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ABSTRACT
This study focuses on the effects of a large Stokes number (St) on the perturbation growth in linear and nonlinear stages of a Richtmyer–Meshkov instability (RMI) in a gas-particle system, which to the best of our knowledge has not been previously reported. A linear growth model is developed by linear stability analysis and numerically verified by the compressible multiphase particle-in-cell (CMP-PIC) method. Additionally, the RMI growth characteristics in the nonlinear stage are also investigated by CMP-PIC. For the linear growth model, two major differences characterize the effects of a large St. The first one is that an RMI with a large St, which performs significantly different from the RMI with a small St, is induced and driven only by the density difference of the gas-phase and totally independent of particle density. Second, due to the significant momentum coupling effects between gas and particle phases, which govern the gas-particle flow, the growth rate experiences exponential decay, even in the linear RMI stage. The decay behavior performs markedly different from any previous RMI models, especially those of the original single-phase RMI and the gas-particle RMI with a small St. Notably, in the nonlinear stage of the RMI with a large particle volume fraction, the decay effects are much more pronounced and lead to a fall in the growth rate to almost zero, which is not found in any other type of RMI. These findings offer the possibility to develop a new method to control the development of hydrodynamic instability.

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I. INTRODUCTION
Shock-driven multiphase flows, such as that in a gas-particle system, often occur in studies involving the Richtmyer–Meshkov instability (RMI) phenomenon, such as in supernova (SN) explosions, particle imaging velocimetry (PIV) measurements, and inertial confinement fusion (ICF), for which one of the critical issues is to suppress the hydrodynamic instability. A primary interest in the study of the RMI is the modeling of the amplitude growth rate of perturbation. In general, the volume fraction of the particle phase (αp) and the Stokes number (St) are two of the most important parameters for the gas-particle flow. The αp is used to determine the pattern of the gas-particle flow. The flow can be classified into three patterns: a dilute gas-particle flow (0 < αp < 0.01), a dense gas-particle flow (0.01 < αp < αp,packed), and a granular flow (αp > αp,packed), as proposed by Zhang et al. The St is defined as the ratio of the responding time of the particles to the characteristic time of flow. A small St (St ≪ 1) indicates that particles can quickly catch up with the flow, and a large St (St ≫ 1) indicates that particles move slowly relative to the characteristic time of flow.

If we classified the research scopes of the previous studies on a map with dimensions of St and αp, as shown in Fig. 1, it would reveal that previous studies on the theoretical growth rate model...
of the multiphase RMI were mainly conducted on the gas-particle system with a small $S_t$.

For region A in Fig. 1 ($0 < \alpha_p < 0.01$ and $S_t \ll 1$), numerous studies have been conducted on theoretical modeling and numerical verification of the linear growth rate for the gas-particle flow.\textsuperscript{17-22} The multiphase Atwood solutions were critical for the growth rate model and the evolution law of the mean velocity of the interface are derived to develop a linear growth rate model. Then, a numerical simulation based on the multiphase particle-in-cell (MP-PIC)\textsuperscript{22,28} is conducted to verify the accuracy of the theoretical model. Moreover, growth rates in the nonlinear stage were also preliminarily investigated by the MP-PIC method. The effects of a large $S_t$ were examined based on the theoretical linear growth model and nonlinear stage simulation. This article is organized as follows: In Sec. II, a detailed theoretical analysis is conducted to obtain the solutions of the Atwood number and mean velocity of the interface. In addition, the RMI linear growth rate model for a large $S_t$ is developed based on the above solutions. In Sec. III, numerical verifications of the linear growth rate model for $S_t \gg 1$ are performed. In Sec. IV, the growth rates in the nonlinear stage simulated by the MP-PIC method were also determined. In Sec. V, the effects of a large $S_t$ are discussed based on the linear theoretical models and numerical results of the nonlinear growth rates. The conclusions drawn in this study are presented in Sec. VI.

II. LINEAR GROWTH MODEL FOR A LARGE STOKES NUMBER

In this section, the governing equations for theoretical derivation are given first. Then, the solutions of the Atwood number and interface velocity are derived by linear stability and magnitude analysis methods. In addition, the linear growth model for RMI with a large $S_t$ is developed.

A. Governing equations for theoretical analysis

The growth model for the dense gas-particle flow is obtained using linear stability analysis, which was the method adopted by Saffman \textsuperscript{1} and Ukai \textsuperscript{11}. The assumptions made in this study are the same as those made in previous studies. \textsuperscript{2} We followed the same approach but used the following governing equations that were applicable to the arbitrary particle volume fraction:

\begin{equation}
\nabla \vec{u}_f = 0,
\end{equation}

Fig. 1, especially for the linear and nonlinear growth of the disturbance amplitude. Numerous experimental studies and numerical simulations have been conducted on gas-particle flow with shock for a large $S_t$. Experimental studies\textsuperscript{17-20} have mainly focused on the interface relaxation of shock waves and particle clouds. McFarland \textsuperscript{11} et al. conducted a very detailed numerical study on the multiphase relaxation time effects. They found that a significant particle lag was created for a large particle size (corresponding to $S_t \gg 1$). Thus, the interface shape evolution was different compared with that of small particle size. Ukai \textsuperscript{11} et al. conducted a numerical simulation for the dilute gas-particle flow with a large $S_t$ and their results showed that the RMI growth rate agreed well with that in the original Richtmyer’s model. However, in their study, there was no corresponding theory to explain that physical phenomenon.

The current study focuses on the amplitude increase of a perturbed interface of air and sulfur hexafluoride ($SF_6$) with particles uniformly distributed at two sides of the interface with a large $S_t$. The growth rates in the linear and nonlinear stages are considered. The analytical solutions of the multiphase Atwood number and the evolution law of the mean velocity of the interface are derived to develop a linear growth rate model. Then, a numerical simulation based on the compressible multiphase particle-in-cell (MP-PIC)\textsuperscript{22,28} is conducted to verify the accuracy of the theoretical model. Moreover, growth rates in the nonlinear stage were also preliminarily investigated by the MP-PIC method. The effects of a large $S_t$ were examined based on the theoretical linear growth model and nonlinear stage simulation. This article is organized as follows: In Sec. II, a detailed theoretical analysis is conducted to obtain the solutions of the Atwood number and mean velocity of the interface. In addition, the RMI linear growth rate model for a large $S_t$ is developed based on the above solutions. In Sec. III, numerical verifications of the linear growth rate model for $S_t \gg 1$ are performed. In Sec. IV, the growth rates in the nonlinear stage simulated by the MP-PIC method were also determined. In Sec. V, the effects of a large $S_t$ are discussed based on the linear theoretical models and numerical results of the nonlinear growth rates. The conclusions drawn in this study are presented in Sec. VI.
The velocity difference of the gas and particle phase, \( \tilde{u}_F \), the mean velocities of the particle and gas are not equal. \( \hat{U}_0, \hat{V}_0 \) represent the mean velocities of the gas and particle phases, respectively, as shown in the following formula:

\[
\tilde{u}_F = \hat{U}_0 + \hat{u}', \quad \tilde{v}_p = \hat{V}_0 + \hat{v}', \quad \text{for } St \ll 1. \tag{6a}
\]

The velocity difference of the gas and particle phase, \( \tilde{v}_p - \tilde{u}_F \), can be expressed as

\[
\tilde{v}_p - \tilde{u}_F = \hat{V}_0 - \hat{U}_0 + \hat{v}' - \hat{u}', \quad \text{for } St \gg 1. \tag{7a}
\]

For the convenience of discussion, the corresponding equation of \( St \ll 1 \) is also given as

\[
\tilde{v}_p - \tilde{u}_F = \hat{V}_0 - \hat{U}_0 + \hat{v}' - \hat{u}', \quad \text{for } St \ll 1. \tag{7b}
\]

Since the magnitude of the mean velocity is larger than the fluctuation, the example for the direction \( x \) is taken, \( O(U_s) > O(\hat{u}') \); \( O(V_s) > O(\hat{v}') \), we assume that

\[
O(V_s - U_s) > O(\hat{v}' - \hat{u}'). \tag{8}
\]

This assumption is based on the physical opinion evidence that the macroscopic quantity is much larger than the fluctuation and the subsequent numerical results will further prove the correctness of this assumption regarding the RMI issue. Actually, different \( St \) is consistent with the different velocity difference of the two phases, which results in the different order of the drag term. For \( St \ll 1 \), the magnitude of the drag term can be estimated as \( O(s(\hat{v}' - \hat{u}')) \). However, the order of the drag term for \( St \gg 1 \) becomes much larger and can be estimated as \( O(s(\hat{V}_s - \hat{U}_s)) \). Similar effects of the \( St \) are also shown in the modeling of the interface velocity in Subsection II C.

With Eqs. (3) and (8), the \( x \)-direction momentum equation of the gas phase can be obtained as follows:

\[
\frac{D\hat{u}_F}{Dt} + U_s \frac{D\hat{u}_F}{Dx} = -\frac{1}{\rho_f} \frac{\partial P_f}{\partial x} + s(U_s - V_s) - g \frac{\partial a}{\partial x}, \tag{9a}
\]

\[
s = \frac{\alpha_0 K}{\alpha_0 V_s \rho_f \gamma}. \tag{9b}
\]

The parameter \( s \), given in Eq. (9b), has a unit of frequency and corresponds to the drag force term. The expression of \( s \) provides an integrated form to represent the effects of the two-phase volume fraction, drag term coefficient, volume of particle, and density of the gas. As the particle volume fraction and the drag coefficient grow larger, the value of \( s \) becomes larger, which means that the momentum coupling effects are very obvious.

The disturbance of each variable is determined with a wave of the form in the \( x-z \) two-dimensional space

\[
[u'(x, z, t), \quad \nu'(x, z, t), \quad \rho'(x, z, t), \quad \alpha'(x, z, t), \quad a(x, t)]
\]

\[
= [\hat{u}(z), \quad \hat{v}(z), \quad \hat{p}(z), \quad \hat{a}(z), \quad \hat{a}(z)] e^{i(kz - ct)}, \tag{10}
\]

where the variables with hat (\( \hat{\cdot} \)) are the amplitude for each variable, \( k \) is the wavenumber, and \( c \) is the phase velocity for each variable. In particular, for the Rayleigh-Taylor instability (RTI) and RMI, the mean velocity in the \( x \)-direction equals to zero. The \( x \)-direction momentum equation [Eq. (9a)] can be linearized as

\[
\hat{p} = \rho_f(c \hat{h}_k - \hat{g} \hat{a}). \tag{11}
\]

Notably, Eq. (11) does not have any items related to the particle phase, which is totally different from the flow of \( St \ll 1 \). In the following derivation, the velocity potential and equation of the interface are introduced to obtain the relationship between \( \hat{u}_k \) and \( \hat{a} \). With the constraint condition at the interface, we obtain the solution of the Atwood number of the gas-particle flow RMI for \( St \gg 1 \), designated as \( A_{LS} \), which is given as

\[
c^2 = -\frac{g}{k} A_{LS}, \tag{12}
\]

\[
A_{LS} = \frac{\rho_2/\rho_1 - \rho_1/\rho_2}{\rho_2/\rho_1}, \quad St \gg 1. \tag{13}
\]

The detailed derivation process is presented in Appendix A. Notably, \( A_{LS} \) performs in the same way as the original Richtmyer’s model \( A \). Also, since we do not set any limit on the volume of the particle phase, the solution of \( A_{LS} \) is effective.
for both the dilute and dense gas-particle flow, i.e., region C in Fig. 1.

**C. Decay law for the mean velocity of the interface with a large \( St \)**

In this subsection, we present the modeling of the interface velocity with varying time. Since the boundary of the gas phases (such as air and SF\(_6\)) is the interface, the governing equation of the interface motion should also obey the same law given by the gas phase momentum equation, i.e., Eq. (4). The interface motion governing equation in the \( x \)-direction is presented as

\[
\frac{dU_{\text{inf}}}{dt} = -\frac{1}{\rho_f} \frac{\partial P_f}{\partial x} + \tilde{\tau}(V_{\text{px}} - U_{\text{inf}}). \tag{14}
\]

Here, \( U_{\text{inf}}, V_{\text{px}} \) are the velocity of the interface and mean velocity of particle phases around the interface, respectively, and \( \tilde{\tau} \) is a parameter related to the drag term.

Further simplification can be made to solve Eq. (14). Once the shock passes the interface, the post-shock pressure is close to a spatially homogeneous shape. Thus, we assume that the order of the gradient of the pressure is smaller than the drag term, \( O\left( \frac{1}{\rho_f \frac{\partial P_f}{\partial x}} \right) > O\left[ \frac{\tilde{\tau}}{\rho_f \frac{\partial P_f}{\partial x}} \right] \). In particular, this assumption is definitely right when the parameter \( \tilde{\tau} \) is large. This may occur for a dense gas-particle flow. In addition, since the \( St \) is large, the particle phase is almost still, that is, \( V_{\text{px}} \approx 0 \). Thus, Eq. (14) can be written as

\[
\frac{dU_{\text{inf}}}{dt} = -\tilde{\tau}U_{\text{inf}}, \quad \text{if} \quad O\left( \frac{1}{\rho_f} \frac{\partial P_f}{\partial x} \right) > O\left[ \frac{\tilde{\tau}}{\rho_f} \frac{\partial P_f}{\partial x} \right]. \tag{15}
\]

With the initial condition, \( t = t_0, U_{\text{inf}} = U_{\text{inf}0} \), the solution of Eq. (15) can be easily obtained as

\[
U_{\text{inf}} = U_{\text{inf}0} e^{-\tilde{\tau}t'}, \quad t' = t - t_0. \tag{16}
\]

For parameter \( \tilde{\tau} \), it is simply modeled as the average value of \( s_1, s_2 \),

\[
\tilde{\tau} = \frac{s_1 + s_2}{2}. \tag{17}
\]

In particular, Eq. (16) can be simplified when the value of \( \tilde{\tau}t' \) is close to zero, given as

\[
U_{\text{inf}} = U_{\text{inf}0}, \quad \text{if} \quad \tilde{\tau}t' \approx 0. \tag{18}
\]

Additionally, the application scope of Eq. (18) can be estimated as follows: For the particles with a diameter of 0.1 ms and a time duration of 0.1 ms, if we set the limit of the velocity decay as 2%, i.e., \( e^{-2\tilde{\tau}} \approx 98\% \), the volume fraction of the particle phase is about 0.01. This result is well consistent with the dilute limit given by Zhang et al.\(^{23}\). Thus, Eq. (18) is applicable for the dilute gas-particle RMI.

**D. Linear RMI growth models for dense and dilute patterns**

In this subsection, the linear RMI growth rate is modeled based on the Atwood number solution [Eq. (13)] and the velocity of the interface [Eqs. (16) and (18)]. The acceleration of the interface motion can be described by the following equation:\(^{20,22}\)

\[
\frac{d^2 a}{dt^2} = agkA_u. \tag{19}
\]

Equation (19) provides the linear growth rate for the RTI and the impulsive model\(^1\) required to extend the scope of the RMI. The mixed zone width \( h \) is twice the amplitude of the perturbation \( a \). The growth rate model of the mixed zone width for the dense pattern is given as

\[
\frac{dh}{dt} = 2kA_u a_0 U_{\text{inf}0} e^{-\tilde{\tau}t'}, \quad \text{dense pattern model.} \tag{20a}
\]

The superscript “+” denotes the post-shock quantities. Particularly, for the dilute gas-particle RMI issue, the growth rate can be simplified by combining Eqs. (20a) and (18), and the growth rate for a dilute pattern can be predicted as

\[
\frac{dh}{dt} = 2kA_u a_0^+ U_{\text{inf}0} a_0^+, \quad \text{dilute pattern model.} \tag{20b}
\]

**III. NUMERICAL VERIFICATION FOR THE LINEAR GROWTH MODELS**

In this section, a numerical verification was conducted by the CMP-PIC method. The basic information of the simulation, such as computational domain and parameter setting, is given first. Then, we present the numerical verifications for the linear theoretical growth models for the dense and dilute patterns developed in Sec. II.

**A. Numerical algorithm and the computational domain**

The numerical simulation was conducted to compare with the theoretical model given in Eqs. (20a) and (20b). The CMP-PIC method was used to solve the multiphase flow. The numerical methods\(^{29–31}\) and validation case\(^{32}\) of the current code program were described in our earlier study.\(^{23}\) The grid independence verification is presented in Appendix B. The computational domain and parameter settings were similar to those used in our previous study.\(^{23}\) The two-dimensional single-mode air/SF\(_6\) RMI surrounded by particles with different volume fraction values was analyzed. The domain configuration used in the current study is depicted in Fig. 2. The incident Mach number was 1.2, and the interface perturbation was applied as

\[
a(y) = a_0 \cos\left( \frac{2\pi}{\lambda} y \right). \tag{21}
\]
The initial amplitude of the perturbation is \( a_0 = 0.5 \) mm, and the wavelength \( \Lambda \) of the perturbation was equal to the transverse length of the domain. Particles represented by small circles, as shown in Fig. 2, were uniformly distributed in the region of unshocked air and SF6 at the start of simulation.

A large \( St \) should be verified, as it is one of the most critical parameters in this study. As mentioned in Sec. I, the parameter \( St \) was defined as the ratio of the responding time of the particles to the characteristic time of the flow. The detailed expression is shown as

\[
St = \frac{\tau_p}{\tau_c}. \tag{22a}
\]

Here, \( \tau_c \) is the characteristic time of the gas phase flow. In this study, we selected \( \tau_c \) as the ratio of the amplitude of the perturbation \( a \) to the velocity of shock when passing the interface, as shown in the following equation:

\[
\tau_c = \frac{a}{U_{shock}}. \tag{22b}
\]

The responding time of the particles can be calculated with\(^{20}\)

\[
\tau_p = \frac{\rho_p V_p}{k}. \tag{22c}
\]

In order to satisfy a large \( St \), a high density and a relatively large diameter of particles were required. Here, the particle density was set at 7800 kg/m\(^2\) and the radius was 0.05 mm. The response time of the particle was about 2 s and \( \tau_c \approx 10^{-5} \) s. The \( St \) was about 200 000.

B. Linear dense pattern model verification

For the dense gas-particle RMI with a large \( St \), we selected six different particle volume fraction values for the simulation. For the convenience of comparison with the numerical results, the width of the mixed zone is also given by integrating with respect to time in Eq. (20a) as follows:

\[
h = \frac{kA^+_1 U_{inf0} a^+_0}{\bar{s}} (e^{-\bar{s}t} - 1) + h_0, \quad \bar{s} = \frac{s_1 + s_2}{2}. \tag{23}
\]

A sum plot of the mixed zone width for different particle volume fraction values is shown in Fig. 3. The plot reveals that with the increase in the particle volume fraction value, the growth rate of the mixed zone decays faster. The parameters related to Eq. (23) to compare with numerical simulation are shown in Table I. For six cases, the parameters \( A^+_1 \) and \( a^+_0 \) remain almost constant, and their values are selected as \( A^+_1 = 0.6967 \) and \( a^+_0 = 0.0004 \) m. Comparisons between the numerical and theoretical predictions of the mixed zone widths of the six cases are presented in Fig. 4. The numerical results agree well with the results of the theoretical model, as given by Eq. (23). There is a small difference for the case of \( \alpha_p = 0.1 \) at the late time in Fig. 4a. The difference may be due to the assumption about the orders of the pressure gradient force and drag force made in Eq. (15), which may not be entirely accurate. Since parameter \( \bar{s} \) is relatively small for the case of \( \alpha_p = 0.1 \), it leads to a relatively small order of the drag term, i.e., \( O(\bar{s}(V_{px} - U_{inf}) \).

C. Linear dilute pattern model verification

In this subsection, we focus on the verification of the growth rate model of the dilute pattern, which corresponds to Eq. (20b), a degradation case of the dense pattern model. Three cases (\( \alpha_p = 0 \), \( \alpha_p = 0.001 \), and \( \alpha_p = 0.01 \)) were simulated, and a plot of the time and the mixed zone widths is shown in Fig. 5.

Clearly, the growth rate differences among the three cases were very small. The results demonstrated that the growth rate of the dilute pattern in the early stage of the linear regime behaved the same and was independent of the volume fraction value of the particle phases. A detailed comparison of the numerical and theoretical results of the growth rate of the mixed zone width is shown in Table II. The theoretical value is calculated by Eq. (20b). Compared with the theoretical predictions, the deviations were less than 3%. The numerical results confirmed the accuracy of the growth rate model of the RMI for the dilute gas-particle flow with \( St \gg 1 \). For the dilute gas-particle RMI with a large \( St \), the linear growth rate in the early stage was consistent with that of the original RMI, that is, the multiphase RMI degenerates into a pure gas phase RMI.

### TABLE I. Values of the parameters of the theoretical model for different volume fraction values.

| \( \alpha_p \) | \( U_{inf0} \) (m/s) | \( h_0 \) (m) | \( \bar{s} \) (Hz) |
|---|---|---|---|
| 0.1 | 57.8 | 1.408 × 10^{-3} | 1312.26 |
| 0.2 | 57.2 | 1.404 × 10^{-3} | 2952.58 |
| 0.3 | 56.7 | 1.400 × 10^{-3} | 5061.56 |
| 0.4 | 55.8 | 1.392 × 10^{-3} | 7880.11 |
| 0.5 | 54.8 | 1.334 × 10^{-3} | 11824.50 |
| 0.6 | 51.4 | 1.284 × 10^{-3} | 17737.64 |
FIG. 4. Comparisons of the numerical results and theoretical predictions of the different particle volume fraction values for the dense pattern. (a)–(f) represent the mixed zone widths corresponding to $\alpha_p = 0.1$, 0.2, 0.3, 0.4, 0.5, and 0.6, respectively.

IV. NONLINEAR GROWTH SIMULATION AND ANALYSIS

In this section, we focus on the nonlinear stage. In particular, we focus on the mixed zone widths and growth rates. The mixed zone width evolutions of the gas-particle RMI with a large $St$ are compared with those of a pure gas RMI and a gas-particle RMI with a small $St$. Then, the flow fields at a typical time in the nonlinear stage of the different types of RMI are compared, and the evolution of the interface corresponding to the RMI with a large $St$ is presented.

A. Comparison of growth rates for different type RMI

The mixed zone widths and growth rates are shown in Fig. 6. Case A is a pure gas case, and case D corresponds to the gas-particle RMI with a small $St$. Cases B and C correspond to the RMI with a large $St$ but with different particle volume fractions. For case B, $\alpha_p = 0.05$, and for case C, $\alpha_p = 0.3$. It should be noted that the dimension in the $x$-direction of the computational domain is extended to 0.08 m for the nonlinear stage simulation. It is found that, for the analysis and comparison of the RMI in different types of flows, the Atwood number solutions and interface velocity are two important factors to determine the evolution of the mixed zone width.

The decay of the mixed zone width growth rate is highlighted in the comparison among the cases with a large $St$ (cases B and C) and the pure gas case (case A). At the initial stage, the growth rates for the three cases are similar and can be estimated as 20.9 m/s. The Atwood solutions for the three cases, as given in Eq. (13), are the same, and the interface velocities are also close in the early stage of linear growth. However, at the later stage, due to significant momentum coupling, the growth rate decay is much more rapid. This phenomenon is more obvious for a larger particle volume fraction. The separation of the curves of cases C and A occurs earlier than that of cases B and C, and the difference is far larger in the later nonlinear stage. In addition, the slope of the curve, i.e., the growth rate of the mixed zone width for $St \gg 1$, experiences remarkable decay. At 0.5 ms, the growth rate of case B is approximately 6.6 m/s and lower than that of case A (7.4 m/s). Notably, for case C, the growth rate
Further comparison was also performed for the gas-particle RMI but with different $St$. For cases B and D, both particle volume fractions are 0.05. However, the mixed zone widths are totally different, as shown in Fig. 6. For the linear stage, the growth rate of case B (about 20.9 m/s) is much larger than that of case D (8.71 m/s), which is mainly due to the difference of the Atwood solutions. For case B, the Atwood solution is given in Eq. (13) and it is independent of the particle density, but for case D, the Atwood solution is given in Eq. (1b) and the equivalent density is influenced by the gas and particle density, which led to the decrease in the Atwood number. However, in the nonlinear stage, such as at 0.5 ms, the growth rate of case B (7.4 m/s) and case D (5.7 m/s) is much closer as the velocity decay of case B began to occur.

B. Comparison of typical flow fields and interface evolution

Here, we present the interface shapes for the RMI in different types of flows. The data presented in Figs. 7(a)–7(c) correspond to case A (pure gas), case B ($St \gg 1$, $\alpha_p = 0.05$), and case D ($St \ll 1$, $\alpha_p = 0.05$), respectively. For cases A and B, the evolution processes of the interface shape are similar. However, the interface of case A propagates slightly faster than that of case B. For case D, the evolution of the interface is much slower due to a smaller Atwood value as defined in Eq. (1b). The gas-phase density distribution is also shown in Figs. 7(d)–7(f) for the three cases. Although cases A and B are similar, a large difference is observed in Fig. 7(f) corresponding to case D. The particle front before the interface is twisted and leads to density changes. The particle volume fraction is also plotted in Figs. 7(g) and 7(h). The data presented in Fig. 7(g) correspond to case B, and the particle volume remains very similar to that under its initial condition. Since the $St$ is very large, the particles almost do not move, but for a small $St$, the particle volume fraction shown in Fig. 7(h) is totally different as the particles can follow the gas phase very well. Thus, a particle cloud front forms before the interface. The evolution of the interface of case B is depicted in Fig. 8. The spike and bubble structures are also found in the flow fields, which are similar to the pure gas RMI. The mushrooms on the two sides of the spike take the initial shape at 0.3322 ms and grow over time.

V. DISCUSSIONS ABOUT THE LARGE $St$ EFFECTS ON MULTIPHASE RMI

In Sec. II, we derived linear growth models for dense and dilute patterns, as expressed in Eqs. (20a) and (20b). In Sec. III, this linear model was verified by the CMP-PIC method. Additionally, the growth rates in the nonlinear stage were also obtained by simulation in Sec. IV. Here, the effects of a large $St$ are discussed based on the above results in the linear and nonlinear stages.

The linear growth model for the dense pattern flow is presented in Eq. (20a). Compared with the growth models of the classical pure gas RMI and the gas-particle RMI with a small $St$, two major differences related to a large $St$ should be emphasized. The first one is that the RMI with a large $St$ is initiated and driven only by the density difference of the gas-phase and totally independent of the particle density. This was expressed in Eq. (13), which is the solution
of the Atwood number, and its performance is markedly different from that of the RMI with a small $St$. A detailed comparison of the Atwood solutions is shown in Table III. The second one is that the growth rate undergoes exponential decay and the parameter $\tilde{\tau}$ determines the rate of decay. As the particle volume fraction grows larger, the decay of the growth rate accelerates. The decay of the growth rate indicates that the multiphase phase effects are very different from those of the original RMI and a gas-particle flow of $St \ll 1$. Here, the effects of the $St$ should be noticeable since a large $St$ induces a large velocity difference and drag effects. A larger magnitude of the drag term causes changes in the modeling of the Atwood number and interface velocity. For the pure gas phase RMI, the growth rate is almost constant during the linear growth stage. In addition, for the gas-particle flow with a small $St$, the particle phase can quickly catch up with the gas phase. Thus, the velocity difference of the two phases is very small and leads to a small order of drag term, and the interface velocity and growth rate are also close to a constant. A detailed comparison of the Atwood number and interface velocity for various cases is presented in Table III.

As the degradation of a dense pattern, the dilute pattern model given in Eq. (20b) is much simpler. Remarkably, it performs in the same form as that of the pure gas RMI due to
the weak momentum coupling and large particle response time. The dilute pattern growth model also provides a theoretical support for previous numerical results obtained in the study by Ukai et al. They pointed out that the RMI growth rate for the dilute gas-particle flow for \( St \gg 1 \) agrees with the original Richtmyer’s model.\(^1\)

The nonlinear results show that the decay of the growth rates related to the large \( St \) effects in the linear stage continues to influence the nonlinear behavior of the interface. As the particle volume fraction grows larger, the decay effects are much more pronounced, especially for the case with a large particle volume fraction (such as the case of \( \alpha_p = 0.3 \)), in which the growth rate may be close to zero in the nonlinear stage.

**VI. CONCLUSION**

The present study focused on the gas-particle RMI growth rates in the linear and nonlinear stages with a large \( St \). A large St"oke number meant a much higher order of magnitude of the drag term (momentum coupling effects) than that for \( St \ll 1 \). The momentum coupling effects dominated the multiphase flow and resulted in a totally different performance of the interface evolution behavior.

For the linear stage, a RMI growth rate model was developed. The theoretical and numerical results of the linear stage proved that the Atwood number for \( St \gg 1 \) was only determined by the parameters of the gas phase and in the same way as the original Richtmyer’s model.\(^1\) Additionally, the growth rate had different evolution properties due to different particle volume fractions. For the dense gas-particle RMI, the interface velocity and growth rate underwent a remarkable exponential decay and a parameter related to the momentum coupling of the two phases determined the rate of decay. The decay rate became faster when the volume fraction of the particle was larger. The decay of the growth rate indicated that the momentum coupling played a critical role in the dense gas-particle RMI. The corresponding numerical results demonstrated the accuracy of the theoretical growth model.

The nonlinear stage results highlighted the effects of the growth rate decay due to lag of particles. In flows with large particle volume fractions, the growth rate may be close to zero in the nonlinear stage due to the large drag coupling and long-process decay. This offers the opportunity to develop a new method involving multiphase effects to control the development of the hydrodynamic instability, especially for that in ICF.

**APPENDIX A: DERIVATION PROCESS FOR OBTAINING THE SOLUTION OF ATWOOD NUMBER**

In the following derivation, the relationship between \( \dot{u}_\kappa \) and \( \dot{\alpha} \) will be determined. The velocity potential introduced is given as follows:

\[
\nabla \phi = \hat{U}_0 + \nabla \phi', \quad (A1)
\]
\[
\nabla \phi' = \hat{u}', \quad (A2)
\]
\[
\phi' = \phi(z)e^{i\theta(x-v_0)}. \quad (A3)
\]

The perturbed velocity potential can also be expressed as a waveform, as shown in Eq. (A3). Combining Eqs. (A2) and (A3), we get

\[
\dot{u}_\kappa = i\kappa\dot{\phi}. \quad (A4)
\]

Considering the mass conservation equation of the gas phase shown in Eq. (2), the equation of the perturbed velocity potential is expressed as

\[
\frac{\partial^2 (\phi'_j)}{\partial x^2} + \frac{\partial^2 (\phi'_j)}{\partial z^2} = 0, \quad j = 1, 2. \quad (A5)
\]

The subscript \( j \) represents the gas type index. Gas phase 1 is located in the zone of \( z > 0 \), and gas phase 2 is located in the zone of \( z < 0 \). With the boundary condition at infinite far points,
We get the solution of the perturbed amplitude of the velocity potential,
\[ \phi_1(z) = \lambda_1 e^{kz}, \]
\[ \phi_2(z) = \lambda_2 e^{kz}. \] (A7)
To solve the variables \( \lambda_1 \) and \( \lambda_2 \), the equation of the fluid motion of the interface should be added. The original equation of the interface can be expressed as
\[ z = a(x,t), \]
which can be rewritten as
\[ F(x,z,t) = z - a(x,t) \equiv 0. \] (A8)
With the total derivation of time \( t \),
\[ \frac{DF}{Dt} = \frac{\partial F}{\partial t} + u_f \frac{\partial F}{\partial x} + u_z \frac{\partial F}{\partial z} = 0. \] (A9)
We get
\[ \frac{\partial a}{\partial t} = \frac{\partial \phi_1'}{\partial z} - U_0 \frac{\partial a}{\partial x}. \] (A10)
Considering that the gas phase includes two types of gas,
\[ \frac{\partial a}{\partial t} = \frac{\partial \phi_1'}{\partial z} - U_1 \frac{\partial a}{\partial x} = \frac{\partial \phi_2'}{\partial z} - U_2 \frac{\partial a}{\partial x}. \] (A11)
Substituting Eq. (A7),
\[-ick\dot{\alpha} = \lambda_1 k e^{ka} - ikU_1 \dot{a}, \]
\[-ick\dot{\alpha} = -\lambda_2 k e^{ka} - ikU_2 \dot{a}. \] (A12)
Noting that the amplitude of the perturbation is a small value, i.e., \( ka \sim 0 \), the solution of the parameters \( \lambda_1 \) and \( \lambda_2 \) is given as
\[ \lambda_1 = i(U_1 - c) \dot{a}, \]
\[ \lambda_2 = -i(U_2 - c) \dot{a}. \] (A13)
Then, the solution of the perturbed velocity potential can be given by Eqs. (A7) and (A15), as shown in the following equation:
\[ \dot{\phi}_1 = i(U_1 - c) \dot{a} e^{kz}, \]
\[ \dot{\phi}_2 = -i(U_2 - c) \dot{a} e^{kz}. \] (A14)
Note that the amplitude of the perturbation is a small value, i.e., \( ka \sim 0 \), and the mean velocity \( U_1, U_2 = 0 \); thus, Eq. (A16) can be simplified as
\[ \dot{\phi}_1 = -ic\dot{a}, \]
\[ \dot{\phi}_2 = ic\dot{a}. \] (A15)
Then, the relationship between \( \dot{a} \) and \( \dot{\alpha} \) is obtained. Substituting Eqs. (A4) and (A17) into Eq. (16), we get
\[ \dot{p}_j = \rho j (ikc \dot{\phi}_j - g \dot{\alpha}_j), \quad j = 1, 2, \] (A16)
where the constraint conditions on the interface are
\[ \dot{p}_1 = \dot{p}_2, \quad \dot{\alpha}_1 = \dot{\alpha}_2. \] (A17)

**APPENDIX B: GRID INDEPENDENCE VERIFICATION**

Four different mesh resolutions were used to determine the grid convergence, and the detailed data are shown in Table IV. The \( Nx \) and \( Ny \) are the resolutions in the \( x \) and \( y \) direction, respectively. A plot of the mixed zone width \( h \), which varied with time, is shown in Fig. 9. The criterion for the selection of the interface position was the same as that in our previous study.\(^{22,33}\) The mixed zone width was measured by the peak and valley of the interface. The results of resolution C agreed well with the results of resolution D. There is a slight difference (less than 0.6%) in the amplitude at the late time of about 0.0002 s due to the difference in the number of grid cells to resolve the initial perturbation shape for different grid sizes. This finding suggests that the refinement does not significantly affect the

| Resolution | \( Nx \) | \( Ny \) | Total grid number | Total parcel number |
|------------|---------|---------|------------------|---------------------|
| A          | 401     | 101     | 40 001           | 121 000             |
| B          | 801     | 201     | 161 001          | 483 000             |
| C          | 1201    | 301     | 361 501          | 1 446 000           |
| D          | 1601    | 401     | 642 001          | 1 926 000           |

**FIG. 9**. Width of the mixed zone of the four resolutions.
simulation. Thus, resolution D was selected for all subsequent simulations. In addition, the message passing interface (MPI) parallel programs were used in the program, and 144 cores were used for the computation.

REFERENCES

1. R. D. Richtmyer, “Taylor instability in shock acceleration of compressible fluids,” Commun. Pure Appl. Math. 13, 297 (1960).
2. E. M. Meshkov, “Instability of the interface of two gases accelerated by a shock wave,” Fluid Dyn. 4, 101 (1969).
3. F. Gao, Y. Zhang, Z. He, L. Li, and B. Tian, “Characteristics of turbulent mixing at late stage of the Richtmyer-Meshkov instability,” AIP Adv. 7, 075020 (2017).
4. G. Xiang and B. Wang, “Numerical investigation on the interaction of planar shock wave with an initial ellipsoidal bubble in liquid medium,” AIP Adv. 8, 075128 (2018).
5. M. A. Skinner, A. Burrows, and J. C. Dolence, “Should one use the ray-by-ray approximation in core-collapse supernova simulations?,” Astrophys. J. 831, 81 (2016).
6. B. J. Balakumar, G. C. Orlicz, J. R. Ristorcelli, S. Balasubramanian, K. P. Prestridge, and C. D. Tomkins, “Turbulent mixing in a Richtmyer-Meshkov fluid layer after reshock: Velocity and density statistics,” J. Fluid Mech. 696, 67 (2012).
7. J. Mcfarland, D. Reilly, S. Creel, C. McDonald, T. Finn, and D. Ranjan, “Experimental investigation of the inclined interface Richtmyer-Meshkov instability before and after reshock,” Exp. Fluids 55, 1640 (2014).
8. E. Koroteeva, I. Mursenkova, Y. Liao, and I. Znamenskaya, “Simulating particle inertia for velocimetry measurements of a flow behind an expanding shock wave,” Phys. Fluids 30, 011702 (2018).
9. H. Y. Gan and Y. C. Lam, “Experimental observations of flow instabilities and rapid mixing of two dissimilar viscoelastic liquids,” AIP Adv. 2, 042146 (2012).
10. D. T. Casey, D. T. Woods, V. A. Smalyuk, O. A. Hurricane, V. Y. Glebov, C. Stoeckl, W. Theobald, R. Wallace, A. Nikroo, and M. Schoff, “Performance and mix measurements of indirect drive Cu-doped Be implosions,” Phys. Rev. Lett. 114, 205002 (2015).
11. K. S. Raman, V. A. Smalyuk, D. T. Casey, S. W. Haan, and J. D. Salomon, “An in-flight radiography platform to measure hydrodynamic instability growth in inertial confinement fusion capsules at the National Ignition Facility,” Phys. Plasma 21, 072710 (2014).
12. J. Ding, T. Si, J. Yang, X. Lu, Z. Zhai, and X. Luo, “Measurement of a Richtmyer-Meshkov instability at an air-SF6 interface in a semianular shock tube,” Phys. Rev. Lett. 119, 014501 (2017).
13. Q. Chen, L. Li, Y. Zhang, and B. Tian, “Effects of the Atwood number on the Richtmyer-Meshkov instability in plastic-media,” Phys. Rev. E 99, 053102 (2019).
14. X. Li, Z. He, Y. Zhang, and B. Tian, “On the role of rarefaction/compression waves in Richtmyer-Meshkov instability with reshock,” Phys. Fluids 31, 054102 (2019).
15. B. Thornber, J. Griffond, O. Poujade, N. Attal, H. Varshochi, P. Bigdelou, P. Ramaprabh, B. Olson, J. Greenough, and Y. Zhou, “Late-time growth rate, mixing, and anisotropy in the multimode narrowband Richtmyer-Meshkov instability: The 0-group collaboration,” Phys. Fluids 29, 105107 (2017).
16. Z. Zhang, D. Frost, P. Thibault, and S. Murray, “Explosive dispersal of solid particles,” Shock Waves 10, 431 (2001).
17. D. H. Michael, “Kelvin-Helmholtz instability of a dusty gas,” Math. Proc. Camb. Philos. Soc. 61, 569 (1965).
18. P. G. Saffman, “On the stability of laminar flow of a dusty gas,” J. Fluid Mech. 13, 120 (1962).
19. L. H. Thomas, “The stability of plane Poiseuille flow,” Phys. Rev. 91, 780 (1953).
20. S. Ukai, K. Balakrishnan, and S. Menon, “On Richtmyer-Meshkov instability in dilute gas-particle mixtures,” Phys. Fluids 22, 297 (2010).
21. K. Balakrishnan and S. Menon, “A multiphase buoyancy-drag model for the study of Rayleigh-Taylor and Richtmyer-Meshkov instabilities in dusty gases,” Laser Part. Beams 29, 201 (2011).
22. B. Meng, J. Zeng, B. Tian, L. Li, Z. He, and X. Guo, “Modeling and verification of the Richtmyer-Meshkov instability linear growth rate of the dense gas-particle flow,” Phys. Fluids 31, 074102 (2019).
23. K. Balakrishnan, “On bubble and spike oscillations in a dusty gas Rayleigh-Taylor instability,” Laser Part. Beams 30, 633 (2012).
24. Y. Vorobieff, M. Anderson, J. Conroy, R. White, C. R. Truman, and S. Kumar, “Vortex formation in a shock-accelerated gas induced by particle seeding,” Phys. Rev. Lett. 106, 184503 (2011).
25. V. Boiko, V. Kiselev, S. Kiselev, A. Papyrin, S. Poplavsky, and V. Fomin, “Shock wave interaction with a cloud of particles,” Shock Waves 7, 275 (1997).
26. K. Xue, K. Du, X. Shi, Y. Gan, and C. Bai, “Dual hierarchical particle jetting of a particle ring undergoing radial explosion,” Soft Matter 14, 4422 (2018).
27. J. A. Mcfarland, W. J. Black, J. Dahal, and B. E. Morgan, “Computational study of the shock driven instability of a multiphase particle-gas system,” Phys. Fluids 28, 297 (2016).
28. M. J. Andrews and P. J. O’Rourke, “The multiphase particle-in-cell (MP- PIC) method for dense particulate flows,” Int. J. Multiphase Flow 22, 379 (1996).
29. X. Liu, S. Osher, and T. Chan, “Weighted essentially non-oscillatory schemes,” J. Comput. Phys. 115, 200 (1994).
30. E. F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics: A Practical Introduction (Springer-Verlag, Berlin, Heidelberg, 1999).
31. S. A. Khan and A. Shah, “Simulation of the two-dimensional Rayleigh-Taylor instability problem by using diffuse-interface model,” AIP Adv. 9, 085312 (2019).
32. X. Rogue, G. Rodriguez, J. F. Haas, and R. Saurel, “Experimental and numerical investigation of the shock-induced fluidization of a particles bed,” Shock Waves 8, 29 (1998).
33. Z. He, B. Tian, Y. Zhang, and F. Gao, “Characteristic-based and interface-sharpening algorithm for high-order simulations of immiscible compressible multi-material flows,” J. Comput. Phys. 333, 247 (2017).