Quantum Secret Authentication Code

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Abstract

This study proposes a quantum secret authentication code for protecting the integrity of secret quantum states. Since BB84 was first proposed, the eavesdropper detection strategy in almost all quantum cryptographic protocols is based on the random sample discussion, in which the probability of eavesdropper detection is depending on the number of check qubits eavesdropped by the eavesdropper. Hence, if the eavesdropper interferes only a few qubits of the transmitted quantum sequence, then the detection probability will be very low. This study attempts to propose a quantum secret authentication code to solve this problem. With the use of quantum secret authentication code, not only is the probability of eavesdropper detection guaranteed to be evenly distributed no matter how many qubits had been eavesdropped, but also can the quantum transmission efficiency be highly enhanced.

keywords: message authentication code, integrity, random sampling

1 Introduction

In 1984, BB84, the first quantum cryptographic protocol was proposed. For the next decade, the eavesdropper detection strategy, following the idea of BB84, for almost all quantum cryptographic protocols is based on the random sample discussion. In this eavesdropper detection method, the check qubits are independent to the message qubits. By scrambling the message qubits with the check qubits together, the strategy assumes

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that an eavesdropper has no idea on discriminating a check qubit from a message qubit. An attempt to eavesdrop the qubit sequence by an outsider would only end up disturbing the check qubit states. And, the only chance for an eavesdropper detection is when the check qubits were altered “inadvertently” by the eavesdropper. Hence, the more check qubits altered by an eavesdropper, the higher probability he/she is detected.

In order to increase the detection probability. The number of check qubits should be large enough, because the eavesdropper detection probability for interfering one check qubit is

\[
\frac{\text{number of check qubits}}{\text{Total number of qubits}} \times \frac{1}{4}
\]

where the \(\frac{\text{number of check qubits}}{\text{Total number of qubits}}\) represents the probability of the eavesdropper to choose a check qubit, and \(\frac{1}{4}\) is the probability that the measurement result of that check qubit is different from what was expected due to the interference of that eavesdropper. Most protocols suggest the number of check qubits should be at least the same as that of the message qubits transmitted [4, 5, 6, 7].

However, there is a problem with the random sampling discussion strategy, especially when an eavesdropper just attempts to eavesdrop only a few qubits in the transmitted quantum sequence. In this situation, some information might be revealed, and the integrity of the message qubits might be jeopardized, but the eavesdropper detection probability could still be very low. Most quantum cryptographic protocols ignore this problem and conversely assume that the eavesdropper always attempts to eavesdrop the entire sequence of qubits. Consequently, the conclusion, the probability of eavesdropper detection can reach 1, is asserted.

This paper aims to design a quantum secret authentication code (QSAC) to solve the above-mentioned problem. This QSAC guarantees the integrity of the secret qubits transmitted from a sender to a receiver. Furthermore, the intentionally designed avalanche effect guarantees the detection of an eavesdropping of even a single qubit in the quantum sequence.

The rest of this paper is organized as follows. Section 2 presents the proposed quantum secret authentication code and the security analysis. Section 3 gives an application
of the proposed QSAC. Finally, conclusions are made in Section 4.

2 Quantum secret authentication code

In this section, a quantum secret authentication code (QSAC) is proposed for a receiver to verify whether or not the received states are indeed from the alleged sender and without being eavesdropped or modified.

2.1 Notations

$K$ the pre-shared key between the sender and the receiver.

$||$ concatenation.

$|M⟩^i$ the $i$th qubit in the quantum sequence $M$.

$\text{CNOT}(a,b)$ Control-Not gate, where $a$ is the control qubit and $b$ is the target qubit. If $a = b$, then performs the identity operation $I$.

2.2 The avalanche effect

A CNOT gate is applied here to create an avalanche effect in the QSAC. Figure 1 demonstrates an avalanche effect of the QSAC. Assume that there is a function $F$ with a sequence of quantum states as the input and a sequence of entangled qubits as the output. The function is composed of a sequence of $\text{CNOT}$ operations. For example, let the input of the function be a three-qubit quantum state $|ψ⟩_{123}$ and the output, $\text{CNOT}(q_1,q_2)\text{CNOT}(q_2,q_3)\text{CNOT}(q_3,q_1)|ψ⟩$. Conversely, the inverse function $F^{-1}$ should be $\text{CNOT}(q_3,q_1)\text{CNOT}(q_2,q_3)\text{CNOT}(q_1,q_2)|ψ⟩$. An avalanche effect is described under a communication model between a sender and a receiver. First, the sender inputs the state, $|000⟩$ for example, to the function $F$ and subsequently, $F$ outputs the state $|000⟩$. Then the sender sends the qubits $|000⟩$ to the receiver via a quantum channel. During the time of transmission, if there is no eavesdropper, then the same state $|000⟩$ will be received and recovered by the receiver, i.e., $|000⟩ → F^{-1} → |000⟩$. Conversely, assume the last qubit is altered by an outsider, i.e., the state received by the receiver becomes $|001⟩$. 
Consequently, the receiver inputs the received state $|001\rangle$ to $F^{-1}$, the inversion of the function $F$. The state that the receiver finally recovers is $|110\rangle$. Obviously, one single qubit modification eventually causes more than one qubits to change.

Figure 1: The avalanche effect of the QSAC.

2.3 The proposed QSAC

Based on the technique described in Sec. 2.2, this section proposes a QSAC design that marries a pre-shared key, $K$, between a sender and a receiver to a message qubits, $|M\rangle$ to be transmitted from the sender to the receiver via a sequence of CNOT operations in such a way that only a few qubits’ interference to the transmitted qubits by an outsider without knowing $K$ would cause an avalanche effect on the qubits recovered by the receiver.

Figure 2 shows the procedure of the QSAC. The sender first inputs the message qubits, $|M\rangle$, and the shared key, $K$, to the QSAC algorithm. The QSAC algorithm generates check qubits from $K$ and outputs a QSAC codeword which strongly entangles the message qubits and the check qubits. Then, the sender sends the QSAC codeword to the receiver. For verification, the receiver inputs the received codeword and the shared key $K$ to the verification algorithm to “extract” the message qubits and the check qubits. Finally, by verifying the correctness of the check qubits, the receiver can verify whether or not the message qubits are indeed from the legitimate participant and are without eavesdropping
on modifications from an outsider.

Figure 2: The procedure of QSAC.

A. QSAC encoding:

The following steps demonstrate the encoding of the proposed QSAC. The inputs are the secret message qubits $|M\rangle$ of length $m$ and the key $K$ of length $k$ bits shared between the sender and the receiver.

Step1  Extend the key $K$ to $K_Q$ and $K_T$, via an extension function, which can be so designed as the key scheduling in DES or AES. The lengths of $K_Q$ and $K_T$ will be understood in the following description.

Step2  Transform $K_Q$ to a quadratic string $S_Q$ of length $n$ and every element of $S_Q$ is in \{0,1,2,3\}, where $n$ is the security parameter determines the number of check qubits. Then, a sequence of check qubits $|C\rangle = S_Q^1 \otimes S_Q^2 \otimes \cdots \otimes S_Q^j$ can be generated according to $S_Q$. The elements in \{0,1,2,3\} of $S_Q$ represents the states \{|0\rangle, |1\rangle, |+\rangle, |−\rangle\} of check qubits, respectively. That is, if the element in $S_Q$ is “0”, then the state “|0\rangle” is produced and so on. For example, if $S_Q$ is “012130”, then the check qubits should be “|0\rangle |1\rangle |+\rangle |1\rangle |−\rangle |0\rangle”.

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Step 3: Attach the message qubits to the check qubits as $|\psi\rangle_{n+m} = |C\rangle_n \otimes |M\rangle_m$.

Step 4: Transform $K_T$ to a digit string $S_T$ of length $(n+m)$ and every element of $S_T$ is in $\{1, 2, 3, \ldots, m+n\}$. Perform $CNOT$ operations among the qubits of $|\psi\rangle_{n+m}$ according to $S_T$. That is, the index of an element in $S_T$ represents the index of the control qubits in $|\psi\rangle_{n+m}$ and the corresponding value of that element in $S_T$ is the index of the target qubit in $|\psi\rangle_{n+m}$. In other words, the $CNOT$ operation is performed on the $i^{th}$ qubit in $|\psi\rangle_{n+m}$, as the control qubit and the $S_T^{i^{th}}$ qubit in $|\psi\rangle_{n+m}$, as the target qubit. For example, if $S_T = 3412$, then the sender first performs $CNOT(|\psi\rangle_1, |\psi\rangle_3)$, then computes $CNOT(|\psi\rangle^2, |\psi\rangle^4)$ and so on.

| Index of $S_T$ | 1 | 2 | 3 | 4 |
|---------------|---|---|---|---|
| Value of $S_T$| 3 | 4 | 1 | 2 |
| Operation     | $CNOT(|\psi\rangle^1, |\psi\rangle^3)$ | $CNOT(|\psi\rangle^2, |\psi\rangle^4)$ | $CNOT(|\psi\rangle^3, |\psi\rangle^1)$ | $CNOT(|\psi\rangle^4, |\psi\rangle^2)$ |

Table 1: Example of Step 4 in QSAC

B. QSAC decoding:

After the execution of Step 4, the message qubits and the check qubits are both strongly entangled. The verification process mainly is the inversion of the QSAC algorithm. The inputs to the decoding function are the received QSAC codeword as well as the shared key $K$. The following steps demonstrate the details of the verification algorithm.

Step 1: The same as the Step 1 in the encoding algorithm, the receiver first produces the sub-keys $K_Q$ and $K_T$ from $K$. Because of the pre-shared key $K$, the sender and the receiver should have the same sub-keys $K_Q$ and $K_T$.

Step 2: Similar to the Step 4 in the encoding algorithm. The receiver preforms the inversion of the encoding function to extract the message qubits as well as the check qubits from the QSAC codeword and $K$. The receiver transforms $K_T$ to a digit string $S_T$ of length $(n+m)$ and every element of $S_T$ is in $\{1, 2, 3, \ldots, m+n\}$. The index of $S_T$ represents the index of the control qubits in $|\psi\rangle_{n+m}$ and the corresponding value of the element in $S_T$ is the
index of the target qubit in $|\psi\rangle_{n+m}$. It should be noted here that in the QSAC decoding, the order to perform the CNOT operations is the reverse of that in the QSAC encoding. For the same example, if $S_T = 3412$, then the sender first performs $CNOT(|\psi\rangle^4, |\psi\rangle^2)$, then $CNOT(|\psi\rangle^3, |\psi\rangle^1)$ and so on.

| Index of $S_T$ | 4 | 3 | 2 | 1 |
|----------------|---|---|---|---|
| Value of $S_T$ | 2 | 3 | 4 | 1 |
| Operation      | $CNOT(|\psi\rangle^4, |\psi\rangle^2)$ | $CNOT(|\psi\rangle^3, |\psi\rangle^1)$ | $CNOT(|\psi\rangle^2, |\psi\rangle^4)$ | $CNOT(|\psi\rangle^1, |\psi\rangle^3)$ |

Table 2: Example of Step2 in verification

Step3 Transform $K_Q$ to a quadratic string $S_Q$ of length $n$ and every element of $S_Q$ is in $\{0, 1, 2, 3\}$. With $S_Q$, the receiver knows the original states of the check qubits the sender prepared. Consequently, the receiver measures the check qubits extracted from the QSAC codeword with the corresponding bases. That is, if the element of $S_q$ is equal to 0 or 1, then Z-basis is used; otherwise X-basis is used.

Step4 If the the measurement result of check qubits in $S_Q$ is the same as the measurement result of the check qubits recovered from the QSAC codeword, then the secret message qubits are authenticated.

2.4 Security analysis and discussions

This section analyzes the features of the proposed QSAC including: (1) the integrity of the secret message qubits, and (2) the originality of the secret message qubits.

The secret message integrity ensures that the received message qubits are not eavesdropped and modified during the time of transmission in an open quantum channel. Conversely, if the message qubits are eavesdropped or modified, then with a high probability the receiver can detect the interference. To analyze the integrity, let us denote the message qubits to be transmitted as $|M\rangle$, the check qubits created from the key, $K$, as $|C\rangle$, and the QSAC codeword state after executing the QSAC encoding algorithm as $|\psi\rangle$. Assume the QSAC codeword state, $|\psi\rangle$, was modified by a malicious user into the other state $|\psi'\rangle$. 

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Upon receiving $|\psi'|$, the receiver executes the verification function and recovers the state to $|\psi''\rangle$. If $|\psi''\rangle \neq |\psi\rangle$, then the only situation that the modification passes the verification process is: (a) a collision occurs, which means that the check qubits $|C''\rangle$ extracted from $|\psi''\rangle$ is the same as the check qubits $|C\rangle$ produced from $K$, or (b) $|C''\rangle \neq |C\rangle$, but their measurement results are the same.

Assume that the probability for the situation (a) to occur is $p_{(a)}$. Hence, the probability for the situation (b) to occur is $(1 - p_{(a)}) \times \epsilon$, where the $\epsilon$ is the probability for $|C''\rangle$ to have the same measurement result as $|C\rangle$, when $|C''\rangle \neq |C\rangle$. Therefore, the total probability for the modification to pass the verification process when $|\psi''\rangle \neq |\psi\rangle$ is:

$$p_{\text{pass}} = p_{(a)} + (1 - p_{(a)}) \times \epsilon$$

where

$$p_{(a)} = \frac{|\text{message space}|}{|\text{codeword space}|}$$

It is noted if the length of check qubits is large enough, then $p_{(a)}$ could approach to zero. The $p_{\text{pass}}$ thus is equal to $\epsilon$. Since $\epsilon$ is the probability for $|C''\rangle$ to have the same measurement result as $|C\rangle$, when $|C''\rangle \neq |C\rangle$, $\epsilon$ could be express as: $\epsilon = |\langle C | C'' \rangle|^2$. Let us assume that only one single qubits (e.g., the $i$th check qubit) was modified. As an example, if $|C\rangle^i = |0\rangle$, then $|C''\rangle^i$ could be $|1\rangle, |+\rangle$ or $|\rangle$. Similarly, if $|C\rangle^i = |1\rangle$, then $|C''\rangle^i = |0\rangle, |+\rangle, |\rangle$ and so on. In any case, $\epsilon = \frac{1}{4} \left( |\langle 0 | 1 \rangle|^2 + |\langle 0 | + \rangle|^2 + |\langle 0 | \rangle|^2 \right) = \frac{1}{4}$. Hence if $j$ check qubits are modified, then $\epsilon = \left( \frac{1}{4} \right)^j$. If the length, $n$, of check qubits is large enough, then due to the avalanche effect, $\epsilon \approx \left( \frac{1}{4} \right)^n \approx 0$.

The message originality means that the receiver can verify whether the message is sent from the same participant whom he/she claimed to be. To impersonate the sender, an eavesdropper must input a guessed key $K'$ to the QSAC encoding algorithm to generate a codeword $|\hat{\psi}\rangle$. If $K' \neq K$, then the situation for $|\hat{\psi}\rangle$ to pass the verification process is exactly the same as the one described above, whose possibility, $P_{\text{pass}}$, can be ignored.

Note that the main difference between the random sampling discussion and the QSAC is in the basic assumption for eavesdropper detection. The former assumes that the num-
ber of check qubits “altered” by an eavesdropper should be large enough, whereas the latter (QSAC) assumes that the number of check qubits “set” by the designer is large enough. Hence, for an eavesdropper to be detected with a high probability in the random sampling discussion, both the number of the check qubits as well as the number of check qubits altered by the eavesdropper should be large enough. Conversely, due to the design of an avalanche effect on the QSAC, for a higher eavesdropper detection rate, it requires only a large enough number of check qubits in the QSAC.

Furthermore, to make an eavesdropper inadvertently alter enough check qubits in the random sampling approach, the positions of those check qubits should be unknown to the eavesdropper before these check qubits are received by the receiver. Then, the sender and the receiver have to communicate back and forth in an authenticated channel later to discuss of the states, positions, and measurement results of these check qubits to judge the existence of an eavesdropper. On the other hand, in the QSAC, these overheads can be removed altogether. The communication between the sender and the receiver can be simplified to just a one-way communication.

3 Applications

To illustrate the usefulness of the proposed QSAC, a quantum secure direct communication with authentication based on the Deng’s two-step QSDC\cite{deng2006secure} is demonstrated. Instead of assuming the existence of authenticated classical channels, we assume that the sender and the receiver pre-share a secret key $K$ via some secure ways.

Step 1 The sender prepares a sequence of EPR states in $|\phi^+\rangle$. The sender encodes his/her classical secret message into a message qubits by performing one of the four operations $\{I, \sigma_x, \sigma_z, i\sigma_y\}$ on the EPR state, which represent the two-bit classical information $\{00, 01, 10, 11\}$, respectively. Subsequently, the sender encodes the message qubits to a QSAC codeword, and then sends it to the receiver.

Step 2 Upon receiving the QSAC codeword, the receiver decodes the QSAC codeword and verifies the correctness of the recovered check qubits. If the code-
word is authenticated, then the receiver performs Bell measurement on the EPR state to recover the secret message; otherwise they abort the communication.

In the original two-step protocol in [2], the EPR pairs should be transmitted separately in two steps in order to protect the security of the message. Furthermore the sender and the receiver have to perform public discussions via an authenticated classical channel to detect the existence of eavesdroppers. However, with the design of the QSAC, these can be done within one step and without any tedious public discussion to detect the existence of an eavesdropper.

4 Conclusions

Using the inherent characteristic in quantum of being very susceptible to be eavesdropped, this paper proposes a QSAC to protect quantum state sequence from being eavesdropped or modified arbitrarily. As compared to the conventionally used strategy – the random sample discussion, the QSAC provides efficiency in quantum sequence communication as well as ease in the design of quantum cryptographic protocols. The assumption of existence of authenticated classical channels in almost all existing quantum cryptographic protocols can be removed. Instead, a secret key is assumed to be shared between a sender and a receiver, which is a more common and practical assumption in modern cryptography. A QSAC-based QSDC is designed to demonstrate the usefulness of the proposed QSAC in the design of various quantum cryptographic protocols. On one hand, though the strongly self-entangled QSAC codeword makes itself very difficult to be forged or interfered without detection. On the other, it is quite susceptible to noises in a quantum channel. Therefore, how to design a QSAC, which is robust under a noisy and lossy quantum channel would be a very interesting future research.

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