Black Swans in Astronomical Data

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ABSTRACT

Astronomy has always been propelled by the discovery of new phenomena lacking precedent, often followed by new theories to explain their existence and properties. In the modern era of large surveys tiling the sky at ever high precision and sampling rates, these serendipitous discoveries look set to continue, with recent examples including Boyajian’s Star, Fast Radio Bursts and ‘Oumuamua. Accordingly, we here look ahead and aim to provide a statistical framework for interpreting such events and providing guidance to future observations, under the basic premise that the phenomenon in question stochastically repeat at some unknown, constant rate, $\lambda$. Specifically, expressions are derived for 1) the \textit{a-posteriori} distribution for $\lambda$, 2) the \textit{a-posteriori} distribution for the recurrence time, and, 3) the benefit-to-cost ratio of further observations relative to that of the inaugural event. Some rule-of-thumb results for each of these are found to be 1) $\lambda < \{0.7,2.3,4.6\} t_1^{-1}$ to $\{50,90,95\}$% confidence (where $t_1$ = time to obtain the first detection), 2) the recurrence time is $t_2 < \{1,9.99\} t_1$ to $\{50,90,95\}$% confidence, with a lack of repetition by time $t_2$ yielding a $p$-value of $1/\{1 + (t_2/t_1)\}$, and, 3) follow-up for $\lesssim 10 t_1$ is expected to be scientifically worthwhile under an array of differing assumptions about the object’s intrinsic scientific value. We apply these methods to the Breakthrough Listen Candidate 1 signal and tidal disruption events observed by \textit{TESS}.

Key words: methods: observational — methods: statistical — surveys

1 INTRODUCTION

The cosmos is a vast enigmatic volume, permeated with mysteries and treasures waiting to be known. History has demonstrated that our studies of the cosmos, and the discoveries integral to that process, have often defied intuition or commonly held expectation (Kuhn 1957). In a Universe where our presuppositions are often wrong, the philosophy of “just look”, with clear eyes and open minds, has often proven to be royal road to success (Lang 2010).

Serendipity has played a particularly important role in this regard (Fabian 2009). Many crucial discoveries in astronomy were simply chance accidents, with no broad expectation for their existence before-hand. Famous examples include the Galilean moons (Galilei 1610), the discovery of Uranus (Herschel 1781), radio emission from the Milky Way (Jansky 1933), cosmic X-rays sources (Giacconi et al. 1962), gamma-ray bursts (Klebesadel, Strong, & Olson 1973), pulsars (Hewish et al. 1968) and hot-Jupiters (Mayor & Queloz 1995).

In most cases, these detections were found to represent the tip of a proverbial iceberg; the inaugural member of a new class of astronomical phenomena. For example, hot-Jupiters are now known to reside around 0.4% of FGK-dwarf stars with new discoveries being somewhat routine today (Zhou et al. 2019). And yet, those “firsts” are perhaps the moments of greatest excitement, when observers naturally wonder about how many analogs might exist, and when the next one will be found. For example, shortly after the discovery of the first interstellar asteroid, ‘Oumuamua (Meech et al. 2017), estimates of the implied number density soon followed (Trilling et al. 2017; Do, Tucker, & Tonry 2018) - estimates which were of course conditioned upon a single data point\textsuperscript{1}. It is sometimes said that “a single data point teaches you nothing”, but truly that describes zero data points. Inference can proceed with a single datum (and evidently do), but naturally the associated uncertainty of such inference will be greater than that conditioned upon $> 1$ data points.

The situation is somewhat analogous to trying to infer the rate of black swans that one expects to observe having thus far seen a singular example. And so, by metaphor, a “black swan” event often refers to an unanticipated, rare event of particular importance or significance (Taleb 2010).

\textsuperscript{1} Other recent examples include Boyajian’s Star (Boyajian et al. 2016) and the Lorimer burst (Lorimer et al. 2007).
In astronomy, the rate calculation of a “black swan” event, like ‘Oumuamua, can take different forms. In Trilling et al. (2017) and Do, Tucker, & Tonry (2018), the authors are principally interested in the true number density of such objects, which requires a careful treatment of the observational selection biases. This naturally depends upon the observing strategy and sensitivity of the telescopes involved and will thus be generally bespoke to that particularly phenomenon. However, a more general case is apparent when one asks - what is the rate at which one should expect to observe these events if my observations continue without any modification? This sidesteps the issue of selection bias since one is not asking about the intrinsic rate, but the observed rate. Similarly, one can ask questions building off this, such as when should I expect to see a recurrence and how long is it worth my time to observe waiting for such analogs?

In this work, a general statistical framework for tackling these questions is sought. The underlying assumption, as already stated, is that the observational selection biases do not change. If they do, this would clearly need to be accounted for. Whilst the idea of a fixed, unchanging observational mode might initially seem somewhat unrealistic or uncommon, it is suggested here that this is indeed becoming much more prevalent with new and upcoming missions. In the modern era of all-sky astronomical surveys, such as Gaia (Gaia Collaboration et al. 2016), TESS (Ricker et al. 2015) and the Vera Rubin Observatory (VRO; Ivezić et al. 2019), the observing strategy and sensitivity are often pre-decided and fixed for many years at a time. In this way, fixed observational biases are becoming common, indeed highly desirable as a means to more straightforwardly invert detection rates into occurrence statistics. Further, the enormous volumes of data being produced by these surveys (e.g. Jurić et al. 2017) has demoted the role of human “by eye” detections (which certainly suffers from time/caffeine dependent sensitivity), such that increasingly homogeneous, automated machine learning approaches are being deployed to detect astronomical phenomena (e.g. Wagstaff et al. 2016; González, Muñoz, & Hernández 2018; Lin, Li, & Luo 2020), including anomalies (e.g. Giles & Walkowicz 2019; Wheeler & Kipping 2019; Storey-Fisher et al. 2020).

This paper is organized as follows. In Section 2, the a-posteriori distribution for the observed anomaly rate is derived, and related summary statistics. In Section 3, the time until the next recurrence is derived in a probabilistic sense, with implications for applying tension to the hypothesis of repeatability. In Section 4, a simple cost-benefit analysis is provided as a way of guiding the economical use of continued time/effort to seek analogs of the black swan event.

2 POSTERIOR DISTRIBUTION FOR THE ANOMALY RATE

2.1 Formalism

Consider observing an object, or a sample of objects, with time series astronomical observations. Let us further consider the simplifying case where the data are homogeneous, regular and no substantial changes to the observing instruments nor strategy are implemented. After a time $t_1$, a highly significant anomaly is detected for which there is no precedent and is confirmed to be of astrophysical origin. If the anomaly is short-lived and transient, it may be impossible to secure any further data about the anomaly, besides the fact it occurred.

In such a scenario, one faces the frustrating situation of seeing something truly remarkable yet for which there is essentially no way of obtaining any further information. Naturally, astronomers may at this point wonder how many analogs of this anomaly exist, and when the next one might be expected to be found. And yet, astronomers are forced to make inferences conditioned upon this sole black swan event.

To make mathematical progress, let us assume that the anomaly (or an event sufficiently similar to be classified as analogous), will indeed repeat at some future time, or within some similar sample, given sufficient observations. Let us further assume that the intrinsic rate at which this anomaly appears in our observations is constant over time. In other words, we do not live at a particularly “special” time (or equivalently did observe a particularly special sample) and thus the anomaly rate is uniform, such that the probability of detection is the same in any given equal duration time interval (or equivalently sample size).

The above describes a Poisson process and hence provides the basis of making analytic progress. In some cases, a Poisson process is not directly appropriate (e.g. repeating FRBs from an individual source can be non-Poisson; Oppermann, Yu, & Pen 2018) but can often be made so by simple re-framing of the problem (e.g. the number of FRBs received over the entire sky from independent sources over a given interval will be Poissonian). For a Poisson process, the probability of observing $N$ events over a time interval $t$ is given by

$$Pr(N|\lambda,t) = \frac{1}{N!} e^{-\lambda t} (\lambda t)^N, \quad (1)$$

where $\lambda$ is the intrinsic rate per unit time of the process.

Note that our formalism is expressed in terms of time, but could equally be replaced with data volume, sample size or even “effort level” more generally. However, time provides a clarifying example and thus we continue with this notation in what follows.

When the first event is detected, all that is known is how long it took to obtain that first detection. We consider that “now” represents the moment of the anomaly detection and impatient astronomers across the world are immediately wondering when the next event will recur. Thus, the only data in-hand is that it took a time $t_1$ to obtain that one event.

A well-known result of Poisson processes is that the time interval between successes follows an exponential distribution (Cooper 2005). Equivalently, it must hold that the probability distribution for the time it takes to obtain our first success when initiating from arbitrary reference time is also an exponential, since a Poisson process is - by definition - an independent process with no “memory” of previous events, much like how your chance of rolling a six on a dice has no dependency on previous rolls. Accordingly, the time for a first success follows

$$Pr(t_1|\lambda) = \lambda e^{-\lambda t_1}. \quad (2)$$
2.2 An a-posteriori distribution for the anomaly rate, $\lambda$

To infer $\lambda$ from $t_1$, one can use Bayes’ theorem (Bayes 1763) to write that

$$\Pr(\lambda|t_1) \propto \Pr(t_1|\lambda) \Pr(\lambda).$$

(3)

For $\Pr(\lambda)$, the prior, a scale-invariant objectively defined distribution in sought - which is provided by using the approach of Jeffreys (1946). Here, the Fisher information (Fisher 1922) of an exponential distribution is $\lambda^{-2}$ and thus the Jeffreys prior is simply $\lambda$ with an expectation value of 1 mode at $1/\langle \lambda \rangle$. Some basic properties of this distribution are that it has a

$$\Pr(\lambda|t_1) = t_1 e^{-\lambda t_1},$$

(4)

whose behaviour is depicted in Figure 1. Note that this should not be applied to the rate of abiogenesis given the timing of the first appearance of life, since our existence is predicated upon a success and thus is severely sculpted by selection bias (Spiegel & Turner 2012). In contrast, it is predicated upon a success and thus is severely sculpted timing of the first appearance of life, since our existence should not be applied to the rate of abiogenesis given the approach of Jeffreys (1946). Here, the Fisher information of an exponential distribution is $\lambda^{-2}$ and thus the Jeffreys prior is simply $\lambda^{-1}$. After normalizing the posterior, one obtains

$$\Pr(t_2|t_1) = \int_0^\infty \Pr(t_2|\lambda)\Pr(\lambda|t_1)d\lambda,$$

(6)

whose behaviour is illustrated in Figure 2.

3.2 Logarithm of the recurrence time

Another useful way to think about the above is in logarithmic time, where one can transform the above to find that $\log t_2$ follows a logistic distribution with a mean of zero and scale-parameter unity:

$$\Pr(\log(t_2) | t_1) = \frac{e^{-\log t_2}}{(1 + e^{-t_2})^2}.$$ (7)

This is convenient because a logistic distribution is quasi-Gaussian (Gauss 1809) in shape (see Figure 2), and thus one can approximate that the logarithm of the time until the next event is approximately Gaussian with a mean of 0 and standard deviation $\pi/\sqrt{3} \approx 1.8$ (recall we are working in units of $t_1$ time). In other words, the recurrence time is approximately log-normal (and correctly log-logistic).

3.3 Upper limits on the recurrence time

Coming back to linear space (Equation 6), the mean of the distribution is infinite and doesn’t present a useful summary statistic (and thus so too is the variance). As with the $\lambda$ posterior, $t_2$ is found to follow a monotonically decreasing function peaking at zero. The cumulative distribution is analytic (simply $t_2/(1 + t_2)$) and thus provides useful quantiles. This allows to state a lower limit that $t_2/t_1 < C/(1 - C)$, to a confidence level of $C$. To give examples, $t_2/t_1 < \{1.99\}$ to $\{50.90, 99\}$% confidence level. An important consequence of the above is that even after waiting 9 times longer than the initial detection time, there is still a respectable 10% probability that one will not have observed a recurrence yet. Black swans demand patience. These results can also be used in testing the hypothesis that the event is indeed repeating. Whilst the black swan event is assumed to be astrophysical nature here with a uniform rate, it could instead by a non-repeating event caused by the instrument itself, for example. In such a scenario, after observing the event for $t_2 = 99t_1$, statistical pressure would bear down on the hypothesis, essentially producing a $p$-value of 1%. Going further, we can write that the $p$-value for this hypothesis is $1/[1 + (t_2/t_1)]$. Rigorous model testing might proceed then through comparison to a suitable instrument outlier model, as $p$-values alone are fraught as a sole means of hypothesis exclusion (Wasserstein & Lazar 2016).

3.4 Constraints from future null detections

Suppose observations of the anomaly’s source continue after its initial detection. Let us assume that these subsequent observations span a time interval $t_{obs}$, but one finds no further anomalies. How does this new information - a lack of subsequent detections - affect our inference on $\lambda$?
Figure 1. Left: The a-posteriori distribution of $\lambda$, the anomaly rate, given one detection of the anomaly in hand. Right: The cumulative density function of the posterior highlighting three useful upper limits.

Figure 2. Left: The a-posteriori distribution of the anomaly recurrence time, $t_2$, given one detection of the anomaly in hand. Right: Same as left, except we show the distribution of the logarithm of $t_2$. 
The probability of observing zero events over a time interval $t_{\text{obs}}$ can be evaluated from the Poisson distribution

$$P(N = 0; \lambda, t_{\text{obs}}) = e^{-\lambda t_{\text{obs}}}. \quad (8)$$

This forms a monotonically decreasing exponential curve, where for any given choice of rate, one can see how “dry spells” much longer than than $1/\lambda$ are improbable - matching our intuition for the problem. And so one may employ this as a likelihood function for $P(t_{\text{obs}}|\lambda)$ and then via Bayes’ theorem, write

$$P(\lambda|t_{\text{obs}}) \propto P(t_{\text{obs}}|\lambda)P(\lambda). \quad (9)$$

One can go further by folding in the information already learned about $\lambda$ via the prior:

$$P(\lambda|t_{\text{obs}}, t_1) \propto P(\lambda|t_{\text{obs}})P(\lambda|t_1),$$

$$P(\lambda|t_{\text{obs}}, t_1) \propto e^{-\lambda t_{\text{obs}}} (1 + e^{-\lambda t_1}). \quad (10)$$

After normalization, this becomes

$$P(\lambda|t_{\text{obs}}, t_1) = (1 + e^{-\lambda t_1}) e^{-\lambda (t_1 + t_{\text{obs}})} \quad (11)$$

By comparison to $P(\lambda|t_1)$ in Equation (11), one can see that this is equivalent to simply setting $t_1 \rightarrow (t_1 + t_{\text{obs}}).$ This make sense since the Poisson process is independent and thus doesn’t “know” that we stopped the clock at $t_1$ before, only that 1 event occurred over said time interval.

4 COST-BENEFIT CONSIDERATIONS OF FUTURE OBSERVATIONS

4.1 Cost-benefit analysis of null results

As shown earlier in Section 3.3, there is a 50% chance of a recurrence by observing over the same time interval as for the first anomaly’s identification. But, going for twice that time only increases the odds to 67% (+17%), and thrice that to 75% (+8%). This implies that the initial observations after the event are worthwhile, but the longer one observes, the slower one’s odds of success grow. In models of diminishing returns, especially those with associated costs, one might anticipate an optimal observing strategy to emerge.

Continuous observations can improve our knowledge in two basic ways. First, a lack of any positive detections is useful in that it constrains the rate parameter, $\lambda.$ Here, one is really interested in how much tighter the $a$-posteriori distribution of $\lambda$ becomes when conditioned upon additional data featuring no detections. Second, a success will, of course, offer the opportunity to not only constrain $\lambda$ much better, but also provide additional objects for which their character and properties can be compared to the original. The “value” of that second possibility is difficult to directly compare to the act of improving our knowledge of $\lambda$, and so it is treated here as a separate problem (see Section 4.2). But in both cases, one can consider a simple cost-benefit optimization to provide guidance to observers.

Let us first consider the possibility of no new detections after observing a time interval $t_{\text{obs}}$ after the first detection. This act naturally improves our constraint on $\lambda$ and one can characterize the “value” associated with that improvement by evaluating the information gained using information theory. This is quantified by the Kullback-Leibler Divergence (KLD; Kullback & Leibler 1951) going from $P(\lambda|t_1) \rightarrow P(\lambda|t_1, t_{\text{obs}}),$ which evaluates the change in Shannon entropy (Shannon 1948) in expending the additional observing time $t_{\text{obs}},$ and is calculated as

$$\text{KLD} = \int_{\lambda=0}^{\infty} P(\lambda|t_{\text{obs}}, t_1) \log \left( \frac{P(\lambda|t_{\text{obs}}, t_1)}{P(\lambda|t_1)} \right) d\lambda,$$

$$= \frac{(1 + t_{\text{obs}})(\log(t_1 + t_{\text{obs}}) - \log t_1) - t_{\text{obs}}}{t_1 + t_{\text{obs}}} \text{ nats.} \quad (12)$$

Information always increases with more time, but of course in many applications that time, or more generally “effort level”, comes at cost. Assuming cost scaling proportional to time, then the information/time provides a direct way of optimizing resources in a cost-benefit analysis - at least under the defined objective of trying to better constrain $\lambda.$ Let us define that the benefit-to-cost ratio of obtaining further null results, $\text{BCR}_{\text{null}},$ is proportional to the KLD in the above divided by the time used $t_{\text{obs}},$ and further work in units of time $t_1$ to yield

$$\text{BCR}_{\text{null}} \propto \frac{(1 + t_{\text{obs}})(\log(1 + t_{\text{obs}}) - t_{\text{obs}})}{t_{\text{obs}}(1 + t_{\text{obs}})}, \quad (13)$$

which is plotted in Figure 3. In this case, we note that the cost-to-benefit ratio has a single maximum located at $t_{\text{obs}} = 2.174 t_1.$

This reveal, then, that if one has observed the source for ~twice as long as the initial run and still not seen a recurrence, it’s becoming uneconomical to continue - in the sense of a null-result providing worthwhile learning on $\lambda.$ Of course, this does not account for the value of a possible detection, which is addressed next.
4.2 Cost-benefit analysis of a fishing expedition

Let’s assume that the first anomaly detection has a “value” $v_1$. Scientific value is somewhat subjective and field/goal specific, but could, for example, be a proxy for scientific impact (such as citation count). Since it took a time $t_1$ to obtain that success, then the benefit-to-cost ratio of the first anomaly is simply $\text{BCR}_1 \approx v_1 / t_1$. Subsequent successes may not be necessarily the same value as the first, and indeed will typically be less scientifically valuable. For the moment, we leave the functional form for these subsequent values undefined, but adopt the notation that the value of the $n^{th}$ anomaly is $v_n$. Accordingly, the total value of finding $m$ new objects (hence $m+1$ objects in total now known) will be

$$V_m = \sum_{n=2}^{m+1} v_n. \quad (14)$$

Consider at time $t_1$, after the announcement of the anomaly, one is planning a new observing program. Consider further that for the new observing plan, a time interval $t_{\text{obs}}$ is selected and fixed. The expected total value, $\langle V_{\text{tot}} \rangle$, of this observing run is found by summing the total value of $m$ successes with the probability of $m$ successes over all $m$ indices (essentially a weighted average):

$$\langle V_{\text{tot}} \rangle = \sum_{m=1}^{\infty} V_m \left( \frac{e^{-\lambda_{\text{obs}}(\lambda_{\text{obs}})_m}}{m!} \right). \quad (15)$$

Note that the above depends on $\lambda$, for which we have a posterior (Equation 4). One can thus include this via marginalization, as before, where we use the bar symbol above our expectation value to denote that it is marginalization:

$$\langle V_{\text{tot}} \rangle = \int_\lambda \sum_{m=1}^{\infty} V_m \left( \frac{e^{-\lambda_{\text{obs}}(\lambda_{\text{obs}})_m}}{m!} \right) t_1 e^{-\lambda t_1} d\lambda. \quad (16)$$

To make progress, one can pull the sum outside, switch to units of time relative to $t_1$, and evaluate the individual integrals as

$$\langle V_{\text{tot}} \rangle = \sum_{m=0}^{\infty} \int_{\lambda=0}^{\infty} V_m \left( \frac{e^{-\lambda_{\text{obs}}(\lambda_{\text{obs}})_m}}{m!} \right) e^{-\lambda} d\lambda$$

$$= \sum_{m=0}^{\infty} V_m \left( \frac{t_{\text{obs}}^m}{(1+t_{\text{obs}})^{m+1}} \right) \left( \frac{\Gamma[m+1]}{m!} \right). \quad (17)$$

Finally, we normalize with respect to the cost by dividing by $t_{\text{obs}}$. This yields a monotonically decreasing function that peaks at zero with a value of $\text{BCR}_1$:

$$\frac{\text{BCR}}{\text{BCR}_1} = \sum_{m=0}^{\infty} \left( \frac{V_m}{v_1} \right) \left( \frac{t_{\text{obs}}^m}{(1+t_{\text{obs}})^{m+1}} \right) \left( \frac{\Gamma[m+1]}{m!} \right). \quad (18)$$

We consider three different toy models of valuing the $n^{th}$ detection. We caution against taking these models too seriously; they are here to guide our intuition and reveal the functional properties in this analysis. The first is a simple power-law of the form $v_n = v_1 n^{-\alpha}$ which implies, via Equation (14), that $v_m = v_1 (H_{m+1}^{(2)} - 1)$ where $H_n^{(2)}$ is the $n^{th}$ Harmonic number of order $\alpha$. A typical choice might be $\alpha = 1/2$, where the second detection is valued at $1/\sqrt{2}$ for the first example. Another reasonable choice might be $\alpha = 1$, which has imbues a quicker decline in value.

The second value model we consider starts from the philosophy of the summed values. If one has a sample of $l$ objects in hand, and one attempts to measure the mean of some property of this sample, our uncertainty on the mean would be proportional to $1/\sqrt{l}$. Let us define our value to be inversely proportional to the error on the mean, which means $v_n = v_1 / \sqrt{n}$, and summing with Equation (14), one finds $V_m = \sqrt{n} / \sqrt{m+1} - 1$.

The third and final value model considered is similar to the second, except we refine the assumption that value is inversely proportional to error on the mean. Instead, this is replaced with the KLD between two Gaussians centered on the same mean, with a standard deviation going from $\sigma \rightarrow \sigma / \sqrt{l}$ for a collection of $l$ objects. This essentially means that our value is directly proportional to the information gained about the properties of the anomaly. Accordingly, one finds $v_n = v_1 (\log(\sqrt{n}) - \log(\sqrt{n-1}))$ and thus $V_m = v_1 \log(m+1)$.

The value of each detection $v_n$, and the summed values $V_m$, are plotted in Figure 4. This reveals that the third valuation system, based on the KLD, yields the most pessimistic valuation over the range shown, where as the power-law of index $\alpha = 1/2$ is the most optimistic. Although we cannot analytically evaluate Equation (18) with these three functions, they are numerically evaluated them up to $t_{\text{obs}} = 10 t_1$ for $m = 10^4$ terms.

All of the functions are monotonically decreasing with $t_{\text{obs}}$ and peak at $t_{\text{obs}} = 0$. However, this does not mean the efforts are “unprofitable”, merely that they are not expected to be as profitable as the initial detection. For example, if the initial run was a Nobel prize winning discovery, the scientific value would be presumed to be extremely high and thus subsequent efforts would still be very scientifically profitable. In this vein, let us assume that $\text{BCR}_1 > 1$ (else ultimately it would probably not be considered interesting enough to warrant follow-up anyway). Let us quantify that $\text{BCR}_1 > 10$ to take this as one order-of-magnitude, then. Accordingly, any campaigns with $\text{BCR}/\text{BCR}_1 > 0.1$ would be deemed profitable and worthwhile. For the three models shown in Figure 4, this occurs at $\sim 100 t_1$ for the most optimistic valuation model, the power-law with $\alpha = 1/2$, and at $\sim 10 t_1$ for the most conservative valuation system, the KLD summation model.

From this, it is concluded that follow-up efforts of a black swan event are expected to remain scientifically worthwhile and “profitable” for the observers for time intervals/sample sizes/effort levels of $\lesssim 10$ times that of the original discovery, but may become unprofitable for longer times.

5 TWO SIMPLE APPLICATIONS

5.1 Case 1: A Current Black Swan

In order to show the derived results in action, we here present two simple applications. In the first case, we take an example of a recent astronomy detection which is currently considered a genuine black swan with no subsequent repetitions. In particular, we take the example of the detec-
tion of Breakthrough Listen Candidate 1 (BLC1) - a narrow bandwidth, high significance radio emission apparently originating from the direction of Proxima Centauri (Breakthrough Listen team, in prep).

BLC1 was detected by the Breakthrough Listen observational survey operated with the 64 m Parkes radio telescope (Price et al. 2018; Lebofsky et al. 2019; Price et al. 2020). The signal was found in archival observations from April to May 2019 during a focussed campaign on the source Proxima Centauri. The signal is likely some form of human radio transmission\(^2\), but could, in principle, originate from alien technology in the Proxima vicinity.

Investigation of the nature and origin of the signal continues (Breakthrough Listen team, in prep), but in what follows we proceed under the assumption it is a) of astrophysical origin, and b) repeats (potentially stochastically) over sufficiently long observing times. Ultimately, our formalism provides a way to ask what kind of observations would put tension on this hypothesis, and thus a pathway to eliminating the candidacy of this signal independent of specific tests, traits or properties of the one recorded signal.

The frequency of BLC1 (982 MHz) is such that it was only detected in one of the four receivers used by the project, namely the “Ultra Wideband Low” (UWL). Proxima Centauri was observed for a total of 1550 minutes over an interval of 1.75 years with UWL up to the point of its detection (A. Siemion, priv. comm.). How long it has been observed since then is fluid, and thus we will adopt these numbers for the purposes of this calculation.

From Equation 6, the recurrence time of BLC1 is expected to occur by a time \(t_2\), where \(t_2\) follows a probability distribution given by \([1 + (t_2/t_1)]^{-2}\). Consider that the source continues to be monitored without repetition. To 95\% confidence, we would expect this repetition to occur after a cumulative observing time of 29,450 minutes, or 19 times the original observing duration. This corresponds to 20.5 days of on-target observations.

The above considers only the on-target observing time. In practice, observing Proxima for 20.5 days could still miss a repeating signal if the signal’s characteristic repetition time is \(\gg 1550\) minutes \(\sim 1\) day. To account for this possibility, the wall-time of the 29,450 minutes (or more) of on-target observations would need to be spread over 19 times longer that the original wall-time, which is 33 years. Accordingly, a short-term repeating signal from Proxima could be excluded to 95\% confidence with \(\gtrsim 20\) days of continuous monitoring, but a long-term repeating signal could not be excluded until \(\gtrsim 33\) years of observations.

We note that our benefit-to-cost calculations do not apply here since the validity of the first signal is ambiguous, and thus the second signal would arguably have greater value by virtue of confirming the nature of the signal.

5.2 Case 2: A Once Black Swan

Let us turn to a case where repetition was observed and see how well the equations presented here perform. Whilst there are many historical examples of black swans later found to repeat, we here try to identify a case where the signal was identified and repeated within a homogeneous, single astronomical survey. As noted earlier, this is an inherent assumption to our formalism, in order neatly sidestep the issues of differing observational biases between surveys. This requirement eliminates most historical examples, but as suggested earlier, the era of large astronomical surveys such as Vera Rubin and the Roman telescope look primed to change this.

To this end, we take tidal disruption events (TDEs) observed photometrically by TESS. TESS pursues a systematic survey of the sky observing approximately the same number of sources at any given time (Ricker et al. 2015), for which the distribution of apparent magnitudes and colors is broadly consistent (Stassun et al. 2019). ASASSN-19bt represents the first TDE observed by TESS, observed simultaneously by the ground-based ASASSN survey (Holoien et al. 2019). The TDE was observed in sectors 7, 8 and 9 peaking at MJD 58550, and beginning to rise 40 days prior. Since

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\(^2\) See this URL

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\(\text{MNRAS} 000, 1-9\) (2020)
TESS observations began MJD 58324, then here we here have \( t_1 = 186 \text{ days} \) - as defined by the start of the slow rise.

Let us ask, when should we expect TESS to observe a second such TDE somewhere on the sky, based on the first. We restrict ourselves to non-repeating TDEs here, since repeating ones (e.g. Payne et al. 2020) could clearly lead to many such events from the same source - rather than really probing the rate at which the sky produces TDEs as seen by TESS (as well as representing an astrophysically distinct scenario). Accordingly, the second TDE observed by TESS was recently reported by Hinkle et al. (2020), ASASSN-20hx. This second TDE was identified in its slow rise phase at MJD 59040 (the peak intensity time has not yet been reported), corresponding to a time TESS’ mission of 716 days, or \( t_2 = 530 \text{ days} \) (2.85 times \( t_1 \)).

Although \( t_2 \) is clearly longer than \( t_1 \) by nearly a factor of 3, using the expressions of Section 3.3, the \( p \)-value of this occurring is 26% and thus hardly surprising. Indeed, the timing is fully consistent with the expectation of the sky producing a Poisson background rate of such TDEs. Further, we can evaluate that the expectation value for the benefit-to-cost ratio at time \( t_2 \) is 0.518, 0.288, 0.286 and 0.181 (in units of the BCR of the first detection) for the power-law 1/2, power-law 1, sigma sum and KLD sum methods, respectively. Thus, by all methods, a continued search for such events in the TESS was very much worthwhile (\( \geq 0.1 \)) by the point of the success.

6 DISCUSSION

In this work, we have considered the statistical implications of the detection of a black swan astronomical event. Although the work is framed largely in terms of time, where the event occurs at some fixed temporal rate, it can be equally applied to other sample types too (e.g. objects surveyed, frequencies scanned, wavelengths monitored), or indeed most generally "effort level", provided that the observational selection biases are homogeneous across these samples, however they are defined.

It is envisioned that this work will be beneficial in the future analysis of large astronomical survey data. Here, the assumption ofunchanging observational bias is most likely to hold, as surveys may adopt fixed strategies for multi-year periods. However, if the black swan event is low signal-to-noise, and was somehow fortunate to have been detected, then software improvements in search strategy would invalidate this assumption. However, large signal-to-noise events will likely not benefit noticeably from software improvements. Furthermore, if they were identifiable in the first place amongst a vast volume of data (as expected from upcoming surveys; Jurić et al. 2017) then they were likely identified with an automated process already, and so the assumption of constant selection effect is sound.

Implications of this work are that black swan events can have unintuitively long recurrence times. Whilst there is a 50% chance of the event repeating within one more unit of effort, 99 times is needed to reach a 99% chance. And thus, if one sought to exclude the hypothesis of repeatability via a \( p \)-value test, even collecting two-orders of magnitude more data only marginally excludes the hypothesis. Nevertheless, black swans detected in the early phases of surveys and then found to not repeat would be good candidates to apply this approach to.

Our work also considers the benefit-to-cost ratio of continuing to monitor black swan sources. Although valuing scientific discoveries is somewhat ill-defined, we find that amongst several possible models, the most conservative models suggest observing for up to 10 times longer than the initial discovery run is generally expected to worthwhile. Since an assumption of this work is a fixed observing mode, one might question the applicability of these results since the survey mode may be unalterable regardless. However, some surveys, such as Breakthrough Listen for example (Isaacson et al. 2017), routinely monitor the same sample of stars with unchanging sensitivities but the dwell-time on each target can be easily changed in response to black swan events. As another example, even if the observing mode is fixed, the effort-level - in particular the computational resources devoted to a search within a data set - are resource-limited and thus could be similarly optimized.

Our work provides a starting point for statistically interpreting black swans, but each event will undoubtedly have its own curiosities and quirks deserving bespoke attention.

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DATA AVAILABILITY

No data was used in the preparation of this manuscript.

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