Neutrino Oscillations: from Standard and Non-standard Viewpoints

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In the standard model of neutrino oscillations, the neutrino flavor states are mixtures of mass-eigenstates, and the phenomena are well described by the neutrino mixing matrix, i.e., the PMNS matrix. I review the recent progress on parametrization of the neutrino mixing matrix. Besides that I also discuss on the possibility to describe the neutrino oscillations by a non-standard model in which the neutrino mixing is caused by the Lorentz violation (LV) contribution in the effective field theory for LV. We assume that neutrinos are massless and that neutrino flavor states are mixing states of energy eigenstates. In our calculation the neutrino mixing parts depend on LV parameters and neutrino energy. The oscillation amplitude varies with the neutrino energy, thus neutrino experiments with energy dependence may test and constrain the Lorentz violation scenario for neutrino oscillation.

Keywords: Fermion mixing; neutrino oscillation; Lorentz violation, unified parametrization

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The mixing between different generations of quarks and leptons is one of the most fundamental and important issues in particle physics. Parametrization of mixing matrices is an important step to understand the mixing of fermions. The Cabibbo\cite{1} Kobayashi and Maskawa\cite{2}(CKM) matrix $V_{\text{CKM}}$ describes the mixing of quarks of three generations, and the mixing could be regarded as a rotation from fermion mass eigenstates to flavor eigenstates. The abundant experimental data on neutrinos convincingly suggest the mixing of different generations of neutrinos, just analogous to that of quarks. To explain neutrino oscillations, the conventional scenario is to assume that neutrinos have masses. From this assumption, there is a spectrum of three or more neutrino mass eigenstates and the flavor state is the mixing state of mass eigenstates, and lepton mixing is described by the Pontecorvo\cite{3}Maki-Nakawaga-Sakata\cite{4}(PMNS) matrix $U_{\text{PMNS}}$.

A commonly used form of mixing matrix for three generations of fermions is
given by \[ V(\text{or } U) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \] (1)

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \) are the mixing angles and \( \delta \) is the CP violating phase. If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases \( \text{diag}(e^{i\alpha_{1}/2}, e^{i\alpha_{2}/2}, 1) \) multiplied to the matrix from right in the above. The CKM matrix is close to the unit matrix in different powers of the parameter \( \lambda \). The parametrization explicitly shows the deviations of the non-diagonal elements from the unit matrix in different powers of the parameter \( \lambda \) with \( \lambda = 0.2257^{+0.0009}_{-0.0010} \). The other parameters are \( \lambda = 0.814^{+0.021}_{-0.022}, \rho(1 - \lambda^{2}/2 + \ldots) = 0.135^{+0.031}_{-0.016}, \) and \( \eta(1 - \lambda^{2}/2 + \ldots) = 0.349^{+0.013}_{-0.017} \). The parameter \( \lambda \) serves as a good indicator of hierarchy of the mixing phenomenon in quark sector.

Quite different from quark mixing matrix, almost all the non-diagonal elements of the neutrino mixing matrix are large, only with the exception of \( V_{\text{ee}} \). So it is impractical to expand the matrix in powers of one of the non-diagonal elements, like the Wolfenstein parametrization of the quark mixing matrix. In practice one may parameterize the neutrino mixing matrix with other bases, such as

\[
\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1/2 & 1/2 & \sqrt{2}/2 \\ 1/2 & -1/2 & \sqrt{2}/2 \end{pmatrix}, \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix},
\]

which are called the bimaximal mixing pattern and the tri-bimaximal pattern respectively. There have been many different forms of parametrizations, and finding one which is simple and convenient to use is important as a tool for further theoretical and experimental studies.

There have been progress to parameterize the lepton mixing matrix based on idea of connection between the quark and lepton mixing matrices. The quark-lepton complementary (QLC) is an interesting example in this direction. The quark-lepton complementarity (QLC) relates quark and lepton mixing angles with

\[
\begin{align*}
\theta_{12}^{Q} + \theta_{12}^{L} &= \frac{\pi}{4}, \\
\theta_{23}^{Q} + \theta_{23}^{L} &= \frac{\pi}{4}, \\
\theta_{13}^{Q} &\sim \theta_{13}^{L} \sim 0,
\end{align*}
\] (3)

where the superscript \( Q \) indicates the mixing angles in the CKM matrix, and the superscript \( L \) indicates mixing angles in the PMNS mixing matrix \( U_{\text{PMNS}} \). The above relations enable one to express the mixing parameters of lepton mixing based on information of quark mixing parameters.
Nan Li and Shi-Wen Li performed the parametrization of the PMNS matrix based on this idea and got the following results in two cases by combining QLC with the Wolfenstein parametrization of CKM matrix. For Case 1, i.e., $\sin^2 \theta = A \lambda^3 (\zeta - i \xi)$, we get the PMNS matrix

$$
U = \begin{pmatrix}
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\
\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{2}}{2}
\end{pmatrix} + \lambda \begin{pmatrix}
\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\
\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{2}}{2}
\end{pmatrix} + \lambda^2 \begin{pmatrix}
-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\
-\frac{1}{2} (A - \frac{i}{2}) & \frac{1}{2} (A - \frac{i}{2}) & -\frac{\sqrt{2}}{2} A \\
-\frac{1}{2} (A + \frac{i}{2}) & \frac{1}{2} (A + \frac{i}{2}) & \frac{\sqrt{2}}{2} A
\end{pmatrix}
+ \lambda^3 \begin{pmatrix}
\frac{1}{2} A (1 - \zeta - i \xi) & \frac{1}{2} A (1 - \zeta - i \xi) & 0 \\
\frac{1}{2} A (1 - \zeta - i \xi) & \frac{1}{2} A (1 - \zeta - i \xi) & 0 \\
\frac{1}{2} A (1 - \zeta - i \xi) & \frac{1}{2} A (1 - \zeta - i \xi) & 0
\end{pmatrix} + O(\lambda^4). \tag{4}
$$

For Case 2, i.e., $\sin^2 \theta = A \lambda^3 (\zeta - i \xi)$, we get

$$
U = \begin{pmatrix}
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\
\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{2}}{2}
\end{pmatrix} + \lambda \begin{pmatrix}
\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\
\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{2}}{2}
\end{pmatrix}
+ \lambda^2 \begin{pmatrix}
-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\
\frac{1}{2} [1 + A (1 + \zeta + i \xi)] & \frac{1}{2} [1 + A (1 + \zeta + i \xi)] & \frac{1}{2} [1 + A (1 + \zeta + i \xi)] \\
\frac{1}{2} [1 + A (1 + \zeta + i \xi)] & \frac{1}{2} [1 + A (1 + \zeta + i \xi)] & \frac{1}{2} [1 + A (1 + \zeta + i \xi)]
\end{pmatrix}
+ \lambda^3 \begin{pmatrix}
A (\zeta - i \xi) & 0 & 0 \\
0 & A (\zeta - i \xi) & 0 \\
0 & 0 & A (\zeta - i \xi)
\end{pmatrix} + O(\lambda^4). \tag{5}
$$

From which we have the following observations: (1). The bimaximal mixing pattern is derived naturally as the leading-order approximation. (2). The Wolfenstein parameter $\lambda$ can characterize both the deviation of the CKM matrix from the unit matrix, and the deviation of the PMNS matrix from the exactly bimaximal mixing pattern. More explicitly, the range of $\lambda$ in PMNS matrix is calculated: $0.11 < \lambda_{\text{PMNS}} < 0.24$. In this unified parametrization, $\lambda$ here is just the Wolfenstein parameter of the CKM matrix, $\lambda_{\text{CKM}} = \sin \theta_{\text{CKM}} = 0.2243$. We can see that the above values are consistent with each other compared with the experimental data. However, we can also take the parameter $\lambda_{\text{PMNS}}$ as being not the same as the Wolfenstein parameter $\lambda_{\text{CKM}}$, and the symmetry between the quark and lepton mixing matrices will break slightly.

Current experimental data show that the $U_{\text{PMNS}}$ matrix is close to the tri-bimaximal pattern. Parametrization of the PMNS matrix around the tri-bimaximal pattern leads to more fast converging expansions. For an unified parametrization of both lepton and quark mixing matrices, Shi-Wen Li and Shi-Wen Li introduced another method of the parametrization of the CKM matrix. A new matrix was introduced instead of the unit matrix as the basis of the CKM matrix.

$$
V_b = \begin{pmatrix}
\frac{\sqrt{2} + 1}{2} & \frac{\sqrt{2} - 1}{2} & 0 \\
\frac{\sqrt{6} - 1}{2} & \frac{\sqrt{6} + 1}{2} & 0 \\
0 & 0 & 1
\end{pmatrix}. \tag{6}
$$

Though this new matrix is a little more complicated than the unit matrix, it is closer to reality. The deviations of the CKM matrix from the new matrix are rather small and the expansion converges very quickly. We have proved that this matrix can be combined with QLC to arrive at new parametrization of PMNS matrix with bases of tri-bimaximal pattern.
The new parametrization of the CKM matrix reads:

\[
V = \begin{pmatrix}
\frac{\sqrt{2} - 1}{\sqrt{6}} & \frac{\sqrt{2} - 1}{\sqrt{6}} & 0 \\
\frac{\sqrt{2} - 1}{\sqrt{6}} & \frac{\sqrt{2} - 1}{\sqrt{6}} & 0 \\
0 & 0 & 1
\end{pmatrix} + \lambda \begin{pmatrix}
-(3 - 2\sqrt{2}) & 1 & 0 \\
-1 & -(3 - 2\sqrt{2}) A & 0 \\
\sqrt{2} - 1 & \sqrt{2} + 1 A & 0
\end{pmatrix} + \lambda^2 \begin{pmatrix}
-(30\sqrt{3} - 21\sqrt{6}) & 0 & 0 \\
\sqrt{2} - 1 & -(30\sqrt{3} - 21\sqrt{6}) - \sqrt{2} + 1 A^2 & 0 \\
(1 - \sqrt{2} + 1 A) (\rho - i\eta) A & 3 - 2\sqrt{2} - \sqrt{2} + 1 A (\rho + i\eta) A & -\frac{1}{2} A^2
\end{pmatrix} + \mathcal{O}(\lambda^3).
\] (7)

For the PMNS matrix, we get the new parametrization with tri-bimaximal pattern in two cases:

\[
U = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} & 0 \\
-\frac{\sqrt{3}}{\sqrt{6}} & -\frac{\sqrt{3}}{\sqrt{6}} & \sqrt{\frac{2}{3}}
\end{pmatrix} + \lambda \begin{pmatrix}
2 - \sqrt{2} & -2\sqrt{2} - 2 & 0 \\
2 - \sqrt{2} - \frac{1}{\sqrt{6}} A & \sqrt{2} - 1 + \frac{1}{\sqrt{3}} A & -\sqrt{2} A \\
-(2 - \sqrt{2}) - \frac{1}{\sqrt{6}} A & -(\sqrt{2} - 1) + \frac{1}{\sqrt{3}} A & \frac{1}{\sqrt{2}} A
\end{pmatrix} + \mathcal{O}(\lambda^2).
\] (8)

\[
U = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} & 0 \\
-\frac{\sqrt{3}}{\sqrt{6}} & -\frac{\sqrt{3}}{\sqrt{6}} & \sqrt{\frac{2}{3}}
\end{pmatrix} + \lambda \begin{pmatrix}
2 - \sqrt{2} - \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}} \right) A & \sqrt{2} - 1 + \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \right) A & z^{\mu} A \\
-(2 - \sqrt{2}) - \frac{1}{\sqrt{6}} A & -(\sqrt{2} - 1) + \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \right) A & \frac{1}{\sqrt{2}} A
\end{pmatrix} + \mathcal{O}(\lambda^2).
\] (9)

More details can be found in Ref.\[13\]

More recently, Xiao-Gang He, Shi-Wen Li and I\[14\] applied the new “triminimal” parametrization method\[15\] to both CKM and PMNS mixing matrices. In this new method, the parameters chosen are not the traditional deviations of the matrix elements around unit matrix, instead, they are the deviations from big mixing angles based on a certain mixing pattern as zeroth-order bases. The method pointed out a new way to parameterize the mixing matrix with all angles small, i.e. the “triminimal” parametrization\[15\]. We thus arrive at a unified description between different kinds of parametrizations for quark and lepton sectors; the standard parametrizations, the Wolfenstein-like parametrizations, and the triminimal parametrizations.

In above, we reported recent progress on unified parametrization of both lepton and quark mixing matrices from standard viewpoint. Now we report on the non-standard attempt to explain neutrino oscillations based on the idea of Lorentz violation (LV). Neutrinos offer a promising possibility to study Lorentz violation that may exist at the low-energy as the remnants of Planck-scale Physics. A number of researchers studied to explain the neutrino oscillations by the non-standard viewpoint of Lorentz violation. Coleman and Glashow pointed out that neutrino oscillation can take place even for massless neutrinos if Lorentz invariance is violated in the neutrino sector\[16\]. There have been more works along this direction\[17\].
Recently, Zhi Xiao, Shimin Yang and 18 studied Lorentz violation contribution to neutrino oscillation. In our calculation we assume that neutrinos are massless and that the neutrino flavor states are mixing states of energy eigenstates. We calculate neutrino oscillation probabilities by the effective theory for Lorentz violation, which is usually called the standard model extension (SME) 19. In our work, the mixing angles for neutrinos are functions of Lorentz violation parameters.

Here we only report our qualitative conclusion from our study: we carried out Lorentz violation contribution to neutrino oscillation by the effective field theory for LV and give out the equations of neutrino oscillation probabilities. In our model, neutrino oscillations do not have drastic oscillation at low energy and oscillations still exist at high energy. The oscillation amplitude varies in different energy scale and will go to zero when the neutrino energy is high enough. Neutrinos may have small mass and both LV and the conventional oscillation mechanisms contribute to neutrino oscillation. However, our calculation at the high energy range where neutrino masses can be neglected is still applicable.

As summary, we give the following conclusions. Standard neutrino oscillations model are well organized by PMNS matrix, which can be unified parameterized with quark mixing matrix by combining with quark-lepton complementarity. Neutrino oscillations can be also obtained by Lorentz violation without neutrino mass. More detailed analysis are needed to check whether Lorentz violation can be a viable model for neutrino oscillations. From the standard mixing viewpoint, the mixing part is independent of energy and neutrino oscillations disappear at high energy. But in Lorentz violation models, mixing part is the function of energy and the oscillation amplitude varies with neutrino energy. Thus neutrino experiments with energy dependence may distinguish between the conventional massive neutrino scenario and the Lorentz violation scenario for neutrino oscillations. Thus we suggest new experiments on neutrino oscillations, such as the ANITA experiment 20, to pay special attention on the energy dependence of high energy neutrino oscillations.

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