Twin Peaks

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The on-shell imaginary part of the retarded selfenergy of massive $\varphi^4$ theory in 1+1 dimensions is logarithmically infrared divergent. This leads to a zero in the spectral function, separating its usual bump into two. The twin peaks interfere in time-dependent correlation functions, which causes oscillating modulations on top of exponential-like decay, while the usual formulas for the decay rate fail. We see similar modulations in our numerical results for a mean field correlator, using a Hartree ensemble approximation.

In our numerical simulations of 1+1 dimensional $\varphi^4$ theory using the Hartree ensemble approximation, we found funny modulations in a time-dependent correlation function. Fig. 1 shows such modulations on top of a roughly exponential decay. The correlation function is the time average of the zero momentum mode of the mean field, $F_{\text{mf}}(t) = \varphi(t)\varphi(0) - \varphi(t)\varphi(0)$, where the over-bar denotes a time average, $\overline{X(t)} = \frac{1}{t_2-t_1} \int_{t_1}^{t_2} dt' X(t + t')/(t_2 - t_1)$, taken after waiting a long time $t_1$ for the system to be in approximate equilibrium. This equilibrium is approximately thermal and $F_{\text{mf}}(t)$ is analogous to the symmetric correlation function of the quantum field theory at finite temperature, $F(t) = \langle \frac{1}{2} (\hat{\varphi}(t), \hat{\varphi}(0)) \rangle_{\text{conn}}$. A natural question is now, does $F(t)$ also have such modulations?

The function $F(t)$ can be expressed in terms of the zero momentum spectral function $\rho(p^0)$,

$$F(t) = \int_{-\infty}^{\infty} \frac{dp^0}{2\pi} e^{-ip^0t} \left( \frac{1}{e^{p^0/T} - 1} + \frac{1}{2} \right) \rho(p^0),$$

and the latter in turn in terms of the retarded selfenergy $\Sigma(p^0)$,

$$\rho(p^0) = \frac{-2\text{Im} \Sigma(p^0)}{m^2 - (p^0 + i\epsilon)^2 + (\text{Re} \Sigma(p^0))^2 + (\text{Im} \Sigma(p^0))^2}.$$  \hspace{1cm} (2)

The selfenergy can be calculated in perturbation theory. The one and two loop diagrams in the imaginary time formalism which have nontrivial energy-momentum dependence are shown in Fig. 2. Diagrams not shown give only rise to an effective temperature dependent mass, which we assume to be the

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mass in the propagators of the diagrams in Fig. 2, after adding a counterterm that sets the real part of $\Sigma$ to zero at $p^0 = m$. The one loop diagram is

![Diagram](image)

Figure 1: Numerically computed correlation $\ln |F_{mf}(t)|$ versus time $t$ in units of the inverse temperature dependent mass $m$. The coupling is weak, $\lambda/m^2 = 0.11$ and the temperature $T/m \approx 1.4$ for the smaller volume (with significant deviations from the Bose-Einstein distribution) and $\approx 1.6$ for the larger volume (reasonable BE).

Figure 2: Diagrams leading to thermal damping.

present only in the ‘broken phase’ (for which $\langle \hat{\phi} \rangle \neq 0$; there is really only a symmetric phase in 1+1 dimensions, but this is due to symmetry restoration by nonperturbative effects which will not obliterate the one-loop damping.) The corresponding selfenergy has been calculated in $\textbullet$, for example. It only leads to damping for frequencies $p^2 > 4m^2$, which are irrelevant for the quasiparticle damping at $p^2 = m^2$. So from now on we concentrate on the two-loop diagram.
After analytic continuation to real time one finds that it is given by the sum of two terms, $\Sigma_1 + \Sigma_2$ (see e.g. 3). The first has an imaginary part corresponding to $1 \leftrightarrow 3$ processes requiring $p_0^2 > 9m^2$, so it does not contribute to plasmon damping. The second is given by

$$\Sigma_2 = -\frac{9\lambda^2}{16\pi^2} \int \frac{dp_2 dp_3}{E_1 E_2 E_3} \frac{(1 + n_1)n_2 n_3 - n_1 (1 + n_2)(1 + n_3)}{p^0 + i\epsilon + E_1 - E_2 - E_3} + \left[ (p^0 + i\epsilon) \rightarrow -(p^0 + i\epsilon) \right],$$

where $\lambda$ is the coupling constant (introduced as $L_1 = -\lambda\phi^4/4$), and $E_i = \sqrt{m^2 + p_i^2}$, $i = 2, 3$, $n_i = [\exp (E_i/T) - 1]^{-1}$, $i = 1, 2, 3$. Its imaginary part corresponds to $2 \leftrightarrow 2$ processes, which contribute in the regions near $p^0 = \pm m$.

Now the usual definition of the thermal plasmon damping rate (at zero momentum) in terms of the retarded selfenergy,

$$\gamma = -\text{Im} \Sigma(m)/2m,$$

leads to a divergent answer (a collinear divergence). A natural way out of this difficulty may be to continue the selfenergy analytically into the lower half of its second Riemann sheet, $p^0 \rightarrow m - i\gamma$, and replace (4) by the improved definition

$$m^2 - (m - i\gamma)^2 + \Sigma(m - i\gamma) = 0.$$

The analytic continuation of the selfenergy into the region $\text{Im} p^0 < 0$ poses the puzzle how to deal with the logarithmic branch point coming from the collinear singularity at $p^0 = m$. However, the ambiguity is present only in the real part of $\Sigma$. For weak coupling $\lambda/m^2 \ll 1$ we get from (5) the equation

$$\frac{\gamma}{m} = \frac{9\lambda^2}{16\pi m^3} \left( e^{m/T} \right)^2 \left[ \ln \frac{m}{\gamma} + c(T) \right].$$

The constant $c$ has to be determined by matching a numerical evaluation of $\Sigma$ to the logarithmic singularity at $p^0 = m$.

We evaluated $\Sigma_2$ in (3) for $T = m$ by numerical integration with $\epsilon/m = 0.02, 0.01$ and linear extrapolation $\epsilon \rightarrow 0$, giving $c \approx -0.51$. For example, Eq. (5) now gives $\gamma/m = 0.061$, for $\lambda/m^2 = 0.4$.

To see how well this $\gamma$ describes the decay of the correlator $F(t)$ we evaluated this function directly from (3) and (5). The divergence in $\text{Im} \Sigma(p^0)$ at $p^0 = m$ leads to a zero in the spectral function $\rho(p^0)$. So is there a peak at all in $\rho(p^0)$? Fig. 3 shows what happens: the ‘usual’ peak has separated into two twins! Fig. 4 shows the resulting $F(t)$. The effect of the double peak is...
indeed an oscillating modulation on top of the roughly exponential decay. The decay corresponding to \( \exp(-\gamma t) \), with \( \gamma \) given by (6), is also indicated in the plot: it does not do a good job in describing the average decay beyond the first interference minimum. The ‘Twin Peaks’ phenomenon implies that the usual definition of damping rate (3) is unreliable in 1+1 dimensions.

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.pdf}
\caption{The spectral function \( \rho(p^0) \) near \( p^0 = m = 1 \) corresponding to the selfenergy shown in Figs. 4, 5 (\( T = m, \lambda = 0.4m^2 \)).}
\end{figure}

\begin{figure}[h]
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\includegraphics[width=0.5\textwidth]{figure4.pdf}
\caption{Plot of \( \ln|F(t)| \) versus \( mt \) for \( T = m, \lambda = 0.4m^2 \). The straight line represents \( \exp(-\gamma t) \).}
\end{figure}