Bilinear discriminant feature line analysis for image feature extraction

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A novel bilinear discriminant feature line analysis (BDFLA) is proposed for image feature extraction. The nearest feature line (NFL) is a powerful classifier. Some NFL-based subspace algorithms were introduced recently. In most of the classical NFL-based subspace learning approaches, the input samples are vectors. For image classification tasks, the image samples should be transformed to vectors first. This process induces a high computational complexity and may also lead to loss of the geometric feature of samples. The proposed BDFLA is a matrix-based algorithm. It aims to minimise the within-class scatter and maximise the between-class scatter based on a two-dimensional (2D) NFL. Experimental results on two-image databases confirm the effectiveness.

Introduction: The nearest feature line (NFL), proposed by Li and Lu in 1999 [1], is a powerful classifier for image classification. Its kernel is a feature line (FL) metric. It measures the distance between a query sample and some class using the distance between the query sample and the FL of the corresponding class, rather than that between the query sample and the prototype sample in the corresponding class. Some NFL-based subspace learning algorithms were designed for feature extraction, including the NFL space (NFLS) [2], the uncorrelated discriminant NFL analysis (UDNFLA) [3] and so on. However, to use most of current NFL-based feature extraction algorithms, the image samples should be transformed to vectors first. This will increase the computational complexity and may lead to loss of the geometric feature of the image samples. In this Letter, a novel image feature extraction algorithm called bilinear discriminant feature line analysis (BDFLA) is proposed. The proposed BDFLA can extract the feature from the image matrix directly.

Uncorrelated discriminant NFL analysis: UNFLA is a subspace learning method based on NFL. Given a prototype sample set, \( \Pi = \{x_1, x_2, \ldots, x_N\} \subseteq \mathbb{R}^D \), denote \( s_{x_m} \) as the project point of sample \( x_m \) to the FL \( \Pi \), spanned by \( x_m \) and \( x_n \). The optimisation function of UNFLA is

\[
W = \arg\min \{\text{tr}(W^T (A - B) W)\}
\]

subject to \( W^T S_W W = I \)

Then the optimisation problem can be transformed to the following eigenvalue problem:

\[
(A - B)W = AS_W W
\]

where

\[
S_W = \frac{1}{N} \sum_{i=1}^{N} (x_i - E X)(x_i - E X)^T
\]

\[
E X = \frac{1}{N} \sum_{i=1}^{N} x_i
\]

\[
A = \frac{1}{N N_i} \sum_{x_n \in P(\Pi)} \sum_{i=1}^{N} (x_i - s_{x_m})(x_i - s_{x_m})^T
\]

Here, \( N_i \) denotes the number of FLs in the same class with \( x_i \) and \( x_m \) in \( P(\Pi) \) means \( x_m \) and \( x_i \) are in the same class

\[
B = \frac{1}{N N_m} \sum_{s_{x_m} \in P(\Pi)} \sum_{i=1}^{N} (x_i - s_{x_m})(x_i - s_{x_m})^T
\]

where \( M_i \) denotes the number of FLs in the different class with \( x_i \) and \( x_m \in \Pi \) means \( x_m \) and \( x_i \) belong to two different classes, respectively.

Proposed algorithm: In the classical NFL classifier, the input samples should be vectors. In this Section, a two-dimensional (2D) NFL is presented using the similar idea of NFL. In 2D NFL, all the matrices with the same size are viewed as the points of the linear space. Given two matrices \( A = [a_{ij}] \) and \( B = [b_{ij}] \), let

\[
\| A \| = \sqrt{\sum_{i,j} |a_{ij}|^2} \quad \text{and} \quad \langle A, B \rangle = \sum_{i,j} a_{ij} b_{ij}
\]

Given a prototype sample set, \( \Pi = \{x_1, x_2, \ldots, x_N\} \subseteq \mathbb{R}^D \), the NFL \( \Pi \) is as follows:

\[
Y = X_i + \mu(X_i - X_0)
\]

Using the same method in NFL, the nearest matrix between query sample \( Q \) and the FL \( \Pi \) is \( Q_p = X_1 + \mu(X_2 - X_1) \) where

\[
\mu = \frac{(Q - X_1)(X_2 - X_1)}{(X_2 - X_1)(X_2 - X_1)}
\]

Let

\[
\text{dis}_{2D NFL} (Q, \Pi) = \| Q - Q_p \|
\]

If there exists an FL \( \Pi_{i,j} \) such that

\[
\text{dis}_{2D NFL} (Q, \Pi_{i,j}) = \min \{ \text{dis}_{2D NFL} (Q, \Pi_{i,j}) \}
\]

then the query image sample \( Q \) will be assigned to the class \( \Pi_{i,j} \).

Given a prototype sample set \( X = X_1, X_2, \ldots, X_N \subseteq \mathbb{R}^D \), the between-class scatter based on 2D NFL \( S_{NF} \) and the within-class scatter based on 2D NFL \( S_{NFL} \) are introduced as follows:

\[
S_{NFL} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N_i} \sum_{x_n \notin P(\Pi)} \| L_i^T X_i - L_i^T X_n \|_R^2
\]

and

\[
S_{NFL} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N_i} \sum_{x_n \in P(\Pi)} \| L_i^T X_i - L_i^T X_n \|_R^2
\]

where \( X_{n_{i}} \) is a matrix, which is also a project point of \( X_i \) to the FL generalised by \( X_m \) and \( X_n \) in the matrix linear space, \( N_i \) is the number of FLs in the same class with \( X_i \), \( N_i(x_{n_{i}}) \) denotes the class label of \( X_m \) and \( N_i(x_{n_{i}}) \) is the number of FLs in the \( X_{n_{i}} \) class. Note that from the definition of 2D NFL, the class label of \( X_m \) equals the class label of \( X_n \). \( S_{NF} \) computes the square sum of the distances between each prototype sample and the 2D FLs in the same class with the corresponding prototype sample. \( S_{NFL} \) calculates the square sum of the distances between each prototype sample and the 2D FLs in the different class. Therefore, \( S_{NFL} \) can evaluate the within-class scatter of the prototype image samples, and \( S_{NF} \) can measure the between-class scatter of the prototype samples.

The proposed BDFLA aims to minimise the within-class scatter based on 2D NFL and maximise the between-class scatter based on 2D NFL. Therefore, to obtain two optimal maps, \( L \in \mathbb{R}^{D \times d} \) and \( R \in \mathbb{R}^{D \times d} \), the criterion of the proposed algorithm is defined as follows:

\[
\max J(L, R) = S_{NFL} - S_{NFL}
\]

then

\[
S_{NFL} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N_i} \sum_{x_n \notin P(\Pi)} \| L_i^T X_i - L_i^T X_n \|_R^2
\]

\[
= \sum_{i=1}^{N} \frac{1}{N_i} \sum_{x_n \notin P(\Pi)} \text{tr}(L_i^T (X_i - X_{n_{i}}) L_i^T (X_i - X_{n_{i}})^T R)
\]

\[
= \text{tr} \left( \sum_{i=1}^{N} \frac{1}{N_i} \sum_{x_n \notin P(\Pi)} [L_i^T (X_i - X_{n_{i}}) L_i^T (X_i - X_{n_{i}})]^T R \right)
\]

\[
= \text{tr} R S_{NFL}^2 R
\]

where \( \text{tr} \) denotes the trace of a matrix, and

\[
S_{NFL}^2 = \sum_{i=1}^{N} \frac{1}{N_i} \sum_{x_n \notin P(\Pi)} [L_i^T (X_i - X_{n_{i}}) L_i^T (X_i - X_{n_{i}})]
\]
At the same time,

\[ S_{\text{BF}} = \frac{1}{N} \sum_{i=1}^{N} \sum_{X_i \in P(X)} \left[ (R^T X_i - R^T X_{\text{m}_i}) L \right] ^2 \]

\[ = \frac{1}{N} \sum_{i=1}^{N} \sum_{X_i \in P(X)} w_i L^T (X_i - X_{\text{m}_i})^T R R^T (X_i - X_{\text{m}_i}) L \]

\[ = \text{tr} L^T S_{\text{BF}}' L \]

where

\[ S_{\text{BF}}' = \frac{1}{N} \sum_{i=1}^{N} \sum_{X_i \in P(X)} (X_i - X_{\text{m}_i})^T R R^T (X_i - X_{\text{m}_i}) L \]

Similar to the above matrix computation

\[ S_{\text{BF}} = \frac{1}{N} \sum_{i=1}^{N} \sum_{X_i \in P(X)} (X_i - X_{\text{m}_i})^T R R^T (X_i - X_{\text{m}_i}) \]

Finally

\[ J(L, R) = \text{tr} L^T (S_{\text{BF}} - S_{\text{BF}}') L = \text{tr} R^T (S_{\text{BF}} - S_{\text{BF}}') R \]

An iterative procedure is presented to solve the problem in (19). For a given \( R^{-1} \in R^{d \times d}, J \) can be rewritten as

\[ J = L^T (S_{\text{BF}}^R - S_{\text{BF}}'^R) L \]

An approximate solution of \( L \) can be calculated using eigenvalue decomposition

\[ J_{\text{opt}} = \lambda_1 \]

That is \( L = [l_1, l_2, \ldots, l_n] \), where \( l_1, l_2, \ldots, l_n \) are eigenvectors corresponding to \( d_1 \) biggest eigenvalues of \( J_{\text{opt}} \). Similarly, for a given \( L \in R^{d \times d} \), \( J \) can be rewritten as

\[ J = L^T (S_{\text{BF}} - S_{\text{BF}}') R \]

Using eigenvalue decomposition again, an approximate solution can be obtained

\[ J_{\text{opt}} = \lambda_1 \]

Denote \( R = [r_1, r_2, \ldots, r_n] \), where \( r_1, r_2, \ldots, r_n \) are eigenvectors corresponding to \( d_1 \) biggest eigenvalues of \( J_{\text{opt}} \). The above procedure is repeated to find the final solution. The procedure of the algorithm is as follows.

Algorithm 1 Proposed BDFLA

Require: The prototype image samples \( X = \{X_1, X_2, \ldots, X_N\} \subset R^{d \times d}, d_1, d_2 \) the iteration number \( T_{\text{max}} \) and the threshold \( \epsilon \).

Ensure: \( L \in R^{d \times d}, \) and \( R \in R^{d \times d} \)

\( t = 0 \)

\( R^t \leftarrow I_d \)

\( L^t \leftarrow I_d \)

while \( t < T_{\text{max}} \) do

\( t \leftarrow t + 1 \)

Compute \( S_{\text{BF}}' \) with (17)

Compute \( S_{\text{BF}} \) with (15)

Compute the projection matrix \( L' \) by solving (20)

Compute the projection matrix \( R' \) by solving (21)

if \( \| L' - L'_{t-1} \|^2 + \| R' - R'_{t-1} \|^2 < \epsilon \) then

Break

end if

end while

Then, for an image sample \( I, F = L^T R \in R^{d \times d} \) is the feature extracted by BDFLA and is used for classification.

Experimental results: In this Section, the COIL20 database [4] and the FKP database [5] are used to evaluate the proposed algorithms. In the following experiments, NFL is used for classification. The system runs 20 times. The average maximum recognition rate (AMRR) with a corresponding feature dimension is given. To evaluate the performance of the proposed algorithms, BDFLA was compared with the principal component analysis (PCA) [6], the linear discriminant analysis (LDA) [7], the 2D-PCA [8], 2D-LDA [7], NFLS, UD NFLA and NFL embedding (NFLE) [9] in the experiments.

To reduce the computation complexity, all the image samples in the COIL20 database were cropped to 48 × 48. About 10 image samples per class were selected randomly for training and the rest were for test. For the FKP database, instead of treating each person’s fingers as one subject, each finger was treated as one subject in this experiment. Some duplicate samples were removed from the database. Each sample was cropped to 40 × 60. Five image samples per class were chosen randomly for training and the other samples were used for test. For vector-based algorithms, PCA was first performed on the FKP database and 97% energy was preserved.

Table 1 shows the experimental results on the COIL20 database and the FKP database. From the Table, BDFLA has higher AMRRs than the other algorithms.

| Algorithm | COIL20 database | FKP database |
|-----------|-----------------|--------------|
|           | AMRR (%) | Dimension | AMRR (%) | Dimension |
| PCA       | 85.91 | 100 | 91.59 | 160 |
| LDA       | 88.23 | 19 | 93.73 | 190 |
| NFLS      | 87.96 | 120 | 90.84 | 160 |
| UD NFLA   | 89.32 | 130 | 90.16 | 150 |
| NFL       | 91.14 | 100 | 92.38 | 140 |
| 2D-PCA    | 90.57 | 15 × 48 | 93.15 | 10 × 60 |
| 2D-LDA    | 92.18 | 12 × 48 | 93.96 | 13 × 60 |
| BDFLA     | 94.48 | 14 × 8 | 95.62 | 15 × 10 |

Conclusion: In this Letter, a novel algorithm called BDFLA is proposed for image feature extraction. It is an NFL-based feature extraction algorithm, which aims to minimise the within-class scatter and maximise the between-class scatter based on the 2D-FL metric. Different from the classical NFL-based approaches, the proposed BDFLA is a matrix-based algorithm. The experimental results on the COIL20 database and the FKP database show the effectiveness of the proposed algorithm.

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