Research Article

Maximum Likelihood Estimation of Parameters for Advanced Continuously Reinforced Concrete Pavement (CRCP) Punchout Calibration Model

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Pavement performance prediction is the essential part of the pavement design, which is very important for highway agencies for the purpose of budget allocating. This study introduces a model of local calibration for punchout, which is the major structural distress of continuously reinforced concrete pavement (CRCP). It is assumed that the number of equivalent single axle loads’ (ESALs) leads to punchout follows a Weibull distribution. The parameters of Weibull distribution were estimated by maximum likelihood estimation (MLE). Additionally, an approach of estimating the initial value of the parameters was also presented before applying the Newton method for solving the likelihood equations. The regression result was found to fit the performance-monitoring data from LTPP very well. The proposed calibration model is capable of describing the punchout and can be employed to predict the failure rate and reliability of CRCP in the pavement design and the arrangement of rehabilitation activities.

1. Introduction

Predicting the pavement performance under various combinations of traffic levels, environmental conditions, pavement structures, and materials are a key component for highway agency to make a proper budget decision of the maintenance and rehabilitation activities [1]. Undoubtedly, the accuracy of the distress prediction depends on the calibration and validation of the mechanistic-empirical (ME) design models with independent datasets. Pavement engineers can definitely gain confidence in the design procedure when the ME models were calibrated by establishing an acceptable correlation between predicted and measured distresses in field. Local calibration is a systematic and mathematical process to minimize the difference between observed and predicted results by modifying, for instance, empirical calibration parameters that eventually would be found to be a function of the key factors as a means to improve the accuracy of the prediction models [2]. No mechanistic pavement design models can be applied in practice for pavement performance prediction without calibration due to the great variety of environmental conditions, pavement structures, materials, and traffic loads. In order to improve the accuracy, reliability, and robustness of the performance prediction model, field investigation data from LTPP was utilized in the calibration process [3]. LTPP database is worthy of the name of the largest pavement performance database, test sections from LTPP were used extensively in the calibration process, which can provide pavement engineers with historically recorded climate information, monitoring of distress and response, materials testing, maintenance, and especially pavement performance.
monitoring data. LTPP data were employed to conduct local and national calibration of the pavement performance by many researchers on both in-service flexible and rigid pavements [4–7], as well as survival analysis for preventive maintenance [8, 9].

However, as mentioned by Prozzi and Madanat [10], the pavement distress is an event of high variety, and it was more reasonable to describe the failure of pavement by a function of probability density rather than by a fixed-point estimation. It is worth noting that the distress investigation was not conducted continuously in LTPP data; moreover, the development of distresses is inconsistent [7]. The recording of punchouts as well as other types of distress is not simultaneous with the occurrence of pavement failure. Instead, only the number of punchouts formulated during the interval is available in the LTPP database. Pavement performance investigation is usually conducted at irregular intervals, ranging from 1–6 years. Therefore, the assumption that all the punchouts occur at the end of each interval will unavoidably underestimate the probability of the occurrence of punchouts.

The number of ESALs leading to punchout for a CRCP panel was assumed to follow Weibull distribution and the maximum likelihood estimation (MLE) of the parameters was introduced to estimate the parameters, due to insufficient data, to build the calibration model. The likelihood equations were solved by the Newton method, and least square regression was proposed to estimate the initial values. As a consequence, investigation data of field performance from LTPP GPS-5 (for CRCP) were extracted in Section 3 to illustrate the reliability and validity of the calibration procedure developed in this study.

2. Calibration Model

2.1. Weibull Distribution. Weibull distribution is the most popular and is widely used in the analysis of lifetime data and reliability since it was first introduced by Weibull in 1950s [11]. It has been proven to be very effective in modelling and analyzing lifetime data in medical, biological, and engineering sciences [12]. For instance, Weibull accelerated the analyzing lifetime data in medical, biological, and engineering sciences [12]. It has been proven to be very effective in modelling and analyzing lifetime data in medical, biological, and engineering sciences [12].

As a consequence, investigation data of field performance from LTPP GPS-5 (for CRCP) were extracted in Section 3 to illustrate the reliability and validity of the calibration procedure developed in this study.

The survival function of Weibull distribution, which is the probability that the pavement cannot be distressed at least up to time \( t \), can be expressed as follows:

\[
S(t) = P(T \geq t) = 1 - F(t) = e^{-(\lambda t)^\beta}, \quad t > 0.
\]

The hazard rate function is corresponding to the probability that failures occur at the short interval \([t, t + \Delta t]\), as \( \Delta t \) approaches zero. The hazard rate function for a Weibull distribution is

\[
h(t) = \lim_{\Delta t \to 0} \frac{Pr(t \leq T < t + \Delta t|T \geq t)}{\Delta t} = \frac{f(t)}{S(t)} = \lambda \beta (\lambda t)^{\beta-1}.
\]

Weibull distribution has a great variety of the shapes and is capable of describing the decreasing or increasing hazard rate of sample failure [14]. The hazard rate can be varied with the shape parameter \( \beta \). A value of \( \beta > 1 \) indicates that the failure rate will increase with time and will decrease if \( \beta < 1 \).

Weibull distribution is believed to be more advantageous to describe the formulation of pavement distress than exponential distribution because the pavement materials’ mechanical properties such as elastic modulus and modulus of rupture are all in a decreasing state with the increasing life of pavement and will increase the probability of concrete failure for CRCP. Therefore, the failure rate could not be a constant value (\( \beta = 1 \)).

Portland cement concrete (PCC) slabs may fail in terms of transverse cracking, longitudinal cracking, faulting, spalling, etc. This research is focused on punchout, the major structural distress of CRCP. It was pointed out by Zollinger and Barenberg [15] that the nature of punchout distress is fatigue related. The performance of CRCP is mainly affected by not only concrete materials but also the base layer [16], noted that poor support conditions coupled with short transverse cracking intervals usually lead to punchout distress. The current ME pavement design guide provides an empirical calibration model as follows [17]:

\[
PO = \frac{a}{1 + b \cdot FD^c},
\]

where \( PO \) denotes the total predicted number of punchouts per mile, \( FD \) is accumulated fatigue damage (due to slab bending in the transverse direction), and \( a, b, \) and \( c \) are calibration constants for the locally or nationally calibrated model.

As suggested by Jung and Zollinger [18], the fatigue-based damage \( FD \) can be determined by

\[
FD = \frac{N_e}{N_f}
\]

where \( N_e \) is the number of equivalent single axle loads (ESALs) and \( N_f \) is allowable equivalent traffic loads to failure.

Therefore, the survival function for CRCP can be expressed as a function of \( N_e \) rather than survival time. \( N_e \) is
assumed to follow the Weibull distribution, and the probability density function is

\[ f(N_e; \lambda, \beta) = \lambda \beta N_e^{\beta - 1} e^{-(\lambda N_e)^\beta}, \quad N_e \geq 0. \]  

(7)

Since the monitoring of distress is not consecutive, only the number \( n_j \) occurred during each interval of \( N_e \) \([N_{j-1}, N_j]\), and the total number of the intervals is \( k+1 \), and \( j = 1, 2, \cdots, k, 0 < N_0 < N_1 < \cdots < N_k < N_{k+1} = \infty \).

The probability of punchout that occurred in the interval \([N_{j-1}, N_j]\) can be expressed by

\[ p_j(N_e) = P(X \in [N_{j-1}, N_j]) = e^{-(\lambda N_j)^\beta} - e^{-(\lambda N_{j-1})^\beta}. \]  

(8)

The total number of punchouts is observed in the most recent investigation is \( n_p \) and then, the number of panel \( n_l \) which denotes the number of concrete panels that was lost to the observation in the site was

\[ n_l = n_t - n_p, \]  

(9)

where \( n_l \) is the total number of panels in the life test for a LTPP test section.

The survival function of the Weibull distribution:

\[ S(N_e) = 1 - F(N_e) = e^{-(\lambda N_e)^\beta}. \]  

(10)

2.2. Maximum Likelihood Estimation of Parameters. Maximum likelihood estimation is widely used to estimate parameters of Weibull distribution [19], and the likelihood function is

\[ L = \left( \prod_{j=1}^{k} p_j^{n_j} \right) \cdot S^{n_l}(N_e > N_k), \]  

(11)

\[ \log L = \sum_{j=1}^{k} n_j \log e^{-(\lambda N_j)^\beta} - e^{-(\lambda N_{j-1})^\beta} - n_l \log (\lambda N_k) \cdot (\lambda N_k)^\beta. \]  

(12)

We differentiate equation (12) with respect to the two unknown parameters and equal the resulting equation to zero as follows:

\[ \frac{\partial L}{\partial \lambda} \frac{n_l \lambda}{\partial \beta} = \sum_{j=1}^{k} n_j \left[ \frac{\beta^{\beta-1} (T_j^\beta e^{-(\lambda N_j)^\beta} - T_{j-1}^\beta e^{-(\lambda N_{j-1})^\beta})}{e^{-(\lambda N_j)^\beta} - e^{-(\lambda N_{j-1})^\beta}} - n_l \lambda N_k \cdot (\lambda N_k)^{\beta-1} \right] = 0, \]  

(13)

\[ \frac{\partial L}{\partial \beta} = \sum_{j=1}^{k} n_j \left[ (\lambda N_j)^\beta \log (\lambda N_j) e^{-(\lambda N_j)^\beta} - (\lambda N_{j-1})^\beta \log (\lambda N_{j-1}) e^{-(\lambda N_{j-1})^\beta} \right] - n_l \lambda N_k \cdot (\lambda N_k)^\beta = 0. \]  

(14)

Equations (13) and (14) can be solved using the Newton method, which can solve the nonlinear system equations by the iteration method [20].

For the given equations,

\[ F(x) = 0, \]

(15)

\[ F(x) = (f_1, f_2, \cdots, f_n)^T. \]

Truncating the Taylor expansion of \( f_i \) at \( x_0 \) after the linear terms gives

\[ f_i(x^0) + \sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j}(x^0) \cdot (x_j - x_j^0). \]  

(16)

The Newton method for the solution of a system of equations is

\[ x^{j+1} = x^j - J^{-1} F(x^j), \]  

(17)

where \( J \) is the Jacobi matrix of \( f \) at \( x^j \), as shown in the following expression:

\[ J_i = F'(x^j) \]

(18)
2.3. Estimation of Initial Values. To ensure the convergence of the iterative methods and make it converge fast, an initial value must be estimated before the application of the Newton method.

Similar to Qian and Correa [21], assuming that the subjects that lost to the follow-up process (just as panels with punchout occurs) live up to at least halfway of the period. It was assumed that all the punchout occurred at the mid for each interval, say $N_j^i$:

$$S(N_j^i) = e^{-(\lambda N_j^i)^\beta},$$

$$N_j^i = \frac{(N_{j-1} + N_j)}{2}$$

(19)

Taking natural logarithm of equation (19), we obtain

$$\ln(S(N_j^i)) = -\left(\lambda N_j^i\right)^\beta$$

$$\ln[-\ln(S(N_j^i))] = \beta \ln \lambda + \beta \ln N_j^i$$

(20)

Make $y = \ln[-\ln(S(N_j^i))]$ and $x = \ln N_j^i$; then, equation (20) can be linearized as follows into the $y = mx + b$ format. Least square regression was employed to estimate the intercept $b$ and the slope $m$, and we get an estimation of the initial values $\beta_0$ and $\lambda_0$:

$$\beta_0 = m,$$

$$\lambda_0 = \exp\left(\frac{b}{m}\right).$$

(21)

2.4. Determination of ESALs. The life of a CRCP panel is quantified as the total number of 80 kN ESALs in the design lane that leads to the formulation of punchout. The distressed-based equivalent single axle load ($N_i$) can be obtained by the methodology proposed by Chen and Zollinger [22].

3. Validation with LTPP Data

3.1. Summary Information of GPS-5 Test Sections. There are 85 test sections in the General Pavement Studies GPS-5 for CRCP performance research, located in 29 states across United States of America. The collection of LTPP data has been under way since 1989 [23]. Two test sections were selected to illustrate the effectiveness of the approach introduced, as shown in Table 1. One section is from South Carolina and the other from Texas.

The number of ESALs of the two sites was determined according to the procedure proposed by Chen et al. [22] and the information in detail is shown in Table 2. Notably, continuously investigations were carried out for both the sections since 1990s to 2000s, and the number of punchouts was increasing in an almost consecutive manner during the period. Unfortunately, investigation data, since the latest investigation, which were not updated, are found and are listed in Table 2.

3.2. Number of Panels in Each Section. About 90% of punchouts were observed on PCC panels between a pair of transverse cracks with crack spacing between 0.3 and 0.9 m [22]; in addition, punchout was defined as the area enclosed by two closely spaced (usually <0.9 m) transverse cracks, a short longitudinal crack, and the edge of the pavement or a longitudinal joint. And, “Y” cracks that exhibit spalling, breakup, or faulting are also included [25]. Furthermore, an advanced model of punchout prediction was proposed by Jung and Zollinger [18], and the number of punchout can be determined by equation (22):

$$N_{PO} = NC_{cs<0.9m} \times P_e \times P_{fc},$$

(22)

where $N_{PO}$ is the number of punchout, $NC_{cs<0.9m}$ is the number of cracks with crack spacing less than 0.9 m, $P_e$ is probability of erosion, and $P_{fc}$ is probability of fatigue cracking.

Consequently, the number of PCC panels between two transverse cracks spaced less than 0.9 m was taken as the total number of samples that placed in the life test, and the distribution of the transverse crack spacing was found to be possibly following the Weibull distribution. Therefore, the probability of the transverse crack spacing between LU and LL can be determined by the following expression [24]:

$$\text{Prob}(L_U \geq L \geq L_L) = 100\left\{\exp\left[-\left(\frac{L_U - L_{\min}}{\alpha}\right)^\gamma\right] - \exp\left[-\left(\frac{L_L - L_{\min}}{\alpha}\right)^\gamma\right]\right\},$$

(23)

And, the probability of crack spacing less than 0.9 m can be determined by

$$\text{Prob}(L_U \geq L) = 100\left\{1 - \exp\left[-\left(\frac{L_U - L_{\min}}{\alpha}\right)^\gamma\right]\right\},$$

(24)

where $\text{Prob}(L_U \geq L \geq L_L)$ is probability of crack spacing between $L_U$ and $L_L$ (%), $L_U$ is the upper limit of the cracking interval (m), $L_L$ is the lower limit of the cracking interval (m), and $L_{\min}$ is minimum length of crack spacing (m); this value is set to be 0.2 m according to field investigation of transverse cracking pattern.

Here, $\alpha$ and $\gamma$ are the scale and shape parameters of the crack spacing distribution, respectively. And, the parameters presented by Selezneva et al. [24] were employed in this paper to determine the number of CRCP panels with the crack spacing less than 0.9 m (Table 3).
4. Result and Discussion

A numerical program in accordance with the methodology in section 2 was written to estimate the values of $\lambda$ and $\beta$. It can be seen from Table 4 that the Newton iteration converged to the root of equations very quickly, and the number of iterations is only 3 to 4.

4.1. The Estimation of the Threshold Value of $N_e$ for Punchout Occurrence. The threshold value of $N_e$ is the equivalent traffic loads under which the probability of the occurrence of punchout is zero; then, the function can be expressed as follows:

$$F(N_e) = 0, \quad N_e \leq N_{e0},$$
$$F(N_e) = 1 - e^{-[\lambda(N_e - N_{e0})]^\beta}, \quad N_e > N_{e0}.$$

(25)

With the values of $\lambda$ and $\beta$ in Table 4, it is advantageous to estimate the value of $N_{e0}$ by the iteration method to find the minimum standard deviation of the predicted number of punchout. The standard deviation of the predicted results under a series of $N_{e0}$ can be determined by equations (26) and (27):

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (POU_{oi} - POU_{pi})^2}{N}},$$

(26)

$$POU_{pi} = N C_{c<0.9m} \times \left(1 - e^{-[\lambda(N_{e0} - N_{e0})]^\beta}\right).$$

(27)

where $\sigma$ is standard deviation, $N$ is the number of pavement performance investigation, $POU_{oi}$ is the number of punchouts recorded at the investigation $i$, $POU_{pi}$ is the predicted number of punchouts for investigation $i$, and $N_{e0}'$ is the assigned value of $N_{e0}$ in the iteration.

The value of $N_{e0}'$ should be in the range of $[0, N_{e1})$, where $N_{e1}$ is the equivalent traffic loads when the 1st investigation with punchouts is conducted. Substitute a series of $N_{e0}'$ into (28) to calculate $\sigma$ for each ease of $N_{e0}'$, as shown in Figures 1 and 2.

4.2. Hazard Function. The shape parameters $\beta$ of both the test sections are greater than 1, which indicates that the hazard rate of occurrence of punchout will increase with the accumulation of ESALs (Figures 3 and 4). In practice, the probability of fatigue cracking as well as that of base erosion with the time elapsing is increasing. The potential of punchout occurrence will increase as a result of the deterioration of the pavement structure. Theoretically, the estimation of the characteristic assuming Weibull distribution agrees well with the engineering practice. The punchout of CRCP is expressed as the product of probability of base erosion and the probability of fatigue cracking, as shown in equation (22). Interestingly, the base types of Section 45_5035 and Section 48_5323 are cement-treated base (CTB) and hot mix asphalt (HMA), respectively. Comparing

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**Table 1: Summary information on LTPP data.**

| State          | SHRP_ID | Date open to traffic | Annual daily truck traffic at beginning year | Mean crack spacing (m) | Steel content (%) | PCC thickness (mm) | Base type                      |
|----------------|---------|----------------------|---------------------------------------------|------------------------|-------------------|-------------------|-------------------------------|
| South Carolina | 45_5035 | 10/1/1975            | 257                                         | 0.75                   | 0.63              | 203               | Cement-treated subgrade soil  |
| Texas          | 48_5323 | 9/1/1980             | 768                                         | 0.71                   | 0.61              | 229               | HMA and lime-treated soil     |

**Table 2: Pavement performance monitoring in LTPP.**

| State and ID | Investigation date | Number of punchouts | ESALs   |
|--------------|--------------------|---------------------|---------|
| 45_5035      | 3/17/1992          | 21                  | 8.42E+06|
|              | 6/8/1993           | 28                  | 9.00E+06|
|              | 6/25/1997          | 30                  | 1.05E+07|
|              | 2/10/1999          | 39                  | 1.16E+07|
|              | 2/27/2002          | 58                  | 1.35E+07|
|              | 2/4/2003           | 75                  | 1.40E+07|
| 48_5323      | 6/11/1991          | 17                  | 5.44E+06|
|              | 5/19/1993          | 22                  | 6.44E+06|
|              | 8/10/1995          | 23                  | 7.44E+06|
|              | 5/14/1997          | 39                  | 8.44E+06|
|              | 6/16/1999          | 71                  | 9.45E+06|
|              | 6/25/2002          | 55                  | 1.10E+07|

**Table 3: number of panels with the length less than 0.9 m**

| Section no. | Count | $\alpha$ | $\beta$ | $N(\alpha, \beta, \gamma)$ |
|-------------|-------|----------|---------|-----------------------------|
| 45_5035     | 199   | 25.887   | 1.165   | 143                         |
| 48_5323     | 210   | 22.860   | 1.812   | 176                         |

**Table 4: Estimation of the shape and scale parameters.**

| Section no. | Initial value | Solutions | Number of iterations |
|-------------|---------------|-----------|----------------------|
| 45_5035     | 0.0434        | 1.3067    | 0.0560               | 1.3193        | 3                   |
| 48_5323     | 0.0326        | 1.0956    | 0.0690               | 1.1868        | 4                   |
Figures 3 and 4, it obviously shows that the hazard rate of CRCP with CTB base increases much more rapidly than that supported by asphalt treated base (ATB) when Ne is less than 10 million. Actually, ATB exhibits much better resistance of erosion than CTB [26, 27].

4.3. Distribution Function. The most concerned application of the pavement performance is the prediction of the level of distress, for instance, number of punchouts for CRCP. The prediction result will help the highway agencies in the decision-making processes related to the maintenance and rehabilitation. In Texas, USA, four levels of treatment are required: (1) preventive maintenance, (2) light rehabilitation, (3) medium rehabilitation, and (4) heavy rehabilitation or reconstruction [28]. The highway agencies may set a limit of number of punchout per unit length, for instance, 10 punchouts/km, at which a preventive pavement maintenance needs to be performed.

As the scale and shape parameters have been determined, it is possible to predict the number of punchouts at a specific traffic level \( N_j \):  

\[
N_{PO} = N_{C5<0.9\text{m}} \cdot \left[ 1 - \exp \left( -\left( \frac{\lambda \cdot N_j}{\beta} \right) \right) \right],
\]  

(28)

where \( N_{PO} \) is the number of punchout at a specific traffic level \( N_j \).

As it is shown in Figures 5 and 6, the predicted number of punchouts matches very well with the values from field investigations, and the proposed approach provides a reliable pavement performance-forecasting model.
4.4. Discussion. The number and accuracy of the LTPP observation datasets play a key role on the reliability and validity of pavements’ survival analysis. Unfortunately, the punchouts of CRCP has not been continuously monitored since latest investigation in 2000s for the two sections. Extensive research need to be conducted when some new datasets can be obtained.

5. Conclusions

In this study, an advanced calibration model for punchout distress in CRCP was proposed. Considering that LTPP database only records the number of punchouts occurred in each interval, which was recognized as grouped data, the number of ESALs was assumed to follow Weibull distribution. The maximum likelihood estimation was proposed to determine the parameters in Weibull distribution. The proposed calibration model was validated with the data from two test sections in LTPP database. The following conclusions can be drawn:

(i) The number of ESALs that leads to punchout distress is found to follow the Weibull distribution
(ii) The maximum likelihood estimation is effective in determining the parameters of the Weibull distribution
(iii) The proposed calibration model is capable of describing the punchout and can be employed to predict the failure rate and reliability of CRCP in the pavement design and arrangement of rehabilitation activities

Data Availability

Some or all data, models, or code generated or used during the study are available from the corresponding author by reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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