) \((\mu p)\) | 3.825  | 3.779   | \(3.8118 \pm 0.0016\) |
| \((ep)\)   | 3.834  | 3.787   | \(3.834 \pm 0.001\)   |
| \(^{120}\mathrm{Sn}\) \((\mu p)\) | 4.653  | 4.627   | \(4.6519 \pm 0.0021\) |
| \((ep)\)   | 4.659  | 4.634   | \(4.659 \pm 0.001\)   |
| \(^{208}\mathrm{Pb}\) \((\mu p)\) | 5.493  | 5.464   | \(5.5012 \pm 0.0013\) |
| \((ep)\)   | 5.499  | 5.470   | \(5.499 \pm 0.001\)   |

D. Absolute values of charge radii

Although we mainly focus on the isotope shifts in this paper, it deserves discussing absolute values of the charge radii of the reference nuclei as well. While the finite-size effects of protons are canceled out in the isotope shifts, absolute values of the charge radii are affected by the ambiguity in the charge radius of an isolated proton. A recent compilation \cite{32} gives two values, \((r^2)_{c}(p) = 0.84087 \pm 0.00039\) by \(\mu p\) Lamb shift and 0.8775 \pm 0.0051 by \(ep\) CODATA. The spherical HFB results on the root-m.s. charge radii \((r^2)_{c}(A^N Z)\) with M3Y-P6 and M3Y-P6a are tabulated in Table I obtained from Eq. \ref{eq:10} by taking both of the \(\mu p\) and the \(ep\) values for \((r^2)_{c}(p)\), in comparison to the experimental data \cite{31}.

We find that the density-dependence in the LS channel tends to reduce the radii, by comparing the M3Y-P6a results to the M3Y-P6 ones. Whereas this effect is weak in \(^{16}\mathrm{O}\) and \(^{40}\mathrm{Ca}\) which are so-called LS-closed nuclei, it is stronger in \(^{60}\mathrm{Ni}\), \(^{120}\mathrm{Sn}\) and \(^{208}\mathrm{Pb}\), with 0.02 – 0.05 fm difference between the two interactions. It is reasonable that the effect is minimal in the LS-closed nuclei in which all the \(ls\) partners (the \(j = \ell \pm 1/2\) orbits) are filled, in contrast to the \(jj\)-closed nuclei in which there is a pair of an occupied \(j = \ell + 1/2\) and unoccupied \(j = \ell - 1/2\) orbits. Since the parameters of the central channels have been more or less fitted to the measured radius of \(^{208}\mathrm{Pb}\), the charge radius is well reproduced with M3Y-P6 for this nucleus, and so for \(^{120}\mathrm{Sn}\). This indicates that M3Y-P6a slightly underestimates the charge radii of \(^{120}\mathrm{Sn}\) and \(^{208}\mathrm{Pb}\), although it gives values closer to the measured ones than M3Y-P6 for \(^{16}\mathrm{O}\) and \(^{40}\mathrm{Ca}\). As the MF approaches are expected to be the more appropriate for the heavier nuclei, there may be a room to readjust the parameters in the central channels. Such readjustment could influence the saturation density by 1%.

The neutron-skin thickness, which is usually represented by the difference of the neutron and proton root-m.s. radii \(\sqrt{(r^2)_{n}} - \sqrt{(r^2)_{p}}\) in a single nucleus, attracts interests because it correlates to variation of the symmetry energy for increasing density \cite{33}. Because of their neutron excess and of experimental accessibility, the neutron-skin thickness is frequently argued for \(^{48}\mathrm{Ca}\), \(^{68}\mathrm{Ni}\), \(^{132}\mathrm{Sn}\) and \(^{208}\mathrm{Pb}\). The density-dependence in the LS channel hardly influences this quantity at \(^{208}\mathrm{Pb}\) and at \(^{132}\mathrm{Sn}\); difference between M3Y-P6 and M3Y-P6a is 0.001 fm or less. On the other hand, \(\sqrt{(r^2)_{n}} - \sqrt{(r^2)_{p}}\) is larger by 0.011 fm with M3Y-P6a than with M3Y-P6 at \(^{68}\mathrm{Ni}\) and smaller by 0.014 fm at \(^{48}\mathrm{Ca}\). This is because the effects of the density-dependence tend to be canceled if both of the \(ls\) partners are occupied. Though not very strong, influence of the density-dependence cannot be discarded for certain nuclei in order to argue their neutron-skin thickness to 0.01 fm accuracy.

IV. SUMMARY

We have extensively studied effects of the density-dependent LS interaction, which has been derived from the chiral 3N interaction, on the isotope shifts of the Ca, Ni and Sn nuclei. Since the density-dependence makes the LS interaction stronger in the nuclear interior relative to its exterior, the s.p. wave functions of the \(j = \ell + 1/2\) orbits shrink while those of the \(j = \ell - 1/2\) orbits become broader, if we keep the \(ls\) splitting.

We have implemented the spherical HFB calculations with M3Y-P6a, which contains density-dependence in the LS channel but is identical to M3Y-P6 in the other channels. It is found that almost equal charge radii between \(^{40}\mathrm{Ca}\) and \(^{48}\mathrm{Ca}\) have been described well, although this property has been difficult to be reproduced in the MF calculations so far. Moreover, the isotope shifts of the Sn nuclei are in excellent agreement with the available data. A kink is predicted at \(N = 82\) in the isotope shifts of Sn, in contrast to those obtained from interactions without density-dependent LS channel. Future experiments on the isotope shifts of Sn beyond \(N = 82\) are awaited, as well as on \(^{56}\mathrm{Ni}\) for which predictions significantly depend on interactions.

Absolute values of the charge radii have also been argued. The density-dependence in the LS channel reduces the absolute values, amounting to about 1% except for the LS-closed nuclei. The density-dependence may influence the neutron-skin thickness in \(^{48}\mathrm{Ca}\) and \(^{68}\mathrm{Ni}\) by about 0.01 fm. Both are small but not necessarily negligible.

ACKNOWLEDGMENTS

This work is financially supported in part as Grant-in-Aid for Scientific Research on Innovative Areas, No. 24105008, by The Ministry of Education, Culture, Sports, Science and Technology, Japan, and as Grant-in-Aid for Scientific Research (C), No. 25400245, by Japan Society for the Promotion of Science. Numerical calculations are performed on HITAC SR16000s at Institute of Management and Information Technologies in Chiba University, Information Technology Center in University of Tokyo, Information Initiative Center in Hokkaido Uni-
versity, and Yukawa Institute for Theoretical Physics in Kyoto University.

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