Recursive parameter estimation: asymptotic expansion

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Abstract This paper is concerned with the asymptotic behaviour of estimation procedures which are recursive in the sense that each successive estimator is obtained from the previous one by a simple adjustment. The results of the paper can be used to determine the form of the recursive procedure which is expected to have the same asymptotic properties as the corresponding non-recursive one defined as a solution of the corresponding estimating equation. Several examples are given to illustrate the theory, including an application to estimation of parameters in exponential families of Markov processes.

Keywords Recursive estimation · Estimating equations · Stochastic approximation · Exponential families of Markov processes

1 Introduction

Let $X_1, \ldots, X_n$ be random variables with a joint distribution depending on an unknown parameter $\theta$. Then an $M$-estimator of $\theta$ is defined as a solution of the estimating equation

$$\sum_{i=1}^{n} \psi_i(v) = 0,$$  \hspace{1cm} (1)

where $\psi_i(v) = \psi_i(X^i_1; v)$, $i = 1, 2, \ldots, n$, are suitably chosen functions which may, in general, depend on the vector $X^i_1 = (X_1, \ldots, X_i)$ of all past and present observations. If $f_i(x, \theta) = f_i(x, \theta|X_1, \ldots, X_{i-1})$ is the conditional probability