Analysis on Effective Coefficient of Porous Continuous Medium in Seepage Field

Xiangyu Luo*, Guoxin Zhang and Yongrong Qiu

State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, China Institute of Water Resources and Hydropower Research, Beijing 100038, China
Email: luo_xy@iwhr.com

Abstract. When a porous continuous medium is in a saturated state, the effect of pore water pressure on its deformation depends on the effective coefficient in the stress equation. Determination of this coefficient is an important subject for analyzing the deformation of porous continuous media in the seepage field. This work considers a porous continuous medium with square pores to determine the effective coefficient by combining theoretical analysis and a numerical test method. The water element analysis method is applied to further verify the value of the effective coefficient and to analyze the influence of pore water pressure on the deformation of a porous continuous medium from a mesoscopic perspective. Then, the correctness of the pore water pressure is verified using the initial strain method. Through the above methods, the action mechanism of pore water pressure on porous continuous media could be further understood. A reasonable and accurate method for calculating the effective coefficient is also provided. The correct method for analyzing the deformation of porous continuous media in the seepage field is also explained.

1. Introduction

A porous continuous medium belongs to the category of porous media, which means that a large number of pores exist in the solid matrix, and these pores are connected by capillary tubes, cracks, etc. However, the pores do not surround the solid medium. Some examples of such porous materials include rocks and concretes. In recent years, many researchers have proven that the generalized effective stress principle proposed by Terzaghi can be applied to porous continuous media, such as rocks, to analyze their deformation characteristics in a seepage field [1, 2]. However, owing to the weak connectivity between pores in the porous continuous medium, an effective coefficient must be added when using the effective stress principle to characterize the extent of the influence of pore water pressure on the deformation of porous media. To characterize the pore water pressure in porous continuous media, researchers [3, 4] have proposed the effective stress equation \( \sigma' = \sigma - \beta p \) for rock mechanics, where \( \beta \) is the effective coefficient. The equation reveals that the deformation characteristics of a porous continuous medium in the seepage field can be reasonably analyzed only by selecting the correct effective coefficient. Therefore, by exploring the action mechanism of pore water pressure in porous continuous media [5], a method for determining the effective coefficient is of great significance for studying the deformation characteristics of porous continuous media in the seepage field [6-9].

To determine the value of the effective coefficient, the factors influencing its value must be determined first to further define its expression. Recently, many studies have been reported on the value of the effective coefficient and its influencing factors. Skemton [3] proposed an equation for solving the effective coefficient, i.e., \( \beta = 1 - C_m/C \), where \( C_m/C \) is the compression coefficient ratio, the compression coefficient of the skeletal material \( C_m = 1/K_m \) is its bulk modulus, and the compression coefficient of the pore structure \( C = 1/K \) is its bulk modulus. Around the same time, ZHU Bofang [10]...
derived the stress–strain relationship of concrete structures in a seepage field and then determined an expression for the effective coefficient. The expression is essentially the same as the aforementioned equation, but it ignores the mechanical properties of the skeletal material in later calculations. Amos Nue [11] obtained a new expression for the effective coefficient through a rigorous derivation using the theory of elastic mechanics based on Biot's consolidation theory. The coefficient was determined to be 1 minus the ratio of the bulk modulus of the rock mass to the bulk modulus of the rock skeleton. After many years of exploration and verification by many researchers, the expression for the effective coefficient has been determined to be \( \beta = 1 - \frac{K}{K_m} \).

After defining the expression for the effective coefficient, its value must be determined according to the materials and the corresponding pore characteristics. However, since the mechanical parameters of the skeletal material of the porous medium are difficult to obtain, to determining the value of the effective coefficient is also difficult. Walsh, Li Chunguang, and Charles et al. [12–14] found that the value of the effective coefficient is related to the pore shape, distribution, and porosity. Among these, porosity is the most important factor affecting the effective coefficient. Recently, Skemton and Li Guangxin [3, 8] and other researchers reported the compression ratios of various materials. For example, concrete has a compression ratio of 0.12, which yields an effective coefficient of 0.88. Similarly, through experimental tests, an effective coefficient of 0.84 for concrete was calculated by the East China Institute of Water Conservancy. K.K. Phan, C.H. ARNS, Y Gueguen, J. Luo, et al. [15-18] studied the K/Km ratio of porous materials through experiments, numerical simulations, and theoretical derivations. Their results showed that the porosity of porous continuous media (rocks, concretes, etc.) is typically less than 0.20, so the obtained compression ratio thereof must be greater than 0.50. Therefore, when the porosity is less than 0.20, the effective coefficient of the porous continuous medium should be less than 0.50. Thus, the value of the effective coefficient for porous media such as concrete remains a controversial.

In summary, the determination of the effective coefficient value is the primary difficulty in analyzing the deformation of porous continuous media in a seepage field. Based on the aforementioned problems, this paper starts from the effect of the pore water pressure on porous continuous media in a seepage field and combines it with the equation for the effective coefficient. The effect of pore water pressure on porous continuous media is analyzed using the initial strain method of pore water pressure, a numerical test method, and a water element analysis method in the seepage field. The value of the effective coefficient is then determined.

## 2. Deformation Analysis of a Porous Continuous Medium in the Seepage Field

A porous continuous medium refers to a structure with randomly distributed pores of different shapes in a continuous medium. This work mainly considered porous continuous medium bodies commonly used in engineering, such as rock mass and concrete. Unlike a granular porous medium (sand), the pores in a porous continuous medium do not surround the solid medium in the structure body. Instead, the continuous solid medium surrounds the pores. In particular, the rock and concrete commonly used in construction have low porosities dense structures, high strength, poor water permeability, and other characteristics. In a water environment, the water in the porous continuous medium is mainly transported to the pores through channels such as capillaries or microcracks, thereby forming a seepage field in the structure. At the same time, the pore water pressure will have a compressive effect on the solid medium, which can be considered a macroscopic deformation of the porous continuous medium in the seepage field. Zhang Guoxin [19] studied and analyzed the deformation of a porous continuous medium in the seepage field in detail. The results showed that the deformation of a porous continuous medium in the seepage field is completely different from that of a granular medium on the mesoscopic level. The macroscopic deformation of a porous continuous medium is mainly due to the compressive effect of the pore water pressure on the skeletal structure, while the macroscopic deformation of a granular medium is mainly caused by the displacement of particles under pore water pressure, and the deformation from granule compression is ignored. Therefore, an effective coefficient value of 1 is not applicable in rock mechanics.
When the porous continuous medium is saturated, the medium mainly deforms under the compressive effect of the pore water pressure on the solid matrix. Thus, the deformation analysis mainly considers this compressive effect, and the effective coefficient of a porous continuous medium is usually less than 1. Zhu Bofang used concrete as an example to derive the stress–strain relationship of a porous continuous medium in a seepage field, as shown in equation (5) (The tangential deformation of the structure is listed here. Please see reference [10] for the detailed derivation process.)

The stress analysis is divided into two parts, as shown in figure 1. Part A shows the stress distribution of the porous continuous medium in the seepage field. The stress can be further decomposed into two parts, B and C. Part B shows the internal pore water pressure on the structure body and the external stress equal to the pore water pressure. Part C illustrates the effective stress of the porous continuous medium. According to the diagram, the stress–strain relationship of a concrete structure in the seepage field can be derived as shown below.

\[
\sigma_{ii}' = \sigma - p, \quad \sigma_{ii} = \sigma_{ii}' + p \quad i=1,2,3
\]  

where \( \sigma_{ii} \) is the total positive stress, \( p \) is the pore water pressure, and \( \sigma_{ii}' \) is the stress after deducting the pore water pressure.

The strain is divided into two parts, namely

\[
\epsilon_{ii}' = \frac{p}{3K_m}
\]

and

\[
\epsilon_{ii}' = \frac{1}{E} \left[ \sigma_{ii} + \mu \left( \sigma_{jj} + \sigma_{kk} \right) \right] - \frac{p}{3K} \quad i, j, k = 1, 2, 3, i \neq j \neq k
\]

where \( E, \mu, \) and \( K \) are the macroscopic elastic modulus, Poisson’s ratio, and bulk deformation modulus, respectively, of the overall material containing pores, cracks, etc. \( K_m \) is the bulk deformation modulus of the skeletal material (matrix material).

\[
\epsilon_{ii} = \frac{1}{E} \left[ \sigma_{ii} + \mu \left( \sigma_{jj} + \sigma_{kk} \right) \right] - p \left( \frac{1}{3K} - \frac{1}{3K_m} \right)
\]

\[
\beta = 1 - \frac{K}{K_m}
\]

The derived equation (5) is the same as the effective coefficient derived by Amos Nue [11]. Therefore, the stress decomposition diagram shown in figure 1 can be used as the theoretical basis for analyzing the deformation of a porous continuous medium in the seepage field.

![Figure 1. Stress decomposition diagram of a porous continuous medium in the seepage field.](image)
3. Determination Method and Verification of the Effective Coefficient

As mentioned above, the effective coefficient expression is determined to be equation (5). The macroscopic mechanical properties of the porous continuous medium can all be obtained using conventional mechanical experiments. However, due to the complex pore characteristics, it is difficult to obtain the mechanical parameters of the matrix material. Therefore, the effective coefficient values could be different for the same material with different porosities. For determining the value of the effective coefficient, in this work, the stress decomposition, shown in figure 1, is considered and the following four methods were combined to further verify the method for calculating and selecting the value of the effective coefficient. Method 1 is the theoretical equation method. Method 2 is the numerical test method. Method 3 is the water element analysis method. Method 4 is the pore water pressure initial strain method.

3.1. Theoretical Analysis Method

The theoretical equation method derives the effective coefficient based on its theoretical principle. At present, there have been many research results on the equation for solving the effective coefficient. The basic idea is to use the Walsh equation to obtain the value of \( K/K_m \) [12], and then obtain the value of the effective coefficient. \( K/K_m \) is a function of pore shape and porosity but ignores the influence of pore distribution on the effective coefficient. For a common porous continuous medium in engineering, since the porosity is relatively low and the connectivity is weak, the influence of the pores on each other could be neglected. Therefore, the equation obtained from the aforementioned idea is applicable for solving the effective coefficient of structures such as rocks and concrete. Furthermore, in this work square pores are used in the numerical analysis to verify the value of the effective coefficient. To synchronize with the numerical analysis process, the following equation is used for solving the effective coefficient based on square pores proposed in [20] (different pore shapes correspond to different effective coefficient equations):

\[
\beta = 1 - \frac{K}{K_m} = 1 - \frac{1}{1 + \alpha \eta} \tag{6}
\]

where \( \eta \) is the porosity, \( \alpha = \frac{2}{5} \frac{1 + \mu_m}{1 - 2 \mu_m} \left[ F(\mu_m, \theta) + H(\mu_m, \theta) \right] \), where \( \mu_m \) is the Poisson’s ratio of the skeletal material (the relationship between the Poisson’s ratio of the matrix and the macro Poisson’s ratio is \( \mu = 0.166 + (1 - \eta/0.604)^{22} (\mu_m - 0.166) \) [21]), \( F(\mu_m, \theta), H(\mu_m, \theta) \) are functions of \( \mu_m \) and \( \theta \). In this work, \( \theta = 0^\circ, 90^\circ \). The specific derivation process of the equation is described in reference [21].

3.2. Numerical Test Method

The numerical test method is based on the theoretical equation of the effective coefficient. A numerical model is established to directly solve the values of \( K \) and \( K_m \) of the porous continuous medium. The effective coefficient is then obtained using equation (5). A continuous structure body containing pores is constructed according to equation (2) and part B in figure 1. The same load \( P \) is applied inside the pores and outside the structure body. After extracting the volume strain from the calculation result, \( K_m \) can then be verified and solved by using equation (2). Similarly, according to equation (7) and part C in figure 1, the value of \( K \) could be verified and solved using the numerical model for solving \( K_m \) and setting the working condition as only applying external load \( P \) on the structure body. Next, the effective coefficient could be calculated using equation (5).

\[
K = \frac{P}{\varepsilon_v} \tag{7}
\]

where \( \varepsilon_v \) is the volume strain, and \( P \) is the external load.
3.3. Water Element

To analyze the influence of pore water pressure on a porous continuous medium, this paper introduces the pore water element, which is a solid element with the mechanical characteristics of water. The specific implementation steps are as follows: (1) Divide the model into solid elements and “pore” elements. (2) Solid element: set the permeability coefficient, i.e., water can seep through, and set the porosity to 0; water element: the permeability coefficient is set as infinitely large, and the porosity is set to 1. (3) During the stress analysis, the pore element is set as an “empty element” before saturation, and as a “water element” after saturation. (4) Weak coupling algorithm: the influence of stress is considered when calculating pore water pressure.

According to the pore-water-pressure simulation method and the variation process described above, when the pores in the structure body are filled with water, the mechanical characteristics of the water in the pores can be expressed as:

$$P = \sigma = \sigma_{xx} = \sigma_{yy} = \sigma_{zz}$$

(8)

where $\sigma$ is the unit stress, and $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are the stresses in three directions $x$, $y$, and $z$, respectively.

The constitutive relation for water is

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ 0 \\ 0 \\ 0 \end{pmatrix} = K_v \cdot \varepsilon_v$$

(9)

where $\varepsilon_v$ is the volume strain of the water element, $\varepsilon_{xx}, \varepsilon_{yy},$ and $\varepsilon_{zz}$ are the strains of the element in three directions, and $K_v=2.18$ GPa is the bulk modulus of water.

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

(10)

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(11)

Equation (11) is the constitutive relation for water.

The above derivation shows how the pore water pressure is considered in the process of numerical modeling. This method can be used to analyze the deformation of a porous continuous medium in a seepage field and to solve the effective coefficient. The specific steps are as follows: First, a porous continuous medium is established. Its elements include solid elements and pore water elements. In a high-pressure water environment, as water continuously permeates the pores of the medium, the volume of the pore water element will first be compressed and then expanded. There are two characteristic volume values during this deformation process: (1) In the initial stage of water pressure loading, the model is subjected to external pressure but no internal pressure (the stress state is the same as part C in figure 1). The volume deformation value of the model is $\varepsilon_v = p/K$; (2) If the external
water pressure is kept constant, as the pores are filled with water and reach the saturation pressure due to water seepage, the model will no longer deform, and the internal and external water pressures are the same. The volume deformation value of the model is \( \varepsilon'_v = p/K_a \). Therefore, by analyzing the deformation of the porous continuous medium in the seepage field using pore water pressure element analysis method, the influencing mechanism of pore water pressure on medium deformation could be explored from the mesoscopic perspective, and the value of the effective coefficient could be determined. The corresponding equation is given below.

\[
\beta = 1 - \frac{K}{K_m} = 1 - \frac{p \cdot K}{p \cdot K_m} = 1 - \frac{\varepsilon'_v}{\varepsilon'_o}
\]  

(12)

3.4. Pore Water Pressure Initial Strain Method

The above three methods can be used to determine the effective coefficient of a porous continuous medium. However, in actual engineering practice, due to the limitations of experimental conditions and the scale of structure body, a porous continuous medium is often considered a non-porous continuous medium when analyzing the model deformation and when establishing the effective stress equation for the analysis. The effective coefficient can be solved by using equation (6). The parameter porosity used in the equation can be obtained from laboratory experiments. To analyze the deformation of a porous continuous medium in the seepage field, the initial strain method proposed in reference [19] is used in this paper. In other words, the effective coefficient is introduced, and the pore water pressure is used as the initial strain for the finite element model, as given by equation (13). By using equation (13), the porous continuous medium can be considered a continuous medium, and the medium deformation in the seepage field can be analyzed.

\[
\{\sigma\} = [D]\{(\varepsilon') - \{\varepsilon_o\}\} = [D][\{B\}][\{\delta\}'] - \gamma \beta pG = [D][\{B\}][\{\delta\}'] - \gamma (1 - \frac{K}{K_m}) pG
\]

(13)

where \( \varepsilon_0 \) is the initial strain, \([D]\) is the elastic matrix, \([B]\) is the strain computing matrix, \(G=(1,1,1,0,0,0)T\), and \(\gamma\) is the volumetric weight of water.

4. Numerical Model and Parameter Selection

The four methods described above can be used to determine the effective coefficient of a porous continuous medium as follows: First, the value of the effective coefficient is determined by using method 1 and 2. Then, the correctness of the effective coefficient is confirmed from a mesoscopic perspective using method 3, and the deformation characteristics are recorded at the same time. Finally, the correctness and applicability of the effective coefficient value are verified using method 4. Methods 3 and 4 in section 2 reveal that by comparing the final deformation \( \varepsilon'_v \) in method 3 and the volumetric deformation in method 4, the correctness of the finite element model in method 4 could be verified.

4.1. Numerical Model Establishment

The numerical calculations in methods 2, 3, and 4 are realized by using the self-developed simulation analysis software SAPTIS [22]. A computational model of the same size is used for all numerical simulations. A 38x38x38 cm\(^3\) cubic model is established for this work. The model is meshed, and the pores are set manually such that the pores are evenly distributed in the model, as shown in figure 2.
4.2. Selection of Mechanical Parameters

In this work, the same numerical model and four methods are used to calculate and determine the value of the effective coefficient. Therefore, the mechanical parameters for each working condition must be the same. First, the mechanical parameters of the skeletal material in the model are determined, as given in Table 1.

| Elastic modulus, $E_m$ (GPa) | Bulk modulus, $K_m$ (GPa) | Poisson’s ratio, $\mu_m$ |
|-----------------------------|--------------------------|--------------------------|
| 66.6                        | 37                       | 0.20                     |

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The steps of the effective coefficient calculation process are as follows:

Method 1: In method 1, the effective coefficient is calculated directly by substituting the porosity obtained from the numerical model and the Poisson’s ratio given in equation (6).

Method 2: Method 2 is implemented in two steps. The first step is to apply a uniform load of 2 MPa both in the pores and on the surface of the structure body. Table 1 lists the mechanical parameters. The numerical calculation result is obtained, and the value of $\varepsilon_v$ is solved. Then, $K_m$ is calculated according to equation (2). The computational results are compared with the input mechanical parameters to verify the correctness. The second step is to apply a uniform load on the surface of the structure body. The mechanical parameters and the numerical result are processed the same way as in the first step. Then, $K$ is calculated according to equation (7). After obtaining the above two results, the effective coefficient can be obtained using equation (5).

Method 3: In method 3, a water element analysis is used, which treats the pores in the model as solid elements. When there is no water present, all the mechanical parameters of these elements are set to 0. When there is water pressure, the element turns into a water element and has the constitutive relation of water. The flow of water in the skeletal material follows Darcy’s law. In the numerical calculation, a special material number (2 given in this paper) is given to the pore water element. The calculation conditions are as follows: The constraint of the bottom z direction (upward) of the model is fixed, and a water level of 200 m is applied to the exterior of the structure. The calculation steps are designed such that the pores are filled with water until they reach a saturation pressure. At this state, the deformation of the model remains unchanged. The numerical calculation results are obtained and the values of $\varepsilon_v^0$ and $\varepsilon_v^s$ are solved. The effective coefficient is then calculated according to equation (11). The effective coefficient is then calculated according to equation (11). The mechanical parameters of the numerical calculation are given in Table 2 below.
Table 2. Mechanical parameters of the model.

| Parameter     | Elastic modulus (GPa) | Poisson’s ratio | Permeability coefficient (m/d) | Porosity | $K_m$ (GPa) | $K_v$ (GPa) |
|---------------|-----------------------|-----------------|--------------------------------|----------|-------------|-------------|
| Matrix element| 66.6                  | 0.20            | 1e−3                           | 0.00     | 37          |             |
| Water element | 0.50                  | 1000            | 1.00                           |          |             | 2.18        |

Method 4: Method 4 is the initial strain method, i.e., the effect of pore water pressure is used as the initial strain and substituted into the stress–strain relationship of the numerical model. In this case, the coefficient before the water pressure item in equation (13) is the effective coefficient. The value of the effective coefficient is obtained according to methods 1, 2, and 3. Then, the final deformation state of the porous continuous medium under pore water pressure is calculated using this method. The volumetric deformation value is extracted from the numerical calculation results and is compared with the calculation results from method 3. Notable, when using method 4 for the numerical simulation, both the calculation model and the mechanical parameters require special treatment, i.e., the pores in the computational model must be filled with solid elements. Thus, the entire medium is a continuous-medium structure, which has the same size as shown in figure 2. Table 3 shows some of the parameters used in the calculation are from the computation results using methods 1, 2, and 3. The calculation conditions are the same as that for method 3, i.e., the constraint on the bottom z direction (upward) is fixed, and a water level of 200 m is applied to the exterior of the structure.

Table 3. Mechanical parameters of the model.

| Elastic modulus (GPa) | Poisson’s ratio | Permeability coefficient (m/d) | Porosity | Effective coefficient |
|-----------------------|-----------------|--------------------------------|----------|-----------------------|
| 54.2                  | 0.193           | 1e−1                           | 0.106    | 0.205                 |

5. Results and Discussion

In this work, the theoretical equation, numerical test, and numerical simulation method are used to solve the effective coefficient from the mesoscopic mechanism of pore water pressure. After obtaining the effective coefficient value, the deformation of the porous continuous medium in the seepage field is analyzed using the initial strain method. The applicability of the effective coefficient for analyzing the deformation of the porous continuous medium in the seepage field is further verified by using the initial strain method in the finite element analysis.

5.1. Analysis of Results

According to the computational model, conditions, mechanical parameters, and results given in section 3, the corresponding values are extracted and combined with the equations given in section 2. The analysis of the results is as follows:

Method 1: The porosity of the numerical model given in figure 2 is 0.106. Combined with Poisson's ratio, the effective coefficient can be obtained: $\beta=0.2098$.

Method 2: $K_m$ is calculated by using the results from step 1. The obtained result is the same as the input parameter $K_m$. By using the numerical calculation result from step 2, $K=29.4$ is obtained, which can be used to obtain $\beta=0.2051$.

Method 3: By substituting the calculation results into equation (12), $\varepsilon_v^0=66.81$, $\varepsilon_v^1=53.34$, and $\beta=0.2017$ are obtained.

Method 4: The effective coefficient value used in method 4 is the average effective coefficient obtained from methods 1, 2, and 3. In the calculation process using the initial strain method, some of the required mechanical parameters are obtained by using methods 1, 2, and 3. The values of $K$ and $K_m$ are obtained in the same way as in methods 1, 2, and 3. Therefore, the mechanical parameters for
method 4 are given in table 3. Table 4 shows the changes in displacement in the x, y, and z directions when using the initial strain method and method 3. The results fully demonstrate that the initial strain method is suitable for analyzing the deformation of the porous continuous medium in the seepage field. Meanwhile, Figure 3 compares the displacement results of the same set of nodes using method 3 and method 4.

![Figure 3](image3.png)

**Figure 3.** Calculation results of method 3 and method 4: Z-direction displacement.

| Displacement (10^{-5}) | Method 3 Water element analysis method | Method 4 Initial strain method |
|------------------------|---------------------------------------|-------------------------------|
| $d_{xx}$               | 3.4055, −3.4055                       | 3.3941, −3.4013               |
| $d_{yy}$               | 3.4055, −3.4008                       | 3.4060, −3.4037               |
| $d_{zz}$               | 0.0000, −6.6523                       | 0.0000, −6.6390               |

The analysis of the results reveals that all calculation methods are essentially based on equation (5). The beginning of method 3 and the first step of method 2 have the same working conditions, i.e., part C in figure 1. In addition, the final state of method 3 and the second step of method 2 have the same working conditions, i.e., part B in figure 1. As the effective values are used during the calculation process, certain errors are inevitable during the process for solving the effective coefficient. The correctness of methods 1, 2, and 3 can be determined from the calculation results. Method 4 is a finite element method based on the continuous medium for analyzing the deformation of the porous continuous medium in the seepage field. The porosity and effective coefficient are introduced during the numerical simulation. This is equivalent to the same pore-water-pressure effect as that in method 3. Thus, the final calculation results of methods 3 and 4 are the same. The stress process curves of the node at the same location in the models of methods 3 and 4 are shown in figure 4. The model where the node is located corresponds to a solid element and a water element in methods 3 and 4, respectively. The final stresses of this node calculated by using methods 3 and 4 are the same.

![Figure 4](image4.png)

**Figure 4.** Stress process curve of node 29660.
5.2. Discussion

In this work, the value of the effective coefficient was calculated and verified based on square pores. The pores in the numerical models are all distributed in an orderly manner. Equation (6) reveals that porosity is one of the main factors influencing the effective coefficient. To verify the applicability and correctness of the method described in this paper, the effective coefficient is calculated with different porosities using the methods described above. All methods gave the same patterns and results as those shown in section 4.1. The porosities of common engineering materials, such as rocks and concrete, are less than 0.20. Therefore, the effective coefficient is mainly verified for a porous continuous medium with a porosity of less than 0.20. Moreover, when the porous continuous medium has low porosity, the porosity and pore shape have little influence on the macroscale Poisson’s ratio $\mu$ and the Poisson’s ratio of the matrix $\mu_m$ [12].

Methods 1 and 2 are both derived based on equation (5). Method 3 is based on the flow and pressure of water in the porous continuous medium[23]. The pore water element method is proposed to solve the effective coefficient from the mesoscopic perspective. The correctness of each method is verified by comparing all three methods. The porosity of a material used in engineering projects can be experimentally obtained in a laboratory setting. The corresponding effective coefficient can then be obtained by using method 1 (different effective coefficient theoretical equations corresponding to different pore shapes), while methods 2 and 3 can be used to further verify the effective coefficient obtained using method 1. However, deformation of a porous continuous medium in a seepage field is analyzed mainly for large-volume structures such as dams, bridge piers, and coasts. The pores in such structures cannot be refined or modeled because of both their scale and quantity of pores. Therefore, a combination of a continuous structure and effective stress is often used to analyze the deformation under the pore water pressure during numerical simulations. Method 4 is the effective stress analysis method in which the numerical model is a continuous medium. The correctness of the initial strain method can be verified from the analysis of calculation results. Therefore, the pore-water-pressure effect on large-volume porous continuous media can be analyzed using method 4.

As the basic equation for calculating the effective coefficient, the correctness of equation (5) has been tested by many researchers. The value of the effective coefficient clearly depends on the value of $K/K_m$, where $K_m$ is the bulk modulus of the skeleton material. When the porosity is equal to zero, the structure body is a solid structure, and $K/K_m=1$; thus, the effective coefficient is zero. There is no effective stress in the structure body, which is consistent with the actual situation. Research has shown that $K/K_m$ and porosity are inversely proportional[11]. When the porosity is low, for example, when the porosity is between 0 and 0.20, the value of $K/K_m$ is usually greater than 0.50. Thus, the effective coefficient must be less than 0.50. The inversely proportional relationship implies that if the porosity gradually increases and approaches 1, the pores will be interconnected. On the other hand, the skeletal structure can be completely divided by the pores at the other extreme. Thus, the entire solid medium becomes uncompressed particles. At this point, the value of $K$ is extremely small and is several orders of magnitude smaller than $K_m$. The value of the effective coefficient can be considered as 1. Therefore, the effective coefficient of porous continuous media that are common in engineering must be less than 0.50.

Normally, the pores in porous continuous media are randomly distributed. In addition to the influence of the pore shape on the effective coefficient, the pore distribution is also an important factor. However, when the porosity is relatively low, the solid medium plays a more dominant role than the pores, and the pore size is smaller than the pore spacing. Therefore, the influence of pore spacing on the effective coefficient can be neglected when the porosity is low. Furthermore, a porous continuous medium is deformed mainly because of the compression of the solid medium is caused by pore water pressure. Thus, the mechanical parameters $K$ and $K_m$ cannot be ignored. In contrast, the deformation of a granular porous medium mainly comes from the displacement between particles. This displacement deformation is much larger than the compression deformation of the particles. Therefore, the effective coefficient is assumed to be 1 by mainly considering the displacement deformation of particles, and $K_m$ can be ignored during the calculation process.
6. Conclusions

Based on the equation for calculating the effective coefficient, four methods are used to analyze and confirm the effective coefficient of a porous continuous medium with certain porosity. The methods of deformation analysis of a porous continuous medium in the seepage field are further explored, and the following conclusions are drawn:

1. This paper proposes a water element analysis method for analyzing the deformation of a porous continuous medium in the seepage field based on previous studies by other researchers. The water element can truly reflect the relationship between the pore water pressure and solid medium in the seepage field, and it can represent the deformation process of the porous continuous medium in the seepage field from the mesoscopic perspective. The state of the pores changes from unsaturated to saturated until a final saturated pressure is reached. In other words, the calculation process evolves from equation (7) to equation (2). The effective coefficient can be solved therefrom.

2. In contrast to a granular porous medium, a porous continuous medium in a seepage field mainly deforms because of the compressive deformation of the solid matrix under water pressure. Common porous continuous media used in engineering, such as rocks and concrete, all have a porosity of less than 0.20. Therefore, the value of the effective coefficient must be less than 0.50 when using the effective stress theory.

3. The initial strain method is a new method for analyzing the deformation of porous continuous media in a seepage field. This method fully considers the deformation characteristics of porous continuous media, and the effective coefficient is introduced during the calculation. By comparing the results, the initial strain method can correctly analyze the deformation of porous continuous media in the seepage field. In engineering applications, numerical simulations of large-volume porous continuous media are limited by their scale and quantity. Thus, they are often considered continuous media. In such cases, the initial strain method is a reasonable analysis method.

This work analyzed and verified the value of the effective coefficient of continuous porous media based on evenly distributed square pores. The pore shape is also an important factor affecting the porous continuous medium. Usually, the pore distribution and shape are both random. Therefore, the value of the effective coefficient for different pore shapes should be further researched and verified in the future.

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