CP-Violation in the Decay $b \rightarrow s\gamma$ in the Left-Right Symmetric Model

H. M. Asatrian$^a$, A. N. Ioannissian$^{a,b}$

$^a$ Yerevan Physics Institute, Alikhanyan Br. 2, Yerevan, Armenia
$^b$ Dept. of Physics, Technion - Israel Inst. of Tech., Haifa, Israel
e-mail: asatryan@vx1.yerphi.am, ioannissyan@vxc.yerphi.am

Abstract
The direct CP-violation in the left-right symmetric $SU(2) \times SU(2) \times U(1)$ model is investigated for the decay $b \rightarrow s\gamma$. The calculated CP-asymmetry for the wide range of parameters can be larger than in standard model and can have an opposite sign.
The experimental and theoretical investigation of the decay $b \to s\gamma$ can give a sign for a new physics in the TeV region [1]. This decay has been extensively studied during the last years. The first experimental evidence was obtained at CLEO for the exclusive decay $\bar{B} \to K^*\gamma$ [2]. The decay $b \to s\gamma$ has been investigated theoretically for the standard model and its extensions [3]-[11]. CP-violation in $B - \bar{B}$ system was considered in [12].

In this paper we consider the CP asymmetry in the decay $b \to s\gamma$ for the left-right symmetric $SU(2) \times SU(2) \times U(1)$ model, which is one of the simplest extensions of the standard model. Calculated value of CP-asymmetry for some range of parameters of the model (the mass of the right W-boson, the ratio of two Higgs doublet vacuum expectation values, phases and mixing angles) is almost 2 times larger than in standard model and can have an opposite sign, while the decay rate is almost the same.

The Lagrangian of interaction of quarks with scalar and $SU(2) \times SU(2) \times U(1)$ gauge fields has the following form:

$$
L = (A_{ik}\bar{\Psi}_L^i \Phi \Psi_R^i + B_{ik}\bar{\Psi}_L^i \tilde{\Phi} \Psi_R^i + c.c) +
ig_R\bar{\Psi}_R^a \tilde{W}_R^a \sigma_a \Psi_R^i + ig_L\bar{\Psi}_L^a \tilde{W}_L^a \sigma_a \Psi_L^i
$$

where $i,k=1,2,3$ and

$$
\Phi = \begin{pmatrix}
\eta^e \\
\eta^\nu \\
\xi^+ \\
-\xi^{o*}
\end{pmatrix},
\tilde{\Phi} = \begin{pmatrix}
\xi^o \\
\eta^+ \\
-\eta^{o*} \\
-\xi^-
\end{pmatrix},
\Psi_{i,L,R} = \begin{pmatrix}
U \\
D
\end{pmatrix}_{i,L,R}
$$

The symmetry $SU(2) \times SU(2) \times U(1)$ can be broken to $SU(2)_L \times U(1)$ by means of vacuum expectation values (vev) of doublet or triplet fields [3] [10] [11]. As for $SU(2) \times U(1)$ symmetry breaking we assume that it takes place when the scalar field $\Phi$ acquires the vev:

$$
\Phi = \begin{pmatrix}
k \\
0 \\
0 \\
-e^{i\delta}k'
\end{pmatrix}
$$

The interaction of quark charged current with W gauge boson and charged Higgs fields has the form:

$$
L^{ch} = \frac{1}{\sqrt{2}}(\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix}
-W_1^+ & -g_L \cos \beta K_L P_- - g_R \sin \beta e^{i\delta} K_R P_+ \\
-g_L \cos \beta K_L P_+ - g_R \sin \beta e^{i\delta} K_R P_- \\
-\frac{g_L}{\sqrt{2}M_W} \left[ -\tan 2\theta K_L M_d + e^{i\delta} \frac{1}{\cos 2\theta} M_u K_R \right] P_+ + \\
+ \left[ \tan 2\theta M_u K_L - e^{i\delta} \frac{1}{\cos 2\theta} K_R M_d \right] P_- + (d) \\
+ (s) \\
(b)
\end{pmatrix}
$$

where $W_1$ is the "light" (predominantly left-handed) charge gauge boson and $\beta$ is the mixing angle between left and right W - bosons,

$$
\tan 2\beta = 2\sin 2\theta \frac{g_R M_W^2}{g_L M_W} \\
\tan \theta = -\frac{k'}{k}
$$
$K_L$ and $K_R$ are Kobayashi-Maskawa mixing matrices for left and right charged currents respectively, $P \pm = (1 \pm \gamma_5)/2$,

\[
M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}
\]

The matrices $K_L$ and $K_R$ can be expressed in such a form where $K_L$ has only one complex phase and $K_R$ has five complex phases [13]. In (3) we omit the term, connected with the interaction with heavy (predominantly right) W- boson, which is not relevant for $b \rightarrow s \gamma$ decay.

The effective lagrangian for $b \rightarrow s \gamma$ decay has the following form [8, 9, 10, 11]:

\[
H_{b \rightarrow s \gamma} = - \frac{e}{16\pi^2} \sqrt{2} m_b \left( K_{ts} L K_{tb} A_{s \gamma} O_{7}^L + e^{i \delta} K_{ts} R K_{tb} \beta_{m_t} A_{s \gamma}^L R O_{7}^L \right) + e^{i \delta} K_{ts} L K_{tb} \sin 2 \theta \cos 2 \theta \frac{m_t}{m_b} A_{s \gamma}^L R O_{7}^L + e^{-i \delta} K_{ts} R K_{tb} \beta_{m_t} A_{s \gamma}^L R O_{7}^L + e^{-i \delta} K_{ts} L K_{tb} \sin 2 \theta \cos 2 \theta \frac{m_t}{m_b} A_{s \gamma}^R O_{7}^L
\]

where

\[
O_{7}^{L,R} = \bar{u}_s \sigma^{\mu \delta} (1 \pm \gamma_5) u_b F_{\mu \nu}, \quad O_{8}^{L,R} = \bar{u}_s \sigma^{\mu \delta} (1 \pm \gamma_5) u_b G_{\mu \nu},
\]

\[
A_{s \gamma}^{W_L} = \eta^{16} \left[ A_{s \gamma}^{W_L} + \frac{8}{3} A_{s g}^{W_L} (\eta^{-\frac{2}{3}} - 1) - \frac{232}{513} (\eta^{-\frac{2}{3}} - 1) \right]
\]

\[
A_{s \gamma}^{W_L,R} = \eta^{16} \left[ A_{s \gamma}^{W_L,R} + \frac{8}{3} A_{s g}^{W_L,R} (\eta^{-\frac{2}{3}} - 1) \right]
\]

\[
A_{s \gamma}^{\phi^+} = \eta^{16} \left[ A_{s \gamma}^{\phi^+} + \frac{5}{3} A_{s g}^{\phi^+} (\eta^{-\frac{2}{3}} - 1) \right]
\]

where $\eta = \frac{\alpha_s(m_W)}{\alpha_s(m_b)}$ and the functions $A_{s \gamma}^{W_L}, A_{s \gamma}^{W_L,R}, A_{s \gamma}^{\phi^+}$ were presented in [8, 9, 10, 11, 12, 13, 14]:

\[
A_{s \gamma}^{W_L} = Q_1 F_1(x) + G_1(x), \quad A_{s \gamma}^{W_L,R} = F_1(x)
\]

\[
A_{s \gamma}^{W_L,R} = Q_1 F_2(x) + G_2(x), \quad A_{s g}^{W_L,R} = F_2(x)
\]

\[
A_{s \gamma}^{\phi^+} = Q_1 F_3(y) + G_3(y), \quad A_{s g}^{\phi^+} = F_3(y)
\]

and

\[
F_1(x) = -\frac{1}{(1 - x)^4} \left[ \frac{x^4}{4} - \frac{3}{2} x^3 + \frac{3}{2} x^2 + \frac{x}{2} + \frac{3}{2} x^2 \log(x) \right]
\]

\[
G_1(x) = -\frac{1}{(1 - x)^3} \left[ \frac{x^4}{2} + \frac{3}{4} x^3 - \frac{3}{2} x^2 + \frac{x}{4} - \frac{3}{2} x^3 \log(x) \right]
\]

\[
F_2(x) = -\frac{1}{(1 - x)^3} \left[ -\frac{x^3}{2} - \frac{3}{2} x + 2 + 3 x \log(x) \right]
\]

\[
G_2(x) = -\frac{1}{(1 - x)^3} \left[ -\frac{x^3}{2} + 6 x^2 - \frac{15}{2} x + 2 - 3 x^2 \log(x) \right]
\]
\[ F_3(y) = -\frac{1}{(1 - y)^3} \left[ \frac{y^3}{2} + 2y^2 - \frac{3}{2}y - y \log(y) \right] \]
\[ G_3(y) = -\frac{1}{(1 - y)^3} \left[ \frac{y^3}{2} + \frac{1}{2}y + y^2 \log(y) \right] \]

where \( Q_t = 2/3 \) is the electric charge of the top quark, \( x = m_t^2/m_W^2 \), \( y = m_t^2/m_{\phi^*}^2 \).

The direct CP- asymmetry for \( b \to s\gamma \) decay arises only when one takes into account the final state interaction effects, when the absorptive parts arise.

Absorptive parts of the decay amplitude arise from rescattering \( b \to su\bar{u} \to s\gamma \), \( b \to sc\bar{c} \to s\gamma \), \( b \to sg \to s\gamma \):

\[ H_{b \to s\gamma}^{\text{absorb}} = \frac{e}{16\pi^2} \frac{2G_F}{\sqrt{2}} m_b \left\{ K_{ts}^L K_{tb}^L A^W_{Ls} O_8^{Ls} \sin 2\theta \frac{m_t}{m_b} A^L_{sg} + \right. \]
\[ + \left. e^{i\delta} K_{ts}^L K_{tb}^R \frac{m_t}{m_b} A^W_{Ls} O_8^{Ls} + e^{i\delta} K_{ts}^L K_{tb}^R \frac{m_t}{m_b} A^L_{sg} + \right. \]
\[ + \left. e^{-i\delta} K_{ts}^R K_{tb}^L \frac{m_t}{m_b} A^W_{Ls} O_8^{Ls} + e^{-i\delta} K_{ts}^R K_{tb}^L \frac{m_t}{m_b} A^L_{sg} \right\} \quad (8) \]

where
\[ A^L_{sg} = \frac{\eta_2}{\eta_{12}} A^L_{sg}, \quad A^W_{Ls} = \frac{\eta_2}{\eta_{12}} \left[ A^W_{Ls} - 0.1687 \right], \quad A^W_{LR} = \frac{\eta_2}{\eta_{12}} A^W_{LR} \tag{9} \]

In the standard model only for the rescattering \( b \to su\bar{u} \to s\gamma \) and \( b \to sc\bar{c} \to s\gamma \) one obtains nonnegligible contribution \[14\]. For the two Higgs doublet extension of the standard model or left-right symmetric model the rescattering \( b \to sg \to s\gamma \) also must be taken into consideration. Taking into account the standard model result \[14\] we obtain the following result for the absorptive part of the decay \( b \to s\gamma \) in left-right symmetric model:

\[ H_{b \to s\gamma}^{\text{absorb}} \approx -\frac{e}{16\pi^2} \frac{2G_F o_s}{\sqrt{2}} m_b \left\{ O_7^L \frac{2}{9} \left( K_{ts}^L K_{tb}^L A^W_{Ls} + e^{i\delta} K_{ts}^L K_{tb}^R K_{lb}^T A^R_{sg} \right) + \right. \]
\[ + \left. \frac{1}{4} \left( K_{as}^L K_{ab}^L + 0.12 K_{cs}^L K_{cb}^L \right) c_1 \right\} \quad (10) \]

where
\[ A^R_{sg} = \beta \frac{m_t}{m_b} A^W_{LR} + \frac{m_t}{m_b} \sin 2\theta \frac{A^L_{sg}}{2} \tag{11} \]

We note that our result is different from the one obtained in \[13\]: the contribution of the rescattering \( b \to sg \to s\gamma \) differs from that in \[13\] by the factor \( 2/9 \). The CP-asymmetry for the decay \( b \to s\gamma \) and \( \bar{b} \to \bar{s}\gamma \) is defined as:

\[ a_{cp} = \frac{\Gamma(b \to s\gamma) - \Gamma(b \to \bar{s}\gamma)}{\Gamma(b \to s\gamma) + \Gamma(b \to \bar{s}\gamma)} \quad (12) \]

The resulting CP-asymmetry is equal to:

\[ a_{cp} = \frac{2\alpha_s}{(|C_{L7}^2| + |C_{R7}^2|)\sin \alpha_{L7}} \left\{ \left( \text{Im}v_{qu}^* v_u + 0.12 \text{Re}v_{qu}^* v_u \right) \times \right. \]
\[ \left( A_{s\gamma}^W + H \cos \alpha A_{s\gamma}^R \right) \frac{c_1}{4} + \left. \left( \text{Re}v_{qL}^* v_u + 0.12 \text{Im}v_{qL}^* v_u \right) \times \right. \]
\[ \left( A_{s\gamma}^R H \sin \alpha \frac{c_1}{4} + \frac{2}{9} H \sin \alpha v_{qL}^* v_u (A_{s\gamma}^W A_{s\gamma}^R - A_{s\gamma}^R A_{s\gamma}^W) \right) \right\} \quad (13) \]
where

\[
He^{i\alpha} \equiv e^{i\delta} \frac{K_{Rtb}}{K_{Ltb}}, \quad A_{s\gamma}^R \equiv \beta \frac{m_t}{m_b} A_{s\gamma}^{WLR} + \frac{m_t}{m_b} \sin 2\theta \frac{1}{2\cos 2\theta} A_{s\gamma}^{R+},
\]

\[
v_t \equiv K_{Lts}^* K_{Ltb}, \quad v_c \equiv K_{Lcs}^* K_{Lcb}, \quad v_u \equiv K_{Lus}^* K_{Lub}
\]

\[
C_L^7 = A_{s\gamma}^{WLR} + \left( e^{i\delta} \frac{K_{Rtb}}{K_{Ltb}} \right) A_{s\gamma}^R, \quad C_R^7 = e^{-i\delta} \frac{K_{Ltb}}{K_{Rtb}} A_{s\gamma}^R.
\]

In the numerical results we take \(\alpha_s = 0.24, c_1 \approx 1.1, m_t = 175\text{GeV}, m_b = 4.5\text{GeV}\), \(m_b \equiv m_b(m_t) = 3\text{GeV}\). The CP-asymmetry \(a_{cp}\) depends on the parameters of Kobayashi-Maskawa mixing matrix in Wolfensteins parametrization: \(\lambda = 0.221, \eta, \rho\), and also it depends on the parameters of left-right symmetric model: \(\alpha, \tan 2\theta, M_{W'}, M_{\varphi^+}, H\). We assume that \(|H| = |K_{tb}/K_{tb}^*| \approx 1\). For the fixed masses we vary the remaining parameters and obtain the allowed region of \(a_{cp}\) values. We take into account that in the left-right symmetric model the decay rate cannot differ sufficiently from those obtained in the standard model. The point is that the standard model predictions for decay rate are reasonable agreement with experiment. We present in Fig 1,2,3 the minimal and maximal values of the asymmetry for the various \(M_{W'}, M_{\varphi^+}\) and \(0 < \tan 2\theta < 3.5\) (the maximal and minimal values of \(a_{CP}\) are symmetric under changing of sign of \(\tan 2\theta\)). It is easy to see that the \(a_{CP}\) can be about 2 times larger than the maximal value of asymmetry in the standard model. For the masses of right sector \(\geq 5\text{TeV}\) the asymmetry is rather sensitive on changes of the Higgs boson mass than the right W-boson mass. In contrary to the standard model, where the asymmetry have a negative sign, in our case the sign of the asymmetry can also be positive. The results for \(a_{CP}\) in Fig 1,2,3 are obtained under the assumption that the decay rate in left-right symmetric model can differ from the standard model predictions no more than \(\Delta = 10\%\). Our calculations show that the results for maximal and minimal values of \(a_{CP}\) practically does not change when we vary \(\Delta\) from 10% to 50%. This fact is illustrated in Fig 4, where we compare the results for \(\Delta = 10\%\) and \(\Delta = 50\%\) for masses \(M_{W'} = M_{\varphi^+} = 10\text{TeV}\).

In conclusion, we have calculated the CP asymmetry in the decay \(b \to s\gamma\) for the left-right symmetric model \(SU(2) \times SU(2) \times U(1)\). We have shown that the CP-asymmetry for the reasonable range of parameters can be larger than in standard model and can have an opposite sign.

The research described in this publication was made possible in part by Grant N MVU000 from the International Science Foundation. A. I. has been supported by Lady Davis Trustship.
The minimal and maximal values of $a_{CP}$ for $\rho=-0.05$, $\eta=0.37$ and some values of $M_{W_R}$, $M_{\phi^+}$ and $\tan 2\theta$.

| $M_{W_R}$ | $M_{\phi^+}$ | $\tan 2\theta$=1 | $\tan 2\theta$=2 | $\tan 2\theta$=3 |
|----------|--------------|-------------------|------------------|------------------|
| 1.5TeV   | 10TeV        | ($-1.51 \div 0.33$)% | (-0.76 \div 0.16)% | - |
| 10TeV    | 1.5TeV       | -                 | -                | -                |
| 10TeV    | 10TeV        | ($-0.90 \div -0.37$)% | ($-1.27 \div 0.25$)% | ($-0.39 \div 0.41$)% |
| 20TeV    | 20TeV        | ($-0.73 \div -0.55$)% | ($-0.93 \div -0.35$)% | ($-1.21 \div -0.06$)% |
| 50TeV    | 10TeV        | ($-0.93 \div -0.35$)% | ($-1.31 \div 0.28$)% | ($-0.48 \div 0.44$)% |
| 10TeV    | 50TeV        | ($-0.65 \div -0.63$)% | ($-0.67 \div -0.61$)% | ($-0.74 \div -0.54$)% |
| 50TeV    | 50TeV        | ($-0.66 \div -0.62$)% | ($-0.70 \div -0.58$)% | ($-0.77 \div -0.51$)% |
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Figure Captions

Fig1. The maximal and minimal values of $a_{CP}$ in % for $M_{WR}=5\text{TeV}, M_{\varphi^+}=5\text{TeV}$ (curves 1 and 2); for $M_{WR}=10\text{TeV}, M_{\varphi^+}=10\text{TeV}$ (curves 3 and 4); for $M_{WR}=20\text{TeV}, M_{\varphi^+}=20\text{TeV}$ (curves 5 and 6); for $M_{WR}=50\text{TeV}, M_{\varphi^+}=50\text{TeV}$ (curves 7 and 8).

Fig2. The maximal and minimal values of $a_{CP}$ in % for $M_{WR}=1.5\text{TeV}$ and $M_{\varphi^+}=10\text{TeV}$ (curves 1 and 2); $M_{WR}=5\text{TeV}$ and $M_{\varphi^+}=10\text{TeV}$ (curves 3 and 4); $M_{WR}=10\text{TeV}$ and $M_{\varphi^+}=10\text{TeV}$ (curves 5 and 6); $M_{WR}=20\text{TeV}$ and $M_{\varphi^+}=10\text{TeV}$ (curves 7 and 8).

Fig3. The maximal and minimal values of $a_{CP}$ in % for $M_{WR}=10\text{TeV}$ and $M_{\varphi^+}=1.5\text{TeV}$ (curves 1 and 2); $M_{WR}=10\text{TeV}$ and $M_{\varphi^+}=5\text{TeV}$ (curves 3 and 4); $M_{WR}=10\text{TeV}$ and $M_{\varphi^+}=10\text{TeV}$ (curves 5 and 6); $M_{WR}=10\text{TeV}$ and $M_{\varphi^+}=20\text{TeV}$ (curves 7 and 8).

Fig4. The maximal and minimal values of $a_{CP}$ in % for $M_{WR}=10\text{TeV}, M_{\varphi^+}=10\text{TeV}$ and allowed difference from standard model 10% (curves 1 and 2); $M_{WR}=10\text{TeV}, M_{\varphi^+}=10\text{TeV}$ and allowed difference from standard model 50% (curves 3 and 4).
Fig. 1
