This paper demonstrates that type Ia supernova observations which appear to provide strong support for time dilation (and thus for an expanding universe) are equally consistent with a model for a static universe.

An important consideration is the Phillips relation, a correlation between the peak luminosity and the width of type Ia supernovae, which is common to both cosmological models. The estimation of the fiducial constant, a quantity which is the same for all supernovae, requires a combined fit over global constants and the properties of each type Ia supernova. This estimate which incorporates the Phillips relation is composed of peak magnitude and width (of the light curve) for each supernova such that the combination is the same for both models but the values of the individual components are different. Because the static model has a different distance modulus, which changes the absolute magnitudes, this change alters the value of the widths so that the contribution from each supernova to each fiducial estimate is essentially the same.

The major result of this paper is to show that the increase of widths of type Ia supernovae with redshift is what would be expected if an expansion model analysis were applied to data from a static universe. In particular this paper shows that the estimation of widths as well as magnitudes is affected by the use of a different distance modulus in conjunction with the Phillips relation.

It is argued that the photometric-redshift relation is really a photometric-width relation that is consistent with a static universe. Similarly, observations of spectroscopic ages are a measure of the light curve width and are consistent with a static universe.

Furthermore as a separate but related issue it is shown that in a static model the density distribution of type Ia supernovae as a function of redshift agrees with the observations.

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I. INTRODUCTION

The main aim of this paper is to show that current type Ia supernovae (for brevity SNe) observations by [1] (hereafter C11) are completely consistent with a static universe.

Modern cosmology is dominated by the Big Bang theory, which attempts to bring together observational astronomy and particle physics. Type Ia supernovae produce consistent peak luminosities because of the uniform mass of white dwarfs that explode via the accretion mechanism. The stability of SNe masses and of their light curves allows these explosions to be used as “standard candles” to measure the distance to their host galaxies.

A fiducial constant is defined to be a property of the supernova that has no dependence on redshift and is, within statistical fluctuations, identical for each supernova. In early work the fiducial constant was taken to be the absolute peak magnitude.

The observed Hubble redshift, \( z \), is defined as the ratio of the observed wavelength to the emitted wavelength minus one. Thus in an expanding universe the ratio of any observed time period to the emitted time period is...
identical to the ratio of the wavelengths, namely \((1 + z)\). This is true for any time interval and is the time dilation. Thus in an expanding universe the stretch factor, defined as the width of the light curve divided by \((1 + z)\), is considered to be an intrinsic property of SNe. An observed characteristic of SNe is the strong positive correlation between the peak luminosity and the width of the light curve so that on average an increase in the observed peak luminosity is accompanied by an increase in the width of the light curve. This relation is referred to as the Phillips relation \([\ref{Phillips}]\) and is used to provide a correction to the magnitude that results in a better estimate for the fiducial constant than the magnitude alone. For example it is shown that the estimated standard deviation of the magnitude of 0.192 mag is reduced to 0.125 mag by using corrections provided by the Phillips relation. A correlation between the colour-measure and the peak magnitude can also be used to make a further improvement in the estimate of the fiducial constant.

The first strong evidence for time dilation in type Ia supernovae was provided by \([\ref{SN1}]\) with one supernova and \([\ref{SN2}]\) with seven SNe. This was quickly followed by multiple SNe results from \([\ref{SNLS}]\). These papers record developments in both SNe observations and analysis, the results of which are asserted to provide strong evidence for an expanding universe.

Nevertheless this paper argues that, with a reanalysis, the observations can be shown to be consistent with a static universe. This reanalysis examines all the observations within the paradigm of a static model, in which there is no time dilation and where redshifts are not due to expansion. Note that in a static model the “stretch factor” has no meaning and the intrinsic property of SNe is width. In the static model considered here, redshifts are due to a tired-light process.

The Phillips relation means that the correlation between width and absolute peak magnitude is such that a change in magnitude due to using a different distance modulus will also bring about a change in width. It is argued that changing the distance modulus from that for an expanding model to that for a static model produces different widths which increase with redshift, in a manner similar to but not because of time dilation. In addition it is shown that if SNe are selected by magnitude and stretch factor limits consistent with an expanding universe, the same limits applied to static universe will select SNe that have widths with a \((1 + z)\) dependence.

In the following analysis it is assumed that the peak apparent magnitude for each supernova is the same for each model. Since the fiducial constant is an intrinsic property of the supernova, then apart from very minor global effects, for each supernova the contribution it makes to the equation used to estimate the fiducial constant in an expanding model is the same as its contribution to the equation used to estimate the fiducial constant for a static model.

Apart from the dependence of width on redshift there are two more findings that appear to support the expanding model. First that is the apparent dependence of photometric-redshift observations on redshift. These are observations that photometric properties of supernova spectra, as distinct from spectral wavelength measurements used to determine redshift, show a redshift dependence. An example is the colour-measure discussed later. It is argued that this relationship is explained by a dependence on width and is therefore consistent with a static model. Second the age of a spectrum is the number of days between the observation of the spectrum and the epoch of the peak magnitude of the supernova. The ability to determine the age from subtle changes in the spectrum provides a method of estimating the width of the light curve. Traditionally this is taken to be strong evidence for time dilation but this paper shows that such estimates of widths are also consistent with a static universe.

In a separate analysis of density of SNe observations it is shown that the static model can predict can predict the density distribution of the Supernova Legacy Survey (SNLS) SNe as a function of redshift without the need for evolution.

Section \([\text{III}]\) of this paper introduces the SNe data set provided by C11 which provides the redshifts, the apparent peak magnitudes, the stretch factors (the light curve width divided by the time dilation), and the colour-measures for each type Ia supernova. This is followed by a discussion of the Phillips relation, including definitions of notations to be used in this paper.

Section \([\text{IV}]\) provides regressions as a function of redshift, \(z\), for fiducial constants, stretch factors, absolute magnitudes and colour measures and discusses the significance of these regressions.

Section \([\text{V}]\) provides the analysis of the C11 data in the context of a static model and shows how the absolute magnitudes and widths for a static universe can be derived from that for an expanding universe by using the Phillips relation and the requirement that apparent magnitudes and fiducial constants are the same in both cosmologies. It is also shown that although the C11 peak absolute magnitude and the stretch factor have significant redshift anomalies, these anomalies are not present in the static model.

Sub-section \([\text{IV A}]\) details the mechanism of selection and its implications. Section \([\text{IV B}]\) considers the basic statistical properties of the C11 data and shows that an inherent statistical property permits the value of the Phillips coefficient to be estimated from the observed rms values. The last sub-section considers the statistical implications of selection and shows how the variance of the magnitudes, widths and fiducial constants depend on the size of the selection windows.

Sections \([\text{VI}]\) and \([\text{VII}]\) examine photometric and spectroscopic indicators of redshift. It is argued that in a static universe the photometric-redshift relation is really a photometric-width relation. It has been posited that spectral age observations provide independent proof of time dilation. However in a static universe, although
spectral changes are a valid measure of width it is shown that this does not necessarily indicate time dilution.

Section VIII shows that in a static universe the observed rate of SNe is independent of redshift and depends only on volume surveyed and the area and duration of the survey. The predicted density of SNe as a function of redshift shows good agreement with the number distribution from the SNLS survey.

Section IX provides a Monte Carlo simulation of widths showing how, in a static universe, an analysis using an expansion model can produce a redshift dependence for the fitted widths.

Section X concludes with a brief discussion and summary of the important findings and summarizes the conclusions.

The distance modulus for the static model is given by equation (A3) and comes from a static cosmology, Curvature Cosmology, briefly described in Appendix A. Note that this distance modulus is theoretical and has no free parameters. The Big Bang distance-modulus used is for the modified Λ-CDM model (the required equations are provided in Appendix B). For both cosmologies the reduced Hubble constant is $h=0.7$. In order to avoid ambiguity all measurements dependent on the expansion model are denoted by the suffix “B” (Big Bang), whereas all measurements dependent on the static model are denoted by the suffix “C” (Curvature Cosmology).

II. THE OBSERVATIONS

A. The SNe data set

Recently C11 [1] have provided a well calibrated list of 472 SNe from the Supernova Legacy Survey (SNLS), including nearby SNe and those revealed by the Hubble Sky Telescope (HST) [5] and the Sloan Digital Sky Survey (SDSS) [6]. For each SNe, C11 provide the redshift, $z$, the apparent B band peak magnitude, $m_B$, the stretch factor, $s_B$, and the colour-measure, $C_B$, all with measurement uncertainty estimates. In this paper the widths are recovered from the stretch factors using the equation $w = (1 + z)s$ where the redshift ($z$) is measured from spectral lines. In order to simplify the analysis a colour-luminosity correction of $-3.16 C_B$ is added to the magnitudes provided by C11 to get corrected apparent magnitudes used here. Then to the first order these modified magnitudes are independent of the colour-measure. The significance of the colour-luminosity relation is discussed in section VII below.

The selection criterion for the stretch factors was that all values are within the range $0.3 \leq s \leq 1.3$. C11 state that magnitude selection was that all magnitudes are within a window about expected magnitude with width $\pm 3.2 \sigma = \pm 0.282$ mag (where $\sigma = 0.088$ is the number-weighted mean square of the intrinsic $\sigma_{int}$ from table 4 in C11).

B. The Phillips relation

The Phillips relation is central to the analysis presented in this paper. Phillips [2] found that the absolute peak luminosity of SNe appear to be tightly correlated with the rate of decline of the B light curve. The correlation may be interpreted as being between magnitude and light curve width. The Phillips relation is intrinsic to the SNe and thus independent of any cosmological model. Rather than using peak luminosity and width the more useful variables are peak magnitude and width. In order to simplify later expressions in this paper the width and stretch factors are measured in magnitude units. Define a new variable, $W$, such that $W = 2.5 \log(w)$ where the width, $w$, is relative to a standard light curve. Thus the reference value of $W$ is zero. Similarly define the stretch factor measured in magnitudes as $S = 2.5 \log(s) \approx 1.086(s - 1)$ and define a new redshift variable $Z = 2.5 \log(1 + z) \approx 1.086z$ which is the redshift measured in magnitudes. Thus the relationship $w = (1 + z)s$ becomes $W = Z + Z$. With this formulation the Phillips relation is equally applicable to the stretch factor and to the width.

The Phillips relation between the peak apparent magnitude, $m$, and $W$ is defined by the equation $m = m_0 - \alpha W$, where $m_0$ is the expected apparent peak magnitude and where $\alpha$ is the slope and by convention is positive. This equation corresponds to a luminosity equation: luminosity $\propto w^\alpha$. C11 provide a range of values from 1.371 to 1.45 for $\alpha$ depending on how the uncertainties are treated whereas [4] have values near 1.39. Allowing for the 1.086 factor a reasonable value to be used here is $\alpha = 1.3$.

In the current notation the use of the Phillips relation at a particular redshift requires that, within statistical fluctuations, $m + \alpha W$ is constant. Early observations showed that all SNe have about the same magnitude. Then if the cosmology is correct $m + \alpha W$ is a better fiducial constant than $m$ and can be used for cosmological investigations.

III. EXPANSION MODEL

Although the literature on the analysis of SNe in an expansion model is comprehensive and extensive [8], the following re-analysis provides a brief summary of results for later comparison with those from a static model. There are two reasons for this reanalysis, first is to put them in the same form as the later results and second to provide results using $S_B$ (the stretch factor measured in magnitudes). Since $W = Z + Z$ any variation in $W$ has the same variation in $S$, the estimate of the fiducial constant $E_B$ is the absolute magnitude corrected for the Phillips relation and for any type Ia supernova with apparent magnitude, $m$, it is

$$E_B = M_B + \alpha S_B = m - \mu_B + \alpha S_B,$$  \hspace{1cm} (1)
where $M = m - \mu_B$ is the absolute magnitude for an expansion cosmology and $\mu_B$ (equation 6 below) is the distance modulus. Because of the Phillips relation the value of the stretch factor depends on the absolute magnitude and therefore depends on the cosmology used. It is $E_B$ that is an estimate of the fiducial constant $M_0$ deemed to be constant for all SNe. Since at any redshift the expected value of the stretch factor, $S$, is zero then the expected value of $E_B$ is $M_0$. The C11 equation (their equation (2)) for the estimate of the fiducial constant which is equivalent to equation (1) is (allowing for the previously applied colour-measure correction)

$$E_B^1 = M_B + 1.397(s_B - 1).$$

The raw data consists of observations of apparent magnitude for each type Ia supernova at distinct epochs. Part of the C11 analysis is to determine a reference light curve that is used to measure the value of the peak luminosity and the width for each supernova. Since in the expansion model the observations will be time dilated, the epoch differences are reduced to a rest frame before they are combined into the reference light curve.

The next step in the C11 analysis is to determine, for each supernova, the values for the peak magnitude, the stretch factor, and the epoch of the peak luminosity, together with global variables $M_0$, $\alpha$, $\beta$ (the colour-measure coefficient), distance modulus parameters, and other auxiliary parameters such as filter gain factors. This is done by a weighted minimization of $(E_B - M_0)^2$ over all the SNe using rest frame epoch differences.

Early work showed that there were systematic variations in the estimates of the fiducial constant that led to the inclusion of the equation of state parameter $w^*$ in the expansion model distance modulus that could reduce the variations. C11 found that the parameter, $w^*$, has a value $w^* = -0.91$, whereas [7] found that $w^* = -1.069$. Since its actual value is not critical for this paper the value of $w^*$ is chosen to be $w^* = -1.11$, so that $E_B$ would be the best fiducial constant with the values for the magnitudes and stretch factors provided by C11.

Table I shows results for important regressions as a function of redshift $(z)$ for both cosmological models. In all rows the regressions were for the complete 472 SNe. For analytic functions of $z$ the regression was done after evaluating them at the 472 redshifts. Row 1 shows the regression for the redshift factor, $s_B$. Row 2 shows the regression for the stretch factor, $S_B = 2.5 \log(s_B)$. Row 3 shows the regression for the width, $W_B = 2.5 \log(w_B)$. Row 4 shows the regression for the colour-measure, $C_B$. Row 5 shows the regression for the magnitude, $M_B$. Row 6 shows the regression for the fiducial constant, $E_B$. Row 7 shows the regression of the C11 expression, $E_B^1$ (equation 2). Row 8 shows the regression for the redshift function, $Z = 2.5 \log(1 + z)$.

Some results, obtained later (section IV), from the static cosmological model are also shown in table I in which row 9 shows the regression for the absolute magnitude, $M_C$, row 10 shows the regression for the fiducial constant, $E_C$. Row 11 shows the regression for the static model stretch factor, $S_C = W_C - Z$; and row 12 shows the regression for the modified static model magnitude, $M_C^*$. Row 6 of table I shows that $E_B$, with its insignificant slope and small variance, is a good fiducial constant that agrees with the C11 fiducial constant $E_B$. Furthermore the colour-measure (row 4) has a significant redshift dependence discussed in section IV below. Both $M_B$ and $S_B$ have significant redshift anomalies. Although these redshift dependencies effectively cancel in producing $E_B$, it is similar redshift dependencies in earlier versions of $E_B$ that have led to modifications to the expansion model distance modulus in order to achieve a better estimate of the fiducial constant. It is shown later that these apparent redshift anomalies for the peak magnitude and for the stretch factor can be explained by the static model.

### IV. STATIC MODEL

The Phillips relation for a static universe and for any type Ia supernova is

$$E_C = M_C + \alpha W_C = m - \mu_C + \alpha W_C,$$

where $M_C = m - \mu_C$ is the observed peak absolute magnitude, $m$ is the apparent peak magnitude, and the static cosmology distance modulus is $\mu_C$ (equation 4). For a static model and for all redshifts the expected value of $W_C$ is zero and the expected value for $E_C$ and $M_C$ is $M_0$. Modification of the C11 analysis to suit a static model would require the use of equation (3) rather than equation (1), the use of the distance modulus $\mu_C$ to determine the light curve, the measurement of the light curve in terms of width rather than stretch factor, and the determination of the coefficients $M_0$, $\alpha$, $\beta$, and all the other auxiliary parameters. This involves extensive modifications to the complex computer program used to do the analysis.

| Row | Variable | Offset | Slope |
|-----|----------|--------|-------|
| 1   | $s_B$    | 0.963  | ± 0.001 | 0.100  | ± 0.004 |
| 2   | $S_B$    | -0.016 | ± 0.001 | 0.090  | ± 0.004 |
| 3   | $W_B$    | -0.008 | ± 0.002 | 0.936  | ± 0.006 |
| 4   | $C_B$    | 0.028  | ± 0.002 | -0.059 | ± 0.007 |
| 5   | $M_B$    | -19.150| ± 0.002 | -0.141 | ± 0.007 |
| 6   | $E_B$    | -19.203| ± 0.003 | 0.008  | ± 0.009 |
| 7   | $E_B^1$  | -19.193| ± 0.003 | -0.002 | ± 0.009 |
| 8   | $Z$      | 0.032  | ± 0.002 | 0.758  | ± 0.003 |
| 9   | $M_C$    | -19.159| ± 0.002 | -0.993 | ± 0.007 |
| 10  | $E_C$    | -19.203| ± 0.002 | 0.008  | ± 0.009 |
| 11  | $S_C$    | -0.061 | ± 0.008 | -0.014 | ± 0.016 |
| 12  | $M_C^*$  | -19.133| ± 0.002 | 0.051  | ± 0.007 |
An alternative approach is to adopt the hypothesis that the universe is static and then determine what would be the results of an analysis based on the expansion model. The connecting point between the two models is that the observations of magnitude as a function of epoch for each type Ia supernova are reduced to a peak observed apparent magnitude that is common to both models.

In an expanding model the reference light curve is determined relative to the epoch difference divided by $1 + z$. In a static model the divisor is the width. Consider a supernova that at an epoch $d$ measured from the epoch of peak luminosity has the same relative luminosity as the reference light curve at the position $\delta$. Clearly in static model the width is $w_C = d/\delta$. However in an expanding model the stretch factor $s_B = d/(1 + z)\delta)$. Hence $w_C = (1 + z)s_B$. Although it appears that the static model width for a supernova is identical the expansion model width there is a subtle difference in that the $1 + z$ is known beforehand but the static model width $w_C$ must be estimated during the fitting procedure. Although this can be done by an iterative process it does mean that there can be small but significant differences between the $w_B$ and $w_C$. Nevertheless since the C11 fiducial estimate equation uses the stretch factors and not the widths a correction for the $1 + z$ factor must be included in order to convert the stretch factor estimate to the static model width. Since the C11 reduces all the epoch differences to the rest frame by dividing them by $1 + z$ the its effects must be removed by adding $\alpha Z$ to the contribution of each supernova to the estimate of the fiducial constant, since the static analysis does not include any $1 + z$ factors that are due to "time dilation".

For simplicity we assume that the analysis for all the global variables has been done, including determination of the reference light curve, so that the only variables to be determined are the magnitude, width, and epoch of peak-light for each SNe and the estimate of the fiducial constant $E_C$. In the expansion model analysis all the essential information about a particular supernova is contained in the fitted values $M_B$ and $S_B$. What is required is the connection between these two variables and the static model equivalents $M_C$ and $W_C$. Thus ignoring very small changes to the global variables the contribution of a particular supernova to the estimate of the fiducial constant should be identical for both cosmologies and therefore from equations 4 and 5 we get

$$m - \mu_C + \alpha W_C = m - \mu_B + \alpha S_B + 2\alpha Z,$$

where the last term allows for the conversion of the stretch factor and for the erroneous conversion of the input epoch differences to the rest frame. Then equating the apparent magnitudes, putting $S_B = W_B - Z$, and rearranging gives

$$W_C = W_B - f(z)/\alpha.$$  

where $f(z)$ is defined to be

$$f(z) = \mu_B - \mu_C - \alpha Z.$$  

What is remarkable is that $f(z)$ is close to zero over the redshift range of the observed SNe. The function $f(z)$ starts at zero and has a maximum value of $\approx 0.15$ mag near $z = 0.8$ and falling to $0.14$ mag at $z = 1.4$. The root mean square of $f(z)$ using the 472 SNe redshifts is $0.055$ mag. In general there is no a priori reason why $f(z)$ should be so small. Note that these properties of $f(z)$ are relevant only for the C11 redshift range.

Then using equation 5 and taking into account that $E_C = E_B$ we get

$$M_C = M_B - \alpha Z + f(z). \tag{7}$$

In order to directly compare the magnitudes, define a modified absolute magnitude by $M_C^*$ = $M_B + f(z)$ which is the static model magnitude without the $\alpha Z$ term. A regression for $M_C^*$ is shown in row 12 of table I. Compared to $M_B$ (row 5) the slope has changed sign and its absolute value is smaller. The value of the slope of $M_C^*$ is sensitive to the value of $\alpha$. If $\alpha = 1.37$ (rather than 1.3) the slope is negligible.

The corrected stretch factor is $S_C = S_B - f(z)/\alpha$ where its regression is shown as row 11 in table I. Since its slope, $-0.014 \pm 0.016$, is insignificant this shows that the static model can explain the anomalous redshift dependence of $S_B$.

The regression for $M_C$ (equation 7) is shown in row 9 of table I and its slope of $(-0.993 \pm 0.007)$ is in agreement with the expected slope $-\alpha Z = -1.3 \times 0.758 = -0.985$ mag (from row 8 in table I). This agreement of the observed slope with the expected slope (slope difference $0.008 \pm 0.007$) is strong support for the static model.

Figure 1 shows a plot of $E_C = E_B$ with $\alpha = 1.3$, as a function of redshift. The individual results for each of 472 SNe were collected for each survey and then binned in increasing order of redshift with 29 or 30 SNe (only 14 SNe for the HST survey) in each bin. The points in the figure show the mean value and the error bars show the estimated standard deviation of the mean for each bin. The regression for $E_B$ is shown in row 10 of table I. The binned regression is $E_C = -19.202 \pm 0.011 - (0.014 \pm 0.021)z$ with the fit to a constant value having $\chi^2_n = 13.82$ (probability of this value or larger is 0.46).

It can be argued that the adjustment of free parameters such as $\Omega_M$ and $w$ in $\mu_B$ has been done to improve the fit of an expansion model to data which actually comes from the static model. As mentioned in Appendix A, the Curvature Cosmology distance modulus calculated from theory, has excellent agreement with all other major cosmological observations and has no free parameters.

### A. SNe selection process

The fiducial constant given by equation 6 is, within statistical variations, an exact relationship. Furthermore in a static universe the expected apparent magnitude at any redshift must be $m_0 = M_0 + \mu_C$. If $m$ is an apparent
where $f$ closely mimics expansion time dilation. Using an expanding model the result is a width selection in a static universe if there is magnitude selection that is done by stretch factor selection or by magnitude selection. The C11 selection (a) if its magnitude is close to the magnitude selection will dominate and assuming a static model. For example equation (8) implies that the widths it is 0.045 mag. Then subtracting (in quadrature) these measurement uncertainties from each rms value produces the corrected values $M_{\text{rms}} = 0.192$ mag, $W_{\text{rms}} = 0.113$ mag, and $E_{\text{rms}} = 0.125$ mag.

Using these values we get estimates of the standard deviations $\hat{\varepsilon} = 0.124 \pm 0.011$ mag, $\hat{\eta} = 0.113 \pm 0.010$ mag and $\hat{\xi} = 0.011 \pm 0.010$ mag where the uncertainties are computed assuming Gaussian distributions. The negligible value for $\hat{\xi}$ implies that all of the intrinsic variation in the width is due to the common component, $\eta$. Thus the width and the magnitude are locked together in accordance with equation (8).

If we assume that $\hat{\xi}$ is zero we can use these rms values to estimate the parameter $\alpha$. In this case $M_{\text{rms}} = \sqrt{\varepsilon^2 + (\alpha \eta)^2} = 0.193$ mag, $W_{\text{rms}} = \eta = 0.113$ mag, and $E_{\text{rms}} = \varepsilon = 0.124$. Solving these equations provides the estimate $\hat{\alpha} = 1.29$ which is in excellent agreement with the assumed value. It might be argued that this analysis might just be returning the value of $\alpha$ used to define the fiducial constant (equation 2) but it is easy to show that this has little effect. For an input $\alpha = 1.2$ the output value is 1.30 and for an input $\alpha = 1.4$ the output value is 1.28, which shows that the output value is almost independent of the input value. The best self consistent estimate is $\alpha = 1.29$.

C. Statistical implications

The observed rms values for width and magnitude are of order 0.1-0.2 mag which appears to be consistent with the large range of widths that are observed in a static model. For example equation (8) implies that $W_C$ can be as large as the maximum value of $\alpha Z$ which is just under one. Obviously from equation (8) the intrinsic magnitudes must have a similar range. It is shown below that the rms values are primarily determined by the width of the selection window.

The actual mechanism for selecting the SNe used by C11 is a combination of stretch factor selection and magnitude selection. The C11 SNe are selected (a) if its magnitude is in the window of size 0.564 mag (section II A) centered on its expected value and (b) the width, $W$, is within the window of size 0.538 mag (section II A) centered on its expected value. Since the size of the magnitude window in width units is 0.564/1.3 = 0.434 mag the magnitude selection will dominate and assuming a uniform distribution over the selection window the expected standard deviation for the fiducial constant $E_C$ is 0.564/\sqrt{2} = 0.16 mag. This value is in reasonable agreement with the expected value $E_C = 1.30$ mag.

B. Statistical properties of this SNe data

The basic statistical properties of the C11 data are investigated in order to determine whether they are in accordance with equation (3) and to show that they can be used to estimate the parameter $\alpha$. A statistical model for the SNe is to express the variation in the magnitude as $\delta M = \varepsilon - \alpha \eta$ and the variation in the width as $\delta W = \xi + \eta$. Then the variation in the fiducial constant is $\delta E = \varepsilon + \alpha \xi$ where $\varepsilon$, $\eta$ and $\xi$ are uncorrelated random variables with standard deviations $\varepsilon$, $\eta$ and $\xi$ respectively. Analysis of 250 SNe with redshifts less than 0.4 provides the expansion model values $M_{\text{rms}} = 0.196$ mag, $W_{\text{rms}} = 0.122$ mag, and $E_{\text{rms}} = 0.138$ mag. In order to reduce any redshift influence each variable in this analysis had a small linear redshift dependence removed before evaluation. Now the square root of the mean squared measurement uncertainty values (provided by C11) for the magnitudes of these SNe is 0.040 mag and for the widths it is 0.045 mag. Then subtracting (in quadrature) these measurement uncertainties from the above rms values produces the corrected values $M_{\text{rms}} = 0.192$ mag, $W_{\text{rms}} = 0.113$ mag, and $E_{\text{rms}} = 0.125$ mag.

FIG. 1. Observed fiducial constant, $E_C = E_B$ as a function of redshift, $z$. The horizontal solid (black) line shows the mean value is 1.30 and for an input $\alpha = 0.45$ the output value is 1.28, which shows that the output value is almost independent of the input value. The best self consistent estimate is $\alpha = 1.29$.
agreement with the observed $E_{\text{rms}} = 0.125$ mag. Note that since the magnitude and the width are locked together there is, apart from the measurement uncertainty, no additional variance coming from width selection.

V. PHOTOMETRIC-REDSHIFTS

Ever since [9] showed that there was a correlation between redshifts of SNe and their colour index B-V there has been a considerable effort [10] to use this correlation in order to develop a predictor of the redshift from photometric measurements. Since, as mentioned, this redshift dependence cannot be distinguished from width dependence, the correct description in a static model is photometric-width dependence.

The colour-measure provided by C11 described by [11] uses a linear relation between the magnitudes measured in four colour bands to get the colour-measure which corresponds to B-V at maximum light. The regression of in four colour bands to get the colour-measure which corresponds to B-V at maximum light. The regression of

\[ C_1 = \sigma(B-V) \]

to estimate the width $w$ which is a good predictor of redshift. However the slope of $C_1$ versus the width $w$ is $-0.52 \pm 0.005$ and since the slope of $w$ versus $z$ is $1.138 \pm 0.006$, the predicted slope is $-0.052 \times 1.138 = -0.059$. This agreement strongly suggests that the photometric-redshift relation is really a relationship between the colour index and width.

For the nearby SNe, $C_2$ has a slope of $-0.062 \pm 0.021$ versus $w$ whereas its slope versus $z$ is $-0.77 \pm 0.15$. Since the redshift range is very small this last slope is anomalous. The critical point is that the relationship between $C_2$ and width, $w$, for the nearby SNe supports the photometric-width relation. Consequently the photometric observations are consistent with a static universe.

VI. WIDTHS FROM SPECTROSCOPIC AGES

SNe show a consistent variation in characteristics of their spectra with the number of days before and after the maximum. This variation is due to changes in composition, changes in the velocity of the ejecta and the depth of penetration of the ejecta. [12] have made a comprehensive analysis of these spectra for both local SNe and 13 high redshift SNe that shows that the age (the position in the light curve from the position of the peak luminosity) of a spectrum can be estimated to within 1-3 days. If there are two or more spectra the aging rate can be estimated. In their figure 8 [12] plot these aging rates as a function of $1/(1+z)$ and find a best-fit

\[ 1/(1+z)^{0.97 \pm 0.10} \]

model that is in good agreement with an expansion cosmology. These results are confirmed by [13].

It is important to note that each template spectrum is corrected for time dilation. The process of determining the spectroscopic redshift is to cross-correlate the target spectra scaled by $(1+z)$ with the template spectra and determine the value of $z$ that gives the best correlation [14]. It has been shown that the widths in the static model agree with $(1+z)$ redshift dependence. Therefore these nominally redshift measurements are consistent with a static universe.

VII. DENSITY OF SNLS SNe

The SNLS and the SDSS surveys both use the technique of wide-field rolling survey in which the same section of the sky is repeatedly observed in a regular manner. Whenever there is sudden brightening a possible supernova is flagged. The magnitude at this position is repeatedly measured and, if it passes selection criteria, a spectrum is taken and the redshift is measured. The important aspect of this technique is that to the first order and within the selected magnitude range there is no selection on redshift. Thus in a static model the relative number of SNe that are observed as a function of redshift depends only on the differential volume at that redshift. Different surveys have different time coverage and cover quite different areas. Therefore this analysis must be applied separately to each survey. Here the analysis is limited to the SNLS survey since it covered the largest redshift range.

In a static universe it is assumed that provided they are brighter than an observational limit all SNe have a width less than a limit, $W_{\text{lim}}$, which is determined by constraints on intrinsic properties such as the local environment. The C11 data show that the three faintest (SNLS) SNe observed have magnitudes 25.23, 25.03, and 24.87 mag which are consistent with a magnitude limit of about 25.2 mag. At the redshift $z = 1.06$ (the highest redshift in the SNLS survey) and with $M_0 = -19.203$ the apparent magnitude in a static universe is

\[ m = 23.97 \]

hence the maximum width that can be observed is

\[ W_C = (25.2 - 23.97)/1.3 = 0.95, \]

\[ W_C = 2.3. \]

Since this is larger than the expected range of widths it means that, for this data in a static universe, the observational magnitude selection is not important.

Assuming that the density of supernova type Ia progenitors and their production rate for a particular survey is independent of redshift, the number expected in a survey is proportional to the density times the observed volume (equation [A3], below). The selection process for the C11 SNe requires that the expected width at any redshift $W_C = (\mu_B - \mu_C)/\alpha$ which is $W_C \approx Z$. Then the probability of observing a type Ia supernova is taken to be the area of a Gaussian distribution with mean $W$ and standard deviation of 0.122 mag (the rms of $W_B$ for all SNe with $z < 0.4$) that is less than $W_{\text{lim}}$. The value of the width limit, $W_{\text{lim}}$, was set by minimizing the $\chi^2$ for the overall fit with the result $W_{\text{lim}} = 0.782$ (i.e. $w_{\text{lim}} = 2.05$) which corresponds to a magnitude change of -1.02 mag and a luminosity increase by a factor of 2.54. Note that if as postulated $W$ has a uniform distribution
then the observed width, \( w \) has a reciprocal distribution, 
\[ p(w) = dw/(w \ln(w_{\text{lim}})). \]
Furthermore with a width selection window of 0.517 mag (section IV C) this implies that at low redshifts about 0.517/0.782 = 0.66 of the SNe are being selected. Thus approximately one third of the SNLS SNe are rejected.

The number of observed SNe in the SNLS survey are plotted as a function of redshift in figure 2. The solid (red) line shows the expected distribution for a static universe with SNLS SNe observed by the C11 selection process. For comparison the results for the expansion model assuming that the magnitudes have a Gaussian distribution with a standard deviation of 0.2 mag, and with apparent magnitude cut off at 25.2 mag is shown as the dashed (blue) line.

In order to illustrate the differences in SNe widths between the two cosmologies a simple Monte Carlo simulation was done with \( \alpha = 1.3 \). The first step is to choose a random redshift in the range 0 < \( z < 1.4 \) so that the probability of selecting that redshift was proportional to the differential volume (equation A5). Then a static model width was chosen so that the fitted width \( W_{\text{C}} = \delta + \eta \) where \( \delta \) is a random variable from the uniform distribution with the range -0.391–0.391 mag and \( \eta \) is a random Gaussian variable with a mean of zero and standard deviation 0.122 mag.

Then if an expansion model is used for the analysis the fitted width is \( W_B = W_C + (\mu_B - \mu_C)/\alpha \). Finally supernova are rejected if \(| s - 1 | > 0.3 \). A scatter plot of both widths for 1000 SNe as a function of redshift are shown in figure 2 where the rejected SNe are shown as inverted green triangles. The black line is a plot of 1 + \( z \). The fraction of SNe rejected varies from 0.23 at low redshifts to 0.31 at high redshifts which agrees with the prediction in section VII of about one third.

In both models the density was chosen to match the observed counts by using a \( \chi^2 \) fit for the first six points (with count \( \geq 5 \) and with \( z < 0.75 \)) where the selection process has negligible effect. The multiplier for equation A3 with a range of \( \pm 0.05 \) about each \( z \) value was 3.10 kpc \(^{-3} \). For all the bins with counts \( \geq 5 \) we get \( \chi^2 = 6.51 \) (probability of this value or large is 0.48).
have been done using Ubuntu Linux and the graphics have been done using the DISLIN plotting library provided by the Max-Plank-Institute in Lindau.

**Appendix A: Curvature Cosmology**

Curvature Cosmology [16] is a complete cosmology that shows excellent agreement with all major cosmological observations without needing dark matter or dark energy. (Note that [17] is an update with corrections of previous work and is essentially identical to the three Journal of Cosmology papers.) It is compatible with both (slightly modified) general relativity and quantum mechanics and obeys the perfect cosmological principle that the universe is statistically the same at all places and times. It was shown in those papers that all the major observations (except supernovae) which have been used as evidence of expansion are in fact consistent with a static universe.

This new analysis of SNe observations is based on two major hypotheses. The first hypothesis is that the Hubble redshift is due to curvature redshift, which is due to an interaction of photons with curved spacetime where they lose energy to other very low energy photons. Thus it is a tired-light model. The second hypothesis is that there is a reaction pressure (curvature pressure) acting on the material causing spacetime curvature from the acceleration of high velocity particles in curved spacetime. Since the acceleration of the particles is normal to their velocity there is no change in their energy. The major effect of curvature pressure is to provide stability in the cosmological model. The basic cosmology is for a simple universal model of a uniform high temperature plasma (cosmic gas) at a constant density in space-time with four spatial dimensions (i.e. identical to the standard expansion model but without expansion).

The theory has a good fit to the background X-ray radiation between the energies of 10–300 keV. The fitted temperature was $2.62 \pm 0.04 \times 10^9$ K (predicted temperature: $2.56 \times 10^9$ K) and the fitted density was equivalent to $N = 1.55 \pm 0.01$ hydrogen atoms per cubic meter ($2.57 \times 10^{-37}$ kg m$^{-3}$). For the simple homogeneous model this density is the only free parameter in the theory of curvature cosmology. The observations recorded in the cited references show that curvature cosmology is consistent with the observations of: Tolman surface brightness, angular size, SNe (superseded by this paper), gamma ray bursts, galaxy luminosity distributions, quasar luminosity distributions, X-ray background radiation, cosmic microwave background radiation, quasar variability, radio source counts, and the Butcher–Oemler effect. In curvature cosmology the cosmic background radiation (CMBR) is produced by the interaction of high energy electrons in the cosmic plasma with curved spacetime. The predicted temperature of the CMBR is 3.18 K to be compared with an observed value of 2.725 K. The prediction does depend on the nuclei mix in the cosm-
mic gas and could vary from this value by several tenths of a degree. It is argued that its black body spectrum arises from the large number of curvature redshift interactions undergone by the CMBR photons. Curvature redshift can explain the velocity dispersion of galaxies in the Coma cluster without requiring dark energy. Finally the anomalous acceleration of Pioneer 10 is explained by the effects of curvature redshift due to inter-planetary dust producing a very small decrease in the radio frequencies sent to and from the spacecraft.

An important result of curvature redshift is that the rate of energy loss by a photon (to extremely low energy secondary photons) as a function of distance, $ds$, is given by

$$\frac{1}{E} \frac{dE}{ds} = -\left(\frac{8\pi G M_H N}{c^2}\right)^{\frac{3}{2}}, \quad (A1)$$

where $M_H$ is the mass of a hydrogen atom and the density in hydrogen atoms per cubic metre is $N = \rho/M_H$. Equation (A1) shows that the energy loss is proportional to the integral of the square root of the density along the photon’s path. The Hubble constant is predicted to be

$$H = -\frac{c}{E} \frac{dE}{ds} = (8\pi G M_H N)^{\frac{3}{2}}$$
$$= 51.69 N^{\frac{3}{2}} \text{ kms}^{-1} \text{ Mpc}^{-1}$$
$$= 64.4 \pm 0.2 \text{ kms}^{-1} \text{ Mpc}^{-1} \quad (N = 1.55 \pm 0.01 m^{-3}).$$

The geometry is that of a three dimensional surface of a four dimensional hyper sphere. For this geometry the area of a three dimensional sphere with radius, $R = R_\chi$ where $\chi = \ln(1+z)/\sqrt{3}$ (work prior to 2009 has $\chi = \ln(1+z)/\sqrt{2}$), is given by

$$A(r) = 4\pi R^2 \sin^2(\chi). \quad (A3)$$

The surface is finite and $\chi$ can vary from 0 to $\pi$. The total volume $v$, is given by

$$v_C(r) = 2\pi R^3 \left[\frac{\chi - \frac{1}{2} \sin(2\chi)}{3}\right] \approx \frac{4\pi}{3} (R\chi)^3$$
$$= \frac{32.648}{h^3} \left[\frac{\chi - \frac{1}{2} \sin(2\chi)}{3}\right] \text{kpc}^3. \quad (A4)$$

The differential volume is

$$\frac{dv_C}{dz} = \frac{2\pi(1 - \cos(2\chi))(2.998\sqrt{3}/h)^3}{\sqrt{3}(1+z)} \text{kpc}^3. \quad (A5)$$

The only other result required here is the equation for the distance-modulus, $\mu_C = m - M$, which is

$$\mu_C = 5 \log \left[\frac{\sqrt{3} \sin(\chi)}{h}\right] + 2.5 \log(1 + z) + 42.384 \quad (A6)$$

where $h = H/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

**Appendix B: Expansion model functions**

The equations needed for the modified Λ-CDM model [18, 20], with $\Omega_M = 0.27$, $\Omega_k = 0$ and where $h$ is the reduced Hubble constant, are listed below. The symbol $w_*$ is used for the acceleration parameter in order to avoid confusion with the width, $w$. These equations depend on the function $E(z)$ defined here by

$$E(z) = \int_0^z \frac{dz}{\sqrt{\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{(1+w_*)}}}.$$ 

The distance modulus is

$$\mu_B(z) = 5 \log(E(z)(1+z)/h) + 42.384 \quad (B2)$$

The co-moving volume is

$$v_B(z) = \frac{4\pi}{3} (2.998 E(z)/h)^3 \text{Gpc}^3. \quad (B3)$$

The differential co-moving volume is

$$\frac{dv_B}{dz} = \frac{(2.998/h)^3 E^2(z)}{\sqrt{\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{(1+w_*)}}}. \quad (B4)$$

[1] A. Conley, J. Guy, M. Sullivan, N. Regnault, P. Astier, C. Balland, S. Basa, R. G. Carlberg, D. Fouchez, D. Hardin, I. M. Hook, D. A. Howell, R. Pain, N. Palanque-Delabrouille, K. M. Perrett, C. J. Pritchet, J. Rich, V. Ruhlmann-Kleider, D. Balam, S. Baumont, R. S. Ellis, S. Fabbro, H. K. Fakhouri, N. Fourmanoit, S. González-Gaitán, M. L. Graham, M. J. Hudson, E. Hsiao, T. Kronborg, C. Lidman, A. M. Mourao, J. D. Neill, S. Perlmutter, P. Ripoche, N. Suzuki, and E. S. Walker, [ApJS 192, 1 (2011)] arXiv:1104.1443 [astro-ph.CO].

[2] M. M. Phillips, [ApJ 413, L105 (1993)]

[3] B. Leibundgut, R. Schommer, M. Phillips, A. Riess, B. Schmidt, J. Spyromilio, J. Walsh, N. Sunzef, M. Hamuy, J. Maza, R. P. Kirshner, P. Challis, P. Garnavich, R. C. Smith, A. Dressler, and R. Ciardullo, [ApJ 466, L21 (1996)] arXiv:astro-ph/9605134.
[4] G. Goldhaber, B. Boyle, P. Buncclark, D. Carter, W. Couch, S. Deustua, M. Dopita, R. Ellis, A. V. Filippenko, S. Gabi, K. Glazebrook, A. Goobar, D. Groom, I. Hook, M. Irwin, A. Kim, M. Kim, J. Lee, T. Matheson, R. McMahon, H. Newberg, R. Pain, C. Pennypacker, S. Perlmutter, and I. Small, Nuclear Physics B Proc. Sup., Vol. SI 51, 123 (1996).

[5] A. Goobar and B. Leibundgut, Ann. Rev. of Nuclear and Particle Science 61, 251 (2011).

[6] R. Tripp, A&A 331, 815 (1998).

[7] [arXiv:1101.0444 [astro-ph.CO]]

[8] [arXiv:0908.4274 [astro-ph.CO]].

[9] D. A. Howell, M. Sullivan, A. Conley, and R. Carlberg, ApJL 667, L37 (2007) [arXiv:astro-ph/0701012].

[10] [arXiv:0908.4274 [astro-ph.CO]].

[11] [arXiv:0709.4488].

[12] [arXiv:1004.3832].

[13] [arXiv:1102.1431 [astro-ph.CO]].

[14] [arXiv:1109.0948 [astro-ph.CO]].

[15] [arXiv:1306.6423 [astro-ph.CO]].

[16] [arXiv:1010.4743 [astro-ph.CO]].

[17] [arXiv:0908.4277 [astro-ph.CO]].

[18] [arXiv:1004.3595].

[19] [arXiv:0804.3595].

[20] [arXiv:0811.4424 [astro-ph.CO]].

[21] [arXiv:1102.1431 [astro-ph.CO]].

[22] [arXiv:1206.0665 [astro-ph.CO]].

[23] [arXiv:1104.1444 [astro-ph.CO]].

[24] [arXiv:1101.0444 [astro-ph.CO]].
[16] D. F. Crawford, Aust. J. Phys. 40, 449 (1987); Aust. J. Phys. 40, 459 (1987); ApJ 377, 1 (1991); ApJ 410, 488 (1993); ApJ 440, 466 (1995); astro-ph/9407029; ApJ 441, 488 (1995); astro-ph/9407028; 52, 753 (1999), astro-ph/9904131. Curvature Cosmology (BrownWalker Press, 2006); ArXiv 0901.4169 (2009), arXiv:0901.4169 [physics.gen-ph].

[17] D. F. Crawford, ArXiv 1009.0953 (2009).

[18] D. W. Hogg, ArXiv Astrophysics e-prints (1999), arXiv:astro-ph/9905116.

[19] M. Goliath, R. Amanullah, P. Astier, A. Goobar, and R. Pain, A&A 380, 6 (2001) arXiv:astro-ph/0104009.

[20] E. M. Barboza and J. S. Alcaniz, Physics Letters B 666, 415 (2008) arXiv:0805.1713.