ESTIMATING THE SPIN OF STELLAR-MASS BLACK HOLES BY SPECTRAL FITTING OF THE X-RAY CONTINUUM

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ABSTRACT

We fit X-ray spectral data in the thermal-dominant, or high-soft, state of two dynamically confirmed black holes, GRO J1655−40 and 4U 1543−47, and estimate the dimensionless spin parameters \( a = a/M \) of the two holes. For GRO J1655−40, using a spectral hardening factor computed for a non-LTE relativistic accretion disk, we estimate \( a \approx 0.75 \) and \( a \approx 0.65−0.75 \), respectively, from ASCA and RXTE data. For 4U 1543−47, we estimate \( a \approx 0.75−0.85 \) from RXTE data. Thus, neither black hole has a spin approaching the theoretical maximum \( a = 1 \).

Subject headings: accretion, accretion disks — binaries: close — black hole physics — stars: individual (4U 1543−47, GRO J1655−40) — X-rays: stars

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1. INTRODUCTION

An astrophysical black hole is completely defined by two numbers that specify its mass and spin. The masses of 20 accreting black holes located in X-ray binary systems have been determined or constrained by dynamical optical studies (McClintock & Remillard 2006, hereafter MR06; Casares et al. 2004; Orosz et al. 2004). However, no reliable measurements have been reported so far of the dimensionless black hole spin parameter \( a = a/M \), where \( a = J/cR_g \), \( J \) is the angular momentum of the black hole, \( M \) is its mass, and \( R_g = GM/c^2 \).

In this Letter, we estimate \( a \), for two black hole binaries by fitting their X-ray thermal continuum spectra using a fully relativistic model of a thin accretion disk around a Kerr black hole (Li et al. 2005). The model includes all relativistic effects, such as frame dragging, Doppler boosting, gravitational redshift, and bending of light by the gravity of the black hole. It also includes self-irradiation of the disk, the effects of limb darkening, and a spectral hardening factor \( f_{\text{col}} \) to relate the color temperature \( T_{\text{col}} \) and the effective temperature \( T_{\text{eff}} \) of the disk emission: \( f_{\text{col}} = T_{\text{col}}/T_{\text{eff}} \) (Shimura & Takahara 1995, hereafter ST95; Merloni et al. 2000; Davis et al. 2005, hereafter D05).

In order to estimate the black hole spin by fitting the broadband X-ray spectrum, one must know the mass \( M \) of the black hole, the inclination \( i \) of the accretion disk (which we assume is the same as the inclination of the binary system; but see Maccarone 2002), and the distance \( D \) to the binary (Zhang et al. 1997). Accordingly, we have selected two black hole binaries, GRO J1655−40 (hereafter J1655) and 4U 1543−47 (hereafter U1543), for which all three of these quantities have been well determined from optical observations: for J1655,

\[
M = 6.30 \pm 0.27 M_\odot, \quad i = 70^\circ 2 \pm 1^\circ 2, \quad D = 3.2 \pm 0.2 \text{ kpc};
\]

for U1543,

\[
M = 9.4 \pm 1.0 M_\odot, \quad i = 20^\circ 7 \pm 1^\circ 5, \quad D = 7.5 \pm 1.0 \text{ kpc}.
\]

We consider only those X-ray data that were obtained in the thermal-dominant, or TD, state (formerly the “high-soft state”), in which more than 75% of the 2−20 keV flux is supplied by the accretion disk (MR06). This state is consistent with a simple multicolor blackbody model (Gierliński & Done 2004), making it amenable to theoretical modeling. We carried out all data analysis and model fits using XSPEC version 12.2.0 and HEASOFT version 5.2.

The continuum X-ray spectrum of J1655 was observed by the Advanced Satellite for Cosmology and Astrophysics (ASCA) on 1995 August 15 (Ueda et al. 1998, hereafter U98) and in 1997 between February 25 and 28 (Yamaoka et al. 2001, hereafter Y01), with total exposure times of \( \approx 90 \) and \( \approx 100 \) ks, respectively. The source was very bright during both observations (2.3 Crab in 1995 and 1.2 Crab in 1997), and we therefore analyzed only the data from the GIS2 and GIS3 detectors. Starting with the unscreened ASCA data files obtained from HEASARC, we followed as closely as possible the data reduction procedures and criteria mentioned in U98 and Y01. As in Y01, we added a systematic error of 2% to the GIS spectra to account for calibration uncertainties. As a check on our reduction procedures, we fitted both the 1995 and 1997 spectra using the disk blackbody models employed by U98 and Y01, and we succeeded in closely reproducing all their published results.

J1655 was also observed with the Rossi X-Ray Timing Explorer (RXTE) in 1997 (Sobczak et al. 1999), including an observation on February 26 that was performed simultaneously with ASCA. Our second source, U1543, was observed with RXTE during its 2002 outburst (Park et al. 2004, hereafter P04). For both RXTE data sets, the data reduction procedures are identical to those described in P04. In brief, we used the “Standard 2 mode” data from PCU-2 only. The event files and spectra were screened and the background spectra and response files created. Systematic errors of 1% were added to all the PCU-2 energy channels. Referring to Figure 2 of Remillard (2005), we selected the contiguous group of 31 observations extending from MJD 50,453.6 to MJD 50,663.7 for which J1655 was in the TD state. Because the data span 210 days, we created and used several different response files. For U1543, from among the 49 observations considered by P04 (see their Table 1), we selected the 34 observations (observations 1–3, 5–19, and 27–42) for which the source was in the TD state (MR06).
resulting in the likely values of $f_{\text{col}}$ according to the D05 and ST95 models.

3. DATA ANALYSIS

RXTE long fits.—First we fitted the RXTE pulse-height spectra of J1655 in the 2.8–25.0 keV range. These fits, which we refer to as the “long fits,” were made using a spectral model comprising three principal components: “kerrbb,” which models a relativistic accretion disk (Li et al. 2005); a standard low-energy absorption component (“phabs”); and a simple power-law component (“power”). In addition, following the work of Y01, we found it necessary to add three edge/line features to obtain acceptable fits: (1) a smeared Fe edge (“smedge”) with edge energy restricted to the interval 6.8–9.0 keV and width fixed at 7 keV; (2) a sharp absorption edge with edge energy restricted to the interval 9–11 keV; and (3) a Gaussian absorption line that was used solely for J1655. The central energy of this line was restricted to the range 6.4–7.0 keV, and its width was fixed at 0.5 keV (the width was determined by our analysis of the ASCA data; see below). We fixed the equivalent neutral hydrogen column density $N_{\text{H}}$ at $0.7 \times 10^{22}$ cm$^{-2}$ for J1655 (Y01) and $0.4 \times 10^{22}$ cm$^{-2}$ for U1543 (P04).

RXTE short fits.—Since we are interested in the disk component of the spectrum, we analyzed the same data over restricted energy ranges that are dominated by the thermal component, thereby generating “short fits.” For all these short fits, we used the three principal components used in the long fits. For the J1655 data we also used the Gaussian line feature mentioned above. We explored several different upper energy limits for the RXTE spectra, finally choosing 2.8–7.5 keV for J1655 and 2.8–7.0 keV for U1543. We determined the upper limit by the highest energy that allowed the exclusion of the smedge component required by the long fits (for which this feature had a mean edge energy of 7.7 keV for J1655 and 7.0 keV for U1543). The 7.5 keV limit for J1655 (vs. 7.0 keV) was required in order to ensure that the parameters of the Fe absorption line feature present between 6.4 and 7.0 keV were well determined. Twenty-six out of 31 of the J1655 short fits (including the 1997 February 26 spectrum featured in Fig. 1) succeeded without a power-law component (i.e., $\chi^2 < 1$); furthermore, its inclusion did not improve these fits significantly. However, in the case of U1543 (which on average had a power-law–to–total flux ratio of 0.15, compared with 0.06 for J1655), the power-law component was essential in fitting all 34 of the spectra. In order to be consistent, the power-law component was included in all short fits of both sources, with parameters fixed to the values determined from the long fits.

ASCA short fits.—The ASCA data for J1655 were analyzed only over a short energy range, which was chosen to be 1.2–7.5 keV, because the bandwidth of the GIS detectors is too limited to constrain the power-law component. For 1997 ASCA data, we modeled the spectra using kerrbb, phabs, and the Gaussian absorption line mentioned earlier. For the 1995 data we used only kerrbb and phabs.

In the analysis of the RXTE data, we found it necessary to correct the fluxes downward as follows: For U1543, the Crab flux calculated using the P04 response file exceeded the flux predicted by the standard Crab spectrum of Koyama et al. (1984) by a factor of 1.174. For J1655, we used a current response matrix and found a much smaller correction factor of 1.034.

In fitting the data using kerrbb, we fixed $M$, $i$, and $D$ to their mean observed values ($\S$ 1). Also, we switched on limb darkening (Iflag = 1) and returning-radiation effects (rflag = 1). We set the torque at the inner boundary of the accretion disk to zero, fixed the normalization to 1 (as appropriate when $M$, $i$, and $D$ are held fixed), and allowed the mass accretion rate $\dot{M}$ to vary freely. Of the two remaining parameters, namely, the spectral hardening factor $f_{\text{col}}$ and the black hole spin $a_*$, we held one or the other fixed and fitted the other.

4. RESULTS

Figure 1 shows the results of analyzing the 1995 and 1997 ASCA data on J1655 with kerrbb. In this analysis, we kept $f_{\text{col}}$ fixed at selected values and fitted $a_*$ and $M$. We see that for a given value of $f_{\text{col}}$, the data are able to determine $a_*$ accurately. The $\chi^2$ values are acceptable, and there is reasonable agreement between the GIS2 and GIS3 results and between the 1995 and 1997 data. There is agreement also with the simultaneous RXTE observation done on 1997 February 26. However, since the $\chi^2$ is acceptable for all the values of $f_{\text{col}}$, it is clear that the data by themselves cannot constrain $f_{\text{col}}$. We thus need an independent theoretical determination of $f_{\text{col}}$ if we wish to estimate $a_*$. ST95 were among the first to calculate model disk atmospheres for black hole binaries, including full radiative transfer and Comptonization. From these models they estimated $f_{\text{col}}$ as a function of the bolometric disk luminosity $L_{\text{bol}}$. Roughly, they found $f_{\text{col}} \approx 1.7 + 0.2(\log l + 1.25)$, where $l = L_{\text{bol}}/L_{\text{Edd}}$ is the Eddington-scaled disk luminosity and $L_{\text{Edd}} = 1.5 \times 10^{38}(M_*/1 M_\odot)^{0.85}$.
ST95 by including metal opacities. Metals tend to reduce the spectral hardening. Therefore, the D05 values of $f_{\text{col}}$ are generally smaller than those of ST95. For the analysis presented in this Letter, we used D05’s code to calculate a grid of values of $f_{\text{col}}$ as a function of $l$ and $a_*$. The calculations were done for the specific inclinations of J1655 and U1543, and for two values of the disk viscosity parameter $\alpha$, 0.01 and 0.1. Figure 1 shows the range of $f_{\text{col}}$ values predicted by the D05 model for the 1995 and 1997 ASCA observations, and the second-to-last column of Table 1 gives the corresponding estimates of $a_*$. While we present in this Letter results for both the ST95 and D05 models of spectral hardening, we view the latter as more reliable.

Figure 2 shows the short-fit results (§ 3) for all 31 RXTE observations of J1655 in the TD state. Here we have assumed various values of $a_*$ and computed for each observation the best-fit values of $f_{\text{col}}$ and $l$ (the latter is obtained from $M$ and $a_*$). We then compare the data-fitted values of $f_{\text{col}}$ with the model predictions of D05 and ST95. The comparison with the D05 model indicates that the spin of J1655 is likely to lie in the range $a_* \sim 0.65–0.75$; this result is entered in Table 1. It is interesting to note that the D05 model gives nearly identical results for $\alpha$-values of 0.01 and 0.1 so long as $\log l \leq -1$ but shows noticeable variations with $\alpha$ when the disk is more luminous. Since the true $\alpha$ of the disk is not known (it is likely to be in the range 0.01–0.2), and moreover, since disks tend to be dominated by radiation pressure at higher luminosities, thereby introducing additional uncertainties, we give greater weight to the observations for which $\log l < -1$. Thus, we ignore the 1995 ASCA data ($\log l < -0.85$) and give more weight to the 1997 data ($\log l < -1.02$). We also favor the GIS2 results because the GIS3 spectra have a prominent feature between 1 and 2 keV that cannot be eliminated by using the XSPEC “gain” command or adding an ad hoc absorption edge to the fit.

Figure 3 shows a similar analysis for U1543. Here we found that seven of the 34 observations in the TD state (MJD 52,444.515 to 52,449.11) yielded poor fits for the short range due to an edgelike feature between 4 and 5 keV that is not present in other observations. The long fits for these data still gave reasonable fits, with $\chi^2 < 2$. Comparing the fitted values of $f_{\text{col}}$ with the D05 model, we estimate $a_* \sim 0.75–0.85$. Once again we focus on the lower luminosity data with $\log l < -1$.

![Fig. 2.—J1655 RXTE spectra fitted for five different fixed values of $a_*$. The squares show the fitted values of the spectral hardening factor $f_{\text{col}}$ and the dimensionless luminosity $l$ obtained with short fits. The ASCA 1997 GIS2 point is shown as a plus sign, and the GIS3 point as a cross. The lines show the calculated values of $f_{\text{col}}$ from the D05 model for $\alpha = 0.01$ and $\alpha = 0.1$ and from the ST95 model. The solid line is for $\alpha = 0.1$, the value we emphasize. The D05 model constrains the $a_*$ of J1655 to lie in the range 0.65–0.75. Error bars on $f_{\text{col}}$ (typically $\sim 0.02$) are not shown. The fits are for 8 dof with $\chi^2$ typically $\sim 0.3$. [See the electronic edition of the Journal for a color version of this figure.]

![Fig. 3.—U1543 RXTE spectra fitted for five different fixed values of $a_*$. The data points and lines have the same meanings as in Fig. 2. The seven dots on the extreme right of each panel correspond to spectra that contained an absorption-edge–like structure between 4 and 5 keV and gave fits with $\chi^2 > 2$. The D05 model constrains the $a_*$ of U1543 to lie in the range 0.75–0.85. The fits are for 7 dof with $\chi^2$ typically $\sim 0.3$.]

### Table 1

| Candidate      | Observation Date | Satellite | Detector | $a_*$ (D05) | $a_*$ (ST95) |
|----------------|------------------|-----------|----------|-------------|-------------|
| GRO J1655-40   | 1995 Aug 15      | ASCA      | GIS2     | $\sim 0.85$ | $\sim 0.8$  |
|                | 1997 Feb 25–28   | ASCA      | GIS2     | $\sim 0.75$ | $\sim 0.70$ |
|                | 1997 Feb 26      | RXTE      | PCA      | $\sim 0.75$ | $\sim 0.7$  |
|                | 1997 (several)   | RXTE      | PCA      | 0.65–0.75   | 0.55–0.65   |
| 4U 1543-47     | 2002 (several)   | RXTE      | PCA      | 0.75–0.85   | 0.55–0.65   |

* Values adopted in this Letter.
The disk parameters obtained from the short and long fits agree very well in all cases, with typical differences in log $l$ and $f_{\text{col}}$ of order 0.003 and 0.005, respectively; the uncertainties in the fitted values are considerably larger ($\sim$0.01 and $\sim$0.02). Also, to check whether the difference in the energy ranges of the ASCA and RXTE data is an issue, we reanalyzed the ASCA data using only the restricted energy range 2.8–7.5 keV and found that the results agree. We also explored the effect of varying the values of $M$, $D$, and $l$ over 1 standard deviation in either direction (§ 1). We find that the derived $a_*$-values lie within the ranges given in Table 1. We note that system parameters need to be measured with high accuracy for this method to succeed. For example, we attempted to analyze the black hole candidate XTE J1550–564 using the methods described here, but since the distance to this source is quite uncertain, $D = 5.9^{+1.4}_{-1.0}$ kpc (Orosz et al. 2002), we were unable to obtain any useful constraint on $a_*$. 

5. SUMMARY AND CONCLUSIONS

The method of estimating spin that we have employed was pioneered by Zhang et al. (1997; see also Gierliński et al. 2001). However, only recently have the necessary data analysis tools (kerrbb; Li et al. 2005) and disk atmosphere models (D05) been developed to the point where the method may be applied with some confidence.

Effectively, in this technique one determines the radius $R_\text{in}$ of the inner edge of the accretion disk and assumes that this radius corresponds to the innermost stable circular orbit ($R_{\text{isco}}$). Since $R_{\text{isco}}/R_* = 2M/c^2$, $M$ and $\dot{M}$ directly gives $a_*$. Provided that (1) $i$ and $D$ are known to sufficient accuracy, (2) the X-ray flux and spectral temperature are measured from well-calibrated X-ray data in the TD state, and (3) the disk radiates as a blackbody, it is clear that $R_\text{in}$ can be estimated. However, the disk emission is not a true blackbody, but a modified blackbody with a spectral hardening factor $f_{\text{col}}$. Therefore, the observations only give the quantity $R_{\text{in}}/f_{\text{col}}$, and we need an independent estimate of $f_{\text{col}}$ in order to estimate $a_*$. We have tailored the state-of-the-art disk atmosphere model of D05 to obtain estimates of $f_{\text{col}}$ for this work.

Our results in brief are as follows: By fitting ASCA and RXTE spectral data on the black hole X-ray binary GRO J1655–40, we estimate the dimensionless spin parameter of the black hole to be $a_* \sim 0.65$–0.75 (Table 1). In the case of 4U 1543–47 we estimate $a_* \sim 0.75$–0.85, though this is based on only RXTE data. To obtain these estimates, we have focused on observations for which the disk luminosity was relatively low (log $l \sim -1$) and we have used the D05 model for $f_{\text{col}}$ (though for completeness we also give results for the ST95 model in Table 1 and in the figures).

Based on these results, we consider it unlikely that either J1655 or U1543 has a spin close to the theoretical maximum for a rotating black hole, $a_* = 1$. Even $a_* = 0.85$, which is the largest value we find, corresponds to a quite moderate spin, as one can see by considering the binding energy per unit mass of a particle in the last stable circular orbit: For $a_*$ in the range 0 to 1, this quantity varies from 5.7% to 42.3%, whereas it is only 13.6% for $a_* = 0.85$. Moreover, most systematic effects that one might consider only push our estimates of $a_*$ down. If $a_*$ is larger than 0.1, as suggested by some studies of white dwarf disks, it would cause our estimates of $a_*$ to decrease (see Figs. 2 and 3). Similarly, if we allow a nonzero torque at the inner edge of the disk or allow the disk to radiate inside $R_{\text{isco}}$ (Krolik 1999), $a_*$ would decrease still further.

What spin might we expect a black hole to accrue due to disk accretion alone? If the hole accretes long enough to achieve spin equilibrium, then the limiting $a_*$ is likely to be in the range $0.9$–$0.998$ (Gammie et al. 2004 and references therein). However, X-ray binaries rarely live long enough for such equilibrium to be established. U1543, for instance, contains a $\approx 2.5 M_\odot$ main-sequence secondary (Orosz et al. 1998), and thus the age of the system is $\lesssim 1$ Gyr. Based on the X-ray fluxes from the 1971, 1983, 1992, and 2002 outbursts, the average mass accretion rate is $\sim 1 \times 10^{-9} M_\odot$ yr$^{-1}$ for $D = 7.5$ kpc (Chen et al. 1997; P04). At this rate, the black hole will accrete at most $1 M_\odot$ during the lifetime of the system, and thus, assuming that its natal spin is zero, the spin today should be $a_* \lesssim 0.35$, considerably less than our estimate of $a_* \sim 0.75$–0.85. This suggests that our measurements are sensitive to the natal spins of these black holes. It is then interesting that neither of the two holes has a spin close to either 0 or 1.

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