A New Method to Estimate Cosmological Parameters Using Baryon Fraction of Clusters of Galaxies

Shin Sasaki
Department of Physics, Tokyo Metropolitan University, Hachioji, Tokyo, 192-03
E-mail: sasaki@phys.metro-u.ac.jp

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Abstract

We propose a new method to estimate cosmological parameters using the baryon fraction of clusters of galaxies for a range of redshifts. The basic assumption is that the baryon fraction of clusters is constant, which is a reasonable assumption when it is averaged within a Mpc scale. We find that the baryon fraction vs. redshift diagram can estimate the cosmological parameters, since the derived value of the baryon fraction from observations depends on the adopted value of cosmological parameters. We also discuss some points in comparing theoretical calculations with observations.

Key words: Cosmology — Galaxies : clusters of — X-rays

1. Introduction

Clusters of galaxies are the largest virialized systems in the universe and their masses can be estimated by X-ray and optical observations. In general, since there is no efficient way of changing the baryon fraction averaged within a Mpc scale, the baryon fraction of a cluster of galaxies \( M_b/M_{tot} \) (\( M_b \) and \( M_{tot} \) are baryon and total masses of a cluster) should take the global fraction in the universe: \( \Omega_b/\Omega_0 \) (\( \Omega_b \) and \( \Omega_0 \) are the baryon density parameter and the total density parameter). White et al. (1993) estimated \( \Omega_0 \) by identifying \( M_b/M_{tot} \) of the Coma cluster with \( \Omega_b/\Omega_0 \) and by using the big bang nucleosynthesis value for \( \Omega_b \), and they found that low \( \Omega_0 \) value is favored: \( \Omega_0 = 0.16h^{-1/2}/(1 + 0.19h^{3/2}) \) (where \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\) is the Hubble constant). However, in this method, the derived \( \Omega_0 \) depends on the Hubble constant and studies of the big bang nucleosynthesis. Furthermore, this method does not probe the value of the cosmological constant \( \lambda_0 \).

In this Letter, we propose a new method to estimate cosmological parameters (both \( \Omega_0 \) and \( \lambda_0 \)) using the baryon fraction of clusters for a range of redshifts.

2. Cosmological parameter dependence on mass estimation

First of all, we briefly summarize the method to estimate gas, galaxy and total masses of a cluster from X-ray and optical observations paying attention to their cosmological parameter dependence.

Firstly, we describe the gas mass in a cluster of galaxies using X-ray data. For simplicity, we assume that the hot gas in a cluster follows a spherically symmetric isothermal \( \beta \)-model (e.g., Sarazin 1988):

\[
n_e(r) = n_{e0} \left( 1 + \frac{r^2}{r_c^2} \right)^{-3\beta/2},
\]

where \( n_e \) is the electron number density, and \( n_{e0} \) and \( r_c \) are the central electron density and the core radius, respectively. Then, using X-ray data, the gas mass \( M_{gas}(< R) \) within a radius \( R \) and the bolometric luminosity \( L_X(< R) \) derived by X-ray observation are written as

\[
M_{gas}(< R) = \int_0^R \rho_{gas}(r)4\pi r^2 dr = \frac{8\pi}{1+X} m_H n_{e0} r_c^3 I_M(R/r_c, \beta), \quad (2)
\]

\[
I_M(y, \beta) = \int_0^y (1 + x^2)^{-3\beta/2} x^2 dx, \quad (3)
\]

\[
L_X(< R) = \int_0^R \frac{\left( \frac{2\pi k_BT_e}{3m_e} \right)^{1/2} \left( \frac{2^4e^6}{3\hbar c^2} \right)}{1+X} dr,
\]

\[
\times \left[ n_e(r) \right]^2 \frac{\bar{g}/T_e}{1+X} \frac{2}{2^4e^6} 4\pi r^2 dr = \frac{\left( \frac{2\pi k_BT_e}{3m_e} \right)^{1/2} \left( \frac{2^4e^6}{3\hbar c^2} \right) \bar{g}/T_e}{1+X} \frac{2}{2^4e^6} \pi n_{e0} r_c^3 I_L(R/r_c, \beta), \quad (4)
\]

\[
I_L(y, \beta) = \int_0^y (1 + x^2)^{-3\beta} x^2 dx, \quad (5)
\]

where \( \rho_{gas} \) is the gas density and other symbols have
their usual meanings. Combining equations (2) and (4) yields the gas mass:

\[ M_{\text{gas}}(< R) = \left( \frac{3\pi \hbar c^2}{2(1 + X)e^\theta} \right)^{1/2} \left( \frac{3mc^2}{2\pi k_B T_c} \right)^{1/4} m_H \]

\[ \times \left[ \frac{I_M(R/r_c, \beta)}{I_L^{1/2}(R/r_c, \beta)} \right] [L_X(< R)]^{1/2} \]

In the above equations, we implicitly assumed that the X-rays are emitted by the thermal bremsstrahlung only (see e.g., Rybicki and Lightman 1979). Note that \( L_X, r_c \text{ and } R \) are not the quantities derived by observations directly, but that they depend on the adopted cosmological parameters. That is, they are estimated as

\[ L_X(< R) = 4\pi [D_L(z; \Omega_0, \lambda_0, H_0)]^2 f_X(< \theta), \]

\[ r_c = \theta_c D_A(z; \Omega_0, \lambda_0, H_0), \]

\[ R = \theta D_A(z; \Omega_0, \lambda_0, H_0), \]

using directly observed quantities: the total bolometric flux \( f_X(< \theta) \) within \( \theta \), the angular core radius \( \theta_c \), and the outer angular radius \( \theta \) (see e.g., Peebles 1993). In the above equations, \( D_L \) and \( D_A \) are the luminosity distance and the angular diameter distance which depend on the redshift \( z \) of the cluster and cosmological parameters: \( \Omega_0, \lambda_0 \text{ and } H_0 \). Using the relation \( D_L = (1+z)^2 D_A \) and paying attention to their cosmological parameter dependences, the gas mass is written as

\[ M_{\text{gas}}(< \theta) \propto h^{-5/2} D_A^{5/2}(z; \Omega_0, \lambda_0), \]

where \( D_A \equiv h^{-1} D_A \). If we estimate the gas mass fixing the value of \( \Omega_0 \) and \( \lambda_0 \), it is proportional to \( h^{-5/2} \), which is a well known relation.

Secondly, we describe the galaxy mass in a cluster. In general, it is estimated as the total blue (absolute) luminosity times the mass-to-luminosity ratio (e.g. White et al. 1993). Note that the total blue luminosity is proportional to \( D_L^2 \) and that the mass-to-luminosity ratio is proportional to \( h \) (since it is derived for nearby galaxies, it does not depend on the cosmological parameters except for the Hubble constant). Thus, the galaxy mass \( M_{\text{gal}} \) in a cluster is written as

\[ M_{\text{gal}} \propto h^{-1} D_A^2. \]

If we fix the value of \( \Omega_0 \) and \( \lambda_0 \), \( M_{\text{gal}} \) is proportional to \( h^{-1} \).

Finally, we estimate the total mass in a cluster of galaxies using X-ray data. We assume that the intracluster gas is in hydrostatic equilibrium. Then, the total mass within the radius \( R \) is

\[ M_{\text{tot}}(< R) = -\frac{k_B T_c R d \ln n_e(r)}{G\mu_{m_H}} \frac{d \ln r}{r = R}, \]

thus,

\[ M_{\text{tot}}(< \theta) \propto h^{-1} D_A. \]

### 3. Method to estimate cosmological parameters

The basic assumption of our new method to estimate cosmological parameters is that the baryon fraction of clusters of galaxies \( M_b/M_{\text{tot}} \) is constant. In this method we do not need to assume that \( M_b/M_{\text{tot}} \) equals the global baryon fraction \( \Omega_b/\Omega_0 \) as in previous works (e.g., White et al. 1993). Based on this assumption, we estimate the cosmological parameters.

Using the results mentioned in the previous section, we can write the baryon fraction \( (M_b = M_{\text{gas}} + M_{\text{gal}}) \) in a cluster as

\[ \frac{M_b}{M_{\text{tot}}} = A D_A + B h^{-3/2} D_A^{3/2}, \]

where \( A \) and \( B \) are some cosmological parameter-independent values. In this Letter, we concern with only cosmological parameter dependence of the baryon fraction, thus precise formula of \( A \) and \( B \) is not needed below. In the above equation, the first term comes from the galaxies and the second from the gas. As shown in the above equation, the derived baryon fraction depends on the cosmological parameters through the angular diameter distance.

The basic and plausible assumption adopted in this Letter is that the value of \( M_b/M_{\text{tot}} \) is constant for all clusters. However, the derived value of \( M_b/M_{\text{tot}} \) depends on the adopted cosmological parameters \( (\Omega_0 \text{ and } \lambda_0) \) in particular at high redshift, since it depends on the angular diameter distance which depends on the adopted cosmological parameters. That is, if we use another cosmological parameters to derive \( M_b/M_{\text{tot}} \), we get another value for it. If we adopt the correct values as \( \Omega_0 \text{ and } \lambda_0 \) to estimate mass of a cluster, we obtain the true value of \( M_b/M_{\text{tot}} \) and it must be constant for all clusters at any redshift. On the other hand, if we adopt incorrect values for \( \Omega_0 \text{ and } \lambda_0 \), we obtain incorrect value of \( M_b/M_{\text{tot}} \) and it varies with redshift. Thus, we can determine \( \Omega_0 \text{ and } \lambda_0 \) from \( M_b/M_{\text{tot}} \) as follows: Estimate \( M_b/M_{\text{tot}} \) for clusters with various redshifts adopting some fixed values as \( \Omega_0 \text{ and } \lambda_0 \) (hereafter \( M_b/M_{\text{tot}} \text{ vs. } z \) diagram). If the derived values of \( M_b/M_{\text{tot}} \) vary with redshift, then, adopted values of \( \Omega_0 \text{ and } \lambda_0 \) are incorrect. If the derived values of \( M_b/M_{\text{tot}} \) are constant, then adopted values of \( \Omega_0 \text{ and } \lambda_0 \) are the correct ones.

It is to be noted that derived \( M_b/M_{\text{tot}} \) depends on \( \Omega_0, \lambda_0 \text{ and } h \) (see eq. (14)). However, the Hubble constant dependence cannot be discriminated unless we know the true value of \( B \) in this method. Although uncertainty of the Hubble constant shifts the absolute value of derived...
M_b/M_{tot}$, it does not change their behavior of the redshift evolution. Thus, we can determine $\Omega_0$ and $\lambda_0$ without knowing the value of $h$. Furthermore, in the above discussion, we took some assumptions for simplicity, but the final result (eq. (4)) is correct even if we relax some assumptions, for example, the gas is isothermal.

In the following, we study $M_{gas}/M_{tot}$ instead of $M_b/M_{tot}$, for simplicity. This is justified when the galaxy mass is negligible compared with the gas mass ($M_{gal}/M_{gas}$ is about $20h^{3/2}$ % in the Coma cluster: White et al. 1993) and/or the value of $M_{gas}/M_{tot}$ is constant for all clusters. In this case, we can use more simple way to estimate $\Omega_0$ and $\lambda_0$: We estimate $M_{gas}/M_{tot}$ using fixed values of $\Omega_0$ and $\lambda_0$. We can predict the artificial redshift evolution of the derived $M_{gas}/M_{tot}$ if the correct values of $\Omega_0$ and $\lambda_0$ are different from the adopted values. Thus, we can also determine $\Omega_0$ and $\lambda_0$ from derived $M_{gas}/M_{tot}$ (using fixed values of $\Omega_0$ and $\lambda_0$) comparing with expected artificial redshift evolutions. In figure 1, we show the expected artificial redshift evolution of $M_{gas}/M_{tot}$ fixing $(\Omega_0, \lambda_0) = (0, 0)$ when the correct values of $(\Omega_0, \lambda_0)$ are $(1, 0)$, $(0, 0)$ and $(0.1, 0.9)$ (dotted). Normalization of $M_{gas}/M_{tot}$ is arbitrary.

![Fig. 1. The expected artificial redshift evolution of the derived $M_{gas}/M_{tot}$. The values of $M_{gas}/M_{tot}$ are estimated fixing $(\Omega_0, \lambda_0) = (0, 0)$ when the correct values are $(1, 0)$ (solid), $(0, 0)$ (dotted) and $(0.1, 0.9)$ (dotted). Normalization of $M_{gas}/M_{tot}$ is arbitrary.](image)

### 4. Discussion and Conclusions

As mentioned above, the $M_{gas}/M_{tot}$ (or $M_b/M_{tot}$) vs. $z$ diagram is a potentially useful method to estimate cosmological parameters. However, to estimate them by this method in practice, we need to check some uncertainties. Firstly, the value of $M_{gas}/M_{tot}$ depends on the outer radius of the analysis except for $\beta = 2/3$. Thus, we must know how large a radius is required to derive $M_{gas}/M_{tot}$. Secondly, we must know whether $M_{gas}/M_{tot}$ ($M_b/M_{tot}$) is constant or not, in fact. Thirdly, we must know how large a scatter there is. To check these uncertainties, we need first study $M_{gas}/M_{tot}$ ($M_b/M_{tot}$) for nearby clusters which are not affected by adopted cosmological parameters. Unfortunately at the present, we do not have large samples enough to study this, but in the near future we will be able to have such samples.

At the present, we do not have large samples of $M_{gas}/M_{tot}$ data enough to study its cosmological parameter dependences, in particular at high redshift. Thus, here, we use inhomogeneous samples which are collected from some literatures, to study the possibility of the method mentioned above. It is to be noted that analysis described below using inhomogeneous samples is not conclusively, but illustrative at best. The data are summarized in table 1. In table 1, we assumed $\Omega_0 = 0, \lambda_0 = 0$ and $h = 1$. When the original data are derived using different cosmological parameters, we translate them using the above relation (eq. (4)) to the values for $(\Omega_0, \lambda_0) = (0, 0)$ and $h = 1$. As mentioned above, this data set is inhomogeneous: For example, outer radius of the analysis (see table 1) and method of mass estimation are different for each cluster and literature. In figure 2, we plot $M_{gas}/M_{tot}$ as a function of redshift with expected artificial redshift evolution curves. In figure 2, the values of data are normalized by the value of the Coma (which is the leftmost one with error bar). Since, here, we are interested in only their redshift evolution and not their absolute values, the normalization is arbitrary. As is seen, there is a large scatter. A part of scatter comes from inhomogeneity of the data set. We think large homogeneous data set will conclude whether this scatter is real or not and how large a scatter there is. If the scatter is real and large, we need some baryon segregation processes for cluster formation and evolution, for example, energy input by galactic winds. Since we think, in general, there is no efficient process of changing the baryon fraction averaged within a Mpc scale, we pay attention to data for which their outer radius is greater than 1Mpc ($\Omega_0 = 0, \lambda_0 = 0$, and $h = 1$) and they are shown as large crosses. The result suggests that high-$\lambda$ model seems to be disfavored. However, any conclusion must be left until large homogeneous data set is available.

In summary, we proposed a new method to estimate cosmological parameters using the baryon fraction of
Table 1. Data for the baryon fraction.

| cluster | redshift | $M_{\text{gas}}/M_{\text{tot}}(< R)$ (%) | $R(h^{-1}\text{Mpc})$ | ref. |
|---------|----------|----------------------------------------|-----------------------|------|
| Coma    | 0.0232   | 4.95                                   | 1.5                   | 1    |
| A2199   | 0.0299   | 3.2                                    | 1.2                   | 3    |
| A85     | 0.0521   | 6.65                                   | 0.71                  | 2    |
| A3266   | 0.0545   | 5.55                                   | 0.71                  | 2    |
| A2319   | 0.0559   | 4.31                                   | 0.70                  | 2    |
| A2256   | 0.0581   | 5.4                                    | 1.2                   | 3    |
| A1795   | 0.0621   | 7.28                                   | 0.71                  | 2    |
| A644    | 0.0704   | 3.71                                   | 0.60                  | 2    |
| A401    | 0.0748   | 5.0                                    | 1.2                   | 3    |
| A2029   | 0.0768   | 5.7                                    | 1.2                   | 3    |
| A1650   | 0.0840   | 4.17                                   | 0.55                  | 2    |
| A478    | 0.0881   | 9.05                                   | 0.98                  | 2    |
| A2142   | 0.0899   | 4.99                                   | 0.97                  | 2    |
| A3186   | 0.1270   | 6.54                                   | 0.75                  | 2    |
| A1413   | 0.1427   | 4.07                                   | 0.86                  | 2    |
| A545    | 0.1530   | 6.40                                   | 0.91                  | 2    |
| A2009   | 0.1530   | 4.81                                   | 0.65                  | 2    |
| A3888   | 0.1680   | 4.91                                   | 0.56                  | 2    |
| A2218   | 0.175    | 6.7                                    | 0.4                   | 5    |
| A1689   | 0.1810   | 4.84                                   | 0.74                  | 2    |
| A605    | 0.1816   | 7.00                                   | 1.2                   | 2    |
| A1763   | 0.1870   | 7.00                                   | 0.91                  | 2    |
| A2163   | 0.2030   | 5.94                                   | 1.1                   | 2    |
| A1246   | 0.216    | 7.0                                    | 1.3                   | 6    |
| Cl0016+16 | 0.5545 | 8.44                                   | 1.7                   | 4    |

The values of $M_{\text{gas}}/M_{\text{tot}}$ and $R$ are estimated for $\Omega_0 = 0$, $\lambda_0 = 0$ and $H_0 = 100\text{km s}^{-1}\text{Mpc}^{-1}$. The last column refer to: 1. White et al. (1993). 2. White and Fabian (1995). 3. Buote and Canizares (1996). 4. Neumann and Bohringer (1996). 5. Squires et al. (1996). 6. Kikuchi (1996).

Clusters of galaxies for a range of redshifts. We found that the $M_b/M_{\text{tot}}$ vs. $z$ diagram provides information about the values of cosmological parameters if we can determine the baryon fraction within $\sim 10\%$ for a cluster at $z \sim 0.4$. Furthermore, if the baryon fraction of a cluster is identified with the global baryon fraction as in many previous works, we can estimate $\Omega_b$ as $\Omega_b \times (M_b/M_{\text{tot}})$ using $\Omega_b$ value derived from the $M_b/M_{\text{tot}}$ vs. $z$ diagram. This is a new $\Omega_b$ estimation method without using big bang nucleosynthesis studies.

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