Shell-model studies of the astrophysical rp-process reactions $^{34}\text{S}(p,\gamma)^{35}\text{Cl}$ and $^{34g,m}\text{Cl}(p,\gamma)^{35}\text{Ar}$

W. A. Richter$^{1,2}$, B. Alex Brown$^3$, R. Longland$^{4,5}$, C. Wrede$^3$, C. Fry$^3$, P. Denisenkov$^6$, F. Herwig$^6$, D. Kurtulgil$^7$, M. Pignatari$^8$ and R. Reifarth$^7$

$^1$ University of Stellenbosch, South Africa
$^2$ iThemba LABS, South Africa
$^3$ Michigan State University, USA
$^4$ North Carolina State University, Raleigh, USA
$^5$ Triangle Universities Nuclear Laboratory, Duke University, Durham NC
$^6$ University of Victoria, Canada
$^7$ Goethe University, Germany
$^8$ University of Hull, UK

E-mail: richter@sun.ac.za

Abstract.

The two rp-reactions $^{34}\text{S}(p,\gamma)^{35}\text{Cl}$ and $^{34g,m}\text{Cl}(p,\gamma)^{35}\text{Ar}$ were studied via a shell-model approach. At energies in the resonance region near the proton-emission threshold many negative-parity states appear. We present results of calculations in a full $(0+1)\hbar\omega$ model space which addresses this problem. Energies, spectroscopic factors and proton-decay widths are calculated for input into the reaction rates. Comparisons are also made with a recent experimental determination of the reaction rate for the $^{34}\text{S}(^3\text{He},d)^{35}\text{Cl}$ reaction. The thermonuclear $^{34g,m}\text{Cl}(p,\gamma)^{35}\text{Ar}$ reaction rates are unknown because of a lack of experimental data. The rates for transitions from the ground state of $^{34}\text{Cl}$ as well as from the isomeric first excited state of $^{34}\text{Cl}$ are explicitly calculated taking into account the relative populations of the two states.

1. Introduction

In a recent experiment [1],[2] the $^{34}\text{S}(^3\text{He},d)^{35}\text{Cl}$ reaction was studied and proton-transfer spectroscopic factors measured for 21 states in an energy region of about 1 MeV above the threshold energy ($S_p = 6.371$ MeV). As a result a new $^{34}\text{S}(p,\gamma)^{35}\text{Cl}$ reaction rate could be determined directly from the experimental data. The product $(2J+1)C^2S$ was measured so that it was not necessary to determine the $J$ values of the resonances explicitly. We have done a theoretical calculation of the rate which takes into account contributions from positive and negative parity states in a full $(0+1)\hbar\omega$ model space based on the interaction SDPFMU [3]. The motivation is to correlate theory and experiment, to determine where differences exist and the reasons for these.

The thermonuclear $^{34g,m}\text{Cl}(p,\gamma)^{35}\text{Ar}$ reaction rates are unknown at nova temperature due to a lack of experimental nuclear physics data for the resonances up to about 800 keV above the $^{35}\text{Ar}$ proton separation energy [4]. Uncertainties in these rates translate to uncertainties in $^{34}\text{S}$ production in models of classical novae on oxygen-neon white dwarfs. $^{34}\text{S}$ abundances have the
potential to aid in the classification of presolar grains. A fast thermonuclear reaction rate leads to the destruction of $^{34}$Cl and bypasses the production of $^{34}$S, the beta decay daughter of $^{34}$Cl ($T_{1/2} = 1.55$ s). The isotopic $^{32}$S/$^{34}$S ratio depends strongly on the $^{34}$Cl(p,$\gamma$)$^{35}$Ar reaction rate.

Estimates based on shell-model calculations are complicated by high level density and the presence of negative-parity states in the resonance region near the proton-emission threshold. We present results of calculations in a full $(0+1)\hbar\omega$ model space which addresses this problem using the interaction SDPFMU [3] and NuShellX [6]. The basis consists of a complete $(0+1)\hbar\omega$ basis made from all possible excitations of one nucleon from 1s-0d to 0p-1f. Such calculations were carried out recently for the first time for the $^{30}$P(p,$\gamma$)$^{31}$S reaction [7]. We explicitly calculate the rates for transitions from the ground state of $^{34}$Cl as well as from the isomeric first excited state of $^{34}$Cl.

In a study by Fry et al. [4] seventeen new $^{35}$Ar levels have been detected in the energy region $E_x = 5.9 - 6.7$ MeV and their excitation energies have been determined, but not spins and parities. Because of the paucity of such information we are obliged to rely on shell-model calculations. We have calculated energies, spectroscopic factors and proton-decay widths for input into the reaction rate.

Uncertainty limits for the total calculated reaction rates have been included based on Monte Carlo techniques of estimating statistically meaningful reaction rates and their associated uncertainties [8].

2. The shell-model calculations
For positive-parity states we use the $(1s_{1/2},0d_{5/2},0d_{3/2})$ ($sd$) model space. For negative-parity states one nucleon is allowed to be excited from the $sd$ shell to the $pf$ shell $(1p_{1/2},1p_{3/2},0f_{7/2},0f_{5/2})$. The spurious states for these $1\hbar\omega$ negative-parity states were removed using the Gloeckner-Lawson method [9]. We start with the SDPFMU Hamiltonian from [3] with its mass-dependent strength evaluated at $A=42$. We then add the Coulomb interaction, and adjust the position of the $\ell = 1$ and $\ell = 3$ proton and neutron single-particle energies to reproduce the position of the first $3/2^-$ and $7/2^-$ states in $^{33}$Cl and $^{33}$S.

3. The $^{34}$S(p,$\gamma$)$^{35}$Cl and $^{34}g,m$Cl(p,$\gamma$)$^{35}$Ar reactions
3.1. Results for the reaction rates
In Fig. 1 in the top panel we show the total rp-process reaction rate versus temperature $T_9$ (GigaK) as well as the contributions from positive and negative parity states for the $^{34}$S(p,$\gamma$)$^{35}$Cl. In the lower panel the contributions of the various dominant resonances are shown. The details of these resonances are shown in Table I. In Ref. [2], Table 4.5 and our result. Evidently our recommended rate is larger in the low temperature region, although it is in agreement within the theoretical reaction rate uncertainty estimates as shown in Fig. 3. The three dominant contributions in the lower temperature region according to our calculations are from the negative parity states $1/2^-(2)$ (6.513 MeV), $3/2^-(4)$ (6.587 MeV) and $3/2^-(5)$ (6.762 MeV). The corresponding energies for Refs. [1], [2] are 6.545 MeV, 6.643 MeV and 6.671 MeV. The $\omega\gamma$ values for the three states correspond reasonably well with the maximum values of Gillespie et al., which correspond to $l=1$ transfer and thus negative parity as in our calculation. In ref. [1] it has been assumed that $\Gamma_{tot}$ is dominated by the contribution from $\Gamma_{\gamma}$, so that $\omega\gamma$ depends only on $\Gamma_p$.

The main difference between experiment and theory resides in the contribution of the $1/2^-(2)$ state through the spectroscopic factor. The theory value $(2J+1)C^2S$ is 0.36 while the experimental value is 0.0028. When we substitute the spectroscopic factor of Gillespie et al. in our calculation the discrepancy at lower temperature is removed. However, the problem may
Figure 1. The total reaction rate versus temperature T9 (GigaK) for positive and negative parity states for transitions from the ground state of $^{34}\text{S}$(top panel) (solid line), and the contribution of each of the final states (lower panel) obtained with the data from Table 1.

Figure 2. The total reaction rate versus temperature T9 (GigaK) for transitions from the ground state of $^{34}\text{S}$ (black line), and the minimum and maximum rates from Ref. [2], as well as a Hauser-Feshbach rate [10].

Table 1. Properties of the rp-resonance states for transitions from the ground state of $^{34}\text{S}$

| n  | $J^\pi$ | $k$ | $E_x$(th) (MeV) | $E_x$(exp) (MeV) | $E_{res}$ (MeV) | $C^2S^+$ $\ell = 0$(1) | $C^2S^+$ $\ell = 2$(3) | $\Gamma_\gamma$ (eV) | $\Gamma_p$ (eV) | $\omega_\gamma$ (eV) |
|----|--------|----|----------------|-----------------|----------------|-------------------|-------------------|----------------|-------------|-------------|
| 39 | 1/2$^-$ | 2  | 6.513          | 0.142           | 3.6×10$^{-1}$   | 2.4               | 2.4×10$^{-9}$     | 2.4×10$^{-9}$ |
| 43 | 3/2$^-$ | 4  | 6.587          | 0.216           | 1.5×10$^{-2}$   | 3.7×10$^{-2}$     | 1.2×10$^{-7}$     | 2.5×10$^{-7}$ |
| 48 | 3/2$^-$ | 5  | 6.761          | 0.390           | 4.1×10$^{-2}$   | 4.1×10$^{-2}$     | 1.7×10$^{-3}$     | 3.3×10$^{-3}$ |
| 53 | 1/2$^+$ | 4  | 7.006          | 0.635           | 6.3×10$^{-3}$   | 1.6               | 3.3×10$^{-1}$     | 2.8×10$^{-1}$ |
| 58 | 1/2$^+$ | 5  | 7.116          | 0.745           | 1.4×10$^{-2}$   | 2.5               | 3.1              | 1.4          |

be related to an incorrect assignment. In a recent measurement of the $^{32}\text{S}(\alpha,p)^{35}\text{Cl}$ reaction rate an assignment of (1/2+) is given to this state [11].

In Fig. 3 the total reaction rate is shown as well as a low rate and a high rate for each temperature according to Monte Carlo estimates, corresponding to the 0.16 and 0.84 quantiles of the cumulative reaction rate distribution [8].

Fig. 4 shows the total reaction rate versus temperature T9 for $^{34}\text{g,m}\text{Cl}(p,\gamma)^{35}\text{Ar}$, including positive and negative parity states for transitions from the ground state of $^{34}\text{Cl}$ and the final states(lower panel) obtained with the data from Table 2. It is evident that the negative parity states dominate the reaction rate by up to three orders of magnitude at the lower temperatures. The rate is mainly due to two resonances, the 3/2$^-$ (3) and 1/2$^-$ (2) states.

Fig. 5 shows the same for positive and negative parity states for transitions from the first excited state of $^{34}\text{Cl}$ (top panel) using the data from Table 3. Again the negative parity states dominate the rate by up to two orders of magnitude, and the rate is mainly due to two resonances, the 5/2$^-$ (5) and 5/2$^-$ (7) states.

Fig. 6 shows the total rate including positive and negative parity and transitions from both the ground and first excited state of $^{34}\text{Cl}$. The relative populations of the two states have been
states of $^{34}$S up to about 1 GK. The rate from the ground state is dominant taken into account through the stellar enhancement factor, which is ratio of the rates from the ground and first excited states and the ground state. The rate from the ground state is dominant up to about 1 GK.

Figs. 7 and 8 show the total reaction rates for transitions from the ground and first excited states of $^{34}$Cl respectively, as well as a low rate and a high rate according to Monte Carlo estimates.

### Table 2. Properties of the rp-resonance states for transitions from the ground state of $^{34}$Cl

| $n$ | $J^\pi$ | $E^*_x$ (th) | $E_{res}$ | $C^2S$ | $C^2S$ | $\Gamma_\gamma$ | $\Gamma_p$ | $\omega_\gamma$ |
|-----|---------|-------------|-----------|---------|---------|---------------|------------|-------------|
|     |         | (MeV)       |           | $\ell = 0(1)$ | $\ell = 2(3)$ | (eV)          | (eV)       | (eV)        |
| 37  | $3/2^-$ | 6.052       | 0.156     | $3.7 \times 10^{-4}$ | $1.9 \times 10^{-4}$ | $1.0 \times 10^{-6}$ | 7.8 $\times 10^{-2}$ | 2.7 $\times 10^{-4}$ |
| 44  | $3/2^-$ | 6.345       | 0.449     | $3.5 \times 10^{-3}$ | $7.8 \times 10^{-2}$ | $2.7 \times 10^{-4}$ | 5.4 $\times 10^{-4}$ | 5.7 $\times 10^{-5}$ |
| 46  | $5/2^-$ | 6.469       | 0.573     | $1.6 \times 10^{-2}$ | $3.5 \times 10^{-1}$ | $2.6 \times 10^{-5}$ | 3.7 $\times 10^{-2}$ | 4.6 $\times 10^{-2}$ |
| 48  | $3/2^-$ | 6.476       | 0.580     | $2.6 \times 10^{-2}$ | $6.0 \times 10^{-2}$ | $3.7 \times 10^{-2}$ | 1.4         | 9.8 $\times 10^{-1}$ |
| 49  | $1/2^-$ | 6.501       | 0.605     | $2.2 \times 10^{-1}$ | $9.8 \times 10^{-1}$ | $5.8 \times 10^{-1}$ | 1.4         | 9.8 $\times 10^{-1}$ |

### Table 3. Properties of the rp-resonance states for transitions from the first excited state of $^{34}$Cl

| $n$ | $J^\pi$ | $E^*_x$ (th) | $E_{res}$ | $C^2S$ | $C^2S$ | $\Gamma_\gamma$ | $\Gamma_p$ | $\omega_\gamma$ |
|-----|---------|-------------|-----------|---------|---------|---------------|------------|-------------|
|     |         | (MeV)       |           | $\ell = 0(1)$ | $\ell = 2(3)$ | (eV)          | (eV)       | (eV)        |
| 41  | $5/2^-$ | 6.278       | 0.236     | $1.9 \times 10^{-4}$ | $6.0 \times 10^{-4}$ | $8.5 \times 10^{-2}$ | 6.1 $\times 10^{-6}$ | 2.6 $\times 10^{-6}$ |
| 45  | $7/2^-$ | 6.395       | 0.353     | $2.6 \times 10^{-2}$ | $3.0 \times 10^{-2}$ | $7.8 \times 10^{-2}$ | 3.0 $\times 10^{-4}$ | 1.7 $\times 10^{-4}$ |
| 46  | $5/2^-$ | 6.469       | 0.427     | $3.3 \times 10^{-2}$ | $1.0 \times 10^{-3}$ | $3.5 \times 10^{-1}$ | 6.4 $\times 10^{-5}$ | 2.7 $\times 10^{-5}$ |
| 48  | $3/2^-$ | 6.476       | 0.434     | $5.3 \times 10^{-2}$ | $4.4 \times 10^{-2}$ | $6.0 \times 10^{-2}$ | 1.9 $\times 10^{-2}$ | 4.1 $\times 10^{-3}$ |
| 55  | $5/2^-$ | 6.695       | 0.653     | $1.9 \times 10^{-1}$ | $7.1 \times 10^{-2}$ | $1.9 \times 10^{-1}$ | 4.4         | 5.6 $\times 10^{-1}$ |
Figure 5. The total reaction rate versus temperature T9 (GigaK) for positive and negative parity states for transitions from the first excited state of $^{34}\text{Cl}$ (top panel) (solid line), and the contribution of each of the final states (lower panel) obtained with the data from Table 3.

Figure 6. The total reaction rate (which includes positive and negative parity with the relative populations of the ground and first excited isomeric states of $^{34}\text{Cl}$ taken into account) versus temperature T9 (GigaK). The contributions from the ground state and the isomeric state are also shown.

Figure 7. The total reaction rate (both parities) versus temperature T9 (GigaK) for transitions from the ground state of $^{34}\text{Cl}$, and the high and low rates according to the Monte Carlo estimates indicated in red and blue respectively.

Figure 8. The total reaction rate (both parities) versus temperature T9 (GigaK) for transitions from the isomeric first excited state of $^{34}\text{Cl}$, and the low and high rates according to the Monte Carlo estimates indicated in red and blue respectively.
4. Nucleosynthesis simulations

Sulfur isotopic ratios were predicted by the MESA/NuGrid multi-zone ONe nova model [12]. Thermal communication between $^{34}\text{g, m}\text{Cl}$ and $^{34}\text{m}\text{Cl}$ was fully incorporated using EM transition rates between all low-lying $^{34}\text{Cl}$ states [13]. Preliminary $^{34}\text{S}/^{32}\text{S}$ ratio uncertainties from $^{34}\text{g, m}\text{Cl}(p, \gamma)^{35}\text{Ar}$ rates are < 13 %. Uncertainties from $^{34}\text{S}(p, \gamma)^{35}\text{Cl}$ rates are < 44 %.

5. Conclusions

In the comparison of our calculations for $^{34}\text{S}(p, \gamma)^{35}\text{Cl}$ with the recent experiment of Gillespie et al. on $^{34}\text{S}(^{3}\text{He},d)^{35}\text{Cl}$ there is general agreement between the calculated total rate and the experimental rate up to 1 GK, taking into account the theoretical uncertainties based on a Monte Carlo analysis. However, as indicated there are also uncertainties associated with the spin-parity assignment of specific experimental states, notably the 6.545 MeV state. The contribution from negative parity dominates except for an intermediate region near $T = 1$ GK. Our theoretical analysis shows that the $^{34}\text{g, m}\text{Cl}(p, \gamma)^{35}\text{Ar}$ reaction rates both for transitions from the ground state of $^{34}\text{Cl}$ and the first excited state are dominated by negative parity states by between two and three orders of magnitude. The contributions to the total rate from the isomeric first excited state of $^{34}\text{Cl}$ become more important above about half of the temperature range considered. The statistical Hauser-Feshbach rate differs from our ground-state rate at lower and temperatures by up to about a order of magnitude, but is closer to our result for higher temperatures. The calculations also identify the most prominent resonances in the reaction rates, and the analysis should serve as a guide for experiments as the spin-parity assignments of the most prominent resonances and their relative strengths are given.

The rates were implemented in a nova nucleosynthesis code including thermal population of the $^{34}\text{m}\text{Cl}$ isomer. Nucleosynthesis uncertainties associated with the shell-model calculations are not very large.

Acknowledgments This work is partly supported by NSF Grant PHY-1811855, the Joint Institute for Nuclear Astrophysics NSF Grant PHY-1430152, RL acknowledges support from U.S. Department of Energy, Grant No. DE-SC0017799 and under Contract No. DE-FG02-97ER41041, and WR the University of Stellenbosch, South Africa.

6. References

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