Small Cosmological Constants from a Modified Randall-Sundrum Model

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Abstract. We study a mechanism, inspired from the mechanism for generating the gauge hierarchy in Randall-Sundrum model, to investigate the cosmological constant problem. First we analyze the bulk cosmological constant and brane vacuum energies in RS model. We show that the five-dimensional bulk cosmological constant and the vacuum energies of the two branes all obtain their natural values. Finally we argue how we can generate a small four-dimensional effective cosmological constant on the branes through modifying the original RS model.
1. Introduction

Since the early works of Kluza and Klein [1] about unifying gravitational and electromagnetic interactions based on five-dimensional spacetime, the topic of extra dimensions and their physical implications has been considered by numerous physicists. The main topic of models with extra dimensions in recent years has been in the form of “brane world” scenarios, in which all matter fields are confined to a “brane” while only gravity propagates along the extra dimensions, with different purposes of solving problems ranging from hierarchy problem to symmetry breaking. Randall-Sundrum (RS) model [2][3] aims at generating the hierarchy between the TeV scale and the Planck scale, with a five-dimensional metric in an $AdS_5$ spacetime

$$ds^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2$$

(1)

The extra dimension is compactified as $S^1/Z_2$, and there are two 3-branes which locate at the orbifold fixed points $\phi = 0, \pi$. The term $e^{-2kr_c|\phi|}$ is called “warped factor”. The action in RS model is

$$S = \int d^4x dy \sqrt{-g} [2M^3R - \Lambda] + \sum_{i=1,2} \int d^4x \sqrt{-g^{(i)}} [\mathcal{L}_i - \Lambda_i]$$

(2)

This action includes a five-dimensional cosmological constant $\Lambda$ and two brane vacuum energies denoted by $\Lambda_i$. $\mathcal{L}_i$ represent the lagrangian of matter fields on the branes, while $g_{\mu\nu}^{(i)}$ represent the induced metric on the branes. Randall and Sundrum showed that the four-dimensional Planck mass $M_{Pl}$ can be related to the five-dimensional fundamental mass scale $M$ as

$$M_{Pl}^2 = \frac{M^3}{k} \left[1 - e^{-2kr_c\pi}\right]$$

(3)

where $r_c$ represents the length of the fifth dimension. This means $M_{Pl}$ depends only weakly on $r_c$ in the large $kr_c$ limit, which is completely different other models involving extra dimensions. Therefore, Randall and Sundrum assumed that $M_{Pl} \approx M$ as the fundamental mass scale. It is also shown in RS model that a hierarchy between a physical mass parameter $m$ and a fundamental mass parameter $m_0$ can be naturally generated

$$m = e^{-kr_c\pi} m_0$$

(4)

If $e^{kr_c\pi}$ is of order $10^{15}$, then this mechanism produces TeV physical mass scales from fundamental mass parameters not far from the Planck scale. In the seconde paper [3] RS discussed how to recover four-dimensional gravity through using an infinite uncompactified extra dimension.

The observed small value of the cosmological constant raised the so called cosmological constant problem [4]. A straightforward analyze shows that this constant should be of order $M_{Pl}^4$, which is nearly 120 times of magnitude larger than its observed value [5]. There are many works on solving the cosmological constant problem, such
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as works based on supergravity [6] and string theory [7]. In the brane-world models, if there is four-dimensional sources in five-dimensional spacetime, the effects of the four-dimensional brane sources can be balanced by a five-dimensional cosmological constant to get a theory in which the effective cosmological constant on the brane would be vanishing. This leads to the so called “warped” extra dimensions. There are many attempts of solving the cosmological constant problem through brane-world models [8].

In this paper we first analyze the vacuum energies and cosmological constant in the conventional RS model. We show that the five-dimensional cosmological constant and the four-dimensional cosmological constants of the two branes are of their “natural” value: \(-\Lambda \sim M^5_{Pl}\) and \(-V_{vis}(= V_{hid}) \sim M^4_{Pl}\). Then in section 3 we discuss the cosmological constant problem, without concerning the hierarchy problem. We analyze a mechanism which can generate a small four-dimensional effective cosmological constant on the brane, not the gauge hierarchy, through modifying the original RS model.

2. Bulk and Brane Cosmological Constants in RS model

It is straightforward to analyze the vacuum energies in RS model. In RS model the four-dimensional Planck mass can be expressed as [2]

\[
M^2_{Pl} = \frac{M^3}{k} \left[1 - e^{-2kr_c \pi}\right], \quad k = \sqrt{\frac{-\Lambda}{24M^3}} \tag{5}
\]

In order to solve the hierarchy problem, according to RS, \(e^{kr_c \pi}\) should be of order \(10^{15}\). Then

\[
M^2_{Pl} \sim \frac{M^3}{k} = M^3 \sqrt{\frac{24M^3}{-\Lambda}} \tag{6}
\]

In RS model, the fundamental mass scale is set by identifying the four-dimensional Planck mass \(M_{Pl}\) with the five-dimensional scale \(M\), then we have

\[-\Lambda \sim M^5_{Pl}\]

which is the natural value of \(\Lambda\) as the five-dimensional cosmological constant. On the other hand, in RS model the vacuum energies, or brane tensions, of the two branes, are determined by the five-dimensional cosmological constant \(\Lambda\) and the five-dimensional mass scale \(M\)

\[-V_{vis} = V_{hid} = 24kM^3\]

Again, if we set \(M_{Pl} \sim M\), we can get

\[-V_{vis} = V_{hid} \sim M^4_{Pl}\]

which is also natural because \(V_{vis}\) and \(V_{hid}\) are the four-dimensional vacuum energies on the visible and hidden 3-branes. Therefore, in RS model the five-dimensional and four-dimensional vacuum energies reach their natural values if the mechanism of solving the hierarchy problem works well.
In RS model the only required fine-tuning is \( kr_c \sim 10 \), so that the magnitude of the extra dimension is
\[
r_c \sim 10^{18} \text{GeV}^{-1} \sim 10^{-34} \text{m}
\]
which is of the magnitude of the Planck length.

3. A Small Cosmological Constant

Notice that in RS model the vacuum energies of the two branes already exist in the setup of the action (2). Here we consider the condition that there are no pre-existing vacuum energies on the branes; the four-dimensional effective cosmological constants on the branes are induced by the five-dimensional bulk cosmological constant. This change does not affect the form of the warped factor in RS model, so Equations (5) still hold (notice that the brane vacuum energies do not exit here, so the discussion about them do not hold anymore). However, we will need an extremely small \( r_c \) here, as well as the constrain that \( M_{Pl} \sim M \), so that from Equation (5) we have
\[
-\Lambda \sim M_{Pl}^5(1 - e^{-2kr_c\pi})^2 = M_{Pl}^5\varepsilon^2
\]
which is different from Equation (7). Here we require that \( \varepsilon \) is extremely small. The reason for this requirement will be seen latter.

Like in RS model, the fifth dimension is \( S^1/\mathbb{Z}_2 \). We start with the action of 5D gravity
\[
S = \int d^4x \int_{-\pi r_c}^{\pi r_c} dy \sqrt{-G(-\Lambda + 2M^3R)}
\]
and the metric
\[
ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2
\]
where \( y = r_c\phi \) represents the extra dimension.

In order to induce the four-dimensional effective cosmological constant, we integrate over the fifth dimension \( y \)
\[
S_\Lambda = \int d^4x \sqrt{-6M^3\Lambda}(1 - e^{-4kr_c\pi})
\]
If we refer the four-dimensional mass scale \( M_{Pl} \) as the fundamental mass scale, i. e. if we set \( M_{Pl} \sim M \), then according to the our previous discussion, the bulk cosmological constant take the value in Equation (11), so that
\[
S_\Lambda \sim \int d^4x \ M_{Pl}^4(1 - e^{-4kr_c\pi})
\]
Compare this result to the four-dimensional effective action on the brane at \( y = r_c\pi \), whose induced metric is \( g_{\mu\nu}^{\text{vis}} = e^{-2kr_c\pi}\eta_{\mu\nu} \)
\[
S_{\text{eff}} = \int d^4x \sqrt{-g_{\text{vis}}(-\Lambda_{(4)}^{\text{vis}} + 2M_{Pl}^2R_{(4)}^{\text{vis}})}
\]
we get
\[ -\Lambda_{(4)}^{vis} \sim M_{Pl}^4 (1 - e^{-2kr_c\pi}) (e^{4kr_c\pi} - 1) \sim M_{Pl}^4 \varepsilon \delta \]

(17)

We can see, if \( \varepsilon \delta \) is of order \( 10^{-120} \), then we can generate a small 4D effective cosmological constant, \( -\Lambda_{(4)}^{vis} \sim 10^{-47}\text{GeV}^4 \), with a slightly larger five-dimensional cosmological constant (11) in the bulk.

Recall that in RS model, in order to generate an exponential hierarchy between TeV scale and Planck scale, they require \( kr_c \sim 10 \) so that \( r_c \sim 10^{-34}\text{m} \). Now in order to generate the huge “hierarchy” between the natural value and observed value of the cosmological constant, \( r_c \) has to be extremely small. In this case, the bulk cosmological constant and the vacuum energies on the two branes are all very small due to the smallness of \( r_c \).

Notice that this small \( r_c \) would be too small to solve the gauge hierarchy problem. However, since our concern here is the cosmological constant problem, not the hierarchy problem, this result only means that we cannot solve both of the two problems at the same time in this modified model. Also, this induced 4D effective cosmological constant is on the “visible” brane which locates at \( y = r_c\pi \). Similarly, for the “hidden” brane at \( y = 0 \), the effective action is (the induced metric on this brane is \( g_{\mu\nu}^{hid} = \eta_{\mu\nu} \))

\[
S_{eff} = \int d^4x \sqrt{-g_{hid}}(-\Lambda_{(4)}^{hid} + 2M_{Pl}^2 R_{(4)}^{hid})
\]

(18)

we find that on this brane the induced cosmological constant can be again very small

\[ -\Lambda_{(4)}^{hid} \sim M_{Pl}^4 \varepsilon (1 - e^{-4kr_c\pi}) \]

(19)

This means that if \( r_c \) is extremely small, we can naturally obtain a small effective cosmological constant on both of the two branes.

4. Conclusion

The mechanism of solving the gauge hierarchy problem in RS model can be used to investigate the cosmological constant problem, if we make some changes to the original RS model. We show that in the original RS model the five-dimensional bulk cosmological constant and the vacuum energies of the two branes obtain their natural value. Through assuming that the four-dimensional cosmological constants on the two branes are the effective vacuum energies induced by the bulk cosmological constant, we are able to obtain two naturally small 4D effective cosmological constants on the two branes from a small five-dimensional cosmological constant in the bulk. This mechanism will put a more restrictive constrain on the size of the extra dimension than the original RS model, that \( r_c \) has to be extremely small. However, like in RS model, this is the only requirement in order to generate a small cosmological constant.
Acknowledgments

This work is supported by the CAS Knowledge Innovation Project (No.KJCX3-SYW-N2, No.KJCX2-SW-N16) and the Science Foundation of China (10435080, 10575123).

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