A Fokker-Planck Code for Laser-Produced Plasmas

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Abstract. A Fokker-Planck code is developed based upon Epperlein’s scheme [Laser Part. Beams 12, 257 (1994)] for the investigation of laser-produced plasmas in relevance to inertial confinement fusion. The equations are integrated implicitly by the time-splitting method. The test problems are simulated to show the versatility of the code. The comparisons among our computational heat flux and the classical Spitzer-H"arm (SH) transport model and non-local transport models have been presented. The result shows that the non-local model of heat flux is in reasonable agreement with the FP simulation in overdense region and invalid in the hot underdense plasmas.

1. Introduction

In the field of inertial confinement fusion (ICF), laser-produced plasmas are generally between the collisionless and collisional regimes and could not simply be regarded in local thermal equilibrium (LTE) state because of the small size and short duration of the plasmas [1]. Electron thermal conduction in ICF plays an important role because it determines ablation pressure and therefore implosion velocity. However, large electron temperature gradient around critical density makes the classical Spitzer-Härm theory failure to describe electron thermal conduction in this region [1]. Electron heat flux is no longer just determined by local temperature gradient, i.e., electron transport becomes non-local. It has been acknowledged that Fokker-Planck (FP) simulations can provide a more accurate description of electron heat flux [2, 3, 4].

FP simulation of a plasma developed steadily after the pioneering work of Chang and Cooper [6], who proposed a practical differencing scheme maintaining particle conservation and positivity at all energy groups. Langdon improved the finite difference scheme for the collision operator to ensure better energy conservation [7]. Based upon the previous researches, Epperlein [8] has advanced an implicit and conservative difference scheme for the FP equation. In this article, we apply Epperlein’s scheme to develop a FP code in standard C language. Our simulation results are well consistent with the existing simulations and theories. The comparisons among our computational heat flux and the classical Spitzer-Härm (SH) transport model and non-local transport models have been discussed.

2. The Fokker-Planck Equations and the Numerical Scheme

The electron FP equation for a laser-produced high—Z plasma can be written as

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{e}{m_e} \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} = C_{ee}(f, f) + C_{ei}(f)
\]
where $E$ is the self-consistent electrostatic field ensuring quasi-neutrality, $-e$ is the electron charge, $m_e$ is the electron mass, $C_{ee}$ is the electron-electron collision operator [9], and $C_{ei}$ is the Lorentz electron-ion collision operator. We have assumed a fixed background of cold ions in one-dimensional planar geometry along $x$–axis. In order to solve Eq. (1), we expand electron distribution function $f(x, v, t)$ in a series of Legendre polynomials $f(x, v, t) = \sum_{i=0}^{N} f_i(x, v, t) P_i(\mu)$, where $\mu = v_x / v = \cos \theta$ is the direction cosine. In the diffusive approximation, the series is truncated at $N = 1$, i.e., only $f_0$ and $f_1$ are kept in the expansion, and the time derivative of $f_1$ is neglected [5]. Substituting $f = f_0 + \mu f_1$ into (1), it is easy to obtain the equation for the isotropic part of the distribution function which is coupled with the anisotropic part $f_1$,

$$\frac{\partial f_0}{\partial t} + \frac{v}{3} \frac{\partial}{\partial x} f_1 = \frac{1}{v^2} \frac{\partial}{\partial v} \left( \frac{v^2 eE}{3 m_e} f_1 \right) + C_{ee}(f_0, f_0)$$

(2)

For a high–$Z$ plasma, the anisotropic part of the electron-electron collision term is negligible in comparison with that of electron-ion collision term. The equation for $f_1$ can be simplified as

$$f_1 = - \frac{1}{\nu_{ei}} \left( v \frac{\partial f_0}{\partial x} - \frac{eE}{m_e} \frac{\partial f_0}{\partial v} \right),$$

(3)

where $\nu_{ei} = 4\pi N_e Z e^4 / m_e^2 \ln \Lambda / v^3$, $Z$ is the ion charge state, $N_e$ is the electron number density. Substituting Eq. (3) into Eq. (2), we obtain a closed equation of $f_0$,

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial x} \left[ \chi \left( \frac{\partial f_0}{\partial x} - \frac{a}{v} \frac{\partial f_0}{\partial v} \right) \right] + \frac{1}{v^2} \frac{\partial}{\partial v} \left[ \chi \left( a^2 \frac{\partial f_0}{\partial v} - av \frac{\partial f_0}{\partial x} \right) \right] + C_{ee}(f_0, f_0) + S_{IB}$$

(4)

where $a = eE / m_e$, $\chi = v^2 / 3\nu_{ei}$. An additional term $S_{IB}$ is added to include the inverse bremsstrahlung (IB) absorption of the laser energy according to Langdon’s theory [10]. The self-consistent electrostatic field $E$ or equivalently $a$ can obtained from the quasi-neutrality condition $\Gamma = \int d^3v \mu e \nu f_1 = 0$.

Phase-space differentiation is carried out with on a fixed Cartesian grid in $(x, v)$. The spatial grid is usually uniform and the grid of the velocity space can be feathered with finer resolution at low velocity grid when the laser-heating effect is taken into consideration in the simulations. Equation (4) is solved implicitly in time by a time-splitting technique. The electron-electron collision operator is calculated with the Chang-Cooper scheme to ensure particle conservation and positivity, and the electron transport term is differenced in a conservative form in configuration space. The numerical implementation is achieved essentially as the same numerical methods as those presented in Epperlein’s work [5].

3. Simulation results and discussion

Here two test problems are solved in order to check our code. The first test problem is the IB absorption in a homogeneous plasma. In the simulation, only the third and fourth terms on the right hand side of Eq. (4) are kept. The Maxwellian electrons with an initial temperature of 50 eV are heated to 2 keV. In the process of IB absorption, due to the competition between laser heating and electron thermalization the electron distribution should approach to a super-Gaussian distribution [10],

$$f_m(v, t) = \frac{N}{4\pi \Gamma(3/m)} v_m^m \exp[-(v/v_m)^m],$$

(5)

with

$$m(\alpha) = 2 + 3/(1 + 1.66/\alpha^{0.724}), \quad v_m^\alpha = \frac{3 k_B T \Gamma(3/m)}{m_e \Gamma(5/m)},$$

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Figure 1. The simulated velocity distribution functions \( f_0(v) \) as a function of electron kinetic energy \( E = (1/2)mv^2 \) (in keV).

Figure 2. Same as the Fig.(1)

where \( \Gamma(x) \) is the gamma function, \( \alpha = Zv_o^2/v_T^2 \), \( v_o \) is the oscillation velocity of the electrons in laser field, and \( v_T = (T_e/m_e)^{1/2} \) is the thermal velocity. Fig. (1) and (2) show the simulated electron distribution functions at the end of simulation with \( \alpha = 6.72 \) and \( 0.233 \) in comparison with the super-Gaussian distribution \( f_m(\alpha) \). The curves with \( m = 2 \) (i.e., without IB absorption) and \( m = 5 \) (i.e., without thermalization) are also plotted. In each case, one can see the super-Gaussian distribution \( f_m(\alpha) \) gives a good approximation to the simulation results.

Another test problem is the ablation of a plasma irradiated with a high-power laser. In our simulation, an inhomogeneous plasma with an initial temperature of 50 eV and a charge state of \( Z = 10 \) is irradiated with a 351 nm laser at a constant intensity of \( 5 \times 10^{14} \) W/cm\(^2\) within 100 ps. Fig. 3(a) shows the temperature and density (normalized to the critical density \( n_c \) ) profiles at the end of the simulation. Fig. 3(b) shows the electron distribution versus the kinetic energy \( E_k \) at the positions of A and B indicated in Fig. 3(a). As seen in Fig. 3(b), in the overdense region A, the electron velocity distribution shows a double-Maxwellian nature [2], where the ‘hot’ tail shares the same temperatures with that of the electron velocity distribution at underdense region B. In Fig. 3, we plot the heat flux given by our FP simulation at the end of the simulation, the Epperlein and Short (ES) model [4], and the SH theory, respectively. We can see the heat flux inhibition occurs and the ES non-local model is in reasonable agreement with the FP simulation in overdense region. However, the non-local model is invalid in the hot underdense plasmas.

4. Summary
In summary, we develop a FP code using diffusive approximation in configuration space to investigate electron transport process in a laser-produced plasma in relevance to inertial
Figure 3. (a) The electron number density (normalized with $n_c$) and temperature (in kev) profiles at the end of the simulation. (b) The electron distribution $f$ (in arbitrary unit) as a function of electron kinetic energy $E_k$. The curves correspond to positions A and B in (a).

![Figure 3](image_url)

confinement fusion. IB absorption and ablation of a plasma are investigated as test problems. The calculated super-Gaussian electron distribution functions in the process of IB absorption, and the typical double-Maxwellian electron distribution functions in the process of target ablation are in good agreement with the existing simulations and theories. It has been found that the non-local model given by Epperlein and Short is in good agreement with our FP simulation except in the hot underdense region of laser-produced plasma.

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