Universal Scaling in the Dynamical Conductivity of Heavy Fermion Ce and Yb Compounds

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Dynamical conductivity spectra $[\sigma(\omega)]$ have been measured for many heavy-fermion (HF) Ce and Yb compounds. A characteristic excitation peak has been observed in the infrared region of $\sigma(\omega)$ for all the compounds, and has been analyzed in terms of a simple model based on conduction (c)-f electron hybridized band. A universal scaling is found between the observed peak energies and the estimated c-f hybridization strengths of these HF compounds. This scaling demonstrates that the model of c-f hybridized band can generally and quantitatively describe the low-energy charge excitations in a wide range of HF compounds.

KEYWORDS: Heavy fermion, optical conductivity

Physics of “heavy fermion (HF)” compounds has attracted a considerable amount of interest during the last few decades. In HF compounds, a strong Coulomb correlation of the f electrons enhances the carrier effective mass ($m^*$). The mass enhancement is manifested, in e.g., the quadratic temperature ($T$) coefficient ($A$) of the resistivity and the linear $T$ coefficient ($\gamma$) of the electronic specific heat.\(^5\) It has been shown that a scaling between $A$ and $\gamma^2$, now widely known as the “Kadowaki-Woods (KW) relation”, holds for a large number of HF compounds having different degrees of mass enhancement and orbital degeneracy.\(^3,4\) Theoretically, the KW relation including effects of electron correlation was successfully derived using the periodic Anderson model (PAM),\(^4\) and later generalized to include effects of orbital degeneracy.\(^3,5\) In addition to the interesting transport, magnetic, and thermal properties of HF compounds, their charge excitation spectra, measured as the dynamical (optical) conductivity $\sigma(\omega)$, have been shown to be highly anomalous compared with those of conventional metals.\(^6,7\) Such anomalies should arise from their peculiar electronic dispersion near the Fermi level ($\epsilon_F$), which is the main interest of this Letter.

The simplest case of a HF compound is that of a Ce (Yb) compound with a $4f^1$ ($4f^{13}$) configuration, where a $4f$ electron (hole) interacts with the conduction electrons.\(^1\) Using the Fermi liquid theory and PAM, it has been shown that the dispersion in such a case may be approximated in terms of “renormalized hybridized bands”, as sketched in Fig. 1(a).\(^8-10\) They result from a conduction (c)-f electron hybridization renormalized by the electron correlation. The $\sigma(\omega)$ spectrum resulting from this dispersion consists of a very narrow Drude peak centered at $\omega=0$\(^11\) and a peak at finite energy due to interband transitions [Fig. 1(b)].\(^12\) Experimentally, $\sigma(\omega)$ of several HF compounds exhibited a narrow Drude peak and a mid-infrared (mid-IR) absorption peak,\(^13-16\) which were consistent with the predictions in Fig. 1(b). Garner et al\(^13\) first pointed out that, according to results of c-f hybridized bands,\(^10\) the position (energy) of the mid-IR peak in $\sigma(\omega)$, $E_{\text{mir}}$, should obey

$$E_{\text{mir}} \propto \sqrt{T_K W},$$  \hspace{0.5cm} (1)

where $W$ is the bare c band width. Degiorgi et al.\(^14\) also discussed $E_{\text{mir}}$ of HF compounds using Eq. (1), while Dordevic et al.\(^15\) reported a scaling of $E_{\text{mir}}$ for a few HF compounds, using a relation essentially equivalent to Eq. (1). Subsequently, Hancock et al.\(^16\) used Eq. (1) to

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analyze $\sigma(\omega)$ spectra of YbIn$_{1-x}$Ag$_x$Cu$_4$. They showed that $E_{\text{mir}}$ indeed scaled with $\sqrt{T_K}$ for $0 \leq x \leq 1$, establishing a scaling of $E_{\text{mir}}$ in terms of Kondo physics for YbIn$_{1-x}$Ag$_x$Cu$_4$. However, since these previous works dealt with a limited range of HF compounds, it was unclear whether such a scaling of mid-IR peak based on c-f hybridized band was universal for many HF compounds.

In this Letter, $\sigma(\omega)$ spectra of many Ce- and Yb-based HF compounds are systematically examined to test whether the mid-IR peaks follow a universal law among these compounds, and to clarify the above question. $\sigma(\omega)$ of CeNi, CeSn$_3$, YbInCu$_4$, YbAl$_2$, YbCu$_2$Si$_2$, YbNi$_2$Ge$_2$, and YbCuAl have been newly measured. $\sigma(\omega)$ spectra are also taken from our previous works on CeNiSn$_3$, CeRu$_2$Sb$_2$, CeOs$_4$Sb$_{12}$, CeRu$_4$Sb$_{12}$, and YbAl$_3$. In addition, several other HF compounds for which $E_{\text{mir}}$ values have been reported in the literature are considered. Table I lists the physical parameters of the studied HF compounds, used below to evaluate their $T_K$ and $W$. Note that these compounds span a wider range of $\gamma$ values compared with previous works, which is important for our purpose. It is shown that their $E_{\text{mir}}$ values scale well in the form of Eq. (1).

The newly measured samples were polycrystals for YbAl$_2$ and YbCu$_2$Al$_2$, and single crystals for CeNi, CeSn$_3$, YbInCu$_4$, YbNi$_2$Ge$_2$, and YbCu$_2$Si$_2$. The optical reflectivity spectra [$R(\omega)$] were measured on as-grown specular surfaces for YbCu$_2$Si$_2$ and YbNi$_2$Ge$_2$, on cleaved surfaces for CeNi and YbInCu$_4$, and on polished surfaces for the others. $\sigma(\omega)$ spectra were obtained from $R(\omega)$ using the Kramers-Kronig analysis. Other details were similar to those described elsewhere.

Figure 2 shows the obtained $\sigma(\omega)$ spectra at 9 K. Since the dispersion sketched in Fig. 1 does not include the $T_K$ and $W$ effects, here we discuss low-$T$ spectra only. All these compounds clearly exhibit a mid-IR peak in $\sigma(\omega)$, which strongly suggests that the appearance of a mid-IR peak is a universal property of the HF compounds. Their $E_{\text{mir}}$ (peak positions), indicated by the arrows in Fig. 2, vary markedly from compound to compound, which indicates a wide range of their $\tilde{V}$ and $T_K$. Our $\sigma(\omega)$ spectra of CeSn$_3$ and CeRu$_4$Sb$_{12}$ are similar to those in Refs. 15, 46 although the overall magnitude of our $\sigma(\omega)$ is larger. Our $E_{\text{mir}}$ of 0.25 eV for YbInCu$_4$, measured on a cleaved sample, agrees with the previously reported value 16 for a polished sample, while the overall magnitude of our $\sigma(\omega)$ is much smaller. 20

Eq. (1) can be derived from the relation $2\tilde{V} \propto \sqrt{T_K W}$, which is a result of PAM, and by setting $E_{\text{mir}} \propto 2\tilde{V}$. Although $E_{\text{mir}}$ is larger than $2\tilde{V}$ as sketched in Fig. 1(b). Hancock et al. 16 have well demonstrated that $E_{\text{mir}} \propto 2\tilde{V}$ holds in fact for YbIn$_{1-x}$Ag$_x$Cu$_4$ ($0 \leq x \leq 1$). To analyze the measured $E_{\text{mir}}$ values using Eq. (1), it should be expressed in terms of experimentally observable quantities. $W$ of a Ce (Yb) compound may be regarded as inversely proportional to $\gamma$ of the non-magnetic, isostructural La (Lu) compound (denoted as $\gamma_0$). Concerning $T_K$, since we need to deal with a large number of compounds, we shall make use of the simple relation

$$T_K = \pi^2 N_A k_B a/(3\gamma).$$

(2)

Here, $N_A$ and $k_B$ are the Avogadro number and Boltzmann constant, respectively, and $a$ is a constant which depends only on the $f$ level degeneracy $N; a=0.21, 0.54$ and 0.59 for $N=2, 6$ and 8, respectively. Then Eq. (1) can be rewritten as

$$E_{\text{mir}} \propto \sqrt{a/(3\gamma)}.$$  

(3)

The constant of proportionality in Eq. (3) is, in the absence of broadening ($E_{\text{mir}} = 2\tilde{V}$), given as

$$S_{\text{th}} = \sqrt{8\pi^3/9 \cdot N_A k_B^2},$$

(4)

which is material independent.

In analyzing the experimental $E_{\text{mir}}$ values using Eqs. (3) and (4), we also include reported $E_{\text{mir}}$'s in the literature for YbAgCu$_4$, YbRu$_2$Si$_2$, CePd$_3$, and CeCu$_6$. Except for CeRu$_4$Sb$_{12}$ and YbRu$_2$Si$_2$, the HF compounds have Fermi liquid (FL) ground state at low $T$. Their $\gamma$ values in Table I are taken from the cited references.

### Table I

| Compound          | $\gamma$ (meV) | $\gamma_0$ (meV) | $\gamma$ (meV) | $\gamma_0$ (meV) |
|-------------------|----------------|------------------|----------------|------------------|
| YbAl$_2$          | 17$^{42}$      | 5.6$^{23}$       | 6.2$^{33}$     |
| YbAl$_3$          | 58$^{41}$      | 4.2$^{25}$       | 38.6$^{35}$    | 3.5$^{36}$       |
| YbCu$_2$Si$_2$    | 134$^{42}$     | 5.2$^{20}$       | 3.5$^{27}$     | 8.2$^{37}$       |
| YbNi$_2$Ge$_2$    | 136$^{42}$     | 12.7$^{47}$      | 5.8$^{48}$     |
| YbCu$_2$Al       | 260$^{42}$     | 5.9$^{48}$       | 2.1$^{37}$     |
| YbInCu$_4$        | 59$^{33}$      | 8.7$^{30}$       | 50-85$^{39}$   |
| YbAgCu$_4$        | 210$^{43}$     | 10.2$^{29}$      | 92$^{40}$      |
| YbRu$_2$Si$_2$    | 100-140$^{31}$ | 7.8$^{32}$       | 180$^{41}$     |

**Fig. 2.** Measured $\sigma(\omega)$ at 9 K, except for YbInCu$_4$ measured at 20 K for technical limitation. The arrows indicate the $E_{\text{mir}}$ values (peak positions) of the mid-IR peaks.
references, which were determined through the usual procedure, i.e., by extrapolating the measured \(C_e/T\) to \(T=0\) \((C_e\) is the electronic specific heat). The \(\gamma\) value thus obtained is the “Kondo contribution” proportional to \(1/T_K\) as discussed above. CeRu\(_4\)Si\(_2\)\(^{39}\) and YbRh\(_2\)Si\(_2\)\(^{31}\) show steep rise in \(C_e/T\) due to non-Fermi liquid (NFL) properties below \(\sim 4\) K and \(\sim 10\) K, respectively. Such a NFL effect is not included in our simple model of Eq. (2). However, since the onset \(T\) of this “NFL contribution” is clearly identified in their \(T\) dependence of \(C_e/T\),\(^{31,39}\) it can be subtracted from the total \(C_e/T\) to estimate a Kondo contribution.\(^{31,39}\) As previously done for, e.g., another NFL compound CeRhSn.\(^{50}\) (Of course, there is no rigorous theoretical justification for this subtraction, since the NFL properties should also arise from the strong electron correlation. However, it is a simple phenomenological scheme to estimate the underlying Kondo coupling of an NFL compound.\(^{50}\) Their \(\gamma\)'s in Table I are the Kondo contributions thus obtained.\(^{31,39}\)

Estimated uncertainty resulting from the subtraction is indicated by the horizontal error bars in Fig. 3, which are rather small since not \(\gamma\) but \(\sqrt{\gamma}\) enters Fig. 3. CeO\(_3\)Si\(_2\)\(^{12}\) shows a gradual increase of resistivity below 50 K, although other physical properties show metallic characteristics including the \(C_e/T\) data.\(^{40,41}\) CeNi and CeRhSb are metals having low carrier densities \((n)\) and a reduced density of states (dos) at \(\epsilon_F\).\(^{37}\) The reported \(E_{\text{mir}}\)'s of YbFe\(_2\)Si\(_2\)\(^{15,48}\) and CeCoIn\(_5\)\(^{49}\) are not included here, since YbFe\(_2\)Si\(_2\) is not an \(f\) electron-derived HF,\(^{48}\) and since CeCoIn\(_5\) shows strong NFL properties up to high \(T\),\(^{51}\) making it difficult to estimate the Kondo contribution to \(\gamma\).

Figure 3(a) shows plots of the measured and reported \(E_{\text{mir}}\) values as a function of \(\sqrt{a/\gamma}\) using the \(\gamma\) and \(\gamma_0\) values in Table I. Here, \(N=8\) (6) has been used for the Yb (Ce) compounds, except for CeCu\(_6\) (see below). Figure 3(a) shows that the \(E_{\text{mir}}\) values scale well, in agreement with the prediction of Eq. (3). Some of the compounds, e.g., YbAl\(_2\), CePd\(_3\), CeSn\(_3\) and CeCu\(_6\), seem to show larger deviations than the others. However, considering the extreme simplicity of our model, the overall scaling is still good. The reported \(\sigma(\omega)\) of CeCu\(_6\) at low \(T\) shows two marked IR peaks: a broad one centered at \(\sim 0.15\) eV and the other at 5 meV.\(^{42}\) Since the energy scale of 0.15 eV (1700 K) is much larger than \(T_K\) of CeCu\(_6\) (\(\sim 1\) K),\(^{42}\) the 5 meV peak rather than the 0.15 eV peak should be associated with the hybridization gap, as previously done.\(^{15}\) Hence only the 5 meV peak is plotted in Fig. 3(a) using \(N=2\) since the Ce ground state in CeCu\(_6\) is a \(N=2\) doublet due to crystal field splitting. (See the discussion below regarding the lowering of degeneracy in other compounds.) From Fig. 3(a), the experimental slope of the scaling is estimated as \(S_{\text{ex}} = (7 \sim 9) \times 10^{-22}\) J\(^2\)/K\(^4\)mol, which is in excellent agreement with the theoretical slope given by Eq. (4), \(S_{\text{th}}=6.0\) \(\times 10^{-22}\) J\(^2\)/K\(^4\)mol. (Note that the actual 2V is slightly smaller than \(E_{\text{mir}}\), as mentioned before.) For comparison, in Fig. 3(b) \(E_{\text{mir}}\) is plotted as a function of \(\sqrt{a/\gamma}\) (\(\propto \sqrt{T_K}\)), without including \(\gamma_0\) (\(\propto W^{-1}\)). Although an overall tendency for scaling is still seen, it is much less satisfactory than that in Fig. 3(a). This is in contrast to the previous result of good scaling without \(W\) observed for YbIn\(_{1-x}\)Ag\(_x\)Cu\(_4.16\) Clearly, the explicit inclusion of \(W (\propto \gamma^{-1})\) in our analysis has been crucial for observing the good scaling over many HF compounds. Note that CeNi and CeRhSb show good scaling of \(E_{\text{mir}}\) despite their small \(n,\)\(^{37}\) while they are completely off the KW relation.\(^{3}\) These contrasting results are reasonable, since \(S_{\text{th}}\) does not depend on \(n\), while the slope of KW relation does depend on \(n.\)\(^{3}\) Figure 3(a) convincingly demonstrates that the universal relation predicted by Eqs. (2) and (3) is in fact followed by these HF compounds, and that the model of \(c-f\) hybridized band has captured the essential physics involved in the low-energy dynamical conductivity of HF compounds.

The Ce or Yb ground state is a doublet also in CeNiS\(_n\),\(^{37}\) YbCu\(_2\)S\(_2\),\(^{3}\) and YbRh\(_2\)Si\(_2.\(^{31}\) Hence the \(T_K\) estimated from their \(\gamma\) at low \(T\) correspond to that of the \(N=2\) ground state. However, except for the 5 meV peak for CeCu\(_6,\) the mid-IR peaks have a large energy scale of the order of 1000 K. Hence the mid-IR peaks should be associated with the “high-temperature \(T_K\)” (\(T_K^h\)) for the entire \(J=5/2\) (7/2) multiplet with \(N=6\) (8). Since \(T_K^h\) is larger than the ground state \(T_K,\) our use of \(\gamma\) at low \(T\) should underestimate \(T_K^h\) relevant to the mid-IR peak. In spite of this, these compounds are scaled well in Fig. 3(a). This is probably because our use of \(N=6\) (8) has partly compensated for the underestimation of \(T_K^h;\) in the relation \(T_K \propto a/\gamma, a=0.54\) (0.59) for \(N=6\) (8) is greater than.
scales with the $\sqrt{f/\sigma}$ of the characteristic IR excitation in a configuration. Although there have been many reports of Culture, Education, Sports, Science and Technology. Field Spin Science in 100 T” (No. 451) from the Ministry of Education, Culture, Sports, Science and Technology. This work has been partly supported by COE Research (No. 2002), and Priority Areas “Skutterudites” (Nos. 15072204, 15072206, 15072207) and “High Field Spin Science in 100 T” (No. 451) from the Ministry of Culture, Education, Sports, Science and Technology.

1. A.C. Hewson, The Kondo problem to heavy fermions (Cambridge University Press, Cambridge 1993).
2. K. Kadawaki and S. B. Woods, Solid State Commun. 58, 507 (1986).
3. N. Tsuji, H. Kontani, and K. Yoshimura, Phys. Rev. Lett. 94, 057201 (2005); Physica B 378-380, 730 (2006).
4. K. Yamada and K. Yoshida, Prog. Theor. Phys. 76, 621 (1986).
5. H. Kontani, J. Phys. Soc. Jpn. 73, 515 (2004).
6. B. C. Webb, A. J. Sievers and T. Mihalisin: Phys. Rev. Lett. 57 (1986) 1951.
7. L. Degiorgi, Rev. Mod. Phys. 71 (1999) 687, and references therein.
8. T. M. Rice and K. Ueda, Phys. Rev. Lett. 55, 995 (1985).
9. Z. Zou and P. W. Anderson, Phys. Rev. Lett. 57, 2073 (1986).
10. D. L. Cox, Phys. Rev. Lett. 58, 2730 (1987).
11. A. J. Millis and P. A. Lee, Phys. Rev. B 35, 3394 (1987).
12. P. Coleman, Phys. Rev. Lett. 59, 1026 (1987).
13. S. R. Garner, J. N. Nancoc, Y. W. Rodriguez, Z. Schlesinger, B. Bucher, Z. Fisk and J. L. Sarrao: Phys. Rev. B 62 (2000) R4778.
14. L. Degiorgi, F. Anders and G. Gruner, Eur. Phys. J. B 19, 167 (2001).
15. S. V. Dordevic, D. N. Basov, N. R. Dilley, E. D. Bauer and M. B. Maple: Phys. Rev. Lett. 86, 684 (2001).
16. J. N. Hancock, T. McKnew, Z. Schlesinger, J. L. Sarrao, and Z. Fisk: Phys. Rev. Lett. 92 (2004) 186405; Phys. Rev. B 73 (2006) 125119.
17. M. Matsunami, H. Okamura, T. Namba, T. Sunemitsu, T. Yoshino, T. Takabatake, Y. Isikawa, and H. Harima: J. Phys. Soc. Jpn. 71 Suppl. (2002) 291.
18. M. Matsunami, H. Okamura, T. Namba, H. Sugawara and H. Sato: J. Phys. Soc. Jpn. 72 (2003) 2722.
19. M. Matsunami, H. Okamura, T. Namba, H. Sugawara, H. Sato: J. Magn. Magn. Mater. 327-3276 (2004) e41.
20. H. Okamura, T. Michizawa, T. Namba and T. Ebihara: cond-mat/0609157 (2006).
21. H. Okamura, T. Michizawa, T. Namba and T. Ebihara: J. Phys. Soc. Jpn. 73 (2004) 2045.
22. N. Tsuji, K. Yoshimura and K. Kosuge: J. Phys.: Condens. Matter 15 (2003) 1993.
23. S. J. Lee, S. Y. Hong, I. R. Fisher, P. C. Canfield, B. N. Harmon, and D. W. Lynch: Phys. Rev. B 61 (2000) 10076.
24. T. Ebihara, S. Uji, C. Terakura, T. Terashima, E. Yamamoto, Y. Haga, Y. Inada and Y. Onuki: Physica B 281 & 282 (2000) 754.
25. E. D. Bauer, C. H. Booth, J. M. Lawrence, M. F. Hundley, J. L. Sarrao, J. D. Thompson, P. S. Riseborough and T. Ebihara: Phys. Rev. B 69, 125102 (2004).
26. C. S. Gee, B. Andrau, J. S. Kim and G. R. Stewart: Phys. Rev. B 43 (1991) 2656.
27. S. L. Bud’ko, Z. Islam, T. A. Wiener, I. R. Fisher, A. H. Lacerda and P. C. Canfield: J. Magn. Magn. Mater. 205 (1999) 53.
28. L. Havela, M. Divic, V. S hochovsky, A. V. Andreev, F. Honda, G. Oomi, Y. Meresse and S. Heathman: J. Alloys and Compounds 322 (2001) 7.
29. A. L. Cornelius, J. M. Lawrence, J. L. Sarrao, Z. Fisk, M. F. Hundley, G. H. Kwei, J. D. Thompson, P. S. Rice and F. Bridges: Phys. Rev. B 56 (1997) 7993.
30. Both In$_{0.7}$Cu$_{0.3}$ and YIn$_{0.7}$Cu$_{0.3}$ are semimetals, so they do not serve as reference to YInCu$_{1-x}$, $\gamma$=8.7 mJ/K$^2$mol is assumed after J. M. Lawrence, P. S. Riseborough, C. H. Booth, J. L. Sarrao, J. D. Thompson and R. Osborn: Phys. Rev. B 63 (2001) 054427.
31. O. Trouvarelli, C. Geibel, S. Mederle, C. Langhammer, F. M. Grosche, P. Gegenwart, M. Lang, G. Sparr and F. Steglich: Phys. Rev. Lett. 85 (2000) 626. This work reports a large $C_2/T$ of 1500 mJ/K$^2$mol below 5 K due to NFL formation. We have estimated a Kondo contribution of 100-140 mJ/K$^2$mol by extrapolating the nearly constant $C_2/T$ above 10 K down to $T=0$.
32. H. S. Jeevan, private communication.
33. I. Umehara, Y. Kurosawa, N. Nagai, M. Kikuchi, K. Satooh and Y. Onuki: J. Phys. Soc. Jpn. 59 (1990) 2848.
34) G. A. Costa, F. Canepa and G. L. Olcese: Sol. State Commun. 40 (1981) 169.
35) M. J. Besnus, J. P. Kappler and A. Meyer: J. Phys. F: Met. Phys. 13 (1983) 597.
36) $\gamma_0$ of YPd$_3$ is used here, C. L. Seaman and M. B. Maple, Physica B 199-200 (1994) 396. LaPd$_3$ has anomalously small $\gamma_0$ of 0.28 mJ/K$^2$mol, and does not serve as reference to CePd$_3$. This is supported by the large difference of the lattice constant between CePd$_3$ and LaPd$_3$ ($\Delta a=0.11$ Å) compared with that between CePd$_3$ and YPd$_3$ ($\Delta a=0.05$ Å).
37) S. Nishigori, H. Goshima, T. Suzuki, T. Fujita, G. Nakamoto, H. Tanaka, T. Takabatake and H. Fujii: J. Phys. Soc. Jpn. 65 (1996) 2614.
38) S. Takayanagi, S. Araki, R. Settai, Y. Ônuki and N. Mori: J. Phys. Soc. Jpn. 70 (2001) 753.
39) N. Takeda and M. Ishikawa, J. Phys. Soc. Jpn. 69 (2000) 868. This work reports a large $C_e/T$ of 190-380 mJ/mol$^2$K below 2 K due to NFL formation. We have estimated a Kondo contribution of 50–85 mJ/mol$^2$K by extrapolating the $C_e/T$ vs $T^2$ plot above 4 K down to 0 K.
40) E. D. Bauer, A. Slebarski, E. J. Freeman, C. Sirvent and M. B. Maple: J. Phys. Condens. Matter 13 (2001) 4495.
41) T. Namiki, Y. Aoki, H. Sugawara and H. Sato: Acta Physica Polonica B 34 (2003) 1161.
42) F. Marabelli and P. Wachter: Phys. Rev. B 42 (1990) 3307, and references therein.
43) T. Fujita, K. Satoh, Y. Ônuki and T. Komatsubara: J. Magn.
44) T. Ebihara, unpublished.
45) N. Tsuchi, in preparation.
46) B. Bucher, Z. Schlesinger, D. Mandrus, Z. Fisk, J. Sarrao, J. F. DiTusa, C. Oglesby, G. Aeppli and E. Bucher: Phys. Rev. B 53 (1996) R2948.
47) S. Kimura, J. Sichelschmidt, J. Fersti, C. Krellner, C. Geibel and F. Steglich: J. Magn. Magn. Mater. 272-276 (2004) 36; Phys. Rev. B 74 (2006) 132408.
48) J. Sichelschmidt, V. Voevodin, H. J. Im, S. Kimura, H. Rosner, A. Leithe-Jasper, W. Schnelle, U. Burkhardt, J. A. Mydosh, Yu. Grin and F. Steglich: Phys. Rev. Lett. 96 (2006) 037406.
49) E. J. Singley, D. N. Basov, E. D. Bauer and M. B. Maple: Phys. Rev. B 65 (2002) 161101.
50) M. S. Kim, Y. Echizen, K. Umeo, S. Kobayashi, M. Sera, P. S. Salamakha, O. L. Sologub, T. Takabatake, X. Chen, T. Tayama, T. Sakakibara, M. H. Jung and M. B. Maple: Phys. Rev. B 68 (2003) 054416.
51) J. S. Kim, J. Alwood, G. R. Stewart, J. L. Sarrao and J. D. Thompson: Phys. Rev. B 64 (2001) 134524.
52) H. Kontani and K. Yamada: J. Phys. Soc. Jpn. 66 (1997) 2232.
53) T. Saso and T. Harima: J. Phys. Soc. Jpn. 72 (2003) 1131.
54) G. Czycholl and H. J. Leder, Z. Phys. B Condens. Matter 44 (1981) 59. When a $k$-independent $V$ is assumed, as in many PAM-based theories on HF compounds, only the $c$ component of the total dos can contribute to $\sigma(\omega)$. 

Magn. Mater. 47 & 48 (1985) 66.