Production of mirror fermions via $e\gamma$ and $ep$ collisions in the littlest Higgs model with T-parity

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Abstract

One of the important features of the littlest Higgs model with T-parity, called the $LHT$ model, is that it introduces the mirror fermions, which are the T-parity partners of the standard model fermions. In this paper, we discuss production of the mirror quark associated with mirror neutrino via $e\gamma$ and $ep$ collisions. We find that, in wide range of the parameter space, the mirror quark can be copiously produced at the International $e^+e^-$ Linear Collider ($ILC$) and $ep$ collider ($THERA$) experiments. The production rates of certain signal events, which are related the main two-body decay modes of the mirror quark, are also calculated.

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1. Introduction

The CERN Large Hadron Collider (LHC) with a center-of-mass (c.m.) energy of 14\,TeV will begin operation in 2008 for breakthrough discoveries in the electroweak-scale physics and in the new physics beyond the standard model (SM). However, the processes and the detection of new physics at the LHC are extremely complicated. Therefore, the lepton-lepton and lepton-hadron colliders with clean environments are required to complement the LHC in drawing a comprehensive and a high-resolution picture of the SM and the new physics models.

It has been shown that the collective symmetry breaking mechanism implemented in little Higgs models provides an interesting solution to the “little hierarchy problem” (for recent review see [1]). The littlest Higgs (LH) model [2] is one of the most economical and interesting models discussed in the literature. However, the low energy electroweak precision tests enforce the symmetry breaking scale $f$ of the LH model to be larger than about 4\,TeV [3], which induces that the fine tuning between the cut-off scale and the electroweak scale is needed again. To alleviate this difficulty, a $Z_2$ discrete symmetry, named “T-parity”, is introduced into the LH model, which forms the so-called LHT model [4]. In the LHT model, T-Parity is an exact symmetry, the SM gauge bosons (T-even) do not mix with the T-odd new gauge bosons, and thus the electroweak observables are not modified at tree level. Beyond the tree level, small radiative corrections to the electroweak observables allow the scale parameter $f$ as low as 500\,GeV [5]. Thus, the LHT model is one of the attractive little Higgs models.

In order to implement T-parity in the fermion sector, one introduces three doublets of “mirror quarks” and three doublets of “mirror leptons”, which have T-odd parity, transform vectorially under $SU(2)_L$. A first study of the collider phenomenology of the LHT model was given in [6]. The possible signals of the mirror fermions have been studied in [7, 8, 9, 10, 11]. In Ref.[12] we have considered pair production of the mirror leptons in an International $e^+e^-$ Linear Collider (ILC) and find that, as long as the mirror leptons are not too heavy, they can be copiously produced via the processes $e^+e^- \rightarrow \bar{L}_iL_j$ in future ILC experiments. In this paper, we will consider production of the
mirror quark associated with mirror neutrino via $e\gamma$ and $ep$ collisions and see whether the mirror fermions can be detected via these collision processes in future $ILC$ and $THERA$ experiments.

In the rest of this paper, we will give our results in detail. In section 2, we briefly review the couplings of the mirror quarks, which are related to our calculation. To discuss the possible signals of the mirror quarks in following sections, their possible partial decay widths are also given in this section. Production of the mirror fermions via $e\gamma$ and $ep$ collisions are studied in sections 3 and 4, respectively. The relevant phenomenology analysis in future $ILC$ and $THERA$ experiments are also given in these two sections. In the last section our conclusions and discussions are given.

2. The couplings and decays of the mirror quarks

Similar to the $LH$ model, the $LHT$ model [4] is based on an $SU(5)/SO(5)$ global symmetry breaking pattern. A subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ of the $SU(5)$ global symmetry is gauged, and at the scale $f$ it is broken into the $SM$ electroweak symmetry $SU(2)_L \times U(1)_Y$. T-parity is an automorphism that exchanges the $[SU(2) \times U(1)]_1$ and $[SU(2) \times U(1)]_2$ gauge symmetries. The T-even combinations of the gauge fields are the $SM$ electroweak gauge bosons $W^a_\mu$ and $B_\mu$. The T-odd combinations are T-parity partners of the $SM$ electroweak gauge bosons.

To avoid severe constraints and simultaneously implement T-parity, one needs to double the $SM$ fermion doublet spectrum [4, 6]. The T-even combination is associated with the $SM$ $SU(2)_L$ doublet, while the T-odd combination is its T-parity partner. To generate mass terms for the T-odd fermions, called the mirror fermions, through Yukawa interactions, one requires additional T-odd $SU(2)$ singlet fermions in the $LHT$ model as suggested in [5, 6]. Assuming universal and flavor diagonal Yukawa coupling $k$, the masses of the up- and down- type mirror fermions can be written as [11]:

$$M_{U_H} \approx \sqrt{2} k f (1 - \frac{\nu^2}{8 f^2}), \quad M_{D_H} \approx \sqrt{2} k f. \quad (1)$$

Being $f \geq 500 GeV$, it is clear from Eq.(1) that there is $M_{U_H} \approx M_{D_H} = M_{Q_H}$. In this paper we will focus our attention on the first and second generation mirror quarks and
assume that the Yukawa coupling constant $k$ is in the range of $0.5 < k < 1.5$.

The couplings of the mirror fermions to other particles, which are related to our analysis, are summarized as [9]:

$$\begin{align*}
B_H U_H^i u^j & : -\frac{ie}{2} \left[ \frac{1}{5C_W} + \frac{x_H}{5W} \right](V_{Hu})_{ij} \gamma^\mu P_L, \\
B_H U_H^i t & : -\frac{ie}{2} \left[ \frac{1}{5C_W} - \frac{x_H}{10C_W} \right](V_{Hu})_{ij} \gamma^\mu P_L, \\
B_H \bar{D}_H^{i\bar{d}} & : -\frac{ie}{2} \left[ \frac{1}{5C_W} - \frac{x_H}{5W} \right](V_{Hd})_{ij} \gamma^\mu P_L, \\
Z_H U_H^i u^j & : \frac{ie}{2} \left[ S_W - \frac{1}{5C_W} \right] x_H \gamma^\mu P_L, \\
Z_H U_H^i t & : \frac{ie}{2} \left[ S_W - \frac{x_H}{2S_W} \right] y^\mu P_L, \\
Z_H \bar{D}_H^{i\bar{d}} & : \frac{ie}{2} \left[ S_W + \frac{x_H}{5C_W} \right] y^\mu P_L, \\
W_H^\mu U_H^{i\bar{d}} & : \frac{ie}{\sqrt{2S_W}} (V_{Hd})_{ij} \gamma^\mu P_L, \\
W_H^\mu \bar{D}_H^{i\bar{d}} & : \frac{ie}{\sqrt{2S_W}} (V_{Hu})_{ij} \gamma^\mu P_L.
\end{align*}$$

Here $\nu \approx 246\text{GeV}$ is the electroweak scale and $x_H = 5S_W C_W / 4(5C_W^2 - S_W^2)$. $S_W = \sin \theta_W$, $C_W = \cos \theta_W$, and $\theta_W$ is the Weinberg angle. $P_L = (1 - \gamma_5)/2$ is the left-handed projection operator. The four $CKM-like$ unitary matrices $V_{Hu}$, $V_{Hd}$, $V_{H\ell}$, and $V_{H\nu}$ satisfy [9, 13]:

$$V_{Hu}^+ V_{Hd} = V_{CKM}, \quad V_{H\nu}^+ V_{H\ell} = V_{PMNS},$$

where the $CKM$ matrix $V_{CKM}$ is defined through flavor mixing in the down-type quark sector, while the $PMNS$ matrix $V_{PMNS}$ is defined through neutrino mixing. To avoid any additional parameters introduced and to simply our calculation, we take $V_{H\ell} = V_{PMNS}$ ($V_{H\nu} = I$) [14], which means that the mirror leptons have no impact on the flavor violating observables in the neutrino sector. For the $CKM-like$ unitary matrices $V_{Hu}$ and $V_{Hd}$, we take $V_{Hd} = I$ and $V_{Hu} = V_{CKM}^\dagger$ [9] in our numerical estimation.

At the order of $\nu^2/f^2$, the masses of the $T$-odd set of the $SU(2) \times U(1)$ gauge bosons are given by [4]:

$$M_{B_H} = \frac{ef}{\sqrt{5C_W}} [1 - 5\nu^2/8f^2], \quad M_{Z_H} \approx M_{W_H} = \frac{ef}{S_W} [1 - \nu^2/8f^2].$$
Comparing Eq.(1) with Eq.(11) we can see that, for $0.5 \leq k \leq 1.5$, the mirror quarks $U_H$ and $D_H$ are always heavier than the T-odd gauge bosons. The gauge boson $B_H$ is the lightest T-odd particle, which can be seen as an attractive dark matter candidate [6]. Thus, the possible two-body decay modes of the mirror quarks $U_H$ and $D_H$ are:

$$U_H : uB_H, \ cB_H, \ tB_H, \ uZ_H, \ cZ_H, \ tZ_H, \ dW_H,$$

$$D_H : dB_H, \ dZ_H, \ uW_H, \ cW_H, \ tW_H.$$  \hspace{1cm} (12) (13)

Figure 1: The branching ratios of the mirror quarks $U_H$ (a) and $D_H$ (b) as functions of the coupling parameter $k$ for the scale parameter $f = 1 TeV$.

From Eq.(2) – Eq.(9) we can see that the couplings of the mirror quark $Q_H$ to ordinary fermion and T-odd gauge boson are all the left-handed couplings. Then the partial decay width can be written in an unified manner as:

$$\Gamma(Q_H \to qV_H) = \frac{M_{Q_H}^3 g_L^2}{32\pi M_{V_H}} \left\{ x^2(1 - 2x^2 + y^2) + (1 - y^2)^2 \right\} \lambda^{1/2}(1, x^2, y^2)$$  \hspace{1cm} (14)

with $x = M_{V_H}/M_{Q_H}$, $y = M_q/M_{Q_H}$, and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, in which $M_{V_H}$ is the mass of the T-odd gauge boson. Obviously, the branchings ratios of the mirror quark $Q_H$ depend on the free parameters $f$, $k$, and $x_L$. Since the mixing parameter $x_L$ contributes the decay widths and the production cross sections relevant for
the top quark \( t \) only at the order of \( \nu^4/f^4 \), we will take \( x_L = 0.5 \) in our following numerical calculation.

In Fig.1, we illustrate the sizes of the branching ratios for the mirror quarks \( U_H \) and \( D_H \) as functions of the coupling parameter \( k \) for the scale parameter \( f = 1 \) TeV. Since the values of the branching ratios \( Br(U_H \rightarrow tB_H), Br(U_H \rightarrow tZ_H), \) and \( Br(D_H \rightarrow tW_H) \) are all smaller than \( 2 \times 10^{-3} \)%, we have not given the relevant curves in Fig.1. Furthermore, we have multiplied the factor 10 to the branching ratio \( Br(U_H \rightarrow cB_H) \) in Fig.1a. One can see that the main decay channels are \( uZ_H \) and \( dW_H (dZ_H \) and \( uW_H) \) for \( U_H (D_H). \)

For \( f = 1 \) TeV and \( 0.6 \leq k \leq 1.2 \), the values of \( Br(U_H \rightarrow dW_H), Br(U_H \rightarrow uZ_H), Br(D_H \rightarrow uW_H), \) and \( Br(D_H \rightarrow dZ_H) \) are in the ranges of \( 55\% \sim 61.3\%, 26\% \sim 29\%, 54.7\% \sim 59.4\%, \) and \( 28.9\% \sim 31.4\%, \) respectively.

3. Production of the mirror quark associated with a mirror neutrino via ep collision

From the above discussion we can see that the mirror quark can be produced associated with a mirror neutrino via the process \( eq \rightarrow \nu_H Q_H \) mediated by the T-odd charged gauge boson \( W_H \), as depicted in Fig.2. The invariant amplitude for the process \( e(P_1)q(P_2) \rightarrow \nu_H(p_3)Q_H(p_4) \) can be written as:

\[
M_{ij}^{eq} = \frac{e^2}{2S_W^2} \frac{V_{ij}(V_{H\ell})_{ii}}{K^2 - M_{WH}^2} \left[ \bar{u}(P_4)\gamma_\nu P_Lu(P_2) \right] \left[ \bar{u}(P_3)\gamma^\nu P_Lu(P_1) \right],
\]

where \( K^2 = (P_1 - P_2)^2 \) and \( q \) represents the SM light quark \( u, c, d, \) or \( s. i \) and \( j \) are the family indexes for the mirror fermions. In this paper, we are only interested in production of the first and second generation mirror fermions, so we will take \( i=1, 2 \) and \( j=1, 2. \)

![Figure 2: Feynman diagram for the process eq → ν_H Q_H.](image-url)
After calculating the cross section $\hat{\sigma}^{ij}(s)$ for the process $e(P_1)q(P_2) \rightarrow \nu^i_H(P_3)Q^j_H(P_4)$, the effective production cross section $\sigma(s)$ of the mirror fermions at the THERA experiment with the center-of-mass (c.m.) energy $\sqrt{S} = 3.7 TeV$ [15] can be obtained by folding $\hat{\sigma}^{ij}(s)$ with the parton distribution function (PDF) $f_{q/p}(x)$:

$$\sigma(s) = \sum_{i,j} \sum_{q} \int_{x_{min}}^{1} f_{q/p}(x, \mu_F)\hat{\sigma}^{ij}(s)dx$$  \hspace{1cm} (16)$$

with $x_{min} = (M_\nu + M_{Q_H})^2/s$ and $s = xs$, in which $M_\nu$ is the mass of the mirror neutrino $\nu_H$. In our numerical calculation, we will use CTEQ6L PDF [16] for the quark distribution function $f_{q/p}(x, \mu_F)$ and assume that the factorization scale $\mu_F$ is of order $\sqrt{s}$.

In the case of $V_{H\ell} = V_{PMNS}$, $V_{Hd} = I$, and $V_{Hu} = V_{CKM}^+$, the production cross section $\sigma(s)$ depends on the parameters $f$, $k$, and $M_\nu$. Our numerical results are given in Fig.3, in which we plot the cross section $\sigma(s)$ as a function of the coupling parameter $k$ for different values of the free parameters $f$ and $M_\nu$. In this figure we have taken the values of the CKM matrix elements $(V_{CKM})_{ij}$ given in [17], in which $V_{CKM}$ is constructed based on PDG parameterization [18]. For the matrix $V_{PMNS}$, we have taken the standard parameterization form parameters given by the neutrino experiments [19]. The production cross section $\sigma(s)$ given by Fig.3 is the total cross section for the first two generation up-type and down-type quarks. One can see from Fig.3 that the value of $\sigma(s)$ decreases as the parameters $f$, $k$, and $M_\nu$ increasing. For $M_\nu = 300 GeV$, $k = 1.0$, and $500 GeV \leq f \leq 2000 GeV$, its value is in the range of $51.2 pb \sim 3.4 \times 10^{-4} pb$. For $f \geq 1500 GeV$ and $k \sim 1$, the mirror fermion is too heavy to be abundantly produced at the THERA experiment with $\sqrt{S} = 3.7 TeV$, and its production cross sections is smaller than $98 fb$, which is very difficult to be detected in future THERA experiments.

For $k > 0.5$, the mirror quark is heavy enough to decay into T-odd gauge boson plus an ordinary fermion. The different chain decays of the mirror quark can given different experimental signatures. To asess the discovery potential of the THERA for the mirror quarks, the production rates and the relevant SM backgrounds for different decay channels need to be discussed.

The decay processes $\nu_H U_H \rightarrow \nu_H u B_H$, $\nu_H c B_H$ and $\nu_H D_H \rightarrow \nu_H d B_H$ can lead to the
jet + $\not{E}_T$ signature. To illustrate the size of the signal production rate, we show in Fig.4 it as a function of the parameter $k$ for different values of the mass parameter $M_\nu$ and scale parameter $f$. One can see from Fig.4 that, for $M_\nu = 200GeV$, $k = 1$ and $f \geq 500GeV$, the production rate can reach $5.2pb$. The SM backgrounds of this kind of signature mainly come from the process $ep \rightarrow \nu q' + X$. Measurement and QCD analysis of the production cross section for the $SM$ process $ep \rightarrow \nu q' + X$ at the HERA collider have been extensively studied [20]. In order to see whether the jet + $\not{E}_T$ signal can be detected in future HERA experiment with the c. m. energy $\sqrt{S} = 3.7TeV$ and a yearly integrated
luminosity $\mathcal{L} = 4 fb^{-1}$, we further calculate its statistical significance $SS$, which is defined as:

$$SS = \frac{\sigma(signal)}{\sqrt{\sigma(SM)}} \sqrt{\mathcal{L}}.$$  

Figure 4: The production rate for the $jet + \slashed{E}$ signature as a function of the parameter $k$ for different values of the free parameters $M_\nu$ and $f$.

To show the sensitivity of the statistical significance $SS$ to the scale parameter $f$, we
plot in Fig. 5 $SS$ as a function of $f$ for a fixed value $M_\nu = 300 GeV$ and three values of the coupling parameter $k$. By varying $f$ from 500GeV to 1300GeV, the value of $SS$ is in the range of $196 \sim 2.9$ for $k = 0.8$. Thus, we expect that, in wide range of the parameter space, the possible signals of the mirror quark can be detected via the process $ep \rightarrow \nu_H Q_H + X$ in future THERA experiments.

Figure 5: The statistical significance $SS$ as a function of the parameter $f$ for $M_\nu = 300 GeV$ and three values of the coupling parameter $k$.

For the decay channel $U_H \rightarrow tB_H$, the process $ep \rightarrow \nu_H U_H + X$ gives rise to the signal event with a single top and large missing energy ($t + E_T$), which can give a characteristic signal. However, its production rate is smaller than $1 fb$ and cannot be detected in future THERA experiments [21]. Same as $U_H \rightarrow tB_H$, the decay channel $U_H \rightarrow tZ_H$ can also generate a signal at unobservable level in future THERA experiments.

For the mirror quark decays into the T-odd gauge boson $Z_H$ and an ordinary fermion, we have the following chain decay processes:

$$
\nu_H U_H \rightarrow \nu_H u Z_H \rightarrow \nu_H u B_H H, \quad \nu_H U_H \rightarrow \nu_H c Z_H \rightarrow \nu_H c B_H H,
$$

(17)
Figure 6: The production rate for the $E + jets$ signature as a function of the parameter $f$ for $M_\nu = 200GeV$ (a), and $M_\nu = 300GeV$ (b), and three values of $k$.

\[ \nu_HD_H \rightarrow \nu_HdZ_H \rightarrow \nu_HdB_HH. \]  

(18)

For $M_H \leq 120GeV$, the dominant decay channel of the Higgs boson $H$ is $H \rightarrow b\bar{b}$. The above chain decay processes can generate the $E + jets$ signature. Its production rate is given in Fig.6, in which we have assumed $Br(Z_H \rightarrow B_HH) \approx 100\%$ [7]. For $M_\nu = 300GeV$, $k = 1$, and $500GeV \leq f \leq 1500GeV$, there will be $5 \times 10^4 \sim 68$ signal events to be generated per year in future HERA with $\sqrt{s} = 3.7TeV$ and $\mathcal{L} = 4fb^{-1}$. The signal events should be distinguished from the SM backgrounds by reconstructing the final states.

If we assume that the mirror quark decays into the T-odd gauge boson $W_H$ and an ordinary fermion, then the production of the mirror quark associated a mirror neutrino can give following chain decay processes:

\[ \nu_HU_H \rightarrow \nu_HdW_H \rightarrow \nu_HB_HdW \rightarrow \nu_HB_Hd\nu_\ell, \]  

(19)

\[ \nu_HD_H \rightarrow \nu_HuW_H \rightarrow \nu_HuWB_H \rightarrow \nu_HB_Hu\nu_\ell, \]  

(20)

\[ \nu_HD_H \rightarrow \nu_HcW_H \rightarrow \nu_HcB_HW \rightarrow \nu_HB_Hc\nu_\ell. \]  

(21)
In the above processes, we have assumed that the T-odd gauge boson $W_H$ mainly decays into $B_H W$ and focus our attention only on the pure leptonic decay modes for the $SM$ gauge boson $W$. These processes can give rise the signal event $E + \ell + \text{jet}$, which is shown in Fig.7. The production rate for the signal $E + \ell + \text{jet}$ is smaller than that for the signal event $E + \text{jets}$. However, for $M_\nu = 300 \text{GeV}$, $k = 0.8$, and $f \geq 500 \text{GeV}$, its value can reach $14.3 \text{pb}$. In wide range of the parameter space, there will be several tens and up to thousands $E + \ell + \text{jet}$ events to be generated per year in future $HERA$ experiments.

![Figure 7: The production rate for the signal event $E + \ell + \text{jet}$ as a function of the parameter $f$ for $M_\nu = 200 \text{GeV}$ (a), and $M_\nu = 300 \text{GeV}$ (b), and three values of $k$.](image)

4. Production of the mirror quark associated with a mirror neutrino via $e\gamma$ collision

It has been shown that, in suitable kinematic region the process $e\gamma \rightarrow \nu q\bar{q}$ can be approximated quite well by the process $eq \rightarrow \nu q'$ [22], where the quark $q$ described by the quark parton content of the photon approach [23]. So production of the mirror quark associated with a mirror neutrino can be induced via $e\gamma$ collision mediated by the T-odd charge gauge boson $W_H$. The hard photon beam of $e\gamma$ collision can be obtained from laser backscattering at the high energy $e^+e^-$ collider experiments. The expression for the total effective cross section of the subprocess $e(P_1)q(P_2) \rightarrow \nu H(P_3)Q_H(P_4)$ at the $ILC$
can be given by
\[ \sigma(s) = \sum_{i,j} \sum_{q} \int \int dx_1 dx_2 f_{\gamma/e}(x_1) f_{q/\gamma}(x_2) \hat{\sigma}_q^{ij}(\hat{s}). \] (22)

Here \( f_{\gamma/e}(x_1) \) is the photon distribution function [24], \( f_{q/\gamma}(x_2) \) is the distribution function for the quark content in the photon. To obtain our numerical results we will use Aurenche, Fontannaz, and Guillet (AFG) distributions [25] for \( f_{q/\gamma} \). Other distributions are available in [26]. Furthermore, we will give the cross section for \( \nu_H Q_H \) production at the ILC experiment with \( \sqrt{S} = 3\text{ TeV} \) and \( \mathcal{L} = 100\text{ fb}^{-1} \) [27].

![Figure 8](image-url)

Figure 8: The production cross section for the subprocess \( eq \rightarrow \nu_H Q_H \) as a function of the parameter \( f \) for \( M_\nu = 200\text{ GeV} \)(a), and \( M_\nu = 300\text{ GeV} \)(b), and three values of the parameter \( k \).

The production cross section \( \sigma \) of the process \( e^+e^- \rightarrow \nu_H Q_H + X \) is shown in Fig.8 as a function of the Yukawa coupling parameter \( k \) for different values of the scale parameter \( f \) and the mirror neutrino mass \( M_\nu \), in which we have assumed \( q = u, c, d, \) and \( s \). The cross section \( \sigma \) is the total production cross section of the first two mirror quarks. From Fig.8 one can see that \( \sigma \) is generally smaller than that for the THERA experiment. The reason is that, compared with the PDF \( f_{q/p} \), the PDF’s factor \( f_{\gamma/e} \) suppresses the cross section for the production of the mirror quark associated with a mirror neutrino.
For \( M_\nu = 200\text{GeV} \), \( k = 1.0 \), and \( 500\text{GeV} \leq f \leq 1400\text{GeV} \), the value of \( \sigma \) is in the range of \( 553\text{fb} \sim 2.2\text{fb} \).

Figure 9: The statistical significance \( SS \) as a function of the parameter \( f \) for \( M_\nu = 200\text{GeV} \)(a), and \( M_\nu = 300\text{GeV} \)(b), and three values of the parameter \( k \).

If we assume that the mirror quarks have the decay channels \( U_H \rightarrow u B_H, U_H \rightarrow c B_H \), and \( D_H \rightarrow d B_H \), then the signal state of the process \( e^+ e^- \rightarrow \nu_H Q_H + X \) is \( \text{jet} + \not{E} \). The SM backgrounds of this signature mainly come from the SM process \( e^+ e^- \rightarrow \nu q + X \). Its statistical significance \( SS \) is plotted as a function of the scale parameter \( f \) for \( M_\nu = 200\text{GeV} \) (Fig.9a), \( 300\text{GeV} \) (Fig.9b), and three values of the coupling parameter \( k \). For \( k \leq 1.2 \), \( M_\nu \leq 300\text{GeV} \), and \( f \leq 1\text{TeV} \), the value of the statistical significance \( SS \) is larger than 2. Thus, for reasonable ranges of the free parameters, the possible signatures of the \( LHT \) model might be detected via the process \( e^+ e^- \rightarrow \nu_H Q_H + X \) with \( Q_H \rightarrow q B_H \) in future ILC experiments.

For the chain decay processes \( U_H \rightarrow u Z_H \rightarrow u B_H H \rightarrow B_H u \bar{b} \), \( U_H \rightarrow c Z_H \rightarrow B_H c \bar{b} \), and \( D_H \rightarrow d Z_H \rightarrow B_H d \bar{b} \), the process \( e^+ e^- \rightarrow \nu_H Q_H + X \) can produce the \( \not{E} + \text{jets} \) signature. In Fig.10 we plot its production rate as a function of the scale parameter \( f \) for different values of the free parameters \( M_\nu \) and \( k \). One can see from Fig.10 that, for \( M_\nu = 200\text{GeV} \), \( k = 1.0 \), and \( 500\text{GeV} \leq f \leq 1400\text{GeV} \), there will be \( 1.36 \times 10^4 \sim 55 \).
$\mathcal{E}+\text{jets}$ events to be generated per year in future $ILC$ experiment with $\sqrt{S} = 3TeV$ and $\mathcal{L} = 100\,fb^{-1}$.

Figure 10: The production rate for the $\mathcal{E} + \text{jets}$ signature as a function of the parameter $f$ for $M_\nu = 200\,GeV$ (a), and $M_\nu = 300\,GeV$ (b), and three values of $k$.

Figure 11: The production rate for the signal event $\mathcal{E} + \ell + \text{jet}$ as the parameter of $f$ for $M_\nu = 200\,GeV$ (a), and $M_\nu = 300\,GeV$ (b), and three values of $k$.

According the chain decay processes Eq.(19) – Eq.(21), the process $e^+e^- \rightarrow \nu_H Q_H + X$ can induce the $\mathcal{E} + \ell + \text{jet}$ final state. Our numerical results show that this signal event can also be abundantly produced in future $ILC$ experiment with $\sqrt{S} = 3TeV$ and $\mathcal{L} =$
100\,fb^{-1}, as illustrated in Fig.11. For \( M_\nu = 200\,GeV \), \( k = 1.0 \) and \( 500\,GeV \leq f \leq 1400\,GeV \), there will be \( 1.12 \times 10^4 \sim 45 \, E + \ell + jet \) events to be generated per year.

5. Conclusion

The LHT model is one of the attractive little Higgs models which not only provides a possible dark matter candidate but also is consistent with electroweak precision tests. In order to implement T-parity in the fermion sector of the model, the heavy T-odd \( SU(2) \) doublet fermions, which are called the mirror fermions of the SM fermions, have to be introduced. These new heavy fermions might produce the observability signatures in future high energy collider experiments.

Recently, we have discussed the possibility of detecting the new charged gauge boson \( W' \) via the process \( eq \rightarrow \nu q' \) and find that one can use this process to distinguish different new physics models in future THERA and ILC experiments [28]. In this paper, we have calculated the effective cross sections of the mirror quark \( Q_H \) production association with mirror neutrino \( \nu_H \) via the subprocess \( eq \rightarrow \nu_H Q_H \) at the THERA experiment with \( \sqrt{S} = 3.7\,TeV \) and the ILC experiment with \( \sqrt{S} = 3\,TeV \). Our numerical results show that the values of the cross sections are strongly dependent on the Yukawa coupling parameter \( k \) and the scale parameter \( f \). This is because the masses of the mirror quarks are written in a unified manner as \( M_{Q_H} = \sqrt{2kf} \). However, in wide range of the parameter space, the mirror fermions can be abundantly produced.

Based on calculating the branching ratios of all possible two-body decay modes of the mirror quarks \( U_H \) and \( D_H \), we further calculate the production rates of certain signal events relevant for the main decay modes. For the mirror quark decays to an ordinary fermion and a T-odd gauge boson \( B_H \), the subprocess \( eq \rightarrow \nu_H Q_H \) can induce the \( jet+E \) signal event, the values of its statistical significance \( SS \) in the THERA and ILC experiments are given in Fig.5 and Fig.9, respectively. As shown, in reasonable ranges of the free parameters, their values can be significantly large. The possible signals of the mirror quarks might be detected via this kind of decay channels in future THERA and ILC experiments. We also calculate the production rates of the signal events \( E+jets \) and \( E + \ell + jet \), which come from the decay channels \( Q_H \rightarrow qZ_H \) and \( Q_H \rightarrow qW_H \), respectively.
One can see from the relevant figures, for certain ranges of the free parameters, there will generally be several tens and up to thousands signal events to be generated per year. However, the SM backgrounds must be further studied.

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