The results on directed and elliptic flow for $Pb+Pb$ at the full energy of the SPS (158 GeV/A) and from the first year of $Au+Au$ at RHIC ($\sqrt{s_{NN}} = 130$ GeV) are reviewed. The different experiments agree well and a consistent picture has emerged indicating early time thermalization at RHIC.

Elliptic flow from non-central collisions has become a prime hadronic signature for pressure at early times in relativistic heavy ion collisions [1, 2]. Parton Cascade Model calculations [3] show that the elliptic flow builds up in the first few $fm/c$ and then remains constant. In this model the amount of elliptic flow produced is proportional to the assumed parton-parton scattering cross section. Thus it is thought that rescattering converts the initial space anisotropy of the overlap region of non-central collision to the momentum anisotropy of elliptic flow. As the initial lens-shaped overlap region expands it becomes more spherical, quenching the driving force that produces the elliptic flow. Thus elliptic flow is sensitive to the number of interactions and is considered to be a measure of the degree of thermalization at early time.

I. INTRODUCTION

Directed and elliptic flow are defined by the $v_1$ and $v_2$ coefficients in the Fourier expansion of the azimuthal distribution of particles relative to the reaction plane:

$$1 + 2v_1 \cos(\phi - \Psi_r) + 2v_2 \cos(2(\phi - \Psi_r)), \quad (1)$$

where $\phi$ is the azimuthal angle of a particle and $\Psi_r$ is the azimuthal angle of the reaction plane. The coefficients are usually evaluated differentially as a function of $y$, $p_t$, and centrality.

Three methods of analysis will be discussed. In the conventional method [4] of correlating particles with an event plane the flow coefficients are given by:

$$v_n^{obs} = \langle \cos(n(\phi_i - \Psi_r)) \rangle,$$  \quad (2)

where $\Psi_n$ is the observed event plane of order $n$. Since the observed event plane is not the true reaction plane, the observed coefficients have to be corrected by dividing by the resolution of the event plane, which is estimated by measuring the correlation of the event planes of subevents.

The flow coefficients can also be obtained by the pairwise correlation [5] of all the particles without referring to an event plane. This two-particle correlation method produces the squares of the coefficients, so that one has to take the square root of the correlation effect:

$$v_n^2 = \langle \cos(n(\phi_i - \phi_j)) \rangle_{i \neq j}. \quad (3)$$

Recently, a multiparticle correlation method has been proposed [6] using cumulants. For four particles, one calculates the four-particle correlation minus twice the square of the two-particle correlations:

$$v_4^n = \langle \cos(n(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle - 2 \langle \cos(n(\phi_i - \phi_j)) \rangle \langle \cos(n(\phi_2 - \phi_4)) \rangle$$
$$+ \langle \cos(n(\phi_i - \phi_j)) \rangle \langle \cos(n(\phi_2 - \phi_3)) \rangle. \quad (4)$$

Here one must take the fourth root of the result, and naturally the statistical errors are larger than for the two-particle analysis. However, this method has the advantage of eliminating two-particle non-flow effects which are not correlated with the reaction plane, such as HBT and resonance decay.

II. SPS

A. NA49

![Graph of NA49 directed and elliptic flow for pions from a minimum bias trigger as a function of rapidity. The open points are reflected. The data are still preliminary.](image)

FIG. 1: NA49 directed and elliptic flow for pions from a minimum bias trigger as a function of rapidity. The open points are reflected. The data are still preliminary.

From NA49, initial flow results from part of the data set were published [6, 8] a few years ago. The full set of
data has now been analyzed [9]. The \( v_1 \) and \( v_2 \) values for pions as a function of rapidity are shown in Fig. 1. The values have been integrated over \( p_t \) by taking mean values weighted with the cross section. The \( v_1 \) values have been corrected for conservation of momentum [10]. Before this correction, \( v_1 \) crossed mid-rapidity at \(+0.6\%\), while after the correction, with no adjustable parameters, it can be seen that the curve now crosses mid-rapidity at zero. Other non-flow effects [11, 12] have been verified to be small using the following tests. Short-range correlations, like HBT, were determined to be not large by using striped sub-events where only physically separated particles were correlated. Coulomb effects were determined to be not large by looking at \( \pi^+ \) and \( \pi^- \) particles separately. Neutral resonances, like \( \rho^0 \), were shown to be not important by correlating \( \pi^- \) with an event plane determined only by other \( \pi^- \) particles. The pion flow values integrated over both \( p_t \) and \( y \) are shown in Fig. 2 as a function of centrality bin.

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FIG. 2: NA49 directed and elliptic flow for pions as a function of centrality bin. The centrality increases to the left. The data are still preliminary.

B. WA98

WA98 has measured [13] the directed flow of pions near target rapidity in the Plastic Ball. These data are plotted in Fig. 3 reflected into the forward hemisphere, together with the NA49 data. It can be seen that at projectile rapidities the directed flow is huge. WA98 also has shown that for protons \( v_1 \) is just as large, but in the positive direction. This indicates that looking at projectile rapidity fragments might be a good way to determine the reaction plane. However, even though \( v_1 \) is large, the resolution of the event plane also depends on the square root of the multiplicity, which in this rapidity region is low.

Using silicon drift chambers NA45 has measured [14] the flow coefficients for charged particles. In Fig. 4 their elliptic flow values at mid-rapidity versus centrality are plotted. The centrality increases to the left because the abscissa is the cumulative most central percentage of all events that are in the bin. The data are preliminary.

C. NA45

FIG. 3: NA49 (circles) and WA98 (squares) pion data are compared. Plotted is the directed flow versus rapidity. The vertical lines are at mid-rapidity and beam rapidity. The data are preliminary.

FIG. 4: NA49 (circles) and NA45 (squares) charged particle data are compared. Plotted is the elliptic flow at mid-rapidity versus the centrality. The centrality increases to the left because the abscissa is the cumulative most central percentage of all events that are in the bin. The data are preliminary.

Using silicon drift chambers NA45 has measured [14] the flow coefficients for charged particles. In Fig. 4 their elliptic flow values at mid-rapidity are plotted versus centrality, together with the same quantity for pions from NA49. The agreement is good. NA45 also has a RICH detector which identifies high \( p_t \) pions. When correlated with an event plane determined from their TPC, the \( p_t \) dependence falls nicely on a line extrapolated from the lower \( p_t \) results of NA49. However, when they correlate pairs of high \( p_t \) pions from the RICH using Eq. 3, their results are somewhat higher [15].
D. NA50

NA50 has analyzed the neutral transverse energy from their calorimeter in the rapidity region from 1.1 to 2.3. NA50 has shown elliptic flow as a function of centrality [16], but it is hard to compare their results with the other experiments because of the different method of analysis.

III. RHIC

A. STAR

STAR results for elliptic flow for charged particles [17] and identified particles [18] have been published. The flow is larger than at the SPS and comes close to the hydrodynamic predictions [19] for the mid-central and central data. The identified particle flow shows a variation with mass as predicted by hydrodynamics. However, the high \( p_t \) behavior [20] exhibits a saturation effect well below where hydro would saturate. It has been suggested that at high \( p_t \) elliptic flow could be due to parton energy loss (jet quenching) [21].

The doubly integrated elliptic flow values as a function of centrality are shown in Fig. 5, using the conventional event plane method (Eq. 2). The asymmetric error bars indicate the systematic uncertainty due to possible non-flow effects. A four-particle correlation analysis [22] using Eq. 4 is also shown in the figure. Indeed, when two-particle non-flow effects are eliminated, the points do come down to about the limits of the systematic errors. It should be noted that part of the decrease could be due to fluctuation effects [23] which also would go in the same direction. However, the agreement with hydro for mid-central and central collisions is not affected.

B. PHENIX

The PHENIX results [24] for charged particle elliptic flow as a function of centrality for a minimum bias trigger is shown together with the STAR results in Fig. 6. PHENIX uses the pair-wise method of Eq. 3. The agreement is excellent. However, at the moment, the doubly-integrated elliptic flow values as a function of centrality are about 50% higher for PHENIX than for STAR. Only about half of this discrepancy can be accounted for by the higher \( p_t \) threshold of PHENIX.

C. PHOBOS

The PHOBOS elliptic flow values [25] as a function of centrality are shown in Fig. 7, together with the STAR values. The agreement is excellent. Fig. 8 shows PHOBOS elliptic flow for a minimum bias trigger measured over a wide range of pseudorapidity. Where there are STAR measurements, in a narrow range near mid-rapidity, the agreement is good with the flat STAR distribution. A 3D hydro calculation [26] gets rather flat distributions over a wide range of pseudorapidity for all reasonable initial conditions. Thus the fast fall off away from midrapidity in Fig. 8 is not understood at present in a model with thermalization.

IV. CONCLUSIONS

Good agreement between the various experiments has been achieved. There might be some indication that the pair-wise method for high \( p_t \) particles indicates non-flow effects. The four-particle method at RHIC indicates that

![Figure 5](image_url)

**FIG. 5:** STAR elliptic flow for charged particles versus centrality. The results using the conventional event plane method (circles) and the four-particle cumulant method (squares) are shown. The later are preliminary.

![Figure 6](image_url)

**FIG. 6:** STAR (circles) and PHENIX (squares) charged particle data for a minimum bias trigger are compared. Plotted is the elliptic flow versus the transverse momentum. The PHENIX data are preliminary.

![Figure 7](image_url)

**FIG. 7:** PHENIX (squares) elliptic flow values as a function of centrality are shown together with the STAR values.
FIG. 7: STAR (circles) and PHOBOS (squares) charged particle data are compared. Plotted is the elliptic flow versus centrality. The PHOBOS data are preliminary and the error bars are only statistical.

FIG. 8: PHOBOS charged particle elliptic flow for a minimum bias trigger plotted versus pseudorapidity. The data are preliminary and the error bars are only statistical.

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there are non-flow effects for peripheral collisions. The elliptic flow increases from the SPS to RHIC, and at RHIC is close to the hydro limit. This is taken to be a signature for early time thermalization at RHIC.
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