Aeroelastic Analysis of Preset Angle for A Typical Airfoil Section with Cubic Nonlinear Stiffness

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Abstract. The influences of the preset angle on the dynamic responses of the typical airfoil section with cubic nonlinear stiffness were investigated under consideration of the aerodynamic flutter onset due to certain angle of attack. The typical airfoil model with cubic nonlinear stiffness is developed to analyze the plunging and the pitching motions for the airfoil section. Based on the Grossman’s quasi steady aerodynamic model, the aerodynamic lift and the moment can be obtained and then the equation of motion for the airfoil section is derived by applying the Lagrange theorem. The effects of the preset angle and the oncoming flow speed on the nonlinear dynamics are analyzed by means of the bifurcation diagram, the frequency-amplitude diagram, the time history, etc. Simulation results show that the preset angle influences the intensity of the vibration, moderate or severe, stable or unstable, even under certain circumstance, the jump and chaos phenomenon occur; and the oncoming flow speed affects the distribution of the vibration energy for the plunging and the pitching motions. The unstable stall region for the responding parameters should be avoided to keep the airfoil safe in engineering practice. Some qualitative and quantitative rules are obtained for the airfoil design and the flutter suppression.

Key words: Aeroelastic; Preset Angle; Airfoil; Cubic Nonlinear Stiffness; Vibration.

1. Introduction
The airfoil flutter is nonlinear phenomenon which is essential instability of the wing structure resulting from a feedback between the airfoil deflection and the corresponding aerodynamic loads. The nonlinear aeroelastic phenomenon arises from the structural factor or the aerodynamic source. The structural factor is associated with the large geometric deflection or the material property of the structures. Pereira et al [1] carried out a study on the aeroelastic response of airfoils with the concentrated structural nonlinearities. Dowell and Tang [2] tested a highly flexible, long aspect ratio wing with a tip concentrated mass; the wing combined structural and aerodynamic nonlinearities. The researchers found supercritical and subcritical LCOs over a range which involved the stall regime. Zakaria et.al [3] exploited the self-excited oscillations of a fluttering composite beam to harvest energy using the piezoelectric transduction; the effects of the preset angle of attack, the wind speed and load resistance
on the levels of harvested power are determined in the research. The beam may reach a self-excited motion with the large geometric deformations associated with a nonlinear aerodynamic loading. Other researchers[4-5] also made great advances on the energy harvesting from the flow-induced vibration. Vasconcellos et al [6] carried out the experiments and obtained the results in which LCOs driven by the dynamic stall effects were observed. Such nonlinear behaviors cause responses that are related to the complex trailing and the leading edge vortex interactions [7-8]. Much effort in flutter suppression was focused on the two-dimensional wing section flutter; and the researchers made much progress in last decades. Karpel[9] proposed an effective design technique for an active flutter-suppression based on a rational approximation of the unsteady aerodynamic loads in the entire Laplace domain. Other researchers[10-11] also made progress in flutter suppression in the past years.

Investigators made much works on the dynamic modeling and experiments of the aerodynamic structures which have the structural nonlinearity or/and the aerodynamic nonlinearity, and on the flutter suppression. However, the effect of the preset angle for a typical airfoil with the cubic nonlinear stiffness has been studied rarely; and the issue is important in engineering practice. In this paper, a numerical investigation on the aeroelastic effect of the two degree of freedom (DOF) typical airfoil section with the cubic nonlinear stiffness was carried out under the consideration of the different preset angles.

2. Aeroelastic Formulation

A spring-restrained rigid wing-section model is illustrated in Figure 1. This configuration represents a two-dimensional section of the aeroelastic airfoil, which has two degrees of freedom, i.e. the plunge motion $h$ and the pitch motion $\alpha$.

$$\alpha = \bar{\alpha}(t) - \alpha_p$$  

(1)

Where $\alpha_p$ is the preset angle; and $\bar{\alpha}(t)$ is the airfoil incidence angle. Many researchers in published articles only considered the linear spring stiffness without considering the effect of the cubic nonlinear spring stiffness. In the model, there are a plunge spring and a pitch spring. The linear spring constants and the cubic nonlinear spring constants in the plunge and pitch directions are $k_h$, $k_\alpha$, $\overline{k_h}$, $\overline{k_\alpha}$, respectively. The distance between the elastic axis and the center of mass is $\alpha$, the chord length of the blade section is $c (c = 2b)$, seen in Figure 1(a).

![Figure 1. (a)The typical aeroelastic sectional model of the coupled two-DOF system; (b)Influences of the oncoming flow speed on the pitching motion and the plunging motion](image)

Applying the Lagrange equation, the equation of motion for the system considering the cubic nonlinear stiffness can be obtained as follows

$$\begin{cases} \quad m\dot{h} + m\alpha \bar{h} \dot{\alpha} - \dot{m} \bar{a} \alpha^2 \dot{a} - m \sin \alpha \dot{\alpha}^2 = L_1 \\
\quad m\alpha^2 \ddot{\alpha} + m \cos \alpha \dot{\alpha} - \dot{m} \bar{a} \alpha^3 = M_1 \end{cases}$$  

(2)
Where \( m \) is the mass of the unit length along the elastic axis, \( c_h, c_\alpha \) denote the structural damping coefficients in the bending and the torsional directions, respectively. According to the Grossman’s quasi steady aerodynamic model, the aerodynamic force and the aerodynamic moment of the system are as follows:

\[
L_1 = -2\pi \rho V^2 \left[ \alpha + \frac{\dot{h}}{\dot{v}} + \left( \frac{\delta}{z} - \bar{a} \right) \frac{\dot{b}}{\dot{v}} \right]
\]

\[
M_1 = 4\pi \rho V^2 \left( \frac{\alpha}{2} - \frac{1}{4} \right) \left[ \alpha + \frac{\dot{h}}{\dot{v}} + \left( \frac{\delta}{z} - \bar{a} \right) \frac{\dot{b}}{\dot{v}} \right] - \frac{1}{2} \pi \rho V b^2 \dot{a}
\]

Equation (2) is nondimensioned as

\[
\ddot{\xi} + x_\alpha \dot{a} - x_\alpha a \dot{a}^2 + c_h b \dot{\xi} + c_h b \xi + k_h b^3 \xi^3 = -\frac{2n}{mb} a \left[ \alpha + \frac{\dot{h}}{\dot{v}} + \left( \frac{\delta}{z} - \bar{a} \right) \frac{\dot{b}}{\dot{v}} \right]
\]

\[
r_\alpha^2 \dot{a} + x_\alpha \ddot{\xi} + 2 \zeta_\alpha r_\alpha^2 \dot{a} + r_\alpha^2 \alpha + R_\alpha r_\alpha^2 \alpha^3 = 4 \frac{\dot{V}^2}{\mu} \left( \frac{\bar{a}}{\delta} - \frac{1}{4} \right) \left[ \alpha + \frac{\dot{h}}{\dot{v}} + \left( \frac{\delta}{z} - \bar{a} \right) \frac{\dot{b}}{\dot{v}} \right] - \frac{1}{4} \bar{V} \dot{a}
\]

Where \( \xi = \frac{h}{r}, x_\alpha = \frac{a}{b}, \sigma_a = \sqrt{k_h / \mu}, \sigma_\alpha = \sqrt{k_\alpha / \mu}, \bar{r}_\alpha = \frac{r_\alpha}{b}, \bar{a} = \sigma_h / \sigma_a \)

\[
R_\xi = \frac{K_h}{K_h}, R_\alpha = \frac{K_\alpha}{K_h}, \mu = \frac{m \pi \rho h^2}{\sigma_a}, \xi_\alpha = \frac{c_h}{2 \sqrt{m \kappa_h}}, \zeta_\alpha = \frac{c_h}{2 \sqrt{\kappa_a \kappa_h}}, \bar{V} = \frac{\dot{V}}{\sigma_a}
\]

Where \( \mu \) is the mass ratio; \( \dot{V} \) is the nondimensional free steam speed; \( \bar{r}_\alpha \) is the frequency ratio; \( \zeta_\alpha \) and \( \zeta_\alpha \) are the plunging natural frequency and the pitching natural frequency without coupling, respectively; \( \bar{R}_\alpha \) is the dimensionless radius of gyration along the elastic axis; \( \bar{K}_\alpha, \bar{K}_a, \bar{\xi}_\alpha \) and \( \bar{\zeta}_\alpha \) are the dimensionless cubic stiffness coefficients and the damping coefficients in the plunging and the pitching directions, respectively. The paper will address the effect of the oncoming flow speed and the preset angle on the dynamic response of the system.

### 3. Results and Discussions

#### 3.1. Oncoming flow speed

| Variables | Description | Values |
|-----------|-------------|--------|
| \( c=2b \) | Chord length (m) | 0.250 |
| \( \rho \) | Air density (kg/m\(^3\)) | 1.079 |
| \( \sum m \) | Airfoil mass (kg) | 1.500 |
| \( \alpha \) | Dimensionless distance between elastic axis and CG of airfoil | 0.660 |
| \( \zeta_\alpha \) | Damping coefficient in the plunging direction | 0.016 |
| \( \zeta_\alpha \) | Damping coefficient in the pitching direction | 0.021 |

The oncoming flow speed is an important parameter for the dynamic response of the system. The influence of the oncoming flow speed on the system responses is analyzed, seen in Figure 1(b). The airfoil section NACA0012 is selected as a calculation example; the corresponding parameters are seen in Table 1. The plunging motion and pitching motion are calculated to analyze the effect of the oncoming
flow speed on the vibration response. In Figure 1(b), the figure (...) is the plunging motion, which keeps stable as the oncoming flow speed increases from 0 to 69. When the oncoming flow \( \overrightarrow{V} \) increases up to 69, the plunging motion suddenly jumps up to a very large value. The jump phenomenon is nonlinear from the flutter; and then the motion increases with the increase of the oncoming flow \( \overrightarrow{V} \); the response of the motion jumps down suddenly and sharply reduces to about 60 and then has a small growth with the increase of the oncoming flow \( \overrightarrow{V} \). Throughout the process, the nonlinear jump phenomena appear twice, where \( \overrightarrow{V} \) is equal to 69 and 120, respectively. The figure (ooo) in Figure 1(b) is the pitching motion, which also has two jumping phenomena at 69 and 125, respectively, the same as the plunging motion. However, the response for the pitching motion is different from that of the plunging motion. When the oncoming flow \( \overrightarrow{V} \) is increasing from 0 to 69, the responses for the two motions remain unchanged, namely very small, close to zero. From 69 to 120, the pitching motion increases, however the response is much less than that of the plunging motion; and the response for the pitching motion jumps to 0 when the oncoming flow speed increases up to 120, while the plunging response reduces by 39.8%. The flutter phenomenon is sensitive to the changing oncoming flow speed.

![Figure 2](image2.png)

**Figure 2.** (a)–(d) Time history and (e)–(h) Phase diagrams of the plunging motion for the nondimensional oncoming flow speed \( \overrightarrow{V} = 12, 27, 29, 35 \), respectively

![Figure 3](image3.png)

**Figure 3.** (a)–(d) Time history and (e)–(h) Phase diagrams of the pitching motion for the nondimensional oncoming flow speed \( \overrightarrow{V} = 12, 27, 29, 35 \), respectively
Different oncoming flow speed has different effect on the response of the aeroelastic system. Figure 2 and Figure 3 are the time history and phase plane diagrams for the oncoming flow $V=12$, 27, 29 and 35, respectively. It can be seen from Figure 2 that the plunging response of the system changes from the static deflection (shown in Figure 2(a)) to the damped oscillation (shown in Figure 2(b)) as the airspeed increases from 12 to 27; with the oncoming flow speed increasing up to 29 and 35, the response tends to be stable (seen in Figure 2(c) and 2(d)). The corresponding phase diagrams for the four flow speed are shown in Figure 2(e)-2(h) where the non-oscillation, the damped vibration and the steady vibration the non-closed loop, the points of convergence and the closed loop, respectively. The pitching response has the same rules as the plunging response, shown in Figure 3(a)-(h).

3.2. Preset angle
The airfoil system may produce high vibration due to the effect of the dynamic stall when the system reach a critical angle of attack; and then leading to fatigue problems or performance loss. Influence of the different preset incidence angle on the stall flutter can be analyzed by the bifurcation diagrams to capture the precision features of the flutter. Five typical preset angle values are chosen, they are 0°, 11.5°, 12.5°, 17.5° and 19.5° respectively (corresponding radian are 0, 0.2006, 0.2181, 0.3053 and 0.3402, respectively). Figure 5 show the bifurcation diagrams for the plunging motion considering all the preset angles cases mentioned above. Figure 4(b) is the view of Figure 4(a) from the preset angle axis perspective. It can be seen from Figure 5 that the oncoming flow speed and the preset angle have effects on the displacements of the flutter. Different preset angle cases lead to different bifurcation characteristics. As the oncoming flow speed increases up to 32, the responses of the flutter for the different preset angle cases all have a slight decline; and in this range, the larger value of the preset angle 0.3042 drops more than others. When the oncoming flow speed increases from 32 to 83, the responses for different preset angel cases show chaos; and the influence of the preset angle on the response shows irregular pattern. The system is under the condition of the instability. The maximum influence of the different preset angles on the amplitude can be changed by 21.7%. In engineering practice, the system should be avoided operating in this range of the oncoming flow speed. As the oncoming flow speed increases from 83, the responses for all cases increase and the response for the larger preset angle case increase more than the other cases; this increase is regular, seen in Figure 4 (b). The effect rule of the preset angle on the pitching motion is similar to the plunging motion, while the amplitude of the effect is different.

![Bifurcation diagram](image)

**Figure 4.** (a) Plunging displacement of airfoil section changing with the oncoming flow speed v.s. preset angle (0, 0.2006, 0.2181, 0.3053 and 0.3402, respectively); (b) View of (a) from the preset angle axis perspective

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4. Conclusion
The influence of the preset angle on the dynamic response for a typical airfoil section with the cubic nonlinear stiffness is investigated under consideration of the aerodynamic flutter onset due to certain angle of attack. Main conclusions are as follows:

- The oncoming flow speed affects the distribution of the vibration energy, the flutter onset and the amplitude of the responses. It is rather remarkable that the plunging motion has increased dozens of times when the flow speed crosses the critical value 69. The flutter phenomenon is sensitive to the changing oncoming flow speed, even to the critical flow speed. The huge jumping behavior of the response should be avoided so as to operate safely and steady in engineering practice.

- The preset angle influences the intensity of the vibration, even under certain range of the preset angle, the jump and chaos phenomena occur. When the oncoming flow speed increases from 32 to 83, the responses for different preset angel cases show chaos, the influence of the preset angle on the response shows irregular pattern. The system is under the condition of the instability. The maximum influence of the different preset angles on the amplitude can be changed by 21.7%. In engineering practice, the interval and amplitude of the flutter can be controlled by designation of the preset angle.

- The research also gives a type of qualitative and quantitative analytical method for the region of the flutter onset. The analytical method can be applied to the similar aerodynamic structures such as the large-rise building, the long-span bridges, and so on.

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