Directly Measured Limit on the Interplanetary Matter Density from Pioneer 10 and 11

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Abstract

The Pioneer 10 and 11 spacecraft had exceptional deep-space navigational capabilities. The accuracies of their orbit reconstruction were limited, however, by a small, anomalous, Doppler frequency drift that can be interpreted as an acceleration of \((8.74 \pm 1.33) \times 10^{-8} \text{ cm/s}^2\) directed toward the Sun. We investigate the possibility that this anomaly could be due to a drag on the spacecraft from their passing through the interplanetary medium. Although this mechanism is an appealing one, the existing Pioneer radiometric data would require an unexpectedly high mass density of interplanetary dust for this mechanism to work. Further, the magnitude of the density would have to be nearly constant at distances \(\sim 20-70\) AU. Therefore, it appears that such an explanation is very unlikely, if not ruled out. Despite this, the measured frequency drift by itself places a directly-measured, model-independent limit of \(\lesssim 3 \times 10^{-19}\) g/cm\(^3\) on the mass density of interplanetary dust in the outer (\(\sim 20-70\) AU) solar system. Lower experimental limits can be placed if one presumes a model that varies with distance. An example is the limit \(\lesssim 6 \times 10^{-20}\) g/cm\(^3\) obtained for the model with an axially-symmetric density distribution that falls off as the inverse of the distance. We emphasize that the limits obtained are experimentally-measured, in situ limits. A mission to investigate the anomaly would be able to place a better limit on the density, or perhaps even to measure it.

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1 Introduction

Due to their long distances from the Sun, their spin-stabilized attitude control, and their long, continuous, radio-tracking Doppler data histories, very precise orbit reconstructions could be obtained for Pioneer 10 and 11. Because of this, the Pioneers were very sensitive detectors for a number of solar system effects; in fact, much more sensitive than any other spacecraft in deep space \[1, 2\]. However, despite their excellent navigational capabilities, the accuracies of the Pioneer orbit reconstructions were limited by a small anomalous, constant, one-way Doppler frequency drift of size \((5.99 \pm 0.01) \times 10^{-9} \text{ Hz/s}\). This frequency drift is clearly present in the data from both Pioneer craft.

Three separate analyses using independent orbit determination codes have confirmed the presence of this anomalous frequency drift in the radiometric Doppler data received from the Pioneer 10 and 11 spacecraft when they were at large heliocentric distances, \(\sim 20-70 \text{ AU} \[1, 2, 3\]. The detected effect can be interpreted as an acceleration \(a_P = (8.74 \pm 1.33) \times 10^{-8} \text{ cm/s}^2\) acting in the approximate direction towards the Sun \[1, 2\].

This interpretation has became known as the Pioneer anomaly.

A possible mechanism to explain the anomaly is that the craft experience a “drag” from their passing through the interplanetary medium \[2, 4, 5, 6, 7\]. Just as for a sailboat, where the relative momentum of the air to the craft’s sail determines the force, so too for a spacecraft. The relative velocity of the dust (and, with the dust’s mass, 1Depending on the particular piece of data and the fitting procedure, the exact size and formal error vary slightly. (This particular number comes from the “experimental” result for \(a_P\) determined with Pioneer 10 data as described in Section VI of Ref. \[2\] and is referenced to the downlink carrier frequency, 2.29 GHz.) But these differences are much less that the size of the anomaly and also significantly less than the systematic error.

2 The precisely analyzed Pioneer 10 data was taken between 3 January 1987 and 22 July 1998 (when the craft was 40 AU to 70.5 AU distant from the Sun) while that from Pioneer 11 was obtained between 5 January 1987 and 1 October 1990 (22.4 to 31.7 AU) \[2\]. Earlier, not thoroughly analyzed data seems to indicate the anomaly may exist as close in as 10 AU from the Sun. (See Figure 7 of \[2\].) The goal is to analyze this data in the future.

3 While the Pioneer Doppler data was being investigated in Refs. \[1, 2\], this possibility was considered. Indeed, one of the motivations to look at the data from the Galileo and Ulysses spacecraft was the possibility that their multifrequency tracking capabilities would allow any “drag” signal for the origin of the Pioneer anomaly to be seen. Unfortunately, because of individual engineering problems with these craft, the results were inconclusive. (For more details see Ref. \[2\].)
its momentum) with respect to the cross-sectional area of the spacecraft determines the force on the craft.

Recently, discoveries of many extra-solar planets with unexpected properties (such as major planets moving in orbits that come close to their parent stars [8]) suggest that the formation of planetary systems may be significantly different than previously believed. Further, infrared observations have found high dust densities around many main sequence stars [9]. (The caveat is that the highest densities tend to be around younger, brighter, and more massive stars.) Even so, and together with our relatively limited knowledge of the outer parts of our own solar system, these discoveries reopen the question of whether there exists as yet undiscovered interplanetary dust in this distant region.

This raises the possibility that momentum transfer between the dust distribution and the moving Pioneers could, in principle, provide enough power to slow the spacecraft down at a nearly constant rate. Here we investigate if a drag force could indeed be the origin of the Pioneer anomaly. In Section 2 we explain the physics of the drag mechanism and review our knowledge of the interplanetary medium in Section 3. We find, in Section 4 that an unexpectedly large amount of interplanetary dust would be needed to cause the anomaly. Even so, our result yields a new in situ, experimental, model-independent limit on the mass density of the medium, since this (expected to be lower) density has not been measured, only modeled. We give our conclusions and an overview of future work in Section 5.

2 The Pioneer anomaly as a drag force

To illuminate how a drag force would yield an acceleration of the Pioneers towards the Sun, consider the dynamics of the situation. The Pioneers are on hyperbolic orbits, roughly in the plane of the ecliptic and parallel to the Sun’s velocity vector in the galaxy. Pioneers 10 and 11 are moving in opposite directions with respect to the Sun, with Pioneer 10’s velocity being opposite to the Sun’s velocity vector. That is, even though
they are traveling away from the Sun on opposite sides of the solar system (see Figure 1), to within the errors the anomalous accelerations of the two Pioneer craft are equal and are both directed towards the Sun.

Figure 1: Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager trajectories. Pioneer 11 is traveling approximately in the direction of the Sun’s orbital motion about the galactic center. The galactic center is approximately in the direction of the top of the figure.

The Pioneers are both moving at about 12 km/s (∼ 2.5 AU/yr) relative to the Sun. Simultaneously, the local galactic rotation velocity is about 220 km/s with respect to the galactic origin and (as we return to below) the Sun is traveling at about 26 km/s relative to the local interstellar medium [10, 11]. Therefore, if the anomaly is due to a drag force, the medium that is causing the drag must be, on average, locally “radially at rest” (no relative radial momentum) about the Sun; for example as a sphere or a disk. This is true whether the medium is composed of normal matter or some unknown “dark matter” [5, 6, 12, 13].

4Here we focus on ordinary matter as the possible origin of spacecraft drag. A dark matter hypothesis is discussed in Refs. [5, 6].
3  The interplanetary medium

The interplanetary medium is known to contain thinly scattered matter in the form of neutral hydrogen, microscopic dust particles, and the hot solar-wind plasma of electrically charged particles (mainly protons and electrons). But the exact composition has long been debated, with many models put forward to describe the medium’s nature and origin. As a result, limits on gas [14, 15] and dust [14, 15, 16] in the deep-space interplanetary medium are not precise, but the amount of gas is well known to be much less than the amount of dust.\(^5\)

As for dust, most of it should be in orbit about the Sun, even though some of the dust originates from the interstellar regime [17]-[20]. Interstellar dust is distinguished from interplanetary dust in situ by its greater impact velocity on deep-space probes. Starting with the Ulysses instruments, it has been measured. It is found in [18] that its total density is only

\[ \rho_{\text{ISD}} \lesssim 3 \times 10^{-26} \text{g/cm}^3. \]  (1)

This careful determination, which we take as an upper bound, is compared to others in Table I of Ref. [18].\(^6\) But in any event, and as will become clear below, the amount and relative velocity of the interstellar dust is much too small to have been seen as a drag on the Pioneers.

The orbiting dust is, for us, mainly in the “Kuiper belt,” a disk-shaped region extending roughly from the orbits of Saturn and Neptune, \(\sim 10-30 \text{ AU} \), out to about \(\sim 80-120 \text{ AU} \) from the Sun. It contains dust and many small icy bodies.\(^7\) The dust will have a variety of eccentricities and inclinations, but for our simple purposes we can average out

\(^5\)The gas is believed to come mainly from the interstellar medium as the Sun revolves around the galaxy [15]. It then has a velocity relative to the solar system of about 26 km/s [10, 14]. The gas drag velocity on a spacecraft is thus the vector sum of the craft’s velocity and this 26 km/s. The constant density of the gas is roughly equal to that of the solar wind at 20 AU, so only perhaps a few hydrogen atoms per 100 cm\(^3\) [15].

\(^6\)These other determinations are up to as much as an order of magnitude smaller.

\(^7\)The Kuiper belt is now considered to be the source of short-period comets whereas long-period comets are believed to be formed further away in the Oort cloud [21, 24].
these different drag components into their circular velocity and have the drag velocity effectively be the radial velocity of the craft.\textsuperscript{8}

There currently is much effort devoted to understanding this region\cite{16,20,25,29}. In particular, the study of the trans-Neptunian asteroids is a rapidly evolving field of research, with major observational and theoretical advances in the last few years\cite{20}.

The real problem is that, although measurements can determine the amount of interstellar dust, the same is not true for the interplanetary dust, which has a much lower relative velocity with respect to deep-space craft. Further, starting with the Pioneers, the instruments that have been sent on missions to deep space have been sensitive only to varying-sized particles.\textsuperscript{9} Instruments used have been combinations of mass spectrometers, dust impact detectors, plasma instruments, energetic particle analyzers, and magnetometers. But even at their best the sensitivities of all the instruments on the deep-space craft so far launched have not been sufficient to detect all the individual effects of all the various mass and energy dust particles.

The net result is that we are dependent on models for the interplanetary dust density, and these models vary greatly. This is especially true for estimates of dust production in the Kuiper belt, which can vary by orders of magnitude\cite{20}. A further complication is that the orbits of dust grains of different sizes will be most significantly influenced by different forces: gravitational, Poynting-Robertson drag, solar wind drag, and electromagnetic. This makes understanding the \textit{total} mass density even harder.

A consensus view is that the \textit{average} interplanetary dust density may be almost two orders of magnitude larger (a factor of order 30) than the interstellar dust density, and probably more\cite{18,28}. Therefore, we can give a secure limit on the interplanetary dust density.

\textsuperscript{8} There is still a sideways drag caused by the average circular motion of the dust. But the side drag velocity is down from the radial velocity. It is lower by a factor $\sqrt{2}$ for a parabolic orbit and down even more for escape hyperbolic orbits.

\textsuperscript{9} For example, the Pioneer 10 and 11 impact detectors were sensitive to particles of masses $>8 \times 10^{-10}$ g and $>6 \times 10^{-9}$ g, respectively, at impact speeds of 20 km/s. The Voyagers had plasma wave instruments that responded to impacts, but which were not calibrated for dust. (See, e.g., Ref. \cite{30}.)
density of
\[ \rho_{\text{IPD}} \gtrsim 10^{-24} \text{g/cm}^3. \]  

The lower bound \( \rho_{\text{IPD}} \) would yield \( \sim 10^{21} \) g in a 100 AU disk. This is not unreasonable since the younger, larger, Vega star is thought to have only approximately 8000 times this amount of dust in its disk \[9\].

However, because of their relative velocities, all the particles discussed above will, as a matter of principle, produce a microscopic effective drag force on a passing spacecraft. Therefore, even though dedicated instruments may be optimized to yield the number density for particles of a certain mass, size, or kinetic energy, the drag on a large-area, low-mass spacecraft provides a way to ask about the total mass density distribution in g/cm\(^3\).

As we now come to, this fact allows \emph{in situ, directly-measured} limits to be placed on the amount of deep-space interplanetary matter by using the Pioneer anomaly to bound overall the interplanetary mass density.

4 Interplanetary density limits from the Pioneers

Drag by the interplanetary medium on a spacecraft causes a deceleration of
\[ a_s(r) = -K_s \frac{\rho(r) v_s^2(r) A_s}{m_s}, \]  
where \( \rho(r) \) is the density of the interplanetary medium, \( K_s \) is the effective reflection/absorption/transmission coefficient of the craft for the particles hitting it, \( v_s(r) \) is the effective relative velocity of the craft with respect to the medium, \( A_s \) is the effective cross-sectional area of the craft, and \( m_s \) is its mass.

In general \( K_s \) is between 0 and 2.\(^{10}\) Here we take \( K_s \) to be a unit constant and the drag velocity to be \( v_s \sim 12 \text{ km/s} \), the radial velocity of the Pioneers.\(^{11}\) We can consider

\(^{10}\) \( K_s \) depends on the sizes and types of the particles and especially on whether the particles are reflected \((K_s = 2)\), absorbed \((= 1)\), or transmitted \((= 0)\) by the spacecraft.

\(^{11}\) The precise hyperbolic velocities of Pioneer 10 and 11 are about 12.2 and 11.6 km/s, respectively.
the effective area to be that of the Pioneers’ antennae (radii of 1.37 m) and the mass (with half the fuel gone) to be 241 kg \(^2\), or \((A/m)_p = 0.245 \text{ cm}^2/\text{g}\). Given this, the critical unknown is \(\rho(r)\).

Below we will be considering densities of the form

\[
\rho_n(r) = \rho_{n0} \left(\frac{r_0}{r}\right)^n,
\]

where the \(\rho_{n0}\) and \(r_0\) are constants, with \(r_0\) set to be at the beginning of the Pioneer data interval, 20 AU.

### 4.1 Uniform density

By assuming that the Pioneers’ entire anomalous acceleration is due to a drag force, we can calculate that, at distances from 20 to 70 AU from the Sun, an axially-symmetric dust distribution with a constant, uniform density

\[
\rho_P(r) \lesssim \rho_{00} = 3 \times 10^{-19} \text{ g/cm}^3
\]

could have produced the constant anomaly. Eq. (5) places an in situ, experimental limit on the density of the interplanetary medium, even though it is larger by a factor of 300,000 than \(\rho_{IPD}\) of Eq. (2), the latter a number more like those usually thought of for deep space. Indeed, the limit of Eq. (5) corresponds to about 200,000 atomic-masses/cm\(^3\).

Ruling against this upper bound being, in fact, a measure of the density is that a drag acceleration from Kuiper belt dust should not have been constant across the Pioneer data range. Not only should there be boundaries from bands of dust, but concentrations of “Kuiper-Belt Objects” (KBO) at 39.4 AU and 47.8 AU, corresponding to Neptune resonances of 3:2 and 2:1 \(^2\;\text{32}\;\text{33}\), have been discovered. The KBO concentrations will at

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\(^2\)Extending the observation in footnote \(\boxed{\text{8}}\), the velocity of the Pioneers are 2 times larger than the orbital velocity at 25 AU, meaning a factor of 4 larger velocity effect for the radial vs. orbital drag. Further, the effective area of the side of the Pioneer main bus and antenna is a factor \(\sim 3\) smaller than the face of the Pioneer antenna. Therefore, we can ignore the side drag in this simple calculation.
least affect dust creation from collisions. But questions remain on exactly how the steady-state mass density of the dust will be affected both by the concentrations themselves and also by the resonances that created them. These density spatial variations are widely discussed in the literature [14, 15, 18, 20, 25, 26, 28, 29].

Therefore, a drag should have shown an increasing effect as the spacecraft approached into belts or concentrations and a decreasing effect as it receded from belts and concentrations, even with an approximate uniform density within the overall belt. However, independent of the lack of a spatial variation in the anomaly, which implies any actual average density that exists is lower than $\rho_P$, Eq. (5) remains a model-independent, in situ, experimental upper bound for the interplanetary density.

4.2 Density varying as $1/r$

Even lower bounds can be placed if one presumes a model density that varies with distance. For example, consider a density that varies as

$$\rho_{1/r}(r) \sim \rho_{01} \left( \frac{T_0}{r} \right).$$

(6)

Conservatively it is limited by the lack of variation seen in the anomaly. The main part of the anomaly then must be due to another origin, but the size of the anomaly’s total error, $\sigma_p = 1.33 \times 10^{-8}$ cm/s$^2$, places a limit on how much matter there is. Between 20 and 70 AU the $1/r$ fall off of the density would imply a change in the acceleration of $\sigma_p$.

This yields a value for $\rho_{01}$ and hence the result

$$\rho_{1/r}(r) \lesssim 6 \times 10^{-20} \left( \frac{20 \text{ AU}}{r} \right) \text{g/cm}^3, \quad 20 \text{ AU} \leq r \leq 70 \text{ AU}.$$  

(7)

13 A related consideration is if the mass in the Kuiper belt could produce a gravitational acceleration that causes the Pioneer anomaly. In Section VII.E and Figure 15 of [2] it is shown that even generous ($\sim 5 \times 10^{-18}$ g/cm$^3$), although reasonable, models of Kuiper belt densities can not produce the Pioneer anomaly by three and more orders of magnitude.

14 One could also argue that what one is seeing in the anomaly is evidence for dark matter causing a drag [5, 6]. But even then one has to explain why this matter is a constant density in the regime penetrated by the Pioneers.

15 The total error is dominated by systematics, many of which are constant or nearly so [2]. Therefore, one could argue that the numbers in Eqs. (7) and (9) below can be reduced accordingly.
4.3 Isothermal density

By the same argument as above, an isothermal density varying as 

\[ \rho_{\text{isoth}}(r) \sim \rho_0 \left( \frac{r_0}{r} \right)^2 \]  

(8)

yields a limit

\[ \rho_{\text{isoth}}(r) \lesssim 5 \times 10^{-20} \left( \frac{20 \text{ AU}}{r} \right)^2 \text{ g/cm}^3, \quad 20 \text{ AU} \leq r \leq 70 \text{ AU}. \]  

(9)

However, there is a caveat with this model. It proposes that \( \sigma_p \) provides a bound on the size of the variation of the unmodeled non-gravitational drag force between 20 and 70 AU. At 20 AU it is of size \( 1.33 \times 10^{-8} \text{ cm/s}^2 \) and it falls off as the square of the distance from there. But at 20 AU there is another non-gravitational force that falls off as the square of the distance, is of similar size (\( \sim 5 \times 10^{-8} \text{ cm/s}^2 \)), but of opposite sign. It is produced by the solar radiation pressure from the Sun on the spacecraft [2]. Therefore, to distinguish this type of drag force from radiation pressure would entail extremely precise modeling and orbit determination.

Also, note that the final two densities above, \( \rho_{1/2}(r) \) and \( \rho_{\text{isoth}}(r) \), must cut off closer in to the Sun. Otherwise they would produce too large a drag.

In Figure 2 we show the above three bounds, \( \rho_P(r) \), \( \rho_{1/2}(r) \), and \( \rho_{\text{isoth}}(r) \), as well as the estimates for interstellar and interplanetary dust, \( \rho_{\text{ISD}} \) and \( \rho_{\text{IPD}} \), quoted in Eqs. (1)-(2).

5 Conclusions and future considerations

In this paper we analyzed the possibility that the Pioneer anomaly is the result of a drag force from dust distributed in the outer solar system. Our analysis showed that for this mechanism to work, one would need the presence of dust with a density on the order of \( \sim(5 - 30) \times 10^{-20} \text{ g/cm}^3 \), in the region 20 to 70 AU from the Sun. This is unexpectedly high. Our present knowledge of dust formation processes in the outer regions of the solar system implies that such a high density is not realistic. Even so, the accuracy of
Figure 2: Plots of the log-to-the-base-10 of density (in g/cm$^3$) vs distance (in AU), for (from top to bottom) the uniform density limit, $\rho_P(r)$, the $1/r$ model density limit, $\rho_{1/r}(r)$, the isothermal model density limit, $\rho_{\text{isoth}}(r)$, the interplanetary dust model estimate, $\rho_{\text{IPD}}$, and the interstellar dust estimation, $\rho_{\text{ISD}}$.

the Pioneer orbit reconstruction allowed us to place both model-independent and model-dependent, *in-situ*, experimental limits on the mass density of dust in the outer solar system.

The results presented in this paper can also be used to help further motivate a mission to explore the origins and evolution of our solar system. Of course, among the objectives of such a mission would be to precisely map the gas and dust distributions in the solar system at various heliocentric distances and latitudes. Gas and dust detectors are typically among the standard set of instruments in deep-space missions, whatever their objectives.

However, we emphasize that any low-mass, deep-space mission with precise radio-science experiments, like a mission that would attempt to explore the Pioneer anomaly
should be prepared to ascertain if any observed effect is due to a drag force, thereby providing an independent limit (or even measurement) of the matter density, especially in the outer regions of the solar system.

Eq. (3) shows that the drag acceleration scales as \( (A_s \frac{v^2_s}{m_s}) \). Comparing this quantity for the Pioneers to that for any other spacecraft quickly yields a figure of merit on the ability of this other spacecraft to obtain a better limit on the interplanetary density (or possibly even a measurement of it). Another spacecraft could also look for an indication of a drag force by using three-dimensional navigation to determine if any anomalous force were directed along the velocity vector of the craft instead of along the vectors towards the Sun, the Earth of along the spin axis. (This point is explained in detail in [7].)

As an example of such a mission, consider a spacecraft design whose architecture is symmetric in the forward and backward directions, having two oppositely facing antennae \([7, 38]\). Its cross section and area are similar to the Pioneers, but its measured effect would be boosted by the square of its (presumed) larger velocity.

A second example is a formation flying concept \([36, 38]\), where the position of a 5 cm radius ball covered with corner-cubes weighing 5 kg is tracked from a mother ship (which in turn is tracked from Earth). The (area/mass) of this concept is 16 times smaller than that of the Pioneers. Therefore, to measure the same drag acceleration the formation flying mission would need to be traveling 4 times faster than the Pioneers.\(^{16}\)

Both of the above concepts are designed to reduce systematic errors by two to three orders of magnitude. This would allow even better limits to be placed on \(r\)-dependent densities. Further, the same idea could be used on a solar-sail mission to deep space if the sail were not jettisoned past Jupiter’s orbit \([31]\), as is usually conceived of.

\(^{16}\)The mother ship (satellite) would probably have an \((A_s/m_s)\) similar to that of the Pioneers. But in the current conception \([36, 38]\) the main point is to eliminate systematics from the sub-satellite ball, the satellite itself not being optimized to have low systematics. This means the two craft would drift apart. Even for a difference in acceleration between the two craft of only \(a_P\), in one month they would separate by 3 km, this distance increasing quadratically with time after that. To maintain laser tracking of the subsatellite, the distance change would periodically have to be compensated for by thrust maneuvers on the mother satellite.
Lastly, if one assumes a drag force is causing the Pioneer anomaly, then a test should also try to determine if the matter starts at about 10 AU.\textsuperscript{17} This is where the early, not yet precisely analyzed, Pioneer 11 data seems to show the anomaly “turning on” as the craft passed by Saturn, turned radially outward, and reached its hyperbolic escape velocity of 11.6 km/s \cite{1,2,10,41}.

The exact architecture for a mission to explore the Pioneer anomaly, including the spacecraft and mission designs, the set of critical instruments, and the launch options, is currently being investigated \cite{7,34-38}. We await progress on this front.

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