Testing the principle of equivalence with Planck surveyor

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We consider the effect of the violation of the equivalence principle (VEP) by the massive neutrino component on the Cosmic Microwave Background angular power spectrum. We show that in the presence of adiabatic and isocurvature primordial density perturbations the Planck surveyor can place limits on the maximal VEP by the massive neutrino component at the level of few × 10⁻³, valid in the general relativity, for the case in which the gravity is the single source of VEP. This work has been performed within the framework of the Planck/LFI activities.

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The atmospheric neutrino experiments \[1, 2\] provide strong evidence for neutrino oscillations implying a non-zero neutrino mass with a lower limit \(\simeq 0.04 - 0.08\) eV. The latest measurements of the oscillation of double beta decay to the neutrino mass matrix [3] place an upper limit on the neutrino mass of \(m_\nu \leq 0.26\)eV. The direct implication of these results is the non-negligible presence of a large neutrino asymmetry the mass-mixing vacuum neutrino oscillations left detectable imprints on the CMB angular power spectrum. In eq. (1) \(\Psi\) plays the role of the gravitational potential in the Newtonian limit [21]. Limits on the VEP for \(\tau = \text{the conformal time}\) and \(\alpha, \gamma = \text{theory dependent parameters}\) in the general relativity \(\alpha = \gamma = 1\) while in alternative theories of relativity satisfying the equivalence principle \(\alpha\) and \(\gamma\) need not to be unity but their values are universal (the same for all kinds of particles).

\[d s^2 = a^2(\tau)(-1 + 2\alpha \Psi)d\tau^2 + (1 - 2\gamma \Phi)dx^idx_i, \tag{1}\]

where \(a(\tau)\) is the cosmic scale factor \((a_0=1\ \text{today})\) and \(\tau\) is the conformal time. \(\alpha, \gamma\) are theory dependent parameters.

\[|\gamma - \gamma_0| < \text{few } \times 10^{-3}, \tag{22}\]

and for neutrinos and antineutrinos \[|\gamma_\nu - \gamma_{\bar{\nu}}| < 10^{-6}. \tag{24}\]

The CMB anisotropy holds the key for understanding the seeds of the cosmological structures and allows the measurement of the most important cosmological parameters. The new generation of CMB experiments such as MAP [41] and Planck [12] will achieve enough precision to reveal the structure formation process up to arcminute angular scales. The aim of this paper is to study the possibility to test with Planck the VEP by the massive neutrino component.

A deviation from the equivalence principle can be parametrized by assuming that the parametrized-post Newtonian (PPN) parameters for a given metric have particle-dependent values \[13\]. In the conformal Newtonian gauge the perturbations in the Robertson-Walker spacetime are characterized by two scalar potentials \(\Psi\) and \(\Phi\) which appear in the line element as \[14\].

The evolution of the density fluctuations from the early universe involves the integration of coupled and linearized Boltzmann, Einstein and fluid equations [21] for all the relevant species (e.g., photons, baryons, cold dark matter, massless and massive neutrinos). The differences introduced in the gravitational potential due to the VEP by the massive neutrino component can generate metric perturbations that affect the evolution of the density fluctuations of all the components, leaving imprints on the CMB angular power spectrum.

A recent work \[26\] analysed the possibility to detect neutrino oscillations with accurate CMB experiments. In the presence of a large neutrino asymmetry the mass-mixing vacuum neutrino oscillations left detectable im-

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prints on the CMB anisotropy and polarization power spectra. For a neutrino flavor $\nu_i$ with energy $E_{\nu_i}$ and mass $m_{\nu_i} \leq 0.26$ eV, the survival probability after the propagation through the distance $L$ in the expanding universe, $P_{\nu_i}(E_{\nu_i}, L) = 1 - \sin^2 2\theta_0 \sin^2(\pi L/\lambda)$, where $\theta_0$ is the vacuum mixing angle, does not exhibit a dependence on the oscillation length $\lambda \sim E_{\nu_i}$, as $L >> \lambda$ and $< \sin^2(L/\lambda) > 1/2$.

For the VEP induced oscillation, the oscillation probability is given by the same equation, replacing $\theta_0$ with the gravitational mixing angle, $\theta_G$, and with the oscillation length given by $\lambda \sim (\Psi \Delta \alpha_{\nu_i} E_{\nu_i})^{-1}$, where $\Psi$ is the gravitational potential and $\Delta \alpha_{\nu_i} = 1 - \alpha_i$ parametrizes the VEP by the neutrino flavor $\nu_i$ [20, 22]. Also in the VEP case, $L >> \lambda$ for neutrinos with masses $m_{\nu_i} \leq 0.26$ eV and the oscillation probability is not sensitive to the VEP parameter $\Delta \alpha_i$. It follows that the mass-mixing vacuum oscillations can not be distinguished from VEP induced oscillations by using the CMB anisotropy.

On the other hand, the imprint of the clusterization process on the CMB anisotropy depends on the dynamics of the involved particles [20] and can be then sensitive to the VEP. We evaluate the effect of the VEP by the massive neutrinos computing the CMB temperature fluctuations generated in the non-linear stages of the universe evolution when clusters and superclusters of galaxies start to form. We place limits on the maximal VEP by the massive neutrinos in the general relativity, valid for the case in which gravity is the only source of VEP.

In [20] we studied the CMB anisotropy induced by the non-linear perturbations in the massive neutrino density associated with the non-linear gravitational clustering process. This process generates metric perturbations leading to a decrease in the CMB anisotropy power spectrum of amplitude $\Delta T/T \approx 10^{-6}$ for angular resolutions between $\sim 4$ and 20 arcminutes, depending on the cluster mass scale $M_{\text{cluster}}$ and the neutrino fraction $f_\nu = \Omega_{\nu c}/(\Omega_b + \Omega_{\text{cdm}})$, where $\Omega_b$, $\Omega_b$, and $\Omega_{\text{cdm}}$ are the energy density parameters for neutrinos, baryons and cold dark matter particles, respectively. In the Newtonian limit, the neutrino gravitational clustering can be described as a deviation from the background by a potential $\Psi$ given by the Poisson equation:

$$\nabla^2 \Psi (\vec{r}, a) = 4\pi Ga^2 \rho_m(a) \delta_m (\vec{r}, a),$$  \hspace{1cm} (2)

where $\vec{r}$ is the position 3-vector, $\rho_m(a)$ is the matter density and $\delta_m (\vec{r}, a)$ is the matter density fluctuation. The equations governing the motion of each particle species $i$ ($i = \text{cold dark matter, baryons, neutrinos}$) in the expanding universe are given by [20, 22]:

$$\frac{dq_i}{da} = -a H(a) \alpha_i \nabla \Psi, \quad \frac{d\vec{r}_i}{da} = q_i [a^3 H(a)]^{-1},$$  \hspace{1cm} (3)

where $q_i$ is the comoving momentum and $H(a)$ is the Hubble expansion rate:

$$H^2(a) = \frac{8\pi G}{3} (\Omega_m/a^3 + \Omega_r/a^4 + \Omega_\Lambda + \Omega_k/a^2).$$

Here $G$ is the gravitational constant, $\Omega_m = \Omega_b + \Omega_{\text{cdm}} + \Omega_\Lambda$ is the matter energy density parameter, $\Omega_r$, $\Omega_\Lambda$ and $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$ are the density parameters for radiation (including the contribution from photons and relativistic neutrinos), vacuum energy and the curvature of the universe. The parameter $\alpha_i$ from the eq. (3) accounts for the coupling of each $i^{th}$ particle species with the gravitational potential. The Newtonian description given by eqs. (2) and (3) applies in the limit of the weak gravitational field if, at each time step, the size of the non-linear structures is much smaller than the causal horizon size (i.e., the background curvature is negligible).

By using numerical simulations [20], we evaluate the effect of the gravitational clustering on the CMB anisotropy power spectrum first assuming $\alpha_i = 1$ for all the particles and then considering that the equivalence principle is violated only by massive neutrinos ($\alpha_i \neq 1$).

We consider a flat $\Lambda$CDM cosmological model with $\Omega_m=0.38$, $\Omega_\Lambda=0.62$, $H_0=62$ Km s$^{-1}$ Mpc$^{-1}$, three massive neutrino flavors giving a neutrino fraction $f_\nu=0.06$ ($m_{\nu_i}=0.78$eV, $\Omega_\nu=0.022$), and adopt a cluster mass $M_{\text{cluster}}=1.5\times 10^{15}M_\odot$. We assume a primordial power spectrum with adiabatic perturbations and a scalar spectral index $n_{s} = 0.98$ and neglect, for simplicity, the contributions from the tensorial modes (gravitational waves) and the reionization effects. This model is consistent with the LSS data and the latest CMB anisotropy measurements, allowing at the same time a pattern of neutrino masses consistent with the results from neutrino oscillation and double beta decay experiments. Panel a) from Fig. 1 presents the CMB anisotropy power spectra obtained from numerical simulations for different values of $\Delta \alpha_\nu = 1 - \alpha_\nu$. The VEP parameter $\Delta \alpha_\nu \neq 0$ induced metric perturbations, leading to a decrease of the CMB temperature anisotropy at small scales as $\Delta \alpha_\nu$ increases.

In order to evaluate to what extent the PLANCK surveyor can test the VEP by the massive neutrino component, we compute the $1-\sigma$ errors on the estimates of the cosmological parameters and on the parameter $\Delta \alpha_\nu$ by using the Fisher information matrix approach. We consider for this computation only the PLANCK "cosmological" channel between 70 and 217 GHz, a sky coverage $f_{\text{sky}} = 0.8$ and neglect for simplicity the foreground contamination [20].

The CMB anisotropies are sensitive to the cosmological parameters through the evolution of the cosmological perturbations since the end of the inflation. The simplest one-field inflation model predicts primordial adiabatic density fluctuations. Isocurvature density perturbations can also arise in multiple field inflation models [21, 22, 23, 24]. In order to test the sensitivity of the CMB anisotropy to the VEP parameter $\Delta \alpha_\nu$ for conditions more general than the pure adiabatic case, we include in our Fisher analysis the parameters for the amplitude of the baryon isocurvature density mode $< I_b I_b >$, neutrino isocurvature density mode $< I_\nu I_\nu >$, their cross-correlation $< I_b I_\nu >$ as well as their cross-correlations with the adiabatic mode $< A I_b >$ and <
FIG. 1: Panel a): the CMB anisotropy power spectra (only adiabatic density mode) including the non-linear gravitational clustering. Different values of \( \Delta \alpha_v = 1 - \alpha_v \) are considered: \( \Delta \alpha_v = 0 \) (solid line), \( \Delta \alpha_v = 2.5 \times 10^{-6} \) (dots), \( 2.5 \times 10^{-5} \) (dash-dots), \( 1.8 \times 10^{-4} \) (dash-three dots), \( 1 \times 10^{-3} \) (long dashes). We report also the case in which the non-linear gravitational clustering and the VEP are not included (dashed line). Panel b): relative differences, in terms of \([1 - C_l^{\alpha} (\Delta \alpha_v > 0)]/C_l^{\alpha} (\Delta \alpha_v = 0)\), between the case \( \Delta \alpha_v = 0 \) and the cases with \( \Delta \alpha_v > 0 \) identified by the same lines as in panel a). Panel c): the CMB anisotropy power spectra of the four considered fiducial models compared to recent CMB anisotropy data: FM1 (solid line), FM2 (dashes), FM3 (dash-dots), and FM4 (dash-three dots). Power spectra normalized to COBE-DMR 4-year data \([30]\) (see also the text).

TABLE I: \( 1 - \sigma \) errors on the cosmological parameters, amplitudes of different modes and VEP parameter \( \Delta \alpha_v \), potentially measurable with \textsc{Planck} (see also the text).

| Parameter | FM1 | FM2 | FM3 | FM4 |
|-----------|-----|-----|-----|-----|
| \( n_{\text{adi}} \) | 0.00182 | 0.00349 | 0.00242 | 0.0153 |
| \( \tau \) | 0.0352 | 0.149 | 0.217 | 0.267 |
| \( \Omega_b h^2 \) | 0.000175 | 0.000231 | 0.000217 | 0.000327 |
| \( \Omega_c h^2 \) | 0.00285 | 0.00361 | 0.00407 | 0.00560 |
| \( \Omega_m h^2 \) | 0.00155 | 0.00199 | 0.00232 | 0.00327 |
| \( \Omega_{\text{iso}} \) | 0.0119 | 0.0152 | 0.0172 | 0.0239 |
| \( \Omega_{\text{bar}} \) | 0.00293 | 0.00361 | 0.00398 | 0.00521 |
| \( h \) | 0.00460 | 0.00582 | 0.00656 | 0.00903 |
| \( n_{\nu}^{\text{iso}} \) | - | 0.0558 | - | 0.285 |
| \( n_{\nu}^{\text{iso}} \) | - | - | 0.264 | 0.368 |
| \( < I_{\nu} I_{\nu} > \) | - | 0.244 | - | 1.58 |
| \( < A I_{\nu} > \) | - | 0.268 | - | 5.27 |
| \( < I_{b} I_{b} > \) | - | - | 0.213 | 5.21 |
| \( < A I_{b} > \) | - | - | 0.346 | 0.570 |
| \( < I_{\nu} I_{b} > \) | - | - | - | 0.368 |
| \( \Delta \alpha_v \times 10^{-5} \) | 0.936 | 1.86 | 2.22 | 2.86 |

\( A I_{\nu} >, \) defining the following parametric fiducial power spectrum \([35]\):

\[
C_l(\tilde{p}) = C_l^A(\tilde{p}) + < I_{\nu} I_{\nu} > C_l^{I_{\nu}}(\tilde{p}) + < A I_{\nu} > C_l^{A I_{\nu}}(\tilde{p}) + < I_{b} I_{b} > C_l^{I_{b}}(\tilde{p}) + < I_{\nu} I_{b} > C_l^{I_{\nu} I_{b}}(\tilde{p}),
\]

where \( \tilde{p} \) represents the cosmological parameters of our \( \Lambda \)CHDM cosmological model, \( C_l^A(\tilde{p}) \), \( C_l^{I_{\nu}}(\tilde{p}) \), \( C_l^{I_{b}}(\tilde{p}) \), and \( C_l^{I_{\nu} I_{b}}(\tilde{p}) \) are the power spectra for the adiabatic, neutrino isocurvature and baryon isocurvature density mode, \( C_l^{A I_{\nu}}(\tilde{p}) \), and \( C_l^{A I_{b}}(\tilde{p}) \) are the power spectra of the cross-correlation between the adiabatic and neutrino isocurvature density mode and adiabatic and baryon isocurvature density mode and \( C_l^{I_{\nu} I_{b}}(\tilde{p}) \) corresponds to the cross-correlation between neutrino isocurvature and baryon isocurvature density mode. The power spectra \( C_l^{A I_{\nu}}, C_l^{I_{\nu} I_{b}} \), and \( C_l^{I_{b} I_{b}} \) are computed by using the CMBFAST code \([33]\) with primordial conditions including also the neutrino isocurvature density mode given by \([34]\). The cross-correlation power spectra are computed by running the CMBFAST code with two modes excited and then subtracting the power spectra computed when each mode was individually excited, following the prescriptions of \([35]\). For each mode we assume a primordial power spectrum with a scalar spectral index \( n_{\text{adi}} = n_{\nu}^{\text{iso}} = n_{b}^{\text{iso}} = 0.98 \). For this computation we do not consider the cold dark matter isocurvature density mode, as its power spectrum is very similar to that obtained for the baryon isocurvature density mode. We consider four fiducial models: only the adiabatic mode (FM1), the adiabatic mode, the neutrino isocurvature density mode and their cross-correlation (FM2), the adiabatic mode, the baryon isocurvature density mode and their cross-correlation (FM3), and the adiabatic and all isocurvature modes as well as their cross-correlations (FM4). Panel c) of Fig. 1 presents the four fiducial power spectra obtained for \( < I_{\nu} I_{\nu} >= 0.1, < I_{b} I_{b} >= 0.1, < A I_{\nu} >= 0.07, < A I_{b} >= 0.07, < I_{\nu} I_{b} >= 0.05 \) and assuming \( \Delta \alpha_v = 0 \): they turn to be in a rough agreement with the current CMB anisotropy data \([37-40]\).
sitive to the considered fiducial model and to the number of parameters involved in the computation, this result shows that the Planck surveyor will be able to place an upper limit on the VEP parameter $\Delta\alpha_\nu$ of few $\times 10^{-5}$, valid in the general relativity, for the case in which the gravity is the only source of VEP.

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