Torsional natural frequencies by Holzer method

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Abstract. In this work is presented the estimation of the torsional natural frequencies, eigenvalues of the equation of motion, as a result of the modal analysis of a mechanical system, by the iterative method Holzer. A numerical model in Workbench ANSYS 14 is used to evaluate the effectiveness of the Holzer method in detecting the torsional natural frequencies. The system consists of a motor coupled to a flywheel by means of a rigid coupling, which has been simplified in a torsional equivalent system of inertial discs and torsional flexible shaft sections. The results obtained by the Holzer method are in good agreement with the results for the numerical case implemented in ANSYS.

1. Introduction

Torsional stresses and vibration have great influence in the performance in servicing of engineering components affecting their strength, expected operational life and dimensional stability. Torsional vibration is gaining the attention of the industrial community due to its high influence in the integrity of the machine components\([1]\). Shafts and rods can be continuously subject not only to torsional load but also to vibration around the rotation axis. This vibration can be masked by the load making difficult to monitoring or track this phenomenon. Under some circumstances, some rotational elements can be failed due to torsional vibration, which can be despised as a fatigue fault.

Monitoring of rotational elements presents some challenges, for instance; mechanical stress variations which are produced along the structure when loads are changing with also produce the torsional vibration\([2]\). The measurement of torsional stresses and torsional vibration are difficult, impractical and often prohibitively expensive. Therefore, it is important to determine in advance the structural dynamics, particularly, the rotational natural frequencies. Numerical and analytical methods have been used to determine frequency and mode shapes in structures\([3]\). In this study, the analytical iterative Holzer method is utilized to determine any number of natural frequencies for a multi-DOF system as illustrated in \([4]\). On the other hand, results are compared with a numerical model implemented in ANSYS.

2. System description

The studied system is composed of a motor coupled to a flywheel through a rigid mechanical coupling, as shown in Figure 1. The system is used to lifting loads. Table 1 presents the inertia moment and the stiffness of the motor, the mechanical coupling, and the flywheel.

The actual system is simplified by an equivalent system constituted of inertia masses and shaft sections, which must represent the inertia and torsional stiffness of motor (disk A), mechanical coupling (disk B) and flywheel (disk C) as represented by Figure 2 \([5]\). All the component is...
considered of the same material, steel, with a density of 7860 kg/m³. Young modulus (E) of 206.84 GPa and Poisson (\(\nu\)) of 0.3. These mechanical properties are employed to build the model in ANSYS of the system[6]. The equivalent system is determined by the equations given by Equations (1) to (4).

**Table 1.** Given data to the system to analyse.

| Variable | Values | Units |
|----------|--------|-------|
| \(J_1\)  | 2      | kg·m² |
| \(J_2\)  | 0.8    | kg·m² |
| \(J_3\)  | 3      | kg·m² |
| \(K_1\)  | 80     | kN·m/rad |
| \(K_2\)  | 60     | kN·m/rad |

**Figure 1.** Real mechanical system.

**Figure 2.** Equivalent system.

\[
G = \frac{E}{2(1 + \nu)} \quad (1)
\]

\[
I = \frac{\pi D^4}{32} \quad (2)
\]

\[
J = \rho \cdot I = \frac{D^4 \cdot \pi \cdot t \cdot D^4}{32} \quad (3)
\]

\[
K = \frac{G \cdot I}{L} \quad (4)
\]

where \(G\), \(I\), \(J\), \(K\), are the shear modulus, the second moment of inertia of a circular section (disk), the polar moment of inertia for the flywheel, and the stiffness of the shaft circular section, respectively.

In Figure 2, the diameters of the motor shaft and flywheel are equals to 0.05 m, with this value the equivalent length of the two shaft sections are determined. The flywheel diameter is calculated supposing a thickness \(t\) of 0.06 m, so substituting the given data in the previous equations the equivalent geometry to be used to model the system in ANSYS is found[7]. In Table 2 a summary of the information utilized to build the equivalent model is presented.

**Table 2.** Geometrical values of the equivalent system.

| Element   | Diameter (m) | T(m)  | L(m)  |
|-----------|--------------|-------|-------|
| Disk 1    | 0.424        | 0.0600| ---   |
| Disk 2    | 0.337        | 0.0600| ---   |
| Disk 3    | 0.470        | 0.0600| ---   |
| Shaft 1   | 0.050        | ---   | 0.610 |
| Shaft 2   | 0.050        | ---   | 0.814 |
3. Modal analysis in workbench

Once the CAD model of the studied equivalent system is built, a dynamical structural analysis is performed with ANSYS 14 which results are shown for the two first modal shapes[8] in Figures 3 and 4. The natural frequencies for these modal shapes are 26.473 Hz and 71.75 Hz.

![Figure 3. First torsional mode shape at 26.473 Hz.](image)

![Figure 4. Second torsional mode shape at 71.75 Hz.](image)

4. Holzer method

This iterative method is used to estimate the torsional natural frequencies and their corresponding modal shapes supposing a frequency and unitary amplitude in one end of the system and progressively calculating torque and angular displacement until reaching another end [4]. The estimated natural frequencies are considered valid if fulfilled the boundary constraints i.e. zero torque if one end is free and zero displacements for the set-in end.

The general expression for the twist angle and the torque for a torsional system of n disks, where \( j=1,2,\ldots, n \) is simply given by Equation (5) [9]:

\[
\theta_j = \theta_{j-1} - \frac{w^2 \sum_{i=1}^{j-1} I_i \theta_i}{K_{j-1}}
\]

(5)

The resultant torque \( T_n \) in the end for n disks is stated in Equation (6) as:

\[
T_n = \sum_{i=1}^{n} I_i w^2 \theta_i
\]

(6)
The iteration process is performed with the software engineering equation solver (EES) following the sequence presented below in Table 3. The rotational natural frequencies are performed for the following given data: \( J_1=2, J_2=0.8, J_3=3, K_1=80000, K_2=60000 \).

**Table 3. Iterative sequence to estimate torsional natural frequencies.**

| Angular Strains | Torque in disks |
|-----------------|-----------------|
| \( \theta_1 = 1 \) | \( T_1 = w^2 J_1 \theta_1 \) |
| \( \theta_2 = \theta_1 - \frac{w^2}{K_1} J_1 \theta_1 \) | \( T_2 = T_1 + w^2 J_2 \theta_2 \) |
| \( \theta_3 = \theta_2 - \frac{w^2}{K_2} (J_1 \theta_1 + J_2 \theta_2) \) | \( T_3 = T_2 + w^2 J_3 \theta_3 \) |

The range of frequencies between 0 and 120 Hz is assumed for practical reasons in the iterative process in EES, for this range, it was found two torsional natural frequencies around of 24.64 and 72.4 Hz. The provided information by the iterative process is are shown in Table 4 (natural frequencies and mode shapes). In Figure 5, it is presented the vibration modes at the estimated natural frequencies and in Figure 6, it is shown the torque variation with respect to the assumed frequency.

**Table 4. Torsional natural frequencies of the studied system by Holzer method.**

| \( f_n[Hz] \) | \( T_n[Nm] \) | \( \theta_n[rad] \) | \( \theta_n[rad] \) | \( \theta_n[rad] \) |
|-------------|-------------|---------------|---------------|---------------|
| 26.64       | 0           | 1             | 0.2996        | -0.7463       |
| 72.4        | 0           | 1             | -4.173        | 0.4438        |

**Figure 5.** Estimated torsional vibration modes by the method of Holzer.

**Figure 6.** Torque variation with the frequency.
5. Results
Comparing the values of the rotational natural frequencies obtained by ANSYS and Holzer method, it is found a good agreement among them with variations in 0.63% and 0.9% for first and second rotational natural frequencies respectively as given in Table 5.

| Table 5. Natural frequencies estimation in Hertz. |
|---------|---------|---------|
| Modal shape | ANSYS14 | HOLZER |
| First     | 26.473  | 26.64   |
| Second    | 71.75   | 72.4    |

6. Conclusions
In this work is studied an iterative method, Holzer, for calculating the rotational natural frequencies of a mechanical system. In order to evaluate its effectiveness in determining the rotational dynamic information, a numerical model of the system is built in ANSYS 14 to estimate the first two rotational natural frequencies in an assumed range between 0 and 120 Hz. This range is expected as an operation frequency range of a mechanical system in a real application. Comparing the obtained values, we can conclude that Holzer method may be an adequate candidate to determine the rotational natural frequencies based on its easy-implementation, low computational cost, and effectiveness in the estimation of the rotational vibration information.

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