Strong-field and attosecond physics in solids

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Abstract

We review the status of strong-field and attosecond processes in bulk transparent solids near the Keldysh tunneling limit. For high enough fields and low-frequency excitations, the optical and electronic properties of dielectrics can be transiently and reversibly modified within the applied pulse. In Ghimire et al (2011 Phys. Rev. Lett. 107 167407) non-parabolic band effects were seen in photon-assisted tunneling experiments in ZnO crystals in a strong mid-infrared field. Using the same ZnO crystals, Ghimire et al (2011 Nat. Phys. 7 138–41) reported the first observation of non-perturbative high harmonics, extending well above the bandgap into the vacuum ultraviolet. Recent experiments by Schubert et al (2014 Nat. Photonics 8 119–23) showed a carrier envelope phase dependence in the harmonic spectrum in strong-field 30 THz driven GaSe crystals which is the most direct evidence yet of the role of sub-cycle electron dynamics in solid-state harmonic generation. The harmonic generation mechanism is different from the gas phase owing to the high density and periodicity of the crystal. For example, this results in a linear dependence of the high-energy cutoff with the applied field in contrast to the quadratic dependence in the gas phase. Sub-100 attosecond pulses could become possible if the harmonic spectrum can be extended into the extreme ultraviolet (XUV). Here we report harmonics generated in bulk MgO crystals, extending to ∼26 eV when driven by ∼35 fs, 800 nm pulses focused to a ∼1 V Å⁻¹ peak field. The fundamental strong-field and attosecond response also leads to Wannier–Stark localization and reversible semimetalization as seen in the sub-optical cycle behavior of XUV absorption and photocurrent experiments on fused silica by Schiffirn et al (2013 Nature 493 70–4) and Schultz et al (2013 Nature 493 75–8). These studies are advancing our understanding of fundamental strong-field and attosecond physics in solids with potential applications for compact coherent short-wavelength sources and ultra-high speed optoelectronics.

Keywords: HHG, attosecond, Bloch oscillations, strong-field, harmonic generation, tunneling

(Some figures may appear in colour only in the online journal)

1. Introduction

Fifty years ago in his seminal theory paper, Keldysh considered both tunneling across the bandgap in insulating crystals as well as tunnel-ionization of free atoms subjected to strong low-frequency electromagnetic waves [1]. It took two decades to obtain a clear confirmation of the tunneling theory...
by experiments in gases [2], but the fairly accurate ADK theory [3] and the strong-field approximation (SFA) [4] and their limits are now well established. In the gas phase the dynamics of an electron after tunneling is often, to a good approximation, described semi-classically by its classical motion in the electromagnetic field, neglecting the effect of the Coulomb potential [5–7] but including the electron wave-packet spreading [8]. The ability to control these strongly driven electron dynamics in the gas phase has been the foundation of much of attosecond science and technology [9–11]. In particular, gas-phase high-order harmonic generation (HHG) has led to synthesis of isolated attosecond pulses [9, 12], imaging of molecular orbitals [13–15] and realization of attosecond transient absorption spectroscopy at short wavelengths [16].

There is a renewed interest in the strong-field response in solids following the remarkable discovery that dielectrics can withstand fields approaching their critical fields when irradiated with long-wavelength pulses that are short compared to the electronic relaxation time scales [17–21]. Despite early theoretical works [1, 22–24], we have only just begun to understand the strong-field response in solids. One complication is that the SFA does not hold in bulk solids as the electrons are never free of the influence of the interatomic potential. At low fields the electron dynamics can be treated in the effective mass approximation as was considered by Keldysh [1]; however, for large bandgaps and strong fields this approximation can break down well before the onset of tunneling [25]. While the full dynamics must take into account both the intraband and interband transitions, the spatial and temporal periodicity of the strong-field Hamiltonian maintains coherence between the electrons on different sites even under strong localization. Nonetheless, for crystalline solids one might expect a regime where subsequent to tunneling, the electron wave-packet dynamics within a single band can be treated semi-classically. For strong enough fields, and when inelastic scattering can be neglected, this gives rise to periodically modulated Bloch oscillations that repeat each half-cycle of the laser [17, 26–32]. This highly non-linear current should radiate high-order harmonics whose spectrum contains a plateau structure with a high-energy cutoff that scales linearly with the electric field [27, 28]. For motion within a single band, it is the Bloch frequency and not the conduction bandwidth that sets the energy scale of the harmonics. In the non-perturbative limit, we expect multiple plateaus extending to,

$$\hbar \omega_{\text{max}} \approx n \hbar \omega_B,$$

where $d$ is the lattice spacing, $\omega_B = eEd/\hbar$ is the Bloch frequency corresponding to the peak electric field, $E$, and $n$ is an integer corresponding to contributions from higher spatial frequencies in the band dispersion [17]. In contrast, the semi-classical three-step model of atomic HHG predicts a high-energy cutoff that is proportional to the square of the electric field from [5, 6, 33]

$$\hbar \omega_{\text{max}} \approx I_p + 3.2U_p,$$

where $I_p$ is the ionization potential and $U_p = e^2E^2/4m\omega^2$ is the ponderomotive energy of an electron in a field of angular frequency $\omega$. Unlike the atomic case, the semi-classical theory for solids predicts that the cutoff is independent of frequency, so long as the Bloch frequency well exceeds the drive frequency. In addition, the effect of increasing field is to localize the electron wave-packet [27, 34, 35] through Bragg scattering from the Brillouin zone boundaries. This leads the wave-packet to reverse direction multiple times per half laser

- In a real space picture this corresponds to transitions between Wannier–Stark (WS) levels at $n$ different sites.
cycle, resulting in a highly non-linear current that will radiate high-order harmonics. This process is illustrated in figure 1.

In table 1, we summarize several selected experiments in bulk solid samples that illustrate different regimes corresponding to various dimensionless parameters characterizing the strength of the electromagnetic field. \( \gamma \) is the well known Keldysh parameter describing the tunneling time with respect to the laser period. In insulating solids \( \gamma = \omega \sqrt{m^* \Delta_{\text{gap}}} / eE \), where \( \Delta_{\text{gap}} \) is the bandgap and \( m^* \) is the electron–hole reduced mass. For periodic solids, we also consider \( \beta = \omega_B / 2 \omega \) which is the ratio of the Bloch frequency to twice laser frequency. In this case \( \beta \gtrsim 1 \) corresponds to the regime where full Bloch oscillations can occur within a single half cycle of the field. Finally, we define \( \delta \) as the ratio of the peak field to the critical field, where \( E_c = \Delta_{\text{gap}} / e \delta d \) is the field required to change the electron energy by the bandgap over the lattice spacing. Thus, \( \delta = \hbar \omega_B / \Delta_{\text{gap}} \) is also the ratio of (\( \beta \times \gamma \)) the Bloch frequency to the bandgap. When \( \delta \gtrsim 1 \), electrons can populate the entire conduction band (CB) via Zener-like transitions [20].

This paper is organized as follows. In section 2 we review experiments focusing on strong-field-induced optical absorption in crystals from the multi-photon to tunneling regimes. In section 3 we review experiments on harmonic generation and wave mixing, including the production of optical side bands and high harmonics into the vacuum ultraviolet (VUV). We present new results in MgO where the harmonics extend to the extreme ultraviolet (XUV). In section 4 we discuss recent advances in the theory and experiment of wide-bandgap materials excited near the critical field by very short, near single-cycle pulses. This includes attosecond XUV transient absorption and photocurrent measurements that suggest reversible semimetallization. Finally, in section 5, we provide some brief concluding remarks.

2. Strong-field-induced below-gap absorption

Franz [22] and Keldysh [38] independently predicted a red-shift in the optical absorption edge in insulators subjected to strong external field (electro-absorption or the Franz–Keldysh Effect). This can be viewed as photon-assisted tunneling from the valence band (VB) to CB and was observed in CdSe for peak dc fields on the order of \( 10^{-3} \text{V/Å} \) [39]. Chin et al [36] observed a substantial red-shift in the electro-absorption in ZnSe and GaAs samples subjected to a maximum field of \( \sim 0.04 \text{V/Å} \) at 3.5 \( \mu \text{m} \). The results were interpreted as being due to the dynamical Franz–Keldysh effect [23] where the tunneling time is comparable to the period of the low-frequency field, representing a transition between photon-assisted tunneling and multi-photon, multi-color absorption.

Ghimire et al [18] performed electro-absorption measurements in strong mid-infrared (MIR)-driven ZnO single crystals, in the limit where the tunneling time is short compared to the period of the low-frequency field (\( \gamma < 1 \)). Figure 2 shows results from this experiment. Weak broadband white light (probe) and strong MIR (pump) pulses are brought into coincidence in a ZnO crystal as shown in figure 2(a). Figure 2(b) shows a dramatic modification of the transmission of the probe as it overlaps in time with the strong MIR pulse inside the crystal at an intensity \( \sim 1 \text{TW/cm}^2 \). The field induces almost complete absorption of the probe light only when the pulses are overlapped temporally inside the crystal. The change in absorption lasts a few picoseconds due to the difference in group velocity of the MIR and probe light and the relatively long crystal. Strong group velocity dispersion near the bandgap causes the different spectral content of the broadband probe to overlap with the MIR pulse at the exit face at slightly different time delays, as the frequency components closest to the band edge travel much slower than the MIR. Negative time delays correspond to the lower energy
portion of the probe arriving before the pump at the exit surface. The measured intensity dependence of the field-induced red-shift in the absorption edge is shown in figure 2(c). The data were fitted to a single power law for $I \lesssim 1 \text{ TW cm}^{-2}$ as $\Delta \omega_0 = 0.38 \text{ eV}(I/\text{TW cm}^{-2})^{1/3}$ (dashed line). For parabolic bands, the photon-assisted transition rate as a function of the electric field $E$ and optical photon energy $\omega$ is

$$w(E, \omega) = C \exp \left\{ -\frac{4\sqrt{2} m^*}{3\varepsilon_0 h} \left( \Delta_{\text{gap}} - \omega \right)^{3/2} \right\},$$

(3)

where the pre-factor, $C$, is a relatively slowly varying function compared to the exponential, such that the red-shift is expected to scale on the order of $\hbar \Delta \omega \sim (e^2\hbar^2E^2/m^*)^{1/3}$ [22, 38]. Thus the measured intensity scaling of the red-shift, up to the peak intensity $1 \text{ TW cm}^{-2}$, is consistent to what is expected from the Franz–Keldysh dc limit in the effective mass approximation, although the measured absolute red-shift is about a factor of five lower than what is expected ($1.6 \text{ eV}$ at $1 \text{ TW cm}^{-2}$). Above $1 \text{ TW cm}^{-2}$ the red-shift saturates. This was interpreted as strong evidence of non-parabolic effects [18, 25] because in this limit of $\beta > 1$ the electron wave-packet is expected to traverse the entire Brillouin zone within a half cycle.

3. Solid-state high harmonic generation

High harmonic generation in insulating crystals subjected to strong laser fields has been studied in both the perturbative and non-perturbative limit. Chin et al [37] observed harmonics of a MIR pump up to seventh order in polycrystalline ZnSe for a peak field strength on the order of $0.1 \text{ VÅ}^{-1}$. The generation was perturbative as indicated by rapidly decreasing harmonic yield as harmonic orders progress. They also reported the generation of side bands on an optical probe that was also found to be consistent with perturbative multi-photon wave-mixing processes [40].

Non-perturbative harmonic generation was reported by Ghimire et al [17, 18] in ZnO and by Schubert et al [21] in GaSe near the Keldysh tunneling limit $\gamma < 1$ and the Bloch limit $\beta > 1$. In the ZnO experiment, up to the 25th order harmonics of the MIR pump were produced well above the bandgap and into the VUV. In the GaSe experiment, up to 22nd order harmonics of the $\sim 30 \text{ THz}$ pump were produced in the visible range. In addition, in the Schubert experiment the harmonics showed clear carrier envelope phase (CEP) dependence, particularly for harmonics above the bandgap.

In the Ghimire experiment, the MIR laser beam is focused on a 500 $\mu$m thick single crystal ZnO sample using a f/3 focusing lens at near normal incidence. Figure 3(a) shows spectra from this experiment at two different input pulse energies. The spectrum for the lower pulse energy ($0.52 \text{ Jcm}^{-2}$) extends further to the 25th order harmonics of the MIR pump were produced well above the bandgap and into the VUV. In the GaSe experiment, up to 22nd order harmonics of the $\sim 30 \text{ THz}$ pump were produced in the visible range. In addition, in the Schubert experiment the harmonics showed clear carrier envelope phase (CEP) dependence, particularly for harmonics above the bandgap.
Bloch frequency. This could be accommodated in a single-band model by allowing for higher spatial frequencies in the dispersion [17, 41] (due to long-range overlap of the wave functions for tight-binding bands). The presence of photon energies above both the bandgap (3.2 eV) and the conduction band model ignores the phase mismatch that limits the efficiency. Figure 4(a) shows the calculated temporal profile of the harmonics as a function of crystal thickness considering the parameters from experiments [17]. The model ignores the finite duration of the pulse and assumes that the carrier density resides at the zone center at the peak of the electric field (as would be expected for tunneling near the peak of the field). This results in an attocycle whereby the highest harmonics are produced at the highest fields and the lower order harmonics are produced symmetrically about that time. The 55 μm modulation period along the direction of crystal thickness is due to the phase mismatch between the drive laser field and harmonics. The temporal profile of harmonics corresponds to a ‘train’ of attosecond pulses, consisting of a main peak every half laser cycle along with side peaks for the optimum thickness. In figure 4(b) we show a lineout of the temporal profile of the harmonics at a crystal thickness of 1 μm. The main peaks have a duration on the order of 650 attoseconds.

Shorter duration and isolated attosecond pulses would require a harmonic spectrum that extends to a much higher photon energy. While the temporal profile of solid-state harmonics remains to be measured, we have been able to extend the cutoff to the XUV photon energy range in previously unreported experiments. Here we focus 800 nm, 35 fs laser pulses, obtained directly from a commercial Ti:sapphire laser amplifier, to a 500 μm thick MgO crystal (Δgap=7.8 eV) mounted inside a vacuum chamber. The samples exhibit no multi-shot damage up to a peak field strength of ~1 VÅ⁻¹ at a kHz repetition rate. We place the...
laser beam waist near the exit surface of the sample to reduce absorption of high-energy photons generated in the crystal. The harmonic beam is dispersed by using a variable line spaced (600 lines/mm) concave grating to an ICCD detector. The diffraction grating covers a fairly wide wavelength range (∼120 nm to ∼20 nm) but the limitation is imposed by the acceptance angle of the detector, which is from ∼65 nm to ∼40 nm. A measured harmonic spectrum near the cutoff is shown in figure 5. It contains 13th, 15th and 17th order harmonics. The color bar is in a linear scale. The higher yield on the 15th harmonic compared to the 13th is consistent with the transmission properties of MgO. The 17th harmonic is weak because it is at the cutoff.

Figure 5. Measured near cutoff harmonics at a maximum field strength of ∼1 V Å−1 in MgO crystals excited by 800 nm ∼35 fs laser pulses. The spectrum contains 13th, 15th and 17th order harmonics. The color bar is in a linear scale. The higher yield on the 15th harmonic compared to the 13th is consistent with the transmission properties of MgO. The 17th harmonic is weak because it is at the cutoff.

4. Wannier–Stark localization and attosecond control of reversible carrier dynamics

The most direct information on the dynamics of strong-field phenomena in condensed matter has been obtained by means of attosecond metrology [20, 44]. In the experiment of Schultze et al [20] a SiO₂ sample is simultaneously excited by powerful sub-4 fs, sub-1.5 cycle near-infrared (NIR) to visible laser pulses and probed by synchronized near-70 attosecond XUV pulses, as schematically illustrated in figure 6. The waveform of the drive pulse obtained by simultaneous XUV streaking is shown in figure 7(a). Figure 7(b) shows a lineout of the recorded transient absorption spectrogram at the band edge, exhibiting an instantaneous, highly reversible transient decrease in the XUV absorptivity that oscillates in synchrony with the strong field. This suggests that this dynamics follows the instantaneous electric field and is dominated by instant changes in local density of the VB and CB states and its virtual population rather than real transfer of population, which would survive for picoseconds (see also the caption to figure 7(c)). The total CB population (sum of the virtual and real populations) is given by equation (10) of the supplemental information of [20]. The dominance of virtual over real population is corroborated by the theoretical prediction shown in figure 7(c). This conclusion appears to contradict the one drawn from the analysis of the second-harmonic emission from dielectrics in a recent study [45] and is of far-reaching importance for potential signal processing and metrology applications.

For fields of strengths approaching the critical field strength (δ ≈ 1) and for low frequencies (ω < < Δgap), the electronic states of the solid are predicted to undergo WS localization [50] within approximately one unit cell of the crystal. For dielectrics such as silica and alumina with Δgap ≈ 9 eV, Eγ ≈ 2 V Å⁻¹, and the low-frequency condition is satisfied for NIR and visible light. The energy of a WS state residing at a certain site l = 1, 2, … of the lattice along the
direction of the (internal) field $E$ becomes

$$E_{i,n} = E_n - edE_l,$$

(4)

following the field variation adiabatically. (Here, $E_n$ with $n = \{c, f\}, \{v, k\}$ is the offset of the $j$th and $k$th sub-band of the CB and VB, respectively; for simplicity, we suppress the sub-band indices in our discussion.) The adjacent levels in each WS ladder are separated by the Bloch frequency. By definition, when $\delta = 1$, $\Delta \approx \Delta_{\text{gap}}$, and hence the Bloch frequency is in the UV. At this point, $E_{i,c}(E)$ and $E_{i,v}(E)$ are equal for $\Delta = 1$, i.e., the WS states residing at neighboring sites in the CB and VB degenerate in energy to undergo an anticrossing corresponding to Zener–Keldysh interband tunneling.

The calculated energy level diagram for silica ([51], shown in figure 8), reveals a large energy gap, $\Delta E_l \gg \omega$ at $\Delta l = 1$. This implies that the WS state belonging to the lower anticrossing level remains fully occupied while adiabatically changing its wave function from VB to CB (the well-known adiabatic exchange of quantum numbers). Consequently, the dynamics is largely reversible. This gradual change can be quantified by projecting the actual filled, time-dependent VB states to zero-field CB states as given by equation (10) of the supplemental information of [20]. This projection results in a transient population of CB electrons (and corresponding VB holes) displayed in figure 7(c). These can be interpreted as virtual carriers similar to those responsible for virtual photoconductivity in semiconductors predicted [46–48] and observed [49] over two decades ago. However, the analogy stops here. In our case, the virtual carriers are produced by a non-resonant (adiabatic), strong-field (and hence, necessarily, non-perturbative) excitation, in contrast with the early experiments, where they were excited via a near-resonant perturbative excitation. The conclusion on the reversibility of the photo-induced changes in dielectrics is generally incompatible with perturbation theory and is at odds with [52]. In any perturbation theory, processes proceed in one direction; thus, the excited-state population should continuously increase with every half-cycle of the field, in contrast with the reversibility seen in both the experiment and theory presented in the works of Schulze et al [20].

In the same regime, reversible (semi)metallization of dielectrics has been discovered by Schiffrin et al [19]. A schematic of this experiment is shown in figure 9; it was conducted in a single-pulse configuration and in a double-pulse (pump-probe) configuration. In the single pulse version, a powerful pulse (1.5 optical oscillations, amplitude $E \approx 1.7 \text{VÅ}^{-1}$) was polarized in the electron-transfer direction (the $x$-polarization), as depicted by the blue waveform, see figure 9. The schematic of figure 9 resembles a metal–oxide–semiconductor field-effect transistor (MOSFET) [53–56] except for a fundamental difference: instead of a semiconductor, it employs a wide-band insulator.

Some principal results of the Schiffrin experiment [19] are illustrated in figure 10(a), which displays the full charge transferred per laser pulse between the electrodes, $|Q_p|$. The charge transfer occurs efficiently only for electric field $E$ polarized in the charge transfer direction; it is practically absent for a perpendicular field. The charge transferred fundamentally depends on the CEP of the pulse, $\Phi_{CE}$, which...
is controlled with a silica wedge introduced into the optical path. As we see, \( Q_p(\phi) \) oscillates, changing from zero to maximum within a CEP change of \( \pi/2 \), i.e. within a quarter of the optical pulse. This strongly suggests that the current induced in the dielectric is controlled by the instantaneous optical field on an attosecond time scale. The peak transferred charge as a function of the optical field amplitude, \( |Q_p| \), is displayed in figure 10(b). It exhibits a highly non-linear, non-perturbative dependence. The maximum transferred charge is \( |Q_p| \approx 2 \) aFs. Given that the peak current is reached within less than 1 fs, this implies that the maximum current in the dielectric reached during the pulse is \( I \gtrsim 2 \) A. From these data, the optical conductivity of silica in the presence of the optical field is estimated to increase by almost 19 orders of magnitude. This is orders of magnitude less than a good metal and is close to the range of semimetals.

In the picture introduced in Schultze et al [19] and Schiffrin et al [20], the strong but deeply adiabatic optical fields create a new state of matter where the original VB and CB states are mixed. The calculated spectrum shows formation of WS ladders and their anticrossings where the population transfer reversibly occurs, see figure 8. The most significant adiabatic population transfer occurs at \( \Delta l = 1 \). As a result, the original bandgap of the dielectric disappears and the occupied and empty states are mixed in energy: the pronounced Fermi surface separating filled and empty states disappears. Due to the collapse of the bandgap, such a non-equilibrium state has a high...
polarizability; in particular the optical conductivity is increased with respect to the original dielectric by >18 orders of magnitude reaching levels typical of semimetals [19]. Such a non-perturbative phenomenon can be called semimetallization. If this $\Delta l = 1$ anticrossing were to be passed without breakdown, the dielectric would experience a full metallization transition.

5. Concluding remarks

Here we reviewed recent results on strong-field and attosecond processes in bulk transparent solids. This is the regime for dynamical Bloch oscillations and WS localization, leading to several non-pertubative effects such as high harmonic generation and reversible semimetallization. As shown by the experiments, in the strong-field and low-frequency excitations the optical and electronic properties of dielectrics can be transiently and reversibly modified within the applied pulse. These processes occur on a time scale that is short compared to the pulse duration and show a CEP dependence suggestive of adiabatic processes. The observation of high-order harmonics, particularly as the spectrum reaches the XUV energy range, implies the presence of attosecond time scale dynamics in strongly driven periodic solids. We note that because the response time of a physical system is determined by its bandwidth: $T_{\text{min}} \lesssim \Delta \omega^{-1}$, to use this bandwidth without major losses, the signal should spectrally be confined within the bandgap. Because dielectrics typically have bandgaps on the scale of a few to 10 eV, they are expected to support electronic responses on the order of several hundred attoseconds. In addition, the reversibility gives promise for a fundamentally new approach to electric and photonic signal manipulation on an unprecedented time scale, which may be largely free from the limitations of contemporary MOSFETs.

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