Recent Studies of the Resonances at a Cell Tune of 0.25 Using the Ibex Paul Trap

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Abstract. We use the IBEX linear Paul trap to study the resonance at a cell tune of $\frac{1}{4}$ with both equal and unequal transverse tunes, at a range on intensities. We compare this experimental result to simulation using the PIC code Warp. We find that the experimental result differs from the simulation, which may be explained by the ion loss in the IBEX experiment, which more closely replicates a real accelerator. Knowledge of the tune corresponding to greatest beam loss is important for the design of future high intensity machines.

1. Introduction

High intensity hadron accelerators are currently used for a range of applications, both within particle physics and more generally across science and industry [1][2]. As beam intensities increase particle beams become harder to control. When a large number of particles are confined by an accelerator, space charge forces between particles lead to a change in the tune, resulting in a depression and spread in the tune footprint of the beam [3].

The space charge forces and resultant tune spread leads to a beam that is more likely to encounter a resonance. Furthermore, there is also the possibility of coherent excitation of the beam [4]. Coherent excitation is thought to be the most dangerous in terms of beam loss, therefore understanding coherent resonances is necessary for the design of machines with increasing intensity.

Without considering space charge effects the resonant condition is

$$Q_0 = \frac{n}{m}, \quad (1)$$

where $Q_0$ is the bare tune, $n$ the harmonic and $m$ the order of the perturbing driving force.

Taking into account the incoherent space charge tune shift, $\Delta Q$, this becomes

$$Q_0 + \Delta Q = \frac{n}{m}. \quad (2)$$

However, further considerations are required. A self consistent solution must be found as all particles interact, and the excitation of a subset of particles will impact the rest of the distribution. Without applying the smooth approximation [5] this gives the resonant condition

$$Q_0 + C_m \Delta Q = \frac{1}{2} \left( \frac{n}{m} \right), \quad (3)$$
for space charge driven resonances, where the factor $C_m$ is due to the coherent excitation of the beam [6]. $C_m$ factors take values between 0 and 1, increasing in magnitude with the order of the resonance [7][8].

High intensity resonance experiments are difficult to perform in accelerators as any beam loss leads to activation of components of the machine and potential damage. Furthermore, the range of tunes that can be explored in accelerators is typically small. Because of this, it is advantageous to study high intensity effects in other ways.

Instead, a Linear Paul Trap (LPT) can be used to experimentally study the transverse dynamics of high intensity machines [9]. A LPT uses an electric quadrupole to confine ions transversely, meaning that the Hamiltonian of ions confined in an LPT is equivalent to that of particles moving in an accelerator. As ions are low energy a large number can be lost without any activation and a wide range of tunes and intensities can be accessed.

In [10] the Simulator for Particle Orbit Dynamics (S-POD) Paul trap is used to study the change in the location of coherent resonances in tune space for equal emittances and tunes in the horizontal and vertical directions. As we cannot know for certain whether resonances are coherent, incoherent or indeed a result of competing orders, we assume that the resonance condition is instead described as

$$Q_0 + A_m \Delta Q = \frac{1}{2} \left( \frac{n}{m} \right),$$

and extract values of $A_m$ using the S-POD experiment. The resonance at a cell tune of $\frac{1}{4}$ is studied in detail. It is found that the $A_m$ factor for this resonance is smaller than the analytically predicted $C_m$ factor due to fourth order incoherent excitation increasing the emittance of the distribution before the coherent resonance is experienced. In this paper we build on this work further, studying the $\frac{1}{4}$ resonance with unequal horizontal and vertical tunes using the Intense Beam Experiment (IBEX) LPT.

2. The IBEX Paul trap

In this experiment we use the IBEX LPT, located at the Rutherford Appleton Laboratory, UK. The structure of IBEX has been described in detail elsewhere [12] [13], so here we will only outline the basic principles.

The structure of the trapping region is shown in Fig. 1, this trapping region sits inside a larger vacuum vessel maintained at an ultra high vacuum. The ions are confined transversely by four cylindrical rods, to which an alternating radio frequency (rf) voltage is applied ($V_{rf}$). The applied voltage is either a sinusoid or a step function. The same voltage is applied to opposing rods and the voltage applied to the second pair of rods has a 180 degree phase difference from that applied to the first pair. A DC offset of 10 V ($V_{DC}$) is then added to this rf voltage to assist with the extraction of ions from the trap. Voltages are created using an arbitrary wavefunction generator and then amplified.

Where equal transverse tunes are studied a voltage of $V_{rf}(t) + V_{DC}$ is applied to the rods and the amplitude of $V_{rf}(t)$ is used to vary the tune. In the unequal tune case a voltage of $V_{rf}(t) + V_{DC} \pm U$ is applied to the rods, where $\pm$ denotes the difference between the rod pairs, and $U$ is a small DC voltage. The voltage $U$ allows different tunes in the horizontal and vertical directions, examples of the voltages applied are shown in Table 1.

The ions are confined longitudinally by the end caps, shorter sections of cylindrical rod not electrically connected to the central rods. A further positive DC voltage (25 V) is added to the alternating rf voltage and applied to the end caps. As the end caps are short compared to the central rods a rectangular longitudinal potential is created, leading to a longitudinally homogeneous ion distribution.
Ions are created through the introduction of argon gas into the vessel. An electron gun, pointing downwards between the central rods, is used to ionise the argon in the trapping region, creating Ar$^+$ ions.

To extract ions from the trap the positive DC voltage is removed from one set of end caps, so that only the rf voltage remains. The ions then exit the trap towards a Faraday cup, which measures the number of ions extracted.

Table 1: Example voltages for unequal cell tunes, where the rf voltage is a sinusoid with a frequency of 1 MHz.

| $V_{rf}$ (V) | U (V) | Vertical tune | Horizontal tune |
|--------------|-------|---------------|-----------------|
| 55.953       | 3.407 | 0.150         | 0.251           |
| 56.249       | 3.501 | 0.150         | 0.253           |
| 56.397       | 3.545 | 0.150         | 0.255           |

3. Resonance measurement

Figure 2: A selection of experimental data showing ion loss against cell tune for the resonance at a cell tune of $\sim \frac{1}{4}$. The remaining data is not shown here for clarity.
To study the location of the resonance at a cell tune of $\frac{1}{4}$ with equal and split transverse tunes experiments must be performed at a range of tunes and intensities. The number of ions in the trap, and therefore the intensity, is controlled by the amount of time for which the electron gun is on. Alternating gradient focusing is achieved by applying a time varying sinusoidal voltage to the confining rods. At a given intensity the ions are accumulated at a cell tune of 0.15 in both the horizontal and vertical directions ($U = 0$ V, frequency =1 MHz). The tunes are then ramped to the tunes of interest over 100 rf periods.

In the case of equal transverse tunes the tune is altered over these 100 rf periods by only varying the voltage, $V_{\text{rf}}$, applied to the rods, so that the tunes in both transverse directions are altered together. However, by changing $V_{\text{rf}}$ and introducing a small DC offset, $U$, the horizontal tune is altered to match the tunes in the equal tune case while the vertical tune is fixed at 0.15. Using this method, at each intensity the ion distribution is ramped to a number of different tunes across the resonance.

At the final tune the ions are stored for 100 ms and then extracted onto the Faraday Cup. To study a different tune the process is repeated, creating new ions using the electron gun. The voltages, after amplification, are recorded using a high precision oscilloscope. Each data point in the waveform is then converted into a transfer matrix and used to calculate the tune in the trap. The ion loss for different cell tunes and a range of intensities is shown in Fig. 2a and Fig. 2b for the unequal and equal tune cases respectively.

![Figure 3: The emittance growth over time at a cell tune of 0.254, the black line shows horizontal emittance and the red line the vertical emittance. Insets show the horizontal phase space at two different rf periods.](image)

4. Simulation
To understand the experiment further we performed a series of simulations using the particle in cell code Warp. We aimed to simulate the experiment over a shorter time frame, as $10^5$ rf periods would take a prohibitive amount of computational time. Instead we use only 700 rf periods and look at the emittance growth of the ions. Over such short time scales any ions lost are most likely due to the creation of a beam halo and do not accurately represent the tune corresponding to the largest beam loss over longer time scales. An ideal Paul trap is simulated, with no misalignments in the confining rods.
Figure 4: Comparison of emittance growth in simulation.

A matched ion distribution is created in the simulated trap at a cell tune of 0.15. These ions are stored for 50 rf periods before the tune is ramped to the value of interest over 100 rf periods. The ions are then stored at the final tune for 500 rf periods, so that the simulation runs for 700 rf periods in total. The phase space and the emittance in both transverse planes is recorded as well as any ion loss.

Figure 3 shows the emittance growth for the unequal tune case at the cell tune corresponding to maximum emittance growth, as well as the horizontal phase space at two points in this simulation. A detailed description of the simulation for the equal tune case can be found in [10]. Figure 4 shows the comparison between the emittance growth for the equal and unequal tune cases. This agrees qualitatively with the analytical prediction of $C^2 = 0.625$ for the unequal tune case and $C^2 = 0.75$ for the equal tune case [6].

5. Discussion

Figure 5: Ion number against the tune corresponding to the greatest ion loss. Red points correspond to equal tunes and black points to the unequal tunes case.

Figure 3 shows that in the unequal tune case the distribution is first excited by a fourth order incoherent resonance, which at the tune of maximum emittance growth becomes a second order coherent excitation at longer time scales. This is also true for equal tunes [10].
Figure 4 shows the maximum emittance growth with unequal tunes is expected to occur at a slightly smaller tune than that of equal tunes, equivalent to a slightly smaller factor $A_m$.

This does not match the experimental results, as shown in Fig. 5. Experimentally resonance in the unequal tune case occurs at a larger tune and with a larger gradient, $A_m$.

However, emittance growth is studied in simulation, not the beam loss. Over the 700 rf periods of the simulation no ions are lost at the location of the coherent resonance. This lack of ion loss may explain the difference between the simulation and the experimental results. The analytically predicted $C_m$ values do not include effect due to beam loss and neither will the simulation. In the unequal tune case fewer ions will be lost from the distribution as the beam is only excited in one dimension, this will effect the emittance change and tune depression of the distribution as a whole.

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