A Cyclic Cosmological Model in the Landscape Scenario

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Abstract

In [1], KKLT give a mechanism to generate de Sitter vacua in string theory. And the scenario, Landscape, is suggested to explain the problem of the cosmological constant. In this paper, adopting a simple potential describing the landscape, we investigate the decay of the vacuum and the evolution of the universe after the decay. We find that the big crunch of the universe is inevitable. But, according to the modified Friedmann equation in [11], the singularity of the big crunch is avoided. Furthermore, we find that this gives a cyclic cosmological model.

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Key words: vacuum decay, big crunch, cyclic model, landscape

1 Introduction

The data from the observation of the first year Wilkinson Microwave Anisotropy Probe (WMAP) [3, 4] and the observation of the SNe Ia [5] make
us almost be sure that the expansion of the universe is accelerating. The simplest explanation is that a small but non-zero cosmological constant, the de Sitter vacuum, dominates the present universe. Recent years, great efforts basing on the string theory have been paid to solve the cosmological constant problem and to construct a complete process of the cosmological evolution. In [1], a mechanism, KKLT mechanism, is given to get de Sitter vacua in string theory. And a scenario named Landscape has been suggested [12] [13] [14]. In this scenario it is argued that string theory has a landscape of vacua. The supersymmetric (SUSY) sector of the landscape has the zero vacuum energy. The non-SUSY sector has a stochastic distribution of vacua energies around the zero vacuum energy, where some vacua are de Sitter vacua with positive vacuum energy and others are anti-de Sitter vacua with negative vacuum energy. One of the de Sitter vacua describes the present acceleration of our universe. So the cosmological constant, as a metastable de Sitter vacuum, must decay into an anti-de Sitter vacuum. Unfortunately, the detailed information of the landscape is absent. Here we simply take the model in [2] to describe it.

On the other hand, the cyclic cosmological model has been suggested as a radical alternative to inflation scenario [16] [15] (For a short review, see Ref.[9]). In this scenario the universe undergoes an endless sequence of cosmic epochs each beginning with a ‘bang’ and ending in a ‘crunch’. It gives a whole process of the evolution of the universe. In the current work, we find that the landscape indicates a cyclic model naturally.

In this paper, we first assume that the de Sitter vacuum of the potential suggested in [2] describes the present acceleration of the universe, and then discuss the evolution of the universe after the decay of the vacuum. We find that the contraction is obtained inside the bubble which is materialized as the decay. So the singularity of the big crunch is encountered. We know the singularity is a long-standing issue in theoretical physics. Here, we find, using the modified Friedmann equation in [11], the singularity of the contraction is avoided and the bounce appears as the end of the contraction. Finally we show this scenario gives a cyclic cosmological model.
2 de Sitter Vacua and anti-de Sitter Vacua in String Theory

In the theory of the $N = 1$ supergravity, the potential is

$$V = e^K \left( \sum_{a,b} G_{ab} D_a W D_b W - 3|W|^2 \right),$$  \hspace{1cm} (1)

where $a, b$ runs over all the modulus fields and $K$ is the Kähler potential, $K = -3 \ln[(\rho + \bar{\rho})]$. The volume modulus $\rho$ is simply taken to be the real field $\rho = \bar{\rho} = \sigma$. In the simplest KKLT model [1], the superpotential $W$ is given by $W = W_0 + A e^{-a\rho}$. When the potential is supplemented by a D-type contribution $D/\sigma^3$ from anti-D3 brane [1] or D7 branes [6], a de Sitter minimum is found. In [2], this model is slightly modified by taking

$$W = W_0 + A e^{-a\rho} + B e^{-b\rho},$$  \hspace{1cm} (2)

in order to be compatible with supersymmetry breaking and inflation. Here, $W_0$ is a tree level contribution which arises from the fluxes, $A, a$ and $b$ are positive constants, and $B$ is a negative constant. Now the potential in equation (1) is written as

$$V = e^{-2(a+b)\sigma} \left( bB e^{a\sigma} + aA e^{b\sigma} \right) \times \left[ B e^{a\sigma} (3 + b\sigma) + e^{b\sigma} (A(3 + a\sigma) + 3e^{a\sigma}W_0) \right].$$  \hspace{1cm} (3)

The resulting potential is shown in Fig. [1]. With the values of the parameters used in this figure, we may find a local minimum, $V_{ds}$, at $\sigma = \sigma_{ds} \approx 62$ and an anti-de Sitter global minimum, $V_{ads}$, at $\sigma = \sigma_{ads} \approx 106$. In fact, the potential value at $\sigma_{ads}$ is negative. But if we add the lifting term $\sim D/\sigma^3$ and fine tune this term, we can always make the value of the local minimum, $V_{ds}$, be equal to the observed cosmological constant $\Lambda \sim 10^{-120}$. (Of course, the curve of the potential would be changed slightly. And the global minimum at $\sigma_{ads}$ is still designed to be an AdS vacuum.) From now on we just suppose this has been achieved.

3 the Decay of the de Sitter Vacua

Here we identify the dark energy in our universe as the de Sitter vacuum of the volume modulus field, $V_{ds}$, and take other components in the universe to
be negligible. The metric of the universe is the FRW metric
\[ ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \]  
(4)

The evolution of the universe is governed only by
\[ H^2 = \frac{1}{3M^2_{pl}} V_{ds}, \]  
(5)

where we take \( 8\pi G = M^{-2}_{pl} \) and \( H = \frac{\partial a}{\partial t}/a = \dot{a}/a \). From this equation, it is obvious that the universe is at a de-Sitter state. But this would not last for ever. As time goes by, due to the quantum mechanics, the decay of the metastable dS vacuum state at \( \sigma_{ds} \) is inevitable. So, at some time, within some region, the volume modulus field \( \sigma \) tunnels through the barrier between the two minimum points (as shown in Fig. 1), and appears at the AdS point, \( \sigma = \sigma_{ads} \). Notably, this means the modulus is still stabilized after the decay. It should be noted that this property is absent in the simplest KKLT potential in [1], where the decay means the decompacting of the modulus \( \rho \).

This vacuum decay may be treated as a first-order phase transition. According to [7], the tunnelling first happens within a region and forms a bubble, and then the bound of the bubble would expand outward at the speed of light. Outside of the bubble, the universe is still at the dS phase. But inside, the universe is at the AdS state. The metric becomes
\[ ds^2 = -dt^2 + a(t)^2(\frac{dr^2}{1 + r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \]  
(6)
The two metrics (4) and (6) are joined at the bound of the bubble. If we choose the center of the bubble at the moment of materialization to be the center of coordinates, the bound of the bubble is at $r = \bar{r}(t)$. Both the initial value of the bound, $\bar{r}(0)$, and the probability of the decay, can be obtained by using the thin wall approximation [7]. But here these values are of no interest to us. We just need verify that the condition for the decay to happen [7],

$$\frac{3S_1^2}{4(V_{ds} - V_{ads})} < 1,$$

is satisfied. Using the potential shown in Fig.1, we get $\frac{3S_1^2}{4(V_{ds} - V_{ads})} \simeq 0.66$. So the condition is satisfied.

### 4 the Evolution after the Decay

Now let us analyze the evolution of the universe after the decay. At first, for cosmological purposes it is convenient to define the canonical variable $\phi = \sqrt{\frac{3}{2}} \ln \sigma$. From now on, we will use the field $\phi$, instead of $\sigma$.

Outside the bubble, everything is the same as before and the evolution is still governed by the equation (5). But inside, the moving equation becomes

$$H^2 = \frac{1}{a^2} - \Lambda^2_a,$$

where $\Lambda_a \equiv \sqrt{-\frac{1}{8M_p^2}V_{ads}}$. This equation has the solution

$$a(t) = \frac{\sin(\Lambda_\alpha t + \delta)}{\Lambda_\alpha},$$

where $\delta$ is the integral constant. Taking account of the expansion of the universe before the decay, we can conclude that the universe in the bubble is expanding at the early stage after the decay. This is ensured by the condition

$$0 < \delta < \pi/2.$$

Obviously, from the solution (8), the contraction would happen after the scaling factor $a$ arrives its maximum $a_m = \Lambda_a^{-1}$. Then the singularity of the big crunch will be encountered at the end of the contraction.
Naively, if the singularity of the big crunch is neglected, it seems that we can expect a cyclic model from Eq. (8). But, it is not the case. In fact, the solution (8) is not valid even during the whole process of the first contraction. The reason is that this equation is an ideal solution by neglecting the effect of the perturbation. In [7], we know, a very small velocity of $\phi$ towards the false vacuum is the necessary condition for the instanton solution to exist. Then, considering the perturbation, there must exist a small value of the kinetic energy term of the field, $\dot{\phi}^2/2$, inside the bubble. On the other hand we know $\dot{\phi}^2/2 \propto a^{-6}$. So, during the contraction, the small kinetic energy increases rapidly and can not be neglected after a while. Indeed, after the sufficiently large time, this kinetic energy would dominates the part of the universe. Then the evolving equation would be

$$H^2 = \frac{1}{a^2} + \frac{1}{3}(\frac{1}{2}\dot{\phi}^2 + V).$$

The solution (8) would be invalid.

However, according to the equation

$$\dot{H} = -\frac{1}{2M^2_{pl}}(1 + w)\rho - \frac{1}{a^2}, p = w\rho$$

we get that $a(t)$ continues to decrease even after the solution (8) invalid. Of course, in this paper we only consider the component with $w \geq -1$. Then the singularity of the big crunch seems to be inevitable.

5 the Bounce of the Universe at the Planck Scale and the Cyclic Scenario

We know that the energy scale of the universe increases as the contraction. So the singularity of the big crunch is a problem about the physics at the Planck energy scale. At the same time it is generally believed that the big crunch singularity should not be a feature of quantum gravity and there might exit some mechanism to avoid the singularity. To solve this problem, many conjectures have been suggested [8, 17, 11]. In this section, to deal with it, we use the result shown in [11] directly. In [11], the Friedmann is modified as

$$H^2 = \frac{1}{3M^2_{pl}}\rho(1 - \frac{\rho}{M^3_{pl}}),$$

$$...$$
where \( \rho \) denotes the energy density in the universe. Of course this equation is the modified Friedmann equation in the flat universe. However, if the curvature term, \( \frac{1}{a^2} \), in (9) is also taken as a component in the universe with the energy density \( \rho = \frac{3M_{pl}^2}{a^2} \), Eq.(10) is applicable inside the vacuum-decay bubble.

It has been shown in [11] that this modified Friedmann equation avoids the catastrophe of the big crunch by giving a bounce at \( \rho = M_{pl}^2 \). After the bounce, the universe begins to expand. Naturally, we may take the bounce as the big bang. Furthermore, we find that a cyclic cosmological model is obtained. Now let's show it.

It has been given in the last section, as the bounce being approached, the kinetic energy term of \( \phi \), \( \frac{1}{2} \dot{\phi}^2 \) scaling as \( a^{-6} \), becomes the dominant component of the universe. Even, the velocity of \( \phi \) is so large that the field can roll up and over the barrier of the potential and continues rolling up along the potential. After the bounce, the energy density of the modulus field, \( \phi \), begins to decrease as the expansion of the universe. The field, \( \phi \) would roll down along the potential, too. Eventually the field might stay at the dS vacuum state, \( V_{ds} \). Notably, the potential used here is just a toy model. Actually, in the scenario of landscape, it is argued that there exist many vacua. Then, after the bounce, it is possible for the field to roll down to any vacuum. This means the universe, after the bounce, may have a different cosmological constant in each cycle [18].

No losing the generality, we assume that, after the bounce, the initial position of the field is appropriate and the rolling-down field can not overshoot to pass barriers between vacua. Then there two different cases after the bounce. One is that, after bounce, the field rolls down to an anti-de Sitter minimum. In this case, the contraction would happen during the oscillating of the field around the minimum. The moving equation is Eq.(9). During the expansion, the potential energy, \( V(\phi) \), becomes negative and the right-hand side would be zero as the decreasing of the kinetic energy term, \( \frac{1}{2} \dot{\phi}^2 \), and the curvature energy term, \( \frac{1}{a^2} \) (Of course the curvature term may be neglected). The turnaround from expansion to contraction is obtained at \( H = 0 \). Then the universe contracts and bounces to evolve into the next new cycle.

The other case is that, after bounce, the field is rolling down to a de Sitter minimum. In this case the evolution of the universe is similar to our universe. The vacuum would decay into an anti-de Sitter vacuum and then contract, bounce to evolve into another new cycle. Notably, in this case, only
one part of the universe in the previous cycle, the part inside the vacuum-decay bubble, contracts, bounces and then develops to be a new observed universe. Then the initial entropy of the observed universe in the new cycle is only one part of the entropy in the last cycle. If we suppose that it is much more possible for the field to roll down to dS vacua than to AdS vacua, the entropy of the observed universe would not become larger and larger as the cycle repeating in our scenario. Of course, the total entropy of the whole universe grows, in accord with the second law of thermodynamics.

Now we can conclude that the contraction/expansion cycles are always obtained in both cases, although there is a difference between them. And the problem of the infinite entropy is avoided in some sense.

6 Summary and Discussion

In this paper, we first assume that the dark energy is the de-Sitter vacuum of the potential in [2] and then analyze the decay of the metastable dS vacuum. We show that the decay of the vacuum is inevitable and inside the vacuum-decay bubble, the modulus field, $\sigma$, tunnels through the barrier of the potential to appear at the anti-de Sitter vacuum state. We find that the universe in the bubble would contract to the singularity. By the modified Friedmann equation in [11], the singularity is avoided and a bounce appears as the end of the contraction. Then we show that this give a cyclic cosmological model.

This model is different from ordinary cyclic models [9,10]. In our scenario, the universe can experience many cycles with different vacua. And the future turnaround happens naturally due to the vacuum decay. This implies another difference that only the part in the vacuum-decay bubble contracts, then bounces and expand to be a new observable universe. Of course, here our model is only a toy model. But we believe it is significant to incorporate the Landscape scenario with the cyclic cosmological model. However, the details of our mechanism need to be explored further. There is much work to do in order to be sure this cyclic scenario to work well.
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