Skyrmion Echo in a system of interacting Skyrmions

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(Dated: September 14, 2022)

We consider helical rotation of skyrmions confined in the potentials formed by nano-disk systems. Based on numerical and analytical calculations we propose the skyrmion echo phenomenon. The physical mechanism of the skyrmion echo formation is also proposed. Due to the distortion of the lattice, impurities, or pinning effect, confined skyrmions experience slightly different local fields, which leads to dephasing of the initial signal. The interaction between skyrmions also can contribute to the dephasing process. However, switching the magnetization direction in the nanodiscs (e.g. by spin transfer torque) also switches the helical rotation of the skyrmions from clockwise to anticlockwise (or vice-versa), and this restores the initial signal (which is the essence of skyrmion echo).

Introduction In 1950 Erwin Hahn discovered the effect that is now known as the spin echo [1]. Due to the inhomogeneity of a local magnetic field in solids, nuclear (or electron) spins precess with slightly different frequencies. Therefore, an initially excited pulse decays after a specific time. However, the application of a properly designed pulse reverses the precession direction from clockwise to anticlockwise (or vice-versa), and this restores the initial signal. The dephasing/rephasing mechanism of precessing spins was subsequently explained by Bloom [2]. In this letter, we show that the echo mechanism is applicable not only to precessing spins but also to more complex objects, such as, for instance, skyrmions. Skyrmions are topological solitons discovered in non-Abelian gauge field theories [3–6], and subsequently in condensed matter physics [7, 8]. They have localized robust shapes and additionally possess a topological charge—the conserved quantity underlying their topological protection. Magnetic films without inversion symmetry can host skyrmions and specific skyrmionic magnetic textures described by the local magnetization \( \mathbf{M}(\mathbf{r}) \) have been studied theoretically and then have been discovered experimentally. Owing to their topological properties and potential applications, skyrmions are currently of great interest, both theoretical and experimental. Skyrmions can exist as independent objects, but can also form regular skyrmion lattices (i.e. skyrmion crystals). For fundamental aspects of skyrmions, we refer to classical handbooks [9, 10]. Concerning modern mathematical aspects of skyrmions we refer to [11]. The key source of the skyrmion formation and non-collinear magnetic textures is either the interfacial Dzyaloshinskii–Moriya interaction (DMI) [12–20], competition between ferromagnetic and antiferromagnetic exchange interactions [21], or bulk DMI in case of antiferromagnetic skyrmions [22, 23]. Individual skyrmions can be pinned by specific confinement potentials, e.g., those created by inhomogeneous magnetic/electric field [24, 25], defects [26], spin transfer torque [27], or magnetic nanodisks [28, 29]. Recent interest in skyrmionics is focused especially on potential applications of skyrmions in data storage and processing technologies [30–35]. In this letter, we explore the formation of the skyrmion echo in a system of interacting skyrmions. In Fig. 1(a) we present the model considered in this paper. The bottom magnetic thin film hosts several skyrmions. On top of this film there are magnetic nanodisks that confine skyrmions in the regions below these nanodisks. Being confined the skyrmions may perform circular clockwise or anticlockwise motion in these regions, and the winding direction depends on the confined field from the nanodisks. By reversing this field one can switch the direction of the skyrmion winding trajectories, and this behavior can play a role similar to that of the second (rephasing) pulse in the Hahn’s spin echo case. Thus, due to dephasing induced by inhomogeneities in the confining magnetic field, one may observe the skyrmion echo—a phenomenon similar to the spin echo. The inhomogeneous field experienced by skyrmions may be formed in various ways, e.g., from lattice distortions or pinning sites. Skyrmion-skyrmion interactions can also contribute to the dephasing process. Material parameters influence the shape and width of the resonance spectra. Consequently, from the skyrmion echo one can extract information on the host material and the skyrmion-skyrmion interactions. To describe the process, we exploit the Landau-Lifshitz-Gilbert (LLG) equation:

\[
\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}. \tag{1}
\]

Here, \( \mathbf{M} = M_s \mathbf{m} \), with \( M_s \) and \( \mathbf{m} \) being the saturation magnetization and unit vector along the magnetization \( \mathbf{M} \), while \( \alpha \) is the phenomenological Gilbert damping constant. The total effective magnetic field, \( \mathbf{H}_{\text{eff}} \), reads:
\( \mathbf{H}_{\text{eff}} = \frac{2A_{\text{ex}}}{\mu_0 M_{\text{s}}} \nabla^2 \mathbf{m} + H_x \mathbf{z} + H_b \mathbf{z} - \frac{1}{\mu_0 M_{\text{s}}} \frac{\delta E_0}{\delta \mathbf{m}} \), where the first term describes the internal exchange field with the exchange stiffness \( A_{\text{ex}} \), the second term corresponds to the external magnetic field \( H_b \) (\( \mathbf{z} \) is a unit vector along the axis \( z \) normal to the film), the third term specifies coupling between the thin film and nanodisks, while the last term is the DM field, with the DM interaction energy density \( E_0 = D_m [(m_x dm_y - m_y dm_x) + (m_x dm_y - m_y dm_x)] \) and \( D_m \) being the strength of the DM interaction. The field \( H_b \) is due to all nanodisks and is nonuniform as contributions from different nanodisks may be different in magnitude. We assume that an individual (say \( i \)-th) nanodisk creates the confined field acting only on a single skyrmion, \( H_b = H_b^i \). The index \( i \) will be omitted in general, and will be included only where necessary. In the supplementary information [36] we show that the main conclusions concerning the skyrmion echo are still valid when the anisotropy and demagnetization fields are included into consideration. In numerical calculations we adopt typical parameters that correspond to Co/heavy-metal multi-layers: \( A_{\text{ex}} = 10 \text{pJ/m}, D_m = 0.2 \text{mJ/m}^2, \) and \( M_s = 1.2 \times 10^6 \text{A/m} \). The bias magnetic field \( H_x = 100 \text{mT} \) is used for stabilization of the skyrmion structure. The skyrmion width is 45 nm (see supplementary information [36]). The size of the ferromagnetic layer is \( 3000 \times 240 \times 3 \text{nm}^3 \). The ferromagnetic layer is discretized with the cells of size \( 3 \times 3 \times 3 \text{nm}^3 \). The radius of the upper nanodisks is 12 nm. These nanodisks exert a coupling field \( H_b \mathbf{z} \) on the skyrmions in the bottom thin ferromagnetic film. The distance between neighboring nanodisks is 270 nm.

**Results:** Let us consider now the dynamics of a skyrmion located in the region below a specific nanodisk. This dynamics is described by the position of the skyrmion center \((q_x, q_y)\) with respect to the center of the corresponding nanodisk, at \((0,0)\). Assume the skyrmion was initially slightly displaced from the center of the nanodisk. When the field exerted by the nanodisk on the skyrmion is negative, \( H_b < 0 \), the skyrmion moves then along the helix trajectory winding clockwise about the nanodisk center, see Fig. 1(b). The corresponding precession frequency is approximately equal to 0.068 GHz. Upon reversing direction of the field \( H_b \) to \( H_b > 0 \), the skyrmion starts to move gradually away from the nanodisk center with the anticlockwise precession. Interestingly, the frequency of an anticlockwise precession (0.071 GHz) is slightly larger than that of the clockwise one (0.068 GHz). This asymmetry in precession under the opposite magnetic fields \( H_b \) and \(-H_b \) is confirmed by detailed calculations for different amplitudes of \( H_b \), see Fig. 1(c).

An interesting question is how fast the system responds to the switching between clockwise and anticlockwise regimes. To explore this problem, we quench the sign of the exerted field \( H_b \) from \(-15 \text{mT}\) to \(15 \text{mT}\). Such a quench can be achieved for instance by reversing the magnetization orientation in the nanodisk, e.g., via a strong spin-transfer torque. We find that the skyrmion reacts almost instantaneously as follows from Fig. 1(b).

We elaborate the experimentally feasible scheme for a skyrmion echo. For the spin echo, the spins initially aligned parallel to the \( z \) axis are rotated by a \( \pi/2 \) pulse applied at \( t = 0 \), and then they start to precess in the \((xy)\) plane with different Larmor frequencies. The mismatch between these frequencies leads to a dephasing of the precessions of different spins. The second \( \pi \) pulse applied at \( t = \tau_0 \) reverses the precession direction and rephases the signal. In particular, the refocusing of the spin orientations occurs at \( t = 2\tau_0 \). Inspired by the idea of the spin echo, we generate a system of \( n = 10 \) separated skyrmions, that are pinned under the ten nanodisks, see Fig. 1. The coupling fields from these nanodisks are slightly different, inducing different precession frequencies of the ten skyrmions. This can be achieved by a slight variation of the spacer thickness between the nanodisks and magnetic film, as the coupling strength is a function of the spacer thickness [37]. In the model above, the pinning field induced by nanodisk is uniformly distributed and its amplitude varies from 10 mT to 20 mT. Variation of the spacer thickness in the range of 1 nm is sufficient to generate such a change in the pinning field [37, 38]. All skyrmions are initially steered away from the centers of nanodisks in the \(-x\) direction. This can be achieved by applying a spin-transfer torque or
FIG. 2. (a) Evolution of the precession phases $\varphi_i = \arctan(q_{xy}^i/q_{xz}^i)$ for 10 skyrmions. After the time interval 150 ns, the direction of magnetic field $H_b$ in all nanodisks is reversed periodically. (b) The time dependence of the amplitude of skyrmion echo, $\langle q_{xy} \rangle = \frac{1}{n}\left(\sum_{i=1}^{n} q_{xy}^i\right)^2 + (\sum_{i=1}^{n} q_{xz}^i)^2)^{1/2}$ (here the average is over all $n = 10$) skyrmions. Blue dashed lines mark the position of the skyrmion echo. (c) The time dependence of the $x$ component of the spin-pumping current $I_{sp}$ into an additional heavy metal layer adjacent to the magnetic layer. (d-e) Skyrmion echo induced by current pulse. Dynamics of the upper nanodisks is included via the spin transfer torque due to a current pulse in an additional magnetic layer adjacent on top of the layer of nanodisks (saturation magnetization $M_s = 1.4 \times 10^6 \text{A/m}$, exchange constant $A_{ex} = 30 \text{pJ/m}$, and uniaxial anisotropy along $z$) with constant $K_u = 1 \times 10^6 \text{J/m}^3$. After a time interval $= 150$ ns, we periodically reverse the magnetization direction of the nanodisks via a strong spin transfer torque, and plot (d) the evolution of $x$ and $y$ coordinates $q_x$ and $q_y$ of the skyrmion position (single skyrmion is plotted while the induced reversal in skyrmion precession applies to all 10 skyrmions), and (e) the averaged amplitude $\langle q_{xy} \rangle$ of all 10 skyrmions.

The skyrmion echo can be detected experimentally by attaching a heavy metal layer (for example a thin Pt layer) below the magnetic layer and exploiting the spin pumping and inverse spin Hall effects. The coherent precession of skyrmions pumps a spin current along the $z$ axis towards the adjacent metallic layer, $I_{sp} = \frac{g_e}{4\pi}(m \times \dot{m})$. The relevant spin mixing conductance is assumed to be $g_e = 7 \times 10^{18} \text{m}^{-2}$. By means of the inverse spin Hall effect, the spin pumping current is converted into an electric current (voltage) $J_{SH} = -\theta_{SH} \tau(I_{sp} \times z)$, where $\theta_{SH}$ is the spin-Hall angle. The $x$ component of $I_{sp}$ (that generates the $y$ component of $J_{SH}$) in the bottom magnetic layer is shown in Fig. 2(c), where one can clearly see the skyrmion echo signal. To prove that the sign of the field $H_b$ exerted on the skyrmions can be reversed in experiment, we analyse the magnetic dynamics of nanodisks due to a spin-transfer torque induced by a current pulse in an additional layer above the nanodisks. This dynamics is described by the LLG equation with the spin-transfer torque term included. Coupling between the additional layer and a particular nanodisk is introduced by the interlayer coupling field $H_c = \frac{1}{\mu_0 M_s} \gamma \tau m$, and a different coupling strength $J_c$ is assumed for each of the nanodisks. Here $\tau$ and $m$ stand for the layer thickness and unit vector along the nanodisk magnetization, respectively. The corresponding numerical results are shown in Fig. 2(d-e).
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FIG. 3. (a) Averaged (over skyrmion area) magnetic free energy density $F_M$ of the skyrmion as a function of the skyrmion position $q$, with respect to the corresponding nanodisk center, $q_0 = 0$. The magnetic free energy is extracted from the simulation results, and the data are fitted to the function $F_M = A r^2$, where $r^2 = q_x^2 + q_y^2$. (b) and (c) Time evolution of the precession phases (b) and amplitude (c). (a) Time evolution of the precession phases $\langle q_{xy} \rangle$ for the system described by the Thiele’s equation and on the micromagnetic simulations. We adjusted the value of $c$ to achieve a good agreement with the results based on the micromagnetic simulations.

Here, $D \approx 1$ stands for the $(x,x)$ and $(y,y)$ components of the dissipative force tensor $D$: $D_{ij} = D$ for $(i,j) = (x,x)$ and $(i,j) = (y,y)$, while $D_{ij} = 0$ otherwise [39]. In turn, $f_{xy}^p = 0$ is the force acting on a particular skyrmion due to the corresponding confining potential $V$, $f_{xy}^p = -\nabla V$. $f_{xy}^p$ and $V$ are determined from $F_M$, see the supplementary information [36]. We calculate numerically the free energy and fitted it to the analytical result obtained for the quadratic confinement function. The confining potential describes the effect of coupling between the skyrmion and the corresponding nanodisk, and generally may be different for different skyrmions. In particular, we consider $V = cr^2$, where $r^2 = q_x^2 + q_y^2$ and the pinning center is set as $(x,y) = (0,0)$. Additionally, the neighbouring skyrmions experience a repulsive interaction. As the skyrmions are confined in the areas below the nanodisks, the distance between them can vary only over a small range. Thus, one may assume that the repulsive force in the confinement region is constant. Accordingly, the energy corresponding to the repulsion of two $(i$-th and $j$-th) neighbouring skyrmions can then be written as $E_{ij} = -c_u r_{ij}$ (the detailed definition of coupling constant $c_u > 0$ and the distance between skyrmions $r_{ij}$ see in the supplementary information [36]). The repulsion force acting on the $i$-th skyrmion is determined as $f_{xy}^{ij} = -\partial E_{ij}/\partial q_{xy}^i$ (and similarly for the $j$-th skyrmion). Note, the absolute value of the force acting on the skyrmions is equal to $c_u$ and is measured in the units of $m/s \cdot m$ (because the force in Thiele’s equation is normalized to $M/\gamma$). In what follows the coefficients $c$ and $c_u$ are phenomenological constants and are tuned to achieve good agreement of the results based on Thiele’s equation and on the micromagnetic simulations.

To explore the role of the skyrmion-skyrmion interaction we first describe the skyrmion precession in the absence of inter-skyrmion coupling, $(c_u = 0)$, and adopt the ansatz $q_x = q_{x0} \exp(i \omega t)$ and $q_y = q_{y0} \exp(i \omega t)$. From the Thiele’s equation we derive the equation for the column vector $q = (q_{x0}, q_{y0})^T$, $\dot{q} = \omega q$. The explicit form of the matrix $\dot{\hat{H}}$ reads:

$$\dot{\hat{H}} = \frac{1}{1 + \alpha^2} \begin{pmatrix} -2i\alpha c & 2ic \\ -2ic & -2i\alpha c \end{pmatrix}.$$  

The eigenfrequencies of the matrix $\dot{\hat{H}}$ read: $\omega_{\pm} = \pm \frac{c}{1 + \alpha^2} \sqrt{\frac{1 + \alpha^2}{\alpha^2}}$. The eigenvectors corresponding to the eigenvalues $\omega_+$ and $\omega_-$ have the forms $(-i,1)$ and $(i,1)$, respectively. The real parts of the eigenfrequencies correspond to the skyrmion precession frequencies, and the imaginary parts describe the attenuation. As one can see in Fig. 3(a), the coefficient of the confinement potential $c$ is positive for a negative field $H_b$. Therefore, the steady clockwise precession is described by the frequency $\omega_+$ and the corresponding vector $(-i,1)$. In turn, for positive field, $H_b > 0$, the parameter $c$ is negative with a larger absolute value. The corresponding frequency $\omega_-$ has larger real part as well and the vector $(i,1)$ corresponds to counter-clockwise precession. This has been also confirmed in numerical simulations. To achieve a good agreement with the results of micromagnetic simulations, we adjusted the value of $c$ for different values of $H_b$, and calculated the time dependence of the precession phase and total oscillations, as plotted in Fig. 3. The analytical results are in good agreement with those obtained from micromagnetic simulations, see Fig. 2. To analyze the influence of coupling between skyrmions, we explored the skyrmion echo as a function of the distance $d_s$ between skyrmions (in the experiment $d_s$ is equal to the distance between nanodisks). The coupling strength between skyrmions increases with decreasing $d_s$ [21]. As shown in Fig. 4, the coupling energy and coupling strength exponentially decay for large $d_s$, while the echo signal increases and saturates. When decreasing $d_s$, the coupling strength increases and the echo signal $\langle q_{xy} \rangle$ becomes reduced. The observed effect has a clear physical explanation: For the short $d_s$, the observed non-monotonic behavior is related to collective oscillations caused by the strong coupling between skyrmions. For $d_s > 150nm$, the interaction between skyrmions is weak, and the echo signal is insensitive to the distance between skyrmions, see Fig. 4. One can extract information on the skyrmion-skyrmion interaction strength by performing echo experiments for $d_s < 150nm$. 
FIG. 4. (a) Dependence of the magnetic free energy density $F_c$ on the distance $d_s$ between neighboring skyrmions. The magnetic free energy is extracted from the simulation results. The energy $F_c(d_s)$ fits to the exponential function $F_c = A_0 \exp(-d_s/\xi)$ with the decaying length $\xi = 6.4$ nm. (b) The coupling strength $c_u$ extracted from linear expansion of the energy profile in the vicinity of the skyrmion center. (c) The amplitude $\langle q_{xy} \rangle$ of the skyrmion echo as a function of the distance $d_s$ between neighboring skyrmions. Amplitudes are fitted to the exponential function $\langle q_{xy} \rangle = I_1 - I_0 \exp(-d_s/\xi)$. The bias field is $H_z = 100$ mT.

Summary and conclusions: The inhomogeneous field leads to a dephasing of an initial signal. Switching the magnetization of the nanodisks (e.g. due to the application of a spin-polarized torque) turns the dephasing process into a rephasing one, and after a certain time, the signal of the skyrmion echo is recovered. The proposed skyrmion echo is experimentally feasible and can be detected by exploiting the spin pumping and inverse spin Hall effects. The skyrmion echo will also be important for the coupled systems of skyrmions and superconducting vortexes [42], i.e. for a hypothetical superconducting vortex echo.

Acknowledgements: The work is supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) (Grant No. FR-19-4049), the National Natural Science Foundation of China (Grants No. 12174452, No. 11704415 and No. 12074437), the National Science Foundation of Hunan Province of China (Grants No. 2022JJ20050 and No. 2021JJ30784), and by the National Science Center in Poland by the Norwegian Financial Mechanism 2014-2021 under the Polish-Norwegian Research Project NCN GRIEG (2Dtrronics) no. 2019/34/H/ST3/00515 (AD,JB), the FWF International Project I 5384, and as a research Project No. DEC-2017/27/B/ST3/ 02881 (VKD).

I. SUPPLEMENTARY INFORMATION

A. Numerical simulation details

The LLG equation is numerically solved employing a fifth-order Runge-Kutta scheme with a fixed time step of 0.5 ps. We adopted the finite difference approximation, and discretized the ferromagnetic layer in the unit simulation cells $s \times s \times s$ with $s = 3$ nm. To quantitatively characterize the skyrmion structure, we used the skyrmion topological charge density $c = (1/4\pi)\mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})$, and the total topological charge $C = \int d^2r c$. The position of $i$-th skyrmion centre, $\mathbf{q}_i = (q_{xi}, q_{yi})$, is weighed by the topological charge: $\mathbf{q}_i = \int d^2r \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m}) r / \int d^2r \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})$ [43]. For each skyrmion the integration range is limited by the area near the corresponding pinning center. When integrating over the whole magnetic layer one finds $\sum_{i=1}^{n_0} \mathbf{q}_i$.

To shift slightly the initial positions of skyrmions one can apply a spin-transfer torque [30] or a magnetic field [44]. For example, the applied spin transfer torque $\tau_{\text{STT}} = \gamma_{c}J \mathbf{m} \times \mathbf{y} \times \mathbf{m}$, with the electron polarization along the $y$ axis, steers the skyrmion center in the $x$ direction. As shown in Fig. S1(a), positive (negative) $c_1$ shifts the skyrmion along $+(-)x$ direction, and the induced displacement $q_x$ depends linearly on $c_1$ (or electric current density). The same can be achieved by spatially inhomogeneous magnetic field $H_z$. The gradient of the field, $\partial_x H_z$, linearly shifts the skyrmion along the axis $\pm x$ and the sign depends on the sign of the gradient $\partial_x H_z$, as it is shown in Fig. S1(b).

To exclude numerical artifacts of the coarse-graining procedure, we performed calculations for the smaller size of the cell, $s = 1$ nm. As it is shown in Fig. S2, the obtained result is almost identical with that obtained for $s = 3$ nm. The difference in precession frequencies is about 5%. The radius 45 nm of the skyrmion is larger than the radius of the nanodisk 12 nm. Confinement of the skyrmion by nanodisk is quite efficient.

We have also analyzed the impact of the Gilbert damping constant $\alpha$ and found that the magnitude of damping parameter $\alpha$ has a significant influence on the skyrmion...
relaxation and on the rephasing of the echo, see Fig. S3. The enhanced damping slows down the rephasing process and also decreases the amplitude of the skyrmion echo.

In the main text, the averaged magnetic energy density is calculated from the formula $F_M = -\mu_0 M_z \int m \cdot H_{\text{eff}} d^2 r$ applied to the region inside of the skyrmion (cross-section S). The spatial profile of the magnetization vector $m$ and the effective field $H_{\text{eff}}$ are obtained from simulation results. To obtain the position-dependent $F_M$ curve (Fig. 3(a) in the main text), we fixed the initial stable skyrmion texture and gradually moved the pinning center along the $x$ axis. Then, from the spatial gradient of $F_M$, one obtains the real phenomenological confining force acting on the skyrmion. However, due to specific normalization of the Thiele equation, to implement the confining force into this equation one needs to normalize accordingly the confining potential. Therefore, we introduce the relevant potential $V$ as $V = c_p F_M = (c_p A)^{2} = cr^{2}$, and in Fig. 1(c) in the main text) we used $c_p = 0.31 \times 10^{-9}\text{m}^5/\text{J s}$. The confining force $f_{xy}^c$ is the give by the gradient of $V$, $f_{xy}^c = -\partial V/\partial x, y$.

**B. Different bias field and distance**

The dephasing and rephasing times are slightly different. This fact can have an impact on the signal of the skyrmion echo. Therefore it is necessary to evaluate the quality of rephasing of the skyrmion echo. Let us define average difference between the skyrmions phases through the equation $\langle \varphi_{pd} \rangle = \sum_{i=1}^{n} \sum_{j=1}^{n} (\varphi_i - \varphi_j)/n^2$. In the case of exact rephasing one finds $\langle \varphi_{pd} \rangle = 0$. In Fig. S4 we see that signal of the skyrmion echo appears when $\langle \varphi_{pd} \rangle$ is minimal. For the bias field $H_x = 100 \text{ mT}$, the minimum of phase differences is $\langle \varphi_{pd} \rangle = 0.25$ at $t = 270 \text{ ns}$. With increasing $H_x$, the asymmetry between dephasing and rephasing times gradually decreases. At $H_x = 180 \text{ mT}$, the minimum of $\langle \varphi_{pd} \rangle \approx 0$ for $t = 300 \text{ ns}$ indicates on a perfect rephasing, and the amplitude of the corresponding echo signal becomes larger. With a further
increase of $H_z$, the rephasing time becomes larger and the minimum of $\langle \varphi_{pd} \rangle$ increases again, as is shown in Fig. S5. The amplitude of the skyrmion echo, $\langle q_{xy} \rangle$, increases until $H_z = 210$ mT, while $\langle \varphi_{pd} \rangle$ is not zero. The difference between behavior of $\langle q_{xy} \rangle$ and $\langle \varphi_{pd} \rangle$ is related to the bigger rephasing time. The skyrmion trajectory becomes larger in the rephasing stage. For example, at $H_z = 210$ mT, the echo (minimum of $\langle \varphi_{pd} \rangle$) is achieved at $t = 330$ ns, and larger rephasing time leads to the greater amplitude $\langle q_{xy} \rangle$.

Furthermore, as discussed in the main text, the skyrmion-skyrmion coupling strength $c_u$ also affects the skyrmion echo strength. We suggest varying the distance $d_u$ between neighboring skyrmions (and nanodisks) to reproduce the experimentally feasible small dispersion of the coupling strength. According to Ref. [45–48], the coupling strength between skyrmions increases with decreasing distance $d_u$. In the analysis, the coupling strength $c_u$ is defined from the linear ansatz. First, from the numerical simulation we extracted the magnetic free energy density $F_c$ as a function of the distance $d_u$ (also the distance between two neighboring skyrmions), which fits to the exponential function $F_c = A_0 \exp(-d_u/\xi)$. For application in the Thiele equation, the repulsion energy is given by the formula, $E_c = c_p A_0 \exp(-r_d/\xi)$, where $r_d = \sqrt{(d_u + q_i^s - q_j^s)^2 + (q_i^s - q_j^s)^2}$ is the distance between two skyrmions. From $f_{s,x} = -\partial E_c/\partial q_i^s$, we find the repulsion forces $f_{s,x} = E_c (d_u + q_i^s - q_j^s)/\xi r_d$ and $f_{s,y} = E_c (q_i^s - q_j^s)/\xi r_d$. As the skyrmion oscillates around the pinning center with a small amplitude, in this small range we adopted the linear ansatz $E_c = c_p A_0 \exp(-r_d/\xi)$.

An interesting question is the change of the echo strength with the pulse duration $\tau_0$. From the simulation results shown in Fig. S6(a), one can see that the amplitude of the echo decreases with the duration of the pulse $\tau_0$ and slightly fluctuates. We analyze this dependence by the numerical solution of Thiele’s equation. When all skyrmions are independent ($c_u = 0$), the echo strength decreases monotonically with $\tau_0$ (Fig. S6(a)). A strong enough coupling ($c_u = 0.12$ m/s) leads to fluctuations similar to those observed in the micromagnetic simulations. With the increase of the skyrmion-skyrmion interaction $c_u$, the fluctuation amplitude increases, but the shape of the curve remains unchanged.

We note that opposite to the conventional spin-echo, the trajectories of the skyrmions are helixes and therefore for individual skyrmions $\langle q_{xy} \rangle = \sqrt{(q_i^s)^2 + (q_j^s)^2}$ are not circles of constant radius [45]. This difference should be taken into account in order to understand behavior of the skyrmion echo amplitude with the coupling strength and pulse duration.

C. Influences of demagnetization field

For a more realistic discussion, we analyze the influences of dipole-dipole interaction on the skyrmion echo. In the simulation, we also include the uniaxial anisotropy field $2K_{u}m_{u}z$ along the $z$ axis with the constant $K_u = 0.905 \times 10^6$ Jm$^{-3}$, as well as the demagnetization field,

$$\mathbf{H}_{\text{demag}}(r) = -\frac{M_s}{4\pi} \int_{V} \nabla \nabla \left( \frac{\mathbf{m}(r')}{|r-r'|} \right) \mathrm{d}r'. \tag{S1}$$

The film is uniformly magnetized along the $z$ axis and the effective field of uniaxial anisotropy $1.2 \times 10^6$ A/m is
FIG. S7. The effect of the demagnetization field. (a) Dynamics of single skyrmion precession around the center (0, 0) of the nanodisk. (b-d) Time evolution of the precession phases, phase difference $\langle \varphi_{pd} \rangle$, and averaged $\langle q_{xy} \rangle$ over ten skyrmions calculated from micromagnetic simulations. The direction of coupling field $H_s$ in the nanodisk is reversed periodically after 100 ns, and the distance between neighboring skyrmions (and thus nanodisks) is $d_s = 540$ nm.

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