A Quantum Phase Transition in the Cosmic Ray Energy Distribution

A. Widom and J. Swain

Physics Department, Northeastern University, Boston MA USA

Y.N. Srivastava

Physics Department, University of Perugia, Perugia IT

We here argue that the “knee” of the cosmic ray energy distribution at $E_c \approx 1$ PeV represents a second order phase transition of cosmic proportions. The discontinuity of the heat capacity per cosmic ray particle is given by $\Delta c = 0.450196 \, k_B$. However the idea of a deeper critical point singularity cannot be ruled out by present accuracy in neither theory nor experiment. The quantum phase transition consists of cosmic rays dominated by bosons for the low temperature phase $E < E_c$ and dominated by fermions for high temperature phase $E > E_c$. The low temperature phase arises from those nuclei described by the usual and conventional collective boson models of nuclear physics. The high temperature phase is dominated by protons. The transition energy $E_c$ may be estimated in terms of the photo-disintegration of nuclei.

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I. INTRODUCTION

We have recently[1] discussed the power law energy exponent values $\{\alpha\}$ in the cosmic ray particle flux distribution[2, 3]

$$\frac{d^4N}{dt dAd\Omega dE} \approx \frac{1.8 \text{ Nucleons}}{\text{sec cm}^2 \text{ sr GeV}} \left( \frac{1 \text{ GeV}}{E} \right)^{\alpha}. \quad (1)$$

The more detailed measured exponent $\alpha$ in reality takes on two values depending upon the cosmic ray particle energy. For energies lower than the crossing energy $E_c \approx 1$ PeV, the exponent takes on a boson value of 2.701178. For energies higher than the crossing energy the exponent takes on the fermion value of 3.151374. The crossing energy is, of course, the location of the so called “knee” in the energy distribution.

In that the energy distribution depends on the heat and thereby entropy of evaporation of bosons from the cosmic ray source, there exists a quantum phase transition of cosmic proportions at the crossing energy. It is evidently a quantum phase transition since the order parameter involves the difference between Bose and Fermi statistical phases. In virtue of the experimental continuity of the energy distribution and thereby the entropy, the phase transition is higher than first order. To the theoretical and experimental present accuracy, the phase transition is of second order with discontinuities in the second derivatives of the entropy although more complicated non-analytic singularities cannot be ruled out. In Sec.II, the thermodynamic properties of ultra-relativistic ideal gases are reviewed. The heat capacity discontinuity is described in Sec.II A.

The boson phase arises from those evaporating nuclei described by the conventional collective Boson models of nuclear physics[4]. There exist pairing correlations in odd odd nuclei made up of deuterons. Correlations between two spin one deuterons lead to spin zero alpha particles and so forth all within the pairing condensate. The condensate resides near the surface of evaporating high baryon number $A \gg 1$ nuclei. For example, a neutron star itself is merely a nucleus of extremely high baryon number ($A \gg \cdots \gg 1$) with superfluidity (and superconductivity) in the neighborhood of the nuclear surface[5]. The “partons” from the pairing condensate are evidently the nucleons. The partons then turn into fermions for the phase that exists above the crossing energy. For energies above the knee, the cosmic rays must be composed mainly of protons. In order to comprehend the phase transition, it is necessary to understand how photons can photo-disintegrate the compound boson odd odd nuclei ultimately into its nucleon parts[6]. In Sec.II A we employ a simple physical kinetics model of photo-disintegration to estimate the crossing energy. There is satisfactory agreement with experiment. A general view of the quantum cosmic ray phase transition is given in the concluding Sec.IV.

II. IDEAL GAS THERMODYNAMICS

In the ultra-relativistic limit wherein the single particle energies are much larger than either the rest mass energy and the source environmental chemical potentials, the energy $\mathcal{E}$, pressure $P$ and volume $V$ of the gas are related by

$$\mathcal{E} = 3PV. \quad (2)$$

Eq.(2) holds strictly true for “massless” gas at “zero” chemical potential. For such a case, the equation of state for the ideal gas reads

$$PV = N k_B \theta \quad \text{wherein} \quad \theta = \left( \frac{\alpha}{3} \right) T, \quad (3)$$

wherein

$$\alpha_{MB} = 3 \quad (\text{Maxwell – Boltzmann Statistics}),$$
\[ \alpha_{BE} \approx 2.701178 \text{ (Bose – Einstein Statistics),} \]
\[ \alpha_{FD} \approx 3.151374 \text{ (Fermi – Dirac Statistics),} \]
i.e.

(Maxwell – Boltzmann Statistics) \Rightarrow
\[ PV = Nk_BT, \]
(Bose – Einstein Statistics) \Rightarrow
\[ PV \approx 0.9003926 Nk_BT, \]
(Fermi – Dirac Statistics) \Rightarrow
\[ PV \approx 1.050458 Nk_BT, \]
wherein \( N \) is the number of particles. One may verify the Bose-Einstein case by computing the pressure of a photon gas, i.e. black body radiation. One may verify the Fermi-Dirac case by computing the pressure of a gas of Weyl neutrinos.

The pressure from the Bose-Einstein gas is slightly lower than that of a Maxwell Boltzmann gas in that the quantum statistics describes an attraction. The pressure from the Fermi-Dirac gas is slightly higher than that of a Maxwell Boltzmann gas in that the quantum statistics describes a Pauli exclusion repulsion. Although the value change in alpha due to quantum statistics is small, the heat capacity discontinuity is thereby
\[ \Delta \alpha = (\alpha_{FD} - \alpha_{BE}) \approx 0.450196, \]
Eq.\((6)\) is entirely responsible for the quantum cosmic ray second order phase transition.

### A. Heat Capacity

The heat capacity at constant volume obeys
\[ C_{N,V} = \left( \frac{\partial E}{\partial T} \right)_{N,V}. \]
In virtue of Eqs.\((2) \) and \((3) \) in a regime in which \( \alpha \) is uniform, the heat capacity per particle \( c = (C_{N,V}/N) \) is related to the energy per particle \( E = (E/N) \)
\[ E = \alpha k_BT. \]
The heat capacity discontinuity is thereby
\[ \Delta c = k_B \Delta \alpha \approx 0.450196 k_B \]
in virtue of Eq.\((6)\).

### B. Entropy

If \( s(E) \) represents the energy per particle, then \((1/T) = (ds/dE)\) determines the temperature. Eq.\((3)\) then turns into a differential equation; It is
\[ E = \alpha k_B \left( \frac{dE}{ds} \right), \]
with the solution
\[ s(E) = \alpha k_B \ln \left( \frac{E}{\bar{E}} \right). \]
In a single phase regime, the entropy of evaporation determines the energy distribution via
\[ e^{-s(E)/k_B} = \left( \frac{\tilde{\gamma}}{E} \right)^\alpha. \]
On either side of the crossover energy, the distribution of energy is determined by Eq.\((12)\).

### III. PHOTO-DISINTEGRATION

The cross-over energy will here be estimated on the basis of the physical kinetics. From this point of view, one goes from the lower temperature boson phase to the higher temperature fermion phase via photo-disintegration processes.

#### A. Relativistic Kinematics

Consider a process wherein an initial compound nucleus \( I \) is hit by a photon \( \gamma \) and disintegrates into final fragments \( F \),
\[ \gamma + I \rightarrow F, \]
with the four momentum conservation
\[ \hbar k_\gamma + p_I = p_F. \]
The invariant mass squared \( s \) of the reaction Eq.\((13)\) obeys
\[ c^2s = -p_F^2 = -(p_I + \hbar k_\gamma)^2 = -p_I^2 - 2\hbar k_\gamma \cdot p_I \]
for a real (not virtual) photon \( k_\gamma^2 = 0 \). Thus
\[ c^2(M_F^2 - M_I^2) = -2\hbar k_\gamma \cdot p_I. \]
In the rest frame of the initial compound nucleus, the photon frequency \( \omega_\gamma \), obeys \(-k_\gamma \cdot p_I = M_I \omega_\gamma \), so that
\[ \hbar \omega_\gamma = (M_F - M_I) \left[ \frac{M_F + M_I}{2M_I} \right], \]
wherein the term in square brackets on the right hand side of Eq.\((17)\) takes into account the total recoil of the final state fragments.

In the rest frame of the cosmic ray source, the photon frequency is given by
\[ \omega_\gamma(1 - \cos \theta) = \omega_\gamma \left[ \frac{c^2 M_I}{E} \right], \]
\[ \hbar \omega_\gamma(1 - \cos \theta) = \hbar^4 \left[ \frac{M_F^2 - M_I^2}{2E} \right], \]
wherein $E$ is the energy of the initial compound nucleus and $\theta$ is the angle between the photon and compound nuclear three momenta. Averaging over angle and types of final fragmentation products yields \[ \bar{\hbar} \omega_\gamma = c^4 \left[ \frac{M_F^2 - M_I^2}{2E} \right]. \] (19)

The central Eq. (19) of this section enables us to estimate the crossing energy.

**B. Crossing Energy**

We have discussed above that $\bar{\hbar} \omega_\gamma \approx 3k_B T_\gamma$, wherein $T_\gamma$ the radiation temperature of the source. The crossing energy can thereby be estimated as

$$E_c \approx c^4 \left[ \frac{M_F^2 - M_I^2}{6k_B T_\gamma} \right].$$  

The orders of magnitude involved in the estimate Eq. (20) are

$$E_c \sim \left( \frac{100 \text{ MeV}}{10 \text{ eV}} \right)^2 \sim 1 \text{ PeV}.$$  

The crossing energy in Eq. (21) is in satisfactory agreement with experiment.

**IV. CONCLUSION**

We have argued that the well observed knee in the cosmic ray energy spectrum corresponds to a quantum phase transition from a lower temperature boson dominated cosmic ray beam to a higher temperature fermion dominated cosmic ray beam. The lower energy boson dominated regime owes its existence to the collective boson nuclei at the sources. These lead to the quantum symmetry energy contribution to the semi-empirical nuclear mass formula [7]. The higher energy fermion phase owes its existence to the free single nucleons which are the partons from which the bosonic nuclei were constructed. As the energy per particle increases there is a de-confinement quantum statistical phase transition as the bosons are disintegrated into constituent fermions.

There are many important applications of this picture of cosmic ray structure. Some light has been shed upon the structure of ordinary nuclei in terms of collective boson models. The notion of a superconducting shell near the surface of neutron stars has been explored. Clearly, nuclear transmutations and the cosmic ray sources of different chemical elements should now be studied anew. With regard to these future developments, we are at the beginnings. However, we hope that our studies of cosmic ray dynamics have clarified their deep connections to fundamental processes through rather precise determinations of the two correct critical indices along with an estimate of the cross-over energy.

**V. ACKNOWLEDGEMENTS**

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