Kinks, energy conditions and closed timelike curves

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Abstract

A link between the possibility of extending a geodesically incomplete kinked spacetime to a spacetime which is geodesically complete and the energy conditions is discussed for the case of a cylindrically-symmetric spacetime kink. It is concluded that neither the strong nor the weak energy condition can be satisfied in the four-dimensional example, though the latter condition may survive on the transversal sections of such a spacetime. It is also shown that the matter which propagates quantum-mechanically in a kinked spacetime can always be trapped by closed timelike curves, but signaling connections between that matter and any possible observer can only be made of totally incoherent radiation, so preventing observation of causality violation.
1 Introduction

Chamblin has recently stressed\(^1\) that the possibility of extending a non-spacelike geodesically incomplete kinked spacetime to a spacetime that is geodesically complete hinges on the energy conditions being satisfied. On the other hand, Gibbons and Hawking proposed\(^2\) that kinked space-times should be related to the existence of closed timelike curves (CTCs). This proposal was subsequently criticized by Chamblin and Penrose\(^3\), who showed that this relation could not be established, at least for classical space-times and matter.

The above alluded connections have been already discussed in the case of de Sitter space kink\(^4\) and other spherically-symmetric gravitational topological defects\(^5,6\). It turned out that in these examples the energy conditions were not satisfied\(^7\) and, however, they all can be maximally extended to a geodesically complete spacetime. Moreover, none of these space-times may contain CTCs when their matter contents are considered classically\(^4,8\), in accordance with the Chamblin-Penrose claim\(^3\).

As a first step to investigate the influence of space-time symmetry on these issues, they will be discussed in the present paper for the case that the kink occurs in a cylindrically-symmetric space-time.

2 Cylindrically-symmetric kink

The typical example of a cylindrically-symmetric kink is the kink in the interior of an extreme cosmic string\(^9,10\). This is characterized by a gravitational coupling \( G\mu = \frac{1}{2} \), where \( \mu \) is the string linear density. In what follows, let us briefly first review the topological properties of the kinked extreme string, and then comment on some aspects of its geometry.

The general concept of a gravitational kink can be introduced by starting with the Lorentz metric \( g_{ab} \) of a four-dimensional spacetime as given by a map, \( P \), from any connected three-manifold, \( \partial M \), of the spacetime four-manifold, \( M \), into the set of timelike directions in \( M \)\(^11\). Metric homotopy can then be classified by the degree of this map, and the kink number (or topological charge) of the Lorentz metric, with respect to a hypersurface \( \Sigma \), can be defined by\(^11\)

\[
\text{Kink}(\Sigma; g_{ab}) = \text{deg}(P),
\]

so that the gravitational kink can be viewed as a measure of how many times the light cones rotate around as one moves along hypersurface \( \Sigma \).

In the case of the spacetime of an extreme cosmic string, whose interior geometry can be visualized as that of a sphere when the corresponding two-metric is embedded in an Euclidean three-sphere\(^12\), the pair \( (\Sigma; g) \) will describe a gravitational kink with topological charge \( \kappa = +1 \) if \( \text{Kink}(\Sigma; g) = 1 \). From the above discussion, one may also visualize the internal geometry of the extreme string by enforcing the constant-time sections, \( \tau = \tau_0 \), of the interior metric of a string\(^9,12\) with uniform density \( \epsilon \), out to
some cylindrical radius,

\[ ds^2 = \frac{dr^2}{1 - \frac{r^2}{r_*^2}} + dz^2 + r^2d\phi^2, \quad (2.1) \]

where

\[ r = r_* \sin \frac{\rho}{r_*}, \quad (2.2) \]

with \( r_* = (8\pi G\epsilon)^{-\frac{1}{2}} \), and \(-\infty < z < \infty, 0 \leq \phi \leq 2\pi, 0 \leq \rho \leq r_* \arccos(1 - 4G\mu), \) to be isometrically embedded in the kinked spacetime. The corresponding cylindrically-symmetric standard, kinked metric is given by \(^9,13\)

\[ ds^2 = -\cos 2\alpha dt^2 \mp 2kdt dr + dz^2 + r^2d\phi^2, \quad (2.3) \]

where the upper/lower sign of the second term corresponds to a positive/negative topological charge, \( k = \pm 1 \), depending on which of the two coordinate patches required for a complete description of the kink is being considered\(^9\), and \( \alpha \) is the tilt angle of the light cones in the kink, \( 0 \leq \alpha \leq \pi \). The isometric embedding will hold if in metric (2.3) we have furthermore

\[ \cos 2\alpha = 1 - \frac{r^2}{r_*^2} \quad (2.4) \]

and

\[ \dot{t} = \tau_0 - k \int \frac{dr}{\cos 2\alpha}. \quad (2.5) \]

Actually, a gravitational kink depends only on \( D-1 \) of the \( D \) spacetime coordinates, and is spherically symmetric on them. However, the cylindric coordinate \( z \) in metric (2.1) and (2.3) is not going to play any role in the analysis to follow and, therefore, one could reduce these metrics just to their hemispherical \( z=\text{const.} \) sections. On the other hand, one can also embed the \( z=\text{const.} \) sections of metric (2.3) in an Euclidean space and, hence re-express that metric in an explicit spherically-symmetric form:

\[ ds^2 = -\cos 2\alpha dt^2 - kdt dr + r_*^2d\Omega_2^2, \]

where we have specialized to the case of a gravitational kink with positive topological charge, \( \kappa = +1 \), \( d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2 \) is the metric on the unit two-sphere, and we have used Eq. (2.5).

The metric (2.3) can be then regarded as the metric for the embedding of metric (2.1), and the kinked time \( \dot{t} \), as the corresponding embedding function. Hence, one can obtain an embedding “rate”

\[ \frac{d^2r}{dt^2} = \frac{2r}{r_*} \left( \frac{r^2}{r_*^2} - 1 \right), \quad (2.6) \]

which tells us that the embedding surface would flare either outward if \( r < r_* \), or inward if \( r > r_* \). The string metric (2.1) should now be interpreted as a kinked boundary in the space with kinked spacetime (2.3).

If the isometric embedding of metric (2.1) in metric (2.3) holds, from (2.2) and (2.4) we have \( \cos^2 \theta = \cos 2\alpha \), with \( \theta = \frac{r}{r_*} \), and if the one-kink is conserved, then \( G\mu \) is
enforced to be $\frac{1}{2}$ and $r$ should be analytically continued beyond $r_*$, up to $\sqrt{2}r_*$. This extension creates a spherical shell filled with broken phase at each $z$-const. section, preventing the extreme string with $G\mu = \frac{1}{2}$ from disappearing, and converts the conical singularity at $r = r_*$ into a de Sitter-like cosmological singularity (horizon)$^9$. All of the topological charge of the kink would then be confined within the shell, that is within a finite compact region beyond the cosmological horizon that extends up to $r = \sqrt{2}r_*$. Inside the horizon all hypersurfaces $\Sigma$ are everywhere spacelike. Thus, as a consequence from the back reaction of the gravitational field of the one-kink, the lost picture of a cosmic string with a core region of trapped energy would be recovered for the extreme string with $G\mu = \frac{1}{2}$.

We have established a consistent and regular embedding of the extreme string metric in each of the two patches of the kinked spacetime whose surfaces would, according to expression (2.6), flare outward at $\sqrt{2}r_*$, with a maximum “rate”

$$\left.\frac{d^2r}{dt^2}\right|_{r=\sqrt{2}r_*} = \frac{2\sqrt{2}}{r_*}.$$ 

To stationary observers at the center of the sphere corresponding to each surface $z$=const., $\tau$=const., the compact shell containing all the topological charge of the kink$^{14}$ locally coincides with a finite region of the exterior of either a de Sitter space when the light cones rotate away from the observers (positive topological charge), or the time-reverse to de Sitter space if the observers see light cones rotating in the opposite direction (negative topological charge). In the latter case, only the region outside the cosmological horizon would be accessible to stationary observers.

The kink metric (2.3) is geodesically incomplete as it shows an apparent horizon at $r = r_*$ on the two coordinate patches. However, it can be converted into a geodesically complete metric by introducing Kruskal coordinates. It was obtained in Ref. 9 that the maximally-extended metric describing the spacetime of an extreme string kink can be written as

$$ds^2 = -\frac{4kr_*^2}{(k - UV)^2}dUdV + dz^2 + r^2d\phi^2, \quad (2.7)$$

where again $k(= \pm 1)$ labels the two coordinate patches required to describe a complete one-kink, and $U$ and $V$ are the Kruskal coordinates$^9$

$$U = \mp e^{\frac{kr_*}{k}}\left(\frac{r_* - r}{r_* + r}\right), \quad V = \pm e^{\frac{kr_*}{k}}, \quad (2.8)$$

in terms of which the radial coordinate can be defined as

$$r = r_*\left(\frac{k + UV}{k - UV}\right), \quad (2.9)$$

with the time $\hat{t}$ given by

$$\hat{t} = t - kr_*\sqrt{2\left(1 - \frac{r^2}{2r_*^2}\right)}.$$
\[
\frac{1}{2} k r_* \ln \left[ \frac{1 + \sqrt{4 \left(1 - \frac{r_*^2}{2r^2}\right)}}{1 - \sqrt{4 \left(1 - \frac{r_*^2}{2r^2}\right)}} \right] (r - r_*),
\]

(2.10)

where \( t \) is the metrical kinked time which is related to the time entering the metric of the kinkless cosmic string.\(^9\)

The interesting feature of metric (2.7) is that its \( z \)-const. sections are exactly the metric which describes a hemispherical section of the de Sitter spacetime kink\(^4\) for a positive cosmological constant \( \Lambda = \frac{3}{r_*^2} \).

### 3 Energy conditions in cylindrically-symmetric kinks

The simplest, general metric describing the spacetime of a cylindrically-symmetric kink can be written as

\[
ds^2 = -\cos 2\alpha (dt^2 - dr^2) - 2 \sin 2\alpha dt dr + dz^2 + r^2 d\phi^2.
\]

(3.1)

In order to investigate the possibility that geodesically incomplete kinks can be extended to geodesically complete ones when the energy conditions are satisfied in a cylindrically-symmetric space-time, let us review these conditions and also physical conditions of the kind considered by Finkelstein and McCollum\(^13\), by using the Hawking-Ellis procedure\(^15\). Thus, we write the eigenvalue equation

\[
(G_{\mu\nu} - \lambda g_{\mu\nu}) \xi^\nu = 0.
\]

(3.2)

For this equation to be implemented in the case of a cylindrically-symmetric space-time kink, we obtain first the nonzero Christoffel symbols for metrics (3.1). These are:

\[
\Gamma^t_{tt} = \sin^2 2\alpha \partial_r \alpha, \quad \Gamma^t_{tr} = \Gamma^r_{tt} = -\sin 2\alpha \cos 2\alpha \partial_r \alpha,
\]

\[
\Gamma^t_{rr} = (1 + \cos^2 2\alpha) \partial_r \alpha, \quad \Gamma^r_{tr} = -\sin^2 2\alpha \partial_r \alpha,
\]

\[
\Gamma^t_{\phi\phi} = r \sin 2\alpha, \quad \Gamma^r_{\phi\phi} = r^{-1}, \quad \Gamma^r_{\phi\phi} = -r \cos 2\alpha.
\]

(3.3)

Hence we can obtain the nonzero components of the Ricci tensor which are:

\[
R_{tt} = -R_{rr} = -\frac{1}{2r} \cos 2\alpha \partial_r (r^2 \Delta),
\]

\[
R_{tr} = -\frac{1}{2r} \sin 2\alpha \partial_r (r^2 \Delta),
\]

\[
R_{\phi\phi} = r^2 \Delta,
\]

(3.4)

where

\[
\Delta = \frac{2}{r^2} \partial_r (r \sin^2 \alpha) = \frac{2}{r^2} \partial_r \mu.
\]

(3.5)
with \( \mu (= r \sin^2 \alpha) \) the function introduced by Finkelstein and McCollum\(^{13} \).

For the curvature scalar we then obtain

\[
R = \triangle + \frac{1}{r} \partial_r (r^2 \triangle), \quad (3.6)
\]

and for the nonzero mixed components of the Einstein tensor

\[
G^t_t = G^r_r = -\frac{1}{2} \triangle = -\frac{1}{r^2} \partial_r \mu
\]

\[
G^z_z = -\frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu
\]

\[
G^\phi_\phi = \frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu. \quad (3.7)
\]

Introducing these Einstein-tensor components in the eigenvalue equation (3.2), we obtain the eigenvalues

\[
\lambda_0 = \lambda_1 = \frac{1}{r^2} \partial_r \mu
\]

\[
\lambda_2 = -\lambda_1 = \frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu
\]

\[
\lambda_3 = \lambda_1 = \frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu. \quad (3.8)
\]

The corresponding eigenvectors are

\[
E_0 = (\cos \alpha, \sin \alpha, 0, 0)
\]

\[
E_1 = (\sin \alpha, -\cos \alpha, 0, 0)
\]

\[
E_2 = (0, 0, 1, 0)
\]

\[
E_3 = \left(0, 0, 0, \frac{1}{r}\right). \quad (3.9)
\]

Since \( E_0 \) is timelike and the \( E_\rho \)'s \((\rho = 1, 2, 3)\) are all spacelike, we have a canonical form of Type I, according to the classification of Hawking and Ellis\(^{15} \). The spacelike eigenvectors \( \{E_\rho\} \) form an orthonormal basis and the tetrad components of the metric tensor are

\[
\bar{g}_{\rho\sigma} = g(E_\rho, E_\sigma) = \text{diag}(-1, 1, 1, 1). \quad (3.10)
\]

Finally, we obtain for the tetrad components of the energy-momentum tensor

\[
\|T^{\rho\sigma}\| = \text{diag}(\epsilon, p_1, p_2, p_3), \quad (3.11)
\]

with

\[
\epsilon = \frac{1}{r^2} \partial_r \mu, \quad p_1 = -\frac{1}{r^2} \partial_r \mu
\]

\[
p_2 = -\frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu, \quad (3.12)
\]

\[
p_3 = \frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu. \quad (3.13)
\]
\[ p_3 = \frac{1}{r^2} \partial_r \mu - \frac{1}{r} \partial^2_r \mu. \] (3.14)

We can now discuss the energy conditions in general cylindrically-symmetric kinked space-times. The weak energy condition states\(^{15,16}\) that the energy density as measured by an observer must be non-negative, and this requires
\[ \epsilon \geq 0, \quad \epsilon + p_\rho \geq 0, \quad \rho = 1, 2, 3. \] (3.15)

Using (3.12)-(3.14), inequalities (3.15) lead to:
\[ \partial_r \mu \geq 0 \] (3.16)
\[ \partial_r \left( \frac{1}{r^2} \partial_r \mu \right) \leq 0 \] (3.17)
\[ \partial^2_r \mu \leq 0. \] (3.18)

On the other hand, for a canonical form of type I there also holds the strong energy condition\(^{15,16}\), provided that
\[ \epsilon + p_\rho \geq 0, \quad \epsilon + \sum \rho p_\rho \geq 0, \quad \rho = 1, 2, 3. \] (3.19)

For (3.12)-(3.14) these inequalities lead to (3.18) again. We note that although the strong energy condition does not imply the weak energy condition, the vice versa is true however.

Physical conditions which function \( \mu(r) \) must satisfy\(^7\) are: (a) \( \partial_r \mu \geq 0 \) for all \( r \), (b) \( \partial_r \left( \frac{1}{r^2} \partial_r \mu \right) \leq 0 \) for all \( r \), (c) \( \partial^2_r \mu \leq 0 \) for all \( r \), (d) \( \mu = O(r^3) \) as \( r \to 0 \) in order for \( |G^\rho_\rho| < \infty \) at \( r = 0 \), and (e) \( 0 \leq \frac{\mu}{r^2} \leq 1 \) in order for \( 0 \leq \sin^2 \alpha \leq 1 \). On the other hand, to ensure the existence of an one-kink, we need to impose the boundary conditions \( \alpha(0) = 0 \) and \( \alpha(0) = \pi \), since in the present case, \( \alpha = \arcsin \left( \frac{r}{\sqrt{2} r^*} \right) \). Hence, recalling that \( \frac{\mu}{r^2} = \sin^2 \alpha \), we have the additional condition (f) \( \lim_{r \to 0} \frac{\mu}{r^2} = 0 \). From this condition it follows \( \mu = O(r^{1+c}) \), with \( c > 0 \) for small \( r \), and hence \( \partial_r \mu = O(r^c) > 0 \) and \( \partial^2_r \mu = O(r^{c-1}) > 0 \), for small \( r > 0 \). The latter inequality violates condition (c) and, therefore, not only the strong energy condition but also the weak energy condition is violated in this space-time. Thus, like for kinked de Sitter space, space-time kinks possessing cylindrical symmetry should violate the above two energy conditions, in spite of them being maximally extendible to geodesically complete space-time. Moreover, it can be readily seen that these violations also imply violation of the dominant energy condition\(^{16}\), \( \epsilon \geq 0 \), \( p_\rho \in [-\epsilon, +\epsilon] \), and the null energy condition\(^{16}\) \( \epsilon + p_\rho \geq 0 \).

Let us finally consider what happens to the energy conditions when we restrict ourselves to a \( z = \text{const.} \) section of metric (3.1). It is easy to see then that the weak energy condition implies only inequalities (3.16) and (3.17) to hold, while the strong energy condition leads to (3.17) again. It follows that the former condition would only be satisfied if \( 0 \leq c \leq 2 \), while the latter one holds when \( 0 \leq c \leq 1 \). Since in the actual case \( c = 2 \), we see that for \( z = \text{const.} \) sections of metric (3.1), the weak energy condition is satisfied but the strong energy condition remains being violated. This conclusion can
be extended to the geodesically complete space-times. Thus, the Chamblin's argument\(^1\) for cylindrically-symmetric kinks should be relaxed so that a non-spacelike geodesically incomplete kinked spacetime would be extendible to a geodesically complete one if the weak (but not the strong) energy condition is preserved on their \(z={\text{const.}}\) sections.

### 4 Kinks and closed timelike curves

We shall analyse next the possible connection between kinks and CTCs, suggested by Gibbons and Hawking\(^2\), for the case of an extreme cosmic-string kink. Both the discussion and conclusion to be obtained can straightforwardly be generalized to any other spacetime kinks.

The existence of CTCs in a given spacetime can be investigated by considering the paths followed by null geodesics on the Kruskal diagrams. For the extreme string kink with Kruskal diagrams given in Fig. 1, it might seem at first glance that since the two coordinate patches are identified on the surfaces \(r=A=\sqrt{2}r_*\), both on the original regions I and II and the new regions III and IV (created by Kruskal extension), because of continuity of the tilt angle at \(\alpha=\frac{\pi}{2}\), one could choose null geodesics that started at \(r=0\) in region I+ and would somewhat loop back through the new regions to finally arrive at their starting point, after traversing both coordinate patches. However, one can easily convince oneself that such itineraries are classically disallowed, since they would require identification of the two patches also on minimal surfaces at \(r=0\) belonging to a physical and a nonphysical region, respectively (see Fig. 1).

Nevertheless, the above conclusion is no longer valid when we consider propagation of quantum fields in the same kinked spacetime. The semiclassical regime for kinked spacetimes with event horizons can simply be achieved by considering the mathematical implications imposed by the fact that \(\hat{t}\) enters the Kruskal coordinates \(U, V\) in the form of the dimensionless exponent \(k\hat{t}/r_*\). The argument of the logarithm in (2.10) becomes then square rooted and, therefore, the expression for the time \(\hat{t}\) entering the definition of coordinates \(U, V\) should be generalized to\(^9\):

\[
\hat{t} \rightarrow \hat{t}_g = \hat{t} + \frac{i}{2}k\kappa(1 - \kappa)\pi r_*,
\]

where \(\hat{t}\) is given by (2.10) and \(\kappa = \pm 1\). For \(\kappa = +1\), \(\hat{t}_g = \hat{t}\), and for \(\kappa = -1\), the points \((\hat{t} - ik\pi r_*, r, z, \phi)\) in each patch are actually the points in the same patch obtained by reflection in the bifurcation point \(U, V = 0\), keeping the Kruskal metric real and unchanged.

We note that one can still recover the standard kink metric (2.3) from a general Kruskal metric if we redefine the Kruskal coordinates as follows:

\[
U = \pm 2bne^{-\frac{i\hat{t}_c}{r_*}}\left(\frac{r_* - r}{r_* + r}\right)
\]

\[
V = \mp kke^{\frac{i\hat{t}_c}{r_*}},
\]

where

\[
\hat{t}_c = \hat{t} + i\pi r_*\kappa k.
\]
This choice leaves the expressions for $UV, F, r$ and the Kruskal metric real and unchanged. For $\kappa = -1$, Eqs. (2.8) become the sign-reversed of (4.2) and (4.3), respectively; i.e.: the points $(\hat{t} - i k \pi r_*, r, z, \phi)$ on the Kruskal diagrams of the two coordinate patches are the points in the new regions $\text{III}_k$ on the same diagrams, obtained by reflecting in the origins of the respective $U, V$ planes, preserving the Kruskal metric real and unchanged. This leads to identification of hyperbolae in the new regions with hyperbolae in the original regions for the same values of $r$; i.e. to identification of hyperbolae $\text{III}_k$ with hyperbolae $\text{II}_k$ and hyperbolae $\text{IV}_k$ with hyperbolae $\text{I}_k$. We note that the existence of such identifications in turn amounts to both the kind of periodicity required by Hawking thermal radiation\textsuperscript{17} in each patch, and the existence of CTCs.

Since in the semiclassical description, we can identify maximum surfaces of the physical regions with those of the nonphysical regions in each coordinate patch separately, we recover allowance for the null geodesics that start at surface $r = 0$ in region $\text{I}_+$ of patch $k = +1$ (Fig. 1) to continue propagating on patch $k = -1$, after the maximum surface of region $\text{III}_-$, first through region $\text{II}_-$ and then through region $\text{IV}_-$, up to the surface at $r = A = \sqrt{2} r_*$ of the latter new region. Because this surface can be identified with the similar surface in region $\text{III}_+$ of patch $k = +1$, the considered null geodesics can thereafter propagate into the region $\text{IV}_+$ and, again by quantum identification of surfaces at $r = 0$, come back to their starting points on the surface at $r = 0$ of region $\text{I}_+$, in patch $k = +1$. Hence, null geodesics starting from original regions at $r = 0$ can still loop back to arrive at their starting points, so completing a CTC, provided such a CTC is involved at a thermal radiation process preventing any information to flow from or to the CTC. Our general conclusion therefore is that CTCs are linked to spacetime kinks if and only if the matter traveling through these spacetimes is considered quantum-mechanically.

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Figure 1: Kruskal diagrams for the two coordinate patches of the one-kink cosmic-string spacetime. The trajectories for some classical null geodesics are shown as straight continuous or dashed lines. Also shown are the geodesic trajectories at time $t = 0$. Pole identifications on the figure are arbitrary.
5 Conclusion

The problem of the relation between geodesic incompleteness and the holding of the energy conditions has been considered for the case of a kink in a cylindrically-symmetric spacetime. We discussed the physical conditions that such a spacetime must satisfy in relation with the weak and strong energy conditions. It was shown that, whereas none of these conditions holds for the four-dimensional case, the weak energy condition can still survive on the $z = \text{const.}$ (transversal) sections of the spacetime. Also considered has been the problem of the connection between spacetime kinks and the existence of closed timelike curves. We obtained that kinked spacetimes do not contain CTCs if the matter propagating on them is dealt with classically, but CTCs become a necessary ingredient in such spacetimes whenever the propagating matter shows quantum behaviour. As it was pointed out first in Ref. [18] and later in Ref. [19] there must be a close connection between CTCs and the thermal processes induced by the presence of an event horizon. Any possible observer of the quantum matter trapped in a CTC could only detect it by means of a totally incoherent radiation carrying no describable information to or from the observer. Thus, the connection of CTCs with the thermal process prevents the existence of any observable causality violating process induced by the CTCs and, therefore, allows one to conjecture a censorship for causality violation, even when CTCs and time machines can exist and be operative. One might illustrate the resulting situation by re-paraphrasing Stephen Hawking [30]: there could perfectly be hords of tourists visiting us from the future, but neither they nor we could know anything about their trip. For them, it would be a touring which costs a lot and rewards nothing; for us, the trip would simply be unnoticed.

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