POMERON NON FACTORIZATION
IN DIFFRACTIVE SCATTERING AT HERA *

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Abstract

Pomeron non factorization for the diffractive scattering is considered and an analysis of experimental data is performed with different cuts in the momentum fraction $\xi$. The results of the analysis are compared with the breaking predicted in the Genovese-Nikolaev-Zakharov model.

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1. Introduction

Factorization theorems \[1\] have been proven, order by order in perturbative QCD, for totally inclusive cross-sections. Factorization of short- and long-distance effects in the hadron- hadron scattering provides all possible corrections to the parton model predictions at high energy. For the diffractive scattering, when one of the initial hadrons changes only its momentum, the factorization theorem fails. The amount of the factorization breaking depends strongly on the kinematical region of the considered process. The case of coherent hard diffraction, where a heavy quark or a jet is produced in addition to the diffraction, has been considered in detail \[2, 3\]. In this case, non-factorization occurs at the lowest relevant order of perturbation theory \[3\].

If we limit ourselves to diffractive DIS at HERA, we can neglect the factorization breaking due to the coherent Pomeron \[3\] and explore different aspects of the problem. According to a conventional wisdom, the diffractive scattering at high energy is dominated by the Pomeron exchange. The Ingelman-Schlein model \[4\] imposes factorization also for semi-inclusive diffractive processes and applies to the hadron-Pomeron scattering. The Pomeron is considered as a bound state of gluons and quarks that interact with hadrons, or leptons in deep inelastic scattering (DIS), and when the modulus of the squared momentum transfer, \(-t\), is small the picture sketched above seems appropriate.

The events we consider in the following are characterized by a large rapidity gap between the unscathed proton and the other final state particles \[5, 6\]. From now on, factorization and its breaking refer more to the factorization property of the residues of the Regge pole exchanged than to the separation of the hard from the soft scattering in the process. This form of factorization has been referred to as ”gap factorization” \[7\]. Since the exchange of a meson gives also rise to rapidity gaps, a natural explanation for the breakdown of gap factorization, observed experimentally
can be found in the exchange of different trajectories \[10\]. Different structures of the Pomeron and mesons and interference terms (that must be present in the diffractive structure function since it is a cross section) break the factorization property \[11, 7\]. Factorization breaking, as it appears experimentally \[8\] and as interpreted with the above mechanism, is limited to values of $\xi$, the momentum fraction lost by the proton, larger than $\approx 0.005$. In the dipole approach \[12, 13\] to the BFKL Pomeron \[14\], instead, the failure of factorization is a phenomenon involving also a pure Pomeron exchange and is present for all values of $\xi$. The microscopic dipole BFKL Pomeron has a two-component structure function and each component has a different coupling to the proton. To each piece, in fact, a different flux of Pomerons in the proton is associated that distinguish between the "valence $q\bar{q}$" and the "valence gluons plus sea" component. A similar effect has been found in Ref. \[15\].

From the published data \[5, 6\] it is difficult to draw definite conclusions about the Pomeron non-factorization. In order to suppress the contribution from meson trajectories, only data with $\xi \leq 0.005$ must be selected and this reduces the already limited statistics of the available data. However it could be possible to distinguish different trends with and without a cut in the $\xi$ variable. From a theoretical point of view it is possible to show that, under quite reasonable conditions, a small departure from factorization of the Pomeron exchange is present already in the Ingelman-Schlein model if the $t$-dependence of the diffractive structure function is taken into account.

In this paper we consider the above points in a Regge model of the diffractive scattering. After a brief discussion of the kinematics, we present an analysis of the experimental data, where different cuts in $\xi$ lead to quite different behaviours of
the Pomeron intercept. We give the proof that, by integrating over $t$ the 4-variable diffractive structure function, the Pomeron factorization can be lost and compare this effect with the breaking predicted in Ref. [12]. Finally we draw the conclusions.

2. Notations and the definition of the $\xi$-slope from experiments

The notation and kinematics for the process

$$e^-(l) + p(p) \rightarrow e^- (l') + p(p') + X_n(k_n)$$

are shown in Fig. 1, where $r$ is the four-momentum of the exchanged Reggeon. Then the spin averaged differential cross section for a diffractive DIS, ignoring spin and the proton mass, is

$$d\sigma = \frac{(2\pi)^{-5}}{2p \cdot l} \frac{d^3 l^*}{2l^0} \frac{d^3 p'}{2p'^0}$$

$$\times \sum_n \left( \frac{d^3 k_n}{2k_n^0} \delta(p + l - l' - k_n - p') |T(p + l \rightarrow p' + l' + k_n)|^2 \right),$$

(1)

where $T$ is the transition amplitude.

If a sufficiently small cut in the variable $\xi$, with $\xi = (r \cdot q)/(p \cdot q)$, is selected, then only the Pomeron trajectory will contribute to the process and $r = p - p'$ represents, in this case, the four-momentum of the Pomeron. In the following, $r^2 \equiv t < 0$ is the squared mass of the Pomeron.

Gap factorization implies that

$$T(p + l \rightarrow p' + l' + k_n) = F(l + r \rightarrow l' + k_n) \Phi(\xi, t),$$

(2)

where the "flux factor" $\Phi(\xi, t)$ has the form

$$\Phi(\xi, t) = (e^{i\pi/2\xi})^{-\alpha_P(t)} \beta(t),$$

(3)

3
Figure 1: The diagram for the diffractive deep-inelastic scattering.
\(\alpha_P(t)\) being the Pomeron trajectory. Since only the Pomeron is exchanged, Eq. \(\text{(1)}\) becomes

\[
d\sigma = \frac{r \cdot l \, d^3p'}{p \cdot l \, 2p_0^0} |\Phi(\xi, t)|^2 d\sigma(l + r \rightarrow l' + k_n) \simeq \\
\frac{\pi}{2} \xi \, d\xi \, dt |\Phi(\xi, t)|^2 d\sigma(l + r \rightarrow l' + k_n),
\]

where the relations \((r \cdot l)/(p \cdot l) \sim (r \cdot q)/(p \cdot q) = \xi\) and \(d^3p'/(2p_0^0) \propto d\xi \, dt\), valid for small \(\xi\), were used.

Let now \(k\) be the four-momentum of the parton interacting with the lepton and \(\beta\) the momentum fraction of the Pomeron carried by the parton:

\[k = \beta r.\]

Then, with the above mentioned approximations, one may write \((Q^2 = -q^2)\)

\[
d\sigma(l + r) = G_{q/P}(\beta, Q^2, t) \frac{d\sigma(k + l)}{dq^2} \, d\beta \, dq^2
\]

in terms of the parton-lepton cross section \(d\tilde{\sigma}\), where \(G_{q/P}(\beta, Q^2, t)\) is the structure function of the Pomeron. By introducing the 4-variable diffractive structure function \(F_2^{D(4)}(\beta, Q^2, \xi, t)\) we can rewrite Eqs. \(\text{(1)}\) and \(\text{(3)}\) as

\[
F_2^{D(4)} = \xi |\Phi(\xi, t)|^2 G_{q/P}(\beta, Q^2, t).
\]

We may consider two rapidity gaps in the whole process: the large rapidity gap determined by \(\ln(1/\xi)\) and the gap \(\ln(1/\beta)\) involved in the creation of that quark from the Pomeron, which will finally interact with the photon. For small \(\beta\) the squared mass \(k_n^2\) of the produced hadrons, besides the proton, is large and the triple Pomeron vertex will play an essential role. For large \(\beta\), instead, the above formalism applies if the quark of four-momentum \(k\) is near its mass-shell. The corresponding \(t\)-dependence of the Pomeron structure function could be quite different in the two \(\beta\)
regions. This point is made clear in the calculation of Ref. [10] where the three pieces contributing to the Pomeron structure function have different $t$ and $\beta$ dependences. Physically they correspond to the quark loop, i.e. the quark diagram important at large $\beta$, the triple Pomeron vertex and the $PPf$ term.

That the Pomeron structure function must depend on the variable $t$ has been emphasized already in earlier papers [16, 17], where this $t$-dependence ultimately disappears since the process is considered near $t = 0$. As we will show later, the $t$-dependence can cause factorization breaking when Eq. (6) is integrated over $t$ to get $F_2^{D(3)}(\beta, Q^2, \xi)$.

We consider now the experimental data. H1 data [5, 8] cover a $\xi$-interval where both the Pomeron and meson trajectories contribute. The analysis of Ref. [8] shows that the effective power $n$ of $\xi$ in $F_2^{D(3)}$ depends on $\beta$. This factorization breaking (Eq. (8) does not hold anymore at $t = 0$) is consistent with a sizable contribution to $F_2^{D(3)}$ from meson exchange [8, 11]. In Fig. 2 this result has been reproduced, with open circles, together with a quadratic fit for $n(\beta)$ (continuous line).

In the above analyses [8, 11] the meson contribution dies out for $\xi \leq 0.005$ leaving only the Pomeron, as given in Eq. (8) in a Regge model. Should the effective slope still depend on $\beta$ in the small-$\xi$ selected sample, then a different interpretation, as for example the one proposed in Ref. [12], could be appropriate. A first hint to the effect we propose to exploit can be found in the published H1 [3] and ZEUS [4] data. Due to the different explored $\xi$-range, we can say that the ZEUS data are essentially determined from Pomeron exchange while H1 data are not. The mean value of the $\xi$ exponent, the same for each $\beta$ and $Q^2$ bin, is larger for ZEUS ($\sim 1.3$) than for H1 ($\sim 1.19$). In a recent analysis [18], with a larger kinematic range $4 \times 10^{-4} < \xi < 0.03$, ZEUS finds a much lower effective $\xi$ slope ($\sim 1.01$) that is
Figure 2: The parameter $n$ versus $\beta$. Open circles represent the result of Ref. [8] while the continuous line comes from a quadratic fit. Open squares and full triangles represent the result according Eq. (7), without and with the cut in $\xi$ respectively.
attributed to a significant component of Reggeon (not Pomeron) exchange.

From the results of the fit in Refs. [8, 11] mesons trajectories become less important at large $\beta$. If this is really the case, the $\xi$ slope for the Pomeron should increase when going to smaller $\beta$ values.

A $\beta$ independent $\xi$-slope, when only the Pomeron is exchanged, would be a quite natural result but other possibilities are not excluded. In order to clarify better this point we consider the data of Ref. [5] for $F_2^{D(3)}$ and parametrize the structure function, for each fixed $\beta$ value, with the simple function

$$F_2^{D(3)}(\beta_i, Q^2, \xi) \simeq (d_0 + d_1 \ln(Q^2/Q_{0}^2))\xi^{-n(\beta_i)}$$

(7)

where $\beta_i = 0.065, 0.175, 0.375, 0.65$.

A FORTRAN program for function minimization and error analysis (MINUIT) determine the parameters, in particular $n(\beta_i)$. The output of the program has been plotted in Fig.2, the open squares, and agrees with the result of Ref. [8] within the errors. When the cut $\xi \leq 0.005$ is applied to the data and the minimization procedure is repeated, $n(\beta)$ changes to the values indicated with full triangles in Fig.2. Results are not reported at $\beta = 0.065$, since the cut leaves only two points, and at $\beta = 0.65$ since practically all the points satisfy the selection criterion. Other parametrizations for $F_2^{D(3)}$, see for example Refs. [19, 20], reproduce the same trend for $n(\beta)$.

While this rather crude analysis cannot be taken too seriously, more intriguing is the prediction of Ref. [12] for $n(\beta)$ shown as a dashed line in Fig.2 and evaluated as follows. We first calculate

$$n(\xi, \beta) = \frac{\partial \ln F_2^{D(3)}(\xi, \beta)}{\partial \ln(1/\xi)}$$

(8)
following the method of Ref. [12], with the values for the parameters appropriate to small $\xi$, and then take the mean value of $n(\xi, \beta)$ in the range $0.0003 \leq \xi \leq 0.005$:

$$n(\beta) = \frac{1}{\Delta \xi} \int_{\xi_L=0.0003}^{\xi_U=0.005} n(\xi, \beta) \, d\xi,$$

where $\Delta \xi = \xi_U - \xi_L$. When averaged over $\xi$, the increase of $n(\beta)$, when $\beta$ decreases, becomes very slow and finally $n(\beta)$ should tend to a constant in the dipole Pomeron model [12].

In order to understand better this effect we will consider, in the next Section, the problem starting from Eq. (6) and a Regge model for the Pomeron flux in the proton [19, 20].

3. The $t$-dependence of $F_2^{D(4)}$ and factorization

We consider a simplified model for the Pomeron flux, defined as

$$F_{P/p}(\xi, t) = |\Phi(\xi, t)|^2,$$

where a linear Pomeron trajectory $\alpha(t) = \alpha(0) + \alpha' t$ gives [19, 20]

$$F_{P/p}(\xi, t) = c \xi^{1-2\alpha(0)} e^{2\alpha'(B-\ln(\xi))t}.$$  

(10)

The following considerations apply also to a non-linear Pomeron trajectory [21] with obvious modifications.

Experimental data [3, 4] are given for

$$F_2^{D(3)}(\beta, Q^2, \xi) = \int_{t_+}^{t_-} F_{P/p}(\xi, t) G_{q/P}(\beta, Q^2, t) \, d\xi,$$

(11)

where $t_\pm$ are given for example in Ref. [21] and $t_- \sim -m_p^2 \xi^2 \gg t_+$, $m_p$ being the proton mass.

The assumption that the structure function $G_{q/P}$ does not depend on the Pomeron invariant mass $t$ is made in Ref. [4] in analogy with the structure function of the
photon and is considered there a "reasonable first approximation". If \( t \) is neglected in \( G_{q/P} \) the rapidity gap variable \( \xi \) is not correlated any more with \( \beta \) and \( Q^2 \).

In general, however, the vertex \( \gamma^* - P - X \), the large black dot in Fig.1, will depend on \( t \) and this dependence is, per se, an interesting problem subject of numerous debates. A source of uncertainty comes from the parametrization of the triple Pomeron vertex (see e.g. Ref. [21] and earlier references therein). The most recent triple Pomeron fits were made as early as the mid’70-ies and neither of these fits accounts for the rising cross sections, i.e. they involve a simple, unit intercept Pomeron pole, inadequate from the present point of view.

The effect we are studying is very small indeed and should be visible at small \( \beta \), that is in the triple Regge region. A detailed analysis would require a refitting of the diffractive hadronic reactions with a modified Pomeron trajectory and a larger sample of data with smaller errors. For the time being we content ourselves to add another argument, in a model calculation, in favour of the behaviour for \( n(\beta) \) predicted in Ref. [12].

The physical mechanism at the basis of the two approaches is quite different but, as far as \( n(\beta) \) is concerned, the result will be the same. In practice, a meaningful comparison should consider only the slope \( dn(\beta)/d\beta \) of \( n(\beta) \). If the two results are to be consistent, \( dn(\beta)/d\beta \) must result negative and small in both calculations. In appendix we give an estimate of the integral over \( t \) of \( F_2^{D(4)} \) that can be used if a series expansion of \( G_{q/P} \) near \( t = 0 \) is known. Setting

\[
\frac{a_1(\beta)}{2\alpha'a_0(\beta)} \equiv r(\beta),
\]

(12)

where \( a_0(\beta) \) and \( a_1(\beta) \) are the first two coefficients of the series expansion (see
Eq. (??), we get from Eq. (??)

\[
\frac{dn(\beta)}{d\beta} = -\frac{1}{\Delta \xi} \left( \frac{dg(\xi_U)}{d\beta} - \frac{dg(\xi_L)}{d\beta} \right),
\]  

(13)

with \( g(\xi) \) defined in Eq. (??). Taking into account that \( \xi \leq 0.005 \) and \( B \simeq 7 \) \( \text{[20]} \), \( dg/d\beta \) can be easily obtained in the convenient form \( \text{[22]} \)

\[
\frac{dg(\xi)}{d\beta} = r'(\beta) \frac{\text{e}^{\text{ln}(\xi)-B}}{B - \text{ln}(\xi) + r(\beta)} \times \\
\left( \sum_{m=1}^{M-1} \frac{(-1)^{m+1} m!}{(B - \text{ln}(\xi) + r(\beta))^m} + O(|B - \text{ln} \xi + r(\beta)|^M) \right)
\]

(14)

that is a continuous and slowly decreasing function of \( \xi \).

The sign of \( r'(\beta) \) can be inferred as follows. We consider again Ref. [10] and, in this scheme, identify the coefficients \( a_0(\beta) \) and \( a_1(\beta) \). The most important contribution to \( a_0(\beta) \) will come from the box diagram, \( G^a_{q/P} \) in Ref. [10], that does not depend on \( t \), while \( G^b_{q/P} \) (with an appreciable \( t \)-dependence) will contribute mainly to \( a_1(\beta) \). An explicit form for \( G^b_{q/P} \) is given only for \(|t| > 0.2 \text{ GeV}^2\), hence, near \( t = 0 \), care must be taken in parametrizing the data for the triple Pomeron contribution to the inclusive differential cross section for the diffraction dissociation. If the \( \beta \) dependence is taken as in Ref. [10], it turns out that \( r(\beta) \) in Eq. (??) is negative with a small positive derivative. Hence \( n(\beta) \) according to Eq. (??) is a decreasing function of \( \beta \) and the prediction of the Regge model approaches the result of the dipole BFKL Pomeron. When other parameters, like \( \alpha' \simeq 0.25 \text{ GeV}^{-2} \), are chosen in a standard way, the two results agree also from a quantitative point of view.

4. Conclusions

In this paper we studied the problem of gap factorization for the Pomeron exchange. Once meson trajectories have been suppressed, by considering values of \( \xi \) such that \( \xi \leq 0.005 \), only the Pomeron contributions remains.
We present first an analysis of experimental data \cite{5,6} and show that the data sample, with a cut in the variable $\xi$, is compatible with a breaking of the gap factorization. In the diffractive structure function $F_{2D}^{(3)}(\beta, Q^2, \xi)$ the $\xi$-dependence allows for an effective power $n(\beta)$ that depends on $\beta$, having a behaviour quite different from the one predicted in Ref. \cite{8} where the breaking comes from the interference of the Pomeron with the exchanged meson trajectories.

We then evaluate $n(\beta)$ in a theoretical model \cite{12} and find a good agreement with our phenomenological analysis. In this model the gap factorization is broken by the presence of two structure functions in the Pomeron, associated with two different fluxes of Pomerons in the proton.

In a first instance it seems that in a Regge model \cite{1,4,5,20,6,16,17}, the unique structure of the proton-Pomeron vertex implies the Pomeron factorization property. The main result of this paper is the finding that the $t$-dependence of the photon-Pomeron vertex may bias the commonly assumed Pomeron factorization. We show in fact that the integration of $F_{2D}^{(4)}$ over $t$, in order to obtain $F_{2D}^{(3)}$, can lead to an effective $\xi$-power $n(\beta)$, with the same behaviour as in Ref. \cite{12}. For this purpose we use the model of Ref. \cite{10} and an approximate estimate of the $t$-integral, but we argue that this effect is really present in any model.

As it appears in Fig.2, the breaking, proportional to the slope of the dashed line, is very small and very difficult to detect experimentally. New unpublished H1 data \cite{8} could already put limits on this breaking and further elucidate the important question of the $t$-dependence in the photon-Pomeron vertex. The knowledge of the variation of this $t$-dependence with $\beta$ could be important in clarifying the dynamics underlying the onset of the triple Pomeron vertex.
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Appendix

Integrating Eq. (11) over $t$ we keep the contribution of the end point $t = t_-$ by considering $2\alpha'(B - \ln \xi)$ as a large parameter. According to the Laplace method

$$\int_{t_-}^{t_+} e^{2\alpha'(B-\ln\xi)t} G_{q/P}(\beta, Q^2, t) \, dt \sim e^{2\alpha'(B-\ln\xi)t_-} \sum_{n=0}^{\infty} \frac{a_n(\beta, Q^2)}{[2\alpha'(B - \ln \xi)]^{n+1}},$$

where

$$a_n(\beta, Q^2) = \left( \frac{\partial G_{q/P}(\beta, Q^2, t_- - u)}{\partial u} \right)_{u=0}^{(n)}.$$

The estimate (??) can be useful if the coefficients $a_n$ tend to zero rapidly enough or, better, if the series can be truncated. The determination of the coefficients $a_n$ will be possible when more accurate data will be available. The saddle point at $t^*$, where $t^*$ is the solution of the equation

$$-2\alpha'(B - \ln \xi) + \frac{\partial \ln G_{q/P}}{\partial |t|} = 0,$$

does not contribute if $G_{q/P}$ is a decreasing function of $|t|$. As an example, suppose to truncate the series in Eq. (??) to the first two terms. Then Eq. (8) gives

$$n(\xi, \beta) = 2\alpha(0) - 1 + 2\alpha't_- - \frac{2}{B - \ln \xi} + \frac{1}{B - \ln \xi} + \frac{a_1(\beta)}{2\alpha^2 a_0(\beta)}.$$  

(A.2)
where no $Q^2$ dependence appears in accordance with the experimental data.

The mean value of $n(\xi, \beta)$ can be evaluated from Eq. (9):

$$n(\beta) = 2\alpha(0) - 1 - \frac{1}{\Delta \xi} [g(\xi_U) - g(\xi_L)] ,$$  \hspace{1cm} (A.3)

where the term

$$\frac{2\alpha'}{\Delta \xi} \int_{\xi_L}^{\xi_U} t \, d\xi \sim \frac{2\alpha'}{3} (\xi_U^2 + \xi_U \xi_L + \xi_L^2)$$

has been neglected and

$$g(\xi) =$$

$$e^B (2E_1[\ln(1/\xi) + B] - e^{a_1(\beta)/(2\alpha'a_0(\beta))} E_1[\ln(1/\xi) + B + a_1(\beta)/(2\alpha'a_0(\beta))] .$$  \hspace{1cm} (A.4)

Here $E_1(z)$ is the exponential integral.
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