Effects of Bulk Viscosity in Non-linear Bubble Dynamics

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The non-linear bubble dynamics equations in a compressible liquid have been modified considering the effects of compressibility of both the liquid and the gas at the bubble interface. A new bubble boundary equation has been derived, which includes a new term resulted from the liquid bulk viscosity effects. The influence of this term has been numerically investigated considering the effects of water vapor and chemical reactions on the bubble evolution. The results clearly indicate that the new term has an important damping role at the collapse, so that its consideration decreases the amplitude of the bubble rebounds after the collapse. This damping feature is more remarkable for higher pressures.

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When a small isolated gas bubble, immersed in a liquid, experiences a high amplitude spherical sound field, it grows and contracts non-linearly. Description of the dynamics of such non-linear motion is an old challenging problem. The radial dynamics of the bubble in an incompressible liquid is described by the well-known incompressible Rayleigh-Plesset equation [1, 2]. The extension of this equation to the bubble motion in a compressible liquid has been studied by many previous authors [3, 4]. The most complete existing description was presented by Prosperetti and Lezzi [5]. They used a singular-perturbation method of the bubble-wall Mach number and derived a one-parameter family of equations describing the bubble motion in the first order approximation of compressibility. This family of equations are written as:

\[
\left(1 - (\eta + 1)\frac{R}{C}\right) R\ddot{R} + \frac{3}{2} \left(1 - \frac{1}{3}(3\eta + 1)\frac{R}{C}\right) \dddot{R} = \frac{R}{\rho C} \frac{d}{dt} (P_l - P_a) + \left(1 + (1 - \eta)\frac{R}{C}\right) \left(\frac{P_l - P_a - P_0}{\rho}\right),
\]

where, \(R\), \(C\), \(P_0\), \(P_a\), and \(\rho\) are bubble radius, liquid sound speed, ambient pressure, driving pressure, and density of the liquid, respectively. Also, \(\eta\) is an arbitrary parameter. Equation (1) must be supplemented by a boundary condition equation at the bubble interface to relate the liquid pressure, \(P_l\), to the gas pressure inside the bubble. Like all previous authors, Prosperetti and Lezzi used the following incompressible equation for this purpose:

\[
P_l = P_g - 4\mu \frac{\dot{R}}{R} - \frac{2\sigma}{R},
\]

where, \(P_g\), \(\mu\), and \(\sigma\) are gas pressure at the bubble interface, liquid viscosity coefficient, and surface tension, respectively. Most of the previously obtained equations belong to this single parameter family of equations, corresponding to different values of \(\eta\). Moreover, \(\eta = 0\) yields results in closest agreement with the numerical simulation of full partial differential equations [5].

In all previous works [1, 2, 3, 4, 5], an important approximation has been used in the derivation of the bubble dynamics equations. That is the incompressibility assumption of the liquid motion at the bubble interface, which has been used in the derivation of Eq’n. (2). Note that, all of the effects of the liquid compressibility in all previous papers have been resulted from the liquid motion around the bubble, but not from the bubble boundary condition equation. In fact, all previous authors, on one hand took into account the compressibility of the liquid motion around the bubble, but on the other hand neglected its consideration at the bubble interface.

In this paper, we have modified the bubble dynamics equations considering the effects of the liquid compressibility at the bubble interface. We have derived a new bubble boundary equation instead of Eq’n. (2). The new equation has new terms resulted from the effects of bulk viscosity of the liquid and the gas.

To derive the compressible bubble boundary equation, the continuity equation and the radial component of the stress tensor under the spherical symmetric condition can be written as:

\[
\frac{1}{\rho} \left[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right] = -\frac{\partial u}{\partial r} - \frac{2u}{r} = -\Delta,
\]

\[
T_{rr} = -p + (\mu_b - \frac{2\mu}{3})\Delta + 2\mu \left(\frac{\partial u}{\partial r}\right).
\]

where, \(\rho\), \(u\), \(p\), and \(\Delta\) are density, velocity, pressure, and divergence of the velocity, respectively. Also, \(\mu_b\) is the bulk viscosity coefficient and is defined by \(\mu_b = \lambda + 2\mu/3\), where \(\lambda\) is second coefficient of viscosity [6]. Inserting
\[ \frac{\partial u}{\partial r} \] from Eq'n. \( \text{[3]} \), into Eq'n. \( \text{[1]} \) yields:

\[ T_{rr} = -p + \left( \mu_b + \frac{4\mu}{3} \right) \Delta - 4\frac{\mu u}{r}. \]  

(5)

The velocity divergence, \( \Delta \), can be written as:

\[ \Delta = -\frac{1}{\rho} \frac{dp}{dt} = -\frac{1}{\rho c^2} \frac{dp}{dt}. \]  

(6)

where, the sound speed, \( c \), is defined as \( c^2 = dp/d\rho \). The boundary continuity requirement at the bubble interface is:

\[ T_{rr}(\text{liquid}) \big|_R = T_{rr}(\text{gas}) \big|_R + 2\frac{\sigma}{R}. \]  

(7)

Applying Eq'n. \( \text{[5]} \) for the gas and the liquid parts of Eq'n. \( \text{[7]} \) leads to:

\[ P_l + 4\frac{\mu \dot{R}}{R} - \left( \mu_b + \frac{4\mu}{3} \right) \Delta_l = P_g + 4\frac{\mu_g \dot{R}}{R} - \left( \mu_{bg} + \frac{4\mu}{3} \right) \Delta_g - 2\frac{\sigma}{R}. \]  

(8)

where, \( \mu_g \) and \( \mu_{bg} \) are the viscosity and the bulk viscosity coefficients of the gas at the bubble interface, respectively. Also, \( \Delta_l \) and \( \Delta_g \) are the divergence of velocity of the liquid and the gas, respectively. Substituting the divergence of velocity for the liquid and the gas from Eq'n \( \text{[6]} \) into Eq'n. \( \text{[8]} \) yields:

\[ P_l + 4\frac{\mu \dot{R}}{R} + \left( \frac{\mu_b}{3\rho c^2} + \frac{4\mu}{3\rho c^2} \right) \frac{dp_l}{dt} = P_g + 4\frac{\mu_g \dot{R}}{R} + \frac{\mu_{bg}}{\rho_g} + \frac{4\mu_g}{3\rho_g} \frac{dp_g}{dt} - 2\frac{\sigma}{R}. \]  

(9)

where, \( \rho_g \) is the gas density at the bubble interface. Equation \( \text{[9]} \) represents the bubble boundary equation containing all effects of the compressibility and the viscosity of both the liquid and the gas. Comparison of Eq'n's. \( \text{[2]} \) and \( \text{[9]} \) indicates the existence of three new terms in Eq'n. \( \text{[9]} \) due to the liquid and the gas compressibility and viscosity effects. Here, we concentrate on the effects of the new term arising from the liquid compressibility. Therefore, we neglect the gas viscosity because of its smallness relative to the liquid viscosity as in previous works \cite{1,2,3,4,5}. Under this circumstance, Eq'n. \( \text{[10]} \) becomes:

\[ P_l + \left( \frac{\mu_b}{3\rho c^2} + \frac{4\mu}{3\rho c^2} \right) \frac{dp_l}{dt} = P_g - 4\frac{\mu \dot{R}}{R} - 2\frac{\sigma}{R}. \]  

(10)

It should be mentioned that, although the effects of compressibility consideration in Eq'n. \( \text{[11]} \) are in the first order approximation, but these effects have been introduced completely in Eq'n. \( \text{[10]} \).

To close the mathematical analysis, the gas pressure evolution at the bubble interface, \( P_g \), must be specified. In the most complete approach, it can be determined from the simultaneous solution of the conservation equations for the bubble interior and the bubble radius equations \cite{7,8,9,10,11,12}. Also, heat conduction and mass exchange between the bubble and the surrounding liquid affect the bubble evolution. In addition, chemical reactions occurring in the high temperature conditions at the end of the collapse, change the bubble content \cite{13,14}. All these complexities have been considered in a complete gas dynamics model by Storey and Szeri \cite{15,16}.

On the other hand, strong spatial inhomogeneities inside the bubble are not remarkably revealed, unless at the end of an intense collapse \cite{10,11}. Therefore, the uniformity assumption for the bubble interior seems to be useful and provides many features of the bubble motion \cite{16,17}. Using this assumption, recently, Lohse and his coworkers presented an ODE model \cite{18,19,20}, in which all effects of heat transfer at the bubble interface, phase change of water vapor, chemical reactions, and diffusion of reaction products have been considered. This model accurately describes various experimental phase diagrams \cite{19} and provides a good agreement with the complete direct numerical simulation of Storey and Szeri \cite{15,16}.

Here, for describing the bubble interior evolution, we have used the Lohse’s group model (the same as what has been presented in Ref. \cite{20}). We do not repeat this model here and for more details we refer to Refs. \cite{18,19,20}. The calculations were carried out under the framework of Eq'n \( \text{[11]} \), \( \eta = 0 \), for both the new compressible (Eq'n. \( \text{[10]} \)) and the old incompressible (Eq'n. \( \text{[2]} \)) boundary conditions. We describe an argon bubble in water at room temperature, \( T_0 = 293.0 \) K, and atmosphere pressure, \( P_0 = 1.0 \) atm, under the conditions of Single Bubble Sonoluminescence \cite{16,17}. The driving pressure was \( P_a(t) = P_0 \sin(\omega t) \), where \( \omega = 2\pi \times 26.5 \) kHz. The constants and the parameters were set accordingly \cite{21}: \( \rho = 998.0 \) kg/m$^3$, \( C = 1483.0 \) m/s, \( \mu = 1.01 \times 10^{-3} \) kg/ms, \( \sigma = 0.0707 \) kgs$^{-2}$. The bulk viscosity of water at room temperature was set to be \( \mu_b = 4.1 \times 10^{-3} \) kg/ms \cite{22}. The constants and parameters of the gas evolution model were set the same as what has been presented in Ref. \cite{20}.

Figures (1) and (2) illustrate the variations of the bubble characteristics (radius, total number of particle species, and temperature), for the two boundary condition cases. It is observed that the addition of the new viscous term in Eq'n. \( \text{[10]} \) considerably changes the bubble evolution after the collapse. The bubble motion is remarkably compressible during the collapse. Therefore, the new viscous term, which has been arisen from the liquid compressibility, is important in this time interval. This term exhibits a damping role and its consideration reduces the amplitude of the bubble rebounds. Also, the period of the rebounds decreases with the addition of...
FIG. 1: (a) Time variations of the bubble radius in one period according to the new compressible (solid) and the old incompressible cases. (b) Details of the bubble rebounds after the collapse for the two cases. (c) Variation of total number of the particle species for the two cases during the bubble rebounds. The equilibrium radius is $R_0 = 3.0 \mu m$ and the deriving pressure is $P_a = 1.35 \ atm$.

The new term. Details of our calculations show that the minimum radius for the new case is about 10% greater than that of the old one. The difference between the two cases also appears on the variations of the total number of particle species (Ar and H$_2$O plus reactions products) after the collapse (Fig. 1c). Note that, the difference gradually disappears as the bubble rebounds weaken.

In Fig. (2), details of the gas temperature evolution around the minimum radius have been demonstrated. Damping feature of the new term is clearly observed by a considerable decrease of the peak temperature (about 50%) and an increase of the temperature pulse width, at the collapse time. Also, the time of the peak temperature about 5 ns changes with the addition of the new term.

Figure (3) represents the effects of variations of $P_a$ on the bubble characteristics at the end of the collapse (peak temperature, mole fraction of H$_2$O and reaction products, and minimum radius), for the two boundary condition cases. The ambient radius was fixed ($R_0 = 5.0 \mu m$). Figure 3(a) shows that the peak temperature in the both cases grows as the driving pressure is increased. However, the rate of increase of the peak temperature for the new case is considerably smaller than that of the old one. This causes that the difference between the two cases becomes remarkable for the higher driving pressures (about 50% for $P_a = 1.5 \ atm$).

The bubble temperature at the end of the collapse is high enough to destroy the chemical bonds of water vapor molecules inside the bubble. The products of the dissociation of water vapor molecules are mainly H$_2$, OH, H, O, and O$_2$. The chemical reactions between the particle species existing inside the bubble affect the bubble content at the collapse and its peak temperature [13, 14, 17, 18, 20]. Here, we have considered the effects of the reactions No. 1-8 of Refs. [14, 19]. The dependence of the mole fraction of H$_2$O plus reactions products, which is defined as $(N_{tot} - N_{Ar})/N_{tot}$, to the driving pressure for the two boundary condition cases has been illustrated in Fig. 3(b). It shows that the mole fraction of H$_2$O plus reactions products is similar for the two cases in low amplitudes. The difference appears for the higher deriving pressures. It is seen that, for the higher deriving pressures, the effect of H$_2$O and reactions products is more important in the new case relative to the old one.

Figure 3(c) shows the variations of the minimum radius as a function of $P_a$, for the two cases. The trend of variations is similar for the two cases. But, the minimum radius for the new equation is more than that of the old equation because of the reduction of the collapse intensity.

A major deficiency of the old bubble dynamics equations is that for strongly driven bubbles, such as sonoluminescence bubbles, large amplitude rebounds are produced after the collapse, so that they often last until the next acoustic cycle of the periodic driving pressure. This
FIG. 3: The bubble characteristics at the time of collapse as a function of driving pressure amplitude for the compressible (solid) and incompressible (dashed) boundary conditions; peak temperature (a), mole fraction of H$_2$O and reaction products (b), and minimum radius (c). The equilibrium radius was fixed ($R_0 = 5.0 \mu$m). Other constants are the same as Figs. (1) and (2).

is in contrast with the experimental results, which show rapidly damped rebounds. By introducing a damping term arisen from the gas compressibility, Moss et. al provided a typical solution for this problem. The effects of the suggested term by Moss et. al is very similar to the damping effects of the new term in this paper, (compare Fig. 1(b) with Figs. (3) and (4) of Ref. [23]). It seems that the damping feature of the bulk viscosity is a better way for solving the mentioned problem. The reason is that Eq’n. [10] has been derived directly from the basic equations of fluid mechanics, on the contrary to Eq’n. (3.2) of Ref. [23], which was derived by an approximate method.

According to the results of this paper, it is expected that the theoretical predictions of the bubble stability limits are affected by the addition of the new term to the bubble dynamics equations.

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