Adaptive Blind Multiuser Detection over Flat Fast Fading Channels using Particle Filtering

Yufei Huang ∗, Jianqiu (Michelle) Zhang †, Isabel Tienda Luna ‡, Petar M Djurić §, Diego Pablo Ruiz Padillo ‡

Abstract—In this paper, we propose a method for blind multiuser detection (MUD) in synchronous systems over flat and fast Rayleigh fading channels. We adopt an autoregressive-moving-average (ARMA) process to model the temporal correlation of the channels. Based on the ARMA process, we propose a novel time-observation state space model (TOSSM) that describes the dynamics of the addressed multiuser system. The TOSSM allows an MUD with natural blending of low complexity particle filtering (PF) and mixture Kalman filtering (for channel estimation). We further propose to use a more efficient PF algorithm known as the stochastic M-algorithm (SMA), which, although having lower complexity than the generic PF implementation, maintains comparable performance.

I. INTRODUCTION

When multiuser detection (MUD) was introduced in the eighties, it received a great deal of attention and since then, numerous detectors have been proposed in the literature including the decorrelating detector, and the minimum mean square error (MMSE) detector [1]. In practice, the distortion in signal strength which is due to the time varying fading channels must be estimated while performing MUD. Recently, for this purpose various blind schemes have been proposed. Blind MUD methods are bandwidth efficient and the approaches proposed include the recursive least square (RLS) method [2], subspace based algorithms [3], expectation-maximization [4], and genetic algorithms [5]. However, most of the approaches cited above assume slow or quasi static fading channels.

In this paper, we focus on blind MUD for fast flat Rayleigh fading channels and in synchronous systems. In particular, we assume to know a priori the second order statistics of the underlying channel, based on which a mathematical tractable approximation using autoregressive-moving-average (ARMA) model is adopted. The approximation enables a dynamic state space modeling (DSSM) of the problem, which lends itself naturally to a Kalman filtering related detection solution. Recently, the combined (mixture) Kalman filtering and sequential importance sampling (particle filtering) algorithms have been applied to blind detection of convolutional codes [6], space-time trellis codes [7], and blind MUD [8] over fading channels. The mixture Kalman filtering (MKF) approach is shown to greatly reduce the error propagation of the decision direct implementations and thus exhibits considerable performance improvement. However, in the proposed use of MKF to blind MUD in [8], particle filtering (PF) was mainly intended for channel tracking and the embedded MUD at a symbol interval is achieved by an optimum detector, which has exponential complexity with the number of users. Consequently, the proposed MKF algorithm becomes prohibitively complex even for systems with moderate number of users.

In this paper, unlike all existing Kalman filtering detectors, we take a completely different viewpoint to multiuser systems and propose a novel time-observation state space model (TOSSM). Even though the TOSSM is equivalent to the common DSSM, it allows the PF based multiuser detection to be naturally blended with the mixture Kalman filtering for channel estimation. The new mixture Kalman filtering algorithm samples one user at a time and therefore permits efficient implementation. We further propose to use a more efficient PF algorithm known as the stochastic M-algorithm (SMA), which has shown to attain additional complexity reduction over the generic PF implementation yet maintain comparable performance.

The rest of the paper is organized as follows: In Section II, the problem of blind MUD is formulated. In Section III, a novel TOSSM is described and in IV, the optimum solution is discussed. A particle filtering and an SMA solutions are proposed in Section V and VI, respectively. The simulation results are presented in Section VII. Section VIII contains some concluding remarks.

II. PROBLEM FORMULATION

Consider a synchronous CDMA system with a processing gain C and K users. At the n-th symbol interval, the set of matched filter outputs at the receiver can be represented in vector-matrix form according to

\[ y_n = RA_n b_n + u_n \]  

where \( y_n = [y_{n,1}, \ldots, y_{n,K}]^T \) with \((\cdot)^T\) denoting matrix transposition, \( R \) is the crosscorrelation matrix whose element \( r_{k_1,k_2} \) represents the crosscorrelation between the signature waveforms of the \( k_1 \)th and the \( k_2 \)th user, \( A_n = \text{diag}\{a_{n,1}, \ldots, a_{n,K}\} \) is the diagonal matrix of the channel state information, \( b_n = [b_{n,1}, \ldots, b_{n,K}]^T \) is the antipodally
modulated user data vector, and $u_n$ is the complex Gaussian noise vector with covariance matrix equal to $\sigma^2I$. The channel for each user is considered as Rayleigh flat fading channel and ARMA processes can be adopted to model its time correlation [9]. Thus, $a_{n,k}$, which we refer to as the channel state information (CSI) of the $k$th user can be represented by an ARMA($r_1, r_2$) model as

$$a_{n,k} + \dot{\phi}_{k,1} a_{n-1,k} + \cdots + \phi_{k,r_1} a_{n-r_1,k} = \rho_{k,0} v_{n,k} + \cdots + \rho_{k,r_2} v_{n-r_2,k}$$

(2)

where $v_{n,k}$ is an i.i.d. random complex Gaussian process, and $\{\phi_{k,1}, \ldots, \phi_{k,r_1}\}$ and $\{\rho_{k,1}, \cdots, \rho_{k,r_2}\}$ are the AR and MA coefficients of the model. We assume that we know a priori the second order statistics of the underlying fading channel, and therefore the coefficients of the ARMA model can be pre-computed by matching the spectrum of (2) with the fading channel process. For convenience, we assume $r_1 = r_2 = r$; otherwise zeros can be added to the coefficients to make the orders equal. Our objective is to perform sequential symbol detection without knowing the CSI $a_{n,k}$.

III. TIME-OBSERVATION STATE SPACE MODELING

The state space representation of CDMA systems in flat fading channels can be found in existing literature [10]. It can be expressed as

$$\begin{align*}
\mathbf{h}_{k,n} &= \mathbf{Q}_k \mathbf{h}_{k,n-1} + \mathbf{g} v_{k,n} \quad \forall k \\
\mathbf{a}_{k,n} &= \rho_k^T \mathbf{h}_{k,n} \quad \forall k \\
\mathbf{y}_n &= \mathbf{R} \mathbf{a}_n \mathbf{b}_n + \mathbf{u}_n 
\end{align*}$$

(3)

where $\mathbf{h}_{k,n} = [h_{n,k} \cdots h_{n-r,k}]$ is a $(r+1) \times 1$ channel state vector, $\rho_k = [\rho_{k,0} \cdots \rho_{k,r}]^T$,

$$Q_k = \begin{bmatrix}
-\phi_{k,1} & \cdots & -\phi_{k,r} & 0 \\
1 & \cdots & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 1 & 0 
\end{bmatrix}, \quad \text{and } \mathbf{g} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. $$

In (3), $\mathbf{h}_{k,n} \forall k$ and $\mathbf{b}_n$ are the unknowns to be estimated. Note that the observation $\mathbf{y}_n$ is not linear in $\mathbf{h}_{k,n} \forall k$ and $\mathbf{b}_n$, and therefore the Kalman filter cannot provide the optimum solution. In fact, the optimum solution can be obtained by a so-called splitting Kalman filter, where, at time $n$, $2^n$ Kalman filters are required. The complexity of the splitting Kalman filter is exponential with both time and users and thus computational prohibited. Instead, particle filtering can be used to obtain good approximations of the optimum solution with reduced complexity. PF algorithms on (3) incorporated with Kalman filtering were proposed in [8]. However, as mentioned in the introduction, due to the structure of (3), particles of $\mathbf{b}_n$ must be sampled jointly, and the complexity becomes exponential with the number of users. The prohibitive complexity on large user systems implies that this PF algorithm is unsuited for practical applications. To circumvent this difficulty, in the following we introduce a time-observation state space model (TOSSM) for the system.

In developing the TOSSM, we start with the Cholesky factorization of the cross-correlation matrix $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{F}^T \mathbf{F},$$

where $\mathbf{F}$ is a uniquely defined $K \times K$ lower triangular matrix. Now, right multiplying $(\mathbf{F}^T)^{-1}$ with the matched filter output, we obtain

$$\mathbf{y}_n = (\mathbf{F}^T)^{-1} \mathbf{y}_n = \mathbf{F} \mathbf{a}_n \mathbf{b}_n + \mathbf{u}_n = \mathbf{F} \mathbf{B}_n \mathbf{a}_n + \mathbf{u}_n$$

(4)

where $\mathbf{B}_n = \text{diag} (b_{n,1}, \ldots, b_{n,K})$ is the diagonal user data matrix, and $\mathbf{a}_n = [a_{n,1}, \ldots, a_{n,K}]$ is the $K \times 1$ vector of CSI. Since the covariance matrix of $\tilde{\mathbf{u}}_n$ becomes $E[\tilde{\mathbf{u}}_n \tilde{\mathbf{u}}_n^T] = \sigma^2 \mathbf{F}^{-1} \mathbf{R} \mathbf{F}^{-1} = \sigma^2 \mathbf{I}$, where $\mathbf{I}$ is an identity matrix, $\mathbf{y}_n$ is called the whitened matched filter (WMF) output. Next, define a tall channel vector of $K(r+1) \times 1$ as $\mathbf{h}_n = [\mathbf{h}_{1,n} \cdots \mathbf{h}_{K,n}]^T$ and the channel transition becomes

$$\mathbf{h}_n = \mathbf{Q} \mathbf{h}_{n-1} + \mathbf{G} v_n$$

(5)

where $\mathbf{Q} = \text{diag}(Q_1, \cdots, Q_K)$ and $\mathbf{G} = \text{diag}(g_1, \cdots, g_K)$ are $K(r+1) \times K(r+1) \times K$ matrices. We can thus express $\mathbf{a}_n$ by $\mathbf{h}_n$ in a compact form by

$$\mathbf{a}_n = \mathbf{P} \mathbf{h}_n$$

(6)

where $\mathbf{P} = \text{diag}(\rho_1, \cdots, \rho_K)$ is of dimension $K \times K(r+1)$. Now by replacing $\mathbf{a}_n$ in (4) by (6), we have

$$\mathbf{y}_n = \mathbf{F} \mathbf{B}_n \mathbf{P} \mathbf{h}_n + \mathbf{u}_n.$$  

(7)

If we denote the $k$th row of $\mathbf{F}$ by $f_{k}^T$, the $k$-th WMF output $\bar{y}_n$ can be written as

$$\bar{y}_n = f_k^T \mathbf{B}_n \mathbf{P} \mathbf{h}_n + \bar{u}_{n,k},$$

(8)

where $\bar{u}_{n,k}$ is the $k$-th element of $\bar{\mathbf{u}}_n$. Now, instead of considering the system evolving only along time, we imagine a system progressing alternately along the path of time and the WMF observations $\bar{y}_{n,k}$. The concept is further illustrated in Figure 1. To describe this new system, we must collapse the time index $n$ and the observation index $k$ into one time-observation index $l$, where $l = (n-1)K + k$. This conversion is reversible, or in other words, we can also calculate $k$ and $n$ from $l$ by $k = \text{mod}(l, K)$ and $n = (l-k)/K + 1$ where mod$(k, K)$ is the $k$ modulo $K$ operation. In the following description of the TOSSM indexed by $l$, all $k$ and $n$ are assumed to be obtained from the corresponding $l$. Now, we introduce a $K \times K$ auxiliary matrix $\mathbf{B}_l = \text{diag}(b_{l,1}, \ldots, b_{l,K}, 0, \cdots, 0)$. The state space representation for the new time-observation system indexed by $l$ can be then constructed as

$$\mathbf{h}_l = \begin{cases}
\mathbf{Q} \mathbf{h}_{l-1} + \mathbf{G} v_l & \text{if } k = 1 \\
\mathbf{h}_{l-1} & \text{if } k \neq 1
\end{cases}$$

(9)

and we call (9) the TOSSM. Note that (9) and (3) describe the same system. There are, however, key differences between
the two models. Unlike (3), the state transitions of $I_1$ in the
TOSSM are time (or index) varying, that is, at different $l,$
different transition is applied. Specifically, when $k = 1,$ or
equivalently, $n$ increases by 1, $I_1$ updates according to the
ARMA channel model, and otherwise when $k \neq 1,$ and $n$
remains unchanged from $l - 1,$ $I_1$ is assumed to be static.
Additionally, in the TOSSM, the number of the unknown user
bits changes with $l,$ and especially, only one new unknown
signal $b_{n,k}$ is included each time when $l$ is incremented by
one. Therefore, if we assume perfect detection at $l - 1,$ i.e.,
$b_{n,1}, \ldots, b_{n,k-1}$ are known exactly, then there is only one
unknown user bit to be detected. Note that in the conventional
DSSM (3), $K$ unknown users bits need to be detected all
together as the system evolves to time $n.$ This is the key of
the model that leads to efficient particle filtering solutions.

We, however, want to stress that the decision on $b_{n,k}$ (except
$k = K$) is not finalized at $l.$ Since the observations from $y_{l+1}$
up to $y_{l+r},$ with $r = K - k$ all contain information about $b_{n,k},$
the final decision is reached only at $l + r,$ or in general, when
$k = K.$

We, therefore, must resort to suboptimum solutions with
manageable complexity. One choice is particle filtering.

V. PARTICLE FILTERING DETECTOR FOR BLIND MUD

Particle filtering belongs to the family of Monte Carlo
sampling which aims at using samples to approximate posterior
distribution. However, particle filtering distinguishes itself
by employing a sequential importance sampling scheme, and
in particular, it is designed for nonlinear and non-Gaussian
systems described through state space modeling such as in
our problem.

In the context of the proposed problem, when $y_N,$ or
equivalently $\bar{y}_N,$ is observed at time $N,$ the objective of
particle filtering is to draw, say, $J$ weighted random samples
$\{b_{1:N}^{(j)}, w_{N,K}^{(j)}\}$ from
$p(b_{1:N}|\bar{y}_{1:N^K}),$ where $w_{N,K}^{(j)}$ is the weight
of the $j$th sample $b_{1:N}^{(j)}.$ With the samples, $p(b_{1:N}|\bar{y}_{1:N^K})$ can be approximated by

$$p(b_{1:N}|\bar{y}_{1:N^K}) \approx \sum_{j=1}^{J} w_{N,K}^{(j)} \prod_{l=1}^{N,K} \delta(b_{n,k} - b_{n,k}^{(j)})$$

and hence the MPM solution of $b$ by a simple weighted
summation as

$$\hat{b}_{N,K}^{MPM} \approx \text{sgn} \left( \sum_{j=1}^{J} w_{N,K}^{(j)} b_{N,K}^{(j)} \right)$$

for $k = 1, \ldots, K.$ By the law of large numbers, the
approximation will converge to the true MPM solution with
the increase of the number of samples $J.$ If these samples
are taken directly from the posterior distribution, then all the
samples have equal weights. However, direct sampling from
$p(b_{1:N}|\bar{y}_{1:N^K})$ is prohibited since all possible combinations
of $b_{1:N}$ must be evaluated on $p(b_{1:N}|\bar{y}_{1:N^K}),$ which again
requires $2^{NK}$ Kalman filters. To circumvent the difficulty,
importance sampling is performed where samples are taken
from a proposal importance function $\pi(b_{1:N}|\bar{y}_{1:N^K})$ and
weighted according to

$$w_{N,K}^{(j)} = \frac{p(b_{1:N}^{(j)}|\bar{y}_{1:N^K})}{\pi(b_{1:N}^{(j)}|\bar{y}_{1:N^K})}, \forall j.$$ 

Notice that $\pi(b_{1:N}|\bar{y}_{1:N^K})$ is a very high dimensional
distribution and it is burdensome to sample the variables and
calculate the weights altogether. Fortunately, the TOSSM
allows a Markovian factorization on the posterior distribution
as

$$p(b_{1:N}, \bar{y}_{1:N^K}) \propto p(\bar{y}_{NK}|b_{1:N}, \bar{y}_{1:N^K-1}) p(b_{N,K})$$

$$\times p(b_{N,K}|b_{1:N-1}, \bar{y}_{1:N^K-1})$$

$$= p(\bar{y}_{NK}|b_{1:N}, \bar{y}_{1:N^K-1}) \times p(b_{N,K}|b_{1:N-1}, \bar{y}_{1:N^K-1}).$$

Then, if we choose the importance distribution as

$$\pi(b_{1:N}|\bar{y}_{1:N^K}) = p(b_{N,K}|b_{1:N-1}, \bar{y}_{1:N^K-1}) \times p(b_{1:N-1}|\bar{y}_{1:N^K-1})$$

IV. OPTIMUM BAYESIAN BLIND DETECTION

In a Bayesian framework, the optimum decision on $b_N$
can be obtained by the marginalized posterior mode (MPM)
criterion, which is expressed as

$$\hat{b}_{N,K}^{MPM} = \text{sgn} \left( \sum_{b_{N} \in \{1, -1\}^K} b_{N,K} \pi(b_{1:N}|\bar{y}_{1:N^K}) \right)$$

(10)

where $p(b_{1:N}|\bar{y}_{1:N^K})$ is the posterior distribution which is essential for computing (10) and the subscript $1:N^K$ denotes a collection of the variable indexed from 1 to $NK,$ e.g.,
$\bar{y}_{1:N^K} = \{\bar{y}_1, \ldots, \bar{y}_{NK}\}.$ Notice that the posterior distribution $p(b_{1:N}|\bar{y}_{1:N^K})$ is independent of $b_{1:(N-1)},$ i.e., the bits transmitted prior to time $n.$ Further, the marginalization in (10) suggests that $\hat{b}_{N,K}^{MPM}$ is also independent of other users’ bits transmitted at $n.$ Therefore, the MPM solution is immune to decision errors on $b_{1:(N-1)}$ and other users’ bits transmitted at $n.$ Nevertheless, one can show that to obtain MPM solution (10), totally $2^{nK}$ Kalman filters are required, which has a complexity that is exponentially increasing with both time $n$ and the number of users $K.$ The MPM solution is apparently a formidable task not possible for real applications.
the weight can be calculated by

\[
w_{jk}^{(j)} = \frac{p(\tilde{y}_{NK}|\bar{b}_{1,N},\tilde{y}_{j:NK-1})p(b_{j:N,K}^{(j)})}{p(b_{j:N,K-1},b_{j:N-1}^{(j)}|\tilde{y}_{j:NK-1})}\times\pi(b_{j:N,K-1},b_{j:N-1}^{(j)}|\tilde{y}_{j:NK-1})
\]

\[
= \frac{p(\tilde{y}_{NK}|b_{j:N,K}^{(j)},\tilde{y}_{j:NK-1})p(b_{j,K}^{(j)})}{p(b_{j,N,K-1},b_{j,N-1}^{(j)}|\tilde{y}_{j:NK-1})}
\]

\[
= \sum_{b_{NK}} p(\tilde{y}_{NK}|b_{j,N},\tilde{y}_{j:NK-1})w_{j,NK-1}^{(j)}
\]

\[
= w_{j,NK-1}^{(j)}w_{jk}^{(j)}
\]

(16)

where \(p_{K,N-1}^{(j)}\) is the incremental weight. Examining (15) and (16), we find that given \(w_{j,NK-1}^{(j)}\) and \(p(b_{j,N,K-1},b_{j,N-1}^{(j)}|\tilde{y}_{j:NK-1})\), the importance function (15) and the weights (16) are known exactly as long as \(p(\tilde{y}_{NK}|b_{j:N,K},\tilde{y}_{j:NK-1})\) can be derived. In fact, \(p(\tilde{y}_{NK}|b_{j:N},\tilde{y}_{j:NK-1})\) can be calculated through the Kalman filter [9]. We can therefore obtain samples and weights using a recursive algorithm. To put the idea in concrete procedure, we assume that at \(l-1\), we have obtained from a previous recursion the trajectories (samples) \(\{b_{l-1}^{(j)}\}_{j=1}^{J}\) appropriately weighted with the weights \(\{w_{l-1}^{(j)}\}_{j=1}^{J}\). Using the recent observations \(\tilde{y}_{l}\), we update the trajectories and weights as follows:

**Algorithm: Particle filtering detector (PFD)**

- For \(j = 1\) to \(J\),

1) **Predictive step:**

- Calculate \(\mu_l^{(j)} = \begin{cases} \hat{Q}_l^{(j)} & \text{if } k = 1 \\ \hat{\eta}_l^{(j)} & \text{if } k \neq 1 \end{cases} \) and \(\Sigma_l^{(j)} = \begin{cases} Q\Sigma_l^{(j)} + \sigma^2G^T & \text{if } k = 1 \\ \Sigma_l^{(j)} & \text{if } k \neq 1 \end{cases} \)

2) **Sampling step:**

   a) For \(i = 1\) and \(-1\), calculate

   \[m_l^{(j)}(i) = c_l^{(j)}(i)\mu_l^{(j)} \text{ and } c_l^{(j)}(i) = c_l(i)\Sigma_l^{(j)} \]

   \[c_l^{(j)}(i) = \begin{cases} f_l^H & \text{if } l = 1 \\ f_l & \text{if } l \neq 1 \end{cases} \]

   \[\lambda_l^{(j)}(i) = \frac{n}{\sum_{i \in [-1,1]} \lambda_l^{(j)}(i)} \]

   b) Sample \(m \in \{-1,1\}\) with probability proportional to \(\lambda_l^{(j)}(i)\) \forall i.

   c) Set \(b_l^{(j)} = m\).

   d) Calculate \(\mu_l^{(j)} = \sum_{i \in [-1,1]} \lambda_l^{(j)}(i)\) and the unnormalized weight \(w_l^{(j)} = \mu_l^{(j)} w_{l-1}^{(j)}\).

3) **Updating step:**

- \(K_l^{(j)} = \Sigma_l^{(j)} c_l^{(j)}(m) / c_l^{(j)}(m)\)
- \(\eta_l^{(j)} = \mu_l^{(j)} + K_l^{(j)}(\tilde{y}_l - c_l^{(j)}(m)\mu_l^{(j)})\)

- \(\hat{\Xi}_l^{(j)} = (I - K_l^{(j)}c_l^{(j)}(m))\Sigma_l^{(j)}\)

- Form the new trajectories \(\{b_l^{(j)}, b_{l+1}^{(j)}\}_{j=1}^{J}\).

- Normalize the weight as \(w_l^{(j)} = \hat{w}_l^{(j)} / \sum_{j=1}^{J} \hat{w}_l^{(j)}\).

This process of recursively obtaining particles is called particle filtering. After each recursion, the mean \(\hat{\eta}_l^{(j)}\) and covariance vectors \(\hat{\Xi}_l^{(j)}\) are passed on to the next recursion. From (12), we also see that to calculate all the elements of \(\{b_N\}_{M,N}^{PM, M} w_{j,NK}\) is required. Therefore the decision on all the elements can only be made after recursion \(l = KN\) and the particles for \(b_{N,k}\) for \(k = 1,2,\cdots,K-1\) must be stored.

In the above derivation of particle filtering, the adopted importance function is known as optimum since it minimizes the variance of the weights. The above particle filtering procedure suffers from particle impoverishment, i.e., after several recursion, some weights of the samples become negligible and stop contributing to the overall evaluation. To prevent it, we insert a resampling step [6] after every fixed recursion. Particularly, during the resampling at recursion \(l\), the particles for \(b_{N,k}\), the mean vectors and covariance matrices must be treated as a set in the resampling process.

**VI. STOCHASTIC M-DETECTOR FOR BLIND MUD**

Recently, a very efficient particle filtering algorithm called stochastic M-algorithm (SMA) was proposed in [11] for problems with discrete unknowns. SMA can provide similar performance as generic particle filtering but with much reduced complexity. SMA can be considered as a particle filtering algorithm with the discrete Delta functions as importance functions. In addition, each trajectory produces two samples (-1 and 1) for the binary case rather than one sample as in the generic PF. A key feature with SMA is that no two trajectories are identical, which is however rarely true with the generic PF. As a result, the SMA can provide more sample diversities with less trajectories than the generic PF. Nonetheless, notice that the number of trajectories doubles after each sampling and therefore a selection step is required to avoid the exponential increase of trajectories. Here, we use the optimal resampling algorithm [12] since it is a sampling-without-replacement algorithm and does not produce replicates of the same trajectories, the feature that is required by SMA. The SMA for the problem concerned at the \(l\)-th recursion is outlined as follows:

**Algorithm: Stochastic M detector (SMD)**

- **Trajectory expansion**

  For \(j = 1\) to \(J\)

  - Perform Predictive step in Algorithm PFD;
  - Perform 2.a in Algorithm PFD;
  - Set \(b_{l-1}^{(j-1)} = 1\) and calculate the the weight by \(w_{l-1}^{(j-1)} = \lambda_l^{(j)}(1)w_{l-1}^{(j-1)}\);
  - Set \(b_{l-1}^{(j)} = -1\) and calculate the weight by \(w_{l-1}^{(j)} = \lambda_l^{(j)}(-1)w_{l-1}^{(j)}\);
  - Form 2\(J\) new trajectories by setting \(b_{l}^{(j)} = \{b_{l-1}^{(j)}\}_{j=1}^{J}\) and \(b_{l}^{(j)} = \{b_{l-1}^{(j)}\}_{j=1}^{J}\).
The structure of the SMA resembles the popular M-algorithm. However, since the SMA is still a PF algorithm, it can provide probability information about the unknowns and thus can be applied to iterative MUD of coded system.

VII. SIMULATION RESULTS

In this section, the bit error rate (BER) performance of the proposed PFDs and SMDs are studied through experiments. In all the experiments, the transmitted signal was differential BPSK modulated. The number of users was 15. For the PFDs, 151 trajectories were maintained, whereas 4 and 32 trajectories were tested for SMDs. Further, an AR model was adopted for the fading process, which was normalized to have a unit power, and thus the signal-to-noise ratio (SNR) was obtained by $10 \log (1/\sigma^2)$.

In figure 2, we provide the BER vs. SNR for the different algorithms on a scenario of $\Omega_d = 0.03$. The Genie-aided detector is included as a lower bound. We notice that the PFDs and SMDs with 32 trajectories are of the same order of magnitude as that of the Genie-aided detector at low SNR (less than 30 dB). On the other hand, the results obtained by the SMDs with 4 and 32 trajectories are very close, especially after 20 dB, and comparable to that of the PFD. The SMD with 4 trajectories is obviously more favorable since it requires only about 1/35 of complexity of the PFD. As a final note, the PFD and SMDs achieve about 7dB gain over the decision directed detectors at $10^{-3}$ BER. In figure 3, we provide the BER vs. SNR performance for a higher Doppler frequency of $\Omega_d = 0.05$. Similar observations can be drawn as for the previous case even though the overall performance of the detectors is worse, which is reasonable considering that the channels are fading faster.

VIII. CONCLUSION

In this paper, we proposed to solve blind MUD over flat fast fading channels. We constructed a novel time-observation state space model, based on which efficient particle filtering and stochastic M detectors were proposed. Particularly, the detectors based on the SMS demonstrated greater potential than those using generic PF. The former can provide comparable performance as the latter but with much smaller complexity.

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