Transition to Quantum Turbulence Generated by an Oscillating Object in Superfluid $^4$He

H Yano, Y Nago, R Goto, K Obara, O Ishikawa and T Hata
Graduate School of Science, Osaka City University, Osaka 558-8585, Japan
E-mail: hideo@sci.osaka-cu.ac.jp

Abstract. The transition to turbulence in boundary flow of superfluid $^4$He is investigated using a vibrating wire. An object in a superfluid is usually accompanied by remanent vortices, enabling to cause turbulence by its oscillation even at very low temperature. However, remanent vortices make it difficult to investigate the transition to turbulence. Recently, we have successfully devised a condition of a vortex-free superfluid, in which an oscillating object cannot generate turbulence. Using a “vortex-free” vibrating wire and a generator of vortex rings, we find that vortex rings can trigger the transition to turbulence for the vortex-free vibrating wire. This result indicates that vortex rings attach to the wire, being stretched and forming turbulence by its oscillation. The propagation time of trigger vortex rings, from their generation to a transition to turbulence for the vortex-free vibrating wire, is the order of 10 ms, suggesting the lower limit of vortex rings in size which enable to trigger the turbulent transition.

1. Introduction
Quantum turbulence generated by an oscillating object in superfluid helium has recently attracted research interest in response to new investigations at very low temperatures (see Ref. [1] and references therein). In experiments, an oscillating object is useful for generating turbulence in superfluid $^4$He at low temperatures [2, 3, 4, 5], since vortices nucleated through the superfluid transition remain firmly attached to boundaries in a superfluid. However, the presence of remanent vortices prevents one from obtaining information about vortex-free state in a superfluid. In a recent study [6], we reported that slow filling by superfluid and using a filter of vortices can prevent vortices from attaching to boundaries, which prevents the generation of turbulence by oscillation, even above an oscillation velocity of 1 m/s. Using this method, we found that vortex rings can trigger the transition to turbulence for a “vortex-free” vibrating wire [7]. In this paper, we report the turbulent transition triggered by vortex rings and discuss the size of trigger vortex rings.

2. Experimental setup
The experimental setup consisted of two vibrating wires, similar to those described in a previous paper [7]. Superconducting wires (NbTi) with a 3 $\mu$m diameter were formed into semicircles. The wires labelled A and B in Fig. 1(a) were located in a small chamber with a 0.1-mm-diameter pinhole. Wire A was oriented so that it pointed towards the centre of wire B, perpendicular to its semicircle face. The distance between the arms of a wire was 1.6 mm for wire A and 1.8 mm for wire B. The distance between the apexes of the wire loops was 1.8 mm. The wires and the chamber were located in a cell in a magnetic field of 25 mT. The Lorentz force oscillates a wire carrying electric current of resonance frequency. The resonance frequencies of wire A and B are 1590 Hz and 1030 Hz in vacuum, respectively. Peak velocity
Figure 1. (a) Schematic drawing of vibrating wires in a chamber with a pinhole. Wire A was used as the detector, and wire B as the generator (see text). (b) Responses of the detector before (●) and after (×) injecting vortex rings. The wire vibration suddenly dropped from a high velocity $v_a$ to a low value $v_b$ after injection of vortex rings at a driving force of 1 nN. The arrows indicate sweep direction.

We filled superfluid $^4$He cooled below 100 mK into the cell over 48 h. After filling, we observed that wire A was unable to generate turbulence and wire B was able to generate turbulence. In a turbulent state, an oscillating object emits free vortex rings [1, 7]. Therefore, we used wire A as a detector of the transition to turbulence and wire B as a generator of vortex rings.

3. Results and discussion

Experiments were performed as following steps: (1) the driving force of the detector was increased to a velocity $v_a$ shown in Fig. 1(b); (2) during keeping driving force, vortex rings were injected from the generator to the detector. After a while, the velocity of the detector suddenly dropped to a low value $v_b$ [7]. This result indicates that vortex rings propagate from the generator to the detector through surrounding superfluid, colliding to the detector and interfering with its oscillation. Even after stopping the vortex-ring generation, the detector still remained in the low velocity $v_b$, indicating that the detector entered a turbulent state. As the driving force was reduced, the velocity of the detector remained low until the detector entered a potential flow state, as shown in Fig. 1(b). While the generator was not vibrating, we further increased the driving force of the detector; however, no transition was observed in the responses of the detector. Though the detector created many vortices in a turbulent state, those vortices were unable to affect the condition of the detector. Thus, one can control the transition to turbulence in superfluid helium, using a vortex-free vibrating wire.

A time difference between the turbulent transition for the generator and that for the detector reflects the motion of vortex rings triggering turbulence. While maintaining the vibration of the detector at a certain driving force, we applied a driving force of 0.54 nN to the generator so that its velocity increased to 0.9 m/s. Turbulence then occurred, as shown in Fig. 2(a). Shortly afterwards, the detector transition to turbulence also started, as shown in Fig. 2(b). The time origin in Figs. 2 (a) and (b) is shifted to the start of the transition for the generator. The time difference $\Delta t$ is determined by the time at the start of the decrease of the wire velocity, indicated by an arrow in Fig. 2(b). At a velocity of 137...
Figure 2. Time series of peak velocities of (a) the generator and (b) the detector at the transition to turbulence. The time is scaled at the start of the transition for the generator and the arrow in Fig. (b) indicates the start of the transition for the detector.

mm/s, $\Delta t$ is estimated to be 16 ms. The generation of turbulence by the generator is expected to be started by remanent vortices forming bridges [1, 6], spreading through the generator wire. Free vortex rings are considered to nucleate from the beginning of the decrease of generator velocity, propagating to the detector and disturbing its vibration. Therefore, the time difference estimated here should equal the propagation time of vortex rings plus the time from when the vortices attach to when they start expanding. Since a vortex attaching to the wire may start to expand within a period of vibration of 0.6 ms, the estimated time difference largely reflects the propagation time of the vortex ring that can trigger turbulence in the boundary flow of the detector.

Since the spatial configuration between the generator and the detector is not simple as shown in Fig. 1(a), we cannot estimate the accurate distance of flight of vortex rings, from which they are generated and to which they collide. We assume that this distance is equal to the distance 1.8 mm between the apexes of the two wires instead. The velocity of trigger vortex rings is estimated to be 110 mm/s from the time of flight 16 ms measured at a detector velocity of 137 mm/s. This result indicates that the velocity of the trigger vortex rings is almost equal to the detector velocity. A vortex ring propagates through a superfluid at a self induced velocity $v$ given by [8]

$$v = \frac{\kappa}{2\pi d} \left( \frac{4d}{\xi} - \frac{1}{4} \right), \quad (1)$$

where $d$, $\kappa$, and $\xi$ are the diameter of the vortex ring, the quantum of circulation, and the radius of the vortex core, respectively. The velocity 110 mm/s of a vortex ring corresponds to ring size of 1.5 $\mu$m. Smaller vortex rings propagate faster; however, the experimental results suggest that these small vortices are unable to cause turbulence.

In a previous paper [7], we reported the motions of vortex rings against an oscillating object by numerical simulations. A vortex ring collides with the oscillator, splitting up into several segments. If no pinning sites on the surface of the oscillator, a vortex loop attached on the oscillator can move at a self induced velocity, which depends on the size of a vortex loop (see Eq. (1)). Small segments may move at a velocity higher than the velocity of the oscillator, splitting into smaller segments and eventually disappearing. In contrast, large segments, moving slowly compared with the oscillation velocity, are stretched and form tangle, because only their attached ends can move with the oscillator motion and the other parts move rather independently with a self induced velocity. If vortex rings are applied continuously, the vortex rings collide with the oscillator or the vortex tangle, eventually forming turbulence in the path of the oscillator. After vortex rings are applied in sufficient quantities, the oscillator can keep a turbulent state, with no supply of further vortices.
These simulations suggest the lower limit of trigger vortex rings in size. Smaller vortex rings can propagate faster but cannot cause turbulence for an oscillator if their velocities are higher than the oscillator velocity. The time of flight of trigger vortex rings is informative for understanding the transition triggered by vortex rings; however, the generation process of vortex rings for the generator such as their sizes and positions is not clear. Assuming that various sizes of vortex rings are created from the beginning of the generation and the distance of flight is 1.8 mm, we estimated the velocity of trigger vortex rings observed here to be 110 mm/s. The velocity 110 mm/s of the trigger vortex rings is almost equal to the peak velocity 137 mm/s of the vibrating wire. This result implies that vortex rings larger than those at the same velocity as an oscillator can trigger the transition to turbulence, which is consistent with the results of the numerical simulations described before.

4. Conclusions
We report the transition to turbulence in superfluid $^4$He using a vortex-free vibrating wire. Slow filling and a filter prevent vortices from attaching to the wire in a superfluid. Using the vortex-free vibrating wire and a generator of vortex rings, we found that vortex rings can trigger the transition to turbulence for the vortex-free vibrating wire. The time of flight of trigger vortex rings suggests the lower limit of vortex rings in size. The time-of-flight experiments using a vortex-free vibrating wire are useful for investigating the trigger vortex rings.

Acknowledgments
The authors are very grateful to M. Tsubota, M. Kobayashi, S. Fujiyama, and R. Hänninen for theoretical discussions, and V. B. Eltsov, and M. Krusius for stimulating discussions. The research was supported by a Grant-in-Aid for Scientific Research on Priority Areas (Grant No. 17071008) from The Ministry of Education, Culture, Sports, Science and Technology of Japan.

References
[1] Hänninen R, Tsubota M and Vinen W F 2007 Phys. Rev. B 75 064502
[2] Jäger J, Schuderer B and Schoepe W 1995 Phys. Rev. Lett. 74 566
[3] Yano H, Handa A, Nakagawa H, Obara K, Ishikawa O, Hata T and Nakagawa M 2005 J. Low Temp. Phys. 138 561
[4] Charalambous D, Skrbek L, Hendry P C, McClintock P V E and Vinen W F 2006 Phys. Rev. E 74 036307
[5] Yano H, Hashimoto N, Handa A, Nakagawa M, Obara K, Ishikawa O and Hata T 2007 Phys. Rev. B 75 012502
[6] Hashimoto N, Goto R, Yano H, Obara K, Ishikawa O and Hata T 2007 Phys. Rev. B 76 020504
[7] Goto R, Fujiyama S, Yano H, Nago Y, Hashimoto N, Obara K, Ishikawa O, Tsubota M and Hata T 2008 Phys. Rev. Lett 100 045301
[8] Donnelly R J 1991 Quantized Vortices in Helium II (Cambridge: Cambridge University Press)