Chiral Corrections to Hyperon Vector Form Factors

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Abstract

We show that the leading $SU(3)$-breaking corrections to the $\Delta S = 1 f_1$ vector form factors of hyperons are $O(m_s)$ and $O(m_s^{3/2})$, and are expected to be $\sim 20$–$30\%$ by dimensional analysis. This is consistent with the Ademollo–Gatto theorem, in a sense that we explain. We compute the $O(m_s)$ corrections and a subset of the $O(m_s^{3/2})$ corrections using an effective lagrangian in which the baryons are treated as heavy particles. All of these corrections are surprisingly small, $\sim 5\%$; combining them, we obtain $\sim 5$–$10\%$ corrections. The pattern of corrections is very different than that predicted by quark models.

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1. Introduction

In this paper, we consider the application of chiral perturbation theory to the $f_1$ vector form factor of octet baryon states. The form factors of the vector current are conventionally defined by

$$\langle B_a|J_{\mu c}(0)|B_b \rangle = \bar{u}(p_a) \left[ f_1^{abc}(q^2) \gamma_\mu + \frac{i f_2^{abc}(q^2)}{M_a + M_b} \sigma_{\mu\nu} q^\nu + \frac{i f_3^{abc}(q^2)}{M_a + M_b} q^\mu \right] u(p_b),$$

(1)

where $q \equiv p_a - p_b$. Our interest in the form factor $f_1$ is due to the fact that it is usually assumed that $SU(3)$ breaking corrections to $f_1$ are small due to the Ademollo–Gatto (AG) theorem [1]. Indeed, nonrelativistic quark model and bag model calculations of $SU(3)$ breaking corrections to $f_1$ typically give corrections of order 1% [6].

In this paper, we point out that the leading corrections to $f_1$ due to the nonvanishing strange quark mass are $O(m_s)$ and $O(m_s^3/2)$. The $O(m_s)$ terms are proportional to

$$\frac{m_s^2}{16\pi^2 f^2} \sim 0.2,$$

(2)

where $f \approx 93$ MeV. The $O(m_s^3/2)$ corrections consist of terms proportional to

$$\frac{m_K \Delta_B}{16\pi f^2} \sim 0.2, \quad \frac{m_s^3}{16\pi^2 \Lambda} \sim 0.3,$$

(3)

where $\Lambda \sim 1$ GeV is the expansion scale in chiral perturbation theory, and $\Delta_B$ is an octet baryon mass splitting. Clearly, it is important to compute these corrections, since dimensional analysis does not guarantee that they are small. For example, they could affect the determination of $D$ and $F$ from semileptonic hyperon decay rates. (In the formalism we employ, $D$ and $F$ are defined as couplings in an effective lagrangian which embodies the low-energy theorems for chiral symmetry. In the $SU(3)$ limit, we recover well-known relations such as $D + F = g_A$, but there are $SU(3)$-breaking corrections to these relations due to nonvanishing quark masses which can be substantial.)

We compute the $O(m_s)$ corrections using an effective lagrangian in which the baryons are treated as heavy particles [2][3]. Using the values $D = 0.61, F = 0.40$ determined using a recent fit to semileptonic hyperon decay [4], we find these corrections to be surprisingly small, $\lesssim 5\%$. These corrections have been computed in ref. [5], and we agree with the results of this paper. The $O(m_s^3/2)$ contributions proportional to $m_K \Delta_B$ have not been previously computed. We find that these corrections are also $\lesssim 5\%$ for all decays. The terms proportional to $m_s^3$ cannot be computed in terms of the lowest order chiral
The predicted corrections are increased significantly if the lowest-order fit values of $D$ and $F$ are used, or if we make the approximation $m_\pi \simeq 0$. Our computation gives some indication that the corrections to $f_1$ are $\lesssim 10\%$, but we cannot exclude the possibility that the remaining $O(m_s^{3/2})$ corrections are $\sim 30\%$ or more.

The plan of this paper is as follows: In section 2, we review the effective lagrangian formalism we use to carry out the computation. In section 3, we discuss the Ademollo–Gatto theorem and how it is manifested in the effective lagrangian framework. The reader eager for the bottom line can skip immediately to section 4, in which we present our results. Section 5 contains our conclusions.

2. The Effective Lagrangian

It has been known for some time that the low-energy theorems of chiral symmetry breaking are equivalent to a description of the low-energy dynamics in terms of an effective lagrangian \cite{7}. Recently, it was realized that baryons could be simply included in an effective lagrangian framework using an heavy particle effective theory \cite{2}\cite{3}. This approach provides significant conceptual and calculational advantages: the non-relativistic limit is incorporated from the start, and the Feynman rules for computing graphs are considerably simplified.

In this section, we define the effective lagrangian and establish our notation.

2.1. Mesons

The field

$$\xi(x) = e^{i\Pi(x)/f},$$

is taken to transform under $SU(3)_L \times SU(3)_R$ as

$$\xi \mapsto L\xi U^\dagger = U\xi R^\dagger,$$

where this equation implicitly defines $U$ as a function of $L$, $R$, and $\xi$. The meson fields are

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ \pi^- \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ K^- \end{pmatrix} \begin{pmatrix} \pi^+ \\ -\frac{1}{\sqrt{2}} K^0 + \frac{1}{\sqrt{6}} \eta \\ K^0 \\ \frac{2}{\sqrt{6}} \eta \end{pmatrix},$$

\footnote{Ref. \cite{5} gives some $O(m_s^{3/2})$ corrections of the form $m_K^3/(16\pi f^2 M_B)$ where $M_B$ is the average octet baryon mass. These contributions are an artifact of the method of computation used in that paper, and including them is not justified.}
We will be interested in matrix elements of the vector currents. We therefore add to the effective lagrangian a source term

$$\delta L = V_\mu J^\mu_V + A_\mu J^\mu_A,$$

where $J^\mu_V$ ($J^\mu_A$) is the vector (axial vector) Noether current. The couplings of $V_\mu$ and $A_\mu$ in the effective lagrangian are then determined by demanding that they transform as gauge fields (see eq. (12)). We therefore define the covariant derivatives

$$D_\mu \xi \equiv \partial_\mu \xi - i L_\mu \xi, \quad D_\mu \xi^\dagger \equiv \partial_\mu \xi^\dagger - i R_\mu \xi^\dagger. \quad (8)$$

(Note that $(D_\mu \xi)^\dagger \neq D_\mu \xi^\dagger$.) Here,

$$L_\mu = \frac{1}{2} (V_\mu + A_\mu), \quad R_\mu = \frac{1}{2} (V_\mu - A_\mu). \quad (9)$$

$V_\mu$ and $A_\mu$ are hermitian. The effective lagrangian is most conveniently written in terms of

$$V_\mu \equiv \frac{i}{2} (\xi D_\mu \xi^\dagger + \xi^\dagger D_\mu \xi), \quad A_\mu \equiv \frac{i}{2} (\xi D_\mu \xi^\dagger - \xi^\dagger D_\mu \xi), \quad (10)$$

which transform under local $SU(3)_L \times SU(3)_R$ transformations as

$$V_\mu \mapsto UV_\mu U^\dagger + i U \partial_\mu U^\dagger, \quad A_\mu \mapsto U A_\mu U^\dagger, \quad (11)$$

since the sources transform as gauge fields:

$$L_\mu \mapsto LL_\mu L^\dagger + i L \partial_\mu L^\dagger, \quad R_\mu \mapsto RR_\mu R^\dagger + i R \partial_\mu R^\dagger. \quad (12)$$

Note that $A_\mu$ and $V_\mu$ are hermitian. We can then define the covariant derivative

$$\nabla_\mu A_\nu \equiv \partial_\mu A_\nu - i [V_\mu, A_\nu], \quad (13)$$

which transforms under local $SU(3)_L \times SU(3)_R$ transformations as

$$\nabla_\mu A_\nu \mapsto U \nabla_\mu A_\nu U^\dagger. \quad (14)$$

The chiral symmetry is broken explicitly by the quark masses. (We neglect the effects of electromagnetism in this paper.) We will ignore isospin breaking, so that the quark mass matrix is taken to be

$$M_q = \begin{pmatrix} \hat{m} & \hat{m} \\ \hat{m} & m_s \end{pmatrix}. \quad (15)$$
It is convenient to define
\[ M \equiv \frac{1}{2} \left( \xi^\dagger M \xi^\dagger + \text{h.c.} \right) \mapsto U M U^\dagger. \tag{16} \]

The simple transformation rules of the fields defined above makes it easy to write down the effective lagrangian. For example, the leading terms can be written
\[ \mathcal{L}_0 = f^2 \text{tr}(A^\mu A_\mu) + a f^3 \text{tr} M. \tag{17} \]

### 2.2. Baryons

We now discuss the inclusion of baryon fields as heavy particles [2][3]. The momentum of a baryon field is written
\[ P = M_B v + p, \tag{18} \]
where \( M_B \) is a \( SU(3) \)-invariant baryon mass and \( v \) is a velocity. The key observation is that for processes involving emission of soft pions, the relevant residual momenta \( p \) are small if \( v \) is chosen appropriately. For a fixed \( v \), we can then write an effective theory in terms of baryon fields \( B \) whose momentum is the residual momentum \( p \) of the baryon [2].

The octet baryon fields \( B \) transform under \( SU(3)_L \times SU(3)_R \) as
\[ B \mapsto U B U^\dagger. \tag{19} \]
Explicitly, we have
\[ B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{2}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}. \tag{20} \]

The lowest order terms in the effective lagrangian involving baryon fields are
\[ \mathcal{L} = \text{tr} (\bar{B} i v \cdot \nabla B) + 2 D \text{tr} (\bar{B} s^\mu \{ A_\mu, B \}) + 2 F \text{tr} (\bar{B} s^\mu [A_\mu, B]) \\
+ \sigma \text{tr} (M) \text{tr} (\bar{B} B) + b_D \text{tr} (\bar{B} \{ M, B \}) + b_F \text{tr} (\bar{B} [M, B]), \tag{21} \]
where \( s^\mu \) is the spin operator [3] and the covariant derivative acts on \( B \) as in eq. (13).

### 2.3. Power Counting

The effective lagrangian described above has a well-defined expansion in inverse powers of \( \Lambda \sim 1 \text{ GeV} \). A typical term in the lagrangian can be written schematically
\[ \mathcal{L} \sim f^2 \Lambda^2 \left( \frac{B}{f \sqrt{\Lambda}} \right)^n_B \left( \frac{\nabla}{\Lambda} \right)^n_D \left( \frac{A}{\Lambda} \right)^n_A \left( \frac{M}{\Lambda} \right)^n_M. \tag{22} \]
If we write
\[ m_\Pi^2 \sim afM_q, \quad \Delta_B \sim bM_q, \tag{23} \]
then topological identities can be used to show that loop corrections are related to tree-level contributions by
\[ \text{loop} \sim \left( \frac{\Lambda}{4\pi f} \right)^{2L} \left( \frac{af}{\Lambda} \right)^{N_m/2} b^{N_\Delta} \times \text{tree}, \tag{24} \]
where \( L \) is the number of loops in the diagram, and \( N_m \) (\( N_\Delta \)) is the number of powers of \( m_\Pi \) (\( \Delta_B \)) in the result of the loop diagram. This expansion is consistent provided that \( \Lambda \lesssim 4\pi f, a \lesssim \Lambda/f \lesssim 4\pi, \) and \( b \lesssim 1. \) This appears to be satisfied in QCD [8].

3. The Ademollo–Gatto Theorem

In this section, we review the Ademollo–Gatto (AG) theorem [1] and discuss how it is realized in the effective lagrangian approach. Much of this section is quite elementary, but we feel that the issues involved deserve a careful treatment.

Suppose that a quantum-mechanical system has a global symmetry \( G \) which is explicitly broken by perturbations whose size is controlled by a parameter \( \lambda. \) We assume \( \lambda \) is sufficiently small so that the explicit symmetry breaking can be treated perturbatively. It is then convenient to expand the physical states of the system in terms of states with definite transformation properties under \( G: \)
\[ |\alpha\rangle = c_\alpha |r_\alpha j_\alpha\rangle + \sum_{r,j} c^r_{\alpha j} |r j\rangle, \quad c^r_{\alpha j} \equiv 0. \tag{25} \]
Here, \( |r j\rangle \) is a state belonging to the irreducible representation \( r; \) \( j \) labels the particular state. The state \( |r_\alpha j_\alpha\rangle \) is the state corresponding to the physical state \( |\alpha\rangle \) in the limit \( \lambda \to 0: \)
\[ c_\alpha \to 1, \quad c^r_{\alpha j} = O(\lambda), \quad \text{as } \lambda \to 0. \tag{26} \]

The AG theorem applies if the symmetry breaking effect is such that it does not mix states from the same irreducible representation, \( i.e. \)
\[ c^r_{\alpha j} = 0. \tag{27} \]
In this case, the AG theorem states that for any charge \( Q \) of \( G, \)
\[ \langle \beta | Q | \alpha \rangle = q_\alpha \delta_{\alpha \beta} + O(\lambda^2), \tag{28} \]
where \( q_\alpha \) is the charge of the unperturbed state:
\[ Q |r_\alpha j_\alpha\rangle = q_\alpha |r_\alpha j_\alpha\rangle. \tag{29} \]
This theorem can be applied to the $f_1$ form factor, since

$$
\langle B_a | Q_c | B_b \rangle = \int d^3x \langle B_a | J_{bc}(x) | B_b \rangle = u^\dagger(p_a)u(p_b)f_1^{abc}(\vec{q} = 0) + O(M_q^2).
$$  \hfill (30)

The conditions of the theorem are satisfied in the case of explicit $SU(3)$ breaking due to the strange quark mass, since the mass matrix eq. (15) has definite isospin and hypercharge, and thus does not mix members of the octet.

The proof of the AG theorem is by direct computation:

$$
\langle \beta | Q | \alpha \rangle = c_\alpha c_\beta \langle r_\beta j_\beta | Q | r_\alpha j_\alpha \rangle + \sum_{r,j} \sum_{s,k} c_{r_\alpha} c_{\beta} \langle r_j | Q | s_k \rangle.
$$  \hfill (31)

"Mixed" terms proportional to e.g. $c_\alpha c_{\beta j}$ are absent by the assumption eq. (27). Demanding that the physical states be normalized to unity gives $c_\alpha = 1 + O(\lambda^2)$, and the result eq. (28) follows immediately.

Usually, simple current algebra arguments such as this are spoiled by nonanalyticity in $M_q$ due to the presence of massless Nambu–Goldstone bosons in the limit $M_q \to 0$. However, we note that if we write

$$
M_q = m_0 1 + \delta m T_8,
$$  \hfill (32)

and consider an expansion in $\delta m$ with $m_0$ held fixed, there are no massless particles, and we expect that physical quantities are analytic in $\delta m$. The AG theorem then guarantees that corrections to the vector form factors are $O(\delta m^2)$. This is not the limit relevant for the real world, where $m_0, \delta m \sim m_s$, but we will use this limit to check whether our calculations are consistent with the AG theorem.

Note that that the AG theorem is not trivially manifest in the effective lagrangian. The lagrangian contains terms which appear to give tree-level corrections to the $f_1$ form factor of order $\delta m$, in violation of the AG theorem:

$$
\delta \mathcal{L} = \frac{c_1}{\Lambda} \text{tr}(M) \text{tr}(\overline{B} i \vec{v} \cdot \nabla B) + \frac{c_2}{\Lambda} \left[ \text{tr}(\overline{B} M i \vec{v} \cdot \nabla B) + \text{h.c.} \right] \\
+ \frac{c_3}{\Lambda} \left[ \text{tr}(\overline{B} i \vec{v} \cdot \nabla B M) + \text{h.c.} \right].
$$  \hfill (33)

(There are other terms which can be related to these by integration by parts.) However, these terms also modify the kinetic term for the baryons so that there is no order $\delta m$ correction to $f_1$. To see this, we make the field redefinition

$$
B' = \left[ 1 + \frac{c_1}{2\Lambda} \text{tr}(M) \right] B + \frac{c_2}{\Lambda} MB + \frac{c_3}{\Lambda} BM.
$$  \hfill (34)
\( B' \) is a good interpolating field for the baryons provided that \( M_q \) does not mix octet states. The lagrangian expressed in terms of \( B' \) does not contain any terms of the form eq. (33), and the AG theorem is manifest.

4. Results

The one-loop graphs contributing to the vector form factor are shown in fig. 1. We write

\[
f_1^{abc}(0) = \alpha_{ab} \left( 1 + \frac{1}{16\pi^2 f^2} \beta_{ab}^{\ell} + \frac{m_K}{16\pi^2 f^2} \gamma_{ab} \right),
\]

where the well-known lowest-order results are

\[
\begin{align*}
\alpha_{p\Lambda}^{4+i5} &= -\sqrt{\frac{3}{2}}, \\
\alpha_{n\Sigma^-}^{4+i5} &= -1, \\
\alpha_{\Lambda\Xi^-}^{4+i5} &= \sqrt{\frac{3}{2}}, \\
\alpha_{\Sigma^0\Xi^-}^{4+i5} &= \frac{1}{\sqrt{2}}.
\end{align*}
\]

For the \( O(m_s) \) corrections, we obtain

\[
\begin{align*}
\beta_{p\Lambda}^{4+i5} &= 2\lambda_1 - D^2\lambda_2 - F(2D + 3F)\lambda_1, \\
\beta_{n\Sigma^-}^{4+i5} &= 2\lambda_1 - D^2\lambda_3 + 3F(2D - F)\lambda_1, \\
\beta_{\Lambda\Xi^-}^{4+i5} &= 2\lambda_1 - D^2\lambda_2 + F(2D - 3F)\lambda_1, \\
\beta_{\Sigma^0\Xi^-}^{4+i5} &= 2\lambda_1 - D^2\lambda_3 - 3F(2D + F)\lambda_1,
\end{align*}
\]

where

\[
\begin{align*}
\lambda_1 &= \frac{3}{16} \left( m_\pi^2 + 2m_K^2 + m_\eta^2 - 2 \frac{m_K^2 m_\pi^2}{m_K^2 - m_\pi^2} \ln \frac{m_K^2}{m_\pi^2} - 2 \frac{m_\eta^2 m_K^2}{m_\eta^2 - m_K^2} \ln \frac{m_\eta^2}{m_K^2} \right), \\
\lambda_2 &= \frac{1}{16} \left( 9m_\pi^2 + 10m_K^2 + m_\eta^2 - 18 \frac{m_K^2 m_\pi^2}{m_K^2 - m_\pi^2} \ln \frac{m_K^2}{m_\pi^2} - 2 \frac{m_\eta^2 m_K^2}{m_\eta^2 - m_K^2} \ln \frac{m_\eta^2}{m_K^2} \right), \\
\lambda_3 &= \frac{1}{16} \left( m_\pi^2 + 10m_K^2 + 9m_\eta^2 - 2 \frac{m_K^2 m_\pi^2}{m_K^2 - m_\pi^2} \ln \frac{m_K^2}{m_\pi^2} - 18 \frac{m_\eta^2 m_K^2}{m_\eta^2 - m_K^2} \ln \frac{m_\eta^2}{m_K^2} \right).
\end{align*}
\]

The combination of masses defined above are easily seen to satisfy the AG theorem in the sense discussed in section 3. Our numerical results are summarized in table 1. The \( O(m_s) \) corrections are \( \lesssim 5\% \) for all decays, significantly smaller than what is expected on the basis of dimensional analysis. We are therefore led to consider the higher order corrections to determine whether they are numerically important.
The $O(m_s^{3/2})$ contributions proportional to $m_K \Delta_B$ are computed from the graphs in fig. 1. Because of the length and unilluminating nature of the resulting formulas, we will give formulas only for the case $m_\pi = 0$, and simplify the results using the Gell-Mann–Okubo relations

\[
\begin{align*}
  m_\eta^2 &= \frac{4}{9} m_K^2, \\
  M_\Xi &= \frac{2}{3} M_\Lambda + \frac{1}{2} M_\Sigma - M_N.
\end{align*}
\]

(We have checked that the full expressions satisfy the AG theorem.) We obtain

\[
\begin{align*}
  \gamma_{p\Lambda}^{4+i5} &= \left[ -\frac{1}{10} (25 - 16\sqrt{3}) D^2 - \frac{1}{5} (39 - 16\sqrt{3}) DF + \frac{3}{10} (25 - 16\sqrt{3}) F^2 \right] M_n \\
  &\quad + \frac{1}{2} D(D - F) M_\Sigma \\
  &\quad + \left[ \frac{2}{5} (5 - 4\sqrt{3}) D^2 + \frac{1}{10} (83 - 32\sqrt{3}) DF - \frac{3}{10} (25 - 16\sqrt{3}) F^2 \right] M_A, \\
  \gamma_{n\Sigma^-}^{4+i5} &= \left[ -\frac{1}{30} (103 - 48\sqrt{3}) D^2 - \frac{1}{5} (39 - 16\sqrt{3}) DF + \frac{1}{10} (103 - 48\sqrt{3}) F^2 \right] M_n \\
  &\quad + \left[ \frac{2}{5} (9 - 4\sqrt{3}) D^2 + \frac{1}{10} (83 - 32\sqrt{3}) DF - \frac{3}{10} (103 - 48\sqrt{3}) F^2 \right] M_\Sigma \\
  &\quad - \frac{1}{6} D(D - 3F) M_A, \\
  \gamma_{\Lambda\Xi^-}^{4+i5} &= \left[ \frac{1}{10} (25 - 16\sqrt{3}) D^2 - \frac{1}{5} (39 - 16\sqrt{3}) DF - \frac{3}{10} (25 - 16\sqrt{3}) F^2 \right] M_n \\
  &\quad + \left[ -\frac{1}{20} (15 - 16\sqrt{3}) D^2 + \frac{2}{5} (11 - 4\sqrt{3}) DF + \frac{3}{20} (25 - 16\sqrt{3}) F^2 \right] M_\Sigma \\
  &\quad + \left[ -\frac{1}{20} (35 - 16\sqrt{3}) D^2 + \frac{1}{5} (17 - 8\sqrt{3}) DF + \frac{3}{20} (25 - 16\sqrt{3}) F^2 \right] M_A, \\
  \gamma_{\Sigma^0\Xi^-}^{4+i5} &= \left[ \frac{1}{30} (103 - 48\sqrt{3}) D^2 - \frac{1}{5} (39 - 16\sqrt{3}) DF - \frac{1}{10} (103 - 48\sqrt{3}) F^2 \right] M_n \\
  &\quad + \left[ \frac{1}{60} (113 - 48\sqrt{3}) D^2 - \frac{2}{5} (11 - 4\sqrt{3}) DF - \frac{1}{20} (103 - 48\sqrt{3}) F^2 \right] M_\Sigma \\
  &\quad + \left[ \frac{1}{60} (319 - 144\sqrt{3}) D^2 + \frac{1}{5} (61 - 24\sqrt{3}) DF + \frac{3}{20} (103 - 48\sqrt{3}) F^2 \right] M_A.
\end{align*}
\]

Our numerical results (including $m_\pi \neq 0$) are summarized in table 1. The main feature of these results is that they are significantly smaller than expected from dimensional analysis. Using the older values of $D$ and $F$ or neglecting the pion mass results in substantially larger corrections.

5. Conclusions

We have computed chiral corrections to the $f_1$ vector form factor for $\Delta S = 1$ semileptonic hyperon decay. We have shown that the leading corrections are $O(m_s)$ and $O(m_s^{3/2})$ and explained how this is consistent with the Ademollo–Gatto theorem. These corrections are $\sim 30\%$ according to dimensional analysis. Explicit calculation shows that the $O(m_s)$ corrections and a computable subset of the $O(m_s^{3/2})$ corrections are $\sim 5\%$ for all decays.
using the values of $D$ and $F$ of ref. [4]. The corrections are significantly larger if the older values of $D$ and $F$ are used, or if the pion mass is neglected.

These results are very different than those obtained from the non-relativistic quark model and bag model [6]. In these models, the corrections to $f_1$ are universal for all decays and are $\simeq -1\%$. The corrections we have computed are much larger and depend on the decay. From the point of view of the chiral expansion, the quark model results are rather hard to understand. Since these model predictions are used in a determination of $V_{us}$ from semileptonic hyperon decay [10], this discrepancy is of more than academic interest. It is possible that the inclusion of the effects of the decuplet [4] may reduce the apparent discrepancy between the chiral corrections and the quark model calculations.

| $\Lambda \to p$ | $O(m_s)$ | $O(m_s^{3/2})$ | total     |
|-----------------|-----------|----------------|-----------|
| $\Sigma^- \to n$ | $0.065 \pm 0.011$ | $0.060 \pm 0.041$ | $0.13 \pm 0.05$ |
| $\Xi^- \to \Lambda$ | $0.021 \pm 0.014$ | $0.060 \pm 0.033$ | $0.081 \pm 0.020$ |
| $\Xi^- \to \Sigma^0$ | $-0.002 \pm 0.031$ | $0.035 \pm 0.034$ | $0.033 \pm 0.013$ |

Table 1: $f_1/f_{1}^{SU(3)}$ for $\Delta S = 1$ hyperon decays using the best-fit values $D = 0.61, F = 0.40$ of ref. [4]. The quoted errors are obtained by (somewhat arbitrarily) assigning a 20% error to the values of $D$ and $F$. Using the older values $D = 0.8, F = 0.5$ [9] increases all of the corrections by $\sim 40\%$. We have kept $m_\pi \neq 0$; taking $m_\pi = 0$ increases all of the corrections, some by as much as 35%.

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Figure Captions

Fig. 1. Graphs contributing to the vector form factor at one loop. The solid lines represent baryons, the dashed lines represent mesons, and cross indicates an insertion of the vector current. The wavefunction graph (b) vanishes identically. Graphs (e) and (f) do not contribute to the \( m_K \Delta_B / 16 \pi f^2 \) corrections.