ZK-SECREC: a Domain-Specific Language for Zero-Knowledge Proofs

DAN BOGDANOV, Cybernetica AS, Estonia
JOOSEP JÄÄGER, Cybernetica AS, Estonia
PEETER LAUD, Cybernetica AS, Estonia
HÄRMEL NESTRA, Cybernetica AS, Estonia
MARTIN PETTAI, Cybernetica AS, Estonia
JAAK RANDMETS, Cybernetica AS, Estonia
VILLE SOKK, Cybernetica AS, Estonia
KERT TALI, Cybernetica AS, Estonia
SANDHRA-MIRELLA VALDMA, Cybernetica AS, Estonia

We present ZK-SECREC, a domain-specific language for zero-knowledge proofs. We present the rationale for its design, its syntax and semantics, and demonstrate its usefulness on the basis of a number of non-trivial examples. The design features a type system, where each piece of data is assigned both a confidentiality and an integrity type, which are not orthogonal to each other. We perform an empiric evaluation of the statements produced by its compiler in terms of their size. We also show the integration of the compiler with the implementation of a zero-knowledge proof technique, and evaluate the running time of both Prover and Verifier.

CCS Concepts: • Theory of computation → Type structures; Functional constructs; • Security and privacy → Privacy-preserving protocols.

Additional Key Words and Phrases: domain-specific languages, type and effect systems, zero-knowledge proofs

1 INTRODUCTION

Zero-knowledge proofs (ZKP) [12] are two-party protocols between Prover and Verifier, where the former attempts to convince the latter that he has a piece of knowledge that validates a statement, while not leaking anything about this knowledge. Here this statement is seen as a binary relation that takes as inputs the instance — common knowledge of Prover and Verifier —, and the witness — Prover’s private knowledge —, and decides whether the latter validates the former. Advances in ZKP [6, 10, 13, 15] allow the creation of ZKP for quite sizable statements, with privacy-preserving distributed ledgers [18, 19] showing a large variety.

All these cryptographic techniques expect the relation to be expressed as an arithmetic circuit over some finite ring (for most techniques, a field, often with additional constraints), handling only a very limited number of operations. Describing and encoding this circuit directly is error-prone; it is difficult to specify the circuit, as well as to understand what it does. One desires to have a domain-specific language and associated compilation tools to specify the relation handled by the cryptographic techniques for ZKP. The specification should mostly be in terms used by common programming languages, exposing only those ZKP-specific details that are highly significant for obtaining a circuit that is handled efficiently by the cryptographic technique. The specified relation should be automatically translated into the arithmetic circuit, while being optimized for the performance profiles of ZKP techniques. Besides the construction of the arithmetic circuit, the language and compilation tools must help in the preparation of common and Prover’s inputs for it. The features of the language have to support the execution by the two parties and a common ZKP protocol between them, with these components not fully trusting each other.
In this paper, we propose ZK-SECREC—a programming language for specifying relations between instances and witnesses, together with a toolchain that produces circuits suitable as inputs for ZKP techniques. In their design, we have aimed to tackle the following issues.

 Execution at multiple locations. A ZK-SECREC program specifies, and the ZK-SECREC compiler produces the description of a circuit, which both Prover and Verifier use as one of the inputs to a cryptographic technique for ZK proofs. Besides running the cryptographic protocol, both parties may need to run some parts of the program locally for the purpose of increasing the efficiency of computations. A ZK-SECREC program can thus specify local computations, and the compiler can produce code that Prover and Verifier will execute with their inputs.

 Compilation into a circuit. Having the arithmetic circuit as an intermediate representation makes our compiler agnostic towards the used cryptographic techniques, and allows the compilation result to be retargeted easily, such that it may be created once and then used multiple times by Prover to convince Verifier that it knows the witnesses for several different instances. Even though the circuit may only contain a very restricted set of operations, targeting only a particular ZKP technology will not significantly increase the supported set. Our type system for ZK-SECREC makes sure that non-supported operations cannot be added to the arithmetic circuit, and that the shape of the circuit and the operations in its nodes are public.

 Witness and instance extension. Adding more inputs to the circuit and verifying that they are correctly related to previous inputs is a pervasive technique for improving the efficiency of computations of the relation. ZK-SECREC allows to freely mix computations on circuit and off circuit. The results of the latter become additional inputs of the circuit.

 Semantics. The statement to be proven is commonly seen as a binary relation, but its meaning is really the left projection of that relation. The semantics of ZK-SECREC precisely defines the meaning of its programs.

 We start this paper in Sect. 2 with an example program in ZK-SECREC, showing off its features. We continue with the description of the syntax of ZK-SECREC in Sect. 3 and its type system in Sect. 4. The execution of a well-typed program can be split between different localities in the manner that we desire, with necessary data available at each locality. An arithmetic circuit, suitable as an input to a ZKP technique, can also be statically extracted from a well-typed program. In Sect. 5 we give a formal semantics of ZK-SECREC, stating the language that is accepted by a ZK-SECREC program. Following up on it, we describe the compilation into an arithmetic circuit in Sect. 6. We continue with the evaluation of expressivity and efficiency of ZK-SECREC, first showing in Sect. 7 how some data structures and methods useful for ZKP can be straightforwardly encoded, and then discussing in Sect. 8 the circuits output by ZK-SECREC compiler for various example problems. We finish the paper in Sect. 9 by comparing ZK-SECREC against other existing languages and means of specifying statements proved in ZK.

2 ZK-SECREC ON AN EXAMPLE

Let us start the description of the language with an example. Suppose that Prover and Verifier both know a large integer $z$. Prover wants to convince Verifier that he also knows a factor $x$ of $z$ such that $1 < x < z$. In ZKP terms, $z$ is the instance and $x$ is the witness. The relation between $z$ and $x$ can be specified in ZK-SECREC as shown in Fig. 1. This program specifies an arithmetic circuit that Prover and Verifier have to execute on top of a ZKP technology of their choice, as well as the local computations they both should perform.

Execution of the program starts from the function `main`. It first loads a public constant `fbw`, determining the size of the inputs handled by the circuit. As next, the two inputs $z$ and $x$ are loaded, with the former being visible to both Prover and Verifier, while the latter is seen by Prover only. Both inputs of the program are made inputs to the arithmetic circuit that the compilation of the
program produces, using the wire construct. The stage $pre$ denotes that the value is only available for the local computations by Prover and Verifier, while $post$ denotes its availability in the circuit. The domain $@prover$ denotes that only Prover knows this value, while $@verifier$ denotes that Verifier also has knowledge of it, and $@public$ means that this value is known at compile time. Note that the domain only informs about the availability of the value in local computations.

The ZKP technology will interpret the operations of the arithmetic circuit over some finite field, and the size of this field may be important in specifying the relation. Hence the data type $\text{uint}[N]$ denotes unsigned integers modulo an integer $N$ (a compile-time parameter). Unbounded integers, available only in $pre$ stage, have the data type $\text{uint}$.

In the next line, Prover expands the witness. In order to show that $\nu_1$ divides $\nu_2$, Prover has to come up with a value $\nu_3$ satisfying $\nu_1 \cdot \nu_3 = \nu_2$. The division operation is only available for local

\begin{verbatim}
fn bitextract_pre[@D](x : $\text{uint}[N]$ $\text{Spre} @D$, fbw : $\text{uint} Spre @public$) -> $\text{list} [\text{bool}[N] Spre @D]$
{
    let rec xx = for i in 0 .. fbw { // generates a list of length fbw
    if (i == 0) { x }
    else { xx[i - 1] / 2 }
    }
    for i in 0 .. fbw { // generates a list of booleans, which is returned
    let b = xx[i] % 2;
    b == 1
    }
}
fn check_bitextract[@D](x : $\text{uint}[N]$ $\text{Spost} @D$, xb : $\text{list} [\text{bool}[N] Spost @D]) {
    let mut s = xb[length(xb) - 1] as $\text{uint}[N]$; // variable s is mutable
    for i in 0 .. length(xb) - 1 { s = 2 * s + xb[length(xb) - i - 2] as $\text{uint}[N]$; }
    assert_zero(x - s);
}
fn bitextract[@D](x : $\text{uint}[N]$ $\text{Spost} @D$, fbw : $\text{uint} Spre @public$) -> $\text{list} [\text{bool}[N] Spre @D]$
{
    let xb_pre = bitextract_pre(x as $\text{Spre} @D$, fbw);
    let xb = for i in 0 .. length(xb_pre) { wire { xb_pre[i] } };
    if (@prover <= @D) { check_bitextract(x, xb); }
    xb
}
fn less_than[@D1, @D2, @D](x : $\text{uint}[N]$ $\text{Spost} @D1$, y : $\text{uint}[N]$ $\text{Spost} @D2$, fbw : $\text{uint} Spre @public$) -> $\text{bool}[N] Spost @D$
{
    let @D1 <= @D, @D2 <= @D {
    let xb = bitextract(x as $\text{Spre} @D$, fbw);
    let yb = bitextract(y as $\text{Spre} @D$, fbw);
    // lexicographic comparison of lists of bits xb and yb omitted
    }
}
fn main() {
    let fbw : $\text{uint} Spre @public$ = get_public("fixed_bit_width");
    let z : $\text{uint}[N] Spost @verifier$ = wire { get_instance("z") };
    let x : $\text{uint}[N] Spost @prover$ = wire { get_witness("x") };
    let y = wire { z as Spre as @prover / x as Spre };
    assert_zero(x * y - (z as @prover));
    assert(less_than(x, z, fbw));
    assert(less_than(y, z, fbw));
}
\end{verbatim}

Fig. 1. A ZK-\text{SECREC} program for verifying that Prover knows a proper factor of a given positive integer
computations. Hence, in the program, the values $z$ and $x$ are turned back into values available for local computations using the \texttt{as $	ext{Spre}$} operation, and divided; the result of the division is turned to another input of the arithmetic circuit. The domain of $z$ as $\text{Spre}$ is still $\text{@verifier}$, because the division operator expects its arguments to have equal types. The type of $y$ is automatically inferred as \texttt{uint[N]} \text{post $\text{@prover}$}. The types of $\text{fbw}$, $z$, and $x$ could have been inferred automatically, too.

The next line in the function \texttt{main} specifies the check that the expression $x \cdot y - z$ is evaluated to zero, i.e., the product of $x$ and $y$ indeed equals $z$. Multiplication and subtraction operations are available in \texttt{Spost} stage (both expect arguments to have equal types), likewise is the check that a number is zero. Even though the previous line has stated that Prover should compute $y$ as $z/x$, Verifier cannot trust that it was computed like this, hence this check is necessary. The computations done in the circuit are trusted by Verifier.

The last two lines in \texttt{main} check that $x < z$ and $y < z$. The comparison is made by obtaining the bitwise representations (of width $\text{fbw}$) of both arguments, which can be straightforwardly compared. We see that the function \texttt{less_than} is polymorphic in its argument and result domains. The size $N$ also matters for booleans, because they are represented as integers when translated to the arithmetic circuit.

The bit representations are computed by the function \texttt{bitextract}. Well, they are actually computed in the stage \texttt{Spre} by the function \texttt{bitextract\_pre}, making use of operations not available in the arithmetic circuit. The result is then added as inputs to the circuit by \texttt{bitextract} that also checks its correctness if needed. The correctness check is necessary only if the argument and the result are in the domain $\text{@prover}$. In the case of domains of lower privacy, Verifier can check correctness of the result directly. Representing some integer as a sequence of bits is a typical instance/witness expansion.

As this example shows, ZK-SECREC (intentionally) enables interleaving circuit computation with local computation and specifying both of them within one language. Although one cannot be sure that computations outside the circuit are performed exactly like in the code, specifying the intended behavior using the same notation is good for readability and reduces the amount of code (as ZK-SECREC allows stage polymorphism). As mingling of circuit computation and local computation can be arbitrarily complex, attempts to keep the corresponding pieces of code separate lead to big difficulties.

3 \ THE SYNTAX OF ZK-SECREC

Figure 2 describes the syntax of a subset of ZK-SECREC that contains its most important features via the extended Backus-Naur forms (EBNF). We use the following conventions:

- The l.h.s. and r.h.s. of productions are separated by the symbol ::=;
- Non-terminals and terminals are written in italic and typewriter font, respectively;
- Zero or more repetitions of a term $t$ is denoted by $t^*$;
- An optional occurrence of a term $t$ is denoted by $t?$.

We also use an auxiliary construct $t^{\text{csl}}$ for comma-separated lists (of length 0 or more) of items $t$.

A \textit{program} in our language consists of \textit{function definitions}. Every function definition contains its \textit{signature} and \textit{body}, the latter of which is an expression. Although ZK-SECREC supports type inference, due to which the types of local variables can be inferred, declaring the types of parameters in function definitions is still required. Explicit signatures of top-level definitions make the code more self-explanating. The return type of a function can be omitted; doing this is equivalent to declaring the unit type () as return type.
The type system of ZK-SECREC supports polymorphic function definitions. The example in Fig. 1 showed the use of type parameters (e.g., the @D in brackets in the function definitions). Type predicates are used for restricting type arguments of polymorphic functions. In Fig. 1, predicates @D1 <= @D and @D2 <= @D were used in the definition of less_than to indicate that the result domain is of as high privacy as either argument domain.

The outermost structure of a type represents it as a (curried) function type with zero or more argument types, each of which is a qualified type. All qualified types in ZK-SECREC are triples consisting of a data type, a stage and a domain (the latter two are jointly called qualifiers). Stage and domain are allowed to be omitted. An omitted stage is inferred, whereas an omitted domain is read as the domain @public. All stages and domains are prefixed with characters $ and @, respectively, to simplify parsing and improve readability.

The only primitive types ZK-SECREC currently supports are Booleans, unsigned integers, and the unit type () consisting of a single value. A list is a data structure of linear shape where all elements have equal type; the element type is given to the list type as parameter. The qualifiers of a list type do not necessarily coincide with those of the element type. For example, a list of domain @public can contain elements of domain @prover, meaning that the shape of the list is known to the compiler while the elements are known to Prover only.

- Top-level structure of ZK-SECREC programs:

prog ::= fundef*

fundef ::= fn sig expr;

sig ::= name typarams? params rettype? typred?

typarams ::= [typaram[!]]

typaram ::= name | $name | @name

params ::= (param[!])

param ::= name : type

rettype ::= -> type

typred ::= where typred[!]

typed ::= domain <= domain

- Types:

type ::= qualified rettype?

qualified ::= datatype stage? domain?

datatype ::= uintty | boolty | () | list[type,?]

uintty ::= uint | uint[mod]

boolty ::= bool | bool[mod]

stage ::= Spre | Spost | $name

domain ::= @public | @verifier | @prover | @name

- “Large” expressions and statements:

blockexpr ::= { stmt* smallexpr? }

stmt ::= vardef ; | smallexpr ; | largeexpr ;?

vardef ::= let rec? mut? name (: type)? = expr

ifexpr ::= if expr blockexpr (else blockexpr)?

forexpr ::= for name in expr .. expr { blockexpr}

wireexpr ::= wire { blockexpr}

- “Small” expressions:

assignexpr ::= lvalexpr = expr

lvexpr ::= name | loadexpr

loadexpr ::= lvexpr[expr]

opexpr ::= smallexpr oper smallexpr

callexpr ::= name (expr[!])

castexpr ::= smallexpr as casttype

casttype ::= type | stage | domain

- General classification of expressions:

expr ::= largeexpr | smallexpr

largeexpr ::= blockexpr | ifexpr | forexpr | wireexpr

smallexpr ::= compexpr | primexpr

compexpr ::= assignexpr | loadexpr | opexpr | callexpr | castexpr

primexpr ::= name | uintlit | boollit | (expr)
We classify expressions as large and small according to whether they must contain a statement block or not. A block consists of a sequence of zero or more statements, optionally followed by a small expression. The value of the block is the value of the expression if it is present and the single representative of the unit type otherwise.

A statement is either an expression or a variable definition. A definition can optionally use modifiers rec and mut; the former allows recursion in the definition and the latter makes the defined variable mutable. Mutability status extends to all constituents, e.g., components of an immutable list are immutable. Immutability has been chosen as the default status to reduce the likelihood of programming errors. The type of the defined variable can be omitted due to type inference.

A for loop of the form \( \text{for } x \text{ in } e_1 \ldots e_2 \{ e_3 \} \) introduces a new variable \( x \), executes the loop body (i.e., \( e_3 \)) for each value of \( x \) between the loop bounds (i.e., the values of expressions \( e_1 \) and \( e_2 \)), and returns all values of the loop body in a list. The bounds are half open, i.e., the lower bound is the smallest value included and the higher bound is the smallest value excluded (among those not smaller than the lower bound). ZK-SecREC also has special syntax for list construction that we skip in this paper. A wire expression transforms local values to circuit inputs.

As the grammar shows, the notions of expressions and statements overlap in ZK-SecREC. For instance, the if-then-else construct can be used both as a statement with side effects in the branches and as a pure expression similar to the ternary conditional operator in C-style languages.

Small expressions are furthermore classified as composed and primary, the latter term incorporating expressions that behave as atoms in the shunting-yard precedence solving algorithm (identifiers, integer and Boolean literals, and arbitrary expressions in parentheses).

We have omitted definitions for names and literals as they are intuitive. Most of the given definitions of composed expressions are intuitive, too. The nonterminal opexpr expands to expressions constructed via binary operator application. Standard operator precedence is assumed which allows some pairs of parentheses to be omitted (it is not made explicit in the grammar). A cast expression enables the programmer to convert between different types. One can provide either a complete type expression or just the domain or the stage instead.

4 STATIC SEMANTICS

The security guarantees of ZK-SecREC are established by its type system. Types are checked (and inferred in certain cases) during compile time, hence the type system is part of the static semantics of ZK-SecREC. In this section, we describe a static semantics that traces also effects (e.g., assertions and mutable variable updates) that expression evaluation can cause. Therefore it is really a type and effect system.

For simplicity, we leave type parameters out of consideration. All features provided by type parameters are orthogonal to ZKP aspects of the language. The high-level language can be converted to an internal monomorphic form using standard monomorphization techniques (code duplication, instantiation) since we do not support polymorphic recursion. For analogous reasons, we skip function definitions and user-defined function calls; a few most important or representative built-in functions are considered.

Assertions in the type rules are of the form \( \Gamma \vdash e : t ! D \). Here, \( \Gamma \) is a type environment, \( e \) is an expression, \( t \) is a qualified type, and \( D \) is an upward closed set of domains, assuming the ordering \( @public <: @verifier <: @prover \) of growing privacy. A type assertion states that under the constraints imposed by \( \Gamma \), the expression \( e \) has type \( t \) and running it can cause effects in domains belonging to the set \( D \). For example, an assignment to a mutable variable whose domain is @prover causes an effect in domain @prover. A type environment consists of the following kinds of components:
Fig. 3. Typing rules of expressions and statements of a subset of ZK-SECREC without parametric polymorphism

- Variable typings written in the form \( x : q \), where \( x \) is a variable and \( q \) is a qualified type;
- Mutability statements in the form \( \text{mut} x : b \) where \( x \) is a variable and \( b \) is its mutability status (0 or 1).

We write \( (x : q), \Gamma \) to denote a new type environment that maps the variable \( x \) to type \( q \) and treats other variables like \( \Gamma \).

The static semantics is presented in Fig. 3. The data type derivation parts of the rules are standard. Hence we mainly comment on stages, domains and effects. There exist four upward closed domain sets linearly ordered by inclusion: \( \emptyset \subset \langle @prover \rangle \subset \langle @verifier \rangle \subset \langle @public \rangle \), where \( \langle d \rangle \) denotes the set consisting of domain \( d \) and all larger domains. Thus the union of upward closed sets always equals the largest set in the union. We also order stages as \( \langle \text{Spost} \rangle : \langle \text{Spre} \rangle \), reflecting that data computed in the circuit are also computed locally in the corresponding domain in order to be able to provide expanded instances/witnesses to the circuit. ZK-SECREC requires conversion to supertype to be made explicit using the \( \text{as keyword.} \)

The first rule in Fig. 3 handles missing expression \( e \); it is needed for the case where the last expression of a block is absent. Integer and Boolean literals are denoted by overlined constants. We show only rules for \( \text{uint}[N] \) and \( \text{bool}[N] \); rules for \( \text{uint} \) and \( \text{bool} \) in stage \( \text{Spre} \) are similar. Literals can be typed with any stage and domain (i.e., no type cast is required). The effect of literals depends on their actual stage: In stage \( \text{Spre} \) they do not have any effect, while in stage \( \text{Spost} \), they have public effect since there they contribute to constructing of the circuit. In general, any operation in the circuit is considered a public effect. To specify the possible effects concisely in the rules, we extend the \( \langle \cdot \rangle \) notation to stages by \( \langle \text{Spre} \rangle = \emptyset, \langle \text{Spost} \rangle = \langle @public \rangle \), and also to data types by \( \langle t \rangle = \emptyset \) if \( t \) is a primitive type and \( \langle t \rangle = \langle t' \rangle \cup \langle s' \rangle \cup \langle d' \rangle \) if \( t = \text{list}[t'; s'; d'] \). (This way, the result of the \( \langle \cdot \rangle \) operation is always an upward closed set of domains.)

Types of variables are read directly from the type environment without an effect. The rule for addition allows this operation to be performed in any domain and stage, but the domain of the arguments and the result must be the same and similarly for stages. The rule for assert establishes the result data type to be the unit type. The unit type, as well as list types, is always in the stage \( \text{Spre} \) since the circuit does not deal with values of these types.

In any domain, the slice of the program available to local computation of that domain can read input data. The ZK-SECREC functions for that are \( \text{get_public}, \text{get_instance} \) and \( \text{get_witness} \) which
read the public constants, the instance, and the witness, respectively (all non-expanded). Here we
denote these functions uniformly by \( \text{get}_d \) where \( d = \text{var} \), \( d = \text{getter} \) and \( d = \text{prover} \),
respectively, so the rule for \( \text{get}_d \) captures all three cases. By \( \text{datatype}(x) \), we denote the entirely
qualifier-free “pure data type” of the value of field \( x \) in the input file; the auxiliary function \( \text{allpre}_d \)
qualifies it pervasively with stage \( \text{pre} \) and domain \( d \). For instance, if \( u = \text{list}(\text{uint}) \) and \( d = \text{getter} \)
then \( \text{allpre}_d(u) = \text{list}(\text{uint} \text{ pre} \text{ verifier}) \text{ getter verifier} \); the formal definition by data type structure
is the following:

\[
\text{allpre}_d(u) = \begin{cases}
    u \text{ pre} d & \text{ if } u \text{ is a primitive type } \\
    \text{list}(\text{allpre}_d(u')) \text{ pre} d & \text{ if } u = \text{list}(u')
\end{cases}
\]

(The “pure data types” need not be expressible in ZK-SECRET. In ZK-SECRET, list element types are
always qualified, and what are called unqualified data types of ZK-SECRET are lacking qualifiers
only at their outermost level.)

The rules for conditional expressions and loops restrict the stage of guards to \( \text{pre} \), meaning that branchings happen only in local computations. For conditional expressions, the restriction
\( \langle d' \rangle \supseteq \langle s \rangle \cup \langle d \rangle \cup D_2 \cup D_3 \) serves the following purposes:

- \( \langle d' \rangle \supseteq \langle s \rangle \) ensures that, if the expression is computed by the circuit, the compiler knows
  which branch the circuit should choose;
- \( \langle d' \rangle \supseteq \langle d \rangle \) (i.e., \( d' < : d \)) guarantees that the value of the branch chosen can give no hint
  about the value of the boolean condition to a less private domain;
- \( \langle d' \rangle \supseteq D_2 \cup D_3 \) does not allow the effects of the branch chosen to give similar hints.

The restriction \( \langle d' \rangle \supseteq \langle s \rangle \cup \langle d \rangle \cup \text{D_2} \cup \text{D_3} \) in the loop rule provides similar guarantees. For example, if the
body of a loop performs assertions then the loop bounds must be in the domain \( \text{var} \). We omit
a rule for conditionals without an else branch as if \( e_1 \{ e_2 \} \) is a syntactic sugar for if \( e_1 \{ e_2 \} \text{ else } \{ \} \).

A type cast can extend the type of a given expression but not reduce it. We present the rule
for the most general variant of type cast as the others are analogous. The condition \( \langle d' \rangle \supseteq \langle t \rangle \) of the
cast rule guarantees a type invariant that prevents list elements from revealing information
about the list structure to domains of lower privacy (via lookups). The type invariant is formally
established in Definition 4.1 and Theorem 4.2.

Let \( \text{var} l \) for any L-value expression \( l \) denote the variable whose mutability permits assignment
to this L-value, i.e., \( \text{var} l = l \) if \( l \) is a variable, and \( \text{var} l = \text{var} l' \) if \( l = l'[e] \). The assignment rule
states that the return value of an assignment expression is of the unit data type and the effect
of an assignment belongs to the domain \( \text{var} \) if the operands are in stage \( \text{post} \), otherwise the
effect belongs to the same domain \( d \) as the operands.

In the list element access rule, the index must be in the same domain as the list structure since
accessing an element via its index may reveal information about the length of the list. Reading
does not introduce new effects.

In the rule for let statements, \( \text{mut} \) stands for \( \text{mut} \) keyword and \( \text{mut} \) means empty string, so the
rule applies to both immutable and mutable variable definitions. Mutability information is reflected
in the type environment when type checking the expression \( e_2 \). The rule assumes that the type of
the defined variable is explicitly denoted; in reality, we allow the type annotation to be omitted if
the type can be inferred. We consider variable definition to be effectful, whence \( \langle d_1 \rangle \) is added into
the set of effects. The last rule similarly handles statement sequences but applies to the case where
the first statement does not define new variables.

The following definition and theorem are essential. Proof of the theorem goes by induction on
the structure of \( e \).

8
**Definition 4.1.** Let \( q = t \cdot s \cdot d \) be a qualified type. We call \( q \) well-structured if either \( t \) is a primitive type, whereby \( s = \text{Spre} \) in the case of \( t = () \), or \( s = \text{Spre} \) and \( t = \text{list}(t' \cdot s' \cdot d') \) such that \( \langle d \rangle \supseteq \langle s' \rangle \cup \langle d' \rangle \) and \( t' \cdot s' \cdot d' \) is well-structured. Call a type environment \( \Gamma \) well-structured if all qualified types occurring in it are well-structured.

**Theorem 4.2.** If \( \Gamma \vdash e : q \!\! : D \) with a well-structured \( \Gamma \) then \( q \) is well-structured.

## 5 Dynamic Semantics

We present dynamic semantics of our language in a denotational style. In fact, the overall setting assumes four different dynamic semantics, loosely corresponding to the views of three domains and the circuit. We call the three semantics corresponding to the domains local since they describe what is computed by different parties locally. For example, Prover’s view contains all computations that are performed as \text{Spre} @\text{prover}, Verifier’s view contains all computations performed as \text{Spre} @\text{verifier}, etc. Values of all expressions and statements in Prover’s domain are unknown from Verifier’s point of view; we denote the unknown value by \( \top \). Likewise, the local semantics for public domain evaluates all expressions and statements in the higher domains to \( \top \). Computations in the public domain are performed by the compiler.

As \text{Spre} and @\text{prover} are the topmost elements of the stage and domain hierarchy, Prover’s view encompasses the whole program. Basically, this view describes what should actually happen, nevertheless ignoring the special way computation is performed in the circuit due to limited supply of operations.

The circuit semantics describes computations performed as \text{Spost} @D for any D, and also everything in \text{Spre} @\text{public}. Although not computed by the circuit, values in \text{Spre} @\text{public} are inevitably needed in performing branching computations as conditions of if expressions and loop bounds belong to stage \text{Spre}. In reality, the compiler unrolls conditionals and loops for the circuit as the latter has no means for branching.

We define all three local semantics via a common set of equations. The differences arise from domain inclusion conditions that can be either true or false depending on the party and can introduce \( \top \). The type system ensures that computing the program parts of the lower domains is not impeded by not knowing the values of the higher domains.

The notation and types are summarized in Fig. 4. The set of values that our semantics can produce consists of non-negative integers, booleans \( \texttt{tt} \) and \( \texttt{ff} \), the only value of the unit type, and finite sequences of (possibly unknown) values. The equation for the value type \( V \) is recursive and defines \( V \) as the least fixpoint satisfying the equation. By \( 1 + A \), we denote the disjoint sum of a singleton set and set \( A \); in the case of \( MV \), we assume \( 1 = \{ \top \} \). Thereby, we refer to the elements of the main summand (i.e., not \( 1 \)) as pure.

The local semantics of an expression takes a value environment and the input of its domain as arguments, and normally produces a triple containing the value of the expression, an updated environment and a finite sequence of values to be delivered to the circuit. Although not made explicit, we assume the local inputs to contain known values only (any party can read its own input completely). In exceptional cases, the semantics can fail, which is shown by the addend \( 1 \) in the equation for \( C_d A \) and means a runtime error. For simplicity, we ignore runtime errors other than assertion failures in the paper; in practice, all other runtime errors are considered semantically equivalent to a failed assertion.

The definition of local semantics is given in Fig. 5. The notation of syntactic objects coincides with that in the type rules (e.g., \( x \) stands for a variable etc.); in addition, \( \gamma, \phi \) and \( o \) denote value environments, inputs and outputs, respectively. To avoid the need to study exceptional cases separately, we use the monad comprehension syntax of the functional programming language Haskell.
This notation was first advocated by Wadler [26] for succinct description of computations that may involve side effects. We only use the notation for the maybe monad $A \mapsto 1 + A$. For example, the sum of values $\delta_1, \delta_2 \in MV$ (the static type system ensures that their types are correct, but either value can be unknown) is written as

$$\text{do}\{i_1 \leftarrow \delta_1; \ i_2 \leftarrow \delta_2; \ \text{pure}(i_1 + i_2)\}.$$  

Here, the first two clauses define $i_1$ and $i_2$ as pure representatives of $\delta_1$ and $\delta_2$, respectively, and the last clause specifies the sum $i_1 + i_2$ as the final outcome. The latter is wrapped into a monadic value (i.e., an element of $MV$) by function pure. Any of the clauses evaluating to $\bot$ turns the final result $\bot$ immediately. In general, monad comprehension can contain any finite number of clauses, all of which except the last one may bind new pure values. Evaluation is strict and progresses from left to right. Note that here and below, we denote monadic values by letters with hat for clarity.

Since $C_d$ also involves exceptional cases, we use monad comprehension for $C_d$, too. So we have a two-layer monadic specification of semantics ($M$ is inside and $C_d$ outside). In the outer layer, we use function guard that on a false condition raises an exception (and jumps out of comprehension) and has no effect otherwise. Due to the two-layer representation, the unknown value $\bot$ causes no exception in the outer layer.

The semantics of + is included as an example of a built-in operator. Arithmetic is implicitly performed modulo some positive integer if that is required by the type (uint[N]).

Concerning type casts, we show only the variant with domain cast as only domain matters here.

The assignment case additionally uses the following auxiliary functions whose formal definitions are omitted:

- $\text{update}$ takes a variable whose value can be a list with 0 or more dimensions (i.e., a primitive value or a list of primitives or a matrix etc.), an index vector, and a value, and returns a new list where the cell indicated by the index vector has been updated with the given value;
- $\text{lhs}$ assumes an expression that can occur as l.h.s. of an assignment and gives it the form of a pair of a variable and an index vector (without evaluating the indices).

We ignore scopes in the semantics for simplicity. Variables once inserted into the environment remain there forever. Provided that variables have unique names, this does not affect the values of variables in scope.

The circuit semantics is defined mostly analogously; the definition is given in Fig. 6. Again, we present only one case of type cast; the other cases are defined in similar lines. The most important difference from local semantics is concerning the wire construct that reads a value from the output streams of local computations. For this reason, the circuit semantics takes a stream triple as a
Fig. 5. Local dynamic semantics of expressions and statements
supplementary argument. Execution of each wire expression removes the first value from the
stream corresponding to the domain of that expression; the updated stream triple is included in
the result.
We can prove Theorems 5.5 and 5.6 below. Theorem 5.5 states that every party can compute all data that belong to its domain or lower domains despite not knowing values of the higher domains, whereby inputs of higher domains do not influence computation results in the lower domains. Theorem 5.6 establishes that if a program succeeds in Prover’s semantics then it succeeds in the circuit semantics, provided that it is given the same input and Prover’s and Verifier’s correct output. Proofs are by induction on the structure of the expression. The theorems rely on a few notions that basically specify, for a fixed domain’s or the circuit’s point of view, what are good correspondences between monadic values and types, and between two (good) monadic values.

**Definition 5.1.** Let a predicate $P$ on qualified types be fixed. For any well-structured qualified type $q = t\;s\;d$ and $\hat{d} \in MV$, we say that $\hat{d}$ is $q$-exact in $P$ if all the following implications hold:

1. If $P(q)$ and $t$ is a primitive type then $\hat{d} = \text{pure}\, v$ where $v \in t$ (e.g., if $t = \text{bool}[N]$ then $v \in \{\text{tt}, \text{ff}\}$);
2. If $P(q)$ and $t = \text{list}(q')$ then $\hat{d} = \text{pure}(\hat{d}_1, \ldots, \hat{d}_n)$ where $n \in N$ and all $\hat{d}_1, \ldots, \hat{d}_n \in MV$ are $q'$-exact in $P$;
3. If not $P(q)$ then $\hat{d} = \top$.

**Definition 5.2.** For any well-structured qualified type $q = t\;s\;d$ and $\hat{d}, \hat{d}' \in MV$, we say that $\hat{d}$ and $\hat{d}'$ are $q$-coincident if $\hat{d} \neq \top$ and $\hat{d}' \neq \top$ together imply one of the following alternatives:

1. $t$ is a primitive type and $\hat{d} = \hat{d}'$;
2. $t = \text{list}(q')$ and there exists $n \in N$ such that $\hat{d} = \text{pure}(\hat{d}_1, \ldots, \hat{d}_n)$, $\hat{d}' = \text{pure}(\hat{d}_1', \ldots, \hat{d}_n')$, whereby $\hat{d}_i, \hat{d}_i'$ are $q'$-coincident for every $i = 1, \ldots, n$.

**Definition 5.3.** Let $\Gamma$ be a well-structured type environment. For any predicate $P$ defined on qualified types, we say that $\gamma \in \text{Env}$ is $\Gamma$-exact in $P$ if, for every $(x : q) \in \Gamma$, $\gamma(x) \in MV$ exists and is $q$-exact in $P$. We say that $\gamma, \gamma' \in \text{Env}$ are $\Gamma$-coincident if $\gamma(x)$ and $\gamma'(x)$ exist and are $q$-coincident for every $(x : q) \in \Gamma$.

**Definition 5.4.** For any fixed domain $d'$, we shall say “$-exact in $d'$” instead of “$q$-exact in $P$” where $P(t\;s\;d) \Rightarrow (d <: d')$. We shall say “$q$-exact in circuit” instead of “$q$-exact in $P$” where $P(t\;s\;d) \Rightarrow (s = \text{public} \lor d = \text{public})$.

**Theorem 5.5.** Let $\Gamma \vdash e : q \;D$ with well-structured $\Gamma$ and $\gamma \in \text{Env}$ be $\Gamma$-exact in $d$ for some domain $d$. If $\| e \|_d \gamma \phi = \text{pure}(\hat{d}, \gamma', o)$ then $\hat{d}$ is $q$-exact and $\gamma'$ is $\Gamma$-exact in $d$, and $\phi'_d = \phi_d$ for every $d' <: d$ implies $\| e \|_d \gamma \phi' = \| e \|_d \gamma \phi$.

**Theorem 5.6.** Let $\Gamma \vdash e : q \;D$ with well-structured $\Gamma$. For every domain $d$, let $\gamma_d \in \text{Env}$ be $\Gamma$-exact in $d$, and let $\gamma \in \text{Env}$ be $\Gamma$-exact in circuit. Assume that, for $d = \text{public}$, there exist $\hat{d}_d, \gamma'_d, o_d$ such that $\| e \|_d \gamma_d \phi = \text{pure}(\hat{d}_d, \gamma'_d, o_d)$. Then for every domain $d$, there exist $\hat{d}_d, \gamma'_d, o_d$ such that $\| e \|_d \gamma'_d \phi = \text{pure}(\hat{d}_d, \gamma'_d, o_d)$. Moreover, if $\rho$ is any triple of stream continuations then $\| e \|_d \gamma_d \phi(o \rho) = \text{pure}(\hat{d}_d, \gamma'_d, \rho)$ for some $\hat{d}'$ and $\gamma'$ such that $\hat{d}'$ is $q$-exact and $\gamma'$ is $\Gamma$-exact in circuit. Thereby, all $\hat{d}_d$ and $\hat{d}'$ are $q$-coincident, and all $\gamma'_d$ and $\gamma'$ are $\Gamma$-coincident. (Here, $o \rho$ denotes the triple containing concatenations $o_d \rho_d$ for every $d$.)

## 6 Compilation

ZK-SECREC programs are compiled into arithmetic circuits corresponding to the circuit semantics defined in Sect. 5. An arithmetic circuit $C$ over a ring $R$ is a directed acyclic graph, the nodes of which are partitioned into input, constant, and operation nodes, such that each operation node has exactly two incoming arcs and other nodes have none. Additionally, $C$ assigns an element of $R$ to each constant node, an operation — either addition or multiplication — to each operation node,
and a domain — either @prover or @verifier — to each input node in it, and specifies a subset of nodes as output nodes. Also, C defines an enumeration of its input nodes of each domain.

Let 'V' be the set of all nodes in C and I ⊆ V the set of all input nodes. An assignment α ∈ R^I of values to the input nodes extends naturally to an assignment α* ∈ R^V to all nodes (the values for the constant nodes are given in the definition of C and the value for each operation node is found by applying the operation in it to the values of its predecessors). We say that C accepts input α ∈ R^I, if α* assigns 0 to all output nodes. If R is a finite field of characteristic N, then such circuits can be evaluated by various ZKP techniques (perhaps with additional restrictions on N).

Denote the set of all circuits by T. Compilation to an arithmetic circuit proceeds in the lines of the dynamic semantics, using a new monad C_C. To get all types right, we replace the sets V, Env, Out defined in Fig. 4 by V_C, Env_C, Out_C defined in Fig. 7. The main difference is that M V is replaced with M V_C × M T at most places, meaning that value-circuit pairs occur here as results of computation. We call these pairs composite values. The compiler still has to carry values along with circuits for making stage casts from Spost to Spre if necessary. The value and the circuit component of a composite value can independently of each other be missing. For instance, if e is in Spre @public then the value component is known but there is no circuit but if e is in Spost @prover then the circuit exists but the value is unknown. In Spost @public, both components are given. The set Out_C contains output streams of subtrees of the circuit under construction rooted at its output nodes (needed to determine the output nodes). The monad C_C has an extra inner state not occurring in dynamic semantics, a triple of natural numbers, for counting how many input nodes of each domain have been introduced.

Our compilation semantics ⌇ : C_C is presented in Figures 8 and 9. We denote by node[t]((c_1, . . . , c_n)) the circuit with a node of the given class t as root and the circuits referenced by c_1, . . . , c_n as subtrees. Here, n is 0 or 2 depending on t, and t can be con(v) (a node of constant value v), op(⊕) (a node of operation ⊕), or in_d(k) (the input node No k of domain d). So the compiler builds up the arithmetic circuit from small pieces as subtrees. As explained above, we consider a node to be an output node iff it is a root of some output subcircuit. Output occurs in the assert case only.

Any Boolean b is replaced with its representative integer [b] where [tt] = 0, [ff] = 1 because the circuit only does (modular) arithmetic. Representing tt by 0 arises from the fact that a ZK-SECRET program succeeds if all arguments of occurrences of assert evaluate to tt whereas the circuit checks if all its outputs are zero.

Input nodes are created if a wire construct is applied to a value in domain @prover or @verifier. This way, Verifier’s and Prover’s private values passed to the circuit via the wire construct form an assignment that the circuit either accepts or not.

In the clause for get, we use an auxiliary operation alltop that lifts a value in M V to M V_C × M T by pairing its all constituents with T:

\[
\text{alltop}(\hat{\delta}) = \begin{cases} 
(\hat{\delta}, \top) & \text{if } \hat{\delta} \text{ is of a primitive type} \\
(\text{pure(alltop}(\hat{\delta}_1'), . . . , alltop(\hat{\delta}_n')), \top) & \text{if } \hat{\delta} = \text{pure}(\hat{\delta}_1', . . . , \hat{\delta}_n') \nonumber
\end{cases}
\]

\[V_C = \mathbb{N} \cup \mathbb{B} \cup (M V_C \times M T)^* \quad \text{the set of composite values}
\]

\[C_C A = \text{Env}_C \to \text{In}^3 \to \mathbb{N}^3 \to 1 + A \times \text{Env}_C \times \text{Out}_C \times \mathbb{N}^3
\]

\[\text{Env}_C = X \to M V_C \times M T \quad \text{composite value environments}
\]

\[\text{Out}_C = T^* \quad \text{streams of subcircuits}
\]

\[\text{⟦e⟧}_C : C_C (M V_C \times M T) \quad \text{the result of compilation of expression } e
\]

Fig. 7. Types of semantic objects
The results must be monadic since the input nodes do not refer to particular values and must be mapped to ⊤. We also need an analogous operation that takes the circuit input into account. For any circuit \( c \) and \( \pi \in (\mathbb{N}^*)^2 \) (a sequence of input values for both Prover and Verifier), we define

\[
\forall \pi \in (\mathbb{N}^*)^2 \rightarrow (\mathbb{N}^*)^2 : C(\pi) = \begin{cases} 
\begin{array}{ll}
n & \text{if } c = \text{node}[\text{con}(n)] \\
\pi_c(\pi) & \text{if } c = \text{node}[\text{op}(\cdot)](c_1, c_2) \\
\pi_d(i) & \text{if } c = \text{node}[\text{in}_d(i)] 
\end{array}
\end{cases}
\]

The results must be monadic since the input nodes do not refer to particular values and must be mapped to ⊤. We also need an analogous operation that takes the circuit input into account. For any circuit \( c \) and \( \pi \in (\mathbb{N}^*)^2 \) (a sequence of input values for both Prover and Verifier), we define

\[
\forall \pi \in (\mathbb{N}^*)^2 : C(\pi) = \begin{cases} 
\begin{array}{ll}
n & \text{if } c = \text{node}[\text{con}(n)] \\
\pi_c(\pi) & \text{if } c = \text{node}[\text{op}(\cdot)](c_1, c_2) \\
\pi_d(i) & \text{if } c = \text{node}[\text{in}_d(i)] 
\end{array}
\end{cases}
\]

In the following, we refer to the following auxiliary operation \( | \cdot | : T \rightarrow M \mathbb{N} \) that evaluates a given circuit:

\[
| c | = \begin{cases} 
\begin{array}{ll}
\text{pure } n & \text{if } c = \text{node}[\text{con}(n)] \\
\text{do } i_1 \leftarrow | c_1 | ; i_2 \leftarrow | c_2 | ; \text{pure}(i_1 \oplus i_2) & \text{if } c = \text{node}[\text{op}(\cdot)](c_1, c_2) \\
\text{pure}(i) & \text{if } c = \text{node}[\text{in}_d(k)] 
\end{array}
\end{cases}
\]

Fig. 8. Compilation to an arithmetic circuit: Small expressions
This operation always results in a pure value. If the output of compilation of an expression of a statement is 0 then the resulting circuit accepts an input π iff \( a_i(\pi) = 0 \) for all \( i \) such that \( a_i \) exists (where \( a_i \) denotes the \( i \)th component of the sequence \( o \)).

We can prove Theorems 6.5 and 6.6 below. Theorem 6.5 implies that well-typed programs can be compiled into a circuit. Theorem 6.6 states that compilation into a circuit preserves semantics. For establishing the claims in a mathematically precise form, we need several new notions. Definition 6.1 introduces one more exactness property that is analogous to those in Sect. 5. Definition 6.2 introduces an operation that calculates a monadic value that represents the given composite value. It uses the first component of the given composite value if it is pure, and finds the value of the circuit (in the second component of the composite value) on the given input if the circuit exists. This operation is needed to establish correspondence between composite values the compiler is computing with and the monadic values in the circuit semantics. Definition 6.3 introduces an equivalence relation on \( MV \) that identifies integer and Boolean values that are encoded the same in the circuit. Definition 6.4 lifts the introduced operations pointwise to value environments.
Definition 6.1. For any well-structured qualified type \( q = t \cdot s \cdot d \) and \((\delta, \hat{c}) \in M_{VC} \times MT\), we say that \((\delta, \hat{c})\) is \(q\)-exact if all the following implications hold:

1. If \( s = \text{spost} \) then there is a circuit \( c \) such that \( \hat{c} = \text{pure} \) \( c \) and either \(|c| = \top = \delta\) or there exists \( n \in \mathbb{N} \) such that \(|c| = \text{pure} \) \( n \) and \( \delta = \text{pure} \) \( n \) if \( t = \text{uint} \);
2. If \( d = \text{@public} \) and \( t \) is a primitive type then \( \delta = \text{pure} \) \( v \) where \( v \in t \);
3. If \( d = \text{@public} \) and \( t = \text{list}[q'] \) then \( \delta = \text{pure}((\hat{\delta}_1, \hat{c}_1), \ldots, (\hat{\delta}_n, \hat{c}_n)) \) where \( n \in \mathbb{N} \) and all \((\hat{\delta}_1, \hat{c}_1), \ldots, (\hat{\delta}_n, \hat{c}_n) \in M_{VC} \times MT\) are \(q'\)-exact;
4. If \( s = \text{spre} \) then \( \hat{c} = \top \);
5. If \( d \not\equiv \text{@public} \) then \( \hat{\delta} = \top \).

Definition 6.2. Define \((\_\_\) \( \bullet \_\_ : (M_{VC} \times MT) \times (\mathbb{N}^n)^2 \rightarrow M V\) as follows: For every pair \((\delta, \hat{c}) \in M_{VC} \times MT\) and circuit input \( \pi \in (\mathbb{N}^n)^2\),

\[
(\delta, \hat{c}) \bullet \pi = \begin{cases} 
\text{pure} \ n & \text{if } \delta = \text{pure} \ n, \ n \in \mathbb{N} \\
\text{pure} \ b & \text{if } \delta = \text{pure} \ b, \ b \in \mathbb{B} \\
\text{pure} \ i & \text{if } \delta = \text{pure} \ i \\
\text{pure}(c(\pi)) & \text{if } \delta = \top \text{ but } \hat{c} = \text{pure} \ c \\
\end{cases}
\]

Definition 6.3. Define a binary predicate \(\sim\) on \(M V\) as follows: For every pair \((\delta, \delta') \in M V \times M V\), we write \(\delta \sim \delta'\) iff one of the following alternatives holds:

1. \( \delta = \text{pure} \ v \) and \( \delta' = \text{pure} \ v' \) where \( v = v' \) or \(|v| = v'\) or \(|v| = |v'|\) (where \(|\cdot|\) is applied to Boolean values but not to integers);
2. \( \delta = \text{pure} \ i = \delta' ;\)
3. \( \delta = \text{pure}(\hat{\delta}_1, \ldots, \hat{\delta}_n) \) and \( \delta' = \text{pure}(\hat{\delta}'_1, \ldots, \hat{\delta}'_n) \) where \( \hat{\delta}_1 \sim \hat{\delta}'_1, \ldots, \hat{\delta}_n \sim \hat{\delta}'_n \);
4. \( \delta = \top = \delta' \).

Definition 6.4. (1) Let \(\Gamma\) be a well-structured type environment. We say that \(\gamma \in \text{Env}_{VC}\) is \(\Gamma\)-exact iff, for every \((x : q) \in \Gamma, y(x) \in M_{VC} \times MT\) exists and is \(q\)-exact.

(2) For \(\gamma \in \text{Env}_{VC} \) and \(\pi \in (\mathbb{N}^n)^2\), we define \(\gamma \bullet \pi \in \text{Env}\) as \((\gamma \bullet \pi)(x) = y(x) \bullet \pi\).

(3) For \(\gamma, \gamma' \in \text{Env}\), we write \(\gamma \sim \gamma'\) iff \(\gamma\) and \(\gamma'\) are defined on the same set of variables, whereby \(\gamma(x) \sim \gamma'(x)\) for each variable \(x\) in this set.

We also extend \(\sim\) pointwise to sequences (of equal length) of monadic values. For any natural number \(n\) and list \(l\), we write \(\text{drop} n l\) to denote the part of list \(l\) remaining if the first \(n\) elements are removed.

Theorem 6.5. Let \(\Gamma \vdash e : q \cdot ! D\) with well-structured \(\Gamma\). Let \(\gamma \in \text{Env}_{VC}\) be \(\Gamma\)-exact. Let \(\phi \in \text{In}^3\) such that \(\phi_{d}(s)\) is a type correct value for every input key \(s\) and \(d = \text{@public}\). Let \(v \in \mathbb{N}^3\). Then \([e]_{C} \gamma v \phi v = \text{pure}((\delta, \hat{c}), \gamma', o, v')\) for some \(\delta, \hat{c}, \gamma', o\) and \(v'\), whereby \((\delta, \hat{c})\) is \(q\)-exact and \(\gamma'\) is \(\Gamma\)-exact.

In the following theorem:

- Letters without tilde denote the original states and those with tilde denote result states;
- Letters without prime denote compiler states while those with prime denote states in circuit semantics.

Theorem 6.6. Let \(\Gamma \vdash e : t \cdot s \cdot d \cdot ! D\) with well-structured \(\Gamma\). Let \(\gamma \in \text{Env}_{VC}\) be \(\Gamma\)-exact, \(\phi \in \text{In}^3\) be a triple of type correct input dictionaries, and \(v \in \mathbb{N}^3\). Let \([e]_{C} \gamma v \phi v = \text{pure}((\delta, \hat{c}), \gamma, \delta, \hat{c})\). Let \(\pi \in (\mathbb{N}^n)^2, \gamma' \in \text{Env}, o' \in \text{Out}^3\) be such that \(\gamma \bullet \pi \sim \gamma'\) and \(\text{drop} v_{d} \pi_{d} o_{d} \sim o'_{d}\) for both \(d_{o} = \text{@prover}\) and \(d'_{o} = \text{@verifier}\). Then:
(1) If \( e \phi' \phi = \text{pure}(\delta', \gamma', \delta') \) then the circuit output by \( e \phi' \phi \) accepts \( \pi \); moreover, \( \gamma' \pi \sim \gamma' \) and drop \( \tilde{\gamma}_d \pi_d \sim \tilde{\delta}'_d \), for both \( d^0 = @\text{prover} \) and \( d^0 = @\text{verifier} \), and \( d = @\text{public} \) or \( s = S_{\text{post}} \impliedby (\delta, \tilde{\gamma}) \bullet \pi \sim \tilde{\delta}' \);

(2) If \( e \phi' \phi = \top \) then the circuit output by \( e \phi' \phi \) does not accept \( \pi \).

The theorems are proved by structural induction on \( e \).

7 USEFUL CONSTRUCTIONS

Our choice of the details of the type system is validated by the ease of implementing certain constructions that often occur in the statements to be proved in ZK. We will describe them next, after two simple extensions to ZK-SECREC.

First, whenever \( s_1, \ldots, s_n \) are identifiers and \( t_1, \ldots, t_n \) are qualified types, then \( \{ s_1 : t_1, \ldots, s_n : t_n \} \) is a qualified type. Records are defined by specifying the values of all fields, and the values of the fields can be read. The record itself is implicitly qualified \( \text{spre} @\text{public} \). ZK-SECREC allows to declare type synonyms for record types using the keyword \text{struct}.

Second, parameters of functions can also be passed by reference. Changes to a by-reference parameter inside the called function are visible in the calling function. A parameter preceded by the keyword \text{ref} in a function declaration is passed by reference.

**Bit extraction.** Arithmetic circuits naturally support addition and multiplication of values. To compare two values, they have to be split into bits as in Fig. 1. We see that each invocation of \text{less_than} causes the bits of both of its arguments being computed. Hence the value of variable \( z \) is split into bits twice, resulting in different lists of bits created by the \text{wire} expression in the function \text{bit_extract}, but with equal values.

It is conceivable that an optimizing compiler is able to detect that these lists of bits have to be equal. However, in a DSL like ZK-SECREC, we prefer to be able to explicitly indicate the availability of such language-specific optimizations. If the value of a variable \( v \) of type \text{uint}[N] \( S_{\text{post}} @D \) could be split into bits, then we could define it with the following type instead (note polymorphism over \( N \), as well as @D):

\begin{verbatim}
struct uint_with_bits[N : Nat, @D] {
    value : uint[N] $post @$D,
    hasBits : bool $spre $public,,
    bits : list[bool][N] $post @$D
}
\end{verbatim}

When creating or updating the value, we define the field \text{value} and set \text{hasBits} to false. The function \text{bit_extract} checks the field \text{hasBits} of its argument (which is passed by reference) and returns the content of the field \text{bits} if it is true. The function performs the actual splitting into bits only if the field \text{hasBits} is false; in this case it updates \text{bits} and sets \text{hasBits}.

**Dictionaries.** Dictionaries generalize arrays, allowing the keys (indices) to come from any set, not just from a segment of integers, and supporting the operations \text{load}(key) and \text{store}(key, value). In ZK-SECREC, lists (which play the role of arrays) have rather restrictive typing rules associated with them, making sure that computations with them can be converted into circuit operations. Having the keys in \text{Spost} and in an arbitrary domain requires the use of Oblivious RAM (ORAM) [11], which has had a number of solutions proposed in the context of ZK proofs [5, 20, 28]. A general method for ORAM in ZKP context [28] performs no correctness checks while the \text{load} and \text{store} operations are executed. Rather, all checks for a particular dictionary will be performed when it is no longer used; these checks involve sorting the list of operations by the values of keys, and checking that the equality of certain keys implies the equality of accompanying values. We refer to [28] for details.
struct keyValuePair[N : Nat,$S,@D] {
    key : uint[N]$S @D,
    val : uint[N]$S @D
}

struct operation[N : Nat,@D] {
    kv : keyValuePair[N,$post,@D],
    op : bool $pre @public
}

struct map[N : Nat,@D] {
    bindings : list[keyValuePair[N,$pre,@D]],
    log : list[operation[N,@D]]
}

fn create[N : Nat,@D] () -> map[N,@D] {
    { bindings = [], log = [] }
}

fn load[N : Nat,@D](ref d : map[N,@D], k : uint[N]$post@D) -> uint[N]$post@D {
    let v : uint[N]$post@D = wire { find(d.bindings, k as $pre) };
    d.log = cons({ kv = { key = k, val = v }, op = false }, d.log);
    v
}

fn store[N : Nat,@D](ref d : map[N,@D], k : uint[N]$post@D, v : uint[N]$post@D) {
    d.bindings = update(d.bindings, k as $pre, v as $pre);
    d.log = cons({ kv = { key = k, val = v }, op = true }, d.log);
}

fn finalize[N : Nat,@D](ops : list[operation[N,@D]]) {
    // sorting and assertions omitted
}

Fig. 10. Implementing dictionaries

The type and the supported operations of a dictionary are given in Fig. 10. During the execution, the performed operations have to be recorded. Both the load and store operations log the key, the value, and the performed operation (where false means loading, and true means storing). When logging, the key and the value (which may be private) are kept in the stage $post, thereby fixing the values in the circuit. The performed operation, however, is public, because load and store can only be invoked in a public context. The functions find and update work with values in the stage $pre only, looking up a binding or updating them; their specification is mundane. The function finalize is run on the field log of a dictionary before that dictionary goes out of scope, it implements the checks in [28]. ZK-SECREC contains syntactic sugar for dictionary creation, and loading and storing of values. It also automatically adds the calls to finalize at the end of the block containing the creation of the dictionary.

Depending on the number of loads and stores, as well as other characteristics of the operations of the dictionary, different ways of performing finalization may be most efficient [17]. If these characteristics can be derived or predicted from public data, the best way can be chosen in finalize.

8 EVALUATION

We have implemented ZK-SECREC compiler in about 20 kLoC of Haskell code, which includes the parser, type-checker, @public precomputation engine, and translator to circuits. With it, we illustrate the expressiveness and efficiency of the ZK-SECREC language on a number of small examples.
Table 1. Size of circuits

| Ex.      | size       | inst. wit. size | operations | exec. | RAM (MB) | n/w MB |
|----------|------------|-----------------|------------|-------|----------|--------|
| F.       | N/A        | 123 124         | 2.1k 845   | 1.47  | 327 329  | 7.3    |
| Millionaires | A1 A2 | 10 10 | 0 1.4k 16k 3.9k | 1.35 | 328 330 | 8.1 |
|          | 10 50      | 0 4.0k 46k 11k  | 1.39 | 332 330 | 8.8 |
|          | 50 50      | 0 6.5k 75k 18k  | 1.39 | 338 332 | 9.5 |
|          | 500 1k     | 0 96k 1.1M 278k | 3.05 | 468 466 | 32 |
|          | 10k 50k    | 0 3.8M 45M 11M  | *  *  *  * |
| Subset check | A1 A2 | 10 10 | 10 1.4k 30k 5.8k | 1.54 | 328 330 | 8.5 |
|           | 10 50      | 50 4.4k 97k 20k  | 1.47 | 334 333 | 10 |
|           | 50 50      | 50 7.4k 168k 36k | 1.63 | 341 338 | 11 |
|           | 500 1k     | 1k 117k 2.9M 703k | 3.47 | 570 525 | 76 |
|           | 10k 50k    | 50k 5.0M 137M 37M | *  *  *  * |
| Curve    | P-256      | 2 1.4k 22k 4.9k  |    |      |        |
|           | P-384      | 2 1.9k 33k 7.2k  |    |      |        |
|           | P-521      | 2 2.5k 44k 9.7k  |    |      |        |
| SSSP     | 10 10      | 0 5.1k 79k 17k  | 1.26 | 331 335 | 8.9 |
|           | 10 50      | 0 13k 175k 38k  | 1.35 | 346 335 | 11 |
|           | 50 50      | 0 40k 662k 139k | 1.69 | 392 370 | 22 |
|           | 500 1k     | 0 769k 13M 2.7M  | *  *  *  * |
|           | 10k 50k    | 0 35M 590M 125M | *  *  *  * |

* did not compile with EMP tools in reasonable time and resources

We have implemented them, and translated them to arithmetic circuits. The ZK-SECREC source of our examples is given in the supplementary material. Table 1 shows the size of generated circuits, including the number of inputs (both for the instance, and the witness), and different arithmetic operations. We distinguish between linear and non-linear (i.e. multiplication) operations, as only the latter are costly for a cryptographic ZKP technique. If our generated circuits compute modulo $2^{61} - 1$, then they can be ingested by EMP toolkit [29], in which case we also report the running time, network traffic, and memory use of Prover and Verifier. The Verifier process, running on a laptop, talks to a Prover process running on a server. They are connected over a link with 5 ms latency and bandwidth of around 450 Mbit/s.

Knowing the factors. Our first example is the program in Fig. 1, with performance reported in row “F” of Table 1.

Historical millionaires. In this example [2], two millionaires are comparing their net worths over time, and want to figure out whose minimum net worth was larger. In [2], this was an example for secure two-party computation. In our setting, we let both arrays of historical net worths (expressed as integers) to be a part of the witness, and check that the smallest element of the first array is larger than the smallest element of the second array. As such, the instance of the relation appears to be empty, and the relation itself trivially satisfied. The setting is still interesting if we consider it to be part of some larger application where Prover has committed to both arrays, the commitments are part of the instance, and the verification that the witness matches the commitments happens elsewhere.

Subset check. In this example, which is part of a larger application developed in ZK-SECREC, the input consists of two arrays, one held by Verifier and the other one by Prover, and Prover wants
to convince Verifier that all elements of the first array are also elements of the second. We show this by sorting the concatenation of two arrays and checking that each element which originated from the first array is followed by an element which is equal to it.

**ECDSA verification.** In this example (again part of a larger application), Verifier has a public key for the ECDSA digital signature scheme [16]. Prover wants to convince Verifier that he has a message digest and a signature that verifies with respect to the public key. We express this by implementing the ECDSA verification procedure in ZK-SECREC. The main part of this verification is the computation of two scalar multiplications in the elliptic curve group, where the scalars originate from the signature and the digest. In one of them, the multiplied point is the public key. We use the standard double-and-add method for computing the scalar multiple. In the second multiplication, the multiplied point is the generator of the group. The second point is public, and we use a windowed method to compute its scalar multiples. The implementation of point doublings and additions follows their standard definitions, which require the inversion of certain elements of the field, which we have implemented with the standard compute-and-verify technique.

In ECDSA verification, computations are performed in two different fields. The computing of scalar multiplications takes place in the field \( \mathbb{Z}/p \), over which the elliptic curve has been defined. But some multiplications and comparisons also take place in the field \( \mathbb{Z}/Q \), where \( Q \) is the cardinality of the elliptic curve group. ZK-SECREC thus supports the use of several different moduli (e.g. \( \text{uint}[P] \) and \( \text{uint}[Q] \)) in the same program. In this case, the compilation procedure creates several circuits. In order to relate the computations by different moduli, there is an operation for asserting the equality of two values in \$\text{post} \$ stage, these values may belong to different types \( \text{uint}[N] \). These assertions are translated into assertions about the equality of values in the wires of different circuits. It is up to the cryptographic ZKP technique to correctly interpret the produced circuits and relationships between them. Techniques like MPC-in-the-head [10, 15] can likely handle them, using the ring conversions developed for secure multiparty computation protocols [4].

In Table 1 we report the sizes of the circuit for different standardized curves [16], adding up the numbers of operations for both moduli. As EMP toolkit does not support fields of this size, we do not report any running times.

**Single-Source Shortest Paths (SSSP).** In this example, the input, known by Prover, is a directed graph with weighted edges. The graph has an initial vertex, from which all other vertices are reachable. The graph is represented in sparse manner, i.e. the representation consists of the number of vertices \( n \), the number of edges \( m \) (also known to Verifier), and, for each edge, its starting vertex, its ending vertex, and its length (all known only to Prover). Prover wants to convince Verifier that the distances from a the initial vertex to all other vertices have certain values.

Here Prover finds the shortest distances by himself, as well as the shortest-path tree represented by giving for each vertex (except the initial) the last edge on the shortest path from the origin to this vertex. He makes both of these sets of data available to the computation, after which the circuit checks that they match with the edges. The check consists of three parts. First, for each vertex we check that its reported distance from the origin is equal to the length of the last edge on the shortest path (found through the shortest-path tree), plus the distance of the source vertex of that last edge. Second, for each edge we check that it is relaxed, i.e. the distance to its starting vertex, plus its length, is not less than the distance to its ending vertex. Third, we check that the shortest-path tree is indeed a tree rooted in the origin vertex. We do this by pointer-jumping [7, Sec. 30.1] the shortest-path tree for \( \lceil \log n \rceil \) times, and check that in the result, all pointers point to the origin vertex.

Due to the sparse representation of both the graph itself and the shortest-path tree, the SSSP example makes extensive use of dictionaries (Sec. 7). It is a nice example of interleaving computation and verification. The shortest distances, and the shortest-path tree are verified in the circuit,
which involves the computation of permutations sorting the keys used in the dictionaries, which are verified by comparing the neighboring elements in the sorted list of keys, which requires the computation of bit extractions of these elements, the correctness of which is again verified.

9 RELATED WORK

The ZKP use-case proliferation has brought with it a number of tools for either generating the circuits for some cryptographic ZK technique, or for directly describing the computation that runs under a ZKP technique. If the toolset contains a domain-specific language, then its features may often be given the following description in terms ZK-SECREC. The included DSL has mainly imperative features, it is strongly typed, and the possible qualifiers of the types are $post @prover and $pre @public. The latter type is used in computations defining the structure of the circuit (e.g. for loop counters), while the computations with the former are translated into the circuit or invoked using the ZKP technique. The instance and witness to the computation both receive the qualifiers $post @prover. The language also allows local computations. Typically, it allows casts from $pre @public to $post @prover, and perhaps to use $post @prover values also in the computations at $pre @public, the results of which must be cast back to $post @prover.

Such languages include CIRCOM [1] and ZoKrates [9]. The latter also includes branchings over $post @prover conditions, translating them into executions of both branches, followed by an oblivious choice. We have purposefully excluded such construction, believing it will confuse the developer. The languages to generate circuits (or rank-1 constraint systems) also include SNARKL [25], a DSL embedded in Haskell. The necessary local computations are introduced during compilation, in the style of PINOCCHIO [21] and Setty et al. [24].

PINOCCHIO is an early example of a system for verifiable computation, aiming to make the correctness of the execution of C programs verifiable in a manner that is cheaper than re-running the program. This approach, refined in [8], as well as in [3, 27] for hardware-like descriptions, uses a common high-level language to define the circuit, with any $pre-stage computation inserted through compiler optimizations.

We consider the language of the xJSnark system [17] to be the closest to ZK-SECREC. It follows the description given above. For local computations, it offers blocks of code which take values with both $post @prover and $pre @public qualifiers, and return the results back to $post @prover. While it is more expressive than [1, 9], we consider our type system to be superior to its method for mixing local computations with those on the circuit. Indeed, our type system offers distinction between $pre @prover and $pre @verifier, and the integrity properties inherent in them. We made use of this distinction in Fig. 1. Additionally, in ZK-SECREC, values in the $pre-stage can be long-lived, the usefulness of which we showed in Sec. 7. Finally, we can be polymorphic over stages, allowing same or similar computations performed either locally or in the circuit to be expressed only once.

Hastings et al. [14] review 11 different MPC suites from the point of view of the language support for the specification of secure MPC protocols. While the proposed languages often distinguish between private and public values, only Wysteria [22, 23] offers constructions to specify the computations done by one or several parties, either locally or using a secure multiparty computation protocol. Their handling of parties is very expressive and general, but the notion of malicious parties is lacking. The recent Viaduct suite [2] expands on Wysteria’s type system with integrity types, and incorporates also ZK proofs and other secure computation techniques besides MPC in its back-end. However, Viaduct’s type system does not distinguish between @verifier and @public, and thus does not support the compilation into a circuit. The distinctions between other qualifiers of ZK-SECREC are present in Viaduct. Additionally, the handling of data structures by
Viaduct’s type system is simplified, with e.g. no distinction between read- and write-access to arrays.

ACKNOWLEDGMENTS

This research has been funded by the Defense Advanced Research Projects Agency (DARPA) under contract HR0011-20-C-0083. The views, opinions, and/or findings expressed are those of the author(s) and should not be interpreted as representing the official views or policies of the Department of Defense or the U.S. Government. This research has also been supported by European Regional Development Fund through the Estonian Centre of Excellence in ICT Research (EXITE).

REFERENCES

[1] 2021. The circom Language. Iden3, https://docs.circom.io/circom-language/signals/.
[2] Cosku Acay, Rolph Recto, Joshua Gancher, Andrew C. Myers, and Elaine Shi. 2021. Viaduct: an extensible, optimizing compiler for secure distributed programs. In PLDI ’21: 42nd ACM SIGPLAN International Conference on Programming Language Design and Implementation, Virtual Event, Canada, June 20-25, 2021, Stephen N. Freund and Eran Yahav (Eds.). ACM, 740–755. https://doi.org/10.1145/3453483.3454074
[3] Eli Ben-Sasson, Alessandro Chiesa, Eran Tromer, and Madars Virza. 2014. Succinct non-interactive zero knowledge for a von Neumann architecture. In 23rd USENIX Security Symposium (USENIX Security 14). 781–796.
[4] Dan Bogdanov, Margus Niitsoo, Tomas Toft, and Jan Willemson. 2012. High-performance secure multi-party computation for data mining applications. Int. J. Inf. Sec. 11, 6 (2012), 403–418. https://doi.org/10.1007/s10207-012-0177-2
[5] Jonathan Bootle, Andrea Cerulli, Jens Groth, Sune K. Jakobsen, and Mary Maller. 2018. Aria: Nearly Linear-Time Zero-Knowledge Proofs for Correct Program Execution. In Advances in Cryptology - ASIACRYPT 2018 - 24th International Conference on the Theory and Application of Cryptology and Information Security, Brisbane, QLD, Australia, December 2-6, 2018, Proceedings, Part I (Lecture Notes in Computer Science, Vol. 11272), Thomas Peyrin and Steven D. Galbraith (Eds.). Springer, 595–626. https://doi.org/10.1007/978-3-030-03326-2_20
[6] Benedikt Bünz, Jonathan Bootle, Dan Boneh, Andrew Poelstra, Pieter Wuille, and Greg Maxwell. 2018. Bulletproofs: Short proofs for confidential transactions and more. In 2018 IEEE Symposium on Security and Privacy (SP). IEEE, 315–334.
[7] Thomas H Cormen, Charles E Leiserson, Ronald L Rivest, and Clifford Stein. 2009. Introduction to algorithms. MIT press.
[8] Craig Costello, Cédric Fournet, Jon Howell, Markulf Kohlweiss, Benjamin Kreuter, Michael Naehrig, Bryan Parno, and Samee Zahrur. 2015. Geppetto: Versatile Verifiable Computation. In 2015 IEEE Symposium on Security and Privacy, SP 2015, San Jose, CA, USA, May 17-21, 2015. IEEE Computer Society, 253–270. https://doi.org/10.1109/SP.2015.23
[9] Jacob Eberhardt and Stefan Tai. 2018. ZoKrates - Scalable Privacy-Preserving Off-Chain Computations. In 2018 IEEE International Conference on Internet of Things (iThings) and IEEE Green Computing and Communications (GreenCom) and IEEE Cyber, Physical and Social Computing (CPSCom) and IEEE Smart Data (SmartData). 1084–1091. https://doi.org/10.1109/Cybermatics_2018.2018.00199
[10] Irene Giacomelli, Jesper Madsen, and Claudio Orlandi. 2016. ZKBoo: Faster Zero-Knowledge for Boolean Circuits. In 25th USENIX Security Symposium, USENIX Security 16, Austin, TX, USA, August 10-12, 2016, Thorsten Holz and Stefan Savage (Eds.). USENIX Association, 1069–1083. https://www.usenix.org/conference/usenixsecurity16
[11] Oded Goldreich and Rafail Ostrovsky. 1996. Software Protection and Simulation on Oblivious RAMs. J. ACM 43, 3 (1996), 431–473. https://doi.org/10.1145/233551.233553
[12] Shafi Goldwasser, Silvio Micali, and Charles Rackoff. 1985. The Knowledge Complexity of Interactive Proof-Systems (Extended Abstract). In Proceedings of the 17th Annual ACM Symposium on Theory of Computing, May 6–8, 1985, Providence, Rhode Island, USA, Robert Sedgewick (Ed.). ACM, 291–304. https://doi.org/10.1145/22145.22178
[13] Jens Groth. 2016. On the Size of Pairing-Based Non-interactive Arguments. In Advances in Cryptology - EUROCRYPT 2016 - 35th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Vienna, Austria, May 8-12, 2016, Proceedings, Part II (Lecture Notes in Computer Science, Vol. 9666), Marc Fischlin and Jean-Sébastien Coron (Eds.). Springer, 305–326. https://doi.org/10.1007/978-3-662-49896-5_11
[14] Marcella Hastings, Brett Hemenway, Daniel Noble, and Steve Zdanowicz. 2019. SoK: General Purpose Compilers for Secure Multi-Party Computation. In 2019 IEEE Symposium on Security and Privacy, SP 2019, San Francisco, CA, USA, May 19-23, 2019. IEEE, 1220–1237. https://doi.org/10.1109/SP.2019.00028
[15] Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, and Amit Sahai. 2007. Zero-knowledge from secure multiparty computation. In Proceedings of the 39th Annual ACM Symposium on Theory of Computing, San Diego, California, USA, June 11-13, 2007; David S. Johnson and Uriel Feige (Eds.). ACM, 21–30. https://doi.org/10.1145/1250790.1250794
Before proving Theorem 4.2, we have to establish that allpre_d produces well-structured types.

**Lemma A.1.** Let d be domain. For any “pure data type” u (in the sense of Sect. 4), the qualified type allpre_d(u) is well-structured.

**Proof.** By induction on the structure of u:

- If u is a primitive type then allpre_d(u) = u $pre d$ which is well-structured by Definition 4.1.
- Let $u = \text{list}(u')$. Then allpre_d(u) = list(allpre_d(u')) $pre d$ where allpre_d(u') is well-structured by the induction hypothesis. We know by the definition of allpre_d that allpre_d(u') = $t'$ $s'$ $d'$ where $s' = $pre and $d' = d$. Hence $\langle d' \rangle = \emptyset \cup \langle d \rangle = \langle s' \rangle \cup \langle d' \rangle$ which completes the proof.

**Theorem A.2 (Theorem 4.2).** If $\Gamma \vdash e : q ! D$ with a well-structured $\Gamma$ then q is well-structured.
PROOF. Let \( q = t s d \). We proceed by induction on the structure of \( e \):

- If \( e = e, e = \overline{e}, e = \overline{e} \) or \( e = (l = e') \) then \( t \) is primitive, where \( t = (\) implies \( s = \overline{\text{Spre}} \). Hence \( q \) is well-structured.
- If \( e = x \) then the claim follows from the assumption that \( \Gamma \) is well-structured.
- If \( e = (s) \) then the claim follows from Lemma A.1.
- Suppose \( e = e_1 \uplus e_2 \). Then \( t = \text{uint}[N] \), and \( \Gamma \vdash e_1 : t s d ! D_1 \) and \( \Gamma \vdash e_2 : t s d ! D_2 \). Hence by the induction hypothesis, \( t s d \) is well-structured, and the desired claim follows.
- Suppose \( e = \text{if} \ e_1 \{ e_2 \} \text{else} \{ e_3 \} \). Then \( \Gamma \vdash e_2 : t s d ! D_2 \) and \( \Gamma \vdash e_3 : t s d ! D_3 \). Hence by the induction hypothesis, \( t s d \) is well-structured, and the desired claim follows.
- Suppose \( e = \text{for} \ x \in e_1 . . . e_2 \{ e_3 \} \). Then \( t = \text{list} \{ t' s' d' \}, s = \overline{\text{Spre}} \) and \( \Gamma \vdash e_1 : \text{uint}[N] \overline{\text{Spre}} d \mid D_1 \) \((i = 1, 2)\). \( \Gamma \vdash e_3 : t' s' d' ! D_3 \), whereby \( \langle d \rangle \supseteq \langle s' \rangle \cup \langle d' \rangle \cup D_3 \). By the induction hypothesis for \( e_1 \) and \( e_2 \), the qualified type \( \text{uint}[N] \overline{\text{Spre}} d \) is well-structured. Therefore, the extended type environment \( \langle x : \text{uint}[N] \overline{\text{Spre}} d \rangle \), \( \Gamma \) is well-structured. Hence the induction hypothesis for \( e_3 \) implies that \( t' s' d' \) is well-structured. This completes checking the assumptions of \( \text{list} \{ t' s' d' \} s d \) being well-structured by Definition 4.1, hence the desired claim follows.
- Suppose \( e = e' \) as \( t s d \). Then \( \Gamma \vdash e' : t s d', s' < : s, d' < : d, \) and \( \langle d \rangle \supseteq \langle t \rangle \). By the induction hypothesis, \( t s d' \) is well-structured. If \( t \) is a primitive type then \( t s d \) is well-structured, too, as \( s' = \overline{\text{Spre}} \) implies \( s = \overline{\text{Spre}} \). Now let \( t = \text{list} \{ t'' s'' d'' \}; \) by \( t s d' \) being well-structured, we have \( s' = \overline{\text{Spre}}, \langle d'' \rangle \supseteq \langle s'' \rangle \cup \langle d'' \rangle \) and \( t'' s'' d'' \) well-structured. Therefore \( s = \overline{\text{Spre}} \). Moreover, using \( \langle d \rangle \supseteq \langle t \rangle \) and the definition of \( \langle t \rangle \), we obtain \( \langle d \rangle \supseteq \langle t'' \rangle \cup \langle s'' \rangle \cup \langle d'' \rangle \supseteq \langle s'' \rangle \cup \langle d'' \rangle \). Hence \( t s d \) is well-structured by Definition 4.1.
- Suppose \( e = \text{let} \ x = e' \). Then \( \Gamma \vdash 1 : t' s' d ! D' \) where \( t' = \text{list}[t s d] \). By the induction hypothesis, \( t' s' d' \) is well-structured. Definition 4.1 now implies \( t s d \) also being well-structured.
- Suppose \( e = \text{let} \ x : t' s' d' = e_1; e_2 \). Then \( \Gamma \vdash e_1 : t' s' d ! D_1 \) and \( \langle x : t' s' d' \rangle, \Gamma \vdash e_2 : t s d ! D_2 \). By the induction hypothesis for \( e_1 \), qualified type \( t' s' d' \) is well-structured. Therefore, the extended type environment \( \langle x : t' s' d' \rangle, \Gamma \) is well-structured. Hence the induction hypothesis for \( e_2 \) implies that \( t s d \) is well-structured.
- If \( e = e_1; e_2 \) then \( \Gamma \vdash e_2 : t s d ! D_2 \), whence the induction hypothesis implies \( t s d \) being well-structured.

\[\square\]

B ZK-SECREC SOURCES OF THE EXAMPLE PROGRAMS

B.1 Knowing the factors

The source code of this example is given in Fig. 1. Below, we give the implementation of the \texttt{less\_than} function, which was elided from that figure.

```haskell
fn \text{less\_than} [N :: \text{Nat}, @(D1, @D2, @D)] (x :: \text{uint}[N] \overline{\text{Spost}} @D1, y :: \text{uint}[N] \overline{\text{Spost}} @D2) :: \text{bool}[N] \overline{\text{Spost}} @D
where \overline{\text{D1}} <= @D, \overline{\text{D2}} <= @D
let \overline{xb} = \text{bitextract}(x, \text{field\_bit\_width}(N));
let \overline{yb} = \text{bitextract}(y, \text{field\_bit\_width}(N));
let rec partialCompare =
  let rec partialCompare =
    for i in 0 .. \text{field\_bit\_width}(N) {
      if (i == 0)
        (!xb[0] as @D & !yb[0] as @D)
      else
        ((!xb[i] as @D & !yb[i] as @D))")
```

25
B.2 Historical millionaires

The code below makes use of *tuples*. These are the same as records introduced in Sec. 7, except that their fields are unnamed, and are referenced as `.0`, `.1`, etc.

```plaintext
fn check_bitextract(x : uint[N] $post @prover, xb : list[bool[N] $post @prover]) {
    let rec partialsums : list[uint[N] $post @prover] =
        for i in 0 .. field_bit_width(N) {
            if (i == 0) {
                xb[field_bit_width(N) − 1] as uint[N]
            } else {
                partialsums[i−1] * 2 + xb[field_bit_width(N) − i − 1] as uint[N]
            }
        };
    assert(x == partialsums[field_bit_width(N) − 1]);
}

fn bitextract_pre(x : uint[N] $pre @prover) -> list[bool[N] $pre @prover] {
    let rec xx = for i in 0 .. field_bit_width(N) {
        if (i == 0) { x } else { xx[i−1] / 2 }
    };
    for i in 0 .. field_bit_width(N) {
        let b = xx[i] % 2;
        b == 1
    }
}

fn bitextract(x : uint[N] $post @prover) -> list[bool[N] $post @prover] {
    let xb = wire {[scalar; field_bit_width(N)]} { bitextract_pre(x as $pre) };
    check_bitextract(x, xb);
    xb
}

fn uint_to_bool[N : Nat, @D](x : uint[N] $post @D) -> bool[N] $post @D {
    let x_pre : uint $pre @D = (x as $pre) as uint;
    let b_pre : bool[N] $pre @D = if(x_pre == 0) { false } else { true };
    let b : bool[N] $post @D = wire { b_pre };
    let b_as_uint = b as uint[N];
    assert(b_as_uint == x);
    b
}

// Argument must have length 3
fn lt_threebitselect[N : Nat, @S, @D](bs : list[uint[N] $S @D]) -> uint[N] $S @D {
    let bs12 = bs[1] * bs[2];
    let s = bs[1] + bs[2];
    s − bs12 + (2 * bs12 − s) * bs[0]
}
```
ZK-SECREC: a Domain-Specific Language for Zero-Knowledge Proofs

// Arguments xb and yb must have the same length
fn less_than[N : Nat, @D](xb : list[bool[N] $post @D], yb : list[bool[N] $post @D]) -> bool[N] $post @D {
    let mut res : uint[N] $post @D = (1 - (xb[0] as uint[N]) ∗ (yb[0] as uint[N]));
    for i in 1 .. length(xb) {
        res = lt_threebitselect([xb[i] as uint[N], yb[i] as uint[N], res]);
    }
    uint_to_bool(res)
}

// Arguments le/f_telem and rightelem must have the same length
fn lessEqual[N : Nat, @D](le/f_telem : list[bool[N] $post @D], rightelem : list[bool[N] $post @D]) -> bool[N] $post @D {
    let mut res = 1 - (1 - (rightelem[0] as uint[N]) ∗ (le/f_telem[0] as uint[N]));
    for i in 1 .. length(le/f_telem) {
        res = lt_threebitselect([le/f_telem[i] as uint[N], rightelem[i] as uint[N], res]);
    }
    uint_to_bool(res)
}

fn updMin(c1 : uint[N] $pre @prover, i1 : uint $pre @prover, c2 : uint[N] $pre @prover, i2 : uint $pre @prover) -> tuple[uint[N] $pre @prover, uint $pre @prover] {
    let g = (c2 < c1) as uint;
    let gn = g as uint[N];
    let s1 = c1 + gn ∗ (c2 - c1);
    let s2 = i1 + g ∗ (i2 - i1);
    (s1, s2)
}

fn get_min(l : list[uint[N] $post @prover] $pre @public) -> list[bool[N] $post @prover] $pre @public {
    let mut currmin : uint[N] $pre @prover = l[0] as $pre;
    let mut minidx : uint $pre @prover = 0;
    let llen : uint $pre @public = length(l);
    for i in 1 .. llen { // this loop does the local computation to find the index of the minimum element in a list
        let c : uint[N] $pre @prover = (l[i] as $pre);
        let u = updMin(currmin, minidx, c, i as @prover);
        currmin = u.0;
        minidx = u.1;
    }
    let mut sumSel = 0;
    let selVec = for i in 0 .. llen {
        let r = wire { ((i as @prover) == minidx) as uint as uint[N] };
        assert(r ∗ r == r);
        sumSel = sumSel + r;
        r
    };
    assert(sumSel == 1); // This assertion, and the assertion in previous loop ensure that selVec is a 1-hot 0/1-vector
    let mut minV = 0;
    for i in 0 .. length(l) {
        minV = minV + selVec[i] ∗ [i];
    }
}
let minVb = bitextract(minV);
for i in 0 .. length(l) {
    assert(lessEqual(minVb, bitextract(l[i])));
}
minVb
}

fn main() {
    let alice_pre = get_witness("alice");
    let bob_pre = get_witness("bob");
    let alice = wire([scalar; get_public("alice.length")]) { alice_pre }; // alice : list[uint[N] $post @prover]
    let bob = wire([scalar; get_public("bob.length")]) { bob_pre }; // bob : list[uint[N] $post @prover]
    let alice_min = get_min(alice);
    let bob_min = get_min(bob);
    assert(less_than(bob_min, alice_min));
}

B.3 Subset check

type bitarray[$S, @D, N : Nat] : Qualified = list[bool[N] $S @D] $pre @public;

fn uintArrayToProver[N : Nat, $S, @D](xs : list[uint[N] $S @D]) -> list[uint[N] $S @prover] {
    for i in 0 .. length(xs) { xs[i] as $prover }
}

fn uintArrayToPre[N : Nat, @D](xs : list[uint[N] $post @D]) -> list[uint[N] $pre @D] {
    for i in 0 .. length(xs) { xs[i] as $pre }
}

fn boolArrayToPost[N : Nat, @D](xs : list[bool[N] $pre @D]) -> list[bool[N] $post @D] {
    wire ([scalar; length(xs)]) { xs }
}

fn boolArrayToProver[N : Nat, $S, @D](xs : list[bool[N] $S @D]) -> list[bool[N] $S @prover] {
    for i in 0 .. length(xs) { xs[i] as $prover }
}

fn uint_to_bool[N : Nat, @D](x : uint[N] $post @D) -> bool[N] $post @D {
    let x_pre = uint $pre @D = (x as $pre) as uint;
    let b_pre = bool[N] $pre @D = if(x_pre == 0) { false } else { true };
    let b = bool[N] $post @D = wire { b_pre };
    let b_as_uint = b as uint[N];
    assert(b_as_uint == x);
    b
}

fn bitextract_pre[@D, N : Nat](x : uint[N] $pre @D, fbw : uint $pre @public) -> bitarray[$pre, @D, N] {
    let rec xx = for i in 0 .. fbw {
        if (i == 0) { x } else { xx[i-1] / 2 }
    };
    for i in 0 .. fbw {
        let b = xx[i] % 2;
    }
}
fn check_bitextract\[N : Nat, @D\](x : uint\[N\] $post @D, xb : list[bool\[N\] $post @D], fbw : uint $pre @public) {
    let rec partialsums : list[uint\[N\] $post @D] =
        for i in 0 .. fbw {
            if (i == 0) {
                xb[fbw − 1] as uint\[N\]
            } else {
                partialsums[i−1] * 2 + (xb[fbw − i − 1] as uint\[N\])
            }
        };
    assert(x == partialsums[fbw − 1]);
}

fn bitextract\[N : Nat, @D\](x : uint\[N\] $post @D, fbw : uint $pre @public) -> list[bool\[N\] $post @D] {
    if @D <= @verifier {
        boolArrayToPost(bitextract_pre(x as $pre, fbw))
    } else {
        let xb = boolArrayToPost(bitextract_pre(x as $pre, fbw));
        check_bitextract(x, xb, fbw);
        xb
    }
}

fn bitextract_array\[N : Nat, @D\](xs : list[uint\[N\] $post @D], fbw : uint $pre @public) -> list[list[bool\[N\] $post @D]] {
    let xbs = for i in 0 .. length(xs) {
        bitextract(xs[i], fbw)
    };
    xbs
}

fn append\[T : Unqualified, $S, @D\](x : list[T $S @D] $pre @public, y : list[T $S @D] $pre @public) -> list[T $S $S $S @D] $pre @public {
    for i in 0 .. length(x) + length(y) {
        if (i < length(x)) {
            x[i]
        } else {
            y[i − length(x)]
        }
    }
}

fn is_even_pre[@D](x : uint $pre @D) -> bool $pre @D {
    x % 2 == 0
}

fn mkWaksmanNetwork(n : uint $pre @public) -> list[uint $pre @public] $pre {
if (n == 1) {
    for i in 0 .. 1 { 0 }
} 
else if (n == 2) {
    for i in 0 .. 4 { i }
} 
else {
    let halfnfloor = n / 2;
    let halfnceil = n - halfnfloor;
    let upperhalf = mkWaksmanNetwork(halfnfloor);
    let lowerhalf = mkWaksmanNetwork(halfnceil);
    let upperlen = length(upperhalf) - halfnfloor;
    let lowerlen = length(lowerhalf) - halfnceil;
    let leftlen = if (is_even_pre(n)) { n } else { n - 1 };
    let rightlen = 2 * n - 2 - leftlen;
    let toallen = upperlen + lowerlen + 2 * n - 2;
    for i in 0 .. toallen + n {
        if (i < leftlen) { i }
        else if (i < leftlen + upperlen) {
            let x = upperhalf[i - leftlen];
            if (x < halfnfloor) {
                n + x * 2
            } 
            else {
                n + leftlen - halfnfloor + x
            }
        } 
        else if (i < leftlen + upperlen + lowerlen) {
            let x = lowerhalf[i - leftlen - upperlen];
            if (x < halfnceil) {
                if ((x == (halfnceil - 1)) & (halfnfloor < halfnceil)) {
                    n - 1
                } 
                else {
                    n + 1 + x * 2
                }
            } 
            else {
                n + leftlen + upperlen - halfnceil + x
            }
        } 
        else { if (i < toallen) {
            let inpc = i - upperlen - lowerlen - leftlen;
            let inpsw = inpc / 2;
            if (is_even_pre(inpc)) {
                let x = upperhalf[inpc + inpsw];
                if (x < halfnfloor) {
                    n + x * 2
                } 
                else {
                    n + leftlen - halfnfloor + x
                }
            } 
            else {
                n + leftlen - halfnceil + x
            }
        } 
    } 
}
let x = lowerhalf[lowerlen + inpsw];
if (x < halfnceil) {
  if (x == (halfnceil - 1)) & (halfnfloor < halfnceil)) {
    n - 1
  }
else {
    n + 1 + x * 2
  }
else {
  n + leftlen + upperlen - halfnceil + x
} }
else {
  if (is_even_pre(n)) {
    if (i < totallen + n - 2) {
      i + 2
    } 
else {
      let x = upperhalf[upperlen + halfnfloor - 1];
      if (x < halfnfloor) {
        n + x * 2
      } 
else {
        n + leftlen - halfnfloor + x
      }
    } 
else {
      let x = lowerhalf[lowerlen + halfnceil - 1];
      if (x < halfnceil) {
        if (x == (halfnceil - 1)) & (halfnfloor < halfnceil)) {
          n - 1
        }
      } 
else {
        n + 1 + x * 2
      }
    } 
else {
      n + leftlen + upperlen - halfnceil + x
    }
  } 
else {
  if (i < totallen + n - 1) {
    i + 1
  } 
else {
    let x = lowerhalf[lowerlen + halfnceil - 1];
    if (x < halfnceil) {
```plaintext
if ((x == (halfnceil - 1)) & (halfnfloor < halfnceil)) {
    n - 1
} else {
    n + 1 + x * 2
} else {
    n + leftlen + upperlen - halfnceil + x
}
}
}
}
}
}
}
}

// The following function returns the values of the switches for the Waksman network, such that the resulting permutation will sort the argument.
// The functions __get_sorting_permutation and __permutation_switches realize parts of this functionality. They work with data in $pre.
fn getSwitchesForSorting[N : Nat, @D](x : list[uint[N] $pre] Spre @D] Spre @public) -> list[bool[N] $post @D] Spre @public {
    let y = for i in 0 .. length(x) { x[i] as uint ];
    let sortPermutation : list[uint Spre @D] = __get_sorting_permutation(y);
    let result : list[bool Spre @D] = __permutation_switches(sortPermutation);
    for i in 0 .. length(result) { wire { result[i] as bool[N] } }
}
}

fn applySwitchingNetwork_uint[N : Nat, @D](x : list[uint[N] $post @D], netDesc : list[uint Spre @public] Spre, switches : list[bool[N] $post @D]) -> list[uint[N] $post @D] {
    // netDesc lists for each switch the index of its upper and its lower input. After the inputs of switches, the last length(x) elements give the elements of the array, from where to read the permuted network. The length of netDesc must be twice the length of switches, plus the length of x
    let ll = length(x);
    let numsw = length(switches);
    let rec content =
        for i in 0 .. length(netDesc) + ll {
            if (i < ll) {
                x[i]
            } else {
                if (i < length(netDesc)) {
                    let ill = i - ll;
                    let isUpper = is_even_pre(ill);
                    let upperval = content[netDesc[i]] if isUpper { ill } else { ill - 1 ]];
                    let lowerval = content[netDesc[i]] if isUpper { ill + 1 } else { ill ]];
                    if (isUpper) {
                        let swaddr = ill / 2;
                        upperval + (switches[swaddr] as uint[N]) * (upperval - lowerval)
                    } else {
                        upperval + lowerval - content[i-1]
                    }
                } else {
                    content[i]
                }
            }
        }
```
ZK-SECRET: a Domain-Specific Language for Zero-Knowledge Proofs

```plaintext
fn impl_xor[N : Nat, @D](x : bool[N] $post @D, y : bool[N] $post @D) -> bool[N] $post @D {
(x & !y) | (y & !x)
}

fn applySwitchingNetwork_bool[N : Nat, @D](x : list[bool[N] $post @D], netDesc : list[uint $pre @public] $pre,
switches : list[bool[N] $post @D]) -> list[bool[N] $post @D] {
// netDesc lists for each switch the index of its upper and its lower input. After the inputs of switches, the last length(x) elements give the elements of the array, from where to read the permuted network. The length of netDesc must be twice the length of switches, plus the length of x
let ll = length(x);
let numsw = length(switches);
let rec content =
  for i in 0 .. length(netDesc) + ll {
    if (i < ll) {
      x[i]
    } else { if (i < length(netDesc)) {
      let ill = i - ll;
      let isUpper = is_even_pre(ill);
      let uperval = content[netDesc[if (isUpper) { ill } else { ill - 1 }]];
      let lowerval = content[netDesc[if (isUpper) { ill + 1 } else { ill }]];
      if (isUpper) {
        let swaddr = ill / 2;
        impl_xor(lowerval, switches[swaddr] & impl_xor(uperval, lowerval))
      } else {
        impl_xor(uperval, impl_xor(lowerval, content[i - 1]))
      }
    } else { content[netDesc[i - ll]]
    }
  }
  for i in 0 .. length(x) {
    content[i + length(netDesc)]
  }
}

fn threebitselect[N : Nat, S, @D](bs : list[uint[N] $S @D], vs_public : list[uint[N] $S @public]) -> uint[N] $S @D {
let vs = for i in 0 .. 8 { vs_public[i] as @D };
let bs12 = bs[1] * bs[2];
```

let $s_0 = v_{s[0]} + (v_{s[2]} - v_{s[0]}) \cdot b_{s[1]} + (v_{s[4]} - v_{s[0]}) \cdot b_{s[2]} + (v_{s[6]} - v_{s[4]} - v_{s[2]} + v_{s[0]}) \cdot b_{s12};$

let $s_1 = v_{s[1]} + (v_{s[3]} - v_{s[1]}) \cdot b_{s[1]} + (v_{s[5]} - v_{s[1]}) \cdot b_{s[2]} + (v_{s[7]} - v_{s[5]} - v_{s[3]} + v_{s[1]}) \cdot b_{s12};$

$s_0 + (s_1 - s_0) \cdot b_{s[0]}$

fn lessEqual[N : Nat, @D](leftelem : list[bool[N] $post @D], rightelem : list[bool[N] $post @D]) -> bool[N] $post @D {
    let mut res = 1 - (1 - (rightelem[0] as uint[N])) \cdot (leftelem[0] as uint[N]);
    for i in 1.. length(leftelem) {
        res = threebitselect([leftelem[i] as uint[N], rightelem[i] as uint[N], res], [0, 0, 1, 0, 1, 0, 1, 1]);
    }
    uint_to_bool(res)
}

fn checkSorted[N : Nat, @D](xb : list[list[bool[N] $post @D]]$pre @D)) -> () {
    for i in 1.. length(xb) {
        assert(lessEqual(xb[i−1], xb[i]));
    }
}

fn is_zero[N : Nat, @D](x : uint[N] $post @D) -> bool[N] $post @D {
    let b : bool[N] $pre @D = ((x as $pre) == 0);
    let w : uint[N] $pre @D = if (b as bool) {
        0
    } else {
        mod_div(1, x as $pre)
    };
    let bp = wire { b };
    let bu = 1 - bp as uint[N];
    let wp = wire { w };
    let xu = x \cdot bu;
    let xuw = xu \cdot wp;
    assert(xu - x == 0);
    assert(xuw - bu == 0);
    bp
}

fn isEqualToPrevious[N : Nat, @D](xb : list[uint[N] $post @D] $pre @D) -> list[bool[N] $post @D] $pre @D) {
    if (@D <= @verifier) {
        for i in 0 .. length(xb) {
            if (i==0) {
                false
            } else {
                wire { (xb[i−1] as $pre) == (xb[i] as $pre) }
            }
        }
    } else {
        for i in 0 .. length(xb) {
            if (i==0) {
                false
            } else {
                wire { (xb[i−1] as $pre) == (xb[i] as $pre) }
            }
        }
    }
}
is_zero((xb[i] - xb[i-1]))

fn mem[N : Nat, @D](x : uint[N] $pre @D, l : list[uint[N] $pre @D]) -> bool $pre @D {
  let rec t = for i in 0 .. length(l)+1 {
    if (i == 0) {
      false
    } else {
      t[i-1] | (x == l[i-1])
    }
  };
  t[length(l)]
}

fn isFromFirstPre[N : Nat](x : list[uint[N] $pre @prover], y : list[uint[N] $pre @prover]) -> list[bool[N] $pre @prover] {
  let x_prover = uintArrayToProver(x);
  for i in 0 .. length(x) + length(y) {
    if (i < length(x)) {
      true
    } else {
      mem(y[i - length(x)], x_prover)
    }
  }
}

fn isFromFirst[N : Nat](x : list[uint[N] $post @verifier], y : list[uint[N] $post @prover]) -> list[bool[N] $post @prover] {
  let permNetwork = mkWaksmanNetwork(length(x) + length(y));
  let allValues = append(uintArrayToProver(x), y);
  let permSws : list[bool[N] $post @prover] = getSwitchesForSorting(uintArrayToPre(allValues));
  let sortedAllValues_uint = applySwitchingNetwork_uint(allValues, permNetwork, permSws);
  let sortedAllValues = bitextract_array(sortedAllValues_uint, 64);
  for i in 0..length(sortedAllValues_uint) {dbg_print(to_string(sortedAllValues_uint[i] as $pre));};
  checkSorted(sortedAllValues);
  let equalValues = isEqualToPrevious(sortedAllValues_uint);
  let is_val_from_first : list[bool[N] $post @prover] = boolArrayToProver(boolArrayToPost(append([true : bool[N] $pre @public]; length(x)], [(false : bool[N] $pre @public]; length(y))));
  let sortedValFromFirst = applySwitchingNetwork_bool(is_val_from_first, permNetwork, permSws);
  let rec sortedGoodVal =
    for i in 0 .. length(x) + length(y) {
      if (i == 0) {
        sortedValFromFirst[0]
      } else {
        (equalValues[i] & sortedGoodVal[i-1]) | (!equalValues[i]) & sortedValFromFirst[i])
      }
  };

for i in 1 .. length(x) + length(y) { // Check that indeed, (equal) values from instance were sorted before values from witness
  let b = (!equalValues[i]) | (!sortedValFromFirst[i]) | sortedValFromFirst[i−1];
  dbg_assert(b as $pre);
  assert(b);
}

let result = boolArrayToPost(isFromFirstPre(uintArrayToPre(x), uintArrayToPre(y)));
let sortedGoodDr_other = applySwitchingNetwork_bool(result, permNetwork, permSws);
for i in 0 .. length(sortedGoodVal) {
  let b = (sortedGoodVal[i] & sortedGoodDr_other[i]) | (!sortedGoodVal[i] & !sortedGoodDr_other[i]);
  dbg_assert(b as $pre);
  assert(b);
}

result

fn main() {
  let numVerifierElements : uint $pre @public = get_public("num_verifier_elements");
  let numProverElements : uint $pre @public = get_public("num_prover_elements");

  let verifierElements : list[uint[N] $post @verifier] = wire ([scalar; numVerifierElements]) { get_instance("verifier_elements") };
  let proverElements : list[uint[N] $post @prover] = wire ([scalar; numProverElements]) { get_witness("prover_elements") };

  let subsetElements : list[bool[N] $post @prover] = isFromFirst(verifierElements, proverElements);

  for i in 0..numProverElements {
    assert(subsetElements[numVerifierElements + i]);
  }
}

B.4 ECDSA verification

type P : Nat = 3940200619639447921227904010014361380507973927046544666794829340424572177149687032290472660882589380018616069731125;
type Q : Nat = 3940200619639447921227904010014361380507973927046544666794829340424572177149687032290472660882589380018616069731125;
type ecpointnz[@D] : /Q_ualified = tuple [uint[P] @D, uint[P] $post @D]
  type digest[@D] : /Q_ualified = uint[Q] $post @D;
  type pkey[@D] : /Q_ualified = ecpointnz[@D];
  type sig[@D] : /Q_ualified = tuple[uint[Q] $post @D, uint[Q] $post @D]

fn boolArrayToPost[N : Nat, @D](xs : list[bool[uint[N] $pre @D]]) -> list[bool[N] $post @D] {
  wire ([scalar; length(xs)] | xs }
}

fn convert_bitarray[M : Nat, N : Nat, @D](bs : list[bool[M] $post @D]) -> list[bool[N] $post @D] {
  for i in 0 .. length(bs) { convert_bit(bs[i]) }
}

fn convert_bit[M : Nat, N : Nat, @D](b : bool[M] $pre @D) -> bool[N] $post @D {
ZK-SECREC: a Domain-Specific Language for Zero-Knowledge Proofs

```haskell
if @D <= @verifier {
    let b1 = b as $pre;
    let b2 = b1 as bool[N];
    let res = wire { b2 };
    res
} else {
    let b1 = b as $pre;
    let b2 = b1 as bool[N];
    let res = wire { b2 };
    assert_eq(b, res);
    res
}
```

```haskell
fn bitextract_pre[N : Nat, @D](x : uint[N] $pre @D, fbw : uint $pre @public) -> list[bool[N] $pre @D] {
    let rec xx = for i in 0 .. fbw {
        if (i == 0) { x } else { xx[i-1] / 2 }
    };
    for i in 0 .. fbw {
        let b = xx[i] % 2;
        b == 1
    }
}
```

```haskell
fn bitextract_pre_uint[@D](x : uint $pre @D, fbw : uint $pre @public) -> list[bool $pre @D] {
    let rec xx = for i in 0 .. fbw {
        if (i == 0) { x } else { xx[i-1] / 2 }
    };
    for i in 0 .. fbw {
        let b = xx[i] % 2;
        b == 1
    }
}
```

```haskell
fn check_bitextract[N : Nat, @D](x : uint[N] $post @D, xb : list[bool[N] $post @D], fbw : uint $pre @public) {
    let rec partialsums : list[uint[N] $post @D] =
        for i in 0 .. fbw {
            if (i == 0) { xb[fbw - 1] as uint[N] }
        } else {
            partialsums[i-1] * 2 + (xb[fbw - i - 1] as uint[N])
        }
    assert(x == partialsums[fbw - 1]);
}
```

```haskell
fn bitextract[N : Nat, @D](x : uint[N] $post @D, fbw : uint $pre @public) -> list[bool[N] $post @D] {
    if @D <= @verifier {
        boolArrayToPost(bitextract_pre(x as $pre, fbw))
    } else {
        let xb = boolArrayToPost(bitextract_pre(x as $pre, fbw));
    }
}
```
fn divide_modulo[N : Nat, @D](x : uint[N] $post @D, y : uint[N] $post @D) $post @D \rightarrow \text{uint}[N] \text{ $post @D} \{
  \text{let } r : \text{uint}[N] \text{ $post @D} = \text{wire \{ mod_div(x as $pre, y as Spre) \}; assert}(r \ast y == x); r
\}

fn choose_in_ecpt[@D](b : uint[P] $post @D, t : ecpointnz[@D], f : ecpointnz[@D]) $post @D \rightarrow \text{ecpointnz}[@D] \{
  (f.0 + b \ast (t.0 - f.0), f.1 + b \ast (t.1 - f.1))
\}

fn ec_add[@D](x : ecpointnz[@D], y : ecpointnz[@D]) $post @D \rightarrow \text{ecpointnz}[@D] \{
  \text{let } s : \text{uint}[P] \text{ $post @D} = \text{divide_modulo}(x.1 - y.1, x.0 - y.0); \text{let } z0 : \text{uint}[P] \text{ $post @D} = s \ast s - x.0 - y.0; \text{let } z1 : \text{uint}[P] \text{ $post @D} = - (x.1 + s \ast (z0 - x.0)); (z0, z1)
\}

fn ec_dbl[@D](x : ecpointnz[@D]) $post @D \rightarrow \text{ecpointnz}[@D] \{
  \text{let } s : \text{uint}[P] \text{ $post @D} = \text{divide_modulo}(3 \ast x.0 \ast x.0 - 3, 2 \ast x.1); // Here "-3" comes from the EC equation \text{let } z0 : \text{uint}[P] \text{ $post @D} = s \ast s - 2 \ast x.0; \text{let } z1 : \text{uint}[P] \text{ $post @D} = - (x.1 + s \ast (z0 - x.0)); (z0, z1)
\}

fn ec_adjY(x : ecpointnz[@prover], b : bool[P] $post @prover) $post @prover \rightarrow \text{ecpointnz}@prover \{
  (x.0, x.1 + 1 - (b as uint[P]))
\}

fn ec_scmult(x : ecpointnz[@prover], pw : list[bool[P] $post @prover]) $post @prover \rightarrow \text{ecpointnz}@prover \{
  \text{let } wp : \text{list[bool}[P] $post @prover] = \text{for } i \text{ in } 0 .. \text{length}(pw) \{\text{pw[length(pw) - i - 1]}; \text{let rec } wp\text{-pfxOR} : \text{list[bool}[P] $post @prover] = \text{for } i \text{ in } 0 .. \text{length}(wp) + 1 \{
    \text{if } (i == 0) \{
      \text{false}
    \} \text{ else } \{
      wp[i-1] | wp\text{-pfxOR}[i-1]
    \}
  \}; \text{let rec } mst : \text{list[ecpointnz}@prover] = \text{for } i \text{ in } 0 .. \text{length}(wp\text{-pfxOR}) \{
    \text{if } (i == 0) \{
      (0,0)
    \} \text{ else } \{
      \text{let } dd : \text{ecpointnz}@prover = ec\_dbl(ec\_adjY(mst[i-1], wp\text{-pfxOR}[i-1]));
    \}
  \});
\}
let aa : ecpointnz[@prover] = choose_in_ecpt((wp_pfxOR[i] as uint[P]) ,  
x, ec_add(dd, x));  
choose_in_ecpt((wp[i-1] as uint[P]), aa, dd)  
};  
mst[length(wp_pfxOR) - 1]  
}  
fn compute_fixpowers() -> list[list[ecpointnz[@public]]] {  
let G_pre : tuple[uint[P] Spre @public, uint[P] Spre @public] = get_public("G");  
let G = ( (wire { G_pre.0 }), (wire { G_pre.1 }) );  
let rec H = for i in 0 .. 128 {  
let rec Hr = for j in 0 .. 7 {  
if (i == 0) {  
if (j == 0) {  
G  
} else {  
if (j == 1) {  
ec_dbl(G)  
} else {  
ec_add(Hr[j-1], G)  
}  
}  
} else {  
if (j == 0) {  
ec_add(H[i-1][6], H[i-1][0])  
} else {  
if (j == 1) {  
ec_dbl(Hr[0])  
} else {  
ec_add(Hr[j-1], Hr[0])  
}  
}  
}  
};  
Hr  
};  
H  
}  
fn ec_add_from_window(is0 : uint[P] $post @prover, st : ecpointnz[@prover], wnd : list[ecpointnz[@public]], s0 :  
bool[P] $post @prover, s1 : bool[P] $post @prover, s2 : bool[P] $post @prover) -> tuple[uint[P] $post  
@prover, ecpointnz[@prover]] {  
let v0 = s0 as uint[P];  
let v1 = s1 as uint[P];  
let v2 = s2 as uint[P];  
let vland2 = v1 * v2; // 1 multiplication  
let xwnd = for i in 0 .. length(wnd) {  
(wnd[i]).0 as @prover  
};  
let ywnd = for i in 0 .. length(wnd) {  
(wnd[i]).1 as @prover  
}
let threezero = (1 - v0) * (1 - v1 - v2 + v1and2); // 1 multiplication

let x_odd = xwnd[0] + v1 * (xwnd[2] - xwnd[0]) + v2 * (xwnd[4] - xwnd[0]) + v1and2 * (xwnd[6] + xwnd[0] - xwnd[2] - xwnd[4]);

let x_even = 1 + v1 * (xwnd[1] - 1) + v2 * (xwnd[3] - 1) + v1and2 * (xwnd[5] + 1 - xwnd[1] - xwnd[3]);

let y_odd = ywnd[0] + v1 * (ywnd[2] - ywnd[0]) + v2 * (ywnd[4] - ywnd[0]) + v1and2 * (ywnd[6] + ywnd[0] - ywnd[2] - ywnd[4]);

let y_even = v1 * ywnd[1] + v2 * ywnd[3] + v1and2 * (ywnd[5] - ywnd[1] - ywnd[3]);

let x = x_even + v0 * (x_odd - x_even); // 1 multiplication

let y = y_even + v0 * (y_odd - y_even); // 1 multiplication

let newpoint = (x,y);

let r = choose_in_ecpt(is0, newpoint, ec_add(st, newpoint)); // 2+3 multiplications

(fn ec_scmult_fixbase(fp : list[list[ecpointnz[@public]]]], pw : list[bool[P] Spost @prover]) -> ecpointnz[@prover]

let mut res = for i in 0 .. 129 {
        if (i == 0) {
            (1,(0,0))
        } else {
            let rp = ec_add_from_window((res[i-1]).0, (res[i-1]).1, fp[i-1], pw[3*(i-1)], pw[3*(i-1)+1], pw[3*(i-1)+2]);
            ((res[i-1]).0 * rp.0, rp.1) // 1 multiplication
        }
    };

// z: digest.
// pk: public key - a point on an elliptic curve.
// S: a signature: two exponents

fn assert_checksig(z : list[bool[Q] Spost @prover], pk : pkey[@prover], S : sig[@prover], fixpowers : list[list[ecpointnz[@public]]]]) -> ()

let mut z_as_exp : uint[Q] Spost @prover = 0;

let mut powtwo : uint[Q] Spre @public = 1;

let numbits_for_zexp = if (length(z) < field_bit_width(Q)) { length(z) } else { field_bit_width(Q) ;

    for i in 0 .. numbits_for_zexp {
        z_as_exp = z_as_exp + (z[numbits_for_zexp - i - 1] as uint[Q]) * ((wire { powtwo }) as @prover);
        powtwo = 2 * powtwo;
    }

    let u_q : uint[Q] Spost @prover = divide_modulo(z_as_exp, S.1);
    let v_q : uint[Q] Spost @prover = divide_modulo(S.0, S.1);

    let u_p : uint[P] Spost @prover = wire { ((u_q as Spre) as uint[P]) };
    let v_p : uint[P] Spost @prover = wire { ((v_q as Spre) as uint[P]) };

    assert_eq(u_p, u_q);
    assert_eq(v_p, v_q);

    let ub : list[bool[P] Spost @prover] = bitextract(u_p, 384);
    let vb : list[bool[P] Spost @prover] = bitextract(v_p, 384);

    let Z = ec_add(ec_scmult_fixbase(fixpowers, ub), ec_scmult(pk, vb));
    assert_eq(Z.0, S.0);
}
fn main() {
  let pubkey_pre : tuple[uint[P] $pre @verifier, uint[P] $pre @verifier] = get_instance("publicKey");
  let pubkey : pkey[@prover] = ( (wire { pubkey_pre.0 }) as @prover, (wire { pubkey_pre.1 }) as @prover);
  let sig_pre : tuple[uint[Q] $pre @prover, uint[Q] $pre @prover] = get_witness("signature");
  let sig : sig[@prover] = ( (wire { sig_pre.0 }), (wire { sig_pre.1 }) );

  let digest_as_bits_pre : list[bool[Q] $pre @prover] = bitextract_pre(get_witness("digest"), 256);
  let digest = for i in 0..length(digest_as_bits_pre) {
    wire { digest_as_bits_pre[255 - i] }
  };

  assert_checksig(digest, pubkey, sig, compute_fixpowers());
}

B.5 Single-Source Shortest Paths
The code for finding and showing the correctness of shortest paths is the following. Here {}# is the syntactic sugar for an empty dictionary, while dict(# k) refers to the (updatable) value stored at position k for the dictionary dict. The type of a dictionary is store[T1,T2] $S @D, where T1 is the data type of keys and T2 is the data type of values. If $S is $post, then both data types must be uint[N]. Dictionaries are passed to functions by reference (indicated in the type).

fn boolArrayToPost[N : Nat, @D](xs : list[bool[N] $pre @D]) -> list[bool[N] $post @D] {
  wire ((scalar; length(xs))) { xs }
}

fn check_bitextract[N : Nat, @D](x : uint[N] $post @D, xb : list[bool[N] $post @D], fbw : uint $pre @public) {
  let rec partialsums : list[uint[N] $post @D] =
    for i in 0 .. fbw {
      if (i == 0) {
        xb[fbw - 1] as uint[N]
      }
      else {
        partialsums[i-1] * 2 + (xb[fbw - i - 1] as uint[N])
      }
    };
  assert(x == partialsums[fbw - 1]);
}

fn bitextract_pre[N : Nat, @D](x : uint[N] $pre @D, fbw : uint $pre @public) -> list[bool[N] $pre @D] {
  let rec xx = for i in 0 .. fbw {
    if (i == 0) { x } else { xx[i-1] / 2 }
  };
  for i in 0 .. fbw {
    let b = xx[i] % 2;
    b == 1
  }
}

fn bitextract[N : Nat, @D](x : uint[N] $post @D, fbw : uint $pre @public) -> list[bool[N] $post @D] {
  if $D <= @verifier {
    boolArrayToPost(bitextract_pre(x as $pre, fbw))
  }
}
) else {
    let xb = boolArrayToPost(bitextract_pre(x as $pre, fbw));
    check_bitextract(x, xb, fbw);
    xb
}

fn uint_to_bool[N : Nat, @D](x : uint[N] $post @D) -> bool[N] $post @D {
    let x_pre : uint $pre @D = (x as $pre)
    as uint;
    let b_pre : bool[N] $pre @D = if (x_pre == 0) { false } else { true };
    let b : bool[N] $post @D = wire { b_pre };
    assert(b_as_uint == x);
    b
}

fn lt_threebitselect[N : Nat, @D] (bs : list[uint[N] $S @D]) -> uint[N] $S @D {
    let bs12 = bs[1] ∗ bs[2];
    let s = bs[1] + bs[2];
    s - bs12 + (2 ∗ bs12 - s) ∗ bs[0]
}

fn lessEqual[N : Nat, @D](le/telem : list[bool[N] $post @D], rightelem : list[bool[N] $post @D]) -> bool[N] $post @D {
    let mut res = 1 - (1 - (rightelem[0] as uint[N]) ∗ (leftelem[0] as uint[N]));
    for i in 1 .. length(leftelem) {
        res = lt_threebitselect([leftelem[i] as uint[N], rightelem[i] as uint[N], res]);
    }
    uint_to_bool(res)
}

fn bellmanford(nn : uint $pre @public, mm : uint $pre @public, ss : list[uint[N] $pre @prover], tt : list[uint[N] $pre @prover], ww : list[uint[N] $pre @prover]) -> tuple[list[uint[N] $pre @prover] @public, list[uint[N] $pre @prover] @public) {
    let mut maxlen = 0;
    for i in 0 .. mm {
        maxlen = if (ww[i] > maxlen) { ww[i] } else { maxlen };
    }
    let infty = (nn as uint[N] as @prover) ∗ maxlen;
    let mut dists : list[uint[N] $pre @prover] @prover = [infty; nn as @prover];
    let mut prevsteps : list[uint[N] $pre @prover] @prover = [0; nn as @prover];
    dists[0 : uint $pre @prover] = 0;
    for idx in 0 .. (nn - 1) {
        for i in 0 .. mm {
            let ssi : uint $pre @prover = ss[i] as uint;
            let tti : uint $pre @prover = tt[i] as uint;
            let newlen : uint[N] $pre @prover = dists[ssi] + ww[i];
            if (newlen < dists[tti]) {
                dists[tti] = newlen;
                prevsteps[tti] = i as uint[N] as @prover;
            }
        }
    }
ZK-SECREC: a Domain-Specific Language for Zero-Knowledge Proofs

```plaintext
let dists_trim = for i in 0 .. nn { dists[i] };
let prevsteps_trim = for i in 0 .. nn { prevsteps[i] };
(dists_trim, prevsteps_trim)

fn check_lengths(nn : uint $pre @public, dists : list[uint][N] $post @prover, ref dists_st : store[uint][N], uint[N] $post @prover, prevsteps : list[uint][N] $post @prover, ref ss : store[uint][N], uint[N] $post @prover, ref tt : store[uint][N], uint[N] $post @prover, ref ww : store[uint][N], uint[N] $post @prover) {
  assert(dists[0] == 0);
  for i in 1 .. nn {
    let idx = (wire[i as uint[N]]) as @prover;
    assert(dists[i] == dists_st{# ss{# prevsteps[i] } } + ww{# prevsteps[i] });
    assert(tt{# prevsteps[i] } == idx);
  }
}

fn check_relaxations(mm : uint $pre @public, ref dists_st : store[uint][N], uint[N] $post @prover, edgestarts : list[uint][N] $post @prover, edgeends : list[uint][N] $post @prover, edgeweights : list[uint][N] $post @prover) {
  for i in 0 .. mm {
    assert(lessEqual(bitextract(dists_st{# edgeends[i] }, field_bit_width(N)), bitextract(dists_st{# edgestarts[i] } + edgeweights[i], field_bit_width(N))));
  }
}

fn check_sptree(nn : uint $pre @public, prevsteps : list[uint][N] $post @prover, ref ss_st : store[uint][N], uint[N] $post @prover) {
  let mut ps_st = {};
  for i in 0 .. nn {
    let idx = (wire[i as uint[N]]) as @prover;
    ps_st{# idx } = ss_st{# prevsteps[i] };
  }
  for rep in 0 .. field_bit_width(nn) + 1 {
    for i in 0 .. nn {
      let idx = (wire[i as uint[N]]) as @prover;
      ps_st{# idx } = ps_st{# ps_st{# idx } };
    }
  }
  for i in 0 .. nn {
    let idx = (wire[i as uint[N]]) as @prover;
    assert(ps_st{# idx } == 0);
  }
}

fn main() {
  let numvertices : uint $pre @public = get_public("numvertices");
  let edgestarts_pre : list[uint][N] $pre @prover Spre @prover = get_witness("edgestarts");
  let edgeends_pre : list[uint][N] $pre @prover Spre @prover = get_witness("edgeends");
  let edgeweights_pre : list[uint][N] $pre @prover Spre @prover = get_witness("edgeweights");
```
let mm = get_public("edges.length");
let edgestarts = wire ([scalar; mm]) { edgestarts_pre }
let edgeends = wire ([scalar; mm]) { edgeends_pre }
let edgeweights = wire ([scalar; mm]) { edgeweights_pre }
let edgestarts_trim = for i in 0 .. mm { edgestarts_pre[i] }
let edgeends_trim = for i in 0 .. mm { edgeends_pre[i] }
let edgeweights_trim = for i in 0 .. mm { edgeweights_pre[i] }
let privateresult = bellmanford(numvertices, mm, edgestarts_trim, edgeends_trim, edgeweights_trim);
let mut ss_st = {#};
let mut tt_st = {#};
let mut ww_st = {#};
let mut dists_st = {#};
for i in 0 .. mm {
    let idx = (wire{i as uint[N]}) as @prover;
    ss_st{#idx} = edgestarts[i];
    tt_st{#idx} = edgeends[i];
    ww_st{#idx} = edgeweights[i];
};
let dists = wire ([scalar; numvertices]) { privateresult.0 }
for i in 0 .. numvertices {
    let idx = (wire[i as uint[N]]) as @prover;
    dists_st{#idx} = dists[i];
};
let prevsteps = wire ([scalar; numvertices]) { privateresult.1 }
check_lengths(numvertices, dists, ref dists_st, prevsteps, ref ss_st, ref tt_st, ref ww_st);
check_relaxations(mm, ref dists_st, edgestarts, edgeends, edgeweights);
check_sptree(numvertices, prevsteps, ref ss_st);
{}
}

Store finalization works as follows. It makes use of the sorting operations given above in Sec. B.3.

fn finalize_store[@D, N : Nat](opers : list[bool Spre @public], keys : list[uint[N] Spost @D], values : list[uint[N] Spost @D]) {
    let contiguous : bool Spre @public = true;
    let writeOnce : bool Spre @public = false;

    // Assuming that all arguments have the same length
    let ops : list[uint[N] Spost @D] = for i in 0..length(opers) {
        (wire{ opers[i] as bool[N] as uint[N] }) as @D
    };

    // Sort lists by (k, i)
    let reorderSwitches : list[bool[N] Spost @D] = getSwitchesForSorting(for i in 0..length(keys) { keys[i] as Spre });
    let permNw = mkWaksmanNetwork(length(keys));

    let sortedOps = if(writeOnce) [] else { applySwitchingNetwork_uint(ops, permNw, reorderSwitches)};
    let sortedKeys = applySwitchingNetwork_uint(keys, permNw, reorderSwitches);
    let sortedKeysBits = if(contiguous) [] else { for i in 0..length(sortedKeys) { bitextract(sortedKeys[i], field_bit_width(N−1)) } }
    let sortedValues = applySwitchingNetwork_uint(values, permNw, reorderSwitches);
let sortedIdxs : list[uint[N] Spost @D] = if(writeOnce) [] else { applySwitchingNetwork_uint(for i in 0..length(keys) { (wire[ i as uint[N] ] as @D }, permNw, reorderSwitches) };
let sortedIdxsBits = if (writeOnce) [] else { for i in 0..length(sortedIdxs) { bitextract(sortedIdxs[i], field_bit_width(length(sortedIdxs))) } }

// op_1 = U
if(!writeOnce) {
    assert(sortedOps[0] == 0);
}

for j in 0..length(opers)–1 {
    let keysEq : uint[N] Spost @D = if(contiguous) { sortedKeys[j] + 1 – sortedKeys[j+1] } else { is_zero(sortedKeys[j] – sortedKeys[j+1]) as uint[N]};
    if(writeOnce) {
        // k_j = k_{j+1} ==> v_j = v_{j+1}
        assert(keysEq * (sortedValues[j] – sortedValues[j+1]) == 0);
        // no checks about the updates. But could probably add it like this:
        // k_j /= k_{j+1} <=> op_{j+1} = U (that will be two assertions)
    }
    else {
        // k_j = k_{j+1} \&/ op_{j+1} = L => v_j = v_{j+1}
        assert(sortedOps[j+1] * keysEq * (sortedValues[j] – sortedValues[j+1]) == 0);
        // k_j /= k_{j+1} => op_{j+1} = U
        assert((1 – keysEq) * sortedOps[j+1] == 0);
    }

    // Sorting checks
    if(contiguous) {
        assert(keysEq * keysEq == keysEq);
    }
    else {
        assert(lessEqual(sortedKeysBits[j], sortedKeysBits[j+1]) : bool[N] Spost @D);
    }
}

// Sorting stability check
if(!writeOnce) {
    let leq : bool[N] Spost @D = lessEqual(sortedIdxsBits[j+1], sortedIdxsBits[j]);
    assert(keysEq * leq as uint[N] == 0);
};
}