Constraints on Generalized Dark Energy from Recent Observations

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Abstract

Effects of a generalized dark energy fluid is investigated on cosmic density fluctuations such as cosmic microwave background. As a general dark energy fluid, we take into consideration the possibility of the anisotropic stress for dark energy, which has not been discussed much in the literature. We comprehensively study its effects on the evolution of density fluctuations along with that of non-adiabatic pressure fluctuation of dark energy, then give constraints on such a generalized dark energy from current observations. We show that, though we cannot find any stringent limits on the anisotropic stress or the non-adiabatic pressure fluctuation themselves, the constrains on the equation of state of dark energy can be affected in some cases by the nature of dark energy fluctuation characterized by these properties. This may have important implications to the strategy to study the nature of dark energy.
1 Introduction

Almost all cosmological observations today suggest that the expansion of the present universe is accelerating. To account for the present cosmic acceleration, one usually assumes an enigmatic component called dark energy. Although many authors have investigated the dark energy to date, we still do not know its nature yet. In most researches of dark energy, one often parameterizes dark energy with its equation of state $w_X = p_X/\rho_X$ where $p_X$ and $\rho_X$ are pressure and energy density of dark energy. Current cosmological observations such as cosmic microwave background (CMB), type Ia supernovae (SNeIa), large scale structure (LSS) and so on can give a constraint on $w_X$. When one puts constraint on the equation of state, it can be assumed to be constant in time for simplicity. However many models of dark energy proposed so far has a time-dependent equation of state. Thus most of recent study of dark energy accommodate its time dependence in some way. Although its time dependence can be complicated in general, it is usually assumed a simple form such as $w_X = w_0 + w_1(1 - a)$ \[1, 2\] when one tries to limit $w_X$ accommodating its time dependence. Some authors have also discussed the constraint on dark energy using the time evolution of the energy density of dark energy itself or Hubble parameter by binning to divide them into several epochs \[3\].

In addition to the time dependence of the equation of state, there is another property of dark energy of which we have to take care. In fact, in various models of dark energy, dark energy can fluctuate and then its fluctuation can affect the cosmic density perturbations such as CMB and LSS. When one specifies a model for dark energy, one can investigate its effects of fluctuation using the perturbation equations in the model. However, because there are many models proposed to date, one may prefer to describe it in a phenomenological way. To describe the nature of fluctuation of dark energy, the (effective) speed of sound $c^2_s$ has been used in many works so far. Some authors have investigated the constraints on the equation of state varying the speed of sound \[4\] and on the speed of sound itself \[5, 6, 7\]. Since the fluctuation of dark energy mostly come into play after when dark energy becomes the dominant component of the universe, the effects of it appear on large scales. Hence, when one tries to limit the speed of sound from observations of CMB, one does not obtain a severe constraint on it because the cosmic variance error is large there. However, some studies have shown that the constrain on the equation of state can vary with the assumption for the perturbation property of dark energy. For example, when one assumes that there is no fluctuation of dark energy, the constraint for the constant equation of state is $w_X = -0.941^{+0.087}_{-0.101}$ \[8\]. However, when one takes into account the perturbation of dark energy, the constraint becomes $w_X = -1.00^{+0.17}_{-0.19}$ where $c^2_s = 1$ is assumed \[8\]. Thus, in this sense, it is important to investigate the properties of dark energy fluctuation from the viewpoint not only from to reveal its fluctuation nature but also from constraining the equation of state for dark energy.

In fact, to parameterize the perturbation property of dark energy, the use of the speed of sound is not enough. If one considers the possibilities of imperfect fluid models for dark energy, one also needs to specify its anisotropic stress in some way. In fact, there are some
models proposed which have such anisotropic stress (for example, [9]). There are also a few works which accommodate such possibilities from a phenomenological point of view [10, 11, 12]. In particular, the authors of Ref. [12] have discussed the implications of the anisotropic stress to observations such as CMB. Although they investigated its effects in some detail, cosmological constrains in such models was not given. Since we have now precise measurements of cosmology, it is worth to investigate constraints for dark energy in such a general setting.

The purpose of the present paper is to discuss a generalized dark energy including the possibilities of anisotropic stress and give the constraints on cosmological parameters assuming such a generalized dark energy.

The structure of this paper is as follows. In the next section, we briefly review the formalism for the investigation of a generalized dark energy. The perturbation equations are also given there. Then, in section 3, we discuss the effects of dark energy fluctuation on CMB, with particular attentions to the anisotropic stress of dark energy. In section 4, we present the constraints on dark energy from current cosmological observations. The final section is devoted to conclusions and discussions.

2 Formalism

In this section, we summarize the formalism to investigate a generalized dark energy. Some detailed descriptions of this issue can be found in Refs. [10, 12].

The background evolution of dark energy can be parameterized by its equation of state $w_X = p_X / \rho_X$. The evolution of the energy density of dark energy is given by solving

$$\rho_X' = -3\mathcal{H}(1 + w_X)\rho_X,$$

where the prime denotes the derivative with respect to the conformal time and $\mathcal{H} = a'/a$ is the conformal Hubble parameter with $a$ being the scale factor. As far as the background evolution is concerned, this equation is enough to specify the effect of dark energy on the evolution of the universe.

However, as mentioned in the introduction, dark energy component itself can fluctuate and affect cosmic density fluctuations. For the fluctuation of dark energy fluid, we have to specify more in addition to the equation of state. Here we briefly summarize what we need to consider a general dark energy. In the rest of the paper, we follow the notation of Ref. [13] and work in the synchronous gauge whose metric perturbations are denoted as $h$ and $\eta$ unless otherwise stated. We also sometimes make use of the gauge-invariant gravitational potentials, which appear as metric perturbations in the conformal Newtonian gauge, to discuss the effects of fluctuation of dark energy.

To specify the nature of dark energy perturbation, we need to give its pressure perturbation and anisotropic stress. For the pressure perturbation, it may be useful to separate it to adiabatic and non-adiabatic parts. When the dark energy fluid is adiabatic, the evolutions of pressure perturbation can be specified with the adiabatic sound speed $c_a^2$ which
is written as

$$c_a^2 \equiv \frac{p'_X}{\rho'_X} = w_X - \frac{w'_X}{3H(1 + w_X)}. \quad (2)$$

Then pressure perturbation is given as \( \delta p = c_a^2 \delta \rho \). If \( w_X < 0 \) and the time evolution of \( w_X \) is small compared with the Hubble expansion rate, \( c_a^2 \) can be negative and the adiabatic pressure can not support density fluctuations. However, in general, non-adiabatic pressure fluctuation may arise. Such a degrees of freedom can be parameterized with the so-called (effective) speed of sound which is defined as

$$c_s^2 \equiv \frac{\delta p_X}{\delta \rho_X} \bigg|_{\text{rest}}. \quad (3)$$

This quantity is usually specified in the rest frame of dark energy and we follow the convention here. A famous example of dark energy models with non-adiabatic pressure fluctuation is a scalar field, called quintessence, which has the speed of sound \( c_s^2 = 1 \). Other models such as k-essence, a scalar field model which can have a non-canonical kinetic term, can have different values for the speed of sound. In the adiabatic case, \( c_s^2 \) and \( c_a^2 \) coincide.

In fact, to consider a general fluid model for dark energy, the speed of sound is not enough to specify its nature. We still have to determine an anisotropic stress for dark energy. Including the anisotropic stress, perturbation equations for density and velocity fluctuations are

$$\delta'_{X} = -(1 + w_X) \left[ k^2 + 9H^2(c_s^2 - c_a^2) \right] \frac{\theta_X}{k^2} - 3H(c_s^2 - w_X)\delta_X - (1 + w_X)\frac{h'}{2} \quad (4)$$

$$\theta'_{X} = -H(1 - 3c_s^2)\theta_X + \frac{c_s^2k^2}{1 + w_X}\delta_X - k^2\sigma_X \quad (5)$$

where \( \delta_X, \theta_X \) and \( \sigma_X \) represent density, velocity and anisotropic stress perturbations for a general fluid, respectively. The anisotropic stress can be presented as the viscosity and damps velocity perturbation in sheer-free frame. Thus, in addition to the sound speed of dark energy \( c_s^2 \), there is another freedom to specify the anisotropic stress \( \sigma_X \) of dark energy. In fact, it may be possible to choose \( \sigma_X \) demanding that it is consistent with viscous damping of velocity perturbation in sheer-free frame. However, in this paper we follow a phenomenological approach of Ref. [10, 12]. Considering the transformation property between frames, \( \sigma_X \) should be gauge invariant thus its evolution can be given by the gauge-invariant combinations of some metric and velocity shear. Taking into account the dissipation, the anisotropic stress can be obtained by solving [10, 12]

$$\sigma'_X + 3H\frac{c_s^2}{w_X}\sigma_X = \frac{8}{3} \alpha \left( \theta_X + \frac{h'}{2} + 3\eta' \right). \quad (6)$$

#1 The relation between \( \sigma_X \) here and \( \pi_g \) in Ref. [10] is

$$\sigma_X = \frac{2}{3} \frac{w_X}{1 + w_X}\pi_g$$
Here $\alpha$ is the viscosity parameter which specifies the nature of an anisotropic stress for dark energy. In fact, in some literatures, another parameter $c^2_{\text{vis}}$ is called the viscosity parameter, which is related to $\alpha$ as

$$\alpha = \frac{c^2_{\text{vis}}}{1 + w_X}.$$

(7)

However, in this paper, we use $\alpha$ as the viscosity parameter. It should be mentioned that $\alpha$ must be kept positive to preserve the physical effects where the viscosity damps the perturbations. Thus, in the following we consider the case with $\alpha > 0$.

We modified CAMB code [14] to include a general dark energy fluid to calculate CMB and matter power spectra. We also checked our results by modifying CMBFAST [15] and found that they both give the same results with good accuracy.

3 Effects of dark energy fluctuation

In this section, we discuss the effects of dark energy fluctuation, paying particular attention to the anisotropic stress of dark energy. Since the dark energy can be a dominant component of the universe only at late times, the effects of dark energy fluctuation is significant on large scales.\textsuperscript{#2} In particular, as we will show in the following, the effects of the anisotropic stress are significant at low multipoles of CMB power spectrum, through the late time integrated Sachs-Wolfe (ISW) effect. Since the ISW effect is driven by the change of the gravitational potential, we start our discussion with investigating the gravitational potential by changing the viscosity parameter $\alpha$. In fact, as it has been discussed in the literature [16, 17], the non-adiabatic pressure fluctuation, which is usually characterized by the effective speed of sound $c^2_s$, can also affect the late time ISW effect. Thus we also discuss the effects of the non-adiabatic pressure fluctuation along with that of the anisotropic stress.

Since we are going to discuss the effects on the ISW effect, first we show time derivative of the gravitational potential $\Phi' + \Psi'$ at present time in Fig. 1 as a function of wave number $k$ for several values of $\alpha$ and $c^2_s$. Here and hereafter gravitational potentials are normalized at super horizon scales such that $\Psi = -10/(4R_\nu + 15)$ with $R_\nu = \rho_\nu/\rho(\gamma + \rho)$ being neutrino fraction, and the conformal time coordinate is in units of Mpc following the convention. The equation of state for dark energy is assumed as $w_X = -0.8$ in this figure. When we vary $\alpha$, $c^2_s$ is assumed to be 0 and vice versa. For illustrative purpose, other cosmological parameters are fixed as $\Omega_m h^2 = 0.128$ (dark matter density), $\Omega_b h^2 = 0.022$ (baryon density), $h = 0.73$ (hubble parameter), $\tau = 0.09$ (optical depth to the last scattering surface) and $n_s = 0.96$ (spectral index of primordial density fluctuation), which are mean

\textsuperscript{#2} The background evolution of dark energy can affect the expansion of the universe, thus the peak positions of CMB power spectrum can depend on the equation of state of dark energy. However, as far as the fluctuation of dark energy concerned, its effects can be large only on large scales unless the equation of state dark energy is close to 0 at early times.
values for ΛCDM model from the analysis using WMAP3 data alone [18]. In this paper, we assume that the universe is flat and no running of scalar spectral index and no tensor mode contribution. Unless otherwise stated, we use the above cosmological parameters except for those of dark energy. As seen from the figure, the change of the gravitational potential increases as α and $c_s^2$ becomes larger. Since α represents the strength of the anisotropic stress, which damps velocity perturbations, the increase of α drives the change of the gravitational potential. Thus we obtain larger ISW effect as α increases. As for the effects of the non-adiabatic pressure fluctuation, $c_s^2$ determines the sound horizon $s = \int c_s dt/a$ under which dark energy can be assumed as “smooth” component. When dark energy is considered to be smooth, fluctuation of dark energy can act as stabilizing force for the gravitational potential, which leads to the decay of the gravitational potential. Above the sound horizon, dark energy can cluster, thus only gravitational force can be relevant to the evolution of the gravitational potential. The transition between “smooth” regime to clustered regime occurs at larger scales as $c_s^2$ increases. Thus the larger values of $c_s^2$ cause the more late time ISW effect, which can be observed in the figure.

![Figure 1: The value of Φ′ + Ψ′ today is shown as a function of k for several values of α (left) and $c_s^2$ (right). The equation of state is fixed as $w_X = -0.8$ in this figure.](image)

For the effects of the non-adiabatic pressure fluctuation, the behavior of the gravitational potential differs depending on $c_s^2$ only around the transition regime but not at smooth or clustered limit. However, for the anisotropic stress, the ISW effect depends on α on large scales. To see this clearly, we show the present value of Φ + Ψ as a function of $k$ in Fig. 2. For the case with α being fixed and varying $c_s^2$, it can be clearly seen that the behaviors on large/small scales are the same and they differ only at transition regime. However, for the case with $c_s^2$ being fixed and varying α, corresponding lines depend on the value of α even on large scale limit. Since the parameter α determines the magnitude of the anisotropic stress $\sigma_X$ through Eq. (6), the change of the gravitational potential is also affected by the magnitude of α. On the other hand, $c_s^2$ only determines the sound horizon which differentiates clustering property of dark energy, thus the magnitude of $c_s^2$ itself is not directly relevant to the amount of the change of the gravitational potential. This is one of the differences between the effects of $c_s^2$ and α although they affect fluctuation on
Figure 2: The value of $\Phi + \Psi$ today is shown as a function of $k$ for several values of $\alpha$ (left) and $c_s^2$ (right). The equation of state is fixed as $w_X = -0.8$ in this figure.

Furthermore, it is well-known that $\Phi$ and $\Psi$ coincide when there is no anisotropic stress, in other words, $\Phi - \Psi$ is sensitive to the anisotropic stress perturbation $\sigma$. This can be understood from the perturbed Einstein equation, as

$$k^2 (\Phi - \Psi) = 12\pi G_N \alpha^2 (\rho + p) \sigma$$

where the quantity in the RHS is for the total matter. In the left panel of Fig. 3, the values of $\Phi - \Psi$ at the present time are shown for several values of $\alpha$ as a function of wave number. As seen from the figure, $|\Phi - \Psi|$ becomes larger as $\alpha$ increases on large scales, which can be easily understood since larger values of $\alpha$ means large anisotropic stresses from dark energy. Another point which should be noticed is that the effect of it becomes larger on large scales. For the generalized dark energy with non-zero $\alpha$, the viscosity acts against the gravitational collapse once the modes come across the horizon (right panel of Fig. 3). Therefore the perturbations including the anisotropic stress $\sigma$ have damped more at smaller scales, which makes the effect significant only at large scales at present.

Now we are going to discuss the effects on CMB TT power spectra. In Figs. 4 and 5, the CMB TT power spectra are depicted for several values of $\alpha$ and $c_s^2$. The value of the equation of state $w_X$ are found in the captions. As noted above, since the dark energy can be dominant component of the universe only at late times, its fluctuation affects the CMB power spectra on large scales. Thus we only show low multipole region in the figures. As seen from Fig. 4, $C_\ell$ increases as $\alpha$ increases for the case with $w_X = -0.8$ and $c_s^2 = 0$. This is expected from the argument on the ISW effect above. However, when the equation of state is less than $-1$, the tendency is the opposite, i.e., as the $\alpha$ increases, the large scale power decreases. This can be understood by looking at the source term in the perturbation equation. The anisotropic stress can affect the evolution of the gravitational potential through the change of density and velocity perturbations. Such an effect is induced by the source term in Eq. (11) which depends on the prefactor $1 + w_X$. Thus the effect should be different depending on the sign of the factor $1 + w_X$. Hence the
tendency becomes the opposite for the cases with $w_X > -1$ and $< -1$.

When the value of the sound speed is fixed as $c_s^2 = 1$, the effects of the anisotropic stress become insignificant for any value of $w_X$. For the case with $c_s^2 = 1$, most scales relevant to low multipole region where dark energy fluctuation can be important are in smooth regime. As shown above, the anisotropic stress can affect the gravitational potential, at least superficially, similarly to smoothed dark energy. Hence when dark energy can be assumed as a smooth component, the effect of the anisotropic stress becomes irrelevant since dark energy is already smoothed.

For comparison, we also show the same plot for the effects of the non-adiabatic pressure fluctuation. In Fig. 5, CMB TT power spectra are shown for several values of $c_s^2$ fixing $\alpha$ and other parameters. Since the effects of the anisotropic stress and non-adiabatic pressure fluctuation are similar in the ISW effect, the tendency should be also the same as that of the effects of $\alpha$. As anticipated, in Fig. 5 we can see almost the same behavior of low multipole region for the case with $c_s^2$ being varied, as for the case with $\alpha$ being varied. Thus we can expect that there are some degeneracy between the effects of $\alpha$ and $c_s^2$ on the fluctuation at low multipoles of CMB. As discussed above, they affect the ISW effect which enhances/reduces the power at low multipoles depending on the strength of these effects characterized by $\alpha$ and $c_s^2$. Although the origin of these effects are different in nature, they seem to affect low multipoles of CMB in the same manner. To see this quantitatively, we show the contours of constant ratio of $\tilde{C}_l \equiv \frac{l(l+1)C_T}{\sigma_k}$ at between $l = 2$ and the first peak in Fig. 6. In the figure, the equation of state of dark energy is assumed as $w_X = -0.8$. Other cosmological parameters are also fixed as the values already noted.
above. Notice that higher multipole region \( l > O(100) \) is unchanged by varying the values of \( c_s^2 \) and \( \alpha \). The ratio represents how low multipoles are enhanced/reduced by the effects of the anisotropic stress and non-adiabatic pressure fluctuation. The figure indicates that some combination of \( c_s^2 \) and \( \alpha \) give the same effect on large scales. Thus when we study the constraint from observations, we can expect some degeneracy between \( c_s^2 \) and \( \alpha \). We discuss this issue in the next section.

Figure 4: CMB TT power spectra for several values of \( \alpha \) for the cases with \( w_X = -0.8, c_s^2 = 0 \) (top left), \( w_X = -1.2, c_s^2 = 0 \) (top right), \( w_X = -0.8, c_s^2 = 1 \) (bottom right) and \( w_X = -1.2, c_s^2 = 1 \) (bottom right).

Here we should make a comment on a nontrivial cancellation between the ISW effect and the cross correlation between the ISW and Sachs-Wolfe (SW) effect. Although we argued that larger values of \( \alpha \) give the larger ISW effect for the case with \( w_X > -1 \), the lines of \( \alpha = 1 \) and 5 in the top left panel of Fig. 4 do not obey this rule. In fact, this is because the ISW and SW effects can nontrivially cancel those effects in some cases. We discuss this issue briefly. For this purpose, we make use of the transfer function on large scales. Formally, \( C_l \) is written as

\[
C_l = 4\pi \int \frac{dk}{k} P_R(k)|\Delta_l(k)|^2.
\]  

(9)

where \( P_R(k) \) is the primordial power spectrum and \( \Delta_l(k) \) represents the transfer function
for a multipole moment \( l \) at present time. On large scales, \( \Delta_l(k) \) are mostly determined by the contribution from the SW and ISW effect, thus it can be written as

\[
\Delta_l(k) = \Delta_l^{\rm SW}(k) + \Delta_l^{\rm ISW}(k) .
\]  

In particular, the contribution from the ISW effect is given by

\[
\Delta_l^{\rm ISW}(k) = \int d\eta e^{-\tau} (\Phi' + \Psi') j_l[k(\eta - \eta_0)] ,
\]  

where \( \tau \) is the optical depth along the line of sight, \( \Phi \) is the gravitational potential and \( j_l \) is the spherical Bessel function. We plot \( |\Delta_l^{\rm ISW}|^2 \) as a function of wave number \( k \) for several values of \( \alpha \) in Fig. 7 for the case with \( w_X = -0.8 \). As seen from the figure, \( |\Delta_l^{\rm ISW}|^2 \) becomes larger as \( \alpha \) increases. In fact, as mentioned above, the quadrupole \( C_2 \) for \( \alpha = 1 \) is larger than that for the case with \( \alpha = 5 \) although the ISW effect is larger for \( \alpha = 5 \). Thus, at first sight, it looks contradictory. However, this comes from a non-trivial cancellation between the contributions from the ISW effect \( |\Delta_l^{\rm ISW}|^2 \) and the cross correlation between the SW and ISW effects \( \Delta_l^{\rm SW} \times \Delta_l^{\rm ISW} \). In Fig. 8 we also plot \( \Delta_l^{\rm SW} \times \Delta_l^{\rm ISW} \) as the same manner as Fig. 7. When \( \alpha \) is larger, the cross correlation becomes also large in amplitude.
as seen from the figure. However, notice that, since the cross correlation of the SW with the ISW effects can give a negative contributions to the transfer function. Thus, for some cases, it gives smaller quadrupole amplitude as a whole effect. This is the reason why the low multipoles for the case with $\alpha = 1$ is larger than that for $\alpha = 5$ even though the contribution from the ISW effect monotonically becomes larger as increasing $\alpha$. For reference, we also show the same plot for the case with $w_X = -1.2$ on the right panel in Figs. 7 and 8.

4 Constraints from current cosmological observations

In this section, we show the constraints from current cosmological observations on the energy density and equation of state of dark energy.

To include the effects of non-adiabatic pressure fluctuation and anisotropic stress of dark energy, we calculate theoretical angular power spectrum by CAMB code modified according to the details in section 2. Because these effects change the power at larger angular
scales, the amplitude and the shape of the power spectrum at low multipoles can be used in principle to put constraints on these fluctuation properties. In fact, however, we cannot find any significant limits on them because of the large errors due to the cosmic variance at low multipoles. Therefore, we conclude that the current cosmological observations cannot put tight constraints on the $\alpha$ or $c_s^2$ parameters. (The forecast of observational constraint on $c_s^2$ from planning galaxy surveys can be found in [19].)

Even though one cannot give limits on these parameters themselves, it does not mean that they are irrelevant when considering the constraints on the nature of dark energy. It can happen that one obtains different limits on the cosmological parameters such as equation of state of dark energy $w_X$ if one takes different parameters on the fluctuation property of dark energy such as effective sound speed $c_s^2$ and/or anisotropic stress $\alpha$. Many observational proposals aim to determine the equation of state parameter of dark energy $w_X$ as precise as possible, even up to $\sim 1\%$ level [20], because the information about $w_X$ is important to pin down a model for dark energy, and also to determine the future of
universe \cite{21}. It is therefore important to investigate how much different assumptions on the fluctuation properties of dark energy can lead the different observational limits on the equation of state of dark energy.

To study this explicitly, we made likelihood analysis to put constraint on $w_X$ with several different assumptions on $c_s^2$ and $\alpha$. The likelihood functions we calculate are based on WMAP three year data \cite{22,23} and the 2dF Galaxy Redshift Survey \cite{24}. To make analysis general enough, one has to vary the other cosmological parameters which also affect the shape of the CMB and matter power spectra. In order to do so effectively, we follow Markov chain Monte Carlo (MCMC) approach \cite{25}, and explore the likelihood in seven dimensional parameter space, namely, $\theta$ (the ratio of the sound horizon to the angular diameter distance at last scattering), $A_s$ (amplitude of primordial perturbation), $\Omega_m h^2$, $\Omega_m h^2$, $n_s$, $\tau$, and $w_X$. Here the Hubble parameter $h$ is derived from $\theta$ \cite{26} and the dark energy density such that the universe is spatially flat.

In the left panel of Fig. 9, marginalized 1 and 2\sigma allowed regions in the $\Omega_m - w_X$ plane from WMAP three year data are shown for the case with $c_s^2 = 0$ and $\alpha = 0$. For comparison, the case with $c_s^2 = 1$ and $\alpha = 0$ is also shown in the figure. As seen from the figure, the constraint on $w_X$ for the case with $c_s^2 = 0$ and $\alpha = 0$ becomes severer compared to the other case. This is due to the fact that the ISW effect becomes significant as $w_X$ decreases to negative value when $w_X < -1$, which makes the fit to the data worse at low multipole region. This results indicates that the perturbation nature of dark energy fluid affects the determination of the equation of state in some cases. The marginalized one-dimensional probability distribution of $w_X$ is also shown in the right panel of Fig. 9.

We found that the confidence region of $w_X$ can differ by $\lesssim 10\%$ between the models with $(\alpha, c_s^2) = (0, 0)$ and $(0, 1)$.

In the left panel of Fig. 10 we show marginalized 1 and 2\sigma allowed regions in the $\Omega_m - w_X$ plane obtained by combining the information from the matter power spectra by 2dF galaxy survey with WMAP three year data. Although the data reduce the allowed region in the $\Omega_m - w_X$ plane, it can still be found that there is a significant difference between the models with different assumptions on the fluctuation properties of dark energy. Similar analysis and conclusion are done and derived by \cite{16} for a fluid dark energy model parameterized by $w$ and $c_s^2$, by cross-correlating the map from WMAP three year data with that from NRAO VLA sky survey data (see also \cite{17}). The present work differs from them in that we analyzed different observational data set, included viscosity parameter $\alpha$, and relaxed the other cosmological parameters to be varied while they fixed cosmological parameters except for the dark energy parameters.

In Fig. 11, marginalized 1 and 2\sigma allowed regions are shown for the case with $(c_s^2, \alpha) = (0, 0.6)$ (left) and $(1, 0.6)$ (right). In fact, large value of $c_s^2/\alpha$ makes any dependence of $w_X$ at low multipoles insignificant. Furthermore, we found that when $\alpha \sim 1$ the result is insensitive to the value of $c_s^2$ because of the fact that the fluctuation amplitude at low multipoles is independent from $c_s^2$ in such cases (see Fig. 6). Thus in this case, the constraint on $\Omega_m - w_X$ plane is almost unchanged. We also show the same plot for the case where we use the data from 2dF in addition to WMAP three year data in Fig. 12. As
Figure 9: (Left) Allowed regions of $1\sigma$ (red region) and $2\sigma$ (purple region) are shown for the case with $c_s^2 = 0$ and $\alpha = 0$. For comparison, allowed regions for the case with $c_s^2 = 1$ and $\alpha = 0$ are also shown in black dashed lines. (Right) Marginalized probability distribution of the equation of state parameter $w_X$ with different assumptions on the fluctuation properties of dark energy. Black line corresponds to the case with $(c_s^2, \alpha) = (1, 0)$ and red dashed line with $(c_s^2, \alpha) = (0, 0)$.

Figure 10: The same as Fig. 9 but we also use the data from 2dF in addition to WMAP three year data.
seen from the figure and easily expected, the constraints become severer but the differences among different assumptions on $c_s^2$ and $\alpha$ are small.

![Figure 11: Allowed regions of $1\sigma$ (blue region) and $2\sigma$ (cyan region) are shown for the case with $c_s^2 = 0$ and $\alpha = 0.6$ (left) and $c_s^2 = 1$ and $\alpha = 0.6$ (right). For comparison, allowed regions for the case with $c_s^2 = 1$ and $\alpha = 0$ are also shown in black dashed lines.](image)

![Figure 12: The same as Fig. 11 but we also use the data from 2dF in addition to WMAP three year data.](image)

5 Conclusions and discussions

We discussed the effects of a generalized dark energy on cosmic density fluctuations and studied cosmological constraints on dark energy in that setting. When one considers a general type of dark energy, one can also include a possible anisotropic stress of dark energy, which has not been investigated much in the literature. We studied the effects of the anisotropic stress characterized by the viscosity parameter $\alpha$ along with the non-adiabatic
pressure fluctuation usually described by the speed of sound $c_s^2$. First we discussed the effects of them on the gravitational potential and argued how the values of $\alpha$ and $c_s^2$ affect it. As shown, in section 3, the anisotropic stress of dark energy can affect the ISW effect in a similar way with the non-adiabatic pressure fluctuation. We also explicitly showed that there is some degeneracy between the effects of $\alpha$ and $c_s^2$ on the power at low multipoles of CMB angular power spectrum.

We also explore constraints on dark energy from recent cosmological observations assuming a generalized dark energy. As shown in Fig. 9, the constraint on the equation of state can be changed depending on the parameters $c_s^2$ and $\alpha$ which describes the perturbation nature of dark energy. This is because fluctuation on large scales are affected by the nature of dark energy fluctuation, i.e., depending on $\alpha$ and $c_s^2$, through the ISW effect. This analysis may indicate that cosmological constraint on dark energy such as the equation of state should be carefully done by taking the perturbation nature of dark energy into account.

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