Systematically Investigating the Structural Variety of Crystalline and Kaleidoscopic Vortex Lattices by Using Laser Beam Arrays

Chengshang Chen, Yuhan Fang, Chichen Jang, Wenchi Chen, Hui-Chi Lin, and Hsingchih Liang

Abstract: We theoretically demonstrate that a family of vortex-lattice structures can be flexibly generated using a multi-beam interference approach. Numerical calculation presents a variety of crystalline and kaleidoscopic patterns. Based on the numerical analysis, we experimentally realized these structure beams by combining an amplitude mask with multiple apertures and a spiral phase plate. The excellent agreement between the experimental and theoretical results not only validates the presented method, but also manifests the structure of vortex lattices.

Keywords: structure light beams; orbital angular momentum; optical vortex array

1. Introduction

An optical vortex is the optical wavefront dislocation with the indeterminate wave phase and zero amplitude at a specific point. Its spiral phase form, exp(iℓφ), can carry an orbital angular momentum (OAM) of ℓℏ per photon [1,2]. Optical vortex beams are attractive for numerous active fields and applications, such as optical tweezers [3,4], phase contrast imaging in microscopy [5], trapping and accelerating cooled atoms [6], quantum computation [7], and controlling the chirality of twisted nanostructures [8]. Despite various works devoted to the investigation of isolated optical vortex beams, the generation of optical vortex arrays or vortex lattices has attracted considerable attention for decades for photonic crystal fabrication, optical metrology, and optical manipulation [9–11]. Optical vortex arrays can, not only be directly generated in diode-pumped solid-state lasers with spherical cavity [12–14], but also be obtained by exploiting external optical devices, such as holographic grating [15,16], astigmatic mode converters [17–19], and spatial light modulators [20–23]. Since J. Masajada et al. demonstrated a regular optical vortex array with crystal structures by a three wave interference [24], the multi-beam interference method has often been exploited to generate optical vortex arrays with crystal and quasicrystal structures [23,25–27]. Recently, a Fourier-transformed method with an amplitude mask was demonstrated for flexibly generating optical beams with crystal and quasicrystal structures [28,29]. However, using the Fourier-transformed method to generate optical vortex arrays has not been explored in detail. It is scientifically interesting as a way to provide a convenient and robust approach for realizing various optical vortex arrays.

In this work, we provide a novel and convenient approach for generating various optical vortex arrays using a combination of an amplitude mask and a vortex phase plate. A collimated coherent laser is employed to illuminate a stencil mask with multiple apertures systematically arranged on a ring. The arrangement of the multiple apertures is based on a numerical calculation. Then, the laser beam passes through a spiral phase plate with variable topology charges from one to eight. It turns out that by changing the vortex geometry of the spiral phase plate, such an optical structure can be converted into various...
optical vortex lattices. The experimental results were found to be in a good agreement with the theoretical calculations. The excellent agreement between the experimental and theoretical results not only validates the present method, but also manifests the structure of vortex lattices.

2. Theoretical Analysis

Based on Fresnel–Huygens theory and considering a mask with \( Q \) apertures arranged on a ring that is illuminated with collimated laser beams, the optical field near the focal region \( z = f(1 + \Delta) \) with \( \Delta \ll 1 \) can be expressed as [30]:

\[
\Phi_Q(\rho, \phi; \theta_n) = e^{ikf}\frac{2}{\pi\omega^2} e^{-\rho^2/\omega^2} \Psi_Q(\rho, \phi; \theta_n)
\]

with

\[
\Psi_Q(\rho, \phi; \theta_n) = \left( \frac{1}{Q} \right)^{1/2} \sum_{s=0}^{Q-1} e^{i\frac{ka\rho}{\omega} \cos\left(\phi - \frac{2\pi s}{Q}\right)} e^{i\theta_n}  \tag{1}
\]

where \((\rho,\phi)\) is the polar coordinate of \((x,y)\), \( f \) is the focal length of lens, \( a \) is the radius of the ring, \( k = 2\pi/\lambda, \lambda \) is the wavelength of laser, \( \omega = \lambda f/(\pi\omega_0), \omega_0 \) is the radius of aperture, and \( \theta_n = 2\pi n/Q \) is the fixed phase difference between adjacent incident beams. Here, \( Q \) and \( n \) are both integer numbers. For a phase difference between adjacent beams greater than \( \pi \), the structure of the optical field will be equal. Therefore, the phase parameter \( n \) is restricted to a range of \( 0 \leq n \leq Q/2 \) for a given \( Q \). The wave function \( \Psi_Q(\rho, \phi; \theta_n) \) represents various two-dimensional crystalline or quasicrystalline structures with different topologic charges. The wave patterns \( |\Phi_Q(\rho, \phi; \theta_n)|^2 \) near the focal region can be confirmed to be nearly propagation invariant over a specific distance range. At the limit of \( Q \to \infty \), a finite sum can be a certain kind of approximation of an integral with a new variable \( \alpha = 2\pi s/Q \) and \( d\alpha = 2\pi/Q \). Consequently, the wave function \( \Psi_N(\rho, \phi; \theta_n) \) can be derived as:

\[
\Psi_Q(\rho, \phi; \theta_n) = J_n\left(\frac{ka\rho}{2\pi}\right) = \sqrt{\frac{1}{\pi}} \int_0^{2\pi} \frac{e^{i\frac{ka\rho}{\omega} \cos(\phi-a)}}{\sqrt{\pi N}} e^{i\theta_n} d\alpha
\]

The result indicates that high-order Bessel Gaussian beams can be achieved by interference of multiple Gaussian beams. From Equation (1), the wave patterns of \( \Phi_N(\rho, \phi; \theta_n) \) can be numerically calculated with \( \omega_0 = 0.13 \text{ mm}, a = 3 \text{ mm}, \) and \( f = 1000 \text{ mm} \). Figure 1 shows the calculated intensity distributions of \( |\Phi_Q(\rho, \phi; \theta_n)|^2 \) for \( Q = 6, 8, 12, 21, 30, \) and 50, with different relative phase \( \theta_n \) from \( n = 0 \) to \( n = 8 \). For \( n = 0 \), the calculated patterns with equal phase \( (n=0) \) are shown in the first column. The patterns exhibit crystalline, quasicrystalline, and kaleidoscopic structures that are consistent with the previous results [28]. For the case of \( Q = 50 \), the central part of the intensity distribution has some resemblance to a Bessel beam of zeroth order. This feature comes from the limiting behavior of \( Q \to \infty \). On the other hand, for the case of \( n \neq 0 \), the intensity distributions show that a variety of rich quasicrystalline wave patterns can be obtained by manipulating the relative phase \( \theta_n \). It can be seen that the wave patterns are repeated as \( n > Q/2, \) which is consistent with the previous prediction. The minimal intensity can be discovered in the center of the wave patterns and the region increases with the increasing value of \( n \) for all cases of the \( Q \)-multiple beams. Moreover, the central area of the wave patterns resembles a high-order Bessel beam at larger values of \( Q \). While, increasing the value of \( n \), a similar behavior can be found in terms of a Bessel Gaussian beam with the order \( n \). The numerical results indicate that the relative phase \( \theta_n \) can benefit the generation of quasicrystalline beams. In the following, we experimentally constructed an optical system to demonstrate the theoretical predictions.
3. Experimental Setup

A schematic illustration of the experimental setup for generating crystalline and quasi-crystalline laser beams with various phases is shown in Figure 2. The laser source was a 20-mW linearly polarized He-Ne laser with a wavelength of 632.8 nm. A beam expander was employed to generate a collimated light and reduce the beam divergence to less than 0.1 mrad. We fabricated several steel masks of $\omega_0 = 0.13\ mm$ and $a = 3\ mm$ by using a laser stencil cutting machine. To systematically control the relative phase, a spiral phase plate (RPC Photonics, VPP-m 633) with variable topology charges, from one to eight, was implemented. The charge map and corresponding vortex geometry are shown in Figure 2b. A spherical lens with a focal length of 1000 mm was placed after the spiral phase plate to focus the laser beams passing through the spiral phase plate. All the experimental patterns were imaged and recorded by using a CCD camera.
4. Results and Discussion

Figure 3 shows the experimental results which have one-to-one correspondence with the numerical results shown in Figure 1. It can be seen that all experimental patterns agree very well with the numerical results. Such excellent agreement, not only confirms the validity of the present analysis, but also presents a distinctive approach for the generation of structured laser beams with novel amplitude and phase. Moreover, the comprehensive agreement ensures that the derived formula can be exploited to investigate the phase structures of those patterns. The phase structures of the optical patterns can be evaluated from $\Theta(\rho, \phi; \theta_n) = \tan^{-1}\left\{\frac{\text{Im}[\Phi_Q(\rho, \phi; \theta_n)]}{\text{Re}[\Phi_Q(\rho, \phi; \theta_n)]}\right\}$, where $\text{Im}[\Phi_Q(\rho, \phi; \theta_n)]$ and $\text{Re}[\Phi_Q(\rho, \phi; \theta_n)]$ are the imaginary and real parts of the wave function $\Phi_Q(\rho, \phi; \theta_n)$.

Figure 4 shows the experimental results for the propagation invariant region for the case with $Q = 6$. The second row of Figure 4 displays the $\Theta(\rho, \phi; \theta_n)$ for the boxed region shown in the first row of Figure 4. For the cases of $n = 0$ and $n = 3$, there were no obvious phase singularities in the phase distribution. These optical fields do not carry a topological charge. On the other hand, the phase distribution displays phase singularities with a clockwise increasing phase for the cases of $n = 1$ and $n = 2$, corresponding to the vortices with a topological charge 1 and 2. Although the intensity distribution for $n = 4$ and $n = 5$ are equal to the cases of $n = 2$ and $n = 1$, the phase singularities show a counterclockwise increasing phase, corresponding to the vortices of topological charge $-1$ and $-2$. With the wave field $\Phi_Q(\rho, \phi; \theta_n)$, the current density $\vec{J} \propto (\Phi_Q^* \nabla \Phi_Q - \Phi_Q \nabla \Phi_Q^*)$ can be calculated to obtain physical aspects of the vortices. The third row of Figure 4 displays the probability current density for the boxed region which is shown in the first row of Figure 4. Again, the current density does not exhibit any vortex structures for $n = 1$ and $n = 3$, but exhibits various features of vortex lattices for the other cases.
Figure 3. Experimental interference patterns that correspond one-to-one with the theoretical results shown in Figure 1.
Figure 4. Experimental crystalline patterns (the first row) for the case of $Q = 6$. The contour plots of the phase fields $\Theta(\rho, \phi, \theta_n)$ (the second row) for the boxed region in the first row. The last row: the probability current density corresponds to the second row.

Figure 5 shows the result for the case of $Q = 21$. The intensity distribution reveals the characteristics of 21-symmetry kaleidoscopic wave patterns. Once again, the complicated structure of phase singularities can be resolved. As $n$ increases, the central area exhibits phase singularities with corresponding topological charges and ends up with the eighth order. The experimental results verify that the numerous structures of vortex lattice can be generated in the presented method.

Figure 5. Experimental interference patterns and the corresponding phase fields for the case of $Q = 21$ with various relative phases.
5. Conclusions

In summary, we have theoretically demonstrated a convenient approach for generating vortex-lattice structures by multi-beam interference. The numerical calculations presented a variety of crystalline and kaleidoscopic patterns. Based on the numerical analysis, we employed a collimated coherent laser to illuminate a stencil mask with multiple apertures systematically arranged on a ring and passing through a spiral phase plate with variable topology charges, from one to eight. The experimental results were found to be in a good agreement with the theoretical calculations. The excellent agreement between the experimental and theoretical results, not only validates the present method, but also manifests the structure of vortex lattices. We believe that the presented approach represents a promising method for generating various optical vortex arrays.

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