1 Introduction

For some time now there has been considerable interest in the natural hyperkahler metric on the moduli space of charge $m$ $SU(2)$ monopoles in $\mathbb{R}^3$. It is known from the work of Taubes that near the boundary of this moduli space the monopole approximates a collection of $m$ particles with internal $U(1)$ phases. It was argued by Manton \cite{Manton} that the geodesics of this metric correspond to scattering of $m$ slow moving monopoles. There are now many interesting examples of scattering of $SU(2)$ monopoles beginning with the calculation of the metric on the moduli space of $SU(2)$ charge two monopoles by Atiyah and Hitchin \cite{Atiyah-Hitchin} and more recently results on the scattering of monopoles with special symmetry \cite{Boulanger, Bonora, Callan, Manton, Murray}.

Monopoles also exist for compact groups $G$ other than $SU(2)$. We will be interested only in the case of maximal symmetry breaking. In this case the particles making up the monopole come in $r$ distinguishable ‘types’ where $r$ is the rank of the group. The $r$ types correspond to the the $r$ different elementary ways of embedding $SU(2)$ into $G$ along simple root directions. The magnetic charge of the monopole is now a vector $m = (m_1, \ldots, m_r)$ where $m_i$ can be thought of as the number of monopoles of type $i$ \cite{Murray}. If any of the $m_i$ vanish the monopole is obtained from an embedded subgroup so that the simplest monopole that is genuinely a monopole for $G$ is one with each $m_i = 1$. We are interested in the structure of the moduli space for this case and its metric. Note that in general the moduli space has dimension $4(m_1 + \cdots + m_r)$ so that the moduli space of $(1,1,\ldots,1)$ monopoles has dimension $4r$.

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For the group $SU(3)$ the rank is 2 and the metric on the moduli space of $(1,1)$ monopoles was studied by Connell [2, 3]. The same result was also obtained independently Gauntlett and Low [4] and Lee, Weinberg and Yi [4, 13]. In these latter two works some special assumptions on the values of the Higg’s field at infinity that simplified the work of Connell are removed. The metric obtained is globally of Taub-NUT type.

For the more general case of an $SU(n+1)$ monopole of charge $(1,\ldots,1)$ Lee, Weinberg and Yi [14] calculate the asymptotic form of the monopole metric and show that it is asymptotically Taub-NUT. They then give an argument that the asymptotic form of the metric can be smoothly extended to the whole moduli space and they conjecture that the monopole metric is indeed exactly this extended metric. I give a partial proof of this result. The reason it is partial is that I construct and describe the natural hyperkahler metric not on the monopole moduli space but on the space of Nahm data. This is indeed of the form conjectured in [14]. Moreover it is known [18, 12] that the moduli space of Nahm data is diffeomorphic to the moduli space of monopoles. In the case of $SU(2)$ is also known that this diffeomorphism is an isometry [17] but for other $SU(n+1)$ groups, while this is believed to be true, it has not yet been proved.

In summary the paper is as follows: In Section 2 I review the hyperkahler quotient construction applied to quaternionic vector spaces. In Section 3 I describe the infinite dimensional hyperkahler quotient that defines $\mathcal{N}$ the moduli space of $(1,\ldots,1)$ Nahm data and show it can be realised as a finite-dimensional hyperkahler quotient. This enables a rigourous definition of the metric on $\mathcal{N}$ as a hyperkahler quotient of a finite dimensional hyperkahler manifold. This is described in Section 4 and in Section 5 it is shown that the moduli space is isometric to a product

$$\mathcal{N} = \frac{\mathcal{N} \times \mathbb{R}^3 \times \mathbb{R}}{\mathbb{Z}}$$

where $\mathcal{N}_c$ is the space of centered Nahm data corresponding to strongly centered monopoles and $\mathbb{R}^3 \times \mathbb{R}$ is given a multiple of the standard metric. Finally in Section 6 we consider the metric on $\mathcal{N}_c$. The space $\mathcal{N}_c$ is just $\mathbb{H}^{n-2}$ where $\mathbb{H} = \mathbb{R}^4$ is quaternionic space. In the case of $SU(3)$ it is possible to give an explicit formula for the metric on this space [5, 13]; in the present case I use a result of Hitchin [5] to show that it has the same form as the metric in [14].
2 Hyperkaehler quotients of vector spaces.

A hyperkaehler manifold \([5]\) is a Riemannian manifold \((M, g)\) with three complex structures \(I, J, K\) which satisfy the quaternion algebra and are covariantly constant.

We need to consider from \([6]\) the hyperkaehler quotient of a hyperkaehler manifold by a group. For our purposes it is enough to consider the case when the manifold that is being quotiented is a vector space. Let \(V\) be a real vector space with three complex structures \(e_1 = I, e_2 = J, e_3 = K\) which satisfy the quaternion algebra. Assume also that \(V\) has an inner product \(\langle \cdot, \cdot \rangle\) which is preserved by each of the \(e_i\). Then \(V\) has three symplectic forms \(\omega_k\) defined by \(\omega_k(v, w) = \langle v, e_kw \rangle\). Because the tangent space at any point of \(V\) is canonically identified with \(V\) itself this makes \(V\) a hyperkaehler manifold.

Assume now that a group \(G\) acts freely on \(V\) in such a way that \(V/G\) is a manifold and \(V \to V/G\) is a principal \(G\) fibration. Assume further that the \(G\) action preserves the inner product on the tangent spaces of \(V\) and also commutes with the action of the \(e_i\). If \(\xi\) is an element of \(LG\), the Lie algebra of \(G\), it defines a vector field \(\iota(\xi)\) on \(V\). The moment map

\[ \mu: V \to \mathbb{R}^3 \otimes LG^* \]

of this group action is then defined by

\[ \mu_k(v) = \int_0^1 \omega_k(\iota(\xi), v)dt \]

\[ = \int_0^1 \langle \iota(\xi), e_k(v) \rangle dt \]  \hspace{1cm} (2.1)

The hyperkaehler quotient of \(V\) is the space \(\mu^{-1}(0)/G\). To see that this is a hyperkaehler manifold let \(\pi\) be the projection \(\pi: \mu^{-1}(0) \to \mu^{-1}(0)/G\). If \(x \in \mu^{-1}(0)/G\) choose \(\hat{x} \in \pi^{-1}(x)\). We can split the tangent space at \(\hat{x}\) into vertical directions tangent to the \(G\) action and horizontal directions which are orthogonal to the vertical directions. The horizontal directions are naturally identified with the tangent space to \(\mu^{-1}(0)/G\) at \(x\) and this enables us to define an inner product and a hyperkaehler structure on that tangent space. This construction is, in fact, independent of the choice of \(\hat{x}\) in \(\pi^{-1}(\pi(x))\) because of the \(G\) invariance. I refer the reader to \([\text{8}]\) for details.

3 The moduli space of Nahm data.

We are interested in \(SU(n+1)\) monopoles or more precisely their Nahm data. In the interests of brevity I will not review the theory of monopoles or
the relationship between monopoles and solutions of Nahm’s equations but refer the reader to [1] and references therein for the \( SU(2) \) theory and to [16, 12, 18] for the \( SU(n+1) \) theory. We will denote by \( \mathcal{N} \) the moduli space of Nahm data which is realised as follows. The Higgs field at infinity of the monopole has eigenvalues \( i\lambda_n, \ldots , i\lambda_0 \) where we assume that

\[ \lambda_n < \lambda_{n-1} < \cdots < \lambda_1 < \lambda_0. \]

In the monopole language this means we have maximal symmetry breaking at infinity.

Denote by \( \mathcal{A} \) the set of all pairs \((T, a)\) where \( a \in \mathbb{H}^{n-1} \) and \( T : [\lambda_n, \lambda_0] \to \mathbb{H} \) with the property that the restriction of \( T \) to each interval \( (\lambda_i, \lambda_{i-1}) \) is smooth and has a smooth extension to \([\lambda_i, \lambda_{i-1}]\). We denote this extension by

\[ T^i : [\lambda_i, \lambda_{i-1}] \to \mathbb{H}. \]

The map \( T \) itself is allowed to have discontinuities at the \( \lambda_i \). It is useful to think of the vector \( a = (a_1, \ldots , a^{n-1}) \) as a function on the set \( (\lambda_n, \ldots , \lambda_0) \) whose value at \( \lambda_i \) is just \( a^i \). We will consider the space \( \mathcal{A} \) as a left quaternionic vector space.

Denote by \( \mathcal{G} \) the group of all continuous maps \( g : [\lambda_n, \lambda_0] \to U(1) \) which are smooth on an open subinterval \( (\lambda_i, \lambda_{i-1}) \) and whose derivatives may be discontinuous at the points \( \lambda_i \) for \( i = 1, \ldots , n-1 \) but such that the restriction of \( g \) to \( (\lambda_i, \lambda_{i-1}) \) has a smooth extension to \([\lambda_i, \lambda_{i-1}]\). We require further that \( g(\lambda_n) = g(\lambda_0) = 1 \). We denote by \( g^i \) the extension to \([\lambda_i, \lambda_{i-1}]\) of the restriction of \( g \) to \((\lambda_i, \lambda_{i-1})\).

The group \( \mathcal{G} \) acts on the left of \( \mathcal{A} \) by

\[
(gT)^j = T^j + \frac{1}{i} \frac{dg^j}{g^j} \\
(ga)^j = a^j g(\lambda^j).
\]

Notice that by continuity \( g(\lambda_i) = g^j(\lambda_j) = g^{i+1}(\lambda_j) \).

We define an inner product on \( \mathcal{A} \) by

\[
\langle (T, a), (S, b) \rangle = \sum_{i=1}^n \int_{\lambda_{i-1}}^{\lambda_i} \text{Re}(T^i S^i) + \sum_{i=1}^{n-1} \text{Re}(a^i b^i).
\]

This inner product makes \( \mathcal{A} \) an (infinite-dimensional) hyperkaehler vector space. We want to consider its hyperkaehler quotient. It is easy to check that the group action preserves the hyperkaehler structure. It is not clear, because
of the infinite dimensionality, that the quotient is nicely behaved. We will avoid confronting this problem by showing that we can replace $A$ by a finite dimensional vector space and form the hyperkaehler quotient of that instead.

To define the moment maps for the action of $G$ we need to consider the infinitesimal action of the Lie algebra $L G$. The Lie algebra $L G$ is the set of all continuous maps $\xi: [\lambda_n, \lambda_0] \to R$ with $\xi(\lambda_n) = \xi(\lambda_0) = 0$ and whose derivative may jump at $\lambda_i$ for $i = 1, \ldots, n - 1$. The derivative has a smooth extension from $(\lambda_i, \lambda_{i-1})$ to $[\lambda_i, \lambda_{i-1}]$ and we denote this smooth extension by $\xi^i$. We fix our conventions by defining the exponentional map for the group $G$ to be $\xi \mapsto \exp(2\pi i \xi)$. The element $\xi \in L G$ then defines a vector field $\iota(\xi)$ on $A$ whose value at $(T, a)$ is

$$(\iota(\xi)(T, a))^j = (2\pi d\xi^j, 2\pi a^j i \xi^j).$$

Here and below we sometimes adopt the notation

$$X^j = (T^j, a^j)$$

to mean

$$X = ((T^1, \ldots, T^n), (a^1, \ldots, a^{n-1})).$$

We can now calculate the moment map from (2.1) and we find that $(T, a)$ is in the kernel of $\mu$ if and only if

$$\operatorname{Re}(dT^j) = 0$$

for each $j = 1, \ldots, n$ and

$$\operatorname{Im}(T^{j+1} - T^j) = \frac{1}{2} a^j i a^j$$

for each $j = 1, \ldots, n - 1$.

It is clear from these equations that to describe the hyperkaehler quotient $\mathcal{N}$ of $A$ by $G$ we could restrict our attention from $A$ to the subset of pairs $(T, a)$ where the imaginary part of $T$ is constant. If we do that and wish to still have a hyperkaehler structure then we will need to restrict attention to $T$ whose real part is also constant. Notice that if we start out with a $T$ which is real then by integrating starting at $\lambda_n$ we can construct a $g: [\lambda_n, \lambda_0] \to U(1)$ such that $gT = 0$ and satisfying all the conditions to be in $G$ except that we may not have $g(\lambda_0) = 1$. But in that case we can find an $h$ such that $dh$ is constant and $h(\lambda_0) = g(\lambda_0)^{-1}$. The composite $gh$ is in $G$ and $gT$ has constant real part. We conclude that every $(T, a)$ in $\mu^{-1}(0)$ can be gauge transformed so that $gT$ is constant.
4 The hyperkaehler quotient.

Denote by \( A_0 \) the set of all triples \((\tau, x, a)\) where \( \tau \in \mathbb{R}^n, x \in \text{Im}(\mathbb{H})^n \) and \( a \in \mathbb{H}^{n-1} \). We identify \( A_0 \) with a subset of \( A \) by mapping each \((\tau, x, a)\) to \((T = \tau + x, a)\) where we think of \( T \) as a step function on \([\lambda_n, \lambda_0]\) whose value on \([\lambda_i, \lambda_{i-1}]\) is \( T^i = \tau^i + x^i \). We shall identify \( x^i \in \text{Im}(\mathbb{H}) \) with the corresponding element of \( \mathbb{R}^3 \) and call it the location of the \( i \)-th monopole.

It follows from the discussion at the end of Section 2 that the hyperkaehler quotient of \( A_0 \) by \( G_0 \) is the same as the hyperkaehler quotient of \( A \) by \( G \) and hence yields \( \mathcal{N} \) the moduli space of Nahm data.

The space \( A_0 \) is a quaternionic vector space and has an inner product induced from \( A \) which is

\[
\langle (\tau, x, a), (\sigma, y, b) \rangle = \sum_{i=1}^n p_i \tau^i \sigma^i + \sum_{i=1}^n p_i \text{Re}(x^i \bar{y}^i) + \sum_{i=1}^n \text{Re}(a^i \bar{b}^i) \tag{4.1}
\]

where \( p_i = \lambda_i - \lambda_{i-1} \).

Consider the subgroup \( G_0 \subset G \) that fixes \( A_0 \). This is the group of all \( g \in G \) such that \( dg \) is a step function on \([\lambda_n, \lambda_0]\). That is each \( dg^i \) is a constant. Such a \( g \) can be written as

\[
g^i(s) = \exp\left(\frac{2i\pi}{p_j} ((W^j_+ - W^j_-) s + W^j_+ \lambda_j - W^j_- \lambda_{j-1})\right).
\]

Notice that \( g^i(\lambda_j) = \exp(2\pi i W^j_+) \) and \( g^i(\lambda_{j-1}) = \exp(2\pi i W^j_-) \) so that the condition for \( g \) to be continuous is that \( W^j_+ - W^j_- \) is an integer. The numbers \( W^j_+ \) and \( W^j_- \) are not uniquely determined by \( g \). They can be changed by adding to both of them the same integer.

The group \( G_0 \) acts on \( A_0 \) by

\[
g(\tau, x, a) = (g\tau, x, ga)
\]

where

\[
(g\tau)^i = \tau^i + \frac{2\pi}{p_i} (W^i_+ - W^i_-) \\
(ga)^i = a^i \exp(2\pi i W^i_+) = a^i \exp(2\pi i W^i_-)
\]

If \( \xi \in L G \) then the vector field \( \iota(\xi) \) it defines on \( A_0 \) is

\[
(\iota(\xi)(\tau, x, a))^j = \left( \frac{2\pi}{p_j} (\xi^j - \xi^{j-1}), 2\pi a^j \xi^j, 0 \right).
\]
The moment map $\mu$ for the action of $G_0$ on $A_0$ can be calculated from (2.1) but it is the restriction of that for $G$ on $A$ and hence we deduce that $(\tau, x, A) \in \mu^{-1}(0)$ if and only if

$$x^{j+1} - x^j = a^j i \bar{a}_j$$

for each $j = 1, \ldots, n - 1$.

Let $\hat{N} = \mu^{-1}(0)$ so the moduli space of Nahm data is $N = \hat{N}/G_0$.

5 The metric on monopoles.

By the Nahm transform [18, 12] the space $N$ is diffeomorphic to the space of monopoles of type $(1, 1, \ldots, 1)$. The monopole corresponding to the orbit of $(\tau, x, a)$ can be interpreted as a collection of $n$ particles, located at each of the points $x^j$ with phases $\exp(ip_j \tau^j)$. Following [13] we define the center of $\tau$ and $x$ by

$$\tau_c = \frac{\sum_{i=1}^n p_i \tau^i}{p}$$

and

$$x_c = \frac{\sum_{i=1}^n p_i x^i}{p},$$

where

$$p = \sum_{i=1}^n p_i.$$  

Define $\hat{N}_c$ to be the subset of $\hat{N}$ consisting of those $(\tau, x, a)$ with $\tau_c = 0$ and $x_c = 0$. We call this the space of centered monopoles. Define also

$$G_c = \{ g \mid \sum_{i=1}^n W^i_+ - W^{i-1}_- = 0 \}.$$  

This is the subgroup of $G$ which fixes $\hat{N}_c$. We define $\mathcal{N}_c = \hat{N}_c/G_c$.

We want to define an isomorphism:

$$\hat{N}/G_c \to \mathcal{N}_c \times \mathbb{R}^3 \times \mathbb{R}.$$  

(5.1)

To construct the isomorphism we first define for any $x \in \mathbb{R}^3$ and $\tau \in \mathbb{R}$ $\hat{x} \in (\mathbb{R}^3)^{n-1}$ and $\hat{\tau} \in \mathbb{R}^{n-1}$ by $\hat{x} = (x, x, \ldots, x)$ and $\hat{\tau} = (\tau, \tau, \ldots, \tau)$. Notice that $\hat{x}_c = x$ and $\tau_c = \tau$. So given a monopole $(\tau, x, a) \in \hat{N}$ we can center it by defining $(\tau - \hat{\tau}_c, x - \hat{x}_c, a) \in \hat{N}_c$. The map in (5.1) is then defined to send $(\tau, x, a)$ to the pair $((\tau - \hat{\tau}_c, x - \hat{x}_c, a), (\tau_c, x_c))$ consisting of the corresponding...
centered monopole and the center of the monopole. This map has inverse
given by \(((\tau,x,a),(s,y))\mapsto(\tau+\hat{s},x+\hat{y},a)\).

The spaces \(\mathcal{N}/\mathcal{G}_c\) and \(\mathcal{N}_c\) inherit inner products by the process described
at the end of Section 2. It is straightforward to calculate that the isomorphism \((5.1)\) is an isometry if we give \(\mathbb{R}^3\times\mathbb{R}\) the standard metric multiplied
by a factor of \(p\).

Finally notice that \(\sum_{i=1}^{n} W_i^+ - W_i^{-1} = \sum_{i=1}^{n-1} W_i^+ - W_i\) is an integer so that \(\mathcal{G}/\mathcal{G}_c\) is isomorphic to \(\mathbb{Z}\). We conclude that there is an isometry
\[\mathcal{N} = \frac{\mathcal{N}_c \times \mathbb{R}^3 \times \mathbb{R}}{\mathbb{Z}}.\]

### 6 The metric on centered monopoles.

If \((\tau,x,a)\) is in \(\mathcal{N}_c\) then the vector \(x\) is determined by the equations \(x^{j+1} - x^j = a^j\hat{a}^j\). So the triple \((\tau,x,a)\) is determined by the pair \((\tau,a)\). It is straightforward to show that the orbit of \((\tau,x,a)\) under \(\mathcal{G}_c\) contains exactly
one triple of the form \((0,x',a')\). It follows that \(\mathcal{N}_c\) has the topology of \(\mathbb{H}^{n-1}\).

In the case that \(n = 2\) Connell calculated explicitly the hyperkahler quotient metric on \(\mathbb{H}\). In the case at hand that calculation is more involved and it is simpler to use an approach due to Hitchin [5]. The \(n-1\) dimensional
torus \(T^{n-1} = U(1)^{n-1}\) acts on the space \(\mathcal{N}_c\) preserving the hyperkaehler metric by rotating each of the \(a^i\). The moment map for the \(i\)th of these actions is given by \(\mu_i(\tau,x,a) = 2\pi(x^{i+1} - x^i)\) for each \(i = 1, \ldots, n - 1\). This action is free if none of the \(a^i\) vanish. Let \(\mathcal{N}_c'\) be the set of \((\tau,x,a)\) such that noe of the \(a^i\) vanish. Denote by \(M\) the image of \(\mathcal{N}_c'\) in \((\mathbb{R}^3)^{n-1}\) under the moment map. The moment map \(\mathcal{N}_c' \to M\) realises \(\mathcal{N}_c'\) as a \(T^{n-1}\) bundle over \(M\). The inner product on \(\mathcal{N}_c\) allows us to define a horizontal subspace orthogonal to the \(T^{n-1}\) action at each point of \(\mathcal{N}_c'\) and hence we can define a connection on \(\mathcal{N}_c' \to M\). This defines a one-form \(\alpha = (\alpha^1, \ldots, \alpha^{n-1})\) corresponding to projecting onto the vertical subspace. By generalising the calculation of Hitchin
in Section IV.4 of [5] it is possible to show that the metric on \(\mathcal{N}_c'\) must have the form
\[g = \sum_{i,j=1}^{n-1} K_{ij}^{-1} \sum_{a=1}^{3} d\mu_i^a d\mu_j^a + \sum_{i,j=1}^{n-1} K_{ij} \alpha^i \alpha^j \] (6.1)
for some matrix valued function \(K_{ij}\) which is constant in the torus directions.
If \(\eta_i\) are the generators of the torus action then Hitchin’s result gives
\[K_{ij} = g(\eta_i, \eta_j).\]
We wish to now calculate the $K_{ij}$.

To calculate the $\eta_i$ we have to split them into a vector $\iota(\xi_i)$ in the direction of the $G_c$ action and an orthogonal vector $\hat{\eta}_i = \eta_i - \iota(\xi_i)$. Then we have that

$$K_{ij} = g(\eta_i, \eta_j) = \langle \hat{\eta}_i, \hat{\eta}_j \rangle$$

where $\langle \ , \ \rangle$ is the inner product defined in (4.1). Using the orthogonality we deduce that

$$K_{ij} = \langle \eta_i, \eta_j \rangle - \langle \iota(\xi_i), \eta_j \rangle.$$

The condition that defines the $\iota(\xi_i)$ is the requirement that $\eta_i - \iota(\xi_i)$ be horizontal, that is:

$$\langle \eta_i - \iota(\xi_i), \iota(\rho) \rangle = 0$$

for all $\rho \in LG_c$. Expanding this we have that

$$\langle \eta_i, \iota(\rho) \rangle = \langle \iota(\xi_i), \iota(\rho) \rangle.$$

The vector $\eta_l$ is

$$\eta_l(\tau, x, T)^j = (0, 0, \delta_{lj}2\pi ia_l^l)$$

and hence we have

$$\langle \eta_l, \iota(\rho) \rangle = 2\pi p_l |a_l|^2.$$

The other inner product is

$$\langle \iota(\xi_i), \iota(\rho) \rangle = 4\pi^2 \sum_{k=1}^{n-1} \frac{1}{p_k} (\xi^k_l - \xi^{k-1}_l)(\rho^k - \rho^k) + 4\pi^2 \sum_{k=1}^{n-1} |a^k|^2 \xi^k_l \rho^k$$

(6.2)

$$= 4\pi^2 \sum_{k=1}^{n} \rho^k \left( \frac{1}{p_k} (\xi^k_l - \xi^{k-1}_l) - \frac{1}{p_{k+1}} (\xi^{k+1}_l - \xi^k_l) + |a^k|^2 \xi^k_l \right).$$

(6.3)

If we equate each coefficient of $\rho^k$ in (6.3) to zero we can put the defining condition for $\iota(\xi_i)$ into the following matrix form. We let $\xi = (\xi^k_l)$ be a matrix with rows labelled by $l$ and columns labelled by $k$. We denote by $X$ the diagonal matrix whose $l$th diagonal entry is $|a^l|^2$. Finally we denote by $P$ the following matrix:

$$P = \left( \begin{array}{cccccccc}
\frac{1}{p_1} + \frac{1}{p_2} & -\frac{1}{p_2} & 0 & 0 & \ldots & 0 \\
-\frac{1}{p_2} & \frac{1}{p_2} + \frac{1}{p_3} & -\frac{1}{p_3} & 0 & \ldots & 0 \\
0 & -\frac{1}{p_3} & \frac{1}{p_3} + \frac{1}{p_4} & -\frac{1}{p_4} & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & 0 \\
0 & 0 & 0 & 0 & \ldots & \frac{1}{p_n} & \frac{1}{p_{n-1}} + \frac{1}{p_n}
\end{array} \right)$$

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Then the condition satisfied by $\xi$ becomes the matrix equation:

$$2\pi \xi (P + X) = X$$

and the matrix we are trying to find, $K$, satisfies

$$K = (1 - 2\pi \xi) X.$$ 

It follows that

$$K^{-1} = P^{-1} + X^{-1}.$$ 

We conclude that the metric on $N_c$ is of the form

$$g = \sum_{i,j=1}^{n-1} \frac{1}{4\pi^2} (P^{-1} + X^{-1})_{ij} \sum_{a=1}^{3} dy^a_i dy^a_j + \sum_{i,j=1}^{n-1} (P^{-1} + X^{-1})_{ij} \alpha^i \alpha^j \quad (6.4)$$

where $y_i = x^{i+1} - x^i = (1/2\pi) \mu^i$.

To finish we want to compare our result (6.4) to formula (7.5) in [14]. Except for rescalings the only question is to show that their matrix $\mu_{ij}$ is the matrix $P^{-1}$. To do this we have to calculate $\mu_{ij}$ in the manner they suggest. We reintroduce the center of mass co-ordinate $x_c$. This means we replace $P$ in (6.4) by $\hat{P}$ where

$$\hat{P}^{-1} = \begin{pmatrix} p_1 & 0 \\ 0 & P^{-1} \end{pmatrix}.$$ 

Then we consider the effect of the co-ordinate change from the co-ordinates $x^i$ to the co-ordinates $(x_c, y^i)$. This is the result of applying the linear transformation

$$X = \begin{pmatrix} p_1 & p_2 & p_3 & \ldots & p_{n-1} & p_n \\ 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{pmatrix}$$

to the co-ordinates $x^i$. Hence the matrix of the metric in terms of the co-ordinates $x^i$ is given by $X^t P^{-1} X$. We leave it to the reader to check that this is the diagonal matrix with entries $p_1, \ldots, p_n$ which agrees with the definition of the constant term in $M_{ii}$ in [14] (their $m_i$ is our $p_i$). So we conclude that the $\mu_{ij}$ in [14] is indeed $P^{-1}$. The metric on $N_c$ is therefore the same asymptotically as the metric on the monopole moduli space calculated in [14].

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