Heterotic M–Theory Vacua with Five–Branes

André Lukas\textsuperscript{1}, Burt A. Ovrut\textsuperscript{2} and Daniel Waldram\textsuperscript{3}

\textsuperscript{1}Department of Physics, Theoretical Physics, University of Oxford
1 Keble Road, Oxford OX1 3NP, United Kingdom

\textsuperscript{2}Department of Physics, University of Pennsylvania
Philadelphia, PA 19104–6396, USA

\textsuperscript{3}Department of Physics, Joseph Henry Laboratories, Princeton University
Princeton, NJ 08544, USA

Abstract: We construct vacua of heterotic M–theory with general gauge bundles and five–branes. Some aspects of the resulting low–energy effective theories are discussed.

M–theory on the orbifold $S^1/Z_2$ is believed to describe the strong coupling limit of the $E_8 \times E_8$ heterotic string \cite{1} and, therefore, constitutes a particularly interesting starting point for M–theory particle phenomenology. At low energy, this theory is described by 11–dimensional supergravity coupled to two 10–dimensional $E_8$ gauge multiplets residing on the two fixed points of the orbifold \cite{2, 4, 3}. Compactifications leading to $\mathcal{N} = 1$ supersymmetry in four dimensions are based on space–times of the structure

$$M_{11} = S^1/Z_2 \times X \times M_4,$$

where $X$ is a Calabi–Yau three-fold and $M_4$ is flat Minkowski space. While most work to date, related to such compactifications of heterotic M–theory, has been limited to the standard embedding, non–standard embedding vacua have previously been addressed in \cite{6, 7, 8}.

Here, we will consider the general configuration leading to $\mathcal{N} = 1$ supersymmetry \cite{9}, where, first, we allow for general gauge bundles, and, second, include five–branes \cite{3}, states which are essentially non-perturbative in heterotic string theory. The important new ingredient is the presence of these five–branes in the vacua. Some effects caused by the presence of the five–branes have been discussed in \cite{7, 10}. Due to gauge and gravity sources on the orbifold planes and the five–brane sources, the space–time \cite{9} receives corrections that can be computed perturbatively as an expansion in \cite{9}

$$\epsilon_S = \left( \frac{\kappa}{4\pi} \right)^{2/3} \frac{2\pi \rho}{v^{2/3}},$$

where $\kappa$, $v$ and $\rho$ are the 11–dimensional Newton constant, the Calabi–Yau volume and the orbifold radius, respectively.

\textsuperscript{*}Talk presented by A. Lukas at the 32nd Symposium Ahrenshoop on the Theory of Elementary Particles, Buckow, Germany, September 1 - 5, 1998
Let us now determine these corrections to linear order in $e_s$. We need to specify the 11-dimensional metric $g_{IJ}$, the three-form $C_{IJK}$ with field strength $F_{IJKL} = 24 \partial_{[I} C_{JKL]}$ (where $I, J, K, \ldots = 0, \ldots, 9, 11$) and the Killing spinor $\eta$. This can be done solving the Bianchi identity

\[
(dG)_{11IJKL} = 4\sqrt{2}\pi \left( \frac{k}{4\pi} \right)^{2/3} \left[ J^{(0)}(x^{11}) + J^{(N+1)}(x^{11} - \pi \rho) + \frac{1}{2} \sum_{n=1}^{N} J^{(n)}(x^{11} - x_n) + \delta(x^{11} + x_n) \right]_{IJKL} .
\]

(3)

where $I, J, K, \ldots = 0, \ldots, 9$, along with the equation of motion for $C$ and the Killing spinor equation $\delta\Psi_I = 0$ for the gravitino to preserve some supersymmetry. The sources on the orbifold planes

\[
J^{(0)} = -\frac{1}{16\pi^2} \left( \text{tr} F^{(1)} - \frac{1}{2} \text{tr} R^2 \right) , \quad J^{(N+1)} = -\frac{1}{16\pi^2} \left( \text{tr} F^{(2)} - \frac{1}{2} \text{tr} R^2 \right)
\]

are given in terms of the $E_8$ gauge field strengths $F^{(i)}$, $i = 1, 2$, and the curvature. In addition, we have considered $N$ five–branes transverse to the orbifold. They induce the sources $J^{(n)}$, $n = 1, \ldots, N$ appearing in the above Bianchi identity. We have chosen the orbifold coordinate $x^{11}$ to be in the range $x^{11} \in [-\pi \rho, \pi \rho]$ with the orbifold planes at $x^{11} = 0, \pi \rho$ and the five–branes at $x^{11} = \pm x_1, \ldots, \pm x_N$. From eq. (3), the sum of all these sources must be cohomologically trivial.

One must also ensure that the theories on the orbifold planes and the five–branes preserve supersymmetry. This is guaranteed by choosing two semi–stable holomorphic (otherwise arbitrary) $E_8$ gauge bundles $V_i$ and by wrapping the five–branes on holomorphic Calabi–Yau two–cycles while stretching them over the four–dimensional uncompactified space. With this choice, all sources $J^{(n)}$, $n = 0, \ldots, N+1$ are $(2,2)$ forms on the Calabi–Yau space. The cohomology condition on the sources can now be expressed as

\[
\left[ \sum_{n=0}^{N+1} J^{(n)} \right] = c_2(V_1) + c_2(V_2) - c_2(TX) + [W] = 0 .
\]

(5)

Here $c_2(V_i)$ and $c_2(TX)$ are the second Chern classes of the vector bundles $V_i$ and the tangent bundle $TX$ and $[W] = \sum_{n=1}^{N} [J^{(n)}]$ is the cohomology class of the total five–brane curve $W$. We write

\[
g_{IJ} = g^{(0)}_{IJ} + g^{(1)}_{IJ} , \quad G_{IJKL} = G^{(1)}_{IJKL} , \quad \eta = \eta^{(0)} + \eta^{(1)} ,
\]

(6)

where $g^{(0)}_{IJ}$ and $\eta^{(0)}$ are the metric and the covariantly constant spinor of the Calabi–Yau space $X$. One can show [3] that $G^{(1)}_{ABCD}$ and $G^{(1)}_{AB11}$ are the only non–vanishing components of the four–form and that the other corrections have the structure

\[
g^{(1)}_{\mu \nu} = b \eta_{\mu \nu} , \quad g^{(1)}_{AB} = h_{AB} , \quad g^{(1)}_{11,11} = \gamma , \quad \eta^{(1)} = \psi \eta^{(0)} .
\]

(7)

Four–dimensional space is indexed by $\mu, \nu, \rho \ldots = 0, \ldots, 3$ while we use $A, B, C, \ldots = 4, \ldots, 9$ for the Calabi–Yau space. Furthermore, holomorphic (anti–holomorphic) Calabi–Yau indices are denoted by $a, b, c, \ldots (\bar{a}, \bar{b}, \bar{c}, \ldots)$. The corrections can be entirely expressed in terms of a $(1,1)$ form $B_{ab}$ on the Calabi–Yau space as

\[
h_{ab} = \sqrt{2i} \left( B_{ab} - \frac{1}{3} \omega_{ab} B \right) , \quad b = -\frac{\sqrt{3}}{6} B
\]

\[
\gamma = -\frac{\sqrt{3}}{3} B , \quad \psi = -\frac{\sqrt{2}}{24} B
\]

\[
G^{(1)}_{ABCD} = \frac{1}{2} \epsilon_{ABCD} \partial B^{EF} , \quad G^{(1)}_{ABC11} = \frac{1}{2} \epsilon_{ABC11} \partial B^{DEF}
\]

(8)
where $B = \omega^{AB} B_{AB}$ and $\omega_{ab} = -i g_{ab}$ is the Kähler form. One can expand this $(1,1)$ form in terms of eigenfunctions of the Calabi–Yau Laplacian as

$$B_{AB} = \sum_i b_i \omega_{AB}^i + \text{massive}.$$  

(9)

Here, we have concentrated on the massless part of this expansion, represented by the harmonic $(1,1)$ forms $\omega_{iab}$ of the Calabi–Yau space, where $i = 1, \ldots, h^{1,1}$. The form of the massive part can be found in \[1\]. The expansion coefficients $b_i$ read explicitly

$$b_i = \frac{\pi \epsilon S}{\sqrt{2}} \left[ \sum_{m=0}^{n} \beta_m^{(i)} (|z| - z_m) - \frac{1}{2} \sum_{m=0}^{N+1} (z_m^2 - 2z_m) \beta_m^{(i)} \right], \quad \beta_m^{(i)} = \int_{\mathcal{C}_{4i}} J^{(n)}. \quad \text{(10)}$$

This expression holds in each interval $z_n \leq |z| \leq z_{n+1}$ for fixed $n$, where $n = 0, \ldots, N$. Here $z = x^{1/4}/\pi \rho$ and $z_n = x_n/\pi \rho$ are normalized orbifold coordinates and $\mathcal{C}_{4i}$ is a basis of four–cycles dual to the harmonic $(1,1)$ forms $\omega_i$. Note that the charges sum up to zero due to the cohomology constraint (3); that is $\sum_{n=0}^{N+1} \beta_m^{(n)} = 0$.

To summarize, eqs. (3)–(10) represent the (massless part) of the background to linear order in $\epsilon S$, given in terms of the topological charges $\beta_m^{(n)}$. Specific consistent models where these charges can be computed explicitly have been constructed recently \[12\]. Eq. (10) shows that the Calabi–Yau space is changing its size and shape across the orbifold. This corresponds to a certain trajectory in the Kähler moduli space.

We would now like to discuss some properties of the low–energy effective actions associated to those vacua. Interesting new features arise from the new degrees of freedom on the five–brane worldvolumes. From a single five–brane wrapped on a holomorphic genus $g$ two–cycle within a Calabi–Yau space one obtains an $\mathcal{N} = 1$ theory in four dimensions with generically $g \ U(1)$ vector multiplets, one universal chiral multiplet (containing the position modulus in the orbifold direction) and a number of additional chiral multiplets parameterizing the moduli space of the curve. In specific cases, the gauge group enhances and becomes non–abelian. For example, $N$ five–branes, wrapped on the same cycle with genus $g$ and positioned at different points in the orbifold lead to a group $U(1)^g N$. Moving all five–branes to the same orbifold point enhances the group to $U(N)^g$.

The five–dimensional effective action of heterotic M–theory obtained by reducing on the Calabi–Yau space is of some interest as the orbifold size could be large. For standard embedding, this action consists of a gauged $\mathcal{N} = 1$ supergravity with vector and hypermultiplets in the bulk coupled to two four–dimensional $\mathcal{N} = 1$ theories on the orbifold planes \[13\]. For the vacua discussed here, the bulk action between each two neighboring five–branes is again given by gauged supergravity. The gauge charges are directly related to the charges $\beta_m^{(n)}$ and depend on the five–brane pair considered. In addition to the two orbifold theories, the bulk is now coupled to $N$ additional four–dimensional $\mathcal{N} = 1$ theories resulting from the five–branes. Each of them carries a field content of the type discussed above.

Upon further reduction to four dimensions, one obtains a theory which consists of the usual “observable” and “hidden” sector originating from $E_8 \times E_8$ and $N$ additional sectors from the five–brane degrees of freedom. In particular, the low–energy gauge group is enhanced to $G^{(1)} \times G^{(2)} \times G$ where $G^{(i)}$ are the unbroken subgroups of $E_8$ and $G$, being typically a product of unitary groups, results from the five branes. In addition to the existence of new sectors, the “conventional” $E_8 \times E_8$ sectors are effected by the presence of the five–branes. The five–brane charges introduce more freedom in the cohomology
condition (5) and, hence, allow for consistent gauge bundles that would otherwise be forbidden. Among other things, this facilitates the construction of three–family model [12]. Also, the conventional part of the four–dimensional effective action receives new corrections caused by the five–branes. They can be computed in leading order using the above results for the explicit form of the vacua. For example, the gauge kinetic functions for $G^{(1)}$ and $G^{(2)}$ take the form

$$f^{(1)} = S + \pi \varepsilon S T^i \sum_{n=0}^{N+1} (1 - z_n)^2 \beta_i^{(n)}, \quad f^{(2)} = S + \pi \varepsilon S T^i \sum_{n=1}^{N+1} z_n^2 \beta_i^{(n)},$$  \hspace{1cm} (11)$$

where $S$ is the dilaton and $T^i$ are the $T$ moduli. Note the dependence on the five–brane charges as well as on the positions $z_n$ of the five–branes in the orbifold.

Acknowledgments
A. L. is supported by the European Community under contract No. FMRXCT 960090. B. A. O. is supported in part by DOE under contract No. DE-AC02-76-ER-03071 and by a Senior Alexander von Humboldt Award. D. W. is supported in part by DOE under contract No. DE-FG02-91ER40671.

References
[1] P. Hořava and E. Witten, *Nucl. Phys.* B460 (1996) 506.
[2] P. Hořava and E. Witten, *Nucl. Phys.* B475 (1996) 94.
[3] E. Witten, *Nucl. Phys.* B471 (1996) 135.
[4] P. Hořava, *Phys. Rev.* D54 (1996) 7561.
[5] T. Banks and M. Dine, *Nucl. Phys.* B479 (1996) 173.
[6] K. Benakli, *Phys. Lett.* B447 (1998) 51.
[7] S. Stieberger, (0,2) heterotic gauge couplings and their $M$ theory origin, CERN-TH-98-228, [hep-th/9807124](http://arxiv.org/abs/hep-th/9807124), to be published in Nucl. Phys. B.
[8] Z. Lalak, S. Pokorski and S. Thomas, Beyond the Standard Embedding in M–Theory on $S^1/Z_2$, CERN-TH/98-230, [hep-ph/9807503](http://arxiv.org/abs/hep-ph/9807503).
[9] A. Lukas, B. A. Ovrut and D. Waldram, Non–Standard Embedding and Five–Branes in Heterotic M–Theory, UPR-815T, [hep-th/9808101](http://arxiv.org/abs/hep-th/9808101), to be published in Phys. Rev. D.
[10] P. Binetruy, C. Deffayet, E. Dudas and P. Ramond, *Phys. Lett.* B441 (1998) 163.
[11] A. Lukas, B. A. Ovrut and D. Waldram, *Nucl. Phys.* B532 (1998) 43.
[12] R. Donagi, A. Lukas, B. A. Ovrut and D. Waldram, Non–Perturbative Vacua and Particle Physics in M–Theory, UPR-823T, [hep-th/9811168](http://arxiv.org/abs/hep-th/9811168); Holomorphic Vector Bundles and Non–perturbative Vacua in M–theory, UPR-827T, [hep-th/9901009](http://arxiv.org/abs/hep-th/9901009).
[13] A. Lukas, B. A. Ovrut, K.S. Stelle and D. Waldram, The Universe as a Domain Wall, UPR-797T, [hep-th/9803233](http://arxiv.org/abs/hep-th/9803233), to be published in Phys. Lett. B; Heterotic M–theory in Five Dimensions, UPR-804T, [hep-th/9806051](http://arxiv.org/abs/hep-th/9806051).