B → η(′)lν decays and the flavor-singlet form factors

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Abstract
We study semileptonic decays $B \rightarrow \eta(\prime)l\nu$, taking into account the flavor-singlet contribution $(F^\text{singlet}_+) \text{to the } B \rightarrow \eta(0) \text{ form factors, which arises from the two-gluon emission in a decaying } B \text{ meson. It has been recently pointed out that, in addition to large weak annihilation effects, the unknown value of } F^\text{singlet}_+ \text{ prevents accurate theoretical estimates in the analysis of } B \rightarrow \eta K \text{ decays in QCD factorization. We present a certain method to determine } F^\text{singlet}_+ \text{ with a reasonable accuracy, using } B \rightarrow \eta(0)l\nu \text{ and } B \rightarrow \pi l\nu \text{ decays. We also investigate the possible effect of } F^\text{singlet}_+ \text{ on the estimated branching ratios (BRs) for } B \rightarrow \eta(0)l\nu \text{ and find that the BR for } B \rightarrow \eta Kl\nu \text{ is particularly sensitive to the effect of } F^\text{singlet}_+.}$

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Semileptonic decays of $B$ mesons have been extensively studied with particular interests. They can serve as useful applications of various non-perturbative theoretical approaches and provide an efficient way for the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, such as $V_{cb}$ and $V_{ub}$. In fact the present best experimental data for $V_{ub}$ come from measurements of the exclusive semileptonic decays $B \to \pi l \nu$ and $B \to \rho l \nu$ (CLEO Collaboration [1]), and the inclusive semileptonic decay $B \to X_u l \nu$ (LEP Heavy Flavor Group [2]):

$$|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55) \times 10^{-3} \quad [\text{CLEO}],$$

$$= (4.09^{+0.36+0.42}_{-0.39-0.47} \pm 0.25 \pm 0.23) \times 10^{-3} \quad [\text{LEP}], \quad (1)$$

as well as the exclusive charmless nonleptonic $B \to D_s \pi$ decay [3]. Recently BELLE [4] and BABAR [5], respectively, have announced preliminary results for $|V_{ub}|$ that are similar to the CLEO result. In the analyses, they also used the exclusive semileptonic processes: $B \to \pi l \nu$ (BELLE) and $B \to \rho l \nu$ (BABAR). Although these measurements currently suffer from large uncertainties due to model-dependence, a dominant background, and so forth, a more accurate value of $|V_{ub}|$ will become available through future studies by using the hadronic invariant mass of the inclusive decay [6].

An analysis of the charmless semileptonic $B$ decays involves the non-perturbative hadronic form factors whose theoretical estimation is usually model-dependent. Over the past few years, there has been considerable progress in the calculations of the $B \to \pi$ form factor, based on various theoretical approaches, such as the lattice QCD calculation [7], the light-cone QCD sum rule (LCSR) [8], and so on. It is known that these approaches produce consistent results for the $B \to \pi$ form factor. For the $B \to \eta^{(')}$ form factors, some theoretical studies have been done by assuming the standard quark content of $\eta^{(')}$ mesons. Under this assumption, the $B \to \eta^{(')}$ form factors can be determined by a phenomenological approach using the semileptonic decays $B \to \eta^{(')} l \nu$ [9], by using the LCSR model [10], or by applying the isospin symmetry to the $B \to \pi$ form factor [11]. Presently these approaches can give consistent results. In particular, in Ref. [9] the branching ratios (BRs) for $B^\pm \to \eta l \nu$ and $B^\pm \to \eta' l \nu$ are estimated to be

$$B(B^\pm \to \eta l \nu) = (4.32 \pm 0.83) \times 10^{-5},$$

$$B(B^\pm \to \eta' l \nu) = (2.10 \pm 0.40) \times 10^{-5}. \quad (2)$$
For last several years the experimental results of unexpectedly large BRs for $B \rightarrow \eta' K$ decays have drawn a lot of theoretical attentions. The observed BRs for $B^{\pm} \rightarrow \eta' K^{\pm}$ in three different experiments are [12, 13, 14]

$$B(B^{\pm} \rightarrow \eta' K^{\pm}) = (80^{+10}_{-9} \pm 7) \times 10^{-6} \ [\text{CLEO}],$$

$$= (77.9^{+6.2+9.3}_{-5.9-8.7}) \times 10^{-6} \ [\text{BELLE}],$$

$$= (67 \pm 5 \pm 5) \times 10^{-6} \ [\text{BABAR}]. \ (3)$$

Many theoretical efforts have been made to explain the large BRs: for instance, approaches using the anomalous $g-g-\eta'$ coupling [15, 16, 17, 18], high charm content in $\eta'$ [19, 20], the spectator hard scattering mechanism [21, 22], the perturbative QCD approach [23] and approaches to invoke new physics [24, 25, 26, 27].

In Ref. [28] Beneke and Neubert (BN) have tried to explain the large BRs for $B \rightarrow \eta' K$ decays through the property of the flavor-singlet component of the $\eta'$ meson as well as large weak annihilation effects in the framework of QCD factorization. In this approach it has been suggested that the form factors for the $B \rightarrow \eta^{(l)}$ transition may have an additional contribution through a singlet mechanism, where the flavor-singlet meson states are produced via the two-gluon emission in a decaying $B$ meson. However, since it is unknown how large this new flavor-singlet contribution ($F_{+}^{\text{singlet}}$) to the $B \rightarrow \eta^{(l)}$ form factors is, the uncertainty in $F_{+}^{\text{singlet}}$ would prevent accurate theoretical estimates of the BRs for $B \rightarrow \eta^{(l)} K$. Indeed, it has been found [28] that the qualitative pattern of the BRs for $B \rightarrow \eta^{(l)} K^{(*)}$ decays can be accounted for in their approach, but within large uncertainties mainly coming from the weak annihilation, and the strange quark mass and the unknown two-gluon contribution $F_{+}^{\text{singlet}}$ to the $B \rightarrow \eta^{(l)}$ form factors. Therefore, in order to improve the theoretical estimations it is essential to develop specific methods for estimating those primary sources of the large uncertainties as accurately as possible. We note that $B \rightarrow \eta^{(l)} l \nu$ decays can be the ideal process to investigate the $B \rightarrow \eta^{(l)}$ transition form factors.

In this work we study $B \rightarrow \eta^{(l)} l \nu$ decays, keeping in mind the possible effect of the additional flavor-singlet term on these decays. Our goal is two-fold. First, we try to present certain phenomenological methods to determine the flavor-singlet contribution $F_{+}^{\text{singlet}}$ to the $B \rightarrow \eta^{(l)}$ form factors with a reasonable accuracy, which is one of the main sources of the large uncertainties involved in BN’s approach. Secondly, we investigate how large the flavor-singlet term $F_{+}^{\text{singlet}}$ can affect the predicted BRs for $B \rightarrow \eta^{(l)} l \nu$ decays, in comparison
with the results previously presented in literature. For this aim, we calculate the BRs for \( B \to \eta^{(0)} l \nu \) decays including the flavor-singlet effect and compare the result with those previously given in literature.

The physical states of \( \eta \) and \( \eta' \) mesons are described as admixtures of the flavor octet state \( |\eta_8\rangle = (|u\bar{u}| + |d\bar{d}| - 2|s\bar{s}|)/\sqrt{6} \) and the flavor singlet state \( |\eta_0\rangle = (|u\bar{u}| + |d\bar{d}| + |s\bar{s}|)/\sqrt{3} \).

An important clue to the solution of the \( B \to \eta^{(0)} \) anomaly may be obtained by examining the flavor-singlet property of the \( \eta^{(0)} \) mesons. The divergence of the axial-vector current is given by

\[
\partial_\mu (q^\mu q_5) = 2i m_\eta q^\mu q_5 - \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} ,
\]

where the last term of the right-handed side is the QCD anomaly term, and \( G^a_{\mu\nu} \) and \( \tilde{G}^{a,\mu\nu} \) are the gluonic field strength tensor and its dual, respectively. The anomalous contribution to the divergence of the axial-vector current does not appear in the flavor-octet state. On the contrary, the flavor-singlet state has the QCD anomaly contribution and the large mass of the \( \eta' \) meson as compared to other light pseudoscalar mesons can be explained by this anomaly contribution.

The transition amplitude for the semileptonic decays of a \( B \) meson to an \( \eta^{(0)} \) meson can be written as

\[
\mathcal{M}(B \to \eta^{(0)} l \nu) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \langle \eta^{(0)}(p_{\eta^{(0)}})|\bar{u}\gamma_\mu (1 - \gamma_5)b|B(p_B)\rangle .
\]

The hadronic matrix element can be parameterized as

\[
\langle \eta^{(0)}(p_{\eta^{(0)}})|\bar{u}\gamma_\mu (1 - \gamma_5)b|B(p_B)\rangle = F_{+}^{B \to \eta^{(0)}}(q^2)(p_B + p_{\eta^{(0)}})_\mu + F_{-}^{B \to \eta^{(0)}}(q^2)(p_B - p_{\eta^{(0)})}_\mu ,
\]

where \( q_\mu = (p_B - p_{\eta^{(0)})}_\mu \). The differential decay width is given by

\[
\frac{d\Gamma(B \to \eta^{(0)} l \nu)}{dq^2 d\cos \theta} = |V_{ub}|^2 \frac{G_F^2 p_{\eta^{(0)}}^3}{32\pi^3} \sin^2 \theta |F_{+}^{B \to \eta^{(0)}}(q^2)|^2 ,
\]

where the mass of the produced lepton has been ignored. Note that the differential decay width depends only on one form factor \( F_{+}^{B \to \eta^{(0)}}(q^2) \). Here \( \theta \) is the angle between the charged lepton direction in the virtual \( W \) rest frame and the direction of the virtual \( W \).

If charm contents and gluonic admixtures of \( \eta^{(0)} \) mesons are ignored, \( \eta^{(0)} \) mesons can be related to the flavor states, \( |\eta_{ud}\rangle \) and \( |\eta_s\rangle \) as

\[
|\eta\rangle = \cos \phi|\eta_{ud}\rangle - \sin \phi|\eta_s\rangle ,
\]

\[
|\eta'\rangle = \sin \phi|\eta_{ud}\rangle + \cos \phi|\eta_s\rangle ,
\]

(8)
FIG. 1: Leading power two-gluon contribution to the $B \to \eta^{(l)}$ transition form factor.

where $|\eta_{ud}\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$ and $|\eta_s\rangle = |s\bar{s}\rangle$. The best fit value of the mixing angle $\phi$ is $39.3^{\circ} \pm 1.0^{\circ}$ \cite{29}. From Eq. (8), the decay constants are related as \cite{29}

$$f_{\eta_{ud}}^{\eta} = f_{ud} \cos \phi, \quad f_{\eta}^{\eta_s} = -f_s \sin \phi,$$
$$f_{\eta_{ud}}^{\eta'} = f_{ud} \sin \phi, \quad f_{\eta'}^{\eta_s} = f_s \cos \phi,$$

(9)

where $f_{ud}$ and $f_s$ are the decay constants obtained from the $\eta_{ud}$ and $\eta_s$ components of the wave functions, respectively. Considering a first order correction due to the flavor symmetry breaking, they can be given by $f_{ud} = f_\pi$ and $f_s = \sqrt{2f_K^2 - f_\pi^2}$, respectively \cite{29}. The phenomenological study about these decay constants yields

$$f_{ud} = (1.07 \pm 0.02)f_\pi, \quad f_s = (1.34 \pm 0.06)f_\pi.$$

(10)

However, for the $B \to \eta^{(l)}$ and $B \to \pi$ transition form factors, the simple relation such as Eq. (10) may not hold, because $\eta^{(l)}$ mesons have the flavor-singlet meson state produced via the two-gluon emission from the light spectator quark \cite{28}, as shown in Fig. 1. This diagram gives a leading power correction to the $B \to \eta^{(l)}$ form factors. We parameterize the new two-gluon contribution, which is proportional to the flavor-singlet decay constant, as $F_{\pi \text{ singlet}}$. From now on, we will call this $F_{\pi \text{ singlet}}$ the flavor-singlet form factor. Including this flavor-singlet contribution $F_{\pi \text{ singlet}}$, the hadronic form factors in the $B \to \eta^{(l)}$ transition can be expressed as \cite{28}

$$F_{\pi \rightarrow \eta}(0) = \frac{1}{\sqrt{2}} f_{\pi}^{\eta_{ud}} F_{\pi \rightarrow \eta}(0) + \frac{\sqrt{2} f_{\pi}^{ud} f_{\eta}^{\eta_s}}{\sqrt{3} f_\pi} F_{\pi \text{ singlet}}(0),$$
$$F_{\pi \rightarrow \eta'}(0) = \frac{1}{\sqrt{2}} f_{\pi}^{\eta_{ud}} F_{\pi \rightarrow \eta}(0) + \frac{\sqrt{2} f_{\pi}^{ud} f_{\eta'}^{\eta_s}}{\sqrt{3} f_\pi} F_{\pi \text{ singlet}}(0),$$

(11)

where $\pi$ in the superscript denotes the charged pion.
In order to determine the flavor-singlet form factor $F^{\text{singlet}}_+(0)$, we present three observables $R_1$, $R_2$ and $R_3$, which can be measured in experiment. These observables, $R_1$, $R_2$ and $R_3$, are defined by ratios of the differential decay rates of the relevant modes measured at maximum recoil point ($q^2 = 0$) as follows:

\[
R_{1(2)} \equiv \frac{d\Gamma(B^- \to \eta'(0)\ell\nu)/dq^2}{d\Gamma(B^- \to \pi^0\ell\nu)/dq^2} \bigg|_{q^2=0} = \frac{(m_B^2 - m_{\eta'(0)}^2)^3}{(m_B^2 - m_{\pi}^2)^3} \frac{f^{ud}_{\eta'(0)}}{f_{\pi}} + \sqrt{2}\tilde{F} \left( \frac{\sqrt{2}f^{ud}_{\eta'(0)} + f^{s}_{\eta'(0)}}{\sqrt{3}f_{\pi}} \right)^2,
\]
\[
R_3 \equiv \frac{d\Gamma(B^- \to \eta'\ell\nu)/dq^2}{d\Gamma(B^- \to \eta\ell\nu)/dq^2} \bigg|_{q^2=0} = \frac{(m_B^2 - m_{\eta'}^2)^3}{(m_B^2 - m_{\eta}^2)^3} \frac{\sqrt{3}f^{ud}_{\eta'} + \sqrt{2}\tilde{F} \left( \frac{\sqrt{2}f^{ud}_{\eta'} + f^{s}_{\eta'}}{\sqrt{3}f_{\pi}} \right)^2}{\sqrt{3}f^{ud}_{\eta} + \sqrt{2}\tilde{F} \left( \frac{\sqrt{2}f^{ud}_{\eta} + f^{s}_{\eta}}{\sqrt{3}f_{\pi}} \right)^2},
\]

where $\tilde{F} \equiv F^{\text{singlet}}_+(0)/F^{B\to\pi}_+(0)$. The differential decay rates $d\Gamma/dq^2$ for the semileptonic $B$ decays at $q^2 = 0$ can be experimentally measured: for example, the $d\Gamma/dq^2$ distribution, including at $q^2 = 0$, for $\bar{B} \to D^*\ell\nu$ decays has been measured by the CLEO Collaboration [30]. We note that the theoretical estimates of $R_1$, $R_2$ and $R_3$ depend only on the ratio, $\tilde{F}$, of the form factors $F^{\text{singlet}}_+(0)$ and $F^{B\to\pi}_+(0)$, besides the relevant meson masses and the decay constants, $f_{\pi}$ and $f_{\eta'}$. In the above observables, the CKM matrix element $|V_{ub}|$ does not appear due to cancellation between the denominator and the numerator, so the large uncertainty involved in $|V_{ub}|$ can be avoided.

The $B \to \pi$ form factor $F^{B\to\pi}_+$ has been studied in many models: for instance, the lattice calculation [7] and the LCSR [8], and so on. The form factor for the $B \to \pi$ transition can be expressed [31] as

\[
F^{B\to\pi}_+(q^2) = \frac{F^{B\to\pi}_+(0)}{(1 - \tilde{q}^2/(1 - \alpha_B\tilde{q}^2)},
\]

where $\tilde{q}^2 = q^2/m_{B^*}^2$, $F^{B\to\pi}_+(0) = c_B(1 - \alpha_B)$ and $m_{B^*}$ is the mass of $B^*$ meson. It is well known that Eq. (13) satisfies most of the known constraints on the form factor, such as

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & Lattice I & Lattice II & LCSR & Average Value \\
\hline
$F^{B\to\pi}_+(0)$ & 0.26 ± 0.05 ± 0.04 & 0.28 ± 0.06 ± 0.05 & 0.28 ± 0.05 & 0.27 ± 0.04 \\
c$_B$ & 0.42 ± 0.13 ± 0.04 & 0.51 ± 0.08 ± 0.01 & 0.41 ± 0.12 & 0.47 ± 0.06 \\
$\alpha_B$ & 0.40 ± 0.15 ± 0.09 & 0.45 ± 0.17 ± 0.06 & 0.32 ± 0.21 & 0.39 ± 0.11 \\
\hline
\end{tabular}
\caption{Parameters of the $B \to \pi$ transition form factor in the lattice I, II, and LCSR models.}
\end{table}
the heavy quark scaling laws predicted by the HQET in the zero recoil region \((q^2 \to q_{\text{max}}^2)\) \cite{32} and by the large energy effective theory in the large recoil region \((q^2 \to 0)\) \cite{33}. To extrapolate parameters of the relevant form factor, we adopt two different lattice calculation models (Lattice I and II) \cite{7, 34} and the LCSR model \cite{8}. Specific values of the \(B \to \pi\) form factor for each model are shown in Table I, where the last column is the weighted-average values of those three models. Those values are in good agreement with each other.

Using the values of \(F_{\pi}^{B \to \pi}(0)\) given in Table I, the observables \(R_1\), \(R_2\) and \(R_3\) are estimated for \(F_{\pi}^{\text{singlet}}(0) = 0, 0.2, 0.4, 0.6\) in Tables II, III and IV. For the values of the relevant decay constants, we have used the relations and the central values given in Eqs. (9) and (10). In the case of \(F_{\pi}^{\text{singlet}}(0) = 0\) there is no uncertainty from the form factors, because \(R_1\), \(R_2\) and \(R_3\) are dependent only on the ratio \(\tilde{F} \equiv F_{\pi}^{\text{singlet}}(0)/F_{\pi}^{B \to \pi}(0)\). It is interesting to observe that \(R_2\) and \(R_3\) are very sensitive to \(F_{\pi}^{\text{singlet}}(0)\). For instance, for \(F_{\pi}^{\text{singlet}}(0) = 0.2\), \(R_2\) and \(R_3\) are \(3.14 \pm 0.70\) and \(3.12 \pm 0.45\), respectively, while for \(F_{\pi}^{\text{singlet}}(0) = 0\), \(R_2 = 0.42\) and \(R_3 = 0.63\). In Fig. 2 we show the predicted \(R_1\), \(R_2\) and \(R_3\) as a function of the ratio \(\tilde{F}\). Again it is clearly seen that \(R_2\) and \(R_3\) change sensitively as the ratio \(\tilde{F}\) varies, while \(R_1\) is rather insensitive to \(\tilde{F}\). Thus, by measuring \(R_2\) and/or \(R_3\) in experiment, one can determine the value of \(F_{\pi}^{\text{singlet}}(0)\). Once the value of the form factor \(F_{\pi}^{B \to \pi}(0)\) becomes more accurately known through both theoretical and experimental studies, the flavor-singlet form factor \(F_{\pi}^{\text{singlet}}(0)\) can be determined more precisely. We should emphasize that in this method there is no ambiguity related to the unknown \(q^2\) dependence of \(F_{\pi}^{\text{singlet}}\), which can cause a large uncertainty to theoretical calculations of quantities of interest, such as the BRs for \(B \to \eta^{(0)}l\nu\) (See below).

Now we examine the possible contribution of the flavor-singlet term to the BRs for \(B \to \eta^{(0)}l\nu\) (See below).

### Table II: \(R_1\) for \(F_{\pi}^{\text{singlet}}(0) = 0, 0.2, 0.4, 0.6\).

| \(F_{\pi}^{\text{singlet}}(0)\) | 0   | 0.2 | 0.4 | 0.6 |
|-----------------|-----|-----|-----|-----|
| Lattice Model I | 0.67| 1.03| 1.47| 2.00|
| Lattice Model II| 0.67| 1.00| 1.41| 1.88|
| LCSR            | 0.67| 1.00| 1.41| 1.88|
| Average         | 0.67| 1.01| 1.42| 1.91|
TABLE III: $R_2$ for $F_{+}^{\text{singlet}}(0) = 0, 0.2, 0.4, 0.6.$

| $F_{+}^{\text{singlet}}(0)$ | 0     | 0.2   | 0.4   | 0.6     |
|-----------------------------|-------|-------|-------|---------|
| Lattice Model I             | 0.42  | 3.38$^{+1.44}_{-0.77}$ | 9.18$^{+4.85}_{-2.51}$ | 17.85$^{+10.23}_{-5.22}$ |
| Lattice Model II            | 0.42  | 3.07$^{+1.75}_{-0.80}$  | 8.19$^{+5.86}_{-2.58}$  | 15.75$^{+12.32}_{-5.32}$ |
| LCSR                        | 0.42  | 3.07$^{+0.90}_{-0.56}$  | 8.19$^{+2.99}_{-1.81}$  | 15.75$^{+6.26}_{-3.74}$  |
| Average                     | 0.42  | 3.14$^{\pm 0.70}_{\pm 2.33}$ | 8.42$^{\pm 2.33}_{\pm 4.90}$ |

TABLE IV: $R_3$ for $F_{+}^{\text{singlet}}(0) = 0, 0.2, 0.4, 0.6.$

| $F_{+}^{\text{singlet}}(0)$ | 0     | 0.2   | 0.4   | 0.6     |
|-----------------------------|-------|-------|-------|---------|
| Lattice Model I             | 0.63  | 3.28$^{+0.90}_{-0.55}$  | 6.23$^{+1.66}_{-1.09}$  | 8.93$^{+2.13}_{-1.48}$  |
| Lattice Model II            | 0.63  | 3.07$^{+1.10}_{-0.60}$  | 5.82$^{+2.07}_{-1.21}$  | 8.38$^{+2.68}_{-1.68}$  |
| LCSR                        | 0.63  | 3.07$^{+0.60}_{-0.41}$  | 5.82$^{+1.15}_{-0.82}$  | 8.38$^{+1.51}_{-1.13}$  |
| Average                     | 0.63  | 3.12$^{\pm 0.45}_{\pm 0.86}$ | 5.93$^{\pm 0.86}_{\pm 1.12}$ |

$\eta^{(i)}l\bar{\nu}$. In order to estimate the BRs, one needs to know the $q^2$ dependence of the flavor-singlet form factor $F_{+}^{\text{singlet}}$. Since it is completely unknown, for illustration we assume that $F_{+}^{\text{singlet}}(q^2)$ has the same $q^2$ dependence as $F_{+}^{B \to \pi}(q^2)$. Since the $B^*$ pole contribution to the form factor could be different for the pion and the singlet state, this assumption could cause an uncertainty to the estimate.

In the numerical calculation, we use the average value of the CLEO and LEP data: $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, which does not include the preliminary results from BELLE and BABAR. The estimated BRs of the decays $B^- \to \eta l^-\bar{\nu}$ and $B^- \to \eta' l^-\bar{\nu}$ as a function of $F_{+}^{\text{singlet}}(0)$ are shown in Figs. 3 and 4 respectively. In the figures, the solid line denotes the case of using $|V_{ub}| = 3.6 \times 10^{-3}$, which is the central value of the average of the CLEO and LEP data. The estimated BRs are

$$B(B^- \to \eta l^-\bar{\nu}) = (4.00 \pm 0.99) \times 10^{-5},$$

$$B(B^- \to \eta' l^-\bar{\nu}) = (1.95 \pm 0.48) \times 10^{-5},$$

(14)

for $F_{+}^{\text{singlet}}(0) = 0$. These values are in good agreement with those in Eq. (2). The dotted
TABLE V: BR of $B \to \eta l\nu$ decay for $F^\text{singlet}_+(0) = 0, 0.2, 0.4, 0.6$

| $F^\text{singlet}_+(0)$ | $\mathcal{B}(B \to \eta l\nu) \times 10^5$ |
|-------------------------|----------------------------------|
| Lattice I               | $3.68^{+1.57}_{-1.29}$          |
|                        | $5.70^{+2.43}_{-2.00}$          |
|                        | $8.16^{+3.48}_{-2.86}$          |
|                        | $11.06^{+4.72}_{-3.88}$         |
| Lattice II              | $4.42^{+1.89}_{-1.55}$          |
|                        | $6.67^{+2.85}_{-2.34}$          |
|                        | $9.37^{+4.00}_{-3.29}$          |
|                        | $12.52^{+5.34}_{-4.39}$         |
| LCSR                    | $4.04^{+1.72}_{-1.42}$          |
|                        | $6.08^{+2.59}_{-2.14}$          |
|                        | $8.53^{+3.64}_{-2.99}$          |
|                        | $11.41^{+4.86}_{-4.01}$         |
| Average                 | $4.00 \pm 0.99$               |
|                        | $6.10 \pm 1.50$               |
|                        | $8.63 \pm 2.13$               |
|                        | $11.60 \pm 2.86$               |

lines denote the case of allowing 1σ error in the CLEO and LEP data: i.e., the upper and lower dotted lines are for $|V_{ub}| = 4.3 \times 10^{-3}$ and $2.9 \times 10^{-3}$, respectively. We also present the case of assuming 10% error in the value of $|V_{ub}|$ as the dashed lines: i.e., the upper and lower dashed lines are for $|V_{ub}| = 3.96 \times 10^{-3}$ and $3.24 \times 10^{-3}$, respectively. For some representative values of $F^\text{singlet}_+(0)$, the BRs of $B \to \eta^{(i)} l\nu$ estimated in each model are shown in Tables V and VI. We see that the estimated BR of $B \to \eta l\nu$ is particularly sensitive to the effect of $F^\text{singlet}_+(0)$: for example, the BR increases from $(1.95 \pm 0.48)$ to $(14.99 \pm 3.70)$ as $F^\text{singlet}_+(0)$ varies from 0 to 0.2. This feature can be easily understood by the fact that the flavor-singlet component of $\eta'$ occupies more contribution than that of $\eta$.

Assuming the $q^2$ dependence of $F^\text{singlet}_+$ is correct, Figs. 3 and 4 could provide another way of determining the value of $F^\text{singlet}_+(0)$. That is, one can measure the BR of $B^- \to \eta^- \bar{\nu}$ and/or $B^- \to \eta' l^- \bar{\nu}$ in experiment and then could determine $F^\text{singlet}_+(0)$ by using Fig. 3 and/or 4. For that purpose, the process $B^- \to \eta' l^- \bar{\nu}$ would be more useful than $B^- \to \eta^- \bar{\nu}$, due to the strong sensitivity of its BR to $F^\text{singlet}_+(0)$ and the smaller uncertainty arising from the error in $|V_{ub}|$, as shown in Fig. 4.

To avoid the large uncertainty in the parameter $|V_{ub}|$, the ratios of the BRs, $\mathcal{B}(B^- \to \eta^{(i)} l\nu)$ and $\mathcal{B}(B^- \to \pi^0 l\nu)$, have been suggested in Ref. [9]. They are now modified after including the flavor-singlet form factor as

$$R_\eta = \frac{\mathcal{B}(B^- \to \eta l\nu)}{\mathcal{B}(B^- \to \pi^0 l\nu)} = \left( \frac{f_{\eta u\bar{d}}}{f_\pi} + \sqrt{2} F_\eta \left( \sqrt{2} f_{\eta u\bar{d}} + f_{\eta s} \right) \right)^2 \frac{\int_0^{m_B-m_\eta} dq_1^2 |F^{\pi \to \eta}(q)|^2 [(m_B^2 + m_\eta^2 - q^2)^2 - 4m_B^2 m_\eta^2]^{3/2}}{\int_0^{m_B-m_\pi} dq_2^2 |F^{\pi \to \eta}(q)|^2 [(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2]^{3/2}}.$$
method mainly comes from the uncertainty in \( F \) factorization scheme. The uncertainty involved in determination of \( F \) contribution, which arises from the effect of the two-gluon emission in a decaying \( B \) meson. Therefore, as the more accurate

\[
\mathcal{R}_\eta \equiv \frac{B(B \to \eta l\nu)}{B(B \to \pi l\nu)} = \frac{\int dq^2 |F_{\pi \to \eta}^B|^2 |F_{\eta \to \pi}^B|^2 \left( |f_{\eta l}^s| \pm |f_{\eta l}^u| \right)^2}{\int dq^2 |F_{\pi \to \pi}^B|^2 |F_{\pi \to \eta}^B|^2 \left( |f_{\eta l}^s| \pm |f_{\eta l}^u| \right)^2} \times \frac{\int dq^2 |F_{\pi \to \eta}^B|^2 |F_{\eta \to \pi}^B|^2 \left( |f_{\eta l}^s| \pm |f_{\eta l}^u| \right)^2}{\int dq^2 |F_{\pi \to \pi}^B|^2 |F_{\pi \to \eta}^B|^2 \left( |f_{\eta l}^s| \pm |f_{\eta l}^u| \right)^2},
\]

where the dependence on \(|V_{ub}|\) cancels out between the numerator and the denominator.

Tables VII and VIII show our predictions of \( \mathcal{R}_\eta \) and \( \mathcal{R}_{\eta'} \), respectively, in Lattice Model I, II and LCSR for \( F_{+}^{\text{singlet}}(0) = 0, 0.2, 0.4, 0.6 \), where we have used the average value of \( F_{\pi \to \pi}^B(q^2) \) with the 1σ error for \( F_{\pi \to \pi}^B(0) \) and \( \alpha_B \) given in Table I. In Figs. 5 and 6, we present the predicted \( \mathcal{R}_\eta \) and \( \mathcal{R}_{\eta'} \) as a function of \( \tilde{F} \equiv F_{+}^{\text{singlet}}(0) / F_{\pi \to \pi}(0) \). Here the uncertainty in \( \alpha_B \) shown in Table I has been also considered, and denoted as the dotted line (\( \alpha_B = 0.50 \)) and the dashed line (\( \alpha_B = 0.28 \)). The dependence of \( \mathcal{R}_{\eta'}(0) \) on the uncertainty in \( \alpha_B \) is rather weak. As expected, \( \mathcal{R}_{\eta'} \) are particularly sensitive to \( \tilde{F} \). Again, assuming the relation (13) holds for the singlet state as well, Figs. 5 and 6 could provide another alternative way of determining the value of \( F_{+}^{\text{singlet}}(0) \), without suffering the large uncertainty in \(|V_{ub}|\): i.e., one could determine the value of \( F_{+}^{\text{singlet}} \) by measuring \( \mathcal{R}_\eta \) and/or \( \mathcal{R}_{\eta'} \) and using Fig. 5 and/or 6.

In conclusion, we studied semileptonic decays \( B \to \eta(0) l\nu \), considering the flavor singlet contribution, which arises from the effect of the two-gluon emission in a decaying \( B \) meson. Using \( B^- \to \eta(0) l\bar{\nu} \) (and \( B^- \to \pi^0 l\bar{\nu} \)) decays, we demonstrated how to determine the flavor-singlet form factor \( F_{+}^{\text{singlet}} \) whose unknown value is one of the main sources of the large uncertainty in theoretical estimates of the BRs for \( B \to \eta' K \) decays in the QCD factorization scheme. The uncertainty involved in determination of \( F_{+}^{\text{singlet}}(0) \) using our method mainly comes from the uncertainty in \( F_{\pi \to \pi}^B(0) \). Therefore, as the more accurate
TABLE VII: $\mathcal{R}_\eta$ for $F^\text{singlet}_+(0) = 0$, 0.2, 0.4, 0.6.

| $F^\text{singlet}_+(0)$ | 0      | 0.2    | 0.4    | 0.6    |
|-------------------------|--------|--------|--------|--------|
| Lattice Model I         | 0.55 ± 0.01 | 0.85$^{+0.10}_{-0.07}$ | 1.22$^{+0.25}_{-0.16}$ | 1.65$^{+0.45}_{-0.24}$ |
| Lattice Model II        | 0.54$^{+0.02}_{-0.03}$ | 0.82$^{+0.13}_{-0.09}$ | 1.15$^{+0.31}_{-0.17}$ | 1.54$^{+0.54}_{-0.28}$ |
| LCSR                    | 0.56$^{+0.01}_{-0.02}$ | 0.84$^{+0.07}_{-0.06}$ | 1.18$^{+0.16}_{-0.12}$ | 1.57$^{+0.29}_{-0.20}$ |
| Average                 | 0.55 ± 0.01 | 0.84 ± 0.05 | 1.19 ± 0.12 | 1.58 ± 0.22 |

TABLE VIII: $\mathcal{R}_{\eta'}$ for $F^\text{singlet}_+(0) = 0$, 0.2, 0.4, 0.6.

| $F^\text{singlet}_+(0)$ | 0      | 0.2    | 0.4    | 0.6    |
|-------------------------|--------|--------|--------|--------|
| Lattice Model I         | 0.27$^{+0.01}_{-0.02}$ | 2.18$^{+0.94}_{-0.53}$ | 5.92$^{+3.14}_{-1.56}$ | 11.49$^{+6.61}_{-3.46}$ |
| Lattice Model II        | 0.26 ± 0.02 | 1.94$^{+1.12}_{-0.55}$ | 5.17$^{+3.72}_{-1.71}$ | 9.95$^{+7.82}_{-3.51}$ |
| LCSR                    | 0.23$^{+0.01}_{-0.03}$ | 2.03$^{+0.61}_{-0.40}$ | 5.41$^{+2.00}_{-1.26}$ | 10.41$^{+4.18}_{-0.59}$ |
| Average                 | 0.26 ± 0.01 | 2.05 ± 0.47 | 5.49 ± 1.54 | 10.59 ± 3.22 |

value of $F^B_+(0)$ becomes available, $F^\text{singlet}_+(0)$ can be more precisely determined in forthcoming studies. We also calculated the BRs for $B^- \to \eta' l \bar{\nu}$ and examined how large the effect of $F^\text{singlet}_+$ on these BRs can be. Our result shows that the estimated BR for $B^- \to \eta' l \bar{\nu}$ strongly depends on the effect of $F^\text{singlet}_+$.

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FIG. 2: The predicted $R_1$, $R_2$ and $R_3$ versus the ratio $\tilde{F} \equiv F_+^{\text{singlet}}(0)/F_{+}^{B \rightarrow \pi}(0)$.

FIG. 3: BR (in $10^{-5}$) of $B^- \rightarrow \eta l^\nu$ versus the flavor-singlet contribution $F_+^{\text{singlet}}(0)$. The solid line is for $|V_{ub}| = 3.6 \times 10^{-3}$ which is the central value of the average, $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, of the LEP and the CLEO data. The region between the dotted lines corresponds to the uncertainty arising from 1σ error in $|V_{ub}|$. For the dashes lines, we have assumed 10% error in $|V_{ub}|$. 
FIG. 4: BR (in $10^{-5}$) of $B^- \to \eta l^- \bar{\nu}$ versus the flavor-singlet contribution $F^\text{singlet}_+(0)$. The definition of the lines are the same as in Fig. 4.

FIG. 5: $R_\eta$ versus $\tilde{F} \equiv F^\text{singlet}_+(0)/F^B_{+\to\pi}(0)$. The solid, the dotted, and the dashed lines correspond to $\alpha_B = 0.39$, $0.50$, $0.28$, respectively.
FIG. 6: $R_{\eta'}$ versus $\tilde{F} = F_+^{\text{singlet}}(0)/F_+^{B \to \pi}(0)$. The solid, the dotted, and the dashed lines correspond to $\alpha_B = 0.39$, 0.50, 0.28, respectively.