CONVEXITY IN LOCALLY CONFORMALLY FLAT MANIFOLDS WITH BOUNDARY

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Let $B_1$ denote the open unit ball of $\mathbb{R}^n$, $n \geq 3$. Given a closed subset $\Lambda \subset B_1$, we will consider a complete Riemannian metric $g$ on $\overline{B_1} \setminus \Lambda$ of constant positive scalar curvature $R(g) = n(n-1)$ and conformally related to the Euclidean metric $\delta$. We will also assume that $g$ has nonnegative boundary mean curvature. Here, and throughout this paper, second fundamental forms will be computed with respect to the inward unit normal vector.

In this talk we prove

**Theorem 0.1.** If $B \subset B_1 \setminus \Lambda$ is a standard Euclidean ball, then $\partial B$ is convex with respect to the metric $g$.

This theorem is motivated by an analogous one on the sphere due to R. Schoen [3]. He shows that if $\Lambda \subset S^n$, $n \geq 3$, is closed and nonempty and $g$ is a complete Riemannian metric on $S^n \setminus \Lambda$, conformal to the standard round metric $g_0$ and with constant positive scalar curvature $n(n-1)$, then every standard ball $B \subset S^n \setminus \Lambda$ is convex with respect to the metric $g$. Schoen used this geometrical result to prove the compactness of the set of solutions to the Yamabe problem in the locally conformally flat case. Later, D. Pollack also used Schoen’s theorem to prove a compactness result for the singular Yamabe problem on the sphere where the singular set is a finite collection of points $\Lambda = \{p_1, \ldots, p_k\} \subset S^n$, $n \geq 3$ (see [2]).

We shall point out that to find a metric satisfying the hypotheses of Theorem 0.1 is equivalent to finding a positive solution to an elliptic equation with critical Sobolev exponent. The idea of the proof is to get geometrical information from that equation by applying the Moving Planes Method as in [1].

**References**

[1] B. Gidas, W. M. Ni and L. Nirenberg, *Symmetry and related properties via the maximum principle.* Comm. Math. Phys. 68 (1979), 209–243.

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[3] R. Schoen, *On the number of constant scalar curvature metrics in a conformal class.* Differential Geometry: A symposium in Honor of Manfredo do Carmo. Pitman Monogr. Surveys Pure Appl. Math 52, Longman Sci. Tech., Harlow (1991) 311–320.

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