Aspects of using a numerical simulator for a robot position-orientation matrix determination

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Abstract. The paper presents the mathematical model of a parallel topology robot of FP₃₊₃·RRS+MP₃ type, used in order to determine the position-orientation matrix of the robot’s mobile reference system. By using a numerical simulator, positions of the characteristic point are plotted in the 3D space.

1. Introduction
Positioning accuracy is a current requirement for the use of robots in various applications [1-4]. This could be estimated by the means of the position-orientation matrix of the reference system attached to the robot’s characteristic point [5-7].

In the case of parallel topology robots, the positioning of the end effector is more complex than in the case of serial topology robots [8-11]. Different structures of parallel topology robots were analyzed in [12-16]. The paper proposes a certain structure of a parallel topology robot and a manner of position-orientation matrix determination by using a numerical simulator.

2. The Kinematical Scheme of the Guiding Device Mechanism
The structure of the guiding device mechanism with parallel topology is FP₃₊₃·RRS+MP₃ [2], [5], [6]. It contains 2 platforms, a fixed one (FP₁), and a mobile one (MP₃), linked by 3 identical open kinematical chains RRS. Every open kinematical chain, or “connexion”, contains 2 binary links, 1 driving rotational joint (R), 1 rotational joint (R) and 1 spherical joint (S).

The concept „connexion” was introduced in Mechanism Theory and Robotics by Professor F.V. Kovacs. According to [17], [18], a ”connexion” is an open linkage interposed between two links, aiming the change of the number of their relative degrees of freedom (DOF).

The kinematical scheme of the guiding device mechanism with parallel topology is presented in Figure 1 [2], [5], [6].

The driving rotational joints are A₁, A₂ and A₃.
Figure 1. The kinematical scheme of the guiding device mechanism of \( F_P + 3\cdot RRS + MP_3 \) type [2], [5], [6]

![Figure 1](image1)

Figure 2. The frames attached to the links of the connexion \( A_j B_j C_j \) [5], [6]

![Figure 2](image2)

The links dimensions are as follows [2], [5], [6]:
- the fixed platform: equilateral triangle with the sides \( A_1 A_2 = A_2 A_3 = A_3 A_1 = 125 \text{ [mm]} \);
- the mobile platform: equilateral triangle with the sides \( C_1 C_2 = C_2 C_3 = C_3 C_1 = 125 \text{ [mm]} \);
- lengths of links 1, 2, and 1: \( A_1 B_1 = A_2 B_2 = A_3 B_3 = 60 \text{ [mm]} \);
- lengths of links 2, 2, and 2: \( B_1 C_1 = B_2 C_2 = B_3 C_3 = 95 \text{ [mm]} \);
- the distance from point \( P \) to plane \( C_1 C_2 C_3 \) is 35,7 [mm].

The points \( A_j, B_j, C_j \) (j = 1, 2, 3) were considered in the geometrical centers of the kinematical joints.

3. Mathematical Modeling of the Mechanism

The mathematical modeling was developed in previous work [5], [6], by using the “Pair of Frames” (PF) concept, defined by Professor F.V. Kovacs in [19], as well as its using for modeling of mechanical systems. Generalized joints and generalized offsets can be mathematically modeled by means of transformation matrices between 3D (or 2D) frames, attached in the first case to each of the links constituting the generalized offset [19].

Figure 2 presents the frames attached to the links of the connexion \( A_j B_j C_j \) (j = 1, 2, 3) from Figure 1 [5], [6].

The frames are referred by means of their origins.

Thus, the origins of the frames attached on the links of the rotational joint \( A_j \) are placed in the geometrical center of joint \( A_j \), and in the point \( O \), chosen on the fixed platform \( F_P \). The frames origins \( O \) and \( A_j \) belong to \( F_P \) and \( A_j \) belongs to link \( 1_j \). So, the rotational joint \( A_j \) is modelled by the \( A_j - A_j' \) pair of frames; the relative rotational displacement is executed around \( A_j y \) axis with an angle \( \hat{\alpha}_{A_j} \) at the considered moment of time \( t \) [5], [6].

The origins of the frames attached on the links of the rotational joint \( B_j \) are chosen in the geometrical center of joint \( B_j \), one on the link \( 1_j \) (\( B_j \)), and the other on the link \( 2_j \) (\( B_j' \)). The rotational joint \( B_j \) is modelled by the \( B_j - B_j' \) pair of frames; the relative rotational displacement is executed around \( B_j y \) axis with an angle \( \hat{\beta}_{B_j} \) [5], [6].
The origins of the frames attached to the spherical joint C_j are chosen on the link 2_j(C_j) and the other on the mobile platform MP_j (C_j'). The points C_j and C_j' are placed in the geometrical center of the spherical joint. The spherical joint C_j is modeled by the C_j-C_j' pair of frames. The 3D rotation is composed by a rotation around C_jx axis with an angle \( \alpha_j \), a rotation around C_jy axis with an angle \( \beta_j \), and a rotation around C_jz axis with an angle \( \gamma_j \). 

In order to simplify the computation, A_y, A_y', B_y, B_y' and C_y axes are chosen parallel [5], [6].

The position-orientation matrix of mobile reference system attached to the characteristic point P can be written [5], [6]:

\[
f_{pp}^{n_{MP}} = \begin{bmatrix}
    n_{p_x} & a_{p_x} & a_{p_x} & p_x \\
    n_{p_y} & a_{p_y} & a_{p_y} & p_y \\
    n_{p_z} & a_{p_z} & a_{p_z} & p_z \\
    0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
    0 \ T_p & 0 \ T_{A_j} & 0 \ T_{C_j} & 0 \ T_{P'}
\end{bmatrix} = \begin{bmatrix}
    \bar{p}_{x}' & \bar{p}_{y}' & \bar{p}_{z}' & 1
\end{bmatrix}
\] (1)

The matrix elements \( n_{p_x}, o_{p_y}, a_{p_z}, p_x, o_{p_y}, a_{p_z}, p_y, o_{p_z}, a_{p_z} \) are the projections of the \( \bar{p}_{x}, \bar{p}_{y}, \bar{p}_{z} \) unit vectors of the Pxyz reference system on the Ox, Oy, Oz axes. The elements \( p_x, p_y, p_z \) are the projections of the OP position vector of the P point on the Oxyz reference system axes [5], [6].

The position-orientation matrix of the characteristic point P is identical for every connexion 1, 2, and 3. Following, 3 relations of type (1) can be equalized, as shown in equation system (2) [5], [6].

\[
\begin{align*}
\begin{bmatrix}
    \bar{p}_{x}' \\
    \bar{p}_{y}' \\
    \bar{p}_{z}'
\end{bmatrix}
&= \begin{bmatrix}
    \bar{p}_{x}' \\
    \bar{p}_{y}' \\
    \bar{p}_{z}'
\end{bmatrix} \\
\begin{bmatrix}
    \bar{p}_{x}' \\
    \bar{p}_{y}' \\
    \bar{p}_{z}'
\end{bmatrix}
&= \begin{bmatrix}
    \bar{p}_{x}' \\
    \bar{p}_{y}' \\
    \bar{p}_{z}'
\end{bmatrix} \\
\begin{bmatrix}
    \bar{p}_{x}' \\
    \bar{p}_{y}' \\
    \bar{p}_{z}'
\end{bmatrix}
&= \begin{bmatrix}
    \bar{p}_{x}' \\
    \bar{p}_{y}' \\
    \bar{p}_{z}'
\end{bmatrix}
\end{align*}
\] (2)

In scalar form, for \( j = 1, 2, 3 \), by equalizing the elements of matrixes in expression (2), it can be obtained a scalar equation system with \( 4 \cdot 4 \cdot 3 = 48 \) equations [5], [6].

Considering that the equalization of position-orientation matrixes for the connexions 1 and 3 leads to redundant equations, and the equalization of the last line of the matrixes leads also to redundant equations, the number of equations which can be used to solving the system is \( 4 \cdot 4 \cdot 2 - 2 \cdot 4 = 24 \) [5], [6].

The parameters \( A_j, \beta_j, \gamma_j \) (\( j = 1, 2, 3 \)) at the time \( t \) are known. From equation system (2), 12 elements with different values of 0 and 1 of the position - orientation matrix \( f_{pp}^{n_{MP_j}} \) can be computed. The variable angles \( \beta_j, \gamma_j \) are unknown. This means that \( 24 - 12 = 12 \) scalar equations of system (5) are redundant [5], [6].

For the connexion 1, \( j = 1 \) and the matrixes from relation (1) are as follows [5], [6]:

\[
0 \ T_{A_1} = \text{Trans}[x(l_{A_1})_x] \cdot \text{Trans}[y(l_{A_1})_y] \cdot \text{Rot}(z, O_{A_1} A_2) =
\begin{bmatrix}
    \cos O_{A_1} A_2 & -\sin O_{A_1} A_2 & 0 & (l_{A_1})_x \\
    \sin O_{A_1} A_2 & \cos O_{A_1} A_2 & 0 & (l_{A_1})_y \\
    0 & 0 & 1 & 0
\end{bmatrix}
\] (3)
\( A_i T_{A_i} = \text{Rot}(y, \beta_{A_i}) = \begin{bmatrix} \cos \beta_{A_i} & 0 & -\sin \beta_{A_i} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta_{A_i} & 0 & \cos \beta_{A_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \) \( 4 \)

\( A_i T_{B_i} = \text{Transl}[z(t_{A_iB_i})] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \) \( 5 \)

\( B_i T_{B_i} = \text{Rot}(y, \beta_{B_i}) = \begin{bmatrix} \cos \beta_{B_i} & 0 & -\sin \beta_{B_i} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta_{B_i} & 0 & \cos \beta_{B_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \) \( 6 \)

\( B_i T_{c_i} = \text{Transl}[z(t_{B_iC_i})] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \) \( 7 \)

\( c_i T_{c_i} = \text{Rot}(x, c_{C_i}) \text{Rot}(y, c_{C_i}) \text{Rot}(z, \gamma_{C_i}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \) \( 8 \)

\( c_i T_{p} = \text{Transl}[x(t_{C_iP})] \text{Transl}[y(t_{C_iP})] \text{Transl}[z(t_{C_iP})] = \begin{bmatrix} 1 & 0 & 0 & \left(t_{C_iP}\right)_x \\ 0 & 1 & 0 & \left(t_{C_iP}\right)_y \\ 0 & 0 & 1 & \left(t_{C_iP}\right)_z \\ 0 & 0 & 0 & 1 \end{bmatrix}; \) \( 9 \)

where \( t_{OA_j} \), \( t_{OA_j} \), etc., are the projections of the segments \( OA_j \), etc., on the axes Ox, Oy (for \( j = 1, 2, 3 \)) [5], [6].

For the connexions 2 and 3, the matrixes from relation (1) can be written in a similar manner, as presented in [5], [6].

4. Scilab Numerical Simulator

Scilab, a free and open source software [20], was used for numerical calculus and plotting results.

For example, for the input parameters \( \beta_{N_t} = 90^{\circ} \); \( \beta_{N_2} = 90^{\circ} \); \( \beta_{N_3} = 90^{\circ} \), the characteristic point P has the coordinates \( p_x = 0\text{[mm]}; p_y = 0\text{[mm]}; p_z = 109.355\text{ [mm]} \); for the input parameters \( \beta_{N_1t} = 0^{\circ} \); \( \beta_{N_2t} = 0^{\circ} \); \( \beta_{N_3t} = 0^{\circ} \), the coordinates of the characteristic point P are
$p_x = 0 [\text{mm}] ; p_y = 0 [\text{mm}] ; p_z = 190,700 [\text{mm}]$. These points and the trajectory of the characteristic point between them, obtained with Scilab software, are presented in Figure 3.

Figure 3. Vertical trajectory of the characteristic point P from the minimum position (0, 0, 109,355) to the maximum position (0, 0, 190,700) on z axis.

Figure 4 shows the trajectory of the characteristic point P from position defined by coordinates (-11.620, -6.709, 152.834) to position (4.066, 2.347, 181.492), for the input parameters $A_1 \beta_{A_1} = 90[^\circ]; A_2 \beta_{A_2} = 30.6[^\circ]; A_1 \beta_{A_1} = 30.6[^\circ]$ and $A_1 \beta_{A_1,t} = 0[^\circ]; A_2 \beta_{A_2,t} = 30.6[^\circ]; A_1 \beta_{A_1,t} = 30.6[^\circ]$ respectively.

Figure 4. Trajectory of the characteristic point P from position (-11.620, -6.709, 152.834) to position (4.066, 2.347, 181.492).

5. Conclusions
By using the SciLab software, the calculus for determination of position-orientation matrix of a certain parallel topology robot structure is performed. This is a part of more complex project regarding the numerical simulation of mechanisms.

The trajectory of the characteristic point can be accomplished for different input parameters.

As further research, this manner of numerical calculus can be applied to other parallel topology mechanism structures.
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