Quark rotation asymmetry and baryon magnetic moments

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Quark rotation asymmetry is proposed in calculating baryon magnetic moments. After taking into account interactions enforced on constituent quarks, assumed to be linear and Coulomb potentials, respectively, and the quark rotation asymmetry, we fit the theoretical values, based on two more hypotheses and several other reasonable assumptions, of baryon magnetic moments with those from experiments. The good fitting results shows the necessity of the consideration of quark rotation asymmetry within baryon.

PACS number(s): 24.85.+p, 14.20.-c, 13.40.e

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I. INTRODUCTION

Problem on the spin of proton has been discussed since a long time ago. Physicists are still still puzzled with where proton gets its spin despite that efforts have been made.

As known, in Naive Quark Model (NQM), proton’s spin is assumed to be carried by three valence quarks. So the magnetic moments of proton is entirely contributed also by three valence quarks. The baryon magnetic moments presented by NQM fit the experimental measurement of baryon magnetic moments well. So for a quite long time our knowledge of proton spin stayed comfortably on the level of NQM.

However, in the last few years, the well-known “Spin-Crisis”, which is arisen by the polarized experiments of muon scattering from proton at CERN by the European Muon collaboration (EMC) [1], has led to the new argument of the origin of proton spin. These experiments presented the surprising conclusion that only a small part of proton spin is carried by the spin of light quarks (and antiquarks) it contains, which was in disagreement with the popular picture for nucleon structure.

Several attempts [2,3] have been made in trying to solve the puzzle mentioned above. Some authors [2] argued that the orbital angular momentum of constituent quarks may have significant contribution to proton spin. But other authors [3] argue that the data obtained by EMC give no definite conclusion on proton spin content and errors, considering the fact that effect resulting from the uncertainty of extrapolation had not been fully considered. The argument is still on. Thus the EMC data and the works [1-3] excite the old question “May the spin of baryon be attributed to the rotation of constituent quarks within baryon ?”. It is worth discussing.

In our opinion it is quite natural to accept the concept of “rotating proton”. Actually that concept has been emphasised by Chou and Yang [4] in 1974. In 1974, Sehgal [5] discussed the importance of constituent quarks rotation. And Ellis-Jaffe sum rule [6] was also given in the same year. Since then many papers, e.g., Refs. [7-9], discussed the origin of proton spin. In Ref. [9] Meng proposed two polarization experiments to test the possible existence for rotating constituents. In a recent work [10] presented by Casu and Sehgal a model with collective quark rotation is used in discussing proton spin and baryon magnetic moments. Their results fit the experimental ones well. Therefore, the above works show the possible contribution from quark revolving to baryon spin and magnetic moments.

In this work we adopt the concept of “revolving quark”. We consider the possible existence of quark rotation asymmetry. We simply assume the interaction potentials enforced on constituents to be linear and Coulomb type respectively. With the formulae for baryon magnetic moments, we fit baryon magnetic moments with those from experiments.

In section II the formulae for contribution of quark moments to baryon magnetic moments are derived quantum mechanically, in section III the contribution from orbital angular momentum to baryon magnetic moments through a simple calculation , and the fitted results, are presented. The last section is a brief conclusion.

II. FORMULAE FOR CONTRIBUTION FROM QUARK MOMENTS TO BARYON MAGNETIC MOMENTS

In this section we will show formulae in calculating baryon magnetic moments without considering the contribution from constituent quarks rotation.

First of all, the $z$ component of polarized quark and antiquark contribution to the spin of a polarized proton is [5]

$$\langle S_z \rangle = (\Delta u + \Delta d + \Delta s)/2,$$

with $\Delta u, \Delta d$ and $\Delta s$ are known as parton spins, which can be related to the values of the axial vector coupling constants $G_A, a^{(8)}$ and $\langle S_z \rangle$ through the following formular

$$\Delta u = \frac{2}{3} \langle S_z \rangle + \frac{1}{2} G_A + \frac{1}{6} a^{(8)},$$

$$\Delta d = \frac{2}{3} \langle S_z \rangle - \frac{1}{2} G_A + \frac{1}{6} a^{(8)},$$

$$\Delta s = \frac{2}{3} \langle S_z \rangle - \frac{1}{3} a^{(8)}.$$  

We take $G_A$ and $a^{(8)}$ their experimentally measured values, 1.26 and 0.60, respectively, and $\langle S_z \rangle$ is used as a fitting parameter.

To a first approximation, the baryon magnetic moments can be expressed as a vector sum of the

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3,$$

where

$$\mathbf{M}_0 \sim \frac{1}{2} \mathbf{S},$$

$$\mathbf{M}_1 \sim \frac{1}{3} \mathbf{S} + G_A \mathbf{A},$$

$$\mathbf{M}_2 \sim \frac{1}{3} \mathbf{S} - G_A \mathbf{A},$$

$$\mathbf{M}_3 \sim \frac{1}{3} \mathbf{S} - \frac{1}{3} a^{(8)} \mathbf{A}.$$
quark moments plus a contribution from any orbital angular moment of the quarks [11]. In this section we will only present the formulae of the contribution from quark moments to baryon magnetic moments. The orbital angular moment will be added in the next section.

The magnetic moments operator \( \hat{\mu} \) of a baryon is given by

\[
\hat{\mu}_B = \sum_{i=1}^{3} \mu_q(i) \hat{\sigma}(i)
\]

, where the sum is over the three quarks in a baryon, and \( \hat{\sigma} \) is the Pauli matrix. The quark moments of any baryon is the expectation value of \( \hat{\mu}_B \) with respect to a baryon wave function \( \Psi_B \) which is maximally polarized along the \( Z \)-axis, that is [11],

\[
\mu_B = \langle \Psi_B | \sum_{i=1}^{3} \mu_q(i) \hat{\sigma}(i) | \Psi_B \rangle
\]

It is hence possible to evaluate \( \mu_B \) in terms of \( \mu_q \) for any baryon once the flavor and spin wave functions of the baryon are specified. Using Eq. (4), after some careful calculation, one can write down the contribution of quark moments to baryon magnetic moments for spin-\( \frac{1}{2} \) baryon Octet as follows [12,13],

\[
\mu(p) = \mu_u \delta u + \mu_d \delta d + \mu_s \delta s,
\mu(n) = \mu_u \delta d + \mu_d \delta u + \mu_s \delta s,
\mu(\Sigma^+) = \mu_u \delta u + \mu_d \delta s + \mu_s \delta d,
\mu(\Sigma^-) = \mu_u \delta s + \mu_d \delta u + \mu_s \delta d,
\mu(\Xi^-) = \mu_u \delta s + \mu_d \delta d + \mu_s \delta u,
\mu(\Xi^0) = \mu_u \delta d + \mu_d \delta s + \mu_s \delta u,
\mu(\Lambda^0) = \frac{1}{6} (\delta u + 4\delta d + \delta s)(\mu_u + \mu_d) + \frac{1}{6} (4\delta u - 2\delta d + 4\delta s)\mu_s,
\mu(\Sigma^0) = \frac{1}{2\sqrt{3}}(\delta u - 2\delta d + \delta s) \times (\mu_u - \mu_d) \tag{5}
\]

, where, \( \delta q \) is defined as

\[
\delta q = \int dx[q_+(x) - q_-(x)]
- \int dx[q_+(x) - \bar{q}_-(x)], \tag{6}
\]

which differs from expression of \( \Delta q \) only in the sign of the antiquark contribution.

Relationship between \( \Delta q \) and \( \delta q \) is set through two reasonable hypotheses. These two hypotheses are based on the consideration of nucleon structure, hence they may be reasonable and acceptable.

**Hypothesis I**

One may expect that sea quarks in a polarized baryon entirely result from a cloud of spin-zero mesons. Such models have been discussed, for instance, in Ref. [14]. In such case \( \Delta q = \delta q \).

**Hypothesis II**

One may also expect that sea quarks in a polarized baryon are entirely produced by gluons splitting \( g \rightarrow q\bar{q} \). For simplicity, one may let \( \bar{u}_+ - \bar{u}_- \approx d_+ - d_- \approx k(\bar{s}_+ - \bar{s}_-) \approx k(s_+ - s_-) \), where \( k \) represents the relative abundance of various antiquarks within the baryon. For \( k = 1 \) it is the case in Ref. [12]. Generally \( \delta u = \Delta u - k\Delta s, \delta d = \Delta d - k\Delta s, \delta s = 0 \). In our work \( k = 0.5 \) is used.

Consider all the above things and the additional two relationship, that is, \( \mu_u = -2\mu_d \) and \( \mu_u = 3\mu_d/5 \), one can reexpress contribution of quark moments to baryon magnetic moments in terms of parameters \( \mu_u, G_A, a^b \) and \( \langle S_z \rangle \), during which only \( \mu_u \) and \( \langle S_z \rangle \) are undetermined.

### III. THE POSSIBLE EXISTENCE OF QUARK ROTATION ASYMMETRY

Formulae in last section for quark moments are given without considering the contribution from the revolving of constituent quarks. In this section we will add the contribution from quark revolving to baryon magnetic moments, considering the possibility of quark rotation asymmetry. Before doing this let us briefly review some other works.

In Ref. [10] the authors proposed a collective rotation method in calculating the spin of proton and baryon magnetic moments. The model they used is a flux-string one. In their model the quarks will tend to be situated at the corners of an equilateral triangle in the plane transverse to proton spin. They assume that the correlated 3-quark structure rotates collectively around the \( z \)-axis, which adds contribution of the revolving quarks to proton spin and to baryon magnetic moments. Some reasonable results are obtained.

As we know the total angular momentum of a polarized proton can be resolved as \( J_z = \langle S_z \rangle + \).
\( \langle L_z \rangle + \Delta G = 1/2 \). \( \Delta G \) is the contribution from gluons. Here \( \langle L_z \rangle \) is related to the motion of quarks within baryon and shared by all the constituents. In Ref. [10] the authors let the revolving radius \( r \) be the same for each quark, so the orbital angular momentum each quark contributes is proportional to its mass.

We appreciate the simplicity of the ideas presented in Ref. [10]. However, we would like to emphasize here we should pay more attention to the structure of baryons. It is most important.

We should note there are three valence quarks within baryons, which mainly contribute to orbital angular momentum. If baryons are made up of three same quarks there will exist symmetry in quark rotation without any doubt. However the three quarks within baryons are not exactly same. In proton there are two \( u \) quarks and one \( d \) quark, for neutron there are two \( d \) quarks and one \( u \) quark. Even for \( \Lambda^0 \) there exists one \( u \), \( d \), and \( s \) quark respectively. At least two quarks within baryons are different. We know that \( u \), \( d \), and \( s \) quark differ from each other in many aspects, during which most important one is that they have different mass. The strange quark is much more heavier than the up and down quarks. Due to the difference, the rotation symmetry is badly breaking. So we propose the possible effect of constituent quark rotation asymmetry. But the concept “asymmetry” is what we should fully understand. The constituents still rotate along the geometry center of the triangle composed by three quarks. But due to the mass difference between constituents the triangle is scalene instead of equilateral, which means the length of radius \( r \) for different quark when they rotate along the axis is different. But another question arises “How can the radius be determined?”.

To answer this question, one has to add some more assumptions. It is obvious that the assumed centripetal force for quark revolving is entirely provided by the interaction between constituents. Since no one can find out the exact form of the interaction potential between quarks inside a hadron, one can and should assume the potential to be in the simplest form. In this paper, the total interactions enforced on a quark from other quarks within baryon is assumed to be proportional or inversely proportional to the revolving radius of the quark. The first interaction form corresponds to a uniform color tube, and the second one is a Coulomb-like one. Sure, they are simplest considerations about internal quark interactions. It can also be assumed that the direction of the force acting on a revolving quark points to the center.

With these assumptions one can easily calculate the revolving quark contribution to baryon magnetic moments.

In the first case we consider a quark in a linear potential, say \( U = cr \), with \( c \) a positive number and \( r \) the radius for quark of mass \( m \) revolving around the axis. So the force acting on the quark is a constant, \( F = -c \). Due to the flavor independence of color interactions, the forces acting on all revolving quarks are the same for every valence quark within the baryon. Then using \( F = m\omega^2 r \) one can get the dependence of \( r \) on \( m \): \( r \propto 1/m \). Hence the orbital angular momentum contributed from quark \( q_i \) of mass \( m_i \) is \( [(1/m_i)/(1/m_1 + 1/m_2 + 1/m_3)]\langle L_z \rangle \). Adding the revolving quark contribution to baryon magnetic moments in Eq. (1) one can obtain the entire formulae of baryon magnetic moments when the interaction potential between constituents is assumed to be a linear one,

\[
\mu_L(p) = \mu(p) + [2\mu_u \times \frac{1}{3}]
\]

\[
\mu_L(n) = \mu(n) + [2\mu_d \times \frac{1}{3}]
\]

\[
\mu_L(\Sigma^+) = \mu(\Sigma^+) + [2\mu_u \times \frac{1}{\lambda + 2}]
\]

\[
\mu_L(\Sigma^-) = \mu(\Sigma^-) + [2\mu_d \times \frac{1}{\lambda + 2}]
\]

\[
\mu_L(\Xi^-) = \mu(\Xi^-) + [2\mu_s \times \frac{\lambda \times 1}{2\lambda + 1}]
\]

\[
\mu_L(\Xi^0) = \mu(\Xi^0) + [2\mu_s \times \frac{\lambda \times 1}{2\lambda + 1}]
\]

\[
\mu_L(\Lambda^0) = \mu(\Lambda^0) + [\mu_u \times \frac{1}{\lambda + 2}]
\]

\[
\mu_L(\Sigma^0) = \mu(\Sigma^0) + [\mu_u \times \frac{1}{\lambda + 2}]
\]

\[
\mu_L(\Xi^0) = \mu(\Xi^0) + [\mu_u \times \frac{1}{\lambda + 2}]
\]
where \( \lambda = \frac{m_d}{m_s} = 0.6 \) and the subscript \( L \) indicates linear potential. We then can fit the corrected baryon magnetic moments with experimental values of baryon. In this paper, we fit the baryon magnetic moments with two different hypotheses presented earlier about sea quark contribution respectively. To every hypothesis we perform two fits. In fit 1, we let \( \mu_u, \langle S_z \rangle \) as fitting parameters with the constraint \( \langle S_z \rangle + \langle L_z \rangle = 1/2 \). This is, in fact, the extreme hypothesis that the “missing” angular momentum of the proton is precisely accounted for by the orbital angular momentum of revolving quarks. In fit 2, we let \( \mu_u, \langle S_z \rangle \) and \( \langle L_z \rangle \) as fitting parameters, where \( \langle S_z \rangle \) and \( \langle L_z \rangle \) are free. The fitting results are given in Table I.

For the second assumed potential, the interaction between constituent quarks is of Coulomb type, \( U = e/r \), and one can get the dependence of \( r \) on \( m \): \( r \propto 1/\sqrt{m} \). Hence the orbital angular momentum carried by the quark \( q_i \) of mass \( m_i \) is \( \left( \sqrt{m_1} / (\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}) \right) \langle L_z \rangle \). With the revolving quark corrections taken into consideration in Eq. (2), one can get the corrected formulae of baryon magnetic moments

\[
\mu_C(\Lambda^0) = \mu(\Lambda^0) + \left[ \mu_u \times \frac{\lambda^*}{2\lambda^* + 1} \right. \\
+ \mu_d \times \frac{\lambda^*}{2\lambda^* + 1} \\
+ \mu_s \times \frac{1}{2\lambda^* + 1} \left. \langle L_z \rangle \right] \\
\]

where \( \lambda^* = \sqrt{m_d/m_s} = \sqrt{0.6} \) and the subscript \( C \) indicates Coulomb potential. Similarly we fit the corrected baryon magnetic moments with experimental values of baryon with hypotheses assumed before respectively. The fitting procedures and constraints are exactly the same as for the case with linear potential. Fitted results are presented in Table II.

**IV. CONCLUSIONS**

In this work we discuss the possible quark rotation asymmetry within baryon. Based on two kinds of hypotheses on sea quark contribution and some reasonable assumptions about the internal interactions, the baryon magnetic moments are calculated. It should be pointed out that contribution from quark moments to baryon magnetic moments can be calculated quantum mechanically, while the contribution from angular momentum is a quasiclassical approximation. We fit the baryon magnetic moments with the experimental ones. The fitted results seem consistent with experimental measurements to certain extent. Thus we may possibly provide a simple method in solving the question “Where does the proton get its spin?” [15].

In our work we assume interactions between constituents are linear or Coulomb potential. To some extent the assumptions are too simple. We know that quarks are confined in the baryon. From the study of hadron spectrum it is known that the potential between quarks can be approximately written as \( AR + B/r \). Such potential can provide some good description of baryon feature. One can see that our two choices are just the two limits of \( AR + B/r \) when parameter \( B \) and \( A \) are assumed very small, respectively. So the two forms of interactions can be thought as reasonable.

In general, a clear picture of revolving quark
within baryon is presented in this paper. Calculation in this work is not too difficult.

One should also note that from our work and others parton quark model can give a good description of baryon magnetic moments, axis vector coupling and the proton spin. It is sure that we need more data to test the feasibility of quark parton model.

By now the inside structure of baryon is still unknown to us. We expect that future experiments on LHC can provide some picture in the nucleon structure, which would be of interest to both theoretical and experimental physicists.

This work was supported by the National Science Foundation of China, the Natural Science Foundation of Hubei, the Foundation of the State Education Commission of China.

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TABLE I. FITS OF MAGNETIC MOMENTS WHEN QUARK INTERACTIONS ARE LINEAR POTENTIAL.

|                  | Experimental Data | Hypothesis I | Hypothesis II |
|------------------|-------------------|--------------|---------------|
|                  | fit 1             | fit 2        | fit 1         | fit 2         |
| $\mu_L(p)$       | 2.79 ± 0.10       | 2.75         | 2.66          | 2.75          | 2.79          |
| $\mu_L(n)$       | -1.91 ± 0.10      | -1.82        | -1.92         | -1.85         | -1.74         |
| $\mu_L(\Sigma^+)$| 2.46 ± 0.10       | 2.71         | 2.61          | 2.63          | 2.58          |
| $\mu_L(\Sigma^-)$| -1.16 ± 0.10      | -1.09        | -1.17         | -1.03         | -1.12         |
| $\mu_L(\Xi^+)$  | -0.65 ± 0.10      | -0.48        | -0.56         | -0.42         | -0.51         |
| $\mu_L(L^0)$    | -1.25 ± 0.10      | -1.24        | -1.34         | -1.34         | -1.37         |
| $\mu_L(\Lambda^0)$| -0.61 ± 0.10      | -0.52        | -0.61         | -0.42         | -0.40         |
| $\mu_L(\Sigma^0)$| -1.61 ± 0.10      | -1.49        | -1.51         | -1.59         | -1.62         |
| $\chi^2$/DoF    | 2.14              | 1.09         | 2.47          | 2.05          |

| Fitted Param     | $\mu_u = 2.27$    | $\mu_u = 2.28$| $\mu_u = 2.40$| $\mu_u = 2.63$|
| $\langle S_z \rangle = 0.26$ | $\langle S_z \rangle = -0.03$ | $\langle S_z \rangle = 0.46$ | $\langle S_z \rangle = -0.33$|
| $\langle L_z \rangle = 0.23$ |                      |                | $\langle L_z \rangle = 1.09$ |                      |

TABLE II. FITS OF MAGNETIC MOMENTS WHEN QUARK INTERACTIONS ARE COULOMB POTENTIAL.

|                  | Experimental Data | Hypothesis I | Hypothesis II |
|------------------|-------------------|--------------|---------------|
|                  | fit 1             | fit 2        | fit 1         | fit 2         |
| $\mu_C(p)$       | 2.79 ± 0.10       | 2.76         | 2.71          | 2.80          | 2.80          |
| $\mu_C(n)$       | -1.91 ± 0.10      | -1.80        | -1.86         | -1.80         | -1.80         |
| $\mu_C(\Sigma^+)$| 2.46 ± 0.10       | 2.62         | 2.57          | 2.62          | 2.61          |
| $\mu_C(\Sigma^-)$| -1.16 ± 0.10      | -1.17        | -1.19         | -1.09         | -1.09         |
| $\mu_C(\Xi^+)$  | -0.65 ± 0.10      | -0.56        | -0.58         | -0.48         | -0.48         |
| $\mu_C(L^0)$    | -1.25 ± 0.10      | -1.32        | -1.38         | -1.37         | -1.38         |
| $\mu_C(\Lambda^0)$| -0.61 ± 0.10      | -0.56        | -0.41         | -0.45         | -0.45         |
| $\mu_C(\Sigma^0)$| -1.61 ± 0.10      | -1.49        | -1.51         | -1.49         | -1.48         |
| $\chi^2$/DoF    | 1.14              | 1.56         | 2.09          | 2.13          |

| Fitted Param     | $\mu_u = 2.22$    | $\mu_u = 2.24$| $\mu_u = 2.01$| $\mu_u = 1.99$|
| $\langle S_z \rangle = 0.17$ | $\langle S_z \rangle = 0.31$ | $\langle S_z \rangle = -0.30$ | $\langle S_z \rangle = 0.86$|
| $\langle L_z \rangle = 0.04$ |                      |                | $\langle L_z \rangle = -0.37$ |                      |