Is the Universe Homogeneous on Large Scales?

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1 Two Competing Visions

1.1 Introduction

This morning’s debate is focused on the question of the large-scale homogeneity of the Universe. I shall present the affirmative position, that there is overwhelming evidence for large scale homogeneity on scales in excess of approximately $50h^{-1}$ Mpc, with a fractal distribution of matter on smaller scales. My worthy opponent, Dr. Luciano Pietronero, will present the counter-argument, that the fractal distribution observed on smaller scales continues to the largest observed scales, and that there is no evidence for homogeneity on any scale.

This is an important question, since the Friedmann-Robertson-Walker metric presumes large scale homogeneity and isotropy for the Universe. This is the simplest cosmological model, and it is a fair question to ask the degree to which it is supported by the observational evidence. We know that the galaxy distribution is far from homogeneous on small scales, and large-scale structure in the form of long filaments and chains extends to lengths in excess of $100h^{-1}$ Mpc. Redshift surveys often show structures nearly as large as the entire survey size; how confident can we be that homogeneity is a valid concept for the large-scale universe?
1.2 The Mean Density of Galaxies

The question of large scale homogeneity revolves around the issue of whether the mean density of the Universe, $\bar{n}$, is a well defined concept. In the standard cosmological model, $\bar{n}$ is perfectly well defined, but in the fractal model of the Universe, $\bar{n}$ has vanishing global value and has relevance only within a given survey volume. In the conventional picture of large scale structure, the $rms$ value of the number density of galaxies as observed at radius $r$ from the vantage point of any randomly chosen galaxy, $n(r)$, is given from the usual definition of the two-point galaxy correlation, $\xi(r)$, by $n(r)_\text{rms} = \bar{n} (1 + \xi(r))$. The measured $\xi(r)$ is well characterized by a power law, $\xi(r) \approx (r/r_0)^{-\gamma}$, ($r_0 \approx 5 h^{-1} \text{ Mpc}$, $\gamma = 1.8$). Alternatively, this can be expressed in terms of the power spectrum of density fluctuations, $P(k) \propto k^{-1.2}$, $$(\delta M/M)_\text{rms} \propto (k^3 P(k))^{1/2} = 1$$ at some well defined scale. In the conventional picture, there exists a distinct transition between “small” and “large” amplitude scales, depending on whether $\xi(r) < 1$ or $\xi(r) > 1$. On large scales, where $\xi(r) \to 0$, one has simply $n(r) \equiv M(r)/V(r) \to \text{constant}$. On scales where the $rms$ density behaves as a power law, the galaxy distribution has fractal properties, but on larger scales the galaxy distribution approaches uniformity.

In contrast, within the fractal model advocated by Pietronero and colleagues [2] [3] [4], there is no constant term in the $rms$ density, the mass within a radius $r$ scales as $M(r) \propto r^D$, $D = 3 - \gamma \approx 1.2 - 1.5$, and $n(r)_\text{rms} = M(r)/V(r) \propto r^{D-3} \to 0$ as $r \to \infty$. In order to preserve the cosmological principle that we are not at the center of the Universe or at any point of special symmetry, such an $rms$ density behavior is possible only if the galaxy distribution viewed from a typical observer is extremely anisotropic, with 100% density fluctuations at any distance $r$ from a typical observer. Such a universe does not approach homogeneity on large scales, and large voids, a constant fraction of the survey radius, would be expected within any survey. Because one must always estimate the mean density from within the same volume used to estimate the correlation function, in the fractal picture one expects the correlation length to scale linearly as the survey size $r$, with $r_0 \approx 0.4 r$. The average density has no meaning in this universe; on a global scale the Universe is empty and its average density approaches zero. There is no transition between linear and nonlinear fluctuations: $$(\delta M/M)_\text{rms} \approx 1$$ on all scales because of the trend of mean density with sample volume.

Such a model is a radical departure from the standard model of a homogeneous large-scale Universe. Does the observational data support such
a conjecture, or can it be decisively ruled out? I shall argue that we have an abundance of observational arguments in support of the standard picture which are incompatible with the fractal model extended to arbitrarily large scale. (The theoretical underpinning of such a radical model is another matter altogether.) The question of real interest is to determine the outer scale of the fractal scaling behavior, and I shall argue that there exist firm bounds on this scale.

2 Evidence for Homogeneity (2-d tests)

The largest data sets available are two-dimensional in nature, and most of these have been well known for decades. The remarkable isotropy of the CMBR, X-ray counts, radio source counts, and the γ-ray bursts all argue that we are either at the center of the Universe or that on the largest scales the Universe is homogeneous. The arguments given by Weinberg [5] are still valid today. We know the CMBR comes to us from redshifts $Z \gg 1$, while the discrete radio sources are distributed to $Z > 1$, with the X-ray background presumably arising from discrete sources at $Z < 3$. If the matter distribution is a pure fractal in space, how does it become so smooth in projection on the sky? The proponents of the fractal universe argue that projection of the 3-d fractals is complicated and that all structure can be lost. It is well known that the projection will dilute the information, but it seems evident that not all the information would be lost and that isotropy at the remarkable levels observed, e.g. 1% precision, is not possible unless the outer scale of the fractal structure is considerably smaller than the redshift limit of the databases. Peebles [6] shows that the large-scale isotropy of the X-ray background radiation constrains the fractal dimension $D$ to be $|3-D| \leq 0.001$ on large scales, which would seem rather definitive. The proponents of the fractal universe have been challenged to produce an example of a fractal that projects to a uniform sky distribution, but to date they have failed to do so.

Similarly, the observed counts of galaxies versus flux $f$, for intermediate magnitudes in the range $14 < m < 18$, scales as $N(> f) \propto f^{-3/2}$, just as expected in a homogeneous, Euclidean Universe. For fainter magnitudes, we observe $N(> f) \propto f^{-1}$, which suggests a combination of evolutionary and expansion effects, while the isotropy of the faint number counts is inconsistent with fractal behavior. Peebles [6][7] presents considerable detail on the constraints these arguments set on the fractal dimension $D$. In the fractal
model, since $M(r) \propto r^{3-\gamma}$, we expect $N(> f) \propto f^{(\gamma-3)/2} \propto f^{-0.6}$, which is very far from the observations. To fit the faint counts, one must adjust $\gamma$, but this is inconsistent with the isotropy of the faint counts and with the fractal dimension found on smaller scales.

The angular correlation function $w(\theta)$ of the large two-dimensional catalogs of galaxies such as the APM catalog obeys the Limber scaling law to a remarkable degree 6. Again this relationship is fully consistent with homogeneity on large scale. In a fractal universe, the angular correlation length of a galaxy catalog should be a constant, large fraction of the angular extent of the survey, and this scale should be independent of the flux. Again, this is exactly contrary to the observations. The proponents of the fractal model argue that the projection of the three dimensional structure to the observed $w(\theta)$ has erased all the information, but years of experience with deconvolution demonstrate that recovery of $\xi(r)$ from $w(\theta)$ is reliable and conceptually straightforward. Recent explicit constructions of fractal models demonstrate this point very clearly– a three dimensional fractal leads to a very anisotropic two-dimensional galaxy distribution.

3 Evidence for Homogeneity (3-d tests)

The past decade has witnessed the explosive growth of redshift surveys of galaxies, from which one can estimate three-dimensional statistics directly. To date, these surveys are necessarily much smaller than the very large databases such as the APM catalog, and they correspondingly show more fluctuations from sample to sample. It is important that one beware that many of the early redshift surveys were too small to represent a fair statistical sample of the Universe (as expected in the orthodox school). Furthermore, many of the existing surveys are based on catalogs with irregular edges, and /or with known non-uniformity in their sample selection. These catalogs are usually flux limited, and some of them are far from complete at any flux level! For example, the CfA2 survey is based on Zwicky magnitudes, where systematic errors from one section of the sky to another are suspected, the Perseus-Pisces redshift survey contains a region of substantial extinction from our galaxy, the selection of Abell clusters of galaxies has known selection effects that depend on the zenith angle of the photographic plates from which the clusters are selected. But at least these catalogs are approximately complete. Worst of all is to use a database such as LEDA or ZCAT which
are repositories of redshifts collected from the literature. There is no way to make a uniform correction for the completeness of these catalogs, since they are constructed in an uncontrolled manner and are most definitely not intended for statistical analysis.

The trouble this can lead to is exhibited in a power spectral analysis of the LEDA sample [11]. A comparison of this measure of $P(k)$ with that obtained from the complete CfA2 survey is shown in the figures which Professor Pietronero prepared for this debate. The LEDA-derived $P(k)$ is an order of magnitude larger than that derived from CfA2 and does not smoothly extrapolate to the Sachs-Wolfe inferred power spectrum on the larger scales measured by COBE (while the CfA2 spectrum can be smoothly connected to the COBE results). Proponents of the fractal picture would cite this instability of $P(k)$ as a demonstration of fractal scaling, but I would counter that it is the LEDA database which is a fractal, not the Universe.

Redshift catalogs which are appropriate for statistical analysis are those for which the selection is well known and well defined. For the purposes of today’s debate, the most suitable redshift catalogs presently available are the survey of a bright subset of the APM galaxies [12], the IRAS catalogs [13], and the recently completed LCRS survey [14].

The notion that the galaxy distribution is a fractal arose from analysis of the two point correlation function $\xi(r)$ in early redshift surveys. Many authors have commented that large coherent structures often appear to be as large as they could be within the sample volume surveyed (e.g. the CfA stickman, [1]). There is widespread agreement that the galaxy distribution approximates a fractal over a considerable range of scales [15]. The early surveys displayed increasing correlation length with increasing sample volume [16], and all surveys continue to show stronger correlation amplitudes for rich clusters, different correlation properties for different types of galaxies [17], and weak evidence for increased correlation strength for more luminous galaxies [18]. In the standard model, these properties are explained by the “bias” of rare events, luminosity bias, and environmental effects.

The defense of the standard interpretation of a homogeneous large-scale universe has never been based on examination of the stability of $\xi(r)$ or on examination of redshift survey maps; its defense up until the recent data has rested on the isotropy of the two-dimensional catalogs and on the stability of the mean density as a function of redshift and direction, $n(z, \omega)$. Local surveys in opposite hemispheres have very similar $n(z)$ curves, and even in the original CfA1 survey, the mean galaxy density derived in the Northern
galactic hemisphere agreed with that in the Southern hemisphere to within a factor of 1.3 \cite{19}. 

In the fractal interpretation, this “bias” explanation for the behavior of $\xi(r)$ is considered a cop-out. Instead, one argues that $n(r)$ is poorly defined and the increased $\xi(r)$ for larger volumes is simply the result of a decreasing mean density, as expected in the fractal model. The problems with this explanation are two-fold. In the fractal model the correlation length $r_0$ should increase linearly with sample depth, but even in the shallowest slices of the CfA1 survey $r_0$ increases only as the square-root of the sample depth \cite{16}. Furthermore, it has been clearly demonstrated that different populations of galaxies drawn from the same volume do exhibit differing correlation properties \cite{18}, so therefore biasing effects must exist.

### 4 Recent Results (using 3-d data)

#### 4.1 IRAS redshift surveys

Pietronero for years has argued that the most suitable volume for statistical analysis is a sphere, since it most efficiently contains the largest fraction of galaxies within the most compact surface. This ideal has now been closely approximated by the IRAS selected galaxy samples, \cite{13} which cover 88\% of the sky to a depth of roughly $180h^{-1}$ Mpc, although the diluteness is quite extreme beyond $80h^{-1}$ Mpc. Full sky maps of the observed galaxy distribution of the 1.2 Jy IRAS survey are shown in Figure 1. Each of these plots are independent slices of redshift with nearly constant aspect ratio, such that $\Delta z/z \approx 1$.

In an unbounded fractal universe, the galaxy distribution must be extremely anisotropic if we are not at a special location. The most elementary aspect of a fractal is that it should be approximately scale invariant, which implies that all four of these figures of nearly constant aspect ratio should appear statistically very similar to each other. But the reality is very different.

Within the IRAS survey, or any optical survey, the galaxy distribution in the nearest shell, $cz < 1600$ km/s, is characterised by 100\% fluctuations from one hemisphere to the other. This is the expected behavior of a fractal distribution. But the more distant shells are progressively more and more isotropic. Here is a clear demonstration of the fractal behavior on small scales.
giving way to homogeneity on large scales, and it is completely contrary to the scale-invariant fractal picture, in which each shell should exhibit similar 100% anisotropies. Plots of \( n(z) \) for one hemisphere compared to one another show complete consistency on large scales, indicating that there is no ambiguity in the definition of the mean density. Analysis of \( \xi(r) \) for four separate volume limited subsets of the 1.2 Jy IRAS survey yields a correlation length \( r_0 \approx 4 h^{-1} \text{Mpc} \) that does not change as the volume limiting radius is increased from 60\( h^{-1} \text{Mpc} \) up to 120\( h^{-1} \text{Mpc} \) \cite{20}. The disagreement with the fractal predicted amplitude \( r_0 \) ranges from a factor of 6 to 12.

Professor Pietronero, in this conference, stated that he agrees that the IRAS survey does not exhibit fractal behavior, and he ascribes this to the diluteness of the sample. But the problem with the fractal model is that the maps shown in Figure 1 are too smooth; dilute sampling could have increased the fluctuations, but how could it have transformed an anisotropic map into a smooth, isotropic map? The maps show that the outer scale of the fractal structure must be at some radius within the second shell, which is much less anisotropic than the first shell. Thus, the conjecture that the Universe is a fractal to the largest observed scales is false, and the diluteness of the IRAS sample cannot change this conclusion.

4.2 Las Campanas Redshift Survey (LCRS)

The LCRS \cite{14} is the first of the next generation of large, deep redshift surveys. It contains over 25,000 galaxies selected in six strips of 80 by 1.5 degrees, to a depth \( cz \approx 45,000 \text{km/s} \). Three of these strips are nearly adjacent in the North, and three are nearly adjacent in the South.

Recently published plots of the LCRS galaxy distribution show an “end of greatness”. There are many structures and voids as large as seen in the shallower surveys such as CfA2, but there is an absence of larger scale structure. It appears as though the survey has crossed the peak in the power spectrum of fluctuations \( P(k) \). The six slices of the LCRS all seem statistically very similar to each, and the observed distributions of \( n(z) \) in the different directions are all the same on large scale, as expected in the standard picture but quite contrary to the idea that the fractal scale extends to the full survey depth. If the fractal did extend to the full LCRS depth, the separate slices should be very different from each other. Thus LCRS, consistent with IRAS, strongly demonstrates the approach to large scale homogeneity.
4.3 QSO Absorption Lines

The spectra of all quasars at redshifts \( z > 1.8 \) contain abundant Ly-\( \alpha \) absorption lines due to intervening clouds of neutral hydrogen. In a fractal universe with no outer scale, most of the quasars would have large voids in front of them, and so exhibit no Ly-\( \alpha \) absorption clouds. A cursory glance at high quality spectra of QSO’s (e.g. as taken by HIRES on Keck [21]) shows that these clouds are ubiquitous, and that the universe is statistically uniform in all directions probed. The interval probed by the clouds is roughly \( 1.8 < z < 4 \), a comoving scale of approximately \( 0.2c/H_0 \).

There are no large holes in the distribution of the Ly-\( \alpha \) clouds, and in fact they are so uniformly distributed that it is very difficult to to measure any spatial correlations in the clouds at all [22]. The Ly-\( \alpha \) clouds appear to be very nearly uniformly distributed in space, and are the most homogeneous tracers yet discovered.

All lines of sight are observed to be statistically equivalent, as expected in a universe homogeneous on large scale. Again, this is completely contrary to the expectations of an unbounded fractal model. It is quite clear that the outer scale of the fractal distribution of matter must be orders of magnitude less than the \( 600h^{-1}\text{Mpc} \) probed by the sight line to the distant QSO’s.

5 Summary

As I have briefly reviewed above, there exist numerous arguments which demonstrate that the outer scale of the fractal distribution is well within the scale of observed volumes. From recent redshift space maps, we detect a characteristic size of voids in the galaxy distribution consistent with a peak in the power spectrum of fluctuations. As emphasized by Kirshner, the new LCRS survey may be seeing the “end of greatness” of large-scale structure. Future surveys such as the AAT redshift survey and the Sloan digital sky survey will lead to much better constraints on the turnover of the large-scale power spectrum and on the amplitude of the large-scale fluctuations. Since I believe that our current surveys are approaching fair sample volumes, I fully expect the correlation amplitudes of the future, massive surveys to be consistent with current measurements.

The measured two-point galaxy correlation function \( \xi(r) \) is a power law over three decades of scale and approximates fractal behavior from scales
of $0.01h^{-1} < r < 10h^{-1}$ Mpc, but on scales larger than $\approx 20h^{-1}$ Mpc, the fractal structure terminates, the $rms$ fluctuation amplitude falls below unity, and the Universe approaches homogeneity, as necessary to make sense of a FRW universe. There clearly exists an outer scale of the fractal behavior, and this outer scale must grow with time (unless $\Omega_0 \ll 1$). The observed galaxy distribution, being a real physical system rather than a mathematical idealization, is a beautiful example of a limited-scale fractal joined onto sensible, dynamically evolving outer boundary conditions.

As testimony to our faith in our respective debating positions, Professor Pietronero and I have agreed to wager a case of the best wine from Italy against the best Californian wine on the following proposition: that the correlation length $r_0$ as ultimately measured from optically selected galaxies in the Sloan digital sky survey will be larger (according to LP) or smaller (according to MD) than $r_0 - CFA2 \left( L_{Sloan}/L_{CFA2} \right)^{1/2}$, where $L_{survey}$ is the radius of the largest inscribed sphere in a given survey. (This splits the harmonic difference of the fractal versus standard prediction.) Neil Turok has agreed to arbitrate this wager. Side bets are welcome.

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Figure 1: Whole sky projection in galactic coordinates of the IRAS 1.2 Jy redshift survey. The different plots represent the galaxy distribution as directly observed in different windows of observed redshift in the Local Group frame. Note the strong anisotropy in the nearby shell, progressively diminishing in the distant shells.