THE THICK DISKS OF SPIRAL GALAXIES AS RELICS FROM GAS-RICH, TURBULENT, CLUMPY DISKS AT HIGH REDSHIFT

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ABSTRACT

The formation of thick stellar disks in spiral galaxies is studied. Simulations of gas-rich young galaxies show formation of internal clumps by gravitational instabilities, clump coalescence into a bulge, and disk thickening by strong stellar scattering. The bulge and thick disks of modern galaxies may form this way. Simulations of minor mergers make thick disks too, but there is an important difference. Thick disks made by internal processes have a constant scale height with galactocentric radius, but thick disks made by mergers flare. The difference arises because in the first case, perpendicular forcing and disk-gravity resistance are both proportional to the disk column density, so the resulting scale height is independent of this density. In the case of mergers, perpendicular forcing is independent of the column density and the low-density regions get thicker; the resulting flaring is inconsistent with observations. Late-stage gas accretion and thin-disk growth are shown to preserve the constant scale heights of thick disks formed by internal evolution. These results reinforce the idea that disk galaxies accrete most of their mass smoothly and acquire their structure by internal processes, in particular through turbulent and clumpy phases at high redshift.

Key words: galaxies: formation – galaxies: high-redshift – Galaxy: disk – ISM: structure

1. INTRODUCTION

In the cosmological model of galaxy formation, gas-rich mergers are necessary to preserve disks and build spiral galaxies (e.g., Robertson et al. 2006). These mergers also scatter disk stars and produce the thick-disk component (Quinn et al. 1993; Walker et al. 1996). There is, however, increasing evidence that most of the baryonic mass enters in cold flows, which are smoother and less disruptive than mergers (Agertz et al. 2009; Ocvirk et al. 2008; Keres et al. 2009). Cold flow accretion is expected to be most active at $z \sim 2$ (Dekel et al. 2009).

Observations of galaxies around that epoch show a large population of rotating disks that are clumpy and turbulent (Elmegreen & Elmegreen 2005; Genzel et al. 2008; Shapiro et al. 2008; Elmegreen et al. 2009a, 2009b). The clumps in these galaxies have masses up to a few $10^{8} M_{\odot}$, sizes of around a kpc, and likely formed internally by gravitational instabilities (Elmegreen et al. 2007; Forster-Schreiber et al. 2009). Cold flows have an advantage over mergers as an explanation for high-redshift clumpy galaxies (Agertz et al. 2009; Ceverino et al. 2009). This is because the formation of the observed clumps requires a relatively high disk velocity dispersion for the ambient Jeans mass to be large. Then, a high fraction of the baryons has to be in the primordial disk to keep the system unstable: only relatively smooth gas accretion can do that (Bournaud & Elmegreen 2009; Dekel et al. 2009).

The origin of thick disks then comes into question. If most of the mass assembly is from smooth gas flows and rapid internal evolution, then thick disks have to form by internal processes, such as stellar scattering off of clumps and disk heating by gravitational instabilities that make clumps (Bournaud et al. 2007, hereafter BEE07). Clumpy disk evolution can form classical bulges (Noguchi 1999; Immeli et al. 2004; Elmegreen et al. 2008), and exponential disks (Bournaud et al. 2007, hereafter BEE07), but there are no distinct features in these remnants that exclusively point to clump-driven evolution. Perhaps thick-disk formation by clump scattering is a more revealing remnant in modern galaxies.

Here, we explore the properties of thick disks formed internally in unstable, gas-rich, clumpy disks. We compare with simulations of merger-induced disk thickening. We find that thick disks formed internally have a nearly constant scale height with radius, as observed for real galaxies. This is unlike the case for merger-induced thick disks, which always flare. We also investigate how much the thick disk changes during later gas accretion when the thin disk forms, and find that the scale height decreases, but remains constant with radius.

2. PROPERTIES OF OBSERVED THICK DISKS

The primary remnants of the earliest stage of galaxy formation are the bulge and thick-disk components. The thick disk in the Milky Way contains no stars younger than 8 Gyr (Gilmore et al. 1985; Reddy et al. 2006). Thick disks are ubiquitous (Dalcanton & Bernstein 2002; Seth et al. 2005), and contain 10%–25% of the baryonic mass of spiral galaxies. In the disk alone, thick-to-thin mass ratios range from 10% to 50% (Youachim & Dalcanton 2006, hereafter YD06); in the Milky Way it is 20%–25% (Gilmore & Reid 1983; Chen et al. 2001; Robin et al. 2003).

The structural properties of thick disks that are relevant for comparisons with our models are as follows.

1. The thick-disk sech$^{2}$ scale height, $z_{0}$, is generally between 1.4 and 2.8 kpc, with a large scatter (Dalcanton & Bernstein 2002, YD06). Equivalent exponential-fit scale heights, $h_{z}$, are about half these values.

2. $z_{0}$ is about constant with radius (YD06). In the Milky Way (Robin et al. 2003) and other galaxies (e.g., Ibata et al. 2009), $h_{z}$ also does not have significant variations until $R_{25}$ and beyond. This constancy of $h_{z}$ is illustrated here in Figure 1, where vertical profiles from the inner and outer parts of NGC 891 are presented. A constant $h_{z}$ implies that thick disks viewed edge-on have disky isophotes. Signs of boxiness were sought but not found (Dalcanton & Bernstein 2002).
3. Thick and thin disks have relatively similar radial scale lengths for most galaxies (YD06).

Bulges and thick disks have enhanced $\alpha$ elements compared to Fe (e.g., Lecureur et al. 2007; Zoccali et al. 2007), suggesting a star formation timescale shorter than 1 Gyr. Other chemical similarities between the bulge and the thick disk of the Milky Way were discussed by Meléndez et al. (2008).

3. SIMULATIONS

3.1. Unstable Disks at High-Redshift and Thin Disk Growth

The simulations used here are similar to those in BEE07, but with an increased resolution to see the vertical disk structure. The $N$-body particle-mesh code uses a sticky particle scheme to model cold gas dynamics. The spatial resolution (softening size) is 35 pc. Two million particles are used for each component of gas, stars, and dark matter. We ran three models:

1. Model 1 starts with a stellar mass of $2 \times 10^{10} M_\odot$ and a gas fraction of 60%.
2. Model 2 starts with a stellar mass of $3 \times 10^{10} M_\odot$ and a gas fraction of 50%.
3. Model 3 starts with a stellar mass of $4 \times 10^{10} M_\odot$ and a gas fraction of 40%.

The models start with idealized, uniform disks having masses that are plausible for $z \sim 2$ progenitors of Milky-Way-like spirals. They rapidly acquire a phase space distribution (morphology and kinematics) that is similar to that in cosmological simulations of clumpy disk formation (Agertz et al. 2009; Ceverino et al. 2009) and also consistent with that of observed clumpy galaxies (see BEE07; Bournaud et al. 2008). Thus, the present models should be adequate for studies of disk evolution during and after the clumpy phase.

The initial disks have constant surface density for both gas and stars. The external radius is 6 kpc and the initial scale height is $h_z = 500$ pc. The ratio of dark matter to baryon mass is 1:2 inside the initial disk radius. The models are first run for 1 Gyr in full isolation in order to identify the effects of internal evolution.

After this time, additional gas is allowed to accrete for another 6 Gyr. This is supposed to model the later growth of the thin disk. The gas accretion rate is $10 M_\odot \text{yr}^{-1}$, which increases the initial galaxy mass by a factor 2–3. Accreted particles are positioned at a large radius (15 kpc), close to the disk plane: they are initially given a vertical distribution with an exponential profile and a scale height of 1.5 kpc, and later on settle in a thinner disk. Accreted particles are not all present in the simulation initially, they are continuously added at the $r = 15$ kpc boundary to model continuous accretion down to low redshifts, as in Bournaud & Combes (2002). Each is given a rotational velocity such that it has the same angular momentum as an average particle in the thick disk at 1 Gyr. This similarity of angular momenta gives the thick and thin disks similar radial scale lengths. Thus, the additional gas is accreted from a thick layer in these models, but is relatively stable and naturally settles to a thin disk.

3.2. Minor Mergers

Many simulations of minor mergers have been published (Walker et al. 1996; Villalobos & Helmi 2008; Read et al. 2009), in addition to cosmological simulations with numerous satellites accreting and merging (Abadi et al. 2003). We performed new merger models to compare with the internal evolution using the same code and assumptions. We ran four idealized simulations of single minor mergers and a cosmological simulation.
where several minor mergers occur with mass ratios and orbits prescribed by a Λ-CDM model.

The four individual minor mergers were chosen to have a mass ratio of 10:1. The primary galaxy is modeled with one million particles for each component (gas, stars, and dark matter). Its stellar mass is $4 \times 10^{10} \, M_\odot$ and its gas mass is $1 \times 10^{10} \, M_\odot$. It starts with an exponential disk of radial scale length 4 kpc, truncated at 15 kpc, and an exponential scale height of 500 pc. Twenty percent of the stellar mass is in a central bulge with a Hernquist profile of scale length 500 pc. The dark matter halo has a Hernquist profile with a scale length of 7 kpc. The dark matter to baryon mass ratio is 1:2 inside the disk radius. The companion galaxy has all masses divided by a factor of 10, and all sizes divided by 3.5. The orbital inclination with respect to the target disk plane is 35°. The impact parameter is 30 kpc. The velocity at an infinite distance is either 150 or 220 km s$^{-1}$, each being simulated for a prograde and a retrograde orbit, thus giving four models. The resolution and softening are the same as in the unstable disk models.

Our second type of merger model includes several satellite galaxies taken from a large-scale cosmological simulation (see Martig et al. 2009a, 2009b). The primary galaxy in this case uses $4.7 \times 10^7$ stellar particles, $3.2 \times 10^8$ gas particles, and $5.7 \times 10^8$ dark matter particles. The softening length for force calculations is 150 pc in this model, larger than in the other simulations, but

![Figure 2. Model 2 of a gas-rich gravitationally unstable disk generating giant clumps of gas and stars, similar to clumpy galaxies at $z > 1$ (see also BEE07). These edge-on views of the gas+star mass density show rapid thickening of the disk by the clumps. The clumps finally coalesce into a central bulge within a Gyr. Subsequent gas infall forms a stable thin disk inside the thick disk. The thick-disk scale height decreases a little when this happens, but the separate thick-disk component remains.](image)

the final thick-disk scale height is still well resolved. The disk is initially thin at redshift $z = 1.2$, and it contains 30% of its mass in the form of gas. Over time, the galaxy interacts with several dwarf satellites, the most massive of which have mass ratios of 11:1, 13:1, 15:1, and 18:1. The final stellar mass in the primary is $8 \times 10^{10} \, M_\odot$. We analyze its thick disk at $z = 0$.

4. RESULTS

4.1. Thick Disks from Instabilities and Giant Clumps at High Redshift

The initial Gyr in our models produces giant star-forming clumps with masses up to a few $10^8 \, M_\odot$. The clumps eventually merge into a bulge (see also BEE07). A thick disk forms quickly because of the same gravitational instabilities that form the clumps. That is, the instabilities scatter stars and gas to high velocity dispersions, and the clumps that are formed by the instabilities continue to scatter stars and increase the dispersion for as long as they are present.

The exponential scale heights in the stellar disks were measured after the clumpy phase, when the clumps have either dissolved or coalesced in a bulge. The scale height is $\sim 2$–2.5 kpc. It is constant with radius (Figure 2), having only a minor flare in the outer regions. That is, at a radius of four times the disk radial scale length, the scale height increases to $\sim 3$ kpc.

The slow addition of gas after 1 Gyr causes the thick disk to shrink toward the mid-plane, as expected (Elmegreen & Elmegreen 2006). In our models, the scale height of the thick disk decreases from $\sim 2$ or 2.5 kpc to $\sim 1.5$ kpc (Figures 3 and 4). In all models, the thick disk remains a separate component and does not become a low-density tail to the thin disk (Figure 3). The thick disk also preserves its constant scale height with radius during the accretion (Figure 4).

4.2. Thick Disks from Minor Mergers

The radial profiles of the thick-disk scale heights in our minor merger models are also shown in Figure 4. They have no large thickening in the inner regions ($r \simeq 0$) and the outer disk becomes extremely thick, with scale heights of 5 kpc and more. Minor mergers produce flared disks instead of uniformly thick disks. This property is not exclusive to our
code or initial conditions: other published simulations of disk thickening by minor mergers show the same thing (Walker et al. 1996; Villalobos & Helmi 2008; Read et al. 2009). The strong thickening by minor mergers show the same thing (Walker et al. code or initial conditions: other published simulations of disk thickening by minor mergers show the same thing (Walker et al. 1996; Villalobos & Helmi 2008; Read et al. 2009). The strong flaring often results in boxy isophotes for edge-on thick disks (Villalobos & Helmi 2008).

Flaring is inevitable in merger models because the dense inner regions have stronger disk forcing compared to companion forcing and the perpendicular response is small. In contrast, low-density outer regions have lower local disk forcing and stronger companion forcing, so the disk responds more. Even if an interaction provides the same amount of kinematic heating to the whole disk, the resultant scale height will vary as the inverse of the surface density. Minor mergers would also add more energy and mass to the outer disk than the inner disk because of the disruption of the small galaxy. Self-driven disk thickening by internal clumps does not produce much flaring because the perpendicular forcing and the disk gravity that resists this forcing are both proportional to the surface density.

5. COMPARISON TO OBSERVED THICK DISKS

Observed thick disks have a relatively constant scale height from the central regions to the outskirts of a galaxy. Minor merger models do not have this property, and they also lack any thick component in the central regions. Models where thick disks form by gravitational instabilities and clump scattering do have this property, however. Therefore, the disky shapes of thick disks are explained exclusively by the instability model.

The scale heights of our thick-disk models are consistent with the observed values of 1–2 kpc for typical bright spirals, even after the thick disk shrinks from the growth of an underlying thin disk. For Milky Way galaxy masses, our models end up with about one-third of the initial baryons in a bulge, one-third in gas that will contribute to the subsequent thin disk. After further thin-disk accretion, the final masses for the bulge, thick disk, and thin disk become $2–3 \times 10^{10} M_\odot$, $2–3 \times 10^{10} M_\odot$, and $8–9 \times 10^{10} M_\odot$, respectively. Thus, the thick-to-thin disk mass ratio is 30%, roughly typical of modern spiral galaxies.

The thick disks in our models are relatively massive, $2–3 \times 10^{10} M_\odot$, but consistent with many observed spiral galaxies. A larger set of lower-resolution simulations in BEE07 showed that the fraction of the clump mass that ends up in the disk can vary largely, so our model can account for a wide range of thick-disk masses. In this case, an anti-correlation between the bulge mass fraction and the thick-disk mass fraction is predicted. That is, clump stars that do not scatter into the thick disk will end up in the bulge. YD06 do observe that late-type spirals (which have low bulge fractions) have higher thick-to-thin disk mass ratios than early-type spirals (which have large bulge fractions).

Our models suggest that minor mergers are not the main formation mechanism for thick disks. Still, they occur for real galaxies over a wide range of redshifts, and they probably also occurred when the thick disks formed. Their effect on the thick disk seems to have been minor though. One problem minor mergers have is that the contribution of thick-disk galaxy is high at high redshift (z > 1; Daddi et al. 2008), and a high gas fraction can prevent minor mergers from thickening the stellar disk (Moster et al. 2009). A problem at low redshift z < 1 is that the thick disk is already in place, so minor mergers cannot impact its structure much (Villalobos & Helmi 2008). Also, the mass of a young disk can be larger than the mass of a minor companion, and the disk is closer to its own stars than the companion is, so the instantaneous gravitational force on the disk from its own stars is generally much larger than the force from a companion. Thus, internal processes should dominate thick-disk formation if young disks are clumpy. This is the essential point of the present models: the gravitational potential in young disks is extremely clumpy and time-variable because of violent instabilities and the presence of spiraling massive clumps.

The stellar disks of spiral galaxies do have some outer flaring, but it begins only at large radii and it affects mainly the thin disk (e.g., Alard 2000; Momany et al. 2006, for the Galaxy). This situation is consistent with a picture in which the thick disk forms mostly by internal processes at high redshift, and the primary effect of interactions is to warp and flare the outer thin disk at lower redshift. Stellar streams within the thick disk could also result from dwarf companions (Gilmore et al. 2002).

An additional consideration is that the young clumpy phase of a galaxy should form only one thick-disk component that has a single large vertical velocity dispersion. If minor mergers form the thick disk, then each merger could form a distinction component. The absence of any significant vertical gradient of the stellar velocity dispersion (Mori Bidin et al. 2009) suggests there is only one thick-disk component.

Figure 4. Radial variations of the thick-disk exponential scale height $h_z$ are shown as a function of the radius $r$ in units of the radial exponential scale length $h_r$. On the left, models of thick-disk formation by internal evolution are shown after 1 Gyr (dashed lines) and after subsequent thin-disk accretion (dotted lines). On the right, models of thick-disk formation by 1:10 mergers are shown in four cases (broken lines) and in a cosmological case (solid line). The scale height is about constant for models with internal disk evolution, but it rapidly increases with radius for models with mergers.
Thick-disk formation by internal forcing is also consistent with the observed elemental abundances. Our models indicate that thick disks and bulges form together within a time span of \(~1\) Gyr. This explains their elevated \(\alpha/\text{Fe}\) abundances and other abundance similarities (Meléndez et al. 2008). If minor mergers were the main drivers of thick disk formation, then thick disks would continue to grow below \(z < 1\) (as illustrated by our cosmological model). In that case, the age range, metallicity range, and formation timescale for the thick disk would differ from the observations. Other models of thick-disk formation by internal evolution, like cluster disruption (Kroupa 2002) or radial mixing (Roškar et al. 2008; Schönrich & Binney 2009), would also imply an unacceptable continuity of stellar ages in the thick disk.

6. CONCLUSION

There is increasing evidence that many disk galaxies at high redshift undergo a clumpy and turbulent phase that can drive rapid internal evolution. To understand if this is the main mode of spiral galaxy formation, remnants of this phase should be identified in the oldest components of today’s galaxies. The two most obvious of these old components are the bulge and thick stellar disk. Previous models showed how the clumps formed by gravitational instabilities could coalesce into a bulge, but bulges can form by galaxy mergers as well. Here, we showed how the clumps, and the instabilities that formed them, can also make the thick disk, while minor mergers fail to explain some observed properties of thick disks.

The standard model in which minor mergers stir an existing thin disk and deposit stars far from the plane has a problem: it inevitably makes flared thick disks because the vertical forcing is independent of the local disk column density. Such flaring is contrary to observations. Our model produces a uniform disk thickness because the forcing is proportional to the column density. This property is preserved during subsequent thin-disk formation by continued slow accretion. In addition, our model reproduces the mass fractions of thick and thin disks and explains, in general terms, the chemical similarities of the Milky Way bulge and thick disk.

The observed properties of thick disks suggest that today’s spiral galaxies went through a clumpy phase with rapid internal evolution, and that clumpy disk galaxies at high redshift are progenitors of spiral galaxies.

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