Semi-leptonic Octet Baryon Weak Axial-Vector Form Factors in the Chiral Constituent Quark Model

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Motivation

- Proton Spin Problem
- Weak decays

Chiral Constituent Quark Model

- $\chi$CQM with Configuration mixing
- Spin polarization functions

Weak form factors

- Generalized Sachs form factors at $Q^2 \approx 0$
- Generalized Sachs form factors in the quark sector

Results

- New Observations
- Application Potential

Summary and Conclusions
The driving question for high-energy physics is "Proton Spin Problem"

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin)
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis"
- Confirmed by SMC, E142-3 and HERMES experiments
- 1991 NMC result: Asymmetric nucleon sea ($\bar{d} > \bar{u}$)
  Recently confirmed by E866 and HERMES
- Non-perturbative effects explained only through the generation of “quark sea”
Outline

1. **Motivation**
   - Proton Spin Problem
   - Weak decays

2. **Chiral Constituent Quark Model**
   - $\chi$CQM with Configuration mixing
   - Spin polarization functions

3. **Weak form factors**
   - Generalized Sachs form factors at $Q^2 \approx 0$
   - Generalized Sachs form factors in the quark sector

4. **Results**
   - New Observations
   - Application Potential

5. **Summary and Conclusions**
Weak axial-vector form factors

“PSP” relates the DIS measurements to the spin-dependent matrix elements via the weak axial-vector form factors of the semi-leptonic baryon decays.

The weak decays and their respective vector ($f_{i=1,2,3}(Q^2)$) and axial-vector form factors ($g_{i=1,2,3}(Q^2)$) important to investigate the dynamics of the hadrons particularly at low energies.

Provide vital information on the interplay between the weak interactions (low-$Q^2$) and the hadronic structure determined by the strong interactions (large-$Q^2$).

Only presently available source for obtaining the two axial-vector coupling parameters $F$ and $D$ separately.
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Only presently available source for obtaining the two axial-vector coupling parameters \( F \) and \( D \) separately.
SU(3) symmetry breaking

- Hyperons are assigned to a SU(3)-flavor octet to deduce the relation between spin densities and weak matrix elements.
- Role of SU(3) symmetry breaking has also been discussed in some phenomenological models and experiments.
- In early eighties CERN WA2 experiments were analysed under the assumptions of exactness of SU(3) symmetry ($g_2 = 0$).
- Some indications that SU(3) symmetry breaking effects are important ($g_2$ is non-zero).
- First observed for the $\Sigma^- \rightarrow n$ decay measuring $| (g_1 - 0.237g_2)/f_1 | = 0.327 \pm 0.007_{stat} \pm 0.019_{syst}$. 

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Weak decays of octet baryons
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Symmetry breaking might play a very important role in the hyperon semileptonic decays.

Further strengthened from the $\Xi^0 \rightarrow \Sigma^+ e^- \nu_e$ decay. KTeV (Fermilab E799) measured $\frac{g_1}{f_1} = 1.32^{+0.21}_{-0.17} \text{stat} \pm 0.05 \text{syst}$ with the assumption of SU(3) symmetry.

In the SU(3) breaking limit it measured $g_1 / f_1 = 1.17 \pm 0.08 \text{stat} \pm 0.05 \text{syst}$.

Very recently, NA48 Collaboration gave $g_1 / f_1 = 1.20 \pm 0.05$.

The KTeV (Fermilab E799) experiment also measured $\frac{g_2}{f_1} = -1.7^{+2.1}_{-2.0} \pm 0.05 \text{syst}$ and $\frac{f_2}{f_1} = 2.0 \pm 1.2 \text{stat} \pm 0.5 \text{syst}$ with the assumption of SU(3) symmetry breaking.

Significant deviations for the axial-vector form factors under SU(3) flavor symmetry for other decays as well.
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On the theoretical side, calculations have been carried out for the form factors in the
- Cabibbo theory
- SU(3) chiral quark-soliton model proposed by Yamanishi
- Relativistic quark model
- Large $1/N_c$ expansion of QCD
- Chiral perturbation theory
- Lattice QCD

Not in agreement with each other on the magnitude as well as sign of these form factors

It would be interesting to estimate the weak form factors and the role of SU(3) symmetry breaking in weak decays
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It would be interesting to estimate the weak form factors and the role of SU(3) symmetry breaking in weak decays
Chiral Constituent Quark Model

- $\chi$CQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NRQM.
- Incorporates \textit{confinement} and \textit{chiral symmetry breaking}.
- “Justifies” the idea of constituent quarks and scope of the model extended in the context of “\textit{proton spin crisis}”
- The $\chi$CQM has been developed further by invoking configuration mixing having its origin in chromodynamic spin-spin forces among quarks.
- $\chi_{\text{CQM}_{\text{config}}}$ improves the predictions of $\chi$CQM in several cases including the spin polarization functions, the octet and decuplet magnetic moments etc..
Methodology

"Quark sea" generation $q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_{\mp}$

$$\mathcal{L} = g_8 \bar{q} \Phi q + g_1 \bar{q} \frac{n'}{\sqrt{3}} q = g_8 \bar{q} \left( \Phi + \zeta \frac{n'}{\sqrt{3}} I \right) q$$

$$\Phi = \begin{pmatrix}
\pi^o \sqrt{2} + \beta \frac{n}{\sqrt{6}} + \zeta \frac{n'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\
\pi^- & -\pi^o \sqrt{2} + \beta \frac{n}{\sqrt{6}} + \zeta \frac{n'}{\sqrt{3}} & \alpha K^0 \\
\alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2n}{\sqrt{6}} + \zeta \frac{n'}{\sqrt{3}}
\end{pmatrix}$$

The parameter $a(= |g_8|^2)$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \rightarrow d'(u) + \pi^+(\pi^-)$, whereas $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively denote the probabilities of transitions of $u(d) \rightarrow s + K^{(-o)}_o$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + \eta'$. 
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\[ \chi \text{CQM with Configuration mixing} \]

Spin polarization functions

Methodology

- "Quark sea" generation \( q_\pm \rightarrow GB^0 + q'_\mp \rightarrow (q\bar{q}') + q'_\mp \)
- \( \mathcal{L} = g_8 \bar{q} \Phi q + g_1 \bar{q} \frac{\eta'}{\sqrt{3}} q = g_8 \bar{q} (\Phi + \zeta \frac{\eta'}{\sqrt{3}} I) q \)
- \( \Phi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^0 \\
\alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}}
\end{pmatrix} \)
- The parameter \( a(=|g_8|^2) \) denotes the transition probability of chiral fluctuation of the splittings \( u(d) \rightarrow d(u) + \pi^+(\pm) \), whereas \( \alpha^2 a \), \( \beta^2 a \) and \( \zeta^2 a \) respectively denote the probabilities of transitions of \( u(d) \rightarrow s + K^{-(o)} \), \( u(d, s) \rightarrow u(d, s) + \eta \), and \( u(d, s) \rightarrow u(d, s) + \eta' \).
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The most general configuration mixing

\[ |B\rangle_{\text{full}} = (|56, 0^+\rangle_{N=0} \cos \theta + |56, 0^+\rangle_{N=2} \sin \theta) \cos \rho + (|70, 0^+\rangle_{N=2} \cos \theta' + |70, 2^+\rangle_{N=2} \sin \theta') \sin \rho \]

In the present context, adequate to consider

\[ |B\rangle_{\text{config}} = \cos \phi |56, 0^+\rangle_{N=0} + \sin \phi |70, 0^+\rangle_{N=2} \]

\[ |56, 0^+\rangle_{N=0} = \frac{1}{\sqrt{2}} (\chi' \phi + \chi'' \phi'') \psi^s(0^+) \]

\[ |70, 0^+\rangle_{N=2} = \frac{1}{2} [(\phi' \chi'' + \phi'' \chi') \psi'(0^+) + (\phi' \chi' - \phi'' \chi'') \psi''(0^+)] \]

with \( \chi, \phi \) and \( \psi \) representing the spin, isospin and spatial part of the wave functions.
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Spin structure of a baryon, $\hat{B} \equiv \langle B|\mathcal{N}|B\rangle$

$\mathcal{N} = n_{u^+}u_+ + n_{u^-}u_- + n_{d^+}d_+ + n_{d^-}d_- + n_{s^+}s_+ + n_{s^-}s_-$

Quark spin polarization $\Delta q = q_+ - q_- + \bar{q}_+ - \bar{q}_-$

$\hat{B} \equiv \langle B|\mathcal{N}|B\rangle = \cos^2\phi \langle 56, 0^+|\mathcal{N}|56, 0^+\rangle_B + \sin^2\phi \langle 70, 0^+|\mathcal{N}|70, 0^+\rangle_B$

$\Delta u = \cos^2\phi \left[\frac{4}{3} - \frac{a}{3}(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2)\right] + \sin^2\phi \left[\frac{2}{3} - \frac{a}{3}(5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2)\right]$

$\Delta d = \cos^2\phi \left[-\frac{1}{3} - \frac{a}{3}(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2)\right] + \sin^2\phi \left[\frac{1}{3} - \frac{a}{3}(4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\right]$

$\Delta s = \cos^2\phi[-a\alpha^2] + \sin^2\phi[-a\alpha^2] = -a\alpha^2$. 
Spin structure of a baryon, $\hat{B} \equiv \langle B|\mathcal{N}|B \rangle$

$\mathcal{N} = n_u^+ u^+ + n_u^- u^- + n_d^+ d^+ + n_d^- d^- + n_s^+ s^+ + n_s^- s^-$

Quark spin polarization $\Delta q = q^+ - q^- + \bar{q}^+ - \bar{q}^-$

$\hat{B} \equiv \langle B|\mathcal{N}|B \rangle = \cos^2\phi \langle 56, 0^+|\mathcal{N}|56, 0^+ \rangle_B + \sin^2\phi \langle 70, 0^+|\mathcal{N}|70, 0^+ \rangle_B$

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Antiquark flavor contents of the “quark sea”

\[
\bar{u} = \frac{1}{12} \left[ (2\zeta + \beta + 1)^2 + 20 \right] a, \\
\bar{d} = \frac{1}{12} \left[ (2\zeta + \beta - 1)^2 + 32 \right] a, \\
\bar{s} = \frac{1}{3} \left[ (\zeta - \beta)^2 + 9\alpha^2 \right] a.
\]
General formalism

Transition matrix element for the semi leptonic hadronic decay process $B_i \rightarrow B_f + l + \bar{\nu}_l$

$M = \frac{G_F}{\sqrt{2}} (V_{qq'}) < B_f | J^\mu_h | B_i > (\bar{u}_l(p_l)\gamma_\mu(1 - \gamma_5)u_\nu(p_\nu))$

$G_F$ is the Fermi coupling constant, $V_{qq'}$ is the $qq'$ element of the CKM mixing matrix.

The weak hadronic current

$< B_f | J^\mu_h | B_i > = < B_f | J^\mu_V - J^\mu_A | B_i >$

$= \bar{u}_f(p_f) \left( f_1(Q^2)\gamma^\mu - i \frac{f_2(Q^2)}{M_i + M_f} \sigma^{\mu\nu} q_\nu + \frac{f_3(Q^2)}{M_i + M_f} q_\nu \right) u_i(p_i)$

$- \bar{u}_f(p_f) \left( g_1(Q^2)\gamma^\mu \gamma^5 - i \frac{g_2(Q^2)}{M_i + M_f} \sigma^{\mu\nu} q_\nu \gamma^5 + \frac{g_3(Q^2)}{M_i + M_f} q_\nu \gamma^5 \right) u_i(p_i)$

$f_1, f_2$ and $f_3$ are the vector, induced tensor or weak magnetism and induced scalar form factors.

$g_1, g_2$ and $g_3$ are the axial-vector, induced pseudotensor or weak electricity and the induced pseudoscalar scalar form factors.
General formalism

- Transition matrix element for the semi leptonic hadronic decay process $B_i \rightarrow B_f + l + \bar{\nu}_l$
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  $< B_f | J^\mu | B_i > = < B_f | J^\mu_V - J^\mu_A | B_i >$
  $= \bar{u}_f(p_f) \left( f_1(Q^2)\gamma^\mu - i \frac{f_2(Q^2)}{M_i+M_f} \sigma^{\mu\nu} q_\nu + \frac{f_3(Q^2)}{M_i+M_f} q^\nu \right) u_i(p_i)$
  $- \bar{u}_f(p_f) \left( g_1(Q^2)\gamma^\mu\gamma^5 - i \frac{g_2(Q^2)}{M_i+M_f} \sigma^{\mu\nu} q_\nu\gamma^5 + \frac{g_3(Q^2)}{M_i+M_f} q^\nu\gamma^5 \right) u_i(p_i)$

- $f_1$, $f_2$ and $f_3$ are the vector, induced tensor or weak magnetism and induced scalar form factors
  $g_1$, $g_2$ and $g_3$ are the axial-vector, induced pseudotensor or weak electricity and the induced pseudoscalar scalar form factors
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Solving in the Breit-frame (Lorentz frame) where $p_i = -p_f = \frac{1}{2}q$ in the non relativistic limit ($q^2 \ll M_i^2, M_f^2$)

\[
\begin{align*}
\langle B_f | J^0_V | B_i \rangle &\equiv N_{B_i} N_{B_f} \chi_f \chi_i^\dagger v_0 \chi_i, \\
\langle B_f | J^i_V | B_i \rangle &\equiv N_{B_i} N_{B_f} \chi_f \chi_i^\dagger (v_V q^i + i \epsilon^{ijk} v_A q^j \sigma^k) \chi_i, \\
\langle B_f | J^0_A | B_i \rangle &\equiv N_{B_i} N_{B_f} \chi_f \chi_i^\dagger a_0 \sigma \cdot q \chi_i, \\
\langle B_f | J^i_A | B_i \rangle &\equiv N_{B_i} N_{B_f} \chi_f \chi_i^\dagger (a_S \sigma^i + a_T q^i \sigma \cdot q) \chi_i, 
\end{align*}
\]

where $N_{B_i}$ ($N_{B_f}$) and $\chi_i$ ($\chi_f$) are the normalization factor and two-component non-relativistic Pauli spinors of the initial (final) baryon state.
The generalized Sachs form factors for the vector functions as

\[ v_0 = f_1 + \frac{\Delta M}{\Sigma M} f_3 \]

\[ v_V = -\frac{\Delta M}{\Sigma M^2 - \Delta M^2} (f_1 + f_2) + \frac{1}{\Sigma M} f_3 \]

\[ v_A = \frac{\Sigma M}{\Sigma M^2 - \Delta M^2} (f_1 + f_2) , \]

For the axial-vector functions

\[ a_0 = \frac{\Delta M}{\Sigma M^2 - \Delta M^2} (g_3 - g_1) - \frac{1}{\Sigma M} g_2 \]

\[ a_S = g_1 - \frac{\Delta M}{\Sigma M} g_2 \]

\[ a_T = \frac{1}{\Sigma M^2 - \Delta M^2} \left( g_3 - \frac{1}{2} \left( g_1 - \frac{\Delta M}{\Sigma M} g_2 \right) \right) , \]

where \( \Sigma M = M_i + M_f \) and \( \Delta M = M_i - M_f \)
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Generalized Sachs form factors at $Q^2 \approx 0$

Generalized Sachs form factors in the quark sector

$$f_1 = v_0 - \Delta M v_V - \frac{\Delta M^2}{\Sigma M} v_A$$

$$f_2 = -v_0 + \Delta M v_V + \Sigma M v_A$$

$$f_3 = \Sigma M v_V + \Delta M v_A$$

$$g_1 = \frac{\Sigma M^2 - \Delta M^2}{\Sigma M^2} \left( -\Delta a_0 + \frac{1}{\Sigma M^2 - \Delta M^2} \left( \Sigma M^2 - \frac{\Delta M^2}{2} \right) a_S + \Delta M^2 a_T \right)$$

$$g_2 = -\frac{1}{\Sigma M} \left( \left( \Sigma M^2 - \Delta M^2 \right) a_0 + \frac{\Delta M}{2} a_S - \left( \Sigma M^2 - \Delta M^2 \right) \Delta M a_T \right)$$

$$g_3 = \frac{1}{2} a_S + \left( \Sigma M^2 - \Delta M^2 \right) a_T$$

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Pseudoscalar field $\Phi'$ present in the $\chi$CQM

$\chi$CQM vector current

$$J_{V,qq'}^\mu = \bar{q}' \gamma^\mu q$$

and the axial-vector current is

$$J_{A,qq'}^\mu = g_a \bar{q}' \gamma^\mu \gamma^5 q - f_\Phi \partial^\mu \Phi$$

$g_a \rightarrow$ quark axial-vector current coupling constant

$f_\Phi \rightarrow$ pseudoscalar (pion) decay constant

Spontaneous Symmetry Breaking gives mass $m_q$ to the $q$ quark and $m_\Phi$ to the pseudoscalar field
The generalized Sachs form factors at $Q^2 \approx 0$ for the vector functions as well as axial-vector functions are

\[
\begin{align*}
    v_0^q & = 1, \\
v_V^q & = -\frac{\Delta m}{\Sigma m^2 - \Delta m^2}, \\
v_A^q & = \frac{\Sigma m}{\Sigma m^2 - \Delta m^2}, \\
a_0^q & = \frac{\Delta m}{\Sigma m^2 - \Delta m^2} \left( g_3^q - 1 \right), \\
a_S^q & = 1, \\
a_T^q & = \frac{1}{\Sigma m^2 - \Delta m^2} \left( g_3^q - \frac{1}{2} \right).
\end{align*}
\]

where $\Sigma m = m_q + m_{q'}$ and $\Delta m = m_q - m_{q'}$.
The Sachs form factors for the quark currents can be used to obtain the corresponding Sachs form factors for the baryons using $f_1(0) \equiv \langle B_f | \lambda_{qq'} \otimes 1 | B_i \rangle$ and $g_1(0) \equiv \langle B_f | \lambda_{qq'} \otimes \sigma^z | B_i \rangle$ where $\sigma^z$ operator measures the spin polarizations of the quarks in the baryons.

The vector form factors can be expressed as

$$f_1 = \left(1 + \frac{\Delta M}{\Sigma m^2 - \Delta m^2} \left( \Delta m - \frac{\Sigma m \Delta M g_1(0)}{f_1(0)} \right) \right) f_1(0),$$

$$f_2 = \left( \frac{1}{\Sigma m^2 - \Delta m^2} \left( \Sigma M \Sigma m \frac{g_1(0)}{f_1(0)} - \Delta M \Delta m \right) - 1 \right) f_1(0),$$

$$f_3 = \frac{1}{\Sigma m^2 - \Delta m^2} \left( \Delta M \Sigma m \frac{g_1(0)}{f_1(0)} - \Sigma M \Delta m \right) f_1(0).$$
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The vector form factors can be expressed as

\[ f_1 = \left( 1 + \frac{\Delta M}{\Sigma m^2 - \Delta m^2} \left( \Delta m - \frac{\Sigma m}{\Sigma M} \Delta M g_1(0) \right) \right) f_1(0), \]
\[ f_2 = \left( \frac{1}{\Sigma m^2 - \Delta m^2} \left( \Sigma M \Sigma m \frac{g_1(0)}{f_1(0)} - \Delta M \Delta m \right) - 1 \right) f_1(0), \]
\[ f_3 = \frac{1}{\Sigma m^2 - \Delta m^2} \left( \Delta M \Sigma m \frac{g_1(0)}{f_1(0)} - \Sigma M \Delta m \right) f_1(0). \]
Similarly, for the axial-vector form factors

\[
\begin{align*}
g_1 & = \left( 1 + \frac{\sum M^2 - \Delta M^2}{\sum M^2} \left( \frac{\Delta M \Delta m}{\sum m^2 - \Delta m^2} - \frac{\Delta M^2}{2} \left( \frac{1}{\sum M^2 - \Delta M^2} + \frac{1}{\sum m^2 - \Delta m^2} \right) + \frac{\Delta M}{\sum m^2 - \Delta m^2} \left( \Delta M - \Delta m \right) g_3^q \right) \right) g_1(0), \\
g_2 & = \frac{1}{\sum M} \left( \frac{\sum M^2 - \Delta M^2}{\sum m^2 - \Delta m^2} \left( \Delta m - \frac{\Delta M}{2} \right) - \frac{\Delta M}{2} \right) + \frac{\sum M^2 - \Delta M^2}{\sum m^2 - \Delta m^2} \left( \Delta M - \Delta m \right) g_3^q \right) g_1(0), \\
g_3 & = \left( \frac{1}{2} \left( 1 - \frac{\sum M^2 - \Delta M^2}{\sum m^2 - \Delta m^2} \right) + \frac{\sum M^2 - \Delta M^2}{\sum m^2 - \Delta m^2} g_3^q \right) g_1(0).
\end{align*}
\]
If \( E \equiv \frac{\Delta M}{\Sigma M} \) and \( \epsilon \equiv \frac{\Delta m}{\Sigma m} \), then higher order terms involving \( E \) and \( \epsilon \) can be neglected.

\[
\begin{align*}
  f_1 &= f_1(0), \\
  f_2 &= \left( \frac{\Sigma M}{\Sigma m} \frac{G_A}{G_V} - 1 \right) f_1(0), \\
  f_3 &= \frac{\Sigma M}{\Sigma m} \left( E \frac{G_A}{G_V} - \epsilon \right) f_1(0), \\
  g_1 &= g_1(0), \\
  g_2 &= \left( \frac{\Sigma M}{\Sigma m} \epsilon - \frac{1}{2} \left( 1 + \frac{\Sigma M^2}{\Sigma m^2} \right) E \right) g_1(0), \\
  g_3 &= \left( \frac{1}{2} \left( 1 - \frac{\Sigma M^2}{\Sigma m^2} \right) + \frac{\Sigma M^2}{\Sigma m^2} g_3^q \right) g_1(0),
\end{align*}
\]

where \( \frac{G_A}{G_V} = \frac{g_1(0)}{f_1(0)} \).
Axial vector coupling parameters $F$ and $D$

\[
\frac{G_A}{G_V}^{np} = F + D = \Delta u - \Delta d,
\]

\[
\frac{G_A}{G_V}^{\Sigma^-\Sigma^0} = F = \frac{1}{2}(\Delta u - \Delta s),
\]

\[
g_1(0)^{\Sigma^\pm\Lambda} = \sqrt{\frac{2}{3}}D = \sqrt{\frac{1}{6}}(\Delta u - 2\Delta d + \Delta s),
\]

\[
\frac{G_A}{G_V}^{\Xi^-\Xi^0} = F - D = \Delta d - \Delta s,
\]

for the $\Delta S = 0$ decays
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Generalized Sachs form factors in the quark sector

\[ \frac{G_A}{G_V}^{\Sigma^- n} = F - D = \Delta d - \Delta s, \]
\[ \frac{G_A}{G_V}^{\Xi^- \Sigma^0} = F + D = \Delta u - \Delta d, \]
\[ \frac{G_A}{G_V}^{\Xi^- \Lambda} = F - \frac{D}{3} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s), \]
\[ \frac{G_A}{G_V}^{\Lambda^p} = F + \frac{D}{3} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s), \]
\[ \frac{G_A}{G_V}^{\Xi^0 \Sigma^+} = F + D = \Delta u - \Delta d, \]

for the $\Delta S = 1$ decays
Results

The calculated values of the weak vector and axial form factors for the strangeness non changing as well as strangeness changing decays.

| decay          | $f_1$ | $f_2$ | $f_3$ | $g_1$    | $g_2$  | $g_3$    |
|----------------|-------|-------|-------|----------|--------|----------|
| $n \to p$      | 1.00  | 2.6119| 0.0025| 1.2695   | -0.00398| -232.921 |
| $\Sigma^- \to \Sigma^0$ | 1.414 | 1.0334| 0.0049| 0.6759   | -0.0096 | -201.313 |
| $\Sigma^- \to \Lambda$ | 0     | 2.2652| 0.0801| 0.6463   | -0.1517 | -271.428 |
| $\Sigma^+ \to \Lambda$ | 0     | 2.2573| 0.0722| 0.6463   | -0.1363 | -245.965 |
| $\Xi^- \to \Xi^0$ | 1.00  | -2.2531| -0.0033| -0.3137 | 0.0069 | 113.86   |
| $\Sigma^- \to n$ | -1.0  | 1.8125| 0.6161| 0.3137   | 0.0166 | -9.1472  |
| $\Xi^- \to \Sigma^0$ | 0.7071| 2.0287| -0.2906| 0.8977  | 0.3101 | -29.1186 |
| $\Xi^- \to \Lambda$ | 1.2247| -0.4502| -0.6582| 0.2622  | 0.0471 | -8.8929  |
| $\Lambda \to p$  | -1.2247| -1.0371| 0.4145| -0.9085 | -0.1699 | 20.7323  |
| $\Xi^0 \to \Sigma^+$ | 1.0   | 2.8535| -0.4140| 1.2695  | 0.4458 | -40.6992 |
Results

Table: The ratio $\frac{G_A}{G_V} = \frac{g_1}{f_1}$ for the decays. * $f_1 = 0$ for $\Sigma^{\pm}\Lambda e^\nu$ decays so only $g_1$ values are mentioned.

| Decay     | Data      | NQM    | $\chi_{\text{CQM}_{\text{config}}}$ |
|-----------|-----------|--------|---------------------------------|
| $G_A^{n\rightarrow p}$ | $1.269 \pm 0.0029$ | 1.667  | 1.269                           |
| $G_A^{\Sigma^- \rightarrow \Sigma^0}$ | -- | 0.667 | 0.478                           |
| $G_A^{\Sigma^- \rightarrow \Lambda}$ | $0.01 \pm 0.1^*$ | 0.816 | 0.646                           |
| $G_A^{\Sigma^+ \rightarrow \Lambda}$ | -- | 0.816 | 0.646                           |
| $G_A^{\Xi^- \rightarrow \Xi^0}$ | -- | $-0.333$ | $-0.314$                        |
| $G_A^{\Sigma^- \rightarrow n}$ | $-0.340 \pm 0.017$ | $-0.333$ | $-0.317$                        |
| $G_A^{\Xi^- \rightarrow \Sigma^0}$ | -- | 1.667 | 1.269                           |
| $G_A^{\Xi^- \rightarrow \Lambda}$ | $+0.25 \pm 0.05$ | 0.333 | 0.214                           |
| $G_A^{\Lambda \rightarrow p}$ | $0.718 \pm 0.015$ | 1.000 | 0.742                           |
| $G_A^{\Xi^0 \rightarrow \Sigma^+}$ | $1.21 \pm 0.05$ | 1.667 | 1.269                           |
### Results

Table: \( \chi_{\text{CQM}} \text{config} \) results for \( f_2/f_1, \ g_2/g_1, \ g_2/f_1 \) and \( f_2/g_1 \). * \( f_1 = 0 \) for \( \Sigma^\pm \Lambda e^\nu \) decays so only \( f_2 \) values are mentioned.

| Decays          | \( f_2/f_1 \) | \( g_2/g_1 \) | \( g_2/f_1 \) | \( f_2/g_1 \) |
|-----------------|----------------|---------------|---------------|---------------|
| \( n \to p \)   | 2.612          | –0.003        | –0.004        | 2.057         |
| \( \Sigma^- \to \Sigma^0 \) | 0.731          | –0.014        | –0.007        | 1.5289        |
| \( \Sigma^- \to \Lambda^* \) | 2.265          | –0.235        | –            | 3.5048(2.4 ±1.7)\(^a\) |
| \( \Sigma^+ \to \Lambda^* \) | 2.257          | –0.211        | –            | 3.492         |
| \( \Xi^- \to \Xi^0 \) | –2.253         | –0.022        | 0.007         | 7.183         |
| \( \Sigma^- \to n \) | –1.812 (–0.97±0.14)\(^a\) | 0.529 | –0.017 | 5.778         |
| \( \Xi^- \to \Sigma^0 \) | 2.869          | 0.3455        | 0.4386        | 2.2599        |
| \( \Xi^- \to \Lambda \) | –0.367         | 0.179         | 0.038         | –1.717        |
| \( \Lambda \to p \) | 0.847          | 0.187         | 0.138         | 1.142         |
| \( \Xi^0 \to \Sigma^+ \) | 2.853 (2.0 ±1.3)\(^a\) | 0.351 | 0.4458(–1.7\(^+2.1\)\(^-2.0\) ±0.5)\(^a\) | 2.248         |
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   • Proton Spin Problem
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   • Spin polarization functions

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5 Summary and Conclusions

H Dahiya
Weak decays of octet baryons
- The vector form factor $f_1$ and the axial-vector form factor $g_1$ are independent of masses. The other form factors $f_2$, $f_3$, $g_2$ and $g_3$ are inversely proportional to the quark masses and their value decreases with increasing quark masses.
- SU(3) symmetry breaking results better as compared to SU(3) symmetry results.
- Axial-vector coupling parameters $F$ and $D$ can be deduced from the weak decays.
- Results consistent with the experimental data as well as with the lattice QCD.
- Substantiated by a measurement of the individual form factors.
- The weak decays important to investigate the dynamics of the hadrons and provide vital information on the interplay between the weak interactions (low-$Q^2$) and the hadronic structure determined by the strong interactions (large-$Q^2$).
Outline

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5. Summary and Conclusions
Understanding the spin structure of the proton will help to resolve the most challenging problems facing subatomic physics which include

- What happens to the spin in the transition between current and constituent quarks in the low energy QCD?
- How can we distinguish between the current quarks and the constituent quarks?
- What is the role played by non-valence flavors in understanding the hadron internal structure?
A small but non-zero value of SU(3) symmetry breaking within the dynamics of chiral constituent quark model, suggests an important role for non-strange and strange quark masses in the non-perturbative regime of QCD.

At leading order, the model envisages constituent quarks, the octet of Goldstone bosons ($\pi, K, \eta$ mesons) as appropriate degrees of freedom.