BINARY ASTROMETRIC MICROLENSING WITH GAIA

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ABSTRACT

We investigate whether or not Gaia can specify the binary fractions of massive stellar populations in the Galactic disk through astrometric microlensing. Furthermore, we study whether or not some information about their mass distributions can be inferred via this method. In this regard, we simulate the binary astrometric microlensing events due to massive stellar populations according to the Gaia observing strategy by considering (i) stellar-mass black holes, (ii) neutron stars, (iii) white dwarfs, and (iv) main-sequence stars as microlenses. The Gaia efficiency for detecting the binary signatures in binary astrometric microlensing events is ~10%–20%. By calculating the optical depth due to the mentioned stellar populations, the numbers of the binary astrometric microlensing events being observed with Gaia with detectable binary signatures, for the binary fraction of about 0.1, are estimated to be 6, 11, 77, and 1316, respectively. Consequently, Gaia can potentially specify the binary fractions of these massive stellar populations. However, the binary fraction of black holes measured with this method has a large uncertainty owing to a low number of the estimated events. Knowing the binary fractions in massive stellar populations helps with studying the gravitational waves. Moreover, we investigate the number of massive microlenses for which Gaia specifies masses through astrometric microlensing of single lenses toward the Galactic bulge. The resulting efficiencies of measuring the mass of mentioned populations are 9.8%, 2.9%, 1.2%, and 0.8%, respectively. The numbers of their astrometric microlensing events being observed in the Gaia era in which the lens mass can be inferred with the relative error less than 0.5 toward the Galactic bulge are estimated as 45, 34, 76, and 786, respectively. Hence, Gaia potentially gives us some information about the mass distribution of these massive stellar populations.

Key words: astrometry – binaries: general – gravitational lensing: micro – methods: numerical

1. INTRODUCTION

Gaia is a space observatory of the European Space Agency (ESA), performing high precision astrometry, multi-color, multi-epoch photometry, and spectroscopy on astronomical objects brighter than 20 G magnitudes. This satellite, surveying throughout the sky, will observe each object 72 times on average during its anticipated 5 yr lifetime (Eyer et al. 2013). The astrometric precision of Gaia depends strongly on the source magnitude and will be 30–600 μas for a single astrometric measurement of stars with a magnitude of 13–19 (Varadi et al. 2009). Different astronomical phenomena could be observed during the Gaia mission, e.g., supernovae (Belokurov & Evans 2003), novae, dwarf novae, eruptive stars, variable stars (Varadi et al. 2009; Eyer et al. 2013), astrometric microlensing events (Belokurov & Evans 2002; Proft et al. 2011), etc. Here, we study some features of detecting astrometric microlensing events with Gaia.

If the light of a background star passes through the gravitation field of a collinear foreground object, according to the Einstein general relativity, the light beam bends toward the center of gravity (Einstein 1936). Consequently, several distorted, (un)magnified images are produced whose angular separations are too small to be resolved even by modern telescopes. Instead, the combined and magnified light of images is received by the observer, in the phenomenon known as gravitational microlensing. One of its features is the displacement between the light centroid of images and the source position while the source star is passing the gravitational field of lens, i.e., astrometric trajectory (Hog et al. 1995; Miyamoto & Yoshii 1995; Walker 1995; Jeong et al. 1999). During a microlensing event, by measuring the astrometric lensing and parallax effect, the mass of the deflector can be inferred (Miralda-Escudé 1996; Paczyński 1998). In microlensing events, the astrometric cross section is much larger than the photometric one (Paczyński 1996). Hence, the optical depth of astrometric microlensing events is larger than the photometric one (Dominik & Sahu 2000).

Detecting astrometric microlensing with the Gaia satellite was first discussed by Dominik & Sahu (2000). Then, Belokurov & Evans (2002) studied this subject in greater detail. The astrometric microlensing due to massive objects in solar orbit beyond the Kuiper Belt was investigated by Gaudi & Bloom (2005). Also, Proft et al. (2011) investigated several catalogs to find several nearby stars with large proper motions that are potential candidates for astrometric microlensing detection in the Gaia era.

Here, we quantitatively investigate the Gaia efficiency for specifying the binary fractions of microlenses in binary astrometric microlensing due to massive stellar populations, i.e., stellar-mass black holes, neutron stars, and white dwarfs in addition to main-sequence stars. To obtain the Gaia efficiency, we perform a Monte Carlo simulation. Then, we evaluate the optical depth and the number of binary events being observed with Gaia with detectable binary signatures, considering different binary fractions for massive stellar populations. Indicating the binary fractions in massive stellar populations are useful for studying gravitational waves and estimating their amounts, see, e.g., Riles (2013); Cutler & Thorne (2002). We find that Gaia can potentially determine the binary fractions of massive stellar populations. Furthermore, we estimate the Gaia efficiency for measuring the mass of massive stellar populations in the Galactic disk through astrometric microlensing of a
single lens to indicate whether or not Gaia can give any information about their mass distributions. Indeed, astrometric microlensing events due to more massive microlenses have longer Einstein crossing times and larger angular Einstein radii. Therefore, the Gaia efficiency for measuring the lens mass in these events is high.

This paper is organized as follows: in Section 2 the basics of astrometric microlensing are explained. In the next section, we simulate binary astrometric microlensing events being observed with Gaia to investigate how much information Gaia gives us about the binary stellar populations in the Galactic disk. In Section 4 we investigate whether or not Gaia can determine the mass distributions of massive stellar populations through detecting astrometric microlensing of a single lens. We conclude in the last section.

2. BASICS OF ASTROMETRIC MICROLENSING

In microlensing events, the light centroid vector of source star images does not coincide with the source position. This astrometric shift in source star position changes with time as a microlensing event progresses. For a point-mass lens, the centroid shift vector of source star images is given by (Hog et al. 1995; Miyamoto & Yoshii 1995; Walker 1995)

$$\delta \theta = \mu_1 \theta_1 + \mu_2 \theta_2 - u \theta_E = \frac{\theta_E}{u^2 + 2} u,$$

where $\theta_1$ and $\mu_1$ are the position and magnification factor of the $i$th image, respectively, $u = p \hat{x} + u_0 \hat{y}$ is the vector of the projected angular position of the source star with respect to the lens normalized by the angular Einstein radius of the lens $\theta_E$ in which $p = (t - t_0)/t_E$, $t_0$ is the time of the closest approach, $t_E$ is the Einstein crossing time, and $\hat{x}$ and $\hat{y}$ are the unit vectors in the respective directions parallel with and normal to the direction of the lens–source transverse motion. The angular Einstein radius of lens is given by

$$\theta_E = \sqrt{\kappa M_l \pi_{\text{rel}}} = 300 \mu\text{as} \frac{M_l}{\sqrt{0.3 \mu l_{\odot}} \sqrt{\pi_{\text{rel}}(\mu\text{as}) 0.036}},$$

where $M_l$ is the lens mass, $\kappa = 4 G / (c^2 \text{AU})$, and $\pi_{\text{rel}} = \text{AU} \left( \frac{1}{D_1} - \frac{1}{D_2} \right)$ where $D_1$ and $D_2$ are the lens and source distances from the observer. Usually, a microlensing parallax is defined as $\pi_{\text{E}} = \pi_{\text{rel}}/\theta_E$, which can be measured from the photometric event. According to Equation (2), by measuring the relative parallax $\pi_{\text{rel}}$ and the angular Einstein radius from astrometric observations, the lens mass can be inferred. Nearby lenses have considerable and measurable relative parallaxes and angular Einstein radii and are suitable candidates for astrometric measurements with Gaia. In that case, if the lens mass is high, the error bars of the lenses mass strongly decrease due to their large angular Einstein radii.

In astrometric microlensing, the threshold amount of impact parameter, i.e., $u_{\text{th}}$, which gives an astrometric centroid shift more than a threshold amount, i.e., $\delta_T$, is given by

$$u_{\text{th}} = \frac{T_{\text{obs}}}{\pi_D^2} = 25.4 \sqrt{\frac{T_{\text{obs}}}{T_{\text{obs}}^2}} \frac{5 \gamma_0 \epsilon}{\delta_T},$$

where $T_{\text{obs}}$ is the lifetime of the satellite and $v_l$ is the relative velocity of source with respect to the lens (Dominik & Sahu 2000). Here, we assume that the astrometric centroid shifts more than $\delta_T = 5 \sqrt{2} \pi_{\text{rel}}$ are measurable with Gaia in which $\pi_{\text{rel}}$ is the Gaia astrometric precision. Since Gaia can measure the astrometric displacement only along the scan, we combine $\pi_{\text{rel}}$ with a factor of $\sqrt{2}$ to give Gaia two-dimensional accuracy (Belokurov & Evans 2002). The amount of the threshold impact parameter for a typical astrometric microlensing event being observed with Gaia with the astrometric precision $\pi_{\text{rel}} = 300 \mu\text{as}$ is equal to $u_{\text{th}} \sim 25$, whereas the threshold impact parameter for the photometric observation of a microlensing event is about 1. Hence, the astrometric cross section, which is proportional to $u_{\text{th}}^2$, is much larger than the photometric one.

2.1. Astrometric Optical Depth

According to the definition of the threshold impact parameter $u_{\text{th}}$, during the observational time $T_{\text{obs}}$ the effective area around each lens that yields an astrometric shift in the projected source positions more than the threshold amount $\delta_T$ is $\mathbf{A}(x, M_l) = 2u_{\text{th}} R E_{\text{obs}}, v_l$. This area is a function of the lens mass $M_l$ and the lens distance $D_1 = x D_2$ from the observer. The astrometric optical depth is defined as the cumulative fraction of these areas in the lens plane, for all lens distances from the observer to the source star, which is given by (Dominik & Sahu 2000)

$$\tau_a = \frac{D_s}{D_1} \frac{1}{\pi_D} \int_0^\infty dM f(M_l) A(x, M_l),$$

where $f(M_l)$ is the distribution function of the lens mass and $\rho_d(x)$ is the stellar density distribution in the Galactic disk. We use the following distribution for the Galactic disk (Rahal et al. 2009):

$$\rho_d(x) = \frac{\Sigma}{2 H} \exp \left[ \frac{-R - R_{\text{h}}}{h} \right] \exp \left[ -\frac{|z|}{H} \right],$$

where $R$ is the radial distance from the Galactic center, $z$ is the altitude with respect to the Galactic plane, $H = 0.325$ kpc is the height scale, $h = 3.5$ kpc is the length scale of the disk, and $\Sigma$ is the column density of the disk at the Sun position. By inserting the value of $A(x, M_l)$, the optical depth is

$$\tau_a = 4 \frac{G}{c^2} D_s \frac{T_{\text{obs}}}{5 \gamma_0 \epsilon} \int_0^\infty dM f(M_l) \frac{A(x, M_l)}{M_l} \sqrt{1 - x} \int_0^\infty dM f(M_l) \frac{A(x, M_l)}{M_l} \sqrt{1 - x}.$$

The optical depth can be used to estimate the number of observed events. If $N_{\text{bg}}$ background source stars brighter than 20 G magnitudes are observed during the Gaia lifetime $T_{\text{obs}}$ the number of source stars with the astrometric shift, in their
projected positions, more than $\delta r$ is given by

$$N_a = \frac{\pi T_{\text{obs}} N_{\text{bg}}}{2} \frac{r_a}{\ell E u_a}. \quad (7)$$

By considering $\epsilon$ as the efficiency for measuring the lens mass of these observed events, the number of astrometric microlensing events being observed with Gaia in which the lens mass can be measured is $N_e = N_a \epsilon$.

3. Binary Astrometric Microlensing During the Gaia Mission

The binary fractions of massive stellar populations have not been explicitly specified yet; however, this issue is very important for studying the gravitational waves because the sources of the gravitational waves are binary systems composed of white dwarfs, neutron stars, and black holes; see, e.g., Riles (2013); Cutler & Thorne (2002). Here, we want to investigate whether Gaia can specify the binary fractions for these stellar populations through binary astrometric microlensing. For this goal, we estimate the number of detectable binary events that have considerable binary signals during the Gaia era by performing Monte Carlo simulation according to the Gaia observing strategy.

In the first step, we produce an ensemble of binary astrometric and photometric microlensing events due to massive stellar populations as microlenses according to their physical distributions explained in Section 3.1. We consider binary stellar-mass black holes (BBHs), neutron stars, white dwarfs, and main-sequence stars as microlenses. Then, corresponding light curves and astrometric trajectories are regenerated according to the Gaia observing strategy. By considering a suitable criterion explained in Section 3.2, we investigate whether the binary signatures can be inferred from the Gaia observations. Finally, by calculating the optical depth (explained in Section 2.1), the number of detectable binary events during the Gaia mission are estimated. The results are gathered in Section 3.2.

3.1. Parameter Space

The distribution functions used to generate the binary astrometric and photometric microlensing events are explained here.

For the source stars, we make an ensemble of the source stars according to their mass, age, metallicity, and magnitude and then uniformly choose the source star from that. We take the ages of the stars in the range of $\log(t/\text{yr}) = [6.6, 10.2]$ with intervals of $\Delta \log(t) = 0.05$ dex. For each age range, we estimate the number of stars according to a positive star formation rate $dn/dt > 0$ (Cignoni et al. 2006; Karami 2010). For each star in this age range, we take the mass of source star, $M_*$, from the Kroupa mass function in the range of $M_* \in \{0.1, 3\} M_\odot$ (Kroupa et al. 1993; Kroupa 2001) and metallicity, Z, according to the age–metallicity relation, and the distribution function for the metallicity from Twarog (1980). Finally, we use the theoretical isochrones for the color–magnitude diagram obtained using the numerical simulation of the stellar evolution from the Padova model (Marigo et al. 2008) to simulate the absolute magnitude of each star. The isochrones are defined for stars with different masses, colors, and magnitudes but with the same metallicities and ages.

The coordinates of the source star toward the Galactic bulge $(l, b, D_*)$, are selected from $dn/dl \propto \rho(l, b, D_*) D_*^2$ in the range of $l \in [10, 10]^\circ$, $b \in [-5, 5]^\circ$, and $D_* \in [10, 10]$ kpc where $\rho(l, b, D_*)$ is the combination of the stellar densities in the Galactic bulge and disk (Rahal et al. 2009). The Galactic bulge has a triaxial structure rotated along the main axes of the bulge according to the orientation angles: $\eta = 78.9^\circ$ represents the angle between the bulge major axis and the line perpendicular to the Sun–Galactic center line, $\beta = 3.5^\circ$ is the angle between the bulge plane and the Galactic plane, and $\gamma = 91.3^\circ$ is the roll angle around the major axis of bulge (Robin et al. 2003).

We obtain the apparent magnitude of the source star $(m_*)$ according to the absolute magnitude, distance modulus, and extinction. The extinction map toward the Galactic bulge and in the Galactic distance is obtained in some references, e.g., Gonzalez et al. (2012); Nataf et al. (2013). But the extinction map at the closer distances is not clear. We know that the extinction can be determined according to the column density of hydrogen content of the Galaxy (Weingartner & Draine 2001). The different fractions of interstellar medium (ISM) in different directions toward the Galactic bulge are due to hydrogen content. Hence, we need to estimate these fractions in different directions toward the Galactic bulge. We do this by comparing (in each direction at the Galactic distance) (i) the extinction map provided by Gonzalez et al. (2012) and (ii) the extinction amount calculated according to its relation with the hydrogen column density by assuming all ISMs are made from the hydrogen content. We use ISM density law introduced by Robin et al. (2003). Also we obtain the $V$-band extinction from its value in near-infrared bands according to the relations introduced by Cardelli et al. (1989). Indeed, we assume the fraction of ISM composed from the hydrogen content in the Galactic scale depends only on the direction but not on the distance. The source stars fainter than 20 G magnitudes are not considered.

For generating the light curve of microlensing events with the finite source size effect, we need the radius of the source star. For the main-sequence stars, the relation between the mass and radius is given by $R = M_*/0.8$ where all parameters normalized to the Sun’s value. We do not consider giant source stars. The Gaia astrometric and photometric precisions are dependent on the apparent magnitude of the source (Varadi et al. 2009). We consider the events in which $\theta_E > 5\sqrt{2} \sigma_u$.

For indicating the trajectory of the source with respect to the $x$-axis, we identify this path with an impact parameter and orientation defined by an angle between the trajectory of the source star and the $x$-axis, $\xi$. We let this angle change in the range of $[0, 2\pi]$. The impact parameter is taken uniformly in the range of $u_0 \in [-u_\text{min}, u_\text{max}]$, and the corresponding time for $u_0$ is chosen uniformly in the range of $t_0 \in [0, 5]$ yr, which is the anticipated lifetime of Gaia.

To simulate the position of the lens in the Galactic disk, by considering the same Galactic latitude $b$ and longitude $l$ as the source, we take its distance with respect to the observer from the probability function of astrometric microlensing detection $dT/dx \propto \rho_d(x)(1 - x)^2$, where $x = D/l$ (Dominik & Sahu 2000).

The mass of the stellar-mass black holes as microlenses is taken from the distribution function introduced by Farr et al. (2010). We take the mass of the neutron stars from a Gaussian function, with an average mass and dispersion of
$M_0 = 1.28 \, M_\odot$ and $\sigma = 0.28 \, M_\odot$, respectively, in the range of $[1, 2.5] \, M_\odot$ (Ozel et al. 2012). The mass of white dwarfs is taken from a combination of two Gaussian functions with the averages and dispersions of $M_0 = 0.59 \, M_\odot$, $\sigma = 0.047 \, M_\odot$ and $M_0 = 0.711 \, M_\odot$, $\sigma = 0.109 \, M_\odot$ for two different samples of white dwarfs in the range of $[0.3, 1.4] \, M_\odot$ (Kepler et al. 2007). We take the mass of main-sequence stars as microlenses from the Kroupa mass function in the range of $[0.1, 2] \, M_\odot$.

The velocities of the lens and the source star are taken from the combination of the global and dispersion velocities of the Galactic disk and bulge (Binney & Tremaine 1987, p. 78; Rahal et al. 2009).

To simulate the parallax effect, we first convert the Galactic coordinates of the source to ecliptic coordinates using the LAMBDA site.\(^3\) Then, the Earth’s position vector with respect to the Sun is calculated (e.g., the appendix section from An et al. 2002). The time when the Earth is in the perihelion is $t_e = -18$ days with respect to the time when Gaia entered its operational orbit, i.e., the start time of the observation.

Here, we review the Gaia scanning law. Gaia has two astrometric fields of view separated by 106:5. Its spin axis makes an angle of 45° with respect to the direction toward the Sun. The satellite spins around this axis with the angular rate of $\sim$60 as s$^{-1}$. Hence, observations occur in pairs whose time interval is 106.5 minutes. The spin axis in turn has a precession motion around a circle. Finally, the spacecraft will move in a Lissajous orbit around the second Lagrange point of the Sun–Earth system. The combination of these motions gives the Gaia scanning law.\(^4\) Each object will be observed 72 times on average with a cadence of $\sim$25 days. This satellite measures the positions of objects set along the scan direction. For simulating the observational astrometric trajectories and photometric light curves, we adopt the Gaussian distribution for the cadence between two pairs of data taken by Gaia with the average and dispersion of 25 and 5 days. Each pair occurs in 106.5 minutes. In each observation we consider a random scanning direction chosen uniformly in the range of $[0, 2 \pi]$.\(^5\)

Here, we explain distribution functions used for binary systems. For all binary systems we take the mass ratio from the microlenses orbit from the Öpik’s law where the distribution function for the primary-secondary distance is proportional to $\rho (s) = dN/ds \propto s^{-3}$ (Öpik 1924) in the range of $s \in [0.6, 30]$ AU. We use the generalized version of the adaptive contouring algorithm (Dominik 2007) for simulating astrometric trajectories in binary microlensing events.

We let the microlenses rotate around their center of mass. Indeed, the perturbations developed with time, such as lens orbital motion, can make considerable deviations in astrometric trajectories that help to discover the second microlens (Sajadian 2014). The parameters of lens orbit are taken as follows: the time of arriving at the perihelion point of orbit $t_p$ is chosen uniformly in the range of $t_p \in [t_0 - P, t_0 + P]$ where $P$ is the orbital period of lenses motion. The projection angles to specify the orientation of orbit of lenses with respect to the sky plane, i.e., $\delta$ and $\vartheta$, are taken uniformly in the range of $[\pi/2, \pi/2]$. We take the eccentricity of the lenses’ orbit uniformly in the range of $e \in [0, 1]$.

### 3.2. Results

To determine the detectability of binary signals we calculate $\chi^2$. Two models are fitted to the astrometric and photometric data: (a) the known rotating binary microlensing model with the parameters used to simulate data, i.e., $\chi^2_b$ and (b) the simple Paczyński microlensing events with the same Paczyński parameters, i.e., $\chi^2_s$. Our criterion for detectability of the binary signal is $\Delta \chi^2 = \chi^2_b - \chi^2_s > \Delta \chi^2_b, \chi^2_s$ (for $j = b, s$) from fitting the astrometric trajectory along the scan $\theta_s$, and the magnification factor $A$ is given by

$$\chi_j^2 = \Sigma_{i=1}^{N} \left[ \frac{(A_i - A_j(t_i))^2}{\sigma_A^2} + \left( \theta_{xi} - \theta_{xj}(t_i) \right)^2 / \sigma_{\theta_x}^2 \right],$$

where $N$ is the number of data points and $\sigma_A$ is the $Gaia$ photometric precision. We assume that from the fitting process and searching all parameter space, the best-fitted solution is the known binary solution. We consider $\Delta \chi^2_b = 200$.

In Figure 1 we show a typical simulated binary microlensing event with a BBH being observed with Gaia. The light curves (left panel), astrometric trajectories (right panel), and the source trajectories with respect to the caustic curve (the inset in the left-hand panel) without and with considering the parallax effect are shown with green dashed and blue dotted lines, respectively. The point-mass lens models (considering the parallax effect) are shown by red solid lines. Data points taken by $Gaia$ are shown with black points. The relevant parameters of the source and lens are $M_s = 1.1 \, M_\odot$, $m_s = 15.8$ mag, $D_s = 2.0$ kpc, $\xi = -20^\circ$, $v_i = 120$ km s$^{-1}$, $u_0 = 0.4$ and $M_l = 10 \, M_\odot$, $q = 0.6$, $D_l = 0.4$ kpc, $d = 0.9$ $R_\odot$, respectively.

Figure 1. A typical binary microlensing event with binary stellar-mass black holes. The light curves (left panel), astrometric trajectories (right panel), and the source trajectories with respect to the caustic curve (the inset in the left-hand panel) without and with considering the parallax effect are shown with green dashed and blue dotted lines, respectively. The point-mass lens models (considering the parallax effect) are shown by red solid lines. Data points taken by $Gaia$ are shown with black points. The relevant parameters of the source and lens are $M_s = 1.1 \, M_\odot$, $m_s = 15.8$ mag, $D_s = 2.0$ kpc, $\xi = -20^\circ$, $v_i = 120$ km s$^{-1}$, $u_0 = 0.4$ and $M_l = 10 \, M_\odot$, $q = 0.6$, $D_l = 0.4$ kpc, $d = 0.9$ $R_\odot$, respectively.

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\(^3\) http://lambda.gsfc.nasa.gov/
\(^4\) http://sci.esa.int/gaia/40846-galactic-structure
Table 1: The Results of the Simulation of the Binary Astrometric Microlensing Events with GAIA

| Population | $\tau_E$ (days) | $\tau_{ae}$ (days) | $\pi_a$ | $\varepsilon_b$ (%) | $\Sigma (M_\odot \text{ pc}^{-2})$ | $\pi \times 10^{-7}$ | $N_b (f = 0.1)$ | $N_b (f = 0.3)$ | $N_b (f = 0.5)$ |
|------------|-----------------|-------------------|--------|---------------------|----------------------------------|-------------------|----------------|----------------|----------------|
| BBHs       | 90.2            | 568.6             | 18.3   | 13.5                | 2.09                             | 8.0               | 5.6            | 16.9           | 28.1           |
| BNSs       | 27.1            | 109.5             | 32.8   | 16.5                | 0.91                             | 8.6               | 11.4           | 34.2           | 57.1           |
| BWDs       | 17.9            | 66.0              | 34.1   | 15.6                | 2.41                             | 34.8              | 76.8           | 230.3          | 383.8          |
| BMSs       | 11.5            | 42.5              | 43.3   | 17.6                | 21.3                             | 434.9             | 1316.1         | 3948.4         | 6580.6         |

Note. The average values of the Einstein crossing time $\tau_E$ (days), the duration of the astrometric event $\tau_{ae}$ (days) threshold impact parameter $\pi_a$, efficiency for detecting binary signatures of microlenses $\varepsilon_b$ (%), the column density $\Sigma (M_\odot \text{ pc}^{-2})$, astrometrical optical depth $\pi$, and number of binary astrometric microlensing events detected with Gaia due to different binary systems: binary stellar-mass black holes (BBHs), binary neutron stars (BNSs), binary white dwarfs (BWDs), and binary main-sequence stars (BMSs) considering different binary fractions $f$. 

$v_t = 120 \text{ km s}^{-1}$, $u_0 = 0.4$ and $M_t = 10 M_\odot$, $q = 0.6$, $D_t = 0.4 \text{ kpc}$, $d = 0.9 R_E$, respectively. In this event, the binary signal is detectable with Gaia; however, in the photometric light curve there is one data point with significant deviation with respect to the simple microlensing light curves, whereas in the astrometric trajectory there are several deviated data points. Hence, although the Gaia cadence is too sparse such that some of the caustic-crossing features may be missed in the photometric light curves, the astrometric measurements can most likely resolve the binary signatures from the second microlenses. Indeed, the astrometric cross section is much larger than the photometric one.

In Table 1 we provide the average values of the Einstein crossing time $\tau_E$, the duration of the astrometric event $\tau_{ae}$, threshold impact parameter $\pi_a$, efficiency for detecting binary signals $\varepsilon_b$ (i.e., the fraction of the simulated binary events in which the binary signals are detectable with $\Delta \chi^2 > \Delta \chi_\text{th}^2$ given in percent), the column densities for different stellar populations $\Sigma$ (taken from Flynn et al. 2006 and Chabrier 2001), and the astrometrical optical depth $\pi$ determined from the second to sixth column data for the lenses resulting from the different binary systems. Note that we calculate the optical depth for different directions toward the Galactic bulge, i.e., $l$ in the range of $[-10, 10]^{\circ}$ and $b$ in the range of $[5, 5]^{\circ}$ and average over all of them.

The expected number of binary microlensing events being observed with Gaia with detectable binary signals, i.e., $N_b$, is given by

$$N_b = N_{b0} f \ varepsilon_b,$$

where $f$ is the fraction of binary objects in different stellar populations. To calculate $N_{b0}$ (Equation (7)) we insert the average values of $\tau_E$, $\pi_a$ for each stellar population. Also, we set the number of background source stars toward the Galactic bulge $N_{bg} = 3.0 \times 10^8$ (Cacciari 2014). Note that the number of background stars brighter than 20 G magnitudes for the whole sky is on the order of billions of objects. The amounts of $N_{b0}$ for three values of $f = 0.1, 0.3$, and $0.5$ are reported in Table 1.

According to this table, (i) the Gaia efficiency for detecting different binary systems as microlenses is almost 10%–20%, which is acceptable according to the sparse cadence of Gaia. It means that Gaia detects the signal from the second lens with a probability of $\sim$10%–20%.

(ii) The Gaia efficiency decreases with increasing the lens mass because, by increasing the lens mass, the normalized distance between two lenses decreases. Hence, binary black holes usually make close binaries for which the probability of caustic crossing is smaller than that in the intermediate binaries due to the less massive microlenses.

(iii) Gaia can potentially determine the binary fraction of massive stellar populations because the numbers of estimated events for different binary fractions are larger than 1. The number of estimated events for binary black holes is small, which means that the uncertainty for specifying their binary fraction is high, especially if the real binary fraction is larger than 0.1.

(iv) The duration of the astrometric events is given by (Dominik & Sahu 2000; Belokurov & Evans 2002) $\tau_{ae} = \tau_E / \left( \frac{5 \sqrt{2} \sigma_a}{\pi} \right)$, which is proportional to the Einstein crossing time. Indeed, the ratio of $\tau_E / 5 \sqrt{2} \sigma_a$ is almost constant for different stellar populations and larger than one. Hence, the more massive microlenses with longer Einstein crossing time have the longer astrometric durations. The longest duration among the astrometric events is that due to BBHs, at $\sim$1.56 yr, which equates to one third of the Gaia lifetime. The more massive microlenses have longer astrometric duration, comparable with the Gaia lifetime, and hence are not suitable for observation by Gaia.

4. MEASURING THE MASS OF MASSIVE STELLAR POPULATIONS

Here we investigate Gaia’s efficiency in measuring the mass of the massive stellar populations through astrometric microlensing of single lenses and specify their mass distributions. In this regard, we quantitatively estimate the Gaia efficiency and the number of high-quality astrometric microlensing events by performing a Monte Carlo simulation.

We consider different stellar populations, i.e., stellar-mass black holes, neutron stars, white dwarfs, and main-sequence stars as microlenses and simulate astrometric microlensing events due to these microlenses according to their distribution functions explained in Section 3.1. Having regenerated the astrometric and photometric curves with synthetic data points according to the Gaia observing strategy, we should indicate whether the lens mass can be inferred from data. In such a case where it can be inferred, we evaluate the uncertainty in the lens mass, i.e., $\sigma_M$, in the same way as Belokurov & Evans (2002), and accept the events with the relative error, i.e., $\sigma_M/M_t$, less than 0.5 as high-quality events.

4.1. Results

The results of this Monte Carlo simulation are summarized in Table 2, which lists the average values of the lens mass $M_t (M_\odot)$, Einstein crossing time $\tau_E$ (days), duration of the
The average values of the lens mass $M_l$ (M$_\odot$), Einstein crossing time $\tau_E$ (days), duration of the astrometric event $\tau_{ae}$ (days), threshold impact parameter $\pi_v$, lens–source transverse velocity $\pi_t$ (km s$^{-1}$), source distance from the observer $D_s$ (kpc), astrometric precision of Gaia $\pi_G$ (mas), detection efficiency for high-quality events $\varepsilon$ (%), and finally the number of high-quality events being observed during the Gaia era for four different stellar populations: stellar-mass black holes (BHs), neutron stars (NSs), white dwarfs (WDs), and main-sequence stars (MSs).

Interpreting this table, (i) the more massive lenses, the larger efficiencies of measuring the lens mass. Indeed, the Einstein crossing time in astrometric microlensing events due to massive lenses is long. As a result, the number of data over the astrometric trajectories and photometric light curves increases. On the other hand, their angular Einstein radius and astrometric signal are high.

(ii) The Gaia astrometric precision for astrometric microlensing due to more massive microlenses is larger than that due to less massive ones. Indeed, we consider two restrictions for simulating astrometric microlensing events: (a) the source star should be brighter than 20 G magnitude and (b) the angular Einstein radius should be larger than $5\sqrt{2}\sigma_s$. Hence, the average value of $\sigma_v$, reported in the seventh column, represents the minimum amount of the angular Einstein radius, which in turn is an increasing function with respect to the lens mass. As a result, the threshold impact parameter, which is proportional to $1/\sqrt{\delta_f}$, decreases with enhancing the lens mass.

(iii) The number of high-quality events due to the massive stellar populations are larger than one. Hence, we can obtain some information about their masses and mass distribution. A significant point is that this method can even give the mass of isolated objects. For the main-sequence stars as microlenses, we conclude that some of the 786 high-quality events toward the Galactic bulge will probably be confirmed during the Gaia era. However, Belokurov & Evans (2002) estimated that number of high-quality events due to main-sequence stars for the whole sky is $\sim$2500. Indeed, we consider the number of background stars brighter than 20 G magnitudes toward the Galactic bulge as 300 million objects, which is far fewer than their consideration, i.e., one billion objects. There are some differences between the distribution functions used in our simulation and those used in the referred paper. However, they estimated just high-quality events due to main-sequence stars.

Note that the mass ranges of these four stellar populations as microlenses overlap on the edges, but we can discern the type of microlenses with the masses in the common ranges (belonging to two populations) through their color and magnitude because these microlenses are most likely located at a distance less than 1 kpc from the observer.

In order to study the dependance of efficiency on the parameters of the model, in Figure 2 we plot the normalized efficiencies, i.e., $\varepsilon_a$, for detecting high-quality events due to stellar-mass black holes (black solid line), neutron stars (red dashed line), white dwarfs (green dotted line), and main-sequence stars (blue dotted–dashed line) in terms of the relevant parameters of the lens and source. To compare the efficiency curves with each other, we normalize all efficiency curves to 1 which means the area under each curve is 1, hence we name them normalized efficiency curves. Since the area under each efficiency curve is proportional to the related value reported in the ninth column of Table 2, i.e., $\varepsilon$, the normalization is done by dividing $Y$ values to $\varepsilon$. Generally, the behaviors of efficiency for different stellar populations are similar, although some differences exist in the slope of curves, peak position, etc. The normalized efficiency function in terms of the lens and source parameters is given as follows.

(i) The first one is the impact parameter, $u_0$. The efficiency of detecting high-quality events is high for astrometric microlensing events with $u_0 < 1$ for which the photometric microlensing as well as the astrometric one can be observed.

(ii) The second parameter is the relative velocity of the source with respect to lens, $v_t$. The efficiency increases with decreasing relative velocity. Indeed, the Einstein crossing time rises with decreasing the relative velocity. Since the Gaia cadence is almost constant for all events toward the Galactic bulge, the number of observational data for long-duration microlensing events (over the astrometric trajectories and photometric light curves) is high. As a result, the errors in parameters for these events are smaller than those due to short-duration ones.

(iii) and (iv) The next two parameters: $M_*$, the mass, and $m_*$, the apparent magnitude of source star. The Gaia astrometric precision depends strongly on the source magnitude and increases as the magnitude decreases. Hence, the Gaia efficiency for measuring the lens mass in astrometric microlensing due to brighter and heavier sources is higher than that due to fainter and less massive ones.

(v) The next parameter is the normalized lens mass, $M_l$. Since mass ranges vary within the stellar population, we normalize the lens mass for each stellar population to its maximum amount so that the range of the normalized lens mass for all populations is in the range of $[0, 1]$. For the massive lenses the Einstein crossing time is long. Consequently, the amount of data over the astrometric trajectory and photometric light curve increases. On the other hand, their angular Einstein radius and astrometric signal are high. Therefore, the Gaia efficiency for measuring the mass of more massive lenses is higher than that of less massive lenses.
Note that all efficiency because for these events the amount of observational data over the astrometric trajectory and light curve is maximum.

The next parameter is the corresponding time to the closest approach, $t_0$. We uniformly choose $t_0$ in the range of $[0, 5]$. However, the events with $t_0 \sim 2.5$ T has the maximum efficiency because for these events the amount of observational data over the astrometric trajectory and light curve is maximum.

(vi) The next parameter is the corresponding time to the closest approach, $t_0$. We uniformly choose $t_0$ in the range of $[0, 5]$. However, the events with $t_0 \sim 2.5$ T has the maximum efficiency because for these events the amount of observational data over the astrometric trajectory and light curve is maximum.

(vii) and (viii) The last two parameters are $x = D_s/D_l$, the relative distance of the lens to the source star from the observer, and $D_s$, the source distance from observer. Indeed there are two kinds of suitable sources for the $Gaia$ detection: (a) nearby sources with $D_s < 1$ kpc and (b) the bright sources often located at the Galactic disk. Correspondingly, there are two peaks in the efficiency diagram versus $D_s$. These two classes have different $x$ values. We know that the Einstein crossing time is proportional to $t_E \propto \sqrt{D_s x (1-x)}$, but the relative parallax and the angular Einstein radius depend on these parameters as $\theta_E \propto \sqrt{\pi rel} \propto \sqrt{\frac{1-x}{D_s}}$. Although the efficiency enhances with increasing each of them, the first is an increasing function versus $x$ and $D_s$, and the others are decreasing functions. Therefore, the efficiency is maximized for the first class when $x \sim [0.6, 0.7]$ and for the second class when $x \ll 1$.

According to the dependence of efficiency on the parameters, we expect that most high-quality events have long Einstein crossing times or small impact parameters or their source is a bright star. Hence, most sources are close to the observer.

We also investigate the dependence of the $Gaia$ (normalized) efficiency on the Galactic latitude, shown in the top panel of Figure 3. The normalized efficiency decreases slowly by increasing the distance from the Galactic plane because there are two effects acting in opposition: (i) the extinction toward the Galactic bulge strongly increases by decreasing the distance from the Galactic plane as shown in the bottom panel of Figure 3, and (ii) the Galactic disk density (Equation (5)), decreases by increasing distance from the Galactic plane. The first effect decreases the probability of finding a source star brighter than 20 G magnitude toward the Galactic center while the abundance of stars is maximum in this direction. Consequently, the efficiency is almost constant over the Galactic bulge. To determine the extinction, we average over the amounts of the extinction due to the Galactic longitudes in the range of $[H0, 10]^2$ for each value of $b$.

5. CONCLUSIONS

One of the astronomical phenomena being observed with $Gaia$ is astrometric microlensing events. In this work, we investigated whether $Gaia$ can specify the binary fractions and mass distributions of massive stellar populations through astrometric microlensing.

We simulated binary astrometric microlensing events being observed with $Gaia$ due to binary black holes, binary neutron stars, binary white dwarfs, and binary main-sequence stars to evaluate the efficiency for detecting the binary signatures. We considered $\chi^2 > 200$ as the detectability criterion and concluded that $Gaia$ detects the signal from the second lens with a probability of $\sim 10\% - 20\%$. Then, we estimated the number of binary astrometric microlensing due to the mentioned populations with detectable binary signals in the $Gaia$ era as 6, 11, 77, and 1316, respectively ($f = 0.1$). Hence, $Gaia$ can potentially determine the binary fraction of massive stellar populations. Since the predicted number of binary astrometric microlensing events of stellar-mass black holes is small, the $Gaia$ accuracy to specify their binary fraction is low. The significant point is that binary systems composed
of white dwarfs, neutron stars, or black holes are sources of detectable gravitational waves. Therefore, indicating the binary fraction of these massive stellar populations helps to assess the amount of the gravitational waves.

Also, we investigated the Gaia efficiency for measuring the mass of massive stellar populations in the Galactic disk, i.e., stellar-mass black holes, neutron stars, and white dwarfs in addition to main-sequence stars as microlenses in astrometric microlensing events of single lenses. The Gaia efficiency for measuring the mass of microlenses enhances with increasing lens mass. By performing Monte Carlo simulations, we concluded that the Gaia efficiencies for detecting high-quality events due to these stellar populations are 9.8%, 2.9%, 1.2%, and 0.8%, respectively. We estimated the number of high-quality events for these populations as 45, 34, 76, and 786, respectively. Hence, Gaia can obtain some information about their masses and the mass distribution of these massive stellar populations.

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