Higgs Factories: Higgs–Strahlung versus W–Fusion

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Higgs factories will be able to measure Higgs properties, and in particular Higgs couplings with high precision. In a follow-up to an earlier analysis we study the impact of the Higgs–strahlung and vector boson fusion processes on the precision of the couplings determination. Provided theoretical uncertainties can be controlled, future Higgs factories will be able to measure Higgs couplings at the sub-percent level. In spite of their different strengths, base-line circular and linear designs can expect a surprisingly similar performance.

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I. INTRODUCTION

The discovery of a light narrow Higgs boson \([1]\) in 2012 \([2]\) has not only established the Higgs mechanism as a key ingredient of the Standard Model, it has also provided a new, powerful handle to test the structure of the Standard Model. The obvious question arises if this fundamental scalar sector really only includes the single Higgs boson predicted by the Standard Model. ATLAS and CMS performed a large number of tests of the observed Higgs resonance, in which no significant deviations from the Standard Model properties were observed, for example in the Higgs production and decay rates \([3–8]\). If we describe the Higgs couplings in terms of gauge-invariant higher-dimensional operators we can relate the level of agreement with the Standard Model predictions to the scale of new physics, for example assuming weakly interacting models of new physics \([9]\). In that case we find that the current indirect limits from the Higgs sector are not significantly exceeding the limits from direct LHC searches \([5, 6, 8]\). Translating for example the measurements of anomalous gauge and Higgs couplings into an underlying mass scale of an effective Lagrangian, weakly interacting modifications of the Standard Model are only constrained to be heavier than \(300\ldots500\,\text{GeV}\) \([5, 6]\). This energy scale is largely determined by the momentum flow through the relevant hard processes which was reached at Run I. Significantly improving this reach beyond the direct searches for example at the LHC is at the heart of the case of an \(e^+e^-\) Higgs factory \([10, 12]\).

The arguably most impressive achievement during the past LHC runs is the extensive Higgs precision analysis program. This is why one cannot estimate the reach of a global Higgs analysis at future Higgs factories without including the projected LHC results. This also means that independent of the energy of a future Higgs factory there will be a measurement of the top Yukawa coupling available for a global analysis. However, a key problem of Higgs coupling measurements at the LHC are the theoretical uncertainties. Higgs coupling measurements are based on experimentally observed event rates, corresponding to cross sections times branching ratios times efficiencies. These depend often on the kinematic configuration of the events. Translating these measurements into parameters of a Lagrangian requires QCD precision calculations which only converge slowly for hadron collider observables. Moreover, the more complex a signature, the harder it is to even assign a reliable theoretical uncertainty to an observation. Altogether, this means that Higgs coupling measurements at the LHC will soon be limited by theoretical uncertainties which are difficult to quantify and even more difficult to reduce. This means that it will be very challenging to arrive at a conclusive statement about possible new physics explanations, if a percent level deviation from the SM-Higgs predictions should be observed.

At an \(e^+e^-\) collider the two main Higgs production mechanisms are Higgs–strahlung and \(W\)-fusion Higgs production \([13]\)
\[
e^+e^- \to ZH \quad e^+e^- \to \nu\bar{\nu}H ,
\]
as illustrated in Fig. 1. They can be complemented by associated production of top quark pairs and a Higgs boson and by Higgs pair production in association with a \(Z\)-boson. Here the level of uncertainties are very different \([11]\): electroweak corrections are typically a factor 10 to 100 smaller than QCD corrections; while at a hadron collider large logarithms are for example induced through the production of strongly boosted particles, this is not an issue for a Higgs factory; analyses in an \(e^+e^-\) environment are less dependent on phase space configurations than at a hadron collider and avoid the definition of fiducial phase space regions; parton densities describing the incoming protons at the LHC with sizeable uncertainties are replaced with very well understood incoming electrons and positrons; initial state radiation of QCD jets is replaced by better-controlled QED radiation and Beam-strahlung. Finally, while at a hadron collider selected final states of the Higgs boson decay are analyzed due to the overwhelming background in some channels, e.g. for the Higgs decay to gluons, an \(e^+e^-\) collider can analyze all standard final states. This includes Higgs decays to charm quarks and gluons, provided we take care in controlling the purity of the event separation. All of this clearly points to an \(e^+e^-\) Higgs factory to be built to test the nature of the Higgs boson.

Two fundamentally different designs for Higgs factories exist to date: the first design is a linear collider (ILC) planned with a center-of-mass energy between 250 GeV and 500 GeV, in principle upgradable up to 1 TeV. The ILC

![Figure 1. Feynman diagrams for Higgs–strahlung and W fusion Higgs production channels at an e^+e^- Higgs factory.](image-url)
Table I. Design parameters for different Higgs factories taken from Refs. [15, 20, 21]. Note that the luminosities are not directly comparable, as the ILC design includes polarized beams.

| collider                  | $\sqrt{s}$ [GeV] | luminosity [ab$^{-1}$] |
|---------------------------|------------------|-------------------------|
| HL-LHC                    | 14000            | 3                       |
| FCCee/CEPC-base           | 240              | 4                       |
| FCCee/CEPC-350            | 240/350          | 4/1                     |
| ILC-stage                 | 250              | 2                       |
| ILC-base                  | 250/350/500      | 0.5/0.2/0.5             |
| ILC-lumi                  | 250/350/500      | 2/0.2/4                 |

can run on the peak of the Higgs–strahlung cross section as well as at larger center–of–mass energy [14, 15]. The latter option is particularly useful to extract precise information from W–fusion Higgs production as well as to measure Higgs production associated with top-quark pairs. One of the aims of this study will be to quantify how much a Higgs couplings analysis will gain from these higher-energy runs. The linear design is also realized in the Compact Linear Collider (CLIC) proposal [16], where the setup is particularly targeted to operate at high energies up to 3 TeV, but will also include runs at lower energies of 380 GeV and 1.5 TeV. At such large energies, a consistent analysis should include a possible momentum-dependence of the couplings in its setup, for example by using dimension-6 operators. Current studies [16, 17] have considered only rate measurements and no distributions, so we do not study the CLIC proposal any further.

The second design is a circular $e^+e^-$ collider that could produce very large numbers of $ZH$ events close to threshold [18, 19]. Its strength is the large luminosity due to the circular setup. However, a circular collider has a limited center-of-mass energy below around 350 GeV. In addition, its luminosity decreases as function of energy, whereas it increases for a linear collider. We give the expected integrated luminosity which we will use in our analysis in Tab. I based on Refs [15, 20, 21]. All values are subject to change as staging, increased running time will impact the scenarios until project approval (and after). The absolute values of the integrated luminosities are not directly comparable, as the ILC is foreseen to be run with polarized beams. The polarization has an impact on the production cross section. While in the Higgs–strahlung process left-chiral and right-chiral fermions couple to the $Z$–boson with different coupling strengths, only left-chiral fermions induce $W$–fusion Higgs production. Throughout the paper we will refer to the circular collider as FCCee, implicitly including the CEPC design with its slightly smaller integrated luminosities. The expected precisions given in [18] do not scale as the square root of the luminosity between the FCCee and CEPC, we take the smaller of the two uncertainties.

This discussion of the fundamental question whether optimizing a collider design for energy or for luminosity will lead to a better new physics reach is technically based on Ref. [11]. We always assume that high-luminosity LHC (HL-LHC) results for 3 ab$^{-1}$ will be available for combination with any of the Higgs factories.

## II. SFITTER SETUP

The setup of this updated analysis closely follows the original analysis of Ref. [11]. Higgs couplings are defined as prefactors of the respective Lagrangian terms coupling the Higgs field to other Standard Model particles [22, 23].

$$g_{xxH} \equiv g_x = (1 + \Delta_x) \ g_{x}^{\text{SM}} \quad \text{and} \quad g_x = \left(1 + \Delta_{x/y}\right) g_{x}^{\text{SM}} / g_{y}^{\text{SM}}.$$  \quad (2)

The loop-induced Higgs-photon (Higgs-gluon) coupling can be modified two ways, through shifted, underlying tree-level Higgs couplings to Standard Model particles and through new particles in the loop,

$$g_{\gamma\gamma H} \equiv g_{\gamma} = (1 + \Delta_{\gamma} + \Delta_{\gamma}^{SM}) \ g_{\gamma}^{\text{SM}} = (1 + \Delta_{\gamma}^{SM+NP}) \ g_{\gamma}^{\text{SM}}.$$  \quad (3)

Equivalent parameters $\kappa_x \equiv 1 + \Delta_x$ have been introduced in Ref. [24], without disentangling modified tree-level couplings and new states in the loop-induced couplings. For deviations of the Higgs width from its SM value we additionally define

$$\Delta_{\Gamma} = \frac{\Gamma_{\text{tot}}(H) - \Gamma_{\text{tot}}^{\text{SM}}(H)}{\Gamma_{\text{tot}}^{\text{SM}}(H)}.$$  \quad (4)

The one Higgs coupling we do not comment on is the self-coupling. Measuring the Higgs self-coupling is a challenge at any collider [13, 18, 23]. For a center–of–mass energy above 450 GeV, i.e. for the ILC setup only, a determination
with 30% accuracy at the end of the high-luminosity stage seems feasible \cite{15}. While its measurement might serve as a test of the structure of a SM-like Higgs potential, it is not obvious where it would give us additional information about relevant new physics models.

A Lagrangian based on the shifted Higgs couplings of Eq. (2) is perfectly fine after electroweak symmetry breaking, once we include a renormalization prescription. The problem is the electroweak renormalizability of the Higgs sector, because this condition fixes each Higgs coupling exactly to its SM prediction. Certainly at a Higgs factory electroweak corrections will be crucial to extract a Higgs coupling measurement \cite{26}. Modified Higgs couplings can be properly defined in different ways. One way is based on a renormalizable ultraviolet completion of the free Higgs couplings model. In such a case we can link the shift in the light Higgs couplings to the well-defined parameters of an ultraviolet completion, like for example an extended Higgs sector \cite{27}.

Second, we can systematically construct an effective dimension-6 Lagrangian of the Higgs sector and identify essentially all $\Delta_x$ with Wilson coefficients $c_x/\Lambda^2$ in the non-linear \cite{6, 28} representation of the Higgs and Goldstone fields. While in the pure Higgs sector at the LHC the linear and non-linear results can be related through re-mapping of the operator basis \cite{6} this is no longer true for the combined Higgs and gauge sectors. At the LHC, additional momentum-dependent couplings play a major role in searching for deviations from Standard Model kinematics, but at a Higgs factory we expect these additional operators not to dominate our analysis. To be compatible with other Higgs factory studies we stick to the modified Higgs couplings or non-linear notation.

All linear collider measurements used in this study are taken from Refs. \cite{10, 12, 29}, with the exception of the measurement of the $W$-fusion process with a $H \rightarrow b\bar{b}$ decay at 250 GeV \cite{30}. The statistical uncertainties are scaled to the corresponding integrated luminosity. The expected error on the luminosity measurement of 0.3\% \cite{31} is added to each measurement, taken as fully correlated between them. For the $e^+e^-$ collider production cross sections we assume a theoretical uncertainty of 0.5\% for the $ZH$ and $W$-fusion production processes and 1\% for $t\bar{t}H$ production. Unlike for the LHC, at an $e^+e^-$ Higgs factory the only limiting theoretical uncertainty is on the partial width $\Gamma(H \rightarrow b\bar{b})$, propagated into the different branching ratios. Whenever we include this theoretical uncertainty we estimate the error on the Higgs branching ratio to be around 2.2\%, consisting of 0.65\% due to missing higher orders, 0.73\% from the quark mass measurement(s), and 0.79\% from the value of $\alpha_s$ \cite{32, 33}, otherwise all these contributions are set to zero. This is consistent with our earlier estimate of a combined 4\% uncertainty for the leading partial width $\Gamma(H \rightarrow b\bar{b})$ \cite{11}.

One well-motivated assumption which is crucial for our analysis is that a Higgs factory is expected to operate after the LHC has analyzed a sizable Higgs data set. Our conservative HL-LHC projections are based on the previous detailed studies \cite{22, 23}. They include the main production and decay channel combinations, which are expected to be measured with $3 \text{ ab}^{-1}$ of integrated luminosity at 14 TeV center-of-mass energy, using results from a single experiment. Since background systematics play an important role at this high-luminosity result, the statistical gain from combining both major experiments is expected to be much smaller than a naive scaling with luminosity. The two LHC channels, which will be most relevant for our $e^+e^-$ study, are inclusive Higgs production with decay into a photon pair and top-quark-associated Higgs production with a Higgs decay to photons. This comparatively clean final state makes it a leading channel to directly probe the top Yukawa coupling, despite the rather low numbers of events originating from an inclusive cross section just above a femtobarn.

For the LHC we scale all statistical errors to the increased integrated luminosity. The statistical component of experimental uncertainties on background rates, which are determined from data, will improve correspondingly. The increase of statistics will also improve the statistical component of the systematic errors. On the other hand, experimental conditions (pile-up) will become significantly more difficult for some of the crucial channels, like $W$-fusion Higgs production and hadronic Higgs decays. Therefore, the same performance of particle identification and $b$-tagging as for lower instantaneous luminosity is assumed, i.e. the relative errors for experimental systematics used in the previous studies are not changed. Theoretical errors on the cross sections and on the Higgs branching ratios are taken from Ref. \cite{32} and included via the profile likelihood Rfit scheme \cite{34, 35}, implying that they are added linearly. Because Higgs analyses start to depend more on exclusive jet observables \cite{36}, rendering the application of fixed-order QCD corrections difficult, we refrain from postulating an improved theoretical uncertainty.

Unless we want to limit our LHC analysis with an overwhelming universal error source, we have to make an assumption about the total Higgs width. We generally assume

$$\Gamma_{\text{tot}} = \sum_{\text{obs}} \Gamma_x(g_x) + \text{2nd generation}. \quad (5)$$

At $e^+e^-$ colliders the total width can be inferred from a combination of measurements. This is mainly due to the measurement of the inclusive $ZH$ cross section based on a system recoiling against a $Z \rightarrow \mu^+\mu^-$ decay. While the simultaneous fit of all couplings will reflect this property, we can illustrate this feature also schematically.

The simpler of two possibilities relies on two measurements,
1. Higgs–strahlung inclusive: $\sigma_{ZH} \propto g_Z^2$.

2. Higgs–strahlung with a decay to $ZZ$: $\sigma_{ZH} \times \text{BR}_{ZZ} \propto g_Z^4/\Gamma_{tot}$.

As the Higgs branching ratio into $ZZ$ is small, this requires a significant amount of luminosity. Another way to determine the Higgs width is based on four measurements \cite{10, 29} of both production processes shown in Fig. 1,

1. Higgs–strahlung inclusive: $\sigma_{ZH}$,

2. Higgs–strahlung with a decay to $b\bar{b}$: $\sigma_{Zbb}$,

3. Higgs–strahlung with a decay to $WW$: $\sigma_{ZWW}$,

4. $W$-fusion with a decay to $b\bar{b}$: $\sigma_{\nu\nu bb}$,

involving the four unknown parameters $\Delta_W$, $\Delta_Z$, $\Delta_b$, and $\Gamma_{tot}$. Schematically, the total width can be extracted as \cite{30}

$$\Gamma_{tot} \leftarrow \frac{\sigma_{\nu\nu bb}/\sigma_{Zbb}}{\sigma_{ZWW}/\sigma_{ZH}} \times \sigma_{ZH}.$$ \hspace{1cm} (6)

A final assumption at the LHC concerns hadronic Higgs decays. Going back to the width definition in Eq.(5), at the LHC the Higgs decay to charm quarks with Standard-Model-like coupling strengths is experimentally challenging. On the other hand, it contributes to the total width at the per-cent level. This is why we usually link the second generation to the third generation via

$$g_c = \frac{m_c}{m_t} g_t^{SM}(1 + \Delta_t).$$ \hspace{1cm} (7)

The leptonic muon Yukawa might be observable at the LHC in weak boson fusion or inclusive searches, depending on the available luminosity \cite{37}. At an $e^+e^-$ Higgs factory such a link is not needed.

III. ENERGY VERSUS LUMINOSITY

We start our comparison of the impact of the different experimental channels on the Higgs couplings analysis following the base scenarios defined in Tab. 1. In Fig. 2 we show the projected coupling measurements, assuming that the total width can be computed as the sum of all observed partial widths plus contributions from second-generation

![Figure 2](image-url)
fermions, as defined in Eq. (5). In particular, we assume that the charm Yukawa scales with the top Yukawa following Eq. (7). In our comparison of FCCee and ILC projections we always implicitly assume a combination with the expected HL-LHC results. Unless we make a significant theory effort [35], the linear and circular designs will both be limited by the theoretical uncertainty in extracting Higgs couplings from observed event rates. Assuming that this will eventually change, we always show results for both the current accuracy and when ignoring theoretical uncertainties for the $e^+e^-$ colliders in lighter and darker bands.

The general pattern we see in Fig. 2 is that for all couplings with the exception of $\Delta_W$ the FCCee base design, the ILC base design, and the ILC staging design give comparable results. As we can see in Tab. 4 the FCCee based design and the initial staging ILC design are similar. This can be understood from the impact of the beam polarization which increases the effective cross section. However, for the full ILC design this outcome is not at all trivial, given that the FCCee foresees eight times the integrated luminosity at the lowest energy, neglecting beam polarization effects. The lower integrated luminosity (total 1.2 ab$^{-1}$) is compensated by the higher center–of–mass energy. The only major difference occurs for $\Delta_W$. Here, the statistical uncertainty on $ZH$ production with a subsequent decay $H \rightarrow WW$ in the FCCee setup is around 0.9%, but the larger error on the indirectly obtained Higgs width limits the achievable precision on $\Delta_W$. We confirm this by only including the $ZH$ measurements in the FCCee base setup. In this case mostly the worse Higgs width estimate increases the error on $\Delta_W$ and other couplings, illustrating how even the FCCee at 240 GeV benefits from W–fusion.

The ILC will probe the $W$–fusion process combined with all main decay modes, allowing for a higher overall precision. This advantage will be reduced once we include electroweak precision measurements testing the custodial symmetry enforcing $\Delta_W = \Delta_Z$. Assuming custodial symmetry also improves the precision on the Yukawa couplings, in particular when we ignore theory uncertainties. For example, the uncertainty on $\Delta_t$ reduces to 2.8% with current theoretical errors and 1.0% without. On the other hand, the accuracy on new-physics contributions to the loop-induced couplings and ratios of couplings is hardly affected by this additional constraint.

Minor differences between the scenarios are the slightly better expected performance of the FCCee for $\Delta_y$, and the sizeable impact of theoretical uncertainties on the FCCee measurement of $\Delta_y$. These theoretical uncertainties are the main source of uncertainty for the branching ratio into both gluons and charm quarks. The direct measurement of Higgs decays to charm quarks also determines the top Yukawa coupling more precisely than the HL-LHC $ttH$ measurements, if we indeed set $\Delta_c = \Delta_t$. When deriving $\Delta_y$, the theoretical uncertainties add up, leading to rather large error bars. When these are set to zero, the total width entering the branching ratios becomes the main source. As this induces a positive correlation between the different decay modes, $\Delta_y$ can now be determined more precisely than $\Delta_c$ and $\Delta_y^{SM+NP}$, which parametrizes effects on the full effective $Hgg$ coupling, leading to this large improvement.

In a second step, we test the impact of the two model assumptions we made in our initial estimate, because they are essentially motivated by the type of measurements available at the LHC. In the left panel of Fig. 3 we show the effect for the two base designs if we separate the charm and top Yukawas instead of using Eq. (7). The corresponding signature is Higgs decays $H \rightarrow cc$, which can be tagged very well in the clean $e^+e^-$ environment. The fact that for both collider designs $\Delta_c$ is essentially unchanged implies that most of the combined $\Delta_t$ and $\Delta_c$ measurement comes

![Figure 3](image-url)

Figure 3. Higgs coupling measurement for relaxed model assumptions in the linear and circular baseline scenarios. In the left panel we allow for $\Delta_c \neq \Delta_t$. In the right panel we include a direct Higgs width measurement in the presence of unobservable Higgs decay modes.
from the charm Yukawa. In contrast, the precision on $\Delta_t$ is reduced by almost a factor three. For the FCCee setup, the determination of $\Delta_t$ now relies solely on the HL-LHC results, while the 500 GeV run of the ILC just above the $t\bar{t}H$ threshold yields a second measurement of similar accuracy. Because the top loop gives a large contribution to the loop-induced couplings means that a poorer measurement of $\Delta_t$ translates into a poorer measurement of $\Delta_\gamma$ and $\Delta_\gamma$. If we combine all information on loop-induced decays into one coupling deviation, $\Delta^{SM+NP}_{\gamma_I\gamma}$ remains unchanged in both cases. At the same time, we also remove the link between the second and third generation in the leptonic sector, allowing $\Delta_\mu \neq \Delta_\tau$. However, the small Higgs branching ratio into muons in the Standard Model prevents this channel from having an effect on our global parameter analysis.

In the right panel of Fig. 3 we relax the assumption on the total width given in Eq. (5), allowing for additional positive contributions. Technically, these can be invisible modes or decay modes which generate final states of Standard Model particles different from the standard Higgs decay channel analyses. Because sizeable contributions to the total width also mean sizeable event rates, for example invisible Higgs decays will be strongly constrained by targeted searches. If a deviation in the total Higgs width cannot be mapped to a single signature, the direct measurement will become difficult and our analysis will benefit from a measurement of the total width. For the FCCee the precision of this actual Higgs width measurement is driven by the first scheme described in Sec. I combining the measurement of the inclusive $ZH$ cross section with the $ZZ$ decay channel. The uncertainty on the total width ranges from 5.6% to 3.8%, depending on the assumed theoretical errors. For the ILC the second scheme, including the $W$-fusion production mode, dominates. It leads to a precision between 4.1% and 2.9%, respectively. Ignoring theoretical uncertainty the FCCee setup matches the expected ILC base setup when we increase the integrated luminosity by a factor 1.7. Interestingly, we also find that only using a direct measurement of the total width leads to very little degradation in our heavily correlated global coupling measurement, independent of slight differences in the actual Higgs width determination.

Going back to the results of Fig. 2 we can see that even ignoring theory uncertainties the difference between the energy-driven ILC base scenario and the luminosity-driven FCCee base scenario is not large. Obviously, a staged ILC at 250 GeV is very similar in performance to the FCCee base scenario, but in comparing the luminosities the beam polarization has to be taken into account. The most significant difference between the energy-driven and luminosity-driven scenarios is the measurement of the Higgs-$W$ coupling, where the $W$-fusion process at higher energies leads to an improvement of the measurement by up to a factor four, but deviations between the Higgs-$W$ and Higgs-$Z$ couplings are also strongly constrained by electroweak precision data. Model assumptions in defining our hypothesis hardly play any role, so we can as well minimize them. This in particular relevant for any assumption on the Higgs width, which is crucial at hadron colliders and simply not needed at an $e^+e^-$ Higgs factory.
IV. CONVERGING UPGRADES

While the FCCee and ILC base design are defined through their focus on luminosity or energy in measuring Higgs couplings, we see in Tab. 1 that the respective upgrade strategies will be much more similar to each other. Compared to the base design, an upgrade of the circular collider option targets the limitations in energy, increasing the center-of-mass energy to 350 GeV. At the same time, the envisioned ILC upgrade will focus on an increased integrated luminosity, at the 250 GeV low-energy run and at the 500 GeV high-energy run.

In Fig. 4 we show the expected precision for all Higgs couplings when we relax both unnecessary conditions on the Higgs width and on the second-generation Yukawa couplings, as studied in Fig. 3. If we focus on the experimental performance and ignore the theory uncertainties, most Higgs couplings will be measured below the percent level at the upgraded Higgs factories. The limiting factor will most likely be the top Yukawa coupling, affecting the higher-dimensional Higgs coupling to photons and gluons. For the combination $\Delta_{\gamma,g}^{\text{SM}+\text{NP}}$, which is fixed directly from the corresponding branching ratios, we also find projected measurements of a few per-cent.

The differences between the upgraded ILC and the FCCee designs are relatively minor: for $\Delta_W$, the ILC with its multitude of $W$-fusion signatures will reduce the errors by a factor two to around 0.27%. In contrast, for the luminosity-driven $\Delta_Z$ measurement the FCCee upgrade will reach a precision of 0.28%, limited by the systematic error on the luminosity measurement. If this uncertainty can be improved, statistics allows for a determination of this coupling below the per-mill level. At the ILC the limit will be slightly weaker. The situation is reversed for the top Yukawa coupling, where FCCee will not be able to improve the bounds obtained from the HL-LHC run, while the accuracy with the upgraded ILC will beat the HL-LHC by more than a factor two. This translates directly into improved bounds on dimension-5 contributions to the effective $Hgg$ and $H\gamma\gamma$ couplings, parametrized by $\Delta_b$ and $\Delta_s$, respectively, and for the full new-physics effects, $\Delta_{\gamma,g}^{\text{SM}+\text{NP}}$. Due to the higher statistics of FCCee the difference is smaller for the parameters describing the effective Higgs-photon coupling.

If we will not be able to improve on the theory uncertainties, the errors on the coupling determination roughly double in size for most of the tree-level Higgs couplings, such that only $\Delta_W$ for the ILC upgrade and $\Delta_Z$ stay below the 1% mark. The only exception is the top Yukawa coupling, where the statistical component clearly dominates and the theoretical uncertainties yield only a small additional contribution. This then translates into the same situation for the dimension-5 contributions to the effective $Hgg$ and $H\gamma\gamma$ couplings, which also worsen only mildly.

Finally, we note that using ratios of couplings allows for better probes for an upgraded FCCee with reduced theoretical errors, as one can see for $\Delta_{b/\tau}$ and $\Delta_{b/W}$ in Fig. 4. The reason is that in this case the precision on absolute values of couplings is limited by the reconstruction of the total Higgs width. In all other instances the limiting factor lies somewhere else, so ratios do not give an advantage there.

One of the most interesting questions concerning indirect measurements is how the accessible mass scales compare to typical mass scales of direct searches. This is the weak point of such analyses at the LHC, where with the exception of higher-dimensional QCD operators the direct and indirect reaches for new physics are comparable. Given that at the Higgs factory large momentum flows through any of the Higgs vertices are unlikely, we can use a non-linearly realized dimension-6 Lagrangian to link the projected precision, 0.2% for the $HZZ$ coupling and 1% for many of the other ones, to a new physics mass scale,

$$|\Delta_\omega| \approx \left| \frac{g^2 v^2}{\Lambda^2} \right| \begin{cases} 10^{-2} & \Lambda \frac{g}{v} > 10 v = 2.5 \text{ TeV} \\ 2 \cdot 10^{-3} & \Lambda \frac{g}{v} > 22 v = 5.5 \text{ TeV} \end{cases} \quad (8)$$

For new physics in the electroweak sector this energy range will hardly be covered by the LHC, with the sole exception of a weak gauge boson directly produced. In case of loop effects the typical mass scale $\Lambda$ will be reduced by a factor $\sqrt{\Lambda}$, still well above the LHC reach for example in the case of supersymmetric electroweakinos.

While we were in the final stage of our paper, a similar study indicated that for a linearly realized dimension-6 Lagrangian an increase in collider energy is more significant. We do not expect this difference to be linked to momentum-dependent couplings in $W$-fusion, because the corresponding virtuality is known to be determined by the $W$ and Higgs masses. In line with Ref. [48], we speculate that the difference between the two analyses is explained by inherent strong correlations in the linearly realized dimension-6 model, particularly between the operators affecting the $VVH$ couplings. Gauge invariance and custodial symmetry link the $Z$-couplings and $W$-couplings, where the much higher statistics for $W$-fusion yields a significant increase in precision.
V. SUMMARY

Measuring the Higgs Lagrangian is the prime motivation for future $e^+e^-$ colliders. One framework, motivated by a non-linearly realized dimension-6 Lagrangian, is the measurement of Higgs couplings. Unlike at hadron colliders, the $e^+e^-$ environment allows us to measure Higgs decays into charm quarks and gluons. Even more importantly, we can directly determine the Higgs width with the help of an inclusive $ZH$ cross section measurement. This means that $e^+e^-$ data determines all parameters of the SM-like Higgs Lagrangian independently.

Two substantially different concepts exist for such a future collider: the FCCee and CEPC, optimized for delivering a high luminosity, and the ILC running at a center-of-mass energies up to 500 GeV. At that energy it probes the $W$-boson fusion production process combined with all major decay channels as well as top-quark-associated Higgs production. A staged ILC running at 250 GeV collider energy and the base-line FCCee are almost identical, the only difference being the beam polarization. In both cases, the statistical uncertainties will be substantially smaller than the current theoretical uncertainties. Therefore, significant progress on the theory side is vital to exploit the full potential of future $e^+e^-$ colliders.

For the circular as well as for the linear designs, the typical experiment-driven coupling uncertainties are going to be below the per-cent level. In the effective theory interpretation they probe energy scales well above what is accessible to the LHC. The ILC design significantly benefits from the $W$-fusion channel, for example in the Higgs width extraction. A direct measurement of the top Yukawa coupling, adding to the expected HL-LHC measurement, increases the precision of all loop-induced Higgs couplings. Possible upgrades of the ILC design include a significant increase of the luminosity, also just above the $ZH$ threshold. The ILC program also foresees a long term plan going to a center–of–mass energy of 1 TeV. Similarly, a proposed upgrade of the circular design targets the energy-related shortcomings with a 350 GeV run.

Altogether, the competition between ILC and FCCee does not see a clear winner. With its higher center-of-mass energy, the ILC proposal has a mild advantage on the Higgs couplings to the $W$ and top quark, which then propagates to the dimension-5 operators. Albeit using very different collider features, the two designs are still similar in terms of their Higgs couplings reach.

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