HELIUM CORE WHITE DWARFS IN GLOBULAR CLUSTERS

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Received 2002 May 24; accepted 2002 November 30

ABSTRACT

We examine the theoretical implications of a population of low-mass helium core white dwarfs in globular clusters. In particular, we focus on the observed population in the core of NGC 6397, where several low-mass white dwarf candidates have been identified as “nonflickerers” by Cool and collaborators. Age and mass estimates from cooling models, combined with dynamical and evolutionary considerations, lead us to infer that the dark binary companions are C/O white dwarfs rather than neutron stars. Furthermore, we find that the progenitor binaries very likely underwent an exchange interaction within the last 10^9 yr. We examine the prospects for detecting a similar population in other globular clusters, with particular attention to the case of 47 Tuc.

Subject headings: binaries: close — globular clusters: general — globular clusters: individual (NGC 6397, 47 Tucanae) — Hertzsprung-Russell diagram — stellar dynamics — white dwarfs

1. INTRODUCTION

Binaries play a central role in the internal dynamical processes that drive globular cluster evolution (e.g., Hut et al. 1992; Bailyn 1995). Of particular importance is the primordial binary star population (Heggie 1975; Hut et al. 1992), which provides a crucial internal energy source for the cluster by virtue of inelastic scattering encounters. Some of the more dramatic products of these encounters include recycled pulsars (Rappaport, Putney, & Verbunt 1989; Phinney 1996), low-mass X-ray binaries (Verbunt & Johnston 1996), cataclysmic variables (CVs; Di Stefano & Rappaport 1994; Cool et al. 1995; Grindlay et al. 2001a, 2001b), and blue stragglers (Sigurdsson, Davies, & Bolte 1994; Lombardi, Rasio, & Shapiro 1996).

Cool et al. (1998, hereafter CGC98) and Edmonds et al. (1999, hereafter EGC99) have reported the detection of a new stellar population near the center of the core-collapsed globular cluster NGC 6397. They named them “nonflickerers” (NFs) and tentatively identified them as low-mass helium core white dwarfs (HeWDs). Similar, but fainter, objects have been discovered by Taylor et al. (2001, hereafter TGE01) in the same cluster. HeWDs are the result of the truncated evolution of low- and intermediate-mass stars in binaries (Kippenhahn, Kohl, & Weigert 1967), in which the hydrogen envelope of an evolving star is removed before the degenerate core is massive enough to burn helium to carbon. Such objects are a generic by-product of mass transfer in close binaries with evolved low-mass donor stars and may offer insight into the formation history of the compact object population and its coupling to the global cluster evolution.

In this paper we use white dwarf cooling models, together with simple binary evolution and dynamical models, to examine the nature of the NFs, extending the initial discussion of EGC99. We confirm that they must be HeWDs with more massive, dark binary companions. We furthermore place these systems in the broader dynamical context appropriate to the underlying binary population and examine the results as a function of globular cluster parameters, with particular application to NGC 6397 and 47 Tuc.

The observations and their immediate implications are briefly summarized in § 2. In § 3 we discuss the formation and cooling of HeWDs. Applications to NGC 6397 and 47 Tuc are presented in §§ 4 and 5, respectively, while other clusters are discussed briefly in § 6.

2. THE NONFLICKERERS

Nonflickerers were discovered by CGC98 during a search for CVs in the core of NGC 6397 (Cool et al. 1995; Grindlay et al. 1995). In the UV, CVs lie between the white dwarf sequence and the main sequence by virtue of emission from the inner parts of the accretion disk. In the redder optical bandpasses, the disk is fainter, and the donor star becomes more apparent, so that the system is found closer to the main sequence. The NFs differ from the CVs in that they show no evidence for a red donor star, and furthermore they show none of the characteristic UV “flicker” associated with the CV accretion disk. On the basis of this, CGC98 coined the term nonflickerers. They advanced the hypothesis that the NFs were hot HeWDs, which lie above the traditional white dwarf sequence by virtue of their smaller masses and thus larger radii. EGC99 presented a spectrum for one of the NFs and derived temperature and gravity constraints (log g = 6.25 ± 1.0, \( T_{\text{eff}} = 17,500 \pm 5000 \) K) consistent with models of low-mass white dwarfs (Hansen & Phinney 1998a; Benvenuto & Althaus 1999).

The masses inferred for these white dwarfs (see § 3.2) are in the range \( M_{\text{wd}} = 0.15-0.25 \, M_\odot \), which is considerably smaller than the main-sequence turnoff mass (~0.8 \( M_\odot \)). If these white dwarfs were single, they would tend to leave the core of the cluster on a timescale comparable to the central two-body relaxation time (Fregeau et al. 2002).
The relaxation time in the core of NGC 6397 is
\[
\tau_{rc} \approx 8 \times 10^5 \text{ yr} \left(\frac{\sigma}{4.5 \text{ km s}^{-1}}\right)^3 \left(\frac{M_{\text{wd}}}{0.3 \, M_\odot}\right)^{-1} \\
\times \left(\frac{M_{\text{to}}}{0.8 \, M_\odot}\right)^{-1} \left(\frac{n}{1.5 \times 10^6 \text{ pc}^{-3}}\right)^{-1},
\]
where we have used the central luminosity density from Djorgovski (1993) and the velocity dispersion \( \sigma \) from Pryor & Meylan (1993), and we have assumed a mass-to-light ratio of 3 and a (conservative—so \( \tau_{rc} \) is an upper limit) average stellar mass of 1 \( M_\odot \) since the core is likely dominated by heavy stellar remnants (King, Sosin, & Cool 1995). For comparison, the central relaxation time estimated by Djorgovski (1993) is \( \approx 8 \times 10^4 \text{ yr} \).

On timescales longer than \( \tau_{rc} \), isolated low-mass white dwarfs should have diffused out of the core as mass segregation develops in the cluster (Fregeau et al. 2002). Yet the NF positions measured by CGC98 are strongly concentrated toward the cluster center. This clearly suggests that the NFs must be members of binary systems with total masses comparable to or greater than the main-sequence turnoff mass. Indeed, the binary companions could have been expected, since low-mass white dwarfs are formed only by truncated stellar evolution in close binaries.\(^4\) A main-sequence companion of sufficient mass to explain the central location (\( \geq 0.6 \, M_\odot \)) would be of comparable or greater luminosity and much redder than the NF. Thus, the total binary mass must be dominated by a dark component, such as a neutron star (as suggested by CGC98) or a C/O white dwarf. This is also consistent with the observation of a Doppler shift of about 200 km s\(^{-1}\) (interpreted as an orbital velocity) for one of the NFs by EGC99.

Since we discuss the role of exchange interactions and the resultant recoils, we also note the half-mass relaxation time is \( \approx 2 \times 10^8 \text{ yr} \) for NGC 6397. On this timescale, massive bodies kicked out of the core (such as hard binaries) will segregate back to the center. Thus, we expect systems older than this to be concentrated toward the core, whereas younger systems could potentially be found in the outer parts.

3. HELIUM CORE WHITE DWARFS

3.1. Origins

Low-mass white dwarfs are the result of binary stellar evolution. In the field they are found as companions to millisecond pulsars (Ryba & Taylor 1991; Phinney & Kulkarni 1994; Hansen & Phinney 1998b) and in double degenerate systems (Bragaglia et al. 1990; Marsh, Dhillon, & Duck 1995; Saffer, Livio, & Yungelson 1998). In binaries with initial separations less than 200 \( R_\odot \), a low-mass star overflows its Roche lobe as it evolves off the main sequence and up the giant branch, but before core-helium ignition starts. The consequent mass loss truncates the stellar evolution and leaves the remnant degenerate core to cool as a HeWD.

\(^4\) Han, Podsiałowski, & Eggleton (1994) note that HeWDs can potentially be formed by the evolution of isolated low-mass (<1 \( M_\odot \)) stars. However, this formation path can only occur for Population I stars and is therefore not relevant to globular clusters. In addition, it appears to produce white dwarfs of mass \( \approx 0.4 \, M_\odot \), i.e., too massive to describe the NFs in any event.

The final binary configuration depends on the stability of the mass transfer process and, in particular, the mass of the companion star.

If the accretor is a neutron star (for which we assume a mass of about 1.4 \( M_\odot \)), then the mass transfer is stable as long as the donor mass is less than about 1.5 \( M_\odot \). This includes the stabilizing effect of nonconservative mass transfer (the critical donor mass for dynamical instability decreases to about 1 \( M_\odot \) for conservative mass transfer; see Kalogera & Webbink 1996). Mass transfer is driven by the radial expansion of the evolving red giant, and the orbit expands until the donor envelope mass is exhausted. The radius of the red giant is primarily determined by its core mass, and consequently the final orbital period should correlate with the white dwarf mass (see, e.g., Rappaport et al. 1995). If instead the accretor is a C/O white dwarf, then there is little possibility of stable mass transfer for donors with mass \( M > 0.8 \, M_\odot \). Indeed Han (1998) and Nelemans et al. (2000) have investigated the formation of double degenerate systems and find that all systems undergo a common envelope phase during the second mass transfer episode, which results in the formation of the low-mass HeWD.\(^5\) This difference in behavior is a consequence of the larger mass of the neutron star and the tendency for the orbit to expand when mass is transferred from a lighter donor to a heavier accretor.

The above considerations apply to isolated binaries and so describe the behavior of the primordial binary population. However, in a dense globular cluster, there are more important dynamical paths that can lead to the production of HeWDs. Of particular importance are exchange interactions involving hard binaries (those with binding energies exceeding the typical kinetic energy of other cluster stars). In most cases it is the least massive of the three stars involved in a strong interaction that will be ejected. This both promotes the formation of HeWDs (by increasing the average secondary mass in binaries with a neutron star or C/O white dwarf primary—thereby increasing the likelihood that there will be a mass transfer episode within a Hubble time) and contributes to their eventual removal from the binary (since the HeWD mass \( \approx 0.1–0.4 \, M_\odot \) is significantly smaller than the masses of typical main-sequence intruders). Although they owe their provenance to dynamical interactions, the products of post-exchange mass-transfer binaries that go through stable mass transfer are still subject to the same basic stellar evolution as primordial binaries. Thus, they should follow the same orbital period–companion mass relation as the primordial systems.

We must also consider those evolutionary pathways in which the dynamical interactions lead directly to close degenerate binaries. These include tidal captures followed by mergers (Ray, Kembhavi, & Antia 1987; McMillan, Taam, & McDermott 1990; Rasio & Shapiro 1991) or direct collisions between red giants and compact objects (Verbunt 1987; Bailyn 1988; Rasio & Shapiro 1991; Davies, Benz, & Hills 1991). In all cases the resulting merger will leave the red giant core (or proto–white dwarf) and the original compact object in a binary, with most of the giant envelope either ejected or accreted onto the more massive companion. The cross section for these processes in globular clusters

\(^5\) The first mass transfer episode, resulting in the formation of the more massive white dwarf, may be stable or may also lead to a common envelope.
is dominated by gravitational focusing and thus proportional to the red giant radius. We may thus determine the distribution of final white dwarf masses by calculating the fraction of the total collision cross section $\Sigma$ associated with stages before the red giant has evolved to the point where the core mass $M_c$ has a particular value $M$, i.e.,

$$\frac{\Sigma(M_c < M)}{\Sigma(M_c < M_f)} = \frac{\int_0^M R[M_c(t)] dt}{\int_{M_f}^0 R[M_c(t)] dt}, \quad (2)$$

where $M_f$ is the core mass at the point of core helium ignition. The result is shown in Figure 1, where we have used the models of Hurley, Pols, & Tout (2000). We see that 75% of the final products have $M_{wd} < 0.3 M_\odot$, a result similar to that of Verbunt (1988). However, the resulting binaries are not particularly hard, as the relative kinetic energy of the two stars is similar to the binding energy of the giant envelope and there is little in-spiral associated with the ejection of the envelope (Rasio & Shapiro 1991). The final orbital separations are therefore comparable to the original giant radius, and these binaries will have lifetimes against further interactions that are similar to those of systems undergoing stable mass transfer.

3.2. Cooling Models

Low-mass white dwarfs fall between the traditional white dwarf cooling sequence and the main sequence because hydrostatic equilibrium with the nonrelativistic degenerate equation of state yields $R \sim M^{-1/3}$, making lower mass dwarfs cooler at fixed luminosity. For relatively young (and thus hot) white dwarfs, there is the additional feature that a small mass fraction of hydrogen on the surface can increase the radius further. Thus, the color-magnitude diagram (CMD) positions of the NFs argue in favor of their low mass. Low-mass white dwarfs also have helium cores, rather than carbon/oxygen cores, because their prior evolution was truncated during the mass transfer episode and they never reached the core helium burning phase. This affects the cooling rate because the specific heat capacity of the white dwarf is thereby increased by a factor of $\sim 3$. Thus, HeWDs will be brighter than contemporaneous C/O dwarfs due to their larger heat reservoirs. In addition, as first noted by Webbink (1975), lower mass white dwarfs have smaller pressures at the base of the envelope, which results in less vigorous nuclear burning during the transition to the white dwarf sequence and consequently larger hydrogen layer masses. The result is that low-mass white dwarfs may possess a significant $pp$-burning contribution to the luminosity over much of the cooling sequence.

The contribution of residual hydrogen burning to the low-mass white dwarf luminosity has been a subject of active investigation in recent years (Webbink 1975; Iben & Tutukov 1986; Alberts et al. 1996; Sarna, Antipova, & Muslimov 1998; Driebe et al. 1999; Althaus, Serenelli, & Benvenuto 2001; Burderi, D’Antona, & Burgay 2002). The amount of residual hydrogen and the consequent $pp$-burning luminosity is found to be dependent on the frequency and strength of shell flashes driven by CNO-cycle burning during the approach to the white dwarf cooling sequence. This in turn is found to depend on the treatment of heavy element diffusion in the proto-white dwarf envelope. Stronger shell flashes mean that more hydrogen is burnt, resulting in smaller hydrogen layer masses on the cooling sequence and consequently more rapid cooling. For instance, the final hydrogen layer mass from Althaus et al. (2001; $M_{H} \sim 4 \times 10^{-4} M_\odot$ on a 0.242 $M_\odot$ star) is an order of magnitude smaller than the closest model from Driebe et al (1999; $M_{H} \sim 5 \times 10^{-3} M_\odot$ on a 0.259 $M_\odot$ star).

Furthermore, the motivations for most of the above calculations stem from the study of the low-mass white dwarf companions to millisecond pulsars, which are the products of dynamically stable mass transfer. As we shall see below, our analysis favors the products of dynamically unstable mass transfer and common envelope evolution, so we need to reexamine what kind of hydrogen layers we might expect.

To estimate the remnant layer mass in the case of a common envelope episode, we note that the in-spiral takes place on timescales much shorter than the envelope thermal time, so that the heat deposition is considerably more rapid than in the stable mass transfer case; i.e., the thermal structure of the envelope has little chance to readjust in response to the in-spiral. Thus, we assume that only that fraction of the original envelope mass that lies within the final Roche lobe will fall back onto the system; i.e., $M_f/M_0 \sim (a_0/a_f)^{3-\alpha}$, where $M_f$ and $M_0$ are the final and initial envelope masses and $\alpha$ is the power-law slope of the envelope density profile ($\alpha = 1.5$ for an isentropic ideal gas). The ratio of pre- and post-in-spiral separations varies from $\sim 25$ to 150, i.e., $10^{-4}$ to $10^{-2}$ of the original envelope mass is retained.

We use the population synthesis models of Rasio, Pfahl, & Rappaport (2000) and Rappaport et al. (2001) to calculate the post-exchange binary population and estimate the hydrogen layer masses as above. The results are shown in Figure 2. We see that NFs with C/O white dwarf companions have considerably smaller hydrogen layer masses than is found for the stable-transfer neutron star case or for the

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This is a conservative assumption as any expansion in response to the dissipation of orbital energy will only reduce the fallback mass still further.
isentropic ($\text{C}_1$) filled dwarf ($\text{open}$) stable and initiated by Roche-lobe overflow. A set of models designed to bracket the likely true situation. The shell flashes and the thickness of hydrogen layer masses ($q$) prompts us to adopt the conservative approach of choosing models from a range of progenitors and for a fixed amount of binding energy. The second is that binaries containing C/O white dwarfs can enter the common envelope stage with much lower mass secondaries than in the neutron star case (see Fig. 2). The cooling of HeWDs with a "moderate" hydrogen surface layer mass fraction of $q_{\text{H}} = 10^{-4}$ can provide the residual hydrogen burning luminosity on the cooling sequence. CNO burning becomes unimportant on the cooling sequence proper once the heavy elements have gravitationally settled.

To verify that our results indeed bracket the more sophisticated treatments, we may compare a representative model to the published results of Driebe et al. (1999). Their model has a mass $0.26 \, M_\odot$ and $q_{\text{H}} = 0.018$. Point G in Figure 3 of Driebe et al. is reached after $1.3 \times 10^8$ yr, with log $L/L_\odot = -0.803$ and log $T_{\text{eff}} = 4.286$. Our $0.25 \, M_\odot$ model, with an initial $q_{\text{H}} = 10^{-2}$, reaches log $L/L_\odot = -0.867$ and log $T_{\text{eff}} = 4.263$ after $10^8$ yr. Thus, our models are in reasonable agreement, even though we do not include the CNO contribution to the nuclear burning. By choosing a wide range of potential layer masses, we examine the range of possible ages that may result owing to variations in the details of CNO diffusion and nuclear burning.

The progenitor models are drawn from the models described in Hurley et al. (2000) and kindly supplied to us by Chris Tout. We assume that the dynamically unstable mass transfer removes the envelope instantaneously, so that our model begins with the core thermal profile of the progenitor appropriate to the particular helium core mass, with an overlying hydrogen layer whose mass we treat as a free parameter. We choose models with $Z = 0.001$ to correspond to the low metallicity of globular cluster stars and consider progenitor masses of 1.1, 1.25, and 1.6 $M_\odot$. The cooling code is described at length in Hansen & Phinney (1998), as well as Hansen (1996, 1999).

Figure 3 shows the cooling of HeWDs with a "moderate" hydrogen surface layer mass fraction of $q_{\text{H}} = 10^{-4}$. For such a layer mass, surface hydrogen burning makes a negligible contribution to the luminosity, and the transition to the cooling sequence is a straightforward contraction. Also shown is the cooling curve for a standard 0.6 $M_\odot$ C/O core white dwarf in order to illustrate how separate this population is from the normal white dwarf population. For these models, the original CGC98 objects have masses 0.17–0.2 $M_\odot$ and ages less than $10^7$ yr, while the TGE01 objects have masses $\sim 0.15 \, M_\odot$ and ages $\sim 3 \times 10^8$ yr. All the progenitor models are drawn from the models described in Hurley et al. (2000) and kindly supplied to us by Chris Tout. We assume that the dynamically unstable mass transfer removes the envelope instantaneously, so that our model begins with the core thermal profile of the progenitor appropriate to the particular helium core mass, with an overlying hydrogen layer whose mass we treat as a free parameter. We choose models with $Z = 0.001$ to correspond to the low metallicity of globular cluster stars and consider progenitor masses of 1.1, 1.25, and 1.6 $M_\odot$. The cooling code is described at length in Hansen & Phinney (1998), as well as Hansen (1996, 1999).

![Figure 2](image.png)

Fig. 2.—Circles show the estimated hydrogen-layer masses for HeWDs in post–common envelope binaries with neutron-star (open) and C/O white dwarf (filled) companions. The estimates have been made assuming an isentropic ($\alpha_{\text{CE}} = 1.5$) pre-CE envelope. The stars correspond to the layer masses found by Driebe et al. (1999) in the case when the mass transfer was stable and initiated by Roche-lobe overflow.

![Figure 3](image.png)

Fig. 3.—Open circles are the data points from CGC98 and TGE01. The filled circle indicates the MSP HeWD companion in 47 Tuc (Edmonds et al. 2001). The solid lines are cooling curves for 0.15, 0.2, and 0.25 $M_\odot$ models drawn from a 1.1 $M_\odot$ progenitor ($Z = 0.001$). The dashed lines are drawn from a 1.25 $M_\odot$ progenitor. The 0.15 and 0.2 $M_\odot$ curves are labeled with ages at the points where they are $10^8$ and $10^7$ yr, respectively. This provides an illustration of the approximate ages of the various observed points. The dotted line shows the location of a standard 0.6 $M_\odot$ cooling sequence.
white dwarfs have masses \(\sim 0.17 - 0.21 \, M_\odot\). Also shown is the optical companion to the millisecond pulsar in 47 Tuc, detected by Edmonds et al. (2001). For these models this object (if it is a white dwarf) has an even lower mass (the Hurley et al. models allow helium cores as low as 0.1 \(M_\odot\) for \(Z = 0.001\) progenitors at the current turnoff mass) and an age similar to the CGC98 NF (the fact that this is much less than the pulsar spin-down age is not a concern, as the spin-down age is a notoriously unreliable measure in the case of millisecond pulsars; e.g., Camilo, Thorsett, & Kulkarni 1994; Hansen & Phinney 1998b).

Figure 4 shows the equivalent diagram but featuring models with an initial surface hydrogen layer mass fraction of \(q_H = 0.03\). For these models, the increased hydrogen burning results in larger radii and thus larger inferred masses. The 0.15 and 0.20 \(M_\odot\) show a temporary reexpansion (or “hook”) due to strong \(pp\)-burning (a weaker version of the CNO-burning shell flashes referred to above). The cooling of the white dwarf is delayed until the hydrogen layer mass is reduced sufficiently that the nuclear burning luminosity does not overwhelm the cooling.7 The CGC98 NF are now \(\sim 0.2 - 0.25 \, M_\odot\) and have ages less than \(10^9\) yr. The TGE01 white dwarfs have slightly larger masses (\(\sim 0.15 - 0.20 \, M_\odot\) and ages \(\sim 10^{10}\) yr). The Edmonds et al. (2001) object is now consistent with a mass \(\sim 0.18 \, M_\odot\).

Thus, if CNO flashes do not remove a lot of material, the residual hydrogen burning can make the TGE01 white dwarfs very old. This is the conclusion reached by Burderi et al. (2002), who find that the TGE01 objects are \(\sim 20\) Gyr old by their scenario (which involves stable mass transfer).

Given that the progenitor mass and hydrogen layer thickness are uncertain, we shall adopt the broadest range of constraints obtained by comparing all the above models.

\[ \tau_{\text{ex}} \sim 10^{11} \, \text{yr} \left( \frac{\sigma}{10 \, \text{km s}^{-1}} \right) \left( \frac{n}{10^5 \, \text{pc}^{-3}} \right)^{-1} \frac{1}{a} \frac{1}{M_1 + M_2 + M_3}, \]

where \(M_1, M_2,\) and \(M_3\) are the masses (in \(M_\odot\)) of the two binary components and of the incoming perturber, respectively, \(a\) is the binary semimajor axis (in \(R_\odot\)), and \(n\) and \(\sigma\) are the number density and velocity dispersion, respectively, of single star perturbers in the core. Converting \(a\) to binary orbital period, assuming \(M_1 + M_2 \approx 2 \, M_\odot\), \(M_3 = 0.8 \, M_\odot\), and adopting the central luminosity density of Djorgovski (1993) with a mass-to-light ratio of 3 and the central velocity dispersion of Pryor & Meylan (1993), we infer the maximum orbital period that a surviving binary is likely to have for a given lifetime to exchange

\[ P_{\text{orb}} \sim 3 \, \text{days} \left( \frac{\tau_{\text{ex}}}{10^8 \, \text{yr}} \right)^{-3/2} \left( \frac{\sigma}{4.5 \, \text{km s}^{-1}} \right)^{3/2} \left( \frac{n}{1.5 \times 10^6 \, \text{pc}^{-3}} \right)^{-3/2}. \]

4. APPLICATION TO NGC 6397

4.1. Young Nonflickerer

In the case of the CGC98 nonflickerer, the relative youth of the HeWDs implies that their progenitors evolved off the main sequence within the last \(\sim 10^6\) yr; i.e., the progenitor mass was not much larger than the current turnoff mass (about \(0.8 \, M_\odot\)). Thus, if the dark companion is a \(1.4 \, M_\odot\) neutron star, the donor/accretor mass ratio is less than 0.6 and the mass transfer is stable. The binary orbit will then expand during mass transfer, and we expect the final system to obey the usual companion mass–orbital period relation. In a dense, core-collapsed cluster such as NGC 6397, it is very likely that the white dwarf will be removed from such a wide binary during a later interaction. To examine this more quantitatively, consider the approximate lifetime of the binary to exchange interactions (e.g., Sigurdsson & Phinney 1993; Davies 1995)

\[ \tau_{\text{ex}} \sim 10^{11} \, \text{yr} \left( \frac{\sigma}{10 \, \text{km s}^{-1}} \right) \left( \frac{n}{10^5 \, \text{pc}^{-3}} \right)^{-1} \frac{1}{a} \frac{1}{M_1 + M_2 + M_3}, \]

where \(M_1, M_2,\) and \(M_3\) are the masses (in \(M_\odot\)) of the two binary components and of the incoming perturber, respectively, \(a\) is the binary semimajor axis (in \(R_\odot\)), and \(n\) and \(\sigma\) are the number density and velocity dispersion, respectively, of single star perturbers in the core. Converting \(a\) to binary orbital period, assuming \(M_1 + M_2 \approx 2 \, M_\odot\), \(M_3 = 0.8 \, M_\odot\), and adopting the central luminosity density of Djorgovski (1993) with a mass-to-light ratio of 3 and the central velocity dispersion of Pryor & Meylan (1993), we infer the maximum orbital period that a surviving binary is likely to have for a given lifetime to exchange

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We have chosen the exchange time to be a nominal lifetime for the NFs. We can furthermore convert this into a relation between white dwarf mass and exchange lifetime, using the orbital period–core mass relation of Rappaport et al. (1995). This yields a range of allowed white dwarf masses

\[ M_{\text{wd}} < 0.2 \, M_\odot \left( \frac{\tau_{\text{ex}}}{10^8 \, \text{yr}} \right)^{-6/25} \left( \frac{\sigma}{4.5 \, \text{km s}^{-1}} \right)^{6/25} \times \left( \frac{n}{1.5 \times 10^6 \, \text{pc}^{-3}} \right)^{-6/25} \]  

(5)

The half-mass relaxation time for NGC 6397 \( \tau_{\text{rh}} \approx 2 \times 10^8 \) yr, so this estimate is not sensitive to whether or not post-exchange systems acquire significant recoil velocities. Those that do will diffuse back into the core on the timescale of interest. These considerations are also consistent with the Doppler velocity of about 200 km s\(^{-1}\) measured by Edmonds et al. (2001), as

\[ P_{\text{orb}} = 2\pi \frac{GM_{\text{tot}}}{V^3} \approx 2 \text{ days} \left( \frac{M_{\text{tot}}}{2 \, M_\odot} \right) \left( \frac{V}{200 \, \text{km s}^{-1}} \right)^{-1} \]  

(6)

However, the white dwarf age is an underestimate in this case because systems with final orbital periods less than 10 days are the result of a competition between nuclear evolution and magnetic braking (Pylyser & Savonije 1988; Ergma 1996; Podsidiadlowski, Rappaport, & Pfahl 2002), a process which takes greater than 10\(^8\) yr. As an example, models B20, C20, and D20 of Pylyser & Savonije (the most appropriate to the probable progenitor systems) spend 1–4 Gyr in the mass transfer phase, 10–20 times the probable lifetime to exchange.

The essential problem is that stable mass transfer leads to post-transfer binaries that are too wide. More compact systems that avoid further exchange interactions can be formed only if the system undergoes common envelope evolution. This can happen if the dark companion is less than the progenitor mass, i.e., if the accretor is a normal C/O white dwarf rather than a neutron star. It also allows for arbitrarily young NFs, since the average C/O white dwarf mass is 0.5–0.6 \( M_\odot \), less than the current turnover mass. The resultant common envelope in-spiral makes the binaries hard enough that they are no longer susceptible to further exchanges. Furthermore, our estimates of the hydrogen layer mass in §3.2 are also consistent with the values required to give the young model ages for the observed systems. Thus, the assumption of a C/O white dwarf companion results in a consistent solution, with both dynamical lifetimes and cooling ages within the observational constraints.

### 4.2. Old NF

The above arguments were built on models in which the hydrogen layer mass was small and the NFs were consequently young. If the NFs had larger hydrogen layer masses, the ages of the CGC98 nonflickerer could be as old as 1 Gyr, and the TGE01 nonflickerer could be a Hubble time old. Hydrogen layers this thick can potentially arise from stable mass transfer in close neutron star binaries, if CNO shell flashes are inefficient at removing hydrogen. However, the lowest masses resulting from stable mass transfer are greater than 0.1 \( M_\odot \), for which equation (5) predicts exchange lifetimes \(< 1.8 \times 10^8\) yr.

The only way that the dark companion could be a neutron star is if the progenitor mass were large enough that the mass transfer was unstable and the system entered a common envelope, i.e., if the progenitor were sufficiently more massive than the neutron star. For a progenitor mass greater than 1.4 \( M_\odot \) and a cluster age \( \approx 12\) Gyr, a white dwarf age greater than 5 Gyr is required. Such ages are possible if we make the hydrogen layer mass large enough. However, progenitor masses greater than 1.4 \( M_\odot \) have cores \(< 0.2 \, M_\odot \) at the main-sequence turnoff point; i.e., to get a helium core of \( \approx 0.15 \, M_\odot \) requires a progenitor less massive than a neutron star, in which case the mass transfer is stable.

Finally, there are dynamical constraints on this scenario too. Very tight binaries will not always survive in the dense cluster core. If the system is too compact, gravitational radiation will merge the binary anyway (thereby destroying the NF). The timescale for this is

\[ \tau_{\text{GR}} \approx 3 \times 10^8 \, \text{yr} \left( \frac{a}{R_\odot} \right)^4 \left( \frac{M_{\text{tot}}}{1.6 \, M_\odot} \right)^{-1} \times \left( \frac{1.4 \, M_\odot}{M_{\text{ns}}} \right) \left( \frac{0.2 \, M_\odot}{M_{\text{wd}}} \right)^{-1/5} \]  

(7)

Thus, by equating (3) and (7), we find the maximum age to which the white dwarf–neutron star binary can survive in the core of NGC 6397 (see Fig. 5):

\[ \tau_{\text{max}} \approx 1 \, \text{Gyr} \left( \frac{M_{\text{wd}}}{0.2 \, M_\odot} \right)^{-1/5} \]  

(8)

![Fig. 5.](image_url) Solid lines show the timescale for exchange and merger for a 1.4 \( M_\odot \) neutron star and a 0.2 \( M_\odot \) white dwarf, under the conditions appropriate for the NGC 6397 core. The dashed lines represent the same, but for a 0.6 \( M_\odot \) white dwarf. An additional constraint on binary survival is that the recoil velocity in an exchange interaction can eject a hard binary from the cluster entirely. The distribution of recoil velocities is quite broad (see Sigurdsson & Phinney 1993) and so is not shown on the diagram. However, binaries with separations in the range \( a \sim 1–10 \, R_\odot \) have a significant chance of retention (for a cluster escape velocity of 50 km s\(^{-1}\)), while those with \( a < 1 \, R_\odot \) are frequently ejected.
This places an upper limit on the age of any NF still surviving in a binary and thereby rules out neutron star companions, for which the systems need to be greater than 5 Gyr old. The ages of the TGE01 nonflicker for the “young” low hydrogen case are approximately a factor of 3 lower than this limit and may represent the oldest NF still in binaries.

This fainter group also has an alternative explanation, suggested by Townsley & Bildsten (2002), in which they could be old, low-luminosity CVs. However, it is important to note that this explanation cannot be applied to the brighter NF objects discussed by CGC98.

4.3. Observational Implications

Given the above considerations, we favor the following model for the observed NF population in NGC 6397. They are HeWDs with moderate ($\sim 10^{-4} M_\odot$) hydrogen surface layers and ages less than $10^8$ yr (for the bright population). They have formed from progenitors of mass $\sim 0.8 M_\odot$ in binaries with C/O white dwarfs, and have undergone common envelope evolution.

Our conclusion that there is no self-consistent and convincing scenario that can allow for neutron star companions has an important observational consequence: none of the NFs should show the X-ray signatures characteristic of millisecond pulsars. Grindlay et al. (2002) have examined the core of NGC 6397 with the Chandra satellite, identifying many sources (including a millisecond pulsar and several potential CVs), none of which are coincident with the NFs. This “surprising” (in their words) finding is consistent with our analysis.

One could also ask about the possibilities for finding fainter NFs in deeper searches. Here the maximum age constraint (eq. [8]) implies that we do not expect to find any NFs fainter than the second population observed by TGE01.

4.4. Surviving Core Collapse

So far we have treated the NGC 6397 core as one of constant density and velocity dispersion. However, NGC 6397 is a “core-collapsed” cluster, where the very definition of a core is somewhat uncertain. We have used the central density and velocity dispersion from Djorgovski (1993) and Pryor & Meylan (1993) respectively, although the true density may be lower (and thus lifetimes longer; A. Cool 2002, private communication). On the other hand, the central densities previously reached at the onset of core collapse were potentially even higher, and the very low binary fraction (Bolton, Cool, & Anderson 1999) suggests that only the very hard binaries have survived. As a result, we feel that our conclusion that the NFs are young, with C/O white dwarfs as companions, is independent of the details of the past cluster evolution.

Since the NFs are $\sim 10^8$ yr old, the progenitor binaries must have either survived core collapse or been produced during the collapse. Within the “young NF” scenario we may constrain the progenitor configuration as follows. The donor mass is required to be $\sim 0.8 M_\odot$ (since the observed NF masses are larger than the $\sim 0.1 M_\odot$ of the initially hydrogen exhausted core of a turnoff star; i.e., the progenitor must have instigated mass transfer after leaving the main sequence), and the observed core mass tells us the giant radius at which point mass transfer started. Thus, we may use the Rappaport et al. (1995) relation to infer the initial orbital separation $a_0$ of a dynamically unstable system. We find

$$a_0 = \frac{0.5 + 2.36 R_\odot(M_{\text{sd}}/0.2 M_\odot)^{4.5}}{0.38 + 0.2 \log(0.8 M_\odot/M_\odot)}$$

where $M_1$ and $M_{\text{sd}}$ are the C/O white dwarf and HeWD masses, respectively. For $M_1 \sim 0.5 M_\odot$ and $M_{\text{sd}} \sim 0.2 M_\odot$, this yields $a_0 \approx 7 R_\odot$, corresponding to a period $P_{\text{orb}} \approx 1.8$ days. The exchange lifetime for such a binary (using the same parameters as before) is $\sim 2 \times 10^8$ yr, similar to the current ages of the white dwarfs. This may also explain why we do not observe HeWDs of greater mass (theoretically up to $0.4 M_\odot$); these more massive HeWDs are prevalent among the field double degenerate systems; see Bergeron, Saffer, & Liebert 1992). The rapid increase in $a_0$ with $M_2$ leads to a rapid decrease in the lifetime to exchange. For $M_2 \sim 0.3 M_\odot$, we find $a_0 \approx 36 R_\odot$ and avoid Roche Lobe overflow (since that would lead to a CE and immediately halt the evolution), it would have to be in such a wide binary that another exchange interaction would quickly ensue. Hence, we infer that, not only did the mass transfer episode occur only $\sim 10^8$ yr ago, but the progenitor binary configuration had a similar lifetime. We therefore expect the progenitor stars to have typically participated in several exchange interactions before forming the final binary configuration that led to the mass transfer.

4.5. An Evolving Population

Since any given binary incarnation cannot survive much longer than 1 Gyr in the core of NGC 6397, compact objects such as white dwarfs and neutron stars may undergo several life cycles with different companions over the course of a Hubble time. Not only do we include soft binaries in this statement but also neutron stars and massive white dwarfs with close companions that undergo gravitational wave inspiral. After disrupting their in-spiraling companions, they will be available again as single objects to exchange into other binaries.

This also implies that the cluster should contain binaries in which the neutron star was recycled by accretion in a previous incarnation and thereafter lost the white dwarf in an exchange interaction, resulting in a millisecond pulsar with a close main-sequence companion. This is most likely the nature of the recently discovered first radio millisecond pulsar in NGC 6397 (D’Amico et al. 2001), which shows a variable optical companion consistent with a main-sequence star (Ferraro et al. 2001a). D’Amico et al. (2001) note that this is potentially the result of an exchange interaction, and Ferraro et al. (2001a) further note that the candidate companion is, in fact, one of the BY Draconis candidates discussed by TGE01. Burderi et al. (2002) discuss the possible history of this system as a neutron star whose pulsar wind is impeding the mass transfer from the companion that contributed to the spin-up and identify the NF in the cluster as potential endpoints of this evolution. However, the assertion that the current companion is responsible for the pulsar spin-up is only an assumption. The post-recycling exchange of a fresh main-sequence star into this binary is also consistent with the data and furthermore consistent with our picture of the dynamically evolving binary population in the
NGC 6397 core. Additional evidence in support of such a picture for the core of NGC 6397 is presented by Grindlay et al. (2001b), who find that another of the BY Dra candidates discovered by TGE01 may actually be a multiply exchanged millisecond pulsar (based on the X-ray properties).

5. APPLICATION TO 47 TUC

We have thus far focused on the NFs discovered in NGC 6397. However, there are now Chandra and HST observations of many globular cluster cores. In what follows, we discuss our expectations for the (currently undetected) NF population in 47 Tuc. The attraction of this cluster is the rich data set spanning X-ray (Grindlay et al. 2001a), UV (Ferraro et al. 2001b; Knigge et al. 2002), optical (Gilliland et al. 2000), and radio (Camilo et al. 2000) wavelengths. 47 Tuc is also interesting in that it is a dense, pre–core-collapse cluster and somewhat more massive than NGC 6397.

The nominal central density of 47 Tuc is $1.5 \times 10^5$ pc$^{-3}$, 10 times smaller than the value we used for NGC 6397. This has the important consequence that much wider binaries can survive for a reasonable length of time without suffering an exchange interaction. If we rescale equation (8) to the 47 Tuc parameters (and for neutron star binaries), we find

$$\tau_{\text{max}} = 10 \text{ Gyr} \left(\frac{M_{\text{wd}}}{0.2 M_{\odot}}\right)^{-0.2}.$$  

Thus, the potential age range of NF binaries in 47 Tuc is much larger than for NGC 6397. This estimate applies to all kinds of binaries. If we consider only those that evolve under stable mass transfer, the orbital period–mass relation implies a tighter limit of

$$\tau < 2.7 \text{ Gyr} \left(\frac{M_{\text{wd}}}{0.2 M_{\odot}}\right)^{-25/6}.$$  

Thus, in 47 Tuc, the products of stable mass transfer can potentially survive for an interesting length of time.

We have also examined the population of binaries that result from exchange interactions, using the binary interaction Monte Carlo code of Rasio et al. (2000; see also Rappaport et al. 2001). Figure 6 shows the NF mass as a function of age for NF with both neutron star and C/O white dwarf companions. Only those binaries that are hardened by a common envelope episode are shown. This includes all the NFs with C/O white dwarfs, and thus a wide range of masses and ages. The population of neutron stars is restricted to older systems with more massive NF, whose progenitors were massive enough to lead to dynamically unstable mass transfer. Thus, the NF population with neutron star companions will be bimodal, with old massive white dwarfs resulting from common envelope evolution and lighter white dwarfs surviving from the tightest of the stable mass-transfer systems.

Indeed, the radio observations find a large population of binary millisecond pulsars in 47 Tuc (Manchester et al. 1991; Camilo et al. 2000), many of which appear to have low-mass white dwarf companions (Rasio et al. 2000). In one case, 47 Tuc U, the NF companion has been detected optically (Edmonds et al. 2001), for which we estimate a mass $\approx 0.14 - 0.2 M_{\odot}$ and an age less than $10^6$ yr, even for a large hydrogen surface layer. This is potentially the product of a stable mass transfer episode, in contrast to the systems observed in NGC 6397. Furthermore, the X-ray and UV searches have uncovered a range of CV candidates, shown in Figure 7. The NF candidates are also expected to be found in this region of the CMD, as shown by the cooling curves in the figure. An attempt to determine what fraction of the CV candidates are actually NFs will elucidate the binary interaction history of this cluster.

6. OTHER CLUSTERS

There are, as yet, no NF candidates in any other clusters. However, we can use the known millisecond pulsar binaries to roughly anticipate the conditions in other clusters. Figure 8 shows the known pulsar binaries in the distribution of lifetime to exchange versus merger lifetime. We have split them up into four groups, depending on cluster density. 47 Tuc occupies a class of its own by virtue of its large number of detections.

What is clear from this diagram is that very few of the cluster millisecond pulsar binaries are likely to be due to an undisturbed primordial configuration; to survive undisturbed for a Hubble time they would need to lie in the top right-hand side quadrant. Thus, nearly all of these clusters may be expected to possess a population of close binaries sculpted by exchange interactions. In Figure 8 we also show the simulations of Rappaport et al. (2001) for the 47 Tuc parameters (which clearly explain the trend of the observed binaries). The majority of the predicted systems lie in the top left-hand quadrant. These are still relatively wide binaries, vulnerable to further exchanges. The observed systems in this area include the NGC 6397 pulsar system,
which is most likely a recent exchange capture of a main-sequence star and the system NGC 6266A, which has very similar parameters. The binary at the extreme upper left of the plot is the one in NGC 6441, where the large companion mass and significant eccentricity argues for a recent white dwarf capture. The bottom right-hand quadrant is occupied by very compact binaries. The lack of systems in this area is most likely because the recoil velocities of post-exchange binaries (Phinney & Sigurdsson 1991) will eject such tight binaries from the cluster (see Fig. 5).

Location of objects on this diagram is based purely on their dynamical properties. The observation of a low-mass white dwarf (NF) allows us to obtain a quantitative measure of the age of the system dating from the end of mass transfer and to thereby test the model that goes into the construction of a figure such as Figure 8. In the case of NGC 6397, all the objects have consistent model ages less than 1 Gyr, in agreement with the location of the NGC 6397 pulsar. The other clusters in class A are also ideally suited for the detection of bright NF.

7. CONCLUSIONS

We have examined the various possible evolutionary pathways for the formation of NF systems, identified as HeWDs by CGC98, in globular cluster cores. We have paid particular attention to the cluster NGC 6397, where these objects were first discovered. We find that most likely the NGC 6397 NFs are HeWDs with CO core white dwarfs as their dark companions. This model satisfies all constraints imposed on the lifetimes and masses of white dwarfs (inferred from the observations and the dynamical properties of the host cluster). The low masses of CO white dwarf companions (relative to neutron-star companions) allow recent common envelope evolution that produces (1) orbits tight enough to avoid disruption due to dynamical interactions and (2) young, bright HeWDs to explain their position in the CMD. Neutron-star dark companions would be too massive and would result in either stable mass transfer (producing systems that are too wide to survive exchange interactions for a significant time in a dense cluster core) or in HeWDs too old and too faint.

The common envelope episode also results in a moderate mass hydrogen envelope on the white dwarf surface, leading to an estimate of the cooling age that is consistent with the dynamical considerations. A massive hydrogen envelope, in which nuclear burning is important, would imply that the observed systems are older than they are likely to be, based on their probability of survival in the cluster core. These results are also consistent with calculations of white dwarf evolution that incorporate detailed calculations of CNO-cycle driven shell flashes (Driebe et al. 1999; Althaus et al. 2001), which limit the amount of hydrogen on the stellar surface.

We have extended our analysis of NGC 6397 to other globular clusters to illustrate how the production of NFs depends on cluster environment. In particular, we have studied the potential NF population in 47 Tuc, which is both less dense and more massive than NGC 6397, and for which a wealth of observational data is becoming available. In this
cluster surviving NFs with neutron star companions are possible, with potentially a bimodal distribution. The majority, resulting from common envelope evolution, will be older and more massive (and correspondingly fainter) than those with C/O white dwarf companions. However, the tightest of the systems resulting from stable mass transfer can survive for several Gyr, making them potentially observable. This is again the consequence of whether binaries can survive intact, otherwise being transformed by exchange interactions or gravitational wave-induced mergers.

We anticipate that these models can be extended to other clusters as observational results accrue. The inferred white dwarf cooling ages will allow us to place constraints on the exchange interaction history of clusters in a manner that is independent of other methods, such as the properties of millisecond pulsar binaries.

We thank Peter Edmonds, Josh Grindlay, Eric Pfahl, and Saul Rappaport for many useful discussions and comments on the manuscript. B. H. acknowledges support from Hubble Fellowship HF-01120.01, which was provided by NASA through a grant from the Space Telescope Science Institute, operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. B. H. also thanks the theoretical astrophysics group at Northwestern University for hospitality. V. K. acknowledges early partial support by a Clay Fellowship at the Harvard-Smithsonian Center for Astrophysics, where this research was initiated. F. R. acknowledges support from NASA ATP grants NAG5-8460, NAG5-11396, and NAG5-12044. The authors also express their thanks to the Aspen Center for Physics, for facilitating many of the discussions that resulted in this work.

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