Generalized Wardrop Equilibrium for Charging Station Selection and Route Choice of Electric Vehicles in Joint Power Distribution and Transportation Networks

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Abstract—This article presents the equilibrium analysis of a game composed of heterogeneous electric vehicles (EVs) and a power distribution system operator (DSO) as the players, and charging station operators (CSOs) and a transportation network operator (TNO) as coordinators. Each EV tries to pick a charging station as its destination and a route to get there at the same time. However, the traffic and electrical load congestion on the roads and charging stations lead to the interdependencies between the optimal decisions of EVs. CSOs and the TNO need to apply some tolling to control such congestion. On the other hand, the pricing at charging stations depends on real-time distribution locational marginal pricing, which is determined by the DSO after solving the optimal power flow over the power distribution network. This article also takes into account the local and the coupling/infrastructure constraints of EVs, along with transportation and distribution networks. This problem is modeled as a generalized aggregative game, and then a decentralized learning method is proposed to obtain an equilibrium point of the game, which is known as variational generalized Wardrop equilibrium. The existence of such an equilibrium point and the convergence of the proposed algorithm to it are proven. We undertake numerical studies on the Savannah city model and the IEEE 33-bus distribution network and investigate the impact of various characteristics on demand and prices.

Index Terms—Decentralized learning, generalized wardrop equilibrium (GWE), transportation and power distribution networks.

I. INTRODUCTION

In recent years, the increasing proliferation of electric vehicles (EVs) in transportation networks and the adoption of distributed energy sources in distribution networks have restructured and tightened the connection between these two types of networks, potentially increasing the complexity of managing both systems simultaneously [1]. While these modifications address climate change and net-zero emission [2], uncoordinated management of large numbers of EVs in cities can have a detrimental effect on the distribution network due to transformer overloads, power flow congestion, and local energy prices [3]. In addition, this might result in higher congestion on both roads and charging stations as price-sensitive users seek out cheaper electricity [4]. Meanwhile, customers now have real-time access to information about road traffic and power costs, thanks to the rapid growth of supporting technologies like Google Maps and PlugShare. Thus, coordinators in a smart city should leverage these new coordination tools to mitigate such negative repercussions and incentivize selfish EVs appropriately, ensuring that the rising charging demand is met through the integration of sustainable energy supplies in a sociotechnical system [5]. In this article, EVs are modeled as self-interested agents that want to find both the best charging station and the best way to get there. They do this by taking into account travel time, electricity price, station crowding, and personal preferences, as well as local constraints and the constraints imposed by the transportation and distribution networks. Furthermore, the distribution network operator aims to minimize the operating cost of distributed generators (DGs) while incentivizing EVs to comply with coupling constraints such as power system capacity and active power balancing constraints, which also depend on EVs’ decisions. This aim can be achieved using market clearing of the power distribution system operator (DSO) based on real-time distributional locational marginal pricing (DLMP). Furthermore, charging station operators (CSOs) and the transportation network operator (TNO) impose and broadcast real-time tolls to ensure that the capacity of stations and roads will not be violated. It is worth mentioning that our proposed framework can be extended to incorporate other scenarios, such as competition.
among charging stations or information asymmetry in EVs. Therefore, considering these aspects, a decentralized method of gradient-based learning of generalized Wardrop equilibrium (GWE) in a holistic model with infrastructure and coupling constraints among users is proposed.

The main contributions of the proposed decentralized game-theoretical interaction of different types of users in coupled transportation and distribution networks are as follows.

1) We present a novel framework for simultaneous routing and charging station selection of selfish EVs by considering the effects of power distribution and transportation networks coordinated by the DSO, CSOs, and the TNO.

2) The LinDistFlow model for distribution power flow is considered to ensure that distribution network constraints are met by imposing DLMPs on the charging stations based on Lagrangian relaxation of the generalized game. Furthermore, we introduce congestion pricing based on the average utilization of heterogeneous stations to manage stations’ crowdedness.

3) We formalize the corresponding interaction as a generalized aggregative game and solve it by exploiting the inertial forward-reflected-backward (I-ForRB) splitting algorithm for nonstrictly monotone games, ensuring the scalability and privacy of all entities. Then, the existence of variational generalized Wardrop equilibrium (v-GWE) and convergence of the decentralized algorithm to a v-GWE is proved.

4) We provide a numerical study of the Savannah city transportation network model and the IEEE 33-bus radial distribution network to explore the effects of DGs and distribution network constraints on system equilibrium as well as users’ preferences.

The rest of this article is organized as follows. We conduct a literature review in Section II. Then, in Section III, we provide the coupled model of user interactions in the transportation and power networks, followed by the decoupled model of the game. Section IV then establishes the existence of a GWE and proposes a decentralized algorithm which converges to a Wardrop equilibrium. Section V investigates a case study. Finally, Section VI concludes this article.

II. LITERATURE REVIEW

Recently, there has been considerable interest in jointly analyzing the system-level impact of the transportation and power networks. We can categorize them into two groups. The first category includes research studies that use centralized optimization methodologies to maximize the social welfare of coupled power and transportation networks. In [6], using Lagrangian relaxations of the related social optimum optimization problem, collaborative pricing based on locational marginal pricing (LMP) and between power and transportation networks is suggested to compute the social optimum of systems. It is also inferred that using marginal pricing of road flows, selfish users would follow the calculated socially optimum flow. The authors in [7] applied the interdependence model to solve the unit commitment problem and proposed mixed-integer linear programming to schedule EV fleets with variable wind power generation. The authors in [8] and [9] conducted a series of works on the interaction between emerging AMoD systems and the power network, in which the fleet of electric self-driving vehicles servicing the on-demand transportation network was controlled centrally in collaboration with the power system.

The second group includes research papers that assume users are selfish agents who want to decide to minimize their desired cost strategically. Using a combined distribution and assignment (CDA) model of user equilibrium, [10] investigated optimal pricing to compute social welfare over joint transportation and power networks. However, the CDA model’s parameters must be empirically calibrated. In [11], the fixed point of the power network decision and transportation network equilibrium was attained by iteratively solving the best response of each transportation and power network. In [12], the socially optimal equilibrium of connected transportation and power networks was achieved using mixed-integer second-order cone programming and the assumption that EVs can be charged wirelessly, eliminating the need for them to drive to charging points. The authors in [13] studied a bilevel model and used reinforcement learning to solve socially optimal charging pricing while mitigating uncertainty in consumers’ origin-destination demand and wind power generation. A recent study [14] presented a trilevel optimization technique for modeling interactions among EVs, charging stations, and electricity networks, using an iterative bilevel algorithm and simulated annealing. The comprehensive literature review on various models of this subject can be found in [15] and [16]. Although the above studies contribute to managing EVs, the following challenges remain. First, in [6], [7], [8], [9], [10], [11], [12], [13], [14] and [17], [18], [19] and [20], the system condition is computed centrally, which means either the EVs implement central coordinator suggestions in optimization formulations or the central operator finds the users’ equilibrium based on complete user information in game-theoretic formulations. But perfect knowledge of users’ preferences is unachievable and ignores privacy. The survey paper [21] comprehensively reviewed recent decentralized coordination mechanisms and their advantages in comparison with centralized approaches in the literature as well.

Second, the user equilibrium models in [6], [7], [8], [9], [10], [11], [12], [13], [14] and [17], [18], [19], [20] are based on the assumption that all users have the same cost function and constraints, i.e., the same monetary value of time and EV compatibility with charging stations. Using Beckman’s potential function, traffic assignment problems result in time value homogeneity among users in studies [6], [7], [8], [9], [10], [11], [12], [13], [14] and [17], [18], [19], [20] because the Hessian matrix of the potential game has to be symmetric. However, our monotone game formulation allows for the heterogeneity of users’ time values. Furthermore, [6], [7], [8], [9], [10], [11], [12], [13], [14] and [17], [18], [19], [20] assume that all EVs are compatible with all charging stations. In [6], although marginal tolling of a transportation network operator would ensure that the users follow a socially optimal solution calculated by the central operator, it was dependent on two strong assumptions: 1) the homogeneity of users and 2) the central operator’s complete

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We consider a transportation network connected to an N-bus radial electrical distribution network at some charging stations. The networks are modeled via directed graphs $G_T = (\mathcal{V}, \mathcal{E})$ and $G_P = (\mathcal{N}, \mathcal{L})$, respectively. Each node $v \in \mathcal{V}$ represents a road intersection, a charging station, or the origin of a user. An edge $e \in \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents a road connecting two nodes in $\mathcal{V}$. In addition, $\mathcal{N}$ and $\mathcal{L}$ denote the set of buses and the set of distribution lines, respectively. The buses in $\mathcal{N}$ may have a distributed generation unit and load demands (such as charging stations and residential loads). Each charging station $d$ from the set of charging stations $\mathcal{D} \subset \mathcal{V}$, which is located at a node $v$ of the transportation network, imposes an electrical load to the corresponding bus node, say $j \in \mathcal{N}$, of the distribution network. Fig. 1 demonstrates the proposed system.

The DSO solves a linearized ac optimal power flow (AC-OPF) problem to minimize distributed generators’ operating costs under power system constraints. By solving the linearized AC-OPF, the DLMP at each bus is obtained, which clears the electricity price at charging stations.

In the transportation network, there are $M$ different EVs, whose set is denoted by $\mathcal{M}$. Each EV $i \in \mathcal{M}$ aims to compute the optimum joint routing and destination strategy by considering the traveling time, the price of electricity, and service at the charging station, as well as the individual and coupling constraints to reach a charging station located on the transportation and distribution networks.

We also consider an aggregator for the transportation network that provides information on traffic conditions to assist EVs in making optimal decisions. From the DSO’s point of view, EVs can be considered as mobile electrical loads. The DSO can partially manage such a load by setting charging prices at the stations. Therefore, we deal with the problem of joint routing and destination planning of EV users in interdependent power distribution and transportation networks. The high-level information flow of the scheme is depicted in Fig. 2.

### III. System Model

#### A. Joint Routing and Destination Planning Of EVs

In this subsection, we introduce each EVs’ decision variables, explicit cost function, as well as local constraints.

1) **Decision Variables:** For the EV $i$ with inelastic energy demand $q_i \in [\underline{q}_i, \overline{q}_i]$, $\mathbf{r}_i = [r_{i,e}]_{e=1}^E$ and $t_i = [t_{i,d}]_{d=1}^D$ are the decision variables of choosing roads and charging stations, where $r_{i,e}$ and $t_{i,d}$ are the probability of selecting road $e$ and charging station $d$, respectively. In addition, $\mathbf{r}_i = [r_{i,e}]_{e=1}^E$ and $\mathbf{t}_i = [t_{i,d}]_{d=1}^D$ are collective vector of decision variables of choosing road $e$ and station $d$, respectively.

2) **Cost Function:** The cost function (1) is composed of the cost of deviating from preferred route and charging station, the traveling cost, and the cost of station’s congestion denoted by $U_i$, $C_i^{\text{travel}}$, and $C_i^{\text{station}}$, respectively. So, we have

$$J_i = U_i + \omega_i C_i^{\text{travel}} + C_i^{\text{station}}$$  \hspace{1cm} (1)
where \( \omega_i \in [\omega, \bar{\omega}] \) is the conversion coefficient of traveling time to a monetary value for user \( i \). The more explicit term for users' preference is

\[
U_i(r_i, t_i) = \frac{\alpha_i}{2} ||r_i - \hat{r}_i||^2 + \frac{\beta_i}{2} ||t_i - \hat{t}_i||^2 .
\]  

(2)

Based on user \( i \)’s previous travel experiences, \( \hat{r}_i = [\hat{r}_{iD}]_{d=1}^D \in [0, 1]^D \), \( \sum_{d=1}^D \hat{r}_{iD}^2 = 1 \) and \( \hat{r}_i = [\hat{r}_{ie}]_{e=1}^E \in [0, 1]^E \) represent the preferred destination and path, respectively, and \( \alpha_i \in [\alpha, \bar{\alpha}] \) and \( \beta_i \in [\beta, \bar{\beta}] \) are the constant weight parameters that represent the monetary value of their preferences. In addition, we assume that each road segment \( e \in E \) has a travel time \( t_e(\cdot) \), which is a continuous and strictly increasing function of the corresponding road’s vehicle flow. A heuristic delay function that is proportionate to the link’s traffic level is used to calculate the travel time along each link. The Bureau of Public Roads (BPR) delay model is a well-known latency function that is often used in the literature [23], which is as follows:

\[
l_e(\sigma_e(r^e)) = \eta_e \left( 1 + \pi \frac{\sigma_e(r^e)}{\kappa_e} \right)^\xi
\]  

(3)

where \( \eta_e \) is the free-flow travel time on the road \( e \), and \( \kappa_e \) is the capacity of road \( e \). The parameters \( \pi \geq 0 \) and \( \xi \geq 0 \) are two positive constants of the BPR latency function [23]. Parameters \( \pi \) and \( \xi \) are commonly set to 0.15 and 4, respectively. Also, \( \sigma_e(r^e) = \sum_{i \in M} r_{ie}^e + s_e \) is the link flow that is equal to the total expected number of EVs (\( \sum_{i \in M} r_{ie}^e \)) and non-EVs (\( s_e \)) on each road segment \( e \in E \). We define the vector \( \sigma(r) = [\sigma_e]_{e=1}^E \). In addition, \( r = [r_{ie}]_{i=1}^I \) and \( t = [t_{ie}]_{i=1}^I \) show collective decision variables of all EVs regarding choosing the route and destination, respectively. Thus, the expected traveling time of the user \( i \in M \) is given by

\[
C_{\text{travel}}(r_i, \sigma(r)) = \sum_{e \in E} l_e(\sigma_e(r^e)) r_{ie}^e .
\]  

(4)

We define congestion pricing at the stations to handle overcrowding at the stations considering the growing number of EVs, the limited supply of EV chargers in the stations, as well as the operation cost of the chargers. Considering the congestion price \( \psi_d \) at station \( d \) proportional to the average utilization of the stations, we have

\[
\psi_d(t^d) = \zeta_d \frac{\delta_d(t^d)}{\gamma_d \times \phi_d}
\]  

(5)

in which for the station \( d, \delta_d(t^d) = \sum_{i \in M} t_{id}^d \) is the arrival rate of EVs, \( \phi_d \) is the number of EV chargers, and each charger gets available with the rate \( \gamma_d \) (1/(average charging time)), which depends on the charging mode at the station \( d \) [24]. Finally, \( \zeta_d \) is a constant value (in $) of the operational expenses that is set by charging stations. Also, the collective vector of stations arrival rate is equal to \( \delta = [\delta_{id}]_{i=1}^I \). Thus, the expected cost of congestion at the stations for user \( i \) is as follows:

\[
C_{\text{station}}(t_i, \delta(t)) = \sum_{d \in D} \psi_d \left( \delta_d(t^d) \right) t_{id}^d .
\]  

(6)

3) Local Constraint: Furthermore, for the user \( i \), the following linear constraint ensures that the user starting from the initial location \( o_i \in V \) reaches the desired destination \( d \in D \) with probability \( t_i^d \) by considering the underlying transportation graph structure \( G_T \):

\[
\sum_{e \in (u,v) \in E} r_{ie}^e - \sum_{e \in (v,w) \in E} r_{ie}^e = \begin{cases} 
-1, & \text{if } v = o_i \\
0, & \text{if } v = d \\
0, & \text{otherwise} 
\end{cases} \quad \forall v \in V, \forall d \in D
\]  

(7)

where \( e = (u, v) \in E \) denotes the link \( e \in E \subseteq V \times V \), which originates at node \( u \) and terminates at node \( v \). As a result, by defining \( x_i = \text{col}(r_i, t_i) \), each user’s individual constraint set can be specified as follows:

\[
\mathcal{X}_i := \{ x_i \in [0, 1]^{E+D} | \text{constraint} \ (7), t_i^d = 0 \forall d \in D \setminus \mathcal{D}_i \}
\]  

(8)

B. OPF Problem of the DSO

The DSO operates the distribution power network and clears the dispatch level of DGs, real-time electricity prices, active and reactive power flow of lines, and voltages of nodes. In detail, \( p_{h}^{\text{gen}}, q_{h}^{\text{gen}} \), and \( \lambda_{\text{dmp}} \) show the active and reactive dispatch level of DG \( h \), and electricity price at bus \( j \), respectively. For each line \((u, j) \in L, P_{u,j} \) and \( Q_{u,j} \) are active and reactive power flowing from bus \( u \) to bus \( j \), respectively. Each line \((u, j) \in L \) has its own internal resistance, denoted by \( R_{u,j} \); reactance, denoted by \( X_{u,j} \); and power limit, denoted by \( S_{u,j}^{\text{max}} \). \( V_j \) stands for the voltage at node \( j \), and we also use \( v_j = |V_j|^2 \) to compute the line voltage drop in the system. In addition, the bus \( 1 \) represents the substation bus, and other buses in \( N \) represent branch buses. The set of DGs is represented by \( \Omega^G = \{1, \ldots, H\} \). For each bus \( j \in N \), the set of DGs connected to the bus \( j \) is denoted by \( \Omega_j^G \subseteq \Omega^G \), and \( \Omega_j^S \subseteq D \) stands for the set of charging stations connected to the bus \( j \). The operational cost of DGs is modeled as a common quadratic function as follows:

\[
C_h^{\text{gen}}(p_{h}^{\text{gen}}) = a_h(p_{h}^{\text{gen}})^2 + b_h(p_{h}^{\text{gen}}) + c_h
\]  

(9)

where \( a_h, b_h \) and \( c_h \) are positive cost coefficients. EVs’ aggregative variable demand in each station \( d \) is obtained as the expected demand of users at the corresponding station

\[
\varphi_d(t^d) = \sum_{i \in M} q_i t_{id}^d .
\]  

(10)

Therefore, power demand at bus \( j \) is determined by the summation of fixed residential load connected to the bus denoted by \( p^{\text{load}}_j \) and variable total expected demand of EVs at stations \( d \in \Omega_j^S \) connected to the bus \( j \). The DSO uses the LinDistFlow model [25], which is a linear approximation of the ac power flow model, to minimize total generation costs while taking operational constraints into account. Hence, the optimization problem of the DSO can be expressed as follows:

\[
\min_y \quad J_0(p_{h}^{\text{gen}}) = \sum_{h \in \Omega^G} C_h^{\text{gen}}(p_{h}^{\text{gen}})
\]  

(11a)
\[ \begin{align*}
\text{s.t.} \sum_{u:(j,u) \in \mathcal{L}} P_{ju} - P_{uj} &= \sum_{h \in \Omega^G} p_{h}^\text{gen} - p_{j}^\text{load} \\
- \sum_{d \in \Omega^S} \varphi_d (t^d), \forall j \in \mathcal{N} &\quad (11b) \\
\sum_{u:(j,u) \in \mathcal{L}} Q_{ju} - Q_{uj} &= \sum_{h \in \Omega^G} q_{h}^\text{gen} - q_{j}^\text{load} \quad \forall j \in \mathcal{N} \quad (11c) \\
v_j &= v_u - 2R_{uj} P_{uj} - 2X_{uj} Q_{uj} \quad \forall (u,j) \in \mathcal{L} \quad (11d) \\
p_{h}^\text{gen} &\leq p_{h}^\text{gen} \leq q_{h}^\text{gen} \quad \forall h \in \Omega^G \quad (11e) \\
q_{h}^\text{gen} &\leq q_{h}^\text{gen} \leq q_{h}^\text{min} \quad \forall h \in \Omega^G \quad (11f) \\
\gamma_j &\leq v_j \leq \bar{v}_j \quad \forall j \in \mathcal{N} \quad (11g) \\
P_{u}^2 + Q_{u}^2 &\leq S_{max} \quad \forall (u,j) \in \mathcal{L} \quad (11h)
\end{align*} \]

We let \( y = \text{col}(p^\text{gen}, q^\text{gen}, P_L, Q_L, v) \) stand for the DSO’s collective decision variable, in which \( p^\text{gen} = [p_{h}^\text{gen}]_{h=1}^{|\Omega^G|}, q^\text{gen} = [q_{h}^\text{gen}]_{h=1}^{|\Omega^G|}, v = [v_{j}]_{j=1}^{|\mathcal{N}|}, P_L = \{P_{uj} \mid \forall (u,j) \in \mathcal{L}\}, Q_L = \{Q_{uj} \mid \forall (u,j) \in \mathcal{L}\}, p_{h}^\text{gen} \) and \( q_{h}^\text{gen} \) are the maximum and minimum active power outputs of DG \( h \). Similarly, \( p_{h}^\text{gen} \) and \( q_{h}^\text{gen} \) are the maximum and minimum reactive power outputs of DG \( h \). Lastly, \( \gamma_j \) and \( \bar{v}_j \) are the maximum and minimum values for voltage constraint of node \( j \). Equation (11b) enforces active power balance at the distribution network, whereas (11c) enforces reactive power balance. Equation (11d) provides a linear voltage drop relationship across the distribution system. Moreover, (11e) and (11f) represent DGs’ active and reactive power output constraints. Lastly, (11g) are nodal voltage constraints and (11h) corresponds to the power constraints for each distribution line.

The equality constraint (11b) is best described as a coupling constraint between users and the DSO, as the decisions of all users affect the power balance equation of the optimal power flow (OPF) problem. The feasible set corresponding to the constraint (11b) can be written as follows:

\[ C^\text{power} = \left\{ t \in \mathbb{R}^{MD} \mid y \in \mathbb{R}^{2H+3|\mathcal{L}|} \text{ constraint (11b)} \right\} \quad (12) \]

where \( \lambda^\text{damp} \) is the Lagrangian multiplier vector of the constraint (12). In the power balance (11b), the effect of EVs’ variable expected demand is shown. Also, for a station \( d \) that is connected to the bus \( j \), the per-unit electricity price \( \lambda^d_{\text{station}} \) for the station is equal to the DLMP price \( \lambda^d_{\text{damp}} \) for the bus \( j \), which is derived from solving problem (11); so we have

\[ \lambda^d_{\text{station}} = \lambda^d_{\text{damp}} \quad \forall d \in \Omega^S \quad (13) \]

If the distribution network is not congested, DLMPs in all buses and, subsequently, prices at all the charging stations will be the same. As the demand for certain buses increases and the power network’s constraints get violated, prices at those buses increase in real-time to encourage users to switch to charging stations with un congested buses or cheaper generation units. The DSO’s private constraint set is the feasible set that corresponds to the constraints (11e)–(11h), which is

\[ \mathcal{Y} = \left\{ y \in \mathbb{R}^{2H+3|\mathcal{L}|} \mid \text{ constraints (11e) – (11h)} \right\} \quad (14) \]

The resulting convex optimization problem (11) can be solved in real-time with acceptable accuracy [26].

### C. Coupling Constraints

We consider the coupling constraints among users related to stations’ power capacity and transportation network vehicles’ flow limit as follows:

\[ C^t = \{ t \in \mathbb{R}^{MD} \mid \varphi_d (t^d) \leq c^d_{\text{dlmp}} \forall d \in \mathcal{D} \} \quad (15a) \]

\[ C^r = \{ r \in \mathbb{R}^{ME} \mid \sigma_e (r^e) \leq c^e_{\text{dlmp}} \forall e \in \mathcal{E} \} \quad (15b) \]

The constraint (15a) with Lagrangian multipliers \( \lambda_t \) ensures that, for each station \( d \), the expected charging demand must be less than the station capacity’s upper bound \( c^d_{\text{dlmp}} \). In addition, constraint (15b) with Lagrangian multipliers \( \lambda_r \) indicates that each road segment \( e \) cannot serve more vehicles than its upper bound \( c^e_{\text{dlmp}} \). Stations’ capacity vectors and roads’ capacity vectors are denoted by \( c^t = [c^d_{\text{dlmp}}]_d=1^{|\mathcal{D}|} \) and \( c^r = [c^e_{\text{dlmp}}]_e=1^{|\mathcal{E}|} \), respectively.

Also, on the basis of what was mentioned in the previous two sections, we define the collective coupling constraint on the system as follows:

\[ C = C^t \cap (C^r \cap C^\text{power}) \quad (16) \]

In fact, Lagrange multipliers \( \lambda^t_{\text{damp}}, \lambda^r_{\text{damp}}, \) and \( \lambda^\text{damp} \) are prices imposed by the DSO, CSOs, and the TNO, respectively, to influence consumers.

### IV. Decentralized Game-Theoretical Analysis

Given the users’ competition for the fastest route to the least crowded stations and the limits on charging stations, roads, and power system, the interaction among EV users and DSO can be formally defined as a game problem.

#### A. Game Formulation

Based on the introduced models, an aggregative game with coupling constraints can be defined among EV users and the DSO–the game \( \mathcal{G} \)–whose elements are as follows:

\[ \mathcal{G} = \begin{cases} \text{Players} : & M \text{ EVs and the DSO} \\
\text{Strategies} : & \{ \text{EV} i : x_i \in \mathcal{X}_i \} \cup \{ \text{DSO} : y \in \mathcal{Y} \} \\
\text{Costs} : & \{ \text{EV} i : J_i (x_i, \Theta (x)) \} \cup \{ \text{DSO} : J_0 (y) \} \\
\text{Coupling Constraints} : & \mathcal{C} \end{cases} \quad (17) \]

We also let \( \mathcal{K} := (\prod_{i=1}^M \mathcal{X}_i \times \mathcal{Y}) \cap \mathcal{C} \) denote the cumulative strategy set of the game \( \mathcal{G} \).

The concept of Wardrop equilibrium is well known in transportation networks [27]. Specifically, we consider the v-GWE [28], [29].
For each user and \( \forall i \in \mathcal{M} \), we have
\[
J_i(x_i, \Theta(x)) \leq J_i(x_i, \Theta(\bar{x}))
\]
\[\forall x_i \text{ s.t. } x_i \in \mathcal{K} (\bar{x}_i, \bar{y}) , \]  
(18)
\[
J_0(y) \leq J_0(y) \forall y \text{ s.t. } y \in \mathcal{K}(\bar{x})
\]
where \( x = [x_i]_{i=1}^M, \bar{x} = [\bar{x}_i]_{i=1}^M, \Theta(x) = \sigma(r), \delta(t) \), and \( \bar{x}_i = (\bar{x}_1, \ldots, \bar{x}_{i-1}, \bar{x}_{i+1}, \ldots, \bar{x}_M) \).

In other words, GWE is a set of decisions in which no agent can reduce its cost function unilaterally by modifying its decision within the feasible constraint set, given that the aggregated strategies \( \Theta(x) \) are fixed.

**Definition 2:** The pseudogradient mapping of game \( \mathcal{G} \) is defined by stacking together the gradient of players’ cost functions with respect to their local decision variables, while \( \Theta(x) \) is fixed, as follows:
\[
F(x, y) := \left[ \frac{\nabla_x J_i(x_i, z)_{z=\Theta(x)}}{\nabla_y J_0(y)} \right]_{i=1}^M .
\]
(19)

In order to ensure the satisfaction of coupling constraint (16), we postulate that the Lagrange multipliers \( \lambda = \text{col}(\lambda_{\text{damp}}, \lambda_r, \lambda_1) \) are the same for all EVs. By this assumption, we restrict the analysis to a subset of equilibrium points of \( \mathcal{G} \), which is known as the v-GWE [29].

**Definition 3:** (Variational Inequality) Given a set \( \mathcal{K} \) and mapping \( F : \mathcal{K} \to \mathbb{R}^n \), the variational inequality problem (VI(\( F, \mathcal{K} \)), is to find a vector \( \text{col}(\bar{x}, \bar{y}) \in \mathcal{K} \) such that
\[
F(\bar{x}, \bar{y})^\top (\text{col}(x, y) - \text{col}(\bar{x}, \bar{y})) \geq 0 \forall \text{col}(x, y) \in \mathcal{K}.
\]
(20)

The set of solutions to (20) is denoted by \( \text{SOL}(F, \mathcal{K}) \).

In this case, the game \( \mathcal{G} \) can be characterized as a variational inequality problem \( \text{VI}(F, \mathcal{K}) \). This representation helps to obtain the \( \text{v-GWEs} \) of \( \mathcal{G} \) by solving the corresponding variational inequality problem. The relation between \( \text{SOL}(F, \mathcal{K}) \) and the \( \text{v-GWEs} \) of \( \mathcal{G} \) is established in the following Proposition.

**Proposition 1:** For game \( \mathcal{G} \), \( \text{SOL}(F, \mathcal{K}) \) of \( \text{VI}(F, \mathcal{K}) \) is also a v-GWE of game \( \mathcal{G} \).

**Proof:** For each user \( i \in \mathcal{M} \) and the DSO, the set \( \mathcal{X}_i \) and \( \mathcal{Y} \) are nonempty, closed, and convex. Also for each \( i \), function \( J_i(x_i, \Theta(x)) \) is continuously differentiable and convex in \( x_i \), and function \( J_0(y) \) is continuously differentiable and convex in \( y \). The set \( \mathcal{C} \) is also nonempty and satisfies Slater’s constraint qualification. Therefore, due to [30, Proposition 12.4], any solution of \( \text{SOL}(F, \mathcal{K}) \) is a v-GWE of \( \mathcal{G} \).

**Definition 4:** Mapping \( F : \mathcal{X} \to \mathbb{R}^n \) is monotone on \( \mathcal{X} \) if, for every \( x, y \in \mathcal{X}, x \neq y, (F(x) - F(y))^\top (x - y) \geq 0. \)

**Lemma 1:** Operator \( F(x, y) \) in (19) is monotone.

**Proof:** We can show operator \( F(x, y) \) as follows:
\[
F = \begin{bmatrix}
\alpha_i(r_i - \tilde{r}_i) + \omega_i l_i(\sigma(r)) \\
\beta_i(t_i - \tilde{t}_i) + \psi_i(\delta(t)) \\
2a_h(p_{h_i}^\text{gen} + b_h) \\
o_{|x_i| > 1}
\end{bmatrix}_{i=1}^M
\]
(21)

where \( l(\sigma(r)) = [l_i(\sigma(r))]_{i=1}^M \) and \( \psi_i(\delta(t)) = [\psi_i(\delta(t))]_{i=1}^M \). Since functions \( U_i(x_i), l_i(\sigma(r)), \) and \( \psi_i(\delta(t)) \) are convex in \( x \) and feasible sets for each \( i \in \mathcal{M} \), and function \( J_0(y) \) is convex in \( y \), the first term is monotone. Consequently, the mapping \( F(x, y) \) is monotone.

**Theorem 1:** The solution set \( \text{SOL}(F, \mathcal{K}) \) is nonempty and compact; therefore, v-GWE exists.

**Proof:** Since for all \( i \in \mathcal{M} \) the sets \( \mathcal{X}_i \) and \( \mathcal{Y} \) are closed, convex, and bounded, the function \( J_i(x_i, \Theta(x)) \) is continuously differentiable and convex in \( x_i \), the function \( J_0(y) \) is continuously differentiable and convex in \( y \) and the set \( \mathcal{C} \) is nonempty; therefore, [31, Corollary 2.2.5] guarantees the existence of an equilibrium of game \( \mathcal{G} \).

### B. Decentralized Algorithm

In this section, we want to solve the game (17) in a decentralized way. The optimization subproblem of each EV can be derived by relaxing coupling constraints \( \mathcal{C} \) as follows:
\[
\min_{x_i \in \mathcal{X}_i} J_i(r_i, t_i, \Theta(x), \lambda) = J_i + \sum_{e=1}^E \sum_{d=1}^D q_i(t_i^d + d_{\text{station}})^\top \phi_d(h^d_i) + \sum_{h \in \Omega_{\text{gen}}} P_{h_i}^\text{gen}
\]
(22)

where \( x = [x_i]_{i=1}^M, \Theta(x) = \sigma(r), \delta(t) \), and \( \lambda = \text{col}(\lambda_{\text{damp}}, \lambda_r, \lambda_1) \).

In summary, the first term in the cost function (22) reflects the explicit cost function of EVs mentioned in (1). The second term denotes the user’s payment of tolls for road usage, the third term shows the cost of energy used to charge the EV.

To solve the problem in a decentralized manner, we introduce the following extended single-valued mapping \( T \) with variables \( z = \text{col}(x, y, \lambda) \):
\[
T = \begin{bmatrix}
\nabla_x (q_i(t_i^\lambda + t_i^\lambda_{\text{station}}) + r_i^\lambda_{\text{load}}) + \lambda^\lambda_{\text{damp}} (\sum_{(j,u) \in \mathcal{L}} P_{j,u} - P_{u,j}) \\
\nabla_y (\sum_{j \in \mathcal{M}} \sum_{u \in \mathcal{X}_j} P_{j,u} - P_{u,j}) \\
\sum_{h \in \Omega} (P_{h}^\text{gen} - \varphi_d(t_i^d)_{j=1}^D)_{i=1}^M \\
\epsilon_c - \varphi_d(t_i^d)_{j=1}^D_{i=1}^M
\end{bmatrix}
\]
(23a)

where \( EC(t, y) \) is the corresponding expression in the constraint (11b) as follows:
\[
EC(t, y) = \sum_{(j,u) \in \mathcal{L}} P_{j,u} - \sum_{h \in \Omega^T_{\text{gen}}} P_{h}^\text{gen} + \sum_{d \in \Omega^T_{\text{load}}} \varphi_d(t_i^d)_{j=1}^D
\]
(24)

The zeros of mapping \( T \) are equal to the equilibrium of the game \( \mathcal{G} \) in (17). Since the mapping \( T \) is decomposable, it can be utilized to seek the v-GWE in a decentralized way. It is worth noting that the pseudogadients associated with EVs in (23a) are gradients of functions in (22).

**Proposition 2:** Two variational inequality problems, \( \text{VI}(T, Z) \) and \( \text{VI}(F, \mathcal{K}) \), are equivalent.
Algorithm 1: Aggregative Game.

Initialization: $k \leftarrow 1$, $\tau_i > 0$, $\mu > 0$, $\vartheta \in [0, 1/3]$, $q_i^{(0),(1)} \in \mathbb{R}_{>0}$, $y_i^{(0),(1)} \in \mathbb{R}^{2H+i\mathcal{L}}$, $\lambda_i^{(0),(1)} \in \mathbb{N}$, $\lambda_i^{(0),(1)} \in \mathbb{R}_{>0}$, $\lambda_i^{(0),(1)} \in \mathbb{R}_{>0}$.

Iteration $k$:

EVs: $\forall i \in \mathcal{M}$:

$$GR_i^{(k)} = 2\nabla z_i f_i(x_i^{(k)}, z) \mid z = \Theta(x_i^{(k)})$$

$$- \nabla z_i f_i(x_i^{(k-1)}, z) \mid z = \Theta(x_i^{(k-1)})$$

$$+ \nabla z_i q_i^{(k)}(x_i^{(k)} + \lambda_i^{(k)} \mathcal{L}_{i}^{T} + \lambda_i^{(k)} \mathcal{L}_{i}^{T})$$

$$z_i^{(k+1)} = \Pi_{\tau_i U(x_i^{(k)} \mathcal{L}_{i}^{T} + \lambda_i^{(k)} \mathcal{L}_{i}^{T})} \Pi_{T} f_i^{(k)}(x_i^{(k)} - \tau_i GR_i^{(k)} + \vartheta(x_i^{(k)} - z_i^{(k-1)}))$$

Power Distribution Network Operator:

$$GR_i^{(k)} = \nabla y_i \Lambda_{\text{damp}} E C(t_i^{(k)}, y_i^{(k)})$$

$$y_i^{(k+1)} = \Pi_{\tau_0 0 + \tau_0} [y_i^{(k)} - \tau_0 GR_i^{(k)} + \vartheta(y_i^{(k)} - y_i^{(k-1)})]$$

$$R_{\text{damp}}^{(k+1)} = 2EC(y_i^{(k+1)}, x_i^{(k+1)} - EC(y_i^{(k)}, x_i^{(k)})) - p_{\text{load}}$$

$$\lambda_i^{(k+1)} = \lambda_i^{(k)} + \mu R_{\text{damp}}^{(k+1)} + \vartheta(\lambda_i^{(k)} - \lambda_i^{(k-1)})$$

$$\lambda_i^{(k+1)} = \lambda_i^{(k-1)} + \vartheta(\lambda_i^{(k)} - \lambda_i^{(k-1)}), \forall d \in \Omega_i^{(k)}$$

Transportation Network Operator: $\forall e \in \mathcal{E}$:

$$R_e^{(k+1)} = 2\sigma_e (1^{(k+1)} - \sigma_e (1^{(k+1)} - c_e^{(k+1)}))$$

$$\lambda_e^{(k+1)} = \text{proj}_{\mathbb{R}^{(k+1)}} [\lambda_e^{(k)} + \mu R_e^{(k+1)} + \vartheta(\lambda_e^{(k)} - \lambda_e^{(k-1)})]$$

Charging Station Operators: $\forall d \in \mathcal{D}$:

$$R_d^{(k+1)} = 2\varphi_d (d^{(k+1)} - \varphi_d (d^{(k+1)}) - c_d^{(k+1)}))$$

$$\lambda_d^{(k+1)} = \text{proj}_{\mathbb{R}^{(k+1)}} [\lambda_d^{(k)} + \mu R_d^{(k+1)} + \vartheta(\lambda_d^{(k)} - \lambda_d^{(k-1)})]$$

$$k \leftarrow k + 1$$

**Proof:** Due to the conditions holding in Proposition 1, and according to [32, Th. 3.1], considering the same Lagrangian multipliers correspond to the coupling constraints $C$ for the users, $\mathcal{L} \in \mathbb{R}^{N+i\mathcal{L}}$ exists such that $\text{col}(C, \mathcal{L}) = \text{SOL}(T, Z)$, if and only if $\mathcal{L} = \text{SOL}(F, K)$, which is a variational equilibrium of the game $\mathcal{G}$.

To find the equilibrium point $\text{VI}(T, Z)$, we use the I-FoRB splitting method [28] and [33]. The suggested algorithm is shown in Algorithm 1. Unlike most decentralized methods, which require strong monotonicity conditions, this algorithm only requires the monotonicity of mapping $F$ (which was proved in Lemma 1) to ensure the convergence to a Wardrop equilibrium point of the game. Furthermore, the majority of the algorithms in the literature use a two-time scale scheme with a vanishing step size, which significantly reduces the convergence rate. Nevertheless, I-FoRB overcomes this problem by splitting two maximally monotone operators, one of which is Lipschitz continuous and single valued. Note that in Algorithm 1, $f_i(x_i, \Theta(x)) = \omega_i C_i^{\text{travel}} + C_i^{\text{station}}$ is a part of the cost function of EV that is affected by the aggregate decision of all users.

**V. SIMULATION RESULTS**

Our simulations are conducted on Savannah city model that includes 58 bidirectional roads, 36 nodes, and six charging stations. We utilize the linear form of the delay function with $\xi = 1$ and $\pi = 4$ [34]. The average vehicle speed on a regular day in Savannah is 30 km/h, from which the free flow travel time on roads $(\text{free flow speed})$ is computed. We assume 20 vehicles per kilometer traveling in an uncongested situation as the capacity of roads is heuristically determined by $\kappa = 30 \text{vehicle km} \times \text{(free-flow speed)}$. In addition, IEEE 33-bus radial distribution network is considered, which is connected to charging stations positioned on the transportation network.

The learning step size $\tau_i$ is set independently and locally by each entity in the Algorithm 1. Each EV updates its strategy using the gradient of its cost function based on the most recent common information provided by the aggregators and then projects its strategy onto its feasible local set using the proximal operator. Then, the DSO collects electrical demand data at distribution buses and updates the optimal power flow variables, including DLMPs, by projecting onto its feasible constraint, considering the deviation of load balance constraint and its operational costs, and updating the dual variables. Following that, the prices are broadcast to CSOs and EVs. Meanwhile, each CSO, along with the TNO, collects the associated aggregative strategies, adjusts the tolls based on their capacity constraints, and broadcasts the shared data to the EVs. This process is repeated until an equilibrium point is reached. In the following proposition, we prove the convergence of the algorithm 1 to a v-GWE of the game.

**Proposition 3:** Algorithm 1 converges to a solution of $\text{VI}(F, K)$.

**Proof:** As we showed before, due to conditions held in Lemma 1, Proposition 1, and Theorem 1, according to [28, Algorithm 5] if the positive step-sizes $\tau_i$, $\tau_0$, and $\mu$ are selected sufficiently small, and inertial parameter $\vartheta$ selected within $\vartheta \in (0, 1/3)$ for each $i \in \mathcal{M}$, the Algorithm 1 would globally converge to $\text{col}(x, \lambda) \in \text{SOL}(T, Z)$ [28, Th. 1].

![Fig. 3. Transportation network of Savannah city, where CSs are located.](image-url)
Fig. 4. IEEE 33-bus distribution network, where CSs and DGs are located.

### TABLE II

| Generation unit | 1   | 2   | 3   | 4   | 5   |
|-----------------|-----|-----|-----|-----|-----|
| $p_{h}^{\text{gen}}$ (MWh) | 5   | 2.3 | 1.5 | 2.2 | 1.6 |
| $q_{h}^{\text{gen}}$ (MVARh) | 5   | 2.3 | 1.5 | 2.2 | 1.6 |
| $\alpha_h$ ($$/\text{MW}^2/\text{h})$ | 0.35 | 0.2 | 0.4 | 0.3 | 0.25 |
| $b_h$ ($$/\text{MWh})$ | 76  | 45  | 88  | 66  | 56  |

### TABLE III

| Line       | 14 | 20 | 22 | 30 |
|------------|----|----|----|----|
| From bus   | 14 | 20 | 3  | 30 |
| To bus     | 15 | 21 | 23 | 31 |
| $s_{\text{max}}$ (kVAh) | 400 | 100 | 200 | 150 |

### TABLE IV

| Station | 1   | 2   | 3   | 4   | 5   | 6   |
|---------|-----|-----|-----|-----|-----|-----|
| $c_d$ (kWh) | 1250 | 1000 | 1400 | 1450 | 1250 | 1350 |
| $\zeta_d$ ($) | 2   | 2   | 2   | 2   | 2   | 2   |
| $\gamma_d$ ($/\text{h}$) | 2   | 2   | 2   | 2   | 2   | 2   |
| $\phi_d$ | 12  | 14  | 8   | 10  | 12  | 8   |

Figs. 3 and 4 illustrate the transportation and power networks, respectively. Fig. 4 also illustrates the locations of distributed generation units and charging stations in the distribution network, where one substation unit and four distributed generation units are assumed. Detailed information on the distribution network resistances and reactants can be found in [35], while the cost function and the generation limits of DGs are presented in Table II, except the parameters $p_{h}^{\text{gen}}$, $q_{h}^{\text{gen}}$, and $c_h$ which are set to 0. We also assume that the substation bus is regulated and has a fixed voltage of 12.66 kV. Lastly, $\pi_j$ and $\varphi_j$ are set to 193.93 and 129.83 at all nodes $j$. In Table III, we also provide the distribution network line limits. In addition, Table IV shows stations’ coefficients. We choose these parameters to demonstrate how different DLMPs could arise in response to distribution line constraints and the heterogeneity of DG cost functions, such that they almost reflect the median electricity price in Savannah city [36].

First, we investigate a case in which there are 125 cars on the road with uniform distributions of electricity demand $q$ (kWh) $\sim U(20, 70)$ and monetary value of time $\omega_i$ ($$/\text{hr}) $\sim U(3.6, 14.4)$, as well as a uniform traffic distribution of $s_e$ (vehicles/ hr) $\sim U(55, 150)$ on roads. Each roadway has a maximum vehicle capacity of 160. We assumed that EVs were dispersed uniformly over the network. In addition, just for simplicity, we assumed that each EV $i$ parameters $\alpha_i$ and $\beta_i$ are equal 0.5$$ and likes to take the shortest route to the nearest station, which has effects on parameters $\tilde{r}$ and $\tilde{t}$.

The first set of analyses investigates the convergence of the algorithm. Fig. 5 shows the convergence of DGs’ output power. We notice that DG 5, which has a lower operating cost than DGs 1, 3, and 4, and is located close to station 3, has reached its generation capacity due to the high demand of station 3. In Fig. 6, we see the convergence of DLMPs at the buses where charging stations are located. The differences in DLMPs across locations are due to different costs associated with the generation units and the fact that all four distribution lines presented in Table III are operating at full capacity. Despite the fact that the generation unit price is cheaper in the area surrounding station 3, its DLMP is the highest. This is due to the fact that user preferences, and consequently energy demand, exceed the capacity of the power network to deliver them. Therefore, the DSO must increase the DLMPs to a level that incentivizes EVs to use stations with lower operational costs. In addition, Fig. 7 depicts the convergence of aggregated demand at stations, and
Fig. 7. Demand for energy at CSs; capacity limits are met.

Fig. 8. Surcharges imposed by CSs to enforce capacity limits.

Fig. 9. Voltage magnitudes of buses throughout the network.

Fig. 10. Effects of users’ parameters on the DSO’s decision.

Simultaneously, the TNO manages the roads by charging users. For example, the southern road of station 4 reached its capacity limits, and the TNO charges 3.44$ for using that roadway. Consequently, users change their route and use the eastern road to reach station 4 instead of the southern road. All computational simulations are performed on Google Colab with 12-GB memory. At each iteration, the average computation time of each EV, the DSO, and the TNO are 0.074, 0.17, and 0.004 s, respectively.

Afterward, we investigate the effects of various elements on the DSO operation. We disregard the limited coupling constraints of stations and roadways for this purpose. First, we analyze the effect of the number of EVs (and thus, the total electricity demand) on the DLMPs. As shown in Fig. 10(a), while demand is low, station 3 has a lower DLMP than stations 1, 2, 4, and 5; nevertheless, increasing the number of EVs raises the DLMP, since in this case stronger persuasion is needed to change the EVs’ preferred decision. Also, we can observe that DLMP at station 6 is increasing since DG 2 and distribution line 14 have reached their limits, and the DSO is unable to supply adequate electricity to that station.

We study the impact of a personal parameter, known as the monetary value of time, on the DSO decision-making. In Fig. 10(b), as the mean value of the time value of money increases, so do the DLMP on popular location like CS 4 because EVs require more persuasion to change their decisions.

VI. CONCLUSION

In using a generalized aggregative game, we modeled and evaluated the interactions of heterogeneous EVs, the DSO, CSO, and the TNO. We presented a decentralized learning technique-based on real-time DLMP by the DSO and pricing and tolling by CSOs and the TNO to satisfy infrastructure restrictions. The decentralized learning technique is scalable and secure. It also leads to a GWE of the holistic model. We study the Savannah city model and the IEEE 33-bus distribution network numerically. We also looked into the influence of station characteristics on pricing and demand. This work can be extended in several directions, including mechanism design for better system-level performance, considering dynamic and time-varying models for more precise modeling of the problem, exploration of multiagent reinforcement learning for solving the problem, and faster equilibrium seeking algorithms for practical implementation of the proposed scheme.
REFERENCES

[1] E. M. Bibra et al., “Global EV outlook 2021: Accelerating ambitions despite the pandemic,” Int. Energy Agency, 2021. [Online]. Available: https://iea.blob.core.windows.net/assets/ed5f4484-1556-4110-8c5c-4ed8ebca637/GLOBALEVOutlook2021.pdf

[2] S. J. Davis et al., “Net-zero emissions energy systems,” Science, vol. 360, no. 6596, 2018, Art. no. eaas9793.

[3] M. Muratori, “Impact of uncoordinated plug-in electric vehicle charging on residential property demand,” Nature Energy, vol. 3, no. 3, pp. 193–201, 2018.

[4] B. G. Bakhshayesh and H. Kebraili, “Decentralized equilibrium seeking of joint routing and destination planning of electric vehicles: A constrained aggregative game approach,” IEEE Trans. Intell. Transp. Syst., vol. 23, no. 8, pp. 13265–13274, Aug. 2022.

[5] J. L. Ratliff, R. Dong, S. Sekar, and T. Fiez, “A perspective on incentive design: Challenges and opportunities,” Ann. Rev. Control, Robot., Auton. Syst., vol. 2, pp. 305–338, 2019.

[6] M. Alizadeh, H.-T. Wai, M. Chowdhury, A. Goldsmith, A. Scaglione, and H. Javidi, “Optimal pricing to manage electric vehicles in coupled power and transportation networks,” IEEE Trans. Control Netw. Syst., vol. 4, no. 4, pp. 863–875, Dec. 2017.

[7] Y. Sun, Z. Chen, Z. Li, W. Tian, and M. Shahidehpour, “EV charging schedule in coupled constrained networks of transportation and power system,” IEEE Trans. Smart Grid, vol. 10, no. 5, pp. 4706–4716, Sep. 2019.

[8] F. Rossi, R. Iglesias, M. Alizadeh, and M. Pavone, “On the interaction between autonomous mobility-on-demand systems and the power network: Models and coordination algorithms,” IEEE Trans. Control Netw. Syst., vol. 7, no. 4, pp. 384–397, Mar. 2020.

[9] A. Estandia et al., “On the interaction between autonomous mobility on demand systems and power distribution networks—an optimal power flow approach,” IEEE Trans. Control Netw. Syst., vol. 8, no. 3, pp. 1163–1176, Sep. 2021.

[10] F. He, Y. Yin, J. Wang, and Y. Yang, “Optimal charging of EVs in urban electricity distribution networks,” IEEE Trans. Power Syst., vol. 21, no. 1, pp. 516–523, Feb. 2006.

[11] W. Wei, L. Wu, J. Wang, and S. Mei, “Network equilibrium of coupled transportation and power distribution systems,” IEEE Trans. Smart Grid, vol. 9, no. 6, pp. 6764–6779, Nov. 2018.

[12] W. Wei, S. Mei, L. Wu, M. Shahidehpour, and Y. Fang, “Optimal traffic-power flow in urban electrified transportation networks,” IEEE Trans. Smart Grid, vol. 8, no. 1, pp. 84–95, Jan. 2017.

[13] T. Qian, C. Shao, X. Li, X. Wang, and M. Shahidehpour, “Enhanced coordinated operations of electric power and transportation networks via EV charging services,” IEEE Trans. Smart Grid, vol. 11, no. 4, pp. 3019–3030, Jul. 2020.

[14] B. Sohet, Y. Hayel, O. Beaude, and A. Jeandin, “Hierarchical coupled driving-and-charging model of electric vehicles, stations and grid operators,” IEEE Trans. Smart Grid, vol. 12, no. 6, pp. 5146–5157, Nov. 2021.

[15] W. Wei, W. Danman, W. Qiuwei, M. Shafie-Khah, and J. P. Catalao, “Spatial Econ., vol. 16, no. 1, pp. 131–145, 2016.

[16] Y. Malitsky and M. K. Tam, “A forward-backward splitting method for monotone inclusions without cocoercivity,” SIAM J. Optim., vol. 10, no. 4, pp. 1097–1115, 2000.

[17] A. Auslender and M. Teboulle, “Lagrangian duality and related multiplier methods for variational inequality problems,” SIAM J. Optim., vol. 10, no. 4, pp. 384–397, Mar. 2020.

[18] M. Baran and F. Wu, “Optimal traffic equilibrium seeking in monotone aggregative games,” IEEE Trans. Autom. Control, vol. 68, no. 1, pp. 140–155, Jan. 2023.

[19] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros, “Nash and Wardrop equilibria in aggregative games with coupling constraints,” IEEE Trans. Autom. Control, vol. 64, no. 4, pp. 1373–1388, Apr. 2019.

[20] J. G. Wardrop, “The user equilibrium concept,” Proc. Inst. Civil Engineers, vol. 2, pp. 32–39, 1952.

[21] J. Wardrop, “Road traffic research,” Prog. Inst. Civil Engineers, vol. 3, pp. 325–362, 1952.

[22] A. Auslender and M. Teboulle, “Lagrangian duality and related multiplier methods for variational inequality problems,” SIAM J. Optim., vol. 10, no. 4, pp. 1097–1115, 2000.

[23] A. V. Fiacco and M. C. Han, “A globally convergent augmented Lagrangean method for optimization with general constraints and equality constraints,” SIAM J. Optim., vol. 1, no. 1, pp. 1–37, Jan. 1991.

[24] R. Cole, Y. Dodis, and T. Roughgarden, “Pricing network edges for heterogeneous selfish users,” in Proc. 35th Annu. ACM Symp. Theory Comput., 2003, pp. 521–530.

[25] Traffic Assignment Manual. Bur. Public Roads, U.S. Dept. Commerce, Urban Planning Division, Washington, DC, USA, 1964.

[26] R. D. Reid and N. R. Sanders, Operations Management: An Integrated Approach. Hoboken, NJ, USA: Wiley, 2019.

[27] M. E. Baran and F. F. Wu, “Optimal capacitor placement on radial distribution systems,” IEEE Trans. Power Del., vol. 4, no. 1, pp. 725–734, Jan. 1989.

[28] H. Zhu and H. J. Liu, “Fast local voltage control under limited reactive power: Optimality and stability analysis,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 3794–3803, Sep. 2016.

[29] G. Belgioioso and S. Grammatico, “Semi-decentralized generalized Nash equilibrium seeking in monotone aggregative games,” IEEE Trans. Autom. Control, vol. 68, no. 1, pp. 140–155, Jan. 2023.

[30] M. Baran and F. Wu, “Network reconﬁguration in distribution systems for loss reduction and load balancing,” IEEE Trans. Power Del., vol. 4, no. 2, pp. 1401–1407, Apr. 1989.

[31] M. Baran and F. Wu, “Network reconﬁguration in distribution systems for loss reduction and load balancing,” IEEE Trans. Power Del., vol. 4, no. 2, pp. 1401–1407, Apr. 1989.

[32] M. Baran and F. Wu, “Network reconﬁguration in distribution systems for loss reduction and load balancing,” IEEE Trans. Power Del., vol. 4, no. 2, pp. 1401–1407, Apr. 1989.

[33] Y. Malitsky and M. K. Tam, “A forward-backward splitting method for monotone inclusions without cocoercivity,” SIAM J. Optim., vol. 30, no. 2, pp. 1451–1472, 2020.

[34] J. G. Wardrop, “Road paper. some theoretical aspects of road traffic,” Proc. Inst. Civil Engineers, vol. 2, pp. 325–362, 1952.

[35] M. Baran and F. Wu, “Network reconﬁguration in distribution systems for loss reduction and load balancing,” IEEE Trans. Power Del., vol. 4, no. 2, pp. 1401–1407, Apr. 1989.

[36] Y. Malitsky and M. K. Tam, “A forward-backward splitting method for monotone inclusions without cocoercivity,” SIAM J. Optim., vol. 30, no. 2, pp. 1451–1472, 2020.

[37] J. G. Wardrop, “Road paper. some theoretical aspects of road traffic,” Proc. Inst. Civil Engineers, vol. 2, pp. 325–362, 1952.