NMR properties of $^3$He-A in biaxially anisotropic aerogel

V V Dmitriev, D A Krasnikhin, A A Senin and A N Yudin
P. L. Kapitza Institute for Physical Problems RAS, Moscow, Russia
E-mail: krasnighind@mail.ru

Abstract. Theoretical model of G.E. Volovik for A-like phase of $^3$He in aerogel suggests formation of Larkin-Imry-Ma state of Anderson-Brinkmann-Morel order parameter. Most of results of NMR studies of A-like phase are in a good agreement with this model in assumption of uniaxial anisotropy, except for some of experiments in weakly anisotropic aerogel samples. We demonstrate that these results can be described in frames of the same model in assumption of biaxial anisotropy. Parameters of anisotropy in these experiments can be determined from the NMR data.

1. Introduction
Properties of A-like superfluid phase of $^3$He in aerogel depend on value and type of aerogel anisotropy. In case of large uniaxial squeezing A-like phase is similar to the bulk A phase, but orbital vector $\mathbf{l}$ is fixed along the axis of squeezing [1]. In other cases anisotropic orbital glass (OG) state is formed in which the spatial distribution of $\mathbf{l}$ corresponds to Larkin-Imry-Ma state with characteristic length $l_{LIM} \sim 1 \mu m$ [2, 3, 4]:

$$\langle \mathbf{l} \rangle = 0 , \quad \langle l_x^2 \rangle + \langle l_y^2 \rangle + \langle l_z^2 \rangle = 1 ,$$

where $\langle \rangle$ means averaging over distances much larger than $l_{LIM}$. The spatial distribution of spin vector $\mathbf{d}$ may correspond to glass state (spin glass, SG) or to spin nematic (SN) state [5]. In the SN state the distribution of vector $\mathbf{d}$ is homogeneous and its equilibrium orientation corresponds to minimum of the dipole energy, while in the SG state $\mathbf{d}$ is randomly oriented in the plane normal to external steady magnetic field. The NMR frequency shift in OG states may be calculated from the following equation:

$$\Delta \omega / \Delta \omega_0 = \frac{7 \cos \beta + 1}{4} \left( \langle (1 \times \mathbf{h})^2 \rangle - \frac{1 + \cos \beta}{2} \langle (\mathbf{l} \cdot \mathbf{d} \times \mathbf{h})^2 \rangle \right) - \cos \beta ,$$

where $\Delta \omega_0 = \Omega_A^2/(2\omega)$ is maximal possible value of the NMR shift, $\Omega_A$ is Leggett frequency, $\mathbf{h}$ is the unit vector oriented along the direction of the external magnetic field and $\beta$ is the tipping angle of magnetization. Note that $\mathbf{d}$ in Eq.2 is the result of averaging over fast precession - it coincides with the original SN vector $\mathbf{d}$ only for $\beta = 0$. Most of the results of NMR studies of A-like phase are in excellent agreement with Eq.2 in assumption of uniaxial anisotropy of aerogel samples [5]. However, some observations (e.g. presented in [6, 7, 8]) have not been explained. Here we demonstrate that these results also can be described by the above mentioned model in assumption of biaxial anisotropy of aerogel.
2. Biaxial anisotropy

Aerogel samples usually grow in cylindrical glass tubes, so the axis of the tube determines one of the axes of anisotropy which we choose to be parallel to z-axis. Then for the case of biaxial anisotropy we can write:

\[
\langle l_x^2 \rangle = \frac{1}{3} + \delta_1 ; \quad \langle l_y^2 \rangle = \frac{1}{3} - \delta_2 ; \quad \langle l_z^2 \rangle = \frac{1}{3} - \delta_1 + \delta_2 ,
\]

where \(-1/3 < \delta_1 < 2/3; -2/3 < \delta_2 < 1/3\) and \(-1/3 < (\delta_2 - \delta_1) < 2/3\). If \(\mathbf{h} \parallel \mathbf{z}\) (as in most experiments) then the normalized dipole energy for the SN state is:

\[
U_D = -\langle (dl)^2 \rangle = \delta_2 - (\delta_1 + \delta_2) \cos^2 \alpha - \frac{1}{3} ,
\]

where \(\alpha\) is the angle between \(\mathbf{d}\) and \(\mathbf{x}\)-axis. If \((\delta_1 + \delta_2) < 0\) then the dipole energy is in minimum for \(\mathbf{d} \parallel \mathbf{y}\) and the NMR frequency shift for OG-SN state follows from Eq.2 after averaging over \(l\):

\[
\Delta \omega / \Delta \omega_0 = \frac{1}{4}[(7\delta_1 - 5\delta_2) \cos \beta + \delta_1 + \delta_2] .
\]

If \((\delta_1 + \delta_2) > 0\) then the dipole energy is in minimum for \(\mathbf{d} \parallel \mathbf{x}\) and we get the same answer. The NMR shift for the OG-SG state is obtained by averaging over \(l\) and \(d\):

\[
\Delta \omega / \Delta \omega_0 = \frac{3}{2}(\delta_1 - \delta_2) \cos \beta .
\]

3. Biaxial anisotropy in experiments

We suggest that in experiments described in [6, 7, 8]) the aerogel samples had biaxial anisotropy. In Ref. [7] two continuous wave (CW) NMR signals (f-state and c-state) were observed in the A-like phase. These signals should correspond to OG-SN and OG-SG state respectively. NMR signal with a smaller frequency shift (c-state) suits to OG-SG state in experiments. NMR data were used to calculate the dipole energy. The NMR shift for the OG-SG state is obtained by averaging over \(l\) and \(d\):

\[
\langle \omega \rangle = \left( \frac{1}{3} + \delta_1 \right) \omega_0 - \delta_2 \omega_0 ,
\]

where \(\omega_0\) is the NMR frequency shift for the OG-SG state. The result of such calculations without fitting parameters is shown in Fig. 1 by solid line while filled circles are the experimental points from [7]. The obtained agreement between the experiment and the calculated curve confirms our suggestion of biaxial anisotropy of the aerogel sample.

In experiments described in [6] and [8] (Sample 3) the frequency shift of FIDS was found to be approximately proportional to \(1 + \cos \beta\). From Eqs. 5 and 6 follow that it is possible only
Figure 1. Frequency shift of FIDS versus the tipping angle of magnetization in mixed "f+c" state. Experimental data from [7] are shown by filled circles. Solid line is the theoretical dependence calculated for these experimental conditions without any fitting parameters and in assumption that $\delta_1 = 0.11$ and $\delta_2 = 0.085$.

for SN state and only if $\delta_1 \approx \delta_2$. Experiments [6] were done with 97.5% aerogel and we can not use the data for $\Omega_{2A}$ from [5] for estimation of values of $\delta_1$ and $\delta_2$. As for the Sample 3 then the observed in [8] value of CW NMR frequency shift implies that $\delta_1 \approx \delta_2 \approx 0.13$.

4. Discussion
We conclude that all known NMR experiments in the A-like phase of $^3$He in aerogel are in a good agreement with the model developed in [2, 3, 4, 5] in assumption of uniaxial or biaxial anisotropy of the aerogel samples. Parameters of anisotropy in the orbital space of the A-like phase order parameter can be calculated from NMR measurements. This anisotropy appears as a result of anisotropy in orientation of aerogel strands. The relationship between these anisotropies, however, has not been established theoretically.

Acknowledgments
We are grateful to G.E. Volovik for stimulating discussions. This work was supported in part by RFBR grant 09-02-01185.

References
[1] Kunimatsu T, Sato T, Izumina K, Matsubara A, Sasaki Y, Kubota M, Ishikawa O, Mizusaki T and Bunkov Yu M 2007 JETP Lett. 86 216
[2] Volovik G E 1996 JETP Lett. 63 301
[3] Volovik G E 2008 J. Low Temp. Phys. 150 453
[4] Elbs J, Bunkov Yu M, Collin E, Godfrin H and Volovik G E 2008 Phys. Rev. Lett. 100 215304
[5] Dmitriev V V, Krasnikhin D A, Mulders N, Senin A A, Volovik G E, Yudin A N 2010 JETP Lett. 91 599
[6] Ishikawa O, Kado R, Nakagawa H, Obara K, Yano H, Hata T, Yokogawa H and Yokoyama M 2006 AIP Conf. Proc. 850 235
[7] Dmitriev V V, Levitin L V, Mulders N and Zmeev D E 2006 JETP Lett. 84 461
[8] Dmitriev V V, Krasnikhin D A, Mulders N, Zavjalov V V and Zmeev D E 2007 JETP Lett. 86 594
[9] Dmitriev V V, Levitin L V and Zmeev D E (not published)