The $\cos 2\phi$ azimuthal asymmetry of unpolarized $p\bar{p}$ collisions at Tevatron

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We calculate the $\cos 2\phi$ azimuthal asymmetry of the unpolarized $p\bar{p}$ Drell-Yan dilepton production process in the Z-resonance region at the Tevatron kinematic domain. Such an azimuthal asymmetry can provide additional information about a spin-related new parton distribution function, i.e., the Boer-Mulders function of the proton, compared to the $pp$ process. Therefore the available data of unpolarized proton-antiproton collision at Tevatron can contribute to our study on the spin structure of the nucleon.

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The study of the intrinsic transverse momentum dependent (TMD) distribution functions has received much attention in recent years [1]. Such new quantities of the nucleon provide us a significant perspective on understanding the spin structure of hadrons and the non-perturbative properties of quantum chromodynamics (QCD). The intrinsic transverse momentum of partons may cause special effects in high energy scattering experiments [2]. It was naively speculated that the polarization of at least one incoming hadron is necessary to investigate the spin-related structure and properties of hadrons, however the situation will change if the transversal motions of quarks inside the hadron will take into account. The Drell-Yan process is an ideal ground for testing perturbative QCD and probing TMD distribution functions, and its cross section is well described by next-to-leading order QCD calculations [3]. Surprisingly, the first measurement of the Drell-Yan angular distribution, performed by NA10 Collaboration for $\pi N$ at 140, 194 and 286 GeV, indicates a sizable $\cos 2\phi$ azimuthal asymmetry [4, 5] which cannot be described by leading and next-to-leading order perturbative QCD [6]. Furthermore, the subsequent result by the Fermilab E615 Collaboration reveals that the Lam-Tung relation [7], which is analogous to the Callan-Gross relation [8] in deep-inelastic scattering, obtained as a consequence of the spin-$1\over 2$ nature of the quarks, is clearly violated [9]. The violation has also been tested in recent $pd$ and $pp$ Drell-Yan dimuon processes measured by E866/NuSea Collaboration [10, 11].

Several attempts were made to interpret this asymmetry, such as the factorization breaking QCD vacuum effect [6] which is possible the helicity flip in the instanton model [12], higher twist effect [13–15] and the coherent states [16]. Boer pointed out that the $\cos 2\phi$ azimuthal asymmetry could be due to a non-vanished TMD distribution function $h^{\perp}_1(x, p_T^2)$ [17], named as the Boer-Mulders function later, as one of the eight leading-twist TMD distribution functions contained in [18, 19]

$$
\Phi = \frac{1}{2} \left\{ f_1 \phi^+ - f_{1\perp} \frac{e_i p_T S T}{M} \phi^+ + \frac{[S_T, \phi^+]_\perp}{2} + \left( S_L g_{1L} + \frac{p_T \cdot S_T}{M} g_{1T} \right) \gamma_5 \phi^+ \right. \\
\left. + \left( S_L h_{1L} + \frac{p_T \cdot S_T}{M} h_{1T} \right) \left[ \frac{[\phi_T, \phi^+]_\perp}{2M} \right. \right. \\
+ \left. \left. i h^\perp \frac{[\phi_T, \phi^+]_\perp}{2M} \right) \right\},
$$

where $\Phi$ is the quark-quark correlation matrix, defined as

$$
\Phi_{ij}(p, P, S) = \int \frac{d^4 \xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) W(0, \xi) \psi_i(\xi) | PS \rangle.
$$

The Boer-Mulders function is another time-reversal odd ($T$-odd) distribution function which characterizes the correlation between quark transverse momentum and quark transverse spin, analogous to the Sivers function $f^{\perp T}_1(x, p_T^2)$ which signifies the correlation between quark transverse momentum and hadron transverse spin [20]. The non-vanished $T$-odd distribution functions can arise from the initial-state or final-state interaction [21–24]. In general, the path-order Wilson line arising from the requirement of QCD gauge invariance for quark correlation functions provides non-trivial phases and leads to non-vanished $T$-odd distribution functions [25–28]. Due to the present of Wilson line, opposite sign of the Boer-Mulders function or Sivers function in semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan processes is expected [29, 30].

$$
h^{\perp}_1(x, p_T^2)|_{\text{SIDIS}} = -h^{\perp}_1(x, p_T^2)|_{\text{DY}}.
$$

The existence of $T$-odd distribution function can cause azimuthal asymmetries in SIDIS at leading twist level [19], and the product of two Boer-Mulders functions of two incoming hadrons may give a sizable $\cos 2\phi$ azimuthal asymmetry in unpolarized Drell-Yan processes by establishing a preferred transverse momentum direction from the spin-transverse momentum correlation.
which is called the Boer-Mulders effect \[17\]. Thus, the measurement of the Boer-Mulders function will promote our understanding of QCD. Many theoretical and phenomenological studies are carried out along this direction \[31\][48].

Recently, the Collider Detector at Fermilab (CDF) Collaboration first measured the angular distribution coefficients of Drell-Yan \(e^+e^-\) pairs in the Z mass region from unpolarized \(pp\) collisions \(p + \bar{p} \rightarrow \gamma^*/Z + X \rightarrow l^+l^- + X\) at \(\sqrt{s} = 1.96\) TeV \[10\]. This indicates that it is feasible to investigate spin physics at Tevatron. In this paper, we calculate the \(\cos 2\phi\) azimuthal asymmetry caused by the Boer-Mulders effect in the Z-pole region with the kinematic conditions at Tevatron.

The angular distribution coefficients are generally frame dependent. We choose the Collins-Soper (CS) frame \[50\] to perform the calculation. It is the center of mass of the lepton pair with the \(z\) axis defined as the bisector of \(p\) and \(\bar{p}\) beams. The polar angular \(\theta\) is defined as the angular of the positive lepton with respect to the \(z\) axis direction, and the azimuthal angular \(\phi\) is defined as the angular of the lepton plane with respect to the proton plane. In this frame the Lam-Tung relation is insensitive to the higher fixed-order perturbative QCD \[51\] or the QCD resummation \[52\][54]. The angular differential cross section for unpolarized Drell-Yan process has the general form:

\[
\frac{1}{\sigma} d\Omega \, = \frac{3}{4\pi} \left(1 + \frac{1}{\lambda + 3} \phi \sin 2\theta \cos \phi \right) \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi \right) + \frac{\nu}{2} \sin^2 \theta \cos 2\phi, \tag{4}
\]

where \(\Omega\) is the solid angle and \(\lambda, \mu,\) and \(\nu\) are angular distribution coefficients. For azimuthal symmetrical scattering, the coefficients \(\mu = \nu = 0\). It can also be written as \[4\][53]:

\[
\frac{d\sigma}{d\Omega} = W_T (1 + \cos^2 \theta) + W_L (1 - \cos^2 \theta) + W_A \sin 2\theta \cos \phi + W_{\Delta A} \sin^2 \theta \cos 2\phi. \tag{5}
\]

When taking into account both virtual photon and Z-boson contribution, the leading order unpolarized Drell-Yan cross section is \[17\]:

\[
\frac{d\sigma(h_1 h_2 \rightarrow l X)}{d\Omega dx_1 dx_2 d^2 q_T} = \frac{\alpha^2}{3 Q^2} \sum_a \left[ K_1(\theta) \mathcal{F}[f_{a\,u}, f_{a\,d}] + K_3(\theta) \cos 2\phi + K_4(\theta) \sin 2\phi \right]
\times \mathcal{F} \left[ (2\mathbf{p}_T \cdot \mathbf{h}_x \cdot \mathbf{k}_T - \mathbf{p}_T \cdot \mathbf{k}_T) \frac{\mathbf{h}_T \cdot \mathbf{h}_{1a}}{M^2} \right], \tag{6}
\]

where \(x_1, x_2\) are the Bjorken variables standing for the longitudinal momentum fractions carried by the partons in the proton and antiproton, and \(\alpha, M, q_T,\) and \(Q\) are the fine structure constant, the mass of proton, the transverse momentum, and invariant mass of \(\gamma^*/Z\) respectively. The structure function notation in this equation is defined as

\[
\mathcal{F}[-\cdots] = \int d^2 p_T d^2 k_T \delta^2 (p_T + k_T - q_T)[-\cdots], \tag{7}
\]

where \(p_T, k_T\) are the transverse momenta of quarks in proton and antiproton, and \(\mathbf{h} \equiv \frac{\mathbf{p}_T}{p_T}\) is the direction of the transverse momentum of \(\gamma^*/Z\). The coefficients \(K_1, K_3,\) and \(K_4\) are expressed as:

\[
K_1(\theta) = \frac{1}{4} (1 + \cos^2 \theta) \left[ e_a^2 + 2 g_V e_a g_A \delta \chi_1 + c_1^2 c_1^2 \chi_2 \right] + \frac{\cos \theta}{2} \left[ 2 g_A e_a g_A \delta \chi_1 + c_3^2 c_3^2 \chi_2 \right], \tag{8}
\]

\[
K_3(\theta) = \frac{1}{4} \sin^2 \theta \left[ e_a^2 + 2 g_V e_a g_A \delta \chi_1 + c_1^2 c_1^2 \chi_2 \right], \tag{9}
\]

\[
K_4(\theta) = \frac{1}{4} \sin^2 \theta \left[ 2 g_V e_a g_A \delta \chi_3 \right], \tag{10}
\]

where \(e_a\) is the charge of quarks (antiquarks), and \(g_V\) and \(g_A\) are the vector and axial-vector coupling constants to the Z-boson. We take their values in Ref.[56]. The \(\mathbf{c}_i\) is defined as:

\[
\mathbf{c}_1 = (g_V^2 + g_A^2), \quad \mathbf{c}_2 = (g_V^2 - g_A^2), \quad \mathbf{c}_3 = 2 g_A g_A^*, \tag{11}
\]

where \(j = l\) or \(a\). The Z-boson propagator \(\chi_i\) is given by:

\[
\chi_1 = \frac{1}{\sin^2 \theta_W} \frac{Q^2 (Q^2 - M_Z^2)}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}, \tag{12}
\]

\[
\chi_2 = \frac{1}{\sin^2 \theta_W} \frac{Q^2}{(Q^2 - M_Z^2)^2}, \tag{13}
\]

\[
\chi_3 = - \frac{\Gamma_Z M_Z}{(Q^2 - M_Z^2) \chi_1}, \tag{14}
\]

where \(\theta_W\) is the Weinberg angle. In Eq.(6), we assume that the TMD distributions for antiquarks (quarks) in the antiproton are the same as those for quarks (antiquarks) in proton, and the summation over the index \(a\) are different flavors with \(a = u, d, \bar{u},\) and \(\bar{d}\).

In our calculation, we take the Boer-Mulders functions extracted from \(pD\) and \(pp\) Drell-Yan data \[40\][43]. The parametrizations for \(h_1^+(x)\) is \[43\]:

\[
h_1^+(x) = H_q x^\alpha (1 - x)^\beta f_{1q}(x), \tag{15}
\]

and the TMD part is parametrized with a Gaussian form:

\[
h_1^+(x, p_T^2) = h_1^+(x) \frac{\exp(-p_T^2/p_{T_m}^2)}{p_{T_m}^2}, \tag{16}
\]

\[
\frac{f_{1q}(x, p_T^2)}{p_{T_m}^2} = f_{1q}(x) \frac{\exp(-p_T^2/p_{T_m}^2)}{p_{T_m}^2}. \tag{17}
\]
This parametrization is based on the assumption that the \( \cos 2\phi \) asymmetry comes only from the Boer-Mulders effect in the region \( q_T^2 \ll Q^2 \), and in this region the following relation hold:

\[
x_1 = \frac{Q}{\sqrt{s}} e^y, \quad x_2 = \frac{Q}{\sqrt{s}} e^{-y},
\]

where \( y \) is the rapidity of the \( \gamma^*/Z \). We can also express the cross section of the Drell-Yan process depending on \( y \) and \( Q^2 \) with an additional Jacobian determinant:

\[
\frac{d\sigma}{dydQ^2d^2q_Td\Omega} = \frac{1}{s} \frac{d\sigma}{dx_1dx_2d^2q_Td\Omega}.
\]

From Eq. (18), the azimuthal dependent terms are the second and the third terms with \( \cos 2\phi \) and \( \sin 2\phi \) forms respectively. However, the \( \sin 2\phi \) term is \( \frac{Q}{\sqrt{s}} \) suppressed, which can be found from (10) and (14). As shown in Ref. [44], we can write the coefficient of \( \cos 2\phi \) term in Eq. (6) \( W_{\Delta\Delta} \) into two parts, the perturbative QCD effect \( W_{\Delta\Delta}^{QCD} \) and the Boer-Mulders effect \( W_{\Delta\Delta}^{BM} \). Then using an approximate Lam-Tung relation \( 2W_{\Delta\Delta}^{QCD} - W_L \approx 0 \), one can give the \( \cos 2\phi \) asymmetry caused by the Boer-Mulders effect:

\[
2\nu^{BM} = \frac{4W_{\Delta\Delta}^{BM}}{W_T + W_L} \approx 2\nu + \lambda - 1.
\]

Comparing (5) and (9), and neglecting the \( W_L \) in the denominator because \( W_L \ll W_T \) at low \( q_T \) region, we can get the following relation:

\[
\nu^{BM}(q_T, y, Q) = \frac{\sum_a \frac{1}{Q^2} K_3(\theta_i) F \left[ (2\hat{h} \cdot p_T \hat{h} \cdot k_T - p_T \cdot k_T) \frac{h_i^+ h_i^-}{M^2} \right]}{\sum a \frac{1}{Q^2} K_1(\theta_i) F [f_{3a}f_{1a}]}.
\]

In the numerical calculation, we choose the values of parameters in the Boer-Mulders function as those in Ref. [43, 53]. There is still an unsettled factor \( \omega \) which might be flavor dependent in the parametrization, because it will be canceled in the product of two Boer-Mulders functions of quark and antiquark. It can range in the region \( 0.48 < \omega < 2.1 \), which is limited by the positivity bounds [40, 43, 58]. However, in the \( p\bar{p} \) Drell-Yan process, it has the product of two Boer-Mulders functions of two quarks or two antiquarks which will not cancel the factor \( \omega \). The cross section has different behavior with different values for \( \omega \). Therefore, we can learn additional information of the Boer-Mulders function from \( p\bar{p} \) Drell-Yan processes. In this work, we choose three different values for \( \omega = 0.5, \omega = 1 \) and \( \omega = 2 \) to calculate \( \nu^{BM} \) and show their different behavior.

In order to give \( \nu^{BM} \) with respect to a parameter \( y \), \( Q \) or \( q_T \), we should integrate for the other parameters of the numerator and the denominator in Eq. (21) respectively. The integral over \( q_T \) need to be cut off at \( q_T = 2 \) GeV, because intrinsic transverse momentum plays a significant role at low \( q_T \) and the fitting for the parameters has excluded the data with \( q_T > 2 \) GeV.

Comparing (5) with the angular distribution form taken by CDF [49],

\[
\frac{d\sigma}{d\phi} \propto 1 + \beta_3 \cos \phi + \beta_2 \cos 2\phi + \beta_7 \sin \phi + \beta_5 \sin 2\phi,
\]

\( \nu^{BM} \) will contribute to \( \beta_2 \) caused by the Boer-Mulders effect at low \( q_T \).

In summary, we calculated the \( \cos 2\phi \) azimuthal asymmetry in the unpolarized \( p\bar{p} \) Drell-Yan dilepton produc-
tion processes in the $Z$ mass region at CDF kinematic domain. It can be measured by experimental detection of the Lam-Tung relation violation. It is possible to study the spin structure of hadrons in unpolarized collision processes around $Z$ mass region at Tevatron. In addition, the $p\bar{p}$ processes can give more significant information of the Boer-Mulders function than $pp$ processes. It can help us to settle the factor $\omega$ in the parametrization, and the prediction that the Boer-Mulders function have different signs in SIDIS and Drell-Yan processes [21] also awaits experimental confirmation. Therefore the available data of Tevatron are ideal to investigate the spin structure of nucleons via the unpolarized $p\bar{p}$ process at the $Z$ pole. Besides, the GSI-PANDA experiment [59] will run unpolarized Drell-Yan processes with $p\bar{p}$ colliding at $s = 30 \text{ GeV}^2$, and PAX experiment [60] may preform unpolarized $p\bar{p}$ Drell-Yan process with the fixed target mode at $s = 45 \text{ GeV}^2$. They will provide us an environment to study the Boer-Mulders effect at $J/\psi$ and $\Upsilon$ peaks and to understand the structure of nucleons. All these $p\bar{p}$ Drell-Yan experiments will give us significant promotion in understanding the hadron structure and non-perturbative QCD properties.

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