Indirect Search for $CP$ Violation in Neutrino Oscillations

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ABSTRACT

We propose the indirect search for the $CP$ violating phase in the long baseline $\nu_\mu \to \nu_e$ oscillation experiment, in which two scenarios of the neutrino mass hierarchy are discussed in the three family model. The $CP$ violating phase effect is clearly found in the scenario: the LSND data plus the atmospheric neutrino deficit. The phase dependence of the oscillation probability is explicitly shown by using typical parameters of the K2K experiment. The matter effect is negligibly small. In order to select the scenario of the neutrino mass hierarchy, the measurement of the $\nu_e \to \nu_\tau$ oscillation is also proposed in the long baseline experiment.

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1 Introduction

Neutrino flavor oscillations provide information of the fundamental property of neutrinos such as masses, flavor mixings and $CP$ violating phase. In these years, there is growing experimental evidences of the neutrino oscillations. The exciting one is the atmospheric neutrino deficit $^{[1]}-^{[3]}$ as well as the solar neutrino deficit $^{[4]}$. Super-Kamiokande $^{[5]}$ also presented the large neutrino flavor oscillation in atmospheric neutrinos. Furthermore, a new stage is represented by the long baseline(LBL) neutrino oscillation experiments $^{[6]}$. The first LBL reactor experiment CHOOZ has already reported a bound of the neutrino oscillation $^{[7]}$, which gives a strong constraint of the flavor mixing pattern. The LBL accelerator experiment K2K $^{[8]}$ is planned to begin taking data in the next year, whereas the MINOS $^{[9]}$ and ICARUS $^{[10]}$ experiments will start in the first year of the next century. Those LBL experiments will clarify masses, flavor mixings and $CP$ violation of neutrinos.

Some authors $^{[11]}$ have already discussed possibilities of observing $CP$ violation in the LBL experiments by measuring the differences of the transition probabilities between $CP$-conjugate channels$^{[12]}$, which originates from the phase of the neutrino mixing matrix, such as $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$. However, the direct measurement is very difficult in the planned LBL experiments since the magnitude of its difference is usually expected at most 0.01 and the difference of energy distributions of neutrino beams $\nu_\mu$ and $\bar{\nu}_\mu$ disturbs this measurement in the order of $O(0.01)$.

In this paper, we propose the indirect search for the $CP$ violating phase in the LBL experiments combined with results of the short baseline(SBL) experiments. The SBL experiments give important constraints for neutrino mixing angles. The tentative indication has been already given by the LSND experiment $^{[13]}$, which is an accelerator experiment for $\nu_\mu \to \nu_e (\bar{\nu}_\mu \to \bar{\nu}_e)$. The CHORUS and NOMAD experiments $^{[14]}$ have
reported the new bound for $\nu_\mu \to \nu_\tau$ oscillation, which has already improved the E531 result \cite{14}. The KARMEN experiment \cite{16} is also searching for the $\nu_\mu \to \nu_e(\bar{\nu}_\mu \to \bar{\nu}_e)$ oscillation as well as LSND. The Bugey \cite{17} and Krasnoyarsk \cite{18} reactor experiments and CDHS \cite{19} and CCFR \cite{20} accelerator experiments have already given bounds for the neutrino mixing parameters as well as E776 \cite{21}.

Taking account of those data, we study the $CP$ violating phase effect of the $\nu_\mu \to \nu_e$ oscillation in the LBL experiments. Bilenky, Giunti and Grimus \cite{22} have already discussed the bound of this transition probability, which is independent of the $CP$ violating phase. By investigating the phase effect of the bound, we provide a possibility to observe the $CP$ violation in the neutrino oscillation indirectly.

2 Mass and mixing patterns and LBL experiments

Let us start with discussing the recent results of atmospheric neutrinos at Super-Kamiokande \cite{5}, which suggests $\nu_\mu \to \nu_\tau$ oscillation with the large mixing. Since the CHOOZ result \cite{7} excludes the large neutrino oscillation of $\nu_\mu \to \nu_e$, the large mixing between $\nu_\mu$ and $\nu_\tau$ is a reasonable interpretation for the atmospheric $\nu_\mu$ deficit. Our starting point as to neutrino mixings is the large $\nu_\mu \to \nu_\tau$ oscillation with

$$\Delta m^2_{\text{atm}} \simeq 5 \times 10^{-3}\text{eV}^2, \quad \sin^2 2\theta_{\text{atm}} \simeq 1, \quad (1)$$

which constrain neutrino oscillations in the LBL experiments. The considerable large oscillations are expected for the $\nu_\mu \to \nu_\tau$ process in K2K \cite{8}, MINOS \cite{9} and ICARUS \cite{10} experiments. This large oscillation hardly depends on other mixings and the $CP$ violating phase. On the other hand, the $\nu_\mu \to \nu_e$ oscillation is small and depends on the phase and other mixing angles, which are constrained by the SBL experiments and the LBL reactor experiments.

In order to give the formulation of the neutrino oscillation probability for the LBL
$\nu_\mu \rightarrow \nu_e$ experiment, we should discuss at first the mass and mixing pattern of neutrinos. Other possible evidences in favor of neutrino oscillations are the solar neutrino and the accelerator experiment LSND. Three different scales of mass-squared differences cannot be reconciled with the three family model of neutrinos unless one of data is disregarded. Therefore, one should two possible scenarios: (1) "sacrifice solar neutrino" and (2) "sacrifice LSND". By stretching the data, the other scenario is still available as discussed by Cardall and Fuller [23]. This scenario appears to emerge naturally as the most likely solution to all oscillation evidences, however, contradicts with the zenith angle dependence of multi-GeV atmospheric data in the Kamiokande and Super-Kamiokande experiments [3]. So, we do not discuss this scenario in our paper. If one would like to sacrifice no data, the sterile neutrino should be included in the analyses, which is out of scope in our paper.

In the scenario (1) "sacrifice solar neutrino", we take $\Delta m_{31}^2 \simeq \Delta m_{21}^2 \simeq \Delta m_{\text{LSND}}^2$ and $\Delta m_{32}^2 \simeq \Delta m_{\text{atm}}^2 (m_3 \simeq m_2 \gg m_1)$. The natural pattern of the neutrino mixing matrix $U$ [25] is

$$U \simeq \begin{pmatrix} 1 & \epsilon_1 & \epsilon_2 \\ \epsilon_3 & c & s \\ \epsilon_4 & -s & c \end{pmatrix},$$

where $c \equiv \cos \theta \simeq 1/\sqrt{2}$ and $s \equiv \sin \theta \simeq 1/\sqrt{2}$ are fixed by taking account of eq.(1), and small values $\epsilon_i$'s are constrained from the SBL experiments. We do not consider the mass hierarchy $m_3 \gg m_2 \gg m_1$, which is unfavourable in constructing the neutrino mass matrix without fine-tuning in the case of the large mixing [25].

In the scenario (2) "sacrifice LSND", we take $\Delta m_{32}^2 \simeq \Delta m_{31}^2 \simeq \Delta m_{\text{atm}}^2$ and $\Delta m_{21}^2 \simeq \Delta m_{\odot}^2$ (solar neutrino). For the MSW small angle solution [26], the mixing pattern is the same one in eq.(2). Another pattern is equivalent to an exchange of the first and second columns in the mixing matrix. In the case of the just-so solution and the MSW large angle solution, the mixing matrix pattern is somewhat complicated.
Taking account of the data of the CHOOZ experiment, Bilenky and Giunti \cite{27} have obtained a typical mixing pattern, on which we will comment later.

Now, in terms of the standard parametrization of the mixing matrix $U$ \cite{28},

$$
U = \begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\
-c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}s_{13}c_{12}e^{i\phi} \\
s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}s_{13}
\end{pmatrix},
$$  \hspace{1cm} (3)

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$ are vacuum mixings, and $\phi$ is the CP violating phase, the neutrino oscillation probability for the LBL $\nu_\mu \to \nu_e$ experiment is given by

$$
P(\nu_\mu \to \nu_e) \approx 2|U_{e1}|^2|U_{\mu 1}|^2 - 4c_{23}s_{23}s_{12}s_{13}\cos \phi \sin^2 \frac{\Delta m_{32}^2 L}{4E} - 2c_{23}s_{23}s_{12}s_{13}\sin \phi S_{CP}
$$  \hspace{1cm} (4)

with

$$
|U_{e1}|^2|U_{\mu 1}|^2 \approx c_{23}^2 s_{12}^2 + s_{23}^2 s_{13}^2 + 2c_{23}s_{23}s_{12}s_{13}\cos \phi,
$$  \hspace{1cm} (5)

for the scenario (1), and

$$
P(\nu_\mu \to \nu_e) \approx 4s_{23}^2 s_{13}^2\sin^2 \frac{\Delta m_{31}^2 L}{4E} - 2c_{23}s_{23}s_{12}s_{13}\sin \phi S_{CP}
$$  \hspace{1cm} (6)

for the scenario (2), where

$$
S_{CP} = \sin \frac{\Delta m_{12}^2 L}{2E} + \sin \frac{\Delta m_{23}^2 L}{2E} + \sin \frac{\Delta m_{31}^2 L}{2E}.
$$  \hspace{1cm} (7)

The oscillatory term $S_{CP}$ is approximately reduced to $\sin(\Delta m_{23}^2 L/2E)$ for the scenario (1), because $\sin(\Delta m_{12}^2 L/2E)$ and $\sin(\Delta m_{31}^2 L/2E)$ can be replaced by the average value 0 in the LBL experiments. For the scenario (2), $S_{CP}$ is suppressed such as $S_{CP} \sim 0.01$, so the second term in eq.(6) can be neglected.

Thus, the probability in eq.(4) strongly depends on the CP violating phase, while the one in eq.(6) hardly depends on it. The CP violating phase effect could be found in the LBL $\nu_\mu \to \nu_e$ experiment if the scenario (1) is true. On the other hand, if the scenario (2) is true, it is impossible to determine the CP violating phase by $\nu_\mu \to \nu_e$. 
3 Constraints from reactor and accelerator experiments

In our analyses, $s_{23} \simeq 1/\sqrt{2}$ is fixed, but $s_{12}$, $s_{13}$ and $\phi$ still remain as free parameters. Those mixing parameters are constrained from other experiments. For the scenario (1), the SBL experiments give strong constraints because the distance $L$ corresponds to the neutrino mass scale $O(1\text{eV})$. Let us consider constraints from disappearance experiments. The Bugey [17] and Krasnoyarsk [18] reactor experiments and CDHS [19] and CCFR [20] accelerator experiments give bounds for the neutrino mixing parameters at the fixed value of $\Delta m_{31}^2$ [29]. In the mixing pattern of eq.(2), the constraint for $U_{e1}$ is given by using

$$P(\nu_e \to \nu_e) \simeq 1 - 4U_{e1}^2(1 - |U_{e1}|^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E} ,$$

(8)

where $\Delta m_{31}^2 \gg \Delta m_{32}^2$ is used. Thus, one obtains the bound $|U_{e1}|^2 \geq a_e$ due to non-observation of neutrino oscillations. For example, we get $a_e = 0.984$ for $\Delta m_{31}^2 = 2\text{eV}^2$. This condition is expressed in terms of mixing parameters as follows:

$$s_{12}^2 + s_{13}^2 - s_{12}^2 s_{13}^2 \leq B_e \equiv 1 - a_e .$$

(9)

Constraints from the appearance experiments are given by LSND, E776 and CHORUS/NOMAD. If the excess of the electron events at LSND are due to the neutrino oscillation, the $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation transition probability is equal to $0.31 \pm 0.09 \pm 0.05\%$ [13]. The plot of the LSND favored region of $\Delta m^2$ vs $\sin^2 2\theta$ allows the mass region to be larger than $\Delta m^2 \simeq 0.04 \text{eV}^2$. However, Bugey[17] has already excluded a mass region lower than $\Delta m^2 = 0.27 \text{eV}^2$. On the other hand, the constraint by E776 [21] has excluded a region larger than $\Delta m^2 = 2.3 \text{eV}^2$. Thus, the mass squared difference
\( \Delta m_{31}^2 \) is obtained in the range \([30]\):

\[
\Delta m_{31}^2 \simeq 0.27 \text{ eV}^2 \sim 2.3 \text{ eV}^2. \tag{10}
\]

The recent CHORUS experiment for \( \nu_\mu \rightarrow \nu_\tau \) has given the bound \( \sin^2 2\theta_{\text{CHORUS}} \leq 1.3 \times 10^{-3} [14] \). Those data lead to constraints of mixing angles by using following equations:

\[
P(\nu_\mu \rightarrow \nu_e) \simeq 4|U_{e1}|^2|U_{\mu 1}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E},
\]

\[
P(\nu_\mu \rightarrow \nu_\tau) \simeq 4|U_{\mu 1}|^2|U_{\tau 1}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}. \tag{11}
\]

Using eq.(5), we get the condition for the LSND data

\[
c_{23}^2 s_{12}^2 + s_{23}^2 s_{13}^2 + 2c_{23}s_{23}s_{12}s_{13}\cos \phi = L_e \equiv \frac{1}{4}\sin^2 2\theta_{\text{LSND}}, \tag{12}
\]

where \( L_e \) is given experimentally for fixed \( \Delta m_{31}^2 \).

For the CHORUS bound of \( \nu_\mu \rightarrow \nu_\tau \), we also obtain a condition

\[
s_{23}^2 s_{12}^2 + c_{23}^2 s_{13}^2 - 2c_{23}s_{23}s_{12}s_{13}\cos \phi \leq C_\tau \equiv \frac{1}{4}\sin^2 2\theta_{\text{CHORUS}}, \tag{13}
\]

which is a weak bound at present since \( \sin^2 2\theta_{\text{CHORUS}} \geq 0.1 \) in the case of \( \Delta m_{31}^2 \leq 2\text{eV}^2 \).

Actually, mixing angles which satisfy eqs.(9) and (12) always satisfy this bound.

The first LBL reactor experiment CHOOZ \([7]\) also constrains mixing angles by the following formula:

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - 2|U_{e1}|^2(1 - |U_{e1}|^2) - 4|U_{e2}|^2|U_{e3}|^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E}. \tag{14}
\]

Taking account of \( s_{12} \ll 1 \) and \( s_{13} \ll 1 \), the probability reduces to

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - 2(s_{12}^2 + s_{13}^2). \tag{15}
\]

Since CHOOZ reported \( \sin^2 2\theta_{\text{CHOOZ}} \leq 0.18 \) for large \( \Delta m^2 \), we get a constraint \( s_{12}^2 + s_{13}^2 \leq 0.18/4 \), which is a loose bound compared with the one in eq.(14).
Next we consider the scenario (2), in which the SBL experiments do not constrain mixing angles because the highest mass difference scale is $\mathcal{O}(0.01\text{eV}^2)$. The constraint follows only from the CHOOZ experiment \[7\], in which the probability is expressed as

$$P(\nu_e \rightarrow \nu_e) \simeq 1 - 4|U_{e3}|^2(1 - |U_{e3}|^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E} \simeq 1 - 4s_{13}^2 (1 - s_{13}^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E} . \quad (16)$$

By using the CHOOZ result $\sin^2 2\theta_{\text{CHOOZ}} \leq 0.12$ for $\Delta m^2 \simeq 5 \times 10^{-3}\text{eV}^2$, we get only a constraint $s_{13}^2 \leq 0.03$. The MSW small angle solution of the solar neutrino \[26\] gives another constraint: $s_{12} = 0.02 \sim 0.06$.

### 4 Phase dependence of $P(\nu_\mu \rightarrow \nu_e)$

In the scenario (1), eq.(4) is written as

$$P(\nu_\mu \rightarrow \nu_e) \simeq 2|U_{e1}|^2|U_{\mu 1}|^2 - 4c_{23}s_{23}s_{12}s_{13} \sin \left( \frac{\Delta m_{32}^2 L}{4E} \right) \sin \left( \frac{\Delta m_{32}^2 L}{4E} - \phi \right) , \quad (17)$$

where the first term in the right hand side is fixed by the LSND data. It is remarked that $c_{23}s_{23}s_{12}s_{13}$ determines this oscillation probability if $\phi$ is fixed. So, the maximal value of $c_{23}s_{23}s_{12}s_{13}$ gives the maximal or minimal $P(\nu_\mu \rightarrow \nu_e)$, which depends on $\phi$.

We show how to extract the maximal value of $s_{12}s_{13}$ from eqs.(4) and (12), taking $s_{23} = 1/\sqrt{2}$ for simplicity. Those conditions turn to

$$s_{12}^2 + s_{13}^2 - s_{12}^2 s_{13}^2 \leq B_e , \quad (18)$$

$$s_{12}^2 + s_{13}^2 + 2s_{12}s_{13}\cos \phi = 2L_e . \quad (19)$$

The maximal value of $s_{12}s_{13}$ under the condition in eq.(13) is obtained by an easy algebraic manipulation as follows:

$$s_{12}s_{13} \leq \frac{1}{1 + \cos \phi} L_e . \quad (20)$$

On the other hand, we get an equation as to $s_{12}s_{13}$ by a subtraction of eq.(18) and eq.(19):

$$s_{12}^2 s_{13}^2 + 2s_{12}s_{13}\cos \phi + (B_e - 2L_e) \geq 0 . \quad (21)$$
Taking a positive solution of $s_{12}s_{13}$, we get

$$s_{12}s_{13} \leq -\cos\phi - \sqrt{\cos^2\phi - (B_e - 2L_e)} .$$

(22)

The true maximum of $s_{12}s_{13}$ should satisfy both conditions of eqs. (21) and (22). In conclusion, we obtain the maximal $s_{12}s_{13}$ as follows:

$$s_{12}s_{13} \leq \frac{1}{1 + \cos\phi}L_e \quad \text{for} \quad \cos\phi \geq -1 + \frac{L_e}{B_e}(1 + \sqrt{1 - B_e}) ,$$

$$s_{12}s_{13} \leq -\cos\phi - \sqrt{\cos^2\phi - (B_e - 2L_e)} \quad \text{for} \quad \cos\phi \leq -1 + \frac{L_e}{B_e}(1 + \sqrt{1 - B_e}) .$$

(23)

By using this maximal value, we can predict the allowed region of the oscillation probability of eq.(17) for the fixed value of $\phi$. We show the allowed region of $P(\nu_\mu \rightarrow \nu_e)$ versus $\phi$ in fig.1, where $\Delta m^2_{32} = 5 \times 10^{-3}\text{eV}^2$ and typical parameters of the K2K experiment $L = 250\text{Km}$, $E = 1.3\text{GeV}$ [8] are taken. We also take $\Delta m^2_{31} = 2\text{eV}^2$, which leads to $B_e = 0.0183$ and $L_e = 6.5 \times 10^{-4}$. The allowed region is shown between the solid curve and the horizontal dashed line. It is noticed that this result is consistent with the one in ref.[22]. Since the condition of eq.(23) is a general one, it is useful even if the LSND data is replaced by the KARMEN [16] data in the future.

As seen in fig.1, the probability increases steeply in the region $\phi \geq 100^\circ$ and to the maximal value around 0.02. Thus, the observation of $P(\nu_\mu \rightarrow \nu_e)$ larger than 0.005 indicates the large $CP$ violating phase although the $CP$ conserved limit $\phi = 180^\circ$ is still allowed. In the case of $\phi = 180^\circ$, the observed $P(\nu_\mu \rightarrow \nu_e)$ fixes both $s_{12}$ and $s_{13}$. Then, one can predict the $CP$ conjugated process $\nu_\mu \rightarrow \nu_e$ for arbitrary neutrino beam energies. Thus, the measurement of the absolute value of $P(\nu_\mu \rightarrow \nu_e)$ is a crucial test of the $CP$ violation. In fig.1, we have also shown the oscillation probability of $\nu_\mu \rightarrow \nu_e$ by taking same values of parameters $L$ and $E$.

In the scenario (2), substituting a bound $s^2_{13} \leq 0.03$ into eq.(8), we obtain

$$P(\nu_\mu \rightarrow \nu_e) \leq 0.06 ,$$

(24)
which is much larger than the maximal value 0.02 in the scenario (1). In this case, the CP phase dependence is not found.

We have taken the MSW small angle solution of the solar neutrino in the scenario (2). In the case of the just-so solution and the MSW large angle solution, the mixing matrix pattern is somewhat complicated. However, the mixing matrix given in ref. [27] hardly changes our result with the MSW small angle solution.

The observation of $0.02 \leq P(\nu_\mu \rightarrow \nu_e) \leq 0.06$ means that the scenario (2) is true, that is to say, the mass difference scale $\Delta m^2 = \mathcal{O}(1\text{eV}^2)$ is rejected. This is the same issue in ref. [31]. What can one say if its observed probability is lower than 0.02? The measurement of the $\nu_\mu \rightarrow \nu_e$ oscillation can select a scenario because the CP violating phase effect is found in the scenario (1). An alternative method is to measure the $\nu_e \rightarrow \nu_\tau$ oscillation in the LBL experiment. For the scenario (1), the probability is different from the $\nu_\mu \rightarrow \nu_e$ oscillation as follows:

$$P(\nu_e \rightarrow \nu_\tau) \simeq 2|U_{e1}|^2|U_{\tau 1}|^2 + 4c_{23}s_{23}s_{12}s_{13}\sin \left( \frac{\Delta m^2_{32}L}{4E} \right) \sin \left( \frac{\Delta m^2_{32}L}{4E} + \phi \right),$$

(25)

while the probability is the same one for the scenario (2) in the case of $s_{23} = 1/\sqrt{2}$. Thus, the $\nu_e \rightarrow \nu_\tau$ oscillation can select a scenario of the mass hierarchy.

It may be useful to remark that the observation of $P(\nu_\mu \rightarrow \nu_e)$ larger than 0.06 indicates new physics.

5 Matter effect

Although the distance travelled by neutrinos is less than 1000 Km in the LBL experiments, those data include the background matter effect which is not CP invariant [32]. In particular, the matter effect should be carefully analyzed for the scenario (1) since the numerical results strongly depend on the CP violating phase.
Hamiltonian in matter $H(x)$ in weak basis is

$$H = U \begin{pmatrix} m_1^2/2E & 0 & 0 \\ 0 & m_2^2/2E & 0 \\ 0 & 0 & m_3^2/2E \end{pmatrix} U^\dagger + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(26)

where

$$a = \sqrt{2} G_F n_e,$$

(27)

with a constant matter density $\rho = 2.34 \text{ g/cm}^3$. For antineutrinos, effective Hamiltonian is given by replacing $a \to -a$ and $U \to U^*$. Minakata and Nunokawa have shown the perturbative treatment of the matter effect $[11]$, which is reliable under the mass scale we consider. In the scenario (1) with $m_3 \simeq m_2 \gg m_1$, the transition probability is given:

$$P(\nu_\mu \to \nu_e) = 2|V_{e1}(a)|^2|V_{\mu1}(a)|^2 - 4\text{Re}[V_{e2}(a)V^*_{e3}(a)V_{\mu2}(a)V_{\mu3}(a)]\sin^2 \frac{1}{2} I_{23}$$

$$+ 2\text{Im}[V_{e2}(a)V^*_{e3}(a)V^*_{\mu2}(a)V_{\mu3}(a)]\sin I_{23},$$

(28)

where

$$V_{ai}(a) = U_{ai} - \sum_{j \neq i} 2aE \frac{U_{aj}U^*_{ej}U_{ei}}{m_j^2 - m_i^2}, \quad I_{ij} = -\frac{\Delta m_{ij}^2 L}{2E} + (|U_{ei}|^2 - |U_{ej}|^2)aL.$$

(29)

The oscillation probability turns to a complicated equation, but it is easy to estimate the matter oscillation effect by using the parameters $E = \mathcal{O}(1\text{GeV})$, $a = \mathcal{O}(10^{-13}\text{eV})$ and $L = 250\text{Km}$. We have found the leading matter corrections for the vacuum oscillation to be dominated by terms of $2aE/\Delta m_{31}^2$, $2aE s_{12}^2/\Delta m_{32}^2$, $2aE s_{13}^2/\Delta m_{32}^2$ and $aL(s_{12}^2 - s_{13}^2)$. Those corrections are at most $\mathcal{O}(10^{-3})$ for the vacuum oscillation probability of $\nu_\mu \to \nu_e$ in the case of $\Delta m_{31}^2 \simeq 1\text{eV}^2$. This result is consistent with the previous numerical one $[11]$. Thus, the matter effect is negligible small.

6 Conclusions

We have proposed the indirect search for the $CP$ violating phase in the LBL experiments combined with results of the SBL experiments in two scenarios of the mass
hierarchy. Taking account of those data, we have investigated the $CP$ violating phase effect of the $\nu_\mu \rightarrow \nu_e$ oscillation in the LBL experiments for the scenario (1): the LSND data plus the atmospheric neutrino deficit. For the scenario (2): the atmospheric neutrino deficit plus the solar neutrino deficit, the phase dependence is negligibly small. As an example, we have shown the phase dependence of the oscillation probability by using typical parameters of the K2K experiment for the scenario (1). The matter effect has been found to be negligibly small. This phase effect on the probability provides a possible method to observe the $CP$ violation in the neutrino oscillation indirectly. In order to select the scenario (1) or (2), the measurement of the $\nu_e \rightarrow \nu_\tau$ oscillation is useful in the LBL experiment as well as $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. Even if our predicted magnitudes are out of sensitivity in the K2K experiment, we can expect the experiments of MINOS and ICARUS.
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Fig. 1: Predicted bound of the $\nu_\mu \to \nu_e$ oscillation in the K2K experiment for the scenario (1), which is denoted by the thick solid curve. The $CP$ conjugated $\bar{\nu}_\mu \to \bar{\nu}_e$ one is shown by the thin solid curve. The horizontal dashed line denotes $2|U_{e1}|^2|U_{\mu 1}|^2$, which is fixed by the LSND data. Here, $\Delta m^2_{31} = 2\text{eV}^2$ and $\Delta m^2_{32} = 5 \times 10^{-3}\text{eV}^2$ are taken.