Relativistic Contributions to Deuteron Photodisintegration in the Bethe–Salpeter Formalism

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Abstract

In plane wave one-body approximation the reaction of deuteron photodisintegration is considered in the framework of the Bethe–Salpeter formalism for two-nucleon system. Results are obtained for deuteron vertex function, which is the solution of the homogeneous Bethe–Salpeter equation with a multi-rank separable interaction kernel, with a given analytical form. A comparison is presented with predictions of non-relativistic, quasipotential approaches and the equal time approximation. It is shown that important contributions come from the boost in the arguments of the initial state vertex function and the boost on the relative energy in the one-particle propagator due to recoil.

25.10+s; 25.20.Dc; 21.45+v; 11.10.St

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I. INTRODUCTION

Recently experiments on elastic and quasi-elastic lepton-nucleus scattering have been considered as one of the fundamental and reliable resources of probing structure of nuclei over a wide range of energy-momentum transfer. A specific place among such reactions is taken by deuteron photodisintegration. The reaction $\gamma + d \rightarrow p + n$ has been thoroughly investigated in the region of small and medium energies of the incoming $\gamma$-quantum (an excellent review of experiments and theoretical frameworks applied to the study of the process can be found in Ref. [1]). The theoretical analysis of experimental data obtained in these kinematic regions produced an important information on deuteron wave function, allowed one to discriminate various contributions to the differential cross section which stem from meson-exchange currents, isobar configurations, spin-orbit and further relativistic corrections.

At the present time deuteron photodisintegration is rated among the leading trends at experimental facilities around the world. Here it is worth-while to point out a performed experiment with the polarized LADON gamma ray beam [2] to investigate the existence of narrow dibaryonic resonances in reaction on nuclei at a low excitation energies, an experimental program carried out on the linearly polarized photon beam at YEREVAN Synchrotron to study the cross section asymmetry of the deuteron photodisintegration process in the energy range 0.9–1.7 GeV for proton center-of-mass angle of 90° [3]. This is connected the problem of the validity of the constituent quark counting rules at energies of a few GeV. Present experiments at SLAC with photons of an energy 2.8 GeV [4] and proposed experimental programs in RCNP Cyclotron with photon beam at an energy up to 8 GeV [5] allow one to focus on investigation of hadronic systems at quark level.

Future studies at the TJNAF are approved to extend measurements on the differential cross section to a wide range of reaction angles at high energies. The first measurements of the cross section from $\gamma + d \rightarrow p + n$ up to 4.0 GeV are in good agreement with previous low energy measurements [6]. A comparison of the high precision Mainz data [7,8] (over the photon energy range 100–800 MeV) with meson-exchange models incorporating relativistic effects [9–11] shows that one still lacks of the ultimate conclusion on the role of non-nucleonic degrees of freedom in nuclei. It is also understood that further development of deuteron photodisintegration theory is needed and consistent treatment of relativistic effects have to be applied. This demands a construction of a genuine relativistic formalism for the description both the deuteron structure and the reaction mechanism.

The formulation of a completely relativistic formalism of hadronic bound states and reaction with them can be developed on the basis field-theoretical Bethe–Salpeter (BS) equation for the nucleon-nucleon (NN) scattering [12]. However approximate methods evolved from the BS formalism due to substantial mathematical and computational problems. We bear in mind the quasipotential (QP) approach, which reduces the 4-dimensional BS equation to a relativistic 3-dimensional equation: the Logunov–Tavkhelidze [13], Blankenbeckler–Sugar (BbS) [14] and Gross [15] and other approximations.

Actually the relativistic description of the reaction with the deuteron is extensively developed in the framework of the BS formalism, which is explicitly Lorentz covariant, provides two-body unitarity and includes nucleon and anti-nucleon degrees of freedom in a hadronic state in a symmetrical manner. So far applicability of the BS equation to reactions with the
deuteron has been bound to: elastic electron-deuteron scattering [12], elastic \(pd\)-backward
scattering [16], inclusive quasi-elastic electron-deuteron scattering [17,18] and description of
the static properties of the deuteron [19,20]. The common feature of these processes is that
the reaction amplitude in the impulse approximation is proportional to the averaging of cur-
rent operator between deuteron states. On the contrary, the reaction amplitude for deuteron
photodisintegration is a non-diagonal matrix element between the incoming deuteron and
outgoing 2N state.

An approach towards a covariant description of the reaction \(\gamma + d \rightarrow p + n\), in which
the basic degrees of freedom are taken to be hadronic, is developed in the framework of
the dispersion relation technique for laboratory photon energies \(E_\gamma < 400\) MeV [21] (this
technique is appropriate for the analysis of partial wave amplitudes and takes into account
final state rescatterings). Other approach describes deuteron photodisintegration as a simple
parameterization of covariant deuteron in terms of a hard component and imposing gauge
invariance on the cross section [22]. This approach looks into the onset of scaling in exclusive
photodisintegration for high energies in the range 1-4 GeV, as it can be brought about by
mechanisms which are different from those of pQCD.

The BS formalism presents a separate view on the problem. Formal application of the BS
formalism to deuteron electro- and photodisintegration can be found in Ref. [23]. Here the
rigorous derivation of the scattering amplitude in terms of BS amplitude of the initial and
final 2N states and Mandelstam electromagnetic (EM) vertex, which comprises the one-body
and two-body parts, is proposed. Although the numerical analysis has not been performed.
In paper [24] it is proposed a framework based on the BS equation approach. This is applied
to both elastic and inelastic electron scattering. But complete calculations are performed
within a QP framework.

The aim of this paper is to apply the fully relativistic analysis of deuteron photodisin-
tegration in the framework BS formalism, to segregate and estimate contribution of various
relativistic effects in the differential cross section.

This paper is organized as follows. In Sec. II we briefly discuss a connection between the
BS formalism and QP approach, and equal time (ET) approximation. All the relativistic
formulations of the 2N dynamics, exploiting relativistic separable interactions, are applied
to the study of the deuteron and its inelastic observables. Basic formulae for definition
of the Minkowski-space BS amplitude for bound and 2N continuum states are given. We
also introduce formulae for the relativistic separable interaction kernel. Solving the BS
equation with the separable interaction, we find the vertex function used in computation of
the unpolarized cross section.

In Sec. III we describe the procedure to derive deuteron photodisintegration cross section.
The EM interaction with 2N system in the framework of the BS formalism is determined
by the Mandelstam vertex, which generally depends on properties of the interaction kernel.
The scheme incorporates two-body part of the Mandelstam vertex in order to guarantee the
gauge independence.

Sec. IV deals with derivation of the simplest contribution to the EM current matrix
elements of deuteron break-up, namely the plane wave one-body approximation (PWOA).
We discuss the transformation properties of the deuteron vertex function between the lab-
oration and c. m. frames. Generally one-body part of the Mandelstam vertex involves half
off-mass-shell \(\gamma\)NN form factors. However, as consequence of gauge invariance the real pho-
ton scattering amplitude does not contain off-shell effects. In the PWOA the EM current matrix elements are proportional to deuteron vertex function which is taken at certain value of the relative energy (relative time) and 3-momentum. These are directly related to the photon energy-momentum transfer. Finally, we write down the expression for the differential photo-disintegration cross section.

Sec. V is devoted to analysis of relativistic effects required by the principles of relativity. These are relativistic kinematics and dynamics, the relative energy dependence (or retardation), the Lorentz contraction and spin precession. We compare results of our fully relativistic analysis with those of conventional non-relativistic (NR) models, the QP approach and the ET approximation.

In Sec. VI we conclude about our results.

II. RELATIVISTIC DESCRIPTION OF TWO-NUCLEON SYSTEM

Formulation of integral equations for amplitudes is commonly taken as the starting point for discussing relativistic scattering and bound state problems in the strongly interacting systems within quantum field theory.

The off-shell $T$-matrix for the elastic scattering of two nucleons with the relative 4-momentum $p, p'$ and the total momentum $P$ satisfies to the inhomogeneous BS equation. In the momentum space this is the four-dimensional (4-D) integral equation with respect to the relative momentum $k = (k_0, \mathbf{k})$

$$T(p, p'; P) = \mathcal{V}(p, p'; P) + \frac{i}{4\pi^3} \int d^4k \mathcal{V}(p, k; P) G_0(k; P) T(k, p'; P),$$

where $\mathcal{V}$ is an interaction kernel obtained by summing of all irreducible 2N Feynman diagrams in a given field-theoretical model of the NN interaction, and $G_0(k; P)$ — the free two-nucleon propagator

$$G_0(k; P) = \frac{[\hat{P}/2 + \hat{k} + m]^{(1)}}{(\frac{P^2}{2} + k^2)^2 - m^2 + i\epsilon} \cdot \frac{[\hat{P}/2 - \hat{k} + m]^{(2)}}{(\frac{P^2}{2} - k^2)^2 - m^2 + i\epsilon}.$$  \hspace{1cm} (2)

The BS amplitude for the 2N scattering state, $P^2 = s > 4m^2$, is expressed in terms of the half off-shell $T$-matrix and the propagator function $G_0$ as follows

$$\chi(k; \hat{p}P) = [4\pi^3 i\delta^{(4)}(\hat{p} - k) - G_0(k; P) T(k, \hat{p}; P)] \chi^{(0)}(\hat{p}; P), \quad \hat{p} \cdot P = 0.$$ \hspace{1cm} (3)

where $\hat{p}$ denotes on-mass-shell relative 4-momentum and $\chi^{(0)}(\hat{p}; P)$ is the amplitude for the motion of free nucleons. The second term comprises rescattering contributions.

When the two-body system has a bound state of mass $M_d$, the $T$-matrix has a pole at $P^2 = M_d^2$

\footnote{For sake of simplicity we omit writing spinor, $\rho$-spin and polarization quantum numbers of amplitudes below}
\[ T(p, p'; P) \propto \frac{\Gamma(p; P)\bar{\Gamma}(p'; P)}{P^2 - M_0^2}, \]  

(4)

where \( \Gamma \) is the vertex function and \( \bar{\Gamma} \) is its conjugate. The vertex function of the 2N bound state satisfies to the homogeneous BS equation with the same interaction kernel

\[ \Gamma(p; P) = i \frac{4}{d_\pi^3} \int d^4k V(p, k; P)\psi(k; P), \quad P^2 = M_0^2, \]  

(5)

where the bound amplitude is defined as:

\[ \psi(k; P) = G_0(k; P)\Gamma(k; P). \]  

(6)

The Eqns. (1) and (5) are manifestly Lorentz covariant and preserve the two-body elastic unitarity. Moreover both equations do not discriminate between the positive and negative energy states, yielding transformation properties of the calculated amplitudes consistent with the charge conjugation and time invariance.

Although the BS calculations are feasible, a rigorous treatment of Eqs. (1) and (5) is rather complicated due to the appearance of a relative energy in loop integrals and the presence of strong singularities in the interaction kernel. The great theoretical effort have been applied to obtain 3-D bound-state equations from the 4-D one. A simple method to obtain an approximate 3-D equation has been developed by the QP approach, in which the BS equation is reduced to a 3-D equation by making use of a new two-nucleon propagator with the internal relative energy variable restricted to a fixed value.

But there are some shortcomings in the relativistic equations obtained from the BS equation via the 3-D reduction. First of all one meets conceptual difficulties with the consistent treatment of both the 2N system and its EM interactions [23]. Secondly by putting particles on mass-shell or using the positive energy projection operators, one results in a equation which violate the charge conjugation and CPT symmetries. Importance of the constraint put by discrete symmetries is discussed in Ref. [26].

An alternative choice, which have been made to study the EM interactions, is the 'instant' or equal time approximation [24,27]. The instant-ET approximation to EM current operator is a 3-D reduction consistent with charge conjugation and unitarity. This approach has been applied to study of the elastic electron scattering case and the deuteron breakup in electrodisintegration process.

In this paper we extend the ET choice in a systematic way to describe the process of deuteron photodisintegration. In the PWOA the ET approximation can be obtained simply by replacing the initial deuteron vertex function by the BbS vertex function. The relative energy variable in the one-particle propagator is prescribed by the condition that the final state describes on-mass-shell particles [24].

The covariant BbS prescription, \( P \cdot \hat{k} = 0 \), puts the relative energy equal to

\[ \hat{k}_0 = \frac{1}{2P_0}(E^2_{\frac{1}{2}P+k} - E^2_{\frac{1}{2}P-k}), \]

where \( E_k = \sqrt{m^2 + k^2} \) and \( \hat{k} \) denotes the restricted 4-vector \( k \). The prescription leads to a relativistic equation of motion for the moving 2N system. By means of the boost it is
transformed to the Schrödinger-type equation in the rest frame[1].

The BbS prescription is simple only in the 2N rest frame, \( P_0 = (\sqrt{s}, 0) \), where the BbS propagator \( G_{BbS}(\hat{k}; P) \) is given by

\[
G_{BbS}(\hat{k}; P_0) = -2\pi i \delta(\hat{k}_0) G(\hat{k}; P_0) \Lambda^{(1)}(k_1) \Lambda^{(2)}(k_2),
\]

where \( \Lambda^{(i)}(k_i) \) is the positive energy projection operators and \( G(\hat{k}; P_0) = \frac{4m^2}{E_k(s - 4E_k^2 + i\epsilon)} \). The delta-function in Eq. (7) sets the relative energy equal to zero putting both nucleons equally off-mass-shell. It is an attractive feature in case of the deuteron because it treats both nucleons in a symmetrical way and, as consequence, it is consistent with the Pauli principle.

The QP wave function is defined in terms of the bound state vertex function \( \hat{\Gamma}(\hat{k}; P) \) defined in terms of the positive energy Dirac spinors [11]:

\[
\phi_{QP}(\hat{k}; P) = \sqrt{Q(\hat{k}; P)} m G(\hat{k}; P) \hat{\Gamma}(\hat{k}; P),
\]

where \( Q(\hat{k}; P) = \frac{\hat{p}_0}{M_d} \sqrt{m^2 - \hat{k}^2} \). The equation satisfied by the vertex function follows from the BbS reduction of the BS equation:

\[
\hat{\Gamma}(\hat{p}; P) = \frac{1}{2\pi^2} \int d^3 k \hat{V}(\hat{p}, \hat{k}; P) G(\hat{k}; P) \hat{\Gamma}(\hat{k}; P),
\]

where \( \hat{V}(\hat{p}, \hat{k}; P) \) is a Lorentz invariant quasipotential with all relative 4-momenta are restricted by the BbS condition. The QP wave function \( \phi_{QP}(k; P_0) \) at the rest frame of the deuteron, \( P_0 = (M_d, 0) \), is expressed in terms of Lorentz invariant \( \sqrt{\frac{T}{M_d}} \phi_{QP}(\hat{k}; P) \). By means of the boost relativistic equation of motion (9) for the moving deuteron transforms into equation in the rest frame. This invariance property yields the NR equation for the wave function:

\[
\frac{M_d^2 - 4E_p^2}{4m_p} \phi_{QP}(p_0; P_0) = \frac{1}{2\pi^2} \int d^3 k V(p_0, k_0; P_0) \phi_{QP}(k_0; P_0),
\]

where 3-momenta \( p \) and \( k \) in the moving frame are mapped by the boost transformation to \( p_0 \) and \( k_0 \) at the rest frame, respectively, and \( V \) is the quasipotential modified by ‘minimal relativity’.

Further the discussion concerns partial decomposition of the BS vertex function of the deuteron \( \Gamma(k; P) \). For definiteness at the rest frame one has (we highlight dependence on the spin projection):

\[
\Gamma_M(p; P_0) = \sum_\alpha g_\alpha(p_0, |p|; \sqrt{s}) \Gamma^\alpha_M(-p), \quad (M = \pm 1, 0), \quad \sqrt{s} = M_d.
\]

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2 A detailed and systematic exposition of the covariant QP formalism for description of the electromagnetic (EM) properties and reactions involving the deuteron is given in Ref. [28].
Here summation index $\alpha$ is determined by the following quantum numbers: $S = 0, 1$ — spin, $L = 0, 1, 2$ — angular momentum, $J = 1$ — total angular momentum and $\rho$-spin — the projection of the total energy spin of the nucleon and anti-nucleon states; $\Gamma^\alpha_M(-p)$ is the spin-angular functions and $g_\alpha$ is the partial amplitudes. Eight partial states contribute to Eq. (11). Apart from two channels with the positive energy intermediate states, viz. $^3S^+_1-^3D^+_1$, six ’extra’ states, which account for anti-nucleon degrees of freedom, come into play. In the spectroscopic notations $^{2S+1}L^\rho_J$ these are

$$^3P^e_1, \quad ^3P^o_1, \quad ^1P^e_1, \quad ^1P^o_1, \quad ^3S^-, \quad ^3D^-,$$

where indexes $e$ and $o$ stands for the even and odd parity relative to the $\rho$-spin functions. The partial amplitudes $^1P^e_1, \quad ^3P^o_1$ are even and $^1P^o_1, \quad ^3P^e_1$ are odd functions in the relative energy variable.

Influence of admixtures of $P$-states and their contribution in interference with the positive energy states to observables in deuteron breakup and elastic proton-deuteron backward scattering calculated within the BS formalism is considered in Refs. [14,29,30]. It is shown that $NN$ pair EM current term in the NR approach can be constructed from the $P$-state in the deuteron BS amplitude. The separate effect of the pair current term, with respect to the one-body, is expected to give constructive interference followed by a destructive interference with increasing photon energy in the range 0.1–0.7 GeV [31].

In the present paper we focus on the positive energy states only. So we have two channels, $^3S^+_1−^3D^+_1$, and the corresponding vertex functions can be written in the matrix form [20]:

$$\sqrt{8\pi} \Gamma^3_{\lambda}(p; P(0)) = \mathcal{N}^2_p(m + \hat{p}_1)(1 + \gamma_0)\hat{e}_\lambda(m - \hat{p}_2)g_0(p_0, |p|; s),$$

$$\sqrt{16\pi} \Gamma^3_{\lambda}(p; P(0)) = -\mathcal{N}^2_p(m + \hat{p}_1)$$

$$\times (1 + \gamma_0) \left( \hat{e}_\lambda + \frac{3}{2}(\hat{p}_1 - \hat{p}_2)\frac{(p \cdot e_\lambda)}{p^2}\right)(m - \hat{p}_2)g_2(p_0, |p|; s),$$

where $s = M^2_d$, $p_1 = (E_p, \mathbf{p})$ $p_2 = (E_p, -\mathbf{p})$ are on-mass-shell 4-momenta, $\mathcal{N}^{-1}_p = \sqrt{2E_p (m + E_p)}$ is the normalization factor and $e_\lambda = (0, e_\lambda)$ is 4-polarization vector of the deuteron:

$$\sum_{\lambda=-1}^{+1} e^\mu_\lambda e^{\nu*}_\lambda = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2_d}, \quad e_\lambda \cdot P = 0. \quad (13)$$

We are interested in calculating matrix elements of the EM current operator between a state containing free nucleons and 2N bound state. To this end we need to take into account change in the state amplitudes, when the BbS prescription is applied to the S-matrix element for this process. We can utilize the normalization condition for the bound state amplitude. Let us suppose that the kernel $\mathcal{V}$ is independent of the total 4-momentum $P$. Then we obtain

$$1 = - \int \frac{d^4 k}{2\pi^2 i} \Gamma(p; P) \frac{\partial G(p; P)}{\partial P^2} \bigg|_{P^2 = M^2_d} \Gamma(p; P). \quad (14)$$

In terms of the radial partial vertex functions $g_L$ this conditions takes the form
\[
\frac{1}{2\pi^2 i M_d} \int_0^\infty dk_0 \int_0^\infty d|k| k^2 \frac{g_L(k_0, |k|; s)^2(E_k - \frac{M_d}{\sqrt{2}})}{((\frac{M_d}{\sqrt{2}} - E_k + i\epsilon)^2 - k_0^2)^2} = P_L, \quad P_0 + P_2 = 1.
\]

We find respectively for the QP vertex function \(\hat{g}_L\):

\[
\frac{2m^2}{\pi^2 M_d} \int_0^\infty d|k| k^2 \frac{\hat{g}_L(0, |k|; s)^2}{E_k(M_d^2 - 4E_k^2)^2} = P_L.
\]

One can deduce out of these Eqns. that the BS and QP vertex functions are formally related to each other as follows

\[
g_L(k_0, |k|; s) \propto \sqrt{\frac{4m^2}{\pi E_k}} \frac{\hat{g}_L(0, |k|; s)}{4E_k^2 - M_d^2}.
\]

In Eq. (17) the QP function (10) is related to the Schrödinger wave function by minimal relativity:

\[
\phi_{\text{QP}}(k; P(0)) \equiv \sqrt{\frac{m}{E_k}} \phi_{\text{NR}}(k).
\]

The presence of strong singularities in the interaction kernel can be avoided by special prescriptions referred to imply analytical properties. The ladder approximation with the interaction kernel of the BS equation \(V\) to be a sum of one boson exchange diagrams is widely used in solving BS equation for the NN scattering. In this case a solution of Eqn. (5) (for example, we refer to papers [16]-[20]), can be found in Euclidean space after a Wick rotation. However that presents a substantial obstacle when one calculates observable in terms of the BS amplitude. Actually an analytical continuation of the solution in the complex \(k_0\)-plane back to the real axis is a procedure which carries ambiguities and extremely laborious. The physical solution can be obtained via the method based on the Perturbation Theoretic Integral Representation of Nakanishi [33]. The BS equation for bound states is solved in terms of a generalized spectral representation directly in Minkowski space [34]. But the approach is developed for bound states in scalar theories.

An alternative way to solve the BS equation is to use a non-local separable interaction kernel [35-37]:

\[
\mathcal{V}(p, p'; P) = \sum_{a,b=1}^{N} \lambda_{ab} v_a(p; P)v_b(p'; P),
\]

where \(\lambda_{ab}\) is a symmetrical matrix. In this case the bound state vertex function and the scattering \(T\)-matrix is obtained in Minkowski space. They exhibit analytical properties determined by form factors \(v_a(p; P)\). The covariant form factors includes a dependence on \(P^2\), \(p^2\) and \(p \cdot P\). Leaving the dependence on \(p^2\) only means a simple procedure to construct relativistic separable interactions to be used in the BS equation. That is done in study of 2N states in the BS formalism with the separable form of interaction, see Refs. [36,38,39]. These form factors are not expected to be genuine separable approximations to a realistic NN interactions.
In this paper we use the separable kernel of rank three ($N = 3$ in Eqn. (18)) for computation of the deuteron photodisintegration cross section. This interaction kernel is a relativistically covariant generalization of the NR Graz-II potential for the description of the phase shifts of the NN scattering in the coupled $^3S_1-^3D_1$ waves (details can be found in [32]). The analytical properties of the radial vertex function is determined by poles in the relative energy:

\[
g_0(k_0, |\mathbf{k}|; s) = A(s) \frac{1 - \gamma_1 k^2}{(k^2 - \beta_{11}^2)^2} + B(s) \frac{k^2}{(k^2 - \beta_{12}^2)^2},
\]

\[
g_2(k_0, |\mathbf{k}|; s) = C(s) \frac{k^2(1 - \gamma_2 k^2)}{(k^2 - \beta_{21}^2)^2(k^2 - \beta_{22}^2)^2}, \quad s = M_d^2,
\]

where $k^2 = k_0^2 - k^2$, the coefficients $A$, $B$, and $C$ are determined by the homogeneous set of three algebraic equations, the parameters $\beta_{ab}$ and $\gamma_a$ are chosen to reproduce $^3S_1-^3D_1$ NN scattering phase shifts up to a laboratory energy of $E_{\text{Lab}} = 500$ MeV, the low-energy NN scattering parameters and the static deuteron properties (the binding energy, the quadrupole and magnetic moments). We adopted the parameters of the interaction kernel corresponding to the value of $^3D_1^+ \text{-state probability}$ $P_2 = 4\%$ and $6\%$. The actual parameters $\beta_{ab}$ and $\gamma_a$ are chosen to have such values that the resulting $T$-matrix $T_{LL'}(p_0, p, p_0', p'; s)$ satisfies the exact two-body unitarity relation at least up to a nucleon kinetic energies $E_{\text{Lab}} = \frac{2}{m} \beta_{11} (m + \beta_{11}^4)$ with $\beta_{11} = 231$ MeV, which is about 500 MeV in the laboratory system.

This is repeated using the QP approximation to the BS equation. The QP vertex function is the solution of the homogeneous BS equation with the BbS version of the Green function (0) and the interaction kernel equivalent to the NR Graz-II potential. The values of the parameters $\lambda_{ab}$ are changed relative to the BS parameters to reproduce resulting deuteron properties, low-energy scattering parameters and the phase shifts. The resulting radial part of the deuteron vertex function $\hat{g}_L$ depends on the relative 3-momentum as follows

\[
\hat{g}_0(0, |\mathbf{k}|; s) = \hat{A}(s) \frac{1 + \gamma_1 k^2}{(k^2 + \beta_{11}^2)^2} + \hat{B}(s) \frac{k^2}{(k^2 + \beta_{12}^2)^2},
\]

\[
\hat{g}_2(0, |\mathbf{k}|; s) = \hat{C}(s) \frac{k^2(1 + \gamma_2 k^2)}{(k^2 + \beta_{21}^2)^2(k^2 + \beta_{22}^2)^2}, \quad s = M_d^2.
\]

The particular feature of the BbS reduction is that all propagators are reduced to the static form. Speaking in language of the meson-exchange model, the BbS reduction completely ignores the retardation or the relative energy dependence in the BS amplitude which arises from non-instantaneous effects in the NN interaction.

### III. DEUTERON PHOTODISINTEGRATION CROSS SECTION

Let us consider disintegration of a deuteron with total 4-momentum $K$ by a photon with 4-momentum $q^\mu$, $q^2 = 0$, into a free neutron-proton (np) pair, characterized with the total and relative 4-momenta $P$ and $p$, respectively. In the rest frame of the np pair, i.e. $P^\mu_{(0)} = (\sqrt{s}, \mathbf{0})$, $p^\mu = (0, \mathbf{p})$, where $\sqrt{s}$ — is the total energy of the pair, the differential absorption cross section of a photon with energy $\omega$ can be written as
\[
\frac{d\sigma}{d\Omega_p} = \frac{\alpha}{16\pi s} \left| \frac{P}{\omega} \epsilon_{\lambda} \cdot \mathcal{M}_{fi} \right|^2
\]

(21)

with \( \alpha = e^2/(4\pi) \) is the fine structure constant, \( \mathcal{M}_{fi}^\mu \) — the invariant amplitude, which is the transition matrix element \( \mathcal{M}_{fi}^\mu = \langle f | \hat{J}^\mu | i \rangle \) of the EM current operator \( \hat{J}^\mu \) between the deuteron bound state and the 2N continuum; \( \epsilon_{\lambda} \) is a photon polarization 4-vector with \( \lambda = \pm 1 \). Momentum conservation at the photon-deuteron vertex gives \( K + q = P \).

Since polarizations of the particles involved in the process will not be considered here, averaging and summing over the photon and nuclear polarizations in the initial and final states, respectively, are assumed. We may choose such a coordinate system where the photon 3-momentum is along a \( Z \) axis: \( q^\mu = (\omega, 0, 0, \omega) \). In the laboratory system, being the rest frame of the deuteron, the deuteron 4-momentum \( K(0) = (M_d, 0) \) and the photon energy is denoted as \( E_\gamma \).

In experiments on two-body photodisintegration of the deuteron the differential cross section \( (21) \) is viewed as a function of the lab photon energy \( E_\gamma \) and angle \( \Theta_p \) between incoming-photon and outgoing-proton 3-momenta in the c. m. system of the np pair. One can obtain the following kinematic relations between the laboratory photon energy and variables in the c. m. frame:

\[
|p| = \sqrt{\frac{s}{4} - m^2}, \quad s = M_d^2 + 2E_\gamma M_d, \quad \omega = \frac{M_d}{\sqrt{s}} E_\gamma.
\]

(22)

Following the Ref. [23] the invariant amplitude \( \mathcal{M}_{fi}^\mu \) can be written in terms of the BS amplitude of the initial (6) and final (3) states as follows:

\[
\mathcal{M}_{fi}^\mu = \frac{1}{4\pi^3} \int d^4k d^4l \bar{\chi}_{Sm_s}(l; pP)\Lambda^\mu(l, k; P, K)\psi_M(k; K),
\]

(23)

where \( S = 0, 1 \) is the total spin of the np pair and \( m_s \) is its projection on to the \( Z \) axis, \( M \) is a projection the total angular momentum of the deuteron; \( \Lambda^\mu \) denotes the Mandelstam vertex which determines the EM interaction with 2N system in the framework of the BS formalism.

Let us make some remarks on current conservation. The Mandelstam vertex consists of one- and two-body parts, \( \Lambda^\mu(p, k; P, K) = \Lambda^{[1]}\mu(p, k; P, K) + \Lambda^{[2]}\mu(p, k; P, K) \). The second part of the Mandelstam vertex determines two-body contributions of the conserved EM current. The specific form of \( \Lambda^{[2]}\mu(p, k; P, K) \) depends on a given model for the interaction kernel in the BS equation and it cannot be associated with the pair and meson exchange currents in the NR approach. The gauge independence of EM current transition matrix element, \( q \cdot \mathcal{M}_{fi} = 0 \), will be fulfilled if the Mandelstam current meets the following relations:

\[
iq \cdot \Lambda^{[1]}(p, k; P, K) = \left\{ \pi_p(1)\delta \left( p - k - \frac{q}{2} \right) \left[ S^{(1)}(\frac{K}{2} + k)^{-1} - S^{(1)}(\frac{P}{2} + p)^{-1} \right] S^{(2)}(\frac{K}{2} - k)^{-1} \right. \\
+ \pi_p(2)\delta \left( p - k + \frac{q}{2} \right) \left[ S^{(2)}(\frac{K}{2} - k)^{-1} - S^{(2)}(\frac{P}{2} - p)^{-1} \right] S^{(1)}(\frac{K}{2} + k)^{-1} \right\}
\]

(24)

(25)
\[ i q \cdot A^{[2]}(p, k; P, K) = \sum_{l=1,2} \left[ \pi_p(l) \mathcal{V} \left( p + (-1)^l \frac{q}{2}, k; K \right) - \mathcal{V} \left( p, k - (-1)^l \frac{q}{2}; K \right) \pi_p(l) \right], \tag{26} \]

where \( S^{(t)}(p) \) is the fermion propagator and \( \pi_p(l) = 1/2[1 + \tau_z(l)] \) is the projector on to proton state. Moreover the BS amplitudes for the initial and final states have to satisfy to the BS equations with the same interaction kernel.

In Ref. [36] it was shown that the gauge independence condition for the elastic electron-deuteron scattering amplitude is fulfilled in the impulse approximation for many-rank separable BS kernels, implying that \( q \cdot M^{[2]}_{fi} = 0 \). In case of the deuteron breakup in the final 2N state isospin \( I = 1 \) states are present, yielding a nonvanishing isovector contribution, \( q \cdot M^{[2]}_{fi} \neq 0 \). Consequently the two-body Mandelstam current operator should be added to guarantee the gauge independence.

IV. PLANE WAVE ONE-BODY APPROXIMATION

The amplitude (23) contains the FSI contributions of the final np pair. This means that the half off mass shell NN scattering \( T \)-matrix is needed, v. s. Eqn. (3). In this paper rescattering contributions and the pair processes in the framework of the BS formalism will be not taken into account. The neglect of FSI is a shortcoming of our present work. At present this is bound up with computational difficulties. In a forthcoming paper will be considered the FSI interactions from \( J = 0, 1 S_0, \) and \( J = 1, 3 S_1 - 3 D_1 \), channels and work on the pair processes is in progress. Full and detailed analysis of the FSI cannot clearly be avoided without reconsidering an entirely different interaction kernel.

In this investigation we confine ourselves to the simplest contribution to the EM current, where the photon couples to one of the two nucleons in the deuteron and the FSI between the outgoing nucleons are dropped — the plane-wave one-body approximation.

A. The BS amplitude for the continuous spectrum

According to the approximations we made the BS amplitude of the final state given in Eqn. (3) is the antisymmetric combination of two free Dirac positive energy spinors \( \chi_{Sm_s}(k_0, k; \sqrt{sp}) = 4\pi^3 \delta(k_0) \times \left[ \chi_{Sm_s}(p)(\eta_0 + \eta_1)\delta^{(3)}(k - p) + (-1)^{S+1}\chi_{Sm_s}(-p)(\eta_0 - \eta_1)\delta^{(3)}(k + p) \right] \),

where \( \chi_{Sm_s}(p) = \sum_{\lambda_p, \lambda_n = \pm \frac{1}{2}} C^{Sm_s}_{\lambda_p \lambda_n} u_{\lambda_p}(p)u_{\lambda_n}(-p) \); \( \eta_0 \) and \( \eta_1 \) stands for isospin singlet and triplet functions respectively. Since the outgoing nucleons are on mass shell, we have constraint \( P \cdot \hat{p} = 0 \) which keeps the relative energy \( p_0 \) to be equal to zero in the rest frame of the np pair.

\(^3\text{We use the covariant normalization of the Dirac spinors, } u^+u = 2E\)
B. The BS amplitude for the bound state

The BS equation for the deuteron is solved in its rest frame. Since the vertex function in Eqn. (23) is referred to a moving frame, it has to be boosted to its rest frame. The Lorentz transformation between the laboratory and c. m. frames is given by

\[ K^\mu = L^\mu_\nu K^\nu_{(0)}, \]

\[ P^\mu = L^\mu_\nu P^\nu_{(0)}, \]

where \( K^\nu_{(0)} \) and \( P^\nu_{(0)} \) — 4-momenta of the deuteron and np-pair in their rest frame, respectively. As only boost along the \( Z \) axis is needed, an explicit expression for the matrix \( L^\nu_\mu \) is given by

\[
L^\nu_\mu = \begin{pmatrix}
\sqrt{1+\eta} & 0 & 0 & -\sqrt{\eta} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\sqrt{\eta} & 0 & 0 & \sqrt{1+\eta}
\end{pmatrix},
\]

where the Lorentz-factor \( \gamma \equiv \sqrt{1+\eta} \) and \( v\gamma \equiv \sqrt{\eta} \), with \( v \) being the velocity of c. m. frame in the laboratory frame are defined in terms of the dimensionless boost parameter \( \eta \):

\[
\sqrt{\eta} = \frac{E\gamma}{\sqrt{s}}, \quad \sqrt{1+\eta} = \frac{E\gamma + M_d\sqrt{s}}{\sqrt{s}}.
\]

Under the boost the vertex function is transformed according to the general rule of the transformation of spinor amplitudes

\[
\Gamma_M(k; K) = \Lambda(\mathcal{L})\Gamma_M(\mathcal{L}^{-1}k; K_{(0)})
\]

with \( \Lambda(\mathcal{L}) = \Lambda^{(1)}(\mathcal{L})\Lambda^{(2)}(\mathcal{L}) \), where \( \Lambda^{(l)}(\mathcal{L}) \) is the boost operator in the spinor space of \( l \)th nucleon corresponding to \( \mathcal{L} \):

\[
\Lambda^{(l)}(\mathcal{L}) = \left( \frac{1 + \sqrt{1+\eta}}{2} \right)^{\frac{l}{2}} \left( 1 + \frac{\gamma_0\gamma_3\sqrt{\eta}}{1 + \sqrt{1+\eta}} \right)^{(l)}.
\]

At \( \eta \to 0 \) the matrix (28) and operator (30) turn to unity, i. e. \( \mathcal{L} \to I \) and \( \Lambda \to I \). This corresponds to the static limit for the BS amplitude and takes place on threshold of deuteron photodisintegration, a case of \( E_\gamma/m \ll 1 \) (v. s. Eqn. (22)). In the Eq. (30) the \( \gamma_0\gamma_3 \)-term affects the spin degrees of freedom of the BS amplitude.

C. The electromagnetic vertex

One-body part of the Mandelstam vertex has the form

\[
\Lambda^{[1]}_\mu(p, k; P, K) = i\delta^{(4)} \left( p - k - \frac{q}{2} \right) \Gamma^{(1)}_\mu \left( \frac{P}{2} + p, \frac{K}{2} + k \right) S^{(2)} \left( \frac{K}{2} - k \right)^{-1}
\]

\[
+ i\delta^{(4)} \left( p - k + \frac{q}{2} \right) \Gamma^{(2)}_\mu \left( \frac{P}{2} - p, \frac{K}{2} - k \right) S^{(1)} \left( \frac{K}{2} + k \right)^{-1},
\]

where \( \Gamma^{(l)}_\mu(p, k) \) is off-mass-shell \( \gamma \)-NN vertex for the \( l \)th nucleon. This is common problem of direct application of the BS equation to the NN interaction. The consequence of gauge
constraints for off-shellness in the EM vertices have been recently considered in Ref. [40] (see also references therein). In our case we deal with half off-mass shell EM vertex. This type of vertex occurs in the \((e, e'N)\) reaction, e.g. nucleon knockout and inclusive electron scattering, when the initial nucleon is taken to be bound (off-mass-shell) and the knocked-out nucleon is assumed to be in physical state. It is shown in Ref. [40] that the off-shell behavior of the EM vertex of the nucleon in a real Compton scattering on a free nucleon does not play a role as consequence of gauge invariance.

Thus we deal with well-known on-mass-shell version of the EM vertex:

\[
\Gamma^\mu_{(l)}(q) = \gamma_\mu \left( F_1^{(s)}(q) + \tau_3^{(l)} F_2^{(v)}(q) \right) + \frac{i}{2m} \sigma_{\mu\nu} q^\nu \left( F_2^{(s)}(q) + \tau_3^{(l)} F_2^{(v)}(q) \right),
\]

(32)

where \(F_{1,2}^{(s,v)}(q)\) is the isoscalar and isovector Pauli-Dirac form factors of the \(l\)th nucleon, normalized as:

\[
F_1^{(s)}(0) = \frac{1}{2}, \quad F_2^{(s)}(0) = \kappa_p + \kappa_n, \quad F_1^{(v)}(0) = \frac{1}{2}, \quad F_2^{(v)}(0) = \frac{\kappa_p - \kappa_n}{2}
\]

with the anomalous part of the proton and neutron magnetic momenta \(\kappa_{p,n}\) respectively.

Employing the written transformation laws, substituting Eqns. (27), (29), (31) to Eqn. (23) and integrating over intermediate 4-momentum, we arrive at an expression in terms of the c. m. vertex function and propagators

\[
\mathcal{M}^\mu_{fi} = \sum_{l=1,2} \chi_{Sm_i}^{(0)}(0, p; \sqrt{s}) \Gamma_{\mu(l)}^{(l)}(q^2 = 0) A(L) S^{(l)}(k; K_0) \Gamma_M(k_{l0}; k_l; K_0),
\]

(33)

where \(k_l = L^{-1}(p + (-1)^l q/2)\). All possible contributions to the transition matrix element in the PWIA are depicted in Fig. 1. Since both the initial and final 2N states are antisymmetric the four diagrams are identical.

In the PWIA it is seen that the matrix element is proportional directly to the deuteron vertex function taken at specific value of energy-momentum. As the relative 4-momentum \(k_l\) is restricted by energy-momentum conservation in a photon-nucleon vertex, the relative energy \(k_{ul}\) and 3-momentum \(k_l\) variables depend on the photon energy in the lab. frame and the dimensionless boost parameter \(\eta\):

\[
k_{ul} = \sqrt{\eta} |p|| + (-1)^l \frac{E_\gamma}{2},
\]

\[
k_{\perp l} = p_\perp, \quad \omega = M_d \sqrt{\eta},
\]

\[
k_{||} = \sqrt{1 + \eta} |p|| + (-1)^l \frac{q}{2}
\]

(34)

where indices \(\parallel\) and \(\perp\) denotes the longitudinal and transverse components of vector \(k\) with respect to direction of the incoming photon 3-momentum \(q\), here \(|q| = E_\gamma\). The situation is illustrated on Fig. 2. For photon energies \(E_\gamma \leq 0.2\) GeV one is probing the energy-momentum distribution of the bound nucleons in the deuteron where the high-momentum ‘tails’ of the nucleonic states in deuteron are especially relevant. For given c. m. angles \(\Theta_p\) it is found that both the relative energy, which accounts for the retardation in the vertex function of the deuteron, and the modulus of the 3-momentum of the proton (neutron) rise strongly with \(E_\gamma\). They are smaller at forward scattering angles, and for other angles covering wide bands from 100 MeV to 2 GeV and from 500 MeV to 2.5 GeV.
Introducing the deuteron state components, the 2N continuum amplitude in the matrix representation (for details we refer to the paper [20]), the transition matrix element can be evaluated calculating traces of $\gamma$-matrix expressions:

$$M^\mu_{fi} = - Sp \left( \bar{\chi}_{Sm_a}(p) \Gamma_{p,\mu} \Lambda(\mathcal{L}) S(\frac{K_0}{2} + k_1; K_0) \Gamma_M(k_1; K_0) \Lambda(\mathcal{L}^{-1}) \right)$$

$$- Sp \left( \bar{\chi}_{Sm_a}(p) \Lambda(\mathcal{L}) \Gamma_M(k_2; K_0) \tilde{S}(\frac{K_0}{2} - k_2; K_0) \Lambda(\mathcal{L}^{-1}) \Gamma_{n,\mu} \right),$$

with

$$\bar{\chi}_{1m_a}(p) = \frac{N^2_p}{2\sqrt{2}} (m - \hat{p}_2) \xi^*_{m_a} (1 + \gamma_0)(m + \hat{p}_1),$$

$$\bar{\chi}_{0\theta}(p) = \frac{N^2_p}{2\sqrt{2}} (m - \hat{p}_2) \gamma_5 (1 + \gamma_0)(m + \hat{p}_1),$$

where $\xi_{m_a}$ is a polarization 4-vector with the following completeness and orthogonality relations

$$\sum_{m_a = -1}^{+1} \xi^\mu_{m_a} \xi^\nu_{m_a} = -g^{\mu\nu} + \frac{P^\mu P^\nu}{s}, \quad \xi \cdot P = 0,$$

and normalization constants $N_p$ and vectors $p_1, 2$ are defined in Eqn. (12).

Since the EM nucleon form factors can be taken by their on-shell form, we have for the charge-current operator

$$\Gamma_{p,\mu} = \gamma_\mu + \frac{i\kappa_p}{2m} \sigma_{\mu\nu} q^\nu, \quad \Gamma_{n,\mu} = \frac{i\kappa_n}{2m} \sigma_{\mu\nu} q^\nu.$$ (38)

The fermion propagator in Eqn. (33)

$$\tilde{S}(k) = \frac{\hat{k} - m}{k^2 - m^2 + i\epsilon}$$

is connected with the propagator $\tilde{S}$ by $\tilde{S} = -C S^T C$, where $C = i\gamma^2 \gamma^0$.

Using the expression (33), we find that the differential photo-absorption cross section can be written as

$$\frac{d\sigma}{d\Omega_p} = \frac{d\sigma_0}{d\Omega_p} + \frac{d\sigma_{SP}}{d\Omega_p},$$

where the $\sigma_0$ is the part of the cross section which makes up the shape of the angular distributions

$$\frac{d\sigma_0}{d\Omega_p} = \frac{\alpha}{4\pi s} \left(1 + \sqrt{1 + \eta} \right)^2 \sum_{S=0,1} |X^S_0|^2$$

and the $d\sigma_{SP}$ accounts for the effect of the boost on the spin degrees of freedom (the spin precession) of the nucleons.
\[ \frac{d\sigma_{SP}}{d\Omega_p} = \frac{\alpha}{4\pi s} \left( \frac{1+\sqrt{1+\eta}}{2} \right)^2 \sum_{s=0,1} |\mathcal{M}_{SP}^s|^2 \]  

(42)

with the square modulus of the amplitude \( |\mathcal{M}_{SP}^s|^2 \) is given by

\[
|\mathcal{M}_{SP}^s|^2 = 2\beta \text{Re}(X_0^S X_1^{S*}) \\
+ \beta^2 \left( |X_1^S|^2 - 2\text{Re}(X_0^S X_1^{S*}) \right) - 2\beta^3 \text{Re}(X_1^S X_2^{S*}) + \beta^4 |X_2^S|^2
\]  

(43)

with \( \beta = \sqrt{\frac{\eta}{1+\sqrt{1+\eta}}} \) and the amplitudes \( X_i^S \) (\( i = 0, 1, 2 \)) are expressed as

\[
X_0^S = Sp \left( \bar{\chi}_{Sm_s}(p) \Gamma_{p,\lambda} S^{(1)}(s_1; K_0) \Gamma_M(k_1; K_0) \right) + p \leftrightarrow n,
\]

\[
X_1^S = Sp \left( \bar{\chi}_{Sm_s}(p) \Gamma_{p,\lambda} \gamma_0 \hat{n}_3 S^{(1)}(s_1; K_0) \Gamma_M(k_1; K_0) \right)
- Sp \left( \bar{\chi}_{Sm_s}(p) \Gamma_{p,\lambda} S^{(1)}(s_1; K_0) \Gamma_M(k_1; K_0) \gamma_0 \hat{n}_3 \right) + p \leftrightarrow n,
\]

(44)

\[
X_2^S = Sp \left( \bar{\chi}_{Sm_s}(p) \Gamma_{p,\lambda} \gamma_0 \hat{n}_3 S^{(1)}(s_1; K_0) \Gamma_M(k_1; K_0) \gamma_0 \hat{n}_3 \right) + p \leftrightarrow n
\]

with \( n_3 = (0, 0, 0, 1) \) denoting a unit vector and \( s_1 = K_0/2 - k_1 \).

These are the general expressions for the deuteron photodisintegration cross section in the PWOA. Since we omit the two-body contribution to the transition matrix element, we do not preserve the gauge independence of the amplitude. Thus averaging over photon polarizations, we make use of Coulomb gauge, \( \varepsilon^0 = 0, \varepsilon \cdot q = 0 \) with the completeness relation of the form

\[
\sum_{\lambda = \pm 1} (\varepsilon_\lambda)_i^*(\varepsilon_\lambda)_j = \delta_{ij} - \frac{q_i q_j}{q^2}, \quad i, j = x, y.
\]  

(45)

All \( \gamma \)-matrix expressions in the matrix elements in Eq. (44) and the square modulus of the amplitude in Eq. (43) are evaluated with the computer algebraic program REDUCE. When summing over nuclear polarizations, we make use of the relations (13) and (37).

V. ANALYSIS OF RELATIVISTIC EFFECTS

Now we are in a position to do the final computations. The results for the angular distributions for deuteron photodisintegration at four different photon energies \( E_\gamma \) in the laboratory frame are depicted in Fig. 3. The cross section is calculated in the framework of the BS equation with the separable interaction kernel (with two different strength of \( D \)-state, \( P_2 = 4 \% \) and \( P_2 = 6 \% \)). This interaction kernel is similar to that employed in the calculations of the deuteron EM elastic form factors [36]. For simplicity, though it might be important, in our calculations we disregard negative-energy partial states of the deuteron vertex function. These will be considered in detail in the BS formalism for a meson exchange interaction kernel.

In Fig. 3 it is seen that the resultant curves reproduce shapes of angular distributions [4]. Above the threshold it is an almost perfect \( \sin^2 \Theta_p \) behavior, which corresponds to \( E1 \).
transition to the $^3P_0$ np continuum state. At higher photon energies the overall magnitude of the cross section rapidly falls off and the maximum is shifted from 90° to 70° and, further, to 60°. The whole distributions, which are dominated by magnetic transitions, become flatten, the ratio of the forward cross section to the maximum decreases. The role of the $D$-state becomes more pronounced. But notwithstanding these similarities, the theory appears systematically less than the experimental distributions. At $E_\gamma=20$ MeV it is less by factor 1.4, at $E_\gamma=100$ MeV by factor 3, in the $\Delta$-resonance region it is expectedly lower by almost factor 10, and at $E_\gamma=500$ MeV by factor 3 with respect to $D$-state weight $P_2 = 6\%$. In the NR approach, incorporating meson-nucleon degrees of freedom, a good deal of discrepancy between the experiment and theory is diminished by contributions from meson-exchange currents and isobar configurations.

A. Approximate calculations

Despite the fact that our theoretical results does not describe the experimental data, we are able to make definitive statements concerning the relative importance of the various relativistic effects. We distinguish the following classes of the relativistic effects in our consistent treatment: relativistic kinematics and EM current operator; relativistic NN dynamics, which forbids instantaneous interactions, and leads to the relative energy dependence of the deuteron vertex function (retardation), the boost transformation affecting the internal variables of the BS amplitude of the deuteron (Lorentz contraction), the part of the cross section denoted as $\frac{d\sigma_0}{d\Omega}$, and its spin degrees of freedom (spin precession), the cross section $d\sigma_{SP}$/d$\Omega$.

We start discussion with evaluation of the size of contributions due to retardation, Lorentz contraction and spin precession. Let us consider a number of approximate calculations with respect to the exact positive-energy BS calculations:

1. First of all we perform the static approximation (BS-SA) to the BS cross sections, which amounts to neglecting the boost on the arguments of the deuteron vertex function and one-particle propagator, see Eq. (33). It is achieved by putting the boost parameter $\eta = 0$ in the deuteron vertex function $\Gamma_L(k_{0l},k_l)|_{\eta=0} = \Gamma_L(p_{0l},p_l)$, where $p_{0l} = (-)^l\frac{E_\gamma}{2}$ and $p_l = p + (-)^l\frac{q}{2}$ ($l = 1,2$). Kinematically this effect is shown in Fig. 4. The booster in Eq. (33) is approximated by $\Lambda(L)|_{\eta=0} = I$. This approximation excludes contributions due to the Lorentz contraction and spin precession.

2. Moreover we pay special attention to investigation of the influence of the boost on the nucleon relative energy in one-particle propagator. As it is shown in Ref. 35 that this is the most important relativistic contributions to the deuteron EM form factors. Here we consider the case (BS-BR), which is the same as BS-SA, but includes the boost on one-particle propagator $S^{(l)}(k_{0l},|p_l|)$ due to recoil, $k_{0l} = \sqrt{\eta}|p| + (-)^l\frac{E_\gamma}{2}$.

3. Finally, in order to find out the relativistic correction associated with the relative energy dependence in the matrix elements, which is brought by the BS vertex function. We consider the zero-order approximation (BS-ZO) for the vertex function, i.e. com-
puting the radial parts of the vertex function in the BS-BR approximation with \( k_0 \) equal to zero.

In Fig. 4 we present the results for the angular distributions at the same lab. photon energies as in Fig. 3. The solid curve corresponds to the exact calculation. We would like to stress the importance of the effects concerning the booster. In Fig. 4 it is clearly seen the role of the \( d\sigma_{SP} \) cross section (3-dot-dash line) with rise of the energy. It becomes pronounced at the scattering in the forward semi-sphere, where it is almost the half of size of the cross section. One should allow for such a contribution starting at the medium photon energies. When added to the BS-BR approximation (dash line), the latter becomes to be plausible for discussed \( E_\gamma \) region. The BS-BR approximation, which includes the boost on the one-particle propagator, gives the shape of the angular distributions close to the exact ones. We can conclude that the BS-BR approximation supplemented with \( d\sigma_{SP} \) contribution accounts for major relativistic effects in the cross section. On the other hand the SA approximation (dot line) ceases to be reasonable for the \( E_\gamma \) above 100 MeV, as it has a wrong position of cross section maxima.

We show the influence of the boost transformation in the arguments of the initial-state vertex function. It is worthwhile to mention that the boost leaves the arguments of the radial part of the vertex function unchanged (the radial function \( g_L(p; K) \) depends on the Lorentz invariants \( K^2, p^2 \) and \( p \cdot K \)). It only has a direct bearing on its spin-orbital part. In Fig. 5 it is displayed the relative deviation of the BS-BR (dash line) and BS-ZO (dot-dash line) approximations from the BS cross section, \( d\sigma_0 \). The deviation of the BS-BR cross section is due to the boost effect on the orbital part of the deuteron vertex function. The respective contribution is quite great especially at forward and backward proton c. m. angles. The effects of retardation is responsible for discrepancy between the two curves in Fig. 5. One can see that this is practically uniform difference reaching up to 5 %. If one ignores the dependence on the relative energy in the deuteron vertex function, one comes up with the cross section which is slightly smaller but keeps the same shape.

B. Comparison with other approaches

The BS formalism consistently accounts for the relativistic effects associated with manifest Lorentz covariance of scattering amplitudes. As far as matter of relativity is concerned, we compare the exact results of the BS framework with the following approaches.

1. In the PWOA the ET approximation can be obtained immediately from the exact expressions replacing the BS deuteron bound state for “++”-channels by the QP vertex function, which is solution of the 3-D QP equation with the BbS propagator (8) and with the refitted version of the separable interaction kernel Graz-II [36]. Essentially this approximation makes use of instantaneous interactions, i. e. with the zero relative time, or respectively, relative energy in the deuteron vertex function \( g_L \) and the one-particle propagator.

2. A minimally relativistic approach employs the same QP vertex function of the deuteron corresponding to solution of the bound state equation the BbS propagator. This
corresponds to the static limit for the ET approximation (ET-SA). The approach incorporates the relativistic kinematics and covariant form of the EM current matrix elements.

3. A purely non-relativistic approach makes use wave function of the deuteron, which is solution of the Schrödinger equation with NR Graz-II separable potential.

In Fig. 6 we compare the above mentioned approaches with the exact relativistic calculation. One can see that NR approach (dot line) is very crude approximation for the photon energies greater that 100 MeV. The minimally relativistic approach (dot-dash line) improves the situation. By no means the NR approach is reasonable to describe deuteron photodisintegration at high photon energies above the pion threshold. At least one should include minimally relativistic corrections. The conclusion is in accordance with the discussion in Ref. [1], where it was shown that should the relativistic effects not included, a theory gave too much peaking of the differential cross section at $\Theta_p=0^\circ$ and $180^\circ$.

As one can see the overall sign of the relativistic contribution to the NR cross section is negative, but it depends on given photon energy in the range $\Theta_p=60^\circ$ to $120^\circ$.

Taking into account of Lorentz deformation (dash line in Fig. 6) produces sizeable effect. It is seen that the ET approximation is a good approximation at low photon energies. Almost equally at all proton c.m. angles, the discrepancy between the exact and ET calculations reaches about 10% at $E_\gamma=300$ MeV and 25 percent at the $E_\gamma=500$ MeV.

C. Expansion of the relativistic model

It is well-known that the usual way to include relativistic effects is $|p|/m$- and $\omega/m$-expansion of the exact relativistic model, if one confines to the lowest order correction $|p|^2/m^2$ and $\omega/m$ beyond the NR amplitudes [1]. This relativistic correction is valid at the photon energies $E_\gamma$ up to few hundred MeV and it seems to correspond to corrections of spin-orbit type to the NR current operator.

We analytically performed approximation of the expression (43) for the angular distributions in the limit $E_\gamma \ll m$. For the c.m. frame variables in the NR limit, corresponding to energies $E_\gamma \lesssim 100$ MeV, we have $|p| \approx \sqrt{m(E_\gamma - \epsilon_d)}$, $\omega \approx E_\gamma$, and the boost parameter $\sqrt{\eta} \approx 0$. Thus the matrix element is expanded in powers of $|p|/m$ and $\omega/m$ with $|p|/m \approx 0.3$ keeping the lowest order terms, recoil effects and boost effects are neglected as well. The result is that of the non-covariant description of deuteron in the framework of the conventional NR models incorporating relativistic effects in a $|p|/m$ expansion of a relativistic model [II].

The differential cross section of a photon with energy $E_\gamma$ in the NR framework is given by

$$\frac{d\sigma_0}{d\Omega_p} = \frac{\alpha |p|}{4\pi E_\gamma} \sum_{S=0,1} |X_0^S|^2,$$

where the deuteron break-up matrix elements are expressed in terms of the NR deuteron wave function $\Psi_M(k)$, the wave function of the 2N scattering states $\Psi_{pSm_s}(k)$ in the following form
\[
X_0^S = - \sum_{l=1,2} \int \frac{d\mathbf{k}}{(2\pi)^3} \bar{\Psi}_p \Psi_{M_l}(\mathbf{k}) F_l^\lambda(\mathbf{q}) \Psi_M(\mathbf{k}_l), \quad (\lambda = \pm 1),
\]

where \( \mathbf{k}_l = \mathbf{p} - (-1)^l \frac{\mathbf{q}}{2} \) and the EM nucleon form factor is given by

\[
F_l^\lambda(\mathbf{q}) = (-1)^{(l+1)} \frac{1 + \tau_z^{(l)}}{2} \frac{\mathbf{p}^\lambda}{m} + \frac{\kappa_s + \tau_z^{(l)} \kappa_v}{2} \frac{i[\sigma^{(l)} \times \mathbf{q}]^\lambda}{2m}
\]

with \( \kappa_s = \frac{1}{2} (\kappa_p + \kappa_n) \) and \( \kappa_v = \frac{1}{2} (\kappa_p - \kappa_n) \).

In the PWOA one can reduce square of the amplitude (47) in Eq. (46) to ‘textbook’ formulae which reproduces the shape of an angular distribution but misses its absolute size:

\[
d\sigma_0 = \frac{\mathbf{p}^2}{2m^2} \sin^2 \Theta_p \left\{ \frac{U_1^2 + W_1^2}{4m^2} \left( \kappa_p^2 (U_1^2 + W_1^2) + \kappa_n^2 (U_2^2 + W_2^2) \right) + \frac{\kappa_p \kappa_n}{3} \left[ 2U_1 U_2 - \frac{U_1 W_2 + U_2 W_1}{\sqrt{2}} \left( 1 + 3 \cos(2\Theta_p) \right) + \frac{W_1 W_2}{2} \left( 5 + 3 \cos(2\Theta_p) \right) \right] \right\},
\]

where indexes 1 and 2 at the \( S \)- and \( D \)-state of the deuteron, denoted as \( U \) and \( W \), respectively, means that they are evaluated at modulus of the proton and neutron 3-momentum in the deuteron, \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \). These are formally related to the BS form factors as

\[
U = \frac{\sqrt{\pi m}}{2E_0 - M_d} g_0(0, |\mathbf{k}|; s = M_d^2), \quad W = \frac{\sqrt{\pi m}}{2E_0 - M_d} g_2(0, |\mathbf{k}|; s = M_d^2).
\]

VI. CONCLUDING REMARKS

The objective of the present study is evaluate the various relativistic contributions to the angular distributions in deuteron photodisintegration process. That can be done in a variety of theoretical frameworks and dynamics. In this paper we have applied the fully relativistic formalism, based on the Bethe–Salpeter equation for the 2N scattering amplitude and deuteron bound state. Beyond the choice of the theoretical framework, which is manifestly covariant at every step of the calculation, the important issue is dynamical model of the nucleon-nucleon interaction. Pursuing the aim to obtain clear understanding and conduct straightforward comparison with the non-relativistic and minimally relativistic approaches, we employ the effective separable interactions in construction the solvable dynamical model of the deuteron.

In order to obtain ultimate results in analytical form we discard channels containing negative-energy states in the Bethe–Salpeter amplitude of the deuteron. We also do not include the two-body contributions to the electromagnetic current operator and neglect the final state interaction in the outgoing 2N state. The last two limitations constitutes the so-called plane wave one-body approximation. The negative-energy states, or P-states, in the deuteron vertex function are not presumably irrelevant. Their contributions are expected to be significant within the considered interval of the photon energies. In this respect our present study is primarily of a comparative character.
Despite to these specifications, the strongest advantage in our investigation concerns the fully covariant and rigorous description of the bound state and the deuteron electromagnetic current. The present approach accounts for wealth of the relativistic effects to the differential cross section of deuteron photodisintegration: the role of relativity of the transition matrix elements between nuclear states, the influence of the retardation in the deuteron vertex function and one-particle propagator and changes in amplitudes due to the Lorentz deformation and spin precession.

Incorporation of these relativistic effects can play a crucial role in theoretical analysis of deuteron photodisintegration even at intermediate laboratory photon energies for the forward and backward scattering. Here the most important contributions comes from the boost in the arguments of the initial state vertex function and the boost on the relative energy in the one-particle propagator due to recoil. As one is concerned with the covariant approaches, the equal time approximation is more or less reasonable approach from a pragmatic point of view.

Further, we can draw the following conclusions out of the present investigation: 1) the Bethe–Salpeter approach allows one to take into account Lorentz invariance and relativistic dynamical structure of the two-nucleon system in the most general form. 2) The novel feature brought by the Bethe–Salpeter approach is the retardation due to the dependence of the bound state amplitude on the relative energy of the nucleons. In the plane wave one-body approximation, the scattering amplitude bears the explicit dependence on this variable and its magnitude is measured by the photon energy. In our opinion, this turns out to be by far the most important fact enabling to study recoil effects due to energy transfer to a nucleon by a photon. 3) The role of the boost transformation of the spin degrees of freedom becomes noticeable in increasing order of the photon energy at forward scattering.

Finally, the region of high photon energies (above $E_\gamma=500$ MeV) calls for a more complete investigation. In this energy region one needs to construct a realistic interaction kernel in the Bethe–Salpeter equation. Moreover, extending the above calculations includes contributions due to P-states and the two-body processes in the EM current operator matrix elements.

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REFERENCES

[1] H. Arenhövel and M. Sanzone, Few-Body Systems Suppl. 3, (1991).
[2] D. Babusci et al. Nucl. Phys. A 633, 683 (1998).
[3] F. Adamain, Abstracts of XIth Particle and Nuclei Int. conference. Uppsawa. Sweden. (1999) 93.
[4] J.E. Belz, Phys. Rev. Lett. 74, 646 (1995).
[5] H. Ejiri, Private communication. XIV Int. Seminar on High Energy Physics Problems. Dubna. 1998.
[6] C. Bocha et al., nucl-ex/9808001 11 Aug. 1998.
[7] R. Crawford et al., Nucl. Phys. A 603, 303 (1998).
[8] S. Wartenberg et al., Few-Body Systems 26, 213 (1999).
[9] P. Wilhelm and H. Arenhövel, Phys. Lett. B 318, 410 (1993).
[10] J.M. Laget, Nucl. Phys. A 579, 333 (1994).
[11] W. Jaus, B. Bofinger and W.S. Woolock, Nucl. Phys. A 562, 477 (1993).
[12] J.M. Zuilhof and J.A. Tjon, Phys. Rev. C 22, 2369 (1980).
[13] A.A. Logunov and A.N. Tavkhelidze, Nuovo Cim. 29, 370 (1963).
[14] R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).
[15] J. Adam, Jr, J.W. Van Orden and F. Gross, Nucl. Phys. A 640, 391 (1998).
[16] L.P. Kaptari, B. Kämpfer, S.M. Dorkin and S.S. Semikh, Phys. Rev. C 57, 1097 (1998).
[17] A.Yu. Umnikov, F.C. Khanna, K.Yu. Kazakov and L.P. Kaptari, Phys. Lett. B 334, 163 (1994).
[18] C. Ciofi degli Atti, L.P Kaptari and S. Scopetta, Eur. Phys. J. A 5, 191 (1999).
[19] N. Honzawa and S. Ishida, Phys. Rev. C 45, 17 (1992).
[20] L.P. Kaptari et al., Phys. Rev. C 54, 986 (1996).
[21] A.V. Anisovich and V.A. Sadovnikova, Eur. Phys. J. A 2, 199 (1998).
[22] A.E.L. Dieperink and S.I. Nagorny, Phys. Lett. B 456, 9 (1999).
[23] A.Yu. Korchin and A.V. Shebeko, Sov. J. Nucl. Phys. 54, 375 (1991).
[24] E. Hummel and J.A. Tjon, Phys. Rev. C 49, 21 (1994).
[25] D.R. Phillips and S.J. Wallace, Few-Body Systems 24, 175 (1998).
[26] V. Pascalutsa and J.A. Tjon, Few-Body Systems Suppl. 10, 105 (2000).
[27] D.R. Phillips, S.J. Wallace and N.K. Devine, arXiv:nucl-th/9906068.
[28] W. Jaus and W.S. Woolock, Helv. Phys. Act. 57, 644 (1984).
[29] S.G. Bondarenko, V.V. Burov, M. Beyer and S.M. Dorkin, Phys. Rev. C 58, 3143 (1998).
[30] L.P. Kaptari, B. Kämpfer, S.M. Dorkin and S.S. Semikh, Few-Body Systems 27, 189 (2000).
[31] M. Anastasio and M. Chemtob, Nucl. Phys. A 364, 219 (1981).
[32] G. Rupp, Nucl. Phys. A 508, 131 (1990).
[33] N. Nakanishi, Graph Theory and Feynman integral, Gordon and Breach, New York, 1971.
[34] K. Kusaka, K.M. Simpson and A.G. Williams, Phys. Rev. D 56, 5071 (1998).
[35] G. Rupp and J.A. Tjon, Phys. Rev. C 41, 472 (1990).
[36] G. Rupp and J.A. Tjon, Phys. Rev. C 45, 2133 (1992).
[37] K. Schwarz, J. Haidenbauer and J. Fröhlich J., Phys. Rev. C 33, 456 (1986).
[38] G. Rupp and J.A. Tjon, Phys. Rev. C 37, 1729 (1988).
[39] S.G. Bondarenko, V.V. Burov and S.M. Dorkin, Proc. XII Int. Seminar on High Energy Physics Problems/Ed Baldin A.M., Burov V.V. Dubna: JINR (1994) 227.
[40] S.I. Nagorny and A.E.L. Dieperink, Eur. Phys. J. A 5, 417 (1999).
FIGURE CAPTION

FIG. 1. Diagrams corresponding to the plane wave one-body approximation to the matrix element of deuteron photodisintegration. Outgoing particles are on their mass-shell.

FIG. 2. Modulus of the energy $|k_0|$ and 3-momentum $|k|$ of a nucleon in the deuteron in its rest frame versus the photon energy $E_\gamma$ (solid lines). Dash lines correspond to solid ones while neglecting the boost parameter, i.e. $\sqrt{\eta} = 0$ (static approximation). A set of fixed proton scattering angles $\Theta_p = 0^\circ$, 90 and 180$^\circ$ in the c.m. frame correspond to curves labeled as 1, 2 and 3, respectively. The same curves are related to the energy and 3-momentum of a spectator nucleon at c.m. frame angles $\Theta_p = 180^\circ$, 90 and 0$^\circ$.

FIG. 3. The differential cross section in the plane wave one-body approximation at different photon energies $E_\gamma$. Curves correspond to different probabilities of $^3D_1^+$ partial state. Solid line, $P_2 = 4\%$, dash line, $P_2 = 6\%$.

FIG. 4. The differential cross section in the plane wave one-body approximation at different photon energies $E_\gamma$. Curves: solid line — the exact positive-energy BS calculation, dotted line — the static approximation (exclusion of the Lorentz contraction), dash line — the static approximation with taking into account the boost on one-particle propagator due to recoil (the Lorentz contraction), 3-dot-dash line — the contribution due to the spin precession, Eqn. (42). Probability of $^3D_1^+$ partial state is $P_2 = 4\%$.

FIG. 5. The relative deviation of the deuteron photodisintegration cross section for the photon energies $E_\gamma = 100$, 300 and 500 MeV for the approximations with respect to the BS result: BS-ZO (dash line) and BS-BR (dot-dash line). On the Y-axis it is plotted $\frac{\sigma_0 - \sigma_{\text{BS-\alpha}}}{\sigma_0} \times 100\%$, where $\alpha = \text{BR}, \text{ZO}$. Probability of $^3D_1^+$ partial state is $P_2 = 4\%$.

FIG. 6. The relative difference of the deuteron photodisintegration cross section for the photon energies $E_\gamma = 100$, 300 and 500 MeV for the following cases: the equal time approximation (dash line), the minimally relativistic approach (dot-dash line) and non-relativistic approaches (dotted line) with respect to the exact positive-energy BS result. On the Y-axis it is plotted $\frac{\sigma_0 - \sigma_\alpha}{\sigma_0} \times 100\%$, where $\alpha = \text{ET}, \text{ET-SA} \text{ and NR}$. Probability of $^3D_1^+$ partial state is $P_2 = 4\%$. 
FIG. 1. Kazakov K.Yu
FIG. 2. Kazakov K.Yu.
FIG. 3. Kazakov K.Yu.
Fig. 4. Kazakov K.Yu.
FIG. 5. Kazakov K.Yu.
FIG. 6. Kazakov K.Yu.