Performance Analysis of Blind Beamforming Algorithms in Adaptive Antenna Array in Rayleigh Fading Channel Model

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Abstract. In this paper, we analyze the performance of adaptive blind algorithms -- i.e. Kaiser Constant Modulus Algorithm (KCMA), Hamming CMA (HAMCMA) -- with CMA in a wireless cellular communication system using digital modulation technique. These blind algorithms are used in digital signal processor of adaptive antenna to make it smart and change weights of the antenna array system dynamically. The simulation results revealed that KCMA and HAMCMA provide minimum mean square error (MSE) with 1.247 dB and 1.077 dB antenna gain enhancement, 75% reduction in bit error rate (BER) respectively over that of CMA. Therefore, KCMA and HAMCMA algorithms give a cost effective solution for a communication system.

1. Introduction
In any communication system, the aim is to reduce interference level to get better quality of service [1]. This can be possible with digital modulation techniques [2] to make the system more spectrally efficient using beamforming concept [3]. Beamforming is either switch or adaptive. Adaptive one is implemented on signal processor with antenna array to make the antenna smart [4]. The windowing techniques have been well addressed in array signal processing to reduce the side lobes in [5]. In [6] three different versions of the CMA are introduced, each one motivated by a different signal processing problem. CMA originally employs an FIR filter which accepts complex input data having complex coefficients [7]. Paper [8] addresses CMA algorithms, their analytical work regarding rate of convergence and behaviour. It also scrutinizes the “capture” problem identified in [7]. In [9], a CM Array and its extension are presented. An analytic method for solving the CM problem is described in [10]. In this article, we analyzed the performance of KCMA and HAMCMA with CMA in a communication system which provides optimum results for adaptive antenna array system. This research is the extension of our previous work reported in [11]. This paper investigates beamforming with windowed CMA (i.e. KCMA and HAMCMA) in a cellular communication system and compared with CMA in terms of directive gain, MSE reduction, BER analysis and convergence.

The next section describes the system model. Section III displays a mathematical model for blinds algorithms. Section IV presents simulation results. Discussion and comments is presented in section V. Finally, section VI concludes the paper.
2. System Model

Consider a wireless cellular communication model as shown in Fig 1. A random digital signal is modulated using 16 point QAM technique. When this modulated signal is passed through Rayleigh fading channel, it becomes faded. As a result, echoes of the modulated signal is produced which arrive at the array of antenna system. This multipath fading phenomena cause fluctuations in the received signal’s amplitude, phase, and angle of arrival. Then received signal is processed through these blind beamforming algorithms one by one. The received signals are filtered by weights of antenna elements. The outputs of each antenna elements are coupled to the summer and generate the array output. The weights of antenna arrays determine the magnitude and phase of signals which helps in beam formation. As a result optimum beam towards desired direction is achieved and minimum energy is transmitted towards interferers, thus creating null towards them. The output of beamformer is demodulated and computes the BER. The output of demodulator has noise parameters and is passed through an affine projection (AP) adaptive filter that extracts desired signal.

![Diagram of wireless cellular communication model](image)

Fig 1. Implementation of blind adaptive algorithm in Rayleigh Fading Channel Model

3. Mathematical Model

3.1. Kaiser Constant Modulus Algorithm

The KCMA algorithm is the unification of CMA and Kaiser Window. Now consider a linear beamformer employing window technique method as shown in Fig. 1. Then its output will be

\[ y_k = X_k (W_k)^T \]  

where \( W_k = \text{kaiser}(N, \beta) W_k \) is the initial estimate weight vector using Kaiser window. \( \beta \) is the Kaiser window parameter that affects the sidelobes attenuation and \( N \) represents the number of elements in an array. The signal array vector having different amplitudes and phases is written by

\[ X_k = [x_1, x_2, \ldots, x_N]^T \]

The error signal for blind algorithm is given by

\[ e_k = y_k - \frac{y_k}{|y_k|} \]
Putting value of \( y_k \) in (3) and differentiate w.r.t. weight \( W \), then we get

\[
\frac{\partial e_k}{\partial W} = [X_k kaiser(N, \beta)] = X_k kaiser(N, \beta)
\] (4)

Putting this value in the gradient estimate of the form giving by

\[
\nabla_k = 2e_k \begin{bmatrix} \frac{\partial e_k}{\partial W_0} \\ \vdots \\ \frac{\partial e_k}{\partial W_k} \end{bmatrix} = 2e_k (X_k kaiser(N, \beta))
\] (5)

Mathematically, the gradient decent rule [12 chap 2 (2.35) and 4 (4.36)] is given by

\[
W_{k+1} = W_k - \mu \nabla_k
\] (6)

Putting value of gradient estimate (5) into (6), we get

\[
W_{k+1} = W_k - 2\mu e_k kaiser(N, \beta)X_k
\] (7)

This is the required weight vector for KCMA using Kaiser Window, where \( \mu \) is the step size.

3.2. Hamming Constant Modulus Algorithm

This proposed algorithm [11] is designed using Hamming window technique. The weight vector for HAMCMA can be derived on the same pattern as KCMA and is given by

\[
W_{k+1} = W_k - 2\mu e_k hamming(N)X_k
\] (8)

The array output signal obtained with the sample weights is given by

\[
y_k = X_k \hat{W}_k^T
\] (9)

where \( \hat{W}_k = hamming(N)W_k \) is the initial estimate weight vector using Hamming window function and \( X_k \) is the input samples. The weight matrix update (8) approaches its true value when the number of samples grows.

3.3. Constant Modulus Algorithm

CMA is a blind algorithm based on the idea to maintain gain on the signal while minimizing the total output energy. The weight update equation of CMA algorithm [4] is given by

\[
W_{k+1} = W_k - 2\mu e_k X_k
\] (10)

where \( e_k \) is the cost function, using (3) and \( \mu \) is the step size. The array output is given by
3.4. Affine Projection Adaptive Filter
The Affine Projection filter is an adaptive filter that has fast convergence property. It is employed to extract a desired signal from a noise-corrupted signal by filtering out the noise. The adaptive filter produces an output which is a true copy of input desired signal.

4. Simulation Results
The QAM signal (16 numbers of bits per symbol) is applied for simulation purpose, to illustrate the effect of various parameters of digital communication system as thrash out in incoming paragraphs:

4.1. Role of Smart Adaptive Antenna in Communication System under Study
Uniform linear array is taken for adaptive antenna array system with $N = 10$ and the distance between two elements is supposed to be as $\lambda/2$. 100 samples are taken for simulation purpose. The angle of arrival (AOA) for desired user is 0 degree. The array factor plot is shown in Fig. 2 for KCMA, CMA and HAMCMA that shows one desired signal arriving from the angle 0 degree and two interferer arriving from angles other than desired angle. The ratio of main lobe to the first side lobe is 4.48 dB, 4.46 dB and 3.63 dB for CMA, KCMA and HAMCMA respectively. It is proved that KCMA is giving 1.247 dB gain improvements than that of CMA and at the same time HAMCMA provides 1.077 dB improvements as compared to CMA. However, the null depth performances of all these algorithms under study are same and cancel the interference by placing a null at the angle of interferer.

4.2. Mean Square Error Performance
The minimum MSE describes the performance of the given system. The minimum MSE for three blind beamforming algorithms are shown in Fig. 3 which indicates that KCMA and HAMCMA have minimum MSE (almost zero) at 5th and at 7th iterations as compared to CMA. It is worth mentioning that minimum MSE for these three blind beamforming algorithms changes and effected by number of iterations. Therefore, performance of KCMA and HAMCMA is better over that of CMA.

4.3. Bit Error Rate Performance
In Fig 4, the theoretical BER curve is taken as a reference value for comparison. The plots as shown in Fig 4 describe that the error rates obtained are smaller as compared to theoretical BER; therefore, the BER performance is greatly improved. The computed BER values for HAMCMA, KCMA and CMA algorithms at SNR=9 are 0.001, 0.001 and 0.004 respectively. The BER values for HAMCMA and KCMA are 25% (0.001 is 25% of 0.004) and 25% (0.001 is 25% of 0.004) as compared to CMA. Then the BER is reduced by 75% by both HAMCMA and by KCMA. Table compares various results obtained from Fig. 2, 3 and 4 for blind beamforming algorithms under study.

\[ y_k = X_k (W_k)^T \]  \hspace{1cm} (11)
Fig 2. Array factor versus AOA for **desired user**

Fig 3. Mean Square Error performance

Fig 4. Comparison between Calculated and Theoretical BER
5. Discussion and Comments
The findings of simulation are:

A. The directional gain of the KCMA beamforming algorithm is more as compared to HAMCMA and CMA.
B. The proposed algorithms save power because a directional gain of proposed model is optimum.
C. KCMA and HAMCMA converge at 5th and at 7th iterations respectively.
D. KCMA is giving 1.247 dB whereas HAMCMA provides 1.077 dB gain improvement than that of CMA.
E. KCMA and HAMCMA reduce BER by 75% as compared to CMA.
F. The convergence rate of HAMCMA is smaller than that of CMA and KCMA as given in Table.

| Table | System Input and Throughput Estimate for Blind Algorithms under study |
|-------|--------------------------------------------------|
|       | Input Parameters       | Algorithms        |
|       | No. of Samples        | KCMA   | HAMCMA | CMA   |
|       | AOA (degree)          | 0      | 0      | 0     |
|       | Element Spacing (d)   | 0.5λ   | 0.5λ   | 0.5λ  |
|       | No. of Elements       | 10     | 10     | 10    |
|       | Step Size μ           | 0.05   | 0.05   | 0.05  |
|       | Output Parameters     |         |        |       |
|       | Beam width (degree)   | 20     | 20     | 20    |
|       | Array Gain (dB)       | 9.589  | 8.99   | 8.342 |
|       | Sidelobe Level (dB)   | 2.15   | 2.47   | 1.86  |
|       | BER at 9 dB SNR       | 0.001  | 0.001  | 0.004 |
|       | Convergence rate (Sec)| 1.0299 | 0.0926 | 0.1211|

6. Conclusion
With respect to CMA, KCMA and HAMCMA provide optimum performance. Therefore, these algorithms can be implemented at base station of mobile communication systems as one of the option.

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