Abstract. In this paper, we construct exact traveling wave solutions of various kinds of partial differential equations arising in mathematical science by the system technique. Further, the Painlevé test is employed to investigate the integrability of the considered equations. In particular, we describe the behaviors of the obtained solutions under certain constraints.

1. Introduction

Most of physical phenomena in mathematical physics and engineering can be described by nonlinear partial differential equations. So, a great deal of attention has been paid towards both exact and numerical solutions of these problems. Thus, it would be useful to obtain a mathematical algorithm for determining the exact solutions of nonlinear partial differential equations. Many powerful methods are proposed to obtain exact traveling wave solutions of nonlinear evolution equations such as differential transform method [1, 2], extended Jacobi elliptic function expansion method [3, 4], factorization method [5], tanh-expansion method [6, 7, 8], \((G'/G)\)-expansion method [9, 10, 11], Kudryashov method [12, 13, 14, 15, 16, 17, 18]. On the other hand, the exact traveling wave solutions have a great significance to reveal the internal mechanism of physical phenomena. Apart from physical importance, the closed-form solutions of nonlinear partial differential equations assist the numerical solvers to compare the correctness of their results and help them in the stability analysis.
In this paper, we employ the system technique to obtain exact solutions for nonlinear partial differential equations that contain exponential function based on a suitable choice of parameters through the Painlevé test [19, 20, 21, 22]. The aim of this paper is to obtain more exact explicit solutions and to analyze the motions of exact solutions as the values of parameters and proper coefficients about the equal width wave equation and the (2+1)-dimensional Maccari’s system.

This paper is organized as follows. In Section 2, we describe the Painlevé test and algorithm for the system technique to find exact solutions of general nonlinear partial differential equations. In Section 3, we represent new exact solutions of the equal width wave equation and the (2+1)-dimensional Maccari’s system and show some graphs of them. Finally, some conclusions are given.

2. Algorithm for the system technique with the Painlevé test

In this section, we present an outline of the proposed approach for finding exact solutions of nonlinear partial differential equations. Suppose that a nonlinear partial differential equation, say in the variables \( x_1, x_2, \ldots, x_n \) and \( t \), is given by

\[
P(u, u_t, u_{x_1}, \ldots, u_{x_n}, u_{tt}, u_{x_1x_1}, u_{x_1x_2}, \ldots) = 0,
\]

where \( u = u(x_1, x_2, \ldots, x_n, t) \) is an unknown function, \( P \) is a polynomial in \( u \) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Combining the independent variables \( x_1, x_2, \ldots, x_n \) and \( t \) into one traveling wave variable \( \eta = k_1x_1 + k_2x_2 + \cdots + k_nx_n + \omega t. \) Using the wave transformation

\[
u(x_1, x_2, \ldots, x_n, t) = u(\eta), \eta = \sum_{i=1}^{n} k_ix_i + \omega t,
\]

we can convert Eq.(1) into an ordinary differential equation for \( u = u(\eta) \)

\[
Q(u, u', u'', u''', \ldots) = 0.
\]

where \( u' = \frac{du}{d\eta}, u'' = \frac{d^2u}{d\eta^2}, u''' = \frac{d^3u}{d\eta^3}, \ldots \)

To find solution \( u \) explicitly, we consider the following two steps:

**Step 1.** Now, we employ the Painlevé test to investigate integrability of nonlinear partial differential equations. The Painlevé test has three
steps. At the first step, we find the pole order of solutions. For this purpose, we substitute the expression

\[ u = \frac{a_0}{\eta^p}, \]

into the leading members of Eq.(3). Here the leading members consist of one highest order derivative term and one highest nonlinear term.

As a result of application of the first step, the Painlevé test of Eq.(3) can be continued if the value of the power of the pole order \( p \) is integer.

At the next step, we obtain the Fuchs indices of the expansion for the solution in the Laurent series. For this purpose, we use the expression

\[ u = \frac{a_0}{\eta^p} + b\eta^{j-p}, \]

where \( b \) is the coefficient of the expansion for the solution in the Laurent series which cannot be determined. The Fuchs indices can be obtained by using the expression (5) into the equation with the leading members again and equating expressions at the same degrees of \( \eta \) to zero and so we obtain the Fuchs indices.

At the final step, we substitute the expansion of the general solution in the Laurent series with undermined coefficient in the form

\[ u(\eta) = \frac{a_0}{\eta^p} + \frac{a_1}{\eta^{p-1}} + \cdots + a_j\eta^{j-p} + \cdots \]

into Eq.(3). Now, we check the existence of the arbitrary constants in the Laurent series of the general solution for the considered equation. More precisely, when there are arbitrary coefficients in the Laurent series, we have the necessary condition for the integrable nonlinear differential equation. Thus, we can obtain arbitrary coefficients in the expansion. Based on the results of the expansion for the solution in the Laurent series, we can have conclusions regarding closed-form solutions by using mathematical tools to find exact solution of nonlinear partial differential equations.

**Step 2.** Firstly, suppose that the solution of Eq.(3) can be expressed by a polynomial in \( \left( \frac{F(\eta)}{G(\eta)} \right) \) as follows:

\[ u(\eta) = \sum_{i=0}^{m} A_i \left( \frac{F(\eta)}{G(\eta)} \right)^i, \]

where \( F(\eta) \) and \( G(\eta) \) are satisfying the following system

\[ F'(\eta) = p F(\eta), \]

\[ G'(\eta) = p F(\eta) + q G(\eta), \]
and $A_m, A_{m-1}, \ldots, A_0, k_1, \ldots, k_n$ and $\omega$ are constants to be determined later, $A_m \neq 0$, the unwritten part in (7) is also a polynomial in $(F(\eta)/G(\eta))$, but the degree of which is generally equal to or less than $m - 1$, the positive integer $m$ can be determined by considering the homogeneous balance between the highest order derivatives of term and nonlinear term appearing in Eq.(3).

The system (8) admits the following solutions

$$
\left( \frac{F(\eta)}{G(\eta)} \right) = \frac{p - q}{p - q \exp\{- (p - q)\eta\}},
$$

where $p$ and $q$ are nonzero constants with $p \neq q$, which shows in appendix.

Secondly, by substituting (7) into Eq.(3), collecting all terms with the same order of $(F(\eta)/G(\eta))$ together, the left-hand side of Eq.(3) is converted into another polynomial in $(F(\eta)/G(\eta))$ using derivatives in Remark 2.2. Equating each coefficient of this polynomial to zero, we get a set of algebraic equations for $A_m, A_{m-1}, \ldots, A_0, k_1, \ldots, k_n$ and $\omega$.

Lastly, assuming that the coefficients $A_m, A_{m-1}, \ldots, A_0, k_1, \ldots, k_n$ and $\omega$ can be obtained by solving the algebraic equations in the above, since the general solutions of the system (8) have been known for us, then by substituting $A_m, A_{m-1}, \ldots, A_0, k_1, \ldots, k_n$ and $\omega$ into (7), we have more new exact solutions of the nonlinear evolution equation (1) by the wave transformation (2).

**Remark 2.1.** The integrability conditions of the system, given by Eq.(8), have been extensively discussed in the literature as the following, and so the relations among the coefficients of the system involve two constraints as follows: $p \neq q$ and $p = q$. Firstly, we consider the solution of Eq.(8) for $p \neq q$. Without loss of generality, assuming $F(0) = 1$ and $G(0) = 1$. In the first differential equation of Eq.(8), the solution $F(\eta)$ is given by

$$
F(\eta) = e^{p\eta}.
$$

Substituting (10) into the second differential equation of Eq.(8), we get the nonlinear ordinary differential equation

$$
G'(\eta) - q G(\eta) = pe^{p\eta}.
$$

Then, we can find the solution of Eq.(11) is given by

$$
G(\eta) = e^{p\eta} \left[ \frac{p}{p - q} e^{-(p-q)\eta} - \frac{q}{p - q} \right].
$$
Combining (10) and (12), we obtain the following proposed function;

$$\left( \frac{F(\eta)}{G(\eta)} \right) = \frac{p - q}{p - q \exp\{- (p - q)\eta\}},$$

where $p$ and $q$ are arbitrary nonzero constants with $p \neq q$.

At the second constraint, we have another proposed function;

$$\left( \frac{F(\eta)}{G(\eta)} \right) = \frac{1}{\eta + 1}.$$  

Remark 2.2. The following higher derivatives are useful for equating the expressions at the same degrees of $\left( \frac{F}{G} \right)$ to zero in Eq.(3); $\left( \frac{F}{G} \right)' = (p - q)(F/G) - p(F/G)^2$, $\left( \frac{F}{G} \right)'' = (p - q)^2(F/G) - 3p(p - q)(F/G)^2 + 2p^2(F/G)^3$, $\left( \frac{F}{G} \right)''' = (p - q)^3(F/G) - 6p(p - q)(F/G)^2 + 2p^2(p - q + 1)(F/G)^3 - 6p^3(F/G)^4$, and so on.

3. Exact solutions of nonlinear evolution equations

3.1. The equal width wave equation

The equal width wave equation

$$u_t + uu_x - u_{xxt} = 0$$

was suggested by Morrison et al [23] to be used as a model partial differential equation for the simulation of one-dimensional wave propagation in a nonlinear medium with a dispersion process. The equal width wave equation is an alternative description of the nonlinear dispersive waves to the more usual Korteweg de Vries equation. It has been shown to have solitary wave solutions and to govern a large number of important physical phenomena such as shallow water waves and plasma waves [24, 25].

By applying the wave transformation

$$u(x, t) = u(z), z = kx + \omega t,$$

Eq.(15) can be written in the form

$$\omega u_z + ku_z - k^2\omega u_{zzz} = 0.$$  

Integrating Eq.(17) with respect to $z$ and then taking the integration constant as zero, we yield

$$\omega u + \frac{1}{2} k u^2 - k^2\omega u_{zz} = 0.$$
For checking the integrability of Eq.(18, we need to choice the leading members in (18) as follows

\[ \frac{1}{2}ku^2 - k^2\omega u_{zz} = 0, \]

and then, firstly, substituting \( u \) from (4) into the leading members (19), we can obtain the values \( a_0 \) and \( p \) as \((a_0, p) = (12k\omega, 2)\). The general solution of Eq.(18) has the pole of the second order.

From the second step of the Painlevé test, substituting

\[ u = \frac{12k\omega}{z^2} + bz^{j-2}, \]

into Eq.(19) we obtain the Fuchs indices in the form

\[ j_1 = -1, j_2 = 6. \]

The index \( j_1 = -1 \) corresponds a coefficient \( a_0 \), but for the further consideration we have to substitute the expansion of the solution in the Laurent series in the form

\[ u = \frac{12k\omega}{z^2} + \frac{a_1}{z} + a_2 + a_3z + a_4z^2 + a_5z^3 + a_6z^4, \]

into Eq.(18). Equating coefficients at different powers of \( z \) to zero, we have the following coefficients of (22);

\[ a_1 = 0, a_2 = -\frac{\omega}{k}, a_3 = 0, a_4 = a_4, a_5 = 0. \]

\[ 2\omega a_4 + ka_3^2 + 2ka_1a_5 + 2ka_2a_4 = 0. \]

where \( k \) and \( \omega \) are none-zero constants.

Eq.(18) passes the Painlevé test if Eq.(24) equals to zero. So, we can obtain exact solutions of Eq.(18) using the proposed technique.

Let us use the system technique (8) for finding the exact solution of Eq.(17) by the balancing property. We are looking for the solution of Eq.(17) in the form

\[ u = A_0 + A_1 \left( \frac{F(\eta)}{G(\eta)} \right) + A_2 \left( \frac{F(\eta)}{G(\eta)} \right)^2, \]

where \( A_0, A_1 \) and \( A_2 \) are the coefficients which are obtained by the system technique (8).

Using the wave transformation, six exact solutions of Eq.(15) can be written as follow:

\[ u_1(x, t) = \pm 4\omega qi \mp \frac{24\omega qi}{1 + \exp\{2qz\}} \pm \frac{24\omega qi}{[1 + \exp\{2qz\}]^2}. \]
where $p = -q$ and a traveling wave variable is $z = \pm \frac{i}{2q} x + \omega t$.

\[
\begin{align*}
  u_2(x, t) &= \pm \frac{2\sqrt{6}}{3} \omega q i \pm \frac{8\sqrt{6}}{9} \frac{\omega q i}{\frac{2}{3} - \exp\left\{\frac{q}{3} z\right\}} \\
               &\pm \frac{8\sqrt{6}}{27} \frac{\omega q i}{\left[\frac{2}{3} - \exp\left\{\frac{q}{3} z\right\}\right]^2}
\end{align*}
\]  

(27)

where $p = \frac{2q}{3}$ and a traveling wave variable is $z = \pm \frac{\sqrt{6} i}{2q} x + \omega t$ and a relation of constants is, and

\[
\begin{align*}
  u_3(x, t) &= \pm \frac{9\sqrt{6}}{4} \frac{\omega q i}{\frac{3}{2} - \exp\left\{-\frac{q}{2} z\right\}}^2
\end{align*}
\]  

(28)

where $p = \frac{3q}{2}$ and a traveling wave variable is $z = \pm \frac{\sqrt{6} i}{3q} x + \omega t$, and

\[
\begin{align*}
  u_4(x, t) &= \mp \sqrt{6} \omega q \pm \frac{9\sqrt{6}}{4} \frac{\omega q}{\frac{3}{2} - \exp\left\{-\frac{q}{2} z\right\}}^2
\end{align*}
\]  

(29)

where $p = \frac{3q}{2}$ and a traveling wave variable is $z = \pm \frac{\sqrt{6} i}{3q} x + \omega t$, and

\[
\begin{align*}
  u_5(x, t) &= \mp \frac{8\sqrt{6}}{9} \frac{\omega q}{\frac{2}{3} - \exp\left\{\frac{q}{3} z\right\}} \pm \frac{8\sqrt{6}}{27} \frac{\omega q}{\frac{2}{3} - \exp\left\{\frac{q}{3} z\right\}}^2
\end{align*}
\]  

(30)

where $p = \frac{2q}{3}$ and a travelling wave variable is $z = \pm \frac{\sqrt{6} i}{2q} x + \omega t$, and

\[
\begin{align*}
  u_6(x, t) &= \pm \frac{24\omega q}{1 + \exp\{2q z\}} \mp \frac{24\omega q}{\left[1 + \exp\{2q z\}\right]^2}
\end{align*}
\]  

(31)

where $p = -q$ and a traveling wave variable is $z = \mp \frac{1}{2q} x + \omega t$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{soliton_profiles.png}
\caption{Profiles of a soliton traveling wave solution (31).}
\end{figure}
Fig. 1 represents the motions of a soliton traveling wave solution of the solution (31) under parameter $q = 0.02$: (a) $\omega = 0.1$, (b) $\omega = 1$, (c) $\omega = 2$, respectively. In Fig. 1, we known that the motions of the solution (31) are traveling as the speed values $\omega$.

### 3.2. New exact solutions of the (2+1)-dimensional Maccari’s system

Consider the following (2+1)-dimensional Maccari’s system in the form

\begin{align}
iu_t + u_{xx} + uv &= 0, \\
v_t + v_y + (|u|^2)_x &= 0. \tag{32}\end{align}

Maccari derived a similar system from the Kadomtsev-Petviashvili equation by using asymptotically exact reduction technique based Fourier expansion and spatiotemporal rescaling [26]. The coupled Maccari’s system is a nonlinear partial differential equations which are used to describe the motion of the isolated wave, location in a small part of space, hydrodynamic plasma physics, nonlinear optics, fluid mechanics, quantum field theory, etc. Zhang used Exp-function method for seeking exact solutions of the Maccari’s system [27].

In order to obtain the traveling wave solutions of Eq.(32), we suppose that

\begin{align}
u(x, y, t) &= U(x, y, t)e^{i\omega(x, y, t)}, \\
v(x, y, t) &= V(x, y, t), \tag{33}\end{align}

where $\omega(x, y, t) = \alpha x + \beta y + \gamma t$ and $\alpha, \beta$ and $\gamma$ are constants to be determined later. By substituting Eq.(33) into Eq.(32), we obtain

\begin{align}
i(U_t + 2\alpha U_x) + U_{xx} - (\alpha^2 + \gamma)U + UV &= 0, \\
V_t + V_y + (U^2)_x &= 0. \tag{34}\end{align}

To look for the travelling wave solutions of Eq.(34), we consider the transformation

\begin{align}
U(x, y, t) &= u(\eta), V(x, y, t) = v(\eta), \eta = x + y - 2\alpha t. \tag{35}\end{align}

Now, Eq.(34) can be converted to the following ordinary differential system

\begin{align}
u'' - (\gamma + \alpha^2)u + uv &= 0, \\
(1 - 2\alpha)v' + (u^2)' &= 0, \tag{36}\end{align}

where $u' = \frac{du}{d\eta}, u'' = \frac{d^2u}{d\eta^2}, v' = \frac{dv}{d\eta}$. 
Integrating the second equation in Eq.(36) with respect to \( \eta \) and neglecting the constant of integration, we obtain
\[
(37) \quad v = -\frac{1}{1 - 2\alpha} u^2.
\]

Substituting (37) into the first equation of Eq.(36), we obtain
\[
(38) \quad (1 - 2\alpha)u'' - (1 - 2\alpha)(\alpha^2 + \gamma)u - u^3 = 0,
\]
where \( u'' = \frac{d^2u}{d\eta^2} \).

At the first hand, by substituting \( u = a_0 \eta^p \) into the equation with the leading members from Eq.(38)
\[
(39) \quad (1 - 2\alpha)u'' - u^3 = 0,
\]
we find the value \( a_0 \) and \( p: (a_0, p) = (\pm \sqrt{2 - 4\alpha}, 1) \). The general solution of Eq.(38) has the pole of the first order.

At the second hand, by substituting
\[
(40) \quad u = \pm \frac{\sqrt{2 - 4\alpha}}{\eta} + b\eta^{j-1},
\]
into Eq.(39), we can obtain the Fuchs indices in the form
\[
(41) \quad j_1 = -1, j_2 = 4.
\]
The index \( j_1 = -1 \) corresponds the arbitrary constant \( a_0 \), but for the further consideration we have to substitute the expansion of the solution in the Laurent series in the form
\[
(42) \quad u \simeq \pm \frac{\sqrt{2 - 4\alpha}}{\eta} + a_1 + a_2\eta + a_3\eta^2 + a_4\eta^3,
\]
into Eq.(38). Equating expressions at different powers of \( \eta \) to zero, we have the following coefficients:
\[
(43) \quad a_1 = 0, a_2 = \mp \frac{(\alpha^2 + \gamma)\sqrt{2 - 4\alpha}}{6}, a_3 = 0,
\]
\[
\pm \alpha^4\sqrt{2 - 4\alpha} \pm \alpha^2\gamma\sqrt{2 - 4\alpha} \pm \gamma^2\sqrt{2 - 4\alpha} \mp \frac{\alpha^5\sqrt{2 - 4\alpha}}{6} \mp \frac{\alpha^3\gamma\sqrt{2 - 4\alpha}}{3} + \alpha^4(2 - 4\alpha)^{3/2} \mp \frac{\alpha^2\gamma(2 - 4\alpha)^{3/2}}{6} \mp \frac{\alpha^2\gamma(2 - 4\alpha)^{3/2}}{3} = 0,
\]
where \( \alpha \) and \( \gamma \) are non-zero constants.

Eq.(38) passes the Painlevé test if Eq.(44) equals to zero. So, there is the expansion of the solution of Eq.(38) in the Laurent series but with
only one arbitrary constant \( a_6 \). So we can find exact solutions of Eq.(38) using the proposed system approach.

Let us find the exact solutions of Eq.(38). By balancing the highest order derivative term \( u'' \) with the nonlinear term \( u^3 \) in Eq.(38), we get the balancing order \( m = 1 \). Suppose that the solution of Eq.(38) can be expressed in the following form

\[
 u = A_0 + A_1 \left( \frac{F(\eta)}{G(\eta)} \right),
\]

(45)

where \( (F(\eta)/G(\eta)) \) is the exponential-type function as follows

\[
 \left( \frac{F(\eta)}{G(\eta)} \right) = \frac{p - q}{p - q \exp\{- (p - q) \eta \}}
\]

(46)

where \( F(\eta) \) and \( G(\eta) \) satisfy the system (8), and \( A_0, A_1, \alpha \) and \( \gamma \) are constants to be determined later.

By taking the expression (45) and the system (8), we can obtain the derivative \( u'' \) expressed via the function \( (F(\eta)/G(\eta)) \) in Remark 2.2. Substituting \( u \) and \( u'' \) into Eq.(38) and equating to zero the expressions with the same degree of \( (F(\eta)/G(\eta)) \), we obtain three exact solutions at different values of parameters of Eq.(38) with substitutions (35) and (45) and then, by a transformation (33), we can obtain the following exact solutions of Eq.(32).

When \( p = -q \) and \( \gamma = -2q^2 - \alpha^2 \), the exact solution of Eq.(32) can be written as

\[
 u_{11}(x, y, t) = \pm q \sqrt{2 - 4\alpha Q_{11}(x, y, t)} e^{\omega(x, y, t)i},
\]

\[
 v_{11}(x, y, t) = -2q^2 Q_{11}(x, y, t) e^{2\omega(x, y, t)i},
\]

(47)

where \( Q_{11}(x, y, t) = \frac{2}{1 + \exp\{2q(x + y - 2\alpha t)\}} + 1 \) and \( \omega(x, y, t) = \alpha x + \beta y - (2q^2 + \alpha^2)t \).

For \( p = \frac{q}{2} \) and \( \gamma = -\frac{q^2}{2} - \alpha^2 \), the exact traveling wave solution of Eq.(32) can be written as

\[
 u_{12}(x, y, t) = \pm q \sqrt{\frac{1}{2} - \alpha Q_{12}(x, y, t)} e^{\omega(x, y, t)i},
\]

\[
 v_{12}(x, y, t) = -\frac{q^2}{2} Q_{12}(x, y, t) e^{2\omega(x, y, t)i},
\]

(48)

where \( Q_{12}(x, y, t) = \frac{-1}{1 - 2\exp\{\frac{1}{2}(x + y - 2\alpha t)\}} + 1 \) and \( \omega(x, y, t) = \alpha x + \beta y - (\frac{q^2}{2} + \alpha^2)t \).
For \( p = 2q \) and \( \gamma = -2q^2 - \alpha^2 \), the exponential-type solution of Eq.(32) is given by
\[
\begin{align*}
\frac{u_{13}(x, y, t)}{v_{13}(x, y, t)} = & \pm q \sqrt{8 - 16 \alpha} Q_{13}(x, y, t) e^{\omega(x, y, t)i}, \\
\omega(x, y, t) = & \alpha x + \beta y - (2q^2 + \alpha^2)t.
\end{align*}
\]

where \( Q_{13}(x, y, t) = \frac{1}{2 - \exp \left(-q(x+y-2\alpha t)\right)} \) and \( \omega(x, y, t) = \alpha x + \beta y - (2q^2 + \alpha^2)t \).

We discuss about the behaviors of the obtained solutions as follows.
When \( y = 0 \), the obtained solutions \((u_{12}, v_{12})\) are solitons traveling wave solutions which keeps its identity upon interacting with other solitons.

Fig. 2 and Fig. 3 represent the regular motions of the solution \((u_{12}, v_{12})\) of Eq.(32) under \( q = 0.85, \alpha = -2.3, y = 0 \) with \(-1 \leq x \leq 1\) and \(0 \leq t \leq 5\) as the real and the imaginary part, respectively. In particular, the solitary traveling wave solutions \((u_{12}, v_{12})\) are represented according to the variation of the physical parameters. Therefore, if we choose the particular values of the physical parameters, some of the obtained solutions coincide with some of the particular solutions obtained by other methods.

\[
\text{Figure 2. Profiles of the exact traveling wave solution } u_{12}(x, 0, t) \text{ of (48) under parameters: } q = 0.85, \alpha = -2.3, y = 0.
\]

4. Conclusion

In this paper, we have focus on finding exact traveling wave solutions of the equal width wave equation and the Maccari’s system easily by the novel system technique, which has been successfully applied to find more general traveling wave solutions of nonlinear partial differential equations. From the obtained solutions, it is noted that when we take
Figure 3. Profiles of the exact traveling wave solution $v_{12}(x,0,t)$ of (48) under parameters: $q = 0.85, \alpha = -2.3, y = 0$.

the particular values for the physical parameters, then we provided the behaviors of new exact solutions. Moreover, the Painlevé test is implemented to investigate the integrability of the considered equations.

References

[1] A. S. V. Ravi Kanth and K. Aruna, Differential transform method for solving linear and non-linear systems of partial differential equations, Physics Letters, A372 (2008) 6896-6898.

[2] A. S. V. Ravi Kanth and K. Aruna, Differential transform method for solving the linear and nonlinear Klein-Gordon equation, Computer Physics Communication, (2009) 708-711.

[3] H. Trikia and A.-M. Wazwaz, Dark Solitons for a Generalized Korteweg-de Vries Equation with Time-Dependent Coefficients, Z. Naturforsch, 66a (2011) 199-204.

[4] Q. Wang, Y. Chen and HQ. Zhang, A new Jacobi elliptic function rational expansion method and its application to (1+1)-dimensional dispersive long wave equation, Chaos Solitons Fract., 23 (2005) 477-483.

[5] H. A. Abdusalam and E. S. Fahmy, Traveling wave solutions for nonlinear wave equation with dissipation and nonlinear transport term through factorizations, Int. J. Comput. Meth., bf 4(4) (2007) 645-651.

[6] W. Malfliet and W. Hereman, The tanh method: I. Exact solutions of nonlinear evolution and wave equations, Physica Scripta, 54 (1996) 563-568.

[7] E. J. Parkes and B. R. Duffy, An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations, Computer Physics Communications, 98 (1996) 288-300.

[8] A. Biswas, Solitary wave solution for the generalized Kawahara equation, Applied Mathematics Letters, 22 (2009) 208-210.

[9] M. L. Wang, X. Li and J. Zhang, The $\left(G'/G\right)$-expansion method and evolution equation in mathematical physics, Phys. Lett. A, 372 (2008) 417-421.

[10] A. Biswas, Solitary wave solution for the generalized Kawahara equation, Appl. Math. Lett., 22 (2009) 208-210.
Some explicit solutions of nonlinear evolution equations

[11] H. Kim and R. Sakthivel, *Travelling wave solutions for time-delayed nonlinear evolution equations*, Applied Mathematics Letters, 23 (2010) 527-532.

[12] K. A. Kudryashov, *Exact solutions of the generalized Kuramoto-Sivashinsky equation*, Phys. Lett. A, 147 (1990) 287-291.

[13] N. A. Kudryashov, *Simplest equation method to look for exact solutions of nonlinear differential equations*, Chaos, Solitons & Fractals, 24 (2005) 1217-1231.

[14] N. K. Vitanov, *Application of simplest equations of Bernoulli and Riccati kind for obtaining exact traveling-wave solutions for a class of PDEs with polynomial nonlinearity*, Communications in Nonlinear Science and Numerical Simulation, 15 (2010) 2050-2060.

[15] H. Kim, J. -H. Bae and R. Sakthivel, *Exact Travelling Wave Solutions of two Important Nonlinear Partial Differential Equations*, Z. Naturforsch, 69a (2014) 155-162.

[16] J. H. Choi, H. Kim and R. Sakthivel, *Exact solution of the wick-type stochastic fractional coupled KdV equations*, J. Math. Chem., 52 (2014) 2482-2493.

[17] M. M. Kabir, A. E. Y. K. Aghdam and A. Y. Koma, *Modified Kudryashov method for finding exact solitary wave solutions of higher order nonlinear equations*, Math. Methods Appl. Sci., 34(2) (2011) 213-219.

[18] S. M. Ege and E. Misirli, *The modified Kudryashov method for solving some fractional-order nonlinear equations*, Adv. Differ. Eq., 135(1) (2014).

[19] N. A. Kudryashov and A. S. Zakharchenko, *Painlevé analysis and exact solutions of a predator-prey system with diffusion*, Math. Metho. Appl. Sci., DOI: 10.1002/mma.3156 (2014).

[20] N. A. Kudryashov, *Painlevé analysis and exact solutions of the fourth-order equation for description of nonlinear waves*, Commun. Nonlinear Sci. Numer. Simulat., 28 (2015) 1-9.

[21] A. M. Abourabia, K. M. Hassan and E. S. Selim, *Painlevé test and some exact solutions for (2+1)-dimensional modified Korteweg-de Vries-Bergers equation*, Int. J. Comput. Meth., 10(3) (2013) 1250058.

[22] Y. Liu, F. Duan and C. Hu, *Painlevé property and exact solutions to a (2 + 1)-dimensional KdV-mKdV equation*, J. of Appl. Math. and Phys., 3(6) (2015) 36083.

[23] P. J. Morrison, J. D. Meiss and J. R. Cary, *Scattering of Regularized-Long-Wave Solitary Waves*, Physica D. Nonlinear Phenomena, (1984) 324-336.

[24] A. Bekir and E. Yusufoglu, *Numerical simulation of equal-width wave equation*, Computers and Mathematics with Applications, 54 (2007) 1147-1153.

[25] K. R. Raslan, *Exact Solitary Wave Solutions of Equal Width Wave and Related Equations Using a Direct Algebraic Method*, International Journal of Nonlinear Science, 6(3) (2008) 246-254.

[26] A. Maccari, *The Kadomtsev-Petviashvili Equation as a Source of Integrable Model Equations*, J. Math. Phys., 37 (1996) 6207-6212.

[27] S. Zhang, *Exp-function method for solving Maccari’s system*, Phys. Lett. A, 371 (2007) 65-71.
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