Superstring Field Theory Action Including Massless Fermions

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Using Berkovits’ Superstring Filed Theory action including the Ramond sector, we calculate the contribution to this action coming from the tachyon and massless fermions from both GSO sectors. Some features of the action are discussed in the end, settling the ground for a more systematic treatment of spacetime fermions in superstring field theory.

1. Introduction

During the 80’s a string field theory was developed for the open bosonic string theory, following the ideas of Witten for the role of non-commutativity in the string theory product (Witten’s midpoint interaction) and the development of BRST techniques in string theory [1].

The natural generalization of these ideas to the case of the superstring theory didn’t achieve the success of the bosonic theory due to the divergences at the tree level for the classical action, a consequence of picture changing operators appearing explicitly in the action [2]. The interest on this kind of theories declined substantially, mainly because of the lack of any significant progress, till Sen’s conjecture on the role of tachyon in a system of unstable D-branes [3], arguing that the tachyon should have a minimum for its potential at the point where the system undergoes a decay, establishing a value for the minimum of the tachyon potential as the value of the tension of the original unstable D-brane system.

String Field Theory is the natural setting for studying the tachyon condensation and so we have seen a renewal of interest on this theory.

About the same time, Berkovits proposed an action for the NS sector of superstring field theory which doesn’t suffer from the divergences of the previous formulation [4]. This action is based in an embedding of $N = 1$ superconformal algebra into an $N = 4$ algebra, and is explicitly supersymmetric in four dimensions. The Ramond sector was included last year [7] in an action with three string fields that will be reviewed latter in this talk.

Calculations of the tachyon potential have been done and they show a good agreement with Sen’s conjecture, but this calculations only require the NS sector [9]. Up to now there’s no study on the Ramond sector.

In this work we are going to consider the action of [7] for the superstring field action with manifest $N = 1$, $D = 4$ supersymmetry. We are going to include the GSO(-) massless sector and the Tachyon and calculate the relevant terms of the action, at the end we recombine the terms in order to write an explicitly $D = 10$ Lorentz invariant action. The pure GSO(+) sector is super-Yang-Mills in 10-dimensions and we are not going to write it.

The Ramond sector is important in order to understand supersymmetry related issues regarding tachyon condensation. If a superstring theory on an unstable D-brane undergoes tachyon condensation either stable D-branes (via solitonic solutions of the Tachyon profile) or a true closed string vacuum (in the annihilation of all the $D-\bar{D}$ system) should appear, these theories are supersymmetric and so the original theory should have a hidden SUSY (in a broken phase), it’s just natural to think of GSO(-) fermions as goldstinos for the broken SUSY.

There are indeed some arguments from Yoneya...
which support these statements \[4\]. Also, if the massless GSO(-) fermions are to be considered as goldstinos, there should be a non-linear super-symmetry realized in the broken phase, in which the goldstinos transform under the usual SUSY transformation with a shift due to a Fayet-Iliopoulos term, which is to be canceled in the tachyonic vacuum, where the linear supersymmetry is restored.

In the next session we review the hybrid formalism, used to describe the superstring field theory action in the following session, emphasizing the issues regarding vertex operators and inclusion of GSO(-) sector. In section 4 we sketch the computations and discuss some perspectives at the end.

2. Hybrid Formalism

2.1. Hybrid Variables

This formalism enables a formulation of superstrings in an explicitly D=4 Super-Poincaré invariant manner (for the GSO(+)) sector. In order to understand some of its features, one should consider breaking of the full 10-dimensional Lorentz invariance of the superstrings, by complexifying 6 of the bosonic coordinates (and analogously to the fermionic variables):

\[
x^{\pm j} = \frac{1}{\sqrt{2}}(x^{2j+2} \pm x^{2j+3}), \quad j = 1, 2, 3.
\]

(1)

Following the lines of \[5\], we use the following set of variables to describe the superstring, including some superfield coordinates \(\theta\) and the conjugate momenta \(p\):

\[
[X^m, \theta^a, \bar{\theta}^\dot{a}, p_{\alpha}, \bar{p}_{\dot{\alpha}}, \rho, \Gamma^{\pm j}, X^{\pm j}] 
\]

(2)

here \(X^m\) is the \(D = 4\) spacetime vector world sheet boson, \(m = 0, 1, 2, 3\);
\(\theta^a\) is the \(D = 4\) spacetime chiral spinor worldsheet fermion, \(a = 1, 2\);
\(\bar{\theta}^\dot{a}\) is the space-time anti-chiral spinor world sheet fermion, \(\dot{a} = 1, 2\);
\(\rho\) is the world sheet chiral boson;
\(\Gamma^{\pm j}\) are internal world sheet fermions, \(j = 1, 2, 3\);
\(X^{\pm j}\) are internal world sheet bosons, \(j = 1, 2, 3\).

All these fields have free field OPEs:

\[
X^n(z)X^m(w) = -g^{mn}\ln(z - w),
\]

(3)

\[
p_\alpha(z)\theta^\beta(w) = \frac{\delta^\beta_\alpha}{z - w},
\]

\[
\rho(z)\rho(w) = -\ln(z - w),
\]

\[
\Gamma^{ij}(z)\Gamma^{-ij}(w) = \delta^{ij}\frac{1}{(z - w)}.\]

(3)

\[
X^{++}(z)X^{-j}(w) = -\delta^{ij}\ln(z - w).
\]

and the relation of these variables to the usual RNS variables, written in the \(SO(3,1)\) invariant form:

\[
\left[ x^m, \psi^m, \bar{b, c, \xi, \eta, \phi}, x^{\pm j}, \bar{\psi}^{\pm j} \right],
\]

(4)

is given by the following set of relations:

\[
X^m = e^{R/2U}(x^m)e^{R-\frac{1}{2}U},
\]

\[
\theta^a = \frac{e^{\frac{1}{2}\Sigma^a}e^{-\frac{1}{2}\bar{\Sigma}^{\dot{a}}}}{e^{R/2U}},
\]

\[
\bar{\theta}^\dot{a} = \frac{\xi e^{-\frac{1}{2}\Sigma^a}\bar{\alpha}^{\dot{a}}}{e^{R-\frac{1}{2}U}},
\]

\[
p_\alpha = e^{R/2U}\left(e^{-\frac{1}{2}\Sigma^a}e^{\frac{1}{2}\bar{\Sigma}^{\dot{a}}}\right)e^{R-\frac{1}{2}U},
\]

\[
\bar{p}_{\dot{\alpha}} = e^{R/2U}\left(b_{\dot{\alpha}}\bar{\Sigma}^{\dot{a}}e^{-\frac{1}{2}\Sigma^a}\right)e^{R-\frac{1}{2}U},
\]

\[
\partial\phi = 3\partial\phi - cb - 2\xi\eta - \partial H,
\]

\[
X^{++} = e^{R/2U}(x^{+j})e^{R-\frac{1}{2}U},
\]

\[
X^{-j} = e^{R/2U}(x^{-j})e^{R-\frac{1}{2}U},
\]

\[
\Gamma^{++} = e^{R/2U}(\xi e^{-\phi}\psi^{+j})e^{R-\frac{1}{2}U},
\]

\[
\Gamma^{-j} = e^{R/2U}(\eta e^{\phi}\psi^{-j})e^{R-\frac{1}{2}U},
\]

(5)

where

\[
R = \int dz\ c\xi e^{-\phi i\psi^{-j}\partial x^{+j}},
\]

\[
U = \int dz\ c\xi e^{-\phi i\psi^{m}\partial x_{m}}
\]

\[+ \frac{1}{2}(\partial\phi + \partial H)c\partial c\xi e^{-2\phi}]
\]

the chiral boson \(H\) is defined by \(\partial H =: \psi^{+j}\psi^{-j}\); and the fields \(\Sigma\) are spin fields made up by bosonization of the 4-dimensional world-sheet fermions, as explained in \[5\].

2.2. Vertex operators

The vertex operators may be constructed directly in the hybrid formalism by using its rich
superconformal structure. The GSO(+)) sector was already contained in [5], and the GSO(−)) vertices for Tachyon and massless fermions were constructed in detail in [6].

The goal of this work is to compute the superstring field theory action contribution due to these GSO(−)) fields, which can be obtained from the RNS vertices by using the mapping in eqs. (5).

Since we’re going to obtain vertices with manifest 4-dimensional Lorentz invariance, is convenient, prior to the exhibition of the vertex operators, to show how the 10-dimensional fields split in different fields.

In 10-dimensions, the superstring spectrum has a massless vector and a chiral fermion in the GSO(+)) sector and the Tachyon and an anti-chiral fermion in the GSO(−)) sector. Using the Lorentz breaking pattern:

\[ \text{SO}(9, 1) \rightarrow \text{SO}(3, 1) \times \text{SO}(6), \]

\[ \text{SO}(9, 1) \rightarrow \text{SO}(3, 1) \times \text{SU}(3) \times \text{U}(1). \]

(6)

the 10-dimensional representations split according to:

\[ 10 \rightarrow 4 \oplus 3 \oplus 3 \]

\[ A_m, \phi^{-i}, \bar{\phi}^{+i} \]

\[ 16 \rightarrow (2, 1, +3) \oplus (2, 3, -1) \oplus (2', 1, -3) \oplus (2', 3, +1) \]

\[ \chi_{(+)} \alpha, (\psi_{\alpha})^{-i}, \bar{\chi}^{(+)}_{\alpha}, (\bar{\psi}^{\alpha})^{+i} \]

\[ 16' \rightarrow (2', 1, +3) \oplus (2', 3, -1) \oplus (2, 1, -3) \oplus (2, 3, +1) \]

\[ \bar{\chi}^{(-)}_{\alpha}, (\bar{\lambda}^{(-)})^{-i}, \chi_{(-)} \alpha, (\lambda_{\alpha})^{+i} \]

where primed quantities mean anti-wei representations, the subscript is the U(1) charge and the number in parenthesis are the dimensions of the representations. The vertex operators, obtained by the mapping are summarized in the table [17].

### 3. SSFT Action

Using these hybrid variables one can construct an \( N = 4, c = 6 \) superconformal algebra with the following generators:

\[ L = -\frac{1}{2} \partial X^m \partial X_m - p_a \theta^a - \bar{p}_a \bar{\theta}^a - \partial \rho \partial \rho \]

In this table \( \Lambda^{++++} \) is a spin field of hybrid variables, from the bosonized \( \theta \) and \( \rho \), as explained in [3].

| RNS | Hybrid |
|-----|--------|
| \( c \partial \bar{c} \partial \bar{c} e^{-H/2} \bar{\phi}^{+i} \phi^{-i} \phi^{-i} \bar{\phi}^{+i} \) | \( \theta^a (\theta)^2 \bar{\lambda}_0^a \) |
| \( c \bar{c} e^{-H/2} \bar{\phi}^e \phi^e \) | \( \bar{\theta}^a (\theta)^2 \lambda_0^a \) |
| \( c \bar{c} e^{-H/2} \bar{\phi}^e \phi^e A_m \) | \( \theta^a \sigma_\alpha^m \delta^a A_m \) |
| \( c \bar{c} e^{-H/2} \bar{\phi}^e \phi^e t \) | \( \Lambda^{++++} t \) |
| \( z^2 c \partial \bar{c} \partial \bar{c} e^{-H/2} \bar{\phi}^{+i} \phi^{-i} \phi^{-i} \bar{\phi}^{+i} \) | \( \theta^a \Lambda_{-a}^0 \) |

| RNS | Hybrid |
|-----|--------|
| \( c \partial c \partial c e^{-H/2} \bar{\phi}^{+i} \phi^{-i} \phi^{-i} \bar{\phi}^{+i} \) | \( e^\Gamma^{-j} \theta^a (\theta)^2 \bar{\lambda}^{-ja}_0 \) |
| \( c \partial \bar{c} \partial \bar{c} e^{-H/2} \bar{\phi}^e \phi^e \) | \( e^\Gamma^{-j} \theta^a (\theta)^2 A_{-j} \) |
| \( z^2 c \partial \bar{c} \partial \bar{c} e^{-H/2} \bar{\phi}^{+i} \phi^{-i} \phi^{-i} \bar{\phi}^{+i} \) | \( e^{-\Gamma^{-j}} \bar{\theta}^a (\theta)^2 \lambda_{ja}^0 \) |
| \( c \partial \bar{c} \partial \bar{c} e^{-H/2} \bar{\phi}^e \phi^e \) | \( e^{-\Gamma^{-j}} \bar{\theta}^a (\theta)^2 A_{-j} \) |
| \( z^2 c \partial \bar{c} \partial \bar{c} e^{-H/2} \bar{\phi}^{+i} \phi^{-i} \phi^{-i} \bar{\phi}^{+i} \) | \( e^{-\Gamma^{-j}} \theta^a (\theta)^2 \Lambda_{-ja}^0 \) |

Table 1

Hybrid vertex operators from RNS.
helps constructing the action, and
\[ d_\alpha(z) = p_\alpha(z) + \frac{i}{2} \bar{\theta} \sigma^m_{\alpha \alpha} \partial X_m(z) \]  
(8)
\[ \frac{1}{4} (\bar{\theta})^2 \partial \theta_\alpha(z) + \frac{1}{8} \theta_\alpha (\partial (\bar{\theta}))^2(z) \]  
(9)
is the current associated to the \( N = 1, D = 4 \) supersymmetric covariant derivative \( D_\alpha \).

Berkovits superstring field theory action, developed in [3] is:
\[ S = \langle (e^{-G}\Phi e^\Phi) + (e^{-G} G e^\Phi) \rangle \]
\[ + \int_0^1 dt (e^{-G} \partial_t e^\Phi) \left( \{ e^{-G} G e^\Phi, e^{-G} G e^\Phi \} \right) \]
\[ - e^{-\Phi} \Omega e^\Phi + \Omega e^\Phi G e^\Phi - e^{-\Phi} G e^\Phi \Omega D \]
\[ - \left( \frac{1}{2} \Omega G_\frac{3}{2} \Omega + \frac{1}{3} \Omega^3 \right)_F \]
\[ + \left( \frac{1}{2} \Omega G_\frac{3}{2} \Omega + \frac{1}{3} \Omega^3 \right)_F \]
where the operators \( G \) appearing in this action are defined in eq. (1). We should also note that there are three different correlators. The D-correlator is defined in the large Hilbert Space of the superstring. The \( F \) and \( \bar{F} \) correlators are chiral (anti-chiral) subspaces defined with the trivial cohomology pieces of the \( G \) operator, \( G_{-1} \) and \( G_{0} \).

The string fields appearing in the action are defined in such a way that \( \Phi \) has zero C-charge, \( \Omega \equiv G_{-1} \Phi \) is a chiral field obtained from the string field \( \Psi \) with C-charge \( 1/3 \) and \( \bar{\Omega} \equiv G_{0} \bar{\Phi} \) is an anti-chiral string field obtained from a string field \( \Psi \) with C-charge \( -1/3 \). We are using the notation \( \hat{\Phi} = t \Phi, \) \( 0 \leq t \leq 1 \) for the WZW part. Note that with this notation all the 4-dimensional fields are in \( \Phi \), while the fields in the other string fields are dependent of the “internal” part.

All products between string fields are the midpoint interaction, calculated according to the following prescription:
\[ \langle V_1 V_2 \cdots V_N \rangle = \left( -\frac{4i}{N} \right)^{\frac{N}{2}} \sum_{k=1}^N h_k \sum_{3=1}^N h_{l(1-1)} \]  
(10)
where \( V_i \) here are conformal primary fields with conformal weight \( h_i \).

The non-vanishing norms for the three subspaces are obtained by background charge cancellation [4]:
\[ \langle \frac{1}{24} \bar{\theta}^2 e^{-\rho \epsilon^{ijk} \Gamma_{-1} \Gamma_j \Gamma_{-1} \Gamma_j} \rangle_D = 1 \]
\[ \langle \frac{1}{24} \bar{\theta}^2 e^{-\rho \epsilon^{ijk} \Gamma_{-1} \Gamma_j \Gamma_{-1} \Gamma_j} \rangle_{\bar{F}} = 1 \]
\[ \langle \frac{1}{4} e^{-3 \rho \bar{\theta}^2 (\Gamma \cdot \partial \Gamma)^3} \rangle_F = 1 \]

There's a small subtlety when including \( GSO(-) \) string fields in the action, since these fields, by their different commutativity properties, may spoil ciclicity of the correlators. The way out of this is considering a suitable tensor product of the string fields with \( 2 \times 2 \) matrices (Pauli Matrices), [5].

\[ \Phi = \Phi^+ \otimes I + \Phi^- \otimes \sigma_1 \]
\[ G = G \otimes \sigma_3 \]

and analogous relations for the other string fields and operators.

The vertex operators discussed in the previous section may be organized in superfields (for the \( GSO(+) \) sector), since the hybrid formalism is \( D=4 \) super-poincare invariant. Doing so we have the following set of string fields for the tachyon and massless fermions:
\[ \Phi^+ = v(x, \theta, \bar{\theta}) \]
\[ \Psi^+ = e^{\theta \gamma^+} \omega^-(x, \theta) \bar{\theta}^2 \]
\[ \Psi^- = e^{-\rho \gamma^-} \bar{\omega}^+(x, \bar{\theta}) \theta^2 \]
\[ \Phi^+ = \Lambda^{++} + T(x) + \Theta^\alpha \chi_\alpha(x) + \bar{\Theta}^\bar{\alpha} \bar{\chi}_{\bar{\alpha}}(x) \]
\[ \Psi^- = e^{\theta \gamma^+} \bar{\Theta}^\alpha \bar{\lambda}_{\alpha}^j(x) \]
\[ \Psi^- = e^{-\rho \gamma^-} \Lambda^\alpha \lambda_{\alpha}^j(x) \]
where \( v(x, \theta, \bar{\theta}) \) is an \( N = 1, D = 4 \) vector superfield, \( \omega(x, \theta) \) is a chiral superfield and \( \Theta \equiv \theta \Lambda^{++} \). Similar relations hold for the barred quantities.

Expanding the non-polynomial string field action up to terms with four string fields \(^3\) and plugging the vertices in this action, it’s possible to compute all the correlators in conformal

\(^3\)Since for the vertices we are considering all other terms vanish due to background charge cancellation
field theory, getting an action with manifest 4-dimensional Lorentz invariance. From the knowledge of the Lorentz breaking pattern in eq (6), it’s still possible to recover a 10-dimensional Lorentz invariant action for these fields. The details of the computation, although a bit straightforward, may be found in [6].

The classical action for the Tachyon and massless fermions in both GSO sectors is:

\[
S = \text{Tr} \int d^{10}x \left[ \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + T \square T + \frac{1}{2} T^2 
+ [T, A^\mu][T, A_\mu] - T^4 
+ \chi^a + \gamma^\mu_{ab} (\partial_\mu \chi^b + [A_\mu, \chi^b]) + \chi^a - \gamma^\mu_{ab} (\partial_\mu \chi^b - [A_\mu, \chi^b]) 
+ 3 \hat{\Delta} T (\chi^a + \chi^- a) + \cdots \right] 
\]

(17)

where \( F_{mn} \) is the field-strength for the gauge-field \( A_m \), \( T \) is the tachyon field, \( \chi^a_+ \), \( \chi^- a \) are the massless fermions coming from the GSO(+) and GSO(−) sector that are 10-dimensional chiral and anti-chiral respectively.

We should comment on some features of this action. First, the GSO(+) part was not calculated in the work; this action have higher derivative terms coming from the OPEs of the space-time fields inserted in different points in the correlators and there exist a trilinear coupling among fermions from both GSO sectors and the Tachyon, signalling for the possibility of the massless GSO(−) fermions to be goldstinos.

4. Conclusions

We performed a first non-trivial calculation with the Berkovits’ superstring field theory action, which includes the Ramond sector, and it’s rather clear that the hybrid formalism, together with this approach to superstring field theory may be very fruitful in the future. The study of supersymmetry breaking related issues regarding tachyon condensation is poorly understood and rely mostly upon guesswork and intuitive arguments. We hope the setup presented here may help the way into a more systematic treatment of this subject.

In particular, it’s possible to construct a Boundary String field Theory [10] in the hybrid formalism, which is currently under development. The insights from both approaches may be as fruitful as it proved to be in the study of the tachyon potential [11]. This formalism is also particularly well suited to discuss Ramond-Ramond backgrounds, which are still far from being well understood.

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