MODELS OF STELLAR STRUCTURE FOR ASTEROSEISMOLOGY

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Abstract

Among the problems still open in the study of stellar structure, we discuss in particular some issues related to the study of convection. We have recently built up complete stellar models, adopting a consistent formulation of convection both in the non gray atmosphere and in the interior, to be used for non adiabatic pulsational analysis, and discuss in some details two problems which have been clarified by these models: the physical interpretation of one of the main sequence “Böhm-Vitense gaps”, and the necessity of parametrizing pre main sequence convection differently from the main sequence convection. We also report preliminary results of the application to the solar model of the non–local turbulence equations by Canuto & Dubovikov (1998) in the down–gradient approximation.

Key words: Stars: structure – Stars: convection – Stars: asteroseismology

1. Introduction

The list of unsolved problems in stellar structure is long: microscopic diffusion; mass loss; rotation, its evolution and its effects on chemical mixing; magnetic fields, and their interaction with rotation and convection; and an adequate model of convection able to describe the overadiabaticity in the shallow layers, and non-local aspects like overshooting. Asteroseismology, and especially asteroseismology from the space, is destined to be very useful in providing us with additional constraints to our models. Here, we only touch the problem of convection in stars, and in particular the problem of envelope convection. The convective envelope in low mass stars has a fundamental role in \delta Scuti, \gamma Doradus and solar-type oscillations. Furthermore, the boundaries between the corresponding domains of instability in the HR diagram seem to be linked to changes in the convection features. The non adiabatic seismological analysis requires knowledge of the entire stellar structure up to the atmosphere, and selfconsistent convection models for the atmosphere and interior integration are necessary for this use. On the other hand, progress in the study of convection is slow enough that today’s main attitude with respect to this problem is the following: parametrize in a rough way the overadiabatic convection, by using constraints either coming from the analysis of stellar spectra or from other structural observational information (e.g. the p–modes solar patterns, or the depletion of light elements in the stellar envelopes, or empirical radii constraints). This can be sufficient for studying solar oscillations, but it is difficult to extend models so much parametric to regions of the HR diagram in which the observational constraints are scarce (or none). We have been attempting to build up complete stellar models to be used for non adiabatic pulsational analysis. These models have clarified some interesting problems in stellar evolution which we discuss here in more detail: \textit{i)}; they have provided an interpretation for the “Böhm Vitense gap” observed in open clusters at B–V$\sim$0.35 (Rachford & Canterna 2000); \textit{ii)}; they have made explicitly clear the necessity of parametrizing pre main sequence convection differently from the main sequence convection, possibly implying that there is an hidden “second parameter” acting to change the convection behaviour during the first phases of stellar evolution. Finally, we show the results of preliminary computation of non local convection in the solar model, which indicate that the overshooting expected at the bottom of the solar convective envelope is $\sim0.02H_P$.

2. Convection in complex evolutionary phases

A good example of complex models of stellar structure are the intermediate mass Asymptotic Giant Branch (AGB) stars, those which evolve through the phase called Hot Bottom Burning (HBB). These stars are extremely important for the chemical evolution of galaxies, and they are probably the key ingredient to develop primordial chemical inhomogeneities among Globular Clusters stars. During 90\% of their life, they are fueled by hydrogen, whose stationary burning in an external shell is interrupted by the sudden ignition of the helium beneath. During the ensuing thermal runaway (the Thermal Pulse –TP– phase), the hydrogen envelope expands, hydrogen burning stops and external convection reaches into the helium mucearly processed layer, provoking the ‘third dredge up’ phase by which nuclearily processed material appears at the stellar surface. In the most massive AGB stars (4–8M\odot), the stationary hydrogen burning shell is partially contained into the convective envelope, that is, the bottom of the
convective envelope attains temperatures so large that nuclear reactions take place there. The CN—or even CNO—for low metallicity stars (Ventura et al. 2001)—nuclearly processed material is convected to the surface and then convected back to the bottom. So the nuclear processing which can be observed in the atmospheres of these stars is due to the combined action of HBB and of the third dredge up. In spite of the fundamental importance of these objects evolution, for sure this phase is not easily modeled!

One of the big problems is the (unknown) mass loss, which affects the yield of the processed chemistry both through the time dependent loss of the envelope into the interstellar medium, and through altering the temperature structure of the envelope. In addition, these stellar models are heavily dependent on the way we model convection it all its aspects:

1. the convective model, that is the convective flux and temperature gradients computation;
2. overshooting, that is mixing outside the formal convection boundaries. For AGBs, this includes both the problem of core—overshooting, which affects the relation between initial mass and mass at the beginning of the TP phase, and the possible overshooting below the convective envelope, which affects the nucleosynthesis;
3. time dependence: mixing must be treated as non instantaneous, and coupled with nucleosynthesis, for all the elements for which the convective mixing timescale is of the same order of the nuclear burning timescale.

3. Convection in more “normal” stars

In the present discussion, we leave entirely aside the “big problems” of convection modelling —unless from the last section. While turbulence in stars is compressible and non local, we must accept that for a long time, for general purposes of computing stellar structure for any mass, chemistry and evolutionary phase, we still will deal with incompressible and local models. 3D radiation hydrodynamics (RHD) simulations still miss the computer power needed to deal with deep envelope convection, although great insight has been obtained in the atmospheric studies. Interesting results are available by 2D simulations. Analytic non–local convection models have recently been applied to stellar atmospheres of A stars (Kupka & Montgomery 2002) and we may foresee important developments also in this area. Among the local models, the Mixing Length Theory (MLT) by Böhm-Vitense (1958) certainly is still dominant, in spite of its limitations. Other local models have become available, among them the Full Spectrum Turbulence (FST) model by Canuto & Mazzitelli (1991) and its variant in Camuto et al. (1996) which have been adopted for a variety of applications in the latest 10 years.

As no available theory is self consistent, the dominant attitude in stellar studies —unless there are particular requirements— is to use the convective model as a ‘black box’ simply to infer the stellar properties in the region in which convection becomes adiabatic. In general, in fact, the main important property which a convection model is required to provide is the stellar radius: whatever is in the ‘black box’ of convection, we first of all wish to know the radius (or the $T_{\text{eff}}$, or, equivalently, the entropy jump between the adiabatic interior and the surface). The solar radius in particular is generally adjusted in the models by varying the convective efficiency. In the MLT this can be done by varying the ‘mixing length’ parameter, that is the ratio of mixing length to pressure scale height $\alpha = l/H_p$. Recently it has been stressed the importance of surpassing the Eddington approximation, or the gray atmosphere approximations to correctly describe the optical atmosphere. For many stellar situations, a non gray model atmosphere—which also must reproduce the observed spectral features—includes as necessary ingredient a treatment of convection, which, in the MLT, will be characterized by a given $\alpha_{\text{atm}}$ down to the optical depth $\tau_{\text{ph}}$ at which the atmosphere will be matched to the interior computation. In the interior, convection will be characterized by a given $\alpha_{\text{in}}$. Thus, actually, a ‘modern’ MLT structure will have three parameters: $\alpha_{\text{atm}}$, $\alpha_{\text{in}}$ and $\tau_{\text{ph}}$. Figure 1 illustrates this classic problem by using our recent models.

![Figure 1. Stratification of temperature vs. pressure in the atmosphere and sub-atmosphere of solar models by Montalbán et al. (2003). Track 1 is the model employing FST convection both in the atmosphere and interior, the match is done at $\tau_{\text{ph}} = 10$; models from 2 to 4 have MLT atmospheres with $\alpha_{\text{atm}} = 0.5$, and MLT interiors with $\alpha_{\text{in}}$ chosen so to fit the solar radius. The differences are due to the different choices of matching points: 2: $\tau_{\text{ph}} = 100$ and $\alpha_{\text{in}} = 6.3$; 3: $\tau_{\text{ph}} = 10$ and $\alpha_{\text{in}} = 2.3$; 4: $\tau_{\text{ph}} = 1$ and $\alpha_{\text{in}} = 1.75$. Model 5 employs the AH97 atmospheres down to $\tau_{\text{ph}} = 100$, and has $\alpha_{\text{in}} = 1.9$ (the model shown in Figure 6). We see that the subatmospheric structure is very different for the different models, in spite of the ‘solar radius’ calibration.](image-url)
which adopt as boundary conditions the NextGen atmospheric grid (Allard et al. 1997 hereinafter referred to as AH97), computed by assuming MLT convection with $\alpha_{\text{atm}} = 1$, down to optical depth $\tau_{\text{ph}} = 100$.

Adopting $\alpha_{\text{in}} = 1.9$ for the interior computations, we obtain a set of tracks, among which the solar track passes through the solar location at the solar age. The solar mass track with $\alpha_{\text{in}} = 1.0$, on the contrary, is $\sim 380$K cooler at the solar luminosity. This well known fact remembers how large is the variation in $T_{\text{eff}}$ allowed by changing the MLT parameter. The meaning of this solar radius adjustment is simply the following: when using the MLT, the entropy jump, necessary to fit the solar $T_{\text{eff}}$, between the adiabatic layers and the surface corresponds to what is contained in the model atmosphere grid. Nevertheless, the model can not tell us anything about the temperature gradient layer by layer. If we only wish to use convection as a 'black box', we just take the $\alpha_{\text{in}} = 1.9$ model for the Sun\(^1\). Of course, our assumption will only be valid for the Sun itself.

An interesting broadening of of this point of view to a wider part of the HR diagram has been given by Ludwig et al. (1999) who performed detailed 2D numerical RHD calculations of time-dependent compressible convection, in the range 4300$< T_{\text{eff}} < 7100$K, 2.54$< \log g < 4.74$ (cgs) for solar composition. They used these models to ‘calibrate’ the effective $\alpha$ for these envelopes, that is the value of $\alpha$ which provides the same specific entropy jump (using a gray atmosphere, therefore this approximation defines a unique average $\alpha$ parameter) than their 2D models. They found values from 1.3$\text{H}_p$ for F dwarfs up to 1.8$\text{H}_p$ for K giants. This calibration of $\alpha$, again, does not tell us anything about the temperature gradient layer by layer.

The approach of keeping convection as an entirely black box is very useful, but it has –obviously– many limitations. In particular, if we wish to study the excitation of oscillation instabilities in stars we must know the full stratification of physical quantities in the star, up to the atmosphere.

4. Convection modelling in full stellar models

We have to recognize that no general purpose convection model is presently available, which can be meaningfully applied to any stellar structure in order to know the layer by layer stratification of physical quantities. Nevertheless, recently there have been several attempts to produce entire stellar models, which satisfy some observational constraints on the atmospheric and envelope convection, and in which the atmospheric and the envelope convection are matched ‘smoothly’ each other. A smooth match is at least technically necessary for stellar stability studies, to avoid discontinuities in the physical quantities. A prototype of these models has been discussed by Schlattl et al. (1997) for the Sun. They built up a non gray 1D model atmosphere with $\alpha_{\text{atm}} = 0.5$ down to $\tau_{\text{ph}} = 20$, and matched it to a subatmosphere and MLT interior, guided by the results of 2D hydro models. To do this, they had to vary the $\alpha_{\text{in}}$ parameter in the interior computation, with the aim to provide the solar radius and obtain a temperature stratification similar to that of the 2D models. The aim of this study was to explore the influence of the physical inputs on the solar p–modes.

Of course, for the Sun we have an enormous number of constraints —from the p–modes themselves, and from the precise knowledge of the solar radius— which help to build up a fully parametric model. But what should we do to extend the analysis to other stars? What kind of model atmosphere can we use, and how do we produce a ‘meaningful’ match of the atmospheric and interior computation? There are not yet many model atmosphere grids available, and many of them have been computed for “atmospheric purposes” only, that is to produce an adequate modelling of the region from which the stellar spectrum emerges, so they are not entirely apt to be used as boundary conditions for full stellar models. For instance, the quoted AH97 models, computed with the PHOENIX code, adopt MLT convection with $\alpha = 1$ and a total of 50 layers down to $\tau = 100$. Heiter et al. (2002) have adopted a new version of Kurucz’s (Kurucz 1993) model atmospheres (the Vienna–ATLAS9 code), in which the convection model is either the MLT with $\alpha = 0.5$ or the FST (both in the Canuto & Mazzitelli 1991 and in the Canuto et al. 1996 versions), and they have increased the number of layers in the latter models to 288. We have recently used all these grids of model atmospheres to explore the meaning of the match between atmosphere and interior, and what happens when one extends to other models (pre main sequence, and giants) the approximations adopted for the solar model. The extensive results of this study are presented elsewhere (Montalban et al. 2003). Here we show, in Figure II that the different solar models which can be built up, all satisfying the constraint of the solar fit, can have very different subatmospheric structures, depending on the choice of the convection efficiency in the different layers.

In Figure II we also see that the model adopting the FST convection both in the atmosphere and in the interior does not show discontinuities in the $T$ vs. $P$ stratification. The structure smoothly passes from a very inefficient convection in the atmosphere, similar to that of the MLT models with $\alpha = 0.5$, to such an efficient convection inside, that the precise solar fit is easily obtained (see e.g. Canuto & Mazzitelli 1991).\(^2\) Computing “full FST”

\(^1\) and generally we forget, or at least do not even mention, that $\alpha$ is actually $\alpha_{\text{atm}} = 1$ for most of the overadiabatic region, which is contained in the model atmosphere grid.

\(^2\) Remember that, whatever the choice of the free parameters in the FST convection, it is not possible to obtain a
structures, then, has as a first a “formal” advantage, as it helps to avoid the problem of discontinuities in the physical quantities for instability studies. In addition, there are other, more physical, reasons to use this model as probe: a series of works on $1) T_{\text{eff}}$ determination from H$_{\alpha}$ and H$_{\beta}$ lines (e.g. Fuhrmann et al. 1993; van't Veer-Menneret et al. 1998a); theoretical predictions of H$_{\alpha}$ and H$_{\beta}$ from 1D models (Gardiner et al. 1999) and from 2D models (Steffen & Ludwig 1999); $3)$: theoretical predictions of $b-y$ and $c$ Strömgren indices (Smalley & Kupka 1997) and abundance determinations (Heiter et al. 1998) indicate that, even if a 1D, homogeneous model can not explain all the spectroscopic and photometric observations, model atmospheres which predict temperature gradients closer to the radiative gradient are in better overall agreement with the observations. These arguments favors those models in which convection in the atmosphere is less efficient than predicted by models having $\alpha > 1)$, and led Heiter et al. (2002) to adopt either the FST model or the $\alpha = 0.5$ MLT models to compute their atmospheric grids. These were also further motivations to produce “full FST” stellar models by use of their atmospheric grids.

We may ask whether the “full FST” models have shown features interesting enough to render it useful to explore the application of the FST convection in different parts of the HR diagram. We give here one positive example (the interpretation of the Böhm–Vitense gap at B–V $\sim 0.35$) and one “negative” example, namely the impossibility of explaining the patterns of Lithium depletion in the Sun and in open clusters with the FST model. This latter result, however, may be telling us something else on the behaviour of convection in young convective stars.

5. The Böhm Vitense gap at B–V $\sim 0.35$

One of the most interesting results in our recent exploration of “full” FST models regards the main sequence: the FST convection, due to its very low efficiency close to the stellar surface, and high efficiency in the inner subatmospheres, yields a very sharp transition between structures which are convective only in the surface layers, and structures which show a well developed convection also in the interior (D’Antona et al. 2002). This is shown in Figure 2 by comparing the stratification of models of different mass in the P–T plane: models down to 1.42$M_{\odot}$ are mostly radiative (or convection is so inefficient that the convective gradient sticks very close to the radiative one), while suddenly, at 1.41$M_{\odot}$, an extended adiabatic convection region appears for a larger part of the envelope. The fast increase in the convective mass fraction ($M_{cc}$) as a function of the main sequence color B–V is shown in Figure 3. This characteristic of the models is reflected in their mass–$T_{\text{eff}}$ relation, which becomes suddenly steeper around $T_{\text{eff}} \approx$...
Figure 4. The Hyades Hipparcos HR diagram is compared with two simulations containing 100 stars between 1.2 and 1.7\(M_{\odot}\), randomly extracted from a Salpeter mass function distribution, following the FST or MLT mass–color isochrone of 600 Myr. The MLT simulation is shifted by +0.7 mag, and the FST one by 1.2 mag for clarity. At the bottom, the FST (dashed) and MLT color histograms are shown (scale at the right). The FST mass–color distribution produces a ‘gap’ at \(B-V\sim0.35\), while the MLT isochrone does not. The good correspondence with the Hyades gap indicates that the transition between structures which are convective only in the surface layers, and structures which show a well developed convection also in the interior is very sharp.

Figures 5. The general HR diagram of the computed tracks, which extend from the PMS to the MS and then to the Giant Branch. The tracks of 1.2, 1.4, 1.45, 1.5, 1.55, 1.6, 1.65\(M_{\odot}\) are shown. The diagonal (green) line on the right shows schematically the separation line between highly convective structures on the right and structures with convection limited to the atmosphere (on the left). The diagonal dotted (black) line indicates the observational red edge of the \(\delta\) Scuti instability strip, according to Pamyatnykh (2000). The two dash–dotted (red) lines indicate the observational boundaries of the \(\gamma\) Dor strip (Zerbi 2001). The MS at 10\(^8\) yr age is also shown.

6800K, and is also apparent in the HR diagram as a sudden change of slope (Figure 3). Using numerical simulations we have shown that this feature produces a stellar depletion which is consistent with the gap seen in the Hyades at \(0.33 \lesssim B-V \lesssim 0.38\), one of the so called “Böhm Vitense gaps” after Böhm-Vitense & Canterna (1974) and Böhm-Vitense (1982) found by Rachford & Canterna (2000) in 6 out of 9 open clusters which have been investigated. The standard MLT models do not show this behavior (see Figure 4).

The very sharp variation of the stellar structures in the HR diagram (or in the \(T_{\text{eff}}\)–gravity plane) is shown as a transition line in the HR diagram of Figure 5. Models on the right of this line have extended convective regions, and models on the left have very small convective regions, independently from their evolutionary phase (pre, on or post the main sequence). This can be seen in Figure 5. The transition line is compared in Figure 5 with the location of the \(\delta\) Scuti and \(\gamma\) Doradus instability strips. We may speculate that our transition line separates HR diagram regions which harbor different modalities of stellar oscillation patterns, as the excitation or driving mechanisms can be very different for stars having so different convective structure. In particular, it could represent the dividing line between coherent pulsations and solar-type oscillations. In this case, we should have expected that also the stars between the red edge of the \(\gamma\) Doradus instability strip and the transition line are pulsating. Notice that the location of the red edge of the \(\gamma\) Doradus strip is based on observations from the ground and might still be uncertain in the theoretical HR diagram. On the other hand, many other structural parameters may have a role in defining these instability strips, and at least two of these —elements diffusion and rotation— should be considered. A further note of caution on the naiveté of this proposal comes also from the complex behavior of the chromospheric and transition layer indicators for the MS stars on the right of the transition line (Böhm-Vitense et al. 2002).

Will the Böhm Vitense gap appear also in the field stars? Here the problem will be much more complicated by the blurring due to the fact that we have a mixture of ages and metallicities! Figure 7 shows simulations for three stellar populations with different metallicities (\(Z=0.01\), 0.02 and 0.03), covering ages from \(10^7\) to \(10^9\) yr, and distributed according to a Salpeter mass function. The diagonal lines separate the location of the models having deep envelope
Figure 6. The radiative (black), adiabatic (red) and superadiabatic (blue) gradient $\nabla - \nabla_{ad}$ in the external layers of the selected models along the $1.5M_\odot$ evolution (shown as open squares in Figure 5) versus the logarithm of the pressure. The first and third model have their convection region limited to the atmospheric layers, while the second one shows an extension of convection down to $\log P \sim 9$, well inside the envelope. The transition occurs for a very small variation of the physical parameters, and is mainly due to the high inefficiency of the FST model in the atmospheric regions.

6. The ‘historical’ problem of Lithium

In the years 1965-1990, we called ‘problem of lithium in the Sun’ the inability of the solar mass evolutionary tracks to burn a substantial fraction of their initial lithium during the Pre Main Sequence (PMS). This result was generally taken as a good proof that additional mechanisms for depletion were required, acting during the long solar MS lifetime, to reduce by a factor $\sim 140$ the initial solar system abundance ($\log N(\text{Li})=3.31\pm0.04$). This interpretation still today is taken as most plausible one, confirmed by the scarce lithium depletion, at the solar mass, in young open clusters (see e.g. Chaboyer 1998). In fact the lithium vs. $T_{\text{eff}}$ relation for the MS stars of young open clusters indicates a lithium depletion by at most a factor two for the solar mass in young clusters, while it is compatible with the solar depletion in some stars of the cluster M67, close to the solar age. For recent reviews see e.g. Jeffries (2000) and Pasquini (2000).

However, a different problem emerges from the most recent computation of solar models: they deplete too much lithium during the PMS evolution. D’Antona & Mazzitelli (1994, 1997) and Schlattl & Weiss (1999) have updated the FST convection. MLT models of the most recent generation, adopting updated equations of state and opacities also deplete too much lithium, D’Antona & Mazzitelli (1994, 1997), and are incompatible with the young open clusters observations. This problem is most severe in models using very efficient convection models, in fact it is more relevant for the D’Antona and Mazzitelli (1994 and 1997) models adopting the FST convection. MLT models of the most recent generation, adopting updated equations of state and opacities also deplete too much lithium, D’Antona & Mazzitelli (2001) and Montalbán et al. (2003). When we compare the lithium depletion predicted by many of our models, computed using AH97 atmospheres ($\alpha_{\text{atm}} = 1$) as boundary conditions, with the Pleiades data by Soderblom et al. (1993) and García Lopez et al. (1994). Only the upper squares, corresponding to the models employing $\alpha_{\text{atm}} = 1$, which do not fit the Sun, are compatible with the data. The HR diagram of the models is shown in Figure 5. The $\alpha = 1$ tracks are on the right, much cooler than what is needed to allow the solar fit! In addition, Figure 5 opens up the complex problem of the location in the HR diagram of PMS tracks (for a review see, e.g. D’Antona et al. 2000) while our full recent computation for this phase can be found in Montalbán et al. 2003. When we compare the location of PMS theoretical tracks with the observed few data of PMS stars for which an independent determination of mass is available, the tracks most consistent with the observations are again those with cooler atmospheres (higher mass for a given spectral type) and thus those which, generally, provide a radius larger than $R_\odot$ for the solar model. In fact, Figure 5 shows that the $\alpha = 1$ tracks are well compatible with the location of the secondary component of the eclipsing spectroscopic binary RXJ 0529.4+0041, one of the best determined PMS masses (Covino et al. 2001).

It is then clear that 1) the HR diagram location of the tracks during the PMS evolution, and 2) PMS lithium depletion, are two problems correlated each other, as it could have been expected, because, the smaller the $T_{\text{eff}}$ of the Hayashi track, the smaller is the temperature at the base of the convective envelope during the possible phase of lithium burning. Both the HR diagram location and the lithium depletion seem to be compatible only with models in which PMS convection is much less efficient than MS convection. Is this simply another proof that we are not able to model convection, or that there are unsolved problems with the opacities? Or does it mean that there is...
some other parameter playing a role in the PMS—and not on the MS? It is probably too early to derive strong conclusions, but we have suggested that PMS convection is inhibited by the thermal role of a dynamo built magnetic field (Ventura et al. 1998a, D’Antona et al. 2000).

7. Non-local convection in the Sun

Always in the framework of stellar evolution for asteroseismology, we performed one more consistency test, of a completely different nature. In this case, our attention is not at the surface, but at the bottom of the solar convective zone (CZ). In fact, a constraint to the thickness of the overshooting layer in this region has been set by helioseismology (e.g. Basu & Antia 1997), and it can not be larger than ~0.05\(H_p\). We constructed a detailed FST solar model matching the correct thickness of the CZ (the surface boundary conditions are in this case of negligible importance) and applied the treatment suggested by Camuto & Dubovikov (1998) (CD98) to a thin region centered around the formal Schwarzschild boundary of convection, to gain insight on what happens when a fully non-local turbulence theory is applied to a stellar structure.

More in detail, we have computed from the local model the starting distribution for the quantities: \(K\) (turbulent kinetic energy in the radial direction), \(\bar{\theta}^2\) (mean quadratic temperature variance), \(J\) (convective flux), \(\bar{\epsilon}\) (mean quadratic velocity variance) and \(\epsilon\) (dissipation rate), according to (42a-c), (43a-c) and (44a-d) in CD98. Then, temporal relaxation to the five above quantities has been allowed, according to the equations (19a-d) and (35a-b), until stationary conditions were reached. For each relaxation step, the gradient \(\beta\):

\[
\beta = \frac{T}{H_p} (\nabla - \nabla_{ad})
\]

has been updated according to:

\[
\beta = \beta_{rad} - \frac{J + F(K)}{\chi}
\]

where \(\chi\) is the thermometric conductivity, \(\beta_{rad} = T (\nabla_{rad} - \nabla_{ad})/H_p\). Further, \(F(K)\) is the kinetic energy flow \(-\nabla r \partial K/\partial r\) and \(\nu_t\) is the turbulent viscosity, \(\nu_t \propto K^{2/3}\).

Eq. (18c) from CD98, relative to the temperature, was not included in the final network since, being superadiabaticity at the bottom of the solar CZ negligible, temperature itself turned out to be nearly stationary already at the beginning of the relaxation.

As for the diffusive terms \(D_f\), the most simple down-gradient approximation has been chosen, namely, for each generic turbulent quantity \(Q\):

\[
D_f(Q) \propto \nu_t \frac{\partial Q}{\partial r}
\]

This is (perhaps) far from the best one can do, but, noticeably, no built-in scale length is present, contrarily to the only other non-local treatment (Xiong 1985) with which extensive stellar models have been computed. In fact, in Xiong case, an explicit, arbitrary scale length was included, and the results almost linearly depended on the value of the scale length. After \(2 \times 3 \times 10^6\) s, all the six quantities (including \(\beta\)) reached a final, stationary distribution, clearly showing a thin overshooting region. Figure 10 shows the behavior of the turbulent flux \(J\); Figure 11 presents the mean quadratic velocity variance \(\bar{\epsilon}^2\). Overshooting is present indeed, but its amplitude does not overcome 0.02\(H_p\), consistently with the solar observational constraints.\(^4\) The same relaxations have been performed for the CZ of the solar mass track at various evolutionary phases, from the first appearance of a radiative nucleus in PMS, up to early red giant. The overall features turn out to be similar in all cases. It is found that the overshooting region is absolutely negligible (< 0.005\(H_p\)) in PMS during the lithium burning phase. This at least ensures us that inclusion of proper overshooting in PMS evolutionary models will not worsen the problems with the exceedingly large solar lithium depletion shown by todays standard models. The thickness of overshooting, then, steadily increases during the evolution, reaching ~ 0.06\(H_p\) at log \(L/L_\odot\) = 1, where the computations have been stopped. In all case, the decay of \(\bar{\epsilon}^2\) is very sharp, putting an end to overshooting.

The only conclusion we can presently draw from these first non-local results is that probably the chosen approximation for the diffusive terms is perhaps not too bad, at least as long as the thickness of overshooting is concerned, since they are consistent with the observational solar constraints, and do not worsen the lithium problem.

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\(^4\) The very sharp decline of the flux is an important constraint for the ‘local’ parametrization of overshooting. In particular, it is different from the approximation of exponential decay adopted, e.g., by Ventura et al. 1998a and Ventura et al. 1998b. These results apply only to overshooting below the CZ, we can not extend them to what happens above the convective cores.
Figure 7. Simulations based on the Montalbán et al. (2003) models of different metallicites. A population of 1500 stars is randomly chosen between ages of $10^7$ and $10^9$ yr defines the Böhm Vitense gap at the boundaries indicated by the diagonal lines.

Figure 8. The Pleiades data by Soderblom et al. (1993) and García Lopez et al. (1994) (open squares) are compared with the depletion predicted by the models in Table 2 (full big squares). The models are placed at the $T_{\text{eff}}$ they would take in an empirical MS, at the Pleiades age. Only the upper squares, corresponding to the models with $\alpha_{\text{atm}} = \alpha_{\text{un}} = 1$, are compatible with the data. The full line shows the depletion from the models by Ventura et al. (1998b) computed including the thermal effect of a magnetic field on the convective temperature gradients. The large open square with the error bar represents the lithium abundance of the secondary component of RXJ 0529.4+0041 ($\log N(\text{Li}) = 2.4 \pm 0.5$, Covino et al. 2001). The $T_{\text{eff}}$ at which the point is located (5500K) is assumed to be the main sequence $T_{\text{eff}}$ of a star of mass $0.925 M_\odot$. 

Figure 7. Simulations based on the Montalbán et al. (2003) models of different metallicites. A population of 1500 stars is randomly chosen between ages of $10^7$ and $10^9$ yr defines the Böhm Vitense gap at the boundaries indicated by the diagonal lines.
Figure 9. The figure shows the tracks of 0.8, 0.9, 1.0 and 1.1M☉ computed with AH97 atmospheres and α_in = 1.9 (full lines, left), or α_in = 1.0 (dash-dotted lines, right). The solar location is shown: it is compatible with the solar model having α_in = 1.9, but ~400K hotter than the solar model with α_in = 1.0. The location of the secondary component of the binary RXJ 0529.4+0041 is also shown. Its mass is 0.925 ± 0.005M☉, so it is best compatible with the α = 1 models.

Figure 10. The non-local turbulent flux J (cgs units) around the local Schwarzschild boundary (vertical dashed line) is shown as a function of the logarithm of the pressure P (cgs units). The flux is obviously negative in the overshooting region and, contrarily to what one could expect according to a thumb rule (perhaps exponential decay), a very sharp decay of the flux is found, suddenly ending the extra-mixed region less than 0.02Hp below the local boundary.

Figure 11. The mean quadratic velocity variance w². Velocity decays outside the local Schwarzschild boundary with the same slope as inside. When it approaches zero, however, the slope becomes steeper, and overshooting comes to a full stop. This suggests that all the evolutionary computations performed up today in the hypothesis of an exponential decay of the diffusive coefficient outside formally convective regions can perhaps (largely?) overestimate the amount of extra-mixed matter.