Mathematical Modeling of a DC Electric Arc—Dimensionless Representation of a DC Arc

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A mathematical model presented in previous publications is used to describe fluid flow, heat transfer and electromagnetic phenomena in the arc region of a Direct Current Electric Arc Furnace (DC-EAF). The effects of the arc current and the arc length on the arc characteristics and arc bath interactions are analyzed in a rather generalized way. This analysis leads to the conclusion that the arc behaves in such a way that when the arc shape (defined with the location of the 10000 K isotherm) is plotted in the appropriate dimensionless form a unique expansion is achieved. This unique expansion (only restricted to arcs burned between graphite electrodes in air and outside the region where the arc jet impinges on the bath surface) gives the possibility to provide single dimensionless arc characteristics regardless the values of the arc current and arc length. These dimensionless correlations represent the main finding of this work and they allow means to estimate the important characteristics of the arc under different operational conditions without the need to run complex numerical calculation. The dimensionless characteristics of arc expressed in simple algebraic forms can be used for this purpose employing a pocket calculator.

KEY WORDS: mathematical modeling; mixing; fluid flow; heat transfer; electric furnace; arc.

1. Introduction

The EAF has been gaining acceptance as a steelmaking process around the world, but the electric energy consumption and electrode consumption are still issues impacting the efficiency of the process. It is a well-known fact that the level of efficiency is reaching a plateau. This fact does not allow achieving further increases in the productivity and in reduction of the operational costs in EAF’s, based on the empirical approaches commonly followed in this industry. It is then clear that acquiring fundamental understanding based on fundamental knowledge should be a critical objective and main motivation to perform research in EAF operations. The first serious attempt to provide a physical description in the arc region was given by Ushio et al. and Szekely and McKelliget using the turbulent Navier–Stokes, turbulent energy conservation and Maxwell equations to describe the arc physics based on heat, mass and momentum transfer equations along with the electromagnetic phenomena. They also predicted for the very first time the four contributions of heat transfer from the arc to the bath. However, their arc model was too simplified to be realistic, especially in representing the electromagnetic phenomena. After this work, only a few attempts have been published in the literature but unfortunately no radically new contributions have been made to this field of modeling since the work of McKelliget and Szekely appeared more than 25 years ago.

This article is the final contribution of a series of works previously published on the same topic. The overall study was based on the development of a rigorous mathematical model based on fundamental laws governing the arc physics. The model involves the simultaneous solution of Maxwell’s equations for the electromagnetic field, and the turbulent fluid flow and heat transfer equations. In the first publication, the effect of the main process variables of the electric arc, i.e. the arc current and the arc length, was studied in relation to the heat transfer from the arc to the bath. Also, the behavior of the main arc characteristics was analyzed as a function of these process parameters. This work was followed by a detailed study and analysis of the arc region based on the behavior of the more important arc characteristics. It was concluded from that analysis that the arc expansion is the main phenomena governing the arc behavior.

Several articles have been found in literature in which the main objective was to develop a global picture of the complex arc structure. The more elegant physical description of a DC electric arc so far was published by Bowman, who described in a rather simple form (based on measurements or on asymptotic solutions) most of the arc characteristics such as arc voltage, arc shapes, radiation heat transfer, and arc temperatures. In earlier work, Jordan et al. tried to understand the behavior of high-power arcs by measuring (using a photographic approach) the arc shapes. Recently, Mendez et al. developed a novel asymptotic technique...
based on the order of magnitude method. Using their technique, they were able to describe the principal forces driving the plasma jet and they were able to give some of the arc characteristics with simple algebraic expressions for arcs found in welding arc processes.

The work presented here summarizes in a rather general, “unique,” and convenient way the physics and structure of the electric arcs found in DC-EAFs. The main finding and contribution of the study is the generation of dimensionless maps of some of the most important arc characteristics. The arc characteristics dimensionless mapped are the temperature, velocity and magnetic flux density. Those “dimensionless maps” are valid for any operational conditions (regardless arc current or arc length), within a rather wide range of process conditions. The only restriction is that those maps are only valid in arcs struck between graphite cathodes in air as a plasma gas. Also, their validity is restricted to the whole arc region, except in the vicinity of the anode where the arc impinges on the bath. The importance of such contribution stands on the fact that the entire arc physics can be explained if the arc expansion is determined, and since the arc expansion is given in a general dimensionless form, there is no need to do computationally expensive numeric calculations, but rather use simple algebraic expressions to obtain detailed information of the arc.

2. Mathematical Model of the Arc Region

For a complete description of the mathematical representation of the arc, the reader is referred to,8) where governing equations, boundary conditions, and special description of the cathode and anode regions are fully and explicitly presented.

3. Results and Discussion

In order to carry out the parametric study, a set of numerical calculations were run varying both arc length and arc current. Five different arc lengths and four different arc currents were employed in order to cover a wide range of practical conditions. Values used for currents are 36 kA, 40 kA, 44 kA, and 50 kA; while arc lengths employed are 0.15 m, 0.2 m, 0.25 m, 0.3 m, and 0.35 m. The commercial CFD code PHOENICS 3.2 was employed to solve iteratively the complete set of governing equations, taking over 3,000 iterations to converge the solutions.

It is clear from previous publications8) that arc parameters, such as arc length and arc current, greatly affect the arc properties and arc–bath interactions. This section has the intention to present a summary of the process by correlating some important arc characteristics with the main arc variables, i.e., arc current, I, and arc length, L. The summary is expected to have a rather general or unique validity and, whenever possible, are presented using dimensionless groups. Generalization of the knowledge gained with the model is, at the end, one of the important objectives of the modeling work.

3.1. Comparison between Numerical Calculations and Analytical Expressions

Figure 1 shows the relationship between the maximum jet velocity (along the symmetry axis) and the arc current. The line in the plot corresponds to the well-known Maecker equation10) (Eq. (1)), analytically derived under the assumption of inviscid flow and isothermal arc conditions, while the points are results from the numerical computations.

\[ V_{\text{max}} = \frac{\mu_0 J_c I}{2 \pi \rho} \] (1)

where \( V_{\text{max}} \) is the maximum velocity, \( \mu_0 \) is the magnetic permeability, \( J_c \) is the current density at the cathode spot, \( I \) is the electric current, and \( \rho \) is the density. The maximum velocities are expected to appear close to the cathode, where the Maecker equation applies.

Close to the cathode temperatures above 19,000 K are found, as can be seen from Figs. 2(a) and 2(b), which show computed results for a typical electric arc (40 kA arc current and 0.25 m arc length). (a) Temperature field; (b) temperature distribution along the symmetry axis, (c) plasma density.
computed temperature field and temperature profile along the symmetry axis respectively, for a 40 kA and 25 cm electric arc. Density of air corresponding to 19000 K is approximately 4.9×10^{-3} kg/m^3, as it can be appreciated in Fig. 2(c), which presents the predicted distribution of density in a 40 kA and 25 cm electric arc. This value of density is a reasonable approximation to predict maximum velocities using the Maeker equation.

The Maeker equation is important because it is an analytical expression derived from the conservation equations and is widely used to provide estimated values. However, this expression is an oversimplification because it assumes a constant radial current density distribution, isothermal plasma and neglects the effect of the cathode surface. Thus, it is known that the Maeker equation overestimates the maximum velocities. The results in Fig. 1 show that the calculated maximum arc velocities follow some functionality with the arc current (V_{max} \propto J^{0.5}) also dictated by the Maeker equation. However, as shown in the figure the maximum velocity is also a function of the arc length, \( L \).

Maeker derived another relation that correlates the arc pressure below the cathode (due to electromagnetic body forces) and the arc current.\(^\text{(10)}\)

\[
P_{\text{max}} = \frac{\mu J L}{4\pi} \quad \text{..............................(2)}
\]

where \( P_{\text{max}} \) is the maximum pressure below the cathode. From the above equations, the pressure is expected to scale linearly with arc current. For a given \( J_{c} \) value of 4.4×10^7 A/m^2, a 4.4 slope value is expected, which is presented in Fig. 3 together with numerical, predicted arc pressures. Fitting the data of 15 cm arc length and 36, 40, 44 and 50 kA to a linear equation gives a 4.98 slope, i.e. a value 13.2% higher than that predicted by Maeker. This means that if Eq. (2) is multiplied by a factor of 1.132 a better fitting of the data is obtained, as shown by the dashed line in Fig. 3. In this figure it is also appreciated that the arc length does not significantly affect the arc pressure. Therefore, the pressure below the cathode is proportional to arc current and is practically independent of arc length.

Figure 4 shows the relation between the maximum bath current density, in dimensionless form \( \left( J_{\text{max}}/J_{c}\right) \) and the dimensionless arc length \( L/R_c \), for all currents and arc lengths conditions used in the calculations of this study. In fact, this dimensionless arc length \( (L/R_c) \) involves the two main process parameters, i.e. the arc length and the arc current, which is related to the arc spot radius, \( R_c \), through the Eq. (3).

\[
R_c = \left( \frac{I}{\pi J_c} \right)^{0.5} \quad \text{..............................(3)}
\]

Since the spot current density \( (J_c) \) is constant, the spot radius is only a function of the arc current, \( I \ (R_c \propto I^{0.5}) \). The figure shows that for small values of \( L/R_c \), i.e. for short arc lengths and high currents, high values of current density at the bath are obtained. In contrast, for large \( L/R_c \) values (high arc lengths and small currents) small currents densities are obtained. The functionality is expressed by the following fitting equation (also presented in Fig. 4):

\[
\frac{J_{\text{max}}}{J_c} = \frac{2.855}{(L/R_c)} \quad \text{..............................(4)}
\]

where \( J_{\text{max}} \) is the maximum current density at the bath surface.

### 3.2 Dimensionless Representation of the Arc

A very interesting finding in this work is presented in Fig. 5. In this plot, the dimensionless arc radius \( (R_c/R_a) \) is plotted as a function of the dimensionless axial position \( L/\pi R_c \) for a large number of predicted conditions. The arc radius, \( R_a\), is determined by the 10 000 K isotherm, which according to Jordan\(^\text{(12)}\) is coincident with the visible arc radius and can be used to define the conduction zone. The dots are experimental points reported by Bowman\(^\text{(13)}\) and the lines are the predicted dimensionless arc radius as a function of dimensionless axial distance for all cases considered in this study. An excellent agreement between predictions and experiments is observed in the plot. It is a very helpful to recognize that all arc shapes lie on the same line when plotted in dimensionless form. It is noted that departure from the fitted line occurs in the vicinity of the bath, where the arc undergoes some additional expansion due to the impingement of the jet on the bath. But the main contribution re-
ported in Fig. 5 is that the arc shape can be represented conveniently by single dimensionless curve for a wide range of arc currents and arc lengths (arc powers). Of course, the unique shape is restricted to electric arcs under air atmosphere and stroke between graphite cathode electrodes. The shape of the arc presents a quadratic relationship with the axial distance from cathode:

\[
\frac{R_a}{R_c} = 0.864 \cdot 0.253 \left(\frac{Z}{R_c}\right)^{0.5}
\]  

(5)

The importance of Fig. 5 is that since the arc region can be expressed in a unique fashion, at least ignoring the immediate region close to the bath, it is possible to express the arc characteristics in analog dimensionless way. To probe the last statement, magnetic flux density radial profiles are plotted in Figs. 6(a) to 6(d) for three different arc conditions, presented in Table 1, at the same dimensionless axial distances, \(Z/R_c\) of 0, 2, 5 and 7. The figure shows similar functionalities in the profiles but the magnitudes in the values of the magnetic flux densities are not the same. If the magnetic flux density radial profiles presented in Fig. 6 are now plotted in the correct dimensionless forms, a single profile can be obtained as indicated in Fig. 7. The magnetic flux density values were normalized by dividing over the maximum value at the specific \(Z/R_c\) position, \(B_{\text{max}}\), while the radius was normalized with the arc radius, \(R_c\). The arc radius is a function of the axial dimensionless distance \(Z/R_c\) as indicated in Eq. (5) and Fig. 5. \(B_{\text{max}}\) is the maximum magnetic flux density value at each axial distance. Therefore, this variable is a function of both the arc current and the dimensionless axial distance \(Z/R_c\). As can be seen in Fig. 7(e), all profiles lie on the same line regardless the axial position and arc current. Then it is possible to give simple expressions for each part of the magnetic flux density profiles. In this way, the region inside the arc can be expressed as a cubic function of the radius (implying a parabolic profile for the axial current density, \(J_z\), since \(B = \rho\), the...
azimuthal magnetic flux density, is obtained by integration of \( J_z \):

\[
B_{\theta} = 1.863 \left( \frac{R}{R_a} \right) - 0.587 \left( \frac{R}{R_a} \right)^2 - 0.191 \left( \frac{R}{R_a} \right)^3 \quad \text{for } R \leq R_a \quad \text{(6)}
\]

Outside the arc, the profile is inversely proportional to the dimensionless radius:

\[
\frac{B_{\theta}}{B^{\text{max}}} = \frac{1}{\left( \frac{R}{R_a} \right)} \quad \text{for } R > R_a \quad \text{(7)}
\]

In order to obtain the \( B_{\theta} \) field, Eqs. (6) and (7) can be employed but \( R_a \) and \( B^{\text{max}} \) values must be first obtained. The arc radius, \( R_a \), can be easily obtained from Fig. 5 or from Eq. (5). However, \( B^{\text{max}} \) can be expressed as a function of the dimensionless distance, \( Z/R_c \), regardless the arc current, if it is expressed in dimensionless form by dividing over the maximum value of \( B_{\theta} \) in the entire domain, \( B^{\text{MAX}} \). \( B^{\text{MAX}} \) is always located at the cathode and can be expressed analytically as:

\[
B^{\text{MAX}} = \frac{\mu_0 I}{2\pi R_a} \quad \text{(8)}
\]

Figure 8 shows a plot of \( B^{\text{max}}/B^{\text{MAX}} \) against \( Z/R_c \) for the three arcs considered. Similar trends are observed for all three arcs considered. The functionality between \( B^{\text{max}}/B^{\text{MAX}} \) and \( Z/R_c \) can be expressed by:

\[
\frac{B^{\text{max}}}{B^{\text{MAX}}} = \exp \left( 0.036 - 0.196 \left( \frac{Z}{R_c} \right)^{0.5} \right) \quad \text{(9)}
\]

This means that before the impingement zone is reached, a unique or dimensionless \( B_{\theta} \) field expressed in dimensionless form can be obtained using Eqs. (6) and (7) since \( B^{\text{max}} \) and \( R_a \) are available through Eqs. (9) and (5), due to the unique arc shape behavior for the range of conditions explored in this study.

Temperature fields can also be presented in the same dimensionless way just as the \( B_{\theta} \) field described above. Again, considering the same arc cases included in Table 1, temperature radial profiles are plotted in Figs. 9(a) to 9(c) at three different dimensionless axial distances of 2, 5, and 7. However, not big differences are observed in the plots for the different arcs. Then, Fig. 9(d) shows the radial tempera-
ture profiles for the 36 kA and 35 cm arc at six axial distances \(Z/R_c/1005\ 1.43, 4.1, 6.62, 9.38, 12.3, 15.067\), and as seen in the figure, arc temperatures decrease as distance from the cathode increases and the profiles become wider as the arc expands.

**Figure 10** presents the same temperature radial profiles as in Fig. 9 but this time in dimensionless form. The temperature \(T\) has been normalized by dividing over its maximum value at each axial position, \(T_{0\max}\) which it is always located at the symmetry axis. The radial distance can be normalized exactly as the previous case, by dividing it over the arc radius, \(R_a\), which in turns can be obtained via Eq. (5). It can be appreciated that the radial temperature profiles are similar in dimensional form but they become even more when these profiles are plotted in dimensionless form.

When all arc cases and axial distances are plotted together (Fig. 10(e)), it is interesting to note that a single line can be observed inside the arc \((R_c<1)\) but outside the arc differences are more evident. However, since the arc region is the most important region in our calculation domain, a simple expression can be given for radial temperature profiles regardless the arc current and axial distance (but only valid in the arc region outside the impingement zone):

\[
\frac{T}{T_{0\max}} = 0.042 + \frac{0.974}{1 + \exp \left(\frac{-(R/R_c - 1.079)}{0.265}\right)}
\]  

\(\text{(10)}\)

In order to obtain the entire temperature field inside the arc region, it is necessary to obtain the axial temperature profiles at the symmetry axis, \(T_{0\max}\), and at the arc radius, \(R_c\). Equation (5) expresses the dimensionless relation between the arc radius and the axial distance. To obtain a unique expression for the temperature at the axis, \(T_{0\max}\), the three numerical computations used under the conditions shown in Table 1 were used to obtain a dimensionless plot for the dimensionless temperature, \(T_{0\max}/T_{MAX}\), along the dimensionless axial distance, \(Z/R_a\). \(T_{0\max}/T_{MAX}\) is the maximum temperature obtained in the domain, which is always located in the vicinity of the cathode. This value must be obtained from the computation, but does not vary too much in the range of conditions analyzed in this work due to the constant value of \(J_c\) employed in the simulations (25 954–26 896 K).

**Figure 11(a)** shows the axial profile of temperature as a function of axial distance for the three arcs in dimensional form. **Figure 11(b)** presents the dimensionless version of Fig. 11(a). It is easily seen that all axial temperature lines converge into a single line. The importance of this finding is that is possible to generate a complete temperature field inside the arc region, just before the impingement point, if \(T_{0\max}\) is available. Consequently, fields of all physical properties can be estimated since they depend exclusively on temperature as was originally stated in this model, under the assumptions considered.

The expression for \(T_{0\max}/T_{MAX}\) versus \(Z/R_c\) is given as:

\[
\frac{T_{0\max}}{T_{MAX}} = 0.435 + 0.544\exp\left(\frac{-(Z/R_c)}{12.974}\right)
\]  

\(\text{(11)}\)
Axial velocity fields are also treated under the same methodology to obtain a unique velocity field. The axial velocity is predominant inside the arc but away from the cathode and bath surfaces where the radial velocities dominate the flow motion. Figure 12 shows radial profiles of axial velocities computed, $V_z$, for the three arcs considered in Table 1 at four dimensionless distances from the cathode of 2, 5, 7 and 11. The axial velocities decrease with increasing axial distance and the radial profiles are wider as the arc expands, as can be seen in Fig. 12(e) that presents a 36 kA and 35 cm arc velocities plotted at four dimensionless distances of 2, 5, 7, 11. The same plots as in Fig. 12 but in dimensionless form are presented in Fig. 13. The axial velocity is normalized by dividing over the maximum velocity computed at each axial distance, $V^{0}_{max}$, while the radius is normalized using the arc radius given by Eq. (5). $V^{0}_{max}$ is always located at the symmetry axis. When all lines are plotted together, a single line again is obtained regardless the arc current, the arc length and the axial position (Fig. 13(f)). This expression may be expressed through the following relationship:

$$\frac{V_z}{V^{0}_{max}} = 1.020 \exp \left(-0.5 \left(\frac{Z}{r_c} - 0.006 \frac{Z}{R_c}\right)^2\right) \ldots (12)$$

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Fig. 10. Dimensionless temperature radial profiles for the arcs presented in Table 1. The profiles correspond to those in Fig. 8. (a) $Z/R_c = 2$, (b) $Z/R_c = 5$, (c) $Z/R_c = 7$. (d) Dimensionless temperature radial profiles are plotted at six different axial distances form cathode for the arc of 36 kA and 35 cm. (e) Lines from Figs. (a), (b), (c) and (d) are plotted together.

Fig. 11. Temperature axial profiles along the symmetry axis for the arcs in Table 1. (a) Dimensional axial profiles. (b) Dimensionless profiles obtained when $T_{max}$ is divided by the maximum temperature in the domain, $T_{MAX}$, while $Z$ is divided by $R_c$. 

$$\frac{V_z}{V^{0}_{max}} = 1.020 \exp \left(-0.5 \left(\frac{Z}{r_c} - 0.006 \frac{Z}{R_c}\right)^2\right) \ldots (12)$$
Fig. 12. Axial velocity profiles along the radius for the arcs presented in Table 1. The profiles correspond to dimensionless distances of (a) $Z/R_c=2$, (b) $Z/R_c=5$, (c) $Z/R_c=7$, (d) $Z/R_c=11$. (e) Axial velocities along the radius at dimensionless distances of 2, 5, 7, and 11 (36 kA and 35 cm arc).

Fig. 13. Dimensionless axial velocity profiles along the radius for the arcs presented in Table 1. The profiles correspond to dimensionless distances of (a) $Z/R_c=2$, (b) $Z/R_c=5$, (c) $Z/R_c=7$, (d) $Z/R_c=11$. (e) Dimensionless axial velocity profiles for the 36 kA and 35 cm arc. (f) All lines are plotted together to show the unique fashion of the profile.
In order to generate a single dimensionless axial velocity field, a unique expression for \( V_{\text{MAX}}^0 \) must be obtained. In order to generate such a relation, a dimensionless axial velocity plot against dimensionless axial distance at the symmetry axis is presented in Fig. 14(b). Axial velocity is normalized by dividing over the maximum arc velocity, \( V_{\text{MAX}} \). The figure presents three lines representing each of the arcs considered in Table 1. Lines are very similar, but differences are important close to the cathode and bath surface (impingement region), which may be explained since radial velocities dominate in those regions. Fig. 14(a) is the same plot presented in Fig. 14(b) but in dimensional form. This plot is presented to point out that despite the differences in the dimensionless curves, still the axial velocity profiles are very similar in dimensionless form when compared against dimensional curves. An interesting observation in the axial profile of the axial velocity is that the maximum velocities are located at the same dimensionless axial distance, \( Z/R_c \), of 5. The complex axial velocity field can only be expressed via a rather complex polynomial function:

\[
\frac{V_{\text{MAX}}^0}{V_{\text{MAX}}} = \frac{3.270}{1+3.270} \left( \frac{Z}{R_c} \right)^{-0.240} \left( \frac{Z}{R_c} \right)^2 + 0.019 \left( \frac{Z}{R_c} \right)^3
\]

\[
= \frac{3.270}{1+3.270} \left( \frac{Z}{R_c} \right)^{-0.240} \left( \frac{Z}{R_c} \right)^2 + 0.052 \left( \frac{Z}{R_c} \right)^3
\]

\[
...........................(13)
\]

The closure would be the acquisition of the maximum arc velocity, \( V_{\text{MAX}} \). This value can be obtained from the actual simulations or by employing the Maecker equation (Eq. (1)) that estimates this value. Then, once \( V_{\text{MAX}} \) is estimated, the complete axial velocity field can be obtained by combining Eqs. (13) and (12).

4. Conclusions

In the light of the analysis performed in this work, the following conclusions can be withdraw to represent the behavior of a DC electric arc, for the range of conditions stated:

1. It was found that the boundary of the arc region could be unambiguously defined as the location of the 10,000 K isotherm.

2. Maecker’s asymptotic expressions for maximum velocity and arc pressure are consistent with our model predictions in both trends and values. The expression obtained for maximum arc pressure at the cathode in this work is as follows:

\[
P_{\text{MAX}} = 1.13 \frac{\mu_{\text{oc}}}{4\pi} \left( \frac{B I}{\pi R_c} \right)^{0.5}
\]

3. The most important contribution of this work, regarding the description of the arc physics, is the definition of a unique arc shape in dimensionless form (valid for any arc current and arc lengths within the range of conditions studied and just before the impingement region close to the bath). The arc shape is the fundamental parameter that permits the prediction of the axial velocity, temperature and magnetic flux density through simple algebraic expressions. These expressions are limited to free-burning arcs with graphite cathode electrodes in air and are presented below:

a) Arc shape or arc radius (before the impingement region):

\[
R_c = \frac{0.864 - 0.253}{\frac{Z}{R_c}}
\]

\[
...........................(5)
\]

where \( R_a \) is the arc radius, \( R_c \) is the cathode spot radius and \( Z \) is the axial distance from the cathode.

b) Magnetic flux density:

\[
\frac{B_{a\theta}}{B_{\text{MAX}}^{a\theta}} = 1.863 \left( \frac{R}{R_a} \right) - 0.587 \left( \frac{R}{R_a} \right)^2 - 0.191 \left( \frac{R}{R_a} \right)^3
\]

\[
...........................(6)
\]

\[
\frac{B_{a\theta}}{B_{\text{MAX}}^{a\theta}} = 1
\]

\[
...........................(7)
\]

\[
for R < R_a
\]

\[
B_{\text{MAX}}^{a\theta} = \frac{\mu_{\text{oc}} I}{2\pi R_c}
\]

\[
...........................(8)
\]
\[
\frac{B^0_{\text{max}}}{B_{\text{MAX}}} = \exp\left(0.036 - 0.196 \left(\frac{Z}{R_e}\right)^{0.5}\right) \quad \cdots (9)
\]

where \(B_0\) is the magnetic flux density, \(B^0_{\text{max}}\) is the maximum magnetic flux density value (located at the arc radius), and \(B_{\text{MAX}}\) is the maximum magnetic flux density value in the domain (always located at the edge of the cathode spot radius).

c) Temperature field:

\[
\frac{T}{T_{\text{max}}} = 0.042 + \frac{0.974}{1 + \exp\left(-\left(\frac{R}{R_e}\right) - 1.079 - \frac{0.265}{R_e}\right)} \quad \cdots (10)
\]

\[
\frac{T^0_{\text{max}}}{T_{\text{MAX}}} = 0.435 + 0.544 \exp\left(-\left(\frac{Z}{R_e}\right) - 12.974\right) \quad \cdots (11)
\]

where \(T\) is the arc temperature, \(T^0_{\text{max}}\) is the temperature along the symmetry axis and \(T_{\text{MAX}}\) is the maximum temperature in the domain (which varies from \(-25,000\) to \(-28,000\) K for the whole range of conditions analyzed in this work).

d) Axial velocity field:

\[
\frac{V_z}{V_{z,\text{max}}} = 1.020 \exp\left(-0.5 \left(\frac{Z}{R_e}\right) - 0.006 \left(\frac{Z}{R_e}\right)^2\right) \quad \cdots (12)
\]

\[
\frac{V^0_{z,\text{max}}}{V_{z,\text{MAX}}} = \frac{3.270 \left(\frac{Z}{R_e}\right) - 0.240 \left(\frac{Z}{R_e}\right)^2 + 0.019 \left(\frac{Z}{R_e}\right)^3}{1 + 3.270 \left(\frac{Z}{R_e}\right) - 0.5 \left(\frac{Z}{R_e}\right)^2 + 0.052 \left(\frac{Z}{R_e}\right)^3} \quad \cdots (13)
\]

where \(V_z\) is the axial velocity, \(V^0_{z,\text{max}}\) is the axial velocity along the axis and \(V_{z,\text{MAX}}\) is the maximum velocity, which can be determined from the Maecker Eq. (1).

All of the physical properties in the arc can be obtained from the temperature field. The magnetic field can be used to obtain the current densities and the electric potential using Ampere’s and Ohm’s laws.

The above expressions can be used to provide estimates of the important arc variables as a function of process conditions and represent a convenient way to summarize results.

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