Students’ difficulties in solving trigonometric equations and identities

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Abstract. This study aims to identify the types of difficulties experienced by high school students in solving equations and trigonometric identities. The method used in this research is descriptive qualitative research method to describe the facts of students' difficulties in solving equations and trigonometric identities. The data collection technique in this study is by using respondents' ability tests and interviews. Students involved in this study are 72 students of grade XI of senior high schools in the city of Bandung. Based on the results of data analysis, there are three aspects of students’ difficulties in solving trigonometric equations and also there are three aspects of students' difficulties in solving trigonometric identity problems. The difficulties of students in solving trigonometric equations, namely the difficulty of students in deciphering the form of the problem, difficulty in factoring in the form of trigonometric quadratic equations, and difficulties using the basic trigonometric equations. Whereas, the difficulties of students in solving trigonometric identity problems, namely the difficulty of students applying general trigonometry formulas, difficulty in describing each of the trigonometric comparison relationships, and difficulties in performing algebraic calculations.

1. Introduction
Mathematics is a scientific discipline that underlies the development of modern technology which has an important role in advancing human thinking, so that mastering and creating technology in the future requires a strong mastery of mathematics from an early age. An important part in learning mathematics itself is the process of learning mathematics. Jaworksy states that the implementation of mathematics learning is not easy because students experience difficulties in learning mathematics [1]. Difficulties in learning mathematics is what causes students to have low abilities in the field of mathematics studies.

One of the material in the field of mathematics study that is studied at the high school level is trigonometry. Trigonometry is one of the material in mathematics that students must understand to develop their mathematical understanding [2]. In learning trigonometry some students often encounter difficulties caused by students' incomprehension in trigonometric concepts. Students tend to memorize the formula given by the teacher or written in the book without understanding the intent and contents, so students often make mistakes in solving the trigonometric problems [3]. One of the trigonometric material that is considered difficult by students is the similarity and proof of trigonometric identity, because it requires an understanding of the right concepts and high accuracy in their application [4-6].
This can be seen from the results of the research of [5], who conducted identification test students with the following question: determine the set of solutions from $2 \sin^2 x = -\sin x + 1$, for $0^\circ \leq x \leq 360^\circ$. One of the answers of students who showed difficulties in answering this question can be seen in Figure 1.

![Figure 1. A student's answer on solving a trigonometric equation](image)

In Figure 1 above, the student tends to solve questions using algebra, he has not been able to connect algebraic reasoning with the trigonometric concepts he has learned. Trigonometric concepts are importance in connecting algebraic and geometric reasoning [7]. Therefore, if students are unable to connect algebraic reasoning and geometry in learning trigonometry, then they will have difficulty in solving trigonometric problems.

Another difficulty experienced by students in Figure 1 is the difficulty in deciphering the form of the problems, understanding angles in trigonometry, and the difficulty of calculating/computing to find a set of solutions. These difficulties if left unchecked will cause low student learning outcomes. Therefore, these difficulties need to be identified and known for their causes so that the right solution can be chosen for use in classroom learning.

2. Methods
The research method used is descriptive qualitative research method, namely research that describes or describes the object of research based on facts that appear or as they are [8]. Qualitative descriptive research seeks to describe all existing symptoms or conditions, namely the state of symptoms according to what they were at the time the research was conducted [9]. The subjects of this study were 72 students class XI science, senior high school in the city of Bandung who had participated in material learning equations and trigonometric identities. While the object in this study is the identification of student difficulties in solving equations and trigonometric identities.

Data collection techniques in this study were the test ability of respondents and interviews. Data obtained by test is the difficulties experienced by students. The data validity checking technique used is the method triangulation. Triangulation method is done by comparing test results and interview data. Data analysis used refers to the analysis of data according to [10], namely data reduction, data presentation, and conclusion drawing. Reduction of data in this study is done by summarizing all the difficulties experienced by students in completing the test, then choosing the things that are the main cause, and focusing on the important things of these difficulties. In presenting data, all important information obtained from data reduction results is presented in the form of a chart. The chart is a presentation of data presentation designed to combine integrated information, so that researchers can analyze what is happening and determine the next steps. Finally, drawing conclusions are drawn from the focus of the study based on the results of data analysis.
3. Result and Discussion

The researcher gave a test to 72 students class XI science, about the problem of equation and trigonometric identity consisting of two items, namely: (1) determine the set of solutions $2 \sin^2 \alpha = - \sin \alpha + 1$, for $0^\circ \leq \alpha \leq 360^\circ$, (2) prove that $\frac{\alpha - \theta}{\alpha + \theta} = \frac{-1 + t}{1 + t}$. The test results show the percentage of students who answered correctly number 1 was 18% and the percentage of students who answered correctly number 2 was 36%. The difficulty of students in completing number 1 questions lies in the difficulty of deciphering the form of the problem, factoring in the quadratic trigonometry equation, and difficulty using the basic trigonometric equation solution. Identifying student difficulties in question number 1 is presented in Figure 2 below:

![Figure 2. Students’ answers to question 1](image)

Based on Figure 2, student A has difficulty in factoring the shape of the trigonometric quadratic equation and the difficulty in determining the $x$ angle that satisfies the $\sin x$ equation, it only writes the result of $\sin x$, not the value from the $x$ angle. In contrast to student B, he understood how to look for $x$ angles but he misconstrued the factoring of the quadratic equation of the trigonometry and the difficulty of using a basic trigonometric equation. After interviews with these students, student A did not understand the trigonometry concept well, he did not understand that looking for the value of $x$ should be sought using the basic trigonometric equation, so it is not the result of the value of $\sin x$. The following interview excerpt provides evidence for this.

I (interviewer): Hello, is this S (A)?
S(A) (student A): Yes ma'am.
I: I want to ask about your answer in question number 1. Try to see the problem!
[S(A) and I see the problem and then S(A) read the problem]
I: You mean the problem is understand or not?
S(A): Understand ma'am.
I: How to find the value of x?
S(A): That's the result, x is -1, 0, 1.
I: Is it x or sin x? [I trying to show the difference again]
S(A): Oh, that's $\sin x$ ma'am.
I: So which x is it?
S(A): Don't know ma'am, I think that's the value of $x$ ma'am, because usually the factoring results are the $x$ value [in algebra]
I: This is a trigonometric quadratic equation, so $x$ is a real number or an angle? [while showing the difference in algebraic and trigonometric equations]
S(A): Angle means yes ma'am? [doubtful in answering]
I: Then how do you find the angle?
S(A): I don't know ma'am, how to find the angle.
The interview excerpt showed that although the student acknowledged that he understood the problem, he did not understand the concept of trigonometry correctly. Instead, student B understood the concept of trigonometry, but he had difficulty in factoring the square of the trigonometry and the difficulty of using the basic trigonometric equation. Interview excerpts with student B as follows:

I: Try checking the answer number 1 again, is it correct or not? 
S(B): [S (B) recheck the answer]. Factorization is confusing this if the coefficient in front of it is not 1. [It means the number 2 in $2 \sin^2 x$]
I: How about looking for the x value? 
S(B): Use that, Ma'am, the formula K multiplied by 360 degrees, K will be replaced by 0, 1, etc. But I don't really understand it, so just try to find the angle.
I: Does this mean $x = 1$ or $\sin x = 1$? [While pointing at students' answers]
S(B): Oh, yes ma'am, the $\sin x$ is the same as 1 [Student smiling]
I: Then why in the settlement set the angles are 0 and 360 degrees? On top of that, you only write 90 and 180 degrees?
S(B): That's because the initial equation is equal to 0 [The initial trigonometric quadratic equation], so there are 0 results as well.
I: Is that also the result of factoring?
S(B): Maybe ma'am, I don't understand ma'am.

In accordance with the results of the study of [11], which states that the trigonometric equation is a material that is difficult to teach and difficult to learn. The difficulty can also be caused because the trigonometric equation material is not liked or not desired by students, so the trigonometric equation material learning becomes more difficult to understand. In addition, many concepts must be mastered by students before learning the trigonometric equation material, for example for the question number 1 above, the concept of reporting quadratic equations must be well-mastered by students.

Furthermore, the difficulty of students in completing question number 2 is located in the difficulty of students applying general trigonometry formulas, difficulties in describing each of the trigonometric comparison relationships, and the difficulty of performing algebraic calculations. Identifying student difficulties in question number 1 is presented in Figure 3 below:

![Figure 3. Student C’s answer to question 2](image)

Based on Figure 3, students have difficulty describing each of the trigonometric comparison relationships, so that achieving trigonometric evidence becomes complicated. After conducting interviews with these students, which made him difficult in the translation of trigonometric forms, he was unable to describe the relationship of trigonometric comparisons to achieve proof. The following are excerpts from interviews with students C.
I: What do you think of your answer number 2? Is it already correct?
S(C): It hasn't been proven yet, ma'am, I keep simplifying the shape, but instead it turns out the
answer back and forth.
I: Just look at the steps of your work in the second step, these are in the form of constants 1 and
-1. Just looking for the tan x right?
S(C): Yes ma'am, to look for the tan x, I simplify the form again.
I: Tan x can be broken down so what? Do you know?
S(C): Forgot ma'am

In the interview above, the student had difficulty in changing the form of trigonometry to tan x
because he did not remember the translation of the tan x form, so he continued to look for simple forms
of proof of the trigonometry. Other students, many are wrong in applying general trigonometric formulas
and wrong in doing algebraic calculations/computations. The most frequent mistakes made by students
in solving problems in trigonometry are mistakes of understanding, transformation errors, and process
skill errors [12]. Most misconceptions occur when students do not understand how to approach a given
trigonometric problem from the concept. Students often misunderstand requests for questions. This may
be due to the lack of emphasis by the teacher in teaching the simplification of concepts that arise, perhaps
also because students only memorize the trigonometry formula. In accordance with the results of [11]
study, one of the causes of students having difficulty in solving trigonometry is because the knowledge
they have is only procedural knowledge, they do not master conceptual knowledge. Therefore, we need
to review how trigonometric learning is done in the classroom, and look for possible errors or
misunderstandings of students before teaching them, so that students’ difficulties in solving trigonometry
problems can be overcome.

4. Conclusion

Based on the results and discussion, student’s difficulty identification in completing the problem of
equation and trigonometry identities are: (a) the difficulties of students in solving trigonometric equation
problems, namely the difficulty of students in describing the form of the problem, difficulty in factoring
the form of the quadratic equation of trigonometry, and the difficulty of using the basic trigonometric
equation solving, (b) students’ difficulties in solving trigonometric identity problems, namely the
difficulty of students applying general trigonometric formulas, difficulties in describing each of the
trigonometric comparison relationships, and the difficulty of doing algebraic calculations/computation.

5. References

[1] Rohimah S M 2017 Jurnal Penelitian dan Pembelajaran Matematika (JPPM) 10 132-141
[2] Eko Y S, Prabawanto S and Jupri A 2018 Journal of Physics: Conf. Series 1097 012146
[3] Ahmad H, Febryanti, Mutmainnah, Yakin A A and Sarbi S 2018 Journal of Physics: Conf. Series
1114 012114
[4] Yulandari T I and Supeno I Pengembangan Lembar Kerja Siswa Materi Trigonometri untuk
Siswa SMA Kelas X dengan Metode Penemuan Terbimbing (Malang: Thesis University Nation
Malang)
[5] Huljannah M, Sugita G, Anggraini 2015 Aksioma: Jurnal Pendidikan Matematika 4 164-176
[6] Aqilah 2012 Analisis Kesalahan Peserta Didik dalam Menyelesaikan Soal Pembuktian Identitas
Trigonometri Kelas X.I SMA Islam Sultan Agung I Semarang Tahun Pelajaran 2011/2012
(Semarang: Thesis IAIN Walisongo)
[7] Koyunkaya M Y 2016 International Journal of Mathematical Education in Science and
Technology 47 1028-1047
[8] Nawawi H and Martini M 1996 Penelitian Terapan (Yogyakarta: Gajahmada University Press)
[9] Mukhtar 2013 Metode Penelitian Deskriptif Kualitatif (Jakarta: GP Press Group)
[10] Miles B M and Huberman M 1992 Analisis Data Kualitatif Buku Sumber tentang Metode-metode
Baru (Jakarta: UIP)
[11] Chigonga B 2016 Proceeding ISTE International Conference on Mathematics, Science and
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Reference

[12] Usman H M and Hussaini M M 2017 *Journal of Mathematics (IOSR-JM)* **13** 01-04