Direct Calculation of Updating Parameters Based on Kriging Model for Bridge Finite Element Model Updating

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Abstract. To improve the calculation efficiency and to avoid the updating results being affected by optimization algorithms, a direct calculation of updating parameters method based on Kriging model for model updating is proposed in this paper. The proposed method first generates the data samples of updating parameters by using Latin hypercube Design (LHD). The corresponding structural response is calculated by initial finite element model. Then, the explicit function between updating parameters and structural responses is obtained based on kriging theory. Finally, the best design parameters are directly solved using the experimentally obtained structural response as the objective value based on the established Kriging model. Model updating of Caiyuanba Yangtze River Bridge is employed to test the proposed method. The results show that the model updating based on direct calculation method avoids the iterative calculation and improves the computational efficiency. The proposed method can provide the approximate accuracy level compared with the traditional Kriging model method.

1. Introduction
For the engineering structure, it is of great significance to establish a finite element model that can represent the actual structure; this finite element model representing the actual structure is called the reference finite element model and can be applied to structural parameter identification, damage identification, health monitoring and performance assessment [1]. However, the initial finite element model established according to the design document often does not represent the actual structure. This is due to factors such as structural design parameter uncertainty, modelling errors, and degradation of the actual structure during use. Therefore, the initial finite element model needs to be calibrated [2]. The finite element model updating uses the measured response to update the design parameters of the initial finite element model, so that the response value calculated by the modified model tends to be consistent with the measured response value. Due to the large geometrical dimensions and design parameters of the civil engineering structure, the finite element model is more complicated. It is very time consuming to continuously call the complex finite element model to calculate the response value during the model updating. The surrogate model replaces the complex and time-consuming finite element model by constructing a simple mathematical metamodel, which improves the computational efficiency. Commonly used surrogate models are Response Surface Method (RSM), Radial Basis Function, Kriging Model, etc. Compared with other surrogate models, the Kriging model is composed of a parametric model and a nonparametric stochastic process. The existence of the stochastic process makes the Kriging model more flexible and more suitable for complex nonlinear structural input and output models.
In the literature [3-4], when using the Kriging surrogate model to update the finite element model, it is to establish the functional relationship between the structural response (output) and the design parameters (input), and calculate the structure of different design parameters through the surrogate model. This method still needs to find the optimal structural parameters through the optimization iterative method. The updating effect depends on the optimization algorithm. Therefore, this paper proposes a model updating method that directly solves the modified parameters by using the Kriging surrogate model. When the structural response is determined by experiments, the design parameters can be directly solved by the Kriging proxy model, thus avoiding the optimization of the iterative process. On the one hand, the computational efficiency can be improved, and on the other hand, the iterative updating process is avoided. The paper first introduces the basic theory of Kriging's model, and briefly describes the process of establishing the functional relationship of design parameters with respect to structural response through experimental design. At last, the method is evaluated with a full-scale bridge model updating.

2. Kriging model

2.1 Test design
The Latin Hypercube Design (LHD) is a space-filled design method proposed by McKay et al. [5] and is a method based on stratified sampling. The Latin hypercube sampling divides the probability distribution function of the variable into equal-probability sub-intervals that are not overlapping each other (for the number of samples), and then performs independent random sampling in each sub-interval to ensure accurate sampling in each sub-interval. Samples of Latin hypercube sampling can be generated by

\[ X_{ij}^{(i)} = \frac{\pi_j^{(i)} - U_j}{N}, (1 \leq i \leq N, 1 \leq j \leq d) \]  

where: \( N \) is the number of sample points, \( \pi \) is an independent random number of 1 to \( N-1 \), \( U \) is a random number independent of \( \pi \) between \([0,1] \), \( i \) is the horizontal number, \( j \) is the dimension.

2.2 Kriging model
The Kriging model is a combination of a deterministic regression model and a stochastic process, as shown in

\[ \hat{Y}(x) = F(x)\beta + Z(x) = \sum_{j=1}^{n} \beta_j f_j(x) + Z(x) \]  

where: \( \hat{Y} \) represents the design parameter matrix, \( n \) is the number of reaction items considered, \( x \) is the response value, the first term \( F(x)\beta \) represents the regression model, provides a global approximation for the model in the design space, \( \beta \) is the regression matrix, \( \beta_j \) is The regression coefficient, \( f_j(x) \) is the polynomial function of the \( j \)th variable \( x \); the second term \( Z(x) \) represents a stochastic process with a mean of zero and a covariance of \( \sigma^2 \), providing a local approximation of the simulation.

3. Engineering examples

3.1 Caiyuanba Yangtze River Bridge
To further test the application effect of the Kriging model directly solving the updating parameter method in practical engineering, the Caiyuanba Yangtze River Bridge is taken as an example. The main bridge of the bridge is a non-thrust rigid frame, steel truss girder and steel box tied arch composite structure. The main bridge has a total length of 800m, of which the main span is 420m. The main span of the bridge is a steel-mixed composite basket-type tied arch, and its layout is shown in Figure 1. The initial finite element model of the bridge is constructed according to the design drawings.
as shown in Figure 2.

In this paper, the bridge arch rib elastic modulus \( E \) and the deck surface density \( \rho \) are taken as the design parameters to be updated, and the two-order natural frequency of the lateral positive symmetry and the vertical positive symmetry which are easily recognized by several orders are taken as the characteristic parameters. The initial elastic modulus \( E \) of the arch rib is 206 GPa, and the initial density \( \rho \) of the bridge deck is 80.06 kg/m\(^3\). The corresponding initial finite element model calculation frequency is 0.321, 1.044 Hz. The selected design parameters are sampled by LHD sampling method to obtain test points and substituted into the finite element model to calculate the corresponding frequency data. The design parameters vary by \( \pm 30\% \), that is, the lower boundary of the arch rib \( E \) is 144.2 GPa, the upper boundary is 267.8 GPa; the lower density of the bridge deck density \( \rho \) is 56.042 kg/m\(^3\), and the upper bound is 104.078 kg/m\(^3\).

The dynamic characteristics of the bridge were measured on site. The vibration signal was measured by computer and analyzed by software. The lateral positive symmetry and vertical positive symmetry modal frequencies measured by the bridge were 0.307 and 0.978 Hz. The modal identification process is not described herein due to limited space.

### 3.2 Model updating and result analysis

To make the established Kriging model and the updating results can be graphically displayed, only the first two frequencies are selected as the target of the updating, that is, only two design variables are guaranteed. The experimental design is substituted into the lateral frequency \( f_1 \) and the vertical frequency \( f_2 \) calculated by the finite element software as the input. The design parameters of rib elastic modulus \( E \) and the bridge deck density \( \rho \) are taken as outputs, and the Kriging model is constructed to obtain the arch. The functional relationship between the rib elastic modulus \( E \) and the frequencies \( f_1 \) and \( f_2 \), and the corresponding root mean square error (MSE) are shown in Figure 3.

In the figure, the curved surface represents the Kriging model of the established design parameters with respect to the structural response. The scatter point indicates the test point sample obtained by LHD. It can be seen that the scatter points are located near the surface, indicating that the Kriging model is better in accuracy. In addition, in the MSE error graph, the maximum root mean square error is 1000, which only \( 10^{-8} \) of \( E \) (200GPa), indicating that the Kriging model of the constructed design parameters with respect to the structural response is highly accurate and can be further used for model updating.

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Figure 1. Caiyuanba Yangtze River bridge elevation layout (unit: mm).

Figure 2. The initial finite element model of Caiyuanba Bridge.
Substituting the measured frequency (target value) into the constructed Kriging model, the updated values of the designed parameters are 173.79 GPa and 96.19 kg/m³, respectively. The corresponding frequencies calculated by substituting this correction value into the finite element program are 0.3073 Hz, 0.9781 Hz. After model updating, the frequency value is very close to the experimental value, and the average maximum error is less than 0.1%, which indicates that the accuracy of the direct solution method meets the requirements, and the correction effect is good.

According to the traditional method, the Kriging function of the structural response with respect to the design parameters is constructed, and then the correction values of the design parameters obtained by the standard PSO optimization are 165.51 GPa and 95.26 kg/m³, respectively. The corresponding frequencies calculated by substituting this correction value into the finite element program are 0.3066, 0.9778 Hz. The results of the two method corrections are compared, as shown in Table 1. From the results of Table 1, it can be seen that both methods can well update the model of the Caiyuanba Bridge. The direct solution method is slightly better in accuracy, and the direct solution correction time is 0.01s, and the Kriging+PSO method is 149.8s, the difference in time between the two methods is mainly due to the fact that the latter requires a large number of iterative operations, while the direct solution method only needs to be solved once. Therefore, the direct solution method is less time-consuming and more efficient for finite element model correction, and can be effectively applied to practical complex engineering projects.

### 4. Conclusion

This paper proposes a structural finite element model correction algorithm based on Kriging model to directly solve the modified parameters. Conclusions can be made as follows.

1) The direct solution method can be used for structural model updating and provides similar accuracy as the traditional Kriging method.

2) The engineering example shows that the direct solution method can be used to update the finite element model of complex practical engineering structures, and to improve the computational efficiency while providing an ideal level of precision.
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