The Applicability of the Astrometric Method in Determining the Physical Parameters of Gravitational Microlenses

Cheongho Han
Department of Astronomy & Space Science, Chungbuk National University, Chongju, Korea 361-763
cheongho@astro-3.chungbuk.ac.kr

Kyongae Chang
Department of Physics, Chongju University, Chongju, Korea 360-764
kchang@alpha94.chongju.ac.kr

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ABSTRACT

In this paper, we investigate the applicability of the astrometric method to the determination of the lens parameters for gravitational microlensing events toward both the LMC and the Galactic bulge. For this analysis, we investigate the dependency of the astrometrically determined angular Einstein ring radius, \( \Delta(\theta_E/\theta_{E,0}) \), on the lens parameters by testing various types of events. In addition, by computing \( \Delta(\theta_E/\theta_{E,0}) \) for events with lensing parameters which are the most probable for a given lens mass under the standard models of Galactic matter density and velocity distributions, we determine the expected distribution of the uncertainties as a function of lens mass. From this study, we find that the values of the angular Einstein ring radius are expected to be measured with uncertainties \( \Delta(\theta_E/\theta_{E,0}) \lesssim 10\% \) up to a lens mass of \( M \sim 0.1 M_\odot \) for both Galactic disk-bulge and halo-LMC events with a moderate observational strategy. The uncertainties are relatively large for Galactic bulge-bulge self-lensing events, \( \Delta(\theta_E/\theta_{E,0}) \sim 25\% \) for \( M \sim 0.1 M_\odot \), but they can be substantially reduced by adopting more aggressive observational strategies. We also find that although astrometric observations can be performed for most photometrically detected Galactic bulge events, a significant fraction (~45\%) of LMC events cannot be astrometrically observed due to the faintness of their source stars.

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1. Introduction

Current microlensing experiments (Alcock et al. 1997a, 1997b; Ansari et al. 1996; Udalski et al. 1997; Alard & Guibert 1997) are detecting massive astrophysical compact objects (MACHOs) by monitoring the light variations of stars undergoing gravitational microlensing. The light curve of a microlensing event is related to the lensing parameters by

\[ A = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}; \quad u = \left[ \beta^2 + \left( \frac{t - t_0}{t_E} \right)^2 \right]^{1/2}, \tag{1.1} \]

where \( A \) is the magnification, \( u \) is the lens-source separation, \( \beta \) is the impact parameter, \( t_0 \) is the time of maximum amplification, and \( t_E \) is the Einstein ring radius crossing time (Einstein timescale). Of these lensing parameters, only the timescale provides information about the lens because it is the only one directly related to the physical parameters of the lens (lens parameters) by

\[ t_E = \frac{r_E}{v}, \quad r_E = \left( \frac{4GM}{c^2} \frac{D_{ol}D_{ls}}{D_{os}} \right)^{1/2}, \tag{1.2} \]

where \( v \) is the lens-source transverse speed, \( r_E \) is the physical size of the Einstein ring radius, \( M \) is the mass of the lens, and \( D_{ol}, D_{ls}, \) and \( D_{os} \) are the separations between the observer, lens, and source star. However, due to the degeneracy in the lens parameters, the nature of the lens population is still very uncertain.

The uncertainties in the lens parameters can be significantly reduced if one can measure the lens proper motion, \( \mu = \theta_E/t_E \), where \( \theta_E = r_E/D_{ol} \) is the angular Einstein ring radius. The lens proper motions can be measured both photometrically (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994) and spectroscopically (Maoz & Gould 1994; Loeb & Sasselov 1995). However, measuring the proper motion with either of these techniques is only possible for several special classes of lensing events.

The lens proper motions can be measured in a general way if the separation between the two source star images can be measured. As this image separation is generally on order 1 \( \mu \)as, it can not be measured with current instrumentation. However, the proposed Space Interferometry Mission (Allen, Shao, & Peterson 1997, hereafter SIM) will have high enough positional accuracy to measure the astrometric displacement in the light centroid caused by gravitational microlensing (Walker 1995; Høg, Novikov & Polnarev 1995; Paczyński 1998; Boden, Shao, & Van Buren 1998). The astrometric shift of the source star image centroid is related to the lensing parameters by

\[ \vec{\delta \theta} = \frac{\theta_E}{u^2 + 2} (T \hat{x} + \beta \hat{y}), \tag{1.3} \]

where \( T = (t - t_0)/t_E \) and the \( x \) and \( y \) axes represent the directions which are parallel and normal to the lens-source transverse motion, respectively. The trajectory of the apparent
source star image traces out an ellipse during the event (astrometric ellipse, Jeong, Han, & Park 1998). With the measured astrometric ellipse, one can determine the impact parameter of the event because the shape (i.e. the axis ratio) of the ellipse is related to the impact parameter \( \beta \). The importance of measuring the astrometric centroid shifts is that one can then determine the angular Einstein ring radius as the size of the astrometric ellipse, i.e., the semimajor axis, is directly proportional to \( \theta_E \). Once the angular Einstein ring radius is determined from the astrometric shifts, one can determine the lens proper motion from \( \theta_E \) combined with the event timescale, which is determined from photometric monitoring of the event.

Since astrometric centroid shifts of microlensing events exhibit various shapes depending on the combination of the lensing parameters, i.e., \( \beta \), \( t_E \), and \( \theta_E \), the uncertainties in the astrometrically determined values of the angular Einstein ring radius, \( \Delta(\theta_E/\theta_{E,0}) \), will depend on the lensing parameters. The values of the lensing parameters \( t_E \) and \( \theta_E \) are related to the physical parameters of lenses, \( D_{ol} \), \( v \), and \( M \), and thus the uncertainties will also depend on the lens parameters. In addition, due to the differences in the lens parameters for different types of lensing events, the expected values of \( \Delta(\theta_E/\theta_{E,0}) \) for LMC events will be different from those of Galactic bulge events. As a result, the expected uncertainties of \( \theta_E \) will have a wide range.

Boden, Shao, & Van Buren (1998) performed a detailed study of the performance of the astrometric observations of gravitational microlensing events in determining various observables, including \( \theta_E \). From this analysis, they demonstrated the usefulness of astrometric observations of lensing events in determining the lens parameters. In their analysis, however, Boden et al. (1998) considered only LMC events, while the majority of events are detected toward the Galactic bulge. In addition, their estimate of the uncertainties is based on the very limited number of test cases they considered. For example, all of the events they considered had a fixed lens location of \( D_{ol} = 8 \) kpc, and their only values for the Einstein timescale and impact parameter were \( t_E = 0.1 \) and \( 0.2 \) yr and \( \beta = 0.4 \) and \( 0.8 \), respectively. They also did not take into account the restriction the source star brightness can impose on astrometric observations. Therefore, one cannot make general conclusions about the usefulness of astrometrically monitoring lensing events based on their analysis alone.

In this paper, we investigate the applicability of the astrometric method to the determination of the lens parameters for events toward both the LMC and the Galactic bulge. For this analysis, we investigate the dependency of \( \Delta(\theta_E/\theta_{E,0}) \) on the lens parameters by testing various types of events. In addition, by computing \( \Delta(\theta_E/\theta_{E,0}) \) for events with the most probable lensing parameters for a given lens mass under the standard models of the Galactic matter density and velocity distributions, we determine the expected distribution of the uncertainties in \( \theta_E \) as a function of a lens mass. From this study, we find that the values of the angular Einstein ring radius are expected to be measured with uncertainties \( \Delta(\theta_E/\theta_{E,0}) \lesssim 10\% \) up to the lens mass of \( M \sim 0.1 \) \( M_\odot \) for both Galactic disk-bulge and
halo-LMC events with a moderate observational strategy. The uncertainties are relatively large for Galactic bulge-bulge self-lensing events, $\Delta(\theta_E/\theta_{E,0}) \sim 25\%$ for $M \sim 0.1\, M_\odot$, but they can be substantially reduced by adopting more aggressive observational strategies. We also find that although astrometric observations can be performed for most photometrically detected Galactic bulge events, a significant fraction ($\sim 45\%$) of LMC events cannot be astrometrically observed due to the faintness of their source stars.

2. Dependency of Lens Parameters

To show the applicability of the astrometric method to the determination of $\theta_E$, we begin our analysis by investigating how the uncertainties in an astrometrically determined $\theta_E$ changes with respect to varying lens parameters. We determine the uncertainties in $\theta_E$ by conducting model fits to the astrometric shifts of simulated events with various lensing parameters. The result of the fit is obtained by computing $\chi^2$, i.e.,

$$\chi^2 = \frac{1}{\sigma_{\delta\theta_c}^2} \sum_{i=1}^{N_{\text{obs}}} \left\{ [\delta\theta_{c,x}(t_i) - \delta\theta_{c,0,x}(t_i)]^2 + [\delta\theta_{c,y}(t_i) - \delta\theta_{c,0,y}(t_i)]^2 \right\},$$

(2.1)

where $N_{\text{obs}}$ is the number of the astrometric observations and $(\delta\theta_{c,0,x}, \delta\theta_{c,0,y})$ and $(\delta\theta_{c,x}, \delta\theta_{c,y})$ are the centroid shifts of the simulated and model events, respectively. Following the mission specifications of SIM, we adopt the positional accuracy of the astrometric measurements to be $\sigma_{\delta\theta_c} = 0.01\,\text{mas}$ (http://huey.jpl.nasa.gov/sim). Since our goal is to see the variation of $\Delta(\theta_E/\theta_{E,0})$ with respect to individual lensing parameters, the uncertainties are determined under fixed but realistic observational conditions. We will discuss the expected uncertainties under other observational conditions in § 4. To determine the uncertainties, we assume that the astrometric centroid shifts of events are measured with a frequency $f = 1\,\text{day}^{-1}$ during $t_{\text{min}} \leq t_{\text{obs}} \leq t_{\text{max}}$, where $t_{\text{min}}$ and $t_{\text{max}}$ are the times of the first and last astrometric measurements. Since the astrometric observations will be performed for events that are photometrically detected, we set $t_{\text{min}} = -0.4\,t_E$ while $t_{\text{max}} = 3.0t_E$. The uncertainties are determined by the $3\sigma$ level, which is equivalent to $\Delta\chi^2 = \chi^2 - \chi_{\text{min}}^2 = 9$, where $\chi_{\text{min}}^2$ is the best-fitting $\chi^2$ value. To illustrate the sensitivity of this uncertainty on the lens parameters, we have tested a sample event with lensing parameters $(\beta, t_E, \theta_E) = (0.5, 11.3\,\text{days}, 0.22\,\text{mas})$. Then the dependency of the uncertainty on each lensing parameter is obtained by varying the parameter of interest while holding the other lensing parameters constant.

In Figure 1, we present the uncertainties of the astrometrically determined angular Einstein ring radius with respect to various lensing parameters. From the figure, one finds that the uncertainties increase significantly with decreasing angular Einstein ring radius and decreasing Einstein timescale. On the other hand, the dependency of $\Delta(\theta_E/\theta_{E,0})$ on the impact parameter is negligible.
3. Dependency on Lens Masses

In the previous section we showed that the uncertainty $\Delta(\theta_E/\theta_{E,0})$ depends strongly on the size of the angular Einstein ring radius and the duration of the Einstein timescale. Since both $\theta_E$ and $t_E$ are related to the mass of the lens, the strong dependency of $\Delta(\theta_E/\theta_{E,0})$ on $\theta_E$ and $t_E$ implies that the uncertainties for events caused by different populations of lenses will be greatly different. Then the naturally arising questions are: “Is the astrometric observation of lensing events a guaranteed method of determining $\theta_E$?” and if not, “To what extent can one determine $\theta_E$ with this method?” To answer these questions, one must determine the expected value of $\Delta(\theta_E/\theta_{E,0})$ as a function of lens mass.

We determine the relation between $\Delta(\theta_E/\theta_{E,0})$ and the lens mass by computing the uncertainties for events with lensing parameters which are the most probable for a given lens mass. Because the values of $\theta_E$ and $t_E$ also depend on the lens parameters $v$ and $D_{ol}$, there is no one-to-one correspondence between the lens mass and the lensing parameters. However, with models of the Galactic matter density and velocity distributions, one can statistically determine the most probable values of the lensing parameters corresponding to individual lens masses.

With models for the Galactic matter density and velocity distributions, the distributions of $t_E$ and $\theta_E$ for a given lens mass $M$ are obtained by

$$f(t_E) = \int_0^\infty dD_{os} \rho(D_{os}) \int_0^{D_{os}} dD_{ol} \rho(D_{ol}) \pi r_E^2 \int_0^\infty \int_0^\infty dv_y dv_z v f(v_y, v_z) \delta\left[t_E - \left(\frac{4GM}{c^2 v^2 D_{ol} D_{os}}\right)^{1/2}\right],$$

$$f(\theta_E) = \int_0^\infty dD_{os} \rho(D_{os}) \int_0^{D_{os}} dD_{ol} \rho(D_{ol}) \pi r_E^2 \int_0^\infty \int_0^\infty dv_y dv_z v f(v_y, v_z) \delta\left[\theta_E - \left(\frac{4GM}{c^2 D_{ls} D_{ol} D_{os}}\right)^{1/2}\right],$$

for the Einstein timescale and the angular Einstein ring radius. Here $\rho(D_{ol})$ and $\rho(D_{os})$ are the density distributions of lens and source stars, $\delta$ is the delta function, $v_y$ and $v_z$ are the components of the transverse velocity $v$, and $f(v_y, v_z)$ is their distribution. In the above equations, the factors $\pi r_E^2$ and $v$ are included because events with larger cross-sections and higher transverse speeds are more likely to occur. For Galactic bulge events (disk-bulge plus bulge-bulge self-lensing events), $v_y$ and $v_z$ represent the velocity components which are normal and azimuthal to the Galactic plane, respectively. On the other hand, since we adopt a non-rotating isotropic velocity model for LMC events (halo-LMC events, see Table 1), these variables represent two velocity components of arbitrary direction in the plane normal to the line of sight.
toward the LMC. We assume that the individual components of the transverse velocities have Gaussian distributions of the form

$$f(v_i) \propto \exp \left[ -\frac{(v_i - \bar{v}_i)^2}{2\sigma_{v_i}^2} \right], \quad i \in \{y, z\}. \quad (3.3)$$

The values of the mean, \(\bar{v}_i\), and the dispersion, \(\sigma_{v_i}^2\), of the velocity distributions, which are listed in Table 1, are adopted from the models of Han & Gould (1995) for the Galactic bulge events and from Han & Gould (1996) for the LMC events. For the matter density distributions of the individual Galactic components, we adopt an axisymmetric model (Kent 1992) for the Galactic bulge, a double-exponential disk model (Bahcall 1986) for the Galactic disk, and an isothermal sphere model with a core radius (Bahcall, Schmidt, & Soneira 1983) for the Galactic halo. The adopted models of the matter density distributions are listed in Table 2. The adopted distances to the Galactic center and to the LMC are 8.0 kpc and 55 kpc, respectively.

In Figure 2, we present the expected distribution of \(f(\theta_E)\) and \(f(t_E)\) for various types of events and different values of lens mass. From the figure, one can see that the expected distribution for a given type of event covers a wide range, considering the variety of lens populations and corresponding lens masses. In addition, even with a fixed lens mass, the distributions for different types of events are substantially different from each other. Therefore, the analysis of \(\Delta(\theta_E/\theta_{E,0})\) based on limited sets of lensing parameters for only a single type of lensing events will lead to erroneous conclusions about the general applicability of the astrometric method to the determination of \(\theta_E\).

Once the most probable values of \(\theta_E\) and \(t_E\) for a given lens mass are obtained from their distributions, the expected values of \(\Delta(\theta_E/\theta_E)\) are determined as before, by carrying out a \(\chi^2\) fit to the astrometric centroid shifts of the event with the most probable lensing parameters. Since the dependency of the uncertainties on the impact parameter is not important, we assume \(\beta = 0.5\) for all simulated events. The astrometric centroid shifts of the events are assumed to be measured under moderate observational conditions with a frequency \(f = 0.5\) day\(^{-1}\) during \(-0.4t_E \leq t_{obs} \leq 3.0t_E\) and with a positional accuracy of \(\sigma_{\delta\theta_c} = 0.01\) mas. Then, the relation between \(\Delta(\theta_E/\theta_E)\) and lens mass is obtained by repeating the uncertainty determinations for different lens masses.

In Figure 3, we present the resulting relation between the lens mass and the expected uncertainty in the astrometrically determined values of \(\theta_E\). We find that the angular Einstein ring radii are expected to be measured with uncertainties \(\Delta(\theta_E/\theta_{E,0}) \lesssim 10\%\) up to the lens mass of \(M \sim 0.1\, M_\odot\) for both Galactic disk-bulge and halo-LMC events. For Galactic bulge-bulge self-lensing events, the uncertainty is relatively large: \(\Delta(\theta_E/\theta_{E,0}) \sim 25\%\) for \(M \sim 0.1\, M_\odot\).
4. Improved Astrometric Observational Strategies

Up to now, we have investigated the uncertainties $\Delta(\theta_E/\theta_{E,0})$ for a fixed observational strategy. In this section, we investigate the dependency of the uncertainties on observational strategies to find the optimal astrometric observational strategies for better determinations of $\theta_E$. The accuracy of the astrometric determination of $\theta_E$ can be broadly improved in three ways. First of all, the uncertainties will be decreased if the astrometric positional accuracy can be improved. However, since the positional accuracy is restricted by the instrumentation and not by the observational strategy, we do not consider this scenario further. Secondly, the accuracy can be improved by increasing the duration of the astrometric observations. Finally, increasing the frequency of astrometric observation will also help to improve the accuracy in determining $\theta_E$.

In Figure 4, we present the uncertainties as functions of the frequency (upper panel) and the duration (lower panel) of the astrometric measurements of $\delta\theta_c$. The uncertainties are obtained for an example event with a set of lensing parameters $(\beta, t_E, \theta_E) = (0.5, 11.3 \text{ days}, 0.22 \text{ mas})$ under various observational conditions. From the figure, one finds that the accuracy improves significantly with increasing observational frequencies. Longer observations of the event also contribute to decrease the uncertainty of $\theta_E$ determination. However, observations longer than $t_{\text{max}} \sim 3t_E$ do not help to a significant reduction of the uncertainty.

As we know that measuring $\delta\theta_c$ with increased frequency will help to improve the accuracy in determining $\theta_E$, we re-determined the relation between $\Delta(\theta_E/\theta_{E,0})$ and the lens mass for a new observational strategy with $f = 3 \text{ day}^{-1}$. Since we already tested the uncertainties for a long enough observational duration of $t_{\text{max}} = 3t_E$, we only increased the observational frequency for this test. In Figure 2, we present the newly determined $\Delta(\theta_E/\theta_{E,0})-M$ relation (represented by a thick solid line) for the Galactic bulge-bulge self-lensing events, which have the largest uncertainties among the various types of events. One finds that the accuracy of the $\theta_E$ determination significantly improves – the uncertainty decreases by approximately one-half over the entire range of lens masses.

5. Restriction by Source Star Brightness

The applicability of the astrometric method in determining lens parameters is additionally restricted by source star brightness because the astrometric observations will only be possible for events with bright source stars. For both the Galactic bulge and LMC fields, the photometric monitoring of source stars is restricted by crowding. Current microlensing experiments toward the Galactic bulge reach the photometric detection limit when the stellar density of the fields arrives at $\sim 10^4 \text{ stars deg}^{-2}$ (C. Alcock 1997, private communication). Based on the model luminosity function of Han (1997), this corresponds
to $V \sim 20.5$, which approximately coincides with the astrometric detection limit of SIM ($V \sim 20$, [http://huey.jpl.nasa.gov/sim](http://huey.jpl.nasa.gov/sim)). Therefore, the astrometric observations of microlensing events can be achieved for most of the photometrically detected Galactic bulge events. By using the $R$-band luminosity function provided by Gould (1998), we also determine the fraction of LMC events with source stars bright enough for astrometric observation. With the photometric determination limit of $V \sim 21$ (Alcock et al. 1997b), which corresponds to $R \sim 21.1$, we find that nearly half ($\sim 45\%$) of LMC source stars are fainter than the astrometric detection limit ($R \sim 20.25$). This implies that for LMC events the major restriction in the general applicability of the astrometric $\theta_E$ determination comes from the faintness of the source stars and not from the large uncertainties in the determined values of $\theta_E$.

6. Conclusion

In this paper, we have considered the effectiveness of astrometric monitoring of gravitational microlensing events in determining the lens parameters. The main results from this analysis can be summarized as follows:

1. The uncertainties of the astrometrically determined angular Einstein ring radii are strongly dependent on the values of $\theta_E$ and $t_E$ for the events. However, the dependency of $\Delta(\theta_E/\theta_{E,0})$ on the impact parameter is not important.

2. The distributions of $\theta_E$ and $t_E$ for events caused by various populations of lenses cover wide ranges. Even with a fixed lens mass, the distributions for different types of lensing events are substantially different from each other. Therefore, the analysis of $\Delta(\theta_E/\theta_{E,0})$ based on limited sets of lensing parameters for only a single type of lensing events will lead to erroneous conclusions about the general applicability of the astrometric method in determining lens parameters.

3. Measurements of $\delta \theta_c$ with increased frequencies during longer times of observations help to improve the accuracy of $\theta_E$ determination. However, measurements longer than $t_{\text{max}} \sim 3t_E$ do not contribute to a significant reduction in $\Delta(\theta_E/\theta_{E,0})$.

4. With a moderate observational strategy, the value of $\theta_E$ can be determined with an uncertainty $\Delta(\theta_E/\theta_{E,0}) \lesssim 10\%$ up to a lens mass of $M \sim 0.1 \, M_\odot$ for the Galactic disk-bulge and halo-LMC events. The uncertainties for the Galactic bulge-bulge self-lensing events is relatively large with $\Delta(\theta_E/\theta_{E,0}) \sim 25\%$ for $M \sim 0.1 \, M_\odot$. However, the uncertainty can be reduced substantially by adopting a more aggressive observational strategy.

5. Astrometric observations will be possible for most photometrically detected Galactic bulge events. However, nearly half of LMC events will not be astrometrically
observable due to the faintness of their source stars. Therefore, for LMC events the major restriction in the general applicability of the astrometric determination of $\theta_E$ comes from the source star brightness and not from the large uncertainties in the determined value of $\theta_E$.

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**TABLE 1**

**The Transverse Velocity Distribution Models**

| event type | $\bar{v}_g$ (km s$^{-1}$) | $\sigma_{\bar{v}_g}$ (km s$^{-1}$) | $\bar{v}_z$ (km s$^{-1}$) | $\sigma_{\bar{v}_z}$ (km s$^{-1}$) |
|------------|-----------------------------|---------------------------------|---------------------------|---------------------------------|
| bulge-bulge | $-220(1 - D)$               | $[100^2(1 + D)^2]^{1/2}$       | 0                         | $[100^2(1 + D)^2]^{1/2}$       |
| disk-bulge  | $220D$                      | $[30^2 + 100^2D^2]^{1/2}$      | 0                         | $[20^2 + 100^2D^2]^{1/2}$      |
| halo-LMC    | 0                           | $250/\sqrt{2}$                | 0                         | $250/\sqrt{2}$                |

**NOTE.**— The transverse velocity distribution models for different types of microlensing events. The distributions are assumed to be Gaussian in form, $f(v_i) = \exp\left[-(v_i - \bar{v}_i)^2/2\sigma_{\bar{v}_i}^2\right], i \in \{y, z\}$, and the values of the mean and the standard deviation of the distributions are listed. Here $D = D_{ol}/D_{os}$.

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**TABLE 2**

**The Model Matter Density Distributions**

| Galactic components | distribution $(M_\odot$ pc$^{-3}$) |
|---------------------|-----------------------------------|
| disk $\rho(R, z)$  | $0.06 \exp\{-[(R - R_0)/3500 + z/325]\}$ |
| bulge $\rho(s)$    | $1.04 \times 10^6(s/0.482)^{-1.85}$ (for $s < 938$ pc) |
|                     | $3.53K_0(s/667)$ (for $s \geq 938$ pc) |
| halo $\rho(r)$     | $7.9 \times 10^{-3}(r_c^2 + R_0^2)/(r_c^2 + r^2)$ |

**NOTE.**— In the Galactic bulge model, $s^4 = R^4 + (z/0.61)^4, R = (x^2 + y^2)^{1/2}$ where $x$ and $z$ represent the axes directed along the line of sight and toward the Galactic pole. The notation $K_0$ represents a modified Bessel function. The adopted values of the solar Galactocentric distance and the core radius of the halo is $R_0 = 8$ kpc and $r_c = 2$ kpc, respectively.
Figure 1: The dependency of the uncertainties of the astrometrically determined $\theta_E$ on various lensing parameters. The astrometric centroid shifts of the event are assumed to be measured with a frequency $f = 1 \, \text{day}^{-1}$ during $-0.4 t_E \leq t_{\text{obs}} \leq 3.0 t_E$ and with a positional accuracy $\sigma_{\delta \theta_c} = 0.01 \, \text{mas}$. To investigate the dependencies, we test an example event with a set of lensing parameters of $(\beta, t_E, \theta_E) = (0.5, 11.3 \, \text{days}, 0.22 \, \text{mas})$. The dependency of the uncertainties on each lensing parameter is obtained by varying the parameter of interest while holding the other lens parameters constant. One finds that the uncertainties $\Delta(\theta_E/\theta_{E,0})$ increase significantly with decreasing angular Einstein ring radius and with decreasing Einstein timescale. On the other hand, the dependency of $\Delta(\theta_E/\theta_{E,0})$ on the impact parameter is negligible.
Figure 2: The distributions of the Einstein ring radii and the Einstein timescales for various types of events and different values of lens mass. From the figure, one finds that the expected distribution for a given type of event covers a wide range, considering the variety of lens populations and corresponding lens masses. In addition, even with a fixed lens mass, the distributions for different types of events are substantially different from each other. Therefore, the analysis of $\Delta(\theta_R/\theta_{E,p})$ based on limited sets of lensing parameters for only a single population of lensing events will lead to erroneous conclusions about the general applicability of the astrometric method to the determination of $\theta_E$. 
Figure 3: The relation between the lens mass and the expected uncertainties in the astrometrically determined values of angular Einstein ring radii for various types of events. The uncertainties are determined by carrying out $\chi^2$ fits to the astrometric centroid shifts of the events with lensing parameters $\theta_E$ and $t_E$, which are the most probable for a given lens mass under the models of Galactic matter density and velocity distributions. Since the dependency of the uncertainty on the impact parameters is not important, we assume $\beta = 0.5$ for all events. For the moderate observational conditions, the astrometric centroid shifts of the events are assumed to be measured with $f = 0.5 \text{ day}^{-1}$ during $-0.4t_E \leq t_{\text{obs}} \leq 3.0t_E$ and a positional accuracy of $\sigma_{\delta}\theta_c = 0.01$ mas. The $\Delta(\theta_R/\theta_{E,p})$-$M$ relation for the Galactic bulge events is re-determined under a new observational strategy with $f = 3 \text{ day}^{-1}$ and it is represented by a thick solid line.
Figure 4: The dependency of the astrometrically determined $\theta_E$ on the observational frequencies (upper panel) and the duration of measurements (lower panel). The uncertainties are determined for an example event with a set of lensing parameter of $(\beta, t_E, \theta_E) = (0.5, 11.3 \text{ days}, 0.22 \text{ mas})$ under different frequencies and durations of astrometric measurements of the centroid shifts. From the figure, one finds that the accuracy of $\theta_E$ determination improves significantly with increasing observational frequencies. Longer observations of the event also contribute to decrease the uncertainty of $\theta_E$ determination. However, measurements of $\delta \theta_c$ longer than $t_{\text{max}} \sim 3t_E$ do not help to a significant reduction of the uncertainty.