Fermion Mass Hierarchy from Symmetry Breaking at the TeV Scale

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Abstract

In this paper we explore models in which we add a singlet scalar Higgs to the Standard Model. All of these models explain the origin of the mass hierarchy amongst the fermion masses and mixing angles. We discuss 24 different variations on this model, and explore the different phenomenological possibilities. We find that the phenomenological implications of all the models are very similar except in the Higgs sector. Higgs decays and signals can be altered very significantly in all the models, but break up into two distinct classes. We also describe a systematic method for generating these models from higher order interactions involving vector-like quarks and flavon scalars.

1 Introduction

One of the major ingredients to the Standard Model (SM) is the mass generating Higgs mechanism. It works remarkably well for describing mass relationships among the electroweak...
bosons. However, the mass relationships among the fermions is more obscure because the
Yukawa couplings don’t have a predicted structure within the SM. They span over five orders of magnitude for no apparent reason.

Many attempts have been made to generate the mass hierarchy of the fermions [1]. Radiative corrections have been used to generate the hierarchy as in [2][3][4]. Warped extra dimensions were used in [5] and [6] where the extra dimensions are creatively “apple-shaped”.

Previous works [7][8][9] describe the Yukawa couplings as dependent on functions of Higgs fields, acting as higher dimensional operators in the Lagrangian. The mass hierarchy of the quarks has a correspondence to the exponents of these operators. The top quark, having dimension four Yukawa interaction, has an exponent of zero on its operator; thus its Yukawa coupling is the same as it is in the SM. For the remaining quarks, the effective Yukawa interactions are successively higher and higher dimensional as we include the lighter quarks, and hence as the exponent increases, the masses get smaller. As an explanation for the origin of these higher dimensional operators, a Froggatt-Nielsen [10] type mechanism was used [8][9].

The operators considered previously were composed from either only a Higgs doublet or only a Higgs singlet. This paper generalizes this to allow the possibility for the operators to consist of both a doublet and a singlet, with the previous works being the limiting cases.

2 Effective Model

2.1 Modeling the Yukawa Couplings

The higher dimensional operators in the Yukawa couplings make the terms non-renormalizable. However, this is only an effective theory below a scale $M$, the mass scale where vector-like quarks exist. The operators in Ref. [7] rely on the SM Higgs doublet $H$, and are of the form

$$\left(\frac{H^\dagger H}{M^2}\right)^n h_u^{ij} Q_L^i u_R^j \bar{H}, \quad \left(\frac{H^\dagger H}{M^2}\right)^n h_d^{ij} Q_L^i d_R^j H,$$

where $\bar{H} = i\sigma_2 H^*$. The exponent $n$ is a non-negative integer that may have a different value for each pair of generation indices $i, j$. The coupling coefficients $h_u, h_d$ are $\mathcal{O}(1)$. The quark...
doublet $q_L$ and quark singlets $u_R$, $d_R$ are the SM quarks. In contrast, the higher dimensional operators in Ref. [9] were made by replacing the doublet operator $H^\dagger H$ with the operator $S^\dagger S$, where $S$ field is a complex scalar singlet under the SM.

$$
\left( \frac{S^\dagger S}{M^2} \right)^n h^u_{ij} \bar{q}^i_L u^j_R \tilde{H},
$$

$$
\left( \frac{S^\dagger S}{M^2} \right)^n h^d_{ij} \bar{q}^i_L d^j_R H.
$$

With this setup, the singlet scalar $S$ acts as the messenger of the EW symmetry breaking for all the quarks and the leptons except the top quark.

In the unitary gauge, the parametrization is

$$
H = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} H_0 \end{pmatrix},
$$

$$
S = \left( v_S + \frac{1}{\sqrt{2}} S_0 \right).
$$

with $v \simeq 174$ GeV. The value of $v_S$ is chosen to also be in the electroweak scale. After assigning vacuum expectation values (vevs) to the Higgs fields, the resulting operators can be written in terms of dimensionless parameters.

$$
\epsilon = \frac{v}{M},
$$

$$
\alpha = \frac{v_S}{v}.
$$

The mass terms resulting from Eq. (1) have a coefficient $\epsilon^{2n} = (v/M)^{2n}$.

$$
\epsilon^{2n} h^u_{ij} \bar{q}^i_L u^j_R v,
$$

$$
\epsilon^{2n} h^d_{ij} \bar{q}^i_L d^j_R v.
$$

Similarly, the mass terms resulting from Eq. (2) have a coefficient $(\alpha \epsilon)^{2n} = (v_S/M)^2$.

$$
(\alpha \epsilon)^{2n} h^u_{ij} \bar{q}^i_L u^j_R v,
$$

$$
(\alpha \epsilon)^{2n} h^d_{ij} \bar{q}^i_L d^j_R v.
$$

Effective dimension four Yukawa couplings, and the mass matrices can then be constructed by using powers of $\epsilon$ and $\alpha$. Since $h^u$, $h^d$ are $\mathcal{O}(1)$, the powers needed must reduce each matrix element by an appropriate order of magnitude below the top mass. By comparing masses of the top and bottom quarks with the smallest allowed choice for the exponent $n$, an estimate on the value of $\epsilon$ can be established.

$$
\frac{m_b}{m_t} \sim \frac{M^d_{33}}{M^u_{33}} = \frac{h^d_{33} \epsilon^2 v}{h^u_{33} v} \sim \epsilon^2 \sim \mathcal{O}(10^{-2}),
$$

and consequently $M$ is around the 1–2 TeV range [8].
Knowing the experimentally determined quark masses and the CKM mixing angles, and the order of magnitude of $\epsilon^2$, many possible mass matrices corresponding to Eq. (1) can be written down. The mass matrices for the up and down quarks used in Ref. [7] were

$$
M^u = \begin{pmatrix}
h_{11}^u \epsilon^6 & h_{12}^u \epsilon^4 & h_{13}^u \\
h_{21}^u \epsilon^4 & h_{22}^u \epsilon^2 & h_{23}^u \\
h_{31}^u \epsilon^4 & h_{32}^u \epsilon^2 & h_{33}^u
\end{pmatrix} v, \quad M^d = \begin{pmatrix}
h_{11}^d \epsilon^6 & h_{12}^d \epsilon^6 & h_{13}^d \epsilon^6 \\
h_{21}^d \epsilon^6 & h_{22}^d \epsilon^4 & h_{23}^d \epsilon^4 \\
h_{31}^d \epsilon^6 & h_{32}^d \epsilon^4 & h_{33}^d \epsilon^2
\end{pmatrix} v. \quad (8)
$$

Similarly, for the mass matrices from Eq. (2), as used in Ref. [9], make the replacement $\epsilon^{2n} \mapsto (\alpha \epsilon)^{2n}$ for each element of Eq. (8).

These two models take the higher dimensional operators to be purely composed of only Higgs doublets or only Higgs singlets. But now consider the general possibility of operators built from combinations of doublets and singlets.

$$
\left(\frac{H^\dagger H}{M^2}\right)^{n-n_S} \left(\frac{S^\dagger S}{M^2}\right)^{n_S} h_{ij}^u q_L^i u_R^j \tilde{H}, \quad \left(\frac{H^\dagger H}{M^2}\right)^{n-n_S} \left(\frac{S^\dagger S}{M^2}\right)^{n_S} h_{ij}^d q_L^i d_R^j H. \quad (9)
$$

Since $\epsilon$ regulates the order of magnitude for the mass matrices, it is reasonable to keep the same $\epsilon$-texture as in Eq. (8). This fixes the value of $n$ that will be used for each matrix element. However, this does not restrict the value of $n_S$. Within the mass matrices $M^u$ and $M^d$, there are 17 matrix elements that may have varying values for the integer $n_S$, only restricted by $0 \leq n_S \leq n$.

For a mass term with a factor of $\epsilon^{2n}$, the allowed powers of $\alpha$ are the even numbers ranging from zero to $2n$. Inspection of the mass matrices shows there are four mass terms with $\epsilon^2$ (two allowed powers of $\alpha$), seven with $\epsilon^4$ (three allowed powers of $\alpha$), and six with $\epsilon^6$ (four allowed powers of $\alpha$). Allowing powers of $\alpha$ to be independent for matrix elements with the same power of $\epsilon$ means there can be $2^4 \cdot 3^7 \cdot 4^6 = 143327232$ possible Lagrangians.

To simplify the situation, all matrix elements that have the same power of $\epsilon$ are restricted to have a common power of $\alpha$. For example, the mass terms $M_{22}^u$ and $M_{22}^d$ are both proportional to $\epsilon^2$. They are restricted to be both be proportional to $\alpha^0$ or both be proportional to $\alpha^2$. They are not allow to have different powers of $\alpha$. This restriction reduces the possible Lagrangians down to $2 \cdot 3 \cdot 4 = 24$, a much more manageable number.
With this restriction, the effective Lagrangian can be written down with common higher dimensional operators factored as leading coefficients.

\[ \mathcal{L}_{\text{Yuk}}^{\text{quark}} = h_{i3}^u \bar{q}_L^i u_R^3 \tilde{H} + \left( \frac{H^1 H}{M^2} \right)^{1-n_1} \left( \frac{S^1 S}{M^2} \right)^{n_1} \left( h_{i3}^d \bar{q}_L^i d_R^3 H + h_{22}^d \bar{q}_L^i u_R^2 \tilde{H} + h_{23}^d \bar{q}_L^i u_R^3 \tilde{H} + h_{32}^d \bar{q}_L^i u_R^2 \tilde{H} \right) + \left( \frac{H^1 H}{M^2} \right)^{2-n_2} \left( \frac{S^2 S}{M^2} \right)^{n_2} \left( h_{i2}^d \bar{q}_L^i d_R^2 H + h_{23}^d \bar{q}_L^i u_R^3 \tilde{H} + h_{32}^d \bar{q}_L^i u_R^2 \tilde{H} \right) + \left( \frac{H^1 H}{M^2} \right)^{3-n_3} \left( \frac{S^3 S}{M^2} \right)^{n_3} \left( h_{i1}^d \bar{q}_L^i d_R^1 H + h_{12}^d \bar{q}_L^i u_R^3 \tilde{H} + h_{21}^d \bar{q}_L^i u_R^2 \tilde{H} \right) + h.c. \]  

The operator exponents from Eq. (9) have been specified. Since \( n \) can takes a fixed value according to the flavors of each coupling, it can be directly set (\( n = 1, 2, 3 \)). Also, since \( 0 \leq n_S \leq n \), the exponent \( n_S \) can be specified as \( n_k \) such that \( 0 \leq n_k \leq k \) where \( k \in \{ 1, 2, 3 \} \).

General expressions for the effective mass matrices and effective Yukawa couplings in the gauge basis of the quarks can be found in terms of the of the dimensionless parameters \( \alpha, \epsilon \) and the exponents \( n, n_S \).

\[ M^u_{ij} = \alpha^{2n_S} \epsilon^{2n} h_{ij}^u v, \quad M^d_{ij} = \alpha^{2n_S} \epsilon^{2n} h_{ij}^d, \]  

(11)

\[ f_{ij}^{hu} = (2(n - n_S) + 1) \alpha^{2n_S} \epsilon^{2n} h_{ij}^u, \quad f_{ij}^{hd} = (2(n - n_S) + 1) \alpha^{2n_S} \epsilon^{2n} h_{ij}^d, \]  

(12)

\[ f_{ij}^{su} = 2n_S \alpha^{2n_S - 1} \epsilon^{2n} h_{ij}^u, \quad f_{ij}^{sd} = 2n_S \alpha^{2n_S - 1} \epsilon^{2n} h_{ij}^d. \]  

(13)

One of the consequences of these higher dimensional operators is flavor changing neutral currents in the Higgs sector. This occurs because the effective Yukawa matrices and the effective mass matrices will not be proportional. The effective mass matrices will all be similar to Eq. (5) with extra factors of \( \alpha \). However, the effective Yukawa matrices will have extra numerical factors corresponding to the coefficients resulting from the binomial expansions of the operators after they acquire vevs. In Eqs. (11)–(13), factors with \( n \) or \( n_S \) will generally be different for each matrix element.
Generalized expressions for the quark masses can be found in terms of $\alpha$, $\epsilon$ and $n_1, n_2, n_3$ using a biunitary transformation $M^x_{\text{diag}} = V^x_L M^x V^x_R$, where $x \in \{u, d\}$ so that $M^x$ is either mass matrix from Eq. (11).

The effective Yukawa matrices can also be found by $Y^x_h = V^x_L f^x_h V^x_R$, where $V^x_L$, $V^x_R$ are the unitary matrices that diagonalized $M^x$. Since the Yukawa matrices $f^x_h$, $f^x_s$ are not proportional to the mass matrices $M^x$, the Yukawa matrices in the mass basis $Y^x_h$, $Y^x_s$ will not be diagonal.

Expansions of the mass and Yukawa matrices were made in powers of $\epsilon$. The masses and Yukawa couplings have been expanded to $\epsilon^6$. The CKM matrix is also given up to $\epsilon^4$. The Calculations were made assuming coupling coefficients are real and symmetric ($h^x_{ij} = h^x_{ji}$).

The masses are listed below. The Yukawa couplings and CKM matrix are in the Appendix.

$$M^u_{\text{diag}11} \approx \left( \alpha^{2n_3} h^u_{11} - \alpha^{2(2n_2-n_1)} \frac{h^u_{12}^2}{h^u_{22}} \right) \epsilon^6, \quad (14a)$$

$$M^u_{\text{diag}22} \approx \alpha^{2n_1} h^u_{22} \epsilon^2 - \alpha^{4n_1} h^u_{23}^2 h^u_{33} \epsilon^4 + \left( \alpha^{2(2n_2-n_1)} \frac{h^u_{12}^2}{h^u_{22}} + \alpha^{6n_1} h^u_{22} h^u_{23}^2 \frac{h^u_{33}^2}{h^u_{22}} \right) \epsilon^6, \quad (14b)$$

$$M^u_{\text{diag}33} \approx h^u_{33} + \alpha^{4n_1} h^u_{23}^2 h^u_{33} \epsilon^4 + \alpha^{6n_1} h^u_{22} h^u_{23}^2 h^u_{33} \epsilon^6, \quad (14c)$$

$$M^d_{\text{diag}11} \approx \alpha^{2n_3} h^d_{11} \epsilon^6, \quad (14d)$$

$$M^d_{\text{diag}22} \approx \alpha^{2n_2} h^d_{22} \epsilon^4 - \alpha^{2(2n_2-n_1)} \frac{h^d_{23}^2}{h^d_{33}} \epsilon^6, \quad (14e)$$

$$M^d_{\text{diag}33} \approx \alpha^{2n_1} h^d_{33} \epsilon^2 + \left( \alpha^{2(2n_2-n_1)} \frac{h^d_{23}^2}{h^d_{33}} - \alpha^{2(6n_2-3n_1-2n_3)} \frac{h^d_{23}^4}{h^d_{13} h^d_{33}} \right) \epsilon^6. \quad (14f)$$

The Yukawa couplings $f^x_h$, $f^x_s$ are couplings of the quarks to the Higgs fields with both field types in the gauge basis. The Yukawa couplings $Y^x_h$, $Y^x_s$ are with the quarks in the mass basis, however, the Higgs fields are still in the gauge basis. To get the Yukawa couplings in the mass basis of both the quarks and the Higgs fields, the rotation of the Higgs fields still needs to be applied (see Sec 2.2). Doing so yields couplings $Y^{',x}_h$, $Y^{',x}_s$ of the form

$$\frac{1}{\sqrt{2}} Y^{',h}_{ij} = \frac{1}{\sqrt{2}} \left( Y^{x}_{ij} \cos \beta - Y^{sx}_{ij} \sin \beta \right) \quad \text{Coupling to Higgs mass eigenstate } h, \quad (15a)$$

$$\frac{1}{\sqrt{2}} Y^{',s}_{ij} = \frac{1}{\sqrt{2}} \left( Y^{x}_{ij} \sin \beta + Y^{sx}_{ij} \cos \beta \right) \quad \text{Coupling to Higgs mass eigenstate } s. \quad (15b)$$

5
### 2.2 Higgs Sector and $Z'$

In the effective Lagrangian, the Higgs boson $S$ only appears in the product $S^\dagger S$. This means it is free to be charged under an additional gauge symmetry while all the SM fields are neutral under this new symmetry. This symmetry is assumed to be a $U(1)_S$ local symmetry. It plays a role—along with extra flavor symmetries—in the mechanism for creating the effective Lagrangian. This will be expanded upon in Section 4.

The general Higgs potential which mixes the SM Higgs doublet $H$ with the $S$ is

$$V(H, S) = -\mu_H^2 H^\dagger H - \mu_S^2 S^\dagger S + \lambda_H (H^\dagger H)^2 + \lambda_{HS} (H^\dagger H)(S^\dagger S) + \lambda_S (S^\dagger S)^2.$$  \hspace{1cm} (16)

Minimization of the potential to find in terms of the vevs yields

$$\mu_H^2 = v^2 (2\lambda_H + \alpha^2 \lambda_{HS}), \quad \mu_S^2 = v^2 (2\alpha^2 \lambda_S + \lambda_{HS}).$$  \hspace{1cm} (17)

The squared mass matrix in the $(H_0, S_0)$ basis is

$$\mathcal{M}^2 = 2v^2 \begin{pmatrix} 2\lambda_H & \alpha \lambda_{HS} \\ \alpha \lambda_{HS} & 2\alpha^2 \lambda_S \end{pmatrix}.$$  \hspace{1cm} (18)

The mass eigenstate basis $(h, s)$ can be written using

$$H_0 = h \cos \theta + s \sin \theta, \quad S_0 = -h \sin \theta + s \cos \theta,$$  \hspace{1cm} (19)

where the Higgs sector mixing angle $\theta$ is expressible with

$$\tan 2\theta = \frac{\alpha \lambda_{HS}}{\lambda_H - \alpha^2 \lambda_S}.$$  \hspace{1cm} (20)

The physical squared masses at the tree level are

$$m_{h,s}^2 = 2v^2 \left( \lambda_H + \alpha^2 \lambda_S \mp \sqrt{(\lambda_H - \alpha^2 \lambda_S)^2 + \alpha^2 \lambda_{HS}^2} \right).$$  \hspace{1cm} (21)

The $Z'$ gauge boson gets its mass from the pseudoscalar component of $S$ when the $U(1)_S$ symmetry is broken. Assuming the $S$ has a charge of 1 under this symmetry, and the coupling constant is $g_S$, then

$$m_{Z'}^2 = 2g_S^2 v_S^2.$$  \hspace{1cm} (22)
The vev $v_S$ is of the order of the EW scale. If the coupling $g_S \sim O(1)$, then the mass of the $Z'$ should also be expected to be near the EW scale. However, since $g_S$ isn’t determined, the mass may be different. There is no significant bound on the mass of the $Z'$ from LEP [11].

Since the $Z'$ does not couple to SM fields directly, its presence can only be determined from interactions with new fields and mixing with the $Z$. Mixing can occur through kinetic mixing or from higher order loop effects. Measurements of the $Z$ properties at LEP1 constrain the mixing to be $\lesssim 10^{-3}$ [12][13], for $m_{Z'} < 1$ TeV.

The $Z'$ will couple to the SM fermions via dimension 6 interactions. No significant bound is placed on $M_{Z'}$ from these interactions as was shown in [14].

3 Phenomenology

There are 24 different models that can be composed under the current scheme. Covering the specific phenomenology of each model would not be very illuminating. The differences can be seen by looking at two different types of signals that can be seen at colliders: Higgs decay signatures and FCNC processes.

3.1 Higgs Decays

In the low Higgs mass range of the SM, allowed by the LEP limit, say 114–130 GeV, the dominant mode of Higgs decay is to two bottom quarks $h \rightarrow b\bar{b}$. The branching ratio for this decay is almost 100%. This is undesirable from an experimental point of view because these signals are difficult to disentangle from a large QCD background. In all of the 24 models, the branching ratio of this decay can be altered significantly. The 24 models break up into two distinct classes. These two classes are characterized by whether we have $(H^\dagger H/M^2)$ or $(S^\dagger S/M^2)$ in the prefactor in the 2nd line of Eq. (10).

For the 12 models coming from $S^\dagger S$ in Eq. (10), $h \rightarrow b\bar{b}$ coupling is

$$h_{33}^d(\alpha \cos \theta - 2 \sin \theta)\epsilon^2 \alpha/\sqrt{2}.$$  (23)
Taking $\alpha \sim 1$ and $\theta \sim 26^\circ$, this coupling is reduced significantly compared to that in the SM making the observation of the $h \rightarrow \gamma\gamma$ signal at the LHC much more favorable compared to the SM. For example, the $h \rightarrow \gamma\gamma$ signal can be enhanced by a factor of 10 as seen in Ref. [9][15][16]. Varying the angle $\theta$ drastically alters the branching ratio of the Higgs. If there is no mixing, $\theta \sim 0^\circ$, the structure of Higgs decays is virtually indistinguishable from the Standard Model.

In the other 12 models, with the prefactor in Eq. (10), line 2 given by $H^\dagger H/M^2$, the coupling of $h \rightarrow b\bar{b}$ is $3\epsilon^2 h_{33}^d \cos \theta$. This causes an enhancement of $9 \cos^2 \theta$ for the $h \rightarrow b\bar{b}$.

The rate for the $h \rightarrow \gamma\gamma$ mode will only change by a factor of $\cos^2 \theta$. Since $h \rightarrow b\bar{b}$ is enhanced by a factor of 9, while the rate for the $h \rightarrow \gamma\gamma$ mode stays the same, the branching ratio for the $\gamma\gamma$ mode is effectively reduced by a factor of 9. Thus in this class of models, the observation of the Higgs signal in the $\gamma\gamma$ mode will be much more difficult than in the SM.

### 3.2 FCNC Processes

#### 3.2.1 $t \rightarrow ch$

The observation of the decay mode $t \rightarrow ch$ will be a clear indication for physics beyond the SM. For the SM, this branching ratio is very tiny $\sim 10^{-14}$ [17]. In the models considered here, this branching ratio can be much larger, and observable at the LHC. For the 12 $H^\dagger H$ models, the coupling for the $t \rightarrow ch$ mode is $h_{32}^u \sqrt{2}\epsilon^2 \cos \theta$, giving rise to a branching ratio $\sim 10^{-4}$, with $h_{32}^u \sim 1$ and $\cos \theta \sim 1$. With large top quark production at the LHC, this decay mode with such a branching ratio will be observable at the LHC. For the other 12 models with $S^\dagger S$, the coupling for $t \rightarrow ch$ mode is $h_{32}^u \sqrt{2}\epsilon^2 \sin \theta$. This BR is also much larger than in the SM, however much smaller than the models with $H^\dagger H$. Furthermore, there is possibility of cancellation leading to further reduction in the BR.

#### 3.2.2 $B_s^0 \rightarrow \mu^+\mu^-$

In all 24 models this process is mediated by $s$ and $h$ exchange. Amongst the 24 models there are 3 different categories of amplitudes for this process. The couplings for the 6 different
the total production cross section for all new quarks will be 60 fb per each new quark [18]. Because there are more than 50 new quarks in each model, the \( \bar{q} \) quarks to be 1 TeV, the production cross section for pair production at the LHC at 14 TeV is Q

While there are differences in the form of each amplitude, they are all proportional to \( \epsilon^8 \). This means the \( \text{BR} \sim 10^{-9} \) is still within \(< 4.7 \times 10^{-8} \), the experimental limit [9].

3.3 Double Higgs Production

It is possible to pair produce the vector-like quarks of our model. The dominant decay mode (95\%) is to a SM quark and a Higgs (e.g. \( Q_L \rightarrow uh \)). The Higgs will then decay to \( b\bar{b} \). The signal for this process will be 4 b-jets and 2 hard jets. Taking the mass of the vector-like quarks to be 1 TeV, the production cross section for pair production at the LHC at 14 TeV is 60 fb per each new quark [18]. Because there are more than 50 new quarks in each model, the total production cross section for all new quarks will be \( \sim 30 \) pb. We place kinematic cuts on the signal and background as follows: the invariant mass of the b-jets, \( m_{bb} > 100 \) GeV, the \( p_T^{\text{jets}} > 100 \) GeV and for the b-jets, \( p_T^b > 30 \) GeV. Using CalcHEP we find that imposing these cuts will reduce the branching ratio of the new quarks to \( \text{BR}(Q_L \rightarrow uh) \sim 0.9 \) fb. If we
take the $b$-tagging efficiency to be 50%, the signal is reduced by a factor of $\frac{1}{16}$. With these cuts and 10 fb$^{-1}$ of luminosity we would expect to see $\sim 30$ events per additional quark in the model.

The SM background for a 6 $b$ final state was calculated in Ref. [19]. With their cuts the background is 60 fb. With a 100 GeV cut on each of the final state non-$b$-jets, we expect that the background for $bbbbjj$ in the SM will be of similar order. With a few extra vector-like quarks from one of the models the signal should be much larger than the background and observable at the LHC with enough luminosity.

4 Mass Generating Mechanism

So far, the gauge symmetries of the SM have been extended by an additional $U(1)_S$ local symmetry. To explain the origin of the operators $H^\dagger H$ and $S^\dagger S$ in the Yukawa couplings, some additional $U(1)_{F_i}$ global symmetries will also be employed in the use of a Froggatt-Nielsen type mechanism. In the one model where none of the effective low energy interactions in the Lagrangian have a coefficient of $(S^\dagger S)^n$, the $U(1)_S$ symmetry is not included; it essentially decouples from the model, so can be ignored.

Each $U(1)_{F_i}$ global symmetry has a flavor scalar boson $F_i$ (called a flavon) that is charged only under its corresponding symmetry, and neutral with all other symmetries. Because the $U(1)_{F_i}$ are global symmetries, there are no gauge bosons associated with them. It should be noted that even though there is no restriction being placed on the number of $U(1)_{F_i}$ symmetries, it is not necessary to have more than two.

The effective Yukawa couplings are created by interactions with new heavy exotic quarks, the new flavons $F_i$, and the Higgs bosons $H$ and $S$. These quarks will be denoted by $Q$ for the doublets, $U$ and $D$ for the singlets. They have the same hypercharges as their SM counterparts $q$, $u$, and $d$.

These extra fields are necessary in a Froggatt-Nielsen mechanism as each field occupies a unique position in the charge space of $U(1)_Y \times U(1)_S \times U(1)_{F_1} \times U(1)_{F_2} \cdots$. The flavons and the Higgs fields provide the interactions that link the fields to their neighbors in the charge space. A sequence of interactions is required to move between non-neighboring fields, such
as the SM quarks.

Within a given model, the sequence of field interactions beginning and ending with SM particles was chosen so that there is only one sequence connecting any two SM quarks (assuming backward steps aren’t taken within the sequence). Although this isn’t strictly necessary, if distinctly different sequences of particle interactions were allowed into a model, then some models may have explicit terms in the Lagrangian with higher powers of \((H^\dagger H)^{n-n_S}(S^\dagger S)^{n_S}\) than are written down.

In order to make different interaction sequences non-interacting with each other, it is necessary to space the non-interacting quark fields at least two quantum numbers away from each other in the charge space. This leads to a large number of quark fields being used as each interaction sequence path through this space must be long enough to go around many other fields and avoid interacting with other paths.

It should be noted that unlike the SM quarks, which only has right handed singlets and left handed doublets, the new heavy quarks occur in left-right pairs and behave vector-like with respect to the gauge groups of the SM and \(U(1)_S\). The quantum numbers of a left-right pair will be identical except for the quantum number of one \(U(1)_F\) symmetry. This quantum number will differ by a value of one. When this symmetry breaks, the vev of the \(F_i\) gives mass to the new heavy quarks. The vev of each \(F_i\) is are assumed to be around the TeV scale.

### 4.1 Couplings in the Lagrangian

The couplings of these heavy quarks in the Lagrangian take on a generalized form. Specifically, the couplings each particle has will depend on their charge assignments within a given model. The hermitian conjugate of each coupling will also be included in the Lagrangian.
For the terms listed in eqs. (25)–(27), no summation over the indices is implied.

\[
\begin{align*}
&f^{FQ}_{ab}Q^b_R F_i \quad f^{FU}_{ab}U^a_L U^b_R F_i \quad f^{FD}_{ab}D^a_L D^b_R F_i \\
&f^{FQ}_{ab}Q^b_R F_i^\dagger \quad f^{FU}_{ab}U^a_L U^b_R F_i^\dagger \quad f^{FD}_{ab}D^a_L D^b_R F_i^\dagger
\end{align*}
\]

Flavon Couplings \hspace{1cm} (25)

\[
\begin{align*}
&f^{SQ}_{ab}Q^b_R S \quad f^{SU}_{ab}U^a_L U^b_R S \quad f^{SD}_{ab}D^a_L D^b_R S \\
&f^{SQ}_{ab}Q^b_R S^\dagger \quad f^{SU}_{ab}U^a_L U^b_R S^\dagger \quad f^{SD}_{ab}D^a_L D^b_R S^\dagger
\end{align*}
\]

Higgs Singlet Couplings \hspace{1cm} (26)

\[
\begin{align*}
&f^{H_U}_{ab}Q^b_R U^b_R \bar{H} \quad f^{H_D}_{ab}Q^b_R D^b_R H \\
&f^{H_U}_{ab}Q^b_R U^b_R \bar{H} \quad f^{H_D}_{ab}Q^b_R D^b_R H
\end{align*}
\]

Higgs Doublet Couplings \hspace{1cm} (27)

All of the coupling coefficients \((f^{FQ}, f^{FU}, f^{FD}, f^{SQ}, f^{SU}, f^{SD}, f^{H_U}, f^{H_D})\) are taken to be \(O(1)\).

Every heavy quark will have one coupling from Eq. (25) where \(a = b\), as this will be a massive left-right pair when the flavon \(F\) breaks the flavor symmetry. Every heavy quark must also have at least one coupling from eqs. (25)–(26) where \(a \neq b\), or from Eq. (27) where \(a\) and \(b\) are indexed over different quark types. For the few heavy quarks that directly couple to the SM quarks, the appropriate replacement should be made to the terms coming from eqs. (25)–(27) (e.g. \(Q_L Q_R \mapsto q_L q_R\)).

### 4.1.1 The Effective Lagrangian

In all model variations, the Yukawa coupling \(h^{u}_{33} \bar{q}_L^3 \bar{u}^3_R \bar{H}\) is the only one that involves only SM particles. All the other model variations have coefficients of \((H^\dagger H)^{n-n_S}(S^\dagger S)^{n_S}\) on terms which would otherwise be SM Yukawa couplings. These terms are larger than four dimensions and come from a process of integrating out the heavy fermions from the tree level diagrams, which correspond with terms of the forms from eqs. (25)–(27).

For example, consider the term \((H^\dagger H/M^2)h^{u}_{33} \bar{q}_L^3 \bar{u}^3_R \bar{H}\). This terms exists in 12 of the 24 variations of the effective Lagrangian. One possible heavy quark model has thirteen terms associated with this process. This process can be represented by the Feynman diagram in
Figure 1: Feynman diagram linking $q_L^2$ to $u_R^3$ in an $H^\dagger H$ model.

Fig. 1 The necessary terms are

$$f_{2,4} H^{\dagger} U_R^4 \tilde{H} + f_{4,4} F U_R^4 U_L F_1 + f_{4,5} F U_R^5 U_L F_1 + f_{5,5} F U_R^5 U_L F_1 + f_{5,6} U_R^5 Q_R^6 H$$

$$+ f_{6,6} Q_R^6 F_1 + f_{6,7} Q_R^7 F_1 + f_{7,7} Q_R^7 F_1 + f_{7,8} Q_R^7 F_1 + f_{7,8} Q_R^7 F_1 + f_{7,8} Q_R^7 F_1 + h.c..$$

(28)

This particular model choice exhibits only a single extra symmetry $U(1)_{F_1}$ in the given terms. The flavon symmetry breaks, and the flavons acquire a vev $\langle F_1 \rangle$. The heavy fermions can be integrated out and an effective expression below the TeV scale is proportional to

$$f_{2,4} F U_R F_1 + f_{4,4} F U_R F_1 + f_{4,5} F U_R F_1 + f_{5,5} F U_R F_1 + f_{5,6} U_R F_1 + f_{6,6} Q_R F_1 + f_{6,7} Q_R F_1 + f_{7,7} Q_R F_1 + f_{7,8} Q_R F_1 + f_{7,8} Q_R F_1 + h.c..$$

(29)

Thus for this particular model choice, the effective coupling parameter in the low energy Lagrangian is

$$h^u_{23} \sim f_{2,4} F U_R F_1 + f_{4,4} F U_R F_1 + f_{4,5} F U_R F_1 + f_{5,5} F U_R F_1 + f_{5,6} F U_R F_1 + h.c..$$

(30)

The couplings $f$ are $O(1)$. The vev of the flavons is the same order as the vector-like quark masses, $\langle F_1 \rangle \sim M$. This means $h^u$ can also be $O(1)$, consistent with the assumption in the effective Lagrangian.

For a comparison, consider the 12 effective Lagrangians that have the same term, except with $(H^\dagger H) \mapsto (S^\dagger S)$. Some possible model choices may again have thirteen terms. The Feynman diagram in Fig. 2 is similar to Fig. 1 but there are noticeable differences in the first 5 interactions.

$$f_{2,4} S Q_R^4 S + f_{4,4} Q_R^4 S F_1 + f_{4,5} Q_R^4 S F_1 + f_{5,5} Q_R^4 S F_1 + f_{5,6} Q_R^4 S F_1 + f_{6,6} Q_R^4 S F_1 + f_{6,7} Q_R^4 S F_1 + f_{7,7} Q_R^4 S F_1 + f_{7,8} Q_R^4 S F_1 + f_{7,8} Q_R^4 S F_1 + h.c..$$

(31)
This expression is similar to the previous case, and likewise when the heavy fermions are integrated out, the effective expression below the TeV scale looks similar.

\[
f^{SQ} f^{FQ} f^{FQ} f^{SQ} f^{FQ} f^{FQ} f^{FQ} f^{FU} f^{FU} f^{FU} f^{FU} \left( \frac{\langle F_1 \rangle}{M} \right)^{10} \frac{S^\dagger S}{M^2} q_L^3 u_R^3 H + h.c.. \tag{32}
\]

As before, the effective coupling parameters in the low energy Lagrangian can be pulled out

\[
h_{23}^u \sim f^{SQ} f^{FQ} f^{FQ} f^{SQ} f^{FQ} f^{FQ} f^{FU} f^{FU} f^{FU} f^{FU} \left( \frac{\langle F_1 \rangle}{M} \right)^{10}, \tag{33}
\]

and once again, \( h_{23}^u \sim O(1) \).

4.2 Charge Assignments for Specific Models

4.2.1 Effective Lagrangian with only powers of \((S^\dagger S/M^2)\)

In the effective Lagrangian where all the interactions of dimension greater than four only have powers of \((S^\dagger S/M^2)\), the \(U(1)_S\) symmetry only needs to be accompanied with only one \(U(1)_F\) symmetry. With only these two extra symmetries, one possible model has a set of 166 heavy quarks (18 \(U^a_L\), 18 \(U^a_R\), 29 \(D^a_L\), 29 \(D^a_R\), 36 \(Q^a_L\), 36 \(Q^a_R\)). The quantum numbers of the SM quarks, Higgs doublet, Higgs singlet, and the vector boson are in Table 4. The quantum numbers of the heavy quarks can be found in Table 4.

4.2.2 Effective Lagrangian with only powers of \((H^\dagger H/M^2)\)

In the effective Lagrangian where there are no interactions with the \(S\), the \(U(1)_S\) symmetry is effectively eliminated.

Some possible choices of fields for a model use only two \(U(1)_F\) symmetries. As indicated previously, this means there will be two new bosons \(F_1\) and \(F_2\). And explicit choice can be made that consists of 124 heavy quarks (17 \(U^a_L\), 17 \(U^a_R\), 20 \(D^a_L\), 20 \(D^a_R\), 20 \(Q^a_L\), 20 \(Q^a_R\)).
Table 1: Charge assignments of the SM quarks, Higgs doublet, Higgs singlet, and new vector boson for an effective Lagrangian with only powers of $(S^\dagger S/M^2)$.

| Fields | $U(1)_S$ | $U(1)_F$ |
|--------|----------|----------|
| $q^3_L$ | 0        | 0        |
| $q^2_L$ | 0        | 16       |
| $q^1_L$ | 0        | 24       |
| $H$     | 0        | 0        |
| $S$     | 1        | 0        |

Table 2: Charge assignments of the SM quarks, Higgs doublet, and new vector bosons, for an effective Lagrangian with only powers of $(H^\dagger H/M^2)$.

| Fields | $U(1)_{F_1}$ | $U(1)_{F_2}$ |
|--------|--------------|--------------|
| $q^3_L$ | 0            | 0            |
| $q^2_L$ | -2           | 0            |
| $q^1_L$ | -4           | -2           |
| $H$     | 0            | 0            |
| $F$     | 0            | 1            |

Quantum numbers of the SM quarks, Higgs doublet, and the vector bosons are in Table 2. The heavy quarks have their quantum numbers listed in Table 6.

4.2.3 Generalized Model

The two previous models were constructed independently from each other. Each of the other 22 models can also be constructed independently from each other. However, constructing a model and assigning appropriate charges to the heavy quarks for each effective Lagrangian doesn’t need to be done 24 times. It is possible to construct a generalized model that will match any of the effective Lagrangians by simply changing specific groups of particles.
Table 3: Charge assignments of the SM quarks, Higgs doublet, Higgs singlet, and the new vector bosons, for a generalized model.

| Fields | $U(1)_S$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields | $U(1)_S$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ |
|--------|----------|-------------|-------------|--------|----------|-------------|-------------|
| $q^3_L$ | 0        | 5           | 12          | $d^3_R$ | 0        | 7           | 18          |
| $q^2_L$ | 0        | 5           | 2           | $d^2_R$ | 0        | 13          | 20          |
| $q^1_L$ | 0        | 6           | 29          | $d^1_R$ | 0        | 0           | 17          |
| $u^3_R$ | 0        | 5           | 12          | $H$     | 0        | 0           | 0           |
| $u^2_R$ | 0        | 11          | 14          | $S$     | 1        | 0           | 0           |
| $u^1_R$ | 0        | 13          | 20          | $F_1$   | 0        | 1           | 0           |
|        |          |             |             | $F_2$   | 0        | 0           | 1           |

An example of this change was already done in Eqs. (28) and (31). In those expressions, the fields $U^4_R, U^4_L, U^5_R,$ and $U^5_L$ were replaced with $Q^4_R, Q^4_L, Q^5_R,$ and $Q^5_L;$ and the corresponding interactions with $H$ were replaced to interactions with $S.$ In terms of their quantum numbers, the change in hypercharge of the $U(1)_Y$ symmetry at either end of the sequence was replaced for a change in the charge of the $U(1)_S$ symmetry.

The generalized model presented here uses two $U(1)_{F_i}$ symmetries and has 282 heavy quarks. The charge assignments of the fields under these symmetries can be found in Tables 3 and 7–9.

As presented, Tables 7 and 9 are for the effective Lagrangian with only powers of $(H^\dagger H/M^2)$. To adjust the table to fit any of the other variations of the Lagrangian, replace an appropriate set of fields with a different set of fields. The choice of replacements is also, in most cases, not unique. A particular choice of replacements is given in Tables 10–15.

It should be noted, in the replacement tables the numbering subscript is changed to avoid possible duplication of names. For example, the replacement $U^2_L \rightarrow Q^2_L$ is made instead of $U^2_L \rightarrow Q^L_L$ because there already exists a quark with the name $Q^L_L$. 

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4.3 Charged Leptons

So far, the mass hierarchy of the quarks has been addressed, but the hierarchy of the leptons hasn’t been mentioned. Fortunately, the same approach of using vector-like leptons can be used. A mass matrix like those from Eqn. (8) can be constructed for the charged leptons. It turns out, the matrix $M^e$ can have the same $\epsilon$ texture as $M^d$.

Thus generally, the mass matrix has and the Yukawa couplings have the form

$$M^e_{ij} = \alpha^{2nS} \epsilon^{2n} h_{ij}^e \frac{\nu_H}{\sqrt{2}},$$

(34)

$$f_{he}^{ij} = (2(n - n_S) + 1) \alpha^{2nS} \epsilon^{2n} h_{ij}^e,$$

(35)

$$f_{se}^{ij} = 2n_S \alpha^{2nS - 1} \epsilon^{2n} h_{ij}^e.$$  

(36)

Again, the values of $n$ and $n_S$ may vary between matrix elements

Because it has the same powers of $\epsilon$ as the down-type quark sector, it is not always necessary to find a set of vector-like leptons from scratch. If a set of vector-like quarks is known, then some simple replacements can be made.

$$D \mapsto E, \quad Q \mapsto L, \quad U \mapsto N.$$  

(37)

Then remove the vector-like leptons that are unnecessary. This will eliminate the unwanted interactions with the light neutrinos.

5 Conclusions

Presented here is a scheme under which the fermion mass hierarchy can be understood by couplings with other massive vector-like fermions. The effective Yukawa couplings are generated by the breaking of global flavor symmetries $U(1)_{F_i}$ at the TeV scale. It should be noted, as an effective model, it may not be valid above the breaking scale. At that point, another mechanism may take over, or the theory may be embedded in a larger symmetry group.

The variations of the effective model have decays and exchange amplitudes that are different, based upon the interactions with Higgs doublets $H$ and singlets $S$. Phenomena
that distinguishes between variations of the effective model have Branching Ratios that should fall within the observable limits of the LHC.

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6 Appendix

6.1 Yukawa Couplings in the Quark Mass Basis

The expansions made in this section use the unitary transformation matrices that diagonalize the mass matrices, $M_{\text{diag}} = V_L^T M V_R$. It is assumed the mass matrices are symmetric. Consequently, the unitary transformation matrices are equal, $V_L = V_R$.

The $uH_0$-Yukawa couplings in the $u$-mass eigenbasis are $Y^{hu} = V_L^u f^{hu} V_R^u$.

\[
Y_{11}^{hu} \approx \left( 4n_2 - 2n_1 - 7 \right) \alpha^{2(n_2-n_1)} \frac{h_{12}^u}{h_{22}^u} + (7 - 2n_3) \alpha^{2n_3} h_{11}^u \right) \epsilon^6 \quad (38a)
\]

\[
Y_{12}^{hu} \approx 2(n_2 - n_1 - 1) \alpha^{2n_2} h_{12}^u \epsilon^4 + 2\alpha^2(n_1+n_2) h_{23}^u \left( 2 - n_2 \right) \frac{h_{11}^u}{h_{33}^u} + (n_1 - 1) \frac{h_{12}^u h_{23}^u}{h_{22}^u h_{33}^u} \right) \epsilon^6 \quad (38b)
\]

\[
Y_{13}^{hu} \approx 2\alpha^{2n_2} \left( 2 - n_2 \right) h_{13}^u + (n_1 - 1) \frac{h_{12}^u h_{23}^u}{h_{22}^u} \right) \epsilon^4
+ 2\alpha^2(n_1+n_2) h_{23}^u \left( 1 + n_1 - n_2 \right) \frac{h_{12}^u}{h_{33}^u} + (1 - n_1) \frac{h_{13}^u h_{23}^u}{h_{22}^u h_{33}^u} + (n_1 - 1) \frac{h_{12}^u h_{23}^u}{h_{22}^u h_{33}^u} \right) \epsilon^6 \quad (38c)
\]

\[
Y_{22}^{hu} \approx (3 - 2n_1) \alpha^{2n_1} h_{22}^u \epsilon^2 + (4n_1 - 5) \alpha^{4n_1} \frac{h_{23}^u}{h_{33}^u} \epsilon^4
+ \left( 6n_1 - 7 \right) \alpha^{6n_1} \frac{h_{22}^u h_{23}^u}{h_{33}^u} + (7 + 2n_1 - 4n_2) \alpha^{2(n_2-n_1)} \frac{h_{12}^u}{h_{22}^u} \right) \epsilon^6 \quad (38d)
\]

\[
Y_{23}^{hu} \approx 2(n_1 - 1) \alpha^{2n_1} h_{23}^u \epsilon^2 + 2(n_1 - 1) \alpha^{4n_1} \frac{h_{22}^u h_{23}^u}{h_{33}^u} \epsilon^4
+ \left( 2(n_1 - 1) \alpha^{6n_1} \frac{h_{22}^u}{h_{33}^u} + 2h_{23}^u \right)
+ 2(n_2 - 2) \alpha^{2(n_2-n_1)} \frac{h_{12}^u h_{13}^u}{h_{22}^u} + (1 - n_1) \alpha^{4n_2-2n_1} \frac{h_{12}^u h_{23}^u}{h_{22}^u} \right) \epsilon^6 \quad (38e)
\]

\[
Y_{33}^{hu} \approx h_{33}^u + (4n_1 - 5) \alpha^{4n_1} \frac{h_{23}^u}{h_{33}^u} \epsilon^4 + (7 - 6n_1) \alpha^{6n_1} \frac{h_{22}^u h_{23}^u}{h_{33}^u} \epsilon^6 \quad (38f)
\]
The $\pi u S_0$-Yukawa couplings in the $u$-mass eigenbasis are $Y_{su} = V_L^{u\dagger} f^{su} V_R^{u}$.

\[ Y_{11}^{su} \approx 2 \left( (n_1 - 2n_2) \alpha^{2n_2-2n_1-1} h_{12}^u + n_3 \alpha^{2n_3-1} h_{11}^u \right) \epsilon^6 \]  
(39a)

\[ Y_{12}^{su} \approx 2(n_1 - n_2) \alpha^{2n_2-2n_1-1} h_{12}^u \epsilon^4 + 2\alpha^{2(n_1+n_2)-1} h_{13}^u h_{23}^u n_2 h_{12}^u - n_1 h_{12}^u h_{22}^u \epsilon^6 \]  
(39b)

\[ Y_{13}^{su} \approx 2\alpha^{2n_2-1} n_2 h_{13}^u h_{22}^u - n_1 h_{12}^u h_{23}^u \epsilon^4 \]  
(39c)

\[ + 2\alpha^{2(n_1+n_2)-1} h_{23}^u \left( (n_2 - n_1) \frac{h_{12}^u}{h_{33}^u} + n_1 \frac{h_{13}^u h_{23}^u}{h_{22}^u h_{33}^u} - n_1 \frac{h_{12}^u h_{23}^u}{h_{22}^u h_{33}^u} \right) \epsilon^6 \]  
(39d)

\[ Y_{22}^{su} \approx 2n_1 \alpha^{2n_1-1} h_{22}^u \epsilon^2 - 4\alpha^{4n_1-1} \frac{h_{22}^u h_{23}^u}{h_{33}^u} \epsilon^4 \]  
(39e)

\[ + \left( -6n_1 \alpha^{6n_1-1} \frac{h_{22}^u h_{23}^u}{h_{33}^u} - 2(n_1 - 2n_2) \alpha^{4n_2-2n_1-1} \frac{h_{12}^u}{h_{22}^u} \right) \epsilon^6 \]  
(39f)

The $\bar{d} d H_0$-Yukawa couplings in the $d$-mass eigenbasis are $Y^{hd} = V_L^{d\dagger} f^{hd} V_R^{d}$.

\[ Y_{11}^{hd} \approx (7 - 2n_3) \alpha^{2n_3} h_{11}^d \epsilon^6 \]  
(40a)

\[ Y_{12}^{hd} \approx 2(n_3 - n_2 - 1) \alpha^{2n_3} h_{12}^d \epsilon^6 \]  
(40b)

\[ Y_{13}^{hd} \approx 2\alpha^{2n_3} \left( (4 + 2n_1 - 2n_3) h_{13}^d + 2(n_2 - n_1 - 1) \frac{h_{12}^d h_{23}^d}{h_{22}^d} \right) \epsilon^6 \]  
(40c)

\[ Y_{22}^{hd} \approx (5 - 2n_2) \alpha^{2n_2} h_{22}^d \epsilon^4 + (4n_2 - 2n_1 - 7) \alpha^{2(n_2-n_1)} \frac{h_{23}^d h_{23}^d}{h_{33}^d} \epsilon^6 \]  
(40d)

\[ Y_{23}^{hd} \approx 2(n_2 - n_1 - 1) \alpha^{2n_2} h_{23}^d \epsilon^4 + 2(n_2 - n_1 - 1) \alpha^{2(n_2-n_1)} \frac{h_{23}^d h_{23}^d}{h_{33}^d} \epsilon^6 \]  
(40e)

\[ Y_{33}^{hd} \approx (3 - 2n_1) \alpha^{2n_1} h_{33}^d \epsilon^2 \]  
(40f)

\[ + \left( 7 + 2n_1 - 4n_2 \right) \alpha^{2(n_2-n_1)} \frac{h_{23}^d h_{23}^d}{h_{33}^d} + (2n_1 - 3) \alpha^{2(6n_2-3n_1-2n_3)} \frac{h_{22}^d h_{23}^d}{h_{13}^d} \epsilon^6 \]  
(40f)
The $\overline{d}dS_0$-Yukawa couplings in the $d$-mass eigenbasis are $Y^{sd} = V_L^{d\dagger} f^{sd} V_R^{d}\dagger$.

\begin{align}
Y^{sd}_{11} &\approx 2n_2\alpha^{2n_3-1}h_{11}^d \epsilon^6 \tag{41a} \\
Y^{sd}_{12} &\approx 2(n_2 - n_3)\alpha^{2n_3-1}h_{12}^d \epsilon^6 \tag{41b} \\
Y^{sd}_{13} &\approx 2\alpha^{2n_3-1}\left((n_3 - n_1)h_{13}^d + (n_1 - n_2)\frac{h_{12}^d h_{23}^d}{h_{22}^d}\right) \epsilon^6 \tag{41c} \\
Y^{sd}_{22} &\approx 2n_2\alpha^{2n_2-1}h_{22}^d \epsilon^4 + 2(n_1 - 2n_2)\alpha^{4n_2-2n_1-1}\frac{h_{23}^d h_{33}^d}{h_{33}^d} \epsilon^6 \tag{41d} \\
Y^{sd}_{23} &\approx 2(n_1 - n_2)\alpha^{2n_2-1}h_{23}^d \epsilon^4 + 2(n_1 - n_2)\alpha^{4n_2-2n_1-1}\frac{h_{22}^d h_{23}^d}{h_{33}^d} \epsilon^6 \tag{41e} \\
Y^{sd}_{33} &\approx 2n_1\alpha^{2n_1-1}h_{33}^d \epsilon^2 \\
&\quad + 2\left((2n_2 - n_1)\alpha^{4n_2-2n_1-1}\frac{h_{23}^d h_{23}^d}{h_{33}^d} - n_1\alpha^{12n_2-6n_1-4n_3-1}\frac{h_{22}^d h_{23}^d h_{33}^d}{h_{33}^d}\right) \epsilon^6 \tag{41f}
\end{align}

### 6.2 CKM Matrix

The Cabbio-Kobayashi-Maskawa matrix is $V^{\text{CKM}} = V_L^{d\dagger} V_L$.

\begin{align}
V^{\text{CKM}}_{11} &\approx 1 - \left(\frac{\alpha^{4(n_2-n_1)}h_{12}^d h_{23}^u}{2 h_{22}^d} + \frac{\alpha^{4(n_3-n_2)}h_{12}^d}{2 h_{22}^d} - \alpha^{2(n_3-n_1)}\frac{h_{12}^d h_{12}^u}{h_{22}^d h_{22}^u}\right) \epsilon^4 \tag{42a} \\
V^{\text{CKM}}_{21} &\approx \left(\alpha^{2(n_3-n_2)}h_{12}^d h_{22}^d - \alpha^{2(n_2-n_1)}h_{12}^u h_{22}^u\right) \epsilon^2 \\
&\quad + \left(\alpha^{4(n_1-n_2)}h_{23}^u h_{23}^u - \alpha^{2(n_3-n_2)}\frac{h_{22}^u h_{23}^u}{h_{22}^d h_{23}^d} - \alpha^{2n_2}\frac{h_{13}^u h_{23}^u}{h_{22}^d h_{23}^d} + \alpha^{4(n_3-n_2)}\frac{h_{12}^d h_{12}^d}{h_{22}^d h_{22}^u}\right) \epsilon^4 \tag{42b} \\
V^{\text{CKM}}_{31} &\approx \left(\alpha^{4(n_1-n_2)}h_{23}^u h_{23}^u - \alpha^{2(n_3-n_2)}\frac{h_{22}^u h_{23}^u}{h_{22}^d h_{23}^d} + \alpha^{2(n_3-n_1)}\frac{h_{12}^d h_{12}^d}{h_{22}^d h_{22}^u} - \alpha^{2(n_3-n_2+n_1)}\frac{h_{12}^d h_{23}^u}{h_{22}^d h_{23}^d}\right) \epsilon^4 \tag{42c}
\end{align}
\[ V_{12}^{\text{CKM}} \approx \left( \alpha^{2(n_2-n_1)} \frac{h_{12}^{u}}{h_{22}^{u}} - \alpha^{2(3n_2-2n_1-n_3)} \frac{h_{12}^{d} h_{23}^{d}}{h_{12}^{d} h_{33}^{d}} \right) \epsilon^2 \\
+ \left( \alpha^{2n_2} \frac{h_{12}^{u} h_{23}^{u} - h_{13}^{u} h_{22}^{u}}{h_{22}^{u} h_{33}^{u}} + \alpha^{2(4n_2-3n_1-n_3)} h_{23}^{u} \frac{2 h_{12}^{d}(h_{23}^{d} - h_{22}^{d}) - h_{13}^{d} h_{22}^{d} h_{23}^{d}}{h_{12}^{d} h_{33}^{d} h_{33}^{d}} \right) \epsilon^4 \] (42d)

\[ V_{22}^{\text{CKM}} \approx 1 - \left( \alpha^{4n_1} \frac{h_{23}^{u} 2}{2 h_{33}^{u}} - \alpha^{2n_2} \frac{h_{23}^{d} h_{33}^{u}}{h_{33}^{d} h_{33}^{u}} + \alpha^{4(n_2-n_1)} \frac{h_{12}^{d} h_{23}^{d}}{h_{33}^{d} h_{33}^{d} h_{33}^{d}} + \alpha^{2(4n_2-3n_1-n_3)} h_{23}^{d} \frac{2 h_{12}^{d} h_{22}^{d} h_{12}^{u}}{h_{12}^{d} h_{33}^{d} h_{33}^{d}} \right) \epsilon^4 \] (42e)

\[ V_{32}^{\text{CKM}} \approx \left( \alpha^{2(n_2-n_1)} \frac{h_{23}^{d}}{h_{33}^{d}} - \alpha^{2n_1} \frac{h_{23}^{u}}{h_{33}^{u}} \right) \epsilon^2 + \left( \alpha^{4(n_2-n_1)} \frac{h_{22}^{d} h_{23}^{d}}{h_{33}^{d} h_{33}^{d}} - \alpha^{4n_1} \frac{h_{22}^{d} h_{23}^{u}}{h_{33}^{d} h_{33}^{u}} \right) \epsilon^4 \] (42f)

\[ V_{13}^{\text{CKM}} \approx \alpha^{2(3n_2-2n_1-n_3)} h_{23}^{d} \frac{h_{12}^{d} h_{23}^{d}}{h_{13}^{d} h_{33}^{d}} \epsilon^2 \\
+ \left( \alpha^{2n_2} \frac{h_{12}^{u} h_{23}^{u} - h_{13}^{u} h_{22}^{u}}{h_{22}^{u} h_{33}^{u}} - \alpha^{4(n_2-n_1)} \frac{h_{12}^{d} h_{23}^{d}}{h_{33}^{d} h_{33}^{d} h_{33}^{d}} + \alpha^{2(4n_2-3n_1-n_3)} h_{12}^{d} h_{13}^{d} \frac{2 h_{23}^{d} h_{22}^{d} h_{22}^{u} h_{23}^{u}}{h_{13}^{d} h_{33}^{d} h_{33}^{d} h_{33}^{d}} \right) \epsilon^4 \] (42g)

\[ V_{23}^{\text{CKM}} \approx \left( \alpha^{2n_1} \frac{h_{23}^{u}}{h_{33}^{u}} - \alpha^{2(n_2-n_1)} \frac{h_{23}^{d}}{h_{33}^{d}} \right) \epsilon^2 \\
+ \left( \alpha^{4n_1} \frac{h_{22}^{u} h_{23}^{u}}{h_{33}^{d} h_{33}^{d}} - \alpha^{4(n_2-n_1)} \frac{h_{23}^{d} h_{23}^{d}}{h_{33}^{d} h_{33}^{d}} + \alpha^{2(4n_2-3n_1-n_3)} \frac{h_{22}^{d} h_{23}^{d} h_{22}^{u} h_{23}^{u}}{h_{13}^{d} h_{33}^{d} h_{33}^{d} h_{33}^{d}} \right) \epsilon^4 \] (42h)

\[ V_{33}^{\text{CKM}} \approx 1 - \left( \alpha^{4n_1} \frac{h_{23}^{u} 2}{2 h_{33}^{u}} - \alpha^{2n_2} \frac{h_{23}^{d} h_{33}^{u}}{h_{33}^{d} h_{33}^{d} h_{33}^{d}} + \alpha^{4(n_2-n_1)} \frac{h_{23}^{d} 2}{2 h_{33}^{d}} + \alpha^{4(3n_2-4n_1-n_3)} \frac{h_{22}^{d} h_{23}^{d} 4}{2 h_{13}^{d} h_{33}^{d} h_{33}^{d}} \right) \epsilon^4 \] (42i)
### 6.3 Tables of Charge Assignments for Exotic Quarks

Table 4: Charge assignments of the heavy $Q$ quarks for a model having an effective Lagrangian with only powers of $(S^\dagger S/M^2)$.

| Fields  | $U(1)_S$ | $U(1)_F$ | Fields  | $U(1)_S$ | $U(1)_F$ | Fields  | $U(1)_S$ | $U(1)_F$ |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $Q^1_{L,R}$ | 3       | 29, 28  | $Q^{13}_{L,R}$ | 1       | 23, 24  | $Q^{25}_{L,R}$ | -2     | 4, 3    |
| $Q^2_{L,R}$ | 2       | 6, 7    | $Q^{14}_{L,R}$ | 1       | 27, 26  | $Q^{26}_{L,R}$ | -2     | 18, 17  |
| $Q^3_{L,R}$ | 2       | 16, 17  | $Q^{15}_{L,R}$ | 0       | 4, 3    | $Q^{27}_{L,R}$ | -2     | 20, 19  |
| $Q^4_{L,R}$ | 2       | 18, 19  | $Q^{16}_{L,R}$ | 0       | 26, 25  | $Q^{28}_{L,R}$ | -2     | 24, 23  |
| $Q^5_{L,R}$ | 2       | 20, 21  | $Q^{17}_{L,R}$ | 0       | 30, 29  | $Q^{29}_{L,R}$ | -2     | 26, 25  |
| $Q^6_{L,R}$ | 2       | 22, 23  | $Q^{18}_{L,R}$ | 0       | 32, 31  | $Q^{30}_{L,R}$ | -2     | 28, 27  |
| $Q^7_{L,R}$ | 2       | 28, 27  | $Q^{19}_{L,R}$ | -1      | 1, 0    | $Q^{31}_{L,R}$ | -3     | 5, 4    |
| $Q^8_{L,R}$ | 1       | 7, 8    | $Q^{20}_{L,R}$ | -1      | 3, 2    | $Q^{32}_{L,R}$ | -3     | 7, 6    |
| $Q^9_{L,R}$ | 1       | 9, 10   | $Q^{21}_{L,R}$ | -1      | 17, 16  | $Q^{33}_{L,R}$ | -3     | 9, 8    |
| $Q^{10}_{L,R}$ | 1     | 11, 12  | $Q^{22}_{L,R}$ | -1      | 21, 20  | $Q^{34}_{L,R}$ | -3     | 11, 10  |
| $Q^{11}_{L,R}$ | 1    | 13, 14  | $Q^{23}_{L,R}$ | -1      | 23, 22  | $Q^{35}_{L,R}$ | -3     | 13, 12  |
| $Q^{12}_{L,R}$ | 1   | 15, 16  | $Q^{24}_{L,R}$ | -1      | 29, 28  | $Q^{36}_{L,R}$ | -3     | 15, 14  |
Table 5: Charge assignments of the heavy $U$ and $D$ quarks for a model having an effective Lagrangian with only powers of $(S\dagger S/M^2)$.

| Fields  | $U(1)_S$ | $U(1)_F$ | Fields  | $U(1)_S$ | $U(1)_F$ | Fields  | $U(1)_S$ | $U(1)_F$ |
|---------|-----------|-----------|---------|-----------|-----------|---------|-----------|-----------|
| $U^1_{L,R}$ | 1         | 1, 0      | $U^7_{L,R}$ | 0         | 8, 9      | $U^{13}_{L,R}$ | -1        | 11, 10    |
| $U^2_{L,R}$ | 1         | 3, 2      | $U^8_{L,R}$ | 0         | 16, 15    | $U^{14}_{L,R}$ | -1        | 15, 14    |
| $U^3_{L,R}$ | 1         | 5, 4      | $U^9_{L,R}$ | 0         | 18, 17    | $U^{15}_{L,R}$ | -2        | 4, 5      |
| $U^4_{L,R}$ | 1         | 7, 6      | $U^{10}_{L,R}$ | 0       | 22, 21    | $U^{16}_{L,R}$ | -2        | 6, 7      |
| $U^5_{L,R}$ | 1         | 19, 18    | $U^{11}_{L,R}$ | 0       | 24, 23    | $U^{17}_{L,R}$ | -2        | 12, 11    |
| $U^6_{L,R}$ | 1         | 21, 20    | $U^{12}_{L,R}$ | -1      | 7, 8      | $U^{18}_{L,R}$ | -2        | 14, 13    |
| $D^1_{L,R}$ | 3         | 29, 30    | $D^{11}_{L,R}$ | 0       | 22, 21    | $D^{21}_{L,R}$ | -2        | 30, 31    |
| $D^2_{L,R}$ | 2         | 6, 5      | $D^{12}_{L,R}$ | 0       | 24, 23    | $D^{22}_{L,R}$ | -3        | 15, 16    |
| $D^3_{L,R}$ | 2         | 30, 31    | $D^{13}_{L,R}$ | -1      | 7, 8      | $D^{23}_{L,R}$ | -3        | 17, 18    |
| $D^4_{L,R}$ | 1         | 5, 4      | $D^{14}_{L,R}$ | -1      | 11, 10    | $D^{24}_{L,R}$ | -3        | 19, 20    |
| $D^5_{L,R}$ | 1         | 19, 18    | $D^{15}_{L,R}$ | -1      | 15, 14    | $D^{25}_{L,R}$ | -3        | 21, 22    |
| $D^6_{L,R}$ | 1         | 21, 20    | $D^{16}_{L,R}$ | -1      | 31, 32    | $D^{26}_{L,R}$ | -3        | 23, 24    |
| $D^7_{L,R}$ | 1         | 31, 32    | $D^{17}_{L,R}$ | -2      | 4, 5      | $D^{27}_{L,R}$ | -3        | 25, 26    |
| $D^8_{L,R}$ | 0         | 8, 9      | $D^{18}_{L,R}$ | -2      | 6, 7      | $D^{28}_{L,R}$ | -3        | 27, 28    |
| $D^9_{L,R}$ | 0         | 16, 15    | $D^{19}_{L,R}$ | -2      | 12, 11    | $D^{29}_{L,R}$ | -3        | 29, 30    |
| $D^{10}_{L,R}$ | 0         | 18, 17    | $D^{20}_{L,R}$ | -2      | 14, 13    |          |           |           |
Table 6: Charge assignments of the heavy $Q$, $U$, and $D$ quarks for a model having an effective Lagrangian with only powers of ($H^\dagger H/M^2$).

| Fields $Q_{L,R}$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields $Q_{L,R}$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields $Q_{L,R}$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ |
|------------------|--------------|--------------|------------------|--------------|--------------|------------------|--------------|--------------|
| $Q^1_{L,R}$      | 5, 4         | -5, -5       | $Q^8_{L,R}$      | 1, 0         | -3, -3       | $Q^{15}_{L,R}$   | -3, -4       | -1, -1       |
| $Q^2_{L,R}$      | 5, 4         | 3, 3         | $Q^9_{L,R}$      | 0, 1         | 4, 4         | $Q^{16}_{L,R}$   | -3, -3       | 1, 0         |
| $Q^3_{L,R}$      | 3, 2         | -5, -5       | $Q^{10}_{L,R}$   | 0, -1        | 6, 6         | $Q^{17}_{L,R}$   | -4, -3       | 6, 6         |
| $Q^4_{L,R}$      | 3, 3         | -1, 0        | $Q^{11}_{L,R}$   | -1, -1       | -1, -2       | $Q^{18}_{L,R}$   | -4, -4       | -4, -3       |
| $Q^5_{L,R}$      | 3, 3         | 3, 4         | $Q^{12}_{L,R}$   | -1, -2       | -5, -5       | $Q^{19}_{L,R}$   | -5, -5       | 1, 0         |
| $Q^6_{L,R}$      | 2, 1         | 6, 6         | $Q^{13}_{L,R}$   | -1, -1       | 1, 0         | $Q^{20}_{L,R}$   | -6, -5       | 4, 4         |
| $Q^7_{L,R}$      | 1, 1         | 1, 0         | $Q^{14}_{L,R}$   | -2, -3       | 4, 4         |                  |              |              |
| $U^1_{L,R}$      | 2, 3         | -1, -1       | $U^7_{L,R}$      | -1, -1       | 2, 1         | $U^{13}_{L,R}$   | -3, -3       | 4, 3         |
| $U^2_{L,R}$      | 0, 1         | -1, -1       | $U^8_{L,R}$      | -1, -1       | 4, 3         | $U^{14}_{L,R}$   | -5, -4       | -2, -2       |
| $U^3_{L,R}$      | 1, 1         | 2, 1         | $U^9_{L,R}$      | -1, -1       | 6, 5         | $U^{15}_{L,R}$   | -5, -4       | 6, 6         |
| $U^4_{L,R}$      | 1, 1         | 4, 3         | $U^{10}_{L,R}$   | -2, -2       | -3, -2       | $U^{16}_{L,R}$   | -5, -5       | 0, -1        |
| $U^5_{L,R}$      | -1, -1       | -4, -5       | $U^{11}_{L,R}$   | -2, -2       | -1, 0        | $U^{17}_{L,R}$   | -5, -5       | 4, 5         |
| $U^6_{L,R}$      | 0, -1        | -3, -3       | $U^{12}_{L,R}$   | -3, -3       | 2, 1         |                  |              |              |

| Fields $D_{L,R}$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields $D_{L,R}$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields $D_{L,R}$ | $U(1)_{F_1}$ | $U(1)_{F_2}$ |
|------------------|--------------|--------------|------------------|--------------|--------------|------------------|--------------|--------------|
| $D^1_{L,R}$      | 3, 3         | 0, 1         | $D^8_{L,R}$      | 0, 0         | -1, 0       | $D^{15}_{L,R}$   | -3, -2       | 6, 6         |
| $D^2_{L,R}$      | 3, 3         | 4, 5         | $D^9_{L,R}$      | 1, 0         | 2, 0         | $D^{16}_{L,R}$   | -2, -3       | -5, -5       |
| $D^3_{L,R}$      | 3, 2         | 2, 2         | $D^{10}_{L,R}$   | -1, 0       | 4, 4         | $D^{17}_{L,R}$   | -4, -4       | -5, -4       |
| $D^4_{L,R}$      | 3, 2         | 6, 6         | $D^{11}_{L,R}$   | -1, -2       | -2, -2       | $D^{18}_{L,R}$   | -3, -4       | -2, -2       |
| $D^5_{L,R}$      | 2, 1         | -5, -5       | $D^{12}_{L,R}$   | -2, -2       | 1, 0         | $D^{19}_{L,R}$   | -3, -4       | 2, 2         |
| $D^6_{L,R}$      | 1, 1         | -4, -3       | $D^{13}_{L,R}$   | -1, -2       | 2, 2         | $D^{20}_{L,R}$   | -5, -5       | 2, 1         |
| $D^7_{L,R}$      | 1, 1         | -2, -1       | $D^{14}_{L,R}$   | -2, -2       | 5, 4         |                  |              |              |
Table 7: Charge assignments for the heavy quark doublets to be used in a generalized model. The given values are for the case where all coefficients are of the form \((H^\dagger H/M^2)^n\). See Tables 10–15 to make the necessary changes for the different Lagrangians.

| Fields  | \(U(1)_{F_1}\) | \(U(1)_{F_2}\) | Fields  | \(U(1)_{F_1}\) | \(U(1)_{F_2}\) | Fields  | \(U(1)_{F_1}\) | \(U(1)_{F_2}\) |
|---------|----------------|----------------|---------|----------------|----------------|---------|----------------|----------------|
| \(Q^1_{L,R}\) | 0, 0 | 3, 2 | \(Q^{21}_{L,R}\) | 2, 3 | 29, 29 | \(Q^{41}_{L,R}\) | 7, 7 | 18, 17 |
| \(Q^2_{L,R}\) | 0, 0 | 5, 4 | \(Q^{22}_{L,R}\) | 3, 4 | 2, 2 | \(Q^{42}_{L,R}\) | 7, 6 | 2, 2 |
| \(Q^3_{L,R}\) | 0, 0 | 9, 8 | \(Q^{23}_{L,R}\) | 3, 4 | 12, 12 | \(Q^{43}_{L,R}\) | 7, 6 | 12, 12 |
| \(Q^4_{L,R}\) | 0, 0 | 11, 10 | \(Q^{24}_{L,R}\) | 4, 5 | 29, 29 | \(Q^{44}_{L,R}\) | 8, 7 | 3, 3 |
| \(Q^5_{L,R}\) | 0, 0 | 15, 14 | \(Q^{25}_{L,R}\) | 5, 5 | 0, 1 | \(Q^{45}_{L,R}\) | 7, 7 | 4, 5 |
| \(Q^6_{L,R}\) | 0, 0 | 17, 16 | \(Q^{26}_{L,R}\) | 7, 6 | 0, 0 | \(Q^{46}_{L,R}\) | 7, 7 | 6, 7 |
| \(Q^7_{L,R}\) | 2, 2 | 1, 2 | \(Q^{27}_{L,R}\) | 9, 8 | 0, 0 | \(Q^{47}_{L,R}\) | 7, 7 | 8, 9 |
| \(Q^8_{L,R}\) | 2, 2 | 3, 4 | \(Q^{28}_{L,R}\) | 11, 10 | 0, 0 | \(Q^{48}_{L,R}\) | 7, 7 | 10, 11 |
| \(Q^9_{L,R}\) | 2, 2 | 5, 6 | \(Q^{29}_{L,R}\) | 13, 12 | 0, 0 | \(Q^{49}_{L,R}\) | 10, 9 | 3, 3 |
| \(Q^{10}_{L,R}\) | 2, 2 | 7, 8 | \(Q^{30}_{L,R}\) | 15, 14 | 0, 0 | \(Q^{50}_{L,R}\) | 11, 11 | 4, 4 |
| \(Q^{11}_{L,R}\) | 2, 2 | 9, 10 | \(Q^{31}_{L,R}\) | 15, 15 | 2, 1 | \(Q^{51}_{L,R}\) | 11, 11 | 6, 5 |
| \(Q^{12}_{L,R}\) | 2, 2 | 11, 12 | \(Q^{32}_{L,R}\) | 15, 15 | 4, 3 | \(Q^{52}_{L,R}\) | 13, 12 | 4, 4 |
| \(Q^{13}_{L,R}\) | 2, 2 | 13, 14 | \(Q^{33}_{L,R}\) | 15, 15 | 6, 5 | \(Q^{53}_{L,R}\) | 13, 13 | 6, 5 |
| \(Q^{14}_{L,R}\) | 2, 2 | 15, 16 | \(Q^{34}_{L,R}\) | 5, 5 | 5, 6 | \(Q^{54}_{L,R}\) | 6, 6 | 31, 30 |
| \(Q^{15}_{L,R}\) | 2, 2 | 17, 18 | \(Q^{35}_{L,R}\) | 5, 5 | 8, 7 | \(Q^{55}_{L,R}\) | 7, 7 | 22, 23 |
| \(Q^{16}_{L,R}\) | 2, 2 | 19, 20 | \(Q^{36}_{L,R}\) | 5, 5 | 14, 15 | \(Q^{56}_{L,R}\) | 7, 8 | 24, 24 |
| \(Q^{17}_{L,R}\) | 2, 2 | 21, 22 | \(Q^{37}_{L,R}\) | 5, 5 | 16, 17 | \(Q^{57}_{L,R}\) | 7, 7 | 26, 27 |
| \(Q^{18}_{L,R}\) | 2, 2 | 23, 24 | \(Q^{38}_{L,R}\) | 5, 5 | 20, 21 | \(Q^{58}_{L,R}\) | 8, 9 | 26, 26 |
| \(Q^{19}_{L,R}\) | 2, 2 | 25, 26 | \(Q^{39}_{L,R}\) | 5, 5 | 22, 23 | \(Q^{59}_{L,R}\) | 8, 7 | 29, 29 |
| \(Q^{20}_{L,R}\) | 2, 2 | 27, 28 | \(Q^{40}_{L,R}\) | 7, 7 | 16, 15 | Cont. on Table** | ** | ** |
Table 8: Charge assignments for the heavy quark doublets to be used in a generalized model. The given values are for the case where all coefficients are of the form $(H^\dagger H/M^2)^n$. See Tables 10–15 to make the necessary changes for the different Lagrangians.

| Fields | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields | $U(1)_{F_1}$ | $U(1)_{F_2}$ | Fields | $U(1)_{F_1}$ | $U(1)_{F_2}$ |
|--------|-------------|-------------|--------|-------------|-------------|--------|-------------|-------------|
| $Q_{L,R}^{60}$ | 10, 9 | 29, 29 | $Q_{L,R}^{67}$ | 9, 9 | 16,15 | $Q_{L,R}^{75}$ | 13, 13 | 12, 11 |
| $Q_{L,R}^{61}$ | 8, 7 | 33, 33 | $Q_{L,R}^{68}$ | 9, 9 | 18, 17 | $Q_{L,R}^{76}$ | 13, 13 | 14, 13 |
| $Q_{L,R}^{62}$ | 10, 9 | 33, 33 | $Q_{L,R}^{69}$ | 11, 11 | 12, 11 | $Q_{L,R}^{77}$ | 13, 13 | 18, 17 |
| $Q_{L,R}^{63}$ | 7, 8 | 20, 20 | $Q_{L,R}^{70}$ | 11, 11 | 14, 13 | $Q_{L,R}^{78}$ | 13, 13 | 20, 19 |
| $Q_{L,R}^{64}$ | 9, 9 | 20, 21 | $Q_{L,R}^{71}$ | 11, 11 | 18, 19 | $Q_{L,R}^{79}$ | 13, 13 | 24, 25 |
| $Q_{L,R}^{65}$ | 9, 9 | 10, 9 | $Q_{L,R}^{72}$ | 11, 11 | 20, 21 | $Q_{L,R}^{80}$ | 13, 13 | 26, 27 |
| $Q_{L,R}^{66}$ | 9, 9 | 12, 11 | $Q_{L,R}^{73}$ | 11, 11 | 24, 25 | $Q_{L,R}^{81}$ | 13, 13 | 30, 31 |
|         |           |           | $Q_{L,R}^{74}$ | 11, 11 | 26,27 | $Q_{L,R}^{82}$ | 13, 13 | 32, 33 |
Table 9: Charge assignments for the heavy quark singlets to be used in a generalized model. The given values are for the case where all coefficients are of the form \((H^\dagger H/M^2)^n\). See Tables 10–15 to make the necessary changes for the different Lagrangians.

| Fields | \(U(1)_{F_1}\) | \(U(1)_{F_2}\) | Fields | \(U(1)_{F_1}\) | \(U(1)_{F_2}\) | Fields | \(U(1)_{F_1}\) | \(U(1)_{F_2}\) |
|--------|---------------|---------------|--------|---------------|---------------|--------|---------------|---------------|
| \(U^1_{L,R}\) | 0, 0 | 2,1 | \(U^1_{L,R}\) | 5, 5 | 17, 18 | \(U^1_{L,R}\) | 11, 10 | 33, 33 |
| \(U^2_{L,R}\) | 1, 2 | 1, 1 | \(U^2_{L,R}\) | 5, 5 | 19, 20 | \(U^2_{L,R}\) | 13, 12 | 33, 33 |
| \(U^3_{L,R}\) | 0, 0 | 6, 5 | \(U^3_{L,R}\) | 5, 5 | 23, 24 | \(U^3_{L,R}\) | 13, 13 | 7, 6 |
| \(U^4_{L,R}\) | 0, 0 | 8, 7 | \(U^4_{L,R}\) | 5, 5 | 25, 26 | \(U^4_{L,R}\) | 13, 13 | 9, 8 |
| \(U^5_{L,R}\) | 0, 0 | 12, 11 | \(U^5_{L,R}\) | 5, 6 | 27, 27 | \(U^5_{L,R}\) | 13, 13 | 11, 10 |
| \(U^6_{L,R}\) | 0, 0 | 14, 13 | \(U^6_{L,R}\) | 6, 6 | 28, 29 | \(U^6_{L,R}\) | 13, 13 | 27, 28 |
| \(U^7_{L,R}\) | 5, 5 | 3, 2 | \(U^7_{L,R}\) | 11, 11 | 7, 6 | \(U^7_{L,R}\) | 13, 13 | 29, 30 |
| \(U^8_{L,R}\) | 5, 5 | 5, 4 | \(U^8_{L,R}\) | 11, 11 | 9, 8 | \(U^8_{L,R}\) | 13, 13 | 21, 22 |
| \(U^9_{L,R}\) | 5, 5 | 9, 8 | \(U^9_{L,R}\) | 11, 11 | 11, 10 | \(U^9_{L,R}\) | 13, 13 | 23, 24 |
| \(U^{10}_{L,R}\) | 5, 5 | 11, 10 | \(U^{10}_{L,R}\) | 11, 11 | 15, 16 |          |          |          |
| \(U^{11}_{L,R}\) | 5, 5 | 13, 14 | \(U^{11}_{L,R}\) | 11, 11 | 17, 18 |          |          |          |
| \(D^1_{L,R}\) | 5, 5 | 12, 13 | \(D^1_{L,R}\) | 9, 9 | 21, 22 | \(D^1_{L,R}\) | 11, 11 | 21, 22 |
| \(D^2_{L,R}\) | 6, 7 | 13, 13 | \(D^2_{L,R}\) | 8, 9 | 24, 24 | \(D^2_{L,R}\) | 11, 11 | 23, 24 |
| \(D^3_{L,R}\) | 7, 7 | 14, 15 | \(D^3_{L,R}\) | 9, 9 | 25, 26 | \(D^3_{L,R}\) | 11, 10 | 29, 29 |
| \(D^4_{L,R}\) | 6, 6 | 32, 31 | \(D^4_{L,R}\) | 9, 9 | 13, 12 | \(D^4_{L,R}\) | 11, 11 | 27, 28 |
| \(D^5_{L,R}\) | 7, 6 | 33, 33 | \(D^5_{L,R}\) | 9, 9 | 15, 14 | \(D^5_{L,R}\) | 13, 13 | 15, 14 |
| \(D^6_{L,R}\) | 7, 7 | 19, 20 | \(D^6_{L,R}\) | 9, 9 | 9, 8 | \(D^6_{L,R}\) | 13, 13 | 17, 16 |
| \(D^7_{L,R}\) | 7, 6 | 27, 27 | \(D^7_{L,R}\) | 10, 11 | 8, 8 | \(D^7_{L,R}\) | 13, 13 | 21, 22 |
| \(D^8_{L,R}\) | 6, 6 | 28, 29 | \(D^8_{L,R}\) | 12, 13 | 8, 8 | \(D^8_{L,R}\) | 13, 13 | 23, 34 |
| \(D^9_{L,R}\) | 8, 9 | 18, 18 | \(D^9_{L,R}\) | 14, 15 | 8, 8 |          |          |          |
| \(D^{10}_{L,R}\) | 8, 7 | 22, 22 | \(D^{10}_{L,R}\) | 15, 15 | 7, 6 |          |          |          |
Table 10: Replacements made to Tables 7–9 when changing the single power coefficient \((H^\dagger H/M^2)\) to \((S^\dagger S/M^2)\). The quantum numbers for the \(U(1)_{F_1}\) and \(U(1)_{F_2}\) symmetries do not change.

| Fields \(U(1)_{S}\) | \(\rightarrow\) | Fields \(U(1)_{S}\) |
|---------------------|----------------|----------------|
| \(D^1_{L,R}\)       | 0              | \(Q^{83}_{L,R}\) | 1 |
| \(D^2_{L,R}\)       | 0              | \(Q^{84}_{L,R}\) | 1 |
| \(D^3_{L,R}\)       | 0              | \(Q^{85}_{L,R}\) | 1 |
| \(U^7_{L,R}\)       | 0              | \(Q^{86}_{L,R}\) | 1 |
| \(U^8_{L,R}\)       | 0              | \(Q^{87}_{L,R}\) | 1 |
| \(U^{18}_{L,R}\)    | 0              | \(Q^{88}_{L,R}\) | 1 |
| \(U^{19}_{L,R}\)    | 0              | \(Q^{89}_{L,R}\) | 1 |
| \(U^{20}_{L,R}\)    | 0              | \(Q^{90}_{L,R}\) | 1 |

Table 11: Replacements made to Tables 7–9 when changing the second power coefficient \((H^\dagger H/M^2)^2\) to \((H^\dagger H/M^2)(S^\dagger S/M^2)\). The quantum numbers for the \(U(1)_{F_1}\) and \(U(1)_{F_2}\) symmetries do not change.

| Fields \(U(1)_{S}\) | \(\rightarrow\) | Fields \(U(1)_{S}\) |
|---------------------|----------------|----------------|
| \(U^{25}_{L,R}\)    | 0              | \(Q^{91}_{L,R}\) | 1 |
| \(U^{26}_{L,R}\)    | 0              | \(Q^{92}_{L,R}\) | 1 |
| \(U^{27}_{L,R}\)    | 0              | \(Q^{93}_{L,R}\) | 1 |
| \(U^{14}_{L,R}\)    | 0              | \(Q^{94}_{L,R}\) | 1 |
| \(U^{15}_{L,R}\)    | 0              | \(Q^{95}_{L,R}\) | 1 |
| \(U^{16}_{L,R}\)    | 0              | \(Q^{96}_{L,R}\) | 1 |
| \(U^{17}_{L,R}\)    | 0              | \(Q^{97}_{L,R}\) | 1 |
| \(D^{21}_{L,R}\)    | 0              | \(Q^{98}_{L,R}\) | 1 |
| \(D^{22}_{L,R}\)    | 0              | \(Q^{99}_{L,R}\) | 1 |
| \(Q^{65}_{L,R}\)    | 0              | \(D^{29}_{L,R}\) | 1 |
| \(Q^{66}_{L,R}\)    | 0              | \(D^{30}_{L,R}\) | 1 |
Table 12: Replacements made to Tables 7–9 when changing the second power coefficient \((H^4H/M^2)^2\) to \((S^4S/M^2)^2\). The replacements from Table 11 must also be made with these replacements. The quantum numbers for the \(U(1)_{F_1}\) and \(U(1)_{F_2}\) symmetries do not change.

| Fields \(U(1)_S\) | \(\rightarrow\) | Fields \(U(1)_S\) |
|-----------------|---------------|-----------------|
| \(D_{25}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{100}^{L,R}\) 1 |
| \(D_{26}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{101}^{L,R}\) 1 |
| \(U_{12}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{102}^{L,R}\) 1 |
| \(U_{13}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{103}^{L,R}\) 1 |
| \(D_{23}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{104}^{L,R}\) 1 |
| \(D_{24}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{105}^{L,R}\) 1 |
| \(Q_{67}^{L,R}\) 0 | \(\rightarrow\) | \(D_{31}^{L,R}\) 1 |
| \(Q_{68}^{L,R}\) 0 | \(\rightarrow\) | \(D_{32}^{L,R}\) 1 |

Table 13: Replacements made to Tables 7–9 when changing the third power coefficient \((H^4H/M^2)^3\) to \((H^4H/M^2)^2(S^4S/M^2)\). The quantum numbers for the \(U(1)_{F_1}\) and \(U(1)_{F_2}\) symmetries do not change.

| Fields \(U(1)_S\) | \(\rightarrow\) | Fields \(U(1)_S\) |
|-----------------|---------------|-----------------|
| \(U_{1}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{106}^{L,R}\) 1 |
| \(U_{2}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{107}^{L,R}\) 1 |
| \(D_{4}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{108}^{L,R}\) 1 |
| \(D_{5}^{L,R}\) 0 | \(\rightarrow\) | \(Q_{109}^{L,R}\) 1 |
| \(Q_{57}^{L,R}\) 0 | \(\rightarrow\) | \(D_{23}^{L,R}\) 1 |
| \(Q_{58}^{L,R}\) 0 | \(\rightarrow\) | \(D_{24}^{L,R}\) 1 |
Table 14: Replacements made to Tables 7–9 when changing the third power coefficient \((H^\dagger H/M^2)^3\) to \((H^\dagger H/M^2)(S^\dagger S/M^2)^2\). The replacements from Table 13 must also be made with these replacements. The quantum numbers for the \(U(1)_{F_1}\) and \(U(1)_{F_2}\) symmetries do not change.

| Fields | \(U(1)_S\) | Fields | \(U(1)_S\) |
|--------|------------|--------|------------|
| \(U_{3 L,R}^3\) | 0 | \(Q_{10 L,R}^{11}\) | 1 |
| \(U_{4 L,R}^4\) | 0 | \(Q_{11 L,R}^{11}\) | 1 |
| \(U_{23 L,R}^23\) | 0 | \(Q_{12 L,R}^{11}\) | 1 |
| \(U_{24 L,R}^24\) | 0 | \(Q_{13 L,R}^{11}\) | 1 |
| \(Q_{55 L,R}^{55}\) | 0 | \(D_{25 L,R}^{35}\) | 1 |
| \(Q_{56 L,R}^{56}\) | 0 | \(D_{26 L,R}^{36}\) | 1 |

Table 15: Replacements made to Tables 7–9 when changing the third power coefficient \((H^\dagger H/M^2)^3\) to \((S^\dagger S/M^2)^3\). The replacements from Tables 13 and 14 must also be made with these replacements. The quantum numbers for the \(U(1)_{F_1}\) and \(U(1)_{F_2}\) symmetries do not change.

| Fields | \(U(1)_S\) | Fields | \(U(1)_S\) |
|--------|------------|--------|------------|
| \(U_{5 L,R}^5\) | 0 | \(Q_{14 L,R}^{114}\) | 1 |
| \(U_{6 L,R}^6\) | 0 | \(Q_{15 L,R}^{115}\) | 1 |
| \(U_{28 L,R}^{28}\) | 0 | \(Q_{16 L,R}^{116}\) | 1 |
| \(U_{20 L,R}^{20}\) | 0 | \(Q_{17 L,R}^{117}\) | 1 |
| \(Q_{63 L,R}^{63}\) | 0 | \(D_{27 L,R}^{27}\) | 1 |
| \(Q_{64 L,R}^{64}\) | 0 | \(D_{28 L,R}^{28}\) | 1 |
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