Robust Transceiver Design for IRS-Assisted Cascaded MIMO Systems

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Abstract—Robust transceiver design against unresolvable system uncertainties is of crucial importance for reliable communication. We consider a MIMO two-hop system, where the source, the relay, and the destination are equipped with multiple antennas. Further, an intelligent reconfigurable surface (IRS) is established to cancel the residual self-interference (RSI) as much as possible. The optimization problem turns out to be non-convex and computationally extensive. We propose a new mathematical method to find a lower bound on the performance of the IRS. This method does not require performing any optimizations and provides an analytical bound that can be used as a benchmark. Further, we use this method to find achievable rates for the worst case RSI, IRS assisted full-duplex MIMO systems.

Index Terms—Intelligent reconfigurable surface (IRS), robust design, relay systems, MIMO.

I. INTRODUCTION

Two of the most important requirements for the next generation of wireless systems are throughput and reliability. In order to enhance the capacity or/and reliability of such networks, optimally relaying signal from a transmitter to a receiver is currently an active research area [1]. Furthermore, relaying is one of the best communication means in disaster scenarios where the direct source-destination link becomes unavailable. Relaying can be either in half-duplex (HD) or full-duplex (FD) mode, in which the relay can only send/receive signals at a moment or can do both, respectively. Further, there are most common relaying techniques namely, decode-and-forward (DF) and amplify-and-forward (AF) relaying. In the former, the signals are being regenerated at the relay while in the latter, the signals are only being amplified before being sent to the destination.

Another emerging candidate for relaying signals are reconfigurable intelligent surfaces (IRSs) [2]. IRS is a device equipped with multiple passive reconfigurable reflectors that can reflect the colliding waves with an adjustable phase. One of the biggest advantages of IRSs is that they work in a real time manner without consuming a noticeable amount of power [3]. However, the characteristics of the IRS (e.g., the lack of signal amplification and DF processes) can potentially limit its functionality. As a result, in cases where reliability and throughput are of greater importance than the power consumption, conventional relays might still be a better option than IRSs. For instance, authors in [4] showed that a simple FD relay can outperform an IRS in terms of throughput under certain conditions.

FD relaying can potentially be a better option than using an IRS only if the self-interference (SI) is negligible. Although, analog and/or digital signal processing at the relay input is utilized to cancel a portion of SI [5]–[8], the remaining portion, the so-called residual self-interference (RSI), is still present at the relay input. In [9], [10] the authors have investigated the distribution of the RSI. The authors in [11] studied the capacity of Gaussian two-hop FD relay in the presence of RSI. By exploiting a MIMO relay, the overall throughput rate from the source to destination can be improved [12], [13]. This also provides the feasibility of SI cancellation by beamforming techniques such that the impact of SI can be mitigated [14].

Robust transceiver design against the worst-case RSI channel is of great importance, as it guarantees that the performance cannot be worse than the provided bounds. This helps establish a very reliable system and adjust all the parameters of the network based on the obtained worst-case results. The authors in [15] have investigated a robust design for multi-user FD relaying with multi-antenna DF relay. In that work, the sources and destinations are equipped with single antennas. Moreover, the authors in [16] applied a robust transceiver design for FD multi-user MIMO systems for maximizing the weighted sum-rate of the network.

Contribution: In this work, a DF two-hop system with multiple antennas at the source, relay and destination is considered. Then we try to maximize the sum rate throughput for the worst case RSI scenario. To the best of our knowledge, this is the first time that the throughput rate maximization against the worst case RSI is evaluated for IRS assisted DF FD relay in MIMO systems. The optimization of maximum achievable rate of the DF FD relaying is cast as a non-convex optimization
II. System Model

In this paper, the communication from a source equipped with $N_s$ antennas to a destination with $N_d$ antennas is considered. It is assumed that the reliable communication is only feasible by means of a relay with $K_r$ transmitter and $K_r$ receiver antennas at the input and output frontends, respectively. An IRS consisting of $M$ elements is established to cancel the RSI. As the IRS is a passive device with potential attenuation, it is assumed that its impact in conveying signals from the source to the destination is negligible in comparison with that of the relay. Thus, the system model is given in Fig. 1. The received signals at the relay and destination are given by
\begin{align}
y_r &= H_1 x_s + (H_r + H_{rI} \Theta H_{rI}) x_r + n_r, \quad (1) \\
y_d &= H_2 x_r + n_d, \quad (2)
\end{align}
respectively. The transmit signal of the transmitter is represented by $x_s \in \mathbb{C}^{N_s}$ whose covariance matrix is $Q_s = \mathbb{E}[x_s x_s^H]$, and the transmit signal of the relay is denoted by $x_r \in \mathbb{C}^{K_r}$ whose covariance matrix is $Q_r = \mathbb{E}[x_r x_r^H]$. The additive noise vectors at the relay and destination are represented by $n_r \in \mathbb{C}^{K_r}$ and $n_d \in \mathbb{C}^{N_d}$, respectively. All noises are assumed to be zero-mean Gaussian distributions where their covariance matrices are $\sigma_r^2 I$ and $\sigma_d^2 I$ respectively, with $I$ being the identity matrix. The source-relay channel is denoted by $H_1 \in \mathbb{C}^{K_r \times N_s}$ and the relay-destination channel is represented by $H_2 \in \mathbb{C}^{N_d \times K_r}$. Similarly, the channels from the relay’s transmitter to the IRS and from the IRS to the relay’s receiver are denoted by $H_{rI} \in \mathbb{C}^{M \times K_r}$ and $H_{Ir} \in \mathbb{C}^{K_r \times M}$ respectively, see Fig. 1. $\Theta \in \mathbb{C}^{M \times M}$ is a diagonal matrix representing the IRS phase profile where for each element we have $|\theta_m| \leq 1$. Finally, the self-interference (SI) channel at the relay is represented by $H_r$, which is assumed to be known only imperfectly. In what follows, we present the achievable throughput rates for the abovementioned system. Notice that in this paper we assumed that the main duty of the IRS is to cancel the RSI. As a result we place the IRS in the vicinity of the relay and face it towards the relay. In such a case, the channels from the source to the IRS and from the IRS to the destination can be neglected as the IRS is far from both of them and also due to its angle, it cannot receive and reflect the signals from source and destination directions respectively.

III. Achievable Rate

A. Overview

We assume that the relay employs a DF strategy. As both source-relay and relay-destination links are active at the same time, the signals from the relay transmitter interfere with the source signals at the relay’s receiver. Even if an estimation of the self-interference (SI) channel $H_r$ is available at the relay denoted by $\hat{H}_r$, there is still some unknown channel estimation error (residual self-interference channel) represented by $\bar{H}_r$ which is given as
\begin{equation}
\bar{H}_r = H_r - \hat{H}_r. \quad (3)
\end{equation}
We assume that some portion of the SI is canceled based on the available estimate $\hat{H}_r$ and only a residual self-interference (RSI) remains. Here, we represent this portion by $\bar{H}_r x_r$.

B. Mathematical Preliminaries

Considering a FD DF relay, the following rates are achievable [17],
\begin{equation}
R^{FD} = \min(R_{sr}^{FD}, R_{rd}^{FD}), \quad (4)
\end{equation}
in which
\begin{align}
R_{sr}^{FD} &= \log_2 \left[ \frac{\sigma_r^2 I_{K_r} + H_1 Q_s H_s^H + H_{tot} Q_s H_{tot}^H}{\sigma_d^2 I_{K_r} + H_{tot} Q_s H_{tot}^H} \right], \quad (5) \\
R_{rd}^{FD} &= \log_2 \left[ I_N + \frac{1}{\sigma_d^2} H_2 Q_s H_2^H \right]. \quad (6)
\end{align}
where \( H_{tot} = (\bar{H}_r + H_{rI} \Theta H_{Ir}) \). Notice that, with perfect channel state information, the SI could be completely removed from the received signal at the relay input-frontend. However, assuming that the RSI remains uncanceled, a robust transceiver against the worst-case RSI channel is required which is formulated as an optimization problem as follows

\[
\begin{aligned}
\max_{Q_r, Q_s} \quad & \min_{H_r} \quad \min \left( P_{s,t}, P_{r,t} \right) \\
\text{subject to} \quad & \Tr(Q_s) \leq P_s, \quad \text{(7a)} \\
& \Tr(Q_r) \leq P_r, \quad \text{(7b)} \\
& \Tr(\bar{H}_r \bar{H}_r^H) \leq T_r, \quad \text{(7c)} \\
& |\theta_m| \leq 1, \forall m \quad \text{(7d)}
\end{aligned}
\]

in which the throughput rate with respect to the worst-case RSI channel is maximized. Two constraints \( P_s \) and \( P_r \) represent the transmit power budgets at the source and the relay respectively.

In constraint (7c), \( T_r \) represents the RSI channel uncertainty or channel estimation error bound corresponding to \( H_r \). Notice that, \( \Tr(\bar{H}_r \bar{H}_r^H) \) represents the sum of the squared singular values of \( H_r \). It should be noted that, using a bounded matrix norm is the most common way for modeling the uncertainty of a matrix [18], [19]. In practice, \( T_r \) can be found using stochastic methods when the distribution of the channel error is known. Otherwise, one may find it using sample average approximation method. Finally, constraints (7d) are due to the unit modulus limitation of the IRS elements.

The problem (7) is non-convex and hard to solve. As a result, for each of the above-mentioned scenarios, we propose a simplified version of the optimization problem and try to solve it instead. Note that as we are interested in finding the throughput corresponding to the worst case RSI, any simplification in the optimization problem should be in favor of the RSI and interference. In the next, we first analyse the performance of the system when the IRS is helping the relay to cancel the RSI. Consequently, the problem (7) can be simplified to the following optimization problem.

\[
\begin{aligned}
\max_{Q_r, Q_s} \quad & \min_{H_r} \quad \min \left( P_{s,t}, P_{r,t} \right) \\
\text{subject to} \quad & \Tr(Q_s) \leq P_s, \quad \text{(8a)} \\
& \Tr(Q_r) \leq P_r, \quad \text{(8b)} \\
& \Tr(\bar{H}_{tot} \bar{H}_{tot}^H) \leq T'(T, \Theta), \quad \text{(8c)}
\end{aligned}
\]

where

\[
\begin{aligned}
T' = \max_{\Theta} \quad & \min_{H_r} \quad \frac{\| \bar{H}_r + H_{rI} \Theta H_{Ir} \|_F^2}{P} \quad \text{(9a)} \\
\text{subject to} \quad & \Tr(H_r H_r^H) \leq T, \quad \text{(9b)}
\end{aligned}
\]

and where \( Vec(\cdot) \) denotes the vector of all non-zero elements of its input matrix. We can equivalently write \( T' \) as

\[
T' = \min_{\Theta} \quad \max_{H_r} \quad \frac{\| \bar{H}_r + (H_{Ir} * H_{Ir}^T)Vec(\Theta) \|_2^2}{2} \quad \text{(10a)}
\]

subject to

\[
\begin{aligned}
\Tr(\bar{H}_r \bar{H}_r^H) & \leq T_r, \quad \text{(10b)} \\
\| Vec(\Theta) \|_2^2 & \leq 1, \quad \text{(10c)}
\end{aligned}
\]

where * denotes column-wise Khatri-Rao product defined as below

\[
A * B = [A_1 \otimes B_1 | A_2 \otimes B_2 | \cdots | A_n \otimes B_n],
\]

and where \( A_i \) is the \( i \)’th column of \( A \) and \( \otimes \) denotes the Kronecker product. Further, one can show that \( T' \leq (\sqrt{T} - \sigma_{\min}(H_{Ir} * H_{Ir}^T))^2 \). As mentioned before, problem (8) is a simplification of the problem (7) where every achievable rate which is inside the feasible set of (8) is also inside the feasible set of (7). Notice that the constraint (9c) is weaker than the original constraint (7d) and it limits the feasible set of the IRS configurations, but as it is mentioned earlier, since this simplification is in favor of the RSI, the resulting bound will still be valid and considered as an achievable rate. The intuition behind that is, in problem (7) the minimization over RSI happens only one time, whereas in (8), first, we do one optimization to find the best \( \bar{H}_r \) against \( \Theta \). After that, another optimization is done to find the best response against \( Q_s \) and \( Q_r \). This makes it more difficult to reach higher throughput in comparison with (7).\(^1\) In the next, we provide the following theorem along with an example.

**Theorem 1.** For the optimization problem (10), one can show that \( T' \leq (\sqrt{T} - \sigma_{\min}(H_{Ir} * H_{Ir}^T))^2 \).

**Proof.** Due to the page limit, the mathematical proof is given in the long version. Here, we provide an intuitive example that shows the geometrical interpretation of the optimization problem. Assume that \( K_s = 1, K_r = 2 \) and \( M = 3 \). Then we have

\[
T' = \max_{\Theta} \quad \min_{H_r} \quad \frac{\| \bar{H}_r + (H_{Ir} * H_{Ir}^T)Vec(\Theta) \|_2^2}{2} \quad \text{(12a)}
\]

subject to

\[
\begin{aligned}
\bar{h}_{11}^2 + \bar{h}_{21}^2 & \leq T, \quad \text{(12b)} \\
\theta_1^2 & \leq 1, \quad \theta_2^2 \leq 1, \quad \theta_3^2 \leq 1. \quad \text{(12c)}
\end{aligned}
\]

\(^1\)The mathematical proof is given in the long version available at https://arxiv.org/abs/2208.02654.
Also, consider the following optimization problem

\[
T'' = \max_{\Theta} \min_{H_r} \|H_r + (H_{Ir} * H_{Ir}^T)Vec(\Theta)\|_2^2
\]

subject to

\[
\begin{align*}
\hat{h}_{11}^2 + \hat{h}_{21}^2 & \leq T, \\
\theta_1^2 + \theta_2^2 + \theta_3^2 & \leq 1,
\end{align*}
\]

(13a) (13b) (13c)

Here, notice that \((H_{Ir} * H_{Ir}^T)\) is a linear map from a three-dimensional into a two-dimensional space. One simple example of such a mapping can be found in Fig 2. Here, an example of mapping from three-dimensional to two-dimensional space is shown. The left shape shows the feasible set for the IRS with three elements in a real valued space. The cube belongs to the case of \(T''\), i.e. constraints \(-1 \leq \theta_m \leq 1, \forall m\), while the sphere shows the constraint \(\theta_1^2 + \theta_2^2 + \theta_3^2 \leq 1\) which belongs to \(T'\). In the right, the feasible sets belonging to two aforementioned cases after a mapping is performed. It can be seen that the first set of constraints (the hexagon) covers the whole area than the second one (the ellipse). Here, the important point is, as the mapping is linear, we have \(A \subset B \Rightarrow f(A) \subset f(B)\), where \(A\) and \(B\) are two arbitrary sets and \(f\) is the mapping.

Now we are ready to deal with the optimization problem. Fig 3 shows the overview of the optimization problem. Due to the constraint (13b), the feasible set of all possible choices of \(H_r\) creates a circle with radius \(\sqrt{T}\) centered at \((0,0)\). The parallelogram and the ellipse show the areas corresponding to the mapping of the cubic and the spherical feasible sets (12c) and (13c) imposed by the IRS constraints respectively. Notice that, the latter is always a subset of the former, i.e., \(S_{\text{ellipse}} \subset S_{\text{parallelogram}}\), where \(S\) represents the area of the set. Now, considering the max-min problem (12), we are able to have a geometrical interpretation of the solution. Note that (12) indicates the distance between \(H_r\) and \(-(H_{Ir} * H_{Ir}^T)Vec(\Theta)\).

While \(H_r\) wants to maximize this distance, the job of \(\Theta\) is to minimize it as much as possible. It can be seen in the figure that the best choice of RSI for \(H_r\) is the point that is farthest from the parallelogram. It is evident that this point is somewhere on the circumference of the outer circle. On the other hand, the best choice for the \(\Theta\) is to be chosen in a way to be as close as possible to \(H_r\). These points are indicated respectively as \(G\) and \(F\) on the figure. In general, as the number of IRS elements or the dimensions of \(H_r\) increase, the mapping of the hypercube becomes more and more complicated and finding the optimal distance becomes more difficult. However, there is an upper bound on this distance. As shown in the figure, if instead of the cube, we limit the feasible set of IRS elements to the sphere inside the cube, i.e. replacing (12c) with (13c), the solution to the problem becomes \(GE \geq GF\). It turns out that finding \(GE\) is very simple as by the definition we have \(\sigma_{\min}(H_{Ir} * H_{Ir}^T) = OE\), and also we know that \(\sqrt{T} = GO\). Therefore, we can conclude \(GE = \sqrt{T} - \sigma_{\min}(H_{Ir} * H_{Ir}^T)\). Finally, we use one last upper bound to make the original problem even easier to solve. Note that, if instead of the ellipse, we consider the circle inscribed in it, we will have \(\max_{H_r} \min_{\Theta} \|H_r + (H_{Ir} * H_{Ir}^T)Vec(\Theta)\|_2 = \sqrt{T} - \sigma_{\min}(H_{Ir} * H_{Ir}^T), \forall H_r\). As a result, we have

\[
\|H_{tot} H_{tot}^H\|_2 \leq T', \quad \forall H_r,
\]

(14)

where \(T' = \left(\sqrt{T} - \sigma_{\min}(H_{Ir} * H_{Ir}^T)\right)^2\).

Eventually, instead of optimization problem (7) we can solve optimization problem (8). The solution to the new problem is guaranteed to be achievable by the original problem as well. Notice that one can readily extend this interpretation into the complex domain, as the constraint (13c) with still be a subset of constraints (12c).

\[
\square
\]

Next, we provide a method to solve (8). Using the following theorem and lemma, we show that for every possible choice of \(H_1\) and \(H_2\), there exists at least one set of simultaneously diagonalizable matrices \(H_r, Q_s, \text{ and } Q_r\) that are the solutions to the problem (7).

![Fig 2: An example of mapping from three-dimensional to two-dimensional space.](image)

![Fig 3: Geometrical representation of optimization problem (12).](image)
Lemma 1. For two positive semi-definite and positive definite matrices $A$ and $B$ with eigenvalues $\lambda_1(A) \geq \lambda_2(A) \geq ... \geq \lambda_N(A)$ and $\lambda_1(B) \geq \lambda_2(B) \geq ... \geq \lambda_N(B)$ respectively, the following inequalities hold,

$$\prod_{i=1}^{N} \frac{1 + \lambda_i(A)}{\lambda_i(B)} \leq \left| I + AB^{-1} \right| \leq \prod_{i=1}^{N} \frac{1 + \lambda_i(A)}{\lambda_{N+1-i}(B)}.$$  \hspace{1cm} (15)

Proof. The proof is given in the long version. \qed

Using the above Lemma and the discussion in [20], we arrive at

$$\max_{\gamma} \min_{\sigma_r} \min \left( \sum_{i=1}^{\min(M, K_r)} \log \left( 1 + \frac{\sigma_i^2 \gamma_{s(i)}}{\sigma_r^2 + \gamma_{r(i)} \sigma_r^2} \right), \sum_{i=1}^{\min(K_1, N)} \log \left( 1 + \frac{\sigma_r^2}{\sigma_d^2 \gamma_{r(i)}} \right) \right),$$

subject to

$$\|\gamma_s\|_1 \leq P_s, \hspace{1cm} (16a)$$
$$\|\gamma_r\|_1 \leq P_r, \hspace{1cm} (16b)$$
$$\|\sigma_r^2\|_1 \leq T', \hspace{1cm} (16c)$$
$$\sigma_i^2 \gamma_{s(i)} \geq \sigma_i^2 \gamma_{s(i+1)} \gamma_{r(i+1)}, \forall i \leq \min(M, K_r), \hspace{1cm} (16d)$$
$$\gamma_{ri} \sigma_{r(i)}^2 \geq \gamma_{ri+1} \sigma_{r(i+1)}^2, \forall i \leq \min(K_1, N). \hspace{1cm} (16e)$$

where the following notions are used for the sake of simplicity,

$$\gamma_{si} = \lambda_i(Q_s), \hspace{1cm} (17)$$
$$\gamma_{ri} = \lambda_i(Q_r), \hspace{1cm} (18)$$
$$\sigma_1^2 = \lambda_i(H_1H_1^H), \hspace{1cm} (19)$$
$$\sigma_r^2 = \lambda_i(H_{tot}H_{tot}^H), \hspace{1cm} (20)$$
$$\sigma_2^2 = \lambda_i(H_2H_2^H). \hspace{1cm} (21)$$

Note that, the two additional constraints (16d) and (16e) need to be satisfied due to the conditions of Lemma 1 (i.e. eigenvalues have to be in decreasing order). Interestingly, these two additional constraints are affine. The above optimization problem can further be simplified using the following lemma.

Lemma 2. The objective function of the optimization problem (16) is optimized when the constraints (16a) and (16c) are satisfied with equality.

Proof. Intuitively, as the objective function is an increasing and decreasing function of each element of $\gamma_s$ and $\sigma_r^2$ respectively, at convergence, the constraints are met with equality. \qed

\footnote{For a full mathematical proof please refer to the long version, Appendix III.}

Algorithm 1 Robust Transceiver Design

1: Define $U = P_r$, $L = 0$, $\bar{P}_r^{(1)} = \frac{P_r}{2}$
2: while $|U - L|$ is large do
3: \hspace{1cm} Determine $\gamma_{s} = \|r_{r} - \frac{1}{\sigma_i^2}\|_1^+$, s.t. $\|\gamma_r\| = \bar{P}_r$
4: \hspace{1cm} Define $\sigma_r^{(0)} = 0$ and $\sigma_r^{(1)} = 1$ and $q = 0$
5: \hspace{1cm} while $\|\sigma_q^{(q)} - \sigma_q^{(q-1)}\|_1$ is large do
6: \hspace{2cm} Obtain $\sigma_r^{(q)}$, using water-filling
7: \hspace{2cm} Obtain $\gamma_{s}^{(q)}$, using Algorithm 2
8: \hspace{1cm} $q = q + 1$
9: \hspace{1cm} end while
10: \hspace{1cm} Calculate $R_{sa}$ and $R_{rd}$
11: \hspace{1cm} if $R_{sr} > R_{rd}$ then
12: \hspace{2cm} $U = \bar{P}_r$
13: \hspace{1cm} else if $R_{sa} < R_{rd}$ then
14: \hspace{2cm} $L = \bar{P}_r$
15: \hspace{1cm} end if
16: \hspace{1cm} $\bar{P}_r = \frac{U + L}{2}$
17: end while

Algorithm 2 The optimal $\gamma_s$

1: Find power allocation $P^0$ using water-filling algorithm
2: while $|P^q - P^{(q-1)}|$ is large do
3: \hspace{1cm} Define temp = 0
4: \hspace{1cm} for $i$ do
5: \hspace{2cm} Calculate $\text{cap}_i = \min_{1 \leq i' \leq i - 1} \{ \frac{\sigma_i^2 \gamma_{s(i')}}{\sigma_i^2} \}$
6: \hspace{2cm} if $P_i > \text{cap}_i$ then
7: \hspace{3cm} $P_i = \text{cap}_i$
8: \hspace{3cm} temp = temp + $P_i - \text{cap}_i$
9: \hspace{2cm} end if
10: \hspace{1cm} end for
11: $P = P + \frac{\text{num. of channels}}{\text{temp}}$
12: end while

C. Algorithm Description

Now, we need to solve the optimization problem (16). It can be readily shown that $R_{rd}^{\text{FD}}$ is a monotonically increasing function of $P_r$. Furthermore, one can show that $R_{rd}^{\text{FD}}$ is an increasing function w.r.t. $P_s$ and decreasing function w.r.t. $T$ and $P_r$. Consequently, the worst-case RSI chooses a strategy to reduce the spectral efficiency, while the relay and the source cope with such strategy for improving the system robustness. That means, on one hand the RSI hurts the stronger eigendirections of the received signal space more than the weaker ones. However, on the other hand the source tries to cope with this strategy adaptively by smart eigen selection. This process clearly makes the optimization problem complicated at the source-relay hop. Unlike the source-relay hop, the resource allocation problem
at the relay-receiver hop is rather easy. Since at the relay-receiver hop there is only one maximization and we can find the sum capacity simply by using the well-known water-filling algorithm.

\[ L = \min(M, K_r) \sum_{i=1}^{\min(M, K_r)} \log_2 \left( 1 + \frac{\sigma_i^2 \gamma_{\rho(i)}}{1 + \gamma_{\rho(i)} \sigma_{\rho(i)}^2} \right) + \lambda \left( \sum_{i=0}^{N} \sigma_i^2 - T \right) \]

(22)

Calculating \( \frac{\partial L}{\partial \sigma_i} = 0 \) we arrive at

\[ \sigma_i^2 = \left[ \frac{\sqrt{(\sigma_i^2 \gamma_{\rho(i)})^2 + 4 \sigma_i^2 \gamma_{\rho(i)} \sigma_{\rho(i)}^2}}{\lambda} - \sigma_i^2 \gamma_{\rho(i)} - 2 \right]^+ \]

where \( \lambda \) is the water level. To maximize the capacity, there are additional constraints \( \gamma_{\rho(i)} \sigma_{\rho(i)}^2 \geq \gamma_{\rho(i)} \sigma_{\rho(i)}^2 \) and must be considered during the minimization process. However, it can be shown that if the constraints \( \gamma_{\rho(i)} \geq \gamma_{\rho(i)} \) and \( \sigma_i^2 \gamma_{\rho(i)} \geq \sigma_{\rho(i)}^2 \gamma_{\rho(i)} \) are met, then the constraint \( \gamma_{\rho(i)} \sigma_{\rho(i)}^2 \geq \gamma_{\rho(i)} \sigma_{\rho(i)}^2 \) becomes redundant.

\[ \text{IV. Numerical Results} \]

We assume that the power budgets are equal at the source and at the relay \( P_s = P_r = 5000 \text{ mW} \). Moreover, the noise is assumed to be AWGN with the power spectral density \(-175 \text{ dBm}\), and with the bandwidth \( BW = 180 \text{ MHz} \) [21]. The full system characteristics are given in Table I. In this section, we investigate the performance of FD relaying with RSI channel uncertainty bound \( T \), i.e., \( \text{Tr}(\mathbf{H}, \mathbf{H}^H) \leq T \). We consider the column vectors of the source-relay and the relay-destination channel matrices to have both small scale fading and path loss effects. For the small scale fading, we consider a Rician distribution with the factor \( \varepsilon = 10 \), and, for the path loss we use \( PL = 32.6 + 36.7 \log_{10}(d) \) where \( d \) is the distance between the transmitter and the receiver [21]. We perform Monte-Carlo simulations with \( L = 10^3 \) realizations from random channels and noise vectors. Hence, the average worst-case throughput rate is defined as the average of worst-case rates for \( L \) randomizations, i.e., \( R_{aw} = \frac{1}{L} \sum_{i=1}^{L} R_i \). This means, for each set of realizations, i.e., \( \{ \mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_{\text{IR}}, \mathbf{H}_{\text{LR}}, \mathbf{n}_r, \mathbf{n}_d \} \), the robust transceiver design is solved as is elaborated in Algorithm I. We run different sets of simulations as described in the following. Fig. 4(a) depicts the convergence behavior of the algorithm. It shows that that more than 97 percent of the realizations will converge after less than 50 iterations. In Fig. 4(b), we evaluated the impact of IRS on the throughput for the case \( \{ N_t, K_t, K_r, N_r \} = \{ 4, 4, 4, 4 \} \). As shown in the figure, the number of IRS elements has a great impact on RSI cancellation to the extend that having an IRS with \( M = 90 \) or above, can almost completely cancel RSI, even if SI cancellation at the relay is so weak that 75 percent of the interference remains as RSI. Further, it can also be shown that that having IRS 16 or less elements is not helpful at all. This is mainly due to the fact that, unlike the average case, for the case of worst case scenario, the number of IRS elements should be at least as large as the dimension of \( \mathbf{H}_r \), otherwise, the IRS feasible set cannot span into all dimensions of \( \mathbf{H}_r \). Therefore, there is always at least one representation for \( \mathbf{H}_r \) in which IRS cannot do any RSI cancellation.

\[ \text{V. Conclusion} \]

In this paper, we investigated a multi-antenna source multi-antenna destination communication link through a multi-antenna decode-and-forward (DF) relay. An IRS is installed to help the relay cope with the RSI. Both the source and the relay’s transceivers are designed to be robust against the worst-case residual self-interference (RSI). The IRS pattern is designed in a way to cancel the RSI as much as possible. To this end, the worst-case achievable throughput rate is maximized. This optimization problem turns out to be a non-convex problem. Hence, the problem is simplified to two different optimization problems. In the first one, we evaluated the effect of IRS on the setup and we proposed a lower bound at which the IRS performance is guaranteed. In the second problem, we dealt with the optimal power allocation at the transmitters, which guarantees robustness against the worst-case RSI singular values. This simplified problem is still non-convex. Based on the intuitions for optimal power allocation at the source and relay, we proposed an efficient algorithm to capture a stationary point.

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\[ \text{3Please refer to the long version for the formal proof.} \]
Fig. 4: (a) Cumulative distribution function (cdf) of convergence in terms of the number of iterations. (b) The sum rate throughput of the system. The transmit power budget at the source and the relay are assumed to be equal, i.e., $P_s = P_t = P = 5000$ mW.

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