THE NAVIER-STOKES EQUATIONS IN NONENDPOINT BORDERLINE LORENTZ SPACES

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ABSTRACT. It is shown both locally and globally that $L^\infty_t(L^{3,q}_x)$ solutions to the three-dimensional Navier-Stokes equations are regular provided $q \neq \infty$. Here $L^{3,q}_x$, $0 < q \leq \infty$, is an increasing scale of Lorentz spaces containing $L^3_x$. Thus the result provides an improvement of a result by Escauriaza, Seregin and Šverák (Russian) Uspekhi Mat. Nauk 58 (2003), 3–44; translation in Russian Math. Surveys 58 (2003), 211–250), which treated the case $q = 3$. A new local energy bound and a new $\epsilon$-regularity criterion are combined with the backward uniqueness theory of parabolic equations to obtain the result. A weak-strong uniqueness of Leray-Hopf weak solutions in $L^\infty_t(L^{3,q}_x)$, $q \neq \infty$, is also obtained as a consequence.