Quantum interference effects in \((\vec{e}, e'p)\) reactions

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Abstract

The response to longitudinally polarized electrons in coincident out-of-plane \((\vec{e}, e'p)\) reaction is discussed to study the role of final state interactions and quantum interference between different reaction channels, i.e. between direct (quasi) elastic proton emission and baryon resonance production. The high-energy suppression of this effect due to colour transparency is also discussed. Results are given for the \(^{12}\text{C}(\vec{e}, e'p)^{11}\text{B}_{\text{g.s.}}\) reaction.
In quasifree nucleon knockout by longitudinally polarized electrons (helicity \( h = \pm 1 \)) the coincidence cross section is separated into helicity independent (\( \Sigma \)) and dependent (\( \Delta \)) contributions [1,2]. The part \( \Sigma \) is responsible for nucleon knockout by unpolarized electrons and has been extensively studied both experimentally and theoretically (see references in [1]). The helicity dependent part \( \Delta \) is proportional to the so-called “fifth” structure function, \( W'_{LT} \), and to \( \sin \alpha \), where the (out-of-plane) angle \( \alpha \) is the angle of rotation about the momentum \( \vec{q} \) of the exchanged virtual photon \( \gamma^* \) to bring the electron scattering plane to coincide with the plane of the final hadron system. The fifth structure function \( W'_{LT} \) is the (imaginary part of the) longitudinal-transverse (L-T) interference of the target currents and vanishes identically in plane-wave-impulse approximation (PWIA) or anyhow when final-state interactions (FSI) are absent. It also vanishes when the reaction proceeds through a channel in which a single phase dominates for all the projections of the target current [3]. As such, \( W'_{LT} \) provides an observable which is highly sensitive to FSI.

The helicity dependent term \( \Delta \) in the cross section breaks the symmetry for particle emission above (\( \alpha > 0 \)) and below (\( \alpha < 0 \)) the electron scattering plane for fixed electron helicity \( h \) or, alternatively, for electrons with \( h = 1 \) and \( h = -1 \) and particles emitted at a fixed \( \alpha \). One may select \( \alpha = \pi/2 \) and define the helicity asymmetry according to one of the following alternative forms:

\[
A = \frac{\Sigma}{\Delta} = \frac{N(1, \frac{\pi}{2}) - N(-1, \frac{\pi}{2})}{N(1, \frac{\pi}{2}) + N(-1, \frac{\pi}{2})} = \frac{N(1, \frac{\pi}{2}) - N(1, -\frac{\pi}{2})}{N(1, \frac{\pi}{2}) + N(1, -\frac{\pi}{2})},
\]

where \( N(h, \alpha) \) is the number of events observed at a given angle \( \alpha \) for a fixed electron helicity \( h \).

In a series of papers [4-7], the effect of the interference between elastic and resonance channels has been studied in quasielastic scattering, with special interest for the colour transparency problem [4,5,7] and for the anomalous production of resonances in nuclei [6]. The idea is that in the hard scattering with \( \gamma^* \) the target
nucleon, besides undergoing elastic scattering, may also be excited into a barionic state $N^*$ [8]. Fermi motion plays an essential role in determining the composition of the hadron wave packet [4,9]. In fact, the wave packet produced by absorption of $\gamma^*$ is a superposition of physical hadronic states with the same energy and direction of motion, but different mass and momentum magnitude. These states propagate through the nuclear medium undergoing the soft FSI which are then responsible for interference between the elastic and inelastic channels. This modifies dramatically the cross section for particle emission.

In this paper, we investigate the effect of this kind of interference on the helicity asymmetry $A$.

The time-independent coupled-channel formalism developed in [4,5] is here extended to consider the case of three channels. The final-state wave function is formed by three coupled components, labelled $i = 1$, $2$ and $3$, which, according to the analysis in [10], are taken to correspond to the nucleon and two excited baryons, i.e. $S_{11}(1535)$ and $F_{15}(1680)$, respectively. These states are simultaneously produced with amplitudes $B_i$ in the hard scattering at a given point $\vec{r}_0$. A common phase $\exp(i\vec{q} \cdot \vec{r}_0)$ is given by the virtual photon $\gamma^*$. Then we calculate a function of the form $\exp(i\vec{q} \cdot \vec{r}_0) \cdot \Sigma_i B_i \Psi_i(\vec{r}, \vec{r}_0)$ ($\Psi_i(\vec{r}_0, \vec{r}_0) \equiv 1$), which is an eigenfunction of the final state Hamiltonian $H_0 + V$ ($3 \times 3$ matrix). All the three channels describe particles propagating in the same direction with the same energy, and $p_i = \sqrt{E^2 - M_i^2}$. $V \equiv \{V_{ij}, i, j = 1, 2, 3\}$ is an optical potential describing FSI. Its nondiagonal terms cause transitions between different channels. $V$ does not conserve flux ($V^\dagger \neq V$), but each of its elements conserves energy. The total wave function $\Psi(\vec{r})$ is a coherent sum of all the waves of the above kind emitted from any nuclear point $\vec{r}_0$.

The formalism in [5] is also modified to take into account the different contributions of the longitudinal and transverse components of the target current. The initial proton is assumed in the $j$-shell with total spin component $m$ resulting from vector coupling of the intrinsic spin with the orbital motion with angular
momentum \( l \). Consequently, the transition matrix, \( M = M^{(+)} + M^{(-)} \), is the sum of the two contributions corresponding to the two initial (orbital) states \( |l, m \pm \frac{1}{2} \rangle \).

According to [5] the amplitudes \( B_i(Q^2, s) \) are calculated as

\[
|B_i(Q^2, s)|^2 = \int d\vec{k} |g_i(Q^2)|^2 R_i(s) n(k),
\]

where \( s \equiv (q + k)^\mu(q + k)_\mu \), \( n(k) \) is the distribution for the bound nucleon momentum,

\[
R_i(s) = \frac{M_i \Gamma_i}{(s - M_i^2) + M_i^2 \Gamma_i^2}.
\]

The functions \( g_i(Q^2) \) are the photocoupling amplitudes for the various intermediate states produced by hard scattering. They were taken all equal in [4-7] according to quark counting rules. In the present approach, however, we are interested in the different spin transitions related to longitudinal/transverse currents. Therefore \( g_i(Q^2) \) must be taken different for \( M^{(\pm)} \) and expressed in terms of the helicity amplitudes [11]:

\[
g^{\pm}_i(Q^2) \equiv \langle i, m_s; e'_h \cdot \vec{J} |1; m_s = \pm 1/2 \rangle,
\]

where \( \vec{J} \) is the target current and \( e'_h \) is the virtual-photon polarization vector expressed in the reference frame with the \( z \)-axis along the outgoing proton momentum \( p' \).

The photocoupling amplitudes are taken from the phenomenological helicity amplitudes according to the analysis of [10,12].

The formalism has been applied to the reaction \(^{12}\text{C}(e, e'p)^{11}\text{B}_{g.s.} \), where the proton is knocked out from the p-shell with \( j = \frac{3}{2} \). This reaction has already been studied under quasielastic conditions in the kinematic range available at Bates with incident electrons of 560 MeV [13]. In this kinematic regime conventional
distorted-wave impulse approximation (DWIA) based upon phenomenological optical potentials is quite successful in explaining the cross section, the asymmetry and the extracted fifth structure function. The sensitivity of $W_{LT}'$ to FSI was proven in [13] to be useful in disentangling different phase-shift equivalent optical potentials which produce different scattering wave functions in the interior nucleus.

In this paper we are interested in quasielastic knockout ($\omega \sim Q^2/2M$) in the region with $|\vec{q}| = 2–6$ GeV. This region is of interest for the planned experiments at CEBAF, and has already been explored partially in the NE18 experiment at SLAC [14]. In the following, results obtained from the full calculation with quantum interference will be compared with PWIA and the conventional (Glauber) DWIA without quantum interference.

Fig. 1 shows the ratio $R$ between the total number of events for unpolarized incident electrons and the PWIA result at different values of the longitudinal component $p_{m\parallel}$ (with respect to $\vec{q}$) of the missing momentum $\vec{p}_m \equiv \vec{p}' - \vec{q}$. The transverse component $p_{m\perp}$ is taken to be 200 MeV, a value comparable with the Fermi momentum ($p_F = 221$ MeV) and suitable to produce significant out-of-plane yield. The figure confirms previously obtained results [5,7], where the role played by Fermi motion in producing the missing-momentum dependence of the ratio $R$ has been emphasized. As already noticed in [4,5], when one is not sensitive to $\vec{p}_m$ or effectively works at $p_{m\parallel} \sim 0$, a flat ratio $R$ is obtained as a function of $|\vec{q}|$ as in the NE18 experiment [14].

The main reason for calculating the ratio $R$ is to correlate it with the results of the full coupled-channel calculation of $A$. In the results for $R$, we can see three different regimes; (1) no remixing at all, leading to curves that are PWIA times a constant damping factor; (2) remixing without interference: at large $\vec{p}_m \cdot \vec{q}$ the traditional quasielastic yield is small, but many $N^*$ are produced by the virtual photon, and converted into a proton by FSI; this leads to the prominent enhancement in $R$ that one can see in Fig. 1, and in [5,7]; (3) remixing with
interference in the transition regions, or at large $Q^2$. Actually, at large $Q^2$ and small $\vec{p}_m \cdot \hat{q}$ all the maxima for resonance production overlap (see Fig. 3 in [5]), producing, hopefully, that “pointlike” hadron configuration that should lead to colour transparency. At lower $Q^2$ remixing and interference have been observed in a large enhancement of $N^*$ production in nuclei, compared with $\Delta$ production in the same conditions [15,6].

In Fig. 2 the helicity asymmetry $A$ multiplied by $Q^2$ is plotted for the conventional DWIA case and with quantum interference. All curves become approximately constant as a function of $|\vec{q}|$. In fact $W'_{LT}$ contains combinations like $G_0G_+$, where $G_\pm$ and $G_0$ are the spin-flip and no-spin-flip helicity amplitudes for $\gamma^* + p \rightarrow p$. At not too small $Q^2$ values, $G_\pm/G_0 \sim Q$, and in the total cross section the pure transverse structure function $W_T$ is the dominating term, so that $A$ becomes proportional to $W'_{LT}/W_T$. The fact that $A$ is roughly proportional to $1/Q^2$ suggests that in the considered range of $Q$ the leading $Q$ dependence of the L-T interference cancels when taking the appropriate combination to build $W'_{LT}$. Otherwise, one would have $A \sim W'_{LT}/W_T \sim G_0G_+/G_+G_+ \sim Q$.

In contrast, the quantum interference effects are evident. While $R$ is especially sensitive to the regime (2) of remixing without interference, the asymmetry $A$ is rather affected by regime (3). Actually, a comparison of Figs. 1 and 2 shows that at small values of $\vec{p}_m \cdot \hat{q}$ we obtain curves that present traditional values of $R$, and for such kinematics $A$ is not much affected by the multichannel effects. On the contrary, at larger values of $\vec{p}_m \cdot \hat{q}$ the behaviour of $A$ is nontrivial. It is remarkable that at large and positive $\vec{p}_m \cdot \hat{q}$, but low $Q^2$, the asymmetry can be enhanced by quantum interference with an excited channel. In the high-$Q^2$ region, where onset of colour transparency could start, the channels enter with relative weights which mutually cancel the non-PWIA contributions, thus restoring the perfect PWIA, i.e. the perfect nuclear transparency. In this regime no term dominates the asymmetry, which goes to zero.

An interesting feature is the changing sign of $A$, which is clearly correlated.
with the dominance of the inelastic channels as a consequence of the choice of phases according to diffractive sum rules [4-7,17]. The elastic and the inelastic channels enter the overall amplitude with opposite signs. The dominance of the elastic or inelastic channels in FSI directly manifests itself in the sign of the asymmetry. This occurs at smaller values of $Q^2$ than in the ratio $R$, because $A$ is insensitive to the dominating PWIA-elastic contribution to the cross section. This feature of $A$ is particularly remarkable, because a direct measurement of transparency at the cross section level cannot test the finite-energy sum rules nor determine the relative phases of elastic and inelastic diffractive N-N scattering in FSI.

On the other side, the absolute values predicted here for $A$ are rather low. As a better guidance for a possible measurement, in Fig. 3 the calculated helicity asymmetry $A$ is shown both in conventional DWIA and with the quantum interference effects. A significant reduction is introduced by such effects. However, the absolute magnitude of the asymmetry is presumably too low to be observed. This low value can be explained with the following argument. In conventional DWIA the fifth structure function $W'_{LT}$ is small because the longitudinal part is also small and decreases with increasing ejectile energy faster than the transverse part. Electromagnetic baryonic excitations are mostly transverse [12]. Thus the L-T interference becomes even smaller when quantum interference is taken into account. The same behaviour can be expected for the nucleon recoil polarization $P_N$ in the direction normal to the hadron plane where under parallel kinematic conditions (i.e. $\vec{p}'$ parallel to $\vec{q}$) a similar L-T structure function determines the observed response (see also [17]).

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References

[1] S. Boffi, C. Giusti and F.D. Pacati, Phys. Rep. 226 (1993) 1.

[2] T.W. Donnelly and A.S. Raskin, Ann. Phys. (NY) 169 (1986) 247; 191 (1989) 78.

[3] T.W. Donnelly, in: Perspectives in Nuclear Physics at Intermediate Energies, eds. S. Boffi, C. Ciofi degli Atti and M.M. Giannini (World Scientific, Singapore, 1983) p. 305.

[4] A. Bianconi, S. Boffi and D.E. Kharzeev, Phys. Lett. B305 (1993) 1.

[5] A. Bianconi, S. Boffi and D.E. Kharzeev, Nucl. Phys. A565 (1993) 767.

[6] A. Bianconi, S. Boffi and D.E. Kharzeev, Phys. Rev. C49 (1994) R1243.

[7] A. Bianconi, S. Boffi and D.E. Kharzeev, Phys. Lett. B325 (1994) 294.

[8] B.K. Jennings and G.A. Miller, Phys. Lett. B236 (1990) 209; Phys. Rev. D44 (1991) 692.

[9] B.K. Jennings and B.Z. Kopeliovich, Phys. Rev. Lett. 70 (1993) 3384.

[10] P. Stoler, Phys. Rep. 226 (1993) 103.

[11] C.E. Carlson, Phys. Rev. D34 (1986) 2704.

[12] Z.P. Li, V. Burkert and Z. Li, Phys. Rev. D46 (1992) 70; V. Burkert and Z. Li, Phys. Rev. D47 (1993) 46.

[13] J. Mandeville et al., Phys. Rev. Lett. 72 (1994) 3325.

[14] N.C.R. Makins et al., Phys. Rev. Lett. 72 (1994) 1986; T.G. O’Neill et al., Phys. Rev. Lett., to be published.

[15] L.B. Weinstein et al., Phys. Rev. C47, 225 (1993).
[16] L. Frankfurt, W.R. Greenberg, G.A. Miller and M. Strikman, Phys. Rev. C46 (1992) 2547; L. Frankfurt, M. Strikman and G.A. Miller, Comm. Nucl. Part. Phys. 21 (1992) 1; N.N. Nikolaev, Int. J. Mod. Phys. E3 (1994) 1.

[17] W.R. Greenberg and G.A. Miller, Phys. Rev. C49 (1994) 2747; C50 (1994) 2643 (E).
**Figure captions**

Fig. 1. The transparency coefficient $R$ for the $^{12}\text{C}(\vec{e}, e'p)^{11}\text{B}_{g.s.}$ reaction as a function of the three-momentum transfer $q = |\vec{q}|$ (in GeV). The transverse component $p_{m\perp}$ (with respect to $\vec{q}$) of the missing momentum is fixed at 200 MeV. The longitudinal component is $p_{m\parallel} = 0, 100, 150, 200$ MeV for the solid, dotted, dashed and dot-dashed curves, respectively.

Fig. 2. The helicity asymmetry $A$ multiplied by $Q^2$ for the $^{12}\text{C}(\vec{e}, e'p)^{11}\text{B}_{g.s.}$ reaction as a function of the momentum transfer $q = |\vec{q}|$ (in GeV). Solid and dotted curves calculated in conventional DWIA, dashed and dot-dashed curves with quantum interference. Solid and dashed curves for a vanishing longitudinal component $p_{m\parallel}$ of the missing momentum, dotted and dot-dashed curves for $p_{m\parallel} = 200$ MeV. The transverse component $p_{m\perp}$ (with respect to $\vec{q}$) of missing momentum is fixed at 200 MeV.

Fig. 3. The helicity asymmetry $A$ for the $^{12}\text{C}(\vec{e}, e'p)^{11}\text{B}_{g.s.}$ reaction as a function of three-momentum transfer $q = |\vec{q}|$ (GeV) for a missing momentum with 200 MeV transverse and 100 MeV longitudinal components with respect to the photon momentum $\vec{q}$. Solid and dashed (dotted and dot-dashed) curves for an electron scattering angle $\theta = 36^\circ$ ($72^\circ$). Dashed and dot-dashed curves in conventional DWIA, solid and dotted curves with quantum interference of intermediate hadron states.
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