Scalaron the healer: removing the strong-coupling in the Higgs- and Higgs-dilaton inflations

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Abstract

We show that introducing $R^2$-term makes the Higgs-inflation and Higgs-dilaton inflation consistent models: the strong coupling energy scales in scalar, gauge and gravity sectors all are lifted up to the Planck scale.

1. The Higgs-inflation [1] is one of the physically motivated inflationary models perfectly consistent with present cosmological observations [2]. It introduces a minimal modification to the Standard Model of particle physics (SM) — additional coupling of the Higgs field to gravity. However, the model suffers from the strong coupling problem [3, 4]: presently (in the electroweak vacuum) the model becomes strongly coupled and loses the perturbative unitarity well below the usual Planck scale, where gravity goes out of control and hence the whole theory. While the relevance of this observation for validity of the inflationary solution [1] is questionable [5, 6], the reheating, originally estimated [7, 8] to be within the perturbative region actually tends to happen earlier [9, 10], when the theory is in the strong coupling regime. Therefore, a healthy modification of the original model free of the strong coupling is desirable. The models suggested so far do not fully address these issues (see Ref. [11] for detailed discussion) leaving the problem unsettled.

In this Letter we put forward an idea that one of the most natural modification of the Higgs-inflation — achieved by adding a quadratic in scalar curvature $R$ term — allows one to push the strong coupling scales in all the model sectors up to the gravity scale $M_P$. 
This modification is a perturbatively tractable inflationary model providing with robust predictions of the cosmological parameters consistent with observations [2].

2. While such a modification is natural within the quantum perturbative theory in a model with non-minimal coupling to gravity [12], its capability of addressing the strong-coupling issue can be illustrated as follows. We start with the action of the Higgs-inflation augmented with the squared scalar curvature term,

\[ S_0 = \int d^4x \sqrt{-g} \left(-\frac{M_P^2 + \xi h^2}{2} R + \frac{\beta}{4} R^2 + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right). \]  

At \( \beta = 0 \) eq. (1) describes the model of Higgs-inflation (\( h \) stands for the Higgs field in unitary gauge). It successfully explains the flatness and homogeneity of the Universe and produces the scalar and tensor perturbations consistent with the present cosmological observations [2], provided non-minimal coupling \( \sim 10^3-10^4 \gg 1 \) [1,13]. However, this large coupling is known to spoil the perturbative unitarity of the model at energy scale \( M_P/\xi \) [3].

With \( \beta \neq 0 \) the action (1) is known to provide with an extra scalar degree of freedom (scalaron) in the gravitational sector [14]. The mass of this particle is given by

\[ m = \frac{M_P}{\sqrt{3} \beta}. \]  

Introducing a Lagrange multiplier \( L \) and auxiliary scalar \( \mathcal{R} \) we obtain from (1),

\[ S = \int d^4x \sqrt{-g} \left( \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 - \frac{M_P^2 + \xi h^2}{2} \mathcal{R} + \frac{\beta}{4} \mathcal{R}^2 - LR + LR \right). \]  

Then the field \( \mathcal{R} \) can be integrated out,

\[ S = \int d^4x \sqrt{-g} \left( \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 + LR - \frac{1}{\beta} (L + \frac{1}{2} \xi h^2 + \frac{1}{2} M_P^2) \right), \]  

and the large coupling \( \xi \) is moved to the potential term. Moreover, the actual coupling constant is \( \xi^2/\beta \) instead of \( \xi \). This constant can well be smaller than unity provided

\[ \beta \gtrsim \frac{\xi^2}{4\pi}. \]  

Thus, we expect that the problem corresponding to the large value of \( \xi \) can be solved in this way. Below we show that this is indeed the case.

\[ ^1 \text{Note that the value } \beta \sim \xi^2 \text{ is a natural choice of this parameter since non-minimal coupling } \xi \text{ induces a loop correction to } \beta \text{ of this order [12].} \]
3. Let us perform the Weyl transformation to the Einstein frame

\[ g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}, \quad \Omega^2 \equiv \frac{2L}{M_P^2}, \]

and replace \( L \) with \( \phi \) (dubbed scalaron) introduced as

\[ \phi \equiv M_P \sqrt{\frac{2}{3}} \log \Omega^2. \]

In terms of \( h, \phi \) and the rescaled metric, action (4) reads [15]

\[ S = \int d^4x \sqrt{-g} \left( -\frac{R}{12} + \frac{1}{2} e^{-2\phi} (\partial h)^2 + \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} e^{-4\phi} \left( \lambda h^4 + \frac{1}{36\beta} (e^{2\phi} - 1 - 6\xi h^2)^2 \right) \right). \]  

(6)

Hereafter we use a convention \( M_P = 1/\sqrt{6} \). In order to make the Higgs field canonical it is convenient to use a different set of variables,

\[ h = e^\Phi \text{th} H, \quad \phi = e^\Phi / \cosh H, \]  

(7)

which leads to the lagrangian in the scalar sector of the theory,

\[ L = \frac{1}{2} \cosh^2 H (\partial \Phi)^2 + \frac{1}{2} (\partial H)^2 - \frac{1}{4} \left( \lambda \sinh^4 H + \frac{1}{36\beta} (1 - e^{-2\Phi} \cosh^2 H - 6\xi \sinh^2 H)^2 \right). \]  

(8)

In this variables, the Higgs field couples to gauge \( W \)-bosons (and similarly to \( Z \)-bosons) as

\[ L_{\text{gauge}} = \frac{g^2 h^2}{4} e^{-2\phi} W_\mu^+ W^-_\mu = \frac{g^2}{4} \sinh^2 H W_\mu^+ W^-_\mu. \]  

(9)

4. Let’s determine positions of the strong coupling scales in the model (1) in the Einstein frame. There are three sectors to be examined.

**Gravity sector.** Action (6) implicitly demonstrates that gravity becomes strongly coupled at the Planck scale, as in General Relativity. Indeed, the gravitons come from the curvature term, which is of the standard form (Hilbert–Einstein, recall our convention \( M_P^2 = 1/6 \)).

**Scalar sector.** Here again, the model action in the form (6) is useful. The interaction between Higgs field \( h \) and scalaron \( \phi \) is originated from the kinetic term (the second term of lagrangian (6)) and the potential. Making use of the series in field \( \phi \) one finds, that it never comes with the large coupling (\( \phi \) always comes as \( \phi / (\sqrt{6} M_P) \)), so the kinetic term in (6) is healthy up to the Planck scale. Similar is true for the potential term of lagrangian (6), provided the inequality (5), even if \( \xi \gg 1 \). This behavior was also found in Ref. [15].
Therefore, we conclude that with model parameters obeying (5) the scalar is free from the strong coupling problem up to the Planck scale, where scalaron becomes strongly coupled.

Gauge sector. In the SM, the self-interaction of gauge bosons produces a part of the \( 2 \to 2 \) scattering amplitude which grows with the particle momenta above the electroweak scale, \( \propto p^2/m_W^2 \). However, this part coming from the scattering of longitudinal modes is canceled by the vertices including the exchange of the Higgs boson, see Fig. 1.

![Figure 1: Scattering of electroweak massive gauge bosons.](image)

If the Higgs sector is modified this compensation doesn’t hold anymore, that is the problem in the Higgs-inflation. The growing part of the amplitude can be written in the form,

\[
\mathcal{A} \sim \frac{p^2}{m_W^2} \left( \frac{4}{g^2} \left( \frac{d m_W(H)}{d H} \right)^2 - 1 \right).
\]

In our model the canonically normalized Higgs field couples to the gauge bosons via the term (9) inducing the mass term of the form \( m_W = g \sinh H/2 \) and hence

\[
\mathcal{A} \propto \frac{p^2}{M_P^2}.
\]

Thus, the unitarity cutoff scale for the scattering of the gauge bosons is the Planck mass (here we restore the Planck mass, according to our convention \( M_P = 1/\sqrt{6} \)).

In the fermionic sector of the Standard model, the scattering amplitude of two fermions to two gauge bosons will also grow linearly with the momentum (see [23]).

\[
\mathcal{A}_f \sim y g \frac{p}{m_W} \left( \frac{2}{g} \left( \frac{d m_W(H)}{d H} \right) - 1 \right) \propto \frac{p}{M_P} \frac{\cosh H - 1}{\sinh H}.
\]

Thus, the corresponding unitarity cutoff scale in the fermionic sector is always higher than the Planck mass.

5. The model predictions for cosmological parameters are similar to those of the original Higgs-inflation\(^2\) provided that the single field approximation is valid for this model [19]. The

\(^2\)Or \( R^2 \)-inflation [16], the difference in predictions of the two models is minuscule, for details see Refs. [17, 18].
inflationary trajectory lies in a deep valley (see Fig. 2) which corresponds to the condition

\[ 1 - e^{-2\Phi} \cosh^2 H - 6\xi \sinh^2 H = 0, \]  

(13)

so that only the first term in potential (8) contributes to the energy density. This condition, however, places a constraint on the two fields \( \Phi \) and \( H \). In general case, to obtain the amplitude of CMB fluctuations of order \( 10^{-5} \), one imposes a normalization condition [15],

\[ \beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9. \]  

(14)

We can speak about the Higgs-scalaron inflation as a UV completion of the Higgs inflation if the CMB amplitude is actually defined by parameters of the Higgs sector, \( \lambda \) and \( \xi \), rather than \( \beta \). The heavy degree of freedom indeed can be integrated out if \( \beta < \xi^2 / \lambda \). In this case, the predictions for the tilt of the scalar perturbation spectrum \( n_s - 1 \) and tensor-to-scalar ratio \( r \) are of the standard form\(^4\) [1],

\[ n_s = 1 - \frac{2}{N_e}, \quad r = \frac{12}{N_e^2}. \]  

(15)

\(^3\)Notice that larger values of \( \beta \) (which correspond to the light scalaron) are not allowed by the normalization condition (14) if \( \xi \) is fixed. However, if \( \xi \) is small the value of \( \beta \) is fixed since it defines the amplitude of the scalar perturbations.

\(^4\)This fact allows to distinguish this model from other UV completions for Higgs inflation suggested in the literature [20–22]. These works consider an addition of the extra scalar field. The cosmological predictions in this case typically depend on the parameters of this hidden scalar.
with $N_e = 50 \div 60$ being a number of e-foldings of inflation which depends slightly on the reheating temperature. These predictions fall right in the ballpark of the region allowed by the Planck experiment [2]. No significant isocurvature and non-gaussianity is expected since the mass of the orthogonal direction is significantly larger than the Hubble scale (see Fig. 3).

![Figure 3: The dependence of the effective mass of the isocurvature mode (orthogonal to the inflaton trajectory) on the value of field $H$. Notice that during inflation this mass is of order $M_P/\sqrt{\xi}$ while at smaller $H$ it becomes $M_P/\sqrt{3\beta} \lesssim M_P/\xi$. This behaviour is similar to the field-dependent cutoff scale in the Higgs inflation [6], something that is expected since this heavy degree of freedom provides a UV completion.](image)

Summarising the bound (14) and perturbativity condition (5) on the parameter $\beta$ we can write,

$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda}. \quad (16)$$

Thus, with typical value of $\lambda \sim 0.01$ at large values of the Higgs field the remaining window for parameter $\beta$ (which determines the scalaron mass (2)) is about three orders of magnitude. Consequently, for the reference value $\lambda = 10^{-2}$, the scalaron mass is in the interval $5 \times 10^{13} \text{ GeV} < m < 1.5 \times 10^{15} \text{ GeV}$.

6. In this part of the Letter, we show that the $R^2$ term can cure the strong coupling problem not only in the Higgs inflation. A possible scale invariant extension of this model known as Higgs-dilaton inflation [24] also suffers from the similar problem with the low cutoff scale. This model yields the Planck mass and naturally small Higgs mass from the spontaneous breaking of the scale symmetry. Under certain choice of parameters, it provides a viable inflationary stage. However, in this model, the Higgs field has to be coupled to gravity with large $\xi$ which again leads to the strong coupling scale about $M_P/\xi$, the same as in the original Higgs inflation.
The action of the Higgs-dilaton model completed with $R^2$ term reads,

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \beta R^2 + (\partial_\mu X)^2 - \xi X^2 R - \xi' h^2 R + (\partial_\mu h)^2 \right] - \frac{\lambda}{4} (h^2 - \alpha^2 X^2)^2$$  \hspace{1cm} (17)$$

In the Einstein frame this action can be written in such variables that the Higgs field direction becomes canonically normalized (see also Ref. [25] for a different choice of field variables where the scalaron field is canonical),

$$L = \frac{1}{2} \left( (\partial H)^2 + \cosh^2 H (\partial \phi)^2 + \cosh^2 H \cosh^2 \varphi (\partial \rho)^2 \right) - \frac{1}{4} \left( \lambda (\sinh^2 H - \alpha^2 \sinh^2 \varphi \cosh^2 H)^2 + \frac{1}{36\beta} (1 - 6\xi \sinh^2 \varphi \cosh^2 H - 6\xi' \sinh^2 H)^2 \right).$$  \hspace{1cm} (18)$$

Here field $\rho$ plays a role of the Goldstone boson of the broken scale invariance. It does not contribute to the potential. The field $\varphi$ corresponds to the scalar degree of freedom coming from gravity. The field $H$ is the only field coupled to the gauge and fermion sectors of the SM with the interaction term exactly of the form (9). Consequently, there is no strong-coupling issue in the gauge sector of the model, as well as in the scalar-gravity sector. The cutoff scale of such model is again pushed up to the Planck scale.

Figure 4: Potential and inflaton trajectory in Higgs-dilaton inflation with $R^2$ term.

The inflaton potential looks similar to the $R^2$-Higgs case. Again, the inflationary stage can be effectively described as the single field rolling inside the valley, under the conditions (16) on $\beta$, see Fig. 4. The predictions for spectral parameters are the same as in the Higgs-dilaton model [24]: the scalar tilt depends on the value of $\xi$,

$$n_s = 1 - 8\xi \coth 4\xi N_e.$$  \hspace{1cm} (19)$$
Therefore, in order to satisfy Planck limits [2], we need $\xi \lesssim 0.004$. The CMB amplitude can be obtained under the same condition as in (14).

7. Finally, as an extra bonus, the introduced $R^2$ term can improve the stability of the Higgs potential. The latter is known to take negative values at large fields if the central value of the top quark mass is considered, for details see [26]. The top Yukawa coupling contributes to the renormalization group running of the Higgs self-coupling $\lambda$, such that it is hard to reach the positive energy density during inflation (see [27]). The scalaron provides a positive one-loop contribution to the beta-function of $\lambda$ [12],

$$
\delta \beta_\lambda = \frac{1}{16\pi^2} \frac{2\xi^2(1 + 6\xi)^2}{9\beta^2}.
$$

(20)

Thus, in presence of the $R^2$-term the stability of the Higgs potential can be secured for larger values of the top quark mass $m_t$ (see Fig. 5).

![Figure 5](image_url)

Figure 5: The dependence of the Higgs self-coupling $\lambda$. Here we plot the running of the parameter $\lambda$ which stands in the potential (6). Due to the matching condition [15], this is exactly the parameter describing the low energy Higgs scattering. on the renormalization scale $\mu$ with the scalaron one-loop impact included. The latter affects the running starting from the scale of order the scalaron mass (2).

Notice also that if $\lambda$ is negative in some region of large fields, the Higgs field would stay in the false vacuum during reheating. However, due to the large reheating temperature, the thermal corrections could finally bring the Higgs to the SM vacuum [28]. While the detailed study of this process in our model is required, we expect that the domain of the top quark masses consisting with viable inflation becomes wider than in the minimal Higgs inflation.

7. To conclude, we show that a new gravitational scalar degree of freedom can improve the models that suffer from the strong coupling problem arising significantly below the Planck scale. With the scalaron added, these models become theoretically self-consistent.
cosmological models with inflation and reheating below the Planck scale. We observe that under certain conditions on the scalaron mass, introduction of this degree of freedom does not spoil the predictions of the Higgs and Higgs-dilation inflation. Moreover, the model allows for a consistent description of the particle production after inflation. We leave the detailed study of the reheating in these models for future work.

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