Vibration stability analysis of cantilever structure based on symbolic regression algorithm and modal analysis method

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Abstract
The vibration stability of cantilever mechanism under high-speed rotation directly affects the positioning accuracy. Modal analysis method is usually used to study the vibration stability. However, the traditional experimental modal analysis (EMA) method needs to measure the impulse excitation, while the operational modal analysis (OMA) method needs to satisfy the assumption of white noise. Therefore, the existing modal analysis methods cannot be applied to the study of vibration stability of high-frequency cantilever mechanism. In this paper, the symbolic regression (SR) algorithm is combined with the EMA method, and the robustness analysis and feasibility verification are carried out under the condition of adding noise. The validation of the new method is divided into two parts. In the first part, a three degree of freedom (DOF) linear model is constructed, and the modal parameters identified by SR method and state space method are compared. In the second part, the method is applied to identify the modal parameters of stepped bar. The results are compared with the results of LMS (Siemens' Testlab). Based on the time-domain response signal only, the modal parameters are extracted by SR, and the main vibration frequency is extracted from the response signal. The system simulation and experimental results show the method provides a possibility to analyze the vibration stability of cantilever structure.

Keywords
Vibration stability, modal analysis, symbolic regression, modal parameters

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Introduction
High frequency rotating cantilever structure is widely used in the field of chip manufacturing. The motion stability of the rotating cantilever structure directly affects the production efficiency of the chips.¹ However, there are few researches on the vibration analysis of the cantilever rotating structure, and few of them can be used in the industrial production site. The research on the vibration characteristics of the cantilever rotating structure at high frequency needs to increase the time and energy input of scientific researchers. With the development of intelligent era and mass mode of big data, it is urgent to adapt to the high-frequency rotation motion of cantilever mechanism controlled by big data.²

In this paper, modal analysis method is used to study the vibration stability of cantilever mechanism under high frequency rotation.³ At present, the research on the vibration of cantilever mechanism adopts the traditional vibration analysis technology,⁴ but also

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stays in the traditional theoretical research stage. Some rules are found, but the laws are closely related to the specific structure types. Therefore, it is important to obtain a general intelligent vibration analysis method of cantilever mechanism adapting to big data mode. At the same time, the current research on the cantilever structure only stays in the static or quasi-static conditions, and the research on the vibration characteristics of the cantilever structure under the high-speed operation condition is very few, and the research on the cantilever motion with emergency stop and commutation motion is not available.

In order to analyze the vibration characteristics of the cantilever structure of the chip sorter under high frequency operation, the modal analysis technology is adopted in this paper. However, in order to adapt to the industrial production in the big data mode, the intelligent algorithm is added to update the traditional algorithm. In order to solve the problem of obtaining dynamic adaptive modal parameters, evolutionary Algorithm is an advanced intelligent and dynamic adaptive technology. Evolutionary algorithm searches for many solutions in the solution space both implicitly and in parallel, and use the difference between different solutions to set the penalty function and obtain the best solution. Evolutionary algorithm refers to the behavior of biological groups, and uses the development mode of survival and reproduction among populations to carry out population evolution. For example, the application of backtracking search algorithm (BSA) in economic dispatch, and artificial cooperative search algorithm for finding the optimal solution of complex optimization problems, and particle swarm optimization (PSO) simulated t67/Nb adhesion strength, and hardness to predict output performance and rainfall optimization algorithm (RFO) is used to obtain the optimal solution of numerical optimization problems. Particle swarm optimization PI (PSO PI) is used to control the rotating d-q current and drive the induction motor. PSO has the same performance as genetic algorithm, but it is faster and simpler.

Most importantly, regression analysis and the most popular evolutionary computation methods assume that the form of function used is set. The disadvantage of these programs is that they consume a lot of computing time. In addition, the function form of the model is selected according to experience, and the natural law may not be able to reflect it well. This means that the complex model function expression is selected for the complex vibration system, but the law implied in the experimental data is very simple, or simple model function expression is selected according to the complex law implied in the experimental data of complex vibration system. But the problem is that it is difficult to determine the number of enough single pulse excitation, and it is difficult to solve the problem of obtaining dynamic adaptive modal parameters. Therefore, it would be advantageous to have an algorithmic approach to determine the best correlation for experimental data without assuming its functional form. The problem is how to analyze the correlation automatically based on the measured data.

At present, due to the emergence of data mining, this problem can be well solved with the help of the current computer technology. Without any physical knowledge, the data mining algorithm finds the Newton’s law, geometry and momentum conservation laws of the reaction system hidden in the data. We always analyze problems in the way that we decompose complex problems into a series of simple problems, to slowly excavate the inherent laws hidden in nature. In the traditional parametric regression analysis, the function model between free variables and coefficients should be assumed in advance to solve the system parameters, and the optimal coefficient of the equation can be obtained by minimizing the error between the estimated value and the experimental value. The difference is that symbolic regression (SR, also known as function identification) does not need to assume the function model in advance. It can obtain the functional relationship between variables by minimizing the error between the predicted value and the experimental value, and automatically search out mathematical laws and function models from the data set. In addition, parametric regression will have pseudo linearity, continuity and other conditions, while SR does not. The other advantages of SR are: (1) depending on the large-scale population number and setting reasonable mutation and crossover probability parameters in the algorithm, the algorithm is not easy to enter the local optimum; (2) the population operation is carried out by using the operator to improve the execution strength of the algorithm; (3) the individuals with hierarchical structure of binary tree can adapt to the nonlinear requirements very well. Therefore, as a good dynamic adaptive algorithm, SR only mines function expression rules from response data, and then obtains dynamic adaptive modal parameters.

At present, there is no research and patent on the application of SR in modal analysis, but the application scope of SR method has developed in recent years. In 2009, Michael Schmidt explained the theoretical usage of SR by mining the classical physical laws implied in model experimental data. In 2016, Wu NT used SR and genetic programming algorithm to mine the variation rules of model parameters of underwater vehicles. Peng applied SR genetic programming algorithm to the field of power quality analysis, which can reduce the loss of time information and improve work efficiency. The main applications of SR are genetic programming, gene expression programming and stepwise regression algorithm for randomly generating candidate
factor sets. Among these algorithms, the most developed one is genetic programming. Genetic programming is a generalized hierarchical computer program that can deal with linear or nonlinear problems adaptively.

In this paper, the SR algorithm is combined with the EMA method. The new EMA method, based on time-domain response and SR method without assuming any pre-existing information and only using mathematical symbols, is introduced to automatically search for motion equations, and finally gets the modal parameters. And the robustness analysis and feasibility verification are carried out under the condition of adding noise.

Modal parameter identification method based on SR

SR algorithm based on genetic programming

As a supervised learning method, SR tries to find some hidden mathematical formula and use characteristic variables to predict target variables. Traditional parametric regression needs to set the parametric model of function in advance, while SR does not need to set the functional model in advance. It can automatically search the mathematical rules (expressed by functional model) from the data set. The advantage of SR is that it doesn’t need to make a function model, and it’s not easy to enter the local optimum. The specific implementation of SR is genetic algorithm (GP). GP algorithm was proposed by Prof. Koza of Stanford University in the early 1990s. The mode operator of genetic programming includes mode individual copy, crossover, mutation and so on. The genetic operator is used to realize the automatic evolution of mode population.

Figure 1 shows the workflow of the GP algorithm, and Figure 2 shows the hierarchical structure of function expressions. Compared with the genetic algorithm (GA), the GP method can avoid the same frequency signal interference, improve the accuracy, and correctly extract the modal parameters. The system response after solving and identifying the GP algorithm.

\[ f(x) = Ae^{-\mu t} \cos(\omega t + \phi) \]

Where \( A_i \) is the amplitude of each linear normal mode, \( B_i \) is the product of damping ratio and frequency, \( \omega_i \) and \( \phi_i \) are the natural frequency and phase of each linear normal mode, respectively.

Fitness function

The correlation between the natural mode and the system vibration obtained from the response time domain experimental data. Evaluate performance criteria to measure how close the observed value is to the value generated using the GP algorithm. Since the goal is to minimize the variance of the error between the prediction and the observed data, fitness is defined as the reciprocal of the variance, which is shown as follows.

\[ F_f = \frac{1}{N-1} \sum_{i=1}^{N} [f^e(x_i) - \mu]^T \cdot [f^p(x_i) - \mu] \]

Among them, \( f^e(x_i), (i = 1, 2, \cdots, N) \) is the experimental response data, \( f^p(x_i), (i = 1, 2, \cdots, N) \) is the predicted value from the correlation of the candidate modalities.
The solution of the objective function is determined by the modal individuals, but the optimal modal individuals need to be screened by fitness function. But there are also some problems. For example, the existence of pseudo modal individuals, their outstanding competitiveness will affect the global optimization of the algorithm. In order to prevent large correlation functions and support more compact correlation functions, the fitness function can be penalized according to the size of each correlation. The equation (3) is as follows:

\[ Q_f = \frac{1}{1 + e^{a_1(t-a_2)}} \cdot \left( \frac{1}{N} \sum_{i=1}^{N} \left( f'(x_i) - \mu \right) \cdot \left( f''(x_i) - \mu \right) \right) \]  

\[ \sigma^1 \cdot \sigma^0 \]  

\[ \text{Where} \ f'(x_i) \text{ and } f''(x_i)(i = 1, 2, \ldots, N) \text{ are the experimental data and the predicted value of the inherent modal of the system respectively; } \sigma^1 \text{ and } \sigma^0 \text{ are the standard differences between the experimental data and the predicted modal model data respectively; } a_1 \text{ and } a_2 \text{ are the parameters of the penalty function; and } L \text{ is the size of the tree.} \]

**Modal parameter identification**

In this paper, the mean square error (MSE) is selected as the fitness evaluation of the model to minimize the average of the squared residuals and assume that the noise follows the normal distribution. According to the characteristics of response signal function structure, the function set is selected as follows:

\[ F = \{ +, -, *, /, \sin, \cos, \exp \} \]

The end point is the time variable “\( t \)” and the constant “\( \Gamma \)”,

\[ T_e = \{ t, \Gamma \} \]

In the model, the input is the time variable “\( t \)” the response “\( h \)” and the constant “\( \Gamma \)” and the output is the coefficients related to the modal frequency and damping ratio. Through the process of SR, we can explore these parameters by searching the expression structure. Without knowing the linear modal theory in advance, just use mathematical notation and function notation end time variable “\( t \)” and constant “\( \Gamma \)” to directly obtain modal parameters before mining. Finally, the following equation (4) is obtained.

\[ h_{ij}^m(t) = \sum_{m=0}^{M} \sum_{r=1}^{N_r} A_r e^{B_r(t + T \cdot m)} \cos(C_r(t + T \cdot m) + D_r) \]  

**Figure 3. 3-DOF mass spring damper system.**

Where, \( N_m \) is the order of the modal; \( M \) is the number of sufficient effective unit pulse excitations; \( A_r \), \( B_r \), \( C_r \), and \( D_r \) are the mining coefficients of the law model, that is, the modal parameters of each mode that are excavated.

**System simulation verification of identification technology**

**3-DOF spring damping system**

In this chapter, 3-DOF spring damping system simulation and step bar experiment are carried out to identify the modal parameters of cantilever structure using SR method. In the simulation experiment, a 3-DOF linear system model is used to extract the modal parameters. At the same time, the result of state space method is used as a reference. The process of identifying modal parameters by SR method is introduced. In addition, noise is added to analyze the robustness of the method.

Figure 3 shows a 3-DOF mass spring damper system and denotes \( m_1 \)–\( m_3 \) as components 1–3. According to the vector modeling method of Newton’s second law, the simplified form is adopted in this paper. The corresponding motion equation in Figure 1 is shown in equation (5).

\[ \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 + c_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \]  

(5)

Where, \( \{m_{1,2,3}\} \) is the mass; \( \{k_{1,2,3}\} \) is the stiffness; \( \{c_{1,2,3}\} \) is the damping coefficient; \( \{x_{1,2,3}\} \) is the output response; and \( \{F_{1,2,3}\} \) is the external excitation force. The selected values of all system parameters are shown in Table 1, and the damping matrix is proportional damping matrix. In addition, the impulse excitation is used to solve the equation, which is equivalent to adding an initial velocity \( v_1 = 1.5mm/s \) to the mass \( m_1 \). The third order damped natural frequencies of the system are \( f_1 = 4.1Hz, f_2 = 8.7Hz, f_3 = 11.0Hz \), and damping ratio are \( \xi_1 = 0.08, \xi_2 = 0.15, \xi_3 = 0.18 \).
Modal parameter identification based on response

Considering that the EMA method is carried out under the condition of no excitation. Only the time domain response signal is used to extract modal parameters by SR. When the system response has been obtained, the SR algorithm is used to analyze the response signal, identify the system modal parameters, and verify the feasibility of the algorithm. In this section, the time-domain solution solved by MATLAB is used as the reference time-domain signal, which is imported into the SR software Eureqa to mine the modal parameters.

Figure 4 shows the time-domain and frequency spectrum distribution diagram of the three-degree-of-freedom mass spring damping system. The first two modes are obvious in the response $x_1$ and $x_3$, and the first and third modes are obvious in the response $x_2$. Figure 5 shows the mining results after the displacement data are imported into the Eureqa platform. The model expression $0.01 \sin(25.8x)e^{-2.1x}$ is the model search result of the first mass block’s response, $0.01$ is the amplitude, $\sin(25.8x)$ is the periodic, and $e^{-2.1x}$ is the index. Similarly, $0.01 \sin(25.8x)e^{-2.1x}$ is the model search result of the second mass block’s response, $0.001$ is the amplitude, $\sin(25.8x)$ is the periodic, and $e^{-2.1x}$ is the exponential. Similarly, $0.0008e^{-2.1x}\cos(1.6 - 25.9x)$ is the model search result of the third mass block’s response, $0.008$ is the amplitude, $\cos(1.6 - 25.9x)$ is the periodic, and $e^{-2.1x}$ is the exponential.

From the results, only the first mode is mined out, and the modal parameters include natural frequency and damping ratio, and the numerical accuracy is very high. Therefore, it is feasible to mine modal parameters through SR algorithm.

Robustness analysis and feasibility test

In the case of random signal interference, the robustness analysis and feasibility test of the modal parameters identified by SR algorithm are carried out. Adding additive noise to the response signal of 3-DOF system, as equation (6).

$$x_{1,2,3} = x_{1,2,3} + 0.0005 \cdot \text{rand}(n,1)$$

Figure 6 shows the displacement response and additive noise spectrum (noise ratio: 0.0005). Figure 7 shows the results of modal parameter mining when the displacement response data of additive noise are imported into Eureqa. The first mode frequency and damping ratio of the three response signals are mined out. The frequency and damping ratio of mining from each response signal are $\tilde{f}_{1,x_1} = 4.1Hz$, $\tilde{f}_{1,x_2} = 4.1Hz$, $\tilde{f}_{1,x_3} = 4.1Hz$, and $\tilde{\xi}_{1,x_1} = 0.08$, $\tilde{\xi}_{1,x_2} = 0.08$, $\tilde{\xi}_{1,x_3} = 0.08$. Table 2 shows the mining results of modal parameters with and without additive noise. The mining errors of frequency and damping ratio are all within 0.1% and 5%, respectively.

Table 1: System model parameter.

| $m_1$ | $m_2$ | $m_3$ | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $\beta$ | $C$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| kg    | kg    | kg    | $10^4$N/m | $10^4$N/m | $10^4$N/m | $10^4$N/m | 0.005 | N/(m/s) |

The results show that the SR method has strong anti-interference ability, and further verifies the robustness and feasibility of the SR method based on the time-domain response signal to identify the modal parameters.
Experimental verification of identification technology

Experimental arrangement

This chapter is based on the analysis results of LMS platform as a reference for experimental verification, and the object is the stepped round bar. The traditional EMA method needs to measure the impulse excitation, considering that the new EMA method proposed in this paper is carried out without measuring the excitation. Only based on the time-domain responses, the SR is used to extract the modal parameters.

Figure 8 shows the experimental arrangement of variable diameter workpieces. The round bar is divided into three sections and two steps. The diameter of the three sections is from large to small. The round bar is fixed on the three grabbing chuck of the lathe. Using the spectrum analysis module of LMS software, the original time-domain signals of excitation and vibration response are collected to obtain the impulse excitation response. Three sensors are arranged on the cylindrical workpiece of cantilever beam. The end and middle part of the workpiece are excited in the vertical direction.

Analysis of main vibration frequency

This section deals with the vibration response under excitation, and the corresponding modal parameters (natural frequency and damping ratio) are extracted, and compares with the modal parameter identification results of LMS software.

Figure 9 is the steady-state diagram of displacement response at position 1–3 (directions: X, Y, Z). Figure 10 is the steady-state response diagram of position 2 at Y-direction. The third-order frequency (blue line) is obtained by removing the pseudo mode with modal
assurance criterion (MAC, a powerful tool for evaluating the quality of the modes). The identified third-order modal frequencies and damping ratios are 353.1, 446.0, 898.1 Hz and 0.023, 0.028, 0.022 Hz, respectively.

Figure 6. Displacement response and spectrum of additive noise (noise ratio: 0.0005).

Figure 7. Results of displacement response data imported into Eureqa platform with additive noise.

...assurance criterion (MAC, a powerful tool for evaluating the quality of the modes). The identified third-order modal frequencies and damping ratios are 353.1, 446.0, 898.1 Hz and 0.023, 0.028, 0.022 Hz, respectively.

Figure 11 presents the mining first mode of position 2 at Y-direction based on Eureqa. Figure 11(a) is the search results, and Figure 11(b) are the experimental values and model fitting values.
Table 2. Mining results of modal parameters under additive noise.

| Modal parameters | Reference value (without noise) | Mining value (with noise) | Absolute error | Relative error (%) |
|------------------|---------------------------------|---------------------------|----------------|--------------------|
| Frequency $f_{1,x_1}$/Hz | 4.1152                          | 4.1169                    | 0.0017         | 0.0413             |
| Damping ratio $\xi_{1,x_1}$ | 0.0843                          | 0.0885                    | 0.0042         | 4.98               |
| Frequency $f_{1,x_2}$/Hz | 4.1152                          | 4.1129                    | 0.0023         | 0.06               |
| Damping ratio $\xi_{1,x_2}$ | 0.0842                          | 0.0834                    | 0.0008         | 0.95               |
| Frequency $f_{1,x_3}$/Hz | 4.1236                          | 4.1554                    | 0.0318         | 0.77               |
| Damping ratio $\xi_{1,x_3}$ | 0.0835                          | 0.0821                    | 0.0014         | 1.68               |

Verification of parameter identification results

The results show that the modal parameters are mined out in eight signal sources, but only the first mode (main vibration mode) is mined out. The main reason is that the programming algorithm of SR software is not mature enough, and the hardware environment needs to be strengthened. Figure 11 shows the mining results of the modal parameters based on displacement responses, the null value at Point 7 is replaced by reference value for drawing convenience. It can be seen from Figure 13 that the maximum error of main vibration frequency is at Points 4 and 6, and the relative error are 0.55% and 0.54%, respectively; the maximum error of damping ratio is at Points 2 and 4, and the relative error are 18.41% and 17.99%, respectively.

The traditional EMA method needs the data of impulse excitation. The new EMA method based on SR is proposed in this paper, which is effective for mining modal parameters from response without excitation.

Figure 8. Experimental arrangement of variable diameter workpiece.

Figure 9. Steady-state response diagram.
Conclusion

The traditional experimental modal analysis method needs to measure the pulse excitation. The new experimental modal analysis method proposed in this paper is carried out without measuring excitation. Based on the time domain response signal, the modal parameters are extracted by SR. In this paper, the SR
Figure 12. Mining results based on Eureqa: (a–c) position 1 at X, Y, Z directions, (d–f) position 2 at X, Y, Z directions, and (g–i) position 3 at X, Y, Z directions.
algorithm is combined with the EMA method, and the robustness analysis and feasibility verification are carried out under the condition of adding noise. Firstly, a 3-DOF linear system model is introduced, and the modal parameters are extracted by SR method, and the results extracted by state space method are used as reference. Then the modal parameter identification of the stepped bar is verified. The results of modal parameter identification based on LMS analysis platform are used as reference. Based on the time-domain response signal only, the modal parameters are extracted by SR, and the main vibration frequency is extracted from the response signal. The system simulation and experimental results show that the method is used to analyze the vibration stability of cantilever structure.

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Figure 13. The mining results of the modal parameters based on displacement responses.

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