Halo mass functions in early dark energy cosmologies

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ABSTRACT

We examine the linear density contrast at collapse time, \( \delta_c \), for large-scale structure in dynamical dark energy cosmologies, including models with early dark energy. Contrary to previous results, we find that as long as dark energy is homogeneous on small scales, \( \delta_c \) is insensitive to dark energy properties for parameter values fitting current data, including the case of early dark energy. This is significant since using the correct \( \delta_c \) is crucial for accurate Press–Schechter prediction of the halo mass function. Previous results have found an apparent failing of the extended Press–Schechter approach (Sheth–Tormen) for early dark energy. Our calculations demonstrate that with the correct \( \delta_c \), the accuracy of this approach is restored. We discuss the significance of this result for the halo mass function and examine what dark energy physics would be needed to cause significant change in \( \delta_c \), and the observational signatures this would leave.

Key words: methods: numerical – large-scale structure of Universe.

1 INTRODUCTION

Observations of Type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999; Kowalski et al. 2008), the cosmic microwave background (CMB; Komatsu et al. 2008) and large-scale structure (Cole et al. 2005) demonstrate that the expansion of the Universe is accelerating. Discovering the physics of the dark energy thought to be driving this phenomenon is a key goal of the modern cosmology. Observations that probe the non-linear growth of structure are sensitive to the entire history of the Universe and are a crucial element in the attempts to measure the evolution of dark energy properties with time.

The abundance of collapsed structures as a function of mass, the halo mass function (HMF), is an important statistic that is measurable through strong lensing statistics (Bartelmann et al. 1998), galaxy redshift surveys (Evrard et al. 2002) and X-ray (Borgani et al. 2001) detection of clusters and future cluster surveys utilizing the Sunyaev–Zel’dovich effect signature in the CMB (Tauber 2005). An accurate estimation of this statistic as a function of cosmology is therefore required in order to extract maximum information from observations.

Determining the HMF of cosmological models can be a computationally expensive task requiring many large \( N \)-body simulations, since non-linear gravitational structure growth cannot be calculated analytically. Therefore, simulation-calibrated tools for rapidly and accurately generating the observational signatures of a wide variety of models are essential. However, given the theoretical uncertainty surrounding the physics of dark energy, tools for predicting the HMF must be valid for dark energy models as generally as possible, to avoid a detailed computation for every one of the plethora of possibilities.

Current methods for estimating the HMF fall into two main categories (see Cooray & Sheth 2002 for a review). The first are methods based on the Press & Schechter (1974) theory that relate the density of the collapsed objects of a given mass to the variance of the density field on scales enclosing that mass in the mean, \( \sigma^2(M) \), and a threshold parameter, \( \delta_c \), determining the linear overdensity required for collapse by a given redshift. The leading approach based on these ideas is the Sheth & Tormen (1999) (hereafter ST) mass function, which incorporates ellipsoidal collapse (rather than the purely spherical collapse of the Press–Schechter theory) and has free parameters that are simulation-calibrated.

The second type of mass function fitting approach is to directly fit the multiplicity function

\[
\frac{M \, dn(M, z)}{\bar{\rho} \, d \ln \sigma^{-1}}
\]

by a universal function of the variance

\[
\sigma^2(M) = \int_0^\infty k^2 P(k) W^2(k, M) \, dk
\]

in which \( P(k) \) is the power spectrum of density fluctuations, \( \bar{\rho} \) is the mean matter density and \( W(k, M) \) is the Fourier transform of a spherical top hat function with a radius that encloses the mass \( M \) at the mean density of the universe. Jenkins et al. (2001) (hereafter J01) found a universal function of \( \sigma \) from simulations of cold dark

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matter (ΛCDM), matter-dominated and open universes that fits the multiplicity function for all of these cosmologies. More recently, Warren et al. (2006) have found a similar formula that fitted their simulation results slightly better.

Since there are many different dark energy models currently proposed, it has not proved practical to include dark energy variation into HMF fitting work, and currently available formulae are calibrated only to ΛCDM and matter-only cosmologies. However, the J01 formula has been demonstrated via N-body simulations to be valid (at least at the 20 per cent level) for an evolving dark energy model (Linder & Jenkins 2003) and the ST formula was also found to agree with the evolving dark energy models in Klypin et al. (2003), indicating that the universality of gravitational collapse appears to extend to at least some dark energy models.

One particularly interesting class of evolving dark energy models is the concept of early dark energy (EDE). In these models (Wetterich 2004; Doran & Robbers 2006), dark energy has a non-negligible energy fraction through the entire course of cosmic history, rather than being important only at late times as in the ΛCDM model. In an investigation of the HMF in the EDE cosmology, Bartelmann, Doran & Wetterich (2006) (hereafter BDW) calculated, using the ST approach, that the presence of EDE leads to a significant enhancement of the abundance of collapsed objects relative to ΛCDM, particularly at a high redshift. However, in a recent study (Francis, Lewis & Linder 2008), we found that the J01 and Warren et al. mass functions, in contrast, predicted much less difference between EDE and ΛCDM and N-body simulations agreed with this result. The ST results using the BDW δc were clearly ruled out by simulation data.

Why did the Press–Schechter (Sheth–Tormen) approach appear to fail? While the multiplicity function is expressed directly in terms of σ, the Press–Schechter approach involves δc/σ, suggesting that δc could be the source of the discrepancy. BDW calculated that δc is significantly altered in the EDE cosmology compared to ΛCDM, and this difference in δc resulted in the difference in mass functions. Grossi & Springel (2008) noted that if instead of using spherical collapse arguments, a constant value of δc = 1.689 is assumed for all cosmologies at all redshifts, the basic agreement between the ST and J01 mass functions is restored. In this work, we investigate this issue further, re-examining the root calculation of δc.

In Section 2, we re-examine the methods for calculating δc and advocate a more accurate technique for dark energy models, especially for EDE. Given our calculations of δc, in Section 3 we determine HMFs from the ST approach and compare to the J01 approach. We also discuss the implications for mass functions in dark energy cosmologies more generally. Finally, in Section 4 we discuss the dependence of δc on dark energy and suggest under what conditions we might expect a significantly different value from that of ΛCDM and the observational consequences.

2 COMPUTING THE LINEAR DENSITY CONTRAST

The picture of non-linear growth of structure involves density perturbations growing in amplitude, initially by a linear growth factor, then achieving the sufficient density contrast to separate from the Hubble expansion and to collapse, increasing its density in a non-linear manner. One can calculate the level to which the density contrast would have grown in the linear theory by collapse time, as a convenient parameter (and an essential ingredient in Press–Schechter formalism), though the true, non-linear density contrast is much larger.

The linear density contrast at collapse, δc, is defined by

\[
\delta_c = \lim_{a \to 0} \frac{D_p(a_c)}{D_\Lambda(a)}
\]

where Δ is the overdensity, ρ/ρ, of some spherical region of the universe that collapses at scalefactor a_c and D_p(a_c) is the linear growth factor. This parameter, therefore, quantifies the linear growth from the early universe until a_c of an overdensity known to collapse under non-linear growth by that time. For matter-dominated cosmologies, δc = 1.686, and for ΛCDM it becomes a weak function of cosmology, retaining the matter-dominated value at a high redshift and dipping only slightly by redshift zero.

In order to calculate δc, we need to solve the linear growth equations to obtain D_p(a_c)/D_\Lambda(a). We also need to find the overdensity in the early universe that leads to the collapse of the perturbation at the exact desired scalefactor a_c. The usual approach (see e.g. Peebles 1980) is to simultaneously solve the Friedmann equations for the background universe and perturbation, treating the perturbation as a closed universe. It is also traditional to normalize these equations to turnaround, when the time derivative of the perturbation radius is zero. Following the notation of BDW, we therefore need to solve the following equations (for simplicity we take a flat-background universe):

\[
x = \sqrt{\frac{\omega x}{x} + \lambda x^2 g(x)}
\]

\[
y = -\frac{\omega \xi}{2 y^2} + \frac{1 + 3 w(x)}{2} \lambda g(x) y,
\]

where \( x \equiv a/a_c \) and \( y \equiv R/R_c \) and \( R \) is the radius of the perturbation (which is assumed to be spherical). The dimensionless density parameters of matter and dark energy at turnaround are \( \omega \) and \( \lambda \), respectively, and \( \xi \) quantifies the overdensity at turnaround; \( w(x) \) is the dark energy equation of state and \( g(x) \) is the dark energy density normalized to the turnaround value. The dots indicate derivatives with respect to the time parameter \( \tau \equiv H_0 \tau_c, \) where \( H \) is the Hubble parameter and \( \tau \) is cosmic time. Finding the value of \( \xi \) that ensures the collapse of the perturbation at the required scalefactor \( a_c \) requires a numerical search.

The overdensity, Δ(x), can be defined via

\[
\Delta(x) \equiv \frac{\xi x^3}{y^3}
\]

and therefore, once \( \xi \) is found, the behaviour of equations (4) and (5) determines the size of the overdensity at early times.

In BDW, an approximate solution for the overdensity at early times was derived of the form

\[
\Delta(x) \approx 1 + \frac{3}{5} F \xi^{1/3} x,
\]

where there are two solutions for \( F \), one for the case when the dark energy equation of state at early times, \( w_1 \equiv \lim_{x \to 0} w(x) < -1/3 \), and one for \( w_1 > -1/3 \). ΛCDM is an example of the first case, and the solution results in

\[
F_{\Lambda CD M} = 1 + \frac{\lambda}{\omega \xi}.
\]

The second case includes the Wetterich (2004) EDE model examined in BDW, with

\[
F_{\text{EDE}} = 1 - \frac{\Omega_\Lambda}{1 - \Omega_\Lambda},
\]

where \( \Omega_\Lambda \) is a parameter of the Wetterich (2004) EDE model quantifying the fractional dark energy density at early times. In this
model, as $\Omega_c \rightarrow 0$, the cosmology converges to $\Lambda$CDM, and hence we should expect that these two solutions also converge. However, this does not occur, in this limit equation (9) converges to unity and not to equation (8).

Moreover, the non-negligible presence of dark energy in the early universe should, relative to $\Lambda$CDM, slow the collapse of the perturbation due to the lower matter clustering source term in the early universe and later the higher expansion rate. We should expect then that for the same collapse scalefactor $a_c$, the addition of EDE should require increasing the overdensity in the early universe in order to compensate for the slower non-linear growth. However, the solution for $\Delta(x)$ in the early universe from BDW predicts a decrease in the overdensity for EDE relative to $\Lambda$CDM. These problems raise doubts about the accuracy of this solution.

We note that a precise determination of the initial $\Delta(a_i)$ is needed in order to accurately determine $\delta_c$. We find typical values of $\Delta(a_i)$ to be $\Delta(a_i) \simeq 1 + 3(a_i/a_c)$, where, to ensure numerical convergence (results independent of the chosen $a_i$), we take $a_i < a_c \times 10^{-4}$. This means that even a small error, say of the order of $10^{-3}$ in $\Delta(a_i)$, is greatly amplified when going to $\Delta(a) - 1$, needed to calculate $\delta_c$ in equation (3). In this example, the error induced in $\delta_c$ would then be of the order of unity.

We employ a different, straightforward approach in computing $\delta_c$ purely numerically. We still solve equations (4) and (5), however rather than searching for the overdensity at turnaround numerically and then scaling this back approximately to early times, we perform the numerical integration starting the integration at early times, and hence search directly for the overdensity $\Delta(a_i)$ that causes collapse of the perturbation ($y \rightarrow 0$) by the desired collapse scalefactor $a_c$. Our approach has several advantages.

(i) In the previous approach, a numerical search must be made to find the overdensity at turnaround before this can be scaled back to the early universe using an approximate solution. Performing this search at the early time instead avoids the need for the approximate scaling back without adding to the computation time.

(ii) Finding the overdensity at turnaround implicitly assumes that the rise and fall times of the radius of the perturbation are equal. This is not true in general for dark energy cosmologies. By avoiding reference to turnover, we avoid this approximation.

(iii) By solving the exact equations, we can quantify the magnitude of any errors introduced by approximations that may lead to a simpler functional form for $\Delta(x)$. Without this solution, we cannot properly test the validity of approximations.

Results from our numerical calculation, compared to the method of BDW, are shown in Fig. 1. The EDE model is model (I) from BDW with parameters $\Omega_m = 0.325$, $w_0 = -0.93$ and $\Omega_C = 2 \times 10^{-4}$. This is compared to a flat $\Lambda$CDM model with $\Omega_m = 0.3$ as in BDW. We also compare to different EDE model, proposed by Doran & Robbers (2006), which, compared to the Wetterich (2004) model, has a greater EDE density at early times relative to its $z \approx 2$ value. We show our results for this model, with $\Omega_m = 0.3$, $w_0 = -1$ and $\Omega_m = 0.05$ (note that current data permit higher values of $\Omega_m$ in the Doran & Robbers 2006 model than the Wetterich 2004 model; see Doran, Robbers & Wetterich 2007 for details.). The key result from Fig. 1 is that we find $\delta_c$ in EDE cosmology to be little changed compared to $\Lambda$CDM. This is significant, and in contrast to the previous results. The implications for the ST mass function are discussed in Section 3. The reasons for breakdown of the BDW solution are detailed in the Appendix. The dependence of $\delta_c$ on EDE is fitted by

$$\delta_c^{\text{EDE}}(a_i) = A + [b(1 + w_0) + c\Omega_m - d]a_c - e\Omega_C,$$

where for the Doran & Robbers (2006) model, $A = 1.6905$, $b = -0.0183$, $c = 0.0264$, $d = 0.0208$ and $e = 0.202$, and for the Wetterich (2004) model, $A = 1.6899$, $b = -0.0170$, $c = 0.0455$, $d = 0.0307$ and $e = 0.753$. Both of these fits are good to $\sim 0.1$ per cent in the range $0.2 < \Omega_m < 0.4$ and $-1.2 < w_0 < -0.8$, $0.1 < a_c < 1.0$ and $0 < \Omega_C < 0.05$ for the Doran & Robbers (2006) model and $0 < \Omega_C < 1 \times 10^{-3}$ for the Wetterich (2004) model. By taking $\Omega_C = 0$ and $w = -1$, equation (10) returns the $\Lambda$CDM result.

### 3 Halo Mass Functions

The results on the linear collapse parameter solve a puzzle from Francis et al. (2008), where a marked difference between the ST and J01 mass functions in the EDE cosmologies was highlighted, and Grossi & Springel (2008) where it was noted that if spherical collapse is ignored and a common value of $\delta_c = 1.689$ is assumed instead the agreement between these methods is restored. For the ST mass function, using spherical collapse arguments for $\delta_c$, both studies relied upon the calculation from BDW. Using instead the method outlined in this study, we have re-examined the ST and J01 mass functions. As found in Francis et al. (2008), the EDE
mass functions are not greatly altered compared to ΛCDM at $z = 0$, however the difference increases with redshift. The EDE mass functions as a ratio to ΛCDM at $z = 1$ for the same models as given in Fig. 1 are shown in Fig. 2. This result demonstrates that the basic agreement between the ST and the J01 mass functions is preserved, even when the spherical collapse-motivated, cosmology-dependent, $\delta_c$ is used. Thus not only the form, but also the conceptual basis of the ST mass function is valid for EDE. We note that at the high-mass end ($M \gtrsim 10^{13} M_\odot h^{-1}$), the choice of calculating $\delta_c$ or holding it fixed makes as big a difference as choosing between the ST and J01 formulae. The simulations from neither Francis et al. (2008) nor Grossi & Springel (2008) have sufficient accuracy at this high-mass end to make any clear judgement about which choice better fits simulation data.

The success of the J01 style mass functions, that are blind to the growth history of the universe (as opposed to the instantaneous growth factor), indicates that indeed the abundance of haloes is insensitive to this. When the growth history is considered, via the alteration of $\delta_c$ in the ST mass functions, small, but not insignificant, differences between the ST and J01 predictions for the relative mass function in EDE and ΛCDM emerge. Future work with simulations containing sufficient volume to accurately probe the high-mass range could discriminate between the two approaches to mass function fitting and determine whether the growth history affects halo abundances.

While the abundance of haloes is unaffected by the growth history, Francis et al. (2008) found that non-linear power at small scales ($k \gtrsim 1$) is increased in EDE cosmologies relative to ΛCDM. Since this part of the power spectrum is dominated by the one-halo term in the halo model (Cooray & Sheth 2002), the internal density profile of haloes in EDE will be different than in ΛCDM. This is also seen in the results of Grossi & Springel (2008).

4 DISCUSSION AND CONCLUSION

In this Letter, we have demonstrated that the ST mass function, when the correct $\delta_c$ is used, agrees with the J01 and Warren et al. (2006) mass functions. For reasonable parameter values, $\delta_c$ in EDE models is not significantly altered compared to ΛCDM. This is in contrast to the previous results for EDE, but agrees with the analyses of other dynamical dark energy models, for instance Mainini et al. (2003). From equation (3), we can see that $\delta_c$ is defined by comparing linear to non-linear growth. If, in some cosmology, we find $\delta_c$ to be significantly altered compared to some other model, then this indicates that the difference between models must be altering the linear and non-linear growth differently.

What kind of cosmology would have a significantly different $\delta_c$ compared to ΛCDM? Some new physics must alter the linear and non-linear growth rates in different ways compared to ΛCDM. One key assumption we have made in our analysis is that dark energy is smoothly distributed on the relevant length-scales. This means that the dark energy density in the background universe and within the perturbation is not evolved independently. If dark energy perturbations were in fact important on small scales, or for instance if dark energy was non-minimally coupled to dark matter then the dark energy density within the perturbation will evolve differently to the background universe. In this case, we would expect a more significant alteration of $\delta_c$, although careful determination of the linear growth as well as the non-linear spherical collapse taking any coupling or dark energy perturbations properly into account would be needed (see e.g. Mainini & Bonometto 2006; Manera & Mota 2006; Basilakos & Voglis 2007; Dutta & Mao 2007; Abramo et al. 2008).

As found in BDW, if $\delta_c$ is postulated to be significantly different to the ΛCDM value, then large differences in the abundance of collapsed objects would be seen, over and above any difference we might anticipate based upon measurements of the linear growth rate from the CMB, weak lensing and the large-scale galaxy power spectrum normalization combined with the knowledge of the expansion history from Type 1a supernova data. Since some discrepancies in structure measurements may exist (Fedeli et al. 2008), it therefore remains a question for further studies as to whether such observations potentially point to the non-linear growth in our universe not following the universal form andhint at added physics for dark energy or dark matter.

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APPENDIX A: THE BDW SOLUTION FOR $\Delta(x)$

The calculation of $\delta_c$ in BDW makes two key assumptions in order to approximate the early time evolution of the overdensity in EDE cosmologies. In equation (4), the second term is neglected on the assumption that it is small compared to the first, and the second-order differential equation (5) is converted to an approximate first-order equation. These approximate equations, for a flat universe, are

$$\dot{x} \approx \left[ \frac{\omega}{x} \right]^{1/2}$$  \hspace{1cm} (A1)

$$\dot{y} \approx \left[ \frac{\omega \zeta}{y} - \omega \zeta + \frac{\zeta \Omega_0 \omega}{(1 - \Omega_0)} \right]^{1/2}$$ \hspace{1cm} (A2)

To examine the effects that these approximations introduce, we first determine $\zeta$ using our numerical solution for $\Delta(a)$, and then numerically integrate equations (A1) and (A2) in the early universe. If the approximations are good then the results should match our exact calculation in the early universe.

There is an important point that must be made using our approach. The boundary conditions of $\dot{y}/y = \dot{x}/x$ at $a_i$ (the perturbation starts off comoving with the Hubble flow) ensure that we have both a growing and a decaying mode initially, such that $\delta_+ = (3/5)\delta_i$, where $\delta \equiv \Delta - 1$. At the initial time $x_i$, we have both growing and decaying modes and hence we need to apply the factor of $3/5$ in order to extricate the growing mode only. The essence of the BDW approach is to approximate the early time trends of equations (4) and (5), which should return simply by the growing mode. As expected from the results of Section 2, the integration using equations (A1) and (A2) does not match our result, however it does reproduce the solution of BDW after the decaying mode has dissipated, indicating that these approximations cause the difference compared to our result (both approximations contribute comparable errors). The subsequent manipulations of these equations in BDW are clever but the damage is already done.

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