Research on design method of double-loop electric servo loading system with high-frequency band based on $H^\infty$ control strategy

Yang Shui, Jianli Wei and Jie Yan

Abstract
In the hardware-in-the-loop simulation, the goal of electric loading is to realize the accurate tracking of the torque signal and test the performance of the aircraft actuator system. For some high dynamic aircraft, it is necessary to reduce the influence of the surplus torque to increase the system frequency band. This paper introduces a new electric loading system which adopts a double-loop servo motor as the torque loading mechanism. It applies two loops to track the position of the rudder and the aerodynamic load spectrum respectively. For the purpose of reducing the disturbance between two loops of the scheme, a two-DOF $H^\infty$ robust controller is designed, which improves the robustness of the system effectively. The simulation results show that the new system increases the upper limit of 25 Hz frequency band of the traditional single-loop system with PID control to the maximum of 40 Hz. The double-loop system thereby meets the technical requirements of the hardware-in-the-loop simulation experiment for high dynamic aircrafts.

Keywords
Electric loading system, high frequency band, surplus force, double-loop structure, $H^\infty$ control

Date received: 8 August 2020; accepted: 8 September 2020

Introduction
Loading system is one of the main equipment for hardware-in-the-loop simulation. It is used to simulate the aerodynamic load on the aircraft control surface under laboratory conditions and test the performance of the aircraft rudder system. The aerodynamic torques acting on the aircraft control surface during flight can be further decomposed into corresponding load spectra. From the engineering point of view, the frequency band of the actual loading system must be higher than that of the load spectrum in order that the system can accurately reproduce the aerodynamic load on the control surface. In engineering, the frequency band of the loading system is usually defined by the maximum operating frequency band that meets the “dual-ten” index, that is, the amplitude error between the torque instruction and the output torque is less than 10%, and the phase difference is less than 10°. Since the beginning of this century, the technology of electric loading system with the frequency band of 20 Hz has been relatively mature. However, with the development of modern air warfare, higher speed requirements are put forward for air-to-air missiles. The torque frequency acting on some small rudders has reached more than 30 Hz. The current loading schemes cannot fully meet the test requirements of high performance systems.

In practice, the displacement output generated by the rudder system and the torque output generated by the loading system will be coupled with each other. This coupling action causes redundant and unexpected mechanical disturbance in the servo motion, which is called “surplus force” in academic circles. Surplus force causes a serious impact on the accuracy, frequency response and stability of the loading system. It is necessary to introduce an effective control method to suppress it.

In general, the restraint of surplus force can be carried out from two aspects, including the selection of system structure and the improvement of system control methods. For system selection, the literature

Northwestern Polytechnical University, Xi’an, P. R. China

Corresponding author:
Yang Shui, Northwestern Polytechnical University School of Astronautics, 127 West Youyi Road, Xi’an, 710072, P. R. China.
Email: synaoh@qq.com

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
introduces the development and current situation of the electric loading system, and points out that at present its actuators mostly adopt single-loop permanent magnet motors. Literature\textsuperscript{11} introduces an elastic element called spring rod into the system, which reduces the surplus force of the system and improves the loading performance from the structural point of view. In reference,\textsuperscript{12} permanent magnet synchronous motor is used as a matched loading motor, and the influence of the mechanical inertia and driving mode of the motor on the system frequency band is discussed. Literature\textsuperscript{13} proposes a low speed and high torque loading scheme using double-stator motor to reduce the influence of surplus force on the loading motor. Nevertheless, the surplus force cannot be completely suppressed due to the synchronous stator’s own electrical inertia. The suppression rate for surplus force is about 90\% at 10 Hz.

In terms of control strategy, literature\textsuperscript{14,15} respectively introduce AC electric loading systems with advanced PID control and fuzzy self-adaptive control, and the operating frequency band reaches 20 Hz when the “dual-ten” index is met. The $H\infty$ controller designed in literature\textsuperscript{16} suppresses the surplus force of the rudder. However, due to the special algorithm and uncertainty of the allowable range, it is complicated to calculate the ideal single-DOF optimal $H\infty$ parameters in engineering.

Traditional single-loop loading systems are designed on the basis of torque index, without paying attention to the position index. They are affected by the surplus torque generated by the position disturbance, which results in a great limitation of system frequency band. In view of the limitation of the single-loop structure, a double-loop loading system is introduced in this paper. The scheme attempts to track the position output of the rudder by using an independent position loop to reduce the influence of the position disturbance on the torque loop. With the double-loop structure, passive loading tracking is approximately converted into active loading. It is where the innovation of this paper lies. In addition, a $H\infty$ controller is designed to reduce the coupling force between two loops and improve the robustness of the system. If the system designed in this paper is successfully applied, it will effectively suppress the surplus force, thus improving the frequency band of the loading system, which is of great significance to the high frequency band actuator loading in hardware-in-the-loop simulation.

Mathematical model of electric loading system

System mathematical model

A typical structural diagram of the electric loading system is as shown in Figure 1, in which, the servo motor is the actuator of the loading system and applies the load according to the torque instruction. The spring rod is used to adjust the specific connection stiffness coefficient between the bearing object and the loading system to dominate the surplus force.\textsuperscript{17} The torque sensor is used to measure the torque applied by the loading system on the rudder and form a closed torque loop. The rotary encoder is used to measure output signals such as the position and speed of the rudder. The rotary inertia simulation mainly simulates the rotary inertia of the control surface so that the load on the rudder is close to the flight state.

The loading system adopts a closed torque loop control mode. The control computer receives the instruction torque signal and outputs it to the servo motor drive controller. Meanwhile, it receives the feedback signal from the torque sensor and the rotary encoder to form a closed torque loop so that the system can apply the loading torque to the rudder according to the instruction in order to realize the torque servo control.\textsuperscript{18}
By analyzing the operating principle of the loading servo motor and the generation mechanism of torque, a mathematical model of the loading system is obtained, as shown in Figure 2.

Where $V_{in}$ is the output voltage of the control system, $K_V$ is the equivalent amplification factor of the preamplifier, $K_P$ is the equivalent amplification factor of the driver, $K_s$ is the equivalent gain of the drive loop, $R$ and $L$ are the operating inductance and resistance of the servo motor, respectively, $K$ is the torque coefficient of the motor, $J$ is the back electromotive force constant of the motor, $D$ and $J_s$ are the rotary inertia and friction coefficient of the motor, respectively, $\theta_t$ is the deflection angle of the motor output, $\theta_m$ is the deflection angle of the rudder, and $T_L$ is the actual output torque of the loading system.

### Analysis on surplus force

By analyzing the mathematical model of the loading system, the actual torque output $T_L(s)$ can be obtained as follows.

$$T_L(s) = G(s) \cdot V_{in}(s) \cdot N(s) \cdot \theta_m(s)$$  \hspace{1cm} (1)

Where $G(s)$ is the equivalent transfer function of the loading system.

$$N(s) = -\frac{s[(R + Ls)(J_s + D) + K_sK_l]}{s[(R + Ls)(J_s + D) + K_sK_l] + K_L(R + Ls)}$$  \hspace{1cm} (2)

The minus in $N(s)$ indicates that the torque direction is opposite to the deflection angle direction of the rudder. Obviously, the torque output corresponding to the rudder deflection angle $\theta_m(s)$ is the surplus force, which affects the actual torque output $T_L(s)$ of the loading system, especially in high frequency loading. Usually, feedforward compensation and decoupling compensation are used to compensate for this effect, while the efficacy is limited. The reason is that the servo motor is disturbed by the position motion of the rudder. On the other hand, the torque generated by the servo motor also causes position disturbance on the rudder’s motion. Therefore, it is necessary to start with the structure in order to decouple the position servo loop and the force servo loop in the loading process.

### Structure and mathematical model of double-loop loading system

**Structure of double-loop loading system**

The main factor affecting the performance of the loading system is the surplus force, which is generated by the motion of the bearing object. The motor of the loading system needs not only to apply load according to the torque instruction, but also to quickly track the motion of the rudder. If the loading motor is divided into two parts, one part is used to carry out loading and the other part is used to follow the motion of the rudder, the surplus force caused by the position disturbance can be minimized. Under the ideal condition that the position output of the system completely follows the loaded rudder, the surplus force will be completely eliminated.

Therefore, the paper puts forward the idea of double-loop loading system. Its schematic structural diagram is as shown in Figure 3.

The structural difference between the double-loop loading system and the single-loop loading system is that the motor uses double rotors, which are respectively called position rotor and torque rotor in this paper. The position rotor receives the position instruction of the loaded rudder to form a closed position loop synchronized with the motion of the rudder. The torque rotor is connected with the position rotor through an elastic connection element and moves along with the position rotor. In which, the function of the elastic element is to connect the torque rotor with the position rotor, so that the position rotor can drive the torque rotor to rotate. At the same time, buffer is added between the position rotor and the torque rotor to reduce the influence of the tracking error of the position loop on the torque loop.

The basic idea of eliminating surplus force is that if the position rotor in Figure 3 can move synchronously with the loaded rudder, the motion of the loaded rudder will not cause additional disturbance to the torque loop in theory. That is to say, when the expected loading torque of the torque rotor angular position motion of the loading motor is zero, and it is synchronous with the rudder angular position motion, there will be no extra force caused by the rudder motion.

The instructions of the double-loop loading system include two items, one of which is the loading...
Establishment of mathematical model

Mathematical model of position loop. Based on the analysis on the mechanism of the double-loop loading system, the main function of the position loop motor in the system is to drive the torque loop motor to move synchronously with the rudder, making it a typical position servo system. Its rotary inertia is equal to the sum of the rotor rotary inertia of a position loop motor and the rotary inertia of a torque loop motor. Therefore, the open loop transfer function of the position loop can be obtained as follows.

\[
\frac{\theta_s}{\mu_0} = \frac{K_{am}K_j}{s(L_sJ_s^2 + R_sJ_s + K_{se}K_{sm})}
\]  

(3)

Where \(\mu_0\) is the input voltage, \(L_s\) is the armature inductance of the position motor, \(R_s\) is the armature resistance of the position motor, \(K_{am}\) is the torque coefficient, \(K_{se}\) is the back electromotive force coefficient, \(K_j\) is the output reduction ratio, and \(J_s\) is the rotary inertia of the load.

The transfer function from the disturbance torque \(T_f\) to the output angle of the position loop \(\theta_s\) is

\[
\frac{\theta_s}{T_f} = \frac{(R_s + L_s\mu_0)K_j^2}{s(L_sJ_s^2 + R_sJ_s + K_{se}K_{am})}
\]

(4)

Mathematical model of torque loop. The torque loading loop is realized by a torque rotor installed on the position rotor. A DC brushless motor is adopted as the loading motor here due to its fast dynamic response,
high flux density and power factor. Unlike traditional loading systems, the loading motor of the torque loop in the double-loop loading system moves synchronously with the rudder along with the motion of the position loop. At the same time, its output shaft is connected with the rudder through a torque sensor to apply the desired load torque. Therefore, the torque between the loading motor and the rudder is

\[ T_l = K_t D_u = K_t \left( \frac{u_l}{C_0} D_u s \right) \]  

Where \( K_t \) indicates the stiffness coefficient of the torque sensor, and \( D_u \) indicates the deformation angle difference of the torque sensor.

Under the traditional loading mode, this angle difference refers to the angle difference between the servo angle and the loading motor angle. While in the system studied in this paper, the value of this angle difference is equal to that of angle difference between the loading motor angle \( \theta_i \) and the synchronous motion angle difference \( \Delta \theta_s \) of position motor and the rudder. Under the action of the driving current of the loading motor, the following balance relationship exists between the loading motor and the rudder:

\[ K_{lm} i = J_l v_r + B v_r + T_l : \]  

Thus, the mathematical model of torque loop can be derived as follows:

\[ T_f = \frac{K_e K_c K_r K_{lm} K_t}{s[(R_t + L_t s)(J_s + B) + K_{se} K_{sm}]} V_c - \frac{K_e K_c K_r K_{lm} K_t}{s[(R_t + L_t s)(J_s + B) + K_{se} K_{sm}]} \Delta \theta_s \]  

System characteristic analysis. In combination with the control block diagrams of the position loop and torque loop, the overall working mechanism model of the double-loop synchronous loading system is obtained as shown in Figure 5.

From the general block diagram, it can be seen that the double-loop synchronous loading system is a kind of multivariable control system. The input signal of this system includes the actual position signal \( \theta_0 \) and the torque instruction signal \( T_0 \) of the rudder, and the output signal includes the angle \( \theta_s \) of the synchronous motor and the actual load torque \( T_l \). Among these parameters, \( \theta_0 \) directly affects the torque loop of traditional single-loop loading systems, causing serious position disturbance. While the position loop of the double-loop loading system can follow \( \theta_0 \) synchronously, making \( \Delta \theta_0 \) acting on the torque loop as small as possible, so as to reduce the influence of the surplus force acting on the system.

There are two coupling relationships between the two loops. (1) the position difference between the angle output by the position loop and the rotation angle of the rudder is coupled to the torque loop, and (2) the loading torque of the torque loop is coupled to the position loop through the speed reduction ratio. The above two coupling relationships make the position loop and torque loop interfere with each other, generating the surplus force which restrict the system performance. Therefore, they should be improved.

The simplified block diagram of double-loop synchronous loading system is as shown in Figure 6.

Let \( F_{11} = C_1(s) K_{sm} \) and \( F_{22} = C_1(s) K_{sm} \) respectively represent the equivalent transfer functions of the position loop controller and the torque loop controller, \( G_{11} = 1/D_1(s) = 1/[L_i J_s s^3 + R_i J_s s^2 + K_{se} K_{sm} s + C_1(s) K_{sm}] \) represent the reciprocal of the characteristic polynomial of the position loop,

\[ G_{22} = 1/D_2(s) = 1/[L_i J_s s^3 + (R_i J_s + L_i B) s^2 + (L_i K_t + BR_i + K_{se} K_{sm} s + K_{lm} K_t)] \] represent the reciprocal of the characteristic polynomial of the torque loop.
The system controller design

In the mechanism model obtained from the above section, for this kind of double-input/output synchronous system, the core design index includes reducing the coupling of the position loop to the torque loop and improving the precision and dynamic range of the torque loop. In this section, $H\infty$ optimization method is employed to design the control system of the double-loop synchronous loading system, and the anti-interference capacity and tracking capability of the system are improved through weighting function.

Definition of two-DOF $H\infty$ controller

When designing the control system, the motor of the position loop is required to quickly and accurately track the angular position output of the rudder, so the control system is designed by adopting a two-DOF controller. The design of traditional single-DOF controller has contradictions in signal tracking and disturbance attenuation of the system. Two indexes cannot be optimized at the same time. The two-DOF controller can control the tracking ability and anti-interference ability respectively rather than single-DOF controller, so that the system performances are significantly improved.

The structure chart of two-DOF $H\infty$ controller is as shown in Figure 7.28

Where $G(s)$ is the nominal model, $H(s)$ indicates the expected reference model, and $\Delta$ is the multiplicative output uncertainty. Denote

$$G_d(s) = G(s)(1 + \Delta)$$

In the two-DOF control, the controller $K_d(s)$ is used to improve the system stability, and the feedforward controller $K_f(s)$ is used to directly introduce the reference input into the controlled object to speed up the system tracking response. The tracking error $e$ is used to evaluate the tracking effect, $r$ indicates the input
The controller output, and z is the evaluation output of the system P(s).

\[
\begin{bmatrix}
    z(s) \\
    v(s)
\end{bmatrix} = P(s) \begin{bmatrix}
    w(s) \\
    u(s)
\end{bmatrix}
\]  

(16)

\[
 u(s) = K(s)v(s) = \begin{bmatrix}
    K_f(s) \\
    K_b(s)
\end{bmatrix} v(s)
\]  

(17)

\[
P(s) = \begin{bmatrix}
    [-H & I] \\
    [0 & 0] & G
\end{bmatrix}
\]  

(18)

\[
w = \begin{bmatrix}
    r \\
    q
\end{bmatrix}, 
 z = \begin{bmatrix}
    e \\
    q
\end{bmatrix}, \quad v = \begin{bmatrix}
    r \\
    y
\end{bmatrix}
\]  

(19)

Where q is the output of the model G(s). The transfer function from w to z is expressed as

\[
T_{zw}(s) = lft(P, K)
\]  

(20)

Substituting (15) to (18) into (19), it can be obtained that

\[
T_{zw}(s) = \begin{bmatrix}
    (I - G_Kb)^{-1}G_Kf - H & (I - G_Kb)^{-1} \\
    (I - G_Kb)^{-1}G_Kf & G_Kb(I - G_Kb)^{-1}
\end{bmatrix}
\]  

(21)

For the generalized system presented in Figure 8, the controller K(s) can be solved by standard H∞ control method to satisfies

\[
\|T_{zw}(s)\|_{\infty} < \rho
\]  

(22)

According to the small gain theorem, \(\rho\) if \(\|T_{zw}(s)\|_{\infty} < \rho\), the controller K(s) guarantees the stability and robustness for \(\|\Delta\|_{\infty} < \rho^{-1}\) of the two-DOF control system presented in Figure 7.

**Two-DOF controller design**

The solved problem of a two-DOF robust controller according to equation (16) can be expressed as Figure 9.

The external disturbance of the system is expressed as

\[
W = \begin{bmatrix}
    R \\
    D
\end{bmatrix}
\]  

(23)

Where \(R = [r_t, r_0]^T\), among which \(r_t\) indicates the torque loading instruction, and \(r_0\) indicates the position instruction of the rudder. \(D = [d_t, d_0]^T\), among which \(d_t\) indicates the torque disturbance, and \(d_0\) indicates the position disturbance.

The evaluation output of the system is expressed as

\[
Z = \begin{bmatrix}
    E \\
    Q
\end{bmatrix}
\]  

(24)

Where \(E = [e_t, e_0]^T\), among which \(e_t\) indicates the difference between the torque output and reference model output, and \(e_0\) indicates the difference between the position output and reference model. \(Q = [q_t, q_0]^T\), among
which $q_t$ indicates the output of the torque, and $q_\theta$ is the output of position tracking.

Controller input is expressed as

$$Y_k = \frac{R}{C_20/C_21}$$

Where $Y = [y_t, y_\theta]^T$, among which $y_t$ indicates the loading torque output, and $y_\theta$ indicates the position tracking output of the rudder.

The generalized control object solved by the controller can be expressed as

$$P = \frac{1}{C_0/C_0 \cdots C_0} W_E \cdot G K_b$$

When the controller is designed as $K(s)$,

$$U = KY_k = [K_b, K_f] \begin{bmatrix} R \\ Y \end{bmatrix}$$

The closed-loop transfer function of the generalized control model is expressed as

$$T_{zw} = \lft(P, K) = \begin{bmatrix} W_E (I - G K_b)^{-1} G K_f - H \\ (I - G K_b)^{-1} G K_f \\ W_T G K_b (I - G K_b)^{-1} \end{bmatrix}$$

The controller $K(s)$ is obtained using the standard $H_\infty$ controller solving method, so that

$$\|T_{zw}\|_\infty = \begin{bmatrix} W_E (I - G K_b)^{-1} G K_f - H \\ (I - G K_b)^{-1} G K_f \\ W_T G K_b (I - G K_b)^{-1} \end{bmatrix} < \rho$$

Thereby ensuring the robustness of the system.

With the previous analysis, it is concluded that the double-loop synchronous loading control system is a two-input and two-output control system, and there is mutual coupling interference among different control channels. The tracking precision of the control system is influenced by the coupling effect of the system. Therefore, when designing the controller, it is necessary to consider the coupling inhibition of the system. The transfer function of the double-loop synchronous loading control system from the instruction to the system tracking output is $T_{zw}$. This closed-loop transfer function can be expressed in blocks as follows.

$$T_{zw} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

$T_{12}$ indicates the coupling disturbance of the position input to the torque loading, and $T_{21}$ indicates the coupling disturbance of the torque input to the synchronous position output of the rudder. The problem of control system disturbance attenuation is to design the controller $K$ to reduce the influence of $T_{12}$ and $T_{21}$ on the system. Therefore, an ideal decoupled closed-loop control system is shown as:

$$T_{zw} = \begin{bmatrix} T_{11} & 0 \\ 0 & T_{22} \end{bmatrix}$$

During the design of two-DOF model reference $H_\infty$ controller, the reference model $H(s)$ is expressed as:

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

The goal of controller solution is to minimize $\|T_{zw} - H\|_\infty$, that is:

$$\begin{bmatrix} T_{11} - H_{11} \\ T_{21} - H_{21} \\ T_{22} - H_{22} \end{bmatrix} < \rho$$

When designing the reference model, it is necessary to set the non-diagonal elements in $H$ to zero to ensure the disturbance attenuation of the system coupling. At the same time, the element $H_{11}$ on the diagonal line represents the expected transfer function from the loading instruction to the torque output, and $H_{22}$ represents the expected transfer function for position tracking of the position motor. The function of the position motor is 

Figure 9. Schematic diagram for design structure of two-DOF $H_\infty$ controller.
to quickly track the rudder position. For the same reference instruction input, it is expected that the output of the position motor is as small as the output of the rudder. Therefore, $H_{22}$ can be designed according to the actual rudder model.

To meet the requirement that the torque loading system has a frequency larger than 40 Hz under the dualten condition, $H_{11}$ is designed into a second-order link with a natural frequency slightly larger than 40 Hz and a damping ratio of 0.7.

The non-diagonal elements of the reference model $H$ are set to 0, for inhibiting the coupling between the two channels in the control system.

$$H = \begin{bmatrix} \frac{(82\pi)^2}{s^2 + 114.8\pi s + (82\pi)^2} & 0 \\ 0 & \frac{(30\pi)^2}{s^2 + 48\pi s + (30\pi)^2} \end{bmatrix}$$  \hspace{1cm} (34)$$

For the selection of weighting function $W_c(s)$, the tracking performance requirements of the system are mainly considered. The low-pass filters with high gain in low frequency band are generally selected. The position motor needs to quickly track the position output of the servo, thereby reducing the interference torque caused by the position difference between the rudder and the torque motor. Therefore, when choosing the weighting function, the attenuation frequency band of the position error signal is larger than that of the torque error signal, and meanwhile, the allowable peak value of the position error signal transfer function is larger than that of the torque error transfer function. The tracking precision is required to be within 5%. The weighting function is selected according to the above requirements $W_c(s)$.

$$W_c = \begin{bmatrix} \frac{0.375s + 100}{s + 1} & 0 \\ 0 & \frac{0.531s + 100}{s + 1} \end{bmatrix}$$  \hspace{1cm} (35)$$

Usually, the inductance, resistance and counter electromotive force of the model in the motor cannot be accurately obtained, and the transformation of these parameters affects the accuracy of the model. When modeling the disturbance, these parameter perturbations are uniformly transformed into the output multiplicative uncertainties of the system. \hspace{1cm} (36)$$

$$G_d = (I + \Delta)G_0$$

$G_0$ indicates the nominal model of the system, $G_d$ indicates the disturbance model, and $\Delta$ indicates the multiplicative uncertain disturbance. To ensure the robustness of the system, it is also necessary to estimate the boundary of multiplicative uncertain disturbance $\Delta$. The output multiplicative uncertainty can be expressed as:

$$\Delta = (G_d - G_0)G_0^{-1}$$  \hspace{1cm} (37)$$

Then

$$\sigma(\Delta) = \sigma((G_d - G_0)G_0^{-1})$$  \hspace{1cm} (38)$$

The parameter samples are randomly selected in the parameter disturbance set and substituted into the nominal model of the system to form a system model set with parameter perturbation. All elements in the model are substituted into equation (31), the singular values are calculated, and the curves are drawn, as shown in Figure 10. When calculating the uncertain model, the disturbance range of inductance, resistance and counter electromotive force is $\pm20\%$. The weighting function $W_T$ is designed according to the boundary of the perturbation curve in the figure. \hspace{1cm} (39)$$

$$W_T = \begin{bmatrix} \frac{s + 6.32}{0.01s + 5.5} & 0 \\ 0 & \frac{s + 6.28}{0.01s + 0.51} \end{bmatrix}$$

**Verification of two-DOF $H\infty$ controller**

The controller $K(s)$ is substituted into the double-loop loading control model for numerical simulation, and the 40 Hz sinusoidal loading tracking curve of the nominal closed-loop control system is provided.

Figures 11 and 12 show that the position motor in the double-loop synchronous loading system with the two-DOF $H\infty$ controller tracks the angular position of the rudder quickly and accurately, thereby reducing the surplus force caused by the angular position difference between the loading motor and the rudder. Therefore, the torque loop can well track the sinusoidal torque with a frequency of 40 Hz.

**System simulation verification**

The digital simulation method is used to compare the designed controller with the traditional composite
controller in terms of control stability and torque loading accuracy. The relevant main performance indicators of the loading system are designed as follows:

1. Maximum loading torque: 60 Nm;
2. Surplus force suppression ratio: 6%;
3. Frequency band: Under the rudder disturbance, the loading system should meet the following conditions:

\[
\Delta A = \left| \frac{M_o(f) - M_i(f)}{M_i(f)} \right| \leq 10\%,
\]

\[
\Delta \phi = |\phi_o(f) - \phi_i(f)| \leq 10^\circ,
\]

\[
f_{TI} \leq 40\text{Hz}, \quad \text{Amp}_{\text{rudder}} \leq 30^\circ, \quad f_{\text{rudder}} \leq 25\text{Hz}
\]

Where \(M_o(f)\) and \(M_i(f)\) are the amplitude of output torque and instruction torque of the loading system respectively, \(\phi_o(f)\) and \(\phi_i(f)\) are phases of output torque and the instruction torque of the loading system respectively. \(f_{TI}\) is the frequency of the instruction torque, \(\text{Amp}_{\text{rudder}}\) is the maximum deflection angle, and \(f_{\text{rudder}}\) is the maximum motion frequency of the rudder. \(\Delta A\) and \(\Delta \phi\) refer to the dual-ten indicators in actual engineering.

**Verification of surplus force suppression capability**

When the torque exerted by the loading system on the output shaft of the rudder is a constant torque, the loading system should be able to effectively overcome the effect of surplus force generated by the movement of the rudder. This surplus force suppression capability allows the output torque to maintain at a stable value.

The simulation is set that the torque instruction is a ramp signal with a maximum value of 60 Nm. Once the loading system applies torque, the rudder keeps a sinusoidal motion with a rudder deflection angle of 1.5° and a frequency of 25 Hz, and then the ability of the two loading modes to suppress the surplus force is examined. The torque loading curve is as shown in Figure 13.

As a comparison, the loading curve of traditional composite control loading system under the same conditions is shown in Figure 14:

As shown in Figures 13 and 14, when the frequency of the rudder increases up to the limit (i.e. 25 Hz), the influence on the double-loop loading system is relatively small. The main reason is that the double-loop synchronous loading system suppresses the surplus force through the position synchronization loop. The bandwidth of the position motor is larger than that of the rudder, thus the position motor can better follow the position of the rudder for movement and reducing the interference of the rudder movement to the torque loading, and the interference ratio of the surplus force is 5%. On the contrary, when the loading system with
composite control is moving at high frequency, the composite control method performs poorly in this case for reasons that the main component of the surplus force is caused by the acceleration speed of the rudder, and that there is no direct observation of the acceleration speed in the system and that the acceleration speed signal obtained by the way of differential and filtering will have noise and phase lags at the same time. The interference ratio of the surplus force is about 30%.

**Verification of torque loading frequency band**

The torque output should rapidly response the changes of the torque instruction in order to realize the torque loading with a high frequency band. The simulation conditions are set as follows: the torque instruction is sinusoidal, the torque amplitude is 60 Nm with the frequency of 40 Hz, the rudder motion is 1.5° with the frequency of 25 Hz. The simulation results are as shown in Figure 15:

It can be seen from Figure 15 that the position motor follows the angle movement of the rudder precisely, with an angular difference of about 0.03°. Under such position synchronous conditions, the torque loading bandwidth can reach 40 Hz. Meanwhile the amplitude difference is 5.8%, with a phase difference of 6.8°. The system meets the dual-ten indicators.
Table 1. Hardware list of test system.

| Device name            | Device model                        |
|------------------------|-------------------------------------|
| Industrial control computer | Advantech 610H                         |
| Motor controller       | Metronix—ARS 2310FS                  |
| Torque sensor          | HEIDENHAN ECN 125 2048 ISS08-C4      |
| Rotary encoder         | HEIDENHAN ECN 125 2048 ISS08-C4      |
| Spectrum analyzer      | CF-920                               |

Verification of physical system

The test verification system mainly includes double-loop DC brushless motor, motor controller, spring rod, torque sensor, rotary encoder, analog actuator, analog actuator driver, strain amplifier, industrial control computer, spectrum analyzer, etc. The hardware devices list is shown in Table 1.

The main components are shown in the Figure 16 and Figure 17.

Figure 16. Industrial control computer.

Figure 17. Overall structure of test system.

The loading control software is composed of two parts, in which the user program runs under windows, the real-time control program is based on the Ardence RTX kernel module and developed in C language, with a control period of 0.5 ms. The flow chart of the whole software is shown in Figure 18.

Figure 18. The software flow chart of the double-loop loading system.

Figure 19. Analyzing result in spectrometer CF-920 with the torque command at 40 Hz.

The loading control software is composed of two parts, in which the user program runs under windows, the real-time control program is based on the Ardence RTX kernel module and developed in C language, with a control period of 0.5 ms. The flow chart of the whole software is shown in Figure 18.

Figure 19 shows the tracking curve of 40 Hz sinusoidal torque command when the rudder is in 10 Hz sinusoidal motion. The curve is acquired by the spectrometer CF-920. The result shows that even the frequency of the torque command is out of sync with the frequency of the rudder motion, the actual load exerted on the rudder is very close to the command torque. The amplitude difference is 9.31% and the phase difference is 9.6° by FFT analysis.
Conclusion

In order to restrain the surplus force and meet the demand of high frequency band loading application, the paper presents an innovative double-loop loading system design. The simulation results show that the new system with $H_{\infty}$ controller solves the issue of unexpected position disturbance caused by the rudder system, therefore effectively suppresses the influence of the surplus torque. It allows the limit operating frequency band to reach 40 Hz, putting forward a new solution to improve the dynamic frequency response of the system. In engineering, it meets the requirements of the hardware-in-the-loop simulations with high frequency band.

Due to the complex structure of double-loop loading system, nonlinear disturbances such as clearance and friction of the transmission mechanism may affect the loading performance of the system. Later work can focus on the combination of disturbance compensation strategy and existing control strategy.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Yang Shui https://orcid.org/0000-0002-4908-2938

References

1. Wang C, Hou Y, Gao Q, et al. Electric load simulator system control based on adaptive particle swarm optimization wavelet neural network with double sliding modes. Adv Mech Eng 2016; 8(8): 168781401666426.
2. Tian W, Yang Z and Zhao T. Nonlinear aeroelastic characteristics of an all-movable fin with freeplay and aerodynamic nonlinearities in hypersonic flow. Int J Non Linear Mech 2019; 116: 123–139.
3. Zhou Y and Zhou X. Modeling and controller design for an experimental test bench for aircraft actuators. Adv Mech Eng 2018; 10(12): 168781401881536.
4. Wang H. Analysis for electric load torque simulator system. Harbin: Harbin Institute of Technology, 2014, pp.3–8.
5. Gurav B, Economou J, Saddington A, et al. Multi-mode electric actuator dynamic modeling for missile fin control. Aerospace 2017; 4(2): 30.
6. Shamisa A and Kiani Z. Robust fault-tolerant controller design for aerodynamic load simulator. Aerosp Sci Technol 2018; 78: 332–341.
7. Zhang B, Li C, Wang T, et al. Design and experimental study of zero-compensation steering gear load simulator with double torsion springs. Measurement 2019; 148: 106930.
8. Yao J, Jiao Z and Yao B. Nonlinear adaptive robust backstepping force control of hydraulic load simulator: theory and experiments. J Mech Sci Technol 2014; 28(4): 1499–1507.
9. Wang X, Hua Q, Jiao Z, et al. Friction and its compensation method in load simulator. China Mech Eng 2003; 14(6): 511–514.
10. Tian J and Zhang K. Development and present status of electric load simulators. Small Special Electric Mach Small Special Electric Machines 2014; 42(5): 70–74.
11. Wang X and Feng DZ. Experimental research on DC load simulator test bed with elastic rod. Electric Mach Control 2012; 16(9): 91–94.
12. Yadav D, Bansal S and Kumar M. Design, development & simulation of fuzzy logic controller to control the speed of permanent magnet synchronous motor drive system. Int J Sci Res Eng Technol 2012; 1(5): 101–106.
13. Feng C, Jing X, Bin G, et al. Double-stator permanent magnet synchronous in-wheel motor for hybrid electric drive system. IEEE Trans Magn 2009; 45(1): 278–281.
14. Ullah N and Shaoping W. High performance direct torque control of electrical aero-dynamics load simulator using adaptive fuzzy backstepping controls. Proc Inst Mech Eng G J Aerosp Eng 2015; 229(2): 369–383.
15. Ullah N, Wang S and Aslam J. Adaptive robust control of electrical load simulator based on fuzzy logic compensation. In: Proceedings of 2011 international conference on fluid power & mechatronics (FPM), 17 August 2011, pp.861–867. IEEE.
16. Wang X, Wang S and Yao B. Adaptive robust torque control of electric load simulator with strong position coupling disturbance. Int J Control Auton Syst 2013; 11(2): 325–332.
17. Lin L, Tian J and Chen Y. The compensation strategy for extraneous torque of electric load simulator. In: 2016 International Conference on Applied Mechanics, Electronics and Mechatronics Engineering (AMEME), Beijing, China, 28–29 May 2016. DEStec Publications, Inc.
18. Wang L, Wang M, Guo B, et al. Analysis and design of a speed controller for electric load simulators. IEEE Trans Industr Inform 2016; 63(12): 7413–7422.
19. Betz RE, Penfold HB and Newton RW. Local vector control of an AC drive system load simulator. IEEE Conf Control Appl 1994; 11(1): 721–726.
20. Rui LI, Jian-fang JI and Rui-feng YA. Overview on control strategies of load simulator. Chinese Hydraul Pneum 2012; 10: 3.
21. Zhao J, Shen G, Yang C, et al. A robust force feed-forward observer for an electro-hydraulic control loading system in flight simulators. ISA Trans 2019; 89: 198–217.
22. Wang C, Jiao Z and Quan L. Nonlinear robust dual-loop control for electro-hydraulic load simulator. ISA Trans 2015; 59: 280–289.
23. Shepovalova OV and Belenov AT. Investigation of DC motors mechanical characteristics with powered by comparable capacity PV array. Energy Procedia 2017; 119: 990–994.
24. Wang C, Jiao Z and Quan L. Adaptive velocity synchronization compound control of electro-hydraulic load simulator. Aerosp Sci Technol 2015; 42: 309–321.
25. Sugiuara E and Hori Y. Vibration suppression in 2- and 3-mass system based on the feedback of imperfect derivative of the estimated torsional torque. IEEE Trans Ind Electron 1996; 43(1): 56–64.
26. Alleyne A, Brennan S, Rasmussen B, et al. Controls and experiments: lessons learned. *IEEE Control Syst* 2015; 23(5): 20–34.

27. Xie Y and Alleyne A. A robust two degree-of-freedom controller for systems with both model and measurement uncertainty. *Control Eng Pract* 2014; 25(25): 55–65.

28. Weiss G and Hafele M. Repetitive control of MIMO systems using $\mathcal{H}_\infty$ design. *Automatica* 1999; 35(7): 1185–1199.

29. Zhou K, Doyle JC and Glover K. *Robust and optimal control*. New Jersey: Prentice Hall, 1996.

30. Aouf N, Boulet B and Botez R. $\mathcal{H}_2$ an $\mathcal{H}_\infty$-optimal gust load alleviation for a flexible aircraft. *Am Control Conf* 2000; 3(3): 1872–1876.

31. Chang X-H, Yang C and Xiong J. Quantized fuzzy output feedback control for nonlinear systems with adjustment of dynamic parameters. *IEEE Trans Syst Man Cybern Syst* 2019; 49(10): 2005–2015.

32. Li C, Li Y and Wang G. $\mathcal{H}_\infty$ output tracking control of electric-motor-driven aerodynamic load simulator with external active motion disturbance and nonlinearity. *Aerosp Sci Technol* 2018; 82–83: 334–349.