BARYON RESONANCE PHENOMENOLOGY

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The Japan Hadron Facility will provide an unprecedented opportunity for the
study of baryon resonance properties. This talk will focus on the chiral nonanalytic
behaviour of magnetic moments exclusive to baryons with open decay channels.
To illustrate the novel features associated with an open decay channel, we consider
the “Access” quark model, where an analytic continuation of chiral perturbation
theory is employed to connect results obtained using the constituent quark model
in the limit of SU(3)-flavour symmetry to empirical determinations.

1. Introduction

The Japan Hadron Facility will present new opportunities for the investigation
of baryon resonance properties. In particular, access to the hyperons of the
baryon decuplet will be unprecedented. This talk serves to highlight the novel
and important aspects of QCD that can be explored through an experimental
program focusing on decuplet-baryon resonance phenomenology.

To highlight the new opportunities, it is sufficient to address the magnetic
moments of the charged Δ baryons of the decuplet. The magnetic moments of
these baryons have already caught the attention of experimentalists and hold
the promise of being accurately measured in the foreseeable future. Experimental estimates exist for the Δ^{++} magnetic moment, based on the reaction
\( \pi^+ p \rightarrow \pi^+ \gamma' p \). The Particle Data Group\(^1\) provides the range 3.7–7.5 \( \mu_N \)
for the Δ^{++} magnetic moment with the two most recent experimental results
of 4.52 ± 0.50 ± 0.45 \( \mu_N \)\(^2\) and 6.14 ± 0.51 \( \mu_N \)\(^3\). In principle, the Δ^+ magnetic moment can be obtained from the reaction \( \gamma p \rightarrow \pi^0 \gamma' p \), as demonstrated
at the Mainz microtron.\(^4\) An experimental value for the Δ^+ magnetic moment
appears imminent.

Recent extrapolations of octet baryon magnetic moments\(^5,6,7\) have utilized
an analytic continuation of the leading nonanalytic (LNA) structure of Chiral
Perturbation Theory (\( \chi PT \)), as the extrapolation function. The unique feature of this extrapolation function is that it contains the correct chiral behaviour
as \( m_q \rightarrow 0 \) while also possessing the Dirac moment mass dependence in the
heavy quark mass regime.
The extrapolation function utilized here has these same features, however we move beyond the previous approach by incorporating not only the LNA but also the next-to-leading nonanalytic (NLNA) structure of χPT in the extrapolation function. Incorporating the NLNA terms contributes little to the octet baryon magnetic moments, however it proves vital for decuplet baryons. The NLNA terms contain information regarding the branch point at $m_\pi = M_\Delta - M_N$, associated with the $\Delta \rightarrow N\pi$ decay channel and play a significant role in decuplet-baryon magnetic moments.

2. Leading and Next-to-Leading Nonanalytic Behavior

We begin with the chiral expansion for decuplet baryon magnetic moments. The LNA and NLNA behaviour is given by

$$ q_i G_K + \frac{2}{3} I_3^i (G_\pi - G_K) ,$$

where $q_i$ and $I_3^i$ are charge and isospin respectively, and $G_j (j = \pi, K)$ is given by

$$ G_j = \frac{-M_N}{32 \pi f_j^2} \left\{ \frac{4}{9} \mathcal{H}^2 \mathcal{F}(0, m_j, \mu_j) + C^2 \mathcal{F}(-\delta_N, m_j, \mu_j) \right\} .$$

$\mathcal{H}$ describes the meson coupling to decuplet baryons and $C$ describes octet-decuplet transitions. We take $\mathcal{H} = -2.2$ and $C = -1.2$. Here we omit the Roper as this transition requires significant excitation energy and is strongly suppressed by the finite size of the meson source. The octet-decuplet mass splitting $\delta_N$ is assigned its average value

$$ \delta_N = M_{10} - M_8 = 1377 - 1151 = +226 \text{ MeV}. $$

We take $f_\pi = 93$ MeV and $f_K = 112$ MeV. The function $\mathcal{F}(\delta, m, \mu)$ has the form

$$ \mathcal{F}(\delta, m, \mu) = -\frac{\delta}{\pi} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{2}{\pi} \sqrt{m^2 - \delta^2} \left( \frac{\pi}{2} - \tan^{-1} \frac{\delta}{\sqrt{m^2 - \delta^2}} \right) , $$

$m > | \delta |$,

$$ \mathcal{F}(\delta, m, \mu) = -\frac{\delta}{\pi} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{1}{\pi} \sqrt{\delta^2 - m^2} \ln \left( \frac{\delta - \sqrt{\delta^2 - m^2}}{\delta + \sqrt{\delta^2 - m^2}} \right) . $$

$m < | \delta |$.

This definition for $\mathcal{F}(\delta, m, \mu)$ corrects a sign error in Ref. 8. It differs by an overall minus sign and suppresses additive constants which are irrelevant in our analysis. As $\delta \rightarrow 0$, $\mathcal{F}(\delta, m_\pi, \mu) \rightarrow m_\pi$. 
Hence the LNA and NLNA behaviour of decuplet magnetic moments is given by
\[ \chi_{\pi} m_{\pi} + \chi_{K} m_{K} + \chi'_{\pi} \mathcal{F}(-\delta_{N}, m_{\pi}, \mu_{\pi}) + \chi'_{K} \mathcal{F}(-\delta_{N}, m_{K}, \mu_{K}), \] (4)

where
\[ \chi_{\pi} = \frac{M_{N} H}{\pi (f_{\pi})^2} \left( \frac{-I_{3}}{108} \right), \quad \chi'_{\pi} = \frac{M_{N} C}{\pi (f_{\pi})^2} \left( \frac{-I_{3}}{48} \right), \]
\[ \chi_{K} = \frac{M_{N} H}{\pi (f_{K})^2} \left( \frac{I_{3}}{108} - \frac{q_{i}}{72} \right), \quad \chi'_{K} = \frac{M_{N} C}{\pi (f_{K})^2} \left( \frac{I_{3}}{48} - \frac{q_{i}}{32} \right). \] (5)

The values for the above chiral coefficients, Eqs. (5), describing the strength of various meson dressings of the Δ baryons, are summarized in Table 1 for the four Δ baryons of the decuplet.

3. Analytic Continuation of χPT

It is now recognized that in any extrapolation from the heavy quark regime (where constituent quark properties are manifest) to the physical world, it is imperative to incorporate the quark-mass dependence of observables predicted by χPT in the chiral limit. However, as results are often obtained using methods ideally suited for heavy quark masses, it is imperative for the extrapolation function to correctly reflect the behaviour of the physical observable in the heavy quark mass regime as well. An extrapolation function for the Δ-baryon magnetic moments satisfying these criteria is
\[ \mu = \frac{\mu_{0}}{1 - \Gamma(m_{\pi})/\mu_{0} + \beta m_{\pi}^{2}}, \] (6)

where \( \mu_{0} \) and \( \beta \) are parameters optimized to fit results obtained near the strange quark mass and \( \Gamma(m_{\pi}) \) is taken from the chiral expansion for decuplet

| Table 1. The baryon chiral coefficients for the four Δ baryons of the decuplet. Coefficients are calculated with \( H = -2.2 \) and \( C = -1.2 \), where \( H \) is the decuplet-decuplet coupling constant and \( C \) is the decuplet-octet coupling constant. We have suppressed the kaon loop contribution by using \( f_{K} = 1.2 f_{\pi} \) and \( f_{\pi} = 93 \text{ MeV} \). |
| --- | --- | --- | --- | --- |
|  | \( \Delta^{++} \) | \( \Delta^{+} \) | \( \Delta^{0} \) | \( \Delta^{-} \) |
| \( \chi_{\pi} \) | -2.33 | -0.777 | +0.777 | +2.33 |
| \( \chi_{K} \) | -1.61 | -1.070 | -0.535 | 0 |
| \( \chi'_{\pi} \) | -1.56 | -0.518 | +0.518 | +1.56 |
| \( \chi'_{K} \) | -1.08 | -0.719 | -0.361 | 0 |
magnetic moments in Sec. 2

\[ \Gamma(m_\pi) = \chi_\pi m_\pi + \chi_K \left( m_K - m_K^{(0)} \right) + \chi_\pi' \left( \mathcal{F}(\delta_N, m_\pi, \mu_\pi) - \mathcal{F}_\pi \right) + \chi_K' \left( \mathcal{F}(\delta_N, m_K, \mu_K) - \mathcal{F}_K \right), \]

(7)

where \( m_K^{(0)}, \mathcal{F}_\pi \) and \( \mathcal{F}_K \) are constants defined to ensure that each term A, B, C and D, vanishes in the chiral limit. Utilizing the following relations provided by \( \chiPT \)

\[ m_K^2 = m_K^{(0)}^2 + \frac{1}{2} m_\pi^2, \]

(8)

\[ m_K^{(0)} = \sqrt{(m_K^{\text{phys}})^2 - \frac{1}{2} (m_\pi^{\text{phys}})^2}, \]

(9)

the four terms of Eq. (7) vanish in the chiral limit provided

\[ \frac{\delta}{\pi} \ln \left( \frac{(2\delta)^2}{\mu^2} \right), \]

\[ \frac{\delta}{\pi} \ln \left( \frac{(m_K^{(0)})^2}{\mu^2} \right) + \frac{2}{\pi} \sqrt{(m_K^{(0)})^2 - \delta_N^2} \left( \frac{\pi}{2} + \tan^{-1} \frac{\delta_N}{\sqrt{(m_K^{(0)})^2 - \delta_N^2}} \right). \]

(10)

Figure 1 presents a plot of each of the four terms of Eq. (7) (without the chiral coefficient pre-factors) as a function of \( m_\pi^2 \). Figure 2 presents a plot of the four terms summed with the appropriate weightings of Table 1 for each of the four charge states of the \( \Delta \).

The extrapolation function of Eq. (6) is designed to reproduce the leading and next-to-leading nonanalytic structure expressed in Eq. (7) for expansions about \( m_\pi = 0 \). Eq. (6) may be regarded as an analytic continuation of Eq. (7), preserving the constraints imposed by chiral symmetry and introducing the heavy quark mass regime behaviour to the extrapolation function. The LNA behaviour of Eq. (7) is complemented by terms analytic in the quark mass with fit parameters \( \mu_0 \) and \( \beta \) adjusted to fit additional constraints on the observable under investigation.

Hence the extrapolation function guarantees the correct nonanalytic behaviour in the chiral limit. Further as \( m_\pi \) becomes large, Eq. (6) is proportional to \( 1/m_\pi^2 \). As \( m_\pi^2 \propto m_q \) over the applicable mass range, the magnetic moment extrapolation function decreases as \( 1/m_q \) for increasing quark mass, precisely as the Dirac moment requires. This extrapolation function therefore provides a functional form bridging the heavy quark mass regime and the chiral limit.
Figure 1. Plots of the four chiral expansion functions (without the chiral coefficient pre-factors) of Eq. (7), labeled A, B, C, D in Eq. (7).

Figure 2. Plots of the sum of all four chiral expansion terms of Eq. (7), for each $\Delta$ baryon.

4. Results

The method employed to obtain our theoretical predictions is analogous to that presented in our previous analysis of octet baryon magnetic moments.\textsuperscript{5} We take the established input parameters, the strange-constituent and strange-current quark masses ($M_s$ and $c\, m_s^{\text{phys}}$ respectively\textsuperscript{b}), obtained by optimizing agree-

\textsuperscript{b}The parameter c is expected to be the order of 1.
ment between the AccessQM\textsuperscript{c} and octet baryon magnetic moments. There, \( M_s = 565 \) MeV and \( c m_s^{\text{phys}} = 144 \) MeV provides optimal agreement.

The constituent quark model (CQM) provides the following formulas that relate the constituent quark masses to the delta magnetic moments.

\[
\mu_{\Delta^+} = 3\mu_u, \quad \mu_{\Delta^0} = 2\mu_u + \mu_d, \quad \mu_{\Delta^-} = \mu_u + 2\mu_d, \quad \mu_{\Delta^0} = 3\mu_d,
\]

(11)

with

\[
\mu_u = \frac{2}{3} M_N \mu_N, \quad \mu_d = \frac{1}{3} M_N \mu_N, \quad \mu_s = \frac{1}{3} M_N \mu_N.
\]

(12)

These formulas are used to obtain two magnetic moment data points near the SU(3)-flavour limit where \( u, d \) quarks take values near the \( s \)-quark mass.

To fit Eq. (6), which is a function of \( m_\pi \), to the magnetic moments given by the CQM in Eq. (11) with constituent-quark masses \( M_u = M_d = M_i \) (\( i = 1, 2 \)), we relate the pion mass to the constituent quark mass via the current quark mass.

Chiral symmetry provides

\[
\frac{m_q}{m_q^{\text{phys}}} = \frac{m_\pi^2}{(m_\pi^{\text{phys}})^2},
\]

(13)

where \( m_q^{\text{phys}} \) is the quark mass associated with the physical pion mass, \( m_\pi^{\text{phys}} \).

From lattice studies, we know that this relation holds well over a remarkably large regime of pion masses, up to \( m_\pi \sim 1 \) GeV. The link between constituent and current quark masses is provided by

\[
M = M_X + c m_q,
\]

(14)

where \( M_X \) is the constituent quark mass in the chiral limit and \( c \) is of order 1. Using Eq. (13) this leads to

\[
M = M_X + c m_q^{\text{phys}} (m_\pi^{\text{phys}})^2 m_\pi^2.
\]

(15)

The link between the constituent quark masses \( M_i \) and \( m_\pi \) is thus provided by

\[
m_i^2 = (m_\pi^{\text{phys}})^2 \left( M_i - (M_s - c m_s^{\text{phys}}) \right) \frac{1}{c m_q^{\text{phys}}} \quad (i = 1, 2),
\]

(16)

where \( M_s - c m_s^{\text{phys}} = M_X \) encapsulates information on the constituent quark mass in the chiral limit, and \( c m_s^{\text{phys}} \) provides information on the strange current quark mass. We use the ratio

\[
\chi_{sq} = \frac{m_s^{\text{phys}}}{m_q^{\text{phys}}} = 24.4 \pm 1.5,
\]

(17)

\( \text{The name indicates the mathematical origins of the model: Analytic Continuation of the Chiral Expansion for the SU(6) Simple Quark Model.} \)
Table 2. Theoretical predictions for the charged ∆ baryon magnetic moments. The fit parameters \( \mu_0 \) and \( \beta \) are given for each scenario. The only known experimental value for the ∆ baryon magnetic moments is the \( \Delta^{++} \) moment, recent measurements provide \( \mu_{\Delta^{++}} = 4.52 \pm 50 \pm 45 \mu_N \) and \( \mu_{\Delta^{++}} = 6.14 \pm 51 \mu_N \).

| Baryon | \( \mu_0 \) | \( \beta \) | AccessQM (\( \mu_N \)) |
|--------|-------------|--------|---------------------|
| \( \Delta^{++} \) | 5.67 | 0.16 | 5.39 |
| \( \Delta^+ \) | 2.69 | 0.20 | 2.58 |
| \( \Delta^- \) | 3.22 | 0.06 | 2.99 |

provided by \( \chi PT \) to express the light current quark mass, \( m_q^{phys} \), in terms of the strange current quark mass, \( m_s^{phys} \), in Eq. (16).

The analytic continuation of \( \chi PT \), Eq. (6), is fit to the CQM as a function of \( m_\pi^2 \). Results are presented in Figs. 3 and 4. The magnetic moments given by the CQM either side of the SU(3)-flavour limit are indicated by a dot (●) and the theoretical prediction is indicated at the physical pion mass by a star (★). These results, along with the parameters \( \mu_0 \) and \( \beta \) are summarized in Table 2.

The interesting feature of these plots is the cusp at \( m_\pi^2 = \delta_N^2 \) which indicates the opening of the octet decay channel, \( \Delta \to N \pi \). The physics behind the cusp is intuitively revealed by the relation between the derivative of the magnetic moment with respect to \( m_\pi^2 \) and the derivative with respect to the momentum transfer \( q^2 \), provided by the pion propagator \( 1/(q^2 + m_\pi^2) \) in the heavy baryon limit. Derivatives with respect to \( q^2 \) are proportional to the magnetic charge radius in the limit \( q^2 \to 0 \),

\[
\langle r_M^2 \rangle = -6\frac{\partial}{\partial q^2}G_M(q^2)|_{q^2=0}.
\]

If we consider for example \( \Delta^{++} \to p \pi^+ \) with \( |j, m_j\rangle = |3/2, 3/2\rangle \), the lowest-lying state conserving parity and angular momentum will have a relative P-wave orbital angular momentum with \( |l, m_l\rangle = |1, 1\rangle \). Thus the positively-charged pion makes a positive contribution to the magnetic moment. As the opening of the \( p \pi^+ \) decay channel is approached from the heavy quark-mass regime, the range of the pion cloud increases in accord with the Heisenberg uncertainty principle, \( \Delta E \Delta t \sim \hbar \). Just above threshold the pion cloud extends towards infinity as \( \Delta E \to 0 \) and the magnetic moment charge radius diverges. Similarly, \( (\partial/\partial m_\pi^2)G_M \to -\infty \). Below threshold, \( G_M \) becomes complex and the magnetic moment of the \( \Delta \) is identified with the real part. The imaginary part describes the physics associated with photon-pion coupling in which the pion is subsequently observed as a decay product.

It is the NLNA terms of the chiral expansion for decuplet baryons that contain the information regarding the decuplet to octet transitions. These
Figure 3. The extrapolation function fit for $\Delta^{++}$ and $\Delta^+$ magnetic moments. The magnetic moments given by the CQM either side of the SU(3)-flavour limit are indicated by dots ($\bullet$) and the theoretical prediction for each baryon is indicated at the physical pion mass by a star ($\star$). The only available experimental data is for the $\Delta^{++}$ and is indicated by an asterisk ($\ast$). The proton extrapolation (dashed line) is included to illustrate the effect of the open decay channel, $\Delta \to N\pi$, in the $\Delta^+$ extrapolation. The presence of this decay channel gives rise to a $\Delta^+$ moment smaller than the proton moment.

Transitions are energetically favourable making them of paramount importance in determining the physical properties of $\Delta$ baryons. The NLNA terms serve to enhance the magnitude of the magnetic moment above the opening of the decay channel. However, as the decay channel opens and an imaginary part develops, the magnitude of the real part of the magnetic moment is suppressed. The strength of the LNA terms, which enhance the magnetic moment magnitude as the chiral limit is approached, overwhelms the NLNA contributions such that the magnitude of the moments continues to rise towards the chiral limit.

The inclusion of the NLNA structure into octet baryon magnetic moment extrapolations is less important for two reasons. The curvature associated with the NLNA terms is negligible for the $N$ and $\Sigma$ baryons and small for the $\Lambda$ and $\Xi$ baryons. More importantly one can infer the effects of the higher order terms of $\chi$PT, usually dropped in truncating the chiral expansion, through the consideration of phenomenological models. If one incorporates form factors at the meson-baryon vertices, reflecting the finite size of the meson source, one finds that transitions from ground state octet baryons to excited state
baryons are suppressed relative to that of $\chi$PT to finite order, where point-like couplings are taken. In $\chi$PT it is argued that the suppression of excited state transitions comes about through higher order terms in the chiral expansion. As such, the inclusion of NLNA terms alone will result in an overestimate of the transition contributions, unless one works very near the chiral limit where higher order terms are indeed small. For this reason octet to decuplet or higher excited state transitions have been omitted in previous studies.\textsuperscript{5,6,7}

In the simplest CQM with $m_u = m_d$ the $\Delta^+$ and proton moments are degenerate. However, spin-dependent interactions between constituent quarks will enhance the $\Delta^+$ relative to the proton at large quark masses, and this is supported by lattice QCD simulation results.\textsuperscript{10} As a result, early lattice QCD predictions based on linear extrapolations\textsuperscript{10} report the $\Delta^+$ moment to be greater than the proton moment. However with the extrapolations presented here which preserve the LNA behavior of $\chi$PT, the opposite conclusion is reached. We predict $\Delta^+$ and proton magnetic moments of 2.58 $\mu_N$ and 2.77 $\mu_N$ respectively. The proton magnetic moment extrapolation\textsuperscript{5} is included in Fig. 3 as an illustration of the importance of incorporating the correct nonanalytic behaviour predicted by $\chi$PT in any extrapolation to the physical world. An experimentally measured value for the $\Delta^+$ magnetic moment would offer important insights into the role of spin-dependent forces and chiral nonanalytic
behaviour in the quark structure of baryon resonances.

5. Conclusion

An extrapolation function for the decuplet baryon magnetic moments has been presented. This function preserves the leading and next-to-leading nonanalytic behaviour of chiral perturbation theory while incorporating the Dirac-moment dependence for moderately heavy quarks. Interesting nonanalytic behavior in the magnetic moments associated with the opening of the $\pi N$ decay channel has been highlighted. It will be interesting to apply these techniques to existing and forthcoming lattice QCD results, and research in this direction is currently in progress.

Experimental value exists only for the $\Delta^{++}$ magnetic moment where the two most recent results are $\mu_{\Delta^{++}} = 4.52 \pm 0.50 \pm 0.45 \mu_N$ and $\mu_{\Delta^{++}} = 6.14 \pm 0.51 \mu_N$. These values are in good agreement with the prediction of 5.39 $\mu_N$ given by our AccessQM as described above. Arrival of experimental values for the $\Delta^+$ and $\Delta^-$ magnetic moments are eagerly anticipated and should be forthcoming in the next few years. More importantly, these techniques may be applied to the decuplet hyperon resonances where the role of the kaon cloud becomes important. We look forward to new JHF results in this area in the future.

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