Communication by 16 scrolls attractors of Chua’s circuit

Sattar A Shahatha1,2*, Mutasim I Malik 1,3*, Eidan A Asi 1,3* and Huda A Jasim 1,3
1Physics department, University of Wasit, college of science, Iraq.
2Thi Qar city
3Wasit city
satara624@uowasit.edu.iq
Mutasim@uowasit.edu.iq
eidan@uowasit.edu.iq
huda624@uowasit.edu.iq

Abstract. This paper analyses the chaotic signals which are always broadband, and similar to noise but they are deterministic. In our paper, we use Chua’s circuit to provide a class of signals (chaotic signals) that can be used in different communications as carrier signals which masking the data. The synchronization feature that occurs between the systems are exploited in our work. On this basis, Chua's chaotic circuits were synchronized and data was transferred between the two circuits. The data were hidden in the output of the first Chua's circuit and transferred to the second chaotic circuit, then these data was showed after the merging between the two chaotic signals. We also doubled the number of Chua’s circuit’s scrolls for about 16 scrolls and showed that by increasing the number of scrolls, the reliability of data transfer was increasing.

1. Introduction
In the past four decades, the study of chaos has grabbed the eyes and minds of mathematicians, engineers, and physicists, and studied physical systems in which chaotic behaviour is evident. One of the most widely studied systems is Chua’s circuit. It’s a simple circuit with its structure but a complex behaviour, after being built by Leon Chua, studies of chaos, their properties and their applications have been poured out of that circuit. which is considered a transition from theory to practice [1]. Alan V. Oppenheim et al studied in 1992 the safe transfer of certain information through chaos. They combined the chaotic signals with the sound signal, then they showed it in the receiver [2]. After that and in 2003, Zhengguo Li et al they introduced new concept of magnifying glass and used the impulse control strategy to make the two circuits in sync [3]. Sergej Celikovsk et al used the synchronization method between two chaotic circuits, which one of them are including certain messages. while ensuring synchronization, the digital data then disappeared, indicating a safe transfer. They called it masking [4].

David Ruelle and Floris Takens developed the concept of the strange attractor in 1971 to try to understand the disorder and how it arose. In dynamic systems, the phase space represents the space in which all possible system states congregate. The state is a description of the system variables, once variable changes, the system moves from one state to another, so another point is added in the phase space. This model depends on the initial conditions of the system, and the parameters (indicators,
variables) 

Phase charts show that the system moves with time, and eventually leads to the same movement each time, whatever its initial conditions, as if systems are attracted to the performance of this movement. At the latter is forming a repetitive pattern called attractors. The attractor is a certain point in the phase space that makes the system as if attracted to it, to perform the same pattern of movement each time. There are simple attractors called points of stability, and other circular curves called Limit cycle, but in chaotic systems, the motion is formed by different attractors called Strange Attractor [6].

2. Modified Chua’s circuit

Please Chua’s circuit is a simple electronic circuit displaying classic chaotic behaviour. This almost means that it is a “nonperiodic oscillator”; it’s output a waveform oscillating, different from the normal electronic oscillator, never "repeated”[7]. Modified Chua’s circuit is made by replacing the inductor with simulating it from resistors and capacitors and op-amps, see figure (1). Many electrical circuits used have a stable behaviour, which occurs when their voltages settle at a fixed value. There are circuits that represent periodic oscillators (usually) where voltage values pass through a period and then repeat after a period of time called the Oscillation Period. These cases represent stable cases as minor changes in voltage will lead us back to the same point where the circuit is started [8].

In chaotic oscillatory circuits, the system's future is entirely dependent on minor changes in voltage. Because the future of that voltage depends heavily on where the circuit stops, where any slight change leads to significant differences in the future, and there are two reasons for this difference:

- Local instability near the oscillation pattern. The place where the circuit settles or goes (in phase space) depends primarily on where it started from.
- The chaotic system does not continue to diverge its paths forever, because non-linear behaviour in the chaotic circuit makes the movement returned to the pattern or general shape that it will draw in the space phase, but will be in a different place than it was previously, and the same period of a time interval.

The points that appear in the phase space are points that represent the variable voltages that determine the circuit, so the pattern is an unpredictable pattern, i.e. the circuit gives a wide range of point values, so outcome structure is rich by chaotic signals and this broadband of spectrum is what helps us in secure communication[9].

\[
I_R = \frac{1}{R} v_R \quad (1)
\]
and nonlinear resistor:

\[ I_R = g(V_R) \quad (2) \]

By using two Kirchhoff’s circuit laws in current and voltage (if you in a loop, then all voltages are gonna end up to Zero) and (if you going into a node, then all current are gonna end up to Zero). According that, the equation of the Chua’s circuit become:

For the right side in the circuit-as in fig1- we have:

\[ \frac{L}{R} \frac{dI}{dt} + V_c = 0 \quad (3) \]

In the middle of circuit (1st node) we have:

\[ I_l - C \frac{dV_c}{dt} + \frac{1}{R} (V_c1-V_c2) = 0 \quad (4) \]

At the left side (side of Chua’s diode, 2nd node):

\[ -C \frac{dV_c}{dt} + \frac{1}{R} (V_c2-V_c1) - g(V_c1) = 0 \quad (5) \]

Where \( g(.) \) is piecewise linear function and it’s equal to:

\[ g(V_c1) = G_B V_c1 + \frac{1}{2} (G_a-G_b) [V_c1 + B_p | V_{c1} - B_p |] \quad (6) \]

Where \( V_{c1} \) = the voltage over the capacitor \( C_1 \), \( V_{c2} \) = the voltage over the capacitor \( C_2 \), \( i_L \) = the current through the inductance, \( C = \) capacitance the capacitor, \( L = \) inductance the inductor, and \( G = \) conductance the linear resistor. \( g(V_{c1}) \) is the non-linear voltage-current characteristic of the nonlinear resistor. The inner segment has a slope (as in fig 2) of \( G_a \), while the outer segments, which begin at the voltage breakpoint \( \pm E \) (that equal to \( m_1 \)), have a slope of \( G_b \).

![Figure 2: Characteristic of Chua's circuit](image)

This line represent a negative resistance in I-V plane, so the system requires active element, that produce energy, and this is an Amplifier, that we need it in our research.
Figure 3: Chua’s circuit and Strange Attractor

Most of studies focused on dimensionless Chua’s circuit equations, and they are:

\[
\begin{align*}
\dot{x} &= \alpha (y-x) - \alpha f(x) \quad (7) \\
\dot{y} &= x - y + z \quad (8) \\
\dot{z} &= -\beta y \quad (9)
\end{align*}
\]

Where \( \pm \) \( E = \pm B_p \), \( G = 1/R \), \( \pm V_{c1}/E \), \( \pm V_{c2}/E \), \( z = I_L/EG \) and \( (\alpha, \beta, \gamma) \) are constant parameters and:

\[
f(x) = bx + \frac{1}{2} (a-b) \left[ |x+1| - |x-1| \right] \quad (10)
\]

These are used for describing the three-segment piecewise-linear characteristic of system with \( a, b \) being the slopes of the inner and outer segments of \( f(x) \) which represent the Chua’s diode. \( \gamma \) is usually ignored, i.e. \( \gamma = 0 [10] \). According to \( B_p \) we designed Chua’s circuit, and showed strange attractor, see image (3), and by multiple \( B_p \) in MultiSIM® program we got on 16 scrolls. This happened by doubling Chua’s diode as in fig(4). The essence of the design is adding Breakpoints \( B_p \) to the piecewise linear function of the nonlinear resistor (NR) in Chua’s circuit. Every Breakpoint represent an attractor. The breakpoints is gotten when we make the op-amp of Chua’s diode in a saturation state. But, the op-amp of NIC must be in linear region[11].

Figure 4: Design 16 scroll attractors by modified Chua’s circuit in MultiSIM®

3. Synchronization in Chua’s circuit

System synchronization is paths’ values equality for two systems[12]. The chaotic systems start from different initial conditions in the phase space, and then begin to vibrate around their points of equilibrium. The paths taken for both systems will be very quickly and completely different from each
other. Both systems will create a strange attractor similar to the other. Because of the extreme sensitivity to changes, even minor ones, to any chaotic system, so thought for a time that there can be no sync between two chaotic systems whatever happens. There are many techniques to synchronize a receiver with a chaotic transmitter as bidirectional way. But the two systems must be tuned to vibrate together by changing the parameters of each system. If precisely configured, arrays of chaotic systems can be synchronized with each other and not just two systems. This can only be done by making all the signals of the two subsystems (transmitter and receiver system) have a positive Lyapunov Exponent[13].

In order to synchronize between the two nonlinear systems, the two systems at first, must be coupled, that is, a link and exchange of information takes place. To create chaotic circuits synchronized must choose the correct and appropriate coupling. In our research, we sent a sine wave from a pulse generator, and we used the chaotic waveform produced by the Chua’s circuit as the data carrier (sinusoidal signal). The two systems can remain synchronized even if a regular message is added to one (called transmitter) and transferred to the other (receiver). Another system must be set at a frequency of the carrier to receive and retrieve that signal. The two systems behave at the same time, their behaviour in the same pattern causes cancelling each other, then the embedded signal remains. This is confirmed by our experience, where sin waves were recovered from the chaotic carrier using a dynamically synchronous and chaotic receiver circuit with the transmission circuit[14][15].

\[
\begin{align*}
\dot{x} &= \alpha (y-x) - f(x) + \delta x (x - \dot{x}) \\
\dot{y} &= x - y + z + \delta y (y - \dot{y}) \\
\dot{z} &= -\beta y + \delta z (z - \dot{z}) \\
\end{align*}
\]

For a driver or master

\[
\begin{align*}
\dot{x} &= \alpha (y-x) - f(x) + \delta x (x - \dot{x}) \\
\dot{y} &= \alpha (\dot{x} - \dot{y}) + \delta y (y - \dot{y}) \\
\dot{z} &= -\beta \dot{y} + \delta z (z - \dot{z}) \\
\end{align*}
\]

For a responder or slave

Where \( \delta \) coupling parameter, and the \( x \) coupled system only \( \delta_x \) is different from zero. In this case we have \( x, y, z \) coupled. we took \( x \) coupled in our experiment so we made two Chua’s circuit connecting by linear resistor from the + terminal of \( N_R \) and link it in the other terminal of other Chua’s circuit[14], [16], as shown in fig(5)and(6), so the state equation is:

\[
\begin{align*}
\dot{x} &= \alpha (y-x) - f(x) + \delta x (x - \dot{x}) \\
\dot{y} &= \alpha (\dot{x} - \dot{y}) + \delta y (y - \dot{y}) \\
\dot{z} &= -\beta \dot{y} + \delta z (z - \dot{z}) \\
\end{align*}
\]

Where \( \delta_x = C_2 R / C_1 R_1 \), \( \beta = C_2 / L G_2 \), and \( \alpha = C_2 / C_1 \).
4. Synchronization of multi-scroll attractors

After we designed multi-scroll attractors by Chua’s circuit, that we designed 16 scrolls, we applied same laws that we mentioned on they, we got sync as you see in fig (6).

We using Hamiltonian forms and an observer approach. Let’s consider the dynamical system described by the master and slave system, so:

\[ \dot{x} = F(x) \quad \forall x \in \mathbb{R}^n \]  
\[ \dot{\xi} = F(\xi) \quad \forall x \in \mathbb{R}^n \]

(17)  
(18)

This two equations represent two chaotic systems (master, slave) described by a set of states \( x_1, x_2, \ldots, x_n \), and \( \xi_1, \xi_2, \ldots, \xi_n \), will synchronize if the following limit fulfills:

\[ \lim_{t \to \infty} |x(t) - \xi(t)| = 0. \]  
(19)

For any initial conditions \( x(0) \neq \xi(0) \). Due to the real limitations of electronic systems, a tolerance value is used in practical applications, where there are some other agents like component mismatching, distortion, noise etc.

\[ |x(t) - \xi(t)| \leq \varepsilon \quad \forall \ t \geq t_f. \]  
(20)

Where \( \varepsilon \) is the allowed tolerance value and a time \( t_f < \infty \) is assumed. Equations and assume the synchronization error[17] defined as

\[ e(t) = x(t) - \xi(t) \]  
(21)

When we made the two multi-scrolls Chua’s circuit synchronized, we noticed that the fast of the synchronization is more fast when the scrolls increase, that because every scroll is a package of states, and the fast of sync depends on these states, because increasing of states of both two Chua’s circuits make their states canceling each other as in this equation \( |x(t) - \xi(t)| = 0 \) i.e. full sync, this concept will be very useful in communication as we’ll see below.

Figure 5: Sync of Modified Chua's Circuit

Figure 6: Sync in 16 attractors between two modified Chua's circuit
5. Data transmission by two synchronous Chua's circuit

Chaotic signals have characteristic properties suitable for safe data transfer and include: sensitive dependence over initial conditions, similarities with noise, unpredictability and difficulty decoding. In the laboratory, we have built several circuits for Chua, which we previously described as nonlinear, because their output is non-linear. Secure chaotic communications can be one of the most prominent phenomena in the physical field and is a very promising phenomenon in the field of communications in various applications. The important feature of chaotic systems is that the output signal can recover the input signal, which means that it is possible to perform a secure connection to a chaotic system. The design of these systems, as explained above, is based on Chua's self-synchronization feature [19].

On the right side, the transmission system is connected to a set of amplifiers in addition to a sine signal generator. This is the modulation circuit and its task are to embed the sinusoidal sign on the chaotic signal (which is the task of the adder as in figure (8)), where the modulation circuit consists of inverting adder and inverter. Through which the signal is sent in a wired or wireless way to the receiver, which it's Chua's circuit with a set of amplifiers acting as a (demodulator), that consists of the same-phase adder. The input of the demodulator is derived from Chua, which means that its input is a chaotic signal, and its output is the error signal of the two chaotic signals (those coming from the transmitter and the receiver signal) so that the two signals cancel each other, leaving the embedded signal on the surface.

![Figure 7: The design of transfer data by two synchronous Chua's circuit](image)

This method is called the mask method, where the sinusoidal signal (it) is mixed (added) through the collector with the chaotic sign x, and then fed to the receiver:

\[ S(t) = x + it \] (22)

In the receiver, the chaotic signal is re-generated x, allowing it to be subtracted from the signal coming from the transmitter, allowing the signal to be retrieved:

\[ (xt + it) - xr \] (23)

So, if \( xr = xt \) and this happens in synchronization, they will cancel each other so that only the (it) which remains, which is the resulting signal [20].
Figure 8: Transmitter of data by tow sync Chua’s circuit

The sync by multi-scrolls is faster as we mentioned, this make the communication faster too, as well as, more Reliability, because each attractor contains an enormous number of chaotic signals, these signals help us to cover up and conceal the signal we want to transmit confidentially, it's mean, greater number of strange attractors, will make more chaotic signals that carry the data, so that the data transmitted in the chaotic signal stack disappears and the hackers are lost.

Figure 9: Transmitter data by Two 16 scroll Chua’s circuit in MultiSIM(R)

6. Conclusion
Chua’s circuit is one of the simplest nonlinear circuits that exhibit complex, chaotic dynamic behavior. It was found through work that the synchronization is a very important phenomenon for communicating safely, without it cannot be done to chaotic circuits. This work aims to knowledge the highly secure communication, and found that chaotic circuits are increasingly being transferred to communications more securely as the number of scrolls increases, and this was clear in transportation experience by 16 scrolls Chua’s circuit. Whenever the chaotic signals become more, the secure communication becomes more reliability.

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