Anomalous D-term, dynamical supersymmetry breaking and
dynamical gauge couplings

Z. Lalak§, ‡

§ Institute of Theoretical Physics
University of Warsaw
00-681 Warsaw

‡ Max Planck Institut für Physik
Heisenberg Institute
D–80805 München, Germany

ABSTRACT

We analyze the structure of the vacuum and supersymmetry breaking pattern in Fayet-Iliopoulos models with dynamical gauge coupling and planck-scale value of the F-I parameter. We show that in this class of models supersymmetry is generically broken, but the mere presence of the D-term is not sufficient to stop the running away of the modulus responsible for the value of the gauge coupling - the dilaton. To stabilize the dilaton, one has to include an additional dilaton-dependent part in the superpotential. The presence of the large D-term gives rise to the mixed dilaton/D-term dominated scenarios of susy breaking, which allow horizontal hierarchy generation. Models which can serve as secluded sectors in gauge mediation scenarios are discussed. It is shown that when the F-I parameter and the gauge coupling are dynamical variables, the D-term dominated Universe does not allow for an inflationary period.
1 Introduction

One of the main problems of unification within the framework of string-derived effective field theories is that of hierarchical supersymmetry breaking and the determination of the expectation value of the dilaton at a suitable scale.

The dilaton puzzle has been reviewed in many papers, [1], and is due to the fact that simplest nonperturbatively induced potentials for the dilaton don’t seem to be able to stabilize it in a phenomenologically relevant region. While the more sophisticated versions of stringy unification, like M-theory, are under development which shall perhaps fix the dynamical gauge coupling in a fundamental way, we would like to explore in greater detail possibilities offered by the standard supergravity models.

From the field theoretical point of view there are in fact two independent “experimental” constraints on the dilaton expectation value: the requirement that the soft mass parameters which parametrize the observable breaking of supersymmetry be in the TeV region, and that the gauge coupling at the string scale be compatible with estimates based on renormalization group evolution of the standard model gauge couplings. The relationship between the gravitino mass and the dilaton vev involves the beta function coefficient of the strongly interacting hidden sector and thus depends on the possible matter content of that sector.

In models derived from the heterotic string at tree level the universal gauge coupling constant \( g_{	ext{string}} \) is determined by the vev of the dilaton field [2] via

\[
S = \frac{4\pi}{g_{	ext{string}}^2} - i \frac{\theta}{2\pi}
\]  

(1)

This normalization of the dilaton is such that under \( \theta \rightarrow \theta + 2\pi \) one has \( iS \rightarrow iS + 1 \). There are nonuniversalities which can appear at the one-loop level and depend on remaining moduli fields \( T, ..., U \) [3]. It follows that the vevs of the dilaton and other moduli should determine gauge couplings in the hidden and in the observable sector. In particular, correct values of the QCD coupling \( \alpha_s \) and the weak mixing angle \( \sin^2 \theta_W \) should follow. These requirements put strong constraints on the expectation value of the dilaton and on \( \alpha_{	ext{string}} \), i.e. one needs

\[
\alpha_{	ext{string}} = \frac{g_{	ext{string}}^2}{4\pi} \approx \frac{1}{20}
\]  

(2)

In the most “realistic” scenarios, where a potential for the dilaton gets generated through gaugino condensation in the hidden sector one needs several condensates, or new symmetries and/or corrections to the Kähler potential of the dilaton in order to
produce a minimum in the dilaton potential away from 0 and ∞. Even then, in the case of models motivated by S-duality the dilaton gets naturally stabilized at a value of the order of unity, clearly outside the favoured region. A similar situation arises in multigaugino condensate models, where the dilaton vev is typically larger than 1, but still smaller than 20. Also, the question of supersymmetry breaking at that nontrivial minimum for the dilaton is a subtle one.

Once a dilaton potential appears, it is natural to ask what the mass of the dilaton is. It is important that the dilaton should receive a mass which is large enough to avoid cosmological problems, and to justify the treatment of the dilaton as a fixed background in low energy models, in other words to suppress the fluctuations around the dilaton background. This requirement is rather hard to fulfill. In the hidden sector susy breaking scenarios soft supersymmetry breaking masses vanish in the limit $M_{pl} \to \infty$, and the natural scale for the moduli fields is $M_{pl}$ itself, hence the usual situation is $m_s \ll <s>$, cosmologically dangerous.

The typical scenario which one has in mind while discussing supersymmetry breaking and fixing the moduli vacuum expectation values relies on the assumption that one can separate from the whole model the sector containing the dilaton and moduli, minimize it on its own, and then substitute fixed vevs of $S,T,U,...$ into the lagrangian describing remaining fields, which will have in turn their own separate dynamics. One always assumes, that the vevs of dilaton and moduli stay frozen at their values obtained at the first step of the above procedure, independently of what is happening in the chiral-gauge sector of the model. This is sometimes justified by the hierarchy of scales in a given model, however in general this point of view doesn’t have to be correct. In particular, the backreaction of the other fields on the dilaton (moduli) can be significant in a class of models where there are from the beginning large terms in the lagrangian which contain both moduli and non-moduli fields. This is precisely the case in models with the anomalous $U(1)$ and the associated D-term, which is generically of the order of the Planck scale, $V_D \approx M_{planck}^4$. With the appearance of the large D-term there open up naturally new possibilities in the supersymmetry breaking mass patterns. Indeed, with the low energy effective lagrangian taking the form

$$L_{eff} = Z_{ij} \partial \phi^i \partial \bar{\phi}^j - m_{ij}^2 \phi^i \bar{\phi}^j + ... - g^2 (\xi + Z_{ij} \phi^i \bar{\phi}^j q_i)^2 \tag{3}$$

the masses that are softly breaking supersymmetry have the F-term contributions

$$\delta_f m_{ij}^2 = m_{ij}^2 Z_{ij} - F^a F^b R_{a\bar{a}ij} + m_{ij}^2 G_i \nabla_j G^k \tag{4}$$

1 To identify the physically meaningful soft terms we assume that at the minimum the cosmological constant vanishes in the underlying supergravity model.
where $R_{\alpha \beta i \bar{j}}$ is the curvature tensor of the Kähler scalar manifold, and the D-term contributions

$$
\delta_d m_{ij}^2 = g^2 G_j G_i D^2 - 2g^2 D(G_i D_j + D_i G_j) + 2g^2 D_i D_j + 2g^2 D D_{ij} - g^2 Z_{ij} D^2
$$

(5)

The (5) is in general nonuniversal, however it simplifies if indices $i, \bar{j}$ correspond to non-messenger fields, i.e. to the fields which do not lie along susy-breaking directions. In this simpler case the (5) reduces to ($M = M_{pl}$)

$$
\delta_d m_{ij}^2 = 2g^2 Z_{ij} q_i < D > - g^2 Z_{ij} \frac{< D^2 >}{M^2}
$$

(6)

Of course, on the rhs of this formulae the first contribution dominates over the second, however the second contribution can in specific cases be comparable to $\delta_f m_{ij}^2$. The omitted terms are important for messengers, the fields which are usually the heaviest ones among the non-moduli fields, however it is the nonmessenger fields, among them hopefully the MSSM fields, which are of immediate interest to us. For non-messengers the F-type contribution tends to be nonuniversal, but the simplified D-type contribution (6) exhibits the natural alignment, and if the D-term is sufficiently large, the aligned terms can dominate soft masses for $U(1)$ charged fields supplying the required amount of universality. Also, in that case, the charged states tend to be heavier than the uncharged ones which can be seen as a source of hierarchy. It should be noted, that the D-type contributions depend, explicitly through the $g^2$ and implicitly through the vev of $D$, on the dilaton.

It is obvious then, that the large F-I term can not only affect the physics of the supersymmetry breaking sector, but also has direct impact on the low-energy effective theory.

In this paper we want to discuss general features of dilaton stabilization and susy breaking in the presence of the F-I term and non-perturbative superpotential for messengers.
2 The dilaton potential in the presence of perturbative superpotential for $U(1)$ charged fields

To start with we will focus on the part of the effective action which involves only the dilaton and a set of fields $X_i$, charged under an anomalous $U(1)$. Let’s consider the low-energy supersymmetric implementation of the stringy anomalous $U(1)$ under which the dilaton superfield $S$ gets shifted by a chiral superfield parameter. Such a shift in the universal gauge kinetic function induces a term which corresponds to a mixed anomaly which includes the $U(1)$ gauge field. To cancel this, we need matter charged both under $U(1)$ and under all other factors of the nonanomalous gauge group, including the strongly coupled hidden sector group. This implies that in principle we shall have hidden matter, and matter superfields condensates, denoted later by $T$, which should be taken into account when writing down the effective lagrangian.

The Kähler potential is

$$K = -M^2 \log(S/M + \bar{S}/M + \delta_{GS}V/M^2) + \sum_i |X_i|^2$$

Here $V$ is the vector superfield of the anomalous $U(1)$, canonical kinetic terms are assumed. The relevant $F$-terms which control the effects of supersymmetry breaking are

$$F^S = e^{\frac{K}{2M^2}}(W_S - \frac{W}{S + \bar{S}})(S + \bar{S})^2 \frac{M^2}{M^2}$$

and

$$F^{X_i} = e^{\frac{K}{2M^2}}(W_{X_i} + \frac{W_{\bar{X}_i}}{M^2})$$

Finally, the scalar potential including the anomalous D-term contribution is

$$V = \frac{1}{M^2}e^{\frac{K}{2M^2}}(|(S + \bar{S})W_S - W|^2 + \sum_i |MW_{X_i} + \frac{W_{\bar{X}_i}}{M}|^2$$

$$- 3|W|^2 + \frac{4\pi M^5}{S + \bar{S}}(\frac{8\pi \delta_{GS}M}{S + \bar{S}} + \frac{1}{M^2} \sum_i q_i |X_i|^2)^2$$

where in the case of the stringy model we have $\delta_{GS} = \frac{TrQ}{192\pi^2}$. As usual, we assume that there exist fields with negative charges with respect to the anomalous $U(1)$ so that the D-term can vanish for some field configuration. It is also obvious, that we need a nontrivial superpotential for X-fields. Let us assume for a moment that there exist a field, $B$, which has no superpotential interactions at all, and is charged only under the
anomalous $U(1)$. Then the relevant part of the potential is

$$V_B = \frac{4\pi M^5}{S+S} \frac{8\pi \delta_{GS} M}{S+S} - \frac{1}{M^2} |q_B||B|^2$$

(11)

The eigenvalues of the mass matrix given by this potential are: 0 for the combination $\phi_0 = -\sqrt{\frac{g^2}{\pi\delta_{GS}}} <s_r> /M^{3/2} \delta s_r + \delta b$ of the fluctuations of the fields around the spontaneously chosen vacuum, and $\sim M^2$ for the orthogonal combination $\phi_m$. After proper normalization the field $\phi_m$ becomes the scalar superpartner of the massive vector multiplet $V$ which contains the gauge boson of the anomalous $U(1)$ with the combination of $s_I$ and $\theta$ - the phase of the field $B$ - as its longitudinal component. The mass of the complete massive vector supermultiplet which decouples in a supersymmetric way from the low-energy model is $g^2 M$. The orthogonal combination $\phi_0$ is massless, which means that the $g^2$ is left undetermined. In this case, which happen to occur in known stringy models and M-theory models, the Fayet-Iliopoulos term plays no role for the low-energy theory.

In reality the potential given by the anomalous D-term is entered by a larger number of fields (dilaton and at least charged chiral fields necessary to cancel anomalies in both hidden and visible sectors) and has flat directions, which precludes dilaton stabilization.

However, one can assume the point of view that the presence of the singlet $B$ is not really guaranteed for all possible string or M-theory compactifications, and continue the investigation under the assumption that it is meaningful to ignore such singlets. This is rather strong assumption, but it can be justified by the interesting phenomenology of the models of the Fayet-Iliopoulos type [2]. It implies that we agree to tolerate vacua with large D-terms as long as the effective scale of these D-terms is hierarchically smaller than the planck scale.

Further to the above assumption one can in imagine, that one constructs a perturbative superpotential for $X$s which together with some generic nonperturbative superpotential for $S$, and perhaps together with gravitational corrections, would fix the dilaton, and this way produce supersymmetry breaking.²

There are two obvious problems with the idea of using purely perturbative superpotential in the X-sector. First, there are in fact many chiral superfields in the model, and as usual in O’Reifertei gh type models one expects many flat directions which generically upset the dilaton stabilization in the manner based on the D-term, and second, and most important, it turns out that we need a new mass scale, say $M_I$, about 2 orders of

²Of course one always needs a nonperturbative part of the superpotential which contains $S$ - one knows that purely perturbative effects cannot break supersymmetry in stringy models.
magnitude below the planck scale, in the X-sector. The most economic and interesting possibility is that the new scale is in fact related to the hidden sector condensation. This scenario is discussed in the remaining part of this paper.

3 Fayet-Iliopoulos model with the dynamical mass scale

Let’s consider more detaily the low-energy supersymmetric implementation of the stringy anomalous $U(1)$.

To proceed assume that there are fields $G_-, G_+$ charged under $U(1)$ and under condensing $SU(N)$ in $\tilde{N}$ and $N$ representations respectively (just one pair of $\tilde{N}$ and $N$). Let’s call another pair of fields, charged under anomalous $U(1)$ but singlet under $SU(N)$, $X_-$ and $X_+$ - these are the would-be messengers, by which we mean the fields whose F-terms can take nonzero expectation values, and which can couple to the observable sector through superpotential couplings. There exists an allowed by symmetries perturbative superpotential coupling of these fields

\[ W_{pert} = \frac{G_- G_+ X_- X_+}{M} \lambda \]  

(We take $\tilde{q} = Q(G_-) = Q(X_-) = q_- = -1$ and define $q = Q(G_+), q = Q(X_+)$, the obvious generalization shall be discussed later). In the presence of matter the nonperturbative superpotential dictated by nonanomalous symmetries is, cf \[9\] and forthcoming sections of this letter,

\[ W_{npert} = U \log \left( \frac{U^{N-1} \det T}{\Lambda^{3N-1}} \right) - \Lambda (N - 1) \]  

where $U$ is gaugino condensate superfield and $T = G_- G_+$. The $W_{pert}$ can be treated as an supersymmetric mass term for $G_- G_+$.

Assuming, as in previous section, that supersymmetry is not broken along the condensate directions, one can integrate out from $W_{pert} + W_{npert}$ the superfields $U$ and $T$ obtaining an effective superpotential for messengers $X_-$ and $X_+$

\[ W_{eff} = \Lambda^3 \left( \frac{X_- X_+}{MA} \right)^\frac{1}{N} \]  

with

\[ < U > = \Lambda^3 \left( \frac{X_- X_+}{MA} \right)^\frac{1}{N} \]  

\[ < T > = \Lambda^2 \left( \frac{X_- X_+}{MA} \right)^\frac{1}{1+N} \]  

(15)
One can build the potential for $X_-$ and $X_+$ using the effective superpotential (14) and known form of the D-term contribution

$$V_D = \frac{g^2}{2} (q_+ |X_+|^2 - |X_-|^2 + \xi)^2$$

(16)

Keeping for a while $g^2$ and $\xi$ as constant parameters one arrives at equations of motion for $X$s. It is straightforward to show that these equations have no solution for any value of $N$, $\xi$ and $g^2$, in sharp contrast to the usual Fayet-Illiopoulos model [7] with perturbative supersymmetric mass analyzed in the paper [8]. The point is that in fact supersymmetry is broken along the $T$ direction in this type of models, so one is not allowed to integrate out $T$ using its supersymmetric equations of motion. This is an interesting point, as at the end it turns out that $F_T$ is subdominant with respect to other $F$-terms throughout the parameter space, however, approximating it by zero prevents one from finding the true ground state.

Next step is to make $S$, the dilaton, a dynamical degree of freedom. Using our conventions, $Re(S) = \frac{4\pi}{g^2}$, and noticing that $\Lambda = e^{-\frac{2\pi}{0^0} S}$ where $b_0 = 3N - 1$ one obtains the effective superpotential for $S, X_-$ and $X_+$ in the form

$$W = M^3 e^{-\frac{2\pi}{3N-1} S} \left( \frac{X_- X_+}{M^2} \right)^{\frac{1}{N}}$$

(17)

and the D-term contribution with variable $S$ and explicit powers of Planck scale $M$ is

$$V_D = \frac{M^4}{S^2 + S} \left( \frac{8\pi \delta_{GS} M}{S + S} + \frac{1}{M^2} (q_+ |X_+|^2 - |X_-|^2)^2 \right)$$

(18)

This superpotential together with the D-term contribution leads to a rather complicated set of equations. However, the numerical study of these equations hints that there appears the standard runaway behaviour in the direction of $S_r$, well known from single-gaugino-condensate models. The next step is to go to the full, $T$-dependent superpotential.

Now, the term

$$\delta W = \frac{\lambda TX_- X_+}{M}$$

(19)

is not required by first principles, so one should check first what happens with the simple superpotential

$$W = M^5 e^{-\frac{\pi S}{3N-1} T}$$

(20)

The answer is that there is the run-away behaviour with $y \to 0, X_- \approx 1/\sqrt{Re(S)}, X \to \infty, s \to \infty$, both in the global and in the local case. Hence, the term (19), which
plays the role of the supersymmetric linear term for $T$, known from non-F-I dynamical supersymmetry braking models - cf. [9], has to be included. Then the situation is at first sight not that obvious. In globally supersymmetric case one can easily convince oneself that there is runaway behaviour also in the presence of (19). But in the local case, the form of the potential hints at the possibility that $T$ stabilizes at $M^2$, which - while $X_-X_+$ is nonzero at the minimum - would imply that $S$ gets stabilized like in gaugino condensate models with a c-number constant in the superpotential (the constant being played by a nonvanishing vev of (19)). However, this scenario doesn’t seem to be realized in a simplest model with the simplest hidden-visible mixing term (19). What happens in this particular model is again the run-away behaviour, largely because of the presence of the additional degree of freedom $T$. In fact, the role of the field $T$ is similar to that played by the “radial” modulus $T$ in heterotic string compactifications, with the crucial difference that in the stringy case the T-modulus has a strong stabilizing superpotential proportional to $1/\eta^0(T)$.

The conclusion of this section is that simple dynamical scenarios which fail to stabilize the dilaton in non-F-I models, fail to do so also in the presence of the large, stringy, F-I term.

4 More general F-I models with dynamical supersymmetry breaking - secluded sector models

So far we have discussed a simple SU(2) model with $N_f = 1$. Let us consider in some detail more general models with $N_f < N$. The nonperturbative superpotential is

$$W = (N-N_f)(\frac{\Lambda^{b_o}}{\det T})^{1/(N-N_f)}$$

where $b_o = 3N - N_f$ and the matrix $T$ is defined as $T_{ij} = Q_i\tilde{Q}_j$. where $\tilde{Q}_j$ transforms as $\bar{N}$. We assume the canonical kinetic terms for the matter superfields. In addition to the F-type potential coming from $W$ one has also nonabelian D-terms of the group $SU(N)$ and the Fayet-Iliopoulos D-term of $U(1)$. In first step we look for the solutions which make the nonabelian D-terms vanish. This is achieved by taking the following form of $T$: $T_{ij} = \delta_{ij}|v_i|^2$ with $i = 1, ..., N_f$. the scalar potential along these directions is

$$V = \frac{2\Lambda^{2b_o}/(N-N_f)}{\prod_i |v_i|^{4/(N-N_f)}} \left( \sum_j \frac{1}{|v_j|^2} \right) + \frac{g^2}{2}(\xi + \sum_k (q_k + \bar{q}_k)|v_k|^2)^2$$

9
Let us look for a symmetric solution of this potential: $|v_1|^2 = |v|^2 = x, q_k + \bar{q}_k = -2\pi \delta_{GS}, \xi = g^2 M^2 \delta_{GS} N_f$. With these assumptions one gets the simpler potential

$$V = \frac{2N_f \Lambda^{2k_0/(N-N_f)}}{x^{2N_f/(N-N_f)}} + \frac{g^2}{2} (\xi - 2\pi \delta_{GS} N_f x)^2$$

(23)

This potential can be easily minimized for $x$ giving

$$x = \frac{g^2 M}{2\pi} (1 + \delta), \delta \approx (\Lambda^2/M^2)^{\frac{3N-N_f}{N-N_f}}$$

(24)

This solution corresponds to

$$F_Q \approx \Lambda^2 \left( \frac{\Lambda}{M} \right)^{\frac{N+N_f}{N-N_f}}, D \approx M^2 \left( \frac{\Lambda^2}{M^2} \right)^{\frac{3N-N_f}{N-N_f}}$$

(25)

From (25) one can see that $F_Q^2 \approx M^2 D$ which implies that $F_Q$ is much larger than $D$, however, the contribution of both sources to the soft masses is similar

$$\delta m^2 \sim F^2/M^2 \sim D \sim \delta \delta m^2$$

(26)

One also finds the general upper limit on the soft masses generated so this way valid for any allowed $N, N_f$: $\delta m^2 < \Lambda^2 (\Lambda/M)^2$. It is easy to check that the actual magnitude of these masses falls down quickly with growing $N, N_f$. At this point we can assume in the spirit of the gauge mediated supersymmetry breaking models that there exist an absolute gauge singlet field $Z$ which couples at the renormalizable level to the hidden sector, and also to the messenger sector. This possibility is of particular interest in the hypothetical models where the observable matter does not carry the F-I $U(1)$ quantum numbers. For simplicity let us assume that the singlet $Z$ couples to the first generation of matter fields

$$\delta W = \lambda Q_1 \bar{Q}_1 Z$$

(27)

Then we have to single out the vev of the first generation and call its square $x_1$ (and the common value of the squared vevs of remaining generations $x$ as previously). The important new terms in the scalar potential are

$$V = ... + x_1 |\lambda z - \Lambda^{k_0} x^{-\frac{N_f}{N-N_f} - \frac{N-N_f+1}{N-N_f}}|^2 + |\lambda|^2 |x_1|^2 + ...$$

(28)

The solution to the potential can be obtained also in this case, and it gives

$$F_{Q_1} \approx 0, F_{Q_1 \neq 1} \neq 0 (and \ dominant)$$

(29)

and

$$|F_Z| \approx |v_1|^2 \neq 0, |z| \approx |v_1|, |v_1|^2 \ll |v|^2 \sim g^2 M^2$$

(30)
These properties of $F_Z$ and $Z$ show that the hidden sector model discussed here could serve as a rather simple “secluded” sector in scenarios with gauge mediation of the supersymmetry breaking between secluded and observable sectors. It is interesting to note at this point that in the potential (22,28) the D-term plays the role analogous to the soft mass terms in explicitly broken SQCD. It induces finite condensates $<\psi_ Q_i \bar{\psi}_ Q_j > \sim < F_{ij} > \neq 0$ although the breaking of supersymmetry is only spontaneous here.

However, one should ask the question what happens in these models when the gauge coupling squared becomes the inverse of the dynamical field $S$. Then the potential computed above obtains a new contribution

\[ V \rightarrow V + \left| \frac{\partial W}{\partial S} \right|^2 \sim \frac{v^2}{M^2} |F_Q|^2 + \frac{|F_Q|^4}{M^4} \]  

(31)

where the second term is the new F-term, taken at the minimum, and the last term comes from the D-term. As expected the $F_Q$ falls down exponentially with $S$ and there is the run-away behaviour towards the ultra-weak coupling region. The situation is not improved when one takes into account gravitational corrections in the full supergravity lagrangian. Hence, the negative conclusion about the stabilization of the dynamical gauge coupling in the presence of the large F-I term remains valid.

Then the question is what happens to the models which are known to give reasonable results without the D-term, like race-track models, cf. [10], or S-dual models of [1].

5 Dynamical F-I models with superpotential which stabilizes the dilaton

The general answer to the question posed at the end of the previous section is that these models work, i.e. stabilize the dilaton, also in the F-I case, but their predictions get affected.

To illustrate the role of the D-term in a model independent way we shall discuss two toy models which however are designed to resemble the typical situations encountered in popular models with dynamical gauge coupling: a) Model I, which corresponds to the situation where the superpotential itself stabilizes the dilaton at some scale equal or larger than the planck (or string) scale (like in S-dual models of [1]), b) Model II, where superpotential alone stabilizes the dilaton at some unacceptably small scale (which can easily happen in race-track models). We shall consider separately the two cases.
model I

We consider the following model

\[ K = -M^2 \log\left(\frac{S + \bar{S}}{M}\right) + X_+X_+ + X_-X_- \]  
\[ W = mX_-X_+ + q(S - p)^2 \]

where \( S \) is the dilaton superfield, and \( X_+ \) and \( X_- \) are “messenger” superfields, charged under \( U(1) \). We assume everywhere that \( m \leq q \ll M (= M_{pl}) \) but, as required by gauge coupling unification, \( p > M^3 \).

If we put aside the charged fields and the D-term, the model has its minimum at \( S = p \), and there is another minimum of the potential at \( S + \bar{S} = 0 \), but with present choice of the Kähler function it is infinitely far away in the field space. In the presence of the D-term, the situation changes. The minimum at small \( x \) appears (comes in from infinity) at \( x_{sm} = \frac{4M}{(\delta_{GS}/(8\pi^2 m^2/q^2 M^2/p^2))^{1/3}} \). Under our assumptions this tends to be smaller than \( M \), hence doesn’t correspond to the phenomenologically required solution, unless \( m \) becomes comparable with \( q \). At this point supersymmetry is broken with \( F_S \gg <D> \), cf. model II in the next subsection. The second minimum appears in the vicinity of \( p \).

The assumed form of the Kähler potential and of the superpotential gives the scalar potential

\[ V = m^2(|X_+|^2 + |X_-|^2) + \frac{(S + \bar{S})^2}{M^2} 4q^2|S-p|^2 + \frac{4\pi M^5}{(S + \bar{S})} \left( \frac{8\pi \delta_{GS} M}{(S + \bar{S})} - |X_-|^2 M^2 + q_+ \frac{|X_+|^2}{M^2} \right)^2 \]

(we assume \( \delta_{GS} \approx 0.01 \), as given by a typical string model, and \( q_+ = 2\pi \delta_{GS} + 1 \) - as explained in Section 7.)

It is more or less obvious that this potential can have a minimum at finite \( S, X_+, X_- \). Also, it is clear that if there is a minimum, it has to correspond to \( X_+ = 0 \). Assuming further that we can restrict ourselves to real values of the scalar components of superfields, we shall solve perturbatively equations

\[ \frac{\partial V}{\partial X_+} = 0, \quad \frac{\partial V}{\partial X_-} = 0, \quad \frac{\partial V}{\partial S} = 0 \]  

(35)

From \( \frac{\partial V}{\partial X_-} = 0 \) we get

\[ X_-^2 = \frac{1}{2M} \left( \frac{8\pi \delta_{GS} M^4}{S} - m^2 \frac{S}{2\pi} \right) \]

(36)

This superpotential should be regarded as a part of a series expansion, cf. Section 7.
or \( X_- = 0 \). The latter possibility cannot correspond to minimum (D-term would be large) so we take the first possibility.

From \( \frac{\partial V}{\partial S} = 0 \) we obtain, after substituting (36), the equation for \( S \) which we can solve perturbatively. To this end we assume \( S = p(1 + e) \). Then in the leading order we get

\[
e = \frac{\pi}{8} \delta_{GS} \frac{m^2 M^5}{q^2 p^5} (<<1)
\]

(37)

With this we can compute the F-terms:

\[
|F_{X_-}| = 0
\]

(38)

\[
|F_{X_+}| = m M \sqrt{\frac{\delta_{GS} M}{p}} \frac{4\pi}{M}
\]

(39)

\[
|F_S| = |(K_{SS})^{-1} W_S| = m M \frac{\delta_{GS} M^2 m}{2} \frac{m}{p^2 q}
\]

(40)

and we can find the contribution to soft scalar masses given by the nonvanishing D-term

\[
\delta m_i^2 = q_i g^2 2 < D > = q_i m^2
\]

(41)

(this is a contribution for the field \( X_i \) with the charge \( q_i \).)

In this case the \( F_S \) is nonzero, but smaller than \( F_{X_+} \), the F-term corresponding to positively charged messenger, unless \( m \) is comparable with \( q \). In general, it is the D-term contribution which is dominant in the soft susy breaking mass parameter of the charged scalars, \( (F_{X_+/M})^2 = 4\pi m^2 (\delta_{GS} M/p) < m^2 \).

**model II**

This is the model which in the absence of the \( U(1) \) and its D-term correspond to strongly coupled vacuum. However, the presence of the planck-scale D-term changes situation dramatically. As before the Kähler function is

\[
K = -M^2 \log\left(\frac{S + \bar{S}}{M}\right) + X_+ \bar{X}_+ + X_- \bar{X}_-
\]

(42)

and the superpotential

\[
W = m X_- X_+ + q S^2
\]

(43)

In these formulae \( S \) is the dilaton superfield, and \( X_+ \) and \( X_- \) are “messenger” superfields, charged under \( U(1) \) - as previously. We assume again that \( m, q \ll M \). The
scalar potential is
\[ V = m^2(|X_+|^2 + |X_-|^2) + \frac{(S + \bar{S})^2}{M^2} 4q^2|S|^2 + \frac{4\pi M^5}{(S + \bar{S})} \left( \frac{8\pi \delta_{GS} M M}{(S + \bar{S})} - \frac{|X_-|^2}{M^2} + q_+ |X_+|^2 \right)^2 \]  
(we assume \( \delta_{GS} \approx 0.01 \), as given by a typical string model, and \( q_+ = 2\pi \delta_{GS} + 1 \) - as explained in Section 7.)

It is more or less obvious that this potential can have a minimum at finite \( S, X_+, X_- \). Also, it is clear that if there is a minimum, it has to correspond to \( X_+ = 0 \). Assuming further that we can restrict ourselves to real values of the scalar components of superfields, we shall solve perturbatively equations
\[ \frac{\partial V}{\partial X_+} = 0, \quad \frac{\partial V}{\partial X_-} = 0, \quad \frac{\partial V}{\partial S} = 0 \]  
(45)

From \( \frac{\partial V}{\partial X_-} = 0 \) we get again
\[ X_-^2 = \frac{1}{2M} \frac{8\pi \delta_{GS} M^4}{S} - m^2 \frac{S}{2\pi} \]  
(46)

From \( \frac{\partial V}{\partial S} = 0 \) we obtain, after substituting (46), the equation for \( S \)
\[ \frac{-m^4}{4M} - \frac{8\pi^2 \delta_{GS} M^3 m^2}{S^2} + 2\pi \frac{64q^2 S^3}{M^2} = 0 \]  
(47)

This we can solve perturbatively assuming \( m^2, q^2 << M^2 \). Knowing that we should get \( S \approx M \) we shall retain in the leading order only last two terms on the LHS of (47) and solve for \( S_o \)
\[ S_o = \frac{\delta_{GS} m^2}{(2\pi^4 q^2)^{1/5}} \]  
(48)

To proceed we assume \( S = S_o (1 + \epsilon) \). Then in the leading order we get
\[ S = (\frac{\delta_{GS} m^2}{2\pi^4 q^2})^{1/5} M (1 + \frac{m^2 (\delta_{GS} m^2/q^2/(8\pi^4)^{2/5}}{2^{2/5}(160\delta_{GS} M^2 - 2^{3/5} m^2 (\delta_{GS} m^2/q^2/(8\pi^4)^{2/5}))} \]  
\]  
(49)

If we substitute this result into (46), we obtain the leading order value of \( X_- \).

With this we can compute the F-terms:
\[ |F_{X_-}| = 0 \]  
(50)
\[ |F_{X_+}| = m((\frac{\delta_{GS} m^2}{8\pi^4 q^2})^{1/5}(2560\delta_{GS} M^4 \frac{\delta_{GS} m^2}{8\pi^4 q^2})^{3/5} q^2 16\pi^4 - 208 \frac{2^{3/5} \delta_{GS} m^4 M^2}{(2^{1/5} \delta_{GS} m^2 M^2)^{1/2}} \]  
(51)
\[ = 2mM (\delta_{GS})^{2/5} (\frac{q}{m})^{1/5} (4\pi^2)^{1/5} \]  
(52)
\[ |F_{S}| = M q (\delta_{GS})^{3/5} (\frac{m}{q})^{6/5} \]  
(53)
It is to be noted that because of the nontrivial Kähler function there is the relation
\[ F_S = \frac{w_S}{K_{SS}} = 4 \frac{s^2}{M} W_S. \]
In the next step we can find the contribution to soft scalar masses given by the nonvanishing D-term (notation as before)
\[ \delta m_i^2 = q_i g^2 2 < D > \approx q_i m^2 \]  
(this is the contribution for the field \( X_i \) with the charge \( q_i \)). In this case one can easily obtain required value of \( s \) taking \( m \) suitably larger than \( q \) (we didn’t assume anything about the ratio \( m/q \) to get the solution). As for the soft terms,
\[ \frac{|F_S|}{|F_{X+}|} = \frac{< s >}{2\pi M} \]  
and
\[ \frac{\delta f m^2}{\delta d m^2} = \frac{\delta_{GS} < s >}{\pi M} \]  
Hence, if the vacuum in this model lies in the weak coupling regime, then the dilaton F-term can be as large as the other auxiliary fields, and its contribution to the soft masses can be comparable with the D-term contribution. This has various consequences. One is that the gaugino masses can be in this case as large as the scalar soft masses, which is very good from the point of view of low-energy phenomenology, the other consequence is, however, that the possibility of creating hierarchy through the assignment of \( U(1) \) charges is essentially lost.

One should note at this point, that we have been a bit cavalier about stringy properties of the effective models which we have discussed. In fact, in the stringy lagrangian all the terms, if we put F-I parameters to zero and omit truly nonperturbative terms, should be multiplied by the common power of \( 1/(S + \bar{S}) \) as all of them come from the tree-level string amplitudes. This feature is not uniquely implementable into the globally supersymmetric lagrangian. We believe, that in principle the correct procedure of taking this feature into account is to minimize not the “naive” globally supersymmetric lagrangian, as we have been doing so far, but to minimize the leading part of the stringy locally supersymmetric lagrangian. The change in the potential would be that all the terms except the D-term would be multiplied by the factor \( e^{K/M^2} \). Now, this factor is
\[ e^{K/M^2} = \frac{M}{S + S} \left( 1 + \frac{1}{M^2} \sum_i |X_i|^2 + \mathcal{O}(\frac{|X|^4}{M^4}) \right) \approx \frac{M}{S + S} \]  
in the leading order. The change in the equations which arises from this modification turns out to be inessential for the overall conclusions, although their actual form becomes more complex. Hence, to keep our discussion simple and quasi-analytical we use the “naive” global models as the illustration.
6 Symmetries of the stringy hidden gauge sector in the presence of the $U(1)$

To narrow down the generality of the discussion to the range of realistic models, let us discuss more carefully the possible dilaton-dependence of the effective superpotential in anomalous $U(1)$ models, in the manner based in symmetries.

It is well known that the supersymmetric $SU(N)$ theories with $N_f$ copies of $\tilde{N} + N$ matter representations, $N_f \leq N - 1$, without dynamical coupling have anomalous axial and R-symmetries. In the case discussed here, $N = 2, N_f = 1$ there are two independent anomalous symmetries: axial $U(1)_A$ (with the associated Konishi current) under which

$$Q \rightarrow e^{i\alpha} Q, \quad \bar{Q} \rightarrow e^{i\alpha} \bar{Q}$$

(58)

and the R-symmetry $U(1)_R$

$$Q \rightarrow e^{2i\alpha} Q, \quad \bar{Q} \rightarrow e^{2i\alpha} \bar{Q}, \quad V \rightarrow e^{-6i\alpha} V, \quad \theta \rightarrow e^{3i\alpha} \theta$$

(59)

( this is the symmetry whose current lies in the supermultiplet with the stress tensor and the supercurrent). The linear combination of these two symmetries forms the nonanomalous R-symmetry $U(1)_{R'}$

$$Q \rightarrow e^{-i\alpha} Q, \quad \bar{Q} \rightarrow e^{-i\alpha} \bar{Q}, \quad V \rightarrow V$$

(60)

One assumes that the nonanomalous R-symmetries should be respected by nonperturbative effects, [9], and this way they constrain the form of the nonperturbative superpotential induced for low-energy gauge invariant degrees of freedom $T = Q\bar{Q}$ giving the well known form of the superpotential $T$. In the presence of dynamical gauge coupling, $L = ... \frac{i}{32\pi}(SW^2)_F + h.c. + ...$ there appears a new anomalous symmetry which consists in an imaginary global shift of $S$

$$S \rightarrow S - i\alpha$$

(61)

This again can be combined with the anomalous $U(1)_R$ to form a nonanomalous R-symmetry involving $S$ and $W^2$ superfields, cf. [11].

The question is whether the presence of the anomalous $U(1)$ in stringy models can restrict the possibilities. The point is that at the scale of the extra gauge boson mass that boson decouples, and below that scale we do not have at our disposal the gauge transformation which shifts $S$ superfield. What is left after anomalous $U(1)$ is the global symmetry with charges equal to the charges under local $U(1)$. This is the
classical invariance of the low energy matter lagrangian, but it is anomalous. Since it
is anomalous, we can combine it with the above global imaginary shift of $S$ to form a
new nonanomalous global symmetry, which we could use to constrain the form of the
Lagrangian. One can easily see that the nonanomalous combination is

$$S \rightarrow S - i\delta_{GS} \alpha, \quad Q \rightarrow e^{-i\theta} Q, \quad \bar{Q} \rightarrow e^{-i\theta} \bar{Q}$$  \hspace{1cm} (62)

where $q + \bar{q} = -2\pi \delta_{GS}$. It is easy to see that (62) gives the following general form of
the superpotential

$$W = \left( \frac{e^{-2\pi S}}{T} \right)^{\gamma}$$  \hspace{1cm} (63)

It is obvious that to fix $\gamma$ we need the nonanomalous R-symmetry. Indeed, when
one combines the imaginary shift with the R-symmetry (59), one obtains one more
independent non-anomalous mixture of anomalous symmetries, which is actually an
R-symmetry $U(1)_{R'}$

$$S \rightarrow S - \frac{\alpha}{\pi} (3N - N_f), \quad Q, \bar{Q} \rightarrow e^{2i\alpha} Q, \bar{Q}, \quad V \rightarrow e^{-6i\alpha} V,$$

$$\theta \rightarrow e^{3i\alpha} \theta$$  \hspace{1cm} (64)

(we use $N = 2, N_f = 1$ in this section). If we impose the symmetry (64) on the effective
superpotential containing just $S$ and $T$ then we obtain the general expression

$$W = M^3 e^{-2\pi S} g \left( \frac{T}{M^2} \right)$$  \hspace{1cm} (65)

where $g$ is any function of $T$ but not $\bar{T}$. When we impose both nonanomalous com-
binations of symmetries, we constrain the form of the superpotential further to the
form

$$W = M^5 \frac{e^{-2\pi S}}{T}$$  \hspace{1cm} (66)

which is exactly the form of the nonperturbative superpotential for $SU(2)$ which we
started the discussion with.

It should be observed, that the crucial role in determining the $S$-dependence of the
superpotential is played by the imaginary shift in $S$. In the above, we have assumed
that that shift should be continuous, but experience with strings tells us, that it should
rather be discreet. With our present normalization of $S$ the discrete shifts allowed by
the string are

$$S \rightarrow S - in$$  \hspace{1cm} (67)

where $n$ is integer. Then the nonanomalous remnant of $U(1)_A$ is

$$S \rightarrow S - in, \quad Q \rightarrow e^{-i\theta_{GS} S} Q, \quad \bar{Q} \rightarrow e^{-i\theta_{GS} S} \bar{Q}$$  \hspace{1cm} (68)
This discrete symmetry is much less restrictive and allows the general superpotential

\[ W = f(e^{-2\pi S}, T) \]  

(69)

where \( f \) is any function restricted by analyticity requirements and by the requirement that the superpotential falls down exponentially in the weak coupling limit (one expects nonperturbative contributions to be proportional to exponentials with negative exponents - like \(-8\pi/g^2\) in one-instanton case). It is easy to see that the discrete version of \( U(1)_{R'} \) doesn’t restrict further the \( S \)-dependence of the superpotential. However, if we impose \( U(1)_{R'} \), then we get

\[ W = \frac{h(e^{-2\pi S})}{T} \]  

(70)

which is the most general form allowed. Hence, for instance, string allows the effective superpotential to be a series \( \sum_i c_i e^{-k_i 2\pi S/T} \) where \( k_i \) are integers, which can lead to stabilization of the dilaton (from here on we are back to the dimensionful version of the superfield \( S \)).

Further to that, the discrete imaginary shift can be viewed as the part of the larger symmetry group, for instance as a subgroup of \( SL(2,\mathbb{Z}) \) S-duality symmetry as discussed in [1]. This leads to the construction of more specific superpotentials, which can be seen as an infinite series of exponentials. An example of the S-dual model is

\[ W(S, T) = \frac{\alpha}{T\eta^2(j(S/M) - 744)^{b_2}} \]  

(71)

where \( \eta \) is the modular \( \eta \)-form, \( j \) denotes the modular invariant \( j \)-function, and \( \alpha, \beta \) are constants, with the gauge coupling given by

\[ g^2 = \frac{8\pi^2}{\log |j(S/M) - 744|} \]  

(72)

Model given with these functions in the absence of the anomalous \( U(1) \) would have a minimum in \( S \) at the self-dual point \( S = 1/4 \) with supersymmetry unbroken in the direction of \( S \).

### 7 Exponential superpotentials

To have some idea of what the actual orders of magnitude of the parameters in the realistic “binding” superpotentials can be, let us consider more detailly the example of

\[ ^4 \text{Of course, to fix } T \text{ we need also here a } T \text{-dependent perturbation of the } S \text{-dual superpotential} \]
superpotentials which can be considered as series of exponential terms, in agreement with the discussion of the previous section

\[ W(S) = \sum_{\gamma} c_{\gamma} e^{-\gamma^{2} \pi^{2} S} \quad (73) \]

where the sum is finite, as in race-track models, or infinite, as in S-dual models of \[.] In any case we shall assume that there is a supersymmetric point \( S_{o} \) in these models, which corresponds to a minimum in the globally supersymmetric model with the assumed superpotential. The choice of the \( S_{o} \) to be the supersymmetric point is dictated by the experience with string-inspired models. In these models at the natural minima of the dilaton supersymmetry is typically unbroken, or only slightly broken, along the dilaton direction. The general expansion is

\[ W(S) = W(S_{o}) + \frac{\partial W}{\partial S} (S - S_{o}) + \frac{\partial^{2} W}{\partial S^{2}} (S - S_{o})^{2} + ... \quad (74) \]

where the second term on the \( rhs \) vanishes by assumption. When we apply this expansion to exponential superpotentials, and truncate it after the second term, we get

\[ W(S) = \Lambda^{3} + \frac{c}{M^{2}} (S - S_{o})^{2} \quad (75) \]

In the above formula the constant \( c \) is of order unity in race-track models, but can be larger, eg. \( O(10) \) or higher, in S-dual models. Of course, the scale of \( \Lambda \) sets the gravitino mass, \( m_{3/2} \approx \frac{\Lambda^{3}}{M^{2}} \), and \( c\frac{\Lambda^{3}}{M^{2}} \) is the scale of the supersymmetric contribution to the dilaton mass.

Hence, in the notation of the previous chapter we obtain

\[ q = \frac{c}{M^{2}} \]

\[ p = S_{o} \quad (76) \]

Now, it is easy to find out the realistic values of \( m \) looking at the generalized version of (72)

\[ W_{pert} = \frac{(G_{-}G_{+})^{n}X_{-}X_{+}}{M^{2n-1}} \lambda \quad (77) \]

In this case the perturbative \( U(1)_{A} \) invariance of the superpotential requires that \( q_{+} + q_{-} = -(q + \bar{q})n = 2\pi \delta_{GS} n \) which means that we should replace in all relevant formulae \( q_{+} \rightarrow 2\pi \delta_{GS} n + 1 \). Now we can identify \( m \) as

\[ m = \frac{< G_{-}G_{+} >= n}{M^{2n-1}} = \frac{< T >}{M^{2n-1}} \quad (78) \]

Now, as we have mentioned before, one can probably achieve \( < T > = M^{2} \) in supergravity models. In such case \( m = M \gg q \). However, the natural value of \( T \), consistent with the observation that formation of condensates due to strong gauge forces is
\[ n = 1 \quad \quad \quad \quad n = 2 \quad \quad n > 2 \]

| \( n \) | \( m \) | \( q \) | \( \frac{m}{q} \) | \( \frac{\left| F_S \right|}{\left| F_X \right|} \) | \( \frac{m^2}{\delta_{GS}^2} \) |
|---|---|---|---|---|---|
| 1 | \( \Lambda \frac{\Lambda}{M} \) | \( c\Lambda(\frac{\Lambda}{M})^2 \) | \( \frac{1}{c}\left(\frac{M}{\Lambda}\right) \gg 1 \) | \( \frac{1}{c}\left(\frac{\Lambda}{M}\right) \ll 1 \) | \( \frac{1}{c}\left(\frac{\Lambda}{M}\right)^2 \gg 1 \) |
| 2 | \( \Lambda(\frac{\Lambda}{M})^3 \) | \( \left(\frac{M}{\Lambda}\right) \ll 1 \) | \( \frac{\left(\frac{M}{\Lambda}\right)^2}{\left(\frac{M}{\Lambda}\right)} \ll 1 \) | \( \frac{\left(\frac{M}{\Lambda}\right)^2}{\left(\frac{M}{\Lambda}\right)} \ll 1 \) | \( \frac{\left(\frac{M}{\Lambda}\right)^2}{\left(\frac{M}{\Lambda}\right)} \gg 1 \) |
| \( n > 2 \) | \( \Lambda(\frac{\Lambda}{M})^{2n-1} \) | \( \left(\frac{M}{\Lambda}\right)^{2n-3} \ll 1 \) | \( \frac{\left(\frac{M}{\Lambda}\right)^{2n-3}}{\left(\frac{M}{\Lambda}\right)^{2n-3}} \ll 1 \) | \( \frac{\left(\frac{M}{\Lambda}\right)^{2n-3}}{\left(\frac{M}{\Lambda}\right)^{2n-3}} \gg 1 \) | \( \frac{\left(\frac{M}{\Lambda}\right)^{2n-3}}{\left(\frac{M}{\Lambda}\right)^{2n-3}} \ll 1 \) |

Table 1: Summary of the models with typical exponential superpotentials. The values of \( c \approx 1 \) and of \( p \leq 1 \) in the column corresponding to \( n = 1 \) are assumed.

rather a field theoretical phenomenon, is \( < T > = \Lambda^2 \). In this case, if \( n=1 \) we obtain \( m = \Lambda \frac{\Lambda}{M} = \frac{M}{\Lambda} q \gg q \). For all \( n > 1 \) the hierarchy between \( q \) and \( m \) gets inverted - \( m \) becomes much smaller than \( q \). It is a straightforward exercise to obtain values of F- and D-terms in all the cases.

The Table (1) summarizes the models with generic exponential superpotentials. It is obvious that the most interesting models correspond to the \( n = 1 \). In this case, for \( p \leq 20 \) and \( \Lambda/M \leq 10^{-5} \) - which is just the interesting range of parameters, the results closely follow those for the generic model II. In fact, to a very good approximation the minimum of the effective potential resulting from integrating out all the degrees of freedom except \( S \) is

\[
< S_o > = M\left(\frac{\delta_{GS}\pi}{16}\right)^{1/5}(\frac{M}{\Lambda})^{2/5}
\]

(79)

If one demands that the assumed minimum corresponds to the value given by the unification conditions, then one obtains a simple relation between the trace of the anomalous \( U(1)_A \) and the condensation scale in the hidden sector

\[
\log(|Tr X|) = \text{const} + \log(\frac{\Lambda}{M})
\]

(80)

It is apparent that for a given strongly interacting hidden sector one can work on the charges of the \( U(1) \) group towards achieving required value of the unification coupling. Once this is done, one obtains highly universal, in the sense discussed in the introduction, soft masses which are generated in similar parts by the non-zero D-term and by the dilaton F-term. In this scenario also the gaugino masses are large and equal in magnitude to scalar masses.
8 Inflation from dynamical F-I term

Some cautious remarks must be made about cosmology of the models we discuss here. As the global supersymmetry is broken in these models spontaneously in the flat limit, they generically have a positive, and unacceptably large, cosmological constant. Cancellation of this constant through the gravitational corrections, if possible at all, would require for instance an addition of the constant piece in the superpotential of the order $< F > M$ at least, which is hardly natural in the present context. The dynamical dilaton in the D-term implies also harmful modifications to scenarios of inflation based on the temporary dominance of the D-term in the potential energy $^{[13]}$. To illustrate the trouble let us consider the epoch when the D-term dominates the energy density. The evolution equations for the scale factor $a(t)$ and the dilaton $s(t)$ are (we put $M = 1$ here)

\[
\frac{\partial^2 s}{\partial t^2} - \left(\frac{\partial s}{\partial t}\right)^2/s + 3H \frac{\partial s}{\partial t} - \frac{192\pi^3 \delta_{GS}^2}{s^2} = 0
\]

\[
\left(\frac{\partial s}{\partial t}\right)^2/(4s^2) + \frac{32\pi^2 \delta_{GS}^2}{s^3} = H^2
\]

where $H$ is the Hubble parameter. The results of the integration of these equations for $\delta_{GS} = 0.1$ is shown in the figure 1. The figure shows that there is no inflationary epoch after a short initial switching-on period which is due to the initial conditions taken for the integration. The dilaton evolves so rapidly, that its kinetic energy very quickly comes to dominate over the potential energy - which doesn’t allow inflation. The large kinetic energy which dilaton would acquire during the stage of such a D-term dominated evolution is very likely to prevent it from settling down in any conceivable minimum which could be generated by gauge dynamics in the intermediate or weak coupling regime.
Figure 1. Evolution of the acceleration of the scale factor, curves (a), (c), and of the ratio of the dilaton kinetic energy and the D-term energy, curves (b) and (d), in the Universe dominated initially by the planck scale dynamical D-term. Thin curves correspond to the solution with the vanishing initial velocity of the dilaton, and the thick curves to the planck scale initial velocity of the dilaton. In both cases the acceleration becomes quickly negative, and the kinetic energy of the dilaton comes to dominate expansion.

9 Discussion and conclusions

In this paper we have analyzed in as much model independent manner as possible the structure of the dilaton vacuum and supersymmetry breaking pattern in Fayet-Iliopoulos models with dynamical gauge coupling and large (in fact - stringy) value of the F-I parameter.

We have found that in this important class of models the supersymmetry is generically broken, but the mere presence of the D-term is not sufficient to stop the running away of the modulus responsible for the value of the gauge coupling - the dilaton.
Some of the models described in section 4 have an interesting vacuum structure resembling softly broken SQCD. They can serve as a secluded sector in models with gauge mediation of the susy breaking once the dilaton gets stabilized.

To stabilize the dilaton, one has to include an additional dilaton-dependent part in the superpotential. Once this is the case, several scenarios are possible, as described in the preceding section. In particular, the models which without the D-term would be considered irrelevant as giving strongly coupled gauge theories at the unification scale, can be saved in the presence of the large D-term. Such models, when asked to give the correct $g_{\text{unification}}^2$, easily give rise to the mixed dilaton/D-term dominated scenarios of susy breaking, which in principle allow horizontal hierarchy generation. The models with purely dilaton dominated supersymmetry breaking could also be thought of, but they would correspond to very weak unified coupling (cf. Table (1), column $n = 1$).

It should be noted, that in general one cannot assume $F^S = 0$ in the class of models discussed here.

Finally, as easily seen from the Table (1) that all interesting effects of the $d$-term are proportional to the value of $\delta_G$ hence quickly become unimportant when the mass scale of the $F$ parameter becomes smaller than $M_{\text{planck}}$. Also, after supersymmetry breaking induced by some third party one can have induced $d$-terms with $F$-I effective parameters $\xi \sim m_{3/2}^2 \log m_{3/2}^2$ which for reasonable values of $m_{3/2}^2 \sim 1 TeV^2$ gives $\frac{\xi}{M^2} \leq 10^{-30}$ resizing all the interesting features discussed above down to nothing.

Some cautious remarks must be made about cosmology of the models we discuss here. As the global supersymmetry is broken in these models spontaneously in the flat limit, they generically have a positive, and unacceptably large, cosmological constant.Cancellation of this constant through the gravitational corrections, if possible at all, would require for instance an addition of the constant piece in the superpotential of the order $< F > M$ at least, which is hardly natural in the present context. We have shown that when the $F$-I parameter and the gauge coupling are dynamical variables, the $D$-term dominated Universe does not allow for an inflationary period.

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Added note We got aware of the eprint [14] which also adresses some of the issues considered here.

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