Fluctuation-dissipation phenomena in the Earth pole oscillations

L D Akulenko 1, V N Pochukaev 2, V V Perepelkin 3, A S Filippova 3

1 Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Prospekt Vernadskogo, 101-1, Moscow, Russia
2 TSNIIMASH, Pionerskaya, 4, Korolyov, Russia
3 Moscow Aviation Institute, Volokolamskoe shosse, 4, Moscow, Russia

E-mail: vadimkin1@yandex.ru

Abstract. Earth pole oscillations at the Chandler frequency are studied taking into account nonlinear fluctuation-dissipative moments of forces and disturbances at the Chandler frequency and those close to it. For the stationary mode, the dependence of the external perturbation amplitude at the Chandler frequency on the dissipation coefficients is found. It is shown that the amplitude of the Chandler wobble is sensitive to the difference between the Chandler and close to it frequencies and nonlinear dissipation coefficients.

1. Pole tide perturbations in the oscillatory process of the Earth pole

The most difficult and poorly studied at the moment is the problem of constructing a model of Chandler wobble[1]. It is associated with the determination of fluctuation-dissipative processes affecting the parameters of the Chandler wobble. In the lack of external perturbations on the Chandler frequency and those close to it on a long interval these processes would lead to damping of the so-called free motion of the Earth instant spin axis [2].

On a short time interval, fluctuations and dissipative perturbations are associated with the changes in the Chandler wobble frequency and amplitude [3-5]. Thus linear and nonlinear dissipative perturbations in the Earth pole oscillations model should be considered when estimating the parameters of the Chandler wobble, and fluctuation-dissipative perturbations lead to small corrections in the frequency and amplitude of the Chandler wobble, as well as to the presence of a self-oscillatory mode at the Chandler frequency.

The pole tide — the response of the deformable layer of the Earth to the displacement of the Earth pole — depends on the properties of the mobile medium and may differ in amplitude and phase from the equilibrium tide at the different points on the planet.

It is known [3] that in some regions on the Earth surface one can observe an anomalously large amplitude pole tide, reaching 3 cm. Historically, this phenomenon has been called the “anomalous pole tide” [3, 6].

In addition to the annual harmonic (changes in the sea level with a period of one year is a typical seasonal fluctuation), the frequency spectrum of the sea level fluctuations in these areas are close to the Chandler $N_1$, and to $v_i$ harmonics with a period of 6.3-6.5 years [3]. You can notice the connection between these harmonics, according to the relation $v_i=1-N_1$, that is also true for the Chandler and
annual frequencies, and the frequency of their amplitude modulation in the oscillatory process of the pole [3].

Fig. 1 A six-year harmonic of sea-level fluctuations, extracted from the PSMSL observational data (blue line) in comparison with the derivative of the amplitude of the Chandler component of the Earth pole oscillations (black line).

In fig. 1 the results of the processing of observations series of sea-level fluctuations for the PSMSL station (Hornbeck, Denmark, North Sea) using the wavelet transformation [3] are shown. On the figure there is a comparison between the six-year harmonic of sea-level fluctuations (blue line) and the rate of change of the Chandler component amplitude (black line). As noted above, along with the "anomalous", close to the Chandler 14-month harmonic, with not very regular behavior, a more regular six-year harmonic is noted with the frequency close to the \( 1 - N_1 \) frequency. The phase of this harmonic turns out to be related to the phase of a six-year amplitude modulated pole. Given that the six-year harmonic is quasi-periodic, this fact turns out to be significant. Its presence may indirectly indicate the process of excitation and changes in the Chandler wobble.

In fact the equations of the Earth pole motion with the right-hand side of the combinational perturbation (obtained as a combination of a six-year frequency and an annual frequency) have the form [3]:

\[
\begin{align*}
\dot{p} + Nq &= j_{qr}^{0} + a(t) \cos(v_1 t)\cos(v_h t), \\
\dot{q} - Np &= -j_{pr}^{0} + (a(t) \cos(v_1 t)\sin(v_h t)).
\end{align*}
\]

(1)

The derivative of the variation of the Chandler wobble amplitude in solving equations (1) will be proportional to the variable amplitude \( a(t) \) of the perturbation at the Chandler frequency \( V_1 - V_h \). This fact is shown on fig. 2.

On the other hand, the six-year harmonic in the noted region may be associated with the non-linear manifestation of the pole tide. Then its presence changes the coefficients in model (1) and the \( \sigma_3 \) value in particular, which can also have a significant impact on the excitation process of the Earth pole oscillations, reducing the required amplitude of external perturbation at the Chandler frequency.
2. Polar tide perturbations in the oscillatory process of the Earth pole

Taking into account the influence of the polar tide by dissipative terms, that are linear and nonlinear in \( p, q \), and also the perturbations at the close to Chandler frequency \( N_1 \), the equations of the Earth pole motion are:

\[
\dot{p} + Nq = f_{qr}^0 + f_p \cos(N_1 t - \beta_p),
\]

\[
\dot{q} - Np = f_{pr}^0 + f_q \cos(N_1 t - \beta_q).
\]

Then the stability conditions for stable oscillations are written as:

\[
\sigma_1 + 2a^2\sigma_3 + 3\frac{\Delta N_1}{N}a^2\sigma_3 > 0,
\]

\[
\left(\sigma_1 + 3a^2\sigma_3 + \frac{9}{2}\frac{\Delta N_1}{N}a^2\sigma_3\right)\left(\sigma_1 + 2a^2\sigma_3 + \frac{3}{2}\frac{\Delta N_1}{N}a^2\sigma_3\right) + \Delta N_1^2 > 0.
\]

Hence with positive values of the dissipation coefficients (in this case, the equation of the stationary oscillations mode for the amplitude has one real root) the stationary mode will be stable. If the coefficient \( \sigma_3 \) is negative, there may be several stationary modes (fig. 2), but the stable solution is the one that satisfies the condition

\[
|a^2\sigma_3| < \frac{1}{3}\sigma_1 + \varepsilon(\Delta N_1), \quad \varepsilon << 1.
\]

The value \( \varepsilon \) is small and is determined by the frequency difference \( \Delta N_1 \).

As a test case the \( \varepsilon = 0 \) case is considered. Denote \( k = -\sigma_1 10^4 \sigma_3 \). In fig. 2 the dependence of the stationary solutions on the coefficient \( k \) at a fixed \( f \) is shown. For negative values of the coefficient \( k \), the stationary solution is always stable, and for \( k \to -\infty \) a solution \( a_1 \) converges to the solution of...
the Earth pole linear model. For nonlinear pole tide terms $\sigma_3 = \sigma_1 10^{-13}$ and $k \sim 10^9$, the solution $a_1$ is stable and differs from the linear system solution $f / \sigma_1$ by a small amount.

In case $\sigma_3 < 0$ ($k > 0$), the solution $a_2$ will be stable with $0 > \sigma_3 > -\sigma_1 / (3a_2)^{-1}$ and ($k > 3a_2 10^{-4}$). For example, stationary mode with an amplitude of $a = 100$ mas will be stable at the value $\sigma_1 = 0.026$ of the linear dissipation coefficient, if $k > 3$.

**Fig. 3** The amplitudes of the Chandler wobble of the Earth pole combinational model (red line) and the deterministic model (blue line), as well as the value of the stationary amplitude of the oscillatory process combinational model (green line).

Now we will consider the equations of fluctuations of the Earth pole motion, taking into account irregular dissipative perturbations, that is written in the coordinate form:

$$
\begin{align*}
\dot{x}_p - N_p x_p &= -(\sigma_1^* + \sigma_1^* V_{1l}) x_p + (\sigma_{12}^* + \sigma_{12}^* V_{2l}) y_p + f_p \cos(\delta) t - \beta p, \\
\dot{y}_p + N_p y_p &= -(\sigma_2^* + \sigma_2^* V_{1r}) x_p + (\sigma_{21}^* + \sigma_{21}^* V_{2r}) y_p + f_q \cos(\delta) t - \beta q.
\end{align*}
$$

(5)

Here $x_p, y_p$ are the Earth pole coordinates; the dissipative terms contain a regular part and nonstationary fluctuations $V_{1l}, V_{1r}$, which are Gaussian broadband processes such as white noise.

To perform the test calculations the following values of the model parameters were chosen: $f_p = f_q = 3$ mas, $\beta_p = \beta_q$, $\delta = N_1 - N = 0.0001$ cycle/year, $\sigma_1^* = \sigma_2^* = 0.0258$, $\sigma_{12}^* = \sigma_{21}^* = 0$, $\sigma_1^* = \sigma_2^* = 5 \cdot 10^{-4}$, $\sigma_{12}^* = \sigma_{21}^* = 5 \cdot 10^{-3}$, white noise intensity: $v = 0.001$. As a result of integrating the system of equations (5), the mathematical expectations, dispersions and covariances of processes $x_p, y_p$ were obtained. In fig. 3 the amplitudes of the Chandler wobble from a random effects model and a deterministic model are compared.

Thus, the sensitivity of the Chandler wobble amplitude to the difference between the Chandler frequency and those close to it, to the linear and nonlinear dissipation coefficients, as well as to the fluctuation perturbations in the pole tide, is shown.

**Acknowledgments**

This work was carried out within the state task no. 9.7555.2017/BCh.

**References**

[1] International Earth Rotation and Reference Systems Service - IERS Annual Reports, http://www.iers.org.

[2] Zlenko A A 2015 Astron. Rep. 59 (1) pp 72 – 87
[3] Markov Yu G, Perepelkin V V, Filippova A S, Rykhlova L V 2017 Astron. Rep. 61 (2) pp 160-168
[4] Markov Yu G, Perepelkin V V, Sinitsyn I N, Korepanov E R, Shi H T 2007 Cosm. Res. 45 (6) pp 513-522
[5] Markov Yu G, Perepelkin V V, Filippova A S 2017 Dokl. Phys. 62 (6) pp 318-322
[6] Perepelkin V V 2016 Mech. Solids. 51 (6) pp 654-659