Digital processing with single electrons for arbitrary waveform generation of current

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We demonstrate arbitrary waveform generation of current using a GaAs-based single-electron pump. In our experiment, a digital processing algorithm known as delta–sigma modulation is incorporated into single-electron pumping to generate a density-modulated single-electron stream, by which we demonstrate the generation of arbitrary waveforms of current including sinusoidal, square, and triangular waves with a peak-to-peak amplitude of approximately 10 pA and an output bandwidth ranging from dc to close to 1 MHz. The developed current generator can be used as the precise and calculable current reference required for measurements of current noise in low-temperature environments.

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P recision measurements of current fluctuations such as shot noise and thermal noise are powerful means of revealing microscopic phenomena in charge transport flowing through nanostructures (for a review, see Refs. 1–3). In most of the works reported to date,2–9) current fluctuations were measured in a frequency range from 100 kHz to several megahertz. In these experiments, researchers first calibrated the system performance, including the transimpedance gain and noise floor, through careful measurements of the thermal noise, whose power density can be calculated using the so-called Johnson–Nyquist formula.8,9) One of the disadvantages of adopting the Johnson–Nyquist thermal noise as a calibration reference is a difficulty in estimating the electron temperature inside the relevant nanostructures, which leads to, for instance, unintentional discrepancies between the measured thermal noise and the result of the Johnson–Nyquist formula at low temperatures.9,10) To overcome this difficulty and achieve more accurate and advanced measurements of current noise so as to elucidate exotic many-body physics10) and to verify nonequilibrium transport theories,4) it is necessary to identify more precise and calculable current noise sources that can be utilized as a calibration reference in such measurements.

Extremely accurate dc current generation by means of single-electron pumping has been demonstrated recently with the goal of realizing quantum current standards.11–13) A single-electron pump can generate a dc current of \( I = e f_{\text{rep}} \), where \( e \) is the elementary charge, and \( f_{\text{rep}} \) is the repetition frequency of the pump. Remarkably, a pump error rate at the 0.2 parts per million (ppm) level has been demonstrated at a driving frequency below 1 GHz using a GaAs-based nonadiabatic single-electron pump.14,15) Furthermore, Yamahata et al. have demonstrated robust single-electron-pumping up to several gigahertz using a single trap level in silicon, allowing dc current in excess of 1 nA to be generated.16) If one extends the operation principle of single-electron pumping to generation of finite-frequency currents around 1 MHz, such a current generator can be used as a source of precise and calculable noise that can replace the Johnson–Nyquist thermal noise used as a calibration reference. Regarding the generation of finite-frequency currents implemented with single-electron pumping, Mirovsky et al. have incorporated the so-called frequency-modulation technique into pump operation to generate \( I(t) = e f(t) \) and pioneered arbitrary waveform generation of current with a period of a few kilohertz.17) Further, Nakamura et al. have mentioned another operation mechanism that generates finite-frequency currents using parallel-integrated single-electron pumps.18) However, the generation of precise finite-frequency currents in a frequency range around 1 MHz (normally targeted in current noise measurements) remains challenging.

In this study, we incorporate on–off digital modulation into the pump operation to generate a density-modulated single-electron stream and demonstrate arbitrary waveform generation of current with an output bandwidth ranging from dc to close to 1 MHz. Using single-electron pumping, dc current can be generated by the clocked transfer of single electrons with a constant repetition frequency \( f_{\text{rep}} \), as shown in Fig. 1(a). The generation of finite-frequency currents, on the other hand, requires modulation of the density of single-electron pulses. In this study, we adapt first-order delta–sigma modulation as an algorithm to implement density

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**Fig. 1.** (a) Schematic of unmodulated single-electron pumping for dc current generation. The rectangular blocks represent single electrons. (b) Schematic of a digitally modulated single-electron stream for arbitrary waveform generation of current. The solid line represents a time-domain waveform to be generated. (c) Block diagram (in \( z \)-domain) of first-order delta–sigma (\( \Delta–\Sigma \)) modulator, which encodes normalized analog input \( i(t) \) into the corresponding digital bit stream.

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**Table 1.** DC current Finite-frequency current
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| Current Type | Description |
|--------------|-------------|
| DC current   | Calculated using Johnson–Nyquist formula. |
| Finite-frequency current | Calculated using the so-called delta–sigma modulation. |
modulation. delta–sigma modulation is one of the authorized analog-to-digital conversion algorithms, by which a time-domain analog waveform can be approximated by a digitally modulated 1 bit pulse stream with a constant bit rate. To implement delta–sigma modulation with single electrons, we control whether or not a single electron is pumped. This “single-electron digital modulation” can yield a density-modulated single-electron stream with a constant sampling frequency, as shown in Fig. 1(b), where bit 1 represents the presence of a single-electron pulse, and bit 0 represents the absence of it. Consequently, a time-domain analog waveform of the current (solid line in the figure) can be generated as a density-modulated single-electron stream. To date, delta–sigma modulation has also been implemented in a Josephson arbitrary waveform generator of voltage. Inspired by the success of implementing delta–sigma modulation in a Josephson waveform generator of voltage, we demonstrate the first experimental implementation of a delta–sigma modulator with single electrons.

First, we describe a recipe for computing a digital bit code that approximates given time-domain waveforms by following the algorithm of first-order delta–sigma modulation. From a given sampling frequency $f_S$, the full-scale current can be defined as $I_{FS} = e f_S$, which is the maximum current to be pumped. In unidirectional current generation, a desired time-domain waveform of the current $i(t)$ should satisfy $0 < i(t) < I_{FS}$. It is also necessary to set $f_S$ sufficiently higher than the frequency component of $i(t)$, which is known as an oversampling condition; otherwise, the quantization noise distorts the resultant waveform. The first-order delta–sigma modulator is composed of a quantizer, an integrator, and a 1 bit delay represented by $z^{-1}$ in the $z$ domain, as depicted in the block diagram in Fig. 1(c). The quantizer outputs bit 1 when the input at the quantizer is larger than 0.5; otherwise, it outputs bit 0. According to this block diagram, the normalized input current $i(t) = I(t)/I_{FS}$ can be converted into the corresponding digital bit stream.

In this experiment, we fix the sampling frequency at $f_S = 62.5$ MHz, which is sufficiently high to obtain an output bandwidth of about 1 MHz. The full-scale current corresponding to this sampling frequency is $I_{FS} = e f_S \approx 10$ pA. The electron pumping in this experiment is unidirectional, so it is impossible to reverse the polarity. Instead, the half-scale current $I_{FS}/2 \approx 5$ pA is set as a dc offset, to which a finite-frequency waveform $I_{AC}(t)$ is added as $I(t) = I_{FS}/2 + I_{AC}(t)$. Hence, a dc current of $I_{FS}/2$ is permanently generated.

Our single-electron pump device is defined into a two-dimensional electron gas system formed in a GaAs/Alo.3Ga0.7As single heterojunction located 80 nm below the surface. On the surface of this wafer, fine Schottky electrodes (Ti/Au = 5/25 nm in thickness) are lithographically defined, as shown in the false-color electron microscope image in Fig. 2(a). Zero-dimensional confinement of electrons can be defined by applying dc gate voltages $V_G$, $V_L$, and $V_R$ to the electrodes labeled C, L, and R in the figure, respectively. In addition, the time-domain waveform $V_{DG}(t)$ generated by a function generator (Keysight 33503A) is superposed on $V_L$ using a bias tee to control the entrance barrier potential that drives single-electron pumping. The measurements are performed in a dilution refrigerator at a base temperature below 10 mK and zero magnetic field. The generated dc current $I_{DC}$ is measured using a room-temperature ammeter. On the other hand, to measure the time-domain waveform in the submegahertz frequency range, the generated current $i(t)$ is fed into a load resistor, $R_L = 1$ kΩ, placed at the lowest temperature. The voltage drop across this load resistor is then amplified by a home-made cryogenic amplifier placed at 3 K, followed by a room-temperature amplifier. The overall trans-impedance gain of this system is $g_m = R_L/A_1/A_2 = 1.8 \times 10^5$ V/A. The details of this setup are also available in Refs. 21 and 22. The amplified signal is then acquired at a sampling rate of 2 MS/s using a digitizer (National Instruments PXP-5922) to obtain the time-domain waveforms $I_{AC}(t)$ and their fast Fourier transform (FFT) amplitudes.

To generate a single-electron pulse stream digitally modulated by a bit code, the gate waveform $V_{DG}(t)$ is prepared. For instance, the $V_{DG}$ waveform shown in Fig. 2(b) can be used to generate a single-electron pulse stream digitally modulated by the bit code “101001”. As illustrated in the right panels, when the bit code is 1, a single electron is captured from the source (upper panel) and then emitted into the drain (lower panel) to generate a single-electron pulse. When the bit is 0, $V_{DG} = 0$, so no electron is emitted. The actual length of the bit code used to generate the waveforms shown in Figs. 3 through 5 is 6250 points. The corresponding $V_{DG}$ waveform prepared from each code is then loaded into the function generator to drive the single-electron pumping.

We begin with a zero finite-frequency current, that is, $I_{AC} = 0$ and $I(t) = I_{FS}/2$, to optimize the gate bias condition for proper single-electron pumping. The bit code corresponding to $I_{FS}/2$ is 1010101·, as depicted in the inset of Fig. 3(b). Using the $V_{DG}(t)$ corresponding to this bit code, we observe the generation of a finite dc current, as shown in Fig. 3(a), where $V_C = -1.21$ V, the peak-to-peak amplitude of $V_{DG}$ = 0.74 V is fixed, and $V_L$ and $V_R$ are swept to optimize the gate bias condition. Figure 3(b) shows a plot of $I_{DC}$ as a function of $V_L$ with $V_R = -0.3$ V, which demonstrates a current plateau whose value is well-quantized at $I_{FS}/2$. From this result, we confirm that single-electron pumping digitally
modulated by the bit code “1010 · · · ” can be properly operated in this gate bias condition.

Next, we advance to the generation of finite-frequency currents. To this end, we replace the bit code and operated in this gate bias condition. The concept of delta–sigma modulation can also be applied to the generation of any arbitrary waveform in currents. To this end, we replace the bit code and operated in this gate bias condition.

In state-of-the-art pump experiments, a pump error rate of the amplitude of the resultant finite-frequency currents. This result suggests that a stochastic pump error is obviously linked to the accuracy of the amplitude of the resultant finite-frequency currents. In state-of-the-art pump experiments, a pump error rate is low as the 0.2 ppm level has been achieved, implying that the generation of finite-frequency current can also reach the same level of precision.

In conclusion, we incorporated delta–sigma modulation into single-electron pump operation using a GaAs-based single-electron pump to demonstrate arbitrary waveform.
generation of current with a period of 80 kHz. The measured FFT spectra demonstrated that the output bandwidth of this current generator ranges from dc to close to 1 MHz. In this experiment, the observed peak-to-peak amplitude of about 10 pA is limited by the sampling frequency (62.5 MHz in the present experiment), but this is potentially scalable up to 1 nA by means of a state-of-the-art single-electron pump with operation frequencies up to several gigahertz.16) This higher operation frequency can also extend the output bandwidth and improve the dynamic range. The functionality and calculability offered by this finite-frequency current generator with single-electron digital modulation are suitable for developing new schemes for precision measurements of finite-frequency current such as shot noise and thermal noise.

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