Multi-layer Random Walks Synchronization for Multi-attributed Multiple Graph Matching

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Abstract

Many applications in computer vision can be formulated as a multiple graph matching problem that finds global correspondences across a bunch of data. To solve this problem, matching consistency should be carefully considered with matching accuracy to prevent conflicts between graph pairs.

In this paper, we aim to solve a multiple graph matching problem in complicated environments by using multiple attributes that are represented in a set of multi-layer structures. The main contribution of this paper is twofold. First, we formulate the global correspondence problem of multi-attributed graphs using multiple layered structures. The formulation is derived by aggregating the multi-layer structures that describe individual pairwise matching problems respectively. Second, we solve the global correspondence problem by using a novel multi-attributed multiple graph matching method that is based on the multi-layer random walks framework. The proposed framework contains additional synchronization steps to lead random walkers to consistent matching candidates. In our extensive experiments, the proposed method exhibited robust and accurate performance over the state-of-the-art algorithms.

1. Introduction

Graph matching is a problem of finding correspondences between two sets of vertices while preserving complex relational information among them. Since the graph structure has a strong capacity to represent objects and robustness to severe deformation and outliers, it is frequently adopted to formulate various correspondence problems in the field of computer vision research [7, 32]. Theoretically, the graph matching problem can be solved by exhaustively searching the entire solution space, but this approach is infeasible in a practical environment because the space expands exponentially with the size of input data. For that reason, previous studies have attempted to solve the problem by using various approximation techniques [5, 6, 8, 10, 13, 14, 15, 26, 33, 34]. These methods provide satisfactory performance in terms of computational complexity at the expense of some accuracy. This exchange between accuracy and complexity is not a serious problem in pairwise graph matching scenarios. However, this can be a crucial problem in multiple graph matching scenarios that find globally consistent correspondences; because fundamental errors due to the approximation can accumulate and conflicts can occur between matching pairs. Therefore, the consistency and accuracy of matching pairs should be considered together to solve multiple graph matching problems.

Conventional multiple graph matching algorithms could be categorized into two types according to the improvement scheme of matching consistency [32]: iterative and one-shot methods. The iterative methods try to enhance the consistency by iteratively updating a set of initial solutions. Yan et al. [29, 30] revised the multiple graph matching formulation to match the graphs jointly, and proposed an iterative framework for its optimization. At the first step, a reference graph is selected, and then each of the graph pairs is replaced with two pairs that bypass through the reference graph. Consequently, the matched vertex pairs automatically satisfy the consistency constraint thanks to the bypass substitution. Yan et al. [27, 28] also proposed a more flexible algorithm that gradually improves consistency over iteration. Park and Yoon [20] proposed an iterative method that encourages the second-order consistency and relaxes the hard consistency constraints. Since these approaches sequentially update solutions from one graph pair to others, the final results are sensitive to the update sequence, and this often causes the error accumulation.

On the other hand, the one-shot approaches directly attempt to achieve overall consistency of whole graph pairs. Bonev et al. [1] proposed a two-step algorithm to filter out inconsistent matches. The algorithm finds all possible vertex isomorphisms between whole graph pairs first, and then iteratively removes the inconsistent matches that violate the cycle-consistency constraint. Solé-Ribalta and Ser-
ratosa [24, 25] proposed a more efficient two-step algorithm that uses an N-dimensional probabilistic matrix called a hypercube. Kim et al. [12] used a fuzzy correspondence function for pairwise matching, before filtering noisy matches by thresholding the consistency scores. These methods can achieve consistency among the multiple graphs by filtering inconsistent matches based on the cycle-consistency constraint. However, since the consistency measure is only applied for a post-processing, these methods cannot rectify inaccurate pairwise matching results during the matching process.

To overcome this limitation, recent algorithms [3, 17, 19, 31, 35] modify the initial solution to consider other pairs for improving the matching consistency. Pachauri et al. [19] formulated a global correspondence problem as a problem of finding projections to a universal graph. Then, the projections are synchronized by using spectral methods. Maset et al. [17] have improved the algorithm robustly to the estimation error of the universal graph size. Chen et al. [3] and Zhou et al. [35] presented different approaches for solving the projection problem by using a semidefinite and low-rank relaxation respectively. Although these algorithms can modify the initial solution to adapt the globally consistent result, still be influenced by the quality of input data. As an answer, Yan et al. [31] presented a novel formulation based on robust principle component analysis (RPCA) [2] to consider affinity information during the synchronization process. By adopting rich information of affinity values, they tried to mitigate the limitation of initial data dependency.

Recently, Park and Yoon [21, 22] reported an issue of attribute integration which is related to representability of attributes under multi-attributed graph matching settings. Many graph matching applications [4, 5, 8, 13, 34] have used multiple attributes and derived a mixed-type attribute by integration. This is because single-type attributes generally do not have enough representability to reflect complex properties of image contents. However, this integration approach have fundamental problems as stated in [21, 22]. First, the distinctive information from multiple attributes can be distorted as a consequence of the attribute integration. Since each attribute can have different representations even for the same object, the distinctive and rich characteristics of one attribute can be weakened by other attributes during the integration process. Second, the integration recipe cannot be updated once it is determined. This can decrease the flexibility of the graph matching algorithm, because integrated attributes cannot be decomposed again and re-customized for different environments. To address this issue, Park and Yoon [21, 22] proposed a multi-layer structure that jointly represents multiple attributes while preserving its unique characteristics. This structure describes each attribute in a separated layer, and connects the layers by using inter-layer edges. By manipulating these inter-layer edges, the relation between attributes can be re-defined during the matching process. They also proposed a multi-attributed graph matching algorithm based on the multi-layer structure [21, 22], and demonstrated the robust performance over state-of-the-art algorithms.

In this paper, we propose a robust multiple graph matching algorithm that considers multiple attributes to deal with more general situations in practical environments. The proposed method jointly considers affinity information of multiple attributes by using several multi-layer association graphs to preserve distinctive information of each attribute; moreover, their relations can be manipulated adaptively during the synchronization process. The main contribution of this paper is twofold. First, we formulate the global correspondence problem of multi-attributed graphs using multi-layered structures. The proposed formulation is derived by aggregating the multi-layer structures that describe individual pairwise matching problems respectively as shown in Fig. 1. To the best of our knowledge, this is the first attempt to solve the global correspondence problem of multi-attributed graphs using the multi-layer structure. Second, we solve the global correspondence problem by using a novel multi-attributed multiple graph matching method that is based on the multi-layer random walks framework. The proposed method contains additional synchronization steps to lead random walkers to consistent matching candidates, which is the main difference between the conventional multi-layer random walks method [21] and our algorithm.

2. Problem formulation

2.1. Pairwise graph matching problems

Suppose two attributed graphs $G_1 (V_1, E_1, A_1)$ and $G_2 (V_2, E_2, A_2)$ are given, where $V$ and $E$ represent a set of vertices and a set of edges respectively, and $A$ is a set of attributes which express each vertex and edge. Then, a correspondence problem between $G_1$ and $G_2$ can be formulated as a Lawler’s quadratic assignment problem (QAP) [5, 8, 13, 21, 22, 34]. In this formulation, the correspondences are represented using an $(N_1 \times N_2)$-dimensional binary assignment matrix $X$, where $N_1$ and $N_2$ represent the number of vertices in each graph. Each element in the assignment matrix $[X]_{ia} \in \mathbb{R}$ represents a correspondence of two vertices $v_{(1)i} \in V_1$ and $v_{(2)a} \in V_2$; if two vertices are matched then the corresponding element is set to 1, otherwise set to 0. The affinity values of correspondences are computed by using the vertex and edge attributes, and are represented by using an $(N_1N_2 \times N_1N_2)$-dimensional affinity matrix $K$, which is called an affinity matrix. For example, a unary affinity of a correspondence between $v_{(1)i}$ and $v_{(2)a}$ is represented in its diagonal element $[K]_{ia,ia}$, and a pairwise affinity of two matching candidates $(v_{(1)i}, v_{(2)a})$ and $(v_{(1)j}, v_{(2)b})$ is...
represented in a non-diagonal element $[K]_{ia,jb}$. Then, the pairwise graph matching problem is formulated as follows:

$$\hat{X} = \arg \max_X \text{vec}(X)^\top K \text{vec}(X),$$  
\[\text{s.t. } X \in \{0, 1\}^{N_1 \times N_2}, X1_{N_2} \leq 1_{N_1}, X^\top 1_{N_1} \leq 1_{N_2},\]  
(1)

where $1_{N_i}$ indicates an $N_i$-dimensional all-ones vector, and the inequality constraints denote the one-to-one matching constraints. As mentioned above, this formulation has the integration issue when using multiple attributes.

To resolve this issue, Park and Yoon [21, 22] proposed a problem formulation to deal with multi-attributed graphs by using a multi-layer association graph. The association graph $G_h$ is represented as a set $\{V_h, E_h, A_h, L_h\}$, where $L_h$ is a set of layer indices. $G_h$ consists of multiple layers which represent multiple attributes respectively, and the layers are linked to each other as shown in Fig. 1. Since each vertex $v_{(k)ia} \in V_h$ has a layer index $\alpha \in L_h$, vertices on different layers, $v_{(k)ia}$ and $v_{(k)ja}$, can be distinguished from each other even if they have the same vertex index. Accordingly, each edge $e_{(k)ia,jb} \in E_h$ should have two layer indices.

The edges can be classified into two types: intra- and inter-layer edges. The intra-layer edge indicates an edge that has the same layer indices such as $e_{(k)ia,jb}$, and the inter-layer edge indicates an edge that has different layer indices such as $e_{(k)ja,ia}$. Therefore, a four-dimensional affinity tensor $\Pi$ is required to describe these two types of relations [21, 22], and $\Pi$ can be flattened to a two-dimensional block affinity matrix $P$, which is called a supra-adjacency matrix [9, 11, 21, 22, 23]. Then, a multi-layer graph matching problem can be formulated by using $P$ as follows:

$$\hat{X} = \arg \max_X \sum_{l,m=1}^{N_G} (L_{lm} \otimes \text{vec}(X_{lm}))^\top (P_{lm} \otimes \text{vec}(X_{lm})),$$  
\[\text{s.t. } X_{lm} \in \{0,1\}^{N_l \times N_m}, L_{lm} \in \{0,1\}^{N_l}, X_{lm}1_{N_m} \leq 1_{N_l}, X_{lm}^\top 1_{N_l} \leq 1_{N_m},\]  
(2)

where, $N_G$ is the number of graphs, and $X'$ represents a collection of the individual assignment matrices, $l$ and $m$ are graph indices satisfying $l < m$. Since this problem formulation also has the attribute integration issue, we generalize this formulation to a problem of multi-layer graph matching in order to utilize the rich information of attributes. Similar to the pairwise graph matching problem, the generalized problem can be formulated as a summation of the individual multi-layer pairwise matching problems as follows:

$$\hat{X} = \arg \max_{X'} \sum_{l,m=1}^{N_G} (L_{lm}^c \otimes \text{vec}(X_{lm}))^\top (P_{lm}^c \otimes \text{vec}(X_{lm})),$$  
\[\text{s.t. } X_{lm} \in \{0,1\}^{N_l \times N_m}, L_{lm} \in \{0,1\}^{N_l}, X_{lm}1_{N_m} \leq 1_{N_l}, X_{lm}^\top 1_{N_l} \leq 1_{N_m},\]  
(3)

2.2. Multiple Graph Matching Problems

The multiple graph matching problem can be formulated as a summation of the individual pairwise matching problems as follows [20, 29]:

$$\hat{X} = \arg \max_{X'} \sum_{l=1}^{N_G} \sum_{m,l=1}^{N_G} (X_{lm}^c)^\top (K_{lm}^c) \text{vec}(X_{lm}),$$  
\[\text{s.t. } X_{lm} \in \{0,1\}^{N_l \times N_m}, L_{lm} \in \{0,1\}^{N_l}, X_{lm}1_{N_m} \leq 1_{N_l}, X_{lm}^\top 1_{N_l} \leq 1_{N_m},\]  
(4)

where $N_G$ is the number of graphs, and $X'$ represents a collection of the individual assignment matrices, $l$ and $m$ are graph indices satisfying $l < m$. Since this problem formulation also has the attribute integration issue, we generalize this formulation to a problem of multi-layer graph matching in order to utilize the rich information of attributes. Similar to the pairwise graph matching problem, the generalized problem can be formulated as a summation of the individual multi-layer pairwise matching problems as follows:

3. Proposed Algorithm

To solve the problem in Eq. (4), we propose a multiple graph matching algorithm based on multi-layer random walks. Multi-layer random walks matching algorithm (MLRWM) [21] is a generalized reweighted random walk matching algorithm (RRWM) [5] to solve the multi-attributed graph matching problems. In the multi-layer random walks framework, random walkers randomly traverse the multi-layer association graph according to the predefined transition probability. Since the traversal does not con-
Algorithm 1 Multi-layer Random Walk Synchronization

**Input:** Supra-adjacency matrices \( \{ \mathbf{P}_{lm} \}_{l,m=1,\ldots,N_G} \), inflation factor \( \rho \), reweighting factor \( \theta \), reweight synchronizing factor \( \omega \), minimum layer confidence value \( \tau \), confidence synchronizing factor \( \mu \)

**Output:** Assignment matrices \( \hat{X} \)

1: Initialize \( \forall l, m \), \( t_{lm} \), \( \hat{X}^{\text{sync}}_{lm} \), \( \mathbf{P}_{lm} \)
2: (Bootstrap process)
3: Perform multi-layer random walks for each graph pair \([21]\)
4: repeat
5: for \( l, m = 1 \) to \( N_G \)
6: (Calculate the next distribution)
7: \( t_{lm} \leftarrow t_{lm}^\top \mathbf{P}_{lm} \)
8: for \( \alpha = 1 \) to \( N_L \)
9: (Reweighting random walks for each layer)
10: \( u_{lm}^\alpha \leftarrow \exp(\rho \cdot t_{lm}^\top / \max(t_{lm}^\top)) \)
11: Bistochastic normalize \( u_{lm}^\alpha \)
12: (Compute Layer-confidence value)
13: \( [s_{lm}]_\alpha \leftarrow C_{\text{layer}}(\alpha) \)
14: end
15: Normalize the layer confidence \( s_{lm} \) into \([\tau, 1]\)
16: (Layer confidence synchronization)
17: \( s_{lm} \leftarrow (1 - \mu) s_{lm} + \mu \, s_{\text{sync}}^{\text{layer}} \)
18: (Aggregate reweighted distribution)
19: \( u_{lm}^{\text{sync}} \leftarrow \sum_{\alpha} [s_{lm}]_\alpha \, u_{lm}^\alpha \)
20: (Reweight synchronization)
21: \( u_{lm}^{\text{sync}} \leftarrow (1 - \omega) u_{lm}^{\text{sync}} + \omega \, u_{lm}^{\text{sync}} \)
22: (Diffuse reweighted distribution to whole layer)
23: \( \forall \alpha \, u_{lm}^\alpha \leftarrow u_{lm}^{\text{sync}} \)
24: (Update information of reweighted jump)
25: \( t_{lm} \leftarrow \theta t_{lm} + (1 - \theta) u_{lm} \)
26: end
27: (Layer confidence and reweight synchronization)
28: Construct synchronized reweight vectors \( u_{lm}^{\text{sync}} \)
29: Construct synchronized the layer confidence \( s_{\text{sync}}^{\text{layer}} \)
30: until \( t \) converges
31: (Integrate the assignment vector)
32: \( \text{vec}(X) \leftarrow \sum_{lm} t_{lm} \)
33: Discretize the assignment matrix \( \hat{X} \)

consider one-to-one matching constraints, the reweighting process is required, which leads random walkers to move into desired matching candidates. MLRWMS uses this reweighting process to not only encourage the one-to-one matching constraint, but also to control relative confidences of layers. The main idea of the proposed algorithm is to achieve the consistency of the matching solution by leading random walkers to consistent matching candidates through the reweighting process.

The proposed algorithm is mainly based on the pairwise multi-layer random walks matching method \([21]\) (please refer \([21]\) for details about the multi-layer random walk process), but there are three differences that are incorporated to consider the consistency of whole matching pairs as represented in Algorithm 1. First, we incorporate a reweighting jump synchronization step into the matching process. This step can allow to lead random walkers to consistent matching candidates during the matching process. Second, we also add a layer confidence synchronization step that synchronizes the values by suppressing the inconsistent confidence values. Last, a bootstrap process at the beginning of the random walk sequence. This process stabilizes the distribution of random walkers before starting the synchronization, thus preventing the entire matching process from falling into the local minima. Details of these steps are introduced in following subsections.

3.1. Reweighting jump synchronization

Suppose all vertices in given graphs belong to a reference graph \( G_{\text{ref}} \), and a permutation matrix \( \mathbf{P}_l \in \{0, 1\}^{N_G \times N_{\text{ref}}} \) represents projections from a graph \( G_l \) to the reference graph \( G_{\text{ref}} \). Then, each assignment matrix can be obtained by multiplying two permutation matrices as follows:

\[
X_{lm} = \mathbf{U}_l \mathbf{U}_m^\top, \quad (5)
\]

By using Eq. (5), whole assignment and permutation matrices can be represented as follows:

\[
\mathbf{X} = \mathbf{U}\mathbf{U}^\top, \quad (6)
\]

where the matrix \( \mathbf{X} \) has rank \( N_{\text{ref}} \), and is a symmetric positive semidefinite matrix. Suppose \( \bar{\mathbf{X}} \) is a block assignment matrix that is estimated from the input data. Then, the permutation synchronization problem can be formulated as follows:

\[
\bar{\mathbf{X}} = \arg \max_{\mathbf{X}} \langle \bar{\mathbf{X}}, \mathbf{X} \rangle
\]

\[
\iff \quad \bar{\mathbf{U}} = \arg \max_{\mathbf{U}} \langle \bar{\mathbf{X}}, \mathbf{U}\mathbf{U}^\top \rangle
\]

\[
= \arg \max_{\mathbf{U}} tr \left( \mathbf{U}\bar{\mathbf{X}}\mathbf{U}^\top \right),
\]

\[
s.t. \quad \mathbf{U}^\top\mathbf{U} = \mathbf{I}_{N_G}, \quad \mathbf{X} = \mathbf{U}\mathbf{U}^\top.
\]

Pachauri et al.[19] proposed a spectral method to solve Eq. (7) by relaxing \( \mathbf{U} \) to a continuous matrix. Then, Eq. (7) can be considered as a generalized Rayleigh problem, and its solution can be obtained by extracting \( N_G \) leading eigenvectors of \( \bar{\mathbf{X}} \). This spectral method, which is called Match-Sync, can solve the permutation synchronization problem in one shot. However, since the method tries to find actual permutations, the size of the reference graph \( N_{\text{ref}} \) should be correctly estimated. To address this limitation, Maset et al.[17] proposed a modified algorithm that directly synchronizes pairwise assignment matrices \( \mathbf{X} \). This method, which
is called MatchEIG, only needs to know the minimum size of the reference graph to ensure proper performance while MatchSync needs to know the exact size. At the first step of MatchEIG, \( N_0 \) leading eigenvectors of \( \hat{X} \) integrated to construct an approximated permutation \( \hat{U} \), and corresponding eigenvalues are collected to construct a diagonal matrix \( D \). Then, the synchronized block assignment matrix \( \hat{X} \) can be obtained as follows:

\[
\hat{X} = \hat{U} D \hat{U}^T.
\]  

(8)

Since \( \hat{X} \) is a continuous matrix, a discretization method, such as Hungarian method [18], should be adopted to obtain a binary solution.

By using this permutation synchronization method, MatchEIG [17], the proposed method synchronizes the reweighting vectors of the current iteration. First, a block assignment matrix \( \hat{X} \) is constructed by integrating the reweighting vectors of whole pairs, and then inserted as the input of MatchEIG. Finally, the synchronized reweighting vectors can be obtained by dividing \( \hat{X} \), and combined with the reweighting vectors of each matching pair.

### 3.2. Layer confidence synchronization

The layer confidence value of each matching pair is defined by computing the gap between clusters of true/false correspondences [21]. According to the assumption that true correspondences have stronger connections with each other than false correspondences [13], the average affinity values of true/false correspondences should be larger that the value of false correspondences. In accordance with this criterion, we define a layer confidence measure using the difference between two mean affinity values of true/false correspondences. However, the true/false correspondences cannot be classified during the matching process. For that reason, we use a discretized reweighting vector of current iteration as an indicator vector of true correspondences. Then, the layer confidence values can be computed as follows:

\[
C_{\text{layer}}(\alpha_{lm}) = \frac{y_{lm}^T P_{lm}^{\alpha_{lm}} y_{lm} - \bar{y}_{lm}^T \bar{P}_{lm}^{\alpha_{lm}} \bar{y}_{lm}}{2 \cdot \text{Std}(P_{lm}^{\alpha_{lm}})}
\]  

(9)

\[s.t. \quad y_{lm} = \text{Hungarian}(u_{lm}^\alpha),\]

where \( u_{lm} \) is a reweighting vector of a matching pair \((G_l, G_m)\), and \( \text{Std}(P_{lm}^{\alpha_{lm}}) \) is a standard deviation of \( P_{lm}^{\alpha_{lm}} \) that is adopted to adjust the scale of each attribute. \( y_{lm} \) is a discretized reweighting distribution that represents the true correspondences, and \( \bar{y}_{lm} \) is the binary complement vector of \( y_{lm} \). Based on this measure, the layer confidence vector \( s_{lm} \) is constructed by normalizing the values as follows:

\[
[s_{lm}] = (1 - \tau) \frac{C_{\text{layer}}(\alpha_{lm})}{C_{\text{max}}} + \tau,
\]  

(10)

\[s.t. \quad C_{\text{max}} = \max_{\alpha} C_{\text{layer}}(\alpha_{lm}),\]

where \( \tau \) is a minimum confidence value to ensure the small leverage for each layer even if it is considered as the most unreliable layer.

In this layer confidence synchronization step, the layer confidence vectors of all pairs are averaged to construct a synchronized confidence vector \( s_{\text{sync}} \) as follows:

\[
s_{\text{sync}} = \frac{1}{N_0 C_2} \sum_{l,m=1}^{N_0} s_{lm}
\]  

(11)

Then, the synchronized vector \( s_{\text{sync}} \) is combined with the layer confidence vector of each matching pair \( s_{lm} \).

### 3.3. Bootstrapping

Since the distribution of random walkers is often unstable at the initial stage of the process, the random walkers can be biased towards unreliable matching candidates if the synchronization step is performed based on the unstable information. For that reason, we insert a bootstrap process at the initial stage. The process is the multi-layer random walks process, but does not synchronize the reweighting vectors and layer confidence values. By waiting the random walks distribution until being stabilized through the bootstrap process, it can prevent the entire matching process from falling into the local minima.

### 4. Experimental results

To evaluate the proposed algorithm, we perform two experiments using the synthetic and WILLOW dataset [4]. We compare our algorithm with the well-known pairwise graph matching algorithms such as reweighted random walks matching (RRWM) [5], max pooling matching (MPM) [6], and multi-layer random walk matching (MLRWM) [21], and multiple graph matching algorithms such as MatchOpt (MOpt) [29, 30], MatchSync (MSync) [19], Composition based affinity optimization (CAO) [28, 27], MatchLift (MLift) [3], MatchALS (MALS) [35], and MatchEIG (MEIG) [17].

In all experiments, we fix the reweighting factor \( \theta \) as 0.2, the minimum layer confidence value \( \tau \) as 0.1, the reweight synchronizing factor \( \omega \) as 0.8, and the layer confidence synchronizing factor \( \mu \) as 0.8. The inflation factor \( \rho \) is set to 30 for all datasets, and each experiment is iteratively performed 50 times. The parameters of compared algorithms are selected as provided in the original papers, and RRWM is adopted as a pairwise solver to generate the input matching results for multiple graph matching algorithms that require initial solutions. Our evaluation framework is based on the open MATLAB programs of [5] and [21]; and the other matching algorithms are used from the authors’ open source codes.

1[http://www.di.ens.fr/willow/research/graphlearning/]
4.1. Performance evaluation for synthetic dataset

In this experiment, we evaluate our algorithm for synthetic graph matching problems. To generate a set of synthetic graphs, we follow the experimental scheme that was represented in [21]. First, we construct a reference graph $G_0$ that has several types of attributes $A_0$. Each attribute in $A_0$ is defined by assigning randomly generated values with different variance values which reflect its distinctive information. Then, we construct each graph $G_m$ by adding randomly defined outlier vertices $v_{out}$ and attribute deformation that follows a Gaussian distribution $N(0,\sigma^2)$. Finally, each affinity value in an inter-layer affinity matrix $[P_{lm}^{\alpha,\beta}]_{ia,jb}$ of a graph pair $(G_l, G_m)$ is defined as follows:

$$[P_{lm}^{\alpha,\beta}]_{ia,jb} = \exp(-|(1 - \beta) + \beta(a_{(l)ij} - a_{(m)ab})^2/\sigma^2|),$$

(12)

where $a_{(l)ij}$ and $a_{(m)ab}$ are randomly assigned attribute values, and $\sigma$ is a scaling factor. $\beta$ is a control parameter that determines the variance of attributes, and is randomly selected in the interval $[0.1, 1]$. To utilize multiple attributes for single-layer graph matching algorithms, we first normalize the affinity matrices individually, and then aggregate the matrices. By normalizing the affinity matrices before integration, it could moderate the negative effects caused by different scales of multiple attributes.

Then, we performed three experiments: deformation, outlier, and graph set size. In the first experiment, each graph is generated with different magnitudes of deformation $\epsilon$, but other parameters are fixed. On the other hand, in the outlier experiment, only the number of outliers $N_{out}$ is changed. In the graph set size experiment, the number of graphs is increased while other parameters are fixed. The details about parameter settings are presented in Table 1.

The proposed method (‘MLSync-multi’) presented the very robust performance to the attribute deformation and outliers than most of other compared algorithms as shown in Fig. 2. In particular, our algorithm showed improved performance than MLRWM when a set of graphs becomes larger, which reflects that the proposed concept of synchronization works well as we intended. Exceptionally, ‘MPM-integrated’ shows better performance than MLSync in the outlier experiment, this is because MPM is extremely robust to outliers under the low attribute deformation setting. However, MPM showed relatively low performance

Figure 2. Synthetic graph matching results. Columns: each type of experiments – deformation, outlier, and graph set size. Rows: number of attributes – five and ten attributes.

Table 1. Parameter setting for the synthetic graph matching experiments

| Experiments     | Varied parameter | Fixed parameters                      |
|-----------------|------------------|---------------------------------------|
| Deformation     | $\epsilon = 0 - 0.3$ | $N_G = 10$, $N_{att} = 5$, $N_{rn} = 10$, $N_{out} = 2$, $\sigma^2 = 0.3$ |
| Outlier         | $N_{out} = 0 - 10$  | $N_G = 10$, $N_{att} = 5$, $N_{rn} = 10$, $\epsilon = 0.1$, $\sigma^2 = 0.3$ |
| Graph set size  | $N_G = 4 - 20$     | $N_{att} = 5, 10$, $N_{rn} = 10$, $N_{out} = 2$, $\epsilon = 0.1$, $\sigma^2 = 0.3$ |
for matching severely deformed graphs while the proposed algorithm still showed the robust performance.

4.2. Performance evaluation of WILLOW dataset

To define multi-attributed multiple graph matching problems, we use nine attributes as represented in Table 2. Th attributes are roughly classified into two types: appearance and geometric attributes. The appearance attributes are defined using a SIFT descriptor [16] or an RGB color histogram. Each prefix represents the style of description. For instance, the 'Pairwise' SIFT descriptor (PSiD) implies that each edge attribute is obtained by calculating the difference between the SIFT descriptors of two interest points. Similarly, the 'Concatenated' color histogram difference (CCoD) means that each attribute is represented by concatenating two SIFT descriptors. The last type of attributes, such as 'Unary' SIFT descriptor difference (USiD), means that each vertex attribute is defined by using a descriptor of each interest point; for this reason, the affinity matrix of this type has only unary affinity values (a diagonal matrix). On the other hand, the geometric attributes are obtained from the HARG [4]. The HARG describes each pair of points by using relative distance and angle histograms. Thus, we define the attributes, relative distance histogram difference (RDHD) and relative angle histogram difference (RAHD) by decomposing the HARG histogram. Then, similar with the synthetic graph matching experiments in Sec. 4.1, we describe each affinity value in a intra-layer affinity matrix $P_{lm \mid l,a,j;b}^{\alpha}$ of a graph pair $(G_l, G_m)$ is defined as follows:

$$
|P_{lm \mid l,a,j;b}^{\alpha}| = \exp(-|a_{(l)ij}^{\alpha} - a_{(m)ab}^{\alpha}|^2/\sigma^2),
$$

(13)

where $a_{(l)ij}^{\alpha}$ and $a_{(m)ab}^{\alpha}$ are attribute values obtained by following the definitions in Table 2, and $\sigma$ is a scaling factor. To construct an integrated attributes, we also normalize the affinity matrices individually, and then aggregated the matrices.

To evaluate the proposed algorithm, we performed two experiments: outlier and graph set size. The outlier experiments change only the number of outliers $N_{out}$ as a control parameter, but this experiment actually evaluates the overall performance of the robustness. Since the attribute deformation is not controllable in the real image dataset, unlike the synthetic graph matching experiments, each test case must contain arbitrary attribute deformations. In the graph set size experiments, the number of graphs is varied while other parameters are fixed. The details about parameter settings are shown in Table 3.

As shown in Fig. 3 and Fig. 4, the proposed algorithm exhibits robust and accurate performance in most categories. Especially, the proposed algorithm outperforms MLRWM in all experiments, as in synthetic graph matching experiments. However, our algorithm shows relatively low accuracy in the face category, but it is still comparable to other algorithms. As stated in the previous research [21], it is because the layer confidence measure is likely to fail to estimate correct confidence values more frequently than other categories, even though the proposed algorithm synchronizes the confidence values. We expect that the confidence measure could be improved by using more sophisticated methods such as machine learning methods. We remains this issue as a future work.

5. Conclusion

In this paper, we proposed a novel multi-attributed multiple graph matching algorithm based on the multi-layer random walks synchronization. The algorithm aims to solve multiple graph matching problems in complicated environments by using multiple attributes that are represented in a set of multi-layer structures. To improve the matching consistency among graphs, we proposed a random walks synchronization process which leads random walkers to consistent matching candidates. In our extensive experiments, the proposed algorithm exhibits robust and accurate performances over the state-of-the-art algorithms.
Table 3. Parameter setting for the real image graph matching experiments

| Experiments | Varied parameter | Fixed parameters |
|-------------|------------------|------------------|
| Outlier     | $N_{out}$ = 0 – 10 | $N_G$ = 10, $N_{att}$ = 9, $N_{in}$ = 10, $\sigma^2 = 0.3$ |
| Graph set size | $N_G$ = 4 – 20 | $N_{att}$ = 9, $N_{in}$ = 10, $N_{out}$ = 2, $\sigma^2 = 0.3$ |

Figure 3. Performance evaluation on WILLOW dataset (Varying the number of outliers).

Figure 4. Performance evaluation on WILLOW dataset (Varying the number of graphs).

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