Robust Imitation Learning from Noisy Demonstrations

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Abstract

Learning from noisy demonstrations is a practical but highly challenging problem in imitation learning. In this paper, we first theoretically show that robust imitation learning can be achieved by optimizing a classification risk with a symmetric loss. Based on this theoretical finding, we then propose a new imitation learning method that optimizes the classification risk by effectively combining pseudo-labeling with co-training. Unlike existing methods, our method does not require additional labels or strict assumptions about noise distributions. Experimental results on continuous-control benchmarks show that our method is more robust compared to state-of-the-art methods.

1 Introduction

The goal of sequential decision making is to learn a good policy that makes good decisions (Puterman, 1994). Imitation learning (IL) is an approach that learns a policy from demonstrations (i.e., sequences of demonstrators' decisions) (Schaal, 1999). Researchers have shown that a good policy can be learned efficiently from high-quality demonstrations collected from experts (Ng and Russell, 2000; Syed et al., 2008; Ziebart et al., 2010; Ho and Ermon, 2016; Sun et al., 2019). However, demonstrations in the real-world often have lower quality due to noise or insufficient expertise of demonstrators, especially when humans are involved in the data collection process (Mandlekar et al., 2018). This is problematic because low-quality demonstrations can reduce the efficiency of IL both in theory and practice (Tangkaratt et al., 2020). In this paper, we theoretically and experimentally show that IL can perform well even in the presence of noises.

In the literature, methods for IL from noisy demonstrations have been proposed, but they still have limitations as they require additional labels from experts or a strict assumption about noise distributions (Brown et al., 2019, 2020; Wu et al., 2019; Tangkaratt et al., 2020). Specifically, methods of Brown et al. (2019, 2020) require noisy demonstrations to be ranked according to their relative performance. Meanwhile, methods of Wu et al. (2019) require some of noisy demonstrations to be labeled with a score determining the probability that demonstrations are collect from experts. On the other hand, the method of Tangkaratt et al. (2020) does not require these labels, but instead it assumes that noisy demonstrations are generated by Gaussian noise distributions. To sum up, these methods require either additional labels from experts or a strict assumption about noise distributions. Due to this, the practicality of these methods is still limited, and IL from noisy demonstrations is still highly challenging.

To overcome the above limitation, we propose a new method for IL from noisy demonstrations called Robust IL with Co-pseudo-labeling (RIL-Co). Briefly, we built upon the recent theoretical results of robust classification (Charoenphakdee et al., 2019), and prove that robust IL can be achieved under a mild assumption by optimizing a classification risk with a symmetric loss. However, optimizing the proposed risk is not trivial because it contains a data density whose data samples are not observed. We show that pseudo-labeling (Chapelle et al., 2010) can be utilized to estimate the data density. However, naive pseudo-labeling may suffer from over-fitting and is not suitable in practice (Kingma et al., 2014). To remedy this issue, we propose co-pseudo-labeling, which effectively combines pseudo-labeling with co-training (Blum and Mitchell, 1998). Compare to prior work, RIL-Co does not require additional labels or assumptions about noise distributions. In addition, RIL-Co does not require an additional hyper-parameter tuning because an appropriate hyper-parameter value can be derived from the theory. Experiments on continuous-control benchmarks show that RIL-Co is more robust against noisy demonstrations when compared to state-of-the-art methods.
2 Imitation Learning and Robustness

In this section, we firstly give backgrounds about reinforcement learning and imitation learning. Then, we describe the setting of imitation learning with noisy demonstrations. Lastly, we discuss the robustness of existing imitation learning methods.

2.1 Reinforcement Learning

Reinforcement learning (RL) aims to learn an optimal policy of a Markov decision process (MDP) [Puterman 1994]. We consider a discrete-time MDP denoted by \( \mathcal{M} = (\mathcal{S}, \mathcal{A}, p_T(s'|s,a), r(s,a), \gamma) \) with state \( s \in \mathcal{S} \), action \( a \in \mathcal{A} \), transition probability density \( p_T \), initial state probability density \( p_1 \), reward function \( r \), and discount factor \( 0 < \gamma < 1 \). A policy function \( \pi(a|s) \) determines the conditional probability density of an action in a state. An agent acts in an MDP by observing a state, choosing an action according to a policy, transitioning to a next state according to the transition probability density, and possibly receiving an immediate reward according to the reward function. An optimal policy of an MDP is a policy maximizing the expected cumulative discounted rewards.

Formally, RL seeks for an optimal policy by solving the optimization problem \( \max_{\pi} \mathcal{J}(\pi) \), where \( \mathcal{J}(\pi) \) is the expected cumulative discounted rewards defined as

\[
\mathcal{J}(\pi) = \mathbb{E}_{p_{\pi}(\tau)} \left[ \sum_{t=1}^{T} \gamma^{t-1} r(s_t, a_t) \right] = \mathbb{E}_{p_{\pi}[r(s_t, a_t)]} / (1 - \gamma).
\]

Here, \( p_{\pi}(\tau) = p_1(s_1) \Pi_{t=1}^{T} p_T(s_{t+1}|s_t, a_t) \pi(a_t|s_t) \) is the probability density of trajectory \( \tau = (s_1, a_1, \ldots, s_{T+1}) \) with length \( T \), \( r_\pi(s, a) = (1 - \gamma) \mathbb{E}_{p_{\pi}(\tau)} [\sum_{t=1}^{T} \gamma^t \delta(s_t - s, a_t - a)] \) is the state-action density determining the probability density of the agent with \( \pi \) observing \( s \) and executing \( a \), and \( \delta(s_t - s, a_t - a) \) is the Dirac delta function. Note that the state-action density is a normalized occupancy measure and uniquely corresponds to a policy by a relation \( \pi(a|s) = r_\pi(s, a) / \rho_\pi(s) \), where \( \rho_\pi(s) = \int_{\mathcal{A}} r_\pi(s, a) \, da \) [Syed et al. 2008]. Also note that the optimal policy is not necessarily unique since there can be different policies that achieve the same expected cumulative discounted rewards.

While RL has achieved impressive performance in recent years [Silver et al. 2017], its major limitation is that it requires a suitable reward function which may be unavailable in practice [Schaal 1999].

2.2 Imitation Learning

Imitation learning (IL) is a well-known approach to learn an optimal policy when the reward function is unavailable [Ng and Russell 2000]. Instead of learning from the reward function or reward values, IL methods learn an optimal policy from demonstrations that contain information about an optimal policy. IL methods typically assume that demonstrations (i.e., a dataset of state-action samples) are collected by using an expert policy that is similar to an optimal (or near optimal) policy, and they aim to learn the expert policy [Ng and Russell 2000] Syed et al. 2008 Ziebart et al. 2010. More formally, the typical goal of IL is to learn an expert policy \( \pi_E \) by using a dataset of state-action samples drawn from an expert state-action density:

\[
\{(s_n, a_n)\}_{n=1}^{N} \overset{i.i.d.}{\sim} \rho_E(s, a),
\]

where \( \rho_E \) is a state-action density of expert policy \( \pi_E \).

The density matching approach was shown to be effective in learning the expert policy from expert demonstrations [Syed et al. 2008] Ho and Ermon 2016 Ghasemipour et al. 2020. Briefly, this approach seeks for a policy \( \pi \) that minimizes a divergence between the state-action densities of the expert and learning policies. Formally, this approach aims to solve the following optimization problem:

\[
\min_{\pi} D(\rho_E(s,a)||\rho_\pi(s,a)),
\]

where \( D \) is a divergence such as the Jensen-Shannon divergence. In practice, the divergence, which contains unknown state-action densities, is estimated by using demonstrations and trajectories drawn from Ghasemipour et al. 2020.

\footnote{For non-symmetric divergence, an optimization problem \( \min_{\pi} D(\rho_\pi(s,a)||\rho_E(s,a)) \) can be considered as well Ghasemipour et al. 2020.}
and Ermon (2016), which minimizes an estimate of the Jensen-Shannon divergence by solving
\[
\min \max_g \mathbb{E}_{\rho_E} \left[ \log \left( \frac{1}{1 + \exp(-g(s,a))} \right) \right] + \mathbb{E}_{\rho_N} \left[ \log \left( \frac{1}{1 + \exp(g(s,a))} \right) \right],
\]
where \( g : S \times A \mapsto \mathbb{R} \) is a discriminator function.

Density matching methods were shown to scale well to high-dimensional problems when combined with deep neural networks (Ho and Ermon 2016; Chasempour et al. 2020). However, an issue of this approach is that it is not robust against noisy demonstrations, as will be described in Section 2.4.

### 2.3 Learning from Noisy Demonstrations

In this paper, we consider a scenario of *IL from noisy demonstrations*, where given demonstrations are a mixture of expert and non-expert demonstrations. We assume that we are given a dataset of state-action samples drawn from a noisy state-action density:
\[
\mathcal{D} = \{(s_n, a_n)\}_{n=1}^{N} \overset{i.i.d.}{\sim} \rho'(s,a),
\]
where the noisy state-action density \( \rho' \) is a mixture of the expert and non-expert state-action densities:
\[
\rho'(s,a) = \alpha \rho_E(s,a) + (1-\alpha) \rho_N(s,a).
\]

Here, \( 0.5 < \alpha < 1 \) is an unknown mixing coefficient and \( \rho_E \) is the state-action density of an expert policy \( \pi_E \). The policy \( \pi_N \) is non-expert in the sense that \( \mathbb{E}_{\rho_N}[r(s,a)] < \mathbb{E}_{\rho_E}[r(s,a)] \), where \( r \) is an unknown reward function of the MDP. Our goal is to learn the expert policy using the dataset in Eq. (5).

We emphasize that \( 0.5 < \alpha < 1 \) corresponds to an assumption that the majority of demonstrations are obtained by the expert policy. This is a typical assumption when learning from noisy data, i.e., the number of good quality samples should be more than that of low quality samples (Angluin and Laird 1988; Natarajan et al. 2013). For notational brevity, we denote a state-action pair by \( x = (s,a) \), where \( x \in \mathcal{X} \) and \( \mathcal{X} = S \times A \).

### 2.4 Robustness of Imitation Learning

It can be verified that the density matching approach is not robust against noisy demonstrations according to the data generation assumption in Eq. (6). Specifically, given demonstrations drawn from \( \rho' \), the density matching approach would solve \( \min_\pi D(\rho'(x)||\rho_E(x)) \). By assuming that the space of \( \pi \) is sufficiently large, the solution of this optimization problem is
\[
\pi^*(x|s) = \pi_E(x|s) \left( \frac{\alpha \rho_E(x)}{\alpha \rho_E(x) + (1-\alpha) \rho_N(x)} \right) + \pi_N(x|s) \left( \frac{(1-\alpha) \rho_N(x)}{\alpha \rho_E(x) + (1-\alpha) \rho_N(x)} \right),
\]
which yields \( D(\rho'(x)||\rho_E(x)) = 0 \). However, this policy is not equivalent to the expert policy unless \( \alpha = 1 \). Therefore, density matching is not robust against noisy demonstrations generated according to Eq. (6).

We note that the data generation assumption in Eq. (6) has been considered previously by Wu et al. (2019). In this prior work, the authors proposed a robust method that learns the policy by solving
\[
\min_\pi D(\rho'(x)||\alpha \rho_E(x) + (1-\alpha) \rho_N(x)).
\]
The authors showed that this optimization problem yields the expert policy under the data generation assumption in Eq. (6). However, solving this optimization problem requires \( \alpha \) and \( \rho_N \) which are typically unknown. To overcome this issue, Wu et al. (2019) proposed to estimate \( \alpha \) and \( \rho_N \) by using additional noisy demonstrations that are labeled with a score determining the probability that demonstrations are drawn from \( \rho_E \). However, this method is not applicable in our setting since labeled demonstrations are not available.

### 3 Robust Imitation Learning

In this section, we propose our method for robust IL. Briefly, in Section 3.1 we propose an IL objective which optimizes a classification risk with a symmetric loss and prove its robustness. Then, in Section 3.2 we propose a new IL method that utilizes co-pseudo-labeling to optimize the classification risk. Lastly, we discuss the choice of a hyper-parameter in Section 3.3 and the choice of symmetric losses in Section 3.4.
3.1 Imitation Learning via Risk Optimization

Classification risks are fundamental quantities in classification \cite{Hastie2001}. We are interested in a balanced risk for binary classification where the class prior is balanced \cite{Brodersen2010}. Specifically, we propose to perform IL by solving the following risk optimization problem:

$$\max_p \min_g \mathcal{R}(g; \rho^\ell, \rho^\lambda, \ell_{\text{sym}}),$$

where $\mathcal{R}$ is the balanced risk defined as

$$\mathcal{R}(g; \rho^\ell, \rho^\lambda, \ell) = \frac{1}{2} \mathbb{E}_{\rho^\ell}[\ell(g(x))] + \frac{1}{2} \mathbb{E}_{\rho^\lambda}[\ell(-g(x))],$$

and $\rho^\lambda$ is a mixture density defined as

$$\rho^\lambda(x) = \lambda \rho_N(x) + (1 - \lambda) \rho_e(x).$$

Here, $0 < \lambda < 1$ is a hyper-parameter, $\pi$ is a policy to be learned by maximizing the risk, $g: \mathcal{X} \rightarrow \mathbb{R}$ is a classifier to be learned by minimizing the risk, and $\ell_{\text{sym}}: \mathbb{R} \rightarrow \mathbb{R}$ is a symmetric loss satisfying

$$\ell_{\text{sym}}(g(x)) + \ell_{\text{sym}}(-g(x)) = c,$$

for all $x \in \mathcal{X}$, where $c \in \mathbb{R}$ is a constant. Appropriate choices of the hyper-parameter and loss will be discussed in Sections 3.3 and 5.2, respectively.

We note that the balanced risk assumes that the positive and negative class priors are equal to $\frac{1}{2}$. This assumption typically makes the balanced risk more restrictive than other risks, because a classifier is learned to maximize the balanced accuracy instead of the accuracy \cite{Menon2013, Lu2019}. However, the balanced risk is not too restrictive for IL, because the goal is to learn the expert policy and the classifier is discarded after learning. Moreover, existing methods such as GAIL can be viewed as methods that optimize the balanced risk, as will be discussed in Section 3.3.

Next, we prove that the risk optimization in Eq. (9) yields the expert policy under a mild assumption.

**Assumption 1** (Mixture state-action density). The state-action density of the learning policy $\pi$ is a mixture of the state-action densities of the expert and non-expert policies with a mixing coefficient $0 < \kappa(\pi) \leq 1$:

$$\rho_\pi(x) = \kappa(\pi) \rho_{\text{E}}(x) + (1 - \kappa(\pi)) \rho_N(x),$$

where $\rho_\pi(x)$, $\rho_{\text{E}}(x)$, and $\rho_N(x)$ are the state-action densities of the learning policy, the expert policy, and the non-expert policy, respectively.

This is a mild assumption that is based on the following observation on a typical optimization procedure of $\pi$: At the start of learning, $\pi$ is randomly initialized and generates data that are similar to those of the non-expert policy. This scenario corresponds to Eq. (13) with $\kappa(\pi) \approx 0$. As training progresses, the policy improves and generates data that are a mixture of those from the expert and non-expert policies. This scenario corresponds to Eq. (13) with $0 < \kappa(\pi) < 1$. Indeed, the scenario where the agent successfully learns the expert policy corresponds to Eq. (13) with $\kappa(\pi) = 1$. We note that a policy corresponding to $\rho_\pi$ in Eq. (13) is a mixture between $\pi_\text{E}$ and $\pi_N$ with a mixture coefficient depending on $\kappa(\pi)$. However, we cannot directly evaluate the value of $\kappa(\pi)$. This is because we do not directly optimize the state-action density $\rho_\pi$. Instead, we optimize the policy $\pi$ by using an RL method, as will be discussed in Section 3.2.

Under Assumption 1, we obtain Lemma 1.

**Lemma 1.** Letting $\ell_{\text{sym}}(\cdot)$ be a symmetric loss that satisfies $\ell_{\text{sym}}(g(x)) + \ell_{\text{sym}}(-g(x)) = c$, $\forall x \in \mathcal{X}$ and a constant $c \in \mathbb{R}$, the following equality holds.

$$\mathcal{R}(g; \rho^\ell, \rho_\pi^\lambda, \ell_{\text{sym}}) = (\alpha - \kappa(\pi)(1 - \lambda)) \mathcal{R}(g; \rho_{\text{E}}, \rho_N, \ell_{\text{sym}}) + \frac{1 - \alpha + \kappa(\pi)(1 - \lambda)}{2} c.$$  

**Proof.** First, we substitute $\rho^\ell := \alpha \rho_{\text{E}}(x) + (1 - \alpha) \rho_N(x)$ and $\rho_e := \kappa(\pi) \rho_{\text{E}}(x) + (1 - \kappa(\pi)) \rho_N(x)$ into the risk $\mathcal{R}(g; \rho^\ell, \rho_\pi^\lambda, \ell)$ as follows.

$$2 \mathcal{R}(g; \rho^\ell, \rho_\pi^\lambda, \ell) = \mathbb{E}_{\rho^\ell}[\ell(g(x))] + \mathbb{E}_{\rho_\pi^\lambda}[\ell(-g(x))]$$

$$= \mathbb{E}_{\rho^\ell}[\ell(g(x))] + (1 - \lambda) \mathbb{E}_{\rho_e}[\ell(-g(x))] + \lambda \mathbb{E}_{\rho_\pi}[\ell(-g(x))]$$

$$= \alpha \mathbb{E}_{\rho_{\text{E}}}[\ell(g(x))] + (1 - \alpha) \mathbb{E}_{\rho_N}[\ell(g(x))] + \kappa(\pi)(1 - \lambda) \mathbb{E}_{\rho_{\text{E}}}[\ell(-g(x))]$$

$$+ (1 - \kappa(\pi))(1 - \lambda) \mathbb{E}_{\rho_N}[\ell(-g(x))] + \lambda \mathbb{E}_{\rho_{\text{E}}}[\ell(-g(x))].$$

(15)
Letting \( \kappa(\pi, \lambda) = \kappa(\pi)(1 - \lambda) \) and \( \delta'(x) = \ell(g(x)) + \ell(-g(x)) \), the risk is expressed as follows.

\[
2\mathcal{R}(g; \rho', \rho^*_\alpha, \ell) = \alpha\mathbb{E}_{p_{\rho}}[\ell(g(x))] + (1 - \alpha)\mathbb{E}_{p_{\rho}}[\ell(-g(x))] + \kappa(\pi, \lambda)\mathbb{E}_{p_{\rho}}[\ell((-g(x))] \\
+ (1 - \kappa(\pi, \lambda))\mathbb{E}_{p_{\rho}}[\ell(-g(x))] \\
= \alpha\mathbb{E}_{p_{\rho}}[\ell(g(x))] + (1 - \alpha)\mathbb{E}_{p_{\rho}}[\delta'(x) - \ell(g(x))] \\
+ \kappa(\pi, \lambda)\mathbb{E}_{p_{\rho}}[\delta'(x) - \ell(g(x))] + (1 - \kappa(\pi, \lambda))\mathbb{E}_{p_{\rho}}[\ell((-g(x))] \\
= (\alpha - \kappa(\pi, \lambda))\mathbb{E}_{p_{\rho}}[\ell(g(x))] + \mathbb{E}_{p_{\rho}}[\ell((-g(x))] \\
+ (1 - \alpha)\mathbb{E}_{p_{\rho}}[\delta'(x)] + \kappa(\pi, \lambda)\mathbb{E}_{p_{\rho}}[\delta'(x)] \\
= 2(\alpha - \kappa(\pi, \lambda))\mathbb{E}_{p_{\rho}}[\delta'(x)] + \kappa(\pi, \lambda)\mathbb{E}_{p_{\rho}}[\delta'(x)].
\]

For symmetric loss, we have \( \delta_{sym}(x) = \ell_{sym}(g(x)) + \ell_{sym}(-g(x)) = c \) for a constant \( c \in \mathbb{R} \). With this, we can express the left hand-side of Eq. (14) as follows:

\[
\mathcal{R}(g; \rho', \rho^*_\alpha, \ell_{sym}) = (\alpha - \kappa(\pi)(1 - \lambda))\mathcal{R}(g; \rho_{E}, \rho_{N}, \ell_{sym}) + \frac{(1 - \alpha)}{2}\mathbb{E}_{p_{\rho}}[c] + \frac{\kappa(\pi)(1 - \lambda)}{2}\mathbb{E}_{\rho_{E}}[c] \\
= (\alpha - \kappa(\pi)(1 - \lambda))\mathcal{R}(g; \rho_{E}, \rho_{N}, \ell_{sym}) + \frac{1 - \alpha + \kappa(\pi)(1 - \lambda)}{2}c.
\]

This equality concludes the proof of Lemma 1. Note that this proof follows Charoenphakdee et al. [2019].

This lemma indicates that minimizing \( \mathcal{R}(g; \rho', \rho^*_\alpha, \ell_{sym}) \) w.r.t. \( g \) yields the same minimizer as that of \( \mathcal{R}(g; \rho_{E}, \rho_{N}, \ell_{sym}) \) for \( \alpha - \kappa(\pi)(1 - \lambda) > 0 \), i.e.,

\[
g^* = \arg\min_{g} \mathcal{R}(g; \rho', \rho^*_\alpha, \ell_{sym}) = \arg\min_{g} \mathcal{R}(g; \rho_{E}, \rho_{N}, \ell_{sym}).
\]

This result enables us to prove that the maximizer of the risk optimization in Eq. (9) is the expert policy.

**Theorem 1.** Given the optimal classifier \( g^* \) in Eq. (18), the solution of \( \max_\pi \mathcal{R}(g^*; \rho', \rho^*_\alpha, \ell_{sym}) \) is equivalent to the expert policy.

**Proof.** By using the definitions of the risk and \( \rho^*_\alpha(x) \), \( \mathcal{R}(g^*; \rho', \rho^*_\alpha, \ell_{sym}) \) can be expressed as

\[
\mathcal{R}(g^*; \rho', \rho^*_\alpha, \ell_{sym}) = \frac{1}{2}\mathbb{E}_{p_{\rho}}[\ell_{sym}(g^*(x))] + \frac{\lambda}{2}\mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] + \frac{\lambda}{2}\mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))].
\]

Since the first and second terms are constant w.r.t. \( \pi \), the solution of \( \max_\pi \mathcal{R}(g^*; \rho', \rho^*_\alpha, \ell_{sym}) \) is equivalent to the solution of \( \max_\pi \mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] \), where we omit the positive constant factor \( (1 - \lambda)/2 \). Under Assumption 1 which assumes \( \rho_{E}(x) = \kappa(\pi)\rho_{E}(x) + (1 - \kappa(\pi))\rho_{N}(x) \), we can further express the objective function as

\[
\mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] = \kappa(\pi)\mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] + (1 - \kappa(\pi))\mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] \\
= \kappa(\pi)\mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] - \mathbb{E}_{p_{\rho}}[\ell_{sym}(g^*(x))] + \mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))].
\]

The last term is a constant w.r.t. \( \pi \) and can be safely ignored. The right hand-side is maximized by increasing \( \kappa(\pi) \) to 1 when \( \mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] - \mathbb{E}_{p_{\rho}}[\ell_{sym}(g^*(x))] > 0 \). Since \( g^* \) is also the optimal classifier of \( \mathcal{R}(g; \rho_{E}, \rho_{N}, \ell_{sym}) \), the expected loss of classifying expert data as non-expert: \( \mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] \), is larger to the expected loss of classifying non-expert data as non-expert: \( \mathbb{E}_{p_{\rho}}[\ell_{sym}(g^*(x))] \). Specifically, \( \mathbb{E}_{p_{\rho}}[\ell_{sym}(-g^*(x))] - \mathbb{E}_{p_{\rho}}[\ell_{sym}(g^*(x))] > 0 \). Thus, the objective can only be maximized by increasing \( \kappa(\pi) \) to 1. Because \( \kappa(\pi) = 1 \) if and only if \( \rho_{E}(x) = \rho_{E}(x) \), we conclude that the solution is equivalent to \( \pi_{E} \).

This result indicates that robust IL can be achieved by optimizing the risk in Eq. (9). We note that this is a significant advance compared to the prior work [Wu et al. 2019], because Theorem 1 shows that robust IL can be achieved without the knowledge of the mixing coefficient \( \alpha \) or additional labels to estimate \( \alpha \). Next, we present a new IL method that optimizes the risk by using co-pseudo-labeling.
3.2 Co-pseudo-labeling for Risk Optimization

While the risk in Eq. (9) leads to robust IL, we cannot directly optimize this risk in our setting. This is because the risk contains an expectation over $\rho_N(x)$, but we are given only demonstration samples drawn from $\rho'(x)$. We address this issue by using co-pseudo-labeling to estimate $\rho_N(x)$ and draw samples from it to approximate the expectation, as described below.

Recall that the optimal classifier $g^*(x)$ in Eq. (18) also minimizes the risk $R(g; \rho_E, \rho_N, \ell_{sym})$. Therefore, given a state-action sample $\tilde{x}$, we can use $g(\tilde{x})$ to predict whether $\tilde{x}$ is drawn from $\rho_E$ or $\rho_N$. Specifically, $\tilde{x}$ is predicted to be drawn from $\rho_E$ when $g(\tilde{x}) > 0$, and it is predicted to be drawn from $\rho_N$ when $g(\tilde{x}) < 0$. Based on this observation, our key idea is to approximate the expectation over $\rho_N$ in Eq. (9) by using samples that are predicted by $g$ to be drawn from $\rho_N$.

To realize this idea, we first consider the following empirical risk which utilizes pseudo-labeling (Chapelle et al., 2010):

$$\hat{R}(g) = \frac{1}{2} \hat{E}_D [\ell_{sym}(g(x))] + \frac{1}{2} \hat{E}_P [\ell_{sym}(-g(x))] + \frac{1}{2} \lambda \hat{E}_P [\ell_{sym}(-g(x))].$$

(21)

Here, $\hat{E}[-]$ denotes an empirical expectation (i.e., the sample average), $D$ is the demonstration dataset in Eq. (9), $P$ is a dataset of trajectories collected by using $\pi$, and $\ell_{sym}$ is the demonstration dataset obtained by choosing demonstrations in $D$ with $g(x) < 0$, i.e., samples that are predicted to be drawn from $\rho_N$. This risk with pseudo-labeling enables us to empirically solve Eq. (9) in our setting. However, the trained classifier may perform poorly, mainly because samples in $P$ are labeled by the classifier itself. Specifically, the classifier during training may predict the labels of demonstrations incorrectly, i.e., demonstrations drawn from $\rho_E(x)$ are incorrectly predicted to be drawn from $\rho_N(x)$. This may degrade the performance of the classifier, because using incorrectly-labeled data can reinforce the classifier to be over-confident in its incorrect prediction (Kingma et al., 2014).

To remedy the over-confidence of the classifier, we propose co-pseudo-labeling, which combines the ideas of pseudo-labeling and co-training (Blum and Mitchell, 1998). Specifically, we train two classifiers denoted by $g_1$ and $g_2$ by minimizing the following empirical risks:

$$\hat{R}_1(g_1) = \frac{1}{2} \hat{E}_{D_1} [\ell_{sym}(g_1(x))] + \frac{1}{2} \hat{E}_{P_1} [\ell_{sym}(-g_1(x))] + \frac{1}{2} \lambda \hat{E}_P [\ell_{sym}(-g_1(x))],$$

(22)

$$\hat{R}_2(g_2) = \frac{1}{2} \hat{E}_{D_2} [\ell_{sym}(g_2(x))] + \frac{1}{2} \hat{E}_{P_2} [\ell_{sym}(-g_2(x))] + \frac{1}{2} \lambda \hat{E}_P [\ell_{sym}(-g_2(x))].$$

(23)

where $D_1$ and $D_2$ are disjoint subsets of $D$. Pseudo-labeled dataset $P_1$ is obtained by choosing demonstrations from $D_2$ with $g_2(x) < 0$, and pseudo-labeled dataset $P_2$ is obtained by choosing demonstrations from $D_1$ with $g_1(x) < 0$. With these risks, we reduce the influence of over-confident classifiers because $g_1$ is trained using samples pseudo-labeled by $g_2$ and vice-versa (Han et al., 2018). We call our proposed method Robust IL with Co-pseudo-labeling (RIL-Co).

We implement RIL-Co by using a stochastic gradient method to optimize the empirical risk where we alternately optimize the classifiers and policy. Recall from Eq. (9) that we aim to maximize $R(g; \rho', \rho_E^2, \ell_{sym})$ w.r.t. $\pi$. After ignoring terms that are constant w.r.t. $\pi$, solving this maximization is equivalent to maximizing $\mathbb{E}_{\rho_E^2} [\ell_{sym}(-g(x))]$ w.r.t. $\pi$. This objective is identical to the RL objective in Eq. (18) with a reward function $r(x) = \ell_{sym}(-g(x))$ and a constant scaling $1/(1 - \gamma)$. Therefore, we can train the policy in RIL-Co by simply using an existing RL method, e.g., the trust-region policy gradient (Wu et al., 2017). We summarize the procedure of RIL-Co in Algorithm 1.

3.3 Choice of Hyper-parameter

We propose to use $\lambda = 0.5$ for RIL-Co, because it makes the equality in Eq. (18) holds which is essential for our theoretical result in Section 3.1. Specifically, recall that Theorem 1 relies on the equality in Eq. (18). At the same time, the equality in Eq. (18) holds if the following inequality holds:

$$\alpha - \kappa(\pi)(1 - \lambda) > 0.$$  

(24)

This inequality depends on $\alpha$, $\kappa(\pi)$, and $\lambda$, where $\alpha$ is unknown, $\kappa(\pi)$ depends on the policy, and $\lambda$ is the hyper-parameter. However, we cannot choose nor evaluate the value of $\kappa(\pi)$ since we do not directly optimize $\kappa(\pi)$ during policy training. Thus, we need to choose an appropriate value of $\lambda$ so that

---

2The expectation over $\rho_*(x)$ can be approximated using trajectories independently collected by the policy $\pi$. 

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Algorithm 1 RIL-Co: Robust Imitation Learning with Co-pseudo-labeling

1: Input: Demonstration dataset \( D \) and hyper-parameter \( \lambda = 0.5 \) (see Section 3.3).
2: Initialize: Policy \( \pi \) and classifiers \( g_1 \) and \( g_2 \).
3: Split \( D \) into two disjoint datasets \( D_1 \) and \( D_2 \).
4: while Not converge do
5: \hspace{1em} while \( |B| < B \) with batch size \( B \) do
6: \hspace{2em} Use \( \pi \) to collect and include transition samples into \( B \)
7: \hspace{1em} end while
8: Co-pseudo-labeling:
9: \hspace{1em} Sample \( \{x_n\}_{n=1}^U \) from \( D_2 \), and choose \( K \) samples with \( g_2(x_n) < 0 \) as \( P_1 \).
10: \hspace{1em} Sample \( \{x_n\}_{n=1}^V \) from \( D_1 \), and choose \( K \) samples with \( g_1(x_n) < 0 \) as \( P_2 \).
11: \hspace{1em} Train classifiers:
12: \hspace{2em} Train \( g_1 \) by using gradient descent to minimize the empirical risk \( \hat{R}_1(g_1) \) using \( D_1 \), \( P_1 \), and \( B \).
13: \hspace{2em} Train \( g_2 \) by using gradient descent to minimize the empirical risk \( \hat{R}_2(g_2) \) using \( D_2 \), \( P_2 \), and \( B \).
14: \hspace{1em} Train policy:
15: \hspace{2em} Train the policy by an RL method with transition samples in \( B \) and rewards \( r(x) = \ell_{\text{sym}}(-g_1(x)) \).
16: end while

the inequality in Eq. (24) always holds. Recall that we assumed \( 0.5 < \alpha < 1 \) in Section 2.3. Under this assumption, the inequality in Eq. (24) holds regardless of the true value of \( \alpha \) when \( \kappa(\pi)(1-\lambda) \leq 0.5 \). Since the value of \( \kappa(\pi) \) increases to 1 as the policy improves by training (see Assumption 1), the appropriate value of \( \lambda \) is \( 0.5 \leq \lambda < 1 \). However, a large value of \( \lambda \) may not be preferable in practice since it increases the influence of pseudo-labels on the risks (i.e., the second term in Eqs. (22) and (23)). These pseudo-labels should not have larger influence than real labels (i.e., the third term in Eqs. (22) and (23)). For this reason, we decided to use \( \lambda = 0.5 \), which is the smallest value of \( \lambda \) that ensures the inequality in Eq. (24) to be always held during training.

Remarks. Choosing \( \lambda = 0 \) corresponds to omitting co-pseudo-labeling, and doing so reduces RIL-Co to variants of GAIL which are not robust. Concretely, the risk optimization problem of RIL-Co with \( \lambda = 0 \) is \( \max_{\pi} \min_p R(g; \rho', \rho_\pi, \ell_{\text{sym}}) \). By using the logistic loss: \( \ell(z) = \log(1 + \exp(-z)) \), instead of a symmetric loss, we obtain the following risk:

\[
2R(g; \rho', \rho_\pi, \ell) = E_{\rho_\pi} [\log(1 + \exp(-g(x)))] + E_{\rho_\pi} [\log(1 + \exp(g(x)))] ,
\]

which is the negative of GAIL’s objective shown in Eq. (13)\(^3\). On the other hand, we can obtain other variants of GAIL by using other losses such as the normalized logistic loss (Ma et al., 2020) and the unhinged loss (van Rooyen et al., 2015). In particular, the risk with the unhinged loss: \( \ell(z) = 1 - z \), corresponds to the negative of Wasserstein GAIL’s objective with an additive constant (Li et al., 2017; Xiao et al., 2019):

\[
2R(g; \rho', \rho_\pi, \ell) = E_{\rho_\pi} [-g(x)] + E_{\rho_\pi} [g(x)] + \frac{1}{2}.
\]

However, even when \( \ell(z) \) is symmetric, we expect that such variants are not robust, because \( \lambda = 0 \) does not make the inequality in Eq. (24) holds when \( \kappa(\pi) > 0.5 \).

3.4 Choice of Symmetric Loss

In our implementation of RIL-Co, we use the active-passive loss (AP loss) (Ma et al., 2020) defined as

\[
\ell_{\text{AP}}(z) = \frac{0.5 \times \log(1 + \exp(-z))}{\log(1 + \exp(-z)) + \log(1 + \exp(z))} + \frac{0.5}{1 + \exp(z)} ,
\]

which satisfies \( \ell_{\text{AP}}(z) + \ell_{\text{AP}}(-z) = 1 \). This loss is a linear combination of two symmetric losses: the normalized logistic loss (the first term) and the sigmoid loss (the second term). It was shown that this loss suffers less from the issue of under-fitting when compared to each of the normalized logistic loss or the sigmoid loss (Ma et al., 2020). However, we emphasize that any symmetric loss can be used to learn

\(^3\)The sign flips because RIL-Co solves the max-min optimization while GAIL solves the min-max optimization.
Table 1: Examples of losses and their symmetric property, i.e., whether \( \ell(z) + \ell(-z) = c \). We denote normalized counterparts of non-symmetric losses by (N). The AP loss in Eq. (27) is a linear combination of the normalized logistic and sigmoid losses.

| Loss name | \( \ell(z) \) | Symmetric |
|-----------|----------------|------------|
| Logistic  | \( \log(1 + \exp(-z)) \) | \( \times \) |
| Hinge     | \( \max(1 - z, 0) \) | \( \times \) |
| Sigmoid   | \( 1/(1 + \exp(z)) \) | \( \checkmark \) |
| Unhinged | \( 1 - z \) | \( \checkmark \) |
| Logistic (N) | \( \sum_{k \in (-1,1)} \log(1 + \exp(-zk)) \) | \( \checkmark \) |
| Hinge (N) | \( \frac{\sum_{k \in (-1,1)} \max(1 - zk, 0)}{\sum_{k \in (-1,1)} \max(1 - zk, 0)} \) | \( \checkmark \) |

Figure 1: The value of losses in Table 1. Non-symmetric losses, i.e., the logistic and hinge losses, become symmetric after normalization.

4 Experiments

We evaluate the robustness of RIL-Co on continuous-control benchmarks simulated by PyBullet simulator (HalfCheetah, Hopper, Walker2d, and Ant) (Coumans and Bai, 2019). These tasks are equipped with the true reward functions that we use for the evaluation purpose. We report the mean and standard error of the performance (cumulative true rewards) over 5 trials.

We compare RIL-Co against the following baselines: RIL-P which is RIL-Co but with naive pseudo-labeling in Eq. (21), FAIRL (Ghasemipour et al., 2020), VILD (Tangkaratt et al., 2020), and three variants of GAIL where each variant uses different losses: logistic, unhinged, and AP. GAIL with the logistic loss denotes the original GAIL that performs density matching with the Jensen-Shannon divergence (Ho and Ermon, 2016), and GAIL with the unhinged loss denotes a variant of GAIL that performs density matching with the Wasserstein distance (Xiao et al., 2019), as discussed in Section 3.3. GAIL with the AP loss corresponds to RIL-Co without co-pseudo-labeling (\( \lambda = 0 \)).

All methods use policy networks with 2 hidden-layers of 64 hyperbolic tangent units. We use similar networks with 100 hyperbolic tangent units for classifiers in RIL-Co and discriminators in other methods. The policy networks are trained by the trust region policy gradient (Wu et al., 2017), where we use the implementation publicly available (Kostrikov, 2018). In each iteration, the policy collect a total of \( B = 640 \) transition samples using 32 parallel agents and we use these transition samples as the dataset \( B \) in Algorithm 1. The total number of transition samples collected by the learning policy is 20 million. For training the classifier and discriminator, we use Adam (Kingma and Ba, 2015) with learning rate \( 10^{-3} \) and the gradient penalty regularizer with the regularization parameter of 10 (Gulrajani et al., 2017). The mini-batch size for classifier/discriminator training is 128. For co-pseudo-labeling in Algorithm 1, we sample \( U = V = 640 \) samples from the split datasets \( D_1 \) and \( D_2 \). Then, we choose \( K = 128 \) samples with the least negative values as pseudo-labeled datasets \( P_1 \) and \( P_2 \).

4.1 Evaluation on Noisy Datasets with Different Noise Rates

Data generation. In this experiment, we evaluate RIL-Co on noisy datasets generated with different noise rates. To obtain datasets, we firstly train policies by RL with the true reward functions. Next, we choose 6 policy snapshots where each snapshot is trained using different numbers of transition samples. Then, we use the best performing policy snapshot (in terms of cumulative rewards) to collect 10000 expert state-action samples, and use the other 5 policy snapshots to collect a total of 7500 non-expert state-action samples. The cumulative rewards achieved by the six snapshots are given in Table 2 where Snapshot 1
Table 2: Cumulative rewards achieved by the six policy snapshots used in the experiment. Snapshot 1 is used as the expert policy. Ant (G) denotes cumulative reward obtained after adding Gaussian noise to expert actions, and is used in the experiment with the Gaussian noise dataset.

| Task         | Snapshot 1 | Snapshot 2 | Snapshot 3 | Snapshot 4 | Snapshot 5 | Snapshot 6 |
|--------------|------------|------------|------------|------------|------------|------------|
| HalfCheetah  | 2500       | 1300       | 1000       | 700        | -1100      | -1000      |
| Hopper       | 2300       | 1100       | 1000       | 900        | 600        | 0          |
| Walker2D     | 2700       | 800        | 600        | 700        | 100        | 0          |
| Ant          | 3500       | 1400       | 1000       | 700        | 400        | 0          |
| Ant (G)      | 3500       | 1500       | 1000       | 800        | 500        | 400        |

Figure 2: Final performance in continuous-control benchmarks with different noise rates. Vertical axes denote the average performance in the last 1000 training iterations. RIL-Co performs well even when the noise rate increases. Meanwhile, the performance of other methods significantly degrades as the noise rate increases.

Results. Figure 2 shows the final performance achieved by each method. We can see that RIL-Co outperforms comparison methods and achieves the best performance in high noise scenarios where \( \delta \in \{0, 0.1, 0.2, 0.3, 0.4\} \) approximately determines the number of non-expert state-action samples. Specifically, a dataset consisting of 10000 expert state-action samples corresponds to a dataset with \( \delta = 0 \) (i.e., no noise), whereas a dataset consisting of 10000 expert and 7500 non-expert state-action samples corresponds to a dataset with \( \delta = 0.4 \) approximately.

We note that the value of \( \delta \) approximately equals to the value of \( 1 - \alpha \) in Eq. (6).

It can be seen that GAIL with the AP loss performs worse than RIL-Co in high noise scenarios. This empirical result supports our theoretical result which indicates that a symmetric loss alone is insufficient for robust IL. We can also see that RIL-P does not perform as well as RIL-Co, which shows that the proposed co-pseudo-labeling is more advantageous than naive pseudo-labeling.

Meanwhile, density matching methods, namely FAIRL and GAIL with the logistic and unhinged losses, do not perform well. This is as expected, because density matching methods would learn a policy that is a mixture of the expert and non-expert policies and would not perform well. Also notice that GAIL with the unhinged loss performs very poorly on the Walker2D and Ant tasks even with noiseless demonstrations where \( \delta = 0 \). This is an intriguing result given that the unhinged loss is also symmetric similarly to the AP loss. We conjecture that the poor performance is due to the unboundedness from below of the unhinged loss (see Figure 1). This unboundedness may lead to a poorly behave discriminator that outputs values with very large magnitudes, as suggested by Charoenphakdee et al. (2019). With such a discriminator, we expect that GAIL with the unhinged loss would require a strong regularization to perform well, especially for complex control tasks.

On the other hand, VILD performs poorly with noisy datasets even with a small noise rate of \( \delta = 0.1 \). We conjecture that this is because VILD could not accurately estimate the noise distributions due to

\[ \hat{\delta} \in \{0, 1000/11000, 2500/12500, 5000/15000, 7500/17500\} \]. We use the approximate value \( \hat{\delta} \) to improve readability.
the violation of its assumption. Specifically, VILD assumes that noisy demonstrations are generated by
adding Gaussian noises to actions drawn from the expert policy, and that expert demonstrations consist
of low-variance actions. However, noisy demonstrations in this experiment are generated by using policy
snapshots without adding any noise. In this case, non-expert demonstrations may consist of low-variance
actions (e.g., non-expert policy may yield a constant action). Due to this, VILD cannot accurately
estimate the noise distributions and performs poorly.

Figure 3 shows the performance against the number of transition samples collected by the learning
policy. RIL-Co achieves better performances and uses less transition samples when compared to other
methods. This result indicates that RIL-Co is data efficient.
4.2 Evaluation on Gaussian Noise Dataset

Data generation. Next, we evaluate RIL-Co in the Ant task with a noisy dataset generated by Gaussian noises according to assumption of VILD (Tangkaratt et al., 2020). That is, we collect non-expert state-action samples by adding Gaussian noises to actions drawn from the expert policy. We use a dataset with 10000 expert and 7500 non-expert state-action samples. The noise rate of is approximately $\delta = 0.4$.

Results. Figure 4 depicts the performance against the number of transition samples. It can be seen that VILD performs the best in this experiment. This is as expected, since the assumption of VILD is correct and VILD is expected to accurately estimate the noise distributions. Still, RIL-Co achieves a performance comparable to that of VILD with 20 million samples, even though RIL-Co relies on a milder data generation assumption (see Section 2.3). Meanwhile, the other methods do not perform as well as RIL-Co and VILD.

Overall, the empirical results in both experiments indicate that RIL-Co is more robust against noisy demonstrations when compared to existing methods.

5 Conclusions

We presented a new method for IL from noisy demonstrations. We proved that robust IL can be achieved by optimizing a classification risk with a symmetric loss, and we proposed RIL-Co which optimizes the risk by using co-pseudo-labeling. We showed through experiments that RIL-Co is more robust against noisy demonstrations when compared to existing methods.

In this paper, we developed the theoretical result of robust IL under an asymptotic setting, i.e., an infinite amount of samples is available and the optimal classifier is learned. In the future, we will study theoretical properties of robust IL in non-asymptotic settings by, e.g., utilizing results of generative adversarial training (Mescheder et al., 2018).

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