GENTLE/4fan v. 2.0
A Program for the Semi-Analytic Calculation of Predictions for the Process $e^+e^- \rightarrow 4f$

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Abstract

We describe version 2.0 of the FORTRAN program GENTLE/4fan for the semi-analytic computation of cross-sections and distributions in four-fermion production in $e^+e^-$ annihilation. GENTLE/4fan covers all charged current and neutral current four-fermion final states with no identical particles, no electrons, and no electron neutrinos in the final state. Initial state radiation representing the most relevant quantum corrections and anomalous triple gauge boson couplings have been included.

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PROGRAM SUMMARY

Title of program: GENTLE/4fan
Version: 2.0 (December 1996)
Catalogue number:
Program obtainable from: http://www.ifh.de/theory or on request by e-mail from the authors
Licensing provisions: non
Computers: all
Operating system: all
Program language: FORTRAN-77
Memory required to execute with typical data: 0.5 Mb
No. of bits per word: 32
No. of lines in distributed program: 8300
Other programs called: none
External files needed: none
Keywords: e+e− annihilation, QED, higher order corrections, four-fermion production, WW, ZZ, Zγ, γγ, ZH production
Nature of physical problem:
Description of two-boson production in e+e− annihilation at high energies with selected possible background contributions; initial state QED corrections (ISR); selected final state corrections; anomalous triple gauge boson couplings
Method of solution:
Semi-analytic; numerical integration of analytic formulae:
(i) Born approximation: Numerical integrations over two virtualities s1, s2 plus optionally over the boson production angle θ;
(ii) QED corrections in the flux function approach (FF): one additional numerical integration over virtuality s';
(iii) QED corrections in the structure function approach (SF): alternatively to (ii), two additional numerical integrations over x1, x2
Restrictions on complexity of the problem:
Only selective experimental cuts are possible; the program calculates total cross-sections and the distribution in the boson production angle; background contributions are available only for selected final state configurations; no inclusion of pure weak corrections
Typical running time:
The running time strongly depends upon the options used; one finds e.g.:
Born cross-section on-shell: below one sec., off-shell: few secs.;
LLA QED corrections (FF) on-shell: few secs.; off-shell: few mins.;
LLA QED corrections (SF) on-shell: few secs.; off-shell: several mins.;
complete ISR (FF): on-shell: few mins., off-shell: several hours;
with background: from few secs. (no QED) to several hours (with QED);
with anomalous couplings: from few secs. (no QED) to several mins. (with QED)
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LONG WRITE-UP

1 Introduction

With the advent of LEP2, now running above the W pair threshold, $e^+e^-$ collisions have entered their multiple heavy boson production era which will continue with a higher energy linear $e^+e^-$ collider (LC). Among the most interesting processes to be studied in high energy $e^+e^-$ annihilation is boson pair production. As heavy bosons decay, their physics is only accessible through their decay products. Thus, at LEP2 or LC, boson pair production must be accessed through the experimental analysis of four-fermion final states,

$$e^+e^- \rightarrow (W^+W^-, ZZ, Z\gamma, \gamma\gamma, ZH, \ldots) \rightarrow f_1\bar{f}_2f_3\bar{f}_4.$$ \hspace{1cm} (1.1)

2 Physics Background

The Standard Model (SM) four-fermion final states produced in $e^+e^-$ annihilation have been classified by several authors [1–4]. Using the semi-analytic approach, we have so far studied the production of four-fermion final states that contain neither identical particles nor electrons or electron neutrinos. These four-fermion final states are printed in **boldface** in tables 1 and 2.

Table 1

| Number of Feynman diagrams for ‘CC’ type final states. |
|---------------------------------|
| \( \bar{d}u \) | \( \bar{s}c \) | \( \bar{e}\nu_e \) | \( \bar{\mu}\nu_\mu \) | \( \bar{\tau}\nu_\tau \) |
| \( \bar{d}\bar{u} \) | 43 | 11 | 20 | 10 | 10 |
| \( e\bar{\nu}_e \) | 20 | 20 | 56 | 18 | 18 |
| \( \mu\bar{\nu}_\mu \) | 10 | 10 | 18 | 19 | 9 |

2.1 Off-shell boson pair production

For semi-analytic tree level calculations (flag \texttt{IBORNF=0}), the four-particle phase space is parametrized by the boson scattering solid angle, the boson decay solid products' angles in the corresponding boson rest frames, and the two final state fermion pair invariant masses \( s_1 \) and \( s_2 \) [5,6]. After analytic
Table 2

Number of Feynman diagrams for NC type final states.

|          | $\bar{d}d$ | $\bar{s}s, \bar{b}b$ | $\bar{u}u$ | $\bar{e}e$ | $\bar{\mu}\bar{\mu}$ | $\bar{\nu}_e\nu_e$ | $\bar{\nu}_\mu\nu_\mu$ | $\bar{\nu}_\tau\nu_\tau$ |
|----------|------------|---------------------|------------|------------|----------------------|---------------------|----------------------|---------------------|
| $\bar{d}d$ | 4·16       | 32                  | 43         | 48         | 24                   | 24                  | 21                   | 10                  | 10                  |
| $\bar{u}u$ | 43         | 43                  | 4·16       | 48         | 24                   | 24                  | 21                   | 10                  | 10                  |
| $\bar{e}e$ | 48         | 48                  | 48         | 43·6      | 48                   | 48                  | 56                   | 20                  | 20                  |
| $\bar{\mu}\bar{\mu}$ | 24 | 24                  | 24         | 4·12      | 24                   | 19                  | 19                   | 10                  |                     |
| $\bar{\nu}_e\nu_e$ | 21 | 21                  | 21         | 56        | 19                   | 19                  | 4·9                  | 12                  | 12                  |
| $\bar{\nu}_\mu\nu_\mu$ | 10 | 10                  | 10         | 20        | 19                   | 10                  | 12                   | 4·3                  | 6                   |

Integration of all angular variables, the total cross-section for double-resonant four-fermion production is given by

$$\sigma^{\text{res}}(s) = \int ds_1 \rho_B(s_1) \int ds_2 \rho_B(s_2) \sigma_0(s; s_1, s_2)$$

(2.1)

with the Breit-Wigner densities

$$\rho_B(s_i) = \frac{1}{\pi} \frac{\sqrt{s_i \Gamma_B}}{|s_i - M_B^2 + i s_i \Gamma_B / M_B|^2} \times \text{BR} \xrightarrow{\Gamma_B \to 0} \delta(s_i - M_B^2) \times \text{BR}$$

(2.2)

of the resonant bosons. Explicit formulae for the twofold differential cross-sections in the double-differential approximation $\sigma_0(s; s_1, s_2)$ are found in reference [7] for $W$ pair production, in [3] for $Z$ pair production, and in [8] for associated Higgs production. We emphasize that these twofold differential cross-sections are given by very simple formulae. The Breit-Wigner functions are smoothed by mapping (see appendix A). Flag IMAP allows for the CC case to use the Breit-Wigner functions directly. In the latter case the program works slower by about a factor of two. We note that in this case the invariant mass distributions are accessible. With flag INCPRC one may select $ZZ$, $\gamma\gamma$, or their common production. For the CC case, the on-shell limit may be calculated with flag IONSHL=0.

The realization of the various generic functions mentioned here and in the following sections is described in section 3.3.

2.2 Background contributions

Although double-resonant amplitudes are dominant in most relevant cases, single- and non-resonant ‘background’ amplitudes contribute non-negligible
to the cross-sections. Including background Feynman diagrams, one can use the numbers from tables 1 and 2 to introduce a transparent notation [3]:

CC Processes with final states of type \( f_1 \bar{f}_1 f_2 \bar{f}_2 \) are called charged current processes: CC11, CC10, CC09; CC20, CC18.

NC Processes with final states of type \( f_1 \bar{f}_1 f_2 \bar{f}_2 \) are called neutral current processes: NC32, NC24, NC10, NC06; NC48, NC20, NC21, NC19; NC4-16, NC4-12, NC4-03, NC12; NC4-36, NC4-09.

mix Processes which may be considered as both CC and NC types are called mixed processes: mix43, mix19; mix56. An example for a mixed process is the production of \( u \bar{d} d \bar{u} \equiv u \bar{d} d \bar{u} \).

In the NC and mix classes, also Feynman diagrams with Higgs bosons contribute. In addition to the two-boson production classes CC03 [7] (W pair production), NC02 [6] (Z pair production), and NC08 [6] (ZZ, Z\(\gamma\), \(\gamma\gamma\) production), GENTLE/4fan is able to perform calculations for CC11, CC10, and CC09 [5] as well as NC32, NC24, NC10, NC06 [9]. This is chosen by flag IPROC. In the above NC cases, also Higgs diagrams are included [8].

Complete tree level four-fermion production cross-sections are given by the generic formula

\[
\sigma^{\text{Born}}(s) = \int ds_1 \int ds_2 \frac{\sqrt{\lambda}}{\pi s^2} \sum_k \frac{d^2 \sigma_k(s; s_1, s_2)}{ds_1 ds_2}
\]  
(2.3)

with \( \lambda \equiv \lambda(s; s_1, s_2) \), \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \) and

\[
\frac{d^2 \sigma_k}{ds_1 ds_2} = C_k(s; s_1, s_2) G_k(s; s_1, s_2)
\]  
(2.4)

The coefficient functions \( C_k \) represent coupling constants and boson propagators, while \( G_k \) is a kinematical function obtained after analytical performance of angular integration. The index \( k \) labels cross-section contributions with both different coupling structure and different Feynman topology. Noticing that often different twofold differential cross-sections \( d^2 \sigma_k / ds_1 ds_2 \) are related by symmetries in their arguments, one realizes that equation (2.3) enables efficient coding of the cross-section. For CC processes, background contributions are switched off with flag IBCKGR=0. With flag ICHNNL one may choose the final state topology in the CC case and with IFERM1 and IFERM2 in the NC case.
2.3 Initial state QED radiative corrections

In $e^+e^-$ annihilation, the most relevant radiative corrections are initial state QED corrections (ISR). The phase space is now parameterized by the center-of-mass photon scattering solid angle, by the boson scattering solid angle in the two-boson (or, equivalently, four-fermion) rest frame, by the boson decay products’ solid angles in the corresponding boson rest frames, by the final state fermion pair invariant masses, and by the four-fermion (or reduced) invariant mass $s'$ [6]. After analytic integration over all angular degrees of freedom, the $\mathcal{O}(\alpha)$ ISR-corrected total four-fermion production cross-section with soft photon exponentiation is determined in the flux function approach (flag ICONVL=0) by

$$
\sigma_{\text{ISR}}(s) = \int ds_1 \int ds_2 \int ds' \sum_k \frac{d^3\sigma_k(s, s'; s_1, s_2)}{ds_1 ds_2 ds'} ,
$$

with

$$
\frac{d^3\sigma_k(s, s'; s_1, s_2)}{ds_1 ds_2 ds'} = C_k \left[ \beta_e^{\beta_k-1} S_k + H_k \right] ,
$$

where $\beta_e = 2 (\alpha/\pi) (L - 1)$, $L \equiv \ln(s/m_e^2)$. The soft+virtual and hard corrections $S_k$ and $H_k$ separate into a universal, factorizing, process-independent and a non-universal, non-factorizing, process-dependent part. The latter is included with flag ITNONU=1. They are given by [6,10]

$$
S_k = \left[ 1 + \bar{S}(s) \right] \sigma_{k,0}(s'; s_1, s_2) + \sigma_{\bar{S},k}(s'; s_1, s_2) ,
$$

$$
H_k = \underbrace{\bar{H}(s, s') \sigma_{k,0}(s'; s_1, s_2)}_{\text{Universal Part}} + \sigma_{H,k}(s, s'; s_1, s_2) \underbrace{\sigma_{\bar{H},k}(s, s'; s_1, s_2)}_{\text{Non-universal Part}}
$$

with $\sigma_{k,0}(s'; s_1, s_2) \equiv \left[ \sqrt{\lambda}/(\pi s'^2) \right] G_k(s'; s_1, s_2)$. The universal $\mathcal{O}(\alpha)$ soft+virtual and hard radiators $\bar{S}$ and $\bar{H}$ read

$$
\bar{S}(s) = \frac{\alpha}{\pi} \left( \frac{\pi^2}{3} - \frac{1}{2} \right) + \frac{3}{4} \beta_e ,
$$

$$
\bar{H}(s, s') = -\frac{1}{2} \left( 1 + \frac{s'}{s} \right) \beta_e .
$$

Flag IZERO influences $\bar{S}(s)$. If the index $k$ labels pure s-channel $e^+e^-$ annihilation, the non-universal contribution vanishes. Non-universal ISR contributions are analytically very complex, but, in contrast to the universal corrections, not
for the CC03 process, a significant QED radiative correction is the so-called Coulomb singularity [11,12,14] originating from the exchange of photons between the two resonant $W$ bosons. It effects in a factor $(1 + \mathcal{F}_{\text{Coulomb}})$ to be multiplied to the twofold differential cross-section of equation (2.4) or to the threefold differential cross-section of equation (2.6). Details are defined with flag ICOLMB.

The leading logarithmic ISR contributions (essentially what we called the universal ISR contributions) may also be determined using the so-called structure function approach [15] (flag ICONVL=1). The basic formula, adapted for our case, is

$$\frac{d^2\sigma_{e^+e^\to 4f\nu\nu}}{ds_1 ds_2} = \int_{x_-^\text{min}}^{1} dx_+ \int_{x_-^\text{min}}^{1} dx_- D(x_+)D(x_-) \sigma_0(s'; s_1, s_2)$$

with the tree level twofold differential cross-section $\sigma_0$ evaluated at $s' = sx_+x_-$. The variables $x_+$ and $x_-$ represent the momentum fractions of the $e^+$ and $e^-$ after the radiation of photons. Their minimum values are given by

$$x_+^\text{min} = (\frac{\sqrt{s_1} + \sqrt{s_2})^2}{s},$$
$$x_-^\text{min} = (\frac{\sqrt{s_1} + \sqrt{s_2})^2}{x_+s}.$$  

An exhaustive discussion of the application of the structure function approach to $4f$ production may be found in [1,15] and in references quoted therein. Our implementation of the structure functions $D(x)$ follows equations (4.18) to (4.21) of reference [5]. Note that, to improve the calculation, $\sigma_0$ can be replaced by a “dressed” tree level cross-section which includes the running QED coupling and/or the Coulomb factor. Since the soft-photon pole is proportional to $(L-1)$ rather than to the leading logarithm $L$ alone there are the so-called BETA and ETA alternatives for the implementation of the structure functions [15]. They are selected by flag IZETTA.

For both descriptions of QED corrections, the details of higher order terms are defined by flag IQEDHS.

In addition to the total cross-section, GENTLE/4fan can compute the angular distribution, bin-integrated cross-sections, and moments for several other physically relevant quantities. These quantities are selected with the variables IDCS and IREGIM, the latter being chosen with flags IRMAX, IRSTP:
(1) the radiative final state four-fermion invariant mass loss $X = (s-s′)/(2\sqrt{s})$
for the CC11 family in the flux function as well as the structure function
approach;

(2) the true radiative energy loss $X = \sqrt{s}/2[(1 - x_+) + (1 - x_-)]$ for the
CC11 family in the structure function approach;

(3) the $W$ mass shift $X = \left[\left(\sqrt{s_1} + \sqrt{s_2}\right)/2 - M_W\right]$ for the CC11 family in
the flux function as well as the structure function approach;

(4) moments $X$ related to the angular distribution are treated in section 2.4.

The running of loop variable IMOMN = $n$ is defined by flags IMMIN, IMMAX. It
is used for the selection of the moment of $n^{th}$ degree $M_{(X,n)}$ of a quantity $X$
calculated with QED corrections:

\[
M_{(X,n)} = \frac{\int X^n \sigma_X}{\int \sigma_X} \tag{2.12}
\]

where $X$ is selected by IREGIM and the integration is over the phase space
variables upon which $X$ depends and $\sigma_X$ is the cross-section differential in
exactly those phase space variables. Note that $M_{(X,1)}$ is an average of the
quantity $X$.

Finally, we mention that an inclusive implementation of QCD corrections is
provided. Firstly, QCD corrections to the total $W$ width have been imple-
mented by a factor $(1 + 2\alpha_s/3\pi)$ to be multiplied to $\Gamma_W$. Secondly, the final
state radiative QCD corrections to the formation of a quark-anti-quark pair
are built in by multiplying the relevant branching ratio with $(1 + \alpha_s/\pi)$.

2.4 Angular distributions

Because of the importance of the CC03 process, GENTLE/4fan is also able
to compute $W$ production angular distributions, bin-integrated cross-sections
and moments for this process. A selection is possible by the flags IDCS and
IREGIM. In the first and last of the three mentioned cases, the last analytical
integration is not carried out. Instead, in Born approximation the analytic
threefold differential cross-section

\[
\frac{d^3\sigma_{CC03}}{ds_1 ds_2 d\cos \theta} = \frac{\sqrt{\xi}}{\pi s^2} \sum_{k=1}^{3} g_{CC03}^{CC03}(s, s_2, s_1; \cos \theta) \tag{2.13}
\]

is used. The angular kinematic functions $g_{CC03}^{CC03}(s, s_2, s_1; \cos \theta)$ were published
in [5]. The cross-section $d\sigma_{CC03}/d\cos \theta$ is obtained upon integration over $s_1$
and $s_2$. In GENTLE/4fan, as a measure for the CC03 angular distribution, the
first four moments of the differential cross-section (2.13) may be calculated as
its convolution with Chebyshev polynomials $T_n(\cos \theta)$. The Born moment of $n^{th}$ degree is given by

$$M_{(\cos \theta, n)} = \int ds_1 \, ds_2 \, d \cos \theta \, T_n(\cos \theta) \, \frac{d^3 \sigma_{\text{CC03}}}{ds_1 \, ds_2 \, d \cos \theta}.$$  \hspace{1cm} (2.14)

The zeroth moment is just the total cross-section.

Initial state QED corrections to the angular distributions and moments have to be calculated in the structure function approach. With photon emission, the $W$ production angle in the laboratory system, $\theta_{\text{lab}}$, and in the center-of-mass system of the four final state fermions, $\theta$, are different. The Lorentz boost relating them depends on both $x_+$ and $x_-$. The QED corrected angular moments with ISR are given by

$$M_{(\cos \theta_{\text{lab}}, n)} = \int ds_1 \int ds_2 \int dx_+ \int dx_- \int d \cos \theta \times D(x_+)^i D(x_-)^i T_n[\cos \theta_{\text{lab}}(\cos \theta)] \, \frac{d^3 \sigma_{\text{CC03}}}{ds_1 \, ds_2 \, d \cos \theta}.$$  \hspace{1cm} (2.15)

With flags IDCS, IBIN, IBINNU, the user may select the calculation of the angular distribution:

$$\frac{d \sigma_{\text{CC03}}}{d \cos \theta_{\text{lab}}} = \int ds_1 \int ds_2 \int dx_+ \int dx_- \times D(x_+)^i D(x_-)^i \sum_i \left| \frac{\partial \cos \theta_i}{\partial \cos \theta_{\text{lab}}} \right| \frac{d^3 \sigma_{\text{CC03}}}{ds_1 \, ds_2 \, d \cos \theta_i}$$  \hspace{1cm} (2.16)

with the threefold differential tree level cross-section $\frac{d^3 \sigma_{\text{CC03}}}{(ds_1 \, ds_2 \, d \cos \theta_i)}$ depending on $s' = sx_+x_-, s_1, s_2$, and $\cos \theta_i(s, s_1, s_2, x_+, x_-, \cos \theta_{\text{lab}})$. Index $i$ indicates that there may be zero, one, or two solutions for the Lorentz boost.

In analogy to (2.15), the bin-integrated cross-section formulae may be written with the boosted angular argument under the integral, thus allowing for relatively simple analytic angular integrations (and avoiding the appearance of the Jacobean) if, of course, the integration limits set in the laboratory frame are transformed into angular limits in the boosted frame. In this respect, the approach to the bin-integrated cross-section differs from that for the angular distribution where we integrate over the laboratory angle (and have to take into account the Jacobean). A choice can be made with flag IBIN.
2.5 Anomalous couplings

Angular distributions as defined in the above section 2.4 are especially useful in the context of anomalous triple boson couplings [16]. In GENTLE/4fan, only those anomalous $\gamma W^+ W^-$ and $ZW^+ W^-$ couplings are implemented which obey Lorentz invariance, conserve CP, and do not modify the electromagnetic interaction. These constraints allow a restricted set of anomalous contributions to the triple gauge boson vertex. These couplings are denoted by $g_3^a(V)$ with $V \in \{\gamma, Z\}$. The $a = s, x, y, z$ denotes the coupling type including the SM triple-boson coupling $g_3^s(V)$. The couplings are derived from the anomalous couplings defined in equations (B.3) to (B.8). The threefold differential CC03 cross-section is then given by [17]

$$
\frac{d^3 \sigma_{\text{CC03}}^\text{ano}}{ds_1 ds_2 d \cos \theta_{\text{lab}}} = \frac{\sqrt{\lambda}}{\pi s^2} \left( C^i G^i + \sum_k C_k^{st} G_k^{st} + \sum_{ij} C_{ij}^{st} G_{ij}^{st} \right) ,
$$

where $i, j, k$ run over $s, x, y, z$. The summation over s-channel photon and Z amplitudes is contained in the coefficient functions $C$. Compared to the SM where only three kinematical functions describe the CC03 process, the inclusion of anomalous couplings adds considerable complexity to the calculation. The user may select a treatment of anomalous couplings with flag IANO. The potential influence of an extra heavy neutral boson $Z'$ is explained in appendix C.

3 Description of the Program

GENTLE/4fan is a FORTRAN77 program for the computation of total and differential cross-sections for four-fermion production in $e^+ e^-$ annihilation with inclusion of the relevant initial state QED corrections. It originated from older versions [18].

3.1 Features of GENTLE/4fan

The cross-sections, angular distributions, and moments presented in equations (2.1), (2.3), (2.5), (2.10), (2.12), (2.14), (2.15), (2.16), and (2.17) are computed in GENTLE/4fan by numerically integrating analytical formulae for expressions such as equation (2.6). The analytical input formulae are found in [3,5,7–9,6,10–12,17] and in references therein. Numerical integrations are performed by a fast and stable self-adaptive Simpson algorithm realized in the
subroutines SIMPS and FDSIMP.

For transparency – and also for historical reasons – version 2.0 of GENTLE/4fan is built from three pieces of code. The corresponding ramification is controlled by the flag IPROC. The first part contains the charged current (CC) reactions including $O(\alpha)$ initial state QED corrections and anomalous couplings. The second part is the 4fan code with NC reactions. The third part, steered by the subroutine NCQED, contains the code for the NC02 and NC08 processes with complete $O(\alpha)$ initial state QED corrections. Subsequently we will call the first part the CC part, the second one will be named the 4fan branch, and the third part will be called NCQED part. The first and second part together are referred to as the CC/NC branch.

At present, the GENTLE/4fan may treat the following four-fermion final states:

1. CC03 (with complete ISR, angular distributions, and anomalous couplings) [6,10,17]
2. NC02, NC08 (with complete ISR) [6]
3. CC09, CC10, CC11 [5]
4. NC06, NC10, NC24, NC32 [9]
5. NC of (4) + Higgs [8]

Initial state radiation is implemented. For total cross-sections universal initial state radiative corrections as described in equations (2.5) to (2.7) are computed. In addition, non-universal ISR is available for the CC03, NC02, and NC08 processes. For the CC11 family, leading logarithmic initial state QED corrections can also be calculated in the structure function approach as indicated in eq. (2.15). Further, the Coulomb correction to CC03 and inclusive QCD corrections (to both the denominators of the Breit-Wigner functions and final states with quarks) have been implemented. The user can choose between the different prescriptions of references [11,12] for the Coulomb correction via the flag ICOLMB.

Cuts may be applied on the invariant masses $s_1$ and $s_2$. In the structure function approach, cuts on the electron and positron momentum fractions after initial state radiation may be imposed. In the NCQED branch of the code, a minimum value for the fraction $s'/s$ may be set as well as for $s_1/s$ and $s_2/s$. A cut on the CC03 $W$ production angle is not directly supported by the code but may be imposed using the option for bin-integration.

In general, final state masses are neglected in GENTLE/4fan, i.e. the program uses the small mass approximation. Where needed, however, masses are retained in the phase space factors. Masses of heavy particles coupled to the

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5 The Fortran code FDSIMP by Yu. Sedykh is an extension of the SIMPS code by I.N Silin.
Higgs boson are taken into account where appropriate. In the NCqed branch of the program, masses may be chosen by altering the values of AM1 or AM2 in the initialization routine NCIN00.

3.2 Program input

The common units throughout the code are GeV and GeV/c². The physical input parameters of GENTLE/4fan are defined in subroutine WUFLAG:

\[\begin{align*}
\text{ALFAI} &= \alpha(0) = 137.0359895, \text{the inverse running fine structure constant at zero momentum transfer} \\
\text{ALFAS} &= \alpha_s(\text{low } q^2) = 0.2 \\
\text{ALPHS} &= \alpha_s(2MW) = 0.12 \\
\text{ALPW} &= \alpha(2MW) = 1/128.07, \text{the running fine structure constant at } 2MW \\
\text{AME} &= m_e = 0.51099906 \times 10^{-3} \text{ GeV, the electron mass} \\
\text{AMZ} &= M_Z = 91.1888 \text{ GeV, the } Z \text{ mass} \\
\text{AMW} &= M_W = 80.230 \text{ GeV, the } W \text{ mass} \\
\text{GAMZ} &= \Gamma_Z = 2.4974 \text{ GeV, the } Z \text{ width} \\
\text{GFER} &= G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \text{the Fermi constant} \\
\end{align*}\]

For the fermion-boson couplings, the value \(\alpha(2MW)\) is used, while for the ISR corrections, the Thompson limit \(\alpha(0)\) of the fine structure constant is applied. Depending on the flag value IINPT, the weak mixing angle is computed as

\[\begin{align*}
\sin^2 \theta_W &= \frac{\pi \alpha(2MW)}{\sqrt{2} M_W^2 G_\mu} \quad \text{for IINPT} = 0 \quad (3.1) \\
\sin^2 \theta_W &= 1 - M_W^2 / M_Z^2 \quad \text{for IINPT} = 1 \quad (3.2) \\
\sin^2 \theta_W &= 0.22591 \text{ (numerical user input)} \quad \text{for IINPT} = 2 \quad (3.3)
\end{align*}\]

For charged current reactions, the following derived quantities are used in

\[\alpha_s(2MW)\] is used for the QCD corrections to the \(W\) width and to the final state quark-anti-quark pairs in CC processes. The low momentum transfer value \(\alpha_s(|q| \ll 2MW)\) is used for the NC32 process [19] in the calculation of gluon exchange diagrams.
GENTLE/4fan:

\[
\Gamma_W = \frac{9}{6\sqrt{2\pi}} \Gamma_\mu M_W^3 \left(1 + \frac{2\alpha_s(2M_W)}{3\pi}\right) \quad \text{for } IGAMW = 0 \quad (3.4)
\]

\[
\Gamma_W = 2.085 \quad \text{(numerical user input)} \quad \text{for } IGAMW = 1 \quad (3.5)
\]

as well as

\[
GAE = -\frac{e}{4sWcW} = -\frac{\sqrt{4\pi\alpha(2M_W)}}{4sWcW}
\]

\[
GVE = GAE(1 - 4s_W)
\]

\[
GWF = \frac{g}{2\sqrt{2}} = -GAE\sqrt{2}c_W
\]

\[
GWWG = \sqrt{4\pi\alpha(2M_W)}
\]

\[
GWWZ = GWWG \frac{c_W}{s_W}.
\]

\text{GVE and GAE are the electron vector and axial vector couplings, GWF is the fermion-W coupling, and GWWG and GWWZ are the triple gauge boson couplings for the photon and the Z boson in the Standard Model, respectively. All other couplings are uniquely determined by the above. Although, for historical reasons, some of the variable names are slightly different, the same derived quantities are used for neutral current reactions. For the neutral current processes, the final state fermion types and their masses are part of the input. For the NC08 family of processes, this input is set in the subroutine NCIN00. The fermion types for the NC32 family are set by the flags IFERM1 and IFERM2, while the corresponding masses are loaded from a DATA statement in the subroutine bbmin. In the subroutine WUFLAG, the Higgs mass AMHIG and the Higgs width GAMHIG as well as the flags which steer the program flow are initialized. The Higgs contribution may be switched on or off by setting flag IHIGGS.}

The flags and, for the CC branch, the physical input may be overridden by

\footnote{Note that the fermion masses are stored in the variables AM1 and AM2 for the NC08 as well as the NC32 families. This is, however, unproblematic, because the routines for each family are completely separate and are never executed together in a single run.}
calls to the subroutine WUFLAG; see also section 3.5. We strongly recommend the user to carefully control that only options are active which are definitely declared to be compatible with each other.

In WUFLAG, the following flags are initialized:

**IANO**

IANO: to set the anomalous couplings defined in equations (B.3) to (B.8) of appendix B
IANO=0: all couplings at their SM value
IANO=±1: XG = x_γ = ±0.5, all other couplings at their SM values
IANO=±2: YG = y_γ = ±0.5, all other couplings at their SM values
IANO=±3: XZ = x_Z = ±0.5, all other couplings at their SM values
IANO=±4: YZ = y_Z = ±0.5, all other couplings at their SM values
IANO=±5: ZZ = z_Z = ±0.1, all other couplings at their SM values
IANO=±6: DZ = δ_Z = ±0.5, all other couplings at their SM values
IANO=±7: “HISZ scenario” [20] with
\[ x_γ = ±0.1, \]
\[ x_Z = x_γ \left( \cos^2 θ_W - \sin^2 θ_W \right) / (2 \sin θ_W \cos θ_W), \]
\[ δ_Z = x_γ / (2 \sin θ_W \cos θ_W), \]
and \( y_γ = y_Z = z_Z = 0 \)
IANO=8: user-set values of \( x_γ, y_γ, x_Z, y_Z, z_Z, δ_γ, \) and \( δ_Z \); for extra \( Z' \) couplings, see appendix C

**IBCKGR**

IBCKGR: only active for IPROC=1
IBCKGR=0: calculations for the double resonant CC03 process only, no CC11 background
IBCKGR=1: inclusion of CC11 background in accordance with ICHNNL not available for IDCS=1

**IBIN**

IBIN: only active for IDCS=1
IBIN=0: calculation of \( dσ/dcosθ \) for IBINNU fixed values of \( cosθ \)
IBIN=1: calculation of \( dσ/dcosθ \) integrated over each of the IBINNU cosθ-bins

**IBINNU**

IBINNU: only active for IDCS=1
IBINNU=n: number of points or bins for the differential cross-section calculation

**IBORNF**

IBORNF=0: Born approximation, no radiative corrections
IBORNF=1: initial state QED corrections are included
ICCHNL
ICCHNL: only active for IPROC=1 and IBCKGR=1
ICCHNL=0: no background taken into account, calculation of inclusive CC03 quantities
ICCHNL=1: calculation of CC09 quantities for a leptonic final state
ICCHNL=2: calculation of CC10 quantities for a semi-leptonic final state with positively charged lepton
ICCHNL=3: calculation of CC10 quantities for a semi-leptonic final state with negatively charged lepton
ICCHNL=4: calculation of CC11 quantities for a hadronic final state
ICCHNL=5: calculation of inclusive quantities for the CC11 family

ICOLMB
ICOLMB: determines, how the Coulomb correction is included in the differential cross-sections of equations (2.4) or (2.6) only relevant for the CC03 process
ICOLMB=0: Coulomb correction not included
ICOLMB=1: Coulomb correction as in equation (5) of reference [11], but without the phase factor
ICOLMB=2: Coulomb correction as in equation (5) of reference [11], but with $\Delta \equiv (s_1 - s_2)/s' = 0$
ICOLMB=3: Coulomb correction as in equation (5) of reference [11]
ICOLMB=4: Coulomb correction as in reference [14]
ICOLMB=5: Coulomb correction as in reference [12]

ICONVL
ICONVL: only active for IPROC=1 and IPROC=2
ICONVL=0: flux function convolution as given in equation (2.5)
ICONVL=1: structure function convolution as indicated in eq. (2.10) attention: extremely CPU time consuming

IDCS
IDCS=0: calculation of total cross-sections and moments
IDCS=1: calculation of the differential cross-section $d\sigma/d\cos\theta$ with anomalous couplings as determined by the flag IANO if used in conjunction with ISR corrections, the structure function approach ICONVL=1 must be used needs IBCKGR=0
**IFERM1, IFERM2**

IFERM1: only active for IPROC=2. IFERM1 is an integer specifying the first final state fermion pair according to the PDG particle numbering scheme [13].

IFERM2: like IFERM1, but for the second final state fermion pair. Remember that it is the second final state fermion pair that couples to the Higgs boson, if a calculation with the Higgs is requested.

**IGAMW**

IGAMW=0: computation of the $W$ width according to (3.4).
IGAMW=1: numerical user input.

**IGAMZS**

IGAMZS=0: constant $Z$ width for $Z$ propagators.
IGAMZS=1: $s$-dependent $Z$ width for $Z$ propagators.

**IHIGGS**

IHIGGS: only active for IPROC=2.
IHIGGS=0: Higgs not included.
IHIGGS=1: Higgs included.

**IINPT**

IINPT=0: weak mixing angle defined by equation (3.1).
IINPT=1: weak mixing angle defined by equation (3.3).
IINPT=2: numerical user input, only active for IPROC=1.

**IIQCD**

IIQCD: only active for IPROC=1.
IIQCD=0: no QCD corrections, i.e. $\alpha_s(2M_W) \equiv 0$.
IIQCD=1: inclusive QCD corrections to the $W$ width and final state QCD corrections to the final state quark pair are taken into account with $\alpha_s(2M_W) = 0.12$.

**IMOMN**

IMOMN: only active for IPROC=1 and IDCS=0.
IMOMN=n: the $n^{th}$ moment of the physical quantity determined by the current IREGIM is computed.

The moments to be computed are determined by the values of IMMIN and IMMAX which control the scope of the FORTRAN loop DO IMOMN=IMMIN,IMMAX.
**INCPRC**
- **INCPRC:** only active for **IPROC=3**
- **INCPRC=0:** NC02 cross-section calculation, i.e. Z pair production only
- **INCPRC=1:** photon pair production only
- **INCPRC=2:** NC08 cross-section calculation

**IONSHT**
- **IONSHT:** only active for **IPROC=1**
- **IONSHT=0:** on-shell limit for $W$ pair production
- **IONSHT=1:** off-shell $W$ bosons

**IPROC**
- **IPROC=1:** CC11 family of processes
- **IPROC=2:** NC32 family of processes
- **IPROC=3:** NC08 family of processes with complete ISR

**IQEDHS**
- **IQEDHS:** determines the soft+virtual and hard radiators $\bar{S}$ and $\bar{H}$ to be used for universal corrections to total cross-sections only active, if **IBORNF=1** and **ICONVL=0**
- **IQEDHS=0:** $O(\alpha)$ exponentiated universal ISR corrections as given in equation (2.6) with the radiators $\bar{S}$ and $\bar{H}$ from equations (2.8) and (2.9)
- **IQEDHS=1:** as **IQEDHS=0**, but terms of $O[(\alpha L)^2]$ added to $\bar{S}$ and $\bar{H}$
- **IQEDHS=2:** as **IQEDHS=1**, but terms of $O(\alpha^2 L)$ added to $\bar{S}$ and $\bar{H}$
- **IQEDHS=3:** as **IQEDHS=2**, but terms of $O(\alpha^2)$ added to $\bar{S}$ and $\bar{H}$

**IREGIM**
- **IREGIM:** only active for **IPROC=1** and **IDCS=0**
- determines the physical quantity to be computed in addition to the total cross-section (see sections 2.3 and 2.4)
- **IREGIM=0:** computation of the total cross-section, used as normalization for next cases
- **IREGIM=1:** computation of moments of the radiative invariant mass loss
- **IREGIM=2:** computation of moments of the radiative energy loss for the structure function approach, i.e. for **ICONVL=1** only
- **IREGIM=3:** computation of moments of the $W$ mass shift for the CC03 process
- **IREGIM=4:** computation of $W$ production angular moments for the CC03 process with Chebyshev polynomials (compare section 2.4)

The physical quantities to be computed are determined by the integer values of the variables **IRMAX** and **IRSTP** which control the scope of **IREGIM** via the Fortran statement **DO IREGIM=0,IRMAX,IRSTP.**
**ITNONU**

ITNONU: only active for IBORNF=1
ITNONU=0: only universal hard ISR corrections are taken into account (compare equations (2.7) and (2.9))
ITNONU=1: complete universal and non-universal hard ISR corrections are taken into account; only in effect for CC03 and for the NC08 family, IBCKGR=IDCS = 0

Attention: very CPU time consuming

**IZERO**

IZERO: only for the flux function convolution, i.e. for ICONVL=0
IZERO=0: the constant term \((\alpha/\pi)(\pi^2/3 - 1/2)\) in the \(\bar{S}\) is neglected (compare equation (2.8))
IZERO=1: the soft+virtual \(\bar{S}\) radiator is given by equation (2.8)

**IZETTA**

IZETTA: only for the structure function convolution, i.e. for ICONVL=1
IZETTA=0: ETA choice, i.e. in the hard photon radiation part of the structure function \((2\alpha/\pi)L\) is used, whereas \((2\alpha/\pi)(L-1)\) is entered into the soft part and the exponentiation
IZETTA=1: BETA choice, i.e. \((2\alpha/\pi)(L-1)\) is used for the whole structure function

Finally, we mention that the relative and absolute precision requirements for the numerical integrations are set at different places in GENTLE/4fan. Sensible default values for the precision requirements are chosen for the different branches of the program. In addition, we collect here the physical input parameters which can be chosen by calls to subroutine WUFLAG:

ALPHS, ALPW, AME, AMPER2, AMHIG, AMW, AMZ, DZ, GAMHIG, GAMW, GAMZ, GFER, SINW2, XG, XZ, YG, YZ, ZZ.

### 3.3 Routines of GENTLE/4fan

In this subsection we give a brief description of the subroutines and functions in the code. GENTLE/4fan uses REAL*8 and COMPLEX*16 variables.

- **FUNCTION alamk** – computes the kinematic \(\lambda\)-function \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\)
- **FUNCTION ALRUN** – may be used in subroutine prop to compute photon propagators with running fine structure constant \(\alpha\)
• **SUBROUTINE** `bbmmin` – constitutes the central routine for the 4fan part of the code, i.e. for computations for the NC32 family; performs the initialization of physical and coupling constants for the 4fan part

• **FUNCTION** `c222` (c322, c322m, c422, c233) – computes the coupling function $C_{222}$ ($C_{322}$, $C_{a322}$, $C_{422}$, $C_{233}$) introduced in references [8,9].

• **FUNCTION** `cg422` (cg233) – computes the coupling function similar to $C_{422}$ ($C_{233}$), but with a gluon exchange replacing the electroweak boson exchange in the final state (compare figure 1e of reference [19]).

• **SUBROUTINE** `check` – checks and prints the input to the 4fan part; produces the integer error flag `ierr`; prints messages if input errors are encountered.

• **FUNCTION** `Coulom` – computes, according to the flag value of `ICOLMB`, the term `Coulom` for the Coulomb correction factor (1 + `Coulom`).

• **FUNCTIONS** `DDILOG`, `DLI2` – provide the dilogarithm function $Li_2(x)$ for real argument; if $x \geq 1$, $x$ is understood to be the real part of the argument with infinitesimal imaginary part, and the real part of the function value is returned.

• **SUBROUTINE** `DELTPR` – computes, for the flux function approach, the three-fold (with respect to $s_1$, $s_2$, and $s'$) differential, non-universal contributions to the CC03 cross-section.

• **SUBROUTINE** `DELVNU` – computes the non-universal soft+virtual contribution to the NCqed cross-section.

• **FUNCTION** `DLI3` – provides the trilogarithm function $Li_3(x)$ for $x \leq 1$.

• **FUNCTION** `DS12` – provides the Nielsen function $S_{1,2}(x)$ for $x \leq 1$.

• **SUBROUTINE** `dsdshsz` – provides the twofold differential cross-section $d^2\sigma/(ds_1 ds_2)$ for the NC32 family as given in references [8,9].

• **FUNCTION** `DXS2` – provides, for the CC/NC branch of the code and according to the flags `IONSHEL`, `IPROC`, `IBORNF`, `IBCKGR`, and `ICHNNL`, twofold differential expressions (with respect to $s_1$ and $s_2$) for the cross-section or for moments by calling `XSEC2` and `XSEC2BG` or `XSEC2E`.

• **FUNCTION** `FACINC` – is the inverse function of `FACNC`.

• **FUNCTION** `FACINV` – represents the inverse function of `FACT`.

• **FUNCTION** `FACNC` – is the mapping function for the $s'$ integrations used in calls to `FDSIMT`.

• **FUNCTION** `FACT` – is the mapping function for the $s'$, $x_+$, and $x_-$ integrations used in calls to `FDSIMP` and `FDSIMS`.

• **FUNCTION** `FCONC` – provides the integrand of the $s'$ integration in `XSNC2E`.

• **FUNCTION** `FCONV1` – provides, according to the flag `ICONVL`, threefold differential (with respect to $s_1$, $s_2$, and $s'$ or $x_+$) expressions for

  1. the ISR-corrected cross-section, the radiative invariant mass loss, and the $W$ mass shift using the flux function approach.

  2. total and differential cross-sections as well as moments in the structure function approach by calling `FCONV2`.

---

8 Beware: The normalization differs from the references in some cases.
• **FUNCTION FCONV2** – provides fourfold differential (with respect to $s_1$, $s_2$, $x_+$, and $x_-$) expressions for all quantities available in the structure function approach

• **SUBROUTINE FDSIMP** – is similar to **SIMPS**, but performs the integration with the help a mapping function which is called in the argument list

• **SUBROUTINES FEYNINT1, FEYNINT2, FEYNINT3** – compute, from the analytical formulae of reference [6], the values of several complicated integrals needed for the non-universal hard ISR correction

• **FUNCTION FINE12 (FI12NC)** – provides the differential cross-section $d\sigma/dx_1, x_i = s_i/s$ for the CC/NC (NC$q\text{ed}$) branch of the code

• **FUNCTION FINE34 (FI34NC)** – provides the differential cross-section $d\sigma/(dx_1 dx_2)$ for the CC/NC (NC$q\text{ed}$) branch of the code by calling the function **DXS2** (**XSBRNC** and **XSNC2E**) which actually provides the value

• **FUNCTION FMAPSH** – performs, for the NC branch, the mapping in the invariant mass squared $s_2$ of the fermion pair defined by the flag **IFERM2**

• **FUNCTION FMAPSZ** – performs, for the NC branch, the mapping in the invariant mass squared $s_1$ of the fermion pair defined by the flag **IFERM1**

• **FUNCTION FMBW** – maps Breit-Wigner resonance peaks

• **FUNCTION FMQED** – maps peaks of the type $1/s$

• **FUNCTION FOTF** – computes the leptonic vacuum polarization for **ALRUN** from lepton and effective quark masses

• **FUNCTION FOTF1** – computes the leptonic vacuum polarization for **ALRUN** from lepton masses and the hadronic vacuum polarization of reference [21]

• **FUNCTION FSZ** – performs, for the NC branch, the integration over $s_2$; this is the complement to the function **FINE12**

• **FUNCTION FTBINV** – represents the inverse function of **FXTB**

• **FUNCTION FXSECB** – provides, in dependence on the flag **IDCS** and for the CC03 process, the threefold differential cross-section $d^3\sigma/(dx_1 dx_2 d\cos\theta)$ or the argument $T_n(cos\theta_{lab}) d^3\sigma_{CC03}/(ds_1 ds_2 d\cos\theta)$ of the moment integral (2.15)

• **FUNCTION FXSECD** – provides, in dependence on the flags **IDCS** and **IBIN** and for the CC03 process, the integral over the threefold differential cross-section $d^3\sigma/(dx_1 dx_2 d\cos\theta)$ for given limits

• **FUNCTION FXTB** – is the mapping function for the $\cos\theta$ integration with the subroutine **FDSIMV**

• **FUNCTION g222 (g322, g322m, g422, g233)** – computes the kinematic function $G_{222}$ ($G_{322}, G_{322}^a, G_{422}, G_{233}$) introduced in references [8,9]

• **SUBROUTINE GFBGD2** – computes the kinematic functions $G_k$ for the pure background for the CC11 process

• **SUBROUTINE GFBGD3** – computes the kinematic functions $G_k$ for the signal-background interferences of the CC11 process

• **SUBROUTINE GFBRN** – computes the kinematic functions $G_k$ for the CC03 process (compare equation (2.4), see reference [7])

• **SUBROUTINE GFBRNA** – computes, in dependence on the flags **IDCS** and **IANO**, the angular kinematic functions $G_k^{CC03}(s; s_1, s_2, \cos\theta)$ (compare eq. (2.13), see
• SUBROUTINE GFBRNB – computes, in dependence on the flags IDCS and IANO, the integrated angular kinematic functions $G_{\kappa s}^{CC03}(s; s_1, s_2, \cos \theta_{\text{min}}, \cos \theta_{\text{max}})$
• SUBROUTINE GFBRNC – computes the kinematic function for the NCqed branch
• MAIN – performs the ramification between the NCqed and the CC/NC branches of the code; carries out some initialization; calls WWIN00 and WWIN01 for further initialization; loops over the center of mass energies; loops over the desired values of IREGIM and IMOMN; prints results
• FUNCTION MYEXP1 (MYEXP2) – is the inverse function of MYLOG1 (MYLOG2) as physics input
• FUNCTION MYLOG1 (MYLOG2) – is the mapping function for the $s_1(s_2)$ integration used in calls to FDSIMP (FDSIMS)
• SUBROUTINE NCGENT – the central routine for the NCqed branch of the code; sets integration precisions; calls NCIN00 and NCIN01 for further initialization; loops over the center of mass energies; provides the total cross-section for the NCqed branch; prints results
• SUBROUTINE NCIN00 – initializes the flags needed for the NCqed branch of the code; initializes the fermion types and couplings and performs other kinematics independent initialization for the NCqed branch
• SUBROUTINE NCIN01 – performs, for the NCqed branch of the code, the initialization of quantities that depend on $s$ only and provides the soft+virtual radiator
• SUBROUTINE NCIN02 – provides the phase space factor and the coupling function for the NCqed branch given in equation (4.5) of first of references [6]
• SUBROUTINE prop – provides the $Z, \gamma$, and gluon propagators for use in the subroutine dsdshsz
• SUBROUTINE PROPAG – computes the neutral gauge boson propagators used in NCIN02
• FUNCTION RHOINV – is the inverse function of RHOPR
• FUNCTION RHOPR – provides, for the CC/NC branch of the code and according to the mapping choice via IMAPPG, the mapping function for the $s_1$ and $s_2$ integrations
• FUNCTION RI3 – computes the scalar two-point function needed in FOTF and FOTF1
• SUBROUTINE SETANO – initializes the anomalous couplings defined by equations (B.3) to (B.8) in dependence on the flags IDCS and IANO
• SUBROUTINE SHORTI – computes the kinematic functions $G_{cc11}^{u,dd}$ and $G_{cc11}^{u,d}$ as given in equations (3.9) and (3.10) of reference [5]
• SUBROUTINE SIMPS, SIMPT – performs the integration of an input function inside given limits and with a given precision by applying a self-adaptive Simpson algorithm
• FUNCTION SPENC – is an auxiliary function used by the complex dilogarithm function XSPENZ
• FUNCTION STRUCF – computes, according to the value of IZETTA, the structure function $D(x)$ from appendix A of reference [15]
• FUNCTION TOTXS – computes the total cross-section for the CC/NC branch and the differential cross-section for the CC03 case

• FUNCTION TRILOG – provides the trilogarithm function $Li_3(x)$ for $0 \leq x \leq 1$

• FUNCTION TRIS12 – provides the Nielsen function $S_{1,2}(x)$ for $0 \leq x \leq 1$

• SUBROUTINE WUFLAG – initializes and changes the default flags as well as physics input

• SUBROUTINE WWIN00 – performs the kinematics independent initialization of mathematical and physical constants for the CC/NC branch of the code; calls the subroutines bmmmin and SETANO for further initialization

• SUBROUTINE WWIN01 – performs, for the CC/NC branch, the initialization of quantities that depend on the center of mass energy or, equivalently, $s$ only; performs initialization in dependence on the flags IPROC, ICONVL, and IQEDHS; provides the soft+virtual radiator for the CC/NC computation

• SUBROUTINE WWIN02 – performs the initialization of tree level quantities that depend on $s$, $s_1$, and $s_2$; for the CC/NC branch

• SUBROUTINE WWIN03 – performs the initialization of radiative quantities that depend on $s$, $s_1$, and $s_2$; for the CC/NC branch

• SUBROUTINE XSBRNC – provides the tree level cross-section for the NCqed branch of the code

• SUBROUTINE XSEC2E – is needed for the computation of ISR-corrected cross-sections, moments, and distributions; performs the $s'$ integration for the flux function approach and the $x_+$ integration for the structure function approach

• SUBROUTINE XSEC3B – computes, in the flux function approach and for the CC11 family with IBCKGR=1, the universally ISR-corrected, threefold (with respect to $s_1$, $s_2$, and $s'$) differential cross-section for background contributions by calling the subroutine XSECBG

• SUBROUTINE XSEC3E – is used in the flux function convolution for the CC03 process, i.e. for IPROC=1, IBORNF=1, and ICONVL=0 only; computes, in dependence on the settings of IITNONU and IQEDHS and according to formula (2.6) and reference [10], the threefold differential CC03-expressions $[\beta_3 v^{n-1}S_k + \mathcal{H_k}]$

• SUBROUTINE XSECB – for the CC/NC branch; provides, in dependence on the flags IPROC, IDCS, and IREGIM, the twofold differential (with respect to $s_1$ and $s_2$) cross-section or moments for the chosen tree level process by calling GFBRN or integrating FXSECB over the $W$ production angle

• SUBROUTINE XSECBG – provides, in dependence on the channel chosen via the flag ICHNL, the twofold (with respect to $s_1$ and $s_2$) differential cross-section for the signal-background interference and the pure background in the CC11 process by calling the subroutines GFBGD2 and GFBGD3

• SUBROUTINE XSNC2E – provides the twofold (with respect to $s_1$ and $s_2$) differential cross-section for the radiative NCqed cross-section by integrating over $s'$
• **SUBROUTINE XSNC3E** – computes, according to the flags ITNONU, the three-fold (with respect to \(s_1\) and \(s_2\)) differential, radiative NC\text{qed} cross-section

• **FUNCTION XSPENZ** – provides the complex dilogarithm function \(\text{Li}_2(z)\) by transforming the argument to the area of fast convergence

### 3.4 Program output

The output of **GENTLE/4fan** strongly depends on the user-chosen branch of the program or, in other words, on the flag settings. Four different output appearances are to be distinguished, namely

1. the output of total cross-sections and moments for the **CC11** family
2. the output of angular distributions for the **CC03** process
3. the output of the **4fan** branch of the code
4. the output of the **NC\text{qed}** branch of the code

In the below subsections, we will provide and explain examples for each of the above four kinds of output. The below outputs will, at the same time, serve as test run output, where we assume that the user has used the flags and physical input as shown in the output. Any other pre-set input should remain unchanged for test run purposes.

#### 3.4.1 Output of total cross-sections and moments for **CC11**

The output of **GENTLE/4fan** for the total cross-section and moments for the **CC11** family of processes appears as follows.

```
***** This is GENTLE/4fan -- Version 2.0 *****

This is the CC branch of GENTLE/4fan for total cross-sections
FLAGS:IPROC ,IINPT ,IONSHT,IBORNF,IBCKGR,ICHNNL= 1 1 1 1 1 2
FLAGS:IGAMZS,IGAMW ,ITNONU,IQEDHS,ICOLMB,ICONVL= 1 0 0 1 2 0
FLAGS:IZERO ,IZETTA,IIQCD ,IDCS ,IANO ,IBIN = 0 1 1 0 0 0
FLAGS:IMAP ,IRMAX ,IRSTP ,IMMIN ,IMMAX = 1 1 1 1 4
EPS1 = 1.0000000000000000E-04
S1MIN,S1MAX= 0.0000E+00 0.2592E+05
S2MIN,S2MAX= 0.0000E+00 0.2592E+05
Energy (GeV) Cross-section (pb)
 161.00     0.1336661D+00
MOMENTS
   1   2   3   4
   1  0.4753809D+00  0.3387719D+01  0.5607608D+02  0.1411261D+04
```
S1MIN,S1MAX= 0.0000E+00 0.3063E+05
S2MIN,S2MAX= 0.0000E+00 0.3063E+05
Energy (GeV) Cross-section (pb)
   175.00     0.4952086D+00
MOMENTS
   1  1  2  3  4
   1  0.1126753D+01 0.8432371D+01 0.1030576D+03 0.1885898D+04
S1MIN,S1MAX= 0.0000E+00 0.3610E+05
S2MIN,S2MAX= 0.0000E+00 0.3610E+05
Energy (GeV) Cross-section (pb)
   190.00     0.6080011D+00
MOMENTS
   1  1  2  3  4
   1  0.2151788D+01 0.2755849D+02 0.4972633D+03 0.1089393D+05
S1MIN,S1MAX= 0.0000E+00 0.4203E+05
S2MIN,S2MAX= 0.0000E+00 0.4203E+05
Energy (GeV) Cross-section (pb)
   205.00     0.6355688D+00
MOMENTS
   1  1  2  3  4
   1  0.3208536D+01 0.5908307D+02 0.1485244D+04 0.4348380D+05

***** This is GENTLE/4fan -- Version 2.0 *****

This is the CC branch of GENTLE/4fan for total cross-sections
FLAGS:IPROC ,IINPT ,IONSUL,IBORNF,IBCKGR,ICHNNL= 1 1 1 0 2
FLAGS:IGAMZS,IGAMW ,ITNONU,IQEDHS,ICOLOMB,ICONV= 1 0 1 2 0
FLAGS:IZERO ,IZETTA,IIQCD ,IDCS ,IANO ,IBIN = 0 1 1 0 0 0
FLAGS:IMAP ,IRMAX ,IRSTP ,IMMIN ,IMMAX = 1 3 3 1 4
EPS1 = 1.0000000000000E-04
S1MIN,S1MAX= 0.0000E+00 0.2592E+05
S2MIN,S2MAX= 0.0000E+00 0.2592E+05
Energy (GeV) Cross-section (pb)
   161.00     0.1332829D+00
MOMENTS
   3  1  2  3  4
   3 -0.3052332D+01 0.2605644D+02 -0.3833234D+03 0.7596405D+04
S1MIN,S1MAX= 0.0000E+00 0.3063E+05
S2MIN,S2MAX= 0.0000E+00 0.3063E+05
Energy (GeV) Cross-section (pb)
   175.00     0.4946375D+00
MOMENTS
   3  1  2  3  4
   3 -0.5666559D+00 0.8922053D+01 -0.9553119D+02 0.1918868D+04

25
First, the program identifies itself and the branch of the code for which output is produced. Then, all relevant flags are printed, before the relative precision \( \text{EPS1} \) of the last computation is given. Finally, physics output is generated. For each center of mass energy the energy and the corresponding total cross-section are printed. Then, in the first column, integers \( k \) indicate the value of \( \text{IREGIM} \) and thus determine the physics quantity for which moments are output. The first line in the moments' output represents the degree \( n \) of the moments in the column below. In the second line, the moment of degree \( n \) for the quantity determined by \( k \) is given. Its value is normalized to the total cross-section.

### 3.4.2 Output of differential cross-sections for CC03

First we give an example of output for \( \text{IBIN} = 0 \), with differential cross-sections computed at fixed values for the scattering angle.

***** This is GENTLE/4fan -- Version 2.0 *****

This is the CC branch of GENTLE/4fan for diff. cross-sections

\[
\begin{align*}
\text{XG} &= 0.100000D+00 & \text{XZ} &= -0.100000D+00 \\
\text{YG} &= 0.000000D+00 & \text{YZ} &= 0.000000D+00 \\
\text{ZZ} &= 0.000000D+00 & \text{DZ} &= 0.500000D-01
\end{align*}
\]

FLAGS: \text{IPROC} , \text{IINPT} , \text{IONSHL} , \text{IBORNF} , \text{IBCKGR} , \text{ICHNNL} = 1 1 1 0 0 2
\]

FLAGS: \text{IGAMZS} , \text{IGAMW} , \text{ITNONU} , \text{IQEDHS} , \text{ICOLMB} , \text{ICONVL} = 1 0 0 1 2 1
\]

FLAGS: \text{IZERO} , \text{IZETTA} , \text{IIQCD} , \text{IDCS} , \text{IANO} , \text{IBIN} = 0 1 1 1 8 0
\]

FLAGS: \text{IMAP} , \text{IRMAX} , \text{IRSTP} , \text{IMMIN} , \text{IMMAX} = 1 3 5 1 1
\]

\( \text{EPS1} = 1.0000000000000000E-04 \)
\begin{verbatim}
S1MIN, S1MAX = 0.0000E+00 0.2592E+05 
S2MIN, S2MAX = 0.0000E+00 0.2592E+05 
Energy (GeV) = 161.0000000000000

\begin{tabular}{ll}
\text{COSW} & \text{Cross-section (pb)} \\
-1.000 & 0.0490713 \\
-0.800 & 0.0536632 \\
-0.400 & 0.0646671 \\
0.000 & 0.0794040 \\
0.400 & 0.1009377 \\
0.800 & 0.1381398 \\
1.000 & 0.1760354 \\
\end{tabular}

S1MIN, S1MAX = 0.0000E+00 0.3063E+05 
S2MIN, S2MAX = 0.0000E+00 0.3063E+05 
Energy (GeV) = 175.0000000000000

\begin{tabular}{ll}
\text{COSW} & \text{Cross-section (pb)} \\
-1.000 & 0.0951087 \\
-0.800 & 0.1123347 \\
-0.400 & 0.1564827 \\
0.000 & 0.2238638 \\
0.400 & 0.3411499 \\
0.800 & 0.5873924 \\
1.000 & 0.8353949 \\
\end{tabular}

S1MIN, S1MAX = 0.0000E+00 0.3610E+05 
S2MIN, S2MAX = 0.0000E+00 0.3610E+05 
Energy (GeV) = 190.0000000000000

\begin{tabular}{ll}
\text{COSW} & \text{Cross-section (pb)} \\
-1.000 & 0.0710393 \\
-0.800 & 0.0891843 \\
-0.400 & 0.1359378 \\
0.000 & 0.2124469 \\
0.400 & 0.3669140 \\
0.800 & 0.7949695 \\
1.000 & 1.3546129 \\
\end{tabular}

S1MIN, S1MAX = 0.0000E+00 0.4203E+05 
S2MIN, S2MAX = 0.0000E+00 0.4203E+05 
Energy (GeV) = 205.0000000000000

\begin{tabular}{ll}
\text{COSW} & \text{Cross-section (pb)} \\
-1.000 & 0.0512161 \\
-0.800 & 0.0678664 \\
-0.400 & 0.1099483 \\
0.000 & 0.1806793 \\
0.400 & 0.3368272 \\
0.800 & 0.8655771 \\
1.000 & 1.7575708 \\
\end{tabular}
\end{verbatim}
After the identification of the active \texttt{GENTLE/4fan} branch, the anomalous couplings (compare page 15 and appendix B) used in the CC03 subprocess are output, before the used flags and the relative precision of the run are printed. Next, below the print of each center of mass energy, the differential cross-sections for IBINNU fixed values of the scattering angle \texttt{COSW} are written to the output.

Below, we present output for IBIN=1, i.e the given differential cross-section values are integrated over bins in the scattering angle. The appearance of the output is identical to the above output for IBIN=0 except that, for each energy, the total cross-section is given right below the prints of scattering angle values and the corresponding cross-sections.

***** This is GENTLE/4fan -- Version 2.0 *****

This is the CC branch of GENTLE/4fan for diff. cross-sections
\begin{verbatim}
XG= 0.10000D+00 XZ= 0.65543D-01
YG= 0.00000D+00 YZ= 0.00000D+00
ZZ= 0.00000D+00 DZ= 0.11957D+00
FLAGS:IPROC ,IINPT ,IONS,IBORN,F,IBCKGR,ICHNNL= 1 1 0 0 2
FLAGS:IGAMZ,IGAMW ,ITNONU,IQEDHS,ICOLMB,ICONVL= 1 0 1 2 1
FLAGS:IZERO ,IZETTA,IIQCD ,IDCS ,IANO ,IBIN = 0 1 1 1 7 1
FLAGS:IMAP ,IRMAX ,IRSTP ,IMMIN ,IMMAX = 1 3 5 1 1
EPS1 = 1.0000000000000000E-04
S1MIN,S1MAX= 0.0000E+00 0.3063E+05
S2MIN,S2MAX= 0.0000E+00 0.3063E+05
Energy (GeV)= 161.0000000000000
\end{verbatim}

\begin{verbatim}
COSW Cross-section (pb)
-1.000 - -0.600 0.020923
-0.600 - -0.200 0.025547
-0.200 - 0.200 0.031763
0.200 - 0.600 0.040919
0.600 - 1.000 0.057338
TOTAL XSEC = 0.176489
\end{verbatim}

\begin{verbatim}
S1MIN,S1MAX= 0.0000E+00 0.3063E+05
S2MIN,S2MAX= 0.0000E+00 0.3063E+05
Energy (GeV)= 175.0000000000000
\end{verbatim}

\begin{verbatim}
COSW Cross-section (pb)
-1.000 - -0.600 0.042294
-0.600 - -0.200 0.060777
-0.200 - 0.200 0.089224
0.200 - 0.600 0.139347
0.600 - 1.000 0.247555
TOTAL XSEC = 0.579198
\end{verbatim}
S1MIN, S1MAX = 0.0000E+00 0.3610E+05
S2MIN, S2MAX = 0.0000E+00 0.3610E+05
Energy (GeV) = 190.0000000000000

COSW Cross-section (pb)
-1.000 - -0.600 0.032453
-0.600 - -0.200 0.051912
-0.200 - 0.200 0.084241
0.200 - 0.600 0.150952
0.600 - 1.000 0.346228
TOTAL XSEC = 0.665786

S1MIN, S1MAX = 0.0000E+00 0.4203E+05
S2MIN, S2MAX = 0.0000E+00 0.4203E+05
Energy (GeV) = 205.0000000000000

COSW Cross-section (pb)
-1.000 - -0.600 0.023983
-0.600 - -0.200 0.041371
-0.200 - 0.200 0.071246
0.200 - 0.600 0.139243
0.600 - 1.000 0.393478
TOTAL XSEC = 0.669320

3.4.3 Output for the 4fan branch

***** This is GENTLE/4fan -- Version 2.0 *****

This is the 4fan branch of GENTLE/4fan
4fan was called for fermions 13 5
with masses 0.500000D-03 0.100000D-01
total cross section is calculated

FLAGS: IPROC, IINPT, IONSHL, IBORNF, IBCKGR, ICHNNL = 2 0 1 0 0 0
FLAGS: IGAMZS, IGAMW, ITNONU, IQEDHS, ICOLMB, ICONVL = 1 0 0 1 0 0
FLAGS: IZERO, IZETTA, IIQCD, IDCS, IANO, IBIN = 1 1 0 0 0 0
FLAGS: IMAP, IRMAX, IRSTP, IMIN, IMAX = 1 0 1 1 1
FLAGS FOR 4FAN: IFERM1, IFERM2, IHIGGS = 13 5 0
EPS1 = 1.0000000000000000E-04
ABS1 = 1.0000000000000000E-04
S1MIN, S1MAX = 0.5805E+04 0.1128E+05
S2MIN, S2MAX = 0.9000E+03 0.2592E+05
3.4.4 Output for the NCqed branch

***** This is GENTLE/4fan -- Version 2.0 *****

This is the NCqed branch of GENTLE/4fan
FLAGS: NINPT,NNCPRC,NGAMZS,NBORNF,NTNONU,NQEDHS
       0 2 1 1 0 0
GFER = 1.166390000000000E-05
AMW = 80.23
AMZ = 91.1888
GAMZ = 2.4974
ALPW = 7.808229874287499E-03
SIN^2(\theta_W) = .2310309124515784
IFERM1,IFERM2 = 2 4
RNCOU1,RNCOU2 = 1.00000000 3.00000000
AM1 ,AM2 = .105658389 4.3
CUTM12,CUTM34 = .211316778 8.6
CUTXPR = .0
EPS1 1.0000000000000000E-05
ABS1 1.0000000000000000E-15
ENERGY (GeV) CROSS-SECTION (fb)
   161.0       .8105861091D+02
   175.0       .6725488126D+02
   190.0       .6303702520D+02
   205.0       .5842607173D+02
Table 3  
*Correspondence between fermion pair indices and fermion types for the NCqed branch of GENTLE/4fan.*

| Fermion Pair Index | 1 | 2 | 3 | 4 |
|--------------------|---|---|---|---|
| Fermion Type       | \(\nu, \nu\) | \(\mu, \tau\) | \(u, c\) | \(d, s, b\) |

After identifying the active branch, the flags relevant to the NCqed branch of the code are printed. The flags are as set in WUFLAG, but the first letter is changed to an \(N\) standing for NCqed. Next, the relevant physical input constants are printed the meaning of which is described in section 3.2. The next paragraph of output begins with the fermion pair indices. The correspondence of fermions and indices is given in table 3. Then, for the two fermion types used in the computation, the color factors (RNCOU1, RNCOU2), the fermion masses (AM1, AM2), and the lower invariant mass cuts (CUTM12, CUTM34) are printed. Closing this paragraph of output, the minimum value CUTXPR of the fraction \(s'/s\) is printed. Finally, together with the relative and absolute required precisions EPS1 and ABS1, the total cross-sections are output for each required center of mass energy.

### 3.5 Use of the program

In this section, we will give a short guide to the compilation and use of GENTLE/4fan.

We recommend to exhaust all means of compiler optimization, because, according to our experience, the CPU time consumption can thus be reduced by a factor of two or more. When available, the use of a compiler option that makes the default size of floating-point constants REAL*8 may be advantageous. For HP computers, we recommend

\[
f77 +O4 +Onolimit +ppu -K -R8 gentle.f .
\]

For Silicon Graphics computers, compilation with

\[
f77 -r8 -O3 -non_shared -G 32 -jmpopt -mips2 -static gentle.f
\]

has proved successful.

Unless the user wants to run GENTLE/4fan with default flags, the user should set the flags described in section 3.2. The setting of a flag IFLAG to a value IVALUE is accomplished by adding the line
CALL WUFLAG('IFLAG',IVALUE)

after the DATA statements in the MAIN of GENTLE/4fan. Please note that the
numerical format of IVALUE is INTEGER. Similarly, for IANO=8, the anomalous
couplings XG, YG, XZ, YZ, ZZ, DG, and DZ may be set by calls like

CALL WUFLAG('XG',VALUE)

at the same place in the MAIN. Please note that the numerical format of VALUE
is DOUBLE PRECISION. In addition, the physical input variables GFER, ALPW,
AME, AMW, AMZ, GAMZ, and ALPHS for the CC branch of the code as well as AMHIG
and GAMHIG for the 4fan branch can be changed by analogous calls to WUFLAG.
Finally, it is possible to change the physics input to the 4fan and NCqed
branches of the code in the subroutines bbmmin and NCIN00 respectively.

To determine the physics quantities and the degrees of moments to be com-
puted and printed, the user must set the range of the loops over IREGIM and
IMOMN (see page 18).

Cuts on the final state invariant pair masses may be applied in the CC and the
4fan branches of the code by altering S1MIN, S1MAX, S2MIN, and S2MAX in the
MAIN. In the NCqed branch, final state invariant pair mass cuts are implemented
by setting X12MIN, X12MAX, X34MIN, and X34MAX in the subroutines NCGENT and
FINC34 (X12 = s1/s, X34 = s2/s).

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A  Numerical Integration in GENTLE/4fan

The integrations over \( s_1, s_2 \) are performed numerically by a successive use of an adaptive one-dimensional Simpson integration routine. One (two) more integrations are needed to calculate initial state QED corrections in the flux function (structure function) approach. Mappings of the integrands may be chosen with flag IMAP. We now describe the mappings.

After the analytical integration over the six angular variables, one has to treat Breit-Wigner resonances arising from the exchange of intermediate bosons:

\[
f_{BW}(V, s) \sim \frac{1}{(s - M^2_V)^2 + \Gamma^2_V M^2_V}, \quad V = Z, W, H. \tag{A.1}
\]

In the case of neutral current processes, additional peaks appear at small momentum transfer due to virtual photons or gluons,

\[
f_{\text{low}}(V, s) \sim \frac{1}{s}, \quad V = \gamma, g. \tag{A.2}
\]

For charged current processes, the standard mapping of the Breit-Wigner resonance of the \( W \) boson is applied in GENTLE/4fan.

In the case of neutral current processes, the peaks at small momentum transfer and the Breit-Wigner resonances of the \( Z \) and the Higgs boson have to be mapped simultaneously. In GENTLE/4fan, the singularity at small momentum transfer is mapped first by the usual transformation to a new variable \( \bar{s} \),

\[
\bar{s} = f_1(s) = \ln s. \tag{A.3}
\]

Then follows a mapping of the \( Z \) resonance, which does not destroy the previous mapping,

\[
\bar{s} = f_2(\bar{s}) = \bar{s} + \frac{c}{[\Gamma_V M_V]} \arctan \frac{\bar{s} - \bar{M}^2_V}{[\Gamma_V M_V]}. \tag{A.4}
\]

The variables \( \bar{M}^2_V \) and \( [\bar{\Gamma}_V \bar{M}_V] \) are calculated taking into account the previous mapping,

\[
\bar{M}^2_V = f_1(M^2_V), \quad [\bar{\Gamma}_V \bar{M}_V] = f_1(M^2_V + \Gamma_V M_V) - f_1(M^2_V). \tag{A.5}
\]

The function \( f_2 \) is inverted numerically by Newton’s Method,

\[
\bar{s}_{i+1} = \bar{s}_i - \frac{f_2(\bar{s}_i)}{f'_2(\bar{s}_i)}, \tag{A.6}
\]

which always converges for the starting value \( \bar{s}_0 = \bar{M}^2_V \). The free constant \( c \)
in equation (A.4) is optimized for every final state separately. However, the quality of the mapping (A.4) is sensitive to the order of magnitude of $c$ only. In the case of Higgs production, a procedure similar to (A.4) is added to map the Higgs resonance.

B Anomalous Triple-Boson Couplings

In addition to the SM Lagrangian, we consider the following CP conserving operators of dimension 6

\[ \Delta \mathcal{L}_{\text{eff}} = g \frac{\mathcal{A}_{B\phi}}{m_W^2} (D_\mu \Phi) \dagger B^{\mu\nu} (D_\nu \Phi) + g \frac{\mathcal{A}_{W\phi}}{m_W^2} (D_\mu \Phi) \dagger \frac{\nabla}{\tau} W^{\mu\nu} (D_\nu \Phi) + g \frac{\mathcal{A}_W}{6m_W^2} \bar{W}_\nu \left( \bar{W}_\rho \times \bar{W}_\mu \right). \] (B.1)

In the unitary gauge, these operators lead to the following effective Lagrangian for the $WWV$ vertex:

\[ \mathcal{L}_{\text{eff}}^{WWV} = ig_{WWV} \left[ g_1^V \left( W^{+\mu} W^{-\mu} - W^{+\mu} W^{-\mu} \right) V^\nu + \kappa_V W^{+\mu} W^{-\mu} V^\nu + \frac{\lambda_V}{m_W^2} W^{+\nu} W^{-\rho} V^{\mu} \right], \] (B.2)

where $V \in \{Z, \gamma\}$. Therefore, one has six relevant anomalous triple-boson couplings, namely $g_1^Z$, $g_1^\gamma$, $\kappa_\gamma$, $\kappa_Z$, $\lambda_\gamma$, and $\lambda_Z$. Electromagnetic gauge invariance requires $g_1^\gamma = 1$. In the HISZ-scenario [20] (IANO = ±7), one sets $\mathcal{A}_{B\phi} = \mathcal{A}_{W\phi}$. Thus, in this scenario, one obtains relations between the parameters $\kappa_\gamma$, $\kappa_Z$, and $g_1^Z$.

For the $WWZ$-vertex, we also include a C and P violating term in the Lagrangian:

\[ \mathcal{L}_Z = \frac{eZ}{m_W^2} \partial_\alpha \hat{Z}_{\rho\sigma} \left( W^{+\alpha} \partial^\rho W^{-\sigma} - W^{+\sigma} \partial^\rho W^{-\alpha} \right), \]

where we have introduced the dual field strength tensor

\[ \hat{Z}_{\rho\sigma} = \frac{1}{2} \epsilon_{\rho\sigma\alpha\beta} Z^{\alpha\beta} \].

To disentangle the anomalous couplings' contributions to the cross-section, we

\footnote{For definitions of anomalous couplings, see e.g. [22,16].}
use redefined parameters:

\[ \delta_Z = (g_1^Z - 1) \cot \theta_W, \]  
\[ x_\gamma = \kappa_\gamma - 1, \]  
\[ x_Z = (\kappa_Z - 1)(\cot \theta_W + \delta_Z), \]  
\[ y_\gamma = \lambda_\gamma, \]  
\[ y_Z = \lambda_Z \cot \theta_W, \]  
\[ z_Z = z_Z. \]

(B.3)  
(B.4)  
(B.5)  
(B.6)  
(B.7)  
(B.8)

The GENTLE/4fan user can set values for the anomalous couplings defined in equations (B.3) to (B.8). In the SM, the anomalous couplings (B.3) to (B.8) vanish. The special case of anomalous couplings induced by a heavy extra neutral gauge boson is described in appendix C.

C Treatment of Extra Neutral Gauge Bosons

The effect of an extra neutral gauge boson $Z'$ can be described with GENTLE/4fan by two anomalous couplings $g_{WW\gamma}$ and $g_{WWZ}$ of the photon and the Standard Model $Z$ boson to $W$ pairs [23],

\[ g_{WW\gamma}^* = 1 + \delta_\gamma \] and \[ g_{WWZ}^* = \cot \theta_W + \delta_Z, \]

(C.1)

where

\[ \delta_\gamma = g_{WWZ_1} \left( \frac{a_1}{a} - \frac{v_1}{v} \right) v(1 + \Delta \chi) \chi_Z + g_{WWZ_2} \left( \frac{a_2}{a} - \frac{v_2}{v} \right) v \chi_2; \]
\[ \delta_Z = -\cot \theta_W + g_{WWZ_1} \frac{a_1}{a}(1 + \Delta \chi) + g_{WWZ_2} \frac{a_2 \chi_2}{a \chi_Z}, \]

(C.2)

and

\[ \chi_Z = \frac{s}{s - M_Z^2}, \quad \Delta \chi = -\frac{2M_Z(M_Z - M_1)}{s - M_Z^2}, \quad \chi_2 = \frac{s}{s - M_2^2}. \]

(C.3)

In equations (C.2) and (C.3), $M_i, v_i, a_i$, $i = 1, 2$ denote the masses and electron couplings of the mass eigenstates $Z_1$ and $Z_2$, which are, in general, a result of a mixing of the gauge symmetry eigenstates $Z$ and $Z'$.
\[ Z_1 = Z \cos \theta_M + Z' \sin \theta_M, \quad Z_2 = -Z \sin \theta_M + Z' \cos \theta_M, \]
\[ v_1 = v \cos \theta_M + v' \sin \theta_M, \quad v_2 = -v \sin \theta_M + v' \cos \theta_M, \]
\[ a_1 = a \cos \theta_M + a' \sin \theta_M, \quad a_2 = -a \sin \theta_M + a' \cos \theta_M, \] (C.4)

where \( v, a (v', a') \) denote the couplings of the symmetry eigenstates \( Z (Z') \) to electrons and \( \theta_M \) is the \( ZZ' \) mixing angle. The couplings \( g_{WWZ_1} \) and \( g_{WWZ_2} \) are fixed by the condition that only the \( Z \) couples to \( W \) pairs,

\[ g_{WWZ_1} = \cot \theta_W \cos \theta_M, \quad g_{WWZ_2} = \cot \theta_W \sin \theta_M. \] (C.5)

The mass shift from \( M_Z \) to \( M_1 \) in the \( Z_1 \) propagator is absorbed into \( \delta_\gamma \) and \( \delta_Z \) through \( \Delta \chi \) assuming \( M_Z - M_1 \ll M_Z \) and \( M_Z - M_1 \ll (s - M_Z^2)/(2M_Z) \). Both approximations are equivalent for \( W \) pair production.

### D Options of GENTLE

Table D.1 shows the observables GENTLE can calculate and the options, which may be chosen. Please note that in the CC11 family anomalous couplings are included only for the CC03 process.

| \( \sigma \) | IREGIM | on-shell | Born | SF | FF | h.o. | Coul. | non-univ. | anom.c. |
|-------------|--------|---------|------|----|----|------|-------|-----------|--------|
| \( \sigma_{CC03} \) | 0 | + | + | + | + | + | + | + | + |
| \( \sigma_{CC03} \) | 1 | + | - | + | + | + | + | + | + |
| \( \sigma_{CC03} \) | 2 | + | - | - | - | + | - | - | + |
| \( \sigma_{CC03} \) | 3 | + | - | - | + | + | + | + | + |
| \( \sigma_{CC03} \) | 4 | + | + | + | - | + | - | - | - |
| \( \frac{d\sigma}{d\cos \theta} \) | - | + | + | + | - | + | - | + | - |
| \( \sigma_{CC11} \) | 0 | - | + | + | + | + | - | - | + |
| \( \sigma_{CC11} \) | 1 | - | - | + | + | + | - | + | + |
| \( \sigma_{CC11} \) | 2 | - | - | + | + | + | - | + | + |
| \( \sigma_{CC11} \) | 3 | - | - | + | + | + | - | + | + |
| \( \sigma_{NC02} \) | - | - | + | + | + | - | + | - | - |
| \( \sigma_{NC08} \) | - | - | + | + | + | - | + | - | - |
| \( \sigma_{NC32} \) | - | - | + | + | + | - | + | - | - |

Table D.1

Options of GENTLE; SF - structure function approach; FF - flux function approach; h.o. - higher order QED corrections; Coul. - Coulomb singularity.
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