Family Universal Anomalous $U(1)$

in Realistic Superstring Derived Models

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Abstract

An important issue in supersymmetry phenomenology is the suppression of squarks contributions to Flavor Changing Neutral Currents (FCNC). Recently it was noted that in some free fermionic three generation models the anomalous $U(1)$ is family universal. It was further shown that if the $D$-term of the $U(1)_{A}$ is the dominant source of supersymmetry breaking, the squark masses are indeed approximately degenerate. In this paper I discuss the properties of the superstring models that give rise to the flavor universal anomalous $U(1)$. The root cause for the universal $U(1)_{A}$ is the cyclic permutation symmetry, the characteristic property of the $Z_{2} \times Z_{2}$ orbifold compactification, realized in the free fermionic models by the NAHE set of boundary condition basis vectors. The properties of the three generation models that preserve this cyclic permutation symmetry in the flavor charges are discussed. The cyclic permutation symmetry of the $Z_{2} \times Z_{2}$ orbifold compactification is proposed to be the characteristic property, of phenomenological interest, that distinguishes it from other classes of superstring compactifications.

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The flavor problem in supersymmetric extensions of the Standard Model is especially interesting. On the one hand the hierarchical pattern of fermion masses clearly indicates the need for flavor dependent symmetries. On the other hand the absence of Flavor Changing Neutral Currents at an observable rate suggests the need for flavor independent symmetries, which force squark mass degeneracy. In supersymmetric field theories the flavor parameters can be chosen to agree with the data. However, in theories that aim at the consistent unification of gravity with the gauge interactions the flavor structure is imposed and cannot be chosen arbitrarily. Superstring theories are examples of such theories and indeed it is in general expected that the squark masses in this context are flavor dependent\(^\text{[1]}\). The question then arises, how can there exist the flavor dependent symmetries, needed to explain the hierarchical fermion mass pattern, while forcing the degeneracy of squark masses, needed to explain the suppression of FCNC\(^\text{[2]}\).

In a recent paper the issue of supersymmetry breaking and squark degeneracy was studied in the three generation free fermionic superstring models\(^\text{[3]}\). The proposed mechanism for supersymmetry breaking is due to the anomalous $U(1)$ $D$–term together with highly suppressed mass terms for some relevant fields. The effect of the mass term is to shift the $D$–term of the anomalous $U(1)$ thereby breaking supersymmetry\(^\text{[4]}\). The interesting feature of some free fermionic models is the fact that the anomalous $U(1)$ is family universal. It was further shown, for a specific choice of flat directions, that the $D$–terms of the flavor dependent $U(1)$’s vanish in the minimum of the vacuum. In these free fermionic models, provided that the dominant component of the squark masses comes from the anomalous $U(1)$ $D$–term, the squark masses will be approximately degenerate.

The purpose of this paper is to study the common properties of the free fermionic models which give rise to a flavor universal anomalous $U(1)$. It is expected that once supersymmetry is broken mass terms of the order of the TeV scale will be generated, irrespective of the dominant source of supersymmetry breaking. Such mass terms

\(^*\)An alternative proposal to the squark mass degeneracy is the alignment mechanism of Ref. \([2]\). A question of interest is whether such alignment can naturally arise in a concrete string model
will then shift the various $D$-terms, resulting in non-vanishing $D$-term contributions to the squark masses. Therefore, in general, the presence of such non-vanishing $D$-terms poses a real danger to the viability of the string models. The generic presence of flavor dependent $U(1)$ symmetries in superstring models then provides an additional criteria in the selection of the viable models. The solutions studied in ref. [3] provide the guideline how string models can on the one hand provide the required symmetries to explain the fermion mass spectrum while on the other hand explain the required squark mass degeneracy.

It should be noted that a universal anomalous $U(1)$ is by no means a general outcome of string solutions [3, 4, 5, 6, 7, 8]. On the contrary, in a generic superstring model, we in general expect that the charges of the chiral generations under the anomalous $U(1)$, like other potential sources for supersymmetry breaking, will be flavor dependent. This is demonstrated, for example, in studies of supersymmetry breaking by moduli fields, which are in general expected to result in flavor dependent soft squark masses [9]. The issue then of flavor universality of the soft squark masses serves as an important guide in the selection of string vacua. Understanding this important issue in a specific class of string vacua, together with other important phenomenological issues, like the proton longevity and the qualitative fermion mass spectrum, then serves as a guide to the properties that an eventual, fully realistic, superstring vacua, might possess.

It should be further emphasized that even in the restricted class of three generation free fermionic models, the emergence of a flavor universal $U(1)$ is by no means the generic situation. Indeed, of the three generation free fermionic models, one can find several examples in which the anomalous $U(1)$ is not family universal [3, 4, 5, 6, 7, 8]. The task then is to try to isolate, in the class of free fermionic three generation models, the properties of the models that do produce a flavor independent anomalous $U(1)$. While the flavor sfermion universality is discussed in this paper only with respect to the anomalous $U(1)$, it should be remarked that the generic properties of this restricted class of models may also result in flavor universal soft SUSY breaking parameters in other sectors of the models, that will not be investigated here, but are
worthy of further investigation.

Let us recall that a model in the free fermionic formulation is defined by a set of boundary condition basis vectors, and the associated one–loop GSO projection coefficients [11]. The massless spectrum is obtained by applying the generalized GSO projections. A physical state defines a vertex operator which encodes all the quantum numbers with respect to the global and gauge symmetries. Superpotential terms are then obtained by calculating the correlators between the vertex operators [12, 13].

The free fermionic models correspond to orbifold models at a fixed point in the Narain moduli space. The same models can be constructed in the orbifold construction by specifying the background fields and fixing the radii of the compactified dimensions at the point which corresponds to the free fermionic construction. This correspondence is important because the free fermionic construction facilitates the study of the string vacua, and the extraction of the properties of the specific orbifold vacua that are important from the phenomenological perspective. The purpose of this paper is partially to highlight one of these properties. Namely, the cyclic permutation symmetry of the $Z_2 \times Z_2$ orbifold compactification, which is one of the basic reasons for the appearance of a flavor universal anomalous $U(1)$ in some free fermionic models.

The free fermionic models studied here are constructed in two stages. The first stage consists of the NAHE set, \{1, S, b_1, b_2, b_3\}. This set of boundary condition basis vectors has been discussed extensively in the literature [5, 14, 15, 16]. As the properties of the NAHE set are important to understand the emergence of a family universal anomalous $U(1)$, for completeness the main features are shortly emphasized.

The basis vectors of the NAHE set are defined by

|   | $\psi^\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\eta^1$, $\eta^2$, $\eta^3$ | $\phi^{1}$, $\phi^{2}$, $\phi^{3}$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1, 1, 1 | 1, 1, 1 |
| S | 1 | 1 | 1 | 1 | 0, 0, 0 | 0, 0, 0 |
| b_1 | 1 | 1 | 0 | 0 | 1, 1 | 1, 0, 0 | 0, 0, 0 |
| b_2 | 1 | 0 | 1 | 0 | 1, 1 | 0, 0, 0 | 0, 0, 0 |
| b_3 | 1 | 0 | 0 | 1 | 1, 1 | 0, 0, 1 | 0, 0, 0 |
with ‘0’ indicating Neveu–Schwarz boundary conditions and ‘1’ indicating Ramond boundary conditions, and with the following choice of GSO phases:

\[ C \begin{pmatrix} b_i \\ b_j \end{pmatrix} = C \begin{pmatrix} b_i \\ S \end{pmatrix} = C \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1. \] (2)

The gauge group after imposing the GSO projections of the NAHE set basis vectors is \( SO(10) \times SO(6)^3 \times E_8 \). The three sectors \( b_1, b_2 \) and \( b_3 \) produce 48 multiplets in the chiral 16 representation of \( SO(10) \). The states from each sector transform under the flavor, right–moving \( SO(6)_j \) gauge symmetries, and under the left–moving global symmetries. This is evident from table (1), as each of the sets of the world–sheet fermions \( \{ y^{3,\ldots,6}, \bar{y}^{3,\ldots,6}, \eta^1 \} \), \( \{ y^{1,2}, \omega^{5,6}, \bar{y}^{1,2}, \omega^{5,6}, \bar{\eta}^2 \} \) and \( \{ \omega^{1,\ldots,4}, \bar{\omega}^{1,\ldots,4}, \eta^3 \} \), has periodic boundary conditions in each of the basis vectors \( b_1, b_2 \) and \( b_3 \), respectively. Also evident from table (1) is the cyclic permutation symmetry between the three sectors \( b_1, b_2 \) and \( b_3 \), with the accompanying permutation between the three sets of internal world–sheet fermions. This cyclic permutation symmetry is the root cause for the emergence of flavor universal anomalous \( U(1) \) in some free fermionic models. If indeed, as argued in ref. [3], flavor universality of the anomalous \( U(1) \) is necessary for the phenomenological viability of a superstring model, the NAHE set may turn out to be a necessary component in a realistic string vacua. This, if correct, is a remarkable outcome, as it serves to isolate the point in the moduli space where the true string vacuum may be located.

The NAHE set corresponds to \( Z_2 \times Z_2 \) orbifold compactification. This correspondence is demonstrated explicitly by adding to the NAHE set the boundary condi-

\[ \begin{array}{c|c|c|c|c|}
   & y^{3,\ldots,6} & \bar{y}^{3,\ldots,6} & y^{1,2}, \omega^{5,6} & \bar{y}^{1,2}, \omega^{5,6} \\
   \hline
   1 & 1,...,1 & 1,...,1 & 1,...,1 & 1,...,1 \\
   S & 0,...,0 & 0,...,0 & 0,...,0 & 0,...,0 \\
   b_1 & 1,...,1 & 1,...,1 & 0,...,0 & 0,...,0 \\
   b_2 & 0,...,0 & 0,...,0 & 1,...,1 & 1,...,1 \\
   b_3 & 0,...,0 & 0,...,0 & 0,...,0 & 1,...,1 \\
\end{array} \]
tion basis vector $X$, with periodic boundary conditions for the world-sheets fermions \{${\bar{\psi}}^{1,\cdots,5}, \bar{\eta}^{1,\bar{\eta}^2}, \bar{\eta}^3$\} and antiperiodic boundary conditions for all others. With a suitable choice of the generalized GSO projection coefficients, the $SO(10)$ gauge group is enhanced to $E_6$. The $SO(6)^3$ symmetries are broken to $SO(4)^3 \times U(1)^3$. One combination of the $U(1)$ symmetries is embedded in $E_6$,

$$U(1)_{E_6} = \frac{1}{\sqrt{3}}(U_1 + U_2 + U_3).$$

This $U(1)$ symmetry is flavor independent, whereas the two orthogonal combinations

$$U(1)_{12} = \frac{1}{\sqrt{2}}(U_1 - U_2);$$

$$U(1)_{\psi} = \frac{1}{\sqrt{6}}(U_1 + U_2 - 2U_3)$$

are flavor dependent. The normalization of the various $U(1)$ combinations is fixed by the requirement that the conformal dimension of the massless states still gives $\bar{h} = 1$ in the new basis. The final gauge group in this case is therefore $E_6 \times U(1)^2 \times SO(4)^3 \times E_8$.

The three sectors $b_j \oplus b_j + X$ ($j = 1, 2, 3$) now produce 24 multiplets in the 27 representation of $E_6$, which are charged under the flavor $U(1)$ symmetries Eq. (3).

The multiplicity of the generations arises from the transformation under the flavor left- and right-moving $SO(4)^3$ symmetries. The same model is constructed in the orbifold formulation by first constructing the background metric and antisymmetric tensor which define the Narain model. Fixing the radii of the six compactified dimensions at $R_I = \sqrt{2}$, produces an $N = 4$ supersymmetric model with $SO(12) \times E_8 \times E_8$ gauge group. Acting with the $Z_2 \times Z_2$ twisting on the compactified dimensions, with the standard embedding, then produces identical spectrum and symmetries as the free fermionic model, defined by the set of boundary condition basis vectors \{1, S, b_1, b_2, b_3, X\} [7]. The set of complex right-moving world-sheet fermions \{${\bar{\psi}}^{1,\cdots,5}, \bar{\eta}^{1,\bar{\eta}^2}, \bar{\eta}^3, \bar{\phi}^{1,\cdots,8}$\}, corresponds to the sixteen dimensional compactified torus of the heterotic string in ten dimensions, whereas the set of left- and right-moving real fermions \{y, $\omega|\bar{y}, \bar{\omega}\}^{1,\cdots,6}$ corresponds to the six compactified dimensions of the $SO(12)$ lattice, and $\chi^{1,\cdots,6}$ are their fermionic superpartners.

In the realistic free fermionic models the $E_6$ symmetry is replaced by $SO(10) \times U(1)$. This can be seen to arise in two ways. The first, which is the one employed
traditionally in the literature, is to substitute the vector $X$ above, with a boundary condition basis vector (typically denoted as $2\gamma$) with periodic boundary conditions for the complex world–sheet fermions $\{\tilde{\psi}^1, \ldots, \tilde{\psi}^5, \tilde{\eta}^1, \tilde{\eta}^2, \tilde{\eta}^3, \tilde{\phi}^1, \ldots, \tilde{\phi}^4\}$. The $E_6$ symmetry is then never realized explicitly and the right–moving gauge group in this case is $SO(10) \times U(1)_A \times U(1)^2 \times SO(4)^3 \times SO(16)$. The three sectors $b_j$ now produce 24 generations in the 16 representation of $SO(10)$ whereas the sectors $b_j+2\gamma$ now produce 24 multiplets in the vectorial 16 representation of the hidden $SO(16)$ gauge group.

Alternatively, the same model is generated by starting with the set $\{1, S, b_1, b_2, b_3, X\}$. However, we have a discrete choice in the GSO phase $c(X, \xi) = \pm 1$, where $\xi = 1 + b_1 + b_2 + b_3$. For one choice we obtain the model with $E_6 \times E_8$ gauge group. For the second choice, the gauge bosons from the sector $X$, in the $16 \oplus \overline{16}$ representation of $SO(10)$ as well as those from the sector $\xi$ in the 128 representation of $SO(16)$, are projected out from the massless spectrum. We then obtain the same model as with the set $\{1, S, b_1, b_2, b_3, 2\gamma\}$. The $E_6 \times E_8$ gauge group in this case is therefore broken to $SO(10) \times U(1)_A \times SO(16)$ where $U(1)_A$ is the anomalous $U(1)$ combination. We therefore see how in this case the anomalous $U(1)$ is just the combination which is embedded in $E_6$ and its flavor universality is fact arises for this reason.

We recall however that the NAHE set and the related $E_6 \times E_8$ and $SO(10) \times U(1)_A \times SO(16)$ models are just the first stage in the construction of the three generation free fermionic models. The next step is the construction of several additional boundary condition basis vectors. These additional boundary condition basis vectors reduce the number of generations to three generations, one from each of the sectors $b_1$, $b_2$ and $b_3$. The additional boundary condition basis vectors break the $SO(10)$ gauge group to one of its subgroups and similarly for the hidden $SO(16)$ gauge group. At the same time the flavor $SO(4)^3$ symmetries are broken to factors of $U(1)$’s. The number of these $U(1)$’s depends on the specific assignment of boundary conditions for the set of internal world–sheet fermions $\{y, \omega | \bar{y}, \bar{\omega}\}^{1, \ldots, 6}$ and can vary from 0 to 6. The additional right–moving $U(1)$ symmetries arise by pairing two of the right–moving real internal fermions from the set $\{\bar{y}, \bar{\omega}\}$, to form a complex fermion. For every right–moving $U(1)$ symmetry, there is a corresponding left–moving global
$U(1)$ symmetry that is obtained by pairing the corresponding two left-moving real fermions from the set $\{y, \omega\}$. Each of the remaining world-sheet left-moving real fermions from the set $\{y, \omega\}$ is paired with a right-moving real fermion from the set $\{\bar{y}, \bar{\omega}\}$ to form an Ising model operator. The different combinations of the real world-sheet fermions into complex fermions, or Ising model operators, are determined by the boundary conditions assignments in the additional boundary condition basis vectors. The allowed pairings are constrained by the requirement that the left-moving world-sheet super-current of the $N = 2$ algebra transforms appropriately under the assignment of boundary conditions. In the models that utilize only periodic and anti-periodic boundary conditions for the left-moving sector, the eighteen left-moving fermions are divided into six triplets in the adjoint representation of the automorphism group $SU(2)^6$ \[ [1, 18] \], typically denoted by $\{\chi_i, y_i, \omega_i\}$ \( i = 1, \ldots, 6 \). The six $\chi^{1\cdots6}$ are paired to form the three complex fermions of the NS/R fermions. The allowed boundary conditions of each of these six triplets depend on the boundary condition of the world-sheet fermions $\psi^\mu_{1,2}$. For sectors with periodic boundary conditions, $b(\psi^\mu_{1,2}) = 1$, i.e. those that produce space-time fermions the allowed boundary condition in each triplet are $(1, 0, 0), (0, 1, 0) (0, 0, 1)$ and $(1, 1, 1)$. For sectors with antiperiodic boundary conditions, $b(\psi^\mu_{1,2}) = 0$, i.e. those that produce space-time bosons, the allowed boundary conditions are $(1, 1, 0), (1, 0, 1) (0, 1, 1)$ and $(0, 0, 0)$. The super current constraint and the various desirable phenomenological criteria then limit the possible complex or Ising model combinations of the left-moving fermions.

In the type of models that are considered here a pair of real fermions which are combined to form a complex fermion or an Ising model operator must have the identical boundary conditions in all sectors. In practice it is sufficient to require that a pair of such real fermions have the same boundary conditions in all the boundary basis vectors which span a given model. The NAHE set of boundary condition basis vectors already divides the eighteen left-moving real fermions into three groups

\begin{align*}
\{(\chi^1, \ , \ ),(\chi^2, \ , \ ),(\ , y^3, \ ),(\ , y^4, \ ),(\ , y^5, \ ),(\ , y^6, \ )\} & \quad (6) \\
\{(\ , y^1, \ ),(\ , y^2, \ ),(\chi^3, \ , \ ),(\chi^4, \ , \ ),(\ , \omega^5, \ ),(\ , \omega^6)\} & \quad (7) \\
\{(\ , \ , \omega^1),(\ , \ , \omega^2),(\ , \ , \omega^3),(\ , \ , \omega^4),(\chi^5, \ , \ ),(\chi^6, \ , \ )\} & \quad (8)
\end{align*}
where the notation emphasizes the division of the eighteen left–moving internal world–sheet fermions into the $SU(2)^6$ triplets. The $\chi^{12,34,56}$ are the complexified combinations which generate the $U(1)$ current of the $N = 2$ left–moving world–sheet supersymmetry $[13]$. We have the freedom to complexify all, some or none of the remaining twelve left–moving world–sheet fermions. These different choices will in turn produce superstring models with substantially different phenomenological implications.

The additional boundary condition basis vectors, beyond the NAHE set, may in fact destroy the cyclic permutation symmetry between the sectors $b_1$, $b_2$ and $b_3$. For example if only one $U(1)$ symmetry remains unbroken from the $SO(4)^3$ symmetries, as for example is the case in the flipped $SU(5)$ model of ref. [3] and in one of the standard–like models in ref. [5], then evidently the cyclic permutation symmetry, which exist at the level of the NAHE set, is lost. Thus, we note the first condition for preservation of the cyclic permutation symmetry of the NAHE set. Namely, the assignment of boundary conditions in the additional basis vectors must be such that the extra $U(1)$’s respect the permutation symmetry. Therefore, it is seen that only models with zero, three or six extra $U(1)$’s that arise from the $SO(4)^3$ group factors, can preserve the permutation symmetry. Furthermore, the vector combination of the additional boundary condition basis vectors, combined with the NAHE set basis vectors, can give rise to additional massless spectrum that contributes to the total trace of the anomalous $U(1)$ charge. Then in the most general case we in fact may expect that the permutation symmetry is not maintained and therefore that the anomalous $U(1)$ is not family universal. Nevertheless, for some choices of the boundary conditions in the additional boundary condition basis vectors the permutation symmetry in the gauge sector is preserved and in these cases the anomalous $U(1)$ is family universal. Our aim is therefore to identify the choices of additional boundary condition basis vectors that preserve the permutation symmetry and the flavor universality of the anomalous $U(1)$.

The free fermionic models that give rise to a flavor universal $U(1)$ are the two classes of models, [19] and [20] that were investigated in ref. [3]. The reason for
referring to those as classes of models is because in addition to the choices of boundary condition assignments for the free world–sheet fermions, we still have the freedom, up to the modular invariance constraints, of the discrete choices of GSO phases. Therefore, each choice of boundary condition basis vectors still spans a space of models that are distinguished by the choices of GSO phases and, in general, may differ in their massless spectrum. The boundary conditions in the basis vectors beyond the NAHE set that define the model of ref. [19] are shown in Eq. (9).

| $\psi^\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\bar{\psi}^{1,\ldots,5}$ | $\bar{\eta}^1$ | $\bar{\eta}^2$ | $\bar{\eta}^3$ | $\bar{\phi}^{1,\ldots,8}$ |
|----------|------------|------------|------------|----------------|-------------|-------------|-------------|----------------|
| $\alpha$ | 0          | 0          | 0          | 1 1 1 0 0     | 0 0 0       | 1 1 1 0 0 0 0 0 |               |
| $\beta$  | 0          | 0          | 0          | 1 1 1 0 0     | 0 0 0       | 1 1 1 0 0 0 0 0 |               |
| $\gamma$ | 0          | 0          | 0          | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ |               |

Interestingly enough there exist also a flipped $SU(5)$ model in which the permutation symmetry between the sectors $b_1$, $b_2$ and $b_3$ is preserved and in which the structure of the anomalous and anomaly free $U(1)$’s is similar to the one found in the model of ref. [19]. This is the flipped $SU(5)$ of ref. [21], and the basis vectors (beyond the NAHE set) defining this model are shown in Eq. (10).

| $y^3 y^6$ | $y^4 y^5$ | $y^5 y^8$ | $y^6 y^8$ | $y^1 \omega^5$ | $y^2 \omega^2$ | $\omega^6 \omega^6$ | $y^1 \omega^5$ | $\omega^2 \omega^4$ | $\omega^1 \omega^1$ | $\omega^3 \omega^3$ | $\omega^2 \omega^4$ |
|------------|------------|------------|------------|----------------|-------------|-------------|----------------|-------------|----------------|----------------|-------------|
| $\alpha$   | 1          | 0          | 0          | 0 0 1 1       | 0 0 1 1     | 0 0 1 1     | 0 0 1 1     |               |
| $\beta$    | 0          | 0          | 1          | 1 0 0 0       | 0 1 0 1     | 0 1 0 1     | 0 1 0 1     |               |
| $\gamma$   | 0          | 1          | 0          | 1 0 0 1       | 1 0 0 1     | 1 0 0 1     | 1 0 0 1     |               |

| $\psi^\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\bar{\psi}^{1,\ldots,5}$ | $\bar{\eta}^1$ | $\bar{\eta}^2$ | $\bar{\eta}^3$ | $\bar{\phi}^{1,\ldots,8}$ |
|------------|------------|------------|------------|----------------|-------------|-------------|-------------|----------------|
| $b_4$      | 1          | 1          | 0          | 1 1 1 1 1     | 1 0 0       | 1 1 1 0 0 0 0 |               |
| $b_5$      | 1          | 0          | 1          | 1 1 1 1 1     | 0 1 0       | 1 1 1 0 0 0 0 |               |
| $\gamma$   | 0          | 0          | 0          | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ |               |
The difference is that this flipped $SU(5)$ superstring model contains two additional pairs of $10 + \overline{10}$ of $SU(5)$, beyond those that arise from the NAHE set basis vectors (i.e. from the sectors $b_4$ and $b_5$). In the flipped $SU(5)$ case there is an additional freedom in the identification of the light generations. The final charges of the generations under the anomalous $U(1)$ will depend on this identification. Consequently, whether or not the universality of the anomalous $U(1)$ can be preserved in flipped $SU(5)$ models depends on a more detailed analysis of flat directions and the fermion mass spectrum. Nevertheless, we can still identify several common features which give rise to a similar structure of the anomalous $U(1)$ in these three models.

Turning to the next common features. There is a large amount of freedom in the allowed pairings of the internal world–sheet fermions, $\{ y, \omega | \bar{y}, \bar{\omega} \}^1, \cdots, 6$. Detailed discussion on the classification of the models by the world–sheet pairings is given in [16]. It is however noted that in all three models that produced the universal $U(1)_A$, the pairing is similar. Namely the choice of pairings is such that all six triplets of the $SU(2)^6$ automorphism group are inter-wind. For example, in the models of ref. [19, 20] the pairing is:

\[
\{(y^3 y^6, y^4 \bar{y}^1, y^5 \bar{y}^5, \bar{y}^3 \bar{y}^6), \\
(y^1 \omega^5, y^2 \bar{y}^2, \omega^6 \bar{\omega}^6, \bar{y}^1 \bar{\omega}^5), \\
(\omega^2 \omega^4, \omega^1 \omega^1, \omega^3 \bar{\omega}^3, \bar{\omega}^2 \bar{\omega}^4)\}. 
\] (11)

In this choice of pairings it is noted that the complexified left–moving fermions mix the six left–moving $SU(2)$ triplets. A similar feature is observed in the choice of pairings in the flipped $SU(5)$ model of ref. [21] (see Eq. (10)). The choice of pairing in this model can in fact to be identical to the one used in the models of ref. [19, 20], with an appropriate change of the boundary conditions in the basis vector $\alpha$, that still produces the same spectrum. In contrast we note that all other choices
of pairings that have been employed in the construction of realistic free fermionic models have not led to a universal $U(1)_A$ (see refs. [3, 4, 5, 8] for several examples of such models).

As we discussed above the NAHE set possesses an inherent cyclic permutation symmetry which is a manifestation of the $Z_2 \times Z_2$ orbifold compactification with the standard embedding. However, to construct the three generation free fermionic models we have to supplement the NAHE set with three additional boundary condition basis vectors, typically denoted as $\{\alpha, \beta, \gamma\}$. In order for the anomalous $U(1)$ to remain universal the additional boundary conditions $\{\alpha, \beta, \gamma\}$ must then preserve the permutation symmetry of the NAHE set, in the charges of the chiral generations under the flavor symmetries.

The standard–like model of ref. [19] and the flipped $SU(5)$ model of ref. [21] exhibit a similar structure of the anomalous $U(1)$ and anomaly free combinations. In these two models the $U(1)$ symmetries, generated by the world–sheet complex fermions $\{\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$ and $\{\bar{y}^3 \bar{y}^6, \bar{y}^1 \bar{\omega}^5, \bar{\omega}^2 \bar{\omega}^4\}$ (or $\{\bar{y}^4 \bar{y}^5, \bar{y}^1 \bar{\omega}^6, \bar{\omega}^2 \bar{\omega}^3\}$ in the case of the flipped $SU(5)$ model of ref. [21]) are anomalous, with: $\text{Tr}U_1 = \text{Tr}U_2 = \text{Tr}U_3 = 24, \text{Tr}U_4 = \text{Tr}U_5 = \text{Tr}U_6 = -12$. The anomalous $U(1)$ combination in both models is therefore given by

$$U_A = \frac{1}{\sqrt{15}}(2(U_1 + U_2 + U_3) - (U_4 + U_5 + U_6)); \quad \text{Tr}Q_A = \frac{1}{\sqrt{15}}180.$$ (12)

One choice for the five anomaly–free combinations is given by

$$U_{12} = \frac{1}{\sqrt{2}}(U_1 - U_2), \quad U_\psi = \frac{1}{\sqrt{6}}(U_1 + U_2 - 2U_3),$$ (13)

$$U_{45} = \frac{1}{\sqrt{2}}(U_4 - U_5), \quad U_\zeta = \frac{1}{\sqrt{6}}(U_4 + U_5 - 2U_6),$$ (14)

$$U_\chi = \frac{1}{\sqrt{15}}(U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6).$$ (15)

The anomalous $U(1)$, containing the sums of $U_{1,2,3}$ and $U_{4,5,6}$ is universal with respect to the three families from the sectors $b_1, b_2$, and $b_3$. This flavor universality of the anomalous $U(1)$ is thus a consequence of the family permutation symmetry of the six $U(1)$–interactions. In the model of ref. [20] only the three $U(1)$ generated by
the world–sheet complex fermions \( \{ \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3 \} \) are anomalous with \( \text{Tr} U_1 = \text{Tr} U_2 = \text{Tr} U_3 = 24 \). The anomalous \( U(1) \) combination in this model is just the combination given in Eq. (3) and the two orthogonal anomaly free combinations are those given in Eqs. (4) and (5).

The next common feature in all three models that yielded a universal anomalous \( U(1) \) is in the structure of the assignment of boundary conditions to the internal world–sheet fermions \( \{ y, \bar{y}, \omega, \bar{\omega} \} \). This common structure is exhibited by the fact that all three models produce a sector, which is a combination of the basis vectors \( \{ \alpha, \beta, \gamma \} \), and possibly with the NAHE set basis vectors, that can produce, depending on the choice of generalized GSO projection coefficients, additional space–time vector bosons. In this vector combination the left–moving sector is completely Neveu–Schwarz. In the model of ref. [19] all the additional space–time vector bosons are projected by the generalized GSO projections, while in the model of ref. [20] the \( SU(3)_C \) gauge group is enhanced to a \( SU(4)_C \) gauge group. Similar phenomena is encountered in the model of ref. [21]. Thus, we see that the additional boundary condition basis vectors in all three models that resulted in a universal \( U(1)_A \) are constructed near an enhanced symmetry point in the moduli space. This type of enhanced symmetries may in fact play an important role in explaining the suppression of proton decay from dimension four and five operators [23].

To conclude, in this paper identified the common features that yielded the appearance of a family universal \( U(1)_A \) in the free fermionic models. These common features are the NAHE set of boundary condition basis vectors, the choice of pairing of the real world–sheet fermions \( \{ y, \omega | \bar{y}, \bar{\omega} \} \), and the assignment of boundary conditions in the basis vectors beyond the NAHE set, to these internal real fermions, which indicates that these models are near an enhanced symmetry point. We further note that family universality of the anomalous \( U(1) \) has also been found to be desirable in recent attempts to fit the fermion mass spectrum by the use of Abelian horizontal symmetries [24]. In this regard we remark that the realistic free fermionic model, and their relation to the underlying \( Z_2 \times Z_2 \) orbifold compactification, possess a phenomenologically appealing structure, which is not shared by other classes of
three generation orbifold models. This include: (a) the natural emergence of three generations \(i.e.\) each generation is obtained from one of the twisted sectors of the \(Z_2 \times Z_2\) orbifold. The existence of three generations together, with the flavor symmetries needed to explain the fermion mass spectrum are correlated with the properties of the underlying orbifold compactification. (b) The standard \(SO(10)\) embedding of the weak–hypercharge. This is an important point that cannot be overemphasized. Although from the point of view of ordinary GUTs it seems rather trivial, this is not the case in string theory. Indeed there exist numerous examples of three generations string models, in which the weak-hypercharge does not have the standard \(SO(10)\) embedding. Standard \(SO(10)\) embedding of the weak-hypercharge means that free fermionic models, despite the fact that a GUT symmetry does not exist in the effective low energy field theory, still predict the canonical value for the weak mixing angle, \(\sin^2 \theta_W = 3/8\). The fact that free fermionic models yield the standard \(SO(10)\) embedding for the weak–hypercharge enables the agreement of these models with the measured values of \(\alpha_s(M_Z)\) and \(\sin^2 \theta(M_Z)\). (c) flavor universal anomalous \(U(1)\). This last feature is again a special feature of free fermionic models that arises due to the permutation symmetry of the underlying \(Z_2 \times Z_2\) orbifold. Furthermore, as these properties are related to the gauge and local discrete symmetries of these models they are expected to survive also in the possible nonperturbative extension of these models. If the NAHE set turns out to be a necessary ingredient for obtaining a universal \(U(1)_A\), and if indeed a universal anomalous \(U(1)\) is necessary to obtain agreement with the phenomenological constraints, it may be an additional indication, that the true string vacuum is in the vicinity of the \(Z_2 \times Z_2\) orbifold, at the free fermionic point in the Narain moduli space, and with the standard embedding of the gauge connection.

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References
[1] L.E. Ibanez and D. Lust, *Nucl. Phys.* **B382** (1992) 305.

[2] Y. Nir and N. Seiberg, *Phys. Lett.* **B309** (1993) 337.

[3] A.E. Faraggi and J.C. Pati, hep-ph/9712516.

[4] P. Fayet, *Nucl. Phys.* **B90** (1975) 104;
   G. Dvali and A. Pomarol, *Phys. Rev. Lett.* **77** (1996) 3728;
   P. Binetruy and E. Dudas, *Phys. Lett.* **B389** (1996) 503.

[5] I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos, *Phys. Lett.* **B231** (1989) 65.

[6] A.E. Faraggi, D.V. Nanopoulos, and K. Yuan, *Nucl. Phys.* **B335** (1990) 347.

[7] I. Antoniadis, G.K. Leontaris and J. Rizos, *Phys. Lett.* **B245** (1990) 161;
   G.K. Leontaris, *Phys. Lett.* **B372** (1996) 212.

[8] J.L. Lopez, D.V. Nanopoulos and K. Yuan, *Nucl. Phys.* **B399** (1993) 654.

[9] S. Chaudhoury, G. Hockney and J. Lykken, *Nucl. Phys.* **B469** (1996) 357.

[10] L.E. Ibanez, J.E. Kim, H.P. Nilles and F. Quevedo, *Phys. Lett.* **B191** (1987) 282;
    D. Bailin, A. Love and S. Thomas, *Phys. Lett.* **B194** (1987) 385;
    A. Font, L.E. Ibanez, F. Quevedo and A. Sierra, *Nucl. Phys.* **B331** (1990) 421.

[11] H. Kawai, D.C. Lewellen, and S.-H.H. Tye, *Nucl. Phys.* **B288** (1987) 1;
    I. Antoniadis, C. Bachas, and C. Kounnas, *Nucl. Phys.* **B289** (1987) 87.

[12] L. Dixon, E. Martinec, D. Friedan and S. Shenker, *Nucl. Phys.* **B282** (1987) 13;
    M. Cvetic, *Phys. Rev. Lett.* **59** (1987) 2829.

[13] S. Kalara, J.L. Lopez and D.V. Nanopoulos, *Nucl. Phys.* **B353** (1991) 650.

[14] A.E. Faraggi and D.V. Nanopoulos, *Phys. Rev.* **D48** (1993) 3288;
    G.B. Cleaver and A.E. Faraggi, hep-ph/9711339.
[15] A.E. Faraggi, *Nucl. Phys.* B387 (1992) 239.

[16] A.E. Faraggi, hep-th/9708112.

[17] A.E. Faraggi, *Phys. Lett.* B326 (1994) 62.

[18] H. Dreiner, J.L. Lopez, D.V. Nanopoulos and D.B. Reiss, *Nucl. Phys.* B320 (1989) 401;
G.B. Cleaver, *Nucl. Phys.* B456 (1995) 219.

[19] A.E. Faraggi, *Phys. Lett.* B278 (1992) 131.

[20] A.E. Faraggi, *Phys. Lett.* B274 (1992) 47.

[21] I. Antoniadis, J. Ellis, S. Kelley and D.V. Nanopoulos, *Phys. Lett.* B272 (1991) 31.

[22] J.C. Pati, *Phys. Lett.* B388 (1996) 532.

[23] A.E. Faraggi, *Phys. Lett.* B339 (1994) 223;
J.C. Pati, *Phys. Lett.* B388 (1996) 532;
A.E. Faraggi, J. Ellis, and D.V. Nanopoulos, hep-th/9709049.

[24] J.K. Elwood, N. Irges and P. Ramond, hep-ph/9705270.