On the electron sheath theory and its applications in plasma–surface interactions

Guangyu SUN (孙光宇)1®, Shu ZHANG (张舒)2®, Anbang SUN (孙安邦)∗® and Guanjun ZHANG (张冠军)∗®

State Key Laboratory of Electrical Insulation and Power Equipment, School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, People’s Republic of China

E-mail: anbang.sun@xjtlu.edu.cn and gjzhang@xjtlu.edu.cn

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Abstract

In this work, an improved understanding of electron sheath theory is provided using both fluid and kinetic approaches while elaborating on their implications for plasma–surface interactions. A fluid model is proposed considering the electron presheath structure, avoiding the singularity in electron sheath Child–Langmuir law which overestimates the sheath potential. Subsequently, a kinetic model of electron sheath is established, showing considerably different sheath profiles in respect to the fluid model due to non-Maxwellian electron velocity distribution function and finite ion temperature. The kinetic model is then further generalized and involves a more realistic truncated ion velocity distribution function. It is demonstrated that such a distribution function yields a super-thermal electron sheath whose entering velocity at the sheath edge is greater than the Bohm criterion prediction. Furthermore, an attempt is made to describe the electron presheath–sheath coupling within the kinetic framework, showing a necessary compromise between a realistic sheath entrance and the inclusion of kinetic effects. Finally, the secondary electron emissions induced by sheath-accelerated plasma electrons in an electron sheath are analysed and the influence of backscattering is discussed.

Keywords: plasma–surface interaction, plasma sheath, secondary electron emission, Child–Langmuir law

(Some figures may appear in colour only in the online journal)

1. Introduction

The sheath is a space-charge region commonly formed at the edge of plasma which breaks up the quasi-neutrality and shields the bulk plasma from a solid boundary. The sheath plays an essential role in confined plasma research, and in particular in plasma–surface interactions, such as in plasma processing, plasma propulsion engines, magnetic-confined fusion, dust particles, plasma diagnostics, etc [1–9]. A classic Debye sheath (also called an ion sheath) appears when a floating slab is injected into a plasma. Initially, more electrons than ions enter the board due to their higher mobility, leaving net positive charges in the gradually formed ‘sheath’ region while depositing negative charges on the solid material, which in turn mitigates the electron flow until the plasma current is balanced at the solid wall.

Although ion-rich sheaths are most frequently encountered, a sheath can be electron-rich in some particular cases. One example is when an electrode is biased above the plasma potential in a Langmuir probe voltage scan, where an electron sheath instead of ion sheath is formed near the electrode at high biased voltage [10, 11]. The electron sheath has increasing potential from the sheath edge to the solid boundary, thus plasma electrons are accelerated by the sheath potential whereas ions are de-accelerated, contrary to the ion sheath. The electron sheath is a relatively local phenomenon and requires a concomitant ion sheath to be present in other plasma-facing components to achieve the global particle balance in a confined plasma system [12]. If the surface areas

1 Current address: Ecole Polytechnique Fédérale de Lausanne, Swiss Plasma Center, CH-1015 Lausanne, Switzerland.
2 Current address: Laboratoire de Physique des Plasma, Ecole Polytechnique, 91120 Palaiseau, France.
∗ Author to whom any correspondence should be addressed.
where the electron and ion sheaths are present are noted as $A_w$ and $A_c$, with subscript w and c representing the electrode and other chamber wall, then the total electron current to all surfaces becomes $I_{e,tot} = e \ln \left( \frac{A_w + A_c}{A_w} \right)$.

Here $I_{e}^{ep}$ is the electron flux at the sheath edge and $\varphi_{pc}$ is the potential difference between the other chamber wall and the plasma (not between the electrode and the plasma), which is positive. Total ion flux is $I_{i,tot} = e \ln \left( \frac{A_w}{A_c} \right)$, with $I_{i}$ the ion flux at the sheath edge. The ion flux reaching the electrode where an electron sheath is present is neglected due to limited ion temperature. Assuming a Maxwellian electron and ion sheath Bohm criterion, the ion and electron current balance gives

$$e \varphi_{pc} = -T_p \ln \left( \frac{2\pi m_e}{m_i} - \frac{A_w}{A_c} \right).$$

Here $m_i$ and $m_e$ are ion and electron mass, respectively. The equation is similar to the floating sheath (with zero net current) potential expression except for the $-\frac{A_{w}}{A_{c}}$ term.

Equation (1) also requires that $A_w \leq A_c \sqrt{\frac{2\pi m_e}{m_i}}$, which stipulates a limited area where the electron sheath is present, i.e., an electron sheath cannot be floated since it requires an ion sheath to appear elsewhere to achieve the global particle balance, and its existence is related to the area ratio between the total ion loss surface and the electrode surface near where electron sheath is formed [13].

Apart from the Langmuir probe sweep in the electron saturation region, electron sheaths also appear in a variety other scenarios, such as in dusty plasma particle circulation, scrape-off layer diagnostics, planet surfaces, spacecraft probes, anode ablation by arc, and some transient processes [14–17]. Nonetheless, there are fewer related studies on electron sheaths than on ion sheaths, and electron sheath counterparts of some well-developed models for ion sheaths have yet to be established.

The current study of electron sheaths is primarily motivated by the pragmatic demand for probe diagnostics. For simplicity, a probe biased more positively than the plasma is assumed to collect the full electron current from the electron sheath, while ions are neglected in the sheath. The electron velocity distribution function (EVDF) of the sheath entrance is trivially regarded as half-Maxwellian and no presheath structure is considered [18]. These assumptions greatly facilitate the implementation of electron sheath physics into probe calibration. However, recent progress in electron sheath theories has shown that the underlying physics is far from simple. A simulation by Yee et al. proved that a presheath exists for electron sheaths, which accelerates the plasma electrons up to thermal velocity at the sheath entrance [19]. The presheath size of an electron sheath is larger than that of ion presheaths and extends deeply into the bulk plasma. Also, a comprehensive presheath theory was proposed by Brett et al. based on fluid equations [20]. It was shown that electrons in presheaths are mainly driven by the pressure gradient, and that the accelerated electrons give rise to fluctuations due to ion acoustic instabilities. Instabilities in electron sheaths were also investigated experimentally by Stenzel et al. [21, 22].

Additionally, the transition between an electron sheath and an ion sheath was reproduced in simulation by varying electrode bias potential [23]. The electron sheath can even be enclosed by an ion presheath by surrounding a metallic electrode with dielectric material [24].

Existing electron sheath theories are mostly based on a fluid model and mainly focus on the presheath region and the sheath is only treated as a boundary condition. A fluid model can be applied in the presheath region where collisionality is sufficiently large to form a near-Maxwellian distribution. However, the scenarios in the electron sheath region are more complex. In the present article, electron sheath theories in low-temperature, low pressure, unmagnetized plasma are investigated. First of all, the newly discovered electron-entering velocity at the sheath edge inevitably yields a truncated EVDF at the sheath entrance. The ion-entering velocity in the ion sheath edge barely matters since the initial ion energy is far lower than the ion sheath potential energy, whereas this is not the case in the electron sheath. In addition, the plasma electrons are accelerated by the sheath potential and collide on the electrode with greater energy compared with the electrons in the ion sheath, inducing secondary electron emission (SEE). This may alter the current balance since both emitted electrons and plasma electrons contribute to the measured current, yet this effect has drawn little attention. The present work attempts to address these issues on theoretical grounds and discusses their pertinent implications in plasma–surface interactions.

The article’s structure is as follows. In section 2, the Child–Langmuir law for electron sheaths is reviewed, and is compared with an updated fluid model of electron sheaths. Section 3 provides comprehensive kinetic modeling of an electron sheath, involving an electron and ion velocity distribution function (IVDF) and the influence of electron–ion temperature ratio. In section 4, the SEE caused by sheath-accelerated electrons is implemented in the electron sheath model. Concluding remarks are given in section 5.

2. Fluid model of an electron sheath

2.1. Review of the Child–Langmuir model in electron sheaths

The Child–Langmuir law is widely used for descriptions of high-voltage sheaths, i.e., plasma-facing electrode is strongly biased such that the sheath potential is much larger than electron temperature as well as the floating sheath potential. For a high-voltage ion sheath, the sheath potential is too high for plasma electrons to penetrate the sheath; therefore, only ion density is counted in the sheath. Different from the matrix sheath model where a uniform ion density is assumed inside the sheath, the Child–Langmuir model considers the acceleration of ions in the sheath and the consequent decreasing ion density towards the boundary. The floating ion sheath approaches the Child law sheath when the electrode potential becomes increasingly negative. The ion sheath potential of a
floating, non-emissive boundary can be easily obtained from the balance of electron and ion fluxes. Here, the electron flux is \( \Gamma_{e} = n_{se} \frac{T_{e}}{2m_{e}} \exp \left( \frac{e\varphi_{e}}{T_{e}} \right) \), with \( \varphi_{e} \) the ion sheath potential and \( T_{e} \) the plasma electron temperature, and the ion flux is \( \Gamma_{i} = n_{i} \frac{T_{i}}{m_{i}} = h_{i}n_{0} \frac{T_{i}}{m_{i}}, h_{i} \) is the presheath plasma density drop and is approximately 0.61, \( n_{0} \) is the bulk plasma density. Here the ions are assumed to be cold. The current balance hence gives:

\[
e^{3/2} = -\frac{T_{e}}{2} \ln \left( \frac{\mu}{2\pi} \right) \tag{2}
\]

with \( \mu = m_{i}/m_{e}, n_{se} \) the plasma density at sheath edge (quasi-neutrality is assumed), and ion sheath potential \( \varphi_{i} < 0 \). It is noted that by convention the potential at sheath edge is assumed to be 0. When raising the biased voltage (absolute value), the flux balance at the electrode is no longer valid and a net current term \( J_{net} \) must be counted to compensate: \( \Gamma_{i} - \Gamma_{e} = \Gamma_{net} = J_{net}/e \). In the high-voltage sheath limit, the electron flux term is dropped and the Child–Langmuir law for the ion sheath as well as the potential distribution are derived as:

\[
J_{net} = 4\pi\frac{\nu_{0}}{9\chi_{i}^{e}} \frac{2e}{m_{i}} (-e\varphi_{i})^{3/2} \tag{3}
\]

\[
x = 0.79 \frac{2e}{\lambda_{De}} \frac{e\varphi_{i}}{T_{e}} \left( \frac{\varphi_{i}}{T_{e}} \right)^{1/4} \tag{4}
\]

where \( x \) is ion sheath length and \( \lambda_{De} \) is electron Debye length. Detailed derivations are available in numerous references and are not repeated here [10, 12]. In the case of an electron sheath, the potential relative to sheath edge is positive and electrons are accelerated while ions are de-accelerated, so the electron sheath is assumed to be ion-free. The Poisson equation is written as:

\[
\frac{d^{2}\varphi}{dx^{2}} = J_{net} \frac{2e}{\varepsilon_{0} m_{i}} \tag{5}
\]

with \( \varepsilon_{0} \) the vacuum permittivity, \( J_{net} = en_{se}(\varphi(2e\varphi/T_{e})^{0.5} \). In the case of a virtual anode, a similar virtual cathode appears [25]. Multiplying both sides in equation (5) by \( \frac{d\varphi}{dx} \) and integrating from the sheath edge to position \( x \) in the electron sheath, the following expression is obtained:

\[
\frac{d\varphi}{dx} = \frac{4J_{net}}{\varepsilon_{0}} \left( \frac{m_{i}}{2e} \right)^{0.5} \varphi^{0.25}. \tag{6}
\]

The potential and electric field are assumed 0 at the sheath edge in deriving equation (6). Integrating the above equation again and normalizing the \( x \) coordinate with regard to the electron Debye length \( \lambda_{De} \), the Child–Langmuir law of the electron sheath and the potential distribution are derived with the current expression \( J_{net} = en_{se}v_{the} \) as follows:

\[
J_{net} = 4\pi \frac{\nu_{0}}{9\chi_{i}^{e}} \frac{2e}{m_{e}} \left( \frac{e\varphi_{i}}{T_{e}} \right)^{3/2} \tag{7}
\]

\[
x = 0.79 \frac{2e}{\lambda_{De}} \left( \frac{e\varphi_{i}}{T_{e}} \right)^{3/4} \tag{8}
\]

For equation (4), the same conclusion was also reported by Brett et al. [20]. A slightly different expression \( x = 0.32 \left( \frac{e\varphi_{i}}{T_{e}} \right)^{3/4} \) was reported previously that uses a different random flux expression [10].

An important assumption on the deduction above is the zero initial electron velocity at the sheath edge. The ion velocity at the sheath edge, in a high-voltage ion sheath, is commonly negligible when deriving the Child–Langmuir law, since the sheath potential is much larger than the electron temperature. As discussed in section 1, the electron presheath is not considered by the conventional electron sheath theory. It was only recently that attention was drawn to the importance of the presheath in electron sheath properties [20, 26]. The electron-entering velocity at the sheath edge was shown to be \( m_{i}/m_{e} \) times larger than the ion Bohm velocity. Meanwhile, the electrode biased voltage is usually smaller in an electron sheath compared with an ion sheath in typical plasma applications. The treatment of electron-entering velocity will be discussed in section 2.2, where it will be shown that such a revision has a remarkable influence on the obtained sheath properties.

Another issue of the above deductions is the neglected EVDF. Equation (4) implicitly regards electrons as a mono-energetic beam. Though this is also used in the derivation of ion sheath Child–Langmuir law, one must realize that the electron temperature is far greater than ion temperature for a typical cold plasma discharge; thus, the omission of electron kinetic effects in an electron sheath will introduce larger discrepancies compared with neglecting the IVDF in an ion sheath. Treatment of the EVDF is discussed in section 3.

2.2 Revised fluid model of an electron sheath

In this section, an electron sheath fluid model that considers the electron presheath is introduced. The derivation below is based on previous studies of the electron-entering velocity in the electron sheath, but a different conclusion about the electron sheath structure is obtained, due to a distinct treatment of Poisson equation [20]. To include the influence of electron-entering velocity, it is intuitive to write down the electron fluid equations in the sheath and regard the entering velocity as a boundary condition. To begin with, the particle and momentum balance of electrons in the sheath is given as...
is the electric

which is reduced to:

follows:

In the above equation, \( u_e \) is the electron fluid velocity, \( P_{\|} \) is the parallel electron pressure and \( E \) is the electric field. In equation (10), the pressure gradient force dominates over the electric field force and their ratio is equal to the electron–ion temperature ratio \( T_e / T_i \), obtained by inserting the ion momentum equation. For the ion sheath it is the opposite, so the pressure gradient can sometimes be ignored in fluid modeling [27, 28]. A similar conclusion was reported before [20]. Source terms are neglected since the sheath is assumed to be collisionless. Combining equations (9) and (10), and solving for \( u_e \) with potential \( \varphi \), the following relation is obtained:

where \( u_{eo} \) is the electron fluid velocity at the sheath edge and is treated as the boundary condition; the potential at the sheath edge is chosen as 0. It is noted that here the temperature gradient term is dropped, and the isothermal relation is adopted. The resulting equation is:

Equation (12) can be solved by rewriting it in the form of a Lambert W function and taking the asymptotic limit at large potential. More details on the solution of this form of equation were given in the work of Brett et al [20], here the result is given directly:

It is then possible to solve the Poisson equation in the following form:

Here the electron continuity equation \( (n_e u_{eo} = n_e u_e) \) is used again and the following normalized terms are adopted for simplicity.

The potential distribution in the electron sheath is then obtained by multiplying equation (14) by \( \frac{d\Phi}{dX} \) and integrating twice with respect to \( X \), which is reduced to:

The potential distributions given by equations (8), (16) and the previous solution in [20] are shown in figure 1 (named separately as the Child–Langmuir model, which is the same as the solution in [20], and the revised fluid model). Generally, the Child–Langmuir solution slightly overestimates the electron sheath potential.

The electron density distributions in the sheath predicted by the Child–Langmuir model and the revised fluid model are shown in figure 2. The Child–Langmuir model generally underestimates the electron density in locations far away from the sheath edge. One major flaw of the Child–Langmuir

\[ \frac{d}{dx}(m_e n_e u_e^2 + P_e) = -en_e E. \]
model is that there is a density singularity at \( X = 0 \) due to neglecting the electron-entering velocity at the sheath edge. Section 3.1 will show that the singularity can also be avoided by considering electron kinetic effects even without involving the electron-entering velocity at the sheath edge.

It is also important to point out that the obtained conclusions from equation (16) seem to be independent of the exact value of the entering velocity. This does not mean that the value of \( u_{e0} \) is irrelevant, rather it is just because the electron sheath cannot achieve self-consistency when isolated from the presheath. For instance, the \( u_{e0} = 0 \) assumption adopted in the Child–Langmuir solution will lead to an unphysical solution for \( u_e = 0 \) everywhere according to equation (13). The \( u_{e0} = 0 \) assumption also leads to a singularity at the sheath edge. Since the plasma electron flux is conserved in the collisionless electron sheath, a zero entering velocity naturally causes infinite density at the sheath entrance. Hence the inclusion of electron-entering velocity is crucial. In addition, the choice of \( u_{e0} \) is not arbitrary and is essentially dictated by the way the presheath matches the sheath. Section 3.2 will show that the entering velocity can always be expressed in the form of \( u_{e0} = \alpha u_{i0} \sqrt{\frac{T_e}{m_e}} \) with \( \alpha u_{i0} \) a coefficient depending on the electron and ion model (fluid or kinetic), their temperatures, and the choice of distribution function if a kinetic model is employed. However, with the given model and plasma parameters, the value of \( u_{e0} \) is definite.

### 3. Kinetic model of an electron sheath

#### 3.1. Kinetic model without a presheath

In section 2, electron fluid equations are used to derive the electron sheath solution. However, the exact EVDF inside the sheath is not thoroughly considered. To illustrate the influence of kinetic effects on the electron sheath solution, the following kinetic model of an electron sheath is constructed. In addition, IVDF is involved to investigate the influence of nonzero ion temperature for more general applications of the theory.

To begin with, the electron and ion densities inside the sheath should be determined. The electrons are accelerated due to increasing sheath potential towards the electrode; hence, a velocity lower bound appears when integrating the half-Maxwellian EVDF, which gives the following electron density:

\[
 n_{e\infty} = n_{e0} \exp \left( \frac{e\varphi}{T_{e\infty}} \right) \text{erfc} \left( \frac{e\varphi}{\sqrt{2} T_{e\infty}} \right). \tag{17}
\]

The ions are repelled by the electron sheath potential, just like the electrons in an ion sheath. If the force posed by the electric field is balanced by the pressure gradient force, the Boltzmann distribution can be used to derive the following ion density:

\[
 n_i = n_{e0} \exp \left( - \frac{e\varphi}{T_{i\infty}} \right). \tag{18}
\]

The Poisson equation can therefore be written as:

\[
 \frac{d^2\Phi}{dX^2} = \exp (\Phi) \text{erfc} (\sqrt{\Phi}) - \exp (-\Theta_T \Phi) \tag{19}
\]

with \( \Theta_T = \frac{T_e}{T_i} \) the ratio of electron and ion temperature. Different from equations (5) and (14), equation (19) cannot be solved fully analytically. A possible solution is to reduce equation (19) into the following form, and then solve it as an initial value problem numerically:

\[
 \frac{d\Phi}{dX} = \sqrt{2} \left[ \exp (\Phi) \text{erfc} (\sqrt{\Phi}) - 1 + 2 \sqrt{\frac{\Phi}{\pi}} \right] + \exp (\Theta_T \Phi)^{0.5}. \tag{20}
\]

The equation above is obtained again by multiplying equation (19) by \( \Phi \) and integrating twice over \( dX \). To solve for the potential numerically, the electrode potential relative to the sheath edge is given as the initial condition. The potential is then solved towards the sheath edge using numerical methods. Here the explicit Euler method is employed:

\[
 \Phi_{n+1} = \Phi_n + g(\Phi_n) \Delta X, \quad n = 1, 2, 3 \ldots \tag{21}
\]

with \( g(\Phi) \) the right-hand side of equation (20), and \( \Delta X \) the position step size. The initial condition at the electrode is the normalized electron sheath potential: \( \Phi_1 = \Phi_n = \Phi_{esh} \). \( \Phi_n \) is the normalized potential at electrode and will constantly appear in the following derivations.

Calculated results with different \( \Phi_n \) values are shown in figure 3(a) and are compared with the Child–Langmuir prediction and the fluid model with a presheath. One can find that the sheath size given by the kinetic model is typically above that given by the Child–Langmuir solution for a smaller electron sheath and is below the Child–Langmuir solution for a larger electron sheath and a higher electrode potential. Considering that an electron presheath makes the electron sheath potential smaller for a fixed electrode potential, it is always below the kinetic model prediction except at very high electrode potential levels. Profiles of sheath potential distribution in space are remarkably different. In general, the kinetic model predicts a shorter sheath size at a fixed electrode potential, and its space potential profile is higher than the Child–Langmuir solution.

The influence of ion temperature is shown in figure 3(b). It is clear that the ion temperature only exerts a minor influence on the shape of the potential profile, and a high ion temperature slightly decreases the size of the electron sheath. The potential profile becomes virtually insensitive to ion temperature after the value of \( \Theta_T \) exceeds around 5. Consequently, in typical low-temperature plasma applications, the influence of ion temperature on potential distribution is marginal. The calculated electron and ion density distributions in the sheath predicted by the kinetic model are compared with the Child–Langmuir model in figure 4. The electron and ion densities are normalized with respect to the sheath edge density:

\[
 N = \frac{n}{n_{e\infty}}. \tag{22}
\]
Generally, the kinetic model gives a lower electron density compared with the Child–Langmuir model and the fluid model considering an electron presheath, suggesting an overestimation of plasma density when using fluid models; the ion density is ignored in fluid models. The kinetic model without an electron presheath (hence zero entering velocity) and the fluid model considering an electron presheath avoids the singularity at $X = 0$. The electron density profile is barely influenced by the change of $\Theta_T$, whereas the ion density profile changes remarkably with $\Theta_T$. Here, the ion Boltzmann distribution is adopted, whereas in practice, some energetic ions might penetrate the electron sheath and arrive at electrodes, which is increasingly obvious for low $\Theta_T$. These ions will not return to the plasma and inevitably leaves an IVDF dissipated at a high-velocity tail [29]. Also, the ion density could be slightly higher than electron density in the vicinity of the sheath edge. This actually contradicts the general Bohm criterion, and is due to the neglect of the electron presheath structure in the kinetic model. Further analyses regarding a more accurate IVDF and the influence of electron presheath structure on the sheath solution will be given in sections 3.2 and 3.3.

3.2. Derivation of the Bohm criterion in the electron sheath

In section 2, the influence of electron-entering velocity on the electron sheath solution has been illustrated, where the inclusion of electron fluid velocity $u_{e0}$ as a boundary condition in the sheath entrance shifts the size and potential distribution of the electron sheath. Judging solely from the sheath side, however, the value of $u_{e0}$ seems arbitrary. In this section, the electron sheath will be matched with the presheath which dictates the choice of $u_{e0}$. The general Bohm criterion will first be applied in the electron sheath, and then the modified Bohm criterion will be proposed considering a more realistic IVDF. The obtained conclusions will be implemented into the following kinetic modeling of the electron sheath under various conditions.

3.2.1. Fluid derivation of the Bohm criterion in an electron sheath

The common procedure for the derivation of the Bohm criterion for an electron sheath is to calculate the electron and ion density at the sheath edge and apply the general Bohm criterion, which is more universal and applies to both electron and ion sheaths. The general Bohm criterion is frequently used directly and was originally proposed in Riemann’s work [30]. However, sometimes its exact form can
be vague in the literature and may cause misunderstanding when applied to ion and electron sheaths. Here the general Bohm criterion is briefly reviewed and then directly applied to the case of an electron sheath.

Starting from the Poisson equation and expanding it at the sheath edge:

$$\nabla^2 \varphi = \frac{-1}{\varepsilon_0} \rho_{v=0} + \frac{d \rho}{d \varphi} \bigg|_{v=0} \varphi + \frac{d^2 \rho}{d \varphi^2} \bigg|_{v=0} \varphi^2 + \ldots$$

(23)

with the charge density \( \rho = e(n_i - n_{ep}) \). Taking up to first order in equation (23) and assuming quasi-neutrality at the sheath edge, then multiplying with \( \frac{d \varphi}{d x} \) and integrating over \( x \), it leads to:

$$E^2 + \frac{1}{\varepsilon_0} \frac{d \rho}{d \varphi} \bigg|_{v=0} \varphi^2 = C$$

(24)

where \( C \) is a constant to be determined. Since the electric field is 0 at the sheath edge, \( C = 0 \) and \( \frac{d \rho}{d \varphi} \bigg|_{v=0} \) should satisfy:

$$\frac{d \rho}{d \varphi} \bigg|_{v=0} = -\left( \frac{E}{\varphi} \right)^2 \leq 0.$$  

(25)

Equation (25) can be further written as \( \frac{d \rho}{d \varphi} = \frac{d \rho}{d x} \frac{d x}{d \varphi} = -E \frac{d \rho}{d x} < 0 \). It leads to the form of the general Bohm criterion frequently used in the literature: \( \frac{d \rho}{d x} > 0 \) for an ion sheath and \( \frac{d \rho}{d x} < 0 \) for an electron sheath. Caution should be taken regarding the sign here.

With equation (25), the electron and ion density gradient at the sheath edge is to be derived. To do that, electron and ion fluid equations in the presheath are given as follows:

$$\frac{d}{d x}(n_i u_i) = S_{ni}$$

(26)

$$\frac{d}{d x}(n_i T_i) = e n_i E + S_{ni}$$

(27)

$$\frac{d}{d x}(n_{ep} u_{ep}) = S_{ne}$$

(28)

$$\frac{d}{d x}(m_i n_{ep} u_{ep}^2 + n_{ep} T_{ep}) = -e n_{ep} E + S_{ne}$$

(29)

where \( u_i \) is ion fluid velocity and \( S_{ni}, S_{ne}, S_{mi} \) and \( S_{me} \) are source terms for electron and ion particle and momentum balance. Comparing equations (27) and (28), one can find that the force on electrons contributed by the electric field is roughly \( \frac{1}{T_i} \) times smaller than the pressure gradient term.

Oppositely, it is the electric field that plays the dominant role in the ion-rich plasma sheath. Similar conclusions have been proposed by Brett et al [20]. At the sheath edge, the source terms near the sheath edge are discarded and the ion and electron density gradients are obtained as follows:

$$\frac{d n_i}{d x} = \frac{e n_i E}{T_i}$$

(30)

It is noted that \( n_{ep} = n_i \) at the sheath edge. Using equation (25), the Bohm criterion for the electron sheath is derived as follows:

$$u_{\text{sheath}} = u_{\text{ifo}} \geq \sqrt{\frac{T_{ep} + T_i}{m_e}}.$$  

(32)

Equation (32) is analogous to the Bohm criterion for an ion sheath except that the electron-entering velocity is \( \sqrt{\frac{m_i}{m_e}} \) times larger than the ion-entering velocity.

3.2.2. Role of kinetic ions in the Bohm criterion in an electron sheath. In the deductions above, ions are assumed to be balanced by the pressure gradient and electric field just as plasma electrons are in an ion sheath. Actually, the EVDF in an ion sheath is commonly known to be depleted at the high-velocity end, since energetic electrons that penetrate the ion sheath will not return to plasma. This phenomenon is known as the electron loss cone [31]. Generally speaking, the loss cone is a 3D phenomenon where electron velocity forms a spherical shell of radius \( w = \frac{1}{2} m_e (v_x^2 + v_y^2 + v_z^2) \) with \( w \) the electron energy. An electron with \( w > e \Delta \varphi \) can have sufficient energy of motion normal to the wall (here in \( x \) direction) to leave the system, with \( \Delta \varphi \) the wall–plasma potential difference [31]. In the present analyses, we focus on the 1D1V model and only consider electron motion normal to the wall. The effects of depleted EVDF at a high-velocity tail in an ion sheath due to the loss cone effect have been addressed in detail before [32–34]. Particularly, Joaquim et al showed that the Bohm velocity of an ion sheath could be smaller than the conventional ion sound velocity \( \frac{T_{io} + T_i}{m_i} \sqrt{\frac{T_i}{m_i}} \) when the electron loss cone is considered [35]. Following a similar logic, it is intuitive to imagine that the Bohm velocity of the electron sheath \( u_{\text{ifo}} \) could also be affected by IVDF depleted at the high-velocity end, though the influence can only be obvious at a high ion temperature.

Similar to the loss cone in the ion sheath, a fast ion may penetrate the potential barrier in an electron sheath and thus will not return, leading to the following truncated velocity distribution function:

$$f_i = n_i(\varphi) \begin{cases} \frac{1}{I(\eta)} \frac{m_i}{2 \pi T_i} \exp \left( -\frac{m_i v_i^2}{2 T_i} \right), & -\infty < v_i < v_{icut}, \\ 0, & v_i \geq v_{icut} \end{cases}$$

(33)

Here the normalized potential term is \( \eta = \frac{e(\varphi_w - \varphi)}{\bar{v}_w} \) with \( \varphi_w \) the potential at electrode (to be distinguished from \( \Phi \) which is normalized with respect to \( T_{ep} \)), ion cutoff velocity is \( v_{icut} = \sqrt{\frac{2e(\varphi_w - \varphi)}{m_i}} = \sqrt{2 \eta} v_{thi}, \) electrode potential relative to sheath edge is \( \varphi_w \), and \( I(\eta) = 0.5[1 + \text{erf}(\sqrt{\eta})] \). Ion flow
velocity is then calculated as $u_i = \frac{\partial u_i}{\partial \eta} \exp(\Lambda - \eta)$, where ion thermal velocity is $v_{thi} = \sqrt{\frac{T_i}{m_i}}$ and $\Lambda = -\ln(\sqrt{2}\pi)$. Its derivation with respect to $\varphi$ is $\partial_\varphi u_i = \frac{e u_i}{T_i} \left( 1 + \frac{\exp(-\eta)}{2\sqrt{2\pi T_i(\eta)}} \right)$.

The above deductions solve the ion flux expression. Now equations (26), (28) and (29) can be rewritten into the matrix form $M \mathbf{X} = \mathbf{S}$ with $\mathbf{X} = (\partial_\eta, \partial_\varphi, \partial_\Lambda, \partial_\eta) = (S_n, S_{ne}, S_{she})^T$ and matrix $M$ in the following form:

$$M = \begin{pmatrix}
    u_i & 0 & n \partial_\eta u_i \\
    u_n & n & 0 \\
    T_{ep} & m_e n u_n & -en \\
\end{pmatrix}$$

(34)

It is noted that in a presheath the quasi-neutrality is satisfied so the electron and ion densities are both $n$.

Since the gradient terms become much larger near the sheath edge, it is assumed that $S = 0$ and $u_i$ is solved by using $|M| = 0$ in order to get a nontrivial set of solutions, giving the following expression:

$$u_e = \sqrt{\frac{T_{ep} + T_e / (1 + \kappa)}{m_e}} = u_{\text{the}} \sqrt{1 + \frac{T_e}{T_{ep}} \frac{1}{1 + \kappa}}.$$ (35)

with $\kappa = \frac{\exp(-\eta_e)}{2\sqrt{2\pi T_e(\eta_e)}}$ (subscript se means sheath edge so $\eta_e = \frac{e\varphi_e}{T_e}$) and electron thermal velocity $v_{\text{the}} = \sqrt{\frac{T_e}{m_e}}$. The electron-entering velocity can be written as $u_{e0} = \alpha u_{e0} u_{\text{the}}$, with the factor $\alpha u_{e0}$ in the following form:

$$\alpha u_{e0} = \sqrt{1 + \frac{1}{\Theta_T} \frac{1}{1 + \kappa}}.$$ (36)

The above equation suggests that the electron sheath can become super-thermal, i.e. the electron-entering velocity at the sheath is higher than the thermal velocity predicted by the Bohm criterion. Calculation results are shown in figure 5(a). Clearly, the electron-flow velocity at the sheath edge is always above the electron thermal velocity $v_{\text{the}}$, and increases with $\eta_{SE}$. It finally saturates at $v_{\text{c,max}} = \sqrt{\frac{T_e + T_u}{m_e}}$ when $\eta_{SE} \to \infty$. The influence of $\eta_{SE}$ is actually unessential if $\Theta_T$ is large. Figure 5(a) also suggests that the presheath acceleration makes the drift velocity dominate the current, since the $u_{e0}/v_{\text{the}}$ is always above 1, and constitutes a current at least around 2.5 times larger than the thermal flux without a presheath. The presheath density drop could slightly reduce the difference of current values but the general trend is not changed. Meanwhile, ion flow velocity is always below its thermal velocity and decreases with $\eta_{SE}$.

One may also estimate ion heat flux $q_i = \frac{1}{2} m_i n_i (v_i - u_i)^3$ by calculating $v_i^2$ and $v_i^3$ according to the distribution function in

Figure 5. Sheath edge properties as a function of term $\eta_{SE} = \frac{e\varphi_e}{T_e}$ and $\Theta_T = \frac{T_e}{T_u}$. (a) Electron-entering velocity at the sheath edge is normalized with respect to electron thermal velocity. Electron velocity increases with $\eta_{SE}$, while its influence becomes smaller at higher $\Theta_T$. (b) ion velocity normalized over ion thermal velocity, which decreases with $\eta_{SE}$. (c) ion heat flux normalized over $\frac{1}{2} m_i n_i v_i^3$.
equation (33). Ion heat flux is expressed as follows:
\[
q_i = \frac{m_e n_e v_{thi}^3}{\sqrt{2\pi I(\eta)}} \left[ \exp(-\eta) \left( \eta - \frac{1}{2} \right) + \frac{3}{2} \sqrt{\frac{\eta}{\pi}} \exp(-2\eta) + \frac{\exp(-3\eta)}{2\pi I(\eta)} \right].
\] (37)

It is noted that both \(v_i\) and \(q_i\) are independent from \(\Theta_T\). Calculation results are given in figures 5(b) and (c). It is reasonable that the ion flux decreases with \(\eta_{\text{sec}}\) since a larger potential barrier obviously inhibits the ion flow in the electron sheath, and the ion flux goes to 0 at infinite \(\eta_{\text{sec}}\). The normalized electron sheath potential \(\eta_{\text{sec}}\) not only affects the ion energy drop when crossing the sheath, but also controls the ion velocity bound via \(v_{i(\text{cut})}\). The resulting ion heat flux first increases and then decreases as \(\eta_{\text{sec}}\) increases. This indicates an ion-erosion peak when an electrode is biased just above the plasma potential and should be avoided in practice.

The trend is similar to the electron heat flux in a subsonic ion sheath and shows interesting symmetry between the two types of sheaths [35].

To summarize the above, various forms of electron-entering velocity are introduced in this section. When neglecting all ions, it is simply \(u_{e0} = \sqrt{\frac{T_e}{m_e}}\). If considering nonzero ion temperature while disregarding the ion loss cone, the expression becomes \(u_{e0} = \sqrt{\frac{T_e + T_i}{m_e}}\). If one further involves the ion loss cone, the entering velocity is expressed as \(u_{e0} = \alpha_{\text{ue0}} u_{\text{the}}\), with \(\alpha_{\text{ue0}} = \sqrt{1 + \frac{1}{\Theta_T} + \frac{1}{1 + \chi}}\). Although the value of the electron presheath potential (or \(u_{e0}\)) does not directly change the calculated electron sheath potential and plasma density profiles, a nonzero \(u_{e0}\) is necessary to avoid the electron density singularity for the fluid model. In addition, the electron presheath structure is affected by the sheath parameters, particularly the electron sheath potential, with the effect more obvious for high ion temperatures. These expressions will be used in the following derivations.

3.3. Application of the Bohm criterion in a kinetic model of an electron sheath

3.3.1. Influence of a truncated EVDF on the kinetic model of an electron sheath. In section 3.2, the electron-entering velocity \(u_{e0}\) is derived with various types of assumption. It is then possible to apply the obtained \(u_{e0}\) to revise the EVDF at the sheath entrance. Note that, in section 3.1, the EVDF at the sheath entrance is in Maxwellian form and no drift velocity is considered. When \(u_{e0}\) is included, however, the EVDF at the sheath edge is inevitably truncated, and a drift velocity should be included.

For simplicity, the original EVDF is assumed to be a half-Maxwellian with no drift velocity, and then a nonzero \(u_{e0}\) is added to see its influence. The comparison of the EVDFs with and without entering velocity is given in figure 6.

The half-Maxwellian EVDF in figure 6 is simply
\[
f_{e,\text{hM}} = n_{se} \sqrt{\frac{2m_e}{\pi T_{ep}}} \exp \left( -\frac{m_e v_x^2}{2 T_{ep}} \right)
\]

The truncated EVDFs in figures 5 are as follows:
\[
f_{e,\text{ue0}} = n_{se} \frac{1}{1 + \text{erf} \left( \frac{u_{e0} - m_e}{2 T_{ep}} \right)} \sqrt{\frac{2m_e}{\pi T_{ep}}} \exp \left[ -\frac{m_e (v_x - u_{e0})^2}{2 T_{ep}} \right], v_x \geq 0.
\] (38)

The additional factor in equation (38) compared with the half-Maxwellian is derived by normalization of the sheath edge density \(n_{se}\). When \(u_{e0} = 0\), equation (38) is reduced to the half-Maxwellian as used in section 3.1, whereas the EVDF approaches a flowing Maxwellian when \(u_{e0}\) is larger than the electron thermal velocity. This suggests that the fluid model employed in section 2.2 is an approximation of infinite electron-entering velocity.

The electron flux at the sheath edge is then calculated as:
\[
\Gamma_{e,\text{ue0}} = \int_0^{+\infty} v_e f_{e,\text{ue0}}(v_e) dv_e = \beta_e \Gamma_{e,\text{phM}},
\] (39)

where \(\Gamma_{e,\text{hM}} = n_{se} \sqrt{\frac{2m_e}{\pi T_{ep}}} \) is the electron flux at the sheath edge for a half-Maxwellian EVDF, and \(\beta_e\) is a coefficient depending on \(u_{e0}\), shown as follows:
\[
\beta_e = \frac{\exp \left( \frac{m_e u_{e0}^2}{2 T_{ep}} \right)}{1 + \text{erf} \left( \frac{u_{e0} - m_e}{2 T_{ep}} \right)} + u_{e0} \sqrt{\frac{\pi m_e}{2 T_{ep}}}.
\] (40)

Equation (40) can be justified by the limit \(u_{e0} = 0\) which yields \(\beta_e = 1\) and is equivalent to the half-Maxwellian case. For
typical low-temperature plasma with $T_e \gg T_i$, $u_0 = \sqrt{\frac{T_e}{m_e}}$ and $\beta_e \approx 1.61$. Note that in section 3.2 the electron-entering velocity is expressed as $u_{0e} = \alpha_{ue0} u_{the}$, where $u_{the} = \sqrt{\frac{T_e}{m_e}}$ and $\alpha_{ue0}$ depends on the electron–ion temperature ratio $\Theta_T$ and $\kappa(\Phi_w)$. The calculated results of factor $\beta_e$ are shown in figure 7.

It is clear that the value of $\beta_e$ increases with $\alpha_{ue0}$. The presheath–sheath matching requires that $\alpha_{ue0} \in [1, 2]$, which dictates a minimum $\beta_e$ of around 1.61 and a maximum of around 1.97. In figure 7(b), one can find that the influence of $\Phi_w$ on $\beta_e$ is obvious at low $\Theta_T$ levels, and the $\Theta_T$ plays a more significant role compared with $\Phi_w$. This is as expected because the ion loss cone is obvious only when sufficient amounts of ions can penetrate the electron sheath potential. At a higher $\Theta_T$ level, the normalized electrode potential $\Phi_w$ is not influential. This is reassuring for low-temperature plasma applications such as Langmuir probes, since if the entering velocity changes with electrode bias potential, the probe measurement must therefore be calibrated accordingly. However, for lower $\Theta_T$ ($\Theta_T < 1$), the influence of $\Phi_w$ is quite remarkable (not shown here), corresponding to, for instance, the plasma in the divertor region of a magnetic confinement fusion device [36]. This might cause concern for probe data processing in edge fusion plasma but is beyond the scope of the present work. Actually, the sheath physics and probe diagnostics in strongly magnetized fusion plasma are far more complex than in the model considered here. First of all, the magnetic field typically interacts with the solid surface at an oblique angle, which produces a magnetic presheath with thickness of a few Larmor radii between the presheath and sheath [37]. In addition, the probe is subject to a more complex plasma environment, such as perpendicular flow, multiple types of bias, etc [38]. Future experimental works on fusion background are expected to further address this issue.

### 3.3.2. An attempt to include the Bohm criterion in the kinetic model of an electron sheath

In section 3.1, a kinetic model is established for an electron sheath involving the corrected EVDF at the sheath edge, which provides a numerical solution of the sheath structure. However, the electron-entering velocity is not counted in that model. It is therefore intuitive to consider if it is possible to modify the form of EVDF at the sheath edge according to the revised electron Bohm criterion and apply similar methods to further improve the kinetic modeling. It will be shown in the following section that this is not as straightforward as anticipated, and not even can a numerical solution be obtained when the entering velocity is considered.

In an arbitrary position of the electron sheath, its EVDF should contain the drift velocity $u_{0e}$ imposed by the Bohm criterion and the electron cutoff velocity $v_{cut}$ due to field acceleration. It is noted that $u_{0e} = \alpha_{ue0} u_{the}$ and $v_{cut} = \sqrt{\frac{2\Phi_e}{m_e}}$. In order to apply the similar methods in section 3.1, the electron density should be evaluated first. The integral of the EVDF depends on the relative position between $u_{0e}$ and $v_{cut}$ in the velocity axis, as shown in figure 8.

In case 1 the electron density is calculated to be:

$$n_{e1} = n_{ue} \exp\left(\frac{e\varphi}{T_{ep}}\right) \frac{1 + \text{erf}\left(\frac{u_{0e} - v_{cut}}{\sqrt{2T_{ep}}} \frac{m_e}{u_{0e}}\right)}{1 + \text{erf}\left(\frac{u_{0e} - m_e}{\sqrt{2T_{ep}}}\right)}.$$  (41)

When $v_{cut} = 0$, $\varphi = 0$, which corresponds to the sheath edge, one arrives at $n_{e1, \infty} = n_{ue}$. For case 2, the integrated electron density is in the following form:

$$n_{e2} = n_{ue} \exp\left(\frac{e\varphi}{T_{ep}}\right) \frac{\text{erfc}\left(\frac{v_{cut} - u_{0e}}{\sqrt{2T_{ep}}} \frac{m_e}{u_{0e}}\right)}{1 + \text{erf}\left(\frac{u_{0e} - m_e}{\sqrt{2T_{ep}}}\right)}.$$  (42)

Here, when $u_{0e} = 0$, the expression is reduced to equation (17) where no entering velocity is considered. Therefore, the two expressions above can be justified by the
limits. Also, it is noted that the above two equations are continuous at $v_{\text{cut}} = u_{\text{e}0}$.

However, when bringing the electron density expression into the Poisson equation, no analytical expression can be obtained from the integral of equations (41) and (42), so not even the numerical solutions of the electron sheath, as in section 3.1, are derivable. This indicates that a compromise has to be made when choosing the electron sheath model: either the electron sheath Bohm criterion is counted and the fluid model is employed (section 2.2) or the kinetic effects (EVDF) are considered while dropping the entering velocity. Future works are expected to propose better solutions and self-consistently combine the truncated EVDF and electron sheath solution.

4. Electron sheath modeling considering the plasma–surface interaction

SEE widely exists in a multitude of scenarios where plasma flux coming from a sheath comes into contact with a solid boundary [31, 33, 34, 39, 40]. Both incident ions and electrons can induce SEE at a solid boundary. The former is generally through Auger neutralization or de-excitation where an ion approaching a solid boundary bends the local field nearby so that the trapped electrons can escape from the solid material [41]. The latter happens when the incident primary electron is de-accelerated after penetrating the material surface while transferring energy to surrounding electrons, some of which could evacuate from the material surface and become true secondary electrons [42].

In low-temperature plasma applications, both types of SEE could happen, but the occurrence of both of them simultaneously is not considered in most if not all plasma simulations. The ion-induced SEE coefficient is independent of the incident ion temperature (only true for cold ion) and is a constant once the wall material is fixed. Recent theories showed that the accumulated surface charges in dielectric materials can modify the ion-induced SEE coefficient [43]. The present study focuses on a metallic boundary, hence the ion-induced SEE coefficient remains constant. On the other hand, the electron-induced SEE depends on primary electron energy. The ion-induced SEE is usually counted if the electron temperature is low and the wall material is metallic, which has a lower electron-induced SEE coefficient. Taking the example of plasma processing using capacitively coupled plasma, it is usually the ion-induced SEE that is counted since electrodes are typically metallic and the electron temperature is low [44], whereas the electron-induced SEE was shown to be influential if the neutral pressure is low and the electrodes are made of a more emissive material such as SiO$_2$ [45].

In the particular case of the electron sheath, the reason why special attention is paid to electron-induced SEE is that the plasma electrons are accelerated by the sheath potential. The electron incident energy on an electrode for an ion sheath is $2T_{\text{ep}}$, whereas the incident energy for an electron sheath is $2T_{\text{ep}} + e\phi_w$. This indicates the possible significance of electron-induced SEE (simply called SEE in the following section) even with metallic electrode.

To include SEE into electron sheath theory, it is important to first clarify the electron dynamics therein. The plasma electrons are accelerated, collide with the electrode and cause SEE. Emitted electrons are repelled by the sheath potential and some of them are returned to the electrode. Since the reflected emitted electron energy is usually much smaller than the energy of sheath-accelerated plasma electrons, it is assumed that the reflected electrons no longer cause SEE. The following derivations are inspired by the highly emissive inverse sheath model [46]. The inverse sheath is fundamentally different from the electron sheath discussed here, since it has no presheath and requires an electron-emission coefficient greater than 1. The inverse sheath does not require a strongly biased electrode and can appear on a large, floating surface (no need for $A_w \ll A_c$). The electron fluxes at the electrode

![Figure 8. Schematic of two cases of EVDF and the influence of the relation between $u_{\text{e}0}$ and $v_{\text{cut}}$. The solid line marks the physical velocity distribution function.](image-url)
for the electron sheath should satisfy the following equation:

\[ \Gamma_{ep} + \Gamma_{eref} - \Gamma_{em} = \Gamma_{net}, \]  

(43)

where \( \Gamma_{eref} \), \( \Gamma_{em} \), and \( \Gamma_{net} \) are reflected electron flux, emitted electron flux and the net electron flux, respectively. \( \Gamma_{net} \) must be balanced with flux at other chamber surfaces where the ion sheath occurs.

The reflected electron flux is in the following form:

\[ \Gamma_{eref} = \Gamma_{em} \left[ 1 - \exp \left( - \frac{e \varphi_{w}}{T_{em}} \right) \right] \]  

(44)

Bringing in the definition of SEE coefficient \( \gamma_e = \Gamma_{em}/\Gamma_{ep} \), the following equation can be derived:

\[ \Gamma_{net} = \Gamma_{em} \left[ \gamma_e^{-1} - \exp \left( - \frac{e \varphi_{w}}{T_{em}} \right) \right] \]  

(45)

The expression of plasma electron flux at the sheath edge is again \( \Gamma_{ep} = n_{sep} \beta_e \sqrt{\frac{2 \pi m_e T_{em}}{\varphi}} \), here the plasma electron density at the sheath edge \( n_{sep} \) is not equal to the sheath edge density \( n_{se} \), since \( n_{se} \) contains both plasma electrons and emitted electrons. In addition, a strong surface emission can lead to the formation of a local potential dip which changes the density drop at the sheath edge. This effect is characterized by the factor \( n_{se} = \exp \left( - \frac{e \Delta \varphi}{T_{em}} \right) \) with \( \Delta \varphi \) the local potential dip and is now not included in the present model for simplicity. Regarding the distribution function of emitted electrons, the half-Maxwellian is frequently chosen to facilitate the calculation, though it was also shown that the choice of EVDF for emitted electrons can influence the sheath solution [47]. The emitted electron flux at the electrode is simply calculated by the equation:

\[ \Gamma_{em} = n_{emw} \sqrt{\frac{2 \pi m_e T_{em}}{\varphi}} \]  

(46)

with \( n_{emw} \) the emitted electron density at the electrode. At the sheath edge, the total electron density is summed up to be \( n_{se} \), which gives the following equation:

\[ n_{emw} \exp \left( - \frac{e \varphi_{w}}{T_{em}} \right) + n_{sep} = n_{se}. \]  

(47)

Combining the above equations, the following expression for electron sheath potential is derived:

\[ \Gamma_{net} = n_{se} \sqrt{\frac{2 \pi m_e T_{em}}{\varphi}} \left[ \gamma_e^{-1} - \exp \left( - \frac{e \varphi_{w}}{T_{em}} \right) \right] \exp \left( - \frac{e \varphi_{w}}{T_{em}} \right) + \Theta_{T_{em}} (\gamma_e \beta_e)^{-1} \]  

(48)

where \( \Theta_{T_{em}} = T_{em}/T_{ep} \). The relation here is essentially \( I-V \) trace of the emissive electron sheath, which means that the electrode current is known once the applied bias potential \( \varphi_{w} \) is given. A change of the sheath \( I-V \) characteristic, if not considered in the probe calibration, can influence the measurement precision. The relation between applied electrode potential and electrode current can be further simplified as follows:

\[ \Phi_{w} = \Theta_{T_{em}} \ln \left( \frac{1 + \xi}{1 - \xi \sqrt{\Theta_{T_{em}} \beta_e}} \right) \]  

(49)

with \( \Phi_{w} \), the normalized electrode potential, and the factor \( \xi \) a normalized term in the following form:

\[ \xi = \frac{\Gamma_{net}}{n_{se} \sqrt{\frac{2 \pi m_e T_{em}}{\varphi}}} \]  

(50)

It has to be pointed out that the newly introduced factor \( \xi \) is not straightforward in physical meaning, but the denominator of the right-hand side of equation (49) dictates a singularity at \( 1 = \xi \sqrt{\Theta_{T_{em}} \beta_e} \), which is simplified as \( \Gamma_{ep} = \Gamma_{net} \). According to equations (43) and (44), \( \Gamma_{ep} = \Gamma_{em} \exp \left( - \frac{e \varphi_{w}}{T_{em}} \right) = \Gamma_{net} \). Therefore, the singularity is never achieved as long as SEE exists. While the upper bound of the possible \( \xi \) value is prescribed by this singularity, the lower bound of \( \xi \) should guarantee that \( \Phi_{w} \geq 0 \) (ensure the existence of electron sheath). The calculation results given by equation (49) are shown in figure 9.

It is obvious that the value of \( \Theta_{T_{em}} \) has a remarkable influence on the range of possible \( \xi \). The value of \( \beta_e \) varies between around 1.61 and 1.97, hence posing a fairly limited influence on the profile of \( \Phi_{w} \) (not shown). The emission coefficient mainly changes the lower bound of \( \xi \) since a higher \( \gamma_e \) allows for smaller \( \xi \) value before the term \( \frac{1 + \xi}{1 - \xi \sqrt{\Theta_{T_{em}} \beta_e}} \) reaches 1.

With equations (43)–(47), the two normalized electron densities \( N_{sep} = n_{sep}/n_w \) and \( N_{emw} = n_{emw}/n_w \), which represent normalized plasma electron density at the sheath edge and emitted electron density at the electrode, can be expressed with given normalized electrode bias potential \( \Phi_{w} \):

\[ N_{sep} = \frac{n_{sep}}{n_{se}} = \frac{\sqrt{\Theta_{T_{em}}/(\gamma_e \beta_e)} \exp \left( - \frac{\Phi_{w}}{\Theta_{T_{em}}} \right)}{1 + \sqrt{\Theta_{T_{em}}/(\gamma_e \beta_e)} \exp \left( - \frac{\Phi_{w}}{\Theta_{T_{em}}} \right)} \]  

(51)

\[ N_{emw} = \frac{n_{emw}}{n_{se}} = \left[ \exp \left( - \frac{\Phi_{w}}{\Theta_{T_{em}}} \right) + \Theta_{T_{em}}/(\gamma_e \beta_e) \right]^{-1}. \]  

(52)

Here \( \gamma_e, \beta_e, \Theta_{T_{em}} \) are regarded as constant once bias potential is fixed. With equations (51) and (52), all electron density components in the electron sheath become functions of space potential:

\[ N_{ep} = N_{sep} \exp (\Phi) \text{erfc}(\sqrt{\Phi}) \]  

(53)

\[ N_{em} = N_{emw} \exp (\Phi - \Phi_{w}) \]  

(54)

\[ N_{eref} = N_{em} \text{erfc}(\sqrt{\Phi}/\Theta_{T_{em}}). \]  

(55)
The Poisson equation is then:

\[
\frac{d^2 \Phi}{dx^2} = N_{s\text{ep}} \exp(\Phi) \text{erfc}(\sqrt{\Phi}) + N_{\text{emw}} \\
	imes \exp(-\Theta_T \Phi) [1 + \text{erf}(\sqrt{\Phi}/\Theta_{\text{Tem}})]. \tag{56}
\]

It is, however, not easy to solve equation (56) directly due to the complex integrand. Further study will be performed to calculate the approximate spatial potential profile.

However, the factor \( \gamma_e \) is not arbitrarily chosen. For thermionic emission or photoemission, the \( \gamma_e \) can be regarded as constant as long as the electrode temperature or light source remains stable \([48, 49]\), but the present research focuses on SEE where the emission coefficient is energy-dependent, and therefore closely linked with plasma properties. More precisely, the emission coefficient should be further expressed as:

\[
\gamma_e = g(2T_{\text{ep}} + e \varphi_{\text{a}}) \tag{57}
\]

where the form of \( g(x) \) in the energy range of low-temperature plasma can be approximated as \( g(x) = x/\varepsilon_{\text{see}} \) or \( g(x) = k / x \) \([34]\), with \( \varepsilon_{\text{see}} \) and \( k \) material-dependent coefficients. For a larger energy range, the SEE coefficient usually first increases with incident energy and then decreases after a certain threshold energy. Such an empirical model is widely employed in a variety of plasma simulations \([50, 51]\). For metallic materials, the value of \( \varepsilon_{\text{see}} \) is usually around several decades eV \([52, 53]\). In practice, the situation is far more complex since the emission coefficient also depends on surface roughness, cleanliness, and other local surface conditions \([54, 55]\). For now, only a conceptual prediction is given based on the simplest form of emission coefficient: \( \gamma_e = \frac{2T_{\text{ep}} + e \varphi_{\text{a}}}{\varepsilon_{\text{see}}} \) with \( \Theta_{T_e} = \frac{T_{\text{ep}}}{\varepsilon_{\text{see}}} \). For cold plasma, the value of \( \Theta_{T_e} \) should be smaller than 1, whereas \( \Theta_{T_e} \) could reach or even surpasses 1 in an attached divertor of magnetically confined fusion device.

In figure 10, the lower bound of \( \xi \) is shifted leftwards as \( \Theta_{T_e} \) increases, for the same reason in figure 9(b). Now the \( \gamma_e \) increases with \( \Phi_{\text{ew}} \) and the obtained profile is somewhat shifted compared with constant \( \gamma_e \) case. At low \( \Phi_{\text{ew}} \) level, the incident electrons might have a larger SEE coefficient due to reflection (also known as backscattering), where the primary electron does not penetrate into the solid material but is instead elastically or inelastically returned. Assuming the reflection probability to be \( R_e \), the expression of the SEE coefficient has to be modified accordingly:

\[
\gamma_{e,R} = R_e + (1 - R_e) \gamma_e \tag{58}
\]

where \( \gamma_{e,R} \) is the electron-emission coefficient involving electron reflection. This is because when a reflection occurs, the ‘effective’ SEE coefficient is 1, and if the true SEE happens, the corresponding contribution to the SEE coefficient is the non-reflection probability multiplied by the true SEE coefficient. However, the nature of such a process is not as
simple as described here and the coefficient $R_t$ is also energy-dependent [56].

A simple formula to estimate the reflection coefficient is based on the reflection of the electron wave function by a rectangular potential well, which predicts $R_t = \left(1 - \frac{1 + \varepsilon_e / \varepsilon_{pe}}{1 + \sqrt{1 + \varepsilon_e / \varepsilon_{pe}}} \right)^2$ [34]. Here $\varepsilon_e = \chi$ for dielectric, and is the sum of the Fermi energy and work function for metal. $\varepsilon_{pe}$ is the incident electron energy. The reflection coefficient is large only for a low electron energy range, e.g. below 5 eV. In numerical simulations, the coefficient $R_t$ is more commonly assumed as constant to facilitate calculation [57]. Here a proof-of-principle study with a constant backscattering coefficient is performed, as shown in figure 11. It is clear that the inclusion of the reflection coefficient slightly increases $\Phi_w$ monotonously. The realistic backscattering process is however much more complex. The reflection is a 3D process, though only one velocity dimension is considered here. The direction of backscattered electrons is also influenced by the electrode surface condition and roughness, potentially reducing the normal velocity component. These surface properties also change the backscattering coefficient.

One major defect of the above discussions on backscattering is the combination of secondary electrons and backscattered electrons in the surface emission flux term. The backscattering and SEE are completely different physical processes. The backscattered electrons usually lose a fraction of their energy when backscattered, with their velocity distribution function (VDF) closely related to the plasma electron VDF at the electrode, and their effective temperature depends on both $e\varepsilon_w$ and $T_{ep}$. The VDF of secondary electrons is however irrelevant to the plasma electron and possesses an independent temperature $T_{em}$. The best treatment is to separate the surface flux and reflected flux contributed by both SEE and backscattering, then rewriting equations (43) and (47), whereas the nonlinear dependence of backscattered electron temperature on $e\varepsilon_w$ causes difficulties in deriving an analytical expression, as shown from equations (48)–(50). The compromised treatment adopted in the present work aims for a qualitative understanding of the influence of backscattering and a better solution is expected in future works.

It is worth mentioning that the model derived above is not promptly applicable in strongly magnetized plasmas, e.g. the discharge plasma in a tokamak [48]. Firstly, the electron-emission mechanism from plasma-facing components (PFCs) is more complex. Apart from SEE and the backscattering mentioned here, thermonic emission from hot PFCs and transient field emission during edge localized mode are also important. Additionally, the prompt re-decomposition of emitted electrons during initial electron gyration is crucial, which depends on the sheath electric field near the surfaces. The application of the proposed work in such plasma conditions therefore requires more detailed investigations.

Last but not least, the influence of SEE on the electron sheath can be verified experimentally by using different types of electrode (thus different $\gamma_c$) biased at the same electrode potential, then checking the electron sheath properties posed by different SEE coefficients of the electrodes. Future experimental works are expected to corroborate the theoretical predictions.

5. Conclusions

The present work is dedicated to an improved understanding of electron sheath theory and its implications in plasma–surface interaction using both fluid and kinetic approaches. A fluid model considering the electron sheath entering velocity is proposed and compared to the classic electron sheath Child–Langmuir model. It is shown that the revised fluid model avoids the singularity at the sheath edge in the Child–Langmuir model and the latter tends to overestimate the electron sheath potential. Subsequently, a kinetic electron sheath model is constructed. Nonzero ion temperature is considered while disregarding the electron presheath structure. The kinetic model is shown to predict a higher sheath potential, and the sheath size is reduced with higher ion temperatures. Electron presheath–sheath matching is developed extensively, utilizing a more realistic truncated ion distribution function due to the loss of the cone effect. The electron-entering velocity at the sheath edge is shown to be dependent on electron–ion temperature ratio and the electron sheath potential, implying a change of probe current prediction in the electron saturation region. Electron-entering velocity is higher than the Bohm criterion prediction when a truncated IVDF is employed. Meanwhile, attempts are made to further include the electron-entering velocity into kinetic models. The exact relation between density and potential is obtained while no analytic or numerical sheath potential solution can be derived, revealing a trade-off between electron presheath structure and kinetic treatment of electron sheath. The electron sheath theory is further generalized to include the SEE on the electrode induced by sheath-accelerated plasma electrons. The $I$–$V$ trace of applied electrode bias potential and electrode
current is obtained, influenced by the boundary emission coefficient. Both constant and energy-dependent electron-emission coefficients are employed in the derivation, and the influence of backscattering is discussed. Further insights are also provided for future verifications of the proposed theories.

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Appendix

The following table gives the key variables adopted in derivations.

| Notation | Meaning | Unit | Remarks |
|----------|---------|------|---------|
| $\kappa_{w0}$ | Normalized factor | 1 | $\kappa_{w0} = \kappa_{w0}/\eta_{be}$ |
| $\beta_{e}$ | Normalized factor | 1 | $\beta_{e0} = \beta_{e}/\Gamma_{p}$ |
| $\gamma_{e}$ | SEE coefficient | 1 | $\gamma_{em}/\gamma_{p}$ |
| $\gamma_{e,R}$ | SEE coefficient | 1 | Consider reflection |
| $A_e$ | Area of chamber wall | m$^{-2}$ | Ion sheath presents |
| $A_{tw}$ | Area of biased electrode | m$^{-2}$ | Electron sheath presents |
| $\Gamma_{p}$ | Plasma electron flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\Gamma_{p,MM}$ | Plasma electron flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\Gamma_{p,te0}$ | Plasma electron flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\Gamma_{i}$ | Ion flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\Gamma_{net}$ | Net plasma flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\Gamma_{ref}$ | Reflected electron flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\Gamma_{em}$ | Emitted electron flux | m$^{-2}$ s$^{-1}$ | Integrated with $f_{w0}$ |
| $\varepsilon_0$ | Vacuum permittivity | C V$^{-1}$ m$^{-1}$ | |
| $\varepsilon_{sec}$ | SEE model factor | eV | |
| $J_{e,MM}$ | Form Factor | m$^{-4}$ s$^{-1}$ | |
| $J_{e,te0}$ | Form Factor | m$^{-4}$ s$^{-1}$ | |
| $f_{i}$ | IVDF | m$^{-4}$ s$^{-1}$ | |
| $E$ | Electric field | V m$^{-1}$ | |
| $b_i$ | Pre sheath density drop | 1 | |
| $h_e$ | Virtual anode density drop | 1 | |
| $\eta$ | Normalized potential | 1 | $\eta = (\varphi_w - \varphi_i)/E$ |
| $\eta_{be}$ | Normalized potential | 1 | $\eta_{be} = \eta$ at sheath edge, $e\varphi_w/T_i$ |
| $\Theta_T$ | Temperature ratio | 1 | $T_{p}/T_i$ |
| $\Theta_{em}$ | Temperature ratio | 1 | $T_{em}/T_{ep}$ |
| $\Theta_{Te}$ | Temperature–energy ratio | 1 | $T_{p}/\varepsilon_{sec}$ |

(Continued.)

| Notation | Meaning | Unit | Remarks |
|----------|---------|------|---------|
| $I(\eta)$ | Normalized factor | 1 | $I(\eta) = 0.5[1 + \exp(-\eta_{be})/2\sqrt{\pi\eta_{be}}I(\eta_{be})]$ |
| $\kappa$ | Normalized factor | 1 | $\exp(-\eta_{be})/2\sqrt{\pi\eta_{be}}I(\eta_{be})$ |
| $\lambda_{De}$ | Electron Debye length | m | |
| $\mu$ | Ion–electron mass ratio | 1 | |
| $J_{net}$ | Net current density | A cm$^{-2}$ | $e\Gamma_{net}$ |
| $m_e$ | Electron mass | kg | |
| $m_i$ | Ion mass | kg | |
| $n_{ep}$ | Plasma electron density | m$^{-3}$ | |
| $n_{em}$ | Emitted electron density | m$^{-3}$ | |
| $n_{e0}$ | Density at sheath edge | m$^{-3}$ | $\varphi_{w}$ |
| $n_i$ | Ion density | m$^{-3}$ | $\varphi_{i}$ |
| $N_{C}$ | Normalized sheath density | 1 | $n_i/n_{i0}$ |
| $R_{sl}$ | Electron parallel pressure | Pa | |
| $q_i$ | Ion heat flux | J m$^{-2}$ s$^{-1}$ | |
| $R_{l}$ | Reflection coefficient | 1 | |
| $S_{p}$ | Electron particle source | m$^{-3}$ s$^{-1}$ | |
| $S_{i}$ | Ion particle source | m$^{-3}$ s$^{-1}$ | |
| $S_{em}$ | Electron momentum source | kg m$^{-2}$ s$^{-2}$ | |
| $S_{i}$ | Ion momentum source | kg m$^{-2}$ s$^{-2}$ | |
| $T_{p}$ | Plasma electron temperature | eV | |
| $T_{em}$ | Electron temperature | eV | |
| $T_{i}$ | Ion temperature | eV | |
| $u_{e0}$ | Electron-entering velocity | m s$^{-1}$ | Velocity at sheath edge |
| $u_{i}$ | Electron fluid velocity | m s$^{-1}$ | |
| $v_{i0}$ | Ion fluid velocity | m s$^{-1}$ | |
| $v_{cut}$ | Ion cutoff velocity | m s$^{-1}$ | |
| $v_{cut}$ | Electron cutoff velocity | m s$^{-1}$ | |
| $v_{the}$ | Electron thermal velocity | m s$^{-1}$ | |
| $v_{the}$ | Ion thermal velocity | m s$^{-1}$ | |
| $\varphi_{the}$ | Electron sheath potential | V | Positive |
| $\varphi_{she}$ | Ion sheath potential | V | Negative |
| $\varphi_{w}$ | Biased electrode potential | V | Positive |
**ORCID iDs**

Guangyu SUN (孙光宇) @ https://orcid.org/0000-0001-6761-6019  
Shu ZHANG (张舒) @ https://orcid.org/0000-0001-9366-7426  
Anbang SUN (孙安邦) @ https://orcid.org/0000-0003-1918-3110  
Guanjun ZHANG (张冠军) @ https://orcid.org/0000-0003-1859-0443

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