Mathematical model of a motor-grader movement in the process of performing working operations

V Shevchenko\textsuperscript{1,3}, O Chaplyhina\textsuperscript{1,3,4}, I Pimonov\textsuperscript{1,3}, O Reznikov\textsuperscript{1,3} and S Ponikarovska\textsuperscript{2,3}

\textsuperscript{1}Department of construction and road machines
\textsuperscript{2}Department of foreign languages
\textsuperscript{3}Kharkiv National Automobile and Highway University, st. Yaroslava Mudrogo, 25, Kharkiv, 61002, Ukraine

\textsuperscript{4}olexandrachaplygina@gmail.com

Abstract. The design features of a motor-grader help it perform a wide range of working operations, which puts it on a par with the main machines in the construction industry. Unlike other earth-moving machines, the motor-grader can perform work operations not only in a cyclic mode, but also in a continuous action mode. Considering these features, not only productivity should be addressed as the main indicator of the efficiency of this machine, but also the indicators of the quality of the working operations performed, in particular, the indicators of road-holding ability in the process of performing continuous working operations. The work has substantiated and developed a mathematical model of the motor-grader movement during working operations. This model makes it possible to predict the expected trajectory of the motor-grader on the basis of deterministic dependencies, which enables to assess the indicators of road-holding ability. The difference between the proposed model and the existing ones is the consideration of a two-stage mode of machine movement: plane motion at the first stage and rotational around the stop point at the second. When describing the external forces acting on the motor-grader, the differences in the formation of resistance forces on the blade during various working operations are taken into account.

1. Introduction

It is typical for a motor-grader to perform working operations with the main blade shifted or turned relative to the longitudinal axis. As a rule, this applies to operations for planning and profiling working surfaces, as well as the operations related to cutting out troughs or cuvettes. In such situations, external working resistances act on the machine, applied asymmetrically with respect to the longitudinal axis of the machine. In addition to them, in situations where the angle of the main blade capture differs from 90\degree, in addition to longitudinal resistance forces transverse forces also act from the side of the developed medium. The specified features of the formation of the force effect in the process of moving the motor-grader during a working operation can cause its deviation from the planned trajectory of movement. The consequence of this is the need for additional machine passes in the working face, which ultimately leads to a drop in the machine performance. In order to understand the processes of the motor-grader deviation from the planned trajectory of movement, it is necessary to substantiate, develop and study a mathematical model of the motor-grader movement in the process of performing working operations.
2. Analysis of publications
The issues of modeling the movement of machines are most fully considered in the works of specialists in such sectors of the national economy as the automobile industry and tractor building, as well as in agricultural machine building. The main issues related to the dynamics of movement and road-holding ability of vehicles are considered in [1-3].

In these works, the dynamic processes are considered that are associated with turning the vehicles, performing maneuvers, movement on a support surface with a transverse slope. Destabilizing forces capable of violating the parameters of road-holding ability are inertial (centrifugal, etc.) forces, as well as the components of gravity when moving on transverse slopes.

The loss of road-holding ability of agricultural machinery is primarily associated with the action of additional lateral resistance forces on the working bodies (plows, cultivators), and is also caused by the design features of most wheeled tractors with articulated frames.

The development and analysis of mathematical models of the motion process for agricultural machines and articulated transport are considered in [4-6].

In the field of earthmoving engineering, the issues related to ensuring the road-holding ability of the machine have not been fully investigated, which is explained by the design variety of both the machines themselves and their working bodies. To date, the works of V.D. Musiyko, A.B. Koval and others are known in which the issues of road-holding ability of a trench excavator with a swinging working body are studied [7]. With regard to motor-graders, the issues of road-holding ability were considered in the works of L.V. Nazarov, A.V. Voronovich and others [8-9]. At the same time, a detailed analysis of the motor-grader road-holding ability in the process of performing various working operations has not been fully performed.

3. The purpose and objectives
The purpose of the article is to substantiate and develop a mathematical model of the motor-grader movement in the process of performing working operations.

To achieve this goal the following tasks must be solved:

• analysis and substantiation of the structure of the mathematical model based on exploratory experimental research;
• drawing up a generalized mathematical model of the motor-grader movement based on the generally accepted concepts of analytical and theoretical mechanics;
• substantiation and presentation in mathematical form of all components included in the mathematical model of the motor-grader movement in the process of performing working operations.

4. The main part
4.1. Substantiating the structure of the mathematical model based on the analysis of the search experiment results
To assess the features of generating the trajectory of the motor-grader movement when performing working operations in the conditions of the KhNAHU test site, an exploratory experimental study was prepared and carried out on the Dzk-251 motor-grader. During the experiments, the process of cutting the soil of category II by the angle of the main blade with the chips of constant cross-section was implemented. The initial speed of the motor-grader and the angle of the blade rotation in the plan, the depth of cut, etc. were chosen as the variable parameters [10]. The digging process was carried out from the sump hole.

The studies have shown that the reasons for the deviation of the real trajectory of the motor-grader motion from the planned one are: asymmetric application of an external load, an increase in the values of external resistances as the motor-grader moves in the result of accumulation of the soil prism in front of the blade, the effect of one-time dynamic loads due to an intensive growth of resistance at the initial stage of digging.

The experiments have shown that the trajectory of a motor-grader movement can be formed in various ways (Figure 1-3).
Figure 1. Single turn at the beginning of the cutting process.

Figure 2. Patch-rotary motion in the process of soil cutting.

Figure 3. Types of trajectories of the motor-grader movement when performing working operations.

- the linear trajectory of the motor-grader movement (Figure 3, a) is formed in cases when the working resistances on the blade are relatively small. In this situation, the deviation of the real trajectory of the motor-grader motion from the planned one is not registered;
- in situations where the coefficient of the propellers’ adhesion with the supporting surface is insignificant, the movement of the motor-grader is recorded when performing working operations along a curved trajectory (Figure 3, b);
- most often in the process of cutting the soil, a patch-linear trajectory of the motor-grader is recorded (Figure 3, c); at the same time, at the initial stage the machine moves straight-line, then, as external resistances grow, it slows down, turns around the blade stop point and then continues the straight-line movement.

The features of formation of the motor-grader movement trajectories in the process of performing working operations make it possible to determine the structure of a generalized mathematical model that describes this process:

- the mathematical model should include two systems of differential equations:
  - mathematical model of the plane motion of the machine in the support plane under the action of external forces;
  - mathematical model of the machine turning relative to the blade stop point under the influence of external forces;
- a generalized mathematical model should include the conditions for the transition from one type of movement to another and back again;
- the model should take into account the redistribution of the values of the support reactions on the driving and driven wheels, as well as the change in the coordinate of application and the magnitude of external resistance forces as a result of the accumulation of the soil prism in front of the blade.
4.2. Development of a mathematical model of the motor-grader movement

The dynamic design diagrams of a motor-grader for the process of performing working operations, corresponding to various types of movement, are shown in Figure 4,5.

Since the object of research is the trajectory of a motor-grader movement, the problem of synthesizing a mathematical model is reduced to analyzing the motion of a free body relative to a fixed coordinate system $xOy$. According to Shal’s theorem, any motion of a body can be represented as a combination of translational motion of an arbitrary point of the body, called a pole, and rotational motion of the body around the same point [11]. To simplify calculations, it is recommended to choose the center of the body mass as a pole ($p.O$, Figure 4). In this case, based on the well-known laws of dynamics, the motion of the body is described by a system of differential equations [11]:

$$
\begin{align*}
mx &= (T_1 + T_2) \cdot \cos \varphi - (W_{f1} + W_{f2}) \cdot \cos \varphi + (P_{l1} + P_{l2}) \cdot \sin \varphi - W_h \cdot \cos \varphi + \\
&+ W_l \cdot \sin \varphi - (W_{f31} + W_{f32}) \cdot \cos \beta_l + \varphi) - (P_{31} + P_{32}) \cdot \sin \beta_l + \varphi); \\
m\ddot{y} &= (T_1 + T_2) \cdot \sin \varphi - (W_{f1} + W_{f2}) \cdot \sin \varphi - (P_{l1} + P_{l2}) \cdot \cos \varphi - W_h \cdot \sin \varphi - \\
&- W_l \cdot \cos \varphi - (W_{f31} + W_{f32}) \cdot \sin \beta_l + \varphi) + (P_{31} + P_{32}) \cdot \cos \beta_l + \varphi); \\
\dot{I} &= (T_2 - T_1 + W_{f1} - W_{f2}) \cdot \frac{I_5}{2} - (P_{l1} + P_{l2}) \cdot l_1 + W_h \cdot l_1 + W_l \cdot l_2 - W_{f31} \cdot r_1 - \\
&- P_{31} \cdot r_2 - W_{f32} \cdot r_2 - P_{32} \cdot r_4.
\end{align*}
$$

Where $x, y, \varphi$ are the generalized coordinates for the conditions of the motor-grader plane motion; $m$ is the mass of the grader; $I$ is the moment of inertia of the motor-grader relative to the center of mass; $T_1, T_2$ is the tractive effort on the right and left leading sides; $N_1, N_2$ are the support reactions on the corresponding sides of the balance carriage; $N_{31}, N_{32}$ are the support reactions on the front driven wheels; $W_{f1}$ is the resistance to rolling of the right leading side; $W_{f2}$ is the resistance to rolling of the left leading side; $W_{f3}$ is the resistance to rolling of the front driving wheels (equal to each other since the front axle is hinged to the center beam and has the ability to freely rotate in the vertical plane at an angle of $\pm 20^\circ$); $P_{l1}$ is the lateral force on the right leading side; $P_{l2}$ is the lateral force on the left leading side; $P_{31}, P_{32}$ is the lateral force on the right and left wheels of the front axle; $W_h$ is the horizontal component of resistance to digging; $W_l$ is the lateral component of digging resistance.
When performing working operations with a motor-grader, it is possible to lock the blade by increasing working resistance forces (cutting resistance of the developed medium and resistance to the movement of the prism). In this case, the experiments recorded the rotation of the motor-grader relative to the blade stop point. It is noteworthy that a change in the nature of the machine's motion causes a change in both the dynamic scheme and the equations of motion. The equation of motion of the motor-grader during the turn around the blade stop point has the following form [12]:

\[
I_{o1}\dot{\phi}_1 = (T_1 - W_{f1}) \cdot (l_1 - \frac{l_2}{2}) + (T_2 - W_{f2}) \cdot (l_2 + \frac{l_1}{2}) - (P_{f1} + P_{f2}) \cdot (l_1 + l_2) - W_{f31} \cdot r_5 - P_{f31} \cdot r_6 - W_{f32} \cdot r_7 - P_{f32} \cdot r_8 - M_{res},
\]

where \( M_{res} \) is the moment of resistance forces from the side of the developed soil.

Formally, the condition for the transition from the first stage of movement to the second is the equality of the sum of the driving forces projections along the longitudinal axis of the motor-grader and the sum of the projections of the resistance forces along the same axis:

\[
\sum P_{dr} \leq \sum P_{res}.
\]

Based on the dynamic scheme (Figure. 4), this condition takes the form

\[
T_1 + T_2 + m \cdot \sqrt{x^2 + y^2} \cdot \cos \left( \arctan \frac{y}{x} \right) \leq W_{f1} + W_{f2} + W_h + (W_{f31} + W_{f32}) \cdot \cos \beta_0.
\]

It is noteworthy that if during the transition to the second stage of movement the blade remains blocked (the motor-grader cannot rotate relative to the stop point), the experiments recorded the development of uncontrolled oscillations of the motor-grader in the longitudinal plane, which have an increasing amplitude. Such loading modes are dangerous and can lead to the destruction of the metal structure of both the working equipment and the main frame of the machine.

4.3. Substantiation and determination of analytical dependences of forces that form the process of a motor-grader moving

Support reactions on the driving and driven wheels determine the magnitude of most of the forces acting in the considered dynamic system. Since the operating speeds of the motor-grader when performing working operations are not high (from 0.7 m/sec to 1.5 m/sec), the inertial forces can be neglected in determining the support reactions due to their smallness. In this case, based on the analysis of the static equilibrium of the machine, taking into account the vertical force \( W_h = \pm k \cdot W_h \) acting on the blade from the soil side, and also considering the possible separation of individual wheels from the support surface, we obtain:

\[
N_{31} = N_{32} = \frac{m \cdot g - (\pm k_2 \cdot W_h) \cdot (l_1 - l_2)}{2(l_1 + l_2)} \quad \text{at} \quad N_{31} + N_{32} \geq 0;
\]

\[
N_{31} = N_{32} = 0 \quad \text{at} \quad N_{31} + N_{32} < 0.
\]

\[
N_2 = \frac{mg - (N_{31} + N_{32})}{2} \quad \text{at} \quad N_2 \geq 0;
\]

\[
N_2 = 0 \quad \text{at} \quad N_2 < 0.
\]

\[
N_1 = mg - (N_{31} + N_{32}) - N_2 - (\pm k_2 \cdot W_h) \quad \text{at} \quad N_1 \geq 0;
\]

\[
N_1 = 0 \quad \text{at} \quad N_1 < 0.
\]
In the presented dependences $g$ is the acceleration of gravity; $k_z$ is the empirical coefficient, adjustable in the range from 0.1 to 0.3. The sign in front of the coefficient determines the direction of the vertical reaction vector from the soil side.

Almost all modern motor-graders are equipped with overpowered engines. The experiments show that even during a sharp increase in the level of drag forces on the blade, the number of engine crank shaft rotations decreases by less than 5%. At the same time, the engine has enough power to put the cars in the mode of 100% slipping of the driving wheels. Considering this fact, in order to determine the values of the traction forces on the driving wheels, the known dependences proposed by Professor N.A. Ulyanov, can be converted to the form [10, 11]:

$$
\begin{align*}
T_1 &= N_1 \cdot \varphi_{ad} \cdot (1 - a\hat{x'} - b\hat{y'}), \\
T_2 &= N_1 \cdot \varphi_{ad} \cdot (1 - a\hat{x'} - b\hat{y'}), \\
\hat{x'} &= \sqrt{\hat{x}^2 + \hat{y}^2} \cdot \cos \left[ \arctg \frac{\hat{y'}}{\hat{x'}} - \varphi \right],
\end{align*}
$$

(8)

where $\varphi_{ad}$ is the coefficient of the wheel adhesion with the supporting surface; $a, b$ are the empirical coefficients.

To determine the rolling resistance forces, the resistance coefficient $f$ is taken into account, which depends on the type of the supporting surface:

$$
W_{f1} = f \cdot N_1; \quad W_{f2} = f \cdot N_2; \quad W_{f31} = f \cdot N_{31}; \quad W_{f32} = f \cdot N_{32}.
$$

(9)

To describe the forces of external resistance acting on the blade on the part of the developed medium, it is advisable to use the well-known theories of cutting and digging soils. In particular, based on the principle of superposition, the horizontal $W_h$ and lateral $W_l$ components of the main resistance vector can be represented as the sums:

$$
W_h = W_c + W_{pr} + W_{up} + W_{al},
$$

(10)

$$
W_l = W_l + W_{pr} + W_{up} + W_{al},
$$

(11)

where $W_c, W_l$ are the longitudinal and transverse components of the soil cutting resistance; $W_{pr}, W_{pr}^l$ are the longitudinal and transverse components of resistance to the movement of the soil prism in front of the blade; $W_{up}, W_{up}^l$ are the longitudinal and transverse components of resistance to the movement of the cut off soil chips up the blade; $W_{al}, W_{al}^l$ are the longitudinal and transverse components of resistance to the movement of the soil prism along the blade.

Analytical dependencies for determining all these forces are given in Table 1.

The following letters are used in the formulas: $k$ is the specific coefficient of soil cutting resistance; $F$ is the cross-sectional area of the cut off chips; $\varphi_{ad}$ is the coefficient depending on the value of the cutting angle; $x, y$ are the displacements along the corresponding axes of the point of application of the external resistances main vector; $\delta'$ is the soil density in natural bedding; $k_{loc}$ is the coefficient of soil loosening; $V_{pr}$ is the volume of the soil prism in front of the blade; $\mu$ is the coefficient of internal friction of the soil; $\mu_s$ is the coefficient of external soil friction.
The analysis of the methods to perform working operations by a motor-grader showed that the values of these forces significantly depend on the cross-sectional area of the cut off soil chips. The values of this parameter for different ways of developing the medium are given in Table 2.

The volume of the soil prism that is formed in front of the blade is variable at the initial stage of digging, after which, upon reaching its limiting value \( V_{ult} \), it remains constant due to the soil loss into the side rolls. To determine this parameter, an analytical dependence of the form is offered:

\[
V_{pr} = \begin{cases} 
F \int_0^t \sqrt{\left(\dot{x}_A\right)^2 + \left(\dot{y}_A\right)^2} \, dt & \text{at } V_{pr} \leq V_{ult} \\
V_{ult} & \text{at } V_{pr} > V_{ult} \end{cases}
\]  

(12)

Lateral forces \( P_{1i}, P_{2i}, P_{3i}, P_{4i} \) determine the ability to keep the motor-grader on the planned trajectory of movement during work operations. The nonlinearity of the lateral force characteristics on the wheel is explained by the fact that with forces lower than the lateral slip forces, the wheel experiences elastic deformation, otherwise it is displaced along the supporting surface. The general dependence for the lateral holding force on the \( i \)-th wheel is as follows:

\[
P_i = \begin{cases} 
C_{ei} \cdot \Delta S_i + \lambda_{ei} \cdot \dot{S}_i \cdot \text{sign}(\dot{S}_i) & \text{at } C_{ei} \cdot \Delta S_i + \lambda_{ei} \cdot \dot{S}_i \cdot \text{sign}(\dot{S}_i) \leq N_i \cdot \varphi_{ad.i} \\
N_i \cdot \varphi_{ad.i} & \text{at } C_{ei} \cdot \Delta S_i + \lambda_{ei} \cdot \dot{S}_i \cdot \text{sign}(\dot{S}_i) \geq N_i \cdot \varphi_{ad.i} \end{cases}
\]  

(13)
Table 2. Analytical dependencies for determining the components of the main vector of resistance on the part of the developed medium.

| Soil digging pattern | Chip cross-sectional area, F | Moment of resistance on the part of the developed medium |
|----------------------|-----------------------------|--------------------------------------------------------|
| Digging with the full width of the blade | $F = B \cdot h$ | $M_{res} = \frac{B}{4} (W_c + W_{pr} + W_{up})$ |
| | | $W_c = \frac{1}{2} - B \cdot h \cdot k \cdot \varphi_a$, |
| | | $V_{pr} = \begin{pmatrix} \frac{1}{2} V_{pr} + \frac{B^2 \cdot h \cdot \varphi_a}{8} \end{pmatrix}$ at $V_{pr} \leq \frac{1}{2} V_{ult}$ |
| | | $\frac{1}{2} V_{ult}$ at $V_{pr} > \frac{1}{2} V_{ult}$ |
| Linear digging pattern | $F = \frac{B \cdot h}{2 \cos \varepsilon}$ | $M_{res} = \left( \frac{W_c + W_{up}}{3} + \frac{W_{pr}}{2} \right) \cdot \frac{B}{2} \cdot \left( \frac{b}{\sin \alpha} + \frac{l_3}{\sin \alpha} \right) \cdot \frac{h}{\tan \varepsilon}$ |
| | | $W_c = \frac{1}{2} \left( \frac{b}{\sin \alpha} + \frac{l_3}{\sin \alpha} \right) \cdot k \cdot \varphi_a \cdot \tan \varepsilon$ |
| | | $V_{pr} = \begin{pmatrix} V_{pr} - \frac{1}{6} \left( \frac{B}{2} \cdot \left( \frac{b}{\sin \alpha} + \frac{l_3}{\sin \alpha} \right) \right) \cdot \varphi_a \end{pmatrix}$ at $V_{pr} \leq V_{ult}$ |
| | | $V_{ult}$ at $V_{pr} > V_{ult}$ |
| Digging with blade angle | $F = \frac{h^2}{\sin 2\varepsilon}$ | $M_{res} = \left( W_c + W_{pr} + W_{up} \right) \cdot \frac{h}{3} \left( \frac{1}{\tan \varepsilon} - \frac{2}{3 \tan 2\varepsilon} \right)$ |
| | | $W_c = k \cdot F \cdot \varphi_a = k \cdot \varphi_a \left( \frac{h^2}{2} \cdot \tan \varepsilon \right)$ |
| | | $V_{pr} = \begin{pmatrix} V_{pr} - \frac{1}{6} \left( \frac{h^2}{\tan \varepsilon} - \frac{2}{3 \tan 2\varepsilon} \right) \cdot \varphi_a \end{pmatrix}$ at $V_{pr} \leq V_{ult}$ |
| | | $V_{ult}$ at $V_{pr} > V_{ult}$ |

In the given dependence: $C_{el}$ is the coefficient of the wheel elasticity in the lateral direction; $\lambda_{el}$ is the coefficient of the wheel damping in the lateral direction; $\varphi_{ad,l}$ is the coefficient of adhesion of the wheel with the support surface in the lateral direction; $\Delta S_i$ is the increment of displacement of the $i$-th wheel in the lateral direction; $\dot{S}_i$ is the speed of displacement of the $i$-th wheel in the lateral direction; $N_i$ is the support reaction on the $i$-th wheel.

Changing the mode of the motor-grader motion from plane to rotational relative to the blade stop point leads to a change in the structure of resistance forces on the part of the developed medium. The studies carried out enabled to record the analytical dependences to determine the value of the main moment of external resistances acting on the blade (Tables 1, 2).

Thus, the mathematical model of the motor-grader movement on a plane support surface in the process of performing working operations is fully specified.
5. Conclusion
The analysis of the conducted experimental studies showed that in the process of performing working operations, at least three options of the motor-grader movement are possible: along a straight-line trajectory without deviations from the planned one; along a curved trajectory simultaneously with lateral sliding and forward movement; along a patch-linear trajectory, when the machine moves straight ahead, then stops, independently turns in place and continues moving straight-line again.

All three variants of motion can be adequately described using a dynamic diagram, which includes two groups of second-degree differential equations of motion. The first group of equations describes the plane motion of the machine along a horizontal support plane. The second equation describes the rotational motion of the machine on the support plane around the blade stop point. The transition from one variant of motion to another is carried out as a result of the analysis of an additional condition in the form of an inequality that determines the balance between the driving forces and the external resistance forces.

The values of the force factors included in the equations of movement depend on the type of working operation being performed and are described using analytical dependencies that are different from each other. The developed mathematical model of the motor-grader movement takes into account all the specified features.

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