Anomalous viscosity of an expanding quark-gluon plasma

M Asakawa\textsuperscript{1}, S A Bass\textsuperscript{2}, and B Müller\textsuperscript{2}

\textsuperscript{1} Department of Physics, Osaka University, Toyonaka, 560-0043, Japan
\textsuperscript{2} Department of Physics, Duke University, Durham, NC 27708, USA

E-mail: yuki@phys.sci.osaka-u.ac.jp

Abstract. We argue that an expanding quark-gluon plasma has an anomalous viscosity, which arises from interactions with dynamically generated colour fields. The anomalous viscosity dominates over the collisional viscosity for large velocity gradients or weak coupling. This effect may provide an explanation for the apparent near perfect liquidity of the matter produced in nuclear collisions at RHIC without the assumption that it is a strongly coupled state.

1. Introduction

Measurements of the anisotropic collective flow of hadrons emitted in noncentral collisions of heavy nuclei at the Relativistic Heavy Ion Collider (RHIC) are in remarkably good agreement with the predictions of ideal relativistic fluid dynamics. The comparison between data and calculations indicates that $\eta$ cannot be much larger than the postulated lower bound $\eta_{\text{min}} = s/4\pi$ \textsuperscript{1, 2, 3}. This have led to speculations that the matter produced at RHIC is a strongly coupled quark-gluon plasma (sQGP). The possible microscopic structure of such a state is not well understood at present \textsuperscript{4, 5}.

Here we present an alternative mechanism that may be responsible for a small viscosity of a weakly coupled, but expanding quark-gluon plasma. The new mechanism is based on the theory of particle transport in turbulent plasmas \textsuperscript{6, 7}. Such plasmas are characterized by strongly excited random field modes in certain regimes of instability, which coherently scatter the charged particles and thus reduce the rate of momentum transport. The scattering by turbulent fields in electromagnetic plasmas is known to greatly reduce the viscosity \textsuperscript{8, 9} of the plasma. Following Abe and Niu \textsuperscript{9}, we call the contribution from turbulent fields to transport coefficients “anomalous”.

The sufficient condition for the spontaneous formation of turbulent, partially coherent fields is the presence of instabilities in the gauge field due to the presence of charged particles. This condition is met in electromagnetic plasmas with an anisotropic momentum distribution of the charged particles \textsuperscript{10}, and it is known to be satisfied in quark-gluon plasmas with an anisotropic momentum distribution of thermal partons \textsuperscript{11, 12, 13}. 


The turbulent plasma fields induce an additional, anomalous contribution to the viscosity, which we denote as $\eta_A$. This anomalous viscosity decreases with increasing strength of the turbulent fields. Since the amplitude of the turbulent fields grows with the magnitude of the momentum anisotropy, a large anisotropy will lead to a small value of $\eta_A$. Because the relaxation rates due to different processes are additive, the total viscosity is given by $\eta^{-1} = \eta_A^{-1} + \eta_C^{-1}$, where $\eta_C$ is the collisional shear viscosity. This implies that $\eta_A$ dominates the total shear viscosity, if it is smaller than $\eta_C$. In that limit, the anomalous mechanism exhibits a stable equilibrium in which the viscosity regulates itself: The anisotropy grows with $\eta$, but an increased anisotropy tends to suppress $\eta_A$ and thus $\eta \approx \eta_A$.

2. Anomalous viscosity

Here we give a heuristic derivation of $\eta_A$. For more systematic derivations, see references [14, 15]. According to classical transport theory, the shear viscosity is given by,

$$\eta \approx \frac{1}{3} n \bar{p} \lambda_f,$$

(1)

where $n$ denotes the particle density, $\bar{p}$ is the thermal momentum, and $\lambda_f$ is the mean free path. For a weakly coupled quark-gluon plasma, $n \approx 5T^3$ and $\bar{p} \approx 3T$. The mean free path depends on the mechanism under consideration. $\eta_C$ is obtained by expressing the mean free path in terms of the transport cross section $\lambda_f^{(C)} = (n \sigma)_{tr}^{-1}$. Using the perturbative QCD expression for the transport cross section in a quark-gluon plasma yields the result, which agrees parametrically with the result for the collisional shear viscosity in leading logarithmic approximation [16]. The anomalous viscosity is determined by the same relation (1) for $\eta$, but the mean free path is now obtained by counting the number of colour field domains a thermal parton has to traverse in order to “forget” its original direction of motion. If we denote the field strength generically by $B^a$ ($a$ denotes the colour index), a single coherent domain of size $r_m$ causes a momentum deflection of the order of $\Delta p \sim gQ^2B^2r_m$, where $Q^a$ is the colour charge of the parton.

If different field domains are uncorrelated, the mean free path due to the action of the turbulent fields is given by

$$\lambda_f^{(A)} = r_m \langle (\bar{p}/\Delta p)^2 \rangle \sim \frac{\bar{p}^3}{g^2Q^2(B^2)r_m}.$$  

(2)

The anomalous shear viscosity thus takes the form:

$$\eta_A \sim \frac{n \bar{p}^3}{3g^2Q^2(B^2)r_m} \sim \frac{9sT^3}{4g^2Q^2(B^2)r_m}.$$  

(3)

The argument now comes down to an estimate of the average field intensity $\langle B^2 \rangle$ and size $r_m$ of a domain. We first note that the size is given by the characteristic wave length of the unstable field modes. Near thermal equilibrium, the parameter describing the influence of hard thermal partons on the soft colour field modes is the colour-electric screening mass $m_D \sim gT$. Introducing a dimensionless parameter $\xi$ for the magnitude
of the momentum space anisotropy \[13\], the wave vector domain of unstable modes is \( k^2 \leq \xi m_B^2 \). Thus \( r_m \sim \xi^{-1/2}(gT)^{-1} \). The exponential growth of the unstable soft field modes is saturated, when the nonlinearities in the Yang-Mills equation become of the same order as the gradient term: \( g|A| \sim k \), which implies that the field energy in the unstable mode is of the order of \( g^2\langle B^2 \rangle \sim k^4 \sim \xi^2 m_B^4 \). The denominator in (3) thus has the characteristic size, at saturation:

\[
g^2Q^2\langle B^2 \rangle r_m \sim \xi^{3/2}m_B^3 \sim \xi^{3/2}(gT)^3.
\]

If our analysis, which requires confirmation by numerical simulations, is correct, it gives the following relation for the anomalous viscosity:

\[
\eta_A \sim \frac{s}{g^3\xi^{3/2}}.
\]

We conclude that \( \eta_A \) will be smaller than \( \eta_C \), if the coupling constant \( g \) is sufficiently small and the anisotropy parameter \( \xi \) is sufficiently large.

Reference \[13\] uses the following parametrization of the anisotropic momentum distribution:

\[
f(p) = f_0 \left( \sqrt{p^2 + \xi(p \cdot \hat{n})^2} \right) \approx f_0(p) - \frac{\xi(p \cdot \hat{n})^2}{2EpT}f_0(1 \pm f_0),
\]

where \( f_0 \) is the thermal equilibrium distribution. For the special case of boost-invariant longitudinal flow, we obtain a relation between \( \eta \) and \( \xi \), which takes the form (for a massless parton gas):

\[
\xi = 10^6 \frac{\eta s}{n} \left| \frac{\nabla u}{T} \right|, \quad \left| \nabla u \right| \equiv \left[ \frac{3}{2} (\nabla u)_{ij}(\nabla u)_{ji} \right]^{1/2} = \frac{1}{\tau}.
\]

Combining (5) and (7) and setting \( \eta = \eta_A \), we obtain:

\[
\frac{\eta_A}{s} = \tilde{c}_0 \left( \frac{T}{g^2|\nabla u|} \right)^{3/5}.
\]

The viscous contribution to the stress tensor is proportional to \( \eta|\nabla u| \). For the collisional viscosity, this implies that the stress tensor grows linearly with \( |\nabla u| \). The anomalous shear viscosity \[8\], on the other hand, is a decreasing function of the velocity gradient; its contribution to the stress tensor grows like \( |\nabla u|^{2/5} \) for our scaling assumptions. The unusual dependence of \( \eta_A \) on \( |\nabla u| \) certainly justifies the term “anomalous viscosity”.

The different dependence of the collisional and the anomalous viscous stress tensors on the velocity gradient is shown schematically in figure \[1\]. For very small gradients the linear dependence of the collisional viscous stress tensor dominates, but for larger velocity gradients the lower power associated with the anomalous shear viscosity will assert its dominance. The precise location of the cross-over between the two domains depends on the value of the numerical constant \( \tilde{c}_0 \), but we can deduce from \[8\] that the cross-over point shifts to lower values of \( |\nabla u| \) with decreasing coupling constant \( g \). We also show in the figure the effect of choosing a different power \( n \) in the scaling law, \( g^2\langle B^2 \rangle = b_0g^4T^4\xi^n \) for the energy density of the turbulent colour fields. Finally, we
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note that the Kovtun-Son-Starinets (KSS) bound $\eta \geq s/4\pi$ [3] does not apply to the anomalous viscosity.

We have discussed the anomalous viscosity in an anisotropically expanding quark-gluon plasma. By reducing the shear viscosity of a weakly coupled, but expanding quark-gluon plasma, this mechanism could possibly explain the observations of the RHIC experiments without the assumption of a strongly coupled plasma state.

Figure 1. Schematic representation of the dependence of the collisional and anomalous viscous stress on the velocity gradient. The collisional viscous stress is shown by the linearly rising line; the anomalous viscous stress is shown by the curved lines for three scaling exponents of the turbulent colour field energy ($n = 1.5, 2, 2.5$). The solid lines indicate the dominant source of viscous stress in different regions of the scaled velocity gradient $|\nabla u|/T$.

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