A Study of Variations in Correlation Between Rotation Residual and Meridional Velocity of Sunspot Groups

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Abstract
We analyzed the combined 142 years sunspot-group data from Greenwich Photoheliographic Results (GPR) and Debrecen Photoheliographic Data (DPD) and determined the yearly mean residual rotation rate and the meridional velocity of sunspot groups in different 5° latitude intervals. We find that there exists a considerable latitude–time dependence in both the residual rotation and the meridional motion. The residual rotation rate is found to be −120 m s⁻¹ to 80 m s⁻¹. In a large number of solar cycles, the rotation is to some extent weaker during maxima than that of during minima. There exist alternate bands of equatorward and poleward meridional motions. The equatorward motion is dominant mostly around the maxima of solar cycles with velocity 8–12 m s⁻¹, whereas the poleward motion is dominant mostly around the minima but with a relatively weak velocity, only 4–6 m s⁻¹. The analysis of the data during Solar Cycles 12–24 that are folded according to the years from their respective epochs of maxima suggests the existence of equatorward migrating alternate bands of slower and faster than average rotation within the activity belt. This analysis suggests no clear equatorward or poleward migrating bands of meridional motions. A statistically significant anticorrelation exists between the meridional motion and residual rotation. The corresponding linear-least-squares best-fit is found to be reasonably good (slope, −0.028 ± 0.008, is about 3.5 times larger than its standard deviation). As per the sign convention used for the meridional motions, the significant negative value of the slope indicates the existence of a strong angular momentum transport toward the equator. The cross-correlation between the slopes determined from the data in 3-year moving time intervals and yearly mean sunspot number (SN) suggests that the slope leads SN by about 4 and 9 years. The Morlet wavelet spectrum of the slope suggests the existence of ≈11-year periodicity in the slope almost throughout the data window, but it was very weak during 1920–1940. We have also done cross-wavelet, wavelet-coherence, and wavelet-phase difference analyses of the slope and SN. Overall the results suggest there exists a strong relationship between the slope and amount of activity during a solar cycle. However, the correlation between the cycle-to-cycle modulations in the slope and the amplitude of solar cycle is found to be insignificant, indicating that there is no relationship between the slope and strength of activity on a long-time scale (longer than 11-year period).

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1. Introduction

Studies of variations in solar activity are important for a better understanding of the basic mechanism of solar activity and solar cycles of various periods to make the long-term forecasts of space-weather and also may be Earth’s climate (Hathaway, 2015). It is well believed that the solar dynamo processes are responsible for solar activity and cycle. The dynamo processes involve the formation of toroidal magnetic field due to the shearing of the poloidal magnetic field by differential rotation and conversion of the toroidal field into poloidal field of opposite polarity over the course of approximately 11-year (Dikpati and Gilman, 2006). The solar meridional circulation may play a vital role to transfer the angular momentum and magnetic flux across the solar latitudes and even can maintain the observed differential rotation (e.g., Schröter, 1985, and references therein). Therefore, the studies of the correlation between the variations in the solar meridional flow and differential rotation are important for understanding the role of meridional flow in the variations of solar differential rotation, and hence the variations in solar activity. The study of correlation between the meridional and rotational flows was started by Ward (1965), who analyzed Greenwich sunspot-group data during the period 1935–1944 and found a significant correlation between the angular and meridional velocities of sunspot groups and interpreted it as meridional flows that transfer angular momentum toward the equator. Later many scientists studied this correlation by using various data and methods. For example, Paternó et al. (1991) from the Greenwich sunspot data during the period 1874–1976 and Howard (1991, 1996) using Mt. Wilson measurements of sunspot groups (1917–1985) and of plages (1967–1985) confirmed the existence of a strong correlation between the meridional and rotation velocities of these solar phenomena. However, some authors found the existence of only small covariance of the meridional and angular velocities derived from the data of different tracers (Nesme-Ribes, Ferreira, and Vince, 1993; Komm, Howard, and Harvey, 1994; Meunier, Nesme-Ribes, and Collin, 1997). Theoretical models (e.g., Gilman, 1986) predicted the observed correlation between the surface latitudinal and longitudinal motions as reflection of equatorial angular momentum transport caused by Reynolds stresses near the surface. There were criticisms of this theoretical explanation on the basis that the observed motions may result from the well-known expansion and contraction of sunspots along the tilted magnetic axes of the sunspot groups. Gilman and Howard (1984) argued that because the effect could be observed for whole sunspot groups, at least some fraction of the observed correlation must be due to Reynolds stresses near the solar surface and that the amount was sufficient to account for the angular momentum transport required to maintain the solar differential rotation. So far most of the authors attributed the observed correlation between meridional and rotational motions to the action of Reynolds stresses that demonstrated the presence of a net transport of angular momentum towards the equator able to maintain the differential rotation. Recently, Sudar et al. (2014) by analyzing Greenwich Photographic Result (GPR) and Solar Observing Optical Network (SOON) sunspot-group data covering the period from 1878 until 2011 and Sudar et al. (2017) by analyzing the Debrecen Photoheliographic Data (DPD) during the period 1974–2016 found a statistically significant correlation between sunspot groups’ meridional velocities and rotation velocity residuals confirming the transfer of angular momentum towards the equator.

Solar meridional flows vary during the solar cycle (Snodgrass, 1987; Komm, Howard, and Harvey, 1993a; Meunier, 2005; Švanda et al., 2008; Hathaway and Rightmire, 2010).
Meridional flows also seem to have a vital role in the cause of solar 11-year period torsional oscillations (see Snodgrass, 1992) and in solar flux transport dynamo process (Dikpati and Gilman, 2006). Javaraiah and Ulrich (2006) analyzed a large set of sunspot group data (1874–2004) and found the existence of correlation (good in the northern hemisphere and weak in the southern hemisphere) between the mean solar-cycle variations of meridional flow and the latitude gradient term of solar rotation. In the present study we analyze the combined updated sunspot-group data reported in GPR during the period 1874–1976 and in DPD during the period 1977–2017 and study variation in the correlation between the residual rotation and meridional motion of sunspot groups and its relationship with solar activity through cross-correlation and continuous- and cross-wavelet analyses.

In the next section we describe the data analysis, in Section 3 we show the results, and in Section 4 we summarize the conclusions and discuss them briefly.

2. Data Analysis

We have downloaded the daily sunspot-group data reported in GPR during the period April 1874 – December 1976 and DPD during the period January 1977 – June 2017 from the website fenyi.solarobs.unideb.hu/pub/DPD/. The details about these data can be found in Győri, Baranyi, and Ludmány (2010), Baranyi, Győri, and Ludmány (2016), Győri, Ludmány, and Baranyi (2017). These data contain, beside other parameters, the date and time of observation, heliographic latitude ($\lambda$) and longitude ($L$), and central meridian distance (CMD) of a sunspot group for each day during its appearance on the solar disk. The solar sidereal angular velocity $\omega$ (in degree day$^{-1}$) and meridional velocity $v_{\text{mer}}$ (in degree day$^{-1}$) of a sunspot group are calculated by using the latitudes and longitudes of the sunspot group measured at times $t_i$ and $t_{i-1}$ during the life time (disk passage) of the sunspot group as follows (here $t$ is the date + fraction of the day corresponding to the time of observation):

$$\omega(\theta) = \frac{L_i - L_{i-1}}{t_i - t_{i-1}} + 14^\circ .18 \text{ and } v_{\text{mer}}(\theta) = \frac{\lambda_i - \lambda_{i-1}}{t_i - t_{i-1}},$$

where $14^\circ .18$ day$^{-1}$ is the Carrington rigid body rotation. In all our earlier analyses we have assigned the velocity value to the mean of $\lambda_{i-1}$ and $\lambda_i$. Following the suggestion by Olemskoy and Kitchatinov (2005) in Javaraiah (2020) and here we assigned the velocity value to the $\theta = \lambda_{i-1}$ (also see Sudar et al., 2014). Each disk passage of a recurrent sunspot group is treated as an independent sunspot group. Hence, we have considered all the sunspot groups that have life times 2–12 days. We have not used the data on the days when $|\text{CMD}| > 75^\circ$. This reduces the foreshortening effect if any. In addition, the data correspond to the absolute latitudinal drifts $> 2^\circ$ day$^{-1}$ and absolute longitudinal drifts $> 3^\circ$ day$^{-1}$ are excluded. This reduces considerably the uncertainty in the derived results (Ward, 1965; Javaraiah and Gokhale, 1995). In the case of meridional velocity, here we have used the following sign convention: in both the northern and southern hemispheres, positive and negative values indicate the poleward and equatorward motions, respectively. We have converted the meridional velocity that measured in degree day$^{-1}$ into m s$^{-1}$ (0.01 degree day$^{-1}$ = 1.4 m s$^{-1}$, cf. Howard, 1991).

The data of sunspot groups in the northern and southern hemispheres are combined. We binned the daily values of $\omega$ and $v_{\text{mer}}$ during each of the years 1874–2017 into different $5^\circ$ latitude intervals within $0^\circ$–40$^\circ$ sunspot latitude (absolute) belt. We determined the mean values $\langle \omega(\theta) \rangle$ and $\langle v_{\text{mer}}(\theta) \rangle$ of the daily values of $\omega$ and $v_{\text{mer}}$, respectively, in each $5^\circ$ latitude interval during each year (note that $\langle \cdot \rangle$ implies the mean over a time interval and $\theta$ represents the middle value of a latitude interval). In several latitude intervals, particularly in a large extent during the years corresponding to the minima of the solar cycles, the numbers of velocity values are found to be zero. In some latitude intervals they are found to
be just one. The data in all such latitude intervals of all the years are excluded. We determined mean and standard error ($\sigma_{mer}$) of $\langle v_{mer}(\theta) \rangle$ of all the remaining latitude intervals and in all years. The $\langle \omega(\theta) \rangle$ and $\langle v_{mer}(\theta) \rangle$ in the latitude intervals which correspond to $\langle v_{mer}(\theta) \rangle > 2.5 \sigma$ are excluded. We fitted the values of $\langle \omega(\theta) \rangle$ in all the remaining latitudes intervals during all the years 1874–2017 into the standard law of differential rotation: $\omega(\theta) = A + B \sin^2 \theta$ degree day$^{-1}$, where $A$ and $B$ represent the equatorial rotation rate and latitude gradient of rotation, respectively, and $\theta$ is the middle value of a $5^\circ$ latitude interval. We obtained the residual ($\Delta \langle \omega(\theta) \rangle$) of the mean angular velocity $\langle \omega(\theta) \rangle$ in each latitude interval (i.e., at each value of $\theta$) by subtracting the mean value of $\langle \omega(\theta) \rangle$ of that latitude interval from the value of $\langle \omega(\theta) \rangle$ of the same latitude interval deduced from the law of the differential rotation. We determined the correlation between $\langle v_{mer}(\theta) \rangle$ and $\Delta \langle \omega(\theta) \rangle$ of all latitude intervals and all years. The corresponding linear-least-squares best-fit is found to be mostly relevant to the values of $\langle v_{mer}(\theta) \rangle \leq 20$ m s$^{-1}$. This is because the values of $\langle v_{mer}(\theta) \rangle$ that are $> 20$ m s$^{-1}$ are few (14 %). Hence, we have excluded the values of $\langle v_{mer}(\theta) \rangle$ and $\langle \omega(\theta) \rangle$ in the latitude intervals which correspond to the $\langle v_{mer}(\theta) \rangle > 20$ m s$^{-1}$. In addition, we have excluded the values of $\langle \omega(\theta) \rangle$ and $\langle v_{mer}(\theta) \rangle$ that correspond to $\Delta \langle \omega(\theta) \rangle > 1^\circ$ day$^{-1}$, which are also very few (0.6 %). We repeated all the aforementioned calculations, i.e., we fitted the remaining values of $\langle \omega(\theta) \rangle$ to the standard law of differential rotation and obtained the differential rotation law $\langle \omega(\theta) \rangle = (14.5 \pm 0.01) – (2.2 \pm 0.07) \sin^2 \theta$ degree day$^{-1}$. By using the values of $\langle \omega(\theta) \rangle$ deduced from this law, we obtained the values of $\Delta \langle \omega(\theta) \rangle$ in all latitude intervals. We have converted the values of the residual rotation $\Delta \langle \omega(\theta) \rangle$ degree day$^{-1}$ into $\Delta \langle v_{rot}(\theta) \rangle$ m s$^{-1}$ to have the same units for both $\Delta \langle v_{rot}(\theta) \rangle$ and $\langle v_{mer}(\theta) \rangle$ for the sake of convenience to study the correlation between these parameters. We determined the correlation between $\langle v_{mer}(\theta) \rangle$ and $\Delta \langle v_{rot}(\theta) \rangle$ and the corresponding linear-least-squares best-fit of these parameters in all latitude intervals during the whole period 1874–2017 and also separately from the values in each of Solar Cycles 12–24. We have used the epochs of minima (1878.958, 1890.204, 1902.042, 1913.623, 1923.623, 1933.707, 1944.124, 1954.288, 1964.791, 1976.206, 1986.707, 1996.624, 2008.958), as well as maxima (1883.958, 1894.042, 1906.123, 1917.623, 1928.290, 1937.288, 1947.371, 1958.204, 1968.874, 1979.958, 1989.874, 2001.874, 2014.288), and the maximum values of these cycles that were determined by Pesnell (2018) by using the 13-month smoothed monthly mean version-2 international sunspot number (SN) series. We determined the average behavior of the correlation during Solar Cycles 12–24 by using the method of superposed epoch analysis, i.e., in order to have a better statistics all the 13 solar cycles data are superposed/combined according to the years from the respective epochs of maxima of the solar cycles. Such analyses were done by many authors (Gilman and Howard, 1984; Balthasar, Vázquez, and Wöhl, 1986; Javaraiah and Komm, 1999; Javaraiah, 2003; Javaraiah and Ulrich, 2006; Brajša, Ruždjak, and Wöhl, 2006; Sudar et al., 2014). We have also binned the values of $\Delta \langle v_{rot}(\theta) \rangle$ and $\langle v_{mer}(\theta) \rangle$ into 3-year moving time intervals (3-year MTIs) successively shifted by one year, namely 1874–1876, 1875–1877, ..., 2015–2017, and determined the corresponding correlation and the linear-least-squares best-fit in each of the 3-year MTIs.

Wavelet transform can be used to analyze time series that contain nonstationary power at many different frequencies. That is, by wavelet analysis it is possible to decompose a time series into time–frequency space. Hence, one can determine both the dominant modes of variability and how those modes vary in time. The cross-wavelet transform between two time series is simply the product of the first complex wavelet transform with the complex conjugate of the second. The cross-wavelet power spectrum can be used as a quantified indication of the similarity of power between two time series. The wavelet-coherency is a normalized time and scale (period) resolved measure for the relationship between two
time series. It is the square of the cross-wavelet power spectrum normalized by the individual wavelet-power spectra. This gives a quantity between zero and one. A value of one means the existence of linear relationship between the two time series around a time on a scale. A value of zero means vanishing correlation (no linear relationship). Measurements of wavelet-phase difference between two time series yield information on the phase delay between oscillations in the time series as a function of frequency (for details, see Torrence and Compo, 1998). We have done Morlet wavelet analysis for the time series of the slopes of the linear relationships in 3-year MTIs. We have also done the wavelet analysis for the yearly values of the version-2 SN. The yearly average version-2 SN time series is downloaded from www.sidc.be/silso/datafiles. The details of changes and corrections in version-2 SN can be found in Clette and Lefévre (2016). Similarities in the Morlet wavelet spectra of the slope and SN are checked from cross-wavelet, wavelet-coherence, and wavelet-phase difference analyses. We have used the IDL-codes of the wavelet analyzes provided by Torrence and Compo (1998) and available at paos.colorado.edu/research/wavelets.

3. Results

3.1. Latitude–Time Dependence of $\Delta\langle v_{rot} \rangle$ and $\langle v_{mer} \rangle$

Figure 1(a) shows the variations in yearly mean residual rotation ($\Delta\langle v_{rot}(\theta) \rangle$) whose values correspond to the $\Delta\langle \omega(\theta) \rangle \leq 1^\circ \text{day}^{-1}$ ($\approx 140 \text{m s}^{-1}$), and Figure 1(b) shows yearly mean meridional velocity ($\langle v_{mer}(\theta) \rangle$) which has values only $\pm 20 \text{m s}^{-1}$, of sunspot groups in different $5^\circ$ latitude ($\theta$) intervals during 1874–2017.

As can be seen in Figure 1(a), there is a considerable latitude–time dependence in $\Delta\langle v_{rot} \rangle$. The values of $\Delta\langle v_{rot}(\theta) \rangle$ are from $-120 \text{m s}^{-1}$ to $80 \text{m s}^{-1}$ (only a few values are beyond this interval). In a large number of solar cycles, the rotation is to some extent weaker during maxima than that of during minima. There is an indication on high-to-low latitude migration in $\Delta\langle v_{rot}(\theta) \rangle$ over a 8–10-year period. Particularly, the band of slower than average rotation seems to be migrating from $\approx 35^\circ$ latitude to around $5^\circ$ latitude during many solar cycles (not clearly visible in the early solar cycles).

As can be seen in Figure 1(b), there is also a considerable latitude–time dependence in $\langle v_{mer} \rangle$. There seem to be alternate bands of equatorial motion ($\langle v_{mer}(\theta) \rangle$ is negative) and poleward motion ($\langle v_{mer}(\theta) \rangle$ is positive). Overall, the equatorward motion is dominant with velocity 8–12 m s$^{-1}$ mostly around maxima of the majority of solar cycles, whereas the poleward motion seems to be relatively weak with velocity only 4–6 m s$^{-1}$ and exists mostly around minima of the majority of solar cycles (also see Javaraiah and Ulrich, 2006). However, in some solar cycles, e.g., 18 and 24, the motion was seem to be equatorward during their whole periods. There is also some indication of equatorward migration in the latitude–time dependence of $\langle v_{mer} \rangle$ during a large number of solar cycles. The beginnings and endings of the equatorward bands are not at exactly the beginnings and endings of solar cycles. Poleward meridional flow with a speed of about 20 m s$^{-1}$ has been well established at the solar photospheric level (Meunier and Zhao, 2009).

Figures 2(a) and 2(b) show the average variations in yearly mean $\Delta\langle v_{rot} \rangle$ and $\langle v_{mer} \rangle$ of sunspot groups in different $5^\circ$ latitude intervals during Solar Cycles 12–24. This is determined by superposing the yearly data (the values shown in Figure 1) of Solar Cycles 12–24 according to the years from their respective epochs of maxima. Northern and southern hemispheres’ data are folded. Figure 2(a) shows the existence of alternate bands of slower and faster than average rotation, i.e., alternate bands of negative and positive values...
Figure 1  Variations in (a) yearly mean residual rotation rate ($\Delta\langle v_{rot}(\theta) \rangle$) and (b) yearly mean meridional velocity ($\langle v_{mer}(\theta) \rangle$) in different $5^\circ$ latitude intervals ($\theta$ is the middle value of a latitude interval) during 1874–2017. Northern and southern hemispheres’ data are folded. Positive and negative values of meridional velocity represent poleward and equatorward motions, respectively.
Figure 2  Average variation in (a) yearly mean residual rotation rate ($\Delta \langle v_{\text{rot}}(\theta) \rangle$) and (b) in yearly mean meridional velocity ($\langle v_{\text{mer}}(\theta) \rangle$) of sunspot groups in different $5^\circ$ latitude intervals during different phase of Solar Cycles 12–24, determined by superposing the yearly data (the values shown in Figure 1) of the Solar Cycles 12–24 (Cycle 24 is incomplete) according to the years from their respective epochs of maxima (the epochs of maxima are taken as zeros). Northern and southern hemispheres’ data are folded. Positive and negative values of meridional velocity represent poleward and equatorward motions, respectively.
of $\Delta (v_{rot}(\theta))$, within the activity belt. The about 10°- wide slow band (residual rotational velocity is $-70$ to $-80$ m s$^{-1}$) seems to be originated around 35° latitude and the narrow one (only about 5° wide) seems to be originated around 15° latitude during the minimum of a solar cycle, and both migrated toward low latitudes. The latter looks to be ended in the declining phase (before end) of solar cycle. The speed of migration of former is much lower up to middle of the declining phases of solar cycles and then suddenly increased. It should be noted here that there is a major drawback in a superposed epoch analysis. The lengths and rise times of solar cycles are considerably different. The values beyond the epochs $-4$ and 4 (cf. Figure 2(a)) represent a few solar cycles rather than an average of all solar cycles. In a similar analysis, Sudar et al. (2014) did not see any regularity in changes of pattern in $\Delta (v_{rot}(\theta))$ over time.

As can be seen in Figure 2(b) there are bands of equatorward meridional motion ($\langle v_{mer}(\theta) \rangle$ is negative), separated by a band of poleward motion in 26°–29° latitude interval. These bands of equatorward or poleward migrations are not clearly visible. The motions of active regions largely represent the plasma motion in the Sun’s subsurface layers (Howard, 1996). During the whole decline phase and in the center of activity belt the motion is mostly equatorward. Around the maxima of solar cycles at the middle and low latitudes, the motion is mostly poleward, and in 21°–25° and 30°–35° latitudes the motion is equatorward. Close to the beginnings (epoch $-4$) of solar cycles, the motion looks to be mostly poleward at all latitudes. Since the magnetic structures of large/long-lived sunspot groups might anchor relatively deeper than those of small sunspot groups (see Javaraiah and Gokhale, 1997), the equatorward motions of sunspot groups during maxima of solar cycles represent the motions in relatively deeper layers of the Sun than the poleward motions of sunspot groups during minima of solar cycles and mostly in low latitudes. The overall pattern of the motion is somewhat consistent with the concept of flux-transport dynamo models (Dikpati and Gilman, 2006). Sivaraman et al. (2010) analyzed the Mt. Wilson and Kodaikanal sunspot-group data and found the equatorward motion in all latitudes. These authors have used only first days’ data of sunspot groups, whereas here we have used the data in all days during the life times of the sunspot groups. Zhao and Kosovichev (2004) by employing a time–distance technique of helioseismology found that the residual meridional flows (the flows subtracted by the mean meridional flow profile of 1996) converged toward the solar activity belts. Recently, Sudar et al. (2014) noted that the meridional motion of sunspot groups is directed towards the zone of solar activity. This result is largely confirmed here around the maxima of solar cycles (see Figure 2(b)), however, the poleward motions (4–5 ms$^{-1}$) are much weaker than the equatorward motions (8–10 ms$^{-1}$). Howard (1991) found that sunspots groups tend to move away from the central latitude of activity. This is not found here.

3.2. **Comparison of Variations in $\Delta (v_{rot}(\theta))$ with Torsional Oscillations**

The so-called torsional oscillation discovered by Howard and LaBonte B. J (1980) from Mt. Wilson velocity data during the period 1967–1980 consists of alternating bands of faster (or slower) than average rotation moving from high latitudes towards the equator in $\approx 22$-year time. The maximum amplitude of the torsional oscillation is about 5 m s$^{-1}$. The faster-than-average rotation band is located on the equatorward side of the magnetic activity belt and a slower-than-average rotation band is located on its poleward side. Here, obviously, both the slower and faster than average rotation, i.e., bands of negative and positive values of $\Delta (v_{rot}(\theta))$, exist within the activity belt. The torsional oscillation pattern was also found from full-disk magnetograms but with the maximum of amplitude of the magnetic pattern is
twice as large as that of the Doppler pattern (Snodgrass, 1991; Komm, Howard, and Harvey, 1993b). The maximum absolute value of $\Delta \langle v_{rot}(\theta) \rangle$, that was determined here from sunspot-group data, is much larger than the amplitude of the velocity as well as the magnetic torsional oscillations. It is well-known that magnetic structures of sunspots are rotating 2–3% faster than the surrounding plasma (see Javaraiah and Komm, 2002). This is commonly interpreted as the magnetic structures anchored at different depths in the convective zone, being coupled to layers rotating at a different velocity (see Javaraiah and Gokhale, 1997). We know from helioseismology that there is a rotation gradient just below the surface (Antia, 2002).

It has been claimed that the torsional pattern is present in sunspot rotation (Godoli and Mazzuconi, 1982; Tuominen, Tuominen, and Kyöläinen, 1983). Tuominen, Tuominen, and Kyöläinen (1983) analyzed longitudinal and latitudinal motions of recurrent sunspot groups using Greenwich data during 1874–1976. Although their time resolution was coarse, they found some evidence for the 11-yr oscillation in the sunspot zones with an amplitude of a few m s$^{-1}$. Gilman and Howard (1984) and Balthasar, Vázquez, and Wöhl (1986) observed faster and slower bands, but with no clear migratory character. Ternullo (1990) found an evidence of equatorward moving bands of torsional oscillation through a very careful study of the sunspot drawings made during Cycle 21 at Catania Astrophysical Observatory. The latitude–time dependent pattern of $\Delta \langle v_{rot}(\theta) \rangle$ that is seen in Figures 1(a) and 2(a) is largely similar to that aforementioned pattern. Ternullo (1990) used only the data of old sunspot groups, i.e., for each sunspot group only the data collected from the 4th day of observation until the last observation available have been taken into account. Here we have used all the available data of sunspot groups whose life times were 2–12 days.

Meunier, Nesme-Ribes, and Collin (1997) analyzed the rotation of photospheric faculae obtained at Meudon throughout Cycle 19 (1954–1964) and found bands of faster and slower rotation rates with an amplitude of a few meters per second similar to the torsional oscillations. Makarov, Tlatov, and Callebaut (1997) studied the long-term variations of the differential rotation of the solar large-scale magnetic field using synoptic Hα maps in the latitude zone from $+45^\circ$ to $-45^\circ$ in the period of 1915–1990. In each solar cycle, they found a band of faster or slower than average rotation moving from high to low latitudes. The slow band roughly corresponds to the location of magnetic activity, which is in a large extent similar to the patterns in $\Delta \langle v_{rot}(\theta) \rangle$ seen here.

### 3.3. Correlation Between $\langle v_{mer}(\theta) \rangle$ and $\Delta \langle v_{rot}(\theta) \rangle$

Figure 3 shows the correlation between $\langle v_{mer}(\theta) \rangle$ and $\Delta \langle v_{rot}(\theta) \rangle$ determined from the whole data gone in Figures 1(a) and 1(b) (the combined data of sunspot groups during Solar Cycles 12–24). As can be seen in this figure, a statistically significant anticorrelation between $\langle v_{mer}(\theta) \rangle$ and $\Delta \langle v_{rot}(\theta) \rangle$ exists (also see Sudar et al., 2014). The correlation coefficient ($r$) is $-0.1$, which is significant at the 99.9% confidence level ($N = 1388$, Students’ t: $\tau = 3.73$, $P = 0.001$). The corresponding linear-least-squares fit is reasonably good in the sense that the slope ($-0.028 \pm 0.008$) is about 3.5 times larger than its standard deviation. Based on the sign convention used for meridional velocities of sunspot groups (see Section 2 above), the negative value of the slope can be considered as a measure of the strength of angular momentum transport toward the equator.

We also determined the correlation between $\langle v_{mer}(\theta) \rangle$ and $\Delta \langle v_{rot}(\theta) \rangle$ at each epoch (year) in Figures 2(a) and 2(b), i.e., during the average solar cycle of the superposed data of Solar Cycles 12–24. We have done linear-least-squares fit to the corresponding data. In Table 1 we have given the values of the corresponding parameters, namely correlation coefficient ($r$), the slope of best-fit linear relationship, etc. In many years, the value of $r$ is found
Figure 3  Scatter plot of meridional velocity $\langle v_{mer} (\theta) \rangle$ from $-20$ to $+20$ m s$^{-1}$ versus residual rotation rate $(\Delta \langle v_{rot} (\theta) \rangle) \approx -140$ to $+140$ m s$^{-1}$ determined by the combined data of sunspot groups during Solar Cycles 12–24 $(r = -0.1$, number of data points $N = 1388$, Student’s $\tau = 3.73$, $P = 0.001$). Positive values of meridional velocity indicate poleward motions and negative values indicate equatorward motions. The continuous line (red) represents the linear best-fit (slope $= -0.028 \pm 0.008$).

Table 1  The values of intercept ($C$) and slope ($D$) of the linear relationship between $\langle v_{mer} (\theta) \rangle$ and $\Delta \langle v_{rot} (\theta) \rangle$ at each epoch (year) determined from the yearly mean values of Solar Cycles 12–24 superposed according to the years from their respective epochs of maxima, i.e., determined from the values shown Figure 2. The corresponding values of the correlation coefficient ($r$), Students’ $t$ ($\tau$), probability ($\text{Prob}$), and number of data points ($N$) are also given. The corresponding values of all these parameters determined from the combined values of $\langle v_{mer} (\theta) \rangle$ and $\Delta \langle v_{rot} (\theta) \rangle$ of all the latitude intervals during the entire period, from $-6$ to $8$, are given in the last row.

| Epoch | $C$  | $D$  | $r$  | $\tau$ | $\text{Prob}$ | $N$ |
|-------|------|------|------|--------|--------------|-----|
| $-6$  | $-0.67 \pm 5.13$ | $-0.32 \pm 0.18$ | $-0.87$  | $1.80$  | $0.838$ | $5$  |
| $-5$  | $0.74 \pm 1.69$  | $0.06 \pm 0.04$  | $0.26$  | $1.30$  | $0.896$ | $27$ |
| $-4$  | $1.93 \pm 1.04$  | $-0.06 \pm 0.03$ | $-0.26$ | $2.13$  | $0.982$ | $69$ |
| $-3$  | $-0.83 \pm 0.77$ | $0.00 \pm 0.02$  | $0.02$  | $0.18$  | $0.573$ | $124$|
| $-2$  | $1.33 \pm 0.70$  | $-0.01 \pm 0.02$ | $-0.03$ | $0.37$  | $0.643$ | $148$|
| $-1$  | $0.19 \pm 0.56$  | $-0.04 \pm 0.02$ | $-0.15$ | $1.90$  | $0.970$ | $169$|
| $0$   | $-0.09 \pm 0.52$ | $-0.07 \pm 0.02$ | $-0.22$ | $2.98$  | $0.998$ | $172$|
| $1$   | $0.72 \pm 0.54$  | $0.00 \pm 0.03$  | $-0.01$ | $0.07$  | $0.530$ | $161$|
| $2$   | $-0.75 \pm 0.58$ | $-0.13 \pm 0.03$ | $-0.39$ | $5.05$  | $1.000$ | $149$|
| $3$   | $-0.76 \pm 0.63$ | $-0.05 \pm 0.03$ | $-0.13$ | $1.46$  | $0.927$ | $127$|
| $4$   | $-1.11 \pm 0.73$ | $-0.08 \pm 0.03$ | $-0.26$ | $2.86$  | $0.997$ | $113$|
| $5$   | $0.32 \pm 0.83$  | $-0.04 \pm 0.04$ | $-0.12$ | $1.06$  | $0.855$ | $84$ |
| $6$   | $0.64 \pm 1.01$  | $0.05 \pm 0.03$  | $0.23$  | $1.95$  | $0.972$ | $70$ |
| $7$   | $1.83 \pm 2.39$  | $0.15 \pm 0.07$  | $0.44$  | $1.98$  | $0.967$ | $20$ |
| Whole | $-0.27 \pm 0.74$ | $-0.085 \pm 0.028$ | $-0.40$ | $3.06$  | $0.998$ | $50$ |

to be significant at above 95% confidence level ($\text{Prob.} \geq 0.95$, i.e., $P \leq 0.05$), obtained from Student’s t-test. The correlation of the combined data of all epochs is also found to be statistically significant (see the last row in Table 1). Figure 4 shows the variation in the slope during the average solar cycle. If we exclude the point at the epoch $-6$, a 11-year period
cycle pattern (anticorrelation with solar cycle) seems to present in the slope, suggesting that there exists a relationship between the slope and activity during solar cycles (also see Sudar et al., 2014). However, the positive value of the slope that corresponds to the minimum epochs of a few long cycles have a large uncertainty (σ, standard deviation).

Figure 5(a) shows the variation in the slopes of the linear relationships of \( \langle v_{\text{mer}}(\theta) \rangle \) and \( \Delta \langle v_{\text{rot}}(\theta) \rangle \) determined from the data in 3-year MTIs during the period 1874–2017. As in some of our earlier analyses (e.g., Javaraiah and Komm, 1999) we have revised the time series of the slopes in 3-year MTIs by replacing those values of the slopes having σ greater than 1.7 times the median-σ with the mean of their respective neighbor values. They are at 1878, 1896, 1907, 1908, 1932, 1952, 1974, 2005, 2006, 2008, 2009, and 2010. (No value of σ is found to be greater than 2.5 times the median-σ.) In Figure 5(a) we have also shown the variation in yearly mean values of SN during 1875–2016 (used the file SN_y_tot_v2.0.txt that was downloaded from www.sidc.be/silso/datafiles). In Figure 5(b) we have shown the cross-correlation between the slope and SN. As can be seen in Figure 5(a), there exist variations of the order of 11 years. In fact, we get a reasonably good anticorrelation \( r = -0.27 \), Student’s \( t = 3.28, p = 0.001 \) between the slope (revised data) and SN, suggesting that there exists a relationship between the slope and activity during solar cycles. The 3–4 cycles (30–40 years) periodic variations are relatively strong during the times of early solar cycles. The existence of strong \( \approx 11 \)-year period variation in the cross-correlation coefficient (see Figure 5(b)) strongly indicates the existence of a strong relationship between the slope and solar cycle. The peak at \( \text{lag} = 4 \) implies that the slope leads SN by about four years. There is also a relatively large peak at \( \text{lag} = 9 \) (largest negative value of the cross-correlation coefficient) suggesting that the slope leads SN by about nine years. The peak at \( \text{lag} = 4 \) suggests that the slope around the preceding minimum of a solar cycle may be related to the strength of activity around the maximum of the same solar cycle. A large negative/positive (less negative) slope at the minimum of a solar cycle probably indicates that the solar cycle will have a small/large maximum. This is to some extent similar to the relationship between the strength of polar fields at minimum of a solar cycle and the solar cycle maximum (Schatten et al., 1978). The peak at \( \text{lag} = 9 \) suggests that a large negative/positive slope at the epoch after 1–2 years from the maximum of a solar cycle \( n \) indicates a small/large amplitude for the next solar cycle \( n + 1 \). This is to some extent similar to the relationship between the sum of the areas of the sunspot groups in the equatorial latitudes just after about one-year from the maximum epoch of Solar Cycle \( n \) and the maximum of Solar Cycle \( n + 1 \) (Javaraiah, 2007). We checked whether we can use the aforementioned relationships of the slope and SN for predicting the maximum of Solar Cycle 25. It was found impossible. That is, we get a very large uncertainty in the predicted value. The aforementioned relations of the slope and SN indicate that large poleward flows
Figure 5 (a) Plot of the values (cross) of the slopes of the linear best-fits of the meridional velocity ($\langle v_{\text{mer}}(\theta) \rangle$) and residual rotation ($\Delta \langle v_{\text{rot}}(\theta) \rangle$) determined from sunspot group data in 3-year MTIs: 1874–1876, 1875–1877, . . ., 2015–2017, versus middle years of these intervals. The dashed curve (red) represents the revised data after those values having $\sigma$ (standard deviation) greater than $1.7 \times$ the median $\sigma$ were replaced with mean of their respective neighbor values. The continuous curve represents the variation in yearly mean values of SN. The Waldmeier numbers of the solar cycles and the corresponding epochs of the minima and the maxima of solar cycles are also shown by the symbols $m$ and $M$, respectively. The horizontal continuous line represents the mean value over the whole period. (b) Plot of the coefficient of cross-correlation (CC) between the slope and SN versus lag.

of magnetic flux (of decaying active regions) during the declining phase of a solar cycle may enhance the strength of polar fields at the following minimum. Large equatorward flows (including down flows at active regions) of magnetic flux may enhance the strength of activity in the equatorial latitudes just after one year from the maximum of the solar cycle.

In Figure 6 we compare the Morlet wavelet spectra of the variations in the slope and SN shown in Figure 5(a). Obviously, 11-year periodicity is strongly present in SN throughout the analyzed data window. This periodicity seems to be present in the slope almost throughout the data window, but it was very weak during 1920–1940. The 16–32-year periodicities were strongly present in the slope during the time of early solar cycles. Figure 7 shows the cross-wavelet and wavelet-coherence spectra of the slope and SN. The cross-wavelet spectrum (Figure 7(a)) suggests that there exists a statistically significant similarity in the temporal behaviors of the slope and SN only during the period 1950–1970, because outside this interval the slope has slightly smaller than 11-year periodicity (see Figure 6(a)). In Figure 7(b) there is a suggestion of the existence of strong coherence between the $\approx 11$-year variations of the slope and SN before 1940 and 1980-onward. There is also a suggestion of the existence of coherence between the 20–30-year periodicities of the slope and SN from 1940-onward. There are episodes of 3–5-year periodicities in both the slope and SN between 1880 and 2016. There is a suggestion of the coherence in 40–50-year periodic variations in the slope and SN, but this signal is within the area of cone-of-influence. That is, the signal of this periodicity is not well resolved, hence this periodicity cannot be detected here due to inadequate data. Figure 8 shows the spectra of the wavelet-phase difference of the slope and SN. As can be seen in this figure, there exists $\approx -180^\circ$ phase difference between the 11-year period variations of the slope and SN before 1900 (the slope seems to be lead SN), during the periods 1940–1960 and 1980–2000. A 20–30 year period variations of the slope and SN seems to be having $\approx -180^\circ$ phase difference throughout 1875–2016. This is con-
Figure 6  (Upper panel) Wavelet power spectrum (left) and global spectra (right) of the slope of linear relationship between the meridional velocity ($\langle v_{mer} (\theta) \rangle$) and residual rotation ($\Delta \langle v_{rot} (\theta) \rangle$) of sunspot groups during 1875–2016 shown in Figure 5(a). (Lower panel) Wavelet power spectrum (left) and global spectrum (right) of the annual mean SN during 1875–2016. The wavelet spectra are normalized by the variances of the corresponding time series. The shadings are at the normalized variances of 1.0, 3.0, 4.5, and 6.0. The dashed curves represent the 95% confidence levels deduced by assuming a white-noise process. The cross-hatched regions indicate the cone of influence where edge effects become significant (Torrence and Compo, 1998).

sistent with the anticorrelation between the slope and SN found above. There seem to be considerable phase differences also in the aforementioned remaining all periodic variations. A 90° difference indicates that there is a considerable phase mixing in the corresponding variations of the slope and SN.

3.4. Cycle-to-Cycle Variation in the Slope

In Table 2 we have given the values of the intercept and the slope of the linear relationship between $\langle v_{mer} (\theta) \rangle$ and $\Delta \langle v_{rot} (\theta) \rangle$ determined from the data during each of Solar Cycles 12–24, and also from the combined data of all cycles. The corresponding values of the correlation coefficient, Student’s t, and probability are given. The cycle interval, number of data points (N), and maximum version-2 sunspot number ($R_M$) are also given. In Figure 9
Figure 7 Panels (a) and (b) show the cross-wavelet power spectrum and wavelet-coherence spectrum, respectively, of sunspot number (SN) and the slope of the linear relationship between meridional velocity ($\langle v_{\text{mer}}(\theta) \rangle$) and residual rotation ($\Delta \langle v_{\text{rot}}(\theta) \rangle$) during the period 1875–2016. The shadings are at levels 1.0, 3.0, 4.5, and 6.0. The cross-hatched regions indicate the cone of influence where edge effects become significant (Torrence and Compo, 1998).

Figure 8 Wavelet phase difference between SN and the slope of the linear relationship between meridional velocity ($\langle v_{\text{mer}}(\theta) \rangle$) and residual rotation ($\Delta \langle v_{\text{rot}}(\theta) \rangle$) in 3-year MTIs during the period 1875–2016. The cross-hatched regions indicate the cone of influence where edge effects become significant (Torrence and Compo, 1998).

We have shown the cycle-to-cycle variations in the slope during Solar Cycles 12–24. As can be seen in this figure, although in several solar cycles the correlation is poor and the coefficients of linear best-fits have large uncertainties (also see Table 2), there is a possibility of a considerable variation in the slope on the time scale of about 3–4 cycles, suggesting the existence of a 3–4-cycle periodicity in the slope. In Solar Cycles 15, 18, 19, 20, and 22, the slopes have significant negative values. The value of Solar Cycle 24 is also to some extent negative (this cycle is incomplete). In Solar Cycles 14, 17, and 21, the slopes have positive values, but these values are not significantly different from zero. The equatorward angular momentum transport may be relatively much less (absent) in Solar Cycles 14, 17, and 21. The correlation ($r = -0.28$) between the slope and the amplitude of solar cycle is found to
Table 2 The values of intercept \(C\) and slope \(D\) of the linear relationship between \(\langle v_{\text{mer}}(\theta) \rangle\) and \(\Delta(v_{\text{rot}}(\theta))\) determined from the data during the whole period of each solar cycle and from the combined data of all Solar Cycles 12–24 (the last row). The corresponding values of the correlation coefficient \(r\), Students’ \(t\) \(\tau\), and probability \(Prob\) are given. Cycle interval (Time), number of data points \(N\), maximum \(R_M\) Version-2 sunspot numbers are also given.

| Cycle | Time      | \(R_M\) | \(C\)          | \(D\)          | \(r\)  | \(\tau\)  | \(Prob\) | \(N\) |
|-------|-----------|---------|----------------|----------------|-------|-----------|----------|------|
| 12    | 1878–1889 | 124.4   | 0.781 ± 0.819  | −0.023 ± 0.027 | −0.09 | 0.85      | 0.802    | 101  |
| 13    | 1890–1901 | 146.5   | −0.398 ± 0.859 | −0.028 ± 0.028 | −0.10 | 0.97      | 0.834    | 107  |
| 14    | 1902–1912 | 107.1   | 0.075 ± 0.871  | 0.029 ± 0.029  | 0.11  | 1.01      | 0.843    | 87   |
| 15    | 1913–1922 | 175.7   | −1.422 ± 0.722 | −0.086 ± 0.030 | −0.29 | 2.85      | 0.997    | 93   |
| 16    | 1923–1932 | 130.2   | −0.381 ± 0.724 | −0.026 ± 0.024 | −0.11 | 1.05      | 0.853    | 103  |
| 17    | 1933–1943 | 198.6   | −0.388 ± 0.689 | 0.006 ± 0.026  | 0.02  | 0.22      | 0.587    | 111  |
| 18    | 1944–1953 | 218.7   | −1.000 ± 0.718 | −0.086 ± 0.029 | −0.27 | 2.97      | 0.998    | 113  |
| 19    | 1954–1963 | 285.0   | 0.074 ± 0.698  | −0.047 ± 0.024 | −0.18 | 1.99      | 0.976    | 119  |
| 20    | 1964–1975 | 156.6   | 1.064 ± 0.648  | −0.048 ± 0.023 | −0.18 | 2.07      | 0.980    | 127  |
| 21    | 1976–1985 | 232.9   | −0.436 ± 0.787 | 0.018 ± 0.031  | 0.05  | 0.56      | 0.712    | 119  |
| 22    | 1986–1995 | 212.5   | 0.909 ± 0.724  | −0.050 ± 0.030 | −0.16 | 1.69      | 0.953    | 113  |
| 23    | 1996–2007 | 180.3   | 1.383 ± 0.664  | −0.017 ± 0.030 | −0.05 | 0.56      | 0.713    | 133  |
| 24    | 2008–2017 | 116.4   | 0.476 ± 0.949  | −0.053 ± 0.043 | −0.13 | 1.23      | 0.889    | 88   |
| All   | 1878–2017 | 0.16 ± 0.20 | −0.028 ± 0.008 | −0.1  | 3.73      | 0.9999   | 1388 |

Figure 9 Cycle-to-cycle variation in the slope (open circle-dotted curve) of the linear best-fit of meridional velocity \(\langle v_{\text{mer}}(\theta) \rangle\) and \(\Delta (v_{\text{rot}}(\theta))\) of sunspot groups during Solar Cycles 12–24. The filled circle-continuous curve (red) represents the cycle-to-cycle variation in solar-cycle amplitude \(R_M\).

be insignificant, indicating that there is no relationship between the slope and strength of activity on a long-time scale (longer than 11-year period).

4. Conclusions and Discussion

We analyzed the combined 142 years GPR and DPD sunspot-group data and found that there is a considerable latitude–time dependence in the yearly mean residual rotation rate of sunspot groups. The yearly average residual rotation rate is roughly from \(-120\) m s\(^{-1}\) to \(80\) m s\(^{-1}\). In a large number of solar cycles, the rotation is to some extent weaker during maxima than that of during minima. In many solar cycles there is an indication that bands of
residual rotation rate migrate from high to low latitudes over 8–10-year periods. The analysis of the data during Solar Cycles 12–24 that are folded according to the years from their respective epochs of maxima suggests the existence of alternate bands of slower and faster than average rotation within the activity belt. A $\approx 10^\circ$-wide slow band ($-70$ to $-80 \text{ m s}^{-1}$) seems to be originated around $35^\circ$ latitude and a narrow one (only $5^\circ$ wide) seems to be originated around $15^\circ$ latitude during the minimum of a solar cycle, and both migrated toward low latitudes. However, their relationship with the well-known torsional oscillations is not clear. The maximum absolute value of the residual rotation rate is much larger than the amplitude of the velocity as well as the magnetic torsional oscillations.

There is also a considerable latitude–time dependence in the yearly mean meridional motion of sunspot groups. There are alternate bands of equatorward and poleward motions. Overall, the equatorward motion is dominant with velocity $8–12 \text{ m s}^{-1}$ mostly around maxima of more solar cycles, whereas the poleward motion seems to be relatively weak with velocity only $4–6 \text{ m s}^{-1}$ mostly around minima of more solar cycles. The analysis of the folded data of Solar Cycles 12–24 suggests no clear equatorward or poleward migrating bands of meridional motions.

There exists a statistically significant anticorrelation between the meridional motion and residual rotation. The corresponding linear-least-squares best-fit is reasonably good in the sense that the slope ($-0.028 \pm 0.008$) is about 3.5 times larger than its standard deviation. As per the sign convention used for the meridional velocities of sunspot groups, the significant negative value of the slope can be considered as a measure of the strength of angular momentum transport toward the equator. This result is in a qualitative agreement with previous results for sunspots (Sudar et al., 2014; Sudar et al., 2017). However, our result has a lower amplitude, which might be a consequence of different selection process. We can conclude that the results of the present work generally confirm earlier results (Sudar et al., 2014; Sudar et al., 2017) and represent, together with them, a strong evidence that Reynolds stress is indeed the dominant mechanism for maintaining solar differential rotation via angular momentum transport towards the equator.

The cross-correlation between the slopes determined from the data in 3-year moving time intervals and yearly mean sunspot number (SN) suggests that the slope leads SN by about 4 and 9 years. The Morlet wavelet spectrum of SN suggests, obviously, 11-year periodicity is strongly present in SN throughout the analyzed data window. The Morlet wavelet spectrum of the slope also suggests the existence of $\approx 11$-year periodicity in the slope almost throughout the data window, but it was very weak during 1920–1940. The cross-wavelet spectrum suggests that there exists a statistically significant similarity in the temporal behaviour of the slope and SN only during the period 1950–1970. The wavelet-coherence spectrum suggests that there exists a strong coherence between the $\approx 11$-year variations of the slope and SN before 1940 and 1980-onward. There exists $\approx -180^\circ$ phase difference between the 11-year period variations of the slope and SN before 1900, during 1940–1960, and during 1980–2000. The 20–30-year period variations of the slope and SN seem to be having $\approx -180^\circ$ phase difference throughout 1874–2017. However, this periodicity is very weak in both the slope and SN. The above mentioned result that the slope leads SN by about 4 and 9 years is consistent with the $\approx 180^\circ$ phase difference between the $\approx 11$-year variations and that between the $\approx 20$-year variations of the slope and SN. Overall these results suggest there exits a strong relationship between the slope and amount of activity during a solar cycle. However, the correlation between the cycle-to-cycle modulations in the slope and the amplitude of solar cycle is found to be insignificant, indicating that there is no relationship between the slope and strength of activity on a long-time scale (longer than 11-year period).

Reynolds stress is produced by interaction of convective elements and the Coriolis force which cause a correlation between longitudinal and latitudinal velocity components. The
motion of sunspots represents the motion of somewhat deeper layer of the Sun (see Javaraiah, 2013). Recent local helioseismic studies suggest that patterns of variations in the large scale flows are the same in both quiet and active regions and the flow patterns in active regions associated with deeper layers than those of quiet regions (Komm, Howe, and Hill, 2020). The anticorrelation between the meridional motion and residual rotation of sunspot groups found here implies the existence of couplings between these motions, somewhere in the convection zone. Since Reynolds stress produced can maintain the equatorial angular momentum transport and thus differential rotation (Gilman, 1986), hence there exists correlation between the strength of activity and the slope of linear relationship between the meridional motion and residual rotation during a solar cycle. The existence of a $\approx 180^\circ$ phase difference between the slope and activity during their 11-year and 20–30-year periodic variations implies the existence of couplings between the strength of magnetic activity and the angular momentum transport toward the equator on these time scales. However, our analysis suggests that there is no such relationship on longer than these time scales.

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Declarations

Conflict of Interest The author declares that he has no conflicts of interest.

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