Dissipative peregrine solitons in fiber lasers

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Abstract

We show both numerically and experimentally that a dissipative type of Akhmediev-breathers (ABs) and Peregrine solitons (PSs) can be formed in a fiber laser, and their features could be well described by the laser Ginzburg-Landau equation. Moreover, we show that laser gain bandwidth limitation effect can arrest the dissipative ABs. Consequently, a stable one-dimensional periodic pulse train or periodic PS crystal structure can be formed. A kind of movable 'PSs' has also been obtained in our fiber laser. The solitons have a PS-like profile but exhibit similar features as those of the dissipative solitons formed in a mode locked fiber laser.

1. Introduction

Recently a type of rational solitons predicted by the nonlinear Schrödinger equation (NLSE), the Peregrine soliton (PS), was experimentally observed in several physical systems. Kibler et al experimentally investigated the properties of the Akhmediev-breathers (ABs) in a fiber transmission system and confirmed that, as the modulation parameter increased, the ABs approached the PS [1]. Bailung et al reported the experimental observation of PSs in multicomponent plasma with negative ions [2]. Chabchoub et al observed PSs in a water wave tank [3]. Moreover, Hammani et al reported the PS generation and breakup in standard telecommunication fibers [4]. The PS, firstly derived by Peregrine in 1983 [5], is the first-order rational solution of the nonlinear Schrödinger equation. It has the characteristic of both spatial and temporal localizations. Due to the feature, it is believed that the formation of PS is related to that of Rogue waves [6], a phenomenon that has attracted great attention of scientific research [7–10].

Theoretically, it can be proved that the PS is the limiting case of either the Ma-breathers [11] or the ABs [12]. Both types of breathers are the exact solutions of the NLSE. While the Ma-breathers are periodic in time and tend to the plane wave as the spatial coordinate goes to infinity, the ABs are periodic in space and approach the plane wave as time goes to infinite. It has been shown that the ABs well describe the modulation instability of light propagation in the anomalous dispersion single mode fibers [12]. Indeed, the experiment of Kibler et al has well shown the relationship between the ABs and the PS. However, it is to note that constrained by the conservative nature of the NLSE, despite of the fact that an AB could possibly approach the PS, it cannot become the PS. In addition, Li et al have theoretically predicted a type of combined PS and ABs in PT-symmetric coupled waveguides whose properties are governed by coupled NLSEs [13]. Dai et al also theoretically predicted the PS and Kuznetsov-Ma solitons in PT-symmetric nonlinear couplers with gain and loss [14]. In this paper we show both numerically and experimentally that a kind of ABs can also be formed in a fiber laser and exhibit similar features as those predicted from the nonlinear Schrödinger equation (NLSE). Moreover, new features are also identified on the observed dissipative ABs, e.g. they can evolve into the PSs by increasing the pump power; The ABs can be arrested by the gain bandwidth limitation effect; A kind of movable 'PSs' have also been experimentally observed in fiber lasers. A fiber laser is intrinsically a dissipative system where the gain-loss balance also plays a role on the dynamics. To distinguish, we have named the breathers and PSs observed in a fiber laser the dissipative ABs.
and PSs. Finally, we shown that the movable ‘PSs’ formed in a fiber laser exhibit similar features as those of the dissipative solitons formed in a mode locked fiber laser.

2. Numerical simulations

The analytic AB solution of the NLSE has the form [1],

$$u(\xi, \tau) = \frac{(1 - 4a) \cosh(b\xi) + \sqrt{2a} \cos(\Omega\tau) + ib \sinh(b\xi)}{\sqrt{2a} \cos(\Omega\tau) - \cosh(b\xi)} e^{i\xi}. \quad (1)$$

Here $\xi$ and $\tau$ are the normalized distance and time, $\Omega$ is the dimensionless modulation frequency, and $0 < a < 1/2$ determines the frequencies that experience gain and $b = [8a(1 - 2a)]^{1/2}$ determines the instability growth. The breathers have the characteristics that, as the modulation parameter increases, the temporal separation between the adjacent peaks increases and the compressed temporal width of each individual peak decrease. As $a$ approaches the limiting value $1/2$, the limiting solution that was first derived by Peregrine and is now known as Peregrine soliton, has the rational form,

$$u(\xi, \tau) = [1 - \frac{4(1 + 2i\xi)}{1 + 4\tau^2 + 4\xi^2}] e^{i\xi}. \quad (2)$$

The Peregrine soliton is described as a 'rational soliton' of the conservative NLSE system. Constrained by the conservative nature of the NLSE, although the AB could possibly approach the PS, it cannot become a PS.

To understand why the NLSE type of ABs could be observed in a fiber laser, we note that under the conditions that the cavity length is far shorter than the dispersion and nonlinear length, and the light field is resonant with the cavity, the dynamics of light circulation in a unidirectional ring fiber cavity are described by the following Ginzburg-Landau equation [15],

$$\frac{i}{\partial z} \frac{\partial u}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 u}{\partial \tau^2} + \gamma |u|^2 u - i \frac{g - \alpha}{2} u - i \frac{g}{2} \frac{\partial^2 u}{\partial \tau^2} = 0 \quad (3)$$

where $z$ is the propagation distance and $t$ is the retarded time in the reference frame of the group velocity. $u$ is the slowly varying amplitude of the light field, $\beta_2$ is the average cavity dispersion parameter, $\gamma$ is the nonlinearity of the fiber, $g$ is the average effective laser gain coefficient, and $\alpha$ is the normalized average loss coefficient in the cavity, $\Omega_\xi$ is the effective bandwidth of the laser gain. The gain saturation of the laser is described by,

$$g = \frac{g_0}{1 + \int |u|^2 \, dt/E_i} \quad (4)$$

where $g_0$ is the effective small signal gain coefficient and $E_i$ is the saturation energy of the gain medium. Moreover, if the gain bandwidth of a laser is sufficiently broad so that it could be considered as infinity, then the last term of the equation (3) could further be neglected, and the equation (3) becomes,

$$\frac{i}{\partial z} \frac{\partial u}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 u}{\partial \tau^2} + \gamma |u|^2 u - i \frac{g - \alpha}{2} u = 0. \quad (5)$$

We note that gain saturation is an intrinsic feature of all lasers. Under steady state operation of a laser, the cavity losses are always balanced by the saturated gain. Hence, the fourth term in equation (5) also automatically disappears. Consequently, the equation (3) is reduced to the well-known NLSE [16]. Hence, despite of the fact that a fiber laser is a dissipative system in nature, under steady state operation and negligible gain bandwidth limitation, its dynamics mimic those of the NLSE.

The soliton formation in fibre lasers was extensively studied in the past. It is now a consensus that soliton operation is a general feature of the mode locked fiber lasers [17]. In a mode locked fiber laser, the mode locking process always generates an optical pulse in the cavity. If the pulse intensity is beyond a certain threshold, as shown in equation (3) above, due to the mutual interactions among the effects of cavity dispersion and nonlinearity, gain and losses, as well as the effective gain bandwidth limitation, the mode locked pulse will be shaped into one or multiple solitons. Strictly speaking, all solitons formed in a fiber laser are dissipative solitons [18]. However, as shown by equation (3), under appropriate conditions solitons that mimic the dynamics of NLSE could also be observed [19–22].

We point out that although conventionally almost all soliton fiber lasers are mode locked lasers, mode locking itself is not a necessary requirement for the soliton formation in a fiber laser. The role of mode
locking in those lasers is just to prepare a suitable initial condition so the solitons will be formed. In fact when a fiber laser is operating in the nonlinear regime, even without mode locking it can also automatically evolve to AB formation and/or soliton formation. To demonstrate it, we have numerically simulated the operation of a unidirectional fiber ring laser.

Our simulation is based on a simple unidirectional fiber ring laser whose cavity is made of single mode erbium-doped fiber and standard single mode fibers. Specifically, we used 3 m gain fiber whose group velocity dispersion (GVD) parameter is $-48 \text{ ps nm}^{-1} \text{ km}$, and 17 m standard single mode fiber whose GVD parameter is $18 \text{ ps nm}^{-1} \text{ km}$, and the laser has 10% output coupling. Although we have used the specific cavity length and the fibers for the calculation, it is to note that the reported phenomena are independent on the concrete cavity length and fiber parameter selections. Different from the conventional mode locked fiber lasers, no any mode locker is inserted in the laser cavity. We used a technical as described in [19] to simulate the operation of our fibre laser. Briefly, we start the simulation with a certain weak initial optical field and let it circulate in the fiber cavity. The light propagation in the gain fiber is described by the Ginzburg-Landau equation as shown in equation (3) except that the fiber parameters are the values given above, rather than the average cavity parameters. The gain saturation of the laser is considered using equation (4). In the passive fibers, we set $g = 0$. In the fibers we set $\alpha = 0$, but when the light encounters the cavity output coupler, 10% of the light intensity is deducted from the optical field. After one cavity roundtrip, the final optical field will then be used as the initial field for the next round of the cavity circulation, until a steady state of the laser operation is obtained, or a very large number of cavity roundtrips have been calculated.

As no mode locking could occur in the laser, its operation always starts with CW emission. The CW power increases with the laser gain and the saturation energy set in equation (4). First, we consider the CW laser emission is perturbed by a weak periodic modulation as $u = A (1 + B \cos(\omega_c t))$, where the parameter $A$ and $B$ are the amplitude of the continuous wave and the small modulation, respectively, and $\omega_c$ is the modulation frequency. Physically, many effects could cause periodic amplitude modulation on an optical field. One of the widely studied effects is the modulation instability [23]. Recently, we have also shown that the cavity detuning could also cause a kind of low frequency modulation instability in a fiber ring laser [24]. Numerically it is observed that when the laser gain bandwidth limitation effect is negligible, numerically this is done by setting a very large gain bandwidth, e.g. $\Omega_g = 40 \text{ nm}$. As a result of the weak periodic modulation, an AB type of laser emission can be automatically formed in the fiber laser, as shown in figure 1. In all our simulations the laser wavelength, which is in resonance with the cavity, is set at the center of the gain profile. At a fixed cavity roundtrip, the laser emission is a periodic pulse train with a pulse repetition rate determined by the modulation frequency. Along the cavity roundtrip each of the pulses exhibits the typical features of the ABs, e.g. at a fixed $E_s$, as the breather period is gradually increased, the pulses at the maximum compression position approach the PS, as shown in figure 2.
The formed ABs also display distinct new features as compared with those formed in a NLSE system. The energy of the breathers formed in a fiber laser varies with the laser gain saturation. Through adjusting pump power, which changes the $g_0$ and $E_s$ values, both the breather intensity and the CW power can be altered. Numerically it is identified that through increasing the gain saturation energy, even at a fixed pulse repetition rate, the dissipative ABs approach the PS, as shown in figure 3. Extended numerical simulations have further shown that as $E_s$ is increased, higher order PSs could also be formed in a fiber laser [25]. Moreover, a high order dissipative PS could split into two, three or multiple pulses in the time domain depending on the $E_s$ value selection. Similar effects were also observed on the conventional ABs [4].

In above numerical simulations we have assumed that the gain bandwidth is very broad so that the gain bandwidth limiting effect could be ignored. In a real fiber laser, if this assumption is valid or not, depends on the concrete laser operation conditions. To determine the influence of effective gain bandwidth on the dissipative breathers, we also numerically simulated the laser operation under different effective gain bandwidths. A typical result is shown in figure 4. As far as the gain bandwidth limitation plays a role, the breather effect of the pulses along the cavity roundtrip will be suppressed, instead of it, all the pulses remain constant both with respect to the time and the cavity roundtrip. In analogy to the breather operation, either as the pulse repetition rate is decreased or the pump power is increased, the pulse width decreases and pulse intensity increases. Eventually each of the pulses in the state is shaped into a dissipative PS.

3. Experimental results

To corroborate the numerical results, we have set up an erbium-doped fiber ring laser to study the dissipative ABs and PS formation. Figure 5(a) shows a schematic of the fiber laser setup. The fiber ring has a length of 22 m that consists of 3 m Erbium doped fiber (OFS-EDF80, $D = -48$ ps km$^{-1}$ nm), 17 m single mode fiber.
Figure 4. (a) ABs arresting by the gain bandwidth limiting effect calculated with the same parameters as in figure 1 except the gain bandwidth $\Omega_g = 10$ nm. (b) A cut of figure 4 along $t = 0$, black line shows the peak power evolution.

(SMF-28, $D = 18$ ps km$^{-1}$ nm), and 2 m dispersion compensation fiber (DCF, $D = -4$ ps km$^{-1}$ nm). All the fibre-pigtailed components (ISO, WDM, OC) used in the cavity are specially selected that they have negligible polarization dependent losses and their functions are polarization independent. The pump source is a 1480 nm Raman fiber laser. It has a maximum output power of 5 W. The fiber laser works in the anomalous dispersion regime with an average group velocity dispersion $\beta_2 = -8$ ps$^2$ km$^{-1}$. The output of the laser was measured with a 25 GHz photodetector (Newport, Model 1014) together with a 33 GHz high-speed real-time oscilloscope (Agilent Technologies, DSA-93204 A). The optical spectrum of the laser emission was measured with an Optical Spectrum Analyzer (Yokogawa, AQ6375), and the pulse width was measured with a commercial autocorrelator (Femtochrome FR-103XL). To help understanding the experimental results easily, we also summarized the two observed scenarios in figures 5(b) and (c).

We emphasize that except the polarization controller, which is used to fine adjust the linear birefringence of the fiber cavity, no other polarization selective components or any saturable absorber exist in our fiber laser. As all the intracavity components are polarization independent, no nonlinear polarization rotation mode locking could occur in the laser. Under relatively low pump power, the fiber laser is always on CW emission. Although there is no any polarization selective component in the cavity, it is found experimentally that the laser emission is linearly polarized. We attribute the single polarization operation of the laser to the polarization instability effect of nonlinear light propagation in the weakly birefringent fibers \[26\]. For a fiber laser with sufficiently small net cavity birefringence, stable single polarization emission is possible. We operated the laser at a pump power of $\sim 27$ dBm. Under strong CW operation, the fiber laser output could easily become modulated. Figure 6 shows for example an experimentally measured laser emission state. Figure 6(a) is a long-time oscilloscope trace record, which shows that the laser emission breathes with a period of about 7 $\mu$s. Figure 6(b) shows the record in two breather periods. The cavity roundtrip time of our fiber laser is about 100 ns. Hence, the breather period is about 70 cavity roundtrips. Figure 6(c) shows a zoom-in of the laser emission. It shows that the laser emission is a periodic pulse train with a pulse repetition rate of $\sim 4$ GHz. In particular, figure 6(c) shows that between two strong pulses there is a weak small pulse, which accords well with the results of numerical simulations shown in figures 2 and 3. Based on the numerical results the small pulse is due to the existence of the finite length background between the breather pulses. Figure 6(d) further shows the optical spectrum of the laser emission. On the broad spectral profile there is clearly a narrow spectral spike located at the peak position of the spectrum, indicating that the periodic pulses are embedded in a CW background.

In our experiment the pulse repetition rate could be varied either by changing the orientation of the intracavity PC or the pump power. Associated with the pulse repetition rate change the breather period also varies, which is in agreement with the features of the ABs. Through carefully adjusting the orientations of the intracavity polarization controller (PC) at a fixed pump strength, another breather state as shown in figure 7 was obtained. Figure 7(a) displays the 3D temporal profile of the pulses in the ABs, the red pulse and violet pulse are obtained at the minimum compression position of the ABs, cyan and green pulse are obtained at the maximum compression position of ABs. Generally, at the maximum compression position, the pulse intensities are greater than the pulse intensities at minimum compression position. In the meanwhile, at maximum compression condition, the twin hole structures become more obvious. Figure 7(b) is the pulse...
obtained at the maximum compression position of ABs. The inset of figure 7(b) shows the pulses within the cavity. It is a periodic pulse train with a pulse repetition rate of $\sim 330$ MHz, which is significantly lower than that of the state shown in figure 6. When the pulse repetition rate is sufficiently low, it becomes obvious that the laser emission is alternating between a finite intensity CW and a narrow pulse. Similar to the case shown in figure 6, all the pulses in the pulse train have identical interference pattern with the CW, in particular, there is always an intensity gap down to the zero intensity between the pulse and the CW background, suggesting that the pulses are approaching the PS [1, 5]. Figure 7(c) shows the measured optical spectrum of the laser emission. It not only has a smooth triangle profile, which is a characteristic of the PSs [1], but also displays Kelly sidebands, showing that they are indeed solitary pulses [27]. We also experimentally compared the Kelly sidebands of the PSs with those of the dissipative solitons formed in mode locked fiber lasers, except that the Kelly sidebands of the PSs is obviously broader, no other detectable difference could be identified. To explain why the Kelly sidebands are broader than the conventional dissipative solitons, we note that Dennis et al have experimentally studied properties of the Kelly sidebands. It is shown that their positions in the soliton spectrum depend on the soliton pulse width and the cavity dispersion value [28]. As the PSs in figure 7 are formed inside the ABs, their pulse widths vary with the ABs. Therefore, their Kelly sideband positions also vary slightly from each other, which leads to the broadening of the observed Kelly sidebands.

Depending on the intracavity PC setting and the pumping power, PS splitting and PS bunching are also experimentally observed, as shown in figure 8. Despite of the existence of the CW background and the generation of new PSs, the relative positions of the solitons and the soliton bunches in the cavity do not change. This is very different from the conventional solitons observed in the mode locked fiber lasers, where...
mediated through the CW background the multiple solitons always show constant relative movements in the laser cavity [29].

However, the kind of ABs could only be obtained under strong pumping and certain appropriate intracavity PC settings. Under general laser operation conditions, instead of the ABs, a kind of one-dimensional periodic laser intensity pulsation is always easily obtained. Again, like the dissipative AB states shown above, either by decreasing the pulse repetition rate or increasing the laser pump power, the periodic intensity pulses could gradually evolve to a one-dimensional PS crystal structure as shown in figure 9. Figure 9(a) is a long-time oscilloscope trace. Figure 9(b) is the corresponding optical spectrum. We note the obvious Kelly sidebands on the pulse spectrum, which shows that the pulses in the pulse train are solitary pulses. Figure 9(c) is a zoom-in of the oscilloscope trace. Limited by the bandwidth of our detection system (25 GHz), the exact intensity profile of the pulses could not be resolved in the oscilloscope trace. Therefore, we further measured it with a commercial autocorrelator, as shown in figure 9(d). Comparing the measured autocorrelation trace with that of the numerical simulated Peregrine soliton, they show reasonable agreement, suggesting that the pulses are approaching the PS. Periodic pulse trains of PSs with a variety of pulse repetition rates have been obtained in our experiment. Once obtained, such a periodic pulsation state is very stable. All the PSs in the train seem frozen in the cavity. Each of them has almost identical temporal interference pattern with the background intensity, suggesting that they are phase synchronized with each other and the CW background.

From a state as shown in figure 9, if the pulse intensity is further increased, the pulses in the cavity will no longer remain static, but start to move freely in the cavity. Consequently, the soliton distribution in the cavity becomes irregular, as shown in figure 10. Once such a state is reached, the pulses then behave as the conventional dissipative solitons formed in a mode locked fiber laser. However, if zooming-in the pulses, they still show the characteristic intensity gap to the CW background, similar to that shown in figure 7(a), although the gap no longer down to the zero intensity.

4. Discussion

We found that the observed experimental results are well in agreement with the numerical simulations. Although in a CW fiber laser there is no such an initial pulse available to trigger the conventional soliton formation, our numerical results clearly demonstrated that a weak periodic modulation could drive a strong...
Figure 7. PS operation of the fiber laser: (a) 3D temporal profile of the pulses in the Abs; (b) Oscilloscope trace of pulse obtained at maximum compression position of Abs; (c) Optical spectrum of the laser emission.

Figure 8. A state of multiple PSs operation of the laser.

CW laser emission to an AB type of laser emission or to a one-dimensional periodic pulse train emission. Specifically, if the effective gain bandwidth limitation effect is ignorable, the dissipative AB emission could be obtained, while if the condition is not fulfilled, a stable periodic pulse train emission will be observed. No matter which of the laser emission states is formed, under an appropriate cavity condition, as the pulse repetition rate is decreased or the pulse intensity is increased, the pulses could evolve to the PSs.

In a practical fiber laser, many factors could affect the effective laser gain bandwidth. Hence it is anticipated that both situations could be observed. Indeed, our experimental results well confirmed it. Specifically, since the ABs are formed as a result of the NLSE-like dynamics, only under certain special conditions such a phenomenon could be obtained. The one-dimensional periodic pulsation state, which is a result of the Ginzburg-Landau equation dynamics, is always easily observed. Different from the breathers in a NLSE system where they can only possibly approach the Peregrine soliton but can never become a Peregrine soliton, in a fiber laser both the breathers and the one-dimensional PS crystals can not only become PSs but also further evolve into movable ‘PSs’. We note that before the movable ‘PSs’ are formed, all the pulses in the cavity are synchronized with each other and the CW background. Their relative positions are fixed in the cavity. Although the Kelly sidebands appear on their optical spectrum, indicating that they are already
solitary pulses, as each individual pulse cannot move freely, they are not a soliton in the conventional sense. Finally, we emphasize the fact that once a freely movable PS is formed, from their dynamics one could hardly distinguish them from the conventional dissipative solitons formed in a mode locked fiber laser. Therefore, special case has to be taken to identify in a fiber laser if a soliton operation state is formed as a result of the mode locking or the Peregrine soliton shaping. Our experimental results clearly demonstrated that under strong nonlinearity, dissipative solitons could be formed in a fiber laser even without mode locking.

5. Conclusion

In conclusion, we have shown numerically that dissipative type of ABs could be formed in a fiber laser under steady state laser operation, and the dissipative breathers could even evolve to a PS. In the case if the effective gain bandwidth limiting could not be neglected, the AB effect will be arrested, consequently a laser will emit a stable one-dimensional periodic pulse train. And under appropriate laser cavity conditions, such a periodic pulse train can also evolve into a one-dimensional PS crystal structure. Based on an erbium-doped fiber ring laser we have also experimentally confirmed the existence of the dissipative AB and their evolution to the dissipative PSs. PS splitting and multiple PS formation are also observed in our fiber laser. Moreover, we have shown experimentally that a dissipative PS could further be shaped into a movable particle-like soliton which behaves just like a conventional dissipative soliton formed in a mode locked fiber laser. Our experimental results have not only shown the existence of dissipative ABs and PSs, but also shown that they are easily accessible in a fiber laser. We anticipate that this experimental result could trigger extended study on the dissipative PSs in the real-world nonlinear systems.

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