The theory of transvers Nernst–Ettingshausen effect and thermopower of heated (hot) charge carriers in nondegenerate semiconductors in the case of high anisotropy of phonons distribution function

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Abstract

The transverse Nernst–Ettingshausen (NE) effect and thermopower of hot charge carriers in nondegenerate impure semiconductors placed at high electric and nonquantized magnetic fields in the nondiffusion approximation is studied. Arbitrary heating and mutual drag of electrons and long wavelength phonons interacting with electrons are considered.

The spectrum of electrons is assumed to be strong nonparabolic in Kane two–band approximation.

The case when the electron concentration is high and frequent interelectronic collisions lead to the equilibrium symmetric part of the electron distribution function with effective electron temperature is considered.

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1 Introduction

The theoretical and experimental interest in thermoelectric power in bulk and recent low dimensional systems has been intensified [1]–[12].

A relatively long survey of literature and some common misunderstandings in the field of thermoelectric power and NE effect under different transport conditions [3, 7, 8, 9, 15]–[23] is given in our recent paper [42].

X. L. Lei [11] showed that the diffusion component of thermoelectric power may be negative within a low enough lattice temperature range at high electric field while the phonon drag component is still positive. It should be noted that such a result is obtained by Babaev and Gassymov [20]. In that paper, the thermoelectric power and transverse Nernst–Ettingshausen (NE) effect in semiconductors at high electric and nonquantizing magnetic fields are theoretically investigated by solving the coupled system of kinetic equations for electrons and phonons by taking into account the heating of electrons and phonons, and the phonon drag. It is shown that when the temperature gradient of hot electrons is produced only by the lattice temperature gradient, $\nabla E = 0$, $\nabla T_e = \frac{\partial T_e}{\partial T} \nabla T$, the electronic parts of the thermoelectric and NE fields reverse their sign. In the case of heated phonons, when $T_p = T_e \gg T$ both the electronic and phonon parts of the thermoelectric and thermomagnetic fields reverse their sign for all cases considered. Here $T_e$, $T_p$ and $T$ are the temperature of electrons, phonons and lattice in energy units, respectively.

The interest in the study of thermoelectric power and NE effect in II–VI semiconductors has been increased [31]–[34]. The earlier investigations of the magnetic field dependence of the longitudinal NE effect in HgSe [35, 36] and lead chalcogenides [37, 38] in the region of comparatively high temperatures ($T \geq 77K$) demonstrated that the thermoeff displays saturation in the classical region of strong magnetic fields ($H$) irrespective of the dominant mechanism of charge carrier scattering in the conduction band. However, the measurements of the longitudinal NE effect in iron–doped HgSe samples at low temperatures ($20 \leq T \leq 60K$) revealed the presence of a maxima in the plot of $\Delta \alpha(H) = |\alpha(H) - \alpha(0)|$. $\Delta \alpha(H)$ first increases quadratically with increasing $H$ for $\omega_H \tau < 1$ (here $\omega_H = \frac{eH}{mc}$ is the cyclotron frequency and $\tau$ is the electron relaxation time), then passes through a maximum for some $H = H_m$, and finally decreases as the field increases further. Another unusual fact is the sign reversal of the transverse NE coefficient $Q_\perp(H)$ with increasing magnetic field in the range $\omega_H \tau > 1$ [33, 34]. The experiments in Ga–doped HgSe showed that at low temperatures the NE coefficients change sign with increasing Ga concentration or the applied magnetic field strength. The unusual features of the NE effect observed in HgSe crystals may be attributed to the effect of mutual drag which can experimentally be detected in semiconductors with high concentration of conduction electrons [33].

In the absence of external magnetic field, the thermopower of hot electrons taking into account the heating of phonons and the thermal drag is considered in [13]. The consideration is made for the deformation potential of interaction between electrons and phonons. The thermopower and transverse NE effect of hot electrons in nondegenerate semiconductors are studied in [11] without taking into account the effect of phonon drag and their heating; and in [20] by taking into account the thermal drag only in transverse magnetic field. Neither in these studies the mutual drag of charge carriers and phonons are considered.
A consistent microscopic theory of transport phenomena in semiconductors and semimetals in high external electric and magnetic fields with due regard for the heating of charge carriers and phonons, their thermal and mutual drags, and the possible phonon generation by the drift charge carriers must be based on the solution of coupled system of kinetic equations for charge carriers and phonons. Such a problem is formulated and solved for the first time by Gassymov[28], see also reference [27]. In the statement of the problem, it should be noted that the traditional approximation of small anisotropy of phonon distribution function (so–called “diffusion approximation”) is applicable to phonons whose drift velocities ($u$) is much smaller than the sound velocity ($s_0$) in crystal. In the presence of external electric and magnetic fields, this condition obviously is not fulfilled. This violation shows up particularly in several ways under the acoustical instability conditions ($u \geq s_0$). Actually, both spherically symmetric, $N_s(q)$, and antisymmetric, $N_a(q)$, parts of the phonon distribution function as well as $\frac{N_a(q)}{N_s(q)}$ grow as $u$ increases. Indeed, $\frac{N_a(q)}{N_s(q)} \rightarrow 1$ as $u \rightarrow s_0$, and $\frac{N_a(q)}{N_s(q)} \gg 1$ when $u \gg s_0$. The general solution of the Boltzmann equation for phonons shows that $N(q)$ is stationary for $u < s_0$, and nonstationary for $u \geq s_0$. These results are obtained by solving the nonstationary kinetic equation for phonons interacting with charge carriers at high electric and arbitrary magnetic fields in the nondiffusion approximation[27, 28, 29].

The theoretical investigation of any thermo and galvanomagnetic effects is usually based on solving the kinetic equations of electrons and phonons in so–called “diffusion approximation (DA)”. It is the approximation of small anisotropy of distribution function (DF) of charge carriers and phonons and it is applicable when the drift velocities of the carriers $v(\varepsilon)$ and phonons $u$ are much less than their thermal (chaotic) velocities $v_T$ and $s_0$ respectively. Here $v_T = (2T/m_n)^{1/2}$ is the thermal velocity of charge carriers, $m_n$ is the effective mass of electrons at the bottom of the conduction band. It was shown in [29] at high external electric field under the conditions of strong mutual drag at low temperatures and near the point of acoustic instability threshold (AIT) for arbitrary temperatures DA is not applicable for the phonons. Because, the drift velocity of phonons under these conditions can be equal or more than the velocity of sound $s_0$ and the condition $u \ll s_0$ for DA is violated. On the other hand, under the same conditions the DA can be applicable for electrons because in this case $v_T/s_0 \approx 10^2, 10^3$.

For the conservation of usual scheme of consideration, we separate the distribution function of phonons, $N(q)$, into its symmetric, $N_s(q)$, and antisymmetric, $N_a(q)$, parts

$$N(q) = N_s(q) + N_a(q).$$

Note that in this case the relative values of $N_s(q)$ and $N_a(q)$ are arbitrary in contrast to DA when we assume that $N_a(q) \ll N_s(q)$, which is equivalent to the assumption $u \ll s_0$. Therefore, in the case considered DA is applicable for electrons (charge carriers) and it is not applicable for phonons. In the light of foregoing discussion we will search the DF of electrons in the form:

$$f(\varepsilon, r) = f_0(\varepsilon, r) + f_1(\varepsilon, r), \quad |f_1(\varepsilon, r)| \ll f_0(\varepsilon, r), \quad (2)$$
where \( f_0(\varepsilon, r) \) and \( f_1(\varepsilon, r) \) are the spherically symmetric and antisymmetric parts of the distribution function of electrons, respectively.

The present work is dealing with theoretical investigation of the transverse Nernst–Ettingshausen (NE) effect and thermopower of hot charge carriers in nondegenerate impure semiconductors placed at high electric \( E \) and nonquantized magnetic fields \( \mathbf{H} \) in nondiffusion approximation. The arbitrary heating and mutual drag of electrons and long wavelength (LW) phonons interacting with electrons are considered. The phonon part of the NE voltage differs from zero only for the nonparabolic spectrum of electrons in that the spectrum of electrons is assumed to be strong nonparabolic in Kane two–band approximation:

\[
p(\varepsilon) = \mu \varepsilon^s,
\]

where \( \mu = \sqrt{2m_n} \), \( s = \frac{1}{2} \) for parabolic, and \( \mu = \sqrt{\frac{2m_n}{\varepsilon_g}} \), \( s = 1 \) for strong nonparabolic spectrum of electrons with band gap \( \varepsilon_g \). Consider the case of high concentration of electrons when the frequent interelectronic collisions lead to the equilibrium symmetric part of electrons distribution function with effective temperature \( T_e \gg T_f(\varepsilon) = \frac{2\pi^2}{3} \hbar^3 n_0^{\frac{2}{3}} T_e^{\frac{1}{3}} \exp \left( -\frac{\varepsilon}{T_e} \right) \),

\[
F_0(T_e) = \frac{s \Gamma(3s)}{\sqrt{2}} \Lambda^\frac{s}{2} \Theta_e^{3(s-\frac{1}{2})}.
\]

where \( \Lambda = \frac{\mu^2 T^2 s-1}{m_n} \), \( \Theta_e = \frac{T_e}{T} \) is the dimensionless temperature of electrons, \( n_0 \) is electron concentration and \( \Gamma(x) \) is the Euler’s gamma function.

### 2 The Main Equations and Their Solutions

From the stationary kinetic equation of phonons in the presence of space nonhomogeneity by taking into account Eqs. (1) and (2), we have the following system of equations for \( N_a(\mathbf{q}) \) and \( N_s(\mathbf{q}) \):

\[
N_a(\mathbf{q}) + \frac{s_0}{\beta(q)} \frac{\mathbf{q} \nabla N_a(q)}{q} = \frac{u}{s_0} N_s(q) \cos \gamma,
\]

\[
N_s(\mathbf{q}) + \frac{s_0}{\beta(q)} \frac{\mathbf{q} \nabla N_a(q)}{q} = N(q, \bar{T}) + \frac{u}{s_0} N_a(q) \cos \gamma,
\]

where \( \gamma \) is the angle between \( \mathbf{q} \) and \( \mathbf{u} \), \( \beta(q) = \beta_e(q) + \beta_{pb}(q) \) is the total collision frequency of phonons with scattering centers, and \( \beta_{pb}(q) = \beta_p(q) + \beta_b(q) \). The subindices of \( \beta \) stand for the scattering of phonons by electrons (e), SW phonons (p), and crystal boundaries (b). \( N(q, \bar{T}) \) is given by
\[ \tilde{N}(q, \tilde{T}) = \frac{\beta_e(q)}{\beta(q)} N(q, T_e) + \frac{\beta_p(q)}{\beta(q)} N(q, T_p) + \frac{\beta_b(q)}{\beta(q)} N(q, T), \]  
\[ \tilde{T} = \frac{\beta_e(q)}{\beta(q)} T_e + \frac{\beta_p(q)}{\beta(q)} T_p + \frac{\beta_b(q)}{\beta(q)} T. \]  

Here \( N(q, T_e), N(q, T_p) \) and \( N(q, T) \) are the Planck distribution functions with \( T_e, T_p \) and \( T \), respectively.

In the absence of space nonhomogeneity, from Eq. (5) we have

\[ N_s(q) = \frac{\tilde{N}(q, \tilde{T})}{1 - \left( \frac{u}{s_0} \right)^2 \cos^2 \gamma}, \quad N_a(q) = \frac{\tilde{N}(q, \tilde{T})}{1 - \left( \frac{u}{s_0} \right)^2 \cos^2 \gamma} s_0 \cos \gamma. \]  

From Eq. (7), for the total distribution function of phonons we obtain

\[ N(q) = N_s(q) + N_a(q) = \frac{\tilde{N}(q, \tilde{T})}{1 - \frac{u}{s_0} \cos \gamma}. \]

The Eqs. (6)–(8) were obtained for the first time in [28].

In the presence of space nonhomogeneity, the symmetric part of the distribution function \( N_s(q) \) is defined by Eq. (7), but the anisotropic (antisymmetric) part has the form

\[ N_a(q) = \frac{u}{s_0} N_s(q) \cos \gamma - \frac{s_0}{\beta(q)} q \nabla N_s(q) \cdot q. \]  

Choosing \( f_1(\varepsilon) = p \nu(\varepsilon) \left( -\frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) \), and substituting Eqs. (1), (2), (7) and (9) into the stationary kinetic equation of electrons, the drift velocity of electrons is found to be

\[ \nu(\varepsilon) - \frac{\omega_H(\varepsilon)}{\nu(\varepsilon, u)} [\hbar \nu(\varepsilon)] + \frac{1}{m(\varepsilon) \nu(\varepsilon, u)} \left\{ eE' + \left( \frac{\varepsilon - \zeta(T_e)}{T_e} \right) \nabla T_e + \frac{4\pi m^2(\varepsilon)}{(2\pi \hbar^3)^{p(\varepsilon)}} \int_0^{2\pi} dq \, W_q \, q^2 \delta(u) \frac{s_0}{\beta(q)} \hbar \omega_q \nabla N(q, T_p) - q u(q) N(q, T_p) \right\}, \]

where, \( H \) \( E' = E + E_T + \frac{1}{e} \nabla \zeta(T_e) \), \( E_T \) is the thermomagnetic field, \( \zeta(T_e) \) is the chemical potential of hot electrons, and \( \nu(\varepsilon, u) = \nu_p(\varepsilon, u) + \nu_i(\varepsilon, u) \) is the total collision frequency of electrons by scatterers. Note that the subscript \( p \) stands for phonons and \( i \) for impurity ions. As it is shown in [28]

\[ \nu_p(\varepsilon, u) = \nu_p(\varepsilon) \delta(u), \quad \delta(u) = \frac{3s_0^2}{u^2} \left( \frac{s_0}{2u} \varphi(u) - 1 \right), \quad \varphi(u) = \ln \frac{1 + \left( \frac{u}{s_0} \right)}{1 - \left( \frac{u}{s_0} \right)}. \]  

Since the isotropic part of the distribution function of electrons \( f_0(\varepsilon) \) is assumed to be the equilibrium one with effective electron temperature \( T_e \), the kinetic equation for the
Later. In other words, the kinetic equation for the $f_0(\varepsilon)$ is a single equation which connects $T_e$ with the amplitude of external electric and magnetic fields. For further calculations let represent $\nu(\varepsilon)$ and $\beta(q)$ in the form:

$$
\nu(\varepsilon) = \nu(T) \left(1 + 2 \frac{\varepsilon}{\varepsilon_g} \right)^{-r} \left(\frac{\varepsilon}{T}\right)^{-r}, \quad \nu(T_p) = \nu_0(T) \left(\frac{T_p}{T}\right)^{\ell},
$$

(12)

$$
\beta(q) = \beta(\varepsilon_g, T) \left(\frac{qs}{T}\right)^{\kappa} \Theta^{-n}, \quad \beta(\varepsilon_g, T) = \beta(T) \left(\frac{T}{\varepsilon_g}\right)^{n(s-1/2)}
$$

here $\kappa = 0, 1, t$, in the case of scattering of phonons by the boundaries of the specimen for the energy and momentum transfer, by SW phonons and electrons, respectively. $n = 2 - s$ for the scattering of LW phonons by electrons and $n = 0$ for other cases; $r$ is the parameter for the scattering of electrons for the momentum transfer and $W_q = W_0 q'$ is a constant of interaction with $t = 1$ for deformation, and $t = -1$ for piezoelectric interaction of electrons with acoustical phonons.

By using the solution of Eq. (10) for $V(\varepsilon)$, we can calculate the current as follows:

$$
\mathbf{J} = \sigma_{1i} \mathbf{E}' + \sigma_{12} [\mathbf{h}\mathbf{E}'] + \sigma_{13} \mathbf{h} [\mathbf{h}\mathbf{E}'] + \beta_{11}^t \nabla T_e + \beta_{12}^t [\mathbf{h}\nabla T_e] + \beta_{13}^t \mathbf{h} [\mathbf{h}\nabla T_e] + \beta_{11}^q \nabla \mathbf{p} + \beta_{12}^q [\mathbf{h} \nabla \mathbf{p}] + \beta_{13}^q \mathbf{h} [\mathbf{h} \nabla \mathbf{p}];
$$

(13)

where,

$$
\sigma_{1i} = \int_0^\infty d\varepsilon \, \Psi(\varepsilon) a_i(\varepsilon), \quad \beta_{1i}^t = \frac{1}{e} \int_0^\infty d\varepsilon \, a_i(\varepsilon) \left(\frac{\varepsilon - \zeta(T_e)}{T_e}\right),
$$

(14)

$$
\beta_{1i}^q = \frac{4\pi s \bar{a}(u)}{(2\pi)^3} \int_0^\infty d\varepsilon \, \Psi(\varepsilon) a_i(\varepsilon) m^2(\varepsilon) \int_0^{2p} dq \, \frac{W_q \hbar \omega_q}{\beta_q} \left|\nabla N(q, T_p)\right|,
$$

$$
\Psi(\varepsilon) = \frac{\nu(\varepsilon, u) p^3(\varepsilon)}{3\pi^2 \hbar^3 (1 - \gamma)(1 + \Omega^2) m(\varepsilon) \left[\omega_H^2(\varepsilon) + \nu^2(\varepsilon, u)\right]}, \quad a_1(\varepsilon) = \left(1 - \frac{\Omega \omega_H(\varepsilon)}{\nu(\varepsilon, u)}\right),
$$

(15)

$$
\gamma = \frac{1}{a_4} \int_0^\infty d\varepsilon \, \lambda(\varepsilon), \quad a_4 = \int_0^\infty d\varepsilon \, a_i(\varepsilon) \left(\frac{\partial f_0(\varepsilon)}{\partial \varepsilon}\right),
$$

(16)
\[
\begin{align*}
\Gamma_0 &= \frac{1}{a_4(1-\gamma)} \int_{\epsilon(q/2)}^{\infty} d\epsilon \lambda(\epsilon) \frac{\omega_H^2(\epsilon)}{\nu^2(\epsilon, u)}, \\
\lambda(\epsilon) &= \frac{B_0(\epsilon)m^2(\epsilon)\nu^2(\epsilon, u)}{\omega_H^2(\epsilon) + \nu^2(\epsilon, u)} \left( -\frac{\partial f_0(\epsilon)}{\partial \epsilon} \right), \\
B_0(\epsilon) &= \frac{1}{2\pi^2\hbar^3} \frac{m(\epsilon)\delta(u)}{\nu(\epsilon, u) p^3(\epsilon) s_0} \int_0^{2p} dq \beta_e(q) \frac{\hbar \omega_q W_q \tilde{N}(q, T_p)}{\beta(q)} q^2.
\end{align*}
\]

We will calculate the thermomagnetic and thermoelectric quantities for the following cases:

**i.** Electrons transfer their momentum to impurity ions, and phonons are scattered preferably by electrons \( k = t \). In this case the thermal drag of electrons with phonons is prevals.

**ii.** Electrons and phonons are scattered preferably by each other. In this case, the mutual drag of electrons and phonons is important (\( k = t, \frac{\beta_{eb}}{\beta} \gg \frac{v_0(\epsilon)}{\nu_0(\epsilon, u)} \)). In the case of heating of electrons and phonons simultaneously under the mutual drag conditions, the energy gained from the external field by the electron–phonon system transferred to environment through the crystal boundaries. It is connected with two facts. First, the channel of energy and momentum transfer to reservoir stands very narrow. Second, it is necessary to fulfill the stationarity condition of the solution. If the thermal drag dominates, this energy is transferred to LW phonons. Each of the cases mentioned consists of two subcases:

**a.** Electrons are heated by the external field, but phonons are nonheated and have the temperature of lattice, \( T_p = T \).

**b.** Both electrons and phonons are heated, \( T_e = T_p(E, H) \).

We consider the NE effect by taking into account the parts of the NE voltage which are related to the interaction of LW phonons with electrons and LW phonons with SW phonons separately. We also consider some special cases which include the part of the thermopower connected to the interaction of heated LW phonons with SW phonons.

The thermopower of cool and heated charge carriers in degenerate and nondegenerate impure semiconductors near AIT is investigated in detail earlier\[44, 45\].

## 3 Thermopower and Thermomagnetic Coefficients

If \( E \parallel H \parallel \hat{y} \perp \nabla T_e \parallel \hat{z} \), then under the condition of \( \nabla_x T_e = 0 \) (the isothermal condition along \( x \) axis) taking into account \( J_{T_x} = J_{T_z} = 0 \), from Eq. (12) for \( E_{T_x} \) and \( E_{T_z} \) we have

\[
E_{T_x} = H(Q_e \nabla_z T_e + Q_p \nabla_z T_p), \quad Q_{e,p} = \frac{1}{H} \frac{\sigma_{11} \beta_{12}^{(e,p)} - \sigma_{12} \beta_{11}^{(e,p)}}{\sigma_{11}^2 + \sigma_{12}^2},
\]

where \( Q_e \) and \( Q_p \) are the electron and phonon parts of the NE coefficient (in this work, \( E_{T_z} \) is studied only when the magnetic field is parallel to \( \nabla T_z \)), respectively.
At weak magnetic fields, when $\omega_H(\varepsilon) \ll \nu(\varepsilon, u)$ from Eqs. (13) and (19) we get

$$Q_e = \frac{1}{eH} \frac{\omega_H}{\nu(\Theta_e, u)} \Lambda^{r-2} \Theta_e^{2s(2r-4)} b_1(s, r),$$

$$Q_p = Q_{p1}(T_e) + Q_{p2}(T, T_e),$$

where,

$$Q_{p1}(T_e) = \frac{8}{3} \frac{b_2(s, r)}{eH} \frac{\omega_H}{\nu(\Theta_e, u)} \frac{\beta_e(\varepsilon, T)}{\beta^2(\varepsilon, T)} \Lambda^{r} \Theta_e^{2s(2r-1)} \delta(u),$$

$$Q_{p2}(T, T_e) = \frac{2^h-t}{(3+h-t)} \frac{\beta_{ph}(T)}{\beta_e(\varepsilon, T)} B^{h-t} \Theta_e^{2s(h-t-1)} \frac{\nabla z T}{\nabla z T_e}. $$

For the scattering of LW phonons by SW phonons $h = 1$, for all other cases $h = 0$; $\omega_H, \nu(\Theta_e, u)$ and $\beta_e(\varepsilon, T)$ have the meaning of usual definitions in the case of parabolic spectrum, $B(T) = \mu s_0 T^{s-1}$.

$$b_1(s, r) = \frac{1}{2r s^2} \frac{\beta_e(\varepsilon, T)}{\beta^2(\varepsilon, T)} \left\{ \int_0^\infty dx x^{5+s(4r-5)} e^{-x} - [3 + s(2r - 1)] \int_0^\infty dx x^{4+s(4r-5)} e^{-x} \right\};$$

$$b_2(s, r) = \frac{1}{2r \Gamma_1} \left\{ \Gamma(3 + s(4r - 1)) - \frac{\Gamma_4}{\Gamma_1} \int_0^\infty dx x^{4+s(4r-5)} e^{-x} \right\}. $$

When $T_p = T$, the term proportional to $\frac{\nabla z T}{\nabla z T_e}$ falls out. However, in $Q_{p1}(T_e)$, the term $\frac{\beta_{ph}(T)}{\beta_e(\varepsilon, T)}$ must be replaced by $\frac{\beta_e(\varepsilon, T)}{\beta(T)}$. Besides, for $s = 1$, $r = \frac{1}{2}$ and $t = 1$, the integrals in Eqs. (20) and (21) diverge, and it is necessary to give either $s < 1$ or lower limit of the corresponding integrals for some variables. For example, from $x_1 = \frac{\varepsilon(q/2)}{T_e}$. In other words, in this case it is necessary to take into account the inelasticity of the scattering of electrons by LW phonons.

At high magnetic field, when $\omega_H^2(\varepsilon) \gg \nu^2(\varepsilon, u)$ we have

$$Q_e = \frac{2^r \varsigma^2}{eH} \frac{\nu(\Theta_e, u)}{\omega_H} \Lambda^{2-r} \Theta_e^{2s(t-r)-2}$$

$$\left\{ 2(r - 2s + 1) + \frac{\beta_e^2}{\Gamma_2 \beta^2} \frac{\nu_p(\Theta_e, u)}{\nu(\Theta_e, u)} \left[ \int_0^\infty dx x^{3s+1} D(x_1) e^{-x} - \int_0^\infty dx x^{3s} D(x_1) e^{-x} \right] \right\};$$

$$\frac{Q_p}{Q_{p0}(T_e)} = 1 + \frac{\beta(T)}{\beta_0(T)} \frac{\Gamma_2}{\Gamma_3} \Gamma(s(7 + h - t) - 1) - \Gamma(s[11 - 2r + h - t] - 3)$$

$$+ C_1(\Theta_e, u) + C_2(\Theta_e, u).$$

In Eq. (24),
\[ Q_p(\Theta_e) = \frac{2^{3+r} b_3(s, t) \Lambda^{4-r} \nu(\Theta_e, u) \beta_e(T) \beta_e(\varepsilon_g, T)}{3 \Gamma_3 e H \omega_H} \beta^2(\varepsilon_g, T) \delta(u) \Theta_e^{(7-2r)-2}, \]

\[ C_1(\Theta_e, u) = \frac{\nu_p(\Theta_e, u)}{b_3(s, r) \nu(\Theta_e, u)} \left( \int_0^\infty x^{7s-2} D(x_1) \text{e}^{-x} dx - \frac{\Gamma(3s-1)}{\Gamma_3} \int_0^\infty x^{3s} D(x_1) \text{e}^{-x} dx \right), \]

\[ C_2(\Theta_e, u) = \frac{3}{3 + h - t} \frac{\beta_p(T)}{\beta_e(T)} B^{h-t}(T) \Theta_e^{3/2} + \frac{\nu_p(\Theta_e, u)}{\nu(\Theta_e, u)} \frac{\nabla_z T}{\nabla_z T_e} \left( \int_0^\infty x^{7h-t} D(x_1) \text{e}^{-x} dx - \frac{\Gamma(s + h - t - 1)}{\Gamma(3)} \int_0^\infty x^{3s} D(x_1) \text{e}^{-x} dx \right), \]

\[ b_3(s, r) = \frac{\Gamma_2 \Gamma(3s-1)}{\Gamma_3} - \Gamma(s(11 - 2r) - 3), \quad D_{x1} = \frac{1}{\alpha_4} \int_{\varepsilon(q/2)}^\infty x^{s(8+t)-4} \text{e}^{-x} dx, \]

\[ \Gamma_1 = \Gamma(3 + s(2r - 1)), \quad \Gamma_2 = \Gamma(s(7 - 2r) - 1), \quad \Gamma_3 = \Gamma(3s + 1), \quad \Gamma_4 = \Gamma(1 + s(3 + 2r)). \]

It is necessary to note that at weak and strong magnetic fields \( \Omega \ll 1 \), but at strong magnetic field \( \Omega \omega_H(\varepsilon) \) is of the order of 1. Therefore, the terms proportional to \( \Omega \omega_H(\varepsilon) \) or \( \nu_p(\Theta_e, u) \nu(\Theta_e, u) \) are essential under the of mutual drag conditions of electrons and phonons, and in these expressions \( r = -t/2 \). As it follows from Eqs. (20)–(22) and (23)–(29), at weak and high magnetic fields \( Q_p = 0 \) for the case \( s = -\frac{1}{2} \) and \( r = t \) is in accordance with the results of [21–23].

When \( E \parallel H \parallel \nabla T_z \parallel \hat{z} \), by taking into account \( J_z = 0 \), \( E_{Tz} \) has the form

\[ E_{Tz} + \frac{1}{e} \nabla_z \zeta(T_e) = \alpha_e \nabla_z T_e + \alpha_p \nabla_z T_p, \quad \alpha_{e,p} = \frac{\beta^{(e,p)}_{11} + \beta^{(e,p)}_{13}}{\sigma_{11} + \sigma_{13}}, \]

where \( \alpha_e \) and \( \alpha_p \) are the electron and phonon parts of the differential thermopower, respectively.

In order to investigate the thermomagnetic effects by taking into account the heating of electrons and phonons, and their arbitrary degree of mutual drag in the region of drift velocities \( 0 < u < s_0 \) or near the acoustical instability threshold \( (u \rightarrow s_0) \), we must first determine the dependences of \( \Theta_e \) and \( u \) on the electric and magnetic fields.

4 The Nernst–Ettingshausen Voltage and Integral Thermopower

The NE voltage \( U \) and the integral thermopower \( V \), which are interesting from the experimental point of view, have the form:
\[ U = - \int_{0}^{L_x} E_{Tz} \, dx = U_e + U_p^{LW} + U_p^{SW}, \]
\[ V = \int_{0}^{L_z} E_{Tz} \, dz = V_e + V_p^{LW} + V_p^{SW}. \]

Here \( L_x \) and \( L_z \) are the length of the specimen in \( x \) and \( z \) directions. For arbitrary \( r \) and \( s \), in the case of heating of LW phonons we have:

\[ V_e = \frac{T}{e} \left\{ (2rs - s + 3) - \frac{(T_e)}{T} \right\} \Theta_e = V_{e0} \Theta_e, \quad (32) \]
\[ V_p^{LW}(T_e) = \frac{8 \Gamma_4 T}{3 \Gamma_1 e (3s + 1) \beta^2 \varepsilon g(T_e) e \Lambda^3} \delta(u) \Theta_e^{3s+1} = V_{p0}^{LW} \delta(u) \Theta_e^{3s+1}, \]
\[ V_p^{SW}(T_e) = \frac{3}{3 + h - t} \frac{\Gamma(1 + (3 + 2r + h - t))}{\Gamma_4} \Delta T \beta_p(T_e) B^{h-t}(T) V_{p0}^{LW} \Theta_e^{2+(h-t)+1}. \]

where \( \Delta T = T(0) - T(L_z) \). In the absence of heating of LW phonons in the expression of \( V_p \), the term \( V_p^{SW} \) is absent and in Eq. (32) the term \( \frac{T}{\beta^2 \varepsilon g(T_e) e \Lambda^3} \delta(u) \Theta_e^{3s+1} \) in \( V_p^{LW} \) must be replaced by

\[ \Delta T = \frac{\beta^0 \varepsilon g(T_e)}{\beta_p(T_e)} \Theta_e^{3s}. \quad (33) \]

### 4.1 The Weak Magnetic Field Case

1. In the case \( i \) and subcase \( a \), from Eqs. (20), (21) and (32), near AIT we have:

\[ U_e = U_{e0}(T) \Theta_e^{3-s/a}, \quad U_p = U_{p0}(\Delta T) \Theta_e^{1+s/a} \delta(u), \quad (34) \]
\[ V_e = V_{e0} \Theta_e^{1/a}, \quad V_p = V_{p0}(T) \Theta_e^{3s/a} \delta(u), \quad \Theta_e = \frac{A_1}{\Lambda^{1/2} B^{h}(T) \nu_p(s, T) / \beta_p(T) \varphi(u)}. \]

Here \( A \), the digital coefficient, is of the order of 1, and \( a = 2 + s(2 + h + t) \). As it follows from Eq. (34), near AIT under the thermal drag condition and in the absence of heating of LW phonons, \( U_{e,p} \) and \( V_{e,p} \) are defined with \( \frac{\nu_p(T)}{\nu_p(s, T)} \) and drag parameter \( \eta = \frac{\beta}{\beta_pb} \gg 1 \). At AIT and at its nearest neighborhood, both \( U_{e,p} \) and \( V_{e,p} \) do not depend on \( E \) and \( H \) or \( \Theta_e \).

Since in the considered case \( u = s_0 \frac{E}{E_1} \Theta_e^{2-s} \), the condition \( u = s_0 = \text{const.} \) is realized only when \( \Theta_e = \left( \frac{E_1}{E} \right)^{1/2-s} \). In other words, as \( u \to s_0, \frac{E}{E_1} \Theta_e^{2-s} \) must tends to 1. Here \( E_1 \) reads

\[ E_1 = \frac{2^{3/2} s^2 \Gamma(x_1, 4s - 1) \beta(T) m_n \gamma \nu_1(T)}{\Gamma(x_1, 3s + 1) \beta_e(T) e} \Lambda^{1/2}. \quad (35) \]
It is clear from the physical point of view that as \( u \to s_0 \), the drift velocity of electrons does not depend on the energy of charge carriers, and \( v \sim u \to \text{const.} \). Thus

\[
U_e \sim \left( \frac{\nu_e}{\nu_p} \varphi(u) \right)^{3-s/a}, \quad U_p \sim \left[ \frac{\nu_i}{\nu_p} \varphi^{s(n+t)}(u) \right]^{1/a}, \tag{36}
\]
\[
V_e \sim \left( \frac{\nu_i}{\nu_p} \varphi(u) \right)^{1/a}, \quad V_p \sim \left[ \frac{\nu_i}{\nu_p} \varphi^{s(h+t+1)}(u) \right]^{1/a}.
\]

Since at AIT point \( u = s_0 \), \( \varphi(u) \) and \( \eta \to 1 \), the quantities \( U_e, U_p, V_e \) and \( V_p \) saturate, and do not depend on \( E \).

2. In the case \( ii \) and subcase \( a \), as \( u \to s_0 \), \( \Theta_e \) has the form:

\[
\Theta_e = \left( \frac{A_2}{B^{s/a}\eta} \right)^{\frac{1}{2+s(h-1)}}, \quad A_2 = \frac{\Gamma^2(x_1, 1-st) \Gamma_2}{\Gamma^2(x_1, 4s-1) \Gamma_1} - 1. \tag{37}
\]

In the case \( E \parallel H \), \( u \) has the form

\[
\frac{u}{s_0}(1-\gamma_0)\delta(u) = \frac{\Gamma^2(x_1, 1-st) \beta_e}{\Gamma^2(x_1, 4s-1) \beta_m s_0 \nu(s, T)} \frac{e E \Theta_e^{-s(2+4t)}}{\Lambda}, \tag{38}
\]

where \( \gamma_0 = \gamma(H = 0) \). As it follows from Eq. (37), if the coupled system of electrons and phonons is scattered by the crystal boundaries, \( (\beta_p \approx \beta_e) \) for arbitrary dispersion law of electrons, they interact with deformation acoustical or piezo acoustical phonons, and \( \Theta_e \geq 1 \). In the case, when the coupled system of electrons and phonons is scattered by SW phonons \( (h = 1) \), the piezo acoustical scattering mechanism is not limited. Since, for \( s = \frac{1}{2}, t = -1, \beta_p \approx \beta_p, \) the coefficient \( A_2 < 0 \) or \( \Theta_e < 0 \), and we have the “cooling” effect. It is connected with so-called the effect of “electrons runaway” near AIT for the case of scattering of electrons by the nonheated piezo acoustical phonons.[13]

From Eqs. (19), (20), (32), (36) and (38), as \( u \to s_0 \) we have:

\[
U_e \sim \eta^{\frac{3+s(h-1)}{2}} \frac{E_2}{E}, \quad U_p = U_{p0}(\Delta T) \Theta_e^{-s(1+t)} \sim \eta^{\frac{3+s(1+t)}{2}}, \tag{39}
\]
\[
V_e = V_{e0}(T) \Theta_e \sim \eta^{\frac{1}{2+s(h-1)}}, \quad V_p \sim \eta^{\frac{3h}{2+s(h-1)}} \frac{E}{E_2}.
\]

Here

\[
E_2 = \frac{s \Gamma(x_1, 4s-1) \beta(T) m_n s_0 \nu_p(s, T)}{\Gamma(x_1, 1-st) \beta_e(T)} \frac{\Lambda B(T)}{e} \tag{40}
\]

As it is follows from Eq. (39), at weak magnetic fields \( U_p \) and \( V_e \) do not depend on \( E \) at AIT point and its neighbourhood; and tends to their constant values as \( u \to s_0 \) while \( V_p \) and \( U_e \) depends on \( E \) linearly and nonlinearly, respectively. Therefore, in the absence of phonon heating the mutual drag leads to the dependence of thermomagnetic voltages on \( E \) and \( \eta \) near AIT when \( \Theta_e = \text{const.} \) in contrast to thermal drag case. In other words, the
3. The investigation of the thermomagnetic voltage \( U \) and thermopower \( V \) in the thermal drag case when \( T_p = T_e \) denoted that for the drift velocities \( 0 < u < s_0 \), the electronic part of the quantities \( U_e \) and \( V_e \) dominates over the phonon parts \( U_p \) and \( V_p \). The results obtained for \( 0 < u < s_0 \) show that in some cases \( V_p \) and \( U_p \) for SW phonons dominate over that for LW phonons. In fact, for the scattering of phonons by phonons (\( t = 1 \)), and scattering of electrons by piezo acoustical phonons (\( t = -1 \)) when \( s = 1 \), \( V_p^{SW} \) and \( U_p^{SW} \) grow proportionally to \( E \), \( \frac{\nu_e(T)}{\beta_p(T)} \gg 1 \) and \( \frac{m_n v_0^2}{T} \gg 1 \) while \( V_p^{LW} \) and \( U_p^{LW} \) decrease. Here \( v_0 = \left( \frac{\varepsilon_g}{2m_n} \right)^{1/2} \). At the nearest neighbourhood of AIT point, the thermomagnetic voltages are preferably obtained by the ratio \( \frac{\nu_e(T)}{\beta_p(T)} \varphi(u) \). In this case \( V_e \) and \( U_e \) decrease as \( u \to s_0 \), and reach their saturation values at \( u = s_0 \). In other words, the electronic part of the thermopower and thermomagnetic voltages as well as the current saturate when \( u = s_0 \). When \( u = s_0 \), the stimulated emission and the absorption of phonons become equal to each other and the dissipation vanishes. At this point, electrons emit all the power gained from the external field spontaneously like phonons, and the stationarity of state is conserved.

4. In the case of ii and subcase b from Eqs. (20), (23) and (24) with taking into account Eq. (39), it is easy to obtain that \( U_p = \text{const.} \), \( V_p \sim \frac{E}{E_2} \), and \( U_e \sim \frac{E_2}{\eta E} \). In other words, in contrast to the case of thermal drag in the case of the mutual drag, the increase of \( U \) and \( V \) with \( \varphi(u) \) near AIT have a upper limit (in the case of \( T_e = T_p \) in Eq. (39), it is necessary to change the degree of \( \Theta \) by \([1 - s(4 + t)]\)). Besides, at \( T_e = T_p \) the scattering mechanism of electrons by piezo acoustical phonons becomes limited. It means that for the case considered, the heating of phonons removes the effect of “electrons runaway”.

5. High Longitudinal (E \( \parallel \) H) Magnetic Field Case

1. In the case \( i \) and subcase \( a \), from Eqs. (24) and (25) with taking into account \( \Theta_e \), from Eq. (34) as \( u \to s_0 \) we have:

\[
U_p \sim \left( \frac{\nu_e(T)}{\nu_p(s, T)} \frac{1}{\eta} \right)^{2(2s-1)\over a} \varphi(u)^{4s+\frac{1}{a}(h+t-2)}, \quad U_e \sim \left( \frac{\nu_e(T)}{\nu_p(s, T)} \frac{\eta}{\varphi(u)} \right)^{2-\frac{1}{a}},
\]

(41)

\( V_e \) and \( V_p \) are determined by Eq. (34) as in the weak longitudinal magnetic field case. As it follows from Eq. (41) in the case of \( s = 1 \) the \( U_e \) coincides with the expression in the linear theory. In this case for \( s > \frac{1}{2} \), \( U_p \) increases proportionally to \( \eta \frac{\nu_e}{\nu_p} \eta \) irrespective of the values \( h \) and \( t \), and it increases with \( \varphi \) as \( u \to s_0 \).

2. As \( u \to s_0 \) in the case ii and subcase \( a \), from Eqs. (23) and (24) it follows:

\[
U_e \sim \Theta_e^{s(4+t)-1} \delta(u), \quad U_p \sim \Theta_e^{s(7+t)-2} \delta^2(u).
\]

(42)
By determining $\varphi(u)$ for the case $u \to s_0$ from Eq. (32), and inserting it in Eq. (42), we obtain:

$$U_e \sim \Theta_e^{(4+t)} \delta (u) \sim \eta \frac{E}{E_2}, \quad U_p \sim \eta \frac{2^{4+t} \delta (u)}{3^{4+t} \eta}. \quad (43)$$

From the comparison of Eq.(43) with Eq. (39), in contrast to the weak magnetic field case, at $T_e = T$ and in the presence of mutual drag at high magnetic field both electron and phonon parts of the NE voltage increase with $E$ and $\eta$ near AIT. In the weak magnetic field case, on the other hand, under the same conditions $U_e$ decreases with $E$, but $U_p$ tends to saturation, see Eq. (39).

3. At the nearest neighborhood of AIT in the case $i$ and subcase $b$, we find:

$$U_e \sim \varphi(u) \frac{1}{3^{4+t} \eta}, \quad U_p^{LW} \sim \varphi(u) \frac{5}{3^{4+t} \eta}, \quad U_p^{SW} \sim \varphi(u) \frac{2^{4+t} \eta}{3^{4+t} \eta}. \quad (44)$$

The thermopower $V$ in this case is obtained from Eq. (39). As it follows from Eq. (44) for $s = 1$, $U_e = \text{const.}$ and does not depend on $\varphi(u)$, but $U_p$ increases by $\varphi(u)$. When $s = \frac{1}{2}$, $U_p = 0$ while $U_e$ grows as $\varphi(u)$ increases. Thus, at high magnetic field for the thermal drag case the thermomagnetic quantities do not have upper limit as $u \to s_0$ in contrast to the mutual drag case.

4. Consider the case $ii$ and subcase $b$. In this case with taking into account $\Theta_e \gg 1$, we have:

$$\Theta_e = \left( \frac{3 \Gamma(3 - 3[1 + t]) \Gamma^2(x_1, 4s - 1)}{\Gamma^2(x_1, 1 - 3t)} \right) A(T) \frac{3 m_n s_0^3 V_p(s, T)}{4 T} \frac{\beta_e(\varepsilon_g, T) A}{\varepsilon_g} \right)^{-\frac{1}{\eta}}. \quad (45)$$

Namely, in the nearest neighborhood of AIT, $\Theta_e = \text{const.}$ Here, $A(T) = \frac{2 \pi^2 \hbar^3 n_0}{(2 m_n T)^{3/2} F_0}$. At AIT, with due regard for Eqs. (37) and (38) from Eqs. (24) and (25) we have:

$$U_e \sim \eta \frac{E}{E_2}, \quad U_p^{SW} \sim U_p^{LW} \sim \left( \eta \frac{E}{E_2} \right) T, \quad V_p^{SW} \sim V_p^{LW} \sim \eta \frac{E}{E_2}. \quad (46)$$

Thus, at weak as well as high magnetic fields, the mutual drag of electrons and phonons, at AIT point and its nearest neighborhood plays the role of retaining mechanism of regulation for the unlimited grow or fall down of the thermomagnetic coefficients as $u \to s_0$. In the absence of mutual drag, the thermal drag of electrons by phonons, the phonon part of the thermomagnetic coefficients in the limit $u \to s_0$ sharply increase as $\varphi(u) \to \infty$. In the absence of heating of electrons and LW phonons, their mutual drag leads to the same dependences of thermomagnetic effects on $E$ and $\eta$ as in Eqs. (39) and (46) for the nondiffusion approximation. Finally, we have the effect of “nonheated nonlinearity” as a result of the mutual drag considered in [27]. The Eqs. (39) and (46) allow us to have some opinion about the behavior of electron–phonon system at AIT. The system of electrons and phonons coupled by the mutual drag (quasiparticle, electron+phonon) heated under high external electric field in the condition $u \ll s_0$ at AIT point has constant drift velocity $u$, temperature $\Theta_e > 1$ and the saturation values of kinetic coefficients. In other words, when $u = s_0$, electrons emit the power gained from the external electric field like acoustical
phonons and as a result have the stationary state. The state \( u = s_0 \) is a dynamical stationary state. This case is characterized by the constant current \( (J = \text{const.}) \) instead of \( J = 0 \) in the ground state.

6 Thermopower At High E \( \perp \) H Magnetic Field

The dependences of \( V_e \) and \( V_p \) on the external electric and magnetic in the diffusion approximation, and \( V_e \) in the nondiffusion approximation are introduced through \( \Theta_e \) and \( u \). In the case \( E \parallel H \), \( \Theta_e \) and \( u \) do not depend on \( H \), and, thus, \( V_e \) and \( V_p \) do not consists \( H \). Only the magnetic field \( (E \perp H) \) influences the thermopower considerably through the dependences on \( \Theta_e \) in the diffusion, and \( \Theta_e \) and \( u \) in the nondiffusion approximations. The corresponding coefficients of thermopower when \( (E \perp H) \) may be greater significantly than in the case \( (E \parallel H) \). \( V_e \) and \( V_p^{LW} \) parts of the thermopower were investigated in detailed in [45]. Here, we consider the part of the thermopower related to the interaction of LW phonons with SW phonons, i.e., \( V_p^{SW} \).

At high magnetic field \( (E \perp H) \) irrespective of the character of the interaction of electrons by scattering centers, their drift velocity \( v = \frac{cE}{H} \), and, therefore, \( u = \frac{\beta_e cE}{H} = \frac{s_0 E}{E_3} \).

In the case \( i \) and subcase \( b \) for \( 0 < u < s_0 \), we find \( \Theta_e \) as

\[
\Theta_e = \left[ \frac{2^{2/3} \Gamma(x_1, (s-1))}{3} \left( \frac{m_n s_0^2}{\varphi(u)} \right)^{1/2} \frac{A(T) \nu_i(T) \beta_p(T)}{\nu_i(u) \beta_p(u)} \left( \frac{E}{E_3} \right)^3 \right]^{1/(1+s)}. \tag{47}
\]

By using Eq. (47) in Eq. (34), we have

\[
V_p^{SW} \sim \Theta_e^{2+s(t+h)} \delta(u) \sim \left( \frac{\nu_i(T)}{\beta_p(T)} \right)^{2+s(2+h-t)} \left( \frac{s_0 H}{cE} \right)^3 \varphi(u)^{1+s(1+t-h)} \left( \frac{E}{E_3} \right)^{3(1+s)}. \tag{48}
\]

For \( u = \frac{\beta_e cE}{H} \to s_0 \), we have

\[
V_p^{SW} \sim \left[ \left( \frac{\nu_i(T)}{\beta_p(T)} \right)^{2+s(2+h-t)} \varphi(u)^{1+s(1+t-h)} \right]^{1/(1+s)}. \tag{49}
\]

As it follows from (32), for \( t = 1 \) and \( h = 1 \), the dependences of \( V_p^{LW} \) and \( V_p^{SW} \) on \( \varphi(u) \) and \( \frac{s_0 H}{cE} \) are identical, but \( V_p^{SW} \) increases by \( \frac{\nu_i(T)}{\beta_p(T)} \gg 1 \) faster than \( V_p^{LW} \). Near AIT, \( V_p^{SW} \) does not depend on \( \varphi(u) \) and becomes a constant at \( s = 1, t = -1, \) and \( h = 1 \).

If the electrons and phonons are scattered preferably by each other for \( 0 < u < s_0 \) in the case \( ii \) and subcase \( b \), we obtain

\[
\Theta_e = \left\{ sB_1(x_1, t, s) \Gamma_2 A(T) \frac{\beta \nu_p(s, T)}{\beta_e \beta_p(T)} \mu T^{s-1} \left( \frac{\beta}{2 \beta_e} s_0 - \frac{1}{\varphi(u)} \frac{cE}{H} \right)^{1/\beta} \right\}. \tag{50}
\]

Here
\[ B_1(x_1, t, s) = 1 - \frac{1}{\Gamma_2} \frac{\beta_e}{\beta} \int_0^\infty x^3 D(x_1) e^{-x} dx. \] (51)

By taking into account Eqs. (50) and (34), we have:

\[ V_{SW}^{p} \sim \left[ \frac{\nu_p(s, T)}{\beta_p(T)} \left( \frac{\beta}{2 \beta_e} s_0 - \varphi^{-1}(u) \frac{cE}{H} \right) \right]^{\frac{2 + s(r + h - t)}{3(1 + s)}} \left( \frac{s_0 H}{cE} \right)^3 \varphi(u), \] (52)

and near AIT

\[ V_{SW}^{p} \sim \left( \frac{\nu_p(s, T)}{\beta_p(T)} \right)^{\frac{2 + s(2 + h - t)}{3(1 + s)}} \varphi(u). \] (53)

Therefore, both \( V_{SW}^{p} \) and \( V_{LW}^{p} \) increase with \( \nu_p(s, T) \beta_p(T) \gg 1 \) and by \( \varphi(u) \) near AIT.

As it follows the foregoing discussion, the thermomagnetic coefficients near AIT, increase or fall down sharply. But these changes have the relative character and have an upper limit (constant value at AIT). These limitations are connected with the following situations: At AIT, or its nearest neighborhood, the equality \( u = s_0 \) may be satisfied only when \( \beta = \beta_e, \beta = \beta_e + \beta_p \). Therefore as \( u \rightarrow s_0, \beta_p \rightarrow 0 \) while \( \varphi(u) \beta_p = const \).

7 Conclusion

We now give a brief resume of the main results obtained in the present work. It is shown that in the absence of heating of phonons near AIT, the scattering mechanism of electrons by piezoelectric potential of acoustic phonons under the condition of mutual drag of electrons and phonons is unlimited, i.e., there is effect of so-called “electrons runaway” for this scattering mechanism [43]. It is shown that this effect is liquidated by taking into account the heating of phonons. At AIT and its nearest neighbourhood, the cases either the longitudinal \( E \parallel H \) or the strong \( E \perp H \), temperature of electrons is saturated, i.e., it does not depend on \( E \) and \( H \), and can be obtained from the drag parameter \( \eta = \frac{\beta(T)}{\beta_p} \).

In the case of weak or strong magnetic fields and in the absence or presence of phonon heating, the phonon parts of the thermopower and NE voltage grow infinitely near the point of AIT under the condition of thermal drag. If the mutual drag of electrons and phonons prevails, then both in the absence and in the presence of phonon heating the thermoelectric quantities near AIT have the same dependence on \( E \) as in the case of absence of heating of electrons and phonons. In other words, at AIT the electrons and phonons have the same temperature \( T_e \) and drift velocity \( u \) (and cools by approaching AIT point). Namely, at the AIT point the joint temperature \( T_e \) and drift velocity (the current \( J \)) of the system of electrons and phonons coupled by the mutual drag are saturated. In contrast to the case of thermal drag, in the case of the mutual drag of electrons and phonons the thermomagnetic quantities are finite, i.e., have upper limit near AIT.

In the absence of phonon heating, the mutual drag leads to the dependence of the thermomagnetic coefficients on \( E \) and \( \eta \) near AIT, when \( \Theta_e = const \). in contrast to the
thermal drag case. In other words, the mutual drag of electrons and phonons but not their heating plays the role of regulation mechanism for unlimited grow or fall down of galvano and thermomagnetic coefficients near AIT, see [27].

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