Majorana-Kondo competition in a cross-shaped double quantum dot-topological superconductor system

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Abstract

We examine the transport properties of a double quantum dot system coupled to a topological superconducting nanowire hosting Majorana quasiparticles at its ends, with the central quantum dot attached to the left and right leads. We focus on the behavior of the local density of states and the linear conductance, calculated with the aid of the numerical renormalization group method, to describe the influence of the Majorana coupling on the low-temperature transport properties induced by the Kondo correlations. In particular, we show that the presence of Majorana quasiparticles in the system affects both the spin-up and spin-down transport channels, affecting the energy scales associated with the first-stage and second-stage Kondo temperatures, respectively, and modifying the low-energy behavior of the system.

Keywords: Kondo effect, Majorana bound states, double quantum dots, numerical renormalization group

1. Introduction

Last decades are marked by intensive expansion in almost every area of physics research. New topics have started, but several other have been revived thanks to the development of theoretical and experimental methods. Two great examples of the latter are the Majorana fermions and the Kondo effect, that have their origin in the 1930s [1, 2]. While the nonmonotonic behavior of metals’ resistance was explained by Kondo and Wilson just a few decades ago [3, 4], Majorana fermions had to wait for proposals from another field of physics, where their signatures as exotic quasiparticles can be searched for [5, 6, 7]. The idea of topological states of matter, which led to possible realizations of Majorana quasiparticles, is currently in the centre of solid-state physics research [8, 9, 10, 11, 12, 13, 14]. This particular field gives promises to use Majorana bound states (MBS), which are the solid-state equivalents of Majorana fermions, for quantum information purposes [15, 16, 17]. This is due to their robustness against local perturbations. The potential use in quantum computing lays e.g. in braiding protocols [18, 19]. One of the first and still very promising setup to realize Majorana bound states is the topological superconducting nanowire [6, 19, 20, 21, 22, 23]. Moreover, several experiments have been done in the pursuit of Majorana quasiparticles in such superconductor-semiconductor devices [24, 25, 26, 27, 28, 29, 30, 31]. Furthermore, studying the properties of quantum dots interacting with MBS is also a very fascinating and stimulating theoretical [32, 33, 34, 35, 36, 37, 38, 39, 40, 41] and experimental [28, 30] field of research. Recently, similar to MBS, quantum dots were reported to potentially serve as a qubit [42, 43].

Taking into account all these similarities, in this paper we focus on the intersection of those fields, analyzing the transport properties of a multi-impurity system, which is a double quantum dot, and the topological superconducting nanowire hosting Majorana bound states at its ends. In particular, we examine the behavior of the spin-resolved spectral functions and the linear conductance, which reveal the interplay between the Majorana and Kondo physics. We show that the coupling to Majorana quasiparticles affects both the spin-up and spin-down local density of states, changing the characteristic Kondo temperatures. Moreover, it destroys the second-stage of Kondo screening by lifting the spectral function at the Fermi energy to half of its maximum value when the usual Kondo effect takes place. These effects are also visible in the behavior of the conductance through the system.

The paper is organized as follows. In Section 2 we describe the Hamiltonian of the system. We introduce the method, which is the numerical renormalization group procedure. Then, in section 3 we present and discuss the numerical results for the spectral functions and conductance. At the end, in section 4 we summarize the paper.

2. Model and method

The model we are considering in this paper is the Anderson-like double quantum dot system, where the first quantum dot is placed in the center of the setup, coupled to two
metallic leads with the strength $\Gamma_r$ and to the second quantum dot by tunneling matrix elements $t$. The second quantum dot, parametrized by $\varepsilon_2$ and $U_2$, is coupled to the first dot by tunneling matrix elements denoted by $t$.

The first quantum dot, described by energy level $\varepsilon_1$ and Coulomb correlations $U_1$, is coupled to two metallic leads by $\Gamma_L$ and to the end of topological superconducting nanowire hosting Majorana quasiparticle $\gamma_1$ by matrix elements $V_M$. The second quantum dot, which describes the Majorana quasiparticle at the end of the nanowire and $V_M$ denotes the relevant tunneling matrix elements [33, 34, 35, 40, 44, 45].

The Majorana quasiparticle can be expressed with the usual fermion operator $f$ as follows $\gamma_1 = (f^\dagger + f)/\sqrt{2}$. For the further calculations we rewrite this part of the Hamiltonian with the fermionic operators, obtaining

$$H_M = V_M(d_1^\dagger - d_1)(f^\dagger + f).$$  

Topological superconducting nanowire hosts a pair of Majorana bound states, however, in our calculations we consider the long nanowire case, where the two Majorana mode wavefunctions do not overlap. Consequently, the second Majorana mode $\gamma_2$ do not show up in our model. We also mention the assumption, that only the spin-down electrons are coupled with the Majorana wire [33].

In this paper we are examining the spectral functions of the considered system, which give insight into the system’s transport behavior. Because in the studied setup the first dot is attached to the leads, we focus on the behavior of the spin-$\sigma$ spectral function of the first quantum dot, which is defined as $A_\sigma(\omega) = -\frac{1}{\pi} \text{Im} G^{\text{R}}_\sigma(\omega)$. Here, $G^{\text{R}}_\sigma(\omega)$ is the Fourier transform of the retarded Green’s function $G^{\text{R}}_\sigma(t) = -i\delta(t)\langle\{d_{1\sigma}(t), d_{1\sigma}^\dagger(0)\}\rangle$. The spectral function is directly related to the linear conductance through the system, which is given by, $G = G_\uparrow + G_\downarrow$, with

$$G_\sigma = \frac{e^2}{\hbar} \frac{4\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \int d\omega \pi A_\sigma(\omega)[-f'(\omega)],$$

where $f'(\omega)$ denotes the derivative of the Fermi function. Due to strong electron correlations in the model, we are using the numerical renormalization group (NRG) procedure introduced by Wilson [4], which allows for obtaining reliable results for the system’s behavior [16, 47].

3. Numerical results and discussion

In this section we present the numerical results obtained with the aid of the NRG. The system was set up...
Figure 2: The spin-up (left column) and spin-down (right column) spectral functions calculated for (a, b) \( t/U = 0.01 \), (c, d) \( t/U = 0.02 \), (e, f) \( t/U = 0.025 \), (g, h) \( t/U = 0.0375 \) and multiple values of \( V_M \), as indicated in the legend. The other parameters are: \( U_1 = U_2 = 0.2, \varepsilon_1 = \varepsilon_2 = -U/2 \), and \( \Gamma = 0.1U \).

with the following parameters: the Coulomb interaction \( U_1 = U_2 = U = 0.2D \), where \( D \) is the half bandwidth used as energy unit, quantum dots' energy levels tuned to the particle-hole symmetry \( \varepsilon_1 = \varepsilon_2 = -U/2 \), \( \Gamma = \Gamma_L + \Gamma_R = 0.1U \), and \( t, V_M \) as described in the figures. The NRG parameters we have used were: the discretization parameter \( \Lambda = 2 \) and the number of states kept during 60 NRG iterations was 3072.

In Fig. 2 we present the spectral functions calculated for fixed value of hopping between the quantum dots and different values of coupling to the Majorana wire. The first row shows the case when a weak hopping between the dots is assumed. Then, the spin-up spectral function shows the usual single-impurity Kondo effect, where the Kondo temperature can be estimated from [38]

\[
T_K \approx \sqrt{\frac{\Gamma U}{2}} \exp \left[ \frac{\pi \varepsilon_1 (\varepsilon_1 + U)}{2 \Gamma U} \right]
\]  

and approximately corresponds to the half width at half maximum of the spectral function. The significant influence of the presence of MBS in the system in the weak hopping regime is the enhancement of \( T_K \) [39]. Much different picture can be observed in the behavior of the spin-down component of the spectral function, see Fig. 2(b). It can be seen that finite coupling to Majorana wire suppresses the low energy spectral function to a finite value of \( 1/2\pi\Gamma \), whereas a local maximum of height \( 1/\pi\Gamma \) remains in certain energy range. When the coupling \( V_M \) becomes considerable (\( V_M \gg t \)), the spectral function becomes fully suppressed to its half value. On the other hand, when the hopping between the dots becomes stronger, the second stage of the Kondo screening occurs in the system, which is revealed through the suppression of the spectral function to zero, see the curve for \( V_M = 0 \) in Figs. 2(c,d). The second-stage Kondo temperature can be defined as [40]

\[
T^* \approx \alpha T_K \exp \left( -\beta T_K/J_{\text{eff}} \right)
\]

with dimensionless constants \( \alpha, \beta \) of the order of unity, and \( J_{\text{eff}} = 4t^2/U \). When the coupling to topological superconductor is turned on, \( T_K \) behaves in a similar way as for the weak hopping regime, being increased as \( V_M \)
is getting stronger. Interestingly, with increasing $V_M$, the second stage of screening becomes suppressed, such that $A_f(0) = 1/\pi \Gamma$. This behavior changes for the spin-down channel, which reveals the half-fermionic character of the coupling to MBS. From the usual two-stage process, the spectral function $A_f(\omega)$ is being lifted in the low-energy scales, and suppressed for higher $\omega$, eventually reaching the half of its maximum value. The above described scenario can also be observed when the hopping between the dots is increased, however, larger values of $V_M$ are needed to suppress the second stage of the Kondo screening.

In Fig. 3 we present the same spectral functions, but now plotted for fixed values of coupling to Majorana quasiparticle, while varying the value of hopping between the quantum dots. The first row shows the case of the bare double quantum dot system, where one can see that the increase of $t$ causes the low-temperature suppression of the spectral function, which results from the second stage of the Kondo effect. Finite interaction between the Majorana wire and DQD system shows the competitive character of these two couplings, where the low-energy value of spectral function is being restored. This behavior depends on the ratio of the two couplings, $V_M$ and $t$, what can be seen particularly for the largest values of either $V_M$ or $t$. When the hopping $t$ does not reach the order of magnitude of $V_M$, the Majorana coupling affects only the spectral function behavior at low energies. However, higher values of coupling to the MBS may give rise to the case when the influence of Majorana quasiparticle in the system is being stronger than the Kondo correlations, destroying the behavior related with the second-stage Kondo effect. These properties again differ between the spin channels, where one can see that not only the second-stage Kondo temperature is being affected, but also the spin-down spectral function becomes half-suppressed as the Majorana coupling is strong enough to exceed the exchange interaction between the quantum dots. It is also important to note that, similarly as in the case of Fig. 2, with an increase of $V_M$, the first-stage Kondo temperature $T_K$ becomes enhanced. This can be especially seen as a shifting of the steep increase (due to Kondo correlations) of the spin-up spectral function to larger energies, which is visible in the left column of Fig. 4.

Finally, in Fig. 4 we present the spin-resolved linear conductance calculated as a function of the coupling to Majorana wire $V_M$ and the hopping between the dots $t$ for different temperatures, as indicated in the figure. Consider first the case of lowest temperature presented in the bottom row of Fig. 4. When the hopping between the dots is relatively weak, $G_\uparrow = e^2/h$ due to the Kondo effect, while $G_\downarrow = e^2/2h$ due to the quantum interference with the Majorana wire. Increasing $t$ results in an enhancement of the second-stage Kondo temperature $T^*$, and once $T^* \gtrsim T$, the second-stage Kondo effect comes into play. As a consequence, the conductance in both spin channels becomes suppressed. Note also that the value of $t$, at which the conductance drops, increases with raising the coupling to the Majorana wire, which clearly indicates that $V_M$ affects the Kondo behavior in the system. When the temperature increases, one observe a change in the conductance for low values of both $t$ and $V_M$, see the case of $T/U = 5 \times 10^{-4}$ and $T/U = 10^{-3}$ in Fig. 4. In the spin-up channel the conductance becomes suppressed, while in
the spin-down channel it gets enhanced. This is due to the fact that now thermal fluctuations smear out the behavior, as described above, resulting from the quantum interference with Majorana wire. Interestingly, in the case when \( T/U = 5 \times 10^{-3} \), shown in the first row of Fig. 4 the behavior of conductance is even more changed. Because in this situation \( T \approx T_K \), \( G_\downarrow \) hardly depends on \( V_M \). This can be understood by analyzing the spectral function behavior presented in the right column of Fig. 2 for \( \omega \approx T \). At this energy, \( A_\downarrow (\omega \approx T) \approx 1/2m \hbar \), and it hardly depends on \( V_M \), which results in \( G_\downarrow \approx e^2/2h \), as long as the hopping between the dots is relatively weak. On the other hand, the spin-up conductance exhibits a different behavior. Clearly, we see an increase of \( G_\uparrow \) with raising \( V_M \) for low values of \( t \). Again, this can be understood by inspecting the dependence of the spin-up spectral function on \( V_M \) at the energy of the order of assumed temperature, see the left column of Fig. 2

4. Conclusions

In this paper we have analyzed the behavior of the Kondo correlated double quantum dot system coupled to the superconducting topological nanowire hosting Majorana bound state in the cross-shaped geometry, where one quantum dot is in the center of the system, being coupled to the leads. We have focused on the transport properties of the system in the Kondo regime, where the two-stage Kondo effect is present. We have presented and discussed the behavior of the spectral functions and the conductance, where the aforementioned effects can be found. The results were obtained with the aid of numerical renormalization group method, which allowed us to analyze the transport properties in an accurate manner.

In our work we have uncovered the most interesting aspects of coupling the DQD system to the MBS. A fierce competition between the Kondo and Majorana physics is present not only in the spin-down electron channel, which is directly coupled to the nanowire, but also the spin-up channel reveals a strong influence of the Majorana coupling in the system. Not only the restoring of the low-energy spectral behavior has been shown, but also the half-fermionic character of the MBS, which demonstrates itself when the spin-down spectral function is restored only to the half of its value. Moreover, the evolution of the characteristic fractional value of conductance with the hopping between the dots, coupling to Majorana wire and the temperature was analyzed. The presented numerical analysis reveals an interesting competing character of interactions driving the Kondo and Majorana physics.

Acknowledgements

This work was supported by the National Science Centre in Poland through the Project No. 2018/29/B/ST3/00037. The computing time at the Poznań Supercomputing and Networking Center is acknowledged.

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