Identifying overlapping communities in social networks using multi-scale local information expansion

Hui-Jia Li\textsuperscript{1,2}, Jun-Hua Zhang\textsuperscript{1,2,3}, Zhi-Ping Liu\textsuperscript{2,4}, Luonan Chen\textsuperscript{2,4,5, a}, and Xiang-Sun Zhang\textsuperscript{1,2, b}

\textsuperscript{1} Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.
\textsuperscript{2} National Center for Mathematics and Interdisciplinary Sciences, Chinese Academy of Sciences, Beijing 100190, China.
\textsuperscript{3} Key Laboratory of Random Complex Structures and Data Science, Chinese Academy of Sciences, Beijing 100190, China.
\textsuperscript{4} Key Laboratory of Systems Biology, Shanghai Institutes for Biological Sciences, Chinese Academy of Sciences, Shanghai 200233, China.
\textsuperscript{5} Collaborative Research Center for Innovative Mathematical Modelling, Institute of Industrial Science, University of Tokyo, Tokyo 153-8505, Japan.

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Abstract. Most existing approaches for community detection require complete information of the graph in a specific scale, which is impractical for many social networks. We propose a novel algorithm that does not embrace the universal approach but instead of trying to focus on local social ties and modeling multi-scales of social interactions occurring in those networks. Our method for the first time optimizes the topological entropy of a network and uncovers communities through a novel dynamic system converging to a local minimum by simply updating the membership vector with very low computational complexity. It naturally supports overlapping communities through associating each node with a membership vector which describes node’s involvement in each community. This way, in addition to uncover overlapping communities, we can also describe different multi-scale partitions by tuning the characteristic size of modules from the optimal partition. Because of the high efficiency and accuracy of the algorithm, it is feasible to be used for the accurate detection of community structure in real networks.

1 Introduction

Since the publication of the seminal works of Barabási and Albert \cite{Barabasi2002}, a lot of real complex systems have been ex-
mined from the viewpoint of complex networks. Having been observed to arise naturally in a vast range of physical phenomena, complex networks can describe complex systems containing massive units (or subsystems) with nodes representing the component units and edges standing for the interactions among them. The social network is a representative complex network and closely related to our life, for example, World Wide Web [2], traffic networks [3], sexual networks [4], and article cite networks [5].

The study on the community structure of social networks has become a very important issue in the field of complex networks. Nodes, which belong to a tight-knit community, are more likely to have particular properties in common. It is significantly important to identify communities in social networks. By taking WWW network as an example, groups of web pages are more likely linking to web pages on related topics. These sets of web pages might correspond to some kinds of communities. Based on this search engines may increase the precision and recall of search results by focusing on narrow but topically-related subsets of the web.

The problem of finding communities in social complex networks has been studied for decades. Recently, several quality functions for community structure have been proposed to solve this problem [6] [7] [8]. Among them, modularity $Q$ is proved to be the most popular [8] [9] [10] and has been pursued by many researchers [10] [11] [23]. However, most of those approaches require knowledge of the entire graph structure to identify global communities based on global information. That means, one needs to access the whole network information. This constraint is impractical for large complex networks, because it is a challenge to know the whole network completely. Moreover, statistical methods can only detect the most significant connectivity community patterns and ignore their multi-scale topology. These identifications don’t have the advantage of providing a coarse-grained representation in the system, thereby they can’t sketch its organization or identify the sets of nodes which are likely to have hidden functions or properties in common.

Because of these limitations, we present a novel algorithm for community detection focused on social networks in this paper. The algorithm does not embrace the universal approach instead of trying to focus on social networks using local information and modeling the multi-scale social interaction patterns occurring in those networks. Our method optimizes the topological entropy that represents the statistic significance of a network for the first time. Although the topological entropy function is not convex and it is unrealistic to expect a standard optimization algorithm to find the global minimum, we develop a novel dynamic system which converges to a local minimum by simply updating the membership vector with low computational complexity. We don’t need to specify the number of communities that need to partition. It naturally supports overlapping communities by associating each node with a membership vector describing node’s involvement in each community. Theoretical analysis and experiments show that the algorithm can uncover communities fast and accurately.
The outline of this paper is as follows. Section 2 introduces the problem of community detection in social networks and the motivation behind our algorithm. In section 3, we present our algorithm through four steps and explain each one respectively. In section 4, we analyze some important properties of our algorithm and in section 5, we run this algorithm in several real social networks which represent a good fraction of the wide body of social networks. Section 6 concludes this paper.

2 Motivation

Given a network $G = (V, E)$ contains $n$ nodes, suppose we can divide them into $a$ groups. For each group suppose that we can select a “leader”. The leaders should be able to have two properties: they should be well connected to the members of their group, and they should also be able to communicate with other leaders when necessary. If the distributed algorithm is carried out in each group separately and the leaders communicate on a higher level, the agents can enjoy faster convergence rate.

It is natural to relate social networks with hierarchical structure. In one such hierarchy there are leader nodes that are more important than some other nodes, hence located on a higher level in the hierarchy. It naturally follows that the leader is located on the highest level within that hierarchy. By taking the DNS network in WWW [2] as an example, the Router server is a natural leader and locates on the highest hierarchy (see Fig. 1) when searching IP address. Since the hierarchies are consequence of the spreading of its correlation, and so are the communities, we believe that the identification of these hierarchies in a network will result in a natural community detection. The area on which a leader has most influence should define its community. So, community detection is performed by finding all natural leaders and all nodes on which they have influence. Partitions obtained in this way can be naturally explained. Also, another intuitive property that a community should possess is satisfied this way, that is shortest paths exist between nodes from a same community.

Given a graph, individual nodes only have local knowledge about its structure, which include information about their neighboring nodes. If any node wants to improve its own performance, it needs to know more about the global picture of the network. This information can be
used by the node to refine its choice of neighbors in order to improve its performance. However, this will cost a lot of computational complexity. The most complete measure of global graph structure is the adjacency matrix. Since each node has limited memory, energy, and computational capacity, it will be difficult to directly use the adjacency matrix. Our goal is to devise a scheme to provide each node with a small vector that includes compact global information on how the node is located with respect to the other nodes. It is desired that the scheme can be disseminated via an implementable distributed manner.

Moreover, a powerful method uncovering the modules in social networks should use a multi-scale way [13] [16]. This identification has the advantage of providing a coarse-grained representation of the system, thereby allowing to sketch its organization and to identify sets of nodes that are likely to have hidden functions or properties in common. Most community detection methods find a partition of the nodes into communities, where most of the links are concentrated within the communities. Each node is assigned to one and only one community, i.e., partitions are not compatible with overlapping communities [17] [18]. At the heart of most partitioning methods, there is a mathematical definition for what is thought to be a good partition. Once this quality function has been defined, different types of heuristics can be used in order to find, approximatively, its optimal partition, i.e., to find the partition having the highest value of the quality function.

3 The algorithm

For a network $G = (V, E)$ with $n$ nodes, we develop a distributed algorithm, which can categorize the node as “leader” or “regular” using local information. Further, the method assigns each regular agent with a membership vector in multi-scale way, indicates that leaders has more influence on it. This provides the nodes with some global picture of the network. The iteration includes three steps described as follows.

3.1 Leadership of nodes

First, we calculate the leadership $f_i$ of every node $i$ in the network. The leadership $f_i$ represents how important is the opinion of node $i$ in the network. Let the node leadership function defined as:

$$f(i) = \sum_{j=1; d_{ij} \leq \lfloor \frac{3\delta}{\sqrt{2}} \rfloor}^{n} e^{-\frac{d_{ij}}{\delta}}$$

where $d_{ij}$ is the shortest distance from vertex $i$ to vertex $j$. $\delta \in (0, +\infty)$ is the influence factor which is used to control mutual action range between nodes. According to the properties of exponential function $e^{-\frac{d_{ij}}{\delta}}$, for a special value of $\delta$, the influence range of every node to other nodes is approximately $[\frac{3\delta}{\sqrt{2}}]$. When $d_{ij}$ larger than $[\frac{3\delta}{\sqrt{2}}]$, the value of exponential function rapidly reduce to 0, so we can use $\delta$ to control the influence range of a node and calculate $f(i)$ only within the range $d_{ij} \leq [\frac{3\delta}{\sqrt{2}}]$. For the dense region of a network, nodes have higher leadership. The nodes with largest leadership mean they have most amount of links with other nodes and can be viewed
as candidate of leader nodes. Therefore, we can use node leadership to represent the importance of a node in the network.

### 3.2 Identifying the leader nodes

Identifying the leader nodes of the community is very important to analyze the properties of the complex networks. Many ways can be used to define the “key node”, such as the nodes with largest degree or betweenness centrality. Here, we use node leadership to search leader nodes. According to the notion of community structure, the density of inner-community links is larger than the rest of nodes. Each community represents a local region with relative higher correlation and the leader node of the community has the highest leadership and is tightly linked by other nodes. Moreover, different communities are divided by local lowest leadership nodes – the boundary nodes.

Note that in the rare cases where two or more leaders are also most influential neighbors between each other, then they are grouping together and are becoming leaders of one group. For example, in a full connected network, all of the nodes are leaders of one community, whereas for a ring network, each node is a leader to its own community. Specifically, if the length of two highest leadership nodes less than \( \left\lfloor \frac{3\delta}{\sqrt{2}} \right\rfloor \), we group them together and consider they are in one group. Finding leader nodes only needs a simple breadth first search and if find, we choose a random node to restart this process until converge. The computational complexity is \( O(m) \), where \( m \) is the number of edges in the network.

### 3.3 Determining the membership using random walk

At this step, our goal is to devise a scheme to provide each node with a small vector that includes compact global information on how the node is located with respect to the other nodes. We provide a definition for the membership vector based on the properties of random walk dynamic on graphs. Consider a graph with a leaders \( l_1, l_2, ..., l_a \) and \( n-a \) regular nodes. Given the leaders and the arbitrary order assigned to them, we describe the algorithm to determine the membership vectors for each regular node. We denote the membership vector of node \( i \) by \( \mathbf{x}_i = (x^1_i, x^2_i, \cdots, x^a_i) \in \mathbb{R}^a \).

The procedure operates as follows. The membership vector of leader \( l_i \) is first assigned to be the unit vector. These \( a \) vectors do not vary. For regular node \( i \), \( x^k_i \) is initialized randomly, distributed uniformly on \([0, 1]\) \( (k = 1, 2, ..., a) \). Then we normalize each row of \( \mathbf{x}_i \) so that for all leader \( k \), the sum of \( x^k_i \) is 1. At each iteration time \( t \), the influence vector of each regular node \( i \) is updated entry-wise \( (k = 1, 2, ..., a) \) using the following rule:

\[
x^k_i(t+1) = \frac{1}{\sum_j a_{ij} + 1} x^k_i(t) + \sum_j a_{ij} x^k_j(t)
\]

where \( A = \{a_{ij}\} \) is the adjacency matrix in which \( a_{ij} = 1 \) if node \( i \) and \( j \) are connected and \( a_{ij} = 0 \) otherwise.
nodes $l_k (k = 1, 2, ..., a)$ on any regular node $i$, $x_i^k$, is the probability that a random walker that starts from $i$ hits $l_k$ before it hits any other leader node [15]. If the underlying graph is connected, the iteration $\lim_{t \to \infty} x_i(t)$ converges to a set of unique vectors and these vectors can naturally be represented as the probability a regular node belongs to the community that a given leader node in. As a result, although leadership of a node only contain local information, we can use random walk dynamic to gain membership containing a global view of the whole graph.

4 Some descriptions of the algorithm

In this section we describe several important properties of the algorithm, including computing the influence factor $\delta$ to recognize multi-scale communities, identifying the leaders using local information, determining the overlapping nodes and estimating the complexity of the algorithm.

4.1 Determining the influence factor $\delta$ to recognize multi-scale communities

According to the definition of leadership, the algorithm is controlled by only one parameter, the influence factor $\delta$. We can naturally use $\delta$ to control the scale of community structure detected by our method. Here, we introduce topological entropy $H$ [24][25] that represents the statistic significance of a network to choose suitable $\delta$: for network $G = (V, E)$, $V = v_1, v_2, ..., v_n$, the leadership of $V$ are $f(1), f(2), ..., f(n)$, the topological entropy is defined as:

$$H = -\sum_{i=1}^{n} \frac{f(i)}{\sum_{i=1}^{n} f(i)} \log\left[\frac{f(i)}{\sum_{i=1}^{n} f(i)}\right]$$

Small $H$ means a stable and suitable partition. For a simple example, we consider network contain 11 nodes shown in Fig.2(a) and calculate topological entropy corresponding to different $\delta$. As shown in Fig.2(b), when $\delta$ increases from 0, the corresponding entropy begins to decrease and reach minimal 2.2805 at specific value of $\delta$ ($\delta = 1.26$). When $\delta$ leaves from optimal value, entropy begin to increase with $\delta$ and finally reach the maximal value.

Fig. 2. (a) A simple network with eleven nodes. (b). Plot of topological entropy $H$ versus influence factor $\delta$.
Therefore, to find an optimal $\delta$, it is equivalent to minimize the single parameter nonlinear function $H(\delta)$ and many algorithms can be used, for example, random search algorithm and simulated annealing algorithm. However, $\delta$ corresponding to a small value of $H$ but not minimal is also meaningful. Specially, according to the property of leadership, the influence range of a node is approximately $\lfloor \frac{3\delta}{\sqrt{2}} \rfloor$.

When $0 < \delta < \frac{\sqrt{2}}{3}$, there is no interaction between two nodes. Because no interaction exists, every node belongs to community contains itself and the number of community is $n$. Similarly, when $\frac{\sqrt{2}}{3} < \delta < 2\frac{\sqrt{2}}{3}$, a node only interacts with its neighborhood. As the value of $\delta$ grows, nodes can influence more and more nodes and thus the number of leaders and communities decreases. Finally, as $\delta \geq \sqrt{D}/3$, $D$ is the diameter of network, every pair of nodes can influence each other no matter how far they are.

To show our method can discover multi-scale community structure with the variation of $\delta$, we have tested the multi-scale modular structure in a classical hierarchical scale-free network with 125 nodes, RB125, proposed by Ravasz and Barabási [13]. In Fig.3(a) we plot the modular structure found with minimal entropy $H = 3.107$ and another small value $H = 3.352$, which shows two different scales that deserve discussion. The value of $H$ versus different $\delta$ is plotted in Fig.3(b). We observe clearly persistent structures in 25 and 5 communities respectively, that accounts for the subdivisions more significant in the process, showing two hierarchical levels for the structure. The

![Image](a)

![Image](b)

Fig. 3. RB 125 corresponds to the hierarchical scale-free network. (a) corresponding to 25, 5 modules are the most reasonable partition in terms of resolution with $H = 3.107$ and 3.352. (b) plots the number of communities versus the value of $\delta$.

partition in 25 modules and the partition in 5 modules are highlighted on the original network.

4.2 Determining the leaders using local information

In the algorithm, leadership of one node can be determined through only local information. By detecting the leader in a community we gain very useful information of the most influential node in its community. By removing the leader it can be expected for the community to suffer serious consequences, like splitting into several smaller
communities. The leader’s hierarchy, or the leader’s community, is the area where the leader’s opinion is the most influential opinion. For example, this can be used for an immunization for epidemic spreading. Thus, the algorithm can naturally determine the number of leaders, that is also, the number of communities. One interesting feature of the algorithm is that although it automatically detects the best leaders, one can manually specify particular nodes as leaders and build community structures around them.

4.3 Determining the overlapping nodes

It is worthwhile to point out that the vast majority of community detection methods assume that communities of complex networks are disjoint, placing each node in only one non-overlapping cluster. Generally, we call these methods ”hard-partition” algorithms. However, in many real networks communities often overlap to some extent.

An important property of our algorithm is the computation of a membership vector for each node. Instead of having one number denoting its membership in a single community, we have a percentage for each community. As a result, we can easily identify nodes that naturally belong to more than one community known as overlapping nodes [26] [27] [18]. So our method is a “soft-partition” algorithm. Additionally, we can find nodes that are good followers of their leader, and also nodes that have no distinguished leader and serve as a proxy between several communities.

4.4 Computational complexity

The overall complexity of the algorithm depends on the highest complexity of the three parts of the algorithm. In the following we analyze each of them sequentially.

The first step is calculating node’s leadership \( f(i) \). We need to calculate the exponential function within length of shortest path \( d_{ij} \leq \lfloor \frac{3}{\sqrt{2}} \rfloor \) between pair of nodes and the complexity of this procedure is at least \( O(m) \), \( m \) is the number of links. Actually, the computation complexity is worst, \( O(n^2) \), for a dense graph. Next step, determining the leader nodes of communities, is proceeded by searching all local highest leadership nodes. This can be done by a simple breadth first search and the complexity is \( O(m) \).

The last operation is very similar to the consensus linear process, whose complexity is \( O(n) \) similar to the random walk process.

To conclude this section, the one with the highest computational complexity is the first step, i.e., calculating the leadership of nodes. Its complexity depends on the degree of connectivity and the graph which is very densely connected needs more complexity. This accounts for the overall complexity of the algorithm is \( O(m) \) at best and \( O(n^2) \) at worst.

5 Experiments

In this section, we respectively apply the algorithm to simulated benchmark networks (LFR networks) [30] and some real social networks: the karate club network of Zachary [20], the scientific collaboration network [22] and
finally a large scale semantic network [21]. Results show that the algorithm can discover multi-scale communities efficiently and accurately.

5.1 The benchmark network

We empirically demonstrate the effectiveness of the algorithm through comparison with other five well-known algorithms on the artificial benchmark networks. These algorithms include: Newman’s fast algorithm [6], Danon et al’s method [9], the Louvain method [28], Infomap [29] and the clique percolation method [17]. We utilize the LFR benchmark proposed by Lancichinetti et al in [30]. This benchmark provides networks with scale-free distributions of node degree and community size and thus poses a much more severe test to community detection algorithms than standard benchmarks. Many parameters are used to control the generated networks in this benchmark: the number of nodes $N$, the average node degree $\langle k \rangle$, the maximum node degree $\text{max}_k$, the mixing ratio $\mu$ (each vertex shares a fraction $\mu$ of its edges with vertices in other communities), the minimum community size $\text{min}_c$ and the maximum community size $\text{max}_c$. The value of $\mu$ varies within $[0, 1]$ and determines the level of the fuzziness of the communities in the network. The larger the $\mu$, the more fuzzy the communities. In our test, we use the default parameter configuration where $N = 1000$, $\langle k \rangle = 15$, $\text{max}_k = 50$, $\text{min}_c = 20$ and $\text{max}_c = 50$.

To evaluate a community detection algorithm, we use the normalized mutual information (NMI) measure [31] to estimate the partition found by each algorithm. The test focuses on whether the intrinsic scale can be correctly uncovered. The experimental results are displayed in Fig. 4, where y-axis represents the value of NMI calculated by the algorithms mentioned above, and each point in curves is obtained by averaging the values obtained on 50 synthetic networks sampled from above model. As we can see, all algorithms work very well when $1 - \mu$ is more than 0.7 with NMI larger than 0.85. Compared with other five algorithms, our algorithm performs quite well and its accuracy is only slightly worse than that of the clique percolation in the case of $0.5 \leq 1 - \mu \leq 0.65$. However, clique percolation is nearly same as the Breath First Search (BFS) and very time consuming. The complexity of clique percolation is almost $O(n^3)$ and much larger than our method.

As real networks may have some different topological properties from synthetic ones, in the following we consider several widely used real-world networks to further evaluate the performance of our method.
5.2 The karate club network of Zachary

Over the course of two years in the early 1970s, Wayne Zachary observed social interactions between the members of a karate club at an American university [20]. He constructed networks of ties between members of the club based on their social interactions both within the club and away from it. By chance, a dispute arose during the course of his study between the club’s administrator and its principal karate teacher over whether to raise club fees, and as a result the club eventually split into two, forming two smaller clubs, centered around the administrator and the teacher.

We minimize the function of $H(\delta)$ and get the optimal value of $\delta = 1.85$ and $H = 3.914$. As it is shown on Fig. 5(a), the partition found by our algorithm not only matches the original partition, but also identifies the exact leaders. Nodes 1 and 33 own the local highest leadership, which respectively represent the administrator and the teacher. In this instance node 3 is detected as an overlapping node because its membership belonging to two communities is nearly equal. Actually, node 3 is on the border between the communities and so it is understandable that it might be an ambiguous case.

Compared with the optimal situation, when decreasing $\delta$ to 1.41, the entropy $H = 4.139$ is also very small. The community structure detected in this situation is shown in Fig. 5(b), which reveals another scale of relationships among the members of the karate club. Node 28 becomes another local highest leadership node and four most unstable nodes including nodes 3, 10, 20, 28 are marked in red. In subgraphs (a), (b) and (c), communities are represented by different shapes and overlapping nodes are enclosed in dashed curves.

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**Fig. 5.** The community structure of the karate club network detected when $\delta$ (a) equals to the optimal value 1.85, (b) decreases to 1.41 from optimal, and (c) further decreases to 0.933. In subgraphs (a), (b) and (c), communities are represented by different shapes and overlapping nodes are enclosed in dashed curves.
a dashed curve. Such members have good friendship with more than one clubs at the same time, so they are overlapping nodes in this situation. And now the number of communities detected in the karate network is three. Furthermore, as decreasing $\delta$ to 0.93 $H$ becomes 4.436, we get a partition with 4 communities shown in Fig.5(c). This partition is identical to [12] described by Newman. Six overlapping nodes are detected which constitute the fuzzy boundaries of the communities. Thus, partitions using different scales of $\delta$ are able to reflect multi-scale property of the real networks.

5.3 The scientific collaboration network

The scientific collaboration network was collected by Girvan and Newman [22] and has been examined in Refs [18] [32]. This network consists of 118 nodes (scientists or authors), and edges between them indicate co-authorship of one or more papers appearing in the archive. The collaborative ties represented in the figure are not limited to papers on topics concerning networks – we were interested primarily in whether people know one another, and collaboration on any topic is a reasonable indicator of acquaintance.

The present method detects eight communities with optimal $\delta = 1.493$ and minimal entropy $H = 5.447$. Fig.6(a) shows the community structure detected at optimal situation which is exactly same as Refs. [22] [18]. This confirms our partition as a good one. However, we believe our method can also make a meaningful “coarse-grained” partition which is visually reasonable. So the value of $\delta$ is amplified to 1.749 from optimal and the corresponding entropy $H = 6.483$. Owing to the amplification of the influence range of nodes with $\delta$, the number of communities decreases. From Fig.6(b) we notice some “uninfluential” communities, like the light blue and yellow ones, are merged by the more powerful red and dark green communities, respectively. Finally we get six communities which can be interpreted readily by the human eye. These multi-scale partitions will be invaluable in helping us to understand the large-scale structure of these network data. Furthermore, overlapping nodes enclosed in dashed curves in Fig.6 are detected according to their membership vectors. These nodes generally locate on the borders of two or more communities and represent authors with multiple research interests or cross-discipline background. Maybe such nodes play a role in bridging two or more communities in a complex network of other types. The ability to find overlapping nodes is a distinguished feature of our method and useful to reveal a natural characteristic in many social networks.

5.4 A large scale semantic network

The semantic network from Ref. [21] contains 7207 phrases and 31784 edges. The weights of edges are calculated in terms of phrase co-occurrences. For visualization purpose, our algorithm outputs a transformed adjacency matrix (in which the vertices within the same communities have been arranged together) with a hierarchical community structure. The distribution of community sizes is shown in Fig.7. Totally, 569 communities are detected by
The community structure of the scientific collaboration network obtained when $\delta$ (a) equals to the optimal value 1.493, (b) is amplified to 1.749 from optimal. In both subgraphs (a) and (b), overlapping nodes are enclosed in dashed curves.

![Diagram](image)

**Fig. 6.**

Setting optimal $\delta = 2.931$ and minimal entropy $H = 5.952$. The maximum size of community is 139, the minimum size is 2, and the average size is 12.57. One can see an approximate power-law phenomenon, that is, most communities are small and only a few are big. Among them, we have selected four interesting communities listed as follows:

**Community 1** = \{Scientist, Inventor, Genius, Gifted, Brilliant, Intelligent, Smart, Science, Intelligence, Musician\};

**Community 2** = \{Violin, Instrument, Cello, Band, Tuba, Clarinet, Orchestra, Trumpet, Trombone, Oboe, Woodwind, Symphony, Flute, Bass, Viola, Fiddle\};

**Community 3** = \{Ovation, Sitting, Low, Descent, Up, Step, Ascend, Elevator, Ascent, Staircase, Stairwell, Climb, Steps, Ladder, Stairs, Wake, Stairway, Rise, Escalator, Stair, Down, Standing, Resting, Using\};

**Community 4** = \{Nails, Hammer, Carpenter, Screw, Screwdriver, Tool, Pliers, Wrench, Sickle, Mechanic, Phillips\}.

These four communities are all reasonable modules listed in Ref. [21] and the elements of each are all have same meaning. Among these elements, \{Musician, Intelligence\} are uncovered as overlapping nodes between communities 1 and 2, and \{Using, Tool, Mechanic\} are the overlapping nodes between communities 3 and 4. We can easily recognize that these overlapping phrases have fuzzy meanings and have high value of phrase co-occurrences.

**Fig. 7.** The distribution of community size in a linear plot.

![Graph](image)
As the inherent community structure for this large semantic network is usually unknown, it is worth to make use of a measure to quantitatively evaluate the performance of our method. Here the popular modularity $Q$ [7] is adopted as a reference, which was proposed by Newman and Girvan and has been heavily used for community detection in recent years. $Q$ is defined as:

$$Q = \sum_{i=1}^{c} \left[ \frac{l_{in}^i}{L} - \left( \frac{d_i}{2L} \right)^2 \right],$$

(4)

Here, $c$ is the number of communities, $L$ is the total number of edges in the network, and $l_{in}^i$ and $d_i = 2l_{in}^i + l_{out}^i$ are the number of edges and the sum of vertex degrees in the $i$th community, respectively. Fig. 8 shows the result that compares modularity $Q$ with the topological entropy $H$ across multi-scale $\delta$. As we can see, the main trend is that the lower value of $H$, the larger value of $Q$. When $\delta$ reaches the optimal value $H = 5.952$, the Modularity $Q$ also reaches the maximal $Q = 0.521$ exactly. The result shows the community structure of the network corresponding to a certain $\delta$ is strong and robust. In conclusion, our algorithm can uncover the most suitable community scale effectively on real-world networks.

\section{Conclusion}

In summary, we have presented a novel community detection method based on local information in social networks. The algorithm does not embrace the universal approach but tries to focus on local social ties and model multi-scales of social interactions that occur on those networks. It identifies leaders and then detects communities located around the leaders using random walk dynamic. Our method not only supports overlapping communities detection using a membership vector to denote node’s involvement in each community, but can also describe different multi-resolution clusters allowing to discover “coarse-grained” modules versus the optimal partition. Applying our algorithm to several typical real-world networks with well defined community structures, we obtained reasonable results. So this method is feasible to be used for the accurate detection of community structures in complex networks. To sum up, from a new perspective, we propose a new community detection algorithm based on local information in this paper. The computational results on real social networks show that the new method not only can detect the accurate communities but also can extract the hierarchical structures of the networks.
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References

1. Barabási.A.L, Albert.R, Science, 286, (1999) 509-512.
2. Albert.R, Barabási.A.L, Jeong.H, Nature, 401, (1999) 130.
3. Li.X.G., Gao.Z.Y, Li.K.P, Zhao.X.M, Phys. Rev. E, 76, (2007) 016110.
4. Liljeros.F, Edling.C.R, Amaral.L.A.N, Stanley.H.E, Aberg.Y, Nature, 411, (2001) 907-908.
5. Sumiyoshi.A, Norikazu.S, Phys. Rev. E, 74, (2006) 026113.
6. Newman.M.E.J, Phys. Rev. E, 69, (2004) 066133.
7. Newman.M.E.J, Girvan.M, Phys. Rev. E, 69, (2004) 026113.
8. Newman.M.E.J, Proc. Natl. Acad. Sci, 103, (2006) 8577-8582.
9. Danon.L, Duch.J, Guillaume.D, Arenas.A, J. Stat. Mech, 29, (2005) P09008.
10. Clauset.A, Newman.M.E.J, Moore.C, Phys. Rev. E, 70, (2004) 066111.
11. Newman.M.E.J, Phys. Rev. E, 74, (2006) 036104.
12. Newman.M.E.J, Eur.Phys.J.B, 38, (2004) 321-330.
13. Ravasz.E, Barabási.A.L, Phys. Rev. E, 67, (2003) 026112.
14. Lambiotte.R, Delvenne.J.C, Barahona.M, (2009), arXiv:0812.1770.
15. Baras.J.S, Hovareshti.P, (2008), Proceedings of 47th IEEE Conference on Decision and Control, 2973-2978.
16. Mucha.P.J , Richardson.T, Macon.K, Porter.M.A, Jukka-Pekka Onnela, Science, 328, (2010) 876-878.
17. Palla.G, Derényi.I, Farkas.I, Vicsek.T, Nature, 435, (2005) 814-818.
18. Li.H.J, Wang.Y, Wu.L.Y, Liu.Z.P, Chen.L, Zhang.X.S, Europhysics Letters, 97, (2012) 48005.
19. Zhang.X.S, Wang.R.S, Wang.Y, Wang.J, Qiu.Y, Wang.L, Chen.L, Europhysics Letters, 87, (2009) 38002.
20. Zachary.W.W, Journal of Anthropological Research, 33, (1977) 452-473.
21. Palla.G, Barabási.A.L, Vicsek.T, Nature, 446, (2007) 664-667.
22. Girvan.M, Newman.M.E.J, Proc. Natl. Acad. Sci, 99, (2002) 7821-7826.
23. Li.Z.P, Zhang.S.H, Wang.R.S, Zhang.X.S, Chen.L, Phys. Rev. E, 77, (2007) 036109.
24. Gfeller.D, Chappelier.J.C, Los Rios P.De, Phys.Rev.E 72(5), (2005) 056135.
25. Bianconi.G, Pin.P, Marsili.M, Proc.Natl.Acad.Sci, 106(28), (2009) 11433-11438.
26. Chen.D.B, Shang.M.S, Fu Y, Physica A, 389, (2010) 4177-4187.
27. Shang.M.S, Chen.D.B, Zhou.T, Chin.Phys.Lett, 27, (2010) 058901.
28. Blondel.V.D, Guillaume.J.L, Lambiotte.R, Lefebvre.E, J. Stat. Mech, 10, (2005) P10008.
29. Rosvall.M, Bergstrom.C.T, Proc.Natl.Acad.Sci, 105(4), (2008) 1118-1123.
30. Lancichinetti A, Fortunato S, Phys. Rev. E, 80, (2009) 016118.

31. Lancichinetti A, Fortunato S, Phys. Rev. E, 80, (2009) 056117.

32. Zhang J.H, Zhang S.H, Zhang X.S, Physica A, 387, (2008) 1675-1682.
