ON USING QUANTUM PROTOCOLS TO DETECT TRAFFIC ANALYSIS

RAINER STEINWANDT, DOMINIK JANZING, and THOMAS BETH

Institut für Algorithmen und Kognitive Systeme, Fakultät für Informatik,
Am Fasanengarten 5, Universität Karlsruhe (TH),
76131 Karlsruhe, Germany

We consider the problem of detecting whether an attacker measures the amount of traffic sent over a communication channel—possibly without extracting information about the transmitted data. A basic approach for designing a quantum protocol for detecting a perpetual traffic analysis of this kind is described.

Keywords: quantum cryptography, traffic analysis

1. Introduction

Within classical cryptography it is a well-known phenomenon that a communication channel can be eavesdropped without affecting the transmitted data. In recent years techniques have been developed which exploit the quantum properties of the microphysical world in order to deal with this problem. One of the most prominent among these methods is the protocol for quantum key distribution described by Bennett and Brassard. By means of such quantum protocols it is possible to ensure that an eavesdropper who tries to get knowledge of the transmitted data can be detected with high probability. However, an interesting aspect of eavesdropping which does not seem to be covered by the quantum protocols suggested so far is traffic analysis: think of an attacker who is only interested in analyzing the amount of traffic sent over a channel, i.e., the attacker is only interested in knowing how much data is transmitted and not necessarily in reading the transmitted information itself.

If the communication channel is part of a network then one approach to thwart such a traffic analysis is to conceal the identity of the recipient of the transmitted data (cf., e.g., Rackoff and Simon and the references given there). In case of an “isolated” communication channel the situation is much worse—if the existence of the channel cannot be kept secret (say by means of steganographic techniques) the classical approach to circumvent an analysis of the amount of meaningful data sent over the channel is to keep the communication channel busy all the time. This means that data is sent continuously, and the legitimate users of the channel have to separate the relevant from the “dummy” data via a suitable (secret) synchronization. No other solution to this problem seems to be known—e.g., Rackoff and Simon state “In other words, any secret regarding the total volume of information sent or received by a party is purchased at the cost of the extra ‘dummy traffic’ generated to disguise it.”
In this contribution we describe a simple quantum protocol which under suitable assumptions enables the legitimate users of a quantum channel to detect with high probability whether the traffic on a quantum channel is analyzed—even if the attacker does not try to read the transmitted data. We are not aware of a classical analogue of such a procedure. However, we emphasize that in the present form, the protocol is not suitable for practical cryptographic use, as e.g., topics like authentication, noisy communication channels, or efficiency are not taken into account. Nevertheless, we think the described protocol to be of interest, as it might point out a new application of quantum physical phenomena for cryptographic purposes.

Roughly speaking, the idea of the protocol is as follows: as carrier of the information to be transmitted we think of using an inner degree of freedom of some particle (like the polarization of a photon) or different types of particles for representing zero and one. Then to transmit a bit, first the wave function of the particle carrying the information is split up into two parts, and one of these parts is transmitted over the communication channel; the other part is kept secret by the sender. At no time more than one wave package is present on the communication channel. The receiving party chooses at random whether the transmitted bit is received or mirrored back to the sender—in the latter case the corresponding bit is sent again in the next step. If no attack takes place and a wave package is returned, then there will be interference between a returned wave package and the wave package retained by the sender. As an observation of the channel causes a partial collapse of the wave function, a traffic analysis destroys this interference.

2. A Quantum Protocol for Detecting Traffic Analysis

Following the established terminology, the communicating parties will be referred to as Alice and Bob in the sequel. Moreover, the attacker who is interested in analyzing the traffic on the communication channel between Alice and Bob will be called Tracy. We avoid the name Eve, as Tracy is not necessarily interested in eavesdropping the channel, i.e., obtaining the transmitted information as such. Instead, she may restrict her interest on learning the number of transmitted bits.

2.1. Statement of the protocol

Let us suppose that Alice wants to send a message \( m = m_1 \ldots m_n \) to Bob (we assume the \( m_i \)'s to be individual bits, but one could as well use a coarser subdivision of \( m \)). Moreover, Alice would like to know whether Tracy is analyzing the communication channel while \( m \) is being sent to Bob. If \( n \) is not too small, then informally the technical apparatus and a protocol for doing so can be described as follows (for a more formal treatment see Section 2.2):
Technical apparatus

- Alice has a source for producing single particles, and she can encode a bit value in an inner degree of freedom of such a particle (as an example we may think of the polarization of a photon). Alternatively one can also think of using different types of particles for encoding zero and one. Moreover, Alice has a 50:50 beam splitter $B_1$ which splits a particle into two wave packages without affecting this inner degree of freedom, and she is able to store one such wave package in her laboratory. Finally, she has a 50:50 beam splitter $B_2$ where a wave package received on the channel and a wave package stored in her laboratory can meet in such a way that in one branch constructive and in the other branch destructive interference occurs. By means of a detector $C$ in the “constructive branch” and a detector $D$ in the “destructive branch” Alice can decide whether a particle is present in one of the branches.

- Bob needs a switchable device which either mirrors a wave package on the channel back to Alice without detecting the presence of the package, or reads out the inner degree of freedom of a particle on the channel without returning anything.

Initialization

We assume that Alice and Bob have agreed upon a time $t_1$ where the transmission of Alice is to begin. By $\Delta \in \mathbb{R}$ we denote the time required by a particle for passing from Alice to Bob back to Alice. Alice and Bob can use $\Delta$ to synchronize their transmissions in such a way that there is at no time more than one wave package present on the channel, and Alice can make use of $\Delta$ to carry out her interference experiments correctly. Finally, Alice initializes a variable $a \leftarrow 0$ for counting the number of transmitted wave packages, and Bob initializes a boolean variable $b \leftarrow true$.

Transmission protocol

For $i \leftarrow [1, \ldots, n]$ Alice proceeds as follows to transmit bit $m_i$ to Bob:

1. Alice produces a single particle $P$ and encodes $m_i$ in an inner degree of freedom of $P$. Then she passes $P$ through the beam splitter $B_1$ and retains one of the two resulting wave packages in her private laboratory. The other wave package is transmitted to Bob over the communication channel at time $t_i = t_1 + a \cdot \Delta$. Then Alice sets $a \leftarrow a + 1$.

2. If $b = true$ then Bob selects $r \in \{true, false\}$ at random. Otherwise, i.e., for $b = false$, the value of $r$ remains unchanged.

3. If $r = true$ then Bob mirrors the wave package back to Alice. Otherwise he reads out the inner degree of freedom of the potentially present particle on the communication channel.

4. Bob sets $(b, r) \leftarrow (not \ b, \ not \ r)$.

5. Using the beam splitter $B_2$ Alice brings the wave package potentially returned by Bob and the wave package stored in her laboratory into interference. Let $C_j$ resp.
\[ D_j \ (1 \leq j \leq a) \] be the random variable describing the number of particles (0 or 1) detected in detector \( C \) resp. \( D \) in Alice’s \( j \)th interference experiment.

6. If \( a \) is even then Alice tests the following hypothesis:

the vector valued random variables \((C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j})\) (where \(1 \leq j \leq a/2\)) are identically independently distributed with the probability distribution

\[
p\left( (C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (1,1,0,0) \right) = 1/4, \\
p\left( (C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (1,0,0,0) \right) = 1/4, \\
p\left( (C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (0,1,0,0) \right) = 1/4, \\
p\left( (C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (1,0,0,1) \right) = 1/8, \\
p\left( (C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (0,1,1,0) \right) = 1/8, \\
p\left( (C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = x \right) = 0 \text{ for all other } x.
\]

If this hypothesis has to be rejected with high probability then Tracy is assumed to analyze the communication channel and the protocol is aborted.

7. If \( C_a + D_a = 0 \) then Alice assumes that the transmission of bit \( m_i \) is complete, i.e., that Bob has succeeded in receiving \( m_i \). Otherwise, i.e., if \( C_a + D_a \neq 0 \), Alice goes back to Step 1—thereby retransmitting \( m_i \).

Before explaining and analyzing the above protocol in more detail we would like to emphasize again that in the above simple form the protocol is not suitable for practical cryptographic use. In particular it does not take the problem of authentication into account; also it is assumed that the communication channel is perfect. Similarly, for ease of presentation, in Step 6 of the above protocol, Alice does not check whether a detected particle indeed encodes the bit \( m_i \).

### 2.2. Explanation and analysis of the protocol

To explain the protocol we introduce some notation. For sake of simplicity for the moment we restrict ourselves to an informal explanation of the individual steps of the protocol; a more rigorous treatment is postponed until later.

By \( \mathcal{G} \) we denote the Hilbert space corresponding to the particle’s inner degrees of freedom and by \( \mathcal{H} \) the Hilbert space spanned by the following three basis states:

1. \( |a\rangle \): the particle \( P \) is in Alice’s laboratory
2. \( |c\rangle \): the particle \( P \) is in the communication channel
3. \( |b\rangle \): the particle \( P \) is in Bob’s laboratory

In the first step of the protocol Alice prepares the state

\[
\frac{1}{\sqrt{2}}(|a\rangle + |c\rangle) \otimes |m_i\rangle
\]

(1)
where $|m_i⟩ ∈ G$ stands for the inner state representing the bit $m_i$.

In the Steps 2–4 the role of Bob’s boolean variables $b$ and $r$ is as follows: the variable $b$ is used to group the transmitted wave packages of Alice into pairs in such a way that for $j ∈ N$ arbitrary either the $(2j − 1)^{\text{st}}$ or the $(2j)^{\text{th}}$ wave package (but never both of them) is mirrored back to Alice: $b = \text{true}$ resp. $b = \text{false}$ means that currently a “first” ($(2j − 1)^{\text{st}}$) resp. “second” ($(2j)^{\text{th}}$) element of a pair is processed by Bob. The random choice of $r$ in Step 2 is used to decide at random whether the first or second wave package of the current pair is mirrored back. Note that the decision whether to return the first or second package is made independently for each individual pair, and we assume that mirroring back the wave package does not alter the state $|\overline{0}⟩$.

In Step 3 if $r = \text{false}$ then Bob attempts to read out the inner degree of freedom of the potentially present particle. For this the wave package enters his laboratory which translates into transforming the state $|\overline{0}⟩$ into

$$
\frac{1}{\sqrt{2}}(|a⟩ + |b⟩) ⊗ |m_i⟩.
$$

Reading out the inner degree of freedom then means to perform a measurement on the state $|\overline{0}⟩$. This measurement is described by the following three projectors:

1. $Q_0$: Projection on the one-dimensional subspace spanned by $|b⟩ ⊗ |0⟩ ∈ H ⊗ G$—Bob receives $m_i = 0$.

2. $Q_1$: Projection on the one-dimensional subspace spanned by $|b⟩ ⊗ |1⟩ ∈ H ⊗ G$—Bob receives $m_i = 1$.

3. $Q_\epsilon := (1 − |b⟩⟨b|) ⊗ 1$—Bob does not detect a particle at all.

In the sequel the corresponding measurement outcomes are denoted by 0, 1, and $\epsilon$.

Finally, to see why Alice’s hypothesis in Step 6 is not rejected with high probability reduces to computing the five non-zero probabilities occurring in the hypothesis:

- $p((C_{2j − 1}, C_{2j}, D_{2j − 1}, D_{2j}) = (1, 1, 0, 0)) = 1/4$: w.l.o.g. we assume that Bob measures the bit transmitted in the $(2j − 1)^{\text{st}}$ transmission. Then the result $C_{2j − 1} = 1$ is only possible if Bob’s measurement yields the result $\epsilon$. In this case we have $C_{2j − 1} = 1$ with probability $1/2$. Since the result $\epsilon$ has probability $1/2$ as well, we obtain $p((C_{2j − 1}, C_{2j}, D_{2j − 1}, D_{2j}) = (1, 1, 0, 0)) = 1/4$ as required.

- $p((C_{2j − 1}, C_{2j}, D_{2j − 1}, D_{2j}) = (1, 0, 0, 0)) = 1/4$: from $C_{2j} = 0$ we conclude that Bob measured the $2j^{\text{th}}$ transmission. This choice occurs with probability $1/2$ and also guarantees $C_{2j − 1} = 1$ and $D_{2j − 1} = 0$. As $C_{2j} = D_{2j} = 0$, the result of Bob’s measurement is different from $\epsilon$; the latter event has probability $1/2$. So in summary we get $p((C_{2j − 1}, C_{2j}, D_{2j − 1}, D_{2j}) = (1, 0, 0, 0)) = 1/4$.

- $p((C_{2j − 1}, C_{2j}, D_{2j − 1}, D_{2j}) = (0, 1, 0, 0)) = 1/4$: the same argument as in the previous case with $2j$ and $2j − 1$ interchanged.
• $p((C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (1, 0, 0, 1)) = 1/8$: from $C_{2j} = 0$ we conclude that Bob measured the $2j^{th}$ transmission. This choice occurs with probability 1/2 and also guarantees $C_{2j-1} = 1$ and $D_{2j-1} = 0$. As $D_{2j} = 1$ the result of Bob’s measurement is $\epsilon$; the latter event has probability 1/2. If Bob does not detect a particle (i.e., the result of his measurement is $\epsilon$) the probability that $C$ (resp. $D$) detects a particle, is 1/2. Consequently, we obtain $p((C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (1, 0, 0, 1)) = 1/8$.

• $p((C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j}) = (0, 1, 1, 0)) = 1/8$: the same argument as in the previous case with $2j$ and $2j - 1$ interchanged.

As the probabilities of these cases sum up to 1 already, no other values of the tuple $(C_{2j-1}, C_{2j}, D_{2j-1}, D_{2j})$ can occur.

Next, we show that measurements of Tracy which are appropriate for detecting whether bits have been transmitted or not change the statistics in Step 6 of the above transmission protocol. For this we use a field-theoretic formulation of the quantum physical situation. The particles used in the protocol can be Bosons or Fermions, here we restrict our attention to Bosons. Hence the field-theoretic description will represent the system in a symmetric Fock space. If an arbitrary particle is described in a Hilbert space the corresponding Bose field is described in the symmetric Fock space $F^+(L)$. Here we take $F^+(L) := L^2(\mathbb{R}^3)$, the set of square integrable functions (cf. the work of Prugovecki, for instance) as one-particle space.

Let $A \subseteq \mathbb{R}^3$ be the area of Alice’s laboratory and $X := \mathbb{R}^3$ its set-theoretic complement. Moreover, for $R \subseteq \mathbb{R}^3$ we set

$$L_R := \{ f \in L \mid \forall y \in (\mathbb{R}^3 \setminus R) : f(y) = 0 \}.$$ 

So in particular we have $L = L_A \oplus L_X$. Moreover, the algebra of operators acting on $F^+(L)$ can be split up into a tensor product of those which can be measured in Alice’s laboratory and those which can be measured outside due to the formula

$$F^+(L_A \oplus L_X) = F^+(L_A) \otimes F^+(L_X). \quad (3)$$

To express this more precisely, we can make use of the concept of a Positive Operator Valued Measure (POVM). A POVM is a family $(a_i)_{i \in I}$ of positive operators with $\sum_{i \in I} a_i = 1$; it describes the most general quantum mechanical measurement. As Tracy does not have access to Alice’s laboratory, every measurement which she can perform is a POVM of the form $(a_i) = (1 \otimes b_i)$ where $b_i$ is an arbitrary positive operator of the right-hand component of the tensor product.$^5$

Now we distinguish between the two possible cases:

1. Alice transmits a particle to Bob: in this case the state vector of the field is contained in the one-particle subspace of $L$. In this subspace the state can be described by a wave function $|\psi\rangle \in L$.

\footnote{This is a locality assumption for the laws of physics: the operators which represent measurements performed inside a certain area act trivially on the Hilbert space corresponding to the fields outside the area. This is formalized by Haag in a rather axiomatic approach.}
At some time $T_1$ one part of the wave function leaves Alice’s Laboratory and returns at a time $T_2$. Hence for $t \in [T_1, T_2]$ the state $|\psi(t)\rangle$ splits into a sum

$$|\psi(t)\rangle := \frac{1}{\sqrt{2}}(|\psi_1(t)\rangle + |\psi_2(t)\rangle)$$

where the $|\psi_i(t)\rangle$ describe the outputs of the two branches of Alice’s beam splitter $B_1$: $\psi_1$ is the part of the wave function which is to remain in Alice’s laboratory, and $\psi_2$ is the part of the wave function which is to be transmitted over the communication channel. If we consider $|\psi(t)\rangle$ canonically as an element of the Fock space $F^+(L_A) \otimes F^+(L_X)$ we obtain

$$|\psi(t)\rangle = |\psi_A(t)\rangle \otimes |0\rangle + |0\rangle \otimes |\psi_X(t)\rangle,$$

where $|0\rangle$ denotes the 0-particle state in $F^+(L_A)$ and $F^+(L_X)$, respectively. Since at no time more than “half of the wave function” is outside the laboratory, the norm of $|\psi_X(t)\rangle$ is not greater than $1/\sqrt{2}$ at any time $t$.

2. Alice does not transmit a particle to Bob: in this case the state of the quantum field is for every time $t$ given by the vector $|\phi(t)\rangle = |0\rangle \otimes |0\rangle$, i.e., the 0-particle state of $F^+(L_A) \otimes F^+(L_X)$.

Tracy’s measurement can only distinguish between the states $|\phi(t)\rangle$ and $|\psi(t)\rangle$ if the corresponding POVM $(a_i)$ contains an operator $a_i$ with the property

$$\langle \psi(t)|a_i|\psi(t)\rangle \neq \langle \phi(t)|a_i|\phi(t)\rangle. \quad (4)$$

On the other hand, a measurement apparatus can only be non-disturbing if the measured states are eigenvectors of every $a_i$ of the POVM$^b$. Hence Tracy’s attack has to be a measurement with the property that $|\psi(t)\rangle$ is an eigenvector of each $a_i = 1 \otimes b_i$. One checks easily that this can only be the case if $|0\rangle$ and $|\psi_X(t)\rangle$ are eigenvectors of $b_i$ with the same eigenvalue:

**Remark 1** With the above notation $|\psi(t)\rangle$ can only be an eigenvector of each $a_i = 1 \otimes b_i$ if $|0\rangle$ and $|\psi_X(t)\rangle$ are eigenvectors of $b_i$ with the same eigenvalue.

**Proof.** The two components $|\psi_A(t)\rangle \otimes |0\rangle$ and $|0\rangle \otimes |\psi_X(t)\rangle$ of $|\psi(t)\rangle$ are contained in the mutually orthogonal vector spaces $|\psi_A\rangle \otimes F^+(L_X)$ and $|0\rangle \otimes F^+(L_X)$, respectively. Both of these vector spaces are invariant under $1 \otimes b_i$, and hence the claim follows. $\square$

Remark 1 implies that the observable $1 \otimes b_i$ cannot distinguish between the state $|0\rangle$ and the state $|\psi(t)\rangle$ in the sense that the right-hand and the left-hand side of equation (4) coincide.

So assuming that whenever the part of the wave function outside Alice’s laboratory is modified then there can be no perfect destructive (constructive) interference in the branch corresponding to Alice’s detector $D$ ($C$), Tracy’s traffic analysis also modifies the statistics

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$^b$In Kraus’ work one can find equations describing the connection between the POVM and the corresponding effect of the measurement on the state.
in Step 6 of the transmission protocol. Qualitatively, this argument also holds for joint attacks (cf., e.g., Biham and Mor\cite{8}): as at no time more than one particle is present on the channel, such an attack had to access single particles in order to store quantum information for a later joint measurement on the memory.

A quantitative analysis of the information-disturbance trade-off\cite{9} had to make use of the fact that there is no time at which the norm of the part outside the laboratory is greater than $1/\sqrt{2}$. However, such an analysis is beyond the scope of this paper, and we do not pursue this topic any further here.

3. Conclusions

We have demonstrated (without giving a quantitative analysis) that in principle quantum mechanical phenomena can be used to detect a perpetual traffic analysis. In other words, it is possible to detect an attacker who perpetually measures the amount of traffic on a communication channel even in the case that the attacker does not read the transmitted data itself.

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