Theoretical Particle Limiting Velocity From The Bicubic Equation: Neutrino Example

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Abstract

There has been a lot of interest in measuring the velocities of massive elementary particles, particularly the neutrinos. Some neutrino experiments at first observed superluminal neutrinos, thus violating the velocity of light $c$ as a limiting velocity. But, after eliminating some mistakes, such as, for the OPERA experiments plugging the cable correctly and calibrating the clock correctly, the measured neutrino velocity complied with $c$. Pursuing the theoretical side of particle limiting velocities, here directly from the special relativistic kinematics, in which all physical quantities are in the overall mathematical consistency with each other, one treats formally the velocity of light $c$ as yet to be deduced particle limiting velocity, and derives the bicubic equation for the particle limiting velocity in the arbitrary reference frame. The Lorentz invariance (LI) of the energy-momentum dispersion relation assumes the velocity of light $c$ to be universal limiting velocity of any particle. This expects the physical solutions of the bicubic equation to be constrained in a sense that any physical limiting velocity solution should equal numerically to $c$, preferably exactly, or at least, in a extremely good approximation. A rather large numerical departure from $c$ means solution which, if it is physical, would indicate the significant degree of Lorentz violation (LV). However, this LV could be false if experimentally particle parameters were read wrongly yielding different from $c$ solution for the physical limiting velocity. Still, one may allow possible anticipation of finding some LV in neutrino physics. The three solutions for the squares of limiting velocities, denoted as $c_1^2$, $c_2^2$ and $c_3^2$, depend on the particle $m$, $E$ and $v$ (mass, energy and ordinary velocity) through inverse sinusoidal functions. As $c_1^2$, $c_3^2 > 0$ and $c_2^2 < 0$, only $c_1^2$ and $c_3^2$ have chances to be physical while $c_2^2$ is unphysical. Furthermore, with the inverse sinusoidal functions principal values dependences, $c_1^2$ and $c_3^2$ being positive are complementary limiting velocity squares, at least one of them physical and presumed luminal, while $c_2^2$ being negative is definitely unphysical. However, $c_2^2$ can become $c_1^2$ and $c_3^2$ when transformed from the principal values region into the multiple values region of the inverse sinusoidal functions. With these solutions one can treat physical limiting
velocities, for any particle, electron, neutrino, photon, etc. The OPERA 17GeV muon neutrino velocity experiments are discussed through the limiting velocity $c_3$ because the calculated neutrino $m_\nu c^2$ of 0.076 eV, being negligible, makes $c_1$ unphysical. Furthermore, because in OPERA experiments, $m_\nu c^2 \ll E_\nu$, one finds out that $c_3 = c(1+\Delta) \simeq c$ because $\Delta$ is negligible (it varies from $O(-10^{-6})$ to $O(10^{-6})$). This implies basically the LI of the neutrino energy-momentum dispersion relation.

1. Introduction

There have been the whole series of neutrino velocity experiments such as the OPERA collaborations with the detector in the CNGS beam [1] (versions 1, 2 and 3), the OPERA detector the CNGS beam using the 2012 dedicated data [2] (versions 1 and 2) as well as the ICARUS detector in the CNGS beam [3] (versions 1 and 2).

These neutrino velocity experiments are not simple to carry out and a lot of them had a variety mistakes. For instance in [1] (version 1) a cable was incorrectly plugged and there was a miscalibrated atomic clock. Both mistakes were found and the OPERA collaborators made proper corrections and after a precise measurement of the neutrino velocity in agreement with $c$, the velocity of light, published the result in [1] (version 3). OPERA collaborators also published the results from a measurement of a special bunched neutrino beam [2] (version 2) giving the precision measurement of the muon neutrino and muon anti-neutrino velocities, in good agreement with $c$, the velocity of light. Other, so called Gran Sasso laboratory repeated the measurements and obtained $c$, the velocity of light, for the neutrino velocity [3] (version 2). Now, the masses of flavor neutrinos, whose velocities one can measure, are not yet known precisely but are calculated as exactly as possible from the provided masses of the mass state neutrinos. Nevertheless, the accepted notion from the special relativity, also in cases like these, expects the neutrino velocity not to exceed the velocity of light $c$, considered in the special relativity as the universal limiting velocity.

In Section 2, from relativistic kinematics one formulates the sixth order bicubic equation for the square of the limiting velocity and, as exposed in [4], can be solved as a cubic equation for $c^2$. The bicubic equation yields three solutions $c_i^2$, $i = 1, 2, 3$, depending on $m, v,$ and $E$ (particle mass, velocity and energy). One expects that at least one solution is physical and luminal and as such supports the LI; that is when evaluated to be numerically, either exactly or practically exactly equal to $c$, so that its substitution in place of $c$, will not change at all the energy-momentum dispersion relation or it will change it insignificantly. The three limiting velocity solutions depend on the inverse sinusoidal function principal values in such a way that the complementary $c_1$ and $c_3$ are real and, at least one of them, physical while $c_2$ is imaginary and as such unphysical. However, for the specific multiple values of the inverse sinusoidal function, $c_2$ can become $c_1$ and $c_3$. The important thing is the fact that $c_1$ and $c_3$ are complementary limiting velocities since they together can cover all the allowed particle parameters, $m, v,$ and $E$, while each of them is limited to particular values.
Besides the exact solutions, Section 2 contains also the perturbative solutions for $c_i^2$, $i = 1, 2, 3$, basically in terms of $(mv^2/E)$. These perturbative solutions are often very convenient for determining as to which of the limiting velocities is physical, $c_1$ or $c_3$, either luminal ($=, \simeq c$) or not ($\neq c$) that is, applicable for the particle in question.

In Section 3, after deducing the three flavor neutrino masses, one finds out that the physical parameter structure of the OPERA [2] muon neutrino velocity experiment is such that the Taylor series expansion, in terms of $(mv^2/E)$, strongly suggests $c_3$ as the luminal solution. Along the same lines, one notices that $c_1$ is unphysical in the OPERA [2] experiments. The same is true for other neutrino velocity experiments [1] and [3]. Furthermore, because the muon neutrino mass being negligible, one finds out that in fact $c_3 \simeq c$. To verify this perturbative result, one performs the calculation also with exact non-perturbative expression for $c_3$. The result is the same as from the perturbative calculation.

Conclusion and final remarks are given in Section 4. Here also the comparisons with other approaches from the literature for discussing the Lorentz invariance and Lorentz violation, either through changes in the Dirac equation or by explicitly changing the relativistic kinematics, are given.

2. Particle limiting velocities from the velocity bicubic equation

The velocity of light, $c$, in the special relativity particle kinematics is considered the universal relativistically invariant limiting velocity. Here, with the desire of having $c$ on an equal basis with other physical parameters, one treats it as yet to be analytically formulated limiting velocity and starts with the same kinematics.

$$\vec{p} = \frac{E}{c^2}, E^2 = \frac{m^2c^4}{1 - \frac{v^2}{c^2}} \quad (1,2)$$

which defines the particle momentum $\vec{p}$, and energy $E$, through its mass $m$ and velocity $\vec{v}$. Momentum and energy from (1) and (2) are related through the mass shell condition,

$$\vec{p}^2c^2 - E^2 = -m^2c^4 \quad (3.1)$$

whose change, if any, caused by replacing $c$ with limiting velocity solutions from the bicubic equation (to be discussed), could indicate either LI or LV, providing that particle parameters, $m, v$, and $E$ are known.

As in the neutrino velocity experiments [1,2,3], one has the known energy and the velocity of fixed direction, it is convenient to continue with just relation (2) by transforming it into the bicubic equation for the particle limiting velocity $c$:

$$m^2c^6 = E^2c^2 - E^2v^2 \quad (3.2)$$

Next, one rewrites it in the mathematically more familiar forms with solutions characterized by the discriminant satisfying $D < 0$,
\[
\left( \frac{c}{v} \right)^2 - \left( \frac{E}{mv^2} \right)^2 \left( \frac{c}{v} \right)^2 + \left( \frac{E}{mv^2} \right)^2 = 0, \quad q = -p = \left( \frac{E}{mv^2} \right)^2,
\]
\[
D = \left( \frac{q}{2} \right)^2 + \left( \frac{p}{3} \right)^3
\]
\[
= \frac{1}{4} \left( \frac{E}{mv^2} \right)^4 \left[ 1 - \frac{4}{27} \left( \frac{E}{mv^2} \right)^2 \right] < 0,
\]
(3.3)

\[
z = \frac{3\sqrt{3}mv^2}{2E}; \quad D < 0: \quad -1 < z < 1 \quad (4)
\]

According to [4], the solutions for (3.2,3) plus (4) can be written as

\[
c_1^2 = 2v^2 \sqrt{\frac{|p|}{3}} \cos \left( \frac{\theta}{3} + \frac{\pi}{6} \right),
\]

\[
c_2^2 = -2v^2 \sqrt{\frac{|p|}{3}} \cos \left( \frac{\theta}{3} - \frac{\pi}{6} \right),
\]

\[
c_3^2 = -2v^2 \sqrt{\frac{|p|}{3}} \cos \left( \frac{\theta}{3} + \frac{\pi}{2} \right)
\]

\[
\cos \left( \theta + \frac{\pi}{2} \right) = -\frac{q}{2 \left( \frac{|p|}{3} \right)^{\frac{2}{3}}} = -\frac{3\sqrt{3}mv^2}{2E},
\]

\[
\theta = -\frac{\pi}{2} + \cos^{-1} \left( \frac{-3\sqrt{3}mv^2}{2E} \right) = \sin^{-1} \left( \frac{3\sqrt{3}mv^2}{2E} \right) \quad (5)
\]

However, in order to see more of the physics, the exact solutions from (5), with the help from relations (3.3) and (4), are rewritten in such a way as to exhibit more explicitly the \( m, v, \) and \( E \) parameters in them:

\[
c_1^2 = \frac{2E}{\sqrt{3}m} \sin \left[ \frac{\pi}{3} - \frac{1}{3} \sin^{-1} \left( \frac{3\sqrt{3}mv^2}{2E} \right) \right] > 0, \quad (6.1)
\]

\[
c_2^2 = -\frac{2E}{\sqrt{3}m} \cos \left[ \frac{1}{3} \sin^{-1} \left( \frac{3\sqrt{3}mv^2}{2E} \right) - \frac{\pi}{2} \right] < 0, \quad (6.2)
\]

\[
c_3^2 = \frac{2E}{\sqrt{3}m} \sin \left[ \frac{1}{3} \sin^{-1} \left( \frac{3\sqrt{3}mv^2}{2E} \right) \right] > 0 \quad (6.3)
\]

Noting that with the variable \( z \) from relation (4) the inverse sinus function, \( \sin^{-1} (z) \), in (6.1,2,3) refer to the principal values that lie in the \((-\pi/2 \; \text{to} \; \pi/2)\)
range where either of the positive $c_1^2$ and $c_3^2$ can be physical while the negative $c_2^2$ is definitely unphysical. However, $c_2^2$ can become physical in the multiple values ranges. Denote $c_i^2, i = 1, 2, 3$ dependence on $z$ as $c_i^2 \left[ \sin^{-1}(z) \right], i = 1, 2, 3$. Then assume that alternately in $c_2^2$ the range of $\sin^{-1}(z)$ is changed from $(-\pi/2$ to $\pi/2)$ to $(3\pi/2$ to $5\pi/2)$ and to $((-5\pi/2$ to $(-3\pi/2))$. This is simply achieved by replacing $c_2^2 \left[ \sin^{-1}(z) \right]$ in (6.2) alternately with $c_2^2 \left[ \sin^{-1}(z) + 2\pi \right]$ and $c_2^2 \left[ \sin^{-1}(z) - 2\pi \right]$. Then the simple evaluations, with the help from (6.1,2,3), shows that

$$c_2^2 \left[ \sin^{-1}(z) + 2\pi \right] = c_2^2 \left[ \sin^{-1}(z) \right]; \quad c_2^2 \left[ \sin^{-1}(z) - 2\pi \right] = c_2^2 \left[ \sin^{-1}(z) \right]$$

(6.4,5)

In (6.4,5) the respective multivalue ranges in $c_2^2$ are $(3\pi/2$ to $5\pi/2)$ and $(-5\pi/2$ to $-3\pi/2)$ while in $c_1^2$ and $c_3^2$ the range is $(-\pi/2$ to $\pi/2)$. As one sees, here the supplementary limiting velocities $c_1$ and $c_3$ are the only ones of the physical significances. Importance of (6.4,5) is in the fact that sometimes one has to change the “coordinates” in order to find out the same or perhaps even the new physics. These different ranges, principle values and multiple values do not change the fact that according to (4) $|z| < 1$. The imaginary $c_2$ moves the “imaginary” physics to the "real" physics with $\sin^{-1}(z)$, being changed to $\sin^{-1}(z) \pm 2\pi$ now with the multiple value ranges.

Next, it is illustrative to perform the Taylor series expansions of $c_i^2, i = 1, 2, 3$ (6.1,2,3). Except for the few first terms, they are done basically in terms of $(mv^2/E)$ with inequalities between $v, E$ and $m$ as indicated in each of the series. Extra $v^2$ factor in each term makes the whole expression to have dimension of $v^2$.

$$c_1^2 = \frac{E}{m} - \frac{v^2}{2} - \frac{3mv^4}{8E} - \frac{m^2v^6}{2E^2} + O \left[ v^2 \left( \frac{mv^2}{E} \right)^3 \right], \quad 0 < v^2 < \frac{2E}{3\sqrt{3}m} \quad (7.1)$$

$$c_2^2 = -\frac{E}{m} - \frac{v^2}{2} + \frac{3mv^4}{8E} - \frac{m^2v^6}{2E^2} + O \left[ v^2 \left( \frac{mv^2}{E} \right)^3 \right], \quad 0 < v^2 < \frac{2E}{3\sqrt{3}m} \quad (7.2)$$

$$c_3^2 = v^2 + \frac{m_2^2v^6}{E^2} + \frac{69m_4v^{10}}{32E^4} + O \left[ v^2 \left( \frac{mv^2}{E} \right)^6 \right], \quad 0, m < \frac{2E}{3\sqrt{5}v^2} \quad (7.3)$$

Already from exact solutions (6.1,2,3) as well as now from the Taylor series, one sees that all three limiting velocities are different from each other in the principle values region. For instance, $c_1^2$ and $c_2^2$ diverge for $m = 0$ but are finite for $v = 0$. So $c_1^2$ and the unphysical $c_2^2$ need to have $m \neq 0$. The different behaviors of $c_1^2$ and $c_2^2$ for either $m \to 0$ or $v \to 0$ emphasizes their complementarity. This small excursion, suggests defining the physical $c_1^2$ and $c_3^2$ in the principle values region satisfying

$$\text{Phyhsical} : c_1^2, c_3^2 \neq 0, \infty \quad (8.1)$$
Here, consistent with (8.1), is the summary of important situations that can occur for $c_1^2$ and $c_3^2$

\[
\begin{align*}
    v &= 0, m \neq 0; \quad \frac{E_0}{m} = c_1^2 = c^2; \quad c_3^2 = 0 \text{ (unphysical)}, \\
    m &= 0, E \text{ finite}; c_3 = v = c \text{ (photon)}; \quad c_1 = \infty \text{ (unphysical)}, \\
    m &\neq 0, E \rightarrow \infty; c_3 \rightarrow v; \quad c_1 \rightarrow \infty \text{ (unphysical)}. 
\end{align*}
\]  

(8.2)

(8.3)

(8.4)

What examples (8.2,3,4) show clearly is that in these particular situations the physically acceptable limiting velocity is either given by $c_1$ or $c_3$ which further indicates to their complementarity. The relation (8.2) is to be understood as a definition of $E(v)$ at $v = 0$ where $c_1^2 = c^2$. As $c_1$ and $c_3$ are the limiting velocity solutions of the bicubic equation (3.3), it is appropriate to see what effect will be caused if one sets either $c_1$ or $c_3$ in place of $c$ in the energy momentum relation (3.1). These substitutions leave the energy momentum relation (3.1) either LI or, to a degree, LV under the Lorentz transformations with the following respective general possible values for $c_1$ or $c_3$:

\[
\begin{align*}
    LI : c_1 &= c \text{ or } c_3 = c; \\
    LV : c_1 &\neq c \text{ or } c_3 \neq c.
\end{align*}
\]  

(8.5)

Here it is assumed that either $c_1$ or $c_3$ is LI but not both of them at the same time. Also, the degree of the LV would depend on how strongly $c_1 \neq c$ or $c_3 \neq c$.

Despite their complementarity, it is necessary to investigate whether it can happen that for a given particle one can have $c_1 = c_3$? Imposing this equality, from (6.1) and (6.2), with $z$ as defined in (4), one arrives at the following sequence of equations,

\[
\begin{align*}
    c_1^2 &= c_3^2 \sin \left[ \frac{\pi}{3} - \frac{1}{3} \sin^{-1}(z) \right] = \sin \left[ \frac{1}{3} \sin^{-1}(z) \right], \\
    \sin^{-1}(z) &= \frac{\pi}{2}; \quad z = 1
\end{align*}
\]  

(9.1)

(9.2)

Since relation (9.2) is in contradiction to the relation (4), which excludes $z = 1$, one concludes that $c_1$ and $c_3$, while complementary, cannot be equal to each other for the same particle, $c_1 \neq c_3$. Hence, if for instance $c_3 = c$ then $c_1 \neq c$ and so on.

### 3. Limiting velocity of the neutrino

Here one is specifically interested in applying the formalism of obtaining the limiting velocity for the muon neutrino, $\nu_\mu$, with the physical parameters from the OPERA experiment [2]. From the perturbative expressions (7), the indication is that $c_1$ and $c_3$ are respectively, the unphysical and physical limiting velocities, in this case. To see whether the physical $c_3$ is also luminal, that is, leading to $c$ and LI, one first expresses $c_3$ perturbatively from (7.3) and then, for verification purposes, also exactly from (6.3).
The formalism in relations (6) and (7) demand, in addition to $E$ and $v$ also the value of the mass, here denoted for the muon neutrino as $m_{\nu}$. As in the reference [2] the value of $m_{\nu}$ is not given, one has to first find which value is presently favored, although in OPERA experiment [2] with the neutrino energy of $E_{\nu}(\mu) = 17$GeV, the calculated neutrino mass even if exact, will be very likely negligible as compared to the energy. Now, there are three flavor neutrinos, denoted as $\nu_e$, $\nu_\mu$ and $\nu_\tau$, the electron, muon and tau neutrino. Their masses $m_{\nu}(e)$, $m_{\nu}(\mu)$ and $m_{\nu}(\tau)$ are derived from the masses of the independent mass-state three neutrinos with masses $m_1, m_2$ and $m_3$. In the discussion of the $\mu - \tau$ symmetry, these masses have been given by S. Gupta et al. [5] as,

\[ m_1c^2 = 0.067 \text{ eV}, \quad m_2c^2 = 0.068 \text{ eV}, \quad m_3c^2 = 0.084 \text{ eV}, \quad (10.1) \]

The flavor neutrino masses are defined in [5] with the help of the Harrison et al. neutrino mixing matrix [6], $U_{\alpha,i} \alpha = e, \mu, \tau; i = 1, 2, 3$, connecting the flavor neutrino states to the mass-state neutrino states (see, also [7]). Hence, using $U_{\alpha,i}$ as in references [6] and [7], according to Gupta et al. [5], the flavor neutrino masses are defined as

\[ \alpha = e, \nu, \tau; \quad i = 1, 2, 3: \quad m_{\nu}(\alpha) = \left[ \sum_i |U_{\alpha,i}|^2 m_i^2 \right]^{\frac{1}{2}}; \]

\[ (U_{\alpha,i}) = \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\frac{1}{\sqrt{2}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (10.2) \]

yielding

\[ m_{\nu}(e)c^2 = 0.067 \text{ eV}, \quad m_{\nu}(\mu)c^2 = 0.076 \text{ eV}, \quad m_{\nu}(\tau)c^2 = 0.076 \text{ eV} \quad (11) \]

With these values and from [2] the collection of data for the OPERA muon neutrino velocity experiment is

\[ E_{\nu}(\mu) = 17\text{GeV}, \quad m_{\nu}(\mu)c^2 = 0.076 \text{ eV}, \quad v_{\nu}(\mu) = c(1 + \Delta) \]

\[ -1.8 \times 10^{-6} \leq \Delta \leq 2.3 \times 10^{-6} \quad (12) \]

Because one has that $m_{\nu}(\mu) < \left( \frac{2E_{\nu}(\mu)}{3\sqrt{3}v_{\nu}^2(\mu)} \right)$ for any $v_{\nu}(\mu)$ from (12), one easily deduces, according to (7.1), that approximate numerical value of $c_1$ is $c_1 \approx 4.73 \times 10^5c$. Such a large value makes $c_1$ unphysical. Expecting that $c_3$ is physical, one calculates it with more precision first perturbatively...
from (7.3),

$$c_3^2 = v_\nu^2(\mu) \left[ 1 + \left( \frac{m_\nu(\mu)c^2}{E_\nu(\mu)} \right)^2 \left( \frac{v_\nu(\mu)}{c} \right)^4 + O \left( \left( \frac{m_\nu(\mu)c^2}{E_\nu(\mu)} \right)^4 \left( \frac{v_\nu(\mu)}{c} \right)^8 \right) \right]$$

Furthermore with \( \left( \frac{m_\nu(\mu)c^2}{E_\nu(\mu)} \right) \simeq 4.5 \times 10^{-12} \) and \(|\Delta| \ll 1\), one obtains for \( c_3 \) the perturbative solution in the form,

$$\frac{c_3}{c} \simeq (1 + \Delta) \left( 1 + \frac{1}{2} \left( \frac{m_\nu(\mu)c^2}{E_\nu(\mu)} \right)^2 (1 + \Delta)^4 \right) = (1 + \Delta)$$

(14.1)

Exact expression for \( c_3 \) from (6.3) with the OPERA physical parameters is as follows

$$\left( \frac{c_3}{c} \right)^2 = \frac{2E_\nu(\mu)}{\sqrt{3m_\nu(\mu)c^2}} \sin \left[ \frac{1}{3} \sin^{-1} \left( \frac{3\sqrt{3}m_\nu(\mu)c^2(1 + \Delta)^2}{2E_\nu(\mu)} \right) \right]$$

$$= (1 + \Delta)^2$$

(14.2)

In deriving (14.2), one simply takes into account that \( \left( \frac{m_\nu(\mu)c^2}{E_\nu(\mu)} \right) \ll 1 \) and that in \( (1 + \Delta) \), \(|\Delta| \ll 1 \) so that only first terms in Taylor expansions of \( \sin \) and \( \sin^{-1} \) functions need to be retained. Hence, both solutions, being equal and basically luminal, yield LI with \( c \) as the solution for \( c_3 \):

$$1 - 1.8 \times 10^{-6}c \leq c_3 \leq (1 + 2.3 \times 10^{-6})c : c_3 \simeq c$$

(14.3)

The result in (14.3) is what Einstein envisioned long time ago.

What one notices here is the fact that for OPERA experiments [2], through a particular collection of neutrino physical parameters, such as mass, ordinary velocity and energy, the bicubic equation yields the luminal limiting velocity solution, that is, with the velocity of light \( c \) and with the LI.

Now, on one example one can show how the luminal limiting velocity solution with the LI, can become the superluminal solution with the LV. Simply, in (14.1) and (14.2) replace the negligible \( \Delta \) with a small but finite and positive \( \Delta \). In doing so, one basically obtains the LV from reference [8], implied by the change in the particle special relativistic velocity, written as \( c / \left( \sqrt{p^2 + m^2c^2} \right)^2 + \Delta c \). It is easily seen that in the situation where the mass is negligible, as is in the OPERA experiments, this \( \Delta \) should be the same as \( \Delta \) in relations (14.1.2), and if negligible, as in relations (14), should allow the LI rather than the LV under the Lorentz transformations. Although so far no verifiable LV showed up in neutrino physics, one should, nevertheless, keep an open mind also for such a possibility with subluminal or superluminal anticipations. The LV formulations with superluminal particles through the Dirac equation have been done, for example, in [9] and [10].
4. Conclusion and final remarks

Identifying the velocity of light $c$ in the relativistic kinematics as a limiting velocity yet to be determined, one is led naturally to the bicubic equation for the limiting velocity. Of the three resulting solutions one, $c_2$, is imaginary while two other solutions, $c_1$ and $c_3$, are real and complementary with different emphasis on particle parameter dependences. Of course, if the particle parameters choose, say $c_1 = c$, then as argued in (8.1) to (8.4) $c_3$ will be unphysical. The remarkable point in determining the limiting velocity of any particle from the bicubic equation is that the particle physical parameters will yield for it most likely $c$, no matter where one measures its mass, energy and the ordinary velocity.

It appears that what one needs are the velocity experiments done with rather a massive particle which allow full participation of the particle mass in determining of its limiting velocity. A natural candidate for such a limiting velocity determination is the electron whose mass is very well known and the energy can be chosen so as not to render the mass negligible.

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