Arrival Directions of Ultrahigh-Energy Cosmic Rays

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Abstract. We discuss some important properties of the arrival directions of ultrahigh-energy cosmic rays detected by the Telescope Array and the Pierre Auger experiments using data released by both collaborations. We also discuss the potential sources and analyse the effect of correcting the arrival directions to take into account the deflections of the cosmic rays by the magnetic field of our galaxy for one specific model of the galactic magnetic field under several assumptions about the composition of the primary cosmic rays. The results show that for a proton-dominated composition the maxima deflections, due to the galactic magnetic field model considered in this paper, are about $20^\circ$ for energies higher the 60 EeV, allowing the approximation of doing astronomy with charged particles, while for medium and heavy compositions the galactic magnetic deflections are much higher and cannot be excluded from any search of the sources of ultrahigh-energy cosmic rays. We also study the validity of the backpropagation method by means of forward propagation of the particles from potential astrophysical sources to the vicinity of the Earth.

1. Introduction
One of the most important results in the field of ultrahigh-energy cosmic rays (UHECRs) of the last few years was the discovery of a hotspot in the arrival directions of the highest-energy cosmic rays detected with the Telescope Array (TA) experiment in the northern sky [1]. In this article we discuss these results along with similar observations in the southern sky made with the Pierre Auger Observatory [2]. We also describe a procedure to backpropagate these cosmic rays from the Earth to the edge of our galaxy, for a particular model of the galactic field and different assumptions about the composition of the primary cosmic rays. For this particular purpose we developed programming code that we wrote in the Julia language. We also elaborate on the possible sources of these events. The results show that for a proton-dominated composition the maxima deflections due to the galactic magnetic field model considered in this paper are about $20^\circ$, allowing the approximation of doing astronomy with charged particles, while for medium and heavy compositions the magnetic deflections are much higher, making the magnetic deflections necessary in any search of the sources of ultrahigh-energy cosmic rays.

The programs used to produce all the plots in this article are available for download [3] and can be used freely by anyone.
2. The hotspot discovered by the Telescope Array Experiment

The hotspot in the arrival directions of ultrahigh-energy cosmic rays discovered by the Telescope Array Collaboration in the northern sky was reported in [1], their analysis was based on events with energies above 57 EeV. They reported a statistical significance of 3.4σ. In this section we use the data released by the TA Collaboration in [1] to reproduce their results with our own analysis software.

2.1. Data Sample

The data for the arrival directions of the UHECRs detected by the TA experiment can be downloaded in a complete form from the electronic edition of the paper. These data consist of 72 events with $E > 57 \text{ EeV}$ and zenith angle $\theta < 55^\circ$ recorded from 2008 to 2013. The following data are provided for each event: date and time of occurrence, measured zenith angle, measured energy, and the equatorial coordinates of the arrival direction given by right ascension (RA or $\alpha$) and declination (DEC or $\delta$).

2.2. Data analysis

The analysis procedure is fully described in [1]. In order to reproduce this analysis we use the program PlotEvents.py to make the plot shown in Fig. 1.a.

In order to follow the same anisotropy analysis described in [1] we perform an oversampling
with a radius of $20^\circ$ around every point in the sky by using a grid of $0.1^\circ \times 0.1^\circ$. We optimize our code through the use of the k-dimensional tree algorithm [4]. The Julia program that we use to oversample the TA events with a radius $R = 20^\circ$ is named {OverSample.jl. Next we use the program {PlotMap.py} to visualise this map and to make the plot shown in Fig. 1.b.

The next step consists in the evaluation of the background noise under the assumption that this noise is isotropic. For this purpose we generate $N_o = 100000$ simulated arrival directions, taking into account the geometrical exposure as a function of zenith angle, $\theta$, for the Telescope Array given by Eq. 1.

$$\frac{d\Omega}{dN} = N_o \sin \theta \cos \theta$$

The program that generates these Monte Carlo events is called {MCSim.jl}. Next we apply the same oversampling procedure that we applied to the data and normalise to a total of 72 events. We therefore run again the program {OverSample.jl}, but this time applied to MC data simulated in the field of view of the TA experiment. This program creates an oversampled map of the MC isotropic events generated in the field of view of the Telescope Array experiment with the same grid of $1800 \times 3600$ points and the same oversampling radius of $20^\circ$ that we used for the data. The map generated is stored in the file named {N_off.dat}.

Since the number of events is higher, the execution of this program takes a few minutes in a typical laptop. One alternative to speed it up is to use MPI [5] to parallelise the execution in case one can use a multiprocessor computing system.

In turn we use the program {PlotMap.py} to visualise this map and to make the plot shown in Fig. 1.c, the scale of its color bar is adjusted to match that of Fig. 1.b.

2.3. Statistical significance based on $S_{LM}$

Next we proceed to use both the oversampled data, $N_{on}.dat$ shown in Fig. 1.b, and the oversampled isotropic background, $N_{off}.dat$ shown in Fig. 1.c, to quantify the deviation from isotropy present in the data.

For this purpose we use the value of the Li-Ma significance [6], denoted by $S_{LM}$, calculated for each pixel by means of Eq. 2, where $\eta = 72/100000$ is the ratio of the number of events in the data sample to the number of isotropic MC events.

$$S_{LM} = \frac{N - N_o}{\sqrt{N_o + \eta}}$$
\[ S_{LM} = \left[2N_{on} \ln \left(\frac{1 + 1/\eta N_{on}}{N_{on} + N_{off}}\right) + 2N_{off} \ln \left(\frac{(1 + \eta)N_{off}}{N_{on} + N_{off}}\right)\right]^{1/2}. \tag{2} \]

We use the program named LiMaSigma.jl to create the file named $S_{LM}.dat$ that in turn can be visualised with the PlotMap.py program to generate the Li-Ma significance map shown in Fig. 2.a.

### 2.4. Statistical significance based on $P_{value}$

An alternative to the use of the Li-Ma significance is the use of $P_{value}$ [7] to measure the deviation from isotropy. The $P_{value}$ is defined in Eq. 3 and Eq. 4.

\[ P_{value} = P_{Poisson}(X \geq N_{on}), \tag{3} \]
\[ P_{value} = 1 - CDF_{Poisson}(\mu = \alpha N_{off}, X = N_{on} - 1), \tag{4} \]

where $CDF_{Poisson}$ is the Poisson cumulative distribution function.

The program LiMaSigma.jl that we run above also generated the $P_{value}$ significance map $P_{value}.dat$ shown in Fig. 2.b.

In order to measure the statistical significance of the hotspot we simulate a large number of sets of MC 72 events generated isotropically and we find out that only a fraction of $3.7 \times 10^{-4}$ have a $S_{LiMa} > 5.1 \sigma$. We also find that the same fraction of sets have a $P_{value} < 10^{-6.4}$ meaning that using $P_{value}$ is equivalent to using $S_{LM}$. These numbers correspond to a statistical significance of $3.4 \sigma$ one-sided.

### 3. The hotspot discovered by the Pierre Auger Observatory in the Southern Sky

The code we have used to reproduce the hotspot discovered with the Telescope Array experiment can be applied straightforwardly to the data released by the Pierre Auger Collaboration in 2015 [2] which also exhibits a hotspot located at $RA = 198.0^\circ$ and $DEC = -25.2^\circ$ in the southern sky, although with a lower statistical significance.

First we copy the data released in [2] and save it in a file, next we use the program PlotEvents.py to make the plot shown in Fig. 3.a. The same procedure and same programs as those described above can be used to measure $S_{LM}$ and $P_{value}$. Fig. 3.b shows the Li-Ma significance map in equatorial coordinates for an oversampling radius of $12^\circ$ and Fig. 3.c shows the same Li-Ma significance map in galactic coordinates. The maximum value $S_{LM} = 4.6 \sigma$ occurs at $RA = 198.0^\circ$ and $DEC = -25.2^\circ$. Application of a penalty factor to account for the search procedure reduces the statistical significance as discussed in [2].

### 4. Potential Sources of Ultrahigh-Energy Cosmic Rays

In search of a correlation with the positions of potential astrophysical sources of the UHECRs we use the Swift BAT 70-Month Hard X-ray Survey, available from [8], which includes 1210 hard X-ray sources with the majority of them classified as AGN.

First we copy these data into a file and proceed to make a map of these potential sources of UHECRs superimposed on the Telescope Array events using the program named PlotEvents.py which produces the plot shown in Fig. 4.a. Likewise, we can make a map of these potential sources superimposed on the Auger events as shown in Fig. 4.b.

The radii displayed for the Swift BAT 70-Month Hard X-ray Survey are arbitrarily scaled. In the absence of a model, we use weights that are directly proportional to the absolute luminosity measured in the 14-195 KeV band, in such a way that the $D^2$ factor in the absolute luminosity cancels out with the $D^2$ in the denominator due to the solid angle that our Galaxy presents as a
Figure 3: Maps using the Hammer projection for the data released by the Pierre Auger Collaboration [2]. The solid curve represents the supergalactic plane and the dashed line represents the galactic plane. The dotted line indicates the limit of the field of view of the Pierre Auger Observatory. (a) Map in equatorial coordinates of the 200 UHECR events detected with $E \geq 54$ EeV and zenith angle $< 80^\circ$. (b) Map in equatorial coordinates of the Li-Ma significance defined in Eq. 2 for an oversampling radius of $12^\circ$. (c) Same as Fig. 3.b but in galactic coordinates.

Figure 4: Maps in equatorial coordinates using the Hammer projection for displaying the arrival directions of UHECR events and the positions of objects from the Swift BAT 70-Month Hard X-ray Survey. The solid curve represents the supergalactic plane and the dashed line represents the galactic plane. The red circles have a radius of $3^\circ$ and represent the detected events. The green circles represent the 202 objects from the catalog located within 100 Mpc from the Earth, their radii are arbitrarily scaled as $Flux/50$ in degrees. (a) Events detected by the Telescope Array experiment previously shown in Fig. 1.a. (b) Events detected by the Pierre Auger Observatory previously shown in Fig. 3.a.
target. Therefore we use $R = \frac{\text{Flux}}{50}$ in degrees with Flux in $10^{-12} \text{ ergs cm}^{-2} \text{s}^{-1}$ for the radii of these astrophysical objects, and $R = 3^\circ$ for the radii of the events.

We arbitrarily limit the redshift of the plotted objects to $< 0.0235$, which corresponds to 100 Mpc when we convert redshift to distance by using [9]. Note that using redshifts as a measure of distance is only a first approximation as it excludes possible motions of the objects within their local gravitational fields, a better approximation would be to use the measured distances available in [10].

Fig. 4 suggests that, assuming that the production of UHECRs is correlated with the production of hard X rays in the energy band measured by BAT-Swift (14-195 KeV), the hotspot observed in the northern sky can be explained as dominated by two sources, namely NGC 4151 and NGC 4388, displayed in the northern sky of Fig. 4.a as green circles with radii $10.8^\circ$ and $5.6^\circ$, respectively, whereas the southern hotspot, reported by Auger with a lower statistical significance, can be attributed to a few sources, namely CenA, NGC 2110, 1C 4329A, NGC 4945, the Circinus Galaxy and NGC 5506, displayed in the southern sky of Fig. 4.b as green circles with radii equal to $27.8^\circ$, $6.4^\circ$, $5.8^\circ$, $5.7^\circ$, $5.4^\circ$ and $4.8^\circ$, respectively. The fact that the northern sky has fewer potential sources in this context may explain its more compact hotspot with respect to the southern sky. Some of the potential sources mentioned above are starburst galaxies, therefore for a more detailed analysis on the correlation of arrival directions of ultrahigh-energy cosmic rays and the directions of starburst galaxies is required.

A similar exercise could be done using the Veron Catalog of Quasars & AGN, 13th Edition [11]. However, this catalog is known to be statistically incomplete and non-uniform, although for sources within 100 Mpc the incompleteness and the non-uniformity may not represent a problem, another inconvenience of the Veron Catalog is the poor precision on the redshifts assigned to the AGN, although this can be easily fixed by using the more precisely measured distances available from [10].

5. Correction due to the Galactic and Extragalactic Magnetic Fields

The dream of doing astronomy with UHECRs, based on the assumption that the deflections of the cosmic rays at these extreme energies would be small enough to point to the sources, has gradually disappeared as we collect more data. This means that either the cosmic rays are light nuclei but the galactic and extragalactic fields are more intense than expected, or the composition is dominated by heavy nuclei at ultrahigh-energies, or both.

As we gradually discover more details about the intensities and structures of the galactic and extragalactic magnetic fields, and improve on the measurement of the composition of the primary cosmic rays, we will, in principle, apply better corrections that may eventually lead to the identification of the most important sources. In addition, the knowledge about the potential sources derived from multi-messenger approaches will lead to a better understanding of the acceleration mechanisms, as well as the ranges of energies and the nature of the primaries.

In this section we describe the use of the required computational tools to correct for the deflections of the detected UHECRs in the presence of a particular model of the galactic magnetic field, called JF2012 and derived from fits to the WMAP7 Galactic Synchrotron Emission map and more than forty thousand extragalactic rotation measurements [12, 13].

In particular we describe the use of a set of programs, written in Julia language and including Bash scripts, to backpropagate charged particles from the Earth to the edge of the Milky Way galaxy. We use the JF2012 model for the galactic magnetic field and use different assumptions about the composition of the UHECRs detected by the TA [1] and the Pierre Auger [2] collaborations.

The program named CRBackProp.jl uses the adaptive Cash-Karp method [14] to solve the Lorentz force equation to obtain the trajectories of the backpropagated UHECRs and the program Propa_JF2012.jl translates the JF2012 model into Julia code. Other alternative programs to
Figure 5: Maps in equatorial coordinates using the Hammer projection for the UHECR events detected by the Telescope Array experiment [1]. The solid curve represents the supergalactic plane and the dashed line represents the galactic plane. The dotted line indicates the limit of the field of view of the TA experiment. The oversampling radius is $20^\circ$ in all plots. In (b), (c) and (d) the galactic magnetic field is approximated by the regular component of the JF2012 model. (a) Original events. (b) Events backpropagated to the edge of the Galaxy assuming pure proton composition. (c) Events backpropagated to the edge of the Galaxy assuming pure oxygen nuclei composition. (d) Events backpropagated to the edge of the Galaxy assuming pure iron nuclei composition.

backpropagate cosmic rays are [17] and [18].

In order to backpropagate all the events detected by the TA experiment we use the Bash script named `Backprop.sh`. This procedure gives rise to the plots shown in Fig. 5 under the assumptions that all the UHECRs are protons, oxygen nuclei or iron nuclei, respectively. Note that we have used only the so-called regular component of the field in the JF2012 model, but one can activate the striated and/or the turbulent component of the model as well. This is done by changing one or the two `false` variables to `true` in the call to the function `Propa_JF2012`

In a similar way we obtain the oversampled maps for the backpropagated events detected by the Pierre Auger Observatory, according to [2] this time we use an oversampling radius = $12^\circ$; to do this we just replace TA by Auger in the sequence of commands given above. The results are shown in Fig. 6.

We can conclude that both hotspots, in the northern and southern sky, are magnified if we take into account the deflections produced by the galactic magnetic field, approximated by the JF2012 model, under the assumption of a composition dominated by medium or heavy nuclei. The positions of the hotspots are near the positions of the uncorrected hotspots, and both are on the same side of the supergalactic plane. We are in the process of adding the deflections due to the extragalactic magnetic field using some of the proposed models for the latter.

Above we suggested the idea that the lower compactness of the southern hotspot may be due to the contribution of more sources from the Swift BAT 70-Month Hard X-ray Survey.
Figure 6: Maps in equatorial coordinates using the Hammer projection for the UHECR events detected by the Pierre Auger Observatory [2]. The solid curve represents the supergalactic plane and the dashed line represents the galactic plane. The dotted line indicates the limit of the field of view of the Auger experiment. The oversampling radius is $12^\circ$ in all plots. In (b), (c) and (d) the galactic magnetic field is approximated by the regular component of the JF2012 model. (a) Original events. (b) Events backpropagated to the edge of the Galaxy assuming pure proton composition. (c) Events backpropagated to the edge of the Galaxy assuming pure oxygen nuclei composition. (d) Events backpropagated to the edge of the Galaxy assuming pure iron nuclei composition.

with respect to the northern sky, this idea may be reinforced if the different potential sources accelerate predominantly different particles at these extreme energies leading to more disperse angular deflections due to the galactic and extragalactic magnetic fields.

It is evident that a more quantitative correlation study is required to reinforce or discard some of these ideas. Likewise, it will be very interesting so study the effect in the positions and compactness of the hotspots due to the inclusion of deflections caused by extragalactic magnetic fields.

6. Maps of Deflection Angles due to the Galactic Magnetic Field for the JF2012 Model

By using the same Julia programs described above we can backpropagate cosmic rays of a fixed energy for different compositions. To do this, we divide the full sky in a grid of $180 \times 5 \times 360 \times 5$ points, i.e., we use the granularity variable equal to 5 instead of 10 as we previously did, and backpropagate protons, oxygen nuclei and iron nuclei arriving at the points of this grid with an energy of 60 EeV. The back propagation to the edge of the Galaxy assumes a galactic magnetic field approximated by the regular component of the JF2012 model.

Given that we need to backpropagate over 1.6 million particles, and since their trajectories are independent, we use MPI [5] in a multiprocessor HPC cluster to speed up the execution of this trivially parallelizable task.
Figure 7: Deflection angles in degrees for particles arriving at each point of these maps with $E = 60$ EeV. These particles are backpropagated to the edge of the Galaxy with the galactic magnetic field approximated by the regular component of the JF2012 model. All the maps are in equatorial coordinates using the Hammer projection. The solid curve represents the supergalactic plane and the dashed line represents the galactic plane. (a) Assuming that particles are protons. (b) Assuming that particles are oxygen nuclei. (c) Assuming that particles are iron nuclei.

Fig. 7 shows the maps of deflections for protons, oxygen nuclei and iron nuclei of 60 EeV backpropagated in equatorial coordinates and Fig. 8 shows the same in galactic coordinates.

7. Forward Propagation
Using the program forwardPropa.jl we can also propagate a beam of cosmic rays directly from an extragalactic source. The method consists in generating a beam of particles incident over the Milky Way, since the distance to the extragalactic source is at least two orders of magnitude larger than the diameter of the Milky Way, we can assume that the trajectories of the simulated particles are parallel among them when they arrive at the Milky Way. We simulate a beam of particles with a given energy and a specific composition. Once the particles arrive at the Milky Way the program generates the trajectory of all the particles through the galactic magnetic field. The results show that only some of these particles cross a sphere of 1 pc centred at the Earth position, making them potentially detectable. In general, the arrival directions of the particles that cross the position of the Earth are different from the directions of the sources and depend on the charge and magnetic field model of the our galaxy. We check the results obtained for forward propagation by using a backward propagation scheme as described above and by checking that the final directions of the trajectories of the particles coincide with their initial directions.

In Fig. 9 six simulations are shown for two sources, M82 and CenA, for a beam of particles with energies of 60 EeV and three different compositions: proton, oxygen and iron. Since the deflection angles depend on the charge of the initial particle, $\theta \sim Z E / B$, we plot the trajectories that reach the Earth for the a proton composition. On the other hand, for heavy nuclei in some cases the deviation is large enough that they are not able to reach the Earth at these energies.
Figure 8: Deflection angles in degrees for particles arriving at each point of these maps with \(E = 60\) EeV. These particles are backpropagated to the edge of the Galaxy with the galactic magnetic field approximated by the regular component of the JF2012 model. All the maps are in galactic coordinates using the Hammer projection. The solid curve represents the supergalactic plane. (a) Assuming that particles are protons. (b) Assuming that particles are oxygen nuclei, (c) Assuming that particles are iron nuclei.

Fig. 10.a shows the direction of the some starburst galaxies and the arrival direction of the cosmic ray protons with 40 EeV coming from these sources.

8. Correlation Analysis

The observation of hotspots implies the existence of anisotropy in the arrival directions of the UHECRs, the positions of these hotspots is near the position of some sources that are good candidates to accelerate particles to ultra high energies, making plausible that some of the ultra high energy cosmic rays detected came from these sources. However, to quantify these possibilities we can use a statistical test method to measure the correlation between the detected data in both observatories and a collection of possible sources of some kind. As in [15] we can build a likelihood function that depends on the contribution of the sources of a catalog and an anisotropic background of UHECRs, the likelihood function is:

\[
L = \prod_{\text{events}} [f_{\text{corr}} P_{\text{src}}(\sigma, \epsilon) + (1 - f_{\text{corr}}) P_{\text{iso}}(\epsilon)],
\]

where \(f_{\text{corr}}\) is the fraction of events correlated with the objects of the catalog, \(P_{\text{src}}\) is the contribution of the source to the events observed, this is a two-dimensional Gaussian function with parameter \(\sigma\). The \(P_{\text{iso}}\) only depends on the relative exposure \(\epsilon\) of the observatories.

We also apply a likelihood ratio test between nested models, for this purpose we define the test statistic as \(TS = 2 \ln \mathcal{L} / \mathcal{L}_0\), where \(\mathcal{L}_0\) is the null hypothesis (isotropy).

In a recent work [16] this method was used to analyse a set of starburst galaxies, the Swift Bat catalog was also included. For starburst galaxies a maximum TS of 24.9 was obtained and 18.2 for the Swift Bat catalog, for a cosmic ray threshold energy of 39 EeV.
Figure 9: The trajectories for two different particle beams from two sources are shown, top: M83, down: CenA, the initial energy of each beam is 60 EeV and the composition is (a) protons, (b) oxygen and (c) iron. The lines in the spiral curves represent the galactic magnetic field disc component for $z=0$, the dot is the position of the Earth, the darker line represent the particles that pass near the position of the Earth.

Figure 10: Map in equatorial coordinates, the empty circles represent the position of the 23 Starburst galaxies, the filled circles are the 23 arrival direction protons with energy of 40 EeV coming from these sources, the radii of the circles are proportional to the particle flux for each source.

In that analysis the effect produced by galactic or extragalactic magnetic fields was not included. We use the same sources and apply forward propagation using the Jansson-Farrar model, with pure proton composition and the same threshold energy. We obtain an average deviation of 5.28° for the Starburst catalog and 6.09° for the Swift Bat catalog. The test statistic for each set of sources is shown in Fig. 11, the maximum of the test statistics for the new arrival directions falls down to $TS_{max} = 19.6$ and $TS_{max} = 17.2$ for the Starburst and Swift Bat catalogs, respectively. As we see the analysis varies wildly after using the deflection correction due to the galactic magnetic field.
Figure 11: The figure shows the profile of the statistical test, above for the sources without deviation (a) Starburst catalog and (b) Swift Bat catalog, below is the statistical test for the sources after having propagated protons through the magnetic field for (c) Starburst catalog and (d) Swift Bat catalog.

9. Conclusions
In this article we have described the use of a number of programs that we have written in Python and Julia that can be used to reproduce data analyses that lead to the discovery of a hotspot in the arrival directions of the highest-energy cosmic rays detected with the Telescope Array experiment in the northern sky, and also to reproduce the findings of the Pierre Auger Collaboration in the southern sky. We do this by using data released by both collaborations.

We have also described the use of programs that can be used to correlate the arrival directions of the detected events with the locations in the sky of possible sources from the Swift BAT 70-Month Hard X-ray Survey.

We have also presented the analysis of the effect of correcting the arrival directions, reported by both experiments, to take into account the deflections of the cosmic rays caused by the magnetic field of our galaxy. We have described the use of these programs for one specific model of the galactic magnetic field under several assumptions about the composition of the primary cosmic rays.

In particular we have shown that both hotspots, in the northern and southern skys, are magnified if we take into account the deflections produced by the galactic magnetic field, approximated by the JF2012 model, under the assumption of a composition dominated by medium to heavy nuclei. The positions of the corrected hotspots are near the positions of the uncorrected hotspots and both are on the same side of the supergalactic plane. It will be interesting to add deflections due to extragalactic magnetic fields using some of the proposed models for the latter that may eventually move the hotspots closer to the supergalactic plane, and closer to the potential sources mentioned above, namely CenA, NGC 2110, IC 4329A, NGC
4945, the Circinus Galaxy and NGC 5506 in the southern sky and NGC 4151 and NGC 4388 in the northern sky. Some of these potential sources are starburst galaxies, therefore a more detailed analysis on the correlation of arrival directions of ultrahigh-energy cosmic rays and the directions of starburst galaxies is required.

We have presented maps, in equatorial and galactic coordinates, of deflections of protons, oxygen nuclei and iron nuclei, of a fixed energy equal to 60 EeV, when they are back propagated to the edge of our galaxy in a galactic magnetic field approximated by the regular component of the JF2012 model.

We also introduced a technique of forward propagation of the particle trajectories that gives us an idea of where ultra-energetic cosmic rays would arrive if they came from a set of sources provided that we take into account the interaction with the galactic magnetic field. We have shown that particle trajectories can exhibit great changes depending on the particle charges and energies.

The statistical test can give us a parameter of how closely the observed events are related to sources of a certain type, however, we can not conclude anything about the results obtained for a certain type of source without modeling the effect of the galactic magnetic fields in the cosmic ray propagation. The results depends on the primary composition and the threshold energy, as well as the magnetic field model employed.

Acknowledgements
The authors thank CONACyT for making possible the creation of the Center for High Performance Computing of UMSNH in Morelia, and the Laboratorio Nacional de Supercomputo del Sureste de México, located in the BUAP Campus in Puebla; the Monte Carlo simulations of this article where performed in these facilities. One of the authors (LV) also thanks CONACyT for the support during his leave of absence from UMSNH.

References
[1] R. U. Abbasi et al. [Telescope Array Collaboration], Astrophys. J. Lett. 790, L21 (2014)
[2] A. Aab et al. [Pierre Auger Collaboration], Astrophys.J. 804 (2015) 1, 15
[3] https://github.com/lvillasen/ToolsCosmicRays
[4] Bentley, J. L., "Multidimensional binary search trees used for associative searching". Communications of the ACM. 18 (1975) 9.
[5] http://dl.acm.org/citation.cfm?id=898758
[6] T. Li and Y. Ma, ApJ, 272 (1983) 317
[7] Bhattacharya, Bhaskar; Habtzghi, DeSale (2002). "Median of the p value under the alternative hypothesis". The American Statistician.
[8] http://swift.gsfc.nasa.gov/results/bs7fmin/
[9] http://roban.github.io/CosmoloPy/
[10] NASA/IPAC Extragalactic database, http://ned.ipac.caltech.edu/forms/byname.html
[11] http://heasarc.gsfc.nasa.gov/W3Browse/galaxy-catalog/veroncat.html
[12] R. Jansson and G. R. Farrar, A New Model of the Galactic Magnetic Field, ApJ 757 (2012) 14.
[13] R. Jansson and G. R. Farrar, The Galactic Magnetic Field, Astrophys. J. 761 (2012) L11.
[14] J. R. Cash, A. H. Karp. "A variable order Runge-Kutta method for initial value problems with rapidly varying right-hand sides", Transactions on Mathematical Software 16: 201-222, 1990.
[15] A. Abreu et al, Update on the correlation of the highest energy cosmic rays with nearby extragalactic matter, Ap Ph 34 (2010) 5.
[16] A. Aab et al., Indication of Anisotropy in Arrival Directions of Ultra-High-Energy Cosmic Rays through Comparison to the Flux Pattern of Extragalactic Gamma-ray Sources, Accepted for publication in: Astrophys. J. Lett. (2018) [arXiv:1801.06160].
[17] M.S. Sutherland, B.M. Baughman, J.J. Beatty, Astroparticle Physics, Volume 34, Issue 4, p. 198-204. (2010)
[18] R.A. Batista, M. Erdmann, C. Evoli, K.-H. Kampert, D. Kuempel, G. M"ujller, G. Sigl, A. van Vliet, D. Walz, T. Winchen, EPJ Web of Conferences 99 (2015) 13004
[19] https://virtualenv.pypa.io/en/stable/
[20] https://www.continuum.io/downloads
[21] http://vanderbiltastro.pbworks.com/w/file/46335397/astroconvert.py
[22] http://julialang.org/downloads/
[23] http://openmp.org/wp/