1. Introduction

When developing the deposits of decorative facing stone, the economic efficiency of mining to a large extent will depend on the effectiveness of management of technological processes. According to the results of research into approaches to the process of extraction of blocks of decorative stone [1–8], the efficiency of application of separate

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technological processes and the productivity of the whole complex of mining equipment is defined by both natural and technological indicators. It should be noted that among the natural factors, fracturing affects the efficiency of extraction of blocks most of all. When planning mining operations, it is necessary to ensure maximum consideration of spatial orientation and distances between the planes of cracks. These factors determine parameters of the system of development of deposit, so the error of their estimation could potentially lead to significant economic losses, which stresses the importance of correct choice of the method of analysis.

A variety of methods are used for the analysis of natural fracturing, which differ in their efficiency and reliability. For the analysis of complex samples when solving various optimization problems, the methods of cluster analysis are often used. This approach in many cases allows obtaining comparatively better results of analysis of systematization of measurement results and increasing significantly the efficiency of prediction.

Thus we can assume that the use of cluster analysis to a large degree of probability will increase efficiency of the process of quality control of block raw materials.

2. Literature review and problem statement

In the course of analysis of effectiveness of the process of extraction of block stone, different approaches were applied. Thus, paper [1] considered the problem of optimization of planning of the work of technological process. Other authors assessed the efficiency of control by the degree of consideration of parameters of the fracturing of deposit [2–6]. In this case, special attention was paid to the methods of measuring the elements of cracks. Thus, article [2] explored the efficiency of determining parameters of fracturing when using a georadar (GPR) and its effect on the quality of array of natural stone. Paper [3] examined influence of natural fracturing on the productivity of technological complexes. It is worthwhile emphasizing a group of articles [4–11], the basic task of which was to assess blockiness of decorative stone deposits. This group of publications is characterized by the diversity of approaches to the definition of blockiness, in part due to different types of rocks that were examined. Thus, papers [4, 5] pay much attention to evaluation of blockiness based on the analysis of geometric ratios between the systems of cracks. The study of effect of fracturing on the quality of decorative stone deposit was the subject of articles [6–11]. Paper [6] examined the method of assessment of decorative stone deposit for obtaining block products based on results of fracturing measurement. Article [7] assessed deposit quality for highly–blocked deposits of marble. Ref. [8], based on the simulation of systems of cracks and splits, assessed deposit blockiness, the peculiarity being the use of cubes instead of the classic parallelepipeds for the model of an artificial block. The authors in [9] conducted geostatic simulation of the efficiency of extracting block stone at cutting, which substantially narrows the scope of application of results of this research, and requires further comprehensive consideration of influence of other technologies when planning development of mining. In paper [10], the estimation of economic efficiency of extracting blocks at different approaches to orientation of the front of work was performed. And in the previously conducted studies of the influence of natural factors on the efficiency of development of decorative stone deposit [11], they used quality assessment of the deposit on the basis of using distance methods of studying fracturing and mathematical modeling of effectiveness of analysis of the received model of the natural separateness depending on the selected direction of development of mining. Summing up the outcome of publications that are devoted to defining the direction of development of mining deposits of block stone, it is possible to conclude that the fracturing of deposit is the dominant indicator of its quality and largely determines the direction of development of mining and the structure of comprehensive mechanization of mining processes. That is why the development of efficient techniques of evaluation of regularities of development and spatial orientation of the systems of natural cracks is a relevant scientific and applied task. In the previous studies, the authors [12] conducted analysis of the patterns of change in structural indicators of deposits of labradorite based on the cluster analysis without research into effectiveness of clustering for different samples, which somewhat limits the use of this approach for a certain part of deposits of labradorite. Paper [13] proposed fuzzy clustering algorithm of K-means that has a capacity to use additional information about splits, as well as their orientation in the course of the search data analysis. But in the course of the study it is not defined under which specific conditions the implementation of this methodology might be promising. Ref. [14] considered several clustering algorithms: Parzen and K-means, and evaluated their effectiveness on a generated sample and on real data. The disadvantages of this research include the use of only one reference sample and analysis of effectiveness of the proposed approach to the analysis of data for only one field. Paper [15] applied clustering algorithm of K-means for the analysis of elements of cracks measured by the results of laser scanning. The main shortcoming of this study is the lack of analysis of the effectiveness of applying other methods of clustering.

Summarising the results of studies of the effectiveness of applying cluster analysis for the fracturing of decorative stone deposits, it is possible to say that, to optimize the process of quality control of block raw materials, it is expedient to improve the existing ones or develop new methods for the analysis of fracturing, which will ensure the effectiveness of their application for different deposits. The most promising area of research to consider is the study of efficiency of different methods of clustering for selecting the systems of cracks for the types of samples that are typical for deposits of decorative stone of the same genetic group. Substantiation of the efficient clustering method and its parameters will simplify implementation of the method of cluster analysis and significantly increase the efficiency of systematization of the results of measurements of the elements of natural cracks. This will make it possible to significantly improve the accuracy of determining parameters of natural separateness and, accordingly, assessment of blockiness of deposit and its separate areas, which will enable us to optimize the process of quality control of block raw materials.

3. The aim and tasks of research

The aim of research is the development of technique of cluster analysis of the fracturing of decorative stone deposits to optimize the process of quality control of block raw materials.
Characteristics of parameters of orientation of the systems of subvertical cracks of labradorite arrays of labradorite deposits, Zhytomyr Oblast

| No. of entry | Deposit name      | Number of measurements | Longitudinal – S | Selected systems of cracks | Diagonal – D |
|--------------|-------------------|------------------------|------------------|-----------------------------|--------------|
|              |                   |                        | Quantity | Dispersion | Quantity | Dispersion | Quantity | Dispersion | Quantity | Dispersion |
| 1            | Andriyivskiy      | 31                     | 17       | 45         | 11       | 45         | 3        | 3          |
| 2            | Brazhenskiy-1     | 83                     | 32       | 28         | 51       | 32         | –        | –          |
| 3            | Verkoluzkiy       | 83                     | 45       | 40         | 38       | 22         | –        | –          |
| 4            | Golovinskiy       | 131                    | 47       | 53         | 77       | 42         | 7        | 36         |
| 5            | Guta-Dobriniskiy  | 43                     | 16       | 44         | 18       | 30         | 9        | 50         |
| 6            | Dobrinskiy        | 30                     | 11       | 40         | 15       | 55         | 4        | 10         |
| 7            | Kovalivskiy       | 27                     | 14       | 44         | 11       | 60         | 2        | 20         |
| 8            | Mykivskiy         | 62                     | 25       | 50         | 31       | 30         | 6        | 25         |
| 9            | Neverivskiy       | 42                     | 24       | 70         | 18       | 45         | –        | –          |
| 10           | Olegovskiy        | 45                     | 23       | 60         | 16       | 50         | 6        | 35         |
| 11           | Osnitskiy         | 60                     | 31       | 33         | 29       | 28         | –        | –          |
| 12           | Ocheretyanskiy    | 35                     | 14       | 36         | 14       | 41         | 7        | 24         |
| 13           | Fedorivskiy       | 100                    | 38       | 60         | 40       | 50         | 22       | 20         |

Analysis of results of the previous studies performed in [12], in which the accuracy of clustering of reference samples is examined, as well as the frequency of application of different methods when exploring the fracturing in papers [13–15], as well as implementation in the applied software packages of data analysis STATISTICA [13], MATLAB and CLUSTERDELTA, allowed us, for substantiation of the optimal method of cluster analysis of fracturing, to select the methods of K-means, EM, Warda as the most promising ones.

To assess the quality of clustering to identify the parameters of a separate system of cracks when using each particular method, it is proposed to use an indicator of clustering quality, which is described by expression (1). This indicator was formed for assessment of the arithmetic average value of relative deviations of the results of clustering of values of strike azimuth, incidence angle of the system of cracks and their deviations from the reference values. We made a decision to accept parameters of the reference samples as the basic values. Accordingly, obvious is the effectiveness of using this indicator for the evaluation of accuracy of clustering. In this case, higher quality will correspond to the lower values of indicator of clustering quality:

\[ Q_i = \frac{1}{4} \left( \left| \frac{A_{Ei} - A_{Ri}}{A_{Ri}} \right| \sum_{d=1}^{D} \frac{d_{Ai} - d_{Ai}}{d_{Ai}} + \left| \frac{E_{Fi} - E_{Fi}}{E_{Fi}} \right| \sum_{d=1}^{D} \frac{d_{Fi} - d_{Fi}}{d_{Fi}} \right) \]  

\( A_{E} \) is the reference value of strike azimuth of the system of cracks, degrees; \( A_{F} \) is the actual value of strike azimuth of the system of cracks, degrees; \( d_{Ai} \) is the reference value of deviation of strike azimuth of the system of cracks, degrees; \( d_{Fi} \) is the actual value of deviation of strike azimuth of the system of cracks, degrees; \( \delta_{E} \) is the reference value of incidence angle of the system of cracks, degrees; \( \delta_{Fi} \) is the actual value of incidence angle of the system of cracks, degrees; \( \delta_{Ei} \) is the reference value of deviation of incidence angle of the system of cracks, degrees; \( \delta_{Fi} \) is the actual value of deviation of incidence angle of the system of cracks, degrees.

Results of clustering for the reference samples of forty, sixty, a hundred and two hundred measurements are displayed in Fig. 1–4, respectively.

Table 1
Initially we examined the reference samples with the set parameters of distribution using STATISTICA 13. Next we chose the 3 most efficient clustering methods that were applied for the analysis.

After analyzing the diagrams of indicator of clustering quality for different samples, we may conclude that the lowest values that correspond to the maximum quality of selection of characteristics of each particular system of cracks fairly widely vary depending on the quantity of measurements and the values of reference samples. That is why it is quite difficult to definitely conclude about the effectiveness of a particular method of clustering. A relevant task, accordingly, is to create an indicator that would accumulate the values of indicators of quality for different systems into one integrated indicator.

Based on these considerations, for quantitative estimation of efficiency of the clustering of the array of data with varying number of systems, we developed the index of measurements quality. The essence of this index is assessing the extent of influence of the quantity of systems of cracks on the value of indicator of clustering quality. At its core, this indicator is the ratio of the sum of values of indicators of clustering quality for each of the selected systems to the quantity of the systems of cracks:

$$ I_Q = \frac{\sum_{i=1}^{n} Q_i}{n} $$

where $n$ is the quantity of the systems of cracks, pcs.

For the generalization analysis of numerical experiment on data clustering, consolidated results are displayed in Fig. 5.
the most efficient method of clustering was the K-means method with the Chebyshev distances; the given method is characterized by minimal fluctuations of the index of measurements quality within 0.22–0.28. As an alternative to it, one may consider the method of K-means with the Euclidean distances. For the sample that consists of 200 measurements, all methods of clustering except for EM yielded similar indices of the measurements quality. This result indicates that the increase in measurements of the elements of natural cracks occurrence leads to minimization of influence from the method of clustering. We obtained quite unexpected results for the EM clustering method: the maximum values of index of measurement error for the samples of 60 and 200 measurements and the lowest value for the sample of 100 measurements. Considering the obtained results, it is recommended for further research into the fracturing of deposits of decorative stone to apply clustering method of K-means with the Chebyshev distances.

5. A study of efficiency of the process of quality control
of block raw materials based on cluster analysis

As a result of conducted cluster analysis of K-means with the Chebyshev distances, we defined main systems of cracks at 13 labradorite deposits of Zhytomyr Oblast (Table 2).

| Deposit               | No. of system | Strike azimuth, degrees | Incidence angle, degrees | Quantity of cracks in the system, pcs. | Quantity of cracks, pcs. | Quantity of systems, pcs. |
|-----------------------|---------------|-------------------------|--------------------------|----------------------------------------|--------------------------|----------------------------|
| Andriyivskiy          | 1             | 184.33                  | 89.750                   | 3                                      | 6                        | 7                          |
|                       | 2             | 315.00                  | 89.250                   | 4                                      |                          |                            |
|                       | 3             | 170.00                  | 78.90                    | 10                                     |                          |                            |
|                       | 4             | 131.00                  | 86.00                    | 5                                      |                          |                            |
| Brazhenskiy           | 1             | 304.00                  | 72.50                    | 6                                      |                          |                            |
|                       | 2             | 36.33                   | 84.31                    | 51                                     |                          |                            |
|                       | 3             | 304.81                  | 83.42                    | 26                                     |                          |                            |
| Verkholuzskiy         | 1             | 295.00                  | 74.00                    | 4                                      |                          |                            |
|                       | 2             | 289.17                  | 86.75                    | 12                                     |                          |                            |
|                       | 3             | 62.00                   | 72.58                    | 12                                     |                          |                            |
|                       | 4             | 56.31                   | 85.29                    | 55                                     |                          |                            |
| Guta-Dobrinskiy       | 1             | 290.00                  | 8.00                     | 1                                      |                          |                            |
|                       | 2             | 31.67                   | 58.67                    | 3                                      |                          |                            |
|                       | 3             | 44.75                   | 82.42                    | 12                                     |                          |                            |
|                       | 4             | 262.50                  | 82.69                    | 26                                     |                          |                            |
| Golovinskiy           | 1             | 120.00                  | 36.00                    | 1                                      |                          |                            |
|                       | 2             | 23.95                   | 89.12                    | 43                                     |                          |                            |
|                       | 3             | 116.92                  | 83.46                    | 48                                     |                          |                            |
|                       | 4             | 27.50                   | 78.52                    | 40                                     |                          |                            |
| Dobryniskiy           | 1             | 41.53                   | 90.00                    | 19                                     |                          |                            |
|                       | 2             | 317.09                  | 90.00                    | 11                                     |                          |                            |
| Kamyanobridskiy       | 1             | 122.20                  | 81.64                    | 25                                     |                          |                            |
|                       | 2             | 172.22                  | 4.78                     | 9                                      |                          |                            |
|                       | 3             | 214.53                  | 83.91                    | 32                                     |                          |                            |
|                       | 4             | 110.00                  | 7.17                     | 6                                      |                          |                            |
| Kovalivskiy           | 1             | 299.45                  | 76.27                    | 11                                     |                          |                            |
|                       | 2             | 182.70                  | 82.00                    | 10                                     |                          |                            |
|                       | 3             | 270.00                  | 58.33                    | 3                                      |                          |                            |
|                       | 4             | 90.00                   | 69.33                    | 3                                      |                          |                            |
| Mykivskiy             | 1             | 16.22                   | 83.56                    | 9                                      |                          |                            |
|                       | 2             | 264.17                  | 80.00                    | 6                                      |                          |                            |
|                       | 3             | 1.67                    | 10.33                    | 3                                      |                          |                            |
|                       | 4             | 191.54                  | 11.69                    | 13                                     |                          |                            |
|                       | 5             | 197.22                  | 81.33                    | 9                                      |                          |                            |
|                       | 6             | 103.86                  | 82.50                    | 22                                     |                          |                            |
| Nevyrivskiy           | 1             | 93.33                   | 31.67                    | 6                                      |                          |                            |
|                       | 2             | 242.50                  | 86.00                    | 12                                     |                          |                            |
|                       | 3             | 125.00                  | 73.71                    | 24                                     |                          |                            |
To assess the quality of clustering, we established the existence of correlation connection between the number of selected systems of cracks and the quantity of cracks, which took part in the analysis for each field. The value of coefficient of correlation is 0.132, which indicates a lack of connection among the specified parameters and confirms efficiency of the proposed approach for selecting the systems of cracks.

Essential to the substantiation of the direction of development of the front of mining works is the quantity of cracks of certain orientation. In the course of carrying out analysis, we established that for all selected systems of cracks there is a weak correlation connection between the orientation of cracks and their quantity, which is characterized by the coefficient of correlation of 0.331. In the course of further analysis, we performed research into the connection between the orientation of cracks and their quantity for the systems of cracks, selected according to classification [23].

Accordingly, for the vertical cracks (incidence angle 85°–90°), the correlation coefficient was 0.56, that allows us to draw a conclusion about the effect of orientation of this type of cracks on their quantity. A similar situation was characteristic for the horizontal systems of cracks (incidence angle 0°–10°): the correlation coefficient was 0.831. The analysis of the previous research into spatial analysis of fracturing allowed us to accept expression (4) to determine the area of fractures, which is described in paper [16] for equation of plane, expressed through the coordinates of the start of crack and angles of its stretch and incidence.

To predict the direction of development of mining and control of the processes of extraction of decorative stone, we obtained analytical expression of dependency of the number of cracks on strike azimuth (Fig. 6) in the form of a polynomial of the second degree:

\[ N = 0.0005\alpha^2 - 0.227\alpha + 38.8532. \]  

The information about the systems of cracks is important for assessing the main criterion of a decorative stone deposit – blockiness. Taking into account the experience of methods of determining the blockiness, which were implemented by the authors in [6–8, 11], the basic task for adequate assessment of blockiness is the construction of appropriate predicting model of angular and linear ratios of the systems of cracks. When determining the angular ratio, the most rational approach is the approach to a crack as the plane in space. Then the angular ratios between the systems of cracks on their quantity, which is confirmed by the correlation coefficients – 0.246 and –0.010, respectively.

To predict the direction of development of mining and control of the processes of extraction of decorative stone, we obtained analytical expression of dependency of the number of cracks on strike azimuth (Fig. 6) in the form of a polynomial of the second degree:

\[ N = 0.0005\alpha^2 - 0.227\alpha + 38.8532. \]
\[ \cos \theta \cdot \tan \alpha \cdot x + \sin \theta \cdot \tan \alpha \cdot y + z - \tan \alpha = 0, \]  
(4)

where \( \theta \) is the strike azimuth of crack, degrees; \( \alpha \) is the incidence angle, degrees; \( x, y, z \) are the coordinates of the start of crack, m.

In this case, the angle between the planes of cracks in [16] is suggested to define from expression (5):

\[ \cos \varphi = \pm \left( \cos \theta_1 \cdot \cos \theta_2 + \sin \theta_1 \cdot \sin \theta_2 \right) \times \]  
\[ \times \sin \alpha_1 \cdot \sin \alpha_2 + \cos \alpha_1 \cdot \cos \alpha_2. \]  
(5)

For the qualitative assessment of a decorative stone deposit, it is necessary to assess the accuracy of angular ratios between the planes of the cracks. For this purpose, we performed transformation of expression (5) for obtaining expression for the angle \( \varphi \) followed by serial differentiation by \( \theta_1, \theta_2, \alpha_1, \alpha_2 \) according to the classic approach of the theory of errors and the least squares method. As a result, we obtained expression for the evaluation of accuracy of determining the angle between the planes of the cracks (6):

\[ m^2 = \left( \frac{\sin(\theta_1) \cdot \sin(\theta_2) \cdot (\cos(\alpha_1) \cdot \sin(\alpha_2) - \cos(\alpha_2) \cdot \sin(\alpha_1))}{\sqrt{(1 - (\sin(\alpha_1) \cdot \sin(\alpha_2) + \cos(\alpha_1) \cdot \cos(\alpha_2))^2)}} \right)^2 \cdot m^2_{\theta_1} \oplus \]  
\[ \oplus \left( \frac{\sin(\theta_1) \cdot \sin(\theta_2) \cdot (\cos(\alpha_1) \cdot \sin(\alpha_2) - \cos(\alpha_2) \cdot \sin(\alpha_1))}{\sqrt{(1 - (\sin(\alpha_1) \cdot \sin(\alpha_2) + \cos(\alpha_1) \cdot \cos(\alpha_2))^2)}} \right)^2 \cdot m^2_{\theta_2} \oplus \]  
\[ \oplus \left( \frac{\cos(\sin(\alpha_1) \cdot \sin(\alpha_2) + \cos(\alpha_1) \cdot \cos(\alpha_2))}{\sin(\theta_1) \cdot \sin(\theta_2) \cdot (\cos(\theta_1) \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \sin(\theta_1)) \cdot \sin(\alpha_1) \cdot \sin(\alpha_2) + \cos(\alpha_1) \cdot \cos(\alpha_2) \cdot \cos(\theta_1) \cdot \cos(\theta_2)} \right)^2 \cdot m^2_{\alpha_1} \oplus \]  
\[ \oplus \left( \frac{\cos(\sin(\alpha_1) \cdot \sin(\alpha_2) + \cos(\alpha_1) \cdot \cos(\alpha_2))}{\sin(\theta_1) \cdot \sin(\theta_2) \cdot (\cos(\theta_1) \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \sin(\theta_1)) \cdot \sin(\alpha_1) \cdot \sin(\alpha_2) + \cos(\alpha_1) \cdot \cos(\alpha_2) \cdot \cos(\theta_1) \cdot \cos(\theta_2)} \right)^2 \cdot m^2_{\alpha_2}, \]

where \( \theta_1, \theta_2 \) are the strike azimuths of a crack, degrees; \( \alpha_1, \alpha_2 \) are the incidence angle of cracks, degrees;

The obtained dependencies allow us to create geometrical-probabilistic model of natural separateness for each particular field or its area.

This result gives the opportunity to develop a new cluster-geometric method of quality control of block stone based on determining the indicator of blockiness, the essence of which lies in the serial execution of the following operations for separate areas of the deposit:

1) measurement of the elements of cracks occurrence and distances between them in an array of decorative stone;
2) selection of the main systems of cracks by strike azimuths and incidence angles based on the use of cluster analysis by the methods of clustering of K-means with the Chebyshev distances or K-means with the Euclidean distances;
3) determining parameters of the systems of cracks: strike azimuths and incidence angle and their dispersion;
4) determining the angular correlations between the systems of cracks;
5) determining the average distances between the systems of cracks;
6) estimation of errors in determining the angles between the systems of cracks;
7) assessment of probability of mutual intersection of the planes of selected cracks;
8) creation of a model of natural separateness in the form of parallelepiped, the angular and linear correlations in which are accepted from results of the previous stages and determining its volume from expression:

\[ V_{\text{art}} = d_Q \cdot d_L \cdot \sin \alpha_{\text{QL}}, \]  
(7)

where \( d_Q \) is the average distance between transverse cracks in the area, m; \( d_L \) is the average distance between the longitudinal cracks in the area, m; \( \alpha_{\text{QL}} \) is the average angle between the system of longitudinal and transverse cracks in the area, deg; \( d_L \) is the average distance between layer cracks in the area, m;
9) determining the volume of artificial rectangular block from expression:

\[ V_{\text{art}} = d_L \cdot d_L \cdot \sin \alpha_{\text{QL}} \cdot \sin \alpha_{\text{QS}} \cdot \sin \alpha_{\text{OS}}, \]  
(8)

where \( \alpha_{\text{QS}} \) is the average angle between the system of longitudinal and layer cracks, degrees; \( \alpha_{\text{QL}} \) is the average angle between the system of transverse and layer cracks, degrees;
10) determining the blockiness of deposit of decorative stone or its separate plot from expression:

\[ V = \frac{1 - \frac{V_{\text{art}} - V_{\text{int}}}{V_{\text{int}}}}{100 \%, \]  
(9)

where \( V_{\text{int}} \) is the volume of natural block, m\(^3\); \( V_{\text{art}} \) is the volume of artificial block, m\(^3\).
11) sequential execution of the above mentioned operations for each of the areas of the field;
12) defining technological solutions for each of the selected areas and the strategy of quality control of block raw materials for maximizing the extraction of blocks from the array.

The article contains an example of implementation of this technique for Nevyrivskiy deposit of labradorite. Results of cluster analysis are given in Table 3.

**Table 3**

| Strike azimuth of the system of cracks, degrees | Quantitative, pcs. | Dispersion | Angle of incidence of the system of cracks | Dispersion | Angle between the system of cracks, degrees | Error of determination, degrees | Probability of joint intersection |
|-----------------------------------------------|--------------------|------------|-------------------------------------------|------------|-------------------------------------------|-------------------------------|-------------------------------|
| 93°19'48"                                    | 6                  | 3671.8     | 31°40'12"                                 | 15.1       | 112°58'37"                               | 30.059                        | 0.06                          |
| 242°30'00"                                   | 12                 | 5600.0     | 86°00'00"                                 | 10.3       |                                           |                               |                               |
| 93°19'48"                                    | 6                  | 3671.8     | 31°40'12"                                 | 15.1       | 48°9'14.4"                               | 6.8663                        | 0.19                          |
| 125°00'00"                                   | 24                 | 6614.1     | 73°42'36"                                 | 284.1      |                                           |                               |                               |
| 125°00'00"                                   | 24                 | 6614.1     | 73°42'36"                                 | 284.1      |                                           |                               |                               |

The probability of formation of the angle of separateness, formed by two systems of cracks, was calculated by the methods of probability theory for the case of joint occurrence of the two events from the following expression:
where \( N \) is the total number of cracks, pcs.; \( n_1, n_2 \) are the quantities of cracks, which are assigned, respectively, to the first and second systems, pcs.

Thus we can conclude that for the Nevyrivsky labradorite deposit, the most probable separateness will be the one formed by the intersection of the systems of cracks 93° and 125°. Finally, the separateness parameters will be as follows: vertical angles are \( \alpha_{Q}=114°39′48″ \), in the horizontal plane the angle is \( \alpha_{S}=48°9′14.4″ \). The probability of formation of separateness with such angular correlations will reach 57%.

Given that the linear correlations between the systems of fracturing are characterized by the following values: \( d_Q=3.2 \) m; \( d_S=3.0 \) m; \( d_l=2.0 \) m and \( K_{pbc}=0.55 \), then the expected deposit’s blockiness will amount to 37.054%, with probability of 57%.

### 6. Discussion of results of the study of efficiency of the methods of clustering in the management of the processes of extraction block stone

A study of efficiency of the methods of clustering has been considerably expanded compared to the previous results through increasing the quantity of reference samples and generalization of research for the whole group of labradorite deposits.

The question of efficiency of the clustering method of K-means with the Chebyshev distances is gaining relevance for other types of deposits of decorative stone. The obtained results allow us to consider as promising for future research the study of regularities of formation of the systems of cracks in rocks, which differ by genesis from labradorites. In addition, the selected systems of cracks have quite a wide range of values, which requires assessment of conditions of their formation. The obtained correlation coefficient of 0.56 for vertical cracks (angle of incidence 85°–90°), for different groups of cracks, the grouping of which was performed by the angles of incidence, indicate genetic affinity of the vertical systems of cracks, the occurrence of which is often associated with the tectonic processes that took place in the vicinity of the deposit location. That is, one may put forth a hypothesis about the possibility to forecast orientation of the systems of vertical cracks based on the analysis of geometry of tectonic faults in the vicinity of the deposit location. Also interesting are the results concerning evaluation of correlation connection between the orientation of cracks and their quantity. The determined orientation of the main systems of cracks for labradorite deposits enables us to select main directions of development of mining works. This considerably increases the efficiency of quality management of technological processes through the refinement of spatial orientation of the planes of separation.

The cluster-geometric method of determining the blockiness, developed in the work, makes it possible to estimate the probability of each of the obtained results, allowing increase in the degree of taking account of the genesis of fracturing and mutual angular correlations between the systems of fracturing and opens up the possibility to increase reliability of quality assessment either the whole deposit or its separate areas, it significantly increases effectiveness of risk assessment when designing mining works and efficiency of quality control of a decorative stone deposit. The proposed technique of quality control of block raw materials based on cluster analysis will increase the accuracy of prediction, but its application requires high competence of the performer. This problem may be solved in future based on the integration of this methodology into the structure of geoinformation systems that will be used when designing the development of deposit of decorative stone and in the process of quality management, based on the assessment of spatial variability of the blockiness.

### 7. Conclusions

1. We selected main varieties of the samples of elements of natural cracks for the deposits of labradorite in Zhytomyr Oblast (Ukraine), which allowed us to form the reference samples for the most common systems:
   - Strike azimuth 45°, dispersion 20°, incidence angle 85°, dispersion 5°;
   - Strike azimuth 135°, dispersion 40°, incidence angle 85°, dispersion 5°;
   - Strike azimuth 215°, dispersion 30°, incidence angle 80°, dispersion 5°;
   - Strike azimuth 305°, dispersion 40°, incidence angle 80°, dispersion 5°.

2. Based on the analysis of the reference samples of forty, sixty, a hundred and two hundred values of azimuths and angles of incidence of cracks, it was proven that the method of clustering of K-means with the Chebyshev distances enables more efficient systematization of fracturing of the labradorite deposits.

3. The selection of systems of cracks for the main deposits of labradorites in Zhytomyr Oblast using the cluster analysis allowed us to define the systems of cracks, specific to each field. In general, for all deposits of labradorites in Zhytomyr Region, 19 systems of vertical cracks were selected, as well as 21 systems of subvertical cracks, 2 systems of subhorizontal cracks and 1 system of horizontal cracks.

4. The evaluation of influence of orientation of cracks on their quantity allowed us to obtain analytical expression of dependency of the number of cracks on the strike azimuth in the form of polynomial of the second degree, which may be used to predict the direction of development of mining and management of the processes of extraction of decorative stone.

5. For the estimation of prospects of development of deposits or separate areas and the development of strategy for quality control, we proposed a new cluster-geometric method of quality control of block stone based on the determination of the indicator of blockiness; the example of its implementation for the conditions of Nevyrivsky deposit of labradorites is presented. The essence of the technique is in selecting the main systems of cracks by strike azimuths and angles of incidence based on the use of cluster analysis by the methods of clustering of K-means with the Chebyshev distances or K-means with the Euclidian distances; defining parameters of the systems of cracks: strike azimuths and incidence and their dispersion and angular and linear correlations between the systems of cracks; estimation of errors in determining the angles between the systems of cracks and probabilities of mutual intersection of planes of the selected cracks; creating a model of natural separateness, angular and linear correlations, which take
account of the results of the previous stages and definition of blockiness, as the ratio of volumes of natural and artificial separatenesses with the assessment of probability of its formation under conditions of the given deposit. The goal of the performed operations is to identify technological solutions for each of the selected areas and the strategy of quality control of block of raw materials for maximizing the extraction of blocks from the array.

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