On power corrections to the event shape distributions in QCD

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Abstract:

We study power corrections to the differential thrust, heavy jet mass and $C$—parameter distributions in the two-jet kinematical region. We argue that away from the end-point region, $e \gg \Lambda_{\text{QCD}}/Q$, the leading $1/Q$—power corrections are parameterized by a single nonperturbative scale while for $e \sim \Lambda_{\text{QCD}}/Q$ one encounters a novel regime in which power corrections of the form $1/(Qe)^n$ have to be taken into account for arbitrary $n$. These nonperturbative corrections can be resummed and factor out into a universal nonperturbative distribution, the shape function, and the differential event shape distributions are given by convolution of the shape function with perturbative cross-sections. Choosing a simple ansatz for the shape function we demonstrate a good agreement of the obtained QCD predictions for the distributions and their lowest moments with the existing data over a wide energy interval.
1. Introduction

Analysis of hadronization effects to the final states in $e^+e^-\to$ annihilation has became the subject of active QCD studies \[1\]. There exist infrared and collinear safe event shape variables for which perturbative QCD can be applied at large center-of-mass energies $s = Q^2$ to calculate their differential distributions and mean values as series in $\alpha_s(Q)$. It has been observed many years ago \[2\] that for some shape variables like thrust, $t$, and heavy jet mass, $\rho$, perturbative QCD predictions deviate from the data by corrections suppressed by powers of the large energy scale $1/Q^p$, with the exponent $p$ depending on the variable and $p = 1$ for $t$- and $\rho$-variables. Such hadronization corrections were measured experimentally \[1\] over a wide energy interval $14 \leq \sqrt{s}/\text{GeV} \leq 189$ and were found to have a different form for the differential event shape distributions, $d\sigma/de$, as compared to their mean values, $\langle e \rangle = \sigma_{\text{tot}}^{-1} \int de \, e \, d\sigma/de$. For the mean value $\langle e \rangle$ the leading power correction is parameterized by a nonperturbative scale $\lambda_p$ of dimension $p$, while hadronization corrections to the differential distribution are described by a function $f_{\text{hadr}}(Q,e)$ depending on both the shape variable and the center-of-mass energy

$$\langle e \rangle = \langle e \rangle_{\text{PT}} + \frac{\lambda_p}{Q^p}, \quad \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{de} = \frac{d\sigma_{\text{PT}}}{de} + f_{\text{hadr}}(Q,e) \quad (1.1)$$

with $e$ denoting a general event shape variable ($e = t, \rho, C, ...$) and the subscript PT referring to perturbative contribution, $\langle e \rangle_{\text{PT}} = \int de \, e \, d\sigma_{\text{PT}}/de$. Obviously, the hadronization corrections to the differential distributions have a richer structure than those to the mean values. For instance, nonperturbative scales $\lambda_p$ parameterizing power corrections to $\langle e \rangle$ are defined by the moment $\int de \, e \, f_{\text{hadr}}(Q,e)$.

Power corrections in (1.1) are associated with hadronization effects in $e^+e^-\to$ final states and, as a consequence, the magnitude of the scales $\lambda_p$ and the function $f_{\text{hadr}}(Q,e)$ cannot be calculated within perturbative QCD approach. However it was recognized some time ago \[3, 4, 5, 6\], that analysis of infrared renormalon ambiguities of perturbative QCD series suggests the value of dimensionless exponents $p$ as well as the dependence of the function $f_{\text{hadr}}(Q,e)$ on the large scale $Q$. Namely, perturbative QCD series generate power corrections of the form (1.1) through IR renormalons contribution but fail to predict uniquely their values – it is only the sum of perturbative and nonperturbative contributions that becomes well-defined \[1\]. To give a meaning to the perturbative series in (1.1) one has to regularize IR renormalon singularities. This can be done in two different ways: one can specify a particular prescription for integrating IR renormalon singularities like principal value prescription \[8\]. Alternatively, one can avoid IR renormalon ambiguities by introducing an explicit IR cut-off $\mu$ on momenta of soft particles in perturbative expressions. In this case, one can either impose a “hard” IR cut-off on momenta of soft particles in the Feynman integrals, $k_\perp > \mu$, \[3\] or replace QCD coupling constant by an effective IR finite coupling constant which coincides with $\alpha_s(k_\perp)$ at large scale $k_\perp$ and deviates from it at $k_\perp < \mu$ \[3, 4\]. Following each of these ways, one specifies perturbative ($\mu$-dependent) contribution to (1.1) including perturbatively induced power corrections. Still, there exists a genuine nonperturbative contribution to the event...
shapes coming from the QCD dynamics at scales below $\mu$. This contribution cannot be
determined from the analysis of perturbative QCD series while its magnitude depends
on the choice of the IR regularization and, as a consequence, on the IR cut-off $\mu$.

For some hadronic observables like mean values of the event shapes and their dif-
ferential distributions away from the end-point region, leading nonperturbative power
corrections can be parameterized using different IR renormalon inspired phenomenological
models. Their predictions agree well with the experimental data and the
extracted values of phenomenological nonperturbative parameters exhibit approximate
universality. Despite a phenomenological success of these models, it remains still unclear
what is the physical meaning of new nonperturbative QCD scales and what is the origin
of the universality property within QCD. In the present paper we address these prob-
lems using the factorization properties of the event shape distributions established in [10].
We shall argue that nonperturbative power corrections to the thrust, heavy jet mass and
$C$–parameter distributions are described by the universal shape function which is a new
nonperturbative QCD distribution measuring the energy flow in the two-jet final states
in $e^+e^−$–annihilation.

The paper is organized as follows. In Sect. 2 we discuss the general properties of
corrections to the event shape distributions in the end-point region. In Sect. 3 we
formulate the factorization procedure and define the shape function. In Sect. 4 we
show that the differential event shape distributions are given by the convolution of the
resummed perturbative cross-sections with universal shape function. Choosing a simple
ansatz for this function we compare QCD predictions with the existing data. In Sect. 5
we apply the obtained expressions to calculate the power corrections to the first two
moments of the distributions. Concluding remarks are given in Sect. 6.

2. Event shape distributions

In this paper we shall consider three event shape variables: thrust $T$, heavy jet mass $\rho$
and $C$–parameter. They are defined in the standard way as

\[ T = \max \frac{\sum_k |\vec{p}_k \cdot \vec{n}_T|}{\sum_k |\vec{p}_k|}, \quad \rho = \max \left( \frac{M_R^2}{Q^2}, \frac{M_L^2}{Q^2} \right), \quad (2.1) \]

where $M_R^2$ and $M_L^2$ denote the total invariant masses flowing into the right and left hemi-
spheres with respect to the plane orthogonal to the thrust axis $\vec{n}_T$. The $C$–parameter
is given by

\[ C = 3(\theta_1 \theta_2 + \theta_2 \theta_3 + \theta_3 \theta_1) \quad (2.2) \]

with $\theta_j$ being eigenvalues of space-like part of the energy-momentum tensor
\[ \Theta^{\alpha\beta} = \sum_k \vec{p}_k \vec{p}_k/|\vec{p}_k|/ \sum_j |\vec{p}_j|. \]

Introducing the new variable $t = 1 - T$ one notices that thus defined event shapes $e =
(t, \rho, C)$ have a number of common features. Lowest order perturbative QCD calculation
leads in all three cases to the following expression for the differential distribution for $e > 0$
\[
\frac{d\sigma_{\text{PT}}}{de} = \frac{\alpha_s(Q)}{2\pi} A_e(e)\theta(e_{\text{max}} - e) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B_e(e) + \mathcal{O}(\alpha_s^3), \tag{2.3}
\]

where \(A_e\) and \(B_e\) are known coefficient functions and normalization is chosen as \(\int d\sigma_{\text{PT}} = 1\). Lowest order correction \(A_e\) gets contribution only from the three-particle final state which populates the kinematic region \(0 \leq e \leq e_{\text{max}}\) with \(t_{\text{max}} = \rho_{\text{max}} = 1/3\) and \(C_{\text{max}} = 3/4\). Away from the end-point region, \(e \gg \Lambda_{\text{QCD}} / Q\), the perturbative expansion (2.3) is well-defined and it describes the final states consisting of particles with relative transverse momentum that scales at large center-of-mass energy as \(\sim Q\). As \(e\) approaches the three-particle upper limit, \(e = e_{\text{max}}\), \(A_t\) and \(A_\rho\) vanish while \(A_C\) takes a finite value \[A_t(1/3) = A_\rho(1/3) = 0, \quad A_C(C) = \frac{256}{243}\pi \sqrt{3} C_F \left[ 1 - \frac{8}{3} \left( C - \frac{3}{4} \right) + \mathcal{O}((C - 3/4)^2) \right]. \tag{2.4}\]

For \(e = (t, \rho, C) \rightarrow \Lambda_{\text{QCD}} / Q\) the final states consist of two narrow jets with invariant mass \(M_{R,L}^2 \sim \Lambda_{\text{QCD}} Q\). Examining (2.3) one finds that \(A_e\) diverges in the end-point region \(e \rightarrow 0\) as \[A_e(e) = \frac{4C_F}{e} \left[ \ln \frac{e_0}{e} - \frac{3}{4} \right] + \mathcal{O}(\ln e) \tag{2.5}\]

with \(e_0 = \rho_0 = 1\) and \(C_0 = 6\). Similar Sudakov-type corrections appear to higher orders, \(\alpha_s^N \ln^{2N-n} e/e\) with \(n \geq 0\), and need to be resummed \([13, 14]\). They originate from the effects of collinear splitting of quarks and gluons inside two narrow energetic jets and their interaction with surrounding cloud of soft gluons. The underlying QCD dynamics depends on two infrared scales, \(Qe\) and \(Q^2 e\), such that \(1/Q \ll 1/(Qe^{1/2}) \ll 1/(Qe)\). The smallest scale \(Qe\) sets up the typical energy carried by soft gluons, while the scale \(Q\sqrt{e}\) defines the transverse momenta of the jets, \(k_{\perp}^2 = Q^2 e\). Applying the standard IR renormalon analysis and examining sensitivity of perturbative expressions with respect to emission of particles on each of these scales, that is soft gluons with energy \(\sim Qe\) and collinear particles with the transverse momentum \(\sim Q^2 e\), one finds that nonperturbative corrections to the differential distribution appear suppressed by powers of both scales. Then, in the end-point region, \(e = \mathcal{O}(\Lambda_{\text{QCD}} / Q)\), we may use the fact that \(Qe = \mathcal{O}(\Lambda_{\text{QCD}})\) and expand the differential distribution in powers of larger scale \(Q^2 e\). Keeping only the leading term of the expansion one gets \[\frac{1}{\sigma_{\text{tot}}} d\sigma = \sigma_0 \left( \alpha_s(Q), \ln e, \frac{1}{Qe} \right) + \mathcal{O}\left( \frac{1}{Q^2 e} \right) \tag{2.6}\]

where \(\sigma_0\) resums perturbative corrections in \(\alpha_s(Q)\) as well as power corrections on the smallest scale \(Qe\)

\[\sigma_0 = \frac{d\sigma_{\text{PT}}}{d\epsilon} + \sum_{k=1}^{\infty} \lambda_k (Qe)^k \Sigma_k(\alpha_s(Q), \ln e). \tag{2.7}\]
Here, $\Sigma_k$ are dimensionless perturbative coefficient functions and $\lambda_k$ are some nonperturbative scales, depending, in general, on the choice of the event shape variable. Using (2.7) we notice that the power corrections have a different form for $e \gg \Lambda_{QCD}/Q$ and $e \sim \Lambda_{QCD}/Q$.

For $e$ away from the end-point region, $e \gg \Lambda_{QCD}/Q$, one may keep in (2.7) only the first term

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{de} = \frac{d\sigma_{pr}}{de} + \frac{\lambda_1}{Qe} \Sigma_1(\alpha_s(Q), \ln e) + O\left(\frac{1}{(Qe)^2}\right).$$

(2.8)

The coefficient function $\Sigma_1$ can be found using the well-known property [4, 15] that the leading $1/Q$–power correction to the differential distribution (2.8) is generated by a shift of perturbative spectrum, $e \to e - \lambda_1/Q$. This leads to

$$\Sigma_1(\alpha_s(Q), \ln e) = -\frac{d}{d \ln e} \left[ \frac{d\sigma_{pr}}{de} \right].$$

(2.9)

Then, it follows from (2.8) that for $e \gg \Lambda_{QCD}/Q$ the leading power corrections to the differential distributions have a rather simple structure: they are parameterized by a single nonperturbative scale $\lambda_1$. The same scale determines $1/Q$–power correction to the mean value $\langle e \rangle$. The QCD predictions (2.8) are in a good agreement with the experimental data and the value of $\lambda_1$ has been fitted for different shape variables [4]. It is worthwhile to note that in the performed analysis of power corrections to the differential distributions [4] the fitting range of event shape variables was restricted to the region $e \gg \Lambda_{QCD}/Q$. At the same time, as we shall argue below, it is in the region $e \sim \Lambda_{QCD}/Q$ where a novel QCD regime is realized and the structure of hadronization corrections is drastically changed.

For $e \sim \Lambda_{QCD}/Q$ one finds that all terms in (2.7) become equally important and, therefore, need to be resummed to all orders in $1/(Qe)$. The resummation is based on the remarkable factorization properties of the differential distributions. As was shown in [10], the nonperturbative corrections to the leading asymptotic term $\sigma_0$ are factorized out into nonperturbative distribution function, the so-called shape function. The general factorized expression for differential distribution looks like [10]

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{de} = \int_0^{eQ} d\varepsilon f_\varepsilon(\varepsilon) \frac{d\sigma_{pr}(e - \frac{\varepsilon}{Q})}{de} + O\left(\frac{1}{Q^2 e}\right).$$

(2.10)

Its explicit form depends on the choice of the event shape variable $e = (t, \rho, C)$ and will be given in Sect. 4 (see Eqs. (4.1), (4.2) and (4.3)). For $e \gg \Lambda_{QCD}/Q$ one can expand the r.h.s. of (2.10) in powers of $1/Q$ to reproduce the expansion (2.7) with

$$\lambda_n = \int d\varepsilon \varepsilon^n f_\varepsilon(\varepsilon), \quad \Sigma_n(\alpha_s(Q), \ln e) = \frac{(-e)^n}{n!} \frac{d^n}{de^n} \left[ \frac{d\sigma_{pr}}{de} \right].$$

(2.11)

Expression (2.10) has a simple physical interpretation – nonperturbative corrections increase invariant masses of jets and effectively shift perturbative spectrum towards larger values of the shape variables with the weight given by nonperturbative distribution $f_\varepsilon(\varepsilon)$. 

4
3. Factorization and Shape functions

Factorization relations (2.10) take a simple form for the radiator functions $R(e)$ defined as

$$R(e) = \int_0^e \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{de'} \equiv \langle \theta(e - e(N)) \rangle. \quad (3.1)$$

Here, $\langle \ldots \rangle$ denotes averaging over all possible final states in $e^+e^-\text{annihilation}$ with the weight given by the differential distribution $1/\sigma_{\text{tot}} d\sigma/\text{d}e$ and $e(N)$ denotes the value of the event shape variable $e$ for a given final state $|N\rangle$. Calculating $R(e)$ in perturbation theory one finds

$$R_{\text{PT}}(e) = 1 - \frac{\alpha_s(Q)}{2\pi} \int_{e}^{e_{\text{max}}} \text{d}e' A_e(e') + O(\alpha_s^2(Q)). \quad (3.2)$$

Close to the two-jet region, $e \to 0$, perturbative expressions for $R(e)$ involve Sudakov logs $\alpha_s^n \ln^{2-n} e$ with $n \geq 0$. In the case of event shapes under consideration, $e = (t, \rho, C)$, these corrections can be systematically resummed to next-to-leading logarithmic (NLL) order and matched into exact two-loop perturbative expressions $\text{[13, 14]}$. To this accuracy the weights $e(N)$ can be expressed in terms of the total invariant masses $M^2_R$ and $M^2_L$ of two jets flowing into the right and left hemispheres, respectively. Moreover, the $t$– and $C$–parameters depend only on the sum of two masses and the corresponding perturbative radiation functions can be expressed to the NLL approximation as $\text{[13, 14]}

$$R_{\text{PT}}^t(e) = \langle \theta(e - t(N)) \rangle_{\text{PT}} = \left\langle \theta \left( e - \frac{M^2_R + M^2_L}{Q^2} \right) \right\rangle_{\text{PT}} \quad (3.3)$$

$$R_{\text{PT}}^C(e) = \langle \theta(e - C(N)) \rangle_{\text{PT}} = \left\langle \theta \left( e - 6 \frac{M^2_R + M^2_L}{Q^2} \right) \right\rangle_{\text{PT}} = R_{\text{PT}}^t(e/6), \quad (3.4)$$

where the subscript PT indicates that the final states in $e^+e^-\text{annihilation}$ are generating by perturbative branching of outgoing quark and antiquark. The radiator function for the $\rho$–parameter depends separately on the masses of two jets. Taking into account that perturbative evolution of two jets is independent on each other to the NLL approximation one gets $\text{[13]}

$$R_{\rho}^\text{PT}(e) = \langle \theta(e - \rho(N)) \rangle_{\text{PT}} = \left\langle \theta \left( e - \frac{M^2_R}{Q^2} \right) \right\rangle_{\text{PT}} \left\langle \theta \left( e - \frac{M^2_L}{Q^2} \right) \right\rangle_{\text{PT}}. \quad (3.4)$$

The perturbative expressions (3.3) and (3.4) are valid in the two-jet kinematical region $\Lambda_{\text{QCD}}/Q \ll e < e_{\text{max}}$ except the end-point region $e \sim \Lambda_{\text{QCD}}/Q$, in which the energy of emitted soft particles scales as $k_\perp \sim eQ \sim \Lambda_{\text{QCD}}$ and perturbation theory is expected to fail.

Calculating the radiator functions $R_e(e)$ one has to combine together perturbative and nonperturbative corrections. In the case of inclusive distributions, like deep inelastic structure functions and Drell-Yan distributions, this can be achieved by applying the
factorisation theorems. They allow to separate short-distance dynamics into perturbatively calculable coefficient functions and absorb large-distance corrections into universal nonperturbative distributions. Specific feature of the differential event-shape distributions is that they are not inclusive quantities but rather weighted cross-sections and, as a consequence, the standard methods are not applicable in this case.

It turns out [10] that IR factorization still holds for the leading term \( \sigma_0 \) in the expansion of the event-distributions (2.6) in the end-point region \( e \sim \Lambda_{QCD}/Q \). Its origin has a simple physical interpretation. In end-point region, the final state in e\(^+\)e\(^-\)–annihilation consists of two narrow jets surrounding by a cloud of soft gluons. Nonperturbative corrections \( \sim 1/(Q^2e) \) and \( \sim 1/(Qe) \) are associated with emission of collinear particles with the transverse momenta \( k^2_\perp \sim Q^2t \) and soft particles on the energy scale \( k^2_\perp \sim Qe \), respectively. Neglecting power corrections to (2.6) on a larger scale, \( \sim 1/(Q^2e) \), we may restrict analysis to soft particles only. Since soft particle cannot resolve the internal structure of narrow jets of transverse size \( k^2_\perp \sim Q^2e \), we may effectively replace two jets by a pair of energetic quark and antiquark moving back-to-back with the energy \( \sim Q/2 \).

The internal dynamics of two jets is governed by perturbative branching of quark and antiquark while effects of their interaction with soft gluons can be factorized out into the eikonal phase \( W_+W_+^\dagger \) with \( W_+ \) and \( W_- \) being the eikonal phases of quark and antiquark, respectively. They are given by Wilson lines \( W_\pm = P \exp(i \int_0^\infty ds n_\pm A(sn_\pm)) \) in which soft gluon field \( A_\mu(x) \) is integrated along the light-like directions \( n_\pm \) defined by the momenta of two outgoing jets. In the end-point region, collinear and soft particles provide additive contributions to the shape variables \( e = (t, \rho, C) \). As a consequence, the radiator functions are given in all three cases by a convolution of perturbative radiators \( R_{\text{PT}} \) and the same universal nonperturbative distribution \( f(\varepsilon_R, \varepsilon_L) \) describing the energy flow into the right and left hemispheres in the final state, \( \varepsilon_R \) and \( \varepsilon_L \), respectively, created by nonperturbative soft gluon radiation. The nonperturbative distribution \( f(\varepsilon_R, \varepsilon_L) \) is defined as follows [10]

\[
f(\varepsilon_R, \varepsilon_L) = \sum_N |\langle 0|W_+W_-^\dagger|N\rangle|^2 \delta(\varepsilon_R - (k_R n_+))\delta(\varepsilon_L - (k_L n_-)).
\]  

(3.5)

Here, sum goes over all possible soft gluon final states \( |N\rangle \) with \( k_R \) and \( k_L \) being the total momentum of soft particles moving into right and left hemispheres, respectively. The quantities \( (k_R n_+) \) and \( (k_L n_-) \) define the projection of the soft gluon momenta onto the directions of two jets, \( n^\mu_\pm = (1, 0, \pm 1) \), propagating into the same hemisphere.

Finally, the factorized expressions for the radiator function for the \( t- \) and \( C- \)variables look like

\[
R_t(e) = \int_0^{\varepsilon Q} d\varepsilon f_t(\varepsilon) R_{t\text{PT}}^{\varepsilon Q} \left( e - \frac{\varepsilon}{Q} \right)
\]  

(3.6)

\[
R_C(e) = \int_0^{\frac{3\pi}{2}\varepsilon Q} d\varepsilon f_C(\varepsilon) R_{C\text{PT}}^{\varepsilon Q} \left( e - \frac{3\pi}{2} \frac{\varepsilon}{Q} \right)
\]  

(3.7)
with nonperturbative distribution \( f_t(\varepsilon) \) defined as
\[
f_t(\varepsilon) = \int d\varepsilon_R \int d\varepsilon_L f(\varepsilon_R, \varepsilon_L) \delta(\varepsilon - \varepsilon_R - \varepsilon_L) = \int_0^\varepsilon d\varepsilon' \ f(\varepsilon - \varepsilon', \varepsilon') . \tag{3.8}
\]

In the case of the \( \rho \)-variable,
\[
R_\rho(e) = \int_0^{eQ} d\varepsilon_R \int_0^{eQ} d\varepsilon_L f(\varepsilon_R, \varepsilon_L) R_{pt}^J(e - \varepsilon_R / Q) R_{pt}^J(e - \varepsilon_L / Q) \tag{3.9}
\]

with \( R_{pt}^J(e) = [R_{pt}^J(e)]^2 \) and \( R_{pt}^J(e) = \langle \theta(e - M_R^2/Q^2) \rangle_{pt} \) being a single jet radiator function. We would like to stress that Eqs. (3.6), (3.7) and (3.9) hold in the region \( \Lambda_{QCD}^2/Q^2 < e < e_{\text{max}} \). They resum all power corrections of the form \( 1/(Qe)^n \) and are valid up to corrections \( \sim 1/(Q^2 e) \). According to (3.6), (3.7) and (3.9), the power corrections have a different form for \( \rho \) and \( e = (t, C) \) variables. In the latter case, the radiator function depends on an overall energy flowing into both hemispheres and described by the integrated distribution (3.8).

Nonperturbative corrections to the radiator functions (3.6), (3.7) and (3.9) are governed by the universal shape function \( f(\varepsilon_R, \varepsilon_L) \). This function is different from the well-known inclusive QCD distributions and its operator definition was given in [10]. Using (3.5) it is straightforward to show that \( f(\varepsilon_R, \varepsilon_L) \) is a symmetric function of its arguments, it does not depend on the center-of-mass energy \( Q \) and is normalized as
\[
\frac{d}{dQ^2} f(\varepsilon_R, \varepsilon_L) = 0, \quad \int d\varepsilon_R \int d\varepsilon_L f(\varepsilon_R, \varepsilon_L) = 1, \tag{3.10}
\]

where the last relation follows from unitarity of the eikonal phase \( W_+ W_-^\dagger \). The matrix element entering (3.3) does not depend on any kinematical scale and, as a consequence, the momenta of soft gluons contributing to (3.3) are not restricted from above. To separate the region of small gluon momenta one has to introduce the factorisation scale \( \mu \). Then, the shape function describes the contribution of gluons with \( k_\perp < \mu \), while the contribution of gluons with \( k_\perp > \mu \), is absorbed into perturbative radiator function \( R(e) \). In this way, both nonperturbative shape function and perturbative radiator become \( \mu \)-dependent while this dependence cancel in their convolution (3.6), (3.7) and (3.9). Since the \( \mu \)-dependence of radiator function \( R_{pt} \) can be calculated perturbatively, the above condition allows to obtain the evolution equations on the nonperturbative distributions [10]. Clearly, there exists an ambiguity in implementing IR cut-off inside perturbative expressions. Different prescriptions correspond to different ways of regularizing IR renormalon singularities and therefore lead to the different expressions for the nonperturbative distributions. In what follows we shall impose a “hard” IR cut-off [10] on gluon momenta inside the perturbative radiator functions entering (3.6), (3.7) and (3.9) as [10]
\[
R_{pt}(e) \to R_{pt}(e; \mu) = \theta(e - \mu) R_{pt}^{\text{NLL}}(e) + \theta(\mu - e) R_{pt}^{\text{NLL}}(\mu/Q) . \tag{3.11}
\]
Throughout the paper we shall substitute $R_{\text{PT}}^{\text{NLL}}(e)$ by its perturbative expression re-summed to the NLL accuracy and matched into two-loop explicit expressions within the modified ln $R$–matching scheme \[13\]. Thus defined radiator function (3.11) depends on two scales, $\Lambda_{\text{QCD}}$ and IR cut-off $\mu$, that we choose as

$$\Lambda_{\text{QCD}} = \mu = 0.25 \text{ GeV}.$$  \hspace{1cm} (3.12)

Within the prescription (3.11), the “regularized” perturbative spectrum $d\sigma_{\text{PT}}(e; \mu)/de = dR_{\text{PT}}^{\text{NLL}}(e)/de$ coincides with the ln $R$–matched perturbative distribution $dR_{\text{PT}}^{\text{NLL}}(e)/de$ for $\mu/Q < e < e_{\text{max}}$ and it vanishes inside the nonperturbative “window” $0 < e < \mu/Q$. Choosing the value of $\mu$ in (3.12) one has to be sure that the end-point of the perturbative distribution, $e = \mu/Q$, belongs to applicability range of the NLL resummed radiator function $R_{\text{PT}}^{\text{NLL}}(e)$ \[13\], $2\beta_0 \alpha_s(Q^2) \ln e < 2$. Despite the fact that the perturbative spectrum is well defined at $e = \mu/Q$ we do not expect that it provides a reasonable description of the physical distribution in the end-point region. Indeed, it is in this region that nonperturbative power corrections become dominant.

### 4. Differential distributions

Differentiating the radiator functions (3.6) and (3.7) we obtain the following expressions for the differential $t$–distribution

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_t}{de} = Q f(Qe; \mu) R_{\text{PT}}^{t}(0; \mu) + \int_0^{Qe} d\varepsilon f_t(\varepsilon; \mu) \frac{d\sigma_{\text{PT}}^{t}(\varepsilon - \varepsilon/Q; \mu)}{d\varepsilon}$$  \hspace{1cm} (4.1)

and $C$–distribution

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_C}{de} = \frac{2}{3\pi} Q f \left( \frac{3\pi}{2} Qe; \mu \right) R_{\text{PT}}^{C}(0; \mu) + \int_0^{2\pi Qe} d\varepsilon f_t(\varepsilon; \mu) \frac{d\sigma_{\text{PT}}^{C}(\varepsilon - \frac{3\pi}{2} \varepsilon/Q; \mu)}{d\varepsilon}.$$  \hspace{1cm} (4.2)

Here, we indicated explicitly the dependence of nonperturbative shape function and perturbative distributions on the factorization scale $\mu$. Two terms entering the r.h.s. of (4.1) and (4.2) have the following interpretation. Since the shape function $f_t(\varepsilon)$ rapidly vanishes for large $\varepsilon$, the first term contributes inside the nonperturbative window $0 \leq e < \mu/Q$. In this region the emission of perturbative real soft gluons is suppressed due to cut-off imposed on soft gluon momenta $k_\perp > \mu$ and the shape of the distribution is governed entirely by nonperturbative function $f_t(\varepsilon)$. Additional Sudakov factor $R_{\text{PT}}^{t}(0; \mu)$ takes into account the contribution of virtual soft gluons with $\mu < k_\perp < Q$ and it rapidly vanishes as $\mu$ decreases. The second term in (4.1) and (4.2) defines the spectrum inside the perturbative window $\mu/Q < e < e_{\text{max}}$. In this region, nonperturbative corrections smear the perturbative spectrum over the interval $\Delta e \sim \Lambda_{\text{QCD}}/Q$.

For the heavy mass distribution one finds

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\rho}}{de} = Q f(\rho Q, \rho Q; \mu) R_{\text{PT}}^{J}(0; \mu) + \int_0^{\rho Q} d\varepsilon f_{\rho}(\varepsilon, \rho Q; \mu) \frac{d\sigma_{\text{PT}}^{J}(\varepsilon - \varepsilon/Q; \mu)}{d\varepsilon},$$  \hspace{1cm} (4.3)
where $d\sigma_{j}^{\text{PT}}/de$ is single jet distribution resummed to the NLL order and defined by the radiator function $R_{j}^{\text{PT}}$, $d\sigma_{j}^{\text{PT}}/de = dR_{j}^{\text{PT}}(e)/de$. The heavy mass nonperturbative distribution is given by

$$f_{\rho}(\epsilon, eQ) = 2 \int_{0}^{\epsilon Q} d\epsilon' f(\epsilon, \epsilon') R_{j}^{\text{PT}}(e - \epsilon'). \quad (4.4)$$

Comparing (4.3) with (4.1) and (4.2) we notice that the factorized expressions for the differential $t-$, $C-$ and $\rho-$distributions have a similar form but the structure of power corrections is different in the case of the heavy mass. In distinction with (3.8), the heavy mass nonperturbative function $f_{\rho}$ depends on the shape variable, $e$, and the center-of-mass energy, $Q$. This dependence is controlled by perturbative radiator function and has the following interpretation. In the two-jet limit, the invariant mass of each jet is given by the sum of perturbative and nonperturbative contributions, $M_{R}^{2} = M_{R,\text{PT}}^{2} + \varepsilon_{R}Q$ and $M_{L}^{2} = M_{L,\text{PT}}^{2} + \varepsilon_{L}Q$. Perturbative radiation leads to $M_{R,\text{PT}}^{2}/Q^{2} \sim M_{L,\text{PT}}^{2}/Q^{2} \sim O(\alpha_{s}(Q))$, while nonperturbative contribution scales as $\varepsilon_{R} \sim \varepsilon_{L} \sim O(\Lambda_{\text{QCD}})$. In contrast with the $t-$variable, which depends on the sum of both masses and therefore is additive with respect to perturbative and nonperturbative contributions, the $\rho-$parameter is defined by the largest mass for which the “additivity” property is lost. Namely, comparing invariant masses flowing into two hemispheres one encounters a situation when masses of two perturbative jets are of the same order, $M_{R,L,\text{PT}}^{2} = O(Q^{2})$, while their difference is much smaller $|M_{R,\text{PT}}^{2} - M_{L,\text{PT}}^{2}| \sim O(Q\Lambda_{\text{QCD}})$.\(^2\)

In this case, nonperturbative correction to the difference of the jet masses becomes comparable with the perturbative contribution $|M_{R,\text{PT}}^{2} - M_{L,\text{PT}}^{2}| \sim Q|\varepsilon_{L} - \varepsilon_{R}|$, and therefore it can invert the perturbative hierarchy of jet masses, $M_{R,\text{PT}}^{2} < M_{L,\text{PT}}^{2}$, into $M_{R}^{2} > M_{L}^{2}$, for instance. Expression (4.3) takes into account this effect through the induced $Q-$dependence of the nonperturbative function (4.4).

According to (4.1), (4.2) and (4.3) the nonperturbative corrections to the $t-$, $C-$ and $\rho-$distributions involve two different nonperturbative functions. They are related however to the same universal nonperturbative shape function (3.3) describing the energy flow into two hemispheres in the final state. We recall, that $f(\varepsilon_{R}, \varepsilon_{L}; \mu)$ depends on the cut-off $\mu$ imposed on the maximal momenta of soft particles but it is independent on the center-of-mass energy $Q$. By the definition, $f(\varepsilon_{R}, \varepsilon_{L}; \mu)$ distinguishes between particles propagating into right and left hemispheres in the final state and therefore it is not completely inclusive with respect to partonic final states. Namely, it takes into account that quarks and gluons produced at short distances $\sim 1/Q$ and moving into one of the hemispheres will eventually decay at large distances $\sim 1/\Lambda_{\text{QCD}}$ and their remnants could flow into opposite hemispheres.\(^3\) This implies that, firstly, in contrast with the well-known inclusive QCD distributions, the shape function $f(\varepsilon_{R}, \varepsilon_{L}; \mu)$ is not related to the short distance QCD dynamics and, in particular, its moments can not be related to hadronic matrix elements of local composite operators. Indeed, according to the

\(^2\)To see that this configuration is not rare it is enough to notice that it corresponds to the vicinity of peak of the perturbative distribution over the difference of the jet masses $|M_{R}^{2} - M_{L}^{2}|/Q^{2}$\(^1\).

\(^3\)Similar effect has been studied using the IR renormalon approach in [16].
operator definition proposed in [10], the shape functions are defined in terms of the so-called “maximally nonlocal” QCD operators [17, 18]. Secondly, non-inclusive corrections to the shape function describe a “cross-talk” between two hemispheres in the final state leading to correlations between $\varepsilon_R$ and $\varepsilon_L$. As a consequence, the shape function is not factorizable into the product of functions depending on the energy flowing into separate hemispheres

$$f(\varepsilon_R, \varepsilon_L) = f_{\text{incl}}(\varepsilon_R)f_{\text{incl}}(\varepsilon_L) + \delta f_{\text{non-incl}}(\varepsilon_R, \varepsilon_L).$$  (4.5)

One should notice that similar property holds for perturbative Sudakov resummed radiator function (3.4). However, one finds that there the factorization holds to the NLL accuracy, Eq. (3.4), and non-inclusive corrections first appear at the NNLL level $\sim \alpha_s^2(\alpha_s L)^N$.

In what follows we shall rely on a particular ansatz for the shape function $f(\varepsilon_R, \varepsilon_L)$ which agrees with general properties of nonperturbative QCD distributions and has been used in previous studies of power corrections to the thrust distributions [10]. Namely, one expects that for small values of $\varepsilon_R$ and $\varepsilon_L$ the shape function should vanish as a power of the energy. Similarly, $f(\varepsilon_R, \varepsilon_L)$ should rapidly vanish as $\varepsilon_R$ or $\varepsilon_L$ becomes large. Taking into account these properties together with (4.5) one chooses the following expression

$$f(\varepsilon_R, \varepsilon_L) = \mathcal{N}(a, b) \frac{\varepsilon_R \varepsilon_L}{\Lambda^2} \left(\frac{\varepsilon_R \varepsilon_L}{\Lambda^2}\right)^{a-1} \exp \left(-\frac{\varepsilon_R^2 + \varepsilon_L^2 + 2b \varepsilon_R \varepsilon_L}{\Lambda^2}\right).$$  (4.6)

It depends on two dimensionless parameters $a$ and $b$ and the scale $\Lambda$. The factor $\mathcal{N}(a, b)$ is fixed by normalization condition (3.10).

The free parameters, $a$, $b$, and $\Lambda$, have the following meaning. The exponent $a$ determines how fast the shape function vanishes at the origin. The scale $\Lambda$ sets up the typical energy of soft radiation. The parameter $b$ controls the non-inclusive contribution to the shape function and its possible values are restricted as $b > -1$ in order for the shape function (4.6) to be normalizable. Non-inclusive corrections vanish at $b = 0$, $\delta f_{\text{non-incl}} = 0$ in (4.5). For $b \gg 0$, the shape function enhances the regions of the phase space $\varepsilon_R \gg \varepsilon_L$ and $\varepsilon_R \ll \varepsilon_L$, in which most of the energy flows into one of the hemispheres. For $b \to -1$ the energies are of the same order, $\varepsilon_R \sim \varepsilon_L$, and strongly correlated to each other. We expect that non-inclusive corrections to the shape function should be important and the configurations in which energy flows mostly into one of the hemispheres to be suppressed. This suggests that the possible values of the $b$—parameter should lie within the interval $-1 < b < 0$.

The parameters $a$, $b$ and $\Lambda$ depend on the factorization scale $\mu$ and are independent on the center-of-mass energy $Q$ as well as the choice of the shape variable $e = (t, \rho, C)$. This allows to fit their values by comparing the event shape distributions, Eqs. (4.1), (4.2) and (4.3), with the most precise available experimental data at $Q = M_Z$. Following this procedure we found that the fit to the heavy jet mass distribution is more sensitive to the choice of the parameters (especially to the non-inclusiveness parameter $b$) then the one to the thrust and the $C$—parameter. Then, fitting the heavy jet mass distribution at $Q = M_Z$ as shown in Fig. 1, we obtain

$$a = 2, \quad b = -0.4, \quad \Lambda = 0.55 \text{ GeV}.$$  (4.7)
Using these values we compare the QCD predictions for the $C$–parameter distribution at $Q = M_Z$ with and without nonperturbative corrections included as shown in Fig. 1b. Similar plot for the thrust distribution can be found in [10]. We observe that the differential distributions (4.3) and (4.2) combined with the shape function, Eqs. (4.4) and (4.7), correctly describe the data throughout the interval $0 < e < e_{\text{max}}$ including the end-point region $e = \mathcal{O}(\Lambda_{\text{QCD}}/Q)$. In addition, the $\rho$–parameter distribution turns out to be very sensitive to the choice of the $b$–parameter. The fact that its value, (4.7), is relatively large indicates that non-inclusive corrections to the shape function (4.5) are important indeed.

![Graphs showing heavy jet mass and C-parameter distributions at Q = M_Z with and without power corrections included.](image)

Figure 1: Heavy jet mass (a) and $C$–parameter (b) distributions at $Q = M_Z$ with and without power corrections included.

Having determined the parameters of the shape function, Eq. (4.7), at the reference energy scale $Q = M_Z$, we can now apply the factorized expressions for the differential distributions, (4.1), (4.2) and (4.3) with the same ansatz for the shape function (4.6) to obtain the QCD predictions at different energy and compare them with the data. The combined plot for the $\rho$– and $C$–parameter distributions over the center-of-mass energy interval $35 \text{ GeV} \leq Q \leq 189 \text{ GeV}$ is shown in Fig. 2a and b, respectively. Similar plot for the thrust distribution can be found in [10]. We observe that the theoretical curves reproduce the data over the whole interval of the shape variables including the end-point region.

5. **Moments of the event shapes**

Recently, the experimental data for the first few moments of various event shape distributions became available [1]. Their analysis indicates a presence of large hadronization
corrections whose form deviates from IR renormalon models describing the nonperturbative corrections to the distributions as the shift of perturbative spectrum.

Let us apply the obtained expressions for differential distributions to calculate the first two moments of the $t$-, $C$- and $\rho$-distributions defined as

$$\langle e^n \rangle = \int_0^{e_{\text{max}}} de e^n \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{de}, \quad (n = 1, 2). \quad (5.1)$$

Here, integration goes only over the part of the available phase space, $0 < e < e_{\text{max}}$, corresponding to the three-particle final states, and it does not take into account the contribution of multi-jet final states, $e > e_{\text{max}}$. Quantitative description of hadronization corrections to such final states is not available yet. Putting an upper limit on the value of the shape variable in (5.1) allows us to avoid the latter contribution and to replace the differential distribution $d\sigma/de$ in (5.1) by the obtained expressions (4.1), (4.2) and (4.3) which are valid for $0 < e < e_{\text{max}}$.

Using general expression (2.10) one calculates the mean value of the event shape as

$$\langle e \rangle = \langle e \rangle_{\text{PT}} + \frac{\langle \varepsilon \rangle}{Q} \left[ 1 - e_{\text{max}} \frac{d\sigma_{\text{PT}}(e_{\text{max}})}{de} \right] + O\left( \frac{1}{Q^2} \right), \quad (5.2)$$

where $\langle ... \rangle_{PT} = \int_0^{e_{\text{max}}} de \langle ... \rangle d\sigma_{\text{PT}}/de$ denotes averaging with respect to perturbative distribution and the scale $\langle \varepsilon \rangle$ is defined as the first moment of the shape function, $\langle \varepsilon \rangle = \int d\varepsilon \varepsilon f(\varepsilon)$. It is important to remember that the factorized expressions for the
differential distributions (4.1), (4.2) and (4.3) are valid up to $\mathcal{O}(1/(Q^2 e))$—corrections which may modify the mean value $\langle e \rangle$ by $\mathcal{O}(1/Q^2)$—terms. The additional factor in front of $\langle \varepsilon \rangle/Q$ takes into account that close to the edge of the three-particle phase space, $e \to e_{\text{max}}$, the perturbative distribution (2.3) could take nonzero values

$$e_{\text{max}} \frac{d\sigma_{\text{PT}}(e_{\text{max}})}{de} = \frac{\alpha_s(Q)}{2\pi} e_{\text{max}} A_e(e_{\text{max}}) + \mathcal{O}(\alpha_s^2).$$  \hspace{1cm} (5.3)

This can be checked using the explicit expressions for perturbative distributions (2.4). We find that $d\sigma_{\text{PT}}/de$ vanishes to one-loop order as $e \to e_{\text{max}}$ for the $t$— and $\rho$—variables while for the $C$—parameter it approaches a finite value. Finally, calculating the mean values $\langle \varepsilon \rangle$ with respect to nonperturbative distributions $f_t(\varepsilon)$ and $f_{\rho}(\varepsilon, eQ)$ defined in (3.8) and (4.4), respectively, we obtain

$$\langle t \rangle = \langle t \rangle_{\text{PT}} + \frac{\lambda_1}{Q} + \mathcal{O}(1/Q^2),$$  \hspace{1cm} (5.4)

$$\langle \rho \rangle = \langle \rho \rangle_{\text{PT}} + \frac{\lambda_1}{2Q} + \mathcal{O}(1/Q^2).$$  \hspace{1cm} (5.5)

Similarly, for the mean value of the $C$—parameter we get

$$\langle C \rangle = \langle C \rangle_{\text{PT}} + \frac{3\pi}{2} \frac{\lambda_1}{Q} \left[ 1 - \frac{\alpha_s(Q)}{2\pi} 5.73 + \mathcal{O}(\alpha_s^2) \right] + \mathcal{O}(1/Q^2).$$  \hspace{1cm} (5.6)

Here, large perturbative coefficient originates from (2.4) and it reduces a magnitude of the nonperturbative scale $\lambda_1$ by 11% at $Q = M_Z$. Relations (5.4) and (5.5) coincide with IR renormalon model predictions [3, 5, 14], while (5.6) differs by perturbative $\alpha_s(Q)$—dependent “boundary” term. Nonperturbative $Q$—independent scale $\lambda_1$ is given by

$$\lambda_1 = \int d\varepsilon_R \int d\varepsilon_L (\varepsilon_R + \varepsilon_L) f(\varepsilon_R, \varepsilon_L) = \int d\varepsilon \varepsilon f_t(\varepsilon).$$  \hspace{1cm} (5.7)

Substituting expression for the shape function, Eq. (4.6), one finds $\lambda_1 = \Lambda \varphi(a, b)$ with $\varphi$ given by $2F_1$—hypergeometric series. Using the values of the parameters (4.7) we find

$$\lambda_1 = 1.22 \text{ GeV}.$$  \hspace{1cm} (5.8)

We would like to recall that this value depends on the factorization scale $\mu$, Eq. (3.12), and its $\mu$—dependence is described by QCD evolution equation [10]. Obviously, the value of $\lambda_1$, and as a consequence $1/Q$—corrections to the mean values (5.4), (5.5) and (5.6) are less sensitive to the choice of the parameters $a$, $b$ and $\Lambda$ as compared with nonperturbative corrections to the corresponding differential distributions.

The comparison of the QCD predictions, (5.4), (5.5) and (5.6), with the data over the energy interval $35 \text{ GeV} \leq Q \leq 189 \text{ GeV}$ is shown in Fig. 3. One should notice that (5.4), (5.5) and (5.6) describe the contribution of the two-jet configurations while experimental data take into account all possible final states. A good agreement observed in Fig. 3 indicates that the contribution to the mean values of the final states with
three and more jets as well as \( \mathcal{O}(1/(Q^2e)) \) subleading corrections to the distributions are subdominant. Indeed, the dominant contribution to the moments (5.1) comes from the vicinity of peak of the differential distribution \( e^n d\sigma/de \). Using existing experimental data one can show \cite{1} that for \( n = 1 \) and \( n = 2 \) the position of the peak is located in the two-jet kinematical region while for higher \( n \) it moves towards larger \( e \) for which the integral (5.1) becomes very sensitive to the choice of the upper integration limit \( e_{\text{max}} \). We shall use this observation calculating the second moment of the event shape distributions.

Let us apply (5.1) to calculate \( \langle e^2 \rangle \). Neglecting \( 1/(Q^2e) \) corrections to the distribution (2.10) we find after some algebra the following general expression

\[
\langle e^2 \rangle = \langle e^2 \rangle_{\text{PT}} + \frac{\langle \epsilon \rangle}{Q} \left[ 2\langle e \rangle_{\text{PT}} - e_{\text{max}}^2 \frac{d\sigma_{\text{PT}}(e_{\text{max}})}{de} \right] +
\]

\[
+ \frac{\langle \epsilon^2 \rangle}{Q^2} \left[ 1 - e_{\text{max}}^2 \frac{d\sigma_{\text{PT}}(e_{\text{max}})}{de} + \frac{1}{2} e_{\text{max}}^2 \frac{d^2\sigma_{\text{PT}}(e_{\text{max}})}{de^2} \right] + \mathcal{O}(1/Q^3). \tag{5.9}
\]

Similar to the mean value, (5.2), the boundary terms vanish for the \( t \)– and \( \rho \)–variables while for the \( C \)–parameter they provide a sizeable contribution. Using the explicit expression for the shape functions, (3.8) and (4.4), we calculate the scales \( \langle \epsilon^2 \rangle \) and take
Figure 4: Comparison of the QCD predictions for the second moments $\langle t^2 \rangle$, $\langle \rho^2 \rangle$ and $\langle C^2 \rangle$ with the data. Dotted lines denote $O(\alpha_s^2)$—perturbative contribution, solid lines take into account power corrections given by Eq. (5.10).

We are grateful to O. Biebel and S. Kluth for providing us $O(\alpha_s^2)$—expressions for the moments of the event shapes.\footnote{We are grateful to O. Biebel and S. Kluth for providing us $O(\alpha_s^2)$—expressions for the moments of the event shapes.}

The $Q$—dependence of the scale $\delta \lambda_2$ is attributed to perturbative prefactor depending on the single jet distribution, $d\sigma_{j}^{PT}/d\rho$, defined in (4.3). Its origin was explained in Sect. 4. We find that the
value of this factor varies from 2.19 at $Q = 10 \text{ GeV}$ to 1.85 at $Q = 100 \text{ GeV}$. It is important to notice that $\langle (\varepsilon_R - \varepsilon_L)^2 \rangle$ vanishes if one does not take into account non-inclusive corrections to the shape function (1.5), $\delta f_{\text{non-incl}} = 0$. Using (1.6) and (4.7) one gets

$$\lambda_2 = 1.70 \text{ GeV}^2, \quad \langle (\varepsilon_R - \varepsilon_L)^2 \rangle = 0.14 \text{ GeV}^2. \quad (5.12)$$

It follows from (5.10) that the boundary terms generate a sizable perturbative corrections to the second moment of the $C-$parameter distribution and diminish the magnitude of scales parameterizing $1/Q-$power corrections. Moreover, one finds from (5.4), (5.5), (5.6) and (5.10) that the variance of the distribution, $\langle e^2 \rangle - \langle e \rangle^2$, does not receive $1/Q-$power corrections for the $t-$ and $\rho-$variables while for the $C-$parameter the boundary terms produce a negative $1/Q-$correction

$$\langle C^2 \rangle - \langle C \rangle^2 = \langle C^2 \rangle_{\text{PT}} - \langle C \rangle_{\text{PT}}^2 - 3.23 \frac{\lambda_1}{Q} \alpha_s(Q) + O(1/Q^2). \quad (5.13)$$

The comparison of the QCD predictions (5.10) with the experimental data is shown in Fig. 4. We would like to recall that the obtained expressions for the moments do not take into account the contribution of multi-jet final states configurations and assume a smallness of $1/(Q^2 e)$—corrections to the distributions (2.6). It is interesting to note that the last assumption is supported by the recent analysis of the power corrections to the first two moments of the thrust distribution in the single dressed gluon approximation [20]. This analysis is complimentary to our studies since it does not resum leading power corrections in the two-jet region and takes into account the contribution coming from the region $e \to e_{\text{max}}$.

6. Conclusions

In this paper we have studied the power corrections to the thrust, $t$, heavy jet mass, $\rho$, and $C-$parameter distributions in the two-jet kinematical region. Our analysis was based on the observation [10] that perturbative and nonperturbative effects can be separated in the differential event shape distributions into calculable Sudakov resummed distribution, $d\sigma_{\text{PT}}/de$, and nonperturbative shape function, $f(\varepsilon_R, \varepsilon_L)$, respectively. Each of them depends separately on the factorization scale $\mu$ but this dependence cancels in their product. The shape function describes the energy flow into two hemispheres in the final state. It does not depend on the center-of-mass energy $Q$ as well as on the choice of the event shape variable $e = t, \rho$ and $C$.

We demonstrated that away from the end-point region, $e \gg \Lambda_{\text{QCD}}/Q$, nonperturbative corrections to the distributions have a simple form (2.8) with the leading $1/Q-$power correction parameterized by a single scale given by the first moment of the shape function. In this region, to which all performed experimental analysis have been restricted so far, our predictions for the thrust and heavy mass distributions and their mean values coincide with those of IR renormalon based models while for the $C-$parameters we find an additional sizeable perturbative contribution modifying the magnitude of the $1/Q-$power correction (5.6).
In the end-point region, \( e \sim \Lambda_{\text{QCD}}/Q \), the obtained factorized expressions for the distributions take into account power corrections of the form \( 1/(Qe)^n \) for arbitrary \( n \). They are controlled by the shape function through (2.11) and are sensitive to the choice of this function. Comparing the QCD predictions with the data we have chosen the simplest ansatz for the shape function (1.6) which is consistent with general properties of nonperturbative distributions and includes nonzero correlations between energy flows into different hemispheres. Examining the dependence of the distributions on the corresponding parameter of the shape function we have observed that these correlations play an important rôle and are not negligible.

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