All-dielectric two-dimensional modified honeycomb lattices for topological photonic insulator and various light manipulation examples in waveguide, cavity and resonator

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Abstract: In this paper, we explore the topological behavior of a two-dimensional honeycomb photonic crystal (PC) based on the presence of double Dirac-cone connected the orbitals $p$ and $d$, due to the $C_6$ point group symmetry of the hexagonal PCs. Indeed, removing the four-folded degeneracies between the bands at the Dirac point can be achieved by adding the small dielectric rods nearby the bigger ones. So the perturbed PCs with the $C_7$ point group symmetry and different topological specific features may be appeared. By proposing the unique structure involves two PCs with different topological effects, one may study the one-way light distribution along the local boundary in spite of the defects, cavities, and disorders. Besides, we investigate the variation of the transmitted intensity values under different defect conditions. It is realized that the size, location and material type have affect on the amount of light to be transmitted. Finally, topological rhombic resonator enables unidirectional filtering of guided mode. The fact that different light manipulation scenarios can be realized provides a unique aspect for topological photonic insulators.

1. Introduction

PCs and their applications for various purposes like optical cavities, waveguides and mirrors based on their dispersion relations, have been investigated for a long time. Then the discovering of the topological behaviour of the materials in condensed matter physics provides an idea to explore the properties of the PC band structures for applying them as photonic topological insulators [1–3]. Although the spontaneous breakdown of symmetries can not justify the band dispersion theory but it can express the innovative topological properties in materials completely [3-7].

The topologically behaviour in PCs can be generated absolutely through two different approaches, breaking the time-reversal (TR) symmetry and without breaking the TR. For illustration, the first approach can be applied in the coupled helical mechanism and the magnetic excitation is required while the second approach may be used in the spin-polarized unidirectional distribution of the surface photons and the bianisotropic photonic metacrystals [8–24].

Regarding the topological behaviour of the PCs, they can be applied for various purposes such as confining of the light. Indeed, putting two PCs with common band gap region but different topological behaviors together, results in appearing the photonic edge modes and their propagation along the interface [25–38]. So the topological PCs made of dielectric material can be used in optical applications instead of metal or magnetic materials, because of their robust and superior edge states [20-26, 39].

2. All-dielectric two-dimensional modified honeycomb lattices
We start with the ordinary 2D honeycomb PC that is constructed with two lattice vectors $\mathbf{a}_1 = (a, 0)$ and $\mathbf{a}_2 = \left(\frac{a}{\sqrt{3}}, \frac{a\sqrt{2}}{2} \right)$ in which $a$ is the lattice constant. Each unit cell composed of six dielectric rods with permittivity $\varepsilon = 12$ and radius $r$ is inserted at a distance $R$ from the centre as shown in Fig. 1(a). By studying the dispersion behaviour of the 2D honeycomb PC, so setting the parameters $R = a/3$ and $r = R/3$, we found that at the double Dirac cone, four degenerate bands cross each other as seen in Fig. 2(a). Indeed, the structure exhibits the $C_6$ point group symmetry which leads to appearing the double Dirac cone at the Brillouin zone center, $\Gamma$ point. This is one of the simplest lattices used to studying the topological transitions without breaking the TR symmetry. Here, we study the band structures for the TM mode ($E_z, H_x, H_y \neq 0$), using the software package *MIT Photonics Bands* (MPB) [40].

Many approaches in terms of increasing/decreasing the radii of the rods or changing the locations have been proposed in the literature to remove the degeneracy at the Dirac point [32,41-42]. In the present work, we propose a new way to modify the band diagrams to investigate topological photonic insulators. By adding three small rods with the same electrical permittivity value and radii of $r_1 = R/7$ at a distance $R_1 = R - r - r_1$ from the center leads to the breaking of the inversion symmetry as the schematic view of the two cases designated as type-A and type-B is presented in Fig. 1(b).

![Fig. 1. (a) The schematic geometry of the 2D honeycomb PC. The vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are the basis lattice vectors. The radii of the circles is $r$ and their distance from the unit cell center is $R$. (b) Two modified PCs named as type A and B, six main cylinders with radii $r$ are embedded at a distance $R$ from the center of the unit cell, while three small circles with radii $r_1$ are embedded at a distance $R_1 = R - r - r_1$.](image)

Figure 2(b) indicates the band diagrams of the two modified honeycomb PCs whose unit cells are seen in Fig. 1(b). Due to lifting of the four-fold degeneracy, there appears a photonic band gaps. Yet there is still two-fold degeneracy. The electric field profiles of the modified honeycomb PCs are designated as symmetric and anti-symmetric and also they are inverted together along the green vectors as given in Fig. 2(c).

We demonstrate that the proposed design may be applied for other radii values other than $r_1 = R/7$. Figure 3 shows that the topological band gap for various radii $r_1$. In all of these figures, the parameters $R = \frac{a}{3}$ and $r = \frac{b}{3}$ are fixed and the distance between the center of the small
Fig. 2. (a) The band diagram of two modified HPCs, types A and B, by adding small circles with radii $r_1 = R/7$ at the distance $R_1 = R - r - r_1$. (b) The dispersion diagram of the HPC for the parameters $R = a/3$ and $r = R/3$. The inset is the Brillouin zone of the lattice. The electric field profiles of the degenerated bands are shown as inset. (c) The electric field profiles of the two-folded degenerate bands for the modified honeycomb PCs type A and B.

circles from the center of the unit cell are adjusted to $R_3 = R - r - r_3$. For example, for radius $r_3 = R$, the orbitals $d$ and $s$ become degenerate and for radii $r_1 = R/2$ and $r_5 = R/3$, the orbital $s$ embeded between orbitals $p$ and $d$. For radii $r_1 = R/4$, $r_2 = R/5$ and $r_1 = R/6$, there is
clear gap between orbitals $p$ and $d$ and for radii $r_1 = R/8$, $r_1 = R/9$, and $r_1 = R/10$, there is small band gap.
field are main characteristic of the topological conversion. They occur via breaking the
symmetry inversion of the honeycomb PC and make a new idea for studying the one-way
propagation of the light along the interface of the trivial and topological PCs. To this aim, we
propose a left-right geometry with the supercell consists of $13 \times 2$ unit cells of both modified
honeycomb PCs to create a photonic topological insulator (PTI) with trivial and topological
PCs acting as the bulk and edge states, respectively. Figure 4 indicates the band diagram of
this configuration with oblique boundary between the two kinds of PCs. The helical edge states
appear at the common band gap frequencies of these PCs. Figures 4(b-d) show that two edge
states A with the spin-up and B with the spin-down, have the same electric field distributions
but possessing opposite Poynting vectors.

Fig. 4. (a) The band diagram of the left-right configuration of both modified honeycomb PCs with oblique boundary,
its supercell composed of $13 \times 2$ unit cells is in the inset, electric field outlines of the (b) spin-up, point A and (c)
spin-down, point B, Poynting vectors of the (d) spin-up, point A and (e) spin-down, point B.

Topological edge states can propagate against the defects, disorders and even local cavities
without experiencing much scattering loss. To confirm this feature, we prepare a Z-shape
interface and cavity configuration as seen in the Figs. 5(b)-5(c). Here the cavity was made by
removing one unit cell of PCs, types A and B, at the interface.

In addition, the scattering-immune propagation is shown in two types of plasmonic defects
[44]. The first defect is designed when one of the dielectric circles with $r = R = 0.1111 \mu m$
is replaced with the silver (Ag) one as seen in Fig. 5(d) and in the second one, an Aluminum
rectangular blocks with the length $1.5 \mu m$ and width $50 nm$ is embedded at $9 \mu m$ from the
Fig. 5. (a) Unidirectional propagation of the circularly polarized source along the interface of two types of HPCs, (b)-(e) helical edge state distributions at the interface, Z shape, against the cavity, Ag circle and Al blocks respectively. Yellow star indicates the point like source at the interface. (f) the intensity versus frequency comparison between the cases without defect, cavity, Al and Ag defects.
source (Fig. 5(e)). We applied Palik and Johnson and Christy models for Ag and Al metals, respectively. Figure 5(f) shows the comparison between the intensities versus frequency for various types of cavity and defects. The decreasing of the maximum intensity is seen clearly in propagating of light through the interfaces involving the Ag circle, Al blocks or cavity. In the following, we study the tunable intensity of the various defects. To illustrate, consider the all-dielectric topological insulator shown in Fig. 5(a). We modify radius of one of the circles (belongs to the type A) embedded at the $x = 11.6667 \mu m, y = -0.144379$ from the source. As seen from Fig. 6(a), we can change the intensity by increasing the radius of the circle. The transmitted intensity can be enhanced via altering defect parameters. The maximum intensity is related to the $r = 0.18 \mu m$ and increased up to the 130% of the reference intensity at $r = 0.1111 \mu m$.

In the following, we change the geometry of the recognized circle ($r = 0.1111 \mu m$ at the $x = 11.6667 \mu m, y = -0.144379$ from the source). So its position may be perturbed in two ways: first, by moving the circle along the $+y$ direction and second one is along the vector $\vec{a}_2$ (60° degree rotation around the x direction). As seen in Figs. 6(b)-(c), the intensity will decrease to 82.08% and 64.64%, respectively.

For the last tuning of the intensity in terms of defects, consider the Al blocks shown in Fig. 5(e) with the length $0.2 \mu m$ and the width in the range 1-25 nm. Similar to the previous case, the intensity decrease by increasing the width of Al rectangular shape. Figures 7(a)-(c) show the intensity versus frequency and normalized decreased intensity versus radius, respectively.

Finally, we design two kinds of topological resonators in a rhombic shape. As seen in Fig. 8(a), the emitted light from the source channel, localizes around the resonator and cannot continue towards the right side of the interface. However, for the other modified resonator configuration, the light couples with the rhombic resonator, then propagates to the left side of the second interface as shown in Fig. 8(b). Besides due to the topological robustness, there is not seen any back-scattering at the interfaces and the sharp corners of the rhombic do not deteriorate efficient light transmission towards the drop channel.

The Intensity of electric field versus frequency of the positions 1, 2 and 3 are shown in Figs. 8(c)-(d). As expected, the intensity of the positions 1 and 2 are unnoticeable in comparison with the intensity at the position 3. A little propagation observed at the right side (position 1), is due to the unidirectional propagation of the topological edge states at the interface. The frequency of the maximum intensity belongs the topological bandgap frequencies.

4. Conclusions

The topological behavior of the perturbed honeycomb PC has been studied by adding three small dielectric rods in the neighborhood of the main rods in each unit cell of the structure. The double Dirac cone which emerges through the four-folded degeneracies at the $\Gamma$ point has been modified and the two-folded degeneracies of the bands are appeared. Closing and opening of the Dirac points which is the main characteristic of the topological transitions provide the useful platform for studying the intriguing phenomena of light like back-scattering immune transmission along the different topological interfaces. We have shown efficient and various light propagation examples as well as tunable field intensity through disorders, defects and cavities.
Fig. 6. The tunable intensity versus frequency for the case of all dielectric defect. (a) by increasing the radius, the intensity will be increased up to 130.62% of the reference number. (b)-(c) by increasing the distance of the defected circle along the directions, \(y\) and vector \(\hat{a}_y\), the intensity will decrease to 82.08% and 64.64% respectively.

Fig. 7. Modification of the intensity against the Al rectangular blocks defect. (a) decreasing the intensity by increasing the width of Al-blocks. (b) decreasing the normalized intensity.
Fig. 8. The localized electric field intensity in the rhombic resonator and robustness of the topological edge states in the sharp corners of the rhombic. The dashed orange line shows the interface of the two types of PCs, A and B. (a) with one topological interface and (b) with two interfaces. (c) the intensity versus frequency at the positions 1, 2 and 3.

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Disclosures

The authors declare no conflicts of interest.

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