The Randall–Sundrum two-brane model admits the flat-brane Lorentz-invariant vacuum solution only if the branes have exactly opposite tensions. We pay attention to this condition and propose a generalization of this model in which two branes are described by actions of the same form and with the same matter content but with opposite signs. In this way, the relation between their tensions (which are their vacuum energy densities) is naturally accounted for. We study a simple example of such a model in detail. It represents the Randall–Sundrum model supplemented by the Einstein scalar-curvature terms in the actions for the branes. We show that this model is tachyon-free for sufficiently large negative values of the brane cosmological constant, that gravitational forces on the branes are of opposite signs, and that physically most reasonable model of this type is the one where the five-dimensional gravity is localized around the visible brane. The massive gravitational modes in this model have ghost-like character, and we discuss the significance of this fact for the quantum instability of the vacuum on the visible brane.

Keywords: braneworld model.

PACS Nos.: 04.50.+h

1. Introduction: the model

The original Randall–Sundrum (RS) two-brane model\textsuperscript{[1]} describes vacuum branes embedded in (or, equivalently, bounding) the five-dimensional AdS spacetime and has the action

\[ S_{\text{RS}} = M^3 \int_{\text{bulk}} (\mathcal{R} - 2\Lambda_b) - \int_{\text{brane1}} 2\sigma + \int_{\text{brane2}} 2\sigma, \tag{1} \]

in which we omit the obvious integration volume elements and the boundary terms with brane extrinsic curvature. Here, \( M \) denotes the bulk Planck mass, \( \Lambda_b \) is the bulk cosmological constant, and the constant \( \sigma \) is the vacuum energy density (brane
of one of the branes (brane1), which is exactly opposite to the brane tension of the other brane (brane2). The model admits the flat-brane Lorentz-invariant vacuum solution only under the constraint

$$\Lambda_{RS} = \frac{\Lambda_b}{2} + \frac{\sigma^2}{3M^6} = 0,$$

which has to be postulated in the theory.\(^a\)

The fact that two brane tensions are exactly equal by absolute value and opposite in sign is also of crucial importance for the existence of flat-brane solutions and requires explanation. In this paper, we generalize the Randall–Sundrum model (1) in such a way that two branes are described by the same form of action with the same matter content but with the opposite signs. In this way, the symmetry relation between their vacuum energy densities is naturally accounted for. Thus, we propose to consider the generic action of the form

$$S = \int_{\text{bulk}} \mathcal{L}_{\text{bulk}}(g_{ab}, \Phi) + \int_{\text{brane1}} \mathcal{L}(h_{ab}, \phi_1) - \int_{\text{brane2}} \mathcal{L}(h_{ab}, \phi_2),$$

where \(\Phi\) denotes the fields in the bulk other than the metric field \(g_{ab}\). It is important to emphasize that the Lagrangians \(\mathcal{L}\) have the same form for two branes, with the field contents \(\{\phi_1\}\) and \(\{\phi_2\}\) being equivalent. Therefore, their vacuum energy densities on the same metric background will be exactly the same.

Were it not for the bulk part in action (3), we would have totally decoupled actions for two non-interacting worlds. The relative signs between the actions in this case would not matter at all. The presence of the bulk means a complicated interaction between the branes and also breaks the equivalence symmetry between them since the physical constants in the bulk Lagrangian are of specific signs.

The Lagrangian \(\mathcal{L}\) in (3) is assumed to be written in the “standard” form. In particular, the kinetic terms of the physical matter fields enter it with the conventional signs. The brane tension \(\sigma\) is then unambiguously defined as the vacuum energy density of the “standard” Lagrangian \(\mathcal{L}\), i.e., on the background of the flat metric \(h_{ab} = \eta_{ab}\),

$$\langle \mathcal{L}(\eta_{ab}, \phi) \rangle_{\text{vac}} = -2\sigma.$$

A particular case of (3) that we are going to consider in this paper is the generalization of the RS model (1) by adding the matter Lagrangians and the Einstein scalar-curvature terms on the branes. In this case, we have the action (again omitting the boundary terms with brane extrinsic curvature)

$$S = M^3 \int_{\text{bulk}} (R - 2\Lambda_b) + \int_{\text{brane1}} [\zeta R - 2\sigma + L(h_{ab}, \phi_1)] - \int_{\text{brane2}} [\zeta R - 2\sigma + L(h_{ab}, \phi_2)],$$

\(^a\)Flat-brane vacuum solutions without Lorentz invariance of the bulk can be constructed without this constraint.\(^3\)
where we have explicitly introduced the vacuum energy density (brane tension) $\sigma$ so that $\langle L(\eta_{ab}, \phi) \rangle_{\text{vac}} = 0$. The sign of the bulk Planck mass $M$ can be chosen arbitrarily by suitably choosing the overall sign in action (5). It is natural to choose $M$ to be positive. Then the brane which appears with positive sign in action (5) (which is brane1 in our case) can be called positive brane, and the one appearing with negative sign can be called negative brane. The scalar-curvature terms for the two branes enter the total action (5) with the same constant $\zeta$ but with opposite signs, according to our general principle expressed by action (3). This can also be quite naturally explained if these terms are regarded as induced by the quantum corrections from the matter Lagrangians $L(h_{ab}, \phi)$. Since the matter actions on the branes have opposite signs but equivalent matter content, the effective gravitational actions that they induce on the respective branes are exactly of the same form but of opposite signs.

Braneworld models with arbitrary relative signs of the brane gravitational actions were recently under consideration in Refs. 4, 5, 6. In this paper, we propose to consider perfect “mirror” symmetry between the actions for the branes, where both the gravitational and matter actions are exactly of the same form but of opposite sign. We repeat, however, that symmetry between the branes is, in general, broken by the presence of the bulk.

We allow for positive as well as negative gravitational coupling $\zeta$ in (5) (its sign is defined with respect to the “standard” matter Lagrangian $L$). The brane tension $\sigma$ a priori can also be of any sign. First of all, we show that the brane cosmological constant $\lambda = \sigma/\zeta$ in the model (5) under consideration must be negative and sufficiently large by absolute value in order that tachyonic gravitational modes be absent in the theory. This will be the issue of the next section, in which we also consider the ghost-like character of the massless radion and massive gravitational modes. In Sec. 3 we study the linearized gravity on the flat two-brane background. We will see that the gravitational forces exerted by matter have opposite signs on the two branes. This selects the brane with attractive gravity as the “visible” one, which turns out to be the negative brane (brane2) in the terminology introduced above. We will also see that it is physically preferable for the model to have positive gravitational constant $\zeta$, hence, negative tension $\sigma$. Thus, the five-dimensional gravity is localized around the physical brane, which makes it the “Planck” brane in the conventional terminology. In Sec. 4 we discuss the quantum instability of the vacuum caused by the ghost-like character of matter on the hidden brane and of the massive gravitational modes. To resolve this problem, one can introduce an explicit Lorentz-violating ultraviolet cutoff as discussed in Refs. 7, 8. In Sec. 5 we present approximate expressions for the gravitational potential on the visible brane induced by a static source on the visible or hidden brane. In Sec. 6 we provide a general discussion of the model.
2. Tachyonic modes and ghosts

Tachyonic modes were studied in our previous paper\textsuperscript{9} in a model quite similar to \textsuperscript{5}, namely, where the matter and induced-gravity actions for two branes were not necessarily the same (the constants $\zeta$ and the Lagrangians $L$ could be different) but entered the total action with the same sign.\textsuperscript{b} The equations for the tachyonic modes in the present case can be derived in exactly the same way as it was done in Ref. \textsuperscript{9} (see also Ref. \textsuperscript{4}).

The background RS solution is described by the metric

$$ds^2 = dy^2 + a^2(y)\eta_{\alpha\beta}dx^\alpha dx^\beta, \quad a(y) = \exp(-ky), \quad k = |\sigma|3M^3_6 = \sqrt{-\frac{\Lambda_b}{6}} ,$$

with the two branes located at $y = 0$ and $y = \rho > 0$, respectively. Thus, throughout the paper, we have $k > 0$ by definition so that the left brane (located at $y = 0$) is the “Planck” brane (the one at which the massless graviton is localized), while the right brane (located at $y = \rho$) is the “TeV” brane in the commonly used terminology. Then, the “Planck” brane is positive (negative) if the brane tension $\sigma$ is positive (negative). Proceeding to the Fourier analysis in terms of the Lorentzian momenta $p^\alpha$ and being interested in the tachyonic case ($p^\alpha p_\alpha > 0$ for our signature convention), we introduce the dimensionless variables

$$\mu = -\frac{\zeta \sigma}{3M^6}, \quad \alpha = e^{k\rho}, \quad s = \frac{\sqrt{p^\alpha p_\alpha}}{k},$$

and arrive at the following equation for the tachyonic modes (see Ref. \textsuperscript{9}):

$$E(s) \equiv D_1(s) - \mu s\left[D(s) - \alpha D_*(s)\right] - \alpha (\mu s)^2 D_2(s) = 0 ,$$

where

$$\begin{align*}
D_1(s) &= I_1(\alpha s)K_1(s) - I_1(s)K_1(\alpha s), \\
D(s) &= I_2(s)K_1(\alpha s) + I_1(\alpha s)K_2(s), \\
D_*(s) &= I_1(s)K_2(\alpha s) + I_2(\alpha s)K_1(s), \\
D_2(s) &= I_2(\alpha s)K_2(s) - I_2(s)K_2(\alpha s).
\end{align*}$$

Since $\alpha > 1$ for $k > 0$, all four functions defined in (9) are strictly positive for positive $s$. Then Eq. \textsuperscript{8} implies that tachyonic modes are absent in the case $\mu = 0$ (which includes the RS model). Thus, we need to consider the case $\mu \neq 0$. The asymptotic limits of the function $E(s)$ in this case can easily be calculated:

$$\lim_{s\to 0} E(s) = (1 - 2\mu) \sinh \rho, \quad \lim_{s\to \infty} E(s) = -\infty , \quad \mu \neq 0 .$$

This immediately implies the existence of tachyonic modes for nonzero $\mu < 1/2$.

\textsuperscript{b}The peculiar relation between the brane tensions was assumed to be the same as in the Randall–Sundrum model, since it is required to have the flat-brane solutions.
Thus, for \( \mu \neq 0 \), one can expect tachyonic modes to be absent only if \( \mu \geq 1/2 \).

We show that this condition is also sufficient. Employing the method used in Ref. 9, we introduce the function

\[
\bar{E}(s, t) = D_1(s) - t \left[ D(s) - \alpha D_*(s) \right] - \alpha t^2 D_2(s), \quad s, t > 0, \tag{11}
\]

so that \( \bar{E}(s, \mu s) \equiv E(s) \) for positive \( \mu \), which is now under consideration. Then, solving the equation

\[
\bar{E}(s, t) = 0 \tag{12}
\]

with respect to \( t \), we obtain the expression for the single positive root:

\[
\bar{t}(s) = \sqrt{\left[ \frac{D(s) - \alpha D_*(s)}{D_2(s)} \right]^2 + \frac{4\alpha D_1(s)D_2(s) - \left[ D(s) - \alpha D_*(s) \right]^2}{2\alpha D_2(s)}}. \tag{13}
\]

It can be verified that this one-parameter function is convex upwards for all values of the parameter \( \alpha > 1 \), and its asymptotic behavior for small and large \( s \) is given by

\[
\bar{t}(s) = \frac{s^2}{2} + o(s), \quad s \to 0; \quad \lim_{s \to \infty} \bar{t}(s) = 1. \tag{14}
\]

Solving Eq. (12) is equivalent to solving the equation

\[
\bar{t}(s) = \mu s, \tag{15}
\]

which, in view of (14), has a positive root for \( s \) if and only if \( 0 < \mu < 1/2 \).

From the asymptotic limits (14) and from Eq. (15) one can see what happens to the tachyon mode as the value of \( \mu \) approaches the boundaries of the interval \((0, \frac{1}{2})\). In the limit \( \mu \to 0 \), the mass of the tachyon goes to infinity and, in the limit \( \mu \to 1/2 \), it goes to zero. We see that the value \( \mu = 0 \) is an isolated point of the theory, and one does not have a physically consistent model in its vicinity.

Thus, model (5) has no tachyonic gravitational modes either if \( \mu = 0 \) (which is an isolated point) or if \( \mu \geq 1/2 \). However, as will be shown shortly, the theory is singular at \( \mu = 1/2 \); hence, this value should also be excluded. The physical condition \( \mu > 1/2 \) can be written as

\[
\zeta \sigma < -\frac{3}{2} M^6. \tag{16}
\]

This constraint implies sufficiently large negative value of the brane cosmological constant

\[
\lambda \equiv \frac{\sigma}{\zeta} < -\frac{3M^6}{2\zeta^2}. \tag{17}
\]

Below, we restrict our investigation to this case.

The issue of ghosts in the gravitational sector of the complementary theory with positive value of \( M \) but arbitrary signs of the brane gravitational constants was considered in Ref. 5, and we apply the results obtained therein to our case.
First, we start with the radion. The radion degree of freedom in our formalism is connected with the possibility of brane bending in the bulk. After identifying the relevant physical degrees of freedom, one can obtain the effective action for the radion $\phi_r$ in our model using the results of Ref. [5]:

$$S_{\text{rad}} = \frac{3M_3}{k(1-2\mu)} \int \phi_r \Box \phi_r \, dx.$$  \hspace{1cm} (18)

One can see that the radion effective action is singular in the case $\mu = 1/2$, which is one of the reasons why this value should be excluded, and, under the condition of absence of tachyonic modes $\mu > 1/2$, the radion field is a ghost from the viewpoint of the positive brane (brane1) and is not a ghost from the viewpoint of the negative brane (brane2).

Now, following Refs. [5, 9], one can show that the massive gravitational modes in the theory under investigation have ghost-like nature. For free metric perturbations described by the transverse traceless modes $\gamma_{\alpha\beta}(x, y)$ in the form

$$h_{\alpha\beta}(x, y) = e^{-2ky} \left[ \eta_{\alpha\beta} + \gamma_{\alpha\beta}(x, y) \right],$$

expanding the perturbation in the physical Fourier modes $\psi(m, y)$ such that $p_{\alpha}p^\alpha = -m^2$ with the corresponding boundary conditions,

$$\gamma_{\alpha\beta}(x, y) = \sum_m \chi_{\alpha\beta}(m, x) \psi(m, y),$$

one arrives at the following quadratic effective action (cf. with Refs. [5, 9]):

$$S = \frac{1}{2} \sum_m C_m \int dx \chi^{\alpha\beta}(m, x) \left( \Box - m^2 \right) \chi_{\alpha\beta}(m, x).$$

(21)

For the massless mode ($m = 0$), we have $\psi(0, y) \equiv \text{const},$ and the constant $C_0$ is given by

$$C_0 = \frac{M_3^3(1-2\mu)}{2k} \left[ (1-e^{-2k\rho}) \right] [\psi(0, 0)]^2,$$

(22)

The case $\mu = 1/2$ was already excluded when considering the radion, and now we see that it can also be excluded by the requirement of the nonzero norm of the zero-mode graviton. Then, the norm of the zero-mode graviton is negative in the region $\mu > 1/2$, where tachyonic modes are absent in the theory. Thus, just as the massless radion, the massless graviton is a ghost from the viewpoint of the positive brane, and is not a ghost from the viewpoint of the negative brane. For the massive modes ($m \neq 0$), one obtains the expression

$$C_m = \frac{M_3^3}{m^2} \int_0^{\rho} dy e^{-4ky}[\psi'(m, y)]^2, \quad m \neq 0,$$

(23)

The gravitational part of model [5] with the additional relation $\mu = 1/2$ was recently under consideration in Ref. [6]. It was observed that the braneworld theory becomes degenerate in this case, which was also previously noted in Ref. [2].

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\textsuperscript{c} The gravitational part of model [5] with the additional relation $\mu = 1/2$ was recently under consideration in Ref. [6]. It was observed that the braneworld theory becomes degenerate in this case, which was also previously noted in Ref. [2].
which means that the massive modes are ghosts from the viewpoint of the negative brane. It is worth noting that matter with ghost-like properties is currently being employed in the literature, although in a somewhat different context.\(^{10}\)

3. Linearized gravity in the zero-mode approximation

The zero-mode approximation developed in Ref.\(^9\) can be directly applied to the theory given by (5). Depending on the signs of the gravitational constant \(\zeta\) and brane tension \(\sigma\) [which should be opposite, as follows from (16) or (17)], the components of the Einstein tensor in the zero-mode approximation on the positive (\(G^+_{\alpha\beta}\)) and negative (\(G^-_{\alpha\beta}\)) brane in the coordinates \(x^a\) are given by the following expressions:

\[
G^\pm_{\alpha\beta} = 8\pi G_N \left[ T^-_{\alpha\beta} - \frac{1}{3} e^{\pm 2k\rho} \left( h^\pm_{\alpha\beta} - \frac{\partial_\alpha \partial_\beta}{\square^\pm} \right) T^- \right] \\
- 8\pi G_N e^{-2k\rho} \left[ T^+_{\alpha\beta} - \frac{1}{3} e^{\pm 2k\rho} \left( h^\pm_{\alpha\beta} - \frac{\partial_\alpha \partial_\beta}{\square^\pm} \right) T^+ \right] + \ldots ,
\]

\(\zeta > 0, \quad \sigma < 0,\)

\[
G^\pm_{\alpha\beta} = 8\pi G_N e^{-2k\rho} \left[ T^-_{\alpha\beta} - \frac{1}{3} e^{\mp 2k\rho} \left( h^\pm_{\alpha\beta} - \frac{\partial_\alpha \partial_\beta}{\square^\pm} \right) T^- \right] \\
- 8\pi G_N \left[ T^+_{\alpha\beta} - \frac{1}{3} e^{\mp 2k\rho} \left( h^\pm_{\alpha\beta} - \frac{\partial_\alpha \partial_\beta}{\square^\pm} \right) T^+ \right] + \ldots ,
\]

\(\zeta < 0, \quad \sigma > 0.\)

Here we used the notation

\[
8\pi G_N = \frac{2k}{M^3(2\mu - 1)(1 - e^{-2k\rho})};
\]

\(T^\pm_{\alpha\beta}, h^\pm_{\alpha\beta},\) and \(\square^\pm\) are the stress–energy tensors, flat induced metrics, and the corresponding D’Alembertians on the positive (“+”) and negative (“−”) branes, respectively, and \(T^\pm\) are the traces of the corresponding stress–energy tensors. The dots in (24) and (25) denote the higher-derivative terms. The D’Alembertians in the denominators of (24) and (25) reflect the presence of the radion degree of freedom in the theory (see Ref.\(^9\) for more details). The gravitational constant (26) is naturally inversely proportional to the norm of the zero-mode graviton (22).

It can be seen from (24) and (25) that matter on the negative brane gravitates attractively, while that on the positive brane gravitates repulsively (in other terms, the low-mass gravitons behave as ghosts with respect to matter on the positive brane). It can also be seen that the general-relativistic part of gravity is stronger by a factor of \(e^{2k\rho}\) on the brane around which the five-dimensional gravity is localized, i.e., on the “Planck” brane (it is positive or negative brane according to the sign of \(\sigma\)), in agreement with the original result by Randall and Sundrum.\(^{11}\) Thus, in all cases, it is physically reasonable to consider the negative brane as “visible” (describing the observable world), and the positive one as “hidden”.\(^{10}\)
In the case of positive constant $\zeta$ and negative $\sigma$, the effect of the hidden-brane matter is exponentially suppressed on the visible (negative) brane as a function of the distance between the branes. Hence, the case of negative $\sigma$ seems to be physically the most appealing.

4. Quantum instability of the vacuum caused by ghosts

The ghost-like character of the matter on the hidden brane and of the massive gravitons in the model under consideration is something to be worried about and requires further investigation. The ghost nature of hidden matter is connected with the fact that the matter Lagrangians enter action (3) with opposite signs. In this respect, we can note that the gravitational influence of the hidden matter in the model (5) is exponentially suppressed by the factor $e^{-2k\rho}$, which significantly alleviates this problem.

The problem of the ghost nature of the massive gravitons is more serious and can lead to quantum instability of the vacuum. To avoid too strong instability of the vacuum caused by the ghosts, one can consider introducing an explicit Lorentz-violating ultraviolet cutoff in the theory (see Ref. 7, 8 for a recent discussion).

As an example, in Fig. 1 we show a typical diagram of the process in which a pair of matter particles $\phi$ are spontaneously created from the vacuum on the visible ("Planck") brane together with a pair of massive gravitons $g$. The coupling of matter fields to massive gravitons in its vertices is determined by the effective action (21) with the constants $C_m$ given by (23). Thus, each vertex of the process contains the constant $\psi(m,0)/|C_m|^{1/2}$, where $\psi(m,y)$ is the wave function of the massive graviton in (20). Introducing also the momentum ultraviolet cutoff $\Lambda$ similarly to Ref. 8, we obtain the production rate for two graviton modes with absolute mass values $m_1$ and $m_2$ on dimensional grounds,

$$\Gamma_{m_1,m_2} \sim \left| \frac{\psi^2(m_1,0)\psi^2(m_2,0)}{C_{m_1}C_{m_2}} \right| \Lambda^8.$$  \hfill (27)

The constant in (27) can be estimated by using the asymptotic expansions of
the wave function in two mass regions as follows:

\[
\left| \frac{\psi^2(m,0)}{C_m} \right| \simeq \frac{1}{\mu^2 \alpha M^3} \times \begin{cases} \frac{k^3/m^2}{m}, & m \gtrsim k, \\ m, & m \lesssim k. \end{cases}
\]  

(28)

The interval between the modes is roughly equal to \( \Delta m \simeq \pi k/\alpha \) (see Ref. 9); thus, summing expression (27) over the massive graviton modes, we estimate the total probability of particle production as

\[
\Gamma \sim \frac{k^2 \Lambda^8}{\mu^4 M^6} \sim \frac{\Lambda^8}{(\zeta \mu)^2} \sim \frac{\Lambda^8}{M_P^4 \mu^2},
\]

(29)

where we have made the assumption that \( (\mu - 1/2) \sim \mu \), i.e., that the value of \( \mu \) is not too close to 1/2, and took into account that \( \zeta = M_P^2 \), where \( M_P \) is the effective Planck mass on the visible brane. Estimate (29) differs from that of Ref. 8 by the factor \( \mu^2 \) in the denominator.

The particles \( \phi \) spontaneously produced from the vacuum in the process in Fig. 1, in particular, be photons. Hence, by the same reasoning as in Ref. 8, we can obtain the upper limit on the cutoff \( \Lambda \). The spectral density of photons at \( E \sim \Lambda \) created in the process of Fig. 1 is roughly given by the expression

\[
\frac{d\mathcal{N}}{dE} \sim \frac{\Lambda^7}{M_P^4 \mu^2 t_0}, \quad E \lesssim \Lambda,
\]

(30)

where \( t_0 \sim \frac{1}{H_0} \) is the age of the universe. On the other hand, the spectrum of photons is constrained by observations of the diffuse gamma ray background by EGRET, which has measured the differential photon flux to be

\[
\frac{dF}{dE} = 7.3 \times 10^{-9} \left( \frac{E}{E_0} \right)^{-2.1} \text{ cm}^2 \text{ sr MeV}^{-1},
\]

(31)

where \( E_0 = 451 \text{ MeV} \). Demanding that (30) does not exceed (31) at \( E \sim \Lambda \), we obtain the upper limit

\[
\Lambda \lesssim 3\mu^q \text{ MeV}, \quad q \equiv \frac{2}{0.1} \approx 0.22.
\]

(32)

This estimate differs from that of Ref. 8 by the presence of the factor \( \mu^q \). In principle, the value of \( \mu \) is unbounded from above; hence, the cutoff upper limit (32) can be made as large as possible.

Of course, these estimates have a preliminary character and should be supported by the detailed analysis of the model. Nevertheless, they give some hope that the ghost character of the invisible matter and massive gravitational modes may not be so serious in the model under consideration.

5. Gravitational potential

In this section, we present approximate expressions for the gravitational potential on the visible brane induced by a static source on the visible or hidden brane in the
physically interesting case where the zero-mode graviton is localized at the visible brane (positive $\zeta$ and negative $\sigma$), i.e., that the visible brane is the “Planck” brane. In doing this, we use the results of our previous paper [9] where the corresponding expressions are derived for a generic two-brane model.

5.1. Matter source on the visible brane

In this case, we obtain the following gravitational potential $V(r)$ induced by a static source of mass $M$ on the visible brane:

$$V(r) = -\frac{GM}{r}, \quad G = G_N \left(1 + \frac{1}{3\alpha^2}\right), \quad kr \gg \alpha, \quad (33)$$

$$V(r) = -\frac{GM}{r} \left(1 - \frac{2}{3(2\mu - 1)(kr)^2}\right), \quad G = G_N \left(1 - \frac{1}{\alpha^2}\right), \quad 1 \ll kr \ll \alpha, \quad (34)$$

where $G_N$ is defined in (26), and $\alpha$ in (7). Note the correspondence with the structure of (24).

For $kr \ll 1$, we have the expression

$$V(r) = -\frac{GM}{r} - \frac{kM}{3\pi^2\zeta} \left(\frac{15}{8} \frac{1}{\mu} \log \left[\frac{15}{8} - \frac{1}{\mu} \right] kr\right), \quad G = \frac{(\mu - 2/3)}{8\pi\zeta(\mu - 1/2)}, \quad (35)$$

where $\mu$ is defined in (7). As $\mu \to \infty$, we recover the Newton’s gravitational law with the standard four-dimensional gravitational coupling $G = 1/8\pi\zeta$. Note that the Newtonian part of potential (35) is repulsive for $\mu$ in the narrow range $1/2 < \mu < 2/3$. The logarithmic correction to this potential is only valid if the expression $15/8 - 1/\mu$ is not very small by absolute value. The gravitational law on small scales is four-dimensional $[V(r) \propto r^{-1}]$ because the allowed values of the brane gravitational constant $\zeta$ are sufficiently large, so that $\mu > 1/2$.

5.2. Matter source on the hidden brane

In a similar way one can consider the case where the stationary matter with mass $\mathcal{M}_*$ resides on the hidden brane. We obtain the following expressions for the gravitational potential on the visible brane:

$$V(r) \approx \frac{4G_N\mathcal{M}_*}{3\alpha^2r}, \quad kr \gg \alpha, \quad (36)$$

$$V(r) \approx \frac{2k^2\mathcal{M}_*}{3\pi^2M^3(2\mu - 1)\alpha^4} \left[\left(1 + \frac{c_1}{\mu} - \frac{c_2}{\mu^2}\right) - \left(c_3 + \frac{c_4}{\mu} - \frac{c_5}{\mu^2}\right) \left(\frac{kr}{\alpha}\right)^2\right], \quad (37)$$

where the constants $c_n$ take the following approximate values:

$$c_1 \approx 1.3, \quad c_2 \approx 0.35, \quad c_3 \approx 0.06, \quad c_4 \approx 1.12, \quad c_5 \approx 0.24. \quad (38)$$

Note that the approximate potential is repulsive and that it does not have Newtonian form for $kr \ll \alpha$.  


6. Discussion

The merit of generalization (3) proposed in this paper is that it naturally takes into account the fact that two branes in the RS setup (1) have exactly opposite tensions. We studied a particular version (5) of the generic model (3) and have shown that it is tachyon-free if the brane cosmological constant \( \lambda = \sigma/\zeta \) is negative and sufficiently large by absolute value, as given by Eq. (17). In this case, the effective gravity is attractive on the negative brane (brane2) and is repulsive on the positive brane (brane1). Thus, a physically viable situation obtains if the visible brane is taken to be the negative brane (brane2) in action (5). If the gravitational constant \( \zeta \) is positive (hence, brane tension \( \sigma \) is negative), then the five-dimensional gravity is localized around the visible brane, where the effects of the hidden brane are exponentially suppressed as functions of the distance \( \rho \) between the branes. The physical consequences of the repulsive gravity of matter of the hidden brane remain to be clarified.

Reversing the overall sign of action (5), one can see that the preferable physical option from the viewpoint of the visible brane (brane2) looks like that of negative bulk gravitational constant, which case was discussed in our previous paper. The negative sign of the bulk gravitational constant with respect to the visible brane is motivated by some braneworld models of dark energy, in particular, the braneworld model of disappearing dark energy. In this model, the expanding universe, after the current period of acceleration, re-enters the matter-dominated regime continuing indefinitely in the future. The model requires negative tension of the visible brane, and the value of the bulk gravitational constant must then be negative for the five-dimensional gravity to be localized around the brane. Negative brane tension is required also for the existence of unusual “quiescent” singularities in the AdS-embedded braneworld models, which occur during the universe expansion and are characterized by the finiteness of the scale factor, Hubble parameter, and matter density. Another instance of an interesting cosmological behavior in the case of negative brane tension is the recently discussed “cosmic mimicry” in braneworld models. The present model of “mirror branes” has also led us to the option of negative-tension visible brane around which the five-dimensional gravity is localized.

The ghost-like character of the matter on the hidden brane and of the massive gravitational modes in the model under consideration is something to be worried about. The estimates obtained in Sec. 4 show that the presence of ghosts may not be a very serious problem. This issue, however, requires further detailed investigation.

Other versions of the generic type (3) can be studied, among which models with curvature terms on the branes and with the additional Gauss–Bonnet term in the bulk, models without the RS constraint, etc. We mention them as a subject for future investigations.

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