Solution of Darcy-Brinkman Flow Over an Irregular Domain by Finite Element Method

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Abstract. The problem of finding the numerical solution for the flow through a sporadic geometry includes a considerable measure of calculation time. In this article a finite element approach which involves sub-parametric finite element technique with bended triangular elements and with less number of degrees of freedom is used for finding solution of Darcy Brinkman flow through a rectangular duct with one bended side. This reduces the computational time considerably. The showing of the viability of the strategy is the target in this article. Results acquired are in great concurrence with the previous works that has been carried out.

Keywords: Finite element method, Darcy Brinkman equation, irregular channel, triangular elements, transport through porous media.

1. Introduction

Numerous investigations that embrace fluid mechanics and biomedical examinations involve Darcy-Brinkman equation. It has been found that it is effectively utilized in demonstrating a thin stringy surface layer covering veins since it is highly porous and a profoundly penetrable medium. The study of permeable medium is not the same as would be expected of rivers and flow of streams. The velocity distribution which is represented by its differential equation has been impacted pore matrix. Porosity relies upon the pressure of the engineering framework like a reservoir pressure. The effect of permeable medium on different engineering applications like warm protections, oil industry, warm exchangers and warmth pipe innovation is well documented in literature. A great deal of research is conducted by Narashimhan and Lage [1], Nield and Kuznetsov [2], Hooman [3] and Shiekh and Vafai [4] in the field of constrained convection for a permeable media. Additionally Greencorn [5] work has carried out considerable work in steady flow through permeable media, where the impacts of various geometrical shapes of permeable media on capillary pressure are identified. Sub parametric finite element approach is giving a decent outcome when contrasted with the test technique in different works. In the present contest, a flow in an irregular flow domain with one bended side porous media and with the help of broad calculation utilizing high end software such as Mathematica 7.0, analysis using finite elements is undertaken. Ergatoudis et al. [6], have presented the utilization of curved triangular elements. A great deal of finite element analysis is discussed in Bhatti [7]. Sub parametric quadratic curves element analysis is present in the works of Rathod et al. [8] and Nagaraja et al. [9].
Detailed application of the present method is demonstrated in the works of Kesavulu et al. [10] and Murali et al. [11]. The present numerical scheme is found to be effective in making a quicker estimation in comparison with the other available methods. Investigation of a steady, linear, uni-dimensional flow is taken in the present study. The flow through the given rectangular porous channel with one curved side (Fig. 1) is solved using the method under discussion.

2. Mathematical formulation (Low Velocity Flow)

The Darcy–Brinkman equation governing the flow for one dimensional (fully developed) steady state flow in a flow domain with porous media (see Figs. 1 & 2) with velocity \( v(x, y) \) is

\[
\frac{\mu}{K} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu}{K} v = \frac{dP}{dz}. \tag{I}
\]

The flow considered the \( z \)-direction. Eq. (I) is non-dimensionalized by substituting.

\[
U = \frac{v}{u_R}, \quad P = \frac{p h}{\mu u_R}, \quad X = \frac{x}{h}, \quad Y = \frac{y}{h}, \quad Z = \frac{z}{h}. \tag{II}
\]

Using Eq. (II) in Eq. (I) yields

\[
\Lambda^2 \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \sigma^2 U = \frac{dP}{dZ}, \tag{III}
\]

where \( \Lambda = \frac{\mu}{mu^\prime} \) is called Brinkman number (viscosity ratio), and \( \sigma^2 = \frac{h^2}{K} \) is Darcy number.

Taking \( U^* = \left( \frac{U}{dP/dZ} \right) \), ignoring the star, Eq. (III) reduces to:

\[
\Lambda^2 \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \Lambda \sigma^2 U = -\Lambda, \quad \text{in the region } \Omega. \tag{IV}
\]

Eq. (IV) is the dimension less version of Eq. (I).

We need to solve Eq. (IV) incorporating the boundary condition:

\[
U = 0 \quad \text{on } \partial \Omega, \tag{V}
\]

where \( \partial \Omega \) represents the boundary of the domain; see Fig. 2.

With a specific end goal to arrive at the solution of governing partial differential Eq. (IV) together with the boundary condition Eq. (V), Galerkin weighted leftover technique is utilized [7]. The domain \( \Omega \) and \( \partial \Omega \) is partitioned into triangular and curved triangular elements respectively [see Fig. 3e].

For the purpose of the study, a computational scheme is undertaken with various curved triangular elements including quartic and quintic ordered elements; Kesavulu et al. [10]. The mapping equations for different order elements and the Jacobean for the transformation are available in Nagaraja et al. [9]. The computational time reduces considerably due to linearity of the Jacobian. Gauss quadrature method and Galerkin finite element technique for differential equations is employed for solving the finite element equations. In the wake of producing the results of the considerable number of elements into record and after that employing the conditions, a direct arrangement of algebraic equations are obtained. Subsequently the obscure nodal point velocities are obtained by solving the equations.
Figs. 3a–d below depicts the mapping between curved triangles used in the computations and straight triangles.

**FIGURE 1.** The domain with one side curved boundary.

**FIGURE 2.** The cross sectional view of the domain.

(i) Second order curved element  (ii) Mapped second order element

**FIGURE 3a.** Mapping of a second order curved element to standard triangular element.
Figure 3b. Mapping of a third order curved element to standard third order triangular element.

Figure 3c. Mapping of a fourth order curved element to standard fourth order triangular element.

Figure 3d. Mapping of a fifth order curved element to standard fourth order triangular element.
3. Results and discussion

Presently, quicker calculation with minimal cost of computation is the need with great importance in all the significant investigation in various fields. The technique involved here demonstrates the exceptionally compelling and exact nature of the method. In classical finite element analysis, the impact of exact boundary conditions is found with lot of computational cost. Therefore, to capture exact impacts, curved triangular element is used in place of linear elements. This necessitates less number of elements. The algorithm is coded in high end software Mathematica7.0 and the contour plots have been plotted using OriginPro8 software. From the results, it is clear that, the accuracy and number and order of elements are directly related. That is an increase in order and number of elements results in better accuracy. Unmistakably, setting up the viability of the strategy is adopted. The graphs of the speed dissemination $U(X,Y)$ in the domain under consideration is presented.

Figs. 4a – 4e demonstrates the changes in velocity profile for various combinations of Darcy and brinkman numbers. The results are in agreement with the ones in the literature [12].

![FIGURE 3e. Partition of the cross section into 12 elements.](image)

![FIGURE 4a. Plot with $\Lambda=1, \sigma^2=0$.](image)
FIGURE 4b. Plot with $\Lambda=1, \sigma^2=5$.

FIGURE 4c. Plot with $\Lambda=1, \sigma^2=10$. 
4. Conclusions

- The method highlights the changes for various combinations of parameters; hence the method can be used for curved domains.
- The flow velocity and friction are inversely proportional and, this concept is clearly demonstrated with the help of unique numerical scheme.
- Better accuracy can be achieved using curved triangular elements of higher order with reduced computational triangles.
References

[1] Narasimhan A and Lage J L 2001 Forced convection of a fluid with temperature-dependent viscosity flowing through a porous medium channel, Numer. Heat Tran.: Part A: Appl. 40 801-820

[2] Nield D A and Kuznetsov A V 2005 Handbook of Porous Media ed. K. Vafai, 2nd ed. (New York: Taylor and Francis) pp143-193.

[3] Hooman K 2005 Fully developed temperature distribution in a porous saturated duct of elliptical cross section, with viscous dissipation effects and entropy generation analysis Heat Tran. – Jpn Res. 36 237-245

[4] Haji-Sheikh A and Vafai K 2004 Analysis of flow and heat transfer in porous media imbedded inside various-shaped ducts, Int. J. Heat Mass Trans. 47 1889-1905

[5] Greenkorn R A 1981 Steady Flow Through Porous Media AIChE J. 27 529-545

[6] Ergatoudis I, Irons B M and Zienkiewicz O C 1968 Curved isoparametric quadrilateral finite element analysis Int. J. Solids Struct. 4 31-42

[7] Bhatti M A 2005 Fundamental Finite Element Analysis and Applications (New York: John Wiley & Sons Inc.)

[8] Rathod H T, Nagaraja K V, Kesavulu Naidu V. and Venkatesudu B 2008 The use of parabolic arcs in matching curved boundaries by point transformations for some higher order triangular elements Fin. Elem. Anal. Des. 44 920–932

[9] Nagaraja K V, Kesavulu Naidu V and Siddheshwar P G 2014 Optimal subparametric finite elements for elliptic partial differential equations using higher-order curved triangular elements Int. J. Comput. Meth. Eng. Sci. Mech. 15 83-100

[10] Kesavulu Naidu V, Siddheshwar P G and Nagaraja K V 2015 Finite element solution of Darcy-Brinkman equation for irregular cross-section flow channel using curved triangular elements. Int. Conf. Comput. Heat and Mass Trans., Procedia Eng. 127 pp 301-308.

[11] Murali K, Kesavulu Naidu V and Venkatesh B 2018 Optimal subparametric finite element approach for a Darcy Brinkman fluid flow problem through a rectangular channel with one curved side, IOP Conf. Ser. Mater. Sci. Eng. 310

[12] Givler R C and Altobelli S A 1994 A determination of the effective viscosity for the Brinkman-Forchheimer flow model, J. Fluid Mech. 258 355-370