Research Article

Analysis of Vertical-Horizontal Coupling Vibration Characteristics of Rolling Mill Rolls Based on Strip Dynamic Deformation Process

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Received 26 October 2013; Revised 21 March 2014; Accepted 21 March 2014; Published 6 April 2014

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Nonlinear dynamic rolling forces in the vertical and horizontal directions are, respectively, established, considering the impact of vertical and horizontal directions vibration of rolls. Then a vertical-horizontal coupling nonlinear vibration dynamic model of rolling mill rolls is proposed, based on the interactions between this dynamic rolling force and mill structure. The amplitude-frequency equations of the main resonance and inner resonance are carried out by using multiple-scale method. The characteristics of amplitude frequency under nonlinear stiffness, damping, and amplitude of the disturbance are obtained by adopting the actual parameters of 1780 rolling mills. Finally, the bifurcation behavior of the system is studied, and it is found that many dynamic behaviors such as period, period-3 motion, and chaos exist in rolling mill, and this behavior could be restrained effectively by choosing proper system parameters.

1. Introduction

The vibration of rolling mill often occurs in rolling process. The occurrence of vibration not only affects the quality of rolling products, but also leads to breakdown of the rolling equipment. In order to understand the vibration behaviors of mills, a number of models have been developed during the past few decades [1–5]. However, in most open literatures, the vibrations in the horizontal direction and in the vertical direction are studied separately. In recent years, the coupling relationship in rolling mill is proposed; Hu et al. studied the linear vibration characteristics in vertical and horizontal direction [6]. Yang et al. studied the stability of coupling dynamic vertical model of cold rolling mill, which consists of the rolling process model, the mill roll stand structure model, and the hydraulic servo system model [7]. In the process of studying rolling mill vibration, modeling of rolling force directly determines the accuracy of vibration model. In the early years, the rolling force in the rolling process is quasistatic, which assumes that only dynamic variations in roll spacing produce variations in force, strip speed, and strip thickness to those occurring under steady-state conditions [8–10]. Yun et al. proposed a dynamic model of rolling mill, which considers the rate variation of change of the roll spacing. But in order to simplify calculation, he only selected the linear section of rolling force by using Taylor formula [11]. In fact, most of the literatures adopted the method of Yun, by taking the rolling force as a linear factor and neglecting nonlinear section.

In this paper, nonlinear dynamic rolling forces in the vertical and horizontal directions are proposed, respectively. Then a vertical-horizontal coupling vibration dynamic model of rolling mill rolls is constructed based on the interactions between this dynamic nonlinear rolling force and mill structure. Then the amplitude-frequency characteristics of the main resonance and inner resonance are analyzed under the nonlinear stiffness, damping, and the amplitude of the disturbance stiffness. Finally, the conditions of different dynamical motions are obtained by analyzing bifurcation behavior of the system, which could provide theoretical base for understanding of vibration mechanism of mill.
2. Nonlinear Rolling Force Based on Dynamic Deformation Process of Strip

2.1. Parameters of Deformation Zone under Vibration Conditions. Considering the influence of vertical and horizontal vibration of roll, the dynamic deformation process of strip is shown in Figure 1.

As shown in Figure 1, the solid line represents the steady-state position of roll and the dashed line is the position of roll under vibration conditions.

Considering the elastic flattening of the rolls, the roll gap is treated as a parabolic curve [11]. Based on Von Karman’s study [10], considering the vertical displacement of rolls, the thickness $h_x$ of the rolling mill at any arbitrary position $x$ is

$$ h_x = h_0 + 2y_c + \frac{(x - x_c)^2}{R}. \quad (1) $$

Assuming that the thickness $h_1$ at entry position is constant, the entry coordinate of strip can be derived from (1) as

$$ x_1 = x_c + \sqrt{R(h_1 - h_0 - 2y_c)}. \quad (2) $$

Then the velocity at entry position along horizontal direction can be derived form (2) as

$$ \dot{x}_1 = \dot{x}_c - (h_1 - h_0 - 2y_c)^{-1/2} R^{1/2} \dot{y}. \quad (3) $$

Considering the influence of roll horizontal vibration, the equivalent velocity at the entry position will be composed of two parts: the entrance velocity $v_1$ of strip and entrance rate $\dot{x}_1$ at entry position, and it can be expressed as

$$ v'_1 = v_1 - \dot{x}_1. \quad (4) $$

From Figure 1, the exit thickness of strip under vibration conditions can be obtained as

$$ h_2 = h_0 + 2y_c. \quad (5) $$

Under vibration conditions, the bulk of metal in deformation zone is not constant, so the equation of constant mass throughput is no longer suitable for vibration conditions. Then a new principal of metal flow per second will be proposed, and the dynamic flow at any arbitrary position $x$ can be expressed as

$$ v_x h_x = v'_1 h_1 - \Delta V, \quad (6) $$

where $v_x h_x$ is equivalent exit flow at any arbitrary position $x$ and $v'_1 h_1$ is equivalent entry flow.

Where the volume in deformation zone range from $x_1$ to $x$ can be expressed as

$$ \Delta V = \frac{1}{2} (x_1 - x) (h_1 + h_x) - 2R^2 \arcsin \frac{l_x}{R} + \frac{1}{4} l_x \sqrt{4R^2 - l_x^2}, \quad (7) $$

where

$$ l_x = \sqrt{(x_1 - x)^2 + (h_1 - h_x)^2}/4. \quad (8) $$

the derivative of (7) can be obtained as

$$ \dot{\Delta V} = \frac{1}{2} \dot{x}_1 (h_1 + h_x) + \frac{1}{2} (x_1 - x) \dot{h}_x $$

$$ - \frac{2R^2 \dot{l}_x}{\sqrt{R^2 - l_x^2}} + \frac{1}{4} l_x \sqrt{4R^2 - l_x^2} - \frac{1}{4} l_x \frac{\dot{l}_x}{\sqrt{4R^2 - l_x^2}}, \quad (9) $$

where

$$ \dot{h}_x = 2\dot{y}_c - \frac{2x_c (x - x_c)}{R}, \quad (10) $$

$$ \dot{l}_x = \frac{4 (x_1 - x) \dot{x}_1 - h_x (h_1 - h_x)}{2 \sqrt{4(x_1 - x)^2 + (h_1 - h_x)^2}}. $$

![Figure 1: The dynamic deformation process of strip.](image-url)
Then the speed \( v_x \) at any arbitrary position can be expressed as

\[
v_x = \frac{v_R}{R} \sqrt{R^2 - (x-x_c)^2},
\]

(12)

at the moment, the position \( x \) is the position of neutral point \( x_n \) in (12). Due to \( R \gg (x-x_c) \) and \(|x-x_c| \ll 1 \) and neglecting high-order item \((x-x_c)^2\), the neutral point can be derived as

\[
x_n = x_1 - \frac{(h_1-h_2)}{8R^2-C^2} (4R^2-2)(C^2-1),
\]

(13)

where

\[
C = v_R h_2 - v_1 h_1 + \frac{1}{2}x_1 (h_1 + h_2) + x_1 y_c.
\]

(14)

2.2. Nonlinear Dynamic Rolling Force in Horizontal and Vertical Direction. According to slab method [10], take a slice of the strip in Figure 2.

Based on force balance theory in deformation zone by Von Karman [10], the force balance expression in horizontal direction can be expressed as

\[
dh_x (P + \tau_x) + h_x \frac{d\tau_x}{dx} + 2F_x = 0,
\]

(15)

where \( F_x = \mu \delta \); when \( x < x_n \), take the negative and when \( x > x_n \), take the positive.

By taking account of the assumption of homogeneous deformation [11], the distribution of stress can be written by von Mises yield criterion as follows:

\[
\frac{d\tau_x}{dx} = \frac{2\delta}{h_x} \left( \frac{\tau_x - 2}{R} \right).
\]

(16)

Integrating (16), the tension \( \tau_x \) can be express as

\[
\tau_x = \tau_b + \int_{x_1}^{x} \frac{2\delta}{h_x} \left( \frac{\tau_x - 2}{R} \right) dx.
\]

(17)

The unit rolling force by (17) can be expressed as

\[
P(x) = 2\delta - \left[ \tau_b + \int_{x_1}^{x} \frac{2\delta}{h_x} \left( \frac{\tau_x - 2}{R} \right) dx \right].
\]

(18)

The rolling force in horizontal and vertical direction can be obtained as follows:

\[
F_x = -\int_{x_1}^{x} P(x) \tan \theta dx + \int_{x_2}^{x_1} \tau_x \mu \delta dx,
\]

(19)

\[
F_y = \int_{x_2}^{x_1} P(x) dx + \int_{x_1}^{x_2} \tau_x \mu \delta \tan \theta dx,
\]

where

\[
\tan \theta = \frac{x - x_c}{\sqrt{R^2 - (x - x_c)^2}}.
\]

(20)

In (19), integrating zone is composed of two sections, namely, \( x_2 \sim x_n \) and \( x_n \sim x_1 \). Integrating (19), we obtained

\[
F_x = -\mu \delta h_2 \left[ \frac{R}{h_0 + 2y_c} \left( 2\tan^{-1} \left( \frac{x_n - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \right) - \tan^{-1} \left( \frac{x_1 - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \right] + \frac{\tau_b}{2} (h_1-h_2) - \delta h_2 \ln \frac{h_1}{h_2},
\]

\[
F_y = \left[ 2\delta \ln \left( \frac{h_1}{h_2} \right) - 2\delta - \tau_b \right] (x_1 - x_c) + 4\delta \sqrt{Rh_0 + 2Ry_c} \tan^{-1} \left( \frac{x_1 - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) + 2\mu \delta \sqrt{Rh_0 + 2Ry_c} (x_1 - x_c) \times \left[ 2\tan^{-1} \left( \frac{x_n - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) - \tan^{-1} \left( \frac{x_1 - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \right] + \mu \delta R \ln \left( \frac{h_1 h_2}{h_n^2} \right).
\]

(21)

3. The Vertical-Horizontal Coupling Dynamic Equation of Mill Rolls

Set \( x_0 \) and \( y_0 \) which are the balance points of rolls in the horizontal and vertical direction, respectively. Under steady conditions, \( x_0 = 0 \) and \( y_0 = 0 \); by using Taylor formula, (21) can be expressed as

\[
F_x(x_c, \dot{x}_c, y_c, \dot{y}_c) = F_x(x_{0}, y_{0}, 0, 0, 0) + \Delta F_x(x_c, \dot{x}_c, y_c, \dot{y}_c),
\]

\[
F_y(x_c, \dot{x}_c, y_c, \dot{y}_c) = F_y(x_{0}, y_{0}, 0, 0, 0) + \Delta F_y(x_c, \dot{x}_c, y_c, \dot{y}_c),
\]

(22)

where \( F(x_{0}, y_{0}, 0, 0, 0) \) is rolling force when there is no vibration and \( \Delta F(x_c, \dot{x}_c, y_c, \dot{y}_c) \) is dynamic parts of rolling force, because too many parameters, for simplify the calculation, take the parts of first and third order as follows:

\[
\Delta F_x(x_c, \dot{x}_c, y_c, \dot{y}_c) = a_1 x_c + a_2 \dot{x}_c + a_3 y_c + a_4 \dot{y}_c + a_5 x_c^3 + a_6 y_c^3,
\]

\[
\Delta F_y(x_c, \dot{x}_c, y_c, \dot{y}_c) = b_1 x_c + b_2 \dot{x}_c + b_3 y_c + b_4 \dot{y}_c + b_5 x_c^3 + b_6 y_c^3.
\]

(23)
where

\[ a_1 = \frac{\partial}{\partial x_c} F_x (x_{c_0}, 0, y_{c_0}, 0), \]
\[ b_1 = \frac{\partial}{\partial x_c} F_y (x_{c_0}, 0, y_{c_0}, 0), \]
\[ a_2 = \frac{\partial}{\partial x_c} F_x (x_{c_0}, 0, y_{c_0}, 0), \]
\[ b_2 = \frac{\partial}{\partial x_c} F_y (x_{c_0}, 0, y_{c_0}, 0), \]
\[ a_3 = \frac{\partial}{\partial y_c} F_x (x_{c_0}, 0, y_{c_0}, 0), \]
\[ b_3 = \frac{\partial}{\partial y_c} F_y (x_{c_0}, 0, y_{c_0}, 0), \]
\[ a_4 = \frac{\partial^3}{\partial x_c^3} F_x (x_{c_0}, 0, y_{c_0}, 0), \]
\[ b_4 = \frac{\partial^3}{\partial y_c^3} F_y (x_{c_0}, 0, y_{c_0}, 0), \]
\[ a_5 = \frac{1}{6} \frac{\partial^3}{\partial x_c^3} F_x (x_{c_0}, 0, y_{c_0}, 0), \]
\[ b_5 = \frac{1}{6} \frac{\partial^3}{\partial y_c^3} F_y (x_{c_0}, 0, y_{c_0}, 0), \]
\[ a_6 = \frac{1}{6} \frac{\partial^3}{\partial y_c^3} F_x (x_{c_0}, 0, y_{c_0}, 0), \]
\[ b_6 = \frac{1}{6} \frac{\partial^3}{\partial y_c^3} F_y (x_{c_0}, 0, y_{c_0}, 0). \]

Based on the assumption that the mass of working rolls are much smaller than that of backup rolls, the mass of the working rolls may be neglected [6]. The vertical-horizontal coupling nonlinear vibration model of rolling mill with nonlinear dynamic rolling force is illustrated in Figure 3.

The dynamic equation in Figure 3 can be written as

\[ m_1 \ddot{x}_c + c_1 \dot{x}_c + k_1 (x_c - x_{c_0}) + F_x (x_c, \dot{x}_c, y_c, \dot{y}_c) = 0, \]
\[ m_1 \ddot{y}_c + c_2 \dot{y}_c + k_2 (y_c - y_{c_0}) + F_y (x_c, \dot{x}_c, y_c, \dot{y}_c) = T_1, \]
\[ m_2 \ddot{x}_c + c_3 \dot{x}_c + k_4 (x_c - x_{c_0}) + F_x (x_c, \dot{x}_c, y_c, \dot{y}_c) = 0, \]
\[ -m_2 \ddot{y}_c - c_4 \dot{y}_c - k_4 (y_c - y_{c_0}) + F_y (x_c, \dot{x}_c, y_c, \dot{y}_c) = T_2. \]  

(25)

Assuming that the structure of rolling mill and vibration are symmetrical in relation to the rolled strip [6], then there exist \( k_1 = k_3, k_2 = k_4, c_1 = c_3, c_2 = c_4, m_1 = m_2, \) and \( T_1 = -T_2, \) and (25) can be simplified as

\[ m_1 \ddot{x}_c + c_1 \dot{x}_c + k_1 (x_c - x_{c_0}) + F_x (x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) = 0, \]
\[ m_1 \ddot{y}_c + c_2 \dot{y}_c + k_2 (y_c - y_{c_0}) + F_y (x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) = T_1. \]  

(26)

Under steady conditions, the external disturbance force \( T_1 = 0, \) there exist \( \ddot{x}_c = \ddot{y}_c = 0, \dot{x}_c = \dot{y}_c = 0, \) and \( x_c = y_c = 0, \) and the balance equation can be obtained as follows:

\[ k_1 x_{c_0} + F_x (x_{c_0}, 0, 2y_{c_0}, 0) = 0, \]
\[ k_2 y_{c_0} + F_y (x_{c_0}, 0, 2y_{c_0}, 0) = 0. \]  

(27)

Substituting (27) into (26), (26) can be expressed as

\[ m_1 \ddot{x}_c + c_1 \dot{x}_c + k_1 x_c + \Delta F_x (x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) = 0, \]
\[ m_1 \ddot{y}_c + c_2 \dot{y}_c + k_2 y_c + \Delta F_y (x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) = T_1. \]  

(28)
4. The Resonance Characteristics of Equation

Assuming that the external disturbance $T = \varepsilon F \cos \omega t$ and the system is a weak nonlinear system, (30) can expressed as

$$\ddot{x}_c + \omega_1^2 x_c + \alpha_1 \dot{x}_c + \beta_1 y_c + \gamma_1 \dot{y}_c = -\varepsilon (\eta_1 x_c^3 + \zeta_1 y_c^3),$$

$$\ddot{y}_c + \omega_2^2 y_c + \alpha_2 \dot{y}_c + \beta_2 x_c + \gamma_2 \dot{x}_c = -\varepsilon (\eta_2 x_c^3 + \zeta_2 y_c^3 + F \cos \omega t).$$  

By using multiple scales method, one has

$$T_n = \varepsilon^n t, \quad n = 0, 1, \ldots,$$

$$\frac{d}{dt} = D_0 + \varepsilon D_1,$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_1 D_0 + \varepsilon^2 (D_1^2 + 2D_0 D_1) + \cdots,$$

where $D_n$ is defined as $\partial / \partial T_n$ and $n = 0, 1$. Set (31) which has solution as follows:

$$x_c = x_0 (T_0, T_1) + \varepsilon x_1 (T_0, T_1) + \cdots,$$

$$y_c = y_0 (T_0, T_1) + \varepsilon y_1 (T_0, T_1) + \cdots.$$  

Substituting (32) and (33) into (31) and separating terms of each order of $\varepsilon$, one has

$$D_0^2 x_0 + \omega_1^2 x_0 = 0,$$

$$D_0^2 y_0 + \omega_2^2 y_0 = 0,$$

$$D_0^2 x_1 + \omega_1^2 x_1 = -2D_0 D_1 x_0 - \alpha_1 D_0 x_0$$

$$- \beta_1 y_0 - \gamma_1 D_0 y_0 - \eta_1 x_0^3 - \zeta_1 y_0^3,$$

$$D_0^2 y_1 + \omega_2^2 y_1 = -2D_0 D_1 y_0 - \alpha_2 D_0 y_0$$

$$- \beta_2 x_0 - \gamma_2 D_0 x_0 - \eta_2 x_0^3 - \zeta_2 y_0^3 - F \cos \omega t.$$  

Set the solution of (34) as

$$x_0 = A (T_1) e^{i\omega_1 T_0} + \text{cc},$$

$$y_0 = B (T_1) e^{i\omega_2 T_0} + \text{cc},$$

where cc represents complex conjugate of former term and $A(T_1)$ and $B(T_1)$ are undetermined complex function. Substituting (36) into (35), the following equation can be expressed as

$$D_0^2 x_1 + \omega_1^2 x_1 = \left( -2i\omega_1 D_1 A - i\omega_1 \alpha_1 A - 3\eta_1 A^2 \bar{A} \right) e^{i\omega_1 T_0}$$

$$- \left( 3\zeta_1 B^3 + \beta_1 B + i\omega_2 Y_1 B \right) e^{i\omega_2 T_0}$$

$$- \eta_1 A^3 \bar{A} e^{3i\omega_1 T_0} - \zeta_1 B^3 e^{3i\omega_2 T_0} + \text{cc},$$

$$D_0^2 y_1 + \omega_2^2 y_1 = \left( -2i\omega_2 D_1 B - i\omega_2 \alpha_2 B - 3\zeta_2 B^2 \bar{B} \right) e^{i\omega_2 T_0}$$

$$- \left( \beta_2 A + i\omega_1 Y_2 A + 3\eta_2 A^2 \bar{A} \right) e^{i\omega_1 T_0}$$

$$- \zeta_2 B^3 e^{3i\omega_2 T_0} - \eta_2 A^3 e^{3i\omega_1 T_0} + \frac{Fe^{i\omega T_0}}{2} + \text{cc}.$$  

Equation (28) can be rewritten as

$$\ddot{x}_c + \omega_1^2 x_c + \alpha_1 \dot{x}_c + \beta_1 y_c + \gamma_1 \dot{y}_c + \eta_1 x_c^3 + \zeta_1 y_c^3 = 0,$$

$$\ddot{y}_c + \omega_2^2 y_c + \alpha_2 \dot{y}_c + \beta_2 x_c + \gamma_2 \dot{x}_c + \eta_2 x_c^3 + \zeta_2 y_c^3 = T.$$  

Equation (28) is vertical-horizontal coupling dynamic equation of mill rolls under vibration conditions, where $\beta, \gamma, \zeta$ are coupling coefficients.
4.1. The Analysis of Main Resonance. In the case of main resonance, set \( \omega = \omega_1 + \varepsilon \sigma \) and eliminating secular term of (37), one can obtain

\[
-2i\omega_1 D_1 A - i\omega_1 \alpha_1 A - 3\eta_1 A^2 \ddot{A} = 0,
\]

\[
-2i\omega_2 D_1 B - i\omega_2 \alpha_2 B - 3\zeta_2 B^2 \ddot{B} + \frac{F e^{i\sigma T_1}}{2} = 0.
\]

(38)

The polar coordinate form of \( A \) and \( B \) in (36) is introduced as follows:

\[
A = \frac{1}{2} a(T_1) e^{i\eta_1(T_1)}, \quad B = \frac{1}{2} b(T_1) e^{i\eta_2(T_1)}.
\]

(39)

By substituting (39) into (38) and separating the real and imaginary parts, the average equation of coupling system can be obtained as follows:

\[
\dot{a} = -\frac{1}{2} \alpha_1 a, \quad a\phi_1 = (\sigma - \sigma_1) a + \frac{3}{8\omega_1} \eta_1 a^3,
\]

\[
b = -\frac{1}{2} \alpha_2 b + \frac{F}{2\omega_2} \sin \theta, \quad b\phi_2 = \sigma b + \frac{3}{8\omega_2} \zeta_2 b^3 - \frac{F}{2\omega_2} \cos \theta,
\]

(40)

where \( \theta = \sigma T_1 - \phi_2 \). In the steady state, existing \( \dot{a} = \dot{b} = 0 \) and \( \dot{\theta}_2 = 0 \) and eliminating \( \dot{\theta}_2 \) from (40), then the amplitude frequency equation of the system can be written as

\[
\frac{9}{16} \zeta_2^2 b^6 + 3\omega_2 \sigma \zeta_2 b^4 + \omega_2^2 (\alpha_2^2 + 4\sigma^2) b^2 - F^2 = 0.
\]

(41)

4.2. The Analysis of Inner Resonance. Assuming that \( \omega_1 = \omega_2 + \varepsilon \sigma_1 \) and \( \omega = \omega_2 + \varepsilon \sigma \), in order to solve the secular term of (37), \( A \) and \( B \) must meet conditions as follows:

\[
-2i\omega_1 D_1 A - i\omega_1 \alpha_1 A - 3\eta_1 A^2 \ddot{A} = 0,
\]

\[
-\left(3\zeta_1 B^2 \ddot{B} + \beta_1 B + i\omega_2 \gamma_1 B\right) e^{-i\sigma T_1} = 0,
\]

\[
-2i\omega_2 D_1 B - i\omega_2 \alpha_2 B - 3\zeta_2 B^2 \ddot{B} = 0.
\]

(42)

Substituting (39) into (42), the average equation can be obtained under the polar coordinate; that is,

\[
\dot{a} = -\frac{1}{2} \alpha_1 a - \frac{2}{\omega_1} \left[\left(\frac{3}{4} \zeta_1 b^2 + \beta_1\right) \sin \theta_1 + \omega_2 \gamma_1 \cos \theta_1\right],
\]

\[
a\phi_1 = (\sigma - \sigma_1) a + \frac{3}{8\omega_1} \eta_1 a^3
\]

\[
+ \frac{b}{2\omega_1} \left(\frac{3}{4} \zeta_1 b^2 + \beta_1\right) \cos \theta_1 - \omega_2 \gamma_1 \sin \theta_1,
\]

\[
b = -\frac{1}{2} \alpha_2 b + \frac{F}{2\omega_2} \sin \theta_2
\]

\[
+ \frac{a}{2\omega_2} \left[\left(\frac{3}{4} \eta_2 a^2 + \beta_2\right) \sin \theta_1 - \omega_1 \gamma_2 \cos \theta_1\right],
\]

\[
b\phi_2 = \sigma b + \frac{3}{8\omega_2} \zeta_2 b^3 - \frac{F}{2\omega_2} \cos \theta_2
\]

\[
+ \frac{a}{2\omega_2} \left[\left(\frac{3}{4} \eta_2 a^2 + \beta_2\right) \cos \theta_1 + \omega_1 \gamma_2 \sin \theta_1\right],
\]

(43)

where \( \theta_1 = \phi_2 - \phi_1 - \sigma_1 T_1 \) and \( \theta_2 = \sigma T_1 - \phi_2 \).

When the system has a periodic motion, (43) will exist, \( \dot{a} = \dot{b} = \dot{\theta}_1 = \dot{\theta}_2 = 0 \); eliminate \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \) from (43) and then the frequency response equation of the coupling system can be obtained as

\[
\omega_1^2 \alpha_1^2 a^2 + 4\omega_1^3 G^2 = b^2 M,
\]

(44)

\[
\frac{3}{4} \eta_2 a^2 + \beta_2 a
\]

\[
+ \left(2\omega_2 \sigma b + \frac{3}{4} \zeta_2 b^3\right)^2 + \omega_1 \gamma_2 a^2 + \omega_1 \gamma_2 a^2 b + \omega_1 \gamma_2 \left(2\omega_2 \sigma + \frac{3}{4} \zeta_2 b^2\right)
\]

\[
+ \left(2\omega_4 a Z - G N\right) - \omega_1 \omega_2 \alpha_1 a^2 M = \left(2\omega_4 a N + G Z\right) F^2
\]

where

\[
M = -\omega_1^2 \gamma_1^2 + \left(\frac{3}{4} \zeta_1 b^2 + \beta_1\right)^2,
\]

\[
N = \frac{3}{4} \zeta_1 b^2 + \beta_1 \left[\frac{3}{4} \eta_2 a^2 + \beta_2\right] - \omega_1 \omega_2 \gamma_1 \gamma_2,
\]

\[
Z = -\omega_1 \gamma_2 \left[\frac{3}{4} \eta_2 a^2 + \beta_2\right] - \omega_1 \gamma_2 \left(\frac{3}{4} \zeta_1 b^2 + \beta_1\right),
\]

(46)

G = (\sigma - \sigma_1) a + \frac{3}{8\omega_1} \eta_1 a^3.

5. Numerical Experiments

Taking the 1780 rolling mills of Chengde Steel Co as an example, the parameters of this mill are listed as follows:

\( h_1 = 0.0141 \) m, \( h_0 = 0.0082 \) m, \( m = 14.4 \) t, \( \nu = 2.5 \) m/s, \( \mu = 0.26, \nu_0 = 3.8 \) Mpa, \( \nu_f = 5.5 \) Mpa, \( R = 0.42 \) m, \( \delta = 30 \) Mpa, \( \zeta_1 = 2 \times 10^3 \) N/s/m, \( k_1 = 2.01 \times 10^{11} \) N/m, \( k_2 = 2.08 \times 10^{11} \) N/m, and \( \zeta_2 = 8.65 \times 10^3 \) N/s/m.

Parameters of rolling force can be listed as follows: \( \varepsilon = 1.0, \eta_1 = 5.8 \times 10^6 \) N/m, \( \alpha_1 = 1.46 \times 10^8 \) N/s/m, \( \alpha_2 = 213.6 \) N/m, \( \alpha_3 = -1131 \) N/s/m, \( \varepsilon a_5 = 4 \times 10^6 \) N/m, \( \varepsilon a_5 = -7.47 \times 10^{11} \) N/m, \( b_1 = 6.978 \times 10^7 \) N/m, \( b_2 = -2593 \) N/s/m, \( b_3 = -2.16 \times 10^9 \) N/m, \( b_1 = 1.4 \times 10^14 \) N/s/m, \( b_2 = 2.65 \times 10^9 \) N/m, and \( b_3 = -3.7 \times 10^{15} \) N/m.

Figures 4–6 show the curve of main resonance amplitude frequency of rolling mill under different parameters.

In Figure 4, the main resonance amplitude-frequency response curves for several values of nonlinear stiffness \( b_2 \) are illustrated by (40). It can be seen that the nonlinear stiffness
\[ b_3 \text{ directly affects the vertical natural frequency of the rolling mill. The natural frequency decreases with the decrease of} b_3, \text{ and the main resonance amplitude becomes larger and the backbone of rolling mill curve deviates toward right, simultaneously.} \]

In Figure 5, the main resonance amplitude-frequency response curves for different values of parameter \( c_2 \) are illustrated by (44). With the increasing of the parameter \( c_2 \), the resonance amplitude of vibration decreases.

Figure 6 shows the curve of the main resonance amplitude frequency in coupling rolling mill system for different nonlinear parameter \( b_6 \). It can be seen that with the decrease of \( b_6 \), the frequency response curve deviates toward right and the jump phenomenon becomes more obvious. When the disturbance frequency \( \sigma \) changes negative to positive, the amplitude changes according to 1-2-3-5 route and jumps in 3-5; when the disturbance frequency \( \sigma \) is changed from positive to negative, the amplitude changes according to 5-4-2 route and jumps in 4-2, which will lead to the jump phenomenon.

In Figure 7, it can be seen that when disturbance amplitude \( \varepsilon F \) is small value, the frequency response curve is single value of system. With the increase of \( \varepsilon F \), the main resonance amplitude increases gradually, and the resonance point of coupling system shifts right, and a clear jump phenomenon arises.

Figures 8 and 9 show the curves of inner resonance amplitude-frequency under different parameters in horizontal and vertical directions, respectively. It can be seen that the system will raise resonance when disturb frequency is near to or equal to natural frequency \( \omega_1 \) or \( \omega_2 \), and it has two resonance zones in coupling system.

In Figure 8, when \( \varepsilon F = 0.3 \) MN, the curve in horizontal direction has two resonance points, and the result is unique and stable. With the increase of disturb amplitude \( \varepsilon F \), when \( \varepsilon F = 0.5 \) MN, the right resonance zone becomes bend toward right, and the jump phenomenon arises in Figure 8(b), and then the system is instable.

In Figure 9, the change of tendency of curve in vertical direction is similar to the curve in horizontal direction.
Figure 8: Curve of inner resonance amplitude frequency in horizontal direction with disturb amplitude $\epsilon F$.

Figure 9: Curve of inner resonance amplitude frequency in vertical direction with disturb amplitude $\epsilon F$.

Figure 10: Bifurcation characteristics of coupling system with disturb amplitude $\epsilon F$ when $\sigma = 50$ Hz.
According to (31), Figure 10 shows the bifurcation diagram with the change of disturb parameter \( \varepsilon F \) in the condition of \( \sigma = 50 \) Hz. It can be seen that the rolling mill may have different motions when it adopts different disturb parameter \( \varepsilon F \). When \( \varepsilon F \) adopts value from range 0.47 to 0.62, the system becomes periodic motion, and then it becomes chaos motion. When \( \varepsilon F \) adopts value from range 0.9 to 0.92, the system becomes period-2 motion. When \( \varepsilon F \) adopts value from range 1.07 to 1.24, the system becomes period-3 motion.

The phase diagrams and Poincare maps are shown in Figures 11–13 when the system adopts different values of \( \varepsilon F \) in Figure 10.

Figure 11 is periodic motion when \( \varepsilon F = 0.529 \) MN, and it can be seen that the phase diagram has one closed curve in Figure 11(a) and the Poincare maps have one single point in Figure 11(b). Figure 12 shows a period-3 motion when \( \varepsilon F = 1.06 \) MN, and it has three single points in Poincare map. Figure 13 illustrates chaos motion when \( \varepsilon F = 0.45 \) MN.

6. Conclusions

(1) The nonlinear rolling force model of rolling mill in the vertical and horizontal directions is built. On this basis, the dynamic model of nonlinear vertical-horizontal coupling vibration model of rolling mill is proposed, considering the influence of mill structure.

(2) By means of multiple-scale method, the amplitude-frequency equations of main resonance and inner resonance of coupling system of rolling mill rolls are carried out. The simulation adopting the actual
parameters of rolling mill is analyzed. It is found that the amplitude of vibration increases with an increase of stiffness and external disturb; but the maximum value of the main resonance will decrease as the increase of structure damp; when changing nonlinear stiffness, jump phenomenon will arise both in main resonance and in inner resonance, so choosing proper parameter will restrain resonance vibration of rolling mill.

(3) The bifurcation characteristics of vertical-horizontal coupling system of rolling mill roll are studied, and it is found that the system has different motions such as period motion, period-3 motion, and chaos, and choosing proper parameters may change the motion state of rolling mill.

Nomenclature

\( x \): Arbitrary distance from the centerline of the rolls
\( x_1 \): Distance of the exit plane from the centerline of the rolls
\( x_2 \): Distance of the entry plane from the centerline of the rolls
\( x_n \): Distance of the neutral plane from the centerline of the rolls
\( x_c \): Variation of the horizontal displacement of rolls
\( \dot{x}_1 \): Rate of change of the horizontal position of roll bite
\( \dot{x}_c \): Rate of change of roll horizontal displacement
\( y \): Arbitrary distance from the asymmetry line of the rolls
\( y_c \): Roll vertical displacement

\( v_0 \): Strip velocity at exit
\( v_1 \): Strip velocity at entry
\( v_\kappa \): Roll velocity
\( v_x \): Strip horizontal velocity at any arbitrary position from the centerline of the rolls
\( \dot{v}_1 \): The equivalent horizontal velocity at entry
\( h_0 \): Variation of the strip thickness at exit
\( h_1 \): Strip thickness at entry
\( h_x \): Strip thickness at any arbitrary distance from the centerline of the rolls
\( \tau_\phi \): Forward tensile stress at exit
\( \tau_\phi \): Backward tensile stress at entry
\( \tau_x \): Horizontal tensile stress at any arbitrary distance from the centerline of the rolls
\( F_P \): Shear stress
\( P \): Interface pressure
\( \mu \): Friction factor
\( \delta \): Shear yield strength
\( F_x \): The rolling force in horizontal direction
\( F_y \): Rolling force in vertical direction
\( R \): Roll radius
\( \Delta V \): Volume flow in deformation zone range from \( x_1 \) to \( x \)
\( \Delta \dot{V} \): The rate of volume flow change in deformation zone range from \( x_1 \) to \( x \)
\( k_1 \): Equivalent stiffness between upper rolls and upper supporting posts
\( k_2 \): Equivalent stiffness between upper rolls and upper beam
\( k_3 \): Equivalent stiffness between lower rolls and lower supporting posts
\( k_4 \): Equivalent stiffness between lower rolls and lower supporting posts
\( c_1 \): Equivalent damping between upper rolls and upper supporting posts
\( c_2 \): Equivalent damping between upper rolls and upper beam

\( (a) \) Phase diagram
\( (b) \) Poincare map

Figure 13: Chaotic motion when \( \sigma = 5 \) Hz and \( eF = 0.45 \) MN.
\( c_3 \): Equivalent damping between lower rolls and lower supporting posts

\( c_4 \): Equivalent damping between lower rolls and lower supporting posts

\( m_1 \): Equivalent mass of upper rolls

\( m_2 \): Equivalent mass of lower rolls

\( T_1 \): External disturbance of upper rolls

\( T_2 \): External disturbance of lower rolls.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**Acknowledgments**

This research is supported by National Natural Science Foundation of China (Grant no. 51105324), Natural Science Foundation of Hebei Province of China (Grant no. E2014501006), and Hebei Province Science and Technology Support Program (Grant no. 13211907D).

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