New Physics contribution to $B \to \rho K, \pi K^*$ decays

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Abstract. In this paper we discuss supersymmetric contributions to the direct CP asymmetries of $B \to \pi K^*$ and $B \to \rho K$ decays. We use Soft Collinear Effective Theory as a framework for our study. We show that within the standard model and including the next leading order QCD corrections, the predicted CP asymmetries can not accommodate the experimental measurements. We show that non-minimal flavor SUSY contributions mediated by gluino exchange can enhance the CP asymmetries significantly to accommodate the experimental measurements.

1. Introduction
The decay modes $B \to \pi K^*$ and $B \to \rho K$ are generated at the quark level in the same way as $B \to K\pi$ decay. As a consequence, studying these decays can shed light of possible new physics that can solve the $K\pi$ puzzle. These decay modes are studied within SM in framework of QCDF [1], PQCD [2] and Soft Collinear Effective Theory (SCET) [6, 7]. In SCET, the direct CP asymmetries of $B^- \to \pi^- K^{*0}$ and $B^- \to \rho^- K^0$ are zero while the CP asymmetries in other channels are small. Recently, in Ref.[8] fits to $B \to \pi K^*$ and $B \to \rho K$ decays are performed where data can be accommodated within the standard model due principally to the large experimental uncertainties, particularly in the CP-violating asymmetries.

Supersymmetry (SUSY) is one of the most interesting candidates for physics beyond the standard model as it naturally solves the hierarchy problem. In addition, SUSY has new sources for CP violation which can account for the baryon number asymmetry and affect other CP violating observables in the B and K decays. The effects of these phases on the CP asymmetries in semi-leptonic $\tau$ decays has been studied in Refs.[9, 10, 11].
Table 1. Branching ratios in units $10^{-6}$ of $B \to \pi K^*$ and $B \to \rho K$ decays. The first uncertainty in the predictions is due to the uncertainties in SCET parameters while the second uncertainty is due to the uncertainties in the CKM matrix elements.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Decay channel & Exp. & SM prediction \\
\hline
$\pi^0 K^{(*)+}$ & 6.9 ± 2.3 & $7.2^{+0.34+1.1}_{-0.2-0.9}$ \\
$\pi^- K^{(*)+}$ & 8.6 ± 0.9 & $7.8^{+0.2+1.1}_{-0.2-1.0}$ \\
$\pi^0 K^{(*)0}$ & 2.4 ± 0.7 & $7.8^{+0.5+1.2}_{-0.5-1.0}$ \\
$\pi^+ K^{(*)0}$ & $9.9^{+0.8}_{-0.9}+0.3$ & $10.3^{+0.7+1.4}_{-0.7-1.4}$ \\
$\rho^0 K^+$ & $3.81^{+0.34}_{-0.36}+0.36$ & $4.8^{+0.6+0.8}_{-0.6-0.7}$ \\
$\rho^+ K^+$ & $8.0^{+1.5}_{-1.4}$ & $10.9^{+0.6+1.7}_{-0.6-1.5}$ \\
$\rho^0 K^0$ & $4.7 ± 0.7$ & $10.2^{+0.6+1.6}_{-0.6-1.4}$ \\
$\rho^- K^+$ & $8.6^{+0.9}_{-1.1}$ & $2.6^{+0.5+0.3}_{-0.4-0.4}$ \\
\hline
\end{tabular}
\end{table}

2. $B \to \pi K^*$ and $B \to \rho K$ decays within Standard Model

Within the framework of the standard model, the branching ratios of $B \to \pi K^*$ and $B \to \rho K$ are in agreements with their corresponding experimental values in most of the decay modes as given in Table 1. In that table, the first uncertainty in the predictions is due to the uncertainties in SCET parameters while the second uncertainty is due to the uncertainties in the CKM matrix elements. On the other hand, the SM predictions for the CP asymmetries of $B \to \pi K^*$ and $B \to \rho K$ are presented in Table 2. Clearly from that Table, the CP asymmetry of $B^+ \to \pi^0 K^*$ has different sign in comparison with the experimental measurement. Moreover, the predicted CP asymmetries in many of the decay modes are in agreement with the experimental measurements due to the large errors in these measurements. We see also from the Table that, the predicted CP asymmetry of $B \to \pi^0 K^*$ and $B^+ \to \rho^0 K^+$ disagree with the experimental results within $1\sigma$ error of the experimental data. This can be attributed to the lack of the weak CP violating phases as SM Wilson coefficients are real and the only source of the weak phase is the phase of the CKM matrix.

3. $B \to \rho K$ and $B \to \pi K^*$ CP asymmetries including Supersymmetry

In this section we analyze the SUSY contributions to the CP asymmetries of $B^- \to \pi^- K^{(*)0}$, $B^- \to \rho^- K^0$, $\bar{B}^0 \to \rho^+ K^-$ and $B^- \to \rho^0 K^-$ as their SM predictions are very small as have been shown in the previous section. In SUSY, Flavor Changing Neutral Current (FCNC) and CP quantities are sensitive to particular entries in the mass matrices of the scalar fermions. Thus it is useful to adopt a model independent- parametrization, the so-called Mass Insertion Approximation (MIA) where all the couplings of fermions and sfermions to neutral gauginos are flavor diagonal [17].
Applying $b$ to have $(B \bar{A})(\bar{A} \pi)$ we find [7] expressions for the gluino and chargino contributions to the Wilson coefficients can be found in due to the uncertainties in the CKM matrix elements.

The predictions is due to the uncertainties in SCET parameters while the second uncertainty is due to the uncertainties in the CKM matrix elements.

At next leading order in $\alpha_s$ expansion, the dominant SUSY contributions to our decay modes are originated from diagrams mediated by the exchange of gluino and chargino. The complete expressions for the gluino and chargino contributions to the Wilson coefficients can be found in Refs. [18] [19] [20] [21].

After including SUSY contributions to the mentioned decays and keeping the dominant terms we find [7]

\[
\begin{align*}
A(B^- \to \pi^- K^{(*)0}) \times 10^7 &\approx -0.0178(\delta_{LL}^d)_{23} - 6.6914(\delta_{LR}^d)_{23} - 1.5857(\delta_{RL}^d)_{23} - (0.0052 + 0.0003i)(\delta_{LR}^u)_{32} \\
&- (0.0046 - 0.0003i)(\delta_{RL}^u)_{32} + (0.3319 - 0.0612i), \\
A(B^- \to \pi^0 K^{(*)-}) \times 10^7 &\approx 0.0125(\delta_{LL}^d)_{23} + 4.7315(\delta_{LR}^d)_{23} + 1.1212(\delta_{RL}^d)_{23} + (0.0056 - 0.0001i)(\delta_{LR}^u)_{32} \\
&- (0.0223 - 0.0001i)(\delta_{RL}^u)_{32} + (0.2508 - 0.1259i), \\
A(B^0 \to \pi^0 K^{(*)0}) \times 10^7 &\approx -0.0127(\delta_{LL}^d)_{23} - 4.7315(\delta_{LR}^d)_{23} - 1.1212(\delta_{RL}^d)_{23} + (0.0094 + 0.0001i)(\delta_{LR}^u)_{32} \\
&- (0.0185 + 0.0001i)(\delta_{RL}^u)_{32} + (0.2949 - 0.0707i), \\
A(B^0 \to \pi^+ K^{(*)-}) \times 10^7 &\approx 0.0178(\delta_{LL}^d)_{23} + 6.6914(\delta_{LR}^d)_{23} + 1.5857(\delta_{RL}^d)_{23} - (0.0106 + 0.0005i)(\delta_{LR}^u)_{32} \\
&- (0.0099 - 0.0005i)(\delta_{RL}^u)_{32} + (0.2695 - 0.1392i), \\
A(\bar{B}^- \to \rho^- K^0) \times 10^7 &\approx 0.0043(\delta_{LL}^d)_{23} + 1.6190(\delta_{LR}^d)_{23} - 1.0851(\delta_{RL}^d)_{23} - (0.0001 + 0.0005i)(\delta_{LR}^u)_{32} \\
&- (0.0021 - 0.0005i)(\delta_{RL}^u)_{32} - (0.3473 + 0.0111i), \\
A(B^- \to \rho^0 K^-) \times 10^7 &\approx -0.0031(\delta_{LL}^d)_{23} - 1.1448(\delta_{LR}^d)_{23} + 0.7673(\delta_{RL}^d)_{23} - (0.0037 + 0.0006i)(\delta_{LR}^u)_{32} \\
&- (0.0120 - 0.0006i)(\delta_{RL}^u)_{32} - (0.2232 + 0.0501i), \\
A(\bar{B}^0 \to \rho^0 K^-) \times 10^7 &\approx 0.0030(\delta_{LL}^d)_{23} + 1.1448(\delta_{LR}^d)_{23} - 0.7673(\delta_{RL}^d)_{23} - (0.0032 + 0.0003i)(\delta_{LR}^u)_{32} \\
&- (0.0108 - 0.0003i)(\delta_{RL}^u)_{32} - (0.3470 + 0.0307i), \\
A(B^- \to \rho^+ K^0) \times 10^7 &\approx -0.0043(\delta_{LL}^d)_{23} - 1.6190(\delta_{LR}^d)_{23} + 1.0851(\delta_{RL}^d)_{23} \\
&- (0.0008 + 0.0010i)(\delta_{LR}^u)_{32} - (0.0037 - 0.0010i)(\delta_{RL}^u)_{32} \\
&- (0.1723 + 0.0386i), \\
\end{align*}
\]

The mass insertions $(\delta_{RL}^u)_{32}$ and $(\delta_{RL}^u)_{32}$ are not constrained by $b \to s\gamma$ and so we can set them as $(\delta_{RL}^u)_{32} = (\delta_{RL}^u)_{32} = e^{i\delta_u}$ where $\delta_u$ is the phase that can vary from $-\pi$ to $\pi$. It should be noted that in order to have a well defined Mass Insertion Approximation scheme, it is necessary to have $|(\delta_{AB}^{LL})_{ab}| < 1$ but here in order to maximize the SUSY CP-violating contributions we take it of order one. Applying $b \to s\gamma$ constraints leads to the following parametrization [22].
Figure 1. CP asymmetries versus the phase of the $(\delta d_{AB})_{23}$ where A and B denote the chirality i.e. L, R. for 3 different mass insertions. The left diagram corresponds to $A_{CP}(B^+ \rightarrow \pi^+ K^{0*})$ while the right diagram corresponds to $A_{CP}(B^+ \rightarrow \pi^0 K^{*+})$. In both diagrams we take only one mass insertion per time and vary the phase of from $-\pi$ to $\pi$. The horizontal lines in both diagrams represent the experimental measurement to $1\sigma$.

$$(\delta_{LL}^d)_{23} = e^{i\delta_d} \quad (\delta_{LR}^d)_{23} = (\delta_{RL}^d)_{23} = 0.01 e^{i\delta}$$

In our analysis we consider two scenarios, the first one with a single mass insertion where we keep only one mass insertion per time and take the other mass insertions to be zero and the second scenario with two mass insertions will be considered only in the cases when one single mass insertion is not sufficient to accommodate the experimental measurement. After setting the different mass insertions as mentioned above, we see from Eq. (1) that, the terms that contain the mass insertions $(\delta_{RL}^u)_{32}$ and $(\delta_{LR}^u)_{32}$ will be small in comparison with the other terms and thus we expect that their contributions to the asymmetries will be small. These terms are obtained from diagrams mediated by the chargino exchange and thus we see that gluino contributions give the dominant contributions as known in the literature.

We start our analysis of the direct CP asymmetries by considering the first scenario in which we take only one mass insertion corresponding to the gluino mediation and set the others to be zero.

After substituting the mass insertions given in eq. (2) in eq. (1) we find that the first and third terms in the amplitudes $B^+ \rightarrow \pi^+ K^{*0}$ and $B^+ \rightarrow \pi^0 K^{*+}$ will be approximately equal and both of them will be smaller than the second term. As a consequence, one predicts that the asymmetries generated by the mass insertions $(\delta_{LL}^d)_{23}$ and $(\delta_{RL}^d)_{23}$ will be equal and in the same time these asymmetries will be smaller than the case of using $(\delta_{LR}^d)_{23}$ which can be seen from Fig. 1. In that Figure, we plot the CP asymmetries, $A_{CP}(B^+ \rightarrow \pi^+ K^{*0})$ and $A_{CP}(B^+ \rightarrow \pi^0 K^{*+})$ versus the phase of the $(\delta_{AB}^d)_{32}$ where A and B denote the chirality i.e. L and R. for 3 different mass insertions. The horizontal lines in both diagrams represent the experimental measurements to $1\sigma$. As can be seen from Fig. 1 left, for all gluino mass insertions, the value of the CP asymmetry $A_{CP}(B^+ \rightarrow \pi^+ K^{*0})$ is enhanced to accommodate the experimental measurement of the asymmetry within $1\sigma$ for many values of the phase of the mass insertions. On the other hand, Fig. 1 right shows that the CP asymmetry $A_{CP}(B^+ \rightarrow \pi^0 K^{*+})$
Figure 2. CP asymmetries versus the phase of the $(\delta_{AB}^d)_{23}$ where A and B denote the chirality i.e. L, R, for 3 different mass insertions. The left diagram corresponds to $A_{CP}(B^0 \rightarrow \pi^0 K^*0)$ while the right diagram corresponds to $A_{CP}(B^0 \rightarrow \pi^- K^*+)$). In both diagrams we take only one mass insertion per time and vary the phase of from $-\pi$ to $\pi$. The horizontal lines in both diagrams represent the experimental measurement to $1\sigma$.

Figure 3. CP asymmetries versus the phase of the $(\delta_{AB}^d)_{23}$ where A and B denote the chirality i.e. L, R, for 3 different mass insertions. The left diagram corresponds to $A_{CP}(B^+ \rightarrow \rho^0 K^0)$ while the right diagram corresponds to $A_{CP}(B^+ \rightarrow \rho^0 K^+)$). In both diagrams we take only one mass insertion per time and vary the phase of from $-\pi$ to $\pi$. 
is enhanced to accommodate the experimental measurement within 1σ for all values of the phase of the mass insertions. The point we stress here is that SUSY Wilson coefficients provide source of large weak phases, which are needed for accommodation of CP asymmetries.

In Fig.1 we plot the two asymmetries, $A_{CP}(B^0 \to \pi^0 K^+ 0)$ and $A_{CP}(B^0 \to \pi^- K^{*+})$ versus the phase of the $(\delta_{AB}^{d})_{32}$ as before. As can be seen from Fig.2 left, $A_{CP}(B^0 \to \pi^0 K^+ 0)$ lies within 1σ range of its experimental value for many values of the phase of the mass insertion $(\delta_{LR}^{d})_{23}$ only. The reason for that is as before, (see eq.(1)) the two mass insertions $(\delta_{LL}^{d})_{23}$ and $(\delta_{RL}^{d})_{23}$ will give equal contributions to the CP asymmetries which will be smaller than the case of using $(\delta_{LR}^{d})_{23}$. On the other hand, Fig.2 right, we see that $A_{CP}(B^0 \to \pi^- K^{*+})$ can be accommodated within 1σ for many values of the phase of the three gluino mass insertions.

Finally we discuss the CP asymmetries of the decay modes $B^+ \to \rho^+ K^0$ and $B^+ \to \rho^0 K^+$. After substituting the mass insertions given in eq. (2) in eq. (1), we find that the first and third terms in the amplitudes $B^+ \to \rho^+ K^0$ and $B^+ \to \rho^0 K^+$ will be no longer equal as previous cases and thus we expect their contributions to the asymmetries will be different which can be seen from Fig.3 where, as before, we plot $A_{CP}(B^+ \to \rho^+ K^0)$ and $A_{CP}(B^+ \to \rho^0 K^+)$ versus the phase of the $(\delta_{AR}^{d})_{23}$. In Fig.4 we do not show the horizontal lines representing the 1σ range of the experimental measurement as the three curves of the $A_{CP}(B^+ \to \rho^+ K^0)$ corresponding to the three gluino mass insertions totally lie in this 1σ range for all values of the phase of the mass insertions. On the other hand, Fig.3 right, we see that $A_{CP}(B^+ \to \rho^0 K^+)$ can not be accommodated within 1σ for any value of the phase of all gluino mass insertions. This motivates us to consider the second scenario with two mass insertions.

In Fig.4 we plot the CP asymmetry, $A_{CP}(B^+ \to \rho^0 K^+)$ versus the phase of the mass insertion for 2 different mass insertions. The left diagram correspond to gluino contributions where we keep the two mass insertions $(\delta_{LL}^{d})_{23}$ and $(\delta_{RL}^{d})_{23}$ and set the other mass insertions to zero. The right diagram correspond to both gluino and chargino contributions where we keep the two mass insertions $(\delta_{LL}^{d})_{23}$ and $(\delta_{RL}^{d})_{32}$ and set the other mass insertions to zero. In both diagrams we assume that the two mass insertion have equal phases and we vary the phase from $-\pi$ to $\pi$. The horizontal lines in both diagrams represent the experimental measurements to 1σ.

![Figure 4](image-url)
The right diagram correspond to both gluino and chargino contributions where we keep the two mass insertions \((\delta d_{LR})_{23}\) and \((\delta d_{RL})_{32}\) and set the other mass insertions to zero. In both diagrams we assume that the two mass insertion have equal phases and we vary the phase from \(-\pi\) to \(\pi\). As before, the horizontal lines in both diagrams represent the experimental measurement to 1\(\sigma\). As can be seen from Fig.4 left, two gluino mass insertions can not accommodate the experimental measurement for any value of the phase of the mass insertion. On the other hand from Fig.4 right, two mass insertions one corresponding to chargino contribution and the other corresponding to gluino contribution can not accommodate the experimental measurements. We find that in order to accommodate the CP symmetry in this case the Wilson coefficient \(\tilde{C}_9\) should be increased at least by a factor \(-6\pi/\alpha\) without violating any constraints on the SUSY parameter space. We show the corresponding diagram in Fig.5.

![Diagram](image)

**Figure 5.** CP asymmetry of \(A_{CP}(B^+ \rightarrow \rho^0K^+)\) versus the phase of the mass insertion for 2 different mass insertions correspond to gluino contributions where we keep the two mass insertions \((\delta d_{LR})_{23}\) and \((\delta d_{RL})_{32}\) and set the other mass insertions to zero. We assume that the two mass insertion have equal phases and we vary the phase from \(-\pi\) to \(\pi\). The horizontal lines in the diagram represent the experimental measurements to 1\(\sigma\).

4. Conclusion
In this paper we studied the decay modes \(B \rightarrow \pi K^*\) and \(B \rightarrow \rho K\) within the framework of Soft Collinear Effective Theory. We have shown that within the standard model, including the next leading order QCD corrections, their direct CP asymmetries can not accommodate the experimental measurements. We have discussed also SUSY contributions to the direct CP asymmetries using the Mass Insertion Approximation. In contrast to SM, we found that direct CP asymmetries can be significantly enhanced by SUSY contributions mediated by gluino exchange and accommodate the experimental measurements.

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