Mathematical modeling of a dynamic thin plate deformation in acoustoelasticity problems

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Abstract. The coupled problem of planar acoustic wave propagation through a composite plate covered with a second damping layer with a large logarithmic decrement of oscillations is formulated. The aerohydrodynamic interaction of a plate with external acoustic environment is described by three-dimensional wave equations and the mechanical behavior of a two-layer plate by the classical Kirchhoff-Love model. An exact analytic solution of the problem is found for the case of hinged support of the edges of a plate. On the basis of this, the parameters of the covering damping layer were found, under which it is possible to achieve a practically complete damping of the plate vibration under resonant modes of its acoustic loading.

1. Introduction
At present, noise pollution of the environment is one of the actual problems for Russia and a lot of technologically advanced countries of the world. The magnitude of the permissible vibration of any design is determined by its influence on the strength characteristics of the structure and its elements, on the performance, people’s health, the work of the installed equipment, etc. In terms of providing strength characteristics, one of the most dangerous modes of dynamic deformation of structures is resonance. It is realized in the construction when the frequencies of its natural oscillations coincide with the frequency of external cyclic action. As it is well known, under this loading regime the amplitude values of the parameters of the dynamic stress-strain state are multiplied. A correct and reliable theoretical definition with the accuracy necessary for practical purposes requires proper consideration in the calculated ratios of the damping properties of structural materials caused by internal friction. The magnitude of the permissible vibration of the structure must be limited by the amount of permissible noise that is formed in the surrounding environment of the acoustic environment as a result of its dynamic interaction with the deforming structure. Vibration problems of mechanical systems are mainly handled by specialists in the mechanics of the deformed body, the dynamics and strength of machinery, instruments and equipment, the strength of aircraft, ships, etc., without paying due attention to the problems created by the construction of noise when deforming them. But questions of formation and propagation of noise are studied by experts in the field of acoustics. The literature devoted to the study of these questions is quite extensive (see, e.g., [1–4]). Nevertheless, studies in the field of the acoustoelasticity of thin-walled structural elements are currently among the topical and priority scientific research. Note that the described problems are very
multifaceted, have a complex character and for their solution further experimental and theoretical research is required. In order to reduce the dynamic tension and noise level, many structural elements are made in the form of multilayer structures [5–11]) consisting of a thickness of rigid carriers and soft damping layers. Composite materials based on high-strength carbon or glass fibers have become widely used construction materials for the manufacture of products of a particular purpose by now. They have a sufficiently low level of impact strength, internal friction parameters and shock energy absorption. In connection with this, one of the directions of increasing these properties of structures is their production by gluing from various composite materials. The basis of such materials are traditional composites with carbon or glass fibers, and the protective layer are various types of reinforced or unreinforced elastomers, which have a high impact strength and a high level of absorption of impact energy. In particular, as a protective layer, it is expedient to use composites made of high-strength high-modulus polyethylene fibers (HHMP) with an elastomer matrix. They differ from other high-strength fibers not only by a higher level of specific mechanical characteristics, but also by a minimum coefficient of friction, a positive effect of the strain rate on strength, a sharp increase in strength in the negative temperature region, and other properties. It should be noted that due to poor adhesion with almost many plastics, HHMP were mainly used for making cords and ropes. However, technologies have recently appeared that allow irradiation with cold plasma to increase substantially the adhesion properties of HHMP fibers and to create composites based on them [12–14]. This paper is devoted to the further development of the described direction of research. The main goal is to construct a mathematical model of dynamic deformation of thin plates for small displacements and deformations and allow for internal damping of the layer materials by the Thompson-Kelvin-Voigt model, and also to use this model to solve the problem of sound transmission Wave through a rectangular two-layer plate. As an energy-absorbing coating, elastomeric materials reinforced with rectilinear fibers from HHMP are considered, which have increased values of elastic characteristics while maintaining high values of internal friction parameters. It will be shown that the pasted layers, having high damping properties, allow significantly reducing the amplitude values of deformations and displacements of structural elements of structures under vibration loading conditions, thus creating a low level of sound pressure in the salons. Moreover, the use of such special coatings with large damping properties leads to a multiple decrease in the level of the cyclic stresses formed in the elements and, as a consequence, to a multiple increase in the life (resource) of the structures.

2. The problem statement

We consider a plate consisting of two thin layers with thicknesses \( t_1, t_2 \) and made of orthotropic materials. We assign it to an orthogonal Cartesian coordinate system \( Ox_1x_2z \) in which the coordinate plane \( z=0 \) coincides with the middle plane of the first layer, and the axes \( x_1, x_2 \) coincide with the orthotropy axes of the layers materials. Let \( u_1, u_2, w \) be the displacements of the points of the plane \( \sigma \) in the directions of the axes \( x_1, x_2, z \) through which, according to the Kirchhoff-Love model, at a middle bending of the plate, the components of displacements and deformations of its arbitrary point are determined by the relations

\[
U_i = u_i - zw_j, \quad \varepsilon_{ij} = \varepsilon_{ij}^0 - zw_{ij}, \quad -t_1/2 \leq (t_1/2 + t_2); \quad 2\varepsilon_{ij}^0 = u_{ij} + u_{ij} + w_{ij}, \quad (1)
\]

where the conventional notations for partial derivatives with respect to coordinates \( x_i \) were used. If we assume that the materials of the layers have viscoelastic properties, then to describe dynamic deformation processes that vary in time \( \tau \) according to a harmonic law with circular frequency \( \omega \) the components of the stresses of the \( k \)-th layer with deformation components (1) can be related by defining relations

\[
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\[\sigma_{11}^{(k)} = \frac{E_1^{(k)}}{1-v_{12}^{(k)} v_{21}^{(k)}} \left[ 1 + \frac{\delta_1^{(k)}}{\pi \omega} \frac{\partial}{\partial \tau} \right] \left( \varepsilon_{11}^{(k)} + v_{21}^{(k)} \varepsilon_{22}^{(k)} - z(w_{11}^{(k)} - v_{12}^{(k)} w_{22}^{(k)}) \right) \frac{v_{22}}{v_{21}}; \]
\[\sigma_{12}^{(k)} = 2G_{12}^{(k)} \left[ 1 + \frac{\delta_2^{(k)}}{\pi \omega} \frac{\partial}{\partial \tau} \right] \left( \varepsilon_{12}^{(k)} - z w_{12}^{(k)} \right).\]

These equalities show a generalization of the relations in [15, 16] used in the articles [17–21] for statement of some acoustoelasticity problems of the plate of isotropic material. Here and below \(-t_1 ≤ t ≤ t_2\) for \(k = 1\); \(-t_1 ≤ t ≤ (t_1 + t_2)\) for \(k = 2\); \(\delta_1^{(k)}, \delta_2^{(k)}\) are the logarithmic decrements of oscillations for axial and shear deformations of the material of the \(k\)-th layer, the determination of which is possible on the basis of the technique [22–24]; \(E_1^{(k)}, E_1^{(k)}, E_1^{(k)}, v_{12}^{(k)}\) are the elastic modules of the first and second kinds, and also the Poisson coefficients. With respect to the internal forces and moments brought to the plane, we have
\[T_{11} = B_1 \varepsilon_{11}^{(0)} + \ddot{B} \varepsilon_{22}^{(0)} - B_1^{(2)} H(w_{11}^{(0)} + v_{21}^{(0)} w_{22}^{(0)})/2, \quad T_{12} = 2B_{12} \varepsilon_{12}^{(0)} - B_{12}^{(2)} H w_{12}^{(0)}; \]
\[M_{11} = -B_1^{(2)} H(\varepsilon_{11}^{(0)} + v_{21}^{(0)} \varepsilon_{22}^{(0)})/2 + D_1 w_{11}^{(0)} + \ddot{D} w_{22}^{(0)}, \quad M_{12} = -B_{12}^{(2)} H \varepsilon_{12}^{(0)} + 2D_{12} w_{12}^{(0)}; \]
where
\[H = t_1 + t_2, B_1^{(k)} = \frac{E_1^{(k)} t_k}{1-v_{12}^{(k)} v_{21}^{(k)}}, B_2^{(k)} = 2G_{12}^{(k)} t_k \left[ 1 + \frac{\delta_2^{(k)}}{\pi \omega} \frac{\partial}{\partial \tau} \right];\]
\[B_1 = B_1^{(1)} + B_1^{(2)}, \quad \ddot{B} = B_1^{(1)} v_{21}^{(1)} + B_1^{(2)} v_{21}^{(2)}, \quad B_{12} = B_{12}^{(1)} + B_{12}^{(2)}, \quad \dot{D}_1^{(1)} = B_{12}^{(1)} t_1^{(1)} / 12, \quad D_s = D_s^{(1)} + D_s^{(2)},\]
\[D_s^{(2)} = B_s^{(2)} t_2^{(2)} \kappa / 12, s = 1, 2, D_{12}^{(1)} = B_{12}^{(1)} t_1^{(1)} / 12, D_{12}^{(2)} = B_{12}^{(2)} t_2^{(2)} \kappa / 12, \quad \ddot{D} = D_1^{(1)} v_{21}^{(1)} + D_1^{(2)} v_{21}^{(2)};\]
\[D_{12} = D_{12}^{(1)} + D_{12}^{(2)}, \quad \kappa = 4 + 3t_1^{(1)} / t_2^{(2)} + 6t_1 / t_2.\]

For the element selected from the plate, we write the motion equations
\[T_{x,11} + T_{x,22} - q \ddot{w}_s = 0, q = \eta t_1 + r_t t_2; \quad s = 1, 2;\]
\[M_{11,11} + 2M_{12,12} + M_{22,22} + (T_{11} w_{11} + T_{12} w_{22})_1 + (T_{12} w_{11} + T_{22} w_{22})_2 + q \ddot{w} - p = 0.\]
where the dots over the letters denote the partial derivatives with respect to \(\tau\), \(r_k\) is the density of the material of the \(k\)-th layer, \(p\) is the external aerodynamic load acting on the plate, to be determined. Accounting for this load, along with taking into account the internal friction of the material, is necessary for correct and reliable determination of the parameters of the stress-strain state of structural elements under dynamic loading processes. Below we restrict ourselves to the investigation of two successive stages of deformation of the plate. In the first stage, which is static, a field of stresses in the plate is formed in coordinates \(x_1, x_2\) and equivalent to the forces \(N_{1j}\). It is inhomogeneous, in general. At the second stage, in the vicinity of the stress-strain state of the first stage, a cyclic deformation process is realized with the formation in the plate of such flexural stresses and deformations that have little effect on the generated forces \(N_{1j}\) of the first stage. By virtue of the assumptions made, from the conditions \(T_{1j} = N_{1j}\), which are valid for small displacements, the last relations (3) can be transformed to the form
\[M_{11} = d_1 w_{11} + \ddot{d}_1 w_{22} + g_1 N_{11} + N_{22} N_{11}, \quad M_{12} = 2d_{12} w_{12} - g_{12} N_{12}; \]
\[1, 2.\]
where
\[ d_1 = D_1 - B_1^{(2)} H^2 (G_1 - v_{21}^2 G_2) / (4 B_d), \quad \tilde{d}_1 = \tilde{D} + B_1^{(2)} H^2 (G_1 - v_{21}^2 G_2) / (4 B_d); \]
\[ G_1 = B_1^{(2)} B_2 - v_{12}^{(2)} B_1^{(2)}, \quad \tilde{G}_1 = B_2^{(2)} B - v_{21}^{(2)} B_1^{(2)} B_2, \]
\[ g_1 = -B_1^{(2)} H^2 (B_2 - v_{21}^2 B) / (2 B_d); \]
\[ \tilde{g}_1 = B_1^{(2)} H (\tilde{B} - v_{21}^2 B) / (2 B_d); \]
\[ d_{12} = D_{12} - (B_1^{(2)} H)^2 / (4 B_d); \quad g_{12} = B_1^{(2)} H / (2 B_{12}). \]

Using relations (7), equation (6) is reduced to the following equation, expressed in terms of a function \( \omega \), which in the case \( T_{ij} = N_{ij} = const \) is written in the form
\[ d_1 w_{1111} + (\tilde{d}_1 + \tilde{d}_2 + 4d_{12}) w_{1122} + d_2 w_{2222} + N_{11} w_{11} + 2N_{12} w_{12} + N_{22} w_{22} - p + q \tilde{w} = 0. \]

The boundary and initial conditions for the obtained equation (9) are formulated in the same form as in the classical theory of plates. In accordance with the assumptions made, the total displacements of the second (dynamic) stage of deformation will be equal \( w^* = w + \Delta w \), \( u_i^* = u_i + \Delta u_i \), where \( \Delta w \), \( \Delta u_i \) are small in comparison with \( w \) and \( u \) increments of displacements. In this case for the deformation components we have the following relations
\[ \kappa_{ij}^* = w_{ij} + \Delta w_{ij}, \]
\[ 2 \epsilon_{ij}^* = u_{i,j} + u_{j,i} + w_{i,j} + \Delta u_{i,j} + \Delta u_{j,i} = 2 \epsilon_{ij} + \Delta \epsilon_{ij}, \]
where
\[ M_{ij}^* = M_{ij} + \Delta M_{ij}, \quad T_{ij}^* = T_{ij} + \Delta T_{ij}. \]
The values of \( \Delta T_{ij} \) have to satisfy the conditions
\[ \Delta T_{ij} = B_1 \Delta \epsilon_{11} + \tilde{B} \Delta \epsilon_{22} - B_1^{(2)} H (\Delta w_{11} + v_{21}^{(2)} \Delta w_{22}) / 2 = 0, \quad \Delta T_{12} = 2B_{12} \Delta \epsilon_{12} + B_1^{(2)} H \Delta w_{12} = 0 \]
\[ \Delta M_{ij} = -B_1^{(2)} H (\Delta \epsilon_{12}^0 + v_{21}^{(2)} \Delta \epsilon_{22}^0) / 2 + D_1 \Delta w_{11} + \tilde{D} \Delta w_{22}, \quad \Delta M_{12} = -B_1^{(2)} H \Delta \epsilon_{12}^0 + 2D_{12} \Delta w_{12}. \]

Determining from the conditions (10) the quantities \( \Delta \epsilon_{11}^0, \Delta \epsilon_{22}^0, 2 \Delta \epsilon_{12}^0 \) and inserting them into relations (11), we arrive at the relations
\[ \Delta M_{11} = d_1 \Delta w_{11} + \tilde{d}_1 \Delta w_{22}; \quad \Delta M_{12} = 2d_{12} \Delta w_{12}, \]
in which the coefficients \( d_1, \tilde{d}_1, d_{12} \) are determined by the formulas (8). It can be shown that in order to determine the function \( \Delta w \) in the approximation under consideration it is necessary that the variational equation of the form
\[ \int_{\tau_0}^{\tau_1} \int_{\sigma} \left[ N_{ij} \Delta w_{ij} \sigma(\Delta w_{ij}) + (d_1 \Delta w_{11} + \tilde{d}_1 \Delta w_{22}) \sigma(\Delta w_{11}) + (\tilde{d}_2 \Delta w_{11} + d_2 \Delta w_{22}) \sigma(\Delta w_{22}) + 4d_{12} \Delta w_{12} \sigma(\Delta w_{12}) - (\rho \sigma q \Delta \tilde{w}) \sigma(\Delta w) \right] dx_1 dx_2 d\tau = 0, \]
is satisfied. From the last equation, under assumption \( N_{ij} = const \), the motion equation (9) follows, if \( \Delta w \) is replaced by \( w \) in it.

3. Construction of solution
Suppose that the plate is rectangular in plan with sizes \( a, b \) in the axes \( x = x_1, \ y = y_1 \) directions, and is surrounded on both sides by the acoustic media "1" and "2" occupying the half-spaces \( V_1 \) and \( V_2 \) bounded by the plane \( z = 0 \). A plane harmonic wave, characterized by pressure \( p_\omega \) and frequency \( \omega \), is incident on the plate. As a result of its interaction with the plate in the surrounding half-spaces \( V_1 \) and
$V_2$, acoustic waves are excited, which are reflected and radiated in the first medium and radiated in the second medium. These waves with respect to the velocity potentials $\Phi_1$, $\Phi_2$ are described by the wave equations, which we write in the approximation (here and below $k=1,2$)

$$\Phi_{1,zz} - c_1^2 \Phi_1 = 0, \quad \Phi_{2,zz} - c_2^2 \Phi_2 = 0,$$

(12)

where $c_k$ is the sound velocities in media "1" and "2". Through functions $\Phi_1$, $\Phi_2$, the pressures $p_1$, $p_2$, and velocity components $v_{z1}$, $v_{z2}$ in half-spaces $V_1$ and $V_2$ are determined by the relations ($\rho_k$ are the densities of media "1" and "2")

$$p_1 = -\rho_1 \Phi_1, \quad v_{z1}^* = \Phi_{1,z}, \quad p_2 = -\rho_2 \Phi_2, \quad v_{z2}^* = \Phi_{2,z}.$$

(13)

In this case, the aerohydrodynamic load $p$ acting on the plate will be equal to

$$p = (p_1 + p_2)|_{z=0},$$

(14)

and the velocity components $v_{z1}^*$, $v_{z2}^*$ for the uninterrupted interaction of acoustic media with the plate, have to satisfy the conditions

$$\dot{w} = (v_{z1}^* + v_{z2}^*)|_{z=0}, \quad \dot{w} = v_{z1}^2|_{z=0}. $$

(15)

For harmonic waves, the following representations ($i$ is the imaginary unit) are valid

$$\Phi_1 = \Phi_1 e^{i\omega t}, \quad \Phi_2 = \Phi_2 e^{i\omega t}, \quad w = \Phi e^{i\omega t},$$

(16)

and for a plate hingedly supported at the edges $x=0$, $x=a$, $y=0$, $y=b$ the solution of equation (9) will have the form

$$w = \sum_{m,n=1,3,...} \tilde{w}_{mn} \sin \lambda_m x \sin \lambda_n y e^{i\omega t},$$

(17)

where $\lambda_m = m\pi/a$, $\lambda_n = n\pi/b$; $m,n = 1,3,...,M,N$. By virtue of (15), the solutions of the last two equations in (12) must be sought in the form

$$\Phi_k = \sum_{m,n=1,3,...} \tilde{\Phi}_{mn} e^{i\omega t},$$

(18)

Taking into account the representations (16)–(18), equations (12) are transformed to the form

$$\Phi_{1,zz} + k_1^2 \Phi_1 = 0, \quad \Phi_{2,zz} - (\kappa_k^{mn})^2 \Phi_{mn} = 0,$$

(19)

where

$$\kappa_k^{mn} = \sqrt{\lambda_m^2 + \lambda_n^2 - k_k^2}, \quad k_k^2 = \omega^2 / c_k^2, $$

(20)

In view of (16), the first equation for the incident wave has a solution $\Phi_1 = A_1 e^{-ik_1 z}$, using which, in accordance with relations (13), we arrive at the dependences

$$p_1|_{z=0} = \tilde{p}_1|_{z=0} e^{i\omega t}, \quad \tilde{p}_1|_{z=0} = -i\rho_1 A_1, \quad v_{z1}^* = \tilde{v}_{z1}^*|_{z=0} e^{i\omega t}, \quad \tilde{v}_{z1}^*|_{z=0} = -ik_1 A_1,$$

(21)

where $A_1$ is an integration constant, uniquely determined for given frequency $\omega$ and sound pressure level $p_1|_{z=0}$. Later, when performing the calculations, we shall consider a parameter $A_1$ to be given, the value of which determines the pressure and the velocity of the incoming wave. The solution of the remaining equations (19) depends on the sign of $(\kappa_k^{mn})^2$. In the case $(\kappa_k^{mn})^2 > 0$ we take them in the form [18–20]

$$\Phi_1^{mn} = B_1^{mn} e^{i\kappa_k^{mn} z}, \quad \Phi_2^{mn} = A_2^{mn} e^{-i\kappa_k^{mn} z} \text{ if } (\kappa_k^{mn})^2 > 0, \quad \Phi_1^{mn} = \Phi_1^{mn} e^{ik_1 z}, \quad \Phi_2^{mn} = A_2^{mn} e^{-ik_1 z} \text{ if } (\kappa_k^{mn})^2 < 0.$$
where \((\widetilde{\kappa}_{mn}^*)^2 = -(\kappa_{mn}^*)^2\), \(B_1^{mn}\), \(A_2^{mn}\) are the integration constants. We expand \(A_\omega\) in a Fourier series

\[
A_\omega = \sum_{m,n=1,3,...}^{M,N} f_{mn} \sin \lambda_m x \sin \lambda_n y \, e^{i \omega \tau}, \quad f_{mn} = 16/(\pi^2 mn)
\]

and the resulting solutions (22) and equation (17), (21) we subject the conditions (15). As a result, to determine the integration constants, we obtain the dependences

\[
B_1^{mn} = i \omega (f_{mn}/c_1 + \mathcal{w}_{mn})/\kappa_{mn}, \quad A_2^{mn} = -i \omega \mathcal{w}_{mn}/\kappa_{mn}^*.
\]

Here and in the future, the quantities \(\kappa_{mn}^*\) are calculated from formulae (20), if \(\lambda_m^2 + \lambda_n^2 - k_k^2 > 0\), and by the formulae \(\kappa_{mn}^{*2} = \sqrt{k_k^2 - \lambda_m^2 - \lambda_n^2}\) for \(\lambda_m^2 + \lambda_n^2 - k_k^2 < 0\). Using the dependences (24) to determine \(p\), in accordance with relations (13), (14), (16), (18), (21)–(23), one can obtain relations

\[
p = \sum_{m,n=1,3,...}^{M,N} \mathcal{p}_{mn} \sin \lambda_m x \sin \lambda_n y \, e^{i \omega \tau},
\]

where \(\mathcal{p}_{mn} = -i \rho_1 \omega f_{mn}[1 + t \omega (c_1 \kappa_{mn})] A_\omega + \phi_{mn} w_{mn}, \quad \phi_{mn} = \rho_1 \omega^2 / \kappa_{mn} + \rho_2 \omega^2 / \kappa_{mn}^*\). If \(N_{12} = 0\), then the substitution of relations (17) and (25) into the motion equation (9) leads to the solution

\[
\mathcal{w}_{mn} = -R_{mn} A_\omega / L_{mn}, \quad R_{mn} = i \rho_1 \omega f_{mn}[1 + t \omega (c_1 \kappa_{mn})], \quad L_{mn} = d_{mn},
\]

where \(d_{mn} = d_1^* \lambda_m + (d_1^* + \bar{d}_2^* + 4d_{12}^*) \lambda_n^2 + d_2^* \lambda_n^2 - N_{11} \lambda_m^2 - N_{12} \lambda_n^2 - q_0 \omega^2, \quad d_1^*, \ d_2^*, \ \bar{d}_1^*, \ \bar{d}_2^*, \ d_1^*\) are calculated by (4), (8) with the replacement of \(B_s^{(k)}, \ B_{12}^{(k)}\) by \(B_s^{(k)\ast} = E_s^{(k)} t_{12}^1 (1 + i \delta_s^{(k)}) / \pi / (1 - \nu_s^{(k)} v_{21}^{(k)}), \ B_{12}^{(k)\ast} = G_{12}^{(k)} t_{12}^1 (1 + i \delta_{12}^{(k)}) / \pi\). In accordance with the obtained solution (26), the amplitude values of the sound pressures at the points of the boundary planes \(z = -t_1 / 2\) and \(z = -t_1 / 2 + t_2\) of the plate will be equal

\[
\mathcal{P}_1 = -i \rho_1 \omega \sum_{m,n=1,3,...}^{M,N} [f_{mn}[1 + t \omega (c_1 \kappa_{mn})] - i \omega R_{mn} / (\kappa_{1mn} L_{mn})] A_\omega, \quad \mathcal{P}_2 = \rho_2 \omega \sum_{m,n=1,3,...}^{M,N} R_{mn} A_\omega / (\kappa_{2mn} L_{mn}).
\]

The given value of \(A_\omega\) and the values of \(\mathcal{P}_1, \mathcal{P}_2\) found, the sound-insulating properties of the plate and the sound pressure levels at the points of the boundary planes will be characterized by the following parameters (\(\mathcal{P}_0\) is the amplitude value of the sound pressure corresponding to the audibility threshold):

\[
R_s^0 = -20 \log |\mathcal{P}_2 / \mathcal{P}_1|, \quad R_s^0 = -20 \log |\mathcal{P}_1 / \mathcal{P}_0|, \quad R_s^0 = -20 \log |\mathcal{P}_1 / \mathcal{P}_0|,
\]

and the amplitude values of the dynamic components of the internal bending moments formed in the plate from the action of the sound wave incident on it will be determined from formulae of the form

\[
M_{11}^{\delta} = \sum_{m,n=1,3,...}^{M,N} [(d_1^* \lambda_m^2 + \bar{d}_2^* \lambda_n^2) R_{mn} / L_{mn}] A_\omega, \ldots, \text{ which follow from relations (7), (17) (26).}
\]

4. Results and discussion

On the basis of the constructed solutions for a plate made of fiberglass and having characteristics \(a = 0.48\, \text{m}, \ b = 0.56\, \text{m}, \ t_1 = 2\, \text{mm}, \ E_1^{(1)} = 18\, \text{GPa}, \ E_2^{(1)} = 14\, \text{GPa}, \ \nu_1^{(1)} = 0.3\, \text{GPa}, \ C_{12}^{(1)} = 2.9\, \text{GPa}, \ \delta_1^{(1)} = 0.03, \ \delta_2^{(1)} = 0.035, \ \delta_{12}^{(1)} = 0.03, \ r_i = 3000\, \text{kg/m}^3, \ \text{at} \ \mathcal{P}_0 = 2 \cdot 10^{-5}\, \text{Pa}, \ A_\omega = 1, \ N_{12} = 0\) and at different frequencies \(f\) of the sound wave, calculations were performed to determine the amplitude
values of the acoustic insulation parameters (27), deflection \( w_0 \), and the dynamic component of the internal bending moment \( M_{11}^0 \). The results of calculations for a single-layer plate \( (t_2 = 0) \) and a double-layer plate \( (t_2 = 5 \text{ mm}) \) with a damping layer of rubber having the characteristics \( E_1^{(2)} = E_2^{(2)} = 500 \text{ MPa}, \nu_{12}^{(2)} = \nu_{21}^{(2)} = 0.4, \delta_1^{(2)} = \delta_2^{(2)} = 1.2, \delta_{12}^{(2)} = 0.9, r_2 = 1500 \text{ kg/m}^3 \) are shown in figure 1, 2. The solid lines correspond to the \( t_2 = 0 \), dashed lines correspond to the \( t_2 = 5 \text{ mm} \). Analyzing the obtained results, we can conclude that the presence of a damping layer on the plate, which has the considerable logarithmic decrements of oscillations values, practically does not affect the characteristic \( R_p^- \). Depending on the frequency of the incident sound wave, the existence of a layer can lead both to a multiple increase, and to a certain decrease (at resonant frequencies) of the sound insulation parameter \( R_p^0 \) (figure 1a). In wide ranges of frequency changes, on the average, it leads to a significant decrease in the characteristic \( R_p^+ \) (and, consequently, the sound pressure level in the half-space \( V_2 \) (figure 1b)). Due to the introduction of a damping layer, it is possible to reduce significantly the values of the vibro-displacement (figure 2a) and the bending moment (and, consequently, the stresses) (figure 2b).

![Figure 1](image1.png)

**Figure 1.** Dependences of the soundproofing parameter (a) and sound pressure parameter (b) on the frequency of oscillation

![Figure 2](image2.png)

**Figure 2.** Dependences of amplitude values of deflection (a) and amplitude values of the bending moment (b) on the frequency of oscillations.

The analysis of the obtained results let us formulate the conclusion that functional rubber-like materials, pasted on power thin-walled structural elements in the form of a second thin layer and possessing high internal damping properties (and, consequently, energy absorbing properties) allow to
reduce significantly the amplitude values of deformations and displacements at vibrational (even resonant) modes of their loading, thus forming a lowered level of radiated sound pressure. The use of such special coatings with large values of the logarithmic decrement of vibrations leads to a multiple decrease in the level of the cyclic stresses formed in the elements and, as a result, to a multiple increase in the life (resource) of the structures.

In conclusion, we note that the obtained results can be used to establish and study practical problems of acousto-elasticity. They are important from environmental safety point of view, and connected, in particular, with the design of noise-shielding screens.

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