AFFINE CONTROL SYSTEMS ON NON-COMPACT LIE GROUP

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Abstract:
In this paper we deal with affine control systems on a non-compact Lie group cx+e group. First we study topological properties of the state space Ef(1) and the automorphism orbit of Ef(1). Affine control system, non-compact Lie group state space Ef(1). Affine control systems on the generalized Heisenberg Lie groups are studied. Affine algebra, automorphism.

Introduction:
The purpose of this paper affine control systems on some specific lie group is called cx+e group by relating to associated bilinear parts.

Related to the affine control system on lie groups, in Ef(1). The authors Ayala and San Martin have the subalgebra of the Lie algebra Ef(G) generated by the vector fields of a linear control system the drift vector field X is an infinitesimal automorphism i.e.,(X_k)k∈M is a one-parameter subgroup of Aut(G); have lifted the system itself to a right-invariant control system on Lie group Ef(1) for compact connected and non-compact semi-simple Lie group.

The affine control systems on a non-compact Lie group cx+e group have been investigated and given characterization.

1. Affine Control Systems On Lie Groups

If G is a connected Lie group with Lie algebra L(G), the affine group Ef(G) of G is the semi-direct product of Aut(G) with G itself i.e.,Ef(G) = Aut(G)× G. The group operation of Ef(G).

The identity element of Aut (G) and e denotes the neutral element of G, then the group identity of Ef(G) is (1, a) and (Φ⁻¹, Φ⁻¹(h⁻¹)) In the invers of (Φ, h) ∈ Ef(G). Hence, h → (1, h) and Φ → (Φ, a) embed G into Ef(G) and Aut(G) into Af(G) respectively. Therefore, G and Aut(G) are subgroups of Ef(G).

The natural transitive action

Ef(G) × G → G

(Φ, h₁), h₂ → h₁ Φ(h₂)
Where \((\Phi, h_1) \in \text{Ef}(G)\) and \(h_2 \in G\).

“Affine in the control” is used to describe class system.

\[
\frac{dx}{dt} = n(x) + h(x)v
\]

is considered affine control.

**Theorem 1**

Let \(\Sigma = (\text{Ef}(1), D)\) be an affine control system. Then, the state space \(\text{Ef}(1)\) is a locally compact Hausdorff space.

**Proof:**

\(\text{Ef}(1)\) is a Hausdorff space is a lie group. The compactness for a given \(x \in \text{Ef}(1)\) and neighborhood \(Z\) of \(x\), the existence of some neighborhood \(Z\) of \(x\) such that. The topology on \(\text{Ef}(1)\) half plane is homomorphic to the standard topology of \(M^2\).

Therefore, \(\forall \ x \in \text{Ef}(1)\), the neighborhood \(Z\) of \(x\) is homeomorphic to an open ball. For each neighborhood \(Z\) of \(x\), there is neighborhood \(W\) of \(x\) such \(x \in W\). Since \(W\) is also homeomorphic to an open ball the closure of \(U\) is a closed ball.

**Theorem 2**

The automorphism orbit of the state space \(\text{Ef}(1)\) is dense.

**Proof:**

The set

\[
J = \exp \left( \text{cf}(1) - [\text{cf}(1), \text{cf}(1)] \right)
\]

\(\text{Aut(\text{Ef}(1))}\)-orbit of \(\text{Ef}(1)\). The exponential mapping from the tangent plane to the surface of diffeomorphism. Then two elements \(h_1, h_2 \in J\) the line segment \(h_1 h_2\) which is parallel to \([\text{Ef}(1), \text{Ef}(1)]\),

\[
\Phi : J \rightarrow J
\]

Defined by

\[
h_1 \rightarrow k_1 h_1 + k_2 = J, k_1, k_2 \in M
\]

Also it is possible to connect those segments with the perpendicular segments. \(\text{Aut(\text{Ef}(1))}\) orbits open the center\([\text{Ef}(1), \text{Ef}(1)]\) for any element \(x \in [\text{Ef}(1), \text{Ef}(1)]\) and every neighborhood \(Q (x, \gamma)\) of \(x\) have some element of \(\text{Ef}(1)\) different then \(x\).

\[
\text{Ef}(1) - [\text{Ef}(1), \text{Ef}(1)] = \text{Ef}(1).
\]

**Theorem 3**

The affine control system \(\Sigma_c\) on the state space \(\text{Ef}(1)\) is not have any equilibrium point and the associated bilinear system
\[ \Sigma_c = (\text{Ef}(1), D_e) \] is control on the \( \text{Aut} (\text{Ef}(1)) \) orbit.

**Proof:**

For the control not having equilibrium point is necessary. Now consider the associated bilinear system

\[ \Sigma_e = (\text{Ef}(1), D_e) \text{ is control on the } \text{Aut}(\text{Ef}(1)) \text{ orbit.} \]

\[ \Phi_\delta : \partial L(G) \times L(G) \rightarrow \partial L(G) \times L(G) \]

\[ \Phi_\delta = \text{Id} \times \frac{1}{\delta} \forall D + X \in \text{cf}(1) = \partial L(G) \times L(G), \text{we have} \]

\[ \Phi_\delta(D+X) = D \frac{1}{\delta} X. \]

Since complete under the small permutations sufficiently large \( \delta \), \( \Phi_\delta(\Sigma_c) \) is control on \( S(1_e,1) - [\text{Ef}(1), \text{Ef}(1)] \). Therefore, since normally control finite system are open on \( S(1_e,1) \). The system \( \Phi_\delta(\Sigma_c) \) is also control on \( B(1_e,1) - [\text{Ef}(1), \text{Ef}(1)] \). Since the state space is connected, the affine system \( \Sigma_c \) is control on \( \text{Ef}(1) \).

**Lemma 1**

For the generalized Heisenberg lie group \( H =: H(W, X, \infty) \), the map \( \varphi_\delta = \sqrt{\delta} \text{Id} \times \delta \text{Id} \), i.e., \( \varphi_\delta (w,g) = (\sqrt{\delta}w,\delta g) \) is an automorphism.

**Proof:**

The mapping \( \varphi_\delta \) is 1-1 and onto its image.

\[ \varphi_\delta ((w_1,g_1) * (w_2,g_2)) = \varphi_\delta (w_1 + w_2, g_1 + g_2 + \frac{1}{2} \alpha(w_1,w_2)) \]

\[ = (\sqrt{\delta} \text{Id}w_1 + \sqrt{\delta} \text{Id}w_2, \delta \text{Id}g_1 + \delta \text{Id}g_2 + \frac{\delta \text{Id}}{2} \alpha(w_1,w_2)) \]

by bilinearity of \( \alpha \)

\[ (\sqrt{\delta} \text{Id}w_1 + \sqrt{\delta} \text{Id}w_2, \delta \text{Id}g_1 + \delta \text{Id}g_2 + \frac{1}{2} \alpha(\sqrt{\delta}w_1,\sqrt{\delta}w_2)) \]

\[ = (\sqrt{\delta} \text{Id}w_1, \delta \text{Id}g_1) * (\sqrt{\delta} \text{Id}w_2, \delta \text{Id}g_2) \]

\[ = \varphi_\delta (w_1,g_1) * \varphi_\delta (w_2,g_2). \]

This proves that \( \varphi_\delta \) is an automorphism.

**Lemma 2**

Let \( H \) be a generalized Heisenberg Lie group. Then there exist a dense \( \text{Aut}(H) \)-orbit.

**Proof:**

The set \( \varphi = : \exp (L(H) - [L(H), L(H)]) = H[H, H] \)
Is an Aut(H)-orbit of H. The exponential map is a global diffeomorphism for simply connected nilpotent Lie groups. Two elements \(X, Y \in \varphi\) the line segment mod XY parallel to \([H, H]\), can be connected via a line segment by taking once X as a initial point so that the function that connection \(f_s : \varphi \to \varphi\) defined by \(X \to k_1 X + k_2 = Y\), where \(k_1, k_2 \in IM\), is an automorphism. Actually it is possible to connect these segments with the perpendicular segments to each other via the same way. That Aut(H)-orbit of H is \(\varphi\) is open. In fact, if \(\dim Z = 1\) the center \([H, H]\) forms a line for any Heisenberg group \([X, Y] = G\), \(X, Y, G \in L(H)\). For the density, any \(x \in [H, H]\) every ball \((B(x, \gamma))\)

\[B(x, \gamma) \cap H - [H, H] \neq \emptyset.\]

Thus, \(H - [H, H] = H\).

**Theorem 4**

Let \(G\) be a non-compact connected Lie group and \(L(G)\) be its Lie algebra. Then, compact subsets of \(G\) are not \(G_\mathcal{Z}\)-invariant, if the control system on \(G\) is an invariant system.

**Proof:**

For \(\forall x \in G, \forall X \in L(G)\) and \(\forall k \in IM\), the differentiable curve \(\rho_x(x) : (c, e) \subseteq IM \to G\) is defined \(\rho_x(k, x) = X_k(x)\). Assume that \(F \subseteq G\) is a compact and \(G_\mathcal{Z}\)-invariant subset. Each vector field \(X \in L(G)\) is complete. Consider any open covering

\[E = \{V_i | i \in / Z^+\} \subseteq K\]

Therefore, \(\forall i \gamma x(k, V_i)\) is an open covering of \(K\), since \(X_k(x), \forall x \in K\). \(K\) is compact, therefore it can be covered by a finite subfamily of \(A_\delta = \{\delta(\gamma(k, V_i)) | i \in / Z^+\}\). Then, inverse images of the elements of \(A_\delta\) covers IM, which is a contradiction.

**References**

1. Helgason S, Differential Geometry, Lie Groups and Symmetric Spaces, Pure Appl. Math., 80, Academic Press, New York, 1978.
2. Sussmann H, Some properties of vector fields systems that are not altered by small perturbations, Journal of Differential Equations, 20 (1976), 292-315.
3. Buliga M, “Sub-Riemannian geometry and Lie groups. Part 1”, arXiv:math.MG/0210189, (2002).
4. Stroppel M, Homogeneous symplectic maps and almost homogeneous Heisenberg groups, Forum Math., 11 (1999), 659-672.
5. Nielsen O. A., “Unitary representations and coadjoint orbits of low – dimensional nilpotent Lie groups”, Queen’s Papers in Pure Appl. Math.e3 (1983).