The influence of the extraneous field on the conductivity tensor in cylindrical quantum wire with electron-optical phonon scattering

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Abstract. This work calculates conductivity tensors in cylindrical quantum wires with parabolic potential, with the presence of an electromagnetic wave field, a laser field, and is only considered for the case of electron-optical phonon scattering. By using the quantum kinetic equation for the electron in the quantum wire with the parabolic potential, combined with the presence of two external fields, the expression of conductivity tensor is computed for the case of optical electron-phonon scattering. This conductivity tensor expression depends on the electromagnetic wave frequency, the laser field frequency, the laser field amplitude, and the quantities that characterize the cylindrical quantum wire with parabolic potential. The figure of the dependence of the conductivity tensor to the external field will be plotted, examined, and analyzed for the quantum wire GaAs / GaAsAl.

1. Introduction

There has been a lot of research on low-dimensional semiconductor [1-15], the quantum wire is a one-dimensional semiconductor system, the electrons in the quantum wire move only along a fixed axis (assuming the oz axis) and is limited in motion on the other two axes (assuming 0y and 0z). In the quantum wire, the electrons are confined in the x0y plane so its energy is quantized [2,3,13,14]. The wave function equations and characteristic parameters for quantum wire are completely different from volume semiconductors and two-dimensional systems such as quantum wells or superlattices [1,2,5-12, 16-19].

Conductivity tensor (CT) is an important concept of materials, from this concept to study other important properties such as electrical properties, electrical conductivity, electrical current density, etc [16]. The presence of the extraneous field greatly affects the conductivity tensor in the material. The extraneous field carries both energy and momentum, so it interacts with the material causing new effects and influencing the parameters of electricity and magnetism of the material [2-7]. In this study, the carrier system is placed in two external fields, the linearly polarized electromagnetic wave field, and the laser field. Both of these fields carry energy and momentum, they will interact with the carriers and change the properties of the material.

Using the quantum kinetic equation, the CT expression is calculated. The method of using quantum kinetic equations is a popular and reliable method. From the Hamiltonian of the electronic system in
the presence of two external fields in the quantum wire, the quantum kinetic equation for the system is established, and finally the conductivity tensor is calculated. This tensor depends on parameters specific to the system and the external field such as temperature, particle density, frequency, amplitude, etc. From the conductivity tensor expression we will examine and plot its dependence on the external field, the quantum wire examined here is the quantum wire GaAs / GaAsAl.

2. Calculating conductivity tensor

Suppose electrons moving freely in the 0z axis and quantized in the x0y plane. The energy of the electron is determined by ( ħ = 1) [14]:

$$\varepsilon_{n,l,p} = \frac{p^2}{2m} + \omega_0(2n + l + 1)$$  (1)

n, n’ = 1, 2, …; l, l’ = 1, 2, … n–1; p is momentum of electron; m is the effective mass of the electron.

The system is placed in two external fields, the electromagnetic wave field and the laser field:

$$\begin{aligned}
\vec{E}(t) &= \vec{E}(e^{-i\omega t} + e^{i\omega t}), \quad \text{and} \quad \vec{H} = [\vec{n}, \vec{E}(t)] \\
\text{and} \quad \vec{F}(t) &= F \sin(\Omega t)
\end{aligned}$$

(2)

here ω is the electromagnetic wave frequency; F is the laser field amplitude; Ω is the laser field frequency. The quantum kinetic equation for system in the presence of an electromagnetic wave and a laser field is ( ħ = 1) [3,4,16,17]:

$$\frac{\partial f_{n,l,p}(t)}{\partial t} + \left( e\vec{E}(t) + \omega_0 \left[ \vec{p}_z, \vec{H}(t) \right] \right) \frac{\partial f_{n,l,p}(t)}{\partial \vec{p}_z} = 2\pi \sum_{n,n',l,l'} M_{n,n',l,l'}(\vec{q}) \sum_{L=-\infty}^{\infty} J_L^z(\vec{a}, \vec{q}) \left[ f_{n,l+1,p'+q}(t) - f_{n,l',p-q}(t) \right] \delta \left( \varepsilon_{n',l',p'} - \varepsilon_{n,l,p} - 1\Omega \right)$$

(3)

$$f_{n,l,p}(t)$$ is distribution function of electrons; \( \vec{q} \) is momentum of phonon;

$$f_{n,l}(\vec{p}_z, t) = f_0(\varepsilon_{n,l,p_z}) + f_1(\vec{p}_z, t); \quad f_0(\varepsilon_{n,l,p_z}) = \theta(\varepsilon_p - \varepsilon_{n,l,p_z}); \quad f_1(\vec{p}_z, t) = -\vec{p}_z \vec{\chi}(t) f_0(\varepsilon_{n,l,p_z});$$

$$f'_0(\varepsilon_{n,l,p_z}) = \frac{\partial f_0(\varepsilon_{n,l,p_z})}{\partial \varepsilon_{n,l,p_z}}; \quad f'_1(\vec{p}_z, t) = f_1(\vec{p}_z) e^{-i\omega t} + f'_1(\vec{p}_z) e^{i\omega t}; \quad f'_1(\vec{p}_z) = -\vec{p}_z f'_0(\varepsilon_{n,l,p_z});$$

$$\vec{\chi}(t) = \vec{\chi}_0 e^{-\omega t} + \vec{\chi}' e^{\omega t}; \quad \vec{\chi}_0 = \frac{eE}{m} \frac{\tau(\varepsilon)}{1 - i\omega \tau(\varepsilon)}; \quad \vec{\chi}' = \frac{eE}{m} \frac{\tau(\varepsilon)}{1 + i\omega \tau(\varepsilon)};$$

$$\frac{\partial f_{n,l}(\vec{p}_z, t)}{\partial t} = \frac{\partial}{\partial t} \left[ f_0(\varepsilon_{n,l,p_z}) + f_1(\vec{p}_z) e^{-i\omega t} + f'_1(\vec{p}_z) e^{i\omega t} \right] = -i\omega f_1(\vec{p}_z) e^{-i\omega t} + i\omega f'_1(\vec{p}_z) e^{i\omega t}$$

(4)
\[ M_{n,l,n',l'}(q) = |C_q|^2 \int |I_{n,l,n',l'}(r)|^2 N_q \]  
\[ \text{(5)} \]

We only consider for the case of electron – optical phonon scattering [15]:

\[ |C_q|^2 = \frac{2\pi e^2 \omega_q}{\varepsilon_0 V_0 q^2} \left( \frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right); \quad \varepsilon_0 = \frac{10^{-9}}{36\pi} \]  
\[ \text{(6)} \]

\[ N_q = \frac{k_B T}{\omega_q} \]  
\[ \text{(7)} \]

\( \chi_\infty \) and \( \chi_0 \) are high-frequency dielectric constant and the static dielectric constant; \( V_0 \) is the volume of the wire; \( \omega_q \) is the frequency of optical phonon; \( k_B \) is Boltzmann constant.

\[ \psi_{n,l}(r) = \frac{e^{ikz}}{\sqrt{\mathcal{L}}} \sqrt{\frac{2n!}{(n+||l||)!}} a_0 \exp \left( -\frac{r^2}{2a_0^2} \right) \]  
\[ \psi^*_{n,l}(r) \]  
\[ \text{is wave function of the electrons [14].} \]

\[ \varepsilon_{n,l,p,s,q} = \frac{\left( \mathbf{p}_z + \mathbf{q} \right)^2}{2m} + \omega_0 (2n + 1 + l); \quad J^2_{\alpha}(\tilde{a}, \mathbf{q}) \]  
\[ \text{is the Bessel function of real argument; } \tilde{a} = \frac{e\mathbf{F}}{m\omega^2}; \]

\[ \omega_H = \frac{eH}{mc}; \quad \mathbf{h}(t) = \frac{\mathbf{H}(t)}{H} \]  
\[ \text{is the unit vector in the magnetic field direction. The simple case: } l = 0, \pm 1: \]

\[ J_0^2 \approx 1; \quad J_1^2 \approx \frac{(\tilde{a}, \mathbf{q})^2}{4} \]  
\[ \text{; multiply both sides Equation (3) by } -\frac{e}{m} \mathbf{p}_z \delta(e - \varepsilon_{n,l,p,s}) \]  
\[ \text{and then sum over } \mathbf{p}_z, \text{ we obtained:} \]

\[ -\frac{e}{m} \sum_{n,l,p,s} \mathbf{p}_z \left[ \frac{\partial f_{n,l}(\mathbf{p}_z, t)}{\partial t} + \left( e\mathbf{E}(t) + \omega_H \mathbf{h}(t) \right) \frac{\partial f_{n,l}(\mathbf{p}_z, t)}{\partial \mathbf{p}_z} \right] \times \]

\[ \times \delta(e - \varepsilon_{n,l,p,s}) = -\frac{e}{m} \sum_{n,l,p,s,q} M_{n,l,n',l'}(q) \sum_{\mathbf{p}_z} \left[ 2\pi \sum_{L=\infty} J^2_{\alpha}(\tilde{a}, \mathbf{q}) \times \right] \]

\[ \times \left[ f_{n,l}(\mathbf{p}_z + \mathbf{q}, t) - f_{n,l}(\mathbf{p}_z, t) \right] \delta(e - \varepsilon_{n,l,p,s} - \varepsilon_{n,l,p,s} - L\Omega) \delta(e - \varepsilon_{n,l,p,s}) \]  
\[ \text{(8)} \]

After calculating, we obtain the expression of the partical current density \( \mathbf{R} \) [17]:

\[ \mathbf{R}(e) = \frac{\tau(e)}{1 - i \omega \tau(e)} \left( \mathbf{Q} + \mathbf{S}(e) \right) \]  
\[ \text{(9)} \]

\[ \mathbf{R}^*(e) = \frac{\tau(e)}{1 + i \omega \tau(e)} \left( \mathbf{Q} + \mathbf{S}(e) \right) \]  
\[ \text{(10)} \]
Here  
\[ Q = \sum_{n,l,p} \left( \delta(\varepsilon - \varepsilon_{n,l,p}) \right) \]  

\[ \hat{S}(\varepsilon) = -\frac{\varepsilon}{m} \sum_{a,n',l',q} M_{a,n',l',q} \langle \tilde{a}, \tilde{q} \rangle^2 \sum_{p_i} \frac{1}{4} f_i(p_i) \times \]  

\[ \times \left[ \delta(\varepsilon_{a,n',l',q} - \varepsilon_{a,n,l,p}) + \delta(\varepsilon_{a,n',l',q} - \varepsilon_{a,n,l,p} + \Omega) \right] \times \]  

\[ \times \left[ \tilde{p}_z + \tilde{q} \right] \delta(\varepsilon - \varepsilon_{n,l,p}) \right] \]  

At time \( t = 0 \), the density of current [7]:  
\[ \bar{j}(t = 0) = \int (\bar{R}(\varepsilon) + \bar{R}'(\varepsilon)) d\varepsilon = i \lambda + i \lambda' \]  

\[ = \frac{4e_0^2 n_0}{m} \frac{\tau(\varepsilon_p)}{1 + \omega^2 \tau^2(\varepsilon_p)} \left[ \varepsilon_p - \omega_0 (2n + 1 + 1) \right] I + \lambda + \frac{\tau(\Omega)}{1 + \omega^2 \tau^2(\Omega)} \left[ 1 - \omega^2 \tau(\Omega) \tau(\varepsilon_p) \right] + \]  

\[ - \frac{\tau(\varepsilon_p)}{1 + \omega^2 \tau^2(\varepsilon_p)} \right] \hat{E} \]  

Here  
\[ \lambda = \frac{e_0^2 F^2}{2m \Omega^3} M_{a,n';l',q} \left( \sqrt{2m \Omega} \right) \sqrt{2m(\varepsilon_p - \omega_0 (2n + 1 + 1))} \]  

\[ \left( \sqrt{2m(\Omega - \omega_0 (2n + 1 + 1))} \right] \left( \sqrt{2m(\Omega - \omega_0 (2n + 1 + 1))} \right) \]  

\[ n_0 \text{ is particle density; } \varepsilon_p \text{ is the Fermi energy; } \]  

Here  
\[ \bar{j}(t = 0) = \sigma \bar{E}(t = 0) = \sigma 2 \bar{E} [3]; \text{ so:} \]  

\[ \sigma_{\lambda} = \frac{2e_0^2 n_0}{m} \frac{\tau(\varepsilon_p)}{1 + \omega^2 \tau^2(\varepsilon_p)} \left[ \varepsilon_p - \omega_0 (2n + 1 + 1) \right] \delta_0 + \]  

\[ \frac{\tau(\Omega)}{1 + \omega^2 \tau^2(\Omega)} \lambda + \]  

\[ - \frac{\tau(\varepsilon_p)}{1 + \omega^2 \tau^2(\varepsilon_p)} \right] \right] \]  

Equation (16) is the CT expression.

3. Numerical results and discussion

Parameters used to plot the figures [3, 4, 14, 15]: \( m = 0.0665m_0 \); \( \varepsilon_p = 50 \text{ meV} \); \( \tau(\varepsilon_p) \sim 10^{-11} \text{ s} \); \( n_0 = 10^{23} \text{ m}^{-3} \); \( \rho = 5.3 \times 10^8 \text{ kg / m}^3 \); \( R = 100 \text{ A} \); \( T = 300 \text{ K} \); \( \chi_\infty = 10.8 \); \( \chi_0 = 13.1 \); \( q = 2.10^7 \text{ m}^{-1} \).

The interaction of photons with material depends on many factors, one of which is the energy of the photon. In the frequency range of the radio wave and the laser field, the photon energy is not large enough to excite the electrons or dislodge the electrons from the atom. The energy of photons can only
cause the electrons to vibrate, which is the cause leads to the tensor of conductivity in the material that changes accordingly.

![Figure 1](image1.png)  
**Figure 1.** The change of $\sigma$ with the electromagnetic wave frequency when $\omega < 9 \times 10^3$

![Figure 2](image2.png)  
**Figure 2.** The change of $\sigma$ with the electromagnetic wave frequency when $10^4 < \omega < 10^{12}$

Figure 1 shows the dependence of $\sigma$ on $\omega$ with $\omega < 9 \times 10^3$, for which CT does not change as the electromagnetic wave frequency increases. Because this is the frequency range of the long radio wave, long radio waves have little interaction with matter, so it does not change CT.

![Figure 3](image3.png)  
**Figure 3.** The change of $\sigma$ with the electromagnetic wave frequency when $\omega > 10^{12}$

![Figure 4](image4.png)  
**Figure 4.** The change of $\sigma$ with the laser field frequency when $\omega < 10^{10}$

At the frequency range $10^4 < \omega < 10^{12}$, which is in the mid, high and very high radio wave regions, the change of $\sigma$ with $\omega$ shown in Figure 2. Look at The figure we see that the CT decreases as the frequency increases, which shows that the electromagnetic waves carrying high energy and momentum interact with the material and change the electrical properties of the material.
When $\omega$ is more than $10^{12}$, the figure of the change of $\sigma$ with $\omega$ is shown Figure 3, the CT continues to decrease but decreases faster. This is explained as follows: As $\omega$ increases, the photon energy is greater, the interaction between the photon and the material is now significant, causing the electrons to vibrate more strongly and changing the electrical properties of the material.

When $\omega$ is less than $10^{10}$, the change of CT with $\Omega$ is shown in Figure 4. Looking at the figure we can be seen that CT increases as $\Omega$ increase, As $\Omega$ increases from $3 \times 10^4$ to $9 \times 10^4$, CT increases by a very small amount of only $10^{-15}$, demonstrating that $\Omega$ has a very small effect on CT of the material. As the frequency of the laser field is larger, the increase is very small and gradually does not increase.

Figure 5 also shows the change of CT with $\Omega$ but when considered at greater than $10^{10}$. CT decreases as $\Omega$ increases, but this decrease is very small, almost negligible. Looking at Figures 4 and 5 we can conclude that although CT is dependent on $\Omega$, this dependence is very small, which can be ignored.

The most obvious manifestation of the dependence of CT the laser field is shown in Figure 6, where CT increases rapidly as $F$ increases. High-intensity laser radiation interacting with matter can lead to significant energy absorption, this energy absorption process causes the electrical conductivity of quantum wire to change.

4. Conclusions

In this work, we have studied CT in a cylindrical quantum wire with parabolic potential and consider only the case of electrons-optical phonon scattering, together with the presence of two extraneous fields, a laser field, and an electromagnetic wave field. The obtained results have shown that the CT is a function of $\Omega$, $F$, $\omega$, and other parameters that characterize the quantum wire. Through the process of surveying and plotting the function with GaAs quantum wire, we see that the CT depends on the frequency of the extraneous field and the amplitude of the extraneous field. The dependence of the CT on the extraneous field frequency is very small, considered insignificant. When the frequency is increased by 3 times, the conductivity tensor changes only a very small amount of about $10^{-5}$. This is also perfectly suitable in theory because the frequency of the electromagnetic waves, which we consider is only in the radio wave range, so it can only cause electrons to vibrate without being able to excite or cause electrons to become free. But when studying the dependence of CT on $F$, CT increases very rapidly as $F$ increases. This is also consistent with the theory because the laser field carries great
momentum and energy, matter will absorb a large amount of this energy, leading to a change in the electrical properties of the material.

Acknowledgement

This research is funded by Thu Dau Mot University, Binh Duong Province, Vietnam.

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