Quantum Symmetries and Conserved Charges of the Cosmological Friedmann-Robertson-Walker Model

B. Chauhan

Department of Physics, Centre of Advance Studies, Institute of Science, Banaras Hindu University, Varanasi - 221 005, (U.P.), India

E-mail: bchauhan501@gmail.com

Abstract: We discuss both the off-shell nilpotent Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST symmetry transformations for the cosmological Friedmann-Robertson-Walker (FRW) model with a differential gauge condition in the extended phase space. In this discussion, the presence of anti-BRST symmetry provides the complete geometrical description of BRST within the ambit of the supervariable approach. We derive the conserved (anti-)BRST charges for the FRW model using the celebrated Noether theorem and show the nilpotency and absolute anti-commutativity properties of these conserved charges within the realm of BRST formalism. Finally, we prove the sanctity of (anti-)BRST symmetries through the derivation of these symmetry transformations within the framework of the (anti-)chiral supervariable approach (ACSA) to BRST formalism.

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# Introduction

The gauge theories provide theoretical description of three out of four fundamental interactions of nature. These theories are characterized by the existence of first-class constrains on them in the terminology of Dirac’s prescription for the classification scheme of constraints [1, 2]. This is the most modern definition of any arbitrary $p$-form ($p = 1, 2, 3, ...$) gauge theory in any arbitrary D-dimension of spacetime. There is a problem of canonical quantization in every gauge theory. To overcome this problem, one of the most intuitive mathematical approaches for the covariant canonical quantization of any gauge invariant field theory is the Becchi-Rouet-Stora-Tyutin (BRST) formalism [3-6]. This approach not only serves as the foundation for renormalization programmes, but it has also been made in a huge variety of applications. Gauge symmetry generalizes to BRST symmetry at the quantum level through the replacement of infinitesimal local gauge parameter by ghost and anti-ghost variables/fields. Therefore, we have two types of quantum symmetries (i.e., BRST and anti-BRST). The study of anti-BRST symmetry is important because it makes the theory complete and ghost free. These symmetries are nilpotent of order two and absolutely anti-commuting which signify the fermionic nature of symmetries and linear independence of both of the symmetries, respectively.

The BRST symmetry is characterized by two main features. The first is that it has a group theoretical basis: applying two different gauge transformations sequentially and then reversing their order does not produce the same outcome (unless the initial symmetry is Abelian), but the two distinct outcomes are connected by a group theoretical rule. The second significant property of the BRST transformations is nilpotency itself. It is obtained from the anticommuting properties of the ghost and anti-ghost fields via the Faddeev-Popov quantization method [7]. This indicates that if we perform the identical change again, we get zero. The Wess-Zumino consistency conditions, a key tool in the investigation of anomalies, are created when the two properties of BRST symmetry are combined. It should be observed that the second property is totally quantum, whereas the first property is classical. In other words, the BRST symmetry is quantum symmetry. The study of BRST formalism is very relevant because of the various applications of the (anti-)BRST symmetry transformations to many practical systems. The various issue connected with the anomalies have been discussed clearly [8-11], and the recent applications of BRST symmetry in M-theory [12, 13] and quantum gravity [14, 15] have also been discussed nicely within the framework of BRST.

The study of different quantum cosmological models is very interesting and challenging towards the development of a quantum theory of gravity where unification of general relativity and quantum mechanics is described [16, 17]. The description of homogeneous and isotropic spacetime symmetry was developed by Friedmann–Robertson–Walker (FRW) and the universe models associated with it are christened as FRW models [18-23]. The FRW models have played a crucial role in the development of the modern cosmology where most of works on quantum cosmology and dark energy in FRW spacetime are based on these models. Nevertheless, it is worthwhile to mention that most of the models of dark energy suffer from some problems related to cosmological constant (i.e., fine-tuning and coincidence problems, etc.). Therefore, it is important to do a more careful study
of the basics of the cosmological FRW models in isotropic and homogeneous spacetime. Some of the BRST analysis has already been looked at for FRW models [24-28], we are highly motivated to carry out this research with some important and significant novel investigations for the cosmological FRW model within the realm of BRST formalism.

In this work, we will take BRST and also anti-BRST symmetries to analyze the fate of time parameterization invariance in a quantum theory for the cosmological FRW model. In fact, the fixing of gauge condition results in the breaking of original gauge symmetry. Nevertheless, the extension of phase space via the introduction of additional variables allows us to recover a formulation with (anti-)BRST symmetries where transition amplitudes do not depend on the choice of gauge-fixing condition. Therefore, for the present investigation, the choice of the differential gauge-fixing condition in extended phase space is appropriate from the theoretical point of view. The extended phase space is formulated by considering the cosmological constant as a variable in the universe model.

The usual supervariable/superfield approach (USFA) to BRST formalism [29-33] uses the horizontality condition (HC) to derive off-shell nilpotent (anti-)BRST symmetry transformations for the gauge, ghost, and anti-ghost variables/fields when full super expansions of the supervariable/superfield [with two Grassmannian coordinates ($\theta, \bar{\theta}$)] are taken into account. In an interaction theory, the USFA does not explain how the (anti-)BRST symmetries for matter fields are derived. To overcome this problem, the idea of HC and gauge invariant restriction(s) (GIR) are combined together and this approach is generalized to obtain the (anti-)BRST symmetries for matter fields [34-36]. The augmented version of the supervariable/superfield (AVSA) technique is the name given to this extended version of USFA. In light of the preceding discussions, we used the newly proposed (anti-)chiral supervariable/superfield formalism (ACSA) to derive the entire set of nilpotent (anti-)BRST symmetries where (anti-)chiral super expansions of the supervariables/superfields (with only one Grassmannian coordinate in the super expansions of the supervariables) have been taken into account (see, e.g. [37-42] for details). In our present endeavor, for the first time, the analysis of the BRST invariant cosmological model (i.e., FRW model) is discussed in the extended phase space within the realm of ACSA. As far as, the application of the superfield method is concerned. The superfield approach nicely applies to the description of consistent anomalies. It has been demonstrated that the superfield formalism not only makes it simple to recreate all the formulas relating to anomalies in any even dimension but also seems to be built specifically for them [8].

The different portions of this paper are organized as follows. In section 2, we discuss the constraints analysis and gauge symmetry associated with the FRW model. The subject matter of section 3 deals with the construction of gauge-fixing condition and (anti-)BRST analysis of the present model. In section 4 of this paper, we derive the Noether conserved (anti-)BRST conserved charges in extended phase space and demonstrate the nilpotency and absolute anti-commutativity properties of these charges. Section 5 is fully devoted to the derivation of the nilpotent (anti-)BRST symmetries within the realm of ACSA to BRST formalism. Finally, in section 6, we summarize our key findings and draw conclusions about the novelty of our work with some future investigations of cosmological models within the ambit of the supervariable/superfield approach to BRST formalism.
2 Preliminaries: Constraints Analysis and Gauge Symmetries of the FRW Model

In this section, we will go over the preliminary of the cosmological FRW model that describes a homogeneous and isotropic universe. We begin with the FRW metric tensor defined in spherical coordinates \((t, r, \vartheta, \phi)\) as follows;

\[
ds^2 = N^2(t)\, dt^2 + a^2(t) \left( \frac{1}{1 - kr^2} \right) dr^2 + a^2(t) \left( r^2 d\vartheta^2 + r^2 \sin^2 \vartheta \, d\phi^2 \right),
\]

where \(N(t)\) denotes the lapse function and \(a(t)\) is the scale factor of the universe that encrypts the size at large scales. The values of \(k = 0, -1, +1\) correspond to a space of zero curvature (i.e., flat universe), negative curvature (i.e., open universe) and positive curvature (i.e., closed universe) of universe, respectively.

We now define the classical Lagrangian* of the FRW model represented in Arnowitt-Desert-Misner (ADM) variables as follows;

\[
L_s = \frac{1}{2} a \dot{a}^2 + \frac{1}{2} k N a,
\]

where \(\dot{a}\) denotes the time derivative of \(a\) w.r.t. time \(t\) (i.e., \(\dot{a} = da/dt\)). This Lagrangian holds the following Euler-Lagrange equations of motion (EL-EOMs):

\[
\dot{a}^2 + k N^2 = 0, \quad a \ddot{a} + k a N^2 + \frac{\dot{a}^2}{2} = \frac{\dot{a}^2}{N} = 0.
\]

Now the canonically conjugate momenta corresponding to the lapse function \(N\) turn out to be zero because there is no time derivative of \(N\) in Lagrangian \(L_s\), we get

\[
\Pi_{(N)} \approx 0.
\]

This canonical conjugate momenta is weakly zero because the first-order time derivative is performed which reflect the primary constraint of the theory. The canonical Hamiltonian of the this theory is given as:

\[
H = p_a \dot{a} - L_f = -N \frac{p_a^2}{2a} - \frac{1}{2} k N a.
\]

Using the time conservation of the primary constraint, we calculate the secondary-constraint of the theory as follows:

\[
\frac{d\Pi_{(N)}}{dt} = \frac{p_a^2}{2a} + \frac{k}{2} a \approx 0.
\]

Both the constraints are first-class as they commute with each other, which confirms that the cosmological FRW model is endowed with gauge symmetry. The gauge symmetry of the variables present in Lagrangian \(L_s\) associated with this model is given by

\[
\delta g N = -N \dot{\eta} - \dot{N} \eta, \quad \delta_g a = -\dot{a} \eta,
\]

*This Lagrangian shows a second-order Lagrangian. However, first-order Lagrangian corresponding to the above Lagrangian \(L_s\) is given, using the Legendre transformation as: \(L_f = p_a \dot{a} + \frac{1}{2} p_a^2 N + \frac{1}{2} k N a\), where \(p_a(= -a \dot{a}/N)\) denotes the canonical conjugate momenta corresponding to the scale factor \(a\).
where \( \eta(t) \) is an infinitesimal gauge transformation parameter. Under the above infinitesimal gauge transformations Lagrangian \( L_s \) transforms to a total time derivative as:

\[
\delta_s L_s = \frac{d}{dt} \left[ - \eta L_s \right].
\]

Hence the Lagrangian (2) remains invariant under the gauge transformations (7) which implies the action integral of this model remains invariant.

### 3 Lagrangian Formulation: Gauge-Fixing Condition

Gauge fixing is a mathematical technique for dealing with extra degrees of freedom for field variables in gauge theory physics. For the canonical quantization of any gauge theory and to remove the redundancy in gauge degrees of freedom, we use the traditional approach of imposing gauge-fixing conditions. The following are the basic prerequisites for a gauge-fixing condition of a gauge theory: (i) it must entirely fix the gauge, i.e., there must be no residual gauge freedom, and (ii) on the application of the transformations, it must be able to bring any configuration given by \( N \) and \( a \) into one satisfying the gauge condition.

Keeping the above parameters in mind, we select the following gauge condition for the present FRW model in extended phase space;

\[
\dot{N} = \frac{d}{dt} F(a),
\]

where \( F(a) \) is an arbitrary function of \( a \). The above gauge condition has been examined in order to provide a well-defined formulation in extended phase space. Since simply extending the phase space by including gauged degrees of freedom is insufficient. Therefore, Lagrangian should additionally include missing velocities terms, too. It is possible to do so using differential gauge condition, which actually extend the phase space. The gauge condition in differential form is

\[
\dot{N} = \frac{dF}{da} \dot{a}.
\]

This gauge condition can be used in quantum theory by including the following gauge-fixing term in the invariant Lagrangian (2);

\[
L_{gf} = \lambda \left( \dot{N} - \frac{dF}{da} \dot{a} \right),
\]

where \( \lambda \) is an auxiliary variable (i.e., Lagrange multiplier) used to linearize the gauge-fixing term. Now the Faddeev-Popov ghost terms of the FRW model corresponding to the above gauge-fixing term is given as

\[
L_{gh} = \dot{\bar{C}} \left( \dot{N} - \frac{dF}{da} \dot{a} \right) C + \bar{C} \dot{N} \dot{C},
\]
where \((\bar{C}, C)\) are anti-ghost and ghost variables having ghost number \((-1, +1)\), respectively. The above ghost terms are used for the consistency of the theory (i.e., to get rid of the unphysical degrees of freedom). Now, the complete extended Lagrangian for the theory \(L_{\text{ext}}\) (i.e., \(L_s + L_{gf} + L_{gh}\)) reads:

\[
L_{\text{ext}} = -\frac{1}{2}a\dot{a}^2 + \frac{k}{2}Na + \lambda \left( \dot{N} - \frac{dF}{da} \dot{\hat{a}} \right) + \dot{\bar{C}} \left( \dot{N} - \frac{dF}{da} \dot{\hat{a}} \right) C + \dot{\bar{C}} \dot{C} N \dot{C}.
\]

(13)

The BRST and anti-BRST symmetry transformations corresponding to the above extended Lagrangian \(L_{\text{ext}}\) are given as,

\[
s_b N = - (\dot{N} C + N \dot{C}), \quad s_b a = - \dot{a} C, \quad s_b C = - \dot{C} C, \quad s_b \hat{C} = - \lambda, \quad s_b \dot{\hat{C}} = 0,
\]

\[
s_{ab} N = - (\dot{N} \bar{C} + N \dot{\bar{C}}), \quad s_{ab} a = - \dot{a} \bar{C}, \quad s_{ab} \bar{C} = - \dot{\bar{C}} \hat{C}, \quad s_{ab} \hat{C} = \lambda, \quad s_{ab} \dot{\hat{C}} = 0.
\]

(14)

It is straightforward to check that the above BRST and anti-BRST symmetry transformations are supersymmetric type (i.e., bosonic variables transform to the fermionic variables and vice-versa), nilpotent of order two (i.e., \(s_b^2 = 0\), \(s_{ab}^2 = 0\)) and absolutely anti-commuting with each other (i.e., \(s_b s_{ab} + s_{ab} s_b = 0\)). The above extended Lagrangian \(L_{\text{ext}}\) is invariant under the (anti-)BRST symmetries up to total time derivative.

The combination of gauge-fixing and ghost terms for both gauges are BRST and anti-BRST exact, therefore, we have the following

\[
L_{gf} + L_{gh} = -s_b \left[ \bar{C} \left( \dot{N} - \frac{dF}{da} \dot{\hat{a}} \right) \right] \equiv -s_{ab} \left[ C \left( \dot{N} - \frac{dF}{da} \dot{\hat{a}} \right) \right],
\]

(15)

which is one of the possible way to derive the gauge-fixing and ghost terms using the above (anti-)BRST symmetry transformations. Under the above fermionic symmetry transformations (14), the extended Lagrangian transform as the total time derivative:

\[
s_b L_{\text{ext}} = -d \frac{dt}{dt} \left[ (\lambda + \dot{\bar{C}} \hat{C}) \left\{ \left( \dot{N} - \frac{dF}{da} \dot{\hat{a}} \right) C + N \dot{\bar{C}} \right\} \right],
\]

\[
s_{ab} L_{\text{ext}} = d \frac{dt}{dt} \left[ (\lambda + \dot{\bar{C}} \hat{C}) \left\{ \left( \dot{N} - \frac{dF}{da} \dot{\hat{a}} \right) \bar{C} - N \dot{\bar{C}} \right\} \right].
\]

(16)

Hence the action integral (i.e. \(S = \int dt L_{\text{ext}}\)) corresponding to the FRW model remains invariant and extended Lagrangian (13) is (anti-)BRST invariant Lagrangian.

4 (Anti-)BRST Charges of the FRW Model

The Noether theorem states that if any Lagrangian or its corresponding action stays invariant under any continuous symmetry transformation, there are conserved current and charges corresponding to that given continuous symmetry. Therefore, for the present FRW model, there exist BRST \((Q_b)\) and anti-BRST \((Q_{ab})\) charges given as:

\[
Q_b = \frac{NC}{2a} \left[ (\lambda + \dot{\bar{C}} \hat{C}) F_a - \frac{a \dot{a}}{N} \right]^2 + \frac{k}{2} NC a + \lambda \dot{N} C - N \dot{\bar{C}} \dot{C} C,
\]

\[
Q_{ab} = \frac{NC}{2a} \left[ (\lambda + \dot{\bar{C}} \hat{C}) F_a + \frac{a \dot{a}}{N} \right]^2 + \frac{k}{2} NC a + \lambda \dot{N} \bar{C} - N \dot{\bar{C}} \hat{C} \hat{C},
\]

(17)
where \( F_a \) is derivative of function \( F \) w.r.t. \( a \) (i.e., \( F_a = dF/da \)). These charges act as the generators for the off-shell nilpotent (anti-)BRST symmetry transformations. The conservation law \((\partial_t Q_b = 0, \partial_t Q_{ab} = 0)\) of these charges can be proven by using the following EL-EoMs evaluated from the extended Lagrangian (13), namely;

\[
\begin{align*}
\dot{\lambda} + \ddot{C} C - \frac{a \dot{a}^2}{2N^2} - \frac{k a}{2} &= 0, \\
\dot{N} - F_a \dot{a} &= 0, \\
\ddot{\bar{C}} N + \dot{\bar{C}} F_a \dot{a} &= 0, \\
2 \ddot{N} + 2 \dot{N} \dot{\bar{C}} - \dot{F}_a \dot{a} C - F_a \dot{a} C - F_a \dot{a} \dot{C} &= 0, \\
2 \dddot{N} - \frac{k \dot{N} \dot{\bar{C}}}{N} - \frac{\dot{\bar{C}} \dot{N}}{2} - \dot{\bar{C}} N + \dot{\bar{C}} F_a \dot{a} &= 0. \\
\end{align*}
\]

(18)

It is straightforward to check that the above (anti-) BRST conserved charges \([Q_{(ab)}]\) are nilpotent of order two (i.e., \(Q_b^2 = Q_{ab}^2 = 0\)) and they follow absolute anti-commutativity property (i.e., \(Q_b Q_{ab} + Q_{ab} Q_b = 0\)) in the extended phase space. These two features are captured by the definition of generator, which is as follows:

\[
\begin{align*}
s_b Q_b &= -i \{Q_b, Q_b\} = 0 \implies Q_b^2 = 0, \\
s_{ab} Q_{ab} &= -i \{Q_{ab}, Q_{ab}\} = 0 \implies Q_{ab}^2 = 0, \\
s_{ab} Q_b &= -i \{Q_b, Q_{ab}\} = 0 \implies Q_b Q_{ab} + Q_{ab} Q_b = 0, \\
s_b Q_{ab} &= -i \{Q_{ab}, Q_b\} = 0 \implies Q_{ab} Q_b + Q_b Q_{ab} = 0. \\
\end{align*}
\]

(19)

The above properties describe the nature and identity of the conserved (anti-)BRST charges of the cosmological FRW model. The nilpotency property of these conserved charges signifies the fermionic nature whereas absolute anti-commutativity property, physically, shows that they are linearly independent with each other.

5 (Anti-)BRST Symmetry Transformations: ACSA

In this section, we provide explicit and step-by-step derivation of the off-shell nilpotent BRST and anti-BRST symmetry transformations for the cosmological FRW model in the extended phase space. Toward this aim in mind, first of all, we generalize our basic variables [i.e., \(N(t), \ a(t), C(t), \bar{C}(t)\)] and auxiliary variable [i.e., \(\lambda(t)\)] of Lagrangian [cf. Eq. (13)] onto (1, 1)-dimensional (anti-)chiral super-subspace of the most general (1, 2)-dimensional superspace by addition of one of the Grassmannian coordinate (i.e. \(\theta, \bar{\theta}\)) into the given time dimension coordinate. We assume that \(s_b \bar{C} = -\lambda\) and \(s_{ab} C = \lambda\) where \(\lambda\) is the Nakanishi-Lautrup type auxiliary variables used to linearize the differential gauge-fixing term. These BRST and anti-BRST symmetry transformations (i.e. \(s_b \bar{C} = -\lambda, \ s_{ab} C = \lambda\)) are the standard assumptions in the realm of BRST formalism.

First of all, we focus on the derivation of BRST symmetry transformations of the FRW model. For this, we generalize all the basic and auxiliary variables present in the
Lagrangian $L_{\text{ext}}$ onto the anti-chiral supervariable as

$$
a(t) \rightarrow A(t, \bar{\theta}) = a(t) + \bar{\theta} b_1(t),
N(t) \rightarrow N(t, \bar{\theta}) = N(t) + \bar{\theta} b_2(t),
C(t) \rightarrow F(t, \bar{\theta}) = C(t) + \bar{\theta} f_1(t),
\lambda(t) \rightarrow \Lambda(t, \bar{\theta}) = \lambda(t) + \bar{\theta} b_3(t),
$$

(20)

where $b_1(t)$, $b_2(t)$, $b_3(t)$ are the bosonic secondary variables and $f_1(t)$ is the fermionic secondary variable which we have to be determined using the standard technique of anti-chiral supervariable approach. The fermionic and bosonic natures of these secondary variables are ensured by the fermionic nature of Grassmannian variable $\bar{\theta}$.

We employ the highly important and intriguing BRST invariant quantities, which are the combinations of the basic and auxiliary variables of the Lagrangian (13), to derive the aforementioned secondary variables, namely;

$$s_b \lambda = 0, \quad s_b (C \dot{C}) = 0, \quad s_b (\dot{C} \dot{\theta}) = 0, \quad s_b (N \dot{C} + \dot{N} C) = 0.
$$

(21)

According to the basic premise of the ACSA to BRST formalism, the above set of BRST invariant restrictions must be independent of the Grassmannian coordinate $\bar{\theta}$ when these invariant quantities are generalized with the coordinate $\bar{\theta}$, we have

$$\Lambda(t, \bar{\theta}) = \lambda(t), \quad F(t, \bar{\theta}) \dot{F}(t, \bar{\theta}) = C(t) \dot{C}(t), \quad F(t, \bar{\theta}) \dot{A}(t, \bar{\theta}) = C(t) \dot{a}(t),
N(t, \bar{\theta}) \dot{F}(t, \bar{\theta}) + \dot{N}(t, \bar{\theta}) F(t, \bar{\theta}) = N(t) \dot{C}(t) + \dot{N}(t) C(t).
$$

Now using the generalization of non-trivial BRST invariant restriction $s_b \lambda = 0$, we have

$$s_b \lambda = 0 \iff \Lambda(t, \bar{\theta}) = \lambda(t) \implies b_3(t) = 0.
$$

(22)

To derive the value of other secondary variables, first of all, we use the trivial BRST invariant quantity $s_b (C \dot{C}) = 0$, we get the following generalization:

$$s_b (C \dot{C}) = 0 \iff F(t, \bar{\theta}) \dot{F}(t, \bar{\theta}) = C(t) \dot{C}(t)
\implies \dot{C}(t) f(t) + C(t) \dot{f}_1(t) = 0
\implies f_1(t) = - C(t) \dot{C}(t).
$$

(23)

Substituting the above value of secondary variable $f_1(t)$ into the anti-chiral super expansion of the supervariable (20), we have

$$C(t) \rightarrow F^{(b)}(t, \bar{\theta}) = C(t) + \bar{\theta} [- C(t) \dot{C}(t)].
$$

(24)

Now using the generalizations of the trivial BRST invariant restrictions $s_b (\dot{C} \dot{\theta}) = 0$ and $s_b (N \dot{C} + \dot{N} C) = 0$ which give the following expression

$$s_b (\dot{C} \dot{\theta}) = 0 \iff F^{(b)}(t, \bar{\theta}) \dot{A}(t, \bar{\theta}) = C(t) \dot{a}(t) \implies b_1(t) = - \dot{a}(t) C(t),
$$

$$s_b (N \dot{C} + \dot{N} C) = 0 \iff N(t, \bar{\theta}) F^{(b)}(t, \bar{\theta}) + \dot{N}(t, \bar{\theta}) F^{(b)}(t, \bar{\theta}) = N(t) \dot{C}(t) + \dot{N}(t) C(t)
\implies b_2(t) = - [N(t) \dot{C}(t) + \dot{N}(t) C(t)].
$$

(25)
where superscript \((b)\) on the supervariable \(F(t, \theta)\) denotes the supervariable has been obtained after the application of the BRST invariant quantity. Thus, finally, we have the following expressions for the anti-chiral super expansions of the supervariable

\[
\begin{align*}
    a(t) & \rightarrow A^{(b)}(t, \theta) = a(t) + \theta \dot{b}_1(t), \\
    N(t) & \rightarrow N^{(b)}(t, \theta) = N(t) + \theta \dot{b}_2(t), \\
    C(t) & \rightarrow F^{(b)}(t, \theta) = C(t) + \theta \dot{f}_1(t), \\
    \lambda(t) & \rightarrow \Lambda^{(b)}(t, \theta) = \lambda(t) + \theta \dot{b}_3(t),
\end{align*}
\]

(26)

where superscript \((b)\), once again, on all the supervariables denote the supervariables have been obtained after the use of BRST invariant restrictions (21). Here we found that the coefficients of the \(\theta\) are nothing but the BRST symmetries of the basic and auxiliary variables of the theory. Therefore, we have a concluding remark that \(s_b\) is connected with the translational generator \(\partial_\theta\) along the \(\theta\)-direction as: \(s_b \leftrightarrow \partial_\theta\) (see, e.g. [29-33]).

Now we are in position to derive the anti-BRST symmetry transformations for various variables of the Lagrangian (13) within the ambit of ACSA to BRST formalism. Towards this aim in mind, first of all, we generalize our basic and auxiliary variables by adding one Grassmannian variable \(\theta\) into the ordinary variables, we get chiral super expansions of the supervariables, namely;

\[
\begin{align*}
    a(t) & \rightarrow A(t, \theta) = a(t) + \theta \dot{b}_1(t), \\
    N(t) & \rightarrow \mathcal{N}(t, \theta) = N(t) + \theta \dot{b}_2(t), \\
    \bar{C}(t) & \rightarrow \bar{F}(t, \theta) = \bar{C}(t) + \theta \dot{f}_1(t), \\
    \lambda(t) & \rightarrow \Lambda(t, \theta) = \lambda(t) + \theta \dot{b}_3(t),
\end{align*}
\]

(27)

where \(\dot{b}_1(t), \dot{b}_2(t), \dot{b}_3(t)\) are the bosonic secondary variables and \(\dot{f}_1(t)\) is the fermionic secondary variable which we is determined by using the standard technique of chiral supervariable approach. For this, once again, we exploit very important and interesting anti-BRST invariant quantities given below,

\[
s_{ab}\lambda = 0, \quad s_{ab}(\bar{C} \dot{C}) = 0, \quad s_{ab}(\bar{C} \dot{a}) = 0, \quad s_{ab}(\bar{N} \dot{C} + \dot{N} \bar{C}) = 0.
\]

(28)

According to the basic premise of ACSA to BRST formalism any anti-BRST invariant quantity must be independent of the Grassmannian variable \(\theta\). Therefore the above anti-BRST invariant restrictions (Eq. (29)) generalize in the following fashion:

\[
\begin{align*}
    \Lambda(t, \theta) = \lambda(t), & \quad \bar{F}(t, \theta) \dot{F}(t, \theta) = \bar{C}(t) \dot{C}(t), \\
    \bar{F}(t, \theta) \dot{A}(t, \theta) = \bar{C}(t) \dot{a}(t), & \quad \mathcal{N}(t, \theta) \dot{F}(t, \theta) \\
    + \mathcal{\dot{N}}(t, \theta) \dot{F}(t, \theta) = \bar{N}(t) \dot{C}(t) + \dot{N}(t) \bar{C}(t).
\end{align*}
\]

(29)

The above generalized anti-BRST invariant quantities lead to the derivation of the following expressions of the secondary variables:

\[
\begin{align*}
    \bar{b}_1(t) & = - \dot{a}(t) \bar{C}(t), \quad \bar{b}_2(t) = -[\dot{N}(t) \bar{C}(t) + \dot{N}(t) \bar{C}(t)], \\
    \dot{f}_1(t) & = - \bar{C}(t) \dot{C}(t), \quad \bar{b}_3(t) = 0.
\end{align*}
\]

(30)
After substituting the above value of secondary variables into the chiral super expansions (28) of the chiral supervariables, we get the following

\[
\begin{align*}
    a(t) & \longrightarrow A^{(ab)}(t, \theta) = a(t) + \theta \left[-\dot{a}(t) \dot{C}(t)\right] = a(t) + \theta \left(s_{ab}a\right), \\
    N(t) & \longrightarrow \mathcal{N}^{(ab)}(t, \theta) = N(t) + \theta \left[-\left\{\dot{N}(t) \dot{C}(t) + \ddot{N}(t) \ddot{C}(t)\right\}\right] = N(t) + \theta \left(s_{ab}N\right), \\
    \dot{C}(t) & \longrightarrow \dot{F}^{(ab)}(t, \theta) = \dot{C}(t) + \theta \left[-\dot{C}(t) \dot{\bar{C}}(t)\right] = \dot{C}(t) + \theta \left(s_{ab}\dot{C}\right), \\
    \lambda(t) & \longrightarrow \Lambda^{(ab)}(t, \theta) = \lambda(t) + \theta \left[0\right] = \lambda(t) + \theta \left(s_{ab}\lambda\right),
\end{align*}
\]

(31)

where superscript \((ab)\) on the chiral supervariables denote the supervariables that have derived after the use of anti-BRST invariant restrictions (29). Here, the coefficients of Grassmannian variable \(\theta\) denote the anti-BRST symmetry transformations for the various basic and auxiliary variables of the cosmological FRW model. We end this section with a concluding remark that \(s_{ab}\) is deeply linked with the translational generator \(\partial_\theta\) along the \(\theta\)-direction of the Grassmannian variable with a mapping: \(s_{ab} \longleftrightarrow \partial_\theta\) (see, e.g. [29-33] for detail). Therefore, the BRST and anti-BRST symmetries acting on the variables are geometrically connected with the translations of supervariables after the application of translational generators \((\partial_{\bar{\theta}}, \partial_\theta)\) along the Grassmannian coordinates \((\bar{\theta}, \theta)\), respectively.

6 Conclusion

The cosmological FRW models are well-known representations of a homogeneous and isotropic universe in the extended phase space where we assume zero cosmological constant, therefore, the fundamental interaction of gravity is the only force at work. These models are of utmost significance in the contemporary cosmology because they are relevant in the behavior and evolution of universe. At present, most of the investigations connected with dark energy are described by these cosmological models in the FRW universe.

In this research, we looked at FRW model that described a closed, flat, and open universe. We found that cosmological FRW model endowed with the first-class constraints in the language of Dirac’s prescription of classification scheme of constraints. We quantize this model using the BRST analysis with differential gauge condition in the extended phase space. We obtained the BRST and anti-BRST symmetry transformations by the replacement of the local gauge parameter through the ghost and anti-ghost variables, respectively. We develop, for the first time, the anti-BRST symmetry transformation for the cosmological FRW model which is useful in explaining the full geometrical description of both the symmetries in the realm of supervariable/superfield approach.

We derived the nilpotent BRST and anti-BRST conserved charges associated with the FRW model using the Noether theorem where global (anti-)BRST invariance of the effective action in the extended phase space play important role. We also demonstrated nilpotency and absolute anti-commutativity properties of these charges within the framework of BRST formalism. To prove the sanctity of the (anti-)BRST symmetry transformations associated with the present FRW model, we have derived the complete set of symmetries within the ambit of (anti-)chiral supervariable approach (ACSA) to BRST...
formalism where only one Grassmannian variable has been taken into account whereas in usual supervariable/superfield approach [29-33] full super expansion of the supervariable/superfield with two Grassmannian coordinates have been taken into account. For the present endeavour, we discovered that BRST symmetry ($s_{\theta}$) is linked to the translational generator ($\partial_{\bar{\theta}}$) along the $\bar{\theta}$-direction, whereas anti-BRST symmetry ($s_{\bar{\theta}}$) is linked to the translational generator ($\partial_{\theta}$) along the $\theta$-direction of the Grassmannian variables.

Our future investigations will focus on expanding the scope of this study to more complex cosmological models to see how well the BRST framework can achieve gauge and diffeomorphism invariance at the quantum level. It would be fascinating to discuss the (anti-)BRST symmetry transformations associated with a more general form of the cosmological FRW model where 1D reparameterization (i.e., diffeomorphism) symmetry plays an important role. The combination of the ACSA and the modified Bonora-Tonin supervariable approach (MBTSA) would be very interesting to discuss and derive the (anti-)BRST symmetry transformations and many more properties associated with for FRW model. It would be very significant step toward the basic understanding and formation of a complete quantum theory of the modern cosmology. In future endeavors, we would also like to work on the physical implications of our results (e.g. anomalies, M-theory and quantum gravity) for the various practical systems/models within the framework of BRST and supervariable/superfield formalism.

**Data Availability**

No data were used to support this study.

**Conflicts of Interest**

The authors declare that there is no conflicts of interest.

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**References**

[1] DIRAC P. A. M., *Lectures on Quantum Mechanics*, Belfer Graduate School of Science, Yeshiva University Press, New York (1964).

[2] SUNDERMEYER K., *Constrained Dynamics: Lecture Notes in Physics*, Vol. 169, Springer-Verlag, Berlin (1982).
[3] BECCHI C., ROUET A., STORA R., Phys. Lett. B, 52 (1974) 344.

[4] BECCHI C., ROUET A., STORA R., Comm. Math. Phys., 42 (1975) 127.

[5] BECCHI C., ROUET A., STORA R., Ann. Phys. (N. Y.), 98 (1976) 287.

[6] TYUTIN I. V., Lebedev Institute Preprint, Report Number: FIAN-39 (1975) (unpublished), arXiv:0812.0580 [hep-th].

[7] FADDEEV L., JACKIW R., Phys. Rev. Lett., 60 (1988) 1692.

[8] BONORA L., MALIK R. P., Universe, 7 (2021) 280.

[9] BONORA L., COTTA-RAMUSINO P., RINALDI M., STASHEFF J., Commun. Math. Phys., 112 (1987) 237.

[10] BONORA L., Nucl. Phys., 912 (2016) 103.

[11] STORA R., Algebraic Structure and Topological Origin of Anomalies, Plenum Press: New York, USA (1984).

[12] FAIZAL M., Comm. Theor. Phys., 57 (2012) 637.

[13] FAIZAL M., Phys. Rev. D, 84 (2011) 106011.

[14] FAIZAL M., J. Phys. A, 44 (2011) 02001.

[15] FAIZAL M., Phys. Lett. B, 705 (2011) 120.

[16] DEWITT B. S., Phys. Rev., 160 (1967) 1113.

[17] WILTSHIRE D. L., Cosmology: The Physics of the Universe, World Scientific, Singapore (1996).

[18] FRIEDMANN A., Zeit. f. Phys., 10 (1922) 377.

[19] FRIEDMANN A., Zeit. f. Phys., 21 (1924) 326.

[20] ROBERTSON H. P., Astrophys. J., 82 (1935) 284.

[21] ROBERTSON H. P., Astrophys. J., 83 (1935) 187.

[22] ROBERTSON H. P., Astrophys. J., 83 (1936) 257.

[23] WALKER A. G., Proc. Lond. Math. Soc. (2), 42 (1937) 90.

[24] UPADHYAY S., Prog. Theor. Exper. Phys., 2015 (2015) 093B06.

[25] UPADHYAY S., Ann. Phys., 356 (2015) 299.

[26] CIANFRANI F., MONTANI G., Phys. Rev. D, 87 (2013) 084025.

[27] SHESTAKOVA T. P., Class. Quantum Grav., 28 (2011) 055009.
[28] HALLIWELL J. J., *Phys. Rev. D.*, **38** (1988) 2468.

[29] THIERRY-MIEG J., *J. Math. Phys.*, **21** (1980) 2834.

[30] QUIROS M., DE URRIES F. J., HOYOS J., MAZON M. L., RODRIGUES E., *J. Math. Phys.*, **22** (1981) 1767.

[31] BONORA L., TONIN M., *Phys. Lett. B*, **98** (1981) 48.

[32] BONORA L., PASTI P., TONIN M., *Nuovo Cimento A*, **64** (1981) 307.

[33] BONORA L., PASTI P., TONIN M., *Ann. Phys.*, **144** (1982) 15.

[34] MALIK R. P., *J. Phys. A: Math. Theor.*, **39** (2006) 10575.

[35] MALIK R. P., *Eur. Phys. J. C*, **51** (2007) 169.

[36] MALIK R. P., *Eur. Phy. J. C*, **60** (2009) 457.

[37] SRINIVAS N., BHANJA T., MALIK R. P., *Adv. High Energy Phys.*, **2017** (2017) 6138263.

[38] CHAUHAN B., KUMAR S., MALIK R. P., *Int. J. Mod. Phys. A*, **33** (2018) 1850026.

[39] SHUKLA A., SRINIVAS N., MALIK R. P., *Ann. Phys.*, **394** (2018) 98.

[40] CHAUHAN B., KUMAR S., *Adv. High Energy Phys.*, **2021** (2021) 5518304.

[41] CHAUHAN B., KUMAR S., MALIK R. P., *Int. J. Mod. Phys. A*, **37** (2022) 2250003.

[42] CHAUHAN B., *Eur. Phys. J. Plus*, **137** (2022) 976.