SU(2) Running Coupling Constant and Confinement in Minimal Coulomb and Landau Gauges

Attilio Cucchieri∗†, Tereza Mendes∗† and Daniel Zwanziger‡

∗IFSC-USP, Caixa Postal 369, 13560-970 São Carlos, SP, Brazil
‡Department of Physics, NYU, New York, USA

We present a numerical study of the space-space and time-time components of the gluon propagator at equal time in the minimal Coulomb gauge, and of the gluon and ghost propagators in the minimal Landau gauge. This work allows a non-perturbative evaluation of the running coupling constant and a numerical check of Gribov’s confinement scenarios for these two gauges. Our simulations are done in pure SU(2) lattice gauge theory at β = 2.2. We consider several lattice volumes in order to control finite-volume effects and extrapolate our results to infinite lattice volume.

1. INTRODUCTION

An essential step for understanding and extracting physical information from gauge theories is the elimination of redundant gauge degrees of freedom. This is usually done by choosing a representative on each orbit of gauge-related fields (gauge fixing). In Ref. [1] Gribov showed that the Coulomb and Landau gauge-fixing conditions do not fix the gauge fields uniquely, i.e. there are many gauge-equivalent configurations satisfying the Coulomb or Landau transversality condition. These Gribov copies do not affect perturbative calculations, but their elimination could play a crucial role for non-perturbative features of gauge theories, such as color confinement and hadronization.

In order to get rid of the problem of spurious gauge copies, Gribov restricted the physical configuration space to the region Ω of transverse configurations, for which the Faddeev-Popov operator is non-negative. This region is delimited by the first Gribov horizon, defined as the set of configurations for which the smallest, non-trivial eigenvalue of the Faddeev-Popov operator is zero. We now know that Ω is not free of Gribov copies and that the physical configuration space has to be identified with the fundamental modular region [2,3]. Nevertheless, the region Ω is of interest in numerical simulations, since it is the space of configurations satisfying the usual lattice Coulomb or Landau gauge condition.

The restriction of the path integral, which defines the partition function, to the region Ω implies a rigorous inequality [4] for the Fourier components of the gluon field. From this inequality, which is a consequence only of the positiveness of the Faddeev-Popov operator, it follows that the region Ω is bounded by a certain ellipsoid E. This bound causes a strong suppression of the (unrenormalized) transverse gluon propagator $D^{tr}$ in the infrared limit [1,4]. More precisely, it was proven that, in the infinite-volume limit, $D^{tr}$ vanishes at zero momentum, although the rate of approach to 0, as a function of the momentum or of L, was not established. This is in marked contrast to the divergence in the infrared limit of the free massless propagator.

Finally, because of entropy considerations [5], the Euclidean probability gets concentrated near the first Gribov horizon where the inverse of the

∗Talk presented by A. Cucchieri.
†Research partially supported by FAPESP, Brazil (Project No. 00/05047-5).
‡Research partially supported by the National Science Foundation, grant no. PHY-0099393.

1In the Coulomb case the transversality condition and the positiveness of the Faddeev-Popov operator are satisfied on each time slice.

2This has been verified numerically in the minimal Landau gauge.
Faddeev-Popov matrix diverges. This causes an enhancement of the ghost propagator \( G(k) \) in the infrared limit \[3\]. This enhancement is a clear indication of a long-range effect in the theory that may result in color confinement.

The confinement scenario is particularly simple in the minimal Coulomb gauge where the ghost propagator determines directly the Coulomb interaction \[4,5\]. In fact, in this case, confinement of color, i.e., the enhancement at long range of the color-Coulomb potential \( V(R) \), is due to the enhancement of \( G(\vec{k}) \) at small momenta. At the same time, the disappearance of gluons from the physical spectrum is manifested by the suppression at \( \vec{k} = 0 \) of the propagator \( D_{ij}(\vec{k}, k_4) \) of 3-dimensionally transverse would-be physical gluons. Remarkably, \( V(R) \) is the instantaneous part of the 4-4 component of the gluon propagator \( D_{44}(\vec{x}, t) \), and is a renormalization-group-invariant quantity \[6\]. Its Fourier transform \( \tilde{V}(\vec{k}) \) may serve to define the running coupling constant of QCD by considering \( x_0 g_2^2(\vec{k}) = \vec{k}^2 V(\vec{k}) = g_0^2 \vec{k}^2 D_{44}(\vec{k}) \), where \( x_0 = 12N/(11N - 2N_f) \) has been calculated in \[7\]. Clearly, if the color-Coulomb potential \( V(R) \) is governed by a string tension at large distances, i.e. \( \tilde{V}(\vec{k}) \) goes like \( 1/\vec{k}^2 \) at small momenta, then \( g_2^2(\vec{k}) \sim 1/\vec{k}^2 \) in the infrared limit.

Similarly, in Landau gauge, one can consider \[8\] the running coupling constant \( g_2^2(k) = g_0^2 [k^2 D(k)] [k^2 G(k)]^2 \). This is also a renormalization-group-invariant quantity since (in Landau gauge) \( Z_g Z_3^{1/2} Z_2 = 1 \). In this case we obtain \( g_2^2(k) \sim k^{-2} \) if, for example, \( D^{tr}(k) \sim \text{const} \) and \( G(k) \sim k^{-4} \) in the infrared limit. On the contrary, if the gluon propagator goes to 0 in the infrared limit and the ghost propagator blows up not faster than \( k^{-4} \) then \( g_2^2(k) \) has an infrared fixed point \[9\].

We test these theoretical predictions with data from a numerical study of \( SU(2) \) lattice gauge theory, without quarks, in the minimal Coulomb and Landau gauge at \( \beta = 2.2 \). Simulations were done at different lattice volumes \( L^4 \), in order to check for finite-size effects and, if possible, to extrapolate to infinite lattice volume. Details of notation and numerical simulations are given, for the Coulomb case, in \[10\] and will be presented, for the Landau case, in \[11\]. For these simulations we have used a cluster of ALPHA work-stations (Coulomb and Landau data) and a PC cluster (Landau data) at the Dept. of Mathematical Physics of the University of São Paulo (DFMA/USP). In the Landau case we have also used a PC cluster at the Institute of Physics of the University of São Paulo, São Carlos (IFSC/USP).

![Figure 1. Plot of the gluon propagators \( D^{tr}(\vec{k}) \) (lower curve) and \( D_{44}(\vec{k}) \) (upper curve) as a function of the square of the lattice momentum \( \vec{k}^2 \) for \( L = 28 \) (symbols * and \( \bigcirc \) respectively) and \( L = 30 \) (symbols \( \bigtriangleup \) and \( \bigtriangledown \) respectively). Notice the logarithmic scale on the y axis.](image)

2. RESULTS

Our data in the minimal Coulomb gauge \[10\] clearly show (see Fig. 1) that the equal-time transverse gluon propagator \( D^{tr}(\vec{k}, L) \) passes...
poles occur at complex \( m^2 \) for \( L = 22 \) (symbols \( * \) and \( \bigcirc \) respectively) and \( L = 26 \) (symbols \( \triangle \) and \( \triangledown \) respectively). Notice the logarithmic scale on the \( y \) axis.

through a maximum and decreases as the momentum \( \vec{k} \) approaches \( \vec{0} \) (for fixed \( L \)), and that \( V(\vec{k}) \) is more singular than \( 1/|k|^2 \) at low \( \vec{k} \), which indeed corresponds to a long-range color-Coulomb potential. We have also obtained \([12]\) an excellent 2-pole fit for \( D_{\perp}\tau(k, L) \). Our fit indicates that the poles occur at complex \( m^2 = x(L) \pm iy(L) \). In the infinite-volume limit we have \( m^2 = 0 \pm iy \), for \( y = 0.375 \pm 0.162 \) in lattice units, or \( y = 0.330 \pm 0.142 \) GeV\(^2\) for the location of the gluon poles in \( k^2 \). (It follows from the Nielsen identities \([13]\) that these poles are independent of the gauge parameters.)

Similarly, in Landau gauge (see Fig. 2), the transverse gluon propagator \( D_{\perp}\tau(k, L) \) is suppressed in the infrared limit, while the ghost propagator \( G(k) \) is more singular than \( 1/|k|^2 \). Also here a 2-pole fit can probably be used to fit the data for \( D_{\perp}\tau(k, L) \). In this case, however, we need to improve the statistics in order to have better control over the fits, and we need to simulate at larger lattice volumes (we went up to \( 26^4 \)) to probe the infinite-volume limit.

Finally, for the running coupling constant \( g^2 \) we consider \([10]\) the fitting formula \( k^2 = \Lambda^2 \exp[(bg^2)^{-1}][(bg^2)^{-r} + (bg^2)^{-\tau}]^{-1} \), which implicitly defines \( g^2 \). Here \( r = 102/121 \), and \( \Lambda, b, \tau \) and \( \alpha \) are fitting parameters. For small \( g^2 \), which corresponds to large momenta, this formula is dominated by the first term in the denominator, whereas for large \( g^2 \), i.e. small momenta, it is dominated by the second term in the denominator. In particular, \( \alpha \) governs the strength of the singularity of \( g^2 \) in the infrared limit; the case \( g^2 \sim 1/k^2 \) corresponds to \( \alpha \sim 1 \).

In the Coulomb case we obtain \([10]\) \( \alpha = 1.9 \pm 0.3 \); this corresponds to \( g_0^2 \sim 1/|k|^2 \), and \( V(\vec{k}) \sim 1/(|k|^2)^3 \) at low momentum. In the Landau case we have \( \alpha = 2.5 \pm 0.3 \), which implies \( g_0^2 \sim 1/k^{6.8} \). In both cases we have fitted the data with a low momentum cut-off at \( \vec{k} = 0.5 \). Of course, an extrapolation in \( \beta \) will be necessary to determine the strength of these singularities in the continuum limit.

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