Evaluating Active Learning Heuristics for Sequential Diagnosis

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Abstract

Given a malfunctioning system, sequential diagnosis aims at identifying the root cause of the failure in terms of abnormally behaving system components. As initial system observations usually do not suffice to deterministically pin down just one explanation of the system’s misbehavior, additional system measurements can help to differentiate between possible explanations. The goal is to restrict the space of explanations until there is only one (highly probable) explanation left. To achieve this with a minimal-cost set of measurements, various (active learning) heuristics for selecting the best next measurement have been proposed.

We report preliminary results of extensive ongoing experiments with a set of selection heuristics on real-world diagnosis cases. In particular, we try to answer questions such as “Is some heuristic always superior to all others?” “On which factors does the (relative) performance of the particular heuristics depend?” or “Under which circumstances should I use which heuristic?”

1 Introduction

If the actual behavior of a system does not match its expected behavior, the task of fault localization is to determine the actual diagnosis, i.e. those system components whose faultiness is responsible for the system’s improper functioning. A major challenge regarding real-world systems such as hardware, software or knowledge bases in this context is the often enormous number of possible explanations (diagnoses) for an observed system failure to begin with. In the presence of the initial observations, each of these diagnoses might correspond to the sought actual one. In this work we focus on one-step lookahead heuristics, called query selection measures (QSMs), that try to optimize measurements formulated as binary true-false tests, called queries [5]. For instance, all of the following can be viewed as queries: a test of a system such as hardware or software [3; 7; 8] (given inputs $I$, do we get the expected outputs $O$?), the inspection of system components [10] (are all tested components normal?), a question to an expert [9] (is statement $X$ true in domain $D$?), or a probe [10] (is the value at measurement point $P$ equal to $v$?)

Popular QSMs currently adopted in SD are the expected information gain [13; 11; 14; 5] and the split-in-half heuristic [5; 12]. A range of new QSMs, most of them originally suggested in the field of active learning [6], have recently been introduced to SD in our previous work [15]. Complementary to the analyses in [15] – mostly addressing the efficient computation of optimal queries wrt. QSMs – we want to bring light to the performance of the QSMs wrt. measurement cost throughout an SD session in the present work.

For this purpose, we are currently conducting extensive evaluations where we investigate the particular QSMs under varying conditions regarding

(a) diagnoses probability distributions,
(b) quality (meaningfulness) of the probabilities,
(c) available diagnostic evidence (size of the diagnoses sample) for query computation, and
(d) diagnostic structure (i.e. system size; number and cardinality of diagnoses; reasoning complexity)

using real-world diagnosis problems. The data of the (already finished) experiments shall be exploited to approach i.a. the following questions:

• Do the factors (a) – (d) have an influence on the (relative) performance of the QSMs?
• Which QSM is preferable under which circumstances?
• Is there a (clear) winner among the QSMs?
• What about the difference (variance) between QSM performances under different conditions?

The rest of the paper is organized as follows. Sec. 2 briefly introduces technical basics wrt. SD. Sec. 5 recapits the QSMs used in the experiments. The evaluation setting is described in Sec. 4, and results discussed in Sec. 5. Sec. 6 concludes.

2 Preliminaries

In this section we briefly characterize the basic technical concepts used throughout this work, based on the framework of [5][12] which is slightly more general [16] than Reiter’s theory [17].

Diagnosis Problem Instance (DPI). A system to be diagnosed, consisting of a set of components \( \{c_1, \ldots, c_n\} \), is described by a finite set of logical sentences \( K \cup B \), where \( K \) (retractable knowledge) characterizes the behavior of the system components, and \( B \) (correct background knowledge) comprises any additional available system knowledge and system observations. More precisely, there is a one-to-one relationship between sentences \( ax_i \in K \) and components \( c_i \), where \( ax_i \) describes the nominal behavior of \( c_i \). E.g., if \( c_i \) is an OR-gate in a circuit, then \( ax_i := \text{out}(c_i) = \text{or}([\text{in}1(c_i), \text{in}2(c_i)]) \); \( B \) in this context might subsume sentences stating, e.g., which components are connected by wires, or observed outputs of the circuit. The inclusion of a sentence \( ax_i \) in \( K \) corresponds to the implicit assumption that \( c_i \) is healthy. Evidence about the system behavior is captured by sets of positive \( (P) \) and negative \( (N) \) measurements [17][13][18]. Each measurement is a logical sentence; positive ones \( p \in P \) must be true and negative ones \( n \in N \) must not be true. We call \( \langle K, B, P, N \rangle \) a diagnosis problem instance (DPI).

Diagnoses. Given that the system description along with the positive measurements (under the assumption \( K \) that all components are healthy) is inconsistent, i.e. \( \bar{K} \cup B \cup P \models \bot \), or some negative measurement is entailed, i.e. \( \bar{K} \cup B \cup P \models n \) for some \( n \in N \), some assumption(s) about the healthiness of components, i.e. some sentences in \( K \), must be retracted. We call such a set of sentences \( D \subseteq K \) a diagnosis for the DPI \( \langle K, B, P, N \rangle \) iff \( (K \setminus D) \cup B \cup P \models \bot \) for all \( x \in N \cup \{\bot\} \). We say that \( D \) is a minimal diagnosis for \( dpi \) iff there is no diagnosis \( D' \subset D \) for \( dpi \). A sample \( D \) of minimal diagnoses, the leading diagnoses, is often used as an input for measurement selection algorithms, serving as a basis for measurement quality assessment. We call \( D^* \) the actual diagnosis iff all \( ax \in D^* \) are faulty and all \( ax \in \bar{K} \setminus D^* \) are healthy.

If component fault probabilities are available – formalized as \( p(ax) \) for \( ax_i \in K \) and defining how likely it is that the nominal behavior description \( ax_i \) of \( c_i \) does not apply – probabilities of diagnoses \( D \in \text{D} \) (of being the actual diagnosis) can be computed as

\[
p(D) = \prod_{ax \in D} p(ax) \prod_{ax \in \bar{K} \setminus D} (1 - p(ax))
\]

and updated by means of Bayes’ Rule (see [12] p. 130) after a new measurement is added. Sometimes \( p(ax) \) for \( ax \in \bar{K} \) might not be directly given, e.g., if components have an internal structure in that they constitute an aggregation of several sub-components \( sc_i \). In such a case, \( p(ax) \) can be computed from fault probabilities \( p(sc_i) \) of sub-components as [5]:

\[
p(ax) = 1 - \prod_{sc \in ax} (1 - p(sc_i))
\]

Queries and Q-Partition. Let \( D \) be a set of leading diagnoses for \( dpi = \langle K, B, P, N \rangle \). A query (wrt. \( D \) is a logical sentence \( q \) that rules out at least one diagnosis in \( D \), both if \( q \) is classified as a positive measurement \( (P \leftarrow P \cup \{q\}) \), and if \( q \) is classified as negative measurement \( (N \leftarrow N \cup \{q\}) \). That is, at least one \( D_1 \in D \) is not a diagnosis for \( \langle K, B, P, N \cup \{q\} \rangle \). The classification of a query \( q \) to either \( P \) or \( N \) is accomplished by an oracle, e.g. an engineer performing measurements or a domain expert answering questions. The oracle is a function class : \( Q \rightarrow \{P, N\} \) where \( Q \) is the relevant query space (a set of logical sentences).

An expedient tool towards the verification and goodness estimation of query candidates \( q \) is the notion of a q-partition. Namely, every logical sentence \( q \) partitions a set of leading diagnoses \( D \) into three subsets:

\( D_q^+ \): includes all \( D \in D \) where \( D \) is not a diagnosis for \( \langle K, B, P, N \cup \{q\} \rangle \) (diagnoses predicting that \( q \) is a positive measurement)

\( D_q^- \): includes all \( D \in D \) where \( D \) is not a diagnosis for \( \langle K, B, P \cup \{q\}, N \rangle \) (diagnoses predicting that \( q \) is a negative measurement)

\( D_q^0 = \{D \in D : D \) is a diagnosis for both \( \langle K, B, P \cup \{q\}, N \rangle \) and \( \langle K, B, P, N \cup \{q\} \rangle \) \) (uncommitted diagnoses, no prediction about \( q \))

A 3-partition \( \mathcal{Q} \) of \( D \) is called q-partition (QP) iff there is a query \( q \) for \( D \) such that \( \mathcal{Q} = \{D_q^+, D_q^-, D_q^0\} \). According to the definition of a query, it holds that \( q \) is a query iff both \( D_q^+ \) and \( D_q^- \) are non-empty sets. This fact can be taken advantage of for query verification.

Coupled with diagnoses probabilities, the QP provides useful hints [13] about query quality in that it enables to:

1. test whether \( q \) is a strong query, i.e. one without uncommitted diagnoses \( D_q^0 = \emptyset \),
2. estimate the impact \( q \)'s classification \( \text{class}(q) \) has in terms of diagnoses elimination (potential a-posteriori change of the diagnoses space), and
3. assess the probability of \( q \)’s positive and negative classification (e.g. to compute the uncertainty of \( q \).

As per [13], given a set of leading diagnoses \( D \), we estimate \( p(\text{class}(q) = P) = p(D_q^+) + \frac{1}{2}p(D_q^0) \) and \( p(\text{class}(q) = N) = p(D_q^-) + \frac{1}{2}p(D_q^0) \) where \( p(D_q^0) = \sum_{D \in D_q^0} p(D) \) for \( X \in \{+, -, 0\} \) and \( p(D) \) for \( D \in D \) is the probability of \( D \) normalized over \( D \) (i.e. \( \sum_{D \in D} p(D) = 1 \)).

Sequential Diagnosis. Formally, the (optimal) SD problem can be stated as follows:

**Problem 1 ((Optimal) SD).** Given: A DPI \( \langle K, B, P, N \rangle \). Find: An (optimal-cost) set of measurements \( P' \cup N' \) such that there is only a single minimal diagnosis for \( \langle K, B, P \cup P', N \cup N' \rangle \).

Query Selection Measures (QSMs). The said query properties [11][3] characterized by the QP are essentially what QSMs take into account to quantitatively rate the query quality. Formally, a QSM is a function \( m : Q \rightarrow \mathbb{R} \) that
assigns a value \( m(q) \) to each query \( q \in Q \). All QSMs are heuristics towards Optimal SD (Problem 1). That is, their goal is to minimize the expected cost \( \sum_D p(D) \text{cost}(D) \) of locating the actual diagnosis \( D^* \). At this, cost(D) is usually conceived of as the sum of individual query costs over all queries required to unambiguously isolate \( D \). For the purpose of this paper we assume cost(\( D \)) represents the number of queries to isolate \( D^* \) (all queries assumed equally costly).

### 3 The Evaluated Heuristics

In this section we briefly revisit and explain the QSMs — originally introduced in other works — we use in our experiments. These include the “classical” ones [13, 5] frequently used in SD (cf. Sec. 1) and the newer ones proposed in [19] and [15] and discussed in-depth in [20]. Since we employ a query computation and selection method [21] that guarantees to produce only (the more favorable, cf. [22], Sec. 2.4.1) strong queries, [15] Tab. 3 tells us that we have to deal with seven (non-equivalent) QSMs in this case. We next illustrate the rough idea behind these heuristics, listed in Tab. 1.

**Information Gain** ENT: Chooses a query with the highest expected information gain or, equivalently, with the lowest expected posterior entropy wrt. the diagnoses set \( D \). As derived in [15], ENT(\( q \)) is the better, the closer the probabilities for positive and negative classification of \( q \) to 0.5 (cf. formula in Tab. 1).

**Split-In-Half** SPL: Chooses a query \( q \) whose QT best splits the diagnoses set \( D \) in half, i.e. where both \( |D^+_q| \) and \( |D^-_q| \) are closest to \( \frac{1}{2} |D| \). Intuitively, an optimal \( q \) wrt. SPL guarantees that a half of the (known) diagnoses are eliminated by querying \( q \)’s classification.

**Kullback–Leibler Divergence** KL: Chooses a query with largest average disagreement between query-classification predictions of single diagnoses \( D \in D \) and the consensus (prediction) of all \( D \in D \), based on an information-theoretic measure of the difference between two probability distributions [6]. As demonstrated in [20] Prop. 26, this QSM can be represented in terms of the formula given in Tab. 1.

**Expected Model Change** EMCb: Chooses a query for which the expected number of invalidated diagnoses in \( D \) is maximized.

**Most Probable Singleton** MPS: Chooses a query \( q \) for which the minimum-cardinality set among \( \{ D^+_q, D^-_q \} \) is a singleton \( \{ D \} \) where \( D \) has maximal probability. Intuitively, MPS seeks to eliminate, with a maximal probability, the maximal possible number of \( |D| - 1 \) diagnoses in \( D \).

**Biased Maximal Elimination** BME: Chooses a query with a bias (probability > 0.5) towards one classification \( P \) or \( N \) such that this more likely classification rules out an as high as possible number of diagnoses in \( D \).

**Risk Optimization** RIO': Chooses a query with optimal information gain (ENT-value) among those that, in the worst case, eliminate (at least) \( n \leq \frac{1}{2} |D| \) diagnoses in \( D \). At this, the parameter \( n \) is learned by reinforcement based on the diagnoses elimination performance achieved so far during an SD session [7].

In addition to these informed QSMs, we used a random QSM in our evaluations as a baseline.

**Random** RDF: Samples one element uniformly at random from the considered query space \( Q \).

We next illustrate these different selection principles:

**Example 1** Consider a DPI (cf. [20] Tab. 1+2|) with \( K = \{1, \ldots, 7\} \) (where numbers \( i \) denote sentences \( ax_i \)) which gives rise to the minimal diagnoses \( D \) given by

\[
\{D_1, \ldots, D_9\} = \{[2, 3], [2, 5], [2, 6], [2, 7], [1, 4, 7], [3, 4, 7]\}
\]
diagnoses probabilities

\[
p(D_1), \ldots, p(D_9) = \{0.01, 0.33, 0.14, 0.07, 0.41, 0.04\}
\]

Let, for simplicity, the possible queries be direct tests of \( ax_i \) (\( i = 1, \ldots, 7 \)) where their QPs are shown in Tab. 2. Then, the query choice of the discussed QSMs is as follows:

- **ENT**: \( q_r \), as \( p(D^{-}_q) \) and \( p(D^{+}_q) \) are closest to 0.5.
- **SPL**: \( q_r \), as \( |D^{+}_q| \) and \( |D^{-}_q| \) are equal to \( \frac{|D|}{2} = 3 \).
- **KL**: \( q_3 \), as \( KL(q_3) = 1.48 \) is maximal over all queries.
- **EMCb**: \( q_r \), as the expected number of eliminated diagnoses is 3 (and lower for all other queries).
- **MPS**: \( q_1 \), as \( |D^{+}_q| = |\{D_1\}| = 1 \) and \( p(D_3) = 0.41 > 0.33 \text{(cf. } q_6 \text{)} > 0.14 \text{(cf. } q_9 \text{)} \).
- **BME**: \( q_7 \), as it has a BME-value of 3 (larger than the value of all other queries).
- **RIO'** (with \( n = 2 \)): \( q_2 \) or \( q_4 \), as these are the queries with best ENT-value among all queries (\( q_2, q_3, q_4 \)) which eliminate \( n \) diagnoses in \( D \) in the worst case.

1Note, we consider the slightly modified version RIO’ of the original QSM RIO [19], as suggested in [15].
the amount of information available for query selection
the type of (sub-component) probability distribution
plausibilities of a specific probability assignment),
the probability of the given probabilistic information
be a query with \( p(\text{class}(q) = P) = x \) (cf. Sec. 3).
The plausibility of the given probabilistic information
were substantially larger due to a change of the DPI after each measurement addition; also, diagnoses sizes can only increase during an SD session (17).

Table 3: Dataset used in experiments.

| KB Kj | |Kj| | reasoning complexity | #D/min/max |
|-------|--------|--------|----------------------|------------|
| 1 University (U) | 50 | SOZ\(N^{(2)}\) | 90/34 |
| 2 MiniTambis (M) | 173 | ALCN\(\langle K_j, \emptyset, \emptyset, 0 \rangle\) | 48/35 |
| 3 Transportation (T) | 1300 | ACG\(\langle T \rangle\) | 1782/69 |
| 4 Economy (E) | 1781 | ACH\(\langle T \rangle\) | 864/48 |

\(\text{eq.}^{(2)}\) Description Logic expressivity, cf. [23, p. 525ff.].
\(\text{eq.}^{(3)}\) Sufficiently complex systems (\#D \(\geq 40\)) used in [3].

4 Experimental Settings

The Dataset. Tab. 3 depicts the (part of the overall) dataset investigated in the already finished experiments. The underlying systems are faulty (inconsistent) real-world knowledge bases (KBs). The DPIs \(dpi_j\) we extracted from these KBs \(K_j\) were \(\langle K_j, \emptyset, \emptyset, 0 \rangle\), i.e. the background \(B\), positive (\(P\)) and negative (\(N\)) measurements were (initially) empty.

Concerning the system components, note that each \(ax \in K\) is (interpreted as) describing a complex component, consisting of sub-components, where the latter are given by the logical operators occurring in \(ax\), as done in [3]. This interpretation helped us to obtain a self-consistent ascription of component and diagnoses probabilities by specifying the probabilities of the sub-components (cf. Sec. 2).

The Factors. To test the behavior and robustness of the discussed QSMs under various scenarios, we – in addition to the DPI – varied the following factors in our experiments:

(F1) the type of (sub-component) probability distribution (non-biased, moderately biased, strongly biased);
(F2) 3 different random choices of assigned probabilities for each distribution type (to average out potential peculiarities of a specific probability assignment),
(F3) the plausibility of the probabilities (simulated by plausible, random, implausible oracle behavior),
(F4) the amount of information available for query selection (number of leading diagnoses \(ld \in \{6, 10, 14\}\)), and
(F5) the actual diagnosis \(D^*\) (i.e. the target solution of the SD sessions).

Ad (F1): Let \(SC\) denote the set of all sub-components occurring in some \(ax \in K\) and \(E_{\lambda}(x_i) = \lambda e^{-\lambda x_i}\) the probability density function of the exponential distribution. Three probability distribution types were modeled, by assigning to each sub-component in \(SC\) . . .

- all-equal (EQ): . . . an equal (random) value \(r \in [0, 1]\)
- moderately biased (MOD): . . . the probability \(E_{\lambda}(x_i)\) for a random \(x_i \in [i - \frac{1}{2}, i + \frac{1}{2}]\) where \(i\) is randomly chosen (without replacement) from \(\{1, \ldots, |SC|\}\) and \(\lambda := 0.5\) (cf. [5]).
- strongly biased (STR): . . . the probability \(E_{\lambda}(x_i)\) for a random \(x_i \in [i - \frac{1}{2}, i + \frac{1}{2}]\) where \(i\) is randomly chosen (without replacement) from \(\{1, \ldots, |SC|\}\) and \(\lambda := 1.75\) (cf. [5]).

Intuitively, one can view both MOD and STR to (1) pre-compute a sequence \(p_1 > \cdots > p_{|SC|}\) of values in \((0, 1)\) where, on average, the ratio between each value \(p_i\) and the next smaller one \(p_{i+1}\) is \(p_i/p_{i+1} = e^\lambda\), i.e. \(\approx 1.6\) for MOD and \(\approx 5.8\) for STR, and (2) assign to each \(sc \in SC\) a randomly chosen probability \(p_i\) from this sequence without replacement. Hence, if sorted from large to small, the sub-component probabilities are completely uniform for EQ (no bias), moderately descending for MOD (moderate bias) and steeply descending for STR (strong bias).

For instance, EQ could model a situation where a novel device gets defect or a novice knowledge/software engineer obtains faulty code, and there is no relevant fault information about device parts or code at hand. On the other hand, MOD can be interpreted to simulate a moderate tendency in the fault information, i.e. a non-negligible number of (sub-)components that have a non-negligible fault probability.

Concerning the system components, note that each \(ax \in K\) is (interpreted as) describing a complex component, consisting of sub-components, where the latter are given by the logical operators occurring in \(ax\), as done in [3]. This interpretation helped us to obtain a self-consistent ascription of component and diagnoses probabilities by specifying the probabilities of the sub-components (cf. Sec. 2).
it might well be the case that the actual solution is a superset of an initial minimal diagnosis.

The Tests. For each of the DPIs \(d_{pi_1}, \ldots, d_{pi_4}\) for each of the 8 QSMs explicated in Sec. 3 and for each of the 3 factors of (F1) – (F4) we performed 20 SD sessions. Factor (F5) was implicitly varied in these 20 runs through the randomized oracle behavior (F3), yielding in most cases a different \(D^*\). In case some \(D^*\) happened to occur repeatedly throughout the 20 sessions, we discarded such duplicate runs.

5 Experimental Results

5.1 Representation

The obtained results are shown by Figures 1 – 4 which graph the number of queries required by the tested QSMs until \(D^*\) could be isolated. At this, the green / yellow / red bars depict the situation of a plausibly / randomly / implausibly answering oracle (F3). Each bar represents an average over (up to) 20 sequential diagnosis sessions (F5) and 3 random choices of probabilities (F2). Each figure summarizes the results for one case in Tab. 3, the plots for the U and T cases are more comprehensive, including all combinations of factor levels for (F1), (F2) and (F4), whereas the depictions of M and E are kept shorter due to space restrictions, showing only the \(ld = 10\) case of (F4) for all settings of (F1) and (F2). Along the x-axes of the figures we have the 8 different QSMs, grouped by manifestations of factor (F4) in Figures 1 and 2 and by instantiations of factor (F1) in Figures 3 and 4.

5.2 Observations

Interesting gained insights are discussed next.

Is there a Clear Winner? This question can be answered negatively pretty clearly. For instance, have a look at the MOD, \(ld = 14\) case in Fig. 1. Here we see that MPS performs really good compared to all other QSMs for all oracle types. In fact, it is better than all others in the plausible and random configurations, and loses just narrowly against RND given implausible answers. However, if we draw our attention to, e.g., the EQ case in the same figure, we recognize that MPS comes off significantly worse than other heuristics under a plausible oracle behavior. Similar arguments apply for all other potential winner QSMs. For \(ld = 10\), Tab. 4 which lists the best QSMs in all the different settings we investigated, confirms that there is no single best QSM.

Sensitivity to Fault Information. That there is no QSM which always outmatches all others is not a great surprise, as we evaluate under various types of given probabilistic information \(p(.)\) and the different measures exploit \(p(.)\) to a different extent when selecting a query. As a result, we can observe probability-independent QSMs such as SPL outperform (lose against) strongly probability-reliant ones such as ENT in situations where the fault information is wrongly (reasonably) biased, e.g., see the implausible (plausible) cases for MOD and STR in Figures 1 and 3. So, e.g., SPL can never benefit from high-quality meta information about faults, but cannot effect a significant overhead given low-quality probabilities either. The behavior of, e.g., ENT, is diametrically opposite. To verify this, check the difference between the green and red bars for both SPL and ENT for MOD and STR; for SPL they are hardly different at all, whereas for ENT they diverge rapidly as we raise the bias (EQ \(\rightarrow\) MOD \(\rightarrow\) STR) in the underlying distribution. In contrast to these extreme cases, there is, e.g., RIO' which incorporates both the diagnoses elimination rate and fault probabilities in its calculations. The consequence is a behavior that mostly lies between the performances of SPL and ENT. Based on the data in the figures, which is quite consistent in this regard, one could impose the following qualitative ordering from most to least probability-sensitive on the QSMs:

\[
\text{\{EMCb, BME, ENT, KL, MPS, RIO', RND, SPL\}}
\]

Impact of the DPI / Diagnostic Structure. Trivially, the overall number of diagnoses to discriminate between impacts the average number of queries required. Thus, for MOD (48 diagnoses), U (90), E (864) and T (1782), respectively, the min/avg/max number of queries over all QSMs and sessions is (rounded) 3/7/18, 4/8/19, 6/10/19 and 4/12/29. The difference between M and E, for instance, can be quite well seen by comparing the length of the bars in Figures 2 and 4 which are placed side by side. On the contrary and as one would expect, there are no indications of the system size \(|K|\) (3rd column, Tab. 3) having a remarkable influence on QSM performance (as the system size has generally no bearing on the diagnoses number). The reasoning complexity (4th column, Tab. 3), in contrast, affects the query computation time, which was, over all runs and QSMs, maximally 0.18/0.13/0.18/0.14 sec (per query) for the cases M/U/E/T. The relative behavior of the QSMs under varying DPI (but otherwise same conditions) appears to be quite stable. To see this, compare, e.g., the EQ, the MOD and the STR cases between Figures 1 and 3, or Figures 2 and 4. From the pragmatic point of view, if this consistency of QSM performances irrespective of the particular DPI generalizes (as needs to be verified using a larger dataset), a nice implication thereof would be the possibility of recommending (against) QSMs independently of (the structure of) the problem at hand.

Impact of the Leading Diagnoses. As Figures 1 and 3 indicate quite well, and numbers confirm, there is no significant average difference in the numbers of queries for varying \(ld \in \{6, 10, 14\}\). This is in line with the findings of [24]. What we can realize, though, is an exacerbation of the discrepancy between the plausible (green bars) and implausible (red bars) cases when \(ld\) increases. The random case (yellow bars), on the other hand, is mostly stable. The reason for this intensification of the effect of good or bad bias with larger diagnoses samples is that more extreme decisions might be made in this case. A simple illustration of this is to compare a “risky” [19] query (one that might invalidate very few diagnoses) wrt. a sample of 3 and 100 diagnoses; in the former case, this would be one eliminating either 1 or 2, in the latter one ruling out either 1 or 99 known hypotheses. We see that the former query is similar to a “risk-less” split-in-half choice, while the latter is far off being that conservative. A practical consequence of this is that it might make sense to try generating a higher number of diagnoses per iteration (if feasible in reasonable time) if a probability-based measure, e.g. EMCb or ENT, is used and the trust in the given (biased) fault information is high (e.g. if reliable historical data is available). Verify this by considering EMCb and ENT in the MOD and STR cases for \(ld \in \{6, 14\}\) in Figures 1 and 3. By contrast, when adopt-
ing a probability-insensitive QSM, say SPL, one seems to be mostly better off when relying on a smaller ld. That is, when the meta information is vague, a good option is to rely on a “cautious” [19] measure such as SPL and a small diagnoses sample. Note, the latter is doubly beneficial as it also decreases computation times.

Importance of Using a Suitable QSM. To quantify the importance of QSM choice we compute the degree of criticality of choosing the right QSM in a scenario as the overhead in oracle cost (number of queries) when employing the worst instead of the best QSM in this scenario, see (the caption of) Tab. 3. At this, a scenario is one factor level combination in (F1)×(F3). We learn from Tab. 3 that, even in the least critical cases (green-colored), we might experience a worst-case overhead in oracle effort of at least 30% when opting for the wrong QSM. This overhead is drastically higher in other cases and reaches figures of over 250%. That is, more than triple the effort might be necessary to locate a fault under an inopportune choice of QSM heuristic. However, we emphasize that even a 30% overhead must be considered serious given that usually oracle inquiries are very costly. Hence, appropriate QSM selection is an important issue to be addressed in all scenarios.

As a predictor of the criticality, the scenario (columns in Tab. 4) appears to be a reasonable candidate, as the colors already suggest. In fact, the coefficients of variance, one computed for each column in Tab. 4 are fairly low, ranging from 3% to 26% (except for the last column with 47%). So, the negative effect of a bad QSM choice is similar in equal scenarios, and does not seem to be dependent on the DPI.

Which QSM to use in which Scenario? To approach this question, we have, for all four DPIs, analyzed all the nine settings in (F1)×(F3) regarding the optimal choice of a QSM. The result is presented in Tab. 5. We now discuss various insights from this analysis.

Overall Picture. SPL is never a (nearly) optimal option. This is quite natural because, intuitively, going for no “risk” at all means at the same time excluding the chance to perform extraordinarily well. All other QSMs appear multiple times among those QSMs which are ≤ 3% off the observed optimal number of queries. Tab. 5 (rows 1+2) lists how often each QSM is (among) the best. It shows that MPS is close to the optimum in a half of the cases, significantly more often than all other heuristics. However, blindly deciding for MPS is not a rational way to go. Instead, one must consider the numbers at a more fine-grained level, distinguishing between the quality of the given fault distribution (blocks in Tab. 4), to get a clearer and more informative picture.

The Implausible Cases: Here RND distinctly prevails. It occurs in all but four optimal QSM sets, and is often much better than other measures, e.g., see the STR setting in Fig. 2. At first sight, it might appear counterintuitive that a random selection outweighs all others. One explanation is simply that the randomness prevents RND from getting misled by the (wrong) fault information. Remarkable is, however, that in quasi all cases RND significantly outperforms SPL, which acts independently of the given probabilities as well. The conclusion from this is that, whenever the prior distribution is wrongly biased, introducing randomness into the query selection procedure saves oracle effort.

The Random Cases: These cases are strongly dominated by MPS which occurs in each set of best QSMs per scenario. Therefore, whenever the given fault information does neither manifest a tendency towards nor against the actual diagnosis, MPS is the proper heuristic. Moreover, the benefit of using MPS seems to increase the more leading diagnoses are available for query selection (see Figures 1 and 3). Since MPS, in attempt to invalidate a maximal number of diagnoses, suggests very “risk”y queries (see above), a possible explanation for this is that acting on a larger diagnoses sample allows to guarantee a higher risk than when relying on a smaller sample (cf. discussion above). However, as all Figures 1 – 4 clearly reveal, MPS is definitely the wrong choice in any situation where we have a plausible, but unbiased probability distribution. In such cases it manifests sometimes significantly worse results than other heuristics do. But, as soon as a bias is given, the performance of MPS gets really good.

The Plausible Cases: Throughout these cases we have the highest variation concerning the optimal QSM. Actually, all QSMs except for RND and SPL do appear as winners in certain cases. The distribution of the number of appearances as (or among) the best QSM(s) over all QSMs is displayed by Tab. 5 (rows 3+4). That, e.g., ENT is rather good in these cases and RND is no good choice (see also the Figures 1 – 4) is in agreement with the findings of [5]. However, we realize that BME is (among) the best QSMs more often than ENT. Comparing only these two, we find that BME outdoes ENT 7 times, ENT wins against BME 4 times, and they are equally good once. A reason for the strength of BME could be the fact that it will in most cases achieve only a minor bias towards one query outcome, as the maximization of the diagnoses elimination rate requires an as small as possible number of diagnoses with a probability sum > 0.5. Hence, there is on the one hand a bias increasing the expected diagnoses invalidation rate, and on the other hand a near 50-50 outcome distribution implying a good entropy value of the query. Unsurprisingly, if we sort the QSMs from most to least times being (among) the best based on Tab. 5 (rows 3+4), the resulting order coincides quite well with Eq. (1). In other words, in the plausible scenarios, probability-sensitive heuristics perform best.

Towards new QSMs / Meta-Heuristics. Exploiting the discussed results, one could endeavor to devise new QSMs that are superior to the investigated ones. For instance, in the implausible cases, only RND, MPS and KL occur as best QSMs. Thus, an optimal heuristic for these cases should likely adopt or unify selection principles of these three QSMs. One idea could be, e.g., to sample a few queries using RND and then choose the best one among them using (a weighted combination of) MPS and/or KL. Generally, one could use a meta heuristic that resorts to an appropriately (possibly dynamically re-)weighted sum of the QSM-functions (Tab. 4 2nd column). Also, a QSM selecting queries based on a majority voting of multiple heuristics is thinkable, e.g., in Ex. 4 the query selected by such a QSM would be q7.

6 Conclusions Results of extensive evaluations on both classical and recently suggested query selection measures (QSMs) for sequential diagnosis (SD) are presented. Main findings are: Using an appropriate QSM is essential, as otherwise SD cost overheads of over 250% are possible. The one and only best QSM does not exist (or has not yet been found). Besides the size of the solution space, main factors influencing SD
Table 4: Shows which QSM(s) exhibited best performance in the various scenarios in (F1)×(F3) for all DPs (1st column) in Tab. and the setting ld = 10 of (F4). The QSM(s) with lowest # of queries (per scenario) are underlined. All stated non-underlined QSMs lay within 3% of the best QSM wrt. # of queries. The number below the QSM(s) gives the possible overhead (\(\#_{\text{worst QSM}}(S)/\#_{\text{best QSM}}(S)\times 100\)) in % incurred by using a non-optimal QSM in a scenario S, where \(\#_{\text{q}}(S)\) refers to the # of required queries of QSM X in scenario S, and \(\text{best QSM}(S)/\text{worst QSM}(S)\) denote the best / worst QSM in scenario S. The colors signify criticality of QSM choice based on the overhead, from lowest=green to highest=red.

| ENT | SPL | KL | MPS | BME | RIO | STR |
|-----|-----|----|-----|-----|-----|-----|
| M   | 63  | 176| 144 |     |     |     |
| U   |     |     |     |     |     |     |
| E   | 64  | 93 | 30  |     |     |     |
| T   | 62  | 125| 174 |     |     |     |

Table 5: Number of times each QSM is (among) the best in Tab. 3.

| ENT | SPL | KL | MPS | BME | RIO | STR |
|-----|-----|----|-----|-----|-----|-----|
| M   | 7   | 0  | 18  | 4   | 16  | 1   |
| U   |     |    |     |     |     |     |
| E   | 5   | 1  | 4   | 3   | 4   | 3   |
| T   | 2   | 0  | 4   | 3   | 3   | 3   |

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Figure 1: Experimental results for the University (U) case.

Figure 2: Experimental results for the MiniTambis (M) case with $ld = 10$ (F4).

Figure 3: Experimental results for the Transportation (T) case.

Figure 4: Experimental results for the Economy (E) case with $ld = 10$ (F4).