Excess entropy and central charge of the two-dimensional random-bond Potts model in the large-$Q$ limit

István A Kovács$^{1,2}$, Jean-Christian Anglès d’Auriac$^3$ and Ferenc Iglói$^{1,2}$

1 Wigner Research Centre, Institute for Solid State Physics and Optics, H-1525 Budapest, PO Box 49, Hungary
2 Institute of Theoretical Physics, Szeged University, H-6720 Szeged, Hungary
3 Institut Néel-MCBT CNRS, BP 166, F-38042 Grenoble, France
E-mail: kovacs.istvan@wigner.mta.hu and igloi.ferenc@wigner.mta.hu

Received 12 June 2014
Accepted for publication 21 July 2014
Published 16 September 2014

Online at stacks.iop.org/JSTAT/2014/P09019
doi:10.1088/1742-5468/2014/09/P09019

Abstract. We consider the random-bond Potts model in the large-$Q$ limit and calculate the excess entropy, $S_\Gamma$, of a contour, $\Gamma$, which is given by the mean number of Fortuin–Kasteleyn clusters which are crossed by $\Gamma$. In two dimensions, $S_\Gamma$ is proportional to the length of $\Gamma$, to which—at the critical point—there are universal logarithmic corrections due to corners. These are calculated by applying techniques of conformal field theory and compared with the results of large scale numerical calculations. The central charge of the model is obtained from the corner contributions to the excess entropy and independently from the finite-size correction of the free-energy as: $\lim_{Q \to \infty} c(Q)/\ln Q = 0.74(2)$, close to previous estimates calculated at finite values of $Q$.

Keywords: conformal field theory, classical phase transitions (theory), finite-size scaling, disordered systems (theory)

ArXiv ePrint: 1406.2913
1. Introduction

Entropy represents a fundamental concept in different domains of science, such as in information theory [1], in quantum systems [2] and in classical statistical mechanics. In quantum systems, the entanglement entropy turned out a very important indicator of new and exotic phases and quantum phase transitions [2–5]. Its analogue in information theory is the mutual information and in classical statistical mechanics the excess entropy. In classical systems, the excess entropy can also be defined as mutual information, where the probabilities are given as Boltzmann weights [6–8]. In these calculations, the system is divided in two (or more) parts, and the excess entropy is basically associated with the interface separating the subsystems. Due to this, the excess entropy is proportional to the surface of the interface, which is called the area law. In critical systems, however, usually there are universal corrections to the area law, which are logarithmic in the linear extent of the interface. In conformally invariant systems, such as in one-dimensional quantum and in two-dimensional classical models, the central charge of the conformal field theory can be deduced from the size-dependence of the critical entanglement entropy [2–5] and mutual information [8], respectively.

Concerning classical statistical mechanics, most of the studies described in the previous paragraph are performed on non-random systems. It is known, however, that systems with quenched disorder are also conformally invariant, provided the properties of averaged quantities (magnetization, correlation function, etc) are concerned. Therefore, it is of interest to study the scaling properties of the excess entropy in random systems, too. For this purpose, we consider the two-dimensional $Q$-state Potts model [9] in the presence of bond disorder [10]. In two dimensions (2D), the phase transition in the random-bond Potts model (RBPM) is of second order [11–13] for any value of $Q$, even in the limit $Q \to \infty$. The critical behavior of this model has been studied by different methods [14–18], in particular the critical exponents and the central charge has been calculated, mainly for $Q > 4$, in which case the phase transition in the non-random model is of first
order \cite{19}. Special attention has been paid to the model in the large-$Q$ limit, in which case the critical parameters are found to be smooth functions of $1/\ln Q$ \cite{20}. For example, from the numerical data calculated at large, but finite values of $Q$ the central charge is conjectured to be \cite{20} $\lim_{Q \to \infty} c(Q)/\ln Q \equiv c' = 1/(2 \ln 2) = 0.7213$.

Later, it has been shown \cite{21} that the model can be studied directly at the limiting value $Q \to \infty$, when in the random cluster representation \cite{22} the partition function of the model is dominated by one term, the so-called optimal graph. This means that thermal fluctuations are negligible compared to disorder fluctuations, thus the critical behavior of the system is controlled by a so-called infinite disorder fixed point \cite{26}. The optimal graph of the RBPM has been calculated by a combinatorial optimization method \cite{23}, which provides the exact value of the partition function for a given sample, i.e. for a given realization of the disorder. From the numerical data, exact values of the critical exponents are conjectured \cite{24,25} through an expected relation with the exactly known infinite disorder fixed point of the random transverse-field Ising chain \cite{27}: $x_m = (3-\sqrt{5})/4$ (bulk magnetization), $x_s = 0.5$ (surface magnetization) and $\nu = 1$ (correlation length).

In this paper, we study the scaling properties of the excess entropy in the RBPM in the large-$Q$ limit. For this we consider a subset of bonds, $\Gamma$, and calculate the corresponding excess entropy, $S_\Gamma$. If $\Gamma$ is a closed loop, separating a subsystem, $\mathcal{A}$, from the rest of the system, $\mathcal{B}$, then $S_\Gamma$ is the mutual entropy $S_\Gamma = S_A + S_B - S_{A\cup B}$. In the following section we show, that in the random cluster representation $S_\Gamma$ is simply given by the mean number of clusters in the optimal sets which are crossed by $\Gamma$. This type of problem has already been considered by two of us in the case of the non-random Potts model both for $Q = 1$, representing percolation \cite{28,29} and for general values of $Q \leq 4$ \cite{30}. Repeating the reasoning applied in these papers, we show that the dominant term of $S_\Gamma$ represents the area law to which there are logarithmic corrections at the critical point due to corners and these are calculated by conformal techniques. The analytical conformal conjectures are then confronted with the results of large scale numerical calculations for different forms of the contour. These results involve the central charge of the RBPM, for which we calculate a precise estimate.

The rest of the paper is organized as follows. The model, its solution in the random cluster representation and the calculation of the excess entropy is presented in section 2. Numerical results for the corner contribution to the excess entropy are presented in section 3. Independent estimates for the central charge of the model through analyzing the finite-size correction of the free-energy is given in section 4. Finally, section 5 contains our conclusions.

2. Random-bond Potts model in the large-$Q$ limit

We consider the $Q$-state Potts model defined by the Hamiltonian \cite{9}

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} \delta(\sigma_i, \sigma_j),$$

(1)

in terms of the Potts spin variables, $\sigma_i = 0, 1, \ldots, Q - 1$. The $J_{ij} > 0$ couplings between nearest neighbour sites are i.i.d. random variables and in the following we restrict ourselves to the square lattice. In the random cluster representation \cite{22}, the partition function of
the model at $T = 1/\beta$ temperature is given by:

$$Z = \sum_G Q^{N_{\text{tot}}(G)} \prod_{ij \in G} [e^{\beta J_{ij}} - 1],$$

(2)

where the sum runs over all bond configurations $G$, and in $G$ the total number of connected components (clusters) are denoted by $N_{\text{tot}}(G)$. The mean number of clusters is given by:

$$\langle N_{\text{tot}} \rangle = \frac{\partial \ln Z(Q)}{\partial \ln Q}.$$  

(3)

where $\langle \cdots \rangle$ denotes thermal averaging and $\cdots$ stands for the average over quenched disorder.

Let us now introduce a subset of bonds, $\Gamma$, and fix all spins on $\Gamma$ (in state 0, say) but leave the couplings unchanged. Then the partition function becomes:

$$Z_{\text{fix}} = \sum_G Q^{N_{\text{tot}}(G) - N_{\Gamma}(G)} \prod_{ij \in G} [Q^{\beta J_{ij}} - 1],$$

(4)

where $N_{\Gamma}(G)$ denotes the number of clusters which intersect $\Gamma$. Consequently:

$$\langle N_{\text{tot}} - N_{\Gamma} \rangle = \frac{\partial \ln Z_{\text{fix}}(Q)}{\partial \ln Q}.$$  

(5)

Now let us take the large-$Q$ limit, in which case the entropy scales as $S \sim \ln Q$, thus it is convenient to use the reduced entropy: $S' = S/\ln Q$ and the reduced temperature: $T' = T \ln Q$ ($\beta' = \beta/\ln Q$). These reduced quantities are of $O(1)$ at the phase transition region.

In terms of $\beta'$, the partition function in equation(2) reads as

$$Z = \sum_G Q^{\phi(G)} \prod_{ij \in G} [Q^{\beta' J_{ij}} - 1],$$

(6)

in which for large-$Q$ we have $Q^{\beta' J_{ij}} \gg 1$, thus

$$Z = \sum_G Q^{\phi(G)}, \quad \phi(G) = N_{\text{tot}}(G) + \beta' \sum_{ij \in G} J_{ij},$$

(7)

which is dominated by the largest term, $\phi^* = \max_G \phi(G)$. Finally, we arrive at

$$Z = n_0 Q^{\phi^*},$$

(8)

where the degeneracy of the optimal set is $n_0 = O(1)$. The free-energy of the system, $F$ is proportional to the mean value of $\phi^*$:

$$\overline{\phi^*} = -\beta' F = S' - \beta' E,$$

(9)

where the reduced entropy of the system is $S' = N_{\text{tot}}(G^*)$ and the mean energy is given by: $E = -\sum_{ij \in G^*} J_{ij}$. Similarly, we obtain for the reduced entropy of the system with a contour of fixed spins as: $S'_{\text{fix}} = N_{\text{tot}}(G^*) - N_{\Gamma}(G^*)$, consequently the excess entropy associated with the contour is given by the mean number of clusters crossed by $\Gamma$:

$$S'_{\Gamma} = \overline{N_{\Gamma}}.$$  

(10)

Using this relation, $S'_{\Gamma}$ can be calculated numerically, which will be performed in the following section.

On the other hand, analytical results on $S'_{\Gamma}$ can be obtained from the difference of $\langle N_{\text{tot}} \rangle$ and $\langle N_{\text{tot}} - N_{\Gamma} \rangle$ in equations (3) and (5), which is the derivative of $\ln Z(Q) -
Excess entropy and central charge of the two-dimensional random-bond Potts model in the large-$Q$ limit

\[ \ln Z_{\text{fix}}(Q). \] At the critical point, this difference is given by: \( \sim L f_s(Q) + C \ln L \), where \( L \) is the linear size of the contour, \( f_s(Q) \) is the surface free-energy density, which is non-universal and the second term represents the corner contribution \[31\]. Thus, we obtain for the excess entropy:

\[ S'_{\Gamma} = -Q f'_s(Q) L + b_{\Gamma}(Q) \ln L, \]

in which the first term corresponds to the ‘area-law’ and in the second term the prefactor is factorized as:

\[ b_{\Gamma}(Q) = \frac{\partial c(Q)}{\partial \ln Q} A_{\Gamma} = c' A_{\Gamma}, \]

where \( c(Q) = c' \ln Q + \text{cst.} \) is the central charge of the RBPM for large \( Q \). \( A_{\Gamma} \) is a geometrical factor, which does not depend on \( Q \) and it follows from the Cardy–Peschel formula \[31\]

\[ A_{\Gamma} = \frac{1}{24} \sum_k \left( \frac{\gamma_k}{\pi} - \frac{\pi}{\gamma_k} + \frac{2\pi - \gamma_k}{\pi} - \frac{\pi}{2\pi - \gamma_k} \right), \]

where \( \gamma_k \) is the interior angle at each corner.

3. Numerical results for the excess entropy

In the numerical calculation, we have considered finite samples of size \( L \times L \) with periodic boundary conditions and the couplings were taken from a bimodal distribution:

\[ P(\beta'J) = \frac{\delta(w + \Delta w - \beta'J) + \delta(w - \Delta w - \beta'J)}{2}. \]

Having \( w = 1/2 \) the critical point is given from self-duality \[32\]: \( \beta'_c = 1 \). In most of the calculations, we have used \( \Delta w = 1/3 \), but to check universality we have also performed some calculations with \( \Delta w = 1/4 \). The linear size of the systems were \( L = 32, 64, 128, 256 \) and 512, and the number of independent samples varied from 80 000 at the smallest sizes to more than 1200 for \( L = 512 \).

The optimal set of a given sample has been calculated by the optimal cooperation algorithm \[23\], which works in polynomial time and has already been used to solve the RBPM in two- \[24, 25\] and three-dimensional \[33\] lattices, as well as in scale-free networks \[34\]. If there are multiple optimal sets in the system, then both the intersection and union of any pair of them yields an optimal set. In order to show that our results are independent of the choice of the representing optimal set in a given system, all our studies are carried out for the two limiting cases, namely for the union and intersection of all the optimal sets. We illustrate the cluster structure of the optimal sets in these limiting cases in figure 1. It is seen that the intersection consists of a large number of smaller clusters, which are partially merged to common clusters having larger masses in the union. Consequently, we have for the averaged number of crossing clusters:

\[ \overline{N}_T(\text{intersection}) \geq \overline{N}_T(\text{union}); \]

however, the corner contributions being dominated by the large clusters are expected to be asymptotically identical in the two cases.

Having the optimal sets of different samples we have calculated the excess entropy for different contours: sheared squares, line segments and crosses, which are illustrated in figure 2. To subtract the corner contribution from the data, we have used the so-called geometric approach \[28, 30, 35\]: for each sample \( N_T \) is calculated in two different geometries, which have the same boundary term, but different corner ones. Thus, the
corner contribution is obtained from their difference. The average in equation (10) is performed over i) different samples and ii) over different (∼1000) positions of the contour in a given sample. For technical details, we refer to our previous investigations on the non-random model [28–30].

We start with sheared squares, having an opening angle γ ≤ π/2 and both its base and altitude is given by L/2. In the numerical method, we have calculated the corner contribution of the excess entropy for different sizes and then finite-size estimates are calculated for the prefactor b_F in equation (12) by two point fit. These are presented in figure 3 for the union and intersection optimal sets. For this contour, the geometrical

Figure 1. Cluster structure of the same sample in the optimal sets for the intersection (upper panel) and for the union (lower panel) at L = 256.
Excess entropy and central charge of the two-dimensional random-bond Potts model in the large-$Q$ limit

Figure 2. Shape of the subsystems used in the numerical calculations: sheared squares, line segments and crosses.

Figure 3. Finite size estimates ($L = 32, 64, 128, 256$ and $512$, from bottom to top) of the prefactor $b$ with sheared squares as a function of $\gamma$ for intersection (top) and union (bottom) data. For larger sizes, the results are approaching the conformal result in equation (14), which is indicated by the dotted (black) line. Inset: finite size estimates of the reduced central charge $c'$ with sheared squares as a function of the system size at $\gamma = \pi/2$ ($\gamma = \pi/4$), indicated by solid (dashed) lines. The estimated value $c' = 0.74$ is shown by the dotted horizontal line.

factor, $A_\Gamma$ in equation (13) reads as

$$A_\Gamma = \frac{1}{12} \left[ 4 - \pi \left( \frac{1}{\gamma} + \frac{1}{\pi - \gamma} + \frac{1}{\pi + \gamma} + \frac{1}{2\pi - \gamma} \right) \right],$$

and we put also the conformal result in figure 3 with an estimated reduced central charge $c' = 0.74$, see equation (16). For opening angles that are not too small, the finite-size
Excess entropy and central charge of the two-dimensional random-bond Potts model in the large-$Q$ limit

Table 1. Numerical estimates for the reduced central charge $c'$ from the corner contribution of the excess entropy using different contour geometries and from the finite-size correction to the free-energy in stripes.

|                  | Union   | Intersection |
|------------------|---------|--------------|
| Excess entropy   | squares | 0.74(3)      | 0.79(6)      |
|                  | lines   | 0.74(3)      | 0.77(5)      |
|                  | crosses | 0.72(2)      | 0.73(1)      |
| Free-energy      |         | 0.75(2)      |

Figure 4. Estimates for the reduced central charge $c'$ for line segments. The estimated value $c' = 0.74$ is indicated by the dotted horizontal line.

estimates are close to the conformal results, for smaller angles the corrections become larger. Extrapolating the prefactors for the two largest angles, $\gamma = \pi/2$ and $\pi/4$ gives approximately the same estimate for the reduced central charge for the intersection and the union, see in table 1.

For line segments with length $\ell = L/2$, we have only the corner contributions of two exterior $\gamma = 2\pi$ angles, so that $A_T = 1/8$. In this case, the corner contribution is simply half of the number of common clusters between two line segments. Finite-size estimates for $c'$ are shown in figure 4 which are extrapolated to the same value (within the accuracy of the calculation) both for intersection and union data, see table 1. We have checked that the effective central charges for the disorder parameter, $\Delta w = 1/4$, are practically indistinguishable from the results in figure 4.

For a cross-like subsystem, the corner contributions are expected to cancel out completely. We have checked that this is indeed the case, see figure 5, showing the validity of the Cardy–Peschel formula. We have also studied subsystems comprised of $n = 1, 2, 3$ or 4 crosses (see in figure 2 of [30]), in which case we have obtained finite-size estimates for $c'$. For 1 and 3 crosses these are presented in figure 6 and the extrapolated values in table 1. In this case, results for the disorder parameter $\Delta w = 1/4$ lead to the same extrapolated value.
Excess entropy and central charge of the two-dimensional random-bond Potts model in the large-$Q$ limit

Figure 5. Difference between the excess entropies for one cross with two different sizes: $2L$ and $L$. There is no size dependence, hence no corner contribution in agreement with the Cardy–Peschel formula.

Figure 6. Finite-size estimates for the reduced central charge, $c'$ obtained by comparing the excess entropies for 1 and 3 crosses, for the intersection (i) and the union (u) data. For the union data, we also present the results obtained for the disorder parameter $\Delta w = 1/4 (u')$. The estimated value $c' = 0.74$ is indicated by the dotted horizontal line.

4. Central charge from the finite-size correction to the free-energy

The traditional way of subtracting the central charge of a two-dimensional lattice model is to study the finite-size correction to the critical free-energy density in the strip geometry [36, 37]. Having the critical RBPM in an infinite strip of width $L$ with periodic boundary conditions, the free-energy density scales as:

$$\beta' f(L) = \beta' f_0 + \frac{\pi c'}{6L^2} + O(L^{-4}) ,$$

where $f_0 \equiv f(L = \infty)$ and $c'$ is the reduced central charge in the large-$Q$ limit. Since the free-energy is the same for all optimal sets, we do not make a distinction here between the
union and intersection data. In a practical adaptation of this method, we have used finite stripes of size $L \times \alpha L$ for $L = 24, 32, 48, 64, \ldots$ and at a fixed $\alpha \geq 1$, we have plotted the free-energy densities as a function of $1/L^2$, see figure 7. As expected, the limiting value, $f_0$, does not depend on the aspect ratio, $\alpha$, but the slopes are different for $\alpha = 1$ and for $\alpha = 4$ and 8. For the latter two values, the slopes are very close, therefore we used the data at $\alpha = 8$ to estimate effective finite-size reduced central charge, $c'(L)$ from two-point fits. These are shown in the inset of figure 7. As expected, the effective $c'(L)$-s have no noticeable size-dependence for $L \geq 32$, since the correction terms are $O(1/L^2)$. Therefore, the extrapolated value given in table 1 is simply the mean value of the estimates in the inset of figure 7.

5. Conclusion

The random-bond Potts model is a basic problem of statistical physics of disordered systems and its large-$Q$ limit is of special interest, when the free energy of the system is dominated by a single diagram, the optimal set. Thermal fluctuations of the $Q \to \infty$ model are negligible compared to disorder fluctuations, therefore its critical properties are controlled by an infinite disorder fixed point. In this paper, we considered this model on the square lattice and studied the properties of the excess entropy, $S_{\Gamma}$, associated with a contour of bonds, $\Gamma$. $S_{\Gamma}$ is found to be proportional to the number of clusters in the optimal set which are crossed by the contour. Using conformal field theory, the excess entropy is shown to have a universal corner contribution at the critical point, which scales with the logarithm of the linear size of the contour and its prefactor is proportional to the central charge of the model. We have performed large-scale numerical calculations and confirmed the validity of the conformal prediction. We have also obtained estimates for
the central charge of the model, which are collected in table 1 for different forms of the contours, as well as for the two extreme forms of the optimal sets. Within the error of the calculation, these all agree with each other, as well as with the results of an independent estimate calculated from the finite-size dependence of the free-energy density. Based on these data we conclude with the estimate:

\[ c' = 0.74(2), \]  

for the reduced central charge of the model. This value is close to (although somewhat larger than) the previous estimate [20]: \( c' = 0.72 \) obtained through transfer matrix calculations for large, but finite values of \( Q \). \( c' \) in equation (16) is expected to be universal for any form of the quenched disorder, for the bimodal distribution it holds for \( 0 < \Delta < 1/2 \). It is different from that at \( \Delta = 1/2 \), which corresponds to bond percolation having a reduced central charge [28]

\[ c'_{\text{perc}} = \frac{5\sqrt{3}}{4\pi} \approx 0.689. \]  

(17)

It remains the subject of other research to study if the central charge of the model is related to some properties of the exactly solved random transverse-field Ising chain [27], as expected from the numerical values of the critical exponents in the two models.

Acknowledgments

This work has been supported by the Hungarian National Research Fund under grant No OTKA K75324 and K109577. The research of IAK was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP 4.2.4. A/2-11-1-2012-0001 ‘National Excellence Program’. J-Ch Ad’A extends thanks to the ‘Theoretical Physics Workshop’ and FI to the Université Joseph Fourier for supporting their visits to Budapest and Grenoble, respectively.

References

[1] Shannon C 1948 Bell Syst. Tech. J. 27 379
[2] Calabrese P, Cardy J and Doyon B (ed) 2009 Entanglement entropy in extended quantum systems J. Phys. A: Math. Theor. 42 (50) (Special issue)
[3] Holzhey C, Larsen F and Wilczek F 1994 Nucl. Phys. B 424 443
[4] Vidal G, Latorre J I, Rico E and Kitaev A 2003 Phys. Rev. Lett. 90 227902
[5] Calabrese P and Cardy J 2004 J. Stat. Mech. P06002
[6] Wilms J, Troyer M and Verstraete F 2011 J. Stat. Mech. P10011
[7] Lau H W and Grassberger P 2013 Phys. Rev. E 87 022128
[8] Stephen J-M, Inglis S, Fendley P and Melko R G 2014 Phys. Rev. Lett. 112 127204
[9] Wu F Y 1982 Rev. Mod. Phys. 54 235
[10] Cardy J L 1999 Physica A 263 215
[11] Imry Y and Wortis M 1979 Phys. Rev. B 19 3580
[12] Aizenman M and Wehr J 1989 Phys. Rev. Lett. 62 2503
Aizenman M and Wehr J 1990 Phys. Rev. Lett. 64 1311(E)
[13] Hui K and Berker A N 1989 Phys. Rev. Lett. 62 2507
Hui K and Berker A N 1989 Phys. Rev. Lett. 63 2433(E)
[14] Cardy J and Jacobsen J L 1997 Phys. Rev. Lett. 79 4063

doi:10.1088/1742-5468/2014/09/P09019
Excess entropy and central charge of the two-dimensional random-bond Potts model in the large-$Q$ limit

[15] Jacobsen J L and Cardy J 1998 *Nucl. Phys. B* **515** 701
[16] Picco M 1997 *Phys. Rev. Lett.* **79** 2998
Chatelain C and Berche B 1998 *Phys. Rev. Lett.* **80** 1670
Chatelain C and Berche B 1998 *Phys. Rev. E* **58** R6899
Chatelain C and Berche B 1999 *Phys. Rev. E* **60** 3853
Olson T and Young A P 1999 *Phys. Rev. B* **60** 3428
[17] Palágyi G, Chatelain C, Berche B and Iglói F 2000 *Eur. Phys. J. B* **13** 357
[18] Chatelain C 2013 *Europhys. Lett.* **102** 66007
Chatelain C 2014 *Phys. Rev. E* **89** 032105
Picco M arXiv:cond-mat/9802092
[19] Baxter R J 1973 *J. Phys. C: Solid State Phys.* **6** L445
[20] Jacobsen J L and Picco M 2000 *Phys. Rev. E* **61** R13
Picco M arXiv:cond-mat/9802092
[21] Juhász R, Rieger H and Iglói F 2001 *Phys. Rev. E* **64** 056122
[22] Kasteleyn P W and Fortuin C M 1969 *J. Phys. Soc. Japan* **26** Suppl. 11
[23] Anglès d’Auriac J-Ch, Iglói F, Preissmann M and Sebő A 2002 *J. Phys. A: Math. Gen.* **35** 6973
Anglès d’Auriac J-Ch 2004 *New Optimization Algorithms in Physics* ed A K Hartmann and H Rieger (Berlin: Wiley-VCH)
[24] Anglès d’Auriac J-Ch and Iglói F 2003 *Phys. Rev. Lett.* **90** 190601
[25] Mercaldo M T, Anglès d’Auriac J-Ch and Iglói F 2004 *Phys. Rev. E* **69** 056112
Iglói F and Monthus C 2005 *Phys. Rep.* **412** 277
[26] Fisher D S 1992 *Phys. Rev. Lett.* **69** 534
Fisher D S 1995 *Phys. Rev. B* **51** 6411
[27] Kovács I A, Iglói F and Cardy J 2012 *Phys. Rev. B* **86** 214203
Kovács I A and Iglói F 2014 *Phys. Rev. B* **89** 174202
Kovács I A, Elçi E M, Weigel M and Iglói F 2014 *Phys. Rev. B* **89** 064421
[28] Kovács I A and Peschel I 1988 *Nucl. Phys. B* **300** 377
Kinzl W and Domany E 1981 *Phys. Rev. B* **23** 3421
[29] Mercaldo M T, Anglès d’Auriac J-Ch and Iglói F 2005 *Europhys. Lett.* **70** 733
Mercaldo M T, Anglès d’Auriac J-Ch and Iglói F 2006 *Phys. Rev. E* **73** 026126
[30] Karsai M, Anglès d’Auriac J-Ch and Iglói F 2007 *Phys. Rev. E* **76** 041107
[31] Kovács I A and Iglói F 2012 *Europhys. Lett.* **97** 67009
[32] Bloete H W J, Cardy J L and Nightingale M P 1986 *Phys. Rev. Lett.* **56** 742
[33] Affleck I 1986 *Phys. Rev. Lett.* **56** 746

*doi:10.1088/1742-5468/2014/09/P09019*