Octet Magnetic Moments with Null Instantons and Semibosonized Nambu-Jona-Lasinio Model

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Abstract

It is shown that the difference between the magnetic moment results in the quark model with null instantons and semibosonized Nambu-Jona-Lasinio model lies in the description of the magnetic moment of the Λ-hyperon.

I. INTRODUCTION

Main features of the baryon magnetic moments have been understood in the frameworks of unitary symmetry [1] and quark models [2] years ago. But nowadays experimental accuracy of the baryon magnetic moments measurement is very high reaching the level of 1% [3]. So theoretician task is no more to describe them reasonably well but to search more elaborated models to account for data. Among many interesting models developed in order to solve this problem from various points of view I cite only few of them [4]- [12]. Recently in [13] we have shown that the baryon magnetic moment description in frameworks of the chiral model [11] and the quark soliton ξQSM one [12] are practically identical. The main difference from the traditional unitary symmetry approach proves to be in terms corresponding to some kind of the meson cloud contribution and in the prediction for the Λ hyperon moment. In this work I want to investigate the unitary symmetry content of two recent rather sophisticated models, that of [9] and of aforementioned [12].

I shall show that the difference between the magnetic moment results in the quark model with null instantons and semibosonized Nambu-Jona-Lasinio model lies principally in the evaluation of the magnetic moment of the Λ-hyperon.
II. MAGNETIC MOMENTS WITH NULL INSTANTONS

First I consider an interesting model with null instantons where a dynamical symmetry breaking for the magnetic moments of baryons was proposed. This effect was attributed to the contribution arising due to the time average of the two quark exchange in spin 0 state, contained in an instanton and anti-instanton ball, into the magnetic moment of the third quark. We remind here the main formulae of [9]:

\[
\begin{align*}
\mu(p) &= (1 + \frac{8}{3}z)\mu - \alpha, \\
\mu(n) &= -\frac{2}{3}\mu(1 + \frac{1}{3}z), \\
\mu(\Sigma^+) &= \frac{1}{3}(8(1 + z)\mu + \mu') - \beta, \\
\mu(\Sigma^-) &= \frac{1}{3}(-4\mu + \mu') + \beta, \\
\mu(\Xi^0) &= \frac{1}{9}(-4\mu' - 2(1 + z)\mu), \\
\mu(\Xi^-) &= \frac{1}{9}(-4\mu' + \mu) + \beta, \\
\mu(\Lambda) &= -\frac{1}{3}\mu - \frac{1}{3}(\alpha - \beta).
\end{align*}
\]

(1)

Now let us analyze the unitary symmetry content of the Eqs. (1). It is straightforward to see that the SU(3) electromagnetic current is no longer a pure octet but is broken with the Gell-Mann - Okubo [14,15] and other terms (I disregard space-time indices):

\[
\begin{align*}
J_{e^-m, symm} &= -F(B_1^\gamma B_2^\gamma - B_1^\gamma B_2^\gamma) + D(B_1^\gamma B_2^\gamma + B_1^\gamma B_2^\gamma) + \\
g_1 B_3^\gamma B_3^\gamma + g_2 B_3^\gamma B_3^\gamma + (T - \frac{2}{3}D)Sp(B_\beta B_\beta) + 3d(B_2 B_2 - B_2 B_2),
\end{align*}
\]

(2)

where

\[
\begin{align*}
\mu(p) &= F + \frac{1}{3}D + g_1 + t, \\
\mu(n) &= -\frac{2}{3}D + g_1 + t, \\
\mu(\Sigma^+) &= F + \frac{1}{3}D + t + d, \\
\mu(\Sigma^-) &= -F + \frac{1}{3}D + t - d, \\
\mu(\Xi^0) &= -\frac{2}{3}D + g_2 + t, \\
\mu(\Xi^-) &= -F + \frac{1}{3}D + g_2 + t, \\
\mu(\Lambda) &= -\frac{1}{3}D + \frac{2}{3}(g_1 + g_2) + t
\end{align*}
\]

(3)

and

\[
\begin{align*}
D &= (\mu_u - \mu_d) - \frac{1}{3}(\alpha - \beta), \\
F &= \frac{2}{3}(\mu_u - \mu_d) - \frac{1}{3}(\alpha + \beta), \\
g_1 &= -\frac{1}{3}(\mu_d - \mu_s) - \frac{1}{3}(\alpha - \beta), \\
g_2 &= -\frac{2}{3}(\mu_d - \mu_s) - \frac{1}{3}(\alpha - \beta), \\
d &= \frac{1}{2}(\alpha - \beta), \\
t &= \frac{1}{3}D = \frac{2}{3}(\mu_u + \mu_d) - \frac{1}{3}\mu_s.
\end{align*}
\]

(4)

Here [9]

\[
\begin{align*}
\mu_u &= \frac{2}{3}(1 + z)\mu, \\
\mu_d &= -\frac{1}{3}\mu, \\
\mu_s &= -\frac{1}{3}\mu
\end{align*}
\]

and \(B_1^\gamma\) is a baryon octet, \(B_1^3 = p, B_3^3 = \Xi^0\) etc. This current is just a sum of the traditional unitary electromagnetic current [9] and the traditional unitary middle-strong baryonic one [14,15] plus a nonlinear in quantum numbers term for \(\Sigma\) -hyperons. The latter term can be
attributed to some kind of meson cloud contribution. With some algebra it can be placed into nucleon magnetic moments or Ξ hyperon ones. In the model based on the phenomenological current Eq.(2) the only relation holds (the numbers under this and following relations are given in nuclear magnetons):

\[2\mu(p) + 2\mu(n) - \mu(\Sigma^+) - \mu(\Sigma^-) + 2\mu(\Xi^-) + 2\mu(\Xi^0) - 6\mu(\Lambda) = 0 \quad (5)\]

\[(10.42 - 10.04 = 0).\]

As due to Eq.(4)

\[5g_1 - g_2 + 4d = 0, \quad (6)\]

it splits into two relations Eq.(7) and Eq.(8) tacitly contained in [9]:

\[10\mu(p) + 20\mu(n) - 5\mu(\Sigma^+) - 13\mu(\Sigma^-) + 18\mu(\Xi^-) - 30\mu(\Lambda) = 0 \quad (7)\]

\[(61.4 - 62.2 = 0),\]

\[4[\mu(\Sigma^-) - \mu(\Xi^-)] - 5[\mu(n) - \mu(\Xi^0)] = 0 \quad (8)\]

\[(-2.04 + 3.30 = 0).\]

The most successful fit of [9] (those named (iv)) reads:

\[\mu = 3.02398909, \quad \mu' = 1.18108, \quad z = -0.1532, \quad \alpha = \beta = -0.18065841.\]

So the main features of the baryon magnetic moment picture in the quark model with instantons of [9] can be comprehended in terms of unitary symmetry approach.

**III. SEMIBOSONIZED NAMBU-JONA-LASINIO MODEL FOR THE BARYON MAGNETIC MOMENTS**

Now I consider another quite distinct model for the magnetic moments of baryons developed in [12] within the chiral quark soliton model. In this model, known also as the semibosonized Nambu-Jona-Lasinio model, the baryon can be considered as \(N_c\) valence quarks
coupled to the polarized Dirac sea bound by a nontrivial chiral background hedgehog field in the Hartree-Fock approximation. Magnetic moments of baryons were written in the form:

\[
\begin{pmatrix}
\mu(p) \\
\mu(n) \\
\mu(\Lambda) \\
\mu(\Sigma^+) \\
\mu(\Sigma^-) \\
\mu(\Xi^0) \\
\mu(\Xi^-)
\end{pmatrix} =
\begin{pmatrix}
-8 & 4 & -8 & -5 & -1 & 0 & 8 \\
6 & 2 & 14 & 5 & 1 & 2 & 4 \\
3 & 1 & -9 & 0 & 0 & 0 & 9 \\
-8 & 4 & -4 & -1 & 1 & 0 & 4 \\
2 & -6 & 14 & 5 & -1 & 2 & 4 \\
6 & 2 & -4 & -1 & -1 & 0 & 4 \\
2 & -6 & -8 & -5 & 1 & 0 & 8
\end{pmatrix}
\begin{pmatrix}
v \\
w \\
x \\
y \\
z \\
p \\
q
\end{pmatrix}
\] (1)

Here parameters \(v, w\) are related linearly with the usual coupling constants of the unitary symmetry approach while parameters \(x, y, z, p, q \approx m_s\) are specific for the model. Upon algebraic transformations the expressions for 6 baryons \(B(qq, q')\) can be rewritten in the form analogous to Eqs.(3) where now

\[
F = -5v + 5w - 9x - 3y - p - 2z, \quad D = -9v - 3w - 13x - 7y + 4q - p, \\
g_1 = 4x + 4y - 4q - z, \quad g_2 = 22x + 10y - 4q + 2p + z, \quad t = \frac{1}{3}(28x + 13y + 8q + 4p). \] (2)

It means that magnetic moments of the octet baryons \(B(qq, q')\) in the semibosonized Nambu-Jona-Lasinio model can also be obtained from the unitary electromagnetic current given by Eq.(2). With this current the magnetic moment of the \(\Lambda\)-hyperon reads:

\[
\mu(\Lambda)^{\null} = -\frac{1}{3}D - (8x + 5y - 8q), \] (3)

which differs from that given by Eq.(4):

\[
\mu(\Lambda)^{\NJJ} - \mu(\Lambda)^{\null} = -\frac{1}{3}(16x - 8y - 7q + p). \] (4)

Besides the relation given by Eq.(3) this is the only important difference in predictions of the two otherwise independent models.

**IV. CONCLUSION**

So the two models with different theoretical foundations that is the quark model with null instanton and anti-instanton balls and semibosonized Nambu-Jona-Lasinio model
[12] give similar algebraic structure for the magnetic moments of the baryon octet. The only difference between them lying effectively in the evaluation of the magnetic moment of the Λ-hyperon. It may have deeper meaning as Λ-hyperon being composed of all different quarks is characterized by zero values of isotopic spin and hypercharge. Without more elaborated theoretical calculations of either instanton ball interactions or profile function of Nambu-Jona-Lasinio model it is difficult to argue for either of these models.

The most important result that can be deduced from [9] and [12] is that the unitary symmetry model with some kind of middle strong interaction and meson cloud contribution lies in the base of both descriptions of the magnetic moments of the baryons.

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