Coherent Parton Showers
with Local Recoils

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ABSTRACT: We outline a new formalism for dipole-type parton showers which maintain exact energy-momentum conservation at each step of the evolution. Particular emphasis is put on the coherence properties, the level at which recoil effects do enter and the role of transverse momentum generation from initial state radiation. The formulated algorithm is shown to correctly incorporate coherence for soft gluon radiation. Furthermore, it is well suited for easing matching to next-to-leading order calculations.

KEYWORDS: QCD, Jets, NLO Calculations
1. Introduction

Parton shower simulation programs like Pythia and HERWIG [1, 2] have been the workhorses for high energy physics experiments for a long time. With the advent of the Large Hadron Collider (LHC) at CERN many new developments have been made in order to refine the existing programs and to extend their applicability. Currently, there are a few major multi purpose event generators in use. The FORTRAN generators have been completely rewritten and extended as Pythia8 and Herwig++ [3–5] and the program Sherpa has been established [6, 7]. All these new generators come with a great effort of new developments.

The classical parton showers with 1 → 2 like branchings themselves have been rewritten and reformulated in order to take into account mass effects and improve the phase space coverage [8] or to incorporate intertwined interactions with partons from additional hard interactions in the underlying event [9]. Other developments try to incorporate quantum interference effects or subleading terms in the expansion in the number of colours [10–13]. Besides many other efforts to improve the event
simulation, e.g. in the area of the underlying event [14–19], one of the most important questions that has been addressed was for the matching with higher order matrix elements or the merging of parton showers with multiple hard emissions at the tree level.

The latter was formulated conceptually by Catani, Krauss, Kuhn and Webber (CKKW) [20, 21] and, in a variant, by Lonnblad [22] and implemented in various parton shower programs [23–27]. An alternative approach to matching has been advocated by Mangano [28, 29] and is known as MLM matching. [29] gives a comprehensive overview of current implementations and shows detailed comparisons. Recent improvements address previous deficiencies of the CKKW algorithm in filling the phase space [30, 31] that had been discussed extensively in [32]. Some problems have been overcome with the help of so-called truncated showers that were introduced earlier in the context of matching with next-to-leading order (NLO) matrix elements [33]. Merging with NLO matrix elements has been recently studied in [34].

First attempts to match parton showers and matrix elements at NLO have been made in a phase space slicing approach [35–37] that suffered from some (numerically small) systematic inconsistencies. The more general and systematic approach has been MC@NLO [38, 39] that has since been extended to include many processes and all possible colour structures [40–42]. As the MC@NLO is closely tied to a particular parton shower algorithm, the subtraction terms that guarantee the consistent matching with the NLO matrix element contributions have to be calculated once for a specific parton shower program. After all initial efforts have been made to match with HERWIG, later also several processes have been matched to Herwig++ [43–45].

While the MC@NLO approach is very successful and was developed for many processes it may be considered to be tied too strongly to the underlying parton shower and to suffer from negative weighted events, which, in practice, never pose a real problem. An alternative approach, now known as POWHEG [33, 46–48], has been formulated by Nason [33]. Here, the matching formalism is based on a modified Sudakov form factor that contains the real emission matrix element. This guarantees that the first emission of the parton shower is the hardest one as well and can, therefore, be described by the full matrix element. In principle, this approach is closely related to the so called hard matrix element corrections [49, 50]. This approach has been further developed into a systematic matching scheme and applied to several processes [51, 52] and was also widely used by other groups, e.g. the Herwig++ collaboration [45, 53–56].

The subtraction based original MC@NLO approach is following the requirement that the underlying parton shower algorithm must not be modified. Already shortly after the publication of the MC@NLO approach it was, however, noted that the intrinsic subtraction in this scheme could be simplified tremendously if the parton shower would follow closely the subtraction terms that are used to regularize the soft and collinear divergences in the NLO calculation to be matched. This was highlighted in [10], where also more formal developments towards new partons were
carried out. Therefore, some groups started to write new parton showers based on subtraction terms. The groups [57, 58] used the Catani–Seymour (CS) [59] terms, while [60] based a shower on Lund dipoles and [61] on antenna subtraction terms.

In this paper we take the same viewpoint and present new theoretical development towards a parton shower, based on CS subtraction kernels as well. After the question for soft coherence effects has been raised by [62] in the context of a simplified toy model, we would like to address this question for a full CS like parton shower. The question of collinear radiation and the implication of the DGLAP evolution was discussed in [63, 64], not the coherence properties, though. In this paper, we formulate a parton shower, based on CS subtraction kernels, in a detail that specifies a full implementation. We show that, with the right choice of evolution variable and initial conditions it is indeed possible to find the correct soft anomalous dimensions with a parton shower, based on CS dipoles. Therefore, it is possible to implement such a shower while incorporating soft colour coherence effect in a way that has always been a vital ingredient of the HERWIG and Herwig++ programs.

2. Local Recoils, Form Factors and Coherence

We consider a single parton emission off a pair of partons with momenta $p_{ij}$ and $p_k$. The probability for this emission is taken to be the sum of two splitting functions, each associated with one leg. Using DGLAP splitting kernels and the Sudakov decomposition for the splitting $p_{ij} \rightarrow q_i, q_j$,

$$q_i = zp_{ij} + \frac{p_{ij}^2}{2p_{ij} \cdot n} n + k_\perp,$$

$$q_j = (1 - z)p_{ij} + \frac{p_{ij}^2}{2p_{ij} \cdot n (1 - z)} n - k_\perp,$$

(2.1)

(2.2)

where $k_\perp^2 = -p_\perp^2$ and $k_\perp \cdot p_{ij} = k_\perp \cdot n \equiv 0$ constitutes the usual collinear approximation, which may be extended to the quasi-collinear approximation for emissions off massive partons, [8]. The light-like vector $n$ defines the collinear direction, and therefore is used as the gauge vector in a light-cone gauge when deriving the collinear-singular behaviour of QCD matrix elements. $n$ needs to be chosen along the colour connected partner $p_k$, the so-called physical gauge, in which interference diagrams are collinearly subleading such that the unregularized splitting kernels are given by cut self-energy diagrams only.\(^1\)

Note that, within this parametrization, the DGLAP splitting kernels are functions of

$$z = \frac{n \cdot q_i}{n \cdot p_{ij}}.$$  

(2.3)

\(^1\)We note that this is a gauge choice for each singular limit of interest. The definition of 'colour-connected' here applies in the large-$N_c$ limit but may be generalized by including the full colour correlations present at finite $N_c$. 
Indeed, there is not a single choice of light-cone gauge, but rather a class of gauge choices which are connected by rescaling the gauge vector \( n \) (i.e. longitudinal boosts along the collinear direction), for which the splitting kernels are left invariant.

We are interested in extending this picture such as not to perform an approximation in the choice of kinematics, thereby introducing exact energy-momentum conservation within the splitting \( p_{ij}, p_k \rightarrow q_i, q_j, q_k \). The choice of the recoil strategy is not unique. However, choosing a spectator to absorb the longitudinal recoil of the splitting,

\[
\begin{align*}
n &= p_k \\
q_k &= \left(1 - \frac{p^2_i}{2p_{ij} \cdot p_k z(1-z)}\right) p_k
\end{align*}
\]

is the only choice compatible with the remaining gauge degrees of freedom in the functional form of the splitting kernels. As we shall also see, this is the only choice which guarantees that the splitting functions in a physical gauge do reproduce the correct soft behaviour.

### 2.1 DGLAP Kernels, ‘Soft Correctness’ and Angular Ordering

As we are primarily interested in soft gluon radiation, we neglect gluon splittings into quark-antiquark pairs in this section.

For final state radiation the spin-averaged DGLAP kernels are given by

\[
P_{gq}(z) = C_F \left(\frac{2z}{1-z} + (1-z)\right), \quad P_{gg}(z) = 2C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right),
\]

such that matrix elements squared, summed over all collinear configurations factorize as

\[
\begin{align*}
n+1\langle \mathcal{M}(q_1, \ldots, q_{n+1})|\mathcal{M}(q_1, \ldots, q_{n+1})\rangle_{n+1} &\rightarrow \\
&\sum_{i=1}^{n} \sum_{j \neq i} \frac{4\pi\alpha_s}{q_i \cdot q_j} P_{ij}(z) n\langle \mathcal{M}(q_1, \ldots, p_{ij}, \ldots, q_{n+1})|\mathcal{M}(q_1, \ldots, p_{ij}, \ldots, q_{n+1})\rangle_n.
\end{align*}
\]

Note that in writing this expression, we do need to include a symmetry factor of 1/2 along with the gluon splitting function.

As each amplitude \( |\mathcal{M}\rangle \) is a colour singlet, i.e.

\[
\sum_{i=1}^{n} T_i^2 + \sum_{i=1}^{n} \sum_{j \neq i} T_i \cdot T_j = 0
\]

we may rewrite collinear factorization within the choice of the physical gauge for single collinear configurations as

\[
\begin{align*}
n+1\langle \mathcal{M}(q_1, \ldots, q_{n+1})|\mathcal{M}(q_1, \ldots, q_{n+1})\rangle_{n+1} &\rightarrow \\
&\sum_{i=1}^{n} \sum_{j,k \neq i} \frac{4\pi\alpha_s}{q_i \cdot q_j} P_{ij}(z)|_{n=p_k} n\langle \mathcal{M}(q_1, \ldots, p_{ij}, \ldots, q_{n})|C_{(ij)k}|\mathcal{M}(q_1, \ldots, p_{ij}, \ldots, q_{n})\rangle_n.
\end{align*}
\]
where
\[ C_{ij} = -\frac{\mathbf{T}_i \cdot \mathbf{T}_j}{T_i^2} \] (2.9)
is the colour correlation operator as introduced in [59].

Within this framework, we have that
\[
\frac{1}{q_i \cdot q_j} \frac{z}{1 - z} \bigg|_{n=p_k} = \frac{q_i \cdot p_k}{q_i \cdot q_j q_j \cdot p_k} \quad \frac{1}{q_i \cdot q_j} \frac{1 - z}{z} \bigg|_{n=p_k} = \frac{q_j \cdot p_k}{q_j \cdot q_i q_i \cdot p_k}
\] (2.10)
such that the single splitting function \( P_{ij}(z) \mid_{n=p_k} \) constitutes the complete, correct soft behaviour for the dipole \( i, k \). Note that the eikonal parts – as well as any other part of a splitting function – is invariant under rescaling of the spectator momentum \( p_k \), which is an even stronger motivation to use the longitudinal recoil strategy defined above.

This will also be a necessary requirement when trying to remove what we call 'soft double counting'. As we will show now, this is closely related to the coherence properties and logarithmic accuracy of a particular shower setup. To be precise, we consider the form factor \( \Delta_{ik}(Q^2, \mu^2) \) associated to a final-final dipole \( i, k \) when evolving from a hard scale \( Q^2 \) to a soft scale \( \mu^2 \). Regarding the leading- (double) and next-to-leading (single) logarithmic contributions, \( \alpha_s^a L^{2n} \) and \( \alpha_s^a L^{2n-1} \) with \( L = \ln(Q^2/\mu^2) \) the correct behaviour can be obtained from the coherent branching formalism [65], reproducing the results of soft gluon resummation, [66], by considering the leading behaviour of the \( z \)-integrated splitting kernel for \( \mu^2 \ll p_{\perp}^2 \ll Q^2 \).

The resulting form factor reads
\[
-\ln \Delta_{ik}(Q^2, \mu^2) = \int_{\mu^2}^{Q^2} \frac{dp_{\perp}^2}{p_{\perp}^2} \frac{\alpha_s(p_{\perp}^2)}{2\pi} \left( \Gamma_i(p_{\perp}^2, Q^2) + \Gamma_k(p_{\perp}^2, Q^2) \right), \quad (2.11)
\]
where the Sudakov anomalous dimensions \( \Gamma_k(p_{\perp}^2, Q^2) \) are given by
\[
\Gamma_q(p_{\perp}^2, Q^2) = C_F \left( \ln \frac{Q^2}{p_{\perp}^2} - \frac{3}{2} \right), \quad (2.12)
\]
\[
\Gamma_g(p_{\perp}^2, Q^2) = C_A \left( \ln \frac{Q^2}{p_{\perp}^2} - \frac{11}{6} \right), \quad (2.13)
\]
receiving contributions both at the LL level from soft collinear, at the NLL level from hard collinear radiation. Note that the latter, \( i.e. \) the non-logarithmic terms in \( \Gamma \) are determined by the average of the soft-suppressed, \( z \)-regular terms of the splitting functions.

### 2.2 Recoils and Soft Coherence

We now want to include the effects of a finite recoil. Within the minimal recoil strategy outlined above this only affects the phase space measure,
\[
\frac{dp_{\perp}^2}{p_{\perp}^2} dz \rightarrow \frac{dp_{\perp}^2}{p_{\perp}^2} dz \left( 1 - \lambda \frac{p_{\perp}^2}{z(1 - z)s_{ik}} \right), \quad s_{ik} = 2p_i \cdot p_k \quad (2.14)
\]
where we introduced $\lambda \to 1$ to explicitly keep track of these effects. Choosing a phase space region related to an ordering in virtuality or transverse momentum,

$$4\mu^2 < \frac{p_\perp^2}{z(1-z)} < Q^2,$$  

(2.15)

we find

$$\Gamma^V_q(p_\perp^2, Q^2) = C_F \left( 2 \ln \frac{Q^2}{p_\perp^2} - \frac{3}{2} - 2\lambda \frac{Q^2}{s_{ik}} \right),$$  

(2.16)

$$\Gamma^V_g(p_\perp^2, Q^2) = C_A \left( 2 \ln \frac{Q^2}{p_\perp^2} - \frac{11}{6} - 2\lambda \frac{Q^2}{s_{ik}} \right).$$  

(2.17)

Note that here, the recoil effects enter at the level of next-to-leading logarithms and the coefficient of the leading logarithms turns out to be twice the correct result. The latter observation has been noted since long [65]. From this example it is very clear that the simple fact that the DGLAP splitting functions reproduce the correct soft behaviour is not enough for the correct soft anomalous dimension. The wrong coefficient of the leading logarithmic contributions may be attributed to a double counting of soft emissions, originating from the fact that the above chosen phase space region does introduce an overlap of the phase space available for emissions off either parton of the dipole.

Choosing angular ordering in the variable $\tilde{q}$ by disentangling soft and collinear limits\(^2\), and imposing phase space constraints through a cutoff on the transverse momentum in the soft limit(s),

$$\tilde{q}^2 = \frac{p_\perp^2}{z^2(1-z)^2} \quad \mu^2 < z^2 \tilde{q}^2 , \quad (1-z)^2 \tilde{q}^2 \quad \tilde{q}^2 < Q^2$$

(2.18)

we recover the correct anomalous dimensions [2.12, 2.13] with recoil effects entering beyond NLL,

$$\Gamma^{AO}_q(p_\perp^2, Q^2) = C_F \left( \ln \frac{Q^2}{p_\perp^2} - \frac{3}{2} \right) + C_F \frac{p_\perp}{Q} \left( 1 - 2\lambda \frac{Q^2}{s_{ik}} \right) + O \left( \frac{p_\perp^2}{Q^2} \right),$$  

(2.19)

$$\Gamma^{AO}_g(p_\perp^2, Q^2) = C_A \left( \ln \frac{Q^2}{p_\perp^2} - \frac{11}{6} \right) + 2C_A \frac{p_\perp}{Q} \left( 1 - \lambda \frac{Q^2}{s_{ik}} \right) + O \left( \frac{p_\perp^2}{Q^2} \right),$$  

(2.20)

the subleading terms giving rise to power corrections in the form factor exponent.

Apart from the recoil effects, this result has a straightforward explanation: The phase space region chosen for the angular ordered evolution provides approximately disjoint regions (exactly disjoint in the case of Herwig++) for emissions off either leg of the dipole, thereby removing the soft double counting observed earlier. Note that this observation would then in principle allow to include local recoils within the angular ordered DGLAP evolution.

\(^2\)Note that in the soft limit(s), $z \to \epsilon , \quad 1-\epsilon , \quad p_\perp^2$ scales as $\epsilon^2$. 


2.3 Catani-Seymour Kernels and New Formalism

As outlined in the previous sections, taking a minimal choice to treat recoils yields a dipole-type picture. Within such a cascade it is however difficult to maintain the strong angular ordering, which is tied to the $1 \rightarrow 2$ nature of independent jet evolution.

Choosing the phase space to be restricted by a cutoff on the transverse momentum, thereby assuming an ordering in $p_\perp$ (or virtuality) is a much more natural picture to consider for a dipole-type evolution. In addition, this also removes complications when implementing matrix element corrections, either stand alone or for the purpose of POWHEG-type NLO matching as the first emission off a dipole then is indeed the hardest emission.

To cure the problem of soft double counting generated by this evolution, one may modify the splitting functions and 'continue' them over the whole available phase space in such a way, that the soft-singular pieces reproduce the correct soft behaviour when adding both modified splitting functions.

More precisely, for each leg $i$ we replace the eikonal part by the radiation pattern associated with collinear emissions of $p_i$

$$\frac{p_i \cdot p_j}{p_i \cdot q \ p_j \cdot q} \rightarrow \frac{p_i \cdot p_j}{p_i \cdot q \ (p_i + p_j) \cdot q}$$

while keeping the collinear parts exactly. Note that this minimal construction, which does not modify the singular properties following from QCD, is nothing but the construction prescription for the subtraction kernels introduced in [59]. This picture of local recoils using a single spectator parton is ideally supplemented with exact factorization of the phase space considering no kinematic approximation. One choice, which so far has been implemented [57, 58] is to invert the kinematic mappings as derived in [59].

For initial state radiation, taking the Catani-Seymour factorization literally does have shortcomings. Most prominently, the choice of keeping the initial state emitter’s momentum collinear to the one before emission leads for example to the fact that a final state singlet as in Drell-Yan lepton pair production, does receive a non-vanishing transverse momentum from the very first shower emission only. Further, an initial-initial system emitting a parton left the spectator parton unchanged, which might not be sufficient for the description of the transverse momentum spectrum of the whole final state. The aim of this work is to provide a formalism, which does overcome these problems. Further, we are interested in the logarithmic accuracy and ordering of soft gluon radiation in our setup reflecting coherence properties.

Starting from the final-state parametrization given above, the outline of our formalism is as follows: We obtain a parametrization of the kinematics for initial state emitters and/or spectators by considering the physical splitting processes while maintaining exact energy-momentum conservation locally to each branching, i.e. involving the emitter-emission system and a single spectator only. The spectator
is restricted to take the longitudinal recoil of the splitting only. For initial state radiation we do allow each initial state emission to generate transverse momentum of the emitting incoming parton in a backward evolution. This transverse momentum is then migrated to the complete final state system by realigning the incoming partons to the beam axes at the end of the evolution.

For final-final dipoles, we find that the anomalous dimensions take the correct form apart from the fact, that the dependence on the arbitrary hard scale $Q^2$ is being replaced by the dipole’s invariant mass $s_{ik}$,

$$\Gamma_q^{CS}(p_2^2, \cdot) = C_F \left( \ln \frac{s_{ik}}{p_2^2} - \frac{3}{2} \right) - C_F \pi \lambda \frac{p_\perp}{\sqrt{s_{ik}}} + O \left( \frac{p_2^4}{Q^2} \right), \quad (2.22)$$

$$\Gamma_g^{CS}(p_2^2, \cdot) = C_A \left( \ln \frac{s_{ik}}{p_2^2} - \frac{11}{6} \right) - C_A \pi \lambda \frac{p_\perp}{\sqrt{s_{ik}}} + O \left( \frac{p_2^4}{Q^2} \right), \quad (2.23)$$

with recoil effects entering beyond NLL.

We note that, in case of DGLAP kernels, the correct coefficient of the leading logarithmic contributions to the anomalous dimension is governed by the choice of boundaries on the momentum fraction for a given (but arbitrary) hard scale $Q^2$, 

$$\int_{1 - \sqrt{\kappa}}^1 \frac{dz}{1 - z} = \frac{1}{2} \ln \left( \frac{Q^2}{p_\perp^2} \right), \quad (2.24)$$

with $\kappa = p_\perp^2/Q^2$.

The above findings for the anomalous dimension can essentially be traced back to the fact that the transition from a DGLAP kernel possessing a soft singularity $\sim 1/(1 - z)$ to the appropriate Catani-Seymour kernel (while keeping track of all recoil effects, i.e. considering the soft limit at fixed $p_2^2$) is the simple replacement

$$\frac{1}{1 - z} \rightarrow \left( 1 - \frac{\kappa_{ik}}{1 - z} \right) \frac{1 - z}{(1 - z)^2 + \kappa_{ik}}, \quad (2.25)$$

where $\kappa_{ik} = p_\perp^2/s_{ik}$. Here, the first factor is the effect of the finite recoil stemming from the exact factorization of the phase space measure.

Within the variables to be outlined in detail in the next section, we find that this pattern generalizes to the cases of initial state emitter or spectator partons, up to a sign on the recoil term owing to timelike or spacelike virtualities of the emitter or whether the relevant dipole scale is a spacelike momentum transfer or invariant mass.

Choosing the $z$ boundaries (in the approximation considered above) to be given by

$$z < 1 - \frac{p_\perp^2}{Q^2} = 1 - \kappa \quad (2.26)$$

it is evident that the recoil contribution only gives rise to power corrections, while the logarithmic contribution is given by

$$\frac{1}{2} \int_{\kappa^2}^1 \frac{d\xi}{\xi + \kappa_{ik}} = \frac{1}{2} \ln \left( \frac{s_{ik}}{p_\perp^2} \right) + \text{power corrections}, \quad (2.27)$$
thereby reproducing the correct coefficient up to the disappearance of the arbitrary hard scale, an immediate consequence of the screening of the soft singularity at fixed transverse momentum.

2.4 Structure of the Evolution

For final state radiation with final state spectator, our findings of the previous section immediately signal a choice of the hard scale for a single dipole originating from a hard process. Choosing an arbitrary hard scale $Q^2 \neq s_{ik}$ will immediately result in the appearance of spurious logarithmic contributions when performing the $p_\perp^2$ integration.

For example, at fixed $\alpha_s$ the leading logarithmic contributions for a dipole $i, k$, with Casimir operators $C_{i,k}$ associated to the partons, take the form

$$- \ln \Delta_{ik} = \frac{\alpha_s}{4\pi} (C_i^2 + C_k^2) \ln \left( \frac{Q^2}{\mu^2} \right) \ln \left( \frac{s_{ik}^2}{\mu^2 Q^2} \right) + \text{NLL}$$

instead of the expected result

$$- \ln \Delta_{ik} = \frac{\alpha_s}{4\pi} (C_i^2 + C_k^2) \ln^2 \left( \frac{Q^2}{\mu^2} \right) + \text{NLL} ,$$

the mismatch being manifest as an ambiguity at the level of next-to-leading logarithms. We are therefore lead to the choice $Q^2 = s_{ik}$, i.e. the hard scale associated to a dipole is the respective invariant mass.

For initial state emitter or spectator partons, we assume that this generalizes to choosing the hard scale in such a way as to fill the complete phase space, modulo the infrared cutoff.

Note that this choice does not determine the ordering per se, but only the choice of hard scale and the shape of the phase space restriction when evolving between two scales. The ordering is to be chosen in such a way, that the leading effects of multiple emissions off each leg of the dipole do exponentiate. Due to the structure of the splitting kernels given above and the additional complications from all finite recoil effects the explicit exponentiation is beyond the scope of this paper.

Having however observed that we can reproduce the correct Sudakov anomalous dimension, while avoiding soft double counting we additionally note that within the variables chosen

$$p_\perp^2 = 2 \frac{p_i \cdot q \cdot q \cdot p_k}{p_i \cdot p_k}$$

for emission of a gluon of momentum $q$ off a dipole $(i, k)$. Ordering emissions in this variable therefore corresponds to an ordering reproducing the most probable history of multiple gluon emission according to the eikonal approximation in the limit of soft gluons strongly ordered in energy.

We therefore conclude that branchings within the physical kinematics outlined above and based on the corresponding CS dipole splitting functions allow us to
construct a parton shower that has the right coherence properties. The final state emissions should in this case be taken as outlined above, i.e. with the hard scale of a single cascade chosen to be the dipole invariant mass and the evolution should be strictly ordered in transverse momentum. However, in the naive adoption of the CS picture to a parton shower not every initial state emission would contribute to the final state transverse momentum. We will formulate a more suitable approach below.

3. Kinematics, Phase space and Splitting Probabilities

The purpose of this section is to provide our new formalism in all details, particularly the kinematic parametrization, phase space convolution properties and boundaries, and the splitting probabilities entering the evolution. The explicit expressions for the splitting kernels can be inferred from [59], as we express the variables chosen there in terms of the physical variables chosen for the shower evolution.

3.1 Final State Radiation

3.1.1 Final State Spectator

Final state radiation with a final state spectator does represent the generic version of the splitting kinematics chosen here. For a splitting \((p_i, p_j) \rightarrow (q_i, q, q_j)\) we choose the standard Sudakov decomposition

\[
q_i = z p_i + \frac{p_{i,\perp}^2}{z s_{ij}} p_j + k_{\perp}
\]

\[
q = (1 - z)p_i + \frac{p_{i,\perp}^2}{(1 - z)s_{ij}} p_j - k_{\perp}
\]

\[
q_j = \left(1 - \frac{p_{i,\perp}^2}{z(1 - z)s_{ij}}\right) p_j
\]

where \(k_{\perp}^2 = -2p_i \cdot p_j \equiv -s_{ij}\) and \(k_{\perp} \cdot p_{i,j} = 0\). The transverse momentum is defined in the dipole’s rest frame to be purely spacelike,

\[
\hat{p}_{i,j} = \left(\frac{\sqrt{s_{ij}}}{2}, \pm \mathbf{p}\right), \quad \hat{k}_{\perp} = (0, \mathbf{p}_{\perp}) , \quad \mathbf{p} \cdot \mathbf{p}_{\perp} = 0
\]

Note that this does preserve the momentum of the emitting system, \(q_i + q + q_j = p_i + p_j\). The parametrization gives rise to the phase space factorization [59]

\[
d\phi_3(q_i, q, q_j|Q) = d\phi_3(p_i, p_j|Q) \frac{1}{16 \pi^2} \frac{d\phi}{2\pi} \frac{dp_{i,\perp}^2}{z(1 - z)} \frac{dz}{z(1 - z)s_{ij}} \left(1 - \frac{p_{i,\perp}^2}{z(1 - z)s_{ij}}\right)
\]

Note that, in the collinear limit, this is the massless version of the kinematics as chosen in [8]. It further constitutes the inversion of the ‘tilde’-mapping, where the variables \(y\) and \(z\) chosen in [59] are given by

\[
y = \frac{p_{i,\perp}^2}{z(1 - z)s_{ij}}, \quad z = \frac{p_j \cdot q_i}{p_i \cdot p_j}
\]
Figure 1: Allowed phase space regions for emissions from a final-final dipole expressed in the Dalitz variables \( x_k = 2Q \cdot p_k/Q^2 \) for a dipole of mass \( s_{ij} = 100 \text{ GeV} \) and infrared cutoff \( \mu = 5 \text{ GeV} \). The shaded region is accessible for emissions off the parton \( i \), whereas the area enclosed by the solid line is accessible for emissions off parton \( j \). The area enclosed by the dotted line is an example of the phase space excluded when starting at a scale lower than \( s_{ij} \). Note that the infrared cutoff is exaggerated for illustrative purposes only. In practice, almost the whole physical phase space will be available.

The allowed phase space region is obtained by considering the limits on the emitter’s virtuality,

\[
4\mu^2 < \frac{p_{\perp}^2}{z(1-z)} < Q_{\text{max}}^2 = s_{ij}
\]

such that

\[
\mu^2 < p_{\perp}^2 < \frac{Q_{\text{max}}^2}{4} , \quad z_\pm = z_\pm = \frac{1}{2} \left( 1 \pm \sqrt{1 - \frac{4p_{\perp}^2}{Q_{\text{max}}^2}} \right).
\]

Averaging over azimuth, the final-final splitting kernels take the form

\[
\frac{8\pi\alpha_s}{2q_i \cdot q} \langle V(p_{\perp}^2, z) \rangle
\]

such that the splitting probability is

\[
dP = \frac{\alpha_s}{2\pi} \langle V(p_{\perp}^2, z) \rangle \left( 1 - \frac{p_{\perp}^2}{z(1-z)s_{ij}} \right) \frac{dp_{\perp}^2}{p_{\perp}^2} dz .
\]

Note that, comparing to the collinear limit, the effect of finite recoils is to act as a damping factor for large-angle hard emissions, provided that \( y < 1 \) which is a consequence of the phase space boundary.
3.1.2 Initial State Spectator

For an initial state spectator we consider the crossing $q_j \rightarrow -q_a$, $p_j \rightarrow -p_a$, such that

$$q_i = z p_i + \frac{p_i^2}{z s_{ia}} p_a + k_\perp$$  \hspace{1cm} (3.11)

$$q = (1 - z) p_i + \frac{p_i^2}{(1 - z) s_{ia}} p_a - k_\perp$$  \hspace{1cm} (3.12)

$$q_a = \left( 1 + \frac{p_a^2}{z(1 - z) s_{ia}} \right) p_a.$$  \hspace{1cm} (3.13)

Note that exact momentum conservation is trivially implemented by just the fact that the parametrization for a final state spectator does respect this constraint. The transverse momentum is defined be purely spacelike in a frame where,

$$\hat{p}_{i,a} = \left( \sqrt{s_{ia}}, \pm \mathbf{p} \right), \quad \hat{k}_\perp = (0, \mathbf{p}_\perp), \quad \mathbf{p} \cdot \mathbf{p}_\perp = 0.$$  \hspace{1cm} (3.14)

The phase space measure then obeys the convolution

$$d \phi^F_2(q_i, q|Q; P_a, q_a, x_a) = d \phi^F_1(p_i|Q; P_a, p_a, x_a) \frac{d \phi}{2 \pi} \frac{x}{16 \pi^2} \frac{dz}{z(1 - z)} dp^2_\perp,$$  \hspace{1cm} (3.15)

where

$$x = \frac{1}{1 + \frac{p^2_i}{(1 - z) s_{ia}}}, \quad z = \frac{p_a \cdot q_i}{p_a \cdot p_i}$$  \hspace{1cm} (3.16)

and it is straightforward to verify that this indeed gives rise to the phase space convolution as given in [59]. Including the parton distributions and the kinematic factor of the partonic flux, the relevant measure is

$$\frac{f_a(x_a)}{4 q_a \cdot n} d \phi^F_2(q_i, q|Q; P_a, q_a, x_a) dx_a =$$

$$\left( \frac{f_a(x_a/x)}{f_a(x_a)} \theta(x - x_a) \frac{d \phi}{2 \pi} \frac{x}{16 \pi^2} \frac{dz}{z(1 - z)} dp^2_\perp \right) \frac{f_a(x_a)}{4 p_a \cdot n} d \phi^F_1(p_i|Q; P_a, p_a, x_a) dx_a.$$  \hspace{1cm} (3.17)

The phase space limits can be obtained as for the final state case,

$$4 \mu^2 < \frac{p^2_\perp}{z(1 - z)} < Q^2_{\text{max}},$$  \hspace{1cm} (3.18)

where, owing to $x > x_a$, the hard scale of a dipole is now given by

$$Q^2_{\text{max}} = s_{ia} \frac{1 - x_a}{x_a}.$$  \hspace{1cm} (3.19)

Averaging over azimuth, the final-initial splitting kernels take the form

$$\frac{8 \pi \alpha_s}{2 q_i \cdot q} \langle V(p^2_\perp, z) \rangle.$$  \hspace{1cm} (3.20)
such that the splitting probability is

\[ dP = \frac{\alpha_s}{2\pi} \langle V(p_\perp^2, z) \rangle \frac{f_a(x_a/x)}{f_a(x_a)} \theta(x - x_a) \frac{dp_\perp^2}{p_\perp^2} \, dz . \]  

(3.21)

Note that the finite recoil enters only in the PDF ratio, reproducing the correct collinear limit when \( x \to 1 \). Once again, the effect of the finite recoil is a damping of hard emissions for \( x \sim x_a \).

### 3.2 Initial State Radiation

A construction of initial state radiation by just crossing prescriptions is not obvious owing to the fact that the shower evolution is formulated as a backward evolution.

The physical variables thus need to be defined from the physical forward kinematics. For the physical emission process \( q_a \to p_a, q \) the relevant Sudakov decomposition for the emission momentum \( q \) is

\[ q_{\text{forward}} = (1 - z)q_a + \frac{p_\perp^2}{2n \cdot q_a(1 - z)} n - k_\perp , \]  

(3.22)

where \( n \) is the backward lightcone direction defining the collinear direction, i.e. the final or initial state spectator’s momentum.

The parametrization above is most conveniently inverted to backward evolution \( p_a \to q_a, q \) by considering the process in a frame where \( q_a = p_a/x \), giving rise to

\[ q_{\text{backward}} = \frac{(1 - z)}{x} p_a + \frac{p_\perp^2}{2n \cdot p_a(1 - z)} n - \frac{1}{\sqrt{x}} k_\perp . \]  

(3.23)

We therefore define the Lorentz invariant physical variables to be given by

\[ x \, q_a \cdot q = \frac{p_\perp^2}{1 - z} \quad x \, n \cdot q = (1 - z) n \cdot p_a . \]  

(3.24)

The parametrization keeping the emitter aligned with the beam axis can then be related to a parametrization where the initial state parton after (backward evolution) emission does acquire a finite transverse momentum while keeping the spectator after emission aligned with the one before emission.

It is this type of splitting kinematics which allows any emission off an initial state parton to contribute transverse momentum to a final state system after having applied a proper realignment boost once the parton shower evolution has terminated. Ideally, this final boost should \textit{not} be related to the parametrization chosen but being determined in a process dependent way such as to leave the interesting kinematic quantities of the hard process invariant.

#### 3.2.1 Final State Spectator

For initial state emissions with final state spectator, \( p_a, p_j \to q_a, q, q_j \), using the variables introduced in [59],

\[ x = \frac{p_a \cdot p_j}{(p_a - p_j) \cdot q_a} \quad u = \frac{q_j \cdot q_a}{(p_a - p_j) \cdot q_a} , \]  

(3.25)
we use the parametrization

\[ q_a = \frac{1-u}{x-u} p_a + \frac{u}{x} \frac{1-x}{x-u} p_j + \frac{1}{u-x} k_{\perp} \]  

\[ q = \frac{1-x}{x-u} p_a + \frac{u}{x} \frac{1-u}{x-u} p_j + \frac{1}{u-x} k_{\perp} \]  

\[ q_j = \left( 1 - \frac{u}{x} \right) p_j \]  

which does preserve the momentum transfer, \( q + q_j - q_a = p_j - p_a \). The transverse momentum obeys

\[ k_{\perp}^2 = -u(1-u) \frac{1-x}{x} s_{aj} \quad s_{aj} = 2 p_a \cdot p_j . \]  

Considering the collinear limit \( u \to 0 \), it is evident that the relevant momentum fraction is \( x \) and we are therefore lead to choose the physical variables to be given by

\[ u = \frac{\kappa}{1-z}, \quad x = \frac{z(1-z) - \kappa}{1-z - \kappa}, \quad \kappa = \frac{\sqrt{s}}{s_{aj}}. \]  

Indeed, the Lorentz transformation

\[ R^{\mu}_{\nu} = \delta^{\mu}_{\nu} + \frac{x}{(1-u)(x-u)} \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{p_a \cdot p_j} + \frac{u(1-x)}{x-u} \frac{K^{\mu} K^{\nu}}{p_a \cdot p_j} + \frac{x}{x-u} \frac{k_{\perp}^{\mu} K^{\nu} - K^{\mu} k_{\perp}^{\nu}}{p_a \cdot p_j} \]  

with \( K = p_a + p_j \) relates the above parametrization to one preserving the direction of the incoming parton,

\[ R q_a = \frac{1}{x} p_a \]  

\[ R q = up_j + (1-u) \frac{1-x}{x} p_a - k_{\perp} \]  

\[ R q_j = (1-u)p_j + u \frac{1-x}{x} p_a + k_{\perp} . \]  

In order to derive the phase space convolution properties associated with the parametrization given above, we employ the formalism outlined in the appendix. Substituting

\[ u = \frac{y}{w+y(1-w)} \quad x = \frac{1}{w+y(1-w)} \]  

the parametrization above is mapped to

\[ (-q_a) = w(-p_a) + (1-w) y p_j - q_{\perp} \]  

\[ q = (1-w)(-p_a) + w y p_j + q_{\perp} \]  

\[ q_j = (1-y) p_j \]
with \( q_\perp^2 = -s_{aj} w (1 - w) \) such that the generalized phase space measure factors as

\[
d\phi_3(q_j, q, -q_a | Q) = \frac{s_{aj}}{16\pi^2} \frac{d\phi}{2\pi} \frac{dx}{x^3} du \ d\phi_2(p_j, -p_a | Q) .
\]  

(3.39)

Having identified \( x \) to be the relevant momentum fraction we consider the hadronic collision in a frame where \( 3 \ P_a \cdot q_a = 1 \ x P_a \cdot p_a , \) \( N \cdot q_a = 1 \ x N \cdot p_a , \) \( (3.40) \)

such that the phase space convolution properties of both parametrizations become equivalent at hadron level,

\[
d\phi_2^F(q_i, q | Q; P_a, q_a, x_a) = d\phi_1^F(p_i | Q; P_a, p_a, x_a) \frac{d\phi}{2\pi} \frac{1}{16\pi^2} \frac{dz}{z(1 - z) - \kappa} dp_\perp^2 ,
\]  

(3.41)

We stress that the crucial difference is related to the fact that, considering the physical forward evolution, our parametrization does generate a finite transverse momentum for the parton entering the hard process after additional parton emission.

Averaging over azimuth, the initial-final splitting kernels take the form

\[
\frac{8\pi\alpha_s}{2q_a \cdot q x} \langle V(p_\perp^2, z) \rangle
\]  

(3.42)

such that the splitting probability is

\[
dP = \frac{\alpha_s}{2\pi} \langle V(p_\perp^2, z) \rangle \frac{f_a(x_a/x)}{f_a(x_a)} \theta(x - x_a) \frac{dp_\perp^2}{p_\perp^2} \frac{(1 - z)dz}{z(1 - z) - \kappa} .
\]  

(3.43)

Note that in the collinear limit, \( \kappa \to 0 \) we have \( x \to z \) such that the collinear behaviour is properly reproduced.\(^4\)

The phase space boundaries are given by the requirement that \( x_a < x, \)

\[
\mu^2 < p_\perp^2 < \frac{(1 - x_a)s_{aj}}{4} , \quad z_\pm = \frac{1}{2} \left( 1 + x_a \pm (1 - x_a) \sqrt{1 - \frac{4p_\perp^2}{(1 - x_a)s_{aj}}} \right) .
\]  

(3.44)

### 3.2.2 Initial State Spectator

Initial state radiation with initial state spectator, \( p_a, p_b \to q_a, q, q_b \) is described by the parametrization

\[
q_a = \frac{1}{v + x} p_a + \frac{1 - v - x}{v + x} p_b + \frac{1}{v + x} k_\perp
\]  

(3.45)

\[
q = \frac{1 - v - x}{v + x} p_a + \frac{1}{v + x} p_b + \frac{1}{v + x} k_\perp
\]  

(3.46)

\[
q_b = \left( 1 + \frac{v}{x} \right) p_b ,
\]  

(3.47)

\(^3\)Note that there is no a priori relation between incoming hadron and parton momenta in our formulation.

\(^4\)For readability we have suppressed indexing a possible flavour change of the incoming parton.
Figure 2: Available phase space for a final-initial dipole with invariant momentum transfer $\sqrt{s_{aj}} = \sqrt{-t} = 100$ GeV and an infrared cutoff of 5 GeV. The shaded region is accessible starting at the hard scale, the region enclosed by the solid line is an example of the phase space excluded when starting at a lower scale. The phase space regions for an initial-final dipole are identical. For a final-initial dipole, the variables are $x_p = x$, $z_p = z$, for the initial-final one $x_p = x$, $z_p = 1 - u$. Note that in the latter case $u \to 1$ and $u \to 0$ correspond to a collinear limit.

preserving $q - q_a - q_b = -p_a - p_b$. The transverse momentum is defined to be purely spacelike in the dipole’s rest frame and obeys

$$k_{\perp}^2 = -(1 - v - x)\frac{v}{x}s_{ab}$$

$$s_{ab} = 2p_a \cdot p_b .$$

(3.48)

The variables $x$ and $v$ are those introduced in [59],

$$x = \frac{p_a \cdot p_a}{q_a \cdot q_b} , \quad v = \frac{q_a \cdot q}{q_a \cdot q_b} ,$$

(3.49)

and we define the physical variables to be given by

$$x = \frac{z(1 - z) - \kappa}{1 - z} , \quad v = \frac{\kappa}{1 - z} , \quad \kappa = \frac{p_{\perp}^2}{s_{ab}} .$$

(3.50)

Note that the Lorentz transformation

$$S_{\mu \nu} =$$

$$\delta_{\nu}^{\mu} + \frac{p_b \cdot p_a}{p_b \cdot q_a q_a \cdot p_a} q_{a,\nu}^{\mu} + \frac{p_b \cdot q_a}{p_b \cdot q_a q_a \cdot p_a} p_{a,\nu}^{\mu} - \frac{1}{q_a \cdot p_a} (q_{a,\mu}^{\nu} p_{\nu} + p_{a,\nu}^{\mu} q_{a,\nu})$$

(3.51)
does transform this parametrization to a parametrization where

\[
S_{q_a} = \frac{1}{x + v} p_a , \quad S_{q_b} = \frac{x + v}{x} p_b .
\]

(3.52)

Following the arguments of the previous section we then find the phase space convolution

\[
\begin{align*}
\frac{\alpha_s}{2q_a \cdot q \ x} \langle V(p^2_{\perp}, z) \rangle 
\end{align*}
\]

(3.54)

such that the splitting probability is

\[
\frac{4\pi}{p^2_{\perp} (1 - z) - \kappa} dp^2_{\perp} .
\]

(3.55)

with

\[
F_{ab} = \frac{f_a(x_a/(x + v))}{f_a(x_a)} \theta(x + v - x_a) \frac{f_b(x_b(x + v)/x)}{f_b(x_b)} \theta \left( \frac{x}{x + v} - x_b \right)
\]

(3.56)

the ratio of incoming parton flux. 5 Note that in the collinear limit, \( v, \kappa \to 0 \) and \( x \to z \) such that we find the correct collinear behaviour.

We remark that it would be possible to keep the spectator unchanged upon properly substituting the integrations over the incoming momentum fractions (the Jacobian being equal to one). This, however, would invalidate the fact that the above given parametrization of the splitting kinematics does preserve energy-momentum locally involving the emitter-emission-spectator system only (in fact, after applying the relevant Lorentz transformation \( S \) this would constitute the inversion of the kinematics as used in the dipole subtraction context). A further argument to not keeping the spectator unchanged is that, following the discussion on soft and collinear factorization, we see no reason why the emission off the \textit{colour connected} system \( p_a, p_b \) should leave \( p_b \) unchanged except for a strictly soft and/or collinear emission.

The phase space limits are now determined from \( x > x_a x_b = \tau \) to be given by

\[
z_{\pm} = \frac{1}{2} \left( 1 + \tau \pm (1 - \tau) \sqrt{1 - \frac{4 p^2_{\perp}}{(1 - \tau)^2 s_{ab}}} \right) , \quad p^2_{\perp} < \frac{(1 - \tau)^2 s_{ab}}{4} .
\]

(3.57)

5As for the final state spectator, we have suppressed indexing a possible flavour change of the incoming parton.
Figure 3: Available phase space for emissions off an initial-initial dipole of mass 100 GeV with \( \tau = 0.02 \) and infrared cutoff 5 GeV. The shaded region is the available phase space when starting from the hard scale, the region enclosed by the solid line is an example of the phase space excluded when starting at a lower scale.

4. Conclusions

We have argued that parton showers, based on Catani–Seymour dipoles have a number of properties that turn out to be useful when one considers the matching with NLO matrix elements. Several other groups \([57, 58]\) have already written a parton shower program, following this motivation. In these approaches, however, the choices of phase space boundaries and evolution variable were made rather intuitively. The question for coherence properties, in particular, was left open.

Taking this as a starting point we have investigated the soft coherence properties of a CS parton shower formalism. We are lead to the transverse momentum of each dipole as the evolution variable of our cascade. Furthermore, we chose a hard scale that allows us to access the whole kinematically allowed phase space. We then explicitly show that such a parton shower can reproduce the expected Sudakov anomalous dimensions and hence include soft coherence effects properly.

We have specified all details of such a parton shower that will be important for its implementation. In particular, we addressed the issue of transverse momentum from initial state radiation that will build up from several emissions in our case. An implementation of such a parton shower is underway.
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A. Treatment of Collinear Factorization

In view of our probabilistic treatment of parton showers [50], we choose to rephrase collinear factorization in a way that fits to the picture of assigning the usual phase space measure

$$d\phi_1(p) = \frac{d^3p}{(2\pi)^32p^0}$$  \hspace{1cm} (A.1)

to each parton in the evolution, and not only final state partons. In turn, this allows us to derive phase space convolution properties using the general phase space factorization inherent to the parametrization of final state splittings with final state spectator.

We therefore rewrite

$$\int_0^1 dx_a = \int dx_a \int \frac{d^3p_a}{(2\pi)^32p^0_a}\delta_F(P_a, p_a, x_a)$$  \hspace{1cm} (A.2)

where

$$\delta_F(P_a, p_a, x_a) = \frac{16\pi^2}{2P_a \cdot P_b} \delta\left(\frac{P_a \cdot p_a}{P_a \cdot P_b}\right) \delta\left(\frac{P_b \cdot p_a}{P_a \cdot P_b} - x_a\right) \theta(x_a(1 - x_a)) .$$  \hspace{1cm} (A.3)

Here, $P_a$ denotes the lightlike momentum of the incoming hadron and the lightlike vector $P_b$ defines the collinear direction, i.e. is taken to be the momentum of the second incoming particle. We note that this extends straightforwardly to dimensional regularization.

The phase space measure with an incoming parton then takes the form

$$d\phi_n^F(p_1, ..., p_n|Q; P_a, p_a, x_a) = d\phi_{n+1}(p_1, ..., p_n, -p_a|Q)\delta_F(P_a, p_a, x_a)$$  \hspace{1cm} (A.4)

in terms of the general phase space measure

$$d\phi_{n+1}(q_1, ..., q_n|Q) = d\phi_1(q_1) \cdots d\phi_1(q_n)(2\pi)^4\delta(q_1 + \cdots + q_n - Q) .$$  \hspace{1cm} (A.5)

The phase space convolutions as given in [59] take the form

$$\int_0^1 dx_a f(x_a) \int_0^1 dz \ d\phi_n(p_1, ..., p_n|Q + zq_a) q_a \cdot p_i J(z) \delta(z - x) dx ,$$  \hspace{1cm} (A.6)
where \( q_a = (1/z)p_a = x_a P_a \) is constrained by collinear factorization. It is then easy to prove that the same convolution within our framework reads

\[
\int \! dx_a f(x_a) \int_0^1 \! \frac{dz}{z} p_a \cdot p_i J(z) \delta(z - x) d\phi_n^F(p_1, \ldots, p_n|Q; P_a, p_a, x_a) dx \quad \text{(A.7)}
\]

and gives rise to the factorization

\[
\int \! dx_a f(x_a) \left( \frac{f(x_a/x)}{f(x_a)} p_a \cdot p_i J(x) \theta(x - x_a) \frac{dx}{x^2} \right) d\phi_n^F(p_1, \ldots, p_n|Q; P_a, p_a, x_a). \quad \text{(A.8)}
\]

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