Goos-Hänchen Displacement and Imbert-Fedorov Displacement on Biaxial Anisotropic Medium Surface

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Abstract. We present both Goos-Hänchen displacement and Imbert-Fedorov displacement on biaxial anisotropic medium surface in this paper. After the total reflection condition of TE wave and TM wave incident from the isotropic medium to biaxial anisotropic medium are derived, based on the steady-state phase method and the energy flux method, we get the expressions of GH and IF displacements. And some unique phenomena are found: total reflection occurs when the incident angle is less than the critical angle, displacement’s tendency changing with the incidence angle is more diverse on biaxial anisotropic mediumsurface.

1. Introduction
The permittivity and permeability of biaxial anisotropic medium are both diagonal tensors with the diagonal elements not equal to each other (Formula 1). Therefore, the electromagnetic wave has different properties when propagates in different directions of this medium.

Scholars have made lots of researches on the propagation of electromagnetic wave in biaxial anisotropic medium. The propagation of plane-wave in biaxial anisotropic medium with single-sheeted hyperboloid dispersion relation is investigated in [1]. Characteristics of surface waves in biaxial anisotropic left-handed materials are discussed in [2]. Negative refraction and cross polarization effects of biaxial anisotropic are summarized in [3]. Application designed using these research also have been proposed [4]. It is worth mentioning that nowadays popular left-handed materials also belong to the category of biaxial anisotropic medium [5].

\[
\begin{bmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{bmatrix}, \quad \begin{bmatrix}
\mu_x & 0 & 0 \\
0 & \mu_y & 0 \\
0 & 0 & \mu_z
\end{bmatrix}
\]

(1)

Based on these studies, this paper focuses on the condition of total reflection and the important physical phenomena under total reflection—Goos-Hänchen (GH) displacement and Imbert-Fedorov (IF) displacement on the surface of biaxial anisotropic medium. It has been found that when totally reflection occurs at the interface, the reflection point will move in the direction parallel to and perpendicular to the incident plane, called GH displacement and IF displacement respectively.
2. Theoretical Analysis

2.1. Total Reflection Condition

Actually, the evanescent wave’s amplitude attenuates exponentially because the vertical interface direction component of the refraction wave vector \( k_x \) becomes an imaginary number. Take the electromagnetic wave refraction and reflection at the interface between two isotropic mediums as an example.

\[
\begin{align*}
\text{(1)} & \quad \sigma_1 \mu_1 > \sigma_2 \mu_2 \\
\text{(2)} & \quad \sigma_1 \mu_1 < \sigma_2 \mu_2
\end{align*}
\]

Figure 1. The k surface when the wave incidents on the interface of two isotropic mediums

Dispersion relation (Formula 2) must be satisfied when electromagnetic waves propagate in the mediums, and phase matching conditions (Formula 3) must be satisfied on the interface.

\[
k^2 = k_x^2 + k_y^2 = \omega^2 \sigma \mu
\]

\[
k_{ix} = k_{ry} = k_{ty}
\]

As show in Figure 2-1, the electromagnetic wave enters medium 2 from medium 1. When the incident angle \( \theta_i \) is greater than the critical angle \( \theta_c \), total reflection occurs, \( k_x \) becomes an imaginary number, evanescent waveforms and produces two kinds of displacement. But in Figure 2-2, \( \sigma_1 \mu_1 < \sigma_2 \mu_2 \), no matter the changing of the incident angle, \( k_x \) is always a real number, so the total reflection cannot occur.

2.2. Total Reflection Condition on the Biaxial Anisotropic Medium

According to Maxwell equations, the dispersion relations of TE wave and TM wave in biaxial anisotropic medium can be obtained [1].

\[
\frac{(k_{ix}^{TE})^2}{\mu_y} + \frac{(k_{ty}^{TE})^2}{\mu_x} = \omega^2 \sigma_z
\]
\[
\frac{(k_{nx}^{TM})^2}{\varepsilon_y} + \frac{(k_{ly}^{TM})^2}{\varepsilon_x} = \omega^2 \mu_z
\]  

(5)

Different from circular in isotropic medium, the k surface in the biaxial anisotropic medium satisfies the conic curve determined by constitutive parameters. (Figure 2).

To make \( k_{tx}^{TE} \) become an imaginary number, need to have Formula 6 established. There are many kinds of constitutive parameters combination can meet this requirement in biaxial anisotropic medium.

| Constitutive parameter | Range of incidence angle |
|------------------------|-------------------------|
| \( \mu_x \mu_y > 01 \) | If \( \varepsilon_x \mu_y < \varepsilon_0 \mu_0 \), when \( \theta_1 > \theta_c \), total reflection occurs, \( \theta_c^{TE} = \arcsin \frac{\varepsilon_x \mu_y}{\varepsilon_0 \mu_0} \) |
| \( \mu_x \mu_y > 01 \) | In any incident angle, total reflection occurs |
| \( \mu_x \mu_y < 01 \) | If \( \varepsilon_x \mu_y > \varepsilon_0 \mu_0 \), in any incident, angle total reflection occurs |

Through numerical calculations, we noticed some unique phenomena. In isotropic mediums, we believe that when electromagnetic waves spread from optically denser medium into the optically thinner medium, case the incident angle is greater than the critical angle will produce total reflection phenomena. But on the surface of biaxial anisotropic medium, the total reflection can occur no matter the changing of the incident angle or occur when the incident angle less than the critical angle, highlight part in Table 1.

2.3. GH Displacement on the Biaxial Anisotropic Medium
According to the dispersion relation, the boundary condition and combined with constitutive parameter of biaxial anisotropic medium, we can get the reflection coefficient \( R \) of TE wave, which reflects the relationship between reflected and incident wave’s amplitude and phase.
Using the steady-state phase method, we get the GH displacement when TE wave total reflected at the biaxial anisotropic medium.

\[
R_{\text{TE}} = \left| R_{\text{TE}} \right| e^{i\phi} = \frac{\mu_x k_x - \mu_y k_y}{\mu_x k_x + \mu_y k_y}
\]  
(6)

Where:

\[
\kappa = \omega \sqrt{\varepsilon_0 \mu_0 \mu_\perp \sin^2 \theta_i / \mu_\parallel - \varepsilon_y \mu_y}
\]  
(7)

2.4. IF Displacement on the Biaxial Anisotropic Medium

The energy flux method is used to derive the Imbert-Fedorov displacement. The energy in the evanescent wave is represented by the Poyntting vector S, shown in the Figure 3.

According to the energy flux method, the IF displacement can be solved as follow:

\[
l_{\text{IF}} = \int_{-\infty}^{+\infty} S_{tz} \cdot dx / S_{rx}
\]  
(9)

From Formula 9 we can get the expression IF displacement.

\[
l_{\text{IF}} = \gamma |T_{\text{TE}}||T_{\text{TM}}| \cos(\beta_1 - \beta_2)(\kappa_1 \sqrt{\varepsilon_0 / \mu_y} + \kappa_2 \sqrt{\varepsilon_0 / \mu_x}) \mu_0 \tan \theta_i
\]  
\[
2\omega \sqrt{\varepsilon_y \mu_\perp (\kappa_1 + \kappa_2)}
\]  
(10)
3. Conclusion
In this paper, we derive the total reflection condition occurs at the interface between isotropic medium and biaxial anisotropic medium. We also get the expressions of Goos-Hänchen displacement and Imbert-Fedorov displacement at the interface. Some unique phenomena of the two displacements on the biaxial anisotropic medium are found and confirmed by simulation. The method adopted in this paper is also applicable to the calculation of total reflection conditions and the two displacements on other complex electromagnetic parameter medium.

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