Testing Einstein’s time dilation under acceleration using Mössbauer spectroscopy

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Abstract

The Einstein time dilation formula was tested in several experiments. Many trials have been conducted to measure the transverse second-order Doppler shift by Mössbauer spectroscopy using a rotating absorber, to test the validity of this formula. Such experiments are also able to test if the time dilation depends only on the velocity of the absorber, as assumed by Einstein’s clock hypothesis, or whether the present centripetal acceleration contributes to the time dilation. We show here that because the experiment requires γ-ray emission and detection slits of finite size, the absorption line is broadened, by geometric longitudinal first-order Doppler shifts immensely. Moreover, the absorption line is non-Lorentzian. We obtain an explicit expression for the absorption line for any angular velocity of the absorber. The analysis of the experimental results in all previous experiments which did not observe the full absorption line itself were wrong and the conclusions doubtful. The only proper experiment was done by Kündig (1963 Phys. Rev. 129 2371), who observed the broadening, but associated it with random vibrations of the absorber. We establish necessary conditions for the successful measurement of a transverse second-order Doppler shift by Mössbauer spectroscopy. We indicate how the results of such an experiment can be used to verify the existence of a Doppler shift due to acceleration and to test the validity of Einstein’s clock hypothesis.

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(Some figures may appear in colour only in the online journal)

1. Introduction

After the discovery of the Mössbauer effect in 1958, quantitative measurements of relativistic time dilation were carried out in the 1960s based on this effect [1–7], and the interest in such measurements lasts to this day [8, 9]. The experiments [1–7] reported full agreement with the time dilation predicted by Einstein’s theory of relativity. In the experiments [1–6], the Mössbauer source was placed at the center of a fast rotating disc and an absorber at the rim of the disc. In the analyses of these experiments, it was assumed that the absorption line of the rotating absorber stays Lorentzian with the same width as at rest, and is shifted only by the time dilation factor.

Based on the generalized principle of relativity and the ensuing symmetry, in [10] it was shown that there are only two possible types of transformations between uniformly accelerated systems. The validity of the clock hypothesis is crucial for determining which one of the two types of transformations is obtained. The clock hypothesis, as stated in [11], maintains that the rate of an accelerated clock is equal to that of a co-moving unaccelerated clock.

If the clock hypothesis is not true, then the transformation is of Lorentz type and implies the existence of a universal maximal acceleration $a_{\text{max}}$ and an additional time dilation due to the acceleration of the clock. By acceleration, we mean the proper acceleration defined [12] as $g = \frac{d^2\mathbf{x}}{dt^2} dr$, where r is the proper time. In this case, it was shown in [13] that a
Doppler-type shift for an accelerated source will be observed. This Doppler-type shift is similar to the Doppler shift due to the velocity of the source. The formulae for this shift are the same as those for the velocity Doppler shift, with $v/c$ replaced by $a/a_m$.

Consider an absorber placed on a disc, rotating with angular velocity $\omega$ at distance $R$ from the center. If this absorber is exposed to radiation of frequency $\nu_0$ through the center of the disc, then, since the velocity of the absorber is perpendicular to the radiation direction, the radiation will undergo a transverse Doppler shift $\nu_0(\omega^2 R^2/2c^2)$ owing to the time dilation of the absorber. If the conjecture about the existence of a shift due to acceleration were true, there will be an additional shift $\nu_0(a/a_m) = \nu_0(\omega^2 R/a_m)$, which is longitudinal, since the acceleration is in the direction of the radiation.

Kündig’s experiment [7] measured the transverse Doppler shift for a rotating disc by means of the Mössbauer effect. In (only) this experiment, the absorption line of the rotating absorber was obtained. This experiment, as reanalyzed by Khomletskii et al [8], showed a significant deviation of about 20% of the shift observed from the one predicted by special relativity. This deviation was explained in [13] by the additional shift due to the acceleration. Moreover, the value of the maximal acceleration $a_m$ was estimated to be about $10^{19}$ m/s$^2$. This value of the maximal acceleration implies that the ratio of the new shift (due to acceleration) to the transversal one due to the velocity is

$$\frac{a/a_m}{v^2/2c^2} = \frac{2c^2}{Ra_m} \approx 0.018/R,$$

where $R$ is the distance ($R = 9.2$ cm in [7]) of the absorber from the center of the disc. Kündig’s experiment was not designed to test the acceleration shift, and the recalculation by Khomletskii et al is not direct. Thus, the corrected Kündig’s result may serve only as an indication of the existence of a maximal acceleration and an estimate of its value.

Note that in other time dilation experiments, the value of $R$ was about 5 cm, implying that the ratio of the new shift to the known one is 35–40%. How was such a deviation not observed in [1–6]? The reason for this is that their analyses did not take into account the broadening of the absorption curves during the rotation, as is explained below.

We show here that the absorption line of a rotating Mössbauer absorber gets broader during the rotation, even in the case when the absorber moves in the perpendicular direction to the radiation. This is due to the fact that the slit for the radiation beam leaving the source and the opening slit of the detector must have a finite width. Therefore, the velocity of the absorber is not perpendicular to all individual rays. Hence, these rays undergo a longitudinal Doppler shift in addition to the expected transversal one. This shift is very significant even for very small slits for the radiation beam and for the detector, and leads to broadened non-Lorentzian absorption lines.

Consider first the case when the absorber moves in the perpendicular direction to the radiation beam, as in figure 1.

Choose the direction of the radiation beam to be in the direction of the $x$-axis, and place the origin at the center of the source and the detector at $L = x_d$. Let $v$ denote the velocity of the absorber. In order for the transversal Doppler shift $v^2/2c^2$ to be observed, the velocity must be at least 100 m/s. A ray leaves the source at the point $(0, y_s)$ and enters the detector at a point $(x_d, y_d)$. The component of the velocity of the absorber in the direction $\mathbf{n}$ of the ray, for $d/L \ll 1$, is $v_\parallel = v(y_d - y_s)/L$, which may be significant even for a very small width of the slits ($2d$ in figure 1). This implies that most rays will also be exposed to a longitudinal Doppler shift and will change completely the shape of the Mössbauer absorption line. One should observe a decrease in the absorption amplitude and a non-Lorentzian broadening of the absorption spectrum.

To analyze the shifts that a ray undergoes in a rotating experiment, consider figure 2.

We choose again the direction of the radiation beam to be in the direction of the $x$-axis. We place the origin at the center of the rotating disc ($2R < L = x_d - x_s$). The radiation leaves the source at the point $(x_s, y_s)$ and enters the detector at a point $(x_d, y_d)$. The equation of the radiation line is

$$y(x_d - x_s) + x(y_s - y_d) = y_s x_d - y_d x_s,$$

and has the direction $\mathbf{n} = (x_d - x_s, y_d - y_s)/\sqrt{(x_d - x_s)^2 + (y_d - y_s)^2}$, and intersects the $y$-axis at the point $B = (0, b)$, where

$$b = \frac{y_s x_d - y_d x_s}{x_d - x_s}.$$
and $b$ obviously may have any value between $-d$ and $d$. Let
us denote the distance of the absorber to the center by $R$ and
denote the intersection point of our ray with the absorber by
$C = (R \cos \phi, R \sin \phi)$. Since this point is on our radiation
line, it satisfies the equation

$$
(x_d - x_s)R \sin \phi + (y_d - y_s)R \cos \phi = y_d x_s - y_s x_d.
$$

The velocity of the absorber at $C$ is $v = R \omega (-\sin \phi, \cos \phi)$, where $\omega$ is the angular velocity of the disc. Thus, from
equation (1), we find that the component of the velocity of the
absorber in the direction of the ray is

$$
v_n = v \cdot n = \omega \frac{y_d x_s - y_s x_d}{\sqrt{(x_d - x_s)^2 + (y_d - y_s)^2}}
= -\omega b \sqrt{\frac{1}{1 + (y_d - y_s)^2/(x_d - x_s)^2}}.
$$

which, for $d/L \ll 1$, is approximately equal to $v_n = -\omega b$, and
even then the velocity $v_n$ is extremely large for $\omega > 10^7$ s$^{-1}$ and
$b \approx 1$ mm, in comparison with the natural full width at half
absorption intensity ($\Gamma = 0.2$ mm s$^{-1}$) of the $^{57}$Fe Mössbauer
absorption single-line spectrum.

A typical absorption curve is obtained by placing the
source on a transducer and moving it with a velocity $v_s$ in
the direction of the $x$-axis. This velocity is of the order of
up to several millimeters per second. For such velocities, we
may assume that the source velocity in the radiation direction
$n$ is approximately equal to $v_s$. The absorption curve $A_0(v_n)$
of an absorber rotating with angular velocity $\omega$ describes the
rate of radiation absorbed as a function of the velocity $v_n$ of
the source. For a thin absorber with no chemical shift, a
typical absorption curve for an absorber at rest is a Lorentzian
function

$$
A_0(v_n) = a \gamma^2 / (v_n^2 + \gamma^2),
$$

of half width $\gamma$, absorption amplitude $a$ and resonance at $v_n = 0$, as the one shown in
figure 3(a) for $\gamma = 0.125$ mm s$^{-1}$.

For a rotating absorber, the total Doppler shift that the ray
undergoes consists of a longitudinal shift due to the relative
velocity of the source and the absorber and a transversal shift
due to the time dilation of the absorber. All together are
equivalent to a longitudinal shift of a source with velocity

$$
\tilde{v}_s = v_s - v_n + \frac{v^2}{2c}.
$$

Since the spread in $v_n$ is increasing with angular velocity, it
contributes significantly to the spread of the observed
absorption Mössbauer line.

Using (2) and (3), the absorption curve of the rotating
absorber will be

$$
A_0(v_n) = \int_A \int \frac{y_d x_s - y_s x_d}{\sqrt{(x_d - x_s)^2 + (y_d - y_s)^2} + R^2 \omega^2} \ dy_s \ dy_d,
$$

where the integral has to be taken over the source emission
intensity at $y_s$ and absorption by the detector at $y_d$. This
curve can be calculated analytically if we assume that the

![Figure 2. Doppler shift with a rotating absorber.](image)

![Figure 3. Transmission line $T_s(v_n)$ for $d = 0.004$ mm and (a) $\omega = 0$, (b) $\omega = 500$ s$^{-1}$, (c) $\omega = 1000$ s$^{-1}$ and (d) $\omega = 3000$ s$^{-1}$.](image)
One must measure the full spectrum of the absorption frequencies: (a) of the non-rotating absorber. This was assumed in the simulated transmission spectra of a Co:Fe source and an measured only in the case when the source and the absorber large enough the absorption line broadens drastically and the same results, shown as the transmission rate spectra line the calculated absorption spectra using equation (b). One can observe clearly the second-order Doppler shift in (d). The spectra resemble those experimentally observed by Kündig. Thus, $T = \int \frac{1}{2} \frac{\partial^2 f}{\partial \omega^2} \text{d} \omega$. Denoting $A(u) = \int f(u+\omega) \text{d} \omega$ and defining a function $f(u)$ such that $f''(u) = A_0(u)$, we obtain

$$A_0(u) = \frac{1}{d^2} \left[ \int_{-d}^{0} f(u+\omega) \text{d} \omega \right] \text{d} \omega + \int_{0}^{d} f''(u+\omega) \text{d} \omega \text{d} \omega = \frac{1}{d^2} \left[ \int_{-d}^{0} f(u+\omega) \text{d} \omega \right] \text{d} \omega + \int_{0}^{d} f''(u+\omega) \text{d} \omega \text{d} \omega.$$ 

Thus,

$$A_0(u) = \frac{f(u-\omega d) - 2f(u) + f(u+\omega d)}{\omega^2 d^2}.$$ 

If $d \to 0$ then $A_0(u) = af''(u)$, implying that the absorption curve of the rotating absorber will be $A_0(v_0) = A_0(v_0 + \frac{R^2 \omega^2}{2c})$, which is a shift of the absorption curve of the non-rotating absorber. This was assumed in the experiments [1–6].

It is obvious that $A_0(u)$ is symmetric and $A_0(v_0)$ is symmetric with respect to $v_0 = -\frac{R^2 \omega^2}{2c}$. The results of the calculated absorption spectra using equation (4), using numerical integration or using the analytic function (5) yield the same results, shown as the transmission rate spectra line $T = 1 - A_0$ in figure 3.

The simulations show that even for small $d$, if $\omega$ becomes large enough the absorption line broadens drastically and even becomes unobservable. These curves have the same type of widening and shift as the absorption curves obtained experimentally in [7].

In the experiments, [1–6, 9], the value of $T(v_0)$ was measured only in the case when the source and the absorber were relatively static, corresponding to $v_0 = 0$. Obviously in their experiments $d > 0$ (they do not specify), and their analysis did not consider the changes in shape of the absorption line as a function of $\omega$. Thus we cannot rely on their conclusions.

If the distributions of source emission intensity is $\xi$ dependent and the detection is $\eta$ dependent, but symmetric in the interval $[-d, d]$, one will still observe under the rotation a symmetric broadening. If the distribution is not symmetric and has non-zero average, we will get an additional shift, but such a shift can be compensated for by reversing the direction of the rotation.

2. Conclusions

In order to measure the transverse Doppler shift by conventional Mössbauer spectroscopy with the source mounted on a static transducer and the absorber moving on a fast rotating disc, one should take care of the following:

(a) One must measure the full spectrum of the absorption line.

(b) One must measure this spectrum in both angular directions of the absorber. But most importantly:

(c) One must put collimators on the source and the detector to reduce the slits to minimal dimensions, which will still allow a spectrum measurement in a reasonable time span.

Only the experiment of Kündig [7] obeyed at least (a) and (b). In his experiment, he used a source of $^{57}$Co : Fe against a $^{57}$Fe-enriched iron absorber. Thus both were almost identical and in both the iron nuclei were exposed to a strong magnetic field (33 T), causing the emission and absorption spectra to be composed of six line patterns, which led to the observed multiline spectra as shown in figure 4 (bottom), for various rotational frequencies of the absorber. Since Kündig scanned the spectra in the range of about $-1$ to $+1$ mm s$^{-1}$, he observed only the central line with reduced intensity and line broadening, as the lines simulated in figure 4 (middle and top). Thus, he actually observed the line broadening due to the finite slits, but interpreted them as due to rotor vibrations affecting the absorber. Nevertheless, this experiment seems to be still the most accurate transverse Doppler shift experiment using the $^{57}$Fe Mössbauer effect. His results from the corrected analysis reported in [8] indicate that the observed shift in this experiment is larger and deviates $\approx 20\%$ from that expected by special relativity theory.

The Einstein time dilation formula was verified by several experiments [14–17]. In the Kündig experiment the absorber was exposed to significant acceleration. In [13] it was indicated that the deviation of the shift in the experiment may be due to a longitudinal Doppler shift caused by the acceleration of the absorber. It led to an estimate of the universal maximal acceleration of the of order $10^{19}$ m s$^{-2}$ and an indication of the non-validity of the Einstein clock hypothesis [11]. For a long time, Mashhoon argued against the clock hypothesis and developed non-local transformations for accelerated observers (see the review article [18]).

Note that the time dilation Mössbauer effect experiments have an advantage in identifying a relatively small shift
due to acceleration, since in these experiments the shift due to the velocity is of second order, while the shift due the acceleration is of first order. Thus, we recommend Mössbauer spectroscopy scientists to perform an accurate experiment measuring the shift of the absorption line for a fast rotating absorber, which may reveal a new fundamental law [13], with a monumental effect on the whole of physics.

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