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The $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decays with the inclusion of Lorentz and CPT violating effects

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Abstract: We study the Lorentz and CPT violating effects on the branching ratio and the CP violating asymmetry of the lepton flavor violating interactions $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, in the model III version of the two Higgs doublet model. Here we consider that the Lorentz and CPT violating effects exist in the QED part of the interactions and enter into expressions in the lepton propagators and in the lepton-photon vertex. We observe that there exists a non-zero CP asymmetry. However, the Lorentz and CPT violating effects on the branching ratio and the CP asymmetry are negligibly small.

Keywords: Non-Commutative Geometry, Standard Model, Rare Decays
1. Introduction

The Lorentz and CPT symmetries are conserved in the standard model (SM). At higher scales, like the Planck scale, there are signals that these symmetries are broken \cite{1}, for instance in the string theory \cite{2}, in the non-commutative theories \cite{3}. In \cite{4} it was emphasized that the spacetime-varying coupling constants can be associated with violations of local Lorentz invariance and CPT symmetry. The discussions on CPT violation on neutrino oscillations are presented in \cite{5}.

At the low energy level, the small violations of these symmetries can appear and with the inclusion of them the general Lorentz and CPT violating extension of the SM is obtained \cite{6,7}. In the extension of the SM the Lorentz and CPT violating effects are carried by the coefficients coming from an underlying theory at the Planck scale \cite{2,3}. There are various studies on the bounds of these coefficients in the literature. They have been constrained by the experiments involving hadrons \cite{8}-{\cite{10}, protons and neutrons \cite{11}, electrons \cite{12,13}, photons \cite{14}, muons \cite{15}. The natural suppression scale for these coefficients can be taken as the ratio of the light one $m_l$ to the one of the order of the Planck mass. Therefore the coefficients which carry the Lorentz and CPT violating effects are in the the range of $10^{-23} - 10^{-17}$ \cite{13}. Here the first (second) number represent the electron mass $m_e$ ($m_{EW} \sim 250$ GeV) scale.

In the recent work \cite{16} the amplitude for vacuum photon splitting in the framework of general Lorentz and CPT violating Quantum Electro Dynamics (QED) extensions have been analyzed and it was observed that radiative corrections arising from Lorentz violation in the fermion sector induce the vacuum photon splitting. In \cite{17} the one loop renormalizability of the general Lorentz and CPT violating extension QED has been showed.

In the present work we study the Lorentz and CPT violating effects on the branching ratio ($BR$) and the CP violating asymmetry $A_{CP}$ for the lepton flavor violating (LFV) interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$, in the model III version of the two Higgs doublet model (2HDM), since these decays do not exist in the SM. Here we consider that the Lorentz and CPT violating effects exist in the QED part of the interactions and enter into expressions in the lepton propagators and the lepton-photon vertex.
In the literature, there are several studies on LFV interactions in different models. Such interactions are analyzed in a model independent way in [18], in the framework of model III 2HDM [19]–[21], in supersymmetric models [22]–[28]. Furthermore the experimental current limits for the BR’s ratios of the processes $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are $1.2 \times 10^{-11}$ [29] and $1.1 \times 10^{-6}$ [30] respectively.

The inclusion of the Lorentz and CPT violating effects in the model III does not bring a detectable correction to the BR of the LFV processes under consideration, since the corresponding coefficients are highly suppressed at the low energy scale. However we try to examine the relative importance of the different coefficients which switch on the Lorentz and CPT violating effects in the BR of the decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. In addition to this, we analyze the possible $A_{CP}$ in these decays and the coefficients which are sources of $A_{CP}$. Notice that the $A_{CP}$ does not exist for the LFV decays $l_1 \rightarrow l_2\gamma$ in the framework of the model III and the possibility of such asymmetry has been studied in [20]. In this work it was assumed that the $A_{CP}$ could be switched on when one considered the model beyond the model III and insert a new parameter into the interactions. The magnitude of the $A_{CP}$ is directly proportional to this new parameter. The inclusion of the Lorentz and CPT violating effects in the model III causes a non-zero $A_{CP}$, however it is too small to be detected, similar to the corrections on the BRs of these processes.

The paper is organized as follows: In section 2, we present the theoretical expression for the matrix element and the $A_{CP}$ of LFV interaction $l_1 \rightarrow l_2\gamma$ with the inclusion of the Lorentz and CPT violating effects. Section 3 is devoted to discussion and our conclusions. In the appendix we present the explicit forms of the functions appearing in the calculation of the matrix element of the decays under consideration.

2. The LFV interactions $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ with the addition of Lorentz and CPT violating effects

This section is devoted to the derivation of the Lorentz and CPT violating effects on the BR and the CP asymmetry of the LFV $l_1 \rightarrow l_2\gamma$ decay. The LFV interactions in the tree level are allowed in the general 2HDM, the so-called model III and the the LFV process can be regulated by the Yukawa interaction,

$$L_Y = \eta_{ij} \bar{l}_{iL}\phi_1 E_{jR} + \xi_{ij} \bar{l}_{iL}\phi_2 E_{jR} + h.c.,$$

(2.1)

where $i, j$ are family indices of leptons, $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $\phi_i$ for $i = 1, 2$, are the two scalar doublets, $l_{iL}$ and $E_{jR}$ are lepton doublets and singlets respectively. Here $\phi_1$ and $\phi_2$ are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2} \chi^+ \\ i\chi^0 \end{pmatrix}; \quad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + iH_2 \end{pmatrix},$$

(2.2)

where only $\phi_1$ has a vacuum expectation value;

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}; \quad \langle \phi_2 \rangle = 0.$$

(2.3)
Figure 1: The CPT and Lorentz violating insertions. (a) Lepton propagator insertion (= $i \Gamma_1^\mu p_\mu$ where $p$ is the 4-momentum vector of the internal lepton). (b) Lepton propagator mass insertion (= $-i m_1$). (c) Lepton-photon insertion (= $-i q \Gamma_1^\mu$ where $q$ is the lepton charge).

Now we consider the gauge and CP invariant Higgs potential which spontaneously breaks SU(2) $\times$ U(1) down to U(1) as:

$$V(\phi_1, \phi_2) = c_1 \left( \phi_1^+ \phi_1 - \frac{v^2}{2} \right)^2 + c_2 (\phi_2^+ \phi_2)^2 +$$

$$+ c_3 \left[ \left( \phi_1^+ \phi_1 - \frac{v^2}{2} \right) + \phi_2^+ \phi_2 \right]^2 + c_4 [(\phi_1^+ \phi_1)(\phi_2^+ \phi_2) - (\phi_1^+ \phi_2)(\phi_2^+ \phi_1)] +$$

$$+ c_5 [\text{Re}(\phi_1^+ \phi_2)]^2 + c_6 [\text{Im}(\phi_1^+ \phi_2)]^2 + c_7 ,$$

(2.4)

with constants $c_i, i = 1, \ldots, 7$. With the choice of $\phi_1, \phi_2$ and the potential $V(\phi_1, \phi_2)$, $H_1$ and $H_2$ are obtained as the mass eigenstates $h^0$ and $A^0$ respectively, since no mixing occurs between two CP-even neutral bosons $H^0$ and $h^0$ in the tree level.

The FCNC is produced by the part of the lagrangian

$$\mathcal{L}_{Y, FC} = \xi_{ij}^E \bar{E}_i L \phi_2 E_j R + \text{h.c.}$$

(2.5)

Here the Yukawa matrices $\xi_{ij}^E$ are responsible for the LFV interactions and, in general, they have complex entries. Notice that in the following we replace $\xi_{ij}^E$ with $\xi_{ij}^E$ where "N" denotes the word "neutral".

At this stage we insert the Lorentz and CPT violating effects with the assumption that they exist in the QED part of the interactions. The fermionic part of the general Lorentz and CPT violating QED lagrangian in 4 space-time dimensions reads

$$L = \bar{\psi} \bar{\Gamma}^\mu D_\mu \psi - \bar{\psi} M \psi ,$$

(2.6)

where

$$\Gamma^\mu = \gamma^\mu + \Gamma_1^\mu$$

$$M = m + m_1$$

(2.7)
Figure 2: One loop diagrams contribute to the LFV interactions $l_1 \to l_2 \gamma$ due to the neutral Higgs bosons $h_0$ and $A_0$ in the 2HDM. Dashed (curly, straight) lines represent $h_0$ and $A_0$ fields (electromagnetic field, lepton), the signs $O$ and $\times$ represent the insertions into the propagator and the photon-fermion vertex.

and

$$
\Gamma_1^\mu = c_\mu \gamma_\alpha + d_\mu \gamma_5 \gamma_\alpha + e_\mu + i f_\mu \gamma_5 + \frac{1}{2} g^{\lambda\nu} \sigma_{\lambda\nu},
$$

$$
m_1 = a_\mu \gamma_\mu + b_\mu \gamma_5 \gamma_\mu + \frac{1}{2} h_{\mu\nu} \sigma^{\mu\nu}.
$$

Here the coefficients $a_\mu$, $b_\mu$, $c_\mu$, $d_\mu$, $e_\mu$, $f_\mu$, $g_{\lambda\nu}$ and $h_{\mu\nu}$ cause the Lorentz violation. Among them $a_\mu$, $b_\mu$, $e_\mu$, $f_\mu$ and $g_{\lambda\nu}$ are responsible for the CPT violation (see [17] for details).

The LFV $l_1 \to l_2 \gamma$ interaction occurs with the help of the neutral Higgs bosons, namely, Higgs bosons $h_0$ and $A_0$, in the model III. The Lorentz and CPT violating effects in $\Gamma_1^\mu$ and $m_1$ (see eq. [2.8]) are taken into account with the insertions in the internal lepton propagator and the additional fermion-photon vertex (see figure [1]). Now we use the on-shell renormalization scheme to calculate the matrix element for the LFV process under consideration. Since, in the on-shell renormalization scheme, the self energy $\Sigma(p)$ can be written as

$$
\Sigma(p) = (\hat{p} - m_{l_2}) \Sigma(p)(\hat{p} - m_{l_1}),
$$

(2.9)
with \( \hat{p} = \gamma_\mu p^\mu \), the corresponding diagrams vanish when \( l_1 ( l_2 ) \)-lepton is on-shell. However, the vertex diagrams (figure 2) give a non-zero contribution and the logarithmic divergences are eliminated by inserting an appropriate the counter term \( \Gamma^C \)

\[
\Gamma^\text{Ren}_\mu = \Gamma^0_\mu + \Gamma^C_\mu ,
\]

(2.10)

where \( \Gamma^\text{Ren}_\mu \) and \( \Gamma^0_\mu \) are renormalized and bare vertex functions respectively. Here the \( \Gamma^\text{Ren}_\mu \) satisfies the equation,

\[
k^\mu \Gamma^\text{Ren}_\mu = 0 ,
\]

(2.11)

where \( k \) is the 4-momentum vector of outgoing photon. Now, the matrix element of the LFV \( l_1 \rightarrow l_2 \gamma \) process with the addition of the Lorentz and CPT violating effects is obtained as,

\[
M = \frac{\sqrt{4 \pi \alpha_e}}{32 \pi^2} \{ A \gamma_\mu + B_\mu \gamma_5 + i H_\alpha \sigma_\mu \alpha + E_\mu + A' \gamma_\mu \gamma_5 + B'_\mu \gamma_\alpha \gamma_5 + H'_\alpha \sigma_\mu \alpha \gamma_5 + E'_\mu \gamma_5 \} .
\]

(2.12)

where \( \alpha_e = 1/137 \) and

\[
A = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) A_i(x,y) ,
\]

\[
B_\mu = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) B_{i,\mu}(x,y) ,
\]

\[
H_\alpha = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) H_{i,\alpha}(x,y) ,
\]

\[
E_\mu = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) E_{i,\mu}(x,y) ,
\]

\[
A' = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) A'_i(x,y) ,
\]

\[
B'_\mu = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) B'_i(\mu)(x,y) ,
\]

\[
H'_\alpha = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) H'_{i,\alpha}(x,y) ,
\]

\[
E'_\mu = \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \left( 1 - x - y \right) E'_{i,\mu}(x,y) ,
\]

(2.13)

and

\[
H_{i,\alpha}(x,y) = C_{i,\alpha}(x,y) + D_i(x,y) k_\alpha ,
\]

\[
H'_{i,\alpha}(x,y) = C'_{i,\alpha}(x,y) + D'_i(x,y) k_\alpha .
\]

(2.14)

Here \( l_1 ( l_2 ) \) is incoming (outgoing) leptons, \( i, Q_i \) denote the internal leptons \( (i = \mu, \tau) \), their charges and we choose the Yukawa coupling \( \xi_{N,il(i\bar{2})} \) real. In eqns. (2.13) and (2.14) the coefficients \( A^{(l)}_i(x,y) , B^{(l)}_i(\mu)(x,y) , C^{(l)}_{i,\alpha}(x,y) , D^{(l)}_i(x,y) , E^{(l)}_{i,\mu}(x,y) \) are given in the appendix.
Finally, using the well known expression,

\[ d\Gamma = \frac{(2\pi)^4}{2m_{l_1}} |M|^2 \delta^4 \left( p - \sum_{i=1}^2 p_i \right) \prod_{i=1}^2 \frac{d^4 p_i}{(2\pi)^3 2E_i}, \]  

(2.15)

the decay width \( \Gamma \) is obtained in the \( l_1 \) lepton rest frame. Here \( p_i, i=1,2 \) is four momentum vector of \( l_1 \) lepton, \( (l_2 \) lepton, outgoing \( \kappa \) photon).

At this stage, we calculate the CP asymmetry \( A_{CP} \) of the process \( l_1 \to l_2 \gamma \). In the model III the CP violation does not exist even with the choice of complex Yukawa couplings \( \xi_{N,i} \). However the addition of the Lorentz and CPT violating terms switch on these effects and these effects are very small since they are proportional to the the Lorentz and CPT violating coefficients. Using the definition of the CP asymmetry \( A_{CP} \)

\[ A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}, \]  

(2.16)

where \( \bar{\Gamma} \) denotes the decay width for the CP conjugate process, we get

\[ A_{CP} = \frac{\int_0^1 dx \int_0^{1-x} dy \Omega(x, y)}{\int_0^1 dx \int_0^{1-x} dy W(x, y)}, \]  

(2.17)

where

\[ \Omega(x, y) = \Omega_g(x, y) + \Omega_a(x, y) + \Omega_b(x, y) + \Omega_e(x, y) \]  

(2.18)

with

\[ \Omega_g(x, y) = -\frac{4\pi\alpha_e}{512\pi^4} \sum_{i=e,\mu,\tau} m_i m_{l_1}^4 Q_i^2 |\xi_{N,i} l_2|^2 |\xi_{N,i}|^2 \sin 2\theta_{l_1 i} g[p_0, k, k] (F(z_h) - F(z_k)) \times \]

\[ \times (1 - x - y) G_1(x, y), \]

\[ \Omega_a(x, y) = -\frac{4\pi\alpha_e}{384\pi^4} \sum_{i=e,\mu,\tau} m_i^4 Q_i^2 |\xi_{N,i} l_2|^2 |\xi_{N,i}|^2 \cos^2 \theta_{l_1 i} a[p_0] (H_1(z_h) - H_2(z_A)) \times \]

\[ \times (1 - x - y) G_2(x, y), \]

\[ \Omega_b(x, y) = -\frac{4\pi\alpha_e}{384\pi^4} \sum_{i=e,\mu,\tau} m_i^4 m_i^4 Q_i^2 |\xi_{N,i} l_2|^2 |\xi_{N,i}|^2 \cos^2 \theta_{l_1 i} b[p_0] (H_1(z_h) - H_2(z_A)) \times \]

\[ \times (1 - x - y) G_3(x, y), \]

\[ \Omega_e(x, y) = -\frac{4\pi\alpha_e}{384\pi^4} \sum_{i=e,\mu,\tau} m_i^4 Q_i^2 |\xi_{N,i} l_2|^2 |\xi_{N,i}|^2 \cos^2 \theta_{l_1 i} e[p_0] (H_1(z_h) - H_2(z_A)) \times \]

\[ \times (1 - x - y) G_4(x, y), \]  

(2.19)

Here the function \( F(z_S), H_i(x, y) \) and \( G_i(x, y) \) read

\[ F(z_S) = \frac{1}{m_S^2 (z_S - 1)^3} (3 - 4 z_S + z_S^2 + 2 \ln z_S), \]

\[ H_1(z_S) = \frac{1}{m_S^2 (z_S - 1)^4} ((z_S - 1) (6 m_i (3 - 4 z_S + z_S^2) + m_i (-4 + 5 z_S + 5 z_S^2)) + \]

\[ + 6 (2 m_i (z_S - 1) + m_i (1 - 2 z_S) z_S) \ln z_S), \]
and we take the Yukawa couplings we do not give the explicit expression for the function

\[ G_1(x, y) = \left( \frac{1}{L_{1,b_0}} - \frac{1}{L_{1,A_0}} \right) x y - \left( \frac{1}{L_{2,b_0}} - \frac{1}{L_{2,A_0}} \right) x (x + y - 2) + 2 \left( \frac{1}{L_{2,b_0}^2} + \frac{1}{L_{2,A_0}^2} \right) m_i m_{l_1} x^2 (x + y), \]

\[ G_2(x, y) = \frac{m_i + m_{l_1} x}{L_{2,b_0}^2} - \frac{m_i - m_{l_1} x}{L_{2,A_0}^2}, \]

\[ G_3(x, y) = \frac{x (1 - 4 x - 4 y)}{4} \left( \frac{1}{L_{1,b_0}^2} - \frac{1}{L_{1,A_0}^2} \right), \]

\[ G_4(x, y) = \frac{x (3 - 4 x - 4 y)}{2} \left( \frac{1}{L_{2,b_0}^2} - \frac{1}{L_{2,A_0}^2} \right) - \frac{1}{L_{1,b_0}^2} x (x+y) + \frac{1}{L_{1,A_0}^2} x (x+y) + \frac{1}{L_{1,b_0}^2} x (x+y) + \frac{1}{L_{1,A_0}^2} x (x+y) \]

\[ f_1^+(x,y) = 2 m_i m_{l_1} x + m_i^2 (-1 + x - y) - m_{l_1}^2 x y (-1 + x + y), \]

\[ f_1^-(x,y) = 2 m_i m_{l_1} x + m_{l_1}^2 (1 - x + y) + m_i^2 x y (-1 + x + y), \]

\[ f_2^+(x,y) = 2 m_i m_{l_1} x + m_i^2 (-1 + x - y) + m_{l_1}^2 x (x+y) (-1 + x + y), \]

\[ f_2^-(x,y) = 2 m_i m_{l_1} x + m_{l_1}^2 (1 - x + y) + m_i^2 x (x+y) (1 - x - y). \]

Notice that the index \( k = 1, 2, 3 \); the parameters \( L_{(1,2)} , z_S \) are given in the appendix and we do not give the explicit expression for the function \( W(x, y) \) since it is very long. Here we take the Yukawa couplings \( \xi_{N,il_1} \) complex with the parametrization

\[ \xi_{N,il_1} = |\xi_{N,il_1}^E| e^{i\theta_{il_1}}, \]

and \( \xi_{N,il_2} \) real. In eq. (2.19), the coefficients \( g[p_0,k,k] = m_i \), \( g[0,0] = m_i \), \( a[p_0] = m_i \), \( b[p_0] = m_i \), \( e[p_0] = m_i \), \( e[0] = m_i \), exist since we study in the rest frame of the incoming lepton \( l_1 \). Here \( g[0,0] \) is CP even and the others, \( a[0], b[0], e[0] \), are CP odd coefficients appearing in the lagrangian eq. (2.7) (see [1]). The \( A_{CP} \) is nonzero due to the complex nature of the couplings for the part \( \Omega_g \) and CP odd nature of the coefficients for the part \( \Omega_a + \Omega_b + \Omega_c \).

3. Discussion

In this section we analyze the Lorentz and CPT violating effects on the \( BR \) and the \( A_{CP} \) for the LFV \( \mu \to e\gamma \) and \( \tau \to \mu\gamma \) decays in the framework of the model III. The Yukawa couplings \( \xi_{N,il_\mu} \) and \( \xi_{N,il_\tau} \) \( (i = e, \mu, \tau) \) are responsible for the LFV decays. They are the free parameters of the theory and they should be restricted by respecting the appropriate experimental measurements. Fortunately, the strength of the Yukawa couplings \( \xi_{N,il_\mu} \) are considered as proportional to the masses of the leptons which are given by the indices and therefore, the contribution of the couplings related to the heavy leptons are dominant. For
the decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ the main contribution comes from the internal $\tau$ lepton and the Yukawa couplings $\xi_{N,\tau\mu}^E$, $\xi_{N,\tau\mu}^F$ and $\xi_{N,\tau\tau}^E \xi_{N,\tau\tau}^F$ play the main role respectively. There are various studies on the strength of these couplings in the literature. The upper limit of the coupling $\xi_{N,\tau\mu}^E$ has been predicted as $\sim 0.15$, by using experimental result of anomalous magnetic moment of muon in [33]. In [23] the coupling $\xi_{N,\tau\mu}^E$ and $\xi_{N,\tau\tau}^E$ has been estimated at the order of 0.03 and 0.15 respectively. For the coupling $\xi_{N,\tau\tau}^E$ the prediction has been done at the order of the magnitude of $10^{-4} - 10^{-3}$ in [19], by using the experimental result of the electric dipole moment of muon [33] and the upper limit of the $BR$ of the process $\mu \rightarrow e\gamma$ [30]. Notice that the couplings $\xi_{N,i\mu}^E$ are complex in general and in the following, we use the parametrization

$$\xi_{N,i\mu}^E = \sqrt{\frac{4G_F}{\sqrt{2}}} \xi_{N,i\mu}^E,$$

where $G_F = 1.6637 \times 10^{-5}$ (GeV$^{-2}$) is the fermi constant.

A possible small violations of Lorentz and CPT symmetry in the extension of the SM arise and those effects could be detected in the existing experiments [34]. In the present work we assume that these effects are only due to the QED part of the interactions and we take the numerical values of the coefficients at the order of the magnitude of $10^{-10}$. We try to understand the relative behavior of different coefficients, violating Lorentz and respecting the existing results [17].

$$|a|, |b| \sim m_\mu(m_\tau) \frac{m_{EW}}{M_P} < 10^{-18} \ (10^{-17}) \text{ GeV},$$

$$|d|, |c|, |e|, |f|, |g| \sim \frac{m_\mu(m_\tau)}{M_P} < 10^{-21} \ (10^{-20}),$$

where $m_{EW}$ ($M_P$) is the electro weak (Planck mass) scale.

In figure 3 we present the magnitude of the coefficient dependence of the Lorentz violating part of the $BR$ for the decay $\mu \rightarrow e\gamma$, for the real Yukawa couplings, $\xi_{N,\tau\mu}^E = 30$ GeV, $\xi_{N,\tau\tau}^E = 0.001$ GeV. Here solid (dashed, small dashed,dotted, dot-dashed) line represents the dependence to the coefficient $|a|, |b|, |c|, |d|, |e|, |f|, |g|$, in the case that the other coefficients have the same numerical value $10^{-20}$. The BR is at the order of the magnitude of $10^{-33} - 10^{-30}$, which is a negligible quantity compared to the current experimental limits $10^{-11}$ [21]. It increases with the increasing values of the coefficients, especially $|f|$ and $|a|$. The BR is much more sensitive to the coefficient $|f|$ compared to the others.

Figure 3 is devoted to the magnitude of the coefficient dependence of the Lorentz violating part of the $BR$ for the decay $\tau \rightarrow \mu\gamma$, for the real Yukawa couplings, $\xi_{N,\tau\mu}^E = 100$ GeV, $\xi_{N,\tau\tau}^E = 30$ GeV. Here solid (dashed, small dashed,dotted, dot-dashed) line represents the dependence to the coefficient $|a|, |b|, |c|, |d|, |e|, |f|, |g|$, in the case that the other coefficients have the same numerical value $10^{-20}$. The BR is at the order of the magnitude of $10^{-29} - 10^{-24}$, which is a negligible quantity compared to the current
Figure 3: The magnitude $\chi$ of the coefficient dependence of the Lorentz violating part of the $BR$ for the decay $\mu \rightarrow e\gamma$, for the real Yukawa couplings, $\xi_{N,\mu}^E = 30 \text{ GeV}$, $\xi_{N,\tau}^E = 0.001 \text{ GeV}$. Here solid (dashed, small dashed,dotted, dot-dashed) line represents the dependence to the coefficient $|a,p|$, $(|b,p|, (c_{\text{Sym}}, d_{\text{Sym}}, |g[p,\beta,\beta]|, |e,p|, |f,p|)$, in the case that the other coefficients have the same numerical value $10^{-20}$.

Figure 4: The same as figure 3 but for the decay $\tau \rightarrow \mu\gamma$.

limits $10^{-6}$ [22], similar to the previous process. Here the increasing values of the coefficients, $|f|$, $|a|$ and $c_{\text{Sym}}$ increases the BR. However the BR decreases with the increasing values the coefficient $|b|$. The BR is much more sensitive to the coefficients $|e|$, $|a|$, $|e|$ and $|b|$ compared to the others.

Now, in figure 3 [23], we present the possible CP violating asymmetry $A_{CP}$ for the decay $\mu \rightarrow e\gamma (\tau \rightarrow \mu\gamma)$, for the Yukawa couplings,$|\xi_{N,\tau\mu}^E| = 30 \text{ GeV}$, $|\xi_{N,\tau\tau}^E| = 0.001 \text{ GeV}$ ($|s_{N,\tau\tau}| = 100 \text{ GeV}$, $|s_{N,\tau\mu}| = 30 \text{ GeV}$), $\sin\theta_{\tau\mu} = 0.5$ ($\sin\theta_{\tau\tau} = 0.5$). Notice that we take $\sin\theta_{\tau\epsilon} = 0$ ($\sin\theta_{\tau\mu} = 0$) for the decays $\mu \rightarrow e\gamma (\tau \rightarrow \mu\gamma)$. Here solid (dashed, small dashed,dotted) line represents the dependence to the coefficient $|g[p_0,k,k]|$, $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$, in the case that the other coefficients have the same numerical value $10^{-20}$. The coefficient $g[p_0,k,k]$ ($a[p_0]$ and $b[p_0]$ and $c[p_0]$) is CP even (odd) and the source of the $A_{CP}$
Figure 5: The magnitude $\chi$ of the coefficient dependence of the $A_{CP}$ for the decay $\mu \rightarrow e\gamma$, for the Yukawa couplings, $|\xi_{N,\gamma\mu}| = 30$ GeV, $|\xi_{N,\gamma\mu}| = 0.001$ GeV, $\sin\theta_{\mu\gamma} = 0.5$, $\sin\theta_{e\gamma} = 0$. Here solid (dashed, small dashed, dotted) line represents the dependence to the coefficient $|g[p_0, k, k]|$, $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$, in the case that the other coefficients have the same numerical value $10^{-20}$.

Figure 6: The magnitude $\chi$ of the coefficient dependence of the $A_{CP}$ for the decay $\tau \rightarrow \mu\gamma$, for the Yukawa couplings, $|\xi_{N,\tau\mu}| = 100$ GeV, $|\xi_{N,\gamma\mu}| = 30$ GeV, $\sin\theta_{\tau\mu} = 0.5$, $\sin\theta_{e\gamma} = 0$. Here solid (dashed, small dashed, dotted) line represents the dependence to the coefficient $|g[p_0, k, k]|$, $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$, in the case that the other coefficients have the same numerical value $10^{-20}$.

is the complex nature of the couplings for the part proportional to the coefficient $g[p_0, k, k]$ and the CP odd nature of the coefficients for the part proportional to coefficients $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$ (see eq. (2.19)). For the $\mu \rightarrow e\gamma$ ($\tau \rightarrow \mu\gamma$) decay $A_{CP}$ is much more sensitive to the coefficients $g[p_0, k, k]$ and $|e[p_0]|$ ($g[p_0, k, k]|$, $|e[p_0]|$ and $|a[p_0]|$) compared to the ones $|a[p_0]|$ and $|b[p_0]|$ ($|b[p_0]|$). It increases with the increasing values of $g[p_0, k, k]$ and $|e[p_0]|$ ($g[p_0, k, k]|$, $|e[p_0]|$ and $|a[p_0]|$). Notice that in the case of real couplings the coefficient $g[p_0, k, k]$ does not give contribution to the $A_{CP}$ However the numerical value of $A_{CP}$ is very small, at the order of the magnitude of $10^{-19}$ for both decays and it seems that it is not possible to detect even in the future experiments.
At this stage we would like to summarize our results:

We analyse the Lorentz and CPT violating effects on the BR and $A_{CP}$ for the LFV decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the framework of the model III. Here we assume that these effects are only due to the QED part of the interactions. By taking the numerical values of the coefficients at the order of the magnitude of $10^{-20} - 10^{-18}$ we study the relative behaviors of different coefficients

- The contribution of the Lorentz and CPT violating part to the BR of the decays $\mu \to e\gamma$ ($\tau \to \mu\gamma$) is at the order of the magnitude of $10^{-32}$ ($10^{-26}$), which is too small to be detected. For the decay $\mu \to e\gamma$ the BR is more sensitive to the coefficient $|e|$ compared to others and for its large values the BR reaches to the order of $10^{-30}$. For the decay $\tau \to \mu\gamma$ the BR is sensitive to the coefficients $|a|, |e|$ and $|b|$ and it can take the values at the order of the magnitude of $10^{-24}$

- We predict the numerical value of $A_{CP}$ at the order of the magnitude of $10^{-19}$ for both decays. The source of the $A_{CP}$ is the coefficients $g[p_0,k,k] = m_{l_1} g[0,k,k]$, $a[p_0] = m_{l_1} a[0]$, $b[p_0] = m_{l_1} b[0]$ and $e[p_0] = m_{l_1} e[0]$. Here $g[0,k,k]$ is CP even and the others, $a[0], b[0], e[0]$, are CP odd appearing in the lagrangian eq. (2.6) (see [17] for details). The $A_{CP}$ is nonzero due to the complex nature of the couplings for the part $\Omega_{\gamma}$ and CP odd nature of the coefficients for the part $\Omega_{\mu} + \Omega_b + \Omega_e$. We observe that the $A_{CP}$ is too small to be measured even in future experiments.

A. The explicit forms of the functions appearing in the matrix element

The coefficients $A_i^{(l)}(x, y), B_i^{(l)}(x, y), C_i^{(l)}(x, y), D_i^{(l)}(x, y), E_i^{(l)}(x, y)$ in eqns. (2.13) and (2.14) reads

$$A_i(x, y) = \left\{ \frac{1}{L_{1, h^0}} + \frac{1}{L_{2, A^0}} \right\} (1 + x + y) + \frac{1}{L_{2, h^0}} + \frac{1}{L_{2, A^0}} f_1 +$$

$$+ \left\{ \frac{1}{L_{2, h^0}} - \frac{1}{L_{2, A^0}} \right\} m_{l_1} m_i x(x + 2y) \right\} a.k +$$

$$+ \left\{ m_i y \left( \frac{1}{L_{1, h^0}} + \frac{1}{L_{1, A^0}} \right) f_2 + 2 \left( \frac{1}{L_{1, h^0}} - \frac{1}{L_{1, A^0}} \right) m_{l_1} m_i x \right\} +$$

$$+ \left( \frac{1}{L_{1, h^0}} + \frac{1}{L_{1, A^0}} \right) m_i y - \left( m_i (2 + x + y) + (m_i - x m_{l_1}) + \frac{1}{L_{2, h^0}} \right) \times$$

$$\times (x + y)(m_{l_1}^2 - x y m_{l_1}^2) - \left( m_i (-2 + x + y) - 4 x (x + y) m_{l_1} \right) \frac{1}{L_{2, h^0}} +$$

$$+ \left( m_i (-2 + x + y) + 4 x (x + y) m_{l_1} \right) \frac{1}{L_{2, A^0}} \right\} e.k +$$

$$+ \left\{ \frac{1}{L_{1, h^0}} - \frac{1}{L_{1, A^0}} \right\} m_{l_1} m_i y (f_3 - m_{l_1}^2 + m_{l_1}^2 x(x + y)) c^{Sym} +$$
\begin{align*}
&+ \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) m_i m_i x^2 (2 + x) \epsilon^{\text{Asym}}[p, p'] + \\
&+ 2 i \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) m_i x^2 (x + y) (y g[p, p', k] + x g[p, p', p]) - \\
&- i \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) x (x + y) (m_i (1 - x - y) + 2 m_i x (1 - y)) g[p, p', k] + \\
&\quad + \frac{1}{2} x^2 (m_i (2 x + y - 2) + 4 m_i x (-1 + y)) g[p, p', p] \right) - \\
&- i \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) x (x + y) (-m_i (x + y - 1) + 2 m_i x (-1 + y)) g[p, p', k] + \\
&\quad + \frac{1}{2} x^2 (m_i (2 x + y - 2) + 4 m_i x (-1 + y)) g[p, p', p] \right),

A_i(x, y) &= \left\{ \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{2,A^0}} \right) f_4 (1 - x - y) - \frac{1}{L_{2,A^0}} \right) x (x - 1) m_i m_i + \\
&+ \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{2,A^0}} \right) (-1 + 3 x - y) - \frac{1}{L_{2,A^0}} x^2 m_i m_i - \\
&- \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) f_4 + \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) (1 + x + y) \right\} b,k - \\
&- \left\{ \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{2,A^0}} \right) m_i y \left( m_i^2 (x + y) - m_i^2 \right) - 3 m_i \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{2,A^0}} \right) - \\
&- \left( \frac{m_i + m_i x}{L_{2,h^0}} + \frac{m_i - m_i x}{L_{2,A^0}} \right) (x + y)(m_i^2 + x y m_i^2) - \\
&- \left( \frac{3 m_i + 4 x m_i}{L_{2,h^0}} + \frac{3 m_i - 4 x m_i}{L_{2,A^0}} \right) (x + y) \right\} f,k + \\
&+ \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) m_i m_i x^2 (1 - 3 x - 3 y) d^{\text{Asym}}[p, p'] \frac{1}{2} \epsilon_{\rho \alpha \beta} \times \\
&\times \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right) m_i x h[\alpha, \beta] k_{\rho} p_{\theta} + \frac{1}{2} \epsilon_{\rho \alpha \beta} \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) \times \\
&\times m_i x (1 - x - y) (x + y) g[\beta, \alpha, k] k_{\rho} p_{\theta},
\end{align*}
\[ B_{i\mu\alpha}(x,y) = -i \left\{ \frac{1}{4} \epsilon_{\rho\alpha\beta\mu} \left( \frac{1}{L_{1,h^0}^2} - \frac{1}{L_{2,A^0}^2} \right) m_{i} m_{l_{1}} x \left( 1 - 4x - 4y \right) + \frac{1}{L_{2,h^0}^2} \times \right. \\
\times \left( f_{4} + 2m_{i} \left( m_{l_{1}} x + m_{l_{1}} y \right) \right) + \frac{1}{L_{2,A^0}^2} \left( f_{4} + 2m_{i} \left( -m_{l_{1}} x + m_{l_{1}} y \right) \right) + \\
\left. + \left( \frac{1}{L_{2,h^0}^2} + \frac{1}{L_{2,A^0}^2} \right) \left( -1 + 3x + 3y \right) \right\} k_{\rho} b_{3} - \\
- \left\{ m_{i} m_{l_{1}} y \left( \left( \frac{1}{L_{1,h^0}^2} - \frac{1}{L_{1,A^0}^2} \right) f_{3} + \left( \frac{1}{L_{2,h^0}^2} + \frac{1}{L_{2,A^0}^2} \right) m_{i} m_{l_{1}} x \right) + \\
+ 3 \left( \frac{1}{L_{1,h^0}^2} - \frac{1}{L_{1,A^0}^2} \right) m_{i} m_{l_{1}} y + m_{i} \left( x + y \right) \left( m_{l_{1}}^{2} + m_{l_{1}}^{2} x y \right) \times \\
\times \left( \frac{1}{L_{2,h^0}^2} \left( m_{i} + m_{l_{1}} x \right) + \frac{1}{L_{2,A^0}^2} \left( -m_{i} + m_{l_{1}} x \right) \right) + m_{i} \times \\
\left. + \left( \frac{1}{L_{2,h^0}^2} \left( 4m_{l_{1}} x \left( x + y \right) + m_{i} \left( 1 + 3x + 3y \right) \right) + \\
+ \frac{1}{L_{2,A^0}^2} \left( 4m_{l_{1}} x \left( x + y \right) - m_{i} \left( 1 + 3x + 3y \right) \right) \right\} e^{\Delta y \left[ \mu, \alpha \right]} - i \epsilon_{\rho\alpha\beta\mu} \times \\
\times \left\{ 2 \left( \frac{1}{L_{1,h^0}^2} - \frac{1}{L_{1,A^0}^2} \right) m_{l_{1}} m_{i} x y^{2} + \left( \frac{1}{L_{2,h^0}^2} - \frac{1}{L_{2,A^0}^2} \right) m_{l_{1}} m_{i} x \times \\
\times \left( x + y \right) \left( y - 1 \right) \right\} k_{\rho} d^{\Delta y \left[ \beta, k \right]} - i \times \\
\times \left\{ \left( \frac{1}{L_{2,h^0}^2} - \frac{1}{L_{2,A^0}^2} \right) m_{l_{1}} \left( x + y \right) - \left( \frac{1}{L_{2,A^0}^2} - \frac{1}{L_{2,A^0}^2} \right) m_{l_{1}} f_{4} - \\
- \left( \frac{1}{L_{2,h^0}^2} + \frac{1}{L_{2,A^0}^2} \right) m_{i} m_{i} x^{2} \right\} h[\mu, \alpha] + \frac{1}{2} \epsilon_{\rho\alpha\beta\mu} \left( \frac{1}{L_{1,h^0}^2} - \frac{1}{L_{1,A^0}^2} \right) \times \\
\times m_{l_{1}} x y^{3} k_{\rho} g[\beta, k, k] + i \left\{ \frac{1}{L_{1,h^0}^2} \left( m_{i} \left( x - 1 \right) - m_{l_{1}} \left( x + 2y + 2xy \right) \right) + \frac{1}{L_{1,A^0}^2} \times \\
\times \left( m_{i} \left( x - 1 \right) + m_{l_{1}} \left( x + 2y + 2xy \right) \right) \right\} \times \\
\times g[k, \alpha, \mu] - 2i \left( \frac{1}{L_{1,h^0}^2} - \frac{1}{L_{1,A^0}^2} \right) m_{l_{1}} x y g[\rho, \mu, \mu] - i \frac{1}{2} \times \\
\times \left( \frac{1}{L_{1,h^0}^2} \left( 2m_{i} \left( x - 1 \right) + m_{l_{1}} \left( x \left( y - 2 \right) - 4y \left( 1 - 3y \right) \right) \right) + \frac{1}{L_{1,A^0}^2} \times \\
\times \left( 2m_{i} \left( x - 1 \right) - m_{l_{1}} \left( x \left( y - 2 \right) - 4y \left( 1 - 3y \right) \right) \right) \right\} g[\mu, \alpha, k] + \]
\[\frac{\Gamma}{2} \left( \frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) m_i x y (g[\mu,k,\alpha] - 2 g[\mu,p,\alpha] + 10 g[\mu,\alpha,p]) - \]
\[\frac{i}{4} \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) m_i^2 m_i x \]
\[\times \left( -4 x (x + y) g[k,\alpha,\mu] + 8 x (x + y) g[p,\alpha,\mu] + 2 \times \right. \]
\[\times (x^2 + y(y - 1) + x(2y - 1)) g[\mu,\alpha,k] - x(-2 + 2x + y) g[\mu,\alpha,p] \right) + \]
\[+ i \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) m_i x (x + y) g[\mu,p,\alpha] + \frac{i}{2} \times \]
\[\times \left\{ \frac{1}{L_{2,h^0}} (m_i (-1 + x + y) + 4m_i (x + x^2 + y - y^2)) + \right. \]
\[+ \frac{1}{L_{2,A^0}} (m_i (-1 + x + y) - 4m_i (x + x^2 + y - y^2)) \right\} g[k,\alpha,\mu] - \]
\[- \frac{i}{2} \left( \frac{1}{L_{2,h^0}} (m_i (-1 + x + y) + m_i (11x^2 + 8y(y - 1) + (19y - 8)x)) - \right. \]
\[- \frac{i}{2} \left( \frac{1}{L_{2,A^0}} (m_i (-1 + x + y) + m_i (-11x^2 - 8y(y - 1) - (19y - 8)x)) \right. \times \]
\[\left. \times \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) \times \right) \]
\[\times \left( m_i \left( 4(x + y - 1) g[p,\alpha,\mu] - (x + y) g[\mu,k,\alpha] + 2(5x + 5y - 4) \times \right. \]
\[\left. \times g[\mu,\alpha,p] \right) \right), \]
\[B_{i\mu\alpha}(x,y) = - i \epsilon_{\rho\alpha\beta\mu} \left\{ \left( \frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) m_i m_i x + \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right) \times \right. \]
\[\times f_3(-1 + x + y) + \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right)(-1 + 3x + 3y) + \right. \]
\[+ \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right)(m_i^2 (y - 1) + m_i^2 x y (-1 + x + y)) + \right. \]
\[+ \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right)(-1 + 3x + 3y) \right\} k_{\rho \alpha \beta} - m_i^3 m_i x \times \]
\[\times \left\{ 2 \left( \frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) y^2 + \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right)(y - 1)(x + y) \right. \times \]
\[\left. \times d^{\text{A Sym}}[\mu,\alpha] - \epsilon_{\rho\alpha\beta\mu} \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right) m_i \times \right) \]
\[ \times \left\{ (2 (x - 1) y \, h[k, \beta] + x (1 + 2 x) h[p, \beta]) \, k_\rho - \\ - x \left( (1 + 2 y) h[k, \beta] \, p_\rho - h[\beta, \alpha] \, k_\rho \, p_\theta + (2 y + 1) h[\mu, \beta] \, k_\rho \, p_\mu \right) \right\} - \\
- \epsilon_\rho \epsilon_\theta \epsilon_\mu \epsilon_\lambda \left\{ \frac{1}{L_{2,h}^2} (2 x (m_i + m_i \, x) h[p, \beta] - (m_i \, x^2 + 2 m_i \, y + 2 m_i \, x^2 \times \\
\times y + 2 m_i \, x^2) \, h[k, \beta]) + \\
+ \frac{1}{L_{2,A}^2} (2 x (m_i - m_i \, x) h[p, \beta] - (m_i \, x^2 + 2 m_i \, y - 2 m_i \, x^2 \times \\
\times y - 2 m_i \, x^2) \, h[k, \beta]) \right\} k_\rho - \\
\epsilon_\rho \epsilon_\theta \epsilon_\mu \left\{ \left( \frac{1}{L_{1,h}^2} - \frac{1}{L_{1,A}^2} \right) (x + y) (x \, g[p, \beta, k] - 2 y \, g[\beta, k, k] - \\
- 2 x \, g[\beta, k, p] - x \, g[\beta, p, k]) + \\
+ \frac{1}{2} (x + y) \, m_i \, y^2 \left( \frac{1}{L_{1,h}^2} + \frac{1}{L_{1,A}^2} \right) \, g[\mu, \alpha, k] \right\} k_\rho + \\
+ \frac{1}{2} \left( \frac{1}{L_{1,h}^2} - \frac{1}{L_{1,A}^2} \right) \, m_i \, x \, y \times \\
\times \left( \epsilon_\alpha \epsilon_\beta \epsilon_\mu \epsilon_\lambda \, (g[k, \beta, \lambda] + 2 \, g[\lambda, \beta, k]) + \epsilon_\rho \epsilon_\theta \epsilon_\mu \epsilon_\lambda \, (2 \, g[\lambda, \beta, \mu] + g[\mu, \beta, \lambda]) \, k_\rho \right) + \\
+ \frac{1}{2} \epsilon_\rho \epsilon_\theta \epsilon_\mu \left( x + y \right) \left( \frac{1}{L_{2,h}^2} (m_i (-1 + x + y) - 4 m_i (y - 1) (x + y)) + \\
+ \frac{1}{L_{2,A}^2} (m_i (-1 + x + y) + 4 m_i (y - 1) (x + y)) \right) \times \\
\times k_\rho \, g[\beta, k, k] - \frac{1}{4} \epsilon_\rho \epsilon_\theta \epsilon_\mu \times \\
\times \left\{ \frac{1}{L_{2,h}^2} (8 m_i \, x (y - 1) (x + y) \, (g[p, \beta, k] - g[\beta, k, p]) + m_i \, x \times \\
\times (-2 + 2 x + y) \, g[\beta, k, p]) - \frac{1}{L_{2,A}^2} \times \\
\times (8 m_i \, x (y - 1) (x + y) \, (g[p, \beta, k] - g[\beta, k, p]) - \\
- m_i \, x (-2 + 2 x + y) \, g[\beta, k, p]) \right\} k_\rho - \\
\frac{1}{4} \left( \frac{1}{L_{2,h}^2} + \frac{1}{L_{2,A}^2} \right) \, m_i^2 \, m_i \, x \, (x + y) \times \\
\times (2 \epsilon_\alpha \epsilon_\beta \epsilon_\mu \epsilon_\lambda \, g[\lambda, \beta, k] + \epsilon_\rho \epsilon_\theta \epsilon_\mu \epsilon_\lambda (1 + x - y) \, g[\lambda, \beta, \mu] \, k_\rho) - \frac{1}{2} \left( \frac{1}{L_{2,h}^2} - \frac{1}{L_{2,A}^2} \right) \times \right\} \]
\[ C_i,\alpha(x, y) = \begin{aligned} & m_i y \left( \left( \frac{1}{L_{1,0}^2} + \frac{1}{L_{2,0}^2} \right) m_i m_1 x + \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{2,0}^2} \right) f_3 \right) + \\
& + 3 m_i y \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{2,0}^2} \right) + (x + y) (m_i^2 + m_1^2 x y) \times \\
& \times \left( \frac{1}{L_{1,0}^2} (m_i + m_1 x) + \frac{1}{L_{2,0}^2} (-m_i + m_1 x) \right) + \\
& + \frac{1}{L_{2,0}^2} (4 m_1 x (x + y) + m_i (1 + 3 x + 3 y)) + \\
& + \frac{1}{L_{2,0}^2} (4 m_1 x (x + y) - m_i (1 + 3 x + 3 y)) \right) \times \\
& \times e^{A_{\text{Sym}}[k, \alpha] - i \times \\
& \times \left( \left( \frac{1}{L_{2,0}^2} + \frac{1}{L_{1,0}^2} \right) m_i m_1 x^2 + \left( \frac{1}{L_{2,0}^2} - \frac{1}{L_{1,0}^2} \right) f_4 - \\
& - \left( \frac{1}{L_{2,0}^2} - \frac{1}{L_{1,0}^2} \right) (x + y) \right) h[k, \alpha] + \\
& + \frac{i}{2} \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{1,0}^2} \right) y \times \\
& \times (2 x g[p, p', \alpha] + (5 x + 12 y) g[k, \alpha, k] + 10 x g[k, \alpha, p] + 4 x g[\alpha, p, k]) - \\
& - \frac{i}{4} m_i m_1 \left( \frac{1}{L_{1,0}^2} + \frac{1}{L_{1,0}^2} \right) x (x - 2 + 2 x + y) g[k, \alpha, p] - 2 (x + y) \times \\
& \times ((1 + x - y) g[\alpha, k, k] - 4 x g[\alpha, p, k]) - i \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{1,0}^2} \right) x \times \\
& \times ((x + y) g[p, p', \alpha] + (5 x + 5 y - 4) g[k, \alpha, p] - \\
& - (7 x^2 + 12 y(y - 1) + x(19 y - 12)) g[k, \alpha, k] - 4 x(-1 + x + y) g[\alpha, p, k] \right) \right) \), \\
\end{aligned} \]

\[ C_i',\alpha(x, y) = m_i m_1^2 \left\{ 2 \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{2,0}^2} \right) y^2 + \left( \frac{1}{L_{2,0}^2} - \frac{1}{L_{1,0}^2} \right) (x + y)(y - 1) \right\} \times \\
\times d^{A_{\text{Sym}}[k, \alpha]} + 3 m_i y \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{1,0}^2} \right) e^{A_{\text{Sym}}[k, \alpha]} + \\
+ \frac{i}{2} \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{1,0}^2} \right) m_i^2 x y^2 g[\alpha, k, k] + \frac{i}{2} \left( \frac{1}{L_{1,0}^2} - \frac{1}{L_{1,0}^2} \right) \times \\
\times (2 x g[p, p', \alpha] - (5 x + 12 y) g[\alpha, k, k] - 10 x g[\alpha, k, p] - 4 x g[\alpha, p, k]) + \\
+ \frac{1}{4} \epsilon_{\rho\alpha\beta} m_i m_1 \left( \frac{1}{L_{2,0}^2} + \frac{1}{L_{2,0}^2} \right) (x^2 + y(y - 1) + x(2 y - 1)) g[\beta, \theta, k] k_{\rho}, \]
\[ D_t(x, y) = \frac{1}{1 - x - y} \left( \frac{1}{L_{1,0}} (m_i (x - 1) + m_i x (y - 1)) - \frac{1}{L_{1,A^0}} (m_i (1 - x) + m_i x (y - 1)) \right) - \]
\[- \left( \frac{1}{L_{2,0}} (m_i (x^2 - 2 x - 2 y + 2 x y) - 2 m_i x y (x + y)) - \frac{1}{L_{2,A^0}} (m_i (x^2 - 2 x - 2 y + 2 x y) + 2 m_i x y (x + y)) \right) \right) a.k - \]
\[- \left( \frac{1}{L_{2,0}} (2x(m_i + m_i x)) - \frac{1}{L_{2,A^0}} (2x(m_i - m_i x)) \right) \right) a.p. + \]
\[ + \frac{1}{4} \left( \frac{1}{L_{1,0}} - \frac{1}{L_{2,A^0}} \right) m_i x(1 - 4 x - 4 y)(b.k - 2 b.p) - \]
\[- \left\{ \frac{f_6}{L_{1,0}} + \frac{f_5}{L_{1,A^0}} - 2 y \left( \frac{1}{L_{1,0}} (3 m_i + 4 x m_i) - \frac{1}{L_{1,A^0}} (3 m_i - 4 x m_i) \right) - \right. \]
\[- \left. \left( \frac{f_8}{L_{1,0}} + \frac{f_7}{L_{2,A^0}} + \frac{2}{L_{2,0}} (m_i (1 - 3 x - 3 y) - m_i x) - 2 \frac{1}{L_{2,0}} \times \right) \right. \]
\[ \times (m_i (1 - 3 x - 3 y) + m_i x) \right\} c^{Sym} + \]
\[ + \left( \frac{2}{L_{1,0}} - \frac{1}{L_{1,A^0}} \right) m_i x y \left( \frac{1}{L_{2,0}} - \frac{1}{L_{2,A^0}} \right) m_i x^3 \right) c^{Asym}[p, p'] - \]
\[- \left\{ y \frac{1}{L_{1,0}} \left( m_i^2 (-1 + x - y) + 2 m_i x + (-1 + x + y) + \right) \right. \]
\[ + \frac{1}{L_{1,A^0}} \left( m_i^2 (1 - x + y) + 2 m_i x (-1 + x + y) - \right) \right. \]
\[- m_i^2 x (x + y) (x + y - 1) \right) - \]
\[ - \left( \frac{1}{L_{1,0}} - \frac{1}{L_{1,A^0}} \right) y (4 x + 4 y - 3) + (x + y) \frac{1}{L_{2,0}} \times \]
\[ \times (m_i^2 (1 - x + y) + 2 m_i x y + m_i^2 x x (x + y - 1)) - (x + y) \times \]
\[ \times \frac{1}{L_{2,A^0}} (m_i^2 (1 - x + y) - 2 m_i x y - m_i^2 x x (x + y - 1)) + \]
\[ + (x + y) \left( \frac{1}{L_{2,0}} - \frac{1}{L_{2,A^0}} \right) (-3 + 4 x + 4 y) \right) e.k - \]

\[ - 17 - \]
\[-\left\{ x \frac{1}{L_{1,h}^2} (m_i^2 (-1 + x - y) - 2 m_i m_{i_1} x + m_{i_1}^2 x (x + y) (-1 + x + y)) + \\
+ x \frac{1}{L_{1,A}^2} \left( m_i^2 (1 - x + y) - 2 m_i m_{i_1} x + m_{i_1}^2 x (x + y) (1 - x - y) - \\
- \left( \frac{1}{L_{1,h}^0} - \frac{1}{L_{1,A}^0} \right) x (4x + 4y - 3)x - \frac{1}{L_{2,h}^0} \times \\
\times (m_i^2 (1 - x + y) - 2 m_i m_{i_1} x + m_{i_1}^2 x y(x + y - 1)) + \\
+ \frac{1}{L_{2,A}^0} (m_i^2 (1 - x + y) - 2 m_i m_{i_1} x + m_{i_1}^2 x y(x + y - 1)) - \\
- x \left( \frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0} \right) (-3 + 4x + 4y) \right) \right\} \text{e.p} - \\
- 2x \left( \frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0} \right) (2y(y - 1) + x(3y - 1)) h[p, p'] - 2i \times \\
\times \left( \frac{1}{L_{1,h}^0} - \frac{1}{L_{1,A}^0} \right) x(x + y)(-2 y^2 g[p, p', k] + x (x - 2 y) g[p, p', p]) + \\
+ i \left( \frac{1}{L_{1,h}^0} - \frac{1}{L_{1,A}^0} \right) x^2 g[k, \beta, \beta] + i \left( \frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0} \right) \times \\
\times (x (-4y(x + y)(x + y - 1)g[p, p', k] + \\
+ 2(-2y + 2y^2 + 3xy - x) g[p, p', p]) - \\
- i \left( \frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0} \right) x(1 - x)g[k, \beta, \beta] \right\}, \\
D_i(x, y) = -\left\{ \frac{1}{4} \frac{1}{L_{1,h}^0} (8 m_i, x y (-1 + x + y) + m_i (4 x^2 - 4 x y + 8 y + x)) + \\
+ \frac{1}{4} \frac{1}{L_{1,A}^0} (8 m_i, x y (-1 + x + y) - m_i (4 x^2 - 4 x y + 8 y + x)) + \\
+ \frac{1}{L_{2,h}^0} (m_i (x^2 + 2 y) + 2 m_{i_1} x y (x + y)) + \\
+ \frac{1}{L_{2,A}^0} (-m_i (x^2 + 2 y) + 2 m_{i_1} x y (x + y)) \right\} \text{b.k} - \\
- \left\{ x \frac{1}{2} \frac{1}{L_{1,h}^0} (-4 m_i, x - m_i (3 - 4 y)) + \frac{1}{2} x \frac{1}{L_{1,A}^0} (-4 m_i, x + m_i (3 - 4 y)) - \\
- 2x \left( \frac{1}{L_{2,h}^0} (m_i + m_{i_1} x) + \frac{1}{L_{2,A}^0} (-m_i + m_{i_1} x) \right) \right\} \text{b.p} + \\
+ \left\{ x y m_i^2 \left( \frac{2}{L_{1,h}^0} (m_i, x (x + y) - m_i (y - x)) + \\
\right. \right\} \text{b.p} + \\
\} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \} \}
\[
\begin{align*}
\frac{2}{L^2_{1,A^0}} (m_i x (x+y) - m_i (x-y)) - \\
x(x+y)(y-1)m_i^2 \left( \frac{1}{L^2_{2,h^0}} (m_i - 2m_i, x) - \frac{1}{L^2_{1,A^0}} (m_i + 2m_i, x) \right) - \\
- 8 \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right) m_i xy - 2 \left( \frac{1}{L^2_{2,h^0}} + \frac{1}{L^2_{2,A^0}} \right) m_i x \times \\
\times (1 - 4x - 4y) \right\} d^8 \text{Sym} - \\
- \left\{ 4xy^2 \left( \frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}} \right) m_i + \left( \frac{1}{L^2_{2,h^0}} - \frac{1}{L^2_{2,A^0}} \right) m_i x \times \\
\times (3x^2 + 2(y-1)y + x(5y-3)) \right\} \times \\
\times q^{A \text{Sym}[p,p']} - ixy^3 \left( \frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}} \right) g[p,p',k],
\end{align*}
\]

\[E_{i,x}(x,y) = m_i \left\{ \frac{1}{L^2_{2,h^0}} (m_i^2 (1 + y) + m_i x y (x + y - 1) + m_i m_i x (x + 2y)) - \\
- \frac{1}{L^2_{2,h^0}} (m_i^2 (1 + y) - m_i x y (x + y - 1) - m_i m_i x (x + 2y)) - \\
- \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) (1 + x + y) \right\} a_{\mu} - \\
- \frac{1}{L^2_{1,h^0}} \text{m}_i p p' b_{\alpha} - \\
- \frac{1}{L^2_{1,h^0}} \text{m}_i x (m_i^2 + m_i m_i x - m_i^2 x (x - y)) + \\
+ \frac{1}{L^2_{1,A^0}} (m_i m_i x - m_i^2 + m_i^2 x (x - y)) - \\
- \frac{1}{L_{1,h^0}} y (m_i + 4m_i x - 2m_i y) + \frac{1}{L_{1,A^0}} (m_i - 4m_i x + 2m_i y) - \\
- (x + y) \left( \frac{1}{L^2_{2,h^0}} (3m_i^3 + 3m_i^2 m_i x + 3m_i^3 x^2 y + m_i m_i^2 x \times \\
\times x (2 - 2x + y)) + \frac{1}{L^2_{2,A^0}} x \times \\
\times (-3m_i^3 + 3m_i^2 m_i x + 3m_i^3 x^2 y - \\
- m_i m_i^2 x (2 - 2x + y)) \right) - \\
\right\}
\]
\begin{align*}
- & \frac{1}{L_{2,0}}(m_i (1 + 5 x + 5 y) + m_{l_i} (6 x^2 + 4 x y - 2 y^2)) + \\
+ & \frac{1}{L_{2,A}}(m_i (1 + 5 x + 5 y) - m_{l_i} (6 x^2 + 4 x y + 2 y^2)) \right) e^{A_{Sym}[k, \mu]} - \\
- & \left\{ 2m_i \left( \left( \frac{1}{L_{1,0}} - \frac{1}{L_{2,0}} \right) f_3 + \left( \frac{1}{L_{1,A}} - \frac{1}{L_{2,A}} \right) m_i m_{l_i} \right) + \\
+ & 6 \left( \frac{1}{L_{1,0}} - \frac{1}{L_{1,A}} \right) m_i y - \frac{1}{L_{2,0}} x \\
\times & \left( 2 m_3^2 (x + y) + 2 m_{l_i} m_i^2 x (x + y) + 2 m_{l_i}^3 x^2 y (x + y) - \\
- & m_{l_i}^2 m_i x (x^2 - 2 x (y - 1) - 2 y^2)) - \\
- & \frac{1}{L_{2,A}}(-2 m_3^2 (x + y) + 2 m_{l_i} m_i^2 x (x + y) + \\
+ & 2 m_{l_i}^3 x^2 y (x + y) + m_i^2 x m_i (x^2 - 2 x (y - 1) - 2 y^2)) - \\
- & 2 \frac{1}{L_{2,0}}(m_i (1 + 3 x + 3 y) + m_{l_i} (4 x^2 + 4 x y)) + \\
+ & 2 \frac{1}{L_{2,A}}(m_i (1 + 3 x + 3 y) - m_{l_i} (4 x^2 + 4 x y)) \right) e^{A_{Sym}[p, \mu]} - \\
- & \left\{ m_i m_{l_i} y \left( \left( \frac{1}{L_{1,0}} + \frac{1}{L_{1,A}} \right) f_2 + 2 \left( \frac{1}{L_{1,0}} - \frac{1}{L_{1,A}} \right) m_i m_{l_i} \right) + \\
+ & \left( \frac{1}{L_{1,0}} + \frac{1}{L_{1,A}} \right) m_i m_{l_i} y - m_{l_i} \times \\
\times & \left( \frac{1}{L_{2,0}}(m_i (x + y - 2) - 4 m_{l_i} x (x + y)) + \\
+ & \frac{1}{L_{2,A}}(m_i (x + y - 2) + 4 m_{l_i} x (x + y)) \right) \right\} e^\mu - \\
- & \left\{ \frac{1}{2} \frac{1}{L_{1,0}}(m_i^2 (1 - x - y) + m_{l_i}^2 (x^2 - x^3 + x y^2) + m_i m_{l_i} x) + \\
+ & \frac{1}{2} \frac{1}{L_{1,A}}(m_i^2 (-1 + x + y) + m_{l_i}^2 (-x^2 + x^3 - x y^2) + m_i m_{l_i} x) + \\
+ & \frac{1}{2} \left( \frac{1}{L_{1,0}} - \frac{1}{L_{1,A}} \right) (x - 3y) - 2 \frac{1}{L_{2,0}}(m_i^2 (y - 1) + m_{l_i}^2 x y + m_i m_{l_i} y) - \\
- & 2 \frac{1}{L_{2,A}}(m_i^2 (1 - y) - m_{l_i}^2 x y + m_i m_{l_i} y) - \\
- & 2 \left( \frac{1}{L_{2,0}} - \frac{1}{L_{2,A}} \right) (3 x + 2 y) \right\} h[k, \mu] + 
\end{align*}
\[
+i \left\{ \frac{1}{L_{2,h^0}} \left( f_4 + m_{i_1} x (m_i + m_{i_1} x y) \right) + \\
+ 2 \frac{1}{L_{2,A^0}} (-f_4 + m_{i_1} x (m_i - m_{i_1} x y)) - \\
- 2 \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) (x + y) \right\} h[p, \mu] - \epsilon_{\rho \theta \alpha \mu} \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) x^2 \times \\
\times k_\rho p_\theta h[p, \alpha] - 2 i \left( \frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) m_{i_1}^2 x (x + y) \times \\
\times \left( y^2 g[\mu, k, k] + x y g[\mu, k, p] + x y g[\mu, p, k] + x^2 g[\mu, p, p] \right) + \\
+ \frac{i}{2} \left( \frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) \times \\
\times \left( 4 (2 + 3 x) y g[k, p, \mu] + 4 y (1 + x + y) g[\mu, k, k] + 4 x (x + y) g[\mu, k, p] - \\
- 8 (y - 3 y) g[\mu, p, k] + 24 x y g[\mu, p, p] + 2 x^2 m_{i_1}^2 g[\mu, \beta, \beta] \right) + \\
+ \frac{i}{2} \frac{1}{L_{2,h^0}} ((x + y) (m_i (-1 + x + y) - 4 m_{i_1} (y - 1) (x + y) g[\mu, k, k] + \\
\quad + 4 m_{i_1} x (y - 1) ((x + y) g[\mu, k, p] - (x + y) g[\mu, p, k] + \\
\quad + x g[\mu, p, p]) - i \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) \times \\
\times ((x + y) (m_i (-1 + x + y) + 4 m_{i_1} (y - 1) (x + y) g[\mu, k, k] - \\
\quad - 4 m_{i_1} x (y - 1) ((x + y) g[\mu, k, p] - (x + y) g[\mu, p, k] + \\
\quad + x g[\mu, p, p]) \right) \times \\
\times \left( 2 (x + 3 x^2 - 2 y^2 + 2 y) g[p, p', \mu] + 4 (-2 x + 4 x^2 - 2 y + 7 x y + 3 y^2) \times \\
\times g[\mu, k, k] + 2 x (3 - 8 x - 7 y) g[\mu, k, p] + \\
\quad + 2 (4 x - 6 x^2 + 4 y - 9 x y - 4 y^2) g[\mu, p, k] - 4 x (2 - 3 x - 3 y) \times \\
\times g[\mu, p, p] - m_{i_1}^2 x (x + 1) g[\mu, \beta, \beta] \right), \\
E'_i(x, y) = i \epsilon_{\rho \theta \alpha \mu} \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) m_{i_1} x k_\rho p_\theta a_\alpha + \\
+ \left\{ -m_{i_1} \left( 2 \left( \frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) m_{i_1} (m_i - 1) x + \\
\quad + \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) f_2 (-1 + x + y) \right) + \left( \frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right) \times \\
\times m_{i_1} (-1 + 3 x - y) - \left( \frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}} \right) m_{i_1} f_4 - \\
\right\}
\]
\[-\left(\frac{1}{L_{2,h}^2} - \frac{1}{L_{2,A}^2}\right) m_i^2 m_i x^2 + \left(\frac{1}{L_{2,h}^0} + \frac{1}{L_{2,A}^0}\right) m_i (1 + x + y)\right\} b_\mu -
\]
\[-\left\{ 2 \left[\left(\frac{1}{L_{2,h}^2} - \frac{1}{L_{2,A}^2}\right) m_i^2 m_i y + 2 \left(\frac{1}{L_{1,h}^2} + \frac{1}{L_{1,A}^2}\right) m_i (2x - y) - 2 \left(\frac{1}{L_{2,h}^0} + \frac{1}{L_{2,A}^0}\right) \right] m_i \left(3x^2 + 4xy + y^2\right)^2\right\} d^\text{ASym}[k, \mu] +
\[+ \left\{ 4 \left(\frac{1}{L_{2,h}^2} - \frac{1}{L_{1,A}^2}\right) m_i^2 m_i x y + \left(\frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0}\right) m_i^2 m_i x \times
\times (3x^2 + 2(y - 1)y + x(5y - 3))\right\} d^\text{ASym}[p, \mu] -
\]
\[-\left\{ \\frac{1}{L_{1,h}^2} + \frac{1}{L_{1,A}^2}\right\} m_i m_i y (m_i^2 x (x + y) - m_j^2) - 3 \left(\frac{1}{L_{1,h}^0} + \frac{1}{L_{1,A}^0}\right) \times
\times m_i m_i y - m_i \left[\frac{1}{L_{2,h}^0} (m_i x + m_i) + \frac{1}{L_{2,A}^0} (-m_i x + m_i)\right] \times
\times (x + y) (m_i^2 x y + m_i^2) - (x + y) m_i \times
\times \left\{ 3 \left(\frac{1}{L_{2,h}^0} + \frac{1}{L_{2,A}^0}\right) m_i + 4 \left(\frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0}\right) m_i x\right\} \times
\times f_\mu + 2x \epsilon_{\rho \delta \alpha \mu} \left(\frac{1}{L_{1,h}^2} - \frac{1}{L_{1,A}^2}\right) \left( y h[k, \alpha] + x h[p, \alpha]\right) k_\rho p_\delta + \epsilon_{\rho \delta \beta \mu} \times
\]
\times \left\{ 2 \left(\frac{1}{L_{2,h}^2} - \frac{1}{L_{2,A}^2}\right) (m_i^2 (-1 + x - y) + m_i x (x + y) (x + y - 1)) k_\rho -
\times - \frac{1}{2} \left(\frac{1}{L_{1,h}^2} + \frac{1}{L_{1,A}^2}\right) \times m_i m_i p_\rho - \frac{3}{2} \left(\frac{1}{L_{1,h}^0} - \frac{1}{L_{1,A}^0}\right) (x + y) k_\rho -
\times - \frac{1}{2} \frac{1}{L_{2,h}^0} (2m_i m_i x + m_i^2 (y + 1) + m_i^2 y (-1 + x + y)) k_\rho +
\times + \frac{1}{2} \frac{1}{L_{2,A}^0} (-2m_i m_i x + m_i^2 (y + 1) + m_i^2 y (-1 + x + y)) k_\rho -
\times - \frac{3}{2} \left(\frac{1}{L_{2,h}^0} - \frac{1}{L_{2,A}^0}\right) (x + y) k_\rho \right\} h[\alpha, \beta] - i \left(\frac{1}{L_{1,h}^2} - \frac{1}{L_{1,A}^2}\right) \times
\times m_i^2 x y^3 g[\mu, p, k] + \epsilon_{\rho \alpha \beta \mu} \times
\]
\times \left(\frac{1}{L_{1,h}^2} - \frac{1}{L_{1,A}^2}\right) xy (g[k, \alpha, \beta] + 2g[\beta, \alpha, k]) -
\times - \frac{1}{2} \left(\frac{1}{L_{2,h}^2} + \frac{1}{L_{2,A}^2}\right) m_i m_i x^2 (x + y) g[\beta, \alpha, k] -
where

\[
L_{1,S} = m_2^2 (x + (1 - x) z_h + x (-1 + x + y) z_{1,h}) ,
\]

\[
L_{2,S} = m_2^2 (x + (1 - x) z_h - x y z_{1,h}) ,
\]

\[
f_1 = m_2^2 (y + 1) - m_{i_1}^2 x y (x + y - 1) ,
\]

\[
f_2 = m_{i_1}^2 + m_{i_1}^2 x (x + y) ,
\]

\[
f_3 = m_{i_1}^2 x (x + y) ,
\]

\[
f_4 = m_{i_1}^2 (1 - y) + m_{i_1}^2 x y (x + y - 1) ,
\]

\[
f_5 = m_{i_1} (f_3 + m_{i_1}^2) y + m_{i_1} x \times
\]

\[
\times (2 m_{i_1}^2 x (y - 1) (x + y) - 2 m_{i_1}^2 y - m_{i_1} m_{i_1}^2 (2 x^2 + 2 y^2 + 2 x y - y)) ,
\]

\[
f_6 = -m_{i_1} (f_3 + m_{i_1}^2) y + m_{i_1} x \times
\]

\[
\times (2 m_{i_1}^2 x (y - 1) (x + y) - 2 m_{i_1}^2 y + m_{i_1} m_{i_1}^2 (2 x^2 + 2 y^2 + 2 x y - y)) ,
\]

\[
f_7 = 2 m_{i_1}^2 (x + y) - 2 m_{i_1}^2 x^3 + m_{i_1} m_{i_1} (x (-2 m_{i_1} (x + y) + m_{i_1} (x + 2 x y + 2 y^2 - 2 y)) ,
\]

\[
f_8 = 2 m_{i_1}^2 (x + y) + 2 m_{i_1}^2 x^3 + m_{i_1} m_{i_1} (x (2 m_{i_1} (x + y) + m_{i_1} (x + 2 x y + 2 y^2 - 2 y)) ,
\]

\[\text{(A.2)}\]

with \(z_h = m_2^2 / m_{i_1}^2, z_{1,h} = m_2^2 / m_{i_1}^2\) and for \(S = h^0, A^0\). Here we use \(h[r, r'] = h_{\alpha \beta} r^\alpha r'^\beta\), \(g[r, r', r''] = g_{\alpha \beta \gamma} r^\alpha r'^\beta r''^\gamma\) and parametrize the coefficients \(c[r, r']\) and \(d[r, r']\) as \(c[r, r'] = c_{\alpha \beta}^\text{Asym} r^\alpha r'^\beta + e_{\alpha \beta}^\text{Sym} r, r'\), \(d[r, r'] = d_{\alpha \beta}^\text{Asym} r^\alpha r'^\beta + d_{\alpha \beta}^\text{Sym} r, r'\), where \(\text{Asym} (\text{Sym})\) denotes the asymmetry (symmetry) in the indices. Here \(h_{\alpha \beta}(g_{\alpha \beta})\) is taken antisymmetric with respect to indices (first two indices) and \(d_{\alpha \beta}(c_{\alpha \beta})\) traceless.

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