Cuts and penalties: a comment on “The clustering of ultra-high energy cosmic rays and their sources”

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In a series of papers we have found statistically significant correlations between arrival directions of ultra-high energy cosmic rays and BL Lacertae objects. Recently, our calculations were partly repeated by Evans, Ferrer and Sarkar \cite{efs} with different conclusions. We demonstrate that the criticism of Ref. \cite{efs} is incorrect. We also present the details of our method.

I. INTRODUCTION

Identification of sources of the ultra-high energy cosmic rays (UHECR) is a key to understanding their nature. The observed small-scale clustering of UHECR \cite{1,2,3,4,5} suggests that already existing data \cite{2,3,4,5} may contain information sufficient to make first steps in this direction. In a series of papers \cite{8,9,10,11,12,13,14} we have shown that there exist significant correlations between the arrival directions of UHECR and BL Lacertae objects (BL Lacs), and therefore BL Lacs are likely to be sources of UHECR. Although present small dataset does not allow to determine with certainty the details of UHECR production and propagation, interesting hints may be obtained \cite{11,12,13,14}.

The first evidence of connection between UHECR and BL Lacs was found in Ref. \cite{8} where we have shown that the combined set of AGASA events with \( E > 4.8 \times 10^{19} \) eV and Yakutsk events with \( E > 2.4 \times 10^{19} \) eV (identified previously as the sets with largest clustering) correlates strongly with most powerful BL Lacs. The statistical significance of this correlation was found to be \( 6 \times 10^{-5} \) with penalty factor included (for explanation of penalty factor see below).

In the recent paper by Evans, Ferrer and Sarkar (EFS) \cite{efs} the validity of this result was called into question. The rational part of the criticism of EFS boils down to the following two issues: i) choice of cuts on BL Lacs and/or calculation of the penalty factor ii) choice of the cosmic ray set. Below we address these issues and demonstrate that criticism of EFS is unjustified. We also clarify some frequently arising questions related to our analysis.

II. STATISTICAL SIGNIFICANCE OF CORRELATIONS: CUTS AND PENALTY FACTORS

A. What is penalty factor

In statistical analysis, one nearly always has to make cuts in order to improve signal-to-noise ratio. The question is how to take them into account correctly. A common wisdom is that cuts should be fixed \textit{a priori}, i.e., based on theoretical considerations. In that case the cuts simply limit the number of data points, but do not alter the calculation of probabilities.

Not always this is possible. For instance, in the case of UHECRs, their acceleration mechanism is not known. How to impose cuts in a catalogue of astrophysical objects in order to select actual UHECR emitters? In which band — radio, optical, X-ray, \( \gamma \), TeV? To which flux limit? In Ref. \cite{8} we proposed an approach to this problem which consists in adjusting cut(s) so as to maximize the signal, and then compensating this cut adjustment by a penalty factor.

A penalty factor takes into account the “number of independent attempts” made when searching for the best signal. For instance, if two independent catalogues were tried, the penalty factor is 2 (in the limit of small probabilities). If \( N \) catalogues tried are not independent, as in the case of cuts in a single catalogue, the penalty factor is smaller than \( N \) and should be calculated by means of the Monte-Carlo simulation.

In correlation analysis of cosmic rays, the quantity of interest is the probability \( p_{\text{data}} \) that the observed excess of cosmic rays around source positions is the result of a chance coincidence. This probability depends on cuts made in the source catalogue; the latter are adjusted so that the probability is minimum (correlations are maximum). The cut adjustment should be compensated by the penalty factor which is calculated as follows. A random set of cosmic rays is generated and treated exactly as the real data: the same cuts are tried and the resulting minimum probability \( p_{\text{MC}} \) is determined. This procedure is then repeated for a large number of random “cosmic ray” sets, and \( p_{\text{MC}} \) is determined each time. The number of occurrences of a value \( p_{\text{MC}} \leq p \) is then counted as a function of \( p \). Divided by the total number of sets, this gives the probability \( P(p) \) that the adjustments of cuts produces \( p_{\text{MC}} \leq p \) for a random set of cosmic rays. \( P(p_{\text{data}}) \) is the correct measure of the significance of observed correlations.

When no adjustment of cuts is made one obviously has \( P(p) = p \), and the quantitative measure of correlation is the probability \( p \) itself. When cuts are adjusted, small
probabilities appear more often by construction, and this relation is modified. The modification can be written in terms of the penalty factor $F(p)$,

$$P(p) = F(p) \cdot p.$$  

(1)

In the presence of cut adjustment, the significance of correlations is determined by the product $F(p_{\text{data}}) \cdot p_{\text{data}}$.

To summarize, all parameters characterizing UHECR sources and UHECRs themselves can be subject to cuts. Some of these cuts can be decided a priori; they reflect (and depend on) physical assumptions made. These cuts do not require penalty. One can call these cuts fixed. Alternatively, in the absence of physical arguments, the cut should be chosen (adjusted) so as to maximize the signal. These cuts can be called adjustable. These are the adjustable cuts which imply non-trivial penalty factors; the latter can be calculated in a way explained above.

**B. UHECR set**

In our analysis [50], the cosmic ray set was fixed on physical grounds using results of previous publications [1], and was not adjusted in search for correlations. The motivation was as follows. It was observed earlier [51, 52, 53] that highest-energy cosmic rays exhibit remarkable property: they auto-correlate on the angular scale consistent with the angular resolution of detectors. The mere existence of these correlations suggests that a relatively small number of point sources of cosmic rays is contributing a considerable fraction of the UHECR flux [54], and that the trace of the original directions to sources is not lost completely. The identification of these sources via cross-correlation analysis may therefore be possible. Within the hypothesis that UHECR clusters are due to point sources, the strongest correlation signal is expected with the UHECR set which has most significant auto-correlations. The latter requirement selects [55] AGASA events with $E > 4.8 \times 10^{19}$ eV and Yakutsk events with $E > 2.4 \times 10^{19}$ eV. The observed angular size of clusters also fixes the angular scale at which correlations with sources are expected.

The fact that AGASA and Yakutsk sets have different energy cuts in this approach is, of course, disturbing. It may, however, be explained by different energy calibration of the two experiments, or smaller number of the Yakutsk events. In either case, within our assumptions it is inconsistent to change energy cuts or to drop Yakutsk events once autocorrelations are found in this set.

The authors of Ref. [1] have rejected the Yakutsk events because Yakutsk array has angular resolution not as good as AGASA. By itself, worse angular resolution does not imply that correlations with sources must be absent in Yakutsk set: even though the angular resolution is worse, the density of UHECR events around actual sources is larger as compared to a random set, and one has an excess in counts even at small angles. The actual presence of autocorrelations in Yakutsk data supports this statement. Therefore, it is not correct that “the correlations found at smaller (than Yakutsk angular resolution) angles cannot be meaningful” [1].

**C. Correlations with BL Lacs**

The authors of Ref. [1] have essentially reproduced our Monte-Carlo simulations: solid curve of Fig. 3 of Ref. [1] agrees with our calculations if Yakutsk events are discarded. They concluded, however, that we have underestimated the penalty factor. This conclusion was based on i) a comparison of correlations in the case when one particular set of cuts was chosen to the case when no cuts on BL Lacs were made at all ii) smaller correlations of BL Lacs with a different set of UHECR. Neither such a comparison, nor the dependence on the UHECR set have anything to do with the penalty factor. The UHECR set was fixed as described above and was not adjusted during the calculation, thus requiring no penalty. The penalty associated with the adjustment of BL Lac set should be calculated as described above. This calculation was performed in our paper [50] as follows.

The most complete catalog of QSO [56] contains 306 confirmed BL Lacs for which the apparent magnitude, redshift (where known) and 6 cm radio flux are listed (for several objects the 11 cm radio flux is also given). In Ref. [50] we considered BL Lacs with $z > 0.1$ as suggested by statistics of clustering. (We also examined a complimentary small set without further cut adjustment. This “independent attempt” adds one to the penalty factor.) We adjusted cuts on magnitude and 6 cm radio flux in the following way. First, the grid of $11 \times 11$ cuts was fixed. Namely, the cut on magnitude was varied from $m < 16$ to $m < 20$ with the step 0.4; simultaneously the cut on 6 cm radio flux was changed from 0.21 Jy to 0.01 Jy with the step 0.02 Jy. Note that further “refining” of this set of cuts would not change the results as the sets of BL Lacs obtained would overlap almost completely. The lowest probability (best correlation) found in the real data in this way equals to $p_{\text{data}} = 4 \times 10^{-6}$ [20].

To calculate the penalty factor associated with cut adjustment, a random configuration of 65 UHECR events was generated. Then its correlation with all $11 \times 11$ subsets of BL Lacs was examined and the minimum probability $p_{\text{min}}$ over 111 cases was found and recorded. The whole procedure was repeated for the next random configuration. A total of $N_{\text{tot}} = 10^5$ random configurations were treated in this way. The number of occurrences $N(p)$ of $p_{\text{min}} < p$ was calculated as a function of $p$. Divided by $N_{\text{tot}}$ it determines the corrected probability $P(p)$ and the penalty factor $F(p)$ as defined in Eq. (1). The resulting penalty factor $F(p)$ is shown in Fig. 1. The error bars correspond to statistical errors in determination of $N(p)$.
Taking the penalty factor at the minimum probability obtained for the real data, \( F(4 \times 10^{-6}) \approx 14 \) and adding 1 as penalty for BL Lacs with \( z < 0.1 \) as explained above, one finds the significance of correlations including penalty,

\[
P(4 \times 10^{-6}) \approx 6 \times 10^{-5}.
\]

The significance is \( \sim 4\sigma \) in terms of Gaussian distribution. It should be pointed out that, in order to get the correct significance, the penalty factor should be multiplied by the minimum probability obtained for the real data with the same set of cuts.

We stress again that the absence of correlations of cosmic rays with the entire sample of known BL Lacs does not mean anything. Physically, cosmic rays correlate with the brightest BL Lacs. Including dimmer BL Lacs in the analysis dilutes the correlations, and this is the only conclusion one draws from the analysis made in Ref. [1].

D. “Correlations” with GRB

The authors of Ref. [1] have chosen GRBs as a control set where correlations are not expected; it has to be compared with the case of BL Lacs. From the fact that correlations with the whole catalogue are absent in both cases while there are many coincidences, the authors of Ref. [1] concluded that GRBs correlate with UHECR “just as well as do BL Lacs!”, implying that correlations with BL Lacs are due to cut adjustment. This conclusion can be tested by Monte-Carlo simulation: one has to check whether cut adjustment leads to apparent correlations with GRBs. It does not, as we now demonstrate.

The Fourth BATSE Gamma-Ray Burst Catalog [17] contains 1637 objects from which we cut out, in sequential order (according to observation date and time), 5 non-overlapping sets of 306 objects each (recall that 306 is the number of confirmed BL Lacs in the catalogue [16]). In each set we have looked for maximum correlations by optimizing cuts. We present two different tests: one-dimensional cut which allows transparent graphical representation, and two-dimensional cut which mimics more precisely the case of BL Lacs. We took the same maximally auto-correlated UHECR data set as in Refs. [1, 8].

In the first test we replace the \( 11 \times 11 \) cuts on magnitude and radio flux by 121 cut on the total number of objects (we take them in sequential order, so that first few cuts correspond to including 3, 6, 9 ... first objects, and the last cut corresponds to including all 306 objects) [21]. Like in the case of BL Lacs, we then calculate the probability of the observed excess of UHECRs around GRBs as a function of the cut. The results are presented in Fig. 2. The smallest probability found in all 5 sets is \( > 4\% \) (which by itself is not statistically significant). In view of the similar number of cuts in two cases, Fig. 1 provides an estimate [22] for the penalty factor, \( F(0.04) \approx 5 \), in a single set of 306 GRBs. Multiplying the best probability, the penalty factor and the number of independent GRB sets one gets a number of order 1.

To mimic the case of BL Lacs we perform a similar test with two independent cuts. The first cut is done as before on the sequential number of the event. The second cut is imposed on the time (in seconds of the day, UT) of the event. Note that these two cuts are virtually uncorrelated since BATSE registered roughly one event per day. We have searched for the best correlation signal on the grid of \( 11 \times 11 \) equidistant cuts from 1 to 306 and from 1 to 86400, respectively. In the above five catalogs consisting of 306 GRBs each, the following lowest probabilities were found: 9.5%, 1.4%, 20%, 1.8%, 24%. Multiplying the
lowest of these probabilities 0.014 by the penalty factor $F(0.01) \approx 8$ and by the number of independent GRB sets, we again obtain a number of order 1.

These examples are a good illustration of compensation between the effects due to cut adjustment and the penalty factor. When compensated by the penalty factor, cut selection does not introduce apparent correlations when in reality they are absent.

To avoid confusion, we note again that we used GRBs here as a control set, and imposed physically unmotivated cuts on purpose.

### III. Subtleties of Correlation Analysis

In this section we discuss some subtleties of correlation analysis which were not mentioned in Ref. [1]. We hope that this will answer a number frequently arising questions regarding our work and correlation analysis in general.

#### A. Why completeness of the BL Lac catalogue is not necessary for establishing correlations with UHECR

A catalogue of astrophysical objects is called complete when it is believed to contain more than, say, 90% of all existing objects of a given type, in a given region of the sky, and down to some fixed level of luminosity in a given waveband(s). The catalogue [16], used in our analysis, is not complete simply because it contains all objects known up to date without further selection.

It is a common wisdom that completeness of the catalogue is crucial for statistical analysis. The reason is obvious: in an incomplete catalogue, the distribution of objects in brightness, position, etc., reflects not only their actual abundance, but also observational bias. When one studies, for instance, the evolution of abundance of some objects with time, one has to make sure that $\log N/\log S$ dependence reflects actual change in spatial density rather than the fact that remote (and therefore dim) objects are more difficult to observe. Indeed, as it is, the catalogue [16] is not suitable for any analysis where statistical properties of BL Lacs are of interest.

In the correlation analysis of UHECR the question is different: given a set of candidate sources (BL Lacs in the case at hand), is the distribution of cosmic rays random, or does it peak around BL Lac positions? Clearly, these are the statistical properties of cosmic rays which are of interest in that case, not the statistical properties of BL Lacs. Technically speaking, when calculating the correlation function by the algorithm of Ref. [8], the cosmic ray directions are simulated, while source positions are held fixed. For this reason the method is applicable without change to studying, for instance, a correlation of UHECR with one particular direction (say, the Galactic center), i.e. in the case when the notion of completeness does not apply.

Completeness is not necessary for establishing the fact of correlations: if statistically significant correlations are found with an incomplete catalogue, this is a real signal. Incompleteness of the catalogue of BL Lacs cannot be a source of correlations with UHECR. Indeed, the objects which have to be added (removed) from a catalogue to make it complete are absent from (present in) the catalogue for reasons not related to cosmic rays. Therefore, correlations with UHECR can only be weakened by such an incompleteness.

#### B. UHECR autocorrelations

The UHECR set used in our calculations is known to contain event clusters — in fact, it was chosen in such a way that auto-correlations are maximum. When studying cross-correlations of such a set with potential sources, one has to be careful to take clusters into account when calculating the probability of chance coincidence. Namely, one has to make sure that the signal observed in the data is not due to chance coincidence of clusters of cosmic ray events with candidate sources. For a given cluster, such a coincidence is roughly as probable as for a given single event, but contributes more into correlation function. This could produce artificial enhancement of correlations if not taken into account in the Monte-Carlo simulation.

In our calculations [8] this problem is solved by introducing in each Monte-Carlo cosmic ray set the same number of doublets and triplets as there are in the real data. Then chance coincidences between clusters and candidate sources happen in simulated sets as often as in the real data, and thus are correctly accounted for. This is confirmed by calculations presented in Sect. [12] where correlations with GRB catalogs do not appear despite auto-correlations in the UHECR dataset.

#### C. Choice of angular scale

The observed angular size of clusters suggests the angular scale at which correlations with sources are expected. In our analysis [8] we have fixed it to the previously published [2] AGASA value of 2.5°. This value is treated as not adjustable.

When the angular resolution of the experiment is known, there exists a preferred choice of the angular scale — the scale at which correlations are expected to be maximum. This angle can be determined by means of the Monte-Carlo simulation as follows. One has to generate cosmic ray sets which are correlated with sources (taking into account the experimental angular resolution) and then measure angle at which the correlation signal is maximum. In the case of AGASA events this procedure gives the result very close to 2.5° (see dotted curve on
Fig. 3 of Ref. [8], so in fact the choice of Ref. [8] was close to optimal.

Alternatively, one may choose not to fix the angular scale and treat it as a free parameter. Then one should adjust it to maximize the correlation and calculate corresponding penalty factor. A serious problem of this approach is that the result would depend on the limits within which the angular scale is varied. So finally one would have to input in one or the other way the information about expected angular scale of correlations in order to obtain a definite answer. We do not follow this approach in our calculations.

D. Hidden penalty

We believe that we have accounted for all contributions to the penalty factor (“effective number of tries”) directly related to our work. Still, the obtained significance may be somewhat overestimated. The point is that we are not the first who are looking for correlations between UHECR and astrophysical objects. Some of these attempts have been published in the literature; the others may have never been reported. All these attempts should contribute, in principle, into the penalty factor. However, it does not seem possible to account correctly for all such contributions.

The way around this problem is obvious. First, only very low values of $P$ should be interpreted as a signal (in our calculations we considered $P < 10^{-4}$ to be sufficiently low to report our results). Second, and more important, the results have to be confirmed with a new independent data set.

IV. FURTHER EVIDENCE

The correlations found in Ref. [10] for a particular set of UHECR and BL Lacs suggest, strictly speaking, only that UHECR and BL Lacs are connected to each other. They say little about the acceleration mechanism, particle nature and other relevant physical parameters. These questions can be fully addressed only with much larger dataset than available now. Some attempts, however, can be made. The final purpose is to arrive at the end at a consistent picture which incorporates all known features of UHECR. Here is a sketch of these attempts, each bringing additional evidence of the connection between UHECR and BL Lacs.

In Ref. [11] it was noted that cuts in BL Lac catalogue, chosen in Ref. [8] so as to maximize correlations with UHECR, select automatically $\gamma$-ray loud BL Lacs. When this observation is consistently elaborated, i.e. a subsample of the catalogue [18] is selected on the basis of a single criterium, namely, the cross-correlation with the EGRET sources, the resulting subset of 14 $\gamma$-ray loud BL Lacs correlates with UHECR at the level of $10^{-7}$ of chance coincidence. This number cannot, of course, be interpreted as the significance of correlations between UHECR and BL Lacs because of a posteriori selection; rather the conclusion is that $\gamma$-ray loudness may be a distinctive feature of those BL Lacs which are UHECR accelerators.

In Ref. [11] an attempt was made to determine the charge composition of UHECR by reconstructing actual arrival directions of UHECR particles bent in the Galactic magnetic field. The idea was that such a reconstruction should improve correlations of UHECR with BL Lacs if the latter are the sources. Substantial improvement was indeed observed for particles with the charge +1, which is an indication of the presence of protons.

In Ref. [13] it was observed that if correlations of UHECR and BL Lacs were due to chance coincidence, the coinciding rays would be distributed over the sky randomly, reflecting only the local density of BL Lacs and exposure of a cosmic ray experiment. Thus, any significant deviation in the distribution of correlating rays over the sky from this expectation speaks in favor of real physical connection between cosmic rays and BL Lacs. In fact, the UHECRs correlating with BL Lacs form two “spots”, with low probability to occur by chance [13]. This non-uniformity of the distribution of correlating rays may be due to several factors: (1) anisotropy of extragalactic magnetic fields at scales of order 500 Mpc; (2) poor knowledge of the Galactic magnetic field in some areas of the sky; (3) fluctuations in the space distribution of the nearest sources.

V. CONCLUSIONS

According to textbooks, any successful statistical analysis consists in formulation of a “null hypothesis” and its subsequent falsification, at some confidence level, by comparing to the experimental data. In the case at hand the “null hypothesis” which is being tested is that BL Lacs (and, in particular, any subset of them) and UHECR are uncorrelated. It takes only one counter-example to disprove a hypothesis, while “pro-examples” do not prove its validity. An illustration of this general rule has been discussed in Sect. II: the absence of significant correlations of UHECR with the whole BL Lac catalogue does not prove that UHECR and BL Lacs are uncorrelated (i.e., that there is no subset of BL Lacs which are sources of UHECR and thus correlate with them).

Having found such counter-example (i.e. the case when correlation is significant), the only thing which can be concluded, to a certain confidence level, is that UHECR and a particular subset of BL Lacs are correlated. The nature and physical implications of these correlations have to be studied separately by formulating and testing different “null hypothesis”. Refs. [11, 12, 13] are first attempts in this direction.
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APPENDIX A: STATISTICS OF CLUSTERING AND THE NUMBER OF UHECR SOURCES.

The authors of Ref. [1] have misinterpreted our earlier paper [15] which concerns the statistics of clustering of UHECR and the number of their sources. In view of the growing confusion we would like to clarify this issue.

In Ref. [1] one reads: “(observed occurrence of clusters of UHECR) ... was used to estimate the spatial density of sources to be $6 \times 10^{-3}$ Mpc$^{-3}$ [15]. This would obviously place stringent constraints on candidate astrophysical sources, e.g. $\gamma$-ray bursts (GRBs) have a spatial density of only $\approx 10^{-5}$ Mpc$^{-3}$. However,

This is merely a more careful analysis [19] shows that the uncertainty is this estimate are very large. The true number is $n = 2.77^{+96.1(916)}_{-2.53(2.79)} \times 10^{-3}$ Mpc$^{-3}$ at the 68% (95%) c.l.; moreover relaxing the assumptions made, viz. that the sources all have the same luminosity and a spectrum $\propto E^{-2}$, increases the allowed range even further, e.g. to $n = 180^{+270(8817)}_{-165(174)} \times 10^{-3}$ Mpc$^{-3}$ ...

The actual situation is different from what is described in this extract. First, intrinsic inaccuracy of the estimate was fully acknowledged in Ref. [15]. Its physical reason is obvious: clustering is not sensitive to the number of dim sources. The estimates of Ref. [15] cited above are a good illustration of the point: upper limits are huge and strongly model-dependent.

Second, the main point of Ref. [15] was to show that there exists a model-independent lower bound on the number of sources. This bound is presented in Table 1 of Ref. [15]: at 90% and 99% confidence levels the number of sources is $n > 2.3 \times 10^{-4}$ Mpc$^{-3}$ and $n > 3.2 \times 10^{-5}$ Mpc$^{-3}$, respectively. This is consistent with the calculations of Ref. [15] performed in particular models.

[1] W. Evans, F. Ferrer and S. Sarkar, astro-ph/0212533
[2] N. Hayashida et al., Phys. Rev. Lett. 77, 1000 (1996).
[3] M. Takeda et al., Astrophys. J. 522, 225 (1999) astro-ph/9902239.
[4] P. G. Tinyakov and I. I. Tkachev, JETP Lett. 74, 1 (2001) [Pisma Zh. Eksp. Teor. Fiz. 74, 3 (2001)], astro-ph/0102101.
[5] M. Takeda et al., Proc. of 27th International Cosmic Ray Conference (Hamburg, Germany, 7-15 Aug 2001) p. 341.
[6] M. Takeda et al., Phys. Rev. Lett. 81, 1163 (1998); M.A. Lawrence, R.J.O. Reid and A.A. Watson, J. Phys. G: Nucl. Part. Phys., 17, 733 (1991); B.N. Afanasiev et al., Proc. Int. Symp. on Extremely High Energy Cosmic Rays: Astrophysics and Future Observatories, Ed. by Nagano, p.32 (1996).
[7] N. Hayashida et al., Astrophys. J. 522, 225 (1999) (Appendix) astro-ph/0008102.
[8] P. G. Tinyakov and I. I. Tkachev, JETP Lett. 74, 445 (2001) [Pisma Zh. Eksp. Teor. Fiz. 74, 499 (2001)], astro-ph/0102476.
[9] P. Tinyakov and I. Tkachev, Compact sources of UHECR, Proc. of 27th International Cosmic Ray Conference (Hamburg, Germany, 7-15 Aug 2001) p. 547; [http://www.copernicus.org/icrc/HE1.05.oral.html].
[10] P. G. Tinyakov and I. I. Tkachev, Talk at TAUP 2001: Topics in Astroparticle and Underground Physics, Assergi, Italy, 8-12 Sep 2001. Published in: Nucl. Phys. Proc. Suppl. 110, 501 (2002).
[11] P. G. Tinyakov and I. I. Tkachev, Astropart. Phys. 18, 165 (2002) astro-ph/0111305.
[12] D. S. Gorbunov, P. G. Tinyakov, I. I. Tkachev and S. V. Troitsky, Astrophys. J. 577, L93 (2002) astro-ph/0204360.
[13] P. Tinyakov and I. Tkachev, Talk at International Workshop on Extremely High Energy Cosmic Rays, Wako, Japan, 5-6 Nov 2002: hep-ph/0212223.
[14] P. Tinyakov and I. Tkachev, Talk at ICRC 2003, Tsukuba, Japan, 31 Jul - 7 Aug 2003: astro-ph/0305363.
[15] S. L. Dubovsky, P. G. Tinyakov and I. I. Tkachev, Phys. Rev. Lett. 85, 1154 (2000).
[16] M.P. Veron-Cetty and P. Veron, Quasars and Active Galactic Nuclei (9th Ed.), ESO Scientific Report (2000). [http://vizier.u-strasbg.fr/viz-bin/Cat?VII/215].
[17] W. S. Paciesas et al., astro-ph/9903205.
[18] This catalogue is regularly updated, current edition is: M.P. Veron-Cetty and P. Veron, Quasars and Active Galactic Nuclei (10th Ed.), Astron. Astrophys. 374, 92 (2001). [http://vizier.u-strasbg.fr/viz-bin/Cat?VII/224].
[19] Z. Fodor and S. D. Katz, Phys. Rev. D 63, 023002 (2001) hep-ph/0007158.
[20] EFS quote a different number, $2 \times 10^{-5}$. This is merely misunderstanding of our paper.
[21] It is not important on which particular parameters the cuts are imposed in the control set. A statement that some parameter is a priori important is already equivalent to the statement that correlations are present.
[22] The exact calculation of penalty factor is, of course, possible along the lines of Sect. IIA, but rough estimate is sufficient in the case at hand.