D-brane in R-R Field Background

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Overview

- System in large field background will have different description with original case.
- For example, when we study the charged particles in large magnetic filed background, we will find the geometry is noncommutative.
- The low energy effective theory of D-brane in constant NS-NS B field background can be described by noncommutative field theory.
- M5 brane in large C field background is described by Nambu-Poisson bracket.
- In this talk, we try to figure out the low energy effective theory of D-brane in large R-R field background.
Noncommutative and NS-NS B field background

Moyal Product and Poisson Bracket

- Noncommutative gauge field theory is governed by Moyal product:

\[ f(x) \ast g(x) = e^{i\frac{1}{2} \theta^{ij} \frac{\partial}{\partial \xi^i} \frac{\partial}{\partial \zeta^j}} f(x + \xi)g(x + \zeta) \bigg|_{\xi = \zeta = 0} \]

\[ = f(x)g(x) + \frac{i}{2} \theta^{ij} \partial_i f(x) \partial_j g(x) + O(\theta^2). \]

- Hence, the geometry is noncommutative \([x^i, x^j] = i \theta^{ij}\).

- Noncommutative effects depend on the reciprocal of B field. \(\theta^{ij} = (\frac{1}{B})^{ij}\).

- In suitable truncation and limit, the effective theory can relate to original effective theory(DBI) with Seiberg-Witten map.
In first $\theta$ order, noncommutative theory describe by Poisson bracket: $\{f, g\}_{p.b.} = \epsilon^{ij} \partial_i f(x) \partial_j g(x)$.

Poisson Bracket is used to generate the 2D area preserving diffeomorphism: $\delta_\alpha \Phi = \{\alpha, \Phi\}$.

Following the same logic, Poisson bracket can generalize to Nambu-Poisson bracket:

$$\{f, g, h\} = \epsilon^{\dot{\mu} \dot{\nu} \dot{\rho}} \partial_{\dot{\mu}} f \partial_{\dot{\nu}} g \partial_{\dot{\rho}} h,$$

to generate the 3D volume preserving diffeomorphism (VPD): $\delta_{\alpha, \beta} \Phi = \{\alpha, \beta, \Phi\}$.

VPD expect to describe the behavior of theory in C-field background as B-field case.

We have practical example of M5-brane in C-field background.
Recently, people apply the Nambu-Poisson bracket into internal 3-manifold of BLG theory, as a realization of Lie-3 algebra. They find the new description of single M5 brane theory in large C field background. It is called by NP M5 theory.

There are many evidences to show the NP M5 theory is M5 in C field background.

For example, the constant term exists in action, the supersymmetry law is nonlinear, the two form gauge field has nonabelian structure, and it reproduces D4 brane in NS-NS B field background, etc.
The NP M5 theory has self-dual 2-form \((b_{\mu\dot{\nu}}, b_{\dot{\mu}\dot{\nu}})\), 5 scalar \((X^i)\) and dimensional reduction 11 dimensional Majorana fermion(\(\Psi\)) with chirality.

In this theory, we use the worldvolume coordinate \((x^\mu, y^{\dot{\mu}}) = (x^0, x^1, x^2, y^1, y^2, y^3)\), here, the \(y^{\dot{\mu}}\) coordinate describe the direction of C-field \(\rightarrow C_{1\dot{2}\dot{3}}\).

The fundamental fields transform under the gauge transformation as

\[
\delta_\Lambda \Phi = g_\kappa^{\dot{\rho}} \partial_{\dot{\rho}} \Phi \quad (\Phi = X^i, \Psi),
\]

\[
\delta_\Lambda b_{\dot{\kappa}\dot{\lambda}} = \partial_{\dot{\kappa}} \Lambda_{\dot{\lambda}} - \partial_{\dot{\lambda}} \Lambda_{\dot{\kappa}} + g_\kappa^{\dot{\rho}} \partial_{\dot{\rho}} b_{\dot{\kappa}\dot{\lambda}},
\]

\[
\delta_\Lambda b_{\lambda\dot{\sigma}} = \partial_\lambda \Lambda_{\dot{\sigma}} - \partial_{\dot{\sigma}} \Lambda_\lambda + g_\kappa^{\dot{\tau}} \partial_{\dot{\tau}} b_{\lambda\dot{\sigma}} + g(\partial_{\dot{\sigma}} \kappa^{\dot{\tau}}) b_{\lambda\dot{\tau}},
\]

where \(\kappa^{\dot{\lambda}} \equiv \epsilon^{\dot{\lambda}\dot{\mu}\dot{\nu}} \partial_{\dot{\mu}} \Lambda_{\dot{\nu}}(x, y)\).

Gauge transformation law of two form fields can be concisely expressed in terms of new variables.
Let’s the two useful variables: \( b^\mu, B_\mu^\mu \)

\[
b^\mu \equiv \frac{1}{2} \epsilon^{\mu\nu\lambda} b_{\nu\lambda},
\]

\[
B_\mu^\mu \equiv \epsilon^{\mu\nu\lambda} \partial_\nu b_{\mu\lambda}
\]

Hence, the gauge transformations of the new variables are:

\[
\delta_\Lambda b^\mu = \kappa^\mu + g \kappa_\nu \partial_\nu b^\mu,
\]

\[
\delta_\Lambda B_\mu^\mu = \partial_\mu \kappa^\mu + g \kappa_\nu \partial_\nu B_\mu^\mu - g (\partial_\nu \kappa_\nu) B_\mu^\nu.
\]

Physics of NP M5 theory can describe by \( (B_\mu^\mu, b^\mu, X^i, \Psi) \), their gauge transformation parameter is only \( \kappa^\nu \).

Gauge transformation can understand as (VPD): \( \delta y^\mu = g \kappa^\mu \), with \( \partial_\mu \kappa^\mu = 0 \).

\( b^\mu \) is the gauge potential for VPD in 3 dimension space which is the direction of C-field background.
$S = S_X + S_\Psi + S_{\text{gauge}}, S_{\text{gauge}} = S_{\mathcal{H}^2} + S_{CS}, \text{where}$

$S_X = \int d^3x d^3y \left[ -\frac{1}{2} (D_\mu X^i)^2 - \frac{1}{2} (\bar{D}_{\dot{\lambda}} X^i)^2 - \frac{1}{2g^2} - \frac{g^4}{12} \{X^{\dot{\mu}}, X^i, X^j\}^2 - \frac{g^4}{12} \{X^i, X^j, X^k\}^2 \right],$

$S_\Psi = \int d^3x d^3y \left[ \frac{i}{2} \bar{\Psi} \Gamma^{\mu} D_\mu \Psi + \frac{i}{2} \bar{\Psi} \Gamma^\dot{\rho} D_{\dot{\rho}} \Psi + \frac{ig^2}{2} \bar{\Psi} \Gamma^{\dot{\mu}} \{X^{\dot{\mu}}, X^i, \Psi\} - \frac{ig^2}{4} \bar{\Psi} \Gamma^{ij} \Gamma_{\dot{1}\dot{2}\dot{3}} \{X^i, X^j, \Psi\} \right],$

$S_{\mathcal{H}^2} = \int d^3x d^3y \left[ -\frac{1}{12} \mathcal{H}^2_{\mu\dot{\nu}\dot{\rho}} - \frac{1}{4} \mathcal{H}^2_{\lambda\dot{\mu}\dot{\nu}} \right],$

$S_{CS} = \int d^3x d^3y \epsilon^{\mu\nu\lambda} \epsilon^{i\dot{\mu}\dot{\nu}\dot{\lambda}} \left[ -\frac{1}{2} \partial_{\dot{\mu}} b_{\mu\dot{\nu}} \partial_{\nu} b_{\lambda\dot{\lambda}} + \frac{g}{6} \partial_{\dot{\mu}} b_{\nu\dot{\nu}} \epsilon^{\dot{\rho}\dot{\sigma}\dot{\tau}} \partial_{\dot{\rho}} b_{\lambda\dot{\rho}} (\partial_{\dot{\lambda}} b_{\mu\dot{\tau}} - \partial_{\dot{\tau}} b_{\mu\dot{\lambda}}) \right].$
Covariant derivatives are defined by \((\Phi = X^i, \Psi)\):

\[
\mathcal{D}_\mu \Phi = \partial_\mu \Phi - gB_\mu \partial_{\dot{\mu}} \Phi, \\
\mathcal{D}_{\dot{\mu}} \Phi = \frac{g^2}{2} \epsilon_{\dot{\mu} \dot{\nu} \dot{\rho}} \{X^\dot{\nu}, X^\dot{\rho}, \Phi\},
\]

where \(X^\mu = \frac{\nu^\mu}{g} + b^\mu\).

Field strengths are defined by:

\[
\mathcal{H}_{\lambda \dot{\mu} \dot{\nu}} = \epsilon_{\dot{\mu} \dot{\nu} \dot{\lambda}} \mathcal{D}_\lambda X^\dot{\lambda} \\
= H_{\lambda \dot{\mu} \dot{\nu}} - g \epsilon^{\dot{\sigma} \dot{\tau} \dot{\rho}} (\partial_\dot{\sigma} b_{\lambda \dot{\tau}}) \partial_{\dot{\rho}} b_{\dot{\mu} \dot{\nu}}, \\
\mathcal{H}_{123} = g^2 \{X^1, X^2, X^3\} - \frac{1}{g} \\
= H_{123} + \frac{g}{2} (\partial_{\dot{\mu}} b^\mu \partial_{\dot{\nu}} b^\nu - \partial_{\dot{\mu}} b^\nu \partial_{\dot{\nu}} b^\mu) + g^2 \{b^1, b^2, b^3\},
\]

where, \(H\) is the normal abelian 2-form field strength.
M5 brane theory can get D4 brane theory after double dimension reduction.

Double Dimension Reduction (DDR) means that we do the dimension reduction on worldvolume and target space at the same time.

We can compact the worldvolume $y^3$ and keep only zero mode, then relate $\{f, g, y^3\} = \epsilon^{\hat{\mu}\hat{\nu}\hat{3}} \partial_{\hat{\mu}} f \partial_{\hat{\nu}} g \equiv \{f, g\}_{p.b.}$

This case is expected to be D4 brane in B-field background, because the background field $B_{12}$ after DDR.

After integrating out the auxiliary field and filed rename, we get $S^{D4}_{boson} =$

\[
\int d^3x d^2y \left[ -\frac{1}{2} (D_a X^i)^2 - \frac{1}{4} (F_{ab})^2 - \frac{g^2}{4} \{X^i, X^j\}^2 - \frac{1}{2g^2} \right]
\]
Another possible D4 brane theory

If we do DDR in another dimension ($x^2$) to keep C field background in D4 theory. We get the new D4 theory:

\[
S[b^{\dot{\mu}}, a_{\dot{\mu}}, B_{\dot{\alpha}}^{\dot{\mu}}] = \int d^2x d^3y \left\{ -\frac{1}{2} \mathcal{H}_{123}^2 - \frac{1}{4} \mathcal{H}_{2\dot{\mu}\dot{\nu}}^2 - \frac{1}{2} (\partial_{\dot{\alpha}} b^{\dot{\mu}} - V_{\dot{\sigma}}^{\dot{\mu}} B_{\dot{\alpha}}^{\dot{\sigma}})^2 
+ \epsilon^{\dot{\alpha}\dot{\beta}} \partial_{\dot{\beta}} a_{\dot{\mu}} B_{\dot{\alpha}}^{\dot{\mu}} + \frac{g}{2} \epsilon^{\dot{\alpha}\dot{\beta}} F_{\dot{\mu}\dot{\nu}} B_{\dot{\alpha}}^{\dot{\mu}} B_{\dot{\beta}}^{\dot{\nu}} \right\},
\]

where

\[a_{\dot{\mu}} \equiv b_{\dot{\mu}2}, \quad B_{\dot{\alpha}}^{\dot{\mu}} \equiv \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}} \partial_{\dot{\nu}} b_{\dot{\alpha}\dot{\rho}}, \quad V_{\dot{\sigma}}^{\dot{\mu}} \equiv (\delta_{\dot{\sigma}}^{\dot{\mu}} + g \partial_{\dot{\sigma}} b^{\dot{\mu}}), \quad \dot{\alpha} = \{0, 1\}.
\]

For simplify, we only show the gauge field terms.
D4 brane in R-R 3-Form Background

- If we want to identify our theory to be D4 brane theory, we need to answer several questions:
- D4 brane should have one-form gauge field $a_A$, where $A = \{\alpha, \dot{\mu}\} = \{0, 1, \dot{1}, \dot{2}, \dot{3}\}$. Where are the gauge fields $a_0, a_1$?
- Even if we find all one form fields, we still need to deal with these additional two-form gauge fields $(b^{\dot{\mu}}, B_{\alpha \dot{\mu}})$ after DDR.
- Gauge transformation of $a_A$ include $U(1)$ and VPD, how to define the field strength or covariant variables?
- After solving these questions, then we can analyze what can we learn from this new D4 brane action.
Dual Transformation

- To find missing one form $a_{\alpha=0,1}$, we use the dual transformation.

- Rewrite the action in another equivalent form:
  \[ S^{(1)}[b^{\dot{\mu}}, a_{\dot{\mu}}, \tilde{B}_{\dot{\alpha}}^{\dot{\mu}}, f_{\alpha\dot{\mu}}, b_{\alpha\dot{\mu}}] = S[b^{\dot{\mu}}, a_{\dot{\mu}}, \tilde{B}_{\dot{\alpha}}^{\dot{\mu}}] - \epsilon^{\alpha\beta} f_{\beta\dot{\mu}}[\tilde{B}_{\dot{\alpha}}^{\dot{\mu}} - \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}} \partial_{\dot{\nu}} b_{\alpha\dot{\rho}}] \] by auxiliary field $f_{\alpha\dot{\mu}}$.

- This new action is same with original action after integrating out $f_{\alpha\dot{\mu}}$.

- On the other hand, we can integrate out $b_{\alpha\dot{\mu}}$ and $\tilde{B}_{\dot{\alpha}}^{\dot{\mu}}$ to get another dual description.

- Firstly, we integrate out $b_{\alpha\dot{\mu}}$, then we find
  \[ \epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}} \partial_{\dot{\mu}} f_{\alpha\dot{\nu}} = 0 \rightarrow f_{\alpha\dot{\mu}} = \partial_{\dot{\mu}} a_{\alpha}. \]

- Gauge transformation of $a_{\alpha}$ is easy to find:
  \[ \delta_{\Lambda} a_{\alpha} = \partial_{\alpha} \lambda + g(\kappa^{\dot{\sigma}} \partial_{\dot{\sigma}} a_{\alpha} + a_{\dot{\sigma}} \partial_{\alpha} \kappa^{\dot{\sigma}}), \] this is same form of $\delta_{\Lambda} a_{\dot{\mu}}$.

- It is original U(1) with additional VPD gauge symmetry.
After integrating out \( b_{\alpha \dot{\mu}} \), it is equivalent to replace action \( S[b^\dot{\mu}, a_{\dot{\mu}}, \tilde{B}_\alpha^\dot{\mu}] \) by this way
\[
\varepsilon^{\alpha \beta} \partial_\beta a_{\dot{\mu}} \tilde{B}_\alpha^\dot{\mu} \rightarrow \varepsilon^{\alpha \beta} F_{\beta \dot{\mu}} \tilde{B}_\alpha^\dot{\mu}, \text{ where } F_{AB} \equiv \partial_A a_B - \partial_B a_A.
\]
To integrate out \( \tilde{B}_\alpha^\dot{\mu} \), we solve the equation of motion of \( \tilde{B}_\alpha^\dot{\mu} \).

After integrating out \( \tilde{B}_\alpha^\dot{\mu} \), we get

\[
S^{(2)}[b^\dot{\mu}, a_{\dot{\mu}}, a_\alpha] = \int d^2 x d^3 y \left\{ -\frac{1}{2} \mathcal{H}_{i23}^2 - \frac{1}{4} (\mathcal{H}_{2\nu \rho})^2 - \frac{1}{2} \partial_\alpha b^\dot{\mu} \partial^\alpha b_{\dot{\mu}}
\right.

\[
+ \frac{1}{2} (\varepsilon^{\alpha \gamma} F_{\gamma \dot{\mu}} + V_{\dot{\mu}} \dot{\sigma} \partial^\alpha b_{\dot{\sigma}})(M^{-1})_{\alpha \beta} \dot{\mu} \dot{\nu} (\varepsilon^{\beta \delta} F_{\delta \dot{\nu}} + V_{\dot{\nu}} \dot{\lambda} \partial^\beta b_{\dot{\lambda}}) \left. \right\},
\]

where, \( \hat{B}_\alpha^\dot{\mu} \equiv (M^{-1})_{\alpha \beta} \dot{\mu} \dot{\nu} (V_{\dot{\nu}} \dot{\sigma} \partial^\beta b_{\dot{\sigma}} + \varepsilon^{\beta \gamma} F_{\gamma \dot{\nu}}) \),
and \( M_{\dot{\mu} \dot{\nu}} \alpha \beta \equiv V_{\dot{\mu} \dot{\rho}} V_{\dot{\nu} \dot{\sigma}} \delta^\alpha \beta - g \varepsilon^{\alpha \beta} F_{\dot{\mu} \dot{\nu}} \).
To find the missing field strength, we start to search covariant variables $\delta_\Lambda \Phi = g \kappa \dot{\mu} \partial_{\dot{\mu}} \Phi$, after some complex calculations, we get:

\[
\begin{align*}
\mathcal{H}_{123} &= \partial_{\dot{\mu}} b^{\dot{\mu}} + \frac{1}{2} g (\partial_{\dot{\nu}} b^{\dot{\nu}} \partial_\rho b^\rho - \partial_{\dot{\nu}} b^{\dot{\rho}} \partial_\rho b^\nu) + g^2 \{ b^{\dot{1}}, b^{\dot{2}}, b^{\dot{3}} \}, \\
\mathcal{F}_{\dot{\mu}\dot{\nu}} &\equiv \mathcal{H}_{2\dot{\mu}\dot{\nu}} = F_{\dot{\mu}\dot{\nu}} + g \left[ \partial_\dot{\sigma} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}} b^{\dot{\sigma}} F_{\dot{\sigma}\dot{\nu}} - \partial_{\dot{\nu}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\sigma}} \right], \\
\mathcal{F}_{\dot{\alpha}\dot{\mu}} &= V^{-1} \dot{\mu} \mathcal{F}_{\dot{\alpha}\dot{\nu}} + g \mathcal{F}_{\dot{\nu}\dot{\sigma}} \hat{B}_{\dot{\alpha}\dot{\sigma}}, \\
\mathcal{F}_{\dot{\alpha}\dot{\beta}} &= F_{\dot{\alpha}\dot{\beta}} + g \left[ - F_{\dot{\alpha}\dot{\mu}} \hat{B}_{\dot{\beta}}^{\dot{\mu}} - F_{\dot{\mu}\dot{\beta}} \hat{B}_{\dot{\alpha}}^{\dot{\mu}} + g F_{\dot{\mu}\dot{\nu}} \hat{B}_{\dot{\alpha}}^{\dot{\mu}} \hat{B}_{\dot{\beta}}^{\dot{\nu}} \right].
\end{align*}
\]

- To construct these covariant variables, we need the combination of $b^{\dot{\mu}}$.
- This situation is not same with original D4 case, because the variable $F_{\dot{A}\dot{B}}$ is not covariant.
- Hence, how to deal with $b^{\dot{\mu}}$ field in final action is an important problem.
After using covariant variables, we can get more simply action form: \( S_{gauge}'[b^\mu, a_A] = \)

\[
\int d^2 x d^3 y \left\{ -\frac{1}{2} \mathcal{H}_{i23} \mathcal{H}^{i23} - \frac{1}{4} \mathcal{F}_{i\dot{\rho}} \mathcal{F}^{i\dot{\rho}} + \frac{1}{2} \mathcal{F}_\beta \mathcal{F}^{\beta\mu} + \frac{1}{2g} \epsilon^{\alpha\beta} \mathcal{F}_{\alpha\beta} \right\}.
\]

- Although, this action does not look like familiar form of original case.
- We can see the interesting structure in the action: \( \frac{1}{2g} \epsilon^{\alpha\beta} \mathcal{F}_{\alpha\beta} \) is Wess-Zumino term for the C-field.
- Order expansion is a good way to more understand the property of action.
- It also can help us to understand the meaning of \( b^\mu \) in the action.
Zeroth Order Analysis

To the 0-th order of \( g \), the action \( S_{gauge}^{(0)}[b^{\dot{\mu}}, a_A] \) can now be expressed as

\[
\int d^2x d^3y \left\{ -\frac{1}{2} H_{i23}^2 - \frac{1}{2} \epsilon^{\alpha\beta} F_{\alpha\beta} H_{i23} - \frac{1}{4} F_{\dot{\mu}\dot{\nu}} F^{\dot{\mu}\dot{\nu}} - \frac{1}{2} F_{\alpha\dot{\mu}} F^\alpha_{\dot{\mu}} \right\}
\]

\[
= \int d^2x d^3y \left\{ -\frac{1}{2} (H_{i23} + F_{01})^2 - \frac{1}{4} F_{AB} F^{AB} \right\},
\]

where \( H_{i23} = \partial_{\dot{\mu}} b^{\dot{\mu}} \) and \( A, B = (\dot{\mu}, \alpha) \).

- Because \( b^{\dot{\mu}} \) has no kinetic term, we can integrate it.
- Finally we get the Maxwell action \( -\frac{1}{4} F_{AB} F^{AB} \).
- To get the Maxwell action, we use the relation \( :H_{i23} = -F_{01} \), the degree of freedom of \( b^{\dot{\mu}} \) is transformed into d.o.f of \( a_\alpha \).
- Hence, the new D4 action only includes the one-form degree of freedom in this sense.
First Order Analysis

The first order corrective action $S^{(1)}_{\text{gauge}}[b^{\dot{\mu}}, a_A]$ is

$$
g \int d^2x d^3y \left\{ \left( \frac{-1}{2} H_{123}^2 + \frac{1}{2} \partial_{\dot{\mu}} b^{\dot{\nu}} \partial_{\dot{\nu}} b^{\dot{\mu}} \right) \left( H_{123} + F_{01} \right) \\
+ H_{123} \left( \frac{-1}{2} F_{\mu\nu} F^{\mu\nu} + \epsilon^{\alpha\beta} F_{\alpha\dot{\mu}} \partial_{\beta} b^{\dot{\mu}} \right) - \frac{1}{2} \epsilon_{\alpha\beta} F_{\mu\nu} F^{\alpha\dot{\mu}} F^{\beta\dot{\nu}} \\
+ F^{\mu\dot{\nu}} F_{\dot{\lambda}\dot{\nu}} \partial_{\dot{\mu}} b^{\dot{\lambda}} + F_{\alpha\dot{\mu}} F^{\alpha\dot{\nu}} \partial_{\dot{\mu}} b^{\dot{\nu}} - F_{\alpha\dot{\mu}} \partial^{\alpha} b_{\dot{\nu}} F^{\mu\dot{\nu}} \right\}.
$$

- We can gauge fix the degree of freedom of $b^{\dot{\mu}}$. Hence, we chose $\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}} \partial_{\dot{\nu}} b_{\dot{\rho}} = 0$ to keep divergence part of $b^{\dot{\mu}}$.
- The divergence part of $b^{\dot{\mu}}$ can be represented by $b^{\dot{\mu}} = \partial^{\dot{\mu}} \partial^{-2} H_{123}$, where the notation $\partial^{-2}$ is the inverse of Laplacian $\partial^2 \equiv \partial_{\dot{\mu}} \partial^{\dot{\mu}}$. 

After these calculations, we get an action as a functional of $H_{123}$ and $a_A$. To integrate out $H_{123}$, we can use $H_{123} = -F_{01} + O(g)$ to find final action $S''_{gauge}[a_A]$ up to first order:

$$\int d^2x d^3 y \left\{ -\frac{1}{4} F_{AB} F^{AB} + g \left[ \frac{1}{2} F_{01} F_{\mu \nu} F_{\mu \nu}^\prime \\
+ \epsilon^{\alpha \beta} F_{01} F_{\alpha \mu} \partial_{\beta} \partial^\mu \partial^{-2} F_{01} - \frac{1}{2} \epsilon_{\alpha \beta} F_{\mu \nu} F_{\alpha \mu} F_{\beta \nu} \\
- F_{\mu \nu} F_{\lambda} \partial_{\mu} \partial_{\lambda} \partial^-2 F_{01} - F_{\alpha \mu} F_{\alpha \nu} \partial_{\mu} \partial_{\nu} \partial^-2 F_{01} \\
+ F_{\alpha \mu} F_{\mu \nu} \partial_{\alpha} \partial_{\nu} \partial^-2 F_{01} \right]\right\},$$

The nonlocal terms appear in this action if we want to describe action only by $a_A$. It should be interesting physics phenomena of D4 brane in C-field background.
Generalize VPD for \((p-1)\)-form

- To generalize D4 to Dp brane, we introduce the \((p-1)\)-bracket which is the generator of VPD in \(p-1\) dimension:

\[
\{f_1, f_2, \cdots, f_{p-1}\} \equiv \epsilon^{\mu_1 \mu_2 \cdots \mu_{p-1}} \partial_{\mu_1} f_1 \partial_{\mu_2} f_2 \cdots \partial_{\mu_{p-1}} f_{p-1}.
\]

- The \((p-1)\) form field strength:

\[
\mathcal{H}_{\dot{\mu}_1 \dot{\mu}_2 \cdots \dot{\mu}_{p-1}} \equiv g^{p-2} \{X^{\dot{\mu}_1}, X^{\dot{\mu}_2}, \cdots, X^{\dot{\mu}_{p-1}}\} - \frac{1}{g} = \partial_{\dot{\mu}} b^\mu + \mathcal{O}(g),
\]

- Field strengths are defined by similar VPD gauge potential: 

\[
b^{\dot{\mu}_1} = \frac{1}{(p-2)!} \epsilon^{\dot{\mu}_1 \dot{\mu}_2 \cdots \dot{\mu}_{p-1}} b_{\dot{\mu}_2 \cdots \dot{\mu}_{p-1}}, \quad X^{\dot{\mu}_i} \equiv \frac{y^{\dot{i}}}{g} + b^{\dot{\mu}_i}.
\]
Gauge Symmetry and Covariant Variables

- To generalize the multi Dp-branes cases, we need to promote gauge fields to matrix, however, it is difficult to modify the gauge transformation of $b^\mu$. One way is that we try to keep this field as U(1) case and promote $a_A$ to be matrix.

- Hence, gauge transformation of $a_A$ is defined by:
  \[ \delta a_A = [D_A, \lambda] + g(\kappa^\mu \partial_\mu a_A + a_\lambda \partial_A \kappa^\mu), \]
  where $D_A = \partial_A + a_A$.

- We choose $\lambda$ is $N \times N$ matrix but $\kappa^\mu$ is $1 \times 1$ parameters.

- Following these definitions, we can also define these covariant variables as same way as before.

- The gauge transformation laws are:
  \[ \delta F_{AB} = [F_{AB}, \lambda - g \kappa^\mu \partial_\mu] \]
Following the results of D4-brane action with covariant variables, we can try to generalize the action for multiple D$p$-branes in R-R $(p - 1)$-form field background:

\[
\int d^2x d^{p-1}y \left\{ -\frac{1}{2} \frac{1}{(p-1)!} \mathcal{H}_{\dot{\mu}_1 \cdots \dot{\mu}_{p-1}} \mathcal{H}^{\dot{\mu}_1 \cdots \dot{\mu}_{p-1}} + \frac{1}{2g} \epsilon^{\alpha \beta} \mathcal{F}^U_{\alpha \beta} \\
- \frac{1}{4} \mathcal{F}^U_{\dot{\nu} \dot{\rho}} \mathcal{F}^U_{\nu \rho} + \frac{1}{2} \mathcal{F}^U_{\beta \dot{\mu}} \mathcal{F}^\beta_{\dot{\mu}} U(1) - \frac{1}{4} \text{tr} \left( \mathcal{F}^{SU(N)}_{AB} \mathcal{F}^{AB}_{SU(N)} \right) \right\}.
\]

From this action, we can see the $\mathcal{F}^{SU(N)}_{AB}$ is still dual with the VPD field strength $\mathcal{H}$. It is easy to read from the 0-th order action:

\[
\int d^2x d^3y \left\{ -\frac{1}{2} (H_{23 \cdots p} + F^{U(1)}_{01})^2 - \frac{1}{4} F^{U(1)}_{AB} F^{AB}_{U(1)} \\
- \frac{1}{4} \text{tr} \left( F^{SU(N)}_{AB} F^{AB}_{SU(N)} \right) \right\}.
\]
Summary

- In this lecture, we construct D4 in R-R three form background from NP M5 theory.
- We solve how to introduce another missing d.o.f of one form, and explain how to deal with the addition two form fields.
- Theory include VPD gauge symmetry.
- The order expansion analysis can reproduce Maxwell action in zeroth order, and we find some nonlocal effects in first order calculation.
- We also generalize the formalism to Dp brane and multi Dp branes in R-R (p-1) form background.
Furthermore Questions

- In this talk, we try to understand the low energy effective action of D brane in R-R field background.
- This formalism has still unknown parts about nonabelian extension of VPD, supersymmetry transformation of dual field, more simple action of high order g expansion with matter fields, etc.
- To focus on supersymmetry’s effect, BPS states of the new theory may be different with previous cases and may be non trivial.
- How to relate our formalism with the research of D brane in R-R field background(non-anticommutative theory) is interesting.
- The story of AdS/CFT are also the application of R-R field background research. What can we relate it to our case?
Thank For Your Attention