Peccei-Quinn invariant singlet extended SUSY
with anomalous $U(1)$ gauge symmetry

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Abstract

Recent discovery of the SM-like Higgs boson with $m_h \simeq 125$ GeV motivates an extension of the minimal supersymmetric standard model (MSSM), which involves a singlet Higgs superfield with a sizable Yukawa coupling to the doublet Higgs superfields. We examine such singlet-extended SUSY models with a Peccei-Quinn (PQ) symmetry that originates from an anomalous $U(1)_A$ gauge symmetry. We focus on the specific scheme that the PQ symmetry is spontaneously broken at an intermediate scale $v_{PQ} \sim \sqrt{m_{SUSY} M_{Pl}}$ by an interplay between Planck scale suppressed operators and tachyonic soft scalar mass $m_{SUSY} \sim \sqrt{D_A}$ induced dominantly by the $U(1)_A$ $D$-term $D_A$. This scheme also results in spontaneous SUSY breaking in the PQ sector, generating the gaugino masses $M_{1/2} \sim \sqrt{D_A}$ when it is transmitted to the MSSM sector by the conventional gauge mediation mechanism. As a result, the MSSM soft parameters in this scheme are induced mostly by the $U(1)_A$ $D$-term and the gauge mediated SUSY breaking from the PQ sector, so that the sparticle masses can be near the present experimental bounds without causing the SUSY flavor problem. The scheme is severely constrained by the condition that a phenomenologically viable form of the low energy operators of the singlet and doublet Higgs superfields is generated by the PQ breaking sector in a way similar to the Kim-Nilles solution of the $\mu$ problem, and the resulting Higgs mass parameters allow the electroweak symmetry breaking with small $\tan \beta$. We find two minimal models with two singlet Higgs superfields, satisfying this condition with a relatively simple form of the PQ breaking sector, and briefly discuss some phenomenological aspects of the model.

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I. INTRODUCTION

Low energy supersymmetry (SUSY) \(^1\) and the QCD axion \(^2\) are compelling candidates for physics beyond the Standard Model (SM) as they not only solve the major fine-tuning problems of the SM, \(i.e.\) the gauge hierarchy problem and the strong CP problem, but also shed a light on different fundamental issues such as dark matter and unification. Furthermore, there are several virtues of having both SUSY and axion together. For instance, the axion scale can be determined by an interplay between SUSY breaking scalar mass \(m_{\text{SUSY}}\) and a Planck scale suppressed higher dimensional operator, which would generate an intermediate axion scale \(v_{\text{PQ}} \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}\) in a natural way \(^3\). The absence of a potentially too large bare \(\mu\) term of the doublet Higgs superfields can be understood also by a Peccei-Quinn (PQ) symmetry, \(U(1)_{\text{PQ}}\) \(^4\) for the QCD axion. Then a right size of the Higgs \(\mu\) parameter can be generated by the spontaneous PQ breaking as \(\mu \sim v_{\text{PQ}}^2 / M_{\text{Pl}} \sim m_{\text{SUSY}}\), solving the \(\mu\) problem for the supersymmetric Higgs sector \(^5\). As another possible virtue, the cosmological PQ phase transition in such model can be preceded by a thermal inflation, thereby solves the cosmological moduli problem \(^6\).

In view of minimizing the fine-tuning for the electroweak symmetry breaking (EWSB), we are most interested in the case that sparticles, particularly the stops, are as light as possible, being close to the present experimental bounds \(^7\). On the other hand, to explain the recently discovered SM-like Higgs boson mass \(m_{h} \simeq 125\) GeV within the framework of the minimal supersymmetric standard model (MSSM), the stops need to have a mass around multi-TeV or even heavier, which is well above the current direct search limit \(^8, 9\). A simple way to avoid this difficulty is to extend the MSSM by adding a singlet Higgs superfield \(S\) which has the superpotential coupling \(\lambda S H_u H_d\) to the doublet Higgs superfields \(H_{u,d}\) \(^10, 11\). In such singlet-extended models, the SM-like Higgs boson mass receives a contribution \(\delta m_{h}^2 = \lambda^2 m_Z^2 \sin^2 2\beta / (g_1^2 + g_2^2)\) from the Higgs quartic coupling \(|\lambda H_u H_d|^2\), and the stops can have a relatively light mass around (or below) TeV, while being compatible with \(m_{h} \simeq 125\) GeV, if \(\lambda \sim 1\) and \(\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle \sim 1\). This is perhaps the most straightforward extension of the MSSM, minimizing the fine-tuning for the EWSB under the known experimental constraints.

The model can be extended further by introducing a PQ symmetry \(^12, 14\), to solve the strong CP problem, together with a PQ sector which breaks the PQ symmetry sponta-
neously at an intermediate scale $v_{PQ} \sim 10^9 - 10^{12}$ GeV generated by $\sqrt{m_{\text{SUSY}}M_{\text{Pl}}}$ without introducing new bare mass parameters \[3, 15\]. One can arrange the model further, so that all the low energy mass parameters of the singlet-extended Higgs sector are generated by the spontaneous PQ breaking, and have a value comparable to $m_{\text{SUSY}}$ in a way similar to the Kim-Nilles mechanism \[3\] for the $\mu$ problem.

An important issue about the axion solution of the strong CP problem is the UV origin of the PQ symmetry which is required to be protected well from quantum gravity effects violating global symmetries in general \[16\]. Note that to solve the strong CP problem, the explicit PQ breaking by quantum gravity effects should be negligible compared to the breaking by the QCD anomaly \[17\]. For the UV origin of a PQ symmetry, an appealing possibility is that $U(1)_{PQ}$ originates from an anomalous $U(1)_A$ gauge symmetry whose gauge boson gains a heavy mass near the Planck scale by the St"uckelberg mechanism \[18–22\]. Then, quantum gravity effects breaking $U(1)_{PQ}$ can be exponentially suppressed.

In this paper we examine the SUSY breaking, as well as some of the phenomenological consequences, in singlet-extended SUSY models involving a PQ symmetry which originates from an anomalous $U(1)_A$ gauge symmetry. We are interested in the scheme to yield flavor conserving soft parameters which lead to the superparticle masses near the present experimental bounds, together with $m_h \simeq 125$ GeV which is largely due to the singlet superpotential term $\lambda S H_u H_d$ with $\lambda \sim 1$ and $\tan \beta \sim 1$. In the next section, we first discuss generic features of SUSY breaking in models with anomalous $U(1)_A$ gauge symmetry broken by the St"uckelberg mechanism, while leaving a global PQ symmetry as a low energy remnant \[22\]. We then examine the specific scheme that the soft SUSY breaking parameters in the PQ sector are dominated by the $U(1)_A$ $D$-term contribution as

$$\epsilon \equiv \frac{m_{\text{MM}}}{\sqrt{D_A}} \ll 1,$$

where $m_{\text{MM}}$ denotes the moduli (or equivalently gravity) mediated soft masses. In this scheme, the PQ symmetry is spontaneously broken at $v_{PQ} \sim (\sqrt{D_A}M_{\text{Pl}})^{1/2}$, or more generically $v_{PQ} \sim (\sqrt{D_A}M_{\text{Pl}}^n)^{1/(n+1)} (n \geq 1)$, by an interplay between the $D$-term induced tachyonic scalar mass and a Planck scale suppressed operator. A notable feature of this scheme is that it leads to a spontaneous SUSY breaking in the PQ sector, showing a hierarchical structure for vacuum expectation values by $\epsilon$. This SUSY breaking in the PQ sector can be transmitted to the MSSM sector by the conventional gauge mediation mechanism, yielding
the gauge mediated soft masses:

\[ m_{GM} \sim \frac{g^2}{8\pi^2} \epsilon \sqrt{D_A} \tag{2} \]

We will focus on a scheme in which \( \epsilon \) amounts to

\[ \epsilon \sim \frac{g^2}{8\pi^2} \tag{3} \]

for which the MSSM soft parameters are determined by the gauge mediated SUSY breaking from the PQ sector and the \( U(1)_A D \)-term, which are comparable to each other.

To complete the scheme, we need to generate a phenomenologically viable form of the low energy operators of the singlet and doublet Higgs superfields through the spontaneous PQ breaking as in the Kim-Nilles solution of the \( \mu \) problem. It turns out that the hierarchical pattern of the SUSY breaking \( F \)-components in the PQ sector makes this nontrivial at least for the relatively simple form of the PQ sector. In Sec. III, we examine this possibility, and find two minimal models with two singlet Higgs superfields. In Sec. IV, we discuss some of the phenomenological consequence of these two minimal models. One of the models is more interesting as it allows the limit that the Higgs sector including the singlet Higgs is parametrically lighter than the other sector of the model, without causing further fine-tuning than the minimal fine-tuning for the EWSB.

II. FEATURES OF PQ SYMMETRY AND SOFT TERMS WITH ANOMALOUS \( U(1)_A \)

A. Peccei-Quinn symmetry and \( D \)-term mediation from an anomalous \( U(1)_A \)

We begin with an observation that a large fraction of phenomenologically viable string compactifications involves an anomalous \( U(1)_A \) gauge symmetry. An anomalous \( U(1)_A \) gauge symmetry can be quantum mechanically consistent by the Green-Schwarz (GS) anomaly cancellation \[23\], which is implemented by introducing the axion-like field \( a_p \), a zero mode of the higher-dimensional \( p \)-form gauge field. In the supersymmetric language, \( a_p \) is a pseudoscalar component of a chiral multiplet for the GS modulus \( T_A \). Then various supermultiplets transform under \( U(1)_A \) as

\[ U(1)_A : \quad V_A \rightarrow V_A + \Lambda + \Lambda^* , \quad T_A \rightarrow T_A + \delta_{GS} \Lambda , \quad \Phi_i \rightarrow e^{-2q_i A} \Phi_i , \tag{4} \]
where $V_A$ is the $U(1)_A$ vector multiplet, $\Lambda$ is a chiral multiplet parametrizing $U(1)_A$ gauge transformation, and a coefficient $\delta_{GS}$ is the $U(1)_A$-QCD-QCD anomaly coefficient given by

$$
\delta_{GS} = \frac{1}{8\pi^2} \sum_i q_i \text{Tr}(T_c(\Phi_i)^2),
$$

(5)

for $T_c(\Phi_i)$ denoting the color charge matrix of $\Phi_i$. From this, one finds that the Kähler potential

$$
K = K_0(t_A, t_b, t_k) + Z_i(t_A, t_b, t_k)\Phi_i^* e^{2q_i V_A \Phi_i},
$$

(6)

depends on $T_A$ through a gauge invariant combination $t_A \equiv T_A + T_A^* - \delta_{GS} V_A$. We note that the Kähler potential Eq. (6) in general contains other moduli $t_k \equiv T_k + T_k^*$, as well as a SUSY breaking modulus $t_b \equiv T_b + T_b^*$, which are not charged under $U(1)_A$.

Let $2 \eta^I = \{-\delta_{GS}, 2q_i \Phi_i\}$ be the holomorphic Killing vector fields generating an infinitesimal $U(1)_A$ gauge transformation of chiral multiplets $\Phi_I = \{T_A, \Phi_i\}$. Then the gauge boson mass and $D$-term of the $U(1)_A$ multiplet of gauge coupling $g_A$ are given by

$$
M_A^2 = 2g_A^2 \eta^K \bar{\eta}^J \partial_I \partial_J K = 2g_A^2 (M_{GS}^2 + M_{\text{matter}}^2),
$$

$$
D_A = -\eta^K \partial_K K = \xi_{\text{FI}} + \bar{M}_{\text{matter}},
$$

(7)

respectively, where the GS modulus contribution

$$
M_{GS}^2 = \frac{\delta_{GS}^2}{4} \partial_{t_{A}} K_0, \quad \xi_{\text{FI}} = \frac{\delta_{GS}}{2} \partial_{t_{A}} K_0,
$$

(8)

and the matter contribution

$$
M_{\text{matter}}^2 = \sum_i \left( q_i Z_i - Q_i \delta_{GS} \partial_{t_{A}} Z_i + \left( \frac{\delta_{GS}}{2} \partial_{t_{A}} Z_i \right)^2 \Phi_i^* e^{2q_i V_A \Phi_i} \right),
$$

$$
\bar{M}_{\text{matter}}^2 = -\sum_i \left( q_i Z_i - \frac{\delta_{GS}}{2} \partial_{t_{A}} Z_i \right) \Phi_i^* e^{2q_i V_A \Phi_i},
$$

(9)

are written separately.

If the underlying string compactification admits a supersymmetric solution with vanishing Fayet-Iliopoulos (FI) term $\xi_{\text{FI}}$\footnote{If it were not the case, we need $q_i |\Phi_i|^2 \sim \xi_{\text{FI}} \sim \delta_{GS} M_{\text{pl}}^2$ for vanishing $D_A$ in the supersymmetric limit. Then the Higgs mechanism contribution ($\sim \delta_{GS} M_{\text{pl}}^2$) dominates over the Stückelberg mechanism contribution ($\sim \delta_{GS} M_{\text{pl}}^2$). This is not appropriate for our purpose to obtain global $U(1)_{PQ}$ symmetry as a remnant of $U(1)_A$.}, matter fields $\Phi_i$ do not develop vacuum expectation
values (VEVs) in the supersymmetric limit in order to make $D$-term vanish. When $\partial_\mu^2 t A K_0 \sim \mathcal{O}(1)$, the $U(1)_A$ gauge boson mass gains a mass $M_A \sim \delta_{\text{GS}} M_{\text{Pl}} \sim 10^{16}$ GeV by eating up an axion-like field $a_p$ in the GS modulus (Stückelberg mechanism), rather than a pseudoscalar in the matter (Higgs mechanism), i.e. $M_A^2 \sim M_{\text{GS}}^2 \gg M_{\text{matter}}^2 \sim |\Phi_i|^2$. After the massive vector field $\tilde{A}_\mu = (A_\mu, a_p)$ is integrated out, the low energy effective theory below $M_A$ involves a global PQ symmetry which can be identified as the global part of $U(1)_A$ without the transformation of $a_p$:

$$U(1)_{\text{PQ}} : \Phi_i \to e^{iq_i \beta} \Phi_i \quad (\beta = \text{constant}).$$

(10)

Because $U(1)_{\text{PQ}}$ differs from the global part of the genuine gauge symmetry $U(1)_A$ only by the absence of the non-linear transformation of $a_p$, any quantum gravity effect which breaks $U(1)_{\text{PQ}}$ explicitly is exponentially suppressed by $e^{-t_p}$, where $t_p$ is the volume modulus of the $p$-cycle which is dual to the zero mode $a_p$. The model then has a sensible limit that the PQ breaking quantum gravity effects are negligible enough for $U(1)_{\text{PQ}}$ to solve the strong CP problem, although it requires an understanding of the dynamics to stabilize the volume modulus $t_p$ at a sufficiently large value [19].

Let us discuss the decoupling of the massive gauge boson in more detail. Since we expect $M_A \gg m_{3/2}$, the massive vector multiplet $V_A$ is integrated out in an almost supersymmetric way. Then, $V_A$ is fixed by the superfield equation of motion

$$\frac{\partial K}{\partial V_A} \simeq 0,$$

(11)
in the supersymmetric limit. The scalar component of Eq. (11) provides a stabilization of $t_A$. On the other hand, if SUSY is mainly broken by the modulus $T_b$ such that $\partial_\mu^2 K |F_{T_b}|^2 \simeq 3|m_{3/2}|^2$, the D-component of Eq. (11) provides the $U(1)_A$ D-term VEV

$$g_A^2 D_A \simeq \frac{2}{\delta_{\text{GS}}} \frac{\partial_{t_A} \partial_{K_0}^2 K_0}{\partial_{t_b}^2 K_0} |F_{T_b}|^2 \sim \left( \frac{\partial_{t_A} \partial_{K_0}^2 K_0}{\partial_{t_b}^2 K_0 \partial_{t_b} \partial_{K_0}^2 K_0} \right) |m_{3/2}|^2 \simeq \epsilon_1 \delta_{\text{GS}} |m_{3/2}|^2,$$

(12)

where $\epsilon_1$ parametrizes the sequestering between a SUSY breaking sector and an $U(1)_A$ sector, which means that $\epsilon_1 = 0$ in the fully sequestered case. The same result is obtained by imposing the $U(1)_A$ invariance condition $\eta^I \partial_I (V_F + V_D) = 0$ to vacuum values [28].

Suppose the sequestering parameter $\epsilon_1$ is of order of $1/8\pi^2 \sim \delta_{\text{GS}}$, which is the case when $\epsilon_1$ represents a mixing between $t_b$ and $t_A$ through loop correction to $t_A$ in the Kähler potential, as observed in APPENDIX A. Then we have $D_A \sim m_{3/2}^2$ and it constitutes soft scalar masses as $m_i^2 = -q_i D_A$. 

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B. Soft terms in the PQ and visible sector

As we have seen, at energy scale below \( M_A \), we have the PQ symmetry as a remnant of \( U(1)_A \). In general, PQ-charged matters and the SM gauge fields are described by Kähler potential, superpotential, and gauge kinetic functions given by

\[
K = K_0(t_b, t_A, t_k) + Z_i(t_i, t_A, t_k)\Phi_i^* e^{2\eta V_A} \Phi_i,
\]

\[
W = W_0(T_b, T_A, T_k) + \frac{1}{3!} \lambda_{ijk}(T_b, T_A, T_k)\Phi_i \Phi_j \Phi_k + \frac{1}{n!} \kappa_{ij_1j_2\ldots j_n}(T_b, T_A, T_k)\Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n},
\]

(13)

\[
f_a = \gamma_a(T_b, T_k) + k_a T_A,
\]

where subscript \( a \) of \( f_a \) runs over the SM gauge group components, \( SU(3)_c, SU(2)_L \) and \( U(1)_Y \). Using basic supergravity (SUGRA) relations

\[
m_{3/2} = e^{K/2}W, \quad F^I = -e^{K/2}K^{IJ}(D_J W)^*, \quad D_I W = W_I + K_I W,
\]

\[
V_F = K_{IJ} F^I F^J - 3e^K |W|^2, \quad V_D = \frac{g_A^2}{2} D_A^2,
\]

we deduce soft terms

\[
\mathcal{L}_{soft} = -\frac{1}{2} M_a \lambda_a \lambda_a - \frac{1}{2} m_t^2 |\phi_t|^2 - \frac{1}{3!} A_{ijk} \hat{\lambda}_{ijk} \hat{\phi}_i \hat{\phi}_j \hat{\phi}_k - \frac{1}{n!} A_{\kappa ij_1j_2\ldots j_n} \hat{\kappa}_{ij_1j_2\ldots j_n} \hat{\phi}_{i_1} \hat{\phi}_{i_2} \cdots \hat{\phi}_{i_n},
\]

(15)

given by

\[
A_{ijk} = -F^I \partial_I \ln \left( \frac{\lambda_{ijk}}{e^{-K_0 Z_i Z_j Z_k}} \right) + \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) \frac{F^C}{C},
\]

\[
A_{\kappa ij_1j_2\ldots j_n} = (n - 3) \frac{F^C}{C} - F^I \partial_I \ln \left( \frac{\kappa_{ij_1j_2\ldots j_n}}{e^{-K_0 Z_i Z_j Z_k Z_3 Z_4 \cdots Z_n}} \right),
\]

\[
m_t^2 = \frac{2}{3} V_F - F^I F^J \partial_I \partial_J \ln (e^{-K_0/3 Z_i}) - (q_i + \eta' \partial_I \ln Z_i) g_A^2 D_A - \frac{1}{4} \partial_i \ln \mu \frac{F^C}{C},
\]

\[
M_a = \frac{1}{2} g_a^2 \frac{1}{8 \pi^2} \sum \text{Tr} \left( T_{a}^2 (\phi_i) \right) F^I \partial_I \ln (e^{-K_0/3 Z_i}) - \frac{b_a}{16 \pi^2} \frac{F^C}{C},
\]

where \( \hat{\phi}_i \) denotes canonically normalized scalar component of the chiral multiplet \( \Phi_i \) and \( \hat{\lambda}, \hat{\kappa} \) are Yukawa couplings in this basis:

\[
\hat{\lambda}_{ijk} = \frac{\lambda_{ijk}}{\sqrt{e^{-K_0 Z_i Z_j Z_k}}}, \quad \hat{\kappa}_{ij_1j_2\ldots j_n} = \frac{\kappa_{ij_1j_2\ldots j_n}}{\sqrt{e^{-K_0 Z_i Z_j Z_k Z_3 Z_4 \cdots Z_n}}}.
\]

(17)

Therefore, so far as gauge mediation is not concerned, we have three origins of soft terms:

- **Moduli mediation (gravity mediation)**
When SUSY is mainly broken by the modulus $T_b$ satisfying $\partial^2_{T_b} |F^{T_b}|^2 \simeq 3|m_{3/2}|^2$, gravity mediation takes a form of moduli mediation [29]. Its effects on soft masses are parametrized by how much the SUSY breaking sector is sequestered from the visible sector:

$$\epsilon_2 m_{3/2} \equiv F^{T_b} \partial_{T_b} \ln(e^{-K/3} Z_i) \quad \text{and} \quad F^I \partial_I f_a. \quad (18)$$

We assume that these two are in the same order, $F^I \partial_I f_a \sim \epsilon_2 m_{3/2}$.

- **Anomaly mediation**

Anomaly mediation [30] is parametrized by the conformal compensator $C$, whose SUSY breaking effect is given by

$$\frac{F^C}{C} = \frac{1}{3} K^F + e^{K/2} W^*. \quad (19)$$

where we have taken the Einstein frame gauge $C = e^{K/6}$.

- **D-term mediation**

Soft scalar masses get contribution from $D$-term mediation [20, 28, 31–38], $-q_i D_A \sim 8\pi^2 \epsilon_1 m_{3/2}^2$. In the specific case of $\epsilon_1 \sim 1/8\pi^2$, we have $D_A \sim m_{3/2}^2$.

In general, the moduli mediation (or gravity mediation) can cause the SUSY flavor problem without some non-trivial assumptions. Thus we will consider the situation that the moduli mediation is somewhat suppressed by some amount of sequestering $\epsilon_2$. If the SUSY breaking modulus $T_b$ contacts with the PQ and visible sector through loop correction in the Kähler potential, it is plausible to take $\epsilon_2 \sim 1/8\pi^2$, as estimated in APPENDIX A. In this case, soft scalar masses are dominated by $D$-term mediation of order of $m_{3/2}$, whereas

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2 In fact, when $f_a = k_a T_A$ we need to consider $F^A$, which is estimated to be $F^A \simeq -e^{K/2} K^{T_A T_b} F_{T_b} \sim e^{K/2} (\partial_{T_A} \partial_{T_b} K_0 / \partial^2_{T_A} K_0) F^{T_b}$. Hence, $F^I \partial_I f_a$ is relevant to $\epsilon_1$, rather than $\epsilon_2$. However, since we will be focusing on the specific choice $\epsilon_1 \sim \epsilon_2 \sim 1/8\pi^2$ in the following discussion, our assumption here is acceptable.
$A$-terms and gaugino masses mainly come from moduli mediation:

$$A_{ijk} \sim F^{\bar{t}_b} \partial_{\bar{t}_b} \ln(e^{-K_0 Z_i Z_j Z_k}) \sim \left(\frac{\partial_{\bar{t}_b} \ln(e^{-K_0/3 Z_i})}{\sqrt{\partial_{\bar{t}_b}^2 K_0}}\right)m_{3/2} \equiv \epsilon_2 m_{3/2},$$

$$A_k^{i_1 i_2 \cdots i_n} \sim F^{\bar{t}_b} \partial_{\bar{t}_b} \ln(e^{-nK_0/3 Z_1 Z_{i_2} \cdots Z_{i_n}}) \sim \epsilon_2 m_{3/2} \quad (n \neq 3),$$

$$\frac{M_a}{g_a^2} \sim \frac{1}{2} F^{\bar{t}_b} \partial_{\bar{t}_b} f_a - \frac{1}{8\pi^2} \sum_i \text{Tr}(T_a^2(\phi_i)) F^{\bar{t}_b} \partial_{\bar{t}_b} \ln(e^{-K_0/3 Z_i}) \sim \frac{1}{8\pi^2} m_{3/2},$$

as well as anomaly mediation.

Concerning the anomaly mediation effects, first consider the case of $F^C/C \sim m_{3/2}$. In this case, gaugino masses, coming from moduli and anomaly mediation, are of order of $(1/8\pi^2)m_{3/2}$. They are one-loop suppressed compared to $\sqrt{D_A}$ with $\epsilon_1 \sim 1/8\pi^2$, the main contribution to soft scalar masses. For gaugino masses of order of TeV, we have the spectrum for split SUSY [39] with soft scalar masses of order of 100 TeV [34]. However, since we are interested in singlet-extended SUSY with a percent level fine-tuning, we look for the situation in which both soft scalar masses and gaugino masses are in the same order, around TeV scale. This is achieved in two ways: one is to take $m_{3/2} \sim 100$ TeV and two sequestering parameters satisfying $\epsilon_2^2 \sim 8\pi^2\epsilon_1 \sim (1/8\pi^2)^2$. Another is to keep $\epsilon_1 \sim \epsilon_2 \sim 1/8\pi^2$ and introduce gauge mediation [40, 41] to give gaugino masses of order of $m_{3/2} \sim \sqrt{D_A} \sim$ TeV [32]. The first way requires peculiar three loop order mixing $\epsilon_1 \sim (1/8\pi^2)^3$ between the SUSY breaking sector and $U(1)_A$ sector, so we will not pursue this possibility. On the other hand, as will be discussed in Sec. 11C with our specific parameter choice $\epsilon_1 \sim \epsilon_2 \sim 1/8\pi^2$, we can realize the latter case by introducing a PQ-charged messenger, with the help of a one-loop suppressed $A$-term $A_k^{i_1 \cdots i_n} \sim \epsilon_2 m_{3/2}$ with $n \neq 3$. In order to obtain such a one-loop suppressed $A$-term, we need to make anomaly mediation negligibly small, because anomaly mediation gives $A$-term with $n \neq 3$ of order of $F^C/C$. If $F^C/C$ is of order of $m_{3/2}$, $A$-term is of order of $m_{3/2}$ as well. Therefore, we need a model in which the SUSY breaking modulus $T_b$ has a no-scale structure [42], $K \simeq -3 \ln t_b$ and superpotential is independent of $T_b$ at the leading order, to give $F^C/C \simeq (1/3)\partial_{\bar{t}_b} K F^{T_b} + m_{3/2} \simeq 0$. For example, in the large volume scenario [43], where the large volume modulus $T_b$ of Calabi-Yau (CY) 3-fold breaks SUSY mainly with a no-scale structure, $F^C/C$ is negligibly small. Moreover, the coupling between $T_b$ and $T_A$ through the loop correction in the form of $(1/t_b^m)(t_A - \alpha_A \ln t_b)^2$ in the Kähler potential, where $m$ is some integer, gives $\epsilon_1 \sim \epsilon_2 \sim 1/8\pi^2$. Detailed calculation can be found in Refs. [44, 45], and briefly described in APPENDIX A. In summary, in the parameter
space we are interested in, $F^C/C$ is negligible and $\epsilon_1 \sim \epsilon_2 \sim 1/8\pi^2$, we have soft terms given by

$$m_i^2 \sim -q_i(8\pi^2\epsilon_1)D_A \sim m_{3/2}^2, \quad A \sim \epsilon_2m_{3/2} \sim \frac{1}{8\pi^2}m_{3/2}, \quad M_a \sim m_{3/2}, \quad (21)$$

where gaugino masses are dominantly given by the gauge mediation to be discussed in the following subsection, and the detailed parametric dependence of gaugino masses on $m_{3/2}$ and $\epsilon_{1,2}$ will be given there.

C. Soft-term-induced spontaneous Peccei-Quinn symmetry breaking and Gauge mediation

Since we are interested in the intermediate PQ breaking scale $v_{PQ}$ obtained through an interplay between $m_{3/2}$ and some cutoff scale $M_*$ (generically either the Planck scale or the GUT scale), we consider a model similar to that discussed in Ref. [3]. In our setup, as a soft scalar mass squared $m_i^2$ from a $D$-term mediation is proportional to an PQ charge $q_i$, we can make some of scalar fields tachyonic by assigning a positive charge. In this regard, our setup provides a natural situation for PQ symmetry breaking through the scenario in Ref. [3].

To begin with, let us consider a non-renormalizable superpotential for PQ charged chiral multiplets $X$ and $Y$,

$$W_{PQ} = yX^{n+2}Y/M_*^n. \quad (22)$$

For this, we assign PQ charges to satisfy $(n + 2)q_X + q_Y = 0$. Together with soft terms, a potential for scalars $X$ and $Y$ is given by

$$V_{PQ}(X, Y) = \left|\frac{y}{M_*^{2n}}\right|^2|X|^{2(n+2)} + \frac{|y|^2}{M_*^{2n}}(n + 2)^2|X|^{2(n+1)}|Y|^2$$

$$+ m_X^2|X|^2 + m_Y^2|Y|^2 + \left(\frac{yA X^{n+2}Y}{M_*^n} + \text{h.c.}\right). \quad (23)$$

Since $X$ and $Y$ have PQ charges in opposite sign, one can assign $q_X > 0$ and $q_Y < 0$ so that $X$ is tachyonic whereas $Y$ is not, at the origin of field space. Then the PQ symmetry is broken as $X$ takes the VEV and it induces non-zero $Y$ VEV:

$$\langle |X| \rangle \simeq \frac{1}{(n + 2)^{1/2(n+1)} |y|^{1/(n+1)}} \sqrt{|m_X|/M_*^n},$$

$$\langle Y \rangle \simeq -\frac{1}{(n + 2)^{1/2(n+1)} |y|^{1/(n+1)}} \frac{A^*|m_X|}{(n + 2)|m_X|^2 + |m_Y|^2} \sqrt{|m_X|/M_*^n}. \quad (24)$$
Note that the linear dependence of superpotential on $Y$ results in the $Y$ VEV proportional to the $A$-term which is suppressed by $\epsilon_2$, so there appears a hierarchy between $X$ and $Y$ VEVs.

$$\frac{|Y|}{|X|} = \frac{|A|m_X}{\sqrt{n + 2}|(n + 2)|m_X|^2 + m_Y^2|} \sim \frac{|A|}{\sqrt{D_A}} \sim \epsilon_2.$$  \tag{25}

We identify the highest scale $X$ VEV as the PQ scale $v_{PQ}$. As $|m_{X,Y}| \sim m_X^3/2 \sim \sqrt{D_A}$, for $M_* = M_{Pl}$, we have

$$v_{PQ} \equiv \langle X \rangle \sim |y|^{-1/(n+1)+1} \sqrt{m_{3/2} M_n} = \begin{cases} |y|^{-1/2}10^{10} \text{ GeV} & n = 1 \\ |y|^{-1/3}10^{12-13} \text{ GeV} & n = 2 \\ |y|^{-1/4}10^{14} \text{ GeV} & n = 3 \end{cases}.$$  \tag{26}

On the other hand, for $M_* = M_{\text{GUT}}$,

$$v_{PQ} \sim \begin{cases} |y|^{-1/2}10^{9} \text{ GeV} & n = 1 \\ |y|^{-1/3}10^{11-12} \text{ GeV} & n = 2 \\ |y|^{-1/4}10^{13} \text{ GeV} & n = 3 \end{cases}.$$  \tag{27}

Therefore, regarding a bound $10^9 \text{ GeV} < v_{PQ} < 10^{12} \text{ GeV}$, we favor $n = 1$ for $M_* = M_{Pl}$ and $n = 1, 2$ for $M_* = M_{\text{GUT}}$.

The soft SUSY breaking terms also induce spontaneous SUSY breaking for the $X,Y$ sector with non-zero VEVs of $F_{X,Y}$:

$$\frac{F_{X}}{|X|} = \frac{|A|m_X|^2}{(n + 2)|m_X|^2 + m_Y^2} \sim A \sim \epsilon_2 m_{3/2},$$

$$\frac{F_{Y}}{|Y|} = \frac{(n + 2)|m_X|^2 + m_Y^2}{|A|} \sim \frac{D_A}{A} \sim \frac{1}{\epsilon_2} m_{3/2}.$$  \tag{28}

In short, we have $X \sim v_{PQ}(1 + \epsilon_2 m_{3/2} \theta^2)$ while $Y \sim v_{PQ}(\epsilon_2 + m_{3/2} \theta^2)$. Here we emphasize that $F_Y/Y$ is enhanced by one loop factor compared to $m_{3/2}$ due to the small $Y$ VEV.

Now we can use non-zero $F$-terms of $X$ and $Y$ to generate the gauge mediation in the KSVZ axion model, in which the PQ breaking field couples to a vector-like quark pair. Since $F_Y/Y$ is enhanced compared to $F_X/X$, let us consider the case where $Y$ couples to a pair of fields which is vector-like under the SM gauge group as

$$W = Y \overline{Q_1} Q_1$$  \tag{29}
as in the KSVZ axion model. The $Q_1Q_1$ pair plays the role of messenger of the gauge mediation whose size is given by

$$m_{GM} \equiv \frac{1}{8\pi^2} \left| \frac{F_Y}{Y} \right| \sim \frac{1}{8\pi^2} \frac{D_A}{A} \sim \frac{1}{8\pi^2} \epsilon_2 \sqrt{DA}. \quad (30)$$

The gauge mediation gives gaugino masses of comparable order with soft scalar masses $\sim \sqrt{D_A}$ for $\epsilon_2 \sim 1/8\pi^2$. We emphasize that a hierarchy between two SUSY breaking scales $A$ and $\sqrt{D_A}$ and the accordingly induced $X$ and $Y$ VEV hierarchy are important aspects for our scheme to realize a low fine-tuned SUSY scenario. Especially the $X$ and $Y$ VEV hierarchy is obtained by the linear dependence of the superpotential on $Y$. In this regard, our choice of the superpotential [22] is quite generic.

III. VIABLE LOW ENERGY MODELS

In the previous section, we have specified the UV origin of a global $U(1)_{PQ}$ symmetry and corresponding SUSY breaking mediation scheme which realizes all superparticle masses around TeV scale for the purpose of minimizing the fine-tuning for EWSB without the SUSY flavor problem. Now we will apply this scheme to singlet extended SUSY models at TeV scale in order to complete a low fine-tuned SUSY scenario. Interestingly, it will turn out that the forms of low energy effective lagrangian are severely constrained within our scheme so that minimally viable low energy models are to be specifically determined.

We will consider singlet extended Higgs sector like the NMSSM models [10] with the $\Delta W = \lambda S H_u H_d$ coupling to obtain the Higgs mass of 125 GeV with TeV scale SUSY, however, with possibly more than one singlet superfields in general. Therefore, the general effective superpotential of the Higgs sector at TeV scale is given by

$$W_{\text{eff}} = \sum_i \lambda_i (1 + \theta^2 A_i) S_i H_u H_d + f(S_i). \quad (31)$$

For convenience, we define a singlet field $S_H$ by $\lambda S_H \equiv \sum_i \lambda_i S_i$. Then, in the field basis that the singlet fields are given by $S_H$ and its orthogonal fields, the general superpotential becomes

$$W_{\text{eff}} = \lambda (1 + \theta^2 A_i) S_H H_u H_d + \theta^2 \sum_j \lambda_j' A_j' S_j H_u H_d + f(S_H, S_j), \quad (32)$$

The bare $\mu$ and $B\mu$ parameters can be always rotated away by choosing an appropriate field basis for $S_i$ at some scale, although there can be non-zero RG induced contribution to $B\mu$ below the chosen scale.
where $S_j$ denotes the singlet fields orthogonal to $S_H$, which do not have a supersymmetric coupling to the doublet Higgs fields $H_u H_d$. In this generalized Higgs sector, the effective $\mu$ and $B\mu$ parameters are given by

$$\mu_{\text{eff}} = \lambda \langle S_H \rangle, \quad (B\mu)_{\text{eff}} = \lambda \langle \partial S_H f^* \rangle + \lambda A_\lambda \langle S_H \rangle + \sum_j \lambda_j A'_j \langle S_j \rangle. \quad (33)$$

The EWSB conditions in terms of these parameters are expressed as

$$\frac{1}{2} m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu_{\text{eff}}^2,$$

$$\sin 2\beta = \frac{2(B\mu)_{\text{eff}}}{2\mu_{\text{eff}}^2 + m_{H_u}^2 + m_{H_d}^2 + \lambda^2 v^2}. \quad (34)$$

We deduce necessary conditions for the low fine-tuned EWSB from (34). First of all, since the Higgs quartic potential $\lambda^2 |H_u H_d|^2$ from $\Delta W = \lambda S_H H_u H_d$ is proportional to $\sin^2 2\beta$, we require low $\tan \beta$ ($\sin 2\beta \sim 1$) to enhance the tree level Higgs mass. Also we have a lower bound for $\mu_{\text{eff}} \gtrsim 100$ GeV by the LEP exclusion on chargino masses [48]. Therefore, in terms of singlet parameters, we find that the required conditions are

$$100 \text{ GeV} \lesssim \mu_{\text{eff}} \lesssim m_{H_{u,d}} \lesssim 1 \text{ TeV}, \quad (B\mu)_{\text{eff}} \sim m_{H_{u,d}}^2, \quad (35)$$

where 1 TeV bound is set for a percent level fine-tuning [7]. The condition $m_{H_{u,d}} \lesssim 1 \text{ TeV}$ is satisfied if we set $\sqrt{D_A} \sim m_{GM} \sim m_{3/2}$ around 1 TeV, because $m_{H_{u,d}}$ are controlled by these quantities according to the previous section. We will denote this scale by $m_{\text{SUSY}}$ hereafter. Then the remaining part of the conditions (35) is that $\mu_{\text{eff}}$ and $(B\mu)_{\text{eff}}$ should be around $m_{\text{SUSY}}$. This will be satisfied if dimensionful parameters in the generalized Higgs sector (32) are around $m_{\text{SUSY}}$. To obtain relevant parameters of order $m_{\text{SUSY}}$, we consider PQ invariant higher dimensional operators of the singlet fields $(S_H, S_j)$ interacting with the PQ breaking fields $(X, Y)$ of the previous section. Below the PQ scale $v_{\text{PQ}}$, $(X, Y)$ fields can be treated as spurion fields of VEVs : $X \sim v_{\text{PQ}}(1 + \epsilon m_{3/2} \theta^2), Y \sim v_{\text{PQ}}(\epsilon + m_{3/2} \theta^2)$. Then using the relation $v_{\text{PQ}}^{n+1}/M_n^a \sim m_{\text{SUSY}}$, dimensionful singlet parameters around $m_{\text{SUSY}}$ will be obtained in the way similar to the Kim-Nilles mechanism [5].

In the following subsections, we are going to examine phenomenologically viable minimal models which fulfill the conditions (35) with the prescribed scheme.
A. One singlet field extension: unviable

The simplest possibility is to consider one singlet field extension by \( f(S_H, S_j) = f(S_H) \) without any other singlet field \( S_j \) interacting with \( S_H \). In this case, the general form of \( f(S_H) \) is given by

\[
f(S_H) = \xi (1 + \theta^2 C) S_H + \frac{1}{2} \mu'(1 + \theta^2 B') S_H^2.
\]

Notice that \( S_H^3 \) is suppressed by a small coupling less than \( (v_{PQ}/M_*)^p \) as \( S_H \) is charged under the PQ symmetry. Solving equations of motion, \( \mu_{\text{eff}} \) and \( (B\mu)_{\text{eff}} \) are found to be

\[
\mu_{\text{eff}} = \frac{\lambda}{2} \frac{-2\xi \mu' - 2C\xi + \lambda(A_\lambda + \mu')v^2 \sin 2\beta}{m_S^2 + \lambda^2 v^2 + \mu'^2 + B'\mu'},
\]
\[
(B\mu)_{\text{eff}} = \lambda \xi + \mu' \mu_{\text{eff}} + A_\lambda \mu_{\text{eff}}.
\]

From these equations, we find that at least two of \( (\xi, C\xi, \mu') \) should be around \( m_{\text{SUSY}} \) to make \( \mu_{\text{eff}} \) and \( (B\mu)_{\text{eff}} \) around \( m_{\text{SUSY}} \), because \( A_\lambda \) is small.\(^4\) However, the sequestering factor \( \epsilon \sim 1/8\pi^2 \) makes it non-trivial. For example, one can obtain \( \xi \sim m_{\text{SUSY}}^2 \) from the following operator,

\[
\Delta W = \frac{X^{2n+2}}{M_*^{2n}} S_H \sim m_{\text{SUSY}}^2 (1 + \theta^2 \epsilon m_{\text{SUSY}}) S_H.
\]

On the other hand, \( C\xi \) is given around \( \epsilon m_{\text{SUSY}}^3 \sim m_{\text{SUSY}}^3 / 8\pi^2 \) from the same operator. Hence we cannot obtain \( \xi \) and \( C\xi \) around \( m_{\text{SUSY}} \) simultaneously from the single operator due to the \( \epsilon \) factor. This situation is actually generic for an arbitrary single operator because of the structure \( X \sim v_{PQ}(1 + \theta^2 \epsilon m_{\text{SUSY}}), \ Y \sim v_{PQ}(\epsilon + \theta^2 m_{\text{SUSY}}) \). Therefore, generally one cannot make two of \( (\xi, C\xi, \mu') \) be around \( m_{\text{SUSY}} \) from a single operator. Remember that the hierarchial factor \( \epsilon \sim 1/8\pi^2 \) makes it possible for the gauge mediation to be comparable to \( m_{\text{SUSY}} \) by \( F^Y / Y \sim m_{\text{SUSY}} / \epsilon \). In this sense, the situation is similar to the \( \mu/B\mu \) problem in the gauge mediation \([49]\).

We are thus led to have at least two higher dimensional operators for the desired EWSB to occur. However, the number of relevant degrees of freedom \( (X, Y, S_H) \) is not enough for the PQ symmetry to control forms of three operators, of which two for the generalized Higgs

\(^4\) Even though \( A \)-term is one loop suppressed compared to \( m_{\text{SUSY}} \) by small moduli mediated contribution, one may consider sizable \( A_\lambda \) by RG mixing effect, dominantly from gaugino masses. However, it turns out to be still not large enough to realize a consistent EWSB.
TABLE I: Possible forms of operators and PQ charge assignments to give each of \( \xi, C\xi, \mu' \) around \( m_{\text{SUSY}} \). For some cases, there appear unavoidable dangerous tadpoles.

| \( \Delta K \) | \( \Delta W \) | \( (q_X, q_Y) \) | Dangerous term |
|----------------|-----------------|-----------------|----------------|
| \( \frac{X^n Y^n S}{M^2} \) | \( \frac{X^{2n+2} S}{M^2} \) | \( \left( -\frac{1}{2(n+1)}, \frac{n+2}{2(n+1)} \right) qS \) | \( X^2 S \) |
| \( \frac{X^n Y^n S}{M^2} \) | \( \left( -\frac{1}{2}, \frac{n+2}{2} \right) qS \) | \( X^{n-1} S \) |
| \( \frac{X^{2n+1} Y S}{M^2} \) | \( \left( -\frac{1}{n-1}, \frac{n+2}{n-1} \right) qS \) |
| \( \frac{X^{n+1} S^2}{M^2} \) | \( \left( -\frac{2}{n+1}, \frac{2(n+2)}{n+1} \right) qS \) |

This fact is also explicitly checked by examining all possible higher dimensional operators to produce each of \( \xi, C\xi, \mu' \) around \( m_{\text{SUSY}} \) and their charge assignments. In Table I we summarize them, where PQ charge assignments are determined by each term and the PQ breaking superpotential \( W_{\text{PQ}} = X^{n+2}Y/M^n \) as described in Sec. II.

Although one singlet field extension turns out to be unviable with the specific scheme given in Sec. III still one may consider more involved spontaneous PQ breaking sectors with more fields than \( X, Y \). Then the increased number of field degrees of freedom allows to
control necessary terms. However, it requires a quite complicated PQ breaking sector as argued in APPENDIX B. Therefore, as the next simplest possibility, we will investigate two singlet fields extension of the Higgs sector in the following subsection.

B. Two singlet fields extension

The next minimal case will be two singlet fields extension with \( f(S_H, S_j) = f(S_H, S_1) \). A generic form of \( f(S_H, S_1) \) is

\[
\begin{align*}
    f(S_H, S_1) &= \frac{1}{2} \kappa (1 + \theta^2 A_\kappa) S_H^2 S_1 + \frac{1}{2} \kappa_1 (1 + \theta^2 A_{\kappa_1}) S_H S_1^2 + M_1 (1 + \theta^2 B_1) S_H S_1 \\
    &\quad + \xi (1 + \theta^2 C) S_H + \frac{1}{2} \mu' (1 + \theta^2 B') S_H^2 + \xi_1 (1 + \theta^2 C_1) S_1 + \frac{1}{2} \mu'_1 (1 + \theta^2 B'_1) S_1^2,
\end{align*}
\]

where \( A \)-terms are still suppressed by \( \epsilon \) compared to \( m_{\text{SUSY}} \), so we will neglect \( A_\kappa \) and \( A_{\kappa_1} \).

Now with one more singlet field, one can control the magnitude of two operators in \( f(S_H, S_1) \) in addition to the PQ breaking sector superpotential \( W_{\text{PQ}} \). Note that we must have at least one of the interaction terms between \( S_H \) and \( S_1 \), like \( S_H^2 S_1 \), \( S_H S_1^2 \), or \( S_H S_1 \), since otherwise the situation is not actually different from the one singlet field extension which turns out to be unviable. Therefore we should find the models in which \textit{two terms} in (39) including one of the interaction terms make \( \mu_{\text{eff}} \sim \langle S_H \rangle \sim O(m_{\text{SUSY}}) \) and \( (B\mu)_{\text{eff}} \sim \langle \partial_{S_H} f \rangle \sim O(m_{\text{SUSY}}^2) \).

There are two ways to stabilize \( S_H \sim \mu_{\text{eff}} \) around \( m_{\text{SUSY}} \). With \( \kappa S_H^2 S_1 \), we get a quartic scalar potential for \( S_H \) so that it can be stabilized by its tachyonic mass. In the absence of this term, one can see from the general form (39) that the scalar potential for \( S_H \) can be quadratic at most, so it needs a tadpole scalar potential of \( S_H \) with a non-negative mass term. Thus we will investigate viable models with/without the \( \kappa S_H^2 S_1 \) term in the following.

1. \textit{With the cubic interaction} \( S_H^2 S_1 \) in \( f(S_H, S_1) \)

The PQ symmetric cubic interaction term \( \kappa S_H^2 S_1/2 \) in \( f(S_H, S_1) \) gives a quartic scalar potential \( \kappa^2 |S_H|^4/4 \), so \( S_H \) can be stabilized by \( |S_H| \sim |m_{S_H}|/\kappa \) if it is tachyonic \( (m_{S_H}^2 < 0) \). Note that the singlet fields get their masses mainly from the \( D \)-term mediation, since the gauge mediation contribution for singlet fields only comes from RG mixing effect which is subdominant. Therefore, if \( S_H \) becomes tachyonic, then \( S_1 \) must be non-tachyonic because
the sign of their soft mass squared is determined by their PQ charges in the \( D \)-term mediation. In this case, the right size of \( (B\mu)_{\text{eff}} \sim \partial_{S_H}f \sim \kappa S_H S_1 + \cdots \) is to be obtained either by stabilizing \( S_1 \) or by another term in \( f(S_H, S_1) \) when \( \langle S_1 \rangle \) vanishes.

First, let us consider the way to obtain \( (B\mu)_{\text{eff}} \) by non-vanishing \( \langle S_1 \rangle \). Since \( S_1 \) is not tachyonic, it needs a tadpole or cubic scalar potential for its stabilization. A cubic scalar potential for \( S_1 \) can be obtained from another superpotential term \( \kappa S_H S_1^2 \), but this term is not allowed by the PQ symmetry unless \( S_H \) and \( S_1 \) are uncharged, because of \( \kappa S_H^2 S_1 \) term. On the other hand, in order to obtain a tadpole scalar potential for \( S_1 \), one can show that there are five possible forms of \( f(S_H, S_1) \) :

\[
f(S_H, S_1) = \frac{1}{2} \kappa S_H^2 S_1 + \left( \theta^2 C_1 \xi_1 S_1, \theta^2 B_1 M_1 S_H S_1, \xi S_H, \frac{1}{2} \mu' S_H^2, \text{ or } \frac{1}{2} \mu'_1 S_1^2 \right).
\] (40)

The first term \( \theta^2 C_1 \xi_1 S_1 \) in the parenthesis is not viable, because it allows unavoidable large tadpole superpotential \( X^{n-1} S_1 \) as shown in Table I so that it induces \( (B\mu)_{\text{eff}} \gg \mathcal{O}(m_{\text{SUSY}}^2) \). The second choice \( \theta^2 B_1 M_1 S_H S_1 \) can be shown to be only realized by the superpotential term \( X^n Y S_1 S_H / M_* \), but this case also suffers from a large tadpole \( X^2 S_H \) which results in \( \mu_{\text{eff}} \gg \mathcal{O}(m_{\text{SUSY}}) \). The third and fourth cases \( \xi S_H, \mu' S_H^2/2 \) require such a PQ charge of \( S_H \) that \( S_H \) is non-tachyonic in order to render \( X \) tachyonic for a spontaneous PQ symmetry breaking, as seen in Table I. Hence it will make \( S_1 \) tachyonic, and then it can be shown that \( S_1 \) is destabilized and \( \langle S_H \rangle \) vanishes to make \( \mu_{\text{eff}} \sim 0 \), so they are excluded. Finally, the last term turns out to give a consistent scenario with \( \mu'_1 \sim X^{n+1}/M_*^n \) which allows non-tachyonic \( S_1 \) and tachyonic \( S_H \). Therefore we have found a working model :

\[
f(S_H, S_1) = \frac{1}{2} \kappa S_H^2 S_1 + \frac{1}{2} x \frac{X^{n+1}}{M_*^n} S_1^2,
\] (41)

where \( x \) is a dimensionless coefficient. We will discuss some phenomenological properties of this model in the next section.

The other way to obtain \( (B\mu)_{\text{eff}} \) is through another term beside \( \kappa S_H^2 S_1 \) in \( f(S_H, S_1) \) assuming that \( \langle S_1 \rangle \) vanishes. This could be achieved by either \( \xi S_H \) or \( \mu' S_H^2/2 \) giving \( \partial_{S_H}f \sim (\xi, \mu' S_H) \sim \mathcal{O}(m_{\text{SUSY}}^2) \) if \( S_H \) is consistently stabilized through its quartic scalar potential. However, these cases are already included in (40), which turned out to be not viable.

Finally, if \( S_H \) is not tachyonic, there should be a tadpole or cubic scalar potential for \( S_H \) to be stabilized even with its quartic scalar potential. Barring the cases already included in
we find that the following superpotentials can give such a scalar potential for $S_H$,

$$f(S_H, S_1) = \frac{1}{2} \kappa S_H^2 S_1 + \left( \theta^2 C \xi S_H, \text{ or } M_1 S_H S_1 \right).$$  \hfill (42)

The first case $\theta^2 C \xi S_H$ suffers from the large tadpole superpotential $X^{n-1} S_H$ as discussed before, and then $(B\mu)_{\text{eff}} \sim \partial_{S_H} f \sim M_s^{3\cdot-n} X^{n-1}$ is much larger than $\mathcal{O}(m_{\text{SUSY}}^2)$. The second case $M_1 S_H S_1$ can be only realized by $M_1 \sim X^{n+1}/M_s^n$. However, this also allows an unavoidable tadpole superpotential $X Y S_H$ similarly. Therefore, we conclude that there is only one working model with the cubic interaction $S_H^2 S_1$ in $f(S_H, S_1)$.

2. Without the cubic interaction $S_H^2 S_1$ in $f(S_H, S_1)$

In the absence of the cubic interaction $S_H^2 S_1$ in $f(S_H, S_1)$, the scalar potential for $S_H$ can be quadratic at most. Thus $S_H$ needs a tadpole scalar potential to be stabilized while being non-tachyonic. The first obvious choice is through soft terms which are linear in $S_H$,

$$f(S_H, S_1) = \left( \theta^2 C \xi S_H, \text{ or } \theta^2 B_1 M_1 S_1 S_H \right) + \text{(something)}. \hfill (43)$$

The $\theta^2 C \xi S_H$ term is not available because of the large tadpole problem as in the previous cases. The second case $\theta^2 B_1 M_1 S_1 S_H$ requires that $S_1$ is stabilized at nonzero value in order to give a tadpole for $S_H$. The stabilization of $S_1$ should be done by another term, and it must be a supersymmetric term to give non-negligible $(B\mu)_{\text{eff}} \sim \partial_{S_H} f$. Then one can find that there is only one possibility, which is through $\kappa_1 S_1^2 S_H/2$ with tachyonic $S_1$. However, this is similar to the second case of (40), so it suffers from a large tadpole $X^2 S_1$ which gives $(B\mu)_{\text{eff}} \gg \mathcal{O}(m_{\text{SUSY}}^2)$.

The second way to get a tadpole scalar potential for $S_H$ is through supersymmetric scalar potentials. Available forms are found to be

$$f(S_H, S_1) = M_1 S_1 S_H + \left( \xi_1 S_1, \text{ or } \frac{1}{2} \mu'_1 S_1^2 \right), \hfill (44)$$

$$f(S_H, S_1) = \frac{1}{2} \kappa_1 S_1^2 S_H + \left( \xi_1 S_1, \frac{1}{2} \mu'_1 S_1^2, \text{ or } \frac{1}{2} \mu'_1 S_1^2 \right). \hfill (45)$$

In the first case $\xi_1 S_1$ in (44), there is no scalar potential for $S_1$ other than its mass term so that $(B\mu)_{\text{eff}} \sim \partial_{S_H} f \sim M_1 S_1$ vanishes or is much larger than $\mathcal{O}(m_{\text{SUSY}}^2)$ depending on whether $S_1$ is non-tachyonic or tachyonic. The second case $\mu'_1 S_1^2/2$ in (44) has only bilinear
scalar potentials for $S_H$ and $S_1$, so they will either have vanishing VEV, or be destabilized, meaning both $\mu_{\text{eff}}$ and $(B\mu)_{\text{eff}}$ cannot have the right size.

The first and second cases of (45) are similar to the third and fourth cases of (40) by just interchanging $S_H \leftrightarrow S_1$, and so now $S_H$ is destabilized and $\langle S_1 \rangle$ vanishes. Thus they result in $\mu_{\text{eff}} \gg O(m_{\text{SUSY}})$ and $(B\mu)_{\text{eff}} \sim 0$. The last case of (45) is also similar to the working model of (41) by interchanging $S_H \leftrightarrow S_1$. The only difference is that now $S_1$ is stabilized by its quartic scalar potential with a tachyonic mass and $S_H$ through its tadpole scalar potential after $S_1$ gets its VEV. Therefore, we conclude that there is one more viable model without the cubic interaction $S_H^2 S_1$ in $f(S_H, S_1)$:

$$f(S_H, S_1) = \frac{1}{2}\kappa_1 S_1^2 S_H + \frac{1}{2}\mu_2' S_2^2,$$

(46)

IV. PHENOMENOLOGICAL IMPLICATIONS OF THE MINIMAL MODELS

In the previous section, we have found two viable models (41) and (46) in the simplest case. Here we discuss some phenomenological consequences of these models. Two models are actually similar to each other, so the extended Higgs sector around TeV scale can be also expressed in a single form:

$$W_{\text{eff}} = \lambda S_{1,2} H_u H_d + \frac{1}{2}\kappa_1 S_1^2 S_2 + \frac{1}{2}\mu_2' S_2^2,$$

(47)

where either $S_1$ or $S_2$ corresponds to $S_H$ in Eq. (52). The mass parameter $\mu_2'$ is given by $\mu_2' = x X^{n+1}/M^*_n$ with $n = 1, 2, \cdots$ depending on the PQ scale. Let us consider the low PQ scale case ($n = 1$) specifically. The higher PQ scale cases will not be essentially different in the following discussion. For $n = 1$, the PQ charge assignment for each field is given in Table III. Here we introduce an additional $Z_2$ symmetry at renormalizable level to forbid a large tadpole $XS_2$. Note that this tadpole is allowed by $U(1)_{\text{PQ}}$ since $\mu_2' S_2^2 = x (XS_2)^2/M^*_n$ is allowed. Thus the tadpole $XS_2$ can be forbidden by a discrete symmetry while allowing $\mu_2' S_2^2 \sim (XS_2)^2$ and is basically different from the previously encountered unavoidable large tadpoles. Also we emphasize that this discrete symmetry is explicitly broken at non-renormalizable level by terms like $(XS_2)^3/M^*_n$ so that cosmologically dangerous domain walls are not generated. For $n = 2$ case, even such a tadpole does not exist.
Now let us investigate the scalar potential for the singlet Higgs fields.

\[ V_{\text{eff}}(S_1, S_2) = m_{S_1}^2 |S_1|^2 + (m_{S_2}^2 + \mu_2^2)|S_2|^2 + \kappa_1^2 |S_1|^2 |S_2|^2 \]
\[ + \frac{1}{4} \kappa_1^2 |S_1|^4 + \frac{1}{2} \kappa_1 \mu_2' (S_1^2 S_2 + \text{h.c.}), \]  

(48)

where \( m_{S_1}^2 \) and \( m_{S_2}^2 \) are soft scalar mass squared of \( S_1 \) and \( S_2 \), respectively, mainly induced from the \( D \)-term mediation while RG mixing effect is subdominant. Note that \( m_{S_1}^2 < 0 \), \( m_{S_2}^2 > 0 \) from the PQ charges in Table II. Hence \( S_1 \) is stabilized from its quartic scalar potential, while \( S_2 \) from the cubic coupling after \( \langle S_1 \rangle \) becomes non-zero. Assuming \( \mu_2^2 (\sim x^2 D_A) \lesssim m_{S_1,2}^2 (\sim D_A) \) with \( x \lesssim \mathcal{O}(1) \), we find that \( S_1 \) and \( S_2 \) are stabilized at

\[ \langle |S_1| \rangle \simeq \sqrt{-2 m_{S_1}^2 \kappa_1^2} \sim \frac{|m_{S_1}|}{\kappa_1}, \]
\[ \langle |S_2| \rangle = \frac{\kappa_1 \mu_2' |S_1|^2}{2 m_{S_2}^2 + \mu_2^2 + \kappa_1^2 |S_1|^2} \sim \frac{\mu_2'}{\kappa_1}. \]  

(49)

Parametric relations between the doublet Higgs sector and singlet sector are quite different depending on which singlet field is \( S_H \). For the first model (model 1) \( S_H = S_1 \), \( \mu_{\text{eff}} = \lambda \langle S_H \rangle \) and \( (B\mu)_{\text{eff}} \simeq \lambda \langle \partial S_H f \rangle \) are given by

\[ \mu_{\text{eff}} = \lambda \langle S_1 \rangle \sim \frac{m_{S_1}}{\kappa_1} \sim \frac{\sqrt{D_A}}{\kappa_1}, \]  
\[ (B\mu)_{\text{eff}} \simeq \lambda \kappa_1 \langle S_1 S_2 \rangle \sim \frac{m_{S_1} \mu_2'}{\kappa_1} \sim \frac{x}{\kappa_1} D_A. \]  

(50)

On the other hand, for the second model (model 2) \( S_H = S_2 \), we have

\[ \mu_{\text{eff}} = \lambda \langle S_2 \rangle \sim \frac{\mu_2'}{\kappa_1} \sim \frac{x}{\kappa_1} \sqrt{D_A}, \]  
\[ (B\mu)_{\text{eff}} \simeq \lambda \left( \frac{1}{2} \kappa_1 S_1^2 + \mu_2' S_2 \right) \sim \frac{m_{S_2}^2}{\kappa_1} \sim \frac{D_A}{\kappa_1}. \]  

(51)

In the model 1, we deduce that the dimensionless coefficients \( \kappa_1, x \) should be \( \mathcal{O}(1) \) with \( \sqrt{D_A} \sim m_{\text{SUSY}} \sim 1 \text{ TeV} \) from the above relations in order to satisfy the conditions \( \mu_{\text{eff}} \lesssim \)}
\( O(m_{\text{SUSY}}) \) and \( (B\mu)_{\text{eff}} \sim O(m^2_{\text{SUSY}}) \). On the other hand, for the model 2, we observe that \( \sqrt{D_A} \) somewhat smaller than \( m_{\text{SUSY}} \sim 1 \text{ TeV} \) is allowed even satisfying \( \mu_{\text{eff}} \lesssim O(m_{\text{SUSY}}) \) and \( (B\mu)_{\text{eff}} \sim O(m^2_{\text{SUSY}}) \) if \( \kappa_1, x \) are smaller than the order unity.\(^7\) Since the singlet masses are governed by the scale of \( \sqrt{D_A} \), this means that the singlet sector of the model 1 must be around \( m_{\text{SUSY}} \sim 1 \text{ TeV} \) similarly with the other SUSY sectors, while the singlet sector of the model 2 could be parametrically lighter.

A caveat must be placed for the possibility of the parametrically lighter singlet sector of the model 2. A relatively small \( \sqrt{D_A} \) compared to \( m_{\text{GM}} \sim m_{\text{SUSY}} \) means that SUSY breaking is dominantly mediated by the gauge mediation. If we assume the minimal gauge mediation for the simplest case, it is known that \( \mu_{\text{eff}} \) cannot be smaller than the scale of \( m_{H_u}(m_{\tilde{t}}) \sim m_{\tilde{t}} \sim m_{\text{SUSY}} \) for the EWSB to occur (see Ref. [50], for instance). This is problematic since the mixing between the singlet scalar \( S_H \) and the SM-like Higgs boson \( h \) is determined by the following off-diagonal element of the mass matrix

\[
m^2_{hS_H} = \lambda v \left( 2\mu_{\text{eff}} - (A_\lambda + \partial^2_{S_H} f(S_1, S_2)) \sin 2\beta \right),
\]

where \( f(S_1, S_2) \) is the singlet sector superpotential. For either \( S_H = S_1 \) or \( S_2 \), we find that \( \partial^2_{S_H} f(S_1, S_2) \sim \mu' \) from [49]. Thus it will be around \( \lambda v \mu_{\text{eff}} \) unless there occurs some fine cancellation between \( \mu_{\text{eff}} \) and \( \mu' \). To ensure the stability of the electroweak vacuum, the diagonal elements of the mass matrix must satisfy \( m^2_{hh}m^2_{S_H S_H} > m^4_{hS_H} \) so that

\[
m^2_{S_H S_H} \sim D_A \gtrsim \mu_{\text{eff}}^2.
\]

Therefore, relatively small \( \sqrt{D_A} \) requires also small \( \mu_{\text{eff}} \), which is impossible in the minimal gauge mediation. It means that, in the simplest case, the singlet sector is expected to be as heavy as the other SUSY sectors around \( m_{\text{SUSY}} \sim 1 \text{ TeV} \) for both the first and second models. Still, there remains a room for lighter singlet sector if the gauge mediation is realized more generally as in Ref. [51], because \( \mu_{\text{eff}} \) can be small in such cases. However, such non-minimal gauge mediation scenarios require to introduce another SUSY breaking field beside the \( Y \) field in Sec. [1C], which can be done via another copy of the spontaneous

\(^7\) Smaller \( \sqrt{D_A} < m_{3/2} \sim m_{\text{SUSY}} \) can be achieved by assuming somewhat larger sequestering \( \epsilon_1 \sim g^2/8\pi^2 < 1/(8\pi^2) \) between the \( U(1)_A \) sector and SUSY breaking modulus in Eq. (12). In this case, if \( \epsilon_2 \) is also similar order with \( \epsilon_1 \), which is actually quite plausible as shown in APPENDIX A, \( m_{\text{GM}} \) in Eq. (30) is still around \( m_{3/2} \).
PQ breaking sector with \( X', Y' \), for example. Thus though it is still possible to realize lighter singlet sector in the model 2, it involves some complication of the model.

Let us more specifically describe the mass spectrum of the relatively light singlet sector of the model 2. From (51), we find that \( x \lesssim \kappa_1 \) to satisfy \( \mu_{\text{eff}} \lesssim \mathcal{O}(m_{\text{SUSY}}) \) and \( (B\mu)_{\text{eff}} \sim \mathcal{O}(m^2_{\text{SUSY}}) \) as well as the condition (53). This gives

\[
\sqrt{D_A} \sim \sqrt{\kappa_1 m_{\text{SUSY}}}, \quad \mu'_2 \sim x \sqrt{\kappa_1 m_{\text{SUSY}}} \lesssim \kappa_1^{3/2} m_{\text{SUSY}},
\]

Hence, for the coupling constants \( x \lesssim \kappa_1 < \mathcal{O}(1) \), there appear hierarchial mass scales \( \mu'_2 < \mu_{\text{eff}} \lesssim \sqrt{D_A} < m_{\text{SUSY}} \). For instance, \( \kappa_1 \sim 0.01 \) gives \( \sqrt{D_A} \sim 0.1 m_{\text{SUSY}} \sim \mathcal{O}(100) \) GeV and \( \mu'_2 \lesssim \mathcal{O}(1) \) GeV. Notice that the limit \( \mu'_2 \to 0 \) corresponds to the PQ symmetric limit in which one pseudoscalar becomes massless, and thus one can find that the pseudoscalar mass is given by \( \sim \mu'_2^2 \) dominantly from the last term in the scalar potential (48). Therefore, with the small coupling constants \( x \lesssim \kappa_1 \), we obtain a lighter singlet scalar sector with \( \sqrt{D_A} \sim \sqrt{\kappa_1 m_{\text{SUSY}}} \) with a lighter singlet pseudoscalar of mass \( \mu'_2 \lesssim \kappa_1^{3/2} m_{\text{SUSY}} \).

The singlet scalars have mixing with the doublet Higgs bosons. Assuming no cancellation between \( \mu_{\text{eff}} \) and \( \partial f(S_1, S_2) \sim \mu'_2 \) in (52), the mixing angle between the SM-like Higgs boson \( h \) and \( S_H \) is

\[
\theta_{hS_H} \simeq \frac{m^2_{hS_H}}{m^2_{S_H} s_H - m^2_{hh}} \sim \mathcal{O} \left( \frac{x}{\kappa_1^{3/2} m_{\text{SUSY}}} \right) \lesssim \mathcal{O} \left( \frac{0.1}{\sqrt{\kappa_1}} \right),
\]

where we use \( x \lesssim \kappa_1 \) for the model 2, while \( x, \kappa_1 \) should be \( \mathcal{O}(1) \) for the model 1. Similarly, the mixing angle between the SM-like Higgs boson \( h \) and another scalar \( S_j \) other than \( S_H \) is estimated to be

\[
\theta_{hS_j} \simeq \frac{m^2_{hS_j}}{m^2_{S_j} s_j - m^2_{hh}} = -\frac{\lambda v \partial f(S_1, S_2) \sin 2\beta}{m^2_{S_j} s_j - m^2_{hh}} \left( \frac{1}{\sqrt{\kappa_1 m_{\text{SUSY}}}} \right) \sim \mathcal{O} \left( \frac{0.1}{\sqrt{\kappa_1}} \right),
\]

where \( \partial f(S_1, S_2) = \partial S_1 \partial S_2 f(S_1, S_2) = \kappa_1 S_1 \sim m_{S_1} \), for either \( S_H = S_1 \) or \( S_2 \). Therefore, the mixing angles can be quite sizable for the model 2 with small \( \kappa_1 \) which results in departure from the SM Higgs boson properties with detectable signatures, while they are always as small as \( \mathcal{O}(0.1) \) in the case of the model 1.
Let us briefly comment about the neutralino sector. The singlino mass matrix in the basis of \((\tilde{S}_1, \tilde{S}_2)\) is given by

\[
M_{\text{singlino}} = \begin{pmatrix}
\kappa_1 \langle S_2 \rangle & \kappa_1 \langle S_1 \rangle \\
\kappa_1 \langle S_1 \rangle & \mu'_2
\end{pmatrix} \sim \begin{pmatrix}
\mu'_2 & m_{S_1} \\
m_{S_1} & \mu'_2
\end{pmatrix},
\]

and they mix with the doublet Higgsinos through the superpotential \(\lambda S_H H_u H_d\) with the off-diagonal elements of \(O(\lambda v)\). Thus the mixing angles between the doublet Higgsinos and singlinos are around \(O(\lambda v/\sqrt{\kappa_1 m_{\text{SUSY}}}) \lesssim O(0.1/\sqrt{\kappa_1})\). When we assume the minimal gauge mediation, we have argued that all mass parameters \(m_{S_1}, \mu'_2\) and \(\mu_{\text{eff}}\) must be around \(m_{\text{SUSY}} \sim 1\) TeV with \(\kappa_1\) and \(x\) of \(O(1)\). In this case, the Higgsinos and singlinos do not mix with each other so much, and they are all heavy around \(m_{\text{SUSY}}\) if there is no fine cancellation between \(\mu'_2\) and \(m_{S_1}\) in the singlino mass matrix, while the bino and winos are lighter than the Higgsinos and singlinos if the gluino mass is not far above the current lower bound at the LHC around 1.3 TeV \([52]\). However, for the model 2 of small \(\kappa_1\) with general gauge mediation scenarios, \(m_{S_1}, \mu'_2\) and \(\mu_{\text{eff}}\) can be much smaller than the typical SUSY scale \(m_{\text{SUSY}}\) with \(\mu'_2 < \mu_{\text{eff}} \lesssim \sqrt{D_A} < m_{\text{SUSY}}\) as discussed above. In this case, there can be large mixing between the Higgsinos and singlinos, and they can be lighter than the gauginos. Finally with small \(\kappa_1\), we point out that the singlinos are almost Dirac-like due to \(\mu'_2 < m_{S_1}\).

\section{Conclusion}

In this paper, we have studied singlet-extended SUSY models in the presence of a PQ symmetry, which originates from an anomalous \(U(1)_A\) gauge symmetry, to realize a TeV scale SUSY scenario with less than a percent level fine-tuning as allowed by the current experimental bounds. An anomalous \(U(1)_A\) symmetry broken by the Stückelberg mechanism provides not only a plausible origin of a PQ symmetry, but also soft scalar masses through the \(D\)-term mediation. Especially, we consider the specific case that the SUSY breaking modulus takes a no-scale form at leading order, and the \(U(1)_A\) and visible sectors are sequestered from the SUSY breaking modulus by one-loop order. As a result, the anomaly mediation is negligible, while the moduli mediation is one-loop suppressed compared to the \(D\)-term mediation. Moreover, a spontaneous PQ breaking at the intermediate scale, occurred by the \(D\)-term induced tachyonic soft scalar mass, also induces a spontaneous SUSY breaking with a hierarchical VEV structure by one-loop factor as in Eq. (28). This SUSY breaking
in the PQ sector is transmitted to the MSSM sector through the gauge mediation, whose effects are comparable to the $D$-term mediation effects. Consequently, superparticle masses around TeV scale without the SUSY flavor problem can be realized.

Having specified the SUSY breaking mediation scheme, we examine its implication to the general singlet-extended Higgs sector, which is considered to explain the observed Higgs mass with TeV scale superpartners. The dimensionful parameters in the generalized Higgs sector are to be obtained around the soft SUSY breaking scale $m_{\text{SUSY}} \sim 1$ TeV, through PQ invariant higher dimensional operators of the PQ breaking fields and Higgs fields, similarly to the Kim-Nilles mechanism for the $\mu$-term. Then the allowed forms of low energy effective operators at TeV scale of the singlet and doublet Higgs fields are quite much restricted by the hierarchical VEV structure of the PQ breaking fields. We show that one singlet field extension like the general NMSSM models cannot be realized within the prescribed scheme with the simplest form of the spontaneous PQ breaking sector, although the conclusion can be evaded if one considers a more complicated PQ breaking sector. As the next minimal possibility, two singlet fields extension is investigated, and we find that interestingly only two forms of low energy models at TeV scale, given in Eq. (47), are viable to consistently realize a low fine-tuned SUSY scenario:

$$W_{\text{eff}} = \lambda S_{1,2} H_u H_d + \frac{1}{2} \kappa_1 S_1^2 S_2 + \frac{1}{2} \mu'_2 S_2^2.$$  \hspace{1cm} (58)

We have investigated some of the phenomenological properties of these two models. For the model 1 (with $\lambda S_1 H_u H_d$ coupling), the singlet Higgs sector must be as heavy as other superparticles around $m_{\text{SUSY}} \sim 1$ TeV scale for a consistent EWSB. On the other hand, the singlet sector can be parametrically lighter than the other sector below TeV scale for the model 2 (with $\lambda S_2 H_u H_d$ coupling) with the small couplings $\kappa_1$ and $\mu'_2$, when the gauge mediation is realized non-minimally. This can lead to a significant departure from the SM Higgs boson properties by singlet mixings with testable signatures. More phenomenological implications of the models will be studied in future works.

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Appendix A: Estimation of the soft terms in the large volume scenario framework

In this appendix, we briefly estimate the soft terms in the large volume scenario (LVS) framework, which provides negligible anomaly mediation and \( \epsilon_1 \sim \epsilon_2 \sim 1/8\pi^2 \). Detailed calculation with an explicit example can be found in Refs. \[44, 45\].

In the LVS, a volume modulus \( T_b \) is stabilized such that \( t_b \equiv T_b + T_b^* \gg 1 \) in the string length unit \( M_{\text{string}} = 1 \) to give a large compactification volume. This can be achieved by introducing another Kähler modulus \( T_s \) determining a volume of a small cycle. When \( T_s \) admits the non-perturbative effect \( e^{-aT_s} \) in the superpotential, the \( \alpha' \)-correction of \( \mathcal{O}(1/t_b^{3/2}) \) in the Kähler potential competes with the effect so that \( T_b \) is stabilized at an exponentially large value \[43\]. For example, in type IIB theory, Calabi-Yau(CY) three-fold volume is given by \( V_{\text{CY}} \sim t_b^{3/2} \sim |e^{aT_s}| \) and this is just \( M_{\text{Pl}}^2/M_{\text{string}}^2 \).

On the other hand, the MSSM sector and an anomalous \( U(1)_A \) sector are supported by the visible sector 4-cycle whose volume is determined by a new modulus \( T_A \). A modulus \( T_A \) cannot be identified with \( T_b \) since it gives too small SM gauge couplings \( g_{\text{SM}}^2 \sim 1/T_b \).

Moreover, \( T_A \) cannot have a D3 instanton superpotential, the essential feature of \( T_s \) to stabilize \( T_b \) \[53\], therefore \( T_s \) cannot play a role of \( T_A \). Since an instanton superpotential \( e^{-aT_A} \) is absent, the modulus \( T_A \) should be stabilized in another way. We consider the case where \( T_A \) is stabilized through the \( D \)-term of the anomalous \( U(1)_A \) gauge multiplet, which results in the \( D \)-term mediation.

Now, consider the dynamics of \( U(1)_A \) and visible sectors. At leading order, \( T_b \) has a no-scale structure satisfying \( K = -3\ln t_b \) and \( \partial W / \partial T_b = 0 \). Subleading effects would appear as expansions in large volume \( 1/t_b \) and quantum correction \( \alpha_{s, A} \ln t_b \). Let \( \tilde{t}_s \equiv t_s - \alpha_s \ln t_b \) and \( \tilde{t}_A \equiv t_A - \alpha_A \ln t_b \) \[54\]. Then, the generic forms of Kähler potential, superpotential, and gauge kinetic functions for \( U(1)_A \) and MSSM sectors are given by

\[
K = K_0(t_b, t_s, t_A) + Z_i \Phi_i^* e^{2q_i V_A} \Phi_i \\
= -3 \ln t_b + \frac{1}{t_b} K_{0,1}(\tilde{t}_s) + \frac{1}{t_b^2} \left[ \Omega_0(\tilde{t}_s, \tilde{t}_A) + \frac{1}{t_b^2} \Omega_1(\tilde{t}_s, \tilde{t}_A) \right] \\
+ \frac{1}{t_b} \left[ \mathcal{Y}_{i,0}(\tilde{t}_s, \tilde{t}_A) + \frac{1}{t_b^2} \mathcal{Y}_{i,1}(\tilde{t}_s, \tilde{t}_A) \right] \Phi_i^* e^{2q_i V_A} \Phi_i, \tag{A1}
\]
\[ W = W_0(T_s, T_A) + \frac{1}{3!} \lambda_{ijk}(T_s, T_A) \Phi_i \Phi_j \Phi_k + \frac{1}{n!} \kappa_{i_1 i_2 \ldots i_n}(T_s, T_A) \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}, \quad (A2) \]

\[ f_A = \gamma_A(T_s) + k_A T_A, \quad f_a = \gamma_a(T_s) + k_a T_A \quad (A3) \]

where \( p, p', p'' \) and \( n \) are some positive integer, especially \( p = 3/2 \) in type IIB string theory.

The matter Kähler metric \( Z_i \) is in the form of \( Z_i = (1/t_b) Y_i \) such that they do not have power-law dependence on the CY3 volume.

More explicitly, we take a Kähler potential and superpotential

\[ K = -3 \ln t_b + \frac{2(t_s^{3/2} - \xi_{\alpha'})}{t_b^{3/2}} + \frac{1}{2t_b} \left( t_A^2 + \mathcal{O}(t_A^3) \right) + Z_X X^* e^{2q_1 V} X + Z_Y Y^* e^{2q_1 V} Y, \quad (A4) \]

\[ W = W_0 + A e^{-a T_s} + y \frac{X^{n+2} Y}{M_s^n}, \]

as investigated in Ref. [44]. In this example, \( t_b \) is stabilized at large value,

\[ \tilde{t}_b^{3/2} = e^{a \lambda t_b/2} \frac{W_0}{a A} \xi_{\alpha'} \left[ 3 - \frac{21 + 8 a \alpha_s}{12 a \tilde{t}_s} + \mathcal{O}\left( \frac{1}{(a \tilde{t}_s)^2} \right) \right], \]

\[ \tilde{t}_s^{3/2} = \xi_{\alpha'} \left[ 1 + \frac{3 - 13 a \alpha_s}{3 a \tilde{t}_s} + \mathcal{O}\left( \frac{1}{(a \tilde{t}_s)^2} \right) \right], \quad (A5) \]

Once \( t_b \) and \( t_s \) are stabilized, we have the effective potential of the PQ sector fields, \( \{ T_A, X, Y \} \) as Eq. (A2) of Ref. [44]. As a result, \( t_A \) is stabilized as \( \tilde{t}_A = \frac{\delta_{GS} v_{PQ}^2}{M_{GS}^2} + \mathcal{O}(\delta_{GS}) \) where \( M_{GS}^2 \) is calculated to be \( (\delta_{GS}/2)^2 (1/t_b^p) \), whereas \( X, Y \) are stabilized at the intermediate scale as discussed in Sec. II C. This example confirms various features used in our setup listed below.

Due to the no-scale structure at leading order, the anomaly-mediation effect is negligibly small,

\[ \frac{F^C}{C} = \mathcal{O}\left( m_{3/2}^2, m_{3/2}^2 \frac{|\phi|^2}{M_{Pl}^2} \right). \quad (A6) \]

On the other hand, FI term is extremely suppressed,

\[ \xi_{FI} \simeq \frac{\delta_{GS}}{t_b} \left( \partial_A \Omega_0 + \mathcal{O}\left( \frac{1}{t_b^p} \right) \right) = \mathcal{O}\left( \frac{|\phi|^2}{M_{Pl}^2 t_b} \right), \quad (A7) \]

where the last relation implies that an almost vanishing \( D \)-term is a result of cancellation between FI term and matter contribution to \( D \)-term. This is explicitly checked in the example Eq. (A4) as

\[ \xi_{FI} = 2 \frac{2 \tilde{t}_A}{\delta_{GS}} = 2 \frac{v_{PQ}^2}{M_{Pl}^2} + \mathcal{O}(\delta_{GS}). \quad (A8) \]
When matter VEVs are developed as a result of SUSY breaking, we can say FI term vanishes in the supersymmetric limit. Actually, $D$-term given by

$$
\frac{\epsilon_1}{\delta_{GS}} \sim \left( \frac{\partial t_A \partial^2 K_0}{\partial^2 K_0 \partial^2 t_A K_0} \right) \frac{|m_{3/2}|^2}{2} \delta_{GS} \equiv \frac{m_{3/2}}{2} \delta_{GS},
$$

is estimated as

$$
\epsilon_1 = \frac{\partial t_A \partial^2 K_0}{\partial^2 K_0 \partial^2 t_A K_0} \sim \frac{1}{\partial^2 t_A} \left( a_1 \partial t_A \Omega_0 + a_2 \alpha_{s,A} \partial^2 t_A \Omega_0 + a_3 \alpha_{s,A}^2 \partial^3 t_A \Omega_0 \right),
$$

where $a_{1,2,3}$ are order one coefficients. Whereas $\partial t_A \Omega_0$ is suppressed due to FI term suppression, $\partial^2 t_A \Omega_0$ can be order one, from e.g. $\Omega_0 = \Omega_A^2$, so we find that $\epsilon_1 \sim \alpha_A \sim 1/8\pi^2$. Finally, using the fact that $t_b$ comes in $\mathcal{Y}_i$ by $-\alpha_{s,A} \ln t_b$ through the combinations $\tilde{t}_{s,A}$, we obtain

$$
\epsilon_2 m_{3/2} \sim F^{t_b} \partial t_b \ln(\mathcal{Y}_{i,0}) \sim m_{3/2} \partial t_b \frac{\partial t_A \mathcal{Y}_i}{\mathcal{Y}_i} \sim m_{3/2} \alpha_{s,A} \frac{\partial t_{s,A} \mathcal{Y}_i}{\mathcal{Y}_i},
$$

and when $\partial t_{s,A} \mathcal{Y}_i / \mathcal{Y}_i \sim \mathcal{O}(1)$, we have $\epsilon_2 \sim \alpha_{s,A} \sim 1/8\pi^2$.

**Appendix B: Spontaneous PQ breaking sector with more than two fields**

In this appendix, we explore the possibility that one singlet field extension is made to be viable through a more involved spontaneous PQ breaking sector with more than two fields. First, let us consider the three fields case with $X, Y, Z$. Notice that one of the three fields must be tachyonic if they are not PQ singlet, because at least one of them should have positive PQ charge to conserve the PQ symmetry. We will call this field $X$ as in the case of two fields in Sec. II C. Likewise, at least one of them should have negative PQ charge so that it is non-tachyonic, and we call this field $Y$. Another point to note is that the PQ breaking sector should consist of only one superpotential term. Remind that we need two operators to obtain two of $(\xi, C\xi, \mu')$ around $m_{SUSY}$ as shown in Sec. III A. With one more field degrees of freedom with $Z$, PQ symmetry can now control up to three terms, and two of them must be for the singlet Higgs field sector. Thus only one term is allowed for the spontaneous PQ breaking sector. Therefore, we write the most general form of the PQ breaking sector with three fields as

$$
W_{PQ} = \frac{X^{n_1} Y^{n_2} Z^{n_3}}{M^n},
$$

(A11)
where \( n_1 + n_2 + n_3 = n + 3 \) and \( n, n_i \geq 1 \). The corresponding scalar potential including soft terms is

\[
V_{\text{PQ}} = m_X^2 |X|^2 + m_Y^2 |Y|^2 + m_Z^2 |Z|^2 + A_3 \frac{X^{n_1} Y^{n_2} Z^{n_3}}{M_*^{2n}} + \frac{|X|^{2(n_1-1)} |Y|^{2n_2} |Z|^{2n_3}}{M_*^{2n}} + \frac{|X|^{2n_1} |Y|^{2(n_2-1)} |Z|^{2n_3}}{M_*^{2n}} + \frac{|X|^{2n_1} |Y|^{2n_2} |Z|^{2(n_3-1)}}{M_*^{2n}},
\]

(B2)

with \( m_X^2 < 0 \) and \( m_Y^2 > 0 \) in our convention as described above.

Now we want that at least one non-tachyonic state is stabilized with a relatively small VEV proportional to the small \( A_3 \) to generate a sizable gauge mediation as discussed in Sec. II C. To this end, one can find that \( n_2 \) should be 1 so that the \( A_3 \)-term is linear in \( Y \). Also we observe that stabilization of \( Y \) by the \( A_3 \)-term can be done only when the other fields \( X, Z \) are somehow stabilized before in order to make a tadpole for \( Y \). In other words, \( X \) and \( Z \) must be stabilized when \( Y = 0 \).

With \( Y = 0 \), we examine the potential in arbitrary field directions in \( X-Z \) plane by parametrizing the fields as \( |X| = |\varphi| \cos \alpha, |Z| = |\varphi| \sin \alpha \) with \( 0 \leq \alpha \leq \pi/2 \). In a field direction with a constant value of \( \alpha \), the potential becomes

\[
V_{\text{PQ}} = (m_X^2 \cos^2 \alpha + m_Z^2 \sin^2 \alpha)|\varphi|^2 + (\cos \alpha)^{2n_1}(\sin \alpha)^{2n_3}\frac{|\varphi|^{2(n+2)}}{M_*^{2n}}.
\]

(B3)

where \( m_X^2 < 0 \). If \( (m_X^2 \cos^2 \alpha + m_Z^2 \sin^2 \alpha) < 0 \), the potential will be minimized at

\[
|\varphi| = \left( \frac{\cos \alpha)^{n_1/(n+1)}(\sin \alpha)^{n_3/(n+1)}}{v_{\text{PQ}}} \left( M_*^{n} \sqrt{|m_X^2 \cos^2 \alpha + m_Z^2 \sin^2 \alpha|} \right)^{1/(n+1)}
\]

(B4)

At this field value, the potential turns out that \( V_{\text{PQ}} \sim -(\cos \alpha)^{-2n_1/(n+1)}(\sin \alpha)^{-2n_3/(n+1)} \) and \( \partial^2 V_{\text{PQ}}/\partial \alpha^2 < 0 \). Thus this minimum point of the potential in the field direction \( \varphi \) with a fixed value of \( \alpha \) actually corresponds to a saddle point in the \( \varphi-\alpha \) (or \( X-Z \)) field space. Also notice that potential is unbounded from below in the field direction \( Z = 0(\alpha = 0) \) unless \( n_3 = 0 \). Therefore, \( \varphi \) cannot be stabilized in any direction in the \( X-Z \) plane but falls away into infinity. It means that generically one cannot make the required pattern of the PQ symmetry breaking with three fields.

We are thus led to consider even more than three fields. The simplest possibility with four fields will be

\[
W_{\text{PQ}} = y_1 \frac{X^{n+2} Y}{M_*^{n}} + y_2 \frac{X^{n+2} Y'}{M_*^{n}},
\]

(B5)
where \((X', Y')\) fields have some different PQ charges with \((X, Y)\) fields in order that any interaction between them is to be suppressed. Then each term will realize the correct PQ symmetry breaking pattern as in the two fields case, and we can use the two kinds of fields of different PQ charges to generate two terms of \((\xi, C\xi, \text{ or } \mu')\) with the PQ charge relations in Table I. In this way, one can realize the low fine-tuned EWSB with one singlet field extension of the Higgs sector, but it requires such a complication of the PQ sector that there must be more than three fields and a non-trivial PQ charge relation among them.

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