A HEAVY HIGGS CAN GIVE STRONG FIRST ORDER ELECTROWEAK PHASE TRANSITION IN THE STANDARD MODEL?

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Abstract

The role of Higgs sector of the standard model in first order phase transition is reexamined. It is found that a solution to the mass gap equations exist which can be used in higher orders. This possible solution leads to a transition which is found to be strongly first order and the ratio of the critical scaler field and the critical temperature is about 1.2 for a Higgs of mass around 400 GeV. This can explain baryogenesis.

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Recently electroweak phase transition at high temperature has been a topic of great interest. Sakharov [1] was the first to point out that the observed baryon asymmetry of the Universe can be produced by processes which violate C, CP and B and occurs out of thermal equilibrium. All the conditions can be met by the Standard Model; C-violation exists, CP violating terms can be accommodated, Sphaleron can induce sufficient B-violating processes at high temperature. A first order phase transition [2] can provide the nonthermal equilibrium. In order to have sufficient departure from equilibrium, it is necessary that this transition be rather strong. The value of the ratio of the Higgs field $\Phi_c$ and the critical temperature $T_c$ should at least be about 1.0. All effective potentials at high temperatures constructed so far obtain this ratio to be rather small. Furthermore it has been shown to decrease when Higgs mass is increased [3–5]. In this letter we propose to construct a plausible theory by solving the gap equation of Ref.[6] to the required order. The ratio $\Phi_c/T_c$ increases with the value of the mass of the Higgs and has the value of 1.0 when the Higgs mass is round 400 GeV.

The most basic quantity to calculate is the one loop effective potential $V_{eff}(\Phi, T)$, improved by appropriate resummation [7] and two loop calculations. We assume that this potential can be cast into the form

$$V_{eff}(\Phi, T) = a\Phi^2 - b\Phi^3 + c\Phi^4 ,$$

plus a constant field independent term which shall be omitted. $a$, $b$ and $c$ are functions of the temperature. $V_{eff}(\Phi, T)$ should be real and imaginary parts possibly do not arise for high temperature. The effective Higgs and Goldstone field dependant masses are

$$m_{eff}^2_{Higgs}(\Phi, T) = \frac{\partial^2 V(\Phi, T)}{\partial \Phi^2}$$

and

$$m_{eff}^2_{Goldstone}(\Phi, T) = \frac{1}{\Phi} \frac{\partial V(\Phi, T)}{\partial \Phi} .$$

The potential should be extremum at the origin.
The classical potential is

\[ V_0 = \frac{\lambda}{4} (\Phi^2 - \sigma^2)^2, \]  

where \( \lambda \) is the coupling constant and \( \sigma = 246 \text{ GeV} \) is the symmetry breaking value of the field. The effective field dependent masses of particles in the standard model are, in obvious notations,

\[ m_H^2(\Phi) = 3\lambda \Phi^2 - \lambda \sigma^2, \]  
\[ m_G^2(\Phi) = \lambda \Phi^2 - \lambda \sigma^2, \]  
\[ m_w^2(\Phi) = \frac{M_w^2}{\sigma^2} \Phi^2, \]  
\[ m_t^2(\Phi) = \frac{M_t^2}{\sigma^2} \Phi^2 \]  
and

\[ m_Z^2(\Phi) = \frac{M_Z^2}{\sigma^2} \Phi^2. \]

We shall omit, for this theory, the contribution of fermions lighter than the top. The zero temperature effective potential as modified by radiation correction up to one loop is [3,4].

\[ V^{(1)}(\Phi, 0) = \sum_{i=H, G, W, Z} n_i \frac{m_i^4(\Phi)}{64\pi^2} \left[ m_i^4(\Phi)(\log \frac{m_i^2(\Phi)}{M_i^2} - \frac{3}{2}) + 2m_i^2(\Phi)M_i^2 \right] + \frac{n_G}{64\pi^2} m_G^4(\Phi)(\log \frac{m_G^2(\Phi)}{M_H^2} - \frac{3}{2}), \]  

where \( n_H = 1, n_G = 3, n_w = 6, n_z = 3, n_t = -12. \) At finite temperature, the one loop effective potential is

\[ V_T = \frac{T^4}{2\pi^2} \left[ \sum_{i=H, G, W, Z} n_i J_+(y_i^2) + n_t J_- (y_t^2) \right], \]  

where \( y_i^2 = \frac{m_i^2(\Phi)}{T^2}, \)
\[ J_{\pm}(y^2) = \int_0^\infty dx x^2 \log[1 \mp e^{-(\sqrt{x^2+y^2})}] . \] (12)

It has been shown by Anderson and Hall [5] that for values of \( \frac{m_i(\Phi)}{T} \) less than 1.6, the high temperature approximation, namely

\[
V_T(\Phi, T) \simeq \sum_{i=H,G,W,Z} n_i \left[ \frac{m_i^2(\Phi)T^2}{24} - \frac{m_i^3(\Phi)}{12\pi} T - \frac{m_i^4(\Phi)}{64\pi^2} \left( \log \frac{m_i^2(\Phi)}{T^2} - 5.4076 \right) \right] 
- n_t \left[ \frac{m_t^2(\Phi)T^2}{48} + \frac{m_t^4(\Phi)}{64\pi^2} \left( \log \frac{m_t^2(\Phi)}{T^2} - 2.6350 \right) \right],
\] (13)

is valid to better than 5%. Our values for the ratio will be less than 1.6 even if the mass of the Higgs will be very high. The \( m^4 \log m^2 \) term of the \( T=0 \) term cancels with the term \( T\neq0 \), leaving the quartic mass term with a constant temperature dependent factor.

There are important corrections due to lollipops, daisies and superdaisies [8]. We shall consider only few of them. The contribution to the \( \Phi^3 \) term in the gauge sector is reduced by a factor of \( \frac{2}{3} \). Since gauge couplings are small, we shall completely neglect the higher order corrections due to them. Reference [3] treats the gauge sector very thoroughly. The important contribution that really matters is a \( \log(\Phi) \) term [9] which does not alter our results.

There are actually three types of expansions, the perturbative expansion in \( \lambda \) at zero temperature, the high temperature expansion in \( \frac{m^2(\phi)}{T^2} \) and the loop expansion which we shall discuss later. At zero temperature the perturbation theory is valid upto \( \lambda=3.5 \) or thereabout [10].

Due to lollipops, daisies and superdaisies the contributions to the effective potential from the Higgs sector only, has been calculated by many authors. From the results of Boyd, Brahm and Hsue [4], it is easily seen that at high temperatures the quadratic part of the potential is enhanced by the addition of \( \lambda \phi^2 T^2 \) due to Higgs. Consistent with high temperature expansion limiting values and to the first order in \( \lambda \) we now collect \( \phi^4, \phi^2 T^2 \) terms and cubic gauge contribution term \( \phi^3 T \). The cubic term of the Higgs sector is left for the detail consideration later. The result is

\[
V(\Phi, T) = \frac{1}{4} \lambda_T \Phi^4 + d(T^2 - T_0^2) \Phi^2 - e\Phi^3 T + V^{(3)}(\phi, T)
\] (14)
\[
\lambda_T = -\frac{3}{16\pi^2\sigma^4} \left[ 2M_w^4 \left( \log \frac{M_w^2}{T^2} - 3.91 \right) + M_z^4 \left( \log \frac{M_z^2}{T^2} - 3.91 \right) - 4M_t^4 \left( \log \frac{M_t^2}{T^2} - 1.14 \right) + M_H^4 \left( \log \frac{M_H^2}{T^2} - 3.91 \right) \right],
\]
(15)

\[
d = \frac{1}{8\sigma^2} \left( M_H^2 + 2M_w^2 + M_z^2 + 2M_t^2 \right),
\]
(16)

\[
T_0^2 = \frac{6M_H^2 - \frac{3}{4\pi^2} \left( 1.5M_H^4 + 6M_w^4 + 3M_z^4 - 12M_t^4 \right)}{3M_H^2 + 6M_w^2 + 3M_z^2 + 6M_t^2} \sigma^2,
\]
(17)

\[
e = e_g + e_H,
\]
(18)

\[
e_g = \frac{1}{6\pi\sigma^3} (2M_w^3 + M_z^3) \simeq 10^{-2}
\]
(19)

and

\[
V^{(3)} = -\frac{T}{12\pi} \left( m_H^2(\phi) + 3m_G^2(\phi) \right).
\]
(20)

\(e_H\) is the contribution from the Higgs sector. We replace \(\lambda\) by \(\lambda_T\) everywhere which we assume to be a partial resummation. The most difficult part has been to deal with the cubic terms of the Higgs and Goldstone sectors. In the lowest order both \(m_H\) and \(m_G\) become imaginary. Most of the earlier works have mainly taken the approximate values

\[
m_H^2 = 3\lambda\phi^2 + 2d(T^2 - T_0^2)
\]
and

\[
m_G^2 = \lambda\phi^2 + 2d(T^2 - T_0^2).
\]

We want to find the value of \(V^{(3)}\) by following an entirely different method. We first calculate the effective masses

\[
m_{H,\text{eff}}^2(\Phi, T) = 3\lambda_T\Phi^2 + 2d(T^2 - T_0^2) - 6e\Phi T + \frac{\partial^2 V^{(3)}}{\partial\phi^2}
\]
(21)

and

\[
m_{G,\text{eff}}^2(\Phi, T) = \lambda T\Phi^2 + 2d(T^2 - T_0^2) - 3e\Phi T + \frac{1}{\phi} \frac{\partial V^{(3)}}{\partial\phi}.
\]
(22)

Let us compare this with the gap equation of Buchmuller et al [3]. To the order \(\lambda^{3/2}\) and \(g^2\), they are in our notation,
\[ m_H^2 = 2d(T^2 - T_0^2) + 3\lambda_T \Phi^2 - 6e_\Phi T \]
\[- \frac{3\lambda_T}{4\pi} [m_H + m_G + \lambda_T \Phi^2 \left( \frac{3}{m_H} + \frac{1}{m_G} \right)] T \] (23)

and

\[ m_G^2 = 2d(T^2 - T_0^2) + \lambda_T \Phi^2 - 3e_\Phi T \]
\[- \frac{\lambda_T}{4\pi} [m_H + 5m_G + 4\lambda_T \Phi^2 \left( \frac{1}{m_H + m_G} \right)] T \] (24)

The last terms of these expressions are \( \lambda^\frac{3}{2} \) corrections and can be related to the cubic mass terms of the full potential. Using the equations (5) and (6) and comparing with the gap equations (23) and (24) we obtain

\[
\frac{\partial^2 V^{(3)}}{\partial \phi^2} = -\frac{T}{12\pi} \left\{ [6m_H(\phi)\left( \frac{\partial m_H(\phi)}{\partial \phi} \right)^2 + 3m_H^2(\phi)\left( \frac{\partial^2 m_H(\phi)}{\partial \phi^2} \right)] + 3[6m_G(\phi)\left( \frac{\partial m_G(\phi)}{\partial \phi} \right)^2 + 3m_G^2(\phi)\left( \frac{\partial^2 m_G(\phi)}{\partial \phi^2} \right)] \right\}
\]
\[- \frac{3\lambda_T}{4\pi} T(m_H(\phi) + m_G(\phi) + \lambda_T \Phi^2 \left( \frac{3}{m_H(\phi)} + \frac{1}{m_G(\phi)} \right)) \] (25)

and

\[
\frac{1}{\phi} \frac{\partial V^{(3)}}{\partial \phi} = -\frac{T}{12\pi \phi} \left\{ [3m_H^2(\phi)\left( \frac{\partial m_H(\phi)}{\partial \phi} \right) + 9m_G^2(\phi)\left( \frac{\partial m_G(\phi)}{\partial \phi} \right)] \right\}
\]
\[- \frac{\lambda_T}{4\pi} T(m_H(\phi) + 5m_G(\phi) + 4\lambda_T \Phi^2 \left( \frac{1}{m_H(\phi) + m_G(\phi)} \right)) \] (26)

Our main interest is to examine the possibility of a strong first order phase transition. Interestingly we find that these equations have a unique property. If \( m_G \) and \( m_H \) are proportional to \( \sqrt{\lambda} \phi \), the dependance of the equations on \( \lambda \) and \( \phi \) cancel out. The cubic term is of order \( \lambda^\frac{3}{2} \) like the correction terms of the gap equation. Let us take \( m_H=a\sqrt{\lambda} \phi \) and \( m_G=b\sqrt{\lambda} \phi \). The equations (25) and (26) reduces to

\[ 2a^3 + 6b^3 + 3\lambda(a + b) + 3\lambda^2 \left( \frac{3}{a} + \frac{1}{b} \right) = 0 \] (27)

and

\[ a^3 + 3b^3 - \lambda(a + 5b) - \frac{4\lambda^2}{a + b} = 0. \] (28)
Using Newton-Ralphson method we find there are several solutions. Ones that interest us is $a=1.732=\sqrt{3}$ and $b=1$. So in evaluating the cubic term of the potential $V^{(3)}$ we shall take $m_H = \sqrt{3}\lambda\phi$ and $m_G = \sqrt{\lambda}\phi$. The value of $e_H$ comes out to be
\[
e_H = \frac{\sqrt{3} + 1}{4\pi} \frac{\lambda^2}{T}
\]
and
\[
V^{(3)} = -(e_g + e_H)\phi^3 T = -e\phi^3 T.
\]

The idea of a strong first order phase transition has been previously ignored. The reason that has been advanced is that the perturbation theory fails at high temperatures for large values of $\lambda$. Let us examine the loop expansion parameter following Buchmuller et al. Inspection of gap equation (23) and (24) shows that if we introduce expansion parameter by
\[
\zeta_H \frac{\lambda T}{4\pi} \left( \frac{3}{m_H} + \frac{1}{m_G} \right) = 1
\]
and
\[
\zeta_G \frac{\lambda T}{\pi} \left( \frac{1}{m_H + m_G} \right) = 1,
\]
the largeness of $\zeta_H$ and $\zeta_G$ will imply convergence of the perturbative expansion. We have introduced the parameter $\zeta_G$, coming from equation (23) as the effective potential can be determined by integrating $\Phi m_{\lambda}^2(\Phi)$.

The occurrence of $m_H$ and $m_G$ imply that the expansion will fail down when they will be imaginary. From equations (24) and (20) it is seen that $m_H^2$ will be negative when $3\lambda\Phi^2 + 2d(T^2 - T_0^2)$ becomes less than $6e\Phi T$. This happens at a value $\Phi = \Phi_b$ and $T = T_b$ which can be obtained by completing square as follows:
\[
m_H^2 = 3\lambda(\Phi - \frac{e}{\lambda} T)^2 - 3\frac{e^2}{\lambda} T + 2d(T^2 - T_0^2).
\]
This gives
\[
\frac{T_b^2 - T_0^2}{T_b^2} = \frac{3}{2d} \frac{e^2}{\lambda} \quad \text{and} \quad \Phi_b = \frac{e}{\lambda} T_b.
\]
At this temperature $T_b$, following Buchmuller et al.

\[ \bar{m}_H \simeq \sqrt{\frac{6e^2}{\lambda_T}}T_b \quad (35) \]

and

\[ \bar{m}_G \simeq \sqrt{\frac{3e^2}{\lambda_T}}T_b \quad (36) \]

Since $e \simeq e_H$, the expansion parameters are obtained as

\[ \zeta_H = (\sqrt{3} + 1)/\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) = 1.52 \quad (37) \]

\[ \zeta_G = (\sqrt{3} + 1)(\sqrt{3} + \sqrt{6})/4 = 2.856 \quad (38) \]

Both the expansion parameters come out independent of the Higgs mass. The convergence of the perturbative expansion is slow but sure and better if effective potential is calculated by integrating $\Phi m_G^2(\Phi)$. We can now write the effective potential as

\[ V(\Phi, T) = \frac{\lambda_T}{4} \Phi^4 + d(T^2 - T_0^2)\phi^2 - e\Phi^3T. \quad (39) \]

Here

\[ e = e_g + \frac{3\sqrt{3} + 3}{12\pi}\lambda_T^{3/2}. \quad (40) \]

We calculate $V(\Phi, T)$ and find the critical temperature which is given in Table I. The graphs at the critical temperature $T_c$ are plotted in Fig.1(a) and (b).

It is to be noted that value of $(\lambda_T - \lambda)/\lambda_T$ is found 0.3 for all cases considered. We must now examine the validity of the high temperature expansion. In general the equation for $V(\phi, T)$ can be recast in the form

\[ V(\Phi, T) = \frac{\lambda_T}{4} \Phi^4 + d(T^2 - T_0^2)\Phi^2 - eT\Phi^3. \quad (41) \]

In the above quantities like $d, T_0$ and $e$ become mildly temperature dependent. At the critical temperature the following equations hold:
\[ T_c^2 = \frac{T_0^2}{1-(e^2/\lambda_T d)} \quad \text{and} \quad \frac{\Phi_c}{T_c} = 2 \frac{e}{\lambda_T}. \] (42)

The highest mass term is to the effective Higgs mass which from the full potential is

\[ m_{H,\text{eff}}^2(\Phi, T) = 3\lambda_T\Phi^2 + 2dT^2 - T_0^2 - 6e\Phi T \] (43)

and at \( T = T_c \)

\[ m_H^2(\Phi_c, T_c) = \frac{\lambda_T}{2} \Phi_c^2 \] (44)

and

\[ \frac{m(\phi)_{\phi=\phi_c}}{T_c} = \sqrt{\frac{\lambda_T}{2}} \frac{\Phi_c}{T_c} \simeq 1.17 \] (45)

at \( M_H=425 \text{ GeV} \). For \( M_H = 425 \text{ GeV} \) \[11\] \( \lambda = 1.49 \) and \( \lambda_T = 2.13 \). This is about the limit where the high temperature expansion holds to better than 5\% \[13\].

An increase in Higgs mass raises the important ratio \( \Phi_c/T_c \). The Higgs mass, of course, can not be arbitrarily large as the high temperature expansion will breakdown possibly for Higgs mass above 500 GeV \[12\].

Thus we have shown that contrary to current results in literature, the first order phase transition can be strong and gets stronger with increase of Higgs mass. It is thus possible to explain Baryogenesis with a very massive Higgs.

The perturbation theory at zero temperature seems to be valid for Higgs mass below 700 GeV \[12\]. However at high temperature with the gap solution conventionally taken, as has been done by Buchmuller et al, the upper limit of Higgs mass is found to be 70 GeV . From the solution of gap equation chosen here the loop expansion parameter at high temperature is independent of Higgs mass.

The result of lattice simulations \[13\] are not conclusive. Some of them e.g. Bunk et al and Kajantie et al have concluded that for intermediate Higgs mass, there is strongly first order phase transition. Moreover all lattice calculations have ignored the fermions which must be playing a role in cancellation of vacuum loops of Higgs sector. A comprehensive
analysis for Higgs mass including top is still wanting. In any event, the lattice calculations have not reached any definite conclusions.

The purpose of this letter is to emphasize the role of the Higgs sector which not only breaks the $SU(2)$ symmetry but may give a strong first order electroweak phase transition.

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REFERENCES

[1] A. D. Sakharov, Pisma Zh. Eksp. Theor. Fiz. 5, 32 (1967).

[2] D. A. Kirzhnits and A. D. Linde, Ann. Phys.(N.Y.) 101, 195 (1976).

[3] J. R. Espinosa, M. Quiros and Zwirner, Phys. Lett. B 314, 206 (1993); P. Arnold and O. Espinosa, Phys. Rev. D47, 3546(1993); M.E.Corrington,Phys.Rev.D45 ,2933(1992); K. Takahashi, Z.Phys.C26,601(1985).

[4] C. Glenn Boyd, D. E. Brahm and Stephen D. H. Hsue, Phys. Rev. D 48, 4952 (1993).

[5] G.W.Anderson and Lawrence J. Hall, Phys. Rev. D 45, 2685 (1992).

[6] W. Buchmuller, Z. Fodor, T. Helbig and D. Walliser, Ann. Phys. 234,260 (1994); W. Buchmuller and O. Philipsen, Nucl. Phys. B443, 47(1995).

[7] M. Dine, R. G. Leigh, P. Huet, A. Linde and D. Linde, Phys. Rev. D46, 550(1992); M. Reuter and C. Wetterich, Nucl. Phys. B408, 91(1993); Z. Fodor and A. Hebecker, Nucl. Phys. B432,127 (1994);A. Jakovac, K. Kajantie and A. Patkos, Phys. Rev. D49, 6810(1994); K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B425, 67(1994); J. A. Adams and N. Tetradis, Nucl. Phys. Lett. B347, 120(1995).

[8] C. Glenn Boyd, David E. Brahm and Stephen D. H. Hsu, Phys. Rev. D 48, 4952 (1993).

[9] J. E. Bagnasco and M. Dine, Phys. Lett. B 303,308 (1993)

[10] R. N. Mohapatra in Unification and Supersymmetry, the frontiers of Quark Lepton Physics, Springer-Verlag, page 57; C. Quigg and H. Thacker, Phys. Rev. D 16,1519(1977) and M. Veltman, Act. Phys. Polon. B 8,475(1977).

[11] P. H. Chaskowski and Stefan Pokorski, Preprint No. MPI-Ph/95-39, FT-95/6.

[12] Ulrich Nieste and Kurt Riesselmann, Phys. Rev. D53,6638 (1996).

[13] B. Bunk, E.M. Ilgenfritz, J. Kripfganz and A. Schiller, Nucl. Phys. B403, 453(1993);
K. Kajantie, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B407, 356(1993); K. Farakos, K. Kajantie, K. Rummukainen and M. E. Shaposhnikov, Phys. Lett. B336, 494(1994); F. Csikor, Z. Fodor, J. Hein, K. Jansen, A. Jaster and I. Montvay, Phys. Lett. B334, 405(1994); Z. Fodor, J. Hein, K. Jansen, A. Jaster and I. Montvay, DESY 94-159; F. Karsh, T. Neuhaus and A. Patkos, preprint B1-TP 94/27(1994); H. G. Evertz, J. Jersak and K. Kanaya, Nucl. Phys. B285, 229(1987).
TABLE I. Values of $\lambda, \lambda_T, \phi_c, T_c$ and $\frac{\phi_c}{T_c}$ against $m_H$

| $m_H$(GeV) | $\lambda$ | $\lambda_T$ | $\phi_c$ (GeV) | $T_c$(GeV) | $\frac{\phi_c}{T_c}$ |
|------------|-----------|-------------|----------------|------------|---------------------|
| 150        | 0.185     | 0.179       | 77             | 164        | 0.46                |
| 200        | 0.33      | 0.337       | 88             | 201        | 0.438               |
| 250        | 0.51      | 0.56        | 110            | 232        | 0.47                |
| 300        | 0.74      | 0.86        | 152            | 263        | 0.58                |
| 350        | 1.01      | 1.25        | 215            | 298.9      | 0.72                |
| 400        | 1.32      | 1.76        | 339            | 362        | 0.93                |
| 425        | 1.49      | 2.13        | 540            | 447        | 1.13                |
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