Detecting Thermal Cloaks via Transient Effects

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Recent research on the development of a thermal cloak has concentrated on engineering an inhomogeneous thermal conductivity and an approximate, homogeneous volumetric heat capacity. While the perfect cloak of inhomogeneous $\kappa$ and inhomogeneous $\rho c_p$ is known to be exact (no signals scattering and only mean values penetrating to the cloak’s interior), the sensitivity of diffusive cloaks to defects and approximations has not been analyzed. We analytically demonstrate that these approximate cloaks are detectable. Although they work as perfect cloaks in the steady-state, their transient (time-dependent) response is imperfect and a small amount of heat is scattered. This is sufficient to determine the presence of a cloak and any heat source it contains, but the material composition hidden within the cloak is not detectable in practice. To demonstrate the feasibility of this technique, we constructed a cloak with similar approximation and directly detected its presence using these transient temperature deviations outside the cloak. Due to limitations in the range of experimentally accessible volumetric specific heats, our detection scheme should allow us to find any realizable cloak, assuming a sufficiently large temperature difference.

The ability to render an object invisible has been a goal since the days of mythology and the Ring of Gyges. It is only recently that invisibility became a plausible subject of inquiry thanks to theoretical advances in electromagnetism1–6. Such cloaks fulfilled the two basic elements of invisibility: anything hidden inside was isolated as if hidden by a perfect insulator (mean values still transmittable) and the perfect insulator had no scattering. This first requirement typically entailed singular, anisotropic materials, while the second required inhomogeneity. These extreme material requirements turned attention to reduced cloaks which merely approximate perfect cloaking7–10 or conditions where these constraints are relaxed11–17. This in turn led to the study the detectability of these cloaks18–22. Concurrently, cloaking was extended to other classes of electromagnetic phenomena23–29, wave equations30–37. and diffusion equations38–51 (refs 52, 53 provide a review of these last categories).

The diffusive cloaks found greatest success with the heat equation:

$$\rho c_p \partial_t T = \nabla \cdot (\kappa \nabla T)$$  \hspace{1cm} (1)

(where $\rho$ is the density, $c_p$ the specific heat capacity, $T$ temperature, and $\kappa$ the thermal conductivity), so we shall confine our attention to thermal cloaks and then generalize our results to other diffusion effects. Because the steady-state temperature is independent of the volumetric heat capacity $\rho c_p$, cloaking has focused on engineering $\kappa$ with $\rho c_p$ constant. This has led to the widespread acceptance of three related classes of diffusive cloaks. First, there are strictly steady-state cloaks that ignored $\rho c_p$ through the assumption that they would only be used in a steady state. While the utility of such a device is limited (truly steady-state conditions are unlikely to be maintained outside of specialized circumstances), they unquestionably work as expected. Their success, however, led to a second class of cloaks, which sought to extend these steady-state designs to time-varying and transient temperatures. For example Schittny et al.49 adopt a theoretical approach from ref. 42 (which assumed constant $\rho c_p$ as an explicit approximation) while still claiming to achieve transient thermal cloaking. Indeed, they even
found experimentally that their specific heat capacity varied between their materials but sought to correct for it to maintain a homogeneous $\rho c_p$. In addition, Ma \textit{et al.} simplify their design by assuming that $\rho c_p$ commutes with spatial derivatives, i.e. that homogeneity of $\rho c_p$ is obtained to good approximation. The third class of diffusive cloaks were chemical diffusion cloaks, which are governed by

$$\partial_t n = \nabla \cdot (D_n \nabla n)$$

for chemical concentration $n$ and diffusion constant $D_n$. There, researchers again assumed that $\rho c_p$ was negligible based upon the performance of steady-state thermal cloaks, and therefore that steady-state thermal cloak designs could be extended to transient effects in chemical diffusion (even though this analogy maps $\rho c_p$ to unity everywhere). Thus, although the steady-state cloak was developed for the limited frequency range of $\omega = 0$, there now exists a broad class of cloaks (which we refer to collectively as steady-state cloaks due to their common origin) which assume that careful engineering of $\kappa$ is sufficient to create a perfect cloak in any regime. Note that an alternative approach to thermal cloaking was also presented in ref. 54, which used scattering cancellation theory. The theoretical justification for this type of cloaking is thus outside of the analysis we develop here, although it is worth noting that they explicitly engineer $\rho c_p$ and $\kappa$. However, as a complementary media method, it only cloaks specific objects (or at most a class of objects all having the same effective parameters), whereas transformation media cloaks work for arbitrary objects. In addition, the technique works by cancelling out finite terms in the multipole expansion of the cloaked object’s scattering cross section, and so remains detectable using higher-order terms in the expansion (e.g. the quadrupole term in their example).

In this paper, we show a homogeneous $\rho c_p$ results in a detectable, transient signal. Under changing boundary conditions, a thermal cloak will flicker and become visible, although it will help to obscure anything hidden inside it. The implications of this imperfection can be seen by considering a faulty cloak with one observer hidden inside and another searching outside (see Fig. 1). The searcher can send out signals and detect the diffuse “scattering” that is reflected back. Furthermore, they can search for signals emanating from the cloak’s interior thereby determining the material composition or temperature distribution hidden inside. Conversely, the observer hiding in the cloak can detect incoming signals to observe any searchers and eavesdrop on the outside. Moreover, by sending out their own signals and detecting them, they can confirm that the cloak is present and functioning.

**Results**

We begin by considering the analytic solution to eq. 1 for the cylindrical perfect cloak (PC) (assuming no $z$ dependence). For a homogeneous medium ($\kappa = \kappa_0$, $\rho c_p = \rho_0 c_p$), source-free medium the solution can be expressed as a linear combination of the fundamental solutions

$$T_j(r, \theta, \omega) = R_l(\sqrt{i} k_0 r) e^{i b r}$$

where $l$ is the rotational symmetry eigenvalue, $R_l$ is a modified Bessel function of the first or second kind ($I_l$ and $K_l$ respectively), $k_0 = \sqrt{\omega \rho c_p / \kappa_0}$ and $\omega$ is a frequency $> 0$ (the steady state of $\omega = 0$ is discussed in the supplement). A PC of interior radius $a$ and exterior radius $b$ (Fig. 1) is constructed from...
is given in the supplement (along with the full analytic solution). The BC is particularly interesting to consider as it is a SSC that was derived directly from the cloak that has radius \(a\), as in Fig. 1.

where \(\kappa_\delta \frac{r-a}{r} = \kappa_0 \frac{r}{r-a}\), \(\kappa_\delta \frac{r-a}{r} = \kappa_0 \frac{r}{r-a}\), and \(\rho_c \frac{r-a}{b-a}\) is the solution becomes

\[
\begin{align*}
T \delta (r, \theta, \omega) &= \mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r} + \frac{1}{\kappa} \mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r} \\
&= \mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r} + \frac{1}{\kappa} \mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r}
\end{align*}
\]

The results of Fig. 3a present a finite element, (i.e. discretized) approximation of the exact inhomogeneity curve. Hence, for a direct comparison, we move from the SSC model used in an impedance mismatched SSC). Therefore, for a steady-state cloak (SSC) \(\kappa = \frac{1}{a}\) is the same as eq. 3 but \(\rho_c \frac{b}{a} = \frac{b}{a}\). Whereas, for a steady-state cloak (SSC) \(\kappa = \frac{1}{a}\), the lowest order perturbation is

\[
\begin{align*}
\mathcal{F}(\mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r} + \frac{1}{\kappa} \mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r} \\
&= \mathcal{F}(\mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r} + \frac{1}{\kappa} \mathcal{F}(\kappa \frac{r-a}{r}) e^{i\theta / r})
\end{align*}
\]

Although the SSC model uses a realistic \(\rho_c \frac{b}{a}\), it still contains an idealized \(\kappa\). The exact inhomogeneity and anisotropy profiles of eq. 3 are not physically realizable. Thus, experimental verification of our predictions for the SSC is impeded by its use of an idealized \(\kappa\). In real thermal cloaks, rings of discretized, constant \(\kappa\) are used in an approximation of this ideal inhomogeneity curve. Hence, for a direct comparison, we move from the SSC model that we have developed to the case of a bilayer cloak (BC) \(\eta \equiv b/(b-a)\), and further experimental data for the BC are in the supplement. The BC is particularly interesting to consider as it is a SSC that was derived directly from

| Cloak | \(\kappa_\delta \frac{r-a}{r}\) | \(\kappa_\delta \frac{r-a}{r}\) | \(\rho_c \frac{r-a}{b-a}\) |
|-------|-----------------|-----------------|-----------------|
| PC    | \(\frac{r-a}{r}\) | \(\frac{r-a}{r}\) | \(\frac{b}{a}\) |
| SSC (M) | \(\frac{r-a}{r}\) | \(\frac{r-a}{r}\) | \(\frac{b}{a}\) |
| SSC (Mis) | \(\frac{r-a}{r}\) | \(\frac{r-a}{r}\) | \(\frac{b}{a}\) |
| BC \([a, b]\) | \(\kappa_\delta \frac{r-a}{r}\) | \(\kappa_\delta \frac{r-a}{r}\) | \(\rho_c \frac{r-a}{b-a}\) |
| BC \([a, b]\) | \(\kappa_\delta \frac{r-a}{r}\) | \(\kappa_\delta \frac{r-a}{r}\) | \(\rho_c \frac{r-a}{b-a}\) |

Table 1. Equations for cloaks considered in this paper. Perfect cloak (PC), impedance matched steady-state cloak (SSC (M)), impedance mismatched SSC (SSC (Mis)) \(\eta \equiv b/(b-a)\), and bilayer cloak (BC). Inner layer of the cloak has radius \(a\), outer layer radius \(b\), as in Fig. 1.

\[
\begin{align*}
\kappa_\delta &= \kappa_0 \frac{r-a}{r}, \quad \kappa_\delta = \kappa_0 \frac{r}{r-a}, \\
\rho_c \frac{b}{a} &= \left(\frac{b}{a}\right)^2
\end{align*}
\]
Figure 2. Simulated temperature snapshots for mismatched SSC ($\eta = b/(b - a)$). Columns correspond to $2.08\tau_D/100$, $2.08\tau_D/10$, and $2.08\tau_D$ respectively. Rows correspond to the homogeneous case (no cloak), SSC, and $T^{(SSC)} - T^{(H)}$. Black circles denote the location of the cloak (for reference in the homogeneous case), colored domains are isotherms, and grey lines are constant separation isotherms.

Figure 3. Temperature deviation $\delta T/\Delta T$ for representative points outside the cloak as a function of time. Black (a), blue (b), and red (c) curves correspond to the PC, impedance matched SSC, and impedance mismatched SSC. Line styles correspond to individual points, as shown in the inset.
Laplace’s equation rather than a coordinate transformation (similar to\(^5\)). In Fig. 4 we plot the normalized temperature deviation for the simulated BC and our experimental realization. This shows a good agreement, with a slight discrepancy near the boundaries of the system. This is due to a slight difference in the experimental temperature gradients applied to the BC and homogeneous cases.

Finally, we turn to the question of detecting objects hidden inside a cloak. For the PC and the SSC \( \kappa \cdot \nabla T = 0 \) at the boundary \((\kappa_c = 0)\), so there should be no heat transferred and therefore no discernable signal (although, as in ref. 18, this is extremely sensitive to deviations of \( \kappa_c \) from 0 and as in refs 23, 24 and 37 even a PC will transmit the mean value of \( T \) at the boundary). However, taking the BC and changing the material hidden inside will effect the temperature distribution. An exterior temperature profile like those considered above must pass through the cloak twice (entering and exiting), so the cloak’s ability to suppress detection is stronger here than in the case of hiding the cloak. In particular, simulations of the BC with different materials inside differ by less than 0.1% (see Fig. 5). Assuming a thermometer of sensitivity of 0.2 K, a gradient of over 200 K would be necessary for the determination of the material hidden within the cloak (whereas merely detecting the presence of the BC requires a temperature difference of 3.64 K).

Figure 4. Comparison of simulations and experimental for the BC. Columns correspond to 1.14\( \tau_D \)/100, 1.14\( \tau_D \)/10, and 1.14\( \tau_D \) respectively. Rows correspond to \( \delta T \) for the simulation and experiment respectively.

Figure 5. Deviation of temperature profiles with changing cloaked object composition \( (T^{\text{cloak+object}} - T^{\text{cloak}})/\Delta T \) for representative points outside the cloak as a function of time. Color corresponds to different points (see inset for key).
On the other hand, one could try to detect the temperature distribution hidden by the cloak, rather than the material. In this case, heat initially confined to the cloak would diffuse out and only pass through the cloak once. Simulations in this case show a detectable signal of 1.5% (see Fig. 6, computational details in the supplement), meaning that a thermometer of 0.2 K sensitivity could be used to detect a temperature difference of at least 13.3 K from the background. It is therefore likely that heat sources can be detected through the cloak, i.e. that it acts like an imperfect insulator. Comparing the efficacy of the BC as an insulator to that of a thermal insulator with properties equal to the insulating layer of the BC (essentially, removing one of the layers) indicates that the BC is no better at suppressing this diffusion of heat into the environment. This suggests that realizable cloaks (i.e. those without a perfectly insulating inner boundary) are no better than conventional insulators for maintaining a temperature difference. This is not entirely surprising if we think of the thermal cloak as a composite or inhomogeneous wall, but it reveals a way in which thermal cloaks deviate from our intuition of what cloaking means (indeed, suggestions that thermal cloaks can hide hot spots or the thermal signatures of objects are somewhat commonplace in the community\(^54\), is one such example). Moreover, even a PC\(^53,24\) with a perfectly insulating inner boundary would still not prevent the matching of the average temperature inside and outside of the cloak, so this setup should still eventually equilibrate. For the PC, though, this should take much longer and be a much smaller signal, so the lack of a perfect insulating layer aids experimental detectability of the internal temperature distribution.

Discussion

We have shown that a SSC can be detected by its transient response. Because the distinction between a PC and a SSC is just \(\rho c_p\), the ability to engineer the volumetric heat capacity is necessary to prevent the \(\omega \neq 0\) response from revealing the cloak. However, the narrow range of \(\rho c_p\) in currently available materials makes it extremely difficult to design this inhomogeneity (indeed, even efforts to construct a “transient” thermal cloak have assumed constant \(\rho c_p\)\(^43,45\)). This is particularly true for other classes of diffusion cloaks where the analog of \(\rho c_p\) is necessarily constant everywhere\(^49,50\). It remains an open question, however, if a diffusive cloak (thermal or otherwise) could be designed to make its time-dependent response undetectable in practice even if the response exists in principle.

Methods

Analytic solutions.

Given the heat equation with homogeneous materials
\[ \rho c_p \partial_t T = \nabla \cdot (\kappa \nabla T) \] (6)
in polar coordinates we take the Fourier transform of time and use a separable solution \(T(r, \theta, t) = R(r) e^{i\omega t} e^{il\theta}\) giving
\[ \frac{k e^{i\omega t} \rho c_p R}{\kappa_0} = \frac{1}{r} \frac{d}{dr} (rR') - l^2 R. \] (7)
This is the differential equation for a modified Bessel function (\(I_l(z)\) or \(K_l(z)\)) of \(z = \frac{k e^{i\omega t} \rho c_p}{\kappa_0} r\) for \(\omega = 0\)\(^56\). The time-dependent solution is therefore
\[ T_l^{(tr)}(r, \theta, \omega) = (a_l I_l(z) + b_l K_l(z)) e^{i\omega t + il\theta} \] (8)
For the steady state of \(\omega = 0\) the solutions become the solution to Laplace's equation
\[ T_l^{(SS)}(r, \theta) = (A_l r^l + B_l r^{-l}) e^{i\theta} \] (9)
for \(l = 0\) and
for \(l = 0\). The general solution is therefore \(T(r, \theta, \omega) = \sum_{l=0}^{\infty} T_l^{(SS)} + T_l^{(tr)}\).

For a perfect cloak
\[
\kappa_r = \kappa_0 \frac{r-a}{r}, \quad \kappa_0 = \kappa_0 \frac{r}{r-a},
\]
\[
\rho c_p = \rho c_p \left( \frac{b}{b-a} \right)^2 \frac{r-a}{r}
\]
we can make the coordinate transformation
\[
r' = \frac{b}{b-a} (r-a)
\]
to reduce the solution in the primed coordinates to the homogeneous case.

For a steady-state cloak (\(\kappa\) as for the perfect cloak, \(\rho c_p = \rho c_p \frac{b}{b-a} \eta\)) i.e. evaluating \(\rho c_p\) at \(r = b\) when \(\eta = 1\) no transformation will reproduce a homogeneous solution. Using \(x = \sqrt{\kappa \rho c_p \eta \omega / \kappa_0 (b-a)} (r-a)\) and separation of variables we find
\[
0 = \partial_\lambda f_\lambda(x) R - \frac{l^2}{l^2} x + x + K a R
\]
where \(K = \sqrt{\kappa \rho c_p \eta \omega / \kappa_0 (b-a)}\). This can be solved by the method of Frobenius \(R_l(x) = \sum b_{nl} x^{\pm l}\) with recurrence relation
\[
b_{nl}^\pm = \frac{1}{n(n \pm 2l)} (K a b_{n-1, l}^\pm + b_{n+1, l}^\pm).
\]
This relation is exact, but additional insight can be gained by expanding the solution by powers of \(K a\). For even terms in the series this is
\[
b_{2m,l}^{\pm(0)} = \frac{1}{2m(2m \pm 2l)} b_{2m-2,l}^{\pm(0)} + O((K a)^2)
\]
which is the same as series expansion for \(I_l\) and \(K_l\) respectively. On the other hand, for odd terms it becomes
\[
b_{2m+1,l}^{\pm(0)} = K a \sum_{n=0}^{m} \frac{(2n-1)!! (2n+2l-1)!!}{(2m+1)!! (2m+2l+1)!!} b_{2n,l}^{\pm(0)} + O((K a)^3)
\]
Because \(b_{2m+1,l}^{\pm(0)}\) is completely determined by \(b_{2m,l}^{\pm(0)}\) the odd terms are therefore a function of the modified Bessel functions. Ergo, we term these components \(R_l(x)|\). A similar derivation can be carried out for a spherical cloak where \(l\) becomes half-integer instead of integer.

**Computational and Experimental Methods.** For the PC and SSC we use COMSOL multiphysics to model a rectangular domain of dimensions \(L = 70\) mm by \(L_z = 50\) mm centered around a cloak of dimension \(a = 13\) mm, \(b = 20\) mm. The background medium is \(\kappa_0 = 71.4\) W/m·K, \(\rho_0 = 21000\) kg/m³, and \(c_0 = 1000\) J/kg·K. This gives a diffusivity of \(D = \kappa_0 / \rho_0 c_0 = 3.4 \times 10^{-2}\) m²/s and diffusion timescale \(\tau_D = L^2 / D = 144.12\) s. The initial temperature was 293.15 K with thermal baths at 300 K, and \(T_0 = 293.15\) K giving a \(\Delta T\) of 6.85 K. After confirming that the simulations were invariant under a change of scale we use the natural units of \(x/L, y/L, t/\tau_D, (T - T_0) / \Delta T\).

For the BC, we follow\(^{48}\) as it rectangular domain of dimensions \(L = 45\) mm by \(L_z = 45\) mm centered around a cloak with hidden region of size \(a = 6\) mm, first layer of \(r_2 = 9.5\) mm, and second layer of \(b = 12\) mm. The background medium is \(\kappa_0 = 2.3\) W/m·K, \(\rho_0 = 2000\) kg/m³, and \(c_0 = 1500\) J/kg·K, the outer layer's medium is \(\kappa_1 = 9.8\) W/m·K, \(\rho_1 = 8440\) kg/m³, and \(c_1 = 400\) J/kg·K, the inner layer's medium is \(\kappa_2 = 0.03\) W/m·K, \(\rho_2 = 50\) kg/m³, and \(c_2 = 1300\) J/kg·K, and the interior medium is \(\kappa_3 = 205\) W/m·K, \(\rho_3 = 27000\) kg/m³, and \(c_3 = 900\) J/kg·K. This gives a diffusivity of \(D_0 = \kappa_0 / \rho_0 c_0 = 7.67 \times 10^{-3}\) m²/s and diffusion timescale \(\tau_D = L^2 / D = 2641.8\) s. The initial temperature was 273.15 K with thermal baths at 333.15 K, and \(T_0 = 273.15\) K giving a \(\Delta T\) of 60 K. The experimental tests of the BC were performed with the same setup, following the procedure outlined in ref. 48.

As for the ability of a cloak to insulate a cloaked object and thus disguise the temperature profile, it is helpful to use different boundary and initial conditions. Instead of applying a thermal gradient across the boundaries, the cloaked region is initially set to 60 K above the background (and cloak) at 273.15 K (These values are then rescaled to 1 and 0). Because the thermal baths are at fixed temperature and perfectly absorb heat flux, energy is not conserved in this simulation and so the steady state should have all the heat removed from the cloak.

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Author Contributions
S.R.S. proposed the project and performed the analytic calculations and computational simulations. X.B. performed the experiment. S.R.S. and B.L. analyzed the results and wrote the manuscript. B.L. and X.Z. supervised the project.

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