Charge transfer in cold collisions of rubidium atoms with calcium and ytterbium ions

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Abstract. Low-energy collisions of the Ca and Yb cations with Rb atoms are investigated theoretically using accurate ab initio potential energy curves and coupling matrix elements to elucidate the dominant charge transfer mechanisms. The cross sections calculated at collision energies above $10^{-5}$ cm$^{-1}$ exhibit the features typical to Langevin ion-atom collision regime, including a rich structure associated with the centrifugal barrier tunnelling (orbiting) resonances. It is shown that the dominant process in Yb$^+$ + Rb collisions is the radiative charge transfer, while in the case of Ca$^+$ + Rb collisions nonadiabatic transitions due to spin-orbit coupling dominate. Theoretical results are in a good agreement with available experimental data.

1. Introduction

With the development of the hybrid traps, which combine laser-cooled atomic ions stored in a radio-frequency trap and ultracold or statistically condensed neutral atoms in a magneto-optical trap, study of the ion-neutral reactions in the energy range between 1 mK and 1 K (usually termed as “cold” conditions) has recently become possible. Due to strong attractive induction interaction, cold ion-neutral collisions do not follow quantum regime being closer to “statistical” Langevin multiple partial wave picture. The effects of the orbiting resonances inherent to Langevin regime on charge transfer, a prototypical process for ion-molecular reaction, can only be understood theoretically since the micromotions of the trapped ions limit experimental collision energy resolution. The same is true for radiative-assisted processes, which are masked experimentally by the cooling laser field. Present work addresses these issues for the charge-transfer dynamics in cold collisions of the Yb$^+$ ions with Rb atom, studied recently using the hybrid trap technique [1], as well as the Ca$^+$ + Rb cold collisions [2], which, despite apparent similarity, exhibit quite distinct nonadiabatic charge transfer mechanism.

2. Interaction and dynamics

2.1. Ab initio data

Potential energy curves and coupling matrix elements used here were obtained ab initio under the scalar-relativistic approximation as described in great detail in [3, 4]. Those relevant to Yb$^+$ + Rb charge transfer are shown in Fig.1 (upper panel). The $A^1\Sigma^+$ state represents the initial channel, from which transition may occur to the ground $X^1\Sigma^+$ state. Nonadiabatic coupling matrix elements (NACMEs), responsible for nonradiative charge transfer mechanism, arise from
the first- and second-order radial couplings and the second-order rotational couplings, presented in lower panel of the same figure.

Figure 1. Potential energy curves and coupling matrix elements for Yb\(^+\)(\(^2S\)) + Rb\(^+\)(\(^2S\)) \rightarrow Yb\(^1S\)) + Rb\(^+\)(\(^1S\)) charge transfer process.

### 2.2. Nuclear dynamics

The nonadiabatic nuclear dynamics was studied within the formalism of the standard adiabatic (Born-Oppenheimer) approach described, for example, in Refs. [5, 6, 7, 8]. Due to the large energy splitting and small values of coupling matrix elements, the nonadiabatic transition probabilities are expected to be small at low collision energies, and the system of the coupled channel equations [7, 8] can be solved within the perturbation theory as shown in [5, 9].

The probabilities for a nonadiabatic transitions \(i \rightarrow f\) represented by the unperturbed (elastic) scattering radial wave functions \(F_{i(f)}^\text{el}(R)\) for the channel \(i(f)\), respectively, are given by the following formulae:

\[
P^\text{rad}_{if} = \frac{1}{4} \int_0^\infty \left| \frac{\partial}{\partial R} \psi_{i\Lambda} \left[ F_{i\Lambda}^\text{el} \frac{dF_{f\Lambda}^\text{el}}{dR} - F_{f\Lambda}^\text{el} \frac{dF_{i\Lambda}^\text{el}}{dR} \right] \right|^2 dR \tag{1}
\]

for the radial couplings and

\[
P^\text{ang}_{if} = \int_0^\infty \frac{1}{R^2} \left| \langle \psi_{i\Lambda} | L_x^2 + L_y^2 | \psi_{f\Lambda} \rangle F_{i\Lambda}^\text{el} F_{f\Lambda}^\text{el} \right|^2 dR, \tag{2}
\]

for the angular coupling. In contrast to Yb\(^+\) + Rb case, where the initial and final charge transfer channels are not coupled by spin-orbit (SO) interaction, it had to be considered for Ca\(^+\) + Rb collisions. The corresponding probability is

\[
P^\text{so}_{if} = 2\mu \int_0^\infty \left| \langle \psi_{i\Lambda} | \hat{H}_{\text{SO}} | \psi_{f\Lambda} \rangle F_{i\Lambda}^\text{el} F_{f\Lambda}^\text{el} \right|^2 dR, \tag{3}
\]

where \(\mu\) is the reduced nuclear mass of the colliding partners.

The nonradiative charge transfer cross sections are computed as a sum over the total angular momentum quantum number \(J\)

\[
\sigma_{if}(E) = \frac{\pi \hbar^2}{2\mu E} p_i^{\text{stat}} \sum_{J=0}^{J_{\text{max}}} P_{if}(J,E)(2J + 1), \tag{4}
\]

where \(p_i^{\text{stat}}\) is the statistical probability for population of the initial channel \(i\).
The dipole coupling between molecular states induces radiative charge transfer mechanism. An expression for the total radiative cross section follows from the optical potential approach [10, 11, 12, 13]. Being combined with the distorted wave approximation, it reads [10]

\[ \sigma_{ij}^{R}(E) = \frac{\pi}{2\mu E} n_{i}^{\text{stat}} \sum_{J} (2J + 1) \left\{ 1 - \exp\left[ -4\eta_{J}(E) \right] \right\} \]

with

\[ \eta_{J}(E) = \frac{2\pi}{3} \sqrt{\frac{\mu}{2E}} \alpha^{3} \int_{0}^{\infty} F_{i}^{J,E} d_{ij}^{2}(R) |U_{i} - U_{f}|^{3} F_{i}^{J,E} dR, \]

where \( F_{i}^{J,E} \) is the unperturbed (elastic) scattering radial wave function, \( d_{ij} \) is the transition dipole matrix element and \( U_{i,f} \) denotes adiabatic potential.

3. Results and conclusions

Cross sections for radiative and nonradiative charge transfer in Yb\(^{+} \) + Rb collisions are shown in Fig.2. Evidently, the latter gives negligible contribution to the charge transfer due to large adiabatic splitting and weak interaction of the \( \text{A}^{1}\Sigma^{+} \) and \( \text{X}^{1}\Sigma^{+} \) states. At the same time, it is worthy of noting that all cross sections exhibit the same resonance structure. Partial wave analysis demonstrate that it entirely corresponds to the orbiting, strong narrow resonances – to the tunneling deep under the centrifugal barriers and weak broad features – to the capture slightly below or above the barriers. Detailed analysis of some resonances [14] clearly shows the occurrence of dynamical trapping of the colliding partners in the entrance channel, causing a correspondingly marked increase of the non-adiabatic transition probabilities.

As far as the kinetic energy in a hybrid trap is controlled by ion micromotions, it is convenient to introduce energy-dependent rate coefficient as \( R(E) = \sqrt{2E/\mu} \sigma_{AX}^{R}(E) \). It is shown in Fig.3 together with the experimental data. Note that the resonance component gives about 10% of the total rate. The mean theoretical rate coefficient is in good agreement with the measured one. It has been demonstrated that account for SO interaction that modifies the potential energy curve for initial channel further improves the coincidence with experiment [4].

In contrast, detailed investigation of Ca\(^{+} \) + Rb collisions [3, 14] showed that the Ca\(^{+}(1\Sigma) + \text{Rb}(3\Sigma) \rightarrow \text{Ca}(3P) + \text{Rb}^{+}(1\Sigma) \) charge transfer at very low collision energies proceeds by the non-radiative mechanism. The states involved, \( \text{A}^{1}\Sigma \) and \( b^{3}\Pi \), are coupled by the SO matrix element, whereas two crossings of the corresponding adiabatic potentials further facilitate the nonadiabatic charge transfer process. Its cross sections, shown in Fig.4, indicate that Langevin
regime onsets at collision energy above $10^{-5}$ cm$^{-1}$. The calculated rate coefficient for Ca$^+$+Rb collisions is equal to $3.5 \times 10^{-12}$ cm$^3$/s at the temperature $T = 2$ K and in good agreement with the experimental data $2 \times 10^{-12}$ cm$^3$/s reported in [2].

To conclude, charge-transfer collisions at the energies typical to hybrid traps follow Langevin regime with its inherent rich orbiting resonance structure. This structure tend to enhance the rate of inelastic processes. The mechanism of charge transfer may vary from system to system being determined by the structure of adiabatic potential energy curves and magnitude of the couplings. The dominant mechanisms in the treated cold collisions are different: It is the radiative charge transfer in Yb$^+$ + Rb collisions, while the main mechanism in Ca$^+$ + Rb collisions corresponds to the nonradiative charge transfer. Spin-orbit interactions are essential for a quantitative description of the cold charge transfer dynamics.

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References
[1] Zipkes C, Palzer S, Ratschbacher L, Sias C and Köhl M 2010 Phys. Rev. Lett. 105 133201
[2] Hall F H J, Eberle P, Hegi G, Willitsch S, Raoul M, Aymar M and Dulieu O 2013 Mol. Phys. 111 2020–32
[3] Tacconi M, Gianturco F A and Belyaev A K 2011 Phys. Chem. Chem. Phys. 13 19156
[4] Sayfutyarova E R, Buchachenko A A, Yakovleva S A and Belyaev A K 2013 Phys. Rev. A 87 052717
[5] Mott N F and Massey H S W 1949 The Theory of Atomic Collisions (Oxford: Clarendon Press)
[6] Macias A and Riera A 1982 Phys. Rep. 90, 299
[7] Grosser J 1986 Z. Phys. D 3 39
[8] Belyaev A K, Egorova D, Grosser J and Menzel T 2001 Phys. Rev. A 64 052701
[9] Belyaev A K 2007 Eur. Phys. J. D 44 497
[10] Zygelman B and Dalgarno A 1988 Phys. Rev. A 38 1877
[11] Cohen J S and Bardsley J N 1978 Phys. Rev. A 18 1004
[12] Tellinghuisen J and Julienne P S 1984 J. Chem. Phys. 81 5779
[13] Gustafsson M, Antipov S V, Franz J and Nyman G 2012 J. Chem. Phys. 137 104301
[14] Belyaev A K, Yakovleva S A, Tacconi M and Gianturco F A 2012 Phys. Rev. A 85 042716