Clamp-tapering increases the quality factor of stressed nanobeams

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Stressed nanomechanical resonators are known to have exceptionally high quality factors (Q), usually limited by clamping loss. Here we study the role of clamp geometry on the Q of nanobeams made of high-stress Si3N4. Contrary to recent studies of tethered-membrane resonators, we find that the Q of the low order modes is enhanced by tapering the beam—and locally increasing the stress—near the clamp. “Tapered-clamping” is complementary to the recently developed “soft-clamping” approach, whereby clamping loss is suppressed by embedding the resonator in a phononic crystal. Unlike soft-clamping, tapered-clamping enhances the Q of the fundamental mode and can be implemented without changing the device size. Both features are advantageous for practical applications.

When tensely stressed, string and membrane-like mechanical resonators exhibit increased quality factors. This effect, known as dissipation dilution, was first predicted and observed three decades ago in fiber suspensions of kilogram Laser-Interferometer Gravitational-Wave Observatory (LIGO) mirrors [1, 2]. More recently, dissipation dilution has been used to realize ultra-high-Q nanomechanical resonators, by exploiting extreme stresses and geometries accessible by modern nanofabrication techniques (especially by patterning Si3N4 thin films [3–8]). Resonator geometries ranging from doubly-clamped beams [9, 10] and tuning forks [11], to membranes [12] and trampolines [13, 14], to 1D [15] and 2D [16] phononic crystals have been studied. In the latter case, mode-shape (“soft-clamping”) and strain localization has been harnessed to achieve room temperature Q × frequency products as high as 1015 Hz [15]. High-Q mechanical resonators have reduced thermal decoherence rates, \( \gamma \approx \Gamma_m \tilde{n}_m \), where \( \Gamma_m = \Omega_m/Q \) is the dissipation rate of the oscillator and \( \tilde{n}_m \approx k_B T/(\hbar \Omega_m) \) is the phononic occupation of the mode with frequency \( \Omega_m \). Therefore, Si3N4 mechanical resonators have been extensively employed within cavity quantum optomechanics, for quantum feedback experiments[17, 18] and sideband-cooling to the mechanical ground state [19].

Dissipation dilution in stressed thin film resonators requires geometric nonlinearity of strain, i.e. that the dynamical strain is nonlinear in the displacement [20]. When this condition is met, the elastic energy can be divided into two parts: a lossy, ‘bending’ term and a lossless ‘tensile’ term. With increasing tensile stress, the fractional contribution of the lossless tensile term is increased, thereby reducing ("diluting") the fraction of energy lost per cycle and thus increasing the Q. Dissipation dilution is particularly large in nanostrings and membranes, mainly because bending loss is inversely proportional to device thickness. An additional insight is that flexural mode curvature (and therefore bending loss) is exaggerated near the clamps, to satisfy boundary conditions [20]. With this in mind, dissipation dilution has recently been investigated in resonators with alternative clamp geometries. Tethered membranes (“trampolines”) [13, 14] and doubly-clamped tuning forks [11] have been found to support low order modes with anomalously high Q, possibly due to reduced clamping loss. A more explicit approach involves patterning a phononic crystal (PuC) shield around the resonator, thereby localizing its motion away from the clamps. This “soft-clamping” approach has proven highly effective for both 1D [15] and 2D [16] geometries, resulting in enhancements of dissi-
pation dilution by an order of magnitude and \( Q \) factors approaching \( 10^9 \) at room temperature \([15]\).

Though powerful, the soft-clamping approach has several drawbacks. First, mode localization can only be achieved with a large number of PnC unit cells, requiring that the device be much larger than the acoustic wavelength of interest—indeed several millimeters for MHz modes of a \( \text{Si}_3\text{N}_4 \) thin film \([15, 16]\), making chip-scale integration highly challenging. Second, soft-clamping can only be applied to high order modes of the extended PnC structure, whereas for practical sensing applications, the fundamental mode (possessing the lowest stiffness and cleanest spectral background) is usually desired.

Here we explore a 1D alternative to soft-clamping that does not have the same limitations: Tapering the width of a beam—and locally enhancing its stress—near its clamping points. Combining a simple analytical model with measurements performed on \( \text{Si}_3\text{N}_4 \) nanobeams, we demonstrate that this “tapered-clamping” strategy can enhance the \( Q \) of low order flexural modes, including the fundamental mode, by a significant factor, limited in our case to 2.4 by the yield stress of \( \text{Si}_3\text{N}_4 \). We also show that commonly implemented convex-filleted (anti-tapered) supports decrease the \( Q \) factor. This finding draws into question the interpretation of ultra-high-\( Q \) modes in tethered membrane resonators \([13]\).

We will now introduce the theoretical framework that informed our choice of clamp geometry. It can be shown analytically that the quality factor of a doubly clamped beam is given by \([15, 20]\)

\[
Q = Q_{\text{int}} \times \left[ 2\alpha_n \lambda + \beta_n (n\pi)^2 \right]^{-1}
\]

where \( \lambda = \frac{h}{L} \sqrt{\frac{E}{12\sigma_{\text{avg}}}} \), \( h \) is the beam thickness, \( L \) is the total beam length, \( Q_{\text{int}} \) is the material’s intrinsic quality factor, \( E \) is the Young’s modulus, \( \sigma_{\text{avg}} \) is the average stress and \( \alpha_n \) and \( \beta_n \) are geometrical factors. The first term in the denominator of Equation 1, which is proportional to \( \lambda \), arises from the high curvature at the clamping points enforced by the boundary conditions. The second term, proportional to \( \lambda^2 \), is due to the distributed mode curvature. Since \( \lambda \ll 1 \) for a highly strained beam, the first term is typically the dominant factor limiting the \( Q \). However, by utilizing an optimized geometry, \( \alpha_n \) can be greatly reduced to attain \( Q \propto 1/\lambda^2 \) scaling \([15]\). In nanofabricated devices, a released thin film device must have a fixed tension force. This implies that by varying the width of a nanobeam one can increase (decrease) the local stress by tapering (anti-tapering) the beam width. It has been shown theoretically that the losses at the clamping points can be reduced by tapering the width towards the clamping region, as long as the length of the taper is short compared to the overall beam length\([20]\).

This geometric change enables the reduction of \( \alpha_n \) by as much as a factor of \( \sqrt{\frac{\sigma_{\text{yield}}}{\sigma_{\text{mat}}(1-\nu)}} \) with a corresponding increase in quality factor, where \( \sigma_{\text{yield}} \) and \( \sigma_{\text{mat}} \) are yield and deposition stress of the material respectively and \( \nu \) is Poisson’s ratio. Additionally, while in-plane (IP) modes typically have lower \( Q \) than out-of-plane (OP) modes, with this method, the \( Q \) of the IP modes can approach the \( Q \) of the OP modes of the same beam (see supplemental information). We study this \( Q \) enhancement by varying the clamping to beam width ratio \( r = \frac{w_{\text{clamp}}}{w_{\text{clamp}}} \) from 0.05 to 10. The \( Q \) of the OP modes is enhanced by as much as a factor of two compared to a normal beam with the tapered design \( r = 0.16 \) and reduced by a factor of three with the anti-tapered design \( r = 10 \).

We now consider the effects of tapering on the frequency of the fundamental mode. First, we introduce the average stress

\[
\sigma_{\text{avg}} = \frac{\sigma_{\text{mat}}(1-\nu)}{\langle \nu(x) \rangle}
\]

where \( \nu \) is Poisson’s ratio \( \nu = 0.23 \) for \( \text{Si}_3\text{N}_4 \), \( \sigma_{\text{mat}} \) is the deposition stress \( \sigma_{\text{mat}} = 1.1 \text{ GPa} \) and \( \nu(x) \) is the normalized width, \( \nu(x) = w(x)/\langle w(x) \rangle \), where \( w(x) \) is the
Corrugation decreases the average stress [20] and thus tapering is predicted to reduce the frequency \( f \propto \sqrt{\sigma_{\text{avg}}}. \)

To implement these ideas and show \( Q \) enhancement due to clamp tapering, we fabricate Si\(_3\)N\(_4\) beams with a range of clamping width ratios \( (r) \) of 0.05 to 10. In each beam, the width in the clamping region is given by \( w(x) \propto \arctan(ax/L_{\text{tran}}) \), which allows a smooth taper combined with a transition slope determined by \( a \) and the transition length \( L_{\text{tran}} \). The geometry is illustrated in Figure 1.

The devices are fabricated in a six step process illustrated in Figure 2. First, 100-nm thick Si\(_3\)N\(_4\) films are deposited using low pressure chemical vapor deposition (LPCVD), with beam designs then patterned by electron beam lithography. The patterns are transferred to Si\(_3\)N\(_4\) layer using fluorine chemistry dry etching. Potassium hydroxide (KOH) is used to release Si\(_3\)N\(_4\) structures from the Si substrate. The KOH etch rate differs for different Si crystalline planes (the \( \langle 111 \rangle \) plane etch rate is 100 times slower than the \( \langle 100 \rangle \)). The clamps, which are wider than the beam by a factor of almost 10, can therefore mask the undercut plane orientation. To avoid incomplete release, we therefore precede the wet release step with a deep etch of the silicon substrate, which allows the \( \langle 111 \rangle \) planes to meet underneath the free-standing nanobeams during the KOH etching step. An upscaled version of the first mask is written using electron beam lithography to protect the structures during the deep etching. The second mask is transferred into the Si substrate using dry etching process. A quick cleaning using buffered HF is performed to remove residual contamination as well as any oxidized Si\(_3\)N\(_4\) layer on the top film, which may arise from oxygen plasma exposure of the film during cleaning [21]. After the release process, the samples are dried using critical point drying (CPD) to avoid collapse of the structures to the underlying silicon substrate.

The frequency and quality factors of the devices are measured using a lensed-fiber-based homodyne interferometer at \( \lambda = 780 \text{ nm} \) (described in detail in [15]). The frequencies are determined by measuring the Brownian motion of the beams and the quality factors are measured using ringdown spectroscopy. For the ringdown spectroscopy, mechanical modes are driven using a piezoelectric slab attached to the bottom of the chip holder on which samples are clamped. The piezoelectric slab is driven using a lock-in amplifier with a frequency swept tone that drives the desired mode. The measurements are performed in a high vacuum chamber (\( P = 10^{-8} \text{ mbar} \)) to avoid gas damping. The samples characterized here are on a chip where \( r \) is swept from 10 to 0.05 using the \( w(x) \propto \arctan(ax/L_{\text{tran}}) \) profile with \( a = 20 \) and \( L_{\text{tran}} = 0.01 \) for two different lengths (\( L = 250 \mu \text{m}, 150 \mu \text{m} \)). The measured \( Q \) data and frequency in Figure 3 and 4 are within 10% agreement with the model for \( Q \) and the solutions of Euler-Bernoulli equations for \( f [20] \). The deposition stress is inferred to be 1.1 GPa from the measurement of resonance frequency for a rectangular \(( r = 1) \) beam.

Our measurement data, in agreement with our model, shows that \( Q \) is increased by tapering at the beam clamping point and decreased by an anti-taper. Since the tension force in the released beam is fixed, tapering the clamps increases the stress in the clamping region. On the other hand, anti-tapering reduces the stress in the clamping region and slightly increases it in the center of the resonator. We can therefore conclude that the enhancement of the \( Q \) factor arises from increased stress in the clamping region, not in the beam bulk. This observation is at odds with previously reported results for trampoline membrane designs [13] that report \( Q \) enhancement by anti-tapered anchor designs due to tensile stress enhancement in the tethers. In agreement with our work, it has been seen in tuning fork resonators that a tuning fork design with a ‘neck’ has enhanced \( Q \) factors [11]. In this design, the necking geometry is essentially equivalent to tapering the clamping point, thus increasing the stress in the clamp.

We also characterize the yield stress of Si\(_3\)N\(_4\). For a tapering parameter of \( r = 0.16 \), the stress in the clamps should be equal to 6.3 GPa—the yield stress of Si\(_3\)N\(_4\)[13]. For both lengths it was observed that all devices with \( r < 0.16 \) were broken at their clamping points (see Fig. 5). For beams with smaller \( r \) the expected stress in the

FIG. 3. Quality factor of Si\(_3\)N\(_4\) nanobeams versus clamping width. Quality factors of the fundamental out-of-plane mode for a 150\( \mu \text{m} \) (blue dots) and 250\( \mu \text{m} \) (red dots) beam, with a theoretical model fit in solid lines for 150\( \mu \text{m} \) (blue) and 250\( \mu \text{m} \) (red). The vertical black dashed line indicates the normal beam geometry \(( r = 1) \). The gray region shows where the stress would be above the yield strength. Schematic beam geometries are plotted in black for tapered, normal and anti-tapered designs.
clamps would be even larger, which is prohibited by the material properties. The parameter region where the stress exceeds the yield stress is indicated in gray in Figures 3 and 4.

Geometrical variations of the beam width affects the average stress ($\sigma_{\text{avg}}$) in accordance with Equation 2, resulting in changed mode frequencies. We observe that the OP mode frequencies are reduced when tapering the clamps due to the reduction in effective stress. We also see some deviation from the theory model for $w_{\text{clamp}}/w_{\text{beam}} < 0.4$, which could be related to the observed buckling of the Si$_3$N$_4$ film at high stresses (see supplementary information). In conclusion we showed a material independent technique to enhance the quality factor of the fundamental and lower order modes of Si$_3$N$_4$ nanobeams by tapering the clamping points. Using this geometry, quality factors can be increased by a factor $\sqrt{\frac{\sigma_{\text{yield}}}{\sigma_{\text{mat}}} (1-\nu)}$, which depends on the deposition and yield stress of the material (a factor of 2.4 for LPCVD Si$_3$N$_4$). Enhancement of the fundamental mode quality factor is of great interest for optomechanics as a sparse mode spectrum can be maintained while increasing the $Q$. Since this method can be applied to short beam lengths, it is also suitable for integration with optical microcavities [5]. Finally, by varying the taper width, we are able to estimate the yield stress of Si$_3$N$_4$ to about 6.3 GPa, consistent with previous studies[13, 22].

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**DATA AVAILABILITY STATEMENT**

Data and data analysis code will be made available through Zonedo upon publication. All other data needed to evaluate the conclusions in the paper are present in the paper or the supplementary materials.

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Supplementary information: “Clamp-tapering increases the quality factor of stressed nanobeams”

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IN-PLANE MODE $Q$ FACTOR ENHANCEMENT

In beams with tapered clamps, $Q$ factors of flexural in-plane modes are enhanced in a similar way to the $Q$ factors of the out-of-plane (OP) modes considered in the main text. Moreover, for IP modes tapering not only makes dissipation around clamping points smaller due to the stress enhancement, but also due to the reduction of the bending moment, and thus the relative enhancement of $Q$ by pinching can be higher for the IP mode family than for the OP modes. This can be shown by considering the dissipation dilution factor $D_Q$ for flexural modes [1]

\[
D_Q = Q/Q_{\text{int}} = \left(2\lambda_{\text{cl}} + n^2 \pi^2 \lambda^2 \right)^{-1}.
\]  

(S1)

Here $Q_{\text{int}}$ is the material quality factor, the dissipation dilution parameter $\lambda$ is given by [2]

\[
\lambda = \frac{h}{L} \sqrt{\frac{E}{12\sigma}},
\]  

(S2)

where $E$ is Young’s modulus and $\lambda_{\text{cl}}$ corresponds to $\lambda$ with the values of thickness $h$ and stress $\sigma$ at the clamping points. In the case where $Q$ is limited by clamping loss,

\[
D_Q \approx (2\lambda_{\text{cl}})^{-1} = \frac{L}{h_{\text{cl}}} \sqrt{\frac{3\sigma_{\text{cl}}}{E}}.
\]  

(S3)

Figure S1. $Q$ factor of IP and OP modes. Dashed lines are simulation results. Note that designs with beam width of 250 nm can result in IP modes with $Q$ factors approaching the $Q$ of the best OP modes (green dashed line).
As discussed in the main text, it follows from Eq. S3 for the fundamental OP mode that $D_Q$ is enhanced as square root of the tapering ratio $w_{\text{beam}}/w_{\text{cl}} = 1/r$, following the enhancement of stress $\sigma_{\text{cl}} \propto 1/r$

$$D_{Q,\text{OP}} \propto r^{-\frac{1}{2}}.$$  \hspace{1cm} (S4)

Additionally, the beam width $w_{\text{cl}}$ should replace thickness in Eq. S2 for the fundamental IP mode, yielding the overall scaling

$$D_{Q,\text{IP}} \propto r^{-\frac{3}{2}}.$$  \hspace{1cm} (S5)

This result means that the $Q$ factors of IP modes are predicted to be enhanced more by tapering than the $Q$s of OP modes.

In the present work the designs of the beams were not suitable to clearly observe the predicted enhancement of $Q$ for the in-plane modes due to the large overall beam width. The beam center width of 5\,\mu m used in this work resulted in $\lambda_{\text{IP}} = 0.1$, which does not satisfy the condition $\lambda \ll 1$ well enough for the IP modes to be clamping-loss limited and the clamping geometry to have strong effect on $Q$. However, we conducted measurements of $Q$ factors of IP modes (shown in Fig. ) which agree with theory and demonstrate a marginal improvement of $Q$ with the reduction of $r$ (except for the vicinity of breaking point at the smallest values of $r$). At the same time our simulations confirm that a stronger enhancement of $Q$ for IP modes is predicted for narrower beams.

**BUCKLING OF Si$_3$N$_4$ FILM**

We observe buckling in the Si$_3$N$_4$ film when reaching high stress ($\sigma > 2$ GPa) in high aspect ratio designs. This effect is caused by the bi-axial relaxation of stress in transverse direction that is developed around the corner points. Compressive stress develops in the transverse direction when the longitudinal stress is increased beyond 2 GPa. We observed this effect in 20-nm thick films when the clamp to beam width ratio was below 0.4.

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