Optimization research on arc height of running surface based on reducing partial wear of straddle-type monorail vehicle running wheel tire

XiaoXia Wen¹, XiXue Du¹ and Liang Chen²

Abstract
This article proposes an ideal of reducing the partial wear of the running wheels by optimizing the arc height of the running surface to improve the wheel-rail contact state. To realize this idea, two kinds of concave and convex running surfaces were designed, the “running wheel-rail beam” finite element model of three kinds of rail surfaces of concave, convex, and plane were established. Taking the arc height of the running surface as the design variable, the total friction work and the friction work deviation (FWD) value as the dual optimization goal, an optimization model of arc height of running surface was established based on finite element model and multidisciplinary optimization platform ModeFrontier. An improved genetic algorithm was used and an co-simulation optimization mode was put forward in the optimization. The optimization results show that when the concave height of the inner running surface is 22.62 mm, the total friction work and the FWD values are reduced by 11% and 11.8% respectively; When the convex height of the outer running surface is 11.81 mm, the objection values are reduced by 4.9% and 32.1% respectively. An ideal running surface was obtained and the life of the running wheel was extended by the research.

Keywords
Straddle-type monorail vehicle, partial wear, arc height of running surface, element model, linkage optimization

Introduction
Straddle monorail is a new urban rail transit system with unique technology and tire-rail coupling drive, which is different from the subway with steel wheel and rail. It has broad application prospects in the public transportation branch lines of major cities and the public transportation trunk lines of medium-sized cities in China. However, straddle-type monorail vehicles have some problems during operation, and a series of studies have been carried out on these problems at home and abroad.

Background
During the operation of the straddle monorail line in Chongqing, China, it was found that the running wheels were severely partial worn and the tires were replaced frequently. Partial wear of the running wheel tires not only leads to the early scrapping of the running wheel, but also brings high maintenance costs. Severe tire wear will also cause a significant decrease in tire adhesion, bringing about some problems such as...
low driving efficiency and long braking distance, which will seriously affect the safety of the vehicle and the energy saving of system operation.

There are many factors that affect the partial wear of pneumatic rubber tires. Domestic and foreign scholars mainly conduct research from tire parameters, tire operating parameters, and road conditions. The straddle-type monorail vehicle (SMV) has a special structure. The running wheels and the rail beam running surface, the two stable wheels and the rail beam side surface, and the four guide wheels and the rail beam side surface are in contact on three different rail surfaces. Pneumatic rubber tires are used for running wheels, guide wheels and stable wheels, as shown in Figure 1(a). Since the running wheels of SMV are supported on the same drive shaft, and there is no differential mechanism between the inner and outer running wheels. There are no wheel alignment parameters such as camber and toe angle for running wheel tires of SMV, the connection between the bogie and the car body is similar to the non-independent suspension system, and the grounding and adaptability of the running wheels and the rail surface both are poor. Therefore, when a SMV passes through a curve beam, additional lateral force will inevitably be generated on the running wheels, causing the running wheel tires have a large side slip, roll angle, and uneven tire pressure distribution, which are the main factors that cause partial wear of the running wheel tires. However, at present, there is little research on the partial wear of running wheel tires at home and abroad.

Since the arc height change of the running surface changes the wheel-rail contact form, and creates a virtual camber to balance the lateral force, authors of the article proposes an ideal of reducing the partial wear of the running wheels by optimizing the arc height of the running surface. In order to realize this idea, two types of concave and convex running surfaces are designed, an optimization model of arc height of running surface was established based on finite element model and multidisciplinary optimization platform Modefrontier. An improved Genetic algorithm was used in the optimization. Through the optimization research, “an ideal running surface” of rail beam is obtained, the contact form between the running wheel and the rail beam was improved when the SMV passes through the curve beam, so as to reduce the partial wear of the running wheel.

The current running surface of the rail beam is a plane (as shown in Figure 1(b)) and the rail beam is a special line for SMV. The arc height of the running surface can be designed and poured according to actual needs. It is feasible to change the arc height of the running surface when building the track beam.

**Research status at home and abroad**

At present, the optimization research on the running surface of monorail rail beam at home and abroad is in a blank. The author of the article is the first to carry out the optimization study on the running surface of monorail rail beam. Those research results of rail profile optimization in the field of railways and wheel-rail coupling dynamics in the field of monorail provide a reference for this study.

In the field of railway vehicles with steel wheels and steel rails, the optimization of rail profile is an effective way to reduce wheel-rail wear. Scholars and research units from various countries have conducted in-depth research on rail profile optimization. Smallwood et al. found that the contact stress between wheels and rails has a greater impact on rail rolling contact fatigue and the initiation and development of cracks. Based on the
change law of contact stress between wheels and rails, a rail profile optimization method with the goal of reducing contact stress was proposed. Magel and Kalousek proposed the evaluation method of optimized rail profile through six aspects of abrasion resistance, fatigue resistance, wave-shaped wear resistance, stability, and noise reduction. Through research and comparison, it was found that the rail service life was effectively extended after the rail was polished. Magel et al. proposed an optimized curved section rail grinding profile based on the difference in rolling circle radius. Persson et al. used the genetic algorithm to obtain the optimization results in the optimization of rail profile. Choi et al. proposed an asymmetric rail profile optimization design method that takes the rail wear index as the objective function and the wheel-rail dynamic performance as the constraint condition, and solves the problem of transitional wear in the curve section in urban rail transit.

In the field of straddle monorails, scholars mainly focus on the research on wheel-rail coupling dynamics. Japanese scholar Lee et al. considered monorail trains and steel as the research objects, analyze coupling dynamic interaction of “vehicle-rail beam.” Japanese scholar Kenjiro et al. used a multi-body dynamics method to establish a SMV dynamics model and simulated the curve passing performance of monorail train. Wen et al., the author of this paper considered the nonlinear characteristics of rubber tires and established a spatial coupling dynamic model of a SMV, which include a three-directional wheel-rail contact pair between running wheel and top surface of rail beam, horizontal wheels, and the side surface of the rail beam.

**The aim of the research and Contributions**

The main purpose of this article is to improve the wheel-rail contact form by optimizing the arc height of the running surface of the rail beam, and to reduce the partial wear of the running wheels. In order to achieve this goal, two kinds of concave and convex running surfaces were designed, an optimization model of arc height of running surface was established based on the finite element model and multidisciplinary optimization platform Modefrontier. The cross-section arc height of the running surface was optimized by an improved genetic algorithm and a co-simulation optimization mode, the ideal running surface with the least partial wear of the running wheel is obtained. The research has mainly the following innovations and contributions:

1. Due to the special structure of SMV, the inner and outer wheels are supported on the same axle, which inevitably produces lateral force. Because of the space limitation of running wheel, it is impossible to design camber angle to balance lateral force like automobile suspension. Therefore, the concept of “ideal running surface” is proposed in this paper. Two kinds of concave and convex running surfaces were designed to improve wheel rail running surfaces and reduce eccentric wear of running wheel.

2. From the interaction between the running wheels and the rail beam, through the optimization of the arc height of the rail beam running surface, a “virtual camber” is generated between the running wheels and the running surface of the rail beam, which reduces the running wheel tires partial wear and realizes theoretical innovation.

3. An integrated optimization simulation mode was put forward. An optimization model of arc height of the running surface was established based on the finite element model and the multidisciplinary optimization platform Modefrontier to realize the active optimization of the arc height of running surface and greatly improve the optimization efficiency.

**“Running wheel-rail beam” finite element model**

The “running wheel-rail beam” finite element model consists of a running wheel model, a rail beam model and a friction model describing wheel-rail contact behavior. In order to reduce the partial wear of the running wheels, the running surface of the rail beam is designed as a curved surface and simulated by an analytical rigid body.

**Running surface design of rail beam**

The partial wear of the running wheel tire in the straight section of the rail beam is small, and the partial wear of the running wheel tires mainly occurs when the vehicle passes through the curved track beam. Therefore, this paper considers to design the running surface in the curved section of the track beam. The current curved rail beam is plane. In order to improve the wheel rail contact state and form a “virtual camber” between the running wheel and the running surface of track beam, the concave running surface, the convex running surface, and the concave convex mixed running surface are designed on the basis of the plane running surface, as shown in Figure 2.

At present, the running surface of the curved section of the rail beam is plane, and there is a certain super-elevation angle at the curve of the track beam. When the SMV passes the curve, there is a roll angle, as shown in Figure 2(a). The concave and convex running surfaces are shown in Figure 2(b) and (c). When the
minimum partial wear occurs on different types of running surfaces, it can be designed as a left-right asymmetric type, that is, a mixed type of concave and convex running surfaces, as shown in Figure 2(d).

In Figure 2, \( a \) is the width of the rail beam; \( h \) is the arc height of running surface; \( r \) is the radius of the curved surface of the “concave-convex” running surface; \( b \) is the distance between running tires.

Circular curve is one of the commonly used curves in engineering, which is widely used in mechanical parts and architectural design. The concave and convex running surfaces of rail beam are described by circular curve equations of equations (1) and (2) respectively:

**Concave running surfaces:**

\[
(x - 425)^2 + (y - r + h)^2 = r^2, \tag{1}
\]

**Convex running surfaces:**

\[
(x - 425)^2 + (y + r - h)^2 = r^2, \tag{2}
\]

**Contact model for the running wheel-rail beam**

When the running wheels roll over the curved rail beam, tangential and normal contact forces are generated in the wheel-rail contact area. Since the running wheel uses pneumatic rubber tires, the contact between the running wheel and the running surface is soft contact, and its normal contact behavior is characterized by exponential characteristics, as shown in Figure 3.

The normal pressure formula\(^{15}\) is:

\[
p = \frac{3F_z}{4a(y)} \left(1 - \left( \frac{x}{a(y)} \right)^2 \right) \left(1 + \left( \frac{x}{a(y)} \right) \eta(y) \right) \tag{3}
\]

Where, \( F_z \) is the normal load of the running wheel, \( a(y) \) is half the length of the impression, \( p \) is the normal pressure, \( \eta(y) \) is the distribution function of the normal pressure along the axial direction of the running wheel, \( x \) is longitudinal coordinate, \( y \) is lateral coordinates.

Rubber tires are viscoelastic bodies. When the tire rolls, there are adhesion areas and slip areas at the same time in the contact area. The friction coefficient of rubber is closely related to the sliding speed and contact state, the classic Coulomb friction model cannot accurately describe the wheel-rail contact state, a friction model with attenuation index related to sliding velocity is proposed.\(^{16}\) Studies have shown that slip speed and contact pressure of tire have a significant effect on the friction coefficient.\(^{17,18}\) In order to establish a friction model consistent with actual friction characteristics, a modified attenuation index model is adopted in the paper.\(^{19}\)

\[
\mu(p, v) = (ae^{bv} + c) [\mu_k + (\mu_s - \mu_k)e^{(v-d_c)}] \tag{4}
\]

\[
\tau = \mu p
\]

where \( p \) is contact pressure, \( v \) is relative slip velocity, \( \mu_s \) and \( \mu_k \) is static friction coefficient and dynamic friction coefficient respectively, \( a, b, \) and \( c \) is constant, \( d_c \) is attenuation index. \( \tau \) is shear stress.
These parameters are obtained by the tread rubber friction test on the MVF-1A friction and wear test machine by Jiangsu University, the model parameters are obtained in the formula (3), $a$ is 0.7796, $b$ is $-1.003$, $c$ is 0.4696, $\mu_r$ is 0.7315, $\mu_k$ is 1.3474, $d_c$ is 0.6422.

In ABAQUS, we defined the exponential normal contact pressure and compiled a subroutine for the modified friction model.

**Constitutive model for running wheel tire**

The mechanical behavior of running wheel tires directly affects tire analysis results. The mechanical behavior of rubber materials is very complex, an accurately running wheel tire finite element model need an accurate constitutive model to describe the complex mechanical properties. Rubber materials of the running wheel are an incompressible hyper-elastic material. The rubber mechanical properties of running wheel tires are often described in the form of strain energy, the commonly used model is the Mooney-Rivilin model, and the strain energy function expression is:

$$U = \sum_{i+j=1}^{N} C_{ij} (\tilde{I}_1 - 3)^i (\tilde{I}_2 - 3)^j + \sum_{i=1}^{N} \frac{1}{D_i} (J^{el} - 1)^{2i}$$  \hspace{1cm} (5)

Assuming $N = 1$: $U = C_{10} (\tilde{I}_1 - 3) + C_{01} (\tilde{I}_2 - 3)$ where, $U$ representative strain energy, $N$ is material parameters, $C_{ij}$ and $D_i$ is material parameters related to temperature, $\tilde{I}_1$, $\tilde{I}_2$ is first and second partial strain constants, $J$ is the elastic volume ratio.

In order to determine model parameters $C_{10}$ and $C_{01}$, the rubber biaxial tensile tests were carried out by Jiatong Tire Manufacturing Company. According to

Figure 3. Stress distribution and critical slip point in wheel rail contact area when SMV passes through a curve: (a) normal stress distribution, (b) tangential stress distribution, and (c) critical slip point.

Figure 4. Stress-strain curve of tire materials.

**Finite element model of “running wheel-rail beam”**

The rail beam of Chongqing straddle monorail transit system mainly adopts precast prestressed reinforced concrete structure (referred to as PC rail beam), which is supported by prestressed steel tendon and poured with concrete on the outer surface. Scholars have extensive research on the FEM in railway track such as, in order to attenuate the transmission of vibration energy of metro and derive a safe distance between building to the metro, a series of finite element models of the track and the surrounding media was developed, which
considered the real train loading conditions as an input and was validated by comparisons of its results with those of a comprehensive field measurement.23–25

In the research, the reinforced concrete rail beam is in contact with pneumatic rubber tires. During the contact between the running wheel and the rail beam, the deformation of the running wheel tire is much greater than that of the rail beam. In order to improve the calculation efficiency, the rigid curved surface is used to simulate the rail beam. The rail beam is 800 m in length and 200 m in width fix at both ends, and curve supper elevation is 0.0733 m. The grid is densified at the wheel-rail contact. The finite element model of “running wheel-rail beam” was established in ABAQUS, as shown in the Figure 5.

Table 1. Rubber material parameters of running wheel tire.

| Material parameters | $C_{10}$/MPa | $C_{01}$/MPa | $D_1$/MPa | Density/ kg/m$^3$ |
|---------------------|-------------|-------------|-----------|------------------|
| Tread rubber        | 0.428       | 0.107       | 0.001     | 1113             |
| Sidewall rubber     | 0.326       | 0.082       | 0.001     | 1107             |
| Cord rubber         | 0.581       | 0.145       | 0.001     | 1168             |
| Triangle rubber     | 0.401       | 0.102       | 0.001     | 1108             |

Table 2. Steel cord parameters.

| Name              | Wire angle/$^\circ$ | Wire spacing/mm | Wire diameter/mm |
|-------------------|---------------------|-----------------|-----------------|
| Carcass cord      | 90                  | 2               | 1.62            |
| Belt 1            | 24                  | 2.22            | 1.36            |
| Belt 2            | 15                  | 2.22            | 1.36            |
| Belt 3            | 15                  | 2.5             | 1.18            |
| Belt 0            | 0                   | 1.8             | 1.38            |

Figure 5. The finite element model of “running wheel-rail beam” for three types of running surfaces: (a) concave contact, (b) plane contact, and (c) convex contact.

Boundary conditions of the finite element model

In The FEM model, the attached contact pair were defined between the running wheel and the running surface, as shown in the Figure 6. The analysis of running tires was completed in five working steps, boundary conditions would change with different working conditions, as shown in Table 3.

Validation of the finite element model of “running wheel-rail beam”

In order to verify the correctness of the “running wheel-rail beam” model, the running wheel stiffness experiment was carried out in the laboratory. In order to make the wheel-rail contact force in the $x$, $y$, $z$ three directions match the actual situation, it is necessary to simulate the contact force as realistically as possible. According to previous studies, different track structures have a greater impact on wheel-rail contact force and vibration transmission characteristics.26 Therefore, in this research, considering the roughness and adhesion state of the running surface of the rail beam, a simulated concrete surface of the rail beam in contact with the running wheel was made. The running wheel stiffness test bench was developed based on MTS electro-hydraulic servo test system of China Automotive Engineering Research Institute. The experiment method is similar to the conventional stiffness experiment, the machine applies the nominal loads to the running wheel by a hydraulic pump actuator, the running wheel’s deflections against the applied loads are drawn by a software as shown in Figure 7(a) and (b).

The special test bench is composed of hydraulic loading equipment, running wheel stiffness test fixture, sensors, and data analysis and processing system, as shown in Figure 7(c).
Different types of sensors are used for the force and displacement measurements, including strain-gauges, matrix based tactile surface sensors, conventional load cells, pressure sensor, LVDTs, and grating sensor. In order to obtain the vibration level of the track at different positions, the LVDTs sensors was used in rail vibration test due to strong adaptability.27,28 In the research, pressure sensor and grating displacement sensor were selected, which are integrated in the MTS test bench and are respectively arranged at the position of the driving actuator. The main experimental test parameters provided by the China Automotive Engineering Research Institute are shown in Table 4.

Based on the special running wheel stiffness test bench, when the sampling frequency is 102.4 Hz, the number of sampled data points is 256, and the resolution is 0.4 Hz, the time-domain history of displacement and force measurement data is shown in Figure 8(a) and (b). After the two discrete sine sequences of dynamic excitation force and deformation of running

**Table 3.** Definition of boundary conditions under different working conditions.

| Working condition       | Constraint                  | Inflation pressure(MPa) | Fz, α, γ, k |
|------------------------|-----------------------------|-------------------------|-------------|
| Inflation condition    | Wheel core: 1, 2, 3, 4, 5, 6| 0.95                    | Fz(h), α(h), γ(h), k(h) |
| Static contact condition| Road: 1, 2, 4, 5, 6; 3: 30 mm | 0.95                    | Fz(h), α(h), γ(h), k(h) |
| Steady condition       | I: 36 km/h; 5: 20.3568 rad/s | 0.95                    | Fz(h), α(h), γ(h), k(h) |
| Slip condition         | I: 36 km/h; 2: 148.36 mm/s  | 0.95                    | Fz(h), α(h), γ(h), k(h) |
| Roll condition         | I: 36 km/h; 4: 0.0566 rad   | 0.95                    | Fz(h), α(h), γ(h), k(h) |

1, 2, and 3 represent the translational freedom along the X-, y-, and Z-axes respectively. 4, 5, and 6 respectively represent the rotational freedom along the x-, y-, and z-axis.
wheel tires are obtained, fast Fourier transform FFT was used to accurately estimate the amplitude and phase angle of the discrete sine sequence. After data processing, the longitudinal, lateral, and vertical stiffness tests of the running wheel were carried out. The comparison results of the test data and the simulation data are shown in Figure 8 and Table 5.

It can be seen from Table 5, the error of the running tire stiffness between the simulated value and test value is within the allowable range, which verifies the correctness and effectiveness of the “running wheel-rail beam” finite element model in this paper.

### Table 4. Experimental test parameter of running wheel stiffness.

| Name                           | The first group of running wheel | The second group of running wheels | The third group of running wheels |
|--------------------------------|---------------------------------|-----------------------------------|----------------------------------|
| Preload/kN                     | 55.5                            | 35.4/38.9/49.5/55.5               | 35.4/38.9/49.5/55.5              |
| Test frequency/Hz              | 2/5/8/12/15/18/22/26/35         | 2/5/8/12/15/18/22/26/35           | 2/5/8/12/15/18/22/26/35          |
| Inflation pressure/kPa         | 1000                            | 950                               | 850                              |

### Figure 8. The comparison between simulation data and test data of running wheel tires stiffness: (a) the time history of deformation of running wheel, (b) the time history of exciting force of the running wheel, (c) longitudinal stiffness curve, (d) lateral stiffness curve, and (e) vertical stiffness curve.

### Table 5. Comparison of simulation and test of running wheel stiffness.

| name          | Longitudinal stiffness (N/mm) | Lateral stiffness (N/mm) | Vertical stiffness (N/mm) |
|---------------|-------------------------------|--------------------------|---------------------------|
| Simulation results | 744.6                         | 508.4                    | 1316.1                    |
| Experimental data        | 759.2                         | 523.8                    | 1286.3                    |
| Error                    | 1.92%                         | 2.94%                    | 2.32%                     |

**Optimization model of the running surface**

The optimization model of the running surface includes design variables, objective function, and constraint
conditions, which need to be determined before optimization.

**Design variables**

Before the optimization study, the influence of plane, concave, and convex types of running surface on the partial wear of running wheel was analyzed. The analysis results show that when SMV passes through the curved rail beam, the running surface contacting with the inner running wheel is properly sunken, and the running surface contacting with the outer running wheel is properly protruding, which can improve the wheel rail contact state and reduce the partial wear of running tires. According to the influence trend of concave and convex running surface on the running wheel tire, the FWD value of running wheel, it can be known that the minimum partial wear can be obtained by using the mixed type running arc surface, as shown in Figure 2(d). Therefore, this paper took the mixed running surface as the basis, selected the optimization variables, established the optimization model, and carried out the optimization research on the running surface.

In Figure 2(d), \( h \) is the arc height of the running surface, \( r \) is the radius of the running arc surface, the two running wheels are symmetrical, and the separation distance \( b \) is 400 mm. Among the cross-section parameters of the running arc surface, the arc height and radius of the running arc surface are two mainly characteristic parameters, the arc height and radius of the running arc surface can be selected as design variables of optimization model.

**Objective function**

According to the theory of friction loss of rubber tire, the amount of tire wear is positively correlated with friction work, the lateral distribution of friction work of the running wheel tire, and the FWD value is used to measure the degree of partial wear. In the finite element model, the local friction work of the running wheel tire surface can be calculated by the Shear force and relative slip of the nodes in the wheel-rail contact area of the tread. The objection function is as follows:

\[
W_{ij} = \sqrt{\tau_{ij,x}^2 + \tau_{ij,y}^2} \cdot \sqrt{\tau_{ij,x}^2 + \tau_{ij,y}^2} \tag{6}
\]

where \( i = (1, 2, 3 \cdots n_i) \), \( n_i \) represents the number of rows, that is, the number of longitudinal nodes; \( j = (1, 2, 3 \cdots n_j) \), \( n_j \) represents the number of columns, that is, the number of horizontal nodes; \( W_{ij} \) represents the friction work of nodes at the row \( i \) column \( j \); \( \tau_{ij,x} \) represents the longitudinal shear force at that location; \( \tau_{ij,y} \) represents the lateral shear force at that location; \( l_{ij,x} \) represents the amount of longitudinal slip at that location; \( l_{ij,y} \) represents the amount of lateral slip at that location.

Partial wear of running tires is mainly manifested as uneven distribution of lateral friction work of the tires. The partial wear of the running tires caused by sudden situations in the tire rolling process is not considered, so the tires will not appear uneven wear at the same circumferential position. In the finite element model of the running tire wheel, each node will experience the friction work state of the longitudinal node sequence in the wheel-rail contact area. Therefore, from a long-term perspective, the lateral distribution of running wheel friction work can be regarded as the lateral distribution of the sum of the friction work of each longitudinal node sequence of the tire. The calculation formula of the sum of the frictional work is as follows:

\[
w_j = \sum_{i=1}^{n_i} w_{ij}, W_{\text{total}} = \sum_{j=1}^{n_j} w_j \tag{7}
\]

where \( w_j \) represents the total friction work of the \( j \)th column in the wheel-rail contact area, \( W_{\text{total}} \) represents the sum of friction work.

The partial wear of running wheel tire can be measured by deviation value of the laterally distributed friction work \( W_j \), the calculation formula is as follows:

\[
f_x = \sqrt{\frac{1}{n_x - 1} \sum_{j=1}^{n_j} (w_j - \bar{w})^2} \tag{8}
\]

where \( f_x \) represents the FWD value, \( \bar{w} \) represents the average friction work of all nodes in the wheel-rail contact area.

**The mathematical optimization model**

The mathematical model for optimizing the running arc surface is expressed as:

\[
\begin{cases}
\min f(x) \\
\text{st. } h^L < h < h^U \\
\quad r^L < r < r^U
\end{cases} \tag{9}
\]

\[
f(x) = \sum_{j=1}^{n_j} w_j + \sqrt{\frac{1}{n_x - 1} \sum_{j=1}^{n_j} (w_j - \bar{w})^2}
\]

\( f(x) \) is the objective function and is composed of the sum of the friction work and FWD value; \( h \) represents the arc height of the running arc surface, \( h^L \) represents the upper and lower boundary values of the arc height of the running arc surface. \( r \) represents the radius of the running arc surface, \( r^L \) represents the upper and lower boundary values of the radius of the running arc respectively.
Constraint conditions
During optimization, it is necessary to determine the value range of design variables and ensure the safety of monorail vehicles. According to the influence law of arc height on the sum of friction work and FWD value, the range of concave arc height $h$ is determined to be 20–25 mm, the range of convex arc height $h$ is 8–15 mm. The safety of monorail vehicles is guaranteed by two indexes: wheel load reduction rate and overturning coefficient.

(1) Wheel load reduction rate

When a monorail vehicle passes a curve, the centrifugal force of the vehicle will change the vertical force of the tires on the inside and outside of the curve. The load reduction rate is used to describe the difference in the vertical load of the tires on the inside and outside of the curve. The calculation formula is as follows:

$$\gamma = \frac{\Delta P}{\bar{P}}$$  \hspace{1cm} (10)

where, $\Delta P$ represents the vertical load difference between the inside and outside running wheel tires; $\bar{P}$ is the average of the vertical load of the running wheel tires. Among them: the first limit requires $\gamma \leq 0.65$, the second limit requires $\gamma \leq 0.6$. This article uses the first limit for optimization.

(2) Overturning coefficient

When a SMV passes through the curve, partial load may cause the vehicle to derail. The overturning coefficient is used to ensure the safety of the SMV when passing through the curve. The calculation formula is as follows:

$$R = \frac{P_d}{P_{st}} = \frac{P_2 - P_1}{P_2 + P_1}$$  \hspace{1cm} (11)

where, $P_d$ represents the difference between the vertical average load on the load-increasing side and the de-loading side; $P_{st}$ is the sum of the vertical load on the load-increasing side and the de-loading side; $P_2$ is the average value of the two running wheel tires on the load-increasing side of the bogie; $P_1$ is the average value of the two running tires on the de-loading side. According to the Chinese National Standard GB/T5599-2019, the overturning coefficient $R < 0.8$.

The relationship between the design variables and the input parameters of running wheel

When optimizing, the arc height $h$ of the concave surface and convex surface may be regard as a independent design variables, the arc radius of running surface can be derived by a function formula $r = (h^2 + 180625)/2h$. Because the input working condition parameters of the running wheel are different corresponding to different arc height $h$ of running surface, it is necessary to establish the relationship between working condition parameters of the running wheel and the height of the arc surface. The process is shown in Figure 9.

Main parameters of the dynamic model of SMV

The dynamic model of SMV contains three attached contact pairs, which had been established in Adams, and the main parameters for the dynamic model of SMV are shown in Table 6.
Table 6. Main parameters for dynamic model of SMV.

| Name                        | Values                  |
|-----------------------------|-------------------------|
| Car body/bogie mass         | 28,900/5600 kg          |
| Car body inertia $I_{xx}/I_{yy}/I_{zz}$ | 20,000/170,000/170,000 kg m² |
| Bogie inertia $I_{xx}/I_{yy}/I_{zz}$ | 2400/3400/9600 kg m²  |
| Running/guide/steady wheel mass | 63.1/28.3/28.3 kg      |
| Running wheel inertia $I_{xx}/I_{yy}/I_{zz}$ | 5.2/5.2/10.2 kg m²  |
| Guide (steady) wheel $I_{xx}/I_{yy}/I_{zz}$ | 2.9/2.9/3.3 kg m²   |
| Vertical/cornering stiffness of running wheel | 1,310,000 N/m/523,800 N/m² |
| Vertical/lateral stiffness of spring | 160,000/10,000 N/m |
| Vehicle running speed       | 36 km/h                |

Table 7. Curve track scenarios.

| Scenario | Radius (m) | Curve supper elevation (m) | Velocity (km/h) | Curve speed limit (km/h) |
|----------|------------|----------------------------|-----------------|--------------------------|
| Zone 1   | 100        | 0.0733                     | 36              | 43                       |

Rail line modeling

By considering transition curve, elevation of curve, joints of rails, and structural characteristics of turnout. The typical rail model composed by linear segments, curve segments, and transition curves which can reflect actual rail characteristics is established. The width of the track is 690 mm. The radius of the rail is composed of three segments.23 Track geometry for the curve with radius 100 m is illustrated in Figure 3 and curve track scenarios are illustrated in Table 7.

Surface roughness simulation of rail beam

According to the straddle-type single-rail PC (Prestress concrete) rail beam structure and casting technology, there are large differences between steel rails for PC rail beams, railway vehicles, and metro vehicles, while it is certain similar to concrete pavement of vehicles. Therefore, the PSD density function is adopted for the unevenness model of the straddle-type monorail vehicle, as the following:

$$G_d(\Omega) = \frac{\alpha}{\Omega^\alpha + \beta^\alpha} = G_d(\Omega_0) \left(\frac{\Omega}{\Omega_0}\right)^{-n}$$  \(12\)

The values of (10) are listed as follows. Surface irregularity samples of contact surface are illustrated in Figure 2.

Rail beam top running surface:

$$\alpha = 0.0004, \beta = 0.31, n = 3.1.$$  

Rail beam lateral guide wheel contact surface:

$$\alpha = 0.0007, \beta = 0.61, n = 2.9.$$  

Rail beam lateral steady wheel contact surface:

$$\alpha = 0.0007, \beta = 0.5, n = 2.7.$$  

The relationship between running wheel operating parameters and design variables

The Points were selected in the upper and lower limits of the arc height of the running surface, and the interval between the points was taken as 0.5 mm. A series of running surfaces were created in the multi-body dynamics software ADAMS, and the working conditions parameters corresponding to different arc height of the running wheel were obtained by simulation. The function formula between the vertical force, slip rate, side slip angle, roll angle, and arc height of running tires was obtained by fitting the data of simulation data. According to the change trend between the running wheel parameters and the arc height of running surface, the relationship between the running wheel vertical force, slip rate, side slip angle, and arc height of running surface is a quadratic function, while the relationship between the running wheel roll angle and the arc height of running surface is a linear function. The functions relationship between working condition parameters of tire and the design variable $h$ is as follow:

(1) The left wheel:

$$Fz = -26h^2 + 1212h + 22474,$$
$$k = 1.08e - 5 \cdot h^2 - 0.0005 \cdot h + 0.0324$$
$$\alpha = 0.0815h^2 - 0.8505h + 10.1531,$$
$$\gamma = -0.1527h + 3.5908$$

(2) The right wheel:

$$Fz = 29h^2 - 689h + 67444,$$
$$k = -0.0001a^2 + 0.0025h - 0.0387,$$
$$\alpha = 0.0243h^2 - 0.5808h + 3.9125,$$
$$\gamma = -0.1248h + 3.1185$$

Co-simulation optimization mode and methodology

ABAQUS and multi-disciplinary optimization software Modefrontier were combined to build a joint simulation
platform for optimization of rail beam running arc surface. In the optimization calculation, the initial population data were generated randomly within the range of values. The second generation improved genetic algorithm NSGA-II was used in the optimization. The initial population was set to 10 and the evolution algebra was set to 100. The optimization flowchart is shown in the Figure 10.

Results analysis and discussion

On the Modefrontier optimization platform, an improved genetic algorithm optimization algorithm is used to optimize the arc height. When the number of iterations is 38, the arc height of the running surface tends to converge. The optimization results and the discussion before and after optimization are as follows.

Optimization results analysis

With the change of arc height and the increase of iteration times, the total friction work and the FWD value of running wheel gradually decrease. After about 38 iterations, it tends to converge gradually. Among them, the front right tire in contact with convex running surface has better convergence trend. The optimization history of the objective function with the iterative process is shown in Figures 11 and 12. The scatter diagram of the total friction work and the FWD value is shown in Figure 13.

In Figure 13, the data in the lower left corner is the Pareto solution. A group of data is found in the Pareto solution. In this paper, the 28th design point is taken as the concave optimal point, and the 38th design point is taken as the convex optimal point. The result comparison before and after optimization is shown in Tables 8 and 9.

Comparison of results before and after optimization

Before and after optimization, the comparison between the footprint and friction work distribution of the running wheel is shown in Table 8. It can be seen from Table 7 that after the optimization of the height of the running arc surface, the partial wear of the front left and front and rear running wheels has been significantly improved from the footprint and friction work distribution of the running wheel. The rear wheel and the front wheel on the same side have a similar distribution of footprint and friction work, so only the footprint and friction work distribution of the front wheel was listed.

The comparison of total friction work and the FWD value before and after optimization of running arc height is shown in Figure 14 and Tables 9 and 10.

Through the optimization research, the following conclusions can be obtained:
When the Pareto solution of the arc height of the inside concave running surface is 22.62 mm, the total friction work of the running wheel tire decreases by 11%, and the FWD value decreases by 11.8%. The total friction work and FWD value are decreased after

Figure 11. Optimization history of total friction work and FWD value on concave surface: (a) optimization history of total friction work and (b) optimization history of FWD value.

Figure 12. Optimization process of total friction work and FWD value on convex surface: (a) optimization history of total friction work and (b) optimization history of deviation value of friction work.

Figure 13. Scatter diagram of total friction work and FWD value: (a) scatter diagram of optimization objective on concave and (b) scatter diagram of optimization objective on convex.

1) When the Pareto solution of the arc height of the inside concave running surface is 22.62 mm, the total friction work of the running wheel tire decreases by 11%, and the FWD value decreases by 11.8%. The total friction work and FWD value are decreased after
optimization, and the tire wear and partial wear were improved to a certain extent.

(2) When the Pareto solution of the arc height of the outer convex running surface is 11.81 mm, the total friction work of the running wheel tires decreases by 4.92%, and the FWD value decreases by 32.1%. The total friction work and the FWD value both decrease. The tire wear and partial wear has been improved to a certain extent.

(3) When the arc height of the inside concave running surface is 22.62 mm, the arc height of the outer convex running surface is 11.81 mm, the total friction work and the FWD value of the running wheel tire are reduced. The tire wear and partial wear can be improved to some extent.

Especially when the outer running wheel tire is in contact with the convex running surface with an arc height of 11.81 mm, the FWD value can drop by 32.1%. Due to the heavier wheel load of the outer running wheel, the tire wear and partial wear of outer running wheel are more serious. Due to the load of the outer running wheel, the degree of tire wear and partial wear decreases by 32.1%. The total friction work and the FWD value both decrease. The tire wear and partial wear has been improved to a certain extent.
wear is more serious. After the convex arc height of the running surface is optimized, the partial wear of the running wheel is obviously reduced.

Conclusions and outlook

In this paper, aiming at the serious wear problem of straddle monorail vehicle’s running wheel tire, two types of concave and convex running surfaces were designed, and the finite element model of “running wheel-rail beam” under three kinds of section types of rail beam, plane, concave, and convex surface, was established. An optimization model of arc height of running surface was established based on finite element model and multidisciplinary optimization platform Modefrontier to realize the active optimization of the arc height of running surface. Based on multidisciplinary platform, taking the total friction work and friction work deviation value of running wheel tire as the optimization control objectives, the improved genetic algorithm was used to optimize the arc height of the running surface.

The research results show that by optimizing the arc height of the running surface, the dual purpose of reducing the wear of the running wheel and improving the eccentric wear of the running wheel can be achieved. The research results can provide a theoretical basis for the design of the rail beam running surface of straddle monorail vehicle, and achieve the goal of reducing the partial wear of the running wheel tires and increasing its service life.

In order to further control the problem of partial wear of running wheel tires, it is necessary to carry out research on the optimization of structural parameters of running wheel tires in the future. The research direction of the future is to integrate the multi-disciplinary knowledge such as vehicle design, structure design of running wheel and tire, and systematically and comprehensively carry out the research on running wheel offset wear control.

Acknowledgements

The data were provided by Changke Rail Vehicle Co., Ltd. and Jiatong Tire Manufacturing Company. Thanks for the photos and data provided by China Automotive Research Institute.

Author contributions

Liang Chen analyzed the data; Zixue Du analysis tools; Zhen Yang conducted experiments; XiaoXia Wen wrote the paper.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the National Natural Science Foundation of China (Grant No. 51475062) and project of Chongqing Key Laboratory of Urban Rail Transit Vehicle Integration and Control (Grant No. GK201806).

ORCID iD

XiaoXia Wen https://orcid.org/0000-0001-6450-7757

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