Quantum topology is powerful in understanding condensed-matter systems that exhibit a winding\textsuperscript{13}. Often, this winding occurs in real space. For example, in a magnetic material, the local magnetization may exhibit a rotating pattern centred around a point in real space, forming a magnetic vortex encoding an integer winding number\textsuperscript{3,6}. Alternatively, the winding may occur in momentum space. For example, in a one-dimensional topological insulator, the quantum-mechanical wavefunctions wind as the momentum scans through the Brillouin zone\textsuperscript{14,9–15,21–25}. Away from the crossing curve, electronic structures in which multiple bands cross along a closed curve in momentum space\textsuperscript{13–15,21–25}. Away from the crossing curve, the bands disperse linearly, so that the node loop consists of a cone dispersion persisting along a loop. Within the paradigm of momentum-space wavefunction winding, node loops are topological, with a quantized-Berry-phase invariant\textsuperscript{9,10,13,24,25}. However, in contrast to other electronic structures studied so far\textsuperscript{9–13}, node loops can link each other, encoding a linking-number invariant (Fig. 1a and Extended Data Fig. 1)\textsuperscript{16–20}. Unlike the traditional paradigms of winding, this linking number is associated with the composite loop structure of quantum-mechanical band crossings of the Hamiltonian. Such linked node loops offer the possibility of a new bridge between physics and knot theory. It has further been proposed that these links are governed by emergent non-Abelian node-loop charges\textsuperscript{26} and that the linking number determines the $\theta$ angle of the axion Lagrangian in certain node-loop phases\textsuperscript{19,26,27}. As the three-dimensional condensed-matter
Fig. 1 | Signatures of linked node loops in Co$_2$MnGa. a, Weyl loops in the electronic structure of Co$_2$MnGa, predicted by DFT. Three distinct Weyl loops are confined to the three mirror planes M$_1$, M$_2$, and M$_3$, in such a way that the loops link one another (further copies of the loops in higher Brillouin zones not shown). b, Element-resolved crystal structure of Co$_2$MnGa along the [001] direction, acquired by atomic-level EDS. Atomic columns consist either entirely of cobalt (green) or of alternating manganese (red) and gallium (blue). Scale bar, 5 Å. c, Bulk Brillouin zone (black truncated octahedron) of Co$_2$MnGa with three mirror planes indicated, M$_1$ (magenta, constant k$_x$), M$_2$ (red, constant k$_y$) and M$_3$ (gold, constant k$_z$). Each mirror plane contains square faces of the Brillouin zone. The high-symmetry momentum-space points at the centre of each square are marked X$_1$, X$_2$, and X$_3$. d, Fermi surface acquired by ARPES at incident photon energy 544 eV, corresponding to M$_1$, H, L, high, low photoemission intensity. Dashed lines mark the energy–momentum slices shown in Fig. 2a. Inset: the M$_1$ mirror plane plotted in an extended bulk Brillouin zone scheme. e, Out-of-plane Fermi surface acquired on the same Co$_2$MnGa sample by an ARPES photon-energy dependence from 500 eV to 800 eV in steps of 2 eV, corresponding to M$_2$. f, Analogous out-of-plane Fermi surface corresponding to M$_3$, again on the same sample.

Brillouin zone is a three-torus (T$^3$), linked node loops also offer the rare possibility of observing links in a space other than ordinary infinite space ($\mathbb{R}^3$). Moreover, the Seifert surface of the bulk link is associated with topological boundary states, opening the possibility of a unique Seifert bulk–boundary correspondence in quantum matter$^{28–32}$.

Ferromagnets with crystalline mirror symmetry naturally give rise to node loops. In this scenario, the ferromagnetic exchange interaction produces spin-split electronic bands that are generically singly degenerate throughout momentum space, and mirror symmetry protects two-fold band degeneracies along closed curves confined to the momentum-space mirror planes$^{21}$. Such node loops are called Weyl loops, by analogy with the two-fold degeneracy of a Weyl point$^{10,11,23–25}$. Weyl loops are effective at concentrating Berry curvature, giving rise to giant anomalous Hall and Nernst effects, up to room temperature and promising for technological applications$^{14,15,33–37}$. In crystallographic space groups with multiple perpendicular mirror planes, different Weyl loops living in different mirror planes can naturally link each other$^{5,20}$. The ferromagnet Co$_2$MnGa exhibits a crystal structure with multiple perpendicular mirror planes and was recently observed to host electronic Weyl loops$^{14,15}$, bringing together the key ingredients for linked node loops.

Linked Weyl loops
Co$_2$MnGa crystallizes in the full Heusler structure, with a face-centred cubic Bravais lattice, space group Fm3m (number 225), octahedral point group O$_h$ and Curie temperature $T_C = 690$ K (Fig. 1b and Extended Data Fig. 2). The point group includes mirror planes normal to $\hat{x}$, $\hat{y}$ and $\hat{z}$ (conventional unit-cell lattice vectors). The real-space mirror planes give rise to momentum-space mirror planes, labeled M$_1$ (normal to $\hat{z}$), M$_2$ ($\hat{y}$) and M$_3$ ($\hat{x}$; Fig. 1c). Motivated by the observation of mirror-symmetry-protected magnetic Weyl loops in Co$_2$MnGa (refs. $^{14,15}$), we explore the electronic structure of our samples on M$_1$, M$_2$ and M$_3$. We perform ab initio calculations of Co$_2$MnGa in the ferromagnetic state, focusing on these three mutually perpendicular mirror planes. We find that each mirror plane hosts a Weyl loop, and that the three Weyl loops link one another (Fig. 1c).

To further understand the loop electronic structures, we examine energy–momentum photoemission spectra slicing through the M$_1$ loop...
Error bars indicate the experimental resolution. Experimental loop trajectory all cone crossings observed along the full electronic structure. The corresponding ab initio calculations also show a double degeneracy at the touching points, indicating a Weyl loop electronicty. As Co2MnGa is ferromagnetic, we expect generically singly degenerate bands throughout the Brillouin zone. This suggests a cone dispersion consisting of singly degenerate branches that exhibit a double degeneracy at the touching points, indicating a Weyl loop electronic structure. The corresponding ab initio calculations also show a Weyl cone with characteristic two-fold degenerate crossing and linear dispersion, in good agreement with the ARPES data (Fig. 2b). To further characterize the Weyl loop experimentally, we systematically track all cone crossings observed along the full M₁-loop trajectory (Fig. 2c and Extended Data Fig. 3) and also examine a deeper constant-energy photoemission slice through the electronic structure typically intersecting a loop crossing only at several discrete points (for example, the cyan dots in Fig. 2d). However, despite this energy dispersion, we find a minimal two-band effective Hamiltonian for a Weyl loop, with all parameters extracted directly from the photoemission spectra (Fig. 2e and Methods). The extracted dispersion reaches the Fermi level within experimental resolution, suggesting that the observed Weyl loops are relevant for low-energy response and consistent with previous reports that Weyl loops play a dominant role in the giant anomalous Hall effect and other exotic transport properties of Co2MnGa (refs. 14,34–36). Our photoemission spectra, ab initio calculations and model suggest that we have observed a magnetic Weyl loop on M₁.

We next investigate the composite structure formed by pairs of Weyl loops, focusing on the Weyl loop crossing itself (degeneracy curve, shown in cyan in Fig. 2e). The loop crossing disperses in energy, so a constant-energy slice through the electronic structure typically intersects a loop crossing only at several discrete points (for example, the cyan dots in Fig. 2d). However, despite this energy dispersion, we find

**Fig. 2** | Weyl loop trajectory in Co2MnGa. a. Energy–momentum photoemission slices through the loop Fermi surface (slice locations marked by the dashed lines in d and Fig. 1d). b. Energy–momentum slices through the Weyl loop from DFT, showing a Weyl loop cone (slice locations marked in Extended Data Fig. 3c). c. Cone locations (magenta squares) systematically extracted from cone dispersions observed in photoemission spectra on M₁. Error bars indicate the experimental resolution. Experimental loop trajectory extracted by fitting to the cone locations (cyan; see text). The binding energy axis is collapsed. d. Constant-energy photoemission slice with the analytical model of the Weyl loop (black lines). This slice intersects the Weyl loop at a discrete set of points (cyan dots). e. Dispersion of an effective Hamiltonian for the Weyl loop, obtained by fitting to the ARPES spectra. In a and d, H, L, high, low photoemission intensity.

**Fig. 3** | Linked Weyl loops in Co2MnGa. a. M₁ and M₂-loop Fermi surfaces plotted in adjacent bulk Brillouin zones, exhibiting a link structure. Inset: M₁ and M₂ plotted across multiple Brillouin zones. b, c. As for a, but for the M₁- and M₂-loop Fermi surfaces (b) and the M₁- and M₂-loop Fermi surfaces (c). The momentum axes are shifted by reciprocal lattice vectors to present the datasets in the vicinity of the first bulk Brillouin zone (Extended Data Fig. 10).
that the typical ‘radius’ of the Weyl loop, $|k|_{avg} = 0.66 \, \text{Å}^{-1}$, is much larger than the typical momentum separation of the two branches of the Weyl cone at the Fermi level, $\eta/v_F = 0.07 \, \text{Å}^{-1}$, where $\eta$ sets a typical energy scale and $v_F$ is the Fermi velocity (Methods). As $|k|_{avg} \gg \eta/v_F$, we can treat the Weyl loop crossing as approximately flat in energy, and we can accurately capture its trajectory by examining a constant-energy slice near the Fermi level. Therefore, to understand the composite structure of the Weyl loops, we can focus on the $M_1$, $M_2$- and $M_3$-Weyl-loop Fermi surfaces. We first consider the $M_1$ and $M_2$ Weyl loops and zoom in on the momentum-space region around $X_1$ (Fig. 3a, inset). By plotting the $M_1$- and $M_2$-Weyl-loop Fermi surfaces simultaneously in this region of three-dimensional momentum space, we observe that these two loops seem to link each other twice (Fig. 3a). Repeating the analogous procedure for $X_2$ and $X_3$, we observe that the $M_1$ and $M_3$ Weyl loops also link twice (Fig. 3b), and similarly for the $M_2$ and $M_3$ Weyl loops (Fig. 3c).

Our three-dimensional momentum-space analysis of the photoemission spectra suggests that each of the $M_1$, $M_2$, and $M_3$ Weyl loops links each other loop twice, forming an interwoven structure (Extended Data Figs. 4 and 5).

**Link diagram in the Brillouin zone**

To further explore the links, we examine three Weyl loops simultaneously using the experimentally extracted loop trajectory (Methods and equation (2)). In an extended zone scheme, we plot the $M_1$, $M_2$, and $M_3$ Weyl loops around six nearby X points, so that two redundant copies of each Weyl loop are included (Fig. 4b). We find that the three Weyl loops all link together, forming a single composite linked structure.

To further characterize the links, we examine energy–momentum photoemission slices tangential to all three loops near their extrema (Fig. 4a and Extended Data Figs. 6–8). All slices exhibit a cone dispersion, consistent with the Weyl loop electronic structure. Moreover, we find quantitative agreement between the Weyl loop extrema expected from equation (2) and the locations of the photoemission cone dispersions, for all three loops.

To better characterize the complete link structure, we construct a link diagram for our Weyl loops. In such a link diagram, we flatten the link from three to two dimensions while preserving the link information using an over/under notation (illustrated for the example of a Hopf link; Fig. 4c). As the Weyl loop link lives in the periodic momentum space of the crystal, it is necessary and sufficient to consider all link components in a single Brillouin zone. To draw a link diagram, we therefore flatten the link into the surface Brillouin zone. Moreover, because our analysis shows that all three Weyl loops are symmetry related, we choose the (111) surface Brillouin zone (Extended Data Fig. 2), which treats $X_1$, $X_2$, and $X_3$ equivalently (Fig. 4d). The resulting link diagram uniquely specifies the Co$_2$MnGa quantum link and shows three loops straddling the edges of the surface Brillouin zone (Fig. 4e and Supplementary Section 4). We observe that the link wraps around $T'$ in all three momentum-space directions. This behaviour suggests that the link is geometrically essential, so that it cannot be smoothly perturbed to live entirely within a local region of the Brillouin zone. The link diagram further shows that each loop is linked with each other loop exactly twice. This gives 2 for the geometric linking number, defined as the minimal number of crossing changes between link components needed to separate the components. The geometric linking number of the
Seifert boundary modes

Having systematically characterized the link structure in the bulk of Co$_2$MnGa, we next consider its topological surface states. Unlinked loop nodes host conventional drumhead surface states, which typically fill a simply connected region of momentum space in the surface Brillouin zone. By contrast, linked loops exhibit an alternating pattern of topologically distinct regions in which surface states are either present or suppressed, and which are pinned together at generic points in momentum space. This topological structure is captured by the Seifert surface, defined as a three-dimensional surface that has the link as its boundary. For a condensed-matter system, we consider a Seifert surface defined in $(k_x, k_y, k_z)$ and bounded by the linked loop nodes, with the energy axis collapsed. For the minimal case of a Hopf link, the Seifert surface exhibits a branched structure that ‘wraps’ around the link (Fig. 5a, left). A two-dimensional projection of the Seifert surface then produces alternating filled and empty regions, which meet at characteristic touching points (Fig. 5a, right). In the case of the Weyl loop link that we observe in Co$_2$MnGa, the Seifert projection on the (111) hexagonal surface Brillouin zone then predicts several alternating regions with and without topological boundary states (blue and white regions, Fig. 5c), exhibiting touching points along $\Gamma$–K. As the Seifert surface encodes the linking number, the topological boundary states are associated with a Seifert bulk–boundary correspondence. In this correspondence, the linking number of the Weyl loops in the bulk is encoded in a Seifert surface, whose projection gives the topological boundary states. A measurement of the bulk link determines the Seifert boundary states, and a measurement of the Seifert boundary states allows a reconstruction of the bulk linking number.

To explore these possible surface states, we examine the (111) surface of our Co$_2$MnGa samples in ab initio calculation and surface-sensitive vacuum ultraviolet (VUV) ARCES. On an energy–momentum cut through the touching point, we observe in calculation a pair of surface modes pinned together at the Weyl loops (Fig. 5d). Moreover, our photoemission spectra are consistent with our ab initio prediction, suggesting the observation of Seifert boundary states approaching the Weyl loop linking point (Fig. 5e and Supplementary Section 2). On iso-energy contours of the electronic structure, we expect to observe arc-like slices of the Seifert states, stretching across the filled regions and connecting the linked Weyl loops. Examining the Fermi surface obtained in calculation, we observe a sharp arc of surface states connecting the linked Weyl loops, consistent with the Seifert projection.
Conclusion

Our photoemission spectra and theoretical analysis suggest the observation of linked node loops with Seifert bulk–boundary correspondence in a quantum magnet. These results establish a new bridge between physics and knot theory, motivating further exploration of links and knots in electronic structures (Supplementary Sections 4 and 5). Moreover, the linked-loop state in Co2MnGa, as well as in other materials, may give rise to an exotic response quantized to the linking number, such as a link-quantized topological magneto-electric effect\(^\text{19,26,27,40}\).

As magnetic and correlated materials are abundant in a wide variety of symmetry classes, these ideas open the way to the discovery of unusual behaviour in many quantum magnets and superconductors, as well as their photonic analogues\(^\text{31,42}\).

Online content

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**Methods**

**Single-crystal growth**

Co₃MnGa single crystals were grown using the Bridgman–Stockbarger method. A polycrystalline ingot was first prepared using an induction melt technique, with a stoichiometric mixture of Co, Mn and Ga metal pieces of 99.99% purity. Then the powdered material was poured into an alumina crucible and sealed in a tantalum tube. Growth temperatures were controlled using a thermocouple attached to the bottom of the crucible. During the heating cycle, the material was melted at temperatures above 1,200 °C and then slowly cooled below 900 °C.

The conventional unit-cell lattice constant is $c = 5.77 \, \AA$. A representative crystallographic mirror plane is shown in orange in Extended Data Fig. 2a. The corresponding momentum-space mirror planes contain the time-reversal-invariant momentum points $X_0$, $X_1$, $X_2$ and $X_3$, sitting at the centres of the square faces of the bulk Brillouin zone (Extended Data Fig. 2c, d). The structure further exhibits three $C_3$ rotation symmetries relating any one of these mirror planes to the other two. Without loss of generality, we consider the crystal cleavage plane in our photoemission experiments to be parallel to $M_0$. We perform a characterization by atomic-level energy-dispersive X-ray spectroscopy (EDS), providing direct structural evidence that our Co₃MnGa samples are crystallographically well ordered, show the expected lattice constant and exhibit these mirror and rotation symmetries⁴¹⁻⁴⁴ (Fig. 1b).

**ARPES**

Soft X-ray ARPES measurements were carried out at the ARUP beamline of the Swiss Light Source in Villigen, Switzerland under a vacuum better than $5 \times 10^{-10}$ torr and a temperature of 16 K (refs. ³⁸⁻⁴⁵). Rod-shaped single crystals of Co₃MnGa oriented along the conventional unit-cell $2\overline{1}$ direction were cleaved in situ at base temperature. The constant-energy cuts were symmetrized about $M_0$, $M_1$, $M_2$ (Fig. 1d), $M_3$, $M_4$ (Fig. 1e) and $M_5$ (Fig. 1f). The high-symmetry momentum–magnetization cuts were similarly symmetrized about $M_0$, $M_1$, $M_2$ or $M_3$, as appropriate and consistent with the nominal symmetries of the crystal (Fig. 4a). A background was removed from the photoemission spectra by a fixed intensity cutoff (raw, unsymmetrized data in Extended Data Figs. 6–8). For the Fermi surfaces acquired at $h\nu = 544$ eV, the nominal energy resolution was $\delta E = 75$ meV; for the photon-energy dependences, the nominal energy resolution varied from $\delta E = 75$ meV at $h\nu = 500$ eV to $\delta E = 125$ meV at $h\nu = 800$ eV. The angular resolution was better than 0.2° in all cases. The Fermi surfaces were fitted to a low-order expansion around $X_1$, consistent with the nominal symmetries of the crystal (Fig. 4a). A background was removed from the photoemission spectra by a fixed intensity cutoff (raw, unsymmetrized data in Extended Data Figs. 6–8). For the Fermi surfaces acquired at $h\nu = 544$ eV, the nominal energy resolution was $\delta E = 75$ meV; for the photon-energy dependences, the nominal energy resolution varied from $\delta E = 75$ meV at $h\nu = 500$ eV to $\delta E = 125$ meV at $h\nu = 800$ eV. The angular resolution was better than 0.2° in all cases. The Fermi surfaces were fitted to a low-order expansion around $X_1$, consistent with the nominal symmetries of the crystal (Fig. 4a). A background was removed from the photoemission spectra by a fixed intensity cutoff (raw, unsymmetrized data in Extended Data Figs. 6–8). For the Fermi surfaces acquired at $h\nu = 544$ eV, the nominal energy resolution was $\delta E = 75$ meV; for the photon-energy dependences, the nominal energy resolution varied from $\delta E = 75$ meV at $h\nu = 500$ eV to $\delta E = 125$ meV at $h\nu = 800$ eV. The angular resolution was better than 0.2° in all cases. The Fermi surfaces were fitted to a low-order expansion around $X_1$, consistent with the nominal symmetries of the crystal (Fig. 4a). A background was removed from the photoemission spectra by a fixed intensity cutoff (raw, unsymmetrized data in Extended Data Figs. 6–8).

**Ab initio calculations**

The electronic structure of Co₃MnGa in the ferromagnetic phase was calculated within the density functional theory (DFT) framework using the projector augmented-wave method as implemented in the VASP package⁴⁶⁻⁴⁷. The generalized gradient approximation (GGA)⁴⁸ and a Γ-centred $k$-point mesh of $12 \times 12 \times 12$ mesh were used. Ga and $p$ orbitals and Mn and Co $d$ orbitals were used to generate a real-space tight-binding model, from which Wannier functions were determined. The Fermi level in DFT was shifted to match the ARPES.

**Scanning transmission electron microscopy**

Thin lamellae for microstructure characterization were prepared from bulk single crystals by focused ion beam cutting using a Helios NanoLab G3 UC dual-beam focused ion beam and scanning electron microscope (FIB/SEM) system. Atomic-resolution high-angle annular dark-field (HAADF) scanning transmission electron microscopy (STEM) imaging and atomic-level energy-dispersive X-ray spectroscopy (EDS) mapping were performed on a double Cs-corrected Titan Cubed Themis 300 scanning/transmission electron microscope (S/TEM) equipped with an X-EDS source operated at 300 kV with a Super-X EDS system.

**Weyl loop trajectory from SX-ARPES**

We fit the Weyl cone crossing points to a two-band effective $k \cdot p$ Hamiltonian for a Weyl loop

$$H = \frac{1}{2} \sum_{k, \alpha \beta} c_{k \alpha}^\dagger h_{\alpha \beta}(k) c_{k \beta}^\dagger,$$

$$h(k) = s(k) \alpha^+ + s^\dagger \beta^+ \gamma_0 + g(k) 1,$$

(1)

Here the $c_{k \alpha}^\dagger$ terms are fermionic creation operators, $\alpha$ is the crystal momentum, $s_\alpha$ and $s_\beta$ are Pauli matrices, $1$ is the $2 \times 2$ identity and $q_\eta = k_\eta - 2\pi / c$ is the $\eta$ component of the momentum measured relative to $M_0$, in which $c$ is the conventional unit-cell lattice constant. This Hamiltonian exhibits a Weyl loop on $q_\eta = 0$ with a trajectory given by $g(k) = 0$, formed from two bands with opposite mirror eigenvalues. From our ARPES spectra, we experimentally extract the full Weyl loop trajectory by fitting to a lower-order expansion around $X_0$, consistent with the symmetries of the system

$$f(k) = y \left[ 1 + a_0 (k_x^2 + k_y^2) + \beta (k_x^2 k_y^2) \right].$$

(2)

Here $a_0$ and $\beta$ fix the Weyl loop trajectory and the scaling factor $y$ sets an energy scale. The train of crossing points observed in ARPES is well captured by $a = -1.23 \pm 0.03 \, \AA^2$ and $\beta = -31.5 \pm 4.1 \, \AA^4$ (Fig. 2c). We also find that our ARPES-extracted Weyl loop trajectory agrees well with the trajectory observed in ab initio calculations (Extended Data Fig. 3).

**Stability of the linked node loops**

To estimate the stability of the link, we can measure how far apart one would need to slide two Weyl loops to unlink them (Extended Data Figs. 4 and 5). From the loop Fermi surfaces, we find that the typical ‘depth’ of the link in momentum space is $d_{\parallel \omega} = 0.58 \pm 0.08 \, \AA^{-1}$, of the same order of magnitude as the radius $|k|_{\parallel \omega}$ of the Weyl loop. This large depth suggests that the system lives well within a linked electronic phase.

**Data availability**

The datasets generated during and/or analysed during the current study are available in the Zenodo repository at https://doi.org/10.5281/zenodo.5793667. Source data are provided with this paper.

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Author contributions M.Z.H. supervised the project. I.B., G. Chang and T.A.C. initiated the project. I.B., T.A.C., Z.-J.C. and M.Z.H. acquired and analysed ARPES spectra with help from X.P.Y., D.M., J.-X.Y., M. Litkevich, N.S. and S.S.Z. ARPES measurements were supported by N.B.M.S., A.C., C.P., B.T., M. Leandersson, J.A. and V.N.S. G. Chang performed the first-principles calculations. I.B. wrote down the \( \mathbf{k} \) model with help from G. Chang and S.-M.H. I.B. developed the linking number theory with help from C.H. G. Cheng and N.Y. performed the scanning transmission electron microscopy measurements. K.M., C.S. and C.F. synthesized and characterized the single crystals. I.B. wrote the manuscript with contributions from all authors.

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Correspondence and requests for materials should be addressed to Ilya Belopolski or M. Zahid Hasan.

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Extended Data Fig. 1 | Topological invariants in physics. a, An example of an order parameter winding in real space: a magnetic vortex. In this case, the order parameter is the local magnetization $m(x)$, confined to a magnetic easy plane in real space ($x, y$). It may happen that $m(x)$ winds around a point in real space, forming a magnetic vortex characterized by a winding number topological invariant, in this example given by $w = 1$. b, An example of a quantum wavefunction winding in momentum space: the one-dimensional topological insulator (Su-Schrieffer-Heeger model). This phase is described by Bloch Hamiltonian $h(k) = \mathbf{d}(k) \cdot \sigma$, where $k$ is the one-dimensional crystal momentum, $\sigma$ refers to the Pauli matrices and $\mathbf{d}(k)$ is a two-component object confined to the $(d_x, d_y)$ plane. The normalized quantity $\hat{d}(k) = \mathbf{d}(k)/|\mathbf{d}(k)|$ (orange arrow) moves around the unit circle (dotted blue) as $k$ varies. The topological invariant is related to how many times $\hat{d}(k)$ winds around the origin as $k$ scans through the one-dimensional Brillouin zone. c, Node loops linking in momentum space: a three-dimensional electronic structure may exhibit multiple node loops (cyan and purple), characterized by $k_n(\theta)$, where $n$ indexes the loops and $\theta$ parametrizes the loop trajectory in momentum space. The loops may link one another, encoding a linking number topological invariant. This example shows a Hopf link. (See also Supplementary Information.)
Extended Data Fig. 2 | Crystal structure and Brillouin zone of Co$_2$MnGa.

a, Conventional unit cell with representative crystallographic mirror plane $M$ (orange). b, The primitive unit cell (grey) includes one formula unit. c, Brillouin zone, with conventional reciprocal lattice basis vectors (black). Brillouin zone edges color-coded to correspond to the mirror planes: magenta $M_1$ plane, (001); red $M_2$ plane, (010), orange $M_3$ plane, (100). d, Slice through $\Gamma$ in an extended zone scheme.
**Extended Data Fig. 3** | Energy dispersion of the Weyl loop.  

**a**, Crossing point energies $E_B$ and **b**, crossing point momenta $(k_x, k_y)$ systematically extracted from cone dispersions observed in the ARPES spectra (magenta squares), same dataset as Fig. 2c ($h\nu = 544$ eV), with fit of the Weyl loop momentum trajectory and energy dispersion (cyan, see main text). The crossing point energies are parametrized by a polar angle $\theta$ defined by $\tan \theta = k_y/k_x$. **c**, Weyl loop trajectory from DFT, with dotted lines indicating the DFT energy-momentum slices shown in Fig. 2b. The binding energy axes in (b) and (c) are collapsed.

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Extended Data Fig. 4 | Link ‘depth’ of the Weyl loops. **a–c,** Distance between the extrema of the Weyl loops and the bulk Brillouin zone W points for the $M_1$, $M_2$ and $M_3$ Weyl loops. We estimate $s_1 = 0.32 \pm 0.1 \text{ Å}^{-1}$, $s_2 = 0.27 \pm 0.1 \text{ Å}^{-1}$ and $s_3 = 0.29 \pm 0.1 \text{ Å}^{-1}$. **d,** The link depth captures how far in momentum space one would need to slide the Weyl loops in order to unlink them, providing a measure of the stability of the link. Based on the loop Fermi surfaces (a–c), we estimate $d_{12} = 0.58 \pm 0.14 \text{ Å}^{-1}$, $d_{23} = 0.55 \pm 0.14 \text{ Å}^{-1}$ and $d_{31} = 0.60 \pm 0.14 \text{ Å}^{-1}$. The average gives a typical link depth extracted from ARPES, $d_{\text{avg}} = 0.58 \pm 0.08 \text{ Å}^{-1}$. **e,** Energy-momentum slice along the high-symmetry path $X_1 - X_2$ from DFT, passing through two linked Weyl loops. We obtain $d_{\text{DFT}} = 0.68 \text{ Å}^{-1}$. 

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Extended Data Fig. 5 | Supplementary measurement of the link depth. a, $M_1$, $M_2$, and $M_3$ Weyl loops, with trajectories obtained from the analytical model (see main text), showing that $M_1$ links $M_2$ twice and $M_3$ twice. Energy-momentum photoemission cuts along the high-symmetry paths b, $X_1 - X_2$ and c, $X_2 - X_3$, obtained at photon energy $h\nu = 642$ eV. We observe $d_{12} = 0.56 \pm 0.1$ Å$^{-1}$ and $d_{31} = 0.61 \pm 0.1$ Å$^{-1}$, consistent with Extended Data Fig. 4. For both cuts, exactly one branch of each Weyl cone exhibits appreciable photoemission cross-section, as expected from the mirror-symmetric measurement geometry$^{59}$. d, Fermi surface acquired at $h\nu = 642$ eV, exhibiting an in-plane Weyl loop contour, $M_1$. We further observe spectral weight emanating along $k_x$ and $k_y$ from the center of $M_1$, corresponding to the linearly dispersive branches in (b, c), again suggesting that $M_1$ is linked by $M_2$ and $M_3$. 
Extended Data Fig. 6 | Unsymmetrized Fermi surfaces. a–c. Left: photoemission spectra displayed in Fig. 1d–f, without symmetrization. Right: the same spectra, with the experimentally-determined Weyl loop trajectory overlaid across multiple Brillouin zones. The irrelevant Γ pocket is consistently observed in all unsymmetrized spectra. Signatures of Weyl loops are observed around all X points.
Extended Data Fig. 7 | SX-ARPES systematics. a–d, Photoemission energy-momentum cuts through the Weyl loop, used to extract Fig. 2c.
Extended Data Fig. 8 | Unsymmetrized energy-momentum cuts. Photoemission spectra displayed in Fig. 4a, without symmetrization.
Extended Data Fig. 9 | Linked Weyl loop Fermi surface. Constant-energy slice of the pockets (navy) making up the linked Weyl loops obtained by ab initio calculation, at binding energy $E_B = -10$ meV below the experimental Fermi level. Plotted a, in an extended zone scheme (only two loops shown for simplicity) and b, the reduced Brillouin zone (all three loops shown). The Fermi surface pockets touch at a set of discrete points, where the Weyl loop disperses through this particular $E_B$. For reference, the full Weyl loop trajectories are indicated, collapsed in energy (magenta around $X_1$, red around $X_2$, orange around $X_3$). The Weyl loop Fermi surface pockets form a linked structure.
Extended Data Fig. 10 | Measured Fermisurfaces in an extended zone scheme. The Brillouin zone corresponds to $\Gamma_{1066}$ in the primitive reciprocal basis.