Calculation of Early Warning Indicators for Small Watersheds Based on Mathematical Statistics

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Abstract. The early warning and forecasting of mountain flood disasters is an indispensable part of mountain flood prevention and control work. Reasonable selection of rainfall warning indicators is the key to mountain flood warning and forecasting work. This paper proposes a method for calculating the critical rainfall warning indicator based on mathematical statistics. That is, the characteristic period is determined based on the confluence time and watershed characteristics of the upstream watershed of the disaster prevention object. The simulated rainfall of each characteristic period and its corresponding antecedent precipitation index is calculated based on the rainfall data of the flooding. Using the regression analysis method to draw the critical rainfall line, calculate the critical rainfall warning indicator and take the typical disaster prevention objects of Chahekou Village in Muping District of Yantai City, Shandong Province as an example, and the historical rainfall records of 2014-2018 were used as verification data to verify the early warning indicators. The results show that the critical rainfall accuracy calculated by this method is about 80%, which can provide reference for the application of early warning indicators for mountain flood disasters.

1. Introduction

The flash floods are swift and powerful, and they are very destructive and harmful. They cause serious losses to the national economy, damage infrastructure, cause great casualties, threaten the safety of life and property of mountain residents, and restrict the sustainable development of mountainous areas.¹ Many domestic and foreign water conservancy experts have also made many contributions to promote the technical level of the calculation of early warning indicators for mountain flood disasters, such as the FFG method developed in the United States [²], the EPIC method [³], domestic Li Changzhi, Guo Liang [⁴] etc. exploring the research method of mountain flood warning and forecasting system by using distributed hydrological model. Generally speaking, the early warning indicators include critical rainfall, critical water level and critical flow. The small river basin has short confluence time, short foresight period, and few automatic monitoring stations. The critical water level and critical flow warning indicators are not applicable to small watersheds in the mountainous area [⁵]. This paper focuses on the determination method of critical rainfall warning indicators. Because China's climate type, geological conditions, underlying surface conditions are complex and diverse, and some areas are in short supply, we should choose appropriate methods to calculate rainfall warning indicators according to different conditions.

Commonly used methods for calculating critical rainfall include statistical induction, hydrology and hydraulics, and empirical analysis [⁶]. Statistical induction method needs to analyze historical data, and combine local rainfall data to obtain critical indicators, which are suitable for areas with less specific characteristics and less accurate prediction. The hydrology and hydraulics method is based on
the rainfall runoff mechanism, which mainly includes the water level flow reverse method, the test algorithm and the dynamic critical rainfall method\cite{7}. This method has wide applicability and high early warning accuracy, which can realize dynamic warning, but Computation is more complicated and requires higher data. The empirical analysis method determines the local critical rainfall by analyzing the rainfall runoff method in a specific area, mainly including single station or regional critical rainfall method, interpolation method and analog method. There is no obvious derivation mechanism for this method\cite{8}. The method for calculating the early warning index based on mathematical statistics is a statistical induction method. The requirements for the underlying surface data of the basin are not high, and it is suitable for areas lacking data.

2. Critical rainfall calculation method based on mathematical statistics
The cause of mountain flooding is the sudden rainstorm in some areas, rapid runoff, increased river flow, water level across the river bank, threatening shore residents and fields. The accumulated rainfall that causes mountain floods, mudslides, landslides and other flash floods in the basin or region is called the stimulated rainfall. The amount of mountain torrents is not only related to rainfall, but also affected by the dry and wet conditions of the soil before the rainfall. When the soil is dry, the rainfall infiltration is large, and the runoff is small; when the soil is wet, the rainfall is Low seepage, large runoff, and more likely to cause flash floods. The comprehensive critical rainfall indicator is composed of two variables: the stimulated rainfall in the upper period of the disaster prevention object and the antecedent precipitation index in the characteristic period.

Among them, the stimulated rainfall during the characteristic period is the stimulating factor, and the influence of the rainfall in the early period of the characteristic period reflects the characteristics of the storage capacity of the underlying surface of the basin. Based on this, the judgment equation for the occurrence of mountain flood disaster can be obtained:

\[ F = (P_{CP}, P_{PE}) \]

- \( P_{CP} \) ---- mountain flood disaster characteristic period stimulated rainfall
- \( P_{PE} \) ---- antecedent influenced rainfall

The method of critical rainfall analysis based on mathematical statistics is to collect the records of mountain flood disasters and long-term rainfall data, and to statistic rainfall during the statistical period and calculate the corresponding antecedent precipitation index. Draw the x-y coordinate system scatter plot by using the antecedent precipitation index as the horizontal axis and the stimulated rainfall as the vertical axis. Based on the regression analysis method to calculate the expression between the simulated rainfall and the impact rainfall in different periods, calculate the critical rainfall value, and select the rainfall data of different years to compare the warning field with the actual disaster field.

2.1. Data collection and organization

2.1.1. Selection and organization of rainfall data
Basic data needs to be collected before the calculation of critical rainfall warning indicators based on mathematical statistics. The data includes the natural geography, hydrological and climatic characteristics of the mountain flood disaster area and its surrounding areas, the basic characteristics of the mountain torrents, and the rainfall data of the rainfall stations near the typical disaster prevention objects. Hydrological and meteorological data of historical flash floods include rainfall intensity, duration, and the interval between the start of the storm and the occurrence of the disaster. If there is a flood disaster in the basin that has not been investigated in time, a supplementary investigation should be conducted. Inquire about the larger flood occurrence schedule to obtain rainfall data for 30 days before the flood disaster. Count the maximum rainfall during each warning period before the flood disaster and calculate the corresponding antecedent precipitation index according to equation 2-6~2-8.
2.1.2 Selection of rainfall station

The mathematical statistics method has no obvious physical mechanism and derivation process. Relatively speaking, the requirements for the underlying surface and flow data of the basin are not high, but the accuracy of the data has a great influence on the formulation of the critical rainfall warning indicator. The choice of rainfall site is also very important. This paper selects the nearest rainfall station as the data source for the typical disaster prevention objects in Muping District.

2.2. The concept and determination of the early warning period

The early warning period refers to the typical rainfall duration used in the mountain flood warning work, which is an important component of the early warning indicator. The factors affecting the early warning period include the size of the upstream watershed by the disaster prevention object, the shape of the watershed, the transfer time, and the rate of flood increase. It is necessary to comprehensively consider various factors and reasonably determine the early warning period. The occurrence of mountain torrent disasters is usually closely related to the rainfall during the confluence time and the rainfall affected in the previous period. Therefore, the convergence time of the upstream basin of the disaster prevention object is taken as the longest warning period, and other periods less than the longest warning period can be based on the storm characteristics of the basin, the characteristics of the underlying surface, the size of the basin, the average ratio, the shape factor, and the historical flash flood conditions to determine. Generally, 2–3 typical warning periods are applicable. If the convergence time is too short, several longer time periods can be added appropriately to improve the safety of early warning work.

The convergence time is calculated using equations (2-1) and (2-2). The longest convergence time and the shortest convergence time are calculated using equations (2-3) and (2-4):

\[ Q_m = 0.278 \frac{h}{\tau} F \]  
\[ \tau = 0.278 \frac{L}{mJ^{\tau/4}Q_m^{1/4}} \]  
\[ \tau_{\text{min}} = 0.278 \frac{L}{mJ^{1/3}Q_{m_{\text{m}}}^{1/4}} \]  
\[ \tau_{\text{max}} = 0.278 \frac{L}{mJ^{1/3}Q_{m_{\text{m}}}^{1/4}} \]

\( Q_m \)—peak flow (m^3/s); h—it represents the maximum net rain of the \( \tau \) period at full confluence, and represents the net rain of a single flood peak at part of the confluence. According to the sum of rainfall and the antecedent precipitation index, check the line 5 in the rainfall-runoff correlation diagram to get it.; \( F \)—basin area (km^2); \( \tau \)—watershed confluence duration (h); \( m \)—convergence parameter, related to \( \theta \) value under no data conditions, \( \theta = \frac{L}{J^{1/2}} \); \( L \)—the longest distance (km) along the main river from the exit section to the watershed; \( J \)—the average ratio of the flow along the process \( L \) (in decimals).

2.3 stimulated rainfall and its antecedent precipitation index calculation

2.3.1 stimulated rainfall

The maximum rainfall of different early warning periods of the disaster prevention objects before the disaster occurrence time is used as the simulated rainfall during the period, and the antecedent precipitation index corresponding to the simulated rainfall is calculated synchronously.

2.3.2 Antecedent precipitation index calculation

The formula for calculating the amount of rainfall in the early stage is:
\[ P_{a,t+1} = K (P_{a,t} + P_t) \]  

\( P_{a,t} \) — the antecedent precipitation index at the beginning of the t-day, mm; \( P_{a,t+1} \) — the antecedent precipitation index at the beginning of the \( t+1 \)th day, mm; \( P_t \) — the rainfall of the t-day, mm; \( K \) — watershed daily regression coefficient or reduction factor.

When the value of \( P_{a,t} \) is greater than the maximum water storage capacity \( WM \) of the basin, \( WM \) is taken as the \( P_{a,t} \) value of the period.

The general form of the linear regression equation for calculating the relationship between the antecedent precipitation index and the critical rainfall is:

\[ \hat{y} = b_0 + b_1 x \]  

\( x \) is the amount of rainfall in the early stage, \( y \) is the rainfall in the period, \( b_0 \) and \( b_1 \) are the parameters.

2.4 Critical rainfall warning indicator verification

Critical rainfall warning indicator is a key indicator of mountain flood disaster prevention work. Since the parameters and empirical formulas used in the calculation are mainly based on local storm flood maps and hydrological manuals, which represent regional integration or design status, there is a certain degree of difference compared with actual mountain torrents in small watersheds in the hilly area. In addition, the rainfall station is not dense enough, and the accuracy of the rainfall used is not high enough, resulting in a certain error in the calculation of the early warning indicators.

3. Applications

3.1 Typical disaster prevention object overview

The floodplain is affected by the uneven spatial and temporal distribution of rainfall, which is characterized by flooding during the rainy season and dryness during the dry season. According to the survey results and the actual needs of flood control, disaster reduction and regional development planning in Muping District, the disaster prevention targets involve 114 villages along the river and 58 small watersheds.

In this paper, we selected Chahekou Village in the Dagujia River Basin as a research example to calculate the rainfall warning index. The drainage area above the control section of the Chahekou Village is 3.57 km², the confluence length is 3.7 km, the basin ratio is 6.3‰, and the river channel is narrow. For natural rivers, there is no obvious remediation, no obvious river beaches, and the distance between residents and rivers is small. 25 historical flood data are selected for the calculation of critical rainfall warning indicators.

The distribution of the research objects in the basin is shown in Figure 1:

![Figure 1. Location of Chahekou Village](image)

3.2 Critical rainfall calculation

The warning period of Luhekou Village is 1 hour, 1.5 hours, 3 hours and 6 hours. After the actual rainfall starts, if the rainfall of 1 hour reaches the rainfall warning indicator, an early warning will be issued, otherwise the rainfall will continue to be monitored. If the accumulated rainfall of 1.5 hours
reaches the early warning indicator, an early warning will be issued, otherwise the rainfall will continue to be monitored, and so on, until the whole rainfall monitoring and warning is completed. Taking 3 hours as an example, this paper uses regression analysis to determine a dividing line as a critical rainfall line on the scatter plot, as shown in Figure 2.

![Figure 2. 3-hour critical rainfall line of Chahekou Village](image)

The rainfall value just on the straight line is used as the warning indicator for the critical rainfall. The warning indicators for the critical rainfall with the antecedent precipitation index of 0.2, 0.5, and 0.8 are shown in Table 1:

| Administrative district (county, township, village) | Confluence time/h | Early warning period/h | antecedent precipitation index/mm |
|---------------------------------------------------|------------------|------------------------|----------------------------------|
| Chahekou Village                                  | 1                | 119                    | Dry(0.2)                         |
|                                                   | 1.5              | 131                    | General(0.5)                     |
|                                                   | 3                | 150                    | Wet(0.8)                         |
|                                                   | 6                | 170                    |                                  |

3.3 Early warning indicator verification
In order to verify the applicability of the early warning indicators, this paper uses the historical rainstorm records of 2014-2018 and the number of disaster victims as verification data, and compares the early warning events with the actual disaster records. The statistical results are shown in Table 2:

| Time     | Rainstorm times | Chahekou Village disaster record times | Chahekou Village disaster warning times |
|----------|-----------------|---------------------------------------|----------------------------------------|
| 2014     | 10              | 4                                     | 4                                      |
| 2015     | 8               | 3                                     | 5                                      |
| 2016     | 9               | 2                                     | 2                                      |
| 2017     | 7               | 3                                     | 4                                      |
| 2018     | 9               | 4                                     | 5                                      |

It can be concluded that the accuracy of the critical rainfall calculated by the regression analysis method is about 80%, and the actual number of disasters is less than the number of warnings, indicating that the critical rainfall indicator is small. The reason is that the characteristics of the underlying surface in the basin have changed due to natural and human factors, resulting in inaccurate calculation of critical rainfall indicators.

4. Conclusion
The area and population of hilly areas in China occupy a large proportion in the whole country. In recent years, sudden floods in small watersheds in hilly areas caused by heavy precipitation frequently occur, which seriously threatens the safety of people's lives and property in hilly areas and restricts the
development of the local economic level. Due to the sparse monitoring network and the fact that the forecasting scheme is not regular, it is difficult to predict and prevent floods in small watersheds in mountainous areas. Therefore, flood warning in small watersheds is still a difficult point in flood control and disaster alleviation.

The mathematical statistics method proposed in this paper for calculating the critical rainfall value is reasonable and can be applied to the small watershed in Muping District of Yantai City. This method does not have a strict physical mechanism, and the results obtained are closely related to the conditions of the underlying surface conditions in the previous watershed. However, the underlying surface of the basin is not static due to natural and human factors. When the underlying surface conditions, especially the characteristics of the river channel, change in the basin, the reliability of the calculation results will be affected. Therefore, in order to ensure the reliability of the prediction results, the data sequence should be supplemented and corrected continuously to further improve the calculation accuracy.

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