The evolution of black–hole mass and angular momentum

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ABSTRACT
We show that neither accretion nor angular momentum extraction are likely to lead to significant changes in the mass $M_1$ or angular momentum parameter $a_*$ of a black hole in a binary system with realistic parameters. Current values of $M_1$ and $a_*$ therefore probably reflect those at formation. We show further that sufficiently energetic jet ejection powered by the black hole’s rotational energy can stabilize mass transfer in systems with large adverse mass ratios, and even reduce the mass transfer rate to the point where the binary becomes transient.

Key words: black hole physics — binaries: close — X–rays: stars — stars: individual (GRO J1655-40, GRS 1915+105, SS433)

1 INTRODUCTION
In a recent paper, Zhang et al. (1997) use the observed strength of the ultrasoft X–ray component in black–hole binaries to estimate the black hole spin. They argue that known systems show a range of spin rate $a_*=a/M_1$ (see their Table 2), where the Kerr parameter $a=cJ_1/GM_1$ with $M_1, J_1$ the black–hole mass and angular momentum. In particular the two superluminal jet sources GRO J1655-40 ($a_*\approx 0.93$) and GRS 1915+105 ($a_*\approx 0.998$) are claimed to spin at rates close to the maximum value $a_*=1$, while systems such as the soft X–ray transient GS 2000+251 have $a_*\approx 0$.

The obvious question is whether this claimed range of spin rates reflects systematic spinup from $a_*=0$, or spindown from $a_*=1$, or the accidents of birth. In this paper we shall show that neither spinup nor spindown is likely to account for the range. The total mass accreted over the lifetime of any binary black hole is too small to increase $a_*$ from 0 to 1. By contrast, spindown of a black hole from an initial state with $a_* = 1$ is a relatively efficient process if the rotational energy is used to power moderately relativistic jets. However, significant spindown requires the hole to have released improbably large amounts of energy into the surrounding interstellar medium.

It seems therefore that binary black holes retain rather similar spin rates to those they had at birth. The same appears true of their masses: although a loss of rotational energy from a black hole implies that its gravitating mass decreases, we shall see that with realistic energy extraction efficiencies this effect is rather small. We conclude that binary black holes essentially retain the masses and spin rates they were born with. A further interesting result emerges from our analysis: moderately relativistic jet ejection is able to stabilize mass transfer in binaries where the mass ratio would otherwise lead to unstable Roche lobe overflow. Sufficiently energetic jet ejection can even lower the mass transfer rate to the point where the system becomes transient.

2 TOTAL MASS TRANSFER IN BLACK–HOLE BINARIES
A black hole can change its spin either by accreting mass and the associated angular momentum, or by giving angular momentum to matter in its close vicinity. If the hole is a member of a binary system both of these processes are limited by the total mass transferred in the system’s lifetime. In this section we estimate this quantity for all types of black–hole binary. We can divide these into high–mass systems, where mass is accreted from the stellar wind of an early–type companion, and low–mass systems, where a late–type star fills its Roche lobe. We may further subdivide the latter group into those where the mass transfer is driven by the nuclear or thermal expansion of the secondary (n–driven), and those where the driving mechanism is angular momentum loss (j–driven).

2.1 Wind–fed binaries
In this type of binary the black hole generally accretes only a small fraction $f = M_{\text{acc}}/M_w$ of the total mass lost by the companion, usually a supergiant, in the form of a wind at a rate $M_w$. A rough estimate for $f$ is

$$f \approx \frac{\dot{r}_w^2}{4a^2} \approx \frac{GM_w^2}{a^2v_w^3} \approx 2 \times 10^{-3} \left( \frac{m_2}{P_1} \right)^{4/3} \left( \frac{v_w}{1000 \text{ km s}^{-1}} \right)^{-4} \tag{1}$$
ties typically transferred mass \( < \) Eddington rate \( \dot{M}_e \) was assumed to enter a common–envelope phase). This mass is transferred on a timescale \( t_{\text{ce}} \), where \( t_{\text{ce}} \) is the evolution time of the binary in the mass–transferring stage. This is given by the angular momentum loss time \( t_j \) for \( j \)-driven systems (donor a main-sequence star and \( M_2 \lesssim 1.5M_\odot \)), and either the thermal timescale for donors crossing the Hertzsprung gap (Kolb et al., 1997; Kolb, 1998) or the nuclear lifetime \( t_n \) if the donor is a low–mass giant or a main–sequence star with mass \( \gtrsim 1.5M_\odot \). Not all of the transferred mass may be accreted by the black hole however, as the accretion rate cannot consistently exceed the Eddington value \( \dot{M}_{\text{Edd}} = 10^{-8}m_1 M_\odot \, \text{yr}^{-1} \), where \( m_1 = M/M_\odot \). An upper limit to the accreted mass is therefore in all cases given by

\[
\dot{M}_{\text{acc}}(\text{Roche}) = \min[M_2, 10^{-8}m_1 t_{\text{ce}} M_\odot] \tag{3}
\]

In practice this means \( \dot{M}_{\text{acc}} \lesssim M_2 < 1.5M_\odot \) for \( j \)-driven (short–period) systems. For \( n \)-driven systems we plot an estimated upper limit for \( \dot{M}_{\text{acc}} \) as a function of initial donor mass in Fig. 1 (case A mass transfer, donor on the main sequence) and Fig. 2 (case B mass transfer, post–core hydrogen but pre–core helium burning phase). The initial black hole mass is \( 8M_\odot \) in all cases; systemic angular momentum losses are assumed to be negligibly small. We used simple fitting formulae to describe the variation of global stellar parameters along single–star tracks as given by Tout et al. (1997). To calculate the case A mass transfer rate the donor’s radius expansion rate \( K = \text{dln}R/\text{df} \) was approximated by that of a single star with the donor’s age and current mass. For case B the core mass growth determines the radius variation with little sensitivity to the total mass (e.g. Webbink et al. 1983 for low–mass stars; Kolb 1998 for stars of higher mass), hence we used \( K(t) \) from a single star with the donor’s initial mass to estimate the transfer rate. The accreted mass \( \dot{M}_{\text{acc}} \) shown in Figs. 1 and 2 represents an upper limit as mass transfer could begin at a later phase than assumed (donor on the ZAMS for case A, on the terminal main sequence for case B). A system formally terminating case A mass transfer when the donor arrives at the terminal main sequence would continue to transfer mass via case B. The case B phase terminates when core helium burning begins or the donor’s envelope is fully lost. Any mass transferred in excess of the Eddington rate was assumed to leave the system with the black hole’s specific orbital angular momentum, otherwise mass transfer was taken to be conservative.

### 3 Changing the Black–Hole Spin by Accretion

Here we consider the effect of accretion in changing the black hole mass and spin. We assume first that black–hole spinup occurs by accretion from a disc terminating at the last stable circular orbit. The accreting matter adds both its rest–mass and its rotational energy to the hole, increasing both the gravitating mass–energy \( M_f \) of the hole and its angular momentum \( J_f \). Bardeen (1970; see also Thorne, 1974) showed that these quantities increase as

\[
\Delta M = 3M_i [\sin^{-1}(M_i/3M_\ast) - \sin^{-1}(1/3)], \tag{5}
\]

\[
a_\ast = \left( \frac{2}{3} \right)^{1/2} \frac{M_i}{M_\ast} \left[ 4 - \left( \frac{18M_i^2}{M_\ast^2} - 2 \right)^{1/2} \right]. \tag{6}
\]

Here \( \Delta M \) is the rest–mass added to the hole from the initial state (assumed to be \( M_i = M_i, a_\ast = 0 \)). Once this is such that \( M_f/M_i = 6^{1/2} \) we see from (5) that \( a_\ast = 1 \). In practice the value of \( a_\ast \) stops slightly short of this maximal value, and further accretion simply maintains this state (Thorne, 1974). From (6) we see that the required additional rest–mass is

\[
\Delta M = 3[\sin^{-1}(2/3)^{1/2} - \sin^{-1}(1/3)]M_i \approx 1.85M_i = 0.75M_\ast \tag{7}
\]

Figure 3 shows \( a_\ast \) as a function of \( \Delta M \) (given by numerically combining eqs. (3) and (6). In order to spin up from \( a_\ast = 0 \) to a value \( \approx 1 \) the black hole must have accreted a rest–mass of order 75% of its current gravitating mass. This is therefore a lower limit to the accreted mass \( \dot{M}_{\text{acc}} \).

The resulting lower limits to \( \dot{M}_{\text{acc}} \) for the two presumed rapidly–spinning systems GRS 1915+105 and GRO J1655-40 can be compared with the upper limits obtained from considerations as in Section 2 (see also Tab. 1):

The binary period and component masses of GRS 1915+105 are not known. Zhang et al. (1997) and Cui et al. (1998) find that consistency between black hole parameters derived from the X–ray spectrum and from the 67 Hz QPO (Morgan, Remillard & Greiner 1997), when interpreted either as a trapped g–mode oscillation or a disc precession from the frame–dragging effect, implies a high–mass near–Kerr black hole \( (M_1 \approx 30M_\odot, a_\ast \approx 0.998) \). If this is true the lower limit on \( \Delta M \approx 22M_\odot \) required for spinup is much larger than what could have been transferred even in the most favourable evolutionary configuration. Repeating the calculations shown in Figs. 1 and 2 for a 30M_\odot primary gives \( \dot{M}_{\text{acc}} \lesssim 4M_\odot \) (case A) and \( \lesssim 1.5M_\odot \) (case B).

Conversely, in GRO J1655-40 the binary parameters are reasonably well determined (Orosz & Bailyn, 1997; van der Hooft et al., 1998; Phillips et al. 1998). To find an upper limit to \( \dot{M}_{\text{acc}} \) we assume that the past evolution was conservative, i.e. that \( M_i^2 M_2^2 P = \text{const} \). Then the present system parameters as given by Orosz & Bailyn (1997), \( M_1 \approx 7M_\odot, M_2 \approx 2.3M_\odot, P = 2.62 \text{ d}, \) imply a minimum period \( P \approx 1.08 \text{ d} \). This is somewhere in the middle of the main sequence band (see e.g. Fig. 1) where the donor’s mass–radius

\[ r = \frac{a}{1 - \sqrt{a}} \tag{8} \]
index $\zeta$ relevant for stability against thermal–timescale mass transfer is $\zeta \simeq 0$ (Hjellming 1989). In this case mass transfer stability demands that initially $(M_2/M_1)_0 < 5/6$, see (2) below, hence initially $M_2 \lesssim 4.2M_\odot$, and therefore $M_{\text{acc}} \lesssim 1.9M_\odot \simeq 0.27M_1$. (If $M_2$ were larger thermal–timescale mass transfer would ensue, at a rate $\simeq M_2/t_{\text{KH}}$ with $t_{\text{KH}} = 10 - 100$; $t_{\text{KH}}$ is the donor’s Kelvin–Helmholtz time. The black hole would accrete only a very small fraction $\simeq 1/f_{\text{edd}}$ of any transferred mass in this phase). Using Phillips et al. (1998) lower limits for the present component masses in GRO J1655–40 ($M_1 = 4.2M_\odot$, $M_2 = 1.4M_\odot$) gives a minimum orbital period of 1.11 d if $a_+ \simeq 0.93$, the value preferred by Zhang et al. (1997), and only barely consistent with their lower limit $a_+ > 0.7$ which requires $\Delta M \gtrsim 0.27M_1 \simeq 1.9M_\odot$.

We conclude that the claimed range of $a_+$ cannot be achieved by spinup of the black hole from an initially non–rotating state.

Much attention has recently been paid to suggestions that quiescent soft X–ray transient (SXT) systems might have higher accretion rates than previously thought, because a large mass flux might be advected into them, i.e. accreted at low radiation efficiency. Advective flows have lower specific angular momentum than the Kepler value and so are even less effective in spinning up the black hole. Advective therefore does not change the conclusion of the last paragraph concerning spinup. However, since the advected specific angular momentum is so low, one might consider the opposite possibility, i.e. reducing $a_+$ to a value close to zero by diluting the original angular momentum. However, even if the advected matter has zero angular momentum we have $a_+ = J_1/M_1^2 = a_+(i)(M_1/M_i)^2$, where $M_i$, $J_i$, $a_+(i)$ specify the hole’s initial mass, angular momentum and Kerr parameter, respectively. Reducing $a_+$ from 1 to $\simeq 0.1$ in this way requires the black hole mass to increase by a factor $\simeq 3$. Thus the transferred mass must satisfy $M_{\text{tr}} \gtrsim 2M_1/3$, again far too large compared with the limits (3)–(4).

4 Changing the black–hole spin by ejection

A rotating black hole can lose angular momentum and rotational energy because of the existence of an ergosphere outside its event horizon. The energy loss implies that the gravitating mass of the hole must decrease, according to

$$dM_{\text{tr}} = \epsilon\Omega_{\text{tr}}dJ$$

For convenience in this Section we use geometricized units, in which $G = c = 1$. Here

$$\Omega_{\text{tr}} = \frac{a_+}{2M_{\text{tr}}[1 + (1 - a_+^2)^{1/2}]}$$

is the apparent angular velocity of the horizon (e.g. Misner et al, 1973), and $\epsilon$ measures the efficiency of energy extraction. $\epsilon = 1$ is the maximum value, and corresponds to a ‘reversible’ transformation, in which the area of the event horizon is held fixed, while efficiencies $\epsilon \lesssim 0.5$ are typical for astrophysically realistic processes (e.g. Blandford & Znajek, 1977). Since

$$\frac{dJ}{d(a_+M_1^2)} = M_1^2da_+ + 2a_+M_1dM_1$$

we get with (5)

$$2dM_{\text{tr}} = \frac{a_+da_+}{1 - a_+^2 + (1 - a_+^2)^{1/2}}.$$  

This can be integrated using the substitution $v = (1 - a_+^2)^{1/2}$, and gives finally

$$M_{\text{tr}}^2 = M_{\text{max}}^2[(1 - a_+^2)^{1/2} + 1]^{n}\left[\left(\frac{(1 - a_+^2)^{1/2} - \epsilon + 1}{1 - \epsilon}\right)^m - 1\right]$$

where

$$n = -\frac{\epsilon}{2\epsilon - 1}$$

$$m = -\frac{\epsilon - 1}{2\epsilon - 1}$$

and the hole starts with $M_1 = M_{\text{max}}, a_+ = 1$. The maximum rotational energy extraction is given by setting $a_+ = 0$ in (4), which gives

$$M_{\text{tr}}^2 = 2^\epsilon(1 - \epsilon)^{-m}M_{\text{max}}^2.$$  

Taking the limit $\epsilon \to 1$ gives $M_{\text{tr}}^2 = M_{\text{max}}^2/2$, i.e. $M_1$ reaches its ‘irreducible’ value $M_{\text{max}}/\sqrt{2}$ (Christodoulou 1970, Christodoulou & Ruffini 1971). This maximal energy yield implies a reversible transformation in which the event horizon area is held constant. However in general the energy yield is considerably smaller than this limit. Fig. 4 shows the extracted fractional energy $\Delta m = (M_{\text{max}} - M_1)/M_{\text{max}}$ as a function of $\epsilon$. We see that for typical efficiencies $\epsilon \lesssim 0.5$ rather less than about 10% of the initial rotational energy is extracted; the hole’s mass thus remains very close to its original value $M_{\text{max}}$. Physically what is happening is that all of the hole’s angular momentum is extracted, but the associated energy is used inefficiently: the extraction process allows much of this to disappear down the hole, reducing the loss of gravitating mass.

The remaining energy must appear in some form outside the hole. We note that both the systems of Table 1 with large claimed $a_+$ are ejecting relativistic jets. These remove energy at the rate $\Gamma M_{\text{ej}}c^2$, where $\Gamma$ is the specific energy of the jet matter, and $M_{\text{ej}}$ is their mass–loss rate. If all the energy of the jet material is in its bulk motion, $\Gamma$ is simply the Lorentz factor $\gamma$ of this motion. However, if the spin energy of the black hole is used in other ways, e.g. to excite relativistic electrons, $\Gamma$ will exceed $\gamma$. Thus spindown wins out over spinup as a way of altering $a_+$ because the effect of transferring rest mass $M_{\text{tr}}$ from the companion is enhanced by a factor $\Gamma$ in the former case. If all the transferred matter is ejected in this way until the hole is spun down, the requirement on the total transferred mass becomes

$$M_{\text{tr}} = \frac{\Delta m}{\Gamma}M_1.$$  

Taking $\Delta m \sim 0.1$, $M_1 = 10M_\odot$ and bounding $\Gamma$ below with the value $\sim 2.55$ inferred for GRO J1655–40 (Hjellming & Rupen 1995) we get a limit $M_{\text{tr}} \lesssim 0.4M_\odot$ on the mass which must be transferred to reduce the spin to zero. Spindown from $a_+ = 1$ to $a_+ = 0$ therefore seems possible for systems with Roche–lobe overflow. However, if this has occurred in a given system, one would expect to see abundant evidence of the effects of the total extracted energy $\Delta m M_1c^2 \sim 10^{54}$ erg on the surrounding interstellar medium. GRO J1655–40
would deposit this energy over a time of $\approx 10^6$ yr, see below. The mean energy output rate $\approx 8 \times 10^6 L_\odot$ is comparable to the luminosity of a cluster of 10 O/B supergiants. The detectability of such an energy deposition depends on the structure and density of the local interstellar medium.

The fact that none of the claimed $a_\star \approx 0$ systems shows such evidence suggests that jets are produced only over a relatively short fraction of the system’s lifetime. In this case $a_\star$ is likely to remain close to its original value ($\approx 1$).

5 BINARY EVOLUTION WITH RELATIVISTIC JETS

We have seen above that the production of relativistic jets by a rotating black hole can lead to a large loss of gravitating mass from this object, i.e. $M_1(\text{jet}) = -\Gamma M_0$, where $M_0$ is the rest mass the jet carries away per unit time. As the jet ejection is a consequence of mass accretion, the ejected rest–mass must be roughly equal to the transferred rest–mass from the companion. Hence we expect that the ejection parameter $\eta$, defined by $-M_0 = \eta M_2 < 0$, is of order unity. (Based on energy considerations for the observed jet ejection, Gliozzi et al. 1998 argue that $\eta$ is close to 1 in GRS 1915+105). In addition the jets will presumably carry off the specific orbital angular momentum $j_\text{jet} = M_2 J/M_1 M$ of the black hole from the binary orbit, where $M, J$ are the total binary mass and orbital angular momentum. The jet ejection process therefore constitutes a ‘consequential angular momentum loss’ or CAML process. The effects of CAML on Roche–lobe–filling systems were studied quite generally by King & Kolb (1995; henceforth KK95), who specified them in terms of mass loss and angular momentum loss parameters $\alpha, \nu$, with

$$M_1 = (\alpha - 1) M_2, \frac{J}{J_{\text{CAML}}} = \frac{\nu}{\nu} \frac{M_2}{M_2}.$$  

Thus we have $\alpha = \eta \Gamma$, $\nu = \frac{\eta \Gamma M_2^2}{M_1 M}$.

The CAML process can only amplify (or damp) an already–existing mass transfer process driven by angular momentum losses $J_\text{sys}$ (j–driven systems; $J_\text{sys}$ is the ‘systemic’ rate given by e.g. gravitational radiation or magnetic braking) or a nuclear expansion rate $R_0 > 0$ (for n–driven systems). Defining the evolution time $t_\text{ev} = |J/J_\text{sys}|$ or $(2R_a/R_2) M = \text{const.}$ and repeating the algebra of KK95 gives

$$M_2 = -\frac{M_2}{D t_\text{ev}},$$  

where

$$D = \frac{1}{2} \left[ \zeta + 2 + \beta_2 - \frac{2 M_2}{M_1} + \frac{M_2}{M_1} (\eta \Gamma - 1) \frac{M_2}{M_1} (\frac{M_2}{M_1} - \beta_2) \right].$$  

(This is a slight generalization of KK95 in the case of n–driven systems.) Here $\beta_2 \equiv \ln f_2/d \ln (M_2/M_1)$ is the logarithmic derivative of the ratio $f_2 = R_L/a$ (the donor’s Roche lobe radius $R_L$ in units of the binary separation) with respect to the mass ratio. If $M_2 \gtrsim M_1$ as in KK95 we have $\beta_2 \simeq -M_1/3M$, hence

$$D = \frac{5}{6} + \frac{\zeta}{2} - \frac{M_2}{M_1} + \eta \Gamma \frac{2 M_2}{3 M},$$  

which is eq. (16) of KK95 with $\alpha, \nu$ form our eq. (18). In general, the donor’s mass–radius index $\zeta = \partial \ln R_2/\partial \ln M_2$ appearing in (19) depends on the secondary’s full internal structure and cannot be expressed in closed form. But in many cases an estimate is available. For low mass transfer and j–driven or n–driven evolution with a main–sequence donor (case A) we have $\zeta \approx 0 \eta \approx O(1)$ ($\zeta_{\text{eq}}$ is the mass–radius exponent evaluated under the assumption of thermal equilibrium). For n–driven case B evolution slow mass transfer corresponds to $\zeta \approx 0$. If mass transfer is rapid $\hat{\zeta}$ approaches $\zeta_{\text{eq}}$, the mass–radius index evaluated with constant entropy profile in the star. Fully convective stars or stars with a deep convective envelope have $\zeta = -1/3$.

Figure 5 shows $D$ (from (19)) as a function of mass ratio, for various values of $\Gamma$ (with $\hat{\zeta} = 0$, $\eta = 1$). Eggleton’s approximation for $f_2$ (Eggleton 1983) was used to calculate $\beta_2$.

Comparing with the case with no CAML (i.e. $\alpha = \nu = 0$; conservative mass transfer)

$$D = D_0 = \frac{5}{6} + \frac{\zeta}{2} - \frac{M_2}{M_1}$$  

(for $M_2 \lesssim M_1$) and neglecting any change in $\zeta$ and $t_\text{ev}$ we see that the mass loss from the black hole always reduces the mass transfer rate (by trying to widen the binary). With CAML the Roche lobe expands faster (or contracts slower) than without CAML for a given transfer rate; hence $|M_2|$ adjusts to a smaller value. For $M_2$ significantly smaller than $M_1$ the reduction in $-M_2$ is small unless

$$\eta \Gamma \gg \frac{3 M}{2 M_2}.$$  

If this holds we have

$$M_2 \simeq -\frac{3 M}{2 \eta \Gamma t_\text{ev}}.$$  

Figure 6 shows the variation of the reduction factor $D(\Gamma)/D_0$ with mass ratio (assuming $\zeta = 0$ = const., $\eta = 1$). The reduction is largest close to where $D \to 0$ with conservative mass transfer. This signals instability against dynamical–timescale (if $\zeta = \zeta_{\text{eq}}$) or thermal–timescale mass transfer (if $\zeta = \zeta_{\text{eq}}$), with ensuing transfer rates much in excess of values indicated by (19) with $\zeta = O(1)$. A very high transfer rate may cause a common envelope phase which could destroy the system (but see King & Ritter 1998). However, jets actually stabilize mass transfer in systems where a large mass ratio $q = M_2/M_1$ would make mass transfer unstable ($D_0 < 0$) without the jet losses. Mass transfer is stable if the mass ratio is smaller than $q_{\text{crit}}$, where $D(q_{\text{crit}}) = 0$. From (21) we obtain the positive root of $D = 0$ as

$$q_{\text{crit}} = \frac{1}{2} \left[ C + \frac{2 \eta \Gamma}{3} - 1 + \sqrt{(C + \frac{2 \eta \Gamma}{3} - 1)^2 + 4 C} \right].$$  

where $C = 5/6 + \zeta/2$. A reasonable approximation for $-1/3 \lesssim \zeta \lesssim 10$ is $q_{\text{crit}} \approx (\eta \Gamma/2.82)^2 + (5 + 3 \zeta)/6$. Systems that are unstable against conservative mass transfer but stabilized by jet–induced CAML will still encounter the instability at the end of the jet phase. The mass loss rate from the hole is much larger than the mass loss rate from the donor, so that the mass ratio $M_2/M_1$ increases during the jet phase.

The maximum duration $\Delta t$ of the jet–induced CAML phase depends only weakly on $\eta \Gamma$, unlike the transfer rate. As shown above, the extractable gravitating mass from a

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Kerr black hole with gravitating mass $M_1$ is limited by $\Delta m M_1$, where $\Delta m$ depends on the efficiency of the extraction process. The corresponding limit $\Delta M_1 < \Delta m M_1/(\eta \Gamma)$ for the transferred rest–mass during the jet phase translates into an upper limit $\Delta t < \Delta M_1/(\eta \Gamma)$ for the duration of this phase. Using (24) as a representative value for $\dot{M}$, and 0.1 as a typical value for $\Delta M$, we have

$$\Delta t \lesssim 0.1 \frac{M_1}{\dot{M}} t_{ev}. \tag{26}$$

As is well known (van Paradijs 1996; King, Kolb & Burderi 1996; King, Kolb & Szuszkiewicz 1997) a low transfer rate is a necessary condition for the system to appear as a soft X–ray transient. This is both empirically true, and expected from the disc instability picture. Black–hole systems with low $M_2$ all have $-\dot{M}_2 \sim \dot{M}_2/t_{ev}$ small enough to satisfy this, whether $j$–driven or $n$–driven (King, Kolb & Szuszkiewicz, 1997), i.e. all low–mass black hole systems are transient, even without the extra effect of the jet losses. Values $\Gamma \gtrsim 10$ could however allow a black hole system with a higher–mass companion nevertheless to appear as a transient.

A possible case in point is GRO J1655–40, whose secondary may have a mass $M_2 \approx 2 M_\odot$ (Orosz & Bailyn 1997; van der Hooft et al. 1998; Phillips et al. 1998) and appears to be crossing the Hertzsprung gap. Although the secondary is close to a regime where its radius expansion temporarily slows, producing a low transfer rate and transient behaviour (Kolb et al., 1997; Kolb 1998), current estimates of its parameters consistently show it to be too hot to be in this regime. Instead one expects $t_{ev} \sim t_{KH} \sim 10^7$ yr, where $t_{KH}$ is the thermal timescale of the secondary’s main–sequence progenitor, so $-\dot{M}_2 \sim \dot{M}_2/t_{ev} \sim \text{few} \times 10^{-7}/D \text{ } M_\odot \text{yr}^{-1}$. Without the CAML effect of the jet losses we would have $D \sim 1$, making $-\dot{M}_2$ far higher than the estimated critical rate $\dot{M}_{c\text{KH}} \lesssim 4 \times 10^{-8} \text{ } M_\odot \text{yr}^{-1}$ (from King, Kolb & Szuszkiewicz, 1997; eqn. 7). However, if the jets are very energetic ($\Gamma \gtrsim 50$) the transfer rate would be reduced by more than the required factor $\approx 10$ for the claimed mass ratio $\approx 0.3$ (Orosz & Bailyn 1997), see Fig. 5. Hence if $\Gamma \gtrsim 50$ the system would appear as a transient. This of course requires the jets to be considerably more relativistic than implied by their bulk motion ($\gamma \approx 2.55$). The occurrence of transients of this type then depends purely on the physics of the jets, and cannot be predicted with current theory. On the other hand, if the donor mass in GRO J1655–40 is close to the lower limit 1.4$M_\odot$ found by Phillips et al. (1998) the transfer rate would be well below the critical rate for transient behaviour even in the absence of jet–induced CAML. However, the luminosity given by the spectral type and orbital period is too high for a 1.4$M_\odot$ donor in the phase of crossing the Hertzsprung gap (e.g. Kolb 1998). This suggests that the actual donor mass is nearer to the upper limit (2.2$M_\odot$) quoted by Phillips et al. (1998). (We note that Regős et al. (1998) suggested that the donor in GRO J1655–40 could be still in the core hydrogen burning phase if the main sequence is significantly widened by convective overshooting in the star. This would also give a transfer rate smaller than the critical rate for any $\Gamma$).

The jet source SS433 could be affected by jet–induced CAML as well. The nature of the compact star in this system is still unclear (e.g. Zwitter & Calvani 1989; D’Odorico et al. 1991), but a black hole cannot be ruled out. The claimed mass ratio of order 3 is certainly above the stability limit for conservative mass transfer. However, the donor might not fill its Roche lobe (Brinkmann et al. 1989), and the jets seem to be less energetic than in the superluminal sources GRO J1655–40 and GRS 1915+105.

6 CONCLUSIONS

We may draw the following conclusions from the arguments of this paper:

1. The mass of a binary black hole changes by only a relatively small amount during the mass transfer process, whether through accretion (fractional increase $\lesssim 2M_2/M_1$ for a low–mass system, and much less for a high–mass system) or rotational energy extraction (fractional decrease $\lesssim 10\%$). Currently measured masses are therefore similar to the formation masses.

2. Too little mass is accreted to spin up a hole from $a_* \approx 0$ to $a_* \approx 1$.

3. Extracting angular momentum from the hole can in principle reduce $a_* \approx 1$ to $a_* \approx 0$, but in practice none of the known systems shows the effects of injecting the extracted rotational energy of about $10^{54}$ erg into the local ISM. In combination with 2. above this suggests that binary black holes also retain a value of $a_*$ close to that at formation.

4. Jet ejection powered by the black hole’s rotational energy can have a major effect in stabilizing mass transfer, particularly in higher–mass systems. Sufficiently energetic jets can reduce the mass transfer rate to values making the system transient.

If we accept 3. above, it would appear that any claimed range of $a_*$ or $M_1$ must represent the range of initial conditions for binary black holes. However, a simple extrapolation of the neutron star case seems to favour values $a_* \approx 1$. In particular even the maximum angular momentum $J_1 = GM_1 c = 3 \times 10^{37} (M_1/10 M_\odot) \text{ cm}^2 \text{ s}^{-1}$ for the hole is far smaller than any plausible value for that of the progenitor star. If confirmed, the range of $a_*$ claimed by Zhang et al. (1997) thus represents a challenge to theory.

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Table 1. Binaries with claimed measurements of black–hole spin $a_*$. 

| system         | orbital period | $a_*$ | type         | $M_{1\text{f}}/M_\odot$ | $M_{\text{spin-up}}/M_\odot$ |
|----------------|---------------|-------|--------------|--------------------------|------------------------------|
| GS1124 – 68    | 10.4 h        | -0.04 | j–driven     | < 1.5                    | -                            |
| GS2000 + 25    | 8.3 h         | 0.03  | j–driven     | < 1.5                    | -                            |
| LMC X-3        | 1.7 d         | -0.03 | n–driven     | $\lesssim 2.5$           | -                            |
|                |               |       | wind–fed     | $\lesssim 0.1$           |                              |
| GRO J1655-40   | 2.62 d        | 0.93  | n–driven     | < 1.9                    | 3.3                          |
|                |               | > 0.7 |             |                          | 1.9                          |
| GRS 1915+105   | ?             | 0.988 | n–driven     | $\lesssim 5$             | 22                           |
|                |               |       | wind–fed     | $\lesssim 0.1$           |                              |

**FIGURE CAPTIONS**

**Figure 1** Top panel: Estimated upper limit $M_{\text{acc}}$ for the mass a black hole can accrete during case A mass transfer, as a function of initial donor mass $M_{2\text{initial}}$ (assuming that the donor is initially on the ZAMS and the black hole has mass $8M_\odot$). Middle panel: Final period $P_{\text{f}}$ (solid, scale on the left) and initial period $P_{\text{i}}$ (dashed, scale on the right). Bottom panel: Total duration $t$ of mass transfer phase (ending when the donor reaches the terminal main sequence; solid, scale on the left) and final secondary mass $M_{2\text{f}}$ (dashed, scale on the right). The donor’s mass–radius index $\zeta$ was fixed at 0.

**Figure 2** As Fig. 1, but for case B mass transfer, with the donor initially on the terminal main-sequence. Mass transfer terminates when core helium burning begins.

**Figure 3** Black hole spin $a_*$ vs. accreted rest–mass $\Delta M$, in units the final mass $M_{\text{max}}$ at maximum spin (solid line, bottom axis), and in units of the initial mass $M(a_* = 0)$ at zero spin (dashed, top axis).

**Figure 4** Extracted fractional energy $\Delta m = (M_{\text{max}} - M_1)/M_{\text{max}}$ as a function of efficiency $\epsilon$ of extraction of a black hole’s rotational energy, cf. (12).

**Figure 5** Stability term $D$ from eq. (21) vs. mass ratio $q = M_2/M_1$, for jets with different specific energy $\Gamma$ (assuming $\zeta = 0$).

**Figure 6** Mass transfer reduction factor $D(\Gamma)/D_0$ vs. mass ratio $q = M_2/M_1$, for jets with different specific energy $\Gamma$ (assuming $\zeta = 0$).
Figure 1
Figure 2: The evolution of black-hole mass and angular momentum.
Figure 3
Figure 4
Figure 5
Figure 6

The evolution of black-hole mass and angular momentum