Quark mass dependence of s-wave baryon resonances

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We study the quark mass dependence of $J^P = \frac{1}{2}^-$ s-wave baryon resonances. Parameter free results are obtained in terms of the leading order chiral Lagrangian. In the 'heavy' SU(3) limit with $m_s = m_K \approx 500$ MeV the resonances turn into bound states forming two octets plus a singlet representations of the SU(3) group. A contrasted result is obtained in the 'light' SU(3) limit with $m_s = m_K \approx 140$ MeV for which no resonances exist. Using physical quark masses our analysis suggests to assign to the $S = -2$ resonances $\Xi(1690)$ and $\Xi(1620)$ the quantum numbers $J^P = 1/2^-$. This reference also use a scheme based on the solution of the BSE with a kernel determined by the WT term, and find just one resonance in the s-wave $S = -2$ sector. The found resonance shows a large decay width and Branching Ratios (BR) which are incompatible with the empirical properties of the $\Xi(1690)$ resonance, and instead it was identified with the one star resonance $\Xi(1620)$. The main difference between the approach of Ref. ¹ and that followed here is the method used to renormalize the BSE. In Ref. ¹, a three-momentum ultraviolet cutoff of natural size was introduced, though some channel dependence of its numerical value was allowed. Such a procedure turns out to work remarkably well in the $S = -1$ sector at low energies, providing a good description of the $\Lambda(1405)$ resonance, but it starts showing limitations at higher energies, where the description of the $\Lambda(1670)$ and $\Sigma(1620)$ is certainly poorer. Indeed, the procedure of ¹ does not work in the $S = 0$ sector at all, and it fails even to produce the lowest lying resonance $\Lambda(1535)$). Our scheme provides reasonable results in the $S = 0, -1$ sectors. In the $S = -2$ sector, we also find, besides the resonance which we identify to the $\Xi(1690)$ and mentioned above, a resonance with the same features as that described in ¹, which can be identified with the one star $\Xi(1620)$ resonance.

I. INTRODUCTION

The question what is the true nature of baryon resonances has attracted considerable attention in recent modern constructions of effective field theories describing meson-baryon scattering. Before the event of the quark-model it was already suggested by Wyld and also by Dalitz, Wong and Rajasekaran that a t-channel vector meson exchange model for the s-wave meson-baryon scattering problem has the potential to dynamically generate s-wave baryon resonances upon solving a coupled channel Schrödinger equation. In a more modern language the t-channel exchange was rediscovered in terms of the Weinberg-Tomozawa (WT) interaction, the leading term of the chiral Lagrangian that reproduces the first term of the vector meson exchange in an appropriate Taylor expansion. This offers a unique opportunity to study the quark mass dependence of baryon resonances, one of the goals of this work. Such studies may be useful to obtain a deeper understanding of baryon resonances. Here we follow a scheme proposed in ¹, based on the solution of the Bethe-Salpeter-Equation (BSE), which incorporates two-body coupled channel unitarity, as other approaches, but also insists on an approximate crossing symmetry. Indeed the latter constraint led to a parameter free description of the $\Lambda(1405)$ and N(1535) resonances in terms of the WT Lagrangian. The goal of this paper is to systematically unravel the SU(3) structure of the lowest lying s-wave baryon resonances. We show that two full octets plus an additional singlet of resonances are dynamically generated within this framework. In the SU(3) limit, with degenerate mesons and baryons, such a structure was already found in Ref. ¹.

Of particular interest is the $S$(Strangeness) = -2 sector where we find a narrow state (with a width of about 5 MeV) with a strong coupling to $K\Sigma$ suggesting the identification with the three star resonance $\Xi(1690)$. For the latter resonance only its isospin quantum number was established experimentally. Our analysis suggests the quantum numbers $J^P = 1/2^-$. This complements the conclusion of the recent work of Ref. ¹. The authors of

II. THEORETICAL FRAMEWORK

We solve the coupled channel BSE with an interaction kernel expanded in chiral perturbation theory as formulated in ¹. The solution for the coupled channel s-wave scattering amplitude, $T(\sqrt{s})$ in the so called on-shell scheme, can be expressed in terms of a renormalized matrix of loop functions, $J(\sqrt{s})$, and an effective on-shell interaction kernel, $V(\sqrt{s})$, as follows

$$T(\sqrt{s}) = \frac{1}{1 - V(\sqrt{s})J(\sqrt{s})}V(\sqrt{s}).$$

Assuming the conservation of isospin and strangeness the scattering problem decouples into 9 different sectors ($(I, S) = (0, 1), (1, 1), (\frac{1}{2}, 0), (\frac{3}{2}, 0), (0, -1), (1, -1), ...$).
scattering amplitudes may be evaluated in standard chiral perturbation theory with the typical parameter $m_K/(4\pi f) < 1$ with $f \approx 90$ MeV. Once the available energy is sufficiently high to permit elastic two-body scattering a further typical dimensionless parameter $m_K^2/(8\pi f^2) \sim 1$ arises. Since this ratio is uniquely linked to two-particle reducible diagrams it is sufficient to sum those diagrams keeping the perturbative expansion of all irreducible diagrams. This is achieved by Eq. (1).

The subtraction points of Eq. (3) are the unique choices that protect the s-channel baryon-octet masses manifestly in the p-wave $J = \frac{1}{2}$ scattering amplitudes. The merit of the scheme (2) lies in the property that for instance the kaon-nucleon and antikaon-nucleon scattering amplitudes match at $\sqrt{s} \sim m_{\Lambda}, m_{\Sigma}$ approximately as expected from crossing symmetry. The subtraction points of Eq. (3) can also be derived if one incorporates photon-baryon inelastic channels in (1). Then additional crossing symmetry constraints arise. For instance the reaction $\gamma \Lambda \rightarrow \gamma \Lambda$, which is subject to a crossing symmetry constraint at threshold, may go via the intermediate state $KN$. Therefore the corresponding loop function must vanish identically at $\sqrt{s} = m_{\Lambda}$ confirming Eq. (3). Here we assume that this reaction is described by a coupled channel scattering equation (11) where the effective on-shell interaction kernel $V$ is expanded in chiral perturbation theory. We use the Leading Order (LO) interaction kernel $V(\sqrt{s})$, as determined by the WT interaction (see Refs. [7, 9, 12]),

$$V_{ab}^{IS}(\sqrt{s}) = D_{ab}^{IS} \frac{2 \sqrt{s} - M_a - M_b}{4 f^2},$$

(4)

where $M_b (M_b)$ is the baryon mass of the initial (final) channel. In Eq. (4) tadpole terms, of subleading chiral order, arising from the on-shell reduction of the interaction kernel (see Ref. [10]) are neglected. A parameter free prediction arises if physical values for the meson and baryon masses are used. This is a direct consequence of the chiral SU(3) symmetry of QCD that predicts the strength of the WT interaction in terms of the parameter $f$ already determined by the pion decay process.

We will also study the quark mass dependence of the baryon resonances that are dynamically generated by using meson and baryon masses that deviate from their chiral SU(3) limit in Eq. (1). We use Goldstone boson masses as determined by the Gell-Mann, Oakes and Renner (GOR) relation (17) in terms of the quark condensate $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = -(280 \text{MeV})^3$, the current quark masses $m_u = m_d = 3.5 \text{MeV}$ and $m_s = 85 \text{MeV}$ and $f = 90 \text{MeV}$ (with these values we get $m_\pi = 137.73 \text{MeV}$ and $m_K = 489.74 \text{MeV}$). The masses of the baryon octet states are described in terms of the chiral parameters $b_0 = -0.346 \text{GeV}^{-1}$, $b_D = 0.061 \text{GeV}^{-1}$ and $b_F = -0.195 \text{GeV}^{-1}$ (in the notation of Ref. [14], $b_{1,2} = \mp (b_F \pm b_D)$). The above values require a baryon octet mass of $823 \text{MeV}$ in the chiral limit with $m_{u,d,s} \approx 0$.

It is well known that at this LO, all baryons and Goldstone boson masses are reproduced quite accurately (5%).

(2, −1), (\frac{1}{2}, −2), (\frac{3}{2}, −2))$. In each sector, there are several coupled channels, for instance, the $S = 0$ sector requires four coupled channels in the $I = 1/2$ sector ($\pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$). The explicit form of the interaction kernel and the loop functions can be found in $[2, 3]$. The latter ones logarithmically diverge and one subtraction is needed to make them finite. Such a freedom can be used to incorporate approximate crossing symmetry in the scheme, by the renormalization condition

$$T(\sqrt{s} = \mu) = V(\mu), \quad \mu = \mu(I, S)$$

where the natural choice

$$\mu(I, 1) = \frac{1}{2}(m_\Lambda + m_{\Sigma}), \quad \mu(I, 0) = m_N,$$

$$\mu(0, -1) = m_\Lambda, \quad \mu(1, -1) = m_{\Sigma}, \quad \mu(I, -2) = m_{\Xi}$$

is used as explained in detail in $[2]$. It is evident that the renormalization condition of Eq. (2) is implemented in a straightforward manner by imposing that the renormalized loop functions $J(\sqrt{s})$ vanish at the appropriate points $\sqrt{s} = \mu(I, S)$. The renormalization condition reflects the fact that at subthreshold energies the
We look for poles in the Second Riemann Sheet (SRS) of our amplitudes. The positions of the poles determine masses and widths of the resonances, while the residues for the different channels define the BR \( \text{SU}(3) \). We find the resonances listed in Table II where only resonances with widths smaller than 250 MeV are included. Since for a resonance placed slightly above a threshold the BR depends strongly on the exact position of the pole, we only quote coupling constants (residues) that are much less sensitive to the pole position. We find a remarkable success predicting rather well the bulk of the features of the four stars \( N(1535), \Lambda(1405) \) and \( \Lambda(1670) \) resonances [24].

We also find a resonance in the \( \Sigma \) channel, though its mass is a bit small [21]. Besides, in the \( S = -2 \) sector we find two resonances, which can clearly be identified to the \( \Xi(1690) \) and \( \Xi(1620) \) resonances. Of particular interest is the signal for the \( \Xi(1690) \) resonance, where we find a quite small (large) coupling to the \( \pi \Xi \) (\( K \Sigma \)) channel, which explains the smallness of the experimental ratio, \( \Gamma(\pi \Xi)/\Gamma( K \Sigma) < 0.09 \) [17], despite of the significant energy difference between the thresholds for the \( \pi \Xi \) and \( K \Sigma \) channels. Thus, this work widely improves the conclusions of Ref. [12], since we also address here the \( \Xi(1690) \) resonance, and determine its spin-parity quantum numbers \( (J^P = \frac{1}{2}^-) \). On the other hand, we also find a third \( \Lambda \) resonance, not included in the PDG yet, placed also below the \( K \) threshold and with a large coupling to the \( \pi \Sigma \) channel. This confirms the findings of the recent work of Ref. [11], where the chiral dynamics of the two \( \Lambda(1405) \) states is studied. The existence of the second \( \Lambda(1405) \) was firstly pointed out in Refs. [12] and [10].

To explore the quark mass dependence of the resonances, we increase the averaged up and down quark masses, but keep fixed the antiquark mass. A parameter \( x \) is introduced in terms of which the pion mass varies as

\[
m^2_{\pi}(x) = m^2_{\pi} + x(m^2_\text{q} - m^2_{\pi}), \quad x \in [0, 1].
\]

A pair \((m^2_\text{b}, m^2_{\Sigma(1690)})\) determines the \( \eta \) meson mass via the GOR relation. Given the SU(3) symmetry breaking parameters \( b_0, b_D \) and \( b_F \), the masses of the baryon octet \((N(940), \Lambda(1115), \Sigma(1190) \) and \( \Xi(1320) \)) are also determined. In the limit \( x = 1 \) our SU(3) pion is as heavy as the real kaon, while when \( x = 0 \) the physical world is recovered, up to some minor mass differences due to imprecisions of the GOR and baryon splitting formulas. For the SU(3) symmetric \( x = 1 \) case, where all baryons (mesons) have a common mass \( M \), the \( T \) matrix has poles in the FRS (bound states). For each \( IS \) channel, the position of the poles, \( s_i \), is such that the dimensionless function \( \beta(s) = 2f^2/(J_s(\sqrt{s}))/\Gamma(\sqrt{s} - M) \), at \( s = s_i \), becomes an eigenvalue of the real and symmetric matrix \( D^{IS} \).

The eigenvalues of the latter matrices are \( 2, -3, -3 \) for both \( IS = (1/2, 0) \) and \( IS = (1/2, -2) \), and \( -2, -6, -3 \) and \( 2, 0, -3, -3 \) for \( IS = (0, -1) \) and \( IS = (1, -1) \), respectively. Since \( \beta(s) \) is negative between \((M - m)^2\) and \((M + m)^2\), only negative eigenvalues can be matched. Thus, we end up, with two degenerate octets of mass \( M_b = 1691.83 \text{ MeV} \) (eigenvalue \(-3\), which has a multiplicity of two in all \( IS \) sectors).
and a singlet of mass $M_0 = 1604.21$ MeV (eigenvalue $-6$ in the $\Lambda$ channel). Thus, we confirm here the findings of Ref. 11 on the nature of the third $\Lambda$ listed in the Table II. Slightly away of the SU(3) symmetric world ($x \approx 0.98$), we can determine the couplings of the baryon-meson states to each baryon-meson channel (last column of Table II). The sum of the squared of the couplings is given by $g^{2}_{\pi,\rho,\sigma} = \lambda_{\pi,\rho,\sigma}/(2M_{\pi,\rho,\sigma}J(M_{\pi,\rho,\sigma})^{3}(M_{\pi,\rho,\sigma}^{2}))$, where $\lambda_{\pi,\rho,\sigma} = -3$ and $-6$ respectively. We find $g^{\pi}_{\pi} = 9.65$ and $g^{\rho}_{\rho} = 15.83$. In Fig. III we show the chiral behavior of each of the members of the two octets and a singlet states. As mentioned above, for $x = 0$ one recovers the physical world, though the results presented in the figure (specially for the widths) do not coincide with those given in the Table II. The differences are produced by relative changes between the position of thresholds and the location of the resonances, due to small differences among the real meson and baryon masses (used in Table II and those predicted by the GOR and baryon splitting formula. Besides, the identification in Fig. IV of the N(1650) and the $\Sigma(1480)$ bump is subject to all uncertainties discussed so far. Finally in the 'light' SU(3) limit with $m_\pi = m_K \approx 140$ MeV, the function $\beta(s)$, defined above, is smaller than $-6$ in the whole interval $[(M - m)^2, (M + m)^2]$ and therefore bound states do no exist.

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20. There exists a second nucleon resonance ($M_R = 1156, \Gamma_R = 415$) MeV. Its large [small] coupling to the $\pi N [\eta N]$ channel ($|g^{2}_{\pi N, \eta N}]^{2}$, $K_{\pi, \eta N}$, $K_{\Sigma, \Sigma}$, $K_{\Lambda, \Lambda}$ $\approx 8.5, 0, 3.3, 0.3$ ) and its SU(3) trajectory, as we will see, make us to think that, despite its mass and width, it might correspond to the four star N(1650) resonance. It is unclear whether next-to-leading chiral corrections take the position of this pole closer to that of the physical N(1650) resonance. A quantitative description may require the inclusion of further inelastic channels, like $\pi \Delta$ and $\rho N$ [19].
21. We define a resonance as pole in an unphysical sheet, usually the SRS (the SRS is determined by continuity to the First Riemann Sheet [FRS] [9]), with an appreciable influence into the physical scattering line. In the $\Sigma$ channel, the pole ($M_R = 1505$ MeV) listed in Table II is above the first three thresholds ($\pi \Lambda, \pi \Sigma$ and $K \bar{N}$, but it appears in the unphysical 11100 sheet, instead of in the 11100 one (each of the five digits counts for the number of turns around each of the branch points). Between the third ($K \bar{N}$) and fourth ($\rho \Sigma$) thresholds, the 11100 sheet maps into the FRS, and thus is this sheet, the one which enters into the definition of SRS of Ref. [9]. Despite of this, it is indeed the narrow ($\Gamma_R \approx 20$ MeV) pole located in the unphysical 11000 sheet, with large couplings to the $\pi \Lambda, \pi \Sigma$ and specially to the $K \bar{N}$ channels, and placed very close above the $K \bar{N}$ threshold, the one which has an important influence on the scattering for energies in the neighborhood of the $K \bar{N}$ threshold. Actually, the modulus of the scattering matrix, for all open channels at these energies, presents a peak, with an appreciable gap in the first derivative (since the pole is placed above the third threshold ($K \bar{N}$), but it is found in the 11000 sheet) which is clearly due to the narrow pole listed in the table. There exists also a pole in the 11100 sheet ($M_R = 1466, \Gamma_R = 574$) MeV. It is above the $K \bar{N}$ threshold and has a large coupling to the $K \Sigma$ channel. This very broad pole is precisely the one quoted in Ref. [11], but it is placed so far from the scattering line, the $K \Sigma$ threshold ($\approx 1810$ MeV) is also so far from the region of about 1500 MeV, that it can not compete with the narrow one found in the 11000 sheet, and it does not influence the physical scattering at all (chiral corrections reduce its width and take its mass closer to the $K \Sigma$ threshold indicating that it is the $\Sigma(1750)$ resonance [8]). There exists a third pole also in the 11000 sheet ($M_R = 1446, \Gamma_R = 343$) MeV, with a large coupling to $\pi \Sigma$, and located also just above the $K \bar{N}$ threshold. Its influence on physical scattering processes is limited due to the presence of the narrower pole listed in the table, but it might be identified to the $\Sigma(1480)$ bump [12].