FlexibleSUSY 2.0: Extensions to investigate the phenomenology of SUSY and non-SUSY models

Peter Athron\textsuperscript{a}, Markus Bach\textsuperscript{b}, Dylan Harries\textsuperscript{c,d}, Thomas Kwasnitza\textsuperscript{b}, Jae-hyeon Park\textsuperscript{e}, Dominik Stöckinger\textsuperscript{b}, Alexander Voigt\textsuperscript{f,*}, Jobst Ziebell\textsuperscript{b}

\textsuperscript{*}ARC Centre of Excellence for Particle Physics at the Terascale, School of Physics, Monash University, Melbourne, Victoria 3800, Australia
\textsuperscript{b}Institut für Kern- und Teilchenphysik, TU Dresden, Zellescher Weg 19, 01069 Dresden, Germany
\textsuperscript{c}ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, The University of Adelaide, Adelaide, South Australia 5005, Australia
\textsuperscript{d}Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University in Prague, V Holešovičkách 2, 180 00 Praha 8, Czech Republic
\textsuperscript{e}Quantum Universe Center, Korea Institute for Advanced Study, 85 Hoejigro Dongdaemun-gu, Seoul 02455, Republic of Korea
\textsuperscript{f}Institute for Theoretical Particle Physics and Cosmology, RWTH Aachen University, 52074 Aachen, Germany

Abstract

We document major new features and improvements of FlexibleSUSY, a Mathematica and C++ package with a dependency on the external package SARAH, that generates fast and precise spectrum generators. The extensions presented here significantly increase the generality and capabilities of the FlexibleSUSY package, which already works with a wide class of models, while maintaining an elegant structure and easy to use interfaces. The FlexibleBSM extension makes it possible to also create spectrum generators for non-supersymmetric extensions of the Standard Model. The FlexibleCPV extension adds the option of complex parameters to the spectrum generators, allowing the study of many interesting models with new sources of CP violation. FlexibleMW computes the decay of the W boson for the generated model and thereby allows FlexibleSUSY to predict the mass of the W boson from the input parameters by using the more precise electroweak input of \( \{G_F, M_Z, \alpha_{em}\} \) instead of \( \{M_W, M_Z, \alpha_{em}\} \). The FlexibleAMU extension provides a calculator of the anomalous magnetic moment of the muon in any model FlexibleSUSY can generate a spectrum for. FlexibleSAS introduces a new solver for the boundary value problem which makes use of semi-analytic expressions for dimensionful parameters to find solutions in models where the classic two-scale solver will not work such as the constrained E6SSM. FlexibleEFTHiggs is a hybrid calculation of the Higgs mass which combines the virtues of both effective field theory calculations and fixed-order calculations. All of these extensions are included in FlexibleSUSY 2.0, which is released simultaneously with this manual.

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*Corresponding author

Email address: Alexander.Voigt@physik.rwth-aachen.de (Alexander Voigt)
New version program summary

Program title: FlexibleSUSY
Licensing provisions: GPLv3
Programming language: C++, Wolfram/Mathematica, FORTRAN, Bourne shell
Journal reference of previous version: Comput.Phys.Commun. 190 (2015) 139-172
Does the new version supersede the previous version?: yes
Reasons for the new version: Program extension including new models, observables and algorithms
Summary of revisions: Extension to non-supersymmetric models (FlexibleBSM), complex parameters (FlexibleCPV), prediction of W boson mass from muon decay (FlexibleMW), calculation of anomalous magnetic moment of the muon (FlexibleAMU), semi-analytic boundary value problem solver (FlexibleSAS), improved hybrid Higgs mass calculation (FlexibleEFTHiggs).
Nature of problem: Determining the mass spectrum, mixings and further observables for an arbitrary extension of the Standard Model, input by the user. The generated code must find simultaneous solutions to constraints that are specified at two or more different renormalization scales, which are connected by renormalization group equations forming a large set of coupled first-order differential equations.
Solution method: Nested iterative algorithm and numerical minimization of the Higgs potential.
Restrictions: The couplings must remain perturbative at all scales between the highest and lowest boundary condition. Tensor-like Lagrangian parameters of rank 3 are currently not supported. The automatic determination of the Standard Model-like gauge and Yukawa couplings is only supported for models that have the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a gauge symmetry group factor. However, due to the modular nature of the generated code, adapting and extending it to overcome restrictions in scope is quite straightforward.
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1. Introduction

Popular and well studied new physics extensions of the Standard Model (SM) are coming under increasing pressure from the searches at the Large Hadron Collider (LHC) as well as other current experiments. At the same time, there remain many outstanding physics problems that are not solved by the SM and require new physics. For example, the origin of dark matter is still unexplained and it requires some new particle to fit the relic density. In addition, the gauge structure and fractional charges of the SM have no explanation, but some observations hint at the possibility of a grand unified theory (GUT) from which these can be derived after the breakdown of the GUT gauge group. Finally, the stability of the weak scale, which is 17 orders of magnitude smaller than the Planck scale, has no explanation within the SM [1–5] and has been the driving motivation for the construction and study of concepts that go beyond the Standard Model (BSM), most notably supersymmetry (SUSY) [6–16] which ensures that the quadratic corrections from fermions and bosons in the loop diagrams cancel at all orders in perturbation theory [17–21].

These issues strongly motivate the development of new ideas and new models. However, in principle there are a huge number of models that can solve some, or all, of these problems and many more that may be motivated by other principles that have not yet gained widespread interest in high energy physics (HEP). Previously, most work has been done on the simplest variants, on scenarios that are easiest to test and on those that individual researchers consider to be the very best motivated models. Examples include the Minimal Supersymmetric Standard Model$^1$ (MSSM), scalar singlet dark matter models (SSDM) [23–25], type II two Higgs doublet models (THDM-II) [26–28], and universal extra dimensions [29]. Now, it is becoming increasingly well motivated to also examine more complicated model variants, explore scenarios that are calculationally difficult or hard to observe, or motivated from an entirely new perspective. Such advanced models can avoid phenomenological or conceptual difficulties of simpler models, and might provide explanations for the lack of experimental evidence for new physics at the LHC.

Faced with this challenge, it is important to reduce the calculational hurdle required to explore a new model and look at its phenomenology as much as possible. This makes it easier for new ideas to get developed and for many more models to be studied together for much more general conclusions.

FlexibleSUSY [30, 31] already made a significant push in this direction, allowing the automatic creation of a spectrum generator for a very wide range of supersymmetric models. FlexibleSUSY uses SARAH [32–36] to obtain Mathematica expressions for model dependent components, the 2-loop renormalization group equations (RGEs), 1-loop self energies, 1-loop tadpoles, mass matrices and electroweak symmetry breaking (EWSB) conditions. FlexibleSUSY then translates these expressions into C++ routines and embeds them inside a code structure for solving the boundary value problem (BVP). It also uses some numerical routines from SOFTSUSY [37, 38] and is heavily unit-tested against the MSSM and Next-to-MSSM versions of SOFTSUSY every night to ensure bugs are avoided in updates.

Spectrum generators determine the pole masses and couplings of a particular model from assumptions about the parameters at some high scale, such as the grand unification scale, or directly from on-shell parameters or running parameters$^2$ given at the new physics scale. This requires computing self energies, tadpole corrections to the EWSB conditions.

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$^1$For a review see Ref. [22].

$^2$Usually defined in the \( \overline{\text{MS}} \) or \( \overline{\text{DR}} \) scheme.
and threshold corrections to SM-like gauge and Yukawa couplings. In the case of high scale assumptions a BVP must be solved, which additionally requires integrating the RGEs for the model. These codes are essential for testing hypotheses about the parameters of the model, and determining if they lead to phenomenologically viable masses.

Spectrum generators are widely used in studies of supersymmetry, and for the MSSM and the Next-to-MSSM\(^3\) (NMSSM) there are a number of them that solve the BVP for particular choices of breaking mechanism inspired boundary conditions (SOFTSUSY [37, 38, 41–45], SuSpect [46], SPheNo [47, 48], ISASUSY [49], NMSPEC [50–52] and SuSeFLAV [53]) and several more that start from \(\overline{\text{DR}}\) parameters at the SUSY scale or on-shell inputs (FeynHiggs [54–61], NMSSMCalc [62, 63] and CPsuperH [64–66]). This means that the parameter space of the MSSM (and to a lesser extent the NMSSM) has been extensively explored in a fast and reliable way.

However, for other models few, if any, public software packages exist. Until recently this meant that one would have to spend a very long time writing and testing code in order to explore the parameter space of the model and do phenomenological investigations, and even then bugs are more likely if one uses a private tool rather than a well tested public one. With the recent development of FlexibleSUSY and generated modules for SPheNo from SARAH [30, 33–36, 47, 48, 67] it is now possible to obtain spectrum generators in a much wider range of models. FlexibleSUSY can generate spectrum generators of various kinds, with boundary conditions at some high scale or purely with low-energy input. This push towards calculators that are not model specific has also happened for other major types of analyses. For example, one may create new models in micrOMEGAS [68–74] using CALCHEP [75, 76] and LANHEP [77–81] to study the relic density and direct and indirect detection of dark matter. FeynRules [82, 83] can also be used to generate the Feynman rules after inputting the Lagrangian, in a similar manner to SARAH and LANHEP. The output of these codes can be used in matrix element generators, such as CALCHEP, MadGraph [84–88], WHIZARD [89, 90], SHERPA [91–93], FeynArts [94–98] and HELAC [99, 100], with showering and hadronization handled by the event generators HERWIG [101, 102], PYTHIA [103–105] and SHERPA. Results from collider experiments can be applied to different BSM models using re-interpretation tools, with some examples being HiggsBounds [106–109], HiggsSignals [110], CheckMate [111, 112], SModels [113–115], Fastlim [116], MadAnalysis [117–119], and the native re-interpretation of ColliderBit [120]. Finally, very recently GAMBIT [121] has been released which uses auto-generated code from FlexibleSUSY [122] and micrOMEGAS [123], along with many other packages, to perform global fits of a user implemented BSM model.

Indeed FlexibleSUSY has been used extensively to study new physics models, on its own or in combination with some of the codes mentioned above, in a large number of physics studies, see, e.g., Refs. [31, 122, 124–143]. Nonetheless, in the first release of FlexibleSUSY there were still a number of limitations that restricted the models and phenomenology that could be studied and the precision of the calculations.

In this paper, we document extensions to FlexibleSUSY, all now available in version 2.0, that make a substantial push further in expanding the number of models that can be explored and the observables that can be calculated within them. Now the list of models that can be investigated with FlexibleSUSY 2.0 also includes non-supersymmetric models (FlexibleBSM), those with complex parameters (FlexibleCPV) and, with the new BVP solver FlexibleSAS, constrained versions of certain non-minimal SUSY models like the

\(^3\)For a review of the model see Refs. [39, 40].
NMSSM [39, 40, 144–148] and the Exceptional Supersymmetric Standard Model (E6SSM) [149–153] that cannot be solved using the two-scale BVP solver [154] approach implemented in public spectrum generators. In addition, FlexibleSUSY 2.0 extends its repertoire of calculations. The user may now calculate the mass of the W boson as a prediction of the model (FlexibleMW) with the Fermi constant treated as an input, as well as the anomalous magnetic moment of the muon (FlexibleAMU) and some electric dipole moments as an application of FlexibleCPV. Furthermore, we document and update our FlexibleEFT-Higgs calculation of the Higgs pole mass, the physics of which has been discussed in Ref. [31]. Besides these new physics features, we also document the new Mathematica interface.

In Section 2 we give a quick start guide explaining how to download and compile the code using basic options. In Section 3 we present the Mathematica interface of FlexibleSUSY’s spectrum generators and in Section 4 we describe the new model file extensions. The following Sections 5–10 document each of the major extensions to FlexibleSUSY in a modular fashion so that readers can easily skip to a particular section. These extensions are as follows. FlexibleBSM, documented in Section 5, allows FlexibleSUSY to work in non-SUSY models as well as SUSY models; the calculation of the anomalous magnetic moment of the muon is given in Section 6; FlexibleCPV, described in Section 7, introduces complex parameters to FlexibleSUSY so that CP violating effects may be considered; FlexibleMW calculates the muon decay, allowing FlexibleSUSY to predict $M_W$, and is documented in Section 8; a new BVP solver FlexibleSAS is introduced in Section 9 and finally the hybrid Higgs mass calculation, FlexibleEFTHiggs, which combines the benefits of both effective field theory and fixed-order calculations is documented in Section 10. After this, we briefly summarize remaining limitations of the code in Section 11 before adding concluding remarks in Section 12. In the appendices we provide a reference of all input parameters and configuration options of FlexibleSUSY 2.0 as well as new models.

2. Quick start

2.1. Requirements

The build process of a custom spectrum generator using FlexibleSUSY requires the following:

- Mathematica, version 7 or higher
- SARAH, version 4.11.0 or higher [http://sarah.hepforge.org]
- C++11 compatible compiler (g++ 4.8.5 or higher, clang++ 3.8 or higher, icpc 15.0 or higher)
- FORTRAN compiler (gfortran, ifort etc.)
- Eigen library, version 3.1 or higher [http://eigen.tuxfamily.org]
- Boost library, version 1.37.0 or higher [http://www.boost.org]
- GNU scientific library [http://www.gnu.org/software/gsl]

Optional:

- an implementation of LAPACK [http://www.netlib.org/lapack] such as ATLAS [http://math-atlas.sourceforge.net] or Intel Math Kernel Library [http://software.intel.com/intel-mkl]
2.2. Downloading FlexibleSUSY and generating a first spectrum generator

FlexibleSUSY is available as gzipped tarball on [http://flexiblesusy.hepforge.org](http://flexiblesusy.hepforge.org) or [https://github.com/FlexibleSUSY/FlexibleSUSY](https://github.com/FlexibleSUSY/FlexibleSUSY) under the version control system git. To download and install FlexibleSUSY 2.0 as a gzipped tarball run at the command line:

```bash
$ wget \
  https://www.hepforge.org/archive/flexiblesusy/FlexibleSUSY-2.0.1.tar.gz
$ tar -xf FlexibleSUSY-2.0.1.tar.gz
$ cd FlexibleSUSY-2.0.1
```

FlexibleSUSY 2.0 is distributed with a huge selection of predefined “models”, including several MSSM and NMSSM scenarios such as the CMSSM (called CMSSM), high-scale SUSY and split-SUSY (called HSSUSY and SplitMSSM; these models have been created for Ref. [131]), the semi-constrained and fully constrained NMSSM (NMSSM, CNMSSM). The distribution also contains BSM models such as the $R$-symmetric MSSM (MRSSM; for a definition of the model see Ref. [155]), the NUHM $E_6$SSM ($E6SSM$, see [149–153, 156]) and the two-Higgs doublet model type II (THDMII). See the contents of model_files/ for all predefined model files.

A spectrum generator for any of these models can be built with just three commands. For example, the CMSSM spectrum generator can be created by running the following shell commands:

```bash
$ ./createmodel --name=CMSSM
$ ./configure --with-models=CMSSM
$ make
```

The `createmodel` command creates the model directory `models/CMSSM/` where the code will be generated and adds the CMSSM model file and an example SLHA input file to it. The `configure` script sets up the `Makefile`, checking the system requirements and dependencies. For more options see `./configure --help`. The last command creates the code for the spectrum generator of this model and compiles it. This generated spectrum generator can then be run from the command line as:

```bash
$ cd models/CMSSM
$ ./run_CMSSM.x --slha-input-file=LesHouches.in.CMSSM \
  --slha-output-file=LesHouches.out.CMSSM
```

The spectrum generator reads the CMSSM input parameters from the SLHA input file `LesHouches.in.CMSSM` and first solves a BVP to find a set of running parameters at the SUSY scale that are consistent with all boundary conditions specified in the model file, and then calculates the pole masses, mixing matrices and potentially further observables. The mass spectrum etc. obtained in this way is output in SLHA format [157, 158] as `LesHouches.out.CMSSM`. See `./run_CMSSM.x --help` for more options.

FlexibleSUSY also provides a Mathematica interface, introduced in version 1.7.0, to call the generated spectrum generators. For each spectrum generator, an example Mathematica script named `models/<model>/run_<model>.m` is created for illustration. For example, the CMSSM spectrum generator can be called from within Mathematica like this:
Get["models/CMSSM/CMSSM_librarylink.m"];  

handle = FSCMSSMOpenHandle[
  fsModelParameters -> {
    m0 -> 125,
    m12 -> 500,
    TanBeta -> 10,
    SignMu -> 1,
    Azero -> 0
  }
];

spectrum = FSCMSSMCalculateSpectrum[handle];
FSCMSSMCloseHandle[handle];
Print[spectrum];

Execute ?FSCMSSMOpenHandle for a list of all allowed options and input parameters. In the example above, the spectrum variable contains the mass spectrum and the running parameters in the form of a list of replacement rules:

```
{CMSSM -> {
  Pole[M[Glu]] -> 1147.35,
  Pole[M[Sd]] -> {957.993, 997.56, 1000.49, 1000.5, 1045.93, 1045.94},
  Pole[M[SV]] -> {350.753, 351.913, 351.917},
  Pole[M[Su]] -> {796.653, 1002.67, 1003.96, 1005.06, 1043.07, 1043.07},
  Pole[M[Se]] -> {222.916, 229.983, 230.008, 360.842, 360.846, 361.978},
  Pole[M[hh]] -> {114.836, 713.119},
  Pole[M[Ah]] -> {88.5864, 712.848},
  Pole[M[Hpm]] -> {77.2642, 717.628},
  Pole[M[Chi]] -> {204.054, 385.012, 629.649, 643.612},
  Pole[M[Cha]] -> {385.017, 643.924},
  Pole[M[VWm]] -> 80.3935, ...}
}
```

More details about the Mathematica interface as well as a neat example of running HSSUSY through it can be found in Sections 3 and 5.5, respectively.

2.3. Spectrum generators for alternative models

If the user instead wants to create a spectrum generator for a model for which there is no pre-existing model file distributed in FlexibleSUSY, then the model file can be written. Before a FlexibleSUSY model file can be written for the spectrum generator, there must exist SARAH model files, which FlexibleSUSY uses to obtain model dependent information. SARAH also comes with many pre-defined models, but if an appropriate model is not available, the users may create their own SARAH model files and add them to the directory sarah/<modelname>/ in FlexibleSUSY. For the writing of a SARAH model file we refer the reader to the extensive SARAH documentation, for example Refs. [30, 33–36, 67].

Creating a new FlexibleSUSY model file is straightforward. Full details are given in the original FlexibleSUSY manual Ref. [30]. Here we just repeat a basic example in the context of the NMSSM that illustrates the main points: The semi-constrained NMSSM (NMSSM) distributed in FlexibleSUSY has all the soft-breaking trilinear scalar couplings set to a unified A0 at the GUT scale. However, often Aλ and Ak are taken to be non-universal in semi-constrained variants of the NMSSM since the non-universality of the soft
singlet mass already violates the standard universality assumptions of constrained models.\(^4\)

To allow separate values for \(A_\lambda\) and \(A_\kappa\) at the GUT scale the \texttt{FlexibleSUSY} model file \texttt{model_files/NMSSM/FlexibleSUSY.m.in} should be changed from

```plaintext
EXTPAR = { {61, LambdaInput} };
HighScaleInput = {
  ... 
  {T[Kappa][Kappa], Azero[Kappa]},
  {T[Lambda][Lambda], Azero LambdaInput}
  ... 
};
```

into

```plaintext
EXTPAR = { {61, LambdaInput},
            {63, ALambdaInput},
            {64, AKappaInput} };
HighScaleInput = {
  ... 
  {T[Kappa][Kappa], AKappaInput[Kappa]},
  {T[Lambda][Lambda], ALambdaInput LambdaInput},
  ... 
};
```

The GUT scale values of \(A_\lambda\) and \(A_\kappa\) can then be specified in the SLHA input file in the \texttt{EXTPAR} block by entries 63 and 64,

```plaintext
Block EXTPAR
  61 0.1 # LambdaInput
  63 -100 # ALambdaInput
  64 -300 # AKappaInput
```

3. Mathematica interface

The spectrum generators created with \texttt{FlexibleSUSY} can be called from within \texttt{Mathematica}. To do that, first the spectrum generator must be built, as described in Section 2.2. Afterwards, the provided \texttt{Mathematica} interface functions for the model must be loaded. For a given model \(<\text{model}>\) this is done by including the following file in the \texttt{Mathematica} session:

```plaintext
Get["models/<model>/<model>_librarylink.m"];
```

This script loads the library \texttt{models/<model>/<model>_librarylink.so} into the \texttt{Mathematica} session (assuming the user is in the \texttt{FlexibleSUSY/} directory). Afterwards, the \texttt{Mathematica} interface functions listed in Table 1 are available.

To run the spectrum generator for a given parameter point, a handle to that point must be created first, using the \texttt{FS<\text{model}>OpenHandle[...]} function. The returned handle

\(^4\)See Section 9 for a new approach that allows the fully constrained NMSSM to be solved.
Function | Description
---|---
FS<model>OpenHandle[...] | Takes all model input parameters as argument and returns a “handle” (a reference) to the given parameter point, the associated mass spectrum and observables.

FS<model>CloseHandle[handle] | Releases the resources associated to a given handle.

FS<model>CalculateSpectrum[handle] | Calculates the mass spectrum for a given handle.

FS<model>CalculateObservables[handle] | Calculates the observables for a given handle.

FS<model>ToSLHA[handle] | Returns a string containing the mass spectrum and observables associated to a given handle in SLHA format.

FS<model>Set[handle, ...] | Changes the input parameters associated to a given handle.

FS<model>GetSettings[handle] | Returns the spectrum generator settings (precision goal, loop orders, etc.) associated to a given handle.

FS<model>GetSMInputParameters[handle] | Returns the SM input parameters associated to a given handle.

FS<model>GetInputParameters[handle] | Returns the model-specific input parameters associated to a given handle.

FS<model>GetProblems[handle] | Returns a list of problems that occurred when calculating the spectrum.

FS<model>GetWarnings[handle] | Returns a list of warnings that occurred when calculating the spectrum.

Table 1: Mathematica interface functions provided for a FlexibleSUSY model with the name <model>.

represents a reference to the given parameter point, the associated mass spectrum and observables. The concept of handles allows the user to calculate mass spectra for different parameter points in parallel using multiple Mathematica kernels: Each kernel can open a handle to a separate parameter point, calculate the mass spectrum and finally close the handle. In this way there is no ambiguity in the parameter point used by each kernel. Example 6 in Section 5.5 illustrates the usage of handles by performing a parallel scan over the MSSM parameter space with HSSUSY. In the most general form, the FS<model>OpenHandle[...] function can take the following three arguments:

```mathematica
handle = FS<model>OpenHandle[
    fsSettings -> {...},
    fsSMParameters -> {...},
    fsModelParameters -> {...}
]
```

The fsSettings symbol can be used to set the spectrum generator options. All possible options are listed in Table B.13 in Appendix B. The fsSMParameters symbol can be used to set the SM input parameters. The possible SM input parameters are listed in Table A.12 in Appendix A. The fsModelParameters symbol can be used to set the BSM model-specific
input parameters. Unspecified model input parameters are set to zero by default. The names of the model input parameters are identical to the ones specified in the MINPAR, IMMINPAR, EXTPAR, IMEXTPAR and FSAuxiliaryParameterInfo variables in the FlexibleSUSY model file. The settings, the SM and the BSM input parameters associated to a given handle can be obtained using the FS<model>GetSettings[], FS<model>GetSMInputParameters[] and FS<model>GetInputParameters[] functions, respectively. The opened handle can then be used to calculate the mass spectrum and the observables:

```
spectrum = FS<model>CalculateSpectrum[handle];
observables = FS<model>CalculateObservables[handle];
FS<model>CloseHandle[handle];
```

Finally, the handle should be closed to release the associated resources by calling the function FS<model>CloseHandle[handle].

**Example 1**

In the CMSSM, the BSM model-specific input parameters are named as m0, m12, TanBeta, SignMu and Azero, see the CMSSM model file provided with FlexibleSUSY 2.0. Thus, an example Mathematica session for the CMSSM could look like:

```
Get["models/CMSSM/CMSSM_librarylink.m"];
handle = FSCMSSMOpenHandle[
 fsSettings -> {
   poleMassLoopOrder -> 2,
   ewsbLoopOrder -> 2,
   thresholdCorrectionsLoopOrder -> 2,
   betaFunctionLoopOrder -> 3
 },
 fsSMParameters -> {
   Mt -> 173.34,
   alphaSMZ -> 0.1184
 },
 fsModelParameters -> {
   m0 -> 125,
   m12 -> 500,
   TanBeta -> 10,
   SignMu -> 1,
   Azero -> 0
 }
];
spectrum = FSCMSSMCalculateSpectrum[handle];
observables = FSCMSSMCalculateObservables[handle];
FSCMSSMCloseHandle[handle];
```

The output of FS<model>CalculateSpectrum[handle] is a list that contains the pole mass spectrum as well as the running masses and parameters at the chosen output scale. The running parameters are named as defined in the SARAH model. For example, g1, g2, g3 usually denote the running gauge couplings and Yu, Yd, Ye the running Yukawa couplings. The running masses are denoted as M(pφ), where φ is the name of the particle as defined
in the **SARAH** model file. All running parameters and masses are given at the parameter output scale, **SCALE**. The pole masses and mixing matrices carry the additional **Pole[]** head. For example, **Pole[M[hh]]** usually denotes the pole mass(es) of the Higgs boson(s).

In the **Mathematica** output, the running parameters, the masses and mixing matrices are defined in the **SARAH** convention, *not* in the SLHA convention. This means in particular that the Yukawa matrices, the soft-breaking squark mass matrices and the soft-breaking trilinear couplings are *not* defined in the (super)-CKM basis. In the **Mathematica** output, the particle masses are always non-negative and mixing matrices are in general complex.

**Example 2**

In the CMSSM, the output of **FSCMSSMCalculateSpectrum[handle]** may look like (skipping some entries for brevity):

```mathematica
{CMSSM -> {1116.4857717819132,
M[Glue] -> {929.5770939936384, 963.6803089181217, 965.7750791635142, 965.7786645820956, 1010.5301444258299, 1010.5317308607175},
M[Sd] -> {770.292836944288, 969.5389940603632, 969.5446720936367, 975.4073015489967, 1007.5435846398124, 1007.544071514123},
M[Su] -> {770.2929836944288, 969.5389940603632, 969.5446720936367, 975.4073015489967, 1007.5435846398124, 1007.544071514123},
M[Se] -> {219.5493971980814, 226.4523005886065, 226.4768407746941, 356.24526186304, 356.25016613376, 357.557672510361},
M[hh] -> {88.16467333922309, 726.260341728729, 726.022988925828, 730.2533306006959, 730.2533306006959, 730.2533306006959},
M[Ah] -> {90.09835220027803, 726.022988925828},
M[Hpm] -> {73.84914789145176, 73.84914789145176, 73.84914789145176, 73.84914789145176, 73.84914789145176, 73.84914789145176},
M[Chi] -> {207.1963755793879, 375.7416364302936, 627.5178023483583, 641.667673271736},
M[Cha] -> {375.56991892585705, 641.3578484531205, ...
ZH -> {{0.10592570722508611, 0.9943740465985952}, {0.9943740465985952, -0.10592570722508611}, ...
Pole[M[Glue]] -> 1147.3536227374905,
Pole[M[Sd]] -> {957.9934299811302, 997.5603867095314, 1000.4932601265115, 1000.496819618583, 1045.9354429433467, 1045.9372472457565},
Pole[M[Su]] -> {796.6536193987982, 1003.9614916607435, 1005.0642702137084, 1043.0672831732345, 1043.067920812505},
Pole[M[Se]] -> {222.90126096766593, 229.9832415178622, 230.00840279144913, 360.84198174065307, 360.8462569384804, 361.9798562942742},
Pole[M[hh]] -> {114.835585197574276, 713.1187313487922},
Pole[M[Ah]] -> {88.5864464630424699},
Pole[M[Hpm]] -> {77.26414997655887, 717.6270882152121},
Pole[M[Chi]] -> {204.05370940499517, 385.0116889026499, 629.6500252267041, 643.6127224069093},
Pole[M[Cha]] -> {385.0164604772902, 643.924798526633},
Pole[ZH] -> {{0.1066307364997843, 0.9942986905520461}, {0.9942986905520461, -0.1066307364997843}, ...
Yd -> {{0.0001399914166255535, 0., 0.},
```
The function `FS<model>CalculateObservables[handle]` returns the observables for a given handle in the form of a list of replacement rules.

**Example 3**

In the CMSSM, the output of `FSCKSSMCalculateObservables[handle]` might look like

```plaintext
{0., 0.0030650771019348657, 0.},
{0., 0.1317656023934078},
Ye -> {{0.000289546231068183, 0., 0.},
{0., 0.00598698727723582, 0.},
{0., 0.100693179859596906}},
Yu -> {{7.267255094462218*^-6, 0., 0.},
{0., 0.0033082824973678956, 0.},
{0., 0.8606532901364391}},
\[Mu\] -> 624.160899893032,
g1 -> 0.4679063156949638,
g2 -> 0.6430285180350706,
g3 -> 1.0655340318624051,
vd -> 25.099612589273388,
vu -> 242.8296409176676,
T[Yd] -> {{-0.19442921534444055, 0., 0.},
{0., -4.256965123994014, 0.},
{0., 0., -171.13078241755716}},
T[Ye] -> {{-0.008660431147847696, 0., 0.},
{0., -1.790668166947902, 0.},
{0., 0., -29.953008563262035}},
T[Yu] -> {{-0.008251796354121698, 0., 0.},
{0., -3.7564603713866354, 0.},
{0., 0., -755.7309107649228}},
B[\[Mu\]] -> 53907.68839928095,
mp2 -> {{1.0178399270038805*^-6, 0., 0.},
{0., 1.0178348758707164*^-6, 0.},
{0., 0., 865711.3590482193}},
ml2 -> {{124853.43144557778, 0., 0.},
{0., 124850.91737364854, 0.},
{0., 0., 124143.31667980803}},
mHd2 -> 109509.1756551005,
mHu2 -> -377534.5544643501,
md2 -> {{932089.8366766714, 0., 0.},
{0., 932084.7501918572, 0.},
{0., 0., 923097.9700749611}},
mu2 -> {{941294.0634724408, 0., 0.},
{0., 941288.914981822, 0.},
{0., 0., 639354.6597906639}},
me2 -> {{49375.97115879859, 0., 0.},
{0., 49370.8410366076, 0.},
{0., 0., 47926.68179837005}},
MassB -> 209.15138268684439,
MassWB -> 387.9365053016521,
MassG -> 1116.4857717819132,
SCALE -> 866.8060753250803
} 
```
The symbol $a_{\mu}$ represents the anomalous magnetic moment of the muon, $a_\mu$, calculated as described in Section 6. The symbols $CpHiggsPhotonPhoton$, $CpHiggsGluonGluon$, $CpPseudoScalarPhotonPhoton$ and $CpPseudoScalarGluonGluon$ denote the effective couplings of the physical $CP$-even and $CP$-odd Higgs boson(s) to two photons and gluons, respectively, as described in Ref. [132]. Note that if the $CP$-even or $CP$-odd Higgs states are multiplets, as is the case for the $CP$-even Higgs in the MSSM, for example, the relevant couplings are calculated for all members of the multiplet and the result is returned as a list.

The calculated spectrum can be printed in an SLHA-compatible format using the FS<model>ToSLHA[] function.

Example 4

For the CMSSM, an example output of FSCMSSMToSLHA[handle] could look like:

```
Block SPINFO
  1 FlexibleSUSY
  2 2.0.1
  5 CMSSM
  9 4.11.0
Block MASS
  1000021  1.14735362E+03 # Glu
     24  8.03935152E+01 # VWm
  1000024  3.85016460E+02 # Cha(1)
  1000037  6.43924799E+02 # Cha(2)
     25  1.14835832E+02 # hh(1)
     35  7.13118731E+02 # hh(2)
...
```

In Section 5.5 several examples can be found that illustrate how to perform parameter scans and uncertainty estimates using the Mathematica interface of FlexibleSUSY’s spectrum generators.
4. FlexibleSUSY model file extensions

4.1. Model-specific higher-order contributions

To improve the accuracy in some specific models, FlexibleSUSY provides a few model file switches to enable further higher-order contributions in the calculation of the running parameters, the \( \beta \) functions or the Higgs pole mass. Table 2 lists all switches available in FlexibleSUSY 2.0, which are explained in the following subsections. These switches can usually be enabled in many models, providing that the corresponding requirements are fulfilled. However, the user should be aware that contributions may be missing if the switches are enabled in models beyond their scope of application.

4.1.1. SM-specific higher-order contributions

2-loop and 3-loop contributions to the SM Higgs mass. In the SM, 2-loop and leading 3-loop contributions to the Higgs pole mass are known in the \( \overline{\text{MS}} \) scheme [159–161]. In FlexibleSUSY the 2-loop contributions of \( O(\alpha_t^2 + \alpha_t \alpha_s) \) [160, 161] and the 3-loop \( O(\alpha_t^3 + \alpha_t^2 \alpha_s + \alpha_t \alpha_s^2) \) contributions [161] can be taken into account to calculate the SM-like Higgs pole mass for non-SUSY models with only one Higgs. In order to enable these loop contributions, the following switches must be set in the model file:

\[
\begin{align*}
\text{UseHiggs2LoopSM} & = \text{True}; \\
\text{UseHiggs3LoopSM} & = \text{True};
\end{align*}
\]

The 2-loop and 3-loop contributions enter the mass of the Higgs boson as

\[
M_h^2 = m_h^2 + (\Delta m_h^2)_{1L}(p^2) + (\Delta m_h^2)_{2L}(p^2) + (\Delta m_h^2)_{3L},
\]

where \( m_h^2 \) and \( (\Delta m_h^2)_{1L}(p^2) \) correspond to the tree-level and a 1-loop expression, respectively. The enabled 2-loop SM contributions of \( O(\alpha_t^2 + \alpha_t \alpha_s) \) to the Higgs mass read

\[
\begin{align*}
(\Delta m_h^2)_{2L}(p^2) &= (\Delta m_h^2)_{2L}^{(\alpha_t^2)} + (\Delta m_h^2)_{2L}^{(\alpha_t \alpha_s)}(p^2), \\
(\Delta m_h^2)_{2L}^{(\alpha_t^2)} &= \frac{2t}{(4\pi)^4} \left[ -3y_t^4 \left( 3 \log^2(t) - 7 \log(t) + 2 + \frac{\pi^2}{3} \right) \right], \\
(\Delta m_h^2)_{2L}^{(\alpha_t \alpha_s)}(p^2) &= \frac{g_3^2 y_t^2}{(4\pi)^4} \left[ \frac{37}{3} p^2 - \frac{122p^4}{135} - 4(5p^2 - 8t) \log(t) - 12(p^2 - 8t) \log^2(t) \right], \\
&\quad p^2 = 0, \quad \frac{2t}{(4\pi)^4} \left[ 16g_3^2 y_t^2 \left( 3 \log^2(t) + \log(t) \right) \right].
\end{align*}
\]

In Eqs. (3) ff. the abbreviations \( t \equiv m_t^2 \) and \( \log(t) \equiv \log(t/Q^2) \) have been used for brevity, analogous to the notation of Ref. [161], where \( m_t \) is the \( \overline{\text{MS}} \) top mass in the SM. By performing a momentum iteration in the computation of the Higgs pole mass, the momentum entering the 1-loop self energy in Eq. (1) consists of a tree-level part and a loop correction. In combination with Eq. (3), the latter yields the complete Higgs mass contribution at \( O(\alpha_t^2 t) \) which is identical to the corresponding correction given in Ref. [159]. The 2-loop self energy at \( O(\alpha_t \alpha_s) \) [161] together with the occurring integral functions from Ref. [162] have been evaluated for a small external momentum argument. Neglecting higher orders, the expansion in powers of the momentum over the \( \overline{\text{MS}} \) top mass up to \( O(p^4/t) \) results in the Eq. (4). Note that the momentum dependence is included in the 2-loop expression.
| Symbol                     | Default value | Allowed values | Description                                                                 |
|----------------------------|---------------|----------------|-----------------------------------------------------------------------------|
| UseHiggs2LoopSM           | False         | True or False  | 2-loop contributions $O(\alpha_t^2 + \alpha_t \alpha_s)$ to $M_h$ in the SM |
| UseHiggs3LoopSM           | False         | True or False  | 3-loop contributions $O(\alpha_t^3 + \alpha_t^2 \alpha_s + \alpha_t \alpha_s)$ to $M_h$ in the SM |
| UseSM3LoopRGEs            | False         | True or False  | 3-loop RGEs in the SM                                                       |
| UseYukawa3LoopQCD         | Automatic     | True or False  | 2-loop and 3-loop QCD contributions $O(\alpha_s^2 + \alpha_s^3)$ to the MS $y_t$ in the SM |
| UseSMAlphaS3Loop          | False         | True or False  | 2-loop and 3-loop QCD threshold corrections $O(\alpha_s^2 + \alpha_s^3)$ to the MS $\alpha_s$ in the SM |

| Symbol                     | Default value | Allowed values | Description                                                                 |
|----------------------------|---------------|----------------|-----------------------------------------------------------------------------|
| UseHiggs2LoopMSSM         | False         | True or False  | 2-loop contributions $O((\alpha_t + \alpha_b)\alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_s^2)$ to $M_h$, $M_H$ and $M_A$ in the MSSM |
| UseHiggs3LoopMSSM         | False         | True or False  | 3-loop contributions $O(\alpha_t \alpha_s^2 + \alpha_b \alpha_s^2)$ to $M_h$ in the MSSM (requires Himalaya) |
| UseMSSM3LoopRGEs          | False         | True or False  | 3-loop RGEs in the MSSM                                                     |
| UseMSSMYukawa2Loop        | False         | True or False  | 2-loop SUSY-QCD contribution $O(\alpha_s^2)$ to the DR $y_b$ and $y_t$ in the MSSM |
| UseMSSMAlphaS2Loop        | False         | True or False  | 2-loop SUSY-QCD contribution $O(\alpha_s^2 + \alpha_s \alpha_t + \alpha_s \alpha_b)$ to the DR $\alpha_s$ in the MSSM |

| Symbol                     | Default value | Allowed values | Description                                                                 |
|----------------------------|---------------|----------------|-----------------------------------------------------------------------------|
| UseHiggs2LoopNMSSM        | False         | True or False  | 2-loop contributions $O((\alpha_t + \alpha_b)\alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_s^2)$ to $M_h$, $M_A$, in the NMSSM |

**Table 2: FlexibleSUSY model file switches to enable/disable model-specific higher-order contributions in the SM, MSSM, NMSSM and split-MSSM.**

in order to generate implicit 3-loop $O(\alpha_t^2 \alpha_s)$ terms. The explicit 3-loop effective potential contributions of $O(\alpha_t^3 + \alpha_t^2 \alpha_s + \alpha_t \alpha_s^2)$ to the Higgs mass included by FlexibleSUSY read

\[
(\Delta m_h^2)_{3L} = \frac{m_t^2}{(4\pi)^6} \left[ g_3^4 y_t^2 \left( 248.122 + 839.197 \log(t) + 160 \log^2(t) - 736 \log^3(t) \right) \right]
\]

18
where $h \equiv m_h^2$ is the squared $\overline{\text{MS}}$ Higgs mass in the SM. Note that the 3-loop Higgs mass calculation in the $\overline{\text{MS}}$ scheme in FlexibleSUSY is only complete at $O(\alpha_t \alpha_s^2)$. The 3-loop contributions of $O(\alpha_t^3 + \alpha_s^3)$ are currently incomplete because they would require the $\overline{\text{MS}}$ top Yukawa coupling to be determined from the top pole mass at the 2-loop $O(\alpha_t^2 + \alpha_t \alpha_s)$. However, these 2-loop contributions to the top Yukawa coupling are currently not available in FlexibleSUSY. Furthermore, the proper inclusion of corrections to the Higgs pole mass at $O(\alpha_t^3)$ would require the extension of Eq. (3) by momentum dependent $O(\alpha_t^2)$ terms. Alternatively, the evaluation of the Higgs mass at the renormalization scale $Q^2 = t$ implies that the neglected 3-loop contributions at $O(\alpha_t^2 t)$ vanish \[161\].

Likewise, Eq. (3) neglects contributions of $O(\alpha_t^2 h)$ which are subdominant in comparison to the implemented $O(\alpha_t^2 t)$ corrections. In contrast, the QCD corrections to the pole mass at 2-loop level, Eq. (4), involve terms proportional to the quartic Higgs coupling $\lambda$ up to $O(\alpha_t \alpha_s^2)$, which are not neglected here.

The 2-loop and 3-loop Higgs mass contributions from above can be enabled at runtime by setting the following flags in the SLHA input file:

| Block FlexibleSUSY | 4 3 | # pole mass loop order |
|--------------------|-----|------------------------|
|                    | 5 3 | # EWSB loop order |
|                    | 7 2 | # threshold corrections loop order |
|                    | 8 1 | # Higgs 2-loop corrections $O(\alpha_t \alpha_s)$ |
|                    | 10 1 | # Higgs 2-loop corrections $O(\alpha_t^2)$ |
|                    | 24 | 122111221 | # individual threshold correction loop orders |
|                    | 26 1 | # Higgs 3-loop corrections $O(\alpha_t \alpha_s^2)$ |
|                    | 28 1 | # Higgs 3-loop corrections $O(\alpha_t^2 \alpha_s)$ |
|                    | 29 1 | # Higgs 3-loop corrections $O(\alpha_t^3)$ |

In FlexibleSUSY’s Mathematica interface, the above SLHA configuration options correspond to

```mathematica
handle = FS[model>OpenHandle[
   fsSettings -> {poleMassLoopOrder -> 3,
   ewsbLoopOrder -> 3,
   thresholdCorrectionsLoopOrder -> 2,
   higgs2LoopCorrectionAtAs -> 1,
   higgs2LoopCorrectionAtAt -> 1,
   thresholdCorrections -> 122111221,
   higgs3LoopCorrectionAtAsAs -> 1,
   higgs3LoopCorrectionAtAtAs -> 1,
   higgs3LoopCorrectionAtAtAt -> 1}]
```

\(^5\)Note that terms of $O(\alpha_t^3 h)$ are neglected here and in Ref. \[161\].
See Appendix B for a list and description of all of FlexibleSUSY’s configuration options. In Figure 1 the impact of the leading 3-loop Standard Model corrections of $O(\alpha_t^3 + \alpha_t^2 \alpha_s + \alpha_t \alpha_s^2)$ on the light CP-even Higgs pole mass in the MSSM in the pure effective field theory (EFT) calculation of HSSUSY is shown with the red dashed-dotted line. The line shows the predicted 3-loop Higgs mass relative to the one calculated at the 2-loop level as a function of the SUSY scale $M_S$, by taking into account the 1-loop threshold correction for $\alpha_{s}^{\text{SM}}(M_Z)$ and the 2-loop QCD correction to $\frac{g_t^2}{y_t^2}(M_Z)$. We find that the explicit 3-loop Standard Model contributions to the Higgs mass lead to a small positive shift by around 30 MeV.

**Figure 1**: Effect of different SM corrections on the Higgs pole mass in the pure EFT calculation of HSSUSY for $\tan \beta = 5$ and $X_t = \sqrt{5} M_S$ as a function of the SUSY scale $M_S$. The orange band marks the region where HSSUSY predicts a Higgs mass compatible with the experimental value of $(125.09 \pm 0.32)$ GeV [163].

3-loop renormalization group running in the SM. In the SM, the 3-loop $\beta$ functions are known [164–168]. FlexibleSUSY allows the user to take these 3-loop $\beta$ functions into account in the running of the $\overline{\text{MS}}$ SM parameters by setting the following switch in the model file:

```
UseSM3LoopRGEs = True;
```

The expression for the 3-loop $\beta$ function of the $\mu^2$ parameter in the SM has been extracted from the file `smh3l.m` available on the arXiv page of Ref. [168]. The remaining 3-loop $\beta$ functions have been kindly provided by the authors of SusyHD 1.0.1 [160]. To use 3-loop running at runtime, the following flag should also be set in the SLHA input file:

```
Block FlexibleSUSY
   6 3 # beta-functions loop order
```

In FlexibleSUSY’s Mathematica interface, the above SLHA configuration corresponds to

```
handle = FS<model>OpenHandle[
   fsSettings -> {betaFunctionLoopOrder -> 3}
   ...
];
```
3-loop QCD corrections to the $\overline{\text{MS}}$ top Yukawa coupling in the SM. The pure SM QCD contribution to the $\overline{\text{MS}}$ top Yukawa coupling is known up to the 3-loop level of $O(\alpha_s^3)$ [169, 170]. This 3-loop QCD expression can be taken into account in FlexibleSUSY to extract the $\overline{\text{MS}}$ top Yukawa coupling in the SM from the top pole mass $M_t$ by setting the following switch in the model file:

```
UseYukawa3LoopQCD = True;
```

By default, `UseYukawa3LoopQCD` is set to `Automatic`, which means that the 3-loop QCD contribution to the top Yukawa coupling is taken into account only if the model is $\overline{\text{MS}}$ renormalized. To take this 3-loop correction into account at runtime, the following flags should be set in the SLHA input file:

```
Block FlexibleSUSY
  7 3 # threshold corrections loop order
  24 12311221 # individual threshold correction loop orders
```

In `FlexibleSUSY`’s `Mathematica` interface, the above SLHA configuration options correspond to

```
handle = FS<model>OpenHandle[
  fsSettings -> {thresholdCorrectionsLoopOrder -> 3,
    thresholdCorrections -> 12311221}
  ];
```

The 3-loop QCD contribution is combined with the full 1-loop and QCD 2-loop contribution as

$$ m_t(Q) = M_t + \text{Re} \Sigma_t^S(p^2 = M_t^2, Q) $$

$$ + M_t \left[ \text{Re} \Sigma_t^L(p^2 = M_t^2, Q) + \text{Re} \Sigma_t^R(p^2 = M_t^2, Q) 
$$

$$ + \Delta m_t^{(1),\text{qcd}}(Q) + \Delta m_t^{(2),\text{qcd}}(Q) + \Delta m_t^{(3),\text{qcd}}(Q) \right], $$

where $\Sigma_t^{S,L,R}(p^2, Q)$ denote the scalar, left- and right-handed parts of the $\overline{\text{MS}}$-renormalized 1-loop top quark self energy, $M_t$ is the top pole mass and

$$ \Delta m_t^{(1),\text{qcd}}(Q) = -\frac{g_3^2}{12\pi^2} \left[ 4 - 3 \log(t) \right], $$

$$ \Delta m_t^{(2),\text{qcd}}(Q) = \left( \Delta m_t^{(1),\text{qcd}}(Q) \right)^2 - \frac{g_3^4}{4608\pi^4} \left[ 396 \log^2(t) - 1452 \log(t) 
$$

$$ - 48 \zeta(3) + 2053 + 16\pi^2 (1 + \log 4) \right]. $$
\[ \Delta m_t^{(3)qcd}(Q) = \frac{g_6^6}{2430 (4\pi)^6} \left\{ 48600 \log^2 (t) - 208980 \log^2 (t) + 540 \left[ -1560 \zeta(3) + 2993 + 40\pi^2(1 + \log 4) \right] \log(t) + 15 \left[ 69120 \text{Li}_4 \left( \frac{1}{2} \right) + 113040 \zeta(3) - 94800 \zeta(5) - 280853 + 2880 \log^4 2 \right] + 4\pi^2 \left[ 129510 \zeta(3) - 388781 + 240(733 + 24\log 2) \log 2 \right] - 10500\pi^4 \right\} \]

\[ \approx \frac{g_6^6}{(4\pi)^6} 20 \left[ \log^3 (t) - \frac{43}{10} \log^2 (t) + 22.8874 \log (t) - 172.937 \right]. \] (10)

In Figure 1 the impact of the 3-loop correction to the \( \overline{\text{MS}} \) top Yukawa coupling in the SM \( y_t^{\text{SM}}(M_Z) \) on the prediction of the light \( CP \)-even Higgs pole mass in the MSSM in the pure EFT calculation of \( \text{HSSUSY} \) is shown as the green dashed-double-dotted line. As already discussed in Ref. [160], we find that the inclusion of the 3-loop correction to \( y_t^{\text{SM}}(M_Z) \) reduces the Higgs mass by up to 500 MeV. Note that this is formally a (partial) 4-loop effect on the Higgs mass, which is beyond the current accuracy of \( \text{HSSUSY} \).

3-loop QCD corrections to the \( \overline{\text{MS}} \) strong coupling in the SM. The pure SM QCD contribution to the \( \overline{\text{MS}} \) strong coupling is known up to the 3-loop level of \( O(\alpha_s^3) \) [171–174]. This 3-loop QCD expression can be taken into account in \( \text{FlexibleSUSY} \) to extract the \( \overline{\text{MS}} \) strong coupling in the SM from the input value \( \alpha_s^{\text{SM}(5)}(M_Z) \) by setting the following switch in the model file:

```
UseSMAlphaS3Loop = True;
```

To take this 3-loop threshold correction into account at runtime, the following flags should also be set in the SLHA input file:

```
Block FlexibleSUSY
  7 3 # threshold corrections loop order
  24 123111321 # individual threshold correction loop orders
```

In \( \text{FlexibleSUSY} \)’s \text{Mathematica} interface, the above SLHA configuration options correspond to

```
handle = FS<model>OpenHandle[
  fsSettings -> {thresholdCorrectionsLoopOrder -> 3,
    thresholdCorrections -> 123111321}
  ...];
```

The 3-loop QCD contributions are combined as

\[ \alpha_s^{\text{SM}}(Q) = \frac{\alpha_s^{\text{SM}(5)}(Q)}{1 - \Delta \alpha_s^{1L}(Q) - \Delta \alpha_s^{2L}(Q) - \Delta \alpha_s^{3L}(Q)}, \] (12)

where \( \Delta \alpha_s^{nL}(Q) \) are the \( n \)-loop threshold corrections, which read [174]

\[ \Delta \alpha_s^{1L}(Q) = \Delta_1, \] (13)
\[
\Delta \alpha_s^{2L}(Q) = \Delta_2 - \Delta_1^2,
\]
\[
\Delta \alpha_s^{3L}(Q) = \Delta_3 + \Delta_1^3 - 2\Delta_1\Delta_2,
\]
\[
\Delta_1 = \left( \frac{\alpha_{s}^{\text{SM}}(5)(Q)}{\pi} \right) \frac{L}{6},
\]
\[
\Delta_2 = \left( \frac{\alpha_{s}^{\text{SM}}(5)(Q)}{\pi} \right)^2 \left[ -\frac{11}{72} + \frac{11}{24}L + \frac{1}{36}L^2 \right],
\]
\[
\Delta_3 = \left( \frac{\alpha_{s}^{\text{SM}}(5)(Q)}{\pi} \right)^3 \left[ \frac{1}{216}L^3 + \frac{167}{576}L^2 + \frac{2645}{1728}L + n_f \left( \frac{1}{36}L^2 - \frac{67}{576}L + \frac{2633}{31104} \right) + \frac{82043}{27648} \right],
\]
\[
\approx \left( \frac{\alpha_{s}^{\text{SM}}(5)(Q)}{\pi} \right)^3 \left[ 0.00462963L^3 + 0.428819L^2 + 0.94907L - 0.54880 \right],
\]

with \( n_f = 5, L = \log(Q^2/(m_t^{\text{SM}}(Q))^2) \) and \( m_t^{\text{SM}}(Q) \) being the \( \overline{\text{MS}} \) top quark mass in the SM at the scale \( Q \).

Figure 1 shows the impact of the 2- and 3-loop QCD threshold corrections for \( \alpha_{s}^{\text{SM}}(M_Z) \) on the light \( CP \)-even Higgs pole mass in the MSSM, as predicted by the pure EFT calculation of \( \text{HSSUSY} \) as a function of the SUSY scale \( M_S \). We find that the inclusion of \( \Delta \alpha_s^{2L} \) (blue dashed line) reduces the Higgs mass by up to 40 MeV, depending on the SUSY scale. Taking into account \( \Delta \alpha_s^{3L} \) (blue dotted line) reduces the Higgs mass further by around 3 MeV.

4.1.2. (N)MSSM-specific higher-order contributions

2-loop contributions to the (N)MSSM Higgs masses. As already described in Ref. [30], the known dominant 2-loop effective potential contributions to the Higgs masses in the (N)MSSM of \( O((\alpha_t + \alpha_b)\alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_s^2) \) [175–180] can be taken into account by FlexibleSUSY. Note, however, that the implemented NMSSM 2-loop contributions of \( O((\alpha_t + \alpha_b)^2 + \alpha_s^2) \) are currently available only in the MSSM-limit. Furthermore, NMSSM-specific 2-loop corrections to the running vacuum expectation value beyond the MSSM limit, which would be required for a consistent treatment of the \( O((\alpha_t + \alpha_b)\alpha_s) \) contributions, are currently not implemented. To take the implemented 2-loop corrections to the Higgs masses in the (N)MSSM into account, the following switches must be set in the model file:

\[
\text{UseHiggs2LoopMSSM} = \text{True};
\]
\[
\text{UseHiggs2LoopNMSSM} = \text{True};
\]

These flags can be enabled in all real (N)MSSM-like models with 2(3) \( CP \)-even Higgs bosons, 1(2) \( CP \)-odd Higgs boson(s) and 1 electrically neutral Goldstone boson. In addition to these flags, the (effective) \( \mu \) parameter in the convention of Ref. [181] and the effective squared \( CP \)-odd Higgs tree-level mass \( m_A^2 \) of the model must be identified. In the NMSSM model file of SARAH they would read for example:

\[
\text{EffectiveMu} = ([\text{Lambda}] \times S) / \text{Sqrt}(2);
\]
EffectiveMASqr = (T[Lambda] vS / Sqrt[2] + 0.5 [Lambda] [Kappa] vS^2) (vu^2 + vd^2) / (vu vd);

These 2-loop contributions can then be enabled at runtime by setting the following flags in the SLHA input file:

```
Block FlexibleSUSY
  4 2 # pole mass loop order
  5 2 # EWSB loop order
  7 2 # threshold corrections loop order
  8 1 # Higgs 2-loop corrections O(alpha_t alpha_s)
  9 1 # Higgs 2-loop corrections O(alpha_b alpha_s)
 10 1 # Higgs 2-loop corrections O((alpha_t + alpha_b)^2)
 11 1 # Higgs 2-loop corrections O(alpha_tau^2)
 24 12211221 # individual threshold correction loop orders
```

In FlexibleSUSY's Mathematica interface, the above SLHA configuration options correspond to

```
handle = FS<model>OpenHandle[
  fsSettings -> {
    poleMassLoopOrder -> 2,
    ewsbLoopOrder -> 2,
    thresholdCorrectionsLoopOrder -> 2,
    higgs2loopCorrectionAtAs -> 1,
    higgs2loopCorrectionAbAs -> 1,
    higgs2loopCorrectionAtAt -> 1,
    higgs2loopCorrectionAtauAtau -> 1,
    thresholdCorrections -> 122111221
  }
];
```

3-loop contributions to the light CP-even MSSM Higgs mass. The 3-loop contributions to the light CP-even Higgs mass in the MSSM have been calculated to $O(\alpha_t\alpha_s^2)$ in the DR and MDR scheme [182–184]. The expressions are available in the public spectrum generator H3m [185], where they are added to the 2-loop on-shell result of FeynHiggs [54–61]. In Ref. [142] the 3-loop contributions of $O(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ have been studied for the first time in a pure DR MSSM spectrum generator and have been made available in the public C++ library Himalaya [186].

The explicit 3-loop contributions of $O(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ to the light CP-even Higgs mass from Himalaya can be taken into account in FlexibleSUSY by setting the following flag in the FlexibleSUSY model file:

```
UseHiggs3LoopMSSM = True;
```

The model is required to be real MSSM-like with two CP-even Higgs bosons and one electrically neutral Goldstone boson. However, in a pure DR calculation, another source of such 3-loop contributions originates from the 2-loop SUSY-QCD contribution to the MSSM DR top Yukawa coupling, which must be included. Then, however, in order to be consistent with respect to the loop orders of the running and decoupling, also 3-loop renormalization group running should be performed and the 2-loop threshold correction $\Delta\alpha_s^{2L}$ of the strong coupling should be included, see below. Therefore, we strongly recommend setting the following flags in addition in the FlexibleSUSY model file:
UseHiggs2LoopMSSM = True; (* 2-loop contribution to Higgs mass *)
EffectiveMu = {[Mu]}; (* specify mu parameter, see above *)
UseMSSM3LoopRGEs = True; (* 3-loop running *)
UseMSSMYukawa2Loop = True; (* 2-loop SUSY-QCD correction to yt *)
UseMSSMAphaS2Loop = True; (* 2-loop threshold correction to alpha_s *)

When building FlexibleSUSY, the path to the Himalaya headers and to the Himalaya library must be specified:

```
$ ./configure --with-models= [...] \
   --enable-himalaya \
   --with-himalaya-incdir=$HIMALAYA_PATH/source/include \
   --with-himalaya-libdir=$HIMALAYA_PATH/build
$ make
```

where $HIMALAYA_PATH$ is the Himalaya directory. To calculate the light $CP$-even Higgs mass in the MSSM at the 3-loop level with FlexibleSUSY, the following flags must be set at runtime: In the SLHA input file we recommend setting at least

| Block FlexibleSUSY |                      |
|--------------------|----------------------|
| 4 3                | # pole mass loop order |
| 5 3                | # EWSB loop order |
| 6 3                | # beta-functions loop order |
| 7 2                | # threshold corrections loop order |
| 24 122111221       | # individual threshold correction loop orders |
| 25 0               | # ren. scheme for 3L corrections ($0 = \text{DR}, 1 = \text{MDR}$) |
| 26 1               | # Higgs 3-loop corrections $0(\alpha_t \alpha_s^2)$ |
| 27 1               | # Higgs 3-loop corrections $0(\alpha_b \alpha_s^2)$ |

In FlexibleSUSY's Mathematica interface, the above SLHA configuration options correspond to

```mathematica
handle = FS<model>OpenHandle[
   fsSettings -> {poleMassLoopOrder -> 3,
                  ewsbLoopOrder -> 3,
                  betaFunctionLoopOrder -> 3,
                  thresholdCorrectionsLoopOrder -> 2,
                  thresholdCorrections -> 122111221,
                  higgs3loopCorrectionRenScheme -> 0,
                  higgs3loopCorrectionAtAsAs -> 1,
                  higgs3loopCorrectionAbAsAs -> 1}
   ...
];
```

Figure 2 shows a comparison of the different light $CP$-even Higgs mass calculations available in FlexibleSUSY. The scenario is chosen such that all soft-breaking $\text{DR}$ mass parameters, the superpotential $\mu$ parameter and the $\text{DR}$ $CP$-odd Higgs mass are equal to the SUSY scale $M_S$ at the scale $Q = M_S$ and all $\text{DR}$ sfermion mixing parameters $X_f$ are set to zero, except for the stop mixing parameter $X_t$. Furthermore, we set $M_t = 173.34 \text{ GeV}$, $\alpha_s^{\text{SM}(5)}(M_Z) = 0.1184$ and $\alpha_{\text{em}}^{\text{SM}(5)}(M_Z) = 1/127.944$. The top row of Figure 2 shows the Higgs mass as a function of $M_S$ for $X_t = 0$ and $\tan \beta = 5$. In the left panel of the top row the classic 2-loop fixed-order calculation with FlexibleSUSY [30] is shown as the blue dashed line and the pure EFT calculation with HSSUSY is shown as the black dotted line.
Figure 2: Comparison of different 2-loop and 3-loop calculations of the lightest $CP$-even Higgs mass in the MSSM with different FlexibleSUSY spectrum generators for $\tan\beta = 5$. In the top row we set $X_t = 0$ and in the bottom row we use $M_S = 2$ TeV.

The reason why these two curves do not deviate from each other logarithmically when $M_S$ increases is an accidental cancellation of large logarithmic contributions in the fixed-order calculation [31]. The green dashed-dotted line shows the improved version of the FlexibleEFTHiggs calculation [31], which interpolates between the fixed-order calculation at low $M_S$ and the pure EFT calculation at large $M_S$. See Section 10 for a discussion of the improvements of FlexibleEFTHiggs. The red solid line shows the 3-loop calculation up to $O(\alpha_t\alpha^2_s + \alpha_b\alpha^2_s)$ with FlexibleSUSY+Himalaya [142]. In this calculation, the full 2-loop SUSY-QCD contributions to the DR top and bottom Yukawa couplings $y_{t,b}(M_Z)$ [187–189] are taken into account. The sum of these contributions and the explicit 3-loop contribution to the $CP$-even Higgs mass matrix leads to a downward shift of the predicted Higgs mass by 1–2 GeV, depending on $M_S$, compared to the classic 2-loop calculation with FlexibleSUSY (blue dashed line). As also discussed in Ref. [142], this brings the prediction closer to the pure EFT calculation, which is expected to lead to a more precise result above
the TeV scale. In the left panel of the bottom row of Figure 2, the calculated Higgs mass is shown as a function of the stop mixing parameter $X_t$ for fixed $M_S = 2$ TeV. Also for non-zero $X_t$, one finds a reduction of the Higgs mass by 1–3 GeV, which brings the prediction closer to the one of the pure EFT calculation.

In the right panels of Figure 2, the effect of the 2-loop SUSY-QCD threshold correction $\Delta \alpha_s^{2L}$ [190–192] to the strong coupling $\alpha_s$ (see below) on the 3-loop Higgs mass calculation is shown for the same scenario as in the corresponding left panels. The threshold correction $\Delta \alpha_s^{2L}$ is included at the scale $Q = M_Z$, the same scale at which all dimensionless MSSM $\overline{\text{DR}}$ parameters are determined from the Standard Model input parameters. The inclusion of $\Delta \alpha_s^{2L}$ is formally a 4-loop effect on the light $CP$-even Higgs mass in the MSSM. However, $\Delta \alpha_s^{2L}$ should be taken into account for a consistent running and decoupling procedure with 3-loop renormalization group running. The red solid lines in the right panels correspond to the 3-loop calculation of Ref. [142], which uses only the 1-loop threshold correction $\Delta \alpha_s^{1L}$. These red solid lines are the same as in the corresponding left panels. The effect of including $\Delta \alpha_s^{2L}$ is shown as the red dashed line. We find that the inclusion of this 2-loop threshold correction leads to an upwards shift of the Higgs mass by up to 2 GeV, depending on $M_S$ and $X_t$. Note, that large 4-loop contributions of multiple GeV hint at a large theoretical uncertainty of the fixed-order calculation of the light $CP$-even Higgs pole mass in parameter regions with multi-TeV stop masses.

3-loop renormalization group running in the MSSM. In the MSSM, the 3-loop $\beta$ functions are also known [193, 194]. FlexibleSUSY allows the user to take these 3-loop $\beta$ functions into account in the running of the $\overline{\text{DR}}$ MSSM parameters by setting the following switch in the model file:

```plaintext
UseMSSM3LoopRGEs = True;
```

To use 3-loop running at runtime, the following flag should also be set in the SLHA input file:

```plaintext
Block FlexibleSUSY
   6 3 # beta-functions loop order
```

In FlexibleSUSY’s Mathematica interface, the above SLHA configuration corresponds to

```plaintext
handle = FS<model>OpenHandle[
   fsSettings -> {betaFunctionLoopOrder -> 3}
];
```

The expressions for the 3-loop $\beta$ functions have been extracted from the official FORM file provided by the authors of Refs. [193, 194]. We have numerically compared the expressions with the ones implemented in SOFTSUSY 3.7.0 [43] and found exact agreement. The effect of the 3-loop RGEs on the Higgs pole mass in the MSSM is of the order of a few 100 MeV as discussed in Ref. [43] and is shown in Figure 3. Due to the complexity of the 3-loop $\beta$ functions, the runtime of the MSSM spectrum generators is increased by a factor of 4–5 if 3-loop running is enabled in the MSSM.

2-loop SUSY-QCD corrections to the $\overline{\text{DR}}$ top and bottom Yukawa couplings in the MSSM. In the MSSM, the full 2-loop SUSY-QCD corrections of $O(\alpha_s^2)$ to the $\overline{\text{DR}}$ top Yukawa

---

6https://www.liverpool.ac.uk/~dij/betas/allgennb.log
coupling [187–189] as well as the 2-loop SUSY-QCD $O(\alpha_s^2)$ and partial electroweak contributions to the DR bottom Yukawa coupling [195] are known. These 2-loop corrections have already been made available in SOFTSUSY 3.7.0 [43]. The 2-loop SUSY-QCD corrections of $O(\alpha_s^2)$ are now also incorporated in FlexibleSUSY 2.0 and can be used by setting the following switch in the model file:\footnote{We kindly thank Alexander Bednyakov for providing the 2-loop SUSY-QCD expressions.}

\begin{verbatim}
UseMSSMYukawa2Loop = True;
\end{verbatim}

To take these 2-loop contributions into account at runtime, the following flags should also be set in the SLHA input file:

\begin{verbatim}
Block FlexibleSUSY
  7 2 # threshold corrections loop order
  24 122111221 # individual threshold correction loop orders
\end{verbatim}

In FlexibleSUSY’s Mathematica interface, the above SLHA configuration options correspond to

\begin{verbatim}
handle = FS<model>OpenHandle[
  fsSettings -> {thresholdCorrectionsLoopOrder -> 2,
    thresholdCorrections -> 122111221}
...];
\end{verbatim}

In Figure 3 we show in blue the effect of the full 2-loop SUSY-QCD corrections on the lightest CP-even Higgs pole mass in the MSSM. The blue dashed line shows the effect in FlexibleSUSY 2.0 and the crosses in SOFTSUSY 4.0.1. As can be seen from the figure, the full 2-loop SUSY-QCD corrections are negative and can affect the Higgs mass by several GeV, as has been observed in Ref. [43]. In the left panel of Figure 3, the shift in the Higgs mass is shown as a function of the SUSY scale. For scales above $\approx 2$ TeV we find a logarithmic shift in $M_h$, which is caused by new large logarithms originating from the 2-loop contribution of the SUSY particles to the DR top Yukawa coupling. We also find that these new logarithms alone would spoil the accidental cancellation of large logarithms described in Ref. [31]. Note that in the MSSM the effect of the 2-loop SUSY-QCD corrections to the top and bottom Yukawa couplings is a partial 3-loop contribution to the light CP-even Higgs pole mass. Thus, these 2-loop SUSY-QCD corrections must be taken into account if the explicit 3-loop Higgs mass contributions of $O(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ from Himalaya are used, see above.

\textit{2-loop SUSY-QCD corrections to the DR strong gauge coupling in the MSSM.} In the MSSM, the full 2-loop SUSY-QCD corrections of $O(\alpha_s^2 + \alpha_s\alpha_t + \alpha_s\alpha_b)$ to the DR strong gauge coupling are known [190–192] and have been made available in SOFTSUSY 3.7.0 [43]. In FlexibleSUSY 2.0, these corrections can be taken into account by setting in the model file:\footnote{We kindly thank Ben Allanach and Alexander Bednyakov for providing the 2-loop SUSY-QCD expressions.}

\begin{verbatim}
UseMSSMAAlphaS2Loop = True;
\end{verbatim}

\footnotetext[7]{We kindly thank Alexander Bednyakov for providing the 2-loop SUSY-QCD expressions.}
\footnotetext[8]{We kindly thank Ben Allanach and Alexander Bednyakov for providing the 2-loop SUSY-QCD expressions.}
Figure 3: Effect of the 3-loop RGEs for all MSSM parameters (red), 2-loop SUSY-QCD contributions to $y_b$ and $y_t$ (blue) and 2-loop SUSY-QCD contributions to $\alpha_s$ (green) on the lightest $CP$-even Higgs pole mass in the MSSM. The lines show the shift in the Higgs pole mass in FlexibleSUSY 2.0 compared to the one obtained with 2-loop RGEs, full 1-loop SUSY-QCD + 2-loop SM-QCD contributions to $y_t$ and 1-loop SUSY-QCD contributions to $\alpha_s$. The crosses show the corresponding shift obtained with SOFTSUSY 4.0.1. In the left panel we use $\tan\beta = 5$ and $X_t = 0$ and in the right panel we fix $\tan\beta = 5$ and $M_S = 2$ TeV.

To take these 2-loop threshold corrections into account at runtime, the following flags should also be set in the SLHA input file:

```
Block FlexibleSUSY
  7 2 # threshold corrections loop order
  24 122111221 # individual threshold correction loop orders
```

In FlexibleSUSY’s Mathematica interface, the above SLHA configuration options correspond to

```
handle = FS[model]OpenHandle[
  fsSettings -> {thresholdCorrectionsLoopOrder -> 2,
                  thresholdCorrections -> 122111221}
  ...
];
```

In Figure 3 we show in green the effect of the 2-loop SUSY-QCD corrections to $\alpha_s$ on the lightest $CP$-even Higgs pole mass in the MSSM. The green dashed-dotted line shows the effect in FlexibleSUSY 2.0 and the crosses in SOFTSUSY 4.0.1. Both implementations agree exactly. We furthermore find that the inclusion of the 2-loop threshold corrections to $\alpha_s$ leads to a logarithmic enhancement of the Higgs mass as a function of the SUSY scale. The enhancement is around $+1$ GeV for $M_S \approx 2$ TeV and maximal stop mixing. Note that in the MSSM the effect of the 2-loop SUSY-QCD corrections to the strong coupling is formally a partial 4-loop contribution to the Higgs pole mass.
4.1.3. Split-MSSM-specific higher-order contributions

3-loop contribution to the Higgs mass in the split-MSSM. In the split-MSSM [196], part of the 3-loop contribution of $O(\alpha_t\alpha_s^2)$ to the SM-like Higgs pole mass is known in the $\overline{\text{MS}}$ scheme [197]. FlexibleSUSY allows the user to take this 3-loop contribution into account by setting the following switch in the model file:

\[
\text{UseHiggs3LoopSplit} = \text{True};
\]

This is used in the distributed SplitMSSM model file. For use in other models, it is a requirement that the model contains a single Higgs boson and a gluino. The enabled 3-loop Higgs pole mass contribution reads

\[
(\Delta m_h^2)_{3L} = \frac{64g_3^4\alpha_s^2}{(4\pi)^6}m_t^2 \log^2 \left( g \right),
\]

where $g = m_{\tilde{g}}^2$ and $m_{\tilde{g}}$ is the $\overline{\text{MS}}$ gluino mass. Furthermore, for consistency the 2-loop gluino contribution in the calculation of the $\overline{\text{MS}}$ top mass from Ref. [197] is taken into account by adding the following term to Eq. (7):

\[
\Delta m_t^{(2)}_{\text{split-qcd}}(Q) = -\frac{g_3^4}{(4\pi)^4}M_t \left\{ \frac{89}{9} + 4 \log(g) \left\{ \frac{13}{3} + \log(g) - 2 \log(t) \right\} \right\}.
\]

To take these contributions into account at runtime, the following flags should also be set in the SLHA input file:

```bash
Block FlexibleSUSY
4 3 # pole mass loop order
5 3 # EWSB loop order
7 2 # threshold corrections loop order
24 122111221 # individual threshold correction loop orders
26 1 # Higgs 3-loop corrections $O(\alpha_t\alpha_s^2)$
```

In FlexibleSUSY’s Mathematica interface, the above SLHA configuration options correspond to

```mathematica
handle = FS<model>OpenHandle[
   fsSettings -> {poleMassLoopOrder -> 3,
                  ewsbLoopOrder -> 3,
                  thresholdCorrectionsLoopOrder -> 2,
                  thresholdCorrections -> 122111221,
                  higgs3loopCorrectionsAtAsAs -> 1}
   ...];
```

4.2. New features for definition of boundary conditions

In FlexibleSUSY 2.0, the expressions to define boundary conditions are allowed to be more complicated and to involve trigonometric functions, dilogarithms, branches and more. This is particularly useful for defining high-scale boundary conditions that match a model to its UV-completion. An example is FlexibleSUSY’s HSSUSY model file, which implements the known 1- and 2-loop high-scale matching condition on the quartic Higgs coupling of the SM against the MSSM at the SUSY scale [137, 160, 196]. The list of special functions and symbols to be used in boundary conditions can be found in Table 3.
| Function       | Description                                                                 |
|---------------|-----------------------------------------------------------------------------|
| Abs[a_]       | Returns the magnitude of a real or complex number a, |a|. If a is a vector, a vector is returned with Abs applied to each element. |
| AbsSqr[a_]    | Returns the squared magnitude of a real or complex number a, |a|².               |
| AbsSqrt[a_]   | Returns the square root of the magnitude of a, \(\sqrt{|a|}\).             |
| ArcSin[a], ArcCos[a] | Returns arcsin a and arccos a, respectively.  |
| ArcTan[a]     | Returns arctan a.                                                         |
| Arg[z]        | Returns the phase angle of a complex number, arg z.                        |
| Cbrt[a]       | Returns the cubic root of a, \(\sqrt[3]{a}\).                             |
| CKM, PMNS     | CKM and PMNS matrices, respectively, as defined in [158].                  |
| Conjugate[a]  | Returns the complex conjugate of a.                                        |
| Exp[a]        | Returns \(e^a\) for real or complex a.                                    |
| FiniteLog[a]  | Returns \(\log a\) if \(\log a\) is well-defined, otherwise returns 0.   |
| FSThrow[msg]  | Throws an exception of type PhysicalError with the message msg.            |
| I             | Imaginary unit.                                                            |
| If[cond_, a_, b_] | If cond is true, a is returned, otherwise b.                           |
| Im[a_]        | Returns the imaginary part of a.                                          |
| IsClose[a_, b_, eps_] | Returns True if Abs[a - b] < eps, otherwise False.                  |
| IsCloseRel[a_, b_, eps_] | Returns True if Abs[(a - b)/a] < eps, otherwise False.                  |
| IsFinite[a]   | Returns True if a is neither nan nor inf.                                  |
| KroneckerDelta[i_, j_] | Returns the Kronecker \(\delta_{ij}\).                                |
| Log[a], ComplexLog[a] | Returns the natural logarithm for real and complex arguments,             |
|               | respectively.                                                             |
| Max[a, ...]   | Returns the maximum of all given arguments.                                 |
| Min[a, ...]   | Returns the minimum of all given arguments.                                 |
| Not[cond]     | Returns the logical negation of cond.                                      |
| PolyLog[2, z_] | Returns the dilogarithm of the real or complex number z.                   |
| Re[a]         | Returns the real part of a.                                                |
| Round[a]      | Returns Floor[a + 0.5] if a ≥ 0, otherwise Floor[a - 0.5].                |
| Print<Type>[msg] | Prints a debug, info, error, warning or fatal message, depending on the <Type>, and returns zero. <Type> can be DEBUG, INFO, WARNING, ERROR or FATAL. PrintFATAL[msg] throws an exception after msg has been printed. |
| SCALE         | Returns the renormalization scale at which the boundary condition is imposed. |
| Sign[a]       | Returns 1 if a ≥ 0, otherwise -1.                                          |
| SignedAbsSqr[a] | Returns Sign[a]*Sqrt[Abs[a]].                                             |
| Sin[a], Cos[a], Tan[a] | Returns sin a, cos a and tan a, respectively.                              |
| Total[vec]    | Returns the sum of all elements of vec, ∑ᵢ vᵢ.                           |
| UnitStep[value] | Returns 0 if value < 0, 1 otherwise.                                      |
| Which[test1_, value1_, test2_, value2_, ...] | If test1 is true, value1 is returned, otherwise if test2 is true, value2 is returned, etc. |
| ZeroSqrt[a_]  | Returns \(\sqrt{a}\) if a > 0, otherwise returns 0.                      |

Table 3: Available special functions and symbols in the boundary conditions.
5. FlexibleBSM extension

Since version 1.1.0, FlexibleSUSY can generate spectrum generators not only for SUSY models, but also for non-SUSY models. We document this feature here for the first time. In Subsections 5.4 and 5.5, we will describe important applications of this feature to the two-Higgs doublet model and to an effective low-energy theory of the MSSM (HSSUSY).

The generated non-SUSY spectrum generators have the same features as the SUSY spectrum generators:

- The running gauge and Yukawa couplings of the non-SUSY model are calculated automatically at the 1-loop level from the known low-energy SM parameters $\alpha_{\text{em}}^\text{SM}(M_Z)$, $\alpha_s^\text{SM}(M_Z)$ and from the known quark and lepton masses as well as $M_Z$ and either $G_F$ or $M_W$. 2-loop and 3-loop QCD corrections can be taken into account to determine the running top Yukawa coupling of the model, see Section 4.1.

- Up to three boundary conditions can be specified to fix the running parameters of the model at different user-defined scales.

- 2-loop renormalization group running is used between the scales at which the boundary conditions are imposed. In the SM and in the MSSM, also 3-loop running is available, see Section 4.1.

- The pole mass spectrum is calculated at the full 1-loop level, taking into account all BSM contributions. Some 2-loop and 3-loop corrections can be added in specific non-SUSY models, see Section 4.1.

5.1. Setting up a FlexibleBSM model

The FlexibleSUSY user interface for creating spectrum generators for non-SUSY models is exactly the same as in the case of SUSY models, except that all non-SUSY parameters are defined in the $\overline{\text{MS}}$ scheme. In particular, at the low-energy scale FlexibleSUSY automatically determines the gauge and Yukawa couplings of the non-SUSY model in the $\overline{\text{MS}}$ scheme. For gauge-dependent quantities like running masses and VEVs, FlexibleSUSY adopts the Feynman gauge, where all gauge fixing parameters $\xi_i$ are set to unity.

5.2. Determination of the $\overline{\text{MS}}$ gauge and Yukawa couplings

If the considered BSM model has a gauge symmetry with the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a factor, then FlexibleSUSY automatically fixes the three corresponding normalized running gauge couplings $g_1$, $g_2$ and $g_3$ at the low-energy boundary condition from the given input parameters $M_Z$, $\alpha_{\text{em}}^\text{SM}(M_Z)$, $\alpha_s^\text{SM}(M_Z)$ and $G_F$ or $M_W$ as

\begin{align*}
g_1(Q) &= N_{g_Y} g_Y(Q), \\
g_Y(Q) &= \sqrt{4\pi\alpha_{\text{em}}(Q)} / \cos \theta_W(Q), \\
g_2(Q) &= N_{g_L} g_L(Q), \\
g_L(Q) &= \sqrt{4\pi\alpha_{\text{em}}(Q)} / \sin \theta_W(Q), \\
g_3(Q) &= N_{g_s} g_s(Q), \\
g_s(Q) &= \sqrt{4\pi\alpha_s(Q)}.
\end{align*}

\[9\] Note that the $\beta$ functions of scalar tadpole terms in non-supersymmetric models [198] are currently not generated by SARAH. For this reason, such tadpole terms do not run in SARAH/SPheno or FlexibleSUSY.
Here, $\alpha_{em}$ and $\alpha_s$ denote the $\overline{\text{MS}}$ electromagnetic and strong coupling constants of the non-SUSY model, respectively, and $\theta_W$ is the $\overline{\text{MS}}$ weak mixing angle. The coefficients $N_i$ denote the potential normalization factors defined in the SARAH model file. The renormalization scale $Q$, at which the gauge couplings are calculated, can be specified using the LowScale variable in the model file. The coupling constants of the model are related to the corresponding ones of the SM with five active quark flavors, $\alpha^{\text{SM}(5)}_{em}(Q)$ and $\alpha^{\text{SM}(5)}_{s}(Q)$, which are input, via the relations

$$\alpha_{em}(Q) = \frac{\alpha^{\text{SM}(5)}_{em}(Q)}{1 - \Delta \alpha_{em}(Q)},$$  \hspace{1cm} (25) \\
$$\alpha_{s}(Q) = \frac{\alpha^{\text{SM}(5)}_{s}(Q)}{1 - \Delta \alpha_{s}(Q)}.$$  \hspace{1cm} (26) 

The threshold corrections $\Delta \alpha_i(Q)$ have the form

$$\Delta \alpha_{em}(Q) = \frac{\alpha_{em}}{2\pi} \sum_i C^{\text{em}}_i \log \frac{m_i}{Q},$$  \hspace{1cm} (27) \\
$$\Delta \alpha_{s}(Q) = \frac{\alpha_{s}}{2\pi} \sum_i C^{s}_i \log \frac{m_i}{Q},$$  \hspace{1cm} (28) 

where the sum runs over all non-SM particles plus the top quark with running $\overline{\text{MS}}$ masses $m_i(Q)$. The constants $C^{\text{em}}_i$ and $C^{s}_i$ depend on the representation of the particle $i$ with respect to the Lorentz and gauge group. The $\overline{\text{MS}}$ weak mixing angle $\theta_W$ in the non-SUSY model is determined either

- from the Fermi constant $G_F$ and $M_Z$ using the iterative approach described in Ref. [199] taking into account the full 1-loop corrections and leading 2-loop SM corrections to $\Delta \hat{\rho}$ and $\Delta \hat{r}$, see Section 8.

- or from the running $W$ and $Z$ masses, which are obtained from the corresponding pole masses via a 1-loop calculation. See Section 8 for more details.

If the considered BSM model does not contain the SM gauge group as a factor, it is of course still possible to fix the gauge couplings at the low-energy boundary condition by defining them to be input parameters.

**Example 5**

In a left-right-symmetric model with the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$, one could for example fix the running BSM gauge couplings $g_3$, $g_L$, $g_R$, $g_{1L}$ and $g_{1R}$ by the running SM-like gauge couplings $g^{\text{SM}}_3$, $g^{\text{SM}}_2$ and $g^{\text{SM}}_1$ which are given as input via the EXTPAR block:

```plaintext
EXTPAR = {
    {100, g1SMInput},
    {101, g2SMInput},
    {102, g3SMInput}
};

LowScaleInput = {
    {g3, g3SMInput},
};
```
Note that, in addition to fixing the BSM gauge couplings, in this example the off-diagonal gauge couplings $g_{1L1R}$ and $g_{1R1L}$ that arise due to $U(1)$ mixing are being set to zero at the low-energy scale.

The $\overline{\text{MS}}$ Yukawa couplings $Y_f(Q)$ of the SM-like fermions $f$ in the non-SUSY model are determined from the corresponding $\overline{\text{MS}}$ masses $m_f(Q)$ using the tree-level relation. For example, in the SM this relation reads

$$y_f^{\overline{\text{MS}}} = \frac{\sqrt{2} m_f^{\overline{\text{SM}}}(Q)}{v^{\overline{\text{SM}}}(Q)},$$

(29)

with $f = u, d, c, s, t, b, e, \mu, \tau$ and the $\overline{\text{MS}}$ vacuum expectation value $v^{\overline{\text{SM}}}(Q)$. The running top quark $\overline{\text{MS}}$ mass in the non-SUSY model, $m_t(Q)$, is calculated from the top pole mass $M_t$ using the full 1-loop self energy plus 2-loop SM QCD corrections as shown in Eq. (7). In the SM, 3-loop QCD contributions can be taken into account as well, see Section 4.1.

The bottom quark $\overline{\text{MS}}$ mass in the non-SUSY model, $m_b(Q)$, is obtained from the $\overline{\text{MS}}$ mass $m_b^{\overline{\text{SM}}}(M_t)$ in the SM with 5 active quark flavors by first evolving $m_b^{\overline{\text{SM}}}(M_t)$ to the scale $Q$ using the 1-loop QED and 3-loop QCD RGEs. Afterwards, $m_b^{\overline{\text{SM}}}(Q)$ is converted to $m_b(Q)$ as

$$m_b(Q) = \frac{m_b^{\overline{\text{SM}}}(Q)}{1 - \Delta m_b},$$

(30)

$$\Delta m_b = \text{Re} \Sigma_b^S (p^2 = (m_b^{\overline{\text{SM}}}(Q))^2, Q) / m_b + \text{Re} \Sigma_b^L (p^2 = (m_b^{\overline{\text{SM}}}(Q))^2, Q) + \text{Re} \Sigma_b^R (p^2 = (m_b^{\overline{\text{SM}}}(Q))^2, Q),$$

(31)

where $\Sigma_b^{S,L,R}$ are the scalar, left- and right-handed parts of the 1-loop bottom quark self energy in the $\overline{\text{MS}}$ scheme, in which all loops that contain only SM(5) particles are omitted. Finally, the $\overline{\text{MS}}$ mass of the $\tau$ lepton, $m_\tau(Q)$, is calculated by first identifying the $\tau$ pole mass, $M_\tau$, with the $\overline{\text{MS}}$ mass in the SM with 5 active quark flavors at the scale $M_\tau$,

$$m_\tau^{\overline{\text{SM}}}(M_\tau) = M_\tau.$$

(32)

In this identification, the 1-loop SM electroweak corrections to $m_\tau^{\overline{\text{SM}}}(M_\tau)$ are neglected. Afterwards, $m_\tau^{\overline{\text{SM}}}(Q)$ is evolved to the scale $Q$ using the 1-loop QED RGE and $m_\tau^{\overline{\text{SM}}}(Q)$ is converted to $m_\tau(Q)$ as

$$m_\tau(Q) = \frac{m_\tau^{\overline{\text{SM}}}(Q)}{1 - \Delta m_\tau},$$

(33)

$$\Delta m_\tau = \text{Re} \Sigma_\tau^S (p^2 = (m_\tau^{\overline{\text{SM}}}(Q))^2, Q) / m_\tau^{\overline{\text{SM}}}(Q) + \text{Re} \Sigma_\tau^L (p^2 = (m_\tau^{\overline{\text{SM}}}(Q))^2, Q) + \text{Re} \Sigma_\tau^R (p^2 = (m_\tau^{\overline{\text{SM}}}(Q))^2, Q),$$

(34)

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where $\Sigma^{S,L,R}_\tau$ are the scalar, left- and right-handed parts of the 1-loop $\tau$ self energy in the $\overline{\text{MS}}$ scheme, from which all loops that contain only SM(5) particles are omitted.

In most models, it is necessary to also fix the running SM-like vacuum expectation value (VEV), $v$, at the low-energy scale. For this purpose FlexibleSUSY provides the symbols $M_Z^{\overline{\text{MS}}}$ and $M_W^{\overline{\text{MS}}}$ in the model file to access the $\overline{\text{MS}}$ $W$ and $Z$ masses $m_W$ and $m_Z$ in the non-SUSY model at the low-energy scale. These running masses can be used to calculate the $\overline{\text{MS}}$ vacuum expectation value $v$, as for example in the SM,

$$v(Q) = \frac{2m_Z(Q)}{\sqrt{3g_1^2(Q)/5 + g_2^2(Q)}}.$$  \hspace{1cm} (35)

5.3. Structure of the generated code

In analogy to SUSY models, the parameters of a non-supersymmetric model are distributed among two classes in the model class hierarchy, see Figure 4: At the top of the model class hierarchy stands the Beta_function interface class, which defines the interface for the RGE integrator of the model parameters. It provides the interface function run_to(), which integrates the RGEs up to a given scale using an adaptive Runge-Kutta algorithm. The Runge-Kutta algorithm makes use of the virtual functions get(), set() and beta() to obtain the model parameters at the current renormalization scale, set the model parameters to new values or calculate the $\beta$ functions. These virtual functions are implemented by the derived classes, `<model>_susy_parameters` and `<model>_soft_parameters`. Dimensionless parameters, like gauge, Yukawa or quartic scalar couplings, are contained in the model class hierarchy.
the `<model>_susy_parameters` class. Parameters with mass dimension greater than zero are contained in the `<model>_soft_parameters` class. The distribution of the model parameters between these two classes reflects the dependency of the $\beta$ functions of the dimensionful parameters upon the dimensionless parameters. Furthermore, it allows the RGEs of the dimensionless parameters to be integrated independently of the dimensionful parameters.

5.4. Application: High-scale MSSM with light Higgs sector

As an application of FlexibleBSM we consider the Higgs pole mass prediction in an MSSM scenario with very heavy sfermions, Higgsinos and gauginos at the SUSY scale $M_S$, but a light Higgs sector. If $M_S$ is larger than a few TeV, an EFT approach should be considered and the heavy SUSY particles should be integrated out at $M_S$. The resulting EFT below $M_S$ is the Two-Higgs-Doublet-Model (THDM). Our aim is to calculate the Higgs pole masses in this effective THDM, where the quartic Higgs couplings are fixed by the MSSM at $M_S$.\(^{10}\) We use a THDM of type II here, for which the full 1- and leading 2-loop threshold corrections at the SUSY scale are known [200–202].

In order to construct such an EFT setup we have to build the THDM-II with SARAH. We start by specifying the gauge group, the field content and the Lagrangian:

```
(* gauge groups *)
Gauge[[1]]={B, U[1], hypercharge, g1,False};
Gauge[[2]]={WB, SU[2], left, g2,True};
Gauge[[3]]={G, SU[3], color, g3,False};

(* field content *)
FermionFields[[1]] = {q, 3, {uL,dL}, 1/6, 2, 3};
FermionFields[[2]] = {l, 3, {vL,eL}, -1/2, 2, 1};
FermionFields[[3]] = {d, 3, conj[dR], 1/3, 1, -3};
FermionFields[[4]] = {u, 3, conj[uR], -2/3, 1, -3};
FermionFields[[5]] = {e, 3, conj[eR], 1, 1, 1};
ScalarFields[[1]] = {H1, 1, {H1p, H10}, 1/2, 2, 1};
ScalarFields[[2]] = {H2, 1, {H2p, H20}, 1/2, 2, 1};

DEFINITION[GaugeEs][Additional] = {
  {LagHC , { AddHC->True }},
  {LagNoHC, { AddHC->False }}
};

LagNoHC = -{M112 conj[H1].H1 + M222 conj[H2].H2 /
  + Lambda1 conj[H1].H1.conj[H1].H1 /
  + Lambda2 conj[H2].H2.conj[H2].H2 /
  + Lambda3 conj[H2].H2.conj[H1].H1 /
  + Lambda4 conj[H2].H1.conj[H1].H2 )

LagHC = -{-M112 conj[H1].H2
  + Lambda5/2 conj[H2].H1.conj[H2].H1
  + Lambda6 conj[H1].H1.conj[H1].H2
  + Lambda7 conj[H2].H2.conj[H1].H2
  + Yd conj[H1].d.q + Ye conj[H1].e.l + Yu H2.u.q};
```

The neutral components of the two Higgs doublets acquire vacuum expectation values $v_1$ and $v_2$:

\(^{10}\)This FlexibleSUSY setup was also used in Ref. [131] to study the vacuum stability at very high SUSY scales in different THDM variants with the MSSM as a supersymmetric UV completion.

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and the Higgs field components mix to $CP$-even, $CP$-odd and charged Higgs mass eigenstates $h_h$, $A_h$ and $H_m$, respectively:

**DEFINITION [EWSB] [MatterSector] =**

{\{phi1, phi2\}, \{hh, ZH\}},
{\{sigma1, sigma2\}, \{Ah, ZA\}},
{\{conj[H1p], conj[H2p]\}, \{Hm, ZP\}}

Now we need a *FlexibleSUSY* model file in which we specify the boundary conditions for all THDM-II parameters. As input we use the $\overline{\text{MS}}$ parameter $\tan\beta_{\text{THDM-II}}(M_t)$, the MSSM DR parameters $\mu(M_S)$ and $A_f(M_S)$ ($f = t, b, \tau$) at the SUSY scale, and the $\overline{\text{MS}} CP$-odd Higgs mass $m_A^{\text{THDM-II}}(M_{\text{EWSB}})$, where $M_{\text{EWSB}}$ is the scale of the electroweak symmetry breaking, at which we calculate the light $CP$-even Higgs pole mass in the end:

**MINPAR =**

{3, TanBeta}

**EXTPAR =**

{0, MSUSY},
{1, MEWSB},
{2, MuInput},
{6, MAInput},
{7, AtInput},
{8, AbInput},
{9, AtauInput}

**LowScale = LowEnergyConstant[MT];**

**LowScaleInput =**

{Yu, Automatic},
{Yd, Automatic},
{Ye, Automatic},
{v1, 2 MZMSbar / Sqrt[GUTNormalization[g1]^2 g1^2 + g2^2] \ Cos[ArcTan[TanBeta]]},
{v2, 2 MZMSbar / Sqrt[GUTNormalization[g1]^2 g1^2 + g2^2] \ Sin[ArcTan[TanBeta]]}

At the scale $M_{\text{EWSB}}$, we fix the $m_{122}$ parameter using the input value of $m_A^{\text{THDM-II}}(M_{\text{EWSB}})$ and we impose the EWSB conditions by fixing $m_{112}$ and $m_{222}:$
Finally, we need to fix the quartic Higgs couplings of the THDM-II at the scale $M_S$. The necessary relations between the MSSM parameters and the quartic Higgs couplings of the THDM-II are known at the full 1-loop and leading 2-loop level [200–202]. We can use expressions from these references to write the boundary conditions on the quartic Higgs couplings at the SUSY scale, shown in lines 23–57 of Appendix D, in terms of the 1- and 2-loop threshold corrections, shown in lines 110–398 of the same listing which displays the complete FlexibleSUSY model file. Figure 5 shows the lightest $CP$-even Higgs pole mass calculated at the 1-loop level with FlexibleSUSY in this EFT setup as a function of $\tan^2 \beta$ and $M_S$ for $m_A^{\text{THDM-II}}(M_{\text{EWSB}}) = 200$ GeV, $M_{\text{EWSB}} = M_t$ and maximal stop mixing. The figure shows that using this setup, FlexibleSUSY can reproduce the results presented in the left panels of Figure 2 of Ref. [202]. This EFT model is distributed with the FlexibleSUSY package under the name THDMIIMSSMBC.

Figure 5: Lightest $CP$-even Higgs pole mass calculated at the 1-loop level in the effective THDM setup as a function of $\tan^2 \beta$ and $M_S$ for $m_A(M_t) = 200$ GeV, $A_t = \mu/\tan \beta + X_t$, $A_0 = A_\tau = A_t$, $\mu = M_S$, $M_t = 173.34$ GeV and $\alpha_s^{\text{SM}(5)}(M_Z) = 0.1184$. The left panel shows the results for $X_t = 0$ and the right panel for $X_t = \sqrt{6} M_S$. The solid line corresponds to a Higgs pole mass of 125 GeV and the dashed lines to 124 GeV and 126 GeV, respectively.

5.5. Application: High-scale MSSM ($\#SSUSY$)

FlexibleBSM has already been applied in Refs. [31, 131, 137] to perform a pure EFT calculation of the lightest $CP$-even Higgs mass in the MSSM, assuming that all SUSY particles are integrated out at a heavy SUSY scale $M_S$. The FlexibleSUSY spectrum
generator constructed for this purpose is called HSSUSY and is based on SARAH’s SM model file (SM). In FlexibleSUSY 1.2.3, HSSUSY implemented the 1-loop and leading 2-loop threshold corrections of $O(\alpha_t \alpha_s + \alpha_t^2)$ to the quartic Higgs coupling of the SM from Refs. [160, 196] at the SUSY scale. In FlexibleSUSY 2.0, the generalized 2-loop expressions of $O(\alpha_t^2)$ for general stop masses as well as the new 2-loop contributions from Ref. [137], which involve the bottom and tau Yukawa couplings, are included. As a result, the version of HSSUSY included in FlexibleSUSY 2.0 uses the 2-loop threshold corrections of $O(\alpha_t \alpha_s + \alpha_t \alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_b \alpha_s + \alpha_t^2)$ for general SUSY spectra.

In the HSSUSY model file (model_files/HSSUSY/FlexibleSUSY.m.in), these threshold corrections are implemented as Mathematica expressions in the high-scale boundary condition:

```mathematica
HighScaleInput = {
  {\Lambda, lambdaTree (* tree-level *)
   + lambda1LReg + lambda1LPhi (* 1-loop *)
   + lambda1LCh1 + lambda1LCh2 (* 1-loop *)
   + lambda1Lbottom + lambda1Ltau (* 1-loop *)
   + ... (* 2-loop *)
  }
};

(* arXiv:1407.4081, Eq. (3) *)
lambdaTree = 1/4 (g2^2 + 3/5 g1^2) Cos[2 ArcTan[TanBeta]]^2;

(* arXiv:1407.4081, Eq. (9) *)
lambda1LReg = 1/(4 Pi)^2 (- 9/100 g1^4 - 3/10 g1^2 g2^2
   - (3/4 - Cos[2 ArcTan[TanBeta]]^2/6) * g2^4);

(* arXiv:1407.4081, Eq. (10) *)
lambda1LPhi = 1/(4 Pi)^2 (3 Yu[3,3]^2 (Yu[3,3]^2
   + 1/2 (g2^2-g1^2/5) Cos[2 ArcTan[TanBeta]]
   ) Log[msq2[3,3]/SCALE^2]
   + 3 Yu[3,3]^2 (Yu[3,3]^2
   + 2/5 g1^2 Cos[2 ArcTan[TanBeta]]
   ) Log[msu2[3,3]/SCALE^2]
   + Cos[2 ArcTan[TanBeta]]^-2/300 (3 (g1^4 + 25 g2^4)
     + Log[msq2[1,1]/SCALE^2]
     + Log[msq2[2,2]/SCALE^2]
     + Log[msq2[3,3]/SCALE^2]
   )
   + ...
   + ...
};
```

The full expressions for the threshold corrections can be found in the HSSUSY model file. In addition, HSSUSY makes use of the known 3-loop SM $\beta$ functions, 3-loop corrections to the running top Yukawa coupling, 3-loop threshold corrections to the strong coupling and up to 3-loop corrections to the Higgs pole mass:

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With all these corrections enabled, **HSSUSY** can be regarded as an improved variant of **SusyHD** [160], the difference being that **HSSUSY** includes generalized 2-loop threshold corrections also involving $\alpha_b$ and $\alpha_\tau$, which are not present in **SusyHD**. In Section 10, **HSSUSY** is also compared to the fixed-order calculation in the full MSSM as well as to **FlexibleEFTHiggs**. The following Examples 6–8 show how **HSSUSY** can be run and how different sources of uncertainty can be estimated.\footnote{The example scripts can be found in the `doc/examples-2.0/` sub-directory of the **FlexibleSUSY** package.}

**Example 6**

For illustration, we show in this example the Higgs mass prediction in the MSSM with **HSSUSY**. In the following example script, a scan over the relative $\overline{\text{DR}}$ stop mixing parameter $X_t/M_S$ is performed for $\tan \beta(M_S) = 5$ and three different values of the SUSY scale $M_S$. This is done in parallel on all available CPU cores.
Example 7

In this example, we slightly extend Example 6 to make a partial uncertainty estimate of the Higgs pole mass predicted by HSSUSY at the 2-loop level. We do this by varying the threshold correction loop orders which determine the running \( \overline{\text{MS}} \) top Yukawa coupling \( y_t \) and the strong coupling in the SM from 2-loop to 3-loop. In addition, the renormalization scale, at which the Higgs pole mass is calculated, is varied by a factor 2. The uncertainty estimated in this way is referred to as “Standard Model uncertainty” in the literature [31, 160] and is one part of the full uncertainty of the EFT calculation of HSSUSY.
\( (\text{Log}[\text{stop}] - \text{Log}[\text{start}])/\text{steps}; \)

\(*\text{ generate logarithmically spaced range } [Q/2, 2Q] *\)
\[\text{GenerateScales}\[Q\_] := \text{LogRange}[Q/2, 2Q, 10];\]
\[\text{CalcMh}[\text{MS}_\_, \text{TB}_\_, \text{Xt}_\_, \text{ytLoops}_\_, \text{asLoops}_\_, \text{Qpole}_\_] :=\]
\[\text{Module}\[\{\text{handle}, \text{spec}\},\]
\[\text{handle} = \text{FSHSSUSYOpenHandle}\[\]
\[\text{fsSettings} -> \{\]
\[\text{precisionGoal} -> 1.\text{^-5},\]
\[\text{calculateStandardModelMasses} -> 1,\]
\[\text{poleMassLoopOrder} -> 2,\]
\[\text{ewsbLoopOrder} -> 2,\]
\[\text{betaFunctionLoopOrder} -> 3,\]
\[\text{thresholdCorrectionsLoopOrder} -> 3,\]
\[\text{poleMassScale} -> \text{Qpole},\]
\[\text{thresholdCorrections} -> 120111021 + \]
\[\text{ytLoops} \times 10^{-6} + \text{asLoops} \times 10^{-2}\]
\],\]
\[\text{fsModelParameters} -> \{\]
\[\text{TanBeta} -> \text{TB},\]
\[\text{MEWSB} -> 173.34,\]
\[\text{MSUSY} -> \text{MS},\]
\[\text{M1Input} -> \text{MS},\]
\[\text{M2Input} -> \text{MS},\]
\[\text{M3Input} -> \text{MS},\]
\[\text{MuInput} -> \text{MS},\]
\[\text{mAInput} -> \text{MS},\]
\[\text{AtInput} -> (\text{Xt} + 1/\text{TB}) \times \text{MS},\]
\[\text{msq2} -> \text{MS}^{-2} \text{IdentityMatrix}[3],\]
\[\text{msu2} -> \text{MS}^{-2} \text{IdentityMatrix}[3],\]
\[\text{msd2} -> \text{MS}^{-2} \text{IdentityMatrix}[3],\]
\[\text{msl2} -> \text{MS}^{-2} \text{IdentityMatrix}[3],\]
\[\text{ms2} -> \text{MS}^{-2} \text{IdentityMatrix}[3],\]
\[\text{LambdaLoopOrder} -> 2,\]
\[\text{TwoLoopAtAs} -> 1,\]
\[\text{TwoLoopAbAs} -> 1,\]
\[\text{TwoLoopAtAb} -> 1,\]
\[\text{TwoLoopAtauAtau} -> 1,\]
\[\text{TwoLoopAtAt} -> 1\]
\],\]
\[\text{spec} = \text{FSHSSUSYCalculateSpectrum}[\text{handle}];\]
\[\text{FSHSSUSYCloseHandle}[\text{handle}];\]
\[\text{If}[\text{spec} =!= \$Failed, \text{Pole}[\text{M}[\text{hh}]] /. (\text{HSSUSY} /. \text{spec}), 0]\]
];

\(*\text{ calculate Higgs mass with uncertainty estimate } *\)
\[\text{CalcDMh}[\text{MS}_\_, \text{TB}_\_, \text{Xt}_\_] :=\]
\[\text{Module}\[\{\text{Mh}, \text{MhYt3L}, \text{MhAs3L}, \text{varyQpole}, \text{DMh}\},\]
\[\text{Mh} = \text{CalcMh}[\text{MS}, \text{TB}, \text{Xt}, 2, 2, 0];\]
\[\text{MhYt3L} = \text{CalcMh}[\text{MS}, \text{TB}, \text{Xt}, 3, 2, 0];\]
\[\text{MhAs3L} = \text{CalcMh}[\text{MS}, \text{TB}, \text{Xt}, 2, 3, 0];\]
\[\text{varyQpole} = \text{CalcMh}[\text{MS}, \text{TB}, \text{Xt}, 2, 2, \#]\& /\#\]
\[\text{GenerateScales}[173.34];\]
\[\text{DMh} = \text{Max}[\text{Abs}[\text{Max}[\text{varyQpole} - \text{Mh}],\]
\[\text{Abs}[\text{Min}[\text{varyQpole} - \text{Mh}]] + \]

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Abs[Mh - MhYt3L] + Abs[Mh - MhAs3L];
{ Mh, DMh }

LaunchKernels[];
DistributeDefinitions[CalcDMh];
data = {
  ParallelMap[{#, CalcDMh[1000 , 5, #]}&, Range[-3.5 , 3.5 , 0.1]],
  ParallelMap[{#, CalcDMh[2000 , 5, #]}&, Range[-3.5 , 3.5 , 0.1]],
  ParallelMap[{#, CalcDMh[10000 , 5, #]}&, Range[-3.5 , 3.5 , 0.1]]
};

In the function CalcDMh[], the three sources of uncertainty are combined linearly. When drawing the uncertainty $\Delta M_{h}^{SM}$, estimated in this way, around the central value as $M_{h} \pm \Delta M_{h}^{SM}$, the following figure results:

![Figure showing the uncertainty $\Delta M_{h}^{SM}$ around $M_{h}$](image)

We find that the uncertainty $\Delta M_{h}^{SM}$ is around or below 500 MeV.

**Example 8**

In this example, we make an estimate of the “EFT uncertainty” for the Higgs pole mass predicted by the pure-EFT calculation of HSSUSY in the MSSM. The pure-EFT calculation performed by HSSUSY neglects terms of $O(v^2/M_S^2)$. We estimate these missing terms by multiplying the individual 1-loop contributions by the term $(1 + \Delta EFT \times v^2/M_S^2)$, where $\Delta EFT$ is an input parameter of HSSUSY. By varying $\Delta EFT$ between 0 and 1 we obtain an estimate of the effect of these missing terms. This method has also been used in Refs. [31, 137, 160]. The following code snippet illustrates how to calculate the Higgs mass with HSSUSY and estimate the “EFT uncertainty” as a function of the SUSY scale for maximal stop mixing, $X_t = \sqrt{6} M_S$ and $\tan \beta = 5$.

Get["models/HSSUSY/HSSUSY_librarylink.m"];
(* generate logarithmically spaced range [start, stop] *)
LogRange[start_, stop_, steps_] :=
    Exp /@ Range[Log[start], Log[stop],
        (Log[stop] - Log[start])/steps];

CalcMh[MS_, TB_, Xt_, deltaEFT_] :=
    Module[{handle, spec},
        handle = FSHSSUSYOpenHandle[
            fsSettings -> {
                precisionGoal -> 1.*^-5,
                calculateStandardModelMasses -> 1,
                poleMassLoopOrder -> 2,
                ewsbLoopOrder -> 2,
                betaFunctionLoopOrder -> 3,
                thresholdCorrectionsLoopOrder -> 3,
                thresholdCorrections -> 122111221
            },
            fsModelParameters -> {
                TanBeta -> TB,
                MEWSB -> 173.34,
                MSUSY -> MS,
                M1Input -> MS,
                M2Input -> MS,
                M3Input -> MS,
                MuInput -> MS,
                mAInput -> MS,
                AtInput -> (Xt + 1/TB) * MS,
                msq2 -> MS^-2 IdentityMatrix[3],
                msu2 -> MS^-2 IdentityMatrix[3],
                msd2 -> MS^-2 IdentityMatrix[3],
                ms12 -> MS^-2 IdentityMatrix[3],
                mse2 -> MS^-2 IdentityMatrix[3],
                LambdaLoopOrder -> 2,
                TwoLoopAtAs -> 1,
                TwoLoopAbAs -> 1,
                TwoLoopAtAb -> 1,
                TwoLoopAtauAtau -> 1,
                TwoLoopAtAt -> 1,
                DeltaEFT -> deltaEFT
            }];
        spec = FSHSSUSYCalculateSpectrum[handle];
        FSHSSUSYCloseHandle[handle];
        If[spec =!= $Failed, Pole[M[hh]] /. (HSSUSY /. spec), 0];
    ];

(* calculate Higgs mass with uncertainty estimate *)
CalcDMh[MS_, TB_, Xt_] :=
    Module[{Mh, MhEFT},
        Mh = CalcMh[MS, TB, Xt, 0];
        MhEFT = CalcMh[MS, TB, Xt, 1];
        {Mh, Abs[Mh - MhEFT]}
    ];

LaunchKernels[];
DistributeDefinitions[CalcDMh];
data = ParallelMap[{#, Sequence @@ CalcDMh[#, 5, Sqrt[6]]} &, ...
In the function \texttt{CalcDMh()}, the difference between the Higgs masses calculated with \texttt{DeltaEFT = 0} and \texttt{DeltaEFT = 1} is used as the uncertainty estimate (\texttt{DeltaEFT = 0} corresponds to the standard HSSUSY calculation). When drawing the uncertainty estimated in this way symmetrically around the central value, the following figure results:

As expected, we find that the “EFT uncertainty” decreases as \( M_S \) increases and falls below 150 MeV for \( M_S \gtrsim 2 \) TeV.

### 5.6. Tests and comparisons with other spectrum generators

We have performed various direct and indirect tests of \texttt{FlexibleBSM} and its components to ensure the correctness of the code:

- We have performed an analytic comparison of the RGEs generated with \texttt{SARAH} 4.5.3 for \texttt{FlexibleSUSY}’s split-MSSM model file (\texttt{SplitMSSM}) against the RGEs presented in Ref. \[197\] and we found exact agreement.

- We have performed a detailed numerical comparison of \texttt{FlexibleSUSY}’s HSSUSY model against \texttt{SusyHD} 1.0.1 and found excellent agreement \[31\]. The small differences between the two programs are of \( O(100 \) MeV) and originate from a different determination of \( y_t \) in the SM at the low-energy scale, a different procedure to calculate the Higgs pole mass in the SM and the inclusion of additional 2-loop corrections in HSSUSY which involve \( \alpha_b \) and \( \alpha_r \).

- The effective THDM shown in Section 5.4 and Appendix D reproduces the results of Ref. \[202\] and \texttt{MhEFT} 1.0 \[203\] for scenarios with heavy Higgsinos and gauginos, see for example Figure 5.

- The \texttt{FlexibleSUSY} package contains various EFT scenarios of the MSSM with boundary conditions from the literature (HSSUSY, SplitMSSM, THDMIIMSSMBC, HTHDMIIMSSMBC, HGTHDMIIMSSMBC). For all these models, we have performed various analytic tests of
the implemented MSSM boundary conditions, checking for example the renormalization scale dependence, relations among the parameters and threshold corrections at the 1-loop level.

- We have checked the numeric equality of the 3-loop MSSM $\beta$ functions implemented in FlexibleSUSY and SOFTSUSY 3.7.0.
- We have checked the correctness of the renormalization scale dependent part of the 2-loop QCD corrections to the $\overline{\text{MS}}$ top Yukawa coupling in the SM by deriving the 2-loop threshold corrections for $y_t$ from the SM to the MSSM and checking that no large logarithms appear [204].
- We have also analytically checked the correctness of the renormalization scale dependent part of the 2-loop and 3-loop QCD corrections to the $\overline{\text{MS}}$ top Yukawa coupling in the SM by proving the renormalization scale invariance of the top quark pole mass in the SM at the 3-loop QCD level.

6. FlexibleAMU extension

FlexibleSUSY 2.0 introduces a calculation of the BSM contributions to the anomalous magnetic moment of the muon, $a_{\mu}^{\text{BSM}}$, in the $\overline{\text{MS}}$/DR scheme at the 1-loop level in the model under consideration plus the universal 2-loop QED contributions [205, 206]. The 1-loop diagram types that are taken into account are shown in Figure 6. The general result

![Figure 6: Generic diagram types implemented in FlexibleSUSY to calculate the 1-loop contribution to $a_{\mu}^{\text{BSM}}$. The solid line in the loop represents any contributing non-SM fermion $F$ and the dashed line any contributing scalar particle $S$.](image_url)

for these two Feynman diagram types reads, in a notation based on Refs. [207–209],

\[
a_{\mu}^{\text{BSM},1L,FFS} = \frac{c}{(4\pi)^2} \frac{M_\mu m_\mu}{m_S^2} \left( \frac{1}{12} A_{SF} F_1^C(x) + \frac{m_F}{3m_\mu} B_{SF} F_2^C(x) \right),
\]

\[
a_{\mu}^{\text{BSM},1L,SSF} = -\frac{c}{(4\pi)^2} \frac{M_\mu m_\mu}{m_S^2} \left( \frac{1}{12} A_{SF} F_1^N(x) + \frac{m_F}{6m_\mu} B_{SF} F_2^N(x) \right),
\]

where $x = m_F^2/m_S^2$ is the squared running mass ratio between $F$ and $S$, $M_\mu$ is the muon pole mass, $m_\mu$ is the muon $\overline{\text{MS}}$/DR mass and $c$ denotes the electric charge of the particle coupling to the photon divided by the muon charge. The $A_{SF}$ and $B_{SF}$ constants are defined as

\[
A_{SF} = z_L z_L^* + z_R z_R^*,
\]
\[ BSF = z_L z_R^* + z_R z_L^*, \]  
(39)

where \( z_L \) and \( z_R \) are the left- and right-handed components of the scalar–fermion–muon vertex, \( \Gamma_{SF\mu} = ie(z_L P_L + z_R P_R) \) with \( e \) being the \( \text{MS}/\text{DR} \) electromagnetic coupling constant. The loop functions \( F_i^C(x) \) and \( F_i^N(x) \) read

\[
F_1^C(x) = \frac{2}{(1-x)^4} \left[ 2 + 3x - 6x^2 + x^3 + 6x \log x \right],
\]
(40)

\[
F_2^C(x) = \frac{3}{2(1-x)^3} \left[ -3 + 4x - x^2 - 2 \log x \right],
\]
(41)

\[
F_1^N(x) = \frac{2}{(1-x)^4} \left[ 1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x \right],
\]
(42)

\[
F_2^N(x) = \frac{3}{(1-x)^3} \left[ 1 - x^2 + 2x \log x \right].
\]
(43)

To calculate \( a_{\text{BSM,1L}}^\mu \), FlexibleSUSY sums over all possible instantiations of these two 1-loop diagram types with at least one non-SM particle in the loop,

\[
a_{\mu}^{\text{BSM,1L}} = \sum_{\text{FFS \ diagrams}} a_{\mu}^{\text{BSM,1L,FFS}} + \sum_{\text{SSF \ diagrams}} a_{\mu}^{\text{BSM,1L,SSF}}.
\]
(44)

The calculation is performed at the renormalization scale \( M_{\text{LCP}} \), which is defined as the running mass of the lightest electrically charged BSM particle contributing to \( a_{\text{BSM,1L}}^\mu \). In addition to this 1-loop BSM contribution, FlexibleAMU adds the universal leading logarithmic 2-loop QED contribution [205],

\[
a_{\mu}^{\text{QED,2L}} = a_{\mu}^{\text{BSM,1L}} \times \left[ 16 \frac{a_{\text{em}}^{\text{BSM}}(M_{\text{LCP}})}{4\pi} \log \frac{M_{\mu}}{M_{\text{LCP}}} \right].
\]
(45)

The overall result for \( a_{\mu}^{\text{BSM}} \) is then given by the sum of 1-loop and the 2-loop QED contribution,

\[
a_{\mu}^{\text{BSM}} = a_{\mu}^{\text{BSM,1L}} + a_{\mu}^{\text{QED,2L}}.
\]
(46)

6.1. Choosing FlexibleAMU in the model file

In order to calculate \( a_{\mu}^{\text{BSM}} \) with FlexibleSUSY, the FlexibleSUSYObservable'aMuon symbol has to be added to an output block, see Table 4 and Example 9.

| Symbol | Default value | Allowed values | Description |
|--------|---------------|----------------|-------------|
| FlexibleSUSYObservable'aMuon | – | – | Represents \( a_{\mu}^{\text{BSM}} \) calculated with FlexibleAMU |
| FlexibleSUSYObservable'aMuonGM2Calc | – | – | Represents \( a_{\mu}^{\text{MSSM}} \) calculated with GM2Calc at 2-loop level |
| FlexibleSUSYObservable'aMuonGM2CalcUncertainty | – | – | Represents the uncertainty \( \Delta a_{\mu}^{\text{MSSM}} \) calculated with GM2Calc |

Table 4: FlexibleAMU model file options.
Example 9

In the following code snippet, the `FlexibleSUSYObservable`'aMuon symbol is added to an output block named `FlexibleSUSYLowEnergy`. The calculated $\alpha^{\text{BSM}}_\mu$ will be written to the entry 21 of this block.

```cpp
ExtraSLHAOutputBlocks = {
    {FlexibleSUSYLowEnergy,
    {21, FlexibleSUSYObservable'aMuon} }
};
```

Note that $\alpha^{\text{BSM}}_\mu$ is treated as an observable in `FlexibleSUSY`. The calculation of observables can be enabled/disabled using the flag 15 of the `FlexibleSUSY` block.

As an alternative to `FlexibleAMU`, `GM2Calc` [210] can be used to calculate $\alpha^{\text{MSSM}}_\mu$ in `FlexibleSUSY` at the 2-loop level employing results from Refs. [206, 209, 211] in MSSM models without flavor violation. This MSSM-specific option was first introduced in `FlexibleSUSY` 1.3.0. In order to let `GM2Calc` calculate the anomalous magnetic moment of the muon, the symbol `FlexibleSUSYObservable`'aMuonGM2Calc must be added to an output block. In addition, the symbol `FlexibleSUSYObservable`'aMuonGM2CalcUncertainty can be used to calculate the estimated corresponding theory uncertainty for $\alpha^{\text{MSSM}}_\mu$.

Example 10

Calculating $\alpha^{\text{MSSM}}_\mu$ with `GM2Calc` in the `CMSSMNoFV` is enabled by defining:

```cpp
ExtraSLHAOutputBlocks = {
    {FlexibleSUSYLowEnergy,
    {1, FlexibleSUSYObservable'aMuonGM2Calc},
    {2, FlexibleSUSYObservable'aMuonGM2CalcUncertainty} }
};
```

`GM2Calc` is incorporated into `FlexibleSUSY` in the form of an addon. In order to enable the `GM2Calc` addon, the `--with-addons=GM2Calc` argument can be passed to the `configure` script during the `FlexibleSUSY` configuration step.

6.2. Structure of `FlexibleAMU` code

The C++ interface for `FlexibleAMU` is defined in the `<model>_a_muon.hpp` file. The interface consists of a single function, which takes a model object as the argument and returns the value of $\alpha^{\text{BSM}}_\mu$.

Example 11

In the `CMSSM`, the interface function reads:

```cpp
namespace CMSSM_a_muon {
  double calculate_a_muon(const CMSSM_mass_eigenstates& model);
}
```
6.3. Tests and comparisons with other calculations

We have tested FlexibleAMU in the MSSM against GM2Calc 1.3.3 [210], which is currently the most precise code available to calculate $a_\mu^{\text{MSSM}}$. We usually find a 1–10\% deviation depending on the parameter choice, see Table 5, except for specific parameter points, where the deviation is much larger due to a large renormalization scale dependence, see below. The 1–10\% deviation is caused mainly by the fact that GM2Calc calculates $a_\mu^{\text{MSSM}}$ in an on-shell scheme and includes all known 2-loop corrections, while FlexibleAMU performs the calculation in the $\overline{\text{MS}}/\overline{\text{DR}}$ scheme at the lightest charged BSM particle scale $M_{\text{LCP}}$ and only includes the leading logarithmic 2-loop QED correction. A comparison of FlexibleAMU with SPheno 4.0.2 shows differences of up to 10\%. These differences are caused by the fact that FlexibleAMU includes the leading logarithmic 2-loop QED contribution and by the different choice of the renormalization scale: SPheno performs the calculation at the $Z$ pole mass scale, while FlexibleAMU performs it at $M_{\text{LCP}}$.

In specific parameter scenarios, where the $\overline{\text{MS}}/\overline{\text{DR}}$ smuon masses show a high sensitivity to the renormalization scale, the predictions of $a_\mu^{\text{BSM}}$ in FlexibleAMU and SPheno are expected to have a large theory uncertainty, see for example BM4' in Table 5 and see the discussion in Ref. [210]. One reason for this large uncertainty are the missing 2-loop corrections in FlexibleAMU and SPheno, which would (if included) cancel the renormalization scale dependence at the 2-loop level. To illustrate the sensitivity on the renormalization scale, we show in the third column of Table 5 the variation of $a_\mu^{\text{BSM}}$ in FlexibleAMU when the scale is varied in the interval $[M_{\text{LCP}}/2, 2M_{\text{LCP}}]$. For BM4', $a_\mu^{\text{BSM}}$ varies by around 66\%, which indicates a very imprecise prediction for this point. Such a large scale uncertainty is avoided in GM2Calc due to the renormalization in an on-shell scheme. Another reason for the larger uncertainty in FlexibleAMU and SPheno is the renormalization scheme used to renormalize the smuon masses: In the $\overline{\text{MS}}/\overline{\text{DR}}$ scheme, smuon self energy contributions which are quadratic in the BSM particle masses can lead to large 1-loop corrections to the smuon masses. Such large corrections are avoided in the on-shell scheme, where they are absorbed into the smuon mass counter term.

| Point | FlexibleAMU | FlexibleAMU scale variation | GM2Calc | SPheno |
|-------|-------------|----------------------------|---------|--------|
| SPS1a | 29.77       | 0.46                       | 29.31 ± 2.36 | 31.00  |
| SPS1b | 32.46       | 0.45                       | 32.38 ± 2.40 | 32.68  |
| SPS3  | 13.80       | 0.12                       | 13.52 ± 2.33 | 14.99  |
| SPS4  | 50.02       | 1.02                       | 52.45 ± 2.64 | 45.64  |
| BM1'  | 42.08       | 1.58                       | 42.34 ± 2.33 | 43.72  |
| BM2'  | 25.79       | 0.10                       | 25.67 ± 2.32 | 26.16  |
| BM3'  | 27.81       | 0.68                       | 27.95 ± 2.34 | 27.98  |
| BM4'  | 8.11        | 5.41                       | 33.11 ± 2.31 | 2.19   |

Table 5: Comparison of $a_\mu^{\text{MSSM}} \cdot 10^{10}$ calculated with FlexibleAMU, GM2Calc 1.3.3 and SPheno 4.0.2 for the CMSSM benchmark points presented in Ref. [212] and the parameter points shown in Table 6. The third column shows the variation of $a_\mu^{\text{BSM}}$ when the renormalization scale is varied between $M_{\text{LCP}}/2$ and $2M_{\text{LCP}}$. For BM4' the value of $a_\mu$ calculated by the $\overline{\text{DR}}$ programs FlexibleAMU and SPheno suffers from a high renormalization scale sensitivity due to the large values of $M_2$ and $(m_{l})_{ii}$, which leads to a very imprecise result and to a huge deviation compared to GM2Calc.
Table 6: Definition of the MSSM benchmark points BM1'–BM4', inspired by the points BM1–BM4 presented in Ref. [211]. All parameters are defined in the DR scheme at the scale $Q = 45.47$ GeV, except for $\tan \beta$, which is defined at $M_Z$. The trilinear couplings and off-diagonal elements of the sfermion mass parameters are set to zero and we have fixed $m_A = 2$ TeV.

7. FlexibleCPV extension

7.1. Setting up a FlexibleCPV model

Since FlexibleSUSY 1.1.0, the model parameters are no longer restricted to be real, but can be complex. Whether a parameter is real or complex is specified in the corresponding SARAH model file. Parameters can be forced to be treated as real in FlexibleSUSY by adding them to the `RealParameters` list in the FlexibleSUSY model file, see Table 7 and the following examples. For compatibility with FlexibleSUSY 1.0, the `RealParameters` list is by default set to `{All}` meaning that all parameters are assumed to be real.

| Symbol   | Default value | Allowed values | Description                          |
|----------|---------------|----------------|---------------------------------------|
| RealParameters | `{All}` | List of model parameters or `{}` or `{All}` | List of parameters to be treated as real |

Table 7: FlexibleCPV model file options.

Example 12

In the MSSM, the $\mu$ parameter, the Yukawa couplings, the soft-breaking trilinear couplings, the soft-breaking scalar mass parameters, the soft-breaking gaugino mass parameters and the $B\mu$ parameter can be complex. In order to choose all of these parameters to be complex, except for $B\mu$, one can set

```plaintext
RealParameters = { B\{M[Mu]\} };
```

Example 13

In order to treat all MSSM parameters defined in the SARAH model file for the MSSM as complex, set `RealParameters` to the empty list:

```plaintext
RealParameters = {};
```
7.2. Application: CMSSM with CP-violation

In SARAH’s MSSM model, the phase factor between the two Higgs doublets is set to zero. Therefore, this model does not allow for CP-violation in the Higgs sector. In order to enable CP-violation in the MSSM Higgs sector, SARAH’s MSSM/CPV model file can be used, which allows for a non-zero relative phase factor $e^{i\eta}$ between the Higgs doublets. In the MSSM/CPV, there are three linearly independent EWSB equations. Therefore, in this model three EWSB output parameters have to be chosen. In FlexibleSUSY’s CMSSMCPV model file, these are chosen to be $\text{Re} B\mu$, $\text{Im} B\mu$ and $|\mu|$ by setting the EWSBOutputParameters variable to

```plaintext
EWSBOutputParameters = { Re[B\[\[Mu\]]], Im[B\[\[Mu\]]], \[Mu\] };
```

Since only the magnitude of the $\mu$ parameter is fixed by the EWSB equations, FlexibleSUSY introduces the phase of $\mu$ as a free parameter, $\text{Phase}[\mu] = e^{i\phi_\mu}$. This phase should be specified in an SLHA-2 compliant way by reading the real and imaginary parts of $e^{i\phi_\mu} = \cos \phi_\mu + i \sin \phi_\mu$ from the MINPAR and IMMINPAR block entries 4 and fixing $e^{i\phi_\mu}$ at the SUSY scale:

```plaintext
MINPAR = { {4, CosPhiMu} };
IMMINPAR = { {4, SinPhiMu} };
SUSYScaleInput = {
    {Phase[\[Mu\]], CosPhiMu + I SinPhiMu}
};
```

In the CMSSMCPV, the phase $\eta$ is read from the EXTPAR block entry 100 and also chosen to be input at the SUSY scale:

```plaintext
EXTPAR = {
    {100, etaInput}
};
SUSYScaleInput = {
    {eta, etaInput}
};
```

The complete FlexibleSUSY CMSSMCPV model file can be found in Appendix C.

7.3. Application: Electric dipole moments of fermions

FlexibleSUSY 2.0 can calculate $d^\text{BSM}_f$, new physics contributions to the electric dipole moment (EDM) of a fermion $f$, in the given model in the DR scheme at the 1-loop level. The procedure is very similar to the calculation of $a^\text{BSM}_\mu$ described in Section 6. This is expected from the following effective Lagrangian,

$$\Delta \mathcal{L}_{\text{eff}} = - \frac{D_f}{2} \bar{f} L \sigma_{\mu\nu} f R F^{\mu\nu} + \text{h.c.},$$  \hspace{1cm} (47)
where the real and the imaginary parts of the Wilson coefficient $D_f$ are proportional to the magnetic (see, e.g., Refs. [213, 214]) and the electric (see, e.g., Ref. [215]) dipole moments of $f$, respectively. More precisely,

$$a_f = -\frac{2M_f}{e}\ Re\ D_f, \quad d_f = \Im\ D_f, \quad (48)$$

where $M_f$ is the pole mass of $f$ and $e$ is the running electromagnetic coupling constant in the BSM model. It is then obvious that the EDM is given by the sum of all FFS-type and SSF-type diagrams,

$$d_{BSM,1L}^f = \sum_{\text{FFS diagrams}} d_{BSM,1L,FFS}^f + \sum_{\text{SSF diagrams}} d_{BSM,1L,SSF}^f, \quad (49)$$

as in Eq. (44). The type of diagram refers to those shown in Figure 6, resulting in the contributions,

$$-\frac{1}{e} d_{f}^{BSM,1L,FFS} = c\left(\frac{4\pi}{2}\right)^2 \frac{m_F}{6m_S} \tilde{B}_{SF}F_C^C(x), \quad \quad (50)$$

$$-\frac{1}{e} d_{f}^{BSM,1L,SSF} = -c\left(\frac{4\pi}{2}\right)^2 \frac{m_F}{12m_S^2} \tilde{B}_{SF}F_N^N(x), \quad \quad (51)$$

which are essentially the second terms of Eq. (36) and Eq. (37), respectively, divided by $2M_f$, except that the coupling factor is instead

$$\tilde{B}_{SF} = 2\Im(z_Lz_R^*). \quad (52)$$

There are no imaginary parts corresponding to the first terms of Eq. (36) and Eq. (37) as can be guessed from Eq. (38). The calculation is performed at the renormalization scale $M_S$ specified in the model file, which is typically set to the stop mass scale in supersymmetric models.

One can have the EDM of particle $f$ calculated by adding to an output block the form: FlexibleSUSYObservable‘EDM[ff]. Then $d_f$ is reported in units of GeV$^{-1}$.

**Example 14**

The output includes the EDMs of the electron, muon, and tau if the following code snippet is inserted into the FlexibleSUSY model file:

```latex
ExtraSLHAOutputBlocks = {
    {FlexibleSUSYLowEnergy,
        {23, FlexibleSUSYObservable‘EDM[Fe[1]]},
        {24, FlexibleSUSYObservable‘EDM[Fe[2]]},
        {25, FlexibleSUSYObservable‘EDM[Fe[3]]} }
};
```

Using a model file thus configured, one can do a quick test of the electron EDM evaluation in the CP-violating MSSM for instance. In this model, only $\arg(\mu M_{1,2,3})$ and $\arg(\mu A_f)$ are physical among the flavor-conserving phases apart from those already present in the SM (see, e.g., Refs. [207, 215]). For simplicity, the gaugino masses are assumed to have an equal (complex) value $M_{1/2}$, and both soft masses of the left- and right-handed selectron as well as the approximate tree-level heavy Higgs masses$^{12}$ shall be $m_0$. The moduli of $\mu$,
$M_{1/2}$, $m_0$, and $A_e$ are specified at the SUSY scale, all of which including the scale are set to 2 TeV. Finally, we set $\tan\beta = 10$ at the $M_Z$ scale.

The resulting electron EDM is displayed as contours in Figure 7. In the left panel, $\phi_{M_{1/2}} \equiv \text{arg} M_{1/2}$ and $\phi_{\mu} \equiv \text{arg} \mu$ are varied while $\text{arg}(\mu A_e)$ is constrained to be zero. From the directions of the contours it is clear that FlexibleSUSY correctly reproduces the behaviour of $d_e$ depending only on the rephasing invariant $\text{arg}(\mu M_{1/2})$. In the right panel, the roles of $\phi_{A_e} \equiv \text{arg} A_e$ and $\phi_{M_{1/2}}$ are swapped so that $\text{arg}(\mu M_{1/2})$ stays at zero. Again, the contours verify that $d_e$ from FlexibleSUSY is determined by the physical phase $\text{arg}(\mu A_e)$. For reference, the experimental upper limit on $|d_e|$ at the 90% confidence level is shown as the thick red lines [216].

7.4. Tests and comparisons with other spectrum generators

In the left panel of Figure 8, we show the lightest Higgs pole mass calculated in the $CP$-violating CMSSM at the 1-loop level with FlexibleSUSY 2.0 and SPheno 4.0.2 as a function of the phase angle of the complex $\mu$ parameter, $\phi_{\mu} \equiv \text{arg} \mu$. We use a low-energy scenario with $m_0 = M_{1/2} = 500$ GeV, $\tan\beta = 10$ and $A_0 = 0$. Even though this scenario is excluded, the figure illustrates that both FlexibleSUSY and SPheno show the same behaviour of the Higgs pole mass as a function of $\phi_{\mu}$. The $\phi_{\mu}$-independent shift of around 0.7 GeV between the Higgs masses calculated by the two programs is mainly caused by the different treatment of higher-order corrections to the running $\overline{\text{DR}}$ top Yukawa coupling, which has been discussed in Refs. [128, 204].

The right panel of Figure 8 shows the renormalization group running of the $(3, 3)$-component of the complex trilinear coupling $T^u$ for the same CMSSM scenario, except that we use $\phi_{\mu} = 0$ and $A_0 = 500 e^{i\pi/4}$ GeV. The lines show the running in FlexibleSUSY 2.0 and the dots the running in SPheno 4.0.2. We find very good agreement of the running of $(T^u)_{33}$ between the two programs over the shown 14 orders of magnitude. The maximum deviation between FlexibleSUSY and SPheno is around 2% for $Q \approx 10^{16}$ GeV and is
mainly caused by a different determination of the dimensionless MSSM parameters at the electroweak scale.

8. FlexibleMW extension

FlexibleMW is a major new feature which is released in FlexibleSUSY 2.0. It allows a more accurate determination of the electroweak gauge couplings $g_1$ and $g_2$, the prediction of the $W$ pole mass and thus enables direct comparisons with electroweak precision data.

The running electroweak gauge couplings are related to the running electromagnetic coupling $e$, the weak mixing angle $\theta_W$ and the normalization factors $N$ by

$$g_1 = N_{g_Y} \frac{e}{\cos \theta_W}, \quad g_2 = N_{g_L} \frac{e}{\sin \theta_W}. \quad (53)$$

The electromagnetic gauge coupling can be obtained from the known value of $\alpha_{\text{em}}(M_Z)$ as described in Section 5 using the known general form of the threshold correction, Eq. (27). The weak mixing angle, together with the $W$ and $Z$ pole masses and the muon decay constant $G_F$, form a set of four electroweak precision quantities. Within the theory, only two of them are independent. Hence, the running weak mixing angle can be calculated in different ways:

1. Using the $W$ and $Z$ pole masses $M_W$ and $M_Z$ as input and calculating the cosine of the running weak mixing angle for example as:

$$\cos^2 \theta_W = \frac{m_W^2}{m_Z^2}, \quad (54)$$

where $m_W$ and $m_Z$ are the running $W$ and $Z$ masses which are obtained from the corresponding pole masses by

$$m_V^2(Q) = M_V^2 + \text{Re} \Sigma_{V,T}(p^2 = M_V^2, Q), \quad (V = W, Z) \quad (55)$$

Figure 8: Left panel: lightest Higgs pole mass calculated in the $CP$-violating CMSSM at the 1-loop level with FlexibleSUSY 2.0 and SPheno 4.0.2 as a function of the phase angle of the $\mu$ parameter for $m_0 = M_{1/2} = 500$ GeV, $\tan \beta = 10$ and $A_0 = 0$. Right panel: renormalization group running of the trilinear coupling $(T_{u33})$ for $m_0 = M_{1/2} = 500$ GeV, $\tan \beta = 10$, $\phi_\mu = 0$ and $A_0 = 500 e^{i\pi/4}$ GeV.
with $\Sigma_{V,T}$ being the transverse part of the vector boson self energy. In models with Higgs triplets having the vacuum expectation value $v_T$ for example, Eq. (54) is adapted accordingly:

$$\cos^2 \theta_W = \frac{m_W^2 - g^2 v_T^2}{m_Z^2}. \tag{56}$$

This approach was implemented in the first release of FlexibleSUSY [30], as a generalization of the calculation in Ref. [217].

2. Using the $Z$ pole mass and the measured muon decay constant $G_F$ as input and determining $\theta_W$ and the $W$ pole mass as described below.

The first approach has the advantage that it can be easily applied to any BSM model with a $W$ and $Z$ boson, because the 1-loop calculation of the running $W$ and $Z$ masses can be fully automatized since only self energies are necessary. However, this approach has the disadvantage that the parametric uncertainty of the calculated electroweak gauge couplings is then limited by that of the measured $W$ pole mass of the order 0.02% [218]. This prohibits a meaningful computation of other electroweak precision observables for which more precise experimental data exist.

The second approach is more complicated to automatize for all BSM models, because also 1-loop vertex and box diagrams contributing to the muon decay have to be taken into account. However, the approach has the advantage that the parametric uncertainty of the calculated electroweak gauge couplings is related to those of $G_F$ and $M_Z$, which are of the order 0.00005% and 0.002%, respectively [218]. As a result, the $W$ pole mass is now a meaningful prediction, which can be used to constrain BSM models.

Before version 2.0, only the first approach could be used in FlexibleSUSY for all BSM models. The second approach was only available in models which are SM-like or (N)MSSM-like, an option introduced in FlexibleSUSY 1.1.0 (as described in Ref. [128]). FlexibleSUSY 2.0 is now able to apply the second approach to all BSM models which have a $W$ and a $Z$ boson and which have the SM gauge group as a gauge group factor.

For the implementation of the second approach, FlexibleSUSY uses a generalization of the procedure presented in Ref. [199] for the SM, which has been adapted to the MSSM in Ref. [219]. The running weak mixing angle is extracted from the relation [199]

$$\sin^2 \theta_W \cos^2 \theta_W = \frac{\pi \alpha_{em}}{\sqrt{2} M_Z^2 G_F \hat{\rho}_{\text{tree}} (1 - \Delta \hat{r})} \tag{57}$$

with the renormalization scale consistently being set to $M_Z$, where $\alpha_{em}$ is the electromagnetic coupling of the BSM model in the $\overline{\text{MS}}/\overline{\text{DR}}$ scheme and

$$\Delta \hat{r} = \Delta \hat{r}_{1L} + \Delta \hat{r}_{2L}^{\text{SM}} \tag{58}$$

with

$$\Delta \hat{r}_{1L} = \frac{1}{1 - \Delta \hat{\rho}} \frac{\text{Re} \Sigma_{W,T}(0)}{M_W^2} - \frac{\text{Re} \Sigma_{Z,T}(M_Z^2)}{M_Z^2} + \delta_{\text{VB}}, \tag{59}$$

$$\Delta \hat{\rho} = \frac{1}{1 + \frac{\text{Re} \Sigma_{Z,T}(M_Z^2)}{M_Z^2}} \left[ \frac{\text{Re} \Sigma_{Z,T}(M_Z^2)}{M_Z^2} - \frac{\text{Re} \Sigma_{W,T}(M_W^2)}{M_W^2} + \Delta \hat{\rho}_{2L}^{\text{SM}} \right]. \tag{60}$$

In the occurring self energies $\Sigma$, the top quark mass is chosen to be the pole mass $M_t$ in order to include partial 2-loop corrections not contained in $\Delta \hat{r}_{2L}^{\text{SM}}$ and $\Delta \hat{\rho}_{2L}^{\text{SM}}$ [220]. Since $\Delta \hat{r}$ and $\Delta \hat{\rho}$ themselves depend on $\theta_W$, an iteration including these equations has to be performed to get a self-consistent solution.
The quantity
\[ \hat{\rho}_{\text{tree}} = \rho_{0} \frac{m_{Z,\text{SM}}^{2}}{m_{Z,\text{mix}}^{2}} \]  
introduces two different generalizations. On the one hand, corrections from higher dimensional Higgs multiplets are included via [221]
\[ \rho_{0} = \frac{\sum_{i} (t_{i}^{2} - t_{3i}^{2} + t_{ii})}{\sum_{i} 2 t_{3i}^{2}} \left| v_{\varphi_{i}} \right|^{2}, \]
where the sums run over all neutral Higgs fields \( \varphi_{i} \) with vacuum expectation value \( v_{\varphi_{i}} \), weak isospin \( t_{ii} \) and its third component \( t_{3i} \). On the other hand, corrections from extra \( U(1) \) gauge groups are included via the ratio of the SM-like tree-level \( Z \) boson mass \( m_{Z,\text{SM}} \) and the tree-level \( Z \) boson mass \( m_{Z,\text{mix}} \) including mixing with additional \( Z' \) bosons [222, 223].

The leading SM 2-loop contributions to \( \Delta \hat{\rho} \) and \( \Delta \hat{\rho} \) [219, 220] are given by\(^{13}\)
\[ \Delta \rho^{\text{SM}}_{2\ell} = \frac{\alpha_{\text{em}} \alpha_{s}^{(5)}}{4\pi^{2}} \frac{\sin^{2} \theta_{W} \cos^{2} \theta_{W}}{2.145} \left[ \frac{M_{t}^{2}}{M_{Z}^{2}} + 0.575 \log \left( \frac{M_{t}}{M_{Z}} \right) - 0.224 - 0.144 \left( \frac{M_{Z}^{2}}{M_{t}^{2}} \right) \right] \]
\[ - \frac{1}{1 - \Delta \hat{\rho}_{1\ell}} \delta_{\text{Higgs}}, \]
\[ \Delta \rho^{\text{SM}}_{2\ell} = \frac{\alpha_{\text{em}} \alpha_{s}^{(5)}}{4\pi^{2}} \sin^{2} \theta_{W} \left[ -2.145 \left( \frac{M_{t}^{2}}{M_{W}^{2}} \right) + 1.262 \log \left( \frac{M_{t}}{M_{Z}} \right) - 2.24 - 0.85 \left( \frac{M_{Z}^{2}}{M_{t}^{2}} \right) \right] + \delta_{\text{Higgs}}, \]  
where the SM strong coupling \( \alpha_{s}^{(5)} \) is taken at the scale \( M_{t} \) and we have generalized the Higgs dependent part as
\[ \delta_{\text{Higgs}} = 3 \left( \frac{G_{F} M_{t} v_{\text{SM}}}{8\pi^{2} \sqrt{2}} \right)^{2} \sum_{i} \left( \left| a_{\varphi_{i}tt} \right|^{2} - \left| b_{\varphi_{i}tt} \right|^{2} \right) \rho^{(2)} \left( \frac{m_{\varphi_{i}}}{M_{t}} \right) \]
\[ \text{(65)} \]
to include corrections from all neutral Higgs fields \( \varphi_{i} \) coupling to the top quark via the vertex \( (1 a_{\varphi_{i}tt} + \gamma_{5} b_{\varphi_{i}tt}) \). For this generalization to work properly, the SM-like vacuum expectation value \( v_{\text{SM}} \) has to be defined and normalized to the value \( \approx 246 \) GeV in the corresponding SARAH model file. The utilized expansions of the function \( \rho^{(2)} \) can be found in Ref. [224].

The model-specific correction in Eq. (59) consists of an SM and a BSM part,
\[ \delta_{\text{VB}} = \delta_{\text{SM}}^{\text{VB}} + \delta_{\text{BSM}}^{\text{VB}} \],  
\[ \text{(66)} \]
where the SM contribution \( \delta_{\text{SM}}^{\text{VB}} \) [199], originating from diagrams with additional internal gauge bosons, is given by Eq. (C.12) from Ref. [219] with the replacement \( \hat{\rho} \rightarrow 1/(1 - \Delta \hat{\rho}) \).\(^{14}\)
The BSM contribution \( \delta_{\text{BSM}}^{\text{VB}} \) contains corrections from 1-loop external wave-function renormalizations, vertex and box diagrams, which are put together as in Eq. (C.13) from the same reference. For the different corrections, FlexibleSUSY considers the diagram types shown in Figure 9 and sums over all possible instantiations of these by inserting the valid combinations of particles into the loop.\(^{15}\)

After the weak mixing angle \( \theta_{W} \) has been calculated via Eq. (57), the \( W \) pole mass can be computed as
\[ M_{W} = \sqrt{\frac{m_{Z}^{2}}{\cos^{2} \theta_{W}} - \frac{\hat{\rho}_{\text{tree}}}{1 - \Delta \hat{\rho}}}. \]
\[ \text{(67)} \]

\(^{13}\)The specific numerical values in these formulas depend on the renormalization scale, which is assumed
Figure 9: Generic diagram types implemented in FlexibleSUSY to calculate the BSM 1-loop contributions to the muon decay, \( \delta^{\text{BSM}}_{\text{VB}} \). The solid and dashed lines in the loops represent any valid combination of fermions and scalars. (a) wave-function renormalization diagrams: \( f \) stands for \( e, \mu, \nu_e \) or \( \nu_\mu \). (b) vertex diagrams: \( e_i \) stands for \( e \) or \( \mu \). (c) box diagrams: the triplet \( (f_1, f_2, f_3) \) stands for any permutation of \( e, \nu_e \) and \( \nu_\mu \).

8.1. Choosing FlexibleMW in the model file

The method to determine the weak mixing angle can be selected by setting the following variable in the FlexibleSUSY model file:

```latex
(* possible values : Automatic, FSFermiConstant or FSMassW *)
FSWeakMixingAngleInput = Automatic;
```

By default, `FSWeakMixingAngleInput` is set to `Automatic`, in which case the weak mixing angle is determined from the muon decay, if all conditions are fulfilled, otherwise the \( W \) mass is used. Further possible values are `FSFermiConstant` and `FSMassW` to explicitly select the muon decay or the \( W \) mass method, respectively.

8.2. Structure of FlexibleMW code

The C++ interface for the determination of the running weak mixing angle \( \theta_W \) and the \( W \) pole mass via muon decay is provided by the class `<model>_weinberg_angle` defined in `<model>_weinberg_angle.hpp`. To construct an object from this class, two arguments have to be provided: a model object, which represents the set of running BSM parameters, and a struct of type `Sm_parameters`. The latter is defined within the `<model>_weinberg_angle` class and contains the required SM parameters, namely the Fermi constant \( G_F \) as well as the pole masses \( M_W, M_Z, M_t \) and \( \alpha_{\text{SM}}^2(M_\ell) \). The values of \( \sin \theta_W \) and \( M_W \) are calculated and returned by the class member function `calculate()`, which includes all of the Eqs. (57)–(67) and the necessary iteration.\(^{16}\)

\(^{14}\) The expression for \( \delta^{\text{SM}}_{\text{VB}} \) implemented in FlexibleSUSY has been extended by a term expressing the scale dependence so that the calculation can be performed consistently at the 1-loop level at scales different from \( M_Z \). However, the 2-loop contributions \( \Delta r^{\text{SM}}_{\text{2L}} \) and \( \Delta \rho^{\text{SM}}_{\text{2L}} \) are omitted if \( Q \neq M_Z \), because their explicit scale dependence is currently not taken into account.

\(^{15}\) Note that self energy, vertex and box diagrams with internal vector bosons are not considered outside of \( \delta^{\text{SM}}_{\text{VB}} \).

\(^{16}\) The input \( W \) pole mass is used as an initial value while the function `calculate()` returns a more fitting one. By updating the utilized value of \( M_W \) during the iteration of the spectrum generator, a self-consistent solution is ensured.
Example 15

In the MSSM, the running weak mixing angle $\theta_W$ and the $W$ pole mass for a given model object can be calculated with the following code:

```cpp
MSSM_weinberg_angle::Sm_parameters sm_pars;
sm_pars.fermi_constant = 1.1663787e-05;
sm_pars.mw_pole = 80.385;
sm_pars.mz_pole = 91.1876;
sm_pars.mt_pole = 173.34;
sm_pars.alpha_s = 0.1079;
MSSM_weinberg_angle weinberg(model, sm_pars);
const auto sw_mw = weinberg.calculate();
const double theta_w = std::asin(sw_mw.first);
const double MW = sw_mw.second;
```

8.3. Tests and comparisons with other spectrum generators

The implementation of the muon decay method for the determination of $\theta_W$ and $M_W$ provided as FlexibleMW has been tested in the SM and CMSSM by comparing to the results obtained using the algorithm from SOFTSUSY, which has been added in FlexibleSUSY 1.1.0. We have found excellent agreement between the two implementations and also added unit tests performing these comparisons to the FlexibleSUSY test suite.

Furthermore, the automatically generated expression for the generalized $\hat{\rho}_{\text{tree}}$ given in Eq. (61) has been analytically checked for many models, such as the UMSSM, MRSSM and $E_6$ SSM.

Finally, we compared numerical results for the $W$ pole mass in the CMSSM and MRSSM obtained with FlexibleMW to the ones from SPheno code generated by SARAH 4.12.2. Figure 10 shows $M_W$ as a function of $M_{1/2}$ (CMSSM, see left panel) or the superpotential parameter $\Lambda_u$ (MRSSM, see right panel) while all the other parameters are specified as described in the caption. In the case of the CMSSM, additionally the results calculated with SPheno 4.0.3 are plotted. For both models, there is a large discrepancy between the FlexibleMW values illustrated by the blue solid line and the results from SARAH/SPheno presented by the red dashed line. A thorough comparison of the two implementations has revealed two major differences. On the one hand, SARAH/SPheno partly uses the $\overline{\text{DR}}$ top mass in the self energies occurring in Eqs. (59) and (60) as well as the SM 2-loop corrections Eqs. (63) and (64) while FlexibleMW always uses the top pole mass, as suggested by Ref. [220]. Not utilizing the top pole mass in all of these formulas spoils the correctness of the included SM 2-loop corrections. On the other hand, the SARAH/SPheno code contains an inconsistency in the final computation of $M_W$, which is similar to Eq. (67) but partly neglects the SM 2-loop correction to $\Delta \hat{\rho}$. This inconsistency is also existent in the original SPheno code that, for the CMSSM, produces the results depicted by the red dashed-double-dotted line in the left panel of Figure 10. After fixing these issues within the SARAH/SPheno and SPheno code, we get the modified results illustrated by the green dotted and dashed-dotted line, respectively. These show good agreement with the values from FlexibleMW for both the CMSSM and MRSSM. The remaining small discrepancies are well understood and mainly caused by minor differences in the implemented formulas and the various utilized $\overline{\text{DR}}$ parameters. In addition, the SARAH/SPheno and SPheno codes
use $\alpha_s^{SM}(M_Z)$ while FlexibleMW uses $\alpha_s^{SM}(M_t)$ as preferred by Ref. [220].

![Graph](image_url)

**Figure 10:** Left panel: W pole mass in the CMSSM as a function of $M_{1/2}$ with $m_0 = 1$ TeV, $\tan \beta = 10$, $\text{sign} \mu = +1$ and $A_0 = 0$. Right panel: W pole mass in the MRSSM as a function of $\Lambda_u$ with the other parameters fixed as for BMP1 in Table 2 from Ref. [124]. For both panels, the differences between the lines are explained in the text.

9. **FlexibleSAS extension**

In this section, we introduce a new BVP solver, a major new feature released in FlexibleSUSY 2.0. The specification of the running parameters of a BSM model at multiple scales connected by RGEs constitutes a BVP that must be solved in order to compute the mass spectrum. FlexibleSUSY was designed with the intention of allowing multiple solvers for this problem. In FlexibleSUSY 1.0 only one supported solver was distributed, the two-scale solver, which uses a fixed point iteration with boundary conditions set at both the high scale and the low scale and is the one used in MSSM spectrum generators. However, additional solvers are important because convergence of valid points is not guaranteed with the two-scale solver and viable regions of the parameter space can be missed if the only option is the two-scale solver. This is already true in the CMSSM where it has also been shown that there can even be multiple solutions to the BVP [225, 226], of which the two-scale solver will only find at most one. Furthermore, the two-scale solver cannot find solutions at all for a large class of models, which includes the fully constrained NMSSM (CNMSSM) [144, 147, 148] and constrained E6SSM (CE6SSM) [153].

A basic problem in these models is that parameters exist that would naturally be computed at the weak scale by EWSB conditions but are at the same time constrained by GUT scale conditions. In the constrained NMSSM, where the soft scalar mass, $m_S$, 17In principle this could be avoided, e.g., in the CNMSSM and CE6SSM by including $\tan \beta$ or $\lambda$ in the set of parameters fixed by minimization conditions. However, $\tan \beta$ affects the top, bottom and $\tau$ Yukawa couplings at tree level, and $\lambda$ appears in the 1-loop RGEs for these couplings and therefore can have a significant impact on the RG evolution. This makes it difficult to obtain a convergent fixed point iteration with such a setup.
is typically an EWSB output, solutions have previously been found by varying one of the input parameters, tan $\beta$, and tuning this until a solution is found where $m_S$ is sufficiently close to the universal scalar mass, $m_0$ [227, 228]. For the constrained $E_6$SSM again the soft scalar masses need to be EWSB outputs. In that case a new BVP solver using semi-analytic solutions was invented to find solutions [152, 153] and extended and improved in further studies [217, 229, 230], though the precision was limited due to analytical approximations used in the code. In general, the challenges in finding solutions to the mass spectrum in BSM models can be quite different to those found in the MSSM and therefore additional solvers can be of great benefit in increasing the scope of FlexibleSUSY.

In FlexibleSUSY 2.0, an additional BVP solver has been added that solves the BVP by using semi-analytic solutions to the RGEs for a subset of the running parameters in the model. The semi-analytic algorithm takes advantage of the hierarchical structure of the RGEs in any model: the parameters can be split into a sequence of sets such that the running of the parameters in each set is independent of all of the parameters in the following sets. For example, in SUSY models the SUSY preserving parameters run independently of the soft breaking parameters; in general renormalizable models the dimensionless parameters run independently of the mass parameters.\(^{18}\) It is clear from the general form of the RGEs [198, 231–241] that further divisions are possible on the basis of the mass dimension of the parameters. The simplest case is the evolution of the mass dimension one parameters $m^i(Q)$, which is described by a system of linear homogeneous differential equations that can be solved in the form

$$m^i(Q) = [c(Q)]_j^i m^j(Q_0),$$

i.e., in terms of a linear combination of initial values and a set of dimensionless coefficients that only depend on dimensionless parameters. The RGEs for higher mass dimension parameters can then be cast as linear, non-homogeneous systems by substituting in the solutions for the lower mass dimension parameters, for which the general solutions can easily be written down.\(^{19}\) In the CMSSM, doing so leads to well-known expressions for the soft masses in terms of the universal parameters $m_0$, $M_{1/2}$, and $A_0$. For instance, the solutions for the soft scalar masses read (see also Eq. (81) below)

$$m_i^2(Q) = a_i(Q)m_0^2 + b_i(Q)M_{1/2}^2 + c_i(Q)M_{1/2}A_0 + d_i(Q)A_0^2,$$

where the dimensionless coefficients $a_i$, $b_i$, $c_i$, and $d_i$ are determined by the running of the gauge and Yukawa couplings, and are computed numerically. The basic idea of the semi-analytic solver is to generalize this approach to other models: first, the analytic forms of the solutions are determined from the boundary conditions, and then the appearing coefficients are determined numerically. Thus the solver obtains a collection of semi-analytic solutions which express parameters at the scale $Q$ directly in terms of parameters at the boundary scale $Q_0$.

To make the discussion concrete, consider a general SUSY model with bilinear and linear superpotential parameters $\mu^{ij}$ and $L^i$, together with a set of soft breaking gaugino

\(^{18}\)This partitioning is already reflected to some extent in the original C++ class structure for the parameters and RGEs, which for SUSY (non-SUSY) models is split up so that the RGEs for the SUSY preserving (dimensionless) parameters can be integrated separately to those for the soft (dimensionful) parameters; see Section 5.

\(^{19}\)See, for example, the derivation given in Ref. [153] for the case of a SUSY model with real soft parameters.
masses $M_a$, scalar trilinear, bilinear and linear couplings $T^{ijk}$, $b^{ij}$ and $t^i$, and a set of soft scalar masses $(m^2)^i_j$. The semi-analytic solutions are obtained from the general 2-loop RGEs given in Refs. [234, 235, 238]. Since the superpotential parameters are only multiplicatively renormalized, the semi-analytic solutions for the parameters $\mu^{ij}$ and $L^i$ are particularly simple, taking the form

$$\mu^{ij}(Q) = [c_{\mu}^{ij}(Q)]_{kp}^{ij} \mu^{kp}(Q_0), \quad (69)$$
$$L^i(Q) = [c_{L}^{ij}(Q)]_{p}^{ij} \mu^{ip}(Q_0), \quad (70)$$

where the coefficients $[c_{\mu}^{ij}(Q)]$ and $[c_{L}^{ij}(Q)]$ satisfy the initial conditions $[c_{\mu}^{ij}(Q_0)]_{kp}^{ij} = \delta_k^i \delta_p^j$, $[c_{L}^{ij}(Q_0)]_{p}^{ij} = \delta_p^i$. Again, the point is that the coefficients $c(Q)$ can be determined once the running dimensionless SUSY parameters are known, and the equations then express parameters at $Q$ in terms of parameters at $Q_0$ with numerically known coefficients.

The semi-analytic solutions for the dimension one soft breaking parameters can be written as

$$T^{ijk}(Q) = [c_{T}^{ijkm}(Q)]_{l}^{ijk} T^{lnm}(Q_0) + [c_{M}^{ij}(Q)]_{lj}^{km} M_{b}(Q_0), \quad (71)$$
$$M_a(Q) = [c_{M}^{ij}(Q)]_{l}^{ij} M_{b}(Q_0). \quad (72)$$

Upon substituting these solutions into the RGEs for the soft breaking bilinears $b^{ij}$ and the soft scalar masses $(m^2)^i_j$, one finds the semi-analytic solutions

$$b^{ij}(Q) = [c_{b}^{ij}(Q)]_{kl}^{ij} b^{kl}(Q_0) + [c_{\mu}^{ij}(Q)]_{l}^{ij} \mu^{kl}(Q_0) T^{mn}(Q_0) + [c_{M}^{ij}(Q)]_{ab}^{ij} \mu^{ab}(Q_0) M_{c}(Q_0), \quad (73)$$

and

$$(m^2)^i_j(Q) = [c_{m}^{ij}(Q)]_{ik}^{il} (m^2)^i_j(Q_0) + [c_{\mu}^{ij}(Q)]_{l}^{ij} M_{a}(Q_0), \quad (74)$$

Finally, the semi-analytic solutions for the soft breaking linear parameters $t^i$ are given by

$$t^i(Q) = [c_{t}^{ij}(Q)]_{l}^{ij} t^i(Q_0) + [c_{LT}^{ij}(Q)]_{l}^{ij} L^i(Q_0) T^{km}(Q_0) + [c_{LM}^{ij}(Q)]_{l}^{ij} L^i(Q_0) M_{a}(Q_0),$$
$$+ [c_{\mu}^{ij}(Q)]_{l}^{ij} \mu^{km}(Q_0) T^{nop}(Q_0) + [c_{\mu}^{ij}(Q)]_{l}^{ij} \mu^{km}(Q_0) T^{nop}(Q_0) + [c_{\mu}^{ij}(Q)]_{l}^{ij} \mu^{km}(Q_0) M_{a}(Q_0),$$
$$+ [c_{\mu}^{ij}(Q)]_{l}^{ij} \mu^{km}(Q_0) M_{a}(Q_0),$$

and

$$t^i(Q) = [c_{T}^{ij}(Q)]_{l}^{ij} T^{km}(Q_0) M_{a}(Q_0),$$
$$+ [c_{T}^{ij}(Q)]_{l}^{ij} T^{km}(Q_0) M_{a}(Q_0),$$
$$+ [c_{T}^{ij}(Q)]_{l}^{ij} T^{km}(Q_0) M_{a}(Q_0),$$
$$+ [c_{T}^{ij}(Q)]_{l}^{ij} T^{km}(Q_0) M_{a}(Q_0),$$

For models in which Dirac gaugino masses $m_{D_a}^i$ are also present, the solutions for the parameters $t^i$ are modified, $t^i \rightarrow t^i + \Delta t^i$, where $\Delta t^i$ is of the form

$$\Delta t^i(Q) = [c_{m}^{ij}(Q)]_{l}^{ij} m_{D_a}^i(Q_0) (m^2)^i_j(Q_0) + [c_{m}^{ij}(Q)]_{l}^{ij} m_{D_a}^i(Q_0) M_{b}(Q_0) M_{c}(Q_0),$$
$$+ [c_{m}^{ij}(Q)]_{l}^{ij} m_{D_a}^i(Q_0) T^{km}(Q_0) M_{a}(Q_0).$$

(75)
The semi-analytic solutions for the Dirac gaugino masses themselves follow from the known general 2-loop RGEs \cite{198}, and can be written in the form

$$m^i_{\text{Da}}(Q) = \left[ c^m_{\text{Da}}(Q) \right]^i m^j_{\text{Da}}(Q_0) .$$  \tag{77}

In non-SUSY models, the semi-analytic solutions follow from the known results for the 2-loop RGEs in a general gauge theory \cite{231-233,237,239}. For a non-SUSY model containing a set of real scalar trilinear couplings $h^{ijk}$ and squared scalar masses $(m^2)^{ij}$, and a set of fermion masses $(M_f)^{ij}$, the semi-analytic solutions for the mass dimension one parameters read

$$h^{ijk}(Q) = \left[ c_h^{M_f}(Q) \right]^{ijk} h^{lmn}(Q_0) + \left[ c_h^{M_f}(Q) \right]^{ijkl}(M_f)^{lmn}(Q_0) + \left[ c_h^{M_f}(Q) \right]^{ij}(M_f)^{lmn}(Q_0) ,$$  \tag{78}

$$(M_f)^{ij}(Q) = \left[ c_{M_f}(Q) \right]^{ij} h^{lmn}(Q_0) + \left[ c_{M_f}(Q) \right]^{ijkl}(M_f)^{lmn}(Q_0) + \left[ c_{M_f}(Q) \right]^{ij}(M_f)^{lmn}(Q_0) .$$  \tag{79}

In general, all of the dimension one parameters must be considered together, unlike in SUSY models where they can be separated into superpotential and soft breaking masses. After substituting the solutions Eq. (78) and Eq. (79) into the RGEs for the squared scalar masses, the semi-analytic solutions for the scalar masses are found to be

$$(m^2)^{ij}(Q) = \left[ c_{m^2}(Q) \right]^{ij} (m^2)^{kl}(Q_0) + \left[ c_{m^2}(Q) \right]^{ijkl} h^{klm}(Q_0) h^{nop}(Q_0)$$

$$+ \left[ c_{h^{M_f}}(Q) \right]^{ijkl} h^{klm}(Q_0) (M_f)^{pq}(Q_0) + \left[ c_{h^{M_f}}(Q) \right]^{ijkl} h^{klm}(Q_0) (M_f)^{pq}(Q_0) + \left[ c_{M_f^2}(Q) \right]^{ijkl} h^{klm}(Q_0) (M_f)^{pq}(Q_0) + \left[ c_{M_f^2}(Q) \right]^{ijkl} h^{klm}(Q_0) (M_f)^{pq}(Q_0)$$

$$+ \left[ c_{M_f^2}(Q) \right]^{ijkl} h^{klm}(Q_0) (M_f)^{pq}(Q_0) + \left[ c_{M_f^2}(Q) \right]^{ijkl} h^{klm}(Q_0) (M_f)^{pq}(Q_0) .$$  \tag{80}

The solver algorithm implemented by FlexibleSAS automatically determines the above semi-analytic solutions in the model, \footnote{In non-SUSY models, SARAH currently does not calculate RGEs for linear scalar couplings $L'$, which have been given in, e.g., Ref. [198], and therefore the semi-analytic solutions for these parameters are also not used in FlexibleSUSY.} given the set of boundary conditions at some scale. Since the required coefficients may be determined knowing only the running of the dimensionless or SUSY preserving parameters, each step of the main fixed point iteration is split up into two parts. Firstly, the BVP for the dimensionless parameters is solved iteratively. The semi-analytic coefficients at any scale can then be calculated. In this second stage, the soft breaking or dimensionful parameters are expanded in terms of the semi-analytic solutions to the RGEs. The low-energy EWSB conditions and masses are thus expressed explicitly in terms of the boundary values at $Q_0$, allowing, for example, unknown quantities at one scale to be directly constrained at another.
9.1. Choosing FlexibleSAS in the model file

The BVP solver algorithms that are applicable in a given model can be specified in the FlexibleSUSY model file using the list FSBVPsSolvers. The elements of this list correspond to the desired BVP solvers that should be enabled for the model, identified using the predefined symbols TwoScaleSolver for the two-scale algorithm and SemiAnalyticSolver for the semi-analytic solver. By default, if a list of BVP solvers is not specified in the model file, only the two-scale algorithm is enabled, as summarized in Table 8.

| Symbol       | Default value | Allowed values | Description                      |
|--------------|---------------|----------------|----------------------------------|
| FSBVPsSolvers| \{ TwoScaleSolver \} | Non-empty list containing TwoScaleSolver or SemiAnalyticSolver or both | List of BVP solvers to be used |
| TwoScaleSolver| –             | –              | Represents the two-scale BVP solver |
| SemiAnalyticSolver| –         | –              | Represents the semi-analytic BVP solver |

Table 8: FlexibleSAS model file options.

In addition to specifying that the semi-analytic solver should be used, the user should ensure that the boundary conditions for the model are compatible with its use. Currently, this requires that those parameters that will be expanded using semi-analytic solutions of the RGEs are fixed in the same boundary condition. So in SUSY models, the boundary values for all of the soft SUSY breaking parameters should be given at a single scale, while in non-SUSY models the same should be done for all of the dimensionful parameters. The expressions for the boundary values must also only be polynomials in any dimensionful parameters, such as universal scalar masses.

Boundary values for the running parameters of the model can be specified in terms of input parameters such as those defined in the MINPAR and EXTPAR variables. In FlexibleSUSY 2.0, the user can also define extra parameters by using the list FSAuxiliaryParameterInfo. Each entry in this list should contain the name of the extra parameter being defined and a list of its properties. The possible properties for the new parameters are specified in Table 9. In particular, the mass dimensions of the new parameters may be specified to allow for simplifying the forms of the semi-analytic solutions, in which dimensionless parameters can be absorbed into the definitions of the semi-analytic coefficients. Input or auxiliary parameters that are not scalars may be defined by setting the ParameterDimensions property to a list of the form \( \{m,n\} \) for an \( M \times N \) matrix or \( \{n\} \) for an \( N \)-dimensional vector. A value of \( \{1\} \) corresponds to a scalar. Note that in versions of FlexibleSUSY prior to version 2.0, this functionality was available for input parameters using the variable FSEExtraInputParameters. However, please note that this variable has been removed in FlexibleSUSY 2.0; definitions that were previously given in FSEExtraInputParameters must now be given in the new list FSAuxiliaryParameterInfo.

Example 16

In the MSSM, input parameters giving the values of the soft SUSY breaking trilinears as 3 × 3 matrices can be defined:
| Symbol             | Default value | Allowed values          | Description                                                                 |
|--------------------|---------------|-------------------------|-----------------------------------------------------------------------------|
| InputParameter     | False         | True or False           | Indicates whether the new parameter is an input parameter                   |
| ParameterDimensions| \{1\}         | A list of the form \{(M,N) or \{N\} | Specifies the dimensions of the parameter                                   |
| MassDimension      | –             | A non-negative integer  | Specifies the mass dimension of the parameter                               |
| LesHouches         | –             | A symbol or string, or a list of the form \{block, entry\} | Specifies the SLHA block from which the parameter should be read if it is an input parameter |

Table 9: Allowed properties for extra parameters

```plaintext
FSAuxiliaryParameterInfo = {
  \{\textit{A}\textsc{e} , \{ \textit{LesHouches} -> \textit{AEIN} ,
    \textit{ParameterDimensions} -> \{3,3\} ,
    \textit{InputParameter} -> \text{True}
  \} ,
  \{\textit{A}\textsc{d} , \{ \textit{LesHouches} -> \textit{ADIN} ,
    \textit{ParameterDimensions} -> \{3,3\} ,
    \textit{InputParameter} -> \text{True}
  \} ,
  \{\textit{A}\textsc{u} , \{ \textit{LesHouches} -> \textit{AUIN} ,
    \textit{ParameterDimensions} -> \{3,3\} ,
    \textit{InputParameter} -> \text{True}
  \}
};
```

Extra parameters defined in this way can be used in the specification of the boundary conditions for running model parameters, or indeed themselves be fixed in the boundary conditions. A special case is if the new parameters are to be fixed by the EWSB conditions. To facilitate this usage, the new variables \textit{EWSBInitialGuess} and \textit{EWSBSubstitutions} may be given in the model file. The former allows for explicit initial guesses to be provided for the EWSB output parameters, in the same format as used for specifying the boundary conditions. The latter, a list of two-component lists, can be used to define any substitutions that should be made in the EWSB equations before attempting to solve them.

**Example 17**

In the so-called VCMSSM [242, 243], the value of the soft breaking bilinear $B\mu$ is fixed at the GUT scale, $M_X$, in terms of the universal soft trilinear $A_0$ and scalar mass $m_0$ according to $B\mu(M_X) = \mu(M_X)(m_0 + A_0)$. Consequently, $B\mu$ can no longer be fixed to ensure proper EWSB, as is usually done in the CMSSM. Instead,
$|\mu|^2$ and $\tan \beta$ are used as EWSB outputs. This can be achieved in a VCMSSM model file through the definitions:

```mathematica
FSAuxiliaryParameterInfo = {
    {TanBeta, {ParameterDimensions -> {1}, MassDimension -> 0 } },
    {MuSq, {ParameterDimensions -> {1}, MassDimension -> 2 } },
    {vMSSM, {ParameterDimensions -> {1}, MassDimension -> 1 } }
};

EWSBOutputParameters = { TanBeta, MuSq };

EWSBSubstitutions = {
    {vd, vMSSM Cos[ArcTan[TanBeta]]},
    {vu, vMSSM Sin[ArcTan[TanBeta]]},
    {\[Mu], Sign[\[Mu]] Sqrt[MuSq]}
};

EWSBInitialGuess = {
    {TanBeta, vu / vd},
    {MuSq, \[Mu]^2}
};

SUSYScaleInput = {
    {vMSSM, Sqrt[vd^2 + vu^2]},
    FSSolveEWSBFor[EWSBOutputParameters]
};
```

Note that `FlexibleSUSY` automatically substitutes the semi-analytic solutions into the EWSB conditions for the soft or dimensionful parameters. Therefore, it is not necessary for the user to explicitly provide them. For the purpose of making use of the semi-analytic algorithm, the only required addition to the model file is the inclusion of `SemiAnalyticSolver` in the list `FSBVPSolvers`.

### Example 18

The CNMSSM is characterized by universal soft scalar masses at the high scale. Typically, this constraint is relaxed somewhat to allow the soft singlet mass $m_S^2$ to differ from the common scalar mass $m_0^2$ at the high scale. This is done to allow fixing $m_0^2$ using the EWSB conditions to ensure correct EWSB, and is the approach taken in the NMSSM model included with `FlexibleSUSY`, as well as in other public spectrum generators such as SOFTSUSY, SPheno and NMSPEC. Universality of the soft scalar masses can alternatively be maintained by using the semi-analytic BVP solver, which is achieved in the CNMSSM model file by specifying the BVP solvers to be used,

```mathematica
FSBVPSolvers = { SemiAnalyticSolver };
```

In this set-up, $m_0$ ceases to be an input parameter and is fixed by the EWSB
conditions. This is achieved in the model file by removing $m_0$ from the list of input parameters and defining it as an additional parameter, as follows:

```plaintext
(* CNMSSM input parameters *)
MINPAR = {
{2, m12},
{3, TanBeta},
{4, Sign[vS]},
{5, Azero}
};

EXTPAR = {
{61, LambdaInput}
};

FSAuxiliaryParameterInfo = {
{m0Sq, { ParameterDimensions -> {1},
          MassDimension -> 2 } },
{LambdaInput, { ParameterDimensions -> {1},
               MassDimension -> 0 } }
};
```

Note that the definition of `LambdaInput` in `FSAuxiliaryParameterInfo` is not compulsory, but allows the semi-analytic solutions to be simplified by using the fact that it is a dimensionless parameter. The value of $m_{0}^2$ is then determined from the EWSB conditions by setting

```plaintext
EWSBOutputParameters = { \[ Kappa \], vS, m0Sq };
```

To impose the universality constraint, the condition $m_{S}^2 = m_{0}^2$ at the GUT scale must be added to the high-scale boundary condition, by defining

```plaintext
HighScaleInput={
{T[Ye], Azero*Ye},
{T[Yd], Azero*Yd},
{T[Yu], Azero*Yu},
{m2, UNITMATRIX[3] m0Sq},
{m12, UNITMATRIX[3] m0Sq},
{md2, UNITMATRIX[3] m0Sq},
{mu2, UNITMATRIX[3] m0Sq},
{me2, UNITMATRIX[3] m0Sq},
{mHu2, m0Sq},
{mHd2, m0Sq},
{ms2, m0Sq},
{\[Lambda\], LambdaInput},
{\[Kappa\], Azero \[Kappa\]},
{MassB, m12},
{MassWB,m12},
{MassG,m12}
};
```

The remainder of the model implementation is otherwise rather similar to that for
the NMSSM solved using the two-scale solver. The full CNMSSM model file is given in Appendix E.

9.2. Structure of the generated code

In keeping with the design goal of FlexibleSUSY to produce generated code that is highly modular in nature, the implementation of the BVP solvers is separated from the details of specific physics models. In FlexibleSUSY, a general BVP solver algorithm is represented by the templated $\text{RGFlow}\langle T \rangle$ class. Particular algorithms are provided as specializations of this class, with the two-scale and semi-analytic solvers corresponding to the classes $\text{RGFlow}\langle \text{Two\_scale} \rangle$ and $\text{RGFlow}\langle \text{Semi\_analytic} \rangle$, respectively. Each realizes an abstract implementation of the appropriate algorithm, with no dependence on the details of any particular model. The required model-dependent information is provided by separate classes representing the model and boundary and matching conditions, which are linked to the desired BVP solver class. New algorithms can easily be added simply by writing additional specializations of the $\text{RGFlow}$ class.

The semi-analytic solver algorithm requires two nested iterations. An inner iteration, carried out at each step, determines consistent values for the SUSY preserving (in SUSY models) or dimensionless parameters (in non-SUSY models) at the low- and high-scale boundaries. Updated estimates for these scales are simultaneously calculated during the iteration if necessary, for example if the high scale $M_X$ is defined in the model file by gauge unification, $g_1(M_X) = g_2(M_X)$. Once this has converged, the resulting estimate for these parameters is used to compute the semi-analytic solutions for the soft SUSY breaking or dimensionful parameters, at which point the EWSB conditions may be solved and the $\overline{\text{DR}}/\overline{\text{MS}}$ mass spectrum calculated. The new values of the soft or dimensionful parameters are then used in the inner iteration for computing the required threshold corrections. This sequence of steps is illustrated in Figure 11. For a single high-scale model such as the CMSSM, the algorithm proceeds as follows:

**Initial guess:** The initial guess involves a first run of the inner iteration. In all of the steps below, threshold corrections are ignored.

1. The known values of the SM gauge couplings at the scale $M_Z$ are used to estimate the values of $g_1$, $g_2$ and $g_3$ at the scale $M_t$, ignoring threshold corrections.
2. The user-defined initial guess at the low scale, as given in $\text{InitialGuessAtLowScale}$, is imposed at the scale $M_t$.
3. The SUSY preserving or dimensionless parameters are run to the initial guess for $M_X$, given by $\text{HighScaleFirstGuess}$, and the high-scale boundary condition for these parameters, defined in $\text{HighScaleInput}$, is imposed. The initial guess at the high scale, defined in $\text{InitialGuessAtHighScale}$, is then applied.
4. The model is run to the guess for the low scale, initially set to the value defined in $\text{LowScaleFirstGuess}$, and the low-scale boundary conditions for the SUSY preserving or dimensionless parameters defined in $\text{LowScaleInput}$ are applied.
5. The model is run to the current guess for $M_X$.
   (a) If necessary, the guess for $M_X$ is updated. For example, in the CMSSM with $M_X$ defined to be the scale at which $g_1(M_X) = g_2(M_X)$, a new estimate for $M_X$ is calculated according to
   $$M_X' = M_X \exp \left( \frac{g_2(M_X) - g_1(M_X)}{\beta_{g_1} - \beta_{g_2}} \right) .$$
(b) The high-scale boundary conditions for the SUSY preserving or dimensionless parameters are applied.

6. If not converged, goto 4.
7. The model is run to the guess for the low scale. The semi-analytic solutions are calculated at this scale.
8. The EWSB equations are solved at tree level.
9. The $\overline{\text{DR/MS}}$ mass spectrum is calculated.

At this stage, initial guesses for all of the model parameters, boundary condition scales and the $\overline{\text{DR/MS}}$ mass spectrum are available. The full iteration now starts, in which the full set of threshold corrections are applied.

**Thresholds iteration:**

1. The SUSY preserving or dimensionless parameters are determined in an inner iteration analogous to that in the initial guess, namely:
   (a) All model parameters are run to the low scale ($\text{LowScale}$) and the $\overline{\text{DR/MS}}$ mass spectrum is calculated.
   (b) The low scale is recalculated if it is not fixed.
   (c) The SM gauge couplings are calculated in the model, including the appropriate threshold corrections.
   (d) The user-defined constraints for the SUSY preserving or dimensionless parameters are applied.
   (e) All model parameters are run to the high scale ($\text{HighScale}$).
   (f) The high scale is recalculated if necessary.
   (g) The user-defined boundary conditions at this scale for the SUSY preserving or dimensionless parameters are applied.
   (h) The model parameters are run to the SUSY scale ($\text{SUSYScale}$) and the SUSY scale is updated if necessary.
   (i) The boundary conditions for the SUSY preserving or dimensionless parameters are applied.
   (j) If not converged, goto 1a.

2. All model parameters are run to the scale at which the EWSB equations are to be solved.
   (a) The coefficients in the semi-analytic solutions are determined at this scale, using the current estimate for the scale at which the relevant boundary conditions are imposed. For example, in the CMSSM, the semi-analytic solutions for the soft gaugino masses, trilinears, scalar masses and bilinear take the form
   \[
   M_i(Q) = p_i(Q)A_0 + q_i(Q)M_{1/2}, \\
   T_i(Q) = e_i(Q)A_0 + f_i(Q)M_{1/2}, \\
   m_i^2(Q) = a_i(Q)m_0^2 + b_i(Q)M_{1/2}^2 + c_i(Q)M_{1/2}A_0 + d_i(Q)A_0^2, \\
   B\mu(Q) = u(Q)B\mu(M_X) + v(Q)u(M_X)M_{1/2} + w(Q)u(M_X)A_0.
   \]
   \(81\)
   The coefficients are determined numerically by varying the values of $M_{1/2}$, $A_0$, $m_0$ and $B\mu(M_X)$ and integrating the RGEs from $M_X$ to $Q$. For example, the coefficients $p_i(Q)$, $e_i(Q)$, $d_i(Q)$ and $w(Q)$ are obtained by keeping only $A_0 \neq 0$. A similar approach is followed to successively obtain all of the remaining coefficients.
(b) The calculated semi-analytic solutions are used to set the values of the soft 
SUSY breaking or dimensionful parameters at this scale. 
(c) The DR/MS mass spectrum is calculated and the scale at which EWSB occurs 
is updated. 
(d) The EWSB conditions are solved at the loop level. 

3. If not converged, goto 1. Otherwise the iteration finishes. 

If the iteration converges, all running parameters in the model are determined between 
the low and high scales. The remainder of the calculation, that is, the calculation of 
the pole mass spectrum and observables, then proceeds in the same way as in the two-scale 
algorithm. Alternatively, the iteration may fail to converge or may encounter problems 
that render the parameter point physically invalid. As for the two-scale solver, the specific 
problems that are encountered for a given parameter space point are stored in the Problems 
class, and may be accessed using the get_problems() function of the model class.21

![Diagram of the semi-analytic algorithm](image)

**Figure 11:** Semi-analytic algorithm for calculating the mass spectrum in a SUSY 
model; in a non-SUSY model, the SUSY parameters are replaced by the dimensionless 
model parameters instead.

### 9.3. Tests and comparisons with other spectrum generators

The semi-analytic algorithm provided by FlexibleSAS has been tested in the CMSSM, 
CNMSSM and the CE6SSM by comparing to the results obtained using the existing two-
scale solver. We have carried out consistency checks between the two by confirming that

---

21Note that in FlexibleSUSY 2.0 a separate class, BVP_solver_problems, is used to store those problems 
that are associated only with failures of the BVP solver algorithm, such as a failure to converge, and which 
do not necessarily mean the parameter point is ruled out. A summary combining all of the problems that 
arise during a run of the spectrum generator can then be obtained by calling the get_problems() method 
of the spectrum generator class.
the same solution can be found in both solvers, provided it is a stable fixed point in both. In each of the models, the existing model solved using the two-scale algorithm has been compared with versions of the model using alternative boundary conditions. For example, in the CMSSM, instead of the traditional approach of fixing $|\mu|^2$ and $B\mu$ using the EWSB conditions, the value of $\mu$ is provided as an input and $m_0^2$ and $B\mu(M_X)$ are determined from the EWSB conditions.\(^{22}\) The benchmark points used as inputs for the semi-analytic solver in each model are displayed in Table 10. In all three models, the running parameters and pole mass spectra are found to differ at or below the level of 0.1%. Unit tests that perform these comparisons have also been added to the FlexibleSUSY test suite.

| Model   | Unit test benchmark points for the semi-analytic solver                                      |
|---------|---------------------------------------------------------------------------------------------|
| CMSSM   | $M_{1/2} = 500$ GeV, $\tan \beta = 10$, $A_0 = 0$ GeV, $\mu(M_S) = 623.36$ GeV             |
| CNMSSM  | $M_{1/2} = 133.33$ GeV, $\tan \beta = 10$, sign $\mu_{\text{eff}} = -1$, $A_0 = -300$ GeV, $\lambda(M_X) = -0.05$ |
| CE0SSM  | $\tan \beta = 10$, $\lambda_3(M_X) = 0.12$, $\kappa(M_X) = 0.2$, $\mu'(M_X) = 10$ TeV, $B'\mu'(M_X) = 0$ GeV$^2$, $s(M_S) = 4$ TeV, $\lambda_{1,2}(M_X) = 0.1$ |

Table 10: Input parameter values used for the unit tests comparing the results of the two-scale and semi-analytic algorithms in the CMSSM, CNMSSM and CE0SSM. The notation for the CNMSSM follows that in Refs. [38, 39], while for the CE0SSM we use the notation of Ref. [153].

In addition to carrying out unit tests on individual benchmark points, we have also performed extensive scans in the CMSSM to confirm that the semi-analytic solver produces results in agreement with the two-scale solver. For most points, this is found to be the case; however, we have also observed important exceptions where non-negligible differences are found between the two solvers. In these cases, one solver may fail to converge to a stable solution, or multiple solutions are present [225] with different stability properties in the two solvers. In this latter case, note that both solvers always return the first solution to which they converge,\(^{23}\) with the iteration stopping immediately once a convergent solution is obtained. That is, neither method attempts to find all possible solutions for the given input parameters or automatically select between multiple fixed points. Since a given solution might not be a stable fixed point of both iterations, the two solvers need not converge to the same solution, leading to the observed differences. We have checked that such points nevertheless satisfy the boundary conditions imposed at each scale and are indeed valid solutions of the BVP. More generally we have checked that solutions found in one solver also correspond to (not necessarily stable) fixed point solutions of the other solver algorithm; that is, they satisfy all of the boundary conditions so that the parameter values remain unchanged after applying a single step of the iteration.

From these tests, we have found that in some cases the agreement between the two solvers can depend quite sensitively on small differences between them. To illustrate this, in Figure 12 we show the percentage changes in the DR mass spectrum in the CMSSM after running points obtained using the two-scale solver through a single step of the semi-analytic

\(^{22}\)This alternative approach is useful in scenarios where one wishes to have direct control over the Higgsino masses, and therefore the composition of the lightest neutralino in the CMSSM, as was done in Ref. [134].

\(^{23}\)When multiple convergent solutions exist, the one first obtained will depend, for instance, on the initial guess used for the iteration.
solver; if the point is also a fixed point of the latter, this change should be negligible. In this scan, the change after a single iteration can be on the level of several percent for a small number of points, reaching between 20% and 30% for some exceptional points. That these points initially appear not to be fixed points of the semi-analytic solver arises primarily from the fact that the semi-analytic coefficients, and hence the EWSB solution, are sensitive to the estimate for the high scale $M_X$, as well as numerical errors in the integration of the RGEs. In particular, for the default convergence criteria imposed by FlexibleSUSY, the two-scale solver’s estimate for $M_X$ is not close enough to convergence, leading to significant differences in the calculated low-energy soft parameters. By requiring convergence in the estimate for $M_X$, together with using a higher-order Runge-Kutta integration and demanding a higher precision for the obtained EWSB solution, the change after one iteration is reduced below the permille level. Thus, it is important to be aware that differences in the convergence properties of the two solvers can have an impact on the solutions found, even if a given point would be a fixed point of both solvers.

The typical runtimes for the two solvers in the CMSSM are compared in Figure 13. The distributions are obtained by randomly sampling the CMSSM input parameters $m_0 \in [0,2]$ TeV, $M_{1/2} \in [0,2]$ TeV, $\tan \beta \in [2,30]$, sign $\mu \in \{-1,+1\}$ and $A_0 \in [-1,1]$ TeV. The runtime of the semi-analytic solver is increased on average by a factor of $\approx 3$ compared to the default Runge-Kutta algorithm provided by FlexibleSUSY and allowing the iteration to stop as soon as the precision goal of $10^{-4}$ is reached (red circles), changes between 1% and 30% are found for a small number of points. By using the 8th order Runge-Kutta integrator and ensuring convergence is reached in the estimate for $M_X$ by forcing 40 iterations in the two-scale algorithm, these differences are reduced below the level of 0.001% (blue diamonds).

Figure 12: Percentage changes in the CMSSM $\overline{\text{DR}}$ mass spectrum after applying a single step of the semi-analytic solver to points obtained using the two-scale solver in a linear scan over $M_{1/2} \in [0,300]$ GeV, $m_0 \in [0,2]$ TeV, $A_0 = 0$ GeV, $\tan \beta = 40$ and $\mu > 0$. When run using the default Runge-Kutta algorithm provided by FlexibleSUSY and allowing the iteration to stop as soon as the precision goal of $10^{-4}$ is reached (red circles), changes between 1% and 30% are found for a small number of points. By using the 8th order Runge-Kutta integrator and ensuring convergence is reached in the estimate for $M_X$ by forcing 40 iterations in the two-scale algorithm, these differences are reduced below the level of 0.001% (blue diamonds).

24By default, FlexibleSUSY makes use of an adaptive 5th order algorithm; to perform this test we have also implemented into FlexibleSUSY 2.0 an 8th order solver that makes use of the Runge-Kutta-Fehlberg method provided by the Boost library odeint. This higher-order solver is available to the user by choosing it at the C++ level.
to the two-scale solver. This increase is mostly due to the increased number of iterations performed by the semi-analytic solver. For each outer iteration of the semi-analytic solver, the inner iteration typically runs through a similar number of steps as for a full run of the two-scale solver, with this number decreasing as convergence is approached on each outer iteration. Consequently, the total number of iterations for the semi-analytic solver tends to be larger than that for the two-scale solver by a similar factor. There is also an additional cost associated with running between scales to compute the semi-analytic coefficients.

While the semi-analytic solver suffers from an increased runtime compared to the two-scale solver, it is also able to provide complementary coverage of the parameter space to that of the two-scale solver. This is demonstrated in the left panel of Figure 14 in the CMSSM, where the solutions found by each solver are plotted in the $m_0 - \mu$ plane. In this case, the use of the semi-analytic solver allows for a large number of solutions to be found in the focus point region $[244–246]$ at small values of $\mu \ll m_0$, where the two-scale solver is unable to find convergent solutions. Conversely, the two-scale solver is more effective for finding solutions with small $m_0$. This highlights the fact that, for some parameter points, the semi-analytic solver generated by FlexibleSUSY might not find a solution where the two-scale solver is able to, and vice versa. As noted above, this can be due to the point in question having different stability properties under the two different algorithms. For example, the CMSSM point with $m_0 = 125\,\text{GeV}$, $M_{1/2} = 300\,\text{GeV}$, $\tan \beta = 10$, $\text{sign} \mu = 1$ and $A_0 = 0\,\text{GeV}$ is successfully solved using the two-scale solver, with $\mu \approx 395\,\text{GeV}$ required for correct EWSB. The same point, when run using the semi-analytic solver with the two-scale solution for $\mu$, fails to converge; for this choice of input parameters and initial guess, the iteration enters a periodic orbit in which the approximations to the solution are close to, but do not correspond to, the fixed point found by the two-scale solver. To avoid the observed cyclic behaviour here, it is necessary to either fine-tune the provided input parameters or otherwise modify the initial guess made by FlexibleSUSY. Due to the differing choice of input and output parameters between the two algorithms in general, certain regions of the parameter space can also be susceptible to high levels of numerical
Figure 14: Left panel: CMSSM solutions found by the two-scale solver (blue diamonds) and the semi-analytic solver (red circles) in the $m_0 - \mu$ plane. For both solvers, $M_{1/2} \in [0, 1] \text{ TeV}$ is randomly sampled, while $\tan \beta = 10$ and $A_0 = 0 \text{ GeV}$. The solutions obtained using the two-scale solver are found by randomly sampling $m_0 \in [0, 6] \text{ TeV}$ with sign $\mu = 1$, and those for the semi-analytic solver are found by randomly sampling $\mu \in [0, 1150] \text{ GeV}$. Right panel: Change in the calculated lightest $CP$-even Higgs pole mass $M_{h_1}$ from a reference value of 121 GeV as a function of $\mu$ at fixed $m_0 = 4 \text{ TeV}$ for the two solvers. The solutions shown correspond to a vertical slice at fixed $m_0$ in the left-hand plot; $M_{1/2}$ is again varied in $[0, 1] \text{ TeV}$, $\tan \beta = 10$ and $A_0 = 0 \text{ GeV}$.

Sensitivity in one approach and not the other, again necessitating significant fine-tuning. In the CMSSM, for instance, large cancellations are typically required in order to produce a small value of $m_0$ when using the semi-analytic solver. In such cases, $\mu$ must be carefully fine-tuned to obtain a valid solution. On the other hand, when using the two-scale solver one has direct control over the value of $m_0$, and it is not necessary to fine-tune in order to obtain the desired small value of $m_0$. The situation is reversed in regions of parameter space with small values of $\mu$ and large $m_0$, where one now has direct control over the fine-tuned value of $\mu$ in the semi-analytic solver but not in the two-scale solver. In general terms, the regions of parameter space in which the two solvers are effective need not overlap, and the use of both in tandem allows for a more complete picture of the parameter space to be obtained. Moreover, in the regions in which both solvers do find solutions, there is excellent agreement between the two algorithms.\(^{25}\) This is illustrated in the right panel of Figure 14, where the lightest $CP$-even Higgs mass, expressed as the difference from a reference value of 121 GeV, is plotted for fixed $m_0 = 4 \text{ TeV}$. If both solvers find a solution for this value of $m_0$, the two values of the Higgs mass agree very well.

It is also evident from Figure 14 that the use of both solvers allows features in the parameter space to be picked up that would be missed by either solver alone. In this case, in Figure 14 the two-scale solver only finds a single solution for each value of $M_{1/2}$, for fixed sign $\mu = 1$, while the semi-analytic solver in some cases finds multiple solutions. These solutions have different values of $|\mu|$ for the same value of $M_{1/2}$, leading to the sharp feature at low values of $\mu$ evident in the right panel of Figure 14. The existence of multiple solutions to the CMSSM BVP, and the inability of the ordinary two-scale fixed point iteration to find all such solutions, are well-known and have previously been studied in Refs. [225, 226]. In Figure 15 we compare the results obtained using the two BVP solvers in FlexibleSUSY 2.0.

\(^{25}\) Provided the same solution is found if multiple solutions exist.
Figure 15: Solutions obtained using the two-scale and semi-analytic solvers in the CMSSM for fixed $M_{1/2} = 660$ GeV, $A_0 = 0$ GeV and $\tan \beta = 40$. For comparison, we also reproduce the curve shown in Figure 5 of Ref. [225], where a modified version of SOFTSUSY 3.3.7 was used to allow multiple solutions to be found to the CMSSM boundary conditions. The inset plot shows the region $2790 \text{ GeV} \leq m_0 \leq 2820 \text{ GeV},$ $-555 \text{ GeV} \leq \mu \leq -520 \text{ GeV}$ in more detail.

To those found using the modified version of SOFTSUSY employed in Ref. [225]. As expected, in this region of the parameter space the two-scale solver produced by FlexibleSUSY finds at most only two solutions, corresponding to the two possible signs of $\mu$. The automatically generated semi-analytic solver enables additional solutions to be found, which are in good agreement with those found using the modified version of SOFTSUSY. Slight differences in the values of $\mu$ and $m_0$, and where solutions are found, arise from small differences in the incorporated corrections between the two codes. In particular, the semi-analytic solver does not find valid solutions for $\mu < -545$ GeV due to the tree-level mass of the $C\!P$-odd Higgs becoming tachyonic. In general, multiple solutions at a given parameter point can have significantly different phenomenological properties [226], so that finding them is important to completely characterize a model. The availability of multiple solvers improves the ability of FlexibleSUSY to locate additional solutions, without requiring extensive modifications to the generated code.

The longer runtime of the semi-analytic solver can also be an acceptable tradeoff if the model of interest cannot easily be handled using the two-scale solver. Constrained models such as the CNMSSM and CE6SSM are examples of this. To demonstrate the applicability of the semi-analytic solver to these models, we have performed scans over the parameter spaces of these models.

First we have performed scans in the CNMSSM to demonstrate that with the semi-analytic solver we can sample the parameter space effectively, making FlexibleSUSY 2.0 the first public spectrum generator that can do this “out of the box”. We performed a four dimensional scan of the CNMSSM, using the model file provided in Appendix E to produce a CNMSSM spectrum generator that was then linked to MultiNest 3.10 for efficient sampling of the parameter space. The input parameters were varied over the
ranges

\[ 2 \leq \tan \beta \leq 50, \]  
\[ 0 \text{TeV} \leq M_{1/2} \leq 5 \text{TeV}, \]  
\[ -5 \text{TeV} \leq A_0 \leq 0 \text{TeV}, \]  
\[ -0.3 \leq \lambda(M_X) \leq 0.3, \]  
\[ \text{sign} \mu_{\text{eff}} = +1, \]

and the log likelihood function provided to MultiNest was defined to be

\[ \log L = -\frac{1}{2}(M_{h}^{\text{SM}} / \text{GeV} - 125.09)^2, \]

where \( M_{h}^{\text{SM}} \) is the SM-like Higgs pole mass. In this way the scan is directed towards solutions with the observed Higgs mass of 125.09 GeV.

![Figure 16: CNMSSM solutions obtained using FlexibleSUSY 2.0 and MultiNest 3.10 shown in the \( m_0 - M_{1/2} \) plane with \( A_0 \) as a color contour (top left panel); \( M_{1/2} - \lambda \) plane showing the SM-like Higgs pole mass \( M_{h}^{\text{SM}} \), as a color contour, with the range restricted to \( M_{h}^{\text{SM}} > 100 \) GeV for clarity (top right panel); \( m_0 - M_{1/2} \) plane with the mass of the lightest up-type squark, which is predominantly stop (bottom left panel), and the \( M_{1/2} - A_0 \) plane with the heaviest up-type squark mass (bottom right panel).](image)

The results are shown in Figure 16, where all points are required to have \( m_0^2 > 0 \). As can be seen in the top left panel, the range of \( m_0 \) values found is considerably smaller than
the range allowed for the universal soft masses, $M_{1/2}$ and $A_0$, which are inputs. The reason for this is that in the CNMSSM a non-zero singlet VEV must be generated by developing the correct shape of the scalar potential. This could be achieved in the usual way through a negative quadratic term if the soft breaking singlet mass squared, $m^2_S$, is negative. In fact, since the NMSSM scalar potential also contains the cubic singlet terms $\kappa A S^3/3 + \text{h.c.}$, the requirement $m^2_S < 0$ can be relaxed so that an approximate condition for generating the correct shape of the potential reads $A^2_S \gtrsim 9m^2_S$ [148, 247]. If $m_0 \gg A_0/3$, this condition is satisfied only if $m^2_S$ is driven to be sufficiently small during the RG evolution from the GUT scale to the SUSY scale. This in turn can be achieved for large enough values of the superpotential cubic singlet coupling $\kappa$ and the singlet-Higgs coupling $\lambda$.

However, in this constrained model, where the soft trilinears are not free input parameters at the SUSY scale, it is also the case that large values of $\lambda$ always generate substantial singlet mixing that can reduce the lightest $CP$-even Higgs mass. As a result, a 125 GeV Higgs mass is obtained with small values of $\lambda$, to avoid this mixing, as well as large $M_{1/2}$, as can be seen in the top right panel of Figure 16. For such small singlet Yukawa couplings, the RG flow between the GUT and SUSY scales leaves $m^2_S(M_S)$ and $A_S(M_S)$ close to their GUT scale values, namely $m^2_0$ and $A_0$. As a result, $m_0$ is heavily constrained if the condition $A^2_S \gtrsim 9m^2_S$ is to be satisfied in the absence of significant RG evolution for $m_S$, as the top left panel of Figure 16 demonstrates. The consequences of such small $m_0$ values can be seen in the bottom left and bottom right panels of Figure 16, where we plot the lightest and heaviest up-type squark mass,

respectively, to illustrate that the squark masses are now predominantly set by the universal gaugino mass, with little influence from $m_0$ or $A_0$. Furthermore, the squarks are always lighter than the gluino, which has a mass $\approx 2 M_{1/2}$.

Compared to the CNMSSM, we find that the situation in the CE6SSM is rather different. The CE6SSM is an alternative to the CNMSSM in which an elementary $\mu$ term is forbidden by a $U(1)_N$ gauge symmetry, and complete $E_6$ matter supermultiplets are included to ensure anomaly cancellation. As with the CNMSSM, the two-scale solver is ineffective for finding solutions because EWSB needs to have a soft mass as an output, but one can make a spectrum generator with the semi-analytic solver where the universal GUT scale masses $m_0$, $M_{1/2}$ and $A_0$ are EWSB outputs. Using the CE6SSM model file provided with FlexibleSUSY 2.0, we carried out a scan in which the singlet-Higgs Yukawa coupling, $\lambda(M_X) \equiv \lambda_3(M_X)$, and the exotic Yukawa coupling, $\kappa(M_X)$, were varied over $[-1, 1]$ and the singlet VEV, $s$, was varied over $[1, 500]$ TeV. The results of this scan are shown in Figure 17.

In contrast to the CNMSSM, in the CE6SSM it is very easy to have large $m_0$ values as there is an exotic Yukawa coupling between the singlet and extra colored matter introduced to avoid gauge anomalies that drives the soft singlet mass negative, providing a radiative symmetry breaking mechanism. Additionally, whereas $m_0 \lesssim M_{1/2}$ in the CNMSSM, in this model it is typically the case that $m_0$ is larger than $M_{1/2}$. This is in qualitative agreement with the literature [152, 153, 217, 230], and arises because the new colored matter results in heavily modified RGEs in which the 1-loop $\beta$ function of the strong coupling now vanishes. The squark masses are mostly set by $m_0$ as a result, as is shown in the top right panel of Figure 17, while the bottom panels show that the range for $\lambda$ and $\kappa$ at the GUT scale is very wide, despite strong constraints on $\lambda$ at the electroweak scale coming from requirements for correct EWSB. We do not make a detailed quantitative comparison to previous work

\footnote{The definitions of the new Lagrangian parameters and GUT scale constraints in the CE6SSM that we use may be found in Ref. [153].}
Figure 17: CE6SSM solutions obtained using FlexibleSUSY 2.0 and MultiNest 3.10 shown in the \( m_0 - M_{1/2} \) plane with \( A_0 \), the lightest up-type squark pole mass \( M_{\tilde{u}_1} \), \(|\lambda|\) and \(|\kappa|\) as color contours.

in the literature here, but note that this is the first time that the CE6SSM results have been presented with the same level of precision (full 2-loop RGEs, 1-loop pole masses) as is standard in the CMSSM and significant quantitative differences are to be expected.

For a more precise comparison between calculations performed at the same level of precision, we have also performed scans in a recently proposed variant of the E6SSM, the so-called CSE6SSM [129, 134]. Here the results obtained using FlexibleSUSY 2.0 have been checked for agreement with those obtained from a hand-written prototype of the semi-analytic solver that was implemented for the studies in Refs. [129, 134]. The solutions found using the generated CSE6SSM spectrum generator are compared with those found in Ref. [134] in Figure 18. The viable solution regions and values of the model parameters are found to be in very good agreement with the results obtained using the earlier code.

10. FlexibleEFTHiggs

FlexibleEFTHiggs is a method to predict the lightest Higgs pole mass in any BSM model accurately for both high and low new physics scales \( M_S \) and was presented first in Ref. [31]. An implementation of this method was first released in FlexibleSUSY 1.7.0. Here we present an upgrade at the next-to-leading order and next-to-leading logarithmic (NLO+NLL) accuracy, which we release in FlexibleSUSY 2.0.
Figure 18: CSeSSM solutions obtained using the spectrum generator automatically generated by FlexibleSUSY 2.0 (left panel) and using the prototype spectrum generator used in the numerical analysis of Ref. [134] (right panel), showing good agreement between the two codes. Note that here we have not applied the limits on the Higgs mass or dark matter relic density that lead to additional restrictions on the parameter space, as discussed in Ref. [134].

FlexibleEFTHiggs combines an EFT approach with a diagrammatic calculation, allowing for an all-order resummation of large logarithms of the ratio $M_S/m_t$, together with the inclusion of all non-logarithmic 1-loop contributions. In particular, all power-suppressed 1-loop contributions of $O(v^2/M_S^2)$ are included in FlexibleEFTHiggs, which would otherwise be neglected in a pure EFT calculation. Thanks to these properties, FlexibleEFTHiggs maintains the accuracy at all scales: For low scales the prediction agrees with a fixed-order calculation; for large scales it agrees with a pure EFT calculation. In the intermediate region, where the $O(v^2/M_S^2)$ terms are small, but still non-negligible, FlexibleEFTHiggs gives the correct fixed-order result plus higher-order logarithms, thus resolving the ambiguity between the fixed-order and the pure EFT approach.

In Ref. [31] already several versions of the FlexibleEFTHiggs approach have been extensively discussed and compared with existing calculations of the lightest Higgs boson mass in the MSSM and other supersymmetric models. The approach has also been implemented recently in SARAH/SPheno [67], including 2-loop corrections in the matching. The version implemented in FlexibleSUSY 2.0 contains additional improvements resulting in a higher accuracy. In the following, we briefly summarize the main idea of FlexibleEFTHiggs, then explain the details of the implemented version and how to use it. For further details of the approach and a detailed comparison of theoretical uncertainties, we refer to Ref. [31].

10.1. Basic matching condition

FlexibleEFTHiggs performs a matching of the BSM model to the SM, thereby determining the quartic Higgs coupling $\lambda$ of the SM. The basic ingredient of FlexibleEFTHiggs to fix $\lambda$ at the matching scale is a Higgs pole mass matching condition

$$
(M_h^{SM})^2 = (M_h^{BSM})^2,
$$

where $M_h^{SM}$ is the Higgs pole mass calculated in the SM at the 1-loop level and $M_h^{BSM}$ is the corresponding SM-like Higgs pole mass in the BSM model, also at the 1-loop level.
Generally, the Higgs pole mass is computed in any BSM model by solving the following equation (or a suitable matrix generalization):

\[
(M_h^{\text{BSM}})^2 = (m_h^{\text{BSM}})^2 - \text{Re} \Sigma_h^{\text{BSM}}(p^2) + t_h^{\text{BSM}} \frac{v}{v},
\]

where \(m_h^{\text{BSM}}\) is the respective tree-level mass and \(\Sigma_h^{\text{BSM}}\) and \(t_h^{\text{BSM}}\) are the Higgs self energy and tadpole in the \(\overline{\text{MS}}/\text{DR}\) scheme. In principle, in an all-order calculation, the Higgs self energy has to be evaluated at the momentum \(p^2 = (M_{SM} h)^2 = (M_h^{\text{BSM}})^2\). From this condition the quartic Higgs coupling of the SM can be extracted as

\[
\lambda = \frac{1}{v^2} \left[ (M_h^{\text{BSM}})^2 + \text{Re} \Sigma_h^{\text{SM}}((M_h^{\text{SM}})^2) - t_h^{\text{SM}} \right].
\]

This matching is equivalent to the one of pure EFT calculations \([160, 196]\) at the 1-loop level, up to power-suppressed terms. Correspondingly, the resulting Higgs boson mass is exact at the 1-loop level and takes into account all leading logarithms \([31]\).

10.2. New matching procedure in \textbf{FlexibleSUSY 2.0}

\textbf{FlexibleSUSY 2.0} has an improved implementation of the approach, which is still exact at the 1-loop level but also correctly resums next-to-leading logarithms. This improvement originates from an amended matching procedure. As mentioned before, the matching procedure in Eq. (89) is equivalent to a pure EFT matching at the 1-loop level. However, depending on implementation details, it can differ by terms of 2-loop or higher order. If these spurious 2-loop terms contain (next-to-leading) large logarithms, they spoil the correct resummation of (next-to-leading) logarithms by RGE running.

By construction, all versions of \textbf{FlexibleEFTHiggs} discussed in Ref. \([31]\) and Ref. \([67]\) are correct at the leading logarithmic level, however not all subleading logarithms are correctly included.

In the following, we discuss the two potential origins of these subleading logarithms and how they are avoided by the improved implementation in \textbf{FlexibleSUSY 2.0}.

\textit{Insertion of 1-loop parameters into 1-loop BSM self energies or tadpoles.} The first potential source of large 2-loop logarithms in the matching procedure is the insertion of parameters, which have been obtained from the SM via a 1-loop matching, into the 1-loop self energies or tadpoles of the BSM model. We illustrate this effect with the most important parameter, the top Yukawa coupling: The running top Yukawa coupling of the BSM model \(y_t^{\text{BSM}}\) is determined by a matching as\(^{27}\)

\[
y_t^{\text{BSM}} = y_t^{\text{SM}} + \Delta y_t,
\]

where \(\Delta y_t\) is of 1-loop order (but without large logarithms). At the same time, the Higgs pole mass calculations on the left-hand side and right-hand side of Eq. (87) are of the form

\[
(M_h^{\text{SM}})^2 = (m_h^{\text{SM}})^2 + \propto \left(\frac{m_h^{\text{SM}}}{Q}\right)^2 \log \frac{m_h^{\text{SM}}}{Q} + \cdots,
\]

\[
(M_h^{\text{BSM}})^2 = (m_h^{\text{BSM}})^2 + \propto \left(\frac{m_h^{\text{BSM}}}{Q}\right)^2 \log \frac{m_h^{\text{BSM}}}{Q} + \cdots,
\]

\(^{27}\)We ignore potential tree-level factors here for brevity.
where the matching scale \( Q \) is of the order \( M_\text{S} \) and we have also introduced \( m_{h}^{\text{SM}}, m_{t}^{\text{SM}} \) and \( m_{t}^{\text{BSM}} \) for the running SM Higgs mass, running SM top mass and running BSM top mass, keeping our convention of using an upper case ‘\( M \)’ for pole masses and lower case ‘\( m \)’ for running tree-level masses. If Eq. (91) and Eq. (92) are set equal and the relation Eq. (90) is inserted, potentially large 2-loop terms for example of the form

\[
\frac{4(v_{\text{SM}})^2(y_{t}^{\text{SM}})^3}{(4\pi)^2} \log \frac{m_{t}^{\text{SM}}}{Q} + \ldots
\]

(93)

remain. Such terms effectively shift the quartic Higgs coupling of the SM by next-to-leading logarithmic 2-loop terms.\(^{28}\)

In order to avoid large higher-order logarithms originating from the insertion of 1-loop parameters into 1-loop BSM self energies and tadpoles, \texttt{FlexibleSUSY} 2.0 maintains two different sets of running BSM parameters: One parameter set which has been obtained from the SM using a tree-level matching, and another set from the SM using 1-loop matching. The tree-level parameter set is used to evaluate the 1-loop self energies and tadpoles on the right-hand side of Eq. (88). In this way, no terms like the ones in Eq. (93) are generated. The 1-loop parameter set is used to evaluate the tree-level Higgs mass (matrix) of the BSM model. In this way, the desired 1-loop corrections to the quartic Higgs coupling \( \lambda \) are generated.

\textit{Momentum iteration.} The second source of large 2-loop logarithms in the matching has to do with the momentum argument of the self energies entering Eqs. (87) and (88). Writing \( p^2 = (m_{h}^{\text{BSM}})^2 + \Delta p^2 \), we see that the momentum argument of Eq. (88) contains the 1-loop term \( \Delta p^2 \) (which also involves large logarithms). The difference between the left-hand side and the right-hand side of the matching condition, Eq. (87), then contains 2-loop terms, which can be expanded as

\[
\left( \frac{\partial}{\partial p^2} \text{Re} \Sigma_{h}^{\text{BSM}}(p^2) - \frac{\partial}{\partial p^2} \text{Re} \Sigma_{h}^{\text{SM}}(p^2) \right) \bigg|_{p^2=(m_{h}^{\text{BSM}})^2} \Delta p^2.
\]

(94)

If the self energies are evaluated at the 1-loop level and \( p^2 \) is determined as described above, these terms do not cancel against anything. Like the terms discussed in Eq. (93), these terms would then lead to large 2-loop next-to-leading logarithms in the determination of \( \lambda \).

To avoid large higher-order logarithmic contributions coming from the momentum argument, \texttt{FlexibleSUSY} 2.0 does not perform the usual momentum iteration when the Higgs pole masses in the SM and in the BSM model are calculated at the matching scale for Eq. (87). Instead, the SM Higgs pole mass at the matching scale is now calculated as

\[
(M_{h}^{\text{SM}})^2 = (m_{h}^{\text{SM}})^2 \frac{1}{\text{Re} \Sigma_{h}^{\text{SM}}((m_{h}^{\text{BSM}})^2)} + \frac{\Delta_M^2}{v},
\]

(95)

where the self energy momentum is set to the tree-level \( \overline{\text{MS}}/\overline{\text{DR}} \) Higgs mass \( m_{h}^{\text{BSM}} \) in the BSM model at the matching scale, which is calculated in terms of running BSM parameters which have been obtained by a tree level matching. A similar expression is used to calculate the Higgs pole mass in the BSM model \( M_{h}^{\text{BSM}} \), where we also insert \( p^2 = (m_{h}^{\text{BSM}})^2 \) as the

\(^{28}\)If the self energies are evaluated at the 2-loop level, the problem repeats itself one order higher, i.e., the term in Eq. (93) is cancelled but similar terms of next-to-leading logarithmic 3-loop order remain.
self energy momentum in order to enable cancellation of momentum-dependent terms. For example, if the BSM Higgs is a singlet, the BSM Higgs pole mass is calculated as

\[(M_h^{BSM})^2 = (m_h^{BSM})^2 - \text{Re} \Sigma_h^{BSM}((m_h^{BSM})^2) + \frac{t_h^{BSM}}{v} \quad . \tag{96}\]

On the other hand, if the BSM Higgs is a multiplet and the \(k\)-th element is the SM-like Higgs, then the SM-like BSM Higgs pole mass is the \(k\)-th eigenvalue of the loop-corrected mass matrix \(M_h\) in the interaction eigenstate basis,

\[(M_h)_{ij} = (m_h^{BSM})_{ij}^2 - \text{Re} \Sigma_h^{BSM}((m_h^{BSM})_{ij}^2) + \frac{t_h^{BSM}}{v_i} \delta_{ij} \quad . \tag{97}\]

By employing this new matching procedure, \texttt{FlexibleEFTHiggs} consistently avoids large higher-order logarithms and thereby resums the leading and next-to-leading logarithms and includes all non-logarithmic 1-loop contributions.

\subsection*{10.3. Comparison of old and improved \texttt{FlexibleEFTHiggs} implementations}

In Figure 19 we show a comparison of the predicted lightest \(CP\)-even Higgs mass in the MSSM between the old \texttt{FlexibleEFTHiggs} implementation of \texttt{FlexibleSUSY} 1.7.4 (red dotted line) and the improved version in \texttt{FlexibleSUSY} 2.0 (red solid line). For small SUSY scales of \(M_S < 300\,\text{GeV}\), we find that the improved version still reproduces the fixed-order calculation. As can be seen in the left panel of Figure 19 for vanishing stop mixing, \(X_t = 0\), both the old and the improved version closely reproduce the 2-loop pure EFT calculation with \texttt{HSSUSY}: For SUSY scales above 10 TeV, the old version deviates from \texttt{HSSUSY-2L} by around 600 MeV while the improved one deviates by around 10 MeV. This is due to the fact that for \(X_t = 0\), the 2-loop threshold correction to the quartic Higgs coupling at the SUSY scale is negligible. However, for maximal stop mixing, \(X_t/M_S = \sqrt{6}\), which is the region where the old implementation showed the largest theoretical uncertainty, we find up to 3 GeV difference between the old and the improved implementation, see the right panel of Figure 19. This difference manifests the consequences of the different treatment of higher-order terms in the two versions, especially the inclusion of large 2-loop logarithms in the old implementation.

The figure shows furthermore that the improved version (which performs a 1-loop calculation) is now able to perfectly reproduce the 1-loop pure EFT calculation of \texttt{HSSUSY} (blue crosses) for arbitrary stop mixing and SUSY scales above \(\approx 1\,\text{TeV}\). This is in contrast to the old version, which shows a stronger deviation of around 2 GeV from the 1-loop pure EFT calculation for large stop mixing, see the right panel of Figure 19. Compared to the 2-loop pure EFT calculation of \texttt{HSSUSY} (blue dashed line), both the old and the improved version deviate by around 1–2 GeV for non-zero stop mixing. This deviation can be attributed to genuine 2-loop contributions. Note that \texttt{HSSUSY} does not reproduce the Higgs mass prediction of the fixed-order calculation for \(M_S \lesssim 400\,\text{GeV}\) in the shown scenario with \(X_t = 0\), because of the neglected terms of \(O(v^2/M_S^2)\). In other scenarios the \(O(v^2/M_S^2)\) terms may be important up to \(M_S \approx 1\,\text{TeV}\).

\subsection*{10.4. Choosing \texttt{FlexibleEFTHiggs} in the model file}

In order to build a \texttt{FlexibleEFTHiggs} spectrum generator, the \texttt{FlexibleEFTHiggs} flag can be set to \texttt{True} in the model file, see Table 11:

\begin{verbatim}
FlexibleEFTHiggs = True;
\end{verbatim}
Figure 19: Comparison of the predicted lightest CP-even Higgs pole mass in the MSSM using the FlexibleEFTHiggs implementations of FlexibleSUSY 1.7.4 and 2.0 for tan β = 5. In the left panel we use $X_t = 0$ and in the right panel $M_S = 2$ TeV.

In FlexibleEFTHiggs spectrum generators, the low-energy boundary condition cannot be modified, because it is fixed internally to perform a matching of the SM(5) to the full SM. The considered BSM model is matched to the SM at the BSM matching scale, $Q_{\text{match}}$, which is set to the susyscale by default. In this matching, the running normalized gauge couplings $g_i^{\text{BSM}}(Q_{\text{match}})$ ($i = 1, 2, 3$), the Yukawa coupling matrices $Y_f^{\text{BSM}}(Q_{\text{match}})$ ($f = u, d, e$) and the SM-like vacuum expectation value $v^{\text{BSM}}(Q_{\text{match}})$ of the BSM model are determined automatically from the following matching conditions on pole masses and running couplings

\begin{align}
(M_V^{\text{BSM}})^2 &= (M_V^{\text{SM}})^2, \quad V = W, Z, \\
M_f^{\text{BSM}} &= M_f^{\text{SM}}, \quad f = e, \mu, \tau, u, d, c, s, t, b, \\
\alpha_{\text{em}}^{\text{BSM}}(Q_{\text{match}}) &= \alpha_{\text{em}}^{\text{SM}}(Q_{\text{match}}) \times (1 + \Delta \alpha_{\text{em}}), \quad (98c) \\
\alpha_s^{\text{BSM}}(Q_{\text{match}}) &= \alpha_s^{\text{SM}}(Q_{\text{match}}) \times (1 + \Delta \alpha_s), \quad (98d)
\end{align}

where $\Delta \alpha_{\text{em}}$ and $\Delta \alpha_s$ are the known 1-loop threshold corrections [248], including potential MS to DR conversion terms [249]. FlexibleSUSY imposes the individual matching conditions in Eqs. (98) at the appropriate loop orders such that no large 2-loop logarithms are generated, as described in Section 10.2. For example, to obtain the correct Higgs mass in the MSSM at the 1-loop level, Eqs. (98a) and (98c) are imposed at the 1-loop level while Eqs. (98b) and (98d) are imposed at the tree level. The SM-like vacuum expectation value $v^{\text{BSM}}(Q_{\text{match}})$ of the BSM model is defined as

\begin{align}
v^{\text{BSM}}(Q_{\text{match}}) &= \frac{2m_Z^{\text{BSM}}(Q_{\text{match}})}{\sqrt{(g_Y^{\text{BSM}}(Q_{\text{match}}))^2 + (g_2^{\text{BSM}}(Q_{\text{match}}))^2}}, \quad (99)
\end{align}

where $m_Z^{\text{BSM}}(Q_{\text{match}})$ is the running Z boson mass and $g_Y^{\text{BSM}}(Q_{\text{match}})$ and $g_2^{\text{BSM}}(Q_{\text{match}})$ are the running electroweak gauge couplings in the BSM model at the matching scale.
The running BSM model parameters can be given as input at either the \texttt{SUSYScale} or at the \texttt{HighScale}. The following example demonstrates how to fix the \( \overline{\text{DR}} \) parameters of the MSSM at the SUSY scale.

**Example 19**

In the \texttt{FlexibleEFTHiggs}/MSSM model (\texttt{MSSMEFTHiggs}) the soft-breaking MSSM parameters, the \( \mu \) parameter and \( \tan \beta \) are input at the SUSY scale, \( M_S \). Thus, the boundary condition at the SUSY scale has the form

\[
\text{SUSYScaleInput} = \{ \\
\{v_u, \sqrt{v_u^2 + v_d^2} \sin[\arctan(\tan\beta)]\}, \\
\{v_d, \sqrt{v_u^2 + v_d^2} \cos[\arctan(\tan\beta)]\}, \\
\{\text{MassB}, \text{M1Input}\}, \\
\{\text{MassWB}, \text{M2Input}\}, \\
\{\text{MassG}, \text{M3Input}\}, \\
\{\text{mq2}, \text{mq2Input}\}, \\
\{\text{mu2}, \text{mu2Input}\}, \\
\{\text{md2}, \text{md2Input}\}, \\
\{\text{ml2}, \text{ml2Input}\}, \\
\{\text{me2}, \text{me2Input}\}, \\
\{\beta_{\{\text{Mu}\}}, \text{MuInput}\}, \\
\{\beta_{\{\text{B}\}}[\{\text{Mu}\}], \text{M1Input}^{-2}/(\tan\beta + 1/\tan\beta)\}, \\
\{\text{T}[\text{Yu}], \text{AuInput Yu}\}, \\
\{\text{T}[\text{Yd}], \text{AdInput Yd}\}, \\
\{\text{T}[\text{Ye}], \text{AeInput Ye}\} \\
\}\;.
\]

The symbols \( \tan\beta, \text{M1Input}, \text{M2Input}, \text{M3Input}, \text{mq2Input}, \text{mu2Input}, \text{md2Input}, \text{ml2Input}, \text{me2Input}, \text{MuInput}, \text{AuInput}, \text{AdInput}, \text{AeInput} \) describe the MSSM input parameters \( \tan\beta_{\text{DR}}(M_S), M_i(M_S), m^2_{\tilde{f}}(M_S) \ (f = q, u, d, l, e), \mu(M_S), m_A(M_S) \) and \( A_f(M_S) \) \( (f = u, d, e) \) in the \( \overline{\text{DR}} \) scheme at the SUSY scale.

Note that no explicit SUSY scale boundary condition for the gauge couplings, \( g_1, g_2 \) and \( g_3 \), and Yukawa couplings, \( Y_u, Y_d \) and \( Y_e \), of the MSSM has to be specified, because they are all fixed automatically at \( Q_{\text{match}} \) using the \texttt{FlexibleEFTHiggs} matching conditions, Eqs. (98).

However, there is a subtlety with the vacuum expectation values: In the above boundary condition, the input value \( \tan\beta_{\text{DR}}(M_S) \) is used to fix the ratio of \( v_u(M_S) \) and \( v_d(M_S) \). However, their magnitude \( \sqrt{v_u^2 + v_d^2} \) is unfixed so far. To fix it, we can use the value of \( v_{\text{MSSM}}(Q_{\text{match}}) \), which is automatically determined by \texttt{FlexibleSUSY} at the matching scale, see Eq. (99). Therefore, we want to set

\[
v_u(Q_{\text{match}}) = v_{\text{MSSM}}(Q_{\text{match}}) \sin \beta_{\text{DR}}^\text{MSSM}(Q_{\text{match}}), \quad (100a) \\
v_d(Q_{\text{match}}) = v_{\text{MSSM}}(Q_{\text{match}}) \cos \beta_{\text{DR}}^\text{MSSM}(Q_{\text{match}}). \quad (100b)
\]

Such a matching is not done automatically by \texttt{FlexibleEFTHiggs}. The user must specify how the VEVs of any Higgs fields that have electroweak interactions are related to the electroweak VEV, \( v_{\text{BSM}}(Q_{\text{match}}) \), which is given above for the MSSM. To do this, the model file has an additional constraint list: \texttt{MatchingScaleInput}. Conditions to relate or fix model parameters at the matching scale \( Q_{\text{match}} \) can be expressed...
using the MatchingScaleInput list. This can actually be used to set any BSM parameter or to override the automatic FlexibleEFTHiggs matching conditions if the user wishes. However, it is only required that the user specifies the matching for the VEVs here. To express the relations of Eqs. (100), we set

```
MatchingScaleInput = {
    {vu, VEV Sin[ArcTan[vu/vd]]},
    {vd, VEV Cos[ArcTan[vu/vd]]}
};
```

The symbol \(\text{VEV}\) is reserved by FlexibleSUSY and represents the running SM-like vacuum expectation value \(v^{\text{MSSM}}(Q_{\text{match}})\) at the matching scale, as defined in Eq. (99).

| Symbol                  | Default value | Allowed values | Description                                      |
|-------------------------|---------------|----------------|--------------------------------------------------|
| FlexibleEFTHiggs        | False         | True or False  | Flag to enable/ disable FlexibleEFTHiggs         |
| VEV                     | –             | –              | SM-like VEV in the BSM model, \(v^{\text{BSM}}(Q_{\text{match}})\) |
| MatchingScaleInput      | {}            | list of 2-tuples | boundary conditions for BSM parameters at the matching scale \(Q_{\text{match}}\) |

Table 11: FlexibleEFTHiggs model file options

Once the model file is written, a spectrum generator can be created and run in the usual way. For example, to build the model described in Example 19, one may run:

```
$ ./createmodel --name=MSSMEFTHiggs
$ ./configure --with-models=MSSMEFTHiggs
$ make
```

These commands create the FlexibleSUSY spectrum generator for the MSSMEFTHiggs model. The generated spectrum generator can then be run from the command line as

```
$ cd models/MSSMEFTHiggs
$ ./run_MSSMEFTHiggs.x --slha-input-file=LesHouches.in.MSSMEFTHiggs
```

The only difference with the SLHA interface is that there are new FlexibleEFTHiggs-specific options in the SLHA file. In FlexibleEFTHiggs, the pole masses of the BSM particles are calculated at the scale \(Q_{\text{pole,BSM}}\), which is set to the SUSYScale by default. The scale \(Q_{\text{pole,BSM}}\) can be changed by setting the entry FlexibleSUSY[17] to a non-zero value in the SLHA input file. Similarly, in the Mathematica interface \(Q_{\text{pole,BSM}}\) can be changed by setting poleMassScale to a non-zero value. The pole masses of the SM particles are calculated at the scale \(Q_{\text{pole,SM}}\), which is set to the top pole mass \(M_t\) by default. The scale \(Q_{\text{pole,SM}}\) can be changed by setting the entry FlexibleSUSY[18] to a non-zero value in the SLHA input file. In the Mathematica interface, \(Q_{\text{pole,SM}}\) can be changed by setting eftPoleMassScale to a non-zero value. The matching scale \(Q_{\text{match}}\) is set to the SUSYScale by default. It can be changed by setting the entry FlexibleSUSY[19] to a non-zero value.
in the SLHA input file. In the Mathematica interface, $Q_{\text{match}}$ can be changed by setting `eftMatchingScale` to a non-zero value.

**Example 20**

This example demonstrates how a partial uncertainty estimate of the lightest Higgs pole mass can be made with `FlexibleEFTHiggs`. The uncertainty is estimated by varying the matching scale $Q_{\text{match}}$ and the scale $Q_{\text{pole,SM}}$, at which the lightest Higgs pole mass is calculated, both by a factor 2.

```mathematica
Get["models/MSSMEFTHiggs/MSSMEFTHiggs_librarylink.m"];
Mtpole = 173.34;

(* generate logarithmically spaced range \([start, stop]\) *)
LogRange[start_, stop_, steps_] :=
  Exp /@ Range[Log[start], Log[stop],
  (Log[stop] - Log[start])/steps];

(* generate logarithmically spaced range \([Q/2, 2Q]\) *)
GenerateScales[Q_] := LogRange[Q/2, 2 Q, 10];

(* run MSSMEFTHiggs spectrum generator *)
RunMSSMEFTHiggs[MS_, TB_, Xt_, Qpole_, Qmatch_] :=
  Module[{handle, spectrum},
    handle = FSMSSMEFTHiggsOpenHandle[
      fsSettings -> {
        precisionGoal -> 1.*^-5,
        maxIterations -> 10000,
        poleMassLoopOrder -> 2,
        ewsbLoopOrder -> 2,
        betaFunctionLoopOrder -> 3,
        thresholdCorrectionsLoopOrder -> 2,
        poleMassScale -> 0,
        eftPoleMassScale -> Qpole,
        eftMatchingScale -> Qmatch,
        eftMatchingLoopOrderUp -> 1,
        eftMatchingLoopOrderDown -> 1,
        calculateBSMMasses -> 0
      },
      fsSMPParameters -> {
        Mt -> Mtpole
      },
      fsModelParameters -> {
        MSUSY -> MS,
        M1Input -> MS,
        M2Input -> MS,
        M3Input -> MS,
        MuInput -> MS,
        mAInput -> MS,
        TanBeta -> TB,
        mq2Input -> MS^2 IdentityMatrix[3],
        mu2Input -> MS^2 IdentityMatrix[3],
        md2Input -> MS^2 IdentityMatrix[3],
        ml2Input -> MS^2 IdentityMatrix[3],
        me2Input -> MS^2 IdentityMatrix[3],
        AuInput -> {{MS/TB, 0, 0},
      }];
```

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{0 , MS/3B , 0},
{0 , MS/3B , 0},
AdInput -> MS TB IdentityMatrix[3],
AeInput -> MS TB IdentityMatrix[3]
}
;
spectrum = FSMSSMEFTHiggsCalculateSpectrum[handle];
FSMSSMEFTHiggsCloseHandle[handle];
spectrum
};
(* extract lightest Higgs pole mass Pole[M[hh]] from spectrum *)
RunMSSMEFTHiggsMh[par..._ :=
(Pole[M[hh]] /. (MSSMEFTHiggs /. RunMSSMEFTHiggs[par...]))[[1]];
(* calculate Higgs mass and perform scale variation *)
RunMSSMEFTHiggsUncertainty[MS_ , TB_ , Xt_ :=
Module[{MhMean , Dm, varyQpole, varyQmatch},
MhMean = RunMSSMEFTHiggsMh[MS , TB , Xt , 0, 0];
 varyQpole = RunMSSMEFTHiggsMh[MS , TB , Xt , #, 0] & /@
  GenerateScales[Mtpole];
 varyQmatch = RunMSSMEFTHiggsMh[MS , TB , Xt , 0, #] & /@
  GenerateScales[MS];
(* combine uncertainty estimates *)
Dm = Max[Abs[Max[varyQpole] - MhMean],
 Abs[Min[varyQpole] - MhMean] +
 Max[Abs[Max[varyQmatch] - MhMean],
 Abs[Min[varyQmatch] - MhMean]];{MhMean, Dm}
];
{Mh, Dm} = RunMSSMEFTHiggsUncertainty[2500 , 20, Sqrt[6]];
Print[“Mh = (" , Mh , " +- " , Dm, ") GeV”];

The output of the script could read

Mh = (125.047 +- 1.52741) GeV

11. Current limitations and workarounds

Currently, the models and scenarios which can be constructed with FlexibleSUSY 2.0 are limited to the following cases:

- The couplings of the model(s) must remain perturbative at all scales between the highest and lowest boundary condition.
- The considered models are required to have a gauge symmetry that has the SM gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ as a factor. This restriction is currently necessary to perform an unambiguous matching of the model to the SM(5) at the low-energy scale. If a model that does not have $G_{SM}$ as a gauge group factor is considered, then the gauge couplings of the model must be fixed by hand in one of the boundary conditions. See Section 5.2 for an example of a model with a left-right symmetry.
• Tensor-like Lagrangian parameters of rank 3, which would arise in $R$-parity violating SUSY models for example, are currently not supported. As a workaround, the rank 3 tensor-like couplings could be decomposed into a sum of terms with rank 2 matrix-like couplings.

• The extraction of the running Yukawa couplings in models where a 4th generation of fermions mixes with the SM fermions is currently not supported. As a workaround, the running Yukawa couplings can manually be fixed by using the running SM fermion masses, which can be accessed in the model file via the `upQuarksDRbar = diag(m_u, m_c, m_t), downQuarksDRbar = diag(m_d, m_s, m_b) and downLeptonsDRbar = diag(m_e, m_μ, m_τ)` symbols. In the $μ$SSM [250] (SARAH/FlexibleSUSY model name: `munuSSM`), for example, the Yukawa coupling matrix $Y_e$ of the down-type leptons can approximately be fixed at the low-energy scale as

$$Y_e(Q) = \frac{\sqrt{3}}{v_d} \text{diag}(m_e, m_μ, m_τ), \quad (101)$$

which is expressed in the FlexibleSUSY model file as

```
LowScaleInput = {
    {Ye, Sqrt[2] downLeptonsDRbar / vd},
    ...
};
```

Due to the modular nature of the generated code, adaptation and extension to overcome restrictions in scope are quite straightforward.

### 12. Conclusions

In order to study the vast zoo of models beyond the SM, tools for each model are necessary to calculate the mass spectrum and observables. FlexibleSUSY is a meta-tool for automatized generation of such tools which reliably operate at high precision and speed for a broad class of BSM models.

In this paper, we have presented all of the substantial updates to FlexibleSUSY available in version 2.0. These include many model-specific higher-order corrections, as well as extensions to support non-SUSY models (FlexibleBSM), models with complex parameters (FlexibleCPV) and a new solver which allows the EWSB outputs to be defined at the high scale (FlexibleSAS). Furthermore, FlexibleSUSY can now calculate in any given model: the anomalous magnetic moment of the muon (FlexibleAMU) as well as the muon decay and the $W$ mass (FlexibleMW), including partial 2-loop contributions. FlexibleSUSY 2.0 also comes with an update of the hybrid EFT/fixed-order calculation of the Higgs mass (FlexibleEFTHiggs) with a higher-order log resummation.

Altogether, these represent a significant extension to the calculations that can be performed in BSM models at high precision. To illustrate the variety of potential applications of FlexibleSUSY 2.0, we have presented many physics examples. These include large-scale parameter scans performed efficiently on multiple CPU cores (see Section 5.5), and the construction of low-energy effective field theories of SUSY models with complicated boundary conditions at the matching scale (see Sections 5.4 and 5.5). Indeed, FlexibleSUSY is already being used extensively for such cases, including global fits by the GAMBIT collaboration and major studies of precision Higgs mass predictions. Furthermore, the modularity...
of the generated spectrum generators allows easy implementation of model-specific higher-order corrections, which has been done in the past to include 3-loop Higgs mass contributions from the Himalaya library (see Section 4.1.2) and to add power-suppressed terms of $O(v^2/M_Z^2)$ to HSSUSY. Furthermore, we have illustrated how to calculate the anomalous magnetic moment of the muon and electric dipole moments with FlexibleSUSY 2.0 in Sections 6–7. Various physics applications for FlexibleSUSY have been presented in Section 9, which include the study of multiple solutions to the boundary value problem of the CMSSM as well as parameter scans in the CNMSSM and CE6SSM.

The FlexibleSUSY system has been extensively tested for correctness against results from the literature and other spectrum generators. In addition, speed tests have been carried out, with results proving its effectiveness in large-scale scans. The auto-generated C++ code is designed in such a way that users can easily read and reuse its components to develop their own analysis tools.

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A. SM input parameters

The list of all SM input parameters is given in Table A.12. In the SLHA interface of FlexibleSUSY, the SM parameters can be given in four different input blocks:

- The block SMINPUTS contains the electromagnetic and strong coupling, the Fermi constant and the masses of SM particles, as defined in the SLHA-2 standard [158]. The definitions of the individual block entries are shown in the first part of Table A.12. If a parameter is omitted from the SMINPUTS block, then it is set to the default value defined in Table A.12.

- The CKM matrix is given as input in the block VCKMIN in the Wolfenstein parametrization, as defined in Ref. [158]. If the VCKMIN block or an entry is missing, the corresponding parameter is set to zero. The given CKM matrix elements can be accessed in the FlexibleSUSY model file via the CKM symbol to set the Yukawa matrices at the low-energy scale. See FlexibleSUSY’s CMSSMCKM model for an example.

- The PMNS matrix is given as input in the block UPMNSIN, as defined in Ref. [158]. If the UPMNSIN block or an entry is missing, the corresponding parameter is set to zero. The given PMNS matrix elements can be accessed in the FlexibleSUSY model file via the PMNS symbol to fix potential neutrino mass parameters at the low-energy scale.

- For special applications such as the calculation of $a_\mu$ with GM2Calc, further input parameters are needed. These can be given in the FlexibleSUSYInput block. The block entries are defined in Table A.12. If the block or a block entry is missing, the input parameters are set to their default values as defined in the table.

In SLHA format the blocks with their respective default values read:

| Block   | SMINPUTS | # Standard Model inputs |
|---------|----------|-------------------------|
| 1       | 1.279160000e+02 | # alpha^(-1) SM MSbar (MZ) |
| 2       | 1.166378700e-05 | # G_Fermi |
| 3       | 1.184000000e-01 | # alpha_s (MZ) SM MSbar |
| 4       | 9.118760000e+01 | # MZ (pole) |
| 5       | 4.180000000e+00 | # mb(mb) SM MSbar |
| 6       | 1.733400000e+02 | # mtop (pole) |
| 7       | 1.776990000e+00 | # mtau (pole) |
| 8       | 0.000000000e+00 | # mnu3 (pole) |
| 9       | 8.038500000e+01 | # MW pole |
| 11      | 5.109889020e-04 | # melectron (pole) |
| 12      | 0.000000000e+00 | # mnu1 (pole) |
| 13      | 1.056683715e-01 | # mmuon (pole) |
| 14      | 0.000000000e+00 | # mnu2 (pole) |
| 21      | 4.750000000e-03 | # md (2 GeV) MS-bar |
| 22      | 2.400000000e-03 | # mu (2 GeV) MS-bar |
| 23      | 1.040000000e-01 | # ms (2 GeV) MS-bar |
| 24      | 1.270000000e+00 | # mc (mc) MS-bar |

| Block   | VCKMIN | # CKM matrix input (Wolfenstein parameters) |
|---------|--------|---------------------------------------------|
| 1       | 0      | # lambda (MZ) SM DR-bar |
| 2       | 0      | # A (MZ) SM DR-bar |
| 3       | 0      | # rhobar (MZ) SM DR-bar |
| 4       | 0      | # etabar (MZ) SM DR-bar |

| Block   | UPMNSIN | # PMNS matrix input |
|---------|---------|---------------------|
| 1       | 0       | # theta_12 |
In FlexibleSUSY’s Mathematica interface, the SM parameters must be passed to the FS<model>OpenHandle[fsSMParameters -> {...}] function in the form of replacement rules. The symbols associated to the SM input parameters are given in Table A.12. Unset parameters are set to their default values defined in the table. Note that in the Mathematica interface of FlexibleSUSY, the CKM matrix parameters are given in the exact parametrization in terms of the angles $\theta_{12}, \theta_{23}, \theta_{13}$ and $\delta$ [218]. A call of FS<model>OpenHandle[] with all parameters set to their respective default values would read:

```mathematica
handle = FS<model>OpenHandle[
  fsSMParameters -> {
    alphaEmMZ -> 1/127.916, (* SMINPUTS[1] *)
    GF -> 1.166378700^-5, (* SMINPUTS[2] *)
    alphaSMZ -> 0.1184, (* SMINPUTS[3] *)
    MZ -> 91.1876, (* SMINPUTS[4] *)
    mbmb -> 4.18, (* SMINPUTS[5] *)
    Mt -> 173.34, (* SMINPUTS[6] *)
    Mtau -> 1.77699, (* SMINPUTS[7] *)
    Mv3 -> 0, (* SMINPUTS[8] *)
    MW -> 80.385, (* SMINPUTS[9] *)
    Me -> 0.000510998902, (* SMINPUTS[11] *)
    Mv1 -> 0, (* SMINPUTS[12] *)
    Mm -> 0.1056583715, (* SMINPUTS[13] *)
    Mv2 -> 0, (* SMINPUTS[14] *)
    md2GeV -> 0.00475, (* SMINPUTS[21] *)
    mu2GeV -> 0.0024, (* SMINPUTS[22] *)
    ms2GeV -> 0.104, (* SMINPUTS[23] *)
    mcmc -> 1.27,
    CKMTheta12 -> 0,
    CKMTheta23 -> 0,
    CKMTheta13 -> 0,
    CKMDelta -> 0,
    PMNSTheta12 -> 0, (* UPMNSIN[1] *)
    PMNSTheta23 -> 0, (* UPMNSIN[2] *)
    PMNSTheta13 -> 0, (* UPMNSIN[3] *)
    PMNSDelta -> 0, (* UPMNSIN[4] *)
    PMNSAlpha1 -> 0, (* UPMNSIN[5] *)
    PMNSAlpha2 -> 0, (* UPMNSIN[6] *)
    alphaEm0 -> 1/137.035999074, (* FlexibleSUSYInput[0] *)
    Mh -> 125.09 (* FlexibleSUSYInput[1] *)
  }
]
| Index | Mathematica symbol | Default | Description |
|-------|--------------------|---------|-------------|
| 1     | alphaEmMZ         | 1/127.916 | electromagnetic coupling, $\alpha_{em}^{SM}(M_Z)$ |
| 2     | GF                 | 1.1663787 $\cdot 10^{-5}$ | Fermi coupling constant, $G_F \times \text{GeV}^2$ |
| 3     | alphaSMZ          | 0.1184 | strong coupling, $\alpha_{SM}^{SM}(M_Z)$ |
| 4     | MZ                 | 91.1876 | $Z$ pole mass, $M_Z/\text{GeV}$ |
| 5     | mbmb               | 4.18 | running bottom mass, $m_b^{SM(5)}(m_b)/\text{GeV}$ |
| 6     | Mt                 | 173.34 | top pole mass, $M_t/\text{GeV}$ |
| 7     | Mtau               | 1.77699 | $\tau$ pole mass, $M_\tau/\text{GeV}$ |
| 8     | Mv3                | 0 | heaviest neutrino pole mass, $M_{\nu_3}/\text{GeV}$ |
| 9     | MW                 | 80.385 | $W$ pole mass, $M_W/\text{GeV}$ |
| 10    | Me                 | 0.000510998902 | electron pole mass, $M_e/\text{GeV}$ |
| 11    | Mv1                | 0 | lightest neutrino pole mass, $M_{\nu_1}/\text{GeV}$ |
| 12    | Mm                 | 0.1056583715 | muon pole mass, $M_\mu/\text{GeV}$ |
| 13    | Mv2                | 0 | 2nd lightest neutrino pole mass, $M_{\nu_2}/\text{GeV}$ |
| 14    | md2GeV             | 0.00475 | running down mass, $m_d(2\text{GeV})/\text{GeV}$ |
| 15    | mu2GeV             | 0.0024 | running up mass, $m_u(2\text{GeV})/\text{GeV}$ |
| 16    | ms2GeV             | 0.104 | running strange mass, $m_s(2\text{GeV})/\text{GeV}$ |
| 17    | mcmc               | 1.27 | running charm mass, $m_c^{SM(4)}(m_c)/\text{GeV}$ |

| Block | VCKMIN |
|-------|--------|
| 1     | CKMTheta12 | 0 | CKM matrix parameter $\theta_{12}$ |
| 2     | CKMTheta23 | 0 | CKM matrix parameter $\theta_{23}$ |
| 3     | CKMTheta13 | 0 | CKM matrix parameter $\theta_{13}$ |
| 4     | CKMDelta  | 0 | CKM matrix parameter $\delta$ |

| Block | UPMNSIN |
|-------|---------|
| 1     | PMNSTheta12 | 0 | PMNS solar angle $\theta_{12}$ |
| 2     | PMNSTheta23 | 0 | PMNS atmospheric angle $\theta_{23}$ |
| 3     | PMNSTheta13 | 0 | PMNS matrix parameter $\theta_{13}$ |
| 4     | PMNSDelta | 0 | PMNS Dirac phase $\delta$ |
| 5     | PMNSAlpha1 | 0 | PMNS 1st Majorana phase $\alpha_1$ |
| 6     | PMNSAlpha2 | 0 | PMNS 2nd Majorana phase $\alpha_2$ |

| Block | FlexibleSUSYInput |
|-------|------------------|
| 0     | alphaEm0         | 1/137.035999074 | $\alpha_{em}$ in the Thomson limit |
| 1     | Mh               | 125.09 | SM Higgs pole mass $M_h/\text{GeV}$ |

**Table A.12:** SLHA input block entries and Mathematica symbols to specify the SM input parameters. The first column represents the index in the corresponding SLHA input block and the second column the symbol used in the Mathematica interface.

**B. FlexibleSUSY configuration options**

FlexibleSUSY provides many configuration options to switch on/off contributions and choose/fine-tune the solver algorithm(s). All runtime configuration options are listed in
Table B.13. In the SLHA interface of FlexibleSUSY, the configuration options are read from the FlexibleSUSY block. In addition, some information is also read from the MODSEL block, see below. In the SLHA format all FlexibleSUSY configuration entries with their respective default values read:

```
| Block MODSEL            | Description                                      |
|-------------------------|--------------------------------------------------|
| 12 0                    | # output scale of running parameters (0 = SUSY scale) |

| Block FlexibleSUSY      | Description                                      |
|-------------------------|--------------------------------------------------|
| 0 1e-04                 | # precision goal                                 |
| 1 0                     | # max. iterations (0 = automatic)                |
| 2 0                     | # solver (0 = all, 1 = two-scale, 2 = semi-analytic) |
| 3 0                     | # calculate SM pole masses                        |
| 4 2                     | # pole mass loop order                           |
| 5 2                     | # EWSB loop order                                |
| 6 3                     | # beta-functions loop order                      |
| 7 2                     | # threshold corrections loop order               |
| 8 1                     | # Higgs 2L corrections O(alpha_t alpha_s)        |
| 9 1                     | # Higgs 2L corrections O(alpha_b alpha_s)        |
| 10 1                    | # Higgs 2L corrections O((alpha_t + alpha_b)^2)  |
| 11 1                    | # Higgs 2L corrections O(alpha_tau)^2             |
| 12 0                    | # force output                                   |
| 13 1                    | # Top quark 2L corrections QCD                   |
| 14 1e-11                | # beta-function zero threshold                   |
| 15 0                    | # calculate observables (a_muon, ...)             |
| 16 0                    | # force positive majorana masses                 |
| 17 0                    | # pole mass renormalization scale (0 = SUSY scale) |
| 18 0                    | # pole mass renormalization scale in the EFT     |
|                         | # (0 = min(SUSY scale, M_t))                     |
| 19 0                    | # EFT matching scale (0 = SUSY scale)            |
| 20 2                    | # EFT loop order for upwards matching            |
| 21 1                    | # EFT loop order for downwards matching          |
| 22 0                    | # EFT index of SM-like Higgs in the BSM model    |
| 23 1                    | # calculate BSM pole masses                       |
| 24 123111321            | # individual threshold correction loop orders    |
| 25 0                    | # ren. scheme for Higgs 3L corrections            |
|                         | # (0 = DR, 1 = MDR)                              |
| 26 1                    | # Higgs 3L corrections O(alpha_t alpha_s)^2      |
| 27 1                    | # Higgs 3L corrections O(alpha_b alpha_s)^2      |
| 28 1                    | # Higgs 3L corrections O(alpha_t^2 alpha_s)      |
| 29 1                    | # Higgs 3L corrections O(alpha_t^3)               |
```

In the Mathematica interface of FlexibleSUSY, the configuration options are passed to the function `FS<model>OpenHandle[fsSettings -> {...}]` in form of replacement rules. The symbols associated to the configuration options are given in Table B.13. Unset options are set to their default values defined in the table. A call of `FS<model>OpenHandle[]` with all configuration options set to their default values would read:

```
handle = FS<model>OpenHandle[  
  fsSettings -> { 
    precisionGoal -> 1.*^-4,    (* FlexibleSUSY[0] *) 
    maxIterations -> 0,         (* FlexibleSUSY[1] *) 
    solver -> 0,                (* FlexibleSUSY[2] *) 
    calculateStandardModelMasses -> 0, (* FlexibleSUSY[3] *) 
    poleMassLoopOrder -> 2,     (* FlexibleSUSY[4] *) 
    ewsbLoopOrder -> 2,         (* FlexibleSUSY[5] *) 
    betaFunctionLoopOrder -> 3, (* FlexibleSUSY[6] *) 
  } ];
```
thresholdCorrectionsLoopOrder -> 2, (* FlexibleSUSY[7] *)
higgs2loopCorrectionAtAs -> 1, (* FlexibleSUSY[8] *)
higgs2loopCorrectionAbAs -> 1, (* FlexibleSUSY[9] *)
higgs2loopCorrectionAtAt -> 1, (* FlexibleSUSY[10] *)
higgs2loopCorrectionAtauAtau -> 1, (* FlexibleSUSY[11] *)
forceOutput -> 0, (* FlexibleSUSY[12] *)
topPoleQCDCorrections -> 1, (* FlexibleSUSY[13] *)
betaZeroThreshold -> 1.*^-11, (* FlexibleSUSY[14] *)
forcePositiveMasses -> 0, (* FlexibleSUSY[16] *)
poleMassScale -> 0, (* FlexibleSUSY[17] *)
eftPoleMassScale -> 0, (* FlexibleSUSY[18] *)
eftMatchingScale -> 0, (* FlexibleSUSY[19] *)
eftMatchingLoopOrderUp -> 2, (* FlexibleSUSY[20] *)
eftMatchingLoopOrderDown -> 1, (* FlexibleSUSY[21] *)
eftHiggsIndex -> 0, (* FlexibleSUSY[22] *)
calculateBSMMasses -> 1, (* FlexibleSUSY[23] *)
thresholdCorrections -> 123111321, (* FlexibleSUSY[24] *)
higgs3loopCorrectionRenScheme -> 0, (* FlexibleSUSY[25] *)
higgs3loopCorrectionAtAsAs -> 1, (* FlexibleSUSY[26] *)
higgs3loopCorrectionAbAsAs -> 1, (* FlexibleSUSY[27] *)
higgs3loopCorrectionAtAsAs -> 1, (* FlexibleSUSY[28] *)
higgs3loopCorrectionAtAsAs -> 1, (* FlexibleSUSY[29] *)
parameterOutputScale -> 0 (* MODSEL[12] *)

The individual configuration options have the following meaning:

FlexibleSUSY[0], precisionGoal: This option describes the numeric precision of the renormalization group running, the mass spectrum calculation, the electroweak symmetry breaking and the calculation of the observables. For most models a precision of $10^{-4}$ is sufficient. For models with various 3-loop corrections, like HSSUSY or MSSM-like models, a precision of $10^{-5}$ might be better.

FlexibleSUSY[1], maxIterations: This option describes the maximum number of iterations for the renormalization group running between the various scales. If it is set to 0, then the maximum number of iterations $N_{\text{max, it}}$ is chosen according to the precision goal $p$ (see above) as

$$N_{\text{max, it}} = -10 \log_{10} p.$$  \hspace{1cm} (B.1)

FlexibleSUSY[2], solver: This option chooses the BVP solver to be used. If set to 0, all solvers that have been enabled in the model file (see Section 9.1) are used. In this case, each of the enabled solvers will be tried in turn, in the same order as given in FSBVPsolve, until a solution is found, at which point FlexibleSUSY will return this solution and no further solvers are tried. In the event that no solver obtains a valid solution, FlexibleSUSY reports the status of the last solver tried. Non-zero values of FlexibleSUSY[2] select a single solver to be used. If set to 1, the two-scale solver is used if it has been enabled in the model file. If set to 2, then the semi-analytic solver is used if it has been enabled in the model file. If a solver that has not been enabled in the model file is chosen, FlexibleSUSY stops with an error.

FlexibleSUSY[3], calculateStandardModelMasses: This option allows the user to enable/disable the calculation of the pole masses of the SM particles. Note that this switch
| Index | Mathematica symbol | Default | Description |
|-------|---------------------|---------|-------------|
| 0     | precisionGoal       | 1.*^-4  | precision goal of RG running and mass spectrum |
| 1     | maxIterations       | 0       | maximum number of iterations for the running between the scales (0 = automatic) |
| 2     | solver              | 0       | BVP solver (0 = all, 1 = two-scale solver, 2 = semi-analytic solver) |
| 3     | calculateStandardModelMasses | 0 | switch to enable/disable calculation of pole masses of SM particles (0 = disabled) |
| 4     | poleMassLoopOrder   | 2       | pole mass loop order |
| 5     | ewsbLoopOrder       | 2       | EWSB loop order (should be set equal to the pole mass loop order) |
| 6     | betaFunctionLoopOrder | 3   | loop order for renormalization group running |
| 7     | threshold CorrectionsLoopOrder | 2 | global switch for loop order of threshold corrections when converting the SM(5) parameters to the BSM parameters |
| 8     | higgs2loopCorrectionAtAs       | 1 | enable/disable 2-loop corrections $O(\alpha_t \alpha_s)$ to $M_{h,H,A}$ |
| 9     | higgs2loopCorrectionAbAs       | 1 | enable/disable 2-loop corrections $O(\alpha_b \alpha_s)$ to $M_{h,H,A}$ |
| 10    | higgs2loopCorrectionAtAt       | 1 | enable/disable 2-loop corrections $O(\alpha_t^2)$ to $M_{h,H,A}$ |
| 11    | higgs2loopCorrectionAttauAttau | 1 | enable/disable 2-loop corrections $O(\alpha_t \alpha_s)$ to $M_{h,H,A}$ |
| 12    | forceOutput          | 0       | force output, even if problems occurred |
| 13    | topPoleQCDCorrections  | 1 | QCD corrections to calculate $M_t$ (0 = 1-loop, 1 = 2-loop, 2 = 3-loop) |
| 14    | betaZeroThreshold    | 1.*^-11 | below this threshold $\beta$ functions are treated as zero |
| 15    | forcePositiveMasses  | 0       | make Majorana masses positive (violates SLHA) |
| 16    | poleMassScale        | 0       | scale at which pole masses are calculated (0 = SUSY scale) |
| 17    | eftPoleMassScale     | 0       | scale at which SM pole masses are calculated in FlexibleEFTHiggs (0 = $M_t$) |
| 18    | eftMatchingScale     | 0       | matching scale in FlexibleEFTHiggs (0 = SUSY scale) |
| 19    | eftMatchingLoopOrderUp | –     | ignored |
| 20    | eftMatchingLoopOrderDown | 1   | loop order for $\Delta \lambda$ in FlexibleEFTHiggs |
| 21    | eftHiggsIndex        | 0       | index of SM-like Higgs in BSM Higgs multiplet in FlexibleEFTHiggs |
| 22    | calculateBSMMasses   | 1       | enable/disable calculation of BSM pole masses |
| 23    | thresholdCorrections  | 123111321 | individual threshold correction loop orders, see Table B.14 |
| 24    | higgs3loopCorrectionRenScheme | 0 | renormalization scheme for 3-loop MSSM Higgs corrections (0 = $\overline{DR}$, 1 = $\overline{MDR}$) |
| 25    | higgs3loopCorrectionAtAsAs       | 1 | enable/disable 3-loop corrections $O(\alpha_t \alpha_s^2)$ to $M_{h,H,A}$ |
| 26    | higgs3loopCorrectionAbAsAs       | 1 | enable/disable 3-loop corrections $O(\alpha_b \alpha_s^2)$ to $M_{h,H,A}$ |
| 27    | higgs3loopCorrectionAtAtAs       | 1 | enable/disable 3-loop corrections $O(\alpha_t^2 \alpha_s)$ to $M_h$ |
| 28    | higgs3loopCorrectionAtAtAt       | 1 | enable/disable 3-loop corrections $O(\alpha_t^3)$ to $M_h$ |

**Table B.13:** SLHA input block entries and corresponding Mathematica symbols to specify the configuration options for FlexibleSUSY’s spectrum generators. The symbols $M_{h,H,A}$ and $M_t$ denote the Higgs and top quark pole masses, respectively.

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Table B.14: Specification of individual loop orders of threshold corrections for extracting the running masses, couplings and Weinberg angle in the BSM model at the low-energy scale using the field FlexibleSUSY[24] in the SLHA interface or the symbol thresholdCorrections in the Mathematica interface, respectively. The digit position is counted from the right, starting at 0. Setting all loop orders to their default values results in the integer 123111321.

| digit position n | default value (prefactor of $10^n$) | parameter |
|------------------|-------------------------------------|-----------|
| 0                | 1 (1-loop)                          | $\alpha_{em}^{BSM}$ |
| 1                | 2 (2-loop)                          | $\sin^{BSM}(\theta_W)$ |
| 2                | 3 (3-loop)                          | $\alpha_s^{BSM}$ |
| 3                | 1 (1-loop)                          | $m_Z^{BSM}$ |
| 4                | 1 (1-loop)                          | $m_H^{BSM}$ |
| 5                | 1 (1-loop)                          | $m_h^{BSM}$ |
| 6                | 3 (3-loop)                          | $m_t^{BSM}$ |
| 7                | 2 (2-loop)                          | $m_b^{BSM}$ |
| 8                | 1 (1-loop)                          | $m_{\tau}^{BSM}$ |

must be set to 1 in HSSUSY to calculate the Higgs pole mass, because in HSSUSY the Higgs pole mass is calculated in the SM.

FlexibleSUSY[4], poleMassLoopOrder: This option allows the user to select the loop order at which the pole masses are calculated. If set to 0, the running tree-level masses are output. If set to 1, the pole masses are calculated at the 1-loop level. If set to 2 or 3, then model-specific 2-loop or 3-loop corrections are taken into account, respectively, if they have been enabled in the model file (see Section 4.1). Important note: In order to obtain a consistent pole mass spectrum, the loop order of the electroweak symmetry breaking (see FlexibleSUSY[5], ewsbLoopOrder) must be set to the same value as the pole mass loop order!

FlexibleSUSY[5], ewsbLoopOrder: This option allows the user to select the loop order at which the electroweak symmetry breaking (EWSB) equations are solved. If set to 0, the EWSB equations are solved at the tree level. If set to 1, the EWSB equations are solved at the 1-loop level. If set to 2 or 3, then model-specific 2-loop or 3-loop corrections are taken into account, respectively, if they have been enabled in the model file (see Section 4.1). Important note: In order to obtain a consistent pole mass spectrum, the loop order of the electroweak symmetry breaking must be set to the same value as the pole mass loop order (see FlexibleSUSY[4], poleMassLoopOrder)!

FlexibleSUSY[6], betaFunctionLoopOrder: With this option the user can select the loop level of the $\beta$ functions used to integrate the RGEs. If set to 1, 1-loop $\beta$ functions are used. If set to 2, 2-loop $\beta$ functions are used. If set to 3, then model-specific 3-loop $\beta$ functions are used (see Section 4.1). Note that SARAH can generate 2-loop $\beta$ functions for all model parameters (except scalar tadpole terms), so 2-loop running can always be used.

FlexibleSUSY[7], thresholdCorrectionsLoopOrder: With this option the user can choose the maximum loop level of the threshold corrections used to determine the running gauge couplings $g_1, g_2, g_3$ and the running Yukawa coupling matrices $Y_u, Y_d, Y_e$ of the

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BSM model at the low-energy scale (LowScale) from the given SM input parameters \( \alpha_{\text{em}}^{\text{SM}(5)}(M_Z), \alpha_s^{\text{SM}(5)}(M_Z), G_F, M_Z, M_t, m_b^{\text{SM}(5)}(m_b), m_c^{\text{SM}(4)}(m_c), \ldots \). See Section 5.2 and 4.1 for a description on how the running gauge and Yukawa couplings are calculated and how model-specific higher-order corrections can be included. If the threshold corrections loop order is set to 1, then no 2-loop threshold corrections or higher are taken into account. If set to 2, then no 3-loop threshold corrections or higher are taken into account. If set to 3, then no 4-loop threshold corrections are taken into account. Note that threshold corrections for individual parameters can be disabled by using FlexibleSUSY[24] or thresholdCorrections, respectively.

FlexibleSUSY[8], higgs2loopCorrectionAtAs: With this option the 2-loop contributions to the Higgs pole mass(es) of \( O(\alpha_t\alpha_s) \) can be enabled/disabled. Note that this option has an effect only if 2-loop contributions have been activated in the FlexibleSUSY model file. See Section 4.1 for details on how to activate 2-loop contributions to the Higgs pole mass(es) of \( O(\alpha_t\alpha_s) \) in the SM, (N)MSSM or split-MSSM.

FlexibleSUSY[9], higgs2loopCorrectionAbAs: With this option the 2-loop contributions to the Higgs pole mass(es) of \( O(\alpha_b\alpha_s) \) can be enabled/disabled. Note that this option has an effect only if 2-loop contributions have been activated in the FlexibleSUSY model file. See Section 4.1 for details on how to activate 2-loop contributions of \( O(\alpha_b\alpha_s) \) to the Higgs pole mass(es) in the (N)MSSM.

FlexibleSUSY[10], higgs2loopCorrectionAtAt: With this option the 2-loop contributions to the Higgs pole mass(es) of \( O(\alpha_t^2) \) or \( O((\alpha_t + \alpha_b)^2) \) can be enabled/disabled. Note that this option has an effect only if 2-loop contributions have been activated in the FlexibleSUSY model file. See Section 4.1 for details on how to activate 2-loop contributions of these orders to the Higgs pole mass(es) in the SM or (N)MSSM.

FlexibleSUSY[11], higgs2loopCorrectionAtauAtau: With this option the 2-loop contributions to the Higgs pole mass(es) of \( O(\alpha_\tau^2) \) can be enabled/disabled. Note that this option has an effect only if 2-loop contributions have been activated in the FlexibleSUSY model file. See Section 4.1 for details on how to activate 2-loop contributions of \( O(\alpha_\tau^2) \) to the Higgs pole mass(es) in the (N)MSSM.

FlexibleSUSY[12], forceOutput: This option allows the user to force an output of FlexibleSUSY, even if a physical problem has occurred (tachyon, non-perturbative parameter, no EWSB, ...). If set to 0, FlexibleSUSY does not give an output if a problem has occurred. If set to 1, an output is always given, even if a problem has occurred. Please be very careful and check for potential warnings/problems when forcing the output!

FlexibleSUSY[13], topPoleQCDCorrections: With this option the user can enable additional loop contributions when the top quark pole mass is re-calculated. Note that the top pole mass is only re-calculated if FlexibleSUSY[3] or calculateStandardModelMasses is set to 1. If set to 0, then 1-loop (SUSY-)QCD contributions are taken into account (but only if FlexibleSUSY[4] or poleMassLoopOrder is set to 1). If set to 1, then 2-loop (SUSY-)QCD contributions are taken into account (but only if FlexibleSUSY[4] or poleMassLoopOrder is set to 2). If set to 2, then 3-loop (SUSY-)QCD contributions are taken into account (but only if FlexibleSUSY[4] or poleMassLoopOrder is set to 3).
FlexibleSUSY[14], betaZeroThreshold: With this option a numerical threshold can be defined below which a $\beta$ function is treated as being exactly zero. A small but non-zero threshold can avoid numerical problems when integrating the RGEs.

FlexibleSUSY[15]: With this option the calculation of the observables ($a_\mu$, EDMs, effective couplings of $h \to \gamma\gamma$ and $h \to gg$) can be enabled/disabled. Note that in the Mathematica interface, the observables are calculated by the function FS\<model\>CalculateObservables, see Section 3.

FlexibleSUSY[16], forcePositiveMasses: With this option the masses of Majorana fermions can be forced to be positive in the SLHA output of FlexibleSUSY. If set to 1, then Majorana masses are always positive, but the corresponding mixing matrices are in general complex (note that this violates the SLHA convention). If set to 0, then the Majorana masses can be positive or negative, but the corresponding mixing matrices are guaranteed to be real (SLHA convention).

FlexibleSUSY[17], poleMassScale: With this option the user can choose the scale (in GeV) at which the pole mass spectrum is calculated. If set to 0, then the value assigned to the SUSYScale variable in the model file is used. In FlexibleEFTHiggs, the pole mass scale is defined to be the scale at which the pole masses in the full BSM model are calculated. To vary the scale at which the pole masses in the effective theory (the SM) are calculated, use FlexibleSUSY[18] or eftPoleMassScale in the SLHA or Mathematica interface, respectively.

FlexibleSUSY[18], eftPoleMassScale: This option applies only to FlexibleEFTHiggs models. With this option the user can choose the scale (in GeV) at which the pole masses in the effective field theory (i.e., in the SM) are calculated. If the scale is set to 0, then $Q = M_t$ is used. This option can be used to estimate a partial uncertainty of the Higgs mass prediction in FlexibleEFTHiggs by varying the pole mass scale around $Q = M_t$, see Example 20.

FlexibleSUSY[19], eftMatchingScale: This option applies only to FlexibleEFTHiggs models. With this option the user can specify the scale at which the matching of the BSM model to the effective theory (i.e., the SM) is performed. If the scale is set to 0, then the value assigned to the SUSYScale variable in the FlexibleSUSY model file is used. This option can be used to estimate a partial uncertainty of the Higgs mass prediction in FlexibleEFTHiggs by varying the matching scale around $Q = M_S$, see Example 20.

FlexibleSUSY[20], eftMatchingLoopOrderUp: This option is ignored in FlexibleSUSY 2.0.

FlexibleSUSY[21], eftMatchingLoopOrderDown: This option applies only to FlexibleEFTHiggs models. With this option the user can select the loop order at which the quartic Higgs coupling $\lambda$ of the SM is fixed when matching the BSM model to the SM in FlexibleEFTHiggs. If set to 0, then $\lambda$ is fixed using only tree-level matching. If set to 1 (recommended), then $\lambda$ is fixed by a 1-loop matching condition.

FlexibleSUSY[22], eftHiggsIndex: This option applies only to FlexibleEFTHiggs models. With this option the user can choose which field in the Higgs multiplet of the BSM model corresponds to the SM-like Higgs. If set to 0, the lightest field in the Higgs multiplet is interpreted as SM-like Higgs. If set to 1, the 2nd lightest field is interpreted as SM-like Higgs.
Higgs, etc. The chosen field is then used in the matching condition $M_{h_i}^{SM} = M_{h_i}^{BSM}$, where $i$ is the index of the chosen field in the Higgs multiplet ($i = 0, 1, 2, \ldots$).

FlexibleSUSY[23], calculateBSMMasses: This option allows the user to enable/disable the calculation of the pole masses of the BSM particles. If set to 0, then the BSM pole masses are not calculated. If set to 1, then the BSM pole masses are calculated. This option is useful in FlexibleEFTHiggs for example: If one is only interested in the prediction of the SM-like Higgs pole mass, then this option can be set to 0 to suppress the calculation of the masses of the heavy Higgs bosons, the charginos, neutralinos and sfermions.

FlexibleSUSY[24], thresholdCorrections: With this option the user has a finer control over the threshold corrections to the individual model parameters. The value assigned to this option is an integer number, where each digit (with respect to base 10) represents the threshold correction loop order for a particular running BSM parameter. The association between the digits and the parameters as well as the default loop orders are shown in Table B.14.

| Example 21 |
|---|
| The following table shows example values for the integer number which specifies the individual threshold correction loop orders, together with the list of included loop corrections. |
| integer | used threshold corrections |
|---|---|
| 0 | no threshold corrections, everything at tree level |
| 1 | only $\Delta \alpha_{em}^{IL}$, everything else at tree level |
| 100 | only $\Delta \alpha_{em}^{IL}$, everything else at tree level |
| 101 | only $\Delta \alpha_{em}^{IL}$ and $\Delta \alpha_{em}^{LL}$, everything else at tree level |
| 3000101 | only $\Delta \alpha_{em}^{IL}$ and $\Delta \alpha_{em}^{IL}$ and $\Delta y_t^{ML}$, everything else at tree level |

If the field FlexibleSUSY[24] or the symbol thresholdCorrections is omitted, then the whole option is set to the default value given in Table B.14. If the field is not omitted, then all loop orders must be given. Note that setting the loop orders larger than the value set in FlexibleSUSY[7] or thresholdCorrectionsLoopOrder has no effect, see above.

| Example 22 |
|---|
| In the model file of HSSUSY, 3-loop QCD corrections to the running top Yukawa coupling $y_t$ are enabled (UseYukawa3LoopQCD = True). Switching between the 2-loop and 3-loop QCD corrections to $y_t$ can be used to estimate a partial uncertainty of the 2-loop Higgs pole mass. In order to do this, FSHSSUSYOpenHandle[] must be called twice, setting |
| thresholdCorrections → 123111321 |
| thresholdCorrections → 122111321 |

respectively, and setting each time thresholdCorrectionsLoopOrder -> 3 to enable the 3-loop corrections globally. Note that the digit at the 6th position...
from the right (the prefactor of $10^6$) has been changed from 3 to 2 to change the threshold correction loop order of $y_t$ from 3-loop to 2-loop. Example 7 makes use of this method to estimate a partial uncertainty of HSSUSY based on changing the threshold correction loop orders for $y_t$ and $\alpha_s$ in the SM.

FlexibleSUSY[25], higgs3loopCorrectionRenScheme: This option applies only to MSSM models in which the 3-loop Higgs pole mass contributions from Himalaya are enabled (the flag UseHiggs3LoopMSSM = True is set in the model file), see Section 4.1. With this option the user can choose between the DR and MDR renormalization scheme. If this option is set to 0, then the DR scheme is used. If set to 1, the MDR scheme is used.

FlexibleSUSY[26], higgs3loopCorrectionAtAsAs: With this option the user can enable/disable 3-loop contributions to the Higgs pole mass(es) of $O(\alpha_t \alpha_s^2)$. Note that this option has an effect only if model-specific 3-loop contributions to the Higgs pole mass(es) of this order have been enabled in the FlexibleSUSY model file. See Section 4.1 on how to enable 3-loop contributions of this order in the SM and in the MSSM.

FlexibleSUSY[27], higgs3loopCorrectionAbAsAs: With this option the user can enable/disable 3-loop contributions to the Higgs pole mass(es) of $O(\alpha_b \alpha_s^2)$. Note that this option has an effect only if model-specific 3-loop contributions to the Higgs pole mass(es) of this order have been enabled in the FlexibleSUSY model file. See Section 4.1 on how to enable 3-loop contributions of this order in the SM.

FlexibleSUSY[28], higgs3loopCorrectionAtAtAs: With this option the user can enable/disable 3-loop contributions to the Higgs pole mass(es) of $O(\alpha_t^2 \alpha_s)$. Note that this option has an effect only if model-specific 3-loop contributions to the Higgs pole mass(es) of this order have been enabled in the FlexibleSUSY model file. See Section 4.1 on how to enable 3-loop contributions of this order in the SM.

FlexibleSUSY[29], higgs3loopCorrectionAtAtAt: With this option the user can enable/disable 3-loop contributions to the Higgs pole mass(es) of $O(\alpha_t^3)$. Note that this option has an effect only if model-specific 3-loop contributions to the Higgs pole mass(es) of this order have been enabled in the FlexibleSUSY model file. See Section 4.1 on how to enable 3-loop contributions of this order in the SM.

MODSEL[12], parameterOutputScale: With this option the scale (in GeV) can be specified, at which the running $\overline{MS}/\overline{DR}$ model parameters are output. If set to 0, then the running parameters are output at the scale assigned to the SUSYScale variable in the FlexibleSUSY model file.

C. CMSSMCPV model file

```plaintext
FSModelName = "@CLASSNAME@";
FSEigenstates = SARAH 'EWSB';
FSDefaultSARAHModel = MSSM/CPV;
MINPAR = { {1, m0},
          {2, m12},
          {3, TanBeta},
```
{{4, CosPhiMu},
{5, Azero},
{100, Phase[Mu]}};

IMMINPAR = {{2, Imm12},
{4, SinPhiMu},
{5, ImAzero}};

EXTPAR = {
{100, etaInput}};

RealParameters = {};

EWSBOutputParameters = {Re[B[Mu]], Im[B[Mu]], [Mu]};

SUSYScale = Sqrt[Product[M[Su[i]]^2 + Abs[ZU[i,6]]^2, i]};

SUSYScaleFirstGuess = Sqrt[m0^2 + 4 m12^2];

SUSYScaleInput = {
{eta, etaInput},
{Phase[Mu], CosPhiMu + I SinPhiMu}};

HighScale = g1 == g2;

HighScaleFirstGuess = 2.0 10^-16;

HighScaleInput = {
{T[Ye], (Azero + I ImAzero) Ye},
{T[Yd], (Azero + I ImAzero) Yd},
{T[Yu], (Azero + I ImAzero) Yu},
{mq2, UNITMATRIX[3] m0^2},
{ml2, UNITMATRIX[3] m0^2},
{md2, UNITMATRIX[3] m0^2},
{mu2, UNITMATRIX[3] m0^2},
{me2, UNITMATRIX[3] m0^2},
{mHu2, m0^2},
{mHd2, m0^2},
{MassB, m12 + I Imm12},
{MassWB, m12 + I Imm12},
{MassG, m12 + I Imm12}};

LowScale = LowEnergyConstant[MZ];

LowScaleFirstGuess = LowEnergyConstant[MZ];

LowScaleInput = {
{Yu, Automatic},
{Yd, Automatic},
{Ye, Automatic},
{vd, 2 MZDRbar / Sqrt[GUTNormalization[g1]^2 g1^2 + g2^2] \ Cos[ArcTan[TanBeta]]},
{vu, 2 MZDRbar / Sqrt[GUTNormalization[g1]^2 g1^2 + g2^2] \ Sin[ArcTan[TanBeta]]}};
InitialGuessAtLowScale = {
    {vd, LowEnergyConstant[vev] Cos[ArcTan[TanBeta]]},
    {vu, LowEnergyConstant[vev] Sin[ArcTan[TanBeta]]},
    {\[Mu\], LowEnergyConstant[MZ]},
    {B[\[Mu\]], LowEnergyConstant[MZ]^2},
    {Yu, Automatic},
    {Yd, Automatic},
    {Ye, Automatic}
};

InitialGuessAtHighScale = {};

UseHiggs2LoopMSSM = False;

ExtraSLHAOutputBlocks = {
    {FlexibleSUSYOutput, NoScale,
        {0, Hold[HighScale]},
        {1, Hold[SUSYScale]},
        {2, Hold[LowScale]} },
    {FlexibleSUSYLowEnergy,
        {{21, FlexibleSUSYObservable'aMuon},
        {23, FlexibleSUSYObservable'EDM[Fe[1]]},
        {24, FlexibleSUSYObservable'EDM[Fe[2]]},
        {25, FlexibleSUSYObservable'EDM[Fe[3]]} },
    {EFFHIGGSCOUPPLINGS, NoScale,
        {{1, FlexibleSUSYObservable'CpHiggsPhotonPhoton},
        {2, FlexibleSUSYObservable'CpHiggsGluonGluon},
        {3, FlexibleSUSYObservable'CpPseudoScalarPhotonPhoton},
        {4, FlexibleSUSYObservable'CpPseudoScalarGluonGluon} },
    {ALPHA, NoScale,
        {{ArcSin[PoleZH[2,2]]}]},
    {HMIX, {{1, Re[\[Mu\]]},
        {2, vu / vd},
        {3, Sqrt[vu^2 + vd^2]},
        {101, Re[B[\[Mu\]]]},
        {102, vd},
        {103, vu} } },
    {ImHMIX,{{1, Im[\[Mu\]]},
        {101, Im[B[\[Mu\]]]} } },
    {Au, {{1, 1, Re[T[Yu][1,1]] / Yu[1,1]]},
        {2, 2, Re[T[Yu][2,2]] / Yu[2,2]]},
        {3, 3, Re[T[Yu][3,3]] / Yu[3,3]]} },
    {Ad, {{1, 1, Re[T[Yd][1,1]] / Yd[1,1]]},
        {2, 2, Re[T[Yd][2,2]] / Yd[2,2]]},
        {3, 3, Re[T[Yd][3,3]] / Yd[3,3]]} },
    {Ae, {{1, 1, Re[T[Ye][1,1]] / Ye[1,1]]},
        {2, 2, Re[T[Ye][2,2]] / Ye[2,2]]},
        {3, 3, Re[T[Ye][3,3]] / Ye[3,3]]} },
    {ImAu, {{1, 1, Im[T[Yu][1,1]] / Yu[1,1]]},
        {2, 2, Im[T[Yu][2,2]] / Yu[2,2]]},
        {3, 3, Im[T[Yu][3,3]] / Yu[3,3]]} },
    {ImAd, {{1, 1, Im[T[Yd][1,1]] / Yd[1,1]]},
        {2, 2, Im[T[Yd][2,2]] / Yd[2,2]]},
        {3, 3, Im[T[Yd][3,3]] / Yd[3,3]]} },
    {ImAe, {{1, 1, Im[T[Ye][1,1]] / Ye[1,1]]},
        {2, 2, Im[T[Ye][2,2]] / Ye[2,2]]},
        {3, 3, Im[T[Ye][3,3]] / Ye[3,3]]} },
    {MSOFT, {{1, Re[MassB]],
        {2, Re[MassWB]}},
        {3, Re[MassG]}},
D. THDM model file

```plaintext
FSModelName = "@CLASSNAME@";
FSEigenstates = SARAH\'EWSB;
AutomaticInputAtMSUSY = False;
FSDefaultSARAHModel = "THDM-II";

MINPAR = {
    {3, TanBeta}
};

EXTPAR = {
    {0, MSUSY},
    {1, MEWSB},
    {2, MuInput},
    {6, MAInput},
    {7, AtInput},
    {8, AbInput},
};
```
{9, AtauInput},
{100, LambdaLoopOrder}
); 

EWSBOutputParameters = \{ M112, M222 \};

(* The high scale where we match to the MSSM *)
HighScale = MSUSY;

HighScaleFirstGuess = MSUSY;

HighScaleInput = {
{Lambda1, 1/2 (1/4 ( (GUTNormalization[g1] g1)^2 + g2^2) + UnitStep[ LambdaLoopOrder -1] (deltaLambda1th1L + deltaLambda1Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda1th2L)},
{Lambda2, 1/2 (1/4 ( (GUTNormalization[g1] g1)^2 + g2^2) + UnitStep[ LambdaLoopOrder -1] (deltaLambda2th1L + deltaLambda2Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda2th2L)},
{Lambda3, 1/4 (- (GUTNormalization[g1] g1)^2 + g2^2) + UnitStep[ LambdaLoopOrder -1] (deltaLambda3th1L + deltaLambda3Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda3th2L},
{Lambda4, -1/2 g2^2 + UnitStep[ LambdaLoopOrder -1] (deltaLambda4th1L + deltaLambda4Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda4th2L},
{Lambda5, 0 + UnitStep[ LambdaLoopOrder -1] (deltaLambda5th1L + deltaLambda5Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda5th2L},
{Lambda6, 0 + UnitStep[ LambdaLoopOrder -1] (deltaLambda6th1L + deltaLambda6Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda6th2L},
{Lambda7, 0 + UnitStep[ LambdaLoopOrder -1] (deltaLambda7th1L + deltaLambda7Phi1L) + UnitStep[ LambdaLoopOrder -2] deltaLambda7th2L}
};

(* The scale where we impose the EWSB conditions and calculate the spectrum *)
SUSYScale = MEWSB;

SUSYScaleFirstGuess = MEWSB;

SUSYScaleInput = {
{M122 , MAInput^2 Sin[ArcTan[v2/v1]] Cos[ArcTan[v2/v1]]}
};

LowScale = LowEnergyConstant[MT];

LowScaleFirstGuess = LowEnergyConstant[MT];

LowScaleInput = {
{Yu, Automatic},
{Yd, Automatic},
{Ye, Automatic},
}
\{v1, 2 \frac{MZMSbar}{\sqrt{GUTNormalization[g1]^2 g1^2 + g2^2}} \cdot \\
\cos[\text{ArcTan}[\text{TanBeta}]]\}
\{v2, 2 \frac{MZMSbar}{\sqrt{GUTNormalization[g1]^2 g1^2 + g2^2}} \cdot \\
\sin[\text{ArcTan}[\text{TanBeta}]]\}\n
\text{InitialGuessAtLowScale = \{}
\{v1, \text{LowEnergyConstant[vev]} \cos[\text{ArcTan}[\text{TanBeta}]]\},
\{v2, \text{LowEnergyConstant[vev]} \sin[\text{ArcTan}[\text{TanBeta}]]\},
\{\text{Yu}, \text{Automatic}\},
\{\text{Yd}, \text{Automatic}\},
\{\text{Ye}, \text{Automatic}\},
\{\text{Ml22}, \text{MAInput}^2 \sin[\text{ArcTan}[\text{TanBeta}]] \cos[\text{ArcTan}[\text{TanBeta}]]\}\n\text{\};}

\text{DefaultPoleMassPrecision = MediumPrecision;}
\text{HighPoleMassPrecision = \{hh\};}
\text{MediumPoleMassPrecision = \{\};}
\text{LowPoleMassPrecision = \{\};}
\text{SMParticles = \{}
\text{Electron, TopQuark, BottomQuark,}
\text{VectorP, VectorZ, VectorG, VectorW, Neutrino,}
\text{gP, gG, gZ, gWm, gWmC}
\text{\};}

\text{(* abbreviations *)}
\text{At = AtInput;}
\text{Ab = AbInput;}
\text{Atau = AtauInput;}
\text{Lambda1WagnerLee = 2 \text{Lambda1};}
\text{Lambda2WagnerLee = 2 \text{Lambda2};}

\text{(* arxiv:1508.00576, Eq. (45) *)}
\text{deltaLambda1th1L = With[\{}
\text{kappa = \frac{1}{(4 \text{ Pi})^2},}
\text{ht = Yu[3,3],}
\text{hb = Yd[3,3],}
\text{htau = Ye[3,3],}
\text{gY = GUTNormalization[g1] g1,}
\text{muMS = MuInput / MSUSY,}
\text{AbMS = Ab / MSUSY,}
\text{AtauMS = Atau / MSUSY,}
\text{\},}
\text{\{\}
\text{- kappap/2 ht^4 muMS^4}
\text{+ 6 kappap hb^4 AbMS^2 (1 - AbMS^2/12)}
\text{+ 2 kappap htau^4 AtauMS^2 (1 - AtauMS^2/12)}
\text{+ kappap (g2^2 + gY^2)/4 (3 ht^2 muMS^2 - 3 hb^2 AbMS^2}
\text{ - htau^2 AtauMS^2)\};}

\text{(* arxiv:1508.00576, Eq. (46) *)}
\text{deltaLambda2th1L = With[\{}
\text{kappa = \frac{1}{(4 \text{ Pi})^2},}
\text{ht = Yu[3,3],}
\text{hb = Yd[3,3],}
\text{htau = Ye[3,3],}
\text{gY = GUTNormalization[g1] g1,}
\[\mu_{\text{MS}} = \frac{\text{MuInput}}{\text{MSUSY}},\]
\[A_{\text{BMS}} = \frac{A_{b}}{\text{MSUSY}},\]
\[A_{\text{tauMS}} = \frac{A_{\tau}}{\text{MSUSY}},\]
\[A_{\text{tMS}} = \frac{A_{t}}{\text{MSUSY}}\}\]
\[
= \frac{6 \kappa \eta_{t}^{4} A_{\text{tMS}}^{2} (1 - A_{\text{tMS}}^{2}/12)}{- \frac{\kappa}{2} \eta_{b}^{4} \mu_{\text{MS}}^{4} - \frac{\kappa}{6} \eta_{\tau}^{4} \mu_{\text{MS}}^{4} - \frac{\kappa}{2} (g_{2}^{2} + g_{Y}^{2})/2 (3 \eta_{t}^{2} A_{\text{tMS}}^{2} - 3 \eta_{b}^{2} \mu_{\text{MS}}^{2} - \eta_{\tau}^{2} \mu_{\text{MS}}^{2})}
\]

\[
\delta \Lambda_{3\text{th}1L} = \text{With}\{
\kappa = \frac{1}{(4 \pi)^{2}},
h_{t} = Y_{u}[3,3],
h_{b} = Y_{d}[3,3],
h_{\tau} = Y_{e}[3,3],
g_{Y} = \text{GUTNormalization}[g_{1}] g_{1},
\mu_{\text{MS}} = \frac{\text{MuInput}}{\text{MSUSY}},
A_{\text{BMS}} = \frac{A_{b}}{\text{MSUSY}},
A_{\text{tMS}} = \frac{A_{\tau}}{\text{MSUSY}}\}
\]
\[
= \left(\frac{\kappa}{6} \mu_{\text{MS}}^{2} (3 \eta_{t}^{4} (3 - A_{\text{tMS}}^{2}) + 3 \eta_{b}^{4} (3 - A_{\text{BMS}}^{2}) + \eta_{\tau}^{4} (3 - A_{\text{tauMS}}^{2}))
+ \kappa \eta_{t}^{2} \eta_{b}^{2} (3 (A_{\text{tMS}} + A_{\text{BMS}})^{2} - (\mu_{\text{MS}}^{2} - A_{\text{tMS}} A_{\text{BMS}})^{2} - 6 \mu_{\text{MS}}^{2})
- \kappa (g_{2}^{2} + g_{Y}^{2})/2 (3 \eta_{t}^{2} A_{\text{tMS}}^{2} - 3 \eta_{b}^{2} \mu_{\text{MS}}^{2} - \eta_{\tau}^{2} \mu_{\text{MS}}^{2})
+ 3 \eta_{b}^{2} (A_{\text{BMS}}^{2} - \mu_{\text{MS}}^{2})
+ \eta_{\tau}^{2} (A_{\text{tauMS}}^{2} - \mu_{\text{MS}}^{2})\right)
\]

\[
\delta \Lambda_{4\text{th}1L} = \text{With}\{
\kappa = \frac{1}{(4 \pi)^{2}},
h_{t} = Y_{u}[3,3],
h_{b} = Y_{d}[3,3],
h_{\tau} = Y_{e}[3,3],
g_{Y} = \text{GUTNormalization}[g_{1}] g_{1},
\mu_{\text{MS}} = \frac{\text{MuInput}}{\text{MSUSY}},
A_{\text{BMS}} = \frac{A_{b}}{\text{MSUSY}},
A_{\text{tMS}} = \frac{A_{\tau}}{\text{MSUSY}}\}
\]
\[
= \left(\frac{\kappa}{6} \mu_{\text{MS}}^{2} (3 \eta_{t}^{4} (3 - A_{\text{tMS}}^{2}) + 3 \eta_{b}^{4} (3 - A_{\text{BMS}}^{2}) + \eta_{\tau}^{4} (3 - A_{\text{tauMS}}^{2}))
+ \kappa \eta_{t}^{2} \eta_{b}^{2} (3 (A_{\text{tMS}} + A_{\text{BMS}})^{2} - (\mu_{\text{MS}}^{2} - A_{\text{tMS}} A_{\text{BMS}})^{2} - 6 \mu_{\text{MS}}^{2})
- \kappa (g_{2}^{2} - g_{Y}^{2})/2 (3 \eta_{t}^{2} A_{\text{tMS}}^{2} - 3 \eta_{b}^{2} \mu_{\text{MS}}^{2} - \eta_{\tau}^{2} \mu_{\text{MS}}^{2})
+ 3 \eta_{b}^{2} (A_{\text{BMS}}^{2} - \mu_{\text{MS}}^{2})
+ \eta_{\tau}^{2} (A_{\text{tauMS}}^{2} - \mu_{\text{MS}}^{2})\right)
\]
\[
+ h_{\text{tau}}^2 \left( A_{\text{tauMS}}^2 - \mu_{\text{MS}}^2 \right) \]
\]

(* arxiv:1508.00576, Eq. (50) *)

\[
\delta \Lambda_{5^{\text{th}}1L} = \text{With}\{ \\
\text{kappa} = 1/(4 \pi)^2, \\
ht = \text{Yu}[3,3], \\
hb = \text{Yd}[3,3], \\
h_{\text{tau}} = \text{Ye}[3,3], \\
\mu_{\text{MS}} = \text{MuInput} / \text{MSUSY}, \\
Ab_{\text{MS}} = \text{Ab} / \text{MSUSY}, \\
A_{\text{tauMS}} = A_{\text{tau}} / \text{MSUSY}, \\
A_{\text{MS}} = A / \text{MSUSY} \\
\}, \\
( - \text{kappa}/6 \mu_{\text{MS}}^2 (3 h_{\text{MS}}^4 A_{\text{MS}}^2 + 3 h_{\text{MS}}^4 A_{\text{MS}}^2 + h_{\text{MS}}^4 A_{\text{MS}}^2) )
\]

(* arxiv:1508.00576, Eq. (51) *)

\[
\delta \Lambda_{6^{\text{th}}1L} = \text{With}\{ \\
\text{kappa} = 1/(4 \pi)^2, \\
ht = \text{Yu}[3,3], \\
hb = \text{Yd}[3,3], \\
h_{\text{tau}} = \text{Ye}[3,3], \\
\mu_{\text{MS}} = \text{MuInput} / \text{MSUSY}, \\
Ab_{\text{MS}} = \text{Ab} / \text{MSUSY}, \\
A_{\text{tauMS}} = A_{\text{tau}} / \text{MSUSY}, \\
A_{\text{MS}} = A / \text{MSUSY}, \\
gbar = ((\text{GUTNormalization}[g1] g1)^2 + g2^2) / 4 \\
\}, \\
( \text{kappa}/6 \mu_{\text{MS}}^2 (+ 3 h_{\text{MS}}^4 \mu_{\text{MS}}^2 A_{\text{MS}}^2 + 3 h_{\text{MS}}^4 \mu_{\text{MS}}^2 A_{\text{MS}}^2 + h_{\text{MS}}^4 \mu_{\text{MS}}^2 A_{\text{MS}}^2) )
\]

(* arxiv:hep-ph/9307201, Eq. (6.13)-(6.14) *)

\[
+ gbar/2 \text{kappa} \mu_{\text{MS}} (+ 3 \text{Ab} \text{hb}^2 - 3 \text{AtMS} \text{ht}^2 + A_{\text{tauMS}} \text{ht}^2) \\
\]

(* arxiv:1508.00576, Eq. (52) *)

\[
\delta \Lambda_{7^{\text{th}}1L} = \text{With}\{ \\
\text{kappa} = 1/(4 \pi)^2, \\
ht = \text{Yu}[3,3], \\
hb = \text{Yd}[3,3], \\
h_{\text{tau}} = \text{Ye}[3,3], \\
\mu_{\text{MS}} = \text{MuInput} / \text{MSUSY}, \\
Ab_{\text{MS}} = \text{Ab} / \text{MSUSY}, \\
A_{\text{tauMS}} = A_{\text{tau}} / \text{MSUSY}, \\
A_{\text{MS}} = A / \text{MSUSY}, \\
gbar = ((\text{GUTNormalization}[g1] g1)^2 + g2^2) / 4 \\
\}, \\
( \text{kappa}/6 \mu_{\text{MS}}^2 (+ 3 h_{\text{MS}}^4 A_{\text{MS}}^2 (A_{\text{MS}}^2 - 6) + h_{\text{MS}}^4 A_{\text{MS}}^2 (A_{\text{MS}}^2 - 6)) \\
+ gbar/2 \text{kappa} \mu_{\text{MS}} (+ 3 \text{Ab} \text{hb}^2 - 3 \text{AtMS} \text{ht}^2 + A_{\text{tauMS}} \text{ht}^2) \\
\)

(* arxiv:hep-ph/9307201, Eq. (6.13)-(6.14) *)
\[ \delta \Lambda_1 \Phi_1 \Delta = \frac{\kappa}{6} \left( \frac{g_2^2 + g_Y^2}{2} \left( 3 h_t^2 \mu_{MS}^2 + 3 h_b^2 A_{MS}^2 + h_{\tau}^2 A_{\tau MS}^2 \right) \right) \];

\[ \delta \Lambda_2 \Phi_1 \Delta = \frac{\kappa}{6} \left( \frac{g_2^2 + g_Y^2}{2} \left( 3 h_t^2 A_{MS}^2 + 3 h_b^2 \mu_{MS}^2 + h_{\tau}^2 \mu_{MS}^2 \right) \right) \];

\[ \delta \Lambda_3 \Phi_1 \Delta = \frac{\kappa}{6} \left( \frac{g_2^2 - g_Y^2}{4} \left( 3 h_t^2 (A_{MS}^2 + \mu_{MS}^2) + 3 h_b^2 (A_{MS}^2 + \mu_{MS}^2) + h_{\tau}^2 (A_{\tau MS}^2 + \mu_{MS}^2) \right) \right) \];

\[ \delta \Lambda_4 \Phi_1 \Delta = \frac{\kappa}{6} \left( \frac{g_2^2 - g_Y^2}{4} \left( 3 h_t^2 (A_{MS}^2 + \mu_{MS}^2) + 3 h_b^2 (A_{MS}^2 + \mu_{MS}^2) + h_{\tau}^2 (A_{\tau MS}^2 + \mu_{MS}^2) \right) \right) \];
\[
\kappa = \frac{1}{(4\pi)^2}, \\
ht = \text{Yu}[3,3], \\
hb = \text{Yd}[3,3], \\
htau = \text{Ye}[3,3], \\
\text{muMS} = \text{MuInput} / \text{MSUSY}, \\
\text{AbMS} = \text{Ab} / \text{MSUSY}, \\
\text{AtauMS} = \text{Atau} / \text{MSUSY}, \\
\text{AtMS} = \text{At} / \text{MSUSY}
\]

\[
\left( \frac{\kappa}{6} g_2^2 \left( 3 \ht^2 (\text{AtMS}^2 + \text{muMS}^2) \\
+ 3 \hb^2 (\text{AbMS}^2 + \text{muMS}^2) \\
+ \htau^2 (\text{AtauMS}^2 + \text{muMS}^2) \right) \right)
\]

\[
(* \text{arxiv:1508.00576, Eq. (57) *})
\]

\[
\delta \Lambda_{5\Phi \text{L}} = 0; \\
\delta \Lambda_{6\Phi \text{L}} = 0; (* \text{wrong in arxiv:hep-ph/9307201, Eq. (6.17) *}) \\
\delta \Lambda_{7\Phi \text{L}} = 0; (* \text{wrong in arxiv:hep-ph/9307201, Eq. (6.17) *})
\]

\[
(* \text{arxiv:1508.00576, Eq. (59) *})
\]

\[
\delta \Lambda_{1\theta^2 \text{L}} = \text{With}\{ \\
\kappa = \frac{1}{(4\pi)^2}, \\
ht = \text{Yu}[3,3], \\
\text{muMS} = \text{MuInput} / \text{MSUSY}\}, \\
\left( -\frac{4}{3} \kappa^2 \ht^4 g_3^2 \text{muMS}^4 \right)
\]

\[
(* \text{arxiv:1508.00576, Eq. (60) *})
\]

\[
\delta \Lambda_{2\theta^2 \text{L}} = \text{With}\{ \\
\kappa = \frac{1}{(4\pi)^2}, \\
ht = \text{Yu}[3,3], \\
\text{muMS} = \text{MuInput} / \text{MSUSY}, \\
\text{AtMS} = \text{At} / \text{MSUSY}\}, \\
\left( 16 \kappa^2 \ht^4 g_3^2 (-2 \text{AtMS} + 1/3 \text{AtMS}^3 - 1/12 \text{AtMS}^4) \right)
\]

\[
(* \text{arxiv:1508.00576, Eq. (61) *})
\]

\[
\delta \Lambda_{3\theta^2 \text{L}} = \text{With}\{ \\
\kappa = \frac{1}{(4\pi)^2}, \\
ht = \text{Yu}[3,3], \\
\text{muMS} = \text{MuInput} / \text{MSUSY}, \\
\text{AtMS} = \text{At} / \text{MSUSY}\}, \\
\left( 4 \kappa^2 \ht^4 g_3^2 \text{AtMS} \text{muMS}^2 (1 - 1/2 \text{AtMS}) \right)
\]

\[
\delta \Lambda_{4\theta^2 \text{L}} = \delta \Lambda_{3\theta^2 \text{L}}; \\
\delta \Lambda_{5\theta^2 \text{L}} = \delta \Lambda_{3\theta^2 \text{L}}; \\
(* \text{arxiv:1508.00576, Eq. (62) *})
\[ \delta \Lambda^{6th2L} = \begin{cases} \frac{kappa}{(4 \pi)^2} \cdot \frac{ht}{Yu[3,3]}, \\ \mu_{\text{MS}} = \frac{\text{MuInput}}{\text{MSUSY}}, \\ \frac{At_{\text{MS}}}{\text{At}} \end{cases} \]

\[ \left( \frac{4}{3} \kappa^{-2} h_t^{-4} g_3^{-2} \mu_{\text{MS}}^{-3} (-1 + At_{\text{MS}}) \right) \]

\[ (* \text{arxiv:1508.00576, Eq. (63) *)} \]

\[ \delta \Lambda^{7th2L} = \begin{cases} \frac{kappa}{(4 \pi)^2} \cdot \frac{ht}{Yu[3,3]}, \\ \mu_{\text{MS}} = \frac{\text{MuInput}}{\text{MSUSY}}, \\ \frac{At_{\text{MS}}}{\text{At}} \end{cases} \]

\[ \left( \frac{4}{kappa^2} h_t^{-4} g_3^{-2} \mu_{\text{MS}} (2 - At_{\text{MS}}^2 + 1/3 At_{\text{MS}}^3) \right) \]

---

E. CNMSSM model file

```plaintext
FSModelName = "@CLASSNAME@";
FSEigenstates = SARAH'EWSS;
FSDefaultSARAHModel = NMSSM;
FSBVPSolvers = { SemiAnalyticSolver };

(* CNMSSM input parameters *)
MINPAR = {
{2, m12},
{3, TanBeta},
{4, Sign[vS]},
{5, Azero}
};
EXITPAR = {
{61, LambdaInput}
};
FSAuxiliaryParameterInfo = {
{m0Sq, { ParameterDimensions -> {1},
MassDimension -> 2 } },
{LambdaInput, { ParameterDimensions -> {1},
MassDimension -> 0 } }
};
EWSBOutputParameters = { \[ Kappa \], vS, m0Sq };
SUSYScale = Sqrt[Product[\[M[Su[i]]\]^(-Abs[ZU[i,3]]^2 + Abs[ZU[i,6]]^2), \{i,6\}]];
```
SUSYScaleFirstGuess = Sqrt[14 m12^2 - 3 m12 Azero + Azero^2];

SUSYScaleInput = {};

HighScale = g1 == g2;

HighScaleFirstGuess = 2.0 10^-16;

HighScaleInput = {
  {T[Ye], Azero*Ye},
  {T[Yd], Azero*Yd},
  {T[Yu], Azero*Yu},
  {mq2, UNITMATRIX[3] m0Sq},
  {m12, UNITMATRIX[3] m0Sq},
  {md2, UNITMATRIX[3] m0Sq},
  {mu2, UNITMATRIX[3] m0Sq},
  {me2, UNITMATRIX[3] m0Sq},
  {mHu2, m0Sq},
  {mHd2, m0Sq},
  {ms2, m0Sq},
  {{Lambda}, LambdaInput},
  {T[\[Kappa]], Azero \[Kappa]},
  {T[\[Lambda]], Azero LambdaInput},
  {MassB, m12},
  {MassWB, m12},
  {MassG, m12}
};

LowScale = LowEnergyConstant[MZ];

LowScaleFirstGuess = LowEnergyConstant[MZ];

LowScaleInput = {
  {Yu, Automatic},
  {Yd, Automatic},
  {Ye, Automatic},
  {vd, 2 MZDRbar / Sqrt[ GUTNormalization[g1]^2 g1^2 + g2^2 ] Cos[ArcTan[TanBeta]]},
  {vu, 2 MZDRbar / Sqrt[ GUTNormalization[g1]^2 g1^2 + g2^2 ] Sin[ArcTan[TanBeta]]}
};

InitialGuessAtLowScale = {
  {vd, LowEnergyConstant[vev] Cos[ArcTan[TanBeta]]},
  {vu, LowEnergyConstant[vev] Sin[ArcTan[TanBeta]]},
  {{Lambda}, LambdaInput},
  {{Kappa}, 0.1},
  {vS, 1000},
  {m0Sq, LowEnergyConstant[MZ]^2},
  {Yu, Automatic},
  {Yd, Automatic},
  {Ye, Automatic}
};

InitialGuessAtHighScale = {};

UseHiggs2LoopNMSSM = True;

EffectiveMu = {{Lambda} vS / Sqrt[2]};
EffectiveMasqr = (T[\[Lambda]] vS / Sqrt[2] + 0.5 \[Lambda] \[Kappa] vS^2) (vu^2 + vd^2) / (vu vd);

PotentialLSPParticles = { Chi, Sv, Su, Sd, Se, Cha, Glu };

DefaultPoleMassPrecision = MediumPrecision;
HighPoleMassPrecision = {hh, Ah, Hpm};
MediumPoleMassPrecision = {};
LowPoleMassPrecision = {};

ExtraSLHABlocks = {
{FlexibleSUSYOutput, NoScale,
 {0, Hold[HighScale]},
 {1, Hold[SUSYScale]},
 {2, Hold[LowScale]} }
},
{EWSBOutputs, NoScale,
 {1, \[Kappa]},
 {2, vS},
 {3, m0Sq} }
},
{FlexibleSUSYLowEnergy,
 {0, FlexibleSUSYObservable\'aMuon} }
},
{EFFHIGGSCOUPLINGS, NoScale,
 {1, FlexibleSUSYObservable\'CpHiggsPhotonPhoton},
 {2, FlexibleSUSYObservable\'CpHiggsGluonGluon},
 {3, FlexibleSUSYObservable\'CpPseudoScalarPhotonPhoton},
 {4, FlexibleSUSYObservable\'CpPseudoScalarGluonGluon} }
},
{NMSSMRUN,
 {1, \[Lambda]},
 {2, \[Kappa]},
 {3, T\[Lambda]/\[Lambda]},
 {4, T\[Kappa]/\[Kappa]},
 {5, \[Lambda] vS / Sqrt[2]},
 {10, ms2} }
}
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