On the massless “just-so” solution to the solar neutrino problem

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We study the effect of the non-resonant, vacuum oscillation-like neutrino flavor conversion induced by non-standard flavor changing and non-universal flavor diagonal neutrino interactions with electrons in the sun. We have found an acceptable fit for the combined analysis for the solar experiments total rates, the Super-Kamiokande (SK) energy spectrum and zenith angle dependence. Phenomenological constraints on non-standard flavor changing and non-universal flavor diagonal neutrino interactions are considered.

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Non-standard neutrino interactions with matter can generate neutrino oscillations. This phenomenon was suggested by Wolfenstein in his seminal 1978 paper \( \text{[1]} \). Applications of this idea to the solar neutrino problem were first suggested in 1991 \( \text{[2,3]} \) when it was observed that resonantly enhanced neutrino oscillations induced by non-standard neutrino flavor changing (FC) as well as non-universal flavor diagonal (FD) neutrino interactions can explain the solar neutrino experimental data \( \text{[4]} \) which clearly indicates a solar neutrino flux smaller than what is predicted by the standard solar models \( \text{[5]} \).

Interestingly enough, such oscillations can be resonantly enhanced even if neutrinos are massless and no vacuum mixing angle exists \( \theta \), as a result of an interplay between the standard electroweak neutrino charged currents and non-universal non-standard flavor diagonal neutrino interactions with matter. In fact, in this mechanism, resonance plays a crucial role in order to provide a viable solution to the solar neutrino problem \( \text{[5]} \).

It should be emphasized that if such nonstandard neutrino FC and FD interactions exist only with electrons, no resonant conversion can happen because the mixing angle in matter is constant, as we will see later, contrary to the case of usual MSW effect \( \text{[6]} \), or the case with \( d,u \)-quarks FC and FD interactions. From this point of view, the oscillation induced by non-standard neutrino interactions with electrons alone is similar to the vacuum oscillation mechanism despite the difference that it occurs only in matter, inside the sun.

This non-resonant neutrino conversion can be useful to explain the solar observations. The first discussion on this possibility appeared in Ref. \( \text{[7]} \) where non-resonant neutrino oscillation induced by FC and FD interactions only with electron in the solar matter was mentioned as a possible solution to the solar neutrino problem.

Nevertheless, so far, no quantitative analysis of this scenario was presented.

In this brief report we investigate this possibility by performing a detailed fit to the most recent solar neutrino data. We conclude that non-resonant neutrino oscillations induced by non-standard neutrino interactions can only provide a rather poor fit to the total rates observed by all the solar neutrino experiments coming from Homestake, Gallex/GNO, Sage and SK \( \text{[8]} \) whereas when we include also the full SK recoil electron spectrum and the zenith angle dependence, the fit become an acceptable one. We find also that this fit requires the new non-standard neutrino interactions parameters to be very large.

Here we assume that neutrinos have non-standard FC as well as FD interactions only with electrons which could be realized in some models such as minimal supersymmetric standard model without R-parity \( \text{[9]} \) or \( SU(3)_C \otimes SU(3)_L \otimes U(1)_Y \) (331) models \( \text{[10]} \). Under this assumption, the evolution equation for massless neutrinos in matter can be expressed as \( \text{[5]} \):

\[
\frac{i}{\hbar} \frac{d}{dr} \begin{pmatrix} A_e(r) \\ A_\mu(r) \end{pmatrix} = \begin{pmatrix} 0 & \epsilon_{el} \\ \epsilon_{el} & 0 \end{pmatrix} \begin{pmatrix} A_e(r) \\ A_\mu(r) \end{pmatrix},
\]

where \( A_\alpha(r) \) and \( A_\mu(r) \) \((\alpha, \mu = e, \mu, \tau)\) are, respectively, the probability amplitudes to detect a \( \nu_e \) and \( \nu_\ell \) at position \( r \) and

\[
\epsilon_{el} = \frac{G_{\nu_e \nu_\ell}}{G_F} \quad \text{and} \quad \epsilon'_{el} = \frac{G_{\nu_\ell \nu_\ell} - G_{\nu_e \nu_\ell}}{G_F},
\]

decribe, respectively, the relative strength of the FC and the FD (but non-universal) interactions of the \( |\alpha, \beta = e, \mu, \tau\rangle \) denotes the effective coupling of the respective interaction.

In this mechanism the mixing angle in matter \( \theta_m \) does not depend on the electron density and is simply given by,

\[
\sin^2 2\theta_m = \frac{4\epsilon^2}{(1 - \epsilon')^2 + 4\epsilon^2}.
\]

We see that no MSW like resonance can occur because the mixing angle in matter is constant and does
not change along the neutrino trajectory (however see Ref. [12]).

Let us introduce the two variables \( r \) and \( \phi \) which define the production point of neutrinos in the sun as shown in Fig. 1. Then, for given values of \( (\epsilon, \epsilon') \) and a given production point in the sun defined by \( r \) and \( \phi \) the survival probability of electron neutrinos at the solar surface can be written as [6].

$$P(\nu_e \rightarrow \nu_e; r, \phi) = 1 - \sin^2 2\theta_m \sin^2 \frac{\Psi(r, \phi)}{2}, \quad \text{(4)}$$

where

$$\Psi(r, \phi) \equiv \sqrt{4\epsilon^2 + (1-\epsilon')^2} \sqrt{2G_F} \int_0^{x_{\text{max}}} N_e(r, \phi, x) dx, \quad \text{(5)}$$

where the \( N_e(r, \phi, x) \) is the electron density profiles along the neutrino trajectory which starts at the creation point \((r, \phi)\) corresponding to \( x = 0 \) and ends at the solar surface corresponding to \( x = x_{\text{max}} \). Note that there is no energy dependence in the probability.

From Eq. (4) we can estimate the oscillation length as

$$L_{\text{osc}} = \frac{2\pi}{\sqrt{2G_FN_e \sqrt{(1-\epsilon')^2 + 4\epsilon^2}}} \approx \frac{2.4 \times 10^2}{\sqrt{(1-\epsilon')^2 + 4\epsilon^2}} \left[ \frac{65 \text{ mol/cc}}{N_e} \right] \text{ km}, \quad \text{(6)}$$

where we take \( N_e = N_e(R \approx 0.1R_\odot) \approx 65 \text{ mol/cc} \) as a reference value. From Eq. (6) we see that if either \( |1-\epsilon'| \) or \( \epsilon \) is of the order of 0.01, the oscillation length is typically less than a few percents of the solar radius in the neutrino production region. This implies that for such values of \( \epsilon \) and \( \epsilon' \) there are many oscillations before neutrinos reach the solar surface and the final survival probability which is averaged over the neutrino production point will be,

$$\langle P(\nu_e \rightarrow \nu_e) \rangle \approx 1 - \frac{1}{2} \sin^2 2\theta_m, \quad \text{(7)}$$

for any values of \( r \) and \( \phi \) and therefore, for any sources of neutrinos [6]. Therefore, such rapid oscillation cannot fit the solar neutrino data well.

As pointed out in Ref. [3], an interesting possibility remains if both \(|1-\epsilon'|\) and \(\epsilon\) is smaller than \( \sim 0.01 \). For such \( \epsilon' \) and \( \epsilon \), if \( \Psi \sim (2n + 1)\pi \) with small \( n \), neutrinos produced as \( \nu_e \) can be almost \( \nu_x \) \((x = \mu, \tau)\) at the solar surface. On the other hand, if \( \Psi \sim 2n\pi \nu_e \) remains as \( \nu_e \) at the solar surface.

Since neutrinos from different nuclear reaction origins have different production distributions, there is a possibility that properly adjusting the parameter \( \epsilon \) and \( \epsilon' \) neutrinos from different reaction origins could be oscillated into another flavor in such a way that the solar neutrino data can be accounted for. In principle, this could happen if neutrinos oscillate only once or a few times before they reach the solar surface, similar to what happen to the case of the vacuum long-wavelength oscillation solution to the solar neutrino problem [3].

In order to settle this issue we have performed a detailed \( \chi^2 \) analysis using the latest standard solar model by Bahcall et al. [3] (BP98 SSM) as well the latest results of the current solar neutrino experiments coming from Homestake, Gallex/GNO, SAGE and SK [1].

The fitting procedure is as follows. We first compute the survival probability of \( \nu_e \) at the solar surface by the formula in Eq. (4) for various different production points defined by \( r \) and \( \phi \) as in Fig. 1. Then we compute averaged survival probabilities for neutrinos from different sources, i.e., \( pp, pep, \ ^7\text{Be}, \ ^8\text{B}, \ ^{13}\text{N} \) and \( ^{15}\text{O} \) (we neglect other minor contributions from \( ^{17}\text{F} \) and \( \text{hep} \) for simplicity) taking into account the neutrino production point distribution from BP98 SSM. After we compute the expected solar neutrino signal for each experiment, using the survival probability obtained above, we perform a \( \chi^2 \) analysis following the prescription given in Ref. [1].

We show in Fig. 2 the allowed parameter region deter-
determined by our $\chi^2$ analysis. We have used only the total observed rates of solar neutrinos by four experiments. The best fit is obtained at $|\langle e \rangle, |\epsilon' - 1| = (2.4, 2.9) \times 10^{-4}$ with $\chi^2_{\min} = 7.5$ for 4 (data) - 2 (free parameters) = 2 d.o.f. which corresponds to 2.4 % C.L. indicating a poor fit. This is because the integrations of the survival probability over the variables $r$ and $\phi$ tend to kill the just-so suppressions of the neutrino fluxes and the final averaged probabilities from different sources end up with rather similar values to each other.

However, the situation is still better than energy independent suppression since the following relation for the probabilities from different sources end up with rather similar values to each other.

Let us to include also the spectrum and zenith angle dependence in our fit. First, let us note that this mechanism does not distort the SK energy spectrum since the conversion probability is completely energy independent. Therefore, contribution in $\chi^2$ from the spectrum is just constant (does not depend on $\epsilon'$ and $\epsilon$) and the allowed parameter region shown in Fig. 2 is not affected by the spectrum. Second, for the range of parameters we are considering in this work, there would be no significant effect of the earth matter because the oscillation length in the earth is much larger than the earth radius. So again, contribution in $\chi^2$ from the zenith angle dependence is also just constant and the allowed parameter region shown in Fig. 2 is not affected by the zenith angle dependence.

The total combined $\chi^2_{\min}$ can be computed by simply adding the two constant contributions from spectrum and zenith angle without affecting the allowed parameter region presented in Fig. 2. In this case, despite the fact that the fit is poor only with the total rates, when we combined SK spectrum as well as zenith angle dependence, we have obtained $\chi^2_{\min} = 25.6$ for 24 d.o.f. which corresponds to 34.4 % C.L.

Here let us consider, as an interesting exercise, that the case when the systematic error of the Homestake is assumed to be 3 times larger than it has been reported. In Fig. 3 we present the region allowed by the rates under this assumption. We have obtained $\chi^2_{\min} = 3.3$, for rates only which indicate the significant improved over the case presented in Fig. 2. We notice that this kind of exercise could be worthwhile to consider to take into account the possibility of some unknown systematic effect of the Homestake experiment as it has not been calibrated with a radioactive source.

The main problem appearing in the solution to the solar neutrino problem based on FC and non-universal FD neutrino interaction with electrons is related with the magnitude of the phenomenologically required non standard parameters. Our statistical analysis shows that although the FC parameter $\epsilon$ does not need to be very high, $(\epsilon_{el} \approx 10^{-3})$, the non-universal FD parameter $\epsilon'_{el}$ is found to be of order of 1.

The value for the FC parameter $\epsilon$ is compatible with the available phenomenological tests to the flavor conservation law. In fact, the most stringent constraints on this parameter are due to the upper bounds on $\mu^- \rightarrow e^- e^+ e^-$ and $\tau^- \rightarrow e^- e^+ e^-$:[15]:

$$\begin{align*}
\text{BR}(\mu^- \rightarrow e^- e^+ e^-) &< 1.0 \times 10^{-12}, \\
\text{BR}(\tau^- \rightarrow e^- e^+ e^-) &< 2.9 \times 10^{-6},
\end{align*}$$

at 90% C.L.. Normalizing the above bounds to the measured rates of the related lepton flavor conserving decays, $\text{BR}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) \approx 100\%$ and $\text{BR}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = 0.1781$[15], we obtain

$$\begin{align*}
\epsilon_{e\mu} &\equiv G_{e\mu}/G_F < 1.0 \times 10^{-6}, \\
\epsilon_{e\tau} &\equiv G_{e\tau}/G_F < 4.2 \times 10^{-3}.
\end{align*}$$

Note, furthermore, that these bounds on $\epsilon$ can be also relaxed by a factor of 5-6 due to the breaking of the $SU(2)_L$ symmetry[14]. Therefore, assuming that the neutrino transitions involve the first and the third families, the required value of $\epsilon$ is compatible with the phenomenological limits.

The challenge to this solution is related with the required value of the parameter $\epsilon'_{el}$ since universality experimental tests in the leptonic sector are very much stringent. In Ref.[16] the constraints involving the second and third lepton families, i.e., interactions involving transitions of the type $\nu_\mu \leftrightarrow \nu_\tau$ are obtained. It was found that [16],

$$\epsilon'_{el} < 3.8 \times 10^{-3}.$$
Note, however, that the parameter relevant for our present analysis of the solution to the solar neutrino problem involves necessarily the first neutrino family ($\nu_e$). Such constraint can be obtained following the same steps of Ref. [3]. No direct limit can be obtained to $\epsilon'_{e\tau}$. Nevertheless, since $\epsilon'_{e\tau} = \epsilon'_{\mu\tau} - \epsilon'_{e\tau}$, limits on this parameter are found considering the experimental constraints of Eq. (11) and limits on $\epsilon'_{\mu\tau}$.

Non-zero values for $\epsilon_{ee}$ ($\epsilon_{\mu\mu}$) gives a additional contribution to $\nu_e e \rightarrow \nu_e e$ ($\nu_\mu e \rightarrow \nu_\mu e$) cross section and can put constraints on $\epsilon_{ee}$ and $\epsilon_{\mu\mu}$. We use the more recent data about the $\nu_e e \rightarrow \nu_e e$ total cross section [17]. This cross section is a function of $\epsilon_{el}$ and $G^A_{\nu_e \nu_e}/G_F$ (the axial part of the effective coupling of the respective interaction). We obtained

$$-2.56 < G_{\nu_e \nu_e}/G_F < 0.63,$$

(12)

at 90% C.L. for arbitrary $G^A_{\nu_e \nu_e}/G_F$. Taking the value quoted by Ref. [12]: $-0.18 < G_{\nu_e \nu_e}/G_F < 0.14$. We obtained that $\epsilon'_{e\mu}$ is bounded to

$$-0.81 < \epsilon'_{e\mu} < 2.70,$$

(13)

where the limit is at 90% C.L. Using the constraint from Eq. (12) and from Eq. (13), we get finally,

$$-1.81 < \epsilon'_{e\tau} - 1 < 1.70,$$

(14)

at 90% C.L. From this constraint, we conclude that is possible to satisfy the experimental constraints of FD couplings and at same time to be compatible with the allowed region of the solar neutrino analysis. A additional caution is necessary because the same FD couplings that induce neutrino oscillations can also change the detection cross section, $\langle \sigma (\nu_e e \rightarrow \nu_e e) \rangle$ used for SK. We check that the absolute values of elastic cross section inside the range given in Eq. (12) are compatible with the assumed theoretical errors used in the solar neutrino analysis. Also the shape of recoil electron in SK is not changed significantly due the FD couplings.

Concluding, we showed here for the first time a quantitative analysis of non-standard flavor changing and non-universal flavor diagonal neutrino interactions with electrons as a possible candidate to solve the solar neutrino problem. If the parameters, $|\epsilon'_{el} - 1| \ll 1$ and $\epsilon_{el} \approx 10^{-4} - 10^{-3}$ then we can get an acceptable fit for the combined analysis of the solar experiments total rates, the SK energy spectrum and the SK zenith angle dependence. We conclude that the constraints on the violation of universality allow us a small $\epsilon'_{e\tau}$ and a large value for $\epsilon'_{\mu\tau}$ and is compatible with the preferred values of our solar neutrino analysis. In this solution, no spectrum distortion, no zenith angle dependence and no seasonal effects are expected. Also only negative results are expected in long-baseline experiments due the very large oscillation length.

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