Event-triggered decentralized robust model predictive control for constrained large-scale interconnected systems

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Ling Lu¹, Yuanyuan Zou¹* and Yugang Niu¹

Abstract: This paper considers the problem of event-triggered decentralized model predictive control (MPC) for constrained large-scale linear systems subject to additive bounded disturbances. The constraint tightening method is utilized to formulate the MPC optimization problem. The local predictive control law for each subsystem is determined aperiodically by relevant triggering rule which allows a considerable reduction of the computational load. And then, the robust feasibility and closed-loop stability are proved and it is shown that every subsystem state will be driven into a robust invariant set. Finally, the effectiveness of the proposed approach is illustrated via numerical simulations.

Subjects: Automation Control; Control Engineering; Dynamical Control Systems; Intelligent Systems

Keywords: constraint tightening method; decentralized control; event-triggered control; input-to-state stability; robust model predictive control

1. Introduction

A class of complex large-scale systems composed of several interconnected subsystems has been receiving an increasing attention due to its various practical applications, e.g. power systems, chemical processes, and transportation systems (Hua, Leng, & Guan, 2012; Yan, Edwards, Spurgeon, &
Bleijs, 2004; Zhang & Liu, 2013; Zhang, Zhang, & Wang, 2014). In the control of large-scale systems, decentralized control structure is often the most appropriate control method for handling the computational complexity. Also, it has the advantages such as easier maintenance, greater reliability, and less computational effort (see e.g. Keviczky, Borrelli, & Balas, 2006; Riverso, Farina, & Trecate, 2013; Yan, Lam, Li, & Chen, 2000, and the references therein).

On the other hand, as a popular control technique, model predictive control (MPC) strategy can effectively handle the uncertainties and hard constraints on states and controls in the process industry. In recent years, many MPC synthesis algorithms that ensure closed-loop stability and robust convergence have been proposed (see e.g. Alessio, Barcelli, & Bemporad, 2011; Magni & Scattolini, 2006; Mayne, Rawlings, Rao, & Scokaert, 2000; Zou & Niu, 2013). Especially, the study of decentralized MPC algorithm for large-scale systems has attracted much attention (Mayne, 2014; Raimondo, Magni, & Scattolini, 2007; Tran & Ha, 2014). Among them, decentralized MPC design was introduced in Tran and Ha (2014) for networks of linear systems with bounded coupling delay. The stability condition was derived for the constrained optimization problem and the issues of input and state constraints had been addressed by adopting decentralized MPC method. In Magni and Scattolini (2006), a stabilizing decentralized MPC algorithm was presented for nonlinear, discrete time systems under the assumption that no information can be exchanged between local control laws. The closed-loop stability was achieved based on the inclusion of a contractive constraint in the optimization problem. Alessio et al. (2011) proposed a decentralized MPC algorithm for constrained large-scale linear system and analyzed the asymptotic stability of closed-loop system. In particular, the decentralized MPC strategy for large-scale nonlinear system with bounded disturbances was considered in Raimondo et al. (2007), where each subsystem was locally controlled with a MPC algorithm ensuring the robust stability. However, it should be pointed out that the main mechanism in the aforementioned decentralized MPC works was based on time-triggered control scheme. That is, at each sampling instant, a finite horizon local optimization problem was solved on-line to determine the local optimal control sequence, in which only the first control signal would be applied to the subsystem. Apparently, this will consume redundant computational and communication resources, and even affect its applications for a case with limited resources and insufficient communication bandwidth. This motivates the research on event-triggered decentralized MPC algorithms.

The key feature of event-triggered control schemes is that the decision for the execution of control laws is not made periodically, but depending on the detailed system behaviors, such as the system state or the performance index (Dimarogonas, Frazzoli, & Johansson, 2012). At present, many developments have been reported on the event-triggered schemes (Dong, Wang, Alsaadi, & Ahmad, 2015; Dong, Wang, Ding, & Gao, 2015; Liu & Hao, 2013). In Liu and Hao (2013), a decentralized event-triggered scheme is proposed for networked control systems in order to reduce network traffic and computation resource. In Dong, Wang, Alsaadi, et al. (2015), an event-triggered robust distributed state estimation problem for sensor networks was studied, and in Dong, Wang, Ding, et al. (2015), an event-triggered H-infinity filter algorithm was presented to alleviate the unnecessary waste of communication resources. For event-triggered MPC, some related works can be found in Eqtamin, Dimarogonas, and Kyriakopoulos (2010), Lehmann, Henriksson, and Hohansson (2013), Eqtami, Dimarogonas, and Kyriakopoulos (2011a), Li and Shi (2014). In Eqtamin et al. (2010), an event-triggered MPC algorithm for discrete-time systems was presented, where the optimization problem was solved only when the triggering condition was violated. Eqtami et al. (2011a) considered the event-triggered robust MPC for both continuous and discrete-time uncertain nonlinear systems with additive disturbances, and derived the triggering rule according to the input-to-state stability (ISS) property. More recently, a class of interconnected large-scale system with bounded disturbances was considered in Eqtami, Dimarogonas, and Kyriakopoulos (2011b), whose key idea was that each subsystem was controlled by a local event-triggered robust model predictive controller. However, it is worthy to note that although the method in Eqtami et al. (2011b) can achieve the reduction on the number of the optimal control updating, there still exists high computational complexity in the optimization problem due to the uncertainties.
In this paper, we investigate the event-triggered decentralized predictive control problem based on constraint tightening approach to reduce both the times of solving optimization problem and computational complexity. By constructing a candidate control sequence and ISS stability, the event-triggered conditions are derived to determine whether the local predictive control optimization problem is solved. Moreover, the robust feasibility and closed-loop stability are proved to show the convergence of subsystem states.

The remainder of the paper is organized as follows. In Section 2, the problem statement for the large-scale system is presented. In Section 3, the main results, including the event-triggered decentralized model predictive controller and the proof of robust feasibility and robust stability are presented. Section 4 provides a numerical example to show the efficiency of the proposed algorithm.

Notations: \( \mathbb{R}^n \) denotes the real \( n \) dimensional Euclidean space, \( \mathbb{R}^+ \) denotes the positive real number. Given two vectors \( x, y \in \mathbb{R}^n \), \( x \geq y \iff x_i \geq y_i, i = 1, 2, \ldots, n. \) For any vector \( x \in \mathbb{R}^n \) and matrix \( Q, ||x||_Q^2 = x^TQx \) \( \lambda_{	ext{max}}(\cdot) \) represents the maximum eigenvalue of a real matrix. Given any two sets \( A, B \) of \( \mathbb{R}^n \), the operator \( ^\sim \) denotes the Pontryagin set difference, i.e. \( A \sim B = \{a | a + b \in A, \forall b \in B\} \), while the operator \( \oplus \) denotes the Minkowski set addition, i.e. \( A \oplus B = \{a + b | a \in A, b \in B\} \).

2. Preliminary

2.1. System description

Consider the linear discrete-time interconnected system composed of \( M \) local subsystems

\[
\begin{align*}
x_i(k+1) &= A_i x_i(k) + B_i u_i(k) + G_i y_i(k) + v_i(k), \\
y_i(k) &= C_i x_i(k), \quad i \in \{1, 2, \ldots, M\},
\end{align*}
\]

where \( x_i(k) \in \mathbb{R}^n \) is the state of the \( i \)th subsystem, \( u_i(k) \in \mathbb{R}^m \) is the control variable, \( y_i(k) \in \mathbb{R}^p \) is the output, \( v_i(k) \in \mathbb{R}^q \) is the additive bounded disturbance, and \( G_i y_i(k) \) denotes the mutual influence of \( M \) subsystems, where \( y(k) \triangleq [y_1(k), y_2(k), \ldots, y_M(k)]^T \in \mathbb{R}^s \) with \( s = \sum_{i=1}^{M} s_i \) is the overall output.

The output, input, and disturbance of the \( i \)th subsystem are assumed to satisfy the following constraints

\[
\begin{align*}
u_i(k) &\in U_i = \{u_i^{\min} \leq u_i(k) \leq u_i^{\max}\}, \\
y_i(k) &\in Y_i = \{y_i^{\min} \leq y_i(k) \leq y_i^{\max}\}, \\
v_i(k) &\in V_i = \{v_i^{\min} \leq v_i(k) \leq v_i^{\max}\}.
\end{align*}
\]

By letting \( \tilde{y}_i(k) \triangleq G_i y_i(k) \), we obtain

\[
\begin{align*}
\tilde{y}_i(k) &\in \tilde{Y}_i = \{\tilde{y}_i^{\min} \leq \tilde{y}_i(k) \leq \tilde{y}_i^{\max}\},
\end{align*}
\]

where \( \tilde{y}_i^{\min} \triangleq \begin{bmatrix} (G_i y_1^{\min})^T, (G_i y_2^{\min})^T, \ldots, (G_i y_m^{\min})^T \end{bmatrix}^T, \tilde{y}_i^{\max} \triangleq \begin{bmatrix} (G_i y_1^{\max})^T, (G_i y_2^{\max})^T, \ldots, (G_i y_m^{\max})^T \end{bmatrix}^T. \)

Define the following augmented vectors

\[
\begin{align*}
x(k) &\triangleq [x_1^T(k), x_2^T(k), \ldots, x_M^T(k)]^T, \\
y(k) &\triangleq [y_1^T(k), y_2^T(k), \ldots, y_M^T(k)]^T, \\
v(k) &\triangleq [v_1^T(k), v_2^T(k), \ldots, v_M^T(k)]^T.
\end{align*}
\]

The whole system can be written as
\[ \begin{align*}
\{ & x(k + 1) = Ax(k) + Bu(k) + Gy(k) + v(k), \\
& y(k) = Cx(k), \}
\end{align*} \tag{7} \]

where
\[ A = \text{diag}\{A_1, A_2, \ldots, A_M\}, \quad B = \text{diag}\{B_1, B_2, \ldots, B_M\}, \quad G = \text{diag}\{G_1, G_2, \ldots, G_M\}, \]
\[ C = \text{diag}\{C_1, C_2, \ldots, C_M\}. \]

### 2.2. Decentralized MPC formulation
In the sequel, we present the decentralized MPC scheme based on the constraint tightened approach, in which each local MPC optimization control problem (OCP) is formulated based on the nominal subsystem corresponding to (1). Moreover, we take the sum of the interaction term \( y_i(k) \) and the additive disturbance \( v(k) \) as the perturbation \( w_i(k) = y_i(k) + v(k) \in W_i, \quad \forall k \) with \( W_i = Y_i \oplus V_i \).

Thus, we obtain the following nominal subsystem
\[ \begin{align*}
\{ & \tilde{x}_i(k + 1) = A_i \tilde{x}_i(k) + B_i u_i(k), \\
& \tilde{y}_i(k) = C_i \tilde{x}_i(k). \}
\end{align*} \tag{8} \]

The following finite horizon optimization problem for the uncertain subsystem (1) is considered
\[ \begin{align*}
J^*_i(k) = \min_{U_i(k)} \left\{ \sum_{j=0}^{N_i} (||y_i(k + j|k)||_Q^2 + ||u_i(k + j|k)||_R^2) + ||y_i(k + N_i + 1|k)||_P^2 \right\},
\end{align*} \tag{9} \]

subject to
\[ \begin{align*}
x_i(k + j + 1|k) &= A_i x_i(k + j|k) + B_i u_i(k + j|k), \\
y_i(k + j|k) &= C_i x_i(k + j|k), \\
x_i(k|k) &= x_i(k), \\
y_i(k + j|k) &\in Y_i(j), \\
u_i(k + j|k) &\in U_i(j), \\
x_i(k + N_i + 1|k) &\in X_{x_i}, \end{align*} \tag{10-15} \]

where \( N_i \) is the prediction horizon, \( Q_i = q_i \cdot I_{s \times s}, \quad q_i \in \mathbb{R}^+, \quad R_i = r_i \cdot I_{m \times m}, \quad r_i \in \mathbb{R}^+. \)

The constraint sets \( U_i(j) \) in (14) are defined by a tightening recursion
\[ \begin{align*}
U_i(0) &= U_i, \\
U_i(j + 1) &= U_i(j) \sim K_i(j)L_i(j)W_i, \quad \forall j \in \{0, 1, \ldots, N_i - 1\}. \tag{16-17} \end{align*} \]

Similarly, the constraint sets \( Y_i(j) \) in (13) are
\[ \begin{align*}
Y_i(0) &= Y_i, \\
Y_i(j + 1) &= Y_i(j) \sim C_i L_i(j) W_i, \quad \forall j \in \{0, 1, \ldots, N_i - 1\}. \tag{18-19} \end{align*} \]

The matrices \( K_i(j) \) and \( L_i(j) \) denote the associated state transmission matrices under the following candidate policy.
In this work, our objective is to propose an event-triggered decentralized MPC algorithm based on the constraint tightened strategy such that system resources can be saved and each local OCP computational complexity caused by uncertainties can be reduced.

3. Event-triggered decentralized robust model predictive controller

In the traditional decentralized MPC strategy, the local optimal control law is usually applied to each subsystem at each sampling instant by solving on-line the local OCP. In this work, we propose an event-triggered decentralized MPC strategy, which determines the updating of control inputs according to a certain triggering condition. In other words, the optimal control law is applied to each subsystem only at its triggered time instant $k^i_t$. During the interval step $k^i_t + m$, $m \in \{1, 2, \ldots, N\}$ of any two successive triggering events $k^i_t$ and $k^{i+1}_t$, a candidate control sequence $\tilde{U}_i(k^i_t + m)$ based on the optimal control sequence $\tilde{U}_i(k^i_t)$ at time $k^i_t$ is applied to the $i$th subsystem. Note that $k^i_t$ is the prior triggering step. Hence, it is important to provide an appropriate control sequence $\tilde{U}_i(k^i_t + m)$ which satisfies specific constraints at time $k^i_t + m$. Based on the analysis of feasibility and robust stability, we further obtain the triggering condition for each subsystem.

3.1. Robust feasibility

In this case, the robust feasibility of the local constrained OCP is analyzed in the following theorem.

**Theorem 1** Suppose that the local OCP has the optimal control sequence $\tilde{U}_i(k^i_t)$ at the triggered time $k^i_t$. The local OCP with the candidate control sequence $\tilde{U}_i(k^i_t + m) = [\tilde{u}_i(k^i_t + m, k^i_t + m), \ldots, \tilde{u}_i(k^i_t + m + N, k^i_t + m)]$ is feasible at time $k^i_t + m$, $m \in \{1, 2, \ldots, N\}$, where
Combining (33) and (34), it can be further written as

\[
\hat{y}_j(k_i^j + 1 + j|k_i^j + 1) \in Y_j(j), j \in \{0, 1, \ldots, N_j\}.
\]

(i) **Constraint (13):** \(\hat{y}_j(k_i^j + 1 + j|k_i^j + 1) \in Y_j(j)\) for \(j \in \{0, 1, \ldots, N_j\}\).

The optimal outputs \(y_j^*(k_i^j + 1 + j|k_i^j)\) satisfies (13) at \(k_i^j\), so we obtain

\[
y_j^*(k_i^j + 1 + j|k_i^j) \in Y_j(j + 1), \quad \forall j \in \{0, 1, \ldots, N_j - 1\}.
\]

(ii) **Constraint (14):** \(\hat{u}_j(k_i^j + 1 + j|k_i^j + 1) \in U_j(j)\) for \(j \in \{0, 1, \ldots, N_j\}\).

Considering that \(U_j^*(k_i^j)\) is the optimal control sequence at \(k_i^j\), it holds that

\[
u_j^*(k_i^j + 1 + j|k_i^j) \in U_j(j), \quad \forall j \in \{0, 1, \ldots, N_j\}.
\]
According to (16–17), we have
\[ \bar{u}_i(k_i^j + 1 + j | k_i^j + 1) \in U(j), \forall j \in \{0, 1, \ldots, N_i - 1\}. \] (37)

Since \( \hat{x}_i(k_i^j + 1 + N_i | k_i^j + 1) \in \Omega \), we obtain
\[ \bar{u}_i(k_i^j + 1 + N_i | k_i^j + 1) = H_i \hat{x}_i(k_i^j + 1 + N_i | k_i^j + 1) \in U(N_i). \] (38)

From (37) and (38), it can be obtained that
\[ \bar{u}_i(k_i^j + 1 + j | k_i^j + 1) \in U(j), \forall j \in \{0, 1, \ldots, N_i\}. \] (39)

(iii) **Constraint (15):** \( \hat{x}_i(k_i^j + N_i + 2 | k_i^j + 1) \in X_p. \)

From (31), we have \( \hat{x}_i(k_i^j + 1 + N_i | k_i^j + 1) \in \Omega \), according to (23), the subsequent state must satisfy
\[ A_i \hat{x}_i(k_i^j + N_i + 1 | k_i^j + 1) + B_i \hat{x}_i(k_i^j + N_i + 1 | k_i^j + 1) + L_i(N_i)w_i \in \Omega. \]

By the definition of \( X_p \) in (22), we have
\[ \hat{x}_i(k_i^j + N_i + 2 | k_i^j + 1) = (A_i + B_i \hat{x}_i(k_i^j + N_i + 1 | k_i^j + 1)) \in X_p, \] (40)

which implies that the terminal constraint (15) at \( k_i^j + 1 \) is satisfied.

From the above it shows that the candidate control law \( \bar{u}_i(k_i^j + 1) \) at time instant \( k_i^j + 1 \) can satisfy the constraints (10–15) and the local OCP is feasible at step \( k_i^j + 1 \). By means of similar arguments, the feasibility of the local OCP at subsequent time \( k_i^j + m_i \), \( m_i \in \{2, 3, \ldots, N_i\} \) can be recursively proved. This completes the proof. \( \square \)

### 3.2. ISS and triggering condition

We choose the cost function in (9) as a candidate Lyapunov function for the \( i \)th subsystem, and define the difference of the feasible cost function as
\[ \Delta J_m^i = J_i^*(k_i^j + m_i) - J_i^*(k_i^j + m_i - 1). \] (41)

Note that \( J_i^*(k_i^j) = J_i^*(k_i^j) \).

Before we discuss the ISS stability and the triggering condition, the following results are presented.

**Theorem 2** Consider the subsystem (1) subject to (3–5) and the control law (27), and suppose the matrix \( P_i \) in (9) satisfies \( C_i^TP_iC_i \geq C_i^TQ_iC_i + H_i^TR_iH_i + (A_i + B_iH_i)^TP_iC_i(A_i + B_iH_i) \). The difference of the feasible cost functions between the time \( k_i^j + m_i \) and \( k_i^j + m_i - 1 \) is bounded by
\[ \Delta J_m^i \leq \sum_{j=1}^{M} (\beta_2^j ||x_i(k_i^j + m_i - 1)||) + \beta_2^i ||v_i(k_i^j + m_i - 1)|| - \alpha_i ||x_i(k_i^j + m_i - 1)||^2, \] (42)

where
\[ \alpha_i \triangleq q_i \cdot ||C_i||^2, \] (43)
\[ \beta_1^i \triangleq \rho_i \cdot ||C_i||, \] (44)
\[ \beta_2^i \triangleq S_i \left( \sum_{j=0}^{N_i-1} ||L_i(j)||^2 \right) + S_i \left( \sum_{j=0}^{N_i-1} ||L_i(j)||^2 \right) + F_i \left( \sum_{j=0}^{N_i-1} ||L_i(j)||^2 \right) + F_i \left( \sum_{j=0}^{N_i-1} ||L_i(j)||^2 \right) + M \] (45)
\[ \beta_i^j \triangleq S_i \left( \sum_{j=0}^{N-1} ||L_i(j)|| \right) + S_0 \left( \sum_{j=0}^{N-1} ||L_i(j)||^2 \right) + F_i^1 \sum_{j=0}^{N-1} ||K_t(j)|| \cdot ||L_i(j)|| + F_i^2 \sum_{j=0}^{N-1} ||K_t(j)||^2 \cdot ||L_i(j)||^2 + M_i^j, \]

with \( \psi_i \triangleq \max \{ ||x_i|| : x_i \in X_i \} \), \( \gamma_i^j \triangleq \max \{ ||y_i|| : y_i \in Y_i \} \), \( j \in \{ 0, 1, \ldots, N_i \} \), \( \gamma_i^j \triangleq \max \{ ||u_i|| : u_i \in U_i(\bar{y}_i) \}, \quad j \in \{ 0, 1, \ldots, N_i \} \), \( \gamma_i \triangleq \max \{ ||y_i|| : y_i \in V_i \} \), \( S_i \triangleq 2q_i \cdot ||C_i|| \cdot ||G_i|| \), \( S_i \triangleq q_i \cdot ||C_i||^2 \cdot ||G_i|| \cdot \gamma_i + ||C_i|| \), \( S_i \triangleq q_i \cdot ||C_i||^2 \cdot ||G_i|| \cdot \gamma_i + ||C_i|| \), \( S_i \triangleq q_i \cdot ||C_i||^2 \cdot ||G_i|| \cdot \gamma_i + ||C_i|| \), \( S_i \triangleq 2q_i \cdot ||C_i|| \cdot ||G_i|| \cdot \gamma_i + ||C_i|| \), \( S_i \triangleq 2q_i \cdot ||C_i|| \cdot ||G_i|| \cdot \gamma_i + ||C_i|| \), \( S_i \triangleq 2q_i \cdot ||C_i|| \cdot ||G_i|| \cdot \gamma_i + ||C_i|| \), \( M_i^j \triangleq \lambda_{\max}(C_i^t P_i C_i) \cdot \cdot \cdot ||L_i(N_i)||^2 \cdot ||G_i|| \cdot \gamma_i + ||L_i(N_i)||^2 \cdot \gamma_i + 2 \psi_i \cdot ||L_i(N_i)||, M_i^j \triangleq M_i^j \cdot ||G_i|| \).

**Proof** For \( m_i = 1 \), we have
\[
\Delta j'_1 = j'_1(k_i^j + 1) - j'_1(k_i^j) \\
= \sum_{j=0}^{N_i} (||y_i||_2(k_i^j + 1) + ||u_i||_2(k_i^j + 1) + ||v_i||_2(k_i^j + 1) + ||\bar{y}_i||_2(N_i + 2|k_i^j + 1|)) - \sum_{j=0}^{N_i} (||y_i||_2(k_i^j) + ||u_i||_2(k_i^j) + ||v_i||_2(k_i^j) + ||\bar{y}_i||_2(N_i + 2|k_i^j + 1|)) \\
\leq (S_i \left( \sum_{j=0}^{N_i} ||Y_i||_2 \right) + S_0 \left( \sum_{j=0}^{N_i} ||Y_i||_2^2 \right)) \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) + \left( F_i^1 \sum_{j=0}^{N_i} ||K_t|| \cdot ||Y_i||_2 + F_i^2 \sum_{j=0}^{N_i} ||K_t||^2 \cdot ||Y_i||_2^2 \right) \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) \\
+ F_i^3 \sum_{j=0}^{N_i} ||K_t||^2 \cdot ||Y_i||_2^2 \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) \leq \beta_i^j \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) - \alpha_i \cdot ||x_i||_2(N_i + 2|k_i^j + 1|)^2.
\]

For \( m_i = 2 \), the difference (41) is
\[
\Delta j'_2 = j'_2(y_i(k_i^j + 1) - j'_2(y_i(k_i^j + 1)) \cdot \frac{n_i}{f_i(m_i - n_i + 1)} = \sum_{j=0}^{N_i} (||y_i||_2(k_i^j + 2 + j|k_i^j + 2|) + ||u_i||_2(k_i^j + 2 + j|k_i^j + 2|) + ||v_i||_2(k_i^j + 2 + j|k_i^j + 2|) + ||\bar{y}_i||_2(N_i + 2|k_i^j + 1|)) - ||y_i||_2(k_i^j + 1) + 2|k_i^j + 1|)^2) \\
\leq (S_i \left( \sum_{j=0}^{N_i} ||Y_i||_2 \right) + S_0 \left( \sum_{j=0}^{N_i} ||Y_i||_2^2 \right) \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) + \left( F_i^1 \sum_{j=0}^{N_i} ||K_t|| \cdot ||Y_i||_2 + F_i^2 \sum_{j=0}^{N_i} ||K_t||^2 \cdot ||Y_i||_2^2 \right) \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) \\
+ F_i^3 \sum_{j=0}^{N_i} ||K_t||^2 \cdot ||Y_i||_2^2 \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) \leq \beta_i^j \cdot ||v_i||_2(N_i + 2|k_i^j + 1|) - \alpha_i \cdot ||x_i||_2(N_i + 2|k_i^j + 1|)^2.
\]

By adopting similar procedures as in (47) and (48), it is easily shown that \( \Delta j'_m \), \( m_i \in \{ 1, 2, \ldots, N_i \} \) yields (42). That leads to the conclusion of Theorem 2.
prove the ISS property for each subsystem. Theorem 3 Consider the subsystem (1) and the event-triggered decentralized robust MPC strategy. The local optimal control law is applied only when the following triggering condition is violated

\[
\sum_{j=1}^{M} (\beta_j^i ||x_j(k^i + m_j - 1)|| + \beta_j^i ||y_j(k^i + m_j - 1)||) \leq \sigma_i \cdot a_i ||x_j(k^i + m_j - 1)||^2, \quad 0 < \sigma_i < 1, m_j \in \{1, 2, \ldots, N_i\}.
\]

(49)

Otherwise, the control sequence given by (27) is applied to the subsystem. Using this event-triggered control scheme, the subsystem is ISS and the subsystem state will be driven to a robust invariant set.

**Proof** Theorem 1 provides a candidate control sequence for the \(i\)th subsystem at \(k^i + m_i\), \(m_i \in \{1, 2, \ldots, N_i\}\) based on the optimal solution computed at triggered step \(k^i\). Theorem 2 presents the bounded difference of the Lyapunov function \(J(y)\) between steps \(k^i + m_i\) and \(k^i + m_i - 1\).

If the disturbance term satisfies (49), we have

\[
\sum_{j=1}^{M} (\beta_j^i ||x_j(k^i + m_j - 1)|| + \beta_j^i ||y_j(k^i + m_j - 1)||) \leq \sigma_i \cdot a_i ||x_j(k^i + m_j - 1)||^2, \quad 0 < \sigma_i < 1,
\]

Substituting (49) into (42), we obtain

\[
\Delta J_{m_i} = \bar{J}(y(k^i + m_i)) - \bar{J}(y(k^i + m_i - 1)) \leq (\sigma_i - 1) \cdot a_i ||x(k^i + m_i - 1)||^2 < 0.
\]

Therefore, the Lyapunov function is strictly decreasing. It also implies the \(i\)th subsystem is ISS. Moreover, the subsystem state \(x(k)\) must enter its robust invariant admissible set \(\Omega_i\) in finite time and remain there in the subsequent time. This completes the proof.

**Remark 3** In this work, the interconnections between subsystems are treated as perturbation. Therefore, the overall system (7) is ISS since the ISS property of each subsystem (1) is derived under the event-triggered decentralized robust MPC strategy.

### 4. Simulation results

Consider the overall system that consists of three linear discrete-time subsystems in the form of (1-4) with the following parameters:

\[
A_1 = A_2 = A_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B_1 = B_2 = B_3 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}^T,
\]

\[
C_1 = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.4 & 0.5 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.6 & 0.6 \end{bmatrix},
\]

\[
G_1 = \begin{bmatrix} 0.0006 & 0.0005 & 0.0001 \\ 0 & 0.0005 & 0.0002 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.0005 & 0 & 0.0004 \\ 0 & 0.0005 & 0.0006 \end{bmatrix},
\]

\[
G_3 = \begin{bmatrix} 0.0005 & 0 & 0.0002 \\ 0.0001 & 0.0005 & 0 \end{bmatrix}.
\]

subject to the following constraints

\[
|u_1| = |u_2| = |u_3| \leq 1.0, \quad |y_1| = |y_2| = |y_3| \leq 10,
\]

\[
|v_{11}| = |0.1\sin(x_{11})| \leq 0.1, \quad |v_{12}| = |0.1\sin(x_{12})| \leq 0.1,
\]

\[
|v_{21}| = |0.1\sin(x_{21})| \leq 0.1, \quad |v_{22}| = |0.1\sin(x_{22})| \leq 0.1.
\]
$|v_{31}| = |0.1\sin(x_{31})| \leq 0.1, |v_{32}| = |0.1\sin(x_{32})| \leq 0.1.$

The cost functions are defined by (10) with $Q_1 = I_{2 \times 2}, R_1 = 0.01, Q_2 = 0.8I_{2 \times 2}, R_2 = 0.03, Q_3 = 0.8I_{2 \times 2}, R_3 = 0.02.$ The prediction horizons of each subsystem are chosen as $N_1 = N_2 = N_3 = 8$, then the candidate matrices can be derived off-line using the LQR nilpotent policy. The constraints (13–14) of each subsystem can be calculated off-line using the tightening recursion (16–19). With these results, it is ready to execute the on-line optimization. The initial states of each subsystem are $x_{10} = [5.2, -5.4]^T$, $x_{20} = [-5.2, 5.0]^T$, $x_{30} = [5.0, -5.4]^T$, and the simulation step is $T = 100$.

The tightened constraints $U_j(j)$, $Y_j(j)$ of each local OCP are as follows:

Subsystem 1:

\[
\begin{align*}
U_1(0) &= [-10, 10], & Y_1(0) &= [-10, 10], \\
U_1(1) &= [-0.7841, 0.7841], & Y_1(1) &= [-9.8798, 9.8798], \\
U_1(2) &= [-0.7131, 0.7131], & Y_1(2) &= [-9.8700, 9.8700], \\
U_1(3) &= [-0.6878, 0.6878], & Y_1(3) &= [-9.8661, 9.8661], \\
U_1(4) &= [-0.6795, 0.6795], & Y_1(4) &= [-9.8647, 9.8647], \\
U_1(5) &= [-0.6770, 0.6770], & Y_1(5) &= [-9.8641, 9.8641], \\
U_1(j) &= [-0.6752, 0.6752], & Y_1(j) &= [-9.8635, 9.8635], \\
& j \geq 6.
\end{align*}
\]

Subsystem 2:

\[
\begin{align*}
U_2(0) &= [-10, 10], & Y_2(0) &= [-10, 10], \\
U_2(1) &= [-0.7845, 0.7845], & Y_2(1) &= [-9.9009, 9.9009], \\
U_2(2) &= [-0.7203, 0.7203], & Y_2(2) &= [-9.8935, 9.8935], \\
U_2(3) &= [-0.6933, 0.6933], & Y_2(3) &= [-9.8892, 9.8892], \\
U_2(4) &= [-0.6844, 0.6844], & Y_2(4) &= [-9.8877, 9.8877], \\
U_2(5) &= [-0.6817, 0.6817], & Y_2(5) &= [-9.8871, 9.8871], \\
U_2(j) &= [-0.6799, 0.6799], & Y_2(j) &= [-9.8866, 9.8866], \\
& j \geq 6.
\end{align*}
\]

Subsystem 3:

\[
\begin{align*}
U_3(0) &= [-10, 10], & Y_3(0) &= [-10, 10], \\
U_3(1) &= [-0.7907, 0.7907], & Y_3(1) &= [-9.8722, 9.8722], \\
U_3(2) &= [-0.7256, 0.7256], & Y_3(2) &= [-9.8682, 9.8692], \\
U_3(3) &= [-0.7000, 0.7000], & Y_3(3) &= [-9.8689, 9.8689], \\
U_3(4) &= [-0.6917, 0.6917], & Y_3(4) &= [-9.867, 9.867], \\
U_3(5) &= [-0.68927, 0.68927], & Y_3(5) &= [-9.8685, 9.8685], \\
U_3(j) &= [-0.6874, 0.6874], & Y_3(j) &= [-9.8679, 9.8679], \\
& j \geq 6.
\end{align*}
\]

The trajectories of states and outputs are given in Figures 1 and 2, which show the convergence of the subsystems under the proposed event-triggered decentralized MPC framework. To verify the reduction on the number of updating control laws, the triggering instants of each subsystem are plotted in Figure 3.
5. Conclusions
In this work, we have provided an event-triggered decentralized robust model predictive controller for a class of constrained linear discrete-time system with additive bounded disturbances. The proposed strategy can not only reduce the on-line computation load, but also achieve the alleviation of computational complexity. It should be pointed out that the systems under consideration in this work are assumed to have full knowledge of states. Actually, it is often difficult to measure the system state in practical application; the event-triggered output feedback MPC strategy will be further considered in future research.
Figure 3. Triggering instants.

Notes: The value 1 means the local OCP is triggered at the corresponding time instant and the value 0 means the local OCP is not triggered.

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