Sunyaev-Zel’dovich anisotropy due to Primordial black holes

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We investigate the Sunyaev-Zel’dovich (SZ) effect caused by primordial black holes (PBHs) on the cosmic microwave background (CMB) temperature fluctuations. The gas accreting on a PBH heats up by the release of the gravitational energy. As a result, the heated gas in the vicinity of the PBH emits the UV and X-ray photons. These photons can ionize and heat the intergalactic medium (IGM) around the PBH. Assuming the simple model of these emitting photons, we compute the profiles of the IGM ionization fraction and temperature around a PBH. Using these profiles, we evaluate the Compton $y$-parameter created by the IGM gas around a PBH. Finally, we estimate the CMB temperature angular power spectrum due to the PBH SZ effect in our model. We show that the SZ temperature anisotropy due to the PBHs has the flat angular power spectrum on small scale, $l \lesssim 2000$ and could dominate the primordial temperature spectrum on smaller scales than the Silk scale. This flat spectrum extends to the scale of the ionized region by the PBH emission. We also discuss the impact of the small-scale CMB measurement on the PBH abundance based on our results.

I. INTRODUCTION

Many astronomical observations including the motion of stars in a galaxy, the large scale structure distribution and the cosmic microwave background (CMB) temperature anisotropy indicate the existence of dark matter (DM) in the Universe. However, its nature is still poorly understood. Currently, since the presence of DM can be probed only through its gravitational interaction, observational information about its nature is very limited. However, there are many works to study the nature of DM and many theoretical models have been suggested so far. Generally, the DM candidates are classified into two types; the non-baryonic particle model and astrophysical compact object model. A lot of models for the non-baryonic particle type are predicted in the physics beyond the standard model such as WIMPs [1], axions [2–5] and axion-like particle [6]. On the other hand, in the latter type, a primordial black hole (PBH) is the most potential candidate [7].

PBHs could have formed from high-density peaks in the very early Universe [8,9]. In the inflation paradigm, the density fluctuations are generated from the quantum fluctuations. When an overdense region which exceeds a critical density threshold enters the horizon scale, the gravitational collapse of this region happens to form a PBH. The resultant mass of the formed PBH corresponds the horizon mass at the horizon-crossing epoch of the overdense region. As a result, the PBH mass range can span very widely. Therefore, the PBH abundance has been studied for a long time not only for DM but also as the relic of the primordial density fluctuations on small scales. Besides, the recent detection of gravitational wave (GW) events draw attention to PBHs as sources of GWs. The analysis of the observed GW data reveals that the detected GW events were produced by binary black hole mergers with masses larger than 20 $M_\odot$ [10,12]. It would be difficult to produce a black hole with such large mass from the stellar evolution in the standard solar metallicity environment [13]. On the other hand, the broad mass range of PBHs can cover the observed black hole masses. Therefore, the PBH abundance is also studied as responsible sources for the detected GW events [14–16].

The existence of PBHs affects various cosmological phenomena, depending on PBH mass. For small mass PBHs, the abundance can be constrained by the effects of their evaporation. As first pointed out by Hawking [17], a black hole emits many kinds of particles with the thermal spectrum. As a result, PBHs with mass smaller than $10^{15}$ g have evaporated by the present epoch. The abundance constraint on evaporated PBHs are obtained by investigating the effect of their evaporation on big bang nucleosynthesis [18], the CMB spectrum distortion [19], the recombination and reionization processes [20,21], and the diffuse gamma-ray background [22,24].

For nonevaporated PBHs, the robust constraint is provided by gravitational lensing observations [25,27]. The black hole merger rate evaluated from the recent detection of GW events also constrains the PBH abundance [15,16]. Recently several works focused on the gas accreting on massive PBHs. Due to the release of the gravitational energy during the accretion, the gas becomes hot and emits X-ray and UV photons [28,29]. Resultantly the surrounding gas around a PBH is heated and ionized. Studying the cosmological effects of such heating and reionization provides the constraint on stellar mass PBHs with the recent CMB measurement [28–30]. There are also some works suggesting that future 21-cm observations can probe these PBH heating and ionizing processes and give the strong constraint on PBHs in this mass range [31,32].

In this paper, we study the CMB anisotropy through the thermal Sunyaev-Zel’dovich (SZ) effect due to PBHs.
The thermal SZ effect is the distortion of the CMB energy spectrum through the inverse Compton scattering by high energy electrons in hot plasma [38, 34]. As mentioned above, PBHs can heat and ionize the surrounding gas. The resultant ionized gas not only contribute to the global optical depth of the Thomson scattering as discussed in Refs. [28, 29], but also create the additional CMB anisotropy on small scales through the SZ effect. Introducing the emission efficiency parameter of the PBH luminosity, we evaluate the ionization fraction and temperature profile through the 1-dimensional radiative transfer computation. Assuming that the PBH distribution follows the dark matter density fluctuations, we calculate the CMB temperature anisotropy due to the SZ effect of PBHs. We investigate the dependence of the generated CMB anisotropy on the PBH fraction and mass. From the comparison with the small-scale measurement of the CMB temperature anisotropy by South Pole Telescope (SPT) [35], we also discuss the impact of the SZ effect due to PBHs on the constraint on the PBH abundance with the emission efficiency parameter.

The rest of this paper is organized as follows. In Sec. II, we compute the gas temperature and ionization fraction around a PBH. Accordingly, we calculate the profile of the thermal SZ effect. In Section III, introducing the PBH fraction to dark matter, we evaluate the CMB temperature angular power spectrum due to the thermal SZ effect around PBHs. We also obtain the PBH abundance constraint from comparing our results with the SPT. Finally, we summarize in Sec. IV. Through out our paper, we take the flat ΛCDM model with the Planck best fit parameters [36]: $(\Omega_m, \Omega_b, h, n_s,\sigma_8) = (0.32, 0.68, 0.049, 0.67, 0.97, 0.81)$.

II. THERMAL SUNYAEV-ZEL’DOVICH EFFECT DUE TO A PBH

In this section, we evaluate the gas temperature and ionization fraction of the intergalactic medium (IGM) around a PBH, assuming the photon energy spectrum emitted from the hot gas in the vicinity of a PBH. Accordingly, we calculate the profile of the thermal SZ effect.

A. The luminosity from a PBH

Since a PBH creates a gravitational potential, surrounding gas accretes on the PBH. During the accretion, as it goes closer to a PBH, the gas becomes hot enough to emit X-ray and UV photons [28, 29]. However, since the gas temperature highly depends on the astrophysical processes and environmental condition, there is a theoretical uncertainty in this luminosity. When the PBH has an accretion disk, the luminosity might become nearly sub-Eddington luminosity similar to the active galactic nuclei. If not, the luminosity is much lower than the Eddington luminosity. Therefore, this emission efficiency can depend on the redshift, the PBH mass and other physical conditions around the PBH. Here, for simplicity, we introduce one free parameter for the emission efficiency, $\epsilon$, which represents the total PBH luminosity, $L_{PBH}$, in terms of the Eddington luminosity.

$$L_{PBH} = \epsilon L_{Edd},$$

where $L_{Edd}$ is the Eddington luminosity for mass $M$

$$L_{Edd} = 3.2 \times 10^8 L_\odot (M/M_\odot).$$

We also assume the power-law type of the luminosity spectrum.

$$L_{PBH,\nu} = A \nu^{-1.5},$$

where $A$ is determined by

$$L_{PBH} = \int_{\nu_L}^{\nu_H} L_{PBH,\nu} d\nu,$$

with the Lyman limit frequency, $\nu_L$. In Eq. (3), we take the frequency spectral index of $-1.5$ similar to that in the case of galactic black holes [37].

B. IGM temperature and ionization profiles around a PBH

Now we consider the profiles of the ionization fraction and gas temperature around a PBH with $L_{PBH}$. For simplicity, we only consider hydrogen as the IGM gas component.

The evolutions of the ionization fraction, $x_e$, and temperature, $T_{gas}$, in the IGM are given by the following equations,

$$\frac{dx_e}{dt} = k_{HI,\gamma} - \alpha_B n_H x_e^2,$$  \hspace{1cm} (5)

$$\frac{dT_{gas}}{dt} = \left(\gamma - 1\right) \frac{\mu m_p}{k_B \rho} \left( k_B T_{gas} \frac{d\rho}{dt} + \Gamma - \Lambda \right),$$  \hspace{1cm} (6)

where $n_H, k_B, \gamma, \mu, m_p, \rho$ and $\alpha_B$ are the number density of hydrogen nucleus, the Boltzmann constant, the IGM gas adiabatic index, $\gamma = 5/3$, the mean molecular weight, the proton mass, the gas mass density, and the case B recombination rate given in Ref. [38]. In Eqs. (5) and (6), $k_{HI,\gamma}, \Gamma$ and $\Lambda$ are the ionization, heating and cooling rates, respectively. For simplicity, we assume that the IGM gas is homogeneous and we set the IGM gas density to the cosmological mean (background) values at a given time $t$.

We consider only photons emitted from a PBH as the ionization and heating source. We solve Eqs. (5) and (6) with the spherically symmetric assumption. Since we evaluate the ionization and temperature evolution for the cosmological timescale, it is useful to introduce
the comoving radial distance from a PBH for representing the spatial position, instead of the physical distance. The ionization and heating rates at the comoving radial distance \( r \) can be written as

\[
k_{\text{HI},\gamma}(r) = (1 - x_e(r)) \int_{v_L}^{\infty} \mathcal{F}_\nu(r) \frac{\sigma_{\text{HI},\nu}}{h\nu} \, dv,
\]

\[
\Gamma(r) = n_{\text{HI}}(r) \int_{v_L}^{\infty} \mathcal{F}_\nu(r) \frac{(\nu - v_L)}{\nu} \sigma_{\text{HI},\nu} \, dv,
\]

where \( h \), \( n_{\text{HI}}(r) \) and \( \sigma_{\text{HI},\nu} \) are the Planck constant, the neutral hydrogen number density given in \( n_{\text{HI}}(r) = (1 - x_e(r)) n_{\text{H}} \), and the absorbed cross section area of ionization photons, \( \sigma_{\text{HI},\nu} = 6.3 \times 10^{-18} (\nu / v_L)^{-3} \text{cm}^2 \). In the above equations, \( \mathcal{F}_\nu(r) \) represents the photon energy flux for a frequency \( \nu \) at a comoving distance \( r \),

\[
\mathcal{F}_\nu(r) = \frac{L_{\text{PBH},\nu}}{4\pi a^2(t)^2} e^{-\tau_{\text{HI},\nu}(r)}.
\]

Here \( a(t) \) is the scale factor normalized as \( a(t_0) = 1 \) at the present epoch, \( t_0 \), and \( \tau_{\text{HI},\nu}(r) \) is the optical depth of HI gas from the central PBH to the comoving distance \( r \),

\[
\tau_{\text{HI},\nu}(r) = \int_0^r a(t) n_{\text{HI}}(r') \sigma_{\text{HI},\nu} \, dr'.
\]

Solving Eqs. (10) and (11) consistently by a spherically symmetric radiative transfer code, we obtain the evolutions of \( x_e \) and \( T_{\text{gas}} \) at each comoving distance \( r \) and cosmic time \( t \).

FIG. 1 shows the radial profile of the ionization fraction as a function of the physical distance, \( R = a(t)r \), from a PBH. In the figure, we set \( M = 10M_\odot \) and \( \epsilon = 0.0001 \) in this figure. The difference of colors represents different redshifts. As the Universe evolves, the ionized region extends outward. This expansion is proportional to \((1 + z)^{-2}\). This behavior is consistent with the redshift evolution of the Strömgren radius with the constant photon flux from a PBH. The Strömgren radius \( R_s \) (in physical) can be related to the number rate of photons emitted from a PBH, \( N_{\text{PBH},\gamma} \), and has the redshift dependence as in

\[
R_s = \left( \frac{3N_{\text{PBH},\gamma}}{4\pi n_e n_p \alpha_B} \right)^{\frac{1}{3}} \propto (1 + z)^{-2},
\]

where \( N_{\text{PBH},\gamma} \) is obtained from

\[
N_{\text{PBH},\gamma} = \int_{v_L}^{\infty} dv \frac{L_{\text{PBH},\nu}}{h\nu}.
\]

Since we assume that the photon emission from a PBH does not have the redshift evolution, the redshift dependence in Eq. (11) comes from the (physical) number density of electrons and protons, \( n_e, n_p \propto (1 + z)^3 \).

FIG. 2 provides the redshift evolution of the IGM gas temperature. We plot the radial profile of the temperature as a function of the physical radial distance from a PBH. The color difference shows the difference of the redshifts. Similarly to the ionization fraction in FIG. 1, the lower the redshift becomes, the more the volume of the heated region increases. In the fully ionized central region, the temperature profile is flat and the redshift dependence of the amplitude is very weak. Since the heating mechanism is the photoionization, the heating becomes efficient in the neutral region. Therefore, the peak of the gas temperature locates at the spatial position where the ionization fraction becomes about \( x_e \approx 0.1 \). Thus, the heated region extends more than the ionized region. In the sufficiently distant region, the gas temperature becomes the background IGM temperature which is represented in the thin dashed lines in FIG. 2.

At the end of this subsection, we comment on the dependence of the ionization fraction and gas temperature on the PBH luminosity, \( L_{\text{PBH}} \propto \epsilon M \). According to Eq. (11) with Eq. (12), the Strömgren radius depends on \( (\epsilon M)^{1/3} \). As shown in FIGs. 1 and 2, the profiles of \( x_e \) and \( T_{\text{gas}} \) is sensitive to the evolution of the Strömgren radius. Therefore, as \( \epsilon \) or \( M \) increases, the ionization front expands towards outside roughly proportional to \( (\epsilon M)^{1/3} \). However, since the temperature is determined by the balance between the heating and cooling, the amplitude of the temperature at the fully ionized region does not have the simple dependence on \( \epsilon M \).

C. Compton \( y \)-parameter induced by a PBH

FIGs. 1 and 2 show that a PBH can make a hot gas plasma around itself. Passing through this plasma, CMB photons suffer from the SZ effect. As a result, the observed brightness temperature of the CMB is shifted by \( \Delta T_{\nu} \) at a frequency \( \nu \) from the background CMB temperature, \( T_{\text{CMB}} \). At the comoving distance \( b \) from the PBH on the sky, this observed shift of the brightness tem-
temperature can be written with the Compton \( y \)-parameter,
\[
\frac{\Delta T_\text{gas}(b)}{T_\text{CMB}} = g(\nu) y(b),
\]
where \( g(\nu) \) is the frequency spectral function of the SZ effect,
\[
g(\nu) = \frac{h \nu}{k_B T_\text{gas}} \tanh^{-1} \left( \frac{h \nu}{2 k_B T_\text{gas}} \right).
\]

The Compton \( y \)-parameter at the comoving distance \( b \) on the sky, \( y(b) \), can be obtained from the integral form along a line of sight with the impact parameter \( b \) from the PBH,
\[
y(b) =\int dx \frac{\sigma_T n_\text{H} x_e(\ell)}{m_e c} k_B T_\text{gas}(\ell),
\]
where \( \sigma_T \) is the Thomson cross section, \( m_e \) is the electron mass and \( c \) is the speed of light. In Eq. (15), \( x \) is the comoving distance projected on the line of sight direction and \( \ell \) is the comoving radial distance from the PBH satisfying, \( \ell^2 = b^2 + x^2 \).

Using Eq. (15) with the profiles of the ionization fraction and temperature shown in FIGs. 1 and 2 we can compute the \( y \)-parameter. We plot the \( y \)-parameter due to a PBH with \( M = 10M_\odot \) and \( \epsilon = 0.0001 \) as a function of the physical distance, \( R_\text{b} = a(t) b \), for different redshifts in FIG. 3. As expected, the \( y \)-parameter profile depends on the ionization fraction profile. Therefore, when \( R_\text{b} \) becomes larger than the Strömgren radius, the \( y \)-parameter quickly falls down. Following the evolution of the Strömgren radius, as the Universe evolves, the flat region in the \( y \)-parameter profile extends outward. In Eq. (14), most of the contribution to the integration come from the region inside the Strömgren radius (the fully ionized region).

The amplitude of \( y \)-parameter decreases as the Universe evolves because the redshift dependence of the electron number density dominates that of the integral distance and gas temperature.

We also comment on the dependence of the \( y \)-parameter profile on \( \epsilon \) and \( M \). As mentioned above, the \( y \)-parameter profile strongly depends on the Strömgren radius. Since the Strömgren radius is proportional to \( (\epsilon M)^{1/3} \), the parameters, \( \epsilon \), and \( M \), can affect the \( y \)-parameter profile. As \( \epsilon \) or \( M \) becomes large, the amplitude of the \( y \)-parameter increases and the tail of the profile moves toward large \( R_\text{b} \).

III. THERMAL SUNYAEV-ZEL’DOVICH ANISOTROPY DUE TO PBHS

As shown in the previous section, a hot plasma is generated around a PBH and can cause the SZ effect. If PBHs account for a significant fraction of the dark matter abundance, they can produce the observable CMB temperature anisotropy through the SZ effect. In this section, we evaluate the angular power spectrum of the CMB temperature fluctuations by the SZ effect. We also discuss the constraint on the PBH abundance from the comparison with the small-scale CMB measurement, SPT.

A. The CMB temperature angular power spectrum by the SZ effect

To calculate the angular power spectrum of the CMB temperature due to the SZ effect caused by PBHs, we take a similar method to the one for galaxy clusters based on the halo formalism [39, 40]. Accordingly, the angular power spectrum can be described as the sum of the two.


\[ C_l^{\text{TT}} = g^2(\nu) \left( C_l^{\text{y}(1P)} + C_l^{\text{y}(2P)} \right), \]

(16)

where \( C_l^{\text{y}(1P)} \) is the “one-PBH” term describing the Poisson contribution and \( C_l^{\text{y}(2P)} \) is the “two-PBH” term arising from clustering of PBHs. Assuming that the PBH mass function is restricted to a single mass \( M \), we can write these two terms as

\[ C_l^{\text{y}(1P)} = \int_{z_i}^{z_{\text{ini}}} d\nu \frac{d^2V}{dz d\Omega} n_{\text{PBH}} |y(\nu)|^2, \]

(17)

\[ C_l^{\text{y}(2P)} = \int_{z_i}^{z_{\text{ini}}} d\nu \frac{d^2V}{dz d\Omega} P \left( \frac{\nu}{|d(\nu)|} \right) n_{\text{PBH}} |y(\nu)|^2, \]

(18)

where \( V \) is the comoving volume, \( d(z) \) is the comoving distance to the redshift \( z \), \( P(k) \) is the matter power spectrum and \( n_{\text{PBH}} \) is the comoving number density of PBHs with mass \( M \). Here we assume that PBHs contribute to the DM abundance with the fraction \( f_{\text{PBH}} \). Therefore, \( n_{\text{PBH}} \) can be written as \( n_{\text{PBH}} = f_{\text{PBH}} \Omega_{\text{DM}} \rho_{\text{crit}} / M \) with the present critical density of the Universe, \( \rho_{\text{crit}} \). We integrate these equations from the redshift \( z_{\text{ini}} \) to \( z_f \) for whose values we will add small discussion later.

In Eqs. (17) and (18), taking the small angle approximation, we get \( y_l \) as the 2-dimensional Fourier transform of the Compton \( y \)-parameter for PBH mass \( M \) at the redshift \( z \) obtained in Sec. 11C:

\[ y_l(\nu) = \int d^2\theta \ y_l(b) \exp(-i \theta \cdot l), \]

(19)

where \( l \) is a vector describing the 2-dimensional Fourier mode with \( l = |l| \), the comoving distance \( b \) on the sky is given in \( b = |\theta| d(z) \) and \( \theta \) is the angular direction on the sky sphere.

Calculating Eqs. (17) and (18) with the profile of \( y(b) \), we can obtain the CMB temperature anisotropy induced by the SZ effect due to PBHs. In FIG. 4, we plot the obtained angular power spectra of the CMB temperature anisotropy for the different PBH parameter sets (\( M \), \( \epsilon \), \( f_{\text{PBH}} \)). Our PBH parameter sets are summarized in Table I. For the comparison, we show the primordial CMB temperature anisotropy in the black dashed-dotted line.

The shape of the spectrum is independent on the PBH parameters. We find out that the contribution from the two-PBH term dominates the one-PBH term. Since the typical scale of the \( y \)-parameter profile in FIG. 3 is the Strömgren radius, \( R_s \), which is much smaller than the CMB observation scales, \( y_l \) is constant on the CMB observation scales. Therefore, according to Eq. (18), the shape of the angular spectrum is determined by the matter power spectrum. In fact, independently on the PBH parameters, FIG. 4 shows that the spectrum has a flat shape on larger multipoles than \( l \sim 2000 \). Because of this flat shape, the SZ temperature anisotropy due to PBHs can dominate the primordial temperature anisotropy on smaller scales than the Silk scale.

\[
\begin{array}{|c|c|c|}
\hline
M [M_\odot] & \epsilon & f_{\text{PBH}} \\
\hline
(A) & 10 & 1.0 \times 10^{-4} & 7.6 \times 10^{-2} \\
(B) & 100 & 1.0 \times 10^{-5} & 7.6 \times 10^{-2} \\
(C) & 1000 & 1.0 \times 10^{-6} & 7.6 \times 10^{-2} \\
(D) & 10 & 1.0 \times 10^{-4} & 7.6 \times 10^{-4} \\
(E) & 100 & 1.0 \times 10^{-2} & 7.6 \times 10^{-4} \\
(F) & 1000 & 1.0 \times 10^{-2} & 7.6 \times 10^{-4} \\
\hline
\end{array}
\]

TABLE I. The PBH parameter sets for the CMB anisotropy due to PBHs in FIG. 3.

On the other hand, the amplitude depends on the PBH parameters. Because of the integration range in Eq. (15), \( y(b) \) is proportional to \( R_s \). FIG. 3 tells us that the typical scale of non-negligible \( y(b) \) is also \( R_s \). Therefore, according to Eq. (19), the angular Fourier component of the \( y \)-parameter, \( y_l \), is proportional to \( R_s^2 \propto \epsilon M \). The PBH number density is \( n_{\text{PBH}} \propto f_{\text{PBH}} / M \). As a result, we can see that the amplitude of \( C_l \) has the dependence given in

\[ C_l \propto |n_{\text{PBH}} y_l|^2 \propto (\epsilon f_{\text{PBH}})^2. \]

(20)

FIG. 4 clearly shows this dependence of \( C_l \) to us. Because of the degeneracy between \( \epsilon \) and \( f_{\text{PBH}} \), the angular spectrum for the case (C) is coincident with that for the case (D).

We also investigate the redshift contribution to the angular spectrum. The observed temperature anisotropy is a resultant effect which is integrated along the line of sight direction. As already mentioned, \( y_l \) is proportional to \( R_s^4 \) which becomes large as the Universe evolves. In particular, we find out that the contribution from the redshift higher than \( z = 50 \) is less than 1 percent. On the other hand, in the low redshift side, the Universe is gradually reionized and finally fully ionized until \( z \sim 7 \). During the epoch of reionization, the regions ionized by PBHs are caught up in the reionization process by stars and galaxies. As a result, in lower redshifts around the epoch of reionization, the SZ effect due to PBHs becomes fainter as the reionization proceeds. Here, for simplicity, we take into account the PBH SZ effect between \( z_{\text{ini}} = 200 \) and \( z_f = 10 \) in the integration of equations (17) and (18).

### B. Application to the PBH constraint

Now we discuss the possible constraint on the PBH abundance from the CMB SZ power spectrum measurement. When we fix the PBH mass \( M \) and the emission efficiency \( \epsilon \), the amplitude of the SZ spectrum is determined by the PBH abundance \( f_{\text{PBH}} \) in our model.

Since the SZ effect due to PBHs can induce the CMB temperature anisotropy on small scales, the small-scale CMB measurement provides the constraint on the PBH abundance. In FIG. 4 we plot the SPT data with the
FIG. 4. The angular power spectrum of the CMB temperature caused by the SZ effects from PBHs. The parameter sets for the case (A)-(F) are as shown in TABLE II. The black dashed-dotted line is the primordial power spectrum of the CMB temperature and the black circles with error bars represent the SPT data.

We found that the constraint from the SZ angular power spectrum is roughly independent on the PBH mass and given in

$$f_{\text{PBH}} < 10^{-3} \left( \frac{\epsilon}{10^{-2}} \right),$$  \hspace{1cm} (21)

for $10^{-2} M_\odot < M < 10^3 M_\odot$. However, as the PBH luminosity (the PBH mass or the emission efficiency $\epsilon$) decreases, the constraint becomes weak. In the case of the low luminosity, the ionization rate is less efficient than the recombination rate and the ionized region becomes smaller than the Str"omgren radius. As a result, the both SZ signal and the resultant constraint decline.

On the small PBH mass side for $M < 1 M_\odot$, the microlensing surveys provide the robust constraint on the PBH abundance. Our constraint highly depends on $\epsilon$. We found that our constraint for $\epsilon > 10^{-3}$ gives tighter constraint than the microlensing one. On the other hand, the CMB measurement of the Thomson optical depth gives the strong constraint on the PBH abundance with a large mass. Similarly to our constraint, the constraint from the Thomson optical depth highly depends on the PBH luminosity. Considering the dependence of the constraint on the PBH luminosity, we conclude that the constraint from the Thomson optical depth is always stronger than our constraint in the mass range $M > 1 M_\odot$, when the PBH luminosity is fixed.

IV. CONCLUSION

In this paper, we have investigated the CMB temperature angular power spectrum due to the SZ effect caused by PBHs. The gas accreting on a PBH heats up because of the release of the gravitational energy. As a result, the heated gas in the vicinity of the PBH emits the UV and X-ray photons. These photons can ionize and heat the IGM around the PBH. The ionized hot IGM causes the secondary CMB temperature anisotropy through the SZ effect.

First assuming the luminosity of photons emitted in the vicinity of a PBH with a free parameter, we have evaluated the profiles of the IGM ionization fraction and temperature around a PBH by solving the 1-dimensional radiative transfer equations. Based on these profiles, we have obtained the SZ Compton $y$-parameter profile around a PBH. Following the halo formalism, finally, we have calculated the CMB temperature angular power spectrum due to the PBH SZ effect with assuming the PBH abundance.

We have shown that the SZ spectrum due to PBHs could dominate the primordial temperature spectrum on smaller scales than the Silk scale. We have found that the shape of the angular power spectrum depends on the matter power spectrum. On the other hand, the amplitude of the spectrum is sensitive to the PBH abundance, mass and the emission efficiency of the gas accreting on PBHs. Therefore, the current small-scale CMB measurement can provide the constraint on the PBH abundance with fixing the PBH mass and the emission efficiency. We have compared our theoretical prediction with the SPT data. Our obtained constraint on the PBH fraction to
dark matter is $f_{\text{PBH}} \lesssim 10^{-3}(\epsilon/10^{-2})$ for the PBH mass range, $10^{-2}M_\odot < M < 10^3M_\odot$. In the case of $\epsilon > 10^{-3}$, our constraint can be stronger than the current constraint from the micro-lensing survey in the above mass range. However, the CMB constraint from the optical depth can provide the much tighter constraint on the PBH with mass, $M > 1M_\odot$. Therefore we can conclude that the SZ measurement gives a new constraint on the PBH abundance for mass $10^{-3}M_\odot < M < 1M_\odot$ in the case of the sub-Eddington luminosity, $\epsilon \gtrsim 10^{-3}$.

Although the constraint from the SZ angular power spectrum is weaker than that from the optical depth for larger mass than $M > 1M_\odot$, it is worth mentioning about the impact of the future small-scale SZ measurements on the probe for the existence of PBHs. We have shown that the CMB temperature spectrum of the SZ effect due to PBHs have the flat spectrum on smaller scales than $\ell \sim 2000$. This flat spectrum extends to the scale of the ionized region by the PBH emission, e.g., roughly 1 kpc in physical scale for $M = 10 M_\odot$ with $\epsilon = 10^{-4}$. The SZ effect in galaxy clusters can produce large CMB anisotropies on small scales. However, its spectrum has the peak around $\ell \sim 4000$ and the amplitude decays on higher $\ell$. Therefore, the detection or non-detection of the flat SZ spectrum on higher $\ell$ than $\ell = 4000$ gives useful information about the existence of PBHs.

In this work, we have assumed the luminosity from the hot gas in the vicinity of a PBH with introducing a free parameter for the emission efficiency. Refs. [28, 29] have studied the emission efficiency with the simple assumptions analytically. However, to obtain the emission efficiency consistently, we need to solve the dynamics of gas accreting on a PBH with considering the backreaction of the luminosity from hot gas. In the next study, we address this issue by numerical simulations and investigate the cosmological effects of the emission from gas accreting on a PBH in more details.

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