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A displacement-doubling prism

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A novel prism has been devised which can be used in place of the ‘flag’ in an optical shadow-sensing type of displacement sensor, for example. In this way, theoretically the displacement sensitivity of the sensor can be doubled. Such a prism has been manufactured, and its displacement-doubling property has been verified.

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1. Introduction

A position-sensitive detector has been under investigation for its potential use in pendulum suspension systems for the Advanced LIGO (aLIGO) upgrade to the initial LIGO gravitational wave detectors [1,2]. In this detector, a movable 3 mm diameter ‘flag’ cuts an infrared beam emitted by an infrared LED, and the flag’s displacement is detected by the change in signal from a facing photodiode detector. The displacement sensitivity attained by this sensor was found to be $1.3 \times 10^{-10}$ m/rt-Hz at a frequency of 1 Hz, and $9.4 \times 10^{-11}$ m/rt-Hz, at 2 Hz. The usable displacement span of the sensor was approximately 0.7 mm.

It seemed, though, that there might still be some way of further improving the displacement sensitivity of the sensor with no increase in the noise level. The use of lenses as optical levers was discounted, however, because of the need for very short focal lengths due to the restricted sensing volume available, and the consequential large variability in displacement gain necessarily due to imperfect alignment. By using a form of reflective optics, on the other hand, it was felt that simply collimated beams of light might be employed in order to achieve a useful doubling of displacement sensitivity, with, comparatively, a much reduced sensitivity to alignment. After some consideration, it was realized that a candidate device in the form of a Dove prism (generally used as an image rotator) might be suitable [3]. In such a prism an incoming light beam is mirrored about one plane—an effect which can be used as the basis for a ‘displacement doubler’, if the prism is displaced.

Fig. 1(a) shows a Dove prism, with internal angles 135°, 45°, 45°, and 135°. If such a prism was realized using BK7 material, for example, with a refractive index of $n_d=1.517$ (the Abbe value $V_d=64.2$), then the height of the prism would be $4.23 \times$ the distance between the parallel sides of its trapezoidal profile. Here, this distance has been taken to be 2.5 mm, leading to an overall prism height of 10.57 mm, as indicated in the figure.

Unfortunately, such a prism would not be suitable in the final application mentioned above, because (i) its height compares very unfavorably with the diameter of the existing flag and (ii) even if it were treated with an AR coating on the face of the prism presented to the input optical beam, it would nonetheless reflect some stray light laterally, which might interfere with the operation of the LIGO interferometers.

However, a new prism design was conceived, shown in Fig. 1(b), and this retains the essential mirroring action of the Dove prism, as depicted in the figure, albeit with an additional lateral offset between the input and output beams. Unlike the Dove prism, this 60°, 120°, 30°, and 150° prism has one of its faces (the longest) mirror-coated; but, in consequence, the incoming/outgoing optical beams are normal to the parallel faces of its entrance/exit windows (so no light-spill), whichever way it may be used. Moreover, as its internal optical path is folded, it is...
altogether more compact transversely than the Dove prism. Indeed, as indicated in the figure, its height can be just 4.33 mm for an aperture width of 2.5 mm—the aperture being equal to that of the Dove prism in [a]. Other angles could have been chosen for the prism, the constraints being that the smallest angle (30°, here) must be < 45° and the two smallest angles (30° and 60°, here) must sum to 90°.

The design target for the improved displacement sensor was a usable span of at least 1 mm, and, in the case of this new prism sensor, the span of the device is determined by the width of the prism’s upper window—the usable displacement-sensing range being one half of this width, or 1.25 mm, here. Furthermore, as the input and output beams are normal to their respective prism faces, the prism is not dispersive, and its relative dimensions remain unchanged, irrespective of the material from which it is made (for any practicable optical material, that is this prism only requires $n_d > 1.16$). In addition, the optical path length through the prism is constant for all light rays normal to its input/output faces, so that focused beams could be used. For the dimension shown, the lateral offset of the central optical beam from input to output is 6.25 mm.

2. Working principle

Fig. 2 demonstrates how the new prism may be used in practice, where it is revealed embedded within a flag made (ideally) from opaque material, the flag being placed between (say) a laser source and an (offset) position-sensing photodiode detector. In Fig. 3 this concept has been extended to that of a shadow-sensor for measuring displacement. The upper sections of views (a) and (b) each show schematically a fixed rectangular photodiode (PD) detector being illuminated by a fixed source of light—via the intermediary of the movable prism. The light source, which is shown in the lower section of each view, is in the form of a knife-edged mask, this being back-lit from below by the collimated beam from a LED. As the prism is moved to the right, the shadow of the knife-edge cast by the source is caused to advance rightwards across the face of the PD. In view (a) the optical paths traced through the prism show the PD to be fully illuminated by the source, the virtual shadow of the mask (not shown in the figure) lying adjacent to, and immediately to the left of, the PD. In view (b) the prism has been displaced to the right by a distance $\Delta$ relative to the fixed source and detector, and the same optical paths linking the exit window to the Prism’s effective input window have been drawn. In this case, however, the shadow of the mask is seen to have advanced across the PD by a distance $2\Delta$.

In Fig. 3, both the PD and the lit section of the source have been taken to be 2.5 mm wide, i.e., equal in width to that of the upper (exit) window of the prism (this is not a necessary condition). Clearly, from the point of view of the photocurrent flowing through the PD, as if it is an occluding flag, casting its shadow onto the PD, had been displaced laterally by a distance $2\Delta$. In summary, the action of the new prism—imagined to have been embedded in a flag (as in Fig. 2)—has been effective to double the displacement of that flag.

3. Practical tests

A small prism, having the dimension shown in Fig. 1(b), and 4 mm wide, was manufactured by BK7 material by Spanoptic Ltd. [4]. A test rig was then built consisting of a prism-holder machined from Perspex, which was clamped to an optical micrometer-driven stage. The new ‘displacement-doubling’ prism was clamped within the holder, such that it could be translated over a test pattern of parallel lines, ruled at 1 mm intervals. The test rig is shown in Fig. 4.

4. Results

In the upper half of Fig. 5 the doubling prism has been placed over the bold line ‘b’ in the test pattern, such that the image of this line appearing in the window of the prism was lined up with another bold line in the test pattern.

In the lower half of the figure the prism holder and prism have been displaced to the right by 1 mm. The image of line ‘b’ is seen to have been displaced relative to the fixed pattern by 2 mm.

5. Discussion

The displacement-doubling prism evidently operates optically as anticipated, and it clearly has the potential to double the
displacement sensitivity of (say) a simple shadow-sensing type of displacement sensor. However, a displacement measuring span of 1.25 mm leads to a prism height of almost 4.5 mm, compared to a current flag width (diameter) of 3 mm in aLIGO [5]. Nevertheless, the existing 1 mm gap on either side of the current 3 mm flag, separating it from its LED source housing on one side, and from its PD detector housing on the other, could be retained straightforwardly for this slightly wider flag/prism by simply moving the LED and associated PD housings 1.5 mm further apart. In addition, of course, the knife-edged mask plus LED source, and the PD

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**Fig. 3.** Potential use of the new prism in a shadow sensor. Comparison of the schematic views (a) and (b) shows that as the prism is moved to the right the shadow of a fixed knife-edged mask, illuminated from below by a collimated LED source, advances across the face of the fixed photodiode (PD) detector. Four light rays emanating from the source are shown in each view, the bold ray coming from the knife-edge. If the prism is displaced laterally by a distance \( \Delta \), the shadow’s edge advances across the PD by \( 2\Delta \).

**Fig. 4.** The displacement doubling test rig. The new prism was mounted in a holder so that it could be moved over a test pattern using the micrometer driven stage. In the foreground is a view of the prism, removed from its holder.

**Fig. 5.** Upper view: the new prism has been positioned over the bold line ‘b’ so that the line’s image appears in the prism’s window aligned with another bold line in the test pattern. Lower view: the prism holder and prism have been displaced to the right by 1 mm. The image of line ‘b’ is seen to have been displaced to the right by 2 mm.
detector, must be offset from each other along the displacement-measuring axis—here, by 6.25 mm. Work is planned to verify in practice the noise power spectral density of this device, and thereby to discover what its achievable displacement sensitivity will be at frequencies in the vicinity of 1 Hz.

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