Theoretical Analysis of the Vibrations in Gas Turbine Rotor

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Abstract

Vibration analysis plays an important role in the dynamics of rotors, which can be afflicted by unbalancing forces that, over the long term, can lead to the initiation and growth of cracks at the rotor shaft. To predict cracking, it is necessary to study the vibration parameters of the rotor and existence of expected cracks. In this study, rotor vibration was studied analytically and numerically using a developed ANSYS code. The rotor under consideration is supported by journal bearings, the stiffness and damping coefficients were calculated analytically and then applied to the vibration model of the rotor. The results of the numerical analysis were compared with those obtained analytically. Good agreement was obtained between both solutions, with maximum error of 7.31\% for the critical speed. These results show that when the depth of the crack is increased, the orbit size and the response are also increased while the critical speed is reduced.

Keywords: Rotor Dynamics, Vibrations, Critical Speed, Crack.

Introduction

Vibration analysis plays an important role in the dynamics of rotors, especially the huge types such as gas turbine rotors. The vibration of rotors can be caused by unbalancing forces, and over the long term this can lead to crack initiation and growth at the rotor shaft. It is possible to have a sudden failure at certain depths of crack, notably 80\% of the radius [1]. Research on the detection of rotor cracking has been conducted since 1954, when it was discovered that the crack generates imbalance in the distribution of masses in the rotor, thus vibration will occur. The commonest type is (1X), where (X) is the rotating speed [2]. The effect of gyroscopic forces and axial loads on the rotor was verified by [3,4], and it was concluded that these forces could be represented by the rotor element. The weight of the rotor will generate the opening and closing states of the crack due to gravity force, where it is in a fully closed state when its position at highest point, and it is in a fully open state when its position at lower point, and
between them is the partial opening or closing partial [5,6]. The least stiffness of rotor occurs when the crack is in a fully open condition, and the greatest extent of stiffness is when it attains a fully closed condition [7]. A relationship has been found between the speed of rotation and the depth of the crack and its location, with the natural frequency of the dynamic system in [8], and the natural frequency appeared in the domain of rotation speed 80–120%. When the crack depth is increased, the stability will decrease, as well as the effect on the vibration parameters of the dynamic system, due to changes in rotor stiffness [9].

The equation of motion for the system having been resolved, it was observed that the critical speed decreases when the depth of the crack is increased, and the vibration parameters at one-third of the critical speed give a good indication of the presence of the crack [10, 11, 12, 13]. The Jeffcott rotor is the simplest type of rotor, and was studied by Ramezanpour and colleagues [15], where a slant crack was fabricated and an equation was found linking the crack angle with deformation displacement. It was concluded that the vibration parameters change with the change of crack angle; this was studied in the presence of a crack in a dynamic rotor, where the finite element method was used.

The method of harmonic equilibrium is also used in this context [16,17]. Tenali and Kadivendi [18] studied the vibration in a steam turbine, where the specific element method was used, and the response was calculated in the bearing for purpose of obtaining the critical velocity value. The analytical results were verified by numerical results using ANSYS and TMS-050 programs. Many scholars have studied two cracks in a rotor, and it has been concluded that the depth and angle and location of the crack affects the vibration parameters and increases the risk of sudden failure [13,19]. In Patil and Vaziri [20], the vibration parameters of a symmetrical rotor were studied. A test rig was constructed to monitor the vibration parameters and it was found that the presence of the crack reduces the flexibility of the rotor at its position.

In this paper, the vibration of a gas turbine of GT 9001E type is studied. The turbine is simulated by a shaft and disk model. In this model, the turbine is studied analytically and numerically. The vibration parameters of the rotor are mainly affected by the features of crack that is initiated on the shaft. Accordingly, studying the vibration parameters for different crack features such as depth, location and its direction will be used in this study as an approach for crack detection in rotor shafts.
Analytical solution

The purpose of analytical solution is to study the behavior of vibration parameters when there is a single crack or two cracks in the rotor, where it is considered an efficient approach for crack detection in real rotor. At first, the value of eccentricity ratio was calculated by solving equation (1), [21].

$$e^8 - 4e^6 + \left(6 - S_x^2(16 - \pi^2)\right)e^4 - \left(4 + \pi^2S_x^2\right)e^2 + 1 = 0 \quad (1)$$

After determining the eccentricity ratio, it becomes possible to find the stiffness and damping parameters of the journal bearings.

Figure (1) is a schematic diagram that shows the cracks in a rotor shaft.

Where E1 and E2 are elements type of cracks, $\zeta$ and $\eta$ are moveable coordinates in the crack zone and the degree of freedom is six on both sides.

When the crack is formed, the area of the cross section is decreased, and this results in decreased shaft stiffness. There are two types of deformations; static and dynamic deformations. The equation of motion can be written as follows:

$$[M][\ddot{y_d}] + [D] + [G][\dot{y}_d] + [[K] - [K_c(\theta, t)]]\{y_s + y_d\} = Q_u \quad (2)$$

$$[K_{cr}] = [K] - [K_c(\theta, t)] \quad (3)$$

Where: ($Q_u$) is the unbalance force, $[K]$ is stiffness matrix for the uncracked rotor, $[K_{cr}]$ is stiffness...
matrix for the cracked rotor, \([K_c (\theta, t)]\) is the decrease in stiffness matrix which results from the crack effect.

From Figure (1), the \(x_r\) is the displacement for rotor in X-axis, and \(x_j\) is the displacement for journal bearing in X-axis. The forces of equilibrium in bearing are given in the equation (4), so it was possible to find the response in the crack location which is close to the disc by solving equation (3) and (4).

\[
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix}
\begin{bmatrix}
x_r \\
y_r
\end{bmatrix}
+ \begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix}
\begin{bmatrix}
\Omega x_r \\
\Omega y_r
\end{bmatrix}
+ \begin{bmatrix}
0 & K \\
K & 0
\end{bmatrix}
\begin{bmatrix}
x_r \\
y_r
\end{bmatrix}
= \begin{bmatrix}
K & 0 \\
0 & K
\end{bmatrix}
\begin{bmatrix}
x_r \\
y_r
\end{bmatrix}
\]  

(4)

When the crack depth is increased, the response increases and the orbit size increases also. The path radius is calculated by the following equation:
\[ |r^*|_{maj} = |r_f^*| + |r_b^*| \quad |r^*|_{min} = |r_f^*| - |r_b^*| \]  

(5)

Where \((r_f^*)\) is the component of forward unbalance response, \((r_b^*)\) is the component of backward unbalance response. The \(|r^*|_{maj}\) is the major radius, and \(|r^*|_{min}\) is the minor radius. When there are two cracks in the rotor, many factors effect on the vibration parameters of which location, depth and angle of crack, but the most influential factor is the angle of the slope for the crack about the horizontal line, where the parameters of the vibration for the rotor have been found with a transverse crack and obelic (oblique) crack, where the angle was changed from 30 to 90 degrees. The equations of motion are as follows, [7]:

\[
\ddot{\xi} + 2\zeta r_o \dot{\xi} + 2 \eta + (r \zeta^2 - 1)\xi + (r \eta^2 - 2\zeta r_o)\eta = e \cos \beta - r o^2 \cos \tau \]  

(6)

\[
\ddot{\eta} + 2\zeta r_o \dot{\eta} + 2 \xi + (r \eta^2 - 1)\eta + (r \eta^2 + 2\zeta r_o)\xi = e \sin \beta + r o^2 \sin \tau \]  

(7)

When:

\[
\omega_n = \frac{K}{M; \omega_\eta = \frac{K\eta}{M}; \omega_\zeta = \frac{K\zeta}{M}; \omega_\zeta \eta = \frac{K\zeta \eta}{M}; \zeta = \frac{c}{2\sqrt{K M}}
\]

\[
r_o = \omega_n \alpha \quad r_\eta = \omega_\eta \alpha \quad r_\zeta = \omega_\zeta \alpha \quad r_\zeta \eta = \omega_\zeta \eta \alpha \quad r_\eta \zeta = \omega_\eta \zeta \alpha
\]

\[
\eta = \frac{\eta_1}{\delta_{st}} \quad \tilde{\zeta} = \frac{\zeta_1}{\delta_{st}} \quad \tau = \Omega t \quad e = \frac{e}{\delta_{st}} \quad \Omega = \frac{a_1}{R} \quad a_2 = \frac{a_2}{R}
\]

Numerical Solution

In order to simulate the rotor numerically, an ANSYS program was built to model the shaft with cracks at a range of depths. The program was built in light of recommendations from rotor dynamic guides for the ANSYS program, which have been relied upon in the composition and linking of program parts. Initially, the model was constructed by determining the dimensions, density, elastic modulus and Poisson's ratio. Then, the elements of the model were selected taking into consideration representation of the gyroscopic forces and consistency with the Coriolis principle. The shaft element is Beam188, the disc element MASS21, and the bearing element COMBI214.

The material was defined and its properties determined as linear or nonlinear, i.e. whether the material had a linear behavior with the forces that influenced it. Then, the speed limits applied in the program and the incremental extents for each step were introduced. The values of stiffness and damping parameters of the bearings are variable with each speed, due to changes the properties of the lubrication layer.
between the bush and the shaft, so they have to be found and inserted into the program as input information.

A meshing of the shaft was created, where the number of elements was 17, and the analysis was applied on the proximal element of the disc. The POST1 and POST26 were used to solve the dynamic rotor model, where POST1 is used for general processes, and POST26 is used to resolve processes which have a relationship with the time history. Figure 3 shows the process of meshing, and Figure 4 shows the boundary conditions.

![Figure 3. The meshing of the dynamic rotor.](image-url)
Results

The analytical results explored two cases, the first case for a single crack with different depths (uncrack, 0.2R, 0.4R, 0.6R, 0.8R) and the second case was for two cracks at different angles and depths ((90, 90) (60,90) and (30,90)). The angles were chosen from 30 degrees and close to the horizon. The corner (90 degree) angle was also taken and is the vertical to take into account all angles.

For these cases the path of the orbit of the rotor was found and can be seen in Figure 5. It was observed that, when the depth of the crack increased, the orbital path would increase in size because the reduction of hardness produced more freedom of motion at the crack zone. This was in addition to a decrease in critical velocity with increased deformation response for the same reason.
The results of the numerical analysis are based on ANSYS software, where one case was considered. In this case a single crack on the shaft with a range of depths (uncrack, 0.2R, 0.4R, 0.6R, 0.8R) is shown in Figure 6. It was stopped at (0.8R) because at this depth, imminent failure was anticipated.

Figure 5. The orbit path for different depths for analytical study.

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Figure 6. The response rotor with multiple depths for single crack for analytical study.

Table (1) The values of response and critical speed with different single crack depths for analytical study.

| CRACK DEPTH RATIO | CRITICAL SPEED (RPM) | RESPONSE (mm) |
|-------------------|----------------------|---------------|
| UNCRACK           | 8125                 | 0.029507      |
| 0.2R              | 7925                 | 0.03069       |
| 0.4R              | 7750                 | 0.03231       |
| 0.6R              | 7370                 | 0.03732       |
| 0.8R              | 6850                 | 0.04125       |
Figures 7, 8 and 9 depict cases where the rotor had two cracks and show that the higher the crack angle the less the critical velocity, and the response increased. This is because the hardness is reduced in a vertical direction when the angle increases.

![Figure 7. The response at (θ1=30°, θ2=90°).](image-url)
Table 2. The value of decreased ratio for critical speed.

| Cracks depths (mm) | Comparison between | Comparison between | Comparison between |
|--------------------|--------------------|--------------------|--------------------|
| θ₁ = 60°, θ₂ = 90° | θ₁ = 60°, θ₂ = 90° | θ₁ = 60°, θ₂ = 90° | θ₁ = 60°, θ₂ = 90° |

Figure 8. The response at (θ₁=60°, θ₂=90°)

Figure 9. The response at (θ₁=90°, θ₂=90°)
\[
\begin{align*}
\theta_1 &= 30^\circ, \theta_1 = 60^\circ, \\
\theta_1 &= 30^\circ, \theta_1 = 90 \\
7.5, 4 & \quad 2.70\% \quad 5.27\% \quad 2.63\% \\
15, 8 & \quad 2.12\% \quad 4.24\% \quad 2.16\% \\
15, 15 & \quad 4.01\% \quad 6.90\% \quad 3.00\%
\end{align*}
\]

Table 3. The value of increased ratio for response.

| Cracks depths (mm) | Comparison between $\theta_1 = 30^\circ, \theta_1 = 60^\circ$ | Comparison between $\theta_1 = 30^\circ, \theta_1 = 90$ | Comparison between $\theta_1 = 60^\circ, \theta_1 = 90$ |
|--------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| 7.5, 4             | 6.75\%                                         | 10.12\%                                         | 3.16\%                                         |
| 15, 8              | 3.75\%                                         | 8.68\%                                         | 4.75\%                                         |
| 15, 15             | 5.16\%                                         | 11.73\%                                         | 5.56\%                                         |

Figure 10 and Table 4 show the numerical solution results. Where critical velocity was decreased, the response was increased with increased depth of crack, as shown in Figure 10.

![Figure 10. The response behavior with different depths.](image-url)
CONCLUSIONS

From the results obtained from this study, it can be concluded that monitoring of the vibration parameters is an efficient approach for detecting crack initiation in rotor shafts. After conducting analytical and numerical analysis, it was concluded that the size of the orbit would increase with increasing crack depth where this leads to a decrease in stability. It also can be concluded that when the depth of the crack is increased, the critical velocity decreases and the rotor response is increased. This can be considered as an efficient indicator for crack detection during the operation of the rotor.

The analysis used in this study represents the system with error ratio of critical speed of analytical to numerical analysis of 0.31% for the uncrack shaft test, and 9.4% for the 0.8R test, where this value of error is acceptable for this type of problem.

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