Multi-orthogonal signal underwater navigation system modeling on moving platform

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Abstract. When the Multi-Orthogonal Signal Underwater Navigation System (MOSUNS) works on a moving platform, the movement of the platform will bring some errors to the positioning of the MOSUNS. To solve this problem, a motion error model is established based on the homogeneous coordinates and the homogeneous rotation matrix, and the relationship between the positioning error and the Rotating velocity of the moving platform (roll rate, pitch rate, yaw rate) is accurately described. The motion error model is verified by the simulation of the MOSUNS on the motion platform through MATLAB.

1. Introduction

Under the support of the national "863 Plan", the author's team proposed a new Underwater Signal Navigation System, called the MOSUNS [1]. MOSUNS is composed of a transmitter and a receiver. The transmitter contains four array elements and simultaneously sends a group of multiple orthogonal signals. A hydrophone is used as the receiver to receive the signals. MOSUNS can provide broadcast positioning information for Underwater users, and can realize the self-concealed positioning of users such as submarines, Unmanned Underwater Vehicle (UUV), divers and Underwater platforms (such as fixed Underwater acoustic monitoring array, etc.), which has a broad application prospect.

The transmitter of the MOSUNS can be installed on the moving platform, which is called the moving platform type, or it can be placed on the sea bottom. When the transmitter is placed on the sea bottom, it has no movement with respect to the earth, so it is called the stationary platform type. Previous research work on the MOSUNS is mainly based on the stationary platform type, in which the positioning accuracy can reach about 0.3% [2-3].

In a stationary platform, there is only translational velocity between the transmitter and the receiver, and the linear velocity of each array element of transmitter is the same, which has no effect on the positioning accuracy of the MOSUNS [3]. In the moving platform, besides the translational velocity, there is the Rotating velocity between the transmitter and the receiver, and the Rotating velocity results in the difference of the linear velocity between each array element of transmitter, which affects the positioning accuracy of the MOSUNS. Aiming on the problems caused by the Rotating velocity of the moving platform, this paper will accurately describe the errors caused by the movement of the platform through mathematical modeling.

The current Underwater Acoustic Positioning System mainly has the following three types: Long Baseline Positioning, Short Baseline Positioning, and Ultra-Short Baseline Positioning. Their positioning principle is to obtain the orientation and distance of the positioning target relative to the fixed beacon of the known reference position through phase measurement and measurement time, and then solve the position of the positioning target [4-8]. In the above positioning system, the transmitting
signal pulse width is short, it is not sensitive to Doppler, so the study of motion error model is not involved.

In this paper, based on the homogeneous coordinates and homogeneous rotation matrix, the relationship between the linear velocity of each array element of transmitter and the rotating velocity of moving platform in the underwater multi-orthogonal system is established. Combined with signal processing, the motion error model is established. Homogeneous coordinates and homogeneous rotation matrix are widely used in Robot Arm control and Unmanned Aerial Vehicle posture control, which have the advantages of less computation and clear physical meaning[9-10].

2. The positioning principle of multi-orthogonal navigation system

In the MOUNS, the depth Z of the receiver can be obtained directly by the depth measuring instrument. In this paper, we mainly discuss the positioning of MOUNS in the x-y plane. As shown in Fig. 1-1, 1, 2, 3 and 4 respectively represent four array element of transmitter, which are located in the same plane, and T is the receiver located in the water. The horizontal and vertical coordinates of the receiver can be solved by using the geometric relationship. The calculation formula is as follows:

$$x = \frac{\lambda \phi_x l}{2\pi d}, \quad y = \frac{\lambda \phi_y l}{2\pi d}$$  \hspace{1cm} (1)

Where, $\lambda$ is the signal wavelength, $d$ is the spacing of between the two array elements of transmitter, $\phi_x$ is the phase difference between two transmitted signals in the direction of x axis and $\phi_y$ is the phase difference between two transmitted signals in the direction of y axis. $l$ is the distance between the receiver and the transmitter. $\lambda, d, l$ is a fixed value. Due to the movement of each array element of transmitter, the received signal will produce a certain Doppler frequency offset relative to the receiver, which will lead to the error of phase difference $\phi_x$ and $\phi_y$ finally lead to the positioning error.

3. Moving platform error modeling

The modeling of movement platform is mainly based on homogeneous coordinate and homogeneous matrix. The homogeneous matrix unifies the posture and position of the movement platform and simplifies the calculation amount[3].

In order to facilitate the expression in theoretical derivation, the following symbolic definitions are made: E represents the geodetic coordinate system, V represents the movement platform coordinate system, and A represents the Transmitting Arrays coordinate system. The homogeneous rotation matrix $E^V_T$ represents the position and posture of the moving platform relative to the earth, the superscript represents the reference coordinate system, and the subscript represents the target coordinate system.
The homogeneous rotation matrix $^V_A T$ represents the position of the Transmitting Arrays coordinate system $A$ relative to the moving platform $V$. The homogeneous coordinate $^A n P$ represents the position of the array element of transmitter in the transmitting Arrays coordinate system $A$, the superscript represents the coordinate system, and the subscript represents the sequence number of array element of Transmitting Array. Where $n=1, 2, 3, 4$. $^V A T$ and $^V T$ can be respectively expressed as:

$$
^V A T = \begin{bmatrix}
^V R & ^V R \cdot A X \\
0 & 1
\end{bmatrix},

^V T = \begin{bmatrix}
^V R & ^V R \cdot A X \\
0 & 1
\end{bmatrix},

^A n P = \begin{bmatrix}
A X \\
0
\end{bmatrix}
$$

Where $^E V R$ represents the posture of the movement platform coordinate system $V$ relative to the geodetic coordinate system $E$; $^V R$ Represents the posture of the Transmitting Arrays coordinate system $A$ relative to the moving platform $V$. $^E X_{V0}$ is the coordinate of the center of the moving platform coordinate system in the geodetic coordinate system $E$, and $^V X_{A0}$ is the coordinate of the center of the Transmitting Arrays coordinate system $A$ in the moving platform coordinate system $V$.

Assuming MOSUNS installed on the motion platform $V$, take the position of the moving platform at time $t_0$ as a reference, after a short time $dt$ , tiny rotational and small translation of motion platform at time $t_0 + dt$ relative to the time $t_0$ can be expressed as: $^{V_{t_0+dt}} T (^V \delta, ^V d)$ . Where, $^V \delta = (^V \delta_x, ^V \delta_y, ^V \delta_z)$ represents the tiny Angle of rotation of the moving platform $V$ around X, Y and Z axes, and $^V d = (^V d_x, ^V d_y, ^V d_z)$ represents the tiny displacement of the moving platform $V$ along X, Y and Z axes.

$t_0 + dt$, the posture of the moving platform $V$ relative to the geodetic coordinate system $E$:

$$
^{E_{t+dt}} T = ^E T \times ^{V_{t_0}} T (^V \delta, ^V d)
$$

Let:

$$
^{V} \Delta (^V \delta, ^V d) = ^{V_{t_0+dt}} T (^V \delta, ^V d) - I
$$

Available:

$$
dT = ^V T \times ^V \Delta (^V \delta, ^V d)
$$

Meanwhile, at the time $t_0$, the position of the array element of transmitting array in the geodetic coordinate system $E$ can be expressed as:

$$
^{E n P} = ^E T \times ^V T \times ^A n P
$$

at the time $t_0 + dt$, the position of the array element of transmitting array in the geodetic coordinate system $E$:

$$
^{E_{t+dt} n P} = (^{E_{t+dt}} T) \times ^V T \times ^A n P
$$

By subtracting Equation (9) from Equation (8), we can obtain:

$$
^{E_{t+dt} n P} = \Delta (^V \delta, ^V d) \times ^V T \times ^A n P
$$
Where, $\Delta^{E}P = E_{n}P - E_{n}P$, divide $dt$ both ends of the equation in Equation (7) at the same time to obtain the linear velocity of each array element of transmitter:

$$E_{n}V = E_{n}T \times \Delta (\nu \omega, \nu v) \times \nu T \times p \Delta$$ (8)

Where, $E_{n}V$ is the linear velocity of the $n$th array element of transmitter in the geodetic coordinate system $E$, $\nu \omega = \frac{dV}{dt}$ is the rotation rate of the moving platform $V$, and $\nu v = \frac{dV}{dt}$ is the translational rate of the moving platform. Specifically $\nu \omega = (\nu \omega_{x}, \nu \omega_{y}, \nu \omega_{z})$ and $\nu v = (\nu v_{x}, \nu v_{y}, \nu v_{z})$ are respectively the rotating rates of the moving platform around $X, Y$ and $Z$ axes, namely, the roll, pitch and yaw rates of the moving platform; $\nu v = (\nu v_{x}, \nu v_{y}, \nu v_{z})$ and $\nu v_{x}, \nu v_{y}, \nu v_{z}$ are respectively the translational rates of the moving platform along $X, Y$ and $Z$. In the (8) formula, the rotation rate $\Delta R(\nu \omega)$ of moving platform $V$ and the translational speed $\nu v$ of moving platform $V$ are described.

The radial velocity of the transmitting signal by the each array element of transmitter relative to the receiver is:

$$\nu_{n} = \left( E_{n}T \times \Delta (\nu \omega, \nu v) \times \nu T \times p \right) \cdot L$$ (9)

Where, $\nu_{n}$ is the scalar, representing the radial rate of the transmitting signal by the each array element of transmitter relative to the receiver in the geodetic coordinate system $E$. and $L = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the $3 \times 1$ vector, is the unit direction vector between the receiver and the transmitter. The rotation rate $\Delta R(\nu \omega)$ of moving platform $V$ and the translational speed $\nu v$ of moving platform $V$ are described.

According to Equation (9), the radial velocity difference between the $i$th and $j$th array element of transmitter can be obtained:

$$\Delta_{ij} \nu = \nu_{i} - \nu_{j}$$

$$\Delta_{ij} \nu = \left( E_{n}T \times \Delta (\nu \omega, \nu v) \times \nu T \times p \right) \cdot L$$ (10)

Where, $\Delta_{ij} p = A_{i}p - A_{j}p$ represents the coordinate vector difference of any two array element of transmitter.

4. Analysis of positioning error caused by platform movement

Because the linear velocity of each transmitting element is different, the positioning deviation of the receiving end is caused. According to the principle of matched filtering, four signals are separated at the receiving end, and the phase deviation of each signal is extracted, and finally the relative $X$ and $Y$ of the receiving end are calculated. Because the linear velocity of each transmitting element is different, the positioning deviation of the receiving end is caused. According to the principle of matched filtering, four signals are separated at the receiving end, and the phase deviation of each signal is extracted, and finally the relative $X$ and $Y$ of the receiving end are calculated in the Transmitting Arrays coordinate system $A$. The deviation is:

The deviation is:
Let the relative error of the receiving end position be: \[
\delta = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{l},
\]
(11)

Substitute (11) to get:

\[
\delta = \frac{T}{4} W
\]
(12)

Where,

\[
W = \left[\frac{\mathbf{R} \times \Delta \mathbf{R}(\mathbf{V}\omega) \times \mathbf{R} \times \frac{2\Delta \mathbf{R}}{d}}{l} \right]^2 + \left[\frac{\mathbf{R} \times \Delta \mathbf{R}(\mathbf{V}\omega) \times \mathbf{R} \times \frac{2\Delta \mathbf{R}}{d}}{l} \right]^2
\]
(13)

According to Equation (13), when the posture of the moving platform and Transmitting Arrays is constant, the relative error \(\delta\) of the system is related to the signal duration \(T\) and the rotation rate \(\Delta \mathbf{R}(\mathbf{V}\omega)\) of the moving platform, independent of the position of the moving platform \(\mathbf{V}\) and transmitter, and independent of the spacing \(d\) of the array element of transmitter.

Table 6-1. Initial data of simulation.

| Arguments | Condition 1 | Condition 2 | Condition 3 | Condition 4 |
|-----------|-------------|-------------|-------------|-------------|
| \(T\) (s) | 0.127       | 0.254       | 0.127       |              |
| SNR (dB)  | 15          |             |             |              |
| \(d\) (m) | 0.5         | 0.6         | 0.4         |              |
| \(\alpha\ \beta\ \lambda\) (rad) | [0.1 0.5 0.3] |             |             |              |
| \(\sigma\ \kappa\ \zeta\) (rad) | [0 0 0]     |             |             |              |
| \(\mathbf{E}_x\mathbf{T}\) (m) | \[0 0 1\] | \[0 0 1\] | \[0 0 2\] |              |
| \(\mathbf{E}_y\mathbf{T}\) (m) | \[0 1 -2\] | \[0 1 -2\] | \[0 1 -3\] |              |
| \(\mathbf{X}_0\) (m) | 50          |             |             |              |
| \(\mathbf{Y}_0\) (m) |             |             |             | 50           |

5. Data Simulation

In order to verify the motion error model of Equation (12), the MOSUNS is simulated by MATLAB, and the relationship between the relative error of the positioning system and various variables in the system is discussed, as shown in Table 6-1. The initial values of various parameters under different conditions are given. In Table 6-1, \(\mathbf{E}_x\mathbf{T}\) and \(\mathbf{E}_y\mathbf{T}\) are respectively the horizontal and vertical coordinate values of the receiving end in the geodetic coordinate system. SNR stands for signal-to-noise ratio.

According to the initial parameters set in Table 6-1, in practice, the MOUNS should work in a relatively stable environment. In this case, the speed of the moving platform is roughly calculated as follows: roll rate \(\omega_x = 0 - 0.15\) rad/s, pitch rate \(\omega_y = 0.05\) rad/s, and yaw rate \(\omega_z = 0.05\) rad/s. According to the above values, \(W\) of Equation (12) is calculated, and then the relation between relative error \(\delta\) and \(W\) is obtained through simulation. \(\alpha\ \beta\ \lambda\ \sigma\ \kappa\ \zeta\) are Euler Angle.
Under condition 1 and condition 2 (only the value of signal pulse width T is different between condition 1 and condition 2), the results are shown in Fig. 6-1 (a), where the blue dotted line represents the theoretical value under condition 1 and the square represents the simulation value under condition 1. The circle represents the simulation value under condition 2, and the solid red line represents the theoretical value under condition 2. It can be seen from Fig. 6-1 (a) that the relative error $\delta$ has a linear relationship with W, and its slope is related to the signal pulse width T. When the value of the signal pulse width T changes, the slope between $\delta$ and W also changes.

Select the signal duration $T=0.127$, change the radius between array element of transmitter the position of the moving platform and the transmitter, i.e. condition 3 and condition 4. Under the above conditions, the result as shown in Figure 6-1 (b) is obtained. It can be seen from the figure that the simulation result 3 is basically consistent with the simulation result 4 and the theoretical value $3,4$, which verifies that the positioning error caused by the rotation of the moving platform has nothing to do with the radius between array element of transmitter and the position of the moving platform and the transmitter.

6. Summarizes
In this paper, the positioning error of MOSUNS on a moving platform is modeled and the corresponding compensation algorithm is given. In the second section, the relation between the linear velocity of the transmitting array element and the rotation rate of the moving platform is established based on the homogeneous rotation matrix and the homogeneous coordinates. In the third section of this paper, the mathematical model of positioning error of MOSUNS is established. In the fourth section, through MATLAB simulation, in the case of small Angle approximation, the simulation results are basically consistent with the motion error model.

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