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Giant negative magnetoresistance in high-mobility 2D electron systems

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We report on a giant negative magnetoresistance in very high mobility GaAs/AlGaAs heterostructures and quantum wells. The effect is the strongest at $B \approx 1$ kG, where the magnetoresistivity develops a minimum emerging at $T \lesssim 2$ K. Unlike the zero-field resistivity which saturates at $T \approx 2$ K, the resistivity at this minimum continues to drop at an accelerated rate to much lower temperatures and becomes several times smaller than the zero-field resistivity. Unexpectedly, we also find that the effect is destroyed not only by increasing temperature but also by modest in-plane magnetic fields. The analysis shows that giant negative magnetoresistance cannot be explained by existing theories considering interaction-induced or disorder-induced corrections.

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Over the past decade, low field magnetotransport in high mobility two-dimensional electron systems (2DESs) became a subject of considerable interest, in part, owing to the discovery of many unexpected phenomena. While the characteristic features of the majority of these phenomena are now understood reasonably well, there are still many unsolved puzzles. One such puzzle is the recently reported giant microwave photoreactivity peak which emerges in the vicinity of the second harmonic of the cyclotron resonance. Therefore, investigating the GNMR effect is not only interesting and important in its own right but may also provide necessary clues to account for other phenomena.

The magnetoresistance can be characterized by the ratio $\rho(B)/\rho_0$, where $\rho(B)$ and $\rho_0$ are the longitudinal resistivities measured with and without perpendicular magnetic field $B$, respectively. In the present study, we focus on the regime of weak magnetic fields where Shubnikov-de Haas oscillations are not yet developed. In this regime, the characteristic feature of $\rho(B)$ is a broad minimum occurring at $B_0 \approx 1$ kG. Quite remarkably, the resistivity at this minimum, $\rho(B_0) \equiv \rho_{\text{min}}$, can be significantly lower than $\rho_0$, i.e. $\rho_{\text{min}}/\rho_0 \ll 1$, in very high mobility samples. In what follows we will use the value of $\rho_{\text{min}}/\rho_0$ to quantitatively describe the GNMR.

While negative magnetoresistance effect has been known for nearly three decades, systematic experimental studies in very high mobility ($\mu \approx 10^7$ cm$^2$/Vs) 2DESs have appeared only recently. More specifically, Bockhorn et al. reported that the effect quickly disappears with increasing density; $\rho_{\text{min}}/\rho_0$ increased from $\approx 0.3$ to $\approx 0.7$ as the carrier density changed from $\approx 2$ to $\approx 3 \cdot 10^{11}$ cm$^{-2}$. In addition, it was found (for the carrier density of $\approx 2.3 \cdot 10^{11}$ cm$^{-2}$) that the minimum resistivity roughly doubles when the temperature is raised from 0.1 to 0.8 K.

In this Rapid Communication we systematically investigated the roles of temperature and in-plane magnetic field on the GNMR effect observed in high mobility GaAs/AlGaAs heterostructures and quantum wells. In all of our samples, the effect manifests itself as a well-defined minimum in the longitudinal resistivity emerging at $B_0 \approx 1$ kG. At low temperatures and low in-plane fields, the resistivity at this minimum is a small fraction of the zero-field resistivity. Remarkably, the GNMR is quickly suppressed not only by temperature but also by modest (a few kG) in-plane magnetic fields. Our analysis of the low-field magnetoresistivity shows that the observed GNMR cannot be explained by existing theories considering either interaction-induced or disorder-induced corrections to the Drude resistivity.

Our samples (A, B, and C) are lithographically defined Hall bars (widths $w_A = 50$ μm, $w_B = 150$ μm, $w_C = 100$ μm). Sample A is fabricated from a GaAs/AlGaAs Sandia-grown heterostructure with density $n_A \approx 1.6 \cdot 10^{11}$ cm$^{-2}$ and mobility $\mu_A \approx 5.4 \cdot 10^6$ cm$^2$/Vs. Sample C (B) is made from a Princeton-grown 24/30 nm-wide GaAs/AlGaAs quantum well with density $n_B \approx 4.3 \cdot 10^{11}$ cm$^{-2}$ ($n_C \approx 3.4 \cdot 10^{11}$) and mobility $\mu_B \approx 1.0 \cdot 10^7$ cm$^2$/Vs ($\mu_C \approx 1.2 \cdot 10^7$ cm$^2$/Vs). Magnetoresistivity $\rho(B)$ was measured in a $^3$He cryostat at temperatures up to $T = 6.0$ K using a standard low frequency lock-in technique.

In Fig. 1(a) (b) we present the magnetoresistivity $\rho(B)$ in sample A (sample B) measured at $T = 0.5$ K to $1.75$ K [from 0.4 K to 1.6 K], in a step of 0.25 K [0.2 K]. In addition to Shubnikov-de Haas oscillations, both samples reveal a GNMR effect marked by a pronounced minimum which occurs at $B_0 \approx 1$ kG and becomes progressively deeper with decreasing $T$; in contrast to the zero-field resistivity, $\rho_0$, which remains nearly temperature-independent, the resistance at this minimum, $\rho_{\text{min}}$, decays rapidly and becomes a small fraction of the zero-field resistivity. For example, in sample A, $\rho_{\text{min}}/\rho_0 \approx 0.2$ at $T = 0.5$ K.

To examine the MR effect at higher $T$, we present in Fig. 1(c) the magnetoresistivity $\rho(B)$ in sample A at temperatures from 2 K to 6 K, in a step of 0.5 K. Here, we no-

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tice that at $T < 4\, \text{K}$, $\rho(B)$ exhibits phonon-induced resistance oscillations, owing to resonant electron scattering on thermally excited $2k_F$-acoustic phonons.$^{2,25,26,35-37}$ The second order maxima of these oscillations occur at $B \approx 1.3\, \text{kG}$, as marked by $\downarrow$ next to the trace at $T = 3.0\, \text{K}$ in Fig. 1(c).$^{38}$ At $T \gtrsim 4\, \text{K}$, the position of the resistivity minimum is shifted to a higher field ($\approx 1.5\, \text{kG}$) and both $\rho_0$ and $\rho_{\text{min}}$ grow at about the same rate, as evidenced by roughly parallel traces in Fig. 1(c). The spacing between adjacent traces remains roughly constant indicating linear temperature dependence of the resistivity over the entire range of magnetic fields.

For a quantitative analysis of the GNMR we present in Fig. 2(a) the zero-field resistivity, $\rho_0$ (open circles), and the resistivity at the minimum, $\rho_{\text{min}}$ (solid circles), measured in sample A for each $T$ studied. The data clearly show that at $T \gtrsim 2.5\, \text{K}$ (to the right of the dashed vertical line), the resistivities are close to each other, $\rho_0 \simeq \rho_{\text{min}}$, both featuring very similar, approximately linear, temperature dependence. Such behavior is consistent with the electron scattering on thermal acoustic phonons.$^{36,39}$

At lower temperatures, $T \lesssim 2.5\, \text{K}$ (to the left of the vertical line), the $T$ dependences of $\rho_0$ and $\rho_{\text{min}}$ become markedly different. The decrease of $\rho_0$ gets considerably slower as the acoustic phonon contribution becomes irrelevant and the resistivity saturates at a value determined by impurity scattering.$^{36,39,40}$ Quite remarkably, in contrast to $\rho_0$, $\rho_{\text{min}}$ not only continues to drop at lower temperatures but also does so at a much faster rate. Such a sudden change of the temperature dependence of $\rho_{\text{min}}$ is totally unexpected. Quantitatively, once the temperature is lowered from $2.5\, \text{K}$ to $0.5\, \text{K}$, $\rho_0$ decreases by about 20% while $\rho_{\text{min}}$ drops by more than a factor of five.$^{41}$

Using $\rho_0$ and $\rho_{\text{min}}$ shown in Fig. 2(a), we calculate $\rho_{\text{min}}/\rho_0$ and present the result (circles) in Fig. 2(b) as a function of temperature. Results for sample B obtained in the same way using the data in Fig. 1(b) are represented by squares. Both samples show a rapid increase of $\rho_{\text{min}}/\rho_0$ with increasing temperature and eventual saturation at $\rho_{\text{min}}/\rho_0 \approx 1$.

In Fig. 2(a) the zero-field resistivity, $\rho_0$ (open circles) and $\rho_{\text{min}}$ (solid circles) versus $T$ in sample A. Vertical line “separates” high and low temperature regimes in sample A. (b) $\rho_{\text{min}}/\rho_0$ versus $T$ in sample A (circles) and in sample B (squares).
Different tilt angles $\theta$ of the sample might originate from the in-plane field-induced positive magnetoresistance effect, recently reported in very high mobility 2DEG. However, according to Ref. 44 an order of magnitude higher netoresistivity precedes the formation of the deep minimum at $B = B_0$. Therefore, further studies are needed to clarify the origin of the $B_\parallel$-induced suppression of the GNMR effect.

In the remainder of this Rapid Communication, we focus on the temperature dependence of the low-field magnetoresistivity preceding the formation of the deep minimum at $B = B_0$. More specifically, we analyze the low $B$ part of the data in terms of

$$\frac{\rho(B)}{\rho_0} = 1 - \beta B^2,$$

and then examine $\beta$ as a function of temperature. In Fig. 4(a) we plot normalized magnetoresistivity, $\rho(B)/\rho_0$, measured in sample A at $T$ from 0.5 K to 2.0 K, in a step of 0.5 K.$^{45}$ To extract $\beta$ we fit the data using Eq. (1) over the range $|B| \leq 0.5$ kG (cf. dashed lines) and observe that the curvature of the low field resistivity $\beta$ decreases with increasing temperature.

After repeating the fitting procedure for all other $T$ studied, we present extracted $\beta$ in Fig. 4(b) and Fig. 4(c) using log-log and log-linear scale, respectively. First, we notice that at $T \lesssim 1$ K, $\beta$ shows a sign of saturation and can be well described by $\beta = 1.45 - T^2/T_0^2$, $T_0 \approx 1.7$ K [cf. solid curve in Fig. 4(b)]. At higher $T$ the data can be described by either $\beta \propto T^{-2.6}$, $T \gtrsim 2.5$ K [cf. solid line in Fig. 4(b)] or by $\beta \propto \exp(-T/T_1 T_2)$, where $T_1 \approx 1.0$ K for $1.0$ K $ \lesssim T \lesssim 3.5$ K and $T_2 \approx 1.9$ K for $3.5$ K $ \lesssim T \lesssim 6.0$ K [cf. solid lines in Fig. 4(c)]. It is clear that the temperature dependence of $\beta$ is rather complex which is likely a result of one or several crossovers between different regimes. In what follows we examine $\beta(T)$ in terms of existing theoretical models and compare the results of our analysis to other experimental studies.$^{13,33}$

**Quasiclassical disorder model.$^{46}$** predicts a parabolic negative magnetoresistance, see Eq. (1), with $\beta$ given by

$$\beta_d = \frac{e^2}{2\pi n_S p_F} \left( \frac{\tau_L}{2\tau_S} \right)^{1/2}, \quad 0 < \tau_L^{-1} < \tau_S^{-1}.$$  

Here, $\tau_L^{-1}$ and $\tau_S^{-1}$ are long- and short-range disorder momentum relaxation rates, $\tau^{-1} = \tau_L^{-1} + \tau_S^{-1},$ and $n_S$ is the areal density of short-range scatterers, and $p_F$ is the Fermi momentum. Equation (2) is valid for $\beta_d B^2 \ll 1$. 

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**FIG. 3.** (Color online) (a) $\rho(B)$ of sample C at $T = 0.3$ K at different tilt angles $\theta$ (as marked). (b) $\rho_{\text{min}}/\rho_0$ versus $1/\cos \theta$ (circles). Solid curve is a guide to an eye.

**FIG. 4.** (Color online) (a) Solid curves represent $\rho(B)/\rho_0$ measured in sample A at $T$ from 0.5 K to 2.0 K, as marked. Dashed curves are fits to the data, $\rho(B)/\rho_0 = 1 - \beta B^2$, at $|B| \leq 0.5$ kG. (b,c) $\beta$ versus $T$. Solid lines are fits to the data (see text) and dashed lines are $\beta_{\text{min}}$ calculated using Eq. (3).
and at higher $B$ the resistivity is expected to saturate at $\rho_{\text{min}} \approx \rho_0 \cdot (T_S/\tau_L) \ll \rho_0$.64,48

While the disorder model can, in principle, lead to GNMR, it clearly fails to explain our experimental findings. First, as shown above, $\beta$ exhibits strong dependence on temperature which does not enter Eq. (2). Second, we believe that the assumption of $\tau_{L}^{-1} \ll \tau_{S}^{-1}$ is not satisfied in our samples. Indeed, the analysis of Hall field-induced resistance oscillations in sample A49 suggests opposite relation, $\tau_{L}^{-1} \approx 5\tau_{S}^{-1}$. We finally notice that while Ref. 13 concluded that the MR in their samples can be consistently described by Eq. (2),50 neither the temperature dependence nor the validity of $\tau_{L}^{-1} \ll \tau_{S}^{-1}$ condition has been examined.

**Electron-electron interaction model**32,51,52 on the other hand, predicts a temperature-dependent magnetoresistance. In the ballistic regime, $h/\tau \ll k_B T$, and for smooth disorder potential this model also leads to Eq. (1), with $\beta$ given by

$$\beta_{\text{sm}}^i = \mu^2 \rho_0 c_0 R_K / \pi \left( h/\tau k_B T \right)^{1/2}, \quad \tau_{S}^{-1} = 0. \tag{3}$$

Here, $R_K = h/e^2$ is the von Klitzing constant and $c_0 = 3c/(3/2)/16\sqrt{\pi} \approx 0.276$. However, Eq. (3) also fails to describe our findings. Indeed, taking $T = 1$ K as an example, our experiment gives $\beta \approx 1.1$ kG$^{-2}$ which is nearly two orders of magnitude larger than $\beta_{\text{sm}}^i \approx 0.014$ kG$^{-2}$ obtained from Eq. (3). Comparison of $\beta_{\text{sm}}^i$ obtained using Eq. (3) [cf. dashed line in Figs. 4(b) and 4(c)] with our data shows that the discrepancy remains significant over the whole range of $T$ studied. Moreover, it this clear that the interaction model fails to explain our data even on a qualitative level. We also notice that significant disagreement with Eq. (3) was found in Ref. 33 reporting low-temperature $\beta$ which is roughly 30 ($n \approx 2 \cdot 10^{11}$ cm$^{-2}$) to 150 ($n \approx 3 \cdot 10^{11}$ cm$^{-2}$) times larger than $\beta_{\text{sm}}^i$.53

We next consider several scenarios for the observed discrepancy. First, in a realistic high-mobility 2DEG, sharp disorder, which is not present in Eq. (3), plays a crucial role in many of the low-field magnetotransport phenomena.3,9–12,22–24,35,46,48,54–58 For the case of mixed disorder potential Eq. (3) is generalized to

$$\beta_{\text{mix}}^i = \left( 4 - 3\tau_{L}/\tau \right) \sqrt{\tau_{L}/\tau} \beta_{\text{sm}}^i. \tag{4}$$

If $\tau_{L}^{-1} \ll \tau_{S}^{-1}$, there appears a parametrically large factor $4(\tau_{L}/\tau)^{1/2} \gg 1$ which leads to $\beta_{\text{mix}}^i \gg \beta_{\text{sm}}^i$. However, in our sample A, as mentioned above, $\tau_{L}^{-1} \approx 5\tau_{S}^{-1}$ from which we estimate $(4 - 3\tau_{L}/\tau)\sqrt{\tau_{L}/\tau} \approx 1.5$. Such a small factor is clearly not sufficient to explain the discrepancy.

Another possible cause for large $\beta$ is the disorder-induced $T$-independent correction, similar to that given by Eq. (2). Assuming that the contributions are additive, one has $\beta = \beta_d + \beta_i$, where $\beta_d (\beta_i) \propto T^0 (T^{-1/2})$. It is clear, however, that the experimentally obtained $\beta(T)$ cannot be described by such dependence.59

Finally, theory should consider a possibility that the low-temperature magnetoresistance originates primarily from the quasiclassical disorder mechanism which, however, is significantly altered by the electron-electron interactions with increasing temperature.60 However, such a theory remains a subject of future work.

In summary, a giant negative magnetoresistance effect in high-mobility GaAs/AlGaAs heterostructures and quantum wells is marked by a pronounced minimum of the longitudinal resistivity appearing at $B \approx 1$ kG. The temperature dependence clearly reveals a crossover between two distinct regimes. In the high temperature regime, the zero-field resistivity and the minimum resistivity both exhibit linear temperature dependence, due to scattering on thermal acoustic phonons. In the low temperature regime, however, zero-field resistivity quickly saturates but the minimum resistivity continues to decrease at an even faster rate eventually becoming a small fraction of the zero-field resistivity. Unexpectedly, we also find that the GNMR is destroyed not only by temperature but also by very modest (a few kG) in-plane magnetic fields. Finally, our analysis of the low-field magnetoresistivity demonstrates that the GNMR effect cannot be understood by existing theoretical models considering either interaction-induced or disorder-induced corrections, even on a qualitative level. Taken together, these findings provide important clues for emerging theories and should help to elucidate the origin of the GNMR in very high mobility 2DES.

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Significant low-field negative magnetoresistance has been observed in several early experiments.\textsuperscript{2,10}

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Ref. 13 assumed $\tau_s / \tau_{\perp} \approx \rho_{\text{min}} / \rho_0$ and using Eq. (2) found that in their samples (with $\mu$ ranging from 0.9$ \cdot 10^{-2}$ cm$^2$/Vs to 3.0$ \cdot 10^{-7}$ cm$^2$/Vs) $n_s$ varies between (2.6 $\mu$m)$^{-2}$ and (8.0 $\mu$m)$^{-2}$. These values correspond to $n_s^{(3D)} \sim n_s / a_B \sim 10^{-3} \cdot c^3$ and $10^{-2} \cdot c^3$ ($a_B \approx 10$ nm is a Bohr radius in GaAs). The same procedure applied to the data from sample A at $T = 0.5$ K leads to $n_s \sim (1.6 \mu$m)$^{-2}$.

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