Time-dependent manufacturing processes lead to a new class of inverse problems

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The control of time-dependent, energy beam manufacturing processes has been achieved in the past through trial-and-error approaches. We identify key research gaps and generic challenges related to inverse problems for these processes that require a multidisciplinary problem-solving approach to tackle them. The generic problems that we identify have a wide range of applications in the algorithmic control of modern manufacturing processes.

For millennia, the main way to manufacture objects has been using solid tools designed for a specific job. The most common shaping principle is simple: move a solid tool so that, by some mechanical principles, material is removed from the workpiece. There is thus a direct geometrical dependence between tool movement and the shape of the object created. Even more sophisticated, numerically controlled machine tools use this basic principle.

Other manufacturing processes using tools that, geometrically, are not fully deterministically defined (i.e., time-dependent tools) have also made their way onto production lines. In these processes, the tool takes the form of a flux of energy emanating from a source (e.g., laser, ion gun, fluid jet), which we will call energy beam processes (EBPs). These time-dependent tools can be used to shape materials that cannot be processed with precision using conventional methods. In contrast to conventional tools, the shape and magnitude of EBP footprints (material removal/deposition rate and/or material transformation rate at the intersection between beam and workpiece) depend not only on the tool path but also on exposure time and energy flux. Furthermore, the energy flux depends on the geometry of the beam, which varies with process setup (e.g., energy losses between source and target). Control of time-dependent tools for generating freeforms is therefore very difficult compared with processes that use traditional, time-independent tools. To make parts using EBPs, the beam moves along a predefined path to convolute the energy footprint with the workpiece, and any variations in the dwell time of the footprint result in errors in the part geometry. Note that the footprint is usually not spatially uniform and it changes with orientation to the target surface.

Taking a predefined tool path and predicting the resulting surface is the direct problem in EBPs, which has been widely documented; this is usually combined with trial and error to achieve the required geometry. However, the real challenge, and the correct approach, is to tackle the inverse problem—that is, for a given part geometry, to determine a path that minimizes errors in its generation using an optimization algorithm. Only very limited research has been carried out in this field.

Our aim here is to disseminate to the wider scientific community this opportunity to study a challenging new set of problems, which requires an interdisciplinary approach. We believe that the lack of research on these inverse problems is due to the difficulty both of accurately formulating the direct problem to take into account the time dependence of the energy footprint and its effect on the workpiece material and subsequently of tackling the inverse problem.

Although inverse problems in general engineering (e.g., heat transfer, acoustics) have been well explored, those related to time-dependent manufacturing processes are novel because they involve optimization of multipass movements of the source of energy relative to the target workpiece. There has, however, been some recent progress. For example, in abrasive jet micromachining, Lari and Papini (1) studied the inverse problem as the solution of an integral equation (see also ref. 2) to determine the feed speed for the generation of shallow periodic and W-shaped forms based on overlapping straight paths for a Gaussian beam footprint. For non-Gaussian footprints, an approximation method based on modeling the beam as a point source was used. In atmospheric plasma etching, Dai et al. (3) dealt with the nonlinear dependence of material removal rate on dwell time by considering a nested pulsed iterative method to evaluate and correct the time-varying nonlinearity of the process and enable the machining of pocket geometries in fused silica. In optical surfacing, Beauchamp and coworkers (4) showed how variable pitch path self-planning strategies, underpinned by a simple, linear model for material removal, can be used to improve the uniformity of a polished surface. Ion beam machining, an approximate solution of the inverse problem involves varying the dwell time of the beam on each pixel of the required surface (5). However, this takes no account of nonlinear effects, and, while it is a plausible starting strategy, for this EBP, we demonstrated (6) how solving the inverse problem for a more sophisticated model using a steepest descent algorithm can significantly improve the accuracy of freeform etching using this tool.

In the past few years, we have defined a series of direct and inverse problems for energy beam manufacturing processes, ranging from abrasive and plain waterjet machining (2) to laser micromachining (7) and focused ion beam machining (6), with a unified description given in ref. 8. If we define the surface of the workpiece to be \( r = \mathbf{R}(\alpha, \beta, t) \), where \( \alpha \) and \( \beta \) parameterize the surface and \( t \) is time, the evolution equation for the surface under the action of the beam is

\[ n \cdot \frac{d\mathbf{R}}{dt} = (\mathbf{n} \mathbf{k}) E, \]

where \( n(\alpha, \beta, t) \) is the outward unit normal to the surface, \( k(t) \) is a unit vector in the direction of the axis of the beam, \( E(r, \theta, t, \mathbf{R}; p) \) is the material removal rate, \( r \) and \( \theta \) parameterize position within the beam, and \( p \) is a vector of parameters that characterize the dynamics of how energy is delivered by the beam. The evolution equation is to be solved subject to \( r = \mathbf{R}_0 \) when \( t = 0 \) for \( 0 < t < T \).

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In the direct problem, the control parameters, \( p \) (a beam path, orientation, and possibly energy output rate and process time, \( T \)), are specified, and the final surface, \( \mathbf{R}(\alpha, \beta, T) \), which will be observed in practice, is computed.

In the inverse problem, we seek parameters, \( p \), that minimize \( ||\mathbf{R}(\alpha, \beta, T) - \mathbf{R}_0(\alpha, \beta)|| \), where \( \mathbf{R}_0(\alpha, \beta) \) specifies the target surface to be created and \( ||.|| \) is an appropriate norm (a weighted sum with \( T \) can be used for process-time minimization). In a model as simple as this, all of the physics is bundled into the material removal rate function, \( E(r, \theta, t, \mathbf{R}, p) \).

Problems like this lie in the broad category of partial differential equation-constrained optimization. Although this is a vast research area, the use of techniques from this field in manufacturing engineering is not common. We do not discuss the effect of process noise and formulate the model in terms of a partial differential equation, but note that modeling material removal as a random process makes direct and inverse problems more mathematically challenging and is an area that would benefit from further development (9).

In addition, we can formulate a set of new problems that presents a wide range of challenges, requires an interdisciplinary approach, and reaches far beyond removal and accretion processes. The following is a simple classification and a nonexhaustive list of manufacturing processes that use time-dependent tools and require consideration of the associated inverse problems (Fig. 1):

- **Material removal**: machining by laser beam, abrasive waterjet/air blasting, focused ion beam, electron beam, electrojet, electrodischarge, and ultrasound

  The material-removal principles vary significantly (vaporization, mechanical abrasion, particle momentum transfer, chemical dissolution, spark erosion) but have in common an energy beam removal footprint with particularities in shape and behavior (e.g., melt redeposition, plastic deformation, loss of energy due to reflection/absorption).

- **Material/surface modification**: mechanical or laser shot peening, laser polishing, localized/selected heat treatment

  No material is added or removed, but the energy beam modifies the (usually superficial) properties of the workpiece by mechanical or thermal means for altering mechanical properties (e.g., hardness, residual stresses) or microstructural constituents of the part.

- **Material accretion**: direct metal deposition, spray coating, fused deposition

  The energy beam deposits similar or dissimilar materials on the workpiece, and the principles are mainly related to energy transfer to individual particles that interact with the base material through different physical mechanisms (e.g., transport of mass, heat, and/or momentum) and results in deposition footprints that are affected by process-specific secondary effects (e.g., nonsticking particles, porosity of the deposited material).

- **Material consolidation**: selective laser sintering, electron beam melting

  This is a generic group of powder bed additive manufacturing processes in which the energy beam causes local consolidation of groups of particles. Each process is influenced by the dwell time and energy distribution of the beam and leads to unique microstructural and mechanical properties of the generated component.

- **Hybrid processes**: laser/ultrasonic/plasma assisted

  The energy beam is used in conjunction with another process that can be nontime dependent (e.g., cutting) or time dependent (e.g., electrodischarge machining). The energy beam is placed either ahead of or exactly at the position where the main process occurs. The aim is to enhance the basic removal process. In the case of two coupled dwell-time processes governed by different phenomena, the solution of the associated inverse problem is particularly challenging.

As an example of a hybrid process, we have recently shown (10) that by using a laser beam scanning ahead of a milling tool on a path obtained by solving the inverse problem, cutting forces can be reduced by around 50%. This enables a significant increase in material removal rate.

In each of the above examples, the energy beam parameters need to be determined in an inverse manner to achieve the required process outcomes. The study of these sets of inverse problems is likely to revolutionize the way EBPs are planned and controlled, eliminating the “craftsmanship” approach. In particular, it is likely that packages specifically designed for time-dependent processes will emerge to replace the current use of computer-aided design/manufacturing systems, which are designed for traditional machine tools and not fit for purpose.
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