Spin-orbit-free topological insulators

A Alexandradinata and B Andrei Bernevig

Department of Physics, Princeton University, Princeton NJ 08544, USA

E-mail: arisa@princeton.edu and bernevig@princeton.edu

Received 7 November 2014
Accepted for publication 12 June 2015
Published 25 August 2015

Abstract
We review a class of translational-invariant insulators without spin–orbit coupling, as may be realized in intrinsically spinless systems, e.g., photonic crystals and ultra-cold atoms. Some of these insulators have no time-reversal symmetry as well, i.e., the relevant symmetries are purely crystalline. Nevertheless, topological phases exist which are distinguished by their robust surface modes. To describe these phases, we introduce the notions of (a) a halved-mirror chirality: an integer invariant which characterizes half-mirror planes in the 3D Brillouin zone, and (b) a bent Chern number: the traditional Thouless–Kohmoto–Nightingale–Nijs invariant generalized to bent 2D manifolds. Like other well-known topological phases, their band topology is unveiled by the crystalline analog of Berry phases, i.e., parallel transport across certain non-contractible loops in the Brillouin zone. We also identify certain topological phases without any robust surface modes—they are uniquely distinguished by parallel transport along bent loops, whose shapes are determined by the symmetry group. Finally, we describe the Weyl semimetallic phase that intermediates two distinct, gapped phases.

Keywords: topological insulator, photonic crystal, cold atom

(Some figures may appear in colour only in the online journal)

1. Introduction
Insulating phases are deemed distinct if they cannot be connected by continuous changes of the Hamiltonian that preserve both the energy gap and the symmetries of the phase; in this sense we say that the symmetry protects the phase. Distinct phases have strikingly different properties—the presence of boundary modes are particularly accessible by experimental techniques such as STM and ARPES. For many well-known topological insulators (TIs), the existence of boundary bands is in one-to-one correspondence with the topology of the bulk wavefunctions, and robust boundary modes indicate a nontrivial phase [1]. Examples include the Chern insulator and the quantum spin Hall insulator; these non-interacting insulators fall under ten well-known symmetry classes which are distinguished by time-reversal, particle–hole and chiral symmetries [2]. Given the completeness of this classification, attention has shifted to identifying topological phases which rely on other symmetries [3]. The symmetries which are ubiquitous in condensed matter are the crystal space groups; among them the point groups involve transformations that preserve a spatial point. For example, the reflection symmetry plays an important role in SnTe, [4–6] where the mirror Chern number [7] characterizes planes in the 3D BZ which are invariant under reflection; in short, we call these mirror planes. The Bloch wavefunctions in each mirror plane may be decomposed according to their representations under reflection, and each subspace may exhibit a quantum anomalous Hall effect [8].

Including SnTe, all experimentally-realized TIs have thus far been strongly spin–orbit-coupled, resulting in a variety of exotic phenomenon, e.g., Rashba spin-momentum locking on the surface of a TI [9]. Considerably less attention has been addressed to spin–orbit-free systems, i.e., electronic insulators and semimetals in which spin–orbit coupling is negligibly weak, [10, 11], as well as intrinsically spinless systems such as photonic crystals and ultra-cold atoms. The topological classifications of spin–orbit-free and spin–orbit-coupled systems generically differ. For example, the mirror Chern numbers are trivially zero for any spin–orbit-free system with the same crystalline symmetries as SnTe [12]. The first proposal of a spin–orbit-free TI relies on a combination of n-fold rotational and time-reversal symmetries, for n = 4 and 6 [10]. Henceforth, we refer to this combined group as Cn + T, where n ∈ {4, 6}. Topological
phys. scr. t164 (2015) 014013

a alexandradinata and b a bernevig

2. \textit{C}_{4v} insulators and semimetals

Given that the mirror Chern number vanishes in spin–orbit-free systems with \textit{C}_{4v} symmetry, how else can surface modes be made robust? Progress is made by identifying two \textit{C}_{4v}-invariant points (\lvec{\Gamma} and \lvec{M}) in the (001) surface BZ; each \textit{C}_{4v}-invariant point is mapped to itself under a four-fold rotation, up to translations by a reciprocal lattice vector, as we illustrate in figure 1(a). In addition, the black lines that intersect \lvec{\Gamma} (resp. \lvec{M}) in figure 1(a) are invariant under reflection, indicating that there are additional symmetries at this point, e.g., the reflection \lvec{M}_1 maps \( (x, y) \rightarrow (y, x) \). The combined symmetries of \( \Gamma \) (resp. \( M \)) form the little group \([16]C_{4v}\). At these points, the symmetry enforces that \( (p_x, p_y) \) and \( (d_{xz}, d_{yz}) \) orbitals are doubly-degenerate. That is, each pair of orbitals transforms in the two-dimensional irreducible representation (irrep) of \textit{C}_{4v}. Indeed, if we choose a basis \( (p_x \pm p_y) \) that diagonalizes \( M_1 \) with eigenvalue \( \pm 1 \), clearly \textit{C}_{4v} transforms \( p_x + p_y \rightarrow -(p_x - p_y) \) and vice versa. We then propose that surface bands of \textit{C}_{4v} systems assume topologically distinct structures on the red line which connects \( \Gamma \) and \( M \), as illustrated in figure 1(b). We call the red line a half-mirror line (HML\(_1\)) because it does not close into a circle (modulo reciprocal lattice vectors), yet all Bloch wavefunctions on the red line may be diagonalized by a \textit{single} reflection operator \( (M_1) \). Suppose we parametrize HML\(_1\) with \( s_1 \in [0, 1] \), where \( s_1 = 0 \) (resp. 1) at \( \Gamma \) (resp. \( M \)). At \( s_1 = 0 \) and 1, we have just established that (001) surface bands form doubly-degenerate pairs with opposite mirror eigenvalues, irrespective of whether the system has TRS. For representations without spin, we may label bands with mirror eigenvalue \( +1 \leftarrow -1 \) as mirror-even (mirror-odd). Given these constraints at \( s_1 = 0 \) and 1, there are \( Z \) ways to connect mirror-even bands to mirror-odd bands, as illustrated schematically in figure 2. We define the halved mirror chirality \( \chi_{\perp} \in \mathbb{Z} \) as the \textit{difference} in number of mirror-even chiral modes with mirror-odd chiral modes; if \( \chi_{\perp} \neq 0 \), the surface bands robustly interpolate across the energy gap. \( \chi_{\perp} \) may be easily extracted by inspection of the surface energy-momentum dispersion: first draw a constant-energy line within the bulk energy gap and parallel to the HML, e.g., the blue line in figure 2\textdegree. At each intersection with a surface band, we calculate the sign of the group velocity \( dE/d\delta_s \), and multiply it with the eigenvalue under reflection \( M_1 \). Finally, we sum this quantity over all intersections along HML\(_1\) to obtain \( \chi_{\perp} \). In figure 2\textdegree, we find two intersections as indicated by red squares, and \( \chi_{\perp} = (1)(1) + (-1)(-1) = 2 \). The \( Z \)-classification of (001) surface bands relies on doublet irreps in the surface BZ; on surfaces which break \textit{C}_{4v} symmetry, the surface bands transform in the singlet irreps, and cannot assume topologically distinct structures.

In addition to HML\(_1\), the blue line in figure 1(b) is a second half-mirror line (HML\(_2\)), and states along HML\(_2\) are characterized by an independent invariant \( \chi_{\perp} \bullet \). HML\(_2\) is the union of two mirror lines, \( \bar{\Gamma}-\bar{X} \) and \( \bar{X}-\bar{M} \), which are related by a \( \pi/2 \) rotation. States along \( \bar{\Gamma}-\bar{X} \) are invariant under the reflection \( M_\perp : y \rightarrow -y \), while along \( \bar{X}-\bar{M} \) the relevant reflection is \( M_\perp : x \rightarrow -x \). The product of these
orthogonal reflections is a $\pi$ rotation ($C_2$) about $\hat{z}$, thus $M_z = C_2 M_x$. Since both ($p_x, p_y$) orbitals are odd under a $\pi$-rotation, an eigenstate of $M_z$ with eigenvalue $\eta$ also diagonalizes $M_y$ with eigenvalue $-\eta$, e.g., $p_y$ is even under $M_y$ but odd under $M_x$. In effect, each Bloch wavefunction along HML$_2$ can be labelled by the quantum number of a single reflection operator $M_i \equiv M_2$. The second invariant $\chi_2$ is then the difference in number of mirror-even ($M_2 = +1$) chiral modes with mirror-odd ($M_2 = -1$) chiral modes, in a straightforward generalization of $\chi_1$.

Thus far we have described the halved chirality $\chi_1$ as a topological property of surface bands along HMP$_i$, but we have not addressed how $\chi_1$ is encoded in the bulk wavefunctions. Taking $\hat{z}$ to lie along the rotational axis, each HML$_i$ in the surface BZ is the $\hat{z}$-projection of a half-mirror-plane (HMP) in the 3D BZ, as illustrated in figure 1(b). Each HMP connects two distinct $C_4$-invariant lines, and all Bloch wavefunctions in a HMP may be diagonalized by a single reflection operator. HMP$_i$ is parametrized by $t_i \in [0, 1]$ and $k_\parallel \in (-\pi, \pi]$, where $t_i = 0$ (1) along the first (second) $C_4$-invariant line. Then the halved mirror chirality has the following expression by bulk wavefunctions: [12]

$$\chi_i = \frac{1}{2\pi} \int_{\text{HMP}_i} dt_i \, dk_\parallel \left[ F_c - F_o \right] \in \mathbb{Z}. \tag{1}$$

$F_c (F_o)$ is defined as the Berry curvature of occupied doublet bands, [17–19] as contributed by the mirror-even (odd) subspace.

The invariants $[\chi_i]$ are well-defined so long as bulk states in the HMP’s are gapped, which is true of $C_4$ insulators. These invariants may also be used to characterize $C_4$ semimetals, so long as the gaps close away from the HMPs. Such band touchings are generically Weyl nodes, [20, 21] though exceptions exist with a conjunction of time-reversal and inversion symmetries [22]. The chirality of each Weyl node is its Berry charge, which is positive (negative) if the node is a source (sink) of Berry flux. By the Nielsen–Ninomiya theorem, the net chirality of all Weyl nodes in the BZ is zero [23]. To make progress, we divide the BZ into ‘unit cells’, such that the properties of one ‘unit cell’ determine all others by symmetry. As seen in figure 1(b), each ‘unit cell’ resembles the interior of a triangular pipe; it is known as the orbifold $T^2/C_{4v}$. The net chirality of an orbifold can be nonzero, and is determined by the Chern number on the 2D boundary of the orbifold. As the boundary resembles the surface of a triangular pipe, we call the invariant a bent Chern number ($C_{12}$). $C_{4v}$ systems are thus described by three invariants ($\chi_1, \chi_2, C_{12}$), which are related by parity $[\chi_1 + \chi_2] = \text{parity} [C_{12}]$. If $\sum_{i=1}^{2} \chi_i$ is odd, $C_{12}$ must be nonzero due to an odd number of Weyl nodes within the orbifold, which implies the system is gapless. If the system is gapped, $\sum_{i=1}^{2} \chi_i$ must be even. However, the converse is not implied. This parity constraint may be understood in light of a Weyl semimetallic phase that intermediates two gapped phases with distinct halved chiralities. There are four types of events that alter the halved chirality $\chi_i$ of HMP$_i$; we explain how Weyl nodes naturally emerge in the process. (i) Suppose the gap closes between two mirror-even bands in HMP$_i$. Around this band touching, bands disperse linearly within the mirror plane, and quadratically in the direction orthogonal to the plane. Within HMP$_i$, the linearized Hamiltonian around the band crossing describes a massless Dirac fermion in the even representation of reflection. If the mass of the fermion inverts sign, $\int_{\text{HMP}_i} F_c/2\pi$ changes by $\eta \in \{\pm 1\}$, implying that $\chi_i$ also changes by $\eta$ through equation (1). This quantized addition of Berry flux is explained by a splitting of the band-touching into two Weyl nodes of opposite chirality, and on opposite sides of the mirror face (figure 3(a)). In analogy with magnetostatics, the initial band touching describes the nucleation of a dipole, which eventually splits into two opposite-charge monopoles; the flux through a plane separating two monopoles is unity. (ii) The same argument applies to the splitting of dipoles in the mirror-odd subspace, which alters $\int_{\text{HMP}_i} F_o/2\pi \eta \in \{\pm 1\}$, and $\chi_i$ by $-\eta$. For (iii) and (iv), consider two opposite-charge monopoles which converge on HMP$_i$ and annihilate, causing $\chi_i$ to change by unity. The sign of this change is determined by whether the annihilation occurs in the mirror-even or odd subspace. The transition between two distinct gapped phases is then characterized by a transfer of Berry charge between two distinct HMPs (figure 3(e)). In the intermediate semimetallic phase,
the experimental implications include Fermi arcs on the (001) surface [21, 24, 25].

3. Cₘ₊T insulators and semimetals

Our case study of the Cₘ₊T insulator is motivated by two issues: (i) in [10], we learned there exists two distinct gapped phases with Cₘ₊T symmetry, which are distinguished by robust surface modes; we refer to one as the trivial phase and the other a strong topological phase. It is unclear if these two phases are physically distinguishable if we experimentally probe the bulk instead of the surface. (ii) In this review, we highlight the existence of a third topological phase which does not manifest robust surface modes. Does this indicate the existence of a third topological phase which is strong? The product Iₐᵯ(0)Iₐᵯ(π) is a strong index that determines the absence or presence of robust surface modes on the 001 surface; we take θ to lie along the principal Cₘ axis. We give these weak indices a physically transparent exploration from the perspective of holonomy. For illustration, we consider a model on a tetragonal lattice that is characterized by two \( \mathbb{Z}_2 \) indices \( \{Iₐ(0), Iₐ(π)\} \) shall be referred to as weak indices. Iₐ(0) = Iₐ(π) = 1 (resp. -1) corresponds to the trivial (resp. weak) phase, and Iₐ(0) = -Iₐ(π) describes a strong phase. The product Iₐ(0)Iₐ(π) is a strong index that determines the absence or presence of robust surface modes on the 001 surface; we take θ to lie along the principal Cₘ axis. We give these weak indices a physically transparent interpretation from the perspective of holonomy. For illustration, we consider a model on a tetragonal lattice that is characterized by two \( \mathbb{Z}_2 \) indices. The Bloch Hamiltonian is

\[
H(k) = \left[ -1 + 8f₁(k) \right] I₀₁ + 2f₂(k)I₁₁ + \alpha f₃(k)I₀₂ + \beta f₄(k)I₁₂ + 2f₅(k)I₁₃,
\]

where

\[
f₁ = 3 \cos(kₓ) - \cos(kᵧ) - \cos(k₂), \quad f₂ = 2 - \cos(kₓ) - \cos(kᵧ), \quad f₃ = \cos(kₓ), \quad f₄ = \cos(kᵧ) - \cos(k₂), \quad f₅ = \sin(kₓ)\sin(kᵧ) - \sin(k₂)\sin(kₘ).
\]

In \( f_{dₖ} \equiv \sigma₀ \bigotimes \tauₙ \) orbitals on one sublattice, and \( f_{dₙ} \equiv \{±1, τ₁ = +1\} \) label \( f_{dₙ} \) orbitals on the other. This Hamiltonian is four-fold symmetric: \( Iₐₜ H(kₙ,kₙ,kₖ)Iₐₜ = H(−kₙ,kₙ,kₖ) \), and time-reversal symmetric: \( I₁₀ H(kₙ)I₁₀ = H(−kₙ) \). The ground state of (2) comprises its two lowest-lying bands. The phase diagram of this model is plotted in figure 5(a) for different parametrizations of equation (2).

To probe the bulk topology, we perform parallel transport along a bent loop that connects two Cₘ⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻˓ fraudulent
one-to-one correspondence with two sectors of ground states in the $k \bar{k}$ plane. Furthermore, we are able to derive the spectrum of $\mathcal{L}[(\bar{k})^4]$ for any number of occupied bands [32]. In a minimal model with $n_{\text{occ}} = 2$, there are only two possibilities $(\mathcal{L}[(\bar{k})^4] = \pm 1)$, which are distinguished by the parity of $d_{\mathcal{L}}(\bar{k})^2$.

Let us demonstrate how to realize both classes of $\mathcal{L}[(\bar{k})^4]$ in the model (2). We consider a family of loops $\mathcal{L}[(\bar{k})^4]$ in planes of constant $k_z$, such that $\mathcal{L}[(\bar{k})^4]$ is the red line in figure 4(a), and all other loops project to $\mathcal{L}[(\bar{k})^4]$ in $\bar{k}$. The absence of surface modes on the line $M - \Gamma - \bar{X}$ is demonstrated in figure 5(b). In comparison, the weak phase is characterized by $\mathcal{L}[(\bar{k})^4] = -1$, which clearly does not exhibit spectral flow. Finally, we point out that a different choice of Wilson loop can identify the strong index $\mathcal{L}[(\bar{k})^4]$, but cannot individually distinguish the weak indices: $\mathcal{L}[(\bar{k})^4]$, $\mathcal{L}[(\bar{k})^4]$.

4. Conclusion

We propose sufficient criteria for surface modes whose robustness rely on a symmetry group. Minimally, (i) the symmetry must be unbroken by the presence of the surface. Additionally, either (ii) a reflection symmetry exists so that mirror-subspaces can display a quantum anomalous Hall effect, or (iii) there exist at least two high-symmetry points in the surface BZ, which admit higher-than-one dimensional irreps of the symmetry group. In addition to predicting new topological materials, these criteria are also satisfied by the well-known SnTe class, [4] and also the $Z_2$ insulators [13, 14, 33, 34, 41–47]. Among the 32 crystallographic point groups, only the $C_2$ and $C_4v$ groups are preserved for a surface that is orthogonal to the rotational axis [16]. Though all $C_{nv}$
groups satisfy (ii), the lack of spin–orbit coupling implies only $C_{3v}$ systems can have nonvanishing mirror Chern numbers [32]. While all $C_{6v}$ groups by themselves only have one-dimensional irreps, the $C_4$ or $C_{2v}$ group satisfies (iii) in combination with TRS, as is known for the topological crystalline insulators introduced in [10]. Finally, only a subset of the $C_{6v}$ groups possess two-dimensional irreps which satisfy (iii): $C_{3v}$, $C_{4v}$, and $C_{6v}$ [12].

Acknowledgments

Chen Fang and Matthew Gilbert played an essential role in formulating the theory of $C_{nv}$ insulators, whose short review we have presented here. AA and BAB were supported by Packard Foundation, 339-4063—Keck Foundation, NSF CAREER DMR-095242, ONR—N00014-11-1-0635 and NSF-MRSEC DMR-0819860 at Princeton University. This work was also supported by DARPA under SPAWAR Grant No.: N66001-11-1-4110.

References

[1] Fidkowski L, Jackson T S and Klich I 2011 Phys. Rev. Lett. 107 036601
[2] Schnyder A P, Ryu S, Furusaki, and A and Ludwig A W W 2009 AIP Conf. Proc. 1134 10
[3] Shiozaki K and Sato M 2014 Phys. Rev. B 90 165114
[4] Hsieh T H et al 2012 Nat. Commun. 3 982
[5] Xu S-Y et al 2012 Nat. Commun. 3 1192
[6] Tanaka Y, Ren Z, Sato T, Nakayama K, Souma S, Takahashi T, Segawa K and Ando Y 2012 Nat. Phys. 8 800
[7] Teo J C Y, Fu L, and Kane C L 2008 Phys. Rev. B 78 045426
[8] Haldane F D M 1988 Phys. Rev. Lett. 61 2015
[9] Winkler R 2003 Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Springer Tracts in Modern Physics) (Berlin: Springer)
[10] Fu L 2011 Phys. Rev. Lett. 106 106802
[11] Liu C-X, Zhang R-X and VanLeeuwen B K 2014 Phys. Rev. B 90 085304
[12] Alexandradinata A, Fang C, Gilbert M J and Bernevig B A 2014 Phys. Rev. Lett. 113 116403
[13] Fu L and Kane C L 2006 Phys. Rev. B 74 195312
[14] Roy R 2009 Phys. Rev. B 79 195321
[15] Thouless D J, Kohmoto M, Nightingale M P and den Nijs M 1982 Phys. Rev. Lett. 49 405
[16] Tinkham M 2003 Group Theory and Quantum Mechanics (Dover: New York)
[17] Berry M V 1984 Proc. R. Soc. A 392 45
[18] Zak J 1982 Phys. Rev. Lett. 48 359
[19] Zak J 1989 Phys. Rev. Lett. 62 2747
[20] Murakami S 2007 New J. Phys. 9 356
[21] Wan X, Turner A, Vishwanath A and Savrasov S Y 2011 Phys. Rev. B 83 205101
[22] Burkov A A, Hook M D and Balents L 2011 Phys. Rev. B 84 235126
[23] Nielsen H B and Ninomiya M 1981 Nucl. Phys. B 185 20
[24] Halasz G B and Balents L 2012 Phys. Rev. B 85 035103
[25] Fang C, Gilbert M J, Dai X and Bernevig B A 2012 Phys. Rev. Lett. 108 266802
[26] Leon R 2011 J. Phys. A: Math. Theor. 44 295301
[27] Ekert A, Ericsson M, Hayden P, Inamori H, Jones J A, Oi D K L and Vedral V 2000 J. Mod. Opt. 47 2501
[28] Recati A, Calarco T, Zanardi P, Cirac J I and Zoller P 2002 Phys. Rev. A 66 0302309
[29] Atala M et al 2013 Nat. Phys. 9 795
[30] King-Smith R D and Vanderbilt D 1993 Phys. Rev. B 47 1651–4
[31] Tahirinejad M, Garrity K F and Vanderbilt D 2014 Phys. Rev. B 89 115102
[32] Alexandradinata A and Bernevig B A 2014 arXiv:1409.3236
[33] Fu L, Kane C L and Mele E J 2007 Phys. Rev. Lett. 98 106803
[34] Roy R 2009b Phys. Rev. B 79 195322
[35] Wilczek F and Zee A 1984 Phys. Rev. Lett. 52 2111
[36] Alexandradinata A, Dai X and Bernevig B A 2014 Phys. Rev. B 89 155114
[37] Qi X L, Wu Y S and Zhang S C 2006 Phys. Rev. B 74 045125
[38] Prodan E 2009 J. Math. Phys. 50 083517
[39] Yu R, Qi X L, Bernevig A, Fang Z and Dai X 2011 Phys. Rev. B 84 075119
[40] Soluyanov A A and Vanderbilt D 2011 Phys. Rev. B 83 235401
[41] Kane C L and Mele E J 2005 Phys. Rev. Lett. 95 226801
[42] Kane C L and Mele E J 2005 Phys. Rev. Lett. 95 146802
[43] Bernevig B A and Zhang S C 2006 Phys. Rev. Lett. 96 106802
[44] Bernevig B A, Hughes T L and Zhang S C 2006 Science 314 1757
[45] König M, Wiedmann S, Brüne C, Roth A, Buhmann H, Molenkamp L, Qi X-L and Zhang S-C 2007 Science 318 766
[46] Moore J E and Balents L 2007 Phys. Rev. B 75 121306
[47] Hsieh D, Qian D, Wray L, Xia Y, Hor Y S, Cava R J and Hasan M Z 2008 Nature 452 970