Application of the maximum entropy method to QCD sum rules

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Abstract. QCD sum rules have long been used to describe the physical properties of hadrons directly from QCD. While this approach was often quite successful, it also has its limitations, the most important one being the need to introduce some specific ansatz for parametrizing the spectral function. For allowing a more general analysis of the sum rules, a new analysis method based on the maximum entropy method has been introduced [1], and has in the meantime been applied to several channels in various environments. In these proceedings, we will discuss some recent results, which have been obtained with the help of this novel approach.

1. Introduction
QCD sum rules already have a long history of studying various hadronic properties [2, 3]. Based on the operator product expansion (OPE) applied to QCD and relying on the analytic properties of hadronic correlators, they provide a class of weighted integrals (the sum rules) over the spectral function, which contain the information on the physical states of the quantum numbers of interest. The problem of analyzing QCD sum rules hence boils down to extracting the properties of the spectral function from these weighted integrals.

The traditional method for tackling this problem is to introduce a phenomenological ansatz for parametrizing the spectral function, which contains a small number of parameters (most commonly used is the “pole + continuum” ansatz) and then to fit these parameters to the sum rules. This analysis has often worked well in cases, in which one already has some prior information on the form of the spectral function, but is clearly not satisfactory if one cannot rely on such prior information. Therefore, it has been argued [1] that the maximum entropy method (MEM) can be used for the analysis of QCD sum rules in order to obtain results, which are less dependent on a-priori assumptions. To verify this claim, the ρ meson channel was studied [1], after which the nucleon channel without and with parity projection [4, 5], and quarkonium spectral functions at both zero and finite temperature [6, 7], have been investigated as well, confirming the usefulness of this novel approach. Recently, the behavior of the φ meson in nuclear matter also has been studied [8]. In these proceedings, we will briefly review the obtained results on quarkonium at finite temperature and discuss our findings on the φ meson in dense matter.

The results presented in these proceedings have been obtained in collaboration with Kei Suzuki, Keisuke Ohtani, Kenji Morita and Makoto Oka.
2. Quarkonia at finite temperature

Let us first discuss the results of the charmonium channels at finite T [6], shown on the left side of Fig. 1. Concentrating firstly on the spectral functions at zero temperature, it is seen that clear peaks are generated, which represent the lowest state of both (pseudoscalar and vector) channels. The positions of these peaks reproduce the experimental values with a precision of about 50 MeV. At finite temperature, we observe that the lowest peaks vanish slightly above the critical temperature $T_c$. The origin of this melting effect is a sudden change of the gluonic condensates around $T_c$, which can be related to the deconfinement transition of the gluonic matter.

Next, let us look at the results for bottomonium [7] (see at right side of Fig. 1). Here, as for charmonium, clear peaks are seen for the pseudoscalar and vector channels at zero temperature. These peaks are found at 150-500 MeV above the experimental values of the respective ground states. This discrepancy is caused by the excited states (for instance $\Upsilon(2S)$ and $\Upsilon(3S)$ in the vector channel), which cannot be resolved by the MEM analysis, and pull the lowest peaks to higher energies than the actual ground state. Turning to the finite temperature curves, it is noted that the bottomonium states are modified much slower than their charmonium counterparts, which is in agreement with phenomenological expectations. Concretely, the spectral functions exhibit a clear peak at $T = 2.0 T_c$ which starts to dissolve at about $2.5 T_c$.

3. The $\phi$ meson at finite density

The study of light vector mesons at finite density has in recent years attracted much interest because the vector mesons potentially contain information on the partial restoration of chiral symmetry, which can be measured in experiments [9]. Specifically, the $\phi$ meson mass is strongly related to the strange quark condensate $\langle \bar{s}s \rangle$, which is modified in matter. This modification is, in the linear density approximation, quantified through the strange sigma term $\sigma_{sN}$:

$$\langle \bar{s}s \rangle_p = \langle 0|\bar{s}s|0 \rangle + \frac{\sigma_{sN}}{m_s} \rho_c$$  \hspace{1cm} (1)
Figure 2. (Left plot) The φ meson mass as a function of density for two typical values of the strange sigma term $\sigma_{sN}$ obtained in recent lattice QCD calculations. The φ meson mass is given relative to its vacuum value, while the density is given in units of the nuclear matter density $\rho_0$.

(Right plot) The φ meson mass at normal nuclear matter density $\rho_0$, as a function of $\sigma_{sN}$.

Studying the mass of the φ meson as a function of density within our approach, it is found that the mass shift caused by the finite density effects depends strongly on the value of $\sigma_{sN}$ [8]. This is shown firstly on the left plot of Fig. 2, where the φ meson mass is shown as a function of density for two typical values of $\sigma_{sN}$ obtained in recent lattice QCD calculations [10, 11]. The dependence becomes even more clear on the right plot of Fig. 2, where the φ meson mass at normal nuclear matter density $\rho_0$ is shown as a function of $\sigma_{sN}$. It is seen in this plot that the φ meson mass shift at $\rho_0$ depends linearly on $\sigma_{sN}$ and is positive for $\sigma_{sN} < 35$ MeV, while becoming negative for $\sigma_{sN} > 35$ MeV.

4. Summary

We have in these proceedings discussed two applications of a novel analysis method for QCD sum rules, based on MEM. This method allows us to study the sum rules without making any strong assumptions on the form of the spectral function and therefore provides a more general framework than the traditional analysis methods for QCD sum rules.

We have firstly reviewed our studies on quarkonium at finite temperatures [6, 7], in which we could for the first time describe the melting of both charmonium and bottomonium ground states in a QCD sum rule analysis. Next, we have presented our results on the behavior of the φ meson at finite density [8] and have discussed the strong relationship between the mass shift of the φ meson at finite density and the strange sigma term $\sigma_{sN} = m_s \langle N \bar{s}s | N \rangle$.

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