Choice Disjunctive Queries in Logic Programming

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SUMMARY  One of the long-standing research problems on logic programming is to treat the cut predicate in a logical, high-level way. We argue that this problem can be solved by adopting linear logic and choice-disjunctive goal formulas of the form $G_0 \oplus G_1$ where $G_0, G_1$ are goals. These goals have the following intended semantics: choose the true disjunct $G_i$ and execute $G_i$, where $i = 0$ or 1, while discarding the unchosen disjunct. Note that only one goal can remain alive during execution. These goals thus allow us to specify mutually exclusive tasks in a high-level way. Note that there is another use of cut which is for breaking out of failure-driven loops and efficient heap management. Unfortunately, it is not possible to replace cut of this kind with use of choice-disjunctive goals.

key words: Game semantics; cut; mutual exclusion

1. Introduction

One of the long-standing research problems on logic programming is to treat the extra-logical primitive in a high-level, logical way. The advances of logic programming – especially structured proof theory – have enriched Horn clauses with additional programming primitives in a high-level way (higher-order programming, modules, local constants, etc) [12], [13], [15]. Nevertheless some key constructs could not be dealt with in a high-level way, in particular when we are concerned with mutual exclusion (and the cut predicate).

Consequently, much attention [10], [16], [17] has been given to finding a semantics that captures the cut predicate. These proposals - based on such notions of if-then-else and until – are quite interesting but somewhat complicated than necessary.

In this paper, inspired by the work in [2]–[5], [7], we propose a purely logical solution to this problem. It involves the direct employment of linear logic [8] to allow for choice-disjunctive goals. A choice-disjunctive goal is of the form $G_0 \oplus G_1$ where $G_0, G_1$ are goals. (A more intuitive name would be \textit{choose}(G_0, G_1).) Executing this goal with respect to a program $\mathcal{P}$ – $ex(\mathcal{P}, G_0 \oplus G_1)$ – has the following intended semantics:

choose a true one between $ex(\mathcal{P}, G_0), ex(\mathcal{P}, G_1)$.

For example, given a program \{\textit{male}(\textit{kim}), \textit{female}(\textit{lee})\}, execution will succeed on the goal \textit{male}(\textit{kim}) $\oplus$ \textit{female}(\textit{kim}) by choosing \textit{male}(\textit{kim}). Similarly, execution will succeed on the goal \textit{male}(\textit{lee}) $\oplus$ \textit{female}(\textit{lee}) by choosing \textit{female}(\textit{lee}). On the other hand, consider a goal \textit{male}(\textit{kim}) $\oplus$ \textit{female}(\textit{lee}). In this case, both disjuncts can lead to a success, and, in our semantics, it does not matter which disjunct to use. For simplicity, we assume from now on that execution always chooses the first successful disjunct if there are many. Thus, back to the above, execution will succeed on the goal \textit{male}(\textit{kim}) $\oplus$ \textit{female}(\textit{lee}) by choosing \textit{male}(\textit{kim}). Note that the class of choice disjunctive goals is a superset of the class of mutually exclusive goals.

Another illustration of this construct is provided by the following definition of the relation $\textit{son}(X,Y)$ which holds if $Y$ is a son of $X$.

$\textit{son}(X,Y) : = (\textit{male}(X) \otimes \textit{father}(Y,X)) \oplus (\textit{female}(X) \otimes \textit{mother}(Y,X))$.

The body of the definition above contains a mutually exclusive goal, denoted by $\oplus$. As a particular example, solving the query $\textit{son}(\textit{tom}, Y)$ would result in selecting and executing the first goal $\textit{male}(\textit{tom}) \otimes \textit{father}(\textit{tom}, Y)$, while discarding the second one. The given goal will succeed, producing solutions for $Y$. Of course, we can specify mutually exclusive goals using cut in Prolog, but it is well-known that cuts complicates the declarative meaning of the program. Our language makes it possible to formulate mutually exclusive goals in a high-level way. The class of choice disjunctive goals is, in a sense, a high-level abstraction for the cut predicate.

As seen from the example above, choice-disjunctive goals can be used to perform mutually exclusive tasks. There are several well-designed linear logic languages [9], [18] in which goals of the form $G_0 \oplus G_1$ are present. A common problem of these works is their treatment of the $\oplus$-goals: these goals are treated as inclusive-OR (or classical disjunctive) goals rather than exclusive-OR ones:

$ex(\mathcal{P}, G_0 \oplus G_1)$ if $ex(\mathcal{P}, G_0) \lor ex(\mathcal{P}, G_1)$

where $\lor$ represents classical disjunction. Hence, it is rather unfortunate that the declarative reading of $\oplus$ – known as \textit{the machine’s choice} – is missing in these
languages.

A satisfactory solution can be obtained by adopting game semantics of [2], i.e., by adding an extra layer of the choice action, as discussed above, to their execution model of ⊕. In this way, the execution respects the declarative reading of ⊕, while maintaining provability. Hence, the main difference is that, once a goal is chosen, the unchosen goal will be discarded in our language, while it will remain alive (typically through a creation of a choicepoint) in those languages.

2. Reconsidering the Foundation in Logic Programming

The computation-as-deduction approach[14], [15] has provided a basis for logic programming. It views the state of a computation as a sequent and computing as the proof search. This approach has proven useful, leading to several extensions. The first such extensions to the Horn clause include hypothetical and universally quantified goal formulas, pioneered in [15]. Additional extensions were made using higher-order quantification and linear logic[9].

Unfortunately, this approach is appropriate only for computation with boolean semantics, i.e., deciding where some formula is true/false. However, this view is too limiting. Instead, we believe that computation should be based on a bigger paradigm, i.e., task/game semantics[2], [6]. That is, computation should be interested in deciding whether some formula can be made true or not. From this viewpoint, the major criterion for judging the adequacy of a logic programming can be explained as follows:

- The first phase – the proof phase – should be sound and complete with respect to the given semantics such as intuitionistic, classical logic or linear logic.
- The second phase – the execution phase – should respect the declarative readings of logical connectives.

Consider, for instance, $P \otimes Q$ in a query. It reads as follows: solve $P$ and $Q$ concurrently. Declarative readings of other connectives are given in the next section.

The sequent calculus for Prolog does not violate a correspondence between the declarative meaning of logical connectives and proof search operations. Unfortunately, there is no such guarantee for new connectives. ⊕ is such an example where the logical connective and the proof search operations do not correspond.

3. Declarative reading of logical connectives

In this section, based on [2], [9], an overview of declarative reading of linear logical connectives is given in the sense of intuitionistic linear logic.

Additive operations The choice group of operations: ⊕, ∃x (and negative occurrences of ∀x) are defined below.

$A_0 \oplus A_1$ is the problem where, in the initial position, only the machine has a legal move which consists in choosing a value 0 or 1. After the machine makes a move $c \in \{0, 1\}$, the problem becomes $A_c$. $\exists x A(x)$ (and negative occurrences of $\forall x A(x)$) is the following: the machine must choose a value $v$ for $x$ and the problem becomes $A(v)$.

Multiplicative operations Playing $A_0 \otimes A_1$ means solving the two problems concurrently. In order to succeed, the machine needs to solve each of two problems.

Reduction $A \supset B$ is the problem of reducing $B$ (consequent) to $A$ (antecedent).

4. Prolog⊕ with the Old Semantics

The language is a version of Horn clauses with choice-disjunctive goals. It is also a subset of Lolli[9]. Note that we disallow linear clauses here, thus allowing only reusable clauses. It is described by $G$- and $D$-formulas given by the syntax rules below:

$$G ::= \top \mid A \mid t = s \mid G \otimes G \mid \exists x \ G \mid G \oplus G$$

$$D ::= A \mid G \supset A \mid \forall x \ D$$

In the rules above, $t$, $s$ represent terms, and $A$ represents an atomic formula. A $D$-formula is called a Horn clause with choice-disjunctive goals. A set of $D$-formulas is called a program.

We will present a machine’s strategy for this language given in [9]. These rules in fact depend on the top-level constructor in the expression, a property known as uniform provability[11], [15]. Note that execution alternates between two phases: the goal-reduction phase and the backchaining phase. In the goal-reduction phase, the machine tries to decompose a goal $G$. If $G$ becomes an atom, the machine switches to the backchaining mode. This is encoded in the rule (2).

Definition 1. Let $\sigma$ be an answer substitution and let $G$ be a goal and let $P$ be a set of $D$-formulas. Then the task of proving $G$ with respect to $\sigma$, $P - pv(\sigma, P \vdash G)$ – is defined as follows:

1. $pv(\sigma, P \vdash \top)$. % success
2. $pv(\sigma, P \vdash t = s)$ if $t$ and $s$ are unifiable.
3. $pv(\sigma, P \vdash A)$ if $A' : - B \in P$ and $A' \theta = A\sigma$ and $pv(\sigma \theta, P \vdash B)$. % DefR (backchaining)
4. $pv(\sigma, P \vdash G_0 \otimes G_1)$ if $pv(\sigma, P \vdash G_0)$ and $pv(\sigma, P \vdash G_1)$. 
5. The Execution Phase

Adding game semantics requires another execution phase beside the proof phase. To be precise, our new execution model – adapted from [2] – actually solves the goal relative to the program using the proof tree built in the proof phase.

In the execution phase, it just follows the path in the proof tree, printing the output values occasionally.

(1) \(ex(\sigma, P \vdash T)\). % success
(2) \(ex(\sigma, P \vdash t = s)\) if \(unify(t, s)\). % invoke unification.
(3) \(ex(\sigma, P \vdash A)\) if \(ex(\sigma \theta, P \vdash B)\), provided that the former is derived from the latter via DefR.
(4) \(ex(\sigma, P \vdash G_0 \otimes G_1)\) if \(ex(\sigma, P \vdash G_0)\) and \(ex(\sigma, P \vdash G_1)\), provided that the former is derived from the latter via \(\land\)-R.

(5) \(pv(\sigma, P \vdash G_0 \ominus G_1)\) if \(pv(\sigma, P \vdash G_0)\).
(6) \(pv(\sigma, P \vdash G_0 \ominus G_1)\) if \(pv(\sigma, P \vdash G_1)\).
(7) \(pv(\sigma, P \vdash \exists xG)\) if \(pv(\sigma \sigma_1, P \vdash [w/x]G)\) where \(w\) is a new free variable, \(\sigma_1 = \{(w, t)\}\) and \(t\) is a term.

Initially, \(\sigma\) is an empty substitution. \(\sigma \theta\) in Rule 3 represents the composition of two substitutions \(\sigma\) and \(\theta\). In the above, most rules are straightforward to read.

As an example of this approach, let us consider the following program \(P\).

\[
\begin{align*}
\{ \text{emp}(\text{tom}) & : = T, \text{emp}(\text{pete}) : = \neg T, \\
\text{harvard}(\text{tom}) & : = \neg T, \text{mit}(\text{pete}) : = \neg T. \}
\end{align*}
\]

Now, consider a goal task \(\exists x((\text{yale}(x) \oplus \text{harvard}(x)) \land \text{emp}(x))\).

The following is a proof tree of this example.

\[
\begin{align*}
\{ (w_0, \text{tom}) \}, P \vdash T & \ % success \\
\{ (w_0, \text{tom}) \}, P \vdash \text{harvard}(w_0) & \ % \text{defR} \\
\{ (w_0, \text{tom}) \}, P \vdash \text{yale}(w_0) & \ % \text{defR} \\
\{ (w_0, \text{tom}) \}, P \vdash \text{yale}(w_0) \oplus \text{harvard}(w_0) & \ % \land-R \\
\{ (w_0, \text{tom}) \}, P \vdash \text{yale}(w_0) \oplus \text{harvard}(w_0) & \ % \land-R \\
\{ (w_0, \text{tom}) \}, P \vdash (\text{yale}(w_0) \oplus \text{harvard}(w_0)) \land \text{emp}(w_0)) & \ % \land-R \\
\emptyset, P \vdash \exists x((\text{yale}(x) \oplus \text{harvard}(x)) \land \text{emp}(x)) & \ % \text{\exists-R}
\end{align*}
\]

The following theorem connects our language to linear logic. Its proof is easily obtained from the analysis of the above algorithm.

**Theorem 2:** Let \(P\) be a program and let \(G\) be a goal. Then, \(ex(P, G)\) terminates with a success if and only if \(G\) follows from \(P\) in intuitionistic linear logic. Furthermore, the interpreter respects the declarative reading of all the logical connectives including \(\oplus\).

6. Some Examples

Let us first consider the relation \(f(X, Y)\) specified by two rules:

(1) \(X < 2\), then \(Y = 0\).
(2) \(X \geq 2\), then \(Y = 3\).

The two conditions are mutually exclusive which is expressed by using the cut in traditional logic programming as shown below:

\[
\begin{align*}
f(X, 0) : = X < 2, !. \\
f(X, 3) : = X \geq 2.
\end{align*}
\]

Using cut, we can specify mutually exclusive goals, but cuts affect the declarative meaning of the program. Our language makes it possible to formulate mutually exclusive goals through the choice-disjunctive goals as shown below:

\[
\begin{align*}
f(X, Y) : = & \ (X \geq 2 \land Y = 3) \lor \\
& \ (X < 2 \land Y = 0).
\end{align*}
\]

The new program, equipped with \(\oplus\)-goals, is more readable than the original version with cuts, while preserving the same efficiency. A similar example is provided by the following “max” program that finds the larger of two numbers.

\[
\begin{align*}
\text{max}(X, Y, Max) : = & \ (X \geq Y \land Max = X) \lor \\
& \ (X < Y \land Max = Y).
\end{align*}
\]

These two goals in the body of the above clause are mutually exclusive. Hence, only one of these two goals can succeed. For example, consider a goal \(\text{max}(3, 9, Max)\). Solving this goal has the effect of choosing and executing the second goal \((3 < 9) \land Max = 9\), producing the
result $Max = 9$.

As another example, we consider the relation $member(X,L)$ for establishing whether $X$ is in the list $L$. A typical Prolog definition of $member(X,L)$ is shown below:

$$member(X,[Y|L]) : - (Y = X) \lor member(X,L)$$

This definition is nondeterministic in the sense that it can find any occurrence of $X$. Our language in Section 2 makes it possible to change $member$ to be deterministic and more efficient: only one occurrence can be found. An example of this is provided by the following program.

$$member(X,[Y|L]) : - (Y = X) \oplus member(X,L)$$

7. Conclusion

In this paper, we have considered an extension to Prolog with choice-disjunctive goals. This extension allows goals of the form $G_0 \oplus G_1$ where $G_0,G_1$ are goals. These goals are particularly useful for replacing the cut in Prolog, making Prolog more concise and more readable.

In the near future, we plan to investigate the connection between Prolog and Japaridze’s Computability Logic (CL)[2],[3]. CL is a new semantic platform for reinterpreting logic as a theory of tasks. Formulas in CL stand for instructions that can carry out some tasks. We plan to investigate whether our operational semantics is sound and complete with respect to the semantics of CL.

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