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Multiscale mixed hydrodynamics in line contacts

Received: 10 July 2021 / Accepted: 30 October 2021 / Published online: 20 November 2021
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Abstract In the hydrodynamic line contact, there is a very thin layer physically adsorbed to the solid surface. When the surface separation is sufficiently small, the Hertzian contact zone will be completely filled with the boundary layer, while in most of the inlet zone still occurs continuum hydrodynamics, which lies between the mated adsorbed layers. The present paper studies this mixed hydrodynamics by a multiscale analysis. The boundary layer flows are simulated by the flow factor approach model. The intermediate continuum fluid flow is simulated by the continuum fluid model. The flow equations are given respectively for the boundary layers and for the intermediate continuum fluid. The final governing equation has been obtained relating the surface separation to the solid surface speeds and the carried load. The calculation results show that for a high rolling speed the hydrodynamic behavior in the contact agrees with the classical hydrodynamic theory; however for a critically low rolling speed it gives the surface separation greatly higher than that calculated from the classical hydrodynamic theory, showing the significant adsorbed layer effect.

Keywords Adsorbed layer · Fluid · Hydrodynamics · Multiscale · Surface separation

List of symbols

- $b$: Half Hertzian contact width
- $C_{y,1}$, $C_{y,2}$: Flow factor
- $\eta_{bf,1}$, $\eta_{bf,2}$: Normalized boundary layer thickness
- $D$: Fluid molecule diameter
- $E_a$, $E_b$: Elastic Young’s moduli of the upper and lower contact surfaces respectively
- $2/E_v$: Elasticity ratio
- $G$: Fluid elasticity
- $h$: Continuum fluid film thickness
- $h_{bf}$: Thickness of the adsorbed boundary layer
- $h_c$: Continuum fluid film thickness at the Hertzian contact center
- $h_{cr,bf,1}$, $h_{cr,bf,2}$: Critical thickness for formulating the rheological properties of the adsorbed layer in the whole surface separation
- $h_{cr,bf,1}$, $h_{cr,bf,2}$: Critical thickness for formulating the rheological properties of the boundary layer confined in the Hertzian contact center
- $h_{tot}$: Surface separation
- $h_{c,tot}$: Surface separation at the Hertzian contact center

Communicated by Andreas Öchsner.

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1 Introduction

Line contact is a popular concentrated contact mode between counter-formal solid surfaces such as occurring on the mechanical elements of gears and roller bearings [1]. Classical hydrodynamic theories [2–4] ignore the effect of the boundary layer physically adsorbed to the solid surface. However, in a line contact the surface separation can be very small and may be on the same scale with the thickness of the adsorbed boundary layer [5,6]. It is still not clear that whether classical hydrodynamic theories stand for this line contact and when and how the hydrodynamics in this line contact deviate from classical hydrodynamics.

Chauveteau et al. [7] experimentally found that for the attractive pore wall the viscosity of the adsorbed layer near the wall was higher than the bulk viscosity in the central region for polymer solutions in the very...
small pore the radius of which is on the same scale with the polymer molecule diameter. They suggested that
the effective viscosity across the pore radius can be easily calculated by assuming the adsorbed layer and the
intermediate continuum fluid respectively with different viscosities. Chan and Horn [8] suggested the ordering
molecule layers of octamethylcyclo-tetrasiloxane (OMCTS) adjacent to the solid mica surface the thickness
of which should be no more than 2.5 nm in their drainage experiment. Similar with Chauveteau et al. [7],
they showed that the effective viscosity across the surface separation can be easily calculated by assuming
the solidified layer on the solid surface and the continuum fluid flow between the two adsorbed layers when
the surface separation was between 50 and 5 nm. They found that by this way the drainage deviation from
the Reynolds continuum lubrication theory can be satisfactorily explained. In the multiscale simulation for
micro channel flows, Zhang [9] showed that the adsorbed boundary layer with the thickness 2 nm will have a
considerable influence on the channel flow even when the channel height was 100 nm, and the channel flow
was actually mainly in the continuum flow rather than in the non-continuum flow.

Obviously, the presence of the adsorbed layer on the contact surface can have a considerable influence on the
hydrodynamics even when the adsorbed layer thickness is far lower than the surface separation. Conventional
hydrodynamic theories for line contacts ignored the effect of the adsorbed layer [2–4]. The author ever analyzed
the performance of line contact hydrodynamics by assuming the solid layers respectively on the two contact
surfaces, and obtained the results qualitatively agreeable with the experimental observations [10]. By assuming
an equivalent solid layer on the contact surface and the equivalent intervening continuum fluid flow, the author
also analyzed the performance of line contact hydrodynamics when the surface separation was very low so
that the effect of the surface plastic deformation was involved [11]. Later, the author further analyzed the
performance of line contact hydrodynamics with very low surface separations by assuming the solidified layer
on the contact surface and the intermediate flowing molecule layers between the two solid layers, when the
effects of the surface plastic and thermal deformations were involved [12].

In the classical modeling of multiscale microchannel flows where the thickness of the adsorbed molecule
layer on the wall was comparable with the channel height, the adsorbed layer flow was simulated by molecular
dynamics simulation (MDS), and the intermediate continuum fluid flow was simulated by the continuum
fluid model [13–18]. The obvious shortcoming of this multiscale approach is that it should normally require
unaffordable computational time and computer storage for engineering flows.

Instead of by assuming the flowing continuum fluids or solid adsorbed layers as done by Chauveteau
et al. [7] and Chan and Horn [8], the author’s earlier study [19] took the adsorbed layer flow as essentially
the molecular scale flow different from the intermediate continuum fluid flow, well corresponding to the
previous MDS-continuum hybrid schemes [11–16]. The density and viscosity increases and the discontinuity
and inhomogeneity effects across the layer thickness can be simultaneously considered in that model. Three
closed-form explicit flow equations for two-dimensional flows have been derived respectively for the two
adsorbed boundary layer flows and for the intermediate continuum fluid flow [19]. By that multiscale approach,
the computational time and computer storage can be largely saved.

By using that multiscale approach, the author has studied the mode of multiscale hydrodynamics in the
line contact where the intermediate continuum fluid exists in the whole contact [20].

The present paper attempts to study a new mode of multiscale hydrodynamics in the line contact where
the continuum fluid film vanishes and consequently there is only the adsorbed boundary layer in the Hertzian
contact zone due to the excessively heavy loads. This is actually the mode of multiscale mixed hydrodynamics,
further progress of the former multiscale hydrodynamic mode [20] with the load increase. In the present study,
the pure boundary layer flows both in the Hertzian contact zone and in the area adjacent to the Hertzian
contact zone are described by the closed-form nanoscale flow equation [21], while the flow in most of the
inlet zone is described by the multiscale flow equations developed previously [19] owing to both the adsorbed
boundary layer flow and the intermediate continuum fluid flow. This multiscale approach is different from
any of the previous ones. The present study is thus further progressive and the obtained results should be of
significant interest to the understanding of the hydrodynamic behavior in the line contact with ultra-low surface
separations.

2 Mixed hydrodynamic line contact

Figure 1a shows the studied mixed hydrodynamics in the line contact where for the surface separation smaller
than \(2h_{bf}\) is only present the adsorbed boundary layer. Here \(h_{bf}\) is the thickness of the adsorbed layer on either
of the solid surfaces. The pure boundary layer area includes the Hertzian contact zone (from \(-1 \leq X \leq 1\)) and
the inlet area (from \( X_0 \leq X \leq -1 \)) close to the Hertzian contact zone. In most of the inlet zone (for \( X < X_0 \)) occurs multiscale hydrodynamics because of both the adsorbed layer effect and the intermediate continuum fluid hydrodynamic effect. In this line contact, \( h_{\text{tot}} \) is the surface separation, \( h_{c,\text{tot}} \) is the surface separation at the Hertzian contact center (where the pressure gradient vanishes), \( h \) is the intermediate continuum fluid film thickness, and \( u_a \) and \( u_b \) are respectively the circumferential speeds of the upper and lower contact surfaces.

When \( h_{c,\text{tot}} > 2h_{bf} \), the line contact will become as Fig. 1b shows, and its hydrodynamics is the first mode of multiscale hydrodynamics in the line contact, which has been studied in [20]. For a full understanding and the purpose of comparison, in the present study the surface separation \( h_{c,\text{tot}} \) covers a wide range which not only includes the hydrodynamics in Fig. 1a but also includes those in Fig. 1b.

3 Analysis

The analysis here is based on the following assumptions:

(a) The adsorbed layers on the two contact surfaces are identical.
(b) The flow is isothermal, and no side leakage occurs i.e. both the contacts in Fig. 1a, b are two dimensional and in the axial direction of the roller there is a negligible flow.
(c) The compressibility of both the continuum fluid and the adsorbed layer due to the film pressure and the contact-fluid interaction is negligible.
(d) The film pressure influence on the fluid film viscosity and the adsorbed layer viscosity is described by the Barus equation.
(e) No interfacial slippage occurs on any interface.
(f) Both the contact surfaces are perfectly smooth.
(g) Both the contact surfaces undergo the elastic deformation.
(h) The fluid is Newtonian.
(i) The contact working condition is steady-state.
3.1 Determination of the coordinate $X_0$

According to the theory [4] of the two-dimensional elastic contact, when the two contact surfaces are in the elastic deformation under the externally applied load $w$, the dimensionless surface separation in the inlet zone (i.e. for $X \leq -1$) in the contact in Fig. 1a is expressed as:

$$H_{\text{tot}}(X) = H_{c,\text{tot}} - \frac{4}{\pi} [X(X^2 - 1)^{0.5} + \ln[-X - (X^2 - 1)^{0.5}]]W$$

(1)

where $H_{\text{tot}} = h_{\text{tot}}/R$, $H_{c,\text{tot}} = h_{c,\text{tot}}/R$, $X = x/b$, and $W = w/(E_aR)$. Here, $R = R_aR_b/(R_a + R_b)$, $R_a$ and $R_b$ are respectively the curvature radii of the upper and lower contact surfaces, $b$ is the half Hertzian contact width, $x$ is the dimensional coordinate, $w$ is the load per unit contact length carried by the contact, $2/E_v = (1 - \nu_a^2)/E_a + (1 - \nu_b^2)/E_b$, $E_a$ and $E_b$ are respectively the elastic Young’s moduli of the upper and lower contact surfaces, and $\nu_a$ and $\nu_b$ are respectively the Poisson’s ratios of the two contact surfaces.

Accordingly, the dimensionless intermediate continuum fluid film thickness in Fig. 1a, which is located in the inlet zone, is expressed as:

$$H(X) = H_0 - \frac{4}{\pi} [X(X^2 - 1)^{0.5} + \ln[-X - (X^2 - 1)^{0.5}]]W, \text{ for } X \leq X_0$$

(2)

where $H = h/R$, $H_0 = H_{c,\text{tot}} - 2H_{bf}$, and $H_{bf} = h_{bf}/R$.

As shown in Fig. 1a, by putting $H(X) = 0$ in Eq. (2), the dimensionless coordinate $X_0$ of the boundary between the pure boundary layer area and the inlet continuum fluid hydrodynamic area is thus solved from the following equation:

$$2H_{bf} = H_{c,\text{tot}} - \frac{4}{\pi} [X_0(X_0^2 - 1)^{0.5} + \ln[-X_0 - (X_0^2 - 1)^{0.5}]]W.$$  

(3)

3.2 Flow equations

3.2.1 For $X < X_0$

In this area, the total flow rate through the contact consists of the adsorbed boundary layer flows and the intermediate continuum fluid flow, and the flow rate conservation equation is [20]:

$$q_{v,bf,A} + q_{v,bf,B} + q_{v,bf} = \frac{u_a + u_b}{2} h_{c,\text{tot}}$$

(4)

where $q_{v,bf,A}$ and $q_{v,bf,B}$ are respectively the volume flow rates per unit contact length of the upper and lower adsorbed boundary layers, and $q_{v,bf}$ is the volume flow rate per unit contact length of the intermediate continuum fluid. The right-hand side of Eq. (4) is the volume flow rate through the contact per unit contact length calculated from the contact center, where the pressure gradient vanishes.

By assuming no interfacial slippage on both the adsorbed layer–fluid interface and the adsorbed layer–contact surface interface and by assuming the adsorbed boundary layer as several molecule layers equivalently ordered normal to the contact surface, according to the earlier study [19], the flow rates $q_{v,bf,A}$ and $q_{v,bf,B}$ are respectively expressed as:

$$q_{v,bf,A} = u_a h_{bf} + \frac{u_b - u_a}{2} h_{bf} \frac{\varepsilon \lambda_{bf}}{2 \lambda_{bf} + C_y \left(1 + \frac{\Delta x}{b}\right)} + \frac{F_{1} h_{bf}^3}{12 h_{bf}^eff_{b,1}} \frac{\partial p}{\partial x}$$

$$- \frac{h_{bf}^3}{2 h_{bf}^eff_{b,1}} \frac{\partial p}{\partial x} \left(1 + \frac{1}{2 \lambda_{bf}} - \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta x}{h_{bf}}\right) \frac{\varepsilon}{1 + \frac{\Delta x}{b}}$$

(5)
and

\[ q_{v, bf} = u_a + u_b h + \frac{u_a - u_b}{2} h^2 \frac{\varepsilon\lambda_{bf}}{2\lambda_{bf} + C_y (1 + \frac{\Delta x}{\lambda})} + \frac{F_1 h_{bf}^2}{12\eta_{bf, 1}} \frac{\partial \rho}{\partial x} + \frac{h_{bf}^2}{2\eta_{bf, 1} \Delta_{bf, 1}} \frac{\partial \rho}{\partial x} \left( 1 + \frac{1}{2\lambda_{bf}} \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h} \right) \frac{\varepsilon}{1 + \frac{\Delta x}{\lambda}} \]  

(6)

where \( p \) is the hydrodynamic pressure, \( D \) is the fluid molecule diameter, \( \lambda_{bf} = h_{bf}/h, \eta_{bf, 1} \) is the effective viscosity of the physical adsorbed boundary layer and \( \eta_{bf, 1} = D h_{bf}/(n - 1)(D + \Delta x)(\Delta_{bf, 1}/\Delta_{bf, n-1}), \) \( \Delta x \) is the separation between the neighboring fluid molecules in the flow direction in the adsorbed layer, \( q_0 = \Delta_{n-1}/\Delta_{n} (\Delta_{n} \text{ is the separation between the } (j + l)\text{th and } j\text{th fluid molecules across the layer thickness}) \) and \( q_0 \) is constant, \( \eta_{line, j}/\eta_{line, j+1} = q_0^n \), \( \varepsilon = (2D_1 + I)/[(h_{bf} (n - 1)(\Delta_{bf, 1}/\Delta_{bf, n-1})]. \)

\[ F_1 = \eta_{bf, 1} (12D^2 \psi + 6D \psi)/h_{bf}^3, \]  

where \( n \) is the equivalent number of the fluid molecules across the adsorbed layer thickness, and \( \Delta_{n-2} \) is the separation between the neighboring fluid molecules across the adsorbed layer thickness just on the adsorbed layer–fluid interface. Here, \( I = \sum_{i=1}^{n-1} i(D_{\Delta_{n-1}}/\Delta_{n-1})_{\Delta_{bf, i}} \), \( \psi = \sum_{i=1}^{n-2} i(D_{\Delta_{n-1}}/\Delta_{n-1})_{\Delta_{bf, i}}, \) \( \phi = \sum_{i=0}^{n-1} i(D_{\Delta_{n-1}}/\Delta_{n-1})_{\Delta_{bf, i}} \), \( h_{bf}^3 = \sum_{i=0}^{n-1} i(D_{\Delta_{n-1}}/\Delta_{n-1})_{\Delta_{bf, i}} \). \( \Delta_{n-1}, i(D_{\Delta_{n-1}}/\Delta_{n-1})_{\Delta_{bf, i}}, \) \( \eta \) is the fluid bulk viscosity, and \( F_2 = 6\eta_{bf, 1} D(n - 1)(\Delta_{n-1}/\Delta_{n-1})_{\Delta_{bf, n-1}}/h_{bf}^2. \) On the right-hand side of Eq. (7), the third term is the coupled term showing the adsorbed layer effect on the intermediate continuum fluid flow.

Based on the Barus’ viscosity–pressure relation \( \eta = \eta_{0} e^{4p}, \) where \( \eta_{0} \) is the fluid bulk viscosity in the ambient condition and \( \alpha \) is the fluid viscosity–pressure index, substituting Eqs. (5)–(7) into Eq. (4) and rearranging gives:

\[ \left\{ \begin{array}{l}
\frac{h^3}{2} - \frac{h^3}{C_{y, 1}} \left[ \frac{F_2 \lambda_{bf}^2}{6} - \frac{\lambda_{bf}}{1 + \frac{\Delta x}{\lambda}} \left( \frac{1}{2} + \frac{\lambda_{bf}}{\lambda} \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \right) \right] - \frac{F_1 h_{bf}^3}{6C_{y, 1}} = \frac{h^3}{C_{y, 1}} (1 + \frac{\Delta x}{\lambda}) \left( 1 + \frac{1}{2\lambda_{bf}} \frac{q_0 - q_0^n}{q_0^{n-1} - q_0^n} \frac{\Delta_{n-2}}{h} \right) \frac{\varepsilon}{1 + \frac{\Delta x}{\lambda}} \\
\end{array} \right. 

\]

(8)

for \( X < \chi_0 \)

where \( C_{y, 1} = \eta_{bf, 1}/\eta. \) Equation (8) is the governing equation for the multiscale hydrodynamic flow in the area where \( X < \chi_0. \)

3.2.2 For \( X_0 \leq X \leq -1 \)

In this area, there is only the pure boundary layer flow, which can be described by the nanoscale flow equation [21]. The flow continuity equation for this area is [21]:

\[ \frac{S h_{tot}^3}{12C_{y, 2}} e^{-\alpha \rho} \frac{dp}{dx} = \frac{(u_a + u_b)\eta_a}{2} (h_{tot} - h_{c, tot}) \]  

(9)

where \( C_{y, 2} = \eta_{bf, 2}/\eta \), \( \eta_{bf, 2} \) is the effective viscosity of the boundary layer confined in the whole surface separation, and \( S \) is the parameter describing the non-continuum effect of the boundary layer. The term on the left-hand side of Eq. (9) is the Poiseuille flow rate of the boundary layer confined in the whole surface separation, which is strongly influenced both by the discontinuity and inhomogeneity effects, i.e. the non-continuum effect of the boundary layer as formulated by the parameter \( S \) and by the solidification effect of the boundary layer due to the confinement as formulated by the parameter \( C_{y, 2} \).
3.3 Solving the differential equations for the contact in Fig. 1a

Define the following dimensionless parameters:

\[ U = (u_a + u_b)h_a/(2E_vR), \quad P = p/p_h, \quad G = \alpha E_v, \quad \lambda = b/(4R) \]

Here, \( p_h \) is the maximum Hertzian contact pressure.

The dimensionless form of Eq. (8) is:

\[
\frac{1}{4} F(\lambda_{bf}) H^3 e^{-PG\lambda} \frac{dP}{dX} = U(H - H_0)
\]

where

\[ F(\lambda_{bf}) = \frac{1}{12} \frac{1 - \frac{1}{C_{y,1}}} {6} \left[ \frac{F_2\lambda_{bf}^2}{1 + \frac{\lambda_{bf}}{2}} \left( \frac{1}{2} + \lambda_{bf} - \frac{q_0 - q_0^n}{q_0^n - q_0^n} \Delta_{n-2\lambda_{bf}} \right) \right] - \frac{F_1\lambda_{bf}^3}{6C_{y,1}}
\]

\[ + \frac{\epsilon\lambda_{bf}^3}{C_{y,1}(1 + \frac{\lambda_{bf}}{2})} \left( 1 + \frac{1}{2\lambda_{bf}} - \frac{q_0 - q_0^n}{q_0^n - q_0^n} \Delta_{n-2\lambda_{bf}} \right) \]

(11)

Since \( P(-\infty) = 0 \), integrating Eq. (10) gives that:

\[
J_2 = \int_{-\infty}^{X_0} \frac{H - H_0}{H^3F(\lambda_{bf})} dX = \int_{0}^{X_0} e^{-PG\lambda} \frac{dP}{4U} = 1 - e^{-P(X_0)G\lambda} - \frac{1}{4UG\lambda}
\]

(12)

Solving Eq. (12) gives that:

\[
P(X_0) = -\frac{\ln(1 - 4UG\lambda J_2)}{G\lambda}
\]

(13)

The dimensionless form of Eq. (9) is:

\[
\frac{SH_{tot}^3}{48C_{y,2}} e^{-PG\lambda} \frac{dP}{dX} = U(H_{c,tot} - H_{tot})
\]

(14)

For the heavy load, as \( P(-1) \) is large enough so that \( e^{-P(-1)G\lambda} \approx 0 \), integrating Eq. (14) gives that:

\[
P(X_0) = -\frac{\ln(-4UG\lambda J_3)}{G\lambda}
\]

(15)

where \( J_3 = \int_{X_0}^{1} C_{y,2}(H_{tot} - H_{c,tot})dX/(SH_{tot}^3) \).

Solving the coupled equations (13) and (15) gives that:

\[
J_2 - 12J_3 = \frac{1}{4UG\lambda} = \frac{1}{UG} \left( \frac{\pi}{8W} \right)^{1/2}
\]

(16)

For a given contact–fluid interaction and given values of \( U, G \) and \( W \), the surface separation \( H_{c,tot} \) can be solved from Eq. (16).

3.4 Solution for the contact in Fig. 1b

The solution for the contact in Fig. 1b has been shown in [20]. For comparison, the equation for solving the dimensionless intermediate continuum fluid film thickness \( H_c \) at the center of the Hertzian contact zone in Fig. 1b is repeated as follows [20]:

\[
-0.2041 \left( \frac{W}{H_c} \right)^2 + 0.8876 \frac{W}{H_c} + \log \left[ \frac{4GU}{\pi F(\lambda_{bf,e})} \left( \frac{8}{\pi} \right)^{1/2} \right] - \frac{3}{2} \log W - 1.2725 = 0
\]

for \( 0.01 \leq \frac{W}{H_c} \leq 200 \)

(17)

where \( H_c = h_c/R \), the function \( F(\lambda_{bf,e}) \) is shown by Eq. (11), and \( \lambda_{bf,e} = h_{bf}/(kh_c) \). The value of \( k \) has been found to be around unity [20]. For a given contact–fluid interaction and given values of \( U, G \) and \( W \), the film thickness \( H_c \) can be solved from Eq. (17). The dimensionless surface separation at the center of the Hertzian contact zone in Fig. 1b is then \( H_{c,tot} = H_c + 2H_{bf} \).
3.5 Solution for the Newtonian fluid

Classical hydrodynamics assumed that there is no adsorbed layer effect and the fluid is Newtonian in the whole contact in Fig. 1a. For this case, the dimensionless surface separation $H_{c,N}$ at the center of the Hertzian contact zone in Fig. 1a is solved from the following regressed equation [20]:

$$-0.2041 \left( \frac{\lg W}{H_{c,N}} \right)^2 + 0.8876 \frac{\lg W}{H_{c,N}} + \lg(24.3825GU) - \frac{3}{2} \lg W - 1.2725 = 0,$$

for $0.01 \leq \frac{W}{H_{c,N}} \leq 200. \quad (18)$

4 Calculation

The calculations were made for the case $G = 4500$ and $W = 1.0E-3$, for a relatively strong contact–fluid interaction. This chosen case corresponds to a normal hydrodynamic lubricated steel contact such as in gears and roller bearings. For a steel–steel contact, the chosen dimensionless load ($W$) gives $p_h = 2.6$ GPa, which is a typically high contact pressure in the modern gear contacts carrying very heavy loads. For the chosen contact–fluid interaction, the effective viscosity of the adsorbed layer is considerably larger than the fluid bulk viscosity due to the strong physical adsorption to the solid surface. The effective viscosity of the pure boundary layer confined in the surface separation in the Hertzian contact zone is significantly increased with the reduction of the surface separation in the Hertzian contact zone owing to the increased solidification of the boundary layer in the confinement. As a representative interaction, here the parameter $C_{y,1}$ is expressed as [20]:

$$C_{y,1}(\tilde{H}_{bf}) = 1.8335 - \frac{1.4252}{\tilde{H}_{bf}} + \frac{0.5917}{\tilde{H}_{bf}^2} \quad (19)$$

where $\tilde{H}_{bf} = h_{bf}/h_{cr,bf,1}$, and $h_{cr,bf,1}$ is the critical thickness and here taken as 20 nm.

The parameter $C_{y,2}$ is formulated as [22]:

$$C_{y,2}(\tilde{H}_{tot}) = 1.8335 - \frac{1.4252}{\tilde{H}_{tot}} + \frac{0.5917}{\tilde{H}_{tot}^2} \quad (20)$$

where $\tilde{H}_{tot} = h_{tot}/h_{cr,bf,2}$, and $h_{cr,bf,2}$ is the critical thickness. On the location $X = X_0$, it should be satisfied that $C_{y,1}(\tilde{H}_{bf}) = C_{y,2}(2h_{bf}/h_{cr,bf,2})$. Thus, $h_{cr,bf,2} = 2h_{cr,bf,1} = 40$ nm.

The parameter $S$ is formulated as [22]:

$$S(\tilde{H}_{tot}) = \begin{cases} -1 & \text{for } \tilde{H}_{tot} \geq 1 \\ [0.4 - 1.374(\tilde{H}_{tot} - 0.035)^{-0.534}]^{-1} & \text{for } 0.035 < \tilde{H}_{tot} < 1 \end{cases} \quad (21)$$

In the present calculation, the other input parameter values were taken as follows [20]:

$$D = 0.5 \text{ nm}, \quad \Delta n_{-2}/D = \Delta x/D = 0.15, \quad m = 1.5, \quad n = 8, \quad q_0 = 1.2$$

These values give that $h_{bf} = 4.32$ nm.

The calculations required the numerical integration for $J_2$ and $J_3$. By dividing the integrated zone into a lot of sub-zones and in each sub zone sufficient discretized points were used, we achieved the high accuracy in calculating $J_2$ and $J_3$. Then, the surface separation $H_{c,tot}$ can be accurately solved from Eq. (16). The solution of $H_{c}$ from Eq. (17) and the solution of $H_{c,N}$ from Eq. (18) are also accurate.
5 Results

Figure 2a shows the variations of the calculated \( h_{c,\text{tot}} \) value with the dimensionless rolling speed \( U \) respectively for \( R = 0.1 \) mm and \( R = 0.01 \) mm. They are respectively compared with those calculated from the conventional Newtonian fluid model for the same cases. For a high rolling speed, the present calculation is nearly equal to the conventional calculation, and this indicates the negligible effect of the adsorbed layer. However, for a critically low rolling speed, the present calculation is much greater than the conventional one, showing the effect of the adsorbed layer very significantly increasing the surface separation. For a very low rolling speed, although the conventional Newtonian fluid model gives the almost vanishing surface separation, the present model still gives the small surface separation, which is important for avoiding the direct surface bumping and efficiently reducing the surface wear. The present results are fairly agreeable with the experimental results [8].

It is interestingly observed that for a given low dimensionless rolling speed, the present calculated dimensional surface separation \( h_{c,\text{tot}} \) is nearly constant, independent on the equivalent curvature radius \( R \) of the contact surfaces. According to the normalized parameter \( U = (u_a + u_b)\eta_a/\left(2E_v R\right) \), this actually means that for the same low dimensional rolling speed \( (u_a + u_b)/2 \), the dimensional surface separation \( h_{c,\text{tot}} \) is greater for a smaller \( R \), which gives a higher dimensionless rolling speed \( U \) for the same operating condition according to Fig. 2a. This may be the significant microscale effect of hydrodynamics, which should be of potential application value.

Figure 2b shows the results similar as in Fig. 2a respectively for \( R = 1 \) mm and \( R = 10 \) mm. Figure 3a plots the \( h_{c,\text{tot}} \) versus \( U \) curves respectively for different \( R \). It is shown that for a given relatively high dimensionless rolling speed \( U \), a higher value of \( R \) gives a significantly greater value of \( h_{c,\text{tot}} \). This qualitatively agrees with the conventional recognition. However, for a given critically low dimensionless rolling speed, the dimensional surface separation \( h_{c,\text{tot}} \) is independent on the equivalent curvature radius \( R \) of the two contact surfaces. This contradicts to the conventional results. It should just be due to the adsorbed layer effect. Figure 3b plots the \( h_{c,\text{tot}} \) Versus \( u \) curves respectively for different \( R \) when \( \eta_a = 0.1 \) Pa s and \( E_v = 209 \) GPa (representative of the steel–steel contact). Here, \( u \) is the dimensional rolling speed and \( u = (u_a + u_b)/2 \). Only for sufficiently high dimensional rolling speeds, the increase of \( R \) results in a higher value of \( h_{c,\text{tot}} \), agreeing with the conventional recognition. However for a critically low dimensional rolling speed such as smaller than 0.1 m/s, by contrary the reduction of \( R \) significantly increases the value of \( h_{c,\text{tot}} \), which is on the scales of 1 nm or 10 nm. This strongly indicates the microscale effect of hydrodynamics due to the adsorbed layer. Conventional hydrodynamics fail to reveal this important effect.

6 Commenting remarks

There are currently no available experimentally measured surface separations for the hydrodynamic line contact with the surface separation on the scales of 1 nm or 10 nm. A direct comparison between the present results and the experiments is still difficult to make. Figure 2a, b show that for sufficiently high rolling speeds the present calculation is equal to the conventional calculation based on the Newtonian fluid model, which has been verified for sufficiently large surface separations. For these high rolling speeds, the present calculation should be physically correct as the adsorbed layer effect is negligible. For critically low rolling speeds, the present calculation very significantly deviates from the conventional calculation showing the residual films remaining in the contact as shown by Fig. 2a, b. This result follows the experimental results for hydrodynamic point contacts with the surface separations no more than 10 nm [8]. The microscale effect of hydrodynamics in line contacts found in the present study should be examined in the future as the experiments on hydrodynamic line contacts with the equivalent curvature radius \( (R) \) of the two contact surfaces on the scales of 0.1 mm and 0.01 mm are really challenging.

7 Conclusions

A multiscale analysis is presented for the mixed hydrodynamics in the line contact where the surface separation in the Hertzian contact zone is so low that it is filled with the adsorbed boundary layer, while in most of the inlet zone the total flow consists of the adsorbed layer flows and the intermediate continuum fluid flow. The flow equations were respectively given for the Hertzian and inlet zones. Solving the coupled flow continuity equations in these two zones fast gives the surface separation for a given operating condition.
Fig. 2 Plots of the value of the dimensional surface separation $h_{c,tot}$ against the dimensionless rolling speed $U$ for different $R$ and its comparison with the conventional calculation result $h_{c,N}$ for the same operating condition.
Fig. 3 Plots of the value of the dimensional surface separation $h_{c,\text{tot}}$ against the dimensionless or dimensional rolling speed respectively for different $R$.

The calculations were made for a steel–steel contact under the load which gives the maximum Hertzian contact pressure 2.6 GPa. A relatively strong fluid–contact surface interaction was taken. It gives the thickness of the adsorbed layer on either of the contact surfaces as 4.32 nm. No interfacial slippage was assumed on any interface. The calculations were also extended to the cases where the surface separation in the Hertzian contact zone is so thick that it is filled with both the adsorbed boundary layers and the intermediate continuum fluid.
The calculated results show that for high rolling speeds, the hydrodynamics in a line contact follows conventional hydrodynamic theory (with the adsorbed boundary layer effect neglected). However for a critically low rolling speed, it largely deviates from the conventional hydrodynamic theory prediction with a much higher surface separation owing to the significant adsorbed layer effect. The results qualitatively agree with the experiments.

The significant microscale effect of hydrodynamics has been observed. That is: for a critically low dimensional rolling speed \( u \), the reduction of the equivalent curvature radius \( R \) of the two contact surfaces significantly increases the value of the surface separation \( h_{c,tot} \) which is on the scales of 1 nm or 10 nm, just because of the adsorbed boundary layer effect.

**Declarations**

**Conflict of interest** The author declares no competing interests.

**References**

1. Kalker, J.J.: On elastic line contacts. J. Appl. Mech. 39, 1125–1132 (1972)
2. Pinkus, O., Sternlicht, B.: Theory of Hydrodynamic Lubrication. McGraw-Hill, New York (1961)
3. Wolveridge, P.E., Baglin, K.P., Archard, J.F.: The starved lubrication of cylinders in line contact. Proc. Inst. Mech. Eng. 185, 1159–1169 (1970)
4. Johnson, K.L.: Contact Mechanics. Cambridge Press, Cambridge (1985)
5. Assay, D.B., Kim, S.H.: Effects of adsorbed water layer structure on adhesion force of silicon oxide nanoasperity contact in humid ambient. J. Chem. Phys. 124, 174712 (2006)
6. Cheng, S., Luan, B., Robbins, M.O.: Contact and friction of nanoasperities: effects of adsorbed monolayers. Phys. Rev. E 81, 016102 (2010)
7. Chauveteau, G., Tirrell, M., Omari, A.: Concentration dependence of the effective viscosity of polymer solutions in small pores with repulsive or attractive walls. J. Colloid Interface Sci. 100, 41–54 (1984)
8. Chan, D.Y.C., Horn, R.G.: The drainage of thin liquid films between solid surfaces. J. Chem. Phys. 83, 5311–5324 (1985)
9. Zhang, Y.B.: Power loss in multiscale mass transfer. Front. Heat Mass Transf. 13, 22 (2019)
10. Zhang, Y.B.: Modeling of molecularly thin film elastohydrodynamic lubrication. J. Balkan Trib. Assoc. 10, 394–421 (2004)
11. Zhang, Y.B.: Lubrication generated between two sliding parallel plane surfaces by micro asperities: part I—performance analysis. J. Comput. Theor. Nanosci. 8, 1111–1119 (2011)
12. Zhang, Y.B.: Lubrication analysis for a line contact covering from boundary lubrication to hydrodynamic lubrication: part I—micro contact results. J. Comput. Theor. Nanosci. 11, 62–70 (2014)
13. Atkas, O., Aluru, N.R.: A combined continuum/DSMC technique for multiscale analysis of microfluidic filters. J. Comput. Phys. 178, 342–372 (2002)
14. Liu, J., Chen, S., Nie, X., Robbins, M.O.: A continuum-atomistic simulation of heat transfer in micro- and nano-flows. J. Comput. Phys. 227, 279–291 (2007)
15. Sun, J., He, Y., Tao, W.Q.: Scale effect on flow and thermal boundaries in micro-/nano-channel flow using molecular dynamics-continuum hybrid simulation method. Int. J. Numer. Methods Eng. 81, 207–228 (2010)
16. Nie, X.B., Chen, S., Robbins, M.O.: A continuum and molecular dynamics hybrid method for micro- and nano-fluid flow. J. Fluid Mech. 500, 55–64 (2004)
17. Yang, X., Zheng, Z.C.: Effects of channel scale on slip length of flow in micro/nano channels. ASME J. Fluids Eng. 132, 061201 (2010)
18. Yen, T.H., Soong, C.Y., Tzeng, P.Y.: Hybrid molecular dynamics-continuum simulation for nano/mesoscale channel flows. Microfluid. Nanofluid. 3, 665–675 (2007)
19. Zhang, Y.B.: Modeling of flow in a very small surface separation. Appl. Math. Mod. 82, 573–586 (2020)
20. Zhang, Y.B.: Multiscale hydrodynamics in line contacts. Mech. Res. Commun. 111, 103658 (2021)
21. Zhang, Y.B.: The flow equation for a nanoscale fluid flow. Int. J. Heat Mass Transf. 92, 1004–1008 (2016)
22. Zhang, Y.B.: Lubrication analysis for a line contact covering from boundary lubrication to hydrodynamic lubrication: part I—micro contact results. J. Comput. Theor. Nanosci. 11, 62–70 (2014)