Implication of U-duality for black holes in M/string theory

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Abstract

U-duality symmetry of M-theory and S and T duality of string theory can be used to study various black branes solutions. We explore some aspect of this idea here. This symmetry can be used to get relations among various components of the metric of the black brane. These relations in turn give relations among various components of energy momentum tensors. We show that, using these relations, without knowing explicit form of form fields we can get the black brane solutions. This features were studied previously in the context of M theory. Here we extensively studied them in string theory (type II supergravity). We also show that, this formulation works for exotic branes.

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1 Introduction

Black Holes are widely studied object in string theory. Various properties of a class of black holes have been successfully described using mutually BPS intersecting configurations of string/M theory branes. Mutually BPS intersecting configurations means that, two or more branes intersect in such a way that they preserve same number of Supersymmetry as the single brane does; for example, that in M theory two stacks of 2 branes intersect at a point; two stacks of 5 branes intersect along three common spatial directions; a stack of 2 branes intersect a stack of 5 branes along one common spatial direction; waves, if present, will be along a common intersection direction; and each stack of branes is smeared uniformly along the other brane directions. See [1, 2] for more details and for other such string/M theory configurations. Black hole entropies are calculated from counting excitations of such configurations, and Hawking radiation is calculated from interactions between them.

Such brane configurations consist of only branes and no antibranes in the extremal limit. In the near extremal limit, they consist of a small number of antibranes also. It is the interaction between branes and antibranes which give rise to Hawking radiation. String theory calculations are tractable and match those of Bekenstein and Hawking in the extremal and near extremal limits. In full non extremal limit it is difficult to control string theory calculation. However, even in the far extremal limit, black hole dynamics is expected to be described by mutually BPS intersecting brane configurations where they now consist of branes, antibranes, and other excitations living on them, at all non zero temperature and in dynamical equilibrium with each other [3] – [12]. From now on, for the sake of brevity, we will refer to such far extremal configurations also as brane configurations even though they may now consist of branes and antibranes, left moving and right moving waves, and other excitations.

The entropy \( S \) of \( N \) stacks of mutually BPS intersecting brane configurations, in the limit where \( S \gg 1 \), is expected to be given by

\[
S \sim \prod_{I} \sqrt{n_{I} + \bar{n}_{I}} \sim \mathcal{E}^{\frac{N}{2}},
\]

where \( n_{I} \) and \( \bar{n}_{I} \), \( I = 1, \cdots, N \), denote the numbers of branes and antibranes of \( I^{th} \) type, \( \mathcal{E} \) is the total energy, and the second expression applies for the charge neutral case where \( n_{I} = \bar{n}_{I} \) for all \( I \). When \( N \geq 3 \), this system describes a proper black hole with horizon. The proof for this expression is given by comparing it in various limits with the entropy of the corresponding black holes [3] – [4], see also [5] – [14]. For \( N \leq 4 \) and when other calculable factors omitted here are restored, this expression matches that for the corresponding black holes in the extremal and near extremal limit and, in the models based on that of Danielsson et al [5], matches up to a numerical factor in the far extremal limit [3] – [14] also.

Note that, in the limit of large \( \mathcal{E} \), the entropy \( S(\mathcal{E}) \) is \( \ll \mathcal{E} \) for radiation in a finite volume and is \( \sim \mathcal{E} \) for strings in the Hagedorn regime. In comparison, the entropy given in (1.1) is much larger when \( N > 2 \). This is because the branes in the mutually BPS intersecting brane configurations form bound states, become fractional, and support very low energy excitations which lead to a large entropy. Thus, for a given energy, such brane configurations are highly entropic.

Another consequence of fractional branes is “fuzz ball” proposal. According to the fuzz ball picture for black holes [15], the fractional branes arising from the bound states formed by intersecting brane configurations have non trivial transverse spatial extensions due to quantum dynamics. The size of their transverse extent is of the order of Schwarzschild radius of the black holes. Therefore, essentially, the region inside the ‘horizon’ of the black hole is not empty but is filled with fuzz ball whose fuzz arise from the quantum dynamics of fractional strings/branes.

This fuzz ball picture of black hole was extended to study early universe [13] [14]. This early universe was further studied in [16] – [18] (See also [19, 20]) using U-duality symmetry of string/M theory.

U-duality is a symmetry of M-theory which consists of T-duality, S-duality of string theory and dimensional reduction and dimension upliftment. In certain cases of supergravity solutions this symmetries can be used to get relations among various metric component. These relations can be used to get relations among various components of energy-momentum tensors. This method was proposed in [16] and further discussed in [17] [18] [19]. We will review this following mainly [18] [19]. We show here this duality technique...
works for black holes as well. Our main goal in this paper is to show this method works for string theory black holes as well.

In this work we also studied whether this duality method works for exotic black brane solutions. Exotic branes are co-dimension 2 object. It is known from study of low dimensional string/M theory that there are certain “exotic” particle whose higher dimensional origin can not be usual brane. These are called exotic states and their higher dimensional branes are called exotic branes [20] – [26]. One should note here that for such branes, metric components are not only function of radial coordinate but also function of angular coordinate. This type of non-geometric backgrounds are called T-fold or more generally U-fold [27]. We discuss here such co-dimension 2 branes and try to see what the U-duality relations come out among various metric components and hence among components of energy momentum tensor.

The relations among the various components of energy-momentum tensors found from duality relations is the key for the cosmological model discussed in [16] – [19]. A similar construction were used to get star like solutions of M theory in [28, 29, 30]. In case of black brane, action is known explicitly, but in case of said cosmological model fields are not known. There duality relations plays most important role. A similar model of early universe in string theory may become useful because in string theory interaction of various branes and strings are more tractable than that of M theory.

This paper is organized as follows. Here we give a brief review of a part of work of [16] – [19] in section 2. Where we show U duality relations for black holes in M theory. In section 3 we show similar relations exist for exotic M brane solutions. In section 4 similar relations for string theory branes were studied using S and t duality. In section 5 we conclude.

2 General Black Brane solutions

In black brane solutions, $T_{AB}$ is obtained from the action for higher form gauge fields. With a suitable ansatz for the metric, equations of motions can be solved to obtain black hole solutions.

To explain the process consider 11-dimensional supergravity action. The bosonic part of the action is

$$S = \frac{1}{16\pi G_{11}} \int d^{11}x \sqrt{-g} \left( R - \frac{1}{2 \times 4!} F_4^2 \right),$$

(2.1)

where $F_4$ is a 4 form field, $F_4 = dC_3$.

The theory given by (2.1) contains a 2 dimensional and a 5 dimensional objects $M2$ and $M5$ brane. $M2$ branes are electrically charged and $M5$ branes are magnetically charged under $F_4$. For this matter field, $F_4$ energy-momentum tensor, $T_{AB}$ is given by

$$T_{AB} = \frac{1}{48} \left[ 4 F_{AMNP} F_{B}^{MNP} - \frac{1}{2} g_{AB} F_4^2 \right].$$

(2.2)

This theory contains black brane solution, which are solutions with solitonic objects. These black holes are actually made of stack of $M2$ or $M5$ branes or BPS intersecting combinations of them. With a suitable ansatz for metric and fields, Einstein equations can be solved to obtain black hole solutions.

To get the solutions, let the spacetime coordinates be $x^4 = (r, x^\alpha)$ where $x^\alpha = (x^0, x^i, \theta^a)$ with $x^0 = t, i = 1, \cdots, q, a = 1, \cdots, m,$ and $q + m = 9$. The $x^i$ directions may be taken to be toroidal, some or all of which are wrapped by branes, and $\theta^a$ are coordinates for an $m$ dimensional space of constant curvature given by $\epsilon = \pm 1$ or 0. The metric and brane fields depend only on $r$ coordinate, and defined by $r^2 = \sum_{q=0}^{q+m} (x^a)^2$. We write the line element $ds$, in an obvious notation, as

$$ds^2 = -\epsilon^{2\lambda} dt^2 + \sum_{i=1}^{q} \epsilon^{2\lambda_i} (dx^i)^2 + \epsilon^{2\sigma} dr^2 + \epsilon^{2\gamma} d\Omega_{m,\epsilon}^2.$$  

(2.3)

Black hole solutions are given by $\epsilon = +1$. But the analysis is true for any maximally symmetric non-compact space.

3
The independent non vanishing components of \( T_{AB} \) are given by \( T_{rr} = P_r \) and \( T_{\alpha\alpha} = P_{\alpha} \). These components can be calculated explicitly using the action \( S_{br} \). For example, for an electric \( p \)-brane along \((x^1, \cdots , x^p)\) directions, they are given by (see equation (2.2))

\[
P_0 = P_{\parallel} = -P_\perp = P_R = \frac{1}{4} F_{01\cdots pr} F^{01\cdots pr},
\]

where \( P_{\parallel} = P_i \) for \( i = 1, \cdots , p \), \( P_\perp = P_i \) for \( i = p+1, \cdots , q \), and note that \( P_R \) is negative. For mutually BPS \( N \) intersecting brane configurations, it turns out [31] – [40] that the respective energy momentum tensors \( T_{AB} \) and \( T_{AB}^{(I)} \) obey conservation equations separately.

\[
T_{AB} = \sum_I T_{AB}^{(I)} , \quad \sum_A \nabla_A T_{AB}^{(I)} = 0 .
\]

(2.5)

In case of configurations with non-BPS intersection, this doesn’t hold. We show this by an example in appendix.

Equations of motion may now be written as

\[
\Lambda_r^2 - \sum_\alpha (\Lambda^\alpha_r)^2 = 2P_R + \epsilon m(m - 1)e^{-2\sigma},
\]

(2.6)

\[
e^{-2\Lambda} [\Lambda^\alpha_r + (\Lambda_r - \lambda_r)\lambda^\alpha_r] = -P_\alpha + \frac{1}{9} \left( P_R + \sum_\beta P_\beta \right) + \epsilon (m - 1)e^{-2\sigma} \delta^{\alpha\alpha},
\]

(2.7)

\[
P_{Rr} + P_R \Lambda_r - \sum_\alpha P_\alpha \lambda^\alpha_r = 0,
\]

(2.8)

where \( \Lambda = \sum_\alpha \lambda^\alpha = \lambda^0 + \sum_i \lambda^i + m \sigma \) and the subscripts \( r \) denote \( r \)-derivatives.

2.1 Example

In this section we give example of black \( M2, M5 \) brane solutions and their intersecting solutions. We see that a relation like (4.17) exists.

\[M2\text{ Branes}\]

Consider a stack of \( M2 \) branes along \((x^1, x^2)\) directions. \( x^1 \) and \( x^2 \) are taken to be compact. \( x^3 \) and \( x^4 \) are also taken to be compact. In this case the solution is known, line element of this solution can be taken to be

\[
ds^2 = -e^{2\lambda^0(r)} dt^2 + \sum_{i=1}^4 e^{2\lambda^i(r)} (dx^i)^2 + e^{2\lambda^r(r)} (dr^2 + r^2 d\Omega_5^2).
\]

(2.9)

Ansatz for field, \( C_{MNP} \) is \( C_{012} = f(r) \) which gives \( F_{012r} = \frac{df(r)}{dr} \). This actually means \( M2 \) branes are electrically charged under the field \( F_4 \). Energy momentum tensor is given by

\[
T^{00} = T^{\parallel\parallel} = -T^{\perp\perp} = T^{rr} = -T^{\alpha\alpha},
\]

(2.10)

where \( \parallel \) and \( \perp \) indicate parallel and perpendicular to the brane directions and \( \alpha \) indicates directions in \( \Omega_5 \) respectively. One can see from equations (2.10) and (2.7) that the constraining relation among scale factor, mentioned before turns out to be

\[
\lambda^0 = \lambda^\parallel = -2\lambda^\perp.
\]

(2.11)
**BPS Intersection of 2 Sets of M2 Branes**

Now consider 2 sets of intersecting M2 branes along \((x^1, x^2)\) and \((x^3, x^4)\). We denote first set by 2 and second set by 2’. Black brane solution of intersecting branes was first identified in [11], then many solutions were quickly constructed and governing rules of their existence were studied. This intersecting configuration follows BPS rules. For these configurations our line element is

\[
ds^2 = -e^{2\lambda_0(r)} dt^2 + \sum_{i=1}^{4} e^{2\lambda_i(r)} (dx^i)^2 + e^{2\lambda(r)} (dr^2 + r^2 d\Omega_5^2) .
\]

Nevertheless, we have now two sets of electrically charged branes, so we have two non-zero components of gauge field, \(C_{012}(r)\) and \(C_{034}(r)\) and their cyclic permutations.

One can see easily from explicit expression of energy momentum tensors that, total energy momentum tensors are just the sum of individual brane configurations, \(T^A_B = \sum_I T^A_{B(I)}\). One can see that conservation equation is satisfied for total energy momentum tensor as well as individual energy momentum tensors. Also like in previous subsection a relation among scale factors comes out, namely,

\[
\begin{align*}
\lambda^1 &= \lambda^2, \\
\lambda^3 &= \lambda^4, \\
2\lambda^1 + 2\lambda^3 + \lambda^0 &= 0. 
\end{align*}
\]

\((2.12)\)

**M5 Branes**

In case of M5 brane, just like M2 brane case metric ansatz is taken of the same form, except now 5 of the 10 spacelike dimensions \((x^1, x^2, x^3, x^4, x^5)\) are compact and M5 branes wrap them. In general we may take some of the other directions are also compact. In that case they will be treated as directions perpendicular to branes and will be in same footing as directions of \(\Omega_4\). Metric ansatz is taken to be

\[
ds^2 = -e^{2\lambda^0(r)} dt^2 + \sum_{i=6}^{10} e^{2\lambda_i(r)} (dx^i)^2 + e^{2\lambda(r)} (dr^2 + r^2 d\Omega_5^2) ,
\]

where now \(r^2 = \sum_{i=6}^{10} (x^i)^2\). M5 branes are magnetically charged under the gauge field \(F_4\). \(C_{NPQ}\) is \(C_{NPQ} = \frac{1}{\prod_{i=1}^{5} f(r)} x^{I}\). So \(F_4\) takes non-zero value only when \(M, N, P, Q \in \Omega_4\) With this ansatz energy momentum tensor turns out to be

\[
T^0_0 = T_{||} = T^0_0 = -T^a_a = -\frac{1}{4},
\]

\((2.14)\)

where index \(||\) indicates parallel to brane directions and \(a\) indicates directions in \(\Omega_4\) respectively. Equations \((2.13)\) and equations of motion imply relations among scale factors. These equations are same as \((2.11)\).

\[
2\lambda^0 = 2\lambda^1 = -\lambda^4 .
\]

\((2.15)\)

**BPS Intersection of M2 Branes and M5 Branes**

In this subsection we give an example of BPS intersecting configuration of a stack of M2 branes, stretched along \((x^1, x^2)\) and that of M5 branes are stretched along \((x^1, x^3, x^4, x^5, x^6)\). As before all these \(x^1 \cdots x^6\) are compact, and the system is localised in common transverse space \(x^7 \cdots x^{10}\). Again ansatz for black brane metric is similar to previous cases. It is taken of the form

\[
ds^2 = -e^{2\lambda^0(r)} dt^2 + \sum_{i=1}^{6} e^{2\lambda_i(r)} (dx^i)^2 + e^{2\lambda(r)} (dr^2 + r^2 d\Omega_3^2) .
\]

\((2.16)\)
Here \( r^2 = \sum_{i=3}^{10} (x^i)^2 \). Under 4-form gauge field \( M2 \) branes are electrically charged and \( M5 \) branes are charged magnetically. Here non-zero gauge potential are \( C_{012}(r) \) and \( C_{NPQ}(r, x^M) \), where \( M, N, P, Q \in \Omega_3 \). The non-zero components of energy momentum tensor for this set of fields are \( T_{ij} \), \( i = 0, 1, \cdots, 6, T_{rr} \) and \( T_{aq}, a \in \Omega_3 \). Use of explicit expression of above components and equation of motion imply constraining relations, like before, among scale factors.

\[
\begin{align*}
\lambda^0 &= \lambda^1 \\
\lambda^3 &= \lambda^4 = \lambda^5 = \lambda^6 \\
\lambda^2 + 2\lambda^3 &= 0
\end{align*}
\] (2.17)

### 2.2 U Duality Relations In M Theory

We now describe the relations which follow from U duality symmetries, involving chains of dimensional reduction and uplifting and T and S dualities of string theory. These relations were found and used in case of cosmological solution previously. To explain the concept let us consider a solution of the form reduction and uplifting and T and S dualities of string theory. These relations were found and used in case of cosmological solution previously. To explain the concept let us consider a solution of the form

\[
ds^2_{11} = -e^{2\lambda^0} dt^2 + \sum_{\mu=1}^g e^{2\lambda^\mu} (dx^\mu)^2 + e^{2\lambda^2} dx^2 + e^{2\sigma} d\Omega^2_{m,e},
\] (2.18)

where we assume for \( \mu = i, j, k \), \( x^i \)'s are compact and Killing direction. That is \( \lambda^\mu = \lambda^\mu(t, X) \), where \( X \) includes space like coordinates except \( x^i, x^j \) and \( x^k \). Let \( \downarrow_k \) and \( \uparrow_k \) denote dimensional reduction and uplifting along \( k^{th} \) direction between M theory and type IIA string theory. To apply \( \downarrow_k \) on the metric given in (2.18) we write \( ds_{11} \) as

\[
ds_{11} = e^{-\frac{2}{3}\lambda^k} ds_{10}^2 + e^{\frac{4}{3}\phi} (dx^k)^2,
\] (2.19)

where \( ds_{10} \) is 10 dimensional line element of type IIA theory. Type IIA string theory metric is given by

\[
ds_{10}^2 = -e^{2\lambda^0} dt^2 + \sum_{\mu \neq k} e^{2\lambda^\mu} (dx^\mu)^2 + e^{2\lambda^j} dx^2 + e^{2\sigma} d\Omega^2_{m,e},
\] (2.20)

\( \phi \) is dilaton and is independent of \( x^i, x^j \) and \( x^k \). It is function of \( t, X \) only. If we integrate over \( x^k \) with above metric we will get type IIA super gravity action. Here comparing equations (2.18), (2.19) and (2.20) one can see \( \phi \) and \( \lambda^\mu \) are given by

\[
\begin{align*}
\phi &= \frac{3}{2} \lambda^k \\
\lambda^\mu &= \lambda^\mu + \frac{1}{3} \phi = \lambda^\mu + \frac{1}{2} \lambda^k \\
\lambda' &= \lambda + \frac{1}{2} \lambda^k \\
\sigma' &= \sigma + \frac{1}{2} \lambda^k
\end{align*}
\] (2.21)

In string theory, application of T duality along a compact direction converts type IIA theory to type IIB theory and back. It also converts a \( Dp \) brane to \( D(p - 1) \) or \( D(p + 1) \) branes depending on whether T duality is applied along brane or perpendicular to brane respectively. Applying this transformation generates new solution. We denote T duality operation along \( i^{th} \) direction by \( T_i \).

Applying a T duality along say, \( x^j \) which we denote by \( T_j \), generates a new solution, given by

\[
ds_{10}^2 = -e^{2\lambda^0} dt^2 + \sum_{\mu \neq \{j,k\}} e^{2\lambda^\mu} (dx^\mu)^2 + e^{-2\lambda^j} (dx^j)^2 + e^{2\lambda^j} dx^2 + e^{2\sigma} d\Omega^2_{m,e},
\]

\[
\phi' = \phi - \lambda^j = \lambda^k - \lambda^j,
\] (2.22)
where equation \(2.21\) has been used. Note that metric along \(x^i\), \(g_{ij}\) goes to \((g_{ij})^{-1}\). This solution is of type IIB theory. Again application of \(T_i\) generate a new solution of IIA theory.

\[
ds'^2_{10} = -e^{2\lambda^0} dt^2 + \sum_{\mu \neq \{1, j, k\}} e^{2\lambda^\mu} (dx^\mu)^2 + e^{-2\lambda^j} (dx^j)^2 + e^{-2\lambda^k} (dx^k)^2 + e^{2\sigma^r} dr^2 + e^{2\sigma^t} d\Omega^2_{m, \epsilon}
\]

\[
\phi'' = \phi - \lambda^i - \lambda^j = \frac{1}{2} \lambda^k - \lambda^i - \lambda^j.
\]

Dimensional upliftment to 11 dimensional theory can be done via

\[
ds'^2_{11} = e^{-\frac{2}{3} \phi''} ds'^2_{10} + e^{\frac{2}{3} \phi''} (dx^j)^2.
\]

Using equation \(2.23\) in \(2.24\) one gets

\[
ds'^2_{11} = -e^{2\lambda''} dt^2 + \sum_{\mu = 1}^9 e^{2\lambda''} (dx^\mu)^2 + e^{2\lambda''} dr^2 + e^{2\sigma''} d\Omega^2_{m, \epsilon},
\]

where these \(\lambda''\)'s are given in terms of \(\lambda\)'s by (using equation \(2.21\)),

\[
\lambda''_{ij} = \lambda^j - \frac{2}{3} (\lambda^i + \lambda^j + \lambda^k)
\]
\[
\lambda''_{ij} = \lambda^i - \frac{2}{3} (\lambda^i + \lambda^j + \lambda^k)
\]
\[
\lambda''_{ik} = \lambda^k - \frac{2}{3} (\lambda^i + \lambda^j + \lambda^k)
\]
\[
\lambda''_{kl} = \lambda^k + \frac{1}{3} (\lambda^i + \lambda^j + \lambda^k) \quad \forall \ t \neq \{i, j, k\}
\]

In general, simplifying notation, we can write, application of U duality operations \(\uparrow_k T_i T_j \downarrow_k\) in \(2.18\), transforms the \(\lambda\)'s in the scale factors to \(\lambda''\)'s given by

\[
\lambda''_{ij} = \lambda^j - 2\lambda, \quad \lambda''_{ij} = \lambda^i - 2\lambda, \quad \lambda''_{ik} = \lambda^k - 2\lambda
\]
\[
\lambda''_{ij} = \lambda^i + \lambda, \quad l \neq \{i, j, k\}, \quad \lambda = \frac{\lambda^i + \lambda^j + \lambda^k}{3}.
\]

### 2.3 Application of U duality relations in Black Holes

Note that, the U duality relations follow as long as the directions involved in the U duality operations are isometry directions. So the relations are valid for the geometry described by \(2.3\). We show now relations among \(\lambda\)'s follow from U duality relations. Consider a solution of \(M2\) brane along \((x^1, x^2)\). Also take \((x^3, x^4, x^5)\) as compact and are isometry direction. An obvious symmetry implies

\[
\lambda^1 = \lambda^2.
\]

It also implies

\[
\lambda^3 = \lambda^4 = \lambda^5.
\]

Directions \(x^\mu\) \((\in \{\Omega_4\) and \(r\)) are also transverse to brane directions. So we may assume

\[
\lambda^3 = \lambda^4 = \lambda^5 = \lambda^6 = \lambda^7 = \lambda^8 = \lambda^9 = \lambda^{10}.
\]

Now apply U duality operations \(\downarrow_5 T_3 T_4 \uparrow_5\). It transforms \(M2\) brane to \(M5\) brane,

\[
M2(12) \xrightarrow{T_5} D2(12) \xrightarrow{T_4} D3(124) \xrightarrow{T_3} D4(1234) \xrightarrow{T_5} M5(12345).
\]
This new solution is $M5$ branes may be given by

$$ds_{11}^2 = -e^{2\lambda_0} dt^2 + \sum_{i=1}^{10} e^{2\lambda_i} (dx^i)^2 + e^{2\lambda_0} dr^2 + e^{2\sigma_2} d\Omega_{m,c}^2 .$$

(2.31)

We can find $\lambda_i$’s using equations (2.24). There are obvious symmetry relation for $M5$ brane too.

$$\lambda^1 = \lambda^2 = \lambda^3 = \lambda^4 = \lambda^5 , \quad \lambda^6 = \lambda^7 = \lambda^8 = \lambda^9 = \lambda^{10} .$$

(2.32)

So now one can write the relations among $\lambda$’s as

$$\lambda^\parallel + 2\lambda^\perp = 0 , \quad 2\lambda^\parallel + \lambda^\perp = 0 .$$

(2.33)

where the superscripts $\parallel$ and $\perp$ denote spatial directions along and transverse to the branes respectively.

Note that, to find these relation we have use duality relations only. Explicit form of $\lambda^\alpha$ can only be known by solving equations of motion and putting proper boundary conditions, like asymptotic flatness.

For the extremal $2255^*$ configuration (12, 34, 13567, 24567) , the transverse space is three dimensional and U duality relation comes out, following above steps, to be

$$\lambda^1 + \lambda^4 + \lambda^5 = \lambda^2 + \lambda^3 + \lambda^5 = 0 .$$

(2.34)

Note that obvious symmetry relations for $2255^*$ black hole are

$$\lambda^5 = \lambda^6 = \lambda^7 , \quad \lambda^8 = \lambda^9 = \lambda^{10} .$$

(2.35)

One can verify from explicit solution that these relations are true. See [19] for detail.

We further illustrate the U duality method by interpreting a U duality relation $\sum_i c_i \lambda^i = 0$ as implying a relation among the components of the energy momentum tensor $T_{\alpha\beta}$. The relations thus obtained are indeed obeyed by the components of $T_{\alpha\beta}$ calculated explicitly.

Consider now the case of 2 branes or 5 branes. We assume that $P_0 = P_\parallel$ which is natural since $\theta^a$ directions are transverse to the branes. Applying the U duality relations in equation (2.33) then implies, for both 2 branes and 5 branes, the relation

$$P_\parallel = P_0 + P_\perp + P_R ,$$

(2.36)

among the components of their energy momentum tensors. See equations (2.10) and (2.11). Note that it is also natural to take $P_0 = P_\parallel$ since $x^0 = t$ is one of the worldvolume coordinates and may naturally be taken to be on the same footing as the other ones $(x^1, \cdots, x^9)$ . Equation (2.36) then implies that $P_\perp = -P_R$. The relation between $P_\parallel$ and $P_R$ is to be specified by an equation of state which is given in equations (2.10) and (2.11).

3 Exotic Branes

The formulation we showed in the previous section can also be applied for exotic branes. In this section we show this. In string theory/M theory exotic branes are always present. They are co-dimension 2 extended object, that is in string theory they are 7 dimensional object and in M theory they are 8 dimensional object. In string theory they are related to D-branes by S and T dualities. In other words if we T or S-dualise D-brane of type IIA or IIB theory we may end up in exotic branes. For example take a $D5(12345)$ branes of type IIB theory stretched along $x^1, x^2, x^3, x^4, x^5$. Again as before our coordinates are $x^0, x^1, \cdots, x^9, x^0$ being timelike. Lets take $x^6$ and $x^7$ are also compact. Now perform a duality operations $ST_6 T_7$. 1

$D5(12345) \xrightarrow{S} NS5(12345) \xrightarrow{T_6} KKM5(12345, 6) \xrightarrow{T_7} 5_2^*(12345, 67)$ .

1Here we follow a notation similar to [22]. That is $A_n^b$ means a $(A+b)$-brane, whose mass linearly depends on $A$ special dimensions, quadratically depends on $b$ special dimensions and subscript $n$ denotes branes mass is proportional to $g_s^n$. First set of numbers in the bracket indicate $A$ spacelike worldvolume direction, second set indicate where $T$ duality is performed, that is $b$ directions.
An S duality on D5 brane generates an NS5 brane. NS5 branes source an NS-NS 2-form field, $B_{\mu\nu}$. NS5 branes couple to $B_{\mu\nu}$ magnetically. So only non-vanishing components of $B$'s are $B_{\mu\nu}$ with $(\mu, \nu) = \{6, 7, 8, 9\}$. In case of solutions which are spherically symmetric in transverse space, in properly chosen gauge only non-vanishing $B$ is $B_{07}$. Now a T duality on $x^6$ generates Kaluza-Klein monopole. This T duality also generates a cross component in metric $g_{\mu\nu}$, but $B$-field becomes zero. Another T duality along $x^7$ generates exotic 5 brane $5^5_2$. Here again cross component of metric is zero but new metric may now depends on the angular coordinate, so that it becomes function of $r$ and $\theta$. Equations of motions becomes more complicated. So it is not obvious what will happen to the relations among various components of metric and energy momentum tensor.

3.1 General Exotic Branes in M Theory

In M theory $M2$ branes or $M5$ branes, while U duality, may end up in exotic branes. We will give an example latter. In such cases, black exotic branes metric have to be function of one angular direction. So line element of general exotic black brane solution can be written as

$$ds^2 = -e^{2\lambda_0(r,\theta)}dt^2 + \sum_{i=1}^{8} e^{2\lambda_i(r,\theta)}(dx^i)^2 + e^{2\sigma(r,\theta)}dr^2 + e^{2\sigma(r,\theta)}d\theta^2. \quad (3.1)$$

We take energy momentum tensor as

$$T^i_i = P_i \quad \forall i = 1, \cdots, 8,$$
$$T^r_r = P_R,$$
$$T^\theta_\theta = P_\theta. \quad (3.2)$$

One can see, there is a non-zero $r - \theta$ component of Einstein tensor and so $T_{r\theta}$ is also non-zero. We take

$$T^r_\theta = P_{R\theta}. \quad (3.3)$$

Note that scale factors are now function or $r$ and $\theta$, so it is natural to treat $r$ and $\theta$ directions separately. Therefore we define $\Lambda$ as $\Lambda = \sum_{i=0}^{8} \lambda_i$. Equations of motion are very similar to equations \[26\], \[28\] and \[31\] with obvious difference. Equations of motion for the ansatz \[31\] turns out to be

$$e^{-2\lambda} \left[ \Lambda_r \sigma_r + \frac{1}{2} \left( A_r^2 - \sum_{i} \lambda_i^2 \right) \right] + e^{-2\sigma} \left[ \Lambda_\theta \sigma_\theta - \Lambda_\theta \sigma_\theta + \frac{1}{2} \left( A_\theta^2 - \sum_{i} \lambda_i^2 \right) \right] = P_R, \quad (3.4)$$

$$e^{-2\lambda} \left[ \Lambda_r - \Lambda_\theta \right] + \frac{1}{2} \left( A_r^2 + \sum_{i} \lambda_i^2 \right) + e^{-2\sigma} \left[ \Lambda_\theta + \frac{1}{2} \left( A_\theta^2 - \sum_{i} \lambda_i^2 \right) \right] = P_\theta, \quad (3.5)$$

$$e^{-2\lambda} \left[ \Lambda_r + \Lambda_\theta \right] + \frac{1}{2} \left( A_r^2 + \sum_{i} \lambda_i^2 \right) + e^{-2\sigma} \left[ \Lambda_\theta + (\Lambda_\theta - \sigma_\theta + \sigma_\theta) \lambda_9 \right] = -P_1 + \frac{1}{2} \left( \sum_i P_i + P_R + P_\theta \right). \quad (3.6)$$

There are two more equations one can write, one for $r\theta$ component and one conservation equation. They are not independent. Solving these equations with proper boundary conditions one can get exotic brane solutions.

3.2 Exotic Branes of M Theory

We illustrate this idea in M theory by one explicit example, black brane solution for $5^3$. An explicit solution may be found in \[28\].

$$ds^2 = H^{-1/3}W^{2/3} \left( -dt^2 + \sum_{i=1}^{5} (dx^i)^2 \right) + H^{2/3}W^{-4/3} \sum_{i=6}^{8} (dx^i)^2 + H^{2/3}W^{2/3} \left( dr^2 + r^2 d\theta^2 \right), \quad (3.7)$$

where $H = \frac{h}{2} + b \ln \frac{R}{R_0}$ and $W = H^2 + b^2 \theta^2$. Here $b$ is the charge, and given in terms of number of branes and radii of $x_5$, $x_7$ and $x_8$ circle as $b = \frac{N_{R_0}}{2\pi l_s^2}$. $\mu$ is a cut off in energy scale which has to be present in any co-dimension 2 brane solutions. 3-form field is given by

$$C_{678} = \frac{b \theta}{W^2}. \quad 9$$
Note that field strength has now two components $F_{678\theta}$ and $F_{678r}$. So there is a non zero $r\theta$-component of energy momentum tensor. Note the non-geometric nature of the above solution. Metric and fields depend on $\theta$ in such a manner that they do not come back when angle goes form $\theta$ to $\theta + 2\pi$. That is metric and fields are not periodic functions of $\theta$.

We see from this explicit calculation that

$$T_{00} = T_{\parallel\parallel} = -T_{\perp\perp} = \frac{b^2}{4r^2 H^{8/3} W^{2/3}}.$$  \hfill (3.8)

Here $\parallel$ and $\perp$ indicate $x^1, \ldots, x^5$ and $x^6, x^7, x^8$ respectively, though strictly speaking $(x^6, x^7, x^8)$-directions are not transverse to the brane. This is an 8-brane in M theory. One also gets from explicit calculation that

$$T_{rr} = -T_{\theta\theta} = -\frac{b^2 (b^2 \theta^4 - 6b^2 \theta^2 H^2 + H^4)}{4r^2 H^{8/3} W^{14/3}}.$$  \hfill (3.9)

Now one need a equations of state which relates $T_{rr}$ and $T_{r\theta}$ to $T_{\perp\perp}$. It turns out these equations of states are dependent on $r$ and $\theta$ by a highly non-linear functions of them. These functions are monotonic functions of $\theta$, which is characteristic of non-geometry.

Equations (3.3), (3.9) and equation of motion (3.6) imply constraining relations among scale factor.

$$\lambda^0 = \lambda^\parallel$$

$$2\lambda^3 + \lambda^\perp = 0.$$  \hfill (3.10)

One can see that above relations are same as in $M5$ brane case. In the next section we show that same relations comes out from U-duality.

### 3.3 U duality Relations In Exotic Branes

We consider two examples, $5^3$ and $2^6$ to illustrate the idea.

#### $5^3$

Lets start with M5-branes, and apply an U duality of the form $\downarrow_1 T_6 ST_7 T_8 S \uparrow_1$. It transform M5 brane into one of the exotic brane of M-theory.

$$M5(12345) \downarrow_1 \rightarrow D4(2345) \rightarrow D5(23456) \rightarrow S \rightarrow NS5(23456)$$

$$5^3(12345, 678) \downarrow_1 \rightarrow 4^5_2 (2345, 678) \rightarrow 5^5_2 (23456, 78) \rightarrow S \rightarrow 5^2_3 (23456, 78)$$

General supergravity solution of black M-brane is given by equation (2.18). In case of black M5 brane we have an obvious symmetry

$$\lambda^1 = \lambda^2 = \lambda^3 = \lambda^4 = \lambda^5 = \lambda^\parallel$$

$$\lambda^6 = \lambda^7 = \lambda^8 = \lambda^9 = \lambda^{10} = \lambda^\perp.$$  \hfill (3.11)

Just like before (see equation (2.24)) doing a dimensional reduction along $x^1$ we find

$$\phi = \frac{3}{2} \lambda^1$$

$$\lambda^{\mu\nu} = \lambda^\mu + \frac{1}{3} \phi = \lambda^\mu + \frac{1}{2} \lambda^1.$$  \hfill (3.11)
that non-geometry is introduced.

\[
\begin{align*}
\phi &= \lambda^6 - \lambda^1 \\
\chi^{\mu
u} &= \left\{ \lambda^0 + \frac{\lambda^6}{2}, \lambda^2 + \frac{\lambda^6}{2}, \lambda^3 + \frac{\lambda^6}{2}, \lambda^4 + \frac{\lambda^6}{2}, \lambda^5 + \frac{\lambda^6}{2},
\end{align*}
\]

\[
-\lambda^1 - \frac{\lambda^6}{2}, \lambda^7, \frac{\lambda^6}{2}, \lambda^8, \frac{\lambda^6}{2}, \lambda^9, \frac{\lambda^6}{2}, \lambda^{10} \right\}.
\]

Last S duality also generates an NS-NS 2-form field. This 2-form field has to be spherically symmetric in transverse space \((x^9, x^{10})\). We write this transverse space metric as \(e^{2\lambda^0} \, dt^2 + e^{\lambda^0} r^2 \, dt^2\). Then in proper gauge 2-form field may have only \(B_{78}\) component, and can be taken as \(B_{78} = b(r) \theta \) in this black exotic brane case \(b(r)\) is constant. Now application of rest of the duality operations \(T_2 T_8 S T_6 \uparrow_5\) uplift the solution to 11-dimension again. This new 11 dimensional solution looks like

\[
ds_{11}^2 = -e^{2\lambda^0} \, dt^2 + \sum_{\mu=1}^{10} e^{2\lambda^\mu} (dx^\mu)^2.
\]

Here \(\chi^{\mu \nu}\)’s are given by

\[
\chi^{\mu} = \left\{ \lambda_0 + \frac{\ln(L)}{6}, \lambda_1 + \frac{\ln(L)}{6}, \lambda_2 + \frac{\ln(L)}{6}, \lambda_3 + \frac{\ln(L)}{6}, \lambda_4 + \frac{\ln(L)}{6}, \lambda_5 + \frac{\ln(L)}{6},
\end{align*}
\]

\[
\lambda_6 - \frac{\ln(L)}{3}, \lambda_7 - \frac{\ln(L)}{3}, \lambda_8 - \frac{\ln(L)}{3}, \lambda_9 + \frac{\ln(L)}{6}, \lambda_{10} + \frac{\ln(L)}{6} \right\}.
\]

where \(L = (b^2 \theta^2 + e^{2(\phi^0 + \lambda^7 + \lambda^5)})\). One can now use obvious symmetry of M5 brane and the equation (3.13) to get a relation among various \(\lambda^i\) for the case of exotic 8-brane, \(5^3\). One can see immediately the symmetries of \(5^3\) from expression of \(\chi^\mu\) of equation (3.14)

\[
\begin{align*}
\lambda_1 &= \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \frac{1}{6} \ln \left( b^2 \theta^2 e^{6\lambda_1} + e^{-6\lambda_1} \right) = \lambda^\parallel, \text{ (say)},
\end{align*}
\]

\[
\begin{align*}
\lambda_6 &= \lambda_7 = \lambda_8 = -\frac{1}{3} \ln \left( b^2 \theta^2 e^{6\lambda_1} + e^{-6\lambda_1} \right) = \lambda^\perp, \text{ (say)},
\end{align*}
\]

\[
\lambda_9 &= \lambda_{10} = \frac{1}{6} \ln \left( b^2 \theta^2 e^{3\lambda_1} + e^{-9\lambda_1} \right) = \lambda^\circ, \text{ (say)}.
\]

Now we can see U-duality implies a relation between \(\lambda^\parallel\) and \(\lambda^\perp\).

\[
\begin{align*}
\lambda^\perp + 2\lambda^\parallel &= 0.
\end{align*}
\]

2\(^6\)

Another exotic brane in M theory is 2\(^6\). Again we start with an \(M5\)-brane, One obtains a 2\(^6\) (12, 345678) brane by applying \(\downarrow_1 T_6 S T_6 S T_6 T_5 T_4 T_3 S \uparrow_1\) on an \(M5\) (12345). As in the case of \(5^3\) brane we start black \(M5\)-brane metric as in equation (2.15) and metric along \((x^9, x^{10})\) as \(e^{2\lambda^0} \, dt^2 + e^{\lambda^0} r^2 \, d\theta^2\). Application of said duality transformations on \(M5\) brane metric we get

\[
ds_{11}^2 = -e^{2\lambda^0} \, dt^2 + \sum_{\mu=1}^{10} e^{2\lambda^\mu} (dx^\mu)^2.
\]

\(^2\)In general for spherically symmetric case \(B_{78} = e^{02345678} r b(r) e^{\mu}\). So field strength is only \(r\)-dependent and has component \(F_{78}\). However one T duality converts \(B_{78}\) in to metric components \(g_{78}\) and one more T duality converts \(g_{78}\) to \(B_{78}\) which is now function of both \(r\) and \(\theta\). So field has components \(F_{78\theta}\) and \(F_{78r}\). They are also function of \(r\) and \(\theta\), so that non-geometry is introduced.
with $\lambda^i$’s are given by

$$
\begin{align*}
\lambda^1 &= \lambda^2 = \frac{1}{3} \ln \left( b^2 \theta^2 e^{6\lambda^i} + e^{-6\lambda^i} \right) = \lambda^||, \quad \text{(say)}, \\
\lambda^3 &= \lambda^4 = \lambda^5 = \lambda^6 = \lambda^7 = \lambda^8 = -\frac{1}{6} \ln \left( b^2 \theta^2 e^{6\lambda^i} + e^{-6\lambda^i} \right) = \lambda^\perp, \quad \text{(say)}, \\
\lambda^9 &= \lambda^{10} = \frac{1}{3} \ln \left( b^2 \theta^2 e^{-3\lambda^3} + e^{-15\lambda^3} \right) = \lambda^\circ, \quad \text{(say)}.
\end{align*}
$$

(3.18)

In deriving above equations we use again obvious symmetry of $M5$ branes and equation (2.33). The relation between $\lambda^||$ and $\lambda^\perp$ turns out to be

$$
2\lambda^\perp + \lambda^|| = 0 \quad \text{(3.19)}
$$

Using equation of motion (3.20) and the above relations among $\lambda$’s gives a relation similar to (2.30)

$$
P_\parallel = P_0 + P_{b\theta} + P_R \quad \text{(3.20)}
$$

But here we know $P_{b\theta} \neq P^\perp$. So equation (3.20) is not of much use. Nevertheless when explicit fields are unknown like in the cosmological case this might turn out to be a very useful relation.

4 Black Branes in String Theory and Dualities

4.1 General Black Branes in String Theory

In this subsection we will show that a similar relation exists in the string theory black brane solutions. In the next subsection we will show just like M-theory case, for certain supergravity solutions $S$ and $T$ duality can be used to get relations among various metric components and dilaton, and hence relations among energy-momentum tensors. To illustrate this, we consider a general solution of string theory (type II supergravity). Line element $ds_{DP}$ in Einstein frame is

$$
ds^2_{DP} = -e^{2\lambda^p} Z dt^2 + \sum_{i=1}^{p} e^{2\lambda^i} (dx^i)^2 + \sum_{i=p+1}^{q} e^{2\lambda^i} (dx^i)^2 + e^{2\lambda^p} (dx^p)^2 + e^{2\sigma} d\Omega^2_{m,e}
$$

(4.1)

and dilaton, $\phi_p = \phi_p(t, r)$. Here we assume $x^1, \cdots, x^q$ are compact and are isometry direction, and $i = 1, \cdots, p$ are directions parallel to $Dp$-branes. Obvious symmetries ensure us to take all $\lambda^i$ parallel to branes are equal and we denote them as before by $\lambda^i_p$, similarly for $\lambda^\perp_p$. Here all $\lambda$s, $Z$ and $\sigma$ are function of $r$. $d\Omega_{m,e}$ is metric of constant curvature $m$-dimensional space.

Bosonic part of the supergravity action for type II supergravity in Einstein frame is

$$
S = \frac{1}{16\pi G_{10}} \int d^{10} x \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \sum_{p} \frac{e^{\phi_p}}{2(p+2)!} F^2_{p+2} \right),
$$

(4.2)

where $F_{p+2} = dA_{p+1}$ is $p + 2$-form field strength coupled to $p$-brane. $T_{AB}$ is given by

$$
T_{AB} = \frac{1}{2} \partial_A \phi \partial_B \phi - \frac{1}{4} g_{AB} (\partial \phi)^2 + \sum_{p} \frac{e^{\phi_p}}{2(p+2)!} \left[ (p+2) F_{AM_1 \cdots M_{p+1}} F_B^{M_1 \cdots M_{p+1}} - \frac{1}{2} g_{AB} F^2_{p+2} \right].
$$

(4.3)

Here $a_p$ is a $p$ dependent factor, its value also depends on the type of brane. For $Dp$ brane, $a_p = \frac{4 - p}{p}$. We also have another “component”, $T_\phi$ coming from $\phi$-variation of action

$$
\delta_\phi S = -\int d^{10} x \sqrt{-g} \left( \nabla^2 \phi + T_\phi \right) \delta \phi,
$$

(4.4)
where $\delta \phi$ denote variation with respect to $\phi$. Equation of motions are now Einstein equations together with
\[ \nabla^2 \phi = -T_\phi. \]  
(4.5)
Again non vanishing components of $T^A_B$ are given by $T^r_r = P_R$ and $T^\alpha_\alpha = P_\alpha$ where $\alpha = (0, i, a)$. These components can be written explicitly as
\[ P_0 = P_\parallel = -\frac{Z}{4} e^{-2\lambda} \phi^2 + \frac{e^{a\phi}}{4} F_{01...pr} F^{01...pr}, \]
\[ P_\perp = P_\alpha = -P_R = -\frac{Z}{4} e^{-2\lambda} \phi^2 - \frac{e^{a\phi}}{4} F_{01...pr} F^{01...pr}. \]

Similarly $T_\phi$ is given by
\[ T_\phi = \frac{a_p}{2} e^{a\phi} F_{01...pr} F^{01...pr}. \]

The equations of motion then may be written as
\[ \Lambda^2 - \sum_\alpha \left( \lambda_\alpha^2 \right)^2 = 2 P_R + \epsilon \ m (m - 1) e^{-2\sigma}, \]  
(4.6)
\[ Ze^{-2\lambda} \left[ \lambda_\alpha \tau + (\Lambda_r - \lambda_\tau) \lambda_\tau \right] = -P_\alpha + \frac{1}{8} \left( P_R + \sum_\beta P_\beta \right) + \epsilon (m - 1) e^{-2\sigma} \delta^{\alpha\alpha}, \]  
(4.7)
\[ Ze^{-2\lambda} \left[ \phi_{rr} + (\Lambda_r - \lambda_\tau) \phi_r \right] = -T_\phi, \]  
(4.8)
\[ P_{rr} + P_R \Lambda_r - \sum_\alpha P_\alpha \lambda_\alpha^2 = 0, \]  
(4.9)
where $\Lambda = \sum_\alpha \lambda_\alpha = \lambda^0 + \sum_i \lambda_i + m\sigma$ and the subscripts $r$ denote $r$-derivatives. In case of intersecting branes $P_\alpha = \sum_I P_{\alpha (I)}$, $P_R = \sum_I P_{R (I)}$ and equation (4.10) may be written as
\[ P_{R (I) r} + P_{R (I)} \Lambda_r - \sum_\alpha p_{\alpha (I)} \lambda_\alpha^2 = 0. \]  
(4.10)
This is because of, as we already claimed, $T^A_B$ obey conservation equation separately.

If we assume $P_{\alpha (I)} = -(1 - u^{\alpha}_I) P_{R (I)}$, then equation (4.10) can be solved. The solution is found to be
\[ P_{R (I)} = -e^{l_I - 2\Lambda}, \quad l_I = \sum_\alpha u^{I}_\alpha \lambda_\alpha^2 + l^I_0. \]  
(4.11)
Now we define the matrices $G_{\alpha\beta}$ and $G^{IJ}$ as
\[ G_{\alpha\beta} = 1 - \delta_{\alpha\beta}, \quad G^{IJ} = \sum_{\alpha,\beta} G_{\alpha\beta} u^{I}_\alpha u^{J}_\beta, \]  
(4.12)
where $G^{\alpha\beta}$ is the inverse of $G_{\alpha\beta}$ and is given by
\[ G^{\alpha\beta} = \frac{1}{8} - \delta^{\alpha\beta}. \]  
(4.13)
If we define a new coordinate $\tau$ by $d\tau = e^{-\Lambda + \lambda} \ dr$, equation (4.7) becomes
\[ e^{-2\lambda} \lambda_\alpha^2 = -P_\alpha + \frac{1}{8} \left( P_R + \sum_\beta P_\beta \right) + \epsilon (m - 1) e^{-2\sigma} \delta^{\alpha\alpha}. \]  
(4.14)
If one multiplies both side by \( u_I^\alpha \) and take a sum over \( \alpha \) and then use equation (4.11) in (4.14) one finds
\[
 l_{\tau \tau}^{I} = - \sum_J G^{IJ} e^{l_J} + \sum_{\alpha \in \Omega} u_I^\alpha \epsilon (m - 1) e^{2(\lambda - \sigma)} .
\] (4.15)
Specific values of \( G^{IJ} \) depend on intersecting configuration. In case of black holes we can calculate them from explicit calculation of energy momentum tensor or by using duality. In section 4.2 we discuss duality method.

If the components of energy momentum tensor follow a relation like
\[
 \sum_{\alpha} c_{\alpha} \left( - P_{\alpha} + \frac{1}{8} (P_{R} + \sum_{\beta} P_{\beta}) \right) = 0
\] (4.16)
then this immediately implies a relation among metric components \( \lambda^\alpha \) and \( \sigma \) from equation (4.14) or (4.14). We will see, for various brane solutions from explicit calculation and also using duality same type of relations come. For example, when \( \alpha \neq a \), then equations (4.14) and (4.16) implies
\[
 \sum_{\alpha} c_{\alpha}' \lambda^\alpha = 0 .
\] (4.17)
We will see some examples in the next section, which shows relation like (4.16) does follow. Also explicit computation shows there is a linear relation among \( P^\alpha \)'s and \( T_\phi \). They in tern implies
\[
 \sum_{\alpha} \tilde{c}_{\alpha} \lambda^\alpha + \tilde{c}_\phi \phi = 0 .
\] (4.18)
Explicit intersecting configurations give values of \( c_{\alpha}, c_{\alpha}', \tilde{c}_{\alpha} \) and \( \tilde{c}_\phi \).

**Example: Extremal D1 Branes**

Let us consider stack of D1 branes wrapped along \( x^1 \) direction and also take \( x^2, \cdots, x^5 \) are compact. The solution is well known. It can be written as
\[
 ds^2 = -e^{2\lambda^0} dt^2 + e^{2\lambda^\parallel} (dx^1)^2 + \sum_{i=2}^{5} e^{2\lambda^\perp} (dx^i)^2 + e^{2\sigma} dr^2 + e^{2\sigma} d\Omega_3^2
\] (4.19)
Dilaton is given by \( \phi(r) \) and non-vanishing components of RR form field are \( C_{01}(r) \). Explicit calculation shows that
\[
 T_0^0 = T^\parallel = \frac{5}{3} T^r = -\frac{5}{3} T^\perp = -\frac{5}{3} T^a .
\] (4.20)
Now equation (4.20) and equation (4.7) implies
\[
 \lambda^\parallel + 3\lambda^\perp = 0 .
\] (4.21)
Also from explicit calculation one can see
\[
 \phi - 4\lambda^\perp = 0 .
\] (4.22)

**Example: Extremal NS5 Branes**

Let us take another example of extremal black NS5-brane solution. This branes are wrapped around \( (x^1, \cdots, x^5) \). Also consider here \( x^6 \) is compact. The line element is given by
\[
 ds^2 = -e^{2\lambda^0} dt^2 + \sum_{i=1}^{5} e^{2\lambda^\parallel} (dx^i)^2 + e^{2\lambda^\perp} (dx^6)^2 + e^{2\lambda^r} dr^2 + e^{2\sigma} d\Omega_5^2 .
\] (4.23)
Again explicit calculation shows that
\[ T^0 = T^{\parallel} = \frac{5}{3} T^r = -\frac{5}{3} T^\bot = -\frac{5}{3} T^a. \] (4.24)

This and explicit calculation with dilaton implies
\begin{align*}
3\lambda^{\parallel} + \lambda^\bot &= 0 \quad (4.25) \\
3\phi - 4\lambda^\bot &= 0 \quad (4.26)
\end{align*}

**Example: BPS Intersection of two sets of D3 branes**

Let’s take two sets of D3 branes along \((x^1, x^2, x^3)\) and \((x^1, x^4, x^5)\). This configuration is BPS configuration. We denote these two sets of D3 branes as \(D3a\) and \(D3b\), and their charges as \(h_a\) and \(h_b\). Let’s also take \(x^6\) direction as compact. The black hole solution for this configuration is given by
\[ ds^2 = -e^{2\lambda_0^p} dt^2 + \sum_{i=1}^{5} e^{2\lambda_i^p} (dx^i)^2 + e^{2\sigma^p} dr^2 + e^{2\sigma^p} d\Omega_3^m, \quad (4.27) \]

Again it is clear from detail of the solution
\[ T^A_{\ B} = \sum_I T^A_{\ B(I)}, \quad (4.28) \]

where \(I\) indicates \(D3a\) and \(D3b\) branes and and conservation equations holds separately for each \(I\). The relation among metric components becomes now
\[ \lambda^1 + \lambda^4 = 0, \quad (4.29) \]

together with obvious symmetry
\begin{align*}
\lambda^1 &= \lambda^2 \\
\lambda^3 &= \lambda^4. \quad (4.30)
\end{align*}

### 4.2 S and T Duality Relations

Here we show above mentioned relations follows form S and T dualities of string theory. To illustrate this, we consider an extremal black \(p\)-brane solution and this is of the form given in equation (4.11). The subscript \(Dp\) indicate metric is for \(Dp\)-branes. This metric is a of general black \(p\)-brane solution. \(q\) is the total number of compact directions, and \(q + m = 8\). This system physically describes geometry created by \(D\)-brane localised in space. In the following analysis \(\lambda\)'s can be function of \(r\) as well as \(t\). So the relations we find here can be used for black holes as well as cosmological solutions.

This solution is in Einstein frame. To apply duality rules we first convert it in string frame. String frame metric is
\[ ds_{Dp}^2 = e^{2\lambda_0^p + \sigma^p} Z dt^2 + \sum_{i=1}^{p} e^{2\lambda_i^p + \sigma^p} (dx^i)^2 + \sum_{i=p+1}^{q} e^{2\lambda_i^p + \sigma^p} (dx^i)^2 \\
+ e^{2\lambda + \sigma^p} dr^2 + e^{2\sigma^p} d\Omega_3^m, \quad (4.31) \]

\(^aA\) cosmological model similar to [16] – [19] can be made in the framework of string theory.
Now if we perform a T-duality along $p^{th}$ direction we get $D(p-1)$ branes solution.

$$\begin{align*}
 ds^2_{s,Dp-1} &= -e^{2\lambda^\|_{p-1}} Z dt^2 + \sum_{i=1}^{p-1} e^{2\lambda^\|_{p-1}} (dx^i)^2 + e^{-2\lambda^\|_{p-1}} (dx^p)^2 \\
 &+ \sum_{i=p+1}^q e^{2\lambda^\|_{p-1}} (dx^i)^2 + e^{2\lambda^\|_{p-1}} dr^2 + e^{2\sigma + \phi_p} d\Omega^2_m.
\end{align*}$$

(4.32)

$$\begin{align*}
 \phi_{p-1} &= \phi_p - \frac{1}{2} \ln\left( e^{2\lambda^\|_{p-1}} + \frac{\phi_p}{8} \right) = \frac{3\phi_p}{4} - \lambda^\|_p.
\end{align*}$$

(4.33)

The Einstein frame metric for $D(2p-1)$ solution can be found by multiplying $e^{-\phi_{p-1}/2}$ to $ds^2_{s,Dp-1}$.

$$\begin{align*}
 ds^2_{Dp-1} &= -e^{\phi_{p-1}/2} ds^2_{s,Dp-1} \\
 &= -e^{2\lambda^\|_{p-1}} Z dt^2 + \sum_{i=1}^{p-1} e^{2\lambda^\|_{p-1}} (dx^i)^2 + e^{-2\lambda^\|_{p-1}} (dx^p)^2 \\
 &+ \sum_{i=p+1}^q e^{2\lambda^\|_{p-1}} (dx^i)^2 + ds^2_2,
\end{align*}$$

(4.34)

where $ds^2_2$ is metric for transverse space. Equations (4.32) and (4.33) have been used to get above expression. This line element can be written as

$$\begin{align*}
 ds^2_{Dp-1} &= -e^{2\lambda^\|_{p-1}} Z dt^2 + \sum_{i=1}^{p-1} e^{2\lambda^\|_{p-1}} (dx^i)^2 + \sum_{i=p+1}^q e^{2\lambda^\|_{p-1}} (dx^i)^2 + ds^2_2,
\end{align*}$$

(4.35)

where $\lambda_p$ can be given in terms of $\lambda_{p-1}$ by

$$\begin{align*}
 2\lambda^\|_{p-1} &= \frac{5\lambda^\|_p}{2} + \frac{\phi_p}{8} \\
 2\lambda^\perp_{p-1} &= -\frac{3}{2} \lambda^\|_p + \frac{7}{8} \phi_p = 2\lambda^\perp_p + \frac{\lambda^\|_p}{2} + \frac{\phi_p}{8}
\end{align*}$$

(4.36)

and

$$\begin{align*}
 \phi_{p-1} &= \frac{3\phi_p}{2} - \lambda^\|_p.
\end{align*}$$

(4.37)

Simplification of equation (4.37) shows that,

$$\begin{align*}
 \lambda^\perp_p + \lambda^\|_p + \frac{\phi_p}{2} &= 0.
\end{align*}$$

(4.38)

Now consider a $D3$ brane, S-duality of $D3$ brane gives $D3$ brane and so $\phi_3 = -\phi_3$, which implies $\phi_3 = 0$. Consider $D2$ brane solution now, $\phi_3 = -\lambda^\|_3$, (using equation (4.38)). Equation (4.39) gives $\lambda^\|_3 = -\lambda^\perp_3$. Denoting $\lambda^\perp_3$ simply by $\lambda^\perp$, one finds

$$\begin{align*}
 \lambda^\perp_3 = \lambda^\perp, \quad \lambda^\|_3 = -\lambda^\perp, \quad \phi_3 = 0 \times \lambda^\perp.
\end{align*}$$

(4.40)

In general using induction, it is easy to show that,

$$\begin{align*}
 \lambda^\perp_p = \frac{p+1}{4} \lambda^\perp, \quad \lambda^\|_p = -\frac{7-p}{4} \lambda^\perp, \quad \phi_p = (3-p) \lambda^\perp,
\end{align*}$$

(4.41)

where $\lambda$ is now the only parameter determining full line element. If one uses different set of duality operations one can also find similar relations for F1-string or NS5-brane. In general

$$\begin{align*}
 \lambda^\perp_p = \frac{p+1}{4} \lambda^\perp, \quad \lambda^\|_p = -\frac{7-p}{4} \lambda^\perp, \quad \phi_p = z(3-p) \lambda^\perp,
\end{align*}$$

(4.42)

where $z = 1$ for $Dp$-brane and $z = -1$ for F1-string or NS5-brane.
Intersecting Branes

We use here above procedure to interesting branes system. This is generally true for any mutually BPS intersecting branes configuration. We show this for D3-D3 system for illustration purpose. Two mutually BPS D3 branes may intersect in one direction. So lets first stack of D3 branes are wrapped around \((x^1, x^2, x^3)\) directions and second stack are wrapped around \((x^1, x^4, x^5)\). So in general we can write this black hole as

\[
ds^2 = -e^{2\lambda^0} dt^2 + \sum_{i=1}^{9} e^{2\lambda^i} (dx^i)^2 .
\]  

(4.43)

Here we have an obvious symmetry

\[
\lambda^2 = \lambda^3 = \lambda^{D3a} \text{ (say)} \\
\lambda^4 = \lambda^5 = \lambda^{D3b} \text{ (say)} .
\]

(4.44)

One S-duality transform dilaton \(\phi \rightarrow -\phi\) but D3-D3 black hole remains same. This implies \(\phi = 0\). Now apply two T-duality operations, \(T_2T_3\). This is a D1-D5 black hole with metric

\[
ds^2 = -e^{2\lambda^0} dt^2 + e^{2\lambda^1}(dx^1)^2 + \sum_{i=2,3} e^{-2\lambda^i}(dx^i)^2 + \sum_{i=4}^{9} e^{2\lambda^i}(dx^i)^2 ,
\]

(4.45)

and dilaton

\[
\phi = -\lambda_2 - \lambda_3 .
\]

(4.46)

Writing above solution in Einstein frame and using obvious symmetry [4.44] we find

\[
ds^2 = -e^{2\lambda^0} dt^2 + \sum_{i=1}^{9} e^{2\lambda^i}(dx^i)^2 ,
\]

(4.47)

where \(\lambda^i\)'s are given by

\[
\lambda^\mu = \left\{ \frac{\lambda^{D3a}}{2} + \lambda^0, \frac{\lambda^{D3a}}{2} + \lambda^1, -\frac{\lambda^{D3a}}{2}, -\frac{\lambda^{D3a}}{2}, \frac{\lambda^{D3a}}{2} + \lambda^{D3b}, \frac{\lambda^{D3a}}{2} + \lambda^{D3b}, \lambda^6, \lambda^7, \lambda^8, \lambda^9 \right\}
\]

(4.48)

Obvious symmetry of D1-D5 black hole implies

\[
\lambda^2 = \lambda^3 = \lambda^4 = \lambda^5 ,
\]

(4.49)

and therefore

\[
\lambda^{D3a} = -\lambda^{D3b} .
\]

(4.50)

This is the constrain one gets for D3-D3 solution which can be verified from the explicit solution. These conclusion can be made just from duality and not using any explicit form of the fields.

Relation among \(\mathbf{P}'s\)

Like in M theory case duality method can be used to interpret S and T duality relation \(\sum_i c_i \lambda^i = 0\) as implying a relation among the components of the energy momentum tensor \(T_{\mathbf{A}\mathbf{B}}\). Using equation (4.42) in equation (4.7) one finds

\[
\frac{p+1}{7-p} = \frac{1}{8} (\mathbf{P}_R + \sum \mathbf{P}_{\beta}) - \mathbf{P}_\perp
\]

(4.51)

which on simplification gives

\[
P_\parallel + (7-q) P_\perp = P_0 + P_R + m P_a .
\]

(4.52)
Similarly use of equation (4.42) in equation (4.8) gives

\[ T_\phi = -\frac{3}{2} p_z (P_0 + P_R + m P_a) - (8 - q) P_\perp \]  

(4.53)

\( \theta^a \) directions are also transverse to brane, so one would expect \( P_\perp = P_a \). Again it is also natural to take \( P_0 = P_\parallel \) since \( x^0 = t \) is one of the worldvolume coordinates and may naturally be taken to be on the same footing as the other ones (\( x^1, \cdots, x^p \)). Equation (4.52) then implies that \( P_\perp = -P_R \). The relation between \( P_\parallel \) and \( P_R \) is to be specified by an equation of state.

For now, however, we take \( P_0 \) and \( P_\parallel \) to be different. we assume the equations of state to be of the form

\[ p_\alpha(I) = - (1 - u_{\alpha}^I) P_R(I) \]

where \( \alpha = (0, i, a) \), \( I = 1, \cdots, N \), and \( u_{\alpha}^I \) are constants. Here we give explicitly \( u_\alpha \)'s for \( D1 \) and \( D5 \) black brane solution.

\[ D1 : u_\alpha = ( u_0, u_\parallel, u_\perp, u_\perp, u_\perp, u_\perp, u_\perp, u_\perp, u_\perp) \]

\[ D5 : u_\alpha = ( u_0, u_\parallel, u_\parallel, u_\parallel, u_\parallel, u_\parallel, u_\perp, u_\perp, u_\perp) \].  

(4.54)

Note here that, \( u_\parallel = u_0 + u_\perp \) which follows from equation (4.52).

Using definition of \( G^{IJ}, l^I \) and \( \tau \), equation (4.13) now becomes

\[ l^I_{\tau \tau} = - \sum_J G^{IJ} e^{l^J} + u_\perp \epsilon \ m(m - 1) \ e^{2(\Lambda - \sigma)} \]  

(4.55)

Since now we know \( u_\alpha \)'s, using equations (4.54) and (4.12), it is now straightforward to calculate \( G^{IJ} \) for \( N \) intersecting brane configurations. Similarly using (4.53) and equation of motion (4.8) dilaton can be determined.  

5 Conclusion

In this paper we discussed general black brane solutions and effect on U-duality on them. We show that U-duality constrains our metric and hence energy momentum tensor. We argued that if explicit form of the fields are unknown, these constrains help us to get a solution. In some previous works this technique was used to get cosmological solution.

We show constraining relations exist in string theory also. We claim that this may becomes useful if we make an early universe model whose matters are \( Dp \) branes, \( NS5 \) branes and fundamental strings. In string theory interactions of branes and string are more tractable than its M theory counterpart. Though, here, problem remains difficult because of dilaton. We see that dilaton is also constructed by duality. This constrain dictate in very high energy density theory becomes strongly coupled.

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A Non-BPS Intersection of Branes

Here we give an example to show for non-BPS intersecting branes energy momentum tensors are not sum of individual brane. We will consider an almost similar configuration of subsection 2.1 except now our configuration is non-BPS. In this case lets take two sets of 2 branes along \( (x^1, x^2) \) and \( (x^2, x^3) \). It is a

\[ \text{Note here that, in black brane case these equations of states are known, but in some more general case they may not be known. These duality relations, on the other hand, are completely general and may help to get equations of states.} \]
non-BPS intersection of branes. We take \( x^1, x^2 \) and \( x^3 \) as compact. Both set are electrically charged. So the 3-form potential in this case are \( C_{012} = f_2(r) \) and \( C_{023} = f_2(r) \). So corresponding fields are

\[
\begin{align*}
F_{012r} &= f'_2(r) = E_2(r) \\
F_{023r} &= f'_2(r) = E_2(r) .
\end{align*}
\]  

(A.1)

For metric we may start with an ansatz like (2.16), but it turns out that this ansatz is inconsistent. The reason is explained below. Because of the first term in the expression of energy momentum tensor (equations (2.16)), \( T_{13} \) is non-zero,

\[
T_{13} = \frac{1}{12} g^{02} g^{22} g^{rr} F_{102r} F_{023r} \times 3 ! .
\]  

(A.2)

But since the metric is diagonal, \( (R_{13} - \frac{1}{2} g_{13} R) \) is zero. So obviously Einstein equations are not satisfied. Therefore one has to take a different ansatz, simplest one is diagonal metric with only \( g_{13} \) non-zero. That is,

\[
ds^2 = -e^{2\lambda(r)} dt^2 + \sum_{i=1}^3 e^{2\lambda_i(r)} (dx^i)^2 + 2 e^{2\lambda} dx^1 dx^3 + + e^{2\lambda(r)} \left( dr^2 + r^2 d\Omega_6^2 \right) .
\]  

(A.3)

With this ansatz it turns out non vanishing components of Einstein tensor are all diagonal components and \( G_{13} \). So now we can equate \( G_{MN} \) and \( T_{MN} \). So we will take the above line element as our ansatz.

We calculate here \( T^M N \). The non-zero components of them turn out to be

\[
\begin{align*}
T_{0}^0 &= \frac{1}{4} \left( g^{00} g^{11} g^{22} g^{rr} F_{012r} F_{012r} + g^{00} g^{22} g^{33} g^{rr} F_{023r} F_{023r} + g^{00} g^{13} g^{22} g^{rr} F_{012r} F_{032r} \right) \\
T_{1}^1 &= \frac{1}{4} \left( g^{00} g^{11} g^{22} g^{rr} F_{012r} F_{012r} - g^{00} g^{22} g^{33} g^{rr} F_{023r} F_{023r} + g^{00} g^{13} g^{22} g^{rr} F_{012r} F_{032r} \right) \\
T_{2}^2 &= \frac{1}{4} \left( g^{00} g^{11} g^{22} g^{rr} F_{012r} F_{012r} - g^{00} g^{22} g^{33} g^{rr} F_{023r} F_{023r} + g^{00} g^{13} g^{22} g^{rr} F_{012r} F_{032r} \right) \\
T_{3}^3 &= \frac{1}{4} \left( -g^{00} g^{11} g^{22} g^{rr} F_{012r} F_{012r} + g^{00} g^{22} g^{33} g^{rr} F_{023r} F_{023r} + g^{00} g^{13} g^{22} g^{rr} F_{012r} F_{032r} \right) \\
T_{r}^r &= \frac{1}{4} \left( g^{00} g^{11} g^{22} g^{rr} F_{012r} F_{012r} + g^{00} g^{22} g^{33} g^{rr} F_{023r} F_{023r} + g^{00} g^{13} g^{22} g^{rr} F_{012r} F_{032r} \right) \\
T_{a}^a &= -\frac{1}{4} \left( g^{00} g^{11} g^{22} g^{rr} F_{012r} F_{012r} + g^{00} g^{22} g^{33} g^{rr} F_{023r} F_{023r} + g^{00} g^{13} g^{22} g^{rr} F_{012r} F_{032r} \right) ,
\end{align*}
\]  

(A.4)

where index \( a \) denotes coordinates in \( \Omega_6 \). There is another component \( T_{13} \). Now because of \( T_{13} = g^{11} T_{13} + g^{13} T_{33} \) and \( T_{31} = g^{33} T_{13} + g^{31} T_{11} \), \( T_{13} \) and \( T_{31} \) are not symmetric. They turned out to be various combination of \( F_{012r} \) and \( F_{023r} \), and are non-zero. From above equations one can see clearly that, total energy momentum tensor is not just the sum of individual energy momentum tensors created by each sets of branes separately. For example in equations (A.1) first two terms in each equation give energy momentum tensor for individual brane configuration, but the third term is extra. Also \( T_{13} \) is a new component, which was not in case of single \( M2 \) brane system. So we may conclude that, for non-BPS intersection equation (2.16) is not satisfied.

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