Fano-Kondo oscillations of the conductance and thermopower in a mesoscopic transistor

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Abstract. We study the conductance \(G\) and Seebeck coefficient \(S\) of a side-coupled double quantum dot system. The transport properties are the result of the competition between the Fano interference and Kondo correlations, being also controlled by the Coulomb blockade of the multilevel side-dot. The external parameters are the gate potential \(V_g\) applied on the side-dot and the temperature. When \(V_g\) is varied continuously the blockade is switched on and off periodically, resulting in oscillations of the transport coefficients. The profile of the oscillations and the temperature dependence are studied. An extended Anderson model and the Keldysh transport formalism are used.

We investigate the transport properties of a double quantum dot system arranged in a transistor-like setup as it is shown in Fig.1. A small interacting dot connected to external leads is pre-prepared in the Kondo state using the gate \(V_{g0}\), while a large side-dot containing many energy levels plays the role of the transistor basis. The functional parameter of the device is the gate potential \(V_g\) applied on the side-dot. When this potential is varied continuously the Coulomb blockade of the side dot is switched on/off periodically giving rise to oscillations of the source-drain current with a period equal to the charging energy. The reason for oscillations is the periodic change of the interference conditions in the double dot device.

Different aspects of the side coupled double dots have already been studied. We mention the two-stage problem [1], the two channel problem [2] and the Fano-Kondo effect [3,4] which are specific to this geometry. Non-equilibrium properties have also been discussed; the possibility of the peak-dip crossover of the differential conductance around the zero bias and questions concerning the scaling properties of the non-equilibrium conductance have been addressed recently [5].

The charge current through the device is the result of the Fano interference between the resonant channel passing through the lateral dot and the continuous channel through the small dot connected directly to the leads. The interference process is strongly affected by the regime (Kondo or non-Kondo) of the small dot, and vice-versa the Kondo effect may be suppressed by destructive Fano interference [4,5]. As a consequence of the competition between the two processes the oscillations of the current should show a Fano-Kondo profile with an asymmetry factor which depends on the weight of the two channels acting in the interference process.

The processes discussed above affect not only the linear conductance of the device but also the thermoelectric coefficient. The energy current should include in principle electron, phonon and interaction contributions. The interactions being essential in mesosystems, their presence
induces specific features of the thermopower as the violation of the Wiedemann-Franz law and the possibility of a high figure of merit $ZT$ [6]. We mention from the very beginning that our study considers only the coherent electronic contribution to thermopower. Inelastic processes have been studied in [7].

For single dots, the calculations by Costi and Zlatic [8] indicate the possibility of a sign changing of the Seebeck coefficient with increasing temperature, in agreement with the experimental facts [9]. For multiple quantum dots the problem has not yet been studied and this paper is a tentative to identify the specific changes in the thermopower of mesoscopic systems resulting from the combined action of correlation and interference processes.

The phenomena occurring in the mesoscopic transistor will be described in terms of an extended Anderson model, using the Keldysh transport formalism and equation of motion technique for the calculation of the Green functions.

![Figure 1](image.png)

Figure 1. The sketch of the double dot system: a small dot in Kondo regime is connected to external leads, and to a non-interacting multilevel lateral dot. The gate potential $V_g$ applied on the lateral dot is the functional parameter of this transistor-like device.

The set-up shown in Fig.1 can be described by the following extended Anderson model:

$$
H = \sum_{k,\sigma,\alpha} (\epsilon_k - \mu_\alpha) c_{k\sigma,\alpha}^\dagger c_{k\sigma,\alpha} + \sum_\sigma E_d d_{\sigma}^\dagger d_\sigma + U_H n_d^\dagger n_d + \sum_{i\sigma} E_i c_{i\sigma}^\dagger c_{i\sigma} + \frac{e^2}{2C} N^2 \sum_{k,\sigma,\alpha} |t_{kd}|^2 \omega^+ - \epsilon_k + \mu_\alpha + \sum_i |\tau_{id}|^2 \omega^+ - E_i - i\Gamma + \sum_i |\tau_{id}|^2 \frac{\omega^+ - E_i}{\omega^+ - E_i^+},
$$

where the terms describe in turn the leads, the Kondo dot with the Hubbard interaction, the lateral dot and Coulomb blockade interaction, and finally the two couplings. The parameters $t$ and $\tau$ describe the lead-dot and interdot tunnel coupling, respectively.

It is important to note that we choose a specific working regime of the system defined by

$$
T < T_K < \delta E < U_C,
$$

situation which can be met experimentally, but does not describe the 2-channel the 2-stage cases ($T$ = temperature, $T_K$ = Kondo temperature; $\delta E$ and $U_C = \frac{e^2}{2C}$ are the interlevel spacing and charging energy of the big dot).

For the considered geometry, the quantity needed to calculate the transport properties is the Green function $G_{dd}(\omega)$ of the small dot (indexed by $d$). For the calculation of $G_{dd}(\omega)$ we use the equation of motion technique in the version of Ref.10 applicable to complex mesostructures. The input quantity is the non-interacting self-energy $\Sigma_0$, which in our case includes two terms corresponding to the two couplings in eq.(1)

$$
\Sigma_0(\omega) = \sum_{k,\alpha} \frac{|t_{kd}|^2}{\omega^+ - \epsilon_k + \mu_\alpha} + \sum_i \frac{|\tau_{id}|^2}{\omega^+ - E_i} = -i\Gamma + \sum_i \frac{|\tau_{id}|^2}{\omega^+ - E_i^+}.
$$
One notices that the two contributions are additive and both of them acts on the shape of the Kondo peak of the density of states of the small dot. The second term is that one which gives rise to the Kondo in a box effect, which was studied in [11] for the case of vanishing dot-leads coupling.

The charge current flowing through the device is given in the Keldysh formalism by the well-known formula:

\[ I(V) = \frac{2e}{h} \int d\omega \left[ f_L(\omega) - f_R(\omega) \right] \Gamma(\omega) \left[ -\frac{1}{\pi} \text{Im} G_{dd}^R(\omega) \right]. \]  

(4)

In the linear approximation with respect to \( \delta V = V_L - V_R \) and \( \delta T = T_L - T_R \), the charge current can be written as

\[ I = L_{11}\delta V + L_{12}\delta T, \]  

(5)

where, assuming that \( \Gamma \) is independent of \( \omega \), the two linear kinetic coefficients \( L_{11} \) (the conductance) and \( L_{12} \) (the thermoelectric coefficient) are given by

\[ L_{11} = \frac{e^2\Gamma}{h} \int d\omega \left( -\frac{df}{d\omega} \right) \text{Im} G_{dd}^R(\omega), \quad L_{12} = \frac{e\Gamma}{hT} \int d\omega \omega \left( -\frac{df}{d\omega} \right) \text{Im} G_{dd}^R(\omega). \]  

(6)

Experimentally, besides the conductance \( G = L_{11} \), one measures the Seebeck coefficient \( S = \delta V/\delta T = -L_{12}/L_{11} \) which is obtained at vanishing current \( I = 0 \).

One should notice that \( L_{11} \) is determined mainly by the magnitude of the Kondo resonance, while the thermoelectric coefficient \( L_{12} \) depends on the shape asymmetry of the resonance around the Fermi energy at the given temperature. The Anderson model and the numerical renormalization group (NRG) method have been used in Ref.8 for the study of thermopower in single quantum dots. Although the equation of motion technique (EOM) which we use in this paper is not reliable in the limit of very low temperatures, the results obtained by this method for single dots are qualitatively similar to those obtained in the aforementioned references. This fact is encouraging for using the EOM method also in the case of double dots.

In the side-coupled double dot the spectral function at the site ‘d’ depends not only on the temperature and the parameters describing the Kondo dot but also on the state of the side-dot which is controlled by the gate potential \( V_g \). Compared to the spectral function of the single dot, the spectral function is strongly modified in this case especially when the energy levels of the side dot are in the vicinity of the d-level of the Kondo dot [5]. Such a situation is shown in Fig.2.

![Figure 2](image-url)

**Figure 2.** The local density of states at the site d, \( DOS = -\frac{1}{\pi} \text{Im} G_{dd}^R \) of the double dot in Kondo regime for the gate potential \( V_g = -0.05 \) \( (E_d = -0.1, \Gamma = 0.02, \tau = 0.005, T = T_k/10; \) the energy unit is the half-width of the leads band) [5].
As we have mentioned, by switching on/off the Coulomb blockade of the big dot, the interference conditions are modified periodically in the meso-transistor device. Accordingly, one expects oscillations of the 'source-drain' conductance as function of the gate potential \( V_g \) on the big dot; the device should behave like a mesoscopic transistor oscillator. The result of the numerical calculation of the conductance is shown in Fig.3, where the oscillations represent a sequence of Fano profiles which repeat itself with the periodicity \( \Delta = \delta E + U_C \). The eventual overlapping of consecutive Fano resonances depends on the ratio \( \tau/\Delta \); if the ratio is small the resonances are independent and the background of the Fano line is quite visible (this is the case in Fig.3). This property is quite understandable since a small interdot coupling \( \tau \) means a narrow resonance, and a large \( \Delta \) means that the resonances are much apart. The Fano asymmetry factor \( q \), which in the non-interacting case \( U_H = 0 \) is simply \( q = E_D/\Gamma \), depends now also on temperature due to the presence of the Kondo channel (which, obviously, is temperature dependent). We have found that when the Kondo channel conducts well, i.e at low temperatures, the asymmetry factor \( q \) is small.

One may notice in Fig.3 that the temperature dependence of the Kondo-type, meaning the increase of the conductance with decreasing temperature, occurs in the middle of the period, while near the Fano zero the Kondo effect is suppressed. This result is in agreement with the experimental data of Sasaki et al [4]. The Fano zeros occur at \( V_g = n\Delta \) ( \( n=\text{integer} \)), and, by calculating the electron occupancy of the big dot, one can check that these values of \( V_g \) correspond to the degeneracy points in the blockade process (where the occupancy of the big dot changes abruptly by one electron).

One may ask the question how the thermoelectric properties are affected by the interference process specific to the side-coupled double dot. The numerical calculation -the only one possible for interacting systems- shows that in the range where the Kondo effect is strong (i.e. in the middle of the oscillation period) the Seebeck coefficient is small indicating that the asymmetry of the density of states is also small. However strong oscillations accompanied by the change of sign occur around the Fano zero, i.e. in the range where the Kondo effect is suppressed (see Fig 4a). The temperature dependence of the Seebeck coefficient shown in Fig 4b shows also the possibility of sign changing, which occurs around the Kondo temperature suggesting significant modifications of the shape of the density of states with vanishing correlation effect. It is interesting to notice that the thermoelectric coefficients \( L_{12} \) and \( S \) do not exhibit a Fano

![Figure 3. Three periods of the oscillations in the Kondo regime exhibiting Fano profile; the curves correspond to different temperatures: \( T/T_K = 1/100, 1/10, 1 \). The number of levels in the big dot is \( N=20 \); the other parameters are: \( E_d = -0.2, \tau = 0.005, \Gamma = 0.04 \) (the energy unit is the half-width of the lead band).](image-url)
profile as the conductance $G$ does.

In conclusion, the transport coefficients of the side-coupled double dot show specific properties as function on the gate potential $V_g$, temperature $T$, and interdot tunnel coupling $\tau$ as an outcome of the competition between Kondo correlation and Fano interference. More than that, the periodic blockade of the side-dot occurring at the variation of the gate gives rise to oscillations of the linear response coefficients. The oscillations have a period equal to the charging energy and shows a Fano profile described by an asymmetry factor which increases with increasing temperature. The Fano zeros occur at values of the gate corresponding to the degeneracy points of the Coulomb blockade process (i.e. when the side-dot is unblocked); it turns out that around these points the Kondo correlations are suppressed, similar to the experimental finding. The Seebeck coefficient depends on the asymmetry of the local density of states around $E_F$, and proves to be suppressed by the Kondo correlations and to get large values around the Fano zeros. Change of sign of the thermopower occurs at values of the gate potential corresponding to the degeneracy points and at temperatures higher than $T_K$.

Figure 4. (a) The Seebeck coefficient as function of the gate potential $V_g$ for $T = T_K/10$. (b) Temperature dependence of the Seebeck coefficient for various $V_g$, chosen in the middle of the oscillation period ($V_g = -17.5, -17.2$; the values are amplified $10 \times$ on the y-axis) and near the Fano zero ($V_g = -16.98$); $V_g$ is measured in units $\Delta$.

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