Eden clusters in three dimensions and the Kardar–Parisi–Zhang universality class

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Abstract. We present large-scale simulations of radial Eden clusters in three dimensions and show that the growth exponent is in agreement with the value $\beta = 0.242$ accepted for the Kardar–Parisi–Zhang (KPZ) universality class. Our results refute a recent assertion proposing that radial Eden growth in $d = 3$ belongs to a universality class distinct from the KPZ one. We associate the previously reported discrepancy with a slow convergence to the asymptotic limit. We also present the skewness and kurtosis in the roughening regime for flat geometry in $2 + 1$ dimensions.

Keywords: critical exponents and amplitudes (theory), kinetic growth processes (theory), self-affine roughness (theory)

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The Kardar–Parisi–Zhang (KPZ) universality class was introduced by means of the equation [1]
\[
\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta,
\]
which describes the non-conservative evolution of an interface \( h(x,t) \) subjected to a white noise \( \eta \). Many analytical [2, 3], numerical [4]–[7] and experimental [8] works have shown that radial (curved) and flat geometries have the same scaling exponents in \( 1 + 1 \) dimensions.

The KPZ class is characterized by an interface width \( w \), defined as the standard deviation of the distance between the interface points and the initial seed, scaling in time as \( w \sim t^{\beta} \) where \( \beta \) is the growth exponent. In \( d = 1 + 1 \) dimensions the KPZ growth exponent is exactly known: as \( \beta = 1/3 \) [1]. The Eden model [9] is a standard KPZ system that generates radial clusters with a rough surface. Off-lattice simulations of Eden clusters immersed in \( d = 2 \) dimensions confirmed the agreement with the KPZ growth exponent [4, 7]. Recently, Kuennen and Wang [10] reported off-lattice simulations of radial Eden clusters in \( d = 3 \) dimensions, for which a growth exponent \( \beta \approx 0.1 \), much smaller than the accepted KPZ value for flat substrates in \( 2 + 1 \) dimensions, \( \beta_{\text{KPZ}} \approx 0.24 \) [11], was observed.

In order to verify the validity of this unexpected result, we have performed large-scale simulations of an off-lattice Eden model. The model is defined as follows. At \( t = 0 \), a single particle stuck to the origin is introduced. New particles are added to the cluster, one at a time, at random positions such that the new particle is adjacent to at least one particle in the cluster, but does not overlap any other added particle. The dynamics runs as follows. A particle of the cluster is randomly chosen and a random direction is uniformly chosen, on a unitary sphere. A new particle is attached to this position if no overlap happens. Otherwise, the simulation proceeds to the next step with the choice of a new particle. Independently of the growth success, the time is incremented by \( \Delta t = 1/N_p \), where \( N_p \) is the total number of particles in the cluster. This version was studied in two dimensions in [7] and we call it Eden D, since there exist three other versions commonly called A, B and C [12]. Notice that we did not concern ourselves with optimizations [13] to avoid uncontrolled sub-leading corrections. A typical surface of a small cluster is shown in figure 1(a).

Figure 2 shows the interface width against time, averaged over 230 clusters. The largest sizes contain more than \( 10^8 \) particles that correspond to a size much larger than \( 5 \times 10^5 \) particles simulated in [10]. Each sample takes about 8 h of simulation using an Intel Xeon 2.53 GHz CPU and 7 GB of RAM memory. The effective exponent \( \beta_{\text{eff}} \) obtained from the logarithmic derivative of \( w \) versus \( t \) is shown in the inset of figure 2. The KPZ growth exponent is approached only in the limit of very large clusters.

The model implementation in [10] is slightly different from the Eden D one used in the present work. There, when a randomly chosen particle has enough nearby space for adding a new particle, the growth always happens. This version is known in the literature as Eden B [14]. In Eden D, frustrated attempts are allowed, which, consequently, amplifies the noise and generates larger interface fluctuations as compared to Eden B.

In order to shed light on this issue, we have also performed flat simulations of both Eden models on square lattices with periodic boundary conditions along directions \( x \) and
Figure 1. (a) Surface of a radial Eden D cluster with $9 \times 10^5$ particles. (b) Surface obtained with the on-lattice Eden D model after the growth of 2000 layers using a flat substrate of size $500 \times 500$.

Figure 2. Main plot: interface width against time for off-lattice simulation of radial Eden D in $d = 3$ dimensions. The dashed line has a slope of 0.24, as a guide to the eyes. Inset: the local slope in a plot of $\ln w$ versus $\ln t$. The dashed line shows the KPZ exponent $\beta = 0.24$.

The initial condition consists in all sites being occupied for $z \leq 0$. In Eden B, particles in the interface (an occupied site with at least one empty nearest neighbour (NN)) are randomly chosen and one of their empty NNs, also chosen at random, is occupied. Eden D

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is equal to Eden B except that any NN (occupied or not) can be selected. If an occupied
neighbour is chosen, the simulation runs to the next step. In both models the time step is
\( \Delta t = 1/N_s \), where \( N_s \) is the number of interface sites. A typical surface for Eden D model
is shown in figure 1(b).

Figure 3 shows the interface width against time for both flat models. Averages were
performed over 400 independent samples. The inset shows the corresponding effective
growth exponent. While the growth exponent in Eden D simulations converges quickly
to a value close to the KPZ universality class, in Eden B an exponent increasing from
\( \beta \gtrsim 0.1 \) but not exhibiting a stationary power regime is observed. These results show that
the Eden B model has very strong corrections to the scaling and that the KPZ exponent
must be observed only in exceedingly long time simulations. It is worth noticing that the
interface width in Eden B is very small (\( w < 2 \) lattice units in figure 3 and \( w < 1.5 \) particle
diameters in [10]), which hinders the observation of the asymptotic regime.

Recent advances in the understanding of the KPZ universality class in 1+1 dimensions
have been based on universal distributions of the interface fluctuations during the
roughening (transient) regime [2]. A basic characterization of the distributions can be
obtained via skewness \( S \) and kurtosis \( K \) defined as the dimensionless cumulant ratios

\[
S = \frac{\langle h^3 \rangle_c}{\langle h^2 \rangle_c^{3/2}}
\]

and

\[
K = \frac{\langle h^4 \rangle_c}{\langle h^2 \rangle_c^2} - 3,
\]

where \( \langle X^n \rangle_c \) denotes the \( n \)th cumulant of \( X \). In our simulations, these quantities did
not exhibit stationary values, either for radial Eden D (figure 4) or for Eden B, on

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flat substrates. For Eden D on flat substrates, the skewness and kurtosis converge to $S = 0.411(2)$ and $K = 0.34(1)$, respectively. Figure 4 shows the skewness and kurtosis against time for radial and flat Eden D simulations. It is important to mention that cumulant ratios for Eden D in flat simulations have finite size effects. So, it was necessary to run shorter times and systems larger than those used in figure 3 to obtain an accurate estimate of the cumulant ratios. The skewness and kurtosis obtained for Eden D agree fairly well with the ratios obtained for other KPZ models in 2 + 1 dimensions [15]. A detailed analysis of the height distributions for several KPZ models in three-dimensional lattices will be reported elsewhere [15].

In conclusion, we have shown that the radial Eden growth in three dimensions has a growth exponent in agreement with the KPZ universality class, at odds with a previous report [10] but in consonance with the common belief. Our results suggest that the low exponent observed in [10] comes from the small simulated sizes and strong corrections to the scaling. In addition, we have presented the stationary values of the skewness and kurtosis during the roughening regime for flat simulations.

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Note. After acceptance, we realized that the finite-time corrections in the skewness and kurtosis of Eden D model (figure 4) were underestimated. Considering extrapolation to an infinite time limit, we have reviewed our estimates and corresponding errors to $S = 0.42(1)$ and $K = 0.36(2)$. Our conclusions do not change in the light of these new estimates.
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