Correlation between flavour violating decay of long-lived slepton and tau in the coannihilation scenario with Seesaw mechanism

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Abstract

We investigate flavour violating decays of the long-lived lightest slepton and the tau lepton in the coannihilation region of the Minimal Supersymmetric Standard Model with a Seesaw mechanism to generate neutrino masses. We consider a situation where the mass difference between the lightest neutralino, as the Lightest Supersymmetric particle (LSP), and the lightest slepton, as the Next-to-LSP, is smaller than the mass of tau lepton. In this situation, the lifetime of the lightest slepton is very long and it is determined by lepton flavour violating (LFV) couplings because the slepton mainly consists of the lighter stau and the flavour conserving 2-body decay is kinematically forbidden. We show that the lifetime can change many orders of magnitude by varying the Yukawa couplings entering the Seesaw mechanism. We also show that branching ratio of LFV tau decays are strongly correlated with the lightest slepton lifetime. Therefore the branching ratios of LFV tau decays can be determined or constrained by measuring the slepton lifetime at the LHC experiment.

Keywords: long-lived slepton, coannihilation scenario, seesaw mechanism, lepton flavour violation

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Neutrino oscillation experiments [1–4] have confirmed that neutrinos are massive and mix with each other. Thus, flavour is violated in the lepton sector similarly to the quark sector flavour violation described by the CKM mixing matrix. These experimental results require that, today, the Standard Model (SM) of particle physics must include a way to accommodate neutrino masses and mixing. In this extension of the SM, Lepton Flavour Violation (LFV) in charged lepton sector should also occur through neutrino mixing in loop corrections, although it has not been found so far. In fact, the rates of LFV of charged leptons induced by neutrino mixing, being proportional to the mass differences of neutrinos, are far below the present and near-future experimental sensitivities. Therefore, the discovery of flavour violating processes in the charged lepton sector would be a clear evidence of new physics beyond the SM.

One of the most attractive mechanisms to realize the observed tiny neutrino masses is the so-called Seesaw mechanism [5–9], where Majorana right-handed neutrinos with heavy masses are introduced. Due to the presence of these right-handed neutrinos, a Yukawa coupling and a Majorana mass term for the right-handed neutrinos are allowed, and left-handed neutrinos acquire masses through an effective dimension-five operator, after the Higgs scalar develops a vacuum expectation value. Then, the masses of left-handed neutrinos are a function of the neutrino Yukawa couplings and the heavy right-handed Majorana masses and they become naturally tiny because they are suppressed by these heavy Majorana masses. Therefore, the observed light neutrino masses and mixing provide information on the heavy right-handed neutrino masses and the neutrino Yukawa couplings in the Seesaw scenario. It was pointed out in [10], however, that apart from the unknown right-handed Majorana masses there exists a complex orthogonal matrix in the parametrization of the neutrino Yukawa coupling, whose six parameters can not be determined by low-energy neutrino oscillation experiments. In a supersymmetric (SUSY) extension of the Seesaw mechanism, flavour mixing among sleptons (scalar partners of leptons) are induced at low scale from the neutrino Yukawa coupling through Renormalization Group Equations (RGEs) [11–13]. Since flavour violating decays and flavour conversions of leptons occur via the slepton mixing [11–13], the parameters can be determined or constrained by measurements of these processes. A lot of theoretical works have been done for this purpose (see some recent works [14–24]), and several experiments to explore LFV processes are ongoing or will start in the near future [25–31].

Apart from the LFV processes of leptons, it is clear that once we are able to produce sleptons in colliders, their decays can also provide information on the flavour violating entries in the slepton mass matrix. In particular, it was shown in [32], that in a scenario of the Minimal Supersymmetric SM (MSSM) where the lightest slepton and the lightest neutralino are nearly degenerate, as it happens in part of the coannihilation region [33], the lifetime of the lightest slepton has a very good sensitivity to small LFV parameters. In this scenario, the Lightest Supersymmetric Particle (LSP) is the lightest neutralino which is almost pure Bino, and the Next-to-LSP (NLSP) is the lightest slepton, which mainly consists of the right-handed stau. When the mass difference between the LSP neutralino and the NLSP
slepton is smaller than the tau mass, the decay of the NLSP slepton into tau and neutralino is kinematically forbidden. Then, flavour conserving decays are only 3-body (or 4-body) decays into a tau neutrino, a pion and the neutralino (a tau neutrino, two leptons and the neutralino). The decay widths of flavour conserving processes are highly suppressed due to the additional Fermi coupling and the small phase space, and therefore the lightest slepton becomes long-lived [34]. On the other hand, flavour violating 2-body decays into an electron (or a muon) and the neutralino are kinematically allowed. They become dominant if the suppression due to LFV couplings is looser than the suppression of the flavour conserving decays. In this situation, the lifetime of the slepton is determined by LFV parameters and is sensitive to very small values of the LFV parameters. In Ref. [32], it was shown that such a small mass difference is realized in the Constrained MSSM (CMSSM) consistent with present constraints from terrestrial experiments and cosmological observations. It was also discussed that determination of the LFV parameters via the lifetime could be possible in the ATLAS detector [32] when the sleptons decay inside the detector. Therefore, the LHC experiment provides a good opportunity to explore the LFV of the sleptons in this nearly degenerate scenario.

In this work, we will analyze this scenario of nearly degenerate LSP and NLSP in the MSSM with the Seesaw mechanism. We assume that slepton mass matrices are perfectly universal at the Grand Unification Theory (GUT) scale and flavour violating entries in these matrices are generated by RGE evolution in the presence of the neutrino Yukawa coupling. Then, the lifetime of the slepton is mainly determined by the mixing of left-handed sleptons, because larger flavour mixing is induced to the left-handed sleptons than to the right-handed sleptons. Similarly, in this scenario, the rates of flavour violating tau decays are strongly related with the lifetime of the slepton because the NLSP slepton mostly consists of the stau. Thus, it is worthwhile to study correlations between the lifetime of the slepton and the branching ratios of the LFV tau decays to obtain information on the parameters of the Seesaw mechanism.

The rest of the paper is organized as follows. In Sec. II, we briefly review the CMSSM with the Seesaw mechanism. In Sec. III, we give expressions for the decay rates of the slepton and LFV tau decays, and then derive relations among branching ratios and lifetimes. The results of our numerical calculation are shown in Sec. IV. Finally, we summarize and discuss our results in Sec. V.

II. THE CMSSM WITH RIGHT-HANDED NEUTRINOS

We start our discussion with a brief review of the Seesaw mechanism in the CMSSM. The CMSSM is defined at the GUT scale, \( M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \), by four parameters and a sign:

\[
\{ M_{1/2}, m_0, A_0, \tan\beta, \text{sign}(\mu) \},
\]

where \( M_{1/2} \) and \( m_0 \) are the universal gaugino and scalar masses, respectively, and \( A_0 \) is the universal trilinear coupling. \( \tan\beta \) is the ratio of the vacuum expectation values of up-type Higgs \( (v_u) \) to down-type Higgs \( (v_d) \), \( \tan\beta = v_u/v_d \), and \( \mu \) is the Higgs mass parameter in the superpotential. It is important to emphasize that in this model, soft SUSY breaking
terms are completely universal at the GUT scale, i.e. soft mass matrices are proportional to the identity matrix and trilinear couplings are proportional to the Yukawa couplings. This universality is broken in the quark sector by radiative corrections due to the presence of the up- and down-type quark Yukawa couplings, but it is preserved in the lepton sector if neutrinos are massless. When Majorana right-handed neutrinos are introduced in the CMSSM, a neutrino Yukawa coupling and a Majorana mass term for the right-handed neutrinos are allowed in the superpotential. In the basis where the charged lepton Yukawa coupling and the right-handed neutrino mass matrix are diagonal, the leptonic part of the superpotential is given by

$$W = \hat{y}_\ell \alpha L_\alpha H_d E^c_\alpha + (y_\nu)_{\alpha i} L_\alpha H_u N^c_i + (\hat{M}_R)_i N^c_i N^c_i + h.c.,$$  

(2)

where $i = 1-3$ and $\alpha = e, \mu, \tau$. The charged lepton Yukawa coupling, the neutrino Yukawa coupling and the diagonal right-handed neutrino mass matrix are denoted as $\hat{y}_\ell$, $y_\nu$, and $\hat{M}_R$ where $\hat{M}_R = \text{diag}(M_{R1}, M_{R2}, M_{R3})$, respectively. The superfields corresponding to the left-handed leptons, right-handed charged leptons and right-handed neutrinos are denoted as $L_\alpha$, $E^c_\alpha$, and $N^c_i$. The superfield of the up-type (the down-type) Higgs doublet is as $H_u$ ($H_d$). Throughout this paper, we use Greek indices to denote flavour eigenstates (= mass eigenstate of charged leptons) and Latin indices to mass eigenstates of both of left- and right-handed neutrinos. After the electroweak symmetry breaking, the effective mass matrix of the left-handed neutrinos, $m_\nu$, is given by

$$m_\nu = -\frac{v_\nu^2}{u} y_\nu \cdot \hat{M}_R^{-1} \cdot y_\nu^T.$$  

(3)

From this equation, we can see that neutrino masses are suppressed by a factor $(v_\nu^2/M_{Ri})$, and hence tiny masses are naturally generated. The mass matrix of the neutrinos, Eq. (3), is a complex symmetric matrix and is diagonalized as

$$\hat{m}_\nu = U^T \cdot m_\nu \cdot U = \text{diag}(m_1, m_2, m_3),$$  

(4)

where $U$ is the neutrino mixing matrix (or the MNS matrix [35]) that contains three mixing angles, one Dirac-type and two Majorana-type CP violating phases. The diagonal matrix, $\hat{m}_\nu$, contains three light neutrino masses, $m_i$. By inverting the seesaw relation Eq. (3), we obtain the following expression for the neutrino Yukawa coupling [10]

$$y_\nu = \frac{2i}{v_\nu} U^* \cdot \sqrt{\hat{m}_\nu} \cdot W \cdot \sqrt{\hat{M}_R},$$  

(5)

where $W$ is a general complex orthogonal matrix. Notice that this orthogonal matrix, $W$, does not contribute to the left-handed neutrino masses and mixing, and hence can not be measured in neutrino oscillation experiments. We can parameterize $W$ as

$$W = \begin{pmatrix} \cos \omega_1 & -\sin \omega_1 & 0 \\ \sin \omega_1 & \cos \omega_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \omega_2 & 0 & \sin \omega_2 \\ 0 & 1 & 0 \\ -\sin \omega_2 & 0 & \cos \omega_2 \end{pmatrix} \begin{pmatrix} \cos \omega_3 & -\sin \omega_3 & 0 \\ \sin \omega_3 & \cos \omega_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

(6)

1 Without loss of generality, we can always take this basis.
where, in principle, the angles $\omega_{1,2,3}$ are complex numbers. In fact, the different elements of the Yukawa coupling, Eq. (5), can change many orders of magnitude due to the imaginary parts of these complex angles, even if all other parameters are fixed. However, as we will see later, these complex angles affect LFV processes, and large values of elements would violate the present bounds. By this reason, in this paper, we will take $-\frac{3}{8}\pi < \text{Im}(\omega) < \frac{3}{8}\pi$ for numerical analysis in Sec.IV.

After introducing the neutrino Yukawa coupling and right-handed Majorana masses in the superpotential, they modify the slepton mass matrix at the loop level and generate lepton flavour violating entries through RGE running in analogy to the quark sector. These flavour violating off-diagonal elements induced in the slepton mass matrix and in the slepton trilinear coupling can be estimated at one-loop in the leading-log approximation as [11, 13, 17]:

$$
\Delta M^2_{\text{LL}} \alpha \beta \approx -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) (y^\dagger_\nu L y_\nu)_{\alpha \beta}, \quad L_{ij} \equiv \log \left( \frac{M_{\text{GUT}}}{M_{Ri}} \right) \delta_{ij},
$$

(7)

$$
\Delta M^2_{\text{RR}} \alpha \beta \approx 0,
$$

(8)

$$
\Delta M^e_{\text{LR}} \alpha \beta \approx -\frac{3}{8\pi^2} A_0 \hat{y}_{\ell \alpha} (y^\dagger_\nu L y_\nu)_{\alpha \beta},
$$

(9)

where $\Delta M^2_{\text{LL}} \alpha \beta$ and $\Delta M^2_{\text{RR}} \alpha \beta$ denote the $3 \times 3$ left- and right-handed slepton soft mass matrices and $\Delta M^e_{\text{LR}} \alpha \beta$ is the $3 \times 3$ left-right mixing slepton matrix that includes a contribution from the trilinear couplings. Notice, that, as we can see in these equations, at one-loop, flavour mixing is not induced in the right-handed slepton mass matrix. However, right-handed slepton mixing is effectively generated from left-handed slepton mixing in the presence of left-right mixing. Therefore, the mixing in right-handed sleptons is suppressed by the left-right mixing. Similarly, flavour mixing is generated in the right-handed sector by two-loop effects [36] but this two-loop induced flavour mixing is strongly suppressed by loop factors and the charged lepton Yukawa coupling. In the following, we neglect the two-loop contributions and consider only the induced flavour mixing by the one-loop RGE. Thus, the slepton mass matrix at the electroweak scale includes only the LFV entries described in Eqs. (7,8,9) that are a function of the neutrino Yukawa coupling.

Moreover, in this paper, we consider a case of the CMSSM where the mass difference between the lightest slepton $\tilde{l}_1$ and the lightest neutralino $\tilde{\chi}^0_1$, $\delta m = m_{\tilde{l}_1} - m_{\tilde{\chi}^0_1}$, is smaller than the tau mass, $m_\tau$. In the CMSSM with this small mass difference, the LSP is the lightest neutralino which is almost pure Bino and the NLSP is the lightest slepton which mainly consists of the right-handed stau $\tilde{\tau}_R$. The LSP and the NLSP remain the neutralino and the slepton in the Seesaw mechanism because the neutrino Yukawa coupling and the Majorana mass term do not change mass spectrum significantly.

In the next section we will discuss the phenomenology of NLSP decays and radiative tau decays in this scenario.

### III. NLSP DECAYS AND LFV TAU BRANCHING RATIOS

In this section, we show the decay rates of flavour violating decays of the lightest slepton and tau radiative decays. We derive explicit relations between the lifetime of the lightest
slepton and the branching ratios of radiative LFV tau decays, \( \tau \to e + \gamma \) and \( \tau \to \mu + \gamma \).

First, we show the decay rates of LFV decays of the lightest sneutrino. The main decay modes of the lightest sneutrino when the mass difference is \( m_e (m_\mu) < \delta m < m_\tau \), are LFV 2-body decays, \( \tilde{l}_1 \to e (\mu) + \chi^0_1 \), unless the induced LFV entries in the sneutrino mass matrix are so small. In these conditions, the sneutrino decay rates are given by [32],

\[
\Gamma(\tilde{l}_1 \to l_\alpha + \chi^0_1) = \frac{g_2^2}{4\pi m_{\tilde{l}_1}} (\delta m)^2 |g_{l_\alpha 1}|^2, \quad (\alpha = e, \mu),
\]

where \( (l_e, l_\mu) = (e, \mu) \) and \( g_2 \) is the \( SU(2) \) coupling constant. \( g_{l_\alpha 1} \) is an effective LFV coupling constant of the neutralino LSP, the lightest sneutrino and the charged lepton interaction. \( g_{l_\alpha 1} \) can be estimated using the Mass Insertion (MI) approximation [37] as shown Fig. 1.(a),

\[
g_{l_\alpha 1} \simeq \frac{1}{2} \tan \theta_W \frac{\Delta M_{LR\alpha}^2}{M_{LR\alpha}^2 - M_{LR\alpha}^2} \frac{M_{LR\alpha}^2}{M_{LR\alpha}^2} (\delta_{L\alpha\gamma}^e)_{\alpha\beta},
\]

where \( \theta_W \) is the Weinberg angle, \( M_{LR\alpha}^e \) are diagonal elements of the sneutrino mass matrix and \( \Delta M_{LR\alpha}^2 = m_\tau (A_0 - \mu \tan \beta) \) is the flavour-diagonal element of the left-right sneutrino mass matrix. The left-left MI, \( (\delta_{L\alpha\gamma}^e)_{\alpha\beta} \), is defined as follows and is generated through RGE running by the neutrino Yukawa coupling,

\[
(\delta_{L\alpha\gamma}^e)_{\alpha\beta} = \frac{\Delta M_{LR\alpha\beta}^2}{M_{LR\alpha}^2 M_{LR\beta}^2} \simeq \frac{(3m_0^2 + A_0^2) (y^e_L y^e_\nu)_{\alpha\beta}}{8\pi^2 M_{LR\alpha}^2 M_{LR\beta}^2}.
\]

Then the lifetime of the sneutrino, \( \tau_{\tilde{l}_1} \), is approximately given by the inverse of sum of decay rates, Eq. (10),

\[
\tau_{\tilde{l}_1} \simeq \left( \sum_{\alpha=e,\mu} \Gamma(\tilde{l}_1 \to l_\alpha + \chi^0_1) \right)^{-1}.
\]

Therefore, the lifetime is inversely proportional to \( (\delta m)^2 (g_{l_\alpha 1})^2 \), and increases as the mass difference and/or the LFV couplings become small.

![FIG. 1: Mass insertion Feynman diagrams.](image)

- Figure (a) depicts the 2-body decays of the lightest slepton in the presence of \( \delta_{L\alpha\gamma}^e \).
- Figure (b) depicts radiative LFV tau decays, \( \tau \to l_\alpha + \gamma \), where \( \alpha = e \) or \( \mu \), from the neutralino and the slepton loop contribution.
the sum of $\Gamma(\tilde{l}_1 \rightarrow \ell_\alpha + \chi_0^0)$ becomes comparable to or smaller than those of flavour conserving 3- or 4-body decay rates, and then the lifetime becomes insensitive to the LFV couplings [32]. The ratio between the 2-body and 3-body decays is given by [34]

$$\frac{\Gamma(\tilde{l}_1 \rightarrow \chi + \nu + e^-)}{\Gamma(\tilde{l}_1 \rightarrow \chi_1^0 + e(\mu))} = \frac{g_2^2 G_F^2 f_\pi^2 \cos^2 \theta_w \tan^2 \theta_w [30(2\pi)^3 m_{11} m_\tau^2]^{-1}(\delta m)^6}{g_2^2 [2\pi m_{11}]^{-1}(\delta m)^2 |g_{1\alpha}^L|^2}$$

$$= 2.31 \times 10^{-16} |g_{1\alpha}^L|^2 \left(\frac{\delta m}{1\text{GeV}}\right)^4,$$

and hence Eq. (13) is valid when $g_{1\alpha}^L \gtrsim 10^{-8}$ for $\delta m = 1 \text{ GeV}$. If $\delta m < m_\tau$ ($m_\tau$ being the mass of charged pions), the 3-body decay is forbidden. The similar ratio with 4-body decay is given by

$$\frac{\Gamma(\tilde{l}_1 \rightarrow \chi_1^0 + \nu + e + \bar{\nu}_e)}{\Gamma(\tilde{l}_1 \rightarrow \chi_1^0 + e(\mu))} = \frac{2g_2^2 G_F^2 \tan^2 \theta_w [3 \cdot 5^3(2\pi)^5 m_{11} m_\tau^2]^{-1}\delta m [(\delta m)^2 - m_\tau^2]^{5/2} [2(\delta m)^2 - 23 m_\tau^2]}{g_2^2 [2\pi m_{11}]^{-1}(\delta m)^2 |g_{1\alpha}^L|^2}$$

$$= 1.80 \times 10^{-24} |g_{1\alpha}^L|^2 \left(\frac{\delta m}{50\text{MeV}}\right)^6,$$

and hence Eq. (13) is valid when $g_{1\alpha}^L \gtrsim 10^{-12}$ for $\delta m = 50 \text{ MeV}$.

Similarly, we can express the decay rates of $\tau \rightarrow e + \gamma$ and $\tau \rightarrow \mu + \gamma$ in terms of the LFV couplings. For simplicity, we assume that $\mu \tan \beta$ is large, which is the interesting case to obtain sizable LFV tau decays. In this case, the lightest slepton has a non-negligible mixture with left-handed stau and the main contribution to LFV tau decays comes from the pure Bino loop with the lightest slepton. Both processes are proportional to $\mu \tan \beta$ and include the same MI, $(\delta_{LL})_{\alpha\tau}$, that contributes to the effective coupling $g_{1\alpha}^L$. Therefore, we can expect these processes to be strongly correlated, and in fact can estimate the decay rates by evaluating Feynmann diagram shown in Fig. 1 (b),

$$\Gamma(\tau \rightarrow \ell_\alpha + \gamma) \simeq \frac{\pi}{2} \left(\frac{\alpha_{em} g_2}{96\pi^2 \cos \theta_w}\right)^2 \frac{m_3^2}{m_{11}^2} |g_{1\alpha}^L|^2,$$

where $\alpha_{em}$ is the fine structure constant. The decay rates have the same dependence on $g_{1\alpha}^L$ as Eq. (10).

Using the above results, we can derive useful relations between the lifetime of the slepton and the branching ratios of the LFV tau decays, $\text{Br}(\tau \rightarrow e (\mu) + \gamma)$. It can be done straightforwardly by replacing the coupling $|g_{1\alpha}^L|^2$ in Eq. (10) with that in Eq. (16). In the case of

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[2] In some parameter space, the Higgsino-Bino mixed loop is also important for LFV tau decays (see Fig. 2 in [13]). However, in the case that the lightest neutralino is almost pure Bino, the mixing is tiny, and as will be shown in numerical results the contribution is smaller. Moreover, it has the same dependence on the flavour-changing MI $(\delta_{LL})_{\alpha\beta}$ as the Bino loop has, and does not modify the correlation in Eq. (17). All of contributions from neutralinos and sleptons are included in the numerical analysis in the next section.
\( m_\mu < \delta m < m_\tau \), the branching ratios are expressed

\[
\text{Br}(\tau \to e + \gamma) + \text{Br}(\tau \to \mu + \gamma) = 2m_\tau^3 \tau \left( \frac{\alpha_e}{96\pi \cos \theta_W} \right)^2 \left( \tau l_1 m_l (\delta m)^2 \right)^{-1}.
\] (17)

From Eq. (17), we can obtain an upper bound on the branching ratios of the radiative tau decay when the lifetime and the mass difference are given. For \( m_e < \delta m < m_\mu \), the lifetime is related only to \( \text{Br}(\tau \to e + \gamma) \),

\[
\text{Br}(\tau \to e + \gamma) = 2m_\tau^3 \tau \left( \frac{\alpha_e}{96\pi \cos \theta_W} \right)^2 \left( \tau l_1 m_l (\delta m)^2 \right)^{-1},
\] (18)

because the slepton decay into \( \mu + \tilde{\chi}_1^0 \) is kinematically forbidden for this mass difference. When the 2-body slepton decays are dominant, the relations, Eqs. (17) and (18), can be expressed in a different way,

\[
\text{Br}(\tau \to l_\alpha + \gamma) = 2m_\tau^3 \tau \left( \frac{\alpha_e}{96\pi \cos \theta_W} \right)^2 \left( \tau l_1 m_l (\delta m)^2 \right)^{-1} \text{Br}(\tilde{l}_1 \to l_\alpha + \tilde{\chi}_1^0),
\] (19)

where \( \text{Br}(\tilde{l}_1 \to l_\alpha + \tilde{\chi}_1^0) \) is the branching ratio of the slepton decay. From Eq.(19), the branching ratios of LFV tau decays can be predicted once the mass difference, the branching ratios and the lifetime of the slepton are determined.

Another relation obtained from Eqs. (10) and (16) is

\[
\frac{\text{Br}(\tau \to e + \gamma)}{\text{Br}(\tau \to \mu + \gamma)} = \frac{\text{Br}(\tilde{l}_1 \to e + \tilde{\chi}_1^0)}{\text{Br}(\tilde{l}_1 \to \mu + \tilde{\chi}_1^0)} \equiv r_{e\mu},
\] (20)

which is valid only for \( m_\mu < \delta m < m_\tau \). The ratio of the branching ratios of the tau decays is determined when \( r_{e\mu} \) is determined through the slepton decays. The ratio can be given in terms of the LFV couplings,

\[
r_{e\mu} \simeq \left( \frac{g_{l_1 e_1}}{g_{l_1 \mu_1}} \right)^2.
\] (21)

This ratio can be predicted once the LFV couplings are given in a specific model. It is important to emphasize here that these relations hold also for models where the LFV entries in the slepton mass matrix are not generated by the RGEs in a Seesaw scenario. Both the lifetime and the branching ratios depend on these LFV parameters and these expressions are still valid in other scenarios if \( \Delta M_{LL_{\alpha\beta}}^2 \gg \Delta M_{RR_{\alpha\beta}}^2 \) and \( \mu \tan \beta \gg m_\text{SUSY} \), \( m_\text{SUSY} \) being a typical mass scale of particles in the loop in Fig. 1.

\section*{IV. NUMERICAL ANALYSIS}

In this section we analyze numerically the lightest slepton lifetime and the radiative tau decays in the CMSSM in the parameter region corresponding to nearly degenerate slepton and neutralino as described above. All these results on the lifetime of the slepton and the
branching ratios of $\tau \to e (\mu) + \gamma$ and also $\mu \to e + \gamma$ are obtained by solving one-loop RGEs and using exact formulae of the decay rates given in [13, 32].

Regarding the CMSSM parameters, we choose three reference points shown in Table. I to see difference of the lifetimes and the branching ratios on $\delta m$. The point 1 corresponds to the case where $\delta m$ is slightly smaller than $m_\tau$ while the point 3 is the one where $\delta m$ is smaller than $m_\mu$, the point 2 is an intermediate value between the point 1 and 3. The masses of the NLSP slepton and the LSP neutralino are almost the same for three points and are approximately 360 GeV. It is shown in [32] that, in the CMSSM parameter space, there exist regions for $\delta m < m_\tau$ where not only the dark matter abundance but also the Higgs mass, the deviation of the muon anomalous magnetic moment and the branching ratios of $b \to s + \gamma$ are consistent with experimental bounds. The points we choose here are included in this region.

In this work, flavour mixing is generated by the neutrino Yukawa coupling through RGE evolution. Therefore, we need to specify the neutrino Yukawa coupling consistent with the observed neutrino masses and mixing, Eq. (5). Mixing angles of the left-handed neutrinos $\theta_{12}, \theta_{23}, \theta_{13}$, have been measured by neutrino oscillation experiments [1–4]. We use following values [38],

$$\tan^2 \theta_{12} = 0.45, \quad \sin^2 2\theta_{23} = 1.0,$$

and we assume that $\theta_{13}$ and the CP violating phases are zero. For the left-handed neutrino masses, we employ the normal hierarchy, $m_2 < m_3$ and $m_1 = 0$, for simplicity. Then $m_2$ and $m_3$ are determined by two squared mass differences, $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$,

$$\Delta m^2_{21} = 8.0 \times 10^{-5} \text{eV}^2, \quad \Delta m^2_{31} = 2.0 \times 10^{-3} \text{eV}^2.$$  

The undetermined parameters in Eq. (5) are the Majorana masses, $\hat{M}_R$, and the complex angles, $\omega_{1,2,3}$. These variables can not be directly determined by near future experiments, perhaps in the future, and could only be indirectly inferred through LFV experiments in a scenario of supersymmetric seesaw mechanism. Thus, for us, they are free parameters in our model.

As mentioned in Sec. II, the flavour violating entries on the slepton mass matrix are strongly dependent on the complex angles $\omega_{1,2,3}$. In this work, we do not intend to do a

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & point 1 & point 2 & point 3 \\
\hline
$m_0$ (GeV) & 325.1 & 275.9 & 351.9 \\
$M_{1/2}$ (GeV) & 859.7 & 870.6 & 821.0 \\
$A_0$ (GeV) & 664.0 & 161.2 & 857.8 \\
$\tan \beta$ & 36.68 & 31.88 & 38.23 \\
$\delta m$ (GeV) & 1.44 & 0.212 & 0.071 \\
\hline
\end{tabular}
\caption{Reference points of the CMSSM parameters. $\delta m$ is the mass difference between the lightest slepton and the lightest neutralino in absence of flavour mixing.}
\end{table}
FIG. 2: Scatter plots of the lifetime of the slepton as a function of $\text{Br}(\tau \to \mu + \gamma)$ and $\text{Br}(\tau \to e + \gamma)$ for three reference points. Values of the lifetime are indicated at colour bar on the side of figures. Figures (a), (b) and (c) correspond to the point 1, 2 and 3.

full analysis of the possible neutrino Yukawa coupling and only present some representative examples. Hence, for simplicity, we fix the Majorana masses assuming a normal hierarchy $^3$, 

$$M_{R1} = 10^{10} \text{ GeV},$$
$$M_{R2} = 10^{11} \text{ GeV},$$
$$M_{R3} = 10^{12} \text{ GeV},$$

and vary $\omega_{1,2,3}$ in the following ranges,

$$0 \leq \text{Re}(\omega_1), \text{Re}(\omega_2), \text{Re}(\omega_3) \leq 2\pi,$$
$$-\frac{3}{2}\pi \leq \text{Im}(\omega_1), \text{Im}(\omega_2), \text{Im}(\omega_3) \leq \frac{3}{2}\pi.$$ 

$^3$ In fact, we performed a similar analysis taking $M_{R3} = 10^{14} \text{ GeV}$ and we did not find significant differences. Moreover, the assumption of the normal hierarchy for the left-handed and the right-handed neutrino masses is not critical in our discussion. A different hierarchy could give different numerical results for the studied lifetimes and branching ratios but would not change the correlations studying here.
Strictly speaking, the imaginary parts of $\omega_{1,2,3}$ can be taken from $-\infty$ to $+\infty$. However, as can be seen from Eq. (5), if we take large values of the imaginary parts, the induced flavour mixing will easily exceed the present experimental bounds. We have checked that, if we restrict Im($\omega_{1,2,3}$) to the above range, the obtained branching ratios do not exceed the present bounds by a large amount.

In Fig. 2, we show the lightest slepton lifetime, $\tau_{\tilde{l}_1}$, in different colours as a function of the branching ratios $\text{Br}(\tau \rightarrow e + \gamma)$ and $\text{Br}(\tau \rightarrow \mu + \gamma)$ as a scatter plot varying the complex angles in the previously defined. Panels (a), (b) and (c) correspond to the CMSSM points 1, 2 and 3, respectively. In all panels, the experimental bound on the branching ratio $\text{Br}(\mu \rightarrow e + \gamma) < 1.2 \times 10^{-11}$ [38], is imposed and it excludes the right-top corner (white region) on each panel. This is because, in this region, both $g_{1\ell 1}^L$ and $g_{1\mu 1}^L$, or both the corresponding mass insertions $(\delta_{LL}^e)_{\ell \tau}$ and $(\delta_{LL}^\mu)_{\mu \tau}$ are large. Then, even if $(\delta_{LL}^e)_{e \mu}$ is small enough, sizable $\mu \rightarrow e + \gamma$ occurs via a double mass insertion which picks up $(\delta_{LL}^e)_{e \tau}$ and $(\delta_{LL}^\mu)_{\mu \tau}$ on the slepton line in the loop. We can also see, in this figure, that the slepton lifetime in the points 1 and 2 are strongly correlated with $\text{Br}(\tau \rightarrow e + \gamma) + \text{Br}(\tau \rightarrow \mu + \gamma)$ as shown in Eq. (17), while the lifetime in the point 3 depends on only $\text{Br}(\tau \rightarrow e + \gamma)$ as shown in Eq. (18). This is due to the fact that the slepton can decay into $e$ or $\mu$ with the neutralino for $m_{\mu} < \delta m < m_\tau$. Therefore the lifetime depends on both of $g_{1\ell 1}^L$ and $g_{1\mu 1}^L$ on which the branching ratios, $\text{Br}(\tau \rightarrow e + \gamma)$ and $\text{Br}(\tau \rightarrow \mu + \gamma)$, also depend. On the other hand, the slepton can decay only into $e$ and the neutralino for $m_e < \delta m < m_\mu$. Therefore the lifetime depends on only $g_{1\ell 1}^L$ and is correlated with only $\text{Br}(\tau \rightarrow e + \gamma)$. In Figs. 2.(a) and (b), for the same values of the branching ratios, the lifetimes become longer as the mass differences become smaller. This is because the lifetime is inversely proportional to $|\delta m|^2$ as shown in Eq. (10).

From Eq. (17), we can see that the experimental upper bound on the branching ratios implies a lower bound on the lightest slepton lifetime. Due to looser experimental bounds on the branching ratios of the radiative tau decays, the branching ratio of $\mu \rightarrow e + \gamma$ provides the lower bound on the lifetime through the double mass insertions. On the other hand, as mentioned in Sec. III, for very small LFV couplings, the lifetime becomes insensitive to $g_{1\ell 1}^L$ for $g_{1\ell 1}^L \lesssim 10^{-8}$, Eq. (14), and the lifetime is given by decay rates of the flavour conserving 3-body and/or 4-decays. In the case of $\delta m < m_\pi$, the lifetime is constant for $g_{1\ell 1}^L \lesssim 10^{-12}$, Eq. (15), and is determined by decay rate of the flavour conserving 4-body decays. The bounds are summarized in Table II. One can see from this Table II that, in the degenerate slepton-neutralino region, the lifetime of the slepton can change in many orders of magnitude in the presence of LFV couplings. The off-diagonal entries in the left-left part

|        | point 1   | point 2   | point 3   |
|--------|-----------|-----------|-----------|
| upper bound (sec.) | $1.05 \times 10^{-7}$ | $2.85 \times 10^{-1}$ | $3.53 \times 10^4$ |
| lower bound (sec.)  | $7.98 \times 10^{-16}$ | $3.13 \times 10^{-12}$ | $4.22 \times 10^{-12}$ |

TABLE II: Upper and lower bounds on the lifetime of the stau. Upper bounds are calculated from flavour conserving 3- and 4-body decays rates given in [34].
of the slepton mass matrix, Eq. (7), depends on the complex orthogonal matrix, $W$, through the Yukawa coupling of neutrinos. Thus, the lifetime depends on $W$ and change in many orders of magnitude by varying the imaginary parts of $\omega_{1,2,3}$. It is important to emphasize here that, as was discussed in [32], the ATLAS detector could measure the lifetimes of the slepton and therefore determine or constrain the LFV couplings. Thus, in the case of the small $\delta m$, the LHC experiment can provide an opportunity to obtain information on the LFV parameters and hence on the parameters in the Seesaw mechanism.

In Figs. 3 (a) and (b) we plot the branching ratios $\text{Br}(\tau \to e (\mu) + \gamma)$ for the points 1 and 2 selecting values of the slepton lifetimes in the ranges, $\tau_{\tilde{l}} = 10^{-9} - 10^{-8.5}$, $10^{-11} - 10^{-10.5}$ and $10^{-13} - 10^{-12.5}$ sec. All the points in this figure are extracted from Fig. 2 (a) and (b). The solid curves in these panels represent the correlation shown in Eq. (17). We can see that Eq. (17) is in very good agreement with the numerical results for the points 1 and 2, and the branching ratios are predicted once the lifetime, the mass of the slepton and the mass difference are given. These correlations between the LFV branching ratios and the lightest slepton lifetime are summarized in Table III for both points 1 and 2. In fact, the LFV branching ratios can be completely determined from slepton decays using $r_{e\mu}$ which is determined by measuring the slepton decays to the different leptonic flavours. In this figure, dashed lines represent Eq. (20) and the corresponding values of $r_{e\mu}$ are shown near the lines. A large value of $r_{e\mu}$ corresponds to larger branching ratio of $\tau \to e + \gamma$ than that of $\tau \to \mu + \gamma$. For example, for $r_{e\mu} = 100$ and the lifetime between $10^{-8.5} - 10^{-9}$ sec. in Fig. 3 (a), $\text{Br}(\tau \to e + \gamma)$ is roughly between $10^{-17}$ and $5 \times 10^{-17}$ and $\text{Br}(\tau \to \mu + \gamma)$ is between $10^{-15}$ and $5 \times 10^{-15}$.

Similarly, Fig. 4 (a) shows $\text{Br}(\tau \to e + \gamma)$ as a function of the slepton lifetime for the point 3. Now, the solid line represents Eq. (18) with $\delta m = 0.071$ GeV. We can see that $\text{Br}(\tau \to e + \gamma)$ is inversely proportional to the lifetime and Eq. (18) is in very good agreement with the numerical analysis. In this case, we can directly determine $\text{Br}(\tau \to e + \gamma)$ by measuring the lightest slepton lifetime. For instance, a lifetime of $10^{-8.5}$ corresponds to a branching ratio of $4.56 \times 10^{-13}$ and for a lifetime of $10^{-10.5}$, the branching ratio is $4.36 \times 10^{-11}$. We also see in this Figure that there are some points which deviate from Eq. (18). This deviation stems from changes of the mass difference. If the neutrino Yukawa coupling is large, relatively large flavour mixing in the slepton mass matrix is induced through RGE and this reduces the mass of the lightest slepton. In Fig. 4 (b), we show $\text{Br}(\tau \to e + \gamma)$ in terms of the mass difference. It can be seen here, that starting from $\delta m = 0.071$ GeV in the absence of LFV mass insertions, the mass difference can become much smaller when $(\delta_{LL}^{e})_{er}$,

| $\tau_{\tilde{l}}$  | point 1                  | point 2                  |
|-------------------|--------------------------|--------------------------|
| $10^{-9} - 10^{-8.5}$ (sec.) | $(1.23 - 3.90) \times 10^{-15}$ | $(0.580 - 1.84) \times 10^{-13}$ |
| $10^{-11} - 10^{-10.5}$ (sec.) | $(1.23 - 3.90) \times 10^{-13}$ | $(0.580 - 1.84) \times 10^{-11}$ |
| $10^{-13} - 10^{-12.5}$ (sec.) | $(1.23 - 3.90) \times 10^{-11}$ | —                       |

**TABLE III:** Upper bounds on $\text{Br}(\tau \to e (\mu) + \gamma)$ for the points 1 and 2.
FIG. 3: Contour plots of branching ratios of $\tau \rightarrow \mu + \gamma$ and $\tau \rightarrow e + \gamma$, and the lifetime of the slepton for the points 1 and 2. The lifetime of $10^{-9} - 10^{-8.5}$, $10^{-11} - 10^{-10.5}$ and $10^{-13} - 10^{-12.5}$ sec. are shown. Solid curves are drawn by Eq. (17) and corresponding lifetimes are shown near the curves. Dashed lines are drawn by Eq. (20) and values of $r_{e\mu}$ are shown near the lines.

FIG. 4: Plots of the branching ratio of $\tau \rightarrow e + \gamma$ as a function of the slepton lifetime, $\tau_{\tilde{l}_1}$, (a) and as a function of $\delta m$, (b) for the point 3. Solid line in the left panel, (a), shows Eq. (18).

i.e. $\text{Br}(\tau \rightarrow e + \gamma)$, is large. Note that for the point 1 and 2, the change on the slepton mass is negligible because the mass difference is not as small as that for the point 3.

V. SUMMARY AND DISCUSSION

In this work, we have studied several lepton flavour violation observables in the supersymmetric Seesaw scenario in the region of nearly degenerate slepton and neutralino. Within the Seesaw scenario, flavour violating entries in the slepton mass matrix are induced only in the left-left and left-right parts at one-loop order, and LFV is transfered to the right-handed sleptons via left-right part. When the mass difference between the NLSP, the lightest slepton, and the LSP, the lightest neutralino, is smaller than the tau mass, the lifetime of the
lightest slepton is inversely proportional to the LFV entries in the slepton mass matrix.

We have seen in Sec. III, that, in the case of $\delta m < m_\tau$, flavour violating 2-body decay rates of the lightest slepton are proportional to the square of the effective couplings $g_{L(\mu)}^{L(\mu)}$. We have also shown that the branching ratios of $\tau \to e (\mu) + \gamma$ are proportional to the square of the effective couplings $g_{L(\mu)}^{L(\mu)}$. Then, using these results, we derived relations between the lifetime of the slepton and the branching ratios of the LFV tau decays. We also found that the ratio of $\text{Br}(\tau \to \mu + \gamma)$ to $\text{Br}(\tau \to e + \gamma)$ is the same as that of $\text{Br}(\tilde{L}_1 \to \mu + \tilde{\chi}_0^0)$ to $\text{Br}(\tilde{L}_1 \to e + \tilde{\chi}_0^0)$. Using these relations, it is possible to determine the branching ratios of LFV tau decays by measuring the lifetime and the branching ratios of the LFV slepton decays. Note that these relations are valid in any scenarios with $\delta m < m_\tau$ when $(\Delta M_{\text{LL}}^2)_{\alpha\tau} \gg (\Delta M_{\text{RR}}^2)_{\alpha\tau}$ and $\mu \tan \beta \gg m_{\text{SUSY}}$ are satisfied.

Then, in Sec. IV, we have checked these relations numerically by calculating the NLSP lifetime and tau LFV branching ratios by varying the complex phases in $W$. We chose three reference points of the CMSSM parameters which correspond to $\delta m = 1.44$, 0.212 and 0.071 GeV, respectively. As summarized in Table II, the lifetime of the lightest slepton, in this scenario, can be reduced in many orders of magnitude when compared with the flavour conserving case due to the LFV entries. These LFV entries are determined by the complex mixings, $\omega_{1,2,3}$, which can not be determined by low-energy experiments. Fixing the right-handed Majorana masses and varying the complex angles $\omega_{1,2,3}$, the slepton lifetimes can be found between $7.98 \times 10^{-16}$ and $1.05 \times 10^{-7}$ sec. for $\delta m = 1.44$ GeV, between $3.13 \times 10^{-12}$ and $2.85 \times 10^{-11}$ sec. for $\delta m = 0.212$ GeV, and between $4.22 \times 10^{-12}$ and $3.53 \times 10^{4}$ sec. for $\delta m = 0.071$ GeV. As shown in Ref. [32], the ATLAS detector will be able to determine the lifetime between $10^{-12}$ and $10^{-5}$ sec. Therefore, it would be possible to measure very small LFV entries through lightest slepton decays at the LHC experiments. In this way, slepton lifetimes for mass differences $m_\mu < \delta m < m_\tau$ are related with both $\text{Br}(\tau \to e + \gamma)$ and $\text{Br}(\tau \to \mu + \gamma)$, while lifetimes for $m_\mu < \delta m < m_\mu$ are related with only $\text{Br}(\tau \to e + \gamma)$. Thus, by measuring at the LHC the lightest slepton lifetime, the branching ratios to the $e$ and $\mu$ channel and the mass difference, we can predict the branching ratios of the radiative tau decays in the slepton-neutralino coannihilation scenario with the supersymmetric Seesaw mechanism.

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