Yukawa corrections to top pair production in photon-photon collision

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ABSTRACT

The $O(\alpha m_t^2/m_W^2)$ Yukawa corrections to top pair production in photon-photon collision are calculated in the standard model (SM), the general two-Higgs-doublet model (2HDM) as well as the minimal supersymmetric model (MSSM). We found that the correction to the cross section can only reach a few percent in the SM, but can be quite significant (>10%) in the 2HDM and MSSM for favorable parameter values, which may be observable at the high energy $e^+e^-$ colliders.

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1. Introduction

Recently, the evidence for top quark production, with a mass of $176 \pm 8_{(\text{stat})} \pm 10_{(\text{syst})}$ GeV and $199^{+19}_{-21_{(\text{stat})}} \pm 22_{(\text{syst})}$ GeV has been reported by the CDF and D0 collaboration, respectively [1]. Due to its large mass, the discovery of the top quark will open a number of new and interesting issues, such as the precision measurement of the mass, width and Yukawa couplings of the top quark through its direct production and subsequent decay at both hadron and $e^+e^-$ colliders. But even with $1000pb^{-1}$ of luminosity, the Fermilab Tevatron could determine the top mass to 5 GeV or better[2]. At the future multi-TeV proton colliders such as the CERN Large Hadron Collider (LHC), $t\bar{t}$ production will be enormously larger than the Tevatron rates, but the accuracy with which the top mass can be measured in proton colliders is limited to about $2 \sim 3$ GeV[2]. Bloude et al.[3], have argued that one must know the top mass to 1 GeV to take full advantage of the constraints that precision electroweak measurements put on the Higgs boson and other massive particles which might contribute to electroweak loops. Beyond this, it would be wonderful to make a precision measurement fo the basic parameter $m_t$ to 0.3GeV or better for looking for new physics beyond the SM by the loop processes which are sensitive to $m_t$. At the next-generation linear collider (NLC) operating at a center-of-mass energy of 500 GeV-2000 GeV with a luminosity of the order of $10^{33}cm^{-2}s^{-1}$, the $e^+e^- \rightarrow t\bar{t}$ events rate would be around $10^4/yr$, comparable with the Tevatron, however the events would be much cleaner and top parameters would be easier to extract. At the NLC a top mass measurement with statistical uncertainty 0.3 GeV from 10$fb^{-1}$ luminosity is expected[2] and it is possible to separately measure all of the various production and decay from factors of the top quark at the level of a few percent[4].

Nowadays, the possibility of transforming a linear $e^+e^-$ collider into a $\gamma\gamma$ collider deserves a lot of attention. With the advent of the new collider technique[5] the collision of high energy, high intensity photon beams , obtained by using the old idea of Compton laser backscattering[6], can be realized in the NLC. The back-scattering of laser photons off the colliding electron and positron beams can yield intense and energetic photon beams which then collide with the high luminosity. There are many uses for such high photon-photon luminosity, one of the most important may be for the production of top quark paris. It has been found[7] that $t\bar{t}$ production in $\gamma\gamma$ collisions realized by laser back-scattering is slightly larger than the direct $e^+e^- \rightarrow t\bar{t}$ production for $m_t < 130$ GeV at $\sqrt{s} = 0.5Tev$ , and at $\sqrt{s} = 1Tev$ the production of $\gamma\gamma \rightarrow t\bar{t}$ is much larger than $e^+e^- \rightarrow t\bar{t}$ for $m_t \sim$
100 − 200 GeV both with and without considering the threshold QCD effect. In the SM, the cross section for top quark pair productions in γγ collisions have been calculated with higher order QCD correction[8]. The radiative corrections to γγt̄t from final state Higgs exchange interactions has also computed in Ref.[9]. The correction is of order O(2−4%) for typical values of the Higgs boson mass and top quark mass. In this paper we calculate the O(αm_t^2/m_W^2) Yukawa correction in a two Higgs doublet model (2HDM)(Model II)[10] and in the minimal supersymmetric model(MSSM) , in which there are three neutral and two charged physical Higgs bosons, \( H, h, A, H^\pm \), of which \( H \) and \( h \) are CP-even and \( A \) is CP-odd. The O(αm_t^2/m_H^2) Yukawa correction arise from the virtual effects of the third family (top and bottom) quarks, charged and neutral Higgs bosons, as well as the Goldstone bosons \( G^0, G^\pm \). The results of the standard model can be obtained from our calculations as a special case. In Sec. II, we present the analytic results in terms of the well-known standard notation of one-loop Feynman integrals. In Sec. III, we present some numerical examples and discuss the implication of our results. And in the apppendix we list the form factors in the cross section.

2. Calculations

The relevant Feynman diagrams are shown in Fig.1 and the Feynman rules can be found in Ref.[10]. In our calculation, we use dimensional regularization to regulate all the ultraviolet divergences in the virtual loop corrections and we adopt the on-mass-shell renormalization scheme[11]. In our calculations we keep the term \( m_b\tan\beta \) in the in the charged Higgs couplings to third family quarks since its effects become rather important for large \( \tan\beta \). Taking into account the O(αm_t^2/m_W^2) Yukawa corrections, the renormalized amplitude for \( \gamma\gamma \to t\bar{t} \) is given by

\[ M_{ren} = M_{ren}^{(t)} + M_{ren}^{(u)}, \]
\[ M_{ren}^{(t)} = M_0^{(t)} + \delta M^{(t)} \]
\[ = M_0^{(t)} + \delta M^{s(t)} + \delta M^{v(t)} + \delta M^{b(t)} + \delta M^{\Delta(t)}, \]
\[ M_{ren}^{(u)} = M_{ren}^{(u)}(p_3 \leftrightarrow p_4, \hat{t} \rightarrow \hat{u}), \]

where \( M_0 \) is the amplitude at tree level, \( \delta M^s, \delta M^v, \delta M^b \) and \( \delta M^{\Delta(t)} \) represent the O(αm_t^2/m_W^2) Yukawa corrections arising from the self energy diagram Fig.1(d), vertex diagram Fig.1(f)-(i) , box diagrams Fig.1(l)-(n) and digrams Fig.1(j),(k), respectively. \( \hat{t} = (p_4 - p_2)^2, \hat{u} = (p_1 - p_4)^2 \)
and \( p_3(p_4) \) denote the momentum of the two incoming photons, and \( p_2(p_1) \) are momentum of the outgoing t quark and its antiparticle. The explicit forms of these matrix elements are given by

\[
M_0^{(t)} = -ie^2 e^\mu(p_4)\epsilon_{\nu}(p_3)\bar{u}(p_2)\gamma^\mu(\gamma \cdot p_3 - \gamma \cdot p_1 + m_t)v(p_1), \tag{4}
\]

\[
\delta M^{s(t)} = ie^2 Q^2 \frac{\alpha m^2_W}{16 \pi^2 s_W^2} e^\mu(p_4)\epsilon_{\nu}(p_3)\bar{u}(p_2)(f^s\gamma_\mu\gamma_\nu + f^s_8 p_2 \gamma_\nu + f^s_{12} p_4 \gamma_\nu)v(p_1), \tag{5}
\]

\[
\delta M^{u(t)} = ie^2 Q^2 \frac{\alpha m^2_W}{16 \pi^2 s_W^2} e^\mu(p_4)\epsilon_{\nu}(p_3)\bar{u}(p_2)(f^u_8 p_2 \gamma_\mu + f^u_6 p_2 \gamma_\nu) + f^u_0 p_4 \gamma_\nu)v(p_1), \tag{6}
\]

\[
\delta M^{b(t)} = ie^2 Q^2 \frac{\alpha m^2_W}{16 \pi^2 s_W^2} e^\mu(p_4)\epsilon_{\nu}(p_3)\bar{u}(p_2)\left[f^b_1 g^{\mu\nu} + f^b_2 \gamma^\mu\gamma^\nu + f^b_3 p_1 \gamma^\nu + f^b_4 p_2 \gamma^\mu + f^b_5 p_2^\dagger p_1^\dagger + f^b_6 p_2^\dagger p_1^\dagger \right]v(p_1), \tag{7}
\]

\[
\delta M^{\Delta(t)} = ie^2 Q^2 \frac{\alpha m^2_W}{16 \pi^2 s_W^2} e^\mu(p_4)\epsilon_{\nu}(p_3)\bar{u}(p_2)\left[f^\Delta_1 g^{\mu\nu} + f^\Delta_2 p_1^\dagger p_1^\dagger + f^\Delta_3 p_2^\dagger p_2^\dagger + f^\Delta_4 p_2^\dagger p_1^\dagger + f^\Delta_5 p_1^\dagger p_2^\dagger \right]v(p_1), \tag{8}
\]

The form factors \( f^s_i, f^u_i, f^b_i \) are presented in Appendix A.

The corresponding amplitude squared can be written as

\[
\sum |M_{ren}|^2 = \sum |M_0|^2 + 2Re \sum |\delta M^{(t)} M_0^{(t)|} + \delta M^{(s)} M_0^{(s)|} + \delta M^{(u)} M_0^{(u)|} + \delta M^{(b)} M_0^{(b)|} + \delta M^{(\Delta)} M_0^{(\Delta)|}, \tag{9}
\]

\[
\sum |\delta M^{(t)} M_0^{(t)|} \text{ is given by}
\]

\[
\sum |\delta M^{(t)} M_0^{(t)|} = |\delta M^{s(t)} M_0^{(t)|} + \delta M^{u(t)} M_0^{(t)|} + \delta M^{b(t)} M_0^{(t)|} + \delta M^{\Delta(t)} M_0^{(t)|}, \tag{10}
\]

where

\[
\sum |\delta M^{s(t)} M_0^{(t)|} = \frac{\pi \alpha^3 m_t^2 Q^2}{4 m_W^2 s_W^2 (t - m_t^2)} (f^s_2 H_2 + f^s_6 H_6 + f^s_{12} H_{12}), \tag{11}
\]

\[
\sum |\delta M^{u(t)} M_0^{(t)|} = \frac{\pi \alpha^3 m_t^2 Q^2}{4 m_W^2 s_W^2 (t - m_t^2)} (f^u_2 H_2 + f^u_3 H_3 + f^u_6 H_6)
\]
Photon luminosity can be obtained by folding the cross section $\hat{\sigma}(s)$ with

$$\sum \delta M^{(t)}(t) M_0^{(t)\dagger} = \frac{\pi \alpha^3 m^2 t^2 Q^2_t}{4 m^2 t_s s W(t - m_t^2)} \sum_{i=1}^{20} f^i_{t} H_i(m_t, p_1 \cdot p_2, p_1 \cdot p_3, p_1 \cdot p_4, p_3 \cdot p_4),$$

$$\sum \delta M^{(s)}(t) M_0^{(t)\dagger} = \frac{\pi \alpha^3 m^2 t^2 Q^2_t}{4 m^2 t_s s W(t - m_t^2)} (f^\Delta_{1} H_1 + f^\Delta_{7} H_7 + f^\Delta_{9} H_9 + f^\Delta_{10} H_{10}),$$

Here the expressions of the $H_i(m_t, p_1 \cdot p_2, p_1 \cdot p_3, p_1 \cdot p_4, p_3 \cdot p_4)$ are given in Appendix B. $\delta M^{(t)}(t) M_0^{(u)\dagger}, \delta M^{(u)} M_0^{(t)\dagger}$ and $\delta M^{(u)} M_0^{(u)\dagger}$ can be obtained by

\[
\begin{align*}
\delta M^{(t)} M_0^{(u)\dagger} &= \delta M^{(t)} M_0^{(t)\dagger}\big|_{p_1 \rightarrow p_4} \\
\delta M^{(u)} M_0^{(t)\dagger} &= \delta M^{(t)} M_0^{(t)\dagger}\big|_{p_4 \rightarrow p_3} \\
\delta M^{(u)} M_0^{(u)\dagger} &= \delta M^{(t)} M_0^{(t)\dagger}\big|_{p_4 \leftrightarrow p_3}
\end{align*}
\]

The cross section of the subprocess is given by

$$\sigma(\hat{s}) = \frac{1}{16\pi^2} \int_{t_-}^{t_+} dt |M_{\text{ren}}(\hat{s}, \hat{t})|^2,$$

where $\hat{t}^\pm = (m^2_t - \frac{1}{2} \hat{s}) \pm \frac{1}{2} \hat{s} \beta_\hat{t}$ and $\beta_\hat{t} = \sqrt{1 - 4m^2_t / \hat{s}}$. The total cross section for top quark pair production can be obtained by folding the cross section $\hat{\sigma}$ for the subprocesses with the photon luminosity

$$\sigma(s) = \int_{2m_t/\sqrt{s}}^{x_{\text{max}}} dz dL_{\gamma\gamma}/dz d\hat{\sigma}(\gamma\gamma \rightarrow t\bar{t} \text{ at } \hat{s} = s^2)$$

where $\sqrt{s}(\sqrt{s})$ is the $e^+e^- (\gamma\gamma)$ center-of-mass energy and the quantity $dL_{\gamma\gamma}/dz$ is the photon luminosity, which is defined as

$$\frac{dL_{\gamma\gamma}}{dz} = 2z \int_{x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x^2} F_{\gamma/e}(x) F_{\gamma/e}(z^2/x)$$

For unpolarized initial electrons and laser, the energy spectrum of the back-scattered photon is given by [12]

$$F_{\gamma/e}(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1 - x} - \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right]$$

where

$$D(\xi) = \frac{1}{2} \frac{4}{\xi} - \frac{8}{\xi^2} \ln(1 + \xi) + \frac{1}{2} \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}$$

and $\xi = 4E_0\omega_0/m^2_e$, $m_e$ and $E_0$ are the incident electron mass and energy, respectively, and $\omega_0$ is the laser-photon energy, $x$ is the fraction of the energy of the incident electron carried
by the back-scattered photon. In our calculation we follow the analysis of Ref. [9], and choose \( \omega_0 \) such that it maximizes the back-scattered photon energy without spoiling the luminosity through \( e^+ e^- \) pair creation. With this choice, we can find \( \xi = 2(1 + \sqrt{2}) \simeq 4.8 \), \( x_{\text{max}} \simeq 0.83 \), and \( D(\xi) \simeq 1.8 \).

3. Numerical results and conclusion

In our numerical calculation, the input parameters[13] are \( m_Z = 91.176 \text{GeV} \), \( \alpha_{\text{em}} = 1/128.8 \), and \( G_F = 1.166372(10^{-5} (\text{GeV})^{-2}) \). \( m_W \) is determined through [4]

\[
m_W^2 (1 - \frac{m_W^2}{M_Z^2}) = \frac{\pi \alpha}{\sqrt{2} G_F} \frac{1}{1 - \Delta r},
\]

(23)

where, to order \( O(\alpha m_t^2/m_W^2) \), \( \Delta r \) is given by [4]

\[
\Delta r \sim -\frac{\alpha N C W^2 m_t^2}{16 \pi^2 s_W^4 m_W^2}.
\]

(24)

The lower limit of the parameter \( \tan \beta \) is 0.6 from perturbative bounds [14]. Reference [15] argues lower values of \( \tan \beta \) from perturbative unitarity which is about 0.25 for top quark mass of 176 GeV. So in our numerical calculations we allow \( \tan \beta \) to take the minimum value of 0.25 in the two Higgs doublet model. In the following we present some numerical examples corresponding to a \( e^+ e^- \) collider with center-of-mass energy of \( \sqrt{s} = 500 \text{ GeV} \).

The numerical results in the SM are presented in Fig.2. The correction to the cross section depends on the Higgs mass and at \( M_h = 300 \text{ GeV} \) the correction reaches its maximum size of \(-2.7\%\). Recently, the correction in the standard model has been calculated in Ref.[16]. But in that work the authors only present the correction to subprocess cross section \( \gamma\gamma \rightarrow t\bar{t} \) and did not give the corresponding results at \( e^+ e^- \) collider. So it is difficult to compare their results with ours.

We present the numerical results in the two-Higgs-doublet model in Fig.3 and Fig.4. In our results we fix the parameters \( \alpha \) and \( \beta \) to be \( \alpha = \beta = 0.25 \) and show the dependence on the masses of Higgs bosons. The correction is sensitive to the Higgs masses and can be quite large for small Higgs masses. Fig.3 shows the dependence on the mass of CP-even Higgs bosons \( h \) and \( H \) for fixed \( M_A \) value. We found that correction can be quite large for small \( M_h \) value. For \( M_h < 100 \text{ GeV} \) the correction can exceed 50\% and makes it necessary to calculate higher order corrections beyond one-loop level. Fig.4 shows the dependence on the mass of CP-odd Higgs boson \( A \) for fixed \( M_{h,H} \) value. For \( M_A = 100 \text{ GeV} \) the correction
reaches -38% and decreases rapidly with the increase of $M_A$. The corrections drops rapidly with the increase of $\tan \beta$ as the case of the minimal supersymmetric model discussed bellow. Here we did not present the numerical results corresponding to large $\tan \beta$.

Figs.5-7 represent some numerical results in the minimal supersymmetric model. The Higgs sector of the minimal supersymmetric model is a special case of the two-Higgs-doublet model. In this model the masses and couplings of the Higgs bosons are controlled by two parameters at tree level, which can be taken to be $M_A$ and $\beta$ for example. In our numerical results presented in Figs.3-5, we show the dependence on $M_A$ for three different values of $\tan \beta$. From these figures one can find that the correction depends strongly on the values of $\tan \beta$. The correction is more significant for smaller $\tan \beta$ values. And for a fixed $\tan \beta$ value the correction can be either positive or negative, depending on the Higgs mass $M_A$. For minimum $\beta$ value $\beta = 0.25$, the correction gets its positive maximum size of 13% at $M_A = 420$ GeV and negative maximum size of -54% at $M_A = 300$ GeV. For $\tan \beta = 1$ the positive and negative maximum size of the correction can only reach 7% and -1.6%, respectively. For larger $\tan \beta$ value $\tan \beta = 5$, the behaviour of plot in Fig.7 is different from small $\tan \beta$ plots in Figs.5,6 since the effect of the coupling $\sim m_b \tan \beta$ becomes significant when $\tan \beta$ gets large and cancel to some extent the effect of the coupling $m_t \cot \beta$.

In conclusion, we calculate the $O(\alpha m_t^2/m_W^2)$ Yukawa corrections to top pair production in photon-photon collision in the standard model (SM), the two-Higgs-doublet model as well as the minimal supersymmetric model. We found that the correction to the cross section can only reach a few percent in the SM, but can be quite significant (>10%) in the 2HDM and MSSM for favorable parameter values. So these corrections are potentially observable at next generation linear collider, and thus could be used to set limits on the parameters of these new models, and the precision study for top pair production in photon-photon collision at NLC will be a powerful indirect probe for new physics beyond the standard model.

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Appendix A

The form factors $f_2^s, f_2^v$ are given by

\[
f_2^s = \frac{m_t}{m_t^2 - t} \left\{ \sum_{i=H^0, h} \eta_i \left[ (F_1 - F_0)(\hat{t}, m_t, m_t) + (F_0 - F_1)(m_t^2, m_t, m_t) \right] \\
+ \sum_{i=A, Z, G} \eta_i \left[ (F_1 + F_0)(\hat{t}, m_t, m_t) - (F_1 + F_0)(m_t^2, m_t, m_t) \right] \\
+ \sum_{i=H^+, W^+} \eta_i \left[ F_1(\hat{t}, 0, m_t) - F_1(m_t^2, 0, m_t) \right] \right\}
\]

\[
f_2^v = 2f_{12}^v = \frac{4}{(m_t^2 - t)^2} \left\{ \sum_{i=H^0, h} \eta_i \left[ (m_t^2 - p_1 \cdot p_3)(F_1(\hat{t}, m_t, m_t) - F_1(m_t^2, m_t, m_t)) \right] \\
+ 2m_t^2 p_1 \cdot p_3(G_0 + G_1)(m_t^2, m_t, m_t) - m_t^2 \left( F_0(\hat{t}, m_t, m_t) - F_0(m_t^2, m_t, m_t) \right) \right\}
\]

\[
f_2^v = \frac{m_t}{t - m_t^2} \left\{ 2 \sum_{i=A, Z} \eta_i \left[ p_2 \cdot p_4 C_{11}(-p_2, p_4, m_i, m_t, m_t) \right] \\
+ \left( m_t^2 C_{21} + p_1 \cdot p_3(C_{11} - 2C_{12} - 2C_{23}) + 2\overline{C}_{24} \right) (p_1, -p_3, m_i, m_t, m_t) \right\}
\]

\[
f_3^v = \frac{1}{t - m_t^2} \left\{ 4 \sum_{i=A, Z} \eta_i \left[ m_t^2 C_{21} - p_1 \cdot p_3(C_{12} + C_{23}) \right] (p_1, -p_3, m_i, m_t, m_t) \\
+ 4 \sum_{i=H^0, h} \eta_i \left[ m_t^2 (2C_{11} + C_{21}) - p_1 \cdot p_3(C_{12} + C_{23}) \right] (p_1, -p_3, m_i, m_t, m_t) \\
+ 2 \sum_{i=H^+, W^+} \eta_i \left[ m_t^2 C_{11} + p_1 \cdot p_3(C_{12} + C_{23}) \right] (p_1, -p_3, m_i, m_t, m_t) \right\}
\]
\[ f_6^v = \frac{1}{t - m_t^2} \left\{ 2 \sum_{i=1,2} \eta_i (2C_{24} - m_t^2 C_{21}) (-p_2, p_4, m_i, m_t, m_t) \right. \\
+ 2 \sum_{i=H^0, h} \eta_i [2C_{24} - m_t^2 (4C_0 + 4C_{11} + C_{21})] (-p_2, p_4, m_i, m_t, m_t) \\
+ \sum_{i=H^+, W^+} \eta_i \left\{ m_t^2 (C_0 + C_{11} + C_{21}) - 2C_{24} \right\} (-p_2, p_4, m_i, m_b, m_b) \\
+ 3 \left[ 2m_t^2 (C_{11} + C_{21}) - 2p_2 \cdot p_4 (C_{12} + C_{23}) + 2C_{24} \right] (-p_2, p_4, m_b, m_i, m_i) \right\} \]

\[ f_{12}^v = \frac{1}{t - m_t^2} \left\{ \sum_{i=1,2} \eta_i \left\{ [2C_{24} + m_t^2 C_{21} - 2p_2 \cdot p_4 (C_{12} + C_{23})] (-p_2, p_4, m_i, m_t, m_t) \right. \\
+ [2C_{24} + m_t^2 C_{21} - 2p_1 \cdot p_3 (C_{12} + C_{23})] (p_1, -p_3, m_i, m_t, m_t) \\
+ \sum_{i=H^0, h} \eta_i \left\{ [2C_{24} + m_t^2 (C_{21} - 4C_0) - 2p_2 \cdot p_4 (C_{12} + C_{23})] (-p_2, p_4, m_i, m_t, m_t) \right. \\
+ \left. \left[ m_t^2 (C_{21} - 4C_0) - 2p_1 \cdot p_3 (C_{12} + C_{23}) + 2C_{24} \right] (p_1, -p_3, m_i, m_t, m_t) \right\} \\
+ \left. \sum_{i=H^+, W^+} \eta_i \left\{ \left[ p_2 \cdot p_4 (C_{12} + C_{23}) + \frac{1}{2} m_t^2 (C_0 - C_{21}) - C_{24} \right] (-p_2, p_4, m_i, m_b, m_b) \right. \\
+ \left. \left[ p_1 \cdot p_3 (C_{12} + C_{23}) - \frac{1}{2} m_t^2 (C_0 + C_{21}) - C_{24} \right] (p_1, -p_3, m_i, m_b, m_b) \right\} \\
+ 3C_{24} (-p_2, p_4, m_b, m_i, m_i) + 3C_{24} (p_1, -p_3, m_b, m_i, m_i) \right\} \right\} \]

\[ f_{13}^v = \frac{m_t}{t - m_t^2} \left\{ 2 \sum_{i=1,2} \eta_i C_{21} (p_1, -p_3, m_i, m_t, m_t) \right. \\
+ 2 \sum_{i=H^0, h} \eta_i (2C_{11} + C_{21}) (p_1, -p_3, m_i, m_t, m_t) \\
- \sum_{i=H^+, W^+} \eta_i [(C_0 + C_{11} + C_{21}) (p_1, -p_3, m_i, m_b, m_b)] \\
+ 3(C_{11} + C_{21}) (p_1, -p_3, m_b, m_i, m_i) \right\} \]

\[ f_{16}^v = \frac{m_t}{t - m_t^2} \left\{ 2 \sum_{i=1,2} \eta_i C_{21} (-p_2, p_4, m_i, m_t, m_t) \right. \\
+ 2 \sum_{i=H^0, h} \eta_i (2C_{11} + C_{21}) (-p_2, p_4, m_i, m_t, m_t) \\
- \sum_{i=H^+, W^+} \eta_i [(C_{11} + C_{21}) (-p_2, p_4, m_b, m_b, m_b) + 3(C_{11} + C_{21}) (-p_2, p_4, m_b, m_i, m_i)] \right\} \]
where

\[ F_n(q, m_1, m_2) = \int_0^1 dy \frac{y^n \log \left[ \frac{-q^2 y(1-y) + m_1^2(1-y) + m_2^2 y}{\mu^2} \right]}{y^n+1(1-y) + m_1^2(1-y) + m_2^2 y}, \]

\[ G_n(q, m_1, m_2) = -\int_0^1 dy \frac{y^n + 1 (1-y) - q^2 y(1-y) + m_1^2(1-y) + m_2^2 y}{y^n+1(1-y) + m_1^2(1-y) + m_2^2 y}, \]

and \( C_{24} \equiv -\frac{i}{2} \Delta + C_{24}, C_0, C_{ij} \) are the three-point Feynman integrals, definition and expression for which can be found in Ref.[17].

The form factor \( f_i^b \) are given by

\[ f_i^b = f_i^{b(1)} + f_i^{b(2)} + f_i^{b(3)} - f_i^{b(4)} + f_i^{b(5)} \]

where

\[ f_1^{b(1)} = 2m_t \sum_{i = H^0, h} \eta_i (2D_{27} - D_{311} + 2D_{312}) \]

\[ f_2^{b(1)} = m_t \sum_{i = H^0, h} \eta_i \left[ m_1^2(D_0 - D_{11} + D_{12} - D_{21} - D_{22} + 2D_{24} + D_{31} - D_{32} - D_{34} + D_{36}) + 2p_3 \cdot p_4(D_{13} - D_{12} - D_{310} + D_{26}) + 2(2D_{311} - 3D_{312} - 2D_{27}) + 2p_2 \cdot p_4(D_{21} - 2D_{24} - D_{25} + D_{26} + D_{38} - D_0) + 2p_1 \cdot p_4(D_{22} - D_{35} + 2p_1 \cdot p_2(D_{34} - D_{36}) \right] \]

\[ f_3^{b(1)} = \sum_{i = H^0, h} \eta_i \left[ m_1^2(D_{13} + 2D_{35} + 2D_{38} - 4D_{39}) - 4p_2 \cdot p_4(D_{12} - D_{13} + D_{23} + D_{24} - D_{25} + D_{26} + D_{31} - D_{32} + 2D_{312} + 3D_{313}) \right] \]

\[ f_4^{b(1)} = 2 \sum_{i = H^0, h} \eta_i \left[ m_1^2(-3D_{12} + 3D_{13} + 2D_{23} + 2D_{24} - 2D_{25} - 2D_{26} + 3D_{31} + 3D_{34} - D_{35} - D_{38}) + 2p_2 \cdot p_4(D_{12} - D_{13} + D_{24} - D_{25} - D_{38} + D_{39}) + 2p_1 \cdot p_2(D_{22} + D_{23} - 2D_{26} + D_{36} - D_{310}) + 2p_2 \cdot p_3(D_{37} + 4(D_{312} - D_{313})) \right] \]

\[ f_5^{b(1)} = 2 \sum_{i = H^0, h} \eta_i \left[ m_1^2(-D_{11} + D_{21} + D_{22} + D_{31} + D_{36} - D_0) + 2p_1 \cdot p_4(D_{12} + D_{13} + D_{24} - 2D_{25} - D_{35}) - 2p_1 \cdot p_3(D_{26} + D_{310}) + 2p_1 \cdot p_2(D_{24} + D_{34}) + 2(2D_{27} + D_{311}) \right] \]

\[ f_6^{b(1)} = 2 \sum_{i = H^0, h} \eta_i \left[ m_1^2(-2D_{11} + 2D_{13} + D_{21} - D_{22} + 2D_{23} + D_0 + 2D_{26} - 2D_{25} + 2D_{39} + 2D_{310}) + 2p_1 \cdot p_4(-D_{12} + D_{13} + D_{25}) - 2p_2 \cdot p_4(D_{23} + D_{310}) \right] \]
\[ f_{7}^{(1)} = 4 m_t \sum_{i = H^0, h} \eta_i(D_{22} - D_{26} + D_{32} - D_{36} - D_{38} + D_{310}) \]

\[ f_{8}^{(1)} = 4 m_t \sum_{i = H^0, h} \eta_i(D_{12} - D_{13} + D_{22} - D_{26} - D_{34} + D_{35} + D_{36} - D_{310}) \]

\[ f_{9}^{(1)} = 4 \sum_{i = H^0, h} \eta_i[m_t(D_{24} - D_{26} - D_{34} + D_{36} - D_{38} + D_{310}) + D_{333}] \]

\[ f_{10}^{(1)} = 4 m_t \sum_{i = H^0, h} \eta_i(D_{11} - D_{13} + D_{24} - D_{26} - D_{31} + D_{34} + D_{35} - D_{310}) \]

\[ f_{11}^{(1)} = -4 \sum_{i = H^0, h} \eta_i(D_{27} + D_{313}) \]

\[ f_{12}^{(1)} = \sum_{i = H^0, h} \eta_i \left[ m_t^2(3D_0 - 2D_{11} + D_{13} - D_{21} - D_{22} - 2D_{23} + 2D_{25} + 2D_{26} + D_{35} + D_{38} - D_{39} + D_{310}) + 2(D_{27} + 3D_{313}) + 2p_1 \cdot p_2(-D_{12} + D_{13} - D_{23} - D_{24} + D_{25} - D_{26} - D_{310}) - 2p_1 \cdot p_3(D_{39} + D_{310}) - 2p_2 \cdot p_3 D_{37} \right] \]

\[ f_{13}^{(1)} = -2m_t \sum_{i = H^0, h} \eta_i(D_{12} + D_{22} - D_{24} + 2D_{39} + 2D_{310})(-p_2, -p_1, p_3, m_t, m_i, m_t, m_t) \]

\[ f_{14}^{(1)} = 2m_t \sum_{i = H^0, h} \eta_i(D_{22} - D_{24} + D_{25} - D_{26}) \]

\[ f_{15}^{(1)} = -2m_t \sum_{i = H^0, h} \eta_i(D_0 + D_{12} - D_{21} + D_{24} + 2D_{39} + 2D_{310}) \]

\[ f_{16}^{(1)} = 2m_t \sum_{i = H^0, h} \eta_i(D_0 + D_{12} - D_{13} - D_{21} + D_{24} + D_{25} - D_{26}) \]

\[ f_{17}^{(1)} = 4 \sum_{i = H^0, h} \eta_i(D_{23} - D_{26} - D_{38} + D_{39}) \]

\[ f_{18}^{(1)} = 4 \sum_{i = H^0, h} \eta_i(-D_{12} + D_{13} + D_{23} - D_{24} + D_{25} - D_{26} + D_{37} - D_{310}) \]

\[ f_{19}^{(1)} = 4 \sum_{i = H^0, h} \eta_i(D_{12} - D_{13} + D_{23} + D_{24} - D_{25} - D_{26} + D_{39} - D_{310}) \]

\[ f_{20}^{(1)} = 4 \sum_{i = H^0, h} \eta_i(D_{23} - D_{25} - D_{35} + D_{37}) \]

\[ f_{1}^{(2)} = \frac{1}{2} m_t \sum_{i = H^+, W^+} \eta_i(2D_{312} - D_{311}) \]

\[ f_{2}^{(2)} = \frac{1}{4} m_t \sum_{i = H^+, W^+} \eta_i \left[ m_t^2(D_{31} - D_{32} - D_{34} + D_{36}) + 2p_2 \cdot p_4(D_{11} - D_{12} + D_{21} - D_{24} + D_{38}) + 2p_1 \cdot p_4(D_{24} - D_{22} - D_{35}) + 2p_1 \cdot p_2(D_{34} - D_{36}) + 2p_3 \cdot p_4(D_{26} - D_{25} - D_{310}) + 2(2D_{311} - 3D_{312}) \right] \]

\[ f_{3}^{(2)} = \frac{1}{2} \sum_{i = H^+, W^+} \eta_i \left[ m_t^2(D_{35} + D_{38} - 2D_{39} - 2D_{310}) + 2p_3 \cdot p_4(D_{26} - D_{23}) \right] \]
\[ f_{4}^{b(2)} = \frac{1}{2} \sum_{i = H^+, W^+} \eta_{i} \left[ m_{t}^{2}(2D_{21} - 2D_{22} + D_{22} - D_{21} + D_{23} + D_{36}) + 2p_{2} \cdot p_{4}(D_{12} - D_{13} + D_{24} - D_{25} - D_{38} + D_{39}) + 2p_{1} \cdot p_{3}D_{23} + 2p_{2} \cdot p_{3}D_{37} + 2p_{1} \cdot p_{2}(D_{22} - 2D_{26} + D_{36} - D_{310}) + 4(D_{312} - D_{313}) \right] \]

\[ f_{5}^{b(2)} = \frac{1}{2} \sum_{i = H^+, W^+} \eta_{i} \left[ m_{t}^{2}(D_{21} + D_{22} + D_{31} + D_{36}) + 2p_{1} \cdot p_{3}(D_{24} + D_{34}) - 2p_{1} \cdot p_{3}(D_{26} + D_{310}) + 2p_{1} \cdot p_{4}(D_{12} - D_{13} + D_{24} - 2D_{25} - D_{35}) + 2(D_{311} + 2D_{27}) \right] \]

\[ f_{6}^{b(2)} = \frac{1}{2} \sum_{i = H^+, W^+} \eta_{i} \left[ m_{t}^{2}(D_{21} - D_{22} - 2D_{25} + 2D_{26} + 2D_{39} + 2D_{310}) + 2p_{1} \cdot p_{4}(D_{13} - D_{12} + D_{24} + D_{25}) - 2p_{1} \cdot p_{3}D_{310} - (2D_{27} + D_{311}) \right] \]

\[ f_{7}^{b(2)} = m_{t} \sum_{i = H^+, W^+} \eta_{i}(D_{22} - D_{24} + D_{25} - D_{26} - D_{34} + D_{35} + D_{36} - D_{310}) \]

\[ f_{8}^{b(2)} = m_{t} \sum_{i = H^+, W^+} \eta_{i}(D_{22} - D_{24} + D_{25} - D_{26} + D_{21} - D_{22} + D_{23} + D_{36}) \]

\[ f_{9}^{b(2)} = \sum_{i = H^+, W^+} \eta_{i}[m_{t}(D_{36} - D_{34} - D_{38} + D_{310}) + D_{313}] \]

\[ f_{10}^{b(2)} = m_{t} \sum_{i = H^+, W^+} \eta_{i}(-D_{21} + D_{24} + D_{25} - D_{26} - D_{31} + D_{34} + D_{35} - D_{310}) \]

\[ f_{11}^{b(2)} = - \sum_{i = H^+, W^+} \eta_{i}(D_{27} + D_{313}) \]

\[ f_{12}^{b(2)} = \frac{1}{4} \sum_{i = H^+, W^+} \eta_{i} \left[ m_{t}^{2}(-2D_{11} + 2D_{13} - D_{21} - D_{22} + 2D_{25} + 2D_{26} - D_{35} + D_{38} - D_{39} + D_{310}) + 2p_{1} \cdot p_{2}(-D_{12} + D_{13} - D_{24} + D_{25} - D_{26} - D_{310}) - 2p_{1} \cdot p_{3}(D_{23} + D_{310}) - 2p_{3} \cdot p_{4}D_{23} - 2p_{2} \cdot p_{3}D_{37} + 2(D_{27} + 3D_{313}) \right] \]

\[ f_{13}^{b(2)} = \frac{1}{2} m_{t} \sum_{i = H^+, W^+} \eta_{i}(-D_{22} + D_{24} - 2D_{39} - 2D_{310}) \]

\[ f_{14}^{b(2)} = \frac{1}{2} m_{t} \sum_{i = H^+, W^+} \eta_{i}(D_{22} - D_{24} + D_{25} - D_{26}) \]

\[ f_{15}^{b(2)} = \frac{1}{2} m_{t} \sum_{i = H^+, W^+} \eta_{i}(D_{11} - D_{12} + D_{21} - D_{24} - 2D_{39} - 2D_{310}) \]

\[ f_{16}^{b(2)} = \frac{1}{2} m_{t} \sum_{i = H^+, W^+} \eta_{i}(-D_{11} + D_{12} - D_{21} + D_{24} + D_{25} - D_{26}) \]
\[\begin{align*}
    f_{17}^{b(2)} &= \sum_{i=H^+,W^+} \eta_i (D_{23} - D_{26} - D_{38} + D_{39}) \\
    f_{18}^{b(2)} &= \sum_{i=H^+,W^+} \eta_i (-D_{12} + D_{13} + D_{23} - D_{24} + D_{25} - D_{26} + D_{37} - D_{310}) \\
    f_{19}^{b(2)} &= \sum_{i=H^+,W^+} \eta_i (D_{12} - D_{13} + D_{23} + D_{24} - D_{25} - D_{26} + D_{39} - D_{310}) \\
    f_{20}^{b(2)} &= \sum_{i=H^+,W^+} \eta_i (D_{23} - D_{25} - D_{35} + D_{37}) \\
    f_1^{b(3)} &= -9m_t \sum_{i=H^+,W^+} \eta_i (-D_{27} - D_{311} + D_{312}) \\
    f_2^{b(3)} &= 0 \\
    f_3^{b(3)} &= 9 \sum_{i=H^+,W^+} \eta_i (D_{27} + D_{312}) \\
    f_4^{b(3)} &= -9 \sum_{i=H^+,W^+} \eta_i (D_{313} - D_{312}) \\
    f_5^{b(3)} &= 9 \sum_{i=H^+,W^+} \eta_i (D_{27} + D_{311}) \\
    f_6^{b(3)} &= -9 \sum_{i=H^+,W^+} \eta_i (D_{313} - D_{311}) \\
    f_7^{b(3)} &= -9m_t \sum_{i=H^+,W^+} \eta_i (D_{13} - D_{12} - D_{24} + D_{25} + D_{32} - D_{36} - D_{38} + D_{310}) \\
    f_8^{b(3)} &= -9m_t \sum_{i=H^+,W^+} \eta_i (D_{13} - D_{12} - 2D_{24} + D_{25} + D_{22} - D_{26} - D_{34} + D_{35} + D_{36} - D_{310}) \\
    f_9^{b(3)} &= -9m_t \sum_{i=H^+,W^+} \eta_i (D_{13} - D_{11} - D_{21} + D_{25} - 2D_{34} + D_{36} - D_{38} + D_{310}) \\
    f_{10}^{b(3)} &= -9m_t \sum_{i=H^+,W^+} \eta_i (D_{13} - D_{11} - 2D_{21} + 2D_{25} + D_{24} - D_{26} - D_{31} + D_{35} - D_{310}) \\
    f_{11}^{b(3)} &= 9 \sum_{i=H^+,W^+} \eta_i D_{313} \\
    f_{12}^{b(3)} &= f_{13}^{b(3)} = f_{14}^{b(3)} = f_{15}^{b(3)} = f_{16}^{b(3)} = 0, \\
    f_{17}^{b(3)} &= -9 \sum_{i=H^+,W^+} \eta_i (D_{23} - D_{26} + D_{39} - D_{38}) \\
    f_{18}^{b(3)} &= -9 \sum_{i=H^+,W^+} \eta_i (D_{23} - D_{26} + D_{37} - D_{310}) \\
    f_{19}^{b(3)} &= -9 \sum_{i=H^+,W^+} \eta_i (D_{23} - D_{25} + D_{39} - D_{310}) \\
    f_{20}^{b(3)} &= -9 \sum_{i=H^+,W^+} \eta_i (D_{23} - D_{25} + D_{37} - D_{35})
\end{align*}\]
\[ f_1^{(4)} = 2m_t \sum_{i=A,Z} \eta_i (2D_{27} - D_{311} + 2D_{312}) \]

\[ f_2^{(4)} = m_t \sum_{i=A,Z} \eta_i \left[ m_t^2 (D_0 - D_{11} + D_{12} + D_{21} + D_{22} + 2D_{24} + D_{31} - D_{32} - D_{34} + D_{36}) + 2p_3 \cdot p_4 (D_{13} - D_{12} - D_{30} + D_{26}) + 2p_2 \cdot p_4 (-D_0 + D_{21} - 2D_{24} - D_{25} - D_{26} + D_{38}) + 2p_1 \cdot p_4 (-2D_{13} + D_{22} - 2D_{25} - D_{35} + 2D_{12}) + 2p_1 \cdot p_2 (2D_{24} + D_{34} - D_{36}) + 2(2D_{311} - 3D_{312}) \right] \]

\[ f_3^{(4)} = \sum_{i=A,Z} \eta_i \left[ m_t^2 (D_{13} + 2D_{35} + 2D_{38} - 4D_{39}) - 4p_2 \cdot p_4 (D_{12} - D_{13} + D_{23} + D_{24} - D_{25} + D_{39}) + 4p_3 \cdot p_4 D_{26} - 4p_2 \cdot p_3 (D_{23} + D_{37} + D_{310}) + 4(D_{27} - D_{312} + 3D_{313}) \right] \]

\[ f_4^{(4)} = 2 \sum_{i=A,Z} \eta_i \left[ m_t^2 (-D_{12} + D_{13} - 2D_{22} - 2D_{23} + 4D_{26} + D_{32} + D_{34} - D_{35} - D_{38}) + 2p_2 \cdot p_4 (D_{12} - D_{13} + D_{24} - D_{25} - D_{38} + D_{39}) + 2p_1 \cdot p_2 (D_{22} + D_{23}) - 2D_{26} + D_{36} - D_{310}) + 2p_2 \cdot p_3 D_{33} + 4(D_{312} - D_{313}) \right] \]

\[ f_5^{(4)} = 2 \sum_{i=A,Z} \eta_i \left[ m_t^2 (D_0 - D_{11} + D_{21} + D_{22} + D_{31} + D_{36}) + 2p_1 \cdot p_4 (D_{12} + D_{13} + D_{24} - 2D_{25} - D_{35}) + 2p_1 \cdot p_2 (D_{24} + D_{34}) - 2p_1 \cdot p_3 (D_{26} + D_{310}) + 2(2D_{27} + D_{311}) \right] \]

\[ f_6^{(4)} = 2 \sum_{i=A,Z} \eta_i \left[ m_t^2 (-D_0 - D_{21} - D_{22} - 2D_{23} - 2D_{24} + 4D_{25} + 4D_{26} + 2D_{39} + D_{310}) + 2p_1 \cdot p_4 (-D_{12} + D_{13} + D_{25}) - 2p_2 \cdot p_4 (D_{23} + D_{310}) - 2p_1 \cdot p_2 D_{24} - (D_{311} + 2D_{37}) \right] \]

\[ f_7^{(4)} = 4m_t \sum_{i=A,Z} \eta_i (-D_{22} - 2D_{23} + 3D_{26} + D_{32} - D_{36} - D_{38} + D_{310}) \]

\[ f_8^{(4)} = 4m_t \sum_{i=A,Z} \eta_i (D_{12} - D_{13} + D_{22} - D_{26} - D_{34} + D_{35} + D_{36} - D_{310}) \]

\[ f_9^{(4)} = 4 \sum_{i=A,Z} \eta_i [m_t (-2D_{23} - D_{24} + 2D_{25} + D_{26} - D_{34} + D_{36} - D_{38} + D_{310}) + D_{313}] \]

\[ f_{10}^{(4)} = 4m_t \sum_{i=A,Z} \eta_i (D_{11} - D_{13} + D_{24} - D_{26} - D_{31} + D_{34} + D_{35} - D_{310}) \]

\[ f_{11}^{(4)} = -4 \sum_{i=A,Z} \eta_i (D_{27} + D_{313}) \]
\[
\begin{align*}
\sum_{i=A,Z} \eta_i \left[ m_t^2 \left( -3D_{13} - D_{21} - D_{22} - 2D_{23} + 2D_{25} + 2D_{26} + D_{35} + D_{38} - D_{39} + D_{310} \right) - 2p_1 \cdot p_3 (D_{39} + D_{310}) - 2p_2 \cdot p_3 D_{37} \\
+ 2p_1 \cdot p_2 (D_{12} + D_{13} - D_{23} - D_{24} + D_{25} - D_{26} - D_{310}) \\
+ 2(D_{27} + 3D_{313}) \right] 
\end{align*}
\]

\[
\begin{align*}
f_{12}^{(4)} &= -2m_t \sum_{i=A,Z} \eta_i (-D_{12} + 2D_{13} + D_{22} - D_{24} + 2D_{39} + 2D_{310}) \\
f_{13}^{(4)} &= 2m_t \sum_{i=A,Z} \eta_i (-2D_{12} + 2D_{13} + D_{22} + 2D_{23} - D_{24} + D_{25} - 3D_{26}) \\
f_{14}^{(4)} &= -2m_t \sum_{i=A,Z} \eta_i (D_0 + D_{12} - D_{21} + D_{24} + 2D_{39} + 2D_{310}) \\
f_{15}^{(4)} &= 2m_t \sum_{i=A,Z} \eta_i (D_0 + D_{12} - D_{13} - D_{21} + 2D_{23} + D_{24} - D_{25} - D_{26}) \\
f_{16}^{(4)} &= 4 \sum_{i=A,Z} \eta_i (D_{23} - D_{26} - D_{38} + D_{39}) \\
f_{17}^{(4)} &= 4 \sum_{i=A,Z} \eta_i (-D_{12} + D_{13} + D_{23} - D_{24} + D_{25} - D_{26} + D_{37} - D_{310}) \\
f_{18}^{(4)} &= 4 \sum_{i=A,Z} \eta_i (D_{12} - D_{13} + D_{23} + D_{24} - D_{25} - D_{26} + D_{39} - D_{310}) \\
f_{19}^{(4)} &= 4 \sum_{i=A,Z} \eta_i (D_{23} - D_{25} - D_{35} + D_{37}) \\
f_{20}^{(4)} &= \sum_{i=A,Z} \eta_i 2m_t D_{312}, f_2^{(5)} = 0, \\
f_{21}^{(5)} &= \sum_{i=H^+,W^+} \eta_i 2m_t D_{312}, f_2^{(5)} = 0, \\
f_{22}^{(5)} &= \sum_{i=H^+,W^+} \eta_i m_t^2 D_{22} - 2D_{27} - 2p_2 \cdot p_4 (D_{24} + D_{12} - D_{13} + D_{34}) \\
&+ 2p_3 \cdot p_4 (D_{25} + D_{35}) - p_2 \cdot p_3 (2D_{26} + D_{310}) + 4D_{311} + m_t^2 D_{36} \\
f_{23}^{(5)} &= \sum_{i=H^+,W^+} \eta_i 2D_{313}, \\
f_{24}^{(5)} &= \sum_{i=H^+,W^+} \eta_i 4(D_{311} - D_{312}) + 2p_3 \cdot p_4 D_{35} - 2p_2 \cdot p_4 D_{34} - p_2 \cdot p_3 D_{310} \\
&+ m_t^2 D_{36} + 2p_2 \cdot p_4 D_{36} + 2p_2 \cdot p_3 D_{38} - p_3 \cdot p_4 D_{310} \\
&- m_t^2 D_{32} + 2p_2 \cdot p_4 (D_{22} + D_{25} - D_{24} - D_{26}) \\
f_{26}^{(5)} &= \sum_{i=H^+,W^+} \eta_i 2(D_{312} - D_{313}), \\
f_{27}^{(5)} &= \sum_{i=H^+,W^+} \eta_i 2m_t (D_{26} + D_{310}), \\
f_{28}^{(5)} &= \sum_{i=H^+,W^+} \eta_i 2m_t (D_{310} - D_{38}),
\end{align*}
\]
\[ f^{b(5)}_9 = \sum_{i=H^+,W^+} \eta_1^2m_i(D_{26} - D_{22} + D_{310} - D_{36}) , \]
\[ f^{b(5)}_{10} = \sum_{i=H^+,W^+} \eta_1^2m_i(D_{32} - D_{38} - D_{310} - D_{36}) , \]
\[ f^{b(5)}_{11} = \sum_{i=H^+,W^+} \eta_1^2(D_{313} - D_{311}) , \]
\[ f^{b(5)}_{12} = \sum_{i=H^+,W^+} \eta_1 D_{27} , \]
\[ f^{b(5)}_{13} = \sum_{i=H^+,W^+} \eta_1 m_i(D_{12} + D_{24}) , \]
\[ f^{b(5)}_{14} = 0 , \]
\[ f^{b(5)}_{15} = \sum_{i=H^+,W^+} \eta_1 m_i(D_{24} - D_{22}) , \]
\[ f^{b(5)}_{16} = 0 , \]
\[ f^{b(5)}_{17} = \sum_{i=H^+,W^+} \eta_1^2(D_{23} - D_{25} + D_{37} - D_{35}) , \]
\[ f^{b(5)}_{18} = \sum_{i=H^+,W^+} \eta_1^2(D_{37} - D_{35} - D_{39}) , \]
\[ f^{b(5)}_{19} = \sum_{i=H^+,W^+} \eta_1^2(D_{23} + 2D_{24} - 2D_{25} - D_{26} + D_{12} - D_{13} + D_{34} + D_{37} - D_{35}) , \]
\[ f^{b(5)}_{20} = \sum_{i=H^+,W^+} \eta_1^2(D_{34} + D_{37} + D_{38} - D_{35} - D_{36} - D_{39} + D_{24} + D_{26} - D_{22} - D_{25}) , \]
\[ f^{\Delta}_1 = \sum_{i=H^9,h} \frac{\eta_i}{(s - m_i^2 + i\text{Im} \Gamma_i)} \{ -12m_i^2[m_i^2 C_0 - p_3 \cdot p_4(2C_{22} - 2C_{23} + C_0) + \frac{1}{2}] \}
(p_4, -p_1 - p_2, m_t, m_i, m_i)
+ \sum_{i=H^+,W^+} \frac{\eta_i}{Q_i^2} (m_i C_{11})(-p_2, p_1 + p_2, m_b, m_i, m_i)
\]
\[ f^{\Delta}_7 = f^{\Delta}_8 = f^{\Delta}_9 = f^{\Delta}_{10} = \sum_{i=H^9,h} \frac{\eta_i}{(s - m_i^2 + i\text{Im} \Gamma_i)} \{ -12m_i^2[C_0 + 4(C_{22} - C_{23})] \}
(p_4, -p_1 - p_2, m_t, m_t) \]

In the above, \(D_0, D_{ij}, D_{ijk}\) are the four-point Feynman integrals[17], and
\(D_0, D_{ij}, D_{ijk}(-p_2, -p_1, p_3, m_t, m_i, m_i, m_t)\) in \(f^{b(1)}_i\) and \(f^{b(4)}_i\)
\(D_0, D_{ij}, D_{ijk}(-p_2, -p_1, p_3, 0, m_t, 0, 0)\) in \(f^{b(2)}_i\)
\(D_0, D_{ij}, D_{ijk}(-p_2, -p_1, p_3, m_i, 0, m_i, m_i)\) in \(f^{b(3)}_i\)
\(D_0, D_{ij}, D_{ijk}(p_4, -p_2, p_3, m_b, m_b, m_i, m_i)\) in \(f^{b(5)}_i\)
and
\[ \hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_3 - p_1)^2, \quad \hat{u} = (p_3 - p_2)^2 \]
\[
p_1 \cdot p_2 = \frac{1}{2}(\hat{s} - 2m_i^2), \quad p_1 \cdot p_3 = p_2 \cdot p_4 = \frac{1}{2}(m_i^2 - \hat{i}),
\]
\[
p_3 \cdot p_4 = \frac{1}{2} \hat{s}, \quad p_1 \cdot p_4 = p_2 \cdot p_3 = \frac{1}{2}(m_i^2 - \hat{u})
\]
\[
\eta_H^0 = \frac{\sin^2 \alpha}{\sin^2 \beta}, \quad \eta_h = \frac{\cos^2 \alpha}{\sin^2 \beta},
\]
\[
\eta_A = \cot^2 \beta, \quad \eta_{H^+} = \cot^2 \beta + \frac{m_i^2}{m_i^2} \tan^2 \beta, \quad \eta_Z = \eta_{H^+} = 1.
\]

**Appendix B**

The expressions of \( H_i(m_i, p_1 \cdot p_2, p_1 \cdot p_3, p_1 \cdot p_4, p_3 \cdot p_4, \) in the amplitude squared are given by

\[
\begin{align*}
H_1 &= -8m_i^3 + 8m_i p_1 \cdot p_2 - 8m_i p_1 \cdot p_3 + 8m_i p_2 \cdot p_3 \\
H_2 &= -32m_i^3 - 16m_i p_1 \cdot p_2 - 32m_i p_1 \cdot p_3 + 32m_i p_2 \cdot p_3 \\
H_3 &= 32m_i^4 + 16m_i^2 p_1 \cdot p_2 - 16m_i^2 p_1 \cdot p_3 - 8m_i^2 p_2 \cdot p_3 \\
H_4 &= 8m_i^4 - 8m_i^2 p_1 \cdot p_2 - 16m_i^2 p_1 \cdot p_3 - 16p_1 \cdot p_2 p_1 \cdot p_3 + 8m_i^2 p_2 \cdot p_3 \\
H_5 &= 32m_i^2 p_1 \cdot p_2 + 16(p_1 \cdot p_2)^2 - 16m_i^2 p_2 \cdot p_3 - 16p_1 \cdot p_2 p_2 \cdot p_3 + 8m_i^2 p_1 \cdot p_3 \\
H_6 &= 8m_i^2 p_1 \cdot p_2 - 8m_i^4 - 16m_i^2 p_2 \cdot p_3 - 8m_i^2 p_1 \cdot p_3 \\
H_7 &= -8m_i^5 + 8m_i^3 p_1 \cdot p_2 + 4m_i^3 p_1 \cdot p_3 - 8m_i p_1 \cdot p_2 p_1 \cdot p_3 + 4m_i^3 p_2 \cdot p_3 \\
H_8 &= -8m_i^3 p_1 \cdot p_2 + 8m_i(p_1 \cdot p_2)^2 + 4m_i^3 p_2 \cdot p_3 - 4m_i^3 p_1 \cdot p_3 \\
H_9 &= -8m_i^3 p_1 \cdot p_2 + 8m_i^5 + 4m_i^3 p_2 \cdot p_3 - 4m_i^3 p_1 \cdot p_3 \\
H_{10} &= -8m_i(p_1 \cdot p_2)^2 + 8m_i^3 p_1 \cdot p_2 + 8m_i p_1 \cdot p_2 p_2 \cdot p_3 - 4m_i^3 p_1 \cdot p_3 - 4m_i^3 p_2 \cdot p_3 \\
H_{11} &= 8m_i^2 p_1 \cdot p_4 - 8m_i^2 p_2 \cdot p_4 + 8m_i^2 p_3 \cdot p_4 - 8p_1 \cdot p_4 p_2 \cdot p_3 \\
&\quad + 8p_1 \cdot p_3 p_3 \cdot p_4 - 8p_1 \cdot p_3 p_2 \cdot p_4 \\
H_{12} &= -16m_i^2 p_1 \cdot p_4 - 32m_i^2 p_2 \cdot p_4 + 32m_i^2 p_3 \cdot p_4 - 32p_1 \cdot p_3 p_2 \cdot p_4 \\
H_{13} &= 16m_i^2 p_1 \cdot p_4 + 32m_i^2 p_2 \cdot p_4 - 8m_i^3 p_3 \cdot p_4 - 16m_i p_1 \cdot p_3 p_2 \cdot p_4 \\
H_{14} &= -8m_i^3 p_1 \cdot p_4 + 8m_i^3 p_2 \cdot p_4 - 16m_i p_1 \cdot p_3 p_1 \cdot p_4 + 8m_i^3 p_3 \cdot p_4 \\
&\quad - 16m_i p_1 \cdot p_2 p_3 \cdot p_4 + 16m_i p_1 \cdot p_4 p_2 \cdot p_3 - 16m_i p_1 \cdot p_3 p_2 \cdot p_4 \\
H_{15} &= 16m_i p_1 \cdot p_2 p_1 \cdot p_4 + 32m_i p_1 \cdot p_2 p_2 \cdot p_4 - 8m_i p_1 \cdot p_4 p_2 \cdot p_3 + 8m_i p_1 \cdot p_3 p_2 \cdot p_4
\end{align*}
\]
\[-8m_{t}p_{1} \cdot p_{2}p_{3} \cdot p_{4} - 16m_{t}p_{2} \cdot p_{3}p_{2} \cdot p_{4}\]

\[H_{16} = -8m_{t}^{2}p_{2} \cdot p_{4} + 8m_{t}^{3}p_{1} \cdot p_{4} - 8m_{t}p_{1} \cdot p_{3}p_{2} \cdot p_{4} + 8m_{t}p_{1} \cdot p_{2}p_{3} \cdot p_{4} - 8m_{t}p_{1} \cdot p_{4}p_{2} \cdot p_{3} - 16m_{t}^{3}p_{3} \cdot p_{4}\]

\[H_{17} = -8m_{t}^{2}p_{1} \cdot p_{3}p_{1} \cdot p_{4} + 4m_{t}^{4}p_{3} \cdot p_{4} + 4m_{t}^{2}p_{1} \cdot p_{3}p_{2} \cdot p_{4} - 4m_{t}^{2}p_{1} \cdot p_{4}p_{2} \cdot p_{3} + 4m_{t}^{2}p_{1} \cdot p_{2}p_{3} \cdot p_{4} + 8m_{t}^{4}p_{1} \cdot p_{4} - 8m_{t}^{4}p_{2} \cdot p_{4}\]

\[H_{18} = 8m_{t}^{2}p_{1} \cdot p_{2}p_{1} \cdot p_{4} - 8m_{t}^{2}p_{1} \cdot p_{2}p_{2} \cdot p_{4} - 4m_{t}^{2}p_{1} \cdot p_{4}p_{2} \cdot p_{3} + 4m_{t}^{2}p_{1} \cdot p_{2}p_{3} \cdot p_{4} - 4m_{t}^{2}p_{1} \cdot p_{3}p_{2} \cdot p_{3} - 4m_{t}^{2}p_{1} \cdot p_{3}p_{1} \cdot p_{4} + 8m_{t}^{2}p_{2} \cdot p_{3}p_{2} \cdot p_{4} - 4m_{t}^{4}p_{3} \cdot p_{4}\]

\[H_{19} = 8m_{t}^{4}p_{2} \cdot p_{4} - 8m_{t}^{4}p_{1} \cdot p_{4} - 4m_{t}^{2}p_{1} \cdot p_{3}p_{2} \cdot p_{4} + 4m_{t}^{2}p_{1} \cdot p_{2}p_{3} \cdot p_{4} - 4m_{t}^{2}p_{1} \cdot p_{4}p_{2} \cdot p_{3} + 4m_{t}^{4}p_{3} \cdot p_{4}\]

\[H_{20} = 8m_{t}^{2}p_{1} \cdot p_{2}p_{2} \cdot p_{4} - 8m_{t}^{2}p_{1} \cdot p_{2}p_{1} \cdot p_{4} - 8m_{t}^{2}p_{2} \cdot p_{3}p_{2} \cdot p_{4} + 4m_{t}^{4}p_{3} \cdot p_{4} + 4m_{t}^{2}p_{1} \cdot p_{4}p_{2} \cdot p_{3} - 4m_{t}^{2}p_{1} \cdot p_{3}p_{2} \cdot p_{4} + 4m_{t}^{2}p_{1} \cdot p_{2}p_{3} \cdot p_{4}\]

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Figure Captions

Fig.1 Feynman diagrams contributive to $O(\alpha m_{\ell}^2/m_W^2)$ Yukawa corrections to $\gamma \gamma \to t\bar{t}$: (a),(b) tree level diagrams; (c)-(e) self-energy diagrams; (f)-(i) vertex diagrams; (j) including neutral Higgs exchange diagrams; (k) including $\gamma \gamma H^+H^+(\gamma \gamma G^+G^+)$-coupling diagrams; (l)-(n) box diagrams. Here we only plot the one-loop diagrams corresponding to tree-level diagram (a). The dashed lines represent $H, h, A, H^\pm, G^0, G^\pm$ for diagrams (c),(d),(e),(f),(h),(l), $H^\pm, G^\pm$ for diagrams (g),(i),(k),(m),(n) and $H, h$ for diagram(j).

Fig.2 Plot $\Delta \sigma/\sigma_0$ versus $M_h$ in the standard model.

Fig.3 Plot $\Delta \sigma/\sigma_0$ versus $M_h$ for $M_A = 600$ GeV in the two-Higgs-doublet model ($\alpha = \beta = 0.25$).

Fig.4 Plot $\Delta \sigma/\sigma_0$ versus $M_A$ for $M_h = 600$ GeV in the two-Higgs-doublet model ($\alpha = \beta = 0.25$).

Fig.5 Plot $\Delta \sigma/\sigma_0$ versus $M_A$ in the minimal SUSY model for $\tan \beta = 0.25$.

Fig.6 Plot $\Delta \sigma/\sigma_0$ versus $M_A$ in the minimal SUSY model for $\tan \beta = 1$.

Fig.7 Plot $\Delta \sigma/\sigma_0$ versus $M_A$ in the minimal SUSY model for $\tan \beta = 5$. 

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