An extendable Space-Time Curvature Mode model

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Abstract

Einstein calculated the curvature of space-time from a Field equation based on an equation of motion derived by the variation of a geodetic path and a second rank Metric tensor. In this study a Space-Time Curvature Mode (STCM) model is calculated from a series of coupled Field equations derived from the variation of a Taylor series expansion of a Lagrangian potential of the STCM. The Taylor series expansion coefficients are Metric tensors of increasing rank. The STCM model can be extended to any number of Field equations. The coupled Field equations of the STCM describe the warping of space-time by masses and energy fields, and the effect of the curved space-time on the motion of masses and energy fields. This results in a description of space coordinates that have locally positive and negative slopes as a function of the time coordinate and can describe a Universe with at times an accelerating expansion and at later times a contracting Universe. This study replaces the postulated concepts of Einstein’s Cosmological constant and postulated concepts of Dark Mass and Dark Energy. It is shown that the Hamiltonian is conserved.

1. Introduction and summary

In this study, an extendable Space-Time Curvature Mode (STCM) model using the variation of a Taylor series expansion of a Lagrangian potential of the STCM was derived. This resulted in a series of Metric tensors of increasing rank to describe the STCM. In the STCM, the spatial coordinates can have at times a positive slope, and at later times a negative slope, as a function of the time coordinate. Here is a brief history which led to the derivation of the STCM.

The 1916 General Relativity Theory (GRT) [1, 2] was derived from basic principles by the variation of a geodetic path using a second rank Metric tensor. It has a parabolic curved space-time solution. This describes a Universe with either a slowing expansion or a shrinking Universe. Since the Universe at that time, was thought to be constant, Einstein added a constant multiplied by the Metric tensor to the GRT Field Equation. Hubble [3] in 1929 observed that the universe is expanding. Thinking this proved his original derivation of the GRT is correct, Einstein removed the Cosmological constant. In 1998 astronomical observations showed that the Universe’s expansion is accelerating and the rotation curve of stars farther than 5 kpc or 16.3078 light years from the central Black Hole of the Milky Way Galaxy is faster than predicted by calculations of the Kepler Newton Theory. This contradicted the GRT which described gravity causing masses being attracted to each other. To explain this surprising observation, Dark Matter (DM) and Dark Energy (DE) [4–6] were postulated. For more than 20 years, experiments were conducted to measure DM or DE. All these experiments failed consistently to find any trace of DM or DE. Perhaps, DM and DE are like the Aether of the early 20th century through which light waves were supposed to propagate. Michelson’s and Morley’s experiment [7], and Einstein’s theory disproved the existence of the Aether.

In this study, an alternate STCM model is derived from basic principles by the variation of a Taylor series expansion of a Lagrangian potential of the STCM to replace the postulated concepts of Einstein’s cosmological constant and the DM and DE. This results in a wave Equation with a series of Metric tensors with increasing rank. The coupled Field Equations of the STCM have solutions of a multi curved space-time. The curvature of
the space coordinates can have negative and positive local slopes as a function of the time coordinate. Therefore, the STCM can describe a Universe with at times an accelerating expansion and at later times a contracting expansion. This mathematical model reverts to the GRT and also Newton’s model of Nature in limits. The Lagrangian potential is not a function of the energies of an object, it is a function of the STCM.

We first hypothesize that the STCM are the coordinates themselves, where time, multiplied by the speed of light, is one of the four coordinates. The STCM consists of an infinite number of sets of values of the four coordinates. The STCM is described by each of the coordinates being a function of the other three coordinates. There are no other external physical coordinates with respect to which the STCM can change. We postulate that the STCM can not be observed from the outside because the observers coordinates must also be part of the STCM.

We secondly hypothesize that if the STCM could change, then according to the theory of relativity the effects causing the STCM to change would also cause the measurements to change the same way, and therefore it would appear that the STCM does not change, it remained constant.

However, the motion inside the STCM can be described by selecting one coordinate of the four, and then observing the change in the remaining three coordinates for different consecutive values of the selected coordinate. If the selected coordinate is the time coordinate, we observe a change in the other coordinates, which are the three spatial coordinates, with time. Here, the function of the spatial coordinates as a function of time is derived from the STCM instead of Newton’s Equations.

We thirdly hypothesize that only actions described in the STCM can cause space-time to warp. Just as there are no coordinates external to the STCM, there are no external interactions. All interactions are described by the STCM. We will need to distinguish between ‘flat space-time’ and ‘curved space-time’. The total STCM is constant. If there are no interactions that will cause space-time to warp then space-time is constant. As described before both the coordinates and interactions are in the STCM.

Our fourth hypothesis is that all derivatives of functions of the coordinates in constant space-time are equal to zero.

The wave Equation is derived from the variation of the STCM which in the second hypothesis was shown to be constant and thus, its variation is equal to zero.

The Taylor series expansion coefficients are higher-order derivatives of the Lagrangian potential. These derivatives are Metric tensors of increasing rank. The odd expansion terms of the Taylor series are zero because of symmetry. The Lagrangian potential expressed by a Taylor series is a sequence of scalar products of Metric tensors and coordinate increments.

The zeroth-order term of the Taylor series is a scalar. The expansion coefficients are derivatives of the scalar Lagrangian potential with respect to the coordinates, and therefore are components of covariant tensors. The second-order expansion coefficient is defined to be equal to the conventional second-rank metric tensor components of Einstein’s GRT [1, 2]. This is a condition that has to be included with the solution. The higher-order expansion terms of the Taylor series are the higher rank Metric tensor components.

The variation of the zero-order term of the Taylor series is the Euler Lagrange equation of Classical Mechanics [8]. The variation of the second-order terms form Einstein’s equation of motion of the GRT [1, 2]. The variation of the higher-order expansion coefficients form higher-order terms including a higher-order Christoffel symbol. The higher-order Christoffel symbol, together with the terms that follow it form a higher-order addition to the GRT. All the calculations in this paper are derived from basic principles by the use of the Calculus of Variation.

The Equation of motion is interpreted here as a wave equation for the portions of the Space-Time Curvature Mode rather than a description of the motion of an object. Here, the function of the spatial coordinates as a function of time coordinate is derived from the STCM instead of Newton’s Equations. It describes the everyday world. These also describe gravity waves.

It would be interesting to calculate the Kretschmann invariant [9] which is a scalar, to observe the effect of the additional terms of the Taylor series would have on the various singularities in the extended GRT.

A Hamiltonian potential for the Lagrangian potential including the effect of the higher-order terms of the Taylor series was derived. The Hamiltonian potential is the Legendre transform of the Lagrangian potential. The change of the Hamiltonian is equal to zero, and thus, the Hamiltonian is conserved.

2. Wave equation and Christoffel symbols

In a flat space, and only in a rectangular coordinate system is each coordinate independent of the other three coordinates. In a flat space, in general curvilinear coordinates, each coordinate can be a function of any number of the other coordinates. To avoid confusion with the curvature of the space-time coordinates, the discussion will be restricted to rectangular coordinates.
This derivation is based on a Lagrangian potential of the Space-Time Curvature Mode (STCM). Our fifth hypothesis is that the Lagrangian potential and its derivatives are continuous and analytic in all of space-time. The Lagrangian potential \( \mathcal{L} \) is a function of the Minkowski space-time coordinates \( x_{\mu} \in \mathbb{M}(4) \). The space-time coordinates \( x_{\mu}(\lambda) \) in turn, are functions of a parameter \( \lambda \in \mathbb{R}(1) \). The Lagrangian potential is not an explicit function of the parameter \( \lambda \). The parameter \( \lambda \) is similar to the way time is used in the theory of Euler and Lagrange. The parameter facilitates the calculation and it does not have a physical meaning, however, here it has a dimension of time. Expanding the Lagrangian potential \( \mathcal{L} \) and a constraint \( C \), in a Taylor series in the coordinates \( x^{\alpha} \in \mathbb{M}(4) \),

\[
\mathcal{L} \equiv \mathcal{L}_0 + \frac{1}{2!} \partial_{\alpha} \partial_{\beta} \mathcal{L}_0 \Delta x^\alpha \Delta x^\beta + \frac{1}{2!} \partial_{\alpha} \partial_{\beta} C \Delta x^\alpha \Delta x^\beta \\
+ \frac{1}{4!} \partial_{\alpha} \partial_{\beta} \partial_{\gamma} \partial_{\delta} \mathcal{L}_0 \Delta x^\alpha \Delta x^\beta \Delta x^\gamma \Delta x^\delta + \frac{1}{4!} \partial_{\alpha} \partial_{\beta} \partial_{\gamma} \partial_{\delta} C \Delta x^\alpha \Delta x^\beta \Delta x^\gamma \Delta x^\delta + \text{even higher order terms (1)}
\]

Because the Lagrangian potential \( \mathcal{L} \) is a scalar, the Taylor series expansion coefficients such as \( \frac{1}{2!} \partial_{\alpha} \partial_{\beta} \mathcal{L}_0 \) are components of a covariant tensors. To be consistent with Einstein’s tensor multiplication convention the \( \Delta x^\alpha \) are contravariant vectors.

1 Dividing and multiplying the expansion coefficients and the coordinate increments \( \Delta x^\alpha \) by the parameter increment \( \Delta \lambda \), and approximating the increment ratio \( \frac{\Delta x^\alpha}{\Delta \lambda} \) by its limit, which is the derivative \( \frac{dx^\alpha}{d\lambda} \).

\[
\mathcal{L} \equiv \mathcal{L}_0 + \frac{1}{2!} \frac{\partial^2 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta} (\Delta \lambda)^2 \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} + \frac{1}{2!} \frac{\partial^2 C}{\partial x_\alpha \partial x_\beta} (\Delta \lambda)^2 \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \\
+ \frac{1}{4!} \frac{\partial^4 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta \partial x_\gamma \partial x_\delta} (\Delta \lambda)^4 \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} \frac{dx^\delta}{d\lambda} + \text{even higher order terms (2)}
\]

The constraint \( C \) in this case, describes that a path length in space-time is constant and uniform.

\[
(a) \ ' \ = \ - \frac{1}{\Delta \lambda^2} x^\alpha x_\alpha \] (b) \ ' \ = \ 0 \quad \text{where (c) } \frac{1}{\Delta \lambda^2} x^4 \equiv - \frac{1}{\Delta \lambda^2} c^2 t^2 \quad (3)
\]

Because only a variation of the constraint is used, the length of the path in space-time is arbitrary. The length of the path in space-time is chosen to be zero, \( x^\alpha x_\alpha = c^2 t^2 = 0 \).

The first term \( \mathcal{L}_0 \) of equation (2) is a scalar. The 2nd and 3rd expansion coefficients of equation (2) are four-dimensional covariant second-rank metric tensor components \( g_{\alpha \beta} \). The 4th term of equation (2) are the components of a four-dimensional fourth-rank covariant metric tensor \( h_{\alpha \beta \gamma \delta} \) and the 5th term is equal to zero.

\[
(a) \ g_{\alpha \beta} \equiv \frac{1}{2!} \frac{\partial^2 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta} (\Delta \lambda)^2 + \frac{\partial^2 C}{\partial x_\alpha \partial x_\beta} (\Delta \lambda)^2 \quad \text{(b) } g_{\alpha \beta} \equiv \frac{1}{2!} \frac{\partial^2 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta} (\Delta \lambda)^2 + \eta_{\alpha \beta} \\
(c) \ h_{\alpha \beta \gamma \delta} \equiv \frac{1}{4!} \frac{\partial^4 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta \partial x_\gamma \partial x_\delta} (\Delta \lambda)^4 \quad \text{(4)}
\]

The \( \eta_{\alpha \beta} \) is a component of the Minkowski tensor. It is possible to have even higher rank metric tensors with components such as \( h_{\alpha \beta \gamma \delta \epsilon \zeta} \) etc.

Because the Lagrangian potential \( \mathcal{L}_0 \) has dimensions of velocity squared, the expansion coefficient \( \frac{1}{2!} \frac{\partial^2 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta} (\Delta \lambda)^2 + \eta_{\alpha \beta} \) is a dimensionless component of a metric tensor. The \( \frac{1}{4!} \frac{\partial^4 \mathcal{L}_0}{\partial x_\alpha \partial x_\beta \partial x_\gamma \partial x_\delta} (\Delta \lambda)^4 \), is a component of a higher-rank metric tensor, with dimensions of reciprocal velocity squared.

Because the form of the Lagrangian potential is not specified at this point of the calculation, it can be required that the tensor with components \( g_{\alpha \beta} \) have properties consistent with the Metric tensor of the GRT [1, 2]. Here a Lorentzian manifold with a signature \(- ++ + +\) is used.

\[
(a) \ g_{\alpha \beta} = g_{\alpha \beta} \quad \text{(b) } g^{\alpha \beta} g_{\beta \gamma} = \delta^{\alpha \gamma} \quad \text{(c) } g^{\alpha \beta} g_{\beta \mu} = 4 \quad \text{(d) } \det(\delta^{\alpha \beta}) = 1 \\
(e) \ \det(g^{\alpha \beta} g_{\alpha \beta}) = \det(\delta^{\alpha \beta}) \cdot \det(g_{\alpha \beta}) = 1 \quad \text{(f) } \det(g^{\alpha \beta} g_{\alpha \beta}) = 1 \quad \text{(h) } \det(g_{\alpha \beta}) \det(g^{\alpha \beta}) = 1 \quad \text{(5)}
\]
Here $\det(g_{ab})$ denotes the determinant of $g_{ab}$. According to our fourth hypothesis that in flat space-time all derivatives with respect to the coordinates are zero the derivatives of the Lagrangian in flat space are zero. Thus the tensor with components $g_{\alpha\beta}$ reverts to the Minkowski tensor with components $\eta_{\alpha\beta}$. The tensor with components $h_{\alpha\beta\gamma\delta}$, and any higher expansion coefficients go to zero.

The Euler–Lagrange equations can be used to calculate the wave equation of the STCM, and it is equivalent to the derivation from the Calculus of Variation. However, here we prefer to use the more basic derivation using the Variation of the Taylor series expansion.

As described before the variation is performed in curved space-time, the STCM. The STCM $\mathcal{I}$ is a function of the coordinates, and the coordinates are functions of the one-dimensional parameter $\lambda \in \mathbb{R}(1)$. The STCM is the integral of the Lagrangian potential $\mathcal{L}$ from $\lambda_a$ to $\lambda_b$.

\[
\begin{align*}
(a) \quad \mathcal{I} &= \int_{\lambda_a}^{\lambda_b} \mathcal{L} d\lambda \mathcal{T} \text{ is a constant} \\
(b) \quad \mathcal{I} &= \int_{\lambda_a}^{\lambda_b} \left[ \mathcal{L}(x^\alpha) + g_{\alpha\beta}(x^\beta) \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} + h_{\alpha\beta\gamma\delta}(x^\gamma) \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} \frac{dx^\delta}{d\lambda} + \ldots \right] d\lambda 
\end{align*}
\]

As previously explained the STCM $\mathcal{I}$ is constant and the variation of the STCM $\mathcal{I}$ is equal to zero. To facilitate this variation a dummy variable $\xi \in \mathbb{R}(1)$ was introduced $x^\alpha = x^\alpha(\lambda, \xi)$. This is a Hamiltonian process applied to the STCM. The variation of the integral of equation (6) is equal to zero. Performing the variation $\frac{\delta \mathcal{I}}{\delta \xi}$ of the Integral of the Lagrangian potential along a path in the one-dimensional $\lambda$ parameter space with some changes in the indices.

\[
\begin{align*}
(a) \quad \frac{\delta \mathcal{I}}{\delta \xi} &= 0 \\
(b) \quad \frac{\delta \mathcal{I}}{\delta \xi} &= \int_{\lambda_a}^{\lambda_b} \left[ \frac{\partial \mathcal{L}}{\partial x^\alpha} \frac{\delta x^\alpha}{\delta \xi} + \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \frac{\delta x^\mu}{\delta \xi} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} + g_{\alpha\beta} \frac{d^2 x^\alpha}{d\lambda d\xi} \frac{dx^\beta}{d\lambda} + g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \frac{d\xi}{d\lambda} \frac{d\xi}{d\lambda} + \ldots \right] d\lambda 
\end{align*}
\]

It is hypothesized that both the Lagrangian potential $\mathcal{L}$ and the tensor components $g_{\alpha\beta}$ and $h_{\alpha\beta\gamma\delta}$ and their derivatives are analytic and continuous in the interval between $\lambda_a$ to $\lambda_b$, of the parameter $\lambda$, and assuming that the variation of the coordinates such $\frac{\delta x^\alpha}{\delta \xi}$ is equal to zero at the ends of the integration at $\lambda$ equal to $\lambda_a$ and to $\lambda_b$.

The 3rd, 4th, 6th, 7th, 8th, and 9th terms of equation (7) must be integrated by parts. Expanding the third term in the bracket of equation (7b)

\[
\begin{align*}
\frac{d}{d\lambda} \left[ g_{\alpha\beta} \frac{\delta x^\mu}{\delta \xi} \frac{dx^\beta}{d\lambda} \right] &= \frac{\partial g_{\alpha\beta}}{\partial x^\lambda} \frac{dx^\alpha}{d\lambda} \frac{\delta x^\beta}{d\lambda} \frac{d^2 x^\lambda}{d\lambda d\xi} \frac{d\lambda}{d\lambda} \\
&= g_{\alpha\beta} \frac{d^2 x^\mu}{d\lambda d\xi} \frac{dx^\beta}{d\lambda} 
\end{align*}
\]

Expanding the fourth term in the bracket of equation (7b)

\[
\begin{align*}
\frac{d}{d\lambda} \left[ g_{\alpha\beta} \frac{\delta x^\mu}{\delta \xi} \frac{dx^\beta}{d\lambda} \right] &= \frac{\partial g_{\alpha\beta}}{\partial x^\lambda} \frac{dx^\alpha}{d\lambda} \frac{\delta x^\beta}{d\lambda} \frac{d^2 x^\lambda}{d\lambda d\xi} \frac{d\lambda}{d\lambda} \\
&= g_{\alpha\beta} \frac{d^2 x^\mu}{d\lambda d\xi} \frac{dx^\beta}{d\lambda} 
\end{align*}
\]

One can, of course use the Euler Lagrange Equation [3] to derive a wave Equation of the STCM. However, here the wave Equation is obtained by basic principles using the Calculus of Variation.
Expanding the sixth term in the bracket of equation (7b)

\[
\frac{d}{d\lambda} \left[ \sum_{\mu} \delta x^{\mu} \frac{d}{d\lambda} \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\xi} \right] = \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\xi} \delta x^{\mu} - \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\lambda} \delta x^{\mu}
\]

\[= \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\xi} \delta x^{\mu} - \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\lambda} \delta x^{\mu}
\]

\[= \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\xi} \delta x^{\mu} - \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\lambda} \delta x^{\mu}
\]

\[= \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\xi} \delta x^{\mu} - \sum_{\mu} \frac{\partial}{\partial \xi} \delta x^{\mu} \frac{d}{d\lambda} \left[ \sum_{\alpha} \delta x^{\alpha} \frac{d x^{\beta} \delta x^{\gamma} \delta x^{\delta}}{d\lambda} \right] \frac{d x^{\gamma} \delta x^{\delta}}{d\lambda} \delta x^{\mu}
\]

The 7th, 8th, and 9th terms of equation (7b) can be expanded similarly to the way the 3rd, 4th and the 6th terms are expanded. Substituting these resulting 6 Equations back into equation (7b).
terms following it are higher-order extensions of the GRT. All these terms were derived from basic principles using the Calculus of Variation.

Collecting terms and factoring out \( \dddot{x} \phi \):

\[
(\partial_\beta g_{\alpha\beta} - \partial_\beta g_{\beta\alpha} - \partial_\alpha g_{\beta\alpha}) \dot{x}^\alpha \dot{x}^\beta - 2g_{\mu\nu} \ddot{x}^\mu
\]

\[+ (\partial_\mu h_{\alpha\beta\gamma} - \partial_\mu h_{\beta\alpha\gamma} - \partial_\beta h_{\alpha\mu\gamma} - \partial_\alpha h_{\beta\mu\gamma} - \partial_{\gamma} h_{\alpha\beta\mu}) \dddot{x}^\alpha \dddot{x}^\beta \dddot{x}^\gamma + 2h_{\gamma\mu\nu} \dddot{x}^\alpha \dddot{x}^\beta + \text{even higher order terms} = 0
\]

(14)

The first bracket with terms in equation (14), is equal to \(-2\) times the Christoffel symbol \[\begin{bmatrix}
\alpha \\ \beta \\ \mu
\end{bmatrix}\] of the first kind [1]. Similarly, the second bracket of equation (14), with five terms, is equal to \(-2\) times the higher-order Christoffel symbol \[\begin{bmatrix}
\alpha \\ \beta \\ \gamma \\ \delta \\ \mu
\end{bmatrix}\] of the first kind. There are four indices \(\alpha, \beta, \gamma,\) and \(\delta\) that appear in pairs in the square bracket. There are 6 ways in which 4 objects can be taken 2 at a time. Therefore, there are 6 terms in the square bracket. The sum of the terms in the square bracket of equation (14) are a second-rank tensor \(\theta_{\mu\phi}\):

\[
\theta_{\mu\phi} \equiv h_{\mu\phi\gamma} \dddot{x}^\gamma \dddot{x}^\phi + h_{\mu\phi\delta} \dddot{x}^\delta \dddot{x}^\phi + h_{\mu\phi\gamma} \dddot{x}^\gamma \dddot{x}^\phi + h_{\mu\phi\gamma} \dddot{x}^\gamma \dddot{x}^\phi
\]

(15)

The tensor with components \(\theta_{\mu\phi}\) which augments the metric tensor with components \(g_{\mu\phi}\) is dimensionless. Substituting equation (15) into equation (14) yields

\[
0 = (\partial_\beta g_{\alpha\beta} - \partial_\beta g_{\beta\alpha} - \partial_\alpha g_{\beta\alpha}) \dot{x}^\alpha \dot{x}^\beta - 2(g_{\mu\phi} + \theta_{\mu\phi}) \dddot{x}^\phi
\]

\[+ (\partial_\mu h_{\alpha\beta\gamma} - \partial_\mu h_{\beta\alpha\gamma} - \partial_\beta h_{\alpha\mu\gamma} - \partial_\alpha h_{\beta\mu\gamma} - \partial_{\gamma} h_{\alpha\beta\mu}) \dddot{x}^\alpha \dddot{x}^\beta \dddot{x}^\gamma + \text{even higher order terms}
\]

(16)

The first two terms of equation (16) less \(\theta_{\mu\phi}\) are Einstein’s Equation of motion of the GRT. By multiplying the first and third terms of equation (16) by the quantity \(1/2(g^{\mu\nu} - \Omega^{\mu\nu})\) one obtains modified Christoffel symbols of the second kind. By multiplying the second term of equation (16) by the quantity \(1/2(g^{\mu\nu} - \Omega^{\mu\nu})\) one obtains the components of the delta function, and where the assumed value \(\Omega^{\mu\nu}\) has yet to be determined.

\[
0 = \frac{1}{2}(g^{\mu\nu} - \Omega^{\mu\nu})(\partial_\beta g_{\alpha\beta} - \partial_\beta g_{\beta\alpha} - \partial_\alpha g_{\beta\alpha}) \dot{x}^\alpha \dot{x}^\beta - (g^{\mu\nu} - \Omega^{\mu\nu})(g_{\mu\phi} + \theta_{\mu\phi}) \dddot{x}^\phi
\]

\[+ \frac{1}{2}(g^{\mu\nu} - \Omega^{\mu\nu})(\partial_\mu h_{\alpha\beta\gamma} - \partial_\mu h_{\beta\alpha\gamma} - \partial_\beta h_{\alpha\mu\gamma} - \partial_\alpha h_{\beta\mu\gamma} - \partial_{\gamma} h_{\alpha\beta\mu}) \dddot{x}^\alpha \dddot{x}^\beta \dddot{x}^\gamma + \text{even higher order terms}
\]

(17)

In order for the term multiplying the quasi acceleration component \(\dddot{x}^\phi\) in equation (17) to be equal to a component of a Minkowski tensor, it is required that the product \(1/2(g^{\mu\nu} - \Omega^{\mu\nu})(g_{\mu\phi} + \theta_{\mu\phi})\) to be equal to a component \(\delta^\phi_{\mu}\) of a delta function.

\[
(a) (g^{\mu\nu} - \Omega^{\mu\nu})(g_{\mu\phi} + \theta_{\mu\phi}) : = \delta^\phi_{\mu}
\]

\[
(b) \eta^\phi_{\mu} - g^{\nu\phi}g_{\nu\phi} - \Omega^{\nu\phi}(g_{\nu\phi} + \theta_{\nu\phi}) = \delta^\phi_{\mu}
\]

\[
(c) g^{\nu\phi}g_{\nu\phi} = \Omega^{\nu\phi}(g_{\nu\phi} + \theta_{\nu\phi})
\]

(18)

The 16 components of the tensor with components \(\Omega^{\mu\nu}\) can be calculated from the 16 Equations represented by equation (18c).

Defining the components of the modified Christoffel symbols of the second kind:

\[
(a) \Gamma^\phi_{\alpha\beta} \equiv \frac{1}{2}(g^{\mu\nu} - \Omega^{\mu\nu})(\partial_\mu g_{\alpha\beta} - \partial_\beta g_{\alpha\mu} - \partial_\alpha g_{\beta\mu})
\]

\[
(b) \Pi^\phi_{\alpha\beta\gamma} \equiv \frac{1}{2}(g^{\mu\nu} - \Omega^{\mu\nu})(\partial_\mu h_{\alpha\beta\gamma} - \partial_\mu h_{\beta\alpha\gamma} - \partial_\beta h_{\alpha\mu\gamma} - \partial_\alpha h_{\beta\mu\gamma} - \partial_{\gamma} h_{\alpha\beta\mu})
\]

(19)

The Christoffel symbol \(\Gamma^\phi_{\alpha\beta}\) is symmetric \(\Gamma^\phi_{\alpha\beta} = \Gamma^\phi_{\beta\alpha}\) in the covariant indices \(\alpha\) and \(\beta\) and has dimensions of reciprocal length \(1\). A higher-order Christoffel symbol, with components \(\Pi^\phi_{\alpha\beta\gamma}\) shown in equation (19b) is also considered in the discussion in the curvature of space-time in the Riemann geometry by Kurt Zwieker [10]. This fifth rank tensor with components \(\Pi^\phi_{\alpha\beta\gamma}\) is symmetric in the covariant indices \(\alpha\) and \(\beta\) and also in the indices \(\gamma\) and \(\delta\). It has dimensions of time squared divided by the reciprocal length to the third power \(T^2/\ell^3\), and it is due to
the fourth-order Taylor series expansion coefficients. Equation (17) can be solved if the values of the components $g_{\alpha\beta}$ and $h_{\alpha\beta\gamma}$ of the Metric tensors are known. Substituting the modified Christoffel symbols with components $\Gamma^\rho_{\alpha\beta}$, and the Christoffel symbol for the higher-order terms with components $\Gamma^\rho_{\alpha\beta\gamma}$ of equation (19) and equation (18a) into equation (17).

\[
0 = \Gamma^\rho_{\alpha\beta} \dot{x}^\alpha \ddot{x}^\beta - \dddot{x}^\rho + \Gamma^\rho_{\alpha\beta\gamma} \ddot{x}^\alpha \ddot{x}^\beta \ddot{x}^\gamma + \text{even higher order terms}
\]  

(20)

Equation (20) is the wave Equation of the Extendable Space-Time Curvature Mode Model. The first two terms of equation (20) are similar to Einstein’s Equation of motion of the GRT. The remaining term is due to the extension of the Space-Time Curvature Mode Model. Here equation (20) is interpreted as a non-linear wave Equation for the coordinates $x^\alpha$ instead of the Equation of a material point as Einstein [1] described. Each coordinate $x^\alpha$ calculated from equation (20) is a function of the parameter $\lambda$. The values of the four coordinates $x^0$, $x^1$, $x^2$, $x^3$ for each value of the parameter $\lambda$ of the nonlinear wave Equation, equation (20), describe the Space-Time Curvature Mode.

The motion in the STCM can be explained by selecting one coordinate, and the remaining three coordinates have different values for each value of the selected coordinate. If a coordinate is selected, it is possible for the remaining coordinates to have several values for each value of the selected coordinate. This would, for example, describe a ‘Worm Hole’. If the selected coordinate is the time coordinate and the remaining three coordinates have multiple values for each particular value of the time coordinate, this could describe ‘Multiverses’.

Equation (20), the wave Equation for the STCM, can be written in a more compact form.

\[
0 = X^\rho_{\alpha\beta} \dddot{x}^\alpha + \dddot{x}^\rho + \text{even higher order terms}
\]  

(21)

where

\[
X^\rho_{\alpha\beta} \equiv \Gamma^\rho_{\alpha\beta} + \Pi^\rho_{\alpha\beta\gamma} \ddot{x}^\gamma
\]  

(22)

3. Riemann Christoffel tensor and field equations

The curvature of space-time is described by the Riemann Christoffel tensor. To derive the Riemann Christoffel tensor two scalar functions $\phi(x_\alpha)$ and $\psi(x_\alpha)$ of the coordinates $\psi(x_\alpha)$ of the curved space-time are hypothesized. The functions $\phi(x_\alpha)$ and $\psi(x_\alpha)$ have dimensions of potential or velocity squared and are not explicit functions of the parameter $\lambda$. As stated, before the parameter $\lambda$ has dimensions of time, but has no physical significance, and only facilitates the calculation. The calculation of the Riemann Christoffel tensor starts with calculating the changes of the functions $\phi(x_\alpha)$ and $\psi(x_\alpha)$ with the parameter $\lambda$:

\[
\begin{align*}
(a) \quad & \frac{d\phi}{d\lambda} = \partial_\alpha \phi \frac{dx_\alpha}{d\lambda} \\
(b) \quad & \frac{d\phi}{d\lambda} = A_{\alpha}\dot{x}^\alpha \\
(c) \quad & \frac{d\psi}{d\lambda} = \partial_\alpha \psi \frac{dx_\alpha}{d\lambda} \\
(d) \quad & \frac{d\psi}{d\lambda} = B_{\alpha}\dot{x}^\alpha
\end{align*}
\]  

(23)

where $\partial_\alpha \phi$ and $\partial_\alpha \psi$ are the components of a first-rank covariant tensor or vector. By taking the derivative with respect to $\lambda$ of equation (23) one obtains the curvatures $\frac{d^2\phi}{d\lambda^2}$ and $\frac{d^2\psi}{d\lambda^2}$ of the scalar functions $\phi$ and $\psi$ in the parameter $\lambda \in \mathbb{R}(1)$ space:

\[
\begin{align*}
(a) \quad & \frac{d^2\phi}{d\lambda^2} = \frac{d}{d\lambda}[(\partial_\beta \phi) \dot{x}^\beta] \\
(b) \quad & \frac{d^2\phi}{d\lambda^2} = (\partial_\alpha V_\beta) \dot{x}^\alpha \dot{x}^\beta + V_\beta \ddot{x}^\beta \\
\text{and similar equations for } & \psi
\end{align*}
\]  

(24)

Substituting equation (21) for the quasi-acceleration term $\dddot{x}^\beta$ into equation (24).

\[
\frac{d^2\phi}{d\lambda^2} = (\partial_\alpha V_\beta + V_\rho X^\rho_{\alpha\beta}) \dot{x}^\alpha \dot{x}^\beta
\]  

(25)

Setting the curvature $\frac{d^2\phi}{d\lambda^2}$ in the $\lambda$ parameter space to be equal to a scalar product of a second-rank tensor and $\lambda$ space quasi-velocities. $\frac{d^2\phi}{d\lambda^2} \Rightarrow F_{\alpha\beta} \dot{x}_\alpha \dot{x}_\beta$. Here $F_{\alpha\beta} \dot{x}_\alpha \dot{x}_\beta = \partial_\alpha (V_\beta \ddot{x}^\beta)$

\[
F_{\alpha\beta} = \partial_\alpha V_\beta + V_\rho X^\rho_{\alpha\beta}
\]  

(26)

The calculations that follow are independent whether equation (26) derived from $\phi$ or $\psi$ is used. In curved space-time the second-rank tensor $F_{\alpha\beta} \equiv \partial_\alpha V_\beta + V_\rho \Gamma^\rho_{\alpha\beta}$ is an extension of the first-rank tensor or vector $V_\beta$. 


As will be shown below, the change with the parameter $\lambda$ of the product of the slopes $\frac{d\phi}{d\lambda}$ and $\frac{d\psi}{d\lambda}$ of the scalar functions $\phi$ and $\psi$ are equal to zero, the covariant derivative reduces to the ordinary derivative. These can be calculated using equation (23).

\begin{align}
(a) \quad \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) &= \frac{d}{d\lambda} (A, x) \quad (B, x) \gamma \\
(b) \quad \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right) &= \frac{d}{d\lambda} (B, x) \gamma 
\end{align}

(27)

In curved space-time the covariant derivatives are not equal for cases where the equivalent conventional derivatives in an equivalent flat space are equal. Therefore, in curved space-time the covariant derivatives obtained from equations (27a) and (27b) are different.

$$\zeta = \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) - \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right)$$

(28)

5Here $\zeta$ is a scalar. Performing the differentiation with respect to the parameter $\lambda$ of equation (27a) where $S^{(1)}_{\alpha\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} = A, x, x^{\alpha} x^{\beta} x^{\gamma}$.

\begin{align}
(a) \quad \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) &= \frac{d}{d\lambda} (S^{(1)}_{\alpha\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma}) \\
(b) \quad \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right) &= (\partial_\alpha S^{(1)}_{\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} + S^{(1)}_{\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} + S^{(1)}_{\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma})
\end{align}

(29)

Changing indices in equation (21) and substituting the results into equation (29b).

\begin{align}
(a) \quad \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) &= \left[ \partial_\alpha S^{(1)}_{\beta\gamma} + S^{(1)}_{\beta\gamma} X^{\alpha}_{\alpha\beta} + S^{(1)}_{\beta\gamma} X^{\alpha}_{\alpha\gamma} \right] x^{\alpha} x^{\beta} x^{\gamma} \\
(b) \quad \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right) &= \left[ \partial_\beta S^{(2)}_{\gamma\alpha} x^{\alpha} x^{\beta} x^{\gamma} + S^{(2)}_{\gamma\alpha} x^{\alpha} x^{\beta} x^{\gamma} + S^{(2)}_{\gamma\alpha} x^{\alpha} x^{\beta} x^{\gamma} \right]
\end{align}

(30)

Where the third-rank tensor with components $U^{(1)}_{\alpha\beta\gamma}$ has the form of a covariant derivative of the second-rank tensor with components $S^{(1)}_{\alpha\beta}$. In the limit of flat space where the $X^{\alpha}_{\beta\gamma}$ are equal to zero, the covariant derivative reduces to the ordinary derivative.

Performing the differentiation with respect to the parameter $\lambda$ of equation (27b) where $S^{(2)}_{\alpha\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} = B, x, x^{\alpha} x^{\beta} x^{\gamma}$.

\begin{align}
(a) \quad \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) &= \frac{d}{d\lambda} (B, x) \gamma \\
(b) \quad \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right) &= (\partial_\beta S^{(2)}_{\gamma\alpha} x^{\alpha} x^{\beta} x^{\gamma})
\end{align}

(31)

Changing indices in equation (21) and substituting the results into equation (31b).

\begin{align}
(a) \quad \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right) &= \left[ \partial_\gamma S^{(2)}_{\alpha\beta} + S^{(2)}_{\alpha\beta} X^{\alpha}_{\alpha\beta} + S^{(2)}_{\alpha\beta} X^{\alpha}_{\alpha\gamma} \right] x^{\alpha} x^{\beta} x^{\gamma} + \text{ even higher terms} \\
(b) \quad \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) &= \left[ \partial_\alpha S^{(2)}_{\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} + S^{(2)}_{\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} + S^{(2)}_{\beta\gamma} x^{\alpha} x^{\beta} x^{\gamma} \right]
\end{align}

(32)

Substituting equation (26) for the $S^{(1)}_{\alpha\beta\gamma}$ into equation (30a)

\begin{align}
\frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) &= (\partial_\beta X^{\alpha}_{\alpha\beta} + (\partial_\beta V_{\alpha}) X^{\alpha}_{\alpha\beta} + V_{\beta}(\partial_\alpha X^{\alpha}_{\alpha\beta}) + (\partial_\beta V_{\alpha}) X^{\alpha}_{\alpha\beta} \\
+ V_{\alpha} X^{\beta}_{\alpha\gamma} + (\partial_\gamma V_{\alpha}) X^{\alpha}_{\alpha\gamma} + V_{\gamma} X^{\alpha}_{\alpha\gamma} \right] x^{\alpha} x^{\beta} x^{\gamma} + \text{ higher order terms}
\end{align}

(33)

5 Equation (28) consists of the sum of two Wronskians $\zeta = \left\{ \frac{d\phi}{d\lambda} \frac{d\psi}{d\lambda} \right\} + \left\{ \frac{d\psi}{d\lambda} \frac{d\phi}{d\lambda} \right\}$.
Substituting equation (26) for the $S^{2}_{\alpha\beta}$ into equation (32a)
\[
\frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right) = \left[ \partial_{\alpha}, \partial_{\beta} \right] V_{\gamma} + (\partial_{\beta} V_{\lambda}) X_{\alpha\gamma}^{\rho} + V_{\rho}(\partial_{\gamma} X_{\alpha\beta}^{\rho}) + (\partial_{\gamma} V_{\lambda}) X_{\alpha\beta}^{\rho}
\]
\[+ V_{\rho} X_{\gamma\alpha\beta}^{\rho} + (\partial_{\gamma} V_{\rho}) X_{\alpha\beta}^{\rho} + V_{\rho} X_{\gamma\alpha\beta}^{\rho} \right] \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + \text{higher order terms} \] (34)

Substituting equations (33) and (34) into equation (28) and collecting terms.

(a) $\zeta = \frac{d}{d\lambda} \left( \frac{d\phi}{d\lambda} \right) - \frac{d}{d\lambda} \left( \frac{d\psi}{d\lambda} \right)$

(b) $\zeta = [V_{\rho}(\partial_{\alpha} X_{\gamma\beta}^{\rho}) - V_{\rho}(\partial_{\beta} X_{\gamma\alpha}^{\rho}) + V_{\rho} X_{\gamma\beta}^{\rho} - V_{\rho} X_{\gamma\alpha}^{\rho}] \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + \text{even higher order terms}$ (35)

The terms $V_{\rho}(\partial_{\alpha} X_{\gamma\beta}^{\rho})$ and $V_{\rho} X_{\gamma\beta}^{\rho}$ in equation (35b) originate from equation (27a). The terms $V_{\rho}(\partial_{\beta} X_{\gamma\alpha}^{\rho})$ and $V_{\rho} X_{\gamma\alpha}^{\rho}$ of equation (35b) originate from equation (27b). Thus, as discussed before, when the components originating from equations (27a) and (27b) are equal, equation (35) is equal to zero.

The term in the square bracket of equation (35b) can be expressed as the scalar product of a vector with components $V_{\rho}$ and a 4th rank mixed covariant-contravariant tensor with components $\tilde{B}_{\alpha\beta\gamma\delta}^{\rho}$.

\[
V_{\rho} \tilde{B}_{\alpha\beta\gamma\delta}^{\rho} = V_{\rho}[\partial_{\alpha} X_{\gamma\beta}^{\rho} - \partial_{\beta} X_{\gamma\alpha}^{\rho} + X_{\gamma\beta}^{\rho} X_{\gamma\alpha}^{\rho} - X_{\gamma\alpha}^{\rho} X_{\gamma\beta}^{\rho}] + \text{even higher order terms} \] (36)

The fourth-rank tensor $\tilde{B}_{\alpha\beta\gamma\delta}^{\rho}$ is only a function of the components of the metric tensors and the coordinates. Substituting equation (22) into equation (36) and sorting the terms.

\[
\tilde{B}_{\alpha\beta\gamma\delta}^{\rho} = \partial_{\lambda} \Gamma_{\beta\gamma}^{\rho} - \partial_{\beta} \Gamma_{\gamma\alpha}^{\rho} + \Gamma_{\beta\gamma}^{\rho} \dot{x}^{\alpha} - \Gamma_{\gamma\alpha}^{\rho} \dot{x}^{\beta} + (\Pi_{\alpha\beta\gamma}^{\mu} - \Pi_{\beta\alpha\gamma}^{\mu} + \Pi_{\alpha\gamma\beta}^{\mu} - \Pi_{\gamma\alpha\beta}^{\mu}) \dot{x}^{\delta} \dot{x}^{\mu}
\]

\[+ (\partial_{\lambda} \Pi_{\beta\gamma\delta}^{\mu} - \partial_{\gamma} \Pi_{\beta\delta\lambda}^{\mu} + \partial_{\gamma} \Pi_{\delta\beta\lambda}^{\mu}) \dot{x}^{\mu} \dot{x}^{\nu} + (\Pi_{\alpha\beta\gamma}^{\mu} \Pi_{\delta\mu\nu} - \Pi_{\beta\alpha\gamma}^{\mu} \Pi_{\delta\mu\nu}) \dot{x}^{\delta} \dot{x}^{\nu} \dot{x}^{\mu} \dot{x}^{\nu} + \text{even higher order terms} \] (37)

where the $\Gamma_{\beta\gamma}^{\rho}$ and the $\Pi_{\alpha\beta\gamma}^{\mu}$ are given by equation (19). One should remember that the tensors $\Gamma_{\beta\gamma}^{\rho}$ and $\Pi_{\alpha\beta\gamma}^{\mu}$ contain $\theta_{\mu\nu}$ given by equation (15).

The first four terms of equation (37) have the form of the Riemann Christoffel tensor of the GRT given in reference 1 'Die Grundlagen der Allgemeinen Relativitats Theorie' Equation (43).

\[
B_{\mu\rho\tau} = -\frac{\partial}{\partial x^{\sigma}} \left\{ \frac{\mu \sigma}{\rho} \right\} + \frac{\partial}{\partial x^{\sigma}} \left\{ \frac{\mu \tau}{\rho} \right\} - \left\{ \frac{\mu \sigma}{\alpha} \right\} \left\{ \frac{\rho \tau}{\alpha} \right\} + \left\{ \frac{\mu \tau}{\alpha} \right\} \left\{ \frac{\alpha \sigma}{\rho} \right\} \right\} \text{ [43 of Ref.1]}
\]

If one changes indices $\rho \rightarrow \tau$, $\mu \rightarrow \sigma$, $\alpha \rightarrow \mu$, and $\beta \rightarrow \nu$ in equation (19a), one obtains Einstein’s formulation of the Christoffel symbol $\left\{ \frac{\mu \sigma}{\rho} \right\}$ which is equal to $-\Gamma_{\mu\rho}^{\nu}$. Einstein’s formulation of the Christoffel symbol is given in Reference 1, Equations (21) and (23). In order to compare the first four terms of equation (37) and reference 1, Equation (43), one makes a change of the indices $\alpha \rightarrow \tau$, $\beta \rightarrow \gamma$, $\nu \rightarrow \mu$, and $\delta \rightarrow \alpha$ and replace $\Gamma_{\mu\rho}$ with $-\Gamma_{\mu\sigma}^{\rho}$, in equation (37), and where $\Gamma_{\mu\rho}^{\nu} = -\Gamma_{\mu\sigma}^{\rho}$ and $\Gamma_{\nu\sigma}^{\mu} = \left\{ \frac{\mu \sigma}{\rho} \right\}$.

\[
\tilde{B}_{\rho\sigma\tau} = -\partial_{\lambda} \Gamma_{\rho\sigma\tau}^{\mu} + \partial_{\lambda} \Gamma_{\rho\sigma\tau}^{\mu} - \Gamma_{\rho\sigma\tau}^{\mu} + \Gamma_{\rho\sigma\tau}^{\mu} + \Gamma_{\rho\sigma\tau}^{\mu} \Gamma_{\mu\tau}^{\nu} + \Gamma_{\rho\sigma\tau}^{\mu} \Gamma_{\mu\tau}^{\nu} + \text{...} \] (38)

Equation (38) agrees with Einstein’s formulation of the Riemann Christoffel tensor of Reference [1] Equation (43).

According to our fourth hypothesis that in constant space-time the derivatives of functions of space-time are equal to zero, the derivatives of the Lagrangian potential are equal to zero and the tensors of equation (4b) revert to a Minkowski tensor. According to the fourth postulate in a rectangular coordinate system for the case when the tensors with components $g_{\alpha\beta}$ and $h_{\alpha\beta\gamma}$ are constant, the tensor with components $\tilde{B}_{\alpha\beta\gamma\delta}^{\rho}$ is zero. Even when this coordinate system is transformed to a curvilinear coordinate system where the $g_{\alpha\beta}$ and $h_{\alpha\beta\gamma}$ are no longer constant, the tensor with components $\tilde{B}_{\alpha\beta\gamma\delta}^{\rho}$ is still zero and describes flat space-time.

The purpose of the Field Equation is to calculate the components $g_{\alpha\beta}$ and $h_{\alpha\beta\gamma}$ of the metric tensors. Contracting the fourth-rank tensor $\tilde{B}_{\alpha\beta\gamma\delta}^{\rho}$ over the indices $\gamma$ and $\rho$ and grouping the Christoffel symbols in equation (37) into three tensors

\[
\tilde{B}_{\alpha\beta\gamma}^{\rho} = g_{\alpha\beta} + H_{\alpha\beta\gamma}^{\delta} \dot{x}^{\delta} + M_{\alpha\beta\gamma\delta} \dot{x}^{\delta} \dot{x}^{\delta} + \text{even higher order terms} \] (39)
where

\[
\begin{align*}
(a) \ G_{\alpha\beta} &= \partial_{\alpha}G^{\gamma}_{\beta\gamma} - \partial_{\beta}G^{\gamma}_{\alpha\gamma} + \Gamma^{\gamma}_{\beta\delta}G_{\alpha\delta} - \Gamma^{\gamma}_{\alpha\delta}G_{\beta\delta} \\
(b) \ H_{\alpha\beta\mu\nu} &= \partial_{\alpha}H^{\rho}_{\beta\mu\nu} - \partial_{\beta}H^{\rho}_{\alpha\mu\nu} + \Gamma^{\rho}_{\beta\delta}H_{\alpha\mu\nu}^{\delta} - \Gamma^{\rho}_{\alpha\delta}H_{\beta\mu\nu}^{\delta} \\
(c) \ M_{\alpha\beta\mu\nu\rho\phi} &= \Pi^{\rho}_{\beta\gamma}H_{\alpha\mu\nu\rho\phi} - \Pi^{\rho}_{\alpha\delta}H_{\beta\mu\nu\rho\phi}
\end{align*}
\]

(40)

The second-rank tensor with components \(G_{\alpha\beta}\) has dimensions of reciprocal length squared \(\frac{1}{L^2}\), the fourth-rank tensor with components \(H_{\alpha\beta\mu\nu}\) has dimensions of \(\frac{T^4}{L^2}\), and the sixth-rank tensor with components \(M_{\alpha\beta\mu\nu\rho\phi}\) has dimensions of \(\frac{T^6}{L^4}\). This suggests forming 3 Field Equations; however, this process can be extended to many more Field Equations.

\[
\begin{align*}
(a) \ G_{\alpha\beta} &= \frac{8\pi G}{c^4}\left(T_{\alpha\beta} + \frac{1}{2}g_{\alpha\beta}T_{\delta}\right) \\
(b) \ \frac{c^6}{8\pi G}H_{\alpha\beta\mu\nu} &= \left(g_{\alpha\beta} - \eta_{\alpha\beta}\right) \otimes \ T_{\mu\nu} \\
(c) \ \frac{c^6}{8\pi G}M_{\alpha\beta\mu\nu\rho\phi} &= \left(g_{\alpha\beta} - \eta_{\alpha\beta}\right) \otimes \left(g_{\mu\nu} - \eta_{\mu\nu}\right) \otimes \ T_{\rho\phi}
\end{align*}
\]

(41)

and even higher order Field equations

Where \(G_{\alpha\beta}\), \(H_{\alpha\beta\mu\nu}\), and \(M_{\alpha\beta\mu\nu\rho\phi}\) are given by equation (40) and the Christoffel symbols are described in equation (19). The \(T_{\alpha\beta}\) is a component of the conventional 4-dimensional stress-energy-density tensor.

For slightly warped space time the value of the tensor with components \(g_{\alpha\beta} - \eta_{\alpha\beta}\) is very small. This results in the tensors with components \(H_{\alpha\beta\mu\nu}\) and \(M_{\alpha\beta\mu\nu\rho\phi}\) to have much smaller values than the tensor with components \(G_{\alpha\beta}\) of equation (41a). Therefore, in the limit of slightly curved space time the coupled Field equations of the STCM model revert to the Field equation of the GRT given in equation (41a).

The three couple Equations 41 describe the reciprocal interaction of the curvature of space-time and the motion of the masses and energy fields. First the curvature of space-time is calculated from the space-time wave Equation, Equation 41a. Equation 41a is similar to the Field Equation of the GRT [11]. The curved space-time coordinates calculated from Equation 41a are used in Equations 41b and 41c to recalculate the coordinates of the masses and energy fields, etc. This results in a multi curved 3-dimensional space as a function of time that can have local positive and negative slopes. This can describe a Universe that at times has an accelerating expansion, and at other times a decreasing expansion.

It would be of interest to calculate the Kretschmann Invariant \(K\), a scalar.

\[
K = B^{\alpha}_{\alpha\beta\gamma}B_{\alpha\beta\gamma}
\]

(42)

Because the quasi-Riemann Christoffel tensor given by equation (37) is much more complicated than the Riemann Christoffel tensor of the GRT, the Kretschmann Invariant is substantially more difficult to calculate, therefore, this will not be attempted here. Nevertheless, it would be interesting to determine the effect of the additional terms of the Taylor series have on the various singularities in the GRT.

4. Hamiltonian potential and conservation of the Hamiltonian potential

The Hamiltonian in this study includes the effect of the fourth order expansion coefficients of the Taylor series. When higher order expansion coefficients are used, the Hamiltonian calculation should accordingly be adjusted.

The Lagrangian potential \(\mathcal{L}\) from equations (1) and (2) has dimensions of velocity squared. It is a function of the Space-Time Curvature Mode. The Lagrangian potential is a function of the space-time coordinates \(x^\alpha(\lambda) \in M(4)\) which are functions of the parameter \(\lambda \in \mathbb{R}(1)\). The Lagrangian potential is not an explicit function of the parameter \(\lambda\). It is also not a function of the quasi-velocities with components \(x^\alpha\). However, the Taylor series expansion introduces the quasi-velocities with components \(x^\alpha\) in the description of the Lagrangian
potential.

\[ \mathcal{L} = \mathcal{L}_0 + g_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta} + h_{\alpha\beta\gamma} x^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + \text{even higher order terms} \quad (43) \]

It is assumed that the tensors with components \( g_{\alpha\beta}, h_{\alpha\beta\gamma} \) and the scalar \( \mathcal{L}_0 \) are not functions of the \( \dot{x}^{\mu} \) therefore:

\[ (a) \, \partial_{\mu} g_{\alpha\beta} = 0 \quad (b) \, \partial_{\mu} h_{\alpha\beta\gamma} = 0 \quad \text{where} \quad \partial_{\mu} \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \dot{x}_{\mu}} \quad (44) \]

The Hamiltonian potential \( \mathcal{H} \) is the Legendre transform of the Lagrangian potential \( \mathcal{L} \) with respect to the \( \lambda \) space velocities \( \dot{x}^{\alpha} \) present in the Taylor series expansion, equation (2).

\[ \mathcal{H} = p_{\mu} \dot{x}^{\mu} - \mathcal{L} \quad (45) \]

where the \( \lambda \) momentum is:

\[ (a) \, p_{\mu} = \partial_{\mu} \mathcal{L} \quad (b) \, p_{\mu} = g_{\alpha\beta} \dot{x}^{\beta} + g_{\mu\alpha} \dot{x}^{\alpha} \]

\[ \quad + h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \]

\[ \quad + h_{\alpha\beta\gamma\delta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + h_{\alpha\beta\gamma\delta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \]

\[ (c) \text{where} \quad \tau_{\mu} = \begin{cases} 1 \text{ if } \dot{x}^{\mu} \text{ and } x^{\nu} \text{ are colinear} \\ 0 \text{ if } \dot{x}^{\mu} \text{ and } x^{\nu} \text{ are not colinear} \end{cases} \quad (46) \]

Taking the scalar product of the metric tensors with the tensors with components \( \tau_{\mu} \) to form the \( \mu \) component of the momentum vector.

\[ p_{\mu} = g_{\mu\beta} \dot{x}^{\beta} + g_{\mu\alpha} \dot{x}^{\alpha} \]

\[ + h_{\mu\beta\gamma} \dot{x}^{\beta} \dot{x}^{\gamma} \]

\[ + h_{\mu\beta\gamma\delta} \dot{x}^{\beta} \dot{x}^{\gamma} - h_{\mu\beta\gamma} \dot{x}^{\beta} \dot{x}^{\gamma} \]

\[ = \mathcal{L}_0 + \text{even higher order terms} \quad (47) \]

To calculate the Hamiltonian, substituting equations (43) and (47) into equation (45)

\[ \mathcal{H} = g_{\mu\beta} \dot{x}^{\beta} \dot{x}^{\mu} + g_{\mu\alpha} \dot{x}^{\alpha} \dot{x}^{\mu} - g_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta} \]

\[ + h_{\mu\beta\gamma} \dot{x}^{\beta} \dot{x}^{\gamma} + h_{\mu\beta\gamma\delta} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} \]

\[ + h_{\mu\beta\gamma\delta} \dot{x}^{\beta} \dot{x}^{\gamma} + h_{\mu\beta\gamma} \dot{x}^{\beta} \dot{x}^{\gamma} \]

\[ - \mathcal{L}_0 + \text{even higher order terms} \quad (48) \]

Calculating the change of the Hamiltonian potential with \( \lambda \).

\[ \frac{d \mathcal{H}}{d \lambda} = \partial_{\mu} g_{\mu\beta} \dot{x}^{\beta} \dot{x}^{\mu} + \partial_{\beta} g_{\mu\beta} \dot{x}^{\beta} \dot{x}^{\mu} - \partial_{\beta} g_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\mu} \]

\[ + g_{\mu\beta} \dot{x}^{\beta} \dot{x}^{\mu} + g_{\mu\beta} \dot{x}^{\beta} \dot{x}^{\mu} + g_{\mu\beta} \dot{x}^{\beta} \dot{x}^{\mu} - g_{\mu\beta} \dot{x}^{\beta} - g_{\mu\beta} \dot{x}^{\beta} \]

\[ + \partial_{\alpha} h_{\alpha\beta\gamma} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\mu} + h_{\alpha\beta\gamma\delta} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} \]

\[ + \partial_{\beta} h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + h_{\alpha\beta\gamma\delta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} \]

\[ + \partial_{\gamma} h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + h_{\alpha\beta\gamma\delta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} \]

\[ + \partial_{\delta} h_{\alpha\beta\gamma\delta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} + h_{\alpha\beta\gamma\delta} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} \]

\[ - \partial_{\mu} h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} + \partial_{\mu} h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} \]

\[ - h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} - h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \dot{x}^{\delta} - h_{\alpha\beta\gamma} \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \]

\[ + \text{even higher order terms} \quad (49) \]
Changing indices in scalar products and collecting terms.

\[ a) \frac{dH}{d\lambda} = \left[ -\left( \partial_\mu g_{\alpha\beta} - \partial_\alpha g_{\mu\beta} - \partial_\beta g_{\mu\alpha} \right) \dot{x}^\alpha \dot{x}^\beta - g_{\mu\alpha} \ddot{x}^\alpha - g_{\mu\alpha} \ddot{x}^\beta \right. \]

\[ \left. -\left( \partial_\mu h_{\alpha\beta\gamma} - \partial_\alpha h_{\mu\beta\gamma} - \partial_\beta h_{\mu\alpha\gamma} - \partial_\gamma h_{\mu\alpha\beta} - \partial_\gamma h_{\mu\alpha\beta} \right) \ddot{x}^\alpha \ddot{x}^\beta \ddot{x}^\gamma \right) \]

\[ b) \frac{dH}{d\lambda} = 0 \tag{50} \]

The results calculated here are only for a Taylor series expansion to fourth order. The results for higher order Taylor series expansion terms have yet to be proven. The terms inside the rectangular bracket of equation (50), with changes in the indices, are equal to the wave equation (12). Because the wave equation (12) is equal to zero, the change in the Hamiltonian potential is equal to zero. Therefore, the Hamiltonian potential is constant and conserved.

5. Conclusion

The spatial part of the solution of the Field Equation of the GRT has a parabolic form for space as a function of time. It describes a Universe with either a slowing expansion or a contracting Universe. In 1916 when the GRT was written it was thought that the Universe was flat. Therefore, Einstein added a constant multiplied by the Metric tensor. In 1929 Hubble [3] observed that the Universe was expanding. He noticed that the farther celestial objects are from the earth, the faster they move away. Thinking this proved that the original derivation of the GRT was correct, Einstein removed the Cosmological constant.

In 1998 a surprising astronomical observation showed that the Universe’s expansion is accelerating. The latest astronomical observations have shown that the rotation curve of stars from the central Black Hole of the Milky Way Galaxy is faster than predicted by calculations from the Kepler Newton Theory. This contradicts the theory of the GRT because the GRT describes masses being attracted to each other by gravity. To explain this surprising observation, Dark Matter (DM) and Dark Energy (DE) were postulated. For more than 20 years, experiments were conducted to measure DM or DE. All these experiments failed consistently to find any trace of DM or DE. Perhaps, DM and DE are like the Ether of the early 20th century through which light waves were supposed to propagate. Michelson’s and Morley’s experiment [7], and Einstein’s theory disproved the existence of the Ether.

In this study, an alternate model is derived from basic principles, to replace the postulated concepts of Einstein’s Cosmological constant, Black Mass and Black Energy. This model is calculated from the variation of a Taylor series expansion of the Lagrangian potential. The expansion coefficients are derivatives of the Lagrangian potential with respect to the coordinate and therefore are covariant Metric tensors. Thus, the Taylor series expansion coefficients are Metric tensors of increasing rank. The Lagrangian is not a function of the energies of an object but is a function of the STCM.

The variation of the Taylor series expansion results in an Equation for the space-time wave mode with a series of Metric tensors of increasing rank. Coupled Field Equations are derived by a method similar to the one used to derive the GRT Field Equation [11], but here, the wave Equation with a series of Metric tensors is used. This study describes the reciprocal interaction of warped space-time and masses and energy fields. The masses and energy fields cause space-time to warp, and the curvature of space-time affects the motion of masses and energy, etc.

The coupled Field Equations of the STCM have solutions of a multi-curved space as a function of time with negative and positive local slopes. Therefore, the STCM model can describe a Universe that can, at times, have an acceleration expansion and can at later times, have a decelerating expansion. This mathematical model reverts to the GRT and also Newton’s model of Nature in appropriate limits.

The variational method used here is similar to the derivation of the Euler Lagrange Equation of Classical Mechanics and Einstein’s Equation of motion in the GRT. All have successfully described real physical events.

Einstein’s Field Equation represents ten simultaneous Equations. These can be solved with considerable effort without the aid of a computer. However, the Field equations of the STCM represent approximately 100 simultaneous Equations. These will have to be solved with the aid of a computer, using Symbolic Algebra Software [12] now in the year 2022.
The Hamiltonian potential for this model was derived. The Hamiltonian potential is the Legendre transform of the Lagrangian potential. The change in the Hamiltonian potential is equal to zero and thus, the Hamiltonian is conserved.

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Data availability statement

No new data were created or analysed in this study.

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