NeuRoRA: Neural Robust Rotation Averaging

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Abstract

Multiple rotation averaging is an essential task for structure from motion, mapping, and robot navigation. The task is to estimate the absolute orientations of several cameras given some of their noisy relative orientation measurements. The conventional methods for this task seek parameters of the absolute orientations that agree best with the observed noisy measurements according to a robust cost function. These robust cost functions are highly non-linear and are designed based on certain assumptions about the noise and outlier distributions. In this work, we aim to build a neural network that learns the noise patterns from the data and predict/regress the model parameters from the noisy relative orientations. The proposed network is a combination of two networks: (1) a view-graph cleaning network, which detects outlier edges in the view-graph and rectifies noisy measurements; and (2) a fine-tuning network, which fine-tunes an initialization of absolute orientations bootstrapped from the cleaned graph, in a single step. The proposed combined network is very fast, moreover, being trained on a large number of synthetic graphs, it is more accurate than the conventional iterative optimization methods. Although the idea of replacing robust optimization methods by a graph-based network is demonstrated only for multiple rotation averaging, it could easily be extended to other graph-based geometric problems, for example, pose-graph optimization.

1. Introduction

Recently, we have witnessed a surge of interest in applying neural networks in various computer vision and robotics problems, such as, single-view depth estimation [12], absolute pose regression [19] and 3D point-cloud classification [25]. However, we still rely on robust optimizations at different steps of geometric problems, for example, robot navigation and mapping. The reason is that neural networks have not yet proven to be effective in solving constrained optimization problems. Some classic examples of

the test-time geometric optimization include rotation averaging [5, 10, 11, 17, 28], pose-graph optimization [21], local bundle adjustment [24] and global structure from motion [29]. These optimization methods estimate the model parameters that agree best with the observed noisy measurements by minimizing a robust (typically non-convex) cost function. Often, these loss functions are designed based on certain assumptions about the sensor noise and outlier distributions. However, the observed noise distribution in a real-world application could be far from those assumptions. A few such examples of noise patterns in real datasets are displayed in Figure 2. Furthermore the nature and the structure of the objective loss function is the same for different problem instances of a specific task. Nonetheless, existing methods optimize the loss function for each instance. Moreover, an optimization during test-time could be slow for a target task involving a large number of parameters, and often forestalls a real-time solution to the problem.

In this work, with the advancement of machine learning,
we address the following question: “can we learn the noise patterns in data, given thousands of different problem instances of a specific task, and regress the target parameters instead of optimizing them during test-time?” The answer is affirmative for some specific applications, and we propose a learning framework that exceeds baseline optimization methods for a geometric problem. We choose multiple rotation averaging (MRA) as a target application to validate our claim. However, we believe that the idea could easily be extended to other robust optimization problems in geometric computer vision, and could lead to a new direction in research for an alternative of robust optimization methods.

In MRA, the task is to estimate the absolute orientations of cameras given some of their pairwise noisy relative orientations defined on a view-graph. There are a different number of cameras for each problem instance of MRA, and usually sparsely connected to each other. Further, the observed relative orientations are often corrupted by outliers. The conventional methods for this task [5, 10, 17, 28] optimize the parameters of the absolute orientations of the cameras that are most compatible (up to a robust cost function) with the observed noisy relative orientations.

We propose a neural network for robust MRA. Our network is a combination of two simple four-layered message-passing neural networks defined on the view-graphs. The first network cleans a view-graph by detecting outlier edges and rectifying noisy observations. An initialization of the absolute orientations is then commenced from a spanning tree of the cleaned view-graph. The second network then fine-tunes the initialization in a single step. Although a recurrent neural network that iteratively fine-tunes an initialization could also be an interesting choice, the proposed single-step refinement is already producing superior results for this task than the baselines optimization methods. The method is also summarized in Figure 1. We name our method Neural Robust Rotation Averaging, which is abbreviated as NeuRoRA in the rest of the manuscript.

**Contribution and findings**

- A graph-based neural network NeuRoRA is proposed as an alternative to conventional optimizations for MRA.
- NeuRoRA requires no explicit optimization at test-time and hence it is much faster (10−50× on CPUs and 500−2000× on GPUs) than the baseline optimizations.
- The proposed NeuRoRA is more accurate than the conventional optimization methods of MRA. The mean/median orientation error of the predicted absolute orientations by the proposed method is 1.45°/0.74°, compared to 2.17°/1.25° by an optimization method [5].
- Being a small size network, the proposed network is fast and can easily be deployed to real-world applications. The combined network (NeuRoRA) size is < 0.5Mb.

Further, the proposed method can potentially be tailored to other geometric problems, for example, pose-graph optimization, with the inclusion of absolute locations of the cameras with additional sensor measurements, e.g. GPS.

**2. Related works**

We separate the related methods into two separate sections—(i) learning based methods as an alternative to optimizations, and (ii) relevant optimizations specific to MRA.

(i) **Learning to optimize** A neural network is proposed as an alternative to non-linear least square optimizations in [8] for camera tracking and mapping. It exploits the least square structure of the problem and uses a recurrent network to compute updated steps of the optimization variables. In a similar direction, [23] relaxes the assumptions made by inverse compositional algorithms for dense image alignment by incorporating data-driven priors. [26] proposes a bundle adjustment layer that learns to predict the damping parameter of the Levenberg-Marquardt algorithm to optimize depth and camera parameters. In contrast to the direct optimization-based methods that explicitly use regularizers to solve an ill-posed problem, [11] implicitly learn the prior through a data-driven method. Aoki et al. [3] proposed an iterative algorithm based on PointNet [25] for point-cloud registration as an alternative to direct optimization. Learning to predict an approximate solution to combinatorial optimization problem over graphs, e.g. minimum vertex cover, traveling salesman problem, etc., is proposed in [20]. Learning methods to optimize general black-box functions [7] have also received a lot of attention recently. These conventional learning-based methods are tailored to some specific problems, where in this work, we are interested in an alternative learning-based solution for geometric problems, e.g., SFM.

(ii) **Robust optimization for rotation averaging** MRA was first introduced in [14] where a linear solution was proposed using quaternion averaging and later in [15] using Lie group based averaging. The solutions were non-robust in both the cases. Recently, there has been progress in designing robust algorithms [6, 16] for rotation averaging. Most of the algorithms are based on iterative methods for optimizing a robust loss function. They initialize the solution from a spanning tree and iteratively fine-tune it by minimizing a robust ℓ₁ or ℓ₂ loss function. The state of the art approaches for robust MRA are listed below:

- Chatterjee and Govindu [5] fine-tune an initialization by first performing an iterative ℓ₁ minimization, followed by another iterative reweighted least squares with a more robust loss function ℓ₁.  
- Hartley et al. [16] propose a straightforward method. It fine-tunes an initialization by the Weiszfeld algorithm of ℓ₁ averaging [4]. At every iteration, the absolute orientations of each camera are updated by the median of those computed from its neighbors.

- DISCO [9] employs a two-step approach. In the first
Relative orientations (first row) and the same of noise (second row) in real datasets (for clarity only 1000 random samples) are displayed. The view-graphs of (a)-(b) are shared by [29] and (c) is shared by [9]. The noise orientation is calculated from the ground-truth absolute orientations and the observed relative orientations. We plotted histograms of the magnitudes of the angles in degrees and the axes of the orientations. Notice that the axes of the sampled relative and noise orientations are distributed mostly along a vertical ring rather than uniformly on a unit ball. These relative orientation patterns (vertical axes) are not utilized by the optimization methods and further the sampled noise orientations (somewhat vertical axes) are far from the typical distribution assumptions regarded by optimization algorithms subject to the cost functions. Samples from such noise distributions are shown in (d).

The angle and axes of sampled observed relative orientations (first row) and the same of noise (second row) in real datasets (for clarity only 1000 random samples) are displayed. The view-graphs of (a)-(b) are shared by [29] and (c) is shared by [9]. The noise orientation is calculated from the ground-truth absolute orientations and the observed relative orientations. We plotted histograms of the magnitudes of the angles in degrees and the axes of the orientations. Notice that the axes of the sampled relative and noise orientations are distributed mostly along a vertical ring rather than uniformly on a unit ball. These relative orientation patterns (vertical axes) are not utilized by the optimization methods and further the sampled noise orientations (somewhat vertical axes) are far from the typical distribution assumptions regarded by optimization algorithms subject to the cost functions. Samples from such noise distributions are shown in (d).

### 3. Multiple rotation averaging

Consider $N$ cameras with $M$ pairwise relative orientation measurements forming a directed view-graph $G = (V, E)$. A vertex $V_i \in V$ corresponds to the absolute orientation $\tilde{R}_i$ (to be estimated) of the $i$th camera and an edge $E_{uv} \in E$ corresponds to the observed relative orientation $\tilde{R}_{uv}$ from $u$th camera to $v$th camera. Conventionally, the task is to estimate the absolute orientations $\{\tilde{R}_i\}$, with respect to a global reference of orientations, such that the estimated orientations are most consistent with the observed noisy relative orientation measurements, i.e., $\tilde{R}_{uv} \approx \tilde{R}_u \tilde{R}_v^{-1}$, $\forall E_{uv} \in E$. Further, the observed measurements are corrupted by outliers, i.e., some of the orientations are far from $\tilde{R}_u \tilde{R}_v^{-1}$. Conventionally, the solution is obtained by minimizing a robust cost function that penalizes the discrepancy between observed noisy relative orientations $\{\tilde{R}_{uv}\}$ and the estimated relative orientations $\{\tilde{R}^\ast_{uv}\} := \{\tilde{R}_u \tilde{R}_v^{-1}\}$. The corresponding optimization problem can then be expressed as

$$\arg\min_{\{\tilde{R}^\ast_{uv}\}} \sum_{E_{uv} \in E} \rho\left(d(\tilde{R}_{uv}, \tilde{R}^\ast_{uv})\right)$$  \hspace{1cm} (1)

where $\rho(.)$ is a robust cost and $d(., .)$ is a distance measure between the orientations. The distance measure include geodesic, quaternion and chordal [17] metrics (described below). The cost function $\rho(.)$ is designed according to the assumption of sensor noise and outlier distribution by the optimization method (described below in detail). The nature of the above optimization is a typical complex multi-variable nonlinear optimization problem with thousands of variables (for thousands of cameras) and there seems to be no direct method (closed-form solution) minimizing the above cost even without outliers [17].

**The choice of distance measure $d(\tilde{R}, R)$** There are three commonly used distance measurements in the rotation group $\text{SO}(3)$: (1) the geodesic or angle metric $d_\theta = \angle(\tilde{R}, R)$, (2) the chordal metric $d_C = \|\tilde{R} - R\|_F$ and (3) the quaternion metric $d_Q = \min\{\|q_{\tilde{R}} - q_R\|, \|q_{\tilde{R}} + q_R\|\}$ where $q_{\tilde{R}}$ and $q_R$ are quaternion representations of $\tilde{R}$ and $R$ respectively, and $\|\cdot\|_F$ is the Frobenius norm. Note that $q_{\tilde{R}}$ and $-q_R$ represent the same orientation. The metrics $d_C$ and $d_Q$ are proven to be $2\sqrt{2}\sin(d_\theta/2)$ and $2\sin(d_\theta/4)$ respectively [17], thus, all the metrics are the same to the first order. In our implementation, we employ the quaternion representations (with non-negative scalars) while designing the network and the loss functions.

**The choice of robust cost $\rho(.)$** In practical applications, e.g., robot navigation, the agent usually ends up with some corrupt measurements (outliers), due to symmetric and repetitive man-made structures, in addition to the sensor noise. To estimate the absolute orientations of the cameras that are immune to those outliers, the conventional methods optimize a robust cost $\rho(.)$ as discussed above. An exhaustive list of such robust functions can be found in [5]. Specifically, Geman-McClure or $\ell_1$ are more robust to outliers than $\ell_2$ or Huber loss, but prone to converge to a local minimum, and require a better initialization. Chatterjee et al. [2] refines the initialization utilizing a weighted $\ell_2$ cost and further fine-tunes the solution employing an $\ell_2$ robust
cost. Hartley et al. [16] employs a successive $\ell_1$ averaging scheme to estimate the orientation of each camera in turn, given its neighbors. The noise and outliers in the observed relative orientations is assumed to follow some distributions and a few such examples are shown in Figure 2. Further, optical axis of most of the cameras are horizontal and hence the axes of the relative orientations are vertical. By training a neural network to perform the task, our aim is for the neural network to capture these patterns while predicting the absolute orientations.

4. Learning to predict absolute orientations

Let $D := \{\mathcal{G}\}$ be a dataset of ground-truth view-graphs. Each view-graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ contains a noisy relative orientation measurement $\hat{R}_{uv}$ for each edge $E_{uv} \in \mathcal{E}$ and a ground-truth absolute orientation $R_v$ for each camera $v \in \mathcal{V}$. The desired neural network learns a mapping $\Phi$ that takes noisy relative measurements $\{\hat{R}_{uv}\}$ as input and predicts the absolute orientations $\{R_v\} := \Phi(\{\hat{R}_{uv}\})$ as output. To train the parameters of such network, one could minimize the discrepancy between the ground-truth $\hat{R}_{uv} := \hat{R}_v \hat{R}_u^{-1}$ and the estimated $R_{uv} := R_v^{-1} R_u^{-1}$ relative orientations (cf. equation (1)), i.e.

$$\arg\min_{\Phi} \sum_{\hat{R}_{uv} \in E} d(R_{uv}^{\Phi}, \hat{R}_{uv})$$

(2)

In contrast to (1), where conventional methods optimize the orientation parameters for each instance of the view-graph $\mathcal{G} \in D$, here in (2), the network parameters are optimized during training that learn the mapping $\Phi$ effectively from observed relative orientations $\{\hat{R}_{uv}\}$ to the target absolute orientations $\{R_v\}$, i.e. $\{R_v\} \approx \Phi(\{\hat{R}_{uv}\})$ over the entire dataset of view-graphs $D$.

Difficulty in direct training of $\Phi$ and gauge freedom

For an arbitrary orientation $R$,

$$R_{uv} := R_v^{-1} R_u^{-1} = (R_v^* R)(R_u^* R)^{-1}, \quad \forall E_{uv} \in \mathcal{E}$$

(3)

Therefore, $\{R_v^*\}$ and $\{R_v^* R\}$ essentially represent the same solution to the MRA problem (1) and there is a gauge freedom of degree 3. The mapping $\Phi$ is thus one-to-many as $\{R_v^*\}$ and $\{R_v^* R\}$ correspond to the same cost (2). This gauge freedom makes it difficult to train such a network. Further, one could choose a direct cost (no associated gauge freedom) to learn an one-to-one mapping $\Phi$, e.g.

$$\arg\min_{\Phi} \sum_{\hat{R}_{uv} \in E} d(R_{uv}^{\Phi}, \hat{R}_{uv})$$

(4)

where the reference orientation is fixed according to the ground-truth. Again, $\{\hat{R}_v\}$ and $\{\hat{R}_v R\}$ represent the same ground-truth where the reference orientations are fixed at different directions. One could fix the issue by fixing the reference orientation to the orientation of the first camera in all the view-graphs in $D$. However, in a graph (set representation), the nodes are permutation invariant. Thus the choice of the first camera, and hence the reference orientation, is arbitrary. Therefore, one needs to pass the reference orientation or the index of the first camera (possibly via a binary encoding) to the network as an additional input to be able to train such a network. However, we employ an alternative strategy adopted from the conventional optimization methods [5, 17], i.e. initialize a solution of the absolute orientations under a fixed reference and pass the initialization to the network to fine-tune the solution. The network gets the reference orientation as an additional input via initialization (see Figure 1(d)) and regres the parameters, i.e. $\{\hat{R}_v\} \approx \Phi(\{\hat{R}_{uv}\}, \{\hat{R}_v\})$. Further, we train the network by minimizing a combined cost where the first term (2) enforces the consistency over the entire graph and the second term (4) enforces a unique solution, i.e.

$$\arg\min_{\Phi} \sum_{\hat{R}_{uv} \in E} d(R_{uv}^{\Phi}, \hat{R}_{uv}) + \beta \sum_{\hat{R}_v \in \mathcal{V}} d(R_{uv}^{\Phi}, \hat{R}_v)$$

(5)

where $\beta$ is a weight parameter. Note that the reference orientation are now fixed at the orientation of a certain camera $c$ in the initialization $\{\hat{R}_v\}$ as well as in the ground-truth absolute orientations $\{\hat{R}_v\}$. Although, the choice of $c$ is not critical in practice, the camera $c$ with most neighboring cameras is chosen as the reference, i.e. $\hat{R}_c = \hat{R}_c = I_{3 \times 3}$.

The above mapping $\Phi$ is now one-to-one. However, it requires an initialization $\{\hat{R}_v\}$ as an additional input. Conventional methods initialize the absolute orientations using a spanning tree of the view graph. However even a single outlier in that spanning tree can lead to a very poor initialization, so it is very important to identify these outliers beforehand. Further, noise in the relative orientation along each edge of the spanning tree will also propagate at the subsequent nodes while computing the initial absolute orientations. Thus, we first clean the view-graph by removing...
the outliers and rectifying the noisy measurements, and then bootstrap an initialization from the cleaned view-graph.

**Cleaning the view-graph** Given the local structure in the view-graph, i.e. measurements of all the edges that the pair of adjacent nodes \( \{v_u, v_v\} \) are connected to (and possibly subsequent edges), an outlier edge \( E_{uv} \) can be detected. To be specific, chaining the relative orientations along a cycle in the local structure of the view-graph forms an orientation close to the identity orientation and an indication of an outlier in the cycle otherwise. The presence of an outliers in multiple such cycles through the current edge indicates that the edge to be an outlier. Furthermore, the amount of noise in a relative measurement \( \hat{R}_{uv} \) can also be estimated. Instead of designing such explicit algorithms, we use another graph neural network to clean the graph. The proposed method can be summarized as follows:

- A graph-based network is employed to clean the view-graph by removing outlier measurements and rectifying noisy measurements (see Section 4.2).
- The cleaned view-graph can then be utilized to initialize the absolute orientations (see Section 4.3).
- The initialization is further fine-tuned using a separate graph-based network (see Section 4.4).

In summary and for clarity of the rest of the paper, the notations are outlined in Table 1.

### 4.1. The network design choice

Generalizing convolution operators to irregular domains, such as graphs, is typically expressed as neighborhood aggregation or a message-passing scheme. The proposed network is built using such Message-Passing Neural Networks (MPNN) [13], directly operating on view-graphs \( G \). A MPNN is defined in terms of message functions \( m_v(t) \) and update functions \( \gamma_v(t) \) that run for \( T \) time-steps (layers). At each time-step, the hidden state \( h_v(t) \) at each node (feature) in the graph is updated according to

\[
h_v(t) = \gamma_v(t)(h_v(t-1), m_v(t))
\]

where \( m_v(t) \) is the condensed message at node \( v \), coming from the neighboring nodes \( u \in N_v \), and can be expressed as follows:

\[
m_v(t) = \square v_u \in N_v \phi_v(t)(h_v(t-1), h_u(t-1), e_{uv})
\]

where \( \square \) denotes a differentiable, permutation invariant symmetric function, e.g. mean, soft-max, etc. \( \phi_v(t) \) and \( \gamma_v(t) \) are concatenation operations followed by 1-D convolutions and ReLUs, \( e_{uv} \) is the edge feature of the edge \( E_{uv} \), \( h_v(t-1) = \phi_v(t)(h_v(t-1), h_u(t-1), e_{uv}) \) is the accumulated message for the edge \( E_{uv} \) at time-step \( t \), and \( N_v \) is the set of all neighboring cameras connected to \( v_u \). A diagram of the elements involved in computing the next-level features of a node is displayed in Figure 3.

### 4.2. View-graph cleaning network

The view-graph cleaning network (CleanNet) is built on a MPNN. The input to CleanNet is a noisy view-graph and the output is a clean one, i.e. the network takes noisy relative orientations \( \hat{R}_{uv} \) as the edge features \( e_{uv} \) and predicts the noise-rectified relative orientations \( R^*_{uv} \) from the accumulated message \( h_v(T) = \phi_v(T)(h_v(T-1), h_u(T-1), e_{uv}) \) at the last layer. It also predicts a score \( \alpha^*_v \) depicting the probability of the edge \( E_{uv} \) to be an outlier, i.e.

\[
R^*_{uv} = \alpha_{uv} = l_{p1}(h_v(t) \rightarrow v) \ast \hat{R}_{uv}
\]

and ground-truth \( \hat{R}_{uv} : \hat{R}_{uv} R^*_{uv} \) relative orientations, and mean binary cross-entropy error \( L_{bce} \) of the predicted \( \alpha^*_v \) and the ground-truth outlier score \( \hat{\alpha}_{uv} \), i.e.

\[
L = \sum \sum (L_{mre}(R^*_{uv}, \hat{R}_{uv}) + \lambda L_{bce}(\alpha^*_v, \hat{\alpha}_{uv}))
\]

where \( \lambda \) is a weight parameter (fixed as \( \lambda = 10 \)). We formulate the orientations using unit quaternions and the predictions are normalized accordingly. The error in the prediction is also normalized by the degree of the node, i.e.

\[
L_{mre}(R^*_{uv}, \hat{R}_{uv}) = \frac{1}{|N_u||N_v|} \sum_{Q \in Q} \left( \frac{R^*_{uv} \cdot Q}{\|R^*_{uv}\|2} \cdot \hat{R}_{uv} \right)
\]

Experimentally, we observed the above loss produces superior performance than the standard discrepancy loss (2). Note that the ground-truth outlier score \( \hat{\alpha}_{uv} \) is generated based on the amount of noise in the relative orientations.
Specifically, if the amount of noise in the relative orientation $\hat{R}_{uv}^{\ast} \hat{R}_{vu} R_v > 20^\circ$, the ground-truth edge label is assigned as an outlier, i.e. $\alpha_{uv} = 1$ and $\alpha_{vu} = 0$ otherwise.

An edge $E_{uv}$ is marked as an outlier edge if the predicted outlier score $\alpha_{uv}^\ast$ is greater than a predefined threshold $\epsilon$. In all of our experiments, we choose the threshold $\epsilon = 0.75^1$.

A cleaned view-graph $G^\ast$ is then generated by removing outlier edges from $G$ and replacing noisy relative orientations $\hat{R}_{uv}$ by the rectified orientations $\tilde{R}_{uv}$. Note that the cleaned graph $G^\ast$ is only employed to bootstrap an initialization of the absolute orientations.

4.3. Bootstrapping absolute orientations

Hartley et al. [16] proposed generating a spanning tree by setting the camera with the maximum number of neighbors as the root and recursively adding adjacent cameras without forming a cycle. The reference orientation is fixed at the camera at the root of the spanning tree. The orientations of the rest of the cameras in the tree are computed by propagating away the rectified orientations $\tilde{R}_{uv}$ from the root node along the edges, i.e. $\tilde{R}_v = \tilde{R}_{vu} \tilde{R}_u$.

As discussed before, the noise in the relative orientation along each edge $R_{uv} \tilde{R}_{vu}^{-1}$ propagates at the subsequent nodes while computing the initial absolute orientations of the cameras. Therefore, the spanning tree that minimizes the sum of depths of all the nodes (also known as shortest path tree [27]) is the best spanning tree for the initialization.

Starting with a root node, a shortest path tree could be computed by greedily connecting nodes to each neighboring node in the breadth-first order. The best shortest path tree can be found by applying the same procedure with each one of the nodes as a root node (time complexity $O(n^2)$) [18]. However, we employed the procedure just once (time complexity $O(n)$) with the root at the node with the maximum number of adjacent nodes (similar to Hartley et al. [16]) and observed similar results as with the best spanning tree. The reference orientation of the initialization and the ground-truth is fixed at the root of the tree. This procedure is very fast and it takes only a fraction of a second for a large view-graph with thousands of cameras. We abbreviate this procedure as SPT and it is the default initializer in all of our experiments.

4.4. Fine-tuning network

The fine-tuning network (FineNet) is again built on a MPNN. It takes the initial absolute orientations $\{ \tilde{R}_v \}$ and the relative orientation measurements $\{ \tilde{R}_{uv} \}$ as inputs, and predicts the refined absolute orientations $\{ R^\ast_v \}$ as the output. The refined orientations are estimated from the hidden states $h^{(T)}_v$ of the nodes at the last layer of the network, i.e.

$$R^\ast_v = lp_3(h^{(T)}_v) \ast \tilde{R}_v$$  \hspace{1cm} (11)

where $lp_3$ is a single layer of linear perceptron. We initialize the hidden states of the MPNN by the initial orientations, i.e. $h^{(0)}_v = \tilde{R}_v$. The edge attributes are chosen as the relative discrepancy of the initial and the observed relative orientations, i.e. $\epsilon_{uv} = \tilde{R}_{uv}^{-1} \hat{R}_{uv} R_u$. The loss for the fine-tuning network is computed as the weighted sum of edge consistency loss and the rotational distance between the predicted orientation $\tilde{R}_v$ and the ground-truth orientation $R_v$, i.e.

$$\mathcal{L} = \sum_{\tilde{E} \in \mathcal{D}} \left( \sum_{E_{uv} \in \mathcal{E}} \mathcal{L}_{mre}(R^\ast_{uv}, \tilde{R}_{uv}) + \beta \frac{1}{|V|} \sum_{v \in V} d_Q(R^\ast_v, \tilde{R}_v) \right)$$  \hspace{1cm} (12)

where $\mathcal{L}_{mre}$ is chosen as the quaternion distance (10). This is a combination of two loss functions chosen according to (5). We value consistency of the entire graph (enforced via relative orientations in the first term) over individual accuracy (second term), and so choose $\beta = 0.1$.

4.5. Training

The view-graph cleaning network and the fine-tuning network are trained separately. For each edge $E_{uv}$ in the view-graph with observed orientation $\tilde{R}_{uv}$, an additional edge $\tilde{E}_{uv}$ is included in the view-graph in the opposite direction with orientation $\tilde{R}_{vu} := \tilde{R}_{uv}^{-1}$. This will ensure the messages flow in both directions of an edge. In both of the above networks, the parameters are chosen as: the number of time-steps $T = 4$, the permutation invariant function $\Box$ as the mean, and the dimensions of the message $m(t)$ and hidden state $h^{(i)}_v$ are 32.

The proposed networks are implemented in PyTorch Toolbox$^2$ and trained on a GTX 1080 Ti GPU with a learning rate of $0.5 \times 10^{-4}$ and weight decay $10^{-4}$. Each of CleanNet and FineNet are trained for 250 epochs (takes $\sim 4 - 6$ hours) to learn the network parameters. To prevent the networks from over-fitting on the training dataset, we randomly drop 25% of the edges of each view-graph along with observed noisy relative orientations in each epoch. During testing all the edges were kept active. The parameters that yielded the minimum validation loss were kept for evaluation. All the baselines including the proposed networks were evaluated on an Intel Core i7 CPU.

5. Results

Experiments were carried out on synthetic as well as real datasets to demonstrate the true potential of our approach.

Baseline methods We evaluated NeuRoRA against the following baseline methods (also described in Section 2):

- Chatterjee and Govindu [5]: The latest implementation of Baseline methods (also described in Section 2):

$^2$https://pytorch-geometric.readthedocs.io$^3$http://www.ee.issc.ac.in/labs/cvl/research/rotaveraging/
### Angular Error Evaluation

| Method          | Angular Error | Runtime (seconds) |
|-----------------|---------------|-------------------|
| **Baseline Methods** |               |                   |
| Chatterjee [4]   | 2.17° ± 1.25° | 4.55 ± 5.38 s (1×) |
| Weiszfeld [15]   | 3.35° ± 1.02° | 9.74 ± 50.92 s (0.11×) |
| **Proposed Methods** |               |                   |
| CleanNet-SPT + [5] | 2.11° ± 1.26° | 4.04 ± 5.41 s (0.99×) |
| CleanNet-SPT + [16] | 1.74° ± 1.01° | 3.53 ± 50.36 s (0.11×) |
| NeuRoRA           | 1.45° ± 0.74° | 3.53 ± 0.21 s (24×) 0.0016 s |
| NeuRoRA-v2        | 1.30° ± 0.68° | 3.28 ± 0.30 s (18×) 0.0021 s |
| **Other Methods** |               |                   |
| CleanNet-SPT     | 2.93° ± 1.47° | 5.34 ± 0.11 s (47×) 0.0007 s |
| SPT-FineNet      | 3.00° ± 1.57° | 6.12 ± 0.11 s (47×) 0.0007 s |
| SPT-FineNet + [5] | 2.12° ± 1.20° | 4.11 ± 5.41 s (0.99×) |
| SPT-FineNet + [16] | 1.78° ± 1.01° | 3.95 ± 50.36 s (0.11×) |
| NeuRoRA           | 2.11° ± 1.26° | 4.04 ± 5.51 s (0.97×) |
| NeuRoRA-v2        | 1.73° ± 1.01° | 3.51 ± 50.46 s (0.10×) |

*mn: mean of the angular error, md: median of the angular error, rms: root mean square angular error, and cpu: the runtime of the method on a cpu.

MethodA + MethodB: MethodB is initialized by the solution of MethodA.

### Synthetic Dataset

We carefully designed a synthetic dataset that closely resembles the real-world datasets. Since the amount of noise in observed relative measurements changes with the sensor type (e.g., camera device), the structure of the connections in the view-graphs and the outlier ratios are varied with the scene (Figure 2). We sampled (1) the number of cameras, (2) the sparsity in the view-graph, (3) the distributions of pairwise relative orientation noise, and (4) the percentage of outlier edges in each view-graph separately. A single view-graph was generated as follows: (1) the number of cameras were sampled in the range 250 – 1000 and their orientations were generated randomly on a horizontal plane (yaw only), (2) pairwise edges and corresponding relative orientations were randomly introduced between the cameras that amounted to (10 – 30)% of all possible pairs, (3) the relative orientations were then corrupted by a noise with a std σ where σ is chosen uniformly in the range (5° – 30°) once for the entire view-graph, and the directions are chosen randomly on the vertical plane (to emulate realistic distributions), and (4) the relative orientations were further corrupted by (0 – 30)% of outliers with random orientations.

Further, no sophisticated rule was employed to determine the connections among cameras, for example, cameras with similar absolute orientations are potentially connected. The good performance of our network depicts that the choice is not critical, and a random connection is employed instead. Our synthetic dataset consisted of 1200 sparse view-graphs. The dataset was divided into training (80%), validation (10%), and testing (10%).

The results are furnished in Table 2 and in Figure 4. The average angular error on all the view-graphs in the dataset is displayed. The proposed method NeuRoRA performs remarkably well compared to the baselines in terms of accuracy and speed. NeuRoRA-v2 further improves the results. Overall, Chatterjee [5] performs well but the performance does not improve with a better initialization. Weiszfeld [16] improves the performance solutions for geometric problems.

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4A detailed comparison can be found in [5]

5https://github.com/pulak09/NeuRoRA
with a better initialization given by CleanNet-SPT, but, it
can not improve the solution further given an even better
initialization by NeuRoRA. This validates our initial claim
about learning the patterns in the data by a neural network
and the proposed NeuRoRA has thus demonstrated a
potential solution to this specific application. Notice that
the proposed NeuRoRA is three orders of magnitude faster
with a GPU than the baseline methods.

Real dataset We summarize the real datasets and display in Table 3. There are a total of 19 publicly available view-
graphs with observed noisy relative orientations and the
ground-truth absolute orientations. The ground-truth orienta-
tions were obtained by applying incremental bundle ad-
justment [2] on the view-graphs. The TNotre Dame dataset
is shared by Chatterjee et al. [5]. The Artsquad and San Francisco
data are provided by DISCO [9]. The rest of the view-
graphs are publicly shared by 1DSFM [29]. In Table 3, #edges represents the number of edges, and #camera represents the number of cameras in the
view-graph. The ground-truth orientations are available for some
of those cameras (indicated in parenthesis) and the training,
validation and testing are performed only on those cameras.

Due to limited availability of real datasets for training, we
employed network parameters pre-trained on the above
synthetic dataset and further fine-tuned on the real datasets
in round-robin fashion. We evaluated (i.e. validated and
tested) each real view-graph in Table 3 one at a time while
the rest of the view-graphs were used to fine-tune the net-
work. Overall, the proposed NeuRoRA outperformed the
baseline optimization methods for this task in terms of ac-
curacy and efficiency. The Artsquad and San Francisco
data sets have larger view-graphs than the rest of the real
datasets, moreover, they potentially have different orienta-
tion patterns (as shared from a different source [9], also see
Figure 2). Thus, the performance of NeuRoRA falls short to
Chatterjee et al. [5] only on those sequences, but still
produces better results than Weiszfeld [16]. The Acropolis
data set is quite clean on which the Weiszfeld [16] algorithm
performs exceptionally well. In the rest of the datasets,
NeuRoRA produces better or similar results. Nonetheless,
the proposed NeuRoRA is much faster than the baselines.

6. Discussion

We have proposed a graph-based neural network for abso-
olute orientation regression of a number of cameras from their
observed relative orientations. The proposed network
is exceptionally faster than the strong optimization-based
baselines while producing better results on most datasets.
The outstanding performance of the current work and the
relevant neural networks for test-time optimization leads to
the following question: “can we then replace all the opti-
mizations in robotics / computer vision by a suitable neu-
ral network-based regression?” The answer is obviously
No. For instance, if an optimization at test-time requires
solving a simpler convex cost with a few parameters to opti-
imize, a naive gradient descent will find the globally op-
timal parameters, while a network-based regression would
only estimate sub-optimal parameters. To date, neural nets
have been proven to be consistently better at solving pattern
recognition problems than solving a constraint optimization
problems. A few neural network-based solutions are pro-
posed recently that can exploit the patterns in the data while
solving a test-time optimization. Therefore the current work

Table 3. Results of MRA on real datasets. The proposed method NeuRoRA is much faster than the baselines while producing overall similar or better results. The numbers of cameras, for which ground-truths are available, is shown within brackets.

| Datasets          | Chatterjee [5] | Weiszfeld [16] | CleanNet-SPT + Weiszfeld [16] | NeuRoRA     |
|-------------------|----------------|----------------|-------------------------------|-------------|
| Name              | # cameras      | # edges        | mn  md  rms  cpu             | mn  md  rms  cpu             |
| Alamo             | 627(577)       | 97206          | 4.16 1.06 12.68 20.47      | 4.89 1.38 15.95 80.04       |
| Ellis Island      | 247(227)       | 20297          | 2.87 0.51 10.36 2.49       | 2.36 1.04 16.50 8.97        |
| Gendermen Market  | 742(677)       | 48143          | 37.65 7.71 67.30 11.08     | 29.37 9.56 49.53 53.71      |
| Madrid Metropol    | 384(341)       | 23784          | 6.97 1.29 17.28 3.28       | 7.52 2.67 17.01 14.53       |
| Montreal NotreDm  | 474(450)       | 52424          | 1.54 0.51 7.45 9.17        | 2.17 0.78 9.32 41.58        |
| NYC Library       | 376(332)       | 20680          | 3.04 1.35 6.90 4.86        | 3.87 2.18 8.65 14.40        |
| Notre Dame        | 553(553)       | 103932         | 3.55 0.65 14.61 23.33      | 4.79 0.88 19.17 80.86       |
| Piazza Del Popolo | 354(338)       | 24710          | 4.06 0.89 8.41 3.30        | 4.87 1.38 11.73 16.78       |
| Piccadilly        | 2568(2152)     | 319257         | 6.97 2.95 19.30 449.09      | 26.44 7.52 46.68 1328.12     |
| Roman Forum       | 1134(1084)     | 70187          | 3.15 1.59 10.21 20.21      | 4.85 1.87 17.22 115.07       |
| Tower of London   | 508(472)       | 23863          | 3.94 2.43 9.06 1.95        | 4.74 2.99 10.58 17.16        |
| Trafalgar         | 5431(5058)     | 680012         | 3.50 2.02 9.85 48.80       | 15.68 11.37 24.95 5572.22    |
| Union Square      | 930(799)       | 25561          | 9.33 3.93 22.44 6.80       | 40.97 10.31 61.46 42.86      |
| Vienna Cathedral  | 918(836)       | 104550         | 8.29 1.28 27.84 48.14      | 11.72 1.99 36.68 158.39      |
| Yorkminster       | 458(437)       | 27299          | 3.51 1.60 8.41 4.03        | 5.73 2.03 17.30 32.04        |
| Acropolis         | 463(463)       | 11421          | 1.14 0.70 1.71 1.59        | 0.66 0.37 1.10 15.04        |
| Arts Qua          | 5503(4978)     | 222014         | 4.82 3.57 8.92 116.18      | 34.44 23.18 49.76 1886.42    |
| San Francisco     | 7868(7866)     | 101532         | 6.88 3.41 4.27 15.27       | 18.84 1.91 21.89 1349.56     |
| TNotre Dame       | 715(715)       | 64678          | 1.07 0.42 3.55 10.60       | 1.40 0.66 4.53 72.50         |

mn: mean of the angular error (in deg), md: median of the angular error (in deg), rms: root mean square angular error, and cpu: the runtime of the method on a cpu (in sec).

6http://www.ee.isc.ac.in/labs/cvl/research/rotationaverage/
7http://vision.soc.indiana.edu/projects/disco/  
8http://www.cs.cornell.edu/projects/1dsfm/
also opens up many questions related to the right tool for a specific application.

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Figure 5. More examples of Figure 2(a)-(c) are displayed. The angle and axes of sampled observed relative orientations (first row) and the same of noise (second row) in real datasets (for clarity only 1000 random samples) are displayed. The view-graphs of (a)-(b) are shared by [29] and (c) is shared by [9]. The noise orientation is calculated from the ground-truth absolute orientations and the observed relative orientations. We plotted histograms of the magnitudes of the angles in degrees and the axes of the orientations. Notice that the axes of the sampled relative and noise orientations are distributed mostly along a vertical ring rather than uniformly on a unit ball.