Heavy ion collisions and lattice QCD at finite baryon density

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Abstract. We discuss a relation between the QCD thermodynamics obtained from a statistical analysis of particle production in heavy ion collisions at SPS and RHIC energies and recent LGT results at finite chemical potential. We show that basic thermodynamic properties obtained from the phenomenological statistical operator of a hadron resonance gas that describes particle yields in heavy ion collisions are consistent with recent LGT results. We argue that for $T \leq T_c$ the equation of state derived from Monte–Carlo simulations of two quark–flavor QCD at finite chemical potential can be well described by a hadron resonance gas when using the same set of approximations as used in LGT calculations. We examine the influence of a finite quark mass on the position of the deconfinement transition in temperature and chemical potential plane.

1. Introduction

The detailed analysis of particle production in heavy ion collisions has shown that in a broad energy range from SIS through AGS, SPS up to RHIC particle yields resemble that of chemical equilibrium population [1]. At SPS and RHIC the freeze-out parameters, the temperature $T_f$ and the energy density $\epsilon_f$, predicted by the presently used thermal model of hadron resonance gas, agree well with recent results from the lattice on critical conditions required for deconfinement [1, 2]. The above quantitative agreement of freeze-out and critical parameters suggests that at SPS and RHIC chemical freeze-out appears in the near vicinity or at the phase boundary [3]. If this is indeed the case then the phenomenological statistical operator of hadron resonance gas $Z_{HG}$ should also provide a consistent with LGT description of QCD thermodynamics in the confined, hadronic phase [2]. Here we show that the basic qualitative properties of $Z_{HG}$ resulting from its dependence on $T$ and baryon chemical potential $\mu_B$ are also present in recent lattice results of two flavor QCD at finite chemical potential. In addition, when imposing the fixed energy density condition for deconfinement the hadron resonance gas partition function $Z_{HG}$ describes the quark mass dependence of the lattice critical temperature at $\mu_B = 0$ and the position of the phase boundary in the $(T, \mu_B)$–plane at small $\mu_B$. || redlich@rose.ift.uni.wroc.pl
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2. Resonances essential degrees of freedom near deconfinement

The phenomenological partition function used in the description of particles production in heavy ion collisions was, following Hagedorn, constructed as a non–interacting hadronic gas which is composed of all hadrons and resonances. In the Boltzmann approximation, suitable for the moderate values of $\mu_B < m_N$ and $T \geq 50$ MeV, there is a factorization of $T$ and $\mu_B/T$ dependence in relevant observables characterizing baryonic sector of the system. The basic quantity is the pressure

$$\Delta P = P(T, \mu_B) - P(T, \mu_B = 0)$$

from which $n_B$ and the baryon number susceptibility $\Delta \chi_B$ are obtained as the first and second order derivatives with respect to $\mu_B$, respectively.

The obvious consequences of the factorization in Eq.(1) is that any ratio of $n_B$, $\Delta P$ and $\Delta \chi_B$ at fixed $\mu_B/T$ should be independent of $T$. This is the property which can be directly checked with recent LGT results. In Fig.(1) we show as an example the ratio of $\Delta P/\Delta (\chi_B T^2)$ for two different values of $\mu_q/T$ as function of $T$. It is clear from Fig.(1) that the factorization predicted by $Z_{HG}$ is also seen in LGT results for $T < T_0$, that is in the confined phase of QCD.

The second transparent feature of $Z_{HG}$ is that the $\mu_B/T$ dependence appears through a cosh-function. The lattice results at finite $\mu_q$ were obtained using a Taylor expansion of the pressure with respect to the quark chemical potential. So far, the Monte Carlo results are available for the second $c_2$, fourth $c_4$ and sixth $c_6$ order coefficient in the Taylor series of $P$ with respect to $\mu_q/T$. If the lattice thermodynamics was consistent with the prediction of $Z_{HG}$ then one would expect $c_2/c_4 = 3/4$ and

$$c_6/c_4 = 3/4$$

* The quark chemical potential in Fig.(1), $\mu_q = \frac{1}{3} \mu_B$.
Figure 2. Left-hand figure: temperature dependence of baryonic pressure for different values of $\mu_q/T$. The lattice results are from [4]. The right-hand figure shows lattice results on phase boundary curve (line with errors) together with phenomenological freeze-out values of $T$ and $\mu_B$ (points) obtained from the analysis of particle production in heavy ion collisions [1]. Short-dashed and dashed-dotted lines are the statistical model results obtained under the condition of fixed $\epsilon \approx 0.6$ GeV/fm$^3$ with $m_\pi \approx 0.77$ GeV and $m_\pi \approx 0.14$ GeV respectively. Also shown (full-line) is the phenomenological freeze-out curve of fixed energy/particle$\approx 1$GeV from [7].

$c_6/c_4 = 0.3$ being the coefficients in the expansion of $\cosh(\mu_B/T)$ in Eq.(1). Fig.(1–right) shows the corresponding ratios obtained on the lattice and the results obtained from Eq.(1). Within statistical error the agreement is indeed seen to be justified.

The pressure calculated on the lattice (see Fig.(2–left) ) increases abruptly when approaching deconfinement transition from the hadronic side. If the phenomenological statistical operator $Z_{HG}$ is of physical significance then this increase could be due to resonance formation. To check the importance of resonances near deconfinement one would need to reproduce lattice results on the $T$–dependence of $P$ at fixed $\mu_q/T$. However, to quantify this dependence one needs to implement the same set of approximations in Eq.(1) as those being used on the lattice. First of all current lattice results in Fig.(2–left) are obtained with quite a large quark mass corresponding to $m_\pi \approx 770$MeV. This also distorts the baryon mass spectrum. Its pion mass dependence can be deduced from lattice calculations at zero temperature. We use the following ansatz for the parametrization of the dependence of baryon masses on the pion mass [2][5],

$$\frac{m^*(m_\pi)}{m} \approx 1 + A \frac{m_\pi}{m^2},$$

where $A = 0.9 \pm 0.1$, $m^*$ is the distorted hadron mass at fixed $m_\pi$ and $m$ is its corresponding physical value.

The lattice results were obtained in 2–flavor QCD, thus there is no contribution of strange baryons in Eq.(1). Due to the Taylor expansion of $P$ in the lattice calculations one also needs to perform a similar approximation in Eq.(1). In Fig.(2–left) the lattice results are compared with Eq.(1). The $T$ dependence of QCD thermodynamics obtained on the lattice is seen in Fig.(2–left) to be consistent with the predictions of $Z_{HG}$. 

\begin{align}
 m^*(m_\pi) &= 1 + A \frac{m_\pi}{m^2},
\end{align}
The value of $T_c$ was shown on the lattice to be dependent on the pion mass. The remarkable feature of this dependence is that $\epsilon$ at $T_c$ is almost constant, independent of $m_\pi$. This would suggest that deconfinement is density driven and can be obtained from the condition of fixed energy density. Fig.(2-right) shows recent lattice results on the position of the phase boundary line in 2–flavor QCD within the Taylor approximation and with the quark mass such that $m_\pi \approx 770\text{MeV}$. The condition of fixed energy density $\epsilon \approx 0.6\text{ GeV}/fm^3$ with $\epsilon$ obtained from Eq.(1) is seen to coincide with lattice results. Decreasing the pion mass to its physical value and including complete set of resonances expected in (2+1)–flavor QCD results in the shift of the position of the phase boundary line towards phenomenological freeze-out condition of fixed energy/particle$\approx 1\text{GeV}$. The splitting of freeze-out and phase boundary line appears when the ratio of meson/baryon multiplicities reaches the unity.

3. Conclusions

We have shown that the statistical operator of hadron resonance gas used to describe particle yields in heavy ion collisions provide satisfactory description of recent lattice results on QCD thermodynamics at finite chemical potential. In particular the basic property of this operator like e.g. factorization of temperature and chemical potential dependence is obviously confirmed by the lattice results. The ratios of the coefficients in the Taylor expansion of thermodynamic pressure are also well described by the expansion of cosh–function predicted by the hadron resonance gas. These results are independent from the particular choice of the quark mass in the lattice calculations and are also to large extent free from lattice artifacts. The phenomenological partition function was also shown to describe quantitatively the temperature and chemical potential dependence of basic thermodynamical observables below deconfinement. This indicates that hadron resonance gas partition function is a good approximation of QCD thermodynamics of the hadronic phase. That is why this partition function is also successful in heavy ion phenomenology.

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