HTL effective action of topologically massive gluons in 3+1 dimensions

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May 7, 2019

Abstract

We construct an effective action for “soft” gluons by integrating out hard thermal modes of topologically massive vector bosons at one loop order. The loop carrying hard gluons (momentum $\sim T$) are known as hard thermal loop (HTL). The gluons are massive in the non-Abelian topologically massive model (TMM) due to a quadratic coupling $B \wedge F$ where a 2-form field $B$ is coupled quadratically with the field strength $F$ of Yang-Mills (YM) field. Due to the presence of infrared cut-off in the model, the color diffusion constant and conductivity can be analyzed in perturbative regime.

PACS numbers: 11.15.-q; 12.38.-t; 12.38.Mh

Keywords: Topologically massive $B \wedge F$ theory; QCD, hard thermal loop; quark gluon plasma; thermal field theory

1 Introduction

Gauge theory plays a crucial role in the standard model of particle physics for the description of fundamental interactions in nature\cite{1,2,3}. Standard model consists of electroweak and strong sectors but excludes gravitational interaction. In the electroweak sector, global $SU(2) \times U(1)$ symmetry is spontaneously broken to $U_{em}(1)$ symmetry. The latter symmetry is responsible for the electromagnetic interaction. The mediators of the weak force, $W^{\pm}$ and $Z$ bosons, become massive via Higgs mechanism which is accompanied by the spontaneous symmetry breaking. The remaining degrees of freedom i.e. the Higgs particle has been discovered in the large hadron collider (LHC)\cite{4,5}.

The strong sector in the standard model has special characteristics which makes it to be significantly different from the electroweak sector. The elementary particles, quarks and gluons, which interact strongly, are not found free isolated in any experiment till date. They are confined within hadrons. The explanation of their confinement is still an open question in quantum
chromodynamics (QCD). Beside this, one of the other characteristics of the strong interaction is the asymptotic freedom which implies the validity of perturbative analysis of quantum SU(3) gauge theory at high energy limit 1. The asymptotic freedom also helps us to realize a deconfined state of matter in QCD known as quark-gluon plasma (QGP) at high density and temperature 13.

QGP is a state of matter the properties of which are governed by quarks and gluons. It can be created in the relativistic heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) 16 and Large Hadron Collider (LHC) 17. In heavy ion collisions a deconfined system of quarks and gluons is formed which may equilibrate thermally in a short time ∼1 fm/c after the collision. This deconfined thermal state of quarks and gluons is called QGP. The hot and dense QGP, created in such collisions expands fast due to high internal pressure. The expansion dynamics is governed by relativistic hydrodynamics 18. The cooling of the QGP due to expansion reverts the system to hadronic phase at the transition temperature, $T_c \sim 160$ MeV. The hadronic system further expands hydrodynamically as long as the interaction rate between hadrons is larger than the expansion rate resulting in reduction of temperature. The temperature ($T_F$) at which the system becomes too dilute to interact, the hadrons fly free from the reaction zone to the detector with momentum value frozen at $T_F$. The transport coefficients of QGP like shear viscosity, bulk viscosity etc. can be used to characterize the QGP by studying its hydrodynamic evolution. In our present endeavor, we are interested in the perturbative aspects of QGP where gluon degrees of freedom dominates. Such a state can be created by colliding nuclei at LHC and higer RHIC energies. The study of QGP is important for understanding the QCD transition in the early universe after a few microsecond of the big bang. This is also important for astrophysics too because compact astrophysical objects like neutron star may contain QGP in its core.

QGP state also provides an opportunity to investigate the non-trivial topological configurations of gauge fields. The non-trivial topological configuration localized in $(3 + 1)$-dimensions of space-time is known to be instanton. This configuration shows that the Yang-Mills theory has infinite vacua. These vacua are designated by a parameter $\theta$. Instanton carries a great importance in producing the chiral magnetic effect in QGP when massless flavoured quarks are considered in it. This effect is a combination of electromagnetic and chromomagnetic phenomena 19,22. This chiral imbalance can help us to investigate the violation of parity $P$ and $CP$ symmetries in QCD 2 (strong $CP$ problem).

QGP is considered often with massless gluons 3 which have non-zero mass at finite temperature i.e. electric and magnetic masses. These masses were shown to be gauge invariant 23,24. The masses carry a great importance in the analysis of QGP 25,26. Electric mass provides the Debye screening whereas the non-zero magnetic mass implies the validity of the application of perturbation technique in the analysis of QGP. Debye mass also plays a pivotal role in the suppression of the effect of large instanton in QGP. On the other hand, it is shown that magnetic mass is absent in massless non-Abelian gauge theory in every loop correction 26 and hence it is treated in non-perturbative regime at the length scale $\sim 1/(g^2 T)$ which is much below the scale of mean free path $\sim 1/(g^4 T)$; here $g(<1)$ is the gauge coupling of the trilinear and quartic interactions among the gluons. It can be shown that the dynamical screening can prevent the infrared singularities in QED plasma, but this won’t work for QCD plasma because the massless gluon fields carry color charges.

In this paper, we are going to construct an effective Lagrangian density by integrating out the hard modes of topologically massive gluons (whose four momentum $\sim T$). The effective action will be useful for the computation of the color conductivity and color diffusion constant 27,30.

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1If the energy of centre of momentum frame of collision be $E$, then here the high energy limit implies $E \gg m$ for any mass $m$ present in the interaction.

2Here $\mathcal{C}$ designates charge conjugation operation.

3Here “massless gauge field” implies the gauge field having “bare mass” mass at zero temperature.
The infrared cut-off can be put in the $(2+1)$-dimensional Chern-Simons theory \[31,32\]. Though this model is super-renormalizable \[33,34\], but the Chern-Simons term violates $\mathcal{CP}$ symmetry. Before going into the construction, we also would like to know the importance of the massiveness of gluon even at $T = 0$ QCD.

At $T = 0$, the massless non-Abelian gauge field has a problem in description of local interaction in quantum field theory (QFT) \[35,36\]. Since the Fock space of the non-Abelian gauge field has positive indefinite metric, the interactions among the massless gluon violates cluster decomposition principle \[35,37,38\], which is not desirable in a Lorentz invariant model. On the other hand, massive gluon can explain the color singlet asymptotic states in physical Hilbert space in QCD \[37,39\] when color symmetry is not broken spontaneously. But requiring mass in the pure non-Abelian gauge theory causes many other problems. Due to non-zero mass, the gauge bosons acquire longitudinal mode whose high energy behaviour violates unitarity in the scattering processes at high energy limit. This can be seen in any massive non-Abelian gauge theory, for example, electroweak sector \[40–42\]. In this sector, these are the Higgs mediated processes which recover the unitarity of scattering matrix. But color symmetry is believed to be exact in the strong sector. Hence, the Higgs mechanism and Proca theory cannot be taken into consideration. We can also think of the non-Abelian Stückelberg model, but it was found to be non-renormalizable \[43,47\]. There is a Curci-Ferrari model which contains Proca-massive gauge field, but it was found to be not unitary in spite of being renormalizable \[48,49\]. There was also attempt for the dynamical generating mass of Yang-Mills field \[50\]. In this sector, these are the Higgs mediated processes which recover the unitarity of scattering matrix. But color symmetry is believed to be exact in the strong sector. Hence, the Higgs mechanism and Proca theory cannot be taken into consideration. We can also think of the non-Abelian Stückelberg model, but it was found to be non-renormalizable \[43,47\]. There is a Curci-Ferrari model which contains Proca-massive gauge field, but it was found to be not unitary in spite of being renormalizable \[48,49\]. There was also attempt for the dynamical generating mass of Yang-Mills field \[50\]. In this sector, these are the Higgs mediated processes which recover the unitarity of scattering matrix. But color symmetry is believed to be exact in the strong sector. Hence, the Higgs mechanism and Proca theory cannot be taken into consideration. We can also think of the non-Abelian Stückelberg model, but it was found to be non-renormalizable \[43,47\]. There is a Curci-Ferrari model which contains Proca-massive gauge field, but it was found to be not unitary in spite of being renormalizable \[48,49\]. There was also attempt for the dynamical generating mass of Yang-Mills field \[50\].

The (3 + 1)-dimensional TMM contains a topological term: 
\[mB \wedge F = \frac{m}{4} \varepsilon^{\mu\nu\rho\lambda} F_{\mu\nu}B_{\rho\lambda}\] . Here $B$ is a two-form field and $F$ is the field strength of the one-form gauge field $A$. This is topological field theory of Schwarz-type \[53,54\]. This term is a key ingredient for the field theories which are to be independent of metric. For example, in the formulation of quantum gravity, this term is used for the action \[55\]. In QFT, considering the kinetic terms of $A$ and $B$ fields, a model can be constructed where observables are related to the local excitations and topological invariants in TMM \[54,56\]. We see, upon introducing the topological term with the kinetic terms, the coupling constant $m$ becomes the pole of gauge field when we integrate out $B$ field from the TMM. Spin representation of the $B$ field is different from the $A$ field. Massless $B$ field has one degree of freedom whereas massive $B$ field behaves like massive one-form field in Lorentz representation \[57\]. Hence, integrating out either $A$ or $B$ in the TMM, we can get an effective field theory of massive vector bosons. We also see that the TMM is invariant under the vector gauge symmetry of $B$ field beside vector gauge symmetry of Yang-Mills field. The presence of infrared cut-off in the non-Abelian generalization of the TMM validate the perturbative analysis in the massive quantum gauge theory.

The contents of our present endeavor are constructed as follows. In section 2, we discuss very briefly about the TMM and its non-Abelian generalization. We provide, in our section 3, the various vertex rules and propagators of the gauge and ghost fields in the TMM and show how the coupling constant “$m$” becomes the pole of the complete propagator of Yang-Mills field. The propagators and the vertex rules are important for the calculation of the contributions
from the one loop diagrams. In section 4, we obtain the thermal mass for one-form massive
\text{gauge field at one loop order. In this section, we also integrate out the hard thermal modes
of one-form, two-form and ghost fields at one loop order and obtain an effective action for soft
massive gluons. Finally, in section 5, we discuss the implications of the results of our present
work.}

\section{(3+1)-dimensional (4D) topologically massive model}

The Lagrangian density of the model is given by \cite{58,60}
\begin{equation}
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{a} F^{a\mu\nu} + \frac{1}{12} \tilde{H}_{\mu\nu\lambda}^{a} \tilde{H}^{a\mu\nu\lambda} + \frac{m}{4} \varepsilon^{\mu\nu\rho\lambda} B_{\mu\nu}^{a} F_{\rho\lambda}^{a},
\end{equation}
where the field strengths corresponding the Yang-Mills field $A_{\mu}^{a}$ and the two-form gauge field $B_{\mu\nu}^{a}$ are respectively given by,
\begin{equation}
F_{\mu\nu}^{a} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g f^{abc} A_{\mu}^{b} A_{\nu}^{c},
\end{equation}
and
\begin{equation}
\tilde{H}_{\mu\nu\lambda}^{a} = (D_{\mu} B_{\nu\lambda})^{a} - g f^{abc} F_{[\mu\nu]}^{b} C_{\lambda}^{c} = \partial_{\mu} B_{\nu\lambda}^{a} + g f^{abc} A_{\mu}^{b} B_{\nu\lambda}^{c} - g f_{\mu\nu} B_{\lambda}^{a} - \tilde{f}_{\mu\nu}^{abc} C_{\lambda}^{c},
\end{equation}
where the fields $A_{\mu}^{a}$, $B_{\mu\nu}^{a}$ and $C_{\mu}^{a}$ are in the adjoint representation of the $SU(N)$ gauge group. Unlike the Abelian model, we have an extra vector field $C_{\mu}^{a}$ in this model. It is an auxiliary field \cite{61} which assures the invariance of the Lagrangian density under the following transformations
\begin{equation}
A_{\mu}^{a} \rightarrow A_{\mu}^{a}, \quad B_{\mu\nu}^{a} \rightarrow B_{\mu\nu}^{a} + (D_{\mu} \theta_{\nu})^{a}, \quad C_{\mu}^{a} \rightarrow C_{\mu}^{a} + \theta_{\mu}^{a},
\end{equation}
where $\theta_{\mu}^{a}$ is a vector field in adjoint representation of $SU(N)$. Including the ghost fields and Nakanishi-Lautrup fields corresponding to the $A_{\mu}^{a}$ and $B_{\mu\nu}^{a}$ fields, we get the full action \cite{60} as
\begin{equation}
S = S_{0} + \int d^{4}x \left[ h^{a} f^{a} + \frac{\xi}{2} h^{a} h^{a} + h_{\mu}^{a}(f^{a\mu} + \partial^{a} h^{a}) + \frac{\beta}{2} (D_{\mu} \beta_{a}^{\mu} - g f^{abc} \omega_{a}^{\mu} \omega_{b}^{\mu}) + \frac{\eta}{2} h_{\mu}^{a} h^{a\mu} - \partial_{\mu} \tilde{\omega}_{a}^{\mu} + \tilde{\omega}_{a}^{\mu} \partial_{\mu} D^{\mu} \omega_{a} + \tilde{\omega}_{a}^{\mu} \{ g f^{abc} \partial_{\mu} (B^{b\mu\nu} \omega_{c}^{\nu}) + \partial_{\nu} (D^{[\mu} \omega_{\nu]}^{\mu}) + \partial_{\nu} (g f^{abc} F^{b\mu\nu} \theta_{c}) \} \right],
\end{equation}
where $S_{0}(= \int d^{4}x \mathcal{L})$ is the action corresponding to the Lagrangian density \cite{1} and $f^{a} = (\partial^{a} A_{\mu})^{a}$, $f_{\mu}^{a} = (\partial^{a} B_{\mu\nu})^{a}$. The parameters $\xi$, $\eta$ and $\zeta$ are the dimensionless gauge-fixing parameters. The auxiliary fields $h^{a}$ and $h_{\mu}^{a}$ play the role of Nakanishi-Lautrup type fields. Here $(\tilde{\omega}_{a}^{a}) \omega_{a}^{a}$ and $(\tilde{\omega}_{a}^{a}) \omega_{a}^{a}$ [with ghost number $(-1) + 1$] are the fermionic scalar and vector (anti-)ghost fields for the vector gauge field $A_{\mu}^{a}$ and tensor field $B_{\mu\nu}^{a}$, respectively. The bosonic scalar fields $(\tilde{\beta}_{a}^{a}) \beta_{a}^{a}$ [with ghost number $(-2) + 2$] are the (anti-)ghost fields for the fermionic vector (anti-)ghost fields and $n^{a}$ is the bosonic scalar ghost field (with ghost number zero). These scalar ghost fields are required for the stage-one reducibility of the 2-form field. Furthermore, $\alpha^{a}$ and $\tilde{\alpha}^{a}$ are the Grassmann valued auxiliary fields (having ghost number $+1$ and $-1$). This model contains massive non-Abelian gauge field and it was shown to be BRST invariant \cite{62,64}. In \cite{63,64}, it is seen that the model is also invariant under the anti-BRST symmetry transformations. The $CP$ symmetry is not violated in this model.
### 3 Vertex rules and propagators of fields

The propagators for the $A$ and $B$ fields are found from the Abelian $B \wedge F$ model. The Lagrangian density for the Abelian model is

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \frac{m}{4} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda},
$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the Abelian gauge field $A_\mu$, $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$ is the field strength for the tensor field $B_{\mu\nu}$ and $m$ is the coupling constant of the topological term which has dimension of mass (in natural units $\hbar = c = 1$). The Lagrangian density is invariant under the two independent gauge transformations, namely:

$$
A_\mu \to A_\mu + \partial_\mu \Lambda, \quad B_{\mu\nu} \to B_{\mu\nu},
$$

$$
A_\mu \to A_\mu, \quad B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu} A_{\nu]},
$$

where $\Lambda(x)$ and $\Lambda_\mu(x)$ are scalar and vector gauge transformation parameters which vanish at infinity. The Euler-Lagrange equations of motion derived from the above Lagrangian density are as follows

$$
\partial_\mu F^{\mu\nu} = -\frac{m}{6} \epsilon^{\nu\mu\lambda\kappa} H_{\mu\lambda\kappa},
$$

$$
\partial_\mu H^{\mu\nu\lambda} = +\frac{m}{2} \epsilon^{\nu\lambda\kappa\rho} F_{\kappa\rho}.
$$

It is interesting to note that one can decouple the above equations for the gauge fields in the following fashion

$$
(\Box + m^2) F_{\mu\nu} = 0, \quad (\Box + m^2) H_{\mu\nu\lambda} = 0,
$$

which shows the well-known Klein-Gordon equations for the massive fields $A_\mu$ and $B_{\mu\nu}$. Thus, $A_\mu$ field has three degrees of freedom as same as the massive $B_{\mu\nu}$ field.

We will consider the elastic scatterings among topologically massive bosons in our present endeavor. This analysis requires the propagators of $A_\mu$ and $B_{\mu\nu}$ fields. We get the propagators of these fields when we introduce the gauge-fixing terms in the Lagrangian density for Abelian fields in Eq. (6). Then we have

$$
\mathcal{L}_{gf} = -\frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \frac{1}{2\eta} (\partial_\mu B^{\mu\nu})^2,
$$

where $\xi$ and $\eta$ are the gauge-fixing parameters. The topological term is also a quadratic term containing both $A_\mu$ and $B_{\mu\nu}$ fields. If we want to calculate the propagator of the fields, we should take all the quadratic terms in the Lagrangian density. For this purpose, first we exclude $B \wedge F$ coupling from our consideration

and get the propagators of $A_\mu$ and $B_{\rho\lambda}$ fields:

$$
i \Delta_{\mu\nu} = -\frac{i}{k^2} \left( g^{\mu\nu} - (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right),
$$

$$
i \Delta_{\mu\nu,\rho\lambda} = \frac{i}{k^2} \left( g_{\mu[\rho} g_{\lambda]\nu} - (1 - \eta) \frac{k_\mu k_\lambda k_{\rho}\nu - k_\nu k_{\lambda} k_{\rho}\mu}{k^2} \right).
$$

Then we consider the quadratic derivative coupling term in the $B \wedge F$ term as an interaction and obtain the vertex rule for the $B - A$ coupling, for the vertex, shown in the Fig. [1]

$$
i V_{\mu\nu,\lambda} = -m \epsilon_{\mu\nu\lambda\rho} k^\rho.
$$
Hence we get the complete propagator of the field, $A_{\mu}$ by taking an infinite number of insertions of the $B - A$ vertex and the $B$ propagator, given in Eq. (13). This process is shown in the Fig. 2 and the sum of diagrams can be written as the infinite sum

$$iD_{\mu\nu} = i\Delta_{\mu\nu} + i\Delta_{\mu\nu}' \frac{1}{2} iV_{\sigma\rho,\mu'} i\Delta_{\sigma\rho,\sigma'} \frac{1}{2} iV_{\sigma',\nu'} i\Delta_{\nu'} + \cdots$$

$$= -i \left[ g_{\mu\nu} - (1 - \xi) \frac{k_{\mu} k_{\nu}}{k^2} - \xi m^2 \frac{k_{\mu} k_{\nu}}{k^4(k^2 - m^2)} \right],$$

which is the propagator of a massive vector boson of mass $m$. The factors of $\frac{1}{2}$ compensate for double-counting due to the anti-symmetrization of the indices. Similarly for the tensor field we have

$$iD_{\mu\nu,\rho\lambda} = \left[ g_{\mu(\rho} g_{\lambda)\nu + (1 - \eta) \frac{k_{\mu} k_{(\lambda} g_{\rho)\nu}}{k^2} + \eta m^2 \frac{k_{(\mu} k_{(\lambda} g_{\rho)\nu)}{k^4(k^2 - m^2)} \right].$$

The kinetic term of the YM fields also in Eq. (5) provides a derivative trilinear and a quartic couplings,

$$\mathcal{L}_{int} = \frac{1}{4} g f^{bca} A^{\mu b} A^{\nu c} \left( \partial_{\mu} A_{\nu} - 2g f^{dea} A_{\mu}^{d} A_{\nu}^{e} \right).$$

The vertex rules corresponding to the couplings are

$$V_{\mu\nu\lambda}^{abc} = -g f^{abc} \left[ (q - r)_{\mu} g_{\nu\lambda} + (r - p)_{\nu} g_{\lambda\mu} + (p - q)_{\lambda} g_{\mu\nu} \right],$$

$$V_{\mu\nu\lambda\rho}^{abcd} = -ig^2 \left[ f^{abe} f^{cde} g_{\mu(\lambda} g_{\rho)\nu} + f^{ace} f^{bde} g_{\mu(\nu} g_{\rho)\lambda} + f^{ade} f^{bce} g_{\mu(\nu} g_{\rho)\lambda} \right],$$

where $f$’s are the structure constants of $SU(N)$ group, which are fully antisymmetric in their indices.

In the derivation of the vertex rules for trilinear coupling the momentum of the legs shown in Fig. 3a. The topological term also provides a trilinear coupling $ABB$ whose vertex rule is

$$iV_{\mu,\nu,\lambda}^{abc} = -ig m f^{bca} \epsilon_{\mu\nu\lambda},$$
We have also noticed in the expression of full action of the model in Eq. (5) that we require the propagators of vector ghost fields, $\omega^\mu$, $\bar{\omega}^\mu$, the ghost fields of the vector ghost fields, $\beta$, $\bar{\beta}$, and the ghost fields corresponding to the one form gauge field, $\omega$, $\bar{\omega}$. We can get the vertex rules for trilinear and quartic couplings $ABB$ and $AABB$ as

$$iV_{\mu,\lambda,\rho,\sigma}^{abc} = g f^{abc} \left[ (p - q)_\mu g_{\lambda[s} g_{\rho]r} + p_{[\sigma} g_{|r|\lambda} g_{\rho]|\mu} - q_{[\lambda} g_{\rho]|\sigma} g_{\mu]|\rho} \right], \tag{21}$$

$$iV_{\mu,\nu,\lambda,\rho}^{abcd} = ig^2 \left[ f^{ace} f^{bde} \left( g_{\mu\nu} g_{\lambda[s} g_{\rho]r} + g_{\mu|\sigma} g_{\rho]|\lambda} g_{\rho]|\nu} \right) \right. \left. + f^{ade} f^{bce} \left( g_{\mu\nu} g_{\lambda[s} g_{\rho]r} + g_{\mu|\lambda} g_{\rho]|\sigma} g_{\mu]|\rho} \right) \right], \tag{22}$$

where the vertices are shown in Fig. 4. We have the propagator of vector ghost field from the Lagrangian density

$$L = \partial_\mu \bar{\omega}^\mu \left( \partial^\mu \omega^{\nu a} - \partial_\nu \omega^{\mu a} \right) + \frac{1}{\xi} (\partial_\mu \omega^{\mu a}) \left( \partial_\nu \bar{\omega}^{\nu a} \right). \tag{23}$$

at the gauge $\bar{\xi} = 1$ which is obtained integrating out $\alpha$ and $\bar{\alpha}$ from the action in Eq. (5).
Apart from the usual trilinear coupling among Fadeev-Popov ghost and YM fields, we can also see the action contains another trilinear coupling among vector ghosts and YM fields

\[ L_{\text{vec-gh}} = -gf^{abc} \partial_\nu \bar{\omega}_\mu A^b_\mu \]  

(24)

The vertex rule corresponding to the coupling in Eq. (24) is

\[ iV^{abc}_{\mu\nu\lambda} = -gf^{abc} (p_\nu g_{\mu\lambda} - p_\lambda g_{\mu\nu}) \]  

(25)

In derivation of the above rule we again take the all four momentums are incoming towards the vertex as shown in Fig. 5(a). There is also a trilinear coupling among YM and ghost of the vector ghost fields. The trilinear vertex is shown in Fig. 5b. The couplings are given by the Lagrangian density as

\[ L_{\text{A\bar{\beta}A\bar{\beta}}} = -gf^{bca} A^b_\mu \partial_\mu \bar{\beta}^a \bar{\beta}^c \]  

(26)

Since the coupling in Eq. (26) also contains derivative coupling, the vertex rule corresponding to the coupling contains momentum. This trilinear coupling is same as the trilinear coupling among YM field and its FP ghosts which provides the vertex rule

\[ iV^\mu_{abc} = gf_{abc} p^\mu. \]  

(27)

4 One loop correction

Using the vertex-rules and the complete propagators we can calculate the one loop correction of the soft modes of massive gluons. Since vector bosons are massive, we should include chemical potential \( \mu \) into our consideration when Matsubara summation will be considered.

We consider a generic form of the loop amplitude for our calculation:

\[ \Pi^{ab}_{\mu\nu} = \frac{g^2 N_C}{n} \delta^{ab} \sum_{\mathbf{p}} \eta_{\mu\nu} \left( a_1 p^2 + a_2 k^2 + a_3 m^2 \right) + \frac{a_4 k_\mu k_\nu + a_5 p_\mu p_\nu + a_6 p_\mu k_\mu + a_7 p_\nu k_\nu + a_8 p_\nu k_\mu + a_9 p_\mu k_\nu}{(p^2 - m^2)} \left( (k - p)^2 - m^2 \right), \]  

(28)
where \( \mathcal{G} = \sum_{p_{on}} T \int \frac{d^d p}{(2\pi)^d}. \) The external legs carries soft momenta \( \sim gT \). We take the Matsubara sum over \( p_{on} \) for the loop momentum \( p \). The integrating out the hard spatial loop momenta, \( P \geq T \), where \(|p| = P\). To carry out further we expand the energy of the external legs \( E_1 \) and \( E_2 \) as:

\[
E_1 = \sqrt{P^2 + m^2} \approx P + \frac{m^2}{2P} + \cdots ,
\]

\[
E_2 = \sqrt{(P - K)^2 + m^2} \approx P - k \cdot v + \frac{m^2}{2P},
\]

where \( v_i = \frac{p_i}{P} \). We will also use the result

\[
\mathcal{G} = T \sum_{p_{on}} \frac{1}{(p_{0n}^2 + E_1^2) \{ (k - p)_{0n}^2 - m^2 \}}
\]

\[
= \frac{1}{4E_1E_2} \left[ \frac{1}{ik_{on} - E_1 - E_2} ( - n_B(E_1) - n_B(E_2) - 1) \\
+ \frac{1}{ik_{on} + E_2 - E_1} ( n_B(E_1) - n_B(E_2) ) \\
+ \frac{1}{ik_{on} + E_1 - E_2} ( n_B(E_2) - n_B(E_1) ) \\
+ \frac{1}{ik_{on} + E_2 + E_1} ( 1 + n_B(E_1) + n_B(E_2) ) \right].
\]

We will neglect the quadratic term \( O(p^2) \) from the numerator in our calculation because they are the soft modes. Using Eq. (29) and Eq. (30), the above expression of \( \mathcal{G} \) in Eq. (31) becomes approximately

\[
\hat{\mathcal{G}} \approx \left[ - \frac{1}{2P} \{ - 2n_B(P) \} + k \cdot v \frac{n_B'(P)}{ik_{on} - k \cdot v} - k \cdot v \frac{n_B'(P)}{ik_{on} + k \cdot v} + \frac{1}{2P} \{ 2n_B(P) \} \right].
\]

In the generic expression of integrand in Eq. (28), the denominator appears due to identical propagators of bosons in loop. Hence the term \( (p_\mu k_\nu + p_\nu k_\mu) \) will be simplified after renaming the variable \( p \to k - p \) one half of this term as

\[
p_\mu k_\nu + p_\nu k_\mu \to \frac{1}{2} [ p_\mu k_\nu + p_\nu k_\mu + (k - p)_\mu k_\nu + (k - p)_\nu k_\mu ] = k_\mu k_\nu
\]

Now we rearrange the terms in the numerator of the integrand in Eq. (28) and neglecting the term \( \sim O(k^2) \), the spatial part reads as,

\[
N_{ij}^{ab} = \frac{g^2 N_C \delta^{ab}}{n} \left[ \eta_{ij} \left( a_1 p^2 + a_3 m^2 \right) + a_5 P_i P_j \right]
\]

\[
= \frac{g^2 N_C \delta^{ab}}{n} \left[ \eta_{ij} \left( a_1 (p^2 - m^2) + a_1 m^2 + a_3 m^2 \right) + a_5 P_i P_j \right]
\]

\[
= \frac{g^2 N_C \delta^{ab}}{n} \left[ \eta_{ij} \left( a_1 (p^2 - m^2) + (a_1 + a_3) m^2 \right) + a_5 P_i P_j \right].
\]

From the above expression, it is clearly seen that the presence of the term \( m^2 \eta_{ij} \) will provide magnetic mass of gluons. In constructing the effective field theory, we neglect the quadratic...
term of $O(k^2)$ from the above expression and substitute $G$ in Eq. (32) to obtain the spatial part of the $\Pi_{\mu\nu}$ after changing the variable $v \rightarrow -v$,

$$\Pi_{ij}^{ab} \approx g^2 N_c \frac{\delta^{ab}}{n} \int \frac{d^4p}{p} \left[ a_1 \frac{n_B(P)}{P} \delta_{ij} + \left( A m^2 \delta_{ij} + a_5 P_1 P_2 \right) \left\{ \frac{n_B(P)}{2P^3} + k \cdot v \frac{n'_B(P)}{2P^2 (ik_0 - k \cdot v)} \right\} \right],$$

(35)

where $A = (a_1 + a_3)$, $\int_p = c(d) \int \frac{d^{d-1}p}{p}$ and $c(d) = \frac{2}{(4\pi)^{d/2} \Gamma(d/2)}$. The angular integration goes over directions of $v_i \equiv \frac{p_i}{P}$ and normalized to unity

$$\int d\Omega_v = 1$$

(36)

and from the rotational invariance, we can write

$$\int d\Omega_v \ v_i \ v_j = \frac{1}{d} \delta_{ij}.$$

(37)

Thus we are going to construct the effective field theory in the energy scale, $E = \sqrt{P^2 + m^2}$, where $m \ll E \lesssim T$. Using the identity

$$\int \frac{d^dP}{P} n'_B(P) = -(d-1) \int \frac{1}{P} n_B(P),$$

(38)

and Eq. (37), we get

$$\Pi_{ij}^{ab} = \delta^{ab} g^2 N_c \frac{\delta^{ab}}{n} \int \frac{d^4p}{p} \left\{ \left( a_1 + \frac{a_5}{d} - \frac{a_5}{d} (d-1) \right) \delta_{ij} - (d-1) a_5 \int d\Omega_v \frac{-iv_i v_j k_0}{ik_0 - k \cdot v} \right\} + A \delta_{ij} \int \frac{k \cdot v}{P^2 (ik_0 - k \cdot v)}.$$  

(39)

The last term of the above integrand can be rearranged as

$$\int \frac{k \cdot v}{P^2 (ik_0 - k \cdot v)} \frac{m^2 n'_B(P)}{P^2 (ik_0 - k \cdot v)} = m^2 I (-1 + k^0 L(K)).$$

(40)

where

$$L(K) = \int d\Omega_v \frac{1}{ik_0 - k \cdot v},$$

(41)

and

$$I = -\beta \int_0^\infty \frac{e^{\beta P}}{(e^{\beta P} - 1)^2} dP,$$

(42)

which can be integrated-out to get:

$$I = \frac{1}{e^{\beta P} - 1} \bigg|_{P=0}.$$  

(43)

The divergence appearing in $I$ here is the artifact of the approximations made in Eq. (29) and Eq. (30).
We can now re-express Eq. (39), using Eq. (43) in $d = 3$, as
\[
\Pi_{ij} = \frac{g^2 N_C}{n} \left[ \delta_{ij} + \frac{T^2}{12} \left( B \frac{T^2}{12} + A m^2 I \left( -1 + k^0 L(K) \right) \right) \right] \delta_{ij} + C \frac{T^2}{12} \left( P_{ij}^T \Pi_T(K) + P_{ij}^E \Pi_E(k) \right),
\]
(44)
where in $d$ spatial dimensions, $B = \left( a_1 + \frac{a_5}{d} - \frac{a_5(d-1)}{d} \right)$ and $C = -(d-1)a_5$. The factor $\frac{T^2}{12}$ appears from the integration $\int_P \frac{1}{P^2 n_B(P)}$, where $\int_P \equiv V(d) \int_0^\infty p^{d-1} dp$; $V(d) = \frac{2}{(4\pi)^2 \Gamma \left( \frac{d}{2} \right)}$.

We have suppressed gauge indices in the above calculations. The coefficients of the projection operators are found as
\[
P_{\mu\nu}^T(k) = \delta_{\mu\nu} \delta_{ij} P_{ij}^T(k),
\]
(45)
\[
P_{\mu\nu}^E = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - P_{\mu\nu}^T(k),
\]
(46)
where $P_{ij}^T(k) = \delta_{ij} - k_i k_j/k^2$ and in the 3-dimension
\[
\Pi_T(K) = \frac{1}{2} \left[ \frac{(ik_{0n})^2}{K^2} + \frac{ik_{0n}}{2K} \right] \left( 1 - \frac{(ik_{0n})^2}{K^2} \right) \ln \frac{ik_{0n} + K}{ik_{0n} - K},
\]
(47)
\[
\Pi_E(K) = \left[ 1 - \frac{(ik_{0n})^2}{K^2} \right] \left[ 1 - \frac{ik_{0n}}{2K} \right] \ln \frac{ik_{0n} + K}{ik_{0n} - K}.
\]
(48)
Hence now we can write the effective Lagrangian density as:
\[
\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \int \frac{\bar{m}^2(K)}{2} A^\mu(K) A_\mu(-K)
\]
\[
+ \frac{m_E^2}{2} \int d\Omega_v \left( \frac{1}{V \cdot D} \mathcal{V}^a F_{a\alpha} \right) \left( \frac{1}{V \cdot D} \mathcal{V}^{\beta} F_{\alpha\beta} \right),
\]
(49)
where $\bar{m}^2 = \frac{g^2 N_C}{n} \left[ B \frac{T^2}{12} + 2A m^2 I \left( -1 + k^0 L(K) \right) \right]$, $m_E^2 \approx \frac{g^2 C N_C T^2}{n} \frac{2}{12}$ and $\mathcal{V}^a \equiv (1, v)$. We have established a generic form of the Debye mass and observed how the "bare" mass of gluon contributes in the construction of effective action. Now we are going to consider relevant contributions to the effective Lagrangian considering various loop diagrams from the topologically massive model. The generic form of the loop integration is
\[
\Pi_{ij} = m^2 \sum_K \frac{(A \delta_{ij} k_i^2 + B k_i k_j)}{k^2 ((p - k)^2 - m^2) (k^2 - m^2)},
\]
(50)
which could be written as
\[
\Pi_{ij} = m^2 \int_K \left( A \delta_{ij} k_i^2 + B k_i k_j \right) \mathcal{G}(E_1, E_2, E_3, k),
\]
(51)
where
\[
\mathcal{G}(E_1, E_2, E_3, k) = \sum_n \frac{1}{(k_{0n} + E_3^2) \left( (p - k_{0n})^2 + E_3^2 \right) (k_{0n}^2 + E_3^2)}
\]
\[
= \frac{1}{E_3^2 - E_1^2} \left[ \mathcal{G}(E_1, E_2, k) - \mathcal{G}(E_2, E_3, k) \right],
\]
(52)
11
where

\[ G(E_1, E_2, k) = T \sum_{p_{0n}} \frac{1}{(p_{0n}^2 + E_1^2) (r_{0n}^2 + E_2^2)} \]

\[ = \frac{1}{4E_1E_2} \left[ \frac{1}{ik_{on} - E_1 - E_2} (-n_B(E_1) - n_B(E_2) - 1) + \frac{1}{ik_{on} + E_2 - E_1} (n_B(E_1) - n_B(E_2)) + \frac{1}{ik_{on} + E_1 - E_2} (n_B(E_2) - n_B(E_1)) + \frac{1}{ik_{on} + E_2 + E_1} (1 + n_B(E_1) + n_B(E_2)) \right]. \]  

(53)

Thus, we get from Eqs. (52) and (53)

\[ G = \frac{1}{m^2} \left[ \frac{1}{4E_1E_2} \left\{ \frac{1}{ik_{on} - E_1 - E_2} (-n_B(E_1) - n_B(E_2) - 1) + \frac{1}{ik_{on} + E_2 - E_1} (n_B(E_1) - n_B(E_2)) + \frac{1}{ik_{on} + E_1 - E_2} (n_B(E_2) - n_B(E_1)) + \frac{1}{ik_{on} + E_2 + E_1} (1 + n_B(E_1) + n_B(E_2)) \right\} \right] - \frac{1}{4E_3E_2} \left[ \frac{1}{ik_{on} - E_2 - E_3} (-n_B(E_2) - n_B(E_3 - \mu) - 1) + \frac{1}{ik_{on} + E_2 - E_3} (n_B(E_3) - n_B(E_2 - \mu)) + \frac{1}{ik_{on} + E_3 - E_2} (n_B(E_2) - n_B(E_1)) + \frac{1}{ik_{on} + E_2 + E_3} (1 + n_B(E_3) + n_B(E_2)) \right] \]  

(54)

where \( \frac{1}{m^2} \) originates from \( E_3^2 - E_1^2 = k_{0n}^2 + m^2 - k_{0n}^2 = m^2 \). Taking the HTL approximation, we can write the above expression

\[ G \approx \frac{n_B(E_1) - n_B(E_3)}{4K^2m^2} \left[ \frac{1}{ip_n - \mathbf{p} \cdot \mathbf{v}} - \frac{1}{ip_n + \mathbf{p} \cdot \mathbf{v}} \right]. \]  

(55)

The whole tedious task will be simplified with the observation that the diagrams in Fig. 5a are to be neglected in HTL approximation. Since in this approximation \( m \ll K \), then \( n_B(E_1) \approx n_B(E_3) \) at leading order. Hence, the contribution in the quantum corrections from the diagrams in Fig. 6a and Fig. 6b, \( \mathcal{D}_{12}^{(E)} \approx 0 \). Similar conclusion can be drawn for Fig. 6c which contains four propagators. Therefore, the \( \mathcal{D}_{12}^{(E)} \) have the Matsubara sum as

\[ G(E_1, E_2, E_3, E_4, k) = \sum_{\mathbf{k}} \frac{1}{(k_{0n}^2 + E_1^2) ((p - k)^2_{0n} + E_2^2) (k_{0n}^2 + E_3^2) ((p - k)^2_{0n} + E_4^2) k}. \]  

(56)

which, after some algebraic manipulation and HTL approximation, becomes

\[ G(E_1, E_2, E_3, E_4, k) \approx \frac{(n_B(E_3) - n_B(E_1))}{4K^2m^4} \left[ \frac{1}{ip_n - \mathbf{p} \cdot \mathbf{v}} - \frac{1}{ip_n + \mathbf{p} \cdot \mathbf{v}} \right] + \left( \frac{1}{ip_n + \mathbf{p} \cdot \mathbf{v}} - \frac{1}{ip_n - \mathbf{p} \cdot \mathbf{v}} \right) k, \]  

(57)
Figure 6: The loops which do not contribute in HTL approximation.

which shows $\Pi_{ij}^i$ does not contribute too.

Only relevant loop diagrams, constructed from $A$ and $B$ fields are shown in the Fig. 7. The rest of the diagrams are from the ghost sectors, where loops are constructed by FP ghost of YM field $\omega$, $\bar{\omega}$; vector ghost $\omega^\mu$, $\bar{\omega}^\mu$ and ghost of the vector ghost $\beta$ and $\bar{\beta}$.

We can easily reach at conclusion from Fig. 7a that the term $m^2/K^4$ in the propagator of massive YM field does not carry relevance in the approximation which we consider, where $K$ is hard momentum. Instead of propagator behaving as $\sim 1/k^2$, now we have to consider
On the other hand, vertex rule of trilinear coupling among the massive gluon field are same as for the massless YM field. This helps the calculation to be easier. We have also noticed that loop amplitude from Fig. 7a in the HTL approximation at the leading order is same as found in the massless YM case. It is because of the structure of the propagator of massive YM field. On the other hand the tri-linear vertex rule among the massive YM field and its massive ghosts is same as found in the massless YM theory. This similarity implies that the thermal loop amplitude for Fig. 9a is same as found in massless YM theory. The contribution from Figs. 7a and 7b are

\[
P_{\mu \nu}^{\text{7a}} = -\frac{g^2 N_c}{2} \sum_{\vec{k}} \frac{-\delta_{\mu \nu}}{(k^2 - m^2^)} \left[ 5p^2 - 2p \cdot k + 2k^2 \right] + (d + 4)p_{\mu}p_{\nu} - (4d - 2)k_{\mu}k_{\nu},
\]

\[
P_{\mu \nu}^{\text{7b}} = -\frac{1}{2} g^2 N_c \delta_{\mu \nu} \sum_{\vec{k}} \frac{2d}{(k^2 - m^2^)}. \tag{58}
\]

Neglecting terms \( \sim \mathcal{O}(p^2) \) in the integrand of numerator of Eq. (58) we get the spatial part

\[
P_{ij}^{\text{7b}} = -\frac{g^2 N_c}{2} \sum_{\vec{k}} \frac{-\delta_{ij}}{(k^2 - m^2^)} \left[ -2p \cdot k + 2k^2 \right] - (4d - 2)k_i k_j
\]

\[
= g^2 N_c \sum_{\vec{k}} \frac{\delta_{ij}}{(k^2 - m^2^)} \left[ -p \cdot k + k^2 \right] + (2d - 1)k_i k_j \tag{59}
\]

Hence, we can see after comparing Eq. (59) with Eq. (44), we have \( a_1 = 1, a_3 = 0, a_5 = 5 \) and \( n = 1 \) for \( d = 3 \). Next, we consider the diagram in 7b, which provide the spatial part of loop amplitude as

\[
P_{ij}^{\text{7b}} = -g^2 N_c \delta_{ij} \sum_{\vec{k}} \frac{d}{(k^2 - m^2^)} \approx -g^2 d N_c \delta_{ij} \int_{K} \frac{1}{2K} (1 + n_B(K)). \tag{60}
\]

Then we consider the diagram in Fig. 7c which is constructed by taking the ABB trilinear coupling. The loop amplitude corresponding to the diagram is

\[
P_{\mu \nu}^{\text{7c}} = \frac{g^2 N_c}{2} \sum_{\vec{k}} \frac{2(d - 2)(k^2 - p \cdot k)\delta_{\mu \nu} + (2d^2 - 3d + 4)(2k_{\mu}k_{\nu} - (p_{\mu}k_{\nu} + k_{\mu}p_{\nu}))}{(k^2 - m^2^)} \tag{61}
\]

which in comparison with Eq. (44) provides \( a_1 = 2(d - 2), a_3 = 0, a_5 = 0 \) and \( n = 2 \). Loop amplitude from the loop diagram Fig. 7d is

\[
P_{ij}^{\text{7d}} = \frac{1}{2} g^2 N_c \delta_{ij} (d^2 - 3d + 2) \sum_{\vec{k}} \frac{1}{(k^2 - m^2^)}
\]

\[
\approx \frac{1}{2} g^2 (d^2 - 3d + 2) N_c \delta_{ij} \int_{K} \frac{1}{2K} (1 + n_B(K)). \tag{62}
\]

There is only one relevant loop diagram involving A and B fields left to be considered, which is shown in Fig. 8. The loop amplitude corresponding to the diagram neglecting the term \( \sim m^2 / k^4 \) from the propagators of the field is given by,

\[
P_{\mu \nu}^{\text{8}} = g^2 m^2 N_c \sum_{\vec{k}} \frac{(2d - 2)\delta_{\mu \nu}}{(k^2 - m^2^)} \tag{63}
\]

\[
\Rightarrow N_{ij}^{\text{8}} = 2g^2 m^2 N_c (d - 1) \sum_{\vec{k}} \frac{\delta_{ij}}{(k^2 - m^2^)} \left[ (p - k)^2 - m^2 \right]. \tag{64}
\]
which gives in $a_3 = 2$, $a_1 = a_5 = 0$ and $n = 1$. Now we consider the ghost sector, which also contributes in the construction of HTL effective Lagrangian. The loops are formed by the FP ghost of YM field in Fig. 9a, vector ghost in Fig. 9b and ghost of the vector ghost in Fig. 9c. The loop amplitude from Fig. 9a is found as

$$\Pi_{\mu\nu}^9 = -g^2 N_c \sum_{\mathbf{k}} \int \frac{\left(\mathbf{k} - \mathbf{p}\right)_\mu \mathbf{k}_\nu}{k^2 (p - k)^2} \mathbf{k}^2 \left(p - k\right)^2 = -g^2 N_c \sum_{\mathbf{k}} \int \frac{2k_\mu k_\nu - p_\mu p_\nu}{k^2 (p - k)^2},$$  \hspace{1cm} (65)$$

where we have used the mechanism given in Eq. (33) in the last step because the loop integration contains the product of two identical propagators. Comparing with Eq. (44), we see that
\(a_1 = a_3 = 0, a_5 = 1\) and \(n = 1\). Loop amplitude from the Fig. 9b is
\[
\Pi_{\mu\nu}^{9b} = -g^2 N_c \sum_k \frac{(d-2)p_\mu k_\nu - k_\mu k_\nu (d-1) + p_\nu k_\mu}{k^2(p-k)^2},
\]
\[
= -\frac{g^2 N_c}{2} \sum_k \frac{(d-1)p_\mu p_\nu - 2k_\mu k_\nu (d-1)}{k^2(p-k)^2},
\]
\[
\Rightarrow \Pi_{ij} \rightarrow g^2 N_c (d-1) \sum_k \frac{k_i k_j}{k^2(p-k)^2}.
\] (66)

In the last step of the above integration, we have used the mechanism shown in Eq. (33). In comparison with Eq. (44), we see this integration contribute in HTL effective Lagrangian with \(a_1 = a_3 = 0\) and \(a_5 = (d-1)\). The contribution from Fig. 9a is same as from Fig. 9c, because of the similarity in the vertex rules of the trilinear couplings \(A\tilde{\omega}\) and \(A\tilde{\beta}\) are same. Hence adding up the contribution from the ghost sectors we get
\[
\Pi^{9a + 9b + 9c}_{ij} = g^2 d N_c \sum_k \frac{k_i k_j}{k^2(p-k)^2} [2 - (d-1)].
\] (67)

Then we can compare the generic expression in Eq. (44), we get \(a_5 = 1\). Hence, we get the effective action from HTL approximation for topologically massive bosons in \(d = 3\) dimensions
\[
L_{eff} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \int K \frac{\tilde{m}^2(K)}{2} A^\mu(K) A_{\mu}(-K)
+ \frac{m_E^2}{2} \int d\Omega_\nu \left( \frac{1}{V \cdot D} V^{\alpha\nu} F_{\alpha\mu}^a \right) \left( \frac{1}{V \cdot D} V^{\beta\mu} F_{a\beta}^a \right),
\] (68)

where
\[
\tilde{m}^2 = 2g^2 N_c \left[ \frac{5 T^2}{12} + 2m^2 I (-1 + k^0 L(K)) \right],
\] (69)
\[
m_E^2 \approx g^2 N_c \frac{5 T^2}{3}.
\] (70)

5 Discussion

We have constructed the HTL effective action for topologically massive gauge theory. It is clearly shown how the Debye mass modified due to presence of bare mass of massive gauge bosons. The bare mass puts a infrared cut-off in QCD at finite temperature. The infrared cut off plays role in the perturbative analysis for transport coefficients, which are merely the response functions. These were believed to be in the non-perturbation regime in QCD at finite temperature. We have not considered any Fermionic interaction with the massive YM gauge bosons. The Fermions will have the same trilinear coupling with massive YM field as it has in massless YM theory. As a consequence they provide same the contribution in the HTL approximated Lagrangian. There is no conserved local current constructed from a trilinear coupling among fermions and \(B_{\mu\nu}\) fields. We have not calculated the transport coefficients from the HTL action for topologically massive gauge bosons when they are coupled with fermions. It will be very interesting to find the response functions from a matter coupled TMM at finite temperature.
We also see the other prospects of the TMM at finite temperature. In the massless YM theory at finite temperature, the phase transition can be explained making it associated with spontaneous broken symmetry. Massless YM field theory is invariant under $SU(N)/Z(N)$ group, where $Z(N)$ is centre of $SU(N)$ group. This symmetry is believed to be spontaneously broken at phase transition which is described by vacuum expectation value of Polyakov loop $L = \frac{1}{N} \text{tr} \mathcal{P} \left( \exp i \oint_C A_0(x, t) \right)$, where $\mathcal{P}$ represents path ordering of the exponent and trace is taken to make $L$, $SU(N)$-invariant. Taking the quarks to be static it can be shown that the implication of phase transition implies the spontaneous breaking of $SU(N)$ symmetry. But in TMM, there are massive gauge fields, which are in the adjoint representation of $SU(N)$ group. In the model, we have more general Polyakov loop $L^{\text{gen}} \sim \text{tr} \mathcal{P} \left( \exp i \oint C A_0(x, t)dx^0 \oint_S B_0 dx^0 dx^3 \right)$, where the closed path $C$ is loop and surface $S$ is taken in space-time. The physical significance and the behavior of $L^{\text{gen}}$ near the critical temperature can be investigated thoroughly. It will be also interesting to consider thermal Bethe Salpeter equations from TMM. This may give the dynamics of the bound state massive gauge bosons at finite temperature.

Appendix A: Calculation of amplitude of Fig.7c

![Diagram](attachment:image.png)

Figure 10: Loop diagram contains the $ABB$ coupling.

The amplitude of the Fig. [10] is given by

$$
\mathcal{M} = \frac{1}{16} \left[ (2k - p)_{\mu} g_{\rho\sigma} g_{\beta\mu} + k_{[\rho} g_{\sigma]_{\mu} + k_{[\alpha} g_{\beta]\mu} - \tilde{k}_{[\alpha} g_{\beta]\mu} \right] \left( g^{[\alpha'} g^{\beta'][\rho]} k^{[\mu} k^{[\sigma} + m^2 k^{[\mu} k^{[\sigma} - k^2 - m^2 \right) 
\times \left( g^{\rho'][\rho'} g^{\sigma'][\sigma} \left( \tilde{k}^2 - m^2 \right) + \frac{m^2 k_{[\rho} k^{[\rho'} g^{\sigma][\sigma]} k_{\sigma']} \left( 2k - p \right)_{\nu} g_{\alpha'} g_{\sigma'} + k_{[\mu} g_{\sigma'][\nu] \right) - \tilde{k}_{[\alpha'} g_{\beta']} g_{\sigma']\nu} \right],
$$

where $\tilde{k}$ is given by

$$
\tilde{k} = p - k.
$$

Now we will use the following property in the amplitude

$$
A^{\mu
\nu} g_{\alpha\mu} g_{\nu\beta} = 2A^{\mu
\nu} g_{\alpha\mu} g_{\nu\beta}.
$$

(A.2)
so that we get

$$\mathcal{M} = \frac{1}{16} \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( \frac{g^{\alpha}[\alpha'] g^{\beta'][\beta]}{k^2 - m^2} + \frac{m^2 k_{[\alpha} k_{[\beta]} g^{\alpha'][\beta]}{k^2 - m^2} \right) \left( \frac{g^{\rho}[\rho'] g^{\sigma'][\sigma]}{k^2 - m^2} + \frac{m^2 k_{[\rho} k_{[\sigma]} g^{\rho'][\sigma]}{k^2 - m^2} \right) \left( 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right) = \frac{4}{16} \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( \frac{g^{\alpha}[\alpha'] g^{\beta'][\beta]}{k^2 - m^2} + \frac{m^2 k_{[\alpha} k_{[\beta]} g^{\alpha'][\beta]}{k^2 - m^2} \right) \left( \frac{g^{\rho}[\rho'] g^{\sigma'][\sigma]}{k^2 - m^2} + \frac{m^2 k_{[\rho} k_{[\sigma]} g^{\rho'][\sigma]}{k^2 - m^2} \right) \left( 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right) \right.$$

Now we ignore $O(m^2/k^4)$ and $O(m^2/k^6)$ terms to get,

$$\mathcal{M} = \frac{4}{16(k^2 - m^2)(k^2 - m^2)} \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right) \times \left( g^{\rho}[\rho'] g^{\sigma'][\sigma] \right) \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right) \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right) \times \left( g^{\rho}[\rho'] g^{\sigma'][\sigma] \right) \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right] = \frac{4}{16(k^2 - m^2)(k^2 - m^2)} \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right) \times \left( g^{\rho}[\rho'] g^{\sigma'][\sigma] \right) \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right].$$

(A.3)

Let us denote the first and second square brackets by I and II respectively i.e.

$$I = \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right).$$

(A.5)

$$II = \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right].$$

(A.6)

The first term in I and first term in II are antisymmetric wrt $\alpha'$ and $\beta'$, so that we have

$$I \times II = \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right) \times \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right].$$

(A.7)

Again the first term in first square bracket and first term in second square bracket above are antisymmetric w.r.t. $\rho$ and $\sigma$, so that we have

$$I \times II = \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right) \times \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right].$$

(A.8)

which implies

$$I \times II = \left[ 2(2k - p)_{\mu} g_{\rho\alpha} g_{\beta\sigma} + 2k_{\rho} g_{\alpha\beta} - 2k_{\alpha} g_{\beta\rho} g_{\sigma\mu} \right] \left( g^{\alpha}[\alpha'] g^{\beta'][\beta] \right) \times \left[ 2(2k - p)_{\nu} g_{\alpha'\rho'} g_{\sigma'\beta'} + 2k_{\rho'} g_{\sigma'[\alpha'} g_{\beta'][\nu} - 2k_{[\alpha'} g_{\beta'][\rho'} g_{\sigma'\nu]} \right].$$

(A.9)
Hence, the amplitude becomes

\[ \mathcal{M} = \frac{1}{(k^2 - m^2)(k^2 - m^2)} \left[ (2k - p)_\mu g_\rho^{\mu \nu} g_\sigma^{\nu \rho} + k_\rho g_\sigma^{[\alpha^\prime \beta^\prime]} - \tilde{k}_\rho g_\sigma^{[\beta^\prime]} g_{\sigma \mu} \right] \times \left[ (2k - p)_\nu g_\rho^{\mu \nu} g_\sigma^{\nu \rho} + k_\rho g_\sigma^{[\rho \sigma]} g_{\rho \nu} - \tilde{k}_\rho g_\sigma^{[\rho \sigma]} g_{\rho \nu} \right]. \quad (A.10) \]

Again, let

\[
I = \left[ (2k - p)_\mu g_\rho^{\mu \nu} g_\sigma^{\rho \nu} + k_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \mu} \right], \quad (A.11)
\]

\[
II = \left[ (2k - p)_\nu g_\rho^{\mu \nu} g_\sigma^{\rho \nu} + k_\rho g_\sigma^{[\rho \sigma]} g_{\rho \nu} - \tilde{k}_\rho g_\sigma^{[\rho \sigma]} g_{\rho \nu} \right]. \quad (A.12)
\]

Also let \( I_i \) denote the \( i \)th term in \( I \), so that we have

\[
I = I_1 + I_2 - I_3, \quad (A.13)
\]

\[
II = II_1 + II_2 - II_3 \quad (A.14)
\]

and

\[
I_1 \times I_1 = (2k - p)_\mu g_\rho^{\mu \nu} g_\sigma^{\rho \nu} \times (2k - p)_\nu g_\rho^{\rho \sigma} g_\sigma^{\nu \rho}
= d^2 (2k - p)_\mu (2k - p)_\nu
\approx d^2 [4k_\mu k_\nu - 2(k_\mu p_\nu + k_\nu p_\mu)], \quad (A.15)
\]

\[
I_1 \times I_2 = (2k - p)_\mu g_\rho^{\mu \nu} g_\sigma^{\rho \nu} \times k_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu}
= (2k - p)_\mu \times k_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu}
= (2k - p)_\mu \times \left( k_\rho g_\sigma^{[\rho \sigma]} - k_\sigma g_\sigma^{[\rho \sigma]} \right) g_{\sigma \nu}
= (1 - d) (2k_\mu k_\nu - p_\mu k_\nu), \quad (A.16)
\]

\[
I_1 \times I_3 = (2k - p)_\mu g_\rho^{\mu \nu} g_\sigma^{\rho \nu} \times \tilde{k}_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu}
= (2k - p)_\mu \tilde{k}_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu}
= (2k - p)_\mu \tilde{k}_\rho (g_\rho^{\rho \nu} - g_\sigma^{\rho \nu})
= (1 - d) (2k_\mu \tilde{k}_\nu - p_\mu \tilde{k}_\nu)
\approx (1 - d) (2k_\mu p_\nu - 2k_\nu k_\mu + p_\mu k_\nu), \quad (A.17)
\]

\[
I_2 \times I_1 = k_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu} \times (2k - p)_\nu g_\rho^{\rho \nu} g_\sigma^{\nu \rho}
= k_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu} \times (2k - p)_\nu
= k_\rho (2k - p)_\nu \left( g_\rho^{\rho \nu} g_\sigma^{\nu \rho} - g_\sigma^{\rho \nu} g_\rho^{\rho \nu} \right)
= (1 - d) k_\rho (2k - p)_\nu
= (1 - d) [2k_\mu k_\nu - k_\mu p_\nu], \quad (A.18)
\]

\[
I_2 \times I_2 = k_\rho g_\sigma^{[\rho \sigma]} g_{\sigma \nu} \times k_\rho g_\sigma^{\rho \sigma} g_{\sigma \nu}
= k_\rho \left( g_\rho^{\rho \nu} g_\sigma^{\nu \rho} - g_\sigma^{\rho \nu} g_\rho^{\rho \nu} \right) \left( k_\rho g_\sigma^{\rho \sigma} - k_\sigma g_\sigma^{\rho \sigma} \right) g_{\sigma \nu}
= (g_\rho^{\rho \nu} g_{\mu \nu} - g_\sigma^{\rho \nu} g_{\mu \nu}) \left( k_\rho g_\sigma^{\rho \sigma} - k_\sigma g_\sigma^{\rho \sigma} \right) g_{\sigma \nu}
= (d - 2) k^2 g_{\mu \nu} + k_\mu k_\nu, \quad (A.19)
\]
\[ I_2 \times I_3 = k_\rho \tilde{g}_\rho^{\alpha\beta} g_\mu^{\alpha\beta} \times \tilde{k}_\alpha g_\beta^{\mu\nu} g_\nu^{\alpha\beta} \]
\[ = k_\rho \tilde{k}_\alpha \left( g_\rho^{\alpha\beta} - g_\alpha^{\beta\rho} \right) \left( g_\mu^{\alpha\beta} - g_\beta^{\mu\alpha} \right) \left( g_\nu^{\alpha\beta} - g_\beta^{\nu\alpha} \right) \]
\[ = \left( k_\rho \tilde{g}_\rho^{\alpha\beta} - \tilde{k}_\rho g_\alpha^{\beta\rho} \right) \left( k_\mu \tilde{g}_\mu^{\alpha\beta} - k_\nu g_\nu^{\alpha\beta} \right) \]
\[ = (d-2)\tilde{k}_\mu k_\nu + \tilde{k}_\nu k_\mu \]
\[ = (d-2)p_\mu k_\nu + p_\nu k_\mu - (d-1)k_\mu k_\nu, \quad (A.20) \]

\[ I_3 \times I_1 = \tilde{k}_\alpha \tilde{g}_\rho^{\alpha\beta} g_\mu^{\alpha\beta} \times (2k-\rho) \nu g_\alpha^{\rho\nu} g_\beta^{\nu} \]
\[ = (2k-\rho)_\nu \tilde{k}_\rho^{\alpha\beta} g_\beta^{\rho\nu} g_\mu^{\alpha\beta} \]
\[ = (1-d)\tilde{k}_\mu (2k-\rho)_\nu \]
\[ \approx (1-d)\left[ 2p_\mu k_\nu - 2k_\mu k_\nu + k_\mu p_\nu \right], \quad (A.21) \]

\[ I_3 \times I_2 = \tilde{k}_\alpha \tilde{g}_\rho^{\alpha\beta} g_\mu^{\alpha\beta} \times k_\mu^{\rho\sigma} g_\nu^{\rho\sigma} g_\beta^{\nu} \]
\[ = \left( k_\rho \tilde{g}_\rho^{\alpha\beta} - k_\alpha \tilde{g}_\rho^{\rho\beta} \right) \left( k_\mu^{\rho\sigma} g_\nu^{\rho\sigma} - k_\sigma^{\rho\mu} g_\nu^{\rho\sigma} \right) g_\sigma \tilde{k}_\beta \]
\[ = \left( k_\beta \tilde{g}_\beta^{\rho\sigma} - k_\sigma \tilde{g}_\beta^{\rho\sigma} \right) \left( k_\mu^{\rho\sigma} g_\nu^{\rho\sigma} - k_\sigma^{\rho\mu} g_\nu^{\rho\sigma} \right) \]
\[ \approx (d-2)\tilde{k}_\beta g_{\mu\nu} + \tilde{k}_\nu \tilde{k}_\beta \]
\[ \approx (d-2)(k_\beta - 2p \cdot k) g_{\mu\nu} + k_\mu k_\nu - (p_\mu k_\nu + k_\mu p_\nu). \quad (A.22) \]

The amplitude is finally given by

\[ \mathcal{M} = \left[ I_1 \times I_2 + I_1 \times I_3 + I_2 \times I_1 + I_2 \times I_3 - I_3 \times I_1 - I_3 \times I_2 + I_3 \times I_3 \right] \]
\[ = 2(d-2)(k^2 - p \cdot k)g_{\mu\nu} + (2d^2 - 3d + 4)[2k_\mu k_\nu - (p_\mu k_\nu + k_\mu p_\nu)]. \quad (A.24) \]

References

[1] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264

[2] A. Salam, Weak and Electromagnetic Interactions, in Elementary Particle Physics: Relativistic Groups and Analyticity, edited by N. Svartholm, (Almqvist and Wiksell, Sweden, 1968).

[3] S. Glashow, Nucl. Phys. 22, 579 (1961)

[4] G. Aad et al., (ATLAS collaboration), Phys. Lett. B 716, 1 (2012)

[5] S. Chatrchyan et al., (CMS collaboration), Phys. Lett. B 716, 30 (2012)

[6] D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973).
[7] H.D. Politzer, Phys. Rev. Lett. 30, 1346(1973).
[8] E. Reya, Phys. Rep. 14C, 129(1974).
[9] S. Coleman and D. J. Gross, Phys. Rev. Lett. 31, 851(1973).
[10] A. Zee, Phys. Rev. D7, 3630(1973).
[11] J. Frankel and J. C. Taylor, Nucl. Phys. B109, 439(1976).
[12] D. J. Gross, Rev. Mod. Phys. 77, 837(2005).
[13] G. ’t Hooft, Nucl. Phys. B254, 11(1985).
[14] C. Patrignani et al. (Particle, Data Group), Chin. Phys. C, 40, 100001(2016).
[15] J. C. Collins and M. J. Perry, Phys. Rev. Lett. 34, 1353(1975).
    G. Baym and S. A. Chin, Phys. Lett B88, 241(1976).
    B. A. Freedman and L. D. Mcerran, Phys. Rev D16, 1196(1977).
    G. Chapline and M. Nauenberg, Phys. Rev. D16, 450(1977).
    E. V. Surayak, Phys Lett. B78, 150(1978).
    O. K. Kalashnikov and V. V. Klimpv, Phys Lett. B88, 328(1979).
    J. I. Kapusta, Nucl. Phys. B148, 461(1979).
[16] J. Adams et al (for STAR collaboration), Nucl. Phys. A757, 102(2005).
    K. Adcox et al (for PHENIX collaboration), Nucl. Phys. A757, 184(2005)
    I. Arsene et al (for BRAHMS collaboration), Nucl. Phys. A757, 1(2005).
    B. B. Back et al (for PHOBOS collaboration), Nucl. Phys. A757, 28(2005).
[17] B. Müller, J. Schukraft and B. Wyslouch, Ann. Rev. Nucl. Part. Sci. 62, 361 (2012).
[18] S. Jeon and U. Heinz, Int. J. Mod. Phys. E 24, 1530010 (2015); P. Kovtun, J. Phys. A 45, 473001 (2012); T. Hirano, Acta. Phys. Pol. B 42, 2811 (2011).
[19] D. Kharzeev and A. Zhitnitski, Nucl. Phys. A797, 67(2007).
[20] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A803, 227(2008).
[21] K. Fukushima, D. Kharzeev and H. J. Kharzeev, Phys.Rev. D78, 074033 (2008).
[22] Y. Akamatsu and N. Yamamoto, Phys.Rev. D90, 125031(2014).
[23] E. Braaten and R. D. Pissarski, Phys. Rev. Lett. 64, 1332(1089)
[24] R. Kobes, G. Kunstatter and A. Rebhan, Phys. Rev. Lett. 64, 2992(1989).
[25] A. D. Linde, Phys. Lett. B96, 289(1980).
[26] T. Furusawa and K. Kikkawa, Phys. Lett. B128, 218(1983).
[27] P. Arnold, Phys. Rev. D 62, 036003 (2000).
[28] D. Bödeker, Phys. Lett. B 426, 351 (1998).
[29] A. Selikov and M. Gyulassy, Phys. Lett. B 316, 373 (1993)
[30] H. Heiselberg, Phys.Rev.Lett. 72, 3013(1994).
[31] R. Jackiw, Topological investigation of quantized gauge theories, Les Houches Summer School, North-Holland(1983).
[32] S. Deser, R. Jackiw and S. Templeton, Annals Phys. 140, 372(1982).
[33] V. P. Nair, Phys. Lett. B 352, 117 (1995).
[34] R. D. Pisarski and S. Rao, Phys. Rev. D 32, 2081 (1985).
[35] F. Strocci, Phys. Lett. B62, 60.(1976).
[36] R. Hagg and D. Kastler, J. Math. Phys. 5, 848(1964).
[37] T. Kugo and I. Ojima, Prog.Theor.Phys.Suppl. 66, 1 (1979).
[38] C. S. Fischer, J.Phys. G32, R253(2006).
[39] M. Chaichian and K. Nishijima, Eur.Phys.J. C47737(2006).
[40] S. Joglekar, Annals. Phys. 83, 427(1973).
[41] C. E. Vayonakis, Lett. Nuovo. Cim 17, 383(1976).
[42] B. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D16, 1519(1977).
[43] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A19, 3265 (2004).
[44] M. Veltman, Nucl. Phys. B7, 637(1968).
[45] M. Veltman, Nucl. Phys. B21, 288(1970).
[46] Ken-Ichi Shizuya, Nucl. Phys. B121, 125(1977).
[47] H. Umezawa, Nucl. Phys. 23, 399(1961).
[48] G. Curci and R. Ferrari, Nuovo Cim. A32, 151(1975).
[49] I. Ojima, Z. Phys. C13, 173(1982).
[50] J. M. Cornwall, Phys. Rev. D26, 1453(1982).
[51] J. M. Cornwall, J. Papavassiliou and D. Binsoi, The Pinch Technique and its Application to Non-Abelian Gauge Theories, Cambridge Univ. Press(UK)(2011).
[52] R. Jackiw, Annals Phys. 140, 372(1982); Annals Phys. 281, 409(2000).
[53] M. Balu and G. Thompson, Ann. Phys. 205, 130 (1991).
[54] D. Birmingham, M. Balu, M. Rakowski and G. Thompson Phys. Rept. 209, 129(1991).
[55] J. C. Baez, Lect. Notes Phys. 543, 25 (2000).
[56] G. Horowitz and M. Srednicki, Commun. Math. Phys. 130, 83(1990).
[57] M. Kalb and P. Ramond Phys. Rev. D9, 2273(1973).
[58] E. Cremmer and J. Scherk Nucl. Phys. B72, 117(1974).
[59] T. J. Allen, M. J. Bowick and A. Lahiri Mod. Phys. Lett. A6, 559(1990).
[60] A. Lahiri Phys. Rev. D63, 105002(2001).
[61] J. Thierry-Mieg and L. Baulieu, Nucl. Phys. B197, 477(1982), Nucl. Phys. B228, 259.
[62] A. Lahiri, Phys. Rev. D55, 5045(1997).
[63] R. Kumar and R. P. Malik, Eur. Phys. J. C71, 1710(2011).
[64] R. Kumar and R. P. Malik, Europhys. Lett. 94, 11001(2011).