MOCCA-SURVEY DATABASE I: ECCENTRIC BLACK HOLE MERGERS DURING BINARY-SINGLE INTERACTIONS IN GLOBULAR CLUSTERS

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ABSTRACT

We estimate the population of eccentric gravitational wave (GW) binary black hole (BBH) mergers forming during binary-single interactions in globular clusters (GCs), using ∼ 800 GC models that were evolved using the MOCCA code for star cluster simulations as part of the MOCCA-Survey Database I project. By re-simulating binary-single interactions (only involving 3 BHs) extracted from this set of GC models using an N-body code that includes GW emission at the 2.5 post-Newtonian level, we find that ∼ 10% of all the BBHs assembled in our GC models that merge at present time form during chaotic binary-single interactions, and that about half of this sample have an eccentricity > 0.1 at 10 Hz. We explicitly show that this derived rate of eccentric mergers is ∼ 100 times higher than one would find with a purely Newtonian N-body code. Furthermore, we demonstrate that the eccentric fraction can be accurately estimated using a simple analytical formalism when the interacting BHs are of similar mass; a result that serves as the first successful analytical description of eccentric GW mergers forming during three-body interactions in realistic GCs.

Keywords: galaxies: star clusters: general – gravitation – gravitational waves – stars: black holes – stars: kinematics and dynamics

1. INTRODUCTION

Gravitational waves (GWs) from binary black hole (BBH) mergers have recently been observed (Abbott et al. 2016c,b,a, 2017a,b), but how the BBHs formed and merged is still an open question. Several merger scenarios have been proposed, from isolated field mergers (Dominik et al. 2015; Belczynski et al. 2016b,a) and dynamically assembled cluster mergers (Portegies Zwart & McMillan 2000; Banerjee et al. 2010; Tanikawa 2013; Bae et al. 2014; Rodriguez et al. 2016a; Amaro-Seoane & Chen 2016; Rodriguez et al. 2016b; Askar et al. 2017; Park et al. 2017), to primordial BH capture mergers (Bird et al. 2016; Cholis et al. 2016; Sasaki et al. 2016; Carr et al. 2016) and mergers forming in active galactic nuclei discs (Bartos et al. 2017; Stone et al. 2017; McKernan et al. 2017), however, how to observationally distinguish these channels from each other is a major challenge.

One of the promising parameters that both can be extracted from the observed GW waveform, and also seems to differ between different merger channels, is the BBH orbital eccentricity at a given gravitational wave frequency (e.g. Samsing et al. 2017a). Generally, one finds that dynamically assembled BBH mergers have a non-negligible probability to appear eccentric at observation, including hierarchical three-body systems (Silsbee & Tremaine 2017; Antonini et al. 2017), strong binary-single interactions (Samsing et al. 2014; Samuel & Ramirez-Ruiz 2017; Samsing et al. 2017a; Samsing et al. 2017; Rodriguez et al. 2017) and single-single interactions (O’Leary et al. 2009; Kocsis & Levin 2012; Cholis et al. 2016; Gondán et al. 2017), whereas all isolated field mergers are expected to be circular due to late time orbital circularization through GW emission (Peters 1964).

The importance of including general relativity (GR) in the equation-of-motion (EOM) for probing the population of eccentric BH mergers forming in globular cluster (GCs), was first pointed out by Samsing & Ramirez-Ruiz (2017); Samsing (2017), who derived that the rate of eccentric BBH mergers (> 0.1 at 10 Hz) forming through binary-single interactions is about ∼ 100 times higher when GR is included in the EOM, compared to using a purely Newtonian solver. By integrating over the dynamical history of a typical BBH Samsing (2017) showed that this implies that at present time up to ∼ 5% of all BBH mergers will have an eccentricity > 0.1 at 10 Hz. As described in Gültekin et al. (2006); Samsing et al. (2014), such eccentric mergers form through two-body GW captures during three-body interactions.

In this paper, we estimate the fraction of eccentric BBH mergers forming through two-body GW captures during binary-single interactions in GCs, using the data from ‘MOCCA-SURVEY Database I’, which consists of nearly 2000 GC models dynamically evolved by the state-of-the-art Monte-Carlo (MC) code MOCCA (Hypki & Giersz 2013; Giersz et al. 2013). Originally, all the binary-single interactions evolved for these GC models were performed with the Newtonian code fewbody (Fregeau et al. 2004); however, for this paper we re-simulate these interactions using a few-body code that includes orbital energy and angular momentum dissipation through GW emission at the 2.5 post-Newtonian (PN) level (Samsing et al. 2017b), with the goal of resolving the eccentric fraction. We further show how the rate of eccentric BBH mergers can be accurately estimated using a simple analytical formalism recently presented in Samsing (2017), which provides valuable insight into the analytical description and understanding of the relativistic few-body problem. Finally, we note that a similar study by Rodriguez et al. (2017) has been done in parallel to our work, but with a completely different code and dataset. This study finds, as well as we do, excellent agreement with the analytical predictions made by Samsing & Ramirez-Ruiz (2017); Samsing (2017).

In Section 2 we introduce the MOCCA code and the extensive GC dataset used for this study; ‘MOCCA-Survey Database I’. In Section 3 we describe our numerical and analytical approaches for estimating the fraction of eccentric
BBH mergers forming in ‘MOCCA-Survey Database I’. Results are given in Section 4, and conclusions in Section 5.

2. CODES AND DATA MODELS

In order to investigate BBH mergers from strong interactions in GCs, we utilize results from star cluster models that were evolved using the MOCCA (MONte Carlo Stimator) code (see Hypki & Giersz 2013; Giersz et al. 2013, and reference therein for details about the MOCCA code and the Monte Carlo method) as part of the MOCCA-Survey Database I project comprising of nearly 2000 GCs (Askar et al. 2017). MOCCA uses the orbit averaged MC method (Hénon 1971; Stodolkiewicz 1986) to carry out the long term evolution of spherically symmetric star clusters. For binary and stellar evolution, MOCCA employs prescriptions provided by the SSE/BSE codes (Hurley et al. 2000, 2002). In order to properly compute strong binary-single and binary-binary interactions, MOCCA uses the fewbody code (Fregeau et al. 2004) which is a direct N-body integrator for small N systems. The MC method is significantly faster than direct N-body codes and MOCCA can simulate the evolution of realistic GCs in a few days. Comparisons between MOCCA and direct N-body results show good agreement for both global parameters and evolution of specific objects in GC models (Giersz et al. 2013; Wang et al. 2016; Madrid et al. 2017).

MOCCA provides as an output every binary-single and binary-binary interaction that was computed using the fewbody code. For this paper, we extracted all the strong binary-single interactions that take place within a Hubble time and involve three BHs that individually have masses less than 100M⊙. There were more than a million such interactions from nearly 800 models in the ‘MOCCA-Survey Database I’. Nearly all of these interactions (99.8%) came from models in which BH kicks were computed according to mass fallback prescription given by Belczynski et al. (2002).

In the output data provided by MOCCA, all the parameters that were used to call the fewbody code for a particular interaction are provided, including the impact parameter, relative velocity, BH masses and initial binary semi-major axis (SMA). For the purpose of this study, we used the input parameters provided to fewbody for a subsample of these million interactions to re-simulate these strong interactions with our 2.5 PN few-body code described in Samsing et al. (2017b), as further explained in Section 3.1.

3. NUMERICAL AND ANALYTICAL METHODS

In this section we describe our numerical and analytical methods used for estimating the rate of eccentric BBH mergers forming through binary-single interactions extracted from ‘MOCCA-SURVEY Database I’.

3.1. Re-Simulating with a PN Few-Body Code

All the binary-single interactions performed for ‘MOCCA-SURVEY Database I’, were originally evolved using the Newtonian few-body code fewbody (Fregeau et al. 2004). To investigate the effects from GR, we re-simulated these binary-single interactions with our 2.5 PN few-body code described in Samsing et al. (2017b). To this end, we first selected all the binary-single interactions from ‘MOCCA-SURVEY Database I’ for which the initial orbital energy is negative (GR effects are only important for hard-binary interactions (Samsing et al. 2014)), and the tidal force exerted on the binary by the incoming single at peri-center assuming a Keplerian orbit is larger than the binding force of the binary itself (dynamical BBH mergers only form through strong interactions). This left us with a total of ~ 500,000 binary-single interactions.

For generating the initial conditions (ICs) for these interactions, we randomly sampled the respective phase angles according to the orbital parameters (Hut & Bahcall 1983), while keeping the initial binary SMA, eccentricity, impact parameter, and relative velocity fixed to the values given by ‘MOCCA-SURVEY Database I’. We did this 5 times for each of the original ~ 500,000 binary-single interactions provided by MOCCA to achieve better statistics, which then resulted in a total of ~ 2.5 × 10⁵ scatterings. Due to computational restrictions, we had to limit each interaction to a maximum of 2500 initial orbital times, which resulted in about 2% unfinished interactions that we chose to discard. Long duration interactions are usually a result of an interaction where one of the three BHs is sent out on a nearly unbound orbit, and represents therefore not any special class of outcome (Samsing & Lilan 2017). All results presented in this paper are based on the completed set of these interactions.

3.2. Analytical Estimate

It has recently been illustrated that the distribution of eccentric BBH mergers forming through binary-single interactions can be estimated analytically (Samsing et al. 2014, 2017a; Samsing 2017), despite the highly chaotic nature of the three-body problem and the complexity of GR. In this section we describe how to apply these recent calculations to estimate the population of eccentric mergers forming in GC data. For the equations below we follow the notation from Samsing (2017), as well as assuming the equal mass limit. This is an excellent approximation, as similar mass objects tend to interact at the same time due to the effect of mass segregation (e.g. Rodriguez et al. 2016a).

To estimate the number of GW capture mergers with measurable eccentricity ε_f at GW frequency f, we first use that a typical binary-single BH interaction generally can be described as a series of temporary BBHs with a bound single BH (Samsing et al. 2014). The single and the BBHs exchange in a semi-chaotic way energy and angular momentum, which makes it possible for the BBHs to occasionally reach very high eccentricities during the interaction (Samsing et al. 2014). Now, if the eccentricity of a given temporary BBH is high enough, the BBH will undergo a two-body GW capture merger while still being bound to the single: this is the population we loosely refer to as three-body GW capture mergers. Although these mergers generally form at very high eccentricity, they do not necessarily have a measurable eccentricity at the time of observation due to circularization during inspiral (Samsing & Ramirez-Ruiz 2017). Luckily, deriving the number of BBH mergers with ε_f at GW frequency f is easier than deriving the full population of three-body GW capture mergers (Samsing 2017), which makes it possible to easily estimate their expected rate, as explained in the following.

Assuming f only depends on peri-center distance (Wen 2003; Samsing 2017), a temporary BBH must form with a specific peri-center distance r_{EM} (‘EM’ is short for ‘Eccentric Merger’), for its orbital eccentricity to be ε_f at frequency f. The value for r_{EM} relates closely to the peri-center distance r_f at which the GW frequency is f, a distance that can be

\[^3\] To simulate a star cluster with a million objects using MOCCA on a present day single CPU, single core processor, it is needed about a day, up to a week, depending on the initial conditions.
Eccentric Black Hole Mergers

![Figure 1](image_url)

Figure 1. Distribution of BBH mergers formed through binary-single interactions. The results are based on ~ 500,000 binary-single interactions extracted from ‘MOCCA-SURVEY Database I’, each of which we simulated 5 times using a 2.5 PN few-body code, as described in Section 3.1. Top plot: Number of BBH mergers formed through binary-single interactions per logarithmic time interval as a function of time. The solid black line shows the BBH mergers originating from the population kicked out of their host cluster through a binary-single interaction. These BBHs are usually referred to as escapers. The dashed orange line shows the BBH mergers with eccentricity $e > 0.1$ at 10 Hz derived from the escaper population (black solid line). For this, we used our analytical framework described in Section 3.2. The solid red line shows the BBH mergers formed through two-body GW captures in binary-single interactions, a population we refer to as three-body GW capture mergers. Such mergers can only be probes using an $N$-body code that includes GW emission in the EOM. The solid blue line shows the BBH mergers with eccentricity $e > 0.1$ at 10 Hz. As seen, this population is ~100 times larger than the eccentric escaper population (dashed orange line), and is therefore completely dominated by three-body GW capture mergers. The dashed blue line shows our analytical estimate of the three-body GW capture mergers with eccentricity $e > 0.1$ at 10 Hz, as described in Section 3.2. Bottom plot: Ratio between the outcomes from the top plot (dashed orange, solid red, solid blue) and the escaper population (solid black). As seen, the three-body GW capture mergers constitute ~10% of all the BBH mergers observable at present time, where 1 ~ 5% will have an eccentricity $e > 0.1$ at 10 Hz. At early times the three-body GW capture mergers seem to even dominate the rate.

shown to fulfill $r_{EM}^2 \approx 2Gm^2\pi^{-2}$, where $m$ is the mass of one of the three (equal mass) BHs (see e.g. Samsing 2017). Using this approximation for $r_f$, the relation between SMA and eccentricity derived by Peters (1964), and that the initial orbital eccentricity at $r_{EM}$ is $\approx 1$ (a limit that follows from that $r_{EM} \ll$ than the initial SMA), one now finds,

$$r_{EM} \approx \left(\frac{2Gm}{f^2 \pi^2}\right)^{1/3} 1.1 \frac{e_f}{2 \frac{1}{12} 19} \left[\frac{425}{304} \left(\frac{1 + 121 e_f^2}{304 e_f^2}\right)^{-1}\right]^{170/2299},$$

as described in greater detail in Samsing (2017). To clarify, our derived $r_{EM}$ is the peri-center distance two BHs have to come within for their eccentricity to be $e_f$ at frequency $f$. Because $r_{EM}$ is a fixed distance, the probability for a single temporary BBH to form with an initial peri-center distance $< r_{EM}$ is simply $\approx 2r_{EM}/a$, where $a$ denotes the SMA of the initial target BBH, a relation that follows from assuming the BBH eccentricity distribution is thermal (Heggie 1975; Samsing 2017). Now, to find the probability for a single binary-single interaction to result in a BBH merger with an initial peri-center distance $< r_{EM}$, referred to as $P_{EM}$, one simply needs to weight with the number of temporary BBHs forming per binary-single interaction, a number we denote by $N_{IMS}$, where ‘IMS’ is short for ‘Intermediate State’ (Samsing 2017). From this finally follows,

$$P_{EM} \approx \frac{2r_{EM}}{a} \times N_{IMS},$$

where $N_{IMS} \approx 20$ in the equal mass case (Samsing et al. 2017a; Samsing 2017). We have here assumed that if two BHs undergo an initial peri-center distance $< r_{EM}$ then they also merge, which is an excellent approximation for sources observable by an instrument similar to the ‘Laser Interferometer Gravitational-Wave Observatory’ (LIGO), but not necessarily for sources in the frequency range of the ‘Laser Interferometer Space Antenna’ (LISA).

We applied this analytical formalism to estimate the number of BBH mergers with eccentricity $> e_f$ at GW frequency $f$, forming in the dataset ‘MOCCA-SURVEY Database I’. To this end, we first calculated $P_{EM}$ for each of the binary-single interactions in the set we extracted for re-simulation (see Section 3.1), assuming that the three interacting BHs all have the same mass equal to their average mass. As $P_{EM}$ effectivly describes the average number of BBH mergers with eccentricity $> e_f$ at GW frequency $f$ forming per interaction, the distribution of such mergers is simply given by the distribution of $P_{EM}$. This approach allows us to instantly derive simple relations between observed eccentricity and GW frequency, that otherwise would take thousand of ‘CPU hours’ and an extensive amount of coding. As shown in the sections below, the estimate from this analytical approach is remarkably accurate.

4. RESULTS

Our main results are presented in Figure 1, where each of the shown outcomes are described in the paragraphs below.

4.1. Escaping Black Hole Mergers

The distribution of BBH mergers originating from the population of BBHs dynamically ejected from their host cluster through binary-single interactions is shown in black. For this estimation, we first identified all the binary-single interactions that resulted in an ejected BBH with a dynamical kick velocity (derived from the output of our re-simulated few-body interactions) larger than the escape velocity of the cluster (derived from the central potential provided by the MOCCA-code output). We then followed this escaped population using the orbital evolution equations given by Peters (1964), from which we derived the final distribution of merger times. This population of BBH mergers have been extensively studied using
both $N$-body (e.g. Bae et al. 2014; Park et al. 2017) and MC (Rodriguez et al. 2016a; Askar et al. 2017) techniques, which all find that this dominates the present day BBH merger rate originating from GCs.

The distribution of BBH mergers with eccentricity $e > 0.1$ at 10 Hz originating from the escaper population (the one shown in black), is shown with an orange dashed line. It was extremely difficult to numerically resolve this population due to its low statistics, so instead we used our analytical framework described in Section 3.2. To this end, we first selected all the binary-single interactions leading to an escaping BBH, after which we calculated the probability for each of these to have $< r_{EM}$ using Equation (2) with $N_{MS}$ set to 1, as there is only one ejected BBH per interaction. As seen, the fraction of escaping BBH mergers with $e > 0.1$ at 10 Hz is extremely low, which have led several cluster studies to conclude that the rate of eccentric BBH mergers forming in GCs is far too low to be observable; however, as described by Samsung & Ramirez-Ruiz (2017); Samsung (2017), the rate of eccentric mergers is not dominated by the escape merger population, but instead by three-body GW capture mergers – a statement recently confirmed by Rodriguez et al. (2017), and further described below.

4.2. Three-Body GW Capture Mergers

The distribution of three-body GW capture mergers is shown in the top panel of Figure 1 with a red solid line. As seen, these three-body GW capture mergers constitute about 10% of all the BBH mergers observable at late times, and seem to even dominate the merger rate at early times. These results are in surprisingly good agreement with recent analytical work by Samsing et al. (2017a); Samsung (2017). The reason why the number of three-body GW capture mergers is surprisingly large, is because all binary-single interactions can contribute to this merger population, and not only the ones leading to BBH escapers. From the data ‘MOCCA-SURVEY Database I’ we found that for every binary-single interaction leading to an escaper, there are of order $10^2$ binary-single interactions each of which potentially can undergo a three-body GW capture merger without leading to an escaper.

The distribution of three-body GW capture mergers with eccentricity $e > 0.1$ at 10 Hz derived using our 2.5 PN few-body code, is shown with a blue solid line. As seen, this population is much larger than the one originating from the escaper population (orange dashed line), which clearly illustrates that the rate of eccentric sources is dominated by three-body GW captures (to exactly which degree binary-binary interactions contribute is topic of current research). By comparing the blue and orange histograms, one finds that the rate of eccentric sources increases by a factor of $\sim 100$ when three-body GW captures are included, which agrees surprisingly well with the recent analytical derivations by Samsung (2017). In short, this enhancement factor is a product of $N_{MS}$ (the three-body system has $N_{MS} \approx 20$ tries during the interaction per single escaper) and a factor that represents the possibility for an eccentric three-body GW capture merger to form in binary-singles that do not lead to an escaper (which is $\approx 5$).

The distribution of three-body GW capture mergers with eccentricity $e > 0.1$ at 10 Hz derived using our analytical framework from Section 3.2 is shown with a dashed blue line. As seen, the agreement with our full numerical estimate (solid blue line) is remarkable, which proves the analytical framework as a highly useful tool for exploring observable relations between eccentricity and GW frequency.

The bottom panel in Figure 1 shows the different outcome distributions from the top plot divided by the distribution of BBH mergers formed through escapers (orange/red/blue histograms divided by the black histogram). As seen, the relative rate of eccentric (dominated by three-body GW captures) to circular mergers (dominated by the escapers) is at present time $1 - 5\%$, which again is in excellent agreement with the analytical predictions by Samsung (2017). This leads to the conclusion that the eccentric fraction is likely to be within observable limits if BBH mergers from GCs contribute notably to the observed population, which highly motives further work on eccentric GW templates (e.g. Harry et al. 2016; Huerta et al. 2016, 2017). Similar encouraging results are discussed in Rodriguez et al. (2017).

5. CONCLUSIONS

We have in this paper presented estimates of the population of GW capture mergers forming during binary-single interactions (three-body GW capture mergers) in GCs evolved using realistic prescriptions. To this end, we re-simulated $\sim 500,000$ strong binary-single interactions extracted from the dataset ‘MOCCA-SURVEY Database I’ derived using the MC code MOCCA, with a few-body code that includes GW emission in the EOM (Samsung et al. 2017b) using the PN formalism (e.g. Blanchet 2014). In addition, we further showed how the analytical framework from Samsung (2017) can be used to make accurate and instant estimates of the rate of BBH mergers that will appear in the observable GW band with a notable eccentricity. This illustration provides an important piece in further developments of analytical GR models for understanding the evolution of dense stellar systems.

Our analytical and numerical results strongly indicate that $\sim 10\%$ of all GC BBH mergers that are observable at present time originate from three-body GW capture mergers (See bottom plot in Figure 1), which is in excellent agreement with the recent analytical study by Samsung (2017); a result also confirmed by Rodriguez et al. (2017). In addition, the population of GC BBH mergers with eccentricity $> 0.1$ at 10 Hz is about $1 - 5\%$ of the total GC merger rate at present time, which strongly suggests that eccentric mergers are within observable limits for an instrument similar to LIGO, given that GCs contribute to the observed rate. This finding opens up for the possibility of using the eccentricity distribution to constrain the fraction of BBH mergers that form dynamically. These promising results indeed motivate further work on the role of GR in the evolution of GCs (e.g. Kupi et al. 2006; Brem et al. 2013), both from the numerical and the analytical sides.

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