Research on Control Technology of Distributed Power Generation Virtual Synchronous Generator

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Abstract. Virtual synchronous generator control technology, as a method that enables distributed inverter power supplies to have similar characteristics to synchronous generators, has an important supporting role in improving the stability of the power system. Based on the rotation equation and electrical equation of the synchronous generator, this paper firstly constructed the virtual synchronous generator topology and mathematical model; then designed the corresponding virtual synchronous generator active-frequency droop control and reactive power-voltage droop control strategies, and through the frequency domain method, this paper analyzes the influence of key control parameters. Finally, the related control strategy is verified in the Matlab/Simulink simulation model. The results show that the proposed control strategy can seamlessly switch between off-grid and grid-connected modes, and has good frequency and voltage support capabilities.

1. Introduction
The basic ideas and concepts of Virtual Synchronous Generator (VSG) control were first proposed in the European VSYNC project [1], which refers to the simulation of the rotor motion equation and voltage regulation of the synchronous generator in the inverter control. Frequency modulation characteristics make it have the same steady-state droop characteristics and transient inertia as synchronous generators to improve the frequency and voltage support capability of the inverter [2-4].

After modeling the dynamic characteristics of the synchronous generator, this paper simulates the inverter as a synchronous generator, and proposes corresponding virtual synchronous generator active-frequency droop control and reactive power-voltage droop control strategies. Control, frequency control and voltage control are analyzed in the frequency domain, and the principle of optimal design of control parameters is given. Finally, using Matlab/Simulink to establish a virtual synchronous generator simulation model and conduct verification experiments, the simulation results verify the effectiveness of the relevant design.

2. Mathematical model of synchronous generator
The output voltage of the generator stator armature satisfies [5]

\[ v = -R_s i - L_s \frac{di}{dt} + e \]  

(1)

In the equation, \( v \) is the terminal voltage, \( e \) is the excitation induced electromotive force, and the remaining two correspond to the armature consumption voltage.

Excitation induced electromotive force \( e \) satisfies
\[ e = M_f i_t \dot{\theta} \sin \theta - M_f \frac{di_t}{dt} \cos \theta \]  

In the equation, \( M_f \) is the amplitude of mutual inductance between the rotor winding and the stator winding, and \( i_t \) is the excitation current of the rotor winding.

The mechanical part of the generator follows the Newton-Euler equation [6],

\[ J \ddot{\omega} = T_m - T_e - D_p \Delta \omega \]  

where \( J \) is the moment of inertia of all components rotating with the rotor, \( T_m \) is the mechanical torque, \( T_e \) is the electromagnetic torque, \( D_p \) is the damping coefficient mainly produced by the rotor damping winding, \( \omega = \dot{\theta} \) is the actual output electrical angular frequency of the generator, \( \Delta \omega = \dot{\theta} - \dot{\theta}_g \), \( \dot{\theta}_g \) is the angular frequency of the public access point.

### 3. Virtual synchronous generator topology and mathematical model

In this paper, VSG adopts a three-phase voltage source inverter structure. Its topology is shown in Figure 1, which mainly includes two parts, power and electronics. The power part is a typical three-phase AC inverter. The DC side of the inverter is supplied with DC voltage by the distributed power supply and energy storage unit, and after PWM modulation and \( LC \) filtering by the switch tube, the output three-phase sinusoidal voltage is connected to the public connection point or the grid via the grid-connected inductor \( L_g \). The electronic part is an electronic controller running in a microprocessor, which is used to control the power electronic switch of the power part. Its core is the virtual synchronous generator control algorithm and voltage and current closed-loop control, \( e_b = [e_a, e_b, e_c]^T \) is the three-phase bridge arm voltage, \( u_o = [u_{oa}, u_{ob}, u_{oc}]^T \) is the three-phase output voltage, \( i_b = [i_{lb}, i_{lc}, i_{lc}]^T \) is the three-phase bridge arm current, \( i_g = [i_{ga}, i_{gb}, i_{gc}]^T \) is the three-phase output current, and \( STS \) is a static transfer switch, which is triggered by the upper-level controller to control the operating mode of the virtual synchronous generator. Since the \( LC \) output filter will inevitably cause reactive power loss, we take \( u_{oa} \) and \( i_{gb} \) as the output of the virtual synchronous generator, instead of the bridge arm voltage \( e_b \) and the bridge arm current \( i_b \).

![Figure 1 Virtual synchronous generator control topology diagram](image-url)

The output power of the virtual synchronous generator can be described by the following formula,

\[
\begin{align*}
P_e &= \left( \frac{E u_o}{Z_o} \cos \delta - \frac{u_o^2}{Z_o} \right) \cos \theta + \frac{E u_o}{Z_o} \sin \delta \sin \theta \\
Q_e &= \left( \frac{E u_o}{Z_o} \cos \delta - \frac{u_o^2}{Z_o} \right) \sin \theta - \frac{E u_o}{Z_o} \sin \delta \cos \theta
\end{align*}
\]  

(4)
In the equation, \( E \) is the internal electromotive force of the virtual synchronous generator, \( \delta \) is the power angle, \( Z_o \) is the equivalent output impedance, and \( \theta \) is the output impedance argument. Equation (4) is mainly used for control analysis, and the power calculation of the output terminal can be conveniently completed in the \( dq \) coordinate system [6],

\[
\begin{align*}
P_e &= 1.5(u_{od}i_{gd} + u_{oq}i_{gq}) \\
Q_e &= 1.5(u_{od}i_{gd} - u_{oq}i_{gq})
\end{align*}
\]  

(5)

Where \([u_{od}, u_{oq}]^T\) is the component of \( u_o \) in the \( dq \) coordinate system, and \([i_{gd}, i_{gq}]^T\) is the component of \( i_g \) in the \( dq \) coordinate system.

To simulate the electrical equation of the synchronous generator, the following virtual synchronous machine voltage equation is established according to equation (1)

\[
u_r = -R_s i_r - L_s \frac{di_r}{dt} + e
\]

(6)

In the equation, \( R_s \) and \( L_s \) are the resistance and inductance components of the virtual armature impedance, \( u_r \) is the voltage reference value of the inner loop control, and \( e \) is the virtual excitation electromagnetic force. Assuming that the rotor winding of the virtual synchronous generator is powered by the variable DC current \( i_t \) instead of the voltage source \( u_r \), then

\[
e = M_i \dot{\theta} \sin \theta
\]

(7)

Considering \( P_m = T_m \omega / p \), where \( p \) is the number of stator pole pairs, assuming that the virtual synchronous generator runs near the rated angular frequency \( \omega_0 \), and \( \omega_q \) and \( \omega_g \) can be approximately equal, the virtual rotation equation of the virtual synchronous power generation machine can be obtained according to equation (3),

\[
\begin{align*}
\frac{d\Delta \omega}{dt} &= \frac{pP_m}{\omega_0} - \frac{P_e}{\omega_0} - D_p \Delta \omega \\
\frac{d\delta}{dt} &= \omega - \omega_0
\end{align*}
\]

(8)

Where \( \Delta \omega = \omega - \omega_0 \), \( P_m \) is the virtual input mechanical power, \( P_e \) is the virtual output electromagnetic power, and \( \delta \) is the power angle.

Combining equation (5), (6), (7) and (8), the core of the virtual synchronous generator model can be obtained. Therefore, the state variables of the virtual synchronous generator include the real current \( i_r \), the virtual angle \( \theta \) and the virtual angular velocity \( \dot{\theta} \), and the control inputs are \( P_m \) and \( M_i \dot{\theta} \). It should be pointed out that the \( \theta \) is the electrical angle of the virtual synchronous machine rotor, that is, the electrical angle of the virtual excitation electromagnetic force.

4. Virtual synchronous generator control strategy

4.1 Active frequency droop control

The rotor speed of the synchronous generator is adjusted by the prime mover. The rotor damping winding will produce a damping effect when it does not match the angular velocity of the stator magnetic field rotation. Its magnitude is proportional to the difference in the rotation angular rate, described by the damping coefficient \( D_p \). This is the synchronous generator machinery reflected by the rotating model (3). In parallel operation of synchronous generators, to achieve uniform load distribution, a frequency droop control mechanism is required, that is, the active power is changed according to the grid frequency. Specifically, when the demand for active power increases, the speed of the synchronous generator will decrease. At this time, the speed control system of the prime mover will increase the mechanical energy output. For example, the throttle valve of a large engine can be opened to achieve a new energy balance, which ensures the stability of the power grid frequency.

The speed regulation characteristic is different, which means that the reference angular speed will decrease with the increase of active power, and the angular speed will not be stabilized at the rated speed again. This characteristic can be expressed as:

\[
P_m = P_{\text{ref}} + k_p (\omega_{\text{ref}} - \omega)
\]

(9)
In the equation, $P_{ref}$ is the reference input mechanical power, $k_p$ is the active power droop control coefficient, and $\omega_{ref}$ is the angular velocity reference value. Under normal circumstances, to ensure the stability of the output frequency, the droop characteristic curve has a small falling slope, which can ensure that the grid frequency changes very little when the active power changes in a wide range, that is, the value of $k_p$ is large. According to the above formula, the control block diagram of the active frequency droop can be drawn as shown in Figure 2.

![Figure 2: The speed control block diagram of the virtual synchronous machine](image)

The active droop coefficient $k_p$ is defined as the ratio of power increment to speed increment, that is:

$$k_p = \frac{\Delta P}{\Delta \omega} = \frac{\Delta P}{P_{mn} \omega_n \Delta \omega}$$

In the equation, $P_{mn}$ is the rated mechanical power, and $\omega_n$ is the rated electrical frequency. $\Delta \omega$ represents the percentage of torque change, $\Delta \omega$ represents the percentage of frequency change. The larger the $k_p$, the smaller the frequency offset.

### 4.2 Reactive voltage droop control

The synchronous generator excitation regulation system plays an important role in maintaining the stability of the system voltage. At the same time, the excitation regulator can ensure the accurate distribution of reactive power between parallel systems. It can be seen from equation (7) that:

$$E = \omega M_{f}\phi$$

In the equation, $E$ is the excitation electromotive force amplitude. The excitation electromotive force amplitude of the synchronous generator is proportional to the excitation control input $M_{f}\phi$. When the output impedance is inductive and the frequency is constant, the amplitude of the excitation electromotive force is proportional to the reactive power.

The magnitude of the virtual synchronous generator excitation induced electromotive force is changed by the virtual excitation regulator to control the excitation current, thereby adjusting the reactive power output of the inverter. To ensure the formation of negative feedback, the excitation induced electromotive force will decrease with the increase of output reactive power under the action of excitation control. This is the reactive voltage droop control strategy. In other words, when the output reactive power is not enough, the excitation will be increased; when the output reactive power is too much, the excitation will be reduced. However, the excitation electromotive force cannot be directly given, it can only be controlled by the excitation current. Therefore, the virtual synchronous voltage regulation mechanism adjusts only the command $E_c$ of the excitation electromotive force. We should note that $E_{ref}$ is the reference value of excitation electromotive force, and $E_c$ is the reference command after reactive power droop. The control block diagram of the reactive power voltage regulation of the virtual synchronous generator is shown in Figure 3.
According to Figure 3,
\[ E_c = E_{\text{ref}} + \frac{1}{k_q} (Q_{\text{ref}} - Q_e) \]  
(13)

In the formula, \( k_q = \frac{\Delta Q}{\Delta E_c} \) describes the proportional relationship of reactive power change with the change of excitation electromotive force.

Let the excitation control link error \( \Delta E = E_c - E \), and substitute it into the above formula, we can know that:
\[ \Delta E = (E_{\text{ref}} - E) + \frac{1}{k_q} (Q_{\text{ref}} - Q_e) \]  
(14)

According to the excitation control error \( \Delta E \), the excitation current increment is generated,
\[ \Delta i_f = \frac{k_q}{k_v} \Delta E \]  
(15)

In the equation, \( k_v \) is the control parameter, and then the excitation current control quantity can be obtained by integration.

### 4.3 Comprehensive analysis of droop control
Integrating the analysis of active frequency droop control and reactive voltage droop control, the overall control block diagram of the virtual synchronous generator can be obtained, as shown in Figure 4.

The virtual synchronous generator in this paper can work in off-grid mode and grid-connected mode. When working in off-grid mode, because there is no grid support, the virtual synchronous generator needs to complete the voltage and frequency support functions. The output power is determined by the load, and the output is controlled by droop. In this case, the set power basically does not work; when working in parallel grid mode, and the grid is stable without frequency and voltage fluctuations, the virtual synchronous generator actually works in PQ control mode and feeds the set...
active and reactive power to the grid. If the grid frequency or voltage fluctuates, its droop control channel will also work. On the one hand, it outputs according to the set value of active power and reactive power, and on the other hand, the output power is drooped and adjusted according to the droop characteristics. It can be seen that the virtual synchronous generator control proposed in this paper can realize seamless switching of operating modes.

5. Influence of key control parameters

First analyze the impact of key parameters on frequency response. Refer to the control block diagram, which can be obtained from (11),

\[(\frac{f_\omega}{p} s + k_p + D_\omega)\Delta \omega = P_{ref} - P_e + (k_p + D_\omega)\Delta \omega_r\]  \hspace{1cm} (16)

Where \(\Delta \omega_r = \omega_{ref} - \omega_0\), and then the transfer function formula can be obtained:

\[\Delta \omega(s) = \frac{k_p + D_\omega}{\frac{f_\omega}{p}s + k_p + D_\omega} \Delta \omega_r(s) + \frac{1}{\tau_v s + 1}\Delta P(s)\]  \hspace{1cm} (17)

Where \(\Delta P = P_{ref} - P_e\). It can be seen from the above formula that the angular frequency deviation of the virtual synchronous generator is mainly caused by two aspects: one is the angular rate setting deviation \(\Delta \omega_r(s)\), which mainly comes from the grid frequency fluctuation in grid-connected mode, and the steady-state gain is 1, that is, the setting deviation will be fully reflected in the system output; the second is the output active power deviation \(\Delta P(s)\), which mainly reflects the influence of droop control on frequency, the steady-state gain is \(1/(k_p + D_\omega)\), the effect of the droop coefficient \(k_p\) and the damping coefficient \(D_\omega\) is the same, the larger the value, the smaller the deviation caused. The dynamics of both are represented as a first-order inertial link, and the time constant is \(\frac{f_\omega}{p(k_p + D_\omega)}\).

This paper will analyze the influence of \(k_v\) and \(k_q\) on the voltage response in the following part. From equation (14) and (15), combined with the control block diagram, it can be seen that:

\[\Delta i_f = \frac{1}{k_v}[k_q(E_{ref} - E) + (Q_{ref} - Q)]\]

Bring the integrated control value into equation (12), there is

\[\frac{se(s)}{\omega_0} = \frac{1}{k_v}[k_q(E_{ref}(s) - E(s)) + (Q_{ref}(s) - Q(s))]\]  \hspace{1cm} (18)

The change of \(\omega\) is ignored here, and \(\omega_0\) is used instead of it. Defining \(\tau_v = \frac{k_v}{\omega_0 k_q}\) and arranging the above equation, we can get,

\[E(s) = \frac{1}{\tau_v s + 1}E_{ref}(s) + \frac{k_q}{\tau_v s + 1}\Delta Q(s)\]  \hspace{1cm} (19)

It can be seen from the above formula that the basic excitation electromotive force of the virtual synchronous generator is established by \(E_{ref}\), and its steady-state gain is 1. \(\Delta Q\) introduces the voltage droop component, and the steady-state deviation of droop control is determined by \(\frac{1}{k_q}\). The larger \(k_q\) is, the smaller the voltage deviation, the smaller the \(k_q\) and the greater the voltage deviation. The dynamics of the two parts are first-order inertial models, and \(\tau_v\) is the time constant, reflecting the speed of the dynamics.

However, in the above analysis, the excitation electromotive force \(E\) is selected for both voltage setting and voltage feedback. In actual control, the excitation electromotive force \(E\) cannot be measured, and only the actual output of the synchronous inverter can be selected. Refer to the inverter circuit, the actual output is the output result of the excitation electromotive force after passing through the \(LC\) filter. In other words, when the output voltage is used to replace the excitation electromotive force, the above analysis also needs to consider the dynamics of the \(LC\) filter. Such a substitution has two consequences. One is that it makes the selection of \(\tau_v\) more complicated, which needs to consider the rapidity and damping characteristics of the voltage control response. If the \(\tau_v\) selection is too small, the rapidity is very good, but under-damping characteristics will appear, and if the \(\tau_v\) selection is too large, the corresponding speed will be too slow but the overshoot is small; the other is that it will cause
a voltage difference, which will cause reactive power control errors. This can be combined with simulation analysis. The grid voltage through the grid-connected inductance will affect the inverter output point voltage, and when this value does not match the reference value, it will cause a continuous voltage deviation.

Finally, analyze the influence of the following key parameters on active power. From (12), we can get

$$\frac{d\omega}{dt} + \frac{k_p}{p} (\omega - \omega_0) = \frac{P_{ref} - P_e + \frac{d\omega}{dt}}{p}$$

(23)

It can be seen from equation (4) that when the output impedance is inductive, the output active power of the virtual synchronous generator satisfies the following relationship

$$P_e = \frac{E_{U_o}}{Z_o} \Delta \omega$$

(20)

Where $E$ is the excitation electromotive force, $U_o$ is the output voltage (at the capacitance), $Z_o$ is the output impedance, and $\delta$ is the power angle. The relationship between the phase angle $\delta$ and the angular frequency is

$$\delta = \int (\omega - \omega_0)$$

So, there is

$$\frac{dP_e}{dt} = \frac{E_{U_o} \omega}{Z_o} \Delta \omega$$

(21)

$$\frac{d^2P_e}{dt^2} = \frac{E_{U_o} \omega}{Z_o} \Delta \omega$$

(22)

Bring the above two formulas into (11),

$$\frac{P_{ref}}{E_{U_o}} \omega^2 \Delta \omega + \frac{(k_p + D_\omega)Z_o \omega}{E_{U_o}} \Delta \omega + P_e = P_{ref} + (k_p + D_\omega) \Delta \omega$$

(23)

Reorganize the transfer function relationship between $P_e$, $P_{ref}$ and $\Delta \omega$

$$P_e(s) = \frac{\frac{pE_{U_o}}{j\omega_o Z_o}}{s^2 + \frac{p(k_p + D_\omega)Z_o}{j\omega_o} + \frac{pE_{U_o}}{j\omega_o}} P_{ref}(s) + \frac{\frac{pE_{U_o}}{j\omega_o}}{s^2 + \frac{p(k_p + D_\omega)Z_o}{j\omega_o} + \frac{pE_{U_o}}{j\omega_o}} \Delta \omega(s)$$

(24)

This is a typical second-order system from which the influence of control parameters on the dynamics of active power can be analyzed. Its natural frequency $\omega_n$ and damping ratio $\xi$ are:

$$\begin{align*}
\omega_n &= \sqrt{\frac{E_{U_o}}{j\omega_o Z_o}} \\
\xi &= \frac{\frac{pZ_o}{2}}{\sqrt{\frac{E_{U_o}}{j\omega_o}}} 
\end{align*}$$

(25)

It can be seen from equation (25) that both the virtual moment of inertia $I$ and the output impedance $Z_o$ will affect $\omega_n$ and $\xi$: the larger $I$ or $Z_o$, the smaller $\omega_n$, and vice versa; the larger $Z_o$, the larger $\xi$, and the larger $I$, the smaller the $\xi$. And $k_p$ and $D_\omega$ only affect the system damping ratio. In addition, from equation (24), the system output active power is mainly affected by two aspects, one is the power setting value $P_{ref}$, and the other is the frequency setting error $\Delta \omega$. In the grid-connected mode, when the grid is stable and the reference frequency $\omega_{ref}$ is equal to the rated frequency $\omega_0$, the inverter works in PQ control mode, and the output power is mainly determined by $P_{ref}$; when the grid is unstable and there is a deviation in frequency, droop control will work. The situation of inverter working in the hybrid mode of PQ control and droop control is consistent with the analysis in the previous section.

6. Simulation verification

To verify the effectiveness of the proposed virtual synchronous generator control strategy, this paper constructs a virtual synchronous generator simulation model based on Matlab/Simulink and conducts two simulation experiments. The simulation experiment process is: 0~0.6s working in off-grid mode, STS is disconnected, and the inverter independently supplies power to the three-phase local symmetrical resistive load $R_{load}$; at 0.6s, STS is closed, and the inverter working mode is switched to
grid-connected mode. Simulation experiment 1 assumes that the grid is stable, and the grid frequency has no fluctuations and deviations. It is mainly used to verify the performance of off-grid droop control and grid-connected PQ control; simulation experiment 2 assumes that the grid frequency fluctuates, which is 49.85Hz, which is mainly used to verify the grid-connected droop control PQ controls mixed mode performance. The parameters used in the simulation are shown in Table 1.

### Table 1 Simulation parameters of virtual synchronous generator

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( L_f \) | 0.45mh | \( L_g \) | 0.45mh |
| \( R_f \) | 0.135\( \omega \) | \( R_g \) | 0.135\( \omega \) |
| \( C \) | 22uf | Frequency | 50hz |
| \( R_{\text{load}} \) | 120\( \omega \) | \( k_p \) | 636.3 |
| \( k_q \) | 605.8 | \( k_q \) | 64.3 |

In the simulation experiment, \( P_{\text{ref}} = 800W, Q_{\text{ref}} = 200W \). The simulation results of Experiment 1 are shown in Figure 5-7, Figure 5 is the output active power and reactive power results, Figure 6 is the output voltage and output current results, and Figure 7 is the output frequency results. It can be seen from Figure 8 that in the off-grid mode, the output has a common power \( P_e = 1240W \), and a reactive power \( Q_e = 0\text{Var} \), which is mainly determined by the local load; after grid-connected, the output active and reactive power can be traced to the reference value, but there is a relatively large error in the output reactive power. Figure 6 shows the smooth transition of the output voltage and output current during the STS closing process; Figure 7 shows the change of the output frequency before and after the grid connection. The output frequency before and after the grid connection is affected by the droop characteristics, and it quickly returns to the grid frequency after the grid connection.

![Figure 5 Output power result of experiment 1](image1)

![Figure 6 Output voltage and current of Experiment 1](image2)

![Figure 7 Output frequency result of Experiment 1](image3)

![Figure 8 Output power result of Experiment 2](image4)
The simulation results of Experiment 2 are shown in Figure 8-10. Figure 8 shows the results of output active power and reactive power. After 0.6s connected to the grid, the power output is automatically adjusted by the droop control of the grid frequency deviation; Figure 9 is the output voltage and output current results, and the STS closing process can be seen from the figure. The output voltage and output current transition smoothly; Figure 10 shows the output frequency results. The output frequency is automatically adjusted by the droop control before grid connection, and the output frequency will quickly track the grid frequency after grid connection.

7. Conclusion
Based on the synchronous generator, this paper constructs the virtual synchronous generator topology and mathematical model, designs the corresponding virtual synchronous generator active-frequency droop control and reactive power-voltage droop control strategy, and analyzes the influence of the key control parameters through the frequency domain method, thus providing guidance for the design of control parameters. The simulation results verify the effectiveness of the proposed virtual synchronous generator control strategy.

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