Mesoscale simulations of polymer dynamics in microchannel flows

L. Cannavacciuolo, R. G. Winkler(a) and G. Gompper

Institut für Festkörperforschung, Forschungszentrum Jülich - 52425 Jülich, Germany, EU

received 29 November 2007; accepted in final form 11 June 2008
published online 28 July 2008

PACS 47.61.-k – Micro- and nano-scale flow phenomena
PACS 47.57.Ng – Polymers and polymer solutions
PACS 82.20.Wt – Computational modelling; simulation

Abstract – The non-equilibrium structural and dynamical properties of flexible polymers confined in a square microchannel and exposed to a Poiseuille flow are investigated by mesoscale simulations. The chain length and the flow strength are systematically varied. Two transport regimes are identified, corresponding to weak and strong confinement. For strong confinement, the transport properties are independent of the polymer length. For weak confinement or sufficiently strong flow and far from the channel center, the local stretching is similar to that of a polymer in simple shear flow, but in the central part the polymer exhibits a different behavior. The analysis of the long-time tumbling dynamics of short polymers yields a non-periodic motion with a sublinear dependence on the flow strength.

Copyright © EPLA, 2008

Introduction. – Confinement fundamentally alters the properties of dilute polymer solutions — compared to the bulk behavior — when either the polymer radius of gyration is on the order of the characteristic dimensions of its proximity [1–5], and/or an external field is applied, e.g., in shear or pressure-driven flow. In the first case, geometrical constraints lead to a stretching of the polymer parallel to the confining surfaces [1–4,6,7]. In the second case, in addition to flow-induced deformations, polymer-surface hydrodynamic interactions determine the polymer dynamics and lead to, e.g., cross-streamline migration [8–13]. The migration effect has been studied intensively for flexible polymers and DNA-like molecules by computer simulations and the relevance of hydrodynamic interactions has been confirmed. Experimental studies of individual DNA molecules in steady shear flow by fluorescence microscopy have provided a wealth of information on single polymer dynamics [14–17]. In particular, large conformational changes have been revealed due to tumbling motion [16,18,19]. Similar non-equilibrium studies for flexible polymers confined between surfaces or in narrow channels have not been conducted so far; however, a similar complex dynamics can be expected. Understanding of single polymer behavior is of paramount importance for the emerging technology of microfluidic devices. Insight into the detailed microscopic conformational, dynamical, and transport properties of polymers, e.g., DNA, will help in the conceptual design of such devices. Moreover, such studies will contribute to the understanding of the transport properties of biological macromolecules in living organisms.

The proper account of hydrodynamic interactions is essential in simulation studies of fluid flows in channels as is emphasized by the appearance of cross-streamline migration. Recently developed mesoscale simulation techniques, such as Lattice Boltzmann [13,20], Brownian dynamics with a hydrodynamic tensor [4], and multiparticle-collision dynamics [21,22] (also called stochastic rotation dynamics), are well suited to study hydrodynamic flows in microchannels and are able to bridge the length- and time-scale gap between the solvent and solute degrees of freedom.

In this letter, we employ mesoscale computer simulations to study the conformational and dynamical properties of confined polymers which are exposed to a Poiseuille flow. The considered polymers fall into the crossover regime from weak to strong confinement. Recent experiments [5,12] and simulations for slit and square-channel geometries yield a distinctly different flow behavior in the two limits. In refs. [4,9], the effect of confinement on the polymer conformations in a square channel has been studied for a broad range of channel widths and a few polymer lengths and shear rates. Similar studies have been performed for polymers confined in narrow slits for various flow rates and widths in refs. [10,13]. Our studies reach beyond these simulations, since we systematically investigate the influence of the flow rate and polymer conformations, and obtain new

(a)E-mail: r.winkler@fz-juelich.de
insight into the polymer migration process. In addition, we discuss the chain orientation and the tumbling dynamics, quantities which have been analyzed experimentally for DNA molecules in shear flow [16,17], but have not been addressed in Poiseuille flow so far.

**Simulation method, model.** – We use a hybrid simulation approach to study the properties of flexible polymers in flow, where molecular-dynamics simulations (MD) are combined with the multi-particle-collision dynamics (MPC) method for the solvent [23,24]. MPC is a particle-based simulation method and proceeds in two steps. In the streaming step, the solvent particles of mass $m$ move ballistically for a time $h$. In the collision step, particles are sorted into the cells of a cubic lattice of lattice constant $a$ and their relative velocities, with respect to the center-of-mass velocity of each cell, are rotated around a random axis by an angle $\alpha$. For every cell, mass, momentum, and energy are conserved. The algorithm is described in detail in refs. [23,25–28]. The fluid is confined in a square channel of side length $L$ with periodic boundary conditions along the channel axis $(L_y)$. No-slip boundary conditions are imposed on the channel walls by the bounce-back rule and virtual wall particles, as described in ref. [29], and flow is induced by a gravitational force ($F = mg$) acting on every fluid particle. The linear polymer is comprised of points of mass $M$, which are connected by a harmonic potential with an equilibrium bond length $b$. Excluded-volume interactions are taken into account by the well-known 12-6 truncated Lennard-Jones potential with the parameters $\sigma$ and $\epsilon$ [30]. Since we consider pressure-driven flows, no gravitational force acts on the polymer. The interaction of a polymer with the solvent is realized by inclusion of its monomers in the MPC collision step [31]. Extensive studies of polymer dynamics confirm the validity of this procedure [21,27,30,31]. Specifically, we employ the parameters $\alpha = 130^\circ$, $h = 0.1 \tau$, with $\tau = \sqrt{ma^2/k_BT}$ ($k_B$ is Boltzmann’s constant and $T$ the temperature), the average number of solvent particles per cell $\rho = 10$, $M = m\rho$, $b = \sigma = a$, the fluid mass density $\rho = pm/a^3$, $k_BT/\epsilon = 1$, and the time step in MD simulations $h_{MD} = 5 \times 10^{-3} \tau$. Polymers with $N = 20$, 40, 80, and 160 monomers are considered. To minimize self-interactions of periodic images, the corresponding channel lengths are $L_\parallel = 40a$, 60a, 100a, and 170a, respectively. Simulations of the pure solvent yield velocity profiles which agree with the solution of the Stokes equation for this geometry. Averages and probability distributions are calculated in the stationary state over time with various independent initial conditions.

**Monomer distributions.** – We expect a pronouncedly different behavior of polymers in Poiseuille flow for radii of gyration either smaller or larger than the width of the channel [12]. Scaling considerations based on the blob model [1] and simulations [4] yield an equilibrium stretch of a flexible polymer in a microchannel when its bulk radius of gyration $r_{g0} \equiv \langle r^2_{g0} \rangle^{1/2} \sim bN^{1/5}$ exceeds the lateral channel dimension, i.e., for $r_{g0} \gg L$. Here, the channel width becomes the characteristic length scale and the blob relaxation time $\tau_B$, which is determined by $L$, the relevant time scale [4,9]. For $r_{g0} \ll L$, the conformations are unaffected by confinement and the characteristic length and time scales are given by the bulk values of the polymer. The ratios $2r_{g0}/L$ for the considered polymer lengths and the channel width $L = 15a$ over the range 0.36–1.3, corresponding to the crossover regime between weak and strong confinement. The snapshots of fig. 1 illustrate the preferred average alignment and extension of a polymer along the flow direction.

Figure 2 shows radial monomer distributions $P_m$, where $r$ is the distance transverse to the flow direction from the channel center, for the polymer length $N = 40$ and various Peclet numbers $Pe_L$. Since the polymer radius of gyration is comparable with the channel width, the effective shear rate $\dot{\gamma}$ is determined by $L$, i.e.,

\[ \dot{\gamma} = g L \dot{\gamma}/(2\eta). \]
The blob diameter yields the blob relaxation time \( \tau_L \) such that

\[
Pc_L \equiv \gamma \tau_L, \quad \text{with} \quad \tau_L = \eta (L/2)^3/(3 k_B T),
\]

where \( \eta \) is the solvent viscosity \([32]\). A similar approach has been adopted in ref. [9]. With increasing Peclet number, the distribution \( P_m \) develops a maximum at a finite distance from the center. At large \( Pc_L \) migration towards the channel center sets in, which leads to a shift of the maximum to smaller radii — a phenomenon already observed in number of other studies \([8–11,13]\). The widths of the radial distribution functions

\[
\langle r^2 \rangle = \int_0^\infty r^2 P_m(r) \, dr
\]

for the various chain lengths are shown as inset in fig. 2. The widths are independent of the Peclet number for small \( Pc_L \) and decay for large \( Pc_L \). For \( N \gtrsim 40 \), there seems to be only a weak dependence on the chain length. The widths start to decrease at approximately the same Peclet number. This is consistent with our assumption that blobs determine the polymer properties. The approach of \( P_m \) towards a universal limiting curve with increasing polymer length is explained as follows. In general, \( P_m \) exhibits the dependence \( P_m(b, r, L, N, g) \) on the parameters and variables of the system, and correspondingly the width is given by \( W^2 = \langle r^2 \rangle - W^2(b, L, N, g) \). Scaling all lengths by \( L \) and using the above definition of the Peclet number yields the dependence \( W^2/L^2 = f(r_m/L, bN/L, Pc_L) \). For strongly confined polymers \( r_m \gg L \) and \( bN \gg L \), and hence

\[
W^2/L^2 = f(Pc_L),
\]

i.e., the width depends on the Peclet number only.

Migration has been addressed in refs. [9,10,33] by studying the center-of-mass distribution of a polymer. These studies predict a migration towards the wall for strongly confined polymers \( r_m \gg 1/2 \). Indeed, our centers-of-mass distributions for the various polymer lengths also show an increase in the probability close to a wall with increasing flow rate in the range \( 10 \lesssim Pc_L \lesssim 100 \). This is accompanied by an increase of the widths of these distributions with a maximum at \( Pc_L \approx 50 \), which grows with increasing polymer length. Hence, we obtain a similar dependence as presented in ref. [9]. However, for larger Peclet numbers, the widths of the centers-of-mass distributions decrease in a similar way as those of the monomer distributions displayed in the inset of fig. 2 as a consequence of the polymer migration towards the channel center. As pointed out before \([9,10]\), the broadening of the center-of-mass distribution is caused by stretching and preferred alignment of the polymer along the flow direction. Since the monomer distribution for sufficiently long and hence strongly confined polymers is only weakly affected by flow for \( Pc_L < 50 \), we conclude that there is only a very weak migration for such Peclet numbers. Flow leads to an alignment of the polymer but hydrodynamic interactions, which are considered fundamental for migration \([8–11,13,34]\), do not contribute to the broadening of the center-of-mass distribution. Only for Peclet numbers \( Pc_L > 50 \), we find a decrease in the width of the monomer distributions for the various chain lengths, which we associate with migration.

Since hydrodynamic interactions are long-range in nature, it is difficult to unravel the system-size dependence of migration. To study the influence of the channel width on migration, we performed simulations for various \( L \) at the same curvature of the flow profile, i.e., \( g r_m^2/a = 0.01 \), and the polymer length \( N = 40 \). As shown in fig. 3, cross-streamline migration is well pronounced for all \( L \), with a shift of the maximum of the monomer distribution to larger radii with increasing \( L \). The position of the maximum exhibits the chain-length dependence

\[
r_m/(r_m)^{4/5} \sim (L/(r_m))^{4/5},
\]

as shown in the inset of fig. 3. Thus, migration and hydrodynamic interactions are more pronounced for polymers confined in wider channels. In narrower channels, hydrodynamic interactions are partially screened by the presence of the surfaces, but there is still a pronounced migration even for the smallest \( L \).

The changes in polymer conformations are reflected in their radii of gyration, which are displayed in fig. 4. Evidently, short polymers \((N < 40)\) experience only minor conformational restrictions at equilibrium, whereas longer polymers are stretched along the channel and are compressed in the transverse direction. Beyond the Peclet number \( Pc_L \approx 10 \), flow induces or enhances stretching parallel to the channel and transverse shrinkage.

Fig. 3: Radial monomer distribution functions \( P_m \) for the channel widths \( L/a = 15, 20, 30 \), and 45 corresponding to the Peclet numbers \( Pc_L = 10^2, 3.3 \times 10^2, 1.7 \times 10^3 \), and \( 8.5 \times 10^4 \), respectively (top to bottom at \( r/r_\theta = 0 \)). The polymer length \( N = 40 \) and \( g r_m^2/a = 0.01 \), i.e., the curvature of the flow profile is independent of \( L \). Inset: position of the maximum as function of the channel width. The straight line represents the power law \( (L/(r_m))^{4/5} \).
Local stretching. — Up to now, we have discussed conformational properties which are averaged over the cross-section of the microchannel. For the parabolic velocity profile, however, we can define a local shear rate which changes linearly with the radial distance. This leads to the local Peclet number

\[ P_{\text{cm}} = \frac{g r g_0 r_{\text{cm}}}{3 k_B T}, \]

where \( r_{\text{cm}} \) is the radial (transverse to the flow direction) chain center-of-mass position, and can be related to \( P_{\text{eL}} \) via \( P_{\text{cm}} = 2(2 r g_0 / L)^3 P_{\text{eL}} r_{\text{cm}} / L \). Note that \( P_{\text{cm}} \) is independent of the system size \( L \); \( P_{\text{eL}} \) is just a label for the curves obtained for the various applied external field strengths. As long as the local radial polymer deformation due to the non-linearity of the flow profile is not too strong, we might expect that the polymer radius of gyration exhibits a dependence on \( P_{\text{cm}} \) similar to the dependence on the Weissenberg number in simple shear flow [19]. Figure 5 shows the mean square radii of gyration parallel to the channel as function of \( P_{\text{cm}} \) for the chain length \( N = 40 \) and \( L = 15 a \). For a given \( P_{\text{eL}} \), an individual curve indicates the conformational changes as a function of the center-of-mass position. For small \( P_{\text{eL}} \), i.e., in the central part of the channel, we find a position-independent stretching of the polymer. The inset of fig. 5 shows that in this regime the radius of gyration increases approximately quadratically with \( g \), within the accuracy of our results, as should be expected due to symmetry reasons (flow reversal), and saturates for strong flows. Interestingly, the stretching seems to be a universal function of \( M g r g_0 / k_B T \), which we adopted as scaling function for the various polymer lengths. Thus, in the central part, the channel flow is different from a simple shear flow — the extended polymer never experience a linear velocity profile only. At larger radial distances, the polymer is further stretched, particularly for large flow velocities [10].

More importantly, the various data sets approach a limiting curve for sufficiently large \( r_{\text{cm}} \) (depending on \( P_{\text{eL}} \)), which is close to the mean square radius of gyration as a function of the shear rate of the same polymer in simple shear flow. The solid line is calculated using a semiflexible polymer model for a polymer in shear flow [19], in close agreement with simulations [35]. Thus, a polymer in Poiseuille flow sufficiently far from the center behaves locally as a polymer in simple shear with a shear rate given by the local velocity gradient, as often assumed in analytical calculations [33,36]. We like to emphasis that the polymers confined in the channels of various widths in fig. 3 exhibit the same \( L \)-independent radial radii of gyration.

Polymer orientation. — Flow leads to an alignment of the polymer, which can be characterized by the orientational distribution function of the end-to-end vector [17,19]. Since the considered system is almost rotationally symmetric with respect to the channel axis, only the distribution function \( P_{\alpha}(\theta) \), where \( \theta \) (\( 0 < \theta < \pi \)) is the angle between the end-to-end vector and the flow direction, needs to be studied. As shown in fig. 6, the distribution function is almost independent of the angle \( \theta \) at small Peclet numbers. The deviation from the bulk value \( P_{\alpha} = 1/2 \) is caused by the confinement of the polymer. As expected, there is no preferred angle; the average is \( \langle \theta \rangle = \pi/2 \). The shape of the distribution strongly depends on the Peclet number. For large Peclet numbers, \( P_{\alpha} \) decays algebraically as \( \theta^{-2} \). A similar algebraic decay is found for the angle between the end-to-end vector and its projection onto the shear plane of a flexible polymer in shear flow [17,19]. However, the distribution \( P_{\alpha} \) also reflects the fundamental difference of polymers in Poiseuille flow in
Polymer dynamics in Poiseuille flow

Fig. 6: Probability distribution of the angle between the end-to-end vector and the flow direction for the Peclet numbers $P_e L = 1, 5, 10, 30, 50, 100$ (bottom to top) and the channel width $L = 15a$. The distribution is normalized such that $\int_0^{\pi} P_\theta(\theta) \sin \theta \, d\theta = 1$. Only half of a period is shown. The solid line represents the power law dependence $P_\theta \sim \theta^{-2}$.

Fig. 7: Ratio of the polymer center-of-mass velocity $v_{cm}$ and the average solvent velocity $v_s$ for the polymer lengths $N = 20$ (squares), 40 (diamonds), 80 (bullets), and 160 (triangles) and the channel width $L = 15a$.

a square channel and in shear flow, respectively (in the latter case, as well as in a slit geometry, there is no axial symmetry with respect to the flow direction). For shear flow, there exists a regime of shear rates, where the distribution $P_\theta$ is Gaussian, only for large shear rates follows the algebraic decay \[17,19\]. In fig. 6 no Gaussian regime is present. Because of the nonlinear flow profile, an algebraic decay is not necessarily surprising, compared to simple shear, where this fact is explained by higher-order correlations and the importance of the whole time history. Nevertheless, the similarity is striking and might be traced back to the same origin.

Polymer transport. – The radial widths of the monomer and center-of-mass distributions decrease with increasing flow strength (cf. fig. 2) and a polymer spends more time in the region of higher axial fluid velocity [9]. This results in an increased polymer transport velocity compared to the average solvent velocity, as shown in fig. 7. For a wide channel $r_{g0} \ll L$, the ratio of $v_{cm}/v_s$ is close to unity at equilibrium. Conformational constraints for $r_{g0} \gg L$ lead to a narrowing of the radial distribution functions [12] accompanied by increased polymer flow velocities even at very weak imposed flow rates. This expected transport behavior is reflected in fig. 7. At small Peclet numbers $P_e L \lesssim 50$, we find for $N \gtrsim 40$ center-of-mass velocities which depend only weakly on $P_e L$. With increasing Peclet number, the average polymer velocity increases, starting at about the same Peclet number $P_e L \approx 10$ for all chain lengths and the ratio reaches almost its maximum value of $v_{cm}/v_s = 2$. We obtain a chain-length dependence of the velocity ratio for short polymers only, which is rather a consequence of the weaker confinement of the shorter polymer than a true molecular-weight dependence. The ratios for the longer polymers seem to approach a universal limiting curve, which is consistent with the assumption that local properties on a blob scale rather than global polymer properties determine their behavior. As a consequence, no separation of highly confined DNA molecules according to their molecular weight is possible by velocity differentiation. Separation would anyhow be difficult in a narrow channel, because individual chains can hardly pass each other.

Polymer tumbling. – Polymers in simple shear flow exhibit large conformational changes due to tumbling motion, i.e., they stretch and recoil in the course of time [16–19]. Visual inspection of the polymer dynamics in Poiseuille flow reveals similar large conformational fluctuations (cf. fig. 1). Tumbling occurs on a characteristic time scale which is denoted as tumbling time $t_T$. There are various ways to extract such a characteristic time, although the dynamics is non-periodic. We tried various strategies and found the following approach most useful. We first determined the distribution of times between consecutive crossings of the end-to-end vector with the plane perpendicular to the flow direction, i.e., $\theta = \pm \pi/2$. As shown in the inset of fig. 8, this yields exponentially decaying distributions, $P_T \sim e^{-t/t_T}$, at large times. The extracted tumbling times $t_T$ are presented in fig. 8. A power law fit yields the dependence

$t_T \sim \tau^{-2/3}. \tag{7}$

Simulations and analytical calculations [16–19] yield the dependence $t_T \sim \tau^{-2/3}$ for flexible polymers in linear shear flow. Hence, the tumbling times of polymers in Poiseuille flow exhibit a weaker dependence on the strength of the external field than in shear flow.

Conclusions. – We have analyzed the flow behavior of flexible polymers confined in a square channel with a side length comparable to the polymer radius of gyration. We have identified two distinct transport regimes were
polymers behave similarly. For $r_{g0} \ll L$ the polymers are unaffected by the channel geometry and the characteristic length and time scales are given by the radius of gyration and the longest polymer relaxation time, respectively. In the strongly confined regime $r_{g0} \gg L$, the polymer structure and dynamics are determined by blobs with a size of the channel cross-section. Polymers with $r_{g0} \approx L/2$ show a rich crossover behavior. We expect that the properties of polymers in a slit geometry [10,13] are less severely affected by confinement than those of polymers in a channel. In the slit geometry, the polymer is always able to relax in the direction parallel to the surfaces and hence the longest relaxation time is always related to the polymer length. The analysis of the radial polymer conformations, the distribution of the orientation angle and the tumbling time reveals differences among Poiseuille flow and simple shear flow for the strongly confined polymers.

***

We thank M. Ripoll for helpful discussions. The financial support of this work by the Deutsche Forschungsgemeinschaft within the Priority Program SPP 1164 is gratefully acknowledged.

REFERENCES

[1] Wall F. T., Seitz W. A., Chin J. C. and de Gennes P. G., Proc. Natl. Acad. Sci. U.S.A., 75 (1978) 2069.
[2] Brochard F. and de Gennes P. G., J. Chem. Phys., 67 (1977) 52.
[3] van Vliet J. H. and ten Brinke G., J. Chem. Phys., 93 (1990) 1436.
[4] Jendrejack R. M., Dimalanta E. T., Schwartz D. C., Graham M. D. and de Pablo J. J., Phys. Rev. Lett., 91 (2003) 038102.
[5] Balducci A., Hsieh C.-C. and Doyle P. S., Phys. Rev. Lett., 99 (2007) 238102.
[6] Jendrejack R. M., Schwartz D. C., Graham M. D. and de Pablo J. J., J. Chem. Phys., 119 (2003) 1165.
[7] Tegenfeldt J. O., Prinz C., Cao H., Reisner W. W., Riehn R., Wang Y. M., Cox E. C., Sturm J. C., Silberzan P. and Austin R. H., Proc. Natl. Acad. Sci. U.S.A., 101 (2004) 10979.
[8] Agarwal U. S., Dutta A. and Mashelkar R. A., Chem. Eng. Sci., 49 (1994) 1693.
[9] Jendrejack R. M., Schwartz D. C., de Pablo J. J. and Graham M. D., J. Chem. Phys., 120 (2004) 2513.
[10] Usta O. B., Butler J. E. and Ladd A. J. C., Phys. Fluids, 18 (2006) 031703.
[11] Khare R., Graham M. D. and de Pablo J. J., Phys. Rev. Lett., 96 (2006) 224505.
[12] Stein D., van der Heyden F. H. J., Koopmans W. J. A. and Dekker C., Proc. Natl. Acad. Sci. U.S.A., 103 (2006) 15853.
[13] Usta O. B., Butler J. E. and Ladd A. J. C., Phys. Rev. Lett., 98 (2007) 098301.
[14] Smith D. E., Babcock H. P. and Chu S., Science, 282 (1999) 1724.
[15] LeDuc P., Haber C., Boa G. and Wirtz D., Nature, 399 (1999) 564.
[16] Schroeder C. M., Teixeira R. E., Shaqfeh E. S. G. and Chu S., Phys. Rev. Lett., 95 (2005) 018301.
[17] Gerashchenko S. and Steinberg V., Phys. Rev. Lett., 96 (2006) 038304.
[18] Delgado-Buscalioni R., Phys. Rev. Lett., 96 (2006) 088303.
[19] Winkler R. G., Phys. Rev. Lett., 97 (2006) 128301.
[20] Ahlrichs P. and Dünweg B., J. Chem. Phys., 111 (1999) 8225.
[21] Webster M. A. and Yeomans J. M., J. Chem. Phys., 122 (2005) 164903.
[22] Noguchi H. and Gompper G., Proc. Natl. Acad. Sci. U.S.A., 102 (2005) 14159.
[23] Malevanets A. and Kapral R., J. Chem. Phys., 110 (1999) 8605.
[24] Ihle T. and Kroll D. M., Phys. Rev. E, 63 (2001) 020201(R).
[25] Kiruchi N., Pooley C. M., Ryder J. F. and Yeomans J. M., J. Chem. Phys., 119 (2003) 6388.
[26] Tüzel E., Strauss M., Ihle T. and Kroll D. M., Phys. Rev. E, 68 (2003) 036701.
[27] Ripoll M., Mussawisade K., Winkler R. G. and Gompper G., Europhys. Lett., 68 (2004) 106.
[28] Ripoll M., Mussawisade K., Winkler R. G. and Gompper G., Phys. Rev. E, 72 (2005) 016701.
[29] Lamura A., Gompper G., Ihle T. and Kroll D. M., Europhys. Lett., 56 (2001) 319.
[30] Mussawisade K., Ripoll M., Winkler R. G. and Gompper G., J. Chem. Phys., 123 (2005) 144905.
[31] Malevanets A. and Yeomans J. M., Europhys. Lett., 52 (2000) 231.
[32] Doi M. and Edwards S. F., The Theory of Polymer Dynamics (Clarendon Press, Oxford) 1986.
[33] Butler J. E., Usta O. B., Kekre R. and Ladd A. J. C., Phys. Fluids, 19 (2007) 113101.
[34] Sender D. and Netz R. R., EPL, 81 (2008) 54006.
[35] Ripoll M., Winkler R. G. and Gompper G., Phys. Rev. Lett., 96 (2006) 185302.
[36] Ma H. and Graham M. D., Phys. Fluids, 17 (2005) 085103.

Fig. 8: Tumbling times for the polymer length $N = 20$ and the channel width $L = 15a$. Inset: distribution of tumbling times for $P e_L = 500$. 

$\sim g^{0.52}$