Is the Gini Index of Inequality Overly Sensitive to Changes in the Middle of the Income Distribution?

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ABSTRACT
The Gini index is the most commonly used measure of income inequality. Like any single summary measure of a set of data, it cannot capture all aspects that are of interest to researchers. One of its widely reported flaws is that it is supposed to be overly sensitive to changes in the middle of the distribution. By studying the effect of small transfers between households or an additional increment in income going to one member of the population on the value of the index, this claim is re-examined. It turns out that the difference in the rank order of donor and recipient is usually the most important factor determining the change in the Gini index due to the transfer, which implies that transfers from an upper income household to a low income household receive more weight that transfers involving the middle. Transfers between two middle-income households do affect a higher fraction of the population than other transfers but those transfers do not receive an excessive weight relative to other transfers because the difference in the ranks of donor and recipient is smaller than the corresponding difference in other transfers. Thus, progressive transfers between two households in the middle of the distribution changes the Gini index less than a transfer of the same amount from an upper income household to a lower income household. Similarly, the effect on the Gini index when a household in either tail of the distribution receives an additional increment is larger than when a middle-income household receives it. Contrary to much of the literature, these results indicate that the Gini index is not overly sensitive to changes in the middle of the distribution. Indeed, it is more sensitive to changes in the lower and upper parts of the distribution than in the middle.

1. Introduction

In his seminal article developing the relationship between measures of income inequality and an underlying social welfare function, Atkinson (1970, p. 255) noted that the Gini index was one of three measures that were sensitive to transfers at all income levels. After analyzing the effect of an “infinitesimal” transfer of income from a household to one with lower income, he concluded (on p. 256) that the Gini index gives more weight to transfers in the center of the distribution than at the tails, that is, the Gini index attaches more weight to transfers affecting the middle class. Alison (1978) provided a thorough review of the properties of several commonly used measures of inequality. Examining the effect of a transfer of an amount from a household with the jth largest income to the ith largest, he concludes that for a typically shaped income distribution the Gini index tends to be most sensitive to transfers around the middle of the distribution and the least sensitive to transfers among the very rich and very poor. Jasso (1979) pointed out that the formula for the effect of a transfer in Alison (1978) did not consider the possibility that a transfer would change the order, so the formula is only valid when the rank-order of the households is unaffected by the transfer (1979). Neither Jasso (1979) nor Alison (1979) comment on the effect of this on the conclusion that the Gini index is most sensitive to transfers in the middle of the distribution. Since the 1970s many authors (Ahn 1997; Jones and Weinberg 2000; Madden 2000 p. 76; Borghi 2005; DeMaio 2007; Callan and Keane (2009); Cobham and Sumner 2013; OECD 2013; Pressman 2013; Schmid and Stein 2013; Chang 2014; Bird and Zolt 2015; Thewissen et al. 2015) have noted the sensitivity of the Gini index to changes in the middle of the distribution. Similarly, Jenkins (2009) and Pak et al. (2016) refer to the Gini index being sensitive to income differences around the mode. Others state that it is relatively insensitive to changes in the top and bottom part of the distribution and more sensitive to changes in the middle (Roberts and Willits 2015) or more sensitive to changes in the middle than in the lower or higher tails of the distribution (Williams and Doessel 2006). Referring to the Gini index, Green et al. (1994, p. 59) wrote “An increase or decrease in the middle of the distribution will have a greater impact on the index than a similar change at either end, since there are more earners in the middle ranks.” Krozer (2015), Cobham et al. (2015), and the Wikipedia (n.d.) entry on income inequality metrics described it as being overly sensitive to changes in the middle of the distribution. The guide by the Australian Bureau of Statistics (2015) states “The Gini coefficient is sometimes criticized as being too sensitive to
relative changes around the middle of the income distribution. This sensitivity arises because the derivation of the Gini coefficient reflects the ranking of the population, and ranking is most likely to change at the densest part of the income distribution, which is likely to be around the middle of the distribution.” In their analysis of the growth in inequality in France, Fremeaux and Piketty (2014) observed, “the increase in income inequality during the 2000s is sharper for indices sensitive to the middle of the distribution like the Gini coefficient.”

Recently, Aaron (2015) and Gale et al. (2015) reached different conclusions about the effect of increasing taxes on the rich in income inequality. Gale et al. (2015) calculated that an increase in the top tax bracket from 39.6% to 50% would only lower the Gini index from about 0.560 to 0.556. Aaron (2015) argued that the Gini index is relatively insensitive to changes at the top and bottom of the distribution and that the ratio of the incomes of taxpayers at the 99th percentile to those at the 10th percentile was preferable to assess the impact of a tax increase on the very rich. He also noted that both sides agreed that the Gini index is more sensitive to changes in the middle of the distribution.

This article re-examines this claim and shows that it is incorrect. Consider a typical income distribution with a density that first increases, reaches its mode, and then decreases. The effect of a change in the distribution due to a transfer or addition to the income of one household will depend on whether it preserves or changes the order or ranks of the households, the difference in the ranks of the donor and recipient of a transfer, and who receives the additional income. In the case of a mean preserving transfer, one from the highest income recipient to the lowest has a larger impact on the value of the Gini index than a transfer of the same amount from any other household to one with less income. Transfers in the middle of the distribution, especially around the mode, do change the relative ranking of a higher proportion of the population. This does not necessarily imply that such a transfer has the largest impact on the numerical value of the Gini index because the change in its numerator also depends on the difference in the ranks of the donor and recipient. When one household receives an additional amount of income and the incomes of all others are unchanged; the average income (the denominator of the Gini index) is slightly increased. The number of households the recipient passes over to reach its higher rank does affect the numerator of the Gini index but in a manner that gives slightly less weight to changes in the middle.

The transfers of primary interest in economics obey the Pigou–Dalton criteria (Thon and Wallace 2004), which states that transfers from a poorer to a richer household increase inequality, while transfers from a richer household to a poorer one that does not reverse their relative ranking decreases inequality. Although the article will emphasize these transfers, in a few illustrative examples this principle will not hold.

Section 2 reviews several useful representations of the Gini index. One of them will be used in Section 3 to examine the effect on the Gini index of a small order-preserving transfer, e.g., an infinitesimal transfer from a high income household to one with less income, discussed by Atkinson (1970) or a small additional increment given to one household (Hoffman 2001). In the first situation, the effect of a transfer depends on the difference between the rank of the donor and the rank of the recipient; the largest decrease in the Gini index occurring when the highest ranked household transfers the small amount to the lowest ranked. The effect of one household receiving a small increment in income, which does not change the rank-order, depends on the rank of the recipient and again the Gini index decreases (increases) most when the poorest (richest) household receives the small increment. Section 4 examines the effect of a transfer of income that does not preserve the order, for example, the recipient of the transfer now has more income than several households whose incomes previously were greater than the recipient was and the rank of the donor might decline. In this situation, the rank order of a larger fraction of the population around a middle-income recipient or donor will change when that household is involved in the transfer; however, the change in the Gini index also depends on the difference in the ranks of the donor and recipient. The relative weight of these two components depends on the magnitude of this difference in ranks. When both the recipient and donor are in the middle of the distribution, the component of the change in the Gini index due to the number of households affected is relatively more important than the difference in the ranks, which should be small. When the donor is in the upper income region, the difference in after transfer ranks will be more important. The trade-off in the relative weights of the two components of change in the value of the Gini index indicates that the Gini index reflects the effect of non-order preserving transfers in various parts of the distribution and is not overly sensitive to those involving the middle. Section 5 focuses on how the Gini index changes when there is a small increase in the income of one household. It turns out that the Gini index decreases (increases) the most when the lowest (highest) income household receives the increment, while the magnitude of the change is smaller when the recipient is in the middle of the distribution. The final section discusses the implications of the results.

The Gini index has also been criticized (Hesse 2016) because different income distributions can have Gini indices with the same value. New proposed indices based on the ratio the share of total income of a fixed percent, for example, 10 or 20, at the upper end of the distribution to the share of income of a fixed percent, for example, 40 or 20 have been used by Palma (2011) and Dorling (2014) in studies of inequality. The Appendix shows that this family of indices can also have the same numerical values for different underlying distributions.

### 2. Formulas for the Gini Index

The Gini index of an income distribution, $F(x)$, is the ratio of the area between the line of equality and the Lorenz curve; however, it will be more convenient to use the fact that it is the ratio of the mean difference ($\Delta$) to twice the mean ($\mu$). The mean difference, $\Delta$, of the distribution is the expected absolute difference of two independent observations from $F(x)$. The empiric estimate ($d$) of $\Delta$ is the average of the absolute value of the differences in all pairs of incomes. Formally, given a sample $x_1, \ldots, x_n$ of observations from $F(x)$ it is defined (Kendall and Stuart 1977, p. 46) as

$$d = \left(\frac{1}{n(n-1)}\right) \sum_{i=1}^{n} |x_i - x_j|, \quad i \neq j.$$  \hspace{1cm} (1)

Sometimes $n(n-1)$ is replaced by $n^2$, but formula (1), which excludes comparisons of an observation with itself, is commonly
used to estimate $\Delta$ (Sudheesh and Dewan 2013). The Gini index, $G$, of the distribution underlying the data is $\Delta/2\mu$. Denoting the sample mean by $\bar{x}$, the estimate of $G$ is $g = d/(2\bar{x})$.

Thon (1982) reviewed several expressions for $G$ and $g$. To assess the impact of transfers and changes, a convenient form first orders the observations by their size, that is, $x_1 < x_2 < \cdots < x_n$ so

$$
g = \left[ \sum_{i=1}^{n} (2i - n - 1)x_i \right] / (n\bar{x}(n - 1)). \tag{2}$$

The numerator of (2) is one-half the $d$ (David 1968; David and Nagaraja 2003) and gives weight $(2i - n - 1)$ to the $i$th observation. For the Gini index the largest observation, $x_n$, receives weight $(n - 1)/n\bar{x}(n - 1)$ and the smallest receives weight $-(n - 1)/n\bar{x}(n - 1)$. Indeed, starting from the smallest, the weight given to each successive order statistic increases by $2/n\bar{x}(n - 1)$. If $n = 2m - 1$, so the median is the $m$th largest observation, it receives zero weight. When $n = 2m$, the median is the average of $x_m$ and $x_{m+1}$ and the numerator of (2) gives weight $-1/n\bar{x}(n - 1)$ to $x_m$ and $+1/n\bar{x}(n - 1)$ to $x_{m+1}$. Thus, the numerator of $g$ gives more weight to both extremes and less weight to the observations in the middle observations. The mean in the denominator weights each observation equally, so the relative weight given to each of the ordered observations increases with their distance from the median. Yitzhaki (2003) and Yitzhaki and Schectman (2013) discussed the properties and a wide variety of statistical applications of the mean difference and Gini index.

### 3. The Effect of a Small Order Preserving Transfer or Increment on the Gini Index

#### 3.1. Transfers to an Individual

Following Atkinson’s (1970) examination of the effect of an “infinitesimal” transfer from a higher income recipient to a lower one, consider the effect of a small transfer of $\varepsilon$ from $x_j$ to $x_i$, where $x_i < x_j$, which is not large enough to change the order of the observations. The mean, $\bar{x}$, is unchanged and only the terms in the numerator of (2) that will change involve $x_i$ and $x_j$. Thus, the change in the numerator is

$$
(2j - n - 1)[(x_j - \varepsilon) - x_j] - (2i - n - 1)[(x_i + \varepsilon) - x_i] = -2(j - i)\varepsilon \tag{3}
$$

and $g$ changes by $-2(j - i)\varepsilon / n\bar{x}(n - 1)$. In particular, the decrease in $g$ due to a transfer of $\varepsilon$ from any observation to the one immediately below, that is, $j - i = 1$ is $-2\varepsilon / n\bar{x}(n - 1)$; so the effect of such a transfer is the same throughout the distribution. From (3), it is clear that the largest decrease occurs when the highest income recipient transfers $\varepsilon$ to the lowest and the magnitude of the change depends only on $\varepsilon$ and twice the difference in the ranks of the donor and recipient. An order-preserving transfer from the highest income holder to the median receives one-half the weight as a transfer to the poorest. These considerations demonstrate that the Gini index is not overly sensitive to small order preserving transfers in the middle of the distribution as transfers to or from the middle do not have as large an effect on $g$ as transfers from the upper end to the lower end of the distribution.

Comment: Schmid (1991) studied the sensitivity of a variety of indices of inequality to small transfers that preserve the rank order. In large samples his result is equivalent to (3), where $j/n$ and $i/n$ approach the fractions $\beta$ and $\alpha$, respectively. Interestingly, the sensitivity of the coefficient of variation for the Theil-Atkinson family depends on the population quantiles, $F^1(\beta)$ and $F^1(\alpha)$ rather than on $\beta$ and $\alpha$.

#### 3.2. Increase to a Single Unit

Next, consider the situation where one unit receives an order preserving increase of size $\varepsilon$, for example, the $j$th so $x_j$ becomes $x_j + \varepsilon$ and the overall mean becomes $(nx + \varepsilon)/n$, where $\bar{x}$ is the mean of the original observations. From formula (2), it follows that the Gini index ($g_1$) of the new data is

$$
g_1 = \frac{\sum_{i=1}^{n} (2i - n - 1)x_i + \varepsilon(2j - n - 1)}{(n - 1)(n\bar{x} + \varepsilon)}. \tag{4}
$$

The part of formula (4) that depends primarily on $j$ is the second term in the numerator, which increases linearly in $j$. When $j = 1$, that is, the small additional income goes to the poorest household, the numerator decreases by $(n - 1)\varepsilon$. When $n = 2j - 1$, the $j$th ranked member of the population is the median, and as a result the contribution to the numerator is zero, and when $j = n$, that is, the richest household receives the additional income, the numerator increases by $(n - 1)\varepsilon$. Since the increase in the denominator of (4) is the same regardless of which household receives the increase, the largest decline in the Gini index occurs when $j = 1$, that is, the poorest household receives the additional income because the numerator decreases the most and the denominator increased. When the median household receives the additional income, the Gini decreases slightly because the mean (the denominator) has slightly increased. Furthermore, considering formula (4) as a function of epsilon, routine calculus shows that for $j > (n + 1)/2$ the increase in the numerator has a greater effect than the increase in denominator, so the largest increase in the Gini index occurs when the additional increment goes to the top-ranked household.

#### 3.3. The Subset of the Population Whose Receipt of an Increment Decreases the Gini Index

It is interesting to determine the percentage of households who can receive an order preserving increment ($\varepsilon$) that leads to a decrease in the Gini index of the population. The first step is to calculate the difference between $g_1$ and the original $g$. From (2), it follows that

$$
g_1 - g = \varepsilon \frac{(2j - n - 1)2\bar{x} + (n - 1)\varepsilon}{(n - 1)nx + (n - 1)\varepsilon}, \tag{5}
$$

where the subscript $\bar{x}$ indicates a function of $\bar{x}$.

Substitution in (4), yields $g_1 = \varepsilon \frac{2\bar{x} + (n - 1)\varepsilon}{(n - 1)2\bar{x} + (n - 1)\varepsilon}$, which implies that

$$
g_1 - g = \frac{\varepsilon(2j - n - 1) - \varepsilon(n - 1)g}{(n - 1)nx + (n - 1)\varepsilon} \varepsilon \left[ \frac{2j}{n+1} - 1 - \frac{n-1}{n+1} \right] n\bar{x}^2 + \varepsilon \frac{2\bar{x}}{n+1} \frac{n-1}{n+1}. \tag{5}
$$
Thus, when the \( j \)th ranked household receives the additional small increment, \( g_i < g \) or inequality decreases when

\[
2j - n - 1 < g(n - 1) \quad \text{or} \quad j < \left( (1 + g)n - 1 - g \right) / 2
\]

(5a)

and \( g_i > g \) or inequality increases if \( j > \left( (1 + g)n - 1 - g \right) / 2 \).

When \( n \) is large, Equation (5a) implies that inequality will still be reduced as long as \( j/n \) is less than \((1 + g)/2\), that is, the recipient (\( j \)th ranked) is at or below the \((50 + 50g)\)th percentile. When the original Gini index is 0.50 (0.30), this implies that as long as the recipient is in the lower 75% (65%) of the distribution, inequality as measured by the index will decrease. On the other hand, if the small increment goes to a household in the upper 25% (35%), inequality will increase by an amount that increases with the rank of the recipient. For any value, \( g \), of the original Gini index, as expected the largest increase in inequality occurs when the household with the largest income receives the small increment and the greatest decrease occurs when the household with the lowest income receives it. Regardless of the original value of the Gini index, it will decrease as long as a household in the lower half of the distribution receives it; however, the magnitude of the decrease is greatest when a household at the lower end receive the increment.

Comment: The result that in large samples the Gini index will decrease as long as the original percentile of the recipient is below the \((50 + 50g)\)th appears in Hoffman (2001, their sec. 2), who considered the effect of an “infinitesimal increment” that does not change the income order. Later, Hoffman presented a similar formula (p. 245, (16)) for the effect of a small increase given to the \( j \)th ranked household in a sample of \( n \) from a continuous distribution. Letting \( j/n \) approach \( \alpha \) as \( n \) increases in the second formula in (5), that is, the recipient is at the 100\( \alpha \)th percentile of the distribution, the change in the Gini index is

\[
e \frac{2\alpha - (1 + G)}{nx + e}.
\]

(6)

Formula (6) is equivalent to Hoffman’s formula in large samples (there is a minor typo in his formula as the denominator should be \( n \) times the mean). In place of \( j/(n + 1) \) in (5) or its limit \( \alpha \) in (6) Hoffman considered \( F(x_j) \) and concludes that the relationship between the change in the Gini index and \( x_j \) depends on the form of the distribution while (6) does not depend on the underlying distribution. Because \( x_j \) is the \( j \)th-order statistic in the sample of \( n \), the sampling distribution of \( F(x_j) \) is the same as the \( j \)th-order statistic in a sample of \( n \) from the uniform distribution (David and Nagaraja 2003, p. 14), so its expected value is \( j/(n + 1) \), which converges to \( \alpha \) as \( n \) increases. As the empiric percentiles of a random sample from a uniform distribution are consistent estimators of the population percentiles, in large samples the formulas are equivalent. However, the change in the Gini index depends on the original percentile of the recipient, rather than the form of the underlying distribution.

Although the analysis leading to formulas (4) and (5) does not require the order preserving transfer or increment to be small, in practice, the size \((n)\) of the population will be reasonably large so the order-preserving requirement restricts the possible magnitude of the transfer or increment. Thus, the results in this section are applicable to the “infinitesimal” transfers considered by Atkinson (1970) and the “small ones” discussed by Allison (1978). In the order-preserving context, neither transfers or an addition to one member of the population in the middle receive excessive weight compared to other parts of the distribution, so the Gini index is not especially sensitive to these types of change in the middle part of the distribution.

### 4. The Effect of a Transfer that Changes the Ranks While Preserving the Mean

Consider the case where the amount, \( a \), transferred by the \( j \)th highest income recipient to the \( i \)th, where \( i < j \), is sufficient to change the ordering of the \( n \) incomes. This means that either \( x_i + a \) is larger than some of the observations, \( x_r \), where \( r > i \) or \( x_j - a \), is now smaller than some of the observations \( x_r \) that previously were below \( x_j \), or both occur.

#### 4.1. The Rank of the Recipient Increases While the Rank of the Donor is Unchanged

In this case, the income of the recipient now equals \( x_i + a \), which becomes the \( k \)th largest. Thus, the \( k - i \), observations, \( x_r \), \( r = i+1, \ldots, k \) that were larger than \( x_i \) were less than \( x_i + a \). After the transfer, the ranks of each of these \( k-i \) observations decrease by one, so each of their contributions to the numerator of \( g \) decreases by \( 2x_r \). The contribution of \( x_i + a \), which is now the \( k \)th largest observation is \( (2k - n - 1)(x_i + a) \), however, this replaces its previous value \( 2i - n - 1 \), so the transfer increases the contribution of \( x_i \) by \( (2k - n - 1)(x_i + a) - (2i - n - 1)x_i = 2(k - i)x_i + a(2k - n - 1) \). Thus, the contribution of the first \( k \) observations to the numerator has changed by

\[
- \sum_{r=i+1}^{k} 2x_r + 2(k - i) x_i + a(2k - n - 1)
\]

(7)

If the rank \( j \) of the donor is unchanged, its contribution to the numerator of \( g \) is reduced by \( 2j - n - 1 \), so the net change in the numerator is

\[
- \sum_{r=i+1}^{k} 2(x_r - x_i) - 2a(j-k).
\]

(8)

As there are \( k - i \) terms in the summand, adding and subtracting \( 2a(k - i) \) implies that the change in the numerator of the Gini index is

\[
2 \sum_{r=i+1}^{k} [a - (x_r - x_i)] - 2a(j - i).
\]

(8a)

It follows that the after transfer Gini index is related to the original one \((g)\) by

\[
g_i = g - \frac{2}{(n - 1) nx} \left( \sum_{r=i+1}^{k} (x_r - (x_i + a)) + (j - i) a \right).
\]

(8b)

Formulas (8a) and (8b) show that the effect of a transfer of size \( a \) from the \( j \)th ranked household that increases the \( i \)th
ranked household to rank $k$, where $i < k < j$ but does not change the rank of the donor, depends on:

1. The size ($a$) of the transfer,
2. The difference ($j - i$) between the original ranks of the donor and recipient and
3. The number ($k - i$) of households in the interval $[x_i, x_j]$. 

The incomes, $x_r$, $r = i + 1, \ldots, k$1 of these $k - i$ households satisfy $x_i < x_r < x_k + a$, that is, $x_r - x_i < a$ implying that each term in the summand in (8a) is positive but less than $a$, so the summand in (8a) is less than $a(k - i)$. Thus, expression (8a) is negative whenever the Pigou–Dalton condition holds (Thon and Wallace 2004), that is, after the transfer the rank of the donor remains greater than the rank of the recipient. Consequently, $(j - i) \geq (k - i)$ and (8b) implies that the Gini index decreases.

Consider the implications of these results when a high-income donor transfers money to a low-income recipient. The Gini index should be less when the recipient is in the lower half of the distribution than when it is in the lower end. This first term, however, reflects the difference ($j - i$) in the ranks of the donor and recipient, which is positive, should be larger when the rank of the donor is in the lower part of the distribution. Thus, the positive summand is much smaller than the absolute value of the second term in (8a), so the term in (8b) reflecting the effect of the transfer will be negative and the Gini index will decrease.

The density function underlying most income distributions has a mode near or below median, so a transfer from a high-income donor to a recipient in the lower half of the distribution is much larger than the difference ($k - i$) in the before and after transfer ranks of the recipient. Thus, the positive summand is much smaller than the absolute value of the second term in (8a), so the term in (8b) reflecting the effect of the transfer will be negative and the Gini index will decrease.

The density function of most income distributions is relatively small in the tails of the distribution, the number ($k - i$) of terms in the summand will be less than the number of terms involved when the recipient is in the central part of the distribution. This means that the difference ($j - i$) in the ranks of the donor and recipient is much larger than the difference ($k - i$) in the before and after transfer ranks of the recipient. Thus, the positive summand is much smaller than the absolute value of the second term in (8a), so the term in (8b) reflecting the effect of the transfer will be negative and the Gini index will decrease.

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not approximate the decrease (8c) in the Gini index resulting from such a transfer.

4.2. Transfers that Increase the Rank of the Recipient and Decrease the Rank of the Donor

Next, assume that the size (a) of the transfer from the jth ranked household in the original distribution is sufficiently large that its rank after the transfer will decrease to t, where t < j; again the recipient’s rank increases from i to k, where k < t. This means that the observations $x_r$ for $r = t, t + 1, \ldots , j - 1$, are less than $x_i$ but larger than $x_t - a$. The contributions of each of these observations to the numerator will increase by $2x_r$, while contribution of $x_j$ will change by $(2t - n - 1)(x_j - a) - (2j - n - 1)x_j = -(2j - t)x_j - a(2t - n - 1)$. This causes the numerator of the Gini index of the new data to differ from the numerator of the Gini index of the original data by

$$2 \sum_{r=t}^{j-1} (x_r - x_j) - a(2t - n - 1) = -2 \sum_{r=t}^{j-1} (x_j - x_r) - a(2t - n - 1). \tag{9}$$

Thus, when the transfer of the amount $a$ from the jth ranked to the ith ranked household increases the rank of the recipient to k, while the donor’s rank is reduced to t, it follows from (7) and (9) that the Gini index, $g_1$ of the new data is

$$g_1 = \frac{g \bar{x} n(n-1) - \sum_{i=t+1}^{k} 2(x_r - x_t) - 2 \sum_{r=i+1}^{j-1} (x_j - x_r) - 2a(t-k)}{\bar{x} n(n-1)} \tag{10}$$

Equivalently,

$$g_1 = g - \frac{2}{n(n-1) \bar{x}} \left( \sum_{i=t+1}^{k} [x_r - (x_r + a)] + \sum_{r=t}^{j-1} [(x_j - a) - x_r] + (j-i)a \right). \tag{10a}$$

When the after transfer rank of the donor is larger than that of the recipient the first two terms in the parenthesis of (10a) are negative and would increase the Gini index; however, their effect is offset by the positive third term, which decreases the Gini index. Recalling that the terms $x_t - x_r$, $r = i + 1, \ldots , k$ and the $x_i - x_r$, $r = t, \ldots , j - 1$ are less than $a$, it follows that

$$2 \sum_{r=i+1}^{k} (x_r - x_t) < 2a(k-i) \quad \text{and}$$

$$2 \sum_{r=t}^{j-1} (x_j - x_r) < 2a(j-t) \quad \text{or} \tag{11}$$

$$0 \geq \sum_{r=i+1}^{k} [x_r - (x_r + a)] \geq -a(k-i) \quad \text{and}$$

$$0 \geq \sum_{r=t}^{j-1} [(x_j - a) - x_r] \geq -a(j-t). \tag{11a}$$

Hence, the magnitude of the contribution of the first two of the three terms in the parentheses in (10a) is less than 2a times the number of households not involved in the transfer whose ranks changed by one. This number, $k - i + j - t = (j - i) - (t - k)$ is less than $(j - i)$ since $t > k$. Thus, the Gini index decreases for transfers from a richer to poorer household that satisfy the Pigou–Dalton criteria. Only when the after transfer ranks are close, that is, $t - k$ is small, will the two terms in (10) or (10a) reflecting the effect of the number of households whose ranks changed by one as a result of the transfer have a major impact on the change in the Gini index.

For the small transfers considered by economists (Atkinson 1970), when the donor is in the upper portion of the distribution while the recipient is in the lower part both $k - i$ and $j - t$ are much less than $(j - i)$, so the magnitude of the effect of the households whose rank changed is relatively small. In this situation, the decrease in the Gini index due to the difference in the ranks of the donor and recipient will have a greater impact and offset the increase in the Gini index due to the number whose ranks changed by one because of the transfer.

When the donor (jth ranked) is in the upper part of the distribution and the recipient (ith ranked) in the modal region of the distribution, for example, near the mode $m$, the number, $k - i$ of households passed over by the recipient is larger and the difference, $j - i$, in the original ranks is smaller than the case of a low-income recipient. Thus, the first of the three terms in (10a), which increase the Gini index, is larger, while the third term, which decreases the index, is smaller. Thus, the net decrease in the index when a donor in the upper region makes a small transfer ($a$) to a recipient is in the modal region is less than when the recipient is at the lower part. When both the recipient and donor are in the middle of the distribution, the higher density of the income distribution in the central region implies that the number of terms in the first and second terms in the parenthesis in (10a) is larger than in the situation where a high-income donor makes a transfer to a middle or low-income one. The term $(j - i)$ reflecting the difference in the original ranks of the donor and recipient is smaller than in the previous two types of transfer. Thus, the net decrease in the Gini index arising from a progressive transfer between two households in the middle will be smaller than that resulting from a transfer from a high-income donor to a low or middle-income household.

Comment: The approximations used in the previous comment imply that for small transfers of size $a$, the change in the Gini index in formula (10a) is approximately

$$- \frac{1}{(n-1) \bar{x}} (f(x_j)a^2 + f(x_t)a^2 - 2f(t^*)[x_j - x_t]a). \tag{12}$$

Again, $t^*$ is in $[x_i, x_j]$ and $x_j - x_t \geq 2a$. When the difference, $x_j - x_t$ between the original incomes of donor and recipient is large, the third term in (12) that decreases the Gini index, is larger in magnitude than the first two terms that increase the index. When the difference, $x_j - x_t$ in the original incomes of donor and recipient is relatively small as in the case of a transfer between two middle-income households, the approximate decrease (12) in the Gini index is greater or equal to

$$- \frac{1}{(n-1) \bar{x}} (f(x_t)a^2 + f(x_t)a^2 - 4f(t^*)a^2). \tag{12a}$$
This lower bound (12a) to the decrease is larger than (8e) because the ranks of more households change when the transfer lowers the donor’s rank and increases the recipient’s.

Some readers might be interested in a simple example where the rank-orders of the population change because of the transfer. Based on the mean incomes of the five quantiles reported by the Current Population Survey, consider the following nine incomes: 11,651, 30,509, 48,322, 50,322, 52,322, 54,322, 83,519, 137,600, and 185,206. Their mean is 72,641.44, Gini index is 0.41678, and mean difference is 60,549.28. Table 1 presents a few examples of the change resulting from a transfer of $10,000,000, from a higher income household to one with a lower income. This large amount will enable us to illustrate the effect of the transfer increasing the recipients rank or reducing the donor’s rank. The transfers between the sixth and fifth as well as the fifth to fourth violate the Pigou–Dalton condition as the donor’s after transfer rank is lower than the recipient’s is. This is why these transfers increase the Gini index. The last transfer in Table 1, from the fourth to fifth is a regressive transfer, which increases the Gini index.

Examining Table 1, the largest change in the Gini index occurs when the richest (rank 9) donates to the poorest (rank 1). As the rank of the recipient of a donation from the richest increases, the magnitude of the change in the Gini index and mean difference decrease as expected. When the sixth ranked household donates to the poorest, the Gini index changes more than when the recipient is the second ranked; however, when the sixth ranked donates to the fifth ranked, the Gini index and mean difference are greater than their values on the original set of nine. The absolute values of the change in the Gini index and mean difference, however, are smaller than when the sixth ranked household transfers money to the lowest ranked. Transfers between the fourth and fifth ranked households also create a slight increase in the Gini index, due to the lowered rank of the donor; however, the absolute value of this increase is smaller than the absolute value of the decrease resulting from transfers from the ninth ranked household to the poorest or the median one. Again, transfers involving the two households in the central part of the distribution do not create the largest changes in the Gini index.

In sum, the change in the Gini index due to a nonorder preserving but mean preserving transfer depends on the difference between the before transfer ranks of the donor and recipient and the number of other households whose rank changed. From (8b) and (10a), the effect of the transfer, \((j - i)a\) is to decrease the Gini index, while the terms in the summands reflecting the change in the rank order and increment given to the recipient increase the Gini index. When an upper income household makes a transfer to one in the lower or central portions of the distribution, the term involving the difference in ranks is the dominant one, while when the transfer is between two households in the middle; the number of other individuals whose rank changed in the process has a larger role. Thus, the decrease in the Gini index is larger when a high-income household makes a transfer to a low or middle-income recipient than when the transfer is between two households in the modal region. Thus, the Gini index is not overly sensitive to transfers in the middle. As Atkinson (1970) noted, the Gini index is sensitive to changes at all levels and a reason for this is that the change has two components, one of which is more sensitive to transfers when the donor and recipient are from different parts of the distribution and the other when they are both in the modal region.

### 5. The Effect on the Gini Index When One Household Receives a Small Increment

Finally, consider the effect of the ith ranked household receiving an additional amount, so the overall mean becomes \((nx + a)/n\). The denominator of \(g_i\) replaces the original mean, \(\bar{x}\) by the new mean and is the same regardless of which household receives the additional amount, \(a\). Thus, both the sensitivity of the numerator and Gini index to which member of the population receives the increment are of interest.

#### 5.1. Increments that do not Change the Rank of the Recipient

If the rank of the recipient is unchanged, which is likely when \(a\) is small, formula (2) implies that the numerator of the Gini index will change by \((2i - n - 1)a\). If \(i = 1\), the numerator changes by \(-a(n - 1)\) and if \(i = n\), it increases by \(a(n - 1)\). When \(n = 2m - 1\) and the median receives the increment the numerator does not change. Indeed, the absolute value of \((2i - n - 1)\) declines from \(n - 1\) to 0 as \(j\) ranges from 1 to \(m\) and then increases to \(n - 1\) as \(j\) ranges from \(m\) to \(n\). Clearly, the numerator of the Gini index changes less when the recipient is in the middle part of the distribution than when a low or upper income household receives the increment.

The after increment Gini index (\(g_1\)) is given by

\[
g_1 = g - \frac{a}{nx + a} \left( \frac{g + n + 1 - 2i}{n - 1} \right). \tag{13}\]

The new index is less than the original when the term in the parenthesis is positive, that is, when

\[(n - 1)g + n + 1 - 2i < 0 \text{ or } 2i < (n - 1)g + (n + 1)\).

For a large population of size \(n\) as \(n \to \infty\) and \(i/n \to p\), that is, the recipient is the 100pth percentile, it follows that the increment will reduce inequality, as measured by the Gini index whenever \(p\) is less than \((1 + g)/2\). Thus, the Gini index will decrease if a small increment that does not change the rank of the recipient if that recipient is below the \((50 + 50g)\)th percentile.

**Table 1.** The effect on the Gini index and mean difference of a transfer of $10,000 from the household \(j\)th ranked to the \(i\)th ranked household.

| Rank of donor | Rank of recipient | Gini index | Change in Gini index | Mean difference |
|---------------|-------------------|------------|----------------------|-----------------|
| 9 (highest)   | 1 (lowest)        | 0.3862     | -0.0306              | 56105           |
| 9             | 2                 | 0.3900     | -0.0268              | 56660           |
| 9             | 5                 | 0.4045     | -0.0122              | 58772           |
| 9             | 8                 | 0.4129     | -0.0038              | 59994           |
| 6             | 1                 | 0.4045     | -0.0122              | 58772           |
| 6             | 2                 | 0.4084     | -0.0084              | 59327           |
| 6             | 5                 | 0.4237     | 0.0069               | 61549           |
| 5             | 4                 | 0.4344     | 0.0076               | 61660           |
| 4             | 5                 | 0.4267     | 0.0099               | 61994           |

NOTE: The mean difference is the expected value of the absolute difference of a random pair chosen from a population. One estimates it from a random sample using formula (1).
5.2. Increments That Increase the Rank of the Recipient

If the increment, \( a \), is sufficient to affect the order, the rank of the recipient increases by the number \( (b) \) of observations between \( x_i \) and \( x_i + a \). Then, the rank of the recipient increases to \( i + b = k \). The reasoning in the previous section shows that the numerator of the Gini index of the new data differs from that of the original data by

\[
- \sum_{r=i+1}^{k} 2(x_r - x_i) + a (2k - n - 1).
\]

(14)

The \( x_r, r = i + 1, \ldots, k \) fall in the interval, \( [x_i, x_i + a] \) so the first term in (14) is negative but \( \geq -2a(k - i) \). The second term is negative if the final rank, \( k \), of the recipient is less than \((n+1)/2\) and positive otherwise. The first term is influenced by the number of observations between \( x_i \) and \( x_i + a \), which is largest when the rank, \( i \), of the recipient is in the modal region of the distribution because the density function is highest there. The second term in (14) tells us that when the final rank, \( k \), of the recipient, is less than \((n+1)/2\), the numerator of the Gini index will decrease as will the index as the mean in the denominator has increased.

From (14), it follows that the Gini index, \( g_1 \), after the increment is

\[
g_1 = g - \frac{n}{n - 1} \cdot \frac{1}{nx + a} \left( \frac{n - 1}{n} \cdot 2g + \frac{2}{n} \sum_{r=i+1}^{k} (x_r - x_i) - a \left( \frac{2k - n - 1}{n} \right) \right)
\]

(15)

Calculations similar to those in Section 4, yield

\[
g_1 = g - \frac{1}{n - 1} \cdot \frac{1}{nx + a} \left( \frac{n - 1}{n} \cdot 2g + \frac{2}{n} \sum_{r=i+1}^{k} (x_r - x_i) - a \left( \frac{2k - n - 1}{n} \right) \right)
\]

(16)

Because the first term in (14) is negative, the numerator of the Gini index will decrease for some values of \( k > (n + 1)/2 \), provided they satisfy

\[
\sum_{r=i+1}^{k} 2(x_r - x_i) > a (2k - n - 1).
\]

(17)

For the remaining values of \( k \), all of which are in the upper half of the distribution, the Gini index will increase. When the household with the lowest income is the recipient both terms in (14) are negative provided the amount \( (a) \) is not so large that their after transfer rank \( (k) \) is in the upper half of the distribution and no longer satisfies (17). Thus, for small increments analogous to the transfers considered by both Atkinson (1970) and Allison (1978), the Gini index will decrease the most when the household with the lowest income is the recipient.

For a large population, as \( n \) increases the term for the change in the Gini index in (16) becomes

\[
- \frac{a}{nx + a} \left( 1 + g + \frac{2}{na} \sum_{r=i+1}^{k} (x_r - x_i) - 2 \frac{k}{n} \right).
\]

(18)

When \( a \) is small, \( 2 \sum_{r=i+1}^{k} (x_r - x_i) \) approximately \( = -a(k - i) \) as the \( x_i \) are approximately uniformly distributed in the small interval \( [x_i, x_i + a] \) so the change in the Gini index is approximately

\[
- \frac{a}{nx + a} \left( 1 + g - \left( \frac{i}{n} + \frac{k}{n} \right) \right).
\]

(19)

Formula (19) implies that the decrease in the Gini index resulting from a small increment is largest when \( \frac{i}{n} + \frac{k}{n} \) is small. From (8d), it follows that \( k/n \sim i/n + f(t^*)a \), where \( t^* \) is in \( [x_i, x_i + a] \). Thus, \( \frac{i}{n} + \frac{k}{n} \sim 2 \frac{i}{n} + f(t^*)a \). Because the density function in the modal region is higher than the density in the very low-income part of the distribution, both \( i/n \) and \( f(t^*) \) are smaller when a low-income household receives the increment than when a household in the modal region receives it.

The above considerations also imply that the Gini index will increase when

\[
\frac{i}{n} + \frac{k}{n} \sim 2 \frac{i}{n} + f(t^*)a > (1 + g).
\]

Thus, the index will decrease when the recipient is below the 50(1 + g - f(t^*)a)th percentile. This result is very similar to the situation when the increment does not change the rank of the recipient, that is, the Gini index decreases when the recipient can be in the (50 + 50g)th percentile. However, it shows that the fraction of households who can receive the increment and the Gini index decrease is slightly smaller when the increment increases the rank of the recipient than when its rank is unchanged.

Formula (19) also shows that when the highest income household receives the small increment the increase in the Gini index is largest, as minus the term in the large parenthesis in (19) becomes \( 1 - g \), which is \( > 0 \). Furthermore, this increase is larger in absolute value than the decrease in the Gini resulting when the median income household receives the increment.

This follows from comparing the changes in the numerator (14) of the Gini as the effect of the second term is \( a(n - 1) \) when the highest income household receives the increment, while it near zero when the median household receives it. While the first term in (14) is negative, it is \( \geq -2a(k - i) \) and \( n - 1 \) is much larger than \( 2(k - i) \) because only a small fraction of households have their ranks changed as a result of a small increment going to a household in the middle region.

The conclusion that additional small increments given to households in the lower (upper) part of the distribution decrease (increase) the Gini index is quite intuitive. The result that the effect on the Gini index of an additional increment given to a household in the middle sufficient to change the ordering is less than the effect of the same increment given to a household in some parts of the distribution, such as the extreme lower and upper, is less obvious. These changes in the Gini index arising from one household receiving additional income are not consistent with the Gini index being more sensitive to changes in the middle of the distribution than elsewhere.

For readers interested in a numerical example, consider the example of nine incomes and assume that one of them receives when

\[
\frac{i}{n} + \frac{k}{n} \sim 2 \frac{i}{n} + f(t^*)a > (1 + g).
\]

Table 2 reports the new values of the Gini index.
and the resulting change and mean difference. The number of households who had more income originally but the recipient now exceeds is the number passed. Recall that the Gini index, mean difference, and mean of the original nine are: 0.41677, 60.549, and $72,641.60, respectively. The ratios of the new Gini index to the original one are 0.8842, 1.0192, and 1.0495 if the first (lowest), median, or ninth (highest) income receives the addition. In percentage terms, the Gini index changes the most when the poorest member of the population is the recipient and the change when the median member receives it is less than the change when the richest member receives it. Thus, the Gini index is more sensitive to additional income going to a household in the two extreme regions of the distribution than it is to a similar change in the middle of the distribution.

6. Summary and Discussion

Although the Gini index yields one number summarizing the entire income distribution or Lorenz curve and cannot capture all the changes in the income distribution that economists or policy makers are interested in, the scenarios studied here indicate that the criticism that it gives undue weight to changes in the middle of the distribution is inaccurate. The recent book by Foster et al. (2013, p. 21) did note that although the Gini coefficient is considered to be most sensitive to changes involving incomes at the middle this "is not entirely accurate." It continues, however, with "The effect of a given-sized transfer on the Gini coefficient depends on the number of people between giver and receiver, not on their respective income levels. Because, empirically, there tend to be more observations bunched together in the middle of the distribution, the effect of a transfer near the middle tends to be larger."

In Section 3, where a transfer or increment preserves the order, for example, when it is small, the opposite is true, that is, transfers from the rich to the poor had a greater effect on the Gini index. When transfers or an increment did not preserve the order, discussed in Sections 4 and 5, in most situations the difference in the ranks of the donor and recipient, before or after the transfer or addition is the main contributor to a change in the Gini index. Transfers or an additional increment involving a middle-income household do change the ranks of a higher fraction of the population because of the density function being higher in that region, however, the ranks of these households decrease by just one. Only when the transfer is between two households in the middle of the distribution did the number of individuals whose rank changes because of the transfer have an important role. This effect on the change in the Gini index does not offset the relatively small difference between the after transfer ranks of the donor and recipient when both are in the central region. Thus, a small transfer or additional increment affecting the middle of the income distribution does not have undue weight on the resulting change in the Gini index. Indeed, in the scenarios examined here a transfer or addition to a middle-income household had a smaller impact on the Gini index than a transfer or addition to a low-income household.

In view of the larger magnitudes of the weights in the numerator of the Gini index in formula (2) assigns to the extreme ordered incomes than to the middle ones, in the context of small transfers or additions, these conclusions are not that surprising.

When the total income of the population and consequently the mean increase, because twice the mean is the denominator of the Gini index it does not fully reflect a shift in favour of the upper end (Gastwirth 2014). Indices such as the ratio of the share of income received by the top 20% to the lower 20%, used by Dorling (2014), the ratio of the share of the top 10% to the lower 40% introduced by Palma (2011) or the median based Gini index (Gastwirth 2014) increase more than the Gini index in response to such a change. Like the Gini index, however, these indices can have the same numerical value for data from two distributions even though the Lorenz curves intersect. A method for constructing two different distributions with same value of a measure that is the ratio of the top 100u% to the bottom 100b%, where $b < 1 - u$, is described in Appendix A.

From a statistical viewpoint, it is unreasonable to expect one summary measure to capture the features of an entire distribution. Thus, the relationship between the choice of measure and its underlying social welfare function, stressed by Atkinson (1970), Newbery (1970), Sheshinski (1972), and Sen (1974) remains very important. Jenkins (2009) noted that the ability to calculate several indices, which focus on changes in different income ranges, is very useful for the analysis of income and earnings distributions. By using different weights than those in the numerator of formula (2), following Mehran (1976) one can create a summary measuring placed increased weight on the part of the income distribution most relevant to the purpose of a study. Like the numerator of the Gini index, these measures are linear combinations of order statistics and there is a large literature deriving their large sample distributions (David and Nagaraja 2003; Greolin et al. 2009; Giorgi and Gigliarano 2016). Similarly, the Lorenz curve and related functions or transformations of it (Sordo et al. 2014; Arnold 2015; Gastwirth 2016) can be used to emphasize the region of the income distribution of primary concern.

Other measures of inequality, for example, the generalized entropy family (Cowell 2011) or Atkinson’s family may be superior to the Gini index for some analytic purposes because the formulas for decomposing them across subpopulations are analogous to the decompositions in classical analysis of variance. Lambert and Decoster (2005), however, showed that the Gini index is useful for some types of inequality decomposition analyses. The Gini index is a well-studied index with a long history (Giorgi 1990) and is associated with the area between the line of equality and the Lorenz curve (Gastwirth 1972); providing a graphical summary of the distribution that economists and policy makers have found useful. Furthermore, it may be difficult.

Table 2. The effect on the Gini index and mean difference from one of the nine households receives an additional $24,000.

| Recipient   | Number passed | Gini index   | Change in the Gini index | Mean difference |
|-------------|---------------|--------------|--------------------------|-----------------|
| First (lowest) | 1             | 0.3685       | −0.0483                  | 55502           |
| Second      | 4             | 0.3802       | −0.0366                  | 57258           |
| Third       | 3             | 0.4064       | −0.0103                  | 61216           |
| Fourth      | 2             | 0.4086       | −0.0081                  | 61549           |
| Fifth       | 1             | 0.4101       | −0.0066                  | 61712           |
| Sixth       | 0             | 0.4109       | −0.0059                  | 61882           |
| Seventh     | 0             | 0.4197       | 0.0029                   | 63216           |
| Eight       | 0             | 0.4286       | 0.0118                   | 64549           |
| Ninth (highest) | 0            | 0.4374       | 0.0207                   | 65883           |
for a government agency producing income statistics to choose between members of a family, which place greater emphasis on different parts of the distribution. While the U.S. Census Bureau reports three different versions of Atkinson’s index and the Theil in addition to the Gini (DeNavas-Walt and Proctor 2014), the Gini index and the income shares of the quintiles are the primary ones used in their discussion of changes in inequality.

Yitzhaki (1982) gave a necessary condition for first- and second-order stochastic dominance of two distributions in terms of the mean and mean difference. This connection between the Gini index and second-order stochastic dominance has an important role in many areas of economics and public policy, for example, in studies of investment decisions under uncertainty (Lutkebohmert 2009; Levy 2016), economic growth and recessions (Bishop et al. 1991) and measuring possible discrimination (Le Breton et al. 2012; Hoy and Huang 2017; Salas et al. 2017).

When Government agencies publish the Gini index along with the mean, median, and quintiles, if they could include the deciles and 95th and 99th percentiles of the distribution and the shares (or average income) of each decile and the top 5 and 1 percent, researchers would be able to accurately estimate most measures of inequality. Providing this information in grouped form would also preserve the privacy and confidentiality of the incomes of the survey respondents.

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Supplementary Materials

Supplemental data for this article can be accessed on the publisher’s website.

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