Thrust control when landing a ship-based aircraft

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Abstract. We consider a landing on an aircraft carrier. We propose a scheme for calculating the probability of go-around due to disengaging the arresting gear and a scheme for calculating the maximum descent of the aircraft trajectory immediately after leaving the deck. The main control parameter is the moment of increase in aircraft's thrust before touching the deck. The requirements imposed on the probability of go-around and the maximum descent of the aircraft trajectory determine the possible range for the moment of increase in aircraft’s thrust. Proposing the scheme for calculating the probability of go-around, we use our previously obtained results related to finding the probability of a successful aircraft landing. We present the numerical results for the real aircraft when landing on the real aircraft carrier.

1. Introduction and problem statement

Landing is the most difficult and important stage of flight. For a ship-based aircraft, it is further complicated by the fact that the landing surface is movable and limited in length. This leads to increased requirements for the accuracy of carrier landing and determines the specifics of such landing. Specifically, the aircraft, with the help of a hook, should engage one of several arresting gear cables stretched across the landing section of the deck. If this engagement does not occur, then the pilot is forced to take off and go-around. The number of such landings go-around is quite large and, according to statistics, is equal to 1-2% of the total number of landings.

In the case of disengaging the arresting gear, the pilot does not have time to create the required angles of pitch and attack at the moment of turning off the deck because of the smallness of time on the deck runs (1.5 - 2 seconds). For this reason, the initial section of the aircraft trajectory has some descent $H_{ds}<0$ (see Fig. 1) after leaving the deck with respect to the deck level (balked landing height loss).

If the angles of attack and pitch are fixed at the time of leaving the deck and the elevator control law is given, then the value $H_{ds}$ is uniquely determined by the velocity of the aircraft at the time of leaving the deck. To prevent a water contact and ensure successful take-off of the aircraft it is necessary to make the value $|H_{ds}|$ as small as possible. And for this it is necessary to increase the aircraft velocity at the time of leaving the deck.

The increase in velocity can be achieved by increasing aircraft’s thrust. This is a lagging action which takes time. Therefore, in the case of disengaging the arresting gear, in order to significantly increase the velocity while the aircraft is moving along the deck, the pilot must increase aircraft’s thrust before the expected moment of touching the deck. This moment is determined automatically by the current aircraft altitude above the deck level. In this case, the following circumstance should be taken into account. If aircraft’s thrust is increased early, then the velocity of leaving the deck will be high and the value $|H_{ds}|$ will be small. But an early increase in aircraft’s thrust will lead to an increase in landing velocity and the probability $P_{ag}$ of disengaging the arresting gear is increased. If the increase in aircraft’s thrust occurs
late, then, in the case of disengaging the arresting gear, the aircraft will have an insufficient velocity of leaving the deck and an unacceptably high value of $|H_{ds}|$.

![Fig. 1. The aircraft trajectory in the case of disengaging the arresting gear.](image)

Thus the following question arises: “What conditions must be satisfied for a range of acceptable moments of thrust increase?” On the one hand, it is necessary to ensure a sufficiently small value of probability $P_{ag}$. On the other hand, in the case of disengaging the arresting gear, it is necessary to guarantee an acceptable value of $H_{ds}$. The purpose of the paper is to answer this question.

2. Scheme of proposed method

As in [1], we consider only the longitudinal motion of the aircraft, i.e., the motion only in the vertical plane. By $s_{ag}$ denote the length of the deck occupied by arresting gear (see Fig. 1). To describe the landing trajectory we introduce the auxiliary planes $A−A$, $B−B$, $C−C$, $C_1−C_1$, $D−D$, and $E−E$ that are perpendicular to the vertical plane. The vertical plane coincides with the plane of Fig. 1. Lines $A−A$, $B−B$, $C−C$, $C_1−C_1$, $D−D$, and $E−E$ are projections of the corresponding planes on the plane of Fig. 1. The plane $A−−A$ is located at a distance of a 3-second flight along the nominal trajectory until touching the deck. The $B−B$ plane corresponds to the moment of thrust increase, i.e., the pilot increases the engine thrust at the moment when the aircraft crosses the $B−B$ plane. The $C−C$ and $C_1−C_1$ planes limit the deck segment occupied by arresting gear. The $D−D$ plane corresponds to the moment of leaving the deck if there was no arresting gear engagement. We assume that the arresting gear engagement does not occur only if there is a flight over zone $s_{ag}$. In the converse case, it is assumed that the arresting gear engagement occurs with probability 1. Finally, the $E−E$ plane corresponds to the moment when the aircraft trajectory has the maximum descent after leaving the deck.

The proposed scheme for solving the problem is as follows. In [1], the method for calculating the probability of a successful aircraft landing was proposed. Random perturbations are an atmospheric turbulence and ship’s motions. This method is based on the theoretical results obtained in [2] and [3] and developed in [4], [5], and [6]. Also, this method uses the results from [7]. This method allows to determine the probability $P_{ag}$ depending on the moment $t_{th}$ of aircraft’s thrust increase. If the moment $t_{th}$ is specified, then, in the case of disengaging the arresting gear, the value $v = |v|$ of the aircraft velocity $v$ at the moment of leaving the deck and the value $H_{ds}$ practically do not depend on the point of initial contact of the aircraft with the deck and are uniquely determined by the moment $t_{th}$ and the angle $\theta$ between the vector $v$ and the horizontal plane at the moment of leaving the deck. This allows us to consider the portion of aircraft’s motion along the deck and after leaving the deck in a deterministic setting and, by numerically integrating the motion equations, to find the function $H_{ds}(t_{th}, \theta)$.

As a result, we get two functions: $P_{ag}(t_{th})$ and $H_{ds}(t_{th}, \theta)$. There are generally accepted restrictions on $P_{ag}(t_{th})$; also there are generally accepted restrictions on $H_{ds}(t_{th}, \theta)$ in the absence of ship’s pitching, i.e., when $\theta = 0$. The range of possible moments $t_{th}$ is determined from the condition of fulfilling these restrictions.

So, the solution to the problem involves the actions:

1) using the method proposed in [1], we calculate the probability $P_{ag}$;
2) integrating the aircraft motion equations until the moment of touching the deck, we determine the aircraft velocity at this moment;
3) integrating the aircraft motion equations along the deck, we determine the aircraft velocity at the moment of leaving the deck;
4) integrating the aircraft motion equations after leaving the deck, we determine the descent $H_{ds}$;
5) we determine the range of possible values for $t_{th}$ from the condition of meeting requirements for $P_{ag}$ and $H_{ds}$.

This algorithm is implemented below in sections 3, 4, 5, 6, and 7, respectively. Numerical results were obtained for the same aircraft (the MiG-29K aircraft) and the same aircraft carrier ("Admiral Kuznetsov") as in [1].

3. Probability of disengaging the arresting gear

We will assume that aircraft’s thrust $F(t)$ increases exponentially from the moment $t_{th}$, i.e.,

$$F(t) = \begin{cases} F_0 & \text{for } t \leq t_{th}, \\ F_0 + \Delta F_0 \left(1 - \exp\left(-\frac{t-t_{th}}{\tau}\right)\right) & \text{for } t > t_{th}, \end{cases}$$

where $F_0$ and $\Delta F_0$ are constant values; $\tau$ is a constant that characterizes the engine acceleration time.

The probability $P_{ag}$ can be found as the difference $1 - P_{MN}$, where $P_{MN}$ is the probability of landing on the deck segment $MN$ (see Fig. 1), i.e., $P_{MN}$ is the probability that the initial contact of the aircraft with the deck takes place at segment $MN$. To find the probability $P_{MN}$ we use the method proposed in [1]. In contrast to [1], in this paper, we consider only atmospheric turbulence and do not take into account possible ship’s motions. We consider the same control law (based on the principle of automatic feedback) as in [1]. We linearize the equations of aircraft motion (equations (9) from [1]) relative to the nominal landing trajectory along which the aircraft moves in the absence of perturbations. The difference from [1] has a purely technical nature and lies in the fact that the nominal trajectory of landing is a straight line only up to the moment $t_{th}$, and from the moment $t_{th}$ this trajectory deviates from the straight line due to the new dependence for $F(t)$; in [1], we considered that $F$ is constant until the moment of landing.

The calculations were performed for the case $\Delta F_0 = F_0$, i.e., the aircraft thrust after the time $t_{th}$ asymptotically increased twice. We considered three values of the constant $\tau$ that characterizes the engine acceleration time: $\tau = 1$ s, $\tau = 1.5$ s, and $\tau = 2$ s. For the considered aircraft and carrier, the results of numerical calculations for probability $P_{ag}$ are presented in Tables 1-3:

| $\Delta t$, s | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 | 1.5 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $P_{ag}$      | 0.0081 | 0.0086 | 0.0090 | 0.0096 | 0.0172 | 0.0545 | 0.0733 | 0.1003 | 0.1917 | 0.3562 | 0.5945 | 0.8312 |

| $\Delta t$, s | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 | 1.5 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $P_{ag}$      | 0.0079 | 0.0082 | 0.0085 | 0.0088 | 0.0129 | 0.0302 | 0.0384 | 0.0498 | 0.0885 | 0.1642 | 0.3015 | 0.5123 |

| $\Delta t$, s | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 | 1.5 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $P_{ag}$      | 0.0078 | 0.0079 | 0.0081 | 0.0083 | 0.0108 | 0.0206 | 0.0248 | 0.0307 | 0.0501 | 0.0876 | 0.1592 | 0.2873 |
The information from Tables 1-3 is presented in graphical form in Fig. 2:

![Graphical representation of the probability of disengaging the arresting gear.](image)

**Fig. 2.** The probability of disengaging the arresting gear.

Here $\Delta t = t_{\text{land}} - t_{\text{th}}$, where $t_{\text{land}}$ is the moment of initial contact of the aircraft with the deck when the aircraft moves along the nominal landing trajectory in the absence of perturbations in the case when aircraft’s thrust does not increase up to the moment of this contact.

4. Aircraft’s velocity at the moment of touching the deck

This velocity is necessary for the subsequent finding the value $H_{ds}$. We assume that the aircraft thrust $F = F(t)$ varies according to law (1) and we find the velocity of the aircraft when it is in the $C_1-C_1$ plane (see Fig. 3).

![Coordinate system Oxy associated with a ship.](image)

**Fig. 3.** The coordinate system $Oxy$ associated with a ship.

We assume that the aircraft motion is described by system (9) from [1] and we solve the problem in a deterministic statement when there is no atmospheric turbulence (i.e., when $w \equiv 0$: see Fig. 1 from [1]). In this case, we get
\[
\begin{align*}
    m \frac{dy}{dt} &= F(t) \cos \alpha - mg \sin(\theta - \alpha) - qSC_x, \\
    mv \frac{d(\theta - \alpha)}{dt} &= qSC_y + F(t) \sin \alpha - mg \cos(\theta - \alpha), \\
    \frac{d\theta}{dt} &= \omega_z, \\
    J \frac{d\omega_z}{dt} &= qSb_A m_z, \\
    \frac{dv}{dt} &= v \sin(\theta - \alpha), \\
    \frac{dz}{dt} &= v \cos(\theta - \alpha) - V. 
\end{align*}
\]

All designations used here have been introduced and described in detail in [1]. These designations are common for aircraft flight dynamics problems. In particular,

\[ C_y = C_{y0} + C_{\alpha} \dot{\alpha} + C_{\delta} \delta, \quad C_x = C_{x0} + AC_x^2, \quad m_z = m_{z0} + m_{z\alpha} \alpha + m_{z\delta} \delta + \frac{b_A}{v_0} m_{z\omega} \omega_z, \quad q = \frac{\rho v^2}{2}, \]

where \( \delta \) is the deviation of a longitudinal body of control; \( C_{y0}, C_{\alpha}, C_{\delta}, A, C_{x0}, m_{z0}, m_{z\alpha}, m_{z\delta} \), and \( m_{z\omega} \) are the aerodynamic coefficients of aircraft; \( \bar{v}_z = \frac{w_0 b_A}{v_0} \); \( C_x \) is the dimensionless coefficient of drag component \( X \) (see Fig. 4) of aerodynamic force: \( C_x = |X|/qS \); \( C_y \) is the dimensionless coefficient of lift component \( Y \) of aerodynamic force: \( C_y = |Y|/qS \); \( b_A \) is the mean aerodynamic wing chord; \( m_z \) is the dimensionless coefficient of the longitudinal moment \( M_z \); \( M_z = m_z qS b_A \); \( I_z \) is the appropriate moment of inertia of the aircraft; \( \dot{\theta} \) is the pitch angle, i.e., the angle between the fuselage axis and the horizontal plane; \( S \) is the wing planform area; \( g = |g| \), where \( g \) is the gravity vector.

**Fig. 4.** Forces, angles, and a velocity in the vertical plane.

We assume that the control law for a longitudinal body of control \( \delta \) has the same form as in [1] (see formula (17) from [1]). In this case, we obtain

\[ \delta = \delta_0 - k_\alpha \alpha_0 + k_\alpha \alpha + k_\omega \omega_z \]

when the aircraft moves along the nominal trajectory in the absence of perturbations; the constants \( \delta_0 \) and \( \alpha_0 \) are the same as in [1]. Equality (3) closes system (2) and this system can be integrated numerically.

Integrating system (2), we find the moment \( t_{C_1} \) such that \( x(t_{C_1}) = s_{ag} \) (see Fig. 3); then we find the velocity \( v(t_{C_1}) \) of the aircraft when it is in the \( C_1 \) plane. Real observations show that the aircraft velocity at the moment of leaving the deck\(^1\) is practically independent of the point of initial contact with the deck and this velocity is uniquely determined by the moment \( t_{th} \). Therefore we can consider that the point of initial contact of the aircraft with the deck is point \( N \) (see Fig. 3), the moment \( t_{ic} \) of initial contact of the aircraft with the deck is the moment \( t_{C_1} \), and the aircraft velocity \( v_{tc} \) at the moment of

\(^1\)Here we mean the observed values of the aircraft velocity at the moment of leaving the deck for real landings of the considered MiG-29K aircraft on the considered aircraft carrier “Admiral Kuznetsov”.

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touching the deck is \( v(t_{ic}) \equiv v(t_{C_1-C_2}) \). For the considered aircraft and carrier, the results of numerical calculations for velocity \( v_{ic} \) are presented in Tables 7-9 in Appendix II.

### 5. Aircraft’s velocity at the moment of leaving the deck

The aircraft motion on the deck is described by the equation

\[
\frac{dv}{dt} = F(t) - \frac{\rho v^2}{2} C_x - fG,
\]

(4)

where \( G = mg \), \( f \) is the friction coefficient, and \( \rho \) is the density of the air. We consider \( \alpha \equiv 0 \) and \( \delta \equiv 0 \) when the aircraft moves on the deck. By definition, put

\[ v(t) = v_{ic} + \Delta v(t), \quad t_{ic} \leq t \leq t_{th}, \]

where \( t_{th} \) is the moment of leaving the deck. Since the difference \( t_{th} - t_{ic} \) (time of motion along the deck) is small, we have \( \Delta v(t) \ll v_{ic} \). Therefore we can linearize equation (4). We obtain for \( \Delta v(t) \):

\[
\frac{d\Delta v}{dt} + \rho \frac{C_x}{m} v_{ic} \Delta v = g \left( \frac{F_0}{G} - f \frac{C_x \rho v^2_{ic}}{2} \right) + g \frac{F(t) - F_0}{G},
\]

where \( C_x = C_{x0} + AC^2_{\gamma0} \) because \( \alpha = \delta = 0 \); \( \Delta v(t_{ic}) = 0 \). Solving this equation, we get

\[
\Delta v(t) = \frac{b_1 + g\Delta F_0/G}{a_1} \left( 1 - \exp(-a_1(t - t_{ic})) \right) + \frac{g\Delta F_0}{G} \frac{\tau}{1 - a_1 \tau} \exp\left( -\frac{t_{ic} - t_{th}}{\tau} \right) \left( \exp\left( -\frac{t - t_{ic}}{\tau} \right) - \exp(-a_1(t - t_{ic})) \right),
\]

where

\[
a_1 = \rho \frac{C_x v_{ic}}{m}, \quad b_1 = \frac{g}{G} \left( \frac{F_0}{G} - f \frac{C_x \rho v^2_{ic}}{2} \right).
\]

Let \( T \) be the time of aircraft’s motion along the deck from moment \( t_{ic} \) to moment \( t_{th} \), i.e., \( T = t_{th} - t_{ic} \), and let \( L \) be the length of the deck from the point of touching the deck to the point of leaving the deck. As we noted in section 4, real observations show that the aircraft velocity at the moment of leaving the deck is practically independent of the point of initial contact with the deck. Therefore we may assume that \( L = NK \) (see Fig. 3). Then

\[
NK = \int_{t_{ic}}^{t_{ic}+T} (v_{ic} + \Delta v(t) - V)dt = \left( v_{ic} - V + \frac{b_1 + g\Delta F_0/G}{a_1} \right) T + \frac{g\Delta F_0}{G} \frac{\tau^2}{1 - a_1 \tau} \exp\left( -\frac{t_{ic} - t_{th}}{\tau} \right) \left( 1 - \exp\left( -\frac{T}{\tau} \right) \right) - \left( \frac{b_1 + g\Delta F_0/G}{a_1} + \frac{g\Delta F_0}{G} \frac{\tau}{1 - a_1 \tau} \exp\left( -\frac{t_{ic} - t_{th}}{\tau} \right) \right) \frac{1 - \exp(-a_1 T)}{a_1}.
\]

From this we obtain the following equation for finding \( T \):

\[
T = \psi(T),
\]

(5)
where 

\[ \psi(T) = \bar{\psi}(T)/\left(v_{tc} - V + \frac{b_1 + g\Delta F_0}{a_1}\right), \]

\[ \bar{\psi}(T) = NK + \left(\frac{b_1 + g\Delta F_0}{a_1} + \frac{g\Delta F_0}{G} \frac{\tau}{1 - a_1 \tau} \exp\left(-\frac{t_{tc} - t_{th}}{\tau}\right)\right) \frac{1 - \exp[-a_1 T]}{a_1} - \frac{g\Delta F_0}{G} \frac{\tau^2}{1 - a_1 \tau} \exp\left(-\frac{t_{tc} - t_{th}}{\tau}\right) \left(1 - \exp\left(-\frac{T}{\tau}\right)\right). \]

The solution of equation (5) is found by the method of successive approximations:

\[ T = \lim_{n \to \infty} T_n, \quad T_n = \psi(T_{n-1}), \quad n = 1, 2, \ldots, \]

where \( T_0 \) is the initial approximation.

The velocity of aircraft at the moment of leaving the deck is defined by the formula

\[ v_{lv} = v_{tc} + \Delta v(t_{tc} + T), \]

where the values \( t_{tc} \) and \( v_{tc} \) are determined as described in section 4. For the considered aircraft and carrier, the results of numerical calculations for velocity \( v_{lv} \) are presented in Tables 10-12 in Appendix III.

6. The maximum descent of the aircraft trajectory after leaving the deck

After leaving the deck, the aircraft motion is considered in a deterministic statement and this motion is described by system (2) of longitudinal motion equations with following initial conditions:

\[ v = v_{lv}, \quad \alpha = 0, \quad \theta = 0, \quad \omega_z = 0, \quad y = 0, \quad x = s_{ag} + NK \text{ (see Fig. 3).} \]

The deviation \( \delta \) of a longitudinal body of control is assumed to be constant and is determined from the balancing condition

\[ m_z \alpha + m_{\alpha} \alpha \alpha + m_{\delta} \delta = 0, \]

where, for the considered aircraft, \( \alpha = \alpha_{bal} = 15^\circ \).

The maximum descent \( H_{ds} \), i.e., the minimum value of coordinate \( y \), is determined as a result of the numerical integration of system (2) with initial conditions (6). For the considered aircraft and carrier, the results of numerical calculations for \( H_{ds} \) are presented in Tables 4-6:

| Table 4. Maximum descent \( H_{ds} \) of the aircraft trajectory if time \( \tau = 1 \) s. |
|---|
| \( \Delta t \), s | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 | 1.5 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
| \( H_{ds} \), m | -2.05 | -1.99 | -1.93 | -1.88 | -1.63 | -1.42 | -1.38 | -1.35 | -1.28 | -1.22 | -1.16 | -1.10 |

| Table 5. Maximum descent \( H_{ds} \) of the aircraft trajectory if time \( \tau = 1.5 \) s. |
|---|
| \( \Delta t \), s | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 | 1.5 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
| \( H_{ds} \), m | -2.33 | -2.27 | -2.20 | -2.14 | -1.87 | -1.63 | -1.59 | -1.55 | -1.47 | -1.40 | -1.33 | -1.26 |

| Table 6. Maximum descent \( H_{ds} \) of the aircraft trajectory if time \( \tau = 2 \) s. |
|---|
| \( \Delta t \), s | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 | 1.5 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
| \( H_{ds} \), m | -2.59 | -2.52 | -2.45 | -2.39 | -2.09 | -1.84 | -1.79 | -1.75 | -1.66 | -1.58 | -1.50 | -1.43 |

The information from Tables 4-6 is presented in graphical form in Fig. 5.
7. Permissible moments of increase in thrust
When choosing permissible moments of thrust increase, the restriction is considered for the probability \( P_{eb} \) of landing on the emergency barrier. The probability of landing on the emergency barrier is equal to the probability of disengaging the arresting gear after \( n \) landing approaches, i.e., \( P_{eb} = P_{ag}^n \). The value \( n \) is determined from the condition of limited fuel supply. As a rule, take \( n = 3 \) and \( P_{eb} = P_{ag}^3 \).

The aircraft can be seriously damaged when landing on the emergency barrier. Standards require that

\[
P_{eb} < 10^{-4}.
\]  

(7)

Therefore \( P_{ag} < 0.0464 \). For the considered aircraft and carrier, this inequality and the results obtained in section 3 (and presented in Tables 1-3 and Fig. 2) lead to the following restriction for time \( \Delta t \):

\[
\Delta t < \Delta t_{\text{max}},
\]

where

\[
\Delta t_{\text{max}} = \begin{cases} 
1.40 \text{ s} & \text{when } \tau = 1 \text{ s}, \\
1.65 \text{ s} & \text{when } \tau = 1.5 \text{ s}, \\
1.85 \text{ s} & \text{when } \tau = 2 \text{ s},
\end{cases}
\]

i.e.,

- at the engine acceleration time \( \tau = 1 \text{ s} \), an increase in thrust should be done no earlier than 1.40 s before the expected moment of touching the deck;
- at the engine acceleration time \( \tau = 1.5 \text{ s} \), an increase in thrust should be done no earlier than 1.65 s before the expected moment of touching the deck;
- at the engine acceleration time \( \tau = 2 \text{ s} \), an increase in thrust should be done no earlier than 1.85 s before the expected moment of touching the deck.

Let us now see what requirements are imposed on \( \Delta t \) due to restrictions on the maximum descent \( H_{ds} \) of the aircraft trajectory after leaving the deck. Above, we found the maximum descent \( H_{ds} \) of the aircraft trajectory without taking into account the pitching of the ship. It is clear that in the presence of ship’s pitching, the value of \(|H_{ds}|\) increases noticeably due to leaving the deck with negative angles \( \theta \).
Nevertheless, the maximum descent $H_{ds}$ is normalized in the absence of pitching, suggesting at the same time its corresponding increase with the appearance of pitching. This rule is proposed to use, e.g., in [8]. If, in accordance with this rule, we require that

$$H_{ds} > -2 \text{ m},$$

then, for the considered aircraft and carrier, we obtain the following restriction for time $\Delta t$ (see Tables 4-6 and Fig. 5):

$$\Delta t > \Delta t_{\text{min}},$$

where

$$\Delta t_{\text{min}} = \begin{cases} 
0.30 \text{ s} & \text{when } \tau = 1 \text{ s}, \\
0.75 \text{ s} & \text{when } \tau = 1.5 \text{ s}, \\
1.20 \text{ s} & \text{when } \tau = 2 \text{ s},
\end{cases}$$

i.e.,

- at the engine acceleration time $\tau = 1 \text{ s}$, an increase in thrust should be done no later than 0.30 s before the expected moment of touching the deck;
- at the engine acceleration time $\tau = 1.5 \text{ s}$, an increase in thrust should be done no later than 0.75 s before the expected moment of touching the deck;
- at the engine acceleration time $\tau = 2 \text{ s}$, an increase in thrust should be done no later than 1.20 s before the expected moment of touching the deck.

Thus, in our case (i.e., for the ship-based aircraft and carrier considered in this paper), the simultaneous fulfillment of conditions (7) and (8) leads to the following allowable range for the moment of increase in aircraft’s thrust:

- at the engine acceleration time $\tau = 1 \text{ s}$, an increase in thrust should be done no earlier than 1.40 s but no later than 0.30 s before the expected moment of touching the deck;
- at the engine acceleration time $\tau = 1.5 \text{ s}$, an increase in thrust should be done no earlier than 1.65 s but no later than 0.75 s before the expected moment of touching the deck;
- at the engine acceleration time $\tau = 2 \text{ s}$, an increase in thrust should be done no earlier than 1.85 s but no later than 1.20 s before the expected moment of touching the deck.

8. Conclusion

In this paper, we propose the algorithm for finding the possible range for moments of increase in aircraft’s thrust when landing on a ship. We propose the scheme that allows us to calculate the probability $P_{ag}$ of disengaging the arresting gear and the maximum descent $H_{ds}$ of the aircraft trajectory after leaving the deck when landing go-around. The probability $P_{ag}$ and the descent $H_{ds}$ are calculated numerically as functions of the moment of increase in thrust. Existing restrictions on $P_{ag}$ and $H_{ds}$ determine the possible range for moments of increase in thrust. The proposed scheme is implemented for the real ship-based aircraft (the MiG-29K aircraft) when landing on the real aircraft carrier (“Admiral Kuznetsov”).

Appendix I
List of used symbols

The main designations introduced and used in the paper are collected and duplicated in the list below:

- $H_{ds}$ is the maximum descent of the aircraft trajectory with respect to the deck level immediately after leaving the deck;
$P_{ag}$ is the probability of go-around due to disengaging the arresting gear;

$P_{eb}$ is the probability of landing on the emergency barrier; in this paper, we consider that $P_{eb} = P_{ag}^3$;

$t_{ih}$ is the moment of increase in aircraft’s thrust before touching the deck;

$t_{land}$ is the moment of initial contact of the aircraft with the deck when the aircraft moves along the nominal landing trajectory in the absence of perturbations in the case when aircraft’s thrust does not increase up to the moment of this contact;

$\Delta t$ is the time from moment $t_{ih}$ to moment $t_{land}$, i.e., $\Delta t = t_{land} - t_{ih}$;

$\tau$ is a constant that characterizes the engine acceleration time in formula (1);

$v_{tc}$ is the aircraft velocity at the moment of touching the deck;

$v_{lv}$ is the aircraft velocity at the moment of leaving the deck;

$T$ is the time of aircraft’s motion along the deck from moment $t_{tc}$ to moment $t_{lv}$, i.e., $T = t_{lv} - t_{tc}$.

Appendix II

The results of numerical calculations for velocity $v_{tc}$

| $\tau=1$ s | $\Delta t, s$ | $v_{tc}, m/s$ |
|------------|---------------|--------------|
| 0.5        | 67.786        |
| 1.0        | 68.930        |
| 1.5        | 70.166        |
| 2.0        | 71.402        |
| 2.5        | 72.585        |

| $\tau=1.5$ s | $\Delta t, s$ | $v_{tc}, m/s$ |
|---------------|---------------|--------------|
| 0.5           | 67.482        |
| 1.0           | 68.407        |
| 1.5           | 69.464        |
| 2.0           | 70.570        |
| 2.5           | 71.674        |

| $\tau=2$ s | $\Delta t, s$ | $v_{tc}, m/s$ |
|-------------|---------------|--------------|
| 0.5         | 67.304        |
| 1.0         | 68.079        |
| 1.5         | 68.997        |
| 2.0         | 69.976        |
| 2.5         | 70.985        |

Appendix III

The results of numerical calculations for velocity $v_{lv}$

| $\tau=1$ s | $\Delta t, s$ | $v_{lv}, m/s$ |
|------------|---------------|--------------|
| 0.5        | 77.705        |
| 1.0        | 78.929        |
| 1.5        | 80.077        |
| 2.0        | 81.142        |
| 2.5        | 82.120        |

| $\tau=1.5$ s | $\Delta t, s$ | $v_{lv}, m/s$ |
|---------------|---------------|--------------|
| 0.5           | 76.674        |
| 1.0           | 77.859        |
| 1.5           | 78.997        |
| 2.0           | 80.075        |
| 2.5           | 81.084        |

| $\tau=2$ s | $\Delta t, s$ | $v_{lv}, m/s$ |
|-------------|---------------|--------------|
| 0.5         | 75.876        |
| 1.0         | 76.991        |
| 1.5         | 78.084        |
| 2.0         | 79.137        |
| 2.5         | 80.140        |

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