Proton Decay and the Dimopoulos-Wilczek Mechanism in
Minimal SO(10) Models

S. Urano and R. Arnowitt

Center for Theoretical Physics, Department of Physics, Texas A&M University, College Station,
TX 77843-4242

Abstract

Proton decay is examined within the framework of certain SO(10) models which have low dimension representations compatible with the recently found constraints coming from string theory, and which use the Dimopoulos-Wilczek mechanism to solve the doublet-triplet splitting problem. It is found that the mass parameter that controls proton decay rate is intimately related to the low energy data via the coupling unification condition, and that a suppression of proton decay is achieved at the cost of large threshold effects. Furthermore, in cases where there are states with intermediate masses, the threshold effects further enhance proton decay. The experimental bound then severely constrains the parameter space of these models. Some possible solutions are suggested.

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SO(10) is an attractive gauge group for grand unification. It may be thought of as arising via gauging the global B-L symmetry of SU(5) models. In SO(10) models, all the quarks and leptons for each generation are unified into a single representation, 16, and since SO(10) observes left-right symmetry, anomaly cancellation is automatic. Also, the Yukawa couplings for the members of a family (e.g., top, bottom, and τ for the third generation) must be the same at the GUT scale and this has allowed a successful prediction for the mass of the top quark, provided that the value of tan β is large, ~ 50. Another nice feature of SO(10) is the existence of a way to avoid the doublet-triplet splitting problem via the so-called “Dimopoulos-Wilczek mechanism.”

A drawback to SO(10) models in the past has been the largeness of the particle content, due to the use of large representations such as 126 and 210. From the viewpoint that a grand unified theory (GUT) is an effective field theory of some Planck scale theory which is perturbative below the Planck scale (e.g., a string model), it is desirable not to have a large number of chiral multiplets since these then would drive the unified coupling constant upwards, exceeding the perturbative bound before reaching the Planck scale. This desire is consistent with recent results in free field heterotic string models, where it has been shown that no massless SO(10) representations larger than 54 can arise, regardless of the affine level at which SO(10) is realized.

There are now several SO(10) models where the Dimopoulos-Wilczek mechanism is employed while using only small particle content. Here, we will refer to these as “minimal” SO(10) models. In this letter, we present two new results that are relevant to proton decay in these minimal SO(10) models, gotten by imposing the coupling constant unification condition. First we show that, contrary to what is found in the current literature, having a mixing via the minimal $2 \times 2$ mass matrix among the color triplet Higgses does not of itself give rise to a new degree of freedom which can be varied in order suppress the proton decay rate. Second, we show that the intermediate ($\sim 10^{10}$ GeV) mass states which often arise in these models, coming from weakly coupling the two gauge breaking sectors, leads to an enhancement of proton decay rate. These results, combined with the fact that proton decay
is already enhanced considerably in SO(10) models due to the largeness of $\tan \beta$, pushes the values of the parameters of these models into unnatural ranges when the experimental bound on proton decay is taken into account. Below, we will demonstrate these points within a general framework. We will show elsewhere a more explicit demonstration of these points using the specific model by Babu and Barr [5].

First we consider proton decay. In minimal SO(10) models, there are two $10$’s of Higgses, each of which decomposes under SU(5) as $5 + \overline{5}$. Generally, if there is more than one $5 + \overline{5}$ pair, there is a possibility of mixing between the triplet partners of the light doublets, that couple to the quarks and leptons, via a mass matrix. When such a mixing occurs, the superpotential may be written schematically as $\tilde{H}_1 J + \tilde{K} H_1 + \mathcal{M}_{IJ} \tilde{H}_I H_J$, where the $5 + \overline{5}$ pairs are denoted by the indices $(I, J)$, and we have chosen a basis where $H_1$ and $\tilde{H}_1$ are the Higgs which couple to the quarks and leptons. The quark and lepton currents are denoted by $J$ and $\tilde{K}$ and $\mathcal{M}$ is the mass matrix. Integrating out $H_1$ then gives $-\tilde{K}(\mathcal{M}^{-1})_{11} J$.

Using a somewhat less bulky notation, $1/M_{PD} \equiv (\mathcal{M}^{-1})_{11}$, we then find two baryon number violating dimension four terms in the superpotential suppressed by $M_{PD}$, identical to the usual minimal SU(5) case with the mass of the color triplet, $M_{H_3}$, replaced with $M_{PD}$.

The actual calculation to estimate the proton decay rate is quite complicated and will not be repeated here. An extensive calculation was presented in [7,8], where the $p \rightarrow \bar{\nu}K^+$ decay rate was given by $\Gamma(p \rightarrow \bar{\nu}K^+) = C(\beta_p/M_{PD})^2 |AB_i|^2$. Here, $C$ and $\beta_p$ depend upon strong interaction parameters, while $A$ depends on electroweak ones. The quantity $B_i$ represents the chargino-squark-squark triangle dressing loop, and includes the SUSY mass dependences. We note here that there is a sin $2\beta$ factor in its denominator. Thus, the the decay rate is proportional to $1/(M_{PD} \sin(2\beta))^2$ which, for large $\tan \beta$, is $\sim (\tan \beta/M_{PD})^2$. An analysis, running over the SUSY mass parameter space gives, from Kamiokande data, $M_{PD} > 1.2 \times 10^{16}$ GeV where the bound involves a lower bound of $\tan \beta$ to be $\sim 1.4$ [8] which comes from the top Yukawa developing a Landau pole at the GUT scale. Noting the $\tan \beta$ dependency in the decay rate, we then require
\[ M_{PD} > \tan \beta \times 0.57 \times 10^{16} \text{ GeV}. \] (1)

Since one expects \( M_{PD} \) to be of order of the GUT scale, \( \sim 2.4 \times 10^{16} \text{ GeV} \), this bound poses a serious problem to SO(10) models for which \( \tan \beta \) is expected to be \( O(50) \), i.e., one must arrange to have \( M_{PD} \) significantly larger than the GUT scale.

In the models we are studying, the Dimopoulos-Wilczek (DW) mechanism is used to create the doublet-triplet splitting. The basic idea is to achieve a breaking of SO(10) via growing a VEV for a 45 of the form: \( \langle A \rangle = \text{diag}(a, a, a, 0, 0) \otimes i\sigma_2 \) where A is the 45 in the 10 \( \times \) 10 anti-symmetric tensor notation. This VEV breaks SO(10) down to the \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \times \text{U}(1) \) subgroup where the extra U(1) is identified with the B−L symmetry. The extra U(1) is broken in minimal models via VEV growth for a 16+\( \overline{16} \) pair. In the one-step unification picture, we would expect \( a \) to be \( O(M_{GUT}) \).

If we now couple the 45 to a pair of 10’s in a natural way and have a mass term only for one of the 10’s, then the superpotential will be \( W_H = M_2H_2^2 + \Lambda H_1AH_2 \) where \( H \)'s are the 10’s. Substituting in the VEV for \( A \), the mass terms for the doublet parts (indicated by superscript \( d \)) and the triplet parts (indicated by superscript \( t \)) are then

\[
\begin{pmatrix}
\bar{H}_1^d & \bar{H}_2^d \\
0 & 0 \\
0 & M_2
\end{pmatrix}
\begin{pmatrix}
H_1^d \\
H_2^d
\end{pmatrix}
\] (2)

and

\[
\begin{pmatrix}
\bar{H}_1^t & \bar{H}_2^t \\
0 & \Lambda a \\
\Lambda a & M_2
\end{pmatrix}
\begin{pmatrix}
H_1^t \\
H_2^t
\end{pmatrix}
\] (3)

Here the bar in the \( \bar{H} \) indicate that it is part of the \( 5 \) under the SU(5) decomposition, \( 10 = 5 + \bar{5} \). Thus one of the doublets is automatically massless. These mass matrices also suggest the possibility of suppressing proton decay by adjusting the parameters \( \Lambda \) and \( M_2 \). The massive parameter which controls Higgsino-mediated proton decay is given by

\[ 1/M_{PD} \equiv (\mathcal{M}^{-1})_{11} = M_2/(\Lambda^2a^2), \] (4)
where $\mathcal{M}$ is the triplet mass matrix. By choosing $M_2$ to be smaller than $\Lambda a$ (expected to be $O(M_{\text{GUT}})$), it appears possible to adjust $M_{\text{PD}}$ so that it is significantly larger than $M_{\text{GUT}}$. However since these parameters are also correlated to the mass spectrum of the model, which affect the coupling constant running, we must check if this suppression does not ruin the coupling unification picture. This is the task we now turn to.

In order to discuss the running of the coupling constants, it is convenient to introduce the variables: $y_i \equiv 1/\alpha_i \equiv 4\pi/g_i^2$ and $x \equiv 1/2\pi \ln(\mu/M_Z)$. Then the RGE’s ($\mu \partial g_i/\partial \mu = \beta_i$) can be written, to two-loop order, as

$$\frac{dy_i}{dx} = b_i + \frac{1}{4\pi} \sum_j b_{ij} 1/y_j,$$

(5)

where the $b_i$ are the one-loop $\beta$ coefficient and $b_{ij}$ are the two-loop coefficients.

The one-loop solutions are just straight lines in the $x$-$y$ plane with the slopes given by the one-loop coefficient. The thresholds corrections in mass-independent renormalization schemes are then just a change in the slope at the corresponding $x$’s. The $\beta$ coefficients for arbitrary gauge theories may be found in [9].

The RGE’s can not be solved exactly at two-loop, but the solution can be approximated by iterating the one-loop result. The error introduced in this approximation is then expected to be the size of the next-order correction. The threshold effects at two-loop are expected to be small. In the numerical results below, we have considered it in two situations: (1) when the threshold involves many particles so that the change in the two-loop $\beta$ coefficients is large, and (2) when the threshold involves a large hierarchy in the energy scales (i.e. when there exists an intermediate scale).

The condition for unification can be stated in the $\overline{\text{DR}}$ scheme as simply the coming together of the coupling constants to one point [10]. The threshold corrections would then make the running of all the coupling constants the same. Calling the running coupling constant in the GUT to be $y_{\text{GUT}}$, the unification condition is then $y_{\text{GUT}}(x) = y_i(x), \quad x \geq x_U$, where $x_U$ is the highest threshold. Adding in the threshold corrections at the one-loop level can be done with just simple analytical geometry, and we arrive at
\begin{equation}
y_{\text{GUT}}(x) = y_i(0) + b_{\text{GUT}}x + \delta y_i^2 - \sum_a b_{ai}x_a,
\end{equation}

where \(\delta y_i^2\) denotes the two-loop level contribution, \(y_i(0)\) are the inverses of the coupling constants measured at \(M_Z\), \(b_{\text{GUT}}\) is the one-loop \(\beta\) coefficient in the unbroken GUT, and \(x_a \equiv \frac{1}{2\pi}\ln\left(\frac{M_a}{M_Z}\right)\) where \(M_a\)'s are the threshold masses.

Eq. 6 represents three constraints on the parameters of a GUT model. Of these, we always need to assign one to solve for \(y_{\text{GUT}}\) whose experimental constraint is less stringent than the constraint we can impose on it by grand unification. Thus there are two constraints upon the remaining parameters of a model. We can describe these constraints in terms of three-dimensional vectors which we may dot into the unification condition. It turns out, from the study of \(\beta\) coefficients, that the two useful vectors are \((1, -3, 2)\) and \((-5, 3, 2)\), where the first one is relevant for \(M_{\text{PD}}\).

We consider first the minimal SU(5) model, the superheavy particles are the \((2\bar{3}, -5/3 + \text{h.c.})\) components of SU(5) gauge fields (vector multiplet) in the \(24\) representation (V), the \((3, 1) + (1, 8)\) components of the Higgs in the \(24\) (A), and the color triplet Higgs \((1, 3) - 2/3 + \text{h.c.}) of the Higgs in the \((5 + \bar{5})\) (H). We have used the notation (SU(2) rep., SU(3) rep.)\text{Hypercharge} to show the representations under the Standard Model gauge group. The three superheavy threshold masses are labeled \(M_V\), \(M_A\), and \(M_{H_3}\). The proton decay mass parameter, \(M_{\text{PD}}\) is here just \(M_{H_3}\). Dotting the \((1, -3, 2)\) vector into the unification condition, Eq. 6 yields the necessary condition for \(M_{\text{PD}}\):

\begin{equation}
\frac{M_{H_3}}{M_{H_3}^{(0)}} = \frac{M_{\text{PD}}}{M_{H_3}^{(0)}} = \left(\frac{M_{\text{SUSY}}}{M_Z}\right)^{5/6}
\end{equation}

where \(M_{H_3}^{(0)}\) is a quantity which is determined mainly by the low energy measurement of the coupling constant, i.e. it is what the mass of the Higgs triplet would be for minimal SU(5) when SUSY threshold corrections are neglected \((M_{\text{SUSY}} = M_Z)\). It can be determined by:

\begin{equation}
\frac{1}{2\pi} \ln \frac{M_{H_3}^{(0)}}{M_Z} = \frac{1}{4\alpha(M_Z)}(6\sin^2(\theta_W) - 1) - \frac{5}{6\alpha_3(M_Z)} + \frac{1}{2} \sum_i (1, -3, 2)\delta y_i^2.
\end{equation}
This quantity is plotted in Fig. 1 in the \(\sin^2(\theta_W) - \alpha_3(M_Z)\) plane, using the estimated two-loop correction by setting the GUT scale to be \(M_{PD} = M_{H_3}\). (We have used \(\alpha(M_Z) = 1/127.9\).) The right hand side of Eq. (7) is not expected to be large, i.e. \(O(1)\), and thus Fig. 1 may be viewed as bounds on \(M_{H_3}\) as a function of \(\sin^2(\theta_W)\) and \(\alpha_3(M_Z)\). The proton decay constraint of \(M_{H_3} > 1.2 \times 10^{16}\) GeV (for the minimal SU(5), i.e. small \(\tan \beta\) indicated by the dotted line can be seen to push the satisfactory \(\alpha_3(M_Z)\) and \(\sin^2(\theta_W)\) values to the corner of the experimentally allowed region. This is in agreement with the well-known result that it is difficult to achieve low values of \(\alpha_3(M_Z)\) in the minimal SU(5) model [11,12].

We are interested in obtaining a similar restriction on \(M_{PD}\) for SO(10). For the models with two 10’s where the doublet-triplet splitting is done by DW mechanism, we have seen that \(M_{PD} = (\Lambda^2 a^2)/M_2\) (Eq. 4). From the mass matrices in Eqs. 2 and 3, we see that \(M_{PD}\) is the product of the triplet masses eigenstates divided by the heavy doublet mass. Or, using \(T_3\) and \(H_3\) to denote the color triplet eigenstates, and \(T_2\) to denote the heavy doublet eigenstate, \(M_{PD} = M_{T_3}M_{H_3}/M_{T_2}\). This turns out to be the same combination of the threshold masses which appear in the unification condition gotten by using the vector \((1, -3, 2)\). Neglecting for the moment the threshold effects due to the splittings among other superheavy particles, we find that the analogue of Eq. 7 is:

\[
\left( \frac{M_{H_3}}{M_{H_3}^{(0)}} \right) = \left( \frac{M_{\text{SUSY}}}{M_Z} \right)^{5/6} \left( \frac{M_{T_2}}{M_{T_2}} \right),
\]

or,

\[
\left( \frac{M_{PD}}{M_{H_3}^{(0)}} \right) = \left( \frac{M_{\text{SUSY}}}{M_Z} \right)^{5/6},
\]

which is exactly the same as in the minimal SU(5) case. Thus, having a DW derived mass matrix mixing does not offer an advantage over the minimal SU(5) case in suppressing proton decay rate since it is precisely the same combination of parameters which affects proton decay that is constrained by the unification condition.

A loosening of the constraint on \(M_{PD}\) can come from threshold effects arising from the splittings amongst other superheavy particles in a model. In general, minimal SO(10) models
contain vector multiplets in the adjoint (45) representation, plus chiral multiplets in 45, 54, 16 + \overline{16}, 10, and singlet representations. Upon breaking to the standard model gauge group, the massive components that can affect the running of the coupling constants are: the vector multiplets in ( (2,3)_{-5/3} + \text{h.c.} ) [V], ( (1,1)_2 + \text{h.c.} ) [V_1], ( (1,3)_{-4/3} + \text{h.c.} ) [V_3], and ( (2,3)_{1/3} + \text{h.c.} ) [V_6] representations; and the chiral multiplets in (3,1)_0 [A_3], (1,8)_0 [A_8], ( (3,1)_2 + \text{h.c.} ) [D_3], ( (2,3)_{1/3} + \text{h.c.} ) [D_6], ( (1,6)_{-4/3} + \text{h.c.} ) [D'_6], ( (1,1)_2 + \text{h.c.} ) [B_1], ( (1,3)_{-4/3} + \text{h.c.} ) [B_3], ( (2,3)_{1/3} + \text{h.c.} ) [B_6], ( (2,1)_1 + \text{h.c.} ) [C_2], and ( (1,3)_{-2/3} + \text{h.c.} ) [C_3] representations where the labels we will use for them are indicated within the square braces. In addition, there are two triplets and a doublet \((H_3, T_3, T_2)\) coming from the two 10’s. The A’s come from the 45, the D’s from the 54, the B’s are a combination of the 45 and the 16 + \overline{16}, and the C’s are from the 16 + \overline{16}. When the unification condition is imposed, we find then that:

\[
\frac{M_{PD}}{M_{H_3}^{(0)}} = \left(\frac{M_{\text{SUSY}}}{M_Z}\right)^{5/6} \left(\frac{M_{V_1} M_{Y_3}^3}{M_{V_6}^3}\right) \left(\frac{M_{A_3}}{M_{A_8}}\right)^{5/2} \times \left(\frac{M_{B_6}^4}{M_{B_1} M_{B_3}^3}\right)^{1/2} \left(\frac{M_{C_2}}{M_{C_3}}\right) \left(\frac{M_{D_4}^7 M_{D_6}^4}{M_{11}^1 D_6^6}\right)^{1/2}. \tag{11}
\]

In the one-step unification picture, the threshold corrections are expected to be small. In certain SO(10) models, on the other hand, the necessity to not badly destabilize the DW form of VEV is satisfied by weakly coupling the two gauge breaking sectors which in turn leads to introduction of of intermediate mass scales \[\overline{E}\]. In these models, the massive states \(B_3\) and \(B_6\) are of intermediate scale while all the other states (including \(B_1\)) are at the GUT scale. Thus for these models, the threshold effect in fact enhances the proton decay rate.

Now we turn to the possible solutions to the problems raised here. The results above point out how the proton decay rate is intimately related to the threshold effects in these SO(10) models. Bringing these models into agreement with experiment hence require large threshold effects. One is then faced with having to explain two large effects: (1) why the value of \(M_{PD} = M_{T_3} M_{H_3}/M_{T_2}\) is much larger than \(M_{GUT}\), and (2) why the threshold effects (the right hand side of Eq. (11) is also large, in the right direction, and of the correct size to explain the ratio \(M_{PD}/M_{H_3}^{(0)}\).
One possible way to explain both of these effects is simply to adjust the values of the threshold masses \[13\]. If we take for granted that item (1) is solved by adjusting \(M_{PD}\) to be large enough to satisfy Eq. [1] for \(\tan \beta \approx 50\), then item (2) can be accomplished by a threshold effect to produce a shift in \(\alpha_3(M_Z) > 13\%\), or \(\gtrsim 5\) std. as can be seen from Fig. 1. If there are intermediate mass states as discussed above, it provides a threshold effect in the wrong direction, and the remnant threshold effect must then be \(\gtrsim 40\%\), or \(\gtrsim 16\) std. in order to bring the ratio \(M_{PD}/M^{(0)}_{H_i}\) into agreement with experiment. These are to be contrasted with the “successful” prediction of \(\alpha_3(M_Z)\) (within \(\lesssim 4\%\) or \(\lesssim 2\) std.) within the framework of supersymmetric grand unification where thresholds are neglected. Thus the naturalness of grand unification in such models is eroded.

Perhaps a more natural solution would be to introduce a mechanism which suppresses proton decay in a different way, so that it is not strongly dependent on threshold masses. For example, with three \(10\)'s, three \(45\)'s and two \(54\)'s, Babu and Barr have shown it is possible to strongly suppress proton decay \[14\]. Such models then require a much larger particle spectrum.

In conclusion, we have shown that the proton decay constraint plays an important role for minimal SO(10) models where the largeness of \(\tan \beta\) requires that there be a mechanism to suppress the Higgsino mediated proton decay. We have shown that the mass matrix mixing among the two pairs of superheavy triplets that occur naturally in the Dimopoulos-Wilczek mechanism does not in itself give rise to such a suppression, since the proton decay rate is constrained by the unification condition in exactly the same manner as without such a mixing. We have also shown that in those models where intermediate mass states exist, proton decay is in fact further enhanced, making the proton decay constraint even more difficult to satisfy.

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† Email: Urano@phys.tamu.edu (internet).

* Email: Arnowitt@phys.tamu.edu (internet).

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FIG. 1. Contour Plot of $M_{H^3}^{(0)}$ in $\sin^2(\theta_W)$–$\alpha_3(M_Z)$ plane. The dashed line corresponds to the lower bound on $M_{PD}$ in minimal SU(5) of $1.2 \times 10^{16}$ GeV. The dot-dashed line corresponds to the lower bound on $M_{PD}$ in minimal SO(10) of $2.7 \times 10^{17}$ GeV. Currently, the measurements are $\sin^2(\theta_W) = 0.2313 \pm 0.0003$ and $\alpha_3(M_Z) = 0.119 \pm 0.003$ [16].