The rms peculiar velocity of galaxy clusters for different cluster masses and radii

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ABSTRACT
We investigate the rms peculiar velocity of galaxy clusters in the Lambda cold dark matter ($\Lambda$CDM) and tau cold dark matter ($\tau$CDM) cosmological models using $N$-body simulations. Cluster velocities for different cluster masses and radii are examined. To identify clusters in the simulations we use two methods: (1) the standard friends-of-friends (FOF) method and (2) the method in which the clusters are defined as the maxima of the density field smoothed on the scale $R \sim 1 \, h^{-1} \text{Mpc}$ (DENSMAX). If we use the DENSMAX method, the size of the selected clusters is similar for all clusters. We find that the rms velocity of clusters defined with the DENSMAX method is almost independent of the cluster density and similar to the linear theory expectations. The rms velocity of FOF clusters decreases with the cluster mass and radius. In the $\Lambda$CDM model, the rms peculiar velocity of massive clusters with an intercluster separation $d_\text{i} = 50 \, h^{-1} \text{Mpc}$ is $\approx 15$ per cent smaller than the rms velocity of the clusters with a separation $d_\text{el} = 10 \, h^{-1} \text{Mpc}$.

Keywords: galaxies; clusters: general -- cosmology: theory -- dark matter -- large-scale structure of Universe.

1 INTRODUCTION
One of the interesting unknowns in cosmology is the large-scale peculiar velocity field in the Universe. The peculiar velocity field can be studied by using galaxies or clusters of galaxies. However, there are some advantages in studying the peculiar velocity field by using galaxy clusters. One of these advantages comes from the fact that, on scales probed by galaxy clusters, velocity fluctuations are largely in the quasi-linear regime and close to the initial state from which large-scale structures developed. In addition, peculiar velocities of clusters can be determined more accurately than peculiar velocities of galaxies since the distance to each cluster can be obtained from a large number of member galaxies, thus considerably reducing the velocity uncertainties of clusters. Cluster motions could therefore provide an important tool in probing the large-scale peculiar velocity field.

Peculiar velocities of clusters of galaxies have been studied in several papers (e.g. Bahcall, Gramann & Cen 1994; Lauer & Postman 1994; Bahcall & Oh 1996; Moscardini et al. 1996; Borgani et al. 1997; Watkins 1997; Dale et al. 1999; Hudson et al. 1999; Borgani et al. 2000; Colless et al. 2001). Watkins (1997) developed a likelihood method for estimating the rms peculiar velocity of clusters from line-of-sight velocity measurements. This method was applied to two observed samples of cluster peculiar velocities: the SCI sample (Giovanelli et al. 1997) and a subsample of the Mark III catalogue (Willick et al. 1997). Watkins (1997) found that the rms one-dimensional cluster peculiar velocity is $256^{+75}_{-100} \, \text{km s}^{-1}$, which corresponds to the three-dimensional rms velocity $459^{+130}_{-150} \, \text{km s}^{-1}$. Dale et al. (1999) obtained Tully–Fisher peculiar velocities for 52 Abell clusters distributed over the whole sky between $\sim 50$ and $\sim 200 \, h^{-1} \text{Mpc}$. They found that the rms one-dimensional cluster peculiar velocity is $341 \pm 93 \, \text{km s}^{-1}$, which corresponds to the three-dimensional rms velocity $591 \pm 161 \, \text{km s}^{-1}$.

Radial peculiar velocities of clusters can be determined to large distances by measuring the Sunyaev–Zel’dovich (1980) (SZ) effect. Rephaeli & Lahav (1991) made one of the first estimates of the possibility of measuring the peculiar velocities by using the SZ effect for a selected sample of galaxy clusters. However, most convincing measurements for individual clusters have been done only recently, using the new generation of sensitive bolometers (Holzapfel et al. 1997; Lamarre et al. 1998). The accuracy of SZ measurements for determining peculiar velocities of clusters have been studied in several papers (Haehnelt & Tegmark 1996; Aghanim, Gorski & Puget 2001; Diego, Hansen & Silk 2002; Holder 2002; Nagai, Kravtsov & Kosowsky 2002). With $\mu K$ sensitivity on arcminute scales at several frequencies it will be possible to measure peculiar velocities to an accuracy of $\sim 130 \, \text{km s}^{-1}$.

In this paper we study the rms peculiar velocity of clusters, $v_{\text{rms}}$, in different cosmological models assuming that the initial density fluctuation field is a Gaussian field. To investigate the nonlinear regime, we use $N$-body simulations. We examine cluster peculiar velocities for different cluster masses. Do cluster velocities depend on their masses? The rms peculiar velocity of peaks in the initial

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2.1 Simulations

We analyse peculiar velocities in \(N\)-body simulations carried out by the Virgo Consortium (1998) for two cosmological models, \(\Lambda CDM (\Omega_0 = 0.3, \Lambda = 0.7)\) and \(\tau CDM (\Omega_0 = 1)\) as described by Jenkins et al. (1998). In these cold dark matter (CDM) models the power spectrum of the initial conditions was chosen to be in the form given by Bond & Efstathiou (1984):

\[
P(k) = \frac{Ak}{1 + [aq + (bq)^{3/2} + (cq)^2]^{3/2}},
\]

where \(q = k/\Gamma, a = 6.4 \ Mpc^{-1}, b = 3 h^{-1} \ Mpc, c = 1.7 h^{-1} \ Mpc, \nu = 1.13\) and \(\Gamma = 0.21\). The normalization constant, \(A\), was chosen by fixing the value of \(\sigma_8\) (the linearly-extrapolated mass fluctuation in spheres of radius \(8 h^{-1} \ Mpc\)); \(\sigma_8 = 0.9\) and 0.51 for the \(\Lambda CDM\) and \(\tau CDM\) models, respectively.

We investigated the linear theory predictions for peculiar velocities of peaks in the \(\Lambda CDM\) and \(\tau CDM\) model. The linear rms peculiar velocity fluctuations on a given scale \(R\) can be expressed as

\[
\sigma(R) = H_0 f(\Omega_0) \sigma_{\cdm}(R),
\]

where the spectral moments \(\sigma_j\) are defined for any integer \(j\) by

\[
\sigma_j^2 = \frac{1}{2\pi^2} \int P(k) W^2(kR) k^{2j+2} \, dk,
\]

\(W(kR)\) is a window function and \(f(\Omega_0)\) is the dimensionless growth rate. The function \(f(\Omega_0) = 0.51\) and 1.0 in the \(\Lambda CDM\) and \(\tau CDM\) models, respectively. (We note that the approximation \(f(\Omega_0) = \Omega_0^{\frac{1}{2}}\) underestimates the dimensionless growth rate by \(\sim 5\) per cent in the flat \(\Omega_0 = 0.3\) model.)

Bardeen et al. (1986) showed that in the linear approximation the rms peculiar velocity at peaks of the smoothed density field differs systematically from \(\sigma(R)\), and can be expressed as

\[
\sigma_p(R) = \sigma(R) \sqrt{1 - \sigma_8^2/\sigma_5^2 \sigma_{\cdm}^2}.
\]

In this approximation, the rms velocities of peaks do not depend on the height of the peaks.

Fig. 1 shows the rms peculiar velocity of peaks, \(\sigma_p(R)\), in the \(\Lambda CDM\) and \(\tau CDM\) models. We have used the top-hat window function. For comparison, we show also the rms peculiar velocity \(\sigma(R)\) for the same models. For the radius \(R = 1 h^{-1} \ Mpc\), \(\sigma_p = 509\) and \(562\) km s\(^{-1}\) in the \(\Lambda CDM\) and \(\tau CDM\) models, respectively. At this radius, \(\sigma_p\) is \(\sim 2\) per cent lower than \(\sigma(R)\) for the models studied. On larger scales, the difference between \(\sigma_p\) and \(\sigma(R)\) increases.

Next we study peculiar velocities in \(N\)-body simulations. The Virgo simulations were created using a parallel adaptive particle–particle–particle–mesh (AP\(^3\)M) code as described by Couchman et al. (1995) and Pearce & Couchman (1997). It supplements the
standard P3M algorithm (Efstathiou et al. 1985) by recursively placing higher-resolution meshes, "refinements", in heavily-clustered regions. The Virgo simulations were done on two large Cray T3D parallel supercomputers at the Edinburgh Parallel Computing Centre and at the Computing Centre of the Max Plank Society in Garching. These simulations are publicly available at http://www.mpa-garching.mpg.de/Virgo/virgoproject.html

In the simulations used here, the evolution of particles was followed in the comoving box of size \(L = 239.5 \, h^{-1} \text{Mpc}\). The number of particles was \(N_p = 256^3\). Therefore, the mean particle separation \(\lambda_p = L/N_p^{1/3} = 0.935 \, h^{-1} \text{Mpc}\). The mass of the particle \(m_p = \rho_p \lambda_p^3 = 6.82 \times 10^{10} \, h^{-1} \text{Mpc}\) and \(m_p = 2.27 \times 10^{13} \, h^{-1} \text{Mpc}\) in the \(\Lambda\text{CDM}\) and \(\tau\text{CDM}\) models, respectively (here \(\rho_p\) is the mean background density). The gravitational softening length is \(\tau_{\text{soft}} = 25\) and \(36 \, h^{-1} \text{kpc}\), respectively (see Jenkins et al. 1998 for a detailed description of the force calculation scheme used in the Virgo simulations). We denote the Virgo \(\Lambda\text{CDM}\) and \(\tau\text{CDM}\) models as models \(\Lambda\text{CDM}1\) and \(\tau\text{CDM}\).

We also investigated the evolution of \(256^3\) particles on a \(256^3\) grid using a particle–mesh (PM) code described by Gramann (1988) and Suhhonenko & Gramann (1999). The PM code achieves a force resolution close to the mean particle separation \(\lambda_p\). The PM method is discussed in detail by Hockney & Eastwood (1981) and Efstathiou et al. (1985). The cosmological parameters for this PM simulation were chosen similar to those of the \(\Lambda\text{CDM}1\) model. We chose the flat \(\Omega_\Lambda = 0.3\) model and used the initial power spectrum \(P(k)\) given in equation (1) (with \(\sigma_8 = 0.9\)). The comoving box size was \(L = 239.5 \, h^{-1} \text{Mpc}\). Therefore, the mean particle separation, \(\lambda_p\), and the mass of the particle, \(m_p\), in this model are the same as used in the \(\Lambda\text{CDM}1\) model. We denote this model as the \(\Lambda\text{CDM}2\) model.

We examined the rms velocity of particles in the simulations studied. In the \(\Lambda\text{CDM}1\) and \(\tau\text{CDM}\) models, the rms velocity of particles was 648 and 636 km s\(^{-1}\), respectively. In the \(\Lambda\text{CDM}2\) model, the rms velocity was 575 km s\(^{-1}\). Owing to the small-scale smoothing inherent in the PM method, the intrinsic velocity dispersions of clusters in the \(\Lambda\text{CDM}2\) model are smaller than in the \(\Lambda\text{CDM}1\) model. The velocity field of clusters in these models is expected to be similar.

In the \(\Lambda\text{CDM}2\) model we used the traditional two-point approximation to calculate the acceleration on the grid. We also studied the evolution of particles using a PM code with the shifted-mesh scheme. The effect of the shifted-mesh scheme on the cluster velocities is discussed in Section 4.

### 2.2 Selection of clusters

We used two different algorithms to identify clusters in simulations: (1) the standard friends-of-friends (FOF) algorithm and (2) the algorithm in which clusters are defined as maxima of the density field smoothed on the scale \(R \sim 1 \, h^{-1} \text{Mpc}\) (DENSMAX).

The friends-of-friends group finder algorithm was applied using the program suite developed by the cosmology group in the University of Washington. These programs are available at http://www-hpcc.astro.washington.edu

The FOF cluster finder depends on one parameter \(b\), which defines the linking length as \(b \lambda_p\). The conventional choice for this parameter is \(b = 0.2\) (see e.g. Götz, Huchra & Brandenberger 1998; Jenkins et al. 2001). In this paper we also define clusters by using the value \(b = 0.2\). We also study velocities of the clusters defined by the parameters \(b = 0.15\) and \(b = 0.3\). In the limit of very large numbers of particles per object, FOF approximately selects the matter enclosed by an isodensity contour at \(1/b^3\).

To test our FOF output data, we found the mass function of clusters in the Virgo simulations. The cluster mass function in these simulations has been studied in detail by Jenkins et al. (2001). Fig. 2 shows the mass function of clusters determined for \(b = 0.2\) and \(b = 0.15\) in the \(\Lambda\text{CDM}1\) model and for \(b = 0.2\) in the \(\tau\text{CDM}\) model. For comparison we show the mass functions given by the approximations obtained by Jenkins et al. (2001) (equations B2 and B1 for the \(\Lambda\text{CDM}\) and \(\tau\text{CDM}\) model, respectively). Jenkins et al. (2001) Studied the mass function at the high-mass end up to the point where the predicted Poisson abundance errors reach 10 per cent. In the simulations studied here, this limit is reached when the number density of clusters is \(n = 7.3 \times 10^{-5} \, h^{-5} \text{Mpc}^{-3}\). Fig. 2 shows that if \(n\) is larger than this value, the agreement between our results and those obtained by Jenkins et al. (2001) is very good.

We studied clusters that contained at least ten particles. The three-dimensional velocity of each cluster was defined as

\[
v_{\text{cl}} = \frac{1}{N_p} \sum_{i=1}^{N_p} v_i,
\]

where \(N_p\) is the number of particles in the cluster and \(v_i\) is the velocity of the particle \(i\) in the cluster. To characterize the size of the cluster, we use the effective radius defined as

\[
R_{\text{eff}} = \frac{1}{N_p} \sum_{i=1}^{N_p} \left[ (x_i - \bar{x})^2 + (y_i - \bar{y})^2 + (z_i - \bar{z})^2 \right]^{1/2},
\]

where \(x_i, y_i, z_i\) are the particle coordinates in the cluster and \((\bar{x}, \bar{y}, \bar{z})\) are the coordinates of the cluster centre. If we use the FOF method, the mean size for the high-mass clusters is larger than the mean size for the low-mass clusters.

We also selected clusters using the DENSMAX method. In this case clusters were identified in the simulations as maxima of the density field that was determined on a \(256^3\) grid using the cloud-in-cell (CIC) scheme. To determine peculiar velocities of clusters, \(v_{\text{cl}}\), we calculated the peculiar velocity field on a \(256^3\) grid using the CIC-scheme, and found peculiar velocities at the grid points where the clusters had been identified. If we use the DENSMAX method, the size of the selected clusters is similar for all clusters, given by the cell size, \(\lambda_p\). In our case, the cell size \(\lambda_p = 0.9355 \, h^{-1} \text{Mpc}\).
To determine the rms peculiar velocities of clusters, we used the equation

\[ v_{\text{rms}}^2 = v_L^2 + \sum_{i=1}^{N_{\text{d}}} v_{\text{ch}}^2, \]  

where the parameter \( v_{\text{rms}} \) describes the dispersion of cluster velocities, \( v_{\text{ch}} \), derived from simulation and the parameter \( v_L \) is the linear contribution from velocity fluctuations on scales greater than the size of the simulation box \( L \) and is given by

\[ v_L^2 = \frac{f^2(\Omega_m)H_0^2}{2\pi^2} \int_0^{\frac{2\pi}{L}} P(k) dk. \]

\( N_{\text{d}} \) is the number of clusters studied. By using equation (4), the linear rms peculiar velocity of peaks can be written as

\[ \sigma_p^2(R) = \sigma_v^2(R) - H_0^2 f^2(\Omega_m) \sigma_R^2(R) / \sigma_7^2(R). \]

The second term in this expression is not sensitive to the amplitude of large-scale fluctuations at wavenumbers \( k < 2\pi/L \). Therefore, the linear rms velocity of peaks can be expressed as

\[ \sigma_p^2(R) = \sigma_v^2(R) + v_L^2, \]

where \( \sigma_v^2(R) \) is determined by the power spectrum at wavenumbers \( k > 2\pi/L \) and \( v_L \) is given by equation (8). For the ΛCDM and rCDM models, we found that \( v_L \approx 220 \) and 245 km s\(^{-1} \) respectively.

If the one-dimensional velocities of clusters, \( v_{\text{ch}} \), follow a Gaussian distribution with a mean \( \bar{v}_i = 0 \) and a dispersion \( \sigma^2 \), then the sum

\[ \chi^2 = \sum_{i=1}^{N_{\text{d}}} \frac{v_{\text{ch}}^2}{\sigma^2} \]

is distributed as a \( \chi^2 \) distribution with the number of degrees of freedom \( \nu = 3N_{\text{d}} \). In this case, the rms error for the variable \( v_{\text{rms}}^2 \) can be determined as

\[ \Delta v_{\text{rms}}^2 = \frac{2}{3N_{\text{d}}} \sigma^2. \]

As a first step, the one-dimensional distribution of cluster velocities can be approximated as a Gaussian distribution (see, for example, Bahcall et al. 1994 for a study of the velocity distribution of clusters in different cosmological models). We used equation (12) to estimate the error bars for the rms velocities of clusters.

### 3 Results

First, we investigated the rms velocity of clusters in different density and mass intervals. The results are presented in Fig. 3. The rms velocities are shown in intervals for which the numbers of clusters are \( N_{\text{cl}} > 10 \).

The clusters defined with the DENSMAX method were divided into subgroups according to their density. We studied the rms velocity of clusters in seven subgroups for which the density \( \rho/\rho_0 \) was in the ranges 1–5, 5–10, 10–50, . . . , 1000–5000. Table 1 shows the number of clusters and the right-hand panel of Table 1 shows the rms velocity of clusters in different density intervals. We see that the rms velocities of clusters in the ΛCDM1 and ΛCDM2 models are similar. In the models studied, the rms velocity of clusters is almost independent of density. The rms velocity somewhat increases at smaller densities. However, this increase is small (≈10 per cent). For the range \( \rho/\rho_0 \approx 100–500 \), the rms velocity is 503 and 570 km s\(^{-1} \) in the ΛCDM1 and rCDM models, respectively. These values are similar to the linear theory expectations. At the radius \( R = 1 \) h\(^{-1} \) Mpc, the rms peculiar velocities of peaks are \( \sigma_v = 509 \) and 562 km s\(^{-1} \) in the ΛCDM1 and rCDM models, respectively. The rms velocity for the low-density clusters is somewhat smaller than predicted by the linear theory.

The clusters determined with the FOF method were divided into subgroups according to their mass. We studied the rms velocity of clusters in eight subgroups for which the mass was in the ranges (5 × 10\(^1\)–10\(^2\)), . . . , (10\(^5\)–5 × 10\(^7\)) h\(^{-1} \) M⊙. Table 2 shows the number of clusters and Fig. 3 demonstrates the rms peculiar velocity of clusters in different mass intervals. The lower left panel in Fig. 3 shows the results for the clusters determined by \( b = 0.2 \). The right panels show the rms velocities of clusters defined by \( b = 0.15 \) and \( b = 0.3 \).

We see that in the ΛCDM1 and ΛCDM2 models the rms velocities are similar. The rms velocity of FOF clusters decreases with cluster mass. The rms velocity of massive FOF clusters is smaller than the rms velocity of high-density clusters determined with the DENSMAX method. In the ΛCDM model for \( b = 0.2 \), the rms velocity of clusters is 525 km s\(^{-1} \) in the mass interval (5 × 10\(^1\)–10\(^2\)) h\(^{-1} \) M⊙ and 430 km s\(^{-1} \) in the mass interval (10\(^5\)–5 × 10\(^7\)) h\(^{-1} \) M⊙. For \( b = 0.15 \) and \( b = 0.3 \), this effect is similar. These results are in good agreement with the results obtained by Sheth & Diaferio (2001). They studied the rms velocities of clusters in different mass intervals and found that the rms cluster velocity decreases with mass.

We also studied the rms velocity of clusters with density (or mass) higher than a given threshold density (or mass). The results are presented in Fig. 4. The DENSMAX clusters were ranked according to their density, and we selected the \( N_{\text{d}} = (L/d_{\text{cl}})^3 \) highest-ranked clusters to produce cluster catalogues with a mean intercluster separation of 10–50 h\(^{-1} \) Mpc. Similarly, the FOF clusters were ranked according to their mass. Table 3 shows the density and mass thresholds used to produce cluster catalogs for different values of the mean cluster separation. For comparison, the number density of observed APM clusters and Abell clusters is \( n_{\text{cl}} \sim 3.4 \times 10^{-3} \) Mpc\(^{-3} \) (\( d_{\text{cl}} \sim 31 \) h\(^{-1} \) Mpc) and \( n_{\text{cl}} \sim 2.5 \times 10^{-5} \) h\(^3 \) Mpc\(^{-3} \) (\( d_{\text{cl}} \sim 34 \) h\(^{-1} \) Mpc), respectively (Dalton et al. 1994; Einasto et al. 1997).

Fig. 4 demonstrates the rms peculiar velocity of clusters with a mean separation \( d_{\text{cl}} = 10–50 \) h\(^{-1} \) Mpc. In this range, the rms velocity of DENSMAX clusters is almost independent of the density of clusters. For clusters with a mean separation \( d_{\text{cl}} = 30 \) h\(^{-1} \) Mpc, \( v_{\text{rms}} \) is 510 and 580 km s\(^{-1} \) in the ΛCDM1 and rCDM models, respectively. These values are similar to the linear rms velocity of peaks at the radius \( R \sim 1 \) h\(^{-1} \) Mpc.

The rms velocity of FOF clusters decreases with cluster richness. For rich clusters, the rms velocity of FOF clusters is smaller than the rms velocity of clusters determined with the DENSMAX method. In the ΛCDM model for \( b = 0.2 \), the rms velocities are 500 and 430 km s\(^{-1} \), if \( d_{\text{cl}} = 10 \) and 50 h\(^{-1} \) Mpc, respectively. For \( b = 0.15 \) and 0.3, this effect is similar. For clusters with \( d_{\text{cl}} = 30 \) h\(^{-1} \) and \( b = 0.2 \), the rms velocity is 475 and 565 km s\(^{-1} \) in the ΛCDM1 and rCDM models, respectively.

In Table 2 we compare the rms velocity of FOF clusters, \( v_{\text{rms}} \), with \( \sigma_p(R) \) for the radius \( R = 1 \) h\(^{-1} \) Mpc. We analysed the rms velocity of clusters for different values of the cluster separation. The results are given for the clusters determined by \( b = 0.2 \). The rms peculiar velocities \( v_{\text{rms}} \) for the clusters determined by \( b = 0.15 \) and 0.3 are similar. In the ΛCDM1 model, the rms peculiar velocity of small clusters is close to linear theory expectations, while the rms peculiar velocity of rich clusters is smaller (≈15 per cent for clusters with a mean intercluster separation \( d_{\text{cl}} = 50 \) h\(^{-1} \) Mpc).

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Figure 3. The rms peculiar velocities of clusters for different densities and masses in the $\Lambda$CDM1 model (solid lines), rCDM model (dot–dashed lines) and in the $\Lambda$CDM2 model (dashed lines). The upper-left panel shows the results for the clusters defined by the DENSMAX method and the lower-left panel shows the results for the clusters selected by the FOF method with $b = 0.2$. The right panels show the rms velocities of clusters defined by the FOF method with $b = 0.15$ and 0.3, respectively.

Table 1. The number of clusters, $N_{cl}$, in different density and mass intervals. $N_{cl}$ for different density intervals is given for the clusters selected with the DENSMAX method. $N_{cl}$ for different mass intervals is given for the clusters determined with the FOF method with $b = 0.2$.

| $\rho/\rho_b$ | $\Lambda$CDM1 | $N_{cl}$ | $\Lambda$CDM2 |
|--------------|---------------|---------|---------------|
| 1–5          | 148496        | 169774  | 103757        |
| 5–10         | 329265        | 35753   | 17369         |
| 10–50        | 31294         | 34705   | 18314         |
| 50–100       | 5246          | 4015    | 3372          |
| 100–500      | 4253          | 1762    | 2619          |
| 500–1000     | 298           | 22      | 107           |
| 1000–5000    | 78            | 0       | 8             |

| $M(\,h^{-1}M_\odot)$ | $\Lambda$CDM1 | $N_{cl}$ | $\Lambda$CDM2 |
|----------------------|---------------|---------|---------------|
| $5 \times 10^{11}$   | 36227         | 0       | 27310         |
| $10^{12}$–$5 \times 10^{12}$ | 51841       | 57460   | 15820         |
| $5 \times 10^{12}$–$10^{13}$ | 7326        | 19298   | 3204          |
| $10^{13}$–$5 \times 10^{13}$ | 6542        | 15489   | 4225          |
| $5 \times 10^{13}$–$10^{14}$ | 856         | 1644    | 659           |
| $10^{14}$–$5 \times 10^{14}$ | 511         | 789     | 507           |
| $5 \times 10^{14}$–$10^{15}$ | 38          | 18      | 24            |
| $10^{15}$–$5 \times 10^{15}$ | 4           | 5       | 3             |

Our results are in good agreement with the results obtained by Colberg et al. (2000). They also studied the rms velocity of clusters in the Virgo $\Lambda$CDM and rCDM models, but used a slightly different method to select clusters. High-density regions were located using an FOF method with $b = 0.05$ and their barycentres were considered as candidate cluster centres. Any candidate centre for which mass within $1.5\,h^{-1}\text{Mpc}$ exceeded the threshold mass $M_t$ was identified as a candidate cluster. The final cluster list was obtained by deleting the lower mass candidate in all pairs separated by less than $1.5\,h^{-1}\text{Mpc}$. The peculiar velocity of each cluster was defined to be the mean peculiar velocity of all the particles within the $1.5\,h^{-1}\text{Mpc}$.

Table 2. Comparison of the rms velocity of FOF clusters, $v_{rms}$, with the linear theory predictions for peculiar velocities of peaks, $\sigma_p$, for the radius $R = 1\,h^{-1}\text{Mpc}$. The results are given for the FOF clusters determined by $b = 0.2$.

| $d_{cl}$ ($h^{-1}\text{Mpc}$) | $\Lambda$CDM1 | $v_{rms}/\sigma_p$ |
|-----------------------------|---------------|-------------------|
| 10                          | 0.98          | 1.03              |
| 30                          | 0.93          | 1.01              |
| 50                          | 0.84          | 0.94              |

Our results are in good agreement with the results obtained by Colberg et al. (2000). They also studied the rms velocity of clusters in the Virgo $\Lambda$CDM and rCDM models, but used a slightly different method to select clusters. High-density regions were located using an FOF method with $b = 0.05$ and their barycentres were considered as candidate cluster centres. Any candidate centre for which mass within $1.5\,h^{-1}\text{Mpc}$ exceeded the threshold mass $M_t$ was identified as a candidate cluster. The final cluster list was obtained by deleting the lower mass candidate in all pairs separated by less than $1.5\,h^{-1}\text{Mpc}$. The peculiar velocity of each cluster was defined to be the mean peculiar velocity of all the particles within the $1.5\,h^{-1}\text{Mpc}$.
Figure 4. The rms peculiar velocities of clusters for different values of the mean cluster separation, $d_{cl}$. The clusters are ranked according to their density (DENSMAX clusters) or mass (FOF clusters). The lines are defined as in Fig. 3.

Table 3. The density and mass thresholds used to produce cluster catalogues with mean separations $d_{cl} = 10$–$50 \, h^{-1}$ Mpc. The mass thresholds are given for the FOF clusters determined by $b = 0.2$.

| $d_{cl}$ (h$^{-1}$ Mpc) | $\Delta$CDM1 $\rho_c/\rho_b$ | rCDM | $\Delta$CDM2 |
|------------------------|-------------------------------|-------|--------------|
| 10                     | 35                            | 27    | 20           |
| 20                     | 213                           | 102   | 136          |
| 30                     | 435                           | 178   | 264          |
| 40                     | 652                           | 254   | 393          |
| 50                     | 850                           | 315   | 505          |
| $d_{cl}$ (h$^{-1}$ Mpc) | $\Delta$CDM1 $M_e (h^{-1} M_\odot)$ | rCDM | $\Delta$CDM2 |
| 10                     | $5.6 \times 10^{12}$          | 1.3 $\times 10^{13}$ | 2.3 $\times 10^{12}$ |
| 20                     | $4.3 \times 10^{13}$          | 6.4 $\times 10^{13}$ | 3.6 $\times 10^{13}$ |
| 30                     | $1.1 \times 10^{14}$          | 1.3 $\times 10^{14}$ | 1.1 $\times 10^{14}$ |
| 40                     | $2.0 \times 10^{14}$          | 2.0 $\times 10^{14}$ | 1.9 $\times 10^{14}$ |
| 50                     | $3.0 \times 10^{14}$          | 2.8 $\times 10^{14}$ | 2.8 $\times 10^{14}$ |

sphere. In this method, the size of the selected clusters is same for all clusters and in this sense, this method is similar to the DENSMAX method.

Colberg et al. (2000) used the value $M_e = 3.5 \times 10^{14} h^{-1} M_\odot$. For this value, the number of clusters was $\approx 70$ in the $\Lambda$CDM1 and rCDM models ($d_{cl} = 58 \, h^{-1}$ Mpc). They found that the rms velocities derived from simulation, $v_s$, are 439 and 535 km s$^{-1}$ in the $\Lambda$CDM1 and rCDM models, respectively (they did not include the dispersion $v^2$). If we use the DENSMAX method, we find that the rms velocity of clusters is almost independent of the number density of clusters and for $d_{cl} = 50 \, h^{-1}$ Mpc, the velocities are $v_s = 450$ and 549 km s$^{-1}$ in the $\Lambda$CDM1 and rCDM models, respectively. These values are very close to the values found by Colberg et al. (2000), only slightly larger ($\approx 2$ per cent). This small difference is probably caused by the fact that in the DENSMAX method we use the smoothing length $\approx 1 \, h^{-1}$ Mpc, which is smaller than the $1.5 \, h^{-1}$ Mpc used by Colberg et al. (2000). For comparison, the rms velocities $v_s$ for the FOF clusters for $b = 0.2$ and $d_{cl} = 50 \, h^{-1}$ Mpc are 368 and 468 km s$^{-1}$ in the $\Lambda$CDM and rCDM models, respectively.

Let us now consider the rms velocities of clusters for different cluster radii. We studied the rms velocity of clusters with the effective radius $R_e$ larger than a given threshold radius. The clusters were ranked according to their effective radius and we selected the $N_{cl} = (L/d_{cl})^3$ highest ranked clusters to produce cluster catalogues with mean separations 10–50 $h^{-1}$ Mpc. Table 4 shows the threshold radii used for different values of the mean cluster separation.

Fig. 5 illustrates the dependence of $v_{rms}$ on cluster radii. We see that the effect of the cluster radius on $v_{rms}$ is similar to the effect of the cluster mass on $v_{rms}$ (compare Figs 4 and 5). In the models studied, the rms velocity of small clusters is higher than the rms.
Table 4. The threshold radii $R_t$ used to produce cluster catalogues with mean separations $d_{cl} = 10$–50 $h^{-1}$ Mpc. The radii are given for the FOF clusters defined by $b = 0.2$.

| $d_{cl}$ ($h^{-1}$ Mpc) | $R_t$ ($h^{-1}$ Mpc) | $\Lambda$CDM1 | $\tau$CDM |
|-----------------------|---------------------|--------------|-------------|
| 10                    | 0.22                | 0.24         |             |
| 20                    | 0.40                | 0.40         |             |
| 30                    | 0.58                | 0.53         |             |
| 40                    | 0.74                | 0.64         |             |
| 50                    | 0.88                | 0.72         |             |

velocity of large clusters. For example, in the $\Lambda$CDM1 model for $b = 0.2$, the rms velocities are 520 and 450 km s$^{-1}$ for clusters with $d_{cl} = 10$ and 50 $h^{-1}$ Mpc, respectively.

4 THE SHIFTED-MESH SCHEME

When we started the study of cluster velocities we first used a PM code with a shifted-mesh scheme. In this scheme, the acceleration on the grid was calculated by using the approximation

$$g_x(i + \frac{1}{2}, j, k) = \varphi(i, j, k) - \varphi(i + 1, j, k),$$

where $\varphi(i, j, k)$ is the gravitational potential on the grid. We note that the shifter-mesh scheme is different from the staggered-mesh scheme proposed by Melott (1986). The traditional two-point finite-difference approximation for the acceleration is given by

$$g_x(i, j, k) = \frac{\varphi(i - 1, j, k) - \varphi(i + 1, j, k)}{2}.$$  

Gramann (1987) studied the evolution of an one-dimensional plane-wave perturbation by using a PM code with differential operators (13) and (14) and found that the approximation (13) leads to smaller deviations from the exact solution than the approximation (14).

Fig. 6 shows the rms peculiar velocities of clusters in a PM simulation in which we used the approximation (13) for a three-dimensional system. The cosmological parameters in this simulation were chosen to be similar to those of the $\Lambda$CDM1 and $\Lambda$CDM2 models. We investigated the evolution of 256$^3$ particles on a 256$^3$ grid in the comoving box of size $L = 239.5$ $h^{-1}$ Mpc. For comparison we show the rms peculiar velocities in the $\Lambda$CDM2 model with a standard finite-difference approximation (14). The clusters are defined by the FOF method with $b = 0.2$. The rms cluster velocities in the $\Lambda$CDM2 model are similar to the $\Lambda$CDM1 model, which achieves a force resolution smaller than the mean particle separation $\lambda_p$ (see Figs 3 and 4).

We see that the shifted-mesh scheme artificially boosts cluster velocities. The effect is particularly strong for rich clusters. The rms velocities are 560 and 735 km s$^{-1}$ if $d_{cl} = 10$ and 50 $h^{-1}$ Mpc, respectively. This effect is probably caused by the fact that, using the scheme (13), we calculate the components $(g_x, g_y, g_z)$ at different locations on the grid and this can lead to artificial enhancement of particle acceleration in one dimension. This effect does not arise in a one-dimensional system, where $g_x = g_y = 0$.

In the $\Lambda$CDM2 model we used the standard two-point finite-difference approximation (14). In this model, the rms velocities of clusters are similar to the $\Lambda$CDM1 model. Therefore, we can use the PM code to study peculiar velocities on scales that are close to the mean particle separation and force resolution. But it is important to use a correct difference operator in simulations.

5 SUMMARY AND DISCUSSION

In this paper we have examined the rms peculiar velocities of galaxy clusters, $v_{rms}$, for different cluster masses and radii. We analysed clusters in the Virgo simulations for two cosmological models,
$\Lambda$CDM and $\tau$CDM (Jenkins et al. 1998). These simulations were carried out using the AP$^*$M code. We used simulations for which the mean particle separation was $\lambda_p \sim 1$ h$^{-1}$ Mpc. We also analysed clusters in an N-body simulation where the evolution was followed using a PM code with the same mass resolution that was used in the Virgo simulations. The cosmological parameters for this simulation were chosen similar to the Virgo $\Lambda$CDM model. We found that the rms velocities of clusters in the PM simulation are similar to the rms velocities in the AP$^*$M simulation. We can use the PM code to study peculiar velocities on scales that are close to the mean particle separation $\lambda_p$.

To identify clusters in the simulations we used two methods: (1) the standard friends-of-friends (FOF) method and (2) the method in which the clusters are defined as maxima of the density field smoothed on the scale $R \sim 1$ h$^{-1}$ Mpc (DENSMAX). The velocity of DENSMAX clusters was determined using the same smoothing scale as for the density field. If we use the DENSMAX method, the size of the selected clusters is similar for all clusters. The velocity of FOF clusters was defined to be the mean velocity of all the particles in the cluster. If we use the FOF method, the size of the high-mass clusters is larger than the size of the low-mass clusters. We studied the velocities of FOF clusters defined by the parameters $b = 0.2$, 0.2 and 0.3 (the parameter $b$ defines the linking length as $b \lambda_p$).

We found that the rms velocity of clusters defined with the DENSMAX method is almost independent of the density of clusters. The rms velocity of FOF clusters decreases with the cluster mass and radius. The effect of the cluster radius on $v_{\text{rms}}$ is similar to the effect of the cluster mass on $v_{\text{rms}}$ (see Figs 4 and 5). The rms velocity of small clusters is higher than the rms velocity of large massive clusters. For different values of $h$, this effect is similar. In the $\Lambda$CDM model, the rms peculiar velocity of massive clusters with an intercluster separation $d_{cl} = 50$ h$^{-1}$ Mpc is $\approx 15$ per cent smaller than the rms velocity of clusters with a separation $d_{cl} = 10$ h$^{-1}$ Mpc.

The rms velocity of massive FOF clusters is smaller than the rms velocity of high-density clusters determined with the DENSMAX method. What is the reason for this difference? Do we select different objects by using different methods or do we define different velocities for the same clusters? We analysed the fraction of DENSMAX clusters, $F$, which match FOF clusters (in terms of their positions). We studied the FOF clusters determined with $b = 0.2$. In the $\Lambda$CDM model, if we compared DENSMAX clusters for $d_{cl} = 50$ h$^{-1}$ Mpc with the FOF clusters for $d_{cl} = 50$ h$^{-1}$ Mpc, we found that the fraction of matched clusters is $F = 0.72$. For $d_{cl} = 30$ h$^{-1}$ Mpc in both methods, the fraction is $F = 0.74$. In the $\tau$CDM model, the fraction $F$ is similar. We select the same objects by using different methods, but we rank them in a somewhat different way. In the DENSMAX method, the clusters are ranked according to their density and in the FOF method according to their mass. If we compared DENSMAX clusters for $d_{cl} = 50$ h$^{-1}$ Mpc with the FOF clusters for $d_{cl} = 40$ h$^{-1}$ Mpc, we found that $F = 0.98$.

But we do assign different velocities for the same clusters by using different methods. Fig. 7 shows the peculiar velocities determined with different methods for the same clusters. We show the velocities of rich clusters that match for $d_{cl} = 50$ h$^{-1}$ Mpc in both methods. We see that the velocities of clusters defined by the FOF method are systematically smaller than the velocities of clusters defined by the DENSMAX method. This is probably caused by the fact that, in the FOF method, the size of rich clusters is larger than the smoothing scale $R \sim 1$ h$^{-1}$ Mpc that was used to define the cluster velocities in the DENSMAX method.

We compared the rms velocities of clusters with the linear theory predictions for the rms peculiar velocities of peaks, $\sigma_p$, for the smoothing radius $R = 1$ h$^{-1}$ Mpc. At this radius, $\sigma_p = 509$ and 562 km s$^{-1}$ in the $\Lambda$CDM and $\tau$CDM models, respectively. We
analysed the rms velocity of FOF clusters for different values of the cluster separation. In the $\Lambda$CDM model, the rms peculiar velocity of small clusters is close to the linear theory expectations, while the rms peculiar velocity of rich clusters is smaller ($\approx 15\%$ for clusters with a mean separation $d_{cl} = 50\ h^{-1}\ Mpc$). The rms velocity of DENSMAX clusters is almost independent of the cluster density and is similar to the linear theory expectations. For the DENSMAX clusters with a mean separation $d_{cl} = 30\ h^{-1}\ Mpc$, $v_{rms} = 510$ and 580 km s$^{-1}$ in the $\Lambda$CDM and $\tau$CDM models, respectively. On scales probed by galaxy clusters, velocity fluctuations are in the quasi-linear regime.

ACKNOWLEDGMENTS

We thank J. Einasto, M. Einasto, P. Heinamäki, G. Hütsi, A. Melott and E. Saar for useful discussions. This work has been supported by the ESF grant 3601. The $N$-body simulations used in this paper are available at http://www.mpa-garching.mpg.de/Virgo/virgoproject.html These simulations were carried out at the Computer Centre of the Max-Planck Society in Garching and at the EPCC in Edinburgh, as part of the Virgo Consortium project. The FOF programs used in this paper are available at http://www-hpcc.astro.washington.edu These programs were developed in the University of Washington.

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