Numerical study of Frohlich and Marchetti monopole creation operator

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The monopole creation operator proposed recently by Frohlich and Marchetti is investigated in the Abelian Higgs model with compact gauge field. We show numerically that the creation operator detects the condensation of monopoles in the presence of the dynamical matter field.

1. INTRODUCTION

The value of the deconfinement temperature is one of the most important prediction of lattice QCD. To study the temperature phase transition we have to investigate the order parameter. For full QCD when dynamical quarks are taken into account, the string tension and the expectation value of the Polyakov line are not the order parameters. If we accept the dual superconductor model of QCD vacuum \cite{1} we have the natural order parameter for confinement – deconfinement phase transition. This is the value of the monopole condensate. It should be nonzero in the confinement phase (the monopoles are condensed as Cooper pairs in ordinary superconductor) and zero in the deconfinement phase. To extract monopole from vacuum non-Abelian fields we have to perform the Abelian projection \cite{2}, after that we can evaluate the value of the monopole condensate using the monopole creation operator.

Originally the gauge invariant monopole creation operator was proposed by Frohlich and Marchetti for compact $U(1)$ gauge theory \cite{3}. The construction is analogous to the Dirac creation operator \cite{4} for a charged particle. The monopole operator was numerically studied in compact Abelian gauge model \cite{5} as well as in the pure $SU(2)$ gauge theory both in usual \cite{6} and spatial \cite{7} Maximal Abelian gauges. It was found that the expectation value of this operator behaves as an order parameter for confinement–deconfinement phase transition: the expectation value is non-zero in the confinement phase and zero in the deconfinement phase. The similar conclusions were made for another types of the monopole creation operators \cite{8}. These results confirm the dual superconductor hypothesis \cite{9} for gluodynamics vacuum.

However, the monopole operator discussed in Ref. \cite{3} exhibits some inconsistency in the presence of charged matter fields, namely the Dirac strings become visible. To get rid of the Dirac string dependence a new monopole operator was proposed recently \cite{9}. Note that even the pure gluodynamics contains electrically charged fields in the Abelian projection: the off–diagonal gluons are (doubly) charged with respect to the diagonal gluon fields. Thus the newly proposed operator \cite{9} is more suitable for the investigation of confinement in $SU(N)$ gauge theories then the older one \cite{3}. The purpose of this paper is to check whether the new monopole creation operator is the order parameter in theories with matter fields. For simplicity we study the Abelian Higgs model in the London limit having in mind the further numerical investigation of the new monopole creation operator in non-Abelian gauge theories.

2. MONOPOLE OPERATORS

The original version of the gauge invariant monopole creation operator \cite{3} in compact $U(1)$ gauge theory is based on the duality of this model to the Abelian Higgs model. The Higgs field $\phi$ is associated with the monopole field and the non-
compact dual gauge field $B_{\alpha}$ represents the dual photon. The gauge invariant operator which creates the monopole in the point $x$, can be written as the Dirac operator $\Phi$ in the dual model:

$$\Phi_{x}^{\text{mon}}(H) = \phi_{x} e^{i(B,H_{x})},$$

(1)

where the magnetic field of the monopole, $H$, is defined in the three–dimensional time slice which includes the point $x$. By definition, the magnetic monopole field satisfies the Maxwell equation, $\delta^{(3)}H_{x} = \delta_{x}$ which guarantees the dual gauge invariance of the operator $\Phi$ ($\delta^{(3)}$ is the three-dimensional co-differential),

$$\phi \rightarrow \phi e^{i\alpha}, \quad B \rightarrow B + d\alpha.$$  

(2)

The monopole creation operator (1) can be rewritten in the original representation in terms of the compact field $\theta$. In lattice notations the expectation value of this operator is (3):

$$\langle \Phi_{x}^{\text{mon}} \rangle = \frac{1}{Z} \int_{-\pi}^{\pi} \! d\theta \exp\{ -S(d\theta + W) \},$$

$$Z = \int_{-\pi}^{\pi} \! d\theta \exp\{ -S(d\theta) \}.$$  

(3)

For compact lattice electrodynamics the general type of the action satisfies the relation: $S(d\theta + 2\pi n) = S(d\theta)$, $n \in \mathbb{Z}$. Besides the Coulomb monopole field $H$ the tensor form $W = 2\pi\delta\Delta^{-1}(H_{x} - \omega_{x})$ depends on the Dirac string $\omega$ which ends at the monopole position, $\delta^{*}\omega_{x} = \Phi^{*}\omega_{x}$, and is not restricted to the 3D time–slice.

The operator (1) is well defined for the theories without dynamical matter fields. However, if an electrically charged matter is added, then the creation operator (2) depends on the positions of the Dirac strings. To see this fact note that in the presence of the dynamical matter the dual gauge field $B$ becomes compact. Indeed, as we mentioned the pure compact gauge model is dual to the non–compact $U(1)$ with matter fields (referred above as the (dual) Abelian Higgs model). Reading this relation backwards we conclude that the presence of the matter field leads to the compactification of the dual gauge field $B$.

The compactness of the dual gauge field implies automatically that the gauge field transformation (2) must be modified:

$$\phi \rightarrow \phi e^{i\alpha}, \quad B \rightarrow B + d\alpha + 2\pi k,$$

(4)

where the compactness of the gauge field, $B \in (-\pi, \pi]$, is supported by the integer–valued vector field $k = k(B, \alpha)$. The role of the field $k$ is to change shape of the dual Dirac strings attached to the electric charges in the dual theory. One can easily check that the operator (1) is not invariant under the compact gauge transformations (4):

$$\Phi_{x}^{\text{mon}}(H) \rightarrow \Phi_{x}^{\text{mon}}(H) e^{2\pi i(k,H_{x})}.$$  

(5)

This inconsistency is studied in Ref. [3]. According to eq. (5) if the field $H$ is integer–valued then operator (1) is invariant under compact gauge transformations (4). This condition and the Maxwell equation require for the field $H$ to have a form of a string attached to the monopole ("Mandelstam string"): $H_{x} \rightarrow j_{x}$, $j \in \mathbb{Z}$. The string must be defined in the three–dimensional time–slice similarly to the magnetic field $H$. However, one can show that for a fixed string position the operator $\Phi$ creates a state with an infinite energy. This difficulty may be bypassed [3] by summation over all possible positions of the Mandelstam strings with a suitable measure $\mu(j)$:

$$\Phi_{x}^{\text{mon,new}} = \phi_{x} \sum_{j_{x} \in \mathbb{Z}} \mu(j_{x}) e^{i(B,j_{x})}.$$  

(6)

If Higgs field $\phi$ is $q$–charged ($q \in \mathbb{Z}$), the summation in eq. (6) should be taken over $q$ different strings each of which carries the magnetic flux $2\pi/q$. The transformation of $\Phi_{x}^{\text{mon,new}}$ to the original representation can be easily performed and we get the expression similar to eq. (4).

3. NUMERICAL RESULTS

The purpose of the present publication is the numerical investigation of the operator $\Phi_{x}^{\text{mon,new}}$. We study the monopole creation operator (3) in the compact Abelian Higgs model with the action:

$$S = -\beta \cos(d\theta) - \kappa \cos(d\varphi + q\theta),$$

(7)

where $\theta$ is the compact gauge field and $\varphi$ is the phase of the Higgs field. For simplicity we consider the London limit of the model in which the radial part of the Higgs field is frozen. We calculate the (modified) effective constraint potential,

$$V_{\text{eff}}(\Phi) = -\ln \left( \langle \delta(\Phi - \Phi_{x}^{\text{mon,new}}) \rangle \right).$$  

(8)
The effective potential \( V_{\text{eff}}(\Phi) \) for positive values of the monopole field is shown in Figure 1. In the confinement phase \( (\beta = 0.9) \) the potential has a Higgs form signalling the monopole condensation. In the deconfinement phase the potential has minimum at \( \Phi = 0 \) which indicates the absence of the monopole condensate.

We conclude that the new operator can be used as a test of the monopole condensation in the theories with electrically charged matter fields. The minimum of the potential, corresponding to the value of the monopole condensate is zero in deconfinement phase (Fig.1(b)) and non zero in the confinement phase (Fig.1(a)).

**ACKNOWLEDGMENTS**

The authors are grateful to F.V. Gubarev for useful discussions. V.A.B. and M.I.P were partially supported by grants RFBR 00-15-96786, RFBR 01-02-17456, INTAS 00-00111 and CRDF award RP1-2103. M.I.Ch. is supported by JSPS Fellowship P01023.

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