Going beyond the entanglement of microscopic objects (such as photons, spins, and ions), here we propose an efficient approach to produce and control the quantum entanglement of three macroscopic coupled superconducting qubits. By conditionally rotating, one by one, selected Josephson charge qubits, we show that their Greenberger-Horne-Zeilinger (GHZ) entangled states could be experimentally demonstrated by effective single-qubit operations followed by high-fidelity single-shot readouts. The possibility of using the prepared GHZ correlations to test the macroscopic conflict between the noncommutativity of quantum mechanics and the commutativity of classical physics is also discussed.

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Experimental measured by quantum-state tomography \[6, 7, 12\]. Above, \(C_{\Sigma j}\) is the sum of all capacitances connected to the \(j\)th box, and other effective capacitances are defined by \(C_{\Sigma 1} = C_{\Sigma 1}/(1 + C_{12}C_{\Sigma 1}/C)\), \(C_{\Sigma 2} = C/(C_{\Sigma 2}C_{\Sigma 1}), C_{\Sigma 3} = C_{\Sigma 3}/(1 + C_{23}C_{\Sigma 1}/C)\), \(C_{12} = C/(C_{\Sigma 2}C_{\Sigma 3}), C_{23} = C/(C_{\Sigma 2}C_{\Sigma 3})\), \(C_{13} = C/(C_{\Sigma 2}C_{\Sigma 3})\), with \(C = \prod_{j=1}^{3} C_{j} - C_{12}^{2} C_{\Sigma 2} - C_{23}^{2} C_{\Sigma 2}\). The pseudospin operators are defined as \(\sigma_{z}^{(j)} = |0_{j}\rangle\langle 0_{j}| - |1_{j}\rangle\langle 1_{j}|\) and \(\sigma_{y}^{(j)} = |0_{j}\rangle\langle 1_{j}| + |1_{j}\rangle\langle 0_{j}|\). As the interbit-couplings are always on, the charge energy \(E_{C}^{(j)}\) of the \(j\)th qubit depends not only on the gate-voltage applied to the \(j\)th qubit, but also on those applied to the other two Cooper-pair boxes. Compared to the coupling \(K_{j,j+1}\) between nearest-neighboring qubits, the interaction of two non-nearest-neighbor qubits (i.e., \(K_{13} = e^{2}/C_{13}\) between the first and the third qubits), is very weak and thus has been safely neglected \[11\]. Indeed, for the typical experimental parameters: \(C_{j} \approx 600 \text{ aF}, C_{m} \approx 300 \text{ aF}\), and \(C_{g} = 0.6 \text{ aF}\) in Ref. \[10\], we have \(K_{13}/K_{12} = K_{13}/K_{23} < C_{m}/C_{1} = 0.05\) and \(K_{12}/2e_{j} = 1/4\).

In principle, the coupled qubits cannot be individually manipulated, as the nearest-neighbor capacitve couplings \(K_{j,j+1}\) are sufficiently strong. However, once the state of the circuit is known, it is still possible to design certain operations for only evolving the selected qubits and keeping the remaining ones unchanged. Our preparation begins with the ground state of the circuit \(|\psi(0)\rangle = |000\rangle\), which can be easily initialized. The expected GHZ state could be produced by the following simple three-step pulse process \[11\]

\[
|\psi(0)\rangle \rightarrow |000\rangle \xrightarrow{\hat{U}_{2}(t_{2})} \frac{1}{\sqrt{2}}(|000\rangle \pm i|010\rangle) \xrightarrow{\hat{U}_{1}(t_{1})} \frac{1}{\sqrt{2}}(|000\rangle \mp |110\rangle) \xrightarrow{\hat{U}_{3}(t_{3})} \frac{1}{\sqrt{2}}(|000\rangle \pm i|111\rangle) = |\psi_{\text{GHZ}}^{+}\rangle. \tag{2}
\]

The first evolution \(\hat{U}_{2}(t_{2})\), with \(\sin[E_{C}^{(2)} t_{2}/(2h)] = \pm 1/\sqrt{2}\), is used to superpose two logic states of the second qubit. This is achieved by simply using a pulse that switches on the Josephson energy \(E_{C}^{(2)} = -2(K_{12} + K_{23})\). The second (or third) evolution \(\hat{U}_{1}(t_{1})\) (or \(\hat{U}_{3}(t_{3})\)) is achieved by switching on the Josephson energy of the first (third) qubit and setting its charging energy as \(E_{C}^{(1)} = -2K_{12}\) (or \(E_{C}^{(3)} = -2K_{23}\)). The corresponding duration is set to satisfy the conditions \(\sin[E_{C}^{(j)} t_{j}/(2h)] = 1\) and \(\cos(\gamma_{j} t_{j}/h) = 1\), with \(\gamma_{j} = \sqrt{(2 K_{12})^{2} + (E_{C}^{(j)})^{2}}/2\), with \(j = 1, 3\), in order to conditionally flip the \(j\)th qubit; that is, flip it if the second qubit is in the \(|1\rangle\) state, and keep it unchanged if the second qubit is in the \(|0\rangle\) state.

The fidelity of the GHZ state produced above can be experimentally measured by quantum-state tomography \[12, 13\]. However, it would be desirable to confirm the existence of a GHZ state without using tomographic measurements on a sufficient number of identically prepared copies. Optical experiments \[2\] have achieved this via single-shot readout and we propose a superconducting-qubit analog of this approach. The single-shot readout of a Josephson-charge qubit has been experimentally demonstrated \[13\] by using a single-electron transistor (SET) \[14\]. Before and after the readout, the SET is physically decoupled from the qubit. The GHZ state generated above implies that the three SETs, if they are individually coupled to each one of the three Cooper-pair boxes at the same time, will simultaneously either receive charge signals or receive no signal. The former case indicates that the circuit is in the state \(|111\rangle\), while the latter one corresponds to the state \(|000\rangle\). However, the existence of these two terms, \(|111\rangle\) and \(|000\rangle\), in these single-shot readouts, is just a necessary but not yet sufficient condition for demonstrating the GHZ entanglement. Indeed, a statistical mixture of those two states may also give the same measurement results. In order to confirm that the state \(|\psi_{\text{GHZ}}^{+}\rangle\), is indeed a coherent superposition of the states \(|000\rangle\) and \(|111\rangle\), we consider the following operational sequence

\[
|\psi_{\text{GHZ}}^{+}\rangle \xrightarrow{\hat{U}_{2}} \frac{1}{2}(|000\rangle - |101\rangle + i|010\rangle + i|111\rangle) \xrightarrow{\hat{P}_{2}} \frac{1}{\sqrt{2}}(|010_{3}\rangle + |11_{13}\rangle) \xrightarrow{\hat{U}_{1} \otimes \hat{U}_{3}} \frac{1}{\sqrt{2}}(|01_{13}\rangle + |10_{3}\rangle), \tag{3}
\]

which is similar to the verification of the optical GHZ correlations \[2\]. Above, \(\hat{P}_{2} = |1_{2}\rangle\langle 1_{2}|\) is a projective measurement of the second qubit. The suffixes are introduced in the second and third steps to denote the order of the qubits. When we finally readout the first and third qubits at the same time, the simultaneous absence of the terms \(|0_{1}0_{3}\rangle\) and \(|1_{13}\rangle\) due to destructive interference indicates the desired coherent superposition of the terms in the prepared GHZ state \(\langle 2\rangle\). The question now is how to realize the required single-qubit operations \(\hat{U}_{j} = \exp[i \pi \sigma_{z}^{(j)}/4], j = 1, 2, 3\), keeping the remaining qubits unchanged, in this circuit with tunable interbit interactions (like the currently available experimental ones).

In order to effectively implement the single-qubit rotation \(\hat{U}_{2}\) performed only on the second qubit, while keeping the first

\[
E_{C}^{(j)} = 2e^{2} \cos(\pi \Phi_{j}/\Phi_{0}) \text{ with } \varepsilon^{(j)} \text{ the Josephson energy of the single-junction and } \Phi_{0} \text{ the flux quantum. The effective coupling energy between the } j \text{th qubit and the } (j + 1) \text{th one is } K_{j,j+1} = e^{2} C_{j,j+1}. \]\n
\[\begin{align*}
\sigma_{z}^{(j)} &= |0_{j}\rangle\langle 0_{j}| - |1_{j}\rangle\langle 1_{j}|, \\
\sigma_{y}^{(j)} &= |0_{j}\rangle\langle 1_{j}| + |1_{j}\rangle\langle 0_{j}|.
\end{align*}\]
and third qubits unchanged, we let the circuit evolve under the Hamiltonian
\[ \hat{H}_2 = -\frac{\epsilon_j}{2}\sigma_j^{(2)} + K_{12}\sigma_1^{(1)}\sigma_2^{(2)} + K_{23}\sigma_2^{(2)}\sigma_3^{(3)}, \]
by only switching on the Josephson energy of the second qubit, e.g., \( E_j = 2\epsilon_j \). Since \( \zeta_{12} = K_{12}/(2\epsilon_j) \), we can treat the second and third terms in \( \hat{H}_2 \) as perturbations of the first one. Indeed, neglecting quantities smaller than the second-order perturbations \( m \), the Hamiltonian \( \hat{H}_2 \) can be effectively approximated to \( 1 \).

\[ \hat{H}^{(2)}_{\text{eff}} = -\frac{\epsilon_j}{2}\left[ 1 + 2\zeta_{12}^2 + 2\zeta_{23}^2 + 4\zeta_{12}\zeta_{23}\sigma_z^{(3)}\sigma_z^{(3)} \right]\sigma_j^{(2)}. \]

In the state (2) the logic states of the first and third qubits are always identical. Thus, by setting the corresponding duration \( \tau_2 = \pi/(4\epsilon_j^2) \), the required single-qubit operation \( \hat{U}_2 = \exp(-i\hat{H}^{(2)}_{\text{eff}}\tau_2/\hbar) = \exp(i\pi\sigma_x^{(2)}/4) \) could be effectively performed on the second qubit in state (2). Similarly, the Hamiltonian \( \hat{H}_{13} = \sum_{j=1,3}[\epsilon_j\sigma_j^{(j)} + K_{2}\sigma_2^{(2)}\sigma_3^{(3)}] \), induced by simultaneously switching on the Josephson energies of the first and third qubits, can be effectively approximated to

\[ \hat{H}^{(13)}_{\text{eff}} = -\sum_{j=1,3}\frac{\epsilon_j}{2}\sigma_j^{(j)}\sigma_x^{(j)} + \frac{\epsilon_j}{2}\sigma_j^{(j)}\sigma_x^{(j)}, \]

by neglecting the higher-order terms of \( \zeta_{j2} = K_{j2}/(2\epsilon_j^2) \). The shifts of the Josephson energies \( \Delta \epsilon_j^{(j)} = 4\epsilon_j^2\zeta_{j2}^2\sigma_z^{(2)} \) depend on the state of the second Cooper-pair box, which collapsed into the state \( \{12\} \) after the projective measurement \( P_2 = |12\rangle|12\rangle \). The measurement tunnels the excess Cooper-pairs into the connected SET. Thus, the effective Hamiltonian \( \hat{H}^{(13)}_{\text{eff}} \) yields the evolution \( \hat{U}_{13}(\tau_{13}) = \exp(-i\hat{H}^{(13)}_{\text{eff}}\tau_{13}/\hbar) = \prod_{j=1,3}\exp(i\tau_{13}\epsilon_j^{(j)}(1 + 2\zeta_{j2}^2)/\hbar) \).

\[ \text{Obviuously, if the duration } \tau_{13} \text{ satisfies the condition } \tau_{13}(\epsilon_j^{(j)}(1 + 2\zeta_{j2}^2))/\hbar = \pi/4, \text{ then the required single-qubit operations } \hat{U}_j = \exp[i\pi\sigma_x^{(j)}/4] \text{ could be simultaneously implemented.} \]

**Possible application.**—The prepared GHZ state, e.g., \( |\psi_{\text{GHZ}}^\pm\rangle \), should allow, at least in principle, to test the macroscopic conflict between the noncommutativity of quantum mechanics and the commutativity of classical physics by definite predictions. Using the EPR’s reality criterion, each observable corresponds to an “element of reality” (even if it is not measured). That is, the quantum operators \( \sigma_\alpha^{(j)} \), \( (\alpha = x, y, z; j = 1, 2, 3) \) are linked to the classical numbers \( m_\alpha^{(j)} \), which have the value +1 or −1. The so-called \( \sigma_\alpha^{(j)} \) -measurement is the projection of the quantum state into one of the eigenstates of \( \sigma_\alpha^{(j)} \). The prepared GHZ state is the eigenstate of the three operators: \( A_{\alpha x} = \sigma_\alpha^{(1)}\sigma_x^{(2)}\sigma_z^{(3)} \), \( A_{\alpha y} = \sigma_x^{(1)}\sigma_\alpha^{(2)}\sigma_z^{(3)} \), and \( A_{\alpha x y} = \sigma_x^{(1)}\sigma_z^{(2)}\sigma_y^{(3)} \), with a common eigenvalue +1. Thus, classical reality implies that

\[ 1 = (m_y^{(1)}m_x^{(2)}m_z^{(3)}) (m_z^{(1)}m_y^{(2)}m_x^{(3)}) (m_z^{(2)}m_x^{(1)}m_y^{(3)}) = m_y^{(1)}m_y^{(2)}m_y^{(3)}. \]

The second formula indicates that, if we perform the \( \prod_{j=1}^4 \sigma_\alpha^{(j)} \) -measurement (i.e., \( yyy\) -experiment) on the state \( |\psi_{\text{GHZ}}^+\rangle \), the eigenstate \( |\pm\rangle \) only shows in pairs. Here, \( |\pm\rangle \) denotes the eigenstate of the operator \( \sigma_x \) with eigenvalue +1 (or −1) and corresponds to the classical number \( m_x = +1 \) (or −1). While, for this \( yyy\) -experiment quantum-mechanics predicts that the state \( |\pm\rangle \) never shows simultaneously in pairs, because the prepared GHZ state can be rewritten as \( |\psi_{\text{GHZ}}^+\rangle = (|++\pm\rangle + |\pm++\rangle + |\pm\pm\pm\rangle)/2 \). Obviously, this contradiction comes from the fact that the observable \( \sigma_z^{(3)} \) anti-commutes with the observable \( \sigma_y^{(3)} \) and the operator identity

\[ (\sigma_y^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}) (\sigma_y^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}) (\sigma_x^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}) = -\sigma_y^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}, \]

which is “opposite” to its classical counterpart.

The protocol described above could be directly (e.g., for the optical system [2]) performed by reading out the eigenstates of the operators \( \sigma_x \) and \( \sigma_y \), respectively. However, in the present solid-state qubit, the eigenstates of \( \sigma_x \) are usually read out. Thus, additional operations, e.g., the Hadamard transformation \( \hat{S}_x = (\sigma_x + \sigma_z)/\sqrt{2} \), and the unitary transformation \( \hat{S}_y = (1 + i\hat{U}) (1 - i\hat{U}) \sum_{\alpha=1,3} \sigma_\alpha/2 \sqrt{2} \), are required to transform the eigenstates of \( \sigma_x \) and \( \sigma_y \) to those of \( \sigma_z \), respectively. These additional single-qubit operations could be implemented by combining the rotations of the selected qubit along the \( x \)-axis (by using the effective Hamiltonian proposed above) and those along the \( z \)-axis (by effectively refocusing the fixed-interactions [13]).

**Conclusion and Discussions.**—The experimental realization of our proposal for producing and testing GHZ correlations is possible, although it may also face various technological challenges, like other theoretical designs [17] for quantum engineering. Of course, the fabrication of the proposed circuit is not difficult, as it only adds one qubit to experimentally-existing superconducting nanocircuits [17]. Moreover, rapidly switching on/off the Josephson energy, to realize the fast quantum manipulations, is experimentally possible. In fact, assuming a SQUID loop size of \( 10 (\mu m)^2 \), changing the flux by about a half of a flux quantum in \( 10^{-10} \) s, requires sweeping the magnetic field at a rate of \( 10^5 \) Tesla/s, almost reachable by current techniques [18].

Also, the prepared GHZ states are the eigenstates of the idle circuit (i.e., no operations on it) without any charge- and Josephson energies (by setting the controllable parameters as \( \Phi_j = \Phi_0/2 \) and \( n_{g1} = 0.5 \)) and thus are relatively long-lived, at least theoretically. Indeed, the couplings \( \hat{V} = \sum_{j=1}^3 \epsilon_j^{(j)}(\sum_{k=1}^3 \lambda_{jk}X_k) \) between the relevant bath and the circuit commute with the non-fluctuating Hamiltonian of the idle circuit \( H_0 = \sum_{j=1,2} K_{j3} + \sigma_j^{(j)}\sigma_j^{(j)+} \). Here, \( \lambda_{jk} \) equals to either 1 if \( j = k \) or \( C_{\lambda_k} \hat{C}_{\lambda_k} \) for \( j \neq k \), and \( X_k = (eC_{\lambda_k} / \hat{C}_{\lambda_k}) \sum_{\omega_k} g_{\omega_k} \tilde{a}_{\omega_k} + g_{\omega_k} \tilde{a}_{\omega_k} \) with \( \tilde{a}_{\omega_k} \), \( \tilde{a}_{\omega_k}^\dagger \).
The operation \( \hat{t}_o (t_2) \) is not influenced by \( K_{13} \). The fidelity of the evolution by the operation \( \hat{U}_1(t_1) (\hat{U}_3(t_3)) \) achieves to 99.95% for the typical parameters: \( K_{13}/K_{12} = K_{13}/K_{23} = 0.05 \). If \( K_{13} \) is considered, the exact evolution could still be obtained by simply modifying the gate voltage \( V_1 \) to satisfy the condition \( E_{12}^{(1)} = 2K_{12} - 2K_{13} (j = 1, 3) \).

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