Reference study to characterize plasma and magnetic properties of ultracool atmospheres

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ABSTRACT
Radio and X-ray emission from brown dwarfs (BDs) suggest that an ionized gas and a magnetic field with a sufficient flux density must be present. We perform a reference study for late M-dwarfs (MD), BDs and giant gas planet to identify which ultracool objects are most susceptible to plasma and magnetic processes. Only thermal ionization is considered. We utilize the DRIFT-PHOENIX model grid where the local atmospheric structure is determined by the global parameters $T_{\text{eff}}, \log(g)$ and $[M/H]$. Our results show that it is not unreasonable to expect Hα or radio emission to originate from BD atmospheres as in particular the rarefied upper parts of the atmospheres can be magnetically coupled despite having low degrees of thermal gas ionization. Such ultracool atmospheres could therefore drive auroral emission without the need for a companion’s wind or an outgassing moon. The minimum threshold for the magnetic flux density required for electrons and ions to be magnetized is well above typical values of the global magnetic field of a BD and a giant gas planet. Na+, K+ and Ca+ are the dominating electron donors in low-density atmospheres (low $\log(g)$), solar metallicity independent of $T_{\text{eff}}$. Mg2+ and Fe2+ dominate the thermal ionization in the inner parts of MD atmospheres. Molecules remain unimportant for thermal ionization. Chemical processes (e.g., cloud formation) affecting the most abundant electron donors, Mg and Fe, will have a direct impact on the state of ionization in ultracool atmospheres.

Key words: plasmas – planets and satellites: atmospheres – stars: atmospheres – brown dwarfs – radio continuum: planetary systems – radio lines: planetary systems.

1 INTRODUCTION
Ultracool objects like brown dwarfs (BDs) and giant gas planets have masses below the hydrogen-burning limit of $\sim 0.08\ M_\odot$ (e.g. Burrows et al. 2001). BDs are born like stars by gravitational collapse, however, they have not sufficient mass to achieve the required core temperature to provide a steady rate of nuclear hydrogen fusion. As a consequence, gravitational collapse provides the only energy source for most of the BD’s lifetime. Cooling and contracting during their entire life, BDs cannot compensate the radiative losses by thermonuclear processes. BDs evolve from a young object with an effective temperature $T_{\text{eff}} \approx 3000$ K that is comparable to late M-dwarfs (MDs) and surface gravities like giant gas planets to high-gravity objects of $\log(g) = 5.0$ with an effective temperature lower than $T_{\text{eff}} \approx 500$ K. BDs are fully convective objects, and they can be fast rotators. BDs and giant gas planets form clouds in their atmospheres which have strong feedback on to the atmospheric structure due to element depletion and due to a high opacity (Helling & Casewell 2014 and references there in). Observations by Biller et al. (2013), Buenzli et al. (2014) and Crossfield et al. (2014), for example, suggest that BD atmospheres show a patchy cloud coverage. Transit spectroscopy from extrasolar planets suggests that giant gas planets are covered in hazes and clouds too (Pont et al. 2008; Gibson et al. 2012; Sing et al. 2015). Model simulations suggest that cloud formation prevails for a large range of effective temperatures up to 2800 K, and for metallicities as low as $[M/H] = -5.0$ (Witte, Helling & Hauschildt 2009). A significant volume of these clouds is susceptible to local discharge events (Helling et al. 2013): large-scale discharges in the upper cloud regions and corona-like small-scale discharges in the inner, denser part of the cloud. These local discharge events may generate atmospheric electrical storms. At the same time, storms may ionize the local gas. Luque et al. (2014) modelled the ionosphere of Saturn and Jupiter finding that the atmospheric electrical storms may produce up to $10^3$ cm$^{-3}$ of free electrons below the ionosphere.

Radio and X-ray emission from ultracool dwarfs are now well established (e.g. 2MASSJ10475385+2124234 by Route & Wolszczan 2012; 2MASS J13153094−2649513AB by Burgasser et al. 2013). Berger (2002) observed 12 sources in radio, X-ray and Hα emission between the spectral types M8 and L3.5. Sorahana, Suzuki &
Yamamura (2014) present mid-IR AKARI observations which suggest the presence of chromospheric activity in BDs. Schmidt et al. (2015) derive a rise in magnetic activity in form of Hα emission from SDSS spectra from 2% for early M-dwarfs (M0) to 88 per cent for early L-dwarfs (L0). 39% of the L dwarfs in their sample observed multiple times are suggested to be variable. Such observations indicate that an appropriately ionized gas is present in the atmospheres of such cool objects that allows chromospheric heating. The prevailing question is how much of such a cool, cloud-forming atmosphere needs to be ionized that a chromosphere could form.

It is suggested that the magnetic field strength of BDs is as high as \(10^5\) G (e.g. Shulyak et al. 2011). Lynch, Mutel & Gäudel (2015) suggest field strengths of \(2.5–2.5\) kG for the surface field on TVLM 0513–46 and 2M 0746+20. A correlation between radio activity and rotation has not been settled for BDs given the wide parameter range occupied by these objects (McLean, Berger & Reiners 2012; Antonova et al. 2013). Williams et al. (2014) demonstrate that ultracool objects do not follow the classical Gudel–Benz relation where the X-ray and the radio luminosity from F – M stars correlate (Güdel & Benz 1993). This relation was shown to persist for solar flares and active rotating binaries (Benz & Güdel 1994). The deviation of ultracool stars from the Gudel–Benz relation beyond approximately M5 may suggest a change in the dynamo mechanism that produces the magnetic field in such ultracool objects (Cook, Williams & Berger 2014). If the atmospheric gas can couple to the strong magnetic field in BDs, the kinetic energy carried by large-scale convective motions may be transported to the top of the atmosphere and released in form of flares, quiescent or quasi-quiescent emissions. The flares are a sudden release of magnetic energy from the deeper convective layers. The quiescent emission is a continuous emission but at lower energy levels than flares. The most likely mechanism to produce this emission is electron cyclotron maser emission from highly relativistic electrons or plasma emission alternatively, incoherent synchrotron or gyrosynchrotron emission from relativistic electrons (Hallinan et al. 2006). Flares and quiescent emission were reported for example by Burgasser & Putman (2005) for two late-type MDs, with a magnetic field strength of 1 kG. Tanaka, Suzuki & Inutsuka (2014) study the mass-loss of hot Jupiters through MHD waves. They suggest the formation of a chromosphere by Alfvén wave heating. Tanaka et al. (2014), however, observe that the gas does not seem to be fully ionized, i.e. degree of ionization \(f_e \ll 1\), for the mechanism to work. Mohanty et al. (2002) analysed magnetic Reynolds number, \(R_m \propto f_e\), for a set of model atmospheres to show why the chromospheric Hα activity should be low in rapid rotating BDs with \(T_{\text{eff}} \gtrsim 1500\) K and solar metallicity. The works by Schmidt et al. (2015) and Sorahana et al. (2014) suggest that L-dwarfs should have a chromosphere (but with a reduced filling factor) despite having low magnetic Reynolds numbers.

Our paper presents a theoretical framework using a set of fundamental parameters to analyse the ionization and magnetic coupling state of objects with ultracool atmospheres. This paper focuses on late MDs, BDs and giant gas planets spanning \(T_{\text{eff}} = 1000–3000\) K. The approach is also applicable to, for example, protoplanetary discs. Only thermal ionization is considered for this reference study against which the effect of additional processes (e.g. dust–dust collisions, cosmic ray ionization, Alfvén ionization, lighting, photoionization) can be tested in future works. Our investigations utilize results from DRIFT-PHOENIX 1D model atmosphere simulations (Helling et al. 2008; Witte et al. 2009, 2011). This allows us to perform an extensive reference study across the late MD, BD and planetary regime on the basis of a consistently calculated model atmosphere grid for a large set of global parameters \((T_{\text{eff}}, \log (g), [\text{M/H}])\). The aim of this study is to identify ultracool objects that are most susceptible to plasma processes by itself or that lead to instabilities that trigger the emergence of a strong plasma. This study does not include any multidimensional atmospheric flows and resulting multi-D radiative transfer effects.

Section 2 describes our approach and the use of the DRIFT-PHOENIX model atmosphere results. Sections 3 and 5 introduce our theoretical framework in form of basic plasma and magnetic parameters, respectively. Our study shows that ultracool atmospheres are composed of an ionized and magnetized gas. The local degree of ionization varies largely amongst the objects and throughout the atmospheres. While a late MD has a considerable degree of ionization throughout the whole atmosphere, the degree of thermal ionization for an L-dwarf is rather low but may well be enough to seed other ionization processes like for example due to Alfvén ionization (Stark et al. 2013). In Section 3.3.2 we demonstrate that electromagnetic interactions can dominate over electron–neutral interactions also in regions of a very low degree of ionization. In Section 3.3.3 we investigate the relevant length-scales effected by electrostatic interactions in the gas phase. Section 4 (with additional material in Appendix A) contains our assessment of the local equilibrium chemistry abundances with respect to electron donors species across the late MDs, BDs and giant gas planets regime.

In Section 5 we demonstrate that 30–50 per cent of the atmospheric volume can be magnetically coupled in L-dwarfs for a magnetic field strength of \(10^3\) G. The atmospheric volume with a degree of thermal ionization above a plasma threshold value of \(>10^{-7}\) is, however, considerably lower. Our results show that it is not unreasonable to expect ultracool atmospheres to emit Hα or even in radio wavelength as in particular the rarefied upper parts of the atmospheres are affected by electromagnetic interactions over many pressure scaleheights despite having low degrees of ionization. Section 6 contains the discussion of our results in view of previous publications. Section 7 presents our conclusions.

2 APPROACH

We aim to assemble a theoretical framework that allows us to assess the plasma and magnetic character in atmospheres of objects across the stellar-planetary boundary, namely for late MDs, BDs and planets. Our approach is not limited to these objects as the plasma parameters used are fundamental properties of a gas rather than of a particular object. We utilize the grid of DRIFT-PHOENIX model atmosphere structures in order to quantify the plasma and magnetic characteristics. DRIFT-PHOENIX is a combination of two complementary codes, DRIFT and PHOENIX (Helling et al. 2008; Witte et al. 2009, 2011). The DRIFT code (Woitke & Helling 2004; Helling et al. 2008) solves a system of equations that describes the stationary dust formation process of mineral clouds (seed formation, growth, evaporation, sedimentation, element depletion) and interaction between the dust grains and gas (Woitke & Helling 2003, 2004; Helling & Woitke 2006; Helling, Woitke & Thi 2008). PHOENIX is a hydrostatic radiative transfer model atmosphere code (Hauschildt & Baron 1999) that determines the resultant thermodynamic structure of the atmosphere (local gas temperature \(T_{\text{gas}}\) [K], local gas pressure \(p_{\text{gas}}\) [bar]) and local electronic pressure \(p_e\) [bar]) from fundamental stellar parameters (effective temperature \(T_{\text{eff}}\) [K], surface gravity \(\log (g)\) [cm s\(^{-2}\)] and metallicity [M/H]). When combined with DRIFT it provides a self-consistent atmospheric model that takes into account cloud formation and its impact on the thermodynamic structure and the resulting spectral energy distribution.
Table 1. DRIFT-PHOENIX atmosphere structures used: **Group 1:** giant gas planets and young BDs. **Group 2:** dependence on log\((g)\). **Group 3:** dependence on metallicity.

| \(T_{\text{eff}}\) [K] | log \((g)\) | [M/H] |
|-----------------|--------|------|
| Group 1         |        |      |
| 1000 – 3000     | 3.0    | 0.0  |
| 1000            |        |      |
| Group 2         |        |      |
| 2000            | 3.0, 4.0, 5.0 | 0.0 |
| 2800            |        |      |
| Group 3         |        |      |
| 2000            | 3.0    | −0.6, −0.3, 0.0, +0.3 |
| 2800            |        |      |

The DRIFT-PHOENIX atmosphere simulations model the kinetic formation of mixed mineral cloud particle made of TiO\(_2\)\(_s\), Al\(_2\)O\(_3\)\(_s\), Fe\(_s\), SiO\(_2\)\(_s\), MgO\(_s\), MgSiO\(_3\)\(_s\), Mg\(_2\)SiO\(_4\)\(_s\) which effects 6 elements (O, Mg, Si, Fe, Al, Ti). Mg, Si, and Fe are the most abundant elements after O and C in a gas with solar element abundances. Using DRIFT-PHOENIX atmospheric models with a range of effective temperatures \(T_{\text{eff}} = 1000\) K – 3000 K, surface gravity \(\log (g) = 3.0, 4.0, 5.0\) and metallicity \([\text{M/H}] = −0.6, −0.3, 0.0, +0.3\), we evaluate the degree of thermal gas ionization, the plasma parameter (Section 3) and magnetic parameters (Section 5). Applying a separate chemical equilibrium code, we evaluate the gas-phase composition of various atmosphere models to assess if the dominating electron donating species changes or remains the same (Section 4.1).

We apply a chemical equilibrium routine to calculate the chemical composition in more detailed than available from the standard DRIFT-PHOENIX output. Our main interest is to evaluate the results regarding the ions in the gas phase. A combination of 155 gas-phase molecules (including 33 complex hydrocarbon molecules), 16 atoms, and various ionic species were used under the assumption of local thermodynamic equilibrium (LTE). The Grevesse, Asplund & Sauval (2007) solar composition is used for calculating the gas-phase chemistry outside the metal depleted cloud layers. No solid particles were included in the chemical equilibrium calculations but their presence influences the gas phase by the reduced element abundances due to cloud formation and the cloud opacity impact on the radiation field, both accounted for in the DRIFT-PHOENIX model simulations.

We utilize DRIFT-PHOENIX model atmosphere \((T_{\text{gas}}, p_{\text{gas}}, p_c)\) structures and the dust element depleted abundances as input for our calculations. We group the DRIFT-PHOENIX atmosphere structures into three groups for an easier presentation of our results (Table 1). These groups, defined by a range of global parameters, cover young and old giant gas planets, young and old BDs and late MDs. Fig. 1 shows the thermodynamic profiles \((T_{\text{gas}}, p_{\text{gas}})\) for each of these groups. As the effective temperature increases, the thermodynamic gas temperature of the atmosphere increases as well for a given log \((g)\) and \([\text{M/H}]\). The effect of dust on the local gas temperature in the atmosphere appears as a step-like temperature change (backwarming) in the atmosphere models.

### 3 Basic Plasma Parameters

In the following section, we lay out the theoretical framework that we use to characterize the plasma of an atmospheric environment with respect to its electrostatic and magnetostatic behaviour. This is inspired by the wealth of radio observations of substellar objects (Berger 2002; Berger et al. 2010; Route & Wolszczan 2012; Burgasser et al. 2013; Williams et al. 2014). To understand these observations, radio wavelength (quiescent emission) and X-ray emission (flares), the atmospheric gas must couple with the background magnetic field. As a consequence, free charged particles produced in the atmosphere would be accelerated along magnetic field lines and released into the upper parts of the atmosphere. A magnetic coupling of the local gas would also be required for Alfvén waves.
to develop and potentially contributing to an acoustic heating of a chromosphere also on such ultracool objects (e.g. Testa, Saar & Drake 2015). We note that ideal and non-ideal MHD simulations require a certain degree of ionization to allow Alfvén wave heating to develop as possible mechanisms for chromospheric heating.

For a plasma to exist, the gas needs to be ionized. The degree of ionization, \( f_e \), measures the extent to which a gas is ionized, and it is defined as

\[
f_e = \frac{p_e}{p_e + p_{gas}}
\]

where \( p_{gas} \) and \( p_e \) are the gas and electron pressure, respectively, both in [bar]. Once we have determined the degree of ionization of the atmospheric gas depending on the global parameters \( (T, \log (g), [M/H]) \) we evaluate the plasma frequency to investigate where in the atmosphere electromagnetic interactions dominate over kinetic collisions between electrons and neutrals,

\[
\omega_{pe} \gg \nu_{ne}.
\]

\( \omega_{pe} \) [rad s\(^{-1}\)] is the plasma frequency (i.e. the frequency at which the plasma reacts to an imposed or perturbed electric fields), \( \nu_{ne} \) [s\(^{-1}\)] is the electron–neutral collision frequency. Only if equation (2) is fulfilled, we can expect the ionized gas to undergo electromagnetic interactions that could lead, for example, to discharge processes. A more refined insight about electrostatic interactions influencing the atmospheric gas can be gained by determining the length-scales beyond which the Coulomb force of a charge does not any more effects its surrounding. On length-scale larger than the Debye length, a gas will be quasi-neutral and no electrostatic forces will affect the gas behaviour. Hence,

\[
\lambda_D \ll L
\]

with \( \lambda_D \) the Debye length and \( L \) the typical length-scale of the considered plasma, both in [m]. Ideally, this would be associated with the atmospheric volume where the ionized gas can couple to an external magnetic field.

In the following subsections we define each of the plasma criteria and evaluate them for our model atmosphere grid. All equations and natural constants are given in \( \text{S} \) units. All results, however, have been converted into cgs unit for an easier representation in the astrophysical context.

### 3.1 Plasma Frequency: \( \omega_{pe} \gg \nu_{ne} \)

In a plasma, if electrons are displaced from their equilibrium position (assuming a uniform, stationary ionic background), a charge imbalance is imposed on the plasma, creating a local, restoring electric field. Consequently, the electrons try to re-establish charge neutrality resulting in them oscillating around their equilibrium position with a particular frequency called plasma frequency. The plasma frequency is defined as

\[
\omega_{pe} = \left( \frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2}
\]

with \( n_e \) the electron number density [m\(^{-3}\)], \( e \) the electron charge [C], \( m_e \) the electron mass [kg]. If the plasma frequency, \( \omega_{pe} \), is greater than the frequency of collisions between the electrons and neutral particles, \( \nu_{ne} \) [s\(^{-1}\)], then, long-range electromagnetic collective effects dominate over short-range binary interactions (see Fig. 3). The collision frequency for neutral particles with electrons is given by \( \nu_{ne} = \sigma_{gas} n_{gas} v_{\text{th.e}} \), where \( v_{\text{th.e}} \) is the thermal velocity of electrons given by \( v_{\text{th.e}} = (k_B T_e/m_e)^{1/2} \) [m s\(^{-1}\)], \( n_{gas} \) the ambient gas density and \( \sigma_{gas} \) the collision, or scattering, cross-section of particles. The latter is assumed to be \( \sigma_{gas} = \pi \times r_{gas}^2 \) with \( r_{gas} = n_1 H_2 \) as the atmospheric gas in late MDs, BDs and most likely in giant gas planets is composed mostly of molecular hydrogen, \( \text{H}_2 \). Therefore, the collision cross-section is approximated by \( \sigma_{gas} \approx \pi \times n_1 H_2^2 \approx 5.81 \times 10^{-26} \text{m}^2 \) \((n_1 = 1.36 \times 10^{-10} \text{m})\).

If the charged particles collide frequently with the ambient neutral gas \( (\omega_{pe} / \nu_{ne} \ll 1) \), their motion will be determined by nearest neighbour interactions and not by collective, long-range electro-magnetic interactions.

### 3.2 Debye length: \( \lambda_D \ll L \)

The Debye length, \( \lambda_D \), is the spatial length-scale beyond which a plasma can be considered quasi-neutral \((n_e \approx n_i \approx n_{gas})\). For length-scales less than the Debye length, a test charge will experience the influence of the charge imbalance inside the Debye sphere. The Debye length, \( \lambda_D \) [m], resulting from the solution of the Poisson equation for a non-zero charge density near test charge, is defined as

\[
\lambda_D = \left( \frac{e \omega_{pe} l_p}{\epsilon_0} \right)^{1/2},
\]

with \( k_B = 1.38 \times 10^{-23} \text{ J K}^{-1} \) and \( \epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1} \). A plasma is quasi-neutral if

\[
\lambda_D \ll L.
\]

For an ionized gas region to exhibit plasma behaviour, it is required that over the length-scale of the region, the electron number density is high enough that \( L \) is greater than the Debye length. The typical length-scale of the plasma, \( L \) [m], considered in the literature (e.g. Mohanty et al. 2002; Tanaka et al. 2014) is the pressure scaleheight which depends on the local gas properties and varies with \( 1/g \). Typical values for the pressure scaleheight are \( 10^2-10^4 \text{ cm for a BD with log (g) = 5} \). Tanaka et al. (2014) base their length-scale on the definition of the Alfvén speed that is of the order of the velocity of sound (their equation 12). Also their approach results in a typical length-scale of the order of the pressure scaleheight. Associate with the Debye length is the number of charges inside a Debye sphere, \( N_D \) (plasma parameter, App. B). If \( N_D \gg 1 \), the ionized gas exhibits plasma behaviour.

### 3.3 Plasma parameters across the star–planet regime

In the following, we evaluate the plasma criteria for late MDs, BDs and giant gas planet atmospheres. All results have been calculated considering thermal ionization only and compose our reference study against which the need for additional ionization processes can be derived. First we examine for which global parameters and \( p_{gas} \) values, the gas is ionized above the threshold value of \( f_e > 10^{-7} \) (Section 3.3.1). In Section 3.3.2 we demonstrate that long-range electromagnetic collective interactions of many charged particles can dominate over short-range binary interactions also in regions of a very low degree of ionization. Recent BD atmospheric investigations have focused on the degree of ionization to characterize plasma behaviour, in this paper we consider multiple parameters to gain a more detailed characterization (e.g. Osten et al. 2015). In Section 3.3.3 we demonstrate for which length-scale ultracool atmospheres will be effected by electrostatic processes and that it is not unreasonable to expect ultracool atmospheres to emit H\( \alpha \) or even in...
radio wavelength as in particular the rarefied upper parts of the atmospheres fulfill plasma criteria easily despite having low degrees of ionization.

3.3.1 Degree of ionization by thermal processes, $f_e$

Fig. 2 shows the degree of thermal ionization evaluated for the same models represented in Fig. 1 (Table 1). Guided by these results we consider $f_e > 10^{-7}$ to be a threshold above which the gas is partially ionized and it may exhibit plasma behaviour. The above choice of a threshold value is supported by results from laboratory experiments and laboratory plasma devices (e.g. Tokamak; Diver 2001; Fridman 2008). For a fluorescent tube, the degree of ionization is $f_e \approx 10^{-5}$ according to Inan & Golkowski (2010). Christophorou & Olthoff (2004) showed that at low temperature ($T_{\text{gas}} \approx 300–600$ K) and low density ($10^{13}–10^{16}$ cm$^{-3}$) the gas is weakly ionized with $f_e \approx 10^{-6}–10^{-5}$. If the density of the charged particles increases towards $f_e \rightarrow 1$ the gas will be fully ionized. For example, a fully ionized gas is assumed in ideal MHD calculations. This threshold, $f_e > 10^{-7}$, allows us to derive the atmospheric volume that can be considered as an ionized gas (Section 3.4, Fig. 9). Deriving such atmospheric volume fractions will enable us to compare the results from different plasma criteria (Equations 2, 3) and to demonstrate that a gas does not need to be fully ionized in order to exhibit collective plasma effects.

(i) Group 1: changing $T_{\text{eff}}$ (Fig. 2, top)

$log(g) = 3.0, [\text{M/H}] = 0.0$

A solar-metallicity MD with $T_{\text{eff}} = 3000$ K achieves $f_e > 10^{-7}$ in almost the entire atmosphere. For cooler atmospheres with $T_{\text{eff}} \leq 2800$ K, $f_e > 10^{-7}$ is only reached for $p_{\text{gas}} > 10^{-4}$ bar. The atmospheric fraction that reached $f_e > 10^{-7}$ increases with increasing $T_{\text{eff}}$.

(ii) Group 2: changing log($g$) (Fig. 2, middle)

$T_{\text{eff}} = 1000, 2000, 2800$ K, $[\text{M/H}] = 0.0$

Values of varying surface gravity log($g$) = 3.0, 4.0, 5.0 are studied here. Models with $T_{\text{eff}} = 2800$ K reach $f_e > 10^{-7}$ for $p_{\text{gas}} > 10^{-4}$ bar; models with $T_{\text{eff}} = 2000$ K for $p_{\text{gas}} > 10^{-2}$ bar. A small part of the upper atmospheric reaches $f_e > 10^{-7}$ for $T_{\text{eff}} = 2800$ K and log($g$) = 4.0, 5.0 due to the increasing contribution of Ca+ with outwards decreasing $p_{\text{gas}}$ (compare Fig. 6). Models with $T_{\text{eff}} = 1000$ K have only a small fraction of the atmospheric gas that reaches the $f_e > 10^{-7}$ threshold. This occurs in the deepest layers of the atmosphere $1 < p_{\text{gas}} < 10^2$ bar. The atmospheric fraction that reached $f_e > 10^{-7}$ increases with decreasing log($g$) at high $p_{\text{gas}}$ and with increasing log($g$) at low $p_{\text{gas}}$.

(iii) Group 3: changing [M/H] = 0.0 (Fig. 2, bottom)

$T_{\text{eff}} = 1000, 2000, 2800$ K, log($g$)

Values of varying metallicity $[\text{M/H}] = -0.6, -0.3, 0.0, +0.3$ are analysed. Models with $T_{\text{eff}} = 2800$ K and $T_{\text{eff}} = 2000$ K satisfy $f_e > 10^{-7}$ for all values of metallicity for $p_{\text{gas}} > 10^{-5}$ bar. A small part of the upper atmospheric reaches $f_e > 10^{-7}$ for $T_{\text{eff}} = 2800$ K due to an increasing electron donation from Ca+. With models with $T_{\text{eff}} = 1000$ K have only a small fraction of the atmospheric gas that can be ionized for $p_{\text{gas}} > 10^{-1}$ bar.

All models of non-irradiated atmospheres show a degree of ionization which increases from a minimum ($p_{\text{gas}} \sim 10^{-8} – 10^{-6}$ bar) with increasing local gas pressure values towards the deeper layers of the atmosphere. Atmosphere models with $T_{\text{eff}} \leq 2800$ K, log($g$) = 3.0, $[\text{M/H}] = 0.0$ can reach $f_e > 10^{-7}$ only for high $p_{\text{gas}}$ (inner parts of the atmosphere). Only one model atmosphere achieves $f_e > 10^{-7}$ throughout nearly the entire atmosphere ($T_{\text{eff}} = 3000$ K, log($g$) = 3.0, $[\text{M/H}] = 0.0$). For atmospheres of late MDs, BDs and giant gas planet atmospheres, the MD atmosphere is easily ionized by thermal processes. Atmosphere of BDs can only be thermally ionized in deeper layers. Top: Group 1. Middle: Group 2. Bottom: Group 3.

Figure 2. The degree of thermal ionization, $f_e = p_{\text{e}}/(p_{\text{e}} + p_{\text{gas}})$ as a measure of free charged particles for MD, BD and giant gas planet atmospheres. The MD atmosphere is easily ionized by thermal processes. Atmosphere of BDs can only be thermally ionized in deeper layers. Top: Group 1. Middle: Group 2. Bottom: Group 3.
Section 4 investigates which atoms or molecules are the most important electron donors in these cold atmospheres, and hence, responsible for the values of the degree of thermal ionization, \( f_e \). We note that the degree of thermal ionization will be influenced by the formation of clouds if the dominating electron donors are amongst the most abundant condensing species. Ca does not fall into this category.

### 3.3.2 Dominating electromagnetic interaction

The criterion \( \omega_{pe} \gg \nu_{ne} \) is used to derive where in an ultracool atmosphere the long-range, electromagnetic, collective interactions of many charged particles dominates over short-range binary interactions in an ionized gas of a certain degree of ionization. Fig. 3 shows the results of this criterion for the three groups of model atmosphere structures (Fig. 1).

(i) **Group 1:** changing \( T_{\text{eff}} \) (Fig. 3, top)

\[ \log (g) = 3.0, \ [M/H] = 0.0 \]

As \( T_{\text{eff}} \) increases, the range of the \( p_{\text{gas}} \) where \( \omega_{pe} \gg \nu_{ne} \) increase too. For models with \( T_{\text{eff}} \geq 2200 \) K the entire atmosphere satisfies this criterion; for \( T_{\text{eff}} = 2000 \) K almost the entire atmosphere; for models with \( T_{\text{eff}} = 1800 \) K in the uppermost and for the innermost parts of the atmosphere and for \( T_{\text{eff}} = 1200 \) K only for \( 10^{-2} < p_{\text{gas}} < 10^{-1} \) bar. The model with \( T_{\text{eff}} = 1000 \) K is too cool to fulfill this criterion.

(ii) **Group 2:** changing \( \log(g) \) (Fig. 3, middle)

\[ T_{\text{eff}} = 1000, 2000, 2800 \text{ K}, \ [M/H] = 0.0 \]

Models with \( T_{\text{eff}} = 2800 \) K satisfy this criterion throughout the whole atmosphere except for \( \log (g) = 5.0 \) at the highest pressures. For \( T_{\text{eff}} = 2000 \) K and \( \log (g) = 3.0 \) almost the entire atmosphere fulfills this criterion; \( T_{\text{eff}} = 2000 \) K and \( \log (g) = 4.0 \) only in the uppermost and for the innermost parts of the atmosphere; for \( T_{\text{eff}} = 2000 \) K and \( \log (g) = 5.0 \) only for \( p_{\text{gas}} < 10^{-6} \) bar. Models with \( T_{\text{eff}} = 1000 \) K do not satisfy this criterion.

(iii) **Group 3:** changing \([M/H] = 0.0\) (Fig. 3, bottom)

\[ T_{\text{eff}} = 1000, 2000, 2800 \text{ K}, \ \log (g) \]

Models with \( T_{\text{eff}} = 2800 \) K and all value of metallicity and \( T_{\text{eff}} = 2000 \) K with \([M/H] = +0.3\) satisfy this criterion in the whole atmosphere. For \( T_{\text{eff}} = 2000 \) K and \([M/H] = +0.3\) the entire atmosphere fulfills this criterion. For \( T_{\text{eff}} = 2000 \) K and \([M/H] = -0.3, -0.6\) only in the uppermost and for the innermost parts of the atmosphere, \( 10^{-7} < p_{\text{gas}} < 10^{-3}\); models with \( T_{\text{eff}} = 1000 \) K and \([M/H] = +0.3\) only for \( 10^{-2} < p_{\text{gas}} < 10^{0}\). Models with \( T_{\text{eff}} = 1000 \) K and \([M/H] = 0.0, -0.3, -0.6\) and do not fulfill this criterion for any atmospheric gas pressure.

Fig. 3 demonstrates that the collective, long-range electromagnetic interactions dominate over short-range binary interactions in atmospheres of low degrees of ionizations, i.e. for \( 2800 \geq T_{\text{eff}} \geq 2000 \) K. As \( T_{\text{eff}} \) and the metallicity increase, \( \omega_{pe} \gg \nu_{ne} \) is easier fulfilled at high \( p_{\text{gas}} \), however, as \( T_{\text{eff}} \) increases and the metallicity decreases, \( \omega_{pe} \gg \nu_{ne} \) is easier fulfilled at low \( p_{\text{gas}} \) for \( T_{\text{eff}} = 2800 \) K, 1000 K. This effect is counteracted by an increase in \( \log(g) \). The lowest value of \( \log (g) \) causes a decrease of \( \omega_{pe}/\nu_{ne} \) in the uppermost parts of the atmosphere and an increase in the innermost parts. Consequently, long-range, electromagnetic, collective interactions of many charged particles do not require a complete ionization of the atmospheric gas, and a moderate gas ionization is sufficient.

---

**Figure 3.** Ratio of plasma frequency of the electrons and the frequency of collisions between neutral particles and electrons. Electromagnetic interactions dominate over electron–neutral interactions if \( \omega_{pe}/\nu_{ne} \gg 1 \). Top: Group 1. Middle: Group 2. Bottom: Group 3.

(i) The hottest model has the highest thermal degree of ionization, \( f_e \).

(ii) The lowest value of surface gravity causes an increase of \( f_e \) at high \( p_{\text{gas}} \); however, the highest value of surface gravity causes an increase of \( f_e \) at low \( p_{\text{gas}} \). Both trends are for a given \( T_{\text{eff}} \) and \([M/H] \).

(iii) The highest metallicity values cause an increase of \( f_e \) at high \( p_{\text{gas}} \) compared to the lowest metallicity models. For \( T_{\text{eff}} = 2800 \) K and \( T_{\text{eff}} = 1000 \) K the lowest value of the metallicity causes an increase of \( f_e \) at low \( p_{\text{gas}} \). Both trends are for a given \( T_{\text{eff}} \) and \( \log (g) \).
3.3.3 Electrostatically effected atmospheric length-scales

The Debye length, $\lambda_D$, is compared to a typical atmospheric length-scale of the order of the pressure scaleheight, $L = 10^3 \text{ m}$ (Helling et al. 2011b). Fig. 4 shows how the Debye length changes depending on the local atmospheric gas pressure, and where $\lambda_D \ll L$ is fulfilled.

(i) Group 1: changing $T_{\text{eff}}$ (Fig. 3, top)

The criterion $\lambda_D \ll L$ is fulfilled throughout the whole atmosphere for $T_{\text{eff}} \geq 1800 \text{ K}$; only for $T_{\text{eff}} = 1600 \text{ K}$ a small atmospheric gas volume for $p_{\text{gas}} < 10^{-10} \text{ bar}$ cannot reach this criterion. For models with $T_{\text{eff}} \leq 1400 \text{ K}$ this criterion is fulfilled for $p_{\text{gas}} > 10^{-6} \text{ bar}$. As $T_{\text{eff}}$ increases, the range of $p_{\text{gas}}$ where $\lambda_D \ll L$ increases.

(ii) Group 2: changing $\log(g)$ (Fig. 3, middle)

Models with $T_{\text{eff}} = 2800$ and $2000 \text{ K}$ satisfy this criterion for all surface gravity values and throughout the whole atmosphere. For $T_{\text{eff}} = 1000 \text{ K}$ and all surface gravity values $\lambda_D \ll L$ is fulfilled only for $p_{\text{gas}} > 10^{-4} \text{ bar}$.

(iii) Group 3: changing $[\text{M/H}] = 0.0$ (Fig. 3, bottom)

Models with $T_{\text{eff}} = 2800$ and $2000 \text{ K}$ satisfy this criterion for all metallicities and $p_{\text{gas}}$. Models with $T_{\text{eff}} = 1000 \text{ K}$ satisfy this criterion in the inner atmospheric regions only where $p_{\text{gas}} > 10^{-4} \text{ bar}$.

Fig. 4 demonstrates that the Debye length is generally very large in the upper atmospheric regions throughout the whole regime of ultracool objects, i.e. late MDs, BDs and giant gas planets. In the upper atmosphere the electron density is low causing an increasing Debye length; whereas deeper in the atmosphere the electron number density is high and so the Debye length is relatively lower. According to Cravens (1997), at the top of the Earth’s ionosphere $\lambda_D \approx 1 \text{ cm}$ for $T_e \approx 1000 \text{ K}$ and $n_e \approx 10^{11} \text{ cm}^{-3}$, compared to a vertical extant of $\approx 300 \text{ km}$ of the ionosphere. For the solar wind at $\approx 1 \text{ au}$, $\lambda_D \approx 700 \text{ cm}$ ($T_e \approx 10^4 \text{ K}$, $n_e \approx 10^{10} \text{ cm}^{-3}$). A comparison of different values of the Debye sphere for different astrophysical environments is presented in Table 2 and in Fig. 5. Duru et al. (2008) investigate the electron density in the upper ionosphere of Mars, Trotignon et al. (2001) the interaction of the Martian’s atmosphere with the solar wind. Both consider the presence of the dust in the plasma environment. Yaroshenko et al. (2011) model a plasma composed of electrons, water group ions and protons with the presence of photoemission due to the UV radiation. Kremer et al. (2006) work with a pure electron plasma.

Appendix B provides supplementary material about $N_D$, the average number of charges in the Debye sphere. The values for $N_D$ are $\gg 10^3$ in the rarefied upper part of the atmospheres ($p_{\text{gas}} < 10^{-4} \text{ bar}$) for all MD, BD and giant gas planet model atmospheres investigated here. Values for the above quoted Debye length for the Earth

Table 2. Debye lengths for different astrophysical environments. Fig. 5 provides a comparison to results of these papers.

\begin{tabular}{|l|l|l|l|}
\hline
Object & $T_e$ [K] & $n_e$ [cm$^{-3}$] & $\lambda_D$ [cm] & References \\
\hline
Martian’s ionosphere & 5000 & $10^{-3}$ & $1.5 \times 10^2$ & Duru et al. (2008) (Mar’s atmosphere) \\
 & 347 & 9.78 & 1.3 & \\
Martian’s ionosphere & 3131 & $4.12 \times 10^{-3}$ & 19 & Trotignon et al. (2001) (Mar’s atmosphere) \\
 & 1222 & $5.82 \times 10^{-2}$ & $10^3$ & \\
 & 86665 & $3.47 \times 10^{-3}$ & $1.1 \times 10^3$ & \\
 & 1.16 $\times 10^4$ & 50 & $10^2$ & \\
Saturn & $3 \times 10^4$ & 30 & $2 \times 10^2$ & Yaroshenko et al. (2011) (electron, water group ions \\
Orbit & $7 \times 10^4$ & 10 & $5.6 \times 10^2$ & \\
Insertion & $1.7 \times 10^4$ & 2 & $1.9 \times 10^3$ & (and protons plasma) \\
 & $4 \times 10^4$ & 0.1 & $5 \times 10^3$ & \\
Laboratory & 46 418 & $7.5 \times 10^6$ & 1.7 & Kremer et al. (2006) (pure electrons plasma) \\
\hline
\end{tabular}
3.4 Comparing different plasma criteria

Radio and X-ray observations from low-mass objects suggest that their atmospheres contain enough free charges to constitute a magnetized plasma (Hallinan et al. 2008). From our evaluation of the thermal degree of ionization (Section 3.3.1), we chose a threshold for the degree of ionization of \( f_e > 10^{-7} \) above which an atmospheric gas can be sufficiently ionized that it may exhibit plasma behaviour. In a plasma, electron–electron interactions dominate over collisions between electrons and neutral particles, \( \omega_{pe} \gg \nu_{ce} \) (Section 3.3.2). Additionally, for a plasma to be considered magnetized, the magnetic field must be sufficiently strong that it significantly influences the electron and ion dynamics, which we address in Section 5.

We now cast the results in terms of atmospheric volumes to allow a comparison between the results for different parameters. Fig. 9 and 10 summarize our findings in terms of the volume fraction, \( V_{gas}^n/V_{atm} \), with \( V_{gas}^n \) the thermally ionized volume of the atmosphere and \( V_{atm} \) the total atmospheric volume. \( V_{gas}^n \) is derived by calculating the fraction of the atmospheric volume for which \( f_e > 10^{-7} \) (Fig. 9). Fig. 10 visualizes the atmospheric volume fraction where \( \omega_{pe} \gg \nu_{ce} \) is fulfilled. The atmosphere volume that reached \( f_e > 10^{-7} \) and satisfied \( \omega_{pe} \gg \nu_{ce} \) is affected by the global parameters as follows.

(i) If \( T_{eff} \) increases, then the thermally ionized atmospheric volume fraction increases for a given \( \log (g) \) and \([M/H]\).

(ii) The \( V_{gas}^n/V_{atm} \), that reaches \( f_e > 10^{-7} \) and \( \omega_{pe} \gg \nu_{ce} \), increases if \( \log (g) \) decreases for a given \( T_{eff} \) and \([M/H]\).

(iii) Higher values of the metallicity, \([M/H]\), cause a larger fraction of the atmosphere volume to have a sufficiently ionized gas that large-scale electromagnetic interactions dominate over electron–neutral collisions for a given \( T_{eff} \) and \( \log (g) \).

The late MDs have the largest atmosphere volume fraction, \( V_{gas}^n/V_{atm} \), that reached \( f_e > 10^{-7} \) that is for model atmosphere structures with \( T_{eff} = 2600–3000 \) \( K \), \( \log (g) = 3.0 \), \([M/H]=0.0\) and \( T_{eff}=2800\) \( K \), \( \log (g)=3.0 \), \([M/H]=+0.3\). For late MD and BDs, the atmospheric gas satisfies \( f_e > 10^{-7} \) only for half of their atmosphere. For planetary objects the fraction of the volume that reaches \( f_e > 10^{-7} \) becomes increasingly small except for those that have the highest value of metallicity and the lowest value of surface gravity.

Models with \( 2200 \leq \log (g) \leq 3000 \) \( K \), \( \log (g) = 3.0 \), \([M/H]=0.0\) have the largest atmosphere volume fraction that reached \( \omega_{pe} \gg \nu_{ce} \) for a given \( \log (g) \) and \([M/H]\). Models with \( T_{eff}=2800 \) \( K \), \( \log (g)=4.0 \), \([M/H]=0.0\); \( T_{eff}=2800 \) \( K \), \( \log (g)=3.0 \), \([M/H]=+0.3\) \( T_{eff}=2800 \) \( K \), \( \log (g)=3.0 \), \([M/H]=+0.3 \) have the largest atmosphere volume fraction that reached \( \omega_{pe} \gg \nu_{ce} \) as well. In young BDs, the atmospheric gas volume that reaches \( \omega_{pe} \gg \nu_{ce} \) is more than 50 per cent. The atmospheric gas volume that reaches \( \omega_{pe} \gg \nu_{ce} \) for planetary objects is smaller than for the rest of the objects, i.e. for \( T_{eff} = 1000 \) \( K \), \( \log (g) = 3.0 \), \([M/H]=+0.3\). Our results show that \( V_{gas}^n/V_{atm} \) \( \omega_{pe} \gg \nu_{ce} \) is larger than \( V_{gas}^n/V_{atm} (f_e > 10^{-7}) \) for \( 1000 \leq T_{eff} \leq 3000 \) \( K \). A general observation is that despite a relatively low degree of ionization, large-scale electromagnetic interactions can dominate a considerably larger atmospheric volume than \( f_e \) evaluation would suggest for all ultracool objects in our sample.

4 MOST ABUNDANT THERMAL IONS IN LATE MD, BD AND GIANT GAS PLANET ATMOSPHERES

We investigate the atmospheric gas-phase composition regarding the most abundant local gas ions to demonstrate which are the dominating electron donors across the star–planet regime based on our non-irradiated DRIFT-PHOENIX model atmosphere grid. This investigation allows us to understand which gas-phase species are responsible for increasing the number of free electrons in ultracool atmospheres and consequently, responsible of satisfying the plasma parameters given in equations (1)–(3). It also allows us to understand how the chemical composition of the gas is linked with the dominating electron donors in the gas-phase. In the case of cloud-forming atmospheres, the abundance of the element depleted by cloud formation has an effect on electron donors at the location of the cloud. This section serve also as reference for future investigations on ionization processes and their effect on the gas composition like in (Rimmer & Helling 2013).

4.1 Dominating ions across the late MD, BD and planetary regime

Fig. 6 demonstrates for a subset of effective temperatures \( T_{eff}=1000–3000 \) \( K \) (\( \log (g)=3.0 \) solar element abundances).
that K+, Na+, Ca+, Mg+ and Fe+ are the dominating thermal electron donors. These ionic species are the most significant contributors to the electron number density from thermal ionization. Species that have sufficiently low first ionization potentials and sufficiently high atmospheric number densities will contribute most effectively to the thermal degree of ionization. Therefore, K+, Na+ and Ca+ provide the majority of thermal electrons. Fig. 6 demonstrates that K+ is the dominating thermal electron donor where \( p_{\text{gas}} < 10^{-2} \) bar for all atmospheres except in an MD atmosphere of \( T_{\text{eff}} = 3000 \) K. The second dominating electron donor is Na+ from \( T_{\text{eff}} = 2000 \) K. Na+ dominates for \( T_{\text{eff}} = 3000 \) K. For increasing gas pressure, \( p_{\text{gas}} > 10^{-2} \) bar, Na+ and K+ are the dominating thermal electron donor for \( T_{\text{eff}} = 2000, 1400, 1000 \) K. For example, Mg+ provides most of the electrons in the \( T_{\text{eff}} = 3000 \) K model for \( p_{\text{gas}} > 10^{-2} \) bar. The detailed results for all model groups are summarize in Tables A1 and A2 in the Appendix A.

Figs 7 and 8 show the distribution of the most important electron donating elements over atoms, molecules and ions. For example, Na+ is the dominating Na-species at high temperature, but the atomic Na followed by NaH and NaOH contain most of the element Na at lower temperatures. Ca, Mg and Fe are involved in the formation of many molecules. In the case of Ca, CaCl2, CaOH and Ca(OH)2 reach the abundances not much lower or even higher than the atomic Ca. Our investigations show that it is not sufficient to determine the local degree of ionization based on one prescribed electron donor species. Such an approach has been chosen in various complex simulations like MHD simulations for protoplanetary discs (e.g. Sano et al. 2000) or atmospheric circulation models (e.g. Perna, Menou & Rauscher 2010).

The exact amount with which K, Na, Ca, Mg and Fe contribute to the local degree of thermal ionization will also depend on the amount of each element that is chemically locked in cloud particles. Element depletion by cloud formation is fully taken into account for Mg and Fe (see Section 2) in the DRIFT-PHOENIX atmosphere simulations. The more extended work by Helling et al. (2008) (Fig. 6) suggests that the effect of cloud formation on the Ca+ abundance is negligible. Morley et al. (2012) (Fig. 3) show that Na2S is thermally stable for \( T_{\text{gas}} < 1100 \) K and KCl for \( T_{\text{gas}} < 900 \) K, hence Na+ and K+ would be effected by cloud formation in a similar temperature window. The solar element abundances for Na (6.17), K (5.08) and Cl (5.5) are lower than for Ca (6.31) and it may therefore be reasonable to expect a similarly negligible effect of cloud formation on the abundances of Na+ and K+.

4.2 Summary on electrostatic parameters

Our reference study suggests that

(i) MD have almost the entire atmospheric gas ionized, BDs present only an ionized gas for \( p_{\text{gas}} > 10^{-2} \) bar and giant gas planets present only a small fraction of an ionized gas for values of \( p_{\text{gas}} > 10^{-1} \) bar.

(ii) Collective, long-range electromagnetic interactions of electrons dominate over short-range binary interactions with neutrals particles increase as \( T_{\text{eff}} \) increases in the atmospheres of MD and
Figure 7. Distribution of electron donor’s abundance over atoms, molecules and ions for a warm model of $T_{\text{eff}} = 2000$ K, log($g$) = 3.0 and solar element composition. The upper left corner contains the element considered.

Figure 8. Same like Fig. 7. Both, Mg and Fe are influenced by dust formation which strongly decreases both elements, resulting into the localize large kink. The upper left corner contains the element considered. H-binding species are shown for comparison.

BDs. Giant gas planets are to cool to fulfill this criterion for large ranges of $p_{\text{gas}}$; only for $T_{\text{eff}} = 1200$ K, log($g$) = 3.0, [M/H] = 0.0 and $T_{\text{eff}} = 1000$ K, log($g$) = 3.0, [M/H] = +0.3 a small faction of the atmospheric gas at most deeper parts of their atmosphere.

(iii) $\lambda_D \ll L$ is fulfilled for MDs throughout their atmospheres. For BDs only for $p_{\text{gas}} > 10^{-8}$ bar and for giant gas planets for $p_{\text{gas}} > 10^{-3}$ bar.

K$^+$, Na$^+$, Ca$^+$, Mg$^+$ and Fe$^+$ are the dominating thermal electron donors, however, K$^+$, Na$^+$ and Ca$^+$ provide the majority of electrons for $T_{\text{eff}} = 1000$–3000 K for log($g$) = 3.0 and solar element abundances. In particular, the degree of ionization is low in the upper atmospheres where the abundances of those ions are low. As their abundances increase, $f_e$ increases as well. Long-range
electromagnetic interactions dominating over collisions (equation 2) and a zero electrostatic forces inside the plasma (equation 3) require a sufficient number of free charged particles. Any process (cloud formation, CR impact) that impacts the element abundance of the dominating electron donors will affect the electric state of the atmosphere, and the potential coupling to a large-scale magnetic field.

5 MAGNETIZED PLASMA PARAMETERS ACROSS THE STAR-PLANET REGIME

In the previous sections we discussed that for a plasma to exists, the gas needs to be ionized to a certain degree ($f_e > 10^{-7}$). The plasma frequency was used to investigate where in the atmosphere, electromagnetic interactions dominate over kinetic collisions between electrons and neutrals.
The Debye length provides insight about the length-scales on which electrostatic interactions influencing the atmospheric gas. We demonstrated that the atmospheric volume fraction, $V_{\text{gas}}/V_{\text{atm}}$, suspected to show plasma behaviour, varies largely through the MD to planetary regime.

A plasma is considered magnetized when the motion and dynamics of the charged particles are influenced by an ambient magnetic field. This requires that the magnetic field is of sufficient magnitude that the charged particles can on average participate in at least one Larmor orbit before colliding with a neutral atom or dust particle. Otherwise, frequent collisions with the ambient neutrals will dominate the dynamical evolution of the plasma particles and the influence of the magnetic field will be negligible. In some instances, because of the differing mass between electrons and ions, the electrons can be magnetized while the ions are not. For magneto-fluid descriptions of plasmas (such as magnetohydrodynamics) both the electrons and ions need to be magnetized. Radio flares (Route & Wolszczan 2012), X-ray flares (Berger et al. 2010) and quiescent radio emission (Williams, Berger & Zauderer 2013) have been observed in BDs, Schmidt et al. (2015) conclude that 45 per cent of their active L-dwarfs are also variable. This fraction of L-dwarfs, which is lower compared to the 60 per cent of active MDs that were found to be variable in their sample, lead Schmidt et al. (2015) to speculate about a BD chromosphere as origin for the observed Hα emission and variability. These observations suggest that there should be a strong magnetic field and considerable coupling between the magnetic field and the atmospheric gas, either to directly accelerate free electrons or to allow plasma waves to travel into the low-density upper atmosphere and deposit their energy causing a chromosphere to develop. It is interesting to note here that old BDs ($\log(g) = 5.0$) have enough time available to build up a chromosphere even if the acoustic heating rates should be low due to an insufficient magnetic coupling of the atmosphere. For young BDs, rapid rotation may favor chromospheric heating by a better coupling to a stronger magnetic field.

5.1 Cyclotron frequency versus collisional frequency $\omega_c \gg \nu_{\text{coll}}$

The cyclotron frequency is the angular velocity with which charged particles gyrate around the magnetic field line (Boyd & Sanderson 2003). $\omega_{\text{cycl}} = v_{\perp,s}/r_{\perp,s} = q_s B/m_s$ in [rad s$^{-1}$], where $m_s$ [kg], $q_s$ [C], $B$ [T] and $v_{\perp,s}$ [m s$^{-1}$] are the mass of species $s$, with charge $q_s$ and speed perpendicular to the magnetic field $v_{\perp,s}$, respectively; and $|B|$ is the magnitude of the external magnetic flux density present in the medium. For a charged particle’s motion to be dictated by a magnetic field, the particle needs to complete on average one gyration before a collision with a neutral atom. Formally, a magnetized plasma requires

$$\omega_{\text{cycl}} \gg \nu_{\text{coll}},$$

where $\nu_{\text{coll}}$ [s$^{-1}$] is the collision frequency for neutral particles with charged species $s$. From equation (7) we obtain the minimum value for the external magnetic field, $|B| = B_{\text{th}}$, that is needed to satisfy this criteria. Applying the definitions for $\omega_{\text{cycl}}$ and $\nu_{\text{coll}}$, we derive the critical magnetic flux density that is required for the dynamics of the charged particle to be influenced by the background magnetic field

$$\frac{eB}{m_s} \gg \sigma_{\text{gas},e} n_{\text{gas}} v_s,$$

where the collision, or scattering, cross-section is $\sigma_{\text{gas},e} = \pi \times r_{\text{gas}}^2$. The atmospheric gas in late MDs, BDs and most likely also in giant gas planets is composed mostly of molecular hydrogen, H$_2$. The collision cross-section is approximated as $\sigma_{\text{gas}} \approx \sigma_{\text{H}_2} \approx \pi \times r_{\text{H}_2}^2 = 5.81 \times 10^{-20}$ m$^2$ ($r_{\text{H}_2} = 1.36 \times 10^{-10}$ m).

Taking the electrons and ions as the particles that are influenced by an external magnetic flux density, equation (9) becomes

$$B_{\text{c}} \gg \frac{m_e \sigma_{\text{gas},e} n_{\text{gas}}}{e} \left( \frac{k_B T_e}{m_e} \right)^{1/2},$$

(10)

$$B_{\text{i}} \gg \frac{m_i \sigma_{\text{gas},i} n_{\text{gas}}}{e} \left( \frac{k_B T_i}{m_i} \right)^{1/2}.$$  

Groping the constants, we rewrite equations 10 and 11 as $B_{\text{c}} \propto n_{\text{gas}} (m_e T_e)^{1/2}$, with $B_{\text{c}}$ as the minimum threshold for the magnetic flux density to ensure that the electrons are magnetized and $B_{\text{i}} \propto n_{\text{gas}} (m_i T_i)^{1/2}$, with $B_{\text{i}}$ as the minimum threshold for magnetic flux density required to ensure that an ion, $i$, is magnetized in Fig. 12. The ion masses, $m_i$, are taken to be for K$^+$, Na$^+$ Ca$^+$, Fe$^+$, and Mg$^+$ according to Section 4 assuming local thermal equilibrium, $T_{\text{gas}} \approx T_i \approx T_e$.

Fig. 11 shows that for a $p_{\text{gas}} < 10^6$ bar and $B = 10^3$ G and for a $p_{\text{gas}} < 10^{-2}$ bar and $B = 10$ G, $\omega_{\text{cycl}} \gg \nu_{\text{coll}}$, (horizontal black line) is reached for all model atmosphere structures. There is almost no dependence on $T_{\text{gas}}$, log ($g$) and the metallicity. Largest values of $\omega_{\text{cycl}}/\nu_{\text{coll}}$ are reached for $B > 10^3$ G representative for MDs or BDs. Fig. 12 shows for which atmospheric gas pressures, $p_{\text{gas}}$, electrons and ions can be magnetized in the atmospheres of MDs, BDs and giant gas planets. For MDs and BDs a background magnetic field density of $B = 10^3$ G is large enough to magnetise the charged particles: for electrons at $p_{\text{gas}} < 10$ bar and for ions at $p_{\text{gas}} < 10^{-3}$ bar. For giant gas planets, the magnetized part of the atmosphere decreases because of a smaller background field ($B \leq 10$ G) compared to MDs and BDs. For electrons this occurs at $p_{\text{gas}} < 10^{-2}$ bar and for ions at $p_{\text{gas}} < 10^{-3}$ bar. Fig. 15 summarizes the results on magnetic coupling in term of the affected atmospheric gas volume, $V_{\text{gas}}/V_{\text{atm}}$, that reach $\omega_{\text{cycl}} \gg \nu_{\text{coll}}$.

- If $T_{\text{eff}} < 7000$ K, the magnetically coupled volume of an atmosphere increases for a given log ($g$) and [M/H].
- If log ($g$) decreases, then the magnetically coupled volume increases for a given $T_{\text{eff}}$ and [M/H].
- Higher values of the metallicity, [M/H], cause an increase of $V_{\text{gas}}/V_{\text{atm}}$ for a given $T_{\text{eff}}$ and log ($g$).

For a fixed value of magnetic flux density, MDs and BD atmospheres have the largest magnetically coupled volume. Unsurprisingly, a smaller fraction of a giant gas planets atmosphere is magnetically coupled when thermal ionization is considered as the only source of gas ionization. However, this fraction can reach 80 per cent also in a planetary atmosphere. The fraction of the atmospheric gas volume, $V_{\text{gas}}/V_{\text{atm}}$, that reaches $f_{\text{esc}} > 10^{-3}$ (Fig. 9) and $\omega_{\text{cycl}} \gg \nu_{\text{coll}}$ (Fig. 10) increases for the same set of global parameters $T_{\text{eff}}$, log ($g$), [M/H] like the atmospheric gas volume that reaches $\omega_{\text{cycl}} \gg \nu_{\text{coll}}$. Fig. 15 demonstrates also that a larger atmospheric gas volume can be expected to be magnetically coupling than the thermal degree of ionization that initially suggested in Fig. 9. This finding is particularly relevant with respect to the effect that the magnetic field geometry might have on the detection of the Hα activity
signatures and on the radio emission: Donati et al. (2008) show that partially-convective MDs host non-axisymmetric large-scale magnetic fields with a strong toroidal component, while fully convective MDs have stronger large-scale field dominated by a mainly axisymmetric poloidal component. A partial ionization of the magnetically coupled gas does influence the magnetic flux density.

5.2 Magnetic Reynolds Number, \( R_m \)

The previous sections outlined the framework quantifying when an ionized gas in a substellar atmosphere behaves like a magnetized plasma. The magnetic Reynolds number is an easy-to-utilize measure of a potentially magnetically coupled ionized gas. Within the context of MHD, the magnetic Reynolds number \( (R_m) \) is the ratio of the convective and diffusive terms from the magnetic field induction equation. It quantifies whether the MHD plasma is in the ideal or resistive regimes. When the magnetic Reynolds number is very large (i.e. in the limit of large length-scales), the MHD plasma is in the ideal MHD regime and the convective term has the dominant influence. In this regime the motion of the plasma fluid is determined by the magnetic field and vice versa. In the resistive MHD regime, the diffusive term is important and dissipative processes as Ohmic dissipation (Perna et al. 2010; Huang & Cumming 2012) become significant. The magnetic Reynolds number is defined through the induction equation

\[
\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \eta \nabla^2 B, \tag{12}
\]

where \( |B| = B \) \( [T] \) is the magnetic flux density, \( u \) is the flow velocity (formed by electrons and ions), and \( \sigma \) \( [S \ m^{-1}] \) is the electric conductivity. The magnetic diffusivity, \( \eta \), is linked to the conductivity by \( \eta = 1/\sigma \) \( [m^2 \ s^{-1}] \) and represents the diffusion of the magnetic field, a measure of the effect of collisions between the electrons and the neutral particles on the magnetic field. The collisions between the neutral particles and the charged particles (electrons or ions) have an influence on the diffusivity of the magnetic field. If the effect of the collisions is sufficient to displace them away from the magnetic lines, the coupling between the magnetic field and the fluid may not be effective. Therefore, the diffusion of the magnetic field depends on the frequency of the collisions between neutral particles and charged particles.

The magnetic Reynolds number, \( R_m \), can be defined as the ratio between the relative strength between the diffusive term and the advective term of the induction equation. It can be used as a measure
Figure 13. Decoupled diffusion coefficient, $\eta_d = c^2 \nu n_e / \omega_{pe}^2$ and the Ohmic diffusion coefficient, $\eta_{\text{Ohm}} = c^2 \nu e_i / \omega_{pe}^2$ for the dominating thermal electron donors for $T_{\text{eff}} = 1000, 2800$, for log($g$) = 3, 0 and solar element abundances; K $^+$, Na $^+$, Ca $^+$, Fe $^+$ and Mg $^+$ (Fig. 6). The Ohmic diffusion coefficient is smaller than the decoupled diffusion coefficient. This result suggests that the binary interactions between the ions and electrons ($\eta_{\text{Ohm}}$) are not significantly compared to the binary interactions between electrons and neutral particles ($\eta_d$) in the case of thermal ionization. The lines in both temperature sets appear from top to bottom in the order $\eta_d$, $\eta_{\text{Ohm}}$, K $^+$, $\eta_{\text{Ohm}}$, Na $^+$, $\eta_{\text{Ohm}}$, Ca $^+$, $\eta_{\text{Ohm}}$, Fe $^+$ and Mg $^+$.

of the magnetic coupling calling the plasma coupled to the magnetic field if $R_m \geq 1$, with

$$ R_m = \frac{\left| \nabla \times (\mathbf{u} \times \mathbf{B}) \right|}{\eta} \frac{1}{\left| \nabla^2 B \right|}. \quad (13) $$

Applying a dimensional analyses, equation (13) reduces to

$$ R_m \approx \frac{v B / L}{\eta B / L^2} \approx \frac{v L}{\eta}, \quad (14) $$

where L [cm] is a typical length-scale of the plasma over which $| B | = B$ varies through a hydrodynamic motion of a velocity $| \mathbf{u} |$, can be approximated by $L = 10^7$ [m] (Helling et al. 2011b). The diffusion coefficient, $\eta$, can be approximated by $\eta \approx \eta_d$ (Fig. 13). Therefore, the expression for the magnetic Reynolds number is rewritten as

$$ R_m \approx 10^{-4} \times \frac{v_{\text{flow}}}{n_e / n_{\text{gas}}} \left( \frac{1}{n_e T_e^{1/2}} \right). \quad (15) $$

The values for the flow velocity chosen are $v_{\text{flow}} = 10^4$ cm s$^{-1}$ and $v_{\text{flow}} = 10^6$ cm s$^{-1}$ guided by values of circulation models (Cooper & Showman 2005; Menou & Rauscher 2009; Rauscher & Menou 2013; Rauscher & Kempton 2014; Heng & Showman 2015). Fig. 14 represents the magnetic Reynolds number for the DRIFT-PHOENIX model atmosphere structures which are comparable to the earlier results by Mohanty et al. (2002) that were based on DUSTY- and COND-PHOENIX.

Reynolds number increases as $T_{\text{eff}}$ increases because of the increasing of $n_e / n_{\text{gas}}$ in globally and locally hotter atmospheres (see Section 3.3.1). The highest $R_m$ is reached for $T_{\text{eff}} = 3000$ K, log($g$) = 3.0, [M/H] = 0.0, $v_{\text{flow}} = 10^6$ cm s$^{-1}$.

Figure 14. Magnetic Reynolds number $R_m$ for the three different model atmosphere groups described in Section 2. $R_m$ is calculated for a flow speed of $v_{\text{flow}} = 10^6$ cm s$^{-1}$. If the flow speed increases, then $R_m$ increases. Top: Group 1. Middle: Group 2. Bottom: Group 3.

1 The diffusion coefficient used in equation (14) is given by $\eta = \eta_d + \eta_{\text{Ohm}}$ being $\eta_d = c^2 \nu n_e / \omega_{pe}^2$ as the decoupled diffusion coefficient and $\eta_{\text{Ohm}} = c^2 \nu e_i / \omega_{pe}^2$ as the Ohmic diffusion coefficient and $\eta_d \gg \eta_{\text{Ohm}}$. Both measure the degree of the dominance of the collisions between electrons–neutral particles and electrons–ions, respectively, over long-range electromagnetic collective interactions. It is easy to relate $\eta_d$ with $\omega_{pe} / \nu e$ (see Fig. 3). If the latter increase, the diffusion coefficient decrease and the magnetic field may be generated and transported by fluid motions allowing the magnetic energy to be released into upper layers of the atmosphere as radio, X-ray and Hα emissions.
(i) Lower gravities cause an increased $R_m$ for a given $T_{\text{eff}}$ and $[\text{M/H}]$ in the inner, high-density part of the atmosphere only. However, higher gravities cause an increased $R_m$ for a given $T_{\text{eff}}$ and $[\text{M/H}]$ in the outer, low-density atmosphere.

(ii) Higher values of metallicity cause an increased of $R_m$ at high values of $p_{\text{gas}}$ for a given $T_{\text{eff}}$ and $\log(g)$. Low-metallicity atmospheres with low effective temperatures, i.e. planet or T- and Y-dwarf atmospheres, have an increasing $R_m$ in the outer, low density. This correlates with a drastically changing gas-phase chemistry as shown in Fig. 6.

Our results suggest that ideal MHD, where a fully ionized gas is assumed, is best suited for models atmospheres with $T_{\text{eff}} \geq 3000$ K which includes MDs and young BDs. For cooler BDs and planetary regime objects only a small fraction of their atmosphere can be considered in ideal MHD.

6 DISCUSSION

6.1 Chromospheres on ultracool objects

Observations in radio, soft X-ray and Hα wavelengths from low-mass objects infer that their atmospheres are populated with magnetized plasmas. Radio and X-ray emission from ultracool dwarfs have been well established by different authors (e.g. Berger 2002, Route & Wolszczan 2012, Burgasser et al. 2013). Sorahana et al. (2014) and Schmidt et al. (2015) suggest the presence of chromospheres in BDs. Sorahana et al. (2014) suggest that weakened H2O (2.7 μm), CH4 (3.3 μm) and CO (4.6 μm) absorption in combination with moderate Hα emission could be linked to chromospheric activity. They represent a potential chromospheric heating by an increased, constant temperature ($p_{\text{gas}} = n k T$) in the upper atmosphere of their UCM 1D model atmosphere which allows a considerably better data fit of their observation. Schmidt et al. (2015) use a comparable approach by replacing the outer atmospheric temperature of BT-settke model atmospheres with a chromospheric temperature where the start of the chromosphere, a chromospheric break and the start of the transition region are used as parameters. Metchev et al. (2015) discuss the likely correlation of magnetic spots and high-amplitude photometric variability in BDs with low surface gravity values. Williams et al. (2014) demonstrate that ultracool objects do not follow the classical Güdel–Benz relationship where the radio luminosity increases proportional to the X-ray luminosity in F–M stars (Güdel & Benz 1993). The deviation of ultracool stars in the Güdel–Benz relationship beyond than approximately M5 may suggest a change in the dynamo mechanism that produces the magnetic field in such ultracool objects (Cook et al. 2014). Another interpretation of radio emission is the concurrence of an auroral region (Nichols et al. 2012). Speirs et al. (2014) describe a theoretical approach for cyclotron radio emission from Earth’s auroral region providing a physical description for the widely used loss cone parametrization (e.g. Osten et al. 2015). Speirs et al. (2014) show that the radiation results from a backward-wave cyclotron-maser emission process. The radio emission is generated by electrons following a horseshoe velocity distribution, instead of a cone, that travel the magnetic field lines downward. The backward travelling waves cause the upward refraction of the radiation which will be further enhanced by density inhomogeneities.

If the atmospheric gas is well coupled with the background magnetic field in BDs and planets, the kinetic energy carried by large-scale convective motions may be transported to the top of the atmosphere and released. Tanaka et al. (2014) suggest that energy from the convective part of the atmosphere might be transported through the upper atmosphere by magneto-convection processes and suggest the formation of a chromosphere by Alfvén wave heating. Mohanty et al. (2002) carried out a study of magnetic field diffusivity to explain why the chromospheric Hα activity in BDs is low in spite of being rapid rotators. Mohanty et al. (2002) based their work on a grid of model atmospheres (mid-M and L dwarfs) in a parameter range $T_{\text{eff}} = 3000 – 1500$ K, $\log(g) = 5.0$ and $[\text{M/H}] = 0.0$. They explained why in these range of $T_{\text{eff}}$ the observation of chromospheric...
levels activity are lower than early MDs, considering mid-M and L dwarfs as rapid rotators. In our work, we extend our model atmosphere grid until $T_{\text{eff}} = 1000 \, \text{K}$ and we include atmosphere structures with different values of $\log(g)$ and [M/H] (Table 1). A linear field diffusion equation, MHD regime and LTE are used in both works.

Results obtained for $R_m$ as a measure of the ideal or resistive MHD atmosphere (see Fig. 14) could be incomplete and therefore misleading. According to equation (14), $R_m \propto 1/\eta$. Our results demonstrate that the regions where $R_m > 1$ satisfy also $f_e > 10^{-7}$. Atmospheres of ultracool objects could be ionized and treated as an ideal MHD gas only at deep layers. Furthermore, $\omega_{ce}/v_{ne} > 1$ measures the coupled between the magnetic field and the atmospheric gas and it depends, mostly, on the strength of the magnetic field (see Fig. 15). Therefore, it is possible to find large volumes of the atmospheric gas that are magnetized, but smaller magnetized volumes that are strongly ionized.

Our results further suggest not only that higher effective temperatures (in agreement with Mohanty et al. (2002)) and higher metallicity atmospheres are the best candidates for forming a magnetized atmospheric plasma in support of radio, X-ray and Hα observations in ultracool objects. Also low surface gravity atmospheres fall in this category which supports the interpretation by Metchev et al. (2015) that high-amplitude photometric variability in L3-L5.5 dwarfs can also be related to magnetic spot appearance. While MDs have been shown to be fully magnetized, L-dwarfs and later BDs have smaller atmospheric volume that can be magnetized in an external magnetic field. This findings relate to the activity-versus-SpecT results in Schmidt et al. (2015) (e.g. their fig. 6). The threshold of $T_{\text{eff}} = 2300 \, \text{K}$ given in their fig. 6 is the Mohanty et al. (2002) threshold that points out the limit from which models $R_m > 1$ using $v = 10^4 \, \text{cms}^{-1}$ and $10^{-2} \leq \tau_J \leq 10^2$ (convection zone) being $\tau_J$ the optical length in the $J$ band. Our paper suggests that this threshold move towards $T_{\text{eff}} = 1400 \, \text{K}$ for the same value of flow velocity and same region. This result suggests that atmospheres cooler than $T_{\text{eff}} = 2300 \, \text{K}$ may be susceptible to be magnetized. Another criterion to consider a gas magnetized is $\omega_{ce} > v_{ne}$ (see Fig. 11). For all models considered in this study (Table 1) the atmospheric gas fulfill $\omega_{ce} \gg v_{ne}$ for $p_{\text{gas}} < 1 \, \text{bar}$. Combining both criteria $\omega_{ce} \gg v_{ne}$ and $R_m > 1$, a large fraction of possible active objects are found for $T_{\text{eff}} = 3000 \, \text{K} - 14000 \, \text{K}$, $\log(g) = 3.0$, [M/H] = 0.0 (Group 1).

### 6.2 Ionization through non-thermal processes

Our work focuses on determining global parameters to provide the suitable local atmospheric conditions for a magnetized plasma to be present. The results suggest that ultracool atmospheres are susceptible to plasma and magnetic processes even if only thermal ionization processes are considered and the influence of dust beyond element depletion is neglected. An atmospheric plasma regime and magnetized gas were found MD atmosphere, cooler BDs and planetary objects require high $p_{\text{gas}}$, hence, their magnetized volume is smaller than in MD atmospheres.

Additional potentially non-thermal ionization processes will enhance the degree of ionization and increase the local volume affected by a magnetic field. Local enhancement can result from dust-dust collisions in large cloud areas (Helling, Jardine & Mokler 2011a) and Alfvén ionization if the local, hydrodynamic wind speed is high enough (Stark et al. 2013). Electric storms that develop inside an atmosphere effects the extent of an ionosphere causing a link between the local ionization processes and the global effects (Luque et al. 2014). Irradiation from a host star for close-in exoplanets or in white dwarfs–BD binaries (Casewell et al. 2013) will increase the local thermal ionization globally. Galactic cosmic rays increase the number of free charge particles in single BDs. Cosmic rays are effective at ionizing the upper atmospheric parts, however, the exact amount is hard to quantify without extensive chemistry simulations (Rimmer & Helling 2013).

### 7 CONCLUSIONS

We present a reference study for late MDs, BDs and giant gas planet to identify which ultracool objects are most susceptible to atmospheric gas-phase plasma processes. Only thermal ionization is considered for this reference study and the influence of dust beyond element depletion is neglected. The effect of additional processes like cosmic ray ionization, irradiation, Alfvén ionization, lighting can be evaluated against the reference results in this paper.

Ultracool atmospheres with high $T_{\text{eff}}$, high [M/H] and low $\log(g)$ have large fraction of atmospheric volume where plasma processes occur, and are therefore the best candidates for radio, X-ray and Hα emissions. MDs have a considerable degree of ionization throughout the whole atmosphere, the degree of thermal ionization for a L-dwarf is low but high enough to seed other local ionization processes like Alfvén ionization or lightning discharges. Electromagnetic interaction dominates over electron–neutral interactions also in regions of a very low degree of ionization in most model atmospheres in our sample. The relevant length-scales affected by electromagnetic interactions in the gas phase are larger in low-density regions of any atmosphere. The minimum threshold for the magnetic flux density required for electrons and ions to be magnetized is smaller than typical values of the magnetic field strengths of a BD and a giant gas planet. A considerably lower magnetic flux density is required for magnetic coupling of the atmosphere in the rarefied upper atmosphere than in the dense inner atmosphere. Na+, K+ and Ca+ are the dominating electron donors in low-density atmospheres (low $\log(g)$, solar metallicity) independent of $T_{\text{eff}}$. Mg2+ and Fe2+ dominate the thermal ionization in the inner parts of MD atmospheres. Molecules remain unimportant for thermal ionization. Chemical processes (e.g. cloud formation, cosmic ray ionization) that affect the abundances of Na, K, Mg, Ca and Fe will have a direct impact on the state of ionization in ultracool atmospheres.

Our results suggest that it is not unreasonable to expect ultracool atmospheres (MDs and BDs) to emit Hα or even in radio wavelength as in particular the rarefied upper parts of the atmospheres fulfill plasma criteria easily despite having low degrees of ionization. Our results therefore suggest that an ionosphere may emerge also in BD and giant gas planet atmospheres, and that the built-up of a chromosphere is likely. Both effects will contribute to atmospheric weather features and to space weather occurrences in extrasolar, planet-like objects. Ultracool atmospheres could also drive auroral emission without the need for a companion’s wind or an outgassing moon.

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APPENDIX A: MOST ABUNDANT THERMAL IONS IN MD, BD AND GIANT GAS PLANET ATMOSPHERES

Varying $T_{\text{eff}}$ and log (g): Table A1 includes model atmospheres where [M/H] is kept at a constant value of 0.0 and $T_{\text{eff}}$ is varied alongside log (g) (see Section 4). The ions shown in the table are the most abundant; it does not include ions that are very prominent but not the most abundant. Note: When there are two ions listed (for example, AI+/Na+), the second ion occurs at the higher pressure.

Higher values of log(g) correspond to higher overall values of pressure, due to the increased surface gravity.

It is worth noting that there are other ions which are highly prominent within these models, however the following tables only show which ones are the most prominent.

Varying $T_{\text{eff}}$ and [M/H] Table A2 shows model atmospheres where log (g) has been kept at a constant value of 3.0 and $T_{\text{eff}}$ is varied alongside [M/H].

APPENDIX B: PLASMA PARAMETER: NUMBER OF PARTICLES IN A DEBYE SPHERE: $N_D \gg 1$

A plasma has the capacity to screen a single charged particle placed at any point. That means that any single charged particle attracts oppositely charged particles producing a screening and repels those who have the same charge. A net space is produced in the neighbourhood of any single charged particle, reducing the electric field generated by it. The effective range of the net force between...
Table A1. First, second and third ions represent the most abundant positive ion in the high, middle and lower pressure regions, respectively. First ion corresponds to the higher pressure (and therefore higher abundances) and vice versa.

| Teff (K) | log(g) | 3.0 | 4.0 | 5.0 |
|---------|--------|-----|-----|-----|
| 1000    | K⁺    | K⁺  | K⁺  | K⁺  |
|         | Na⁺   | Na⁺ | Na⁺ | Na⁺ |
| 1200    | K⁺    | K⁺  | K⁺  | K⁺  |
| 1400    | K⁺    | K⁺  | K⁺  | K⁺  |
| 1600    | Na⁺   | K⁺  | K⁺  | K⁺  |
| 1800    | Na⁺   | K⁺  | K⁺  | K⁺  |
| 2000    | Na⁺   | K⁺  | Na⁺ | K⁺  | K⁺  |
|         | Ca⁺   | Na⁺ | Na⁺ | Na⁺ |
| 2200    | Na⁺   | K⁺  | K⁺  | K⁺  |
| 2400    | Na⁺   | K⁺  | K⁺  | K⁺  |
| 2600    | Mg⁺   | K⁺  | K⁺  | K⁺  |
| 2800    | Mg⁺   | K⁺  | Na⁺ | K⁺  | K⁺  | Na⁺  |
|         | Na⁺   | Na⁺  | Mg⁺ | Na⁺  | Mg⁺ |
| 3000    | Na⁺   | K⁺  | Na⁺ | K⁺  | H⁺  |

Table A2. First, second and third ions represent the most abundant positive ion in the high, middle and lower pressure regions respectively. First ion corresponds to the higher pressure (and therefore higher abundances) and vice versa.

| M/H | Teff (K) | −0.6 | −0.3 | 0.0 | +0.3 |
|-----|---------|------|------|-----|------|
| 1000| K⁺      | K⁺   | K⁺   | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 1200| K⁺      | K⁺   | K⁺   | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 1400| K⁺      | K⁺   | K⁺   | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 1500| K⁺      | K⁺   | K⁺   | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 1600| K⁺      | K⁺   | K⁺   | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 1800| Na⁺     | K⁺   | Na⁺  | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 2000| Na⁺     | K⁺   | Na⁺  | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 2200| Na⁺     | K⁺   | Na⁺  | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 2400| Na⁺     | K⁺   | Na⁺  | K⁺  | K⁺   |
|     | Na⁺     | Na⁺  | Na⁺  | Na⁺ | Na⁺  |
| 2600| Mg⁺     | K⁺   | Na⁺  | K⁺  | K⁺   |
|     | Mg⁺     | Na⁺  | Na⁺  | Na⁺  | Mg⁺  |
| 2800| Na⁺     | K⁺   | Na⁺  | K⁺  | K⁺   |
|     | Ca⁺     | Na⁺  | Na⁺  | Na⁺  | Na⁺  |
| 3000| Na⁺     | K⁺   | Na⁺  | K⁺  | H⁺   |

Particles are restricted to the order of the Debye length (see Section 3). As a consequence, a test particle in the Debye sphere interacts only with particles that lie within this sphere. \( N_D \) measured the efficiency of this screening and allows us to calculate how many gas particles are required to participate. Hence, only particle inside this screening areas (\( \lambda_D \)) can be considered as electrostatically active.

All particles have a thermal velocity due to the temperature of the plasma. The deflected angle due to the electrostatic interactions is bigger if the number of particles around of the screened particle in the Debye sphere is small. The movement of the screened particle will not be smooth, unlike in the situation when the number of particles in the Debye sphere is sufficiently large to reduce it. That is why the Debye length increases as the number of particles in the...
screened sphere decreases. This is demonstrated in Fig. 4 where all Debye length increase with height in the atmosphere, i.e. with the outwards decreasing local gas pressure.

The change in velocity due to the interactions with the particles produces a non-negligible net electrostatic force inside the Debye sphere. Therefore, large numbers of particles that are uniformly distributed inside the Debye sphere are required to avoid a large-angle deflection on a test particle. Hence the Debye length will be small in comparison to the length-scale of the plasma. In this case, the plasma is dominated by many long-range interactions, rather than the short-range binary collisions of a neutral gas. A measure of the efficiency of the screening is the plasma parameter $N_D$.

The plasma parameter is defined as

$$N_D = \left(\frac{4}{3}\pi n_e \lambda_D^3\right),$$

(B1)

the number of particles in a Debye sphere with radius $\lambda_D$ and centred on a single charge particle that produced the charge imbalance. When there are many plasma particles in a Debye sphere ($N_D \gg 1$) and long-range collective interactions are dominant over short-range collisions, the plasma frequency is much larger than the electron-ion collision frequency. Fig. B1 shows $N_D \gg 1$ for all model atmosphere structures. This indicates that thermal electrons interact over large distances in atmosphere of ultracool atmospheres.

Figure B1. Number of particles inside of Debye sphere measure the efficiency of the screening given by the Debye sphere in the plasma. $N_D \gg 1$ results in the collective interactions dominate over short-range collisions in the gas. Top: Group 1. Middle: Group 2. Bottom: Group 3.