Pairing in exotic neutron rich nuclei around the drip line and in the crust of neutron stars

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Exotic and drip-line nuclei as well as nuclei immersed in a low density gas of neutrons in the outer crust of neutron stars are systematically investigated with respect to their neutron pairing properties. This is done using Skyrme density-functional and different pairing forces such as a density-dependent contact interaction and a separable form of a finite-range Gogny interaction. Hartree-Fock-Bogoliubov and BCS theories are compared. It is found that neutron pairing is reduced towards the drip line while overcast by strong shell effects. Furthermore resonances in the continuum can have an important effect counterbalancing the tendency of reduction and leading to a persistence of pairing at the drip line. It is also shown that in these systems the difference between HFB and BCS approaches can be qualitatively large.

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I. INTRODUCTION

Superfluid Fermionic systems, in most of their realizations in physics, are either infinite-size and uniform, like, e.g., superfluid 3He or neutron matter in stars, or confined in a finite volume like, e.g., nuclei, cold atoms in traps, or metallic grains. In the former case, the pairing gap at the Fermi energy is a function of the density of matter while in the latter case, confinement induces a variation of the density on a scale which may be smaller or of the order of the coherence length of the Cooper pairs. For example the size of the Cooper pair in nuclei may vary locally by a big factor going from the size of the order of the nucleus in the interior to something like 2 fm in the surface region [2]. In cold atoms, the size of Cooper pairs can be varied with the help of Feshbach resonances and one can cover the whole range from the BCS to Bose-Einstein condensation (BEC) regimes. In nuclear physics, the local density approximation (LDA) is at its very limit of applicability because the size of the Cooper pairs is at the best, locally, of the size of the surface thickness [2]. Nevertheless pairing correlations at the surface of finite nuclei may be expected to show some remembrance of what happens in infinite nuclear matter [3, 5, 6] where the 1S0 pairing gap is strongly peaked at a density close to ρ0/5, where ρ0 is the saturation density of nuclear matter. Pairing correlations are, therefore, expected to be slightly enhanced at the surface of nuclei. In particular, one may naively think that in exotic or drip line nuclei neutron pairing is enhanced in such situations. This would be due to the fact that the neutron density extends more smoothly out to low density and forms a more or less thick neutron skin which may resemble a piece of low density neutron matter. This, however, is not the case as we will see in this work. The reason lies in the fact that the relation between infinite matter and surface of nuclei is, as just mentioned, not one-to-one, because of the long coherence length of the Cooper pairs [1]. As a consequence, finite-nuclei mostly reflect the pairing properties at a density that is the average density of the systems [2]. The effect of the change in density on the pairing gap at the surface of nuclei is still difficult to analyze and to pin down from experiment [3, 6]. This, for instance, concerns the question whether and to what extent the pairing force may show a surface peaking [10].

The main objective of the present work is, thus, the study of what happens to neutron pairing of nuclei around the neutron drip. This also concerns neutron rich nuclei at the frontier of stability, with a large neutron skin, as well as neutron rich nuclei embedded in a low density neutron gas as it can be in the inner crust of neutron stars. In this paper we are interested in what happens just at the overflow point where the neutron gas is about to appear, or has appeared at a very low density. Pairing correlations are built there in two rather different systems, the nuclear cluster and the shallow superfluid gas. Considered alone, these systems are paired, and put together, a mutual effect of pairing between the two components of the system might eventually change the pairing properties of the whole system. In cold atomic gases, it is possible to fabricate a trapping potential which goes over from a narrow container to a much wider one at a certain energy [11, 12]. Filling up this potential with atoms one may also reach drip and overflow situations. Our study may, therefore, be of interest for other fields of physics as well.

Since the limit of a nucleus embedded in a vanishing dilute gas is a drip-line nucleus, some properties of the latter type of nuclei, such as the existence of resonant
states in the continuum, can have an important impact on nuclei immersed in a low density gas \[13\]. On the experimental side, nuclei at the border of stability cannot be created at the present time, but systematics towards neutron rich systems can be extracted from known nuclei masses. These systematics will therefore also be analyzed in the first part of the work.

In this work an important issue is related to the extrapolation of the pairing properties of nuclear systems towards the limits of a very dilute external gas of neutrons. This limit can be obtained either by increasing the size of the box at a fixed number of neutrons, or by varying the number of neutrons at a fixed size of the box. However, at the end, we will also study realistic Wigner Seitz cell scenarios in the context of the inner crust of neutron stars, where the neutron gas reaches non negligible densities.

In addition, we also systematically investigate the mentioned pairing properties using two different forces: a density-dependent contact interaction (DDCI) and a separable finite-range interaction (SFRI) within the Hartree-Fock-Bogoliubov framework which is appropriate for inhomogeneous systems. The appropriateness of these two pairing interactions will be assessed. In inhomogeneous systems where the change in density is of the order of the coherence length or smaller, DDCI might be at its limit, since the interaction depends on the density via a local density approximation. It has been shown that, given a DDCI that reproduces the gaps of SFRI in both symmetric nuclear matter (SNM) and pure neutron matter (PNM), the two interactions behave in a similar way also in inhomogeneous systems as nuclear clusters in the inner crust of neutron stars \[13\].

In the present article, we discuss the signature of the superfluid state comparing two different theoretical prescriptions: we take the pairing gap, in canonical basis, closer to the Fermi energy, also called pairing gap of the Lowest Canonical State (LCS) \( \Delta_{\text{LCS}} \), and the pairing gap \( \Delta_{\text{UV}} \), averaged over all the states with the pairing tensor. We show that in finite systems like nuclei or Wigner-Seitz cells, the pairing gap \( \Delta_{\text{LCS}} \) can be suppressed at overflow while the pairing energy and the pairing gap \( \Delta_{\text{UV}} \) may persist at overflow. The average pairing gap \( \Delta_{\text{UV}} \) and the pairing energy can, for certain superfluid features, be more appropriate quantities concerning the properties of inhomogeneous systems than the pairing gap \( \Delta_{\text{LCS}} \).

The paper is organized as follows: in Sec. II we present the equations we use to do our calculations and the methods to solve them; the results concerning nuclei around the drip-line are given in Sec. III, while in Sec. IV we discuss the phenomenon of overflow in the passage outer/inner crust of a neutron star. Finally we give our conclusions in Sec. V.

### II. THE HARTREE-FOCK BOGOLIUBOV (HFB) THEORY

The self-consistent HFB equations, see Eqs. [2], are solved in a box on a spherical mesh with radius \( R_{\text{box}} \). Using the standard notation \((nlj,q)\) for the spherical single-particle states with radial quantum number \( n \), orbital angular momentum \( l \), total angular momentum \( j \), and isospin \( q = n, p \), the single-particle wave functions \((U,V)^{nlj,q}(r)\), are expanded on a basis of spherical Bessel functions,

\[
(U,V)^{nlj,q}(r) = \sum_{\alpha}(U,V)_{\alpha}^{nlj,q}u_{\alpha,l}(r),
\]

where \( u_{\alpha,l}(r) = C_{\alpha,l}j_l(k_{\alpha,l}r) \), \( C_{\alpha,l} \) is the normalization factor in the box, and \( j_l \) are Bessel functions of the first kind with integer index \( l \). The index \( \alpha \) runs over a set of zeros of the Bessel function \( j_l(k_{\alpha,l}R_{\text{box}}) \), going from the lower value \( k_{\alpha=1,l} \) up to the momentum cutoff \( k_{\text{max}} = 4 \text{ fm}^{-1} \). This corresponds to an HFB model-space energy cutoff of about \( h^2k_{\text{max}}^2/2m \approx 320 \text{ MeV} \) (see Ref. \[18\] and references therein for more details).

We use a Skyrme functional to build the single-particle Hamiltonian \( \hat{h} \) and then we let the particles interact pairwise in the pairing channel. The two-body matrix elements of the pairing interaction in the \( J = 0, T = 1 \) channel enter the neutron-neutron and proton-proton gap equations, whose solutions provide the matrix elements of the state-dependent gap matrix \( \Delta \). The latter, in turn, enters the HFB equations,

\[
\sum_{\alpha'}(h^{l,j,q}_{\alpha'} - \mu_F)V^{nlj,q}_{\alpha'} + \sum_{\alpha'}\Delta^{l,j,q}_{\alpha\alpha'}V^{nlj,q}_{\alpha'} = E^{nlj,q}U^{nlj,q}_{\alpha},
\]

\[
\sum_{\alpha'}\Delta^{l,j,q}_{\alpha\alpha'}U^{nlj,q}_{\alpha'} - \sum_{\alpha'}(h^{l,j,q}_{\alpha'} - \mu_F)V^{nlj,q}_{\alpha'} = E^{nlj,q}V^{nlj,q}_{\alpha},
\]

where \( \mu_F \) is the chemical potential and \( U^{nlj,q}_{\alpha} \) and \( V^{nlj,q}_{\alpha} \) are the Bogoliubov amplitudes for the quasiparticle of energy \( E^{nlj,q} \). For the pairing channel we used two pairing interactions: a density dependent contact force and a finite range interaction in its separable approximation:

(i) The two-body Density-Dependent Contact Interaction (DDCI) between particles at positions \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) reads \[19, 20\]

\[
v(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left[ 1 - \eta \left( \frac{\rho_0 V(\mathbf{R})}{\rho_0} \right)^{\alpha} \right] \delta(\mathbf{r}),
\]

\[\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2\] is the center of mass of the two interacting particles and \( \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \) is their mutual distance. We choose \( V_0 = -530.0 \text{ MeV fm}^3 \), \( \eta = 0.7, \alpha = 0.45, \rho_0 = 0.16 \text{ fm}^{-3} \). We use a smooth cut-off acting in quasiparticle space at \( E^a \geq 20 \text{ MeV} \), that is defined by an gaussian factor \( \exp(-(E^a - 20)^2/100) \), where \( a \) is a shorthand
notation for $a = (nlj, q)$. The parameters of this interaction are adjusted such that it mimics the gaps obtained using a Gogny force in SNM.

(ii) A Separable Finite-range pairing Interaction (SFRI) $^{22}$ that reproduces the $^{1}S_{0}$ Gogny D1S pairing gap at the Fermi surface in infinite nuclear matter (INM) $^{23}$,

$$v(r_1, r_2, r'_1, r'_2) = \gamma P(r) P(r') \delta(R - R') \frac{1}{2} (1 - P^2).$$

(4)

The operator $1/2(1 - P^2)$ restricts the interaction to total spin $S = 0$. Strength and form factor are $\gamma = -728$ MeV fm$^{-3}$ and $P(r) = 1/(4\pi b^3)^{3/2} \exp(-r^2/(4b^2))$, where $b = 0.644$. The finite-range interaction in the pairing channel is added on top of a single-particle spectrum obtained with Skyrme interaction (see for instance Ref. $^{24}$).

A slight correction of the strength of the SFRI is therefore necessary and we fix $\gamma \to 0.9\gamma$ $^{23}$.

Since we are interested in systems at or beyond the neutron drip (i.e. a nucleus surrounded by a gas), we consider the same boundary conditions used in the calculation of Wigner Size cells in the inner crust of neutron stars. We thus impose the following Dirichlet-Neumann mixed boundary conditions $^{23}$: (i) even-parity wave functions vanish at $R = R_{box}$; (ii) the first derivative of odd-parity wave functions vanishes at $R = R_{box}$. When presenting the results for the HFB neutron pairing gap, we use two different definitions for the pairing gap. The first one, $\Delta_{LCS}$, is defined as the diagonal pairing matrix element corresponding to the canonical single-particle state $^{13}$, whose quasi-particle energy,

$$E^a = \sqrt{(\varepsilon^a - \mu^a)^2 + (\Delta^a)^2},$$

(5)

is the lowest. Here $\varepsilon^a$ stands for the diagonal matrix element of the single-particle field $h_{lj,q}$ in canonical basis and $\Delta^a$ the corresponding diagonal pairing-field matrix element.

The second definition of the pairing gap, $\Delta^q_{UV}$, is related to the average of the state dependent gaps over the pairing tensor, i.e.

$$\Delta^q_{UV} = \sum_{nlj} (2j + 1) \sum_{a} U_{\alpha a}^{nlj,q} \Delta^q_{\alpha a} V_{\alpha a}^{nlj,q} \sum_{nlj} (2j + 1) \sum_{a} U_{\alpha a}^{nlj,q} V_{\alpha a}^{nlj,q}. $$

(6)

The pairing energy is defined in terms of the pairing tensor as,

$$E^q_{pair} = \frac{1}{2} \sum_{nlj} (2j + 1) \sum_{a} U_{\alpha a}^{nlj,q} \Delta^q_{\alpha a} V_{\alpha a}^{nlj,q}. $$

A comparison of the pairing gaps $\Delta_{LCS}$ and $\Delta_{UV}$ with experimental value can be made considering some limitations due to other effects, such as the one induced by the time-odd term of the Hamiltonian. The experimental gap for odd nuclei can, however, be deduced from the binding energies using a three-point formula centered on the odd nucleus, see the discussion in Ref. $^{25}$ and references therein, as

$$\Delta_{exp}^{odd}(N) = \frac{1}{2} [E_b(N + 1) - 2E_b(N) + E_b(N - 1)], $$

(8)

where $E_b$ is the binding energy taken from Audi’s database $^{26}$. The experimental pairing gap for even nuclei are deduced from the average of the three-point formula $^{8}$ applied to the two closest even nuclei as $^{27}$,

$$\Delta_{exp}^{even}(N) = \frac{1}{2} [\Delta_{odd}^{exp}(N - 1) + \Delta_{odd}^{exp}(N + 1)]. $$

(9)

Definitions $^{8}$ and $^{9}$ are closer to $\Delta_{LCS}$ $^{23}$ than to $\Delta_{UV}$ and, therefore, a comparison of the pairing gap $\Delta_{LCS}$ and the experimental gap $^{9}$ is shown in Fig. $^{1}$ for calcium, nickel, tin and lead isotopes. The error bars on the experimental gaps are estimated to be $\pm 200$ keV. This takes into account a small contribution coming from the experimental error bars on the masses, and a large contribution due to other contributions than the pairing gap on the experimental gap, such as, for instance, the time odd terms in the mean field $^{24, 31}$, the use of the 3-point formula $^{16, 32}$, 3-body terms in the pairing channel $^{22, 33}$, and the particle-vibration coupling $^{4, 14, 35}$.

On the left side of Fig. $^{1}$ are shown the theoretical predictions based on SLy4 $^{24}$ $^{36, 37}$ Skyrme interaction and SFRI in the pairing channel while on the right, the results are obtained with the F+ interaction $^{38}$ and SFRI. The evolution with the neutron number $N$ of the experimental gap $^{9}$ shown in Fig. $^{1}$ (top panels) is well reproduced by the HFB model with both the SLy4 or F+ interaction in the mean-field and same interaction SFRI in the pairing channel. The theoretical calculations have been performed up to the drip line, beyond the domain where experimental information are known. The limitations of the experimental data is quite visible in Fig. $^{1}$ (bottom panels) where they are represented versus the asymmetry parameter $(N - Z)/A$. The experimental data hardly reach $(N - Z)/A \approx 0.25$. It is interesting to notice that at the edge of the experimental data the gaps (experimental and theoretical) tend to decrease with an important slope in the asymmetry direction $(N - Z)/A$, while beyond, the theoretical gaps go up again for larger $(N - Z)/A \geq 0.3$. The decrease of the experimental gaps $^{9}$ is mostly due to the shell closure at the boundary of experimental measurements. As a consequence, the asymmetry dependence of the experimental pairing gap, which has been fitted as

$$\Delta = \left[ 1 - 7.74 \left( \frac{N - Z}{A} \right)^2 \right] \frac{6.75}{A^{1/3}}$$

(10)

in the work of Ref. $^{39, 40}$, as well as in the former work of Ref. $^{41}$, tends to overestimate the asymmetry.
dependence of the pairing gap. This is clearly illustrated in Figs. (c) and (d) where the fit (10) is shown for Sn isotopes, i.e. Z=50 (solid line). Completing the unknown experimental data for large \((N-Z)/A\) by the theoretical calculations shown in the upper panels of Fig. 1, we can obtain a new set of parameters

\[
\Delta = \left[ 1 - 2 \left( \frac{N-Z}{A} \right)^2 \right] \frac{6.75}{A^{1/3}}, \quad (11)
\]

where the coefficient in front of the asymmetry is lower than in Eq. (10). The fit (11) is also shown for Sn isotopes in Figs (c) and (d) (dashed lines).

III. NUCLEAR SYSTEMS BEYOND THE DRIP-LINE

In the following, the predictions for overflowing nuclear systems based on different pairing forces are analyzed.

A. Global properties around the drip-line

To investigate the behavior of different systems when crossing the drip line, we perform a systematic study of different isotopic chains, namely the Calcium, Nickel, Molybdenum, Tin, and Lead isotopes. The calculations have been done in a box of 40 fm radius and with the Dirichlet-Neumann mixed boundary conditions.

In Fig. 2 we display the representative neutron gaps...
\[\Delta_{\text{LCS}}\] and \[\Delta_{\text{UV}}\] (left panel) as well as the pairing energy per neutron (right panel) corresponding to Calcium, Nickel, and Molybdenum isotopic chains computed using the SFRI interaction for the pairing channel and the mean-field provided by the SLy4, LNS1, F+, F−, and F0 Skyrme forces. In the left panel, the experimental neutron gaps are also displayed together with their associated error bars. The same quantities are shown in the two panels of Fig. 3 for Tin and Lead isotopes. The isotopes represented in Figs. 2 and 3 have been selected from their behavior at the drip line \[43\]: in Fig. 2 the isotopes are not magic at the drip line and some pairing correlations persist, while in Fig. 3 the isotopes are magic at the drip line and pairing correlations are strongly reduced at and beyond the drip.

From Fig. 2 it can be observed that the two definitions of the pairing gap: \[\Delta_{\text{LCS}}\] [15] and \[\Delta_{\text{UV}}\] of Eq. (6) give quite similar predictions for bound nuclei within the experimental error-bars, while they show a noticeable difference at the drip-line and beyond. In particular it is

FIG. 2: (Colors online) The evolution of pairing properties is shown along different isotopic chains: from top to bottom Calcium, Nickel and Molybdenum isotopes are represented. On the left are compared the two definitions of pairing gaps for \[\Delta_{\text{LCS}}\] and \[\Delta_{\text{UV}}\] using Skyrme SLy4 interaction for the \[\text{ph}\] channel and the SFRI [4] for the pairing channel. On the right, the pairing energy per neutron (7) is compared using different interactions in the \[\text{ph}\] channel such as F+, F0, F−, LNS1 and SLy4. The vertical dashed line stands for the neutron drip-line nucleus using the same color as the associated interaction.
found that the gap $\Delta_{\text{LCS}}$ is almost zero at the drip line for all nuclei analyzed in Figs. 2 and 3, while the gap $\Delta_{\text{UV}}$ can persist with non-zero values in some cases, namely for the Ca, Ni, and Mo isotopic chains shown in Fig. 2. When it is zero for other cases such as the Sn and Pb chains represented in Fig. 3. At the drip line, since the LCS-gap is the gap at the Fermi energy, the LCS-gap changes from its value in a bound state to the value corresponding to a delocalized unbound state at low density. It is known that the $1S_0$ pairing gap in neutron matter is essentially zero at vanishing density [44]. Therefore, the LCS-gap is, indeed, expected to be quite suppressed at the drip-line.

The pairing energies go to zero at the drip-line for Sn and Pb isotopes, see Fig. 3, while they persist in the case of Ca, Ni, and Mo as shown in Fig. 2. In Ref. [13], it has been argued that these differences are due to the presence of resonance states lying near the Fermi level of drip-line nuclei with non-negligible occupancy. In this case pairing correlations can persist and the pairing energy remains non-zero, as it happens in the Ca, Ni, and Mo isotopic chains. However, if there is a large energy gap between the last fully occupied bound state at the drip line and the first unoccupied resonant unbound state, as it is the case in the Sn and Pb isotopic chains, pairing correlations are strongly reduced at the drip-line.

Since the pairing gaps $\Delta_{\text{UV}}$ displayed on the left of Figs. 2 and 3 (see also Fig. 4) represent an average of the pairing correlations over all the states, it behaves similarly to the pairing energy see Eqs. (6) and (7). The qualitative difference between the pairing gaps $\Delta_{\text{LCS}}$ and $\Delta_{\text{UV}}$ shown in Fig. 2 is, therefore, simply related to the fact that in overflowing systems, there could be a significant difference between the average pairing properties and the pairing gap corresponding to the last occupied state. The pairing gap which enters the ground state energy is the pairing gap $\Delta_{\text{UV}}$, since it behaves like the pairing energy, while the pairing gap $\Delta_{\text{LCS}}$ provides information on the last occupied state which, e.g., influences the gap in the level density and quantities which depend on it. The strong reduction of the pairing gap $\Delta_{\text{LCS}}$, being related to pairing property of a single state, does not necessary induce the suppression of the pairing energy, as shown in Fig. 2. Let us notice again that in stable nuclei, the pairing gaps $\Delta_{\text{LCS}}$ and $\Delta_{\text{UV}}$ are very similar (like in the nuclei represented on the left panels of Figs. 2 and 3). It was, therefore, at first a surprise to observe a qualitative difference between the pairing gaps $\Delta_{\text{LCS}}$ and $\Delta_{\text{UV}}$ in overflowing systems. However, since $\Delta_{\text{UV}}$ is an average of the gaps over the pairing tensor, it is clear that in regions where the individual $\Delta_i$’s vary rapidly, as it happens around the drip, an average will be different from the gap $\Delta_{\text{LCS}}$ of the last occupied level. On the contrary, for stable nuclei, the individual gaps are smoothly varying and then $\Delta_{\text{LCS}} \sim \Delta_{\text{UV}}$. 

FIG. 3: (Colors online) As Fig. 2 but for Tin and Lead isotopes.
For both interaction, compared to $F_0$, its occupation number is closer to 1 at a nucleus at the drip line, a neutron effective mass $66 \text{SLy4}$ (bottom). The Skyrme interaction $F_0$ predicts for neutron matter. In this case the effective mass is defined it is shown in Tab. II for $68 \text{Ca}$ and for $F_0$ (top) and $\text{SLy4}$ (bottom). The Skyrme interaction $F_0$ predicts for $66 \text{Ca}$, the nucleus at the drip-line, a bound state $g_{9/2}$ in the canonical basis, close to the continuum with an energy $\approx -0.2 \text{MeV}$, while $\text{SLy4}$ predict for $66 \text{Ca}$, the nucleus at the drip line, a bound state partially unfilled so to gain extra pairing energy. This is what is observed in Fig. 2 not only at the drip line, but also beyond. The structure of the drip line nuclei is mostly conserved even in the presence of a gas. Despite these differences, it is interesting to remark that the global level occupation picture around the drip line is similar for the Skyrme interactions $F_0$ and $\text{SLy4}$: the unbound states are occupied before the drip line is reached at variance with the usual claim that the drip occurs when the first unbound particle is produced \cite{15}. In fact, among unbound states there are resonant states which play an important role around the drip line. They have large spatial overlaps with the single-particle levels of the nucleus and give a significant contribution to the pairing correlations. In most of the cases, resonant states are populated before unbound scattering states. They are very important to understand the transition from isolated nuclei to overflowing systems.

In Figs. 2 and 3 are also shown the pairing energies for several mean-field models, namely $\text{SLy4}$ \cite{24, 37}, $F_+$, $F_-$, $F_0$ \cite{38} and $\text{LNS1}$ \cite{42}. It is observed that the reduction of the gaps at the drip-line and beyond is a property which is independent of the considered models, while the absolute value of the pairing energy can vary from one model to another. The main difference among these models is related to the effective mass in symmetric and in neutron matter. In this case the effective mass is defined as \cite{38},

$$m = m_q + q I \left( \frac{m}{m_q^*} - \frac{m}{m_p^*} \right),$$  

(12)

where $I = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ and $q$ is the isospin charge ($q = +1$, $-1$ respectively for neutrons and protons). In Eq. (12) the effective mass in symmetric matter is given by the isoscalar effective mass $m_q^*$ and the isovector effective mass $m_p^*/m$ is related to the isospin splitting in asymmetric matter. These quantities, as well as the effective mass splitting $\Delta m^* = m_q^*/m - m_p^*/m$ and the neutron effective mass $m_n^*/m$, both computed in neutron matter, are given in Tab. I for the Skyrme interactions represented in Figs. 2 and 3.

For calcium isotopes, the pairing energy at the drip-line and beyond shown in Fig. 2 depends on the Skyrme interaction. There is indeed a group of Skyrme interactions for which the pairing gap is large at the drip line ($F_+, F_0, F_-$), and another group for which it is reduced approximatively by a factor 2 at the drip line ($\text{SLy4}, \text{LNS1}$). This qualitative difference is due to the structure of the drip line nuclei around the Fermi energy as it is shown in Tab. I for $66, 68, 70 \text{Ca}$ and for $F_0$ (top) and $\text{SLy4}$ (bottom). The Skyrme interaction $F_0$ predicts for $66 \text{Ca}$, the nucleus at the drip-line, a bound state $g_{9/2}$, in the canonical basis, close to the continuum with an energy $\approx -0.2 \text{MeV}$, while $\text{SLy4}$ predict for $66 \text{Ca}$, the nucleus at the drip line, a $g_{9/2}$ state with lower energy $\approx -0.9 \text{MeV}$. Since the $g_{9/2}$ is lower in energy with $\text{SLy4}$ compared to $F_0$, its occupation number is closer to 1 at the drip point and this state participates to a lower extent to the pairing correlations. For both interaction, the system gains energy if instead of realizing the shell closure at $N=50$ and becoming non-superfluid, it leaves the $g_{9/2}$ state partially unfilled so to gain extra pairing energy. This is what is observed in Fig. 2 not only at the drip line, but also beyond. The structure of the drip line nuclei is mostly conserved even in the presence of a gas. Despite these differences, it is interesting to remark that the global level occupation picture around the drip line is similar for the Skyrme interactions $F_0$ and $\text{SLy4}$: the unbound states are occupied before the drip line is reached at variance with the usual claim that the drip occurs when the first unbound particle is produced \cite{15}. In fact, among unbound states there are resonant states which play an important role around the drip line. They have large spatial overlaps with the single-particle levels of the nucleus and give a significant contribution to the pairing correlations. In most of the cases, resonant states are populated before unbound scattering states. They are very important to understand the transition from isolated nuclei to overflowing systems.

We also analyzed the difference between the two kinds of pairing interactions which we used in this work: SFRI and DDCI. It is important to analyze the influence of the density dependence of the DDCI in inhomogeneous systems such as Wigner-Seitz cells at neutron overflow. A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells is shown in Fig. 3 for SFRI (left) and DDCI (right). The drip-line nucleus predicted by $\text{SLy4+SFRI}$ is $N = 84$ while it is $N = 88$ for $\text{SLy4+DDCI}$. Despite this small difference, the behavior of the pairing gaps $\Delta_{\text{LCS}}$ and $\Delta_{\text{UV}}$, as well as the pairing energy is very similar at the drip line and beyond. We conclude that provided that the DDCI reproduce the same pairing gaps in symmetric and neutron matter, DDCI give similar results compared to SFRI in inhomogeneous systems such as the Wigner-Seitz cells \cite{14}. Otherwise the same scenario as in Figs. 2 and 3 is recovered also for the Zirconium isotopes.

### Table I: In this table we show the isoscalar and isovector masses $m_q^*/m$ and $m_p^*/m$, as well as the difference in neutron matter $\Delta m^* = m_q^*/m - m_p^*/m$ and the neutron effective mass $m_n^*/m$ for $\text{SLy4}$ \cite{24, 37}, $F_+$, $F_-$, $F_0$ \cite{38} and $\text{LNS1}$ \cite{42}.

| Force | $m_q^*/m$ | $m_p^*/m$ | $\Delta m^*$ | $m_n^*/m$ |
|-------|-----------|-----------|--------------|-----------|
| $F_+$ | 0.700     | 0.625     | 0.170        | 0.795     |
| $F_0$ | 0.700     | 0.700     | 0.001        | 0.700     |
| $F_-$ | 0.700     | 0.870     | -0.284       | 0.586     |
| $\text{SLy4}$ | 0.695 | 0.800 | -0.186     | 0.614     |
| $\text{LNS1}$ | 0.604 | 0.478 | 0.342     | 0.820     |

A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells at neutron overflow. A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells at neutron overflow. A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells at neutron overflow. A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells at neutron overflow. A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells at neutron overflow. A comparison of the pairing properties of isolated zirconium isotopes and in Wigner-Seitz cells at neutron overflow.

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**B. The limit of nuclei immersed in a vanishing dilute gas**

Below, we will explore what happens in the outer crust of neutron stars around the point where the neutrons start to drip into the free space between the lattice sites built by the nuclei. The question we want to answer is whether a very low density gas of superfluid neutrons in a large container can have any major influence on the superfluidity of the nuclei at the lattice sites. In this section, we, thus, explore, in a schematic study, the limit of nuclei immersed in a dilute gas with the density of the gas going to zero.

We first show in Fig. 4 a zoom of Figs. 2 and 3 focused on the overflowing nuclear systems. In calcium and nickel isotopes pairing correlations persist at overflow since the gap $\Delta_{\text{UV}}$ do not vanish, while pairing correlations are almost suppressed at overflow in tin and lead isotopes.
| Isotope | $e_F = -0.02$ MeV | $e_F = 0.04$ MeV |
|---------|------------------|------------------|
| $^{66}$Ca | 1.892 0.051 2 5 | 1.813 0.057 2 5 | 1.811 0.058 2 5 |
| $^{68}$Ca | 0.994 0.054 0 1 | 0.554 0.091 0 1 | 0.466 0.105 0 1 |
| $^{70}$Ca | -0.206 0.567 4 9 | -0.240 0.600 4 9 | -0.231 0.600 4 9 |
| $^{82}$Ca | -4.639 0.971 3 5 | -4.740 0.974 3 5 | -4.735 0.974 3 5 |

TABLE II: Canonical single particle energies and canonical occupation probabilities for some calcium isotopes.

**FIG. 4**: (Colors online) Similar to Fig. 2 but for Zr isotopes. On the first four panels the behavior using SLy4 Skyrme interaction and different pairing forces, DDCI (panel (a)-(b)) and SFRI (panel (c)-(d)), are compared.
The solid line corresponds to the value of the pairing gap in uniform neutron matter for the densities of the gas. By construction, the gap $\Delta_{LCS}$ follows quite well the trend of the uniform neutron matter gap. However, in a realistic system, composed of a nucleus plus a gas, one has $\Delta_{UV} \neq \Delta_{LCS}$ and, thus, $\Delta_{UV}$ shows clear differences with the gas due to the influence of the resonance states in calcium and nickel isotopes. The understanding of overflowing systems, therefore, requires a better study of the pairing properties of nuclei immersed inside a gas at the limit of very low density. In the following we aim at decreasing the density of the gas to the lowest possible value.

Starting from an overflowing system with an external gas, the low density limit can be reached in two different manners: the first one is by increasing the size of the box for a fixed number of neutrons and the second one is by decreasing the total number of neutrons at fixed box size. However, the numerical calculations cannot be performed in boxes with sizes larger than 80 fm with SFRI and 150 fm with DDCI. These limitations are due to the increasing number of partial waves as well as of the level density as the size of the box is increased. The larger size of the box reached with DDCI is related to the lower CPU time and memory request to perform calculations compared to SFRI.

The effect of increasing the size of the box is illustrated in Fig. 6 for the case of $^{166}$Zr. The total density and anomalous density profiles are represented with a linear and logarithmic scale. There is an important reduction of the gas density for boxes going from 20 to 100 fm, while the reduction of the density going from 100 to 150 fm is quite marginal. This is a limitation of this method which imposes to work with very large boxes to reach the low density limit.

From the behavior of the density in $^{166}$Zr as a function of the box radius represented in Fig. 6, two regions can roughly be distinguished: one is the ”bulk” and the other the gas. Fixing an arbitrary limit $R_{\text{lim}} = 10$ fm to separate the ”bulk” from the ”gas”, the number of neutrons in the bulk can be estimated as,

$$N_{\text{bulk}} = \int_{0}^{R_{\text{lim}}} \rho_{n}(r) \, d^{3}r.$$  \hspace{1cm} (13)

We obtain that for $R_{\text{box}} = 20$ fm, $N_{\text{bulk}} \approx 99$ neutrons and for $R_{\text{box}} = 100$ fm, $N_{\text{bulk}} \approx 87$. The number of neutrons in the bulk decreases as a function of the box size having as a limit the isolated nucleus at the drip line.
From a box of 20 fm to 40 fm for these two models. In such a box, the external density of the neutron gas, \( \rho_g \), is different in case of F0+DDCI model since a different mean field produces a different single particle structure. Nevertheless, judging from the last numerical values, one may assume that the limit eventually goes to zero.

Since we perform constrained HFB calculation conserving the total number of neutrons, the particles evaporated from the bulk appear in the scattering states.

In Fig. 7 we display by dots the difference between the neutron pairing energy of the drip-line nucleus and that of the overflowing nuclear system \( ^{166}\text{Zr} \), \( E_{\text{pair}}^{n}(^{X}\text{Zr}) - E_{\text{pair}}^{n}(^{166}\text{Zr}) \), as a function of the box size for the models SLy4+DDCI, SLy4+SFRI and F0+DDCI. The considered drip-line nuclei in our models are \(^{126}\text{Zr} \) for SLy4+DDCI, \(^{124}\text{Zr} \) for SLy4+SFRI, \(^{122}\text{Zr} \) for F0+DDCI. The values of the pairing energy in these nuclei are extracted from Tab. III. The convergence to the asymptotic value is different for the models SLy4+DDCI and SLy4+SFRI. We observe a fast convergence when going from a box of 20 fm to 40 fm for these two models. In such a case we have seen that the particle density is sufficiently high to completely fill the resonances and thus such states do not contribute to pairing superfluidity. The excess of pairing energy in those small boxes comes mainly from scattering states. In fact when we go from \( R_{\text{box}} = 40 \text{ fm} \) to \( R_{\text{box}} = 50 \text{ fm} \), the resonant state \( f_{7/2} \) starts to depopulate and thus we can form Cooper pairs using such state. This explains why we have an increase of superfluidity. Going from \( R_{\text{box}} = 50 \text{ fm} \) to \( R_{\text{box}} = 150 \text{ fm} \), the convergence becomes very slow. In this case the box is sufficiently large to decouple bound and scattering states, the residual pairing energy comes from the superfluidity of neutrons trapped into resonant states. The behavior is different in case of F0+DDCI model since a different mean field produces a different single particle structure.

The asymptotic value is not reached in the larger boxes used in our calculations. Nevertheless, judging from the last numerical values, one may assume that the limit eventually goes to zero.

We also explore the alternative scenario where the size of the box is kept fixed and the number of neutrons is varied from 88 to 160. The results computed for boxes of size ranging from 30 fm to 50 fm are displayed by solid lines in Fig. 8. These results should be compared with the ones shown in Fig. 7. To this end we show by points the pairing energy of the nucleus \(^{166}\text{Zr} \) computed within boxes of different sizes as in Fig. 7. Again we observe in Fig. 8 that for the case \( R_{\text{box}} = 30 \text{ fm} \) we have a quick drop of superfluidity at \( \rho_g \approx 1.4 \times 10^4 \text{ fm}^{-3} \) and then an increase. This is the same phenomenon observed in Fig. 7 and it is due to a depopulation of the resonant state \( f_{7/2} \), that when occupied does not contribute to superfluidity and when is half filled gives an important contribution to superfluidity. In conclusion, the two
methods employed in this work to reach the low density limit in the gas show that the overflowing systems tends to the limit of the drip-line nucleus. The drip-line nucleus is therefore an important reference to understand and analyze the properties of the overflowing systems.

In Fig. 9, we show the evolution of the canonical single particle states for neutrons as a function of the box and analyze the properties of the overflowing systems. The inset shows the behavior of the transition among bound and unbound nuclei. SLy4+SFRI.

Mo isotopes as a function of the neutron chemical potential $\mu_n^p$, using SLy4+SFRI model. The calculations have been performed for a fixed number of protons (Z=42) and fixed box radius ($R_{box} = 40$ fm), similarly to what has been done by Grasso et al. [47]. We can observe in such a way the transition from bound nuclei to the gas+nucleus system. The results are compared with the analogous calculation done in neutron matter. We observe that for small positive values of $\mu_n^p$ the cluster+gas system has a bigger pairing energy than the homogeneous system. Such difference is due to the resonance structure as explained in the previous sections. When we reach high density regions ($\mu_n^p \gtrsim 2$MeV), we see that the presence of the cluster reduces the pairing correlation. Such phenomenon has been already discussed by many authors concerning Wigner-Seitz calculations [3, 14, 21, 48, 50].

C. Detailed analysis of the resonant states

In the previous sections, the role of resonant states have been stressed in order to understand the transition between nuclei and overflowing systems. To better describe within a theoretical framework the resonant states we decided to solve the HFB equations in r-space treating in a proper way the continuum (without discretization).

Defining, from the fully converged solution of Eq. (2), the hamiltonian $h(R)$ and the pairing field $\Delta^\nu(R)$ in the
FIG. 11: (Colors online) We show the occupation probability \( V_{lj}^2(E) \) for neutrons as a function of the quasi-particle energy for some given quantum numbers for \( ^{128}\text{Zr} \) (left) and \( ^{166}\text{Zr} \) (right) calculated using SLy4+DDCI functional starting from Eq. (18) and for 4 given values of the size of the box in which we perform the calculations. See text for more details.

The following way,

\[
\Delta^q(R) = \frac{V_0}{2} \left[ 1 - \eta \left( \frac{\rho_b(R)}{\rho_0} \right)^\alpha \right] \times \sum (2j+1)U^{nj\alpha}(R)V^{nj\alpha}(R),
\]

where \( W^q(R) \) is the central potential and \( m^*_q(R) \) is the effective mass. Eq. (15) is valid only in the case of a DDCI while in the case of a finite range interaction, the pairing field is, in principle, a function of two variables, see for instance Eqs. (A4) and (B1).
indeed, that the occupation of the resonant state appearing in Eq. (18), have been obtained. As expected, we notice, of particle that stays trapped in the resonances, that is the reason for the slow convergence of the evaporating gas to the value of the pairing gap of the last bound nucleus.

From Fig. 11 we clearly see the resonant character of the state \( f_{7/2} \). Its centroid is located around \( \approx 0.85 \) MeV and its width \( \approx 300 \) KeV for both \( ^{128}\text{Zr} \) and \( ^{166}\text{Zr} \). The result is in good agreement with the canonical basis result shown in Fig. 9. Since this level is located very close to the Fermi energy we can use the approximate formula (13): 

\[
E \approx \sqrt{(e - \lambda)^2 + \Delta^2} \approx \Delta 
\]

The position of this peak mostly depends on the strength of the pairing force. It explains the stability of the resonant state \( f_{7/2} \) in \( ^{166}\text{Zr} \) as the size of the box increases.

### IV. PAIRING IN THE CRUST OF NEUTRON STARS

The inner crust of neutron stars provides an excellent frame to apply the self-consistent mean-field theory [21, 50, 53]. The inner crust extends from the drip density \( \rho_{\text{drip}} \approx 4 \times 10^{11} \) g cm\(^{-3}\), where the neutrons start to leave from the nuclei into free space, till \( \rho \approx 1.4 \times 10^{14} \) g cm\(^{-3}\), where the transition to uniform matter takes place. The inner crust of neutron stars is believed to be formed by a crystal lattice of nuclear clusters embedded in a low-density neutron gas and ultra-relativistic electrons. To describe crust matter, the Wigner-Seitz (WS) approximation is widely used. In this approximation the crust is divided into spherical cells, each one representing an inner crust region of a given average density. The WS cells are electrically neutral and the interaction among them is neglected in many cases. Since the seminal calculation of Negele and Vautherin in the inner crust of neutron stars [53], more refined quantal calculations at HF or HFB level [3, 13, 53, 52] of different degrees of complexity within the WS approximation have been performed. Also semiclassical models as the Constrained Liquid Drop Model [60] and Thomas-Fermi (TF) calculations including pairing correlations [11, 61, 63] have been used to study the crust of neutrons stars.
In the following, we will analyze the pairing properties of the lattice in the crust of neutron stars, based, for the outer crust, on the Douchin-Haensel equation of state \([52]\) and, for the inner crust, on the Negele-Vautherin one \([53]\).

The properties of the WS cells are listed in Table IV for the outer crust and Table V for the inner crust. We have not recalculated the WS configurations which minimize the energy for each of the densities that we considered and we have preferred, as a first step, to build the pairing correlations on WS configurations obtained from previous minimizations. Our choice is motivated by two reasons: first, we want to compare our results with other published previously, as in Refs. \([3, 13, 55]\), and second, we did not want to introduce self-consistency by varying at the same time the pairing interaction and the energy which would have introduced an important non-linear effect in the search scheme.

In Fig. 12 we represent the local pairing gap \(\Delta_{\text{LOC}}(R)\), defined in Eq. (16), for some WS cells representative of the inner crust, which are: \(^{250}\text{Zr}\), \(^{500}\text{Zr}\), \(^{1100}\text{Sn}\), and \(^{1800}\text{Sn}\). On the top panel of this figure we show the results obtained by solving the full HFB equations given in Eq. (2), using the SLy4+SFRI model, while on the bottom panel we display the results obtained using the HF+BCS approximation, where only the diagonal coupling among pairing matrix elements in Eq. (2) are considered, by solid lines and the results computed with the TF+BCS approach \([61]\) by dashed lines. For a presentation of the Thomas-Fermi BCS approximation, see Appendix B and references therein. We first discuss the HFB results shown on the top panel of Fig. 12. The pairing field in the external region of the WS increases as the mass number of the WS cells increases. This is a well known phenomenon which can qualitatively be understood from a local density approximation in the very low density regime: The density of the external neutron
gas increases as the mass number of the cell goes up, and from uniform matter calculations, it is known that the average pairing gap increases as a function of the neutron density, for the cells considered in Fig. 12. We observe that in the gas region both, HFB, HF+BCS, and TF+BCS approaches give the same value for the pairing field \( \Delta_{\text{LOC}}(R) \). It is indeed expected that HFB and BCS theories coincide in uniform matter \[17, 64, 65\].

The peak of the pairing field at the surface of the clusters can also be roughly justified from a LDA and it corresponds to the maximum of the pairing gap in neutron matter \[56\]. However, the peak in LDA is quantitatively much higher than in the HF+BCS and TF+BCS calculations \[4, 66\].

The behavior of the pairing field inside the cluster is more complex to understand. From the HFB predictions, it is almost independent of the WS cells that we have considered, except for \( ^{250}\text{Zr} \). In the case of \( ^{250}\text{Zr} \), the reduction of the pairing field compared to the other calculations is induced by a shell effect: a small increase of the pairing strength makes the pairing field inside this cluster identical to that of the other WS cells. The independence of the pairing field inside the cluster with respect to the outer gas density is typical of HFB theory as it can be seen in the upper panel of Fig. 12. For the BCS approximations, shown on the bottom panel of Fig. 12, the pairing field inside the cluster increases as the mass number of the cell goes up. In the BCS approximations, the pairing properties of the gas and of the cluster are strongly coupled, as observed in the bottom part of Fig. 12, since pairing amplitudes \( U \) and \( V \) are diagonal in the HF basis. The non-diagonal pairing matrix elements in HFB theory strongly reduce the coupling between the gas and the cluster, as far as the pairing correlations are concerned.

In Wigner-seitz cells, off-diagonal couplings in the pairing field play an important role in HFB theory, while they are neglected in the BCS approximation. Such a feature has been already remarked in finite nuclei \[67\] where it was found that the use of the BCS approximation leads to a reduction of the gap compared to a full HFB solution. We leave a better analysis concerning the difference between HFB and BCS to a future work.
The pairing persistence phenomenon concerns only the first neutrons that start to drip out, as we have seen previously. It is now clear that a better investigation of the transition from the outer to the inner crust will be necessary in the future, using a smaller discretization on the values of the density of the equation of state. The effect emphasized in the previous sections, e.g. the coupling to resonant states close to the drip-line, is not clearly seen in Fig. 13. The reason is that the change in neutron number is too sharp in the existing tables IV and V passing from $^{88}$Ni ($\mu_{F}^{n} < 0$) to $^{180}$Zr ($\mu_{F}^{n} > 0$). The pairing persistence phenomenon concerns only the first neutrons that start to drip out, as we have seen previously. It is now clear that a better investigation of the transition from the outer to the inner crust will be necessary in the future, using a smaller discretization on the values of the density of the equation of state.

V. CONCLUSIONS

In this work, we made a quite exhaustive study of pairing properties of nuclei around the neutron drip. In a LDA picture, one could have expected that neutron pairing is enhanced in such situations, since the neutron skin could be considered as a low density piece of neutron matter where pairing is at its maximum. However, nuclei are too small and the pairing force too weak for LDA being a good description. The reality is more complex as our investigations show. Globally pairing is certainly not enhanced going from stability to the drip, rather it is reduced. However, the general feature is strongly hidden by nuclear shell effects. The isovector dependence of the pairing gap extrapolated from measured nuclei masses is too strong, and results as the consequence of an accumulation of closed shell nuclei at the border of the present experimental knowledge. From our theoretical calculations, we predict that exotic neutron rich nuclei beyond this border shall exhibit a new raise of the pairing correlations compared to the present extrapolations yielding a much weaker average decrease to the drip than previously assumed 10.

We should, moreover, mention that in this work our studies were restricted to spherical nuclei and that in this case and in most examples, the drip occurred at a magic, or close to magic neutron number with, naturally, reduced pairing correlations. The reason for this is not entirely clear. Either the nuclei search to gain binding energy in approaching magicity towards the drip because gain in energy by pairing is weakened, or it is the other way round, i.e., pairing is reduced because anyway (spherical) nuclei drive to magicity at the drip. Furthermore, large scale nuclear mass calculations, such as for instance Gogny D1S web mass table (http://www-phynu.cea.fr/HFB-Gogny.htm) show that only about half of the nuclei at the neutron drip are deformed, a ratio much more in favor of sphericity than in the case of stable nuclei. In the deformed cases pairing acquires more usual values. Nevertheless, looking at the values given in Gogny D1S web mass table, pairing is certainly not enhanced with respect to the stable region in such cases either. These conclusions are based on theoretical predictions but it can be surmised that reality is not entirely different. In any case, the situation of pairing properties of nuclei around the neutron drip is overcast by very large shell fluctuations as can be seen from the various figures given in the main text. An additional feature which makes the situation complicated is the fact that there are resonances in the continuum which can be populated and which can have, in some cases, a sensible influence on the pairing properties of drip nuclei. This depends, for instance, on the precise position of the resonances. In particular, for cases where a strongly degenerate resonance level becomes located very closely to the chemical potential, pairing can become quite important like this is, e.g., the case for the Ca isotopes in Fig. 2.

We have also analyzed the pairing correlations in the crust of neutron stars described in the Wigner-Seitz (WS) approach, which allowed us to study a scenario formed by nuclear clusters embedded in a low-density neutron gas. We see that in this situation the gap in the clusters is only weakly affected by the pairing in the neutron gas and that the cluster and the gas behave as almost independent systems. This result obtained from the state-of-the-art...
HFB theory is not reproduced by its BCS approximation, since BCS is less adapted for such systems as WS cells as compared with stable nuclei, at least in the case of low neutron density. When the density of the gas decreases in approaching the transition to the outer crust where all the neutrons are bound, the average pairing gap and the pairing energy in the WS cell decrease following the trend of the neutron matter. The pairing correlations, though reduced at the transition between the inner-crust and the outer-crust, remain, however, non-zero. Our studies have revealed that in this situation there is a large qualitative difference between HFB theory and semiclassical TF+BCS approximations. We postpone a detailed investigation of this feature to a future work.

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Appendix A: Pairing field

In this appendix we briefly describe the numerical procedure to obtain the pairing field in coordinate space. Following ref. [4] we can write it as

\[
\Phi^q(\vec{r}_1, \vec{r}_2) = \sum_{\alpha\alpha'nlj} \frac{2j + 1}{2} (I_{\alpha}^{nlj,q} V_{\alpha'}^{nlj,q}) \psi^{00}_{\alpha\alpha'lj}(\vec{r}_1, \vec{r}_2),
\]

(A1)

where

\[
\psi^{00}_{\alpha\alpha'lj}(\vec{r}_1, \vec{r}_2) = [\phi_{\alpha lj}(\vec{r}_1)\phi_{\alpha' lj}(\vec{r}_2)]_{00}
\]

is the wave-function of two neutrons coupled to \(L = S = 0\).

\[
\psi^{00}_{\alpha\alpha'lj}(\vec{r}_1, \vec{r}_2) = \frac{1}{4\pi} \phi_{\alpha lj}(\vec{r}_1)\phi_{\alpha' lj}(\vec{r}_2) P_l(\cos \theta_{12}) \chi_{00}.
\]

(A2)

where \(\chi_{00}\) is the total spin function of two particles coupled to \(S=0\). The wave function of the basis \(\phi_{\alpha lj}(\vec{r}_1)\) is defined as

\[
\phi_{\alpha lj}(\vec{r}) = \sum_{m_1m_s} C^{jm}_{lm_1m_s} u_{\alpha,l}(r) Y_{lm_1}(\hat{r}) \chi_{m_s}.
\]

(A3)

where \(u_{\alpha,l}(r)\) are defined in Eq. (1).

As recently discussed in ref. [69] the \(1^1S_0\) component is by far the dominant one concerning calculations of pairing gaps at subnuclear densities. We can thus immediately obtain the pairing field as

\[
\Delta^q(\vec{r}_1, \vec{r}_2) = -v(|\vec{r}_1 - \vec{r}_2|) \Phi^q(\vec{r}_1, \vec{r}_2)
\]

(A4)

where \(v(|\vec{r}_1 - \vec{r}_2|)\) is the pairing interaction. Whose matrix elements can be written as

\[
\Delta^q_{\alpha\alpha'} = - \int d^3r_1 \int d^3r_2 \psi^{00}_{\alpha\alpha'lj}(\vec{r}_1, \vec{r}_2) v(|\vec{r}_1 - \vec{r}_2|) \Phi^q(\vec{r}_1, \vec{r}_2).
\]

(A5)

To define a local pairing field, we need to apply the Wigner transformation \([17]\) and write the pairing field as \(\Delta^q(R, k)\), where \(R\) is the center of mass coordinate and \(k\) is the relative momentum among two particles.

Appendix B: Thomas-Fermi BCS approximation

We present briefly the Thomas-Fermi BCS approximation \([61, 68]\) at a momentum equal to the Fermi momentum as \([62]\)

\[
\Delta(R, k_F) = - \frac{d k'}{(2\pi)^3} v(k_F - k') \kappa(R, k').
\]

(B1)

Within the TF approach to the pairing problem, the above formula can still be written in a different way. Quantally the anomalous density matrix in phase space \([61, 68]\) is given by \(\kappa(\vec{r}, \vec{r'}) = \sum_n \kappa_n(\vec{r}|n)\langle n|\vec{r'}\rangle\). Therefore, after Wigner transformation, in the TF (\(\hbar \to 0\)) limit, one obtains

\[
\kappa(\vec{r}, \vec{p}) = \int dE f_E^TF(E) \kappa(E) f_E(\vec{r}, \vec{p}),
\]

(B2)

where \(f_E^TF(\vec{r}, \vec{p})\) is the normalized distribution function \([70]\). Inserting Eq. (B2) in Eq. (B1), using the fact that in this case we can write the distribution function as \(f_E(R, \vec{p}) = \delta(E - H_d)/g(E)\) with \(H_d\) the classical Hamiltonian, and performing the angular average of the pairing force \(\vec{v}(\vec{p}, \vec{p'}) = \frac{1}{4\pi} \int \vec{v}(\vec{p} - \vec{p'})d\Omega\) (assuming that \(\Delta(R, \vec{p})\) and \(\kappa(\vec{r}, \vec{p})\) are spherically symmetric in momentum space), the gap \(\Delta(R, k = k_F)\) can be finally recast as

\[
\Delta(R, k_F) = - \frac{1}{4\pi^2} \left( \frac{2m^*(R)}{\hbar^2} \right)^2 \int dE \kappa(E) |k_F(\vec{r})\vec{v}(k_F, E(R))|
\]

(B3)

where \(k_F(\vec{r}) = \frac{2m^*(R)}{\hbar^2}(E - V(R))^{1/2}\) is the local Fermi momentum at energy \(E\).
