Y(2175): Distinguish Hybrid State from Higher Quarkonium

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Abstract

The possibility of Y(2175) as a $^2D_1 s\bar{s}$ meson is studied. We study the decay of $^2D_1 s\bar{s}$ from both the $^3P_0$ model and the flux tube model, and the results are similar in the two models. We show that the decay patterns of $1^{--}$ strangeonium hybrid and $^2D_1 s\bar{s}$ are very different. The experimental search of the decay modes $KK$, $K^*K^*$, $K(1460)K$, $h_1(1380)\eta$ is suggested to distinguish the two pictures. Measuring the $K^*K^*$ partial width ratios is crucial to discriminate the $^2D_1$ from the $^3S_1 s\bar{s}$ assignment.

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I. INTRODUCTION

Recently the Babar Collaboration has observed a structure at 2175 MeV in $e^+e^- \rightarrow \phi f_0(980)$ via initial-state radiation, which is consistent with $1^{-+}$ resonance with a mass $m = 2175 \pm 0.010 \pm 0.015$ GeV/c$^2$ and width $\Gamma = 58 \pm 16 \pm 20$ MeV [1]. Henceforth, this structure is denoted as $Y(2175)$. Furthermore, the Babar collaboration demonstrates that this structure is not due to the dominant $K^*(892)K\pi$ intermediate states, and there is no known meson resonance with $I = 0$ near this mass.

In Ref. [2], we suggested that this structure is a $1^{-+}$ strangeonium hybrid $(s\bar{s}g)$ and the decay properties are studied from both the flux tube model and the constituent gluon model. Both the mass and decay width are consistent with the current experimental data in the hybrid scenario. Moreover, we suggested that the tetraquark hypothesis for $Y(2175)$ is not favored by the current data in [1], although this picture cannot be completely excluded.

The mass of $Y(2175)$ which is assigned as a tetraquark state, has been calculated in QCD sum rule [3]. To confirm that $Y(2175)$ is hybrid or another exotic state, it is necessary to examine the radial excitation of quarkonium in the quark model to determine whether they imitate the decay and production properties of the hybrid state. The aim of this work is to study the decay of $1^{-+}$ strange quarkonium $(2^3D_1)$ and reveal possible experimental signals that can discriminate between the hybrid and the higher quarkonium.

Its quantum numbers $J^{PC} = 1^{-+}$ implies that the possible quarkonium states are $3S_1$, $3D_1$, $2^3S_1$, $2^3D_1$, $3^3S_1$, $3^3D_1$ (in the notation $n^{2S+1}L_J$, denoting the $n$th state with spin $S$, orbital angular momentum $L$, and total angular momentum $J$) and so on. Among these states, only the $3^3S_1$ and $2^3D_1$ strange quarkonium states have masses consistent with $Y(2175)$ within the experimental error [4]. The decay of $3^3S_1$ strange quarkonium has been studied in detail in the $3P_0$ model by T.Barnes et al., [5, 6], and this state is predicted to be a rather broad resonance, $\Gamma \approx 380$ MeV in the $3P_0$ model, and it mainly decays into $K^*K^*$, $KK^*(1414)$, $KK_1(1273)$. However, the $1^{-+}$ strangeonium hybrid that we predicted is much narrower $\Gamma \approx 100 - 130$ MeV, and the $K^*K^*$ mode is forbidden [2]. Since the width of $3^3S_1$ strange quarkonium is much larger than that of $Y(2175)(\Gamma = 58 \pm 16 \pm 20$ MeV), it is very difficult to identify $Y(2175)$ as a $3^3S_1$ strange quarkonium.

In this paper, we will calculate the partial widths of $2^3D_1$ strange quarkonium to all Okubo-Zweig-Iizuka-(OZI) allowed two- body final states allowed by phase space. To assess
the correctness of our analysis, we calculated the widths by using both the $^3P_0$ decay model\cite{7,8,9,10,11} and the flux tube breaking model\cite{12,13,14}. Our goal is to shed some light on the nature of $Y(2175)$ and reveal some promising signals that can discriminate the $1^{--}$ strangeonium hybrid picture from the $2^3D_1$ strange quarkonium.

In this paper, we will study the decay of $2^3D_1$ strange quarkonium from the $^3P_0$ model in Sec. II and from the flux tube model in Sec. III. We use the simple harmonic approximation in both models so that the amplitudes can be derived analytically. Finally, we present our summary and some discussions.

II. THE DECAY OF $2^3D_1 s\bar{s}$ IN $^3P_0$ DECAY MODEL

The $^3P_0$ model (quark pair creation model), which describes the process that a pair of quark-antiquark with quantum number $J^{PC} = 0^{++}$ is created from the vacuum was first proposed by Micu\cite{7} in 1969. In the 1970s, this model was developed by Yaouanc et al.\cite{8} and applied to study hadron decays extensively. The $^3P_0$ model is applicable to OZI-allowed strong decay of a meson into two other mesons, as well as the two body-decay of baryons and other hadrons\cite{15}. In the $^3P_0$ model, the created quark pairs with any color and any flavor can be generated anywhere in space, but only those whose color-flavor wave functions and spatial wave functions overlap with those of outgoing hadrons can make a contribution to the final decay width.

It is widely assumed that $^3P_0$ model is successful because it provides a good description of many of the observed decay amplitudes and partial widths of open flavor meson strong decays. Several published papers study the decay of light mesons, open charm mesons and charmonium using different wavefunction and phase space normalization\cite{16,17}. We will use the diagrammatic technique developed in Ref.\cite{11} to derive the amplitudes and the $^3P_0$ matrix elements. In this formalism, the $^3P_0$ model describes decay matrix elements using a $q\bar{q}$ pair production Hamiltonian, which is the nonrelativistic limit of,

$$H_I = g \int d^3x \, \bar{\psi}(x) \, \psi(x)$$

where $\psi$ is a Dirac quark field, $g = 2m_q\gamma$, $\gamma$ is the strength of the conventional $^3P_0$ mode, and $m_q$ is the mass of both created quarks. To determine a decay rate, we evaluate the
matrix element of the decay Hamiltonian between the initial and final states,

\[ \langle BC|H_I|A \rangle = h_{fi} \delta^3(p_A - p_B - p_C) \quad (2) \]

To compare with the experiment, we transform the helicity \( h_{fi} \) into the partial wave amplitude \( M_{LBC,SBC} \) by the recoupling calculation\[18\]. Then the decay width is:

\[ \Gamma(A \rightarrow B + C) = 2\pi P E_B E_C \sum_{L,S} |M_{LS}|^2 \quad (3) \]

where relativistic phase space normalization has been taken, and \( P \) is the momentum of the final states in the rest frame of \( A \), i.e., \( P = \frac{((M_A^2-(M_B+M_C)^2)(M_A^2-(M_B-M_C)^2))^{1/2}}{2M_A} \). The decay amplitudes for \( 2^3D_1 \rightarrow 1^S_0 + 1^S_0, 2^3D_1 \rightarrow 3^S_1 + 1^S_0 \) etc can be derived analytically under the simple harmonic oscillator wavefunction approximation. The mass difference \( m_u, d \neq m_s \) is ignored in the wavefunctions as Ref.[5, 6]. However, this approximation may not be good in some channels, and we will clearly see the effect of large strange quark mass in the flux tube model in Sec.III. In Ref.[5, 6], the authors assume that the harmonic oscillation parameter \( \beta_A \) of the initial state is the same as the \( \beta \) of the final states. In the present paper, we relax this assumption by allowing \( \beta_A \) to be different from \( \beta \).

We assume that the harmonic oscillation parameter of the final states and the pair-production amplitude respectively are \( \beta = 0.4 \text{GeV}, \gamma = 0.4 \) as in Ref.[5, 6]. It is theoretically expected that the magnitude of \( \beta_A \) for higher excited states are smaller than those of lower states by \( \leq 0.1 \text{ GeV} \), so we take \( \beta_A = 0.35 \text{GeV} \) in our numerical calculations of the partial decay widths. Meson mass are taken from the Particle Data Group(PDG)-2006\[19\]. If a state was not included in the Meson Summary Table of PDG, we use an estimated mass motivated by the spectroscopy predictions\[4\], adjusted in the absolute value relative to the known masses. For the pseudoscalar \( \eta \) and \( \eta' \), we assume perfect mixing, and the flavor structures are as follows,

\[ \eta = \frac{1}{2}(u\bar{u} + d\bar{d}) - \frac{1}{\sqrt{2}} s\bar{s} \]
\[ \eta' = \frac{1}{2}(u\bar{u} + d\bar{d}) + \frac{1}{\sqrt{2}} s\bar{s} \quad (4) \]

The mixing angle is consistent with the angle obtained from the \( \eta - \eta' \) mass matrix. Using the analytical decay amplitudes and including the flavor factors, summing over all final flavor states, we obtain the numerical value of the partial decay width for \( 2^3D_1 \ s\bar{s} \) decay in the \( ^3P_0 \) model, which are listed in the second column of the Table I.
TABLE I: The decay of $Y(2175)$ as $2^3D_1$ strange quarkonium in the $^3P_0$ model and the flux tube model. $\Gamma_{L,J}$(MeV) is partial decay width, where $L$ represents the relative angular momentum between two final states, and $J$ is the total angular momentum of the final states. We choose $\beta = 0.4$GeV, $\beta_A = 0.35$GeV. The starred amplitudes vanish exactly with the simple harmonic oscillator wavefunction. The large difference in some channels is due to the large strange quark mass of the nodal suppression effect. For comparing the above predictions for $Y(2175)$ as a $2^3D_1$ $s\bar{s}$ state with those of the possible hybrid interpretation in [2] and of the possible $3^3S_1$ quarkonium assignment in [6], the corresponding results of [2] and [6] are listed in the final two columns of the table. Note, the mass of $3^3S_1$ $s\bar{s}$ state in [6] was set to be 2050MeV.

| Decay modes       | $Y(2175)$ as $2^3D_1$ $s\bar{s}$ quarkonium | $Y(2175)$ as $ss$ hybrid [2] | $Y(2175)$ as $3^3S_1$ $ss$ quarkonium [6] |
|-------------------|-----------------------------------------------|-----------------------------|-----------------------------------------------|
|                   | $\Gamma_{L,J}$ in $^3P_0$ Model | $\Gamma_{L,J}$ in Flux Tube Model | in Flux Tube Model | in $^3P_0$ Model |
| $KK$              | $\Gamma_P = 9.8$ | $\Gamma_P = 23.1$ | 0 | 0 |
| $K^*K$            | $\Gamma_P = 1.3$ | $\Gamma_P = 11.7$ | 3.7 | 20 |
| $\phi\eta$       | $\Gamma_P = 0$ | $\Gamma_P = 0$ | 1.2 | 21 |
| $\phi\eta'$      | $\Gamma_P = 2.9$ | $\Gamma_P = 2.8$ | 0.4 | 11 |
| $K^*K^*$          | $\Gamma_P = 0.76$ | $\Gamma_P = 0$ | 0 | 102 |
|                   | $\Gamma_P = 0^*$ | $\Gamma_P = 0^*$ | | |
|                   | $\Gamma_P = 0.15$ | $\Gamma_P = 0$ | | |
|                   | $\Gamma_F = 17.2$ | $\Gamma_F = 23.5$ | | |
| $K(1460)K$        | $\Gamma_P = 58.3$ | $\Gamma_P = 50.2$ | 0 | 29 |
| $K^*(1410)K$      | $\Gamma_P = 31.9$ | $\Gamma_P = 26.0$ | 23 | 93 |
| $h_1(1380)\eta$   | $\Gamma_S = 3.6$ | $\Gamma_S = 3.5$ | 0 | 8 |
| $K_1(1270)K$      | $\Gamma_S = 2.3$ | $\Gamma_S = 20.5$ | 35.3 | 58 |
|                   | $\Gamma_D = 19.6$ | $\Gamma_D = 25.9$ | | |
| $K_1(1400)K$      | $\Gamma_S = 3.0$ | $\Gamma_S = 0.8$ | 70.1 | 26 |
|                   | $\Gamma_D = 5.6$ | $\Gamma_D = 8.6$ | | |
| $K_2(1430)K$      | $\Gamma_D = 10.8$ | $\Gamma_D = 15.3$ | 15.0 | 9 |
| $\Gamma_{tot}$   | 167.21 | 211.9 | 148.7 | 378 |
The $2^3D_1 s\bar{s}$ state is predicted to be rather narrow in the $^3P_0$ model, $\Gamma \approx 167.2$MeV, so we cannot exclude the possibility that $Y(2175)$ could be a $2^3D_1 s\bar{s}$ state. In the case of $\beta_A = 0.35$GeV, the dominant decay modes are:

$$2^3D_1 \rightarrow K(1460)K, \ K^*(1410)K, \ K_1(1270)K, \ K^*K^*$$

(5)

All these lead to the important $KK\pi\pi$ final state. We suggested in [2] that if $Y(2175)$ is assigned as a hybrid, the decay modes of $K(1460)K$ and $K^*K^*$ are forbidden due to the famous selection rule [20] and since $K(1460)$ is $^1S_0$ state, and $K^*$ is $S$-wave state. However, when $Y(2175)$ is as $2^3D_1$ strange quarkonium discussed in this present paper, in contrast with the strangeonium hybrid picture, the widths of decay modes $K(1460)K$ and $K^*K^*$ are significantly larger (please see Table I). This remarkable feature is a criterion to distinguish the quarkonium interpretation from the hybrid assignment for $Y(2175)$. Therefore, experimentally observing the $K(1460)K$ and $K^*K^*$ decay modes of $Y(2175)$ is crucial for exploring the nature of $Y(2175)$. The $K^*K$, $\phi\eta$ and $\phi\eta'$ modes are near the nodes of the decay amplitudes, as a result, the decay widths corresponding to them are predicted to be rather small. Furthermore, we can see another interesting property that $2^3D_1 s\bar{s}$ prefers to decay into $2S + 1S$ final states (Table I). Namely, $K^*(1410)K$ has a large branch ratio if the problematical $K^*(1410)$ is a $2^3S_1$ state.

The $K^*K^*$ mode is especially interesting and there are four partial widths, i.e., $\Gamma_{P0}$, $\Gamma_{P1}$, $\Gamma_{P2}$ and $\Gamma_{F2}$. If $Y(2175)$ is a pure $2^3D_1 s\bar{s}$ state, we predict $\Gamma_{P1} = 0$, $\Gamma_{P2}$ is the largest, and the ratio $\Gamma_{P2}/\Gamma_{P0} = 1/5$, which is independent of the radial wavefunction (in Table I $\Gamma_{P2}/\Gamma_{P0} = 0.197$, because we keep only two effective numbers in numerical results of partial widths there). However, if $Y(2175)$ is assigned as a $3^3S_1 s\bar{s}$ state, both $\Gamma_{P1}$ and $\Gamma_{F2}$ are predicted to be zero, and $\Gamma_{P2}/\Gamma_{P0} = 20$. Determining the $K^*K^*$ decay width ratios is very important to examine whether $Y(2175)$ is a $2^3D_1$ or $3^3S_1 s\bar{s}$ state.

To test the robustness of the these conclusions, we should study the stability of these results with respect to independent variations in $\beta_A$ and the mass of the initial state. We show the $\beta_A$ dependence of the partial widths and total width respectively in Fig.1 and Fig.2. We can see that the width of $2^3D_1 \rightarrow K_2(1430)K$ depends weakly on $\beta_A$, however, the partial width of the modes $K(1460)K$, $K^*(1410)K$, $K_1(1400)K$, $K^*K^*$ and $h_1(1380)\eta$ vary dramatically with $\beta_A$. For small $\beta_A (\beta_A \simeq 0.3 \sim 0.35)$, $2^3D_1 s\bar{s}$ dominantly decays into $K(1460)K$, $K^*(1410)K$, $K_1(1270)K$, $K^*K^*$ and $KK$, while it dominantly decays into
$K_1(1400)K$, $h_1(1380)\eta$, $K^*K^*$, $K_1(1270)K$ for large $\beta_A$ ($\beta_A \approx 0.4 \sim 0.5$). Experimental observations of $K(1460)K$, $K^*K^*$ or $KK$ modes would be strong indications of a $2^{3}D_1$ $s\bar{s}$ component. Since a $1^{--}$ strangeonium hybrid decay into $h_1(1380)\eta$ is forbidden due to the ”spin selection” rule[20], the $h_1(1380)\eta$ mode is also very important in determining the nature of $Y(2175)$.

The variation of the partial decay width and total width with the mass of the initial state are shown in Fig.3 and Fig.4, respectively, where the initial state mass is denoted as $M_A$, and we choose $\beta_A = 0.35$ GeV. The total width becomes large with increasing $M_A$. Interestingly, the partial decay widths $2^{3}D_1 \rightarrow K_1(1400)K$ and $2^{3}D_1 \rightarrow h_1(1380)\eta$ decrease when the mass of $2^{3}D_1$ state increases. This is because the modes $K_1(1400)K$ and $h_1(1380)\eta$ are closer to the nodes of the decay amplitude with increasing $M_A$. Moreover, both the $K(1460)K$ and $K^*K^*$ modes always have a sizable branch ratio in the mass region $2.05 \sim 2.25$ GeV.

### III. DECAY OF $2^{3}D_1$ $s\bar{s}$ IN THE FLUX TUBE MODEL

The flux tube model is extracted from the strong coupling limit of the QCD lattice Hamiltonian[12, 21]. In flux tube mode, a meson consists of a quark and antiquark connected by discretized quantum string. For conventional mesons, the string is in its ground state. Vibrational excitation of the string corresponds to the hybrid mesons[12, 21]. The flux tube
model extends the $^3P_0$ model by including the dynamics of the flux tube. This is done by including a factor that represents the overlap of initial meson flux tube with those of the two final mesons. Though the two models are not identical, their quantitative futures are similar\cite{12}, and the flux tube model coincides with the $^3P_0$ model in the limit of infinitely thick flux-tube. In the rest frame of $A$, the decay amplitude of an initial meson $A$ into two final mesons $B$ and $C$ is,

$$
\mathcal{M}(A \rightarrow B + C) = \int d^3r_A \int d^3y \psi_A(r_A) \exp(i \frac{M}{m + M} \mathbf{p}_B \cdot \mathbf{r}_A) \gamma(r_A, y) \\
\times (i \nabla_{r_B} + i \nabla_{r_C} + \frac{2m \mathbf{p}_B}{m + M}) \psi_B^*(\mathbf{r}_B) \psi_C^*(\mathbf{r}_C) + (B \leftrightarrow C) \quad (6)
$$

where both the flavor and spin overlaps have been omitted in the above amplitude, and $\gamma(r_A, y)$ is the flux-tube overlap function, which measures the spatial dependence of the pair creation amplitude. The initial quark and antiquark of the initial meson $A$ are assumed to be of the same mass $M$, and $m$ is the mass of the created quark pair. $y$ is the pair creation position, $r_A$, $r_B$ and $r_C$ are respectively the quark-antiquark axes of $A$, $B$, and $C$ mesons, they are related by $r_B = r_A/2 + y$, $r_C = r_A/2 - y$. For the conventional meson decay, the flux-tube overlap function is usually chosen as the following form\cite{12},

$$
\gamma(r_A, y) = A_0^0 \sqrt{\frac{f_b}{\pi}} \exp\left(-\frac{f_b}{2} y_\perp^2\right) \quad (7)
$$

here $y_\perp = -(y \times \hat{r}_A) \times \hat{r}_A$. With these elements, the decay amplitude can be calculated analytically under the simple harmonic oscillator wavefunction approximation following the
procedure for the calculation of widths in Ref.\[12\]. The amplitudes for $2^3D_1 \rightarrow ^1S_0 + ^1S_0$, $2^3D_1 \rightarrow ^3S_1 + ^1S_0$ etc are derived by using the harmonic oscillation parameter $\beta$ of the outing mesons. The overall normalization factor $\gamma_0$ was phenomenologically found to be equal to 0.64 for creating light quark pairs\[12, 14, 20\]. We take the string tension $b = 0.18$, and the constitute quark mass $m_u = m_d = 0.33$GeV, $m_s = 0.55$GeV. As usual, the estimate value $f = 1.1$ and $A_{00}^0 = 1.0$ is used, a detailed discussion about these quantity can be found in Ref.\[12, 22\]. The mesons masses are chosen in the same way as in the above $^3P_0$ model case. The numerical values for the partial decay width and total width are shown in the third column of Table I, where we assume $\beta_A = 0.35$GeV.

We see that the overall behaviors of the decay modes in flux tube model is similar to those in the $^3P_0$ model. $K(1460)K$, $K_1(1270)K$, $K^*(1410)K$, $K^*K^*$, $KK$ are still the dominant decay modes for $\beta_A = 0.35$ GeV. Although there is large difference comparing with the $^3P_0$ model in some channels, such as $2^3D_1 \rightarrow KK$, $2^3D_1 \rightarrow K^*K$, $2^3D_1 \rightarrow K(1270)K$, which is due to the large strange quark mass and the dynamical nodal suppression. This is strongly supported by the fact that the partial decay width of $2^3D_1 \rightarrow \phi\eta$, $2^3D_1 \rightarrow \phi\eta'$ and $2^3D_1 \rightarrow h_1(1380)\eta$ are similar to those in the $^3P_0$ model within 3%, even when considering variations in $\beta_A$ and $M_A$, where a $s\bar{s}$ pair is created from the vacuum. Additional evidence would be that the widths for the $S + S$ and $P + S$ final states($KK, K^*K, \phi\eta$ etc) in the flux tube model would be the same as those in the $^3P_0$ model within 3%, if we set $m_{u,d} = m_s$. For simplicity, the effect of $m_{u,d} \neq m_s$ is ignored in the above $^3P_0$ model, following Ref.\[5, 6\], whereas this effect is included explicitly in the flux tube model. A more delicate study of radial strange quarkonium with the mass difference between the original quark and the created quark considered may be necessary to improve the overall agreement between the predictions and experimental data, which has been done for the higher charmonium decay in the $^3P_0$ model\[23\]. For the interesting $K^*K^*$ mode, $\Gamma_{F2}$ is predicted to the dominant as well, and the partial widths ratios are the same as the $^3P_0$ model results.

In order to illustrate the parameter dependence of the model prediction, we show the $\beta_A$ dependence of partial width and total width for $2^3D_1 s\bar{s}$ decay respectively in Fig.5 and Fig.6, and the $M_A$ dependence is displayed in Fig.7 and Fig.8. We can see that the $\beta_A$ and $M_A$ dependence is very similar in the $^3P_0$ model and the flux tube model, we expect the agreement will be improved further if the effect $m_{u,d} \neq m_s$ is considered. The width is predicted to be around 200MeV, which is large than the width of $1^{--}$ strangeonium hybrid\[2\]. Although the
predicted decay width is larger than present experimental value ($\Gamma = 58 \pm 16 \pm 20\text{MeV}$), we cannot exclude the $2^3D_1 s\bar{s}$ hypothesis considering the uncertainties of the model and the experimental errors. The main conclusions remain to be similar to the $^3P_0$ model’s. Namely, the modes $K(1460)K$, $K^*K^*$, $KK$ and $h_1(1380)\eta$ are crucial in distinguishing between the $2^3D_1 s\bar{s}$ interpretation and the strangeonium hybrid, which has a large branch ratio in the former picture, but are forbidden instead in the latter.

FIG. 5: The variation of $Y(2175)$ partial decay widths with $\beta_A$ as a $2^3D_1 s\bar{s}$ state in the flux tube model.

FIG. 6: $Y(2175)$ total width dependence on $\beta_A$ as a $2^3D_1 s\bar{s}$ state in the flux tube model.

FIG. 7: Partial decay widths of $Y(2175)$ as a $2^3D_1 s\bar{s}$ state at different initial state mass $M_A$ in the flux tube model.

FIG. 8: Total widths of $Y(2175)$ as a $2^3D_1 s\bar{s}$ state at different initial state mass $M_A$ in the flux tube model.
IV. SUMMARY AND DISCUSSIONS

$Y(2175)$ has mass that is consistent with $2^3D_1\ s\bar{s}$ and $3^3S_1\ s\bar{s}$ meson. In this paper, we examine the possibility that $Y(2175)$ is a $2^3D_1\ s\bar{s}$ meson. We would like to mention that the $3^3S_1\ s\bar{s}$ state is predicted to be a rather broad state by T.Barnes et al. (please see the last column of Table I) so that it cannot be identified with $Y(2175)$ [6]. We have studied $2^3D_1\ s\bar{s}$ decay from both the $^3P_0$ model and the flux tube model, and the results are similar in the two models, despite the large difference in some channels. This is due to large strange quark mass and dynamic nodal suppression. The agreement will be better, if the mass difference between the origin quarks in the initial state and the created quarks is considered in the $^3P_0$ model.

It has been found that the difference between the decays of $1^{--}$ strangeonium hybrid and that of $2^3D_1\ s\bar{s}$ is significant. For the hybrid suggestion of [2], $Y(2175) \rightarrow K_1(1400)K, \ K_1(1270)K$ are the main decay channels, and the decay modes of $Y(2175) \rightarrow KK, \ K^*K^*, \ K(1460)K, h_1(1380)\eta$ are forbidden. For the $2^3D_1\ s\bar{s}$ scenario discussed in this present paper, however, the decay modes of $Y(2175) \rightarrow KK, \ K^*K^*, \ K(1460)K, h_1(1380)\eta$ should be visible and the corresponding decay widths are large in contrast to the hybrid picture of $Y(2175)$. Therefore, we conclude that according to the studies in [2] and the present paper the experimental search of the $Y(2175)$’s decay modes $KK, \ K^*K^*, \ K(1460)K, h_1(1380)\eta$ is a criteria to identify the structure of $Y(2175)$. In the other words, if these signals would be observed in the experiment, $Y(2175)$ as a $2^3D_1\ s\bar{s}$ quarkonium is favored, otherwise the interpretation of $Y(2175)$ as a hybrid is preferred. At the present stage, because of the lack of such experimental data, we cannot exclude the possibility that $Y(2175)$ is $2^3D_1\ s\bar{s}$ meson. Obviously, it is crucial and significant to detect the $Y(2175)$’s decay modes $KK, \ K^*K^*, \ K(1460)K, h_1(1380)\eta$ experimentally in order to identify whether $Y(2175)$ is a $2^3D_1\ s\bar{s}$ quarkonium or an exotic $s\bar{s}g$ hybrid meson.

The measurement of the ratios of the $K^*K^*$ partial decay widths is a very important test of whether $Y(2175)$ is a $2^3D_1$ or $3^3S_1\ s\bar{s}$ state. Both $^3P_0$ model and the flux tube model predict that $\Gamma_{F2}$ is the largest, and $\Gamma_{P2}/\Gamma_{P0} = 1/5$, provided that $Y(2175)$ is a pure $2^3D_1\ s\bar{s}$ quarkonium. However, $\Gamma_{F2}$ is predicted to be zero, and $\Gamma_{P2}/\Gamma_{P0} = 20$ for the $3^3S_1$ assignment.
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