Crack-tip Stress Fields on Anisotropic Plate

Purong Jia¹, Yongyong Suo²

¹ School of Mechanics and Civil-Engineering & Architecture, Northwestern Polytechnical University, Xi’an, Shaanxi, 710129, PR China
² School of Mechanics and Civil-Engineering & Architecture, Northwestern Polytechnical University, Xi’an, Shaanxi, 710129, PR China

*Corresponding author’s e-mail: prjia@nwpu.edu.cn

Abstract. The basic equation of elastic mechanics has been established according to the anisotropic material properties. The crack-tip stress problem has been investigated for the mixed loading condition. In order to determine the stress fields in an anisotropic plate, the pan-complex variable means must be utilized. Typical analytic function including the material parameter was analyzed fully. The special functions have been determined to be satisfactory for the stress boundary conditions. The pan-complex variable means as well as polar coordinate transform method were adopted so as to solve the mixed loading problems on cracked anisotropic plate. Finally, the explicit expressions of the singular stress fields were obtained with the real variables. The means in the paper ought to be of benefit to the stress fields and fracture character analysis of anisotropic plates.

1. Introduction
Fracture mechanics of non-homogeneous materials or anisotropic materials has more applications in new engineering structure materials. Conventional fracture theory discusses on the homogeneous solid body in thorough investigation and has been of rich experience in much engineering application [1-3]. However, the crack problems of anisotropic plates must be actually relative to a lot of engineering components [4-5]. The only feasible method to solve the stress problems in the anisotropic body may be to use the complex variable function theory, and the results have been reported [6-7]. Even though, the general solution of crack-tip stress fields may be not accomplished successfully. Thus, it is necessary to make further improvements in the method of solving stress fields to anisotropic materials. This paper is aimed at the stress field investigation in the cracked plate for anisotropic materials.

2. Basic equation and pan-complex function
Engineering configurations with some flaws must be discussed about the stresses at the crack zone. The crack-tip stress fields of composite material structure ought to be solved for the sake of actual application. Particularly, the investigation of the mixed loading cracked plate shown in Figure 1 must have much meaning in the plane failure analysis for anisotropic plates. The analytic method on stress fields will be explained as below.

2.1 Stress and strain equations
It is common knowledge that the equilibrium equations in the x-y-plane (no body forces) are written in the following form:
Figure 1. The centre crack plate with mixed-mode loading.

\[ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad , \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \]  

(1)

For the sake of the demands of the equilibrium equations, the stress components may be expressed generally by the function \( F = F(x, y) \) in the following:

\[ \sigma_x = \frac{\partial^2 F}{\partial y^2} \quad , \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} \quad , \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \]  

(2)

The linear elastic physical relation of the stresses with the strains in the plane can be given by means of the material characteristics. It is known that the strain-stress relationship of the anisotropic plates can be indicated by the compliances \( (a_{ij}) \) as below:

\[ \begin{align*}
\varepsilon_x &= a_{11} \sigma_x + a_{12} \sigma_y + a_{16} \tau_{xy} \\
\varepsilon_y &= a_{12} \sigma_x + a_{22} \sigma_y + a_{26} \tau_{xy} \\
\gamma_{xy} &= a_{16} \sigma_x + a_{26} \sigma_y + a_{66} \tau_{xy}
\end{align*} \]  

(3)

The three strains \( (\varepsilon_x, \varepsilon_y, \gamma_{xy}) \) are of restrict relativity. Besides, we know that the plane harmonic demand of the strains in the x-y plane must be necessary. Thus it is introduced that:

\[ \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \]  

(4)

By means of above stress function and constitutive equations, the governing equation of the harmonic condition can be expressed by the stress function \( F(x, y) \), that is:

\[ \frac{\partial^4 F}{\partial y^4} + A_1 \frac{\partial^4 F}{\partial x \partial y^3} + A_2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + A_3 \frac{\partial^4 F}{\partial x^3 \partial y} + A_4 \frac{\partial^4 F}{\partial x^4} = 0 \]  

(5)

In which,

\[ A_1 = -\frac{2a_{16}}{a_{11}} \quad , \quad A_2 = \frac{a_{66} + 2a_{12}}{a_{11}} \quad , \quad A_3 = -\frac{2a_{26}}{a_{11}} \quad , \quad A_4 = \frac{a_{22}}{a_{11}} \]
Thus, solving the typical problem of anisotropic plates can be reduced to find the solution of above governing equation, and the solution must satisfy the given boundary conditions.

2.2 Pan-complex variables and relative derivations

In order to obtain the solution of the governing equation, we must have the aid of the pan-complex function. For solving the complete stresses in the cracked anisotropic plate, the variables \((w, \bar{w})\) with complex numbers may be defined as:

\[
\begin{align*}
    w &= x + qy = x + gy + ihy, \\
    \bar{w} &= x + \bar{q}y = x + gy - ihy
\end{align*}
\]

In which, \(q = g + ih\), \(\bar{q} = g - ih\), and \(i = \sqrt{-1}\), that denotes the imaginary unit. Notice that two constants, \(g\) and \(h\) are arbitrary real numbers. Besides, let \(h > 0\) to be necessary. Two common complex numbers \((q, \bar{q})\) have some relations, that is:

\[
q + \bar{q} = 2g, \quad q - \bar{q} = 2hi, \quad q\bar{q} = g^2 + h^2
\]

Two variables \((w, \bar{w})\) can be of the normal result as below:

\[
\begin{align*}
    \|w\|^2 &= \|\bar{w}\|^2 = w\bar{w} = (x + qy)(x + \bar{q}y) \\
    &= (x + gy)^2 + h^2y^2 = x^2 + 2gxy + g^2y^2 + h^2y^2
\end{align*}
\]

Generally, \(F(x, y)\) can be replaced by the variables, \(w, \bar{w}\), namely, \(F(w, \bar{w})\). Then partial derivations of \(F\) with \(x\) or \(y\) can be given in the following expressions:

\[
\begin{align*}
    \frac{\partial F(w, \bar{w})}{\partial x} &= \frac{\partial F}{\partial w} \frac{\partial w}{\partial x} + \frac{\partial F}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial x} = \frac{\partial F}{\partial w} + \frac{\partial F}{\partial \bar{w}} \\
    \frac{\partial F(w, \bar{w})}{\partial y} &= \frac{\partial F}{\partial w} \frac{\partial w}{\partial y} + \frac{\partial F}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial y} = q \frac{\partial F}{\partial w} + \bar{q} \frac{\partial F}{\partial \bar{w}}
\end{align*}
\]

(7)

And

\[
\begin{align*}
    \frac{\partial^2 F}{\partial x^2} &= \frac{\partial^2 F}{\partial w^2} + 2 \frac{\partial^2 F}{\partial w \partial \bar{w}} + \frac{\partial^2 F}{\partial \bar{w}^2} \\
    \frac{\partial^2 F}{\partial y^2} &= q^2 \frac{\partial^2 F}{\partial w^2} + 2(g^2 + h^2) \frac{\partial^2 F}{\partial w \partial \bar{w}} + \bar{q}^2 \frac{\partial^2 F}{\partial \bar{w}^2} \\
    \frac{\partial^2 F}{\partial x \partial y} &= q \frac{\partial^2 F}{\partial w \partial \bar{w}} + 2g \frac{\partial^2 F}{\partial w \partial \bar{w}} + \bar{q} \frac{\partial^2 F}{\partial \bar{w}^2}
\end{align*}
\]

(8)

By the reference axes as drawing in Figure 1, the crack with the length, \(2a\), two variables must be written in the following form:

\[
\begin{align*}
    w &= x + qy = a + r \cos \theta + gr \sin \theta + ihr \sin \theta \\
    \bar{w} &= x + \bar{q}y = a + r \cos \theta + gr \sin \theta - ihr \sin \theta
\end{align*}
\]

(9)

For the sake of the simplification of pan-complex functions, the real functions must be determined by the following relations:

\[
\begin{align*}
    \cos \theta + g \sin \theta &= J \cos \beta \\
    h \sin \theta &= J \sin \beta
\end{align*}
\]

(10)

In which, \(J\) is positive. Therefore, some relations may be derived by:
\[
\cos \theta + g \sin \theta + ih \sin \theta = J(\cos \beta + i \sin \beta) = Je^{i\beta}
\]
\[
J = \sqrt{(\cos \theta + g \sin \theta)^2 + (h \sin \theta)^2}, \quad \tan \beta = \frac{h \sin \theta}{\cos \theta + g \sin \theta}
\]  

(11)

It is well seen that both \( J \) and \( \beta \) are relative to the real numbers (\( g, h \)). Also it is very clear that the real variable \( J \) is of the norm property, and \( \beta \) varies with \( \theta \) in certain regularity.

3. Solution for centre crack plate

In order to find the stress problems in the cracked plate, and the real function \( F(x,y) \) may be written in the pan-complex function by the following form:

\[
F = B\Psi(w) + \overline{B}\overline{\Psi}(\overline{w}) + c_0(w - \overline{w})^2
\]

(12)

According to above given formulas (8) and (12), the stresses in the x-y plane may be indicated by the pan-complex variable functions. Also from the expressions (2), the stresses are given by:

\[
\sigma_x = \frac{\partial^2 F}{\partial y^2} = Bq^2 \frac{d^2\Psi}{dw^2} + \overline{Bq} \frac{d^2\overline{\Psi}}{d\overline{w}^2} - 8h^2 c_0 = 2 \text{Re}(Bq^2\Phi) - 8h^2 c_0
\]

\[
\sigma_y = \frac{\partial^2 F}{\partial x^2} = B\frac{d^2\Psi}{dw^2} + \overline{B} \frac{d^2\overline{\Psi}}{d\overline{w}^2} = 2 \text{Re}(B\Phi)
\]

\[
\tau_w = -\frac{\partial^2 F}{\partial x \partial y} = -(Bq \frac{d^2\Psi}{dw^2} + \overline{Bq} \frac{d^2\overline{\Psi}}{d\overline{w}^2}) = -2 \text{Re}(Bq\Phi)
\]

(13)

Where, \( \Phi = \Phi(w) = \Psi''(w) = \frac{d^2\Psi}{dw^2} \). By using the equation (8), and also substituting the complex function expression (12) into the harmonic equation (5), then it must be given that:

\[
(q^4 + A_1q^3 + A_2q^2 + A_3q + A_4)B \frac{d^2\Phi}{dw^2} + (\overline{q}^4 + A_1\overline{q}^3 + A_2\overline{q}^2 + A_3\overline{q} + A_4)\overline{B} \frac{d^2\overline{\Phi}}{d\overline{w}^2} = 0
\]

(14)

Obviously, we can obtain two characteristic formulas as below:

\[
q^4 + A_1q^3 + A_2q^2 + A_3q + A_4 = 0
\]

\[
\overline{q}^4 + A_1\overline{q}^3 + A_2\overline{q}^2 + A_3\overline{q} + A_4 = 0
\]

(15)

The solutions of the equations can be determined in the following:

\[ q_1 = g_1 + ih_1, \quad q_2 = g_2 + ih_2, \quad q_3 = \overline{g}_1 = g_1 - ih_1, \quad q_4 = \overline{g}_2 = g_2 - ih_2 \]

Where, the real numbers \( h_1 \) and \( h_2 \) are positive, and assume that: \( h_1 > h_2 > 0 \). Therefore, two pan-complex variables \( w_1 \) and \( w_2 \) can be expressed as:

\[ w_1 = x + q_1y = x + g_1y + ih_1y, \quad w_2 = x + q_2y = x + g_2y + ih_2y \]

For the anisotropic plate with a central crack, the complex functions \( \{\Phi_1(w_1), \Phi_2(w_2)\} \) should be selected reasonably by the following form:
\[ \Phi_1 = \sqrt{\frac{w_1^2}{w_1^2 - a^2}} , \quad \Phi_2 = \sqrt{\frac{w_2^2}{w_2^2 - a^2}} \]  

(16)

The stresses in the central cracked plate may be indicated by two functions \( \Phi_1 \) and \( \Phi_2 \). In brief, the stresses can be then expressed as:

\[
\begin{align*}
\sigma_x &= 2 \text{Re}(q_1^2 B_1 \Phi_1 + q_2^2 B_2 \Phi_2) + T \\
\sigma_y &= 2 \text{Re}(B_1 \Phi_1 + B_2 \Phi_2) \\
\tau_{xy} &= -2 \text{Re}(q_1 B_1 \Phi_1 + q_2 B_2 \Phi_2)
\end{align*}
\]  

(17)

Where, \( T \) is undetermined temporarily. The other constants in the expressions may be found certainly by the stress boundary requirement. To solve the problem in centre cracked plate, the stress boundary conditions must be given by the following:

\[
\begin{align*}
\sigma_x &= 0 , \quad \sigma_y = \sigma , \quad \tau_{xy} = \tau \quad \text{at} \quad x^2 + y^2 \to \infty \\
\sigma_y &= 0 , \quad \tau_{xy} = 0 \quad \text{at} \quad x^2 < a^2 \quad \text{and} \quad y = 0 \quad (\text{crack surface})
\end{align*}
\]  

(18)

Therefore it can be seen that some relations are given as below:

\[
\begin{align*}
2 \text{Re}(q_1^2 B_1 + q_2^2 B_2) + T &= 0 \quad , \quad 2 \text{Re}(B_1 + B_2) = \sigma \quad , \quad -2 \text{Re}(q_1 B_1 + q_2 B_2) = \tau \\
\text{Im}(B_1 + B_2) &= 0 \quad , \quad \text{Im}(q_1 B_1 + q_2 B_2) = 0
\end{align*}
\]

The values can be determined by solving the equations, the constants \( \text{Re} B_1, \text{Im} B_1, \text{Re} B_2, \text{Im} B_2, T \) shall certainly be determined. And finally, the above constants can be derived as follows:

\[
\begin{align*}
T &= \sigma(g_1 g_2 - h_1 h_2) + \tau(g_1 + g_2) \\
B_1 &= \sigma \left( \frac{D_1 i D_3}{2D_{12}} + \tau \frac{g_2 - g_1 + i(h_1 - h_2)}{2D_{12}} \right) \\
B_2 &= \sigma \left( \frac{D_2 - i D_3}{2D_{12}} + \tau \frac{g_1 - g_2 - i(h_1 - h_2)}{2D_{12}} \right)
\end{align*}
\]  

(19)

Where,

\[
\begin{align*}
D_1 &= g_2(g_2 - g_1) + h_2(h_1 - h_2) \quad , \quad D_2 = g_1(g_1 - g_2) + h_1(h_1 - h_2) \\
D_{12} &= D_1 + D_2 = (g_1 - g_2)^2 + (h_1 - h_2)^2 \quad , \quad D_3 = g_2 h_1 - g_1 h_2
\end{align*}
\]

In the vicinity region of the crack tip \( (r << a) \) in the plate, the complex variables can be simplified by using the coordinate transformation \( (x = a + r \cos \theta , \; y = r \sin \theta) \) as below:

\[
\begin{align*}
w + a &\approx 2a \quad , \quad \frac{w^2}{w^2 - a^2} \approx \frac{a}{2(w - a)} = \frac{1}{2r(\cos \theta + g \sin \theta + i h \sin \theta)} = \frac{a}{2r J e^{i \beta}}
\end{align*}
\]

Hence, the pan-complex variable functions in above equations may be written as:
$$\Phi_1 = \sqrt{\frac{w_1^2}{w_1^2 - a^2}} = \sqrt{\frac{a}{2r} \frac{e^{-\beta/2}}{J_1}} = \sqrt{\frac{a}{2r} \frac{1}{\sqrt{J_1}} (\cos \beta_1 - i \sin \beta_1/2)}$$

$$\Phi_2 = \sqrt{\frac{w_2^2}{w_2^2 - a^2}} = \sqrt{\frac{a}{2r} \frac{e^{-\beta/2}}{J_2}} = \sqrt{\frac{a}{2r} \frac{1}{\sqrt{J_2}} (\cos \beta_2 - i \sin \beta_2/2)}$$

(20)

In which,

$$\beta_1 = \arctan \frac{h_1 \sin \theta}{\cos \theta + g_1 \sin \theta}, \quad J_1 = (\cos \theta + g_1 \sin \theta)^2 + (h_1 \sin \theta)^2$$

$$\beta_2 = \arctan \frac{h_2 \sin \theta}{\cos \theta + g_2 \sin \theta}, \quad J_2 = (\cos \theta + g_2 \sin \theta)^2 + (h_2 \sin \theta)^2$$

And finally, substituting above derivative results into equation (17) and making the proper simplification, the unusual stress fields may be determined by:

$$\sigma_x = \frac{\sigma}{D_{12}} \sqrt{\frac{a}{2r} \left[ \frac{C_1}{\sqrt{J_1}} \cos \frac{\beta_1}{2} + \frac{C_2}{\sqrt{J_2}} \cos \frac{\beta_2}{2} + \frac{C_3}{\sqrt{J_1}} \sin \frac{\beta_1}{2} + \frac{C_4}{\sqrt{J_2}} \sin \frac{\beta_2}{2} \right] + T}$$

$$\sigma_y = \frac{\sigma}{D_{12}} \sqrt{\frac{a}{2r} \left[ \frac{D_1}{\sqrt{J_1}} \cos \frac{\beta_1}{2} + \frac{D_2}{\sqrt{J_2}} \cos \frac{\beta_2}{2} + \frac{D_3}{\sqrt{J_1}} \sin \frac{\beta_1}{2} + \frac{D_4}{\sqrt{J_2}} \sin \frac{\beta_2}{2} \right] + T}$$

$$\tau_{xy} = \frac{\sigma}{D_{12}} \sqrt{\frac{a}{2r} \left[ \frac{C_5}{\sqrt{J_1}} \cos \frac{\beta_1}{2} + \frac{C_6}{\sqrt{J_2}} \cos \frac{\beta_2}{2} + \frac{C_7}{\sqrt{J_1}} \sin \frac{\beta_1}{2} + \frac{C_8}{\sqrt{J_2}} \sin \frac{\beta_2}{2} \right] + T}$$

$$+ \frac{\tau}{D_{12}} \sqrt{\frac{a}{2r} \left[ \frac{D_5}{\sqrt{J_1}} \cos \frac{\beta_1}{2} + \frac{D_6}{\sqrt{J_2}} \cos \frac{\beta_2}{2} - \frac{D_7}{\sqrt{J_1}} \sin \frac{\beta_1}{2} + \frac{D_8}{\sqrt{J_2}} \sin \frac{\beta_2}{2} \right] + T}$$

(21)

Where, the variable $r$ is the radial distance from the crack tip. The different constants must have the given relations in the following:

$$C_1 = D_1 (g_1^2 - h_1^2) - 2g_1 h_1 D_3 \quad , \quad C_2 = D_2 (g_2^2 - h_2^2) + 2g_2 h_2 D_3$$

$$C_3 = 2g_1 h_1 D_1 + D_3 (g_1^2 - h_1^2) \quad , \quad C_4 = 2g_2 h_2 D_2 - D_3 (g_2^2 - h_2^2)$$

$$C_5 = (g_2 - g_1)(g_1^2 - h_1^2) - 2g_1 h_1 (h_1 - h_2) \quad , \quad C_6 = (g_1 - g_2)(g_2^2 - h_2^2) + 2g_2 h_2 (h_1 - h_2)$$

$$C_7 = (h_1 - h_2)(g_1^2 - h_1^2) - 2g_1 h_1 (g_1 - g_2) \quad , \quad C_8 = 2g_2 h_2 (g_1 - g_2) - (h_1 - h_2) (g_2^2 - h_2^2)$$

$$D_4 = -g_1 D_1 + h_1 D_3 = g_2 D_2 + h_2 D_3 = g_1 g_2 (g_1 - g_2) + g_2 h_2 ^2 - g_1 h_1^2$$

$$D_5 = h_1 D_1 + g_1 D_3 = -h_2 D_2 + g_2 D_3 = h_1 g_2^2 - h_2 g_1^2 - h_1 h_2 (h_1 - h_2)$$

The constants are in relation to one another as below:

$$C_1 + C_2 = (h_1 h_2 - g_1 g_2) D_{12} \quad , \quad C_3 + C_4 = -(g_1 h_1 + g_2 h_2) D_{12}$$

$$C_5 + C_6 = -g_1 h_1 (h_1 - h_2) D_{12} \quad , \quad C_7 + C_8 = g_2 h_2 (g_1 - g_2) D_{12}$$

$$D_4 + D_5 = g_2 h_2 (g_1 - g_2) D_{12} \quad , \quad D_4 + D_5 = (h_1 h_2 - g_1 g_2) D_{12}$$
\[ C_5 + C_6 = -(g_1 + g_2)D_{12} \quad , \quad C_7 + C_8 = -(h_1 + h_2)D_{12} \]

The constant stress \( T \) can be given in the following:
\[
T = -\frac{\sigma}{D_{12}}(C_1 + C_2) - \frac{\tau}{D_{12}}(C_5 + C_6) = \sigma(g_1g_2 - h_1h_2) + \tau(g_1 + g_2)
\]

According to usual practice in some books, the stress intensity factors \( K_I \) and \( K_{II} \) are:
\[
K_I = \sigma \sqrt{\pi a} \quad , \quad K_{II} = \tau \sqrt{\pi a}
\]

Therefore, the stress expressions can be written into:
\[
\begin{align*}
\sigma_x &= \frac{K_I}{\sqrt{2\pi rD_{12}}} \left[ \frac{C_1}{J_1} \cos \frac{\beta_1}{2} + \frac{C_2}{J_2} \cos \frac{\beta_2}{2} + \frac{C_3}{J_1} \sin \frac{\beta_1}{2} + \frac{C_4}{J_2} \sin \frac{\beta_2}{2} \right]
\quad + \frac{K_{II}}{\sqrt{2\pi rD_{12}}} \left[ \frac{C_5}{J_1} \cos \frac{\beta_1}{2} + \frac{C_6}{J_2} \cos \frac{\beta_2}{2} + \frac{C_7}{J_1} \sin \frac{\beta_1}{2} + \frac{C_8}{J_2} \sin \frac{\beta_2}{2} \right] + T \\
\sigma_y &= \frac{K_I}{\sqrt{2\pi rD_{12}}} \left[ \frac{D_1}{J_1} \cos \frac{\beta_1}{2} + \frac{D_2}{J_2} \cos \frac{\beta_2}{2} + \frac{D_3}{J_1} \sin \frac{\beta_1}{2} - \frac{D_4}{J_2} \sin \frac{\beta_2}{2} \right]
\quad + \frac{K_{II}}{\sqrt{2\pi rD_{12}}} \left[ \frac{g_1 - g_2}{J_1} \cos \frac{\beta_1}{2} + \frac{g_1 - g_2}{J_2} \cos \frac{\beta_2}{2} + \frac{h_1 - h_2}{J_1} \sin \frac{\beta_1}{2} - \frac{h_1 - h_2}{J_2} \sin \frac{\beta_2}{2} \right] \\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi rD_{12}}} \left[ \frac{D_4}{J_1} \cos \frac{\beta_1}{2} - \frac{D_4}{J_2} \cos \frac{\beta_2}{2} - \frac{D_3}{J_1} \sin \frac{\beta_1}{2} + \frac{D_3}{J_2} \sin \frac{\beta_2}{2} \right]
\quad + \frac{K_{II}}{\sqrt{2\pi rD_{12}}} \left[ \frac{D_2}{J_1} \cos \frac{\beta_1}{2} + \frac{D_2}{J_2} \cos \frac{\beta_2}{2} - \frac{D_1}{J_1} \sin \frac{\beta_1}{2} + \frac{D_1}{J_2} \sin \frac{\beta_2}{2} \right]
\end{align*}
\]

It is very clear that the stress components can reach to infinity at the crack tip \( (r \to 0) \), thus it is usually to be called the stress singularity. Obviously, the singular stress fields have been given fully in basic real variables. The complex variables are no longer present in the equations.

4. Conclusion

It is known for certain that the fracture mechanics must be very relative to the engineering structures with the anisotropic materials at present days. Especially, the study on the case of the mixed-mode loading may be full of meaning for the plane fracture mechanics. For the sake of determining the solution of the stress fields in the anisotropic plate, some equations of elastic mechanics have been given by using the anisotropic material properties. The crack-tip stress problem has been discussed under the mixed loading. The pan-complex variable functions have been determined properly for the needs of the stress boundary conditions. By using the polar coordinate transformation method, the singular stress fields near the crack-tip region have been confirmed eventually.

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