DO WE NEED TWO POMERONS?

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Summary. We show that one single Pomeron compatible with the Froissart limit, can account for all the present HERA data.
High energy diffraction, popular some twenty years ago in hadronic physics, has been rejuvenated after many years of almost total neglect by the so-called low-$x$ physics i.e. by the measurement at HERA of the proton structure function $\nu W_2$ at small $x$ [1,2]. A terminology which had become nearly obsolete is essentially being rediscovered and of great interest is presently the connection between this new physics and the traditional high energy hadronic physics. The main issue at stake is whether QCD may shed light on the origin and the nature of the Pomeron, the entity which, in the conventional language of high energy physics determines the asymptotic behavior of the hadronic total cross sections. More specifically, the question is the precise determination of the Pomeron structure function following the original suggestions of Ingelman and Schlein [3] and of Donnachie and Landshoff [4]. It is not our aim in this paper neither to review the (by now fairly large) literature on this subject [5], nor to debate how much precisely gluonic or partonic components the data seem to attribute to the Pomeron according to the various analyses [6] nor how well the data are accounted for by the various models [7]. Similarly, it is not our goal to review and update the old fashioned terminology (see for instance Ref. 8). What we want to do in this paper is to challenge the rather widespread belief that two Pomerons are necessary to describe the physical situation (even though the philosophies in these two papers are profoundly different, the reader could benefit from reading, for instance, the papers quoted in Ref. 9a, and 9b).

We will try to reduce the formulation of our problem to its bare minimum at the risk of oversimplifying it (the kinematic and the variables to be used are perfectly conventional from Deep Inelastic Scattering (DIS) and summarized in Fig. 1 for the reaction $\ell(k) + N(p) \rightarrow \ell'(k') + N(p') + X$ where $\ell$ is a lepton, $N$ is a nucleon and $X$ is all the remaining hadronic debris over whose variables a summation is implied).

i) When $Q^2 \rightarrow 0$, $\nu W_2$ is related to the total cross section for real-photon proton scattering according to :

$$\sigma_{tot}^{\gamma p} = \frac{4\pi^2\alpha}{Q^2} \nu W_2|_{Q^2=0} ,$$

(where $\alpha$ is the electromagnetic coupling constant) as a consequence $\nu W_2$ must vanish linearly with $Q^2$.

ii) When the Bjorken variable of DIS $x$ is very small (say, typically $x \leq 10^{-3}$), $\nu W_2$ will be dominated by its gluonic component and we are going to assume this even when comparing our form with data at considerably larger $x$, say of order $10^{-2}$. In this kinematical range we will run perilously close to where our approximation may break down; on the other hand the complications we would have to introduce to avoid this danger would make our analysis much more muddy and, consequently, much less
conclusive.

iii) According to the conventional Regge theory, the asymptotic behavior of hadronic cross sections as $s \to \infty$ should be up to logarithms of the form:

$$
\sigma_{\text{tot}} \xrightarrow{s \to \infty} s^{(\alpha(0)-1)},
$$

where $\alpha(0)$ is the intercept of each contributing Regge trajectory (of which, when the quantum numbers are those of the vacuum, the dominant, $\alpha_P(0)$ is known as the Pomeron intercept).

iv) For a diffractive process (such as the one analyzed at HERA, $e+p \to e'+p'+X$ where $X$ has the quantum numbers of a vector meson), the dominant contribution comes from the Pomeron for which the intercept is allowed to attain its maximum value compatible with unitarity $\alpha_P(0) = 1$. In this case, however, logarithmic contributions are expected in Eq.(2) but, let us stress,

v) unitarity guarantees that it must be:

$$
\alpha_P(0) \leq 1.
$$

In particular, Froissart’s bound [10] states that a hadronic total cross section cannot grow faster than $\ln^2 s$. Translated into the language of structure functions, owing to the correspondence:

$$
W^2 = M^2 + Q^2 \frac{1-x}{x},
$$

(where $W^2$, the total squared energy of the system $\gamma^* p$ is the equivalent of $s$ in a hadronic reaction), Froissart bound states that, asymptotically, as $W^2 \to \infty$, i.e. as $1/x \to \infty$:

$$
\left( \frac{\nu W^2}{Q^2} \right) \propto \ln^2 (1/x).
$$

In what follows, we will show that one can indeed accommodate the HERA data to this limiting logarithmic behavior (or to a $\ln(1/x)$ one), in the line of thought of Ref. [11], instead of the power one discussed below (eq.(8)).

Concerning this latter point, it was, in fact, shown long ago by Donnachie and Landshoff [12] that an effective Pomeron intercept of:

$$
\alpha_P(0) = 1.08,
$$

i.e. an effective form of the total cross sections:

$$
\sigma_{\text{tot}}(s \to \infty) \propto s^\epsilon \quad \text{where} \quad \epsilon \approx 0.08,
$$
accounts very well for a large quantity of data. Eq. (7) formally violates Froissart bound but the idea is that this will occur only at fantastically high energies\(^*\) which will probably never be reached and where, presumably, higher order corrections (such as multi-Pomeron cuts) will restore the validity of Froissart’s bound. Be it as it may, the point is that the form (7) is phenomenologically quite adequate and with a minimum of parameters accounts, qualitatively, for a large set of data. Moreover, as shown by the same authors, the combination (1+7) extrapolates well the photoproduction cross section to the HERA energy domain. More precisely, one can say that it accounts well for the early HERA data (in Figure 2 which is taken from Ref. 9a, these data, not shown would lie along the curve up to \(x\) not smaller than some \(10^{-2}\)). Actually, the form which is shown in Figure 2 corresponds to including subasymptotic corrections suggested by the Regge pole analysis, i.e. the curve is:

\[
\nu W_2 = 0.32 x^{-0.08} \left(\frac{Q^2}{Q^2 + a}\right)^{1.08} + 0.10 x^{0.45} \left(\frac{Q^2}{Q^2 + b}\right)^{0.55},
\]

(8)

where

\[
a = (750 \text{ MeV})^2 \quad b = (110 \text{ MeV})^2.
\]

As one sees from Fig. 2, however, Eq. (8) while reproducing well the data for \(x\) not smaller than \(\approx 10^{-2}\) and \(Q^2\) small, fails quite badly when extrapolated to much smaller \(x\) values where the latest HERA data show a much sharper rise.

Two problems arise at this point. One, conceptual, is, could this treatment be extended to the case in which the Froissart bound is respected (i.e. could we use a form which would behave as (5) in the proper domain)? and the second, practical one, is, can this treatment be made compatible with the ensemble of HERA data small \(x\) but large \(Q^2\) which, on the contrary, deviate drastically from the form (8)?

These questions are central to our present paper. Concerning the second, practical point, this is precisely the reason why, in the literature, one introduces [9,13] something which we will call a hard Pomeron [14] in order to recover agreement with the data. On the other hand, always concerning this point, doubts about the real necessity of doing so are raised by some recent findings [15].

It is our contention that the conclusion that two Pomerons, a hard Pomeron\(^*\), and a soft Pomeron, to simplify somehow the issue are necessary, is not really required by the

\(^*\) One should worry, however, not only about the violation of Froissart’s bound but also of the S-wave unitarity.

\(^+\) The intercept of a hard Pomeron would be somewhere between 0.3 and 0.5 i.e. much larger than the value 0.08 of Eq. (6). This is why the case of Eq. (8) is also referred to as a soft Pomeron in the literature.
data and that one can live without this somewhat disturbing if not directly unpleasant possibility.

A very interesting way out was suggested recently by Capella et al.[16], that the Pomeron intercept could have a $Q^2$ dependence. In Ref. 16, however, this possibility was exploited to obtain a soft Pomeron i.e. à la Donnachie-Landshoff starting from a hard Pomeron à la Lipatov et al. In this paper, rather than using two components for the Pomeron (describing its small $Q^2$ and large $Q^2$ contributions to the structure functions as in Ref. 11b) we wish to suggest that both points, the conceptual violation of unitarity by the soft Pomeron of Eq. (6) and the practical one, i.e. a good reproduction of HERA data, could be offered by an extension of the method suggested in Ref. 16 by allowing the Pomeron intercept to vary with $Q^2$ in such a way that in the limit $Q^2 \to 0$ the Froissart Pomeron i.e. a ln$^2(1/x)$ form (see Eq. (5)) is obtained.

To make our point, we propose a specific small $x$ form for $\nu W_2$ which i) fits well all the small $x$ HERA data and ii) reduces to a form (5) (or, alternatively to a ln$(1/x)$) limit when $Q^2 \to 0$. Specifically, we propose, as an example (certainly other examples could be offered):

$$\nu W_2(x, Q^2) \simeq A_P \left[ \frac{\tilde{x}^{\epsilon(Q^2)} - (1 + \epsilon(Q^2) \ln(\tilde{x}))}{\frac{1}{2} \epsilon^2(Q^2)} \right] \ln \left( 1 + \frac{Q^2}{Q^2 + a_{Pom}^2} \right), \quad (9)$$

or, alternatively:

$$\nu W_2(x, Q^2) \simeq A_P \left[ \frac{\tilde{x}^{\epsilon(Q^2)} - 1}{\epsilon(Q^2)} \right] \ln \left( 1 + \frac{Q^2}{Q^2 + a_{Pom}^2} \right), \quad (10)$$

where $\tilde{x} = W^2/s_0$, with the hadronic scale taken as $s_0 = 1$ GeV$^2$.

These forms reduce to the wanted cases if $\epsilon(Q^2)$ vanishes as $Q^2 \to 0$ because $\ln(\tilde{x}) \simeq \ln(1/x)$ if $W^2 \gg Q^2$. Again as an example, in both cases, we choose for the intercept $\epsilon(Q^2)$ the specific (and arbitrary) form:

$$\epsilon(Q^2) = \frac{\lambda}{\ln 2} \ln \left( 1 + \frac{Q^2}{Q^2 + b^2} \right), \quad (11)$$

which we borrow from Ref.[11b]. Then Eq. (9) leads to a ln$^2(1/x)$ behavior and Eq. (10) to a ln$(1/x)$.

In Eqs. (10,11) the parameters $A_P$ and $a_{Pom}^2$ are fixed by the requirement that the total photoproduction cross section comes out correct. We take the values obtained from previous results on $\sigma_{tot}^{\gamma p}$ [11b].

So, with the specific choice (11) of $\epsilon(Q^2)$ there are just two adjustable parameters $\lambda$ and $b^2$. Fitting the small $x$ (specifically up to $x \leq 5 \times 10^{-3}$), the result is shown for the case
of Eq. (9) in Fig.3 and the best fit to the parameters gives $A_P = 5.72 \times 10^{-3}$, $a_{Pom}^2 = 1.12$ GeV$^2$, $\lambda = 0.254$, and $b^2 = 0.198$ GeV$^2$ with a $\chi^2$/d.o.f.($/58$ HERA data) of about 1.2.

The result of Fig. 3 is quite spectacular and deserves some comments (the NMC data [17], not fitted, are shown for completeness). First, recall that the data with $x \geq 5 \times 10^{-3}$ are not the result of a best fit; in spite of this, it is only for very high $Q^2$ that the curve deviates considerably from the data. Second, had we used Eq. (10) instead of Eq. (9), the result would have been quite similar. Third and perhaps most interesting, notice that the asymptotic value of $\epsilon$ as $Q^2$ grows to $\approx 2000$ GeV$^2$ is, roughly $= 0.3$ i.e. reaches the lower limit of what are considered the range of values appropriate for the hard Pomeron (the value of the soft Pomeron à la Donnachie and Landshoff, 0.08, being reached for $Q^2$ between 1 and 5 GeV$^2$). Notice also, that no evolution à la Altarelli-Parisi has been taken into account to get the previous results (to perform a correct evolution, the whole machinery of structure functions, of their gluonic and of their partonic contributions would have to be properly taken into account. This, however, would obviously improve the fit but would make the result depend on so many additional facts and parameters that the main point of the paper would be lost in the details of the parametrization).

In order to see what happens when a factor correcting for $x$ not being so small is inserted into Eq. (9) (or (10)), we show in Fig. 4 the result obtained repeating the previous procedure with the form:

$$
\nu W_2(x, Q^2) \simeq A_P \left[ \frac{\tilde{x}(Q^2) - (1 + \epsilon(Q^2) \ln(\tilde{x}))}{\frac{1}{2} \epsilon^2(Q^2)} \right] \ln \left( 1 + \frac{Q^2}{Q^2 + a_{Pom}^2} \right) (1 - x)^{\beta(Q^2)} ,
$$

where,

$$
\beta(Q^2) = \beta_0 + \beta_1 t \quad \text{with} \quad t = \ln \left( \frac{\ln \left( (Q^2 + Q_0^2)/\Lambda^2 \right)}{\ln \left( Q_0^2/\Lambda^2 \right)} \right) ,
$$

(the same form (11) has been used for $\epsilon(Q^2)$). Fig. 4a (obtained with the form (12)) shows the equivalent of Fig. 3 i.e. the structure function as a function of $x$ for the various available bins in $Q^2$ whereas Fig. 4b shows the converse i.e. the variation in $Q^2$ for the various bins in $x$. Compared with the previous result, the $\chi^2$/d.o.f. ($/67$ HERA data) is now 1.55 and the various parameters are now given by: $A_P = 5.72 \times 10^{-3}$, $a_{Pom}^2 = 1.12$ GeV$^2$, $\lambda = 0.256$, and $b^2 = 0.21$ GeV$^2$, $\beta_0 = 7.0$, $\beta_1 = 5.6$. As expected, the overall picture has further improved proving that the large $x$ disagreement in Fig. 3 was largely due to the lack of an appropriate treatment of the not so small $x$ data (in Fig. 4 HERA data for $x \leq 10^{-2}$ have been fitted, not just those below $x \approx 5 \times 10^{-3}$ as in the previous case). Notice also that the parameters already present in the previous fit have practically
remained the same since they were determined to reproduce the small-$x$ data; only the parameters involved in $\beta(Q^2)$ are sensitive to including larger $x$-values in the fit.

Once again, $\epsilon(Q^2)$ is closed to 0.3 at the highest $Q^2$ values and crosses the soft value 0.08 for $Q^2$ somewhere between 1 and 5 GeV$^2$.

Some general conclusions are in order. We have shown that we can live well without two Pomerons and, furthermore, that a form compatible with the Froissart limit, which we call Froissart Pomeron is quite acceptable. The form we offered is quite ad hoc but this is true of basically all the parametrizations used in this game. No doubt more clever and elaborate forms could be offered and, no doubt, the analysis could be largely ameliorated, for example by using the whole machinery in which not only gluon distributions are taken into account but also partons together, of course, with their correct $Q^2$ evolution. This, however, raises the issue of how well one could fit the ensemble of all data on structure functions with a parametrization of the kind proposed here. We hope to come back to these questions in the future.
References

[1] H1 Collaboration, I. Abt et al., Nucl. Phys. B407 (1993) 515.

[2] ZEUS Collaboration, M. Derrick et al., Phys. Lett. B316 (1993) 412.
ZEUS Collaboration Measurement of the Proton Structure Function $F_2$ from the 1993 HERA Data DESY 94-113 August 1994.

[3] G. Ingelman and P. Schlein, Phys. Lett. B152 (1985) 256.

[4] A. Donnachie and P. V. Landshoff, Phys. Lett. B191 (1987) 309 and Nucl. Phys. B303 (1988) 634.

[5] See, for instance, P.V Landshoff, XXXVII Rencontres de Moriond, March 1992 Ed. by J. Tran Than Van (Edition Frontmires) and references there in; see also G. Ingelman and K. Prytz, Z. Phys. C58 (1993) 285; G. Ingelman, J. Phys G 19 Workshop on HERA - the New Frontier for QCD (1994) 1631.

[6] UA1 Collaboration K. Eggert, 2th Blois workshop on Elastic and Diffractive Scattering, Rockefeller University, New York, USA, 1987; UA8 Collaboration, P.E Schlein, Nucl. Phys. 33A, B (Proc. Suppl.) 1993 (41) ; Phys. Lett. B332 (1994) 126.

[7] P.V Landshoff and O. Nachtman, Z. Phys. C35 (1987) 405; D.A. Ross, J.Phys G 15 (1989) 1175; N.N Nikolaev and B.G Zakharov, Z. Phys. C53 (1992) 331; E. Gotsman, E.M Levin and U. Maor, Z. Phys. C57 (1993) 677; J.R Cudell and B.U Nguyen, Nucl. Phys. B420 (1994) 669.

[8] E. Predazzi, Perspectives in High Energy Physics Lectures delivered at the IIIth G. Wataghin School in Phenomenology, Campinas, July 1994.

[9] a) P. V. Landshoff, The Two Pomerons Lecture delivered at the PSI school at Zuoz, August 1994; A. Donnachie and P.V. Landshoff, Z. Phys. C61 (1994) 139.

b) M. Genovese, N. N. Nikolaev and B. G. Zakharov, Diffractive DIS from the Generalized BFKL Pomeron. Predictions for HERA KFA-IKO(Th)-1994-37 DFTT 42/94, October 1994.

[10] M. Froissart, Phys. Rev. 123 (1961) 1053. For the proof of this theorem starting from axiomatic field theory, see A. Martin, Nuovo Cimento 42 (1966) 930. The unexperienced reader who would like a more exhaustive picture on this and related subjects, may profitably consult R. J. Eden, High Energy Collisions of Elementary Particles Cambridge Press (1967).
[11] a) P. Desgrolard et al. Phys. Lett. B309 (1993) 191;  
b) M. Bertini, M. Giffon and L. Jenkovszky, Small-x behaviour of the Proton Structure Function to be published in the proceedings of VIth Rencontre de Blois, The heart of the matter, France (June 1994), Edition Frontihres.

[12] A. Donnachie and P. V. Landshoff, Nuc. Phys. B 244 (1984) 322 and Nuc. Phys. B 267 (1986) 690.

[13] A.D. Martin, W.J. Stirling and R.G. Roberts, Parton Distributions of the Proton RAL-94-055 DTP/94/34, June 1994 (and references therein).

[14] E.A. Kuraev, L.N. Lipatov and V.S Fadin, Sov. Phys. JETP 45 (1977) 199;  
Ya. Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822;  
J. C. Collins and J. Kwiecinski, Nucl. Phys. B316 (1989) 307;  
J. Kwieciński, A.D. Martin and W.J. Stirling, Phys. Rev. D42 (1990) 3645;  
J.Bartels, Nucl. Phys. B (Proc. Suppl.) 12 (1990) 201;  
J. C. Collins and P. V. Landshoff, Phys. Lett. B 276 (1992) 196;  
A.H Mueller, J. Phys G 19 Workshop on HERA - the New Frontier for QCD (1994) 1463.

[15] R.D Ball and S. Forte, Phys. Lett. B335 (1994) 77;  
R.D Ball and S. Forte, The Rise of $F_2^p$ at HERA to be published in the proceedings of VIth Rencontre de Blois The heart of the matter, France (June 1994), Edition Frontihres.

[16] A. Capella, A. Kaidalov, C. Merino, and J. Tran Thanh Van, Phys. Lett. B337, (1994), 358. For related approaches to this problem, the reader could profitably refer to: H. Abramowicz, E.M. Levin, A. Levy and U. Maor Phys. Lett. B269 (1991) 465; A. Levy The energy behaviour of the real and virtual photon-proton cross sections, DESY Report 95-003; C. Bourrely, J. Soffer and T.T. Wu Phys. Lett. B339 (1994) 322.

[17] NMC Collaboration, P. Amaudruz et al. Phys. Lett. B295 (1992) 159.
Figures captions

Fig. 1 Kinematic and variables of the process $l + N \rightarrow l' + N' + X$ used in the text.

Fig. 2 The fit of Eq. (7) (obtained from Ref. 11a) to the early HERA data extrapolated to the very small $x$ values.

Fig. 3 Small-$x$ structure function $F_2^p$ from H1 data [1] (triangulated dots) and ZEUS data [2] (closed points and stars) plotted as function of $x$ at fixed $Q^2$ compared with the fit of Eq. (9) (solid line). Only data with $x \leq 5 \times 10^{-3}$ have been used in the fit. The NMC data [17] (open points) are not fitted.

Fig. 4 a,b Structure function with the same data of Fig.3 plotted as a function of $x$ at $Q^2$ fixed (a) and as a function of $Q^2$ at $x$ fixed (b). The solid line is obtained with Eq. (12). Only data with $x \leq 10^{-2}$ have been used in the fit.