Quantum heat transport in condensed matter systems

Jukka P. Pekola\textsuperscript{1,2} and Bayan Karimi\textsuperscript{1}
\textsuperscript{1}Pico group, QTF Centre of Excellence, Department of Applied Physics, Aalto University School of Science, P.O. Box 13500, 00076 Aalto, Finland
\textsuperscript{2}Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

(Dated: July 28, 2021)

In this Colloquium recent advances in the field of quantum heat transport are reviewed. This topic has been investigated theoretically for several decades, but only during the past twenty years have experiments on various mesoscopic systems become feasible. A summary of the theoretical basis for describing heat transport in one-dimensional channels is first provided. Then the main experimental investigations of quantized heat conductance due to phonons, photons, electrons, and anyons in such channels are presented. These experiments are important for understanding the fundamental processes that underly the concept of a heat conductance quantum for a single channel. Then an illustration on how one can control the quantum heat transport by means of electric and magnetic fields, and how such tunable heat currents can be useful in devices is given. This lays the basis for realizing various thermal device components such as quantum heat valves, rectifiers, heat engines, refrigerators, and calorimeters. Also of interest are fluctuations of quantum heat currents, both for fundamental reasons and for optimizing the most sensitive thermal detectors; at the end of the review the status of research on this intriguing topic is given.

CONTENTS

I. Introduction

II. Thermoelectric transport in a one-dimensional (1D) channel

III. Thermal conductance - measurement aspects
A. Principles of measuring heat currents
B. Thermometry and temperature control

IV. Experimental setups, background information
A. Thermal conductance of a superconductor
B. Heat transport in tunneling
C. Hamiltonian of a quantum circuit
D. Quantum noise of a resistor

V. Phonons

VI. Electrons and fractional charges

VII. Photons
A. A ballistic photon channel
B. Circuit limitations of the ballistic picture
C. Experiments on heat mediated by microwave photons

VIII. Tunable quantum heat transport
A. Electronic quantum heat interferometer
B. Cooling a quantum circuit

IX. Quantum heat transport mediated by a superconducting qubit
A. Quantum heat valve
B. Thermal rectifier

X. Heat current noise
A. FDT for heat in tunneling

XI. Summary and outlook

XII. Acknowledgments

References

I. INTRODUCTION

In this review we present advances on fundamental aspects of thermal transport in the regime where quantum effects play an important role. Usually this means dealing with atomic scale structures or low temperatures, or combination of the two. The seminal theoretical work by Pendry \cite{Pendry1983} presented, almost 40 years ago, the important observation that a ballistic channel for any type of a carrier can transport heat at the rate given by the so-called quantum of thermal conductance $G_Q$. During the present millennium the theoretical ideas have developed into a plethora of experiments in systems involving phonons, electrons, photons and recently also particles obeying fractional statistics. We give an overview of these experiments backed by the necessary theoretical framework. The question whether a channel is ballistic or not, and under what conditions, is interesting as such, but it has also more practical implications. If one can
control the degree of ballisticity, i.e. the transmission coefficient of the channel, one can turn the heat current on and off. Such quantum heat switches, or heat valves as they are often called, will be discussed in this review as well. Furthermore heat current via a quantum element in an asymmetric structure can violate reciprocity in the sense that rectification of heat current becomes possible. The bulk of the review deals with the time average (mean) of the heat current. Yet, the fluctuations of this quantity are interesting and they provide a yardstick for the minimal detectable power and for the ultimate energy resolution of a thermal detector. We discuss such a noise and its implications in ultrasensitive detection.

The review opens after the present introductory section by a theoretical discussion of thermoelectric transport in one-dimensional channels in Section II. In Section III we present the concept and method of how to measure heat currents in general. Section IV reviews the central elements of the experimental setups. After these general sections, we move to heat transport in different physical systems: phonons in Section V, electrons and fractional charges in Section VI, and photons in Section VII including some detailed theoretical discussion in the subsections. Section VIII presents experimental results on heat current by external fields. In Section IX we move to the discussion of a superconducting qubit as a tunable element in quantum thermodynamics. Section X gives an account of both theoretical expectations and experimental status on heat current noise and associated fluctuations of effective temperature. Section XI concludes the review by a summary and outlook including prospects on useful thermal devices and some interesting physical questions related to quantum heat transport.

II. THERMOELECTRIC TRANSPORT IN A ONE-DIMENSIONAL (1D) CHANNEL

Consider two infinite reservoirs with temperature $T_i$ and chemical potential $\mu_i$, which are connected adiabatically via a conductor as shown schematically in Fig. 1. Here the subscripts $i=L$, $R$ represent the left and right, respectively. Based on Landauer theory (Landauer 1981), (Sivan and Imry 1986), (Butcher 1990) the charge and energy currents, $I$ and $J$, between the two reservoirs (from $L$ to $R$) are given for a 1D conductor by

$$I = q \sum_n \int_0^\infty \frac{dk}{2\pi} v_n(k)(\vartheta_L - \vartheta_R)T_n(k)$$

$$J = \sum_n \int_0^\infty \frac{dk}{2\pi} \varepsilon_n(k)v_n(k)(\vartheta_L - \vartheta_R)T_n(k),$$

where $q$ is the particle charge, $\sum_n$ presents the sum over independent modes in the conductor, and $\varepsilon_n(k)$ and $v_n(k)$ indicate the energy and the velocity of the particles with wave vector $k$, respectively. $T_n(k)$ indicates the particle transmission probability through the conductor via the channel; for ballistic transport $T_n(k) \equiv 1$, and $\vartheta_{L,R}$ represents the statistical distribution functions in each reservoir. Changing the variable from wave vector to energy via the definition of the velocity $v_n(k) = \frac{1}{\hbar} \frac{\partial \varepsilon_n(k)}{\partial k}$, we have

$$I = \frac{q}{\hbar} \sum_n \int_{\varepsilon(0)}^{\infty} d\varepsilon [\vartheta_L(\varepsilon) - \vartheta_R(\varepsilon)]T_n(\varepsilon)$$

$$J = \frac{1}{\hbar} \sum_n \int_{\varepsilon(0)}^{\infty} d\varepsilon \varepsilon [\vartheta_L(\varepsilon) - \vartheta_R(\varepsilon)]T_n(\varepsilon),$$

where $\varepsilon(0) \equiv \varepsilon$ for $k = 0$. These equations constitute the basis of thermoelectrics, with linear response for electrical and thermal conductance and for Seebeck and Peltier coefficients.

Now we solve analytically these equations for a ballistic contact $T_n(\varepsilon) \equiv 1$ with the most common carriers, that is fermions and bosons. For fermions $\vartheta_i(\varepsilon) \equiv f_i(\varepsilon - \mu_i) = 1/(1 + e^{\varepsilon(\varepsilon-\mu_i)})$ is the Fermi distribution function for each reservoir, with inverse temperature $\beta_i = 1/(k_BT_i)$. Note that we have taken the Fermi energy as the zero of $\varepsilon$, meaning that $\varepsilon(0) \rightarrow -\infty$. In this case at temperature $T$, with only chemical potential difference $eV$ across the contact, the charge current is

$$I = Ne\frac{e}{\hbar} \int_{-\infty}^{\infty} d\varepsilon [f(\varepsilon) - f(\varepsilon - eV)] = Ne^2V.$$  

Here $N$ replacing the sum represents the number of current carrying modes in the conductor with $q = e$. The electrical conductance $G = dI/dV$ is then

$$G = Ne^2/\hbar,$$

which is the famous quantization of electrical conductance. The thermal conductance for fermions can be obtained from the heat flux $\dot{Q} = J$ when both reservoirs have the same chemical potential. The heat current across the ballistic contact is then

$$\dot{Q} = \frac{1}{\hbar} \sum_n \int_{-\infty}^{\infty} d\varepsilon \varepsilon [f_L(\varepsilon) - f_R(\varepsilon)].$$
The subtle differences between energy and heat currents are briefly discussed in Section IV.B. In this review we focus mainly on thermal conductance in equilibrium, $T_L = T_R = T$, i.e. on $G_{th}(T) \equiv dQ/dT_L|_T$. The thermal conductance is then

$$G_{th}^{(f)}(\varepsilon) = \frac{N}{\hbar} \frac{1}{k_B T^2} \int_{-\infty}^{\infty} d\varepsilon \ 2\pi \ f(\varepsilon)[1 - f(\varepsilon)]$$

$$= N \frac{\pi^2 k_B^2 T^2}{3h} \equiv NG_Q,$$

(6)

where the superscript $(f)$ stands for fermions and

$$G_Q \equiv \frac{\pi^2 k_B^2 T^2}{3h}$$

is the thermal conductance quantum. The ratio of the thermal and electrical conductances satisfies the Wiedemann-Franz law $G_{th}^{(f)}/G_L = \mathcal{L} T$, where the Lorenz number is \( \mathcal{L} = \frac{\pi^2 k_B^2}{3e^2} \) (Ashcroft and Mermin 1976).

We obtain the thermal conductance for bosons, \( G_{th}^{(b)} \), with the same procedure but with the distribution function $\vartheta_{R,L}(\varepsilon) \equiv n_{R,L}(\varepsilon) = 1/(e^{\varepsilon/T} - 1)$ in Eq. (2):

$$G_{th}^{(b)} = \frac{\hbar}{2\pi k_B T^2} \int_0^\infty d\omega \omega^2 e^{\beta\hbar\omega} \vartheta_{th}^2(\omega).$$

(8)

Here $\varepsilon = \hbar\omega$ is the energy of each boson. For a single fully transmitting channel $\vartheta_{th}(\omega) = 1$, we then obtain again

$$G_{th}^{(b)} = G_Q.$$

(9)

Fermions and bosons form naturally the playground for most experimental realizations in the quantum regime. Yet the result above for a ballistic channel, $G_{th} = G_Q$, is far more general. As demonstrated in (Blencowe and Vitelli 2000; Rego and Kirchmow 1999), this expression is invariant even if one introduces carriers with arbitrary fractional exclusion statistics (Wu 1994). A few years back, (Banerjee et al. 2017) experimented on a fractional quantum Hall system addressing this interesting universality of thermal conductance quantum for anyons.

### III. THERMAL CONDUCTANCE - MEASUREMENT ASPECTS

#### A. Principles of measuring heat currents

For determining thermal conductance one needs in general a measurement of local temperature. Suppose an absorber, like the one in Fig. 2(a), is heated at a constant power $\dot{Q}$. By continuity, the relation between $\dot{Q}$ and temperature $T$ of the absorber with respect to the bath temperature $T_0$ can often be written as

$$\dot{Q} = K(T^n - T_0^n),$$

(10)

where $K$ and $n$ are constants characteristic to the absorber and the process of thermalization. For instance, for the most common process in metals, coupling of absorber electrons to the phonon bath, the standard expression is $\dot{Q} = \Sigma V(T^5 - T_0^5)$ (Gantmakher 1974; Roukes et al. 1989; Schwab et al. 2000; Wang et al. 2019; Wellstood et al. 1994), where $\Sigma$ is a material specific parameter and $V$ the volume of the absorber. It is often the case that the temperature difference $\delta T \equiv T - T_0$ is small, $|\delta T/T| \ll 1$, and we can linearize Eq. (10) into

$$\dot{Q} = G_{th}\delta T,$$

(11)

where $G_{th} = nK(T_0^n - 1)$ is the thermal conductance between the absorber and the bath. For the electron-phonon coupling above, we then have $G_{th}^{(ep)} = 5\Sigma VT_0^4$. It is worth pointing out that electron-electron relaxation in metals is fast enough to secure a well defined electron temperature $\vartheta_{th}$ (Pothier et al. 1997).

For the ballistic channel discussed widely in the current manuscript, $G_{th} = G_Q = \frac{\pi^2 k_B^2}{3h} T_0$, and we have for a general temperature difference exactly

$$\dot{Q} = \frac{\pi^2 k_B^2}{6h} (T^2 - T_0^2) = \frac{\pi^2 k_B^2}{3h} T_m \delta T,$$

(12)

where $T_m \equiv (T + T_0)/2$ is the mean temperature.

In some experiments a differential two-absorber setup is preferable (see Fig. 2(b)). This allows for measurements of the temperatures of the two absorbers, $T_1$ and $T_2$, separately, and to determine the heat flux between the two without extra physical (wiring) connections for thermometry across the object of interest. In this case equations in this section apply if we replace $T$, $T_0$ by $T_1$, $T_2$, respectively. Such a setup offers more flexible calibration and sanity check options for the system, and also for tests of reciprocity (thermal rectification) by inverting the roles of source and drain, i.e., by reversing the temperature bias.

#### B. Thermometry and temperature control

Here we comment very briefly on thermometry and temperature control in the experiments to be reported in this paper. The control of the local temperature is typically achieved by Joule heating applied to the electronic system. But depending on the type of the reservoir, this heat is either directly acting on the quantum conductor or indirectly, e.g. via the phonon bath. The simplest heating element is a resistive on-chip wire. For heating (and local cooling), and in particular for thermometry, a hybrid normal metal-insulator-superconductor tunnel junction (NIS junction) is a common choice (Courtois et al. 2014; Giazotto et al. 2006; Muhonen et al. 2012). We defer discussion of this technique to Section IV.B.
used as a local heater. For thermometry one may use a similar wire and measure its thermal noise (Schwab et al., 2000). Another option used in some recent experiments is to measure the current noise of a quantum point contact (Banerjee et al., 2017; Jezouin et al., 2013).

IV. EXPERIMENTAL SETUPS, BACKGROUND INFORMATION

A. Thermal conductance of a superconductor

A superconductor obeying Bardeen-Cooper-Schrieffer theory (BCS theory) (Bardeen et al., 1957) forms an ideal building block for thermal experiments at low temperatures. A basic feature of a BCS-superconductor is its zero resistance, but in our context an even more important property is its essentially vanishing thermal conductance (Bardeen et al., 1959). In bulk superconductors both electronic and the non-vanishing lattice thermal conductances play a role.

In small structures the exponentially vanishing thermal conductance at low temperatures can be exploited effectively to form thermal insulators that can at the same time provide perfect electrical contacts. In quantitative terms, according to the theory (Bardeen et al., 1959), the ratio of the thermal conductivity \(\kappa_{e,S}\) in the superconducting state and \(\kappa_{e,N}\) in the normal state of the same material is given by

\[
\frac{\kappa_{e,S}}{\kappa_{e,N}} = \frac{\int_{-\Delta}^{\infty} \text{d}e 2f'(e) /\int_{-\infty}^{\infty} \text{d}e 2f'(e)},
\]

where \(\Delta \approx 1.76k_B T_C\) is the gap of the superconductor with critical temperature \(T_C\). For temperatures well below \(T_C\), i.e., for \(\Delta/k_B T \gg 1\), we obtain an approximate answer for Eq. (13) as

\[
\frac{\kappa_{e,S}}{\kappa_{e,N}} \approx \frac{6}{\pi^2} \frac{\Delta}{k_B T} e^{-\Delta/k_B T}.
\]

Since the normal state thermal and electrical conductivities are related by the Wiedemann-Franz law, we obtain

\[
\kappa_{e,S} \approx \frac{2\Delta^2}{e^2 \rho T} e^{-\Delta/k_B T},
\]

where \(\rho\) is the normal state resistivity of the conductor material. As usual, for the basic case of a uniform conductor with cross-sectional area \(A\) and length \(\ell\), we may then associate the thermal conductance \(G_{th}\) to thermal conductivity \(\kappa\) as \(G_{th} = (A/\ell) \kappa\).

Aluminum and niobium are the most common superconductors used in the experiments described here. In many respects, Al follows BCS-theory accurately. In particular it has been shown (Saira et al., 2012a) that the density of states (DoS) at energies inside the gap is suppressed at least by a factor \(\sim 10^{-7}\) leading for instance to the exponentially high thermal insulation discussed here. The measured thermal conductivity of Al follows closely Eq. (15), shown in (Feshchenko et al., 2017; Peltonen et al., 2010). At the same time Nb films suffer from non-vanishing subgap DoS, leading to power-law thermal conductance in \(T\), i.e. poor thermal insulation at the low temperature regime. In conclusion of this subsection we want to emphasize that Al is a perfect thermal insulator at \(T \lesssim 0.3T_C\), except in the immediate contact with a normal metal leading to inverse proximity effect; this proximity induced thermal conductivity affects typically only within few hundred nm distances from a clean normal metal contact (Peltonen et al., 2010).

B. Heat transport in tunneling

One central element in this review is a tunnel junction between two electrodes L (left) and R (right). The charge and heat currents through the junction can be obtained by perturbation theory where the coupling Hamiltonian between the electrodes is written as the tunnel Hamiltonian (Bruus and Flensberg, 2004)

\[
\hat{H}_c = \sum_{l,r} (t_{lr} \hat{a}^\dagger_{l} \hat{a}_{r} + t^*_{lr} \hat{a}_{l} \hat{a}^\dagger_{r}).
\]

Here \(t_{lr}\) is the tunneling amplitude, and \(\hat{a}^\dagger_{l}\) and \(\hat{a}_{l}\) are the creation and annihilation operators for electrons in the left (right) electrode, respectively.

To have the expression for number current from R to L one first obtains operator for it as \(\hat{N}_L = \frac{\text{i}}{\hbar} [\hat{H}_c, \hat{N}_L]\), where \(\hat{N}_L = \sum_{l} \hat{a}^\dagger_{l} \hat{a}_{l}\) is the operator for the number of electrons in L. Then one can write the charge current operator as \(I = -e \hat{N}_L\). In order to obtain the expectation value of the current that is measured in an experiment, \(I = \langle \hat{I} \rangle\), we employ linear response theory (Kubo formula (Kubo, 1957)) on the corresponding current operator, where \(I = -i/\hbar \int_{-\infty}^{0} dt' \langle \hat{I}(0), \hat{H}_c(t') \rangle_0\), with \(\langle \cdot \rangle_0\)
the expectation value in the unperturbed state. Assuming that the averages are given by the Fermi distributions in each lead, we have at voltage bias $V$

$$I = \frac{1}{\epsilon R_T} \int \, d\epsilon n_L(\epsilon) n_R(\epsilon) [f_L(\epsilon) - f_R(\epsilon)],$$

(17)

where $\dot{\epsilon} = \epsilon - eV$. Here the constant prefactor includes the inverse of the resistance $R_T$ of the junction such that $1/R_T = 2\pi |t|^2 \nu_L(0) \nu_R(0) / \hbar$, with $|t|^2 = |t_{\epsilon l}|^2$ constant, and $\nu_L(0), \nu_R(0)$ the density of states (DoS) in the normal state at Fermi energy in the left and right electrodes, respectively. Under the integral, $n_L(\epsilon), n_R(\epsilon)$ are the normalized $(\nu_L(0), \nu_R(0)$ respectively) energy dependent DoSes, and $f_L(\epsilon), f_R(\epsilon)$ the corresponding energy distributions that are Fermi-Dirac distributions for equilibrium electrodes.

For heat current we use precisely the same procedure but now for the operator of energy of the left electrode $\hat{E}_L = \sum_i \epsilon_i a_i^{\dagger} a_i$, instead of the number operator, where $\epsilon_i$ is the energy of a single particle state in L. Then we find the expectation value of the heat current out from L electrode, $\dot{Q}_L = -\langle \hat{H}_L \rangle$ as

$$\dot{Q}_L = \frac{1}{\epsilon^2 R_T} \int \, d\epsilon \dot{\epsilon} n_L(\epsilon) n_R(\epsilon) [f_L(\epsilon) - f_R(\epsilon)].$$

(18)

Here we may briefly comment on the relation of energy and heat currents, $\mathcal{J}$ and $\dot{Q}$ introduced in Section II. Inserting $\dot{\epsilon} = \epsilon - eV$, we find immediately $\dot{Q}_L = \mathcal{J} - 4V$, where $\mathcal{J} \equiv (\epsilon^2 R_T)^{-1} \int \, d\epsilon \epsilon n_L(\epsilon) n_R(\epsilon) [f_L(\epsilon) - f_R(\epsilon)]$. Writing the equation for the heat from the right electrode in analogy to Eq. (18), we find $\dot{Q}_R = -\mathcal{J}$. Thus we have $\dot{Q}_L + \dot{Q}_R = -4V$, which presents energy conservation: the total power taken from the source goes into heating the two electrodes. This is natural since in steady state conditions heat, as the internal energy of the system is constant.

As the most basic example of both the electrodes being normal metal (NIN junction, I for insulator), we have $n_L(\epsilon) = n_R(\epsilon) = 1$. Equations (17) and (18) then yield under quite relaxed conditions $I = V/R_T$ and $\dot{Q}_L = -V^2/2R_T$, i.e. the junction is ohmic and the Joule power is dissipated equally to the two electrodes.

Another important example is a normal-superconductor junction (NIS junction, L=N, R=S), Fig. 3. Its usefulness in thermometry (see Fig. 3(a)) is based on the superconducting gap $\Delta$ that leads to non-linear, temperature dependent current-voltage characteristics. This feature probes the temperature of the normal side of the contact. Such a temperature dependence is universal, $d\ln(I/I_0)/dV = e/(\hbar k_B T)$, where $I_0 = \sqrt{\pi \Delta_0^2 k_B T / (\epsilon R_T)}$, making NIS a primary thermometer in principle. This is strictly speaking true only for an ideal junction with low transparency. Therefore the common practice is to use it as a secondary thermometer (Lounasmaa 1974), meaning that one measures a thermometric response of it near equilibrium, for instance the voltage at a small fixed current, against the independently measured temperature of the cryostat (heat bath). The other important feature of the NIS junction lies in its thermal properties. When biased at a voltage of about $\Delta/e$, heat is carried away from the N side (and S is heated), i.e., it acts as a refrigerator (see Fig. 3(b)). At $V \gg \Delta/e$ the junction provides usual Joule heating. This is how a NIS junction can be used both as a cooler and heater of a mesoscopic reservoir. Numerically calculated current-voltage and cooling power characteristics, together with a schematic energy diagram have been depicted in the figure. The main characteristics of a NIS junction, based on analytical approximations at low temperatures are: $I \approx I_0 e^{-\Delta/k_B T}$ at voltages below the gap, and the maximal cooling of normal metal at $eV \approx \Delta$ is $\dot{Q}_L^{\text{max}} \approx +0.59(\Delta^2/e^2 R_T)/(k_B T/\Delta)^{3/2}$.

Microrefrigeration by electron transport is a technique that has been reviewed elsewhere (Courtois et al. 2014; Glazotto et al. 2006; Muhonen et al. 2012). For an in-
C. Hamiltonian of a quantum circuit

Another key element in our context is a harmonic oscillator, and in some cases a non-linear quantum oscillator, usually in form of a Josephson junction (Tinkham 2004). To avoid dissipation the linear harmonic oscillator in a circuit is commonly made of a superconductor, often in form of a coplanar wave resonator (Krantz et al. 2019). The Hamiltonian of such an LC-oscillator, shown in Fig. 4(a) is composed of the kinetic $q^2/(2C)$ and potential $\Phi^2/(2L)$ energies, respectively, where $q$ is the charge on the capacitor and $\Phi$ the flux of the inductor.

The charge is the conjugate momentum to flux as $\dot{q} = \hbar \partial E/\partial \Phi$. In the second Josephson relation, yielding the standard harmonic oscillator Hamiltonian

$$H = \hbar \omega_0 (\hat{c}^\dagger \hat{c} + \frac{1}{2}),$$

where $\omega_0 = 1/\sqrt{LC}$, and $Z_0 = \sqrt{L/C}$ are the (angular) frequency and impedance of the oscillator.

For a Josephson tunnel junction, shown in Fig. 4(b), the Josephson relations (Josephson 1962) are

$$\hbar \dot{\Phi} = 2eV, \quad I = I_c \sin \phi,$$

where $\phi$ is the phase difference across the junction related to flux by $\Phi = (2e/\hbar)\Phi$. In the second Josephson relation, $I$ is the current through the junction. The sinusoidal current-phase relation applies strictly for a tunnel junction, with critical current $I_c$. For different types of weak links, sinusoidal dependence does not necessarily hold (Tinkham 2004). The energy stored in the junction (= work done by the source) is then obtained for a current biased case from $I = \partial E/\partial \Phi$ as

$$E = \int_0^\Phi I \ d\Phi = -E_J \cos \phi,$$

where $E_J = \hbar/(2eI_c)$ is the Josephson inductance. Therefore, in the “linear regime” a Josephson junction can be considered as a harmonic oscillator such that Eqs. (19)–(21) apply with $L$ replaced by $L_J$. Yet the actual non-linearity of a Josephson junction makes it an invaluable component in quantum information processing and in quantum thermodynamics.

A magnetic flux tunable Josephson junction, for instance in form of two parallel junctions with a superconducting loop in between, is the renowned superconducting quantum interference device (SQUID) to be discussed in later sections.

D. Quantum noise of a resistor

The quantum noise of a resistor is an important quantity as it determines the heat emission and absorption in form of thermal excitations. In later sections it becomes obvious how this noise yields the Joule power in a circuit.

Consider that the resistor in the quantum circuit is formed of a collection of harmonic oscillators with ladder operators $\hat{b}_i$ and $\hat{b}_i^\dagger$ with frequencies $\omega_i$. The phase...
operator in the interaction picture reads
\[
\phi(t) = \sum_i \lambda_i (\hat{b}_i e^{-i\omega_i t} + \hat{b}_i^\dagger e^{i\omega_i t}),
\]
with coefficients \( \lambda_i \). The voltage fluctuations are related to phase as \( v(t) = \frac{\hbar}{e} \phi(t) \)
\[
v(t) = i \frac{\hbar}{e} \sum_i \lambda_i \omega_i (\hat{b}_i^\dagger e^{i\omega_i t} - \hat{b}_i e^{-i\omega_i t}).
\]

Then the spectral density of voltage noise \( S_v(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle v(t)v(0) \rangle \) is given by
\[
S_v(\omega) = \frac{2\pi \hbar^2}{e^2} \nu(\Omega) \lambda(\omega)^2 \Omega \left\{ [1 + n(\Omega)]\delta(\omega - \Omega) + n(\Omega)\delta(\omega + \Omega) \right\},
\]
where \( \nu(\Omega) \) is the oscillator density of states. Now we consider both positive and negative frequencies, which correspond to quantum emission and absorption processes. For positive frequencies only the first term survives as
\[
S_v(\omega) = \frac{2\pi \hbar^2}{e^2} \nu(\omega) \lambda(\omega) \omega^2 [1 + n(\omega)]
\]
(28)

Considering similarly the negative frequencies, we find
\[
S_v(-\omega) = e^{-\beta \hbar \omega} S_v(\omega),
\]
which is the famous detailed balance condition.

We know that the classical Johnson-Nyquist noise \( \left( \text{Johnson } 1928, \text{Nyquist } 1928 \right) \) of a resistor at \( k_B T \gg \hbar \omega \) reads
\[
S_v(\omega) = 2k_B T R.
\]
(30)

This is the classical fluctuation-dissipation theorem (FDT \( \text{(Callen and Welton } 1951) \)) applied to the resistor. In this limit by using Taylor expansion, we have \( (1 - e^{-\beta \hbar \omega})^{-1} \simeq (\beta \hbar \omega)^{-1} \), so that using Eq. \( 28 \) we have connection between the oscillator properties and the physical resistance as \( \text{(Karimi and Pekola } 2021) \)
\[
\lambda^2 = \frac{R e^2}{\pi \hbar \nu(\omega_i) \omega_i}.
\]
(31)

Substituting this result in Eq. \( 27 \), we obtain at all frequencies
\[
S_v(\omega) = 2R \frac{\hbar \omega}{1 - e^{-\beta \hbar \omega}}.
\]
(32)

V. PHONONS

Quantized thermal conductance was demonstrated experimentally for the first time by \( \text{(Schwab et al. } 2000) \). In their setup, as shown in the inset of Fig. 3, the "phonon cavity" consists of a \( 4 \times 4 \mu m^2 \) block of silicon nitride membrane with 60 nm thickness suspended by four legs of equal thickness. Each leg has catenoidal waveguide shape whose diameter at the narrowest point is less than 200 nm. This waveguide shape as a 1D channel is the ideal profile to achieve unit transmissivity between the suspended cavity and the bulk reservoir \( \text{(Rego and Kirzczewin } 1998) \). Two Au-film resistors with 25 nm thickness were patterned on the suspended central block; one of them serves to apply the Joule heating to generate the temperature gradient along the legs and the other one worked as a thermometer to measure the phonon cavity temperature. The electron temperature of the resistor was measured by a low noise amplifier (dc SQUID) operating with nearly quantum-limited energy sensitivity by measuring the electrical Johnson noise of the resistor.

The measurement of \( \text{(Schwab et al. } 2000) \) probes the thermal conductance by phonons across the four silicon nitride bridges as a function of bath temperature. These data are shown in the main frame of Fig. 5. The result exhibits the usual phononic thermal conductance, \( \propto T^3 \), at temperatures above 1 K. Below this temperature there is a rather abrupt leveling off of \( G_{th} \) to the value \( 16G_Q \) (here the notation is such that \( g_0 = G_Q \)). The authors
argue that the coefficient 16 arises from the trivial factor 4 due to four independent bridges in the structure and the less trivial factor 4 due to four possible acoustic vibration modes of each leg in the low temperature limit: one longitudinal, one torsional, and two transverse modes. In later theoretical works the somewhat meandering behavior of $G_{th}/G_Q$ below the crossover temperature was explained to arise from remaining scattering of phonons in the bridges, i.e. from non-ballistic transport, whose effect is expected to get weaker in the low temperature limit (Santamore and Cross 2001).

Over the years, there have been a few other experiments on thermal conductance by phonons in restricted geometries. The one by Leivo (Leivo and Pekola 1998) employed 200 nm-thick silicon nitride membranes in various geometries (see Fig. 6). The experiments were performed by applying Joule heating on a central membrane in an analogous way as in the experiment of (Schwab et al., 2000), and the resulting temperature change to obtain the thermal conductance was then read out by measuring the temperature dependent conductance of NIS probes processed on top of the same membrane. In this case the wiring running along the bridges was made of aluminum which is known to provide close to perfect thermal isolation at temperatures well below the superconducting transition at $T_c \approx 1.4$ K, see Section V.A. In general, there are many conduction channels in the wide bridges, as demonstrated in the figure caption. Yet this number for a single $w = 4 \mu m$ wide bridge is $N = 14$ at $T = 100$ mK, which is already in the same ballpark as the prediction of $N = 4$ of (Rego and Kirczenow 1998). Naturally the ballisticity of these 15 $\mu m$ long bridges is unknown though. Yet these experiments provide evidence of thermal conductance close to the quantum limit.

The experiment of (Schwab et al., 2000) was followed by several measurements in different temperature ranges and materials. Experiments on GaAs phonon bridges of sub-$\mu m$ lateral dimensions were earlier performed at temperatures above 1 K (Tighe et al., 1997) and later down to 25 mK bath temperature (Yung et al., 2002). The latter experiment measuring the temperature of the GaAs platform in the middle using NIS tunnel junctions demonstrated Debye thermal conductance at $T \gg 100$ mK, but tended to follow the expected quantum thermal conductance at the lowest temperatures. In the more recent experiments by (Tavakoli et al., 2017, 2018) the measurement on sub-micron wide silicon nitride bridges was made differential in the sense that there was no need to add superconducting leads on these phonon-conducting legs. The results at the lowest temperatures of $\sim 0.1$ K fall about one order of magnitude below the quantum value, and the temperature dependence of thermal conductance is close to $T^2$. The authors of (Tavakoli et al., 2017, 2018) propose non-ballistic transmission in their bridges to be the origin of their results. Finally experiments by (Zen et al.) 2014 demonstrate that thermal conductance can be suppressed strongly even in two-dimensions by proper patterning of the membranes into a nanostructured periodic phononic crystal.

VI. ELECTRONS AND FRACTIONAL CHARGES

Charged particles play a special role in assessing quantum transport properties, since they provide a straightforward access to both particle number current and heat current. For instance, in the case of electrons, we can count the carriers by measuring directly the charge current and the associated conductance. When the mean free path of the carriers is much larger than the physical dimensions of the contact, transport can become ballistic. According to Eq. (4), the electrical conductance then assumes only integer multiple values of elementary conductance quantum. The very first experiments on quantized conductance of a point contact in a two-dimensional electron gas as a function of gate voltage. The conductance demonstrates plateaus at multiples of $2e^2/h$.

FIG. 7 Measured quantized conductance of a point contact in a two-dimensional electron gas as a function of gate voltage. The conductance demonstrates plateaus at multiples of $2e^2/h$.

The layout of the point contact is shown schematically in the inset. Adapted from (van Wees et al., 1988).

30 years after the experiments on quantized transport properties, since they provide a straightforward access to both particle number current and heat current. For instance, in the case of electrons, we can count the carriers by measuring directly the charge current and the associated conductance. When the mean free path of the carriers is much larger than the physical dimensions of the contact, transport can become ballistic. According to Eq. (4), the electrical conductance then assumes only integer multiple values of elementary conductance quantum. The very first experiments on quantized conductance of a point contact in a GaAs-AlGaAs heterostructures were performed by (van Wees et al., 1988; Wharam et al., 1988). In (van Wees et al., 1988) the point contact was formed by a top metallic gate with a width $W \approx 250$ nm opening in a tapered geometry to form a voltage-controlled narrow and short channel in the underlying electron gas. The layout of the gate electrode is shown in the inset of Fig. 7. At negative gate voltages electrons are repelled under the gate and the width of the channel for carriers is $\lesssim 100$ nm which is well below the mean free path of $l \approx 8.5$ $\mu m$. The measured conductance of the point contact shown in Fig. 7 exhibits well defined plateaus at the expected positions $N2e^2/h$ as a function of applied gate voltage (van Wees et al., 1988). The factor 2 with respect to Eq. (4) arises from spin degeneracy.

Thirty years after the experiments on quantized electrical conductance by electrons (van Wees et al., 1988)
The number of quanta carrying the heat out of the plate and the two-dimensional electron gas underneath is expected to be sufficient electrical and thermal contact between the metal phonon bath at rate $\dot{Q}$, which both have fixed temperature $T_0$. This power is then transmitted via the thermometer. This power is then transmitted via the phonon bath at rate $\dot{Q}$, which both have fixed temperature $T_0$. Both these measurements were performed on GaAs-based 2DEGs, and in both of them, thermal conductance was obtained by measuring the Seebeck coefficient (thermopower) and extracting the corresponding temperature difference. In [Molenkamp et al., 1992], they then determined $G_{th}$, which is within a factor two agreement with the assumption that Wiedemann-Franz law applies on their conduction plateaus of the QPC. In [Chiatti et al., 2006], they made a similar experiment with the same philosophy but with improved control of the structure and system parameters. With these assumptions they come to a good agreement between thermal conductance and electrical conductance via the Wiedemann-Franz law.

In recent years, it has become possible to measure quantized thermal conductance even at room temperature [Cui et al., 2017; Mosso et al., 2017]. The experiments are performed on metallic contacts of atomic size with scanning thermal microscopy probes. The material of choice is typically Au, although experiments on Pt have also been reported [Cui et al., 2017]. The setup and experimental observations of [Cui et al., 2017] are presented in Fig. 9. The electrical conductance plateaus at multiples of $2e^2/h$ are typically seen when pulling the contact to the few conductance channel limit. The remarkable feature in the data is that the simultaneous thermometric measurement confirms the Wiedemann-Franz law for electric transport within 5 - 10% accuracy, thereby demonstrating quantized thermal conductance [Cui et al., 2017].

In the measurement performed by [Banerjee et al., 2017], the value of the quantum of thermal conductance for different Hall states including integer and fractional states was verified. First, they confirmed the observations of [Jezouin et al., 2013] in a similar setup in the integer states with filling factors $\nu = 1$ and 2. Figure 10(a) demonstrates the validity of quantized heat conductance at $\Delta N G_Q$ for $\Delta N = 1, 2, ..., 6$ channels with about 3% accuracy (inset). The main result of...
VII. PHOTONS

In this section we discuss transport by thermal microwave photons, presenting another important bosonic system to study in this context.

A. A ballistic photon channel

The concept of microwave photon heat transport becomes concrete when describing it on a circuit level [Schmidt et al. 2004]. We start from a setup familiar from the century-old discussion by Johnson and Nyquist [Johnson 1928; Nyquist 1928]. There two resistors $R_1$, $R_2$ are directly coupled to each other as shown in Fig. 11(a). They are generally at different temperatures $T_1$ and $T_2$. Each resistor then produces thermal noise with the spectrum $S_\nu(\omega)$ of Eq. (32), i.e., they can be considered as photon sources. First we consider that $R_1$ generates noise current $i_1$ on resistor $R_2$ as $i_1 = \nu_1/(R_1 + R_2)$. Then the spectral density of current noise is $S_{i_1}(\omega) = (R_1 + R_2)^{-2}S_{\nu_1}(\omega)$. The voltage noise produced by resistor $R_i$, $i = 1, 2$ is $S_{v_i}(\omega) = 2R_i\hbar\omega/(1 - e^{-\hbar\omega/}\omega)$, for $i = 1, 2$. The power density produced by noise of $R_1$ and dissipated in resistor $R_2$ is then $S_{p_2}(\omega) = \frac{R_2}{R_1 + R_2} S_{v_1}(\omega)$. The corresponding total power dissipated in resistor $R_2$ due to the noise of resistor $R_1$ is

$$P_2 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_{p_2}(\omega)$$

\[= \frac{4R_1R_2}{(R_1 + R_2)^2} \int_{0}^{\infty} \frac{d\omega}{2\pi} \hbar\omega|i_1(\omega)| + \frac{1}{2}. \quad (33)\]

The net heat flux from 1 to 2, $P_{net}$, is the difference between $P_2$ and $P_1$, where $P_1$ is the corresponding power produced by $R_2$ on $R_1$ by the uncorrelated voltage (cur-
as has been done, e.g., in (Pascal et al., 2011; Thomas et al., 2013). The necessary key ingredients for “quantumness” are that the experiment combines low temperatures and physically small structures. More quantitatively, the realistic circuit is never presented fully by the simple combination of two resistors, but, instead the full picture of it includes also inevitable reactive elements. A way of describing a more realistic circuit (Golubev and Pekola, 2015) is to include a parallel capacitance and series inductance in the basic circuit, as shown in Fig. 11(b). The point is that electromagnetics tells us that an order of magnitude estimate for capacitance is given by \( C = \varepsilon \ell k_B T / \hbar \), the thermal cut-off of the resistor at temperature \( T \). In form of simple inequalities we then need to require \( \omega_0 L \ll R \ll (\omega_0 C)^{-1} \), and based on our arguments above this transforms into

\[ \varepsilon k_B T R / \hbar \ll 1, \quad \mu_0 \ell k_B T / (h R) \ll 1. \]  

C. Experiments on heat mediated by microwave photons

We modelled in Section VII the heat emitted by a resistor and absorbed by another one in an otherwise dissipationless circuit. It was shown (Schmidt et al., 2004) that this heat carried by microwave photons behaves as if the two resistors were coupled by a contact whose ballisticity is controlled by the impedances in the circuit. Ideally two physically small and identical resistors at low temperatures can come very close to the ballistic limit with thermal conductance approaching \( G_Q \). Certainly Johnson-Nyquist work (Johnson, 1928; Nyquist, 1928) was out of this domain, as well as an elegant more recent experiment by Ciliberto (Ciliberto et al., 2013).
They were all performed essentially in the same scenario: the resistors are normal metallic thin film strips with sufficiently small size such that their temperature varies significantly from typical changes of power affecting them. The electrical connection between the resistors is provided by superconducting (aluminum) leads, whose electronic heat conductance is vanishingly small at the temperature of operation, see Section IV.A. In one of the experiments (Meschke et al. 2006), the superconducting lines are interrupted by a SQUID that acts as a tunable inductor providing a magnetic flux controlled valve of photon mediated heat current. All these experiments are performed at $T \sim 0.1 \text{ K}$, far below $T_C \approx 1.4 \text{ K}$ of aluminum. Temperatures are controlled and monitored by biased NIS tunnel junctions.

The experiment of Timofeev et al. (Timofeev et al. 2009) was designed to mimic as closely as possible the basic configuration of Fig. 11(a) with a superconducting (Al) loop. In this case the distance between the resistors was about $50 \mu m$ and the temperatures of both the heated (or cooled) source and the drain resistor were measured. The experiment (Fig. 12(a-c)) demonstrates thermal transport via the electronic channel, i.e. quasiparticle thermal transport (Bardeen et al. 1959) described in Section IV.A at temperatures exceeding $\sim 250 \text{ mK}$. The result in this regime is well in line with the basic theory, taken the dimensions and material parameters of the aluminum leads. More importantly, below about $200 \text{ mK}$ the photon contribution kicks in. In the loop geometry it turns out that the temperatures of the two resistors follow each other closely at lowest bath temperatures, yielding thermal conductance given by $G_Q$. Some uncertainty remains about the absolute value of $G_Q$ since the precise magnitude of the competing electron-phonon heat transport coefficient $\Sigma$ remained somewhat uncertain. The measurement was backed by a reference experiment, where a similar sample as that described above was measured under the same conditions and fabricated in the same way. This reference sample lacked intentionally one arm of the loop leading to poor matching of the circuit in the spirit discussed in Section VII.B. In this case the quasiparticle heat transport prevails as in the matched sample, but the photon $G_p$ is vanishingly small, confirming, one could say even quantitatively, the ideas presented about the heat transfer via a non-vanishing (reactive) impedance.

The experiment described above was performed on a structure with physical dimensions not exceeding $100 \mu m$. A natural question arises: is it possible to transport heat over macroscopic distances by microwave photons, like radiating the heat away from the whole chip? This could be important for example in quantum information applications (for superconducting qubit realizations, see Kjaergaard et al. 2020). This question was addressed experimentally by Partanen et al. 2016. FIG. 12 Quantum-limited heat conduction over microscopic and macroscopic distances. (a) The scanning electron micrograph (SEM) of two AuPd resistors at a distance of $50 \mu m$ are connected via Al superconducting lines into a loop to match the impedance between them to reach the full quantum of heat conductance. NIS probe junctions are used to apply Joule heat and to measure the island temperature. (b) Measured local island temperature, $T_I$ (blue) and remote one $T_B$ (red) as functions of applied bias voltage on cooler/heater NIS junction pairs at different bath temperatures $T_B$. The drop for $T_I$ is naturally stronger than that for $T_B$. (c) The measured relative temperature drops (symbols) $\Delta T_B/\Delta T_I$ against $T_B$ at the optimum cooling bias voltage obtained from data like those in (b). The descending solid and dashed black lines are obtained from the linearized thermal model, which is simply given by $\Delta T_B/\Delta T_I = G_p/(G_p + G_{th,2})$ (see Fig. 2(b)) at low temperatures. The thermal model is that shown in Fig. 2(b). The electron-phonon constant considered to be $\Sigma_{AuPd} = 2 \times 10^6 \text{ WK}^{-3}\text{m}^{-3}$ and $\Sigma_{AlPd} = 4 \times 10^6 \text{ WK}^{-3}\text{m}^{-3}$ for the solid and dashed lines, respectively. The remaining red solid lines are the results of the numerical thermal model. (d) A superconducting transmission line terminated at the two ends by normal-metal resistances $R_A$ and $R_B$ at different electron temperatures $T_A$ and $T_B$, respectively, is schematically shown. (e) SEM image of an actual device, where the length of the coplanar waveguide (transmission line), made out of Al, is either $20 \text{ cm}$ or $1 \text{ m}$ and has a double-spiral structure. The SEM image of the zoom-out of one of the resistors (made out of either AuPd or Cu) with a simplified measurement scheme is shown in the bottom right of the figure. (a-c) Adapted from Timofeev et al. (2009) and (d-e) adapted from Partanen et al. (2016).
(Fig. 12d-e)), who placed the two resistors at a distance of about 10 mm, i.e. about 100 times further away from each other as compared to what was done earlier. Furthermore, the connecting line between the two baths was a 1 m long meander made of a superconductor, acting as a transmission line. Such a coplanar line has typically an impedance of about 50 Ω irrespective of its length, thus potentially supporting the heat transport even over large distances. The thermal conductance was measured as in Timofeev et al. (Timofeev et al. 2009), with similar results proving the hypothesis of photon transport over macroscopic distances. These experiments may open the way for practical heat transport schemes in microwave circuits.

VIII. TUNABLE QUANTUM HEAT TRANSPORT

In this section we describe quantum systems where heat transport is controlled by either magnetic or electric field to achieve useful functional operation. These devices include heat valves, heat interferometers, thermal rectifiers and circuit refrigerators. Mesoscopic structures provide an option to control currents by external fields. Concerning charge currents, SQUIDs (Tinkham 2004) and single-electron transistors (Averin and Likharev 1991) provide hallmark devices in this context, where magnetic field (flux in a superconducting loop) and electric field (gate voltage), respectively, are the parameters that control the current.

The first experiment on heat transport by thermal microwave photons (Meschke et al. 2006) was realized in a setup where a SQUID is used as a heat valve. The experiment depicted in Fig. 13 shows two metallic (AuPd) resistors at the distance of few tens of μm from each other, connected by superconducting (Al) lines. The loop is interrupted in each arm by a SQUID, whose flux can be controlled by the common external field for both of them. The thermal model of Fig. 2(b) applies to this circuit. In the experiment only the heated resistor’s temperature $T_{e1}$ was measured. The panel at bottom right displays the magnetic flux dependent variation of temperature $T_{e1}$ at different bath temperatures $T_0$ under a constant level of heating. At bath temperatures well above 100 mK, the flux dependence vanishes, since the inter-resistance thermal conductance by photons, $\propto T$, is much weaker than the conductance to the phonon bath, $\propto T^3$. On the contrary, towards low temperatures below 100 mK, the electron temperature $T_{e1}$ varies with magnetic flux as the inter-resistor coupling becomes comparable to the bath-coupling, demonstrating the photonic thermal conductance. Moreover, the magnitude of the thermal conductance was shown to follow from the circuit model presented in Section III.A quantitatively, when applied to the current setup. Among other things, the data and this calculation predicted that at $T_0 = 60$ mK the maximum value of thermal conductance with zero flux in the SQUID (i.e. with minimum Josephson inductance) was $\sim 50\%$ of $G_Q$.

Photonic heat current was controlled by magnetic field in the previous example. A dual method is to apply electric field as a control as indicated in Fig. 14(a). This procedure was realized in the experiment recently (Maillet et al. 2020), where the superconducting loop is interrupted by a Cooper-pair transistor (“charge qubit” Nakamura et al. 1999). In this setup, the Josephson coupling is tuned by the gate voltage. The
A. Electronic quantum heat interferometer

Another quantum interference experiment on heat current by electrons was performed by Giazotto and Martínez-Pérez (2012), shown in Fig. 15. They used a magnetic field controlled SQUID as an interferometer. They could independently measure the electrical and heat transport via the device. For the latter, the SQUID was placed between two mesoscopic heat baths and the heat current was measured with the principle that was depicted in Fig. 2(b). The measurement was performed in a temperature regime exceeding that described in Section IV.A such that $\delta T$ is high enough for the superconductor to have a substantial equilibrium quasiparticle population (i.e., not all electrons are paired). In this regime the superconductor as such can support heat current, and heat interference across the Josephson junctions of the SQUID becomes possible. This work addressed experimentally for the first time a half-a-century old proposal and theory (Maki and Griffin, 1965) which has been followed by several works more recently (Golubev et al., 2013; Guttman et al., 1998, 1997; Zhao et al., 2003). It also demonstrated potential of electronic caloritronics in superconducting circuits.

B. Cooling a quantum circuit

In the experiment performed by (Tan et al., 2017) photon-assisted tunneling serves the purpose of decreasing the number of microwave quanta in a superconducting quantum circuit, namely a coplanar wave resonator (harmonic oscillator). The optical micrograph of the sample presented in Fig. 16(a) shows resistive elements inserted at the two ends of the resonator, acting as heat sinks for it. Figure 16(b) displays the temperature of one of these resistors (Quantum Circuit Refrigerator, QCR), measured and controlled by NIS tunneling, effectively lowering and elevating the electronic temperature of it depending on the biasing of the cooler junction, see Section IV.B. The other resistor (“Probe”) is passive but its temperature is likewise monitored. This temperature reacts weakly to the QCR temperature changes. The authors in (Tan et al., 2017) develop a thermal model based on which they extract the average number of photons in the resonator and the corresponding temperature $T_{\text{res}}$. In this experiment $T_{\text{res}} \approx 800 \, \text{mK}$ far exceeds all other temperatures, most notably the electronic temperatures of the two resistors, $T_{\text{QCR}} \approx T_{\text{probe}} \approx 150 \, \text{mK}$ even under no bias on the QCR. The model then predicts cooling of the resonator down to about 400 mK under optimal conditions.
biasing conditions of the QCR (Fig. 11(c)). Based on the parameters given in the manuscript, one would estimate the resonator to have $T_{\text{res}} \approx 200$ mK when the QCR is not biased. Indeed, in a later work (Masuda et al., 2018) resonator temperatures in the 200 mK regime were reported at zero bias. When biased, the NIS junctions operate as an incoherent microwave source. Then the mode temperature of the resonator can be driven even beyond 2.5 K, far above the temperature of the phonon andelectron reservoirs of the system (Masuda et al., 2018). This phenomenon was theoretically modelled by Silveri et al. (2017) using photon-assisted tunneling of the biased, NIS-junctions as the environment. The effective temperature of the resonator is expected to be lifted to $\sim eV/(2k_B)$ at bias voltage $V$.

**IX. QUANTUM HEAT TRANSPORT MEDIATED BY A SUPERCONDUCTING QUBIT**

In this section we introduce a superconducting qubit as an element that mediates heat by microwave photons between two baths. Different types of superconducting quantum bits, e.g. flux, charge and transmon qubits to mention a few common ones, are options in such devices (Clarke and Wilhelm, 2008). They feature different coupling options and strengths, as well as different degrees of anharmonicity in the Josephson potential to be discussed below. In the experiments of (Ronzani et al., 2018) transmon type qubits (Koch et al., 2007) were employed. This kind of a qubit has levels whose positions can be controlled by magnetic flux through the SQUID-loop. Transmon qubit is only weakly anharmonic, meaning that one typically needs to consider not only the two lowest levels (that form the actual qubit) but also the higher levels in this nearly harmonic potential. One should point out an important difference: although even weak anharmonicity is enough to address only the two lowest levels (that form the actual qubit) but also the higher levels in this nearly harmonic potential. Yet in a typical experiment to be described here, the separation of the levels is of the order of 0.5 K, meaning...
that the thermal population of the third level is already quite small at the low temperatures of the experiment, say, below 0.2 K ($\sim e^{-5} < 0.01$).

In this Section we present thermal transport experiments under the conditions where the qubit is not driven. Coherent properties of the qubit do not then play any important role. In the future the same devices will be driven by RF-fields and evolution of off-diagonal elements of the density matrix will develop as well.

### A. Quantum heat valve

Figure 17(a) shows on the top a typical experimental configuration of heat control with qubit, taken from [Ronzani et al., 2018]. The energy separation of the transmon qubit (center) can be controlled by external magnetic flux $\Phi$. The qubit is coupled capacitively (coupling $g$) to two nominally identical superconducting coplanar wave resonators that act as $LC$ resonators with a resonant frequency of $\sim 5$ GHz each. For thermal transport experiments the $\lambda/4$ resonators are terminated by on-chip resistors which form the controlled dissipative elements in the circuit (Chang et al., 2019). The dissipation is then given by the inverse of the quality factor of the resonator and can be quantified by another coupling parameter $\gamma$. In this circuit, which is called a quantum heat valve, heat is carried wirelessly (via capacitors) by thermal microwave photons over a distance of few millimeters from one bath to another. A schematic model of such a coupled circuit is shown in panels (b) and (c) on top.

It turns out that the measurement of heat transport in such a circuit addresses some fundamental questions of open quantum systems (Aurell and Montana, 2019; Chiara et al., 2018; Donvil et al., 2020, 2018; Hewgill et al., 2020; Holer et al., 2017; Levy and Kosloff, 2014; Magazzù and Grilli, 2019; Rivas et al., 2010). There are (at least) two possible ways of viewing the circuit, namely the local view (Fig. 17(b)) and the global view (Fig. 17(c)). In the local picture as we define it, the environment of the qubit is formed of the dissipative $LC$ resonator with Lorentzian noise spectrum centered around its resonance frequency. In this regime, which occurs when $\gamma \gg g$, the system acts indeed as a valve admitting heat current through only when the qubit frequency matches (within the range determined by the quality factor) the frequency of the resonators. This results in Lorentzian peaks in power centered at flux positions corresponding to the said matching condition, demonstrated by both experiment and theory, shown in the bottom panel of Fig. 17(b). In the opposite limit, in the global regime, when $\gamma \ll g$, the situation is different. The combined system composed of the resonators and the qubit then make up a hybrid that interacts with bare environment formed of the two resistors. In this limit the hybrid quantum system has the energy spectrum shown in the bottom panel of Fig. 17(a) exhibiting multilevel structure. This is shown by the basic calculated spectrum and the spectroscopic measurement on a structure similar to that in the top panel but in the absence of the resistive loads. The data in Fig. 17(c) demonstrates results in the global regime, with experiment and theory developed in (Ronzani et al., 2018) in agreement with each other. This experiment is the first one to assess
In a symmetric structure as in Fig. 17, there is naturally no directional dependence of heat transport between the two baths. However, heat current rectification becomes possible if one breaks the symmetry of the structure (Segal and Nitzan, 2005). Heat rectification (Hindawi et al., 2021; Goury and Sánchez, 2019; Iorio et al., 2021; Karga et al., 2019; Motz et al., 2018; Riera-Campeny et al., 2019; Ruokola et al., 2009; Sánchez et al., 2015; Segal and Nitzan, 2005; Sothmann et al., 2012) can be quantified in different ways, but in general finite rectification means that the magnitudes of forward and reverse heat currents differ under identical but opposite biasing conditions. There exist a few experiments on heat current rectification, e.g. on phonons in carbon nanotubes (Chang et al., 2000), and electrons in quantum dots (Scheibner et al., 2008), mesoscopic tunnel junctions (Martínez-Pérez et al., 2015) and suspended graphene (Wang et al., 2017). In (Senior et al., 2020) rectification was realized in a structure similar to that in Fig. 17 but by making the two resonators of unequal length: the two resonators had in this case frequencies 3 GHz and 7 GHz. An additional feature necessary for heat rectification is the non-linearity of the central element, which arises from the anharmonicity of the transmon Josephson potential. Figure 18 shows data from (Senior et al., 2020) where heat current through the structure is measured in forward and reverse directions under the same but opposite temperature biasing, respectively. Complicated flux dependence can be observed, but the main feature is that one reaches 10% rectification at best and it depends strongly on the flux position determining the coupling asymmetry to the two baths. Quantitative analysis of the flux dependence is challenging and experiments in simpler setups would be welcome.

X. HEAT CURRENT NOISE

In this section we focus on fluctuations of currents, which are generally considered to be harmful, and something to get rid of. The synonym of fluctuations, noise, proposes this negative side of the concept of fluctuations. Noise typically determines the minimal detectable signal in a measurement, i.e. the resolution. Here we do not consider noise caused by the measurement apparatus or from other extrinsic sources, but we focus on intrinsic noise, due to fundamental quantum and thermal fluctuations. This noise, for instance in form of electrical current or heat current fluctuations determines the ultimate achievable measurement accuracy. But besides being a limiting factor of a measurement, noise can also serve as a signal to build on in order to realize a sensor: for instance, measurement of thermal current noise of a conductor provides one of the most popular and fundamental thermometers in use (Fleischmann et al., 2020).

We already discussed current and voltage noise of a linear dissipative element in Section IV.D. Here we review the heat current noise, both classical and quantum, see e.g. (Crépieux, 2021; Karimi and Pekola, 2021; Moskalets, 2014; Pekola and Karimi, 2018; Sánchez and Büttiker, 2012). For simplicity, we consider first the tunneling as an example. Besides presenting the classical fluctuation-dissipation theorem for heat current, which we review in a general case after it, we also observe the intriguing quantum expression of heat current noise including the frequency dependent component due to zero point fluctuations surviving down to T = 0. Next we focus on the temperature dynamics of a finite system coupled to a bath, which yields the experimentally accessible fundamental fluctuations of the effective temperature of this subsystem. Finally we review the experimental situa-
tion, with up to now only a small number of examples, on fluctuations in heat transport of quantum and classical systems.

A. FDT for heat in tunneling

We consider tunneling where the average heat current out from lead $L$ was given by Eq. (18). Taking for simplicity the normal conductors (NIN junction) with $n_L(\epsilon) = n_R(\epsilon) = 1$, we have the average heat current at $eV = 0$

$$\dot{Q}_L = \frac{1}{e^2 R_T} \int de \epsilon [f_L(\epsilon) - f_R(\epsilon)].$$

(39)

The thermal conductance for tunneling, $G_{th} = \frac{dQ_L}{dT}$, at $T_L = T$, is then given by $G_{th} = G_{T} T$, where $G_T = 1/R_T$ is the conductance of the tunnel junction. Like the fully transmitting channels in Section II, the tunnel junction satisfies the Wiedemann-Franz law.

The heat current operator $\hat{H}_T$ to obtain the average heat current of Eq. (18) was calculated using the tunnel coupling operator of Eq. (16) and commuting this with the Hamiltonian of the left lead. We may use this operator to find the two-time correlator of it and Fourier-transform to find the spectral density of noise of heat current of Eq. (18) was calculated using the tunneling dissipation theorem for heat current as

$$S_Q(\omega) = \frac{G_T}{6e^2} [(2\pi k_B T)^2 + (h\omega)^2] \frac{h\omega}{1 - e^{-h\omega/k_B T}}.$$  

(40)

For the symmetrized noise, $S_Q^{(s)}(\omega) = \frac{1}{2} [S_Q(\omega) + S_Q(-\omega)]$, we then have

$$S_Q^{(s)}(\omega) = \frac{G_T}{12e^2} [(2\pi k_B T)^2 + (h\omega)^2] h\omega \coth(\frac{h\omega}{2k_B T}).$$  

(41)

Now there are two important limits to consider. First of all, for $\omega \to 0$, we obtain the classical fluctuation-dissipation theorem for heat current as

$$S_Q^{(s)}(0) = 2k_B T^2 G_{th}.$$  

(42)

On the other hand, the finite frequency noise does not vanish at zero temperature, but

$$S_Q^{(s)}(\omega) = \frac{G_T}{12e^2} |h\omega|^3, \quad T = 0.$$  

(43)

B. FDT for heat for a general system

The previous sub-section serves as an illustration of how noise and dissipation are related. Here we extend the discussion to a general setup beyond the tunneling case. This would allow us to treat other mechanisms as well, for instance phonons, photons and electron-phonon coupling relevant for the current review. In general the FDT for heat applies in the form introduced in Eq. (42) for low frequency noise. To see this we may write the Hamiltonian

$$\mathcal{H} = \hat{H}_s + \hat{H}_b + \hat{H}_c \equiv \hat{H}_0 + \hat{H}_c,$$  

(44)

where the unperturbed Hamiltonian $\hat{H}_0 = \hat{H}_s + \hat{H}_b$ is composed of the system and bath and $\hat{H}_c$ is again the coupling. Then in linear response, we have the expecta-
tion value of the heat current to the system
\[ \dot{Q} = (\dot{\mathcal{H}}_s) = \frac{i}{\hbar} \int_{-\infty}^{0} dt' \langle [\dot{\mathcal{H}}_s(0), \dot{\mathcal{H}}_s(t')] \rangle_0, \tag{45} \]

The expectation value of a general operator $\mathcal{O}$ in the non-interacting system is written as $\langle \mathcal{O} \rangle_0 = \text{Tr}(e^{-\beta_s \mathcal{H}_s} e^{-\beta B \mathcal{O}})/\text{Tr}(e^{-\beta_s \mathcal{H}_s} e^{-\beta B})$, where $\beta_s = 1/k_B T_s$ and $\beta = 1/k_B T$ are the corresponding inverse temperatures of the system and bath, respectively. By definition, the thermal conductance is given by
\[ G_{th} = -\frac{d\dot{Q}}{dT_s} \bigg|_{T_s = T} = \frac{1}{k_B T^2} \langle \delta \mathcal{H}_s \dot{\mathcal{H}}_s \rangle_0, \tag{46} \]

where we used $\dot{\mathcal{H}}_s = \langle \dot{\mathcal{H}}_s \rangle_0 = \delta \mathcal{H}_s$. On the other hand, the spectral density of noise for the heat current at zero frequency is given by
\[ S_Q(0) = \int_{-\infty}^{\infty} dt' \langle \dot{\mathcal{H}}_s(t') \dot{\mathcal{H}}_s(t') \rangle_0 \tag{47} \]
analogously to what was introduced in tunneling case. After some algebra and a careful comparison of Eqs. (46) and (47) we find the FDT given in Eq. (42).

C. Effective temperature fluctuations

Here we consider a system with varying temperature $T(t)$. This setup, shown in Fig. 2(a), presents an absorber of a calorimeter or bolometer coupled via thermal conductance $G_{th}$ to a heat bath at fixed temperature $T_0$. If we further assume that the small system has heat capacity $C$, the energy balance equation reads for the heat current $\dot{Q}(t)$ between the bath and the absorber
\[ \dot{Q}(t) = C \dot{T}(t) + G_{th} \delta T(t), \tag{48} \]

where $\delta T(t)$ is the difference between the absorber temperature and that of the bath. In order to calculate thermal noise, we again evaluate the two-time correlator as
\[ \langle \dot{Q}(t) \dot{Q}(0) \rangle = C^2 \langle \delta T(t) \delta T(0) \rangle + G_{th}^2 \langle \delta T(t) \delta T(0) \rangle, \tag{49} \]

leading to
\[ S_Q(\omega) = (\omega^2 C^2 + G_{th}^2) S_T(\omega). \tag{50} \]

Since we typically consider frequencies well below the temperature, $S_Q(\omega)$ is essentially frequency independent, (which was shown by Eq. 41 for tunneling) and the classical FDT holds for $S_Q(0)$ in form of Eq. 42 in equilibrium. Thus we have
\[ S_T(\omega) = \frac{2k_B T_0^2}{G_{th}} \frac{1}{1 + (\omega \tau)^2}, \tag{51} \]

where $\tau = C/G_{th}$ is the thermal relaxation time. This means that at very low frequencies $S_T(0) = 2k_B T_0^2/G_{th}$. The root-mean-square (rms) fluctuation of temperature is obtained as inverse Fourier transform of the noise spectrum at $t = 0$ as
\[ \langle \delta T^2 \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_T(\omega) = \frac{k_B T_0^2}{C}, \tag{52} \]

which is the well-known textbook result of temperature fluctuations (van den Berg et al. 2015; Heikkilä and Nazarov 2009; Lifshitz and Pitaevskii 1980). The results of Eqs. (51) and (52) are directly accessible in experiments.

D. Progress on measuring fluctuations of heat current and entropy

The previous discussion applies for systems and processes in or very near equilibrium. Over the past decades relations holding also far from equilibrium and for finite times have been developed (Bochkov and Kuzovlev 1981; Crooks 1999; Jarzynski 1997; Seifert 2012). During the past 20 years they have also become experimentally feasible mainly thanks to advances in production and manipulation of nanostructures. The best known non-equilibrium fluctuation relations governing entropy production $\Delta S$ are given by $P(\Delta S)/P(-\Delta S) = e^{\Delta S/k_B}$ and its corollary $(e^{-\Delta S/k_B}) = 1$. Here $\langle \cdot \rangle$ refers to the average over many experimental realizations or to the expectation value for the measurement. For macroscopic systems near equilibrium these relations simplify into second law of thermodynamics.

Here we give a brief summary of such non-equilibrium experiments on electrical systems. Fluctuations of entropy production and heat currents have been actively studied experimentally for more than a decade in the classical regime, but mainly via indirect means of detection since entropy is a tricky quantity for a direct measurement (Klecorin et al. 2019). Two main classes of systems under study have been those in the seminal experiments on molecules (Collin et al. 2005) and on electrical circuits (Bérut et al. 2016; Ciliberto et al. 2013; Küng et al. 2012; Pekola 2015; Saira et al. 2012b). These works go beyond FDT by addressing far-from-equilibrium fluctuation relations (Bochkov and Kuzovlev 1981; Campisi et al. 2011; Crooks 1999; Jarzynski 1997; Pekola and Khaymovich 2011; Seifert 2012). In the work by Ciliberto et al. (2013) as shown in Fig. 19(a), the setup of two macroscopic resistors was examined at temperatures around the ambient. An indirect measurement of entropy was facilitated in Ciliberto et al. (2013) via the detection of instantaneous electrical power $IV$, integrated over time and divided by the corresponding temperature of the macroscopic resistor. This way several fluctuation relations for entropy production under non-equilibrium conditions (Seifert 2005, 2012) could be verified together with the standard FDT in linear response regime. Similarly in the setup of Saira...
et al., 2012b) of Fig. 19(b), detecting single electrons making non-equilibrium transitions across a junction in a single-electron box provides indirect means of observing the dissipated energy and entropy production quantitatively (Averin and Pekola, 2011; Koski et al., 2013). These experiments were performed at temperatures three orders of magnitude lower than in (Ciliberto et al., 2013). The Crooks (Crooks, 1999) and Jarzynski (Jarzynski, 1997) relations as well as generalized relations incorporating the role of information in Maxwell demon setup (Sagawa and Ueda, 2010) could be tested accurately in these experiments (Pekola and Khaymovich, 2019).

The reason for using indirect measurement of heat by detailed electrical characterization is the fact that the powers are far too small to resolve, e.g. by direct thermometry (Section III.B). Next, we focus on progress of direct measurement of heat current fluctuations.

E. Energy sensitivity of a calorimeter

The ultimate energy resolution of a thermal detector, see Fig. 20, is determined by the coupling of it to the heat bath associated with the fluctuations of heat current. Taking a wide band thermometer on a calorimeter, the rms fluctuations of the effective temperature due to this intrinsic noise are given by Eq. (52). In order to find the energy resolution of the detector one needs to compare this noise to the impact of the absorption of energy \( E \) on the temperature of the detector, which can be evaluated by solving Eq. (48) for an instantaneous absorption of a photon at energy \( E \) at time instant \( t = 0 \), meaning \( \dot{Q}(t) = E \delta(t) \) with the solution \( \delta T(t) = \left( E/\mathcal{C} \right) e^{-t/\tau} \theta(t) \), where the time constant \( \tau = \mathcal{C}/G_{\text{th}} \) and \( \theta(t) \) is the Heaviside step function. Thus at \( t = 0^+ \), the immediate rise of \( T \) is \( \delta T(0) = E/\mathcal{C} \). The signal-to-noise ratio \( \text{SNR} = \delta T(0)/\sqrt{\langle \delta T^2 \rangle} \) is

\[
\text{SNR} = E/\sqrt{k_B T_0^2 \mathcal{C}},
\]

meaning that the energy resolution of the detector in this regime is

\[
\delta E = \sqrt{k_B T_0^2 \mathcal{C}}.
\]

For convenience, let us write \( \mathcal{C} = \eta k_B \), where \( \eta \) is a dimensionless constant that we assess below. Then we find that \( \delta E = \sqrt{\eta k_B T_0} \). As an example, related to the experiment of (Karimi et al., 2020), we take a metallic calorimeter where \( \mathcal{C} = \gamma VT_0 \) at low temperatures (see Fig. 20(a),(b)). Here \( \gamma \sim 100 \text{ JK}^{-2}\text{m}^{-3} \) for copper and \( V < 10^{-21} \text{ m}^3 \) is the volume of the absorber, yielding \( \eta \sim 100 \) and energy resolution \( \delta E/k_B \sim 0.1 \text{ K at } T_0 = 0.01 \text{ K} \) (Karimi and Pekola, 2020).

FIG. 20 Quantum calorimeter. (a) A normal-metal absorber is coupled to the phonon heat bath at fixed temperature \( T_0 \) via electron-phonon collisions, shown by many arrows. These collisions lead to stochastic exchange of heat between the absorber and the bath, and to fluctuating temperature \( \delta T \) in the absorber. The core of the device is the calorimeter including a thermometer, which measures the temporal temperature variations. An example of a temperature trace is shown next to the absorber. (b) Noise equivalent temperature of the calorimeter. The solid symbols are the measured signal of temperature fluctuations in equilibrium \( \text{NET} \equiv \sqrt{\delta T^2} = \sqrt{\langle \delta T^2 \rangle}/(2\Delta f) \) obtained via the measurement of \( \langle \delta T^2 \rangle \) with \( \Delta f = 10 \text{ kHz} \). The solid and dashed lines represent the noise-equivalent temperature in equilibrium \( \text{NET} \), of the normal-metal absorber in the presence and absence of an extra photon contribution, respectively. Red lines (the lower two) display \( \text{NET} \) in equilibrium \( \text{NET}_{\exp} = \sqrt{2k_B T_0^2/G_{\text{th}}} \) while the blue lines (the upper two) show \( \text{NET} = \delta T/\sqrt{C G_{\text{th}}} \), which is the required \( \text{NET} \) of a detector to observe a photon with energy \( \delta \epsilon = 1 \text{ K} \times k_B \). The prohibited range bordered by the fundamental temperature fluctuations in equilibrium is indicated by the shaded area. (Inset) Scanning electron micrograph of part of the actual sensor, the SNIS structure, where Cu is used as a normal metal N and Al as a superconductor S. The SNIS junction is the dissipative element in the RLC circuit operating at \( f_0 \approx 650 \text{ MHz} \). (c) Simulation of the expected thermometer signal of a calorimeter in response to the absorption of an incoming photon depicted in the inset. The parameters of the simulation of the detector correspond to the experiment shown in (b), with \( \eta \equiv C/k_B = 100 \) and \( h\omega_q/(k_B T_0) = 100 \), where \( h\omega_q \) is the qubit energy, and \( T_0 = 10 \text{ mK} \). The noise seen in the traces is the result of temperature fluctuations due to the coupling of the absorber to the phonon bath. Upper panel shows a jump occurring at \( t/\tau = 1 \) (the time instant set arbitrarily) exceeding the noise level of equilibrium fluctuations, with signal-to-noise ratio of about 10. The red lines (truncated by filtering) show the results at different cut-off frequencies of the thermometer, parametrized by the ratio of the electron-phonon time and the detector response time. (a) Adapted from (Karimi et al., 2020) and (b) adapted from (Karimi and Pekola, 2020).

The fluctuations in power have direct impact on the performance of calorimeters and bolometers (Gildemeister et al., 2001; Irwin, 1995), i.e. thermal detectors of radiation. This noise determines the energy resolution of a calorimeter, and also the noise-equivalent power under continuous irradiation, like in the measurement of the
cosmic microwave background (Mather 1982). Direct measurements of fluctuating temperature are rare ((Chui et al. 1992, Karimi et al. 2020). The pioneering measurement of (Chui et al. 1992) employed a macroscopic calorimeter working at the so-called lambda point of liquid helium, i.e. its superfluid transition temperature at \( T = 2.17 \) K. They managed to verify Eqs. (51) and (52) thanks to a very high resolution of the thermometer measuring the magnetization of a paramagnetic salt (copper ammonium bromide) with the help of a SQUID down to \( 10^{-10} \) K/√Hz noise equivalent temperature.

The nanofabricated detector of (Karimi et al. 2020) worked in the regime where the measurement cut-off frequency was 10 kHz, which falls somewhat below \( 1/r \). Furthermore the metallic absorber was proximitized by a superconducting contact further decreasing \( C \) and \( G_h \) (Heikkilä and Giazotto 2009; Nikolic et al. 2020) and this way improving its performance. The experiment (Fig. 20(b)), utilizing a SNIS (superconductor-normal metal-insulator-superconductor) thermometer (Karimi and Pekola 2018) demonstrated noise of the effective temperature of the calorimeter that is very close to the expected fundamental fluctuation limit of Eq. (51) at low frequencies, at the same time promising a SNR of \( \sim 10 \) in measuring an absorption event with photon energy \( E/k_B = 1 \) K. Figure 20(c) demonstrates by simulation the validity of the analysis above. The wide-band detector would present

\[
\frac{\Delta T}{\Delta \nu} = \frac{E}{k_B T_0} \approx \frac{10}{\sqrt{10^3}} = 10 \text{ mK}.
\]

There exist several other concepts of ultrasensitive thermal detectors, either metallic (Govenius et al. 2016, Kuzmin et al. 2019) or those utilizing graphene or semiconductors (Kokkonen et al. 2020, 2019, Lara-Avila et al. 2019, Lee et al. 2020, Roukes 1999), or those based on, e.g., temperature dependent magnetization (Ens 2005, Kempf et al. 2018). The advantage of graphene is its supposedly very low heat capacity that could make the thermal response time shorter than in metal detectors. Yet at the time of writing this paper, none of the proposed detectors has demonstrated detection of quanta in said microwave regime.

**XI. Summary and Outlook**

In this review we have focused on fundamental aspects of quantum heat transport, moreover with main emphasis on experiments carried out during the past 20 years. It is probably fair to say that in many respects the physics of heat transport in quantum nanostructures is by now well understood, and experiments tend to confirm the theoretical predictions. In some systems clean experiments are, however, more difficult to realize than in others from practical point of view, and more experiments are needed: one example among others is presented by the one dimensional phonon structures where beautiful pioneering experiments were performed already long ago (Schwab et al. 2000), but where precise conditions of how to realize ballistic contacts are still under debate. The experiments on quantum heat transport serve also as tools to understand quantum matter itself, like the recent experiments in the fractional quantum Hall regime demonstrate (Banerjee et al. 2017, Dutta et al. 2021). On the other hand they show us ways of realizing new kinds of devices and of how to nail down and achieve their ultimate limits of performance. We discussed the latter issue in Section VII.C. It could thus serve as a way to reset quantum circuits rapidly. There is, however, a trade-off to be considered. Rapid thermalization is almost a synonym for low quality factor and fast decoherence of a quantum system, which are naturally not desirable properties. Therefore tunable coupling is a possible way to go, to switch on and off the coupling to a heat bath on demand. Variations of the many heat valves presented in this article could in principle serve the purpose. Tests of such an idea have been proposed and experimented on in (Partanen et al. 2018).

Quantum heat engines and cyclic refrigerators are presently under intensive study, see, e.g. (Alicki and Kosloff 2018, Benenti et al. 2017, Brandner et al. 2017, Campisi and Fazio 2016, Defner et al. 2014, Humphrey and Linke 2005, Joesfsson et al. 2018, Quan et al. 2007, Raja et al. 2021). Experiments fully in the quantum regime are up to now practically nonexistent, although there are proposals that address realistic setups (Abah et al. 2012, Karimi and Pekola 2016). For instance, a so-called quantum Otto cycle can be realized by coupling a superconducting qubit alternately to two different heat baths (Karimi and Pekola, 2016). If this is done by varying the energy level separation of the qubit, like was done in the photonic heat valve or rectifier above, but now cyclically at RF frequencies, one can extract heat from the cold bath and dump it to the hot one, when system parameters are chosen properly. We expect devices of this type or analogous ones to work in the near future. Interesting questions arise on whether one can boost up the powers and/or efficiencies by exploiting quantum dynamics, and by what kind of protocols one can speed up the cycles for higher powers in general (Funo et al. 2019, Menczel et al. 2019, Solfanelli et al. 2020).

We want to briefly note here that topological mat-
ter (Hasan and Kane, 2010) [Qi and Zhang, 2011], specifically topological superconductors and Josephson junctions, have been proposed as potential novel elements in quantum thermodynamics and heat transport experiments due to their unconventional physical properties, see e.g., non-exhaustive list of some recent work in (Bauer and Sothmann 2019) [Pan et al. 2021] [Rivas and Martin-Delgado 2017] [Schart et al. 2020]. Due to the focus of the current paper, mainly on experiments, we do not discuss this topic further.

In this review we have alluded to the connections of heat transport and quantum thermodynamics mainly regarding concrete device concepts, including thermal detectors, heat engines and refrigerators. On a more fundamental level, quantum heat transport is in the heart of open quantum systems physics (Breuer and Petruccione 2002) with non-Hermitian dynamics governed by the quantum noise (Gardiner and Zoller 2010) widely discussed in this review. True thermodynamics counts on observations of heat currents and temperatures, and power consumption of the sources. Adopting this view, one can pose many questions like how to measure work and heat in an open quantum system, for which the measurement apparatus cannot be viewed as an innocent witness of what is happening in the quantum system itself. The calorimeter can eventually become the microscope of quantum dynamics on the level of exchange of energy by individual quanta emitted or absorbed by the quantum system. This would give us the optimal tool to investigate stochastic thermodynamics in true quantum regime. Many other fundamentally and practically important questions arise and can potentially be answered by heat transport experiments. For instance, how does a quantum system thermalize, and does it find an equilibrium thermal state even in the absence of a heat bath?

To conclude, investigations and exploitation of quantum heat transport will play an important role in the currently active field of quantum thermodynamics and in future quantum technologies in general.

XII. ACKNOWLEDGMENTS

This work was supported by Academy of Finland grant 312057, the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie actions (grant agreement 766025), the Russian Science Foundation (Grant No. 20-62-46026), and Foundational Questions Institute Fund (FQXi) via Grant No. FQXi-IAF19-06.

REFERENCES

Abah, O, J, Roßnagel, G, Jacob, S, Deffner, F, Schmidt-Kaler, K, Singer, and E, Lutz (2012), “Single-ion heat engine at maximum power,” Phys. Rev. Lett. 109, 203006

Alicki, Robert, and Ronnie Kosloff (2018), Thermodynamics in the Quantum Regime: Fundamental Aspects and New Directions edited by Felix Binder, Luis A. Correa, Christian Gogolin, Janet Anders, and Gerardo Adesso (Springer).

Altimiras, C, H, le Sueur, U, Gennser, A, Cavanna, D, Mailly, and F, Pierre (2010), “Tuning energy relaxation along quantum hall channels,” Phys. Rev. Lett. 105, 226804

Ashcroft, Neil W, and N, David Mermin (1976), Solid State Physics (Holt, Rinehart and Winston).

Aurell, Erik, and Federica Montana (2019), “Thermal power of heat flow through a qubit,” Phys. Rev. E 99, 042130

Averin, D V, and K, K, Likharev (1991), Mesoscopic phenomena in solids, Single Electronics: A correlated transfer of single electrons and Cooper pairs in systems of small tunnel junctions, edited by B, L, Althüler, P, A, Lee, and R, A, Webb (Elsevier Science Publishers B, V.).

Averin, D V, and J, P, Pekola (2011), “Statistics of the dissipated energy in driven single-electron transitions,” EPL 96, 67004.

Averin, Dmitri V, and Jukka P. Pekola (2010), “Violation of the fluctuation-dissipation theorem in time-dependent mesoscopic heat transport,” Phys. Rev. Lett. 104, 220601

Banerjee, Mitali, Moty Heiblum, Amir Rosenblatt, Yuval Oreg, Dima E. Feldman, Ady Stern, and Vladimir Umansky (2017), “Observed quantization of anyonic heat flow,” Nature 545, 75–79.

Bardeen, J, L, N, Cooper, and J, R, Schrieffer (1957), “Theory of superconductivity,” Phys. Rev. 108, 1175–1204

Bardeen, J, G, Rickayzen, and L, Tewordt (1959). “Theory of the thermal conductivity of superconductors,” Phys. Rev. 113, 982–994

Bauer, Alexander G, and Björn Sothmann (2019), “Phase-dependent heat transport in josephson junctions with p-wave superconductors and superfluids,” Phys. Rev. B 99, 214508.

Benenti, Giuliano, Giulio Casati, Keiji Saito, and Robert S, Whitney (2017), “Fundamental aspects of steady-state conversion of heat to work at the nanoscale,” Physics Reports 694, 1 – 124

van den Berg, Tineke L, Fredrik Brange, and Peter Samuelsson (2015), “Energy and temperature fluctuations in the single electron box,” New Journal of Physics 17 (7), 075012.

Bhandari, Bibeik, Paolo Andrea Erdman, Rosario Fazio, Elisabetta Paladin, and Fabio Taddei (2021), “Thermal rectification through a nonlinear quantum resonator,” Phys. Rev. B 103, 155434

Blencowe, Miles P, and Vincenzo Vitelli (2000), “Universal quantum limits on single-channel information, entropy, and heat flow,” Phys. Rev. A 62, 052104.

Bochkov, GN, and Yu,E, Kuzovlev (1981), “Nonlinear fluctuation-dissipation relations and stochastic models in nonequilibrium thermodynamics: I. generalized fluctuation-dissipation theorem,” Physica A: Statistical Mechanics and its Applications 106 (3), 443 – 479

Brandner, Kay, Michael Bauer, and Udo Seifert (2017), “Universal coherence-induced power losses of quantum heat engines in linear response,” Phys. Rev. Lett. 119, 170602

Breuer, H-P, and F, Petruccione (2002), The theory of open quantum systems (Oxford University Press).
Bérut, A. A. Imparato, A. Petrosyan, and S. Ciliberto (2016), “The role of coupling on the statistical properties of the energy fluxes between stochastic systems at different temperatures,” J. Stat. Mech., 054002.

Butcher, P N (1990), “Thermal and electrical transport formalism for electronic microstructures with many terminals,” Journal of Physics: Condensed Matter 2 (22), 4869-4887.

Campisi, Michele, and Rosario Fazio (2016), “The power of a critical heat engine,” Nat. Commun. 7, 11895.

Campisi, Michele, Peter Hänggi, and Peter Talkner (2011), “Colloquium: Quantum fluctuation relations: Foundations and applications,” Rev. Mod. Phys. 83, 771–791.

Chang, C W, D. Okawa, A. Majumdar, and A. Zettl (2006), “Quantum thermal conductance of electrons in a one-dimensional wire,” Phys. Rev. Lett. 97, 056601.

Chau, T C P, D. R. Swanson, M. J. Adriaans, J. A. Nissen, Chiatti, O, J. T. Nicholls, Y. Y. Proskuryakov, N. Lumpkin, Chiara, Gabriele De, Gabriel Landi, Adam Hewgill, Brendan Crooks, Gavin E (1999), “Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences,” Phys. Rev. E 60, 2721–2726.

Donvil, Brecht, Paolo Muratore-Ginanneschi, and Dmitry Golubev (2020), “Exactly solvable model of calorimetric measurements,” Phys. Rev. B 102, 245401.

Donvil, Brecht, Paolo Muratore-Ginanneschi, Jukka P. Pekola, and Kay Schwieger (2018), “Model for calorimetric measurements in an open quantum system,” Phys. Rev. A 97, 052107.

Dutta, B. J. T. Peltonen, D. S. Autenkon, M. Meschke, M. A. Skvortsov, B. Kubala, J. König, C. B. Winkelmann, H. Courtois, and J. P. Pekola (2017), “Thermal conductance of a single-electron transistor,” Phys. Rev. Lett. 119, 077701.

Dutta, Bivas, Wenmin Yang, Ron Aharon Melcer, Hemantha Kumar Kundu, Moty Heiblum, Vladimir Umansky, Yuval Oreg, Ady Stern, and David Mross (2021), “Novel method distinguishing between competing topological orders,” arXiv:2101.01419 https://arxiv.org/abs/2101.01419.

Feschenko, A V. J. V. Koski, and J. P. Pekola (2014), “Experimental realization of a coulomb blockade refrigerator,” Phys. Rev. B 90, 201407.

Feschenko, A V, O.-P. Sara, J. T. Peltonen, and J. P. Pekola (2017), “Thermal conductance of uh thin films at sub-kelvin temperatures,” Sci. Rep. 7, 41728.

Gardiner, C W, and P. Zoller (2010), Quantum Noise (Springer-Verlag Berlin Heidelberg).

Giazotto, Francesco, Tero T. Heikkilä, Arttu Luukanen, Alexander M. Savin, and Jukka P. Pekola (2006), “Opportunities for mesoscopics in thermometry and refrigeration: Physics and applications,” Rev. Mod. Phys. 78, 217–274.

Giazotto, Francesco, and María José Martínez-Pérez (2012), “The josephson heat interferometer,” Nature 492, 401–405.

Goury, Donald, and Rafael Sánchez (2019), “Reversible thermal diode and energy harvester with a superconducting quantum interference single-electron transistor,” Appl. Phys. Lett. 115, 092601.

Gøvenius, J. R., É. Lake, K. Y. Tan, and M. Möttönen (2016), “Detection of zeptojoule microwave pulses using electrothermal feedback in proximity-induced josephson junctions,” Phys. Rev. Lett. 117, 030802.

Callen, Herbert B, and Theodore A. Welton (1951), “Irreversibility and generalized noise,” Phys. Rev. 83, 34–40.

Campisi, Michele, and Rosario Fazio (2016), “The power of a critical heat engine,” Nat. Commun. 7, 11895.
Granger, G. J. P., Eisenstein, and J. L. Reno (2009), “Observation of chiral heat transport in the quantum Hall regime,” Phys. Rev. Lett. 102, 086803.

Guttmann, Glen D, Eshel Ben-Jacob, and David J. Bergman (1998), “Interference effect heat conductance in a josephson junction and its detection in an rf squid,” Phys. Rev. B 57, 2717–2719.

Guttmann, Glen D, Benny Nathanson, Eshel Ben-Jacob, and David J. Bergman (1997), “Phase-dependent thermal transport in josephson junctions,” Phys. Rev. B 55, 3849–3855.

Halbertal, D. J., Cuppens, M. Ben Shalom, L. Embon, N. Shadmi, Y. Anahory, H. R. Naren, J. Sarkar, A. Uri, Y. Ronen, Y. Myasoedov, L. S. Levitov, E. Joselevich, A. K. Geim, and E. Zeldov (2016), “Nanoscale thermal imaging of dissipation in quantum systems,” Nature 539, 407.

Halbertal, Dorri, Moshe Ben Shalom, Aviram Uri, Kousik Bagani, Alexander Y. Meltzer, Ido Marcus, Yuri Myasoedov, John Birkbeck, Leonid S. Levitov, Andre K. Geim, and Eli Zeldov (2017), “Imaging resonant dissipation from individual atomic defects in graphene,” Science 358, 1303.

Hasan, M. Z., and C. L. Kane (2010), “Colloquium: Topological insulators,” Rev. Mod. Phys. 82, 3045–3067.

Heikkilä, T.T., and Francesco Giazotto (2009), “Phase sensitive electron-photon coupling in a superconducting proximity structure,” Phys. Rev. B 79, 094514.

Heikkilä, T. T., and Yuli V. Nazarov (2009), “Statistics of temperature fluctuations in an electron system out of equilibrium,” Phys. Rev. Lett. 102, 136605.

Hewgill, Adam, Gabriele De Chiara, and Alberto Imunde, (2017), “Markovian master equations for qubit design derived from the cooper pair box,” Phys. Rev. Applied 10, 054050.

Hoffmann, Sofia Fahlvik, Claes Thelander, Martin Leijnse, Mikko Möttönen (2019), “Nanobolometer with ultralow noise equivalent power,” Commun. Phys. 2, 124.

Kempf, S. A., Fleischmann, L. Gastaldo, and C. E. Nest (2018), “Physics and applications of metallic magnetic calorimeters,” J. Low Temp. Phys. 193, 363–379.

Kjaergaard, Morten, Mollie E. Schwartz, Jochen Baumüller, Philip Krantz, Joel I.-J., Wang, Simon Gustavsson, and William D. Oliver (2020), “Superconducting qubits: Current state of play,” Annual Review of Condensed Matter Physics 11 (1), 369–395.

Kleedorf, Yaakov, Holger Thierschmann, Hartmut Buhmann, Antoine Georges, Laurens W. Molenkamp, and Yigal Meir (2019), “How to measure the entropy of a mesoscopic system via thermoelectric transport,” Nat. Commun. 10, 5801.

Koch, Jens, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf (2007), “Charge-insensitive qubit design derived from the cooper pair box,” Phys. Rev. A 76, 042319.

Kokkoniemi, R., J.-P. Girard, D. Hazra, A. Laitinen, J. Gove nius, R. E. Lake, I. Sallinen, V. Vesterinen, M. Partanen, J. Y. Tan, K. W. Chan, K. Y. Tan, P. Hakonen, and M. Möttönen (2020), “Bolometer operating at the threshold for circuit quantum electrodynamics,” Nature 586, pages 47–51.

Kokkoniemi, Roope, Joonas Govenius, Visa Vesterinen, Russell E. Lake, András M. Gunyhö, Kuan Y. Tan, Slawomir Simbierowicz, Leif Grönberg, Janne Lehtinen, Mika Prunila, Juha Hassel, Antti Lamminen, Olli-Pentti Saara, and Mikko Möttönen (2019), “Nanobolometer with ultralow noise equivalent power,” Commun. Phys. 2, 124.

Koski, J. V. T., Sagawa, O.-P. Saira, Y. Yoon, A. Kuvveten, P. Solinas, M. Möttönen, T. Ala-Nissila, and J. P. Pekola (2013), “Distribution of entropy production in a single-electron box,” Nat. Phys. 9, 644–648.

Krantz, P. M., Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver (2019), “A quantum engineer’s guide to superconducting qubits,” Appl. Phys. Rev. 6, 021318.

Kubala, Björn, Jürgen König, and Jukka Pekola (2008), “Violation of the wiedemann-franz law in a single-electron transistor,” Phys. Rev. Lett. 100, 066801.

Kubo, Ryogo (1957), “Statistical-mechanical theory of irreversible processes. i. general theory and simple applications to magnetic and conduction problems,” J. Phys. Soc. Jpn.
Künneth, L. and P. Pekola (2012), “Micrometre-scale refrigerators,” Reports on Progress in Physics 75, 444–476.

Levin, A. J., M. A. A. Mabesoone, C. T. Foxon, and M. M. Leivo (1992), “Peltier effect and thermal conductance of a quantum point contact,” Phys. Rev. Lett. 68, 3765–3768.

Makela, P., U. Drechsler, F. Menges, P. Nirmalraj, M. Ranzani, K. Y. Tan, T. Ohki, T. Masuda, and J. F. Noggle (2020), “Graphene-based Josephson junction microwave bolometer,” Nature Nanotechnology 15, 226801.

Moleskamp, I. W., D. Boum, and J. A. van Houten (1992), “Peltier coefficient and thermal conductance of a quantum point contact,” Phys. Rev. Lett. 68, 3765–3768.

Mott, P. M., J. T. Stockbarger, and L. Ankerhold (2018), “Rectification of heat currents across nonlinear quantum chains: a versatile approach beyond weak thermal contact,” New Journal of Physics 20, 113020.

Muhonen, J. T. M. Eiles, and J. M. Martinis (1994), “Electron microrefrigerator based on a normal-insulator-superconductor tunnel junction,” Appl. Phys. Lett. 65, 3123.

Nakamura, Y., M. Pashkin, and J. S. Tsai (1999), “Coherent control of macroscopic quantum states in a single-photon-pair box,” Nature 398, 786–788.

Nam, Y. G., H. P. Lee, and S. W. Hong (2013), “Thermoelectric detection of chiral heat transport in graphene in the quantum hall regime,” Phys. Rev. Lett. 110, 226801.

Nguyen, H. Q., T. Aref, V. J. Aukppula, M. Mchekhe, C. Winkelm, H. Courtois, and J. P. Pekola (2013), “Trapping hot quasi-particles in a high-power superconducting electronic cooler,” New Journal of Physics 15, 085013.

Nikolic, D. M., D. Basko, and M. Belzgin (2020), “Electron cooling by phonons in superconducting proximity structures,” Phys. Rev. B 102, 214514.

Nyquist, H. (1928), “Thermal agitation of electric charge in conductors,” Phys. Rev. 32, 110–113.

Pan, Haining, Jay D. Sau, and S. Das Sarma (2021), “Threeterminal nonlocal conductance in majorana nanowires: Distinguishing topological and trivial in realistic systems with disorder and inhomogeneous potential,” Phys. Rev. B 103, 014513.

Partanen, K. Y., T. M. Eiles, and J. P. Pekola (2012), “Micrometre-scale refrigerators,” Reports on Progress in Physics 75, 444–476.

Mott, P. M., J. T. Stockbarger, and L. Ankerhold (2018), “Rectification of heat currents across nonlinear quantum chains: a versatile approach beyond weak thermal contact,” New Journal of Physics 20, 113020.

Muhonen, J. T. M. Eiles, and J. M. Martinis (1994), “Electron microrefrigerator based on a normal-insulator-superconductor tunnel junction,” Appl. Phys. Lett. 65, 3123.

Nakamura, Y., Yu. A. Pashkin, and J. S. Tsai (1999), “Coherent control of macroscopic quantum states in a single-photon-pair box,” Nature 398, 786–788.

Nam, Seung-Geol, E. H. Hwang, and Hu-Jong Lee (2013), “Heat transport through atomic contacts,” Nature Nanotechnology 12, 335–339.

Munk, A., and J. A. van Houten (1992), “Peltier coefficient and thermal conductance of a quantum point contact,” Phys. Rev. Lett. 68, 3765–3768.

Mott, P. M., J. T. Stockbarger, and L. Ankerhold (2018), “Rectification of heat currents across nonlinear quantum chains: a versatile approach beyond weak thermal contact,” New Journal of Physics 20, 113020.

Muhonen, J. T. M. Eiles, and J. M. Martinis (1994), “Electron microrefrigerator based on a normal-insulator-superconductor tunnel junction,” Appl. Phys. Lett. 65, 3123.

Nakamura, Y., Yu. A. Pashkin, and J. S. Tsai (1999), “Coherent control of macroscopic quantum states in a single-photon-pair box,” Nature 398, 786–788.

Nam, Seung-Geol, E. H. Hwang, and Hu-Jong Lee (2013), “Thermoelectric detection of chiral heat transport in graphene in the quantum hall regime,” Phys. Rev. Lett. 110, 226801.

Nguyen, H. Q., T. Aref, V. J. Aukppula, M. Mchekhe, C. Winkelm, H. Courtois, and J. P. Pekola (2013), “Trapping hot quasi-particles in a high-power superconducting electronic cooler,” New Journal of Physics 15, 085013.

Nikolic, D. M., Denis M. Basko, and Wolfgang Belzig (2020), “Electron cooling by phonons in superconducting proximity structures,” Phys. Rev. B 102, 214514.

Nyquist, H. (1928), “Thermal agitation of electric charge in conductors,” Phys. Rev. 32, 110–113.

Pan, Haining, Jay D. Sau, and S. Das Sarma (2021), “Three-terminal nonlocal conductance in majorana nanowires: Distinguishing topological and trivial in realistic systems with disorder and inhomogeneous potential,” Phys. Rev. B 103, 014513.
Partanen, M. K. Y. Tan, S. Masuda, J. Govenius, R. E. Lake, M. Jenei, L. Grönb erg, J. Hassel, S. Simbiewicz, V. Vesterinen, J. Tuorila, T. Ala-Nissila, and M. Möttönen (2018), “Flux-tunable heat sink for quantum electric circuits,” Sci. Rep. 8, 6325.

Pascal, L. M. A. H. Courtois, and F. W. J. Hekking (2011), “Circuit approach to photonic heat transport,” Phys. Rev. B 83, 125113.

Pekola, J. P., and I. M. Khaymovich (2019), “Thermodynamics in single-electron circuits and superconducting qubits,” Annual Review of Condensed Matter Physics 10 (1), 193–212.

Pekola, Jukka P. (2015), “Towards quantum thermodynamics in electronic circuits,” Nat. Phys. 11, 118–123.

Pekola, Jukka P., and Bayan Karimi (2018), “Quantum noise of electron–phonon heat current,” J. Low Temp. Phys. 191, 373–379.

Pekola, Jukka P., and Bayan Karimi (2020), “Qubit decay in circuit quantum thermodynamics,” arXiv:2010.11122.

Peltonen, J. T. P., Virtanen, M., Meschke, J. V., Koski, T. T., Heikkilä, I., and J. P. Pekola (2010), “Thermal conductance by the inverse proximity effect in a superconductor,” Phys. Rev. Lett. 105, 097004.

Pendry, J. (1983), “Quantum limits to the flow of information and entropy,” Journal of Physics A 16, 2161–2171.

Pothier, H. S., Guéron, Nora, O., Birge, D., Esteve, and M. H. Devoret (1997), “Energy distribution function of quasiparticles in mesoscopic wires,” Phys. Rev. Lett. 79, 3490–3493.

Prance, J. R., C. G. Smith, J. P. Griffiths, S. J. Borley, D. Anderson, G. A. C. Jones, I. Farrer, and D. A. Ritchie (2009), “Electronic refrigeration of a two-dimensional electron gas,” Phys. Rev. Lett. 102, 146602.

Qi, Xiao-Liang, and Shou-Cheng Zhang (2011), “Topological insulators and superconductors,” Rev. Mod. Phys. 83, 1057–1110.

Quan, H. T., Yu-xi Liu, C. P. Sun, and Franco Nori (2007), “Quantum thermodynamic cycles and quantum heat engines,” Phys. Rev. E 76, 031105.

Raja, S. Hamedani, S. Maniscalco, G. S. Paraoanu, J. P. Pekola, and N. Lo Gullo (2021), “Finite-time quantum stirring heat engine,” New Journal of Physics 23 (3), 033034.

Rego, Luis G C, and George Kirchzenow (1998), “Quantized thermal conductance of dielectric quantum wires,” Phys. Rev. Lett. 81, 232–235.

Rego, Luis G C, and George Kirchzenow (1999), “Fractional exclusion statistics and the universal quantum of thermal conductance: A unifying approach,” Phys. Rev. B 59, 13080–13086.

Riera-Campeny, Andreu, Mohammad Mehboudi, Marisa Pons, and Anna Sanpera (2019), “Dynamically induced heat rectification in quantum systems,” Phys. Rev. E 99, 032126.

Rivas, Ángel, and Miguel A. Martin-Delgado (2017), “Topological heat transport and symmetry-protected boson currents,” Sci. Rep. 7, 6350.

Rivas, Ángel, A. Douglas K. Plato, Susana F. Huelga, and Martin B. Plenio (2010), “Markovian master equations: a critical study,” New Journal of Physics 12 (11), 113032.

Ronzani, Alberto, Bayan Karimi, Jorden Senior, Yu-Cheng Chang, Joonas T. Peltonen, ChiDong Chen, and Jukka P. Pekola (2018), “Tunable photonic heat transport in a quantum heat valve,” Nat. Phys. 14, 991.
tunneling,” Phys. Rev. B 96, 094524.
Sivan, U, and Y. Imry (1986), “Multichannel landauer formula for thermoelectric transport with application to thermopower near the mobility edge,” Phys. Rev. B 33, 551-558.
Sivre, E, A. Anthore, F. D. Parmentier, A. Cavanna, U. Gennser, A. Ouerghi, Y. Jin, and F. Pierre (2018), “Heat coulomb blockade of one ballistic channel,” Nat. Phys. 14, 145.
Solfanelli, Andrea, Marco Falsetti, and Michele Campisi (2020), “Nonadiabatic single-qubit quantumotto engine,” Phys. Rev. B 101, 054513.
Sothmann, Björn, Rafael Sánchez, Andrew N. Jordan, and Markus Büttiker (2012), “Rectification of thermal fluctuations in a chaotic cavity heat engine,” Phys. Rev. B 85, 205301.

le Sueur, H, C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre (2010), “Energy relaxation in the integer quantum hall regime,” Phys. Rev. Lett. 105, 056803.
Tan, Kuan Yen, Matti Partanen, Russell E. Lake, Joonas Govenius, Shumpei Masuda, and Mikko Möttönen (2017), “Quantum-circuit refrigerator,” Nat. Commun. 8, 15189.
Tavakoli, Adib, Christophe Blanc, Hossein Ftouni, Kunal J. Lulla, Andrew D. Fefferman, Eddy Collin, and Olivier Bourgeois (2017), “Universality of thermal transport in amorphous nanowires at low temperatures,” Phys. Rev. B 95, 165411.
Tavakoli, Adib, Kunal Lulla, Thierry Crozes, Natalio Mingo, Eddy Collin, and Olivier Bourgeois (2018), “Heat conduction measurements in ballistic 1d phonon waveguides indicate breakdown of the thermal conductance quantization,” Nat. Commun. 9, 4287.
Thomas, George, Yuki Pekola, and Dmitry S. Golubev (2019), “Photonic heat transport across a josephson junction,” Phys. Rev. B 100, 094508.
Tighe, T S, J. M. Worlock, and M. L. Roukes (1997), “Direct thermal conductance measurements on suspended monocristalline nanostructures,” Appl. Phys. Lett. 70, 2687.
Timofeev, Andrey V, Meri Helle, Matthias Meschke, Mikko Möttönen, and Jukka P. Pekola (2009), “Electronic refrigeration at the quantum limit,” Phys. Rev. Lett. 102, 200801.
Tinkham, Michael (2004), Introduction to Superconductivity, 2nd ed. (Dover Publications).
Wang, Haidong, Shiqian Hu, Koji Takahashi, Xing Zhang Hiroshi Takamatsu, and Jie Chen (2017), “Experimental study of thermal rectification in suspended monolayer graphene,” Nat. Commun. 8, 15843.
Wang, L B, O.-P. Saira, D. S. Golubev, and J. P. Pekola (2019), “Crossover between electron-phonon and boundary-resistance limits to thermal relaxation in copper films,” Phys. Rev. Applied 12, 024051.
van Wees, B J, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon (1988), “Quantized conductance of point contacts in a two-dimensional electron gas,” Phys. Rev. Lett. 60, 848–850.
Wells, F C, C. Urbina, and John Clarke (1994), “Hotelectron effects in metals,” Phys. Rev. B 49, 5942–5955.
Wharam, D A, T J Thornton, R Newbury, M Pepper, H Ahmed, J E F Frost, D G Hasko, D C Peacock, D A Ritchie, and G A C Jones (1988), “One-dimensional transport and the quantisation of the ballistic resistance,” Journal of Physics C: Solid State Physics 21 (8), L209–L214.
Wu, Yong-Shi (1994), “Statistical distribution for generalized ideal gas of fractional-statistics particles,” Phys. Rev. Lett. 73, 922–925.
Yung, C S, D. R. Schmidt, and A. N. Cleland (2002), “Thermal conductance and electron-phonon coupling in mechanically suspended nanostructures,” Appl. Phys. Lett. 81, 31.
Zen, Nobuyuki, Tuomas A. Puurutinen, Tero J. Isotalo, Saumyadip Chaudhuri, and Ilari J. Maasilta (2014), “Engineering thermal conductance using a two-dimensional phononic crystal,” Nat Commun 5, 3345.
Zhan, Fei, Sergey Denisov, and Peter Hänggi (2013), “Power spectrum of electronic heat current fluctuations,” Phys. Status Solidi B 250 (11), 2355.
Zhao, Erhai, Tomas Löfwander, and J. A. Sauls (2003), “Phase modulated thermal conductance of josephson weak links,” Phys. Rev. Lett. 91, 077003.