Structure Functions from Chiral Soliton Models

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Abstract. We study nucleon structure functions within the bosonized Nambu–Jona–Lasinio (NJL) model where the nucleon emerges as a chiral soliton. We discuss the model predictions on the Gottfried sum rule for electron–nucleon scattering. A comparison with a low–scale parametrization shows that the model reproduces the gross features of the empirical structure functions. We also compute the leading twist contributions of the polarized structure functions $g_1$ and $g_2$ in this model. We compare the model predictions on these structure functions with data from the E143 experiment by GLAP evolving them from the scale characteristic for the NJL-model to the scale of the data.

The purpose of this investigation is to provide a link between two successful although seemingly unrelated pictures of baryons. On one side we have the quark parton model which successfully describes the scaling behavior of the structure functions in deep inelastic scattering (DIS) processes. The deviations from these scaling laws are computable in the framework of perturbative QCD. On the other side we have the chiral soliton approach which is motivated by the large $N_C$ expansion of QCD, $N_C$ being the number of color degrees of freedom. For $N_C \to \infty$, QCD is known to be equivalent to an effective theory of weakly interacting mesons. Although this theory is not explicitly known it can be modeled by assuming that at low energies only the light mesons (pions, kaons, $\rho$, $\omega$) are relevant. When modeling the meson theory one requires the symmetry structure of QCD. In particular besides Pioncaré invariance we require chiral symmetry and its spontaneous breaking. Baryons emerge as non–perturbative (topological) configurations of the meson fields, the so–called solitons. The link between these two pictures can be established by computing structure functions.

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within a chiral soliton model for the nucleon from the hadronic tensor

\[ W^{ab}_{\mu\nu}(q) = \frac{1}{4\pi} \int d^4\xi \, e^{iq\cdot\xi} \langle N(P) | \left[ J^a_{\mu}(\xi), J^b_{\nu}(0) \right] | N(P) \rangle , \]  

(1)

which describes the strong interaction part of the DIS cross-section. In eq (1) \( |N(P)\rangle \) refers to the nucleon state with momentum \( P \) and \( J^a_{\mu}(\xi) \) to the hadronic current suitable for the process under consideration. In most soliton models – due to the non-perturbative nature of the soliton configuration – the current commutator (1) remains intractable. However, the Nambu and Jona–Lasinio (NJL) model [1] of quark flavor dynamics, which can be bosonized by functional integral techniques [2], contains simple current operators. Most importantly, the bosonized version of the NJL–model contains soliton solutions [3]. This paves the way to compute structure functions in the soliton approach.

In order to extract the leading twist contributions to the structure function one computes the hadronic tensor in the Bjorken limit

\[ q_0 = |q| - M_N x \quad \text{with} \quad |q| \to \infty \quad \text{and} \quad x = -q^2/2P \cdot q \quad \text{fixed} . \]  

(2)

Here we confine ourselves to presenting the key issues of the calculation, details may be traced from refs. [4–6].

**THE NUCLEON FROM THE CHIRAL SOLITON IN THE NJL MODEL**

In this section we briefly summarize the basic features of the chiral soliton in the NJL–model and discuss how states with nucleon quantum numbers are generated. For more details see refs. [3,7] and quotations therein.

The NJL–model Lagrangian contains a quartic quark interaction which is chirally symmetric. Derivatives of the quarks fields only appear in form of a free Dirac Lagrangian, hence the current operator is formally free. Upon bosonization the action may be expressed as [2]

\[ A = \text{Trln}_\Lambda (i\slashed{\partial} - mU\gamma_5) + \frac{m_0m}{4G} \text{tr} (U + U^\dagger - 2) \]  

(3)

where we have confined ourselves to the interaction in the pseudoscalar channel. The associated pion fields \( \pi \) are contained in the non-linear realization \( U = \exp(i\slashed{\tau} \cdot \pi/f_\pi) \). In eq (3) \( \text{tr} \) denotes discrete flavor trace while \( \text{Tr} \) also includes the functional trace. The parameters of the model are the coupling constant \( G \), the current quark mass \( m_0 \) and the UV cut–off \( \Lambda \). The constituent quark mass \( m \) arises as the solution to the Schwinger–Dyson (gap) equation and characterizes the spontaneous breaking of chiral symmetry. A Bethe–Salpeter equation of the pion field can be derived from eq (3) which allows one to express the pion mass \( m_\pi = 135\text{MeV} \) and decay constant \( f_\pi = 93\text{MeV} \) in terms of the model parameters. Fixing these quantities leaves one parameter undetermined which
maybe expressed in terms of the constituent quark mass \( m \). Subsequently an energy functional for non–perturbative but static field configurations \( U(r) \) can be extracted from (3). It can be expressed as a regularized sum of single quark energies \( \epsilon_\mu \). For the hedgehog ansatz, \( U_H = \exp(i\tau \cdot \hat{r}\Theta(r)) \) the associated one–particle Dirac Hamiltonian becomes

\[
h = \alpha \cdot p - \beta m \exp(i\gamma_5 \tau \cdot \hat{r}\Theta(r)), \quad h\Psi_\mu = \epsilon_\mu \Psi_\mu .
\] (4)

The distinct level \((v)\), which is bound in the background of \( U_H \), is referred to as the valence quark state. Its explicit occupation guarantees unit baryon number. The chiral angle \( \Theta(r) \) of the soliton is determined by self–consistently minimizing the energy functional. This soliton configuration does not yet carry nucleon quantum numbers. To generate them the (unknown) time dependent field configuration is approximated by elevating the zero modes to time dependent collective coordinates \( U(r,t) = A(t)U_H(r)A^\dagger(t), \ A(t) \in SU(2) \). Upon canonical quantization the angular velocities, \( \Omega = -2i\text{tr}(\tau \cdot \hat{r}\dot{A}) \), are replaced by the spin operator \( J \) via \( \Omega = J/\alpha^2 \) with \( \alpha^2 \) being the moment of inertia while the nucleon states \(|N\rangle \) emerge as Wigner \( D \)–functions. To compute nucleon properties the action (3) is expanded in powers of \( \Omega \) corresponding to an expansion in \( 1/N_C \). In particular the valence quark wave–function \( \Psi_v(x) \) acquires a linear correction

\[
\Psi_v(x,t) = e^{-i\epsilon_v t}A(t) \left\{ \Psi_v(x) + \sum_{\mu \neq v} \Psi_\mu(x) \frac{\langle \mu | \tau \cdot \Omega | v \rangle}{2(\epsilon_v - \epsilon_\mu)} \right\} = e^{-i\epsilon_v t}A(t)\psi_v(x). \] (5)

Here \( \psi_v(x) \) refers to the spatial part of the body–fixed valence quark wave–function with the rotational corrections included.

**STRUCTURE FUNCTIONS IN THE VALENCE QUARK APPROXIMATION**

The starting point for computing the unpolarized structure functions is the symmetric part of hadronic tensor in a form suitable for localized fields [9],

\[
W_{\{\mu\nu\}}^{lm}(q) = \zeta \int \frac{d^4k}{(2\pi)^4} S_{\mu\rho\sigma} k^\rho \text{sgn}(k_0) \delta(k^2) \int_{-\infty}^{+\infty} dt e^{i(k_0 + q_0)t} \times \int d^3x_1 \int d^3x_2 \exp[-i(k + q) \cdot (x_1 - x_2)] \times \langle N | \left\{ \hat{\Psi}(x_1,t)t_l t_m \gamma^\sigma \hat{\Psi}(x_2,0) - \hat{\Psi}(x_2,0)t_m t_l \gamma^\sigma \hat{\Psi}(x_1,t) \right\} |N \rangle. \] (6)

Note that the quark spinors are functionals of the soliton. Here \( S_{\mu\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma} \) and \( \zeta = 1(2) \) for the structure functions associated

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2) Generalizing this treatment to flavor SU(3) indeed shows that the baryons have to be quantized as half–integer objects. For a review on solitons in SU(3) see e.g. [8].
FIGURE 1. The unpolarized structure functions obtained after extracting the collective part of the nucleon matrix elements. Here we used \( m = 350 \text{MeV} \).

with the vector (weak) current and \( t_m \) is a suitable isospin matrix. The matrix element between the nucleon states \((|N\rangle)\) is taken in the space of the collective coordinates. In deriving eq. (6) the free correlation function for the intermediate quark fields has been assumed. In the Bjorken limit (2) the momentum, \( k \), of the intermediate quark is highly off–shell and hence not sensitive to momenta typical for the soliton configuration. Thus the use of the free correlation function is a valid treatment in this kinematical regime.

The valence quark approximation ignores the vacuum polarization in (6), e.g. the quark field operator \( \hat{\Psi} \) is substituted by the valence quark contribution (5). For small constituent quark masses \( m \sim 400 \text{MeV} \) this is well justified since this level provides the dominant share to static observables [3,7]. The structure function \( F_2(x) \) can be obtained from (6) by an appropriate projection\(^3\). After computing the collective coordinate matrix elements all physical relevant processes are described in terms of four structure functions \( f_{\pm}^0, f_{\pm}^1 \). The superscript denotes the isospin combination of \( t_l t_m \) while the subscript refers to forward and backward moving intermediate quarks in (6). In figure 1 the predictions for these four structure functions are displayed. Although the problem is not formulated Lorentz–covariantly these structure functions are reasonably well localized in the interval \( x \in [0, 1] \). Furthermore the contributions of the backward moving quarks are quite small, however, they increase with \( m \). Note that for consistency with the Adler sum rule also the moment of inertia must be restricted to the valence quark contribution [4,5]. For \( m = 350 \text{MeV} \) this, however, is almost 90%.

We continue by presenting the numerical results for the structure functions for physical processes. In figure 2 we display the linear combination relevant for the Gottfried sum rule

\[
(F_2^{ep} - F_2^{en}) = -x (f_+^1 - f_-^1) / 3
\]

\(^3\) In the Bjorken limit the Callan–Gross relation \( F_2(x) = 2xF_1(x) \) is satisfied.
and compare it to the low–scale parametrization of the empirical data [10]. This is obtained from a next–to–leading order QCD evolution of the experimental to a low–energy regime, where soliton models are valid. The agreement improves with increasing constituent quark masses. Apparently the model reproduce the gross features of the low–scale parametrization. Moreover the integral of the Gottfried sum rule

$$S_G = \int_0^\infty \frac{dx}{x} (F_{ep}^2 - F_{en}^2) = \begin{cases} 0.29, & m = 400\,\text{MeV} \\ 0.27, & m = 450\,\text{MeV} \end{cases} \quad (8)$$

agrees reasonably well with the empirical value $S_G = 0.235 \pm 0.026$ [11]. In particular the deviation from the naïve value $(1/3)$ [12] is in the direction demanded by experiment.

Figure 2 also shows the comparison of the model prediction for the polarized structure function $g_1(x)$ with the corresponding low–scale parametrization [10]. In this case the agreement improves with decreasing $m$.

No low–scale approximation is available for the polarized structure function $g_2(x)$. We have therefore projected the predicted structure function onto the interval $x \in [0, 1]$ [13] and subsequently performed a leading order QCD evolution to the scale of the experiment, see ref. [6] for details. The resulting polarized structure functions are displayed in figure 3. Apparently the model reproduces the empirical data quite well, although the associated error bars are sizable.

**CONCLUSIONS**

In this talk we have presented a calculation of nucleon structure functions within a chiral soliton model. We have argued that the soliton approach to the bosonized version of the NJL–model is most suitable since (formally) the required current operator is identical to the one in a free Dirac theory. Hence
there is no need to approximate the current operator by e.g. performing a gradient expansion. Although the calculation contains a few (well–motivated) approximations it reproduces the gross features of the empirical structure functions at low energy scales. This happens to be the case for both the polarized as well as the unpolarized structure functions.

Future projects will include to extend the valence quark approximation, improvements on the projection issue and the extension to flavor SU(3).

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