WHITHER HADRON SUPERSYMMETRY?

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Abstract. A dynamically broken hadron supersymmetry appears to exist as a consequence of QCD. The reasons for the supersymmetry appear most transparently in the framework of the constituent quark model with a diquark approximation to two quarks. Applications of the supersymmetry have led to relations between meson and baryon masses and to predictions that certain kinds of exotic hadrons should not be observed. I summarize the successful applications and discuss possible future directions for this research.

INTRODUCTION

Physicists have applied the concept of supersymmetry to a number of different areas. To particle physicists, the most familiar supersymmetry is a spontaneously broken supersymmetry between particles and sparticles, for which at present no experimental evidence exists. However, there is experimental evidence for dynamically broken supersymmetries in the areas of atomic physics, nuclear physics, and hadron physics.

As far as I know, the oldest of these applications of supersymmetry was to hadron physics, discussed first by Miyazawa [1] in 1966. Almost a decade later, Catto and Gürsey [2] made plausible that dynamically broken hadron supersymmetry is a consequence of QCD. They also showed that one consequence of the supersymmetry is that Regge trajectories of mesons and baryons have approximately the same slope.

The reason for hadron supersymmetry is most transparent in the approximation to QCD known as the constituent quark model. In this model, the reason for hadron supersymmetry can be seen as follows: According to QCD an antiquark belongs to
a $\bar{3}$ multiplet of color SU(3). A two-quark system, which I call a diquark, can be in either a $6$ or $3$ multiplet. Any two constituent quarks in a baryon must belong to the $3$ so that the baryon can be an overall color singlet. Now a meson contains a constituent quark and a constituent antiquark. If we replace the antiquark (a fermion) by a $3$ diquark (a boson), we make a supersymmetric transformation of a meson into a baryon. This transformation does not change the color configuration. Because, in first approximation, the QCD interaction depends only on the color configuration, the force between the quark and diquark in a baryon should be approximately the same as the force between quark and antiquark in a meson. Hence, we should be able to use supersymmetry to relate the properties of baryons to the properties of mesons.

If we replace antiquarks by diquarks in normal hadrons, we can obtain exotic hadrons. For example, if we replace $Q\bar{Q}$ (a bar on the symbol for a particle denotes the antiparticle) by $D\bar{D}$, where $Q$ is a quark and $D$ is a diquark, we obtain an exotic meson from a normal one. Making use of supersymmetry, we can relate properties of exotic hadrons to similar properties of normal ones. In exotic hadrons, a diquark can be either in a $3$ or a $6$ multiplet of color. The interactions of the $6$ cannot be related by supersymmetry to the interactions of an antiquark, and so we must neglect the $6$. We justify this neglect as follows: When two quarks are close together, QCD says that their Coulomb-like interaction is attractive in a $\bar{3}$ and repulsive in a $6$. It is then plausible that the $\bar{3}$ lies lower in energy than the $6$. If we confine ourselves to low-mass exotics we hope that we may safely neglect the contribution of color-$6$ diquarks.

The difficulty with applying supersymmetry to hadrons is that the supersymmetry is badly broken, or the pion and proton would have the same mass. Miyazawa [1] was already aware of this difficulty in 1966. Supersymmetry breaking arises from at least three differences between a diquark and a quark (or antiquark): 1) they have different sizes; 2) they have different masses; and 3) they have different spins. We briefly discuss these differences.

1) Obviously, a diquark is not a point particle, but neither is a constituent quark, as it consists of a pointlike quark surrounded by a cloud of gluons and quark antiquark pairs. I have not seen any paper discussing how supersymmetry is broken by size differences between quark and diquark, and I have made no progress on this problem myself, so I have to neglect the effects of diquark size.

2) Mass effects can be taken into account in several ways. One particularly simple method is to relate mass differences between mesons to mass differences between baryons in such a way that the effects of the diquark-quark mass difference is most likely to cancel out. Another method is to make use of the fact that the quark-antiquark binding energy in mesons depends smoothly on the constituent quark masses [3]. In this method, the binding energy of a quark with a diquark can be estimated by treating the diquark as a fictitious antiquark with the diquark mass.

3) There are spin-dependent forces in QCD. One way to minimize their effect is to take appropriate averages over spin. Another way is to assume that the spin-dependent interaction energy between two quarks in a diquark is independent of the hadrons in which the diquark is embedded. This assumption is not strictly correct
but it is a good approximation. Then the spin-dependent contribution to the interaction energy can be approximately extracted \[4\] from the experimentally known masses of baryons. In both spin averaging and extracting spin-dependent forces from baryons, it is assumed that the spin-dependent force in ground-state hadrons is the usual chromomagnetic force arising from one-gluon exchange \[5\]. This assumption has been challenged by Glozman and Riska \[6\], and I have discussed the arguments in favor of one-gluon exchange in a talk at the last Orbis meeting \[7\].

I have been working on the consequences of broken hadron supersymmetry for several years and have spoken about it at two previous Orbis meetings \[8,9\]. In the present talk I shall update the conclusions of my two earlier talks and discuss possible directions for future work in hadron supersymmetry.

**RELATIONS BETWEEN MESON AND BARYON MASSES**

From here on, I will sometimes call an antiquark a quark and an antidiquark a diquark. In this language, for example, a meson is a two-quark state and an exotic meson is a four-quark or two-diquark state.

We use the notation that $Q$ denotes any quark, $q$ denotes a light $u$ or $d$ quark, and $D$ ($QQ$) denotes a color 3 diquark. Also $M$ ($Q\bar{Q}$) is a normal meson, $M_E$ ($QQ\bar{Q}\bar{Q}$ or $D\bar{D}$) is an exotic meson, $B$ ($QQQ$ or $QD$) is a normal baryon, $B_E$ ($QQQQ\bar{Q}$ or $DD\bar{Q}$) is an exotic baryon, and $B_2$ ($QQQQQQ$ or $DDD$) a dibaryon.

As a consequence of hadron supersymmetry, we can make the transformations

\[Q \rightarrow D, \quad Q \rightarrow \bar{D}.\]  

Applying either the first or second of eqs. (1) one or more times, we obtain

\[M = Q\bar{Q} \rightarrow B = QD,\]  
\[B = QD \rightarrow M_E = \bar{D}\bar{D},\]  
\[B = QQQ \rightarrow B_E = D\bar{D}\bar{Q},\]  
\[\bar{B} = Q\bar{Q}\bar{Q} \rightarrow B_2 = DDD.\]  

We next consider how to take into account supersymmetry breaking. One way to minimize the effects of spin-dependent forces is to average over spins in such a way that perturbatively the spin-dependent forces cancel out. In order to do this, we must make an assumption about the nature of these spin-dependent forces. Following De Rújula et al. \[5\], we assume that the spin-dependent forces arise from one-gluon exchange. Then the spin averaging of ground-state hadrons is given by the prescription of Anselmino et al. \[4\]. One way to minimize the effects of mass differences between quarks and diquarks is to let one quark in the diquark be a light quark $q$. We do this by confining ourselves (in this section) to the transformations

\[\bar{Q} \rightarrow D_q = Qq, \quad Q \rightarrow \bar{D}_q = \bar{Q}\bar{q}.\]
We also take differences in masses such that the effect of the extra light quark in the diquark will tend to cancel out.

In the following, we let the symbol for a hadron denote its mass, and we write the constituent quarks of a hadron in parentheses following the hadron symbol. We are led by the considerations of the previous paragraph to consider the difference of two meson masses: $M(Q_2q) - M(Q_1q)$. Applying the transformation of eq. (6), we get

$$M(Q_2q) - M(Q_1q) = B(Q_2qq) - B(Q_1qq).$$

(7)

The masses in eq. (7) are to be thought of as spin averages, i.e.

$$M(qq) = (3\rho + \pi)/4, \quad M(sq) = (3K^* + K)/4,$$

$$M(cq) = (3D^* + D)/4, \quad M(bq) = (3B^* + B)/4,$$

$$B(qqq) = (\Delta + N)/2, \quad B(sqq) = (2\Sigma^* + \Sigma + \Lambda)/4,$$

$$B(cqq) = (2\Sigma^*_c + \Sigma_c + \Lambda_c)/4, \quad B(bqq) = (2\Sigma^*_b + \Sigma_b + \Lambda_b)/4,$$

(8)

(9)

where the symbols for the mesons and baryons are those of the Particle Data Group [10]. Using eqs. (8) and (9) in (7), we obtain the sum rules [8]

$$(3K^* + K)/4 - (3\rho + \pi)/4 = (2\Sigma^* + \Sigma + \Lambda)/4 - (N + \Delta)/2,$$

(10)

$$(3D^* + D)/4 - (3K^* + K)/4 = (2\Sigma^*_c + \Sigma_c + \Lambda_c)/4 - (2\Sigma^* + \Sigma + \Lambda)/4,$$

(11)

$$(3B^* + B)/4 - (3D^* + D)/4 = (2\Sigma^*_b + \Sigma_b + \Lambda_b)/4 - (2\Sigma^*_c + \Sigma_c + \Lambda_c)/4.$$

(12)

These same sum rules were obtained earlier [11] by a method not using hadron supersymmetry. However, the assumption of one-gluon exchange was needed for averaging over spin states.

We can test the sum rules with the experimental values of the known hadron masses [10]. The left-hand side of eq. (10) is $182 \pm 1$ MeV, while the right-hand side is $184 \pm 1$ MeV, in good agreement with experiment. Similarly, the left-hand side of eq. (11) is $1179 \pm 1$ MeV, while the right-hand side is $1174 \pm 1$ MeV, also in satisfactory agreement with the data. In a 1996 talk [8], I noted that eq. (12) was consistent with preliminary data on baryons containing $b$ quarks, but the 1998 tables of the Particle Data Group [10] do not confirm those data. Therefore, the sum rule of eq. (12) remains to be tested by experiment.

The fact that the sum rules of eqs. (10) and (11) agree with the data constitutes evidence in support of spin-dependent forces arising from one-gluon exchange. These sum rules do not follow from the spin-dependent forces postulated by Glozman and Riska [6]. In their work, the spin-dependent forces in baryons containing only light quarks arise from pseudoscalar meson exchanges. However, I don’t see how the same mechanism can apply to mesons or to baryons containing heavy quarks. If I would need two or three different mechanisms to account for the spin-dependent forces in hadrons (or a linear combination of them), then I would not know how to obtain sum rules.
EXOTIC HADRONS

We do not need to restrict ourselves to spin-averaged hadron masses or to diquarks containing at least one light quark, as we can explicitly take into account mass and spin effects. I discussed this problem at a previous Orbis [9], and so will only briefly review the method.

We start with the spin-averaged hadron masses, but include spin effects explicitly at a later stage. We assign constituent masses to the quarks such that the binding energy of a quark and antiquark in a meson is a smooth function of the reduced mass of the two constituents [3]. We can use this “meson curve” to read off the binding energy of a fictitious hadron made of a fictitious quark and antiquark of any given masses.

We consider a spin-averaged baryon made of a quark and a diquark, treating the diquark as a fictitious antiquark. Our first guess for the diquark mass is that it equals the sum of its two constituent quark masses. We obtain the reduced mass of the quark and diquark and read off the binding energy from the meson curve. We add this binding energy to the masses of the quark and diquark to obtain a calculated spin-averaged baryon mass. In general, this mass does not equal the experimental mass of a baryon, averaged over spin. However, by repeatedly adjusting the mass of the diquark, we can obtain the correct spin-averaged baryon mass. We are thus able to obtain the spin-averaged diquark masses for constituent quarks of any flavors.

Next we obtain diquark properties from observed baryon masses rather than from spin-averaged masses. We extract the spin-dependent interaction energies of two quarks in a diquark from the observed baryon masses [4]. Adding these terms to the spin-averaged diquark mass, we obtain the masses of spin-one and spin-zero diquarks.

We are now ready to calculate the masses of ground-state exotic hadrons. We first consider exotic mesons containing at least one diquark of spin zero. In such mesons, there are no spin-dependent forces between the diquarks. Therefore, we only have to calculate the reduced mass of the constituents and add the binding energy from the meson curve to the diquark masses in order to obtain the exotic meson mass. (If both diquarks have spin one, there are additional spin-dependent forces, but their effects can be calculated.)

The results of these calculations is that diquark-antidiquark exotic mesons have sufficiently large masses to decay rapidly into two normal mesons. Because we expect production cross sections to be small and decay widths large, it is unlikely that such exotic mesons will be observed. A possible exception is that an exotic meson containing a $bb$ diquark might be stable against strong decay, but its production cross section will be extremely small. Our conclusion is in agreement with the fact that no exotic mesons composed of a diquark and antidiquark have yet been seen.

The same method can be applied to exotic baryons and to dibaryons. However, there is the complication that, except in the limit of point-like diquarks, the Pauli principle is not strictly satisfied for quarks in different diquarks. The results are similar to the results for mesons: exotic baryons and dibaryons (other than the deuteron) are not likely to be observed. Again, this conclusion is in agreement with
observations to date.

THE FUTURE

The predictions of the previous sections follow from broken hadron supersymmetry plus spin-dependent forces arising from one-gluon exchange. It is gratifying that we have not obtained any predictions in serious disagreement with experiments done so far, but it is disappointing that our model says that diquark exotics will probably not be observed.

Although enough has been established so far to give me confidence that hadron supersymmetry is a useful concept, open questions remain to be answered. Among them are:

(1) A diquark may be almost as large as the hadron that contains it. How do we correct for the non-negligible size of a diquark?

(2) Is there any way to take into account the contribution from color-sextet diquarks to exotic hadrons?

(3) If the spin-dependent forces in some hadrons are not given by one-gluon exchange but rather by the mechanism of Glozman and Riska, how do the results change? Are the changes large enough to destroy the good agreement with experiment?

(4) How can we take the Pauli principle into account in exotic baryons and dibaryons?

(5) Exotic hadrons containing diquarks can mix with other hadrons having the same quantum numbers. For example, quantum numbers permitting, a diquark-antidiquark meson can mix with normal mesons, hybrids, and glueballs. Can we take this mixing into account?

(6) Are there any other useful predictions to be obtained from broken hadron supersymmetry?

In conclusion, if physicists can successfully tackle the preceding open questions, hadron supersymmetry will rest on a much sounder foundation than it does now. However, if answers are not forthcoming, it may be time for physicists to store in their minds that broken hadron supersymmetry exists and go on to other topics.

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