Belief Propagation Methods for Intercell Interference Coordination

Sundeep Rangan, Member, IEEE, Ritesh Madan, Member, IEEE

Abstract—We consider a broad class of interference coordination and resource allocation problems for wireless links where the goal is to maximize the sum of functions of individual link rates. Such problems arise in the context of, for example, fractional frequency reuse (FFR) for macro-cellular networks and dynamic interference management in femtocells. The resulting optimization problems are typically hard to solve optimally even using centralized algorithms but are an essential computational step in implementing rate-fair and queue stabilizing scheduling policies in wireless networks. We consider a belief propagation framework to solve such problems approximately. In particular, we construct approximations to the belief propagation iterations to obtain computationally simple and distributed algorithms with low communication overhead. Notably, our methods are very general and apply to, for example, the optimization of transmit powers, transmit beamforming vectors, and sub-band allocation to maximize the above objective. Numerical results for femtocell deployments demonstrate that such algorithms compute a very good operating point in typically just a couple of iterations.

Index Terms—Interference coordination, cellular systems, wireless communications, belief propagation, femtocells.

I. INTRODUCTION

Interference coordination has re-emerged as a fundamental challenge for next-generation cellular wireless systems. Traditional macrocellular deployments are likely to be supplemented with smaller femtocells and relays, with mixtures of restricted and open access, often deployed in an ad hoc manner [1], [2]. Such deployments may create much stronger and highly variable (in time and space) interference conditions than those experienced in current macrocellular networks, and traditional cellular power and rate control may not be adequate [3]. To address this challenge, a key focus of the current 3GPP LTE-Advanced standardization efforts is on the design of an interference coordination framework for such unplanned cellular deployments of base-stations with widely different transmission powers [4]. Current release of the LTE specification [5] provides simple methods for inter-cell interference coordination (ICIC), see e.g., [6]. Along with the design of mechanisms for ICIC, algorithms to exploit such mechanisms are an active area of research as well, see for example, [7], [8].

Mathematically, interference coordination is a complex distributed optimization problem involving scheduling decisions at the transmitters of multiple interfering links. In this work, we consider a general linear mixing interference model where the scheduling decisions in each link are represented as vector (e.g., transmission powers on different sub-bands in frequency, beamforming weights) and the interference on each link is a linear combination of the scheduling vectors on the other links. Associated with each link at a given time is a utility function which describes the benefit to a link as a function of the scheduling vector from the serving transmitter and interference from the other transmitters. The linear mixing model is extremely general and can apply to a large class of interference models and objectives. Computing maximum weighted matching for queue stability [9] and maximization of sum utility of average rates for fairness [10] are special cases of this formulation.

In the past few years, algorithms for special cases of the above problem have been extensively studied. In many cases, algorithms with provable desired properties have been obtained – for example, there is a rich literature on distributed power control methods to achieve a desired SINR for each link [11] and to maximize a certain class of utility functions of SINRs [12], approximation algorithms for maximum weight matching for combinatorial interference model were obtained in [13], [14], efficient methods to compute optimal beamforming vector for multiuser downlink [15] and maximizing sum utility of SINRs on uplink [16], stabilizing policies for collision sense multiple access (CSMA) type of models based on simulated annealing were derived in [17]. More generally, heuristic algorithms have been constructed to solve certain specific problems approximately in, for example, [18], [19]. While these algorithms perform well in practice in spite of no provable guarantees, the insights and approximations used to obtain these algorithms are very specific to the problem under consideration.

In this paper, we make the following contributions: BP Framework: We consider a belief propagation (BP) framework for a very general wireless scheduling and interference coordination optimization problem. The underlying optimization problem is posed as a problem of estimating marginals of a joint probability distribution. BP provides a systematic and general approach to obtain distributed algorithms; it can be used with arbitrary nonlinear utility functions and scheduling vectors sets, which enable the algorithm to be applied a range of complex scheduling problems including power control, subband scheduling and distributed beamforming. Also, while we do not obtain any theoretical guarantees, in practice a few iterations of BP generate a good operating point. Thus, it typically has faster convergence than gradient based algorithms (e.g.,10s of iterations in [16]) or simulated annealing [20].

Approximation Algorithms: It is well known that implementing BP for distributed optimization problems entails high compu-
tational complexity and communication overhead. Exploiting the linear mixing interference model and applying Gaussian and first-order approximations similar to [21]–[25], we develop an approximate BP method that has low complexity, distributed implementation and minimal messaging. Along many links, messages can be carried in small payloads and can be broadcast without separate unicast transmissions, which is particularly crucial for wireless systems. Moreover, the resulting algorithm has a natural interpretation has a “soft” RTS / CTS type handshaking. The approximate BP algorithm is also similar to the recent approximate message passing (AMP) algorithm in [24], [26] and this connection may be useful for further analysis.

Numerical Results: Through simulations for femtocell deployments, we demonstrate that approximate BP provides good performance for sub-band and power allocation to maximize utilities of rates, sub-band and power allocation to maximize a weighted sum of rates, and beamforming optimization to maximize utilities of average rates. Also, although the BP algorithm requires multiple exchange of messages before each scheduling decision, our simulations indicate good performance with only two rounds of messaging. Thus, the approximate BP approach is a promising paradigm for new emerging cellular deployments with large interference variations in time and space compared to current predominantly macro-only deployments.

A. Previous Work

Algorithms for ICIC in LTE macrocells have been considered in a large number of works in both the uplink and downlink [27]–[29]. These works are generally based on adaptive subband scheduling and fractional frequency reuse (FFR) methods [18], [30] and exploit statistics over large numbers of mobiles per macrocell. Interference mitigation in femtocells has focused on similar techniques as well as frequency planning, power control [31], or semi-static resource allocation [7], [8], [32]. As we will demonstrate in the simulations, the BP methods presented here can also be used for adaptive subband scheduling as a special case of the linear mixing interference model. Also, much of the ICIC work has considered slowly varying allocations that don’t change over few 10s to 100s of milliseconds. Due to the low messaging overhead, it is possible that approximate BP can also be used for more dynamic interference management in femtocell deployments, where there is high variability in load and interference from one timeslot to another.

For scheduling based on more dynamic traffic statistics such as queue lengths and head-of-line delays, variants of maximum weight scheduling can be used [33], [34]. Unfortunately, computing a maximum weight schedule is generally NP-hard, and much work has thus focused on approximate algorithms. In addition to the works mentioned above, the works [33] and [35] proposed randomized linear complexity (but centralized) algorithms, and [36]–[38] present simple distributed algorithms for combinatorial interference models. Greedy maximal weight matching for such interference models has been considered in [39]–[41]. However, many of the above works apply a hard constraint interference models, where neighboring links cannot transmit simultaneously. Cellular systems in contrast permit multiple interfering links to transmit simultaneously and then use rate control to adapt to the resulting signal-to-interference and noise ratio (SINR). Thus, the degradation in rate with interference is gradual, and are difficult to capture in the combinatorial interference model. In contrast, “soft” interference effects can be easily modeled in the BP utility framework.

However, we note that there is an important theoretical connection between the methods in [36], [37] and the BP method considered here. As we will discuss below, BP arrives at the scheduling decision by estimating the marginals of a certain joint probability distribution function given in [7]. The CSMA-type methods in [36], [37] can be seen as a simulated annealing (SA) method for selecting a random scheduling vector from precisely the same distribution in the context of a constraint combinatorial interference model. SA can be seen as an asymptotically exact but slow method [20] for solving the optimization problem. In contrast, BP is approximate, but potentially faster.

General overviews of BP can be found in a number of works including [42], [43]. In the context of wireless scheduling, theoretical guarantees have been obtained for off-channels and the combinatorial contention graph model [44], [45]. The methods here can be seen as a generalization of these methods to soft interference models with larger class of scheduling vectors.

II. PROBLEM FORMULATION

We consider a wireless scheduling problem with \( n \) links. The transmitter of each link \( j = 1, \ldots, n \), denoted \( TX_j \), must select some scheduling vector \( x_j \in \mathcal{X} \subseteq \mathbb{R}^{n_s} \), which contains \( n_s \) parameters related to link \( i \). Examples of these parameters will be given below. The selection of the scheduling vectors results in an interference vector \( z_i \in \mathbb{R}^{n_s} \) at the receiver of each link \( i \), denoted \( RX_i \). The interference is assumed to be a linear function of the scheduling vectors of the other links,

\[
    z_i = \sum_{j=1}^{n} A_{ij} x_j
\]

for some matrices \( A_{ij} \in \mathbb{R}^{n_s \times n_s} \). We assume that \( A_{ii} = 0 \), so that link \( i \) does not interfere with itself. We will let \( x \) and \( z \) be the column vectors with entries \( x_j \) and \( z_i \),

\[
    x = [x_1 \cdots x_n]' \in \mathbb{R}^{n \times n_s} \quad \text{and} \quad z = [z_1 \cdots z_n]' \in \mathbb{R}^{n \times n_s},
\]

and write \( z = Ax \) where \( A \) is the block matrix with entries \( A_{ij} \). We call \( A \) the interference matrix. Also, we let \( A_i \) denote the ith column of the \( A \) so that \( z_i = A_i x \).

Associated with each link \( i \), is some utility function \( f_i(x_i, z_i) \) of the scheduling vector \( x_i \) and interference vector \( z_i \). The scheduling problem is to maximize the overall utility

\[
    \max_{x} F(x), \quad F(x) = \sum_{i=1}^{n} f_i(x_i, z_i). \tag{2}
\]
We will sometimes call the optimization problem (2), an optimization with linear mixing to stress the linear dependence of the interference on the transmit vectors.

III. LINEAR MIXING UTILITY EXAMPLES

The linear mixing formulation above is extremely general and can incorporate a large class of utility functions and interference models.

A general treatment of utility functions for wireless scheduling can be found in [46], [47]. In our simulations, we will consider utility maximization for both static and time-varying problems. For static optimization, the scheduling vectors \( x_i \) are selected once for a long time period and the utility function is typically of the form

\[
B_i(x_i, z_i) = U_i(R_i(x_i, z_i)),
\]

where \( R_i(x_i, z_i) \) is the long-term rate as a function of the TX vector \( x_i \) and interference \( z_i \), and \( U_i(R) \) is the utility as a function of the rate. The problem formulation above can incorporate any of the common utility functions including: \( U_i(R) = \log(R) \) which results in a sum rate optimization; \( U_i(R) = \log(R) \) which is the proportional fair metric and \( U_i(R) = -\beta R^{-\beta} \) for some \( \beta > 0 \) called an \( \beta \)-fair utility. Penalties can also be added if there is a cost associated with the selection of the TX vector \( x_j \) such as power.

To accommodate time-varying channels and traffic loads, many cellular systems enable fast dynamic scheduling in time slots in the order of 1 to 2 ms. For these systems, the utility maximization can be re-run in each time slot. One common approach is that in each time slot \( t = 0, 1, 2, \ldots \), the scheduler uses a utility of the form

\[
w_i(t) = w_i(t) R_i(t, x_i, z_i),
\]

where \( w_i(t) \) is a time-varying weight given by the marginal utility

\[
w_i(t) = \frac{\partial U_i(R_i(t))}{\partial R},
\]

and \( R_i(t) \) is exponentially weighted average rate updated as

\[
R_i(t+1) = (1-\alpha) R_i(t) + \alpha R_i(t, \bar{x}_i(t), \bar{z}_i(t)),
\]

where \( \bar{x}_i(t) \) is the TX vector and \( \bar{z}_i(t) \) is the interference for link \( i \) at time \( t \). Any maxima of the optimization (2) with the weighted utility (3) is called a maximal weight matching. A well-known result of stochastic approximation [10] is that if \( \alpha \to 0 \), and the scheduler performs the maximum weight matching with the marginal utilities (5), then for a large class of processes, the resulting average rates will maximize the total utility \( \sum_i U_i(R_i(t)) \).

The above utilities are designed for infinite backlog queues. For delay sensitive traffic, one can take the weights \( w_i(t) \) to be the queue length or head-of-line delay. Maximal weight matching performed with these weights generally results in so-called throughput optimal performance [9]. These results also apply to multihop networks with the so-called backpressure weights.

In addition to incorporating general utilities, an appealing feature of the linear mixing framework is that a large class of interference models can also be considered, including, for example:

- **Flat fading with power control:** In this case, \( x_j \) is a scalar representing the transmit power, and \( A_{ij} \) is the gain from TX \( j \) to RX \( i \), so that \( z_{ij} \) is the total interference at RX \( i \). The rate, \( R_i(x_i, z_i) \) can then be described as a function of the SINR \( g_{ij}(z_{ij}) \), where \( g_i \) is the channel gain along link \( i \). Arbitrary SINR to rate mappings may be used. Note that a special case of on-off channels where \( x_{ij} \) is zero or a maximum transmit power can be used.

- **Multiple subbands:** The above example is easily extended to the case of multiple subbands. As described in the Introduction, subband scheduling is one of the key motivating features of LTE, but the optimization is difficult. To handle multiple subbands, we simply let \( x_j \) and \( z_i \) be the vectors of transmit and interference powers in each subband and \( A_{ij} \) be a diagonal matrix with channel gains in each subband.

- **Beamforming and linear precoding:** The linear mixing formulation can also incorporate problems with transmit beamforming or linear precoding. For example, suppose a link has \( N \) transmit antennas and one receive antenna. If each transmitter TX \( j \) uses a beamforming vector \( b_j \in \mathbb{C}^N \), and \( g_{ij} \in \mathbb{C}^N \) is the channel from TX \( j \) to RX \( i \), the interference at RX \( i \) is given by

\[
z_i = \sum_{j \neq i} g_{ij} b_j b_j^* g_{ij},
\]

which is linear in the rank one matrices \( b_j b_j^* \). Hence, if we let \( x_j \in \mathbb{C}^{N^2} \) be the column vector with entries of the matrix \( b_j b_j^* \), the interference \( z_i \) can be represented as a linear combination of the vectors \( x_j \). The idea can also be generalized to precoding matrices with multiple transmit streams.

IV. BELIEF PROPAGATION

A. Standard BP

We begin by briefly reviewing how we would apply standard BP to the optimization (2). Let \( u > 0 \) and define the probability distribution

\[
p(x) = \frac{1}{Z} \exp(u F(x)) = \frac{1}{Z} \prod_{i=1}^{n} \exp(u f_i(x_i, z_i)),
\]

where \( Z \) is a normalization constant called the partition function (it is a function of \( u \)). BP can be seen as a method to estimate the marginal distributions of the distribution \( p(x) \) with respect to the variables \( x_i \). From these marginals, one can compute the marginal expectations \( \hat{x}_j = \mathbb{E}(x_j) \). A standard result of large deviations [8] is that as \( u \to \infty \), under suitable conditions, \( p(x) \) concentrates around the maxima of \( F(x) \) and

\[
\lim_{u \to \infty} \hat{x}_j = \arg \max_x F(x).
\]

So, if we can estimate the marginal expectations of the probability distribution (7) for large \( u \), we can recover a good estimate for the maximization of (2).
To compute the marginal distributions, BP associates with the interference matrix $A$ a bipartite graph $G = (V, E)$ called the factor or Tanner graph. The vertices $V$ consists of $n$ transmitter nodes associated with the transmitters $TX$ $j$, and $n$ receiver nodes associated with the receivers $RX$ $i$. There is an edge $(i,j) \in E$ if and only if $i = j$ or $A_{ij}$ is non-zero – that is $TX$ $j$ has some influence on the interference or signal at $RX$ $i$. We let $N_{tx}(i)$ and $N_{rx}(j)$ be the neighbors sets of the nodes $RX$ $i$ and $TX$ $j$ in graph $G$, respectively.

With this graph, BP iteratively passes beliefs along the edges of the graph that represent estimates of the marginal distributions of $p(x)$ with respect to the variables $x_j$. In the context of the wireless scheduling problem, we can interpret the iterations as rounds, where computations are first performed at the receivers and then at the transmitters. We index the round by $t$, and let $p_{i\rightarrow j}(t, \cdot) : \mathcal{X} \mapsto \mathbb{R}$ denote the belief message from $RX$ $i$ to $TX$ $j$ in the receiver half of the round. The reverse belief message from $TX$ $j$ to $RX$ $i$ is denoted $p_{j\rightarrow i}(t, \cdot) : \mathcal{X} \mapsto \mathbb{R}$. $p_{i\rightarrow j}(t, x_j)$ and $p_{j\rightarrow i}(t, x_j)$ denote the values of the beliefs at $x_j$. After some fixed number of rounds, the algorithm is stopped and a final scheduling decision, meaning a selection of the $TX$ vectors $x_j$, is made by the transmitters. The steps for BP are as follows:

1) **Initialization:** Set $t = 0$ and for all $(i,j) \in E$, let $p_{i\rightarrow j}(t, x_j)$ be some initial distribution on $x_j$. This distribution could be, for example, the uniform distribution on the set $\mathcal{X}$.

2) **RX node update:** In the RX half of the round, each RX $i$ sends a belief message to the transmitters $TX$ $j$ with $j \in N_{tx}(i)$ given by

$$p_{i\rightarrow j}(t, x_j) = \mathbb{E}[\exp(u_{fi}(x_i, z_i)) | x_j],$$

where $z_i = A_i x$ as in (1) and the expectation is over independent $x_k \sim p_{t\rightarrow k}(t, x_k), \forall k \in N_{tx}(i)$.

3) **TX node update:** In the TX half of the round, each TX $j$ sends a belief message back to the receivers RX $i$ with $i \in N_{rx}(j)$ given by

$$p_{i\rightarrow j}(t + 1, x_j) = \frac{1}{Z} \prod_{t' \in N_{tx}(i) \neq i} p_{t\rightarrow j}(t, x_j),$$

where $Z$ is a normalization constant and the product is over all $t' \in N_{tx}(i)$ with $t' \neq i$. The iteration number is incremented, $t = t + 1$, and we return to step 2 until a sufficient number of rounds have been performed.

4) **Final solution:** The final estimate for the marginal distribution of $x_j$ is given by

$$p_j(t + 1, x_j) = \frac{1}{Z} \prod_{i \in N_{rx}(j)} p_{i\rightarrow j}(t, x_j).$$

The scheduling vector can be selected as the maximum of this marginal distribution.

In the case when the graph $G$ has no cycles, it can be shown that the $p_j(t, x_j)$ converges to the true marginal distribution of $p(x)$ in (1) with respect to the variable $x_j$. However, for general $G$, BP is approximate. A complete treatment of BP is beyond the scope of this work – the reader is referred to the references above.

Implementation of the above BP algorithm in wireless networks is challenging due to two reasons:

- **High computational complexity:** The expectation in (8) requires integration over all the variables $x_r$ with $r \in N_{tx}(i)$ and $r \neq j$. This computation grows exponentially in $|N_{tx}(i)|$, which is the number of transmitters interfering with $RX$ $i$. If this set is large, the computation is prohibitive.

- **High messaging overhead:** Passing the beliefs requires unicast messages between each RX and each TX (which are neighbors as per graph $G$) as opposed to single broadcast message. Also, in each round $t$, the messages comprise of the beliefs $p_{i\rightarrow j}(t, x_j)$ and $p_{j\rightarrow i}(t, x_j)$ for all values $x_j \in \mathcal{X}$. If $\mathcal{X}$ is large, the messaging overhead may be significant, and it grows with the number of transmitters interfering at a receiver.

### B. Gaussian Approximation

We first consider the simplification of the RX node update (5). We describe the simplification in log domain. Let $\Delta_{i\rightarrow j}(t, \cdot)$ and $\Delta_{i\rightarrow j}(t, \cdot)$ be log likelihood functions, meaning any functions such that

$$\Delta_{i\rightarrow j}(t, x_j) := \frac{1}{u} \log [p_{i\rightarrow j}(t, x_j)] + \text{const}, \quad (11a)$$

$$\Delta_{i\rightarrow j}(t, x_j) := \frac{1}{u} \log [p_{i\rightarrow j}(t, x_j)] + \text{const}. \quad (11b)$$

where the constants do not depend on $x_j$ (although they may depend on $t$ and the indices $i$ and $j$). Observe that we can recover the probabilities from the log likelihoods by the relation

$$p_{i\rightarrow j}(t, x_j) \propto \exp [u\Delta_{i\rightarrow j}(t, x_j)]. \quad (12a)$$

$$p_{i\rightarrow j}(t, x_j) \propto \exp [u\Delta_{i\rightarrow j}(t, x_j)]. \quad (12b)$$

We now consider the simplification of the RX node update (5) under two cases: when $i = j$ and when $i \neq j$. We begin with the case when $i = j$. The expectation in (5) is to be evaluated with $z_i$ given by (1) with the variables $x_j$ being independent and $x_j \sim p_{i\rightarrow j}(t, x_j)$. Let $\tilde{s}_{i\rightarrow j}(t)$ and $Q_{i\rightarrow j}(t)$ be the mean and $1/u$ times the variance of the distribution $p_{i\rightarrow j}(t, \cdot)$. Then, under the simplifying assumption that the summation in (1) consists of a large number of independent terms, we can apply the Central Limit Theorem and approximate the distribution of $z_i$ with

$$z_i = N(\tilde{s}_{i\rightarrow j}(t), Q_{i\rightarrow j}(t))/u, \quad (13)$$

where

$$\tilde{s}_{i\rightarrow j}(t) = \sum_{j \in N_{tx}(i)} A_{ij} \tilde{x}_{i\rightarrow j}(t) \quad (14a)$$

$$Q_{i\rightarrow j}(t) = \sum_{j \in N_{tx}(i)} A_{ij} Q_{i\rightarrow j}(t) A_{ij}^T \quad (14b)$$

which have the interpretation as a mean and variance of the interference at $RX$ $i$. Applying the Gaussian approximation (13) to the expectation (5) shows that we can write $\Delta_{i\rightarrow i}(\cdot)$ in (10a) as

$$\Delta_{i\rightarrow i}(t, x_i) \approx \frac{1}{u} \log [Z_{ii}(x_i, \tilde{s}_{i\rightarrow i}(t), Q_{i\rightarrow i}(t))], \quad (15)$$
where $Z_{i0}(\cdot)$ is the partition function

$$Z_{i0}(x_i, s_i, Q_{ii}^s) = \int \exp \left[uL_{i0}(x_i, z_i, s_i, Q_{ii}^s)\right] dz_i,$$

(16)

and

$$L_{i0}(x_i, z_i, s_i, Q_{ii}^s) = f_i(x_i, z_i) - \frac{1}{2}(z_i - \hat{s}_i)^T Q_{ii}^{-s}(z_i - \hat{s}_i)$$

(17)

and $Q_{ii}^{-s}$ is the matrix inverse of $Q_{ii}$.

Next consider the case $i \neq j$. A similar argument as above shows that, conditional on $x_j$, the distribution of $z_i$ can be approximated by the Gaussian

$$z_i = N(\hat{s}_{ij}(t), Q_{ij}^r(t)/u),$$

(18)

where

$$\hat{s}_{ij}(t) = \sum_{r \in N_x(i) \neq j} A_{ir} \hat{x}_{i-r}(t) + A_{ij} x_j$$

(19a)

and

$$Q_{ij}^r(t) = \sum_{r \in N_x(i) \neq j} A_{ir} Q_{r-r}(t) A_{ir}^\prime,$$

(19b)

Then, applying the Gaussian approximation (18) along with (19) to the expectation [8] shows that we can write $\Delta_{i-j}(\cdot)$ in (11a) as

$$\Delta_{i-j}(t, x_j) \approx \frac{1}{u} \log \left[Z_i(\Delta_{i-i}(t, \cdot), \hat{s}_{ij}(t), Q_{ij}^r(t))\right],$$

(20)

where

$$Z_i(\Delta(\cdot), \hat{s}_{ij}, Q_{ij}^r) = \int \exp \left[uL_i(x_i, z_i, \hat{s}_{ij}, Q_{ij}^r)\right] dx_i dz_i,$$

and

$$L_i(x_i, z_i, \hat{s}_{ij}, Q_{ij}^r) = L_{i0}(x_i, z_i, \hat{s}_{ij}, Q_{ii}^s) + \Delta(x_i).$$

(21)

Then, the RX and TX node update steps of the BP algorithm with Gaussian approximation of the interference at the RX are:

1. **RX node update**: The above equations are used to simplify the standard BP algorithm as follows. Each RX $i$ receives mean and variances $\hat{x}_{i-r}(t)$ and $Q_{r-r}^{-}(t)$ of the vectors from the transmitters TX $j$ with $j \in N_x(i)$. RX $i$ also receives the function $\Delta_{i-i}(t, \cdot)$ from its serving transmitter, TX $i$. RX $i$ then computes the interference means and variances in (14) and (19). Then, for each TX $j$ it sends back the log likelihoods $\Delta_{i-j}(t, \cdot)$ by evaluating the log partition functions (15) and (20).

2. **TX node update**: It can be verified that converting the update (9) to log domain yields

$$\Delta_{i-j}(t+1, x_j) = \sum_{\ell \in N_x(j) \neq i} \Delta_{\ell-j}(t, x_j),$$

(22)

Each TX $j$ can first compute the log likelihoods $\Delta_{i-j}(t+1, x_j)$ from the log likelihoods $\Delta_{\ell-j}(t, x_j)$ from the receivers RX $\ell$ with $\ell \in N_x(j), \ell \neq i$. Then, using the log likelihoods $\Delta_{i-j}(t+1, x_j), TX$ $j$ computes the mean and variance $\hat{x}_{i-j}(t+1)$ and $Q_{i-j}^{-}(t+1)$ from the probability distribution $p_{i-j}(t+1, x_j)$ in (12b). Then, TX $j$ sends messages to the receivers as described in the RX node update above.

After a sufficient number of rounds, TX $j$ can compute the final log likelihood

$$\Delta_j(t+1, x_j) = \sum_{\ell \in N_x(j)} \Delta_{\ell-j}(t, x_j),$$

(23)

and then compute the final scheduling vector as the one which maximizes the above function.

We have therefore simplified the standard BP algorithm by eliminating the exponential complexity of the RX update (8), and replaced this computation with a Gaussian approximation.

### C. First Order Approximations

Unfortunately, the Gaussian approximation above does not significantly reduce the messaging overhead. Each RX and TX must still send separate unicast messages to every TX or RX in its neighbor set. Also, although the TX must only send a mean and a covariance matrix $\hat{x}_{i-j}(t)$ and $Q_{i-j}^{-}(t)$, the receivers must send the entire log likelihood functions $\Delta_{i-j}(t, x_j)$.

The messaging overhead can be reduced via selective use of first order approximations as follows: Divide the edges $(i, j) \in E$ with $i \neq j$ into two sets – weak and strong – depending on whether $A_{ij}$ is small or large. Along a strong edge $(i, j)$, RX $i$ and TX $j$ exchange the full unicast messages described above. However, for the weak edges, the messages can be replaced with a first order approximation described below. The precise classification rule between weak and strong edges is an algorithm parameter that can be used to trade off complexity and accuracy.

To describe the first order approximation along the weak edges, suppose $A_{ij}$ is small for some edge $(i, j)$. Let $x_j(t)$ be the mean value of the distribution corresponding to the log likelihood $\Delta_j(t, \cdot)$ in (23). Now consider the log likelihood $\Delta_{i-j}(t, x_j)$ in (20). Applying (19)

$$\Delta_{i-j}(t, x_j) \approx \frac{1}{u} \log \left[Z_i(\Delta_{i-i}(t, \cdot), \hat{s}_{ij}(t), Q_{ij}^r(t))\right]\left(a\right) \approx \frac{1}{u} \log \left[Z_i(\Delta_{i-i}(t, \cdot), \hat{s}_{ij}(t) + A_{ij} (x_j - \hat{x}_j(t)), Q_{ij}^r(t))\right]\left(b\right) \approx u_i(t)x_j + \text{const}$$

(24)

where (a) follows from (19) and the approximation that $Q_{ij}^r(t) \approx Q_{ii}^s(t)$ and $\hat{x}_{i-j}(t) \approx \hat{x}_j(t)$ when $A_{ij}$ is small; and (b) follows from taking a Taylor’s approximation with

$$u_i(t) = A_{ij} \hat{D}_i(t) - A_{ij} \hat{D}_i(t) A_{ij} \hat{x}_j(t),$$

(25)

where for $r = 1, 2$, $D_r(t)$ are the derivatives

$$D_r(t) := \frac{1}{u} \frac{\partial^r}{\partial s^r} \log \left[Z_i(\Delta_{i-i}(\cdot), \hat{s}_{ij}(t), Q_{ij}^r(t))\right].$$

(26)

The constant in (24) is independent of $x_j$. Using standard properties of the cumulant function [43], it can be shown that the derivative is given by

$$D_1(t) = E \left[ \frac{\partial}{\partial \hat{x}_i(t)} f_i(x_i, z_i) \right] \hat{s}_{ii}(t), Q_{ii}^s(t), \Delta_{i-i}(t, \cdot)\right].$$

(27)
where the expectation is with respect to the conditional distribution

\[
p_{x|z_i,t}(x_i,z_i | \hat{s}_{ii}(t), Q_{ii}^z(t)) := \frac{1}{Z_t} \exp \left[ uL_i(x_i,z_i, \hat{s}_{ii}(t), Q_{ii}^z(t)) \right],
\]

and \( L_t(\cdot) \) is defined in (21) and (17). The dependence on \( \Delta_{i \rightarrow j}(t) \) in (22) is implicit in (21). Note that the derivatives \( D_{ij}(t) \) in (22) can be interpreted as a sensitivity of the expected utility \( f_i(x_i,z_i) \) to changes in the interference \( z_i \).

The computation of \( x_{i \rightarrow j}(t) \) can also be simplified as follows: Recall that \( \hat{x}_{i \rightarrow j}(t) \) and \( x_j(t) \) are the expectation with respect to the likelihood functions \( \Delta_{i \rightarrow j}(t, \cdot) \) in (22) and \( \Delta_j(t, \cdot) \) in (23), respectively. Therefore, we can write them as

\[
\hat{x}_{i \rightarrow j}(t) = E[x_j | \Delta_{i \rightarrow j}(t, \cdot)] \quad (29a)
\]

\[
x_j(t) = E[x_j | \Delta_j(t, \cdot)], \quad (29b)
\]

where we have used the notation \( E(g(x)|\Delta(x)) \) to denote the expectation of \( g(x) \) with respect to the probability distribution \( p_\Delta(x) \propto \exp \left[ u\Delta(x) \right] \).

It is easy to check that for small perturbations of \( \epsilon(x) \) of \( \Delta(x) \), we have the first order approximation

\[
E(g(x)|\Delta + \epsilon) \approx E(g(x)|\Delta) + uE(\epsilon(x)|\Delta)E(\epsilon(x)|\Delta).
\]

Therefore,

\[
\hat{x}_{i \rightarrow j}(t + 1) \overset{(a)}{=} E[x_j | \Delta_j(t + 1, \cdot) - \Delta_{i \rightarrow j}(t, \cdot)] \\
\overset{(b)}{=} E(x_j | \Delta_j(t + 1, \cdot)) - uE(x_j | \Delta_{i \rightarrow j}(t, x_j) | \Delta_j(t + 1, \cdot)) \\
- uE(x_j | \Delta_j(t, \cdot))E(\hat{x}_{i \rightarrow j}(t, x_j) | \Delta_j(t + 1, \cdot)) \\
\overset{(c)}{=} \hat{x}_j(t + 1) - Q^x_{ij}(t + 1)u_{ij}(t) \quad (31)
\]

where (a) follows from (29a) along with (22) and (23); (b) follows from (30); and (c) follows from (29a) and (22).

We can use the above relations to define the following algorithm:

1) **Initialization**: Set \( t = 0 \). Each TX \( j \) broadcasts an initial scheduling vector \( \hat{x}_j(t) \) and variance \( Q^x_{ij}(t) \). These can be based on the mean and variance of \( x_j \) over the set \( \mathcal{X} \). The receivers RX \( i \) sets \( \hat{x}_{i \rightarrow j}(t) = x_j(t) \) and \( Q^x_{i \rightarrow j}(t) = Q^x_{ij}(t) \) for all \( j \), and \( \Delta_{i \rightarrow j}(t, x_j) = 0 \) for all \( x_j \).

2) **RX node update**: In the RX half of the round, each RX \( i \) first computes the interference means and variances \( \hat{s}_{ii}(t) \) and \( Q^z_{ii}(t) \) in (14) and \( \hat{s}_{ij}(t) \) and \( Q^z_{ij}(t) \) in (19). Then, RX \( i \) computes the log likelihood function \( \Delta_{i \rightarrow j}(t, x_j) \) in (15) and sends it as a unicast message to its serving transmitter RX \( i \). Also, for each strong edge \( (i, j) \), RX \( i \) computes the log likelihood functions \( \Delta_{i \rightarrow j}(t, x_j) \) in (20) and sends it as a unicast message to the interfering TX \( j \). For the weak edges, RX \( i \) simply computes the sensitivity \( D_{ij}(t) \) in (27) and broadcasts it to all other transmitters TX \( j \). The receiver also computes \( u_{ij}(t) \) in (25) and stores it for the next round.

3) **TX node update**: In the TX half of the round, each transmitter TX \( j \) would have received the log likelihoods \( \Delta_{i \rightarrow j}(t, x_j) \) from the receivers RX \( i \), along the edges \( (i, j) \) that were strong. For any weak edge \( (i, j) \), TX \( j \) can approximately compute the log likelihood from the sensitivity \( D_{ij}(t) \) using (24). TX \( j \) can then compute the log likelihoods \( \Delta_{i \rightarrow j}(t + 1, x_j) \) for all \( i \in N_{tx}(j) \) and the log likelihood \( \Delta_j(t + 1, x_j) \) in (23). TX \( j \) sends the receiver RX \( j \) that it is serving the entire log likelihood \( \Delta_{i \rightarrow j}(t + 1, x_j) \). For the receivers RX \( i \) such that \( (i, j) \) is a strong edge, TX \( j \) computes the mean \( \hat{x}_{i \rightarrow j}(t + 1) \) and variance \( Q^x_{i \rightarrow j}(t + 1) \) from the log likelihood \( \Delta_{i \rightarrow j}(t + 1, x_j) \) and sends it as a unicast message to RX \( i \). Each TX \( j \) also computes the mean and variance \( \hat{x}_j(t + 1) \) and \( Q^x_{ij}(t + 1) \) from the log likelihood \( \Delta_j(t + 1, x_j) \) and broadcasts the quantities to the other receivers. Any receiver RX \( i \) that is along a weak edge \( (i, j) \) can then approximately compute \( \hat{x}_{i \rightarrow j}(t) \) from (31).

The round number is incremented, \( t = t + 1 \), and we return to Step 2 until a fixed number of rounds have been performed.

4) **Final solution**: After the final round, each transmitter TX \( j \) takes the scheduling vectors to be the vector \( x_j \) that maximizes the log likelihood \( \Delta_j(t, x_j) \).

The message flow along the weak edges has an appealing interpretation. Consider Fig. 1 where a transmitter TX1 attempts to send data to a receiver RX1. The receiver RX1 experiences interference from a second transmitter TX2, while the transmitter TX1 causes interference onto a victim receiver RX3. Fig. 1 shows the messages along the weak edges in one round of the BP algorithm to coordinate the interference. The transmitters TX1 and TX2 will broadcast the mean and variance \( \hat{x}_j(t) \) and \( Q^x_{ij}(t) \) of their intended transmit vectors. These transmissions can be interpreted as “soft” request to sends (RTS). They are soft since the intended transmit vectors are signaled by a distribution. Based on the transmit vector distribution from the interfering TX2, the receiver RX1 replies to the serving TX1 with an estimate of the interference \( \hat{s}_{11}(t) \) and \( Q^z_{11}(t) \) which can be interpreted as a “soft” channel quality indication (CQI). As victim receivers, RX1 and RX3...
also compute the sensitivities to the interference level by the derivatives \( D_{ir}(t) \). These values can be interpreted as soft clear to send (CTS) indications, since instead of a binary go/no go type CTS, they signal a soft cost on changes in the interference from other transmitters.

**D. Communication and Computation Complexity**

We summarize the complexity of one round of the three variations of BP in Tables I and II. Specifically, we focus on RX \( i \) and TX \( j \) and consider how the complexity grows with \( X \), \( N_{tx}(i) \) and \( N_{tx}(j) \).

**V. Numerical Simulation**

The BP algorithm was simulated on a a simplified version of an industry standard model for LTE femtocell evaluation in [3]. The simulation parameters are shown in Table III. The network consists of a 3 x 3 grid of 10m x 10m apartments with active links in 5 of the 9 apartments. Each link consists of one femto BS transmitting to one femto mobile (called a UE, or user equipment, in 3GPP terminology). Due to restricted association, UEs connect to the femto BS in their apartment even if it is not the BS with the minimum path loss. As mentioned in the introduction, this scenario exposes many links to strong interference, and thus presents a good test scenario for advanced interference coordination algorithms.

In the first simulation, we considered a time-varying simulation with a simple on-off model where, in each time slot, each link either transmits at the max power or is completely off. As described in Section III for the time-varying problem, the utility maximization was rerun in each time slot with the weighted sum rate utility \( U_i(R) = \log(R) \). We generated independent flat fades on the links in each slot, and took a filter time constant \( \alpha = 0.1 \). For each random realization, or “drop”, of the femto network, we ran the simulation over 100 time slots and measured the average rate. The wall loss in the femto model was 10 dB.

The top panel of Fig. 2 plots the cumulative distribution function (CDF) of the time-averaged rates for 5 links and 100 drops comparing various optimization methods for computing the maximum weighted matching optimization (2). The curve “reuse 1” is the case when all links transmit at max power. Two cases of BP are simulated: (i) 4 rounds of BP in each time slot using the Gaussian approximation in Section I-V-B but no first order approximations; and (ii) using only two rounds of BP with both the Gaussian approximations and first order approximations on all interfering links less than 0 dB below the serving link. We see that even the linear approximated BP

| Parameter                  | Value                                      |
|----------------------------|--------------------------------------------|
| Network topology           | 3 x 3 apartment model, with active links in 5 of the 9 apartments. |
| Carrier freq               | 2 GHz                                      |
| Bandwidth                  | 5 MHz                                      |
| Wall loss                  | 0 or 10 dB                                 |
| Lognormal shadowing        | 10 dB std. dev.                            |
| Path loss                  | \(-38.10 + 20 \log_{10}(R) + 0.7R \) dB, \( R \) distance in meters. |
| Femto BS TX power          | 0 dBm                                      |
| Femto UE noise figure      | 4 dB                                       |

**TABLE III**

**SIMULATION PARAMETERS.**
with 2 rounds does significantly better than reuse 1, and is not that far from the 4 round BP. The gap between BP and reuse 1 particularly large at low rates. For example, for the 20% worst links, BP offers almost a factor of 5 improvement in rate over reuse 1.

Also plotted is the rate CDF with optimal matching in each time slot based on an exhaustive centralized search. BP performs reasonably close to this curve, although there is still an obvious gap, again at low rates.

The bottom panel of Fig. 2 shows the CDF of the total system utility over the 100 drops. Since the optimization used a log utility, the optimization is equivalent to maximizing the harmonic mean rate over the 5 links. We see in this plot that the approximate BP algorithms are close to optimal and significantly better than reuse 1.

As a second simulation, we considered the same problem but with a static single optimization and $K = 4$ subbands. Random fading was generated in each subband, so the simulation would be applicable in the case with frequency selective fading with coherence bandwidth roughly equal to the subband bandwidth. The optimization was over scheduling vectors $x_j \in \mathbb{C}^K$ with each component being on or off, so there are $2^K - 1 = 15$ possible non-zero scheduling vectors in each link. Optimal subband allocation is a well-known challenging but important problem for OFDMA systems like LTE.

Under these assumptions, Fig. 3 shows the CDF of the rates under various optimization methods. Similar to the dynamic single subband case, we see that BP provides significant gains over simple reuse 1, even when we use linear approximations and only four rounds. Also, for most links, BP achieves a rate reasonably close to the optimal subband allocation found by exhaustive search over all subband allocations over all rates. However, for the lowest rate links, there is a significant gap between BP and optimal. Thus, even though BP outperforms reuse 1 significantly in this regime, there is significant room for improvement.

The BP method can also be applied to beamforming (BF) problems. As a simple simulation, we consider again the femtocell deployment with transmit beamforming with two antennas with half wavelength spacing spacing and one receive antenna. We neglect scattering so the channel appears as a linear phase across the two antennas on all links. For the transmit vectors $x_j$, we optimize beamforming angles over 10 angles uniformly spaced between 0 and $\pi$. A performance comparison of the rate CDFs under various optimization algorithms is shown in Fig. 4. In this case, we took a wall loss of 0 dB. The curve labeled “opt serving link only” is the case when the BF vector is chosen to maximize the signal strength from the serving link only without regard to interference. This simple method provides the baseline. We see that BP provides some gains over serving link optimization. For example, the median rate with BP is approximately 50% higher than using optimal BF on the serving link only. Also, BP appears to be reasonably close to the optimal BF selection based on exhaustive search.

VI. CONCLUSIONS

We formulated a general wireless scheduling and interference coordination problem as an optimization problem with linear mixing utilities. This was cast in a BP framework where the goal is to compute the marginal distributions of a joint probability function. Using Gaussian and linear approximations, we obtained a distributed interference coordination algorithms with low overhead. The algorithm has a natural interpretation as a soft RTS/CTS scheme. Numerical simulations demonstrated that the resulting algorithm is close to an optimal scheme for dynamic and static sub-band optimization as well as for beamforming coordination across cells. Moreover, the results show that the algorithm computes a good operating point in just two to four iterations making it very attractive to be used in practical wireless cellular systems. In the future, we plan to explore connections between our work and the AMP framework in [22] to obtain performance bounds for large random networks.
