The electro-magnetic Form Factors of the Proton in chiral Soliton Models

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Abstract

The electro-magnetic form factors of the proton are calculated in a chiral soliton model with relativistic corrections. The magnetic form factor $G_M$ is shown to agree well with the new SLAC data for spacelike $Q^2$ up to 30 (GeV/c)$^2$ if superconvergence is imposed. The direct continuation through a Laurent series to the timelike region above the physical threshold is in fair agreement with the presently available set of data. The electric form factor $G_E$ is dominated by a zero in the few (GeV/c)$^2$ region which appears to be in conflict with the SLAC data.

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1 Relativistic soliton form factors

The new SLAC data\textsuperscript{1,2} for electro-magnetic form factors (FF) of the proton to high $Q^2$ pose a challenging test for the relativistically corrected FFs of chiral soliton models.

It has repeatedly been demonstrated for various versions of chiral lagrangians that the nucleon e.m. FFs are rather well accounted for low $Q^2$ with nucleons as nonrelativistic solitons in coupled $\pi$, $\rho$, and $\omega$ fields\textsuperscript{3,4}. The implementation of relativistic corrections is especially easy for solitonic nucleons due to the Lorentz covariance of the field equations (in contrast to the corresponding problem in bag models\textsuperscript{5}). The corrections reflect the Lorentz boost from the soliton rest frame to the Breit frame, in which the soliton moves with velocity $v$ which satisfies

$$\gamma^2 = (1 - v^2)^{-1} = 1 + \frac{Q^2}{(2M)^2}$$

for momentum transfer $Q^2$ ($Q^2 > 0$ in the spacelike region) and soliton mass $M$. The classical result for the magnetic FF is\textsuperscript{6}

$$G_M(Q^2) = \frac{1}{1 + \frac{Q^2}{(2M)^2}} G_{nr}^M \left( \frac{Q^2}{1 + \frac{Q^2}{(2M)^2}} \right)$$

where $G_{nr}$ is the nonrelativistic FF evaluated in the soliton restframe. The electric FF $G_E$ does not contain the factor $\gamma^{-2}$ on the right-hand side\textsuperscript{6}:

$$G_E(Q^2) = G_{nr}^E \left( \frac{Q^2}{1 + \frac{Q^2}{(2M)^2}} \right)$$

(this is in contrast to bag models\textsuperscript{5} where the wave functions of the spectator quarks supply the factor $\gamma^{-2}$ also for $G_E$.)

According to the derivation of (2,3) within the tree approximation of the soliton model $M$ is the classical soliton mass $M_S$, although ideally, of course, $M$ should coincide with the physical nucleon mass $M_N$. From (2,3) the asymptotic limit of $G(Q^2)$ for $Q^2 \to \infty$ is given by $G_{nr}(4M^2)$. For commonly used chiral lagrangians the first zeros of the nonrelativistic FFs occur at masses $M_0$

$$G_{nr}(4M_0^2) = 0$$

which are of the order of the nucleon mass, with $M_0 < M_N$ for $G_{nr}^E$ and $M_0 > M_N$ for $G_{nr}^M$. This implies that the asymptotic behaviour of $Q^4G(Q^2)$ is very sensitive to the precise value of $M$ used in (2,3):

$$\lim_{Q^2 \to \infty} Q^4G(Q^2) = \pm \infty \quad \text{for} \quad M \lesssim M_0.$$
The actual values of $M_0$ for which $G^{nr}(4M_0^2)$ vanishes, depend on the choice of the parameters in the effective lagrangian; furthermore both, $M_S$ and $G^{nr}$ are subject to quantum corrections. It is therefore unrealistic to expect reliable predictions from the model itself for the high-$Q^2$ behaviour of $Q^4G(Q^2)$.

This ambiguity in the high-$Q^2$ behaviour of $Q^4G(Q^2)$ can be used to impose superconvergence ($Q^2G_M(Q^2) \to 0$ for $Q^2 \to \infty$) on $G_M(Q^2)$ by choosing $M = M_0$ in (2), or, to put it more generally, to check the functional form of (2) against the experimentally observed behaviour of $Q^4G(Q^2)$ for large $Q^2$ by choosing $M$ as an adjustable parameter. Due to the lack of the factor $\gamma^{-2}$ on the right hand side in (3), superconvergence cannot be imposed on $G_E$ by any choice of $M$. For a specific effective lagrangian (and due to possibly different quantum corrections) we also should not expect $M$ to be necessarily the same for different formfactors.

The low-$Q^2$ behaviour is not strongly affected by these variations in $M$, although due to the factor $\gamma^{-2}$ in front of $G^{nr}_M$ in (2), even the magnetic radius receives a small contribution from finite values of $M$.

2 The minimal $\pi$-$\rho$-$\omega$ model

In order to study the implications of a simple effective lagrangian we choose the minimal model which comprises $\rho$ and $\omega$ mesons together with the pionic field $U$ in chiral covariant way:

$$L_{VM} = L^{(2)} + L^{(\rho)} + L^{(\omega)}$$

with

$$L^{(2)} = \frac{f^2}{4} \int (-trL_\mu L^\mu + m^2_\pi tr(U + U^\dagger - 2))d^3x,$$  

$$L^{(\rho)} = \int \left( -\frac{1}{8} tr\rho_{\mu\nu} \rho^{\mu\nu} + \frac{m^2_\rho}{4} tr(\rho_\mu - i \frac{g}{2}(l_\mu - r_\mu))^2 \right) dx,$$  

$$L^{(\omega)} = \int \left( -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{m^2_\omega}{2} \omega_\mu \omega^\mu + 3g_{\omega\omega} B^\mu \right) dx,$$

the Maurer-Cartan forms

$$L^\mu = U^\dagger \partial^\mu U = L^\mu_a \tau_a,$$

topological baryon current $B_\mu$

$$B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} tr L^\nu L^\rho L^\sigma$$
and \( l_\mu = \xi^\dagger \partial_\mu \xi \), \( r_\mu = \partial_\mu \xi \xi^\dagger \) with \( \xi = U^\dagger \). In the gauge transformation of the vector mesons \( V_\mu \)

\[
V^\mu \rightarrow e^{(i\sigma Q_0 + i\epsilon V Q_V)}(V^\mu + \frac{Q_V}{g} \partial^\mu \epsilon_V + \frac{Q_0}{g_0} \partial^\mu \epsilon_0) e^{(-i\sigma Q_0 - i\epsilon V Q_V)}
\]

(12)

(with \( Q_0 = 1/6 \), \( Q_V = \tau_3/2 \)) through which the electromagnetic currents are defined, the gauge parameter \( g_0 \) need not coincide with \( g_\omega \) because the contribution of the neutral \( \omega \)-mesons to the isoscalar part of the e.m. current is not necessarily fixed through the electric charge \( e(=1) \).

With the experimental values for \( f_\pi \), the meson masses \( m_\pi, m_\rho, m_\omega \), and \( g \) fixed by the KSRF relation \( g = m_\rho/(2\sqrt{2}f_\pi) = 2.925 \), \( L_{VM} \) contains \( g_\omega \) as the only free parameter; we use it to fit the magnetic moment of the proton \( \mu_p = G_M(0) = 2.79 \); the resulting value is \( g_\omega = 4.125 \).

### 3 Results

For this choice of the effective \( L_{VM} \) the low-\( Q^2 \) pattern of the FFs is still sensitive to the value of \( g_0 \) in the isoscalar part of the e.m. current. Agreement with the data for \( G_E \) in the region \( Q^2 < 1 \) (GeV/c)^2 can be achieved for \( g_0 \geq 2.5g_\omega \). For \( g_0 = 2.5g_\omega \) superconvergence for \( G_M \) requires \( M = 1.12 \) GeV in (2). The resulting e.m. FFs for the proton (divided through the standard dipole \( G_D = (1 + Q^2/0.71)^{-2} \)) are shown by the full lines in figs. 1 and 2, plotted against the logarithm of \( Q^2 \). Both, \( G_E \) and \( G_M/\mu_p \), are calculated for the same value of \( M(=1.12 \) GeV). The rapid decrease of \( G_E/G_D \) above \( Q^2 \sim 1 \) (GeV/c)^2 which is in apparent contradiction to the SLAC data, has its origin in the first zero of \( G^\pi_E \) which is pushed up to \( Q^2 \approx 3.7 \) (GeV/c)^2 by the boost to the Breit frame in (3). It can be shifted to higher \( Q^2 \) by decreasing \( M \) but then \( G_E \) overshoots the dipole near \( Q^2 \approx 1 \) (GeV/c)^2 (the dash-dotted line in fig.2 is calculated for \( M = 0.94 \) GeV). Because the rapid decrease of \( G_E/G_D \) is due to a zero in \( G^\pi_E \) it cannot be removed by an additional factor \( \gamma^{-2} \) in front of \( G^\pi_E \) which may appear in bag models.

For the high-\( Q^2 \) part of \( G_M \) the choice \( g_0 = g_\omega \) seems preferable, which then requires \( M = 1.13 \) GeV for superconvergence (dashed line in fig.1). However, this impairs the quality of agreement at low \( Q^2 \) for \( G_E \). Only with a value of \( M \) smaller than the nucleon mass the zero in \( G_E \) can be pushed up to about 10 (GeV/c)^2 so that the SLAC data can be accommodated. But then the overall pattern of \( G_E \) is very unsatisfactory (the dashed line in fig.2 is calculated with \( M = 0.76 \) GeV).

3
Although details depend on the choice of parameters in the effective lagrangian and in
the isoscalar part of the e.m. current it is evident from figs.1 and 2 that the functional
form (2) is able to describe the general pattern of the observed magnetic FF over the whole
range of measured $Q^2$ values if superconvergence is imposed, without any further "QCD"

corrections. The electric FF is dominated by a zero in the few (GeV/c)$^2$ region which is
very difficult to avoid and appears to be in conflict with the SLAC data. For $g_0 = 2.5g_\omega$
it is possible to satisfy the scaling relation $G_M/\mu_p = G_E$ with very good accuracy up to
$Q^2 \approx 1$ (GeV/c)$^2$ which is quite remarkable for a model in which the Besselfunctions $j_0$ and
$j_1$ determine the electric and magnetic FFs, respectively, (which naively implies for the ratio
of the radii $<r^2_M>/<r^2_E> \sim 3/5$). Clearly, more experimental information on the proton
electric FF in the few (GeV/c)$^2$ region would be very helpful for a critical assessment of
these implications of the soliton model.

4 Extension to large timelike $Q^2$

We have seen that, with superconvergence imposed, the expression (2) reproduces the es-
sential features of $G_M(Q^2)$ up to the highest measured values of spacelike $Q^2$. A peculiar
consequence of (2,3) is that the argument of $G_{nr}$ in (2,3) is positive for $Q^2 < -(2M)^2$, i.e.
$G(Q^2)$ is real for timelike $Q^2$ beyond the $N\bar{N}$ threshold. This may indicate an unphysical
feature of (2,3) which maps $G_{nr}(q^2)$ for large $q^2$ onto $G(Q^2)$ with timelike $Q^2$ just beyond
the $N\bar{N}$ threshold. (It should also be noted that $G_M(Q^2)$ does not have a pole at $Q^2 = -(2M)^2$,
because the divergent factor in front of $G_{nr}^M$ in (2) is compensated by the vanishing of $G_{nr}^M(q^2)$
for $q^2 \rightarrow \infty$.) But as a speculation, it is tempting to accept the transformation (2) also for
large timelike $Q^2$ (corresponding to $q^2 > 4M^2$ in $G_{nr}^M(q^2)$) as a prediction for (at least the
real part of) $G_M(Q^2)$. The connection between large space- and timelike values of $Q^2$ then
may be established through a Laurent expansion of $G_M(Q^2)$ for $|Q^2| \rightarrow \infty$:

$$G_M(Q^2) = \frac{1}{\pi} \int_{t_0}^{(2M)^2} \frac{\Gamma(t')}{t' + Q^2} dt' = \sum_{i=0}^{\infty} M^{(i)} \cdot (Q^2)^{-1-i}$$

(13)

with moments $M^{(i)}$ of the spectral function

$$M^{(i)} = \frac{1}{\pi} \int_{t_0}^{(2M)^2} \Gamma(t') (-t')^i dt'.$$

(14)

The continuation to the timelike region beyond the $N\bar{N}$ threshold then is simply a matter
of changing the sign of $Q^2$ in the Laurent series (13). With a sufficiently accurate set of data
such an analysis could be done in a model independent way.

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Table 1 shows two fits A and B of the series (13) to the function $Q^4 G_M(Q^2)/\mu_p$ for large spacelike $Q^2$, with 5 and 7 moments, respectively.

Tab. 1. The moments $M^{(i)}/\mu_p$ in units of $[\text{GeV}^2]^{1+i}$ as obtained from two fits of the Laurent series (13) to $Q^4 G_M^p(Q^2)$ (for $g_0 = g_\omega$, i.e. corresponding to the dashed line in fig.1) for large spacelike $Q^2$ with 5 (7) nonvanishing moments in fit A(B).

| i = | 1    | 2    | 3   | 4   | 5   | 6   | 7   |
|-----|------|------|-----|-----|-----|-----|-----|
| A   | 0.2937 | 1.91 | -12.06 | 28.4 | -25 | 0   | 0   |
| B   | 0.2937 | 1.91 | -12.06 | 27.6 | -22 | 55  | -190|

The formfactors $G_M^p$ resulting from the series (13) with the moments of table 1, for timelike $-Q^2 > 3.5$ GeV$^2$ are plotted in fig.3. ($G_M$ is negative in this region, fig.3 shows $|G_M|$ together with the present worldwide set of data for this quantity$^9$). The fact that the experimental data for $|G|$ show a slower decrease may be an indication of the imaginary part missing in the expression (2) for $G_M(Q^2)$. It appears that $|G|$ is not affected by the higher moments above $-Q^2 > 5$ (GeV/c)$^2$, and it is not very sensitive in the region from 3.5 to 5 (GeV/c)$^2$ as long as we exclude the possibility of extremely large higher moments in (13), or strong singularities close to the physical threshold $-Q^2 = 4M^2$. In this respect it is interesting that this continuation of (13) to timelike $Q^2$ reproduces at least the order of magnitude of the form factor above the physical threshold.
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Fig. 1. The magnetic form factor of the proton, $G_M^p/\mu_p$ (divided through the standard dipole $G_D = (1 + Q^2/0.71)^{-2}$) plotted against the logarithm of the spacelike momentum transfer $Q^2$ as obtained from the model defined in section 2. Full line: $g_0 = 2.5g_\omega$ with $M = 1.12$ GeV; dashed line: $g_0 = g_\omega$ with $M = 1.13$ GeV; dots and triangles denote the SLAC data of ref.\textsuperscript{1} and refs.\textsuperscript{2}, respectively; open circles show the data compilation of ref.\textsuperscript{7}.

Fig. 2. The electric form factor of the proton, $G_E^p/G_D$ plotted against the logarithm of the spacelike momentum transfer $Q^2$ as obtained from the model defined in section 2. Full line: $g_0 = 2.5g_\omega$ with $M = 1.12$ GeV, (dash-dotted line: $M = 0.94$ GeV); dashed line: $g_0 = g_\omega$ with $M = 0.76$ GeV; triangles denote the SLAC data of refs.\textsuperscript{2}; open circles show the data compilation of ref.\textsuperscript{7}.

Fig. 3. The Magnetic formfactor of the proton $|G|$ for timelike momentum transfer $t = -Q^2$ above the $NN$ threshold. The dotted (dashed) curve is the series (13) with the 5 (7) moments of fit A (B) given in table 1. The dots with error bars show the present worldwide data set for $|G|$ from ref.\textsuperscript{9}.