Threshold frequency of standing wave at hydrodynamic separation of particles

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Abstract. In this paper, the question of the separation of particles in a standing wave in closed resonator at first resonance frequency is considered. The threshold values of the frequency of the standing wave, as well as the densities and radii of the particle, which allow to separate the particles depending on their radius and density are studied.

In [1-5] a numerical study of the particle drift in a standing wave was made. To investigate the separation of particles, a 1D resonator with length \( L \) filled with air is considered. In a Cartesian coordinate system, equations of particle motion are expressed as

\[
\frac{dx_2}{dt} = v_2, \quad \frac{dv_2}{dt} = \frac{v_1(x_2,t) - v_2}{\tau} + D \left( \frac{\partial v_1}{\partial t} + \frac{\partial v_1}{\partial x} \right),
\]

where \( x \) – Cartesian coordinate, \( t \) – time, \( x_2 \) – particle coordinate, \( v_1 = U \sin \omega t \) – air velocity, \( v_2 \) – particle velocity, \( U = U_0 \sin \omega t \), \( U_0 = \sin kx \) – amplitude of the standing wave, \( \omega = U_{\text{max}}^2 / 4c_0 \) – resonant frequency, \( c_0 \) – unperturbed velocity of sound in the air, \( \tau = (1 + 0.5\tilde{\rho})\tau_v \) – relaxation parameter, \( \tau_v = 2/9\tilde{\rho}r^2 / \nu \) – relaxation time of a spherical particle, \( D = 3\tilde{\rho} / (2 + \tilde{\rho}) \) – density parameter, \( \tilde{\rho} = \rho_1 / \rho_2 \) – relative density, \( \rho_1 \) – air density, \( \rho_2 \) – particle density, \( r \) – particle radius, \( \nu \) – coefficient of kinematic viscosity of the air.

To study the particle drift the evolutionary equations of a particle are written [1]:

\[
\frac{d\xi}{dr} = \eta, \quad \frac{d\eta}{dr} = - \frac{\eta}{\tau} - \frac{U(\xi)U'(\xi)}{2} A, \quad \text{where} \quad A = (1-D)(1+D-D/\mu_p^2) \sqrt{\mu_p^2(1-\mu_p^2)},
\]

where \( \xi \) – period average position of the particle, \( \eta \) – velocity of particle drift, \( \mu_p = \left(1 + (\omega \tau)^2\right)^{-0.5} \) – drag coefficient of the particle with account of additional mass forces.

From (1) the local equilibrium velocity of particle drift can be introduced as
\[\eta_x = -0.5U(\xi)U'(\xi)(1-D)[(1+D)\mu_p^2 - D] \tau.\]

At a fixed \(D\) maximum is reached \(A\) at
\[
\mu_p^2 = 0.25 \left( \frac{1+9D}{1+D} + 1 \right). 
\]

In the resonator, a standing wave is received, therefore the direction and speed of the drift is determined by the parameter \(A\). Formula to acceleration shows that for any particle there is a threshold frequency \(\omega_c\). At frequencies lower than this frequency \(\omega < \omega_c\) the acceleration of the particle drift is determined by the Stokes force and is directed to the node of the velocity wave. At frequencies of higher than threshold frequency \(\omega > \omega_c\), the acceleration of particle drift is determined by the inertia forces of the carrier medium (i.e. by the forces of the added masses and the dynamic force of Archimedes) and is directed to the antinode of the wave.

1. Separation of particles from the same material
Let us consider a particle drift with radii \(\eta=95 \ \mu m\), \(r_2=143 \ \mu m\) in standing wave. By adopting Styrofoam density equal to \(\rho_2=20 \ \text{kg/m}^3\) the frequency of standing wave in air resonator is calculated.

Each radius corresponds to its own threshold frequency, namely: for radius \(\eta=95 \ \mu m\) \(\omega_c^{(1)} = 1524.75 \ \text{rad/s} \) (\(f_1=242.68 \ \text{Hz}\)), for \(r_2=143 \ \mu m\) \(\omega_c^{(2)} = 677.67 \ \text{rad/s} \) (\(f_2=107.86 \ \text{Hz}\)). A large particle has a larger relaxation time, which corresponds to lower threshold frequency. If the frequency of the standing wave \(\omega\) lays between threshold frequencies \(\omega_c^{(2)} < \omega < \omega_c^{(1)}\), then for a larger particle, this frequency will be greater than its threshold value, and the particle will drift into the antinode of the wave. For a smaller particle, the frequency will be less than its threshold value, and the particle will drift to the node. We choose as the frequency of the standing wave value \(\omega=1016.5 \ \text{rad/s}\) (between frequency thresholds for two Styrofoam particles). Period is \(T = 2\pi / \omega=0.006 \ \text{s}\), closed resonator length \(L = \pi c_0 / \omega=1.06 \ \text{m}\). The speed of sound in the air is taken \(c_0=343 \ \text{m/s}\), wave amplitude \(U_0=1 \ \text{m/s}\). Initial conditions: \(x_2(0)=0.75L, v_2(0)=0, \xi(0)=0.75L, \eta(0)=0.\)

Figure 1. The trajectory and evolution of two particles of Styrofoam at \(\omega=1016.5 \ \text{rad/s}\)
1–η = 95 μm 2–r₂ = 143 μm (a–trajectory, b–evolution)

Figure 1 shows the trajectories and evolution of particles of different radii in the range \( t/T \in [0,20000] \). Figure 1 shows good agreement between trajectories and evolution for each of the two particles. The smaller particle drifts toward the node, and the larger toward the antinode. In this case, the average drift velocity of particles can be determined from the corresponding angles of inclination. Modulo it is about the same for both particles and has the order \( |v_d| = 166 \mu m/s \).

\[
l = \frac{\pi}{T} \quad R_c = \frac{\pi}{2} \quad L_c = \frac{\lambda}{2}
\]

In figure 2 depicts the evolution of three particles of Styrofoam radii \( \eta = 95 \mu m, r_c = 116 \mu m \) and \( r_2 = 143 \mu m \). The larger particle drifts toward the antinode of the wave, and the smaller toward the node. In this case, the absolute values of the particle drift velocity are close to each other. Particle with a threshold radius \( r_c = 116 \mu m \) stays at rest and has zero drift velocity.

2. Separation of particles of the same size

Consider the drift of two particles of the same radius \( r = 100 \mu m \): one of Styrofoam \( \rho_2^{(1)} = 20 \text{ kg/m}^3 \), and the other out of the water \( \rho_2^{(2)} = 10^3 \text{ kg/m}^3 \). For these particles the threshold frequencies \( \omega_c^{(1)}, \omega_c^{(2)} \) of standing wave for air resonator \( \rho_1 = 1.29 \text{ kg/m}^3 \) is calculated.

Different particles correspond to different threshold frequency field: for the Styrofoam \( \omega_c^{(1)} = 1390 \text{ rad/s} \) \( (f_1 = 220 \text{ Hz}) \), for water: \( \omega_c^{(2)} = 189.4 \text{ rad/s} \) \( (f_2 = 30 \text{ Hz}) \). The threshold frequency for these particles will be \( \omega = (\omega_c^{(1)} + \omega_c^{(2)})/2 = 789.7 \text{ rad/s} \). Period is \( T = 2\pi / \omega = 0.0079 \text{ s} \), resonator length \( L = \pi c_0 / \omega = 1.36 \text{ m} \). Threshold particle density \( \rho_c \) for frequency \( \omega = 789.7 \text{ rad/s} \). Threshold density is \( \rho_c = 60 \text{ kg/m}^3 \), what corresponds to foam rubber.
Figure 3. Trajectories and evolution of two particles of radius $r = 100 \, \mu m$ при $\omega = 789.7 \, \text{rad/s}$

1 – Styrofoam, 2 – water ($a$ – trajectory, $b$ – evolution).

Figure 3 shows the trajectories and evolution of particles of Styrofoam and water of radius $r = 100 \, \mu m$ at $\omega = 789.7 \, \text{rad/s}$. Figure 5 shows good agreement between trajectories and evolution for each of the two particles. The particle of the Styrofoam drifts to the node, and the water – to the antinode.

Figure 4 shows the evolution of three particles: Styrofoam, foam rubber and water of radius $r = 100 \, \mu m$ at $\omega = 789.7 \, \text{rad/s}$. Figure 4 shows good agreement between trajectories and evolution for each of the two particles. In this case, the absolute values of the particle drift velocity are close to each other. Foam rubber particle with a threshold density $\rho_c = 60 \, \text{kg/m}^3$ practically does not drift.
3. Appendix

From the evolution equation, a formula for dimensionless vibration acceleration of a particle is obtained. The existence of threshold frequencies at which the direction of particle drift changes is determined. The existence of radii and densities corresponding to the threshold frequencies at which the particles practically do not drift is defined.

References

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