Feature Extraction Method of Ball Mill Load Based on Adaptive Variational Mode Decomposition and Improved Power Spectrum Analysis

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Abstract. The real-time working conditions of the ball mill in the grinding process are complicated, which makes it difficult to accurately obtain the internal load status of the ball mill. In this paper, the energy difference between the original cylinder vibration signal and the intrinsic mode function is proposed as the evaluation parameter of the adaptive variational mode decomposition (VMD) layer number, and a new autocorrelation function is constructed. The intrinsic mode function is processed by introducing the energy centrobaric method of the Nuttall self-convolution window. Accordingly, a ball mill load feature extraction method based on adaptive VMD and improved power spectrum estimation is proposed, and the ball mill load identification system based on LabVIEW is developed. The number of layers of intrinsic mode function could be adaptively determined. And the algorithm's ability to resist mode aliasing and false components of this method is improved, which improves the accuracy of ball mill load detection. The measured results show that internal load features of ball mill during the grinding process are effectively extracted, and the mill load status is accurately identified, which provide an accurate and reliable basis for grinding optimization control and efficiency.

1. Introduction
Ball mill is the most important core production equipment in grinding, whose efficiency directly affects the production of the entire concentration plant [1]. Ball mill load refers to the total materials inside the cylinder, including ore, grinding media, water, mineral pulp, etc. Understanding the load status accurately is an important basis for the optimal control of the ball mill. However, in actual production, ball mill is time-varying, non-linear, and randomly interfered, making it hard to accurately acquire its load features [2].
In recent years, researchers at home and abroad have put forward various methods and advanced technologies in order to accurately obtain the load status of a ball mill. The Commonwealth Scientific and Industrial Research Organisation (CSIRO) proposed an analysis method based on the ball mill vibration signals, which enjoys great sensitivity and anti-interference ability, and has more direct measurement compared with the traditional power method, differential pressure method and noise method [3, 4]. Reference [5] built a soft-sensing model of ball mill load after extracting the features of the ball mill vibration signal. Reference [6] divided the vibration spectrum to extract the features of the material level of the ball mill. Reference [7] proposed a spectral clustering method based on the prior knowledge and the grinding mechanism, automatically dividing the vibration spectrum into sub-bands with different physical meanings. Unfortunately, none of these methods effectively decomposed the vibration signals from the perspective of generation mechanism, but the empirical mode decomposition (EMD) is able to adaptively decompose the non-stationary signals into intrinsic mode function (IMF) based on the local features of the signals. Reference [8] adopted EMD and Ensemble-EMD (EEMD) to adaptively decompose the vibration signals, and then extract the load features. But, both EMD and EEMD are recursive, with the problems of mode aliasing and endpoint effects, leading to non-obvious load features extracted from the mode components and low load status identification rate.

This paper studies the accurate extraction of ball mill load features during the grinding process. Based on the energy difference between the original cylinder vibration signals and the IMFs, the paper adopts adaptive VMD to decompose the vibration signal into $k$ IMFs and constructs a new autocorrelation function. It introduces the energy centrobaric method of the Nuttall self-convolution window to extract the frequency corresponding to the maximum value of each IMF power spectrum layer as the ball mill load feature. Accordingly, a feature extraction method of ball mill load based on adaptive VMD and improved power spectrum estimation is proposed, a LabVIEW-based load status identification system is also developed, and the on-site data measured in the industry is utilized to verify the accuracy and effectiveness of the proposed method.

2. Adaptive VMD

2.1. Brief Introduction of VMD

Variational Mode Decomposition (VMD) [9] is a new adaptive and quasi-orthogonal signal processing method based on Wiener filter, one-dimensional Hilbert transform and mixing. It can effectively overcome mode aliasing and endpoint effects compared with EMD and EEMD.

The essence of VMD is to construct and solve variational problems [10, 11]. The corresponding constrained variational model of original signal $f(t)$ is

$$\min_{\{u_k\}, \{\omega_k\}} \left\{ \sum_{k=1}^{K} \left\| \delta(t) * u_k(t) \right\|^{2} \right\}$$

where $\delta(t)$ represents the impulse function, ‘*’ is the convolution symbol, $\{u_k\} = \{u_1, u_2, \ldots, u_k\}$ means eigenmode function quantity $k$, and $\{\omega_k\} = \{\omega_1, \omega_2, \ldots, \omega_k\}$ refers to the center frequency of each mode.

The binding condition will be changed into a non-binding variational problem after the Lagrange multiplier operator $\lambda$ and the penalty factor $\alpha$ are introduced [12]. The extended Lagrangian formula is

$$L(\{u_k\}, \{\omega_k\}, \lambda) = \alpha \sum_{k=1}^{K} \left\| \left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right\|^{2} + \left\| f(t) - \sum_{k=1}^{K} u_k(t) \right\|^{2} + \left( \lambda(t), f(t) - \sum_{k=1}^{K} u_k(t) \right)$$

The alternating direction method of multipliers is used to iteratively solve the saddle point of formula (2). The mode component $u_k$ and the center frequency $\omega_k$ are respectively
After solving the mode function and center frequency, the Lagrange multiplier operator \( \lambda \) is updated through the following formula:

\[
\hat{\lambda}^{n+1} = \hat{\lambda}^n + \sigma \left( f(\omega) - \sum_k \hat{u}_k^n(\omega) \right)
\]

where \( \sigma \) is an update factor.

The update and iteration will continue until the following convergence conditions are met:

\[
\sum_k \| \hat{u}_k^n - \hat{u}_k^m \|_2^2 / \| \hat{u}_k^n \|_2^2 < \epsilon
\]

where \( \epsilon \) is the tolerance of the convergence criterion using the default value of standard VMD. The process of VMD is completed after the above calculation, obtaining \( k \) eigenmode components \( \{u_k\} \).

2.2. Adaptive VMD mode quantity \( k \)

The number of eigenmode components decomposed needs to be set in advance when the VMD algorithm is applied. A too-small number will cause information loss while a too-large one will lead to frequency aliasing [13].

This paper proposes an energy difference-based adaptive VMD algorithm considering the above factors. The energy calculation formula of the original signals or each mode component signals is as follows:

\[
E = \frac{1}{N} \sum_{i=1}^{N} u^2(i)
\]

where \( E \) represents the energy value of the original signals or mode component signals; \( u(i) \) the signal sequence; and \( N \) the sequence length. The sum of energy values with different mode decomposition layers is varied. The energy difference parameter defining the energy sum of mode components and the original signal energy is as follows:

\[
\rho = \frac{\sum_{j=1}^{k} E_j - E_s}{E_s}
\]

where \( E_j \) is the energy of the \( j \)-th mode component and \( E_s \) is the energy of the original signal.

The energy difference parameter \( \rho \) is used to determine the decomposition quantity. \( \rho \) increases as the number of decomposition mode goes up. Parameter \( \rho \) has a small rise before the mode over-decomposition and increases significantly after the over-decomposition.

The VMD mode quantity is determined based on the above principle. The energy difference parameter \( \rho \) is calculated layer by layer from \( k=2 \) until \( k \) reaches the preset maximum number of decomposition layers \( K \) and the change of \( \rho \) on each layer is observed. Then the corresponding \( k \) before \( \rho \) increases significantly is selected as the layer number of VMD.
3. VMD and improved power spectrum feature extraction

The vibration signals of the ball mill cylinder are random signals that change with time. The distribution of random signals containing a large number of sample sets must be studied as a whole. This paper uses the improved power spectrum estimation to obtain the power spectrum of each mode component of the vibration signal, hence the features.

3.1. Improved autocorrelation function

The autocorrelation function should be first obtained for the power spectral density of random signals [14]. The autocorrelation function of the discrete signal \( u(i) \) is

\[
r(\tau) = \frac{1}{N} \sum_{i=1}^{N} u(i)u(i+\tau)
\]

(9)

where \( \tau \) is the time delay. Formula (9) shows that the sum of items of \( r(\tau) \) gradually decreases as the time delay \( \tau \) increases, resulting in the attenuation of waveform amplitude, which makes it difficult to detect the peak point. To overcome the shortcoming, an improved autocorrelation function is proposed, unchanging the sum of terms and improving the amplitude attenuation during the operation. The improved processes are:

1) The length of the autocorrelation function is set as \( N \);
2) The sequence length of the discrete signal \( u(i) \) is extended from \( N \) to \( 2N \);
3) When \( i > N \), \( u(i) = u(i-N) \);
4) Formula (9) is used to calculate the autocorrelation function.

3.2. The energy centrobaric method of the Nuttall self-convolution window

The energy centrobaric method corrects the frequency considering that the energy center of gravity of the window discrete spectrum is infinitely close to the coordinate origin. The correction formula of the harmonic signal frequency \( f_k \) of the energy centrobaric method is

\[
f_k = x_0 \cdot \frac{f_s}{N} \frac{\sum_{i=-n}^{n} P(k_0+i) \cdot (k_0+i) \cdot f_i}{\sum_{i=-n}^{n} P(k_0+i)}
\]

(10)

where \( x_0 \) is the spectral line of the corrected power spectrum peak, \( f_s \) is the sampling frequency, \( N \) is the number of sampling points, the peak spectral line of the harmonic signal obtained by the discrete Fourier transform is \( k_0 \), and \( P(k_0+i) \) is the power value of the windowed power spectrum of the \( i \)th spectral line.

The spectral analysis accuracy of the energy centrobaric method is easily affected by the window function [15, 16]. Comparing the performance of the existing window function, this paper constructs a Nuttall self-convolution window with a lower sidelobe peak sidelobe and a faster attenuation rate. The energy centrobaric method of Nuttall self-convolution window is proposed to improve the analysis accuracy.

The discrete function of the Nuttall window is

\[
w(n) = \sum_{m=0}^{M-1} (-1)^m b_m \cos\left(2\pi n \cdot m / N \right)
\]

(11)

where \( n=0,1,2,\ldots,N-1; \ M \) is the term number of the Nuttall window function; \( b_m \) is the window coefficient, which satisfies the constraint condition of \( \sum_{m=0}^{M-1} (-1)^m b_m = 0 \). The sidelobe performance of Nuttall windows with varying coefficients is different, as shown in Table 1. Considering the sidelobe peak sidelobe and attenuation rate comprehensively, this paper adopts 4-term third derivative Nuttall windows.

So, the spectral mode function \( W_S(\omega) \) is
\[
W_N(\omega) = \sum_{m=0}^{M-1} (-1)^m \frac{b_m}{2} \left[ W_r \left( \omega - \frac{2\pi m}{N} \right) + W_r \left( \omega + \frac{2\pi m}{N} \right) \right]
\]

where \( W_r \) is the spectral function of the rectangular window.

### Table 1 The sidelobe performance of Nuttall

| Window Type of Nuttall | peak sidelobe (dB) | attenuation rate (dB/oct) |
|------------------------|-------------------|--------------------------|
| 3-term first derivative | -64.19            | 6                        |
| Minimum 3-term         | -71.49            | 6                        |
| 4-term first derivative | -93.3             | 18                       |
| 4-term third derivative | -82.6             | 30                       |
| Minimum 4-term         | -98.2             | 6                        |

With the definition of the cosine self-convolution window, the time-domain function of the Nuttall self-convolution window is

\[
w_{N-2}(n) = [w(n)]^2
\]

According to the convolution theorem, the convolution in the time domain of the function is equivalent to the product in the frequency domain. The spectral mode function of the Nuttall self-convolution window is

\[
W_{N-2}(\omega) = \left[ W_N(\omega) \right]^2
\]

When \( P_{N:2}(\omega) \) is the power spectrum function of Nuttall, then its window spectrum energy function is

\[
P_{N:2}(\omega) = W_{N-2}^2(\omega) = \left\{ \sum_{m=0}^{M-1} \frac{b_m}{2} \left[ W_r \left( \omega - \frac{2\pi m}{N} \right) + W_r \left( \omega + \frac{2\pi m}{N} \right) \right] \right\}^4
\]

Through mathematical derivation, the Nuttall self-convolution window meets the application criteria of the energy centrobaric method, i.e., the energy center of gravity of the discrete window spectrum is near the coordinate origin.

In practical applications, the value of \( n \) cannot be infinite. The value in the main lobe of the window spectrum can be selected for correction, so the correction accuracy is not a theoretical solution, but an approximate solution related to the type of window function and the point number \( n \). When the signal sequence length \( N \) is fixed, the main lobe of the self-convolution window is wider.

In order to utilize the larger spectral lines in the main lobe and reduce the effect of the side lobes on the results, this paper uses the spectral line \( n=7 \) for spectrum correction. Through function (10), the correction formula of the wave signal frequency \( f_k \) for the energy center of gravity of the Nuttall self-convolution window is

\[
f_k = \frac{\sum_{i=1}^{7} P_{N:2}(k_0+i)(k_0+i) \ f_k}{N}
\]

To compare the spectrum correction accuracy of the energy centrobaric method of Nuttall self-convolution window, the paper adopts a single-frequency sinusoidal signal with the sampling frequency \( f_s=1024\text{Hz} \) and the data length \( N=4096 \). Besides, 51 frequency points of frequency \( f \) are selected at equal intervals in the resolution range of \([49.5\text{Hz}, 50.5\text{Hz}]\). The normalized frequency correction errors under different frequency deviations are calculated for the energy centrobaric method of each common cosine combination window and of Nuttall self-convolution window respectively. The simulation results are shown in Figure.1.
Figure 1 presents that compared with the energy centrobaric methods of cosine combined windows such as Hanning window and Blackman window, the frequency correction of Nuttall self-convolution window is significantly more accurate. When the frequency deviation increases, the correction results of Hanning window and Nuttall window are affected, while Nuttall self-convolution window remains highly accurate.

3.3. Load feature extraction based on adaptive VMD and improved power spectrum

The specific steps of the ball mill load feature extraction method based on adaptive VMD and improved power spectrum estimation are as follows:

1) Obtain the vibration signals of the ball mill cylinder, initialize the VMD number to \( k = 2 \), set the maximum decomposition mode number \( K \), and set other two parameters of the VMD, namely the penalty factor \( \alpha \) and the update factor \( \sigma \) to the default values, and calculate the energy value \( E_s \) of the vibration signals according to formula (7);

2) Perform the VMD of ball mill vibration signals, calculate the sum of the energy value of each mode under the decomposition mode number according to formula (7), and calculate whether the energy difference parameter \( \rho \) significantly deviates. If there is no obvious deviation, then \( k = k + 1 \) and \( \rho \) calculation should continue; if there is a significant deviation or the number of decomposition levels reaches the maximum mode number \( K \) that has been preset, then the number of decomposition modes is determined as \( k = K - 1 \) (when it reaches the maximum decomposition mode number, the number is \( K - 1 \));

3) Decompose the vibration signals of the ball mill cylinder into \( k \) eigenmode functions \( \{u_i\}, i = 1, 2, \ldots, k \) with a sequence length \( N \), extend the sequence length \( \{u_i\} \) to \( 2N \), calculate the improved autocorrelation function \( \{r_i(\tau)\} \) based on function (9);

4) Conduct Fourier transform of the Nuttall self-convolution window with a window length \( N \) for \( \{r_i(\tau)\} \) to get the power spectrum:

\[
P_{N-2}(\omega) = \int_{-\infty}^{\infty} r_i(\tau) W_{N-2}(\omega) d\omega
\]

5) Determine the maximum spectral line \( \{k_{0i}^j\} \) in the power spectrum, and calculate the center frequency of the main lobe corrected by the energy centrobaric method through formula (17).

4. Experiment and analysis

To prove the accuracy and effectiveness of the load feature extraction method proposed in this paper, the overflow ball mill (MQYΦ5.5 × 8.5m) in the grinding and floating workshop of the Jiaojia Gold
Mine Concentrator of Shandong Gold Group was used for the experiment. An acceleration sensor on the ball mill cylinder was installed with the frequency $f_s$ set to 18750Hz; the vibration signals of the ball mill cylinder under three load status (underload, normal load, and overload) were collected. The sequence length of each vibration signal was $N = 82000$. A total of 150 groups of experimental signal samples were extracted on three load status, with 50 from each status. The time-domain waveforms of the three statuses are shown in Figure 2.

Figure 2 shows that the waveforms of the vibration signals under different load statuses are very similar to a spindle shape with middle large and ends small, and the peak positions are basically the same. The high peak in the middle is because that the acceleration sensor is on the bottom lining of the ball mill, and objects such as steel balls inside are dropped on the lining, resulting in greater acceleration. Overall, the regularity is consistent, but the time domain waveform of the vibration signals is severely disturbed by noises. It is difficult to judge the internal load status only by the time domain features. In this paper, the vibration signals are processed through the ball mill load feature extraction method. VMD is adopted to decompose the original vibration signals, and the number of decomposition modes is initialized as $k=2$. Considering a large amount of data, if the maximum mode number $K$ is set too large, it will result in low efficiency and too much calculation. Therefore, the maximum number $K$ is set to 12 in this paper to calculate the total energy of the original vibration signals and the energy sum of each mode signal under different decomposition mode numbers, and the corresponding energy difference parameter $\rho$. The specific results are shown in Table 2.

| Decomposition mode quantity $k$ | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|-------------------------------|----|----|----|----|----|----|----|----|----|----|----|
| Energy difference parameter $\rho$ | 0.010 | 0.023 | 0.036 | 0.045 | 0.065 | 0.081 | 0.095 | 0.103 | 0.442 | 0.523 | 0.635 |
| Points                       | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
The energy difference parameter $\rho$ fluctuates slightly on each layer when $k \leq 9$. The energy difference parameter $\rho$ changes significantly when $k=10$. Based on the method proposed in this paper, $k=9$ is the most reasonable value of the VMD number. IMFs of vibration signals are obtained by VMD under various load status, and then the improved power spectrum estimation is used to correct the frequency on each IMF to obtain the power spectrum. The IMF of vibration signals and the improved power spectrum are shown in Figure.3 and Figure.4, respectively.

Figure.3 The IMFs of vibration signal
Figure 4 The improved power spectrum of IMFs

VMD is performed on the vibration signals of three load status, and the improved power spectrum estimation of IMFs is applied to obtain the frequency \( f_i \) as the feature vector corresponding to the maximum power spectrum. The corresponding frequency under three load statuses is shown in Table 3.

Table 3 The feature vector table of 3 kinds of load status

| IMF No. | Frequency corresponding to the maximum power spectrum of IMFs in 3 load (Hz) |
|---------|--------------------------------------------------------------------------------|
|         | Underload | Normal load | Overload       |
| 1       | 224.78     | 246.30      | 240.98     |
| 2       | 799.18     | 642.19      | 563.61     |
| 3       | 1335.09    | 1065.50     | 891.21     |
| 4       | 1899.95    | 1754.98     | 1555.78    |
| 5       | 2522.80    | 2967.75     | 2216.61    |
| 6       | 3055.23    | 4094.86     | 3293.84    |
| 7       | 3664.48    | 5146.63     | 4068.97    |
| 8       | 4099.62    | 6936.56     | 6703.59    |
| 9       | 8798.56    | 8725.41     | 7973.60    |

In order to verify if the proposed method can accurately extract the load features in a ball mill, it is compared with the EMD and EEMD to decompose the extracted feature quantities (the maximum frequency of the IMF power spectrum is extracted accordingly). Feature vectors extracted through different methods are imported into a support vector machine (SVM) for testing. The 50 samples of each load status are randomly divided into two groups, one as the training set of 30 samples, and the other as the test set of 20 samples. Then the training set and test set of each status constitute a general training set and a general test set respectively. The former one has 90 samples, and the latter one has 60 samples.
The recognition results of the test samples under EMD, EEMD and the proposed method in this paper are shown in Figure.5, Figure.6 and Figure.7, respectively.

Figure.5  The SVM prediction results of EMD method

Figure.6  The SVM prediction results of EEMD method
In Figures 5-7, category 1 represents the status of underload, 2 is the normal load, and 3 the overload. The prediction accuracy rate under the proposed method based on adaptive VMD and improved the power spectrum estimation is 90%, higher than that of EMD and EEMD.

5. Identification system of ball mill load status based on LabVIEW

Based on the method proposed, an identification system of ball mill load status based on LabVIEW is established, and its overall block diagram is shown in Figure.8.

The vibration signals of the ball mill cylinder were collected by the MEMS acceleration sensor ADXL001 of Analog Devices, Inc. After processed by high-precision ADC and the microcontroller C8051F06X, the signals were wirelessly transmitted to the ground through the low-power data transmission module nRF24L01+. Since the ball mill has a large-scale rotating structure, the solar power supply is applied for the vibration signal acquisition unit. For the power supply unit, the common DC-DC converter is replaced by a low-dropout linear voltage regulator, which has fewer external components, smaller static current, and more stable power supply. After the signals transmitted by the acquisition unit is received by the ground wireless receiver, they were collected and transmitted to the host computer system through myDAQ of NI company, and the feature parameters of input signals were extracted based on adaptive VMD and improved power spectrum estimation in the host computer system, so as to realize the load status identification.
This system can display the vibration signal waveform, feature vector and corresponding load status of the ball mill in real-time. It has the functions of online analysis, data storage, user management, etc., and has a good human-computer interaction interface. Figure.9 is the identification interface system of the ball mill load status.

![Identification System of Ball Mill Load Status](image)

**Figure.9** The identification system interface of ball mill load

6. **Conclusion**
Considering the difficulties in obtaining the ball mill load status accurately during the grinding process, this paper proposes a ball mill load feature extraction method based on adaptive VMD decomposition and improved power spectrum estimation. Theoretical analysis and actual measurement results show that applying VMD to ball mill vibration signal effectively avoids the mode aliasing and endpoint effects of current EMD and EEMD methods; the energy difference method is proposed to effectively implement the optimal decomposition as for determining the decomposition mode number, which is more accurate and reliable than subjective decision-making; the autocorrelation function is improved by shift and addition and the energy centrobaric method of the Nuttall self-convolution window proposed effectively improves the frequency correction accuracy of the mode components of the vibration signal. Compared with the existing EMD and EEMD feature extraction methods, the method proposed in this paper can more accurately extract the load features of the ball mill and can effectively identify and classify the load status, providing an accurate and reliable basis for optimal control and efficiency improvement of grinding.

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