Second Order Corrections to QED Coupling at Low Temperature

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We calculate the second order corrections to vacuum polarization tensor of photons at low temperatures, i.e; \( T \ll 10^{10} \) K \((T \ll m_e)\). The thermal contributions to the QED coupling constant are evaluated at temperatures below the electron mass that is \( T < m_e \). Renormalization of QED at these temperatures has explicitly been checked. The electromagnetic properties of such a thermal medium are modified. Parameters like electric permittivity and magnetic permeability of such a medium are no more constant and become functions of temperature.

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I. INTRODUCTION

In quantum field theory, thermal background effects are incorporated through the radiative corrections. Renormalization of gauge theories at finite temperature requires the renormalization of gauge parameters of the corresponding theory. The propagation of particles and the electromagnetic properties of media are also known to modify in this framework. One of the ways to obtain these changes is through the renormalization techniques. Masses of particles are enhanced at one-loop \([1-6]\), two-loop \([7]\) and presumably to all loop levels. However, the gauge bosons acquire a dynamically generated mass due to plasma screening effect \([8,9]\) including the first order radiative corrections in gauge theories. It helps to determine the changes in electromagnetic properties \([10]\) of a hot medium. In hot gauge theories \( m_e \) is the electron mass and corresponds to \( 10^{10} \) K. The vacuum polarization tensor in order \( \alpha \) does not acquire any hot corrections from hot photons in the heat bath \([8]\) because of the absence of self-interaction of photons in QED. This effect has already been studied in detail at one-loop level and shown explicitly that electric permittivity and magnetic permeability of a medium are modified in real particle background only at higher temperatures. However, the higher order corrections to vacuum polarization tensor of photons are nonzero in the same background. Electric charge therefore increase in such a situation and consequently leads to the modifications in electromagnetic properties of a medium due to enhancement in QED coupling. These statistical contributions are calculated either in Euclidean or Minkowski space by using imaginary or real-time formalisms, respectively. In Euclidean space the covariance breaks and time is included as an imaginary parameter. On the other hand, in real-time formalism, an analytical continuation of energies along with the Wick’s rotation restores covariance in Minkowski space. It has been noticed earlier that the breaking of Lorentz invariance can lead to non-commutative nature of gauge theories \([11]\). The manifest covariance is incorporated through the 4-component velocity of background heat bath which is \( u^\mu = (1, 0, 0, 0) \).

The particle propagators include temperature dependent (hot) term in addition to the temperature independent (cold) term \([1]\). At the higher loop level, the loop integrals involve an overlap of hot and cold terms in particle propagators. This makes the situation cumbersome and getting rid of these singularities is a much more involved process. Sometimes \( \delta(0) \) type pinch singularities may appear in Minkowski space. This problem has been earlier resolved, while calculating \([7]\) the electron self energy at the two loop level at high temperature where hot fermion loops contribute. One can get rid of this type of singularities in thermo-field dynamics \([12]\) by doubling the degrees of freedom. However, we do not come across this type of situation at low temperatures so we do not need to use the techniques required to handle these singularities due to hot fermion loops. We work with real-time propagators for our calculations since we are dealing with the heat bath of real particles for physical systems and keep ourselves restricted to on mass shell. The renormalization of QED in this scheme was checked in detail at one loop level for all ranges of temperatures and chemical potentials \([5,6,8]\) of interest. At the higher-loop level, the loop integrals have a combination of cold and hot terms which appear due to the overlapping propagator terms in the matrix element. In this paper
we calculate the photon self-energy and restrict ourselves to the low-temperatures, for simplicity, and explicitly prove the renormalizability of QED at the two-loop level through the order by order cancellation of singularities.

In the next section we give the detailed calculations of vacuum polarization tensor at low temperatures. Section III is comprised of the calculation of second order contributions to QED coupling constant. The electromagnetic properties of hot media are presented in section IV. Discussions of these results are given in section V.

II. VACUUM POLARIZATION TENSOR IN QED

Vacuum polarization tensor of photon at the two-loop level gives the second order hot corrections to charge renormalization constant of QED at low temperature. This contribution basically comes from the self mass and vertex type electron loop corrections inside the vacuum polarization tensor, as in vacuum, the counter term has to be included to cancel singularities.

![Diagrams](image)

**FIG. 1:** Self energy of photon at two loop level

The diagrams in Fig.1 give the main thermal contributions to the vacuum polarization tensor up to the order $\alpha^2$ at low temperature, i.e., $T \ll m_e$. In this scheme of calculations, the vacuum polarization tensor of photon in Fig.(1a) is given by

$$\Pi^a_{\mu\nu}(p) = e^4 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \text{tr} \left[ \gamma_\mu S_\beta(k) \gamma_\rho D_\rho^\sigma(l) S_\beta(k+l) \right] \gamma_\nu S_\beta(k+l-k) S_\beta(k-p),$$

(1)

while that in Fig.(1b) is

$$\Pi^b_{\mu\nu}(p) = e^4 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \text{tr} \left[ \gamma_\mu S_\beta(k) \gamma_\rho D_\rho^\sigma(l) \right] S_\beta(k+l) \gamma_\sigma S_\beta(k) S_\beta(p-k).$$

(2)

The hot terms give the overlapping divergent terms and the removal of this divergence to establish the renormalization is done by using specific integration techniques in the rest frame of heat bath. However, accuracy of the results depend on the order of integration. This order of integration is so important that the theory can only be proved renormalizable if the correct order of integration is maintained. The integration of thermal integrals has to be done before the temperature independent integrals of cold fermion momenta. We need to get rid of hot divergences appearing due to the presence of the hot boson loops without distributing them over the cold fermion loops where Lorentz invariance does not break. Another justification of this order could be found in the fact that the hot terms in this technique correspond to the contribution of real background particles on mass-shell and incorporates thermal equilibrium. However, the preferred frame of heat bath affects the energy integration of the loop. So the renormalization can only be proven with the right order of integration. For on-shell contributions, we do not have to include the off-diagonal elements of the propagator matrices in our calculations. Moreover, the other components of the matrices can be related to 1-1 components in thermofield dynamics.

We restrict ourselves to low temperatures where hot fermion contribution in background is suppressed and only the hot photons are contributing from the background heat bath. Therefore, cold fermion and hot photon propagators are included. The calculations are simplified if the temperature dependent integrations are performed before the temperature independent ones. The cold loops can then be integrated using the standard techniques of Feynman parametrization and dimensional regularization as is done in vacuum and are discussed in the standard textbooks [12]. As an illustration of the importance of the correct order of integration, we just compare the results for one of these terms. Let us consider the singular terms when the hot loop in Fig.(1b) is evaluated before the cold one, we simply get

$$g^{\mu\nu} \Pi^b_{\mu\nu}(p, T) = \frac{\alpha^2 T^2}{3} \left( 1 - \frac{2}{\varepsilon} \right),$$

(3)
whereas, in the same term, the evaluation of the hot loop after the cold one gives

\[
g^{\mu\nu}\Pi_{\mu\nu}^{b}(p, T) = -\frac{\alpha^2}{\pi^2} \frac{4\pi^2 T^2}{3\varepsilon} \frac{\pi^2 T^2}{5} \\
-\frac{2T^3}{5m^2} \zeta(3) (3|p| + \frac{49}{3} p_0 + \frac{52p_0^3 T^4}{5m^2|p|}).
\]  

(4)

The details of the calculations for Eqs. (3) and (4) are given in Appendix. The \(\frac{1}{3}\) contribution is due to the cold loop momentum integration overlap and is cancelled by the counter term from Fig. (1c). Calculation of thermal corrections to self energy of photons becomes much more cumbersome at high temperatures. However, it has to be done to determine the corresponding changes in electromagnetic properties of hot media. First order contributions have already been obtained even for dense media in this scheme [5,6,10].

Due to the imposed covariance in the propagators, physically measurable couplings can be evaluated through the contraction of vacuum polarization tensor \(\Pi_{\mu\nu}\) with the metric in Minkowski space \(g^{\mu\nu}\) and the bath velocities \(u^\mu u_\nu\) [8]. This helps to evaluate the longitudinal and transverse components of vacuum polarization tensors in order to obtain the thermal contribution to electric permittivity and magnetic permeability. The gauge invariant finite contributions of the sum of all the diagrams of Fig.(1) are

\[
u^\mu u^\nu \Pi_{\mu\nu}(p, T) = -\frac{2\alpha^2 T^2}{3} \left(1 + \frac{p_0^2}{2m^2}\right),
\]  

(5)

and

\[
g^{\mu\nu}\Pi_{\mu\nu}(p, T) = \frac{\alpha^2 T^2}{3}.
\]  

(6)

Moreover, the longitudinal and the transverse components of the vacuum polarization tensor that can be calculated from Eqs. (5) and (6) are

\[
\Pi_L(p, T) = -\frac{p^2}{|p|^2} u^\mu u^\nu \Pi_{\mu\nu}(p, T) \\
= \frac{2\alpha^2 T^2 p_0^2}{3|p|^2} \left(1 + \frac{p_0^2}{2m^2}\right),
\]  

(7)

and

\[
\Pi_T(p, T) = -\frac{1}{2} [\Pi_L(p, T) - g^{\mu\nu}\Pi_{\mu\nu}^{b}(p, T)] \\
= \frac{\alpha^2 T^2}{3} \frac{1}{2} \left[\frac{p^2}{|p|^2} \left(1 + \frac{p_0^2}{2m^2}\right)\right].
\]  

(8)

respectively. These components of the vacuum polarization tensor are used to determine the electromagnetic properties of a medium with hot photons and will be discussed in the next section in a little more detail. It is worth-mentioning that Fig.(1b) gives major contributions to the finite terms in the above equations. However, the other two diagrams are needed to establish the cancelation of singularities even at low temperature. It is also interesting to note that the order of singularities is the same as the order of \(\alpha\) in the matrix element. It is not an accidental occurrence of the matching order of singularity. It is due to the fact that the number of loops and the number of maximum delta functions associated with photon propagators in a term in the matrix element are the same and would correspond to the highest order of hot divergences.
III. CHARGE RENORMALIZATION

Vacuum polarization in a medium gives modification to electric charge and the coupling constant in QED. The electric charge couples with the medium through vacuum polarization and picks up thermal corrections accordingly. This change leads to the enhancement in coupling constant. Using Eq. (8) from the last section and the standard method of evaluation of charge renormalization constant of QED [12], the electron charge renormalization up to the order $\alpha^2$ can be expressed as

$$Z_3 = 1 + \frac{\alpha^2 T^2}{6m^2}. \quad (9)$$

The first order temperature dependent term in the above equation is not present indicating the absence of corrections from one loop as calculated in detail in Ref. [8]. The corresponding value of the QED coupling constant comes out to be

$$\alpha_R = \alpha(T = 0)(1 + \frac{\alpha^2 T^2}{6m^2}). \quad (10)$$

It can be clearly seen in the above equations that the electron charge and hence the QED coupling constant goes smaller with the increased order of loops which clearly assures the renormalization of electron charge. This change in the coupling constant leads to changes in the electromagnetic properties of a medium which we will discuss in the next section.

IV. ELECTROMAGNETIC PROPERTIES OF A MEDIUM

This is a well-known fact that the electromagnetic properties of media depend [8] on the coupling of charge with the medium, that is determined through the statistical properties. One-loop corrections do not change it at low temperatures whereas the second order corrections contribute to them. It can be seen that the electric permittivity of a medium at the two loop level modifies to

$$\varepsilon(p, T) = 1 - |p|^2 \Pi_L(p, T) = 1 - \frac{2\alpha^2 T^2 p^2}{3}(1 + \frac{p_0^2}{2m^2}), \quad (11)$$

whereas the magnetic permeability is given by

$$\frac{1}{\mu(p, T)} = 1 + \frac{1}{p^2}[\Pi_T(p, T) - \frac{p_0^2}{|p|^2} \Pi_L(p, T)] = 1 + \frac{\alpha^2 T^2}{3} \left[ \frac{1}{2p^2} - \frac{1}{|p|^2}(1 + \frac{p_0^2}{2m^2})(1 + \frac{2p_0^2}{|p|^2}) \right]. \quad (12)$$

Equations (11) and (12) show that the second order radiative emission and absorption of hot photons from the heat bath leads to deviations from unity in the values of the dielectric constant and the magnetic susceptibility. This particular feature is not present at the one-loop level at low temperature because of the absence of thermal corrections to $\Pi_L$ and $\Pi_T$ for cold fermions. There are two ways to approach limit. If $p_0 = |p|$ in the rest frame of the heat bath and then the limit $|p| \rightarrow 0$ is taken, we get

$$\kappa_L^2 \rightarrow \lim_{|p| \rightarrow 0} \Pi_L(0, |p|, T) = \frac{2\alpha^2 T^2}{3}, \quad (13)$$

$$\kappa_T^2 \rightarrow \lim_{|p| \rightarrow 0} \Pi_T(0, |p|, T) = \frac{\alpha^2 T^2}{2}. \quad (14)$$
On the other hand if we set \( p_0 = 0 \) with \( |p| \to 0 \) then we obtain

\[
\omega_L^2 \to \lim_{|p| \to 0} \Pi_L(|p|, |p|, T) = 0, \tag{15}
\]

\[
\omega_T^2 \to \lim_{|p| \to 0} \Pi_T(|p|, |p|, T) = \frac{\alpha^2 T^2}{6}. \tag{16}
\]

The difference of the longitudinal and transverse components of the vacuum polarization tensor in \( p^2 \to 0 \) limit measures the dynamically generated mass of the photon. Electromagnetic properties of a medium change due to this perturbative mass.

\section*{V. RESULTS AND DISCUSSIONS}

We have previously noticed [8] that the low temperature effects on vacuum polarization tensor due to hot photon background at one loop level are zero because of the absence of self couplings of photons. However, we found in this paper that thermal corrections to vacuum polarization tensor are non-zero at the higher loop level, even at low temperature. It occurs due to the overlap of hot photon loop with the cold fermions loops. It is effectively the mutual interaction of propagating hot photons through the cold fermions of the medium. This leads to the modifications in electromagnetic properties of a medium itself and are expected to give larger contribution at higher loop levels. However, most of these terms are finite and the order by order cancellation of singularities can be observed through the addition of all the same order diagrams. The renormalization of the theory can only be proved if covariant hot integrals are evaluated before the cold divergent integrals on mass shell. Once the hot loop energies are integrated out, the usual vacuum techniques of Feynman parameterization and dimensional regularization can be applied to get rid of vacuum singularities. It is also worth-mentioning that all the hot corrections give a similar \( T^2 \) dependence. The incorrect order of integration gives increasing order of \( T \) (See Eq.(4)) due to the overlap with the vacuum divergences. This unusual behaviour of hot integrals appear due to the overlap of hot and cold terms. Whereas, the usual regularization techniques of vacuum theory like dimensional regularization would only be valid in a covariant framework of a lorentz invariant system. Higher order terms may even give a stronger dependence on \( T \) with this inverted order of integration.

At the two loop level, it has been checked in detail by integration that the vertex type corrections to the virtual electrons vanish and the self energy type corrections to the electron loops contribute to QED coupling constant at low temperatures. It indicates the mutual interaction of photons at the higher loop level. The dominant temperature corrections in this hot medium are of the order \( \alpha \) which implies the convergence of the perturbative series. Since the thermal effects are incorporated as perturbative effects, at sufficiently low energies \( (E < < \mu_e) \), the finite temperature corrections become negligible and one can recover the terms in vacuum by taking \( T = 0 \) in these results. The presence of the statistical contribution of photon propagator modifies the vacuum polarization and hence the electron charge which leads to changes (though small) in the electromagnetic properties of the hot medium even at low temperatures. Mass, wavefunction, and charge of electron are renormalized in the presence of heat bath. These renormalized values give the dynamically generated mass of photon and its effective charge in such a background. Equations (11) and (12) give the estimation of dielectric constant and magnetic permeability of a hot medium. Both of these quantities deviate from unity whereas their value was unity in QED at the one loop level [13].

It may be noted that as \( p^2 \to 0 \), Eq. (8) reduces to

\[
\Pi_T(p) = \frac{\alpha^2 T^2}{6}, \tag{17}
\]

which corresponds to the dynamically generated thermal mass of photon in the hot background. This type of effect has been earlier observed for self-mass of gluon even at the one-loop level [9] due to the self-coupling of gluons. With rising temperature this photon mass drastically change the behavior of this coupling, especially when \( T \geq m \). Further we obtain the respective propagation vectors and frequencies when we take the limiting values for the longitudinal and transverse component of the vacuum polarization tensor in Eqs. (7) and (8). In particular, in Eq. (13), \( \frac{2\alpha^2 T^2}{3} \) represents the Debye screening length in such a medium. This helps to evaluate the decay rates and the scattering crosssections of particles in such media.

It is worth-mentioning that in the real-time formalism, the propagator has two additive terms, the vacuum term and the temperature dependent hot term. Therefore, in the second order perturbation theory we get purely hot term
(\alpha^2 T^4/m^4), purely cold terms (T^0) and the overlapping hot and cold terms (\alpha^2 T^2/m^2). These terms can only be obtained in this formalism at the two-loop level. So the overall result comes out to be a combination of all of these terms. For example, eq.(10) can be written as

$$\alpha_R = \alpha(T = 0)(1 + \frac{\alpha^2 T^2}{6m^2} + O(\frac{T^4}{m^4})).$$

Since the last term (\alpha^2 T^4/m^4), is much smaller than the second term (\alpha^2 T^2/m^2), so we can ignore this term at the moment. We got similar results in Ref. [7] regarding the self-mass of electron in QED at two loops level. However, for T > m, the last term has to be evaluated. It was correctly speculated in Ref. [16] that the overall effect is of the order T^4/m^4. In some of the earlier results, the effective potential approach has been used to obtain two loop thermal corrections as T^4/m^4 [14,15]. Here in this paper, the dominant contribution of the two hot loops is calculated for temperature sufficiently smaller than the electron mass. The leading order contribution at low T is based on the perturbative expansion in QED and the first term should be proportional to T^2/m^2 in this expansion. The damping factor exp(−m/T) appears in the effective action when all the contributing terms are included simultaneously. We look at all these terms one by one and work for sufficiently small values of temperature to get the simple and approximate results to evaluate some physically measurable parameters. We also ignore the magnetic field effect in this calculation. It is again a reasonable approximation to demonstrate the renormalizability of QED at low temperatures. Next term in this calculation is proportional to T^4/m^4 with a smaller coefficient. However, this term is non-ignoreable and is needed to be included at high temperatures. We are already working on the evaluation of such terms which have some extra high T singularities. The perturbative analysis in QED is important to show the order by order cancellation of singularities in real-time formalism. In this formalism, the leading order terms can be evaluated ignoring the rest of it. We are now working on the evaluation of the next term. Even though it is a lengthy procedure, a good estimate of the background contribution can be obtained from it at high T also. In QCD, the effective potential approach has to be used since perturbative analysis becomes unmanageable due to the self-coupling of gluons in the presence of hot and dense media. Such estimations are done in literature [17]. However, this effective potential approach has to be used in QED and other theories, especially for the calculation of the effective action.

VI. APPENDIX

The hot contribution at low temperature, comes from the photon background only. Eq. (2) for Fig. (1b) therefore simplifies to

$$\Pi^b_{\mu\nu}(p,T = 0) = -2\pi i e^4 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} N^b_{\mu\nu}(l) \delta(l^2)n_B(l)$$

$$\times \frac{1}{(k^2 - m^2 + i\varepsilon)^2[(p - k)^2 - m^2 + i\varepsilon][(l + k)^2 - m^2 + i\varepsilon]},$$

where

$$N^b_{\mu\nu} = 8 \left[ (3m^2 - k^2 - 2k.l) \right] k_{\mu}(p - k)_{\nu} + k_{\nu}(p - k)_{\mu} - g_{\mu\nu}k.(p - k) + (k^2 - m^2) \{ l_{\mu}(p - k)_{\nu} + l_{\nu}(p - k)_{\mu} - g_{\mu\nu}l.(p - k) \} + 2g_{\mu\nu}m^2(m^2 - k.l),$$

$$g^{\mu\nu}\Pi^b_{\mu\nu}(p,T) = \frac{16e^4}{(2\pi)^6} \int \frac{(2p.k - k^2 - m^2)}{(k^2 - m^2)^2(k^2 - m^2 - 2p.k)} \int |l|d|l|n_B(l),$$

with

$$\int |l|d|l|n_B(l) = \frac{\alpha^2 T^2}{6}.$$

Now integrating over the cold loop by using Feynman parametrization and dimensional regularization one gets Eq. \(3\). On the other hand, first doing Feynman parametrization, integrating by dimensional regularization, and simplifying one gets

\[
g_{\mu\nu}^{\Pi_b}(p, T) = \left(\frac{e^4}{8\pi^3}\right) \int d^4l \delta(l^2) n_B(l) \int_0^1 dx \int_0^x dy \int_0^y dz \left\{ \frac{1}{\left\{-m^2 - 2pl.y(y + z)\right\}^2} \left\{-2(p.l)^2(2y^4 + 2y^2z^2 + 4y^3z - y^2 - yz) + m^2(p.l)(6y^2 + 6yz - y + z) + 4m^4\right\} + \frac{p.l}{m^2 - 2pl.y(y + z)} \right\} \left[ -12 \left( \frac{1}{\eta} - \gamma - \ln\left(\frac{4\pi}{m^2 - 2pl.y(y + z)}\right) \right) \right\},
\]

(23)

and

\[
g_{\mu\nu}^{\Pi_b}(p, T) = \left(\frac{e^4}{8\pi^3}\right) \int d\mu \int |d|d|n_B(l)\int_0^1 dx \int_0^x dy \int_0^y dz \left\{ 2(2y^4 + 2y^2z^2 + 4y^3z - y^2 - yz) |d|^2 \left( \frac{p_o - |p|\mu}{a_+^2} \right) + \left( \frac{p_o + |p|\mu}{a_-^2} \right) \right\} + m^2|d|(6y^2 + 6yz - y + z) \left( \frac{p_o - |p|\mu}{a_+^2} \right) - \left( \frac{p_o + |p|\mu}{a_-^2} \right) \right\} + m^2|d|(24y^2 + 24yz - 18y - 12z) \left[ \frac{p_o - |p|\mu}{a_+^2} \right] - \left[ \frac{p_o + |p|\mu}{a_-^2} \right] \right\} + 4m^4 \left[ \frac{1}{a_+^2} + \frac{1}{a_-^2} \right] - 6m^2 \left[ \frac{1}{a_+} + \frac{1}{a_-} \right] 
- 12 \left[ 2 \left( \frac{1}{\eta} - \gamma - \ln(4\pi) \right) - \ln(a_+) - \ln(a_-) \right] \right\},
\]

(24)

where

\[
a_\pm = m^2 \pm 2y(y + z) (p_o \pm |p|\mu).
\]

(25)

After a bit lengthy calculation of the integrals, these relations finally give Eq. \(4\).

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