Sample-Efficient Reinforcement Learning via Conservative Model-Based Actor-Critic

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Abstract

Model-based reinforcement learning algorithms, which aim to learn a model of the environment to make decisions, are more sample efficient than their model-free counterparts. The sample efficiency of model-based approaches relies on whether the model can well approximate the environment. However, learning an accurate model is challenging, especially in complex and noisy environments. To tackle this problem, we propose the conservative model-based actor-critic (CMBAC), a novel approach that achieves high sample efficiency without the strong reliance on accurate learned models. Specifically, CMBAC learns multiple estimates of the Q-value function from a set of inaccurate models and uses the average of the bottom-k estimates—a conservative estimate—to optimize the policy. An appealing feature of CMBAC is that the conservative estimates effectively encourage the agent to avoid unreliable “promising actions”—whose values are high in only a small fraction of the models; (2) the distribution approximation to optimize the policy. Specifically, they explicitly quantify the uncertainty of the Q-value via the discrepancy of ensemble models and use it as a penalty to learn a lower bound of the true Q-value. However, many works have shown that the uncertainty quantification of existing methods can be unreliable, which acts as a bottleneck to their sample efficiency (Ovadia et al. 2019; Yu et al. 2020; Pan et al. 2020) prevent the agent from exploiting model errors by an uncertainty-based penalty. Specifically, they explicitly quantify the uncertainty of the Q-value via the discrepancy of ensemble models and use it as a penalty to learn a lower bound of the true Q-value. However, many works have shown that the uncertainty quantification of existing methods can be unreliable, which acts as a bottleneck to their sample efficiency (Ovadia et al. 2019; Yu et al. 2020). We also empirically show that many existing uncertainty quantification methods can not well approximate the errors of Q-function in Section 5.3.

In this paper, we propose the conservative model-based actor-critic (CMBAC), a novel approach that approximates a posterior distribution over Q-values based on the ensemble models and uses the average of the left tail of the distribution approximation to optimize the policy. Specifically, CMBAC alternates between learning multiple estimates of the Q-value from the ensemble models and uses a conservative estimate, i.e., the average of the bottom-k estimates, to avoid unreliable “promising actions”—whose values are high in only a small fraction of the models; (2) the distribution approximation over Q-values can produce reasonable uncertainty estimates of the Q-value. We empirically show that the uncertainty quantification of CMBAC approximates the errors of Q-function more accurately than previous uncertainty quantification methods, which plays a crucial role...
in the impressive performance of CMBAC (please refer to Figure 3). Experiments show that CMBAC significantly outperforms state-of-the-art methods in terms of sample efficiency on several challenging control tasks (Brockman et al., 2016; Todorov, Erez, and Tassa, 2012). Moreover, experiments demonstrate that CMBAC is more robust to model imperfections than previous methods in noisy environments.

2 Related Work

In this section, we discuss related work, including model-based reinforcement learning, uncertainty in reinforcement learning, and conservatism in reinforcement learning.

2.1 Model-Based Reinforcement Learning

Roughly speaking, model-based approaches fall into three categories according to the way of model usage: (1) dyna-style methods (Sutton, 1990; Todorov, Erez, and Tassa, 2012), which use the model to generate imaginary samples as additional training data; (2) shooting algorithms (de Boer et al., 2005; Chua et al., 2018; Wang and Ba, 2020), which use the model to plan to seek the optimal action sequence; (3) policy search with backpropagation (Nguyen and Widrow, 1990; Fairbank and Alonso, 2012; Heess et al., 2015; Clavera, Fu, and Abbeel, 2020; Amos et al., 2021) through time, which exploits the model derivatives and computes the analytic policy gradient. Our work falls into the first category, i.e., the dyna-style algorithm, which has recently shown the potential to achieve high sample efficiency (Janner et al., 2019).

2.2 Uncertainty in Reinforcement Learning

Uncertainty estimation plays a crucial role in many reinforcement learning methods (Sutton, 1990; Osband et al., 2016; O’Donoghue et al., 2018; Zhou, Li, and Wang, 2020; Janner et al., 2019), which use the model to generate imaginary samples as additional training data; (2) shooting algorithms (de Boer et al., 2005; Chua et al., 2018; Wang and Ba, 2020), which use the model to plan to seek the optimal action sequence; (3) policy search with backpropagation (Nguyen and Widrow, 1990; Fairbank and Alonso, 2012; Heess et al., 2015; Clavera, Fu, and Abbeel, 2020; Amos et al., 2021) through time, which exploits the model derivatives and computes the analytic policy gradient. Our work falls into the first category, i.e., the dyna-style algorithm, which has recently shown the potential to achieve high sample efficiency (Janner et al., 2019).

2.3 Conservatism in Reinforcement Learning

Previous methods introduce the conservatism into policy optimization by underestimating the true value to improve the robustness (Doyle, 1996; Lim, Xu, and Mannor, 2013; Rajeswaran et al., 2017) and alleviate the overestimation bias (Fujimoto, van Hoof, and Meger, 2018; Kuznetsov et al., 2020; Kumar et al., 2020). In model-based reinforcement learning, some methods (Doyle, 1996; Lim, Xu, and Mannor, 2013; Rajeswaran et al., 2017) leverage robust policy optimization, which learns a policy that performs well across models. However, the learned policies tend to be over-conservative (Clavera et al., 2018). In contrast, we use the average of the bottom-k estimates instead of the minimum to optimize the policy, controlling the degree of conservatism. In model-free reinforcement learning, many methods incorporate conservatism into policy learning to alleviate the overestimation bias that comes from function approximation errors (Fujimoto, van Hoof, and Meger, 2018; Kuznetsov et al., 2020; Kumar et al., 2020). In contrast, CMBAC leverages conservatism to alleviate the overestimation that comes from model errors (please refer to Section 5.3).

3 Background

In this section, we present the notation and provide a brief introduction to the state-of-the-art model-based algorithm, i.e., Model-Based Policy Optimization (Janner et al., 2019).

3.1 Preliminaries

We here introduce notation which we will use throughout the paper. We consider an infinite-horizon Markov decision process (MDP) denoted by a tuple \((S, A, P^*, r, \gamma, \rho_0)\), where \(S\) and \(A\) are the sets of states and actions, respectively, \(P^*: S \times A \times S \rightarrow [0, \infty)\) is the transition probability density function with \(P^*(s, a)\) representing the conditional distribution of the next state given the current state \(s\) and action \(a\), \(r: S \times A \rightarrow \mathbb{R}\) is the reward function, \(\rho_0: S \rightarrow [0, \infty)\) is the starting state distribution, and \(\gamma\) is the discount factor. Let \(\pi: S \rightarrow \mathcal{P}(A)\) be a stationary policy, where \(\mathcal{P}(A)\) is a set of probability distribution over \(A\). Let \(\pi(\cdot|s)\) denote the probability distribution over \(A\) at state \(s\). In model-based reinforcement learning, we learn a dynamics model \(P(\cdot|s, a)\) using data collected from interaction with the true MDP. For simplicity, we assume that the reward function \(r(s, a)\) is known throughout the paper, but in practice, we learn a reward function. Let \(S_0\) be the random variable for the initial state. Let \(Q^\pi, P\) be the state-action value function on the model \(P\) and policy \(\pi\) defined by:

\[
Q^\pi, P(s, a) = \mathbb{E}_{\pi, P}[\sum_{t=0}^{\infty} \gamma^t r(S_t, A_t)|S_0 = s, A_0 = a].
\]

We define \(\eta(\pi, P) = \mathbb{E}_{\pi}[Q^\pi, P(S_0, A_0)]\) as the expected reward-to-go. Our goal is to maximize the reward-to-go on the true model, that is, \(\eta(\pi, P^*)\), over the policy \(\pi\).

3.2 Model-Based Policy Optimization

Model-based policy optimization (MBPO) is a state-of-the-art model-based algorithm that has achieved impressive performance (Janner et al., 2019). MBPO has three ingredients:
Algorithm 1: Pseudo code for CMBAC

1. Initialize an ensemble of models \{P_{\psi_i}\}_{i=1}^N, environment dataset \mathcal{D}_{env}, and model dataset \mathcal{D}_{model}.
2. Initialize policy \pi_{\phi}, multi-head Q-network \{Q_{\theta_j}\}_{j=1}^K.
3. for \(N\) epochs do
4. Train models \{P_{\psi_i}\}_{i=1}^N on \mathcal{D}_{env}.
5. for \(E\) steps do
6. Take action in environment using \pi_{\phi}; add to \mathcal{D}_{env}.
7. for \(M\) model rollouts do
8. Sample \(s_t\) uniformly from \mathcal{D}_{env}.
9. Perform \(k\)-step model rollouts starting from \(s_t\) using policy \pi_{\phi}; add to \mathcal{D}_{model}.
10. end for
11. for \(G\) gradient updates do
12. Train value functions on model data: \(\theta_j \leftarrow \theta_j - \lambda_Q \nabla J^{\phi}(\theta_j)\) for \(j \in \{1, \ldots, K\}\).
13. Conservative policy optimization on model data: \(\phi \leftarrow \phi - \lambda_\pi \nabla J^{\pi}(\phi)\).
14. end for
15. end for
16. end for

1. **Ensemble models** MBPO trains a bootstrap ensemble of dynamics models via maximum likelihood technique (Chua et al. 2018) on dataset \mathcal{D}_{env}, collected from interactions with the true environment. Each member of the set is modeled as a Gaussian with mean and diagonal covariances given by neural networks. Our work also learns an ensemble of probabilistic models.

2. **Model usage** MBPO selects a model uniformly at random from the ensemble and generates a prediction from the selected model. To reduce compounding model errors introduced by long rollouts, MBPO generates many short rollouts as additional training dataset \mathcal{D}_{model}.

3. **Policy optimization** MBPO uses soft actor-critic (SAC) (Haarnoja et al. 2018)—a state-of-the-art model-free algorithm based on the maximum entropy reinforcement learning framework—to optimize the policy.

4. Conservative Model-Based Actor-Critic

In this section, we present a detailed description of CMBAC. CMBAC alternates between (1) learning multiple estimates of the Q-value function from the ensemble models and (2) using the average of the bottom-k estimates to optimize the policy. We provide an illustration of CMBAC in Figure 1 and summarize the procedure of CMBAC in Algorithm 1.

4.1 Capturing the Uncertainty of the Q-value

To capture the uncertainty of the Q-value, CMBAC directly approximates the posterior distribution over Q-values based on the posterior distribution approximation over models. The uncertainty of the Q-value comes from the unknown true environment, and model-based approaches usually estimate it using a possible set of models.

To approximate the posterior distribution over models, CMBAC learns an ensemble of probabilistic neural networks \{P_{\psi_i}\}_{i=1}^N, which has been shown the potential to capture the uncertainty of models (Lakshminarayanan, Pritzel, and Blundell 2017; Chua et al. 2018). Each probabilistic neural network model the transition probability density as a Gaussian with mean and diagonal covariances given by neural networks. That is, \(p_{\psi_i}(s'|s,a) = \mathcal{N}(\mu_{\psi_i}(s,a), \sigma_{\psi_i}(s,a))\). To control the granularity of discrepancy between models, CMBAC constructs a set of models \(\mathcal{M}\) using these probabilistic neural networks. Each element in \(\mathcal{M}\), denoted by \(M_j\), is a set consists of \(M\) different networks \((M < N)\) and thus the size of \(\mathcal{M}\) is \(K = \binom{N}{M}\). We view each \(M_j\) as a model, and \(M_j\) generates next state \(s'\) given current state-action pair \((s,a)\) under the distribution \(P_j(s'|s,a) = \sum_{p \in (\{P_{\psi_i}\}_{i=1}^N \cap \mathcal{M}_j)} \frac{1}{M} p(s'|s,a)\).

The set of models \(\mathcal{M}\) has the ability to well approximate the posterior distribution of the true environment using non-parametric bootstrap with random initialization (Efron and Tibshirani 1993; Osband et al. 2016; Chua et al. 2018). Given fixed \(\mathcal{M}\), CMBAC achieves more fine-grained granularity of discrepancy between models in \(\mathcal{M}\) by increasing the total number of networks \(N\). Given fixed \(N\), the discrepancy between models drops with increasing \(M\), i.e., the number of networks in each \(M_j\). If \(M = N\), then CMBAC reduces to MBPO. Moreover, representing each model by an ensemble of neural networks significantly improves the model accuracy (Kurutach et al. 2018; Chua et al. 2018).

To approximate the posterior distribution over Q-values, CMBAC learns multiple estimates via a multi-head Q-network from the distribution approximation over models \(\mathcal{M}\). Similar to bootstrapped DQN (Osband et al. 2016), the multi-head Q-network is a shared neural network architecture with \(K\) “heads” branching off independently (please refer to Appendix D for details). Each “head” \(\hat{Q}_{\theta_j}\) provides an estimate of the Q-value for a policy \(\pi\), which corresponds to a model \(M_j \in \mathcal{M}\). That is, CMBAC aims to approximate the \(Q^{\pi,\theta_j}\) via the “head” \(\hat{Q}_{\theta_j}\).

The target value of each “head” \(\hat{Q}_{\theta_j}(s,a)\) is given by
\[
y_j(s,a) = r(s,a) + \gamma(\hat{Q}_{\theta_j}(s',a') - \alpha \log(\pi(a'|s'))),
\]
where \(j = 1, \ldots, K\), \(s'\) is sampled from \(P_j(\cdot|s,a)\), \(a'\) is sampled from \(\pi(\cdot|s')\), \(\alpha\) is the temperature parameter, and \(\hat{Q}_{\theta_j}\) is the target value network with \(\theta_j\) being an exponentially moving average of the value network weights, which has been shown to stabilize training (Mnih et al. 2015). For each \(j \in \{1, \ldots, K\}\), the parameter \(\theta_j\) can be trained to minimize the Bellman residual
\[
J_{\theta_j}(\theta_j) = \mathbb{E}_{(s,a) \sim D_{model}} \frac{1}{2} (\hat{Q}_{\theta_j}(s,a) - y_j(s,a))^2.
\]

CMBAC naturally approximates the posterior distribution over Q-values, as it learns an estimate from a model sampled from the approximated posterior distribution over models, respectively. We use the clipped double Q-learning as proposed by Fujimoto, van Hoof, and Meger (2018). In practice, we use two multi-head Q-network and train each “head” using the minimum of the corresponding two “heads”.

\[
P_j(s'|s,a) = \sum_{p \in (\{P_{\psi_i}\}_{i=1}^N \cap \mathcal{M}_j)} \frac{1}{M} p(s'|s,a).
\]
4.2 Conservative Policy Optimization

To prevent the agent from exploiting model errors, CMBAC uses a conservative estimate of the Q-value function to optimize the policy, named conservative policy optimization. Previous model-based methods aim to learn a conservative estimate that is an approximate lower bound of the true Q-value $Q(s, a)$. For example, robust policy optimization (Doyle 1996; Lim, Xu, and Mannor 2013) considers a set of possible models and learns the worst-case Q-value, i.e., $Q^*(s, a) = \min_{P \in \mathcal{M}} Q_{P}(s, a)$. However, these methods tend to learn an over-conservative policy, which severely degrades their sample efficiency and asymptotic performance (Clavera et al. 2013). Unlike these methods, CMBAC introduces conservatism to alleviate the overestimation that comes from the model errors and does not aim to learn a lower bound. We observe that if we learn multiple estimates using the approach in Section 4.1, a small fraction of the “heads” severely overestimate the Q-value (see details in Section 4.3) while the others provide a relatively reasonable estimation. Based on the observation, we hypothesize that the actions—whose values are high in only a small fraction of the models—are unreliable in the true environment. To explicitly capture the uncertainty of Q-value, we propose to drop the top-k estimates and use the average of the bottom-2 estimates to optimize the policy.

Figure 1: Illustration of CMBAC with $N = 3$, $M = 2$, and $L = 1$. It first learns three probabilistic neural networks and constructs each model using the arbitrarily two probabilistic neural networks. It then alternates between learning three estimates of the Q-value function from the ensemble models and using the average of the bottom-2 estimates to optimize the policy.

4.3 Discussion

We discuss some advantages of CMBAC in this subsection.

Capture global uncertainty CMBAC naturally captures the global uncertainty (O’Donoghue et al. 2018; Zhou, Li, and Wang 2020), which considers the compounding model error and its effect on the critic learning. The global uncertainty allows CMBAC to deal with the overestimation that comes from the long-term prediction errors of the model. Otherwise, conservatively optimizing the policy via local uncertainty (i.e., the one-step prediction error) may mislead the agent into exploiting the regions where the one-step prediction is accurate, but multi-step prediction errors are large.

Capture uncertainty with various granularity CMBAC approximates a posterior distribution over Q-values to implicitly capture the uncertainty of Q-value. To achieve granularity in uncertainty capturing, CMBAC controls the granularity of distribution approximation by varying $M$, i.e., the number of neural networks in each model. The number of estimates $K$ depends on $M$, and different $K$ correspond to the different granularity of distribution approximation.

Flexibly control the degree of conservatism CMBAC can flexibly control the degree of conservatism by varying $M$ and $L$. By varying $M$, CMBAC can provide fine-grained control over the degree of conservatism, as it achieves granularity in capturing uncertainty. By varying $L$, the degree of conservatism increases with $L$.

5 Experiments

Our experiments have four main goals in this section: (1) Test whether CMBAC can significantly outperform state-of-the-art methods. (2) Perform carefully designed ablation study of CMBAC. (3) Perform visualization experiments of CMBAC to explain its effectiveness. (4) Test the robustness of CMBAC in noisy environments.
which has achieved comparable sample efficiency to MBPO.

We evaluate CMBAC and these baselines on MuJoCo (Todorov, Erez, and Tassa 2012) benchmark tasks as used in MBPO. We use two multi-head Q-networks with three hidden layers of 512 neurons each. For all environments except Walker2d, we use the number of dropped estimates $L = 1$. On Walker2d, we use $L = 0$. For our method, we select the hyperparameter $M$ for each environment independently via grid search. The best hyperparameter for Humanoid, Hopper, Walker2d, and the rest is $M = 1, 3, 4, 2$, respectively. The details of the experimental setup are in Appendix B.

Figure 2 shows that CMBAC significantly outperforms these baselines in terms of sample efficiency on several challenging control tasks. For the most challenging Humanoid environment, CMBAC learns substantially faster than state-of-the-art methods. Specifically, the performance of CMBAC on the Humanoid task at 200 thousand steps matches that of MBPO at 300 thousand steps and SAC at 3 million steps. MOPO-Online achieves poor results on several challenging tasks, which may suggest that its uncertainty-based penalty is inappropriate for the online setting (please refer to Figure 7). The model-free method REDQ has recently achieved comparable sample efficiency to MBPO, which raises the question of whether model-based methods carry the promise of being data efficient. Our results demonstrate that model-based methods can be more sample efficient than their model-free counterparts with careful model usage.

To demonstrate the hyperparameter insensitivity of CMBAC, we replot Figure 2 using unified hyperparameters. Detailed results are in Appendix E. To demonstrate the scalability of CMBAC, we compare CMBAC and the baselines on two additional environments, i.e., Walker2d-NT and Hopper-NT (Wang et al. 2019) (See Appendix E).

5.1 Comparative Evaluation

For model-based methods, we compare our method to model-based policy optimization (MBPO) (Janner et al. 2019), a state-of-the-art algorithm. In addition, we compare to an online variant of model-based offline policy optimization (MOPO) (Yu et al. 2020)—a state-of-the-art offline model-based algorithm—which quantifies the uncertainty of the models and uses the uncertainty as a penalty for policy optimization. For a fair comparison, we implement CMBAC and MOPO-Online, i.e., the online variant of MOPO, both on top of the MBPO. Although masked model-based actor-critic (M2AC) (Fan et al. 2020) outperforms MBPO by using the uncertainty of models, its core component is a masking mechanism, which is orthogonal to our method. Therefore, we do not compare to M2AC. For model-free methods, we compare to soft actor-critic (SAC) (Haarnoja et al. 2018), which is used for policy learning in our method; randomized ensembled double Q-learning (REDQ) (Chen et al. 2021),
both environments. A possible reason is that it may incor-
port more information of the model into the multi-head Q-network and thus improves the value estimation. For the Ant environment, conservative policy optimization provides an additional performance improvement. For the Walker2d environment, we find that our method without conservative policy optimization performs better than that with it (please refer to Figure 5). One possible reason is that the agent may require an optimistic value estimate to promote exploration, and thus avoid suboptimal policies on Walker2d.

**Sensitivity to hyperparameters** In this part, we analyze the sensitivity of CMBAC to the number of neural networks in each model $M$ and the number of dropped estimates $L$. Please refer to Appendix C for additional results.

The number of neural networks in a model $M$ We vary the number $M \in \{1, 2, 3, 4\}$ with $N = 5$ on the Ant and Walker2d environments. Figure 4 shows that (1) the number $M$ is essential as it controls the granularity of uncertainty capturing, and (2) there is an optimal number, i.e., $M = 2$.

The number of dropped estimates $L$ We vary the number of dropped estimates for each ensemble multi-head Q-network $L \in \{0, 1, 2\}$. The total number of estimates dropped is $2L$. Figure 5 suggests that dropping $L$ topmost estimates provides a performance improvement on the Ant environment but degrades the performance on the Walker2d.

**Analysis of CMBAC variants** To provide further insight into CMBAC, we discuss the performance of some CMBAC variants. These variants implement the two core components of CMBAC, i.e., capturing the uncertainty of the Q-value and conservative policy optimization, in different ways.

**LMEQ variants** To capture the uncertainty of the Q-value, we propose to increase the number of ensemble Q-value functions, i.e., deep ensembles [Lakshminarayanan, Pratzel, and Blundell 2017]. Thus, we propose Model-Based Policy Optimization with Ensemble Q (MBPOEQ), which uses the same multi-head Q-networks as our method. In contrast to CMBAC, MBPOEQ learns all “heads” using the same target, i.e., the average of these “heads”.

**CPO variants** We additionally propose three possible conservative policy optimization variants, named CMBAC Uncertainty Penalty (CMBACUP), MINimum CMBAC (MINCMBAC), and REDQ-CMBAC, respectively. CMBACUP uses the standard deviation of the Q-value estimates as a penalty. MINCMBAC uses the minimum of all

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**Figure 4:** Varying the number of neural networks in a model. The results demonstrate that $M$ is essential, and there is evidence that $L$ is essential, and $L = 1$ provides significant performance improvement on Ant.

**Figure 5:** Varying the number of dropped estimates $L$. The results show that increasing ensemble Q-values does not boost the performance, and different implementations of conservative policy optimization significantly impact performance.

**Figure 6:** Learning curves of CMBAC and its variants on the Ant and Walker2d environments. The results show that increasing ensemble Q-values does not boost the performance, and different implementations of conservative policy optimization significantly impact performance.
estimates to optimize the policy. Similar to REDQ (Chen et al. 2021), REDQ-CMBAC produces a conservative estimate via each multi-head Q-network and uses the mean of the two conservative estimates to optimize the policy. Due to limited space, we provide the details in Appendix D.

The results in Figure 6 suggest the following conclusions. CMBACUP achieves comparable performance to CMBAC in both environments. However, we find it sensitive to the penalty coefficient (please refer to Appendix C). Increasing the number of ensemble Q-values does not improve performance, indicating that our uncertainty capturing technique is critical to performance improvement. Incorporating conservatism is significant for performance improvement, and different implementations of conservative policy optimization have a large impact on performance.

5.3 Visualization

Uncertainty estimation In this part, we regard the standard deviation of the multiple heads as our estimated uncertainty and compare it to previous uncertainty estimation used in state-of-the-art model-based algorithms. The uncertainty estimation methods include Global, i.e., the cumulative discounted sum of prediction errors similar to that used in model-based offline policy optimization (Yu et al. 2020), M2AC that is used in masked model-based actor-critic (Pan et al. 2020), and SLBO that is used in stochastic lower bounds optimization (Luo et al. 2019). We visualize the prediction error of the Q-value and the estimated uncertainty via scatters. The results in Figure 7 show that our estimated uncertainty can approximate the errors of the Q-function more accurately than previous methods. The superiority of our uncertainty estimation demonstrates that the multi-head Q-networks used in CMBAC can provide a reasonable approximation of the posterior distribution of the true Q-value. Moreover, we find that the cumulative discounted sum of prediction errors can hardly approximate the errors of the Q-function. We provide possible reasons in Appendix D.

Conservative policy optimization We visualize the estimates of the Q-value in each model versus the true return on the 2D point environment. To better understand the superiority of the proposed conservative policy optimization, we use a simplified version of CMBAC (please refer to Appendix D for details). Figure 8 (left) shows that there are a small fraction of models in which the value estimates significantly overestimate due to model errors. Moreover, Figure 8 shows that dropping several topmost estimates effectively encourages the agent to avoid the unreliable “promising actions” — whose values are high in only a small fraction of models.

5.4 Robust Analysis

To understand the robustness of CMBAC, we compare it to MOPO-Online and MBPO on noisy Walker2d and HalfCheetah environments. In these noisy environments, we add Gaussian white noises with the standard deviation $\sigma = 0.1$ to the agent’s action at every step. The added noise will decrease the accuracy of the learned models. The results in Figure 9 show that CMBAC performs robustly and significantly outperforms the baselines in terms of sample efficiency in noisy environments.
6 Conclusion

In this paper, we present conservative model-based actor-critic, a novel approach that approximates a posterior distribution over Q-values based on the ensemble models and uses the average of the left tail of the distribution approximation to optimize the policy. Experiments show that CM-BAC significantly outperforms state-of-the-art methods in terms of sample efficiency on several challenging control tasks. Moreover, experiments demonstrate that the proposed method is more robust than previous methods in noisy environments. We believe that our proposed approach will bring new insights into model-based reinforcement learning.

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A Algorithm Implementation Details

A.1 Details of models

To enhance the reproducibility of our method, we provide additional details of model learning and model usage. As mentioned in Section 4.1, we represent the model as a set of probabilistic neural networks. The probabilistic neural network (NN) outputs a Gaussian distribution over the next state and reward given the current state and action. Following model-based policy optimization (MBPO) [Janner et al. 2019], we train an ensemble of 7 such probabilistic neural networks and pick the best 5 networks based on the validation prediction error on a held-out set.

We represent the model as a set of \( M \) probabilistic neural networks with \( M < 5 \). Note that \( M \) is a fixed hyperparameter in our experiments. We use an ensemble of such models, denoted by \( M \), and the number of models is \( K = \binom{5}{M} \). For example, if \( M = 2 \), then we use an ensemble of 10 models. During model rollouts, we randomly pick one dynamics model \( M_j \) from \( M \) and randomly pick one probabilistic neural network from \( M_j \) to generate model dataset \( D_{model} \), which is the same with MBPO. Moreover, to learn the Q-value estimate in each model, we preserve next states \( D \) denoted by \( s' \). Since we can generate these next states in parallel, the increased computational cost is low as shown in Figure 11.

A.2 Difference between CMBAC and robust policy optimization

As mentioned in Section 4.2, the conservative policy optimization aims to seek a policy that has high Q-values in most models. Specifically, CMBAC aims to learn the Q-value in every model \( M_j \), denoted by \( Q^{x_j} \). Then given the state \( s \) and action \( a \), CMBAC uses the average of the bottom-k Q-value estimates pointwisely to optimize the policy. In contrast to robust policy optimization [Doyle 1996, Lim, Xu, and Mannor 2013; Rajeswaran et al. 2017], the Q-value estimate, i.e., the average of the bottom-k estimates, used in CMBAC may correspond to different models at different state-action pairs. Therefore, the Q-value estimate may not correspond to any individual transition function. Even if CMBAC uses the minimum estimate to optimize the policy, it is still different from robust policy optimization.

B Experimental Settings and Hyperparameters

B.1 Reproducing the baselines

We first list the implementations of our baselines, including model-based policy optimization (MBPO) [Janner et al. 2019], soft actor-critic (SAC) [Haarnoja et al. 2018], and randomized ensembled double Q-learning (REDQ) [Chen et al. 2021]. We then present the implementation details of MOPO-Online, i.e., the online variant of model-based offline policy optimization (MOPO) [Yu et al. 2020].

MBPO We use the official data provided by the authors of MBPO in our figures.

Table 1: Common parameters used in the comparative evaluation and ablation study.

| Parameter                           | Value     |
|-------------------------------------|-----------|
| environment steps per epoch         | 1000      |
| policy updates per environment step | 20        |
| optimizer                           | Adam      |
| discount (\( \gamma \))             | 0.99      |
| probabilistic nn ensemble size      | 7         |
| value network architecture          | [256,256,256] |
| policy network architecture         | [256,256] |

SAC We use the PyTorch implementation of SAC in the rlkit repository, which is recommended by the authors of SAC, to reproduce the results.

REDQ We use the official code provided by the authors of REDQ to reproduce the results.

MOPO-Online We implement MOPO-Online on top of the MBPO. We quantify the uncertainty as proposed in [Yu et al. 2020], i.e.,

\[
  u(s, a) = \max_{i=1,...,7} \| \Sigma_i(s, a) \|_F, \tag{2}
\]

where \( \Sigma_i(s, a) \) is the diagonal covariance given by each probabilistic neural network. We then use the uncertainty as a reward penalty

\[
  r_p(s, a) = r(s, a) - c \cdot u(s, a), \tag{3}
\]

where \( c \) is a constant.

B.2 Hyperparameters

We use the same hyperparameter as that of MBPO [Janner et al. 2019] if possible. First, we list the common CMBAC parameters used in the comparative evaluation and ablation study in Table 1. Second, we list additional CMBAC parameters, which are different across different environments.

Probabilistic NN architecture Four hidden layers of size 400 for Humanoid and four hidden layers of size 200 for the other five environments, which follows MBPO.

Model horizon As [Janner et al. 2019], \( x \rightarrow y \) over epochs \( a \rightarrow b \) denotes a thresholded linear function, i.e., at epoch \( e \), \( f(e) = \min(\max(x + \frac{y - x}{b-a} \cdot (y - x), x), y) \). HalfCheetah and InvertedPendulum: 1; Hopper: 1 \( \rightarrow \) 15 over epochs 20 \( \rightarrow \) 100; The rest: 1 \( \rightarrow \) 25 over epochs 20 \( \rightarrow \) 300.

The number of probabilistic NN in a model \( M \) As mentioned in Section 5, we select \( M \) via grid search in \([1, 2, 3, 4]\). The best number for Walker2d, Hopper, Humanoid, and the rest is 4, 3, 1, and 2, respectively.

The number of dropped estimates \( L \) As mentioned in Section 5, we select the number of dropped estimates via grid search in \([0, 1, 2]\). The best number for Walker2d and the rest is 0 and 1, respectively.
C Additional Experimental Results

C.1 Sensitivity analysis of CMBACUP

To test the sensitivity to the penalty coefficient of CMBACUP, we design two different kinds of experiments.

1. First, we test the performance of CMBACUP with the same penalty coefficient in different environments. We select the penalty coefficient via grid search in \([0.01, 0.1, 0.5, 1]\) on the Walker2d and Ant environments. We find that 0.1 is the best. Note that Section 5.2 reports the best performance of CMBACUP. However, Figure 10 (left two) shows that CMBAC significantly outperforms CMBACUP on the Humanoid and Hopper environments. This result suggests that we need to carefully tune the penalty coefficient of CMBACUP in different environments to achieve high performance. However, CMBAC uses the same number of dropped estimates \(L\) in all environments, except Walker2d.

2. Second, we test the performance of CMBACUP with different penalty coefficients in the same environment, i.e., Walker2d. The results in Figure 10 (right) demonstrate that CMBACUP is sensitive to the penalty coefficient.

Overall, these results suggest that CMBACUP tends to be sensitive to the penalty coefficient.

C.2 Computational analysis

We compare the computational complexity of CMBAC to MBPO. Specifically, we report the wall clock time versus the average return on the Ant environment when running the experiments on a single NVIDIA GeForce RTX 2080Ti GPU card in Figure 11. The results in Figure 11 demonstrate that our method requires less computing time than MBPO to reach the maximum performance on the Ant environment.

C.3 Additional ablation study results

We provide additional ablation study results on the Hopper and HalfCheetah tasks in this subsection.

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D Additional Details of CMBAC

D.1 Multi-head Q-network architecture

To approximate the posterior distribution over Q-values, we use a multi-head Q-network similar to Osband et al. (2016). As shown in Figure 14, the multi-head q-network outputs \(K\) different values. In our experiment, each “head” provides an estimate of the Q-value. Osband et al. (2016) show that this
As CMBAC, MBPOEQ uses two multi-head Q-networks, denoted by \{Q_1, Q_2\}_{i=1}^K. Given \((s, a, s')\) sampled from \(D_{\text{model}}\), the target of each “head” is given by

\[ y(s, a) = r(s, a) + \gamma \min_{j=1,2} \left( \frac{1}{K} \sum_{i=1}^K Q_i(s', a') \right), \]  

where \(a' \sim \pi(\cdot|s')\).

**CMBACUP** We define the standard deviation of the estimates given by the multi-head Q-network as our estimated uncertainty \(u_Q(s, a)\). Given state \(s\) and action \(a\), we use the uncertainty \(u_Q(s, a)\) as a penalty instead of dropping several topmost estimates.

**MINCMBAC** Given the state \(s\) and action \(a\), MINCMBAC uses the minimum estimate of all possible estimates to optimize the policy. As discussed in Section A.2, MINCMBAC is different from robust policy optimization.

**REDQ-CMBAC** Both CMBAC and REDQ-CMBAC learn two multi-head Q-networks and obtain a conservative estimate by dropping several topmost estimates for each multi-head Q-network. CMBAC then uses the minimum of the two conservative estimates to optimize the policy similar to the clipped double Q-learning trick (Fujimoto, van Hoof, and Meger 2018). In contrast, REDQ-CMBAC uses the average of the two conservative estimates instead of the minimum to optimize the policy as Chen et al. (2021).

**D.3 Details of visualization experiments**

We present detailed experimental settings and analyses of the visualization experiments in Section 5.3.

**Uncertainty estimation** In Section 5.3, we find that the cumulative discounted sum of prediction errors can hardly approximate the errors of the Q-function. Here we provide possible reasons. One possible reason is due to compounding model errors. Similar to Yu et al. (2020), we approximate the prediction error at each step by the uncertainty estimated using Eq. We compute the cumulative discounted sum of prediction errors by generating a 1000-step rollout in the model. In this case, the cumulative discounted sum
of prediction errors can be large due to compounding model errors. Another possible reason is that the uncertainty quantification in [Yu et al. (2020)] is inappropriate for the online setting, as it is designed for the offline setting.

**Conservative policy optimization** We provide details of the 2D point environment and the simplified CMBAC.

### 2D point environment

Our 2D point environments is similar to that defined by [Clavera et al. (2018)]. The state space is $[-2, 2]^2$. The action space is $[-1, 1]^2$. The initial state distribution is an uniform distribution over the state space. The transition function is defined by

$$f(s, a) = \text{clip}(s + a, -2, 2),$$

where clip represents a pointwise clipping of $s + a$ to bound the next state. The goal the agent is to reach the goal state, i.e., $g = (0, 0)$. Following [Haarnoja et al. (2017)], we define the reward function by

$$r(s, a) = c \cdot \exp\left(- \frac{\|f(s, a) - g\|^2}{\alpha}\right),$$

where $c = 0.05$ and $\alpha = 5$. The horizon is 50.

### A simplified CMBAC

To disentangle different components in CMBAC, we implement a simplified CMBAC, named SCMBAC, in this visualization experiment. To analyze the errors of the true Q-value in the model-based setting, SCMBAC removes the entropy and does not use the clipped double Q-learning trick.

### E Additional results of CMBAC

#### E.1 Evaluation with unified hyperparameters

For high-dimensional tasks, i.e., Humanoid and Ant, we use $M = 1$. For the rest, we use $M = 3$. For all tasks, we use $L = 1$. Figure 15 shows that CMBAC using the unified hyperparameters significantly outperforms MBPO on several challenging tasks.

#### E.2 Results on two additional environments

We conduct experiments using the unified hyperparameters in two additional challenging benchmark environments, i.e., Walker2d-NT and Hopper-NT, which are different from Walker2d and Hopper in the terminal functions and reward functions ([Wang et al. (2019)]). Figure 16 shows that CMBAC consistently outperforms MBPO on Walker2d-NT and Hopper-NT.