Exploiting the violation of Lorentz symmetry for the planar Hall effect

Muhammad Imran and Selman Hershfield
Department of Physics, University of Florida, Gainesville, Florida 32611, USA
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The low energy Dirac and Weyl spectra are allowed to violate the Lorentz symmetry and thereby have a tilted energy dispersion. The tilt in the energy dispersion induces a Hall voltage in the plane spanned by the electric field and the tilt velocity. In the presence of a magnetic field the planar Hall conductivity and resistivity show Shubnikov de Haas oscillations. The oscillations in the planar Hall effect can become a fingerprint to spot the anomalous transport in Dirac and Weyl semimetals.

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INTRODUCTION

The Dirac equation predicts three elementary fermions. These are well known by the names of Dirac, Majorana, and Weyl fermions. Two of them, the Majorana and Weyl fermions, have not been experimentally observed as a fundamental particle. However, this family of fermions has been experimentally detected in the low energy physics.1–3

The speed of electrons and holes in the energy bands of solids is always less than the speed of light. Therefore it is not a surprise that these fermions violate the fundamental Lorentz symmetry, which is not possible for the elementary particles.4 There are two types of fermions that violate the Lorentz symmetry in the low energy physics.5

This classification can be explained by the energy dispersion equation. The constant energy surfaces for Dirac and Weyl fermions is a spheroid. With type-I tilt constant energy surfaces are hyperboloids. This classification is also made from the boundary where the conduction and valance bands meet. The type-I Dirac and Weyl fermions bands converge to a single point. The type-II Dirac and Weyl fermions bands meet and form a non-closed isoenergetic orbit. The Weyl fermions emerge from the Dirac fermions provided any or both of the two the time reversal and the inversion symmetries are absent.6 Both types of Weyl fermions are topologically protected excitations. The type-I and type-II Dirac and Weyl fermions have been observed in the angle resolved photo emission spectroscopy and the magnetotransport measurements.7–9

The tilt in energy dispersion of the Dirac and Weyl fermions is parameterized by the tilt velocity. In general, the tilt velocity modifies the energy spectrum. This propagates in the energy density of states and the magnetotransport properties. It should be noted that a strong magnetic field can not quantize the motion of charged particles in a hyperboloid surface. Therefore no quantized solutions exists for magnetic field applied perpendicular to the direction of type-II tilt velocity.10,11 In this study we show that the tilt velocity mixes different elements of the conductivity matrix. This induces the Hall voltage in the plane formed by the perpendicular electric and magnetic fields, provided the tilt velocity makes finite angle in this plane. Although the planar Hall voltage survives in the absence of the magnetic field, the magnetic field induces quantization signatures, the Shubnikov de Haas oscillations.

The semiclassical theory of the planar Hall effect in the three dimensional(3D) Weyl semimetals has been studied10,11 and experimentally observed.12 Those semiclassical theories of the planar Hall effect rely on the chiral anomaly, a Berry curvature effect in magneto-transport. According to this theory for an anomalous semimetal longitudinal magnetoresistivity decreases quadratically in the direction of magnetic field. Although there are experimental results that observe the planar Hall effect, the anomalous magnetoresistivity expected with the chiral anomaly was not observed.12 In general the planar Hall effect can arise even without magnetic field.13 We are studying the planar Hall effect both in the quantum and semiclassical regions of magnetic field. In this study we show the origin of the planar Hall effect in the Dirac and Weyl semimetals can also be a tilted energy dispersion even without chiral anomaly.

We shall discuss the theoretical formulation and model Hamiltonian for a general 3D Dirac and Weyl fermions in section-II, the numerical results and discussions are presented in section -III, and finally conclusions are given in section- IV.

THEORETICAL FORMULATION

The Hamiltonian for a tilted Dirac and Weyl semimetals is

\[ H = \chi (v_T p_i \sigma^0 + v_F p_i \sigma^i), \]

where \( \sigma^i \) and \( p_i \) denotes the \( x, y, \) and \( z \) components of the Pauli matrix and the momentum. Summation is assumed in the repeated indexes. \( \chi = \pm \) is the chirality of the Weyl fermions, and \( v_T \) is Fermi velocity. The eigenvalues of this Hamiltonian are \( \epsilon(s, p) = \chi (v_T p_i \cos \theta + sv_F p_i) \), where \( s = \pm \) denotes the band index and \( \theta \) is the angle between the tilt velocity and the momentum. The energy
density of states for a tilted Dirac and Weyl spectrum has a parabolic energy dependence in the normalized multiplicity constant.

\[
D(E) = \begin{cases} 
\frac{g_1}{(1-\frac{v_T}{v_F})^2} v_F & v_F > v_T \text{ Type-I} \\
\frac{g_2}{(1-\frac{v_T}{v_F})^2} (1 + (\frac{v_T}{v_F})^2) & v_F > v_T \text{ Type-II} 
\end{cases}
\]

(2)

In the above formula the tilt is assumed along z-axis, \( \tilde{v}_T = v_T \hat{z} \). Here \( D_0(E) = \frac{E^2}{2\pi^2 \hbar^2 v_F} \) shows the energy density of states for the Dirac and Weyl spectra without any tilt velocity. The symbol \( g_{1(2)} \) refers to the number of type-I(II) Dirac and Weyl nodes. The energy density of states diverges for \( \frac{\tilde{v}_T}{v_F} \rightarrow 1 \) (see Ref. [15] for discussions on this topic).

We assume the z-axis is along the magnetic field \( \bar{B} = B_z \hat{z} \). \( v_T^z \) is the tilt in the direction of magnetic field, and \( v_T^x \) in the direction perpendicular to the magnetic field. We take \( v_T^x \) to be nonzero only for type-I Dirac and Weyl semimetals because one can not quantize the Hamiltonian with our approach for type-II semimetals. Thus the Hamiltonian including tilt is

\[
H = \chi (v_T^x p_x + v_T^y p_y + v_F \sigma \cdot \bar{p}).
\]

(3)
The energy eigenvalues and energy eigenstates are found after transforming the above Hamiltonian by hyperbolic transformation in the \( \tilde{x} \) direction,

\[
(\epsilon - \hat{H}) |\psi\rangle \rightarrow \tilde{\epsilon} - \hat{H} |\tilde{\psi}\rangle.
\]

(4)
The bar on top of the operators and the eigenstates show hyperbolic transformation taken along the \( \tilde{x} \) axis.

\[
\tilde{O} = N^2 \exp[\sigma_z \theta \frac{E}{2}] \exp[\sigma_z \theta \frac{E}{2}] |\tilde{\psi}\rangle = \frac{1}{N} \exp[-\sigma_z \theta \frac{E}{2}] |\psi\rangle.
\]

(6)

Here \( N \) is the normalization constant of the hyperbolic transformation. For convenience we use the Landau gauge for solving this problem: \( p_y \rightarrow \tilde{p}_y - eA_y \), with \( A_y = B_z x \). The energy eigenvalues are

\[
\epsilon_n^s(p_z) = \chi (v_T^x p_x + v_T^y p_y + v_F \sigma \cdot \bar{p}) + \frac{s}{2} \sqrt{v_T^2 p_z^2 + \frac{2\hbar^2 v_T^2}{\gamma l_B^2}}.
\]

(7)

Here \( s = \pm \), and \( \gamma^{-1} = \sqrt{1 - (\frac{v_T}{v_F})^2} \). The corresponding energy eigenstates are

\[
\psi^+_n = \frac{1}{\sqrt{2}} \begin{bmatrix} a_n \phi_n(x - x_0) \\
a_{n-1} \phi_{n-1}(x - x_0) \end{bmatrix}
\]

\[
\psi^-_n = \frac{1}{\sqrt{2}} \begin{bmatrix} b_n \phi_n(x - x_0) \\
b_{n-1} \phi_{n-1}(x - x_0) \end{bmatrix}.
\]

(8)
The energy eigenstates are shifted by the displacement \( x_0 = \frac{p_z l_B}{\hbar} \). The symbol \( l_B \) is used for the magnetic length, and it is related with the magnetic momentum \( p_B \) by \( p_B = \frac{\sqrt{2} \hbar}{l_B} \). The normalization constants are

\[
a_n = \frac{1 + \sqrt{v_T^2 p_z^2 + \frac{2\hbar^2 v_T^2}{\gamma l_B^2}}}{\gamma \sqrt{v_T^2 p_z^2 + \frac{2\hbar^2 v_T^2}{\gamma l_B^2}}},
\]

\[
b_n = \frac{1 - \sqrt{v_T^2 p_z^2 + \frac{2\hbar^2 v_T^2}{\gamma l_B^2}}}{\gamma \sqrt{v_T^2 p_z^2 + \frac{2\hbar^2 v_T^2}{\gamma l_B^2}}},
\]

(11)

Ground state, \( n = 0 \), is the gapless state with eigenvalue

\[
\epsilon_n^{s=0}(p_z) = \chi (v_T^x + s \sigma \cdot \bar{p}) v_F p_z.
\]

(12)
The energy spectra of the first two bands is plotted in Fig. [1]. The ground state is chiral for type-I Dirac and Weyl spectra, whereas the ground state for type-II Dirac and Weyl spectra is independent of the chirality.

The quantum theory of the planar Hall effect

The energy density of states for tilted Dirac and Weyl semimetals in quantizing magnetic field is

\[
D(E) = \frac{g}{4\pi^2 l_B^2 v_F E} \left\{ 1 - \frac{v_T^2}{v_F^2} (1 + n_{max}(n_{max} + 1)) \right. \\
+ 2 \sum_{n=1}^{n_{max}} \frac{E \gamma}{\sqrt{E^2 \gamma^2 - (n v_T^2 - \frac{p_B^2}{\gamma})^2}} \big| 1 - \gamma^2 (\frac{v_T}{v_F})^2 \big| \bigg\}.
\]

(13)

The above equation shows oscillations in the energy density of states with the changing magnetic field or the energy \( E, E^2 \gamma^2 = v_T^2 p_B n (1 - \gamma^2 (\frac{v_T}{v_F})^2) \). The magnetoconductivity, and the magnetoresistivity also show these oscillations. The above formula for the energy density of states reproduces the result of Refs. [13,14] by turning off the tilt parameters \( \gamma = 1, v_T^x = 0, v_T^y = 0 \) and setting \( g = 1 \).

![Fig. 1. The energy spectra of Weyl gases. We assume the Lorentz factor \( \gamma = 1 \), the type-I tilt velocity \( \frac{v_T}{v_F} = 0.9 \), and the type-II tilt velocity \( \frac{v_T}{v_F} = 1.1 \).](image-url)
For completeness we derive the formulas for all the components of the magnetoconductivity and the magnetoresistivity tensors. We use the Kubo linear response theory for studying the quantum magnetotransport. The current current correlation function is given below:\[12\]

$$Q_{ij}(\Omega) = 2e^2T \sum_\omega \int \frac{dp_x}{2\pi\hbar} \int \frac{dp_y}{2\pi\hbar} \int dx' \quad Tr[\bar{v}_iG(p_y, p_z, \Omega + \omega, x, x')\bar{v}_jG(p_y, p_z, \omega, x', x)]$$

(14)

In the above formula $\omega$ and $\Omega$ are Matsubara frequencies. The retarded Green's function is

$$G^r(\omega, x, x', p_z, p_y) = \sum_{n,s} \frac{\psi_{x,s}^\dagger(\bar{x})\psi_{x,s}(\bar{x}')}{\omega - \epsilon_{n,s}(p_z) + i\Gamma}.$$  

(15)

Here $\Gamma = \frac{\hbar}{T}$ is used for the finite broadening of the Landau levels. The components of the velocity after the hyperbolic transformation are

$$\bar{v}_x = \frac{1}{\gamma} v_F \sigma_{x}, \quad \bar{v}_y = v_F \sigma_{y}, \quad \bar{v}_z = \gamma v_F (\sigma_0 + \frac{v_F}{\hbar} \sigma_{z}) + v_F \sigma_{z}.$$  

(16)

At this point we find it convenient to introduce a new tensor, $s_{ij}$, that only involves the Pauli matrices.

$$s_{ij}(\Omega) = 2e^2T \sum_\omega \int \frac{dp_x}{2\pi\hbar} \int \frac{dp_y}{2\pi\hbar} \int dx' \quad Tr[\sigma_iG(p_y, p_z, \Omega + \omega, x, x')\sigma_jG(p_y, p_z, \omega, x', x)]$$

(17)

For finding static response we set $s_{ij} = \lim_{\Omega \to 0} s_{ij}(\Omega)$. The different elements of the tensor $s_{ij}$ are given below.

$$s_{xx} = \frac{e^2 \beta T}{16\pi l_D^2} \sum_n \int \frac{dp_z}{2\pi\hbar} \left[ \frac{\hbar}{\tau^2(\epsilon_{n,s}(p_z) + \epsilon_{n-1,-}(p_z))^2 + \hbar^2} \right]$$

$$\times \left[ a_{n,s}^2 a_{n-1,-}^2 \left( sech^2 \left( \frac{\beta\epsilon_{n,s}(p_z)}{2} \right) + sech^2 \left( \frac{\beta\epsilon_{n-1,-}(p_z)}{2} \right) \right) + b_{n,s}^2 b_{n-1,-}^2 \left( sech^2 \left( \frac{\beta\epsilon_{n,s}(p_z)}{2} \right) + sech^2 \left( \frac{\beta\epsilon_{n-1,-}(p_z)}{2} \right) \right) \right]$$

$$+ \tau^2(\epsilon_{n,s}(p_z) - \epsilon_{n-1,-}(p_z))^2 + \hbar^2$$

$$\times \left[ a_{n,s}^2 b_{n-1,-}^2 \left( sech^2 \left( \frac{\beta\epsilon_{n,s}(p_z)}{2} \right) + sech^2 \left( \frac{\beta\epsilon_{n-1,-}(p_z)}{2} \right) \right) + a_{n,s}^2 b_{n-1,-}^2 \left( sech^2 \left( \frac{\beta\epsilon_{n,s}(p_z)}{2} \right) + sech^2 \left( \frac{\beta\epsilon_{n-1,-}(p_z)}{2} \right) \right) \right]$$

(18)

$$s_{zz} = \frac{e^2 \beta T}{16\pi l_D^2} \sum_n \int \frac{dp_z}{2\pi\hbar} \left[ a_{n,s}^2 + b_{n,s}^2 \right]$$

$$+ 2a_{n,s}^2 b_{n-1,-}^2 \tau^2(2\epsilon_{n,s}(p_z))^2 + \hbar^2$$

$$\times \left[ sech^2 \left( \frac{\beta\epsilon_{n,s}(p_z)}{2} \right) + sech^2 \left( \frac{\beta\epsilon_{n-1,-}(p_z)}{2} \right) \right]$$

(19)

$$s_{xy} = \frac{\hbar e^2}{4\pi l_D^2} \sum_n \int \frac{dp_z}{2\pi\hbar} \left[ \frac{1}{(\epsilon_{n,s}(p_z) + \epsilon_{n-1,-}(p_z))^2} \right]$$

$$\times \left[ \frac{\tanh(\frac{\beta\epsilon_{n,s}(p_z)}{2}) - \tanh(\frac{\beta\epsilon_{n-1,-}(p_z)}{2})}{2} \right]$$

$$+ b_{n,s}^2 b_{n-1,-}^2 \left[ \frac{\tanh(\frac{\beta\epsilon_{n,s}(p_z)}{2}) - \tanh(\frac{\beta\epsilon_{n-1,-}(p_z)}{2})}{2} \right]$$

$$+ \frac{1}{(\epsilon_{n,s}(p_z) - \epsilon_{n-1,-}(p_z))^2}$$

$$\times \left[ \frac{\tanh(\frac{\beta\epsilon_{n,s}(p_z)}{2}) - \tanh(\frac{\beta\epsilon_{n-1,-}(p_z)}{2})}{2} \right]$$

(20)

The remaining elements of the function $s_{ij}$ are found by exploiting symmetry of the Pauli matrices: $s_{ij} = s_{zx}$, and $s_{yx} = -s_{xy}$. All the elements of the conductivity matrix $\sigma_{ij}$ are found by using the $s_{ij}$ tensor.

$$[\sigma_{ij}] = \left[ \begin{array}{ccc} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{array} \right]$$

(21)

$$\sigma_{xx} = s_{xx} \frac{v_F^2}{\gamma^2}, \quad \sigma_{xy} = s_{xy} \frac{v_F^2}{\gamma}, \quad \sigma_{xz} = s_{xx} v_F^2 v_T^2$$

(22)

$$\sigma_{yx} = -s_{xy}, \quad \sigma_{yy} = s_{yy} v_F^2, \quad \sigma_{yz} = s_{yx} v_F^2 v_T^2$$

(23)

$$\sigma_{zx} = s_{xz}, \quad \sigma_{zy} = \sigma_{yz}, \quad \sigma_{zz} = s_{zz}(v_F^2 + (v_T^2)^2) + s_{xx}(v_T^2)^2 \gamma^2(v_F^2)^2$$

(24)

The new components of the conductivity matrix ($\sigma_{xx}$, $\sigma_{xy}$, $\sigma_{yz}$, $\sigma_{xyz}$) are directly proportional to the tilt velocity $v_T^2$, $v_T^2$. In the absence of any one component of the tilt velocity ($v_T^2 = 0$ or $v_T^2 = 0$), the structure of the conductivity matrix does not show any signature of the tilted Dirac and Weyl spectra because it is the product $v_T^2 v_T^2$ which enters into these components. The transverse conductivity $\sigma_{xx}$, and the Hall conductivity $\sigma_{xy}$ are renormalized by the Lorentz factor $\gamma$. The longitudinal conductivity $\sigma_{zz}$ is enhanced by the parallel component of the tilt velocity. The response of the transverse conductivity $s_{xx}$ is also added up with the longitudinal conductivity $\sigma_{zz}$. For comparison the equation of conductivity matrix in the absence of the perpendicular component of tilt velocity is

$$[\sigma_{ij}] = \left[ \begin{array}{ccc} s_{xx} v_F^2 & s_{xy} v_F^2 & 0 \\ s_{yx} v_F^2 & s_{yy} v_F^2 & 0 \\ 0 & 0 & s_{zz}(v_F^2 + (v_T^2)^2) \end{array} \right].$$

(25)

In general the resistivity matrix

$$[\rho_{ij}] = \left[ \begin{array}{ccc} \rho_{xx} \rho_{xy} \rho_{xz} \\ \rho_{yx} \rho_{yy} \rho_{yz} \\ \rho_{zx} \rho_{zy} \rho_{zz} \end{array} \right].$$

(26)
and the tilt velocity. In the above formula the presence of both components of the tilt velocity makes the planar Hall effect \( \rho_{xx} \) finite. A schematic picture of the planar Hall effect in a slab of the 3D Dirac and Weyl gases is shown in the Fig. 2. In general a sample of 3D Dirac and Weyl semimetals with tilted spectrum supports the planar Hall effect. The tilt velocity can be set in the plane formed by the perpendicular electric and magnetic fields. This accumulates the Hall voltage in the plane of the electric and magnetic fields. The planar Hall voltage is present even in the absence of the magnetic field because with the tilt the energy spectrum is asymmetric in the \( \hat{z} \) and \( \hat{x} \) direction.

The remaining elements of the resistivity matrix are reducible by symmetry: \( \rho_{yx} = -\rho_{xy}, \quad \rho_{zx} = \rho_{xz}, \quad \rho_{yz} = \rho_{zy} = 0 \). The transverse resistivity matrix \( \rho_{xz} \) contains all elements of the tensor \( s_{ij} \). The Hall and longitudinal resistivities, \( \rho_{xy}, \rho_{zz} \), are renormalized by the Lorentz factor \( \gamma \) and the tilt velocity. In the above formula the presence of both components of the tilt velocity makes the planar Hall effect \( \rho_{xx} \) finite. A schematic picture of the planar Hall effect in a slab of the 3D Dirac and Weyl gases is shown in the Fig. 2. In general a sample of 3D Dirac and Weyl semimetals with tilted spectrum supports the planar Hall effect. The tilt velocity can be set in the plane formed by the perpendicular electric and magnetic fields. This accumulates the Hall voltage in the plane of the electric and magnetic fields. The planar Hall voltage is present even in the absence of the magnetic field because with the tilt the energy spectrum is asymmetric in the \( \hat{z} \) and \( \hat{x} \) direction.

The Semiclassical theory of the planar Hall effect

For the semiclassical study of magnetotransport in the Dirac and Weyl semimetals we solve the Boltzmann equation in the relaxation time approximation

\[
\frac{\partial f_x}{\partial t} + \vec{v}_T \cdot \nabla_x f_x + \frac{1}{\hbar} \vec{F} \cdot \nabla_k f_x = -\frac{\delta f_x}{\tau_{tr}},
\]

where

\[
\vec{F} = \frac{1}{1 + \frac{e}{\hbar} B \cdot \Omega} (e \vec{E} + e \vec{v} \times \vec{B}) + \frac{e^2}{\hbar} (\vec{E} \cdot \vec{B}) \vec{\Omega}
\]

\[
\vec{x} = \frac{1}{1 + \frac{e}{\hbar} B \cdot \Omega} (e \vec{v} \times \vec{B} + e (\vec{\Omega} \cdot \vec{v}) \vec{B}).
\]

\( \vec{F} \) is the force acting on Dirac and Weyl fermions, \( \vec{x} \) is the group velocity, \( \tau_{tr} \) is the transport time, and \( \vec{\Omega} = -\frac{\hbar}{2e} \vec{B} \) is the Berry curvature. The apparent difference in equations of velocity and force for the Dirac and Weyl gases enters due to the Berry curvature. For the study of planar Hall effect we ignore the effect of the Lorentz force and assume a constant in time and space distribution function.

\[
\delta f_x(x, k, t) \equiv \delta f_x^0(k)
\]

We use the linear order in electric field approximation for calculating current.

\[
\delta f_x^\parallel = A \tau_{tr} e (\vec{E} + \frac{e}{\hbar} \vec{v} \times \vec{B} \vec{\Omega}) \cdot \vec{v} (-\frac{\partial f_{eq}(\epsilon_{k,\chi})}{\partial \epsilon_{k,\chi}})
\]

\[
\tau_{\parallel} = e \int \frac{d^3k}{(2\pi)^3} A^{-1} \vec{x} \delta f_x^\parallel
\]

In the formula for the current we have included the phase space correction factor \( A = \frac{1}{1 + \frac{e}{\hbar} B \cdot \Omega} \). The planar Hall conductivity is

\[
\sigma_{xx} = e^2 \tau_{tr} \sum_{\chi=\pm} \int \frac{d^3k}{(2\pi)^3} A \delta f_x^\parallel (\vec{v}_x + \frac{e}{\hbar} B_\chi (\vec{\Omega} \cdot \vec{v}))
\]

\[
\tau_{\parallel} = \sum_{\chi=\pm} \int \frac{d^3k}{(2\pi)^3} A^{-1} \vec{x} \delta f_x^\parallel (\vec{v}_x + \frac{e}{\hbar} B_\chi (\vec{\Omega} \cdot \vec{v})) (-\frac{\partial f_{eq}(\epsilon_{k,\chi})}{\partial \epsilon_{k,\chi}}).
\]

The energy dispersion and the velocities of the tilted Dirac and Weyl semimetals are

\[
\epsilon_{k,\chi} = \chi (hk_x v^e_T + hk_z v^e_z + sh|k| v_F),
\]

\[
v_x = \chi (v^e_T = sv_F \cos \phi \sin \theta)
\]

\[
v_y = \chi s v_F \sin \phi \sin \theta
\]

\[
v_z = \chi (v^e_z = sv_F \cos \theta).
\]

The sum of the particles densities \( (N^+, N^-) \) and the currents \( (J^+, J^-) \) are conserved in the presence of the chiral anomaly and the tilt velocity.

\[
\sum_{\chi=\pm} \int \frac{d^3k}{2\pi^3} A^{-1} \delta f_x^\parallel \tau_{tr} = e \sum_{\chi=\pm} \chi \int \frac{d^3k}{2\pi^3} \frac{\partial f_{eq}(\epsilon_{k,\chi})}{\partial \epsilon_{k,\chi}}
\]

\[
[(v^e_T E_x + v^e_z E_z) + \frac{e}{\hbar} (\vec{E} \cdot \vec{B})(\vec{\Omega} \cdot \vec{v})] = 0
\]
\begin{equation}
\frac{\partial}{\partial t}(N^+ + N^-) + \vec{\nabla} \cdot (\vec{j}^+ + \vec{j}^-) = 0 \tag{41}
\end{equation}

However, the chiral anomaly and tilt velocity creates imbalance between population densities of different chirality particles

\[
\delta N = N^+ - N^- = \int \frac{d^3k}{2\pi^3} A^{-1}(\delta f_+^\parallel - \delta f_-^\parallel).
\tag{42}
\]

In the absence of a magnetic field this imbalance is created by the tilt velocity

\[
\delta N = N^+ - N^- = -e\tau_T(v^x_F E_x + v^z_F E_z) \int \frac{d^3k}{2\pi^3} \left( \frac{\partial f_{eq}(\epsilon_{k,+})}{\partial \epsilon_{k,+}} + \frac{\partial f_{eq}(\epsilon_{k,-})}{\partial \epsilon_{k,-}} \right).
\tag{43}
\]

This imbalance in turn creates planar Hall effect

\[
\sigma_{xz} = e^2\tau_T v_F^x v_F^z \int \frac{d^3k}{2\pi^3} \left( \frac{\partial f_{eq}(\epsilon_k)}{\partial \epsilon_{k,+}} + \frac{\partial f_{eq}(\epsilon_k)}{\partial \epsilon_{k,-}} \right),
\tag{44}
\]

\[
\frac{\partial f_{eq}(\epsilon_k)}{\partial \epsilon_{k,+}} = \frac{\partial f_{eq}(\epsilon_{k,+})}{\partial \epsilon_{k,+}} + \frac{\partial f_{eq}(\epsilon_{k,-})}{\partial \epsilon_{k,-}}.
\tag{45}
\]

The planar Hall effect arising due to chiral anomaly changes its functional dependence on magnetic field in the presence of tilt velocity, \(n_{xz} \sim B_z\) since the planar Hall effect without tilt velocity is proportional to \(B_x B_z\). Because we have allowed for both \(v^x_F\) and \(v^z_F\) to be non zero, the planar Hall effect derived here exists even for zero magnetic field. For magnetic field along one axis like \(B_z\) the planar Hall effect due to chiral anomaly will vanish. Therefore the tilt velocity provides another fingerprint to detect chiral anomaly via the planar Hall effect.

**RESULTS AND DISCUSSIONS**

In the region of strong magnetic field when the Landau levels are well resolved, the spacing between the two adjacent Landau levels \(\Delta E = \sqrt{2\hbar v_F^2 k_F^2} / eB \sim 10 \text{ meV}\) is much greater than the thermal energy \(k_B T / \Delta E \sim 0.01\) and the impurity broadening \(\Gamma_b / \Delta E \sim 0.01\). In our calculations for the current-current correlation function we have assumed the quasiparticle lifetime \(\tau_q\) is of the same order as the transport lifetime \(\tau_{tr} \sim \tau_q\). In general the calculations of the transport time requires the self-consistent Born approximation, and this can differ by order of magnitudes with the quasiparticle lifetime. This introduces the significant new physics by updating the quasiparticle lifetime with the transport time. However, in the domain of a strong magnetic field where the Landau levels spacing is the largest magnetotransport parameter, the transport and the quasiparticle lifetimes are reasonably of the same order. The case of overlapping Landau levels and intermediate temperature region, where the interaction corrections are important, is discussed in the Refs. [13, 14].

Experimentally the quasiparticle lifetime, Fermi velocity and the effective mass is measured by fitting the De Haas Van Alphen oscillations or the Shubnikov-de Haas oscillations with the Lifshitz-Kosevich formula. By using the experimental data of \(TaP\) for the Fermi velocity \(v_F \sim 0.125 \text{ (eV nm)}\) and Fermi energy \(E_F \sim 0.039 \text{ eV}\), impurity broadening \(\Gamma \sim 0.0023 \text{ eV}\), and the mobility \(\mu \sim 0.36 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}\), we found the quasiparticle lifetime \(\tau_q \sim 2.8 \times 10^{-13} \text{ s}\) and the transport time \(\tau_{tr} \sim 4 \times 10^{-13} \text{ s}\) are of the same order \(\tau_q \sim \tau_{tr}\). The longitudinal and planar Hall resistivities \(\rho_{xz}\), \(\rho_{zz}\) are plotted in the Figs. 3 and 4. The oscillations in the longitudinal resistivity is also shown in the Ref. [15,20]. The oscillations in the longitudinal resistivity is a unique feature of the 3D Dirac and Weyl semimetals. The planar Hall resistivity enters into the magnetoresistivity tensor due to the tilt velocity \(v_F^x\) and \(v_F^z\). The planar Hall resistivity is directly proportional to the longitudinal resistivity \(\rho_{zz} = -v_F^z v_F^x \rho_{zz}\). Therefore, the oscillations in the planar Hall resistivity provide another probe for the detection of the chiral anomaly. The planar Hall effect can be detected in materials like \(TaP\) and \(WTe_2\). By allowing the tilt velocity to make a finite angle in the plane formed by the perpendicular electric and magnetic fields, the planar Hall effect and the Shubnikov de Haas

**FIG. 3.** The longitudinal resistivity \(\rho_{zz}\). The solid black curve is the plot for no tilt in the Dirac and Weyl spectrum, whereas the solid red curve is the plot in the presence of the tilt parameters, \(v_F^x / v_F^z = 0.2\), and \(v_F^z / v_F^x = 0.2\). Here \(\rho_0 = 32 e^2 \beta^2 / v_F^z\), \(\beta = 100 \text{ meV}^{−1}\), and \(\sqrt{\tau_{tr}/\tau_q} = 1 \text{ meV}\).

**FIG. 4.** The planar Hall resistivity \(\rho_{xz}\). The parameters used here are the same as used in the \(\rho_{zz}\).
oscillations can be detected.

CONCLUSIONS

In this study we have investigated the role of tilt velocity in magnetotransport properties. We have shown that all the components of the magnetoconductivity and the magnetoresistivity matrix are modified by the tilt velocity. The longitudinal $\rho_{zz}$, transverse $\rho_{xx}$, Hall $\rho_{xy}$, and planar Hall $\rho_{xz}$ magnetoresistivities resonate whenever the chemical potential crosses the energy band $\mu = \epsilon_n$.

The tilt velocity shifts the position and renormalizes the weight of these peaks. This also mixes the different elements of the resistivity matrix. The planar Hall effect is directly proportional to the tilt velocity.

The planar Hall effect can become fingerprint to detect nontrivial magnetotransport in the 3D Dirac and Weyl gases with the tilted spectrum. The planar Hall effect is directly proportional to the longitudinal resistivity, which oscillates with magnetic field. The oscillations in the planar Hall effect can identify a material that support the tilted Dirac and Weyl spectra with the unique anomalous transport features.

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