Spin-Isospin Response Functions
and the Effects of the $\Delta$-Hole Configurations
in Finite Nuclei

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Effects of the delta-isobar ($\Delta$) mixing on the spin-isospin response functions in finite nuclei are studied in the quasi-elastic region.

A method to calculate the response function for a finite system composed of nucleon ($N$) and $\Delta$ is formulated in a ring approximation. It is designed to treat the $\Delta$-related Landau-Migdal parameters, $g'_{N\Delta}$ and $g'_{\Delta\Delta}$, and the nucleon parameter $g'_{NN}$, independently, so that the universality ansatz, $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta}$, is removed.

We calculated the isovector spin-longitudinal and -transverse response functions, $R_L$ and $R_T$, with and without the $\Delta$-mixing. Inclusion of $\Delta$ enhances $R_L$ but reduces $R_T$ for ordinary interactions. Dependence of $R_{L,T}$ on $g'_{NN}$ and $g'_{N\Delta}$ is investigated. Decomposition into the process-decomposed response functions, $R_{[NN]}$, $R_{[N\Delta]}$ and $R_{[\Delta\Delta]}$, is very elucidative to see the $\Delta$ effects, which are found to be mainly governed by $R_{[N\Delta]}$ and sensitive to $g'_{N\Delta}$.

The isovector spin-transverse response function $R_T^{(e,e')}$ obtained by $(e,e')$ is calculated by various effective interactions and compared to each other as well as experimental data.

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I. INTRODUCTION

The spin-isospin properties of nuclei are a long standing and still very interesting subject of nuclear physics [1,2]. New theoretical and experimental developments are seen in the study of nuclear spin-isospin response functions over the last decade. They promoted many interesting problems and active researches in this fields [2].

In the analysis of the responses in the quasi-elastic region, nuclear currents with the spin and isospin degrees of freedom are usually separated into the isovector spin-longitudinal and -transverse components. They are characterized by the operators, $\tau(\sigma \cdot \hat{q})$ and $\tau(\sigma \times \hat{q})$, respectively, where $q$ is the transferred momentum to the nucleus and $\hat{q} \equiv q/q$. The spin-transitional response function $R_T(q, \omega)$ has long been observed in a wide range of $q$ and the transferred energy $\omega$ by electron scattering [3–7]. However, hadronic probes are needed to study the spin-longitudinal response function $R_L(q, \omega)$.

Owing to the great progress of the experimental technique, complete measurement of the polarization transfers $D_{ij}$ was first carried out at LAMPF [8,9] for the quasi-elastic $(\vec{p}, \vec{p}')$ scattering, from which they extracted the ratio $R_L/R_T$ by use of an eikonal approximation. Measurement of $(\vec{p}, \vec{n})$ reaction is more difficult but more preferable because it can exclusively extract the isovector part. Recently LAMPF group [10–12] observed $D_{ij}$ of the quasi-elastic $(\vec{p}, \vec{n})$ reaction and extracted $R_L/R_T$ in a similar way. An interesting finding of these experiments is that the ratio is close to or less than unity in the quasi-elastic region. This was surprising because the standard nuclear model predicted that the ratio was much larger than unity at relatively low $\omega$. Based on the random-phase approximation (RPA) in nuclear matter, Alberico et al. [13] pointed out that $R_L$ is enhanced and its peak position is shifted downwards (softening), whereas $R_T$ is quenched and its peak is shifted upwards (hardening) around the transferred momentum $q = 1.75 \text{ fm}^{-1}$. Therefore the ratio $R_L/R_T$ extremely exceeds unity especially at relatively low $\omega$. However, such behavior has hardly been seen in the experiments.

This contradiction has been challenged from various aspects, such as finite size effect of nucleus [14–17], relativistic RPA approach [18], nuclear correlations beyond RPA [19–23], effects of absorptions and distortions [17,24–27], etc.. Among them here we investigate effects of $\Delta$-mixing and dependence on the nucleon particle-hole $(ph)$ and $\Delta$-hole $(\Delta h)$ effective interactions.

The analysis of Alberico et al. [13] took account of the $\Delta$ degree of freedom and used the $(\pi + \rho + g')$ model for the effective interaction, namely one-pion exchange + one-rho-meson exchange + the contact interaction specified by the Landau-Migdal parameters $g'$s. There appear three different $g'$s relevant to the interactions between $ph$ and $ph$, between $ph$ and $\Delta h$ and between $\Delta h$ and $\Delta h$. They are denoted by $g'_{NN}$, $g'_{N\Delta}$ and $g'_{\Delta\Delta}$, respectively.

For computational simplicity, most of previous works adopted the universality ansatz [28–29], $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta}$. However it has no theoretical foundation nor was supported by
various estimations [2]. For instance, the phenomenological analysis [30,31] yields $g'_{NN} \approx 0.6 \sim 0.7$. G-matrix calculations suggested $g'_{NN} \approx 0.5$ and $g'_{N\Delta} \approx 0.4$, but the induced interaction increases and the finite size effect decreases them [32–37]. A recent estimation by Brown et al. [38] gives $g'_{NN} \approx 1.0$, $g'_{N\Delta} \approx 0.33$ and $g'_{\Delta\Delta} \approx 0.5$.

Considering this situation, we developed a RPA formalism for the finite nucleus composed of $N$ and $\Delta$, in which the universality ansatz is not adopted. Then we investigated the effects of $\Delta$ and the $g'$ dependence of $R_L$ and $R_T$. Similar analysis was performed in the Fermi gas model by Shiino et al. [39]. However the model gives unreasonably large enhancement and softening of $R_L$ [17] and hence it is somewhat misleading. Here we perform a more realistic and detailed analysis for finite nuclei.

The response functions including $N$ and $\Delta$ constitute of three different components, $R^{[NN]}$, $R^{[N\Delta]}(= R^{[\Delta N]})$ and $R^{[\Delta\Delta]}$, which correspond to the processes depicted in Fig. 1. $R^{[NN]}$ represents the process in which the nuclear current first creates a $ph$ state and annihilates it at the end. $R^{[N\Delta]}$ corresponds to the process which start with the $\Delta h$ creation and end with the $ph$ annihilation, and $R^{[\Delta N]}$ corresponds to the inverse one. $R^{[\Delta\Delta]}$ represents those which start from the $\Delta h$ excitation and end with its annihilation. We call them the process-decomposed response functions.

We will see that the $\Delta h$ configuration is crucial for the enhancement of $R_L$ and plays some role for the quenching of $R_T$ for ordinary interactions. These effects come mainly from $R^{[N\Delta]}$, which is sensitive to $g'_{N\Delta}$. This indicates that we must avoid the universality ansatz and treat $g'$'s independently.

In many of the previous works with the universality ansatz, the decomposition did not appear explicitly. The response functions have implicitly been calculated by

$$R_{L,T} = R^{[NN]}_{L,T} + 2\frac{f_\Delta}{f_N} R^{[N\Delta]}_{L,T} + \left( \frac{f_\Delta}{f_N} \right)^2 R^{[\Delta\Delta]}_{L,T}$$

(1)

where $f_N$ and $f_\Delta$ are the $\pi NN$ and $\pi N\Delta$ coupling constants, respectively, and $f_\Delta = 2f_N$ is usually used. However, observed response functions depend on probes. For instance, the isovector spin-transverse response function $R_T^{(e,e')}$ observed by $(e, e')$ is approximated by

$$R_T^{(e,e')} = R^{[NN]}_T + 2\frac{f_{\gamma NN}}{f_{\gamma N\Delta}} R^{[N\Delta]}_T + \left( \frac{f_{\gamma N\Delta}}{f_{\gamma NN}} \right)^2 R^{[\Delta\Delta]}_T$$

(2)

where $f_{\gamma NN}^{IV}$ and $f_{\gamma N\Delta}$ specify the isovector magnetic coupling strength of $\gamma NN$ and $\gamma N\Delta$ vertices, respectively. We also check this coupling constant dependence.

In Sec. II we present a formalism for calculating the response functions under the ring approximation in the finite system which is composed of $N$ and $\Delta$. The finiteness is handled by the continuum RPA with the orthogonality condition [40,17,41]. The universality ansatz for $g'$ is removed. In Sec. III we show the effects of the $\Delta$ mixing on the response functions, and analyze them in the form of the process-decomposed components. The energy-weighted
and non-weighted sums are also discussed. In Sec. IV we investigate the effective interaction dependence of $R_{L,T}$ and $R^{[a,b]}_{L,T}$. We compare the results obtained by various values of $g'$. We also show some results with lighter $\rho$-meson effective mass $m^*_\rho$. In Sec. V we present our calculation of $R^{(e,e')}_{\gamma,NN}$ with experimental data. Its dependence on $g'$ and the ratio $f_{\gamma NN}/f_{\gamma N\Delta}$ is shown. The summary is given in Sec. VI.

II. FORMALISM

In this section, we formulate a method to calculate the response functions for a finite system composed of $N$ and $\Delta$ in the ring approximation. For simplicity, we only consider doubly (sub-)closed shell nuclei, so that the particle and the hole states are well defined and the spin of the ground-state is zero.

We express the spin and the isospin operators of $N$ ($\sigma$ and $\tau$) and the transition operators between $N$ and $\Delta$ ($S$ and $T$) in unified spherical tensor forms, $\sigma^s_{\mu}(ab)$ and $\tau^t_{\nu}(ab)$, as

$$\sigma^0_{0(ab)} \equiv \delta_{ab} \quad \text{for } s = 0,$$
$$\sigma^1_{1(NN)} \equiv \sigma^1_s, \quad \sigma^1_{1(\Delta N)} \equiv S_{\mu}, \quad \sigma^1_{1(N\Delta)} \equiv (-)^{\mu}S_{-\mu} \quad \text{for } s = 1,$$

with $a, b = N$ or $\Delta$, and in a similar way for $\tau^t_{\nu}(ab)$.

A. Spin-isospin polarization propagator

We define the spin-isospin (transition) current operators as

$$j_{s\mu}^{t\nu(ab)}(r) \equiv \sum_{k=1}^A j_{s\mu}^{t\nu(ab)}(r; r_k),$$
$$j_{s\mu}^{t\nu(ab)}(r; r_k) \equiv \left[\left(\tau^t_{\nu(ab)}\right)_k \left(\sigma^s_{\mu(ab)}\right)_k\right] \delta^3(r - r_k),$$

where $r_k$ is the position vector of the $k$-th particle in the nucleus and $j^{(ab)}$ operates only on the type $(b)$ particle. They are separated into the angle and radial parts of $r$ as

$$j_{s\mu}^{t\nu(ab)}(r) = \sum_{\ell J M} \sum_m \left(\ell m s \mu | J M\right) Y^\ell_m(\Omega_r) \left(\tau^t_{\nu(ab)}\right)_k \left(\sigma^s_{\mu(ab)}\right)_k \delta^3(r - r_k) \times j_{s\ell J M}^{t\nu(ab)}(r),$$

where

$$j_{s\ell J M}^{t\nu(ab)}(r) \equiv \sum_{k=1}^A j_{s\ell J M}^{t\nu(ab)}(r; r_k),$$
$$j_{s\ell J M}^{t\nu(ab)}(r; r_k) \equiv \frac{\delta(r - r_k)}{rr_k} \left(\tau^t_{\nu(ab)}\right)_k \left[\delta Y^\ell(\Omega_{r_k}) \otimes \left(\sigma^s_{\mu(ab)}\right)_k\right]_{s\ell J M}^{t\nu(ab)}.$$
We then introduce the spin-isospin polarization propagator \([4]\) as

\[
\Pi_{\ell s JM, \ell' s' J'M'}^{t\ell', t'\ell' (ab, cd)} (r, r'; \omega) = \langle \Psi_0 | \left[ \frac{-t\ell' (ab)}{J_{lsJM} (r)} - \frac{t'\ell' (dc)}{J_{lsJM} (r')} \right] \left[ \delta_{JJ'} \frac{t\ell' (ab)}{\omega - (H - E_0) + i\eta} + \frac{-t'\ell' (dc)}{\omega + (H - E_0) - i\eta} \right] | \Psi_0 \rangle
\]

where \(\tilde{j} \equiv j - \langle \Psi_0 | j | \Psi_0 \rangle\) is the current fluctuation and \(| \Psi_0 \rangle\) and \(E_0\) denote the ground-state of the nuclei and its energy, respectively. The second identity comes from the assumption that the total angular momentum of the ground-state is zero.

In the pure shell model without the residual interaction, the total Hamiltonian \(H\) is replaced by the uncorrelated one \(H^{(0)}\), the sum of the single-particle Hamiltonian. Then the uncorrelated polarization propagator is given by

\[
\Pi_{\ell s JM, \ell' s' J'M'}^{(0)t\ell', t'\ell' (ab, cd)} (r, r'; \omega) = \delta_{\ell\ell'} \delta_{s s'} \delta_{JJ'} \delta_{MM'} \delta_{\nu\nu'} \delta_{t t'} \Pi_{\ell s JM, \ell' s' J'M'}^{t\ell', t'\ell' (ab, cd)} (r, r'; \omega),
\]

where \(| \Phi_0 \rangle\) is the ground state of \(H^{(0)}\) and \(|p(\Delta)h\rangle\) denotes a \(p\hbar(\Delta h)\) state. Here \(\epsilon_h\) denotes the single-hole energy and \(\epsilon_p(\Delta)\) is the single-particle energy of the type \((a)\) particle.

In a continuum RPA \([42]\), the sum over the discrete and continuum particle states is carried out by use of a single-particle Green’s function,

\[
g^{(a)} (r, r'; \epsilon) = \langle r | \frac{1}{\epsilon - h^{(a)} + i\eta} | r' \rangle
\]

\[
= \sum_{\ell j m m'} Y_{\ell sjm}(\Omega_r) \eta_{m t} Y_{\ell sjm}^\dagger(\Omega_{r'}) \left[ Y_{\ell sjm}(\Omega_r) \eta_{m t} \right]^\dagger,\]

where \(h^{(a)}\) is the single-particle Hamiltonian of the type \((a)\) particle and \(Y_{\ell sjm}(\Omega_r) = \left[ Y^J(\Omega_r) \otimes \chi^s \right]^j_m\) with the spin function \(\chi^s\) and the isospin function \(\eta_{m t}\).

Using \(g^{(a)} (r, r'; \epsilon)\) and the single-hole wave function \(u_{n\beta} (r)\), we get \([4, 17]\)
where \( \alpha = (\ell_\alpha s_\alpha j_\alpha t_\alpha m^t_\alpha) \) and \( \beta = (\ell_\beta s_\beta j_\beta t_\beta m^t_\beta) \), and
\[
\mathcal{B}_{\ell s J}^{t\nu(ab)}(\alpha_1, \alpha_2) \equiv \left( t_1 m^t_1 \ | \ r^{t\nu(ab)}_\nu \ | \ t_2 m^t_2 \right) \sqrt{2j_1 + 1} \sqrt{2j_2 + 1} \\
\times \left\{ \begin{array}{ll}
\ell_1 & s_1 j_1 \\
\ell & s & J \\
\ell_2 & s_2 j_2
\end{array} \right\} \left( s_1 \| \sigma^{s(ab)} \| s_2 \right) \right.
\] (15)

with
\[
\left( s_1 \| \sigma^{s(ab)} \| s_2 \right) \equiv \left\{ \begin{array}{ll}
\delta_{s_1 s_2} \sqrt{2s_1 + 1} & \text{for } s = 0 \\
\sqrt{6} & \text{for } s = 1, \ s_1 = s_2 = \frac{1}{2} \\
2 & \text{for } s = 1, \ s_1 = \frac{3}{2}, \ s_2 = \frac{1}{2} \\
-2 & \text{for } s = 1, \ s_1 = \frac{1}{2}, \ s_2 = \frac{3}{2}
\end{array} \right.
\] (16)

B. Ring approximation

We take into account the nuclear correlation by the ring approximation. We assume that
the effective interaction is the charge-independent local two-body force, which is written as
\[
V^{(ab,cd)}(\mathbf{r} - \mathbf{r}'; \omega) = \sum_{ssts'j_Mj_M} \int_0^\infty r_1^2 dr_1 r_2^2 dr_2 j_M^{(ab)}(j_M^{(ab)}(r_1; \mathbf{r})W^{t_{(ab,cd)}_{j_Mj_M}}(j_M^{(ab,cd)}(r_1, r_2; \omega)j_M^{t_{(ab,cd)}(r_2; \mathbf{r}')}. \right.
\] (17)

Then the polarization propagator satisfies the ring equation [13],
\[
\Pi^{t\nu}_{j_Mj_M'j_M''}(r, r'; \omega) = \Pi^{(0)t\nu}_{j_Mj_M'j_M''}(r, r'; \omega) \\
+ \sum_{t_1 t_1 s_1 t_2 s_2} \int_0^\infty r_1^2 dr_1 r_2^2 dr_2 \Pi^{(0)t_{j_Mj_M}}(r, r_1; \omega)W^{t_{j_Mj_M}}(r_1, r_2; \omega)
\times \Pi^{t\nu}_{j_Mj_M'j_M''}(r_2, r''), \right.
\] (18)

where \( \Pi \) and \( W \) are the matrices with respect to the particle types,
\[
\Pi \equiv \begin{bmatrix}
\Pi^{(NN,NN)} & \Pi^{(NN,N\Delta)} & \Pi^{(N\Delta,N\Delta)} \\
\Pi^{(N\Delta,NN)} & \Pi^{(N\Delta,N\Delta)} & \Pi^{(N\Delta,N\Delta)} \\
\Pi^{(\Delta N,NN)} & \Pi^{(\Delta N,N\Delta)} & \Pi^{(\Delta N,\Delta N)}
\end{bmatrix}, \right.
\] (19)
\[
W \equiv \begin{bmatrix}
W^{(NN,NN)} & W^{(NN,N\Delta)} & W^{(NN,N\Delta)} \\
W^{(N\Delta,NN)} & W^{(N\Delta,N\Delta)} & W^{(N\Delta,N\Delta)} \\
W^{(\Delta N,NN)} & W^{(\Delta N,N\Delta)} & W^{(\Delta N,\Delta N)}
\end{bmatrix}. \right.
\] (20)
Using the symmetry of $W$, we introduce the grouped notation $W^{[\alpha\beta]}$ for $W^{(ab,cd)}$ as

$$W^{[NN]} = W^{(NN,NN)},$$
$$W^{[N\Delta]} = W^{[\Delta N]} \equiv W^{(NN,N\Delta)} = W^{(N\Delta,NN)} = W^{(\Delta N,NN)},$$
$$W^{[\Delta\Delta]} \equiv W^{(N\Delta,N\Delta)} = W^{(N\Delta,\Delta N)} = W^{(\Delta N,\Delta N)},$$

and correspondingly the grouped representation of $\Pi$ as

$$\Pi^{[NN]} \equiv \Pi^{(NN,NN)},$$
$$\Pi^{[N\Delta]} \equiv \Pi^{(NN,N\Delta)} + \Pi^{(NN,\Delta N)},$$
$$\Pi^{[\Delta N]} \equiv \Pi^{(N\Delta,NN)} + \Pi^{(\Delta N,NN)},$$
$$\Pi^{[\Delta\Delta]} \equiv \Pi^{(N\Delta,N\Delta)} + \Pi^{(N\Delta,\Delta N)} + \Pi^{(\Delta N,N\Delta)} + \Pi^{(\Delta N,\Delta N)}.$$  

Then we can reduce the dimension of $\Pi$ and $W$ in Eq. (18), and express them as

$$\Pi \equiv \begin{bmatrix} \Pi^{[NN]} & \Pi^{[N\Delta]} \\ \Pi^{[\Delta N]} & \Pi^{[\Delta\Delta]} \end{bmatrix}, \quad W \equiv \begin{bmatrix} W^{[NN]} & W^{[N\Delta]} \\ W^{[\Delta N]} & W^{[\Delta\Delta]} \end{bmatrix}. $$

The ring equation (18) is now symbolically written as

$$\Pi = \Pi^{(0)} + \Pi^{(0)} W \Pi,$$

with

$$\Pi^{(0)} = \begin{bmatrix} \Pi^{(0)}[NN] & 0 \\ 0 & \Pi^{(0)}[\Delta\Delta] \end{bmatrix}. $$

We also get the symmetry, $\Pi^{[N\Delta]} = \Pi^{[\Delta N]}$, since $W^{[N\Delta]} = W^{[\Delta N]}$.

C. Spin-longitudinal and -transverse modes

In this paper we restrict ourselves to the response functions for the isovector spin-dependent currents ($s = t = 1$). So we use the abbreviations,

$$j^{\nu(ab)}_{LJM} \equiv j_{t=1s=1JM}, \quad \Pi^{\nu(ab,cd)}_{LL'} \equiv \Pi^{t=1s=1t'=1s'=1}_{JJ'L'L''}. $$

We define the spin-longitudinal and -transverse currents, $j_L$ and $j_T$, in the momentum representation as

$$j_L^{\nu(ab)}(q) \equiv \sum_{k=1}^{A} j_L^{\nu(ab)}(q; r_k),$$
$$j_T^{\mu\nu(ab)}(q) \equiv \sum_{k=1}^{A} j_T^{\mu\nu(ab)}(q; r_k),$$

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From Eqs. (36), (37) and (42), they can be rewritten as

\[ j_L^{\nu(ab)}(q; r_k) = \sum_{\ell J M} F_{\ell J M}(\Omega_q) j_{\ell J M}^{\nu(ab)}(q; r_k), \]

\[ j_T^{\nu(ab)}(q; r_k) = \sum_{\ell J M} F_{\ell J M}(\Omega_q) j_{\ell J M}^{\nu(ab)}(q; r_k), \]

where

\[ F_{\ell J M}(\Omega_q) = [4\pi a_{\ell J} Y_{M}^J(\Omega_q)]^*, \]

\[ F_{\ell J M}(\Omega_q) = \sum_{KQ} 4\pi i \cdot b_{JK\ell} (-)^{M+\ell} \begin{pmatrix} J & K & 1 \\ M & Q & -\mu \end{pmatrix} Y_{Q}^{K}(\Omega_q) \]

\[ a_{\ell J} \equiv \langle J010 | \ell J \rangle, \]

\[ b_{JK\ell} \equiv \sqrt{6(2J+1)(2K+1)} a_{JK\ell} \begin{pmatrix} 1 & 1 & 1 \\ J & K & \ell \end{pmatrix}. \]

The current \( j(q; r_k) \) is related with \( j(r; r_k) \) defined in Eq. (3) as

\[ j_{\ell J K\ell}(q; r_k) = \int_{0}^{\infty} r^2 dr j_{\ell J K\ell}(r; r_k) j_{\ell}(qr), \]

with the \( \ell \)-th order spherical Bessel function \( j_{\ell}(qr) \).

In terms of these currents, we define the spin-longitudinal and -transverse polarization propagators as

\[ \Pi_L^{\nu(ab,cd)}(q, q'; \omega) = \langle \Psi_0 | j_L^{\nu(ab)}(q) \frac{1}{\omega - (H - E_0) + i\eta} j_L^{\nu(cd)\dagger}(q') \]

\[ -j_L^{\nu(cd)\dagger}(q') \frac{1}{\omega + (H - E_0) - i\eta} j_L^{\nu(ab)}(q) | \Psi_0 \rangle, \]

\[ \Pi_T^{\nu(ab,cd)}(q, q'; \omega) = \langle \Psi_0 | j_T^{\nu(ab)}(q) \frac{1}{\omega - (H - E_0) + i\eta} j_T^{\nu(cd)\dagger}(q') \]

\[ -j_T^{\nu(cd)\dagger}(q') \frac{1}{\omega + (H - E_0) - i\eta} j_T^{\nu(ab)}(q) | \Psi_0 \rangle. \]

From Eqs. (36), (37) and (42), they can be rewritten as...
\[ \Pi_{L}^{\nu(ab,cd)}(q, q'; \omega) = \sum_{JM'\ell'\nu} F_{JM'\ell'\nu}(\Omega_q) F_{JM}^{\ell}(\Omega_{q'}) \Pi_{JM'\ell'\nu}^{\nu(ab,cd)}(q, q'; \omega), \]
\[ \Pi_{T}^{\nu(ab,cd)}(q, q'; \omega) = \sum_{JM'\ell'\nu} F_{JM'\ell'\nu}(\Omega_q) F_{JM}^{\ell}(\Omega_{q'}) \Pi_{JM'\ell'\nu}^{\nu(ab,cd)}(q, q'; \omega), \]

where \( \Pi(q, q'; \omega) \) is related with \( \Pi(r, r'; \omega) \) defined in Eq. (9) as
\[ \Pi_{J\ell\nu}^{\nu(ab,cd)}(q, q'; \omega) = \int_{0}^{\infty} r^2 dr \ n_r(q) \Pi_{JM'\ell'\nu}^{\nu(ab,cd)}(r, r'; \omega) j_{\ell}(qr). \]  

D. \((\pi + \rho + g')\) model

For the effective interaction \([17]\), the \((\pi + \rho + g')\) model is commonly adopted. It gives
\[ V_{(ab,cd)}^{(r_1 - r_2; \omega)} = V_{L}^{(ab,cd)}(r_1 - r_2; \omega) + V_{T}^{(ab,cd)}(r_1 - r_2; \omega), \]

with
\[ V_{L}^{(ab,cd)}(r_1 - r_2; \omega) = \int \frac{d^3 q}{(2\pi)^3} e^{i\hat{q} \cdot (r_1 - r_2)} W_{L}^{(ab,cd)}(q, \omega) \cdot [\tau_1^{(ab)} \cdot \tau_2^{(cd)}] \cdot [\sigma_1^{(ab)} \cdot \hat{q}] \cdot [\sigma_2^{(cd)} \cdot \hat{q}], \]
\[ V_{T}^{(ab,cd)}(r_1 - r_2; \omega) = \int \frac{d^3 q}{(2\pi)^3} e^{i\hat{q} \cdot (r_1 - r_2)} W_{T}^{(ab,cd)}(q, \omega) \cdot [\tau_1^{(ab)} \cdot \tau_2^{(cd)}] \cdot [\sigma_1^{(ab)} \times \hat{q}] \cdot [\sigma_2^{(cd)} \times \hat{q}]. \]

In the grouped representation \([21)-(23)\), \( W_{L} \) and \( W_{T} \) are given \([13,39]\) as
\[ W_{L}^{\alpha\beta}(q, \omega) = \frac{\tilde{f}_\alpha f_\beta}{m_\pi^2} \left[ g_{\alpha\beta}(q) + \Gamma_\alpha(q, \omega) \Gamma_\beta(q, \omega) \frac{q^2}{\omega^2 - q^2 - m_\pi^2} \right], \]
\[ W_{T}^{\alpha\beta}(q, \omega) = \frac{\tilde{f}_\alpha f_\beta}{m_\pi^2} \left[ g_{\alpha\beta}(q) + C_{\alpha\beta}^\rho \Gamma_\alpha(q, \omega) \Gamma_\beta(q, \omega) \frac{q^2}{\omega^2 - q^2 - m_\rho^2} \right], \]

where \( \alpha \) and \( \beta \) denote \( N \) or \( \Delta \), and \( m_\pi \) and \( m_\rho \) are the pion and the \( \rho \)-meson masses, respectively. The coefficient \( C_{\alpha\beta}^\rho \equiv \frac{f_\rho f_\beta}{m_\pi^2} / f_\alpha f_\beta \) is the ratio of the \( \rho \)-meson exchange coupling to the \( \pi \)-meson one. The following vertex form factors are used,
\[ \Gamma_\alpha(q, \omega) = \frac{m_\pi^2 - \Lambda_\pi^2}{\omega^2 - q^2 - \Lambda_\pi^2}, \quad \Gamma_\alpha(q, \omega) = \frac{m_\rho^2 - \Lambda_\rho^2}{\omega^2 - q^2 - \Lambda_\rho^2}. \]

The Landau-Migdal parameters \( g'(q) \)’s depend on \( q \) so weakly that we neglect their \( q \) dependence in the region of \( q \leq 3 \text{ fm}^{-1} \) \([37,38]\). They are treated as free parameters.

The effective interaction \([18]\) is expressed in angular-momentum representation as
\[ V_{(ab,cd)}^{(r_1 - r_2; \omega)} = \sum_{JM'\ell'\nu} \frac{2}{\pi} \int_0^{\infty} q^2 dq J_{JM'\ell'\nu}^{(ab,cd)}(q; r_1) W_{JM'\ell'\nu}^{(ab,cd)}(q, \omega) J_{JM'\ell'\nu}^{(ab,cd)}(q; r_2), \]
where
\[ W_{J\ell\ell'}^{(ab,cd)}(q, \omega) = W_{LJ\ell\ell'}^{(ab,cd)}(q, \omega) + W_{TJ\ell\ell'}^{(ab,cd)}(q, \omega), \] (55)

with
\[ W_{LJ\ell\ell'}^{(ab,cd)}(q, \omega) \equiv a_{J\ell} W_{L}^{(ab,cd)}(q, \omega) a_{J\ell'}, \] (56)
\[ W_{TJ\ell\ell'}^{(ab,cd)}(q, \omega) \equiv W_{T}^{(ab,cd)}(q, \omega) [\delta_{\ell\ell'} - a_{J\ell} a_{J\ell'}]. \] (57)

The coordinate representation of the effective interaction in Eq. (17) is given by
\[ W_{J\ell\ell'}^{(ab,cd)}(r_1, r_2; \omega) \equiv \frac{2}{\pi} \int_{0}^{\infty} q^2 dq \ j_\ell(qr_1) W_{J\ell\ell'}^{(ab,cd)}(q, \omega) j_{\ell'}(qr_2). \] (58)

The ring equation (18) reads as
\[ \Pi_\nu^{\ell\ell'}(r, r'; \omega) = \Pi_\nu^{(0)\ell\ell'}(r, r'; \omega) + \sum_{\ell_1 \ell_2} \int_{0}^{\infty} r_1^2 dr_1 r_2^2 dr_2 \Pi_\nu^{(0)\ell_1\ell_2}(r, r; \omega) W_{J\ell_1\ell_2}(r_1, r_2; \omega) \Pi_\nu^{\ell_2\ell'}(r_2, r'; \omega). \] (59)

E. Spin-longitudinal and -transversal response functions

We define the spin-longitudinal and -transversal response functions as
\[ R_{L}^{\nu(\alpha\beta)}(q, \omega) \equiv \frac{1}{A} \sum_{n \neq 0} \langle \Psi_0 | j_L^{\nu(\alpha\beta)}(q, \omega) | \Psi_n \rangle \langle \Psi_n | j_L^{\nu(\alpha\beta)}(q, \omega) | \Psi_0 \rangle \times \delta[\omega - (E_n - E_0)], \] (60)
\[ R_{T}^{\nu(\alpha\beta)}(q, \omega) \equiv \frac{1}{A} \sum_{n \neq 0} \sum_{\mu} \frac{1}{2} \langle \Psi_0 | j_T^{\nu(\alpha\beta)}(q, \omega) | \Psi_n \rangle \langle \Psi_n | j_T^{\nu(\alpha\beta)}(q, \omega) | \Psi_0 \rangle \times \delta[\omega - (E_n - E_0)]. \] (61)

These are rewritten in the grouped representation as
\[ R_{L}^{\nu[\alpha\beta]}(q, \omega) = -\frac{1}{A} \frac{1}{\pi} \text{Im} \Pi_{L}^{\nu[\alpha\beta]}(q, \omega), \] (62)
\[ R_{T}^{\nu[\alpha\beta]}(q, \omega) = -\frac{1}{A} \frac{1}{\pi} \text{Im} \Pi_{T}^{\nu[\alpha\beta]}(q, \omega), \] (63)

with the momentum diagonal parts of the polarization propagators (53) and (54),
\[ \Pi_{L}^{\nu[\alpha\beta]}(q, \omega) \equiv \Pi_{L}^{\nu[\alpha\beta]}(q, q; \omega) = 4\pi \sum_{J\ell\ell'} (2J + 1) a_{J\ell} a_{J\ell'} \Pi_{J\ell\ell'}^{\nu[\alpha\beta]}(q, q; \omega), \] (64)
\[ \Pi_{T}^{\nu[\alpha\beta]}(q; \omega) \equiv \frac{1}{2} \sum_{\mu} \Pi_{T}^{\nu[\alpha\beta]}(q, q; \omega) = 2\pi \sum_{J\ell\ell'} (2J + 1) (\delta_{\ell\ell'} - a_{J\ell} a_{J\ell'}) \Pi_{J\ell\ell'}^{\nu[\alpha\beta]}(q, q; \omega), \] (65)
where Eqs. (45), (46), (38) and (39) are used.

As to the responses for nucleon probes, we assume that the ratio of the scattering amplitudes, the $NN \rightarrow N\Delta$ to the $NN \rightarrow NN$, is $f_\Delta/f_N$, as is commonly done implicitly [13] and explicitly [39]. Then relevant response functions are given by

$$R_{L,T}^{\nu}(q, \omega) = R_{L,T}^{\nu[NN]}(q, \omega) + 2 \frac{f_\Delta}{f_N} R_{L,T}^{\nu[N\Delta]}(q, \omega) + \left(\frac{f_\Delta}{f_N}\right)^2 R_{L,T}^{\nu[\Delta\Delta]}(q, \omega),$$

(66)

as was shown in Eq. (1). The uncorrelated ones are

$$R_{L,T}^{(0)\nu}(q, \omega) = R_{L,T}^{(0)\nu[NN]}(q, \omega) + \left(\frac{f_\Delta}{f_N}\right)^2 R_{L,T}^{(0)\nu[\Delta\Delta]}(q, \omega).$$

(67)

In the quasi-elastic region, the second term does not contribute since real $\Delta$ production does not occur.

For $(e,e')$ scattering, the cross section is expressed in the one-photon-exchange approximation as

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[ \left(\frac{q^4}{q^2}\right)^2 S_L(q, \omega) + \left(\tan^2 \frac{\theta}{2} - \frac{q^2}{2q^2}\right) S_T(q, \omega) \right],$$

(68)

with the Mott cross section $\sigma_M$, a transferred four-momentum $q_\mu = (\omega, q)$, $q^2 = \omega^2 - q^2$ and a scattering angle $\theta$. The dynamic structure factor [13], $S_L$ and $S_T$, are given by

$$S_L(q, \omega) = \sum_{n \neq 0} \left| \langle \Psi_n | \rho_C(\mathbf{q}, \omega) | \Psi_0 \rangle \right|^2 \delta(\omega - (E_n - E_0)],$$

(69)

$$S_T(q, \omega) = \sum_{n \neq 0} \left| \langle \Psi_n | J_T(\mathbf{q}, \omega) | \Psi_0 \rangle \right|^2 \delta(\omega - (E_n - E_0)],$$

(70)

where $(\rho_C, J_T)$ is the nuclear electromagnetic current operator. In this paper, we restrict ourselves to the transverse part $J_T$, which is given by the sum of the one-body convection and magnetic currents, and the exchange current, $J_T = J_T^{\text{conv}} + J_T^{\text{mag}} + J_T^{\text{exch}}$. We neglect the convection current since its contribution is small [13]. Although the exchange current contributes to a certain extent, we do not take into account of this term since we only consider the responses of one-body operators. The magnetic current is given by

$$J_T^{\text{mag}}(q) = -\frac{i}{2M_N} \sum_k e^{-i\mathbf{q}\cdot\mathbf{r}_k} \left[ \left( G_{\gamma NN}^{IV}(q_\mu^2) + G_{\gamma NN}^{IV}(q_\mu^2) \left(\tau_0^{(NN)}\right)_k \right) \left(\sigma_k^{(NN)} \times q\right) \right.$$

$$\left. + G_{\gamma N\Delta}(q_\mu^2) \left(\tau_0^{(N\Delta)}\right)_k \left(\sigma_k^{(N\Delta)} \times q\right) + \left(\tau_0^{(\Delta\Delta)}\right)_k \left(\sigma_k^{(\Delta\Delta)} \times q\right) \right] ,$$

(71)

Some papers use the notation $(4\pi/M_T)S_{L,T}$ instead of the present $S_{L,T}$, where $M_T$ stands for the target mass.
with the nucleon mass $m_N$. We take the magnetic form factors in the form of

$$G_{\gamma\alpha\beta}(q^2) \equiv f_{\gamma\alpha\beta} \left[1 - \frac{q^2}{\lambda^2}\right]^2,$$

(72)

where $\lambda = 855 \text{ MeV}/c$ and $f's$ are the magnetic strengths. Then we get

$$J_{T}^{\text{mag}}(q) = -\frac{i}{2m_N}G_{\gamma NN}^{IV}(q^2) \sum_k e^{-iqr_k} \left\{ \left( \frac{f_{\gamma NN}^{IS}}{f_{\gamma NN}^{IV}} \right) \left( \sigma_k^{(NN)} \times q \right) + \left( \frac{f_{\gamma NN}^{IV}}{f_{\gamma NN}^{IV}} \right) \left( \sigma_k^{(N\Delta)} \times q \right) \right\},$$

(73)

where

$$f_{\gamma NN}^{IS} = (\mu_p + \mu_n) / 2, \quad f_{\gamma NN}^{IV} = (\mu_p - \mu_n) / 2,$$

(74)

with $\mu_p = 2.79$ and $\mu_n = -1.91$. When we neglect the isospin-mixing and consider only $T = 0$ target nuclei, the isoscalar and isovector parts do not interfere. Then we can neglect the isoscalar part since $\left( \frac{f_{\gamma NN}^{IS}}{f_{\gamma NN}^{IV}} \right)^2 \approx 0.04$ is very small. Inserting Eq. (73) into Eq. (70) and using Eqs. (61) and (35), we get

$$S_T(q, \omega) = \sum_{n \neq 0} \left| \langle \Psi_n | \bar{J}_{T}^{\text{mag}}(q, \omega) | \Psi_0 \rangle \right|^2 \delta[\omega - (E_n - E_0)]$$

$$= 2A \left| \frac{q}{2m_N} G_{\gamma NN}^{IV}(q^2) \right|^2 R_T^{(e,e')}(q, \omega),$$

(75)

where

$$R_T^{(e,e')}(q, \omega) \equiv R_T^{p=0[NN]}(q, \omega) + 2\frac{f_{\gamma NN}^{IS}}{f_{\gamma NN}^{IV}} R_T^{\nu=0[N\Delta]}(q, \omega) + \left( \frac{f_{\gamma NN}^{IS}}{f_{\gamma NN}^{IV}} \right)^2 R_T^{\nu=0[N\Delta]}(q, \omega).$$

(76)

(77)

III. EFFECTS OF THE $\Delta$-HOLE MIXING

In the following two sections we present our numerical calculations of the isovector spin-longitudinal and -transverse response functions, $R_L(q, \omega)$ and $R_T(q, \omega)$, for the doubly (sub-)closed shell nuclei, $^{40}$Ca, $^{16}$O and $^{12}$C, and analyze them from various points of view.

In this section we compare $R_{L,T}$ with and without $\Delta$ to see effects of $\Delta$-mixing, and investigate relative importance of the process-decomposed response functions $R_{L,T}^{[\alpha\beta]}$. This manifests the $\Delta$ effects more clearly. We also discuss the energy-weighted and energy-non-weighted sums.

Calculations are carried out by the ring approximation, the Tamm-Dancoff approximation (TDA) and without any residual interactions. These results will be called the RPA, TDA and uncorrelated response functions, respectively.
A. Technical comments and choice of parameters

Before presenting the numerical calculations, we make some technical comments and summarize the values of the parameters.

In the previous section, $H$ and $\Psi$ include the center-of-mass motion, however, it is better to replace them by the intrinsic ones to isolate the structure part. Then the transferred energy $\omega$ should be replaced by that to the intrinsic state, $\omega_{\text{int}} = \omega - \omega_{\text{recoil}} = \omega - q^2/(2AmN)$. Similarly the transferred momentum $q$ is replaced by that to the relative motion between the active nucleon and the remaining $(A-1)$-nucleon core, $q_{\text{int}} = [(A-1)/A]q$. For simplicity we suppress the script “int” in Sects. [III] and [IV].

We took the single-particle potential for $N$ and $\Delta$ as

\[
U(r) = -(V + iW) \frac{1}{1 + \exp \left(\frac{r-R}{a}\right)} - 2 \frac{1}{m^2} \frac{V_{\ell s}}{a} \frac{\exp \left(\frac{r-R}{a}\right)}{r^2} \cdot (\ell \cdot s) + V_{\text{coul}},
\]

with $R = r_0A^{1/3}$. $V_{\text{coul}}$ is the Coulomb potential of the uniformly charged sphere with the radius parameter $r_c$. The shape parameters are fixed to be $r_0 = r_c = 1.27$ fm and $a = 0.67$ fm [14]. For the nucleon the spin-orbit potential depth $V_{\ell s}$ are fixed to be 6.5 MeV for $^{12}$C, 10.4 MeV for $^{16}$O and 10.0 MeV for $^{40}$Ca. The real potential depth $V$ is so determined as to give the observed separation energy of the outermost occupied state. The imaginary potential depth $W$ are fixed to be zero for the occupied (hole) states and 5.0 MeV for the particle states. For $\Delta$ we set $V = 30$ MeV and $W = V_{\ell s} = 0.0$ MeV since we do not have enough information for such virtual $\Delta$ appearing in the quasi-elastic region.

The masses are chosen to be $m_N = 940$ MeV, $m_\Delta = 1236$ MeV, $m_\pi = 139$ MeV, and $m_\rho = 770$ MeV unless explicitly mentioned. The coupling constants are fixed to be $f_N^2/4\pi = 0.081$, $f_\Delta/f_N = 2.00$, and $C_{\alpha\beta}^\rho = 2.18$. The cutoff parameters are set to be $\Lambda_\pi = 1300$ MeV and $\Lambda_\rho = 2000$ MeV [13].

B. Effects of the $\Delta$ components

Here we present the energy spectra of the response functions at $q = 1.70$ fm$^{-1}$ for the charge exchange mode related with the $(p,n)$-like reactions ($\nu = -1$). From now on we suppress $\nu$ on $R$. The Landau-Migdal parameters are taken to be $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.4, 0.5)$.

In Fig. 2 we show the RPA response functions $R_{L,T}$ of $^{40}$Ca (a) without and (b) with $\Delta$. The uncorrelated response functions $R_{L,T}^{(0)}$ are also shown, which are very close to each other for this $LS$ closed shell. Slight difference comes from the spin-orbit force. Fluctuations seen at lower $\omega$ are somewhat artificial because the spreading widths of the hole states are not included in the present calculation.
When $\Delta$ is not included, $R_L$ is enhanced below about 85MeV but quenched above that. On the other hand, $R_T$ is quenched below about 96MeV and enhanced above that. Consequently $R_L$ is softened but $R_T$ is hardened.

Once $\Delta$ is introduced, $R_L$ increases but $R_T$ decreases for the whole quasi-elastic region as seen in Fig. 2(b). This is attributed to the coupling interaction $W^{[N\Delta]}$, which brings down the spin-longitudinal strength from the $\Delta h$ to the $ph$ sector but brings up the spin-transverse strength in the opposite direction. This is the essential effect of $\Delta$.

Fig. 3 shows the response functions with $\Delta$ for (a) $^{16}\text{O}$ and (b) $^{12}\text{C}$. Qualitative features are common for all the nuclei and the RPA effects are stronger for larger $A$. The difference between $R_L^{(0)}$ and $R_T^{(0)}$ is larger for $^{12}\text{C}$ than for other two nuclei, because $^{12}\text{C}$ is not the spin-saturated nucleus.

To see the situation in more detail, we separate the response functions into the process-decomposed ones. The RPA response functions $R_L$ and $R_T$ of $^{40}\text{Ca}$ are decomposed into $R_L^{[\alpha\beta]}$ in Fig. 4(a) and $R_T^{[\alpha\beta]}$ in Fig. 4(b). The main contribution comes from $R_L^{[NN]}$, but that from $R_L^{[N\Delta]}$ is also significant. The contribution from $R_T^{[\Delta\Delta]}$ is negligibly small. An important point is that $R_L^{[NN]}$ and $R_L^{[N\Delta]}$ contribute constructively, whereas $R_T^{[NN]}$ and $R_T^{[N\Delta]}$ do destructively. This explains the shift of the spin-longitudinal and -transverse strengths seen above.

To see the backward effect we show the TDA response functions without $\Delta$ in Fig. 5. Softening of $R_L$ and hardening of $R_T$ are well developed, but their magnitudes do not change so much. Comparing with Fig. 2(a), we can say that the backward amplitudes in the ring approximation induce further enhancement of $R_L$ and quenching of $R_T$.

These behaviors are qualitatively understood in the following way. The formal solution of the RPA equation (29) is given by

$$\Pi = \left[1 - \Pi^{(0)} W^{[\alpha\beta]}(q, \omega) \right]^{-1} \Pi^{(0)}, \quad \Pi^{(0)} = \Pi^{(0)}_{FW} + \Pi^{(0)}_{BK}. \tag{79}$$

where $\Pi^{(0)}_{FW}$ and $\Pi^{(0)}_{BK}$ are the forward and the backward part of the uncorrelated polarization propagator. The fact to be kept in mind is that around the present momentum ($q = 1.7 \text{ fm}^{-1}$) the spin-longitudinal effective interaction $W^{[\alpha\beta]}_L(q, \omega)$ is negative if $g'_{\alpha\beta} < 0.7$ but the spin-transverse one $W^{[\alpha\beta]}_T(q, \omega)$ is positive if $g'_{\alpha\beta} > 0.25$ for the present parameters (see Fig. 8).

First let us consider the cases without $\Delta$. For simplicity we treat $\Pi$ and $W$ as c-numbers instead of matrices like in a Fermi gas model and consider a real single-particle potential. Then response functions are expressed as

$$R = \left[1 - \Pi^{(0)} W^{[\alpha\beta]} \right]^{-2} R^{(0)}. \tag{80}$$

The real part of $\Pi^{(0)}_{FW}$ changes the sign in the middle of the energy range concerned, negative at lower side of $\omega$ but positive at higher side, as is seen from the first term of r.h.s. of Eq. (79).
Hence $\Pi^{(0)}_{FW} W_T$ is negative but $\Pi^{(0)}_{FW} W_L$ positive at lower $\omega$, and thus $R_T$ is quenched but $R_L$ enhanced if $W_L$ is not so strong ($\Pi^{(0)}_{FW} W_L < 2$). At higher $\omega$ the situation is opposite. As the results the TDA response functions are softened for the spin-longitudinal mode but hardened for the spin-transverse mode. On the other hand $\Pi^{(0)}_{BK}$ is always negative (see Eq. (11)). Therefore $R_L$ is enhanced but $R_T$ quenched by the backward amplitude in the whole energy region.

Next let us consider the cases with $\Delta$. In the first order of $W$ the RPA response functions $R^{[N\Delta]}_{L,T}$ are given by

$$R^{[N\Delta]}_{L,T} = \Pi^{(0)}[\Delta\Delta] W_{L,T}^{[N\Delta]} R^{(0)[NN]}_{L,T}.$$ (81)

The uncorrelated $\Delta h$ polarization propagator $\Pi^{(0)}[\Delta\Delta]$ is real negative in the quasi-elastic region (see Eq. (11)). Therefore $R^{[N\Delta]}_L$ is positive but $R^{[N\Delta]}_T$ is negative. This is the reason why $R^{[NN]}_L$ and $R^{[N\Delta]}_L$ contribute constructively, whereas $R^{[NN]}_T$ and $R^{[N\Delta]}_T$ do destructively.

### C. Sum rules

Next we consider the energy-non-weighted and energy-weighted sums defined by

$$X^0_{L,T}(q) = \int R_{L,T}(q, \omega) d\omega,$$ (82)

$$X^1_{L,T}(q) = \int \omega R_{L,T}(q, \omega) d\omega.$$ (83)

When $\Delta$ is not included, the former behaves as

$$X^0_{L,T}(q) \to 1 \quad (q \to \infty).$$ (84)

Here we used $\tilde{j} = j$ for isovector currents since the isospin of the ground state is assumed to be zero.

In the Fermi gas model there is the definite upper limit of the integral $\omega_{\text{max}}$. In the case without $\Delta$, it is

$$\omega_{\text{max}}^F = \frac{q^2}{2m} + \frac{qp_F}{m},$$ (85)

with the Fermi momentum $p_F$. For the finite nucleus the upper limit extends to infinity in principle, because there is no sharp cutoff of the momentum distribution. However the response functions is small and decrease rapidly beyond $\omega_{\text{max}}^F$. Therefore we assume exponential damping beyond $\omega_{\text{max}}^F$ to evaluate the integrations.

In the case with $\Delta$, $R(q, \omega)$ is sizable both in the $ph$ and $\Delta h$ sectors. For the sake of comparison, however, we make the same prescription as in the case without $\Delta$, because we are interested in the strength distributed only in the quasi-elastic region in this paper.
We present the energy-non-weighted sums \( X^0_{L,T}(q) \) in Fig. 6(a) for \(^{40}\text{Ca}\). We took \( p_F = 1.20 \text{ fm}^{-1} \) to estimate \( \omega_F^{\text{max}} \). The sums \( X^0_L \) and \( X^0_T \) of the uncorrelated responses are too close to distinguish. We found that they are also very close to the Fermi gas model value

\[
X^0_{\text{FG}}(q) = \begin{cases} 
\frac{3}{4} Q \left( 1 - \frac{Q^2}{12} \right) & \text{for } Q \leq 2 \\
1 & \text{for } Q > 2 
\end{cases}
\]  

with \( Q \equiv q/p_F \). For the uncorrelated responses reduction from unity reflects the Pauli blocking effect.

The RPA correlation without \( \Delta \) slightly increase \( X^0_L \) but largely decreases \( X^0_T \) as were seen in Fig. 2(a). Once \( \Delta \) is included, \( X^0_L \) is drastically enhanced, while \( X^0_T \) is more quenched in the low \( q \) region but the quenching becomes smaller as \( q \) increases.

In Fig. 6(b), the energy-weighted sums \( X^1_{L,T} \) are shown. We found that the RPA correlation without \( \Delta \) hardly changes them from the uncorrelated cases, therefore we did not show the results for this case. Once \( \Delta \) is introduced, the sum of the spin-longitudinal mode \( X^1_L \) is enhanced very much, whereas that of the spin-transverse mode \( X^1_T \) is only slightly affected.

The sum rule says that the sums of the response function for an operator \( \hat{O} \) are given by

\[
X^0(q) = \langle \Psi_0 | \hat{O} \hat{O}^\dagger | \Psi_0 \rangle / A, \\
X^1(q) = \langle \Psi_0 | [\hat{O} , [H, \hat{O}]] | \Psi_0 \rangle / A,
\]

where \( \hat{O} \) is assumed to be hermitian in Eq. (88). Pandharipande et al. [23] calculated \( | \Psi_0 \rangle \) exactly by using realistic nuclear force within the nucleon degree of freedom. Then they evaluated \( X^0 \) and \( X^1 \) from these sum rules. It is found that their sum rule values are significantly larger than our results without \( \Delta \). Fig. 7(a) and (b) compare their values with ours for \(^{16}\text{O}\). For instance, their energy-non-weighted sums are about 19% (longitudinal) and 10% (transverse) and their energy-weighted sums are about 75% (longitudinal) and 55% (transverse) larger than ours, at \( q = 1.70 \text{ fm}^{-1} \). Such larger difference strongly indicates importance of the correlations beyond RPA. Note that in their calculation the effects of \( \Delta \) are implicitly included in part through the \( \Delta \) mediated three body force, but the processes expressed by \( R^{(N\Delta)} \) and \( R^{(\Delta\Delta)} \) are not included.

Thouless [46] proved that one gets the energy-weighted sum of the RPA response function by replacing \( | \Psi_0 \rangle \) in Eq. (88) by \( | \Psi_0^{\text{HF}} \rangle \), the Hartree-Fock ground state wave function. This theorem explains why the RPA energy-weighted sums without \( \Delta \) are very close to uncorrelated ones. It also supports that the difference between our results and those of Pandharipande et al. must be due to the nuclear correlations beyond RPA. This should also be reflected in the energy spectra of the response functions. The importance of the 2\( p \)-2\( h \) configuration mixing has been also pointed out by several authors [19,22].
IV. DEPENDENCE ON EFFECTIVE INTERACTIONS

In this section we investigate the effective interaction dependence of $R_{L,T}$ and their process-decomposed components $R_{L,T}^{[\alpha\beta]}$. Some of the effective interactions are shown in Fig. 3 in the form of $W_{L,T}^{[\alpha\beta]}/(m_F^2)$. Their $[\alpha\beta]$ dependence comes only through $g_{\alpha\beta}$.

We present the $g'_{NN}$ dependence of $R_L$ and $R_T$ in Fig. 9(a) and (b), and their $g'_{NN}$ dependence in Fig. 3(c) and (d), respectively, for $^{40}$Ca at $q = 1.70$ fm$^{-1}$. We fixed $g'_{NN} = 0.4$ and $g'_{\Delta\Delta} = 0.5$ in the study of $g'_{NN}$ dependence and $g'_{NN} = 0.6$ and $g'_{\Delta\Delta} = 0.5$ for $g'_{NN}$ dependence. The response functions so weakly depend on $g'_{\Delta\Delta}$ that we do not discuss about it.

Both $R_L$ and $R_T$ considerably depend on $g'_{NN}$ as well as $g'_{NN}$. As $g'_{NN}$ decreases, $R_L$ becomes more enhanced at lower $\omega$ but less at higher $\omega$, while $R_T$ becomes less quenched at the lower side and less enhanced at the higher side. The $g'_{NN}$ dependence is more simply summarized. As $g'_{NN}$ decreases, both $R_L$ and $R_T$ increase, namely, $R_L$ is more enhanced but $R_T$ less quenched. These features are qualitatively common for all nuclei $^{40}$Ca, $^{16}$O and $^{12}$C we analyzed.

Such dependence is more clearly seen through the process-decomposed response functions. The $g'$ dependence of $R_{L,T}^{[\alpha\beta]}$ is shown in Fig. 10. Since $g'_{NN}$ controls the coupling strength between $N$ and $\Delta$, $R_{L,T}^{[NN]}$ is more sensitive to $g'_{NN}$ than $g'_{NN}$, whereas opposite is true for $R_{L,T}^{[NN]}$.

Their behaviors are well understood by the interpretation given at the end of Subsec. III_B. As $g'_{NN}$ decreases the effective interaction $W_L^{[NN]}$ becomes more attractive as shown in Fig. 3. Consequently $R_{L,T}^{[NN]}$ is more enhanced at lower $\omega$ but more reduced at higher $\omega$. On the other hand, $W_T^{[NN]}$ becomes less repulsive and therefore $R_{T}^{[NN]}$ is less quenched at lower $\omega$ but less enhanced at higher $\omega$. These behaviors reflect in the $g'_{NN}$ dependence of $R_L$ and $R_T$ seen in Fig. 3.

As $g'_{NN}$ decreases, $W_L^{[N\Delta]}$ becomes more negative but $W_T^{[N\Delta]}$ does less positive. Therefore both $R_L^{[N\Delta]}$ and $R_T^{[N\Delta]}$ are increased, consequently more enhancement of $R_L$ and less quenching of $R_T$ are resulted in.

G.E. Brown and his collaborators [47] advocated the scaling of effective masses of nucleon and mesons (except for pion) in the nucleus; e.g. $m^*_{\rho}/m_{\rho} \approx m^*_{N}/m_{N}$. Correspondingly they claimed necessity of large $g'_{NN}$ [48]. To see an implication of this proposal, we show in Fig. 11 the transverse response functions $R_T$ with smaller $m^*_{\rho}(=0.75m_{\rho})$ and larger $g'_{NN} (=0.8)$ together with the uncorrelated and the RPA responses with $m^*_{\rho} = m_{\rho}$ and $g'_{NN} = 0.6$ as reference. We did not change the nucleon effective mass because our computer program cannot take into account its density dependence at the present and it must be the free nucleon mass at infinity. So the present calculation aims only to get feeling about the interaction dependence.

Let us compare two cases, (a) a previous parameter set $[m^*_{\rho} = m_{\rho}, (g'_{NN}, g'_{NN}, g'_{\Delta\Delta})}$
\( (0.6, 0.4, 0.5) \), and (b) the new one \( m^*_\rho = 0.75m_\rho, (g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.8, 0.4, 0.5) \). In Fig. 3(b) we also show \( W_T \) with smaller \( m^*_\rho (= 0.75m_\rho) \). It shows that \( W_T^{[NN]} \) is almost the same for the both cases around \( q \approx 1.70 \text{ fm}^{-1} \) accidentally, and hence similar hardening is expected. However the positive \( W_T^{[N\Delta]} \) in the case (a) becomes very weak negative in the case (b), and hence \( R_T^{[N\Delta]} \) changes the sign. Consequently we see in Fig. 11 that the hardening of \( R_T \) stays similar for the both cases but quenching is very much reduced in the latter. We must note that such change strongly depends on the momentum \( q \) as is seen in Figs. 8, 14 and 15. We remark that \( R_L \) is also affected through the change of \( g'_{NN} \). Its enhancement is reduced at lower \( \omega \) because of large \( g'_{N\Delta} \).

Next we show the collectivity ratio \( R_L/R_T \) in Fig. 12, for (a) \(^{40}\text{Ca}\) and (b) \(^{12}\text{C}\) at \( q = 1.70 \text{ fm}^{-1} \). In the uncorrelated case, the ratio is, of course, close to unity, deviation from which is only due to the single-particle spin-orbit force. In the RPA calculation without \( \Delta \), the ratio is larger than unity at lower \( \omega \) but smaller at higher \( \omega \). It is because \( R_L \) is enhanced at lower \( \omega \) and quenched at higher \( \omega \), but \( R_T \) behaves in the opposite way as was shown in Fig. 3(a). In the case with \( \Delta \), the RPA correlation drastically increases the ratio and makes it larger than unity almost for whole \( \omega \). The case with \( (g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.4, 0.5) \) gives larger ratio at lower \( \omega \) than the almost universality case with \( (0.6, 0.6, 0.5) \) and changes more steeply as \( \omega \) changes. Smaller \( g'_{N\Delta} \) brings down more \( R_L \) strength from the \( \Delta h \) region. In the case with \( m^*_\rho = 0.75m_\rho \), enhancement of \( R_L \) is suppressed due to larger \( g'_{NN} = 0.8 \), thus the ratio is smaller than the above two cases.

The RPA effects became larger as the mass number increases. We note that these results of \( R_L/R_T \) cannot be compared with the \((p, n)\) data at the present stage, because effects of distortion and absorption have not yet been considered.

V. ELECTRON SCATTERING

In this section we study the transverse response functions \( R_T^{(e,e')} \) obtained by electron scattering (see Eq. (77)).

To analyze the electron scattering, it has been known that we must take account of not only RPA correlation with \( \Delta \) but also the mixing of \( 2p-2h \) or more complicate configurations \cite{19,22} and the exchange currents \cite{13,49}, etc.. Here we do not intend to reproduce the experimental data, but want to see to what extent the RPA results depend on the magnetic transition ratio \( f_{\gamma N\Delta}/f_{\gamma NN}^{IV} \) and the effective interactions.

As was mentioned in Sec. I, most of previous analyses used the ratio \( f_{\gamma N\Delta}/f_{\gamma NN}^{IV} = 2.0 \). However, the SU(6) quark model \cite{50} gives \( f_{\gamma N\Delta}/f_{\gamma NN}^{IV} = 6\sqrt{2}/5 \approx 1.70 \). Phenomenological analyses by Koch et al. \cite{11} and Kumano \cite{52} gave the ratio 2.20 and 2.26, respectively. A reason for the discrepancy is that the \( \pi N \) background scattering is renormalized in the phenomenological analyses but should be treated explicitly in the quark model \cite{53}.
In Fig. 13, we compare the results with different values of $f_{\gamma N\Delta}/f_{\gamma NN}^{IV}$. The smaller the ratio the smaller the quenching, because the effect of $\Delta$ is essentially determined by the product $2(f_{\gamma N\Delta}/f_{\gamma NN}^{IV})R_{T}^{[N\Delta]}$. We see some dependence on $f_{\gamma N\Delta}/f_{\gamma NN}^{IV}$ when $R_{T}^{[N\Delta]}$ is sizable.

We compare the results with various effective interactions for $^{12}$C at $q = 300$ MeV/c in Fig. 14(a) and at 400 MeV/c in (b), and for $^{40}$Ca at 330 MeV/c in Fig. 15(a) and at 410 MeV/c in (b). Here we fixed $f_{\gamma N\Delta}/f_{\gamma NN}^{IV} = 2.20$ [51]. Experimental data are shown as a reference. Compared with the energy spectrum of the uncorrelated case, the experimental spectrum is very much hardened, but the magnitudes are comparative.

The RPA result with $(g_{NN}', g_{N\Delta}', g_{\Delta\Delta}') = (0.6, 0.6, 0.5)$, which is practically the same as that of the universality ansatz (0.6, 0.6, 0.6), is quenched and hardened. As a result, the peak moves closer to the observed position, but the magnitude becomes much smaller than the data.

The calculation with (0.6, 0.4, 0.5), which we used as the standard in Sec. III, places the peak at slightly higher energy and now at almost the right position. It also increases the magnitude though it is still smaller than the data.

If we take $m_{\rho}' = 0.75m_{\rho}$ and $g_{NN}' = 0.8$ as Brown and Rho [17] suggested, the magnitude is increased very much and the peak comes very close to the experimental data. A good fit is seen in Fig. 14(b) but overshooting in Fig. 15(b). We must note that the good fit does not necessarily mean that the effective interaction is good because there must be other contributions.

Qualitative features of our analysis are consistent with previous calculation of Alberico et al. [15], in which the RPA correlation gives reasonable hardening but underestimates the magnitude, the deficiency of which may be fulfilled by nuclear correlations beyond RPA (the 2p-2h effects, etc.) and exchange currents.

Since $R_{T}^{(e,e')}$ eminently depends on the effective interactions, it must be a good tool to discriminate them if the reliable estimation is possible of the other contributions such as 2p-2h configuration mixing and exchange currents, etc.. Their estimation is beyond the scope of the present paper.

**VI. SUMMARY**

We studied the effects of the $\Delta$-hole configurations on the spin-isospin response functions in finite nuclei in the quasi-elastic region. We removed the universality ansatz for the Landau-Migdal parameters and treated $g_{NN}', g_{N\Delta}'$ and $g_{\Delta\Delta}'$ independently. For this sake we formulated the response function method for a finite system consisting of $N$ and $\Delta$ in the ring approximation.

We showed that the $\Delta$-mixing is crucial for the enhancement of $R_{L}$ in the whole range.
of the quasi-elastic region, and it promotes the quenching of $R_T$. If $\Delta$ is not included, $R_L$ and $R_T$ are both partially enhanced and partially quenched. We emphasize that reliable estimation of the effects of $\Delta$ is definitely needed for comparison with experimental data.

Detailed analysis was carried out by dividing the response functions $R_{L,T}$ into the process-decomposed ones, $R_{L,T}^{[NN]}$, $R_{L,T}^{[N\Delta]}$ and $R_{L,T}^{[\Delta\Delta]}$. The main contribution comes from $R_{L,T}^{[NN]}$ but contribution from $R_{L,T}^{[N\Delta]}$ is also significant, whereas $R_{L,T}^{[\Delta\Delta]}$ is negligible. The effects of $\Delta$ is mostly represented by $R_{L}^{[N\Delta]}$.

We showed that $R_{L}^{[NN]}$ and $R_{L}^{[N\Delta]}$ contribute constructively, whereas $R_{T}^{[NN]}$ and $R_{T}^{[N\Delta]}$ do destructively. This is the reflection that the negative interaction $W_{L}^{[N\Delta]}$ between $ph$ and $\Delta h$ brings down the spin-longitudinal strength from the $\Delta h$ to $ph$ region, but the positive interaction $W_{T}^{[N\Delta]}$ brings up the spin-transverse strength in the opposite direction. Thus $R_{L}^{[\Delta\Delta]}$ plays an important role for strong enhancement of $R_L$, and $R_{T}^{[\Delta\Delta]}$ does some role for quenching of $R_T$.

Analysis of $g'$ dependence of $R_{[\alpha\beta]}^{[\alpha\beta]}$ tells that $R_{[NN]}^{[N\Delta]}$ is very sensitive to $g'_{NN}$ but not to $g'_{N\Delta}$, whereas $R_{[NN]}^{[NN]}$ is sensitive to $g'_{NN}$ but not to $g'_{N\Delta}$. As $g'_{N\Delta}$ decreases, both $R_L$ and $R_T$ increase, thus $R_L$ is more enhanced but $R_T$ less quenched. As $g'_{NN}$ decreases, $R_L$ becomes more enhanced at lower $\omega$ side but reduced at higher side, while $R_T$ becomes less quenched at lower side but more at the higher side. Consequently the choice of $g'_{NN}$ and $g'_{N\Delta}$ is crucial to determine the behavior of the response functions $R_{L,T}$, such as the collectivity ratio $R_L/R_T$. Thus we should relax the universality condition for $g'$'s. Effect of change of the $\rho$-meson effective mass is also presented.

We further studied the interaction dependence of the isovector transverse response functions $R_{T}^{(e,e')}\,(e,e')$ scattering in comparison with the experimental data. The comparison must be useful to investigate the effective interaction at large $q$, though we need reliable estimation of the exchange currents, 2p-2h configuration mixing, etc..

In this paper we present the detailed analysis for $^{40}$Ca. The similar analysis for $^{12}$C is given in Ref. [54].

At the end we itemize some of the remaining problems we have to investigate. 1) For analysis of data of hadronic probes we must investigate reaction mechanisms such as distortions, multistep processes, etc.. This will be discussed in the forthcoming paper. 2) As we often mentioned nuclear correlations beyond RPA have to be evaluated. 3) We used Woods-Saxon shell model with the free nucleon mass. Hartree-Fock field should be used to keep consistency between the mean field and the residual interactions. 4) Nucleon effective mass with position dependence should be incorporated. 5) Spreading widths of $ph$ propagation should be properly taken into account. At the present we only included that of the particles by an energy independent complex potential.
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FIGURE CAPTIONS

FIG. 1. Process-decomposed response functions.

FIG. 2. Isovector spin-response functions, $R_L$ and $R_T$, for $^{40}$Ca at $q = 1.70$ fm$^{-1}$. (a) Without and (b) with $\Delta$. The dotted and full lines denote $R_L$ and $R_T$ with RPA correlation, respectively. The dot-dashed and dashed lines represent $R_L^{(0)}$ and $R_T^{(0)}$, respectively. $(g_{NN}', g_{N\Delta}', g_{\Delta\Delta}') = (0.6, 0.4, 0.5)$ are used in RPA.

FIG. 3. $R_L$ and $R_T$ for (a) $^{16}$O and (b) $^{12}$C at $q = 1.70$ fm$^{-1}$ with $\Delta$. The notations of the lines and the values of $g'$ are same as those in Fig. 2.

FIG. 4. Process-decomposed response functions, (a) $R_L^{[\alpha\beta]}$ and (b) $R_T^{[\alpha\beta]}$, for $^{40}$Ca at $q = 1.70$ fm$^{-1}$. The dotted, dashed and dot-dashed lines denote $R_L^{[NN]}$, $2(f_{\Delta}/f_N)R_L^{[N\Delta]}$, and $(f_{\Delta}/f_N)^2R_L^{[\Delta\Delta]}$, respectively, appeared in the Eq. (66). $R_L$ and $R_T$ are shown by the full lines. The values of $g'$ are same as those in Fig. 2.

FIG. 5. Response functions for $^{40}$Ca at $q = 1.70$ fm$^{-1}$ without $\Delta$ calculated in TDA ($g_{NN}' = 0.6$). The dotted and full lines show $R_L$ and $R_T$, respectively. The dot-dashed and dashed lines represent $R_L^{(0)}$ and $R_T^{(0)}$, respectively.

FIG. 6. (a) Energy-non-weighted sums $X^0$ and (b) energy-weighted sums $X^1$ for $^{40}$Ca. The full line denotes the sums of the uncorrelated transverse response. The dotted line represents the RPA transverse results without $\Delta$. The RPA transverse and longitudinal ones with $\Delta$ are shown by the dashed and dot-dashed lines, respectively. The uncorrelated longitudinal sum and the RPA longitudinal sum without $\Delta$ are very close to the uncorrelated transverse sum and not shown.
FIG. 7. (a) Energy-non-weighted sums $X^0$ and (b) energy-weighted sums $X^1$ for $^{16}$O. The dashed and dot-dashed lines denote the transverse and longitudinal sums, respectively, calculated by Pandharipande et al. [23]. Our results without $\Delta$ are shown by the full (transverse) and dotted (longitudinal) lines.

FIG. 8. Effective interactions $W^{\{\alpha\beta\}}_{L,T}/(f_\alpha f_\beta/m_\pi^2)$ at $\omega = 80$ MeV; (a) longitudinal and (b) transverse part. At $q = 0$ fm$^{-1}$ their values become equal to their $g'_\alpha\beta$. The interactions with $m^*_\rho = m_\rho$ and with $m^*_\rho = 0.75m_\rho$ are denoted by the full and dotted lines, respectively.

FIG. 9. The $g'$ dependence of the response functions for $^{40}$Ca at $q = 1.70$ fm$^{-1}$. Left side: the $g'_{NN}$ dependence of (a) $R_L$ and (b) $R_T$ with $g'_{N\Delta} = 0.4$ and $g'_{\Delta\Delta} = 0.5$. $g'_{NN}$ is set to be 0.5 (dotted line), 0.6 (dashed line) and 0.7 (dot-dashed line). Right side: the $g'_{N\Delta}$ dependence of (c) $R_L$ and (d) $R_T$ with $g'_{NN} = 0.6$ and $g'_{\Delta\Delta} = 0.5$. $g'_{N\Delta}$ is set to be 0.4 (dotted line), 0.5 (dashed line) and 0.6 (dot-dashed line). The full line denotes the uncorrelated responses.

FIG. 10. The $g'$ dependence of the process-decomposed response functions for $^{40}$Ca at $q = 1.70$ fm$^{-1}$. The $g'_{NN}$ dependence of (a) $R^{[\alpha\beta]}_L$ and (b) $R^{[\alpha\beta]}_T$, and the $g'_{N\Delta}$ dependence of (c) $R^{[\alpha\beta]}_L$ and (d) $R^{[\alpha\beta]}_T$. The notations of lines and the values of $g'$ are same as those in Fig. 9. The uncorrelated responses are not shown here.

FIG. 11. Effective $\rho$-meson mass dependence of the transverse response function for $^{40}$Ca at $q = 1.70$ fm$^{-1}$ with the large $g'_{NN}$. The dot-dashed line represents the RPA result with $m^*_\rho = 0.75m_\rho$ and $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.8, 0.4, 0.5)$. As reference, the uncorrelated response (full line) and the RPA one (dotted line) with $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.4, 0.5)$ are also shown.

FIG. 12. Collectivity ratio $R_L/R_T$ for (a) $^{40}$Ca and (b) $^{12}$C at $q = 1.70$ fm$^{-1}$. The thin full line denotes the ratio without the correlation. The dotted line represents the RPA results without $\Delta$ [$g'_{NN} = 0.6$]. The RPA results with $\Delta$ are shown by the dashed line [(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.6, 0.5)], the thick full line [(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.4, 0.5)], and the dot-dashed line [$m^*_\rho = 0.75m_\rho$ and (g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.8, 0.4, 0.5)], respectively.
FIG. 13. Dependence on $f_{\gamma N\Delta}/f_{\gamma NN}^{IV}$ of $S_T$ for $^{12}\text{C}(e,e')$ at $q = 300 \text{ MeV}/c$. $f_{\gamma N\Delta}/f_{\gamma NN}^{IV}$ is set to be 1.70 (dotted line), 2.00 (dashed line) and 2.20 (full line). $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta})$ is set to be (0.6, 0.6, 0.5).

FIG. 14. $S_T$ for $^{12}\text{C}(e,e')$ with various effective interactions at (a) $q = 300 \text{ MeV}/c$ and (b) $400 \text{ MeV}/c$. $f_{\gamma N\Delta}/f_{\gamma NN}^{IV}$ is set to be 2.20. The dotted and dashed lines denote the RPA results with $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.4, 0.5)$ and $(0.6, 0.6, 0.5)$, respectively. The dot-dashed line represents the RPA result with $(g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta}) = (0.8, 0.4, 0.5)$ and $m^*_\rho = 0.75m_\rho$. The uncorrelated one is shown by the full line. The experimental data are taken from Ref. [4].

FIG. 15. $S_T$ for $^{40}\text{Ca}(e,e')$ with various effective interactions at (a) $q = 330 \text{ MeV}/c$ and (b) $410 \text{ MeV}/c$. The notations of the lines are same as those in Fig. [14]. The experimental data are taken from Ref. [15].