Turbulence in exciton–polariton condensates

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Nonequilibrium condensate systems such as exciton-polariton condensates are capable of supporting a spontaneous vortex nucleation. The spatial inhomogeneity of pumping field or/and disordered potential creates velocity flow fields that may become unstable to vortex formation. This letter considers ways in which turbulent states of interacting vortices can be created. It is shown that by combining just two pumping intensities it is possible to create a superfluid turbulence state of well-separated vortices, a strong turbulence state of de-structured vortices, or a weak turbulence state in which all coherence of the field is lost and motion is driven by weakly interacting dispersive waves. The decay of turbulence can be obtained by replacing an inhomogeneous pumping by a uniform one. We show that both in quasi-equilibrium and during the turbulence decay there exists an inertial range dominated by four-wave interactions of acoustic waves.

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Introduction. The phenomenon of turbulence – chaotic motion of vortices of many different length scales – is ubiquitous in nature, and quantitative understanding of it is a notoriously difficult problem of classical physics. Turbulence occurs in many usual fluid flows as well as in exotic systems such as plasmas and superfluids. Vorticity in superfluids is quantized in units of $\hbar/m$, where $m$ is the mass of the boson in contrast with continuously distributed vorticity of a classical Navier-Stokes fluid. In superfluids quantized vorticity is considered to be an evidence for a macroscopically occupied quantum state that can be described by a classical complex-valued wave function $\psi(x,t)$. Quantization of velocity circulation in superfluids leads to significant differences between superfluid turbulence (ST) and classical turbulence. On the other hand, at large Reynolds numbers the motion of well-separated vortices in an incompressible classical flow may have similar features to ST. In this case the vortex dynamics in superfluids is almost classical in accordance with the Biot-Savart law (BSL). The decay of the turbulence (loss of the vortex line density) occurs due to dissipative effects induced by interactions with a normal fluid component (with a thermal cloud).

Recently by introducing an external oscillatory perturbation in a trapped atomic BEC it became possible to obtain a disordered system of many topological defects [1]. The dynamics of this matter field differs from both dynamics of vortices in classical turbulence and in superfluid helium turbulence. Firstly, the characteristic distance between vortices is comparable to their core sizes, so the chaotic behavior is seen on the level of a single vortex, secondly, these vortices are not structured, so they do not obey BSL, finally, the system is in a strongly non-equilibrium state. These creates a novel nontrivial regime of a classical complex matter field — "strong turbulence" state – whose evolution is quite different from that of ordered condensate. In analogy with other nonlinear systems such as plasmas, fluids and nonlinear optics, apart from the regime of strong turbulence there exists the regime of weak turbulence where all phases of the complex amplitudes of the matter field are random. Recently [2] these three regimes (superfluid, strong turbulence and weak turbulence) have been observed at different temperatures in 2D cold atomic gases, showing a universal scaling. The weak turbulence plays crucial role in kinetics of Bose-Einstein condensation [3]. It was shown that a strongly non-equilibrium Bose gas evolves from the regime of weak turbulence to superfluid turbulence, via states of strong turbulence in the long-wavelength region of energy space. An important question remains whether it is possible to force a condensate system to pass through these stages in a reverse order. It has been suggested [4] that if a sufficiently strong external perturbation is applied to the trap, it is in principle possible to obtain the weak turbulence state. When this is done it will lead to a discovery of nontrivial transitional regimes of classical matter fields in atomic systems [5].

In the last few years the Bose-Einstein condensation has been achieved in solid state systems [6], such as microcavities, ferromagnetic insulators and within superfluid phases of $^3$He. Microcavity exciton-polaritons are quasi-particles that consist of superpositions of photons in semiconductor microcavities and excitons in quantum wells. The Bragg reflectors confining photon component are imperfect, so exciton-polariton have finite life time and have to be continuously re-populated. Such combination of pumping and decay leads to quasi-particle flow even at steady states of the system. At sufficiently low densities these quasi-particles can form a Bose-Einstein condensate, so the many particles quantum system can be described by a classical equation in a form of the complex Ginzburg-Landau equation (cGLE)
Vortex formation. To illustrate the basic mechanism that drives the formation of vortices we first consider a pumping field in a form of a step function in 1D, so that \( \alpha = \alpha_1 + \alpha_0, \sigma = \sigma_1, \eta = \eta_1 \) for \( x < 0 \) and \( \alpha = \alpha_0, \sigma = \sigma_0, \eta = \eta_0 \) for \( x > 0 \). The steady state mass continuity and Bernoulli equations resulting from the Madelung transformation \( \psi = \sqrt{\rho} \exp i S \) applied to Eq. (1) are \( \mu = u^2 + \rho - d^2 / \sqrt{\rho} / 2 \sqrt{\rho} dx^2 \) and \( d(\rho u)/dx = (\alpha - \eta \rho - \sigma \rho) \rho \) where \( \rho \) is the number density, \( u = S'(x) \) is the velocity and the chemical potential \( \mu \) is introduced by \( 2i \partial_t \psi = \mu \psi \). Away from large density fluctuations we can drop the quantum pressure term \( d^2 / \sqrt{\rho} / 2 \sqrt{\rho} dx^2 \). We expect that \( u \to 0 \) as \( x \to -\infty \), so \( \mu \to (\alpha_1 + \alpha_0) / (\sigma_1 + \eta_1) \). As \( x \to \infty \), therefore, there will be a steady current \( u = (\alpha_0(\eta_0 + \sigma_0) + \alpha_1(\eta_0 - \eta_1 + \sigma_0 - \sigma_1) / \sigma_0(\eta_1 + \sigma_1))^{-1/2} \) generated by the step. The presence of boundaries or other sources of outflow generate interference fringes seen, for instance, in recent experiments in 1D [11]. In 2D the fringes that meet at nonzero angles evolve into a pair of vortices of opposite circulation as seen on the left panel of Fig. 1. The mechanism leading to vortex formation in this case is analogous to the transverse instability of a density depletion in a conservative Gross-Pitaevskii equation [17]: the motion of grey solitons is inversely proportional to their depth, so modulation in the transverse direction forces different parts of the front to move with different velocities leading to vortex pair formation. This suggests that the several sources of such flows may continuously generate a large number of vortices leading to a turbulent flow. Another possibility to create a turbulent flow is related to the formation of vortex lattice in a harmonic trapping potential due to an instability of a
non-rotating solution $\alpha=0$. By removing the circular symmetry of either the trapping potential or pumping field it is possible to create a turbulent flow of vortices instead of a regular vortex lattice (see the right panel of Fig. 1).

**Numerical set-up.** In order to engineer a turbulent formation and interaction of vortices we shall consider an inhomogeneous pump $\alpha(x)$ that can be obtained by passing the laser beam through a spatial phase (light) modulator. This will be even further simplified by assuming that only two laser intensities are allowed: the background with a homogeneous pump and interaction of vortices we shall consider an inhomogeneous pump $\alpha(x)$ that can be obtained by passing the laser beam through a spatial phase (light) modulator.

As the density of vortices decreases, the system reaches the superfluid turbulence regime of well-separated vortices with a logarithmic decay $^{20}$. These decay rates are in contrast with a power-decay rates of the order $t^{-3/4}$ in classical 2D viscous fluids and in the limit of the cGLE equation with zero dispersion $^{21}$.

By tuning the nonuniformity of the pumping field it is possible to reach different turbulent regimes. If the difference between intensities, $\alpha_1$, is below a threshold or the distance between the spots of higher intensity is large, no vortices will be created. In a case of a moderate $\alpha_1$ and only few spots a set of several well-formed well-separated vortex pairs is created and the system is in a superfluid turbulence state (see the left panel of Fig. 1 and the left inset of Fig. 2). By increasing the difference between intensities $\alpha_1$ it is possible to create the state of strong turbulence (where vortex cores start to overlap; see the top inset of Fig. 2). It is, therefore, tempting to see if the system can be driven even further to enter the regime of weak turbulence in which all coherence is lost and all Fourier amplitudes have random phases. To verify this we calculated the second moment of the correlation function $g_2 = \langle |\psi|^4 \rangle / \langle |\psi|^2 \rangle^2$. By Wick’s theorem the state of the weak turbulence corresponds to $g_2 = 2$. As shown on Fig. 2 by raising $\alpha_1$ it is possible for the system to reach the weak turbulence state. Note that the relaxation $\eta$ increases $g_2$. This occurs because the relaxation increases the rate at which vortex pairs annihilate by bringing the vortex cores closer to each other; this effect can be seen on Fig. 2 showing the number of vortices in quasi-equilibrium. The energy released from vortex annihilation becomes converted into acoustic energy therefore increasing $g_2$.

In order to describe the turbulence in the Eq. 1 we shall assume that there exists an inertial range in the momentum space and that the role of pumping and dissipation is insignificant there. The evolution equation for the wave spectrum defined by $\langle a_k^* a_k \rangle = n_k \delta(k_1 - k_2)$, with $a_k$ being the Fourier transform of $\psi$ and $k_i$ are discrete wave vectors, can be obtained by using a random phase approximation and expanding in small nonlinearity $^{22}$. The equation takes the form $\partial_t n_k(t) = \int d^2 k_i d^2 k_i d^2 k_i W_{k_1, k_2, k_3, k_4} \times (n_{k_3} n_{k_4} n_{k_1} + n_{k_1} n_{k_3} n_{k_2} - n_{k_2} n_{k_3} n_{k_4} - n_{k_1} n_{k_2} n_{k_3})$, where $W_{k_1, k_2, k_3, k_4} = \frac{\pi}{(2\pi)^2} \delta(k_1 + k_2 - k_3 - k_4) \delta(k_1^2 + k_2^2 - k_3^2 - k_4^2)$. Two solutions of this evolution equation correspond to a thermodynamic equipartition of the total kinetic energy $E = \int k^2 n_k \, dk$, so that $n_k \sim k^{-2}$ and to an equipartition of the total number of particles $N = \int n_k \, dk$, so that $n_k \sim \text{const}$. These correspond to the two limits of the Rayleigh-Jeans distribution $T/(k^2 + \mu)$, where $T$ is the temperature.

We verified the existence of the inertial range in our simulations. Although the system is in a quasi-

![Figure 2](image-url)
A. Amo, C. Ciuti, J. Keeling and B. Svistunov.

...regimes fundamentally different from the classical fluid of complex matter field with turbulence that may span are new and exciting systems with a nontrivial evolution of coherence. The nonequilibrium condensates, therefore, vortices or the weak turbulence state with a complete loss of coherence. By designing a nonuniform pumping field that leads to sufficiently strong turbulence with well separated quantised vortices, the strong turbulence with overlapping and de-structured vortices or the weak turbulence state with a complete loss of coherence. The nonequilibrium condensates, therefore, new and exciting systems with a nontrivial evolution of complex matter field with turbulence that may span regimes fundamentally different from the classical fluid turbulence.

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[1] E. A. L. Henn, et al. Phys. Rev. Lett., 103, 045301 (2009)
[2] C.-L. Hung et al arXiv:1009.0016 (2010)
[3] N.G. Berloff and B.V. Svistunov, Phys. Rev. A 66, 013603 (2002); and references therein; C.N. Weiler et al Nature 455, 948 (2008).
[4] N.G. Berloff and B.V. Svistunov Physics 2, 61 (2009)
[5] It is feasible that the granulated state observed in J.A. Seman et al arXiv:1007.4953 (2010) is the weak turbulence state.
[6] J.Kaspzak et al Nature, 443, 409 (2006); R. Balili et al Science, 316, 1007 (2007); A.Amo et al Nature, 457, 291 (2009); S. Utsunomiya et al Nature Phys., 4, 700 (2008); A. Amo et al Nature Phys. (2009); S. O. Demokritov et al Nature, 443, 430 (2006); V. E. Demidov et al Phys. Rev. Lett., 100, 047205 (2008); O. Dzyapko et al Phys. Rev. B, 80, 060401(R) (2009); A.V.Chumak et al Phys. Rev. Lett.,102, 187205 (2009); Y. M. Bunkov and G. E. Volovik Phys. Rev. Lett. 98, 265302 (2007); G. E. Volovik J. Low Temp. Phys., 153, 266 (2008).
[7] K.G. Lagoudakis et al Nature Phys., 4, 706 (2008).
[8] J. Keeling and N. G. Berloff, Phys. Rev. Lett., 100, 250401 (2008)
[9] M. Wouters and I. Carusotto, Phys. Rev. B, 80, 195332 (2007).
[10] M. H. Szymańska et al, Phys. Rev. B, 75, 195331 (2007).
[11] E. Wertz et al, arXiv:1004.4084 (2010)
[12] T. C. H. Liew et al, arXiv:1008.5320 (2010).
[13] M. Wouters and V. Savona, arXiv:1007.5431 (2010)
[14] L.P.Pitaevskii Sov. Phys. JETP, 8, 282 (1959).
[15] N.G. Berloff, J. Phys. A: Math. Gen. 37, 1617 (2004).
[16] T. Frisch, Y. Pomeau, and S. Rica Phys. Rev. Lett.69 1644 (1992); N.G. Berloff and R.H. Roberts J. Phys. A: Math. Gen., 33, 4025 (2000); T. Waniecki et al, J. Phys.}
B: At. Mol. Opt. Phys. 33, 4069 (2000).

[17] E. A. Kuznetsov and S. K. Turitsyn Sov. Phys. JETP 76, 1583 (1988); E. A. Kuznetsov and J. J. Rasmussen Phys. Rev. E 51, 4479 (1995); N.S. Ginsberg, J. Brand, L.V. Hau, Phys. Rev. Lett. 94, 040403 (2005); N.G. Berloff and C.F. Barenghi, Phys. Rev. Letts. 93 090401 (2004).

[18] S. Pigeon et al, arXiv:1006.4755 (2010).

[19] When we refer to a difference of pumping intensities as a parameter that defines different turbulent regimes, it should be understood that it is the difference between chemical potentials that the system is trying to establish in different regions that is driving the turbulence. For numerical simulations we used a fourth–order finite differences in space and fourth–order Runge-Kutta integration in time. The number of grid points in physical space was set to 512² for the physical domain [−20, 20]² with doubly periodic boundaries. The initial state is always taken to be constant \( \psi = \alpha_0 / (\sigma + \eta) \), \( c = 4, \alpha_i = 10, T = 10 \). Numerically, the inhomogeneous pump consisting, for instance, of one spot will be represented by \( \alpha(x) = \alpha_0 + \alpha_1 (1 - \tanh(x^2 + y^2 - c))/2 \).

[20] S. Nazarenko and M. Onorato Physica D, 219, 1 (2006).

[21] G.B. Weiss and J. McWilliams Phys. Fluids A, 5, 608 (1993); G. Huber and P. Alstrom, Physica A, 195, 448 (1993).

[22] Z.E. Zakharov et al “Kolmogorov Spectra of Turbulence”, Springer-Verlag (1992).