TESTING EMERGENT GRAVITY WITH MASS DENSITIES OF GALAXY CLUSTERS.

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Draft version July 5, 2018

ABSTRACT

We use a sample of 23 galaxy clusters to test the predictions of an Emergent Gravity (EG) Verlinde (2017) as alternative to dark matter. Our sample has both weak-lensing inferred total mass profiles as well as X-ray inferred baryonic gas mass profiles. Using nominal assumptions about the weak-lensing and X-ray mass profiles, we find that the EG predictions (based on no dark matter) are acceptable fits only near the virial radius. In the cores and in the outskirts, the mass profile shape differences allow us to rule out EG at > 5σ. However, when we account for systematic uncertainties in the observed profiles, we find good agreement for the EG predictions. For instance, if the weak-lensing total mass profiles are shallow in the core and the X-ray gas density profiles are steep in the outskirts, EG can predict the observed dark matter profile from 0.3 ≤ r ≤ 1R200, where R200 is the radius which encloses 200 × the critical density of the Universe. The required X-ray and lensing shapes are within the current observational systematics-limited errors on cluster profiles. We also show that EG itself allows flexibility in its predictions, which can allow for good agreement between the observations and the predictions. We conclude that we cannot formally rule our EG as an alternative to dark matter on the cluster scale and that we require better constraints on the weak-lensing and gas mass profile shapes in the region 0.3 ≤ r ≤ 1R200.

Keywords: Emergent Gravity: galaxy clusters: cosmology.

1. INTRODUCTION

Galaxy clusters provide a unique opportunity to study gravity in the weak-field regime. They are the only astrophysical objects which provide three simultaneous measures of gravity. We can observe the dynamical properties of clusters through the line-of-sight movement of their member galaxies. We can measure their gas content via the Bremsstrahlung X-ray emission. We can observe the distortion of spacetime through the shearing of the shapes of background galaxies. In turn, each of these needs to produce a consistent picture of the underlying gravitational theory. Our standard cosmological paradigm is based on general relativity (GR) in a de Sitter spacetime with a positive cosmological constant, where the majority of the gravitating mass is in a dark form (Frieman et al. 2008). Clusters should be able to test this theory on a case-by-case basis.

This paper is concerned with one of the biggest mysteries in modern cosmology: the origin of the dark matter, which was introduced to explain the deviation from Newtonian dynamics for galaxy rotation curves (Zwicky 1933; Rubin & Ford 1970). Current particle theory favors options such as weakly interacting massive particles, neutrinos and axions (Freese 2017). Alternatively, modified Newtonian dynamics (MOND) has been shown to provide a phenomenological explanation (Milgrom 1983; Famaey & McGaugh 2012).

Recently, there has been an advance in the theory of gravity as an emergent property of the universe. It was shown by Jacobson (1995) that general relativity is an emergent theory and it is possible to derive Einstein’s equations from the concept of entropy of black holes and thermodynamic concepts such as temperature, heat and entropy. The revised emergent gravity (EG) proposal emphasizes the entropy content of space, which could be due to excitations of the vacuum state that manifest as dark energy (Verlinde 2011, 2017). Briefly, this new EG defines the spacetime geometry as due to the quantum entanglement of structure at the microscopic level. Entropy then describes the information content of a gravitating system and its amount is reflected by the number of microscopic degrees of freedom. In Verlinde (2011), anti-de Sitter space was used to derive the surface entropy contribution around matter. In Verlinde (2017), de Sitter spacetime was implemented in the theory which resulted in an additional bulk volume component to the entropy. This volume contribution grows as the scale-size of a system increases. The excess entropy (over the surface component) results in a scale dependence for gravity as manifested through the elastic spacetime, which in turn mimics an apparent dark matter. This apparent dark matter is a result of the presence of baryonic matter.

Given the observational signature of the gas content as the dominant baryonic component in clusters, as well as the observational signature of the spacetime metric through lensing, galaxy clusters provide a rare opportunity to test EG’s predictions. However, the current model proposed in Verlinde (2017) makes some important simplifying assumptions, such as that objects need to be spherically symmetrical, isolated, and dynamically “relaxed”. In addition to that, Verlinde (2017) assumes that the universe is totally dominated by the dark energy which implies that Hubble parameter H(z) is a constant. Working in a small redshift regime is a good approximation to this assumption as it implies small changes to the Hubble parameter, which makes it close to being con-
stant, as well as adds negligible corrections to the measurements due to the small change in the cosmological evolution. The real galaxy clusters which are used in the current work fit well into these assumptions as we do not include merging systems in our sample, such as the Bullet cluster, and clusters with high redshifts. Moreover, we assume that the gravitational potential in the EG framework affects the pathway of photons the same way as it is affected in GR.

In section 2 we introduce the theoretical framework of the EG model. Description of the observational data are presented in section 3. In section 4 the testing procedure is described as well as constraints of the EG model are presented. Discussion of the results and the conclusions are described as well as constraints of the EG model are presented. Description of the observational data are presented in sections 5 and 6.

For the observational data we assume a flat standard cosmology with \( \Omega_M = 0.3, \Omega_\Lambda = 1 - \Omega_M \) and \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\) with \( h = 0.7 \). Throughout the paper we refer to the following quantities and relations. The radius and the mass of the clusters at the point when the density drops to 200\(\rho_c, z\), where \( \rho_c, z = 3H^2/(8\pi G) \) is the critical density of the universe at redshift \( z \) and \( H^2 = H_0^2(\Omega_\Lambda + \Omega_M(1 + z)^3) \).

2. THEORETICAL FRAMEWORK

The full emergent gravity theory is presented in the Verlinde (2017) and here we point out the main ideas of the EG model as well as present the equation which provides connection between baryon matter distribution of the spherically symmetrical isolated non-dynamical system and the apparent dark matter. To do so we adopt the EG description presented in Tortora et al. (2018).

While the original model is derived for an \( n \)-dimensional surface area, we work in four dimensional spacetime and in a spherically symmetric approximation, such that the surface mass density is

\[
\Sigma(r) = \frac{M(r)}{A(r)},
\]

where \( A(r) = 4\pi r^2 \) and \( M(r) \) is the total mass inside a radius \( r \)

\[
M(r) = \int_0^r 4\pi r'^2 \rho(r')dr'.
\]

By incorporating quantum entanglement entropy in a de Sitter spacetime, Verlinde (2017) identified a thermal volume law contribution to the entropy of the universe \( (S_{DE}) \). Heuristically, one can think of emergent gravity as modifying the law of gravity due to the displacement of \( S_{DE} \) in the presence of matter. Tortora et al. (2018) emphasizes the "strain" as the ratio of entropy from the baryonic matter in some volume compared to the entropy from the vacuum expansion of the universe:

\[
\epsilon_M(r) = \frac{S_M}{S_{DE}} = \frac{8\pi G \Sigma_M(r)}{a_0},
\]

where \( a_0 = cH_0 \) is the acceleration scale (Milgrom 1983). In regions of normal matter density with a large number of microscopic states \( \epsilon_M(r) > 1 \), the theory recovers the simple Newtonian equations as a limit to the theory of general relativity. However, as the number of microscopic states becomes small (i.e., in low density regions of the Universe) \( \epsilon_M(r) < 1 \), not all of the de Sitter entropy \( (S_{DE}) \) is displaced by matter. The remaining entropy modifies the normal gravitational laws in the GR weak-field limit (i.e., the Newtonian regime). This gravitational effect can be described by an additional surface density component,

\[
\Sigma_{DM} = \frac{a_0 \epsilon_{DM}}{8\pi G}.
\]

where the subscript \( DM \) refers to the apparent dark matter.

To get the "mass" of the apparent DM one needs to estimate the elastic energy due to the presence of the baryonic matter. The calculations (see Verlinde (2017)) lead to the following inequality

\[
\int_B \epsilon_{DM}^2 dV \leq V_{M_b}(B),
\]

where \( \epsilon_{DM} \) is defined in formula 3 and \( B \) is the spherical region with the area \( A(r) = 4\pi r^2 \) and radius \( r \). The r.h.s. of the inequality is the volume which contains an equal amount of entropy with the average entropy density of the universe to the one which is removed by the presence of baryons,

\[
V_{M_b}(r) = \frac{8\pi GrM_b(r)}{3a_0},
\]

where \( M_b(r) \) is the total mass of the baryonic matter inside some radius \( r \).

Tortora et al. (2018) notes that most of the recent papers on the EG theory focus on the equality in the expression but there is no particular reason to choose this case as it places the upper bound on the amount of the apparent DM. However, if we work at the maximum, we can combine equations 4 and 5 with equality in 3 to get:

\[
M_b(r) = \frac{6}{a_0 r} \int_0^r \frac{GM_D(r')}{r'^2} dr'.
\]

To find the apparent dark matter we can differentiate both sides of the equation to get:

\[
M_{DM}(r) = \left[ \frac{a_0 r^2}{6G} \left( M_b(r) + r \frac{\partial M_b(r)}{\partial r} \right) \right]^{0.5}.
\]

Equations 7 and 8 provide predictions from the theory to test the data against. We use the observed baryonic matter density through the emitting X-ray gas combined with a total (dark matter plus baryonic) inferred from weak lensing to make these tests.

3. DATA

We require inferred total mass and baryonic mass profiles for a large set of galaxy clusters. The weak lensing data are given in the NFW formalism (Navarro et al. 1996). The baryonic data are given via a\( \beta \) profile (Vikhlinin et al. 2006). Because we are going to focus on the virial region of clusters, we simplify the analysis by using a single analytical form for all of the mass profiles. There has been much recent work (Merritt et al. 2006; Miller et al. 2016) on the dark matter mass profiles of clusters in simulations which show that the preferred profile is close to an Einasto form (Einasto 1965). A great advantage of the Einasto parametrization over the NFW or the \( \beta \) form in the context of gravitational studies is
that it predicts a fixed mass of a cluster, i.e. $M(r)$ converges to a particular number. The Einasto profile is described by

$$\rho(r) = \rho_0 \exp(-s^{1/n})$$  \hspace{1cm} (9)

where $s = r/c$, $r_0$ is the scale radius, $\rho_0$ is the normalization and $n$ is the power index. Below, we discuss how we convert between the Einasto and the NFW or $\beta$ models, as well as the implication of this profile homogenization.

3.1. Total Mass Profiles

We are using Sereno meta catalog (Sereno 2015) as a source of weak lensing data of the galaxy clusters. The weak lensing parameters are presented in the NFW form (Navarro et al. 1997)

$$\rho_{\text{NFW}} = \frac{\rho_s}{r_s(1 + r_s/c)^2},$$  \hspace{1cm} (10)

where $\rho_s$ and $r_s$ are two parameters of the model and we can define concentration parameter $c_{200} = r_{200}/r_s$ which describes the overall shapes of the density profiles. Sereno (2015) uses the following relationship between $M_{200}$ and $c_{200}$

$$c_{200} = A \left( \frac{M_{200}}{M_{\text{pivot}}} \right)^B (1 + z)^C,$$  \hspace{1cm} (11)

where $A = 5.71 \pm 0.12$, $B = 0.0834 \pm 0.006$, $C = 0.47 \pm 0.04$, $M_{\text{pivot}} = 2 \times 10^{12} M_\odot/h$ (Duffy et al. 2008).

We convert the NFW profiles to the Einasto form \cite{Sereno2016}. Sereno et al. (2016) has already showed that both the NFW and the Einasto density profiles are nearly identical outside the core region of clusters up to $R_{200}$. We confirm this and find that the Einasto parametrization can recreate a given NFW profile in the region $0.15 \leq r \leq R_{200}$ to less than 1% accuracy. This defines the statistical floor of our total mass profiles. We include additional error on the total mass profiles from the published errors in Sereno (2015).

The use of a specific mass versus concentration relationship adds a systematic uncertainty from the observations. The average concentration of our sample is $<c_{200}> = 3.15$ with specific concentrations in the range $2.57 < c_{200} < 3.58$. We also explore the effect of an additional systematic error in the concentrations on our conclusions.

3.2. Baryon profiles

In what follows we are using only gas density profile as a source of baryon density while neglecting stellar mass contribution as it is around or less than 10% of the overall baryon mass for the clusters with the masses of the clusters we use in our analysis (Giodini et al. 2009; Andreon 2010; Laganà et al. 2013). We will test the assumption of neglecting stellar contribution later in the text. The gas density profiles are taken from several sources \cite{Giles2017; Vikhlinin2006; Giacintucci2017}. Unlike the weak lensing data, the baryon density data do not have uncertainties.

\cite{Giles2017; Vikhlinin2006} use beta profile to infer the baryon density distribution,

$$n_p n_e = n_0^2 \left( \frac{r}{r_c} \right)^{α} \left( \frac{1 + r^2/r_c^2}{1 + r^2/r_c^2} \right)^{β} \left( \frac{1}{1 + r^2/r_c^2} \right)^{γ} \left( \frac{1}{1 + r^2/r_c^2} \right)^{α/2} \left( \frac{1}{1 + r^2/r_c^2} \right)^{β/2},$$  \hspace{1cm} (12)

where $n_p$ and $n_e$ are the number densities of protons and electrons in a gas, $r_c$ is the characteristic radius and $n_0$ is the central density. \cite{Giles2017} uses the same profile but without the second term in the sum, i.e. without $\frac{n_0^2}{(1 + r^2/r_c^2)^{α/2}}$.

To get the actual baryon matter density distribution, relation 12 is used (Vikhlinin et al. 2006)

$$\rho_b = 1.624 m_p (n_p n_e)^{0.5},$$  \hspace{1cm} (13)

where $m_p$ is the proton mass. \cite{Giacintucci2017} uses so called double beta model which provides the number density of the electrons in the gas,

$$n_e = \frac{n_0}{1 + f \left( 1 + \frac{r^2}{r_c^2} \right)^{−1.5 β_1} + f \left( 1 + \frac{r^2}{r_c^2} \right)^{−1.5 β_2}},$$  \hspace{1cm} (14)

where $n_0$ is the central density, the rest of the parameters are free parameters and in order to infer the baryon matter profile the following relation is used (Schellenberger & Reiprich 2017)

$$M_b(r) = 4.576 \pi m_p \int_0^r n_e(r') r'^2 dr'.$$  \hspace{1cm} (15)

We transform the beta profiles into Einasto profiles in the identical manner as the NFW profiles what was described in the previous subsection. The Einasto profile recreates the beta profile with a high precision in the region from around the core until $R_{200}$ (see fig. 3.2). While we chose to transfer beta to the Einasto profile in the region up to $R_{200}$, we could do this procedure with almost identical accuracy in the region up to $2 R_{200}$.

**Figure 1.** Partial difference between Einasto and beta profiles. Blue lines are the partial differences of individual clusters. Red solid line is the mean value and dashed lines are 68% error bars around the mean. As we can see they are almost identical all the way until $R_{200}$ and starts to deviate outside this range. Moreover, the beta profile at average tends to overestimate the mass $M(r)$ since the partial difference is smaller than zero after $R_{200}$. 


We note that like for the case of the weak lensing profiles, the shapes of the baryon profiles are systematically limited. In equation 12 the parameter $\epsilon$ governs the shape of the baryon profile in the outskirts. Large values indicate steeper slopes. Vikhlinin et al. (2006) applies an upper limit of $\epsilon = 5$ and his original sample has a $\langle \epsilon \rangle = 3.24$. On the other hand, the fits to our subset of the cluster data by equation 12 have significantly shallower slopes at $\langle \epsilon \rangle = 1.69$. Uncertainties on $\epsilon$ are not available, and so like concentration in weak lensing NFW fits, we explore systematic errors in this parameter later on.

### 3.3. Dark Matter profiles

In what follows, we treat the weak lensing masses as total masses of the galaxy clusters and the dark matter mass is calculated as

$$M_{DM} = M_{tot} - M_b,$$

where $M_{tot}$, $M_{DM}$ and $M_b$ are the total mass, the dark matter mass and the baryon mass of a cluster.

### 3.4. The Clusters

We list all the 23 clusters in the table 1. The average mass of our set of 23 observed galaxy clusters is

$$\langle M \rangle > 1.14 \times 10^{15} M_\odot$$

while individual masses are in rather broad range ($5.4 \times 10^{14} M_\odot$–$1.89 \times 10^{15} M_\odot$). All of the clusters have rather small redshifts ($< 0.289$) and that fits well into approximation made by the EG theory, i.e. constant Hubble parameter. However, we will still test this assumption later in the current manuscript.

| Cluster name | Redshift | Weak lensing | $M_{200, w}$ ($10^{14} M_\odot$) | $\rho_{w, r}$ ($10^{13} M_\odot$) | $r_{w, r}$ ($10^{-3}$ Mpc) | $n_w$ | Baryon $\rho_{b}$ ($10^{15} M_\odot$) | $r_{b, b}$ ($10^{-3}$ Mpc) | $n_b$ |
|--------------|----------|--------------|--------------------------------|-------------------------------|-----------------|--------|--------------------------------|-----------------|--------|
| A1682        | 0.227    | P07          | 6.05                           | 6.1                           | 6.58             | 4.21   | G17                             | 1.62             | 898    |
| A1423        | 0.214    | OK15         | 6.7                             | 5.8                           | 7.19             | 4.19   | G17                             | 40.5             | 2.08   |
| A2029        | 0.077    | CO4          | 10.28                          | 5.2                           | 8.63             | 4.19   | V06                             | 54.0             | 11.16  |
| A2219        | 0.226    | OK10 / OK15 / A14 | 15.33                  | 4.46                          | 12.27            | 4.13   | G17                             | 4.63             | 634.78 |
| A520         | 0.201    | H15          | 12.75                          | 4.63                          | 11.16            | 4.17   | G17                             | 0.46             | 9710   |
| A773         | 0.217    | OK15 / D06   | 15.45                          | 4.43                          | 12.37            | 4.13   | G17                             | 8.36             | 167    |
| ZwCl3146     | 0.289    | OK10         | 7.94                           | 5.63                          | 8.66             | 4.23   | G17                             | 1170.0           | 0.18   |
| RXJ1720      | 0.16     | OK10         | 5.38                           | 5.46                          | 8.12             | 4.18   | G17                             | 250.0            | 0.71   |
| RXCJ1504     | 0.217    | OK15         | 8.26                           | 5.38                          | 8.35             | 4.17   | G17                             | 1280.0           | 0.09   |
| A2111        | 0.229    | H15          | 8.08                           | 5.38                          | 8.35             | 4.17   | G17                             | 9.49             | 53.5   |
| A611         | 0.287    | OK10         | 8.68                           | 5.19                          | 9.22             | 4.15   | G17                             | 260.0            | 0.63   |
| A697         | 0.281    | OK10         | 15.16                          | 4.47                          | 12.59            | 4.12   | G17                             | 3.16             | 1150   |
| A1689        | 0.184    | U15          | 18.86                          | 4.2                           | 13.72            | 4.12   | G17                             | 311.0            | 0.39   |
| A1914        | 0.166    | H15          | 11.2                           | 4.89                          | 9.9              | 4.16   | G17                             | 74.51            | 17.4   |
| A2261        | 0.224    | OK15         | 18.01                          | 4.25                          | 13.57            | 4.12   | G17                             | 526.0            | 0.11   |
| A1835        | 0.251    | H15          | 16.88                          | 4.35                          | 13.13            | 4.12   | G17                             | 568.0            | 0.49   |
| A267         | 0.229    | OK15         | 9.07                           | 4.35                          | 8.77             | 4.17   | G17                             | 383.0            | 0.22   |
| A1763        | 0.251    | H15          | 14.13                          | 4.48                          | 12.09            | 4.12   | G17                             | 2.19             | 1100   |
| A963         | 0.204    | OK15         | 10.66                          | 4.95                          | 9.79             | 4.15   | G17                             | 2.36             | 1463.4 |
| A383         | 0.189    | OK15         | 8.06                           | 5.54                          | 7.82             | 4.19   | V06                             | 450.0            | 0.19   |
| A2142        | 0.09     | OK08         | 13.63                          | 4.74                          | 10.44            | 4.16   | G17                             | 333.0            | 0.11   |
| RXCJ1229     | 0.234    | OK15         | 7.24                           | 5.07                          | 7.58             | 4.18   | G17                             | 23.8             | 44.3   |
| A2631        | 0.277    | OK15         | 12.34                          | 4.7                           | 11.25            | 4.13   | G17                             | 1.11             | 3680   |

*The original papers are cited above, but actual weak lensing masses (and their respective errors) we use in our analysis were taken from the Sereno (2015) meta catalog. More specifically, Sereno (2015) standardizes the $M_{200}$ masses for the clusters shown above (as inferred from each reference listed in the “weak lensing” column) for the fiducial cosmology mentioned in our introduction.

*The abbreviations in this column refer to the following papers: G17 = Giles et al. (2017), V06 = Vikhlinin et al. (2006), Gi17 = Giacintucci et al. (2017).

*Index w stands for weak lensing in the Einasto parameters

*Index b stands for baryon gas in the Einasto parameters

4. TESTING EMERGENT GRAVITY

We have two ways of comparing the EG model with the data. The first one is based on equation 7 such that we compare the observed baryon mass profile to the one predicted from the “observed” dark matter profile. Recall from Section 3.3 that the observed dark matter profile is actually the total mass profile from weak lensing minus the observed baryon profile. The second approach is based on equation 8 which represents opposite situation. In this case, we use the observed baryon profile to make a prediction for the dark matter profile and compare that to the “observed” dark matter profile.

4.1. Qualitative assessment of the EG model

Figure 2 shows the results of applying equation 7 which makes a prediction for the baryon profile from the dark matter profile. The red line is the observed baryon profiles using the X-ray data and including a 10% additional stellar component. The blue line comes from applying equation 7 using the dark matter mass profile from equation 16. We normalize each cluster baryon profile to the value at the observed weak lensing $R_{200}$ in order to conduct a combined analysis. The solid lines represent the means of the samples and the dashed lines the observed $\sigma$ scatter from the 23 systems. We find that the data (red) and the model (blue) agree at $\sim R_{200}$ and beyond. However, EG predicts that the majority of the baryon mass is...
baryons are enclosed within the cluster core. Specifically, EG predicts that 50% of the baryons are within \( \sim 0.2 \times R_{200} \). However, the observed baryons do not reach 50% until \( \sim 0.5 \times R_{200} \).

Figure 3 shows the results of applying equation \( 8 \) which makes a prediction for the dark matter profile from observed baryon profile. The red line is from the observed dark matter profiles. The blue line comes from applying equation \( 8 \) to the observed baryon profiles. The solid lines represent the means of the samples and the dashed lines the observed 1σ scatter from the 23 systems. We normalize each of the cluster’s dark matter profile to the value at the weak-lensing inferred \( R_{200} \) in order to conduct a combined analysis.

From figures 2 and 3 we find a qualitative agreement between the observations and EG theory. A key success of the theory is the amplitude it predicts, which is close to what we observe near the virial radius. In other words, using just the observed baryons, EG predicts the observed dark matter mass at \( \sim R_{200} \). Likewise, the difference between the total weak-lensing inferred mass and the baryon mass at \( \sim R_{200} \) is what is predicted from EG using just the baryons alone. However, differences become apparent at smaller and larger radii. Unfortunately, the observed baryon profiles are not highly constraining in the core regions and in the outskirts of clusters. The cores of clusters are active environments with varying levels of astrophysical processes which could alter the profiles. Likewise, X-ray surface brightnesses drop steeply beyond \( R_{500} \), to the point where it becomes impossible to constrain the gas density profile out beyond the virial radius. We discuss these issues in the next subsections. In the meantime, we can first apply a more stringent quantitative comparison in the region where the data is more certain.

4.2. Data analysis and statistical constraint of the EG model

To compare the EG model with the data we apply fitting procedure which is based on minimization of \( \chi^2 \)

\[
\chi^2 = \sum_i \frac{(M(r_i) - M_{th}(r_i))^2}{\sigma(r_i)^2}, \tag{17}
\]

where \( M_{th}(r_i) \) is given by the r.h.s. of the equation \( 8 \) (the apparent dark matter prediction by the EG model) while \( M(r_i) \) and \( \sigma(r_i) \) are provided by the weak lensing data. The relevant quantity to compare the model with the data is a reduced \( \chi^2 \) which is calculated as \( \chi^2_{d.o.f.} = \chi^2/N_{d.o.f.} \), where \( N_{d.o.f.} \) is the number of degrees of freedom.

As shown previously, the best qualitative agreement is the radial region around the virial radius. In what follows, we measure each of the cluster mass profiles with a step 0.1\( R_{200} \) and for example in the range from 0.3\( R_{200} \) to \( R_{200} \) that gives us 9 data points per clusters and 184 data points in total as we have 23 clusters in our data sample. The total \( N_{d.o.f.} = 181 \) since the Einasto matter

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3 One can notice strange behaviour in EG predictions at high radii which is especially noticeable on the figure 2 where \( M_h(r) \) starts to decrease at \( \sim 1.5 \times R_{200} \). This result can be derived analytically: equation 7 leads to \( M_h(r) \propto \frac{1}{r^2} \) assuming convergence of \( M_{DM}(r) \) to a constant number at high radii.
density model has three free parameters.

In spite of the fact that at \( R_{200} \) the predicted by the EG model the apparent dark matter is similar to the observed dark matter, quantitatively we find that the profiles predicted by EG differ from the observed profiles by > 5\( \sigma \). The best agreement we find is within the narrow range 0.55\( R_{200} \leq r \leq 0.75 R_{200} \), where the EG model is only ruled out at the 2\( \sigma \) level.

Having uncertainties of the baryon density profiles could not easing the level of the precision of the constraint of the EG model. To confirm this statement we add some error of the baryon profiles by treating \( \sigma(r_i) \) in the formula as a sum of the squares of the errors of the weak lensing and baryon masses, i.e. \( \sigma(r_i)^2 = \sigma_{\text{weak}}(r_i)^2 + \sigma_{\text{bar}}(r_i)^2 \). Placing uncertainties on the baryon matter even half of the uncertainties of the weak lensing data (i.e. \( \sigma_{\text{bar}}(r_i) = 0.5 \sigma_{\text{weak}}(r_i) \)) does not decrease the level of precision in the range 0.3\( R_{200} \leq r \leq R_{200} \) as it is still \( \sim 5 \sigma \). However, with these baryon matter uncertainties the EG model is compatible with the observations at almost 1\( \sigma \) level in the "narrow" range.

Given that the amplitude predicted by EG is reasonably well represented by the model, we focus our comparison on the shape. Fig. 4 shows the mass ratio \( \frac{M_{\text{GR}}}{M_{\text{EG}}} \) of the observed dark matter (\( M_{\text{GR}} \)) and the apparent dark matter (\( M_{\text{EG}} \)) which is predicted by the EG model. One can see that the observed dark matter is almost two times higher than the apparent dark matter in the area close to the cores (0.1\( R_{200} \)) of the galaxy clusters (around 40% higher at 0.3\( R_{200} \)) and it also can be seen that the mass profiles of the dark matter and the apparent dark matter are very different. EG underestimates the dark matter mass in the regions closer to the core while overestimating the mass in the regions beyond approximately 0.9\( R_{200} \). At the current stage we must claim that the EG model is unable to describe the real observational data at Mpc scales.

4.3. Systematic uncertainty from concentration

As it was discussed above (see subsection 3.1), the mass-concentration relation of the galaxy clusters is a source of systematic uncertainty. We can include these systematics in the following way: \( \sigma(r_i) \) in the formula is now a sum of statistical and systematical uncertainties, i.e. \( \sigma(r_i)^2 = \sigma_{\text{weak}}(r_i)^2 + \sigma_{\text{sys}}(r_i)^2 \). We neglect \( \sigma_{\text{sys}}(r_i) \) here as discussion of the baryon uncertainty was done in the previous subsection. We define \( \sigma_{\text{sys}}(r_i) \) as the difference between true value of the \( M_{\text{DM,true}} \), i.e. at the concentration which is given by the data and \( M_{\text{DM,new}} \) at the concentration motivated by [38] (2016),

\[
\sigma_{\text{sys}}(r_i) = M_{\text{DM,true}} - M_{\text{DM,new}}.
\]

Through this technique, we allow the systematic uncertainty in the concentration to impact the uncertainty on the amplitude of the profiles, but not the shape. We consider the effect of systematic uncertainties by concentrations up to \( c_{200,new} = 10 \). We focus our analyses only on the range (0.3\( R_{200} \) \leq r \leq \( R_{200} \)) where the mass densities are measured with the step 0.1\( R_{200} \). The effect of the systematic uncertainty starts to be noticeable at \( c_{200,new} \approx 4.1 \) were the median \( \sigma(r_i)/\sigma_{\text{sys}}(r_i) \approx 5 \). This effect pushes the constraint level down to \( \sim 3 \sigma \) and at \( c_{200,new} = 10 \) the EG model is compatible with the observations at 1\( \sigma \).

4.4. Systematic shape bias from concentration

An alternative approach to simply increasing our mass measurement errors as a result of systematic uncertainties in our \( \chi^2 \) analysis, we can fix the mass measurement
The mass ratio $\frac{M_{GR}}{M_{EG}}$ of the observed dark matter ($M_{GR}$) and the apparent dark matter ($M_{EG}$) which is predicted by the EG model. Solid lines and shaded regions around them are the mean and 68% error bars around the mean. Baryon matter distribution in our sample have rather small steepness which is described by $\epsilon$ in the form $\frac{\epsilon}{\epsilon_{\text{mean}}} = 1.69$ for 20 clusters and zero $\epsilon$ for the three clusters with double beta profiles [14]. However, in general steepness parameter is higher (for example it is $\epsilon_{\text{mean}} = 3.24$ in Vikhlinin et al. 2006). To take that into account we have increased $\epsilon$ of the 20 clusters by 1 (green) and by 2 (red), which made steepness parameter to be $< \epsilon > = 2.69$ and $< \epsilon > = 3.69$ respectively. Blue color corresponds to the implementation of the data with the original steepness parameters.

with our current errors but allow the profiles shapes to be more uncertain. As we can see from the figure 3 if we assume that the cluster weak-lensing inferred masses are unbiased, the EG model becomes more consistent with the data for $c_{200} = 2$. While small, this average value for the NFW concentration of the weak-lensing mass profiles of massive clusters is close to those obtained in simulations (Groener et al. 2016; Klypin et al. 2016; Correa et al. 2015).

4.5. Baryon profile bias

Three clusters from Giacintucci et al. (2017) utilize double beta profile [14] which does not take into account steepness parameter $\epsilon$ in equation 12. The remaining 20 clusters in our sample have average steepness parameter $< \epsilon > = 1.69$ which is significantly smaller than the average steepness parameter $< \epsilon _U> = 3.24$ of Vikhlinin et al. (2006) data set. Increasing $\epsilon$ in our data rotates the apparent DM distribution curve and shifts it upwards which makes the EG prediction of the apparent DM more consistent with the observation of DM (see figure 6). Recent results from Ettori & Balestra (2009); Eckert et al. (2012) suggest that the baryon profiles are in fact much steeper than the original beta profile and in agreement with the high $\epsilon$ values from Vikhlinin et al. (2006).

4.6. Other Systematics

One of the assumptions which was discussed above in the introduction was the fixed value of the Hubble parameter which implies no dependence on the redshifts of the data. To test this assumption we divided by redshifts our data sample of 23 galaxy clusters into two bins, i.e. one bin contained 11 clusters with the lowest redshifts ($< z > = 0.17$) and the second bin contained 12 clusters with the highest redshifts ($< z > = 0.25$). Utilization of

5. DISCUSSION

In this section, we discuss the consequences of the current EG predictions in the context of the observation data. We also explore alternatives to our fiducial analysis which could bring the EG predictions and the data into better agreement.

5.1. Effect on the baryon fraction

Figure 6. The ratio of baryon mass to the total mass of the galaxy cluster as a function of radius of the observed data set of 23 galaxy clusters. Red line and red shaded region represent the baryon fraction of the observed clusters, i.e. $M_b/M_{\text{tot,GR}}$, where $M_b$ is the observed baryon mass, $M_{\text{tot,GR}}$ is the total mass from the weak lensing data and this result correlates with other results (Giodini et al. 2009; Andreon 2010) as we expect to see higher baryon fraction for heavier galaxy clusters and the average mass of the clusters in our sample is high ($< M_{200} = 1.14 \times 10^{15} M_{\odot}$). Green line and green shaded region correspond to the effective baryon fraction which is predicted by the EG model, i.e. $M_b/M_{\text{tot,EG}}$, where $M_{\text{tot,EG}}$ is the total mass predicted by the EG model, i.e. the sum of the apparent dark matter and the baryon matter. Solid lines are the mean values and shaded regions are 68% error bars around the mean. One can observe that the EG model prediction diverge from the observed baryon fraction starting from the cores of the clusters up to $\sim 0.6 R_{200}$ which means that the EG model predicts that the baryon fraction is the biggest in the regions around the core of the clusters while the observations predict the baryon fraction to increase with a distance from the core. Interestingly, the baryon fraction prediction of the EG model agrees well with the baryon fraction which is observed from the CMB (Ade et al. 2016) (see blue flat line) at around $R_{200}$.

Figure 7. The mass ratio $M_{GR}/M_{EG}$ of the observed dark matter ($M_{GR}$) and the apparent dark matter ($M_{EG}$) which is predicted by the EG model. Solid lines and shaded regions around them are the mean and 68% error bars around the mean. Baryon matter distribution in our sample have rather small steepness which is described by $\epsilon$ in the form $\frac{\epsilon}{\epsilon_{\text{mean}}} = 1.69$ for 20 clusters and zero $\epsilon$ for the three clusters with double beta profiles [14]. However, in general steepness parameter is higher (for example it is $\epsilon_{\text{mean}} = 3.24$ in Vikhlinin et al. 2006). To take that into account we have increased $\epsilon$ of the 20 clusters by 1 (green) and by 2 (red), which made steepness parameter to be $< \epsilon > = 2.69$ and $< \epsilon > = 3.69$ respectively. Blue color corresponds to the implementation of the data with the original steepness parameters.

both bins produced almost completely identical results which supports the assumption made.

The second assumption which was made on the data is that the hot gas represents the total baryon mass of the clusters which is not totally true as stars contribute as well. However, stellar mass is less than 10% (Giodini et al. 2009; Andreon 2010; Laganá et al. 2013) of the hot gas for the clusters with the masses we use in this paper ($< M_{200} = 1.14 \times 10^{15} M_{\odot}$). To check this assumption, we increased the baryon mass by 10% which shifted the mass ratio $\frac{M_{GR}}{M_{EG}}$ in figure 4 only by approximately 0.05 − 0.08 or changed this ratio by around 6%. This small shift in the mass ratio not only does not change the precision of constraining the EG model, but also does not change at all the main conclusion of incompatibility of the EG model with the galaxy clusters. So, the assumption of neglecting stellar masses is totally valid.
One of the consequences of the EG model is in the distribution of the baryons in clusters. We can define the effective baryon fraction which is predicted by the EG model by introducing the following ratio

\[ f_{b,EG} = \frac{M_b}{M_{tot,EG}}, \quad (19) \]

where \( M_b \) is the observed baryon mass and \( M_{tot,EG} \) is the total mass which is predicted by the EG model.

The results of the fig. 7 imply that the EG effective baryon fraction is different in many aspects from the observed baryon fraction with the total mass \( M_{tot,GR} \) defined by the weak lensing data. The first difference is the shape of the lines in fig. 7, the EG model has a monotonically decreasing behaviour while the data shows that the baryon fraction is an increasing with the radius function. This means that the EG predicts baryons to be concentrated in the region around the cores of the galaxy clusters while the observations imply that the baryons are actually spread in the broader regions with highest fraction in the outskirts of the clusters. Secondly, the effective baryon fraction is almost twice as high close to the core (at \( r \approx 0.1R_{200} \)) which should be detected as it implies brighter cluster cores than we would observe in GR. In spite of these differences, the EG model predicts correctly the baryon fraction at the distances approximately 0.4R_{200} \( \leq r \leq 0.8R_{200} \). Additionally, the EG model predicts the effective baryon fraction to be close to 15.6% (the number which is expected from the CMB observations [Ade et al. 2016]) at the distances close to \( R_{200} \).

One of the tenets of EG is that there is no particle-like dark matter. In the case of a flat universe, the global baryon contribution to the energy density then becomes of order 5-10%, which dark energy providing the rest. We can build a toy model for how the baryons should be distributed in EG such that at the core of a virialized system one finds \( \sim 100\% \) of the baryons while in the outskirts the EG baryon fraction falls to the global value of 5-10%. This toy model is shown in figure 8 right. If this toy model were to describe how the real baryons are distributed in our Universe, we would find a high level of consistency between what we observe with weak lensing predicts for the dark matter profiles and what EG predicts for the apparent dark matter. This is just a toy model, but it is an example of how one could achieve closer agreement between the EG predictions and the current observations.

5.2. Modifying EG

As opposed to reconsidering the distribution of the baryons inside clusters, one could alter the maximal strain of the EG model as described in Section 3 in equation 5. Recall that we chose equality in the inequality of the EG model in equation 5. We could have chosen some form away from its maximum value. As a new toy model, we propose a modification to the EG model which consists in changing \( \rho^2 \rightarrow \rho_0 \rho^2 \) in the denominator on the r.h.s. of the equation 7. For \( r_0 = 1.2 \text{ Mpc} \), the l.h.s. is smaller than its maximum value until beyond this radius. In the case \( r_0 = 1.2 \text{ Mpc} \) the result is consistent with the observations (see fig. 9). While the modification is based purely on phenomenological ground it might help in developing the theory of the EG model as we can see that the data favor the proposed form instead of the original form. This results leads to the conclusion that while by default equality is chosen in most of the works related to the testing and development of the EG theory, it is not necessarily the right or only choice.

5.3. Combining Systematics

As it was mentioned in the section 4, concentration parameter \( (\epsilon_{200}) \) of the weak lensing and the steepness parameter \( (\epsilon) \) could be changed to make EG be more compatible with the observed data. Moreover, by adjusting both of these parameters at the same time the prediction of the EG model correlates nicely with the observed data (see figure 9).

6. CONCLUSIONS

The EG model is in good agreement with the galaxy data [Brouwer et al. 2017] while it is less successful in describing galaxy clusters [Ettori et al. 2017] where the EG theory was tested only with two clusters. In this
Figure 9. The mass ratio \( \frac{M_{\text{EG}}}{M_{\text{GR}}} \) of the observed dark matter \( M_{\text{GR}} \) and the apparent dark matter \( M_{\text{EG}} \). Solid lines and shaded regions are the means and 68.3\% error bars around the means. Green color corresponds to the phenomenological modification of EG prediction (see subsection 5.4 for motivation of this modification) and baryon matter distribution (increasing steepness parameter by \( \Delta \epsilon = 1.5, c_{200} = 1.5 \)). It can be seen that both modifications presented in the figure make EG model to be consistent with the observed data.

Our results lead to the conclusion that the EG model is a viable alternative to dark matter, given our current level of systematic errors in the observed shape profiles. However, under the nominal assumptions (i.e., without systematics), EG favors a radially decreasing baryon fraction profile which peaks in the cluster core. This is a different baryon fraction profile when compared the standard dark matter model (see [Ade et al. 2016]).

The EG model predicts a flatter shape of the dark matter mass distribution than the observed data, as well as steep X-ray gas density profiles. One of the successes of the model is that the observed weak lensing data and the predicted apparent dark matter are almost identical in the region close to \( R_{200} \).

Finally, we investigate the level of systematic errors needed to reach good agreement between EG and the data. We find that within the current systematic limits, there are combinations of shape profiles which can match EG to the data. Likewise, we investigate whether the EG model itself has the flexibility to better match the data and we find that it does through a lowering of the maximal strain. Given the level of systematic uncertainties in the data, as well as the depth of the theoretical framework, we are unable to formally rule out EG as an alternative to dark matter in galaxy clusters.

7. ACKNOWLEDGMENTS

VH and CJM are supported by the Department of Energy grant [de-sc0013520]. This research has made use of the VizieR catalogue access tool, CDS, Strasbourg, France.

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