Predicted Modulated Differential Rates for Direct WIMP Searches at Low Energy Transfers*

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Abstract The differential event rate for direct detection of dark matter, both the time averaged and the modulated one due to the motion of the Earth, are discussed. The calculations focus on relatively light cold dark matter candidates (WIMP) and low energy transfers. It is shown that for sufficiently light WIMPs the extraction of relatively large nucleon cross sections is possible. Furthermore for some WIMP masses the modulation amplitude may change sign, meaning that, in such a case, the maximum rate may occur six months later than naively expected. This effect can be exploited to yield information about the mass of the dark matter candidate, if and when the observation of the modulation of the event rate is established.

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1 Introduction

The combined MAXIMA-1,[1] BOOMERANG,[2] DASI[8] and COBE/DMR Cosmic Microwave Background (CMB) observations[4] imply that the Universe is flat[5] and that most of the matter in the Universe is dark,[6] i.e. exotic. These results have been confirmed and improved by the recent WMAP data.[7] Combining the data of these quite precise experiments one finds:

Ω_b = 0.0456 ± 0.0015,
Ω_{CDM} = 0.228 ± 0.013,
Ω_A = 0.726 ± 0.015.

Since any “invisible” non exotic component cannot possibly exceed 40% of the above Ω_{CDM},[8] exotic (non baryonic) matter is required and there is room for cold dark matter candidates or WIMPs (Weakly Interacting Massive Particles).

Even though there exists firm indirect evidence for a halo of dark matter in galaxies from the observed rotational curves, see e.g. the review,[9] it is essential to directly detect such matter. Until dark matter is actually detected, we shall not be able to exclude the possibility that the rotation curves result from a modification of the laws of nature as we currently view them. This makes it imperative that we invest a maximum effort in attempting to directly detect dark matter in the laboratory. Furthermore such a direct detection will also unravel the nature of the constituents of dark matter. The possibility of such detection, however, depends on the nature of the dark matter constituents and their interactions.

Since the WIMP’s are expected to be extremely non relativistic, with average kinetic energy \langle T \rangle ≈ 50\,keV(m_{WIMP}/100\,GeV), they are not likely to excite the nucleus, even if they are quite massive m_{WIMP} > 100\,GeV. So they can be directly detected mainly via the recoiling of a nucleus (A,Z) in elastic scattering. The event rate for such a process can be computed from the following ingredients: i) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of the prevailing particle theory. In supersymmetry the dark matter candidate is the LSP (Lightest Supersymmetric Particle).[10–16] In this case the effective Lagrangian is constructed as described, e.g., in Refs. [10]–[18]. ii) A well defined procedure for transforming the amplitude thus obtained, using the previous effective Lagrangian, from the quark to the nucleon level. To achieve this one needs a quark model for the nucleon, see e.g.[18–21] This step is particularly important in supersymmetry or other models dominated by a scalar interaction (intermediate Higgs etc.), since, then, the elementary amplitude becomes proportional to the quark interaction and the content of the nucleon in quarks other than u and d becomes very important. iii) knowledge of the relevant nuclear matrix elements,[22–23] obtained with as reliable as possible many body nuclear wave functions. iv) knowledge of the WIMP density in our vicinity and its velocity distribution.

From steps i) and ii) one obtains the nucleon cross sections. These can also be extracted from the data of event rates, if and when such data become available. From lim-
its on the event rates, one can obtain exclusion plots on
the nucleon cross sections as functions of the WIMP mass.
The extracted cross sections depend, of course, on inputs
from steps iii)-iv).

In the standard nuclear recoil experiments, first pro-
posed more than 30 years ago,[24] one has to face the prob-
lem that the reaction of interest does not have a charac-
teristic feature to distinguish it from the background. So
for the expected low counting rates the background is a
formidable problem. Some special features of the WIMP-
nuclear interaction can be exploited to reduce the back-
ground problems. Such are:

i) the modulation effect: this yields a periodic signal
due to the motion of the earth around the sun. Unfortu-
nately this effect, also proposed a long time ago[25] and
subsequently studied by many authors,[26–34] is small and
becomes even smaller than 2% due to cancelations arising
from nuclear physics effects,

ii) backward-forward asymmetry expected in direc-
tional experiments, i.e. experiments in which the direction
of the recoiling nucleus is also observed. Such an asym-
metry has also been predicted a long time ago,[35] but it has
not been exploited, since such experiments have been con-
sidered very difficult to perform, but they now appear to
be feasible.[35–47] In such experiments the event rate de-
pends on the direction of observation. In the most favor-
able direction, opposite to the sun’s direction of motion,
is comparable to the standard event rate. The sensitivity
of these experiments for various halo models has also been
discussed.[40–41] Furthermore we should mention that in
such experiments[36,39,47] all events are counted. If some
interesting events can be found, they can be established
by further analyzing them by the direction of the observed
recoils.

iii) transitions to excited states: in this case one needs
not measure nuclear recoils, but the de-excitation γ rays.
This can happen only in very special cases since the aver-
age WIMP energy is too low to excite the nucleus. It has,
however, been found that in the special case of the tar-
et $^{127}$I such a process is feasible[48] with branching ratios
around 5%.

iv) detection of electrons produced during the WIMP-
nucleus collision: it turns out, however, that this produc-
tion peaks at very low energies. So only gaseous TPC
detectors can reach the desired level of 100 eV. In such
a case the number of electrons detected may exceed the
number of recoils for a target with high $Z$.[49–50]

v) detection of hard X-rays produced when the inner
shell holes are filled: it has been found[51] that in the
previous mechanism inner shell electrons can be ejected.
These holes can be filled by the Auger process or X-ray
emission.

In connection with nuclear structure aspects, in a se-
ries of calculations, e.g. in [32], [52–53] and references
there in, it has been shown that for the coherentcontri-
bution, due to the scalar interaction, the inclusion of the
nuclear form factor is important, especially in the case
of relatively heavy targets. They also showed that the
nuclear spin cross sections are characterized by a single,
i.e. essentially isospin independent, structure function
and two static spin values, one for the proton and one for
the neutron, which depend on the target.

As we have already mentioned an essential ingredi-
ent in direct WIMP detection is the WIMP density in
our vicinity and, especially, the WIMP velocity distri-
bution. Some of the calculations have considered vari-
ous forms of phenomenological non symmetric velocity
distributions[33,39–40] and some of them even more exotic
dark matter flows like the late infall of dark matter into
the galaxy, i.e. caustic rings,[54–58] dark matter orbiting
the Sun[42] and Sagittarius dark matter.[59]

In addition to computing the time averaged rates,
these calculations studied the modulation effect. They
showed that in the standard recoil experiments the modu-
lation amplitude in the total rate may change sign for
large reduced mass, i.e. heavy WIMPs and large $A$.

In the present paper we will expand the above calcu-
lations and study the differential event rates, both time
averaged and modulated, in the region of low energy
transfers, as in the DAMA experiment,[60–61] focusing
our attention on relatively light WIMPs,[62–64] Such light
WIMPs can be accommodated in some SUSY models.[65]
We will employ here on the standard Maxwell–Boltzmann
(M–B) distribution for the WIMPs of our galaxy and we
will not be concerned with other distributions,[66–69] even
though some of them may affect the modulation. The lat-
ter will be studied elsewhere. We will explicitly show that
the modulation amplitude, entering both the differential
and the total rates, changes sign for certain WIMP masses.
As a result such an effect, if and when the needed data
become available, may be exploited to infer the WIMP
mass.

2 Formalism for WIMP-Nucleus Differential
Event Rate

This formalism adopted in this work is well known (see
e.g. the recent reviews[18,70]). So we will briefly discuss
its essential elements here. The differential event rate can
be cast in the form:

$$\left.\frac{dR}{dQ}\right|_A = \left.\frac{dR_0}{dQ}\right|_A + \left.\frac{d\tilde{H}}{dQ}\right|_A \cos \alpha,$$  \hspace{1cm} (1)

where the first term represents the time averaged (non
modulated) differential event rate, while the second gives
the time dependent (modulated) one due to the motion of the Earth (see below). Furthermore

\[
\frac{dR_0}{dQ}|_A = \frac{\rho_X}{m_X} \frac{m_t}{m_p} A_{\text{m}} \sigma_n \Big( \frac{\mu_t}{\mu_p} \Big)^2 \sqrt{\langle v^2 \rangle A^2} \; \frac{1}{Q_0(A)} \frac{dt}{du},
\]

\[
\frac{dH}{dQ}|_A = \frac{\rho_X}{m_X} \frac{m_t}{m_p} A_{\text{m}} \sigma_n \Big( \frac{\mu_t}{\mu_p} \Big)^2 \sqrt{\langle v^2 \rangle A^2} \; \frac{1}{Q_0(A)} \frac{dh}{du},
\]

(2)

with \( \mu_t \) (\( \mu_p \)) the WIMP-nucleus (nucleon) reduced mass, \( A \) is the nuclear mass number and \( \sigma_n \) is the elementary WIMP-nucleon cross section. \( m_X \) is the WIMP mass and \( m_t \) the mass of the target. \( \alpha \) is the phase of the earth \((\alpha = 0 \text{ on June 2nd})\). Sometimes we will write the differential rate as:

\[
\frac{dR}{dQ}|_A = \frac{\rho_X}{m_X} \frac{m_t}{m_p} A_{\text{m}} \sigma_n \Big( \frac{\mu_t}{\mu_p} \Big)^2 \sqrt{\langle v^2 \rangle A^2} \; \frac{1}{Q_0(A)} \frac{dt}{du} (1 + H(a\sqrt{u}) \cos \alpha).
\]

(3)

In this formulation \( H(a\sqrt{u}) \), the ratio of the modulated to the non modulated differential rate, gives the relative differential modulation amplitude.

Furthermore one can show that

\[
\frac{dt}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) \psi_0(a\sqrt{u}),
\]

\[
\frac{dh}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) \psi_1(a\sqrt{u}),
\]

(4)

with \( a = (\sqrt{2} \mu p b v_0)^{-1} \), \( v_0 \) the velocity of the sun around the center of the galaxy and \( b \) the nuclear harmonic oscillator size parameter characterizing the nuclear wave function. \( u \) is the energy transfer \( Q \) in dimensionless units given by

\[
u = \frac{Q}{Q_0(A)}, \quad Q_0(A) = [m_p A b^2]^{-1} = 40 A^{-4/3} \text{ MeV},
\]

(5)

and \( F(u) \) is the nuclear form factor. Note that the parameter \( a \) depends both on the WIMP mass, the target and the velocity distribution. Note also that for a given energy transfer \( Q \) the quantity \( u \) depends on \( A \). The functions \( \psi_0(a\sqrt{u}) \) and \( \psi_1(a\sqrt{u}) \) can be obtained as follows:

- One starts with a Maxwell-Boltzmann distribution in the galactic frame with a characteristic velocity \( v_0 \) equal to the suns velocity around the center of the galaxy.

Strictly speaking, since an upper cutoff is introduced to the velocity distribution, equal to the escape velocity, the velocity distribution should be renormalized. However the normalization integral is close to one, namely

\[
norm = \frac{\sqrt{\pi} \text{erf} \left( \frac{y_{\text{esc}}}{v_0} \right) - 2 e^{-y_{\text{esc}}^2} y_{\text{esc}}}{\sqrt{\pi}}, \quad y_{\text{esc}} = \frac{v_{\text{esc}}}{v_0},
\]

(6)

i.e. \( \norm \approx 0.9989 \) for \( y_{\text{esc}} = 2.84 \)

- one transforms to the local coordinate system:

\[
y = y + \hat{\nu}_s + \delta (\sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{z}),
\]

\[
y = \frac{v}{v_0},
\]

(7)

with \( \hat{\nu}_s \) a unit vector in the suns direction of motion and \( \delta \) is the ratio of the Earth's velocity around the sun divided by \( v_0 \). The above formula assumes that the motion of both the sun around the galaxy and of the Earth around the sun are uniformly circular. The exact orbits are, of course, more complicated[33,71] but such deviations are not expected to significantly modify our results.

- One integrates over the velocity integration over the angles and the result is multiplied the velocity \( y = v/v_0 \) due to the WIMP flux.

- The result is obtained from a minimum value, which depends on the energy transfer \( y = a\sqrt{u} \), to a maximum

\[
y = y_{\text{esc}}, \quad y_{\text{esc}} = \frac{v_{\text{esc}}}{v_0}, \quad y_{\text{esc}} \approx 2.84.
\]

The result is

\[
J(x) = \frac{1}{\delta \cos \alpha - 2} \left[ \text{erf} \left( - x + \frac{1}{2} \delta \cos \alpha + 1 \right) + \text{erf} \left( x + \frac{1}{2} \delta \cos \alpha + 1 \right) \right.
\]

\[
+ \text{erfc} \left( - y_{\text{esc}} + \frac{1}{2} \delta \cos \alpha + 1 \right) + \text{erfc} \left( y_{\text{esc}} + \frac{1}{2} \delta \cos \alpha + 1 \right) - 2 \left], \quad x = a\sqrt{u},
\]

(8)

where \( \text{erf}(x) \) and \( \text{erfc}(x) \) are the error function and its complement respectively. Furthermore since \( \delta = 0.135 \) we can expand in powers of \( \delta \) and obtain:

\[
J(a\sqrt{u}) \approx \Psi_0(a\sqrt{u}) + \Psi_1(a\sqrt{u}) \cos \alpha + \Psi_2(a\sqrt{u}) \cos 2\alpha,
\]

(9)

with

\[
\Psi_0(x) = \frac{1}{2} \left( \text{erf}(1 - x) + \text{erf}(x + 1) + \text{erf}(1 - y_{\text{esc}}) - \text{erf}(y_{\text{esc}} + 1) - 2 \right),
\]

(10)

\[
\Psi_1(x) = \frac{1}{4} \left( - \text{erf}(1 - x) - \text{erf}(x + 1) - \text{erf}(1 - y_{\text{esc}}) - \text{erf}(y_{\text{esc}} + 1) \right.
\]

\[
+ \frac{2 e^{-(x+1)^2}}{\sqrt{\pi}} + \frac{2 e^{-(x+1)^2}}{\sqrt{\pi}}
\]

\[
- \frac{2 e^{-y_{\text{esc}}^2}}{\sqrt{\pi}} - \frac{2 e^{-y_{\text{esc}}^2}}{\sqrt{\pi}} + 2 \left). \quad \right]
\]

(11)

The function \( \Psi_2(x) \) is small, of order \( \delta^2 \) and it can be ignored. If, however, the experiments, which attempt to measure the modulation, want to go beyond the \( \cos \alpha \) term, they should consider terms \( \cos 2\alpha \) rather than \( \sin \alpha \) as some of them have done. The functions \( \Psi_0(x) \) and \( \Psi_1(x) \) characterize both the coherent and the spin induced mode.[72] We should note that the function \( \Psi_1(x) \) changes sign at some value of \( x \), which has implications on the total modulated rate, a point often missed (see Fig. 1). The functions \( \Psi_0(a\sqrt{u}) \) and \( \Psi_1(a\sqrt{u}) \), which exhibit the general characteristics of the differential rates, are exhibited in Figs. 2 and 3, while the function \( H(a\sqrt{u}) \) is shown
in Fig. 4. These functions are independent of the nuclear physics. They only depend on the reduced mass and the velocity distribution. They are thus the same for both the coherent and the spin mode. Note that \( \Psi_1(a\sqrt{u}) \) and, consequently, \( H(a\sqrt{u}) \) can take both positive and negative values, which affects the location of the maximum.

**Fig. 1** The generic functions \( \Psi_0(x) \) and \( \Psi_1(x) \) entering the differential rate, time averaged (a) and modulated (b). Note in (b) the change in sign at some point which depends on the target, the recoil energy and the WIMP mass.

**Fig. 2** The function \( \Psi_0(a\sqrt{u}) \) entering the differential rate as a function of the recoil energy for a heavy target, e.g. \(^{127}\text{I}\), without the form factor (a) and including the form factor (b). The solid, dotted, dot-dashed, dashed, long dashed and thick solid lines correspond to 5, 7, 10, 20, 50 and 100 GeV WIMP masses.

**Fig. 3** The function \( \Psi_1(a\sqrt{u}) \) entering the modulated differential rate as a function of the recoil energy for a heavy target, e.g. \(^{127}\text{I}\), without the form factor (a) and including the form factor (b). The solid, dotted, dot-dashed, dashed, long dashed and thick solid lines correspond to 5, 7, 10, 20, 50 and 100 GeV WIMP masses.

**Fig. 4** The same as in Fig. 3 for function \( H(a\sqrt{u}) \) entering the modulated differential rate as a function of the recoil energy for a heavy target, e.g. \(^{127}\text{I}\). Note that this is independent of the form factor. The solid, dotted, dot-dashed, dashed, long dashed and thick solid lines correspond to 5, 7, 10, 20, 50 and 100 GeV WIMP masses.

### 3 Some Results on Differential Rates

We will apply the above formalism in the case of NaI, a target used in the DAMA experiment.\(^{60-61}\) The results for the Xe target are similar.\(^{62}\) The differential rates \( dR/dQ|_A \) and \( d\tilde{H}/dQ|_A \), for each component \( (A = 127 \) and \( A = 23) \) are exhibited in Figs. 5–6. Following the practice of the DAMA experiment we express the energy transfer in keVee using the phenomenological quenching factor,\(^{73-74}\) shown in Fig. 7. The nuclear form factor has been included (for the \(^{127}\text{I}\) its effect is sizable even for an energy transfer of 10 keV, see Fig. 8). The differential rate for the spin mode for low energy transfers is similar to those exhibited in Figs. 5–6, since the spin form factors are similar with those of the target I. They
are, of course, simply scaled down by $A^2$, if one takes the spin cross section, a combination of the nuclear spin ME and the nucleon spin amplitudes, to be the same with the coherent nucleon cross section, i.e. $\sigma^{\text{spin}}_{\text{nuclear}} = 10^{-7}$ pb. For the actual spin nucleon cross sections extracted from experiment see [72] and [75]–[77].

The functions $H(\sqrt{u})\cos\alpha$ for each target component are shown in Figs. 9–13 as a function of $\alpha$ for various low energy transfers. The corresponding quantities for the spin mode are almost identical. We see that for certain values of the WIMP mass the modulation amplitude changes sign. This may perhaps be exploited to extract information on the WIMP mass from the data. A similar behavior has been found by considering various halo models and different minimum WIMP velocities.[33–34]

**Fig. 5** The differential rate $dR/dQ$, as a function of the recoil energy for a heavy target, e.g. $^{127}$I (a) and the amplitude for the modulated differential rate $d\tilde{H}/dQ$ (b), assuming a nucleon cross section of $10^{-7}$ pb. The solid, dotted, dot-dashed, dashed, long dashed and thick solid lines correspond to 5, 7, 10, 20, 50 and 100 GeV WIMP masses. Note that $d\tilde{H}/dQ$ is given in absolute units.

**Fig. 6** The same as in Fig. 7 for the target $^{23}$Na.

**Fig. 7** The quenching factor used in this work to transform keV $\rightarrow$ keVee.

**Fig. 8** The square of the nuclear form factor used in this work For $^{127}$I (a) and $^{23}$Na (b).
Fig. 9 The modulation $H(\sqrt{u}) \cos\alpha$ with an energy transfer of 1 keVee (a) and 2 keVee (b) for a heavy target (I or Xe). The solid, dotted, dot-dashed, dashed, long dashed and thick solid lines correspond to 5, 7, 10, 20, 50 and 100 GeV WIMP masses. Note that for some wimp masses on June 2nd the amplitude becomes negative (location of minimum rate). Note that the modulation is given relative to the time averaged rate.

Fig. 10 The same as in Fig. 9 with an energy transfer of 3 keVee (a) and 4 keVee (b).

Fig. 11 The same as in Fig. 9 with an energy transfer of 5 keVee (a) and 6 keVee (b).

Fig. 12 The same as in Fig. 9 for a light target (Na or F).

Fig. 13 The same as in Fig. 10 with a light target (Na or F).
Sometimes, as is the case for the DAMA experiment, the target has many components. In such cases the above formalism can be applied as follows: \( \frac{dR}{dQ} \big|_{A_i} \rightarrow \sum_i X_i \frac{dR}{dQ} \big|_{A_i}, \ u \rightarrow u_i, \ X_i = \text{the fraction of the component } A_i \text{ in the target.} \) Thus we get the results shown in Figs. 15 and 16–17. The corresponding ones for the spin mode are not expected to be the same.

**Fig. 14** The same as in Fig. 11 with a light target (Na or F).

**Fig. 15** The same as in Fig. 7 for the target NaI.

**Fig. 16** The same as in Fig. 9 for a NaI target.

**Fig. 17** The same as in Fig. 10 with a target of NaI.
4 Some Results on Total Rates

For completeness and comparison we will briefly present our results on the total rates. Integrating the differential rates discussed in the previous section we obtain the total time averaged rate $R_0$, the total modulated rate $\tilde{H}$ and the relative modulation amplitude $h$ given by:

$$R = R_0 + \tilde{H} \cos \alpha, \quad R = R_0(1 + h \cos \alpha).$$  \hspace{1cm} (13)

These are exhibited for zero threshold as functions of the WIMP mass in Figs. 19 and 20 respectively. Some special results in the case of low WIMP mass are exhibited in Tables 1–2. From Table 2 it becomes clear that, for low mass WIMPs, large nucleon cross sections can accommodate the data. A similar interpretation holds for the data. In the case of non zero threshold one notices the strong dependence of the time averaged rate on the WIMP mass. Also in this case the relative modulation $h$ substantially increases, the difference between the maximum and the minimum can reach 20%. This however occurs at the expense of the number of counts, since both the time averaged and the time dependent part decrease, but the time averaged part decreases faster. So their ratio increases. This can be understood by noticing that the cancellation of the negative and positive parts in the differential modulated amplitude, see Fig. 1, becomes less effective in this case.
between 20% and 40% for a heavy target, but it is a bit less between the maximum and the minimum could reach be-
on the energy transfer, especially at low transfers.

establish the location of the maximum on the

in particular the DAMA experiment, along these lines to

targets. We thus suggest an analysis of the experiments,

June). This effect is more pronounced in the case of heavy

behavior, but for large masses it changes sign (minimum in

For WIMP masses less than 10 GeV, the difference

between the maximum and the minimum could reach be-
 tween 20% and 40% for a heavy target, but it is a bit less

for a light target, depending on the energy transfer.

The relative modulation amplitude for NaI is the
weighted average of its two components, and in the low
energy regime, between 1 and 6 keVee, it does not change
much with the energy transfer.

Once it is established that one actually observes the
modulation effect, the sign of the modulation may be ex-
plotted to infer the WIMP mass.

For low WIMP mass the total rates depend strongly
on the threshold energy, especially for a heavy target. The
relative modulation in the presence a threshold gets quite
large (\(h \approx 0.2\)), but, unfortunately, this occurs at the
expense of the number of counts. It is important to com-
pare the relative total modulation in a least one light and
one heavy target. For very low energy thresholds, if the
signs are opposite, one may infer that the WIMP is heavy,
\(m_{\text{WIMP}} \geq 100\) GeV.

Table 1 Some total event rates for some special WIMP masses and energy thresholds. The coherent nucleon cross section
of \(\sigma_n = 10^{-7}\) pb was employed.

| \(E_{\text{th}}/\text{keV}\) | \(m_{\text{WIMP}}/\text{GeV}\) | \(R_0(1)/\text{kg-y}\) | \(H(1)/\text{kg-y}\) | \(h(1)/\text{kg-y}\) | \(R_0(\text{Na})/\text{kg-y}\) | \(H(\text{Na})/\text{kg-y}\) | \(h(\text{Na})/\text{kg-y}\) | \(R_0(\text{NaI})/\text{kg-y}\) | \(H(\text{NaI})/\text{kg-y}\) | \(h(\text{NaI})/\text{kg-y}\) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 80 | 16.3 | -0.311 | -0.019 | 1.518 | 0.028 | 0.019 | 14.0 | -0.259 | -0.018 |
| 0 | 20 | 25.8 | 0.285 | 0.011 | 2.35 | 0.050 | 0.021 | 22.2 | 0.249 | 0.019 |
| 0 | 10 | 18.4 | 0.356 | 0.019 | 2.045 | 0.046 | 0.022 | 15.9 | 0.309 | 0.019 |
| 5 | 80 | 7.90 | -0.042 | -0.006 | 1.133 | 0.038 | 0.034 | 6.11 | -0.030 | -0.005 |
| 5 | 20 | 2.72 | 0.247 | 0.091 | 1.07 | 0.065 | 0.060 | 2.47 | 0.219 | 0.089 |
| 5 | 10 | 0.008 | 0.001 | 0.187 | 0.303 | 0.031 | 0.103 | 0.053 | 0.006 | 0.114 |

Table 2 The same as in Table 1 for \(\sigma_n = 2 \times 10^{-4}\) pb relevant for the DAMA region. One sees that, for very low mass WIMPs,
large nucleon cross sections are required to obtain the rates claimed by the DAMA experiment.\[^{[61]}\]

| \(E_{\text{th}}/\text{keV}\) | \(m_{\text{WIMP}}/\text{GeV}\) | \(R_0(1)/\text{kg-y}\) | \(H(1)/\text{kg-y}\) | \(h(1)/\text{kg-y}\) | \(R_0(\text{Na})/\text{kg-y}\) | \(H(\text{Na})/\text{kg-y}\) | \(h(\text{Na})/\text{kg-y}\) | \(R_0(\text{NaI})/\text{kg-y}\) | \(H(\text{NaI})/\text{kg-y}\) | \(h(\text{NaI})/\text{kg-y}\) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 80 | \(4.07 \times 10^4\) | -776 | -0.019 | 3.80 \(\times 10^3\) | 70.2 | 0.019 | 3.50 \(\times 10^4\) | -647 | -0.018 |
| 0 | 20 | \(6.43 \times 10^4\) | 712 | 0.011 | 5.87 \(\times 10^3\) | 126 | 0.021 | 5.54 \(\times 10^4\) | 622 | 0.011 |
| 0 | 10 | \(4.61 \times 10^4\) | 891 | 0.019 | 5.11 \(\times 10^3\) | 115 | 0.022 | 3.98 \(\times 10^4\) | 772 | 0.019 |
| 5 | 80 | \(1.75 \times 10^4\) | -105 | -0.006 | 4.83 \(\times 10^3\) | 95.0 | 0.034 | 1.53 \(\times 10^4\) | -74.6 | -0.005 |
| 5 | 20 | \(6.80 \times 10^3\) | 617 | 0.091 | 2.69 \(\times 10^3\) | 162 | 0.060 | 6.17 \(\times 10^3\) | 547 | 0.089 |
| 5 | 10 | 19.4 | 3.62 | 0.187 | 757 | 78.1 | 0.103 | 132 | 15.0 | 0.114 |

Fig. 22 The total modulated event rate in kg-y for a kg of target of \(^{127}\)I (a), of \(^{23}\)Na (b) and of NaI (c) assuming
a coherent nucleon cross section \(\sigma_n = 10^{-7}\) pb and a threshold energy of 5 keVee.

5 Discussion

In the present paper we obtained results on the differ-
cential event rates, both modulated and time averaged,
focusing our attention on small energy transfers and rela-
tively light WIMPs. We found that:

The relative modulation amplitude crucially depends
on the WIMP mass. For small masses it exhibits normal
behavior, but for large masses it changes sign (minimum in
June). This effect is more pronounced in the case of heavy
targets. We thus suggest an analysis of the experiments,
in particular the DAMA experiment, along these lines to
establish the location of the maximum on the \(\alpha\)-axis.

The relative modulation amplitude depends somewhat
on the energy transfer, especially at low transfers.

For WIMP masses less than 10 GeV, the difference
between the maximum and the minimum could reach be-
tween 20% and 40% for a heavy target, but it is a bit less
for a light target, depending on the energy transfer.

The relative modulation amplitude for NaI is the
weighted average of its two components, and in the low
energy regime, between 1 and 6 keVee, it does not change
much with the energy transfer.

Once it is established that one actually observes the
modulation effect, the sign of the modulation may be ex-
plotted to infer the WIMP mass.

For low WIMP mass the total rates depend strongly
on the threshold energy, especially for a heavy target. The
relative modulation in the presence a threshold gets quite
large (\(h \approx 0.2\)), but, unfortunately, this occurs at the
expense of the number of counts. It is important to com-
pare the relative total modulation in a least one light and
one heavy target. For very low energy thresholds, if the
signs are opposite, one may infer that the WIMP is heavy,
\(m_{\text{WIMP}} \geq 100\) GeV.
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