ULTRAQUANTUM DYNAMICS

James Baugh\textsuperscript{a}, David Ritz Finkelstein\textsuperscript{a}, Andrei Galiautdinov\textsuperscript{b}, Mohsen Shiri-Garakani\textsuperscript{a}

\textsuperscript{a} School of Physics, Georgia Institute of Technology, Atlanta, GA 30332-0430
\textsuperscript{b} Department of Mathematics and Science, Brenau University, Gainesville, GA 30501

Abstract

Segal proposed ultraquantum commutation relations with two ultraquantum constants $\hbar', \hbar''$ besides Planck’s quantum constant $\hbar$ and with a variable $i$. The Heisenberg quantum algebra is a contraction — in a more general sense than that of Inönü and Wigner — of the Segal ultraquantum algebra. The usual constant $i$ arises as a vacuum order-parameter in the quantum limit $\hbar', \hbar'' \to 0$. One physical consequence is a discrete spectrum for canonical variables and space-time coordinates. Another is an interconversion of time and energy accompanying space-time meltdown (disorder), with a fundamental conversion factor of some kilograms of energy per second.

Keywords: ultraquantum, group contraction, chronon,

\textsuperscript{*}Based on a talk presented to IARD 2002, The Third Biennial Meeting of the International Association for Relativistic Dynamics, June 24 - 26, 2002 at Howard University. A fuller account of the work is in preparation.
1 Segal Doctrine

A group that is not semisimple we call compound; a theory based on a compound group we also call compound. Compound theories have idols in the sense of Francis Bacon, false absolutes, which couple into other entities with no reciprocal coupling.

A group is unstable if any neighborhood of its structure tensor — defining the product on its Lie algebra — contains structure tensors of a non-isomorphic group. Then almost every group (structure tensor) in the neighborhood is non-isomorphic to the unstable group. Since measurements of structure tensors have error bars, they assign probability 0 to unstable groups. Unstable groups in physics are not based on experimental results as much as on faith in an idol.

Lie products $\times : V \otimes V \to V$ — also called structure tensors — form a quadratic submanifold $\Xi$ of the space of tensors $V \otimes V \to V$. By a contraction of a Lie algebra we mean the endpoint $\times(0)$ of a homotopy $\times(p)$ in $\Xi$ with $\times(1) = \times, \times(p) \cong \times$ for $0 < p \leq 1$, and $\times(0) \not\cong \times$. The concept is Segal’s, the nomenclature ours. Segal’s concept includes the Inönü-Wigner contraction as a very special case.

Segal proposed that groups of a physical theory should be stable. He further pointed out that stability requires semisimplicity: Compounds are unstable. The Segal Doctrine suggests that any compound physical theory is a contraction of a more stable, more accurate, semisimple theory, which we call its expansion.

2 Examples

The standard example of group contraction and expansion is the Galileo group, in which

$$B_x \times B_y = 0, \quad R_z \times B_x = B_y, \quad R_z \times B_y = -B_x, \quad \ldots$$

(1)
Here $B_x$ is the boost along the $x$ axis, $R_z$ is the rotation about the $z$ axis, and $A \times B := [A, B] = AB - BA$ is a Lie product. The Galileo algebra is unstable, its idol the absolute time $t$. Its familiar expansion is the Lorentz group, in which now

$$B_x \times B_y = c^{-2}R_z, \quad R_z \times B_y = B_x, \quad R_z \times B_y = -B_x, \quad \ldots. \quad (2)$$

This expansion is now stable against itself: a further small change in $c$ makes no difference to the group. The expansion parameter is $1/c^2 \to 0$.

Every bundle in physics implies a non-reciprocity, an idol, an instability, and a compound group. Indeed, the Galileo group is a bundle, the Lorentz is not.

For example, the point $p = (x, y)$ is a simple object, the chord $(p_1, p_2)$ is a semisimple object, the Cartesian product of two simples, but the limiting chord, the tangent vector $(p, dp)$, is a compound, containing the simple $p$ but not as a factor in a Cartesian product. The differential calculus is thus unstable, and so may be an unsuitable language for a supposedly empirical fundamental physics.

In consequence the Heisenberg Lie algebra defined by

$$p \times x = -\hbar i, \quad \hbar i \times p = x \times \hbar i = 0 \quad (3)$$

is not simple or semisimple but compound and unstable. Its idol is $i$. In the seminal paper [9] that stimulated the paper of Inönü and Wigner [7, 6], Segal proposed an expansion that simplified the Heisenberg algebra. For homogeneity we introduce skewsymmetric operators $\hat{x} := ix, \hat{p} = -ip$ to go with $i$ and designate their ultraquantum expansions by $\hat{x}, \hat{p}, \hat{i}$. The Segal ultraquantum commutation relations are essentially

$$\hat{p} \times \hat{x} = -\hbar \hat{i}, \quad \hat{i} \times \hat{p} = -\hbar' \hat{x}, \quad \hat{x} \times \hat{i} = -\hbar'' \hat{p}. \quad (4)$$
The compound Heisenberg algebra has become the simple SO(3) algebra. (SO(2,1) is another possibility, more promising in several respects.) This ultraquantum theory has two ultraquantum constants \( \hbar', \hbar'' \) in addition to the quantum constant \( \hbar \). It also goes beyond the quantum theory in that it quantizes — gives a discrete spectrum to — the canonical variables themselves. The space and time coordinates obey essentially the same commutation relations with the momentum and energy, and therefore undergo the same expansion and quantization.

3 Quantum Principles

By the relativity group \([2]\) of a system we mean the group of all admissible frame transformations. In classical mechanics this is the canonical group of the phase space. In quantum theory it is the unitary group of the Hilbert space of the quantum system. The Segal stability criterion suggests a fundamental quantum principle \([5]\):

*The relativity group of the system is a simple Lie group.*

We recover the part of quantum logic expressed by the ortholattice of predicates or projection operators as the lattice of simple Lie subgroups of the relativity group of the system; the orthogonality of two predicates is the commutativity of two orthogonal subgroups, element by element. It is well-known how to build the rest of the standard quantum kinematics on this ortholattice foundation. The ortholattice in question being non-distributive, the simplicity principle implies all the well-known quantum paradoxes.

Just as one may associate the unit energy intervals of the harmonic oscillator with an elementary quantum or phonon, we identify the unit intervals of the discrete ultraquantum spectrum of time with an elementary operation which we call the chronon \([4,5]\), characterized by a fundamental unit of time \( X \) (Chi).
4 Standard Model

We apply the Segal criterion to the Standard Model. The elements of the theory are

- Complex numbers $\mathbb{C}$
- Charges: hypercharge, isospin, color $U(1) \times SU(2) \times SU(3)$
- Spin $SL(2, \mathbb{C})$
- Space-time
- Fermions
- Bosons

We have italicized the unstable compounds. The instabilities both arise from the Heisenberg commutation relations: for $x$ and $\partial_x$ in the case of space-time, for annihilators $\phi$ and creators $\phi^\dagger$ in the case of Bose statistics. The key idol is the $i$ (or 1) on the right-hand side of the canonical commutation relation. The fermion algebra is a Clifford algebra, which is stable.

The diffeomorphism group of general relativity is likewise unstable and has the same idol. Thus present physics is infested with a cluster of idols all of the same tribe.

The expansion parameters $\bar{h}', \bar{h}''$ stabilize all these compounds if we replace every occurrence of the unstable Heisenberg quantum commutation relations by the stable ultraquantum commutation relations of Segal. Stability thus leads us to a real quantum theory rather than complex, with a large orthogonal group rather than a unitary one. The representations of this group may be found within its Clifford algebra.

Associativity is just as destabilizing as commutativity. It seems likely that experiment will eventually reveal non-associative processes underlying the present associative ones. Physically, non-associativity is a kind of binding. A powerful language for non-associative combination is set theory. There binding is represented by the Cantor brace $\{x\}$, which maps any set to a unit set. This defines...
a non-associative product: \( \{xy\}z \neq \{x\{yz\}\} \). We do not pursue non-associative physics at present. We introduce an ultraquantum set theory only to expand the classical set theory incorporated in the present theory of space-time.

We form the mode vector space of ultraquantum set theory by expanding the mode vector space of Fermi-Dirac statistics, a Grassmann algebra. The result is a free Clifford algebra \( \text{CLIFF} V \) over a quadratic space \( V \). We write this as an exponential

\[
\text{CLIFF} V = 2^V, \tag{5}
\]

the quantum correspondent of the classical power-set exponential

\[
PX = 2^X. \tag{6}
\]

We designate the Clifford algebra over a quadratic space with dimension \( n = n_+ + n_- \) and signature \( s = n_+ - n_- \) by \( \text{Cliff}(n_+, n_-) \). Our \( \text{Cliff}(n_+, n_-) \) has \( n_+ \) generators \( \gamma \) with \( \gamma^2 = -1 \) and \( n_- \) with \( \gamma^2 = +1 \). In particular \( \text{Cliff}(1, 0) = \mathbb{C} \).

In principle we need not give a separate meaning to the sub-symbol \( 2 \), since we use it only in the combination \( 2^V \). In fact we think of \( 2 \) as a two-dimensional Clifford algebra, either the complex numbers \( \text{Cliff}(1, 0) \) or the duplex numbers \( \text{Cliff}(0, 1) \), depending on the signature of \( V \). \( 2^V \) is a Clifford product of \( 2 \)'s, complex or duplex, one for each dimension of \( V \), with all its \( i \)'s anticommuting. For even \( \dim V \), we designate the underlying spinor module by \( \Sigma := \sqrt{2^V} \). This power notation for Clifford algebras and spinors incorporates several useful combinatorial identities.
5 Ultraquantum Dirac Algebra

We unify in one Clifford algebra Cliff(3, 3)^N of dimension 2^{6N} the variables of space-time-momentum-energy-spin and the imaginary unit i \[4, 5\], a six-fold unity, putting off for the moment the unitary charges. We index Clifford generators \(\gamma^\omega(n) \in \text{Cliff}(3N,3N) \sim \text{Cliff}(3,3)^N\), \(\omega = 1, \ldots, 6\), \(n = 1, \ldots, N\). The index \(n\) enumerates processes along the world line, a proper time variable. The usual Dirac spin \(\gamma^\mu\) is the “growing tip” of the world line, \(\gamma^{\mu5}(N)\).

The ultraquantum variables for one spin-1/2 particle are then

\[
\hat{i} := \frac{1}{N} \sum_{n=1}^{N} \gamma^{56}(n),
\]

\[
\hat{x}^\mu := -X \sum_{n=1}^{N} \gamma^{\mu5}(n),
\]

\[
\hat{p}^\nu := E \sum_{n=1}^{N} \gamma^{\nu6}(n),
\]

\[
\hat{\gamma}^\mu := -\gamma^{\mu5}(N).
\] (7)

In the contraction to the quantum limit of the ultraquantum theory, \(N \to \infty\), \(X, E \to 0\), \(i^2 \approx -1\), we impose \(X E N = \hbar^2/2\). Then Alg(\(\hat{i}, \hat{x}, \hat{p}\)) contracts to the Heisenberg algebra Alg(\(i, x, p\)) and Alg(\(\hat{i}, \hat{x}, \hat{p}, \hat{\gamma}\)) contracts to Dirac’s extension of the Heisenberg algebra, Alg(\(i, x, p, \gamma\)).

6 Ultraquantum Dirac Equation

A natural expansion of the Dirac equation, with expanded symmetry group, is \[4\]

\[
\hat{D}\psi = 0
\]
\[
\hat{D} := \frac{2E}{\hbar^2} \hat{S}^{\omega\rho} \hat{L}_{\omega\rho}
\]
\[
\rightarrow \gamma^\mu \partial_\mu + m_X \quad \text{as } N \to \infty, E \to 0, \text{and } X \text{ remains finite.} \quad (8)
\]

Here,
\[
\hat{S}^{\omega\rho} := \frac{\hbar}{2} \gamma^{\omega\rho}(N + 1),
\]
\[
\hat{L}_{\omega\rho} := \frac{\hbar}{2} \sum_{n=1}^{N} \gamma_{\omega\rho}(n), \quad (9)
\]

with \(\omega, \rho = 1, \ldots, 6\), and \(\mu, \nu = 1, \ldots, 4\).

We guess that \(m_X \sim \) top-quark mass and \(NX \lesssim \) age of universe:

\[
m_X \sim 10^2 \text{ GeV}
\]
\[
X \sim 10^{-25} \text{ sec}
\]
\[
N \lesssim 10^{41}.
\]

7 Cosmos Theory

Quantum theory treats of sharp filtration processes. No experimenter can carry out sharp filtration processes on the entire cosmos. How can one make a sensible cosmos theory?

Actually field theorists do this all the time. Laplace already did it. Their method was to ignore the philosophical problem and invent an imaginary exocosmic observer. We do the same.

The Segal criterion helps here: The purely operationalist theory is naive, and makes the absolute experimenter into a destabilizing idol.

Thus the first step is to renounce operationalism. Instead of the usual idealized version of the physical experimenter \(\hat{S}\) on the system we introduce an imaginary Cosmic Experimenter \(\hat{U}\) who
inputs, propagates, and outtakes the cosmos; a quantum version of Laplace’s fictitious omniscient intellect. The CE does experiments described by unitary operators $U$ with matrix elements of the form

$$\langle \tau = +T | U | \tau = -T \rangle, \quad (10)$$

where $2T$ is the duration of the experiment in the proper time of the CE. We imagine $\hat{U}$ measures what $\hat{S}$ measures; and ignores (traces over) what $\hat{S}$ ignores: especially the proper time $\tau$ of the CE.

Then cosmic energy $E = (i/\hbar)d/d\tau$ is a central (“superselection”) operator.

8 Q/Q Kinematics

Field theory began with classical fields on classical space-time (C/C), evolved to quantum fields on classical space-time (Q/C), and we can now formulate a Q/Q theory (quantum fields on quantum space-time).

Question: How do we define a field space

$$F = f^b$$

with quantum fiber and base defined by mode spaces $f, b$? By correspondence we demand that these spaces admit a concept of dimension with $\text{Dim} f^b = (\text{Dim} f)^{\text{Dim} b}$.

Answer \(\text{II}\): Set $f = (\sqrt{2})^\phi$.

Then

$$F = f^b := ((\sqrt{2})^\phi)^b = (\sqrt{2})^{\phi^b}. \quad (12)$$

For this construction the field $f$ must be spinorial, of course. So it is.
9 Qubits to Qunats

We propose to use the new algebraic structure created by the X expansion to make the observed bosons out of our fundamental fermions. Fermion occupation numbers are quantum binary numbers with eigenvalues $b = 0, 1$: qubits. A boson occupation number is a variable natural number with eigenvalues $n = 0, 1, 2, 3, \ldots$: a qunat. We have to make qunats out of qubits.

It is easy to make “nats” (natural numbers) out of bits; evidently 1 nat = $\infty$ bits. Likewise 1 qunat = $\infty$ qubits. Here is one way of making a qunat $(q, p)$ from $N$ qubits $\gamma(i, n)$, $i = 0, 1, 2$, $0 \leq n \leq N$ in the limit $N \to \infty$:

$$\hat{q} = Q \sum_n \gamma^{01}(n),$$
$$\hat{p} = P \sum_n \gamma^{02}(n),$$
$$\hat{\imath} = N^{-1} \sum_n \gamma^{12}(n),$$
$$\hat{q} \times \hat{p} = \hat{\imath},$$
$$\hat{\imath} \times \hat{q} = \hbar' \hat{p},$$
$$\hat{p} \times \hat{\imath} = \hbar'' \hat{q},$$
$$\hat{\imath}^2 = -1.$$

$$2NQP = 1, \quad 2Q = N\hbar, \quad 2PQ = \hbar'.$$

(13)

Here $\hbar', \hbar'' \ll 1$ are Segal ultraquantum constants and the value of the qunat is the usual oscillator Hamiltonian $H = \frac{\hbar}{2}(p^2 + q^2) - H_0$ with minimum eigenvalue $H_0$ reset to 0 by subtraction. For finite $J, Q, P$, however — and this is supposedly the actuality — these are not oscillators but rotators, namely spins, and the nat is bounded by $2l + 1$. 
10 Covariant Finite Toy Q/Q Kinematics

The above Cliff(3, 3) is a toy: the least Clifford algebra expanding and stabilizing the Poincaré and Heisenberg Lie algebras. It doesn’t accommodate the standard model. In quantum set theory, after the Dirac algebra $C^2 = \text{Cliff}(1, 3)$ comes $C^3 = \text{Cliff}(10, 6)$, a Clifford algebra over a 16-dimensional space instead of a 6-dimensional one. The additional 10 dimensions might accommodate the GUT, and 16 resonates with the Cartan-Atiyah-Singer-Botts octal periods.

Let us apply the Q/Q field construction (12) to the Cliff(3, 3) toy:

\[
\phi = \sqrt{\text{Cliff}(3, 3)} \sim 8, \\
\rightarrow = (\sqrt{2})^\phi \sim (\sqrt{2})^8 \sim 16.
\]

The number of qubits per cell in this toy is 16. The main thing is that the number is finite and the theory is covariant.

ACKNOWLEDGMENTS

One of the authors (A.G) acknowledges support from grant no. NICHD HD39787-02. We thank Larry Horwitz for helpful suggestions.

References

[1] J. Baugh, D. Finkelstein, A. Galiautdinov, and H. Saller, *J. Math. Phys.* 42, 1489 (2001)

[2] D. Finkelstein, *Quantum Relativity* (Springer-Verlag, New York, 1996)

[3] D. Finkelstein and A. Galiautdinov, *J. Math. Phys.* 42, 3299 (2001).

[4] A.A. Galiautdinov, *IJTP* 41, 1423 (2002)
[5] A.A. Galiautdinov and D.R. Finkelstein, *J. Math. Phys.* **43**, 4741 (2002)

[6] E. Inönü, Contraction of Lie Groups and their Representations, in F. Gürsey, ed., *Group Theoretical Concepts and Methods in Elementary Particle Physics*, pp. 391–402 (Gordon and Breach, Science publishers, New York, 1964)

[7] E. Inönü and E.P. Wigner, *Proc. Nat. Acad. Sci.* **39**, 510 (1952)

[8] C. Nayak and F. Wilczek, *Nucl. Phys.* **B479**, 529 (1996)

[9] I.E. Segal, *Duke Math. J.* **18**, 221 (1951)

[10] F. Wilczek, *Nucl.Phys.Proc.Suppl.* **68**, 367 (1998). Also [hep-th/9710135](http://arxiv.org/abs/hep-th/9710135)

[11] F. Wilczek, [hep-th/9806228](http://arxiv.org/abs/hep-th/9806228) [LANL]