Unsteady-state moisture behavior calculation for multilayer enclosing structure made of capillary-porous materials

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Abstract. The paper studies unsteady-state moisture behavior of multilayer enclosing structure using discrete-continuous method. Modification of Gagarin’s differential moisture transfer equation is proposed. The proposed moisture transfer equation within one layer of a multilayer enclosing structure is a second-order differential parabolic equation with constant coefficients. Third-kind boundary conditions of moisture exchange are set for enclosing structure boundaries. Fourth-kind boundary conditions represented by moisture potential flows equality are set at the joint of two different materials. An analytical expression for moisture potential calculation in any enclosing structure section, at any time, under continuous control for temperature distribution, has been derived using discrete-continuous method. Calculations for multilayer enclosing structure consisting of clay brick base and lime brick lining have been made. The obtained results have been compared to calculation results obtained by well-known Gagarin’s unsteady-state method and Kozlov’s engineering method. It has been shown that the discrete-continuous method allows calculating unsteady-state moisture behavior of multilayer enclosing structures by analytical expression, and obtaining moisture distribution similar to Gagarin’s unsteady-state method, thus it can be used by design engineers in practice.

1. Introduction

Moisture behavior of enclosing structure is extremely important for construction. Error in construction material moisture condition assessment can result in premature deterioration of the enclosing structure [1, 2], high heat loss of the building [3, 4], and negative humidity effect on human life [5].

Moisture behavior calculations for multilayer enclosing structures are the most important and challenging ones [6]. In multilayer enclosing structure moisture transfer speed is different for every material due to its porous structure and heat-transfer properties. Mathematical models for capillary-porous bodies are considered in the given paper [7]. In such construction materials vapor moisture moves according to vapor permeability mechanism in sorption wetting zone, and condensed moisture also moves by means of capillary pressure gradient in over-sorption wetting zone.

Gagarin’s mathematical model [8] based on moisture potential is known:

\[ F(w, t) = E_i(t) \cdot \varphi(w) + \frac{1}{\mu_0} \int_0^w \beta(\zeta) d\zeta. \]  

(1)
where $F$ – moisture potential, $Pa$; $E_t$ – maximum water vapor tension, $Pa$; $\varphi$ – relative air humidity; $\mu$ – vapor permeability coefficient, $kg/(m \cdot s \cdot Pa)$; $\beta$ – moisture conductivity coefficient, $kg/(m \cdot s \cdot kg/kg)$, depending on humidity.

Differential thermal conductivity equation governs temperature distribution of a multilayer enclosing structure:

$$c \cdot \gamma_0 \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x} (\lambda \cdot \frac{\partial t}{\partial x}).$$

(2)

where $t$ – temperature, °C; $\tau$ – time, s; $x$ – coordinate, m; $\gamma_0$ – enclosing structure dry material density, $kg/m^3$; $c$ – specific material heat capacity, $J/(kg \cdot \text{C})$.

Differential moisture transfer equation governs moisture potential distribution of a multilayer enclosing structure:

$$\gamma_0 \cdot \left( \frac{1}{\mu} \beta(w) + \frac{\partial \varphi(w)}{\partial w} E_t(t) \right)^{-1} \frac{\partial F(w,t)}{\partial \tau} = \frac{\partial}{\partial x} \left( \mu \frac{\partial F(w,t)}{\partial x} \right).$$

(3)

Total moisture flow in materials of a multilayer enclosing structure is a sum of liquid and vapor moisture flows:

$$g_{tot} = g_v + g_L.$$  

(4)

where $g_{tot}$ – total moisture flow, $kg/(m^2 \cdot s)$, $g_v$ – vapor moisture flow, $kg/(m^2 \cdot s)$, $g_L$ – liquid moisture flow, $kg/(m^2 \cdot s)$.

Unsteady-state moisture behavior calculation by means of (1) – (3) equations demands computer use, thus its use in construction practice is complicated.

For engineering calculation, it is much more convenient to use Kozlov’s quasistationary method, which appears to be a study of temperature and moisture potential steady-state distribution within every month.

Steady-state temperature distribution takes the following form for temperature pattern of a multilayer enclosing structure:

$$\frac{\partial}{\partial x} \left( \lambda \cdot \frac{\partial t}{\partial x} \right) = 0.$$  

(5)

where $\lambda$ – thermal conductivity coefficient, $W/(m \cdot \text{C})$.

Steady-state moisture potential distribution takes the following form for moisture potential pattern of a multilayer enclosing structure:

$$\frac{\partial}{\partial x} \left( \mu \cdot \frac{\partial F}{\partial x} \right) = 0.$$  

(6)

Equations (5) and (6) are much more convenient for engineering calculation, however this method has much less potential as compared to unsteady-state calculation.

2. The problem

To develop calculation method for unsteady-state moisture behavior of a multilayer enclosing structure, which, on one hand, is easy to use, and, on the other hand, allows unsteady-state processes calculation to a high precision.
3. Materials and methods

Modified differential moisture transfer equation for multilayer enclosing structure is proposed:

\[
\gamma_0 \frac{\xi_{F0}(w,t)}{E_i(t)} \frac{\partial F(w,t)}{\partial \tau} = \frac{\partial}{\partial x} \left( \mu \frac{\partial F(w,t)}{\partial x} \right).
\]  

(7)

where \( \xi_{F0} \) – “material relative potential capacity coefficient”, \( \text{kg} / \text{kg} \).

“Relative potential capacity coefficient” is calculated by formula:

\[
\xi_{F0} = \frac{dw}{d\phi_p}.
\]  

(8)

where \( \phi_p \) – “relative elasticity of moisture potential”.

The term “heat-to-humidity material property coefficient” has been introduced:

\[
\kappa = \frac{\mu}{\gamma_0 \cdot \xi_{F0}}.
\]  

(9)

Product of “heat-to-humidity material property coefficient” by maximum water vapor elasticity is a potential capacity coefficient.

“Relative elasticity of moisture potential” is calculated by formula:

\[
\phi_p = \phi + \frac{1}{\mu} \int_0^\phi \beta(\sigma) d\sigma / E_i.
\]  

(10)

The equation (7) takes the following form for a separate layer:

\[
\frac{\partial F(w,t)}{\partial \tau} = \kappa(w,t) \cdot E_i(t) \frac{\partial^2 F(w,t)}{\partial x^2}.
\]  

(11)

Temperature pattern is a steady-state equation (5) within a month. Temperature pattern effects on equation (7) by means of maximum elasticity of water vapor determined by Clausius-Clapeyron correlation:

\[
E_i = 1.84 \cdot 10^{11} \cdot \exp(-5330 / (273 + t)).
\]  

(12)

As temperature distribution appears to be steady within a month (5), distribution of maximum elasticity of water vapor is also constant within a month.

Average values of relative potential capacity coefficients are calculated by equation (9) for every layer of a multilayer enclosing structure for every month. These coefficients are taken to be constant within an assessment month.

Thus, moisture transfer equation (11) is a differential parabolic equation with constant coefficients.

Third-kind conditions for moisture exchange of surface and ambient air are set for the enclosing structure boundaries.

Moisture exchange boundary condition for outside air and structure surface takes the form:

\[
-\mu_4 \frac{\partial F}{\partial x}
\bigg|_{x=1} = \beta_{ext} \left( F_{ext} - F_i \right).
\]  

(13)

where \( F_{ext} \) – outside air moisture potential, \( \text{Pa} \); \( F_i \) – moisture potential of the enclosing structure section which contacts with outside air, \( \text{Pa} \); \( \beta_{ext} \) – moisture exchange coefficient of outside air and
enclosing structure surface, $kg/(m^2 \cdot s \cdot Pa)$, $\mu_i$ – vapor permeability coefficient of the enclosing structure layer, which is the closest to the outer surface of the building, $kg/(m \cdot s \cdot Pa)$.

Boundary condition of moisture exchange between inside air and structure inner surface takes the form:

$$
\mu_2 \frac{\partial F}{\partial x} \bigg|_{x=N} = \beta_{in} (F_{in} - F_N).
$$

(14)

where $F_{in}$ – inside air moisture potential, Pa; $F_N$ – moisture potential of the enclosing structure surface which contacts with inside air, Pa; $\beta_{in}$ – moisture exchange coefficient of inside air and enclosing structure surface, $kg/(m^2 \cdot s \cdot Pa)$, $\mu_2$ – vapor permeability coefficient of the enclosing structure layer, which is the closest to the inner surface of the enclosing structure, $kg/(m \cdot s \cdot Pa)$.

Boundary condition of flow continuity is set for the joint between enclosing structure layers:

$$
-\mu_1 \frac{\partial F}{\partial x} \bigg|_{x=0} = -\mu_2 \frac{\partial F}{\partial x} \bigg|_{x=0}.
$$

(15)

where $\nu$ – multilayer enclosing structure section with material joint.

Let’s consider solution of the equation (7) with boundary conditions (13), (14), (15) using discrete-continuous method [9, 10]. Let’s carry out standard procedure of spatial-time domain discretization for double-layer enclosing structure [Fig. 1].

![Figure 1. Spatial-time domain discretization for double-layer enclosing structure](image)

In this case initial boundary value problem (7), (13), (14), (15) can be represented by the system of simultaneous equations:
\[
\frac{\partial F_i}{\partial \tau} = \frac{\kappa_1}{h_1} \cdot E_{i,1} \cdot (-1 + \frac{\beta_{\mu_{1}}}{\mu_{1}} \cdot h_1) \cdot F_i + F_2 + \frac{\kappa_1}{h_1} \cdot E_{i,1} \cdot \frac{\beta_{\mu_{1}}}{\mu_{1}} \cdot h_1 \cdot F_{\text{in}}, \quad i = 1
\]
\[
\frac{\partial F_i}{\partial \tau} = \frac{\kappa_2}{h_2} \cdot F_{i-1} \cdot (F_{i-1} - 2 \cdot F_i + F_{i+1}), \quad i = 2, 3, 4, \ldots, v - 1
\]
\[
\frac{\partial F_i}{\partial \tau} = d_{v-1} \cdot F_{v-1} - d_v \cdot F_v + d_{v+1} \cdot F_{v+1}, \quad i = v
\]
\[
\frac{\partial F_i}{\partial \tau} = \frac{\kappa_2}{h_2} \cdot E_{i,1} \cdot (F_{i-1} - 2 \cdot F_i + F_{i+1}), \quad i = v + 1, v + 2, \ldots, N - 1
\]
\[
\frac{\partial F_{N-1}}{\partial \tau} = \frac{\kappa_2}{h_2} \cdot E_{N-1} \cdot (F_{N-1} - (1 + \frac{\beta_{\mu_{2}}}{\mu_{2}} \cdot h_2) F_N) + \frac{\kappa_2}{h_2} \cdot E_{N-1} \cdot \frac{\beta_{\mu_{2}}}{\mu_{2}} \cdot h_2 \cdot F_{\text{in}}, \quad i = N
\]
\[
F(x, 0) = u(x), \quad 0 \leq x \leq l
\]

where \( i \) – enclosing structure section number; \( h_1 \) – first layer partition step (closer to outer surface) of the enclosing structure in coordinate \( x, m \); \( h_2 \) – second layer partition step (closer to inner surface) of the enclosing structure in coordinate \( x, m \); \( \gamma_{01} \) – dry material density in the first layer of an enclosing structure, \( \text{kg} / \text{m}^3 \); \( \gamma_{02} \) – dry material density in the second layer of an enclosing structure, \( \text{kg} / \text{m}^3 \); \( \xi_{F01} \) – relative potential capacity of the first layer material of an enclosing structure, \( \text{kg} / \text{kg} \); \( \xi_{F02} \) – relative potential capacity of the second layer material of an enclosing structure, \( \text{kg} / \text{kg} \); \( \kappa_1 \) – “averaged heat-to-humidity material property coefficient” of the first layer of an enclosing structure, \( \text{m}^2 / (\text{s} \cdot \text{Pa}) \); \( \kappa_2 \) – “averaged heat-to-humidity material property coefficient” of the second layer of an enclosing structure, \( \text{m}^2 / (\text{s} \cdot \text{Pa}) \); \( u \) – initial moisture potential distribution, Pa.

Inside air moisture potential appears to be constant during the whole calculation period, and is equal to partial pressure of inside air water vapor:
\[
F_{\text{in}} = e_{\text{in}}.
\]

where \( e_{\text{in}} \) – partial pressure of inside air water vapor, Pa;

Outside air moisture potential is taken to be equal to partial pressure of outside air water vapor:
\[
F_{\text{ext}} = e_{\text{ext}}.
\]

where \( e_{\text{ext}} \) – partial pressure of outside air water vapor, Pa;

Assume that outside air moisture potential changes continuously within every month in linear fashion:
\[
F_{\text{ext}} = m \cdot \tau + n.
\]

where \( m \) – boundary conditions slope ratio within a month; \( n \) – boundary conditions graph rise within a month, Pa;

Taking into account (20), (21), and (22), simultaneous equations (16) can be represented in matrix form:
\[
\begin{align*}
\mathbf{F}_t &= (G + K \cdot E_1 \cdot A) \cdot \mathbf{F} + p \cdot \tau \cdot \mathbf{L} + \mathbf{B}, \\
\mathbf{F}_0 &= \mathbf{F} \cdot 0, \quad 0 \leq x \leq l.
\end{align*}
\]

where \( p = \kappa_i \cdot E_{ni} \cdot \beta_{ext} \cdot h_i \cdot m / (h_i^2 \cdot \mu_i) \).

\[
\begin{align*}
F_1'(\tau) &= 0 \quad 0 \quad 0 \quad 0, \\
F_2'(\tau) &= 0 \quad 0 \quad 0 \quad 0, \\
\ldots &= 0 \quad 0 \quad 0 \quad 0, \\
F_{N-1}'(\tau) &= 0 \quad 0 \quad \ldots \quad 0, \\
F_N'(\tau) &= 0 \quad 0 \quad 0 \quad 0.
\end{align*}
\]

\[
G = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
K = \begin{bmatrix}
\kappa_i / h_{i}^2 & 0 & 0 & 0 \\
0 & \kappa_i / h_{i}^2 & 0 & 0 \\
0 & 0 & \kappa_i / h_{i}^2 & 0 \\
0 & 0 & 0 & \kappa_i / h_{i}^2
\end{bmatrix}.
\]

\[
E_i = \begin{bmatrix}
E_{ni} & 0 & 0 & 0 \\
E_{ni} & 0 & 0 & 0 \\
\ldots \ldots & \ldots \ldots & \ldots \ldots & \ldots \ldots \\
0 & 0 & E_{(N-1)} & 0 \\
0 & 0 & 0 & E
\end{bmatrix},
\]

\[
A = \begin{bmatrix}
-1 + \beta_{ext} \cdot h_i / \mu_i & 1 & 0 & 0 \\
1 & -2 & 1 & 0 \\
0 & \ldots & \ldots & 0 \\
0 & 0 & 1 & -2 \\
0 & 0 & 0 & 1 - (1 + \beta_{ext} \cdot h_i / \mu_i)
\end{bmatrix}.
\]

\[
\mathbf{F} = \begin{bmatrix}
F_1, F_2, \ldots, F_{N-1}, F_N
\end{bmatrix},
\]

\[
\mathbf{L} = \begin{bmatrix}
1, 0, \ldots, 0
\end{bmatrix},
\]

\[
\mathbf{B} = \begin{bmatrix}
\kappa_i \cdot E_{ni} \cdot \beta_{ext} \cdot h_i \cdot m / (h_i^2 \cdot \mu_i) \\
0 \\
\ldots \\
0 \\
\kappa_i \cdot E_{ni} \cdot \beta_{ext} \cdot h_i \cdot m / (h_i^2 \cdot \mu_i)
\end{bmatrix},
\]

\[
\mathbf{F}_0 = \begin{bmatrix}
F_{01}, F_{02}, \ldots, F_{0(N-1)}, F_{0N}
\end{bmatrix},
\]

\[
\mathbf{\bar{F}} = \begin{bmatrix}
F_{10}, F_{20}, \ldots, F_{(N-1)0}, F_{N0}
\end{bmatrix}.
\]

The equation (23) is a Cauchy problem, which can be solved in matrix form:

\[
\mathbf{F} = p \cdot \left( (G + K \cdot E_1 \cdot A)^{-1} \cdot e^{(G + K \cdot E_1 \cdot A) \cdot \tau} - (G + K \cdot E_1 \cdot A)^{-1} - \cdot (G + K \cdot E_1 \cdot A)^{-1} \right) \cdot \mathbf{L} + \\
+ (G + K \cdot E_1 \cdot A)^{-1} \cdot e^{(G + K \cdot E_1 \cdot A) \cdot \tau} - \mathbf{B} + e^{(G + K \cdot E_1 \cdot A) \cdot \tau} \cdot \mathbf{F}_0.
\]

where \( E \) – unit matrix.

4. Results and discussion

Multilayer enclosing structure consisting of subbase made of clay brick 0.38 m thick and lining made of lime brick 0.12 m has been examined. Construction area is Moscow. Inside air temperature 22 °C and relative inside air humidity 60% have been maintained constant in the building. Calculation results of the given multilayer enclosing structure moisture behavior obtained using different moisture potential theory methods have been compared. The following moisture potential theory methods have been used: the proposed discrete-continuous method, Gagarin’s unsteady-state method, and Kozlov’s engineering method.

An analytical formula (6) has been used for calculation of moisture potential distribution along multilayer enclosing structure thickness according to Kozlov’s engineering method. Numerical finite difference method using explicit difference scheme has been used for calculation of moisture potential distribution along multilayer enclosing structure thickness according to Gagarin’s unsteady-state method.

Kozlov’s experimental data [8] on vapor permeability coefficient, static moisture conductivity and sorption coefficient has been used for the proposed calculation method by equation (24). This data is
the basis for moisture potential scales [Fig. 2, Fig. 3] for all materials of a multilayer enclosing structure by expression (1).

![Figure 2. Moisture potential scale of lime brick](image_url)

(1 – -10 °C temperature; 2 – -5 °C temperature; 3 – 0 °C temperature; 4 – 5 °C temperature; 5 – 10 °C temperature; 6 – 15 °C temperature; 7 – 20 °C temperature)

![Figure 3. Moisture potential scale of clay brick](image_url)

(1 – -10 °C temperature; 2 – -5 °C temperature; 3 – 0 °C temperature; 4 – 5 °C temperature; 5 – 10 °C temperature; 6 – 15 °C temperature; 7 – 20 °C temperature)

The proposed material moisture dependence on “relative elasticity of moisture potential” has been shown on the basis of equation (10) for all layers of a multilayer enclosing structure [Fig. 4, Fig. 5].
Moisture potential distribution at month end has been calculated by formula (24). Material moisture has been determined by obtained moisture potential distribution according to moisture potential scales [Fig. 2, Fig. 3]. Moisture distribution at month start has been taken as equal to moisture distribution at the end of the previous month. Comparison of calculation results of moisture potential distribution along multilayer enclosing structure thickness obtained by means of different moisture potential theory methods for the coldest month – January – is given in [Fig. 6]. Comparison of calculation results of moisture distribution along multilayer enclosing structure thickness obtained by means of different moisture potential theory methods for the coldest month – January – is given in [Fig. 7].
Figure 6. Comparison of moisture potential theory methods for moisture potential distribution along multi-layer enclosing structure thickness in January

(1 – maximum water vapor pressure, 2 – moisture potential distribution in enclosing structure according to Gagarin’s unsteady-state method, 3 – moisture potential distribution in enclosing structure according to discrete-continuous method, 4 – moisture potential distribution in enclosing structure according to Kozlov’s engineering method, 5 – border between layers)

Figure 7. Comparison of moisture potential theory methods for moisture regime calculation of single-layer enclosing structure in January

(1 – moisture distribution in enclosing structure according to Gagarin’s unsteady-state method, 2 – moisture distribution in enclosing structure according to discrete-continuous method, 3 – moisture distribution in enclosing structure according to Kozlov’s engineering method, 4 – border between layers)

The graphs show that moisture distribution in multilayer enclosing structure obtained by means of discrete-continuous method gives quantitative and qualitative results similar to moisture distribution
results, obtained by means of Gagarin’s unsteady-state method. However, this distribution is calculated from analytical expression, thus, the calculation is easier.

5. Conclusion

Unsteady-state moisture behavior determination method using discrete-continuous approach has been developed for multilayer enclosing structures. It has been shown that moisture behavior calculation results obtained by the proposed method are close to results obtained by existing Gagarin’s unsteady-state method. Moisture behavior is finally calculated from equation (24) without numerical method application. Due to this fact, the proposed method is convenient for design engineers in practice.

The proposed method can be used for calculation of multilayer enclosing structures consisting of two layers and more.

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