Deliberative Democracy
with Dilutive Voting Power Sharing

Dimitrios Karoukis

April 5, 2022

Abstract

We present a deliberation model where a group of individuals with heterogeneous preferences iteratively forms expert committees whose members are tasked with the updating of an exogenously given status quo change proposal. Every individual holds some initial voting power that is represented by a finite amount of indivisible units with some underlying value. Iterations happen in three stages. In the first stage, everyone decides which units to keep for themselves and where to distribute the rest. With every ownership mutation, a unit’s underlying value diminishes by some exogenously given amount. In the second stage, the deliberative committee is formed by the individuals with the most accumulated voting power. These experts can author corrections to the proposal which are proportional to their accumulated power. In the third stage, if an individual outside of the committee disagrees with a correction, she can vote against it with their remaining voting power. A correction is discarded if more than half of the total voting power outside of the committee is against it. If either the committee or the proposal remain unchanged for two consecutive iterations, the process stops. We show that this will happen in finite time.

Keywords: deliberation, liquid democracy, social choice, consensus

1 Introduction

In a representative democracy, meaningful and frequent interaction between the citizens and their representatives during the various stages of the legislative process, from the drafting of a law to its final vote, safeguards the quality of the democracy’s institutions from deterioration. However, if the members of the legislative bodies remain constant over extended periods of
time, the risk of them lacking expertise as individuals in the subject of some law that they are called to decide upon during their term increases overtime. In order for representation to work properly, there needs to exist a mechanism that lets society pick its representatives according to its perception of their expertise on each subject in question. This mechanism could also allow for updating of these representatives during the process in order to protect the electorate from corrupt individuals.

Suppose that a status quo change proposal has been drafted exogenously and the affected group of individuals wants to manipulate its text according to their preferences. In this model, the group forms a deliberative committee consisting of its elected experts, whose function is to make corrections to the proposal prior to it becoming permanent. This committee could be seen as a type of parliament. The committee members are assumed to be iteratively updated through a modified version of liquid democracy that is used as a mechanism to help the group dynamically identify its experts for the subject in question.

Liquid democracy \cite{1}, or more accurately liquid voting, is an extension of proxy voting \cite{8} where a voter $A$ can either vote for themselves or delegate their voting power to some other voter $B$. Voter $B$ can decide to vote with their increased voting power or delegate it all to another voter $C$ and so on, making these delegations transitive. Liquid voting was designed to help in situations where a voter realises that they do not possess sufficient knowledge to make an informed decision and they feel more comfortable assigning their democratic right to another voter who they deem more capable of making an informed decision.

In our approach, we use a generalisation of liquid voting which was briefly discussed in \cite{10}. In this context, voter $A$ can delegate shares of their voting power to one or more voters who can also transitively delegate shares of their voting power to others and so on. The voters may delegate shares that add up to less than 100\% of their voting power and keep the rest for themselves. It could be used as a vote hedging strategy in the sense that it may reduce the risk induced by incomplete information about the knowledge possessed by the receiving voters on the subject in question. We represent these shares as indivisible voting power units whose underlying value suffers a predefined amount of dilution with each change in ownership.

There are numerous other variations of liquid voting in the literature. Many of them exist because a shortcoming of pure liquid voting has been shown in \cite{7} to be the tendency of some voters to become super-delegates by accumulating a large part of the total voting power in the society, while the majority of voters never receive any delegations. This accumulation of voting power has been shown in \cite{6} to be harmful for social welfare, even if
these super-delegates are competent individuals.

A remedy that was suggested in [5] is to let voters choose more than one candidates to potentially receive their voting power and let a centralized mechanism assign the voting power to the right candidates according to the law of communicating vessels. This law of physics suggests that a homogeneous fluid in a set of connected containers will always balance out to the same level regardless of the shape of the containers. In our context, we can think of the homogeneous fluid as the voting power and the containers as the candidate voters. The centralized mechanism that follows this law will ensure that no voter among the candidates will end up with excess voting power relative to the other candidates.

The inspiration for the deliberative committee of experts and the updating of the proposal is drawn from from the DeGroot model of social influence as described in [3], [2] and [4]. In this model, weighted averaging is used in order to update the belief that an agent has about the state of the world depending on the weight that they assign to the beliefs of their acquaintances. In our model, instead of subjective probability distributions over the state of the world, status quo change proposals are represented by vectors in the $[0, 1]^s$ set, with $s$ being the number of subjects that will be affected and $[0, 1]$ the range of values that each subject can attain. Every individual in the society has an opinion regarding the status quo change that depends on her information set and is represented by a preference over every possible status quo change that can affect these $s$ subjects.

The choices of every individual in the society regarding the distribution of their voting power in every iteration, which happens in three stages, are as follows. If they take no action in the first stage of the initial iteration $t = 0$, they have chosen to keep all of their voting power units to themselves. If they want to distribute them, they announce the units to be transferred and their recipients. In the first stage of every subsequent iteration, inaction means that they will stick with their previous distribution. In the second stage of every period there is no choice regarding the use of voting power to be made. In the third stage, a vote needs to be cast only if the individual disagrees with a correction that was proposed by a deliberative committee member. If at any $t > 0$ they announce a change in their voting power distribution, the units that are going to be redistributed will lose a percentage of their value.

The members of the dynamic deliberative committee of experts are the individuals with the most accumulated voting power at the second stage of every iteration. Their accumulated power is represented by the units that they kept for themselves and the dilution that these units have suffered until that point. Each member can update the proposal’s text to be closer to their opinion by replacing or adding text whose percentage of the corpus is less
than or equal to the percentage of the total available voting power of the committee that they hold at $t$. If more than half of the voting power of the society outside the committee at $t$ is against a specific correction, it will be discarded.

If either the committee members and their voting power or the proposal’s text remain unchanged for two consecutive time periods, the deliberation stops. We show that at least one of the two conditions will be satisfied in finite time, so there is no need for predefined restrictions on the number of iterations that need to take place before the process stops. The assumption that voting power units depreciate in value with every mutation in their “chain of custody” ensures this assertion. It also ensures that an individuals with many directly transferred units has better chances to be in the expert committee compared to a peer with more indirectly transferred units.

2 Model

Suppose society $N$ of $n < \infty$ individuals and deliberative committee of experts at iteration $t < \infty$ consisting of $k < n$ members, denoted by $K_t \subset N$.

Definition 2.1. The status quo is represented by the vector

$$\ell_{-1} \in [0, 1]^s,$$

where $s$ indicates the subjects of interest and $[0, 1]$ the range of values that each subject can attain. The initial status quo change proposal is exogenously given and represented by $\ell_0 \in [0, 1]^s$ such that $\ell_0 \neq \ell_{-1}$. Every subsequent proposal at $t$ is represented by $\ell_t \in [0, 1]^s$.

The voting power of an individual is represented by voting power units, which are elements in the set $U \subseteq [0, 1] \times N^\omega$, with $1 \leq \omega \leq n$ and $|U| = u < \infty$.

Definition 2.2. At some iteration $t < \infty$, a voting power unit with index $id \in [0, u]$

$$u_{id}(t) \in U$$

has value represented by a real number

$$u_{id}^{value}(t) \in [0, 1]$$

and chain of custody represented by a vector

$$u_{id}^{chain}(t) = \{i\}_{i \in N}^\omega \in N^\omega$$

of size $\omega \leq n$, ordered from their original owner to the latest.
The last element of the chain of custody of a unit at $t < \infty$ is represented by $u_{id,\omega}^{\text{chain}}(t)$. The original owner is immutable, but every other owner in the chain of custody can be mutated by owners who come before them.

**Definition 2.3.** The voting power of an individual $i \in N$ at iteration $t < \infty$ is measured by the sum of all units’ values whose last owner in the chain of custody is $i$, or

$$P_i(t) = \sum_{id=0}^{u} \{ u_{id}^{\text{value}}(t) | u_{id,\omega}^{\text{chain}}(t) = i \}. \quad (1)$$

At $t = 0$, every $i \in N$ is assigned a finite number of units with value equal to one and chain of custody only including the original owner, or

$$u_{id}^{\text{value}}(0) = 1 \text{ and } u_{id}^{\text{chain}}(0) = i.$$ 

The conceptually simplest case would be for every individual to be assigned 100 units of voting power at $t = 0$ but the model can also accommodate scenarios where individuals start the process with different amounts.

In every iteration $t$, there are three stages to the deliberative process. In the first stage, each individual $i \in N$ decides whether to keep their voting power units to themselves or to delegate some or all of them to one or more individuals. The receivers can transitively delegate some or all of these units to other individuals etc. In the second stage, the distribution of voting power stops and the top $k$ individuals with respect to their total voting power are the experts chosen by the society to form the deliberative committee at $t$. The third stage of the process is where the proposal is updated but it will be described in detail in section 3.

If, at any $t < \infty$, $i \in N$ changes their voting power distribution, the distributed units’ values are diminished by an exogenously given dilution factor $c \in (0, 1)$.

**Definition 2.4.** The distribution of voting power units from some $i \in N$ to some $j \in N \setminus \{i\}$ mutates the set $U$ by diluting the value of the units by an exogenously given factor of $c \in (0, 1)$ and by making $j$ the last element of the chain of custody right after $i$. In functional form, the distribution is a map $T_{i,j} : U \rightarrow [0,1] \times \omega$ such that

$$T_{i,j}^{\text{value}}(u_{id}(t)) = (1 - c)u_{id}^{\text{value}}(t),$$

$$T_{i,j,\omega}^{\text{chain}}(u_{id}(t)) = u_{id,\omega}^{\text{chain}}(t) = i \text{ and } T_{i,j,\omega}^{\text{chain}}(u_{id}(t)) = j. \quad (2)$$
The experts that comprise the committee in the second stage of period \( t \) are the top \( k \) individuals in the society with respect to their voting power. To break possible ties in voting power amount, we compare the various-order appeals of the competing experts, meaning that the winner of the tie is the one to whom the most voting power was transferred to directly or indirectly but at the same order.

**Definition 2.5.** The first-order appeal of \( i \in N \) at time \( t < \infty \) is the sum of voting power units that were assigned to her directly by \( j \in N \setminus \{i\} \), or whose \( u_{id}^{\text{chain}}(t) = [j, i] \).

**Definition 2.6.** The \( \omega \)-th-order appeal of \( i \in N \) at time \( t < \infty \) is the sum of units that were assigned to her from their \((\omega - 1)\)th owner, or whose \(|u_{id}^{\text{chain}}(t)| = \omega \) and \( u_{id,\omega}^{\text{chain}}(t) = i \).

If no individual in the society has transferred any voting power units to another by the end of the first stage, the process stops and the status quo change proposal remains unchanged. If less than \( k \) individuals in the society can successfully break the tie then the deliberative committee consists of only those individuals who hold more than the tie-inducing voting power amount.

## 3 Proposal Minting

At the end of the second stage of each time period there are at most \( k \) candidate corrections to the proposal \( \ell_t \), since the members of the committee are the only ones that can author corrections but they are not obliged to exercise their right. The size of each correction relative to the corpus of \( \ell_t \) is proportional to the percentage of the total voting power of the committee that each \( \sum_{i \in k_t} P_i(t) \) that each expert holds.

The updating of the corpus happens in the third stage of each time period and it happens in two steps. First, the members of the committee propose corrections and the citizens that are not in the committee vote for or against them. Second, amendments to some corrections are made and the citizens outside of the committee vote on them.

Assume that there is a function

\[
f : [0, 1]^s \times [0, 1]^s \rightarrow [0, 1]
\]

denoting the percentage of the corpus of \( \ell_t \) that needs to change due to a correction brought forward by a member of the committee.
Definition 3.1. A correction to the corpus of \( \ell_t \) brought forward by a member of the committee \( i \in k_t \) is denoted by \( d\ell^i_{t+1} \) and it needs to satisfy the constraint

\[
f(d\ell^i_{t+1}, \ell_t) \leq \frac{P_i(t)}{\sum_{j \in k_t} P_j(t)}. \tag{3}
\]

A member of the committee \( i \in k_t \) can also build on the correction of another member of the committee \( j \in k_t \setminus \{i\} \) and their correction also needs to satisfy (3).

If an individual \( i \notin k_t \) with \( P_i(t) \neq 0 \) does not agree with a correction to the proposal at \( t \), they can vote against it. If more than half of the total voting power of the rest of the society is against a correction then it is blocked. In order to require minimum engagement by those outside of \( k_t \), we assume that their vote is positive by default unless they explicitly disagree with the correction.

Definition 3.2. Each individual \( i \in N \) has an opinion regarding status quo changes, which is described by a total order \( \succsim_i \) over the set \([0, 1]^s\) and an optimal proposal \( \ell_i \in [0, 1]^s \) such that \( \ell_i \succsim_i \ell \) for any \( \ell \in [0, 1]^s \).

Definition 3.3. The vote of an individual \( i \in N \setminus k_t \) on a \( j \in k_t \) committee member’s correction \( d\ell^i_{t+1} \) is a function \( v : [0, 1]^s \to \mathbb{R}_+ \) that is equal to \( P_i(t) \) if the corrected proposal is preferable to the status quo and to the latest state of the proposal and zero otherwise. This function is of the form

\[
v_j(d\ell^i_{t+1}) = P_j(t) \* \mathbf{1}\{d\ell^i_{t+1} \succsim_j \ell_i \succsim_j \ell_{-1}\}. \tag{4}
\]

The condition that a correction \( d\ell^i_{t+1} \) needs to satisfy in order to become part of \( \ell_{t+1} \) is that more than half of all the voting power outside of the committee is favorable to it, or

\[
\sum_{j \in N \setminus k_t} v_j(d\ell^i_{t+1}) > \frac{1}{2} \sum_{j \in N \setminus k_t} P_j(t). \tag{5}
\]

\(^1\)A total order is a binary relation \( \succsim \) on a set \( X \) which satisfies, for any \( a, b, c \in X \), the following:

1. \( a \succsim a \) (reflexivity)
2. if \( a \succsim b \) and \( b \succsim a \) then \( a = b \) (antisymmetry)
3. if \( a \succsim b \) and \( b \succsim c \) then \( a \succsim c \) (transitivity)
4. \( a \succsim b \) or \( b \succsim a \) (totality).
If two conflicting corrections \(d_{t+1}^i \) and \(d_{t+1}^j \) of some \(i, j \in k_t \) both get accepted, \(i \) and \(j \) need to generate a new correction at the second step of the process, which needs to pass (5) or it will not be included in \(\ell_{t+1} \).

If \(d_{t+1}^j \) builds upon \(d_{t+1}^i \) for any \(i, j \in k_t \) and either \(j \)'s correction or both corrections get rejected we do not have any ambiguous outcomes. If both corrections are accepted then the richest one, in the sense that it builds upon the most corrections, is added to \(\ell_{t+1} \). If one or more corrections upon which a correction \(d_{t+1}^j \) is built are voted down then \(j \) can propose a different correction, whose inclusion in \(\ell_{t+1} \) is subject to (5).

4 Results

The process is assumed to stop naturally if no corrections have been made to the proposal between \(t-1 \) and \(t \). This can mean that either no member of \(k_t \) brought forward a correction to the proposal at \(t \) or that every correction that was brought forward was voted down by the rest of the society. The process is also assumed to stop if the resulting committee’s members and their voting power remain unchanged between \(t-1 \) and \(t \). A final assumption that was mentioned earlier is that if at any time \(t < \infty \) it is true that \(P_i(t) = P_j(t) \) for all \(i, j \in N \) then the deliberation stops.

Theorem 4.1. There exists some \(T < \infty \) when the deliberative process stops.

Proof. By assumption, the process stops if the proposal remains unchanged for two consecutive iterations or if the members of the committee and their voting power remain unchanged for two consecutive iterations. If the experts and their voting power change then some \(i \in N \) has redistributed their voting power units, which means that these units’ value has diluted by \(c \in (0, 1) \). As the process continues, more and more citizens will have diluted voting power units. Since the number of individuals in the society and the number of total available voting power units are finite, and \(c \in (0, 1) \), eventually all citizens will hold zero voting power and this will happen in finite time. By the assumptions, this will stop the deliberation. □

5 Future research

In the future, work on economic-theoretic questions would shed light on matters such as the existence of Nash equilibrium deliberative committees, their stability and their representation of citizens’ preferences. We will then be able to answer questions such as the optimal size \(k \) of the deliberative
committee with respect to the size of the society or the optimal penalty $c$ etc. We could also introduce the possibility to purchase voting power at some point in the process, maybe with some quadratic cost as in [9].

An assumption that can be relaxed is the fully connected societal graph. In practice, people would not transfer their voting power units to anyone in the society that is outside of their social network, unless they were a "celebrity". The proximity of citizens in the social graph could be a restricting factor for the number of other citizens that a citizen can assign their voting power to.

6 Conclusion

We introduced a form of governance that can bridge the gap between the will of the legislators and the voters. If the legislators remain invariant over time periods, we proposed the formation of a sort of dynamic "parliament", a deliberative committee of experts, that negotiates status quo change proposals in order to ensure that the opinions of the voters are taken into account during the legislative process. By making this committee dynamic, we also give the voters the power to mitigate possible adverse effects of corruption of its members. By getting constant signals on the sentiment about the corrections of the proposal, the chosen experts can make informed decisions about what type of changes to the status quo do the voters want implemented. It is our hope that the proposed system will help improve representation of peoples’ preferences in collective decision making settings.

References

[1] Christian Blum and Christina Isabel Zuber. Liquid democracy: Potentials, problems, and perspectives. *Journal of Political Philosophy*, 24(2):162–182, 2016.

[2] Samprit Chatterjee and Eugene Seneta. Towards consensus: Some convergence theorems on repeated averaging. *Journal of Applied Probability*, pages 89–97, 1977.

[3] Morris H DeGroot. Reaching a consensus. *Journal of the American Statistical Association*, 69(345):118–121, 1974.

[4] Benjamin Golub and Matthew O Jackson. Naive learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics*, 2(1):112–49, 2010.
[5] Paul Gölz, Anson Kahng, Simon Mackenzie, and Ariel D Procaccia. The fluid mechanics of liquid democracy. In International Conference on Web and Internet Economics, pages 188–202. Springer, 2018.

[6] Anson Kahng, Simon Mackenzie, and Ariel Procaccia. Liquid democracy: An algorithmic perspective. Journal of Artificial Intelligence Research, 70:1223–1252, 2021.

[7] Christoph Kling, Jérôme Kunegis, Heinrich Hartmann, Markus Strohmaier, and Steffen Staab. Voting behaviour and power in online democracy: A study of liquidfeedback in germany’s pirate party. In Proceedings of the International AAAI Conference on Web and Social Media, volume 9, 2015.

[8] James C Miller. A program for direct and proxy voting in the legislative process. Public choice, 7(1):107–113, 1969.

[9] E Glen Weyl. Quadratic vote buying. unpublished, University of Chicago, 2012.

[10] Yuzhe Zhang and Davide Grossi. Tracking truth by weighting proxies in liquid democracy. arXiv preprint arXiv:2103.09081, 2021.