Theory of two-dimensional magneto-photonic crystals using complex Fourier factorization

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Abstract. The plane wave expansion method for calculating magneto-photonic crystals (MPCs) is modified via complex Fourier factorization whose basic principles are briefly explained. The new approach is used to simulate the photonic band structure of a two-dimensional MPC made as Bi:YIG rods arranged with square periodicity. The considerable improvement of the convergence properties is demonstrated for selected points of the bands.

1. Introduction
The plane wave expansion (PWE) method for calculating photonic crystal structures [1] uses the procedures of coupled wave theory treating the optics of periodic structures [2, 3]. In the case of periodic discontinuities (typical for photonic crystals) the method suffered from poor convergence before the discovery of the correct Fourier factorization (FF) rules [4], which have then been successfully applied to anisotropic [5], slanted [6], arbitrary-relief [7], and two-dimensional (2D) [8] periodic systems.

We have recently developed a method of applying the FF rules to 2D diffraction gratings made as 2D-periodic cylindrical rods of an isotropic material [9], following and generalizing the techniques previously developed in Refs. [10, 11, 12]. In this paper we use this approach to a 2D magneto-photonic crystal (MPC) made as 2D-periodic cylindrical rods of magnetic material with a uniaxial anisotropy forced by an external magnetic field [13]. We call our approach the complex Fourier factorization (CFF) method because our polarization transformation employs generally elliptic polarization, simply by using complex-valued Jones matrices.

2. Theoretical approach
The PWE method is based on solving the equation for eigenmodes of a MPC, \( \varepsilon^{-1} \nabla \times (\nabla \times \mathbf{E}) = (\omega/c)^2 \mathbf{E} \), in the Fourier (reciprocal) space, where \( \omega \) is the eigenfrequency, \( c \) the light velocity in vacuum, \( \mathbf{E} \) the vector of electric field, and \( \varepsilon \) the tensor of anisotropic permittivity, whose components depend on the orientation of the magnetization vector \( \mathbf{M} \). In the Fourier space the multiplication of the components of the permittivity tensor and the electric field can be treated by the Laurent rule \( [\varepsilon_{jk}E_k] = [\varepsilon_{jk}][E_k] \), where \( [f] \) denotes a column vector of the Fourier coefficients of a function \( f \), while \( [g] \) denotes a Toeplitz-like matrix (a 2D analogy of the Toeplitz matrix) composed of the Fourier coefficients of a function \( g \). By applying the Laurent rule for the entire tensor of permittivity \( \varepsilon_{jk} \), we obtain a numerically ineffective method (here referred to as model A), because the functions \( \varepsilon_{jk} \) and \( E_k \) have, in general, concurrent discontinuities.
On the other hand, there are various methods of FF which transform those quantities to obtain pairs of functions without concurrent discontinuities, for which the Laurent rule would be valid. The principle of the CFF method follows the normal vector method [10, 11, 12], where the polarization basis was distributed within the periodic cell so that the normal and tangential components of electric field were treated independently [Fig. 1(a)]. This enabled the correct application of the FF rules as $[\varepsilon E_n] = [1/\varepsilon][E_n]$, and $[\varepsilon E_t] = [\varepsilon'][E_t]$, where $E_n$ and $E_t$ denote the normal and tangential components of the electric field (and where permittivity is for clarity assumed isotropic). However, this approach (generalized to arbitrarily anisotropic medium [14], it is here referred to as model B) only deals with linear polarizations, which ignores the fact that the transformation matrix between the Cartesian and the normal/tangential component bases of polarization becomes discontinuous at the center and on the boundaries of the periodic cell, which slows down the convergence of the numerical implementation.

To avoid the discontinuities, the CFF method (here referred to as model C) uses generally elliptic polarization bases, so that the matrix of transformation between polarization bases becomes completely continuous. The distribution of one of the polarization vectors is displayed in Fig. 1(b), where linear polarizations (oriented normal to the cylinder) are only on the cylinder’s edges, the right-hand circular polarization is at the center and on the boundaries of the periodic cell, and intermediate elliptical polarizations are continuously varying everywhere else.

3. Numerical example
An example of the photonic band structure of a MPC, made as a 2D square lattice of bismuth-substituted yttrium iron garnet (Bi:YIG) cylinders, calculated with model C for the maximum Fourier harmonics (retained inside the crystal) chosen $N = 4$, is displayed in Fig. 2, assuming material and geometrical parameters same as in Ref. [13] (diagonal permittivity of the cylinder is 5.1, the magneto-optical parameter is 0.1, and the ratio of the lattice constant to the cylinder’s radius is 1:0.4). The inset shows the geometry of the 1st Brillouin zone of the MPC and the orientation of the magnetization vector along the $x$-direction. The convergence properties at the point $\Gamma$ are compared with the models A and B in Fig. 3 for the 3rd (a), 4th (b), 5th (c), and 6th (d) band, respectively. Here we see that the speed of the convergence is similar to all the three models, but the limits to which the curves are approaching are considerably different. A particular difference is between the models A and C, which is probably due to numerical errors accumulated during the linear-algebraic operations using different matrices. Fig. 4 shows the
Figure 2. Example of the band structure, with the 1st BZ in the inset.

Figure 3. Convergence properties at the point $\Gamma$ for the 3rd–6th band (a–d).

convergence properties of the three models at the point $X$ for the 1st (a), 2nd (b), 3rd (c), and 4th (d) band, respectively. Here we see that all the three models converge to the same limit, but with considerably different convergence speeds. Obviously the model C yields the best performance, which suggests that the correct limits in Fig. 3 also correspond to the model C.
4. Conclusion
In summary, the principle of applying CFF into the PWE method for calculating 2D MPC band structures was briefly explained, and a numerical example was demonstrated. By comparing with previous models, it was shown the the CFF method yields the best numerical performance, including the correct limit and the fastest speed of the convergence curves.

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References
[1] Ho K M, Chan C T and Soukoulis C M 1990 Phys. Rev. Lett. 65 3152–3155
[2] Petit, R (ed) 1980 Electromagnetic Theory of Gratings (Springer)
[3] Maystre D 1984 Prog. Optics 21 1–67
[4] Li L 1996 J. Opt. Soc. Am. A 13 1870–1876
[5] Li L 1998 J. Mod. Opt. 45 1313–1334
[6] Chernov B, Neviere M and Popov E 2001 Opt. Commun. 194 289–297
[7] Popov E and Neviere M 2000 J. Opt. Soc. Am. A 17 1773–1784
[8] Li L 1997 J. Opt. Soc. Am. A 14 2758–2767
[9] Antos R 2009 Opt. Express 17 7269–7274
[10] Schuster T, Ruoff J, Kerwien N, Rafler S and Osten W 2007 J. Opt. Soc. Am. A 24 2880–2890
[11] Gotz P, Schuster T, Frenner K, Rafler S and Osten W 2008 Opt. Express 16 17295–17301
[12] David A, Benisty H and Weisbuch C 2006 Phys. Rev. B 73 075107
[13] Khanikaev A B, Inoue M and Granovsky A B 2006 J. Magn. Magn. Mater. 300 104–107
[14] Visnovsky S and Yasumoto K 2001 Czech. J. Phys. 51 229–247