Electrodynamics under a Possible Alternative to the Lorentz Transformation

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Abstract

A generalization of the classical electrodynamics for systems in absolute motion is presented using a possible alternative to the Lorentz transformation. The main hypothesis assumed in this work are: a) The inertial transformations relate two inertial frames: the privileged frame $S$ and the moving frame $S'$ with velocity $v$ with respect to $S$. b) The transformation of the fields from $S$ to the moving frame $S'$ is given by $H' = a(H - v \times D)$ and $E' = a(E + v \times B)$ where $a$ is a matrix whose elements depend of the absolute velocity of the system. c) The constitutive relations in the moving frame $S'$ are given by $D' = \epsilon E'$, $B' = \mu H'$ and $J' = \eta E'$. It is found that Maxwell’s equations, which are transformed to the moving frame, take a new form depending on the absolute velocity of the system. Moreover, differing from classical electrodynamics, it is proved that the electrodynamics proposed explains satisfactorily the Wilson effect.

1 Introduction

The extension of Maxwell’s electrodynamics from systems at rest to those in motion was the fundamental problem of the beginning of the twentieth century. In those days, it seemed impossible to detect the absolute motion of the earth (that is, its motion with respect to the preferred reference frame called ether) by means of electromagnetic experiments. For the purpose of explaining the negative results of these experiments, Lorentz’s idea was simple: if a system is in uniform translational motion and no fundamental phenomenon is modified, this means that the

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electrodynamic equations are form invariant under a certain transformation. Thus, Lorentz was the first to discover that Maxwell’s equations admit a transformation that leaves them covariant.

However, it was a well-known fact that the mechanical laws did not have the covariance property under the Lorentz transformation. Einstein gave the decisive step introducing the so-called principle of relativity as a restriction scheme for natural laws. Mathematically, this principle can be expressed as “the Lorentz covariance of all the basic laws of physics under a change of inertial reference frame”. This covariance expresses the group property of the transformations since the velocity of every inertial frame is considered as relative. In this way, it was understood why it had been impossible to determine the “absolute” velocity by some physical experiment.

Nowadays, physicists agree that the Lorentz transformation describes a fundamental symmetry of all natural phenomena. However, it would be interesting to know if there is any alternative to such transformation. In other words, is there any transformation between two inertial frames which both agree with the experimental evidence (that is, kinematically equivalent to the special relativity) and generalize the Lorentz transformation? Or, on the contrary, is the Lorentz transformation an unavoidable consequence of nature?

In this sense, Robertson [1] proposed to replace the Einstein’s postulates of relativity with hypotheses suggested by certain typical optical experiments as a way of testing the relativity theory. Without these hypotheses, the transformations so obtained describe a family of transformations which are determined except for three functions depending on the “absolute” velocity of the reference frame. However, an implicit hypothesis underlying these transformations is the postulate of equality of the one-way velocity of light in all directions. A generalization of the Robertson transformations that eliminates this last postulate was studied by Vargas [2, 3]. This new family of transformations, therefore, depends on four arbitrary functions of the velocity of the reference frame. Each member of this family is fixed with a choice of these four functions and thus it describes a possible alternative to the Lorentz transformation.

The most interesting member of this family of transformations was found by Tangherlini [4] and studied by Mansouri & Sexl [5], Chang [6, 7] and Rembieliński [8] among others. Particularly, in this transformation the four-dimensional line element is considered as an invariant [2, 6, 7, 8, 9, 10, 11] and, therefore, the Minkowski metric appears modified in the moving system losing its diagonality. As a consequence, the co-variant and contra-variant components have different properties: if the contra-variant component of the temporal coordinate is only dilated, its co-variant component mixes space and time; while the opposite happens with the spatial coordinate.
The expression of Maxwell’s equations under this transformation was deduced by Chang \[6, 7\] and by Rembieliński \[8\]. However, a certain ambiguity for the definition of the fields $\mathbf{E}$ and $\mathbf{B}$ appears as a consequence of using a non-diagonal metric with a 4-line element invariant. In order to solve this ambiguity, Rembieliński \[8\] has supplemented Maxwell’s equations with the equation of motion for the test charge. As we shall see, this ambiguity is absent under the so-called “inertial transformations” and, therefore, no operational definition of electromagnetic field via equation of motion for charged particles is necessary.

The inertial transformations proposed by Selleri have the same mathematical form as those of Tangherlini, but they admit a different interpretation since the four-dimensional line element is not defined and, therefore, cannot be an invariant \[12, 13\]. This means that the geometrical structure of the four-dimensional space-time is not required into the framework of the inertial transformation and, consequently, the geometrical structure is given by the three-dimensional Euclidean geometry. Moreover, time is (in one sense) similar to that of classical physics because the concept of “absolute simultaneity” of the Galilean physics remain valid. \(^1\) Given these transformations, the one-way velocity of light is the same in all directions only in the privileged frame. Therefore, in a moving reference frame the one-way velocity of light will be different in every direction. This situation, however, is not in conflict with the known experimental results since the one-way velocity of light has never been measured (see ref.\[13, 14, 15\] for a discussion about the non-invariant one-way velocity of light). Another characteristic, shared with the Tangherlini transformations, is that the inertial transformations do not form a group. As a consequence, the absolute velocity will appear in any physical law expressed in an inertial frame (i.e. the velocity with respect to the preferred frame). \(^2\) In spite of these facts, the inertial transformations provide a suitable framework within which the effects of time dilatation, Fitzgerald contraction, the relativistic kinematics of particles, the phenomenon of aberration of light and Doppler effect can be satisfactorily explained \[13, 14, 15, 16\]. Thus, inertial transformations have become a viable alternative to the Lorentz transformation.

In this paper we generalize the classical Maxwell electrodynamics for moving systems using the inertial transformations. We show that such an electrodynamics may be obtained assuming that the fields transform in a “form similar” to that

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\(^1\) Absolute simultaneity means that two events simultaneous in $S$ (i.e. taking place at the same $t$) are judged also simultaneous in $S'$ (and vice-versa). This property, being a consequence of the absence of space variables in the transformation of time, does not imply that time is absolute. On the contrary, time-dilation phenomena similar to those of the special relativity theory \[13, 14\] can be explained thanks to the $\beta$-dependent factor in the transformation of time (see eq.\[11\]).

\(^2\) It must be stressed, however, that the Lorentz covariance and the absolute reference frame can coexist under the Tangherlini transformations, as has been shown by Rembieliński \[8, 9\].

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of special relativity. Finally, we prove that the electrodynamics proposed explains satisfactorily the Wilson effect.

2 Inertial transformations

In this section we shall present the inertial transformations in their original form as given by Selleri [12, 13] and generalize them in a vectorial form independent of the orientation of the axes.

The inertial transformations relate two inertial frames: $S$ (privileged system) and $S'$ (moving system), and are given by

\[
\begin{align*}
    t' &= \gamma^{-1} t \\
    x' &= \gamma(x - vt) \\
    y' &= y \\
    z' &= z
\end{align*}
\]

where $\gamma^{-2} = 1 - \beta^2$, $\beta^2 = v^2/c^2$, with $v$ the velocity of $S'$, and $c$ the two-way velocity of light. It is important to note that the inertial transformations are a direct generalization of the Galilean transformation, as it can be seen doing $\gamma = 1$.

The inertial transformations as given in (1) are restricted to the case of parallel axis of the two coordinate systems and the motion of $S'$ is in the direction of the $x$ axis. It is easy to obtain a generalization which is independent of the orientation of the spatial axes [6]. To do so, we observe that the spatial component will be given by the same relativistic expression, while the temporal component will be the same as (1). That is,

\[
\begin{align*}
    t' &= \gamma^{-1} t \\
    r' &= r + v \left( \frac{\gamma^{-1}}{v^2} r \cdot v - \gamma t \right)
\end{align*}
\]

(2)

In order to obtain the inverse transformation, we cannot proceed as in the relativistic case, that is, inverting the sign of the velocity $v$. This is due to the difference in form of the spatial component of the direct and inverse transformations

\[
\begin{align*}
    t &= \gamma t' \\
    x &= \gamma^{-1} x' + v \gamma t' \\
    y &= y' \\
    z &= z'
\end{align*}
\]

(3)

at variance with the Lorentz transformation. To obtain such a transformation in vectorial form, we write the spatial component as

\[
    r = r' + v \Theta,
\]

(4)
where, in order to determine \( \Theta \), we must multiply scalarly the equation (4) by \( \mathbf{v} \) and replace the scalar product \( \mathbf{r} \cdot \mathbf{v} \) by the expression

\[
\mathbf{r} \cdot \mathbf{v} = \gamma^{-1}(\mathbf{r}' \cdot \mathbf{v}) + v^2 \gamma t',
\]

in the resultant equation. Note that the equation (5) is obtained from (2) multiplying scalarly \( \mathbf{r}' \) by \( \mathbf{v} \), and then solving \( \mathbf{r} \cdot \mathbf{v} \). Finally, substituting \( \Theta \) we find

\[
\begin{aligned}
\begin{cases}
t = \gamma t' \\
r = \mathbf{r}' + \mathbf{v} \left[ \frac{\gamma^{-1}-1}{v^2} \mathbf{r}' \cdot \mathbf{v} + \gamma t' \right]
\end{cases}
\end{aligned}
\]

A difference that appears in the spatial component of this inverse transformation with respect to those obtained with the Lorentz transformation should be noted: the exponent of \( \gamma \) is \(-1\), while in the inverse Lorentz transformation is \(+1\).

### 3 Generalized classical electrodynamics

As it is well-known, the general framework to deal with electromagnetic phenomena is given by Maxwell’s equations. In a system at rest in the privileged frame \( S \), the Maxwell equations are given by:

\[
\begin{align*}
\partial_t \mathbf{B} &= -\nabla \times \mathbf{E}, \\
\partial_t \mathbf{D} + \mathbf{J} &= \nabla \times \mathbf{H}.
\end{align*}
\]

Moreover, these equations are supplemented by

\[
\begin{align*}
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \cdot \mathbf{D} &= \rho, \\
\partial_t \rho + \nabla \cdot \mathbf{J} &= 0,
\end{align*}
\]

with the constitutive relations (for isotropic bodies),

\[
\begin{align*}
\mathbf{D} &= \epsilon \mathbf{E}, \\
\mathbf{B} &= \mu \mathbf{H}, \\
\mathbf{J} &= \eta \mathbf{E},
\end{align*}
\]

where \( \epsilon \) is the dielectric constant, \( \mu \) is the permeability, and \( \eta \) is the electric conductivity and these will be supposed constant throughout the medium. Moreover, in empty space we have \( \eta = 0 \), \( \mathbf{D} = \epsilon_0 \mathbf{E} \) and \( \mathbf{B} = \mu_0 \mathbf{H} \), where \( \epsilon_0 \mu_0 = 1/c^2 \) is the two-way velocity of light.

\[\text{3} \text{We will use the MKSQ system of units.}\]
In order to obtain Maxwell’s equations in the moving frame \( S' \), first we must write the transformation formulae for the spatial and temporal derivatives. In the simplest case, given by the transformation, (1), we have:

\[
\begin{aligned}
\partial_t &= \gamma^{-1} \partial'_t - \gamma v \partial'_x \\
\partial_x &= \gamma \partial'_x \\
\partial_y &= \partial'_y \\
\partial_z &= \partial'_z
\end{aligned}
\] (10)

and for the vectorial case given by (2), we have:

\[
\begin{aligned}
\partial_t &= \gamma^{-1} \partial'_t - \gamma \mathbf{v} \cdot \nabla' \\
\mathbf{\nabla} &= (\gamma - 1) \frac{\mathbf{v}}{v^2} \mathbf{v} \cdot \nabla' + \nabla' .
\end{aligned}
\] (11)

We note that the Galilean case is obtained with \( \gamma = 1 \). It will be convenient to work with a form of this equation similar to (10), but without the distinctive role of the \( x \) axis. To obtain it, we write the gradient operator of eq. (11) as

\[
\mathbf{\nabla} = \frac{\gamma - 1}{v^2} [\mathbf{v} (\mathbf{v} \cdot \nabla') - v^2 \nabla'] + \gamma \nabla' ,
\]

using the decomposition \( \mathbf{\nabla} = (\mathbf{\nabla}_\parallel, \mathbf{\nabla}_\perp) \), with the subscripts \( \parallel \) and \( \perp \) denoting parallel and perpendicular to the velocity \( \mathbf{v} \). Then, we find

\[
\begin{aligned}
\partial_t &= \gamma^{-1} \partial'_t - \gamma \mathbf{v} \cdot \mathbf{\hat{\nabla}}' \\
\mathbf{\nabla} &= \gamma \mathbf{\hat{\nabla}}'
\end{aligned}
\] (12)

in which we have defined \( \mathbf{\hat{\nabla}}' \equiv (\mathbf{\nabla}_\parallel, \gamma^{-1} \mathbf{\nabla}_\perp) \) and replaced \( \mathbf{v} \cdot \nabla' \) with \( \mathbf{v} \cdot \mathbf{\hat{\nabla}}' \) in the first equation.

We can see that for any vector \( \mathbf{A} \), the temporal derivative will be given in the moving frame \( S' \) by

\[
\partial_t \mathbf{A} = \gamma^{-1} \partial'_t \mathbf{A} - \gamma (\mathbf{v} \cdot \mathbf{\hat{\nabla}}') \mathbf{A} .
\]

Applying vectorial algebra, it becomes

\[
\partial_t \mathbf{A} = \gamma^{-1} \partial'_t \mathbf{A} - \gamma \mathbf{\hat{\nabla}}' \times (\mathbf{A} \times \mathbf{v}) - \gamma \mathbf{v} \mathbf{\hat{\nabla}}' \cdot \mathbf{A} .
\] (13)

Now it is easy to obtain the Maxwell equations in terms of the coordinates of the moving frame \( S' \). Substituting both the expression that is equivalent to (13) and the transformation of the gradient (12) in the equations (7), we obtain

\[
\begin{aligned}
\gamma^{-1} \partial'_t \mathbf{B} - \gamma \mathbf{\hat{\nabla}}' \times (\mathbf{B} \times \mathbf{v}) - \gamma \mathbf{v} \mathbf{\hat{\nabla}}' \cdot \mathbf{B} &= -\gamma \mathbf{\hat{\nabla}}' \times \mathbf{E} , \\
\gamma^{-1} \partial'_t \mathbf{D} - \gamma \mathbf{\hat{\nabla}}' \times (\mathbf{D} \times \mathbf{v}) - \gamma \mathbf{v} \mathbf{\hat{\nabla}}' \cdot \mathbf{D} + \mathbf{J} &= \gamma \mathbf{\hat{\nabla}}' \times \mathbf{H} .
\end{aligned}
\] (14)
The supplementary equations transform if we apply (12) directly:

\[ \gamma \tilde{\nabla}' \cdot B = 0 , \]
\[ \gamma \tilde{\nabla}' \cdot D = \rho , \]
\[ \gamma^{-1} \partial'_t \rho - \gamma (v \cdot \tilde{\nabla}') \rho + \gamma \tilde{\nabla}' \cdot J = 0 . \]  

(15)

By considering the first two equations (15), we may simplify the expressions (14) to obtain

\[ \gamma^{-1} \partial'_t B = -\gamma \tilde{\nabla}' \times (E + v \times B) , \]
\[ \gamma^{-1} \partial'_t D + J - \rho v = \gamma \tilde{\nabla}' \times (H - v \times D) . \]  

(16)

It is important to note that we have assumed no transformation form for the fields yet. If we assume that the fields are the same in both frames \( S \) and \( S' \), that is, if the following relations are valid,

\[ H(r, t) = H'(r', t') , \quad B(r, t) = B'(r', t') , \]
\[ E(r, t) = E'(r', t') , \quad D(r, t) = D'(r', t') , \]

the equations (16) become, for low velocities (i.e., for \( \gamma = 1 \)), an electrodynamics “invariant” under Galilean transformations. That is, those equations express the electromagnetism in the privileged frame as described from a moving frame with velocity \( v \). However, it is well-known that such an electrodynamics does not agree with the experimental results. Therefore, in order to obtain an electrodynamics in agreement with the experiment, we shall assume that the fields do not remain invariant in both reference frames. That is, we assume that the expressions into the rotor on the right hand side of the equations (16) represent (except for a matrix-coefficient) the fields in the moving frame. In other words, we shall postulate the following transformation laws:

\[ H' = a(H - v \times D) , \]
\[ E' = a(E + v \times B) , \]  

(17)

where the matrix \( a \) has the form

\[ a = \begin{pmatrix} a_{\parallel}(v) & 0 \\ 0 & a_{\perp}(v) \end{pmatrix} \]

with \( a_{\parallel} \) and \( a_{\perp} \) non nulls for any \( v \), and to be determined. The simple form assumed for the matrix \( a \) is due to the following: (a) It has to be of 2x2-dimension due to the
decomposition of the fields in their parallel and perpendicular components. (b) It could have their four elements not equals and depending of the value of the absolute velocity of the system. (c) However, the non-diagonal elements are zeros due to the invariance of the systems under rotations, because if the parallel and perpendicular components are mixed (as it happens when the non-diagonal elements are non nulls), this invariance would disappear. Thus, it remains only two elements not necessarily equals to be determined. Moreover, it should be noted that we have assumed the same matrix for both $H'$ and $E'$.

Now we can obtain the transformation law for the fields $B'$ and $D'$. To do this, we use the constitutive relations in vacuum, valid not only in the privileged frame $S$ (i.e., $D = \epsilon_0 E$ and $B = \mu_0 H$) but also in the moving frame $S'$ (i.e., $D' = \epsilon_0 E'$ and $B' = \mu_0 H'$). Multiplication by $\mu_0$ and $\epsilon_0$ in the first and second eq. (17) and the use of $\mu_0 \epsilon_0 = 1/c^2$ yield

$$B' = a[B - \frac{1}{c^2}(v \times E)]$$

$$D' = a[D + \frac{1}{c^2}(v \times H)] .$$

(18)

The inverse transformation can be obtained with the expressions of $E$ and $H$ of the equations (17):

$$H = a^{-1}H' + \epsilon_0 v \times (a^{-1}E' - v \times B) ,$$

$$E = a^{-1}E' - \mu_0 v \times (a^{-1}H' + v \times D) ,$$

where $a^{-1}$ is the inverse matrix of $a$ given by

$$a^{-1} = \left( \begin{array}{cc} a_{\parallel}^{-1}(v) & 0 \\ 0 & a_{\perp}^{-1}(v) \end{array} \right) .$$

From the above equations, we can obtain the components which are parallel and perpendicular to the velocity $v$:

$$H_{\parallel} = a_{\parallel}^{-1}(H' + v \times D')_{\parallel} , \quad H_{\perp} = \gamma^2 a_{\perp}^{-1}(H' + v \times D')_{\perp} ,$$

$$E_{\parallel} = a_{\parallel}^{-1}(E' - v \times B')_{\parallel} , \quad E_{\perp} = \gamma^2 a_{\perp}^{-1}(E' - v \times B')_{\perp} ,$$

(19)

where clearly $(v \times D')_{\parallel} = 0$ and $(v \times B')_{\parallel} = 0$. Using the constitutive relations in vacuum, we have

$$B_{\parallel} = a_{\parallel}^{-1}(B' + \frac{1}{c^2}v \times E')_{\parallel} , \quad B_{\perp} = \gamma^2 a_{\perp}^{-1}(B' + \frac{1}{c^2}v \times E')_{\perp} ,$$

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\[ D_\parallel = a_\parallel^{-1}(D' - \frac{1}{c^2}v \times H')_\parallel, \quad D_\perp = \gamma^2 a_\perp^{-1}(D' - \frac{1}{c^2}v \times H')_\perp. \] (20)

Now we can obtain the Maxwell equations valid in the moving frame \( S' \). We calculate the second member of (16) by substituting the equations (17):

\[ -\gamma \hat{\nabla}' \times (E + v \times B) = -\gamma a^{-1} \hat{\nabla}' \times E' \]

\[ = (-a^{-1}_\parallel \hat{\nabla}' \times E'_\perp, -a^{-1}_\perp \gamma \hat{\nabla}' \times E'_\perp - a^{-1}_\parallel \hat{\nabla}' \times E'_\parallel) \]

\[ \gamma \hat{\nabla}' \times (H - v \times D) = \gamma a^{-1} \hat{\nabla}' \times H' \]

\[ = (a^{-1}_\parallel \hat{\nabla}' \times H'_\perp, a^{-1}_\perp \gamma \hat{\nabla}' \times H'_\perp + a^{-1}_\parallel \hat{\nabla}' \times H'_\parallel). \] (21)

where, in the final expressions, the first component of the vector is the parallel component.

Substituting now the expressions (20) and (21) in the equations (16), we obtain

\[ \begin{cases} 
\gamma^{-1} a^{-1}_\parallel \partial'_t B'_\parallel = -a^{-1}_\parallel \hat{\nabla}' \times E'_\perp \\
\gamma a^{-1}_\parallel \partial'_t (v \times E')_\perp = -a^{-1}_\perp \gamma \hat{\nabla}' \times E'_\perp - a^{-1}_\parallel \hat{\nabla}' \times E'_\parallel \\
\gamma a^{-1}_\parallel \partial'_t D'_\parallel + (J - \rho v)_\parallel = a^{-1}_\parallel \hat{\nabla}' \times H'_\perp \\
\gamma a^{-1}_\parallel \partial'_t (v \times H')_\perp + (J - \rho v)_\perp = a^{-1}_\perp \gamma \hat{\nabla}' \times H'_\perp + a^{-1}_\parallel \hat{\nabla}' \times H'_\parallel 
\end{cases} \]

Moreover, in \( S' \) the first two supplementary equations become,

\[ a^{-1}_\parallel \gamma \hat{\nabla}' \cdot B'_\parallel + a^{-1}_\perp \gamma^2 \hat{\nabla}'_\perp \cdot B'_\perp + a^{-1}_\parallel \frac{1}{c^2} \hat{\nabla}'_\parallel \cdot (v \times E')_\parallel + a^{-1}_\perp \frac{1}{c^2} \hat{\nabla}'_\perp \cdot (v \times E')_\perp = 0, \]

\[ a^{-1}_\parallel \gamma \hat{\nabla}' \cdot D'_\parallel + a^{-1}_\perp \gamma^2 \hat{\nabla}'_\perp \cdot D'_\perp + a^{-1}_\parallel \frac{1}{c^2} \hat{\nabla}'_\parallel \cdot (v \times H')_\parallel - a^{-1}_\perp \gamma^2 \frac{1}{c^2} \hat{\nabla}'_\perp \cdot (v \times H')_\perp = \rho. \]

The above equations acquire a simple form if we take

\[ \gamma^{-1} a^{-1}_\parallel = a^{-1}_\perp. \] (22)

Thus, we can write the final vectorial form of Maxwell’s equations in the system \( S' \):

\[ \partial'_t B' + \frac{1}{c^2} \partial'_t (v \times E') = -\hat{\nabla}' \times E', \]

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∂'_D' - \frac{1}{c^2} \partial'_t (v \times H') + J' = \nabla' \times H' , \quad (23)

where \( J' \) has components \( J'_\parallel \overset{df}{=} a_\perp (J - \rho v)_\parallel \) and \( J'_\perp \overset{df}{=} a_\parallel (J - \rho v)_\perp \). The supplementary equations can be written in a unified form by the use of (22),

\[ \nabla' \cdot B' + \frac{1}{c^2} \nabla' \cdot (v \times E') = 0 , \]

\[ \nabla' \cdot D' - \frac{1}{c^2} \nabla' \cdot (v \times H') = a_\parallel \rho' , \]

\[ \partial'_t \rho' + \gamma \nabla'_\parallel (J - \rho v)_\parallel + \nabla'_\perp (J - \rho v)_\perp = 0 . \quad (24) \]

where we have defined, \( \rho' \overset{df}{=} \gamma^{-1} \rho \). Comparing the last equation (24) with the definitions of \( J'_\parallel \) and \( J'_\perp \), we can see that considering \( a_\parallel = 1 \) and \( a_\perp = \gamma \), we can write finally the equation for the conservation of charge in the frame \( S' \):

\[ \partial'_t \rho' + \nabla' \cdot J' = 0 . \]

To complete our equations we need to know the constitutive relations for a material medium. Here we shall assume that the following constitutive relations are valid in the frame \( S' \):

\[ D' = \epsilon E' , \quad B' = \mu H' , \quad J' = \eta E' \quad (25) \]

That the constitutive equations have the same form of eq.(19) with the same material constants \( \epsilon, \mu \) and \( \eta \) in both \( S \) and \( S' \) is an assumption which can be tested indirectly testing their consequences. In other words, if this hypothesis is true must be inferred from the way this framework describes the accepted knowledge and predicts results amenable of empirical control.

With the expression of \( J' \) and the transformation of \( E' \) (eq.(17)), the Ohm law in the moving frame, \( J' = \eta E' \), becomes in the same relativistic form:

\[ \gamma (J - \rho v)_\parallel = \eta (E + v \times B)_\parallel , \]

\[ (J - \rho v)_\perp = \eta \gamma (E + v \times B)_\perp . \quad (26) \]

Now we can determine the transformation laws for the fields \( B' \) and \( D' \) in a material medium by following the same steps used for the fields in vacuum. That is, we multiply by \( \mu \) and \( \epsilon \) in the first and second equation (17), and then using \( D = \epsilon E \) and \( B = \mu H \), and the new assumption (25) we obtain:
\[ B' = a(B - \mu \epsilon v \times E) \]
\[ D' = a(D + \mu \epsilon v \times H) \quad (27) \]

As a consequence, the transformation properties of the fields \( B' \) and \( D' \) will depend on the properties of the medium, which is reasonable in the present context of absolute motion.\(^4\)

In summary, we have obtained Maxwell’s equations, the supplementary equations and the transformations of the fields for a frame in motion with respect to the privileged frame. It is important to note that the equations (23) and (24) express electromagnetism as described in the moving frame \( S' \) with absolute velocity \( v \). It should be noted, moreover, that the final form of the equations has been obtained thanks to the relation (22) and to the particular choice of the coefficients \( a_\parallel = 1 \) and \( a_\perp = \gamma \). With these, it is easy to see from the eqs. (17) and (19), that the fields transform in the same way as under the Lorentz transformation. However, a such choice of the coefficients could be wrong and the equations could take a different form. In the following section, we shall prove that our electrodynamics is both theoretically consistent and in agreement with the experiment (and with the special relativity) only if the relation (22) is valid.

## 4 The Rowland effect

From the expressions (23) we can see a term (known as “convection current”) depending on the absolute velocity of the inertial reference frame \( S' \). This convection current \( \rho v \) produces a magnetic field (Rowland’s effect), that was used by Roentgen in 1888 in an attempt to measure the so-called “wind of ether” (for a discussion of this effect see ref.[17, 18]). In Roentgen’s experiment, the free charges which occur at the surface of a dielectric material submitted to an external electric field produce a magnetic field when the dielectric is set into motion. A variant of the Roentgen experiment was proposed by Wilson. In this experiment, a uniform magnetic field is applied parallel to the plates of a condenser. When the dielectric is moved

\[^4\text{Certainly this situation is different to the relativistic case in which the constitutive relations (25) in } S' \text{ take a new form when is transformed to the unprimed frame } S \text{ (see ref. [17]). This is due principally to the different hypothesis assumed: In the relativistic theory we begin from the expression for the transformation law of } B' \text{ and } H' \text{ (for example), and then we use the relation } B' = \mu H' \text{ valid in } S' \text{ to obtain the constitutive relation in } S. \text{ Here, in contrast, we begin from the eq. (17) and then, with the constitutive relations in } S \text{ and } S', \text{ we obtain the transformations for } B' \text{ and } D'.\]
perpendicularly to the magnetic field and parallel to the plates, the condenser is charged.

The theory of relativity predicts a value for the surface charge density which appears on the plates of

$$\sigma_{\text{rel}} = -\gamma^2 v H_0 (\epsilon \mu - \epsilon_0 \mu_0) \simeq -v H_0 (\epsilon \mu - \epsilon_0 \mu_0) \ ,$$

in agreement with the experimental result [18]. On the other hand, it is not difficult to see that a Galilean electrodynamics described by the equations (16) (i.e. without transformation of the fields) predicts:

$$\sigma_{\text{gal}} = -v H_0 \epsilon \mu \ .$$

In this section, we shall see that the surface charge density predicted by our generalized electrodynamics correctly explains the Wilson effect. In order to do that, let us consider a dielectric material with dielectric constant $\epsilon$, permeability $\mu$ and with a velocity $v$ (in the direction of the $x$ axes) with respect to the two conductor plates (fixed in the frame $S$). Let us suppose that the external magnetic field $H_0$ is parallel to the plates of the condenser and perpendicular to the velocity. The surface charge density $\sigma$ which appears on the conducting plates may be calculated in the privileged frame $S$. We calculate the perpendicular components of $E'$ and $H'$ outside the dielectric. As we know, these are given by the relations (17):

$$H'_\perp = a_\perp H_0 \ ,$$

$$E'_\perp = a_\perp \mu_0 v H_0 \ ,$$

$$D'_\perp = \epsilon_0 E'_\perp = a_\perp \mu_0 \epsilon_0 v H_0 \ .$$

We can now calculate the expressions inside the dielectric material. Using the second supplementary equation (24) in integral form (in $S'$):

$$\int_{V'} \rho' dV' = a_\perp^{-1} \int_{C.S.'} [D' - \frac{1}{c^2} (\mathbf{v} \times \mathbf{H}')] \cdot d\mathbf{S}'$$

we can obtain:

$$\sigma' = a_\perp^{-1} (a_\perp \mu_0 \epsilon_0 v H_0 - D'_\perp) \ ,$$

and with the use of the relation $D'_\perp = \epsilon E'_\perp$ we obtain
Hence we can find the corresponding relation in the privileged system $S$. Substituting (30) in the expression (19),

$$E_\bot = \frac{a_\bot \mu_0 \epsilon_0 v H_0 - a_{\parallel} \sigma'}{\epsilon} .$$

(30)

Since the condenser plates are connected by a wire so that $E_\bot = 0$ and taking into account that the surface charge density, which is computed in $S'$, satisfies the relation $\sigma' = \gamma^{-1} \sigma$, we finally obtain 5

$$\sigma = -a_\parallel^{-1} a_\bot \gamma v H_0 (\epsilon \mu - \epsilon_0 \mu_0) .$$

(31)

Therefore, this electrodynamics is in agreement with the experiment (and with the special relativity) if we take $a_\parallel^{-1} a_\bot = \gamma$. It should be noted that these values have been obtained independently from the relation (22). In summary, we have found the same relativistic values for the coefficients of the matrix $a$ as those proposed in the previous section under the theoretical criterion of symmetry in our equations.

## 5 Conclusions

We have obtained a generalization of the classical Maxwell’s electrodynamics for systems in (absolute) motion with respect to the privileged frame. As a main hypothesis, we have assumed that the fields transform in a “similar” way (as given in (17)) to the relativistic case, i.e. mixing electric and magnetic fields, but being proportional to some coefficients which are dependent on the velocity of the reference frame.

From a theoretical point of view, we have found that the most convenient choice of those coefficients is given by $a_\parallel = 1$ and $a_\bot = \gamma$. These values determine the transformation of the fields in the same relativistic form. However, as it is expected from transformations that do not satisfy the principle of relativity, the Maxwell equations lose their form invariance by a term depending on the absolute velocity of the reference frame.

As a way of testing our theory, we have solved the Wilson and we have obtained the same relativistic result if we take $a_\parallel^{-1} a_\bot = \gamma$, as was assumed under the theoretical basis of symmetry in our equations. In conclusion, historically the Wilson

5Note that this result can also be obtained from eq. (29) replacing $D'_\bot$ by the expression given in eq. (27).
experiment has been considered as one of the crucial tests to reject the ether theories. However, it is interesting to note that this effect may be explained into the Galilean framework (as it should be expected from an effect produced at low velocities) if certain transformation of the fields is admitted. Therefore, the property of transformation of the fields (with $\gamma = 1$ in (17)) does not appear as a relativistic effect, but as an effect of the motion with respect to the privileged frame.

Finally, it should be stressed that the aim of this work was to investigate if it is possible to obtain an electrodynamics with different transformation properties as those of the relativistic theory. A previous study in this line was realized by Chang [6, 7] and by Rembieliński [8] who derived the Maxwell equations under the Tangherlini transformations (in which the 4-line element is an invariant). Rembieliński [8, 9], moreover, have shown that the Tangherlini transformations form a subclass of the $v$-dependent transformations of the Lorentz group. As an interesting consequence, the Maxwell equations proposed by Chang [6] are obtained by Rembieliński [8] by means of this $v$-transformation. Thus, into this framework the absolute reference frame and the Lorentz covariant can coexist. In contrast, the geometrical properties of the space and time are very different in the inertial transformations because the 4-line element and the metric tensor are not defined. This is the reason why Maxwell’s equations in the moving frame $S'$ have a different form in our formulation compared with those obtained by Chang [6, 7] and by Rembieliński [8]. For example, the first Maxwell equation (7) and the first supplementary equation (8) remain invariant in ref. [6, 8] whereas these equations change in our approach. It could be interesting to investigate other possible similarities and differences between these two frameworks. Moreover, it is mandatory to exhibit the concordance (if any) of these theories with other experiments. In any case, these investigations will provide an improvement in the understanding of the electromagnetic phenomena.

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