Better be third then second in a search for a majority opinion

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Abstract: Monte Carlo simulations of a Sznajd model show that if a near-consensus is formed out of four initially equally widespread opinions, the one which at intermediate times is second in the number of adherents usually loses out against the third-placed opinion.

If several opinions compete against each other via mutual persuasion, and finally a consensus or very large majority emerges, then usually (and also in the present work) the final winner is that opinion which at some intermediate stage had already a narrow majority. If at the end a tiny minority of dissenters remains, it seems plausible that they are the remnants of the second-most-important opinion at some intermediate stage. However, we present here simulations in a four-opinion model, where usually the tiny minority of dissenters was on third and not on second place halfway through the process, while the opinion which then was on second place finally died out. This model is the Sznajd model on the dilute square lattice with diffusing agents of four possible opinions.

The Sznajd model [1] (see [2] for a recent review) is put onto a square lattice. Each lattice site initially is either empty, with probability 1/2, or has one of four possible opinions 1, 2, 3, 4, with probability 1/8 each. Then at each time step every occupied site tries to move to an empty neighbour. Afterwards randomly selected pairs of nearest neighbours, who share the same opinion, convince all those neighbours of the pair’s opinion, which differ by at most one unit [3]. If this is done for each lattice site, one time step is completed, and we start again with diffusion and convincing. In this way the rigidity of the standard Sznajd lattices is avoided; in principle everybody can exchange opinions with everybody else. The opinions no longer can change if they have settled onto the choices 1 and 3, or 2 and 4, or 1 and 4, or if one opinion covers everybody.

In all ten simulations of 301 \times 301 sites, an opinion fixed point was reached after about 4,000 to 100,000 time steps. In one case, only opinion 3 survived; in all other cases, of the two opinions which survived at the end, one had only 12 to 332 adherents compared with the about 45,000 of the winner. But this tiny surviving minority was on third place half way through the process, while the opinion which at half time had much more (4676 to about 22500) adherents finally had none. (In one case the leading and second opinion at half time had about the same number of votes. The opinion ranked fourth at half time always died out.) So to be first or third is good, while the second place is dangerous. (With 10000 samples of 101 \times 101 sites, an opinion fixed point was always reached, and the at half time second-ranked opinion finally vanished in about 92 percent of the cases; for 31 \times 31 exception were less rare.)
The explanation is based on the discreteness of the four opinions and the fact that opinion 1 is not regarded as similar to opinion 4. The two extreme opinions 1 and 4 thus can convince only one neighbouring opinion each, while the two centrist opinions 2 and 3 have two neighbours each. After some time, most of the opinions will be centrist (2 or 3). These two centrist opinions then fight for a clear majority, one is winning and also will convince the extremist opinion close to it, while the other centrist opinion is losing out completely and thus leaves its neighbouring extremist opinion untouched. This mechanism should also work in other models like Potts spins at low temperatures, as long as the opinions are discrete: Being second means to lose completely; being third allows a small chance of survival.

In the final fixed configuration for the 4 possible states, in 10000 simulations of $101 \times 101$ sites each, opinions 1 and 3 survived in 41 percent of the cases, opinions 2 and 4 in another 41 percent, while 9 percent each had only opinion 2 or only opinion 3 surviving. For three instead of four possible opinions, nearly always at the end everybody shared the centrist opinion 2; with five possible opinions, usually a small number of opinions 1 and 5 together with a big majority for opinion 3, and without any opinions 2 and 4, survived. For $31 \times 31$ lattices more exceptions occur.

In summary, for survival of an opinion among four choices it may be better to hide on third place then to be nearly the winner.

[1] K. Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys. C 11, 1157 (2000).
[2] D. Stauffer, Journal of Artificial Societies and Social Simulation 5, No.1, paper 4 (2002) (jasss.soc.surrey.ac.uk).
[3] G. Deffuant, D. Neau, F. Amblard and G. Weisbuch, Adv. Complex Syst. 3, 87 (2000); R. Hegselmann and M. Krause, for Journal of Artificial Societies and Social Simulation 5 (2002).