Two particle rapidity, transverse momentum, and azimuthal correlations in relativistic nuclear collisions and transverse radial expansion

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At the very first stage of an ultra-relativistic nucleus-nucleus collision new particles are produced in individual nucleon-nucleon collisions. In the transverse plane, all particles from a single $NN$ collision are initially located at the same position. The subsequent transverse radial expansion of the system creates strong position-momentum correlations and leads to characteristic rapidity, transverse momentum, and azimuthal correlations among the produced particles.

1. Introduction

The physics of the high energy heavy ion collisions attracts strong attention of the physics community as creation of a new type of matter, the Quark-Gluon Plasma, is expected in such collisions. During the last few years of Au+Au collisions at the BNL Relativistic Heavy Ion Collider (RHIC) many new phenomena has been observed, such as strong elliptic flow \[ 1 \] and suppression of the high transverse momentum two particle back-to-back correlations \[ 2 \]. These observations strongly indicate that a dense partonic matter has been created in such high energy nuclear collisions. Parton reinteractions lead to pressure build-up and the system undergoes longitudinal and transverse expansion, the latter leading to an increase in the particle final transverse momenta. Usually the transverse expansion is studied via detailed analysis of single particle transverse momentum spectra, most often using thermal parameterization suggested in \[ 3 \]. In this paper we note that the transverse radial expansion should also lead to characteristic rapidity, transverse momentum, and azimuthal angle two-particle correlations.

At the first stage of a $AA$ collision many individual nucleon-nucleon collision happen. The subsequent transverse expansion of the system creates strong position-momentum correlations in the transverse plane: further from the center axis of the system a particle is produced initially, on average the larger push it gets from other particles during the system evolution. The ‘push’ is in the transverse direction, and on average does not affects the longitudinal momentum component. As all particles produced in the same $NN$ collision have initially the same spatial position in the transverse plane, they get on average the same push and thus become correlated. As discussed below this picture leads to many distinctive phenomena, most of which can be studied by means of two (and many-) particle correlations. The correlations can extend over wide rapidity range, but are not boost invariant, as transverse expansion itself depends on rapidity.
2. Two particle transverse momentum correlations

The single particle spectra are affected by radial flow in such a way that the mean transverse momentum is mostly sensitive to the average expansion velocity squared \( \langle p_t \rangle_{AA} \approx \langle p_t \rangle_{NN} + \alpha \langle v^2 \rangle \), (see Fig. 1 below) and to much lesser extend to the actual velocity profile (dependence of the expansion velocity on the radial distance from the center axis of the system). The two-particle transverse momentum correlations \( \langle \delta p_{t1} \delta p_{t2} \rangle \equiv \langle \delta p_{t1} \delta p_{t2} \rangle_{NN} \), would be sensitive mostly to the variance in collective transverse expansion velocity, and thus are more sensitive to the actual velocity profile:

\[
\langle \delta p_{t1} \delta p_{t2} \rangle_{AA} \approx D_{Nc} \left( \langle \delta p_{t1} \delta p_{t2} \rangle_{NN} + \alpha^2 \sigma_{v}^2 \right); D_{Nc} = \frac{\langle n(n-1) \rangle_{NN}}{(N_c-1)\langle n \rangle_{NN}^2 + \langle n(n-1) \rangle_{NN}}. \tag{1}
\]

The factor \( D \) takes into account the dilution of the correlations due to a mixture of particles from \( N_c \) uncorrelated \( NN \) collisions, and that in an individual \( NN \) collision the mean number of particle pairs, \( \langle n(n-1) \rangle_{NN} \), on average is larger than \( \langle n \rangle_{NN}^2 \); at ISR energies in central rapidity region \( \langle n(n-1) \rangle_{NN} \approx 1.66 \langle n \rangle_{NN}^2 \) [12].

We employ a thermal model [3] for further calculations. In this model particles are produced by freeze-out of the thermalized matter at temperature \( T \), approximated by a boosted Boltzmann distribution. Assuming boost-invariant longitudinal expansion and freeze-out at constant proper time, one finds

\[
\frac{dn}{dp_t} \sim \int d\rho_t d\phi_b p_t^{2/n-1} J(p_t; T, \rho_t, \phi_b); \quad J(p_t; T, \rho_t, \phi_b) \equiv m_t K_1(\beta_t) e^{\alpha_t \cos(\phi_b - \phi)}, \tag{2}
\]

where \( \rho_t \) is the transverse flow rapidity, \( \phi_b \) is the boost direction, \( \alpha_t = (p_t/T) \sinh(\rho_t) \), and \( \beta_t = (m_t/T) \cosh(\rho_t) \). It also assumes a uniform matter density within a cylinder, \( r < R \), and a power law transverse rapidity flow profile \( \rho_t \propto r^n \). Two particle spectrum for particles originating from the same \( NN \) collision would be

\[
\frac{dn_{pair}}{dp_{t1} dp_{t2}} \sim \int d\rho_t d\phi_b p_t^{2/n-1} J(p_{t1}; T, \rho_t, \phi_b) J(p_{t2}; T, \rho_t, \phi_b). \tag{3}
\]

It additionally assumes that during the expansion time (before the freeze-out) the particles produced originally at the same spatial position do not diffuse far one from another compared to the system size.

The results of the numerical calculations based on the above equations are presented in Fig. 1 as function of \( \langle \rho_t^2 \rangle = \langle p_t \rangle^2/(4n+4)/(2+n)^2 \). The results are shown for two different velocity (transverse rapidity) profiles, \( n = 2 \), and \( n = 0.5 \). One observes that indeed for all the particle types presented, \( \langle p_t \rangle \) depends very weakly on the actual profile. On opposite, the correlations are drastically different for two cases studied.

In Fig. 2 we compare our estimates with preliminary STAR data [7] on two particle \( p_t \) correlations. We use the mean expansion velocity and temperature parameters from [13] and assume \( N_c = \text{N part}/2 \) and \( \langle \delta p_{t1} \delta p_{t2} \rangle/\langle p_t \rangle^2 = 0.011 \) (about 20\% smaller than measured at ISR [8]). It is observed that the transverse expansion with linear velocity profile produces too strong correlations.
Two particle correlations in nuclear collisions

Figure 1. (color online) Mean transverse momentum and two particle $p_t$ correlations in the blast wave calculations. $T = 110$ MeV.

Figure 2. Comparison of Blast Wave calculations for two different velocity profiles with preliminary STAR data [7]. Relation \( \langle n(n-1) \rangle_{NN} = 1.66 \langle n \rangle_{NN}^3 \) has been used.

Figure 3. (color online) Two pion $\Delta \phi$ distribution as function of $\langle \rho_t^2 \rangle$ in the Blast Wave model. Linear velocity profile and $T = 110$ MeV have been assumed.

3. Rapidity and azimuthal correlations. Charge balance functions.

The rescatterings during transverse expansion lead not only to the increase of the transverse momentum but also to the particle diffusion in the rapidity space. As we neglect such a diffusion in the current analysis, the narrowing of the charge balance function [8] observed in [10] would be described just by increase in mean $p_t$ as [9]:

$$\Delta p_z = m_t \sinh(\Delta y) \approx m_t \Delta y \approx \text{const.}$$  \hspace{1cm} (4)

This effect is consistent with experimentally observed narrowing for about 15 – 20% of the balance function width with centrality [10] and with centrality dependence of the net charge fluctuations [11].

As all particles from the same $NN$ collisions are pushed in the same direction (radially in the transverse plane) they become correlated in azimuthal space. The correlations can become really strong for large transverse flow as shown in Fig. 2 (again, for particles
originated from the same NN collision). The second harmonic in the azimuthal correlations generated by radial expansion is of a particular interest as it would contribute to the measurements of elliptic flow. The numbers from Fig. 3 corresponding to $\langle \rho_t^2 \rangle \sim 0.3$ are comparable with the estimates of the strength of non-flow type azimuthal correlation estimates \[14\]. Thus the azimuthal correlations generated by transverse expansion could be a major contributor to the non-flow azimuthal correlations.

The magnitude of the correlations due to transverse expansion should be sensitive to the system thermalization time and the particle diffusion in the thermalized matter during the expansion; the azimuthal correlations would be the most interesting/useful for such a study. Another application can be in “jet tomography” to infer information about how “deep” in the system the hard collision has occurred. For that, one has to correlate the jet (high $p_t$ hadron) yield with mean transverse momentum of particles taken at different rapidity (but better at similar azimuthal angle). In the same $NN$ collision where the high $p_t$ particle is emitted, the soft particles are produced as well. Those soft particles experience the transverse 'push' corresponding to the spatial position in the transverse plane where the original $NN$ collision happens to be. Then the mean transverse momentum of the associated particles would provide the information on how close to the center of the system the collision occurred.

The above described picture of $AA$ collisions has many interesting observable effects, only a few mentioned in this paper. The picture become even richer if one looks at the identified particle correlations. Many questions require a detailed model study, but the approach opens a potentially very interesting possibility to address the initial conditions and the subsequent evolution of the system created in an $AA$ collision.

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