TMDs in the bag model*

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Abstract. Leading and subleading twist transverse momentum dependent parton distribution functions (TMDs) are studied in a quark model framework provided by the bag model. A complete set of relations among different TMDs is derived, and the question is discussed how model-(in)dependent such relations are. A connection of the pretzelosity distribution and quark orbital angular momentum is derived.

1 Introduction

TMDs are a generalization of parton distribution functions (PDFs) promising to extend our knowledge of the nucleon structure far beyond what we have learned from PDFs about the longitudinal momentum distributions of partons in the nucleon. In addition to the latter, TMDs carry also information on transverse parton momenta and spin-orbit correlations. TMDs enter the description of leading-twist observables in deeply inelastic reactions on which data are available like: semi-inclusive deep-inelastic scattering (SIDIS), Drell-Yan process or hadron production in $e^+e^-$ annihilations. The interpretation of these data is not straight-forward. In SIDIS one deals with convolutions of a priori unknown transverse momentum distributions in nucleon and fragmentation process, and in practice is forced to assume models such as the Gaussian Ansatz. In the case of subleading twist observables, one moreover faces the problem that several twist-3 TMDs and fragmentation functions enter the description of one observable (we recall that presently factorization is not proven for subleading-twist observables).

In this situation information from models is valuable for several reasons. Models can be used for direct estimates of observables. An equally interesting aspect concerns relations among different TMDs observed in models. Such relations, especially when supported by several models, could be helpful — at least for qualitative interpretations of first data. Model results also allow to test assumptions made in literature, such as the Gaussian Ansatz for transverse momentum distributions or certain approximations. In addition to that model studies are of interest also because they provide important insights into non-perturbative properties of TMDs.

The purpose of this talk is to review the main results for relations among TMDs derived in the MIT bag model. In this model, quark-quark correlation functions in the nucleon [1] can be expressed in terms of a quark wave-functions, which have an $S$-wave

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component proportional to the function $t_0(k)$ and a $P$-wave component proportional to $t_1(k)$, where $k = |k|$ is the quark momentum. The presence of these components is a minimal requirement for the modelling of T-even TMDs. Since there are no explicit gluon degrees of freedom, T-odd TMDs vanish in this model. We restrict ourselves to T-even TMDs. More details and references can be found in [2].

2 TMDs in the bag model

We assume $SU(6)$ spin-flavor symmetry of the proton wave function. In $SU(6)$ spin-independent TMDs of definite flavor are given in terms of ‘flavor-less’ expressions multiplied by a ‘flavor factor’ $N_q$ with $N_u = 2$, $N_d = 1$. Spin-dependent TMDs of definite flavor follow from multiplying ‘flavor-less’ expressions by a ‘spin-flavor factor’ $P_q$ with $P_u = \frac{2}{3}$, $P_d = -\frac{1}{3}$.

Defining $k = k/k$ and $\hat{M}_M = M_N/k$, the results for T-even leading twist TMDs read

$$f^q_t(x, k_\perp) = N_q A [t_0^q + 2\hat{k}_z t_0 t_1 + t_1^2],$$

$$g^q_t(x, k_\perp) = P_q A [t_0^q + 2\hat{k}_z t_0 t_1 + (2\hat{k}_z^2 - 1) t_1^2],$$

$$h^q_1(x, k_\perp) = P_q A [t_0^q + \hat{k}_z t_0 t_1 + \hat{k}_z^2 t_1^2],$$

$$g^q_{1T}(x, k_\perp) = P_q A [2\hat{M}_N(t_0 t_1 + \hat{k}_z t_1^2)],$$

$$h^q_{1T}(x, k_\perp) = P_q A [-2\hat{M}_N t_0 t_1 + \hat{k}_z t_1^2],$$

and for the subleading twist TMDs we obtain

$$e^q(x, k_\perp) = N_q A [t_0^q - t_1^2],$$

$$f^{\perp q}(x, k_\perp) = N_q A [2\hat{M}_N t_0 t_1],$$

$$g^{\perp q}_L(x, k_\perp) = P_q A [t_0^q - \hat{k}_z^2 t_1^2],$$

$$g^{\perp q}_{1T}(x, k_\perp) = P_q A [2\hat{M}_N \hat{k}_z t_1^2],$$

$$h^{\perp q}_1(x, k_\perp) = P_q A [t_0^q + (1 - 2\hat{k}_z^2) t_1^2],$$

$$h^{\perp q}_{1T}(x, k_\perp) = P_q A [-2\hat{M}_N \hat{k}_z t_1^2],$$

where $A$ is a common normalization factor [2].

3 Relations among TMDs in bag model

In the bag model, there are 9 linear relations among the 14 (twist-2 and 3) T-even TMDs, which can be written as follows (where $j^{(1)q}(x, k_\perp) = \frac{k^2}{2M_N} f^q(x, k_\perp)$ and the ‘dilution factor’ $D^q = \frac{P_q}{M_N}$),

$$D^q f^q_t(x, k_\perp) + 2g^q_t(x, k_\perp) = 2h^q_1(x, k_\perp),$$

$$D^q e^q(x, k_\perp) + h^q_1(x, k_\perp) = 2g^q_t(x, k_\perp),$$

$$D^q f^{\perp q}(x, k_\perp) = h^{\perp q}_{1T}(x, k_\perp),$$

$$g^{\perp q}_{1T}(x, k_\perp) = -h^{\perp q}_{1T}(x, k_\perp),$$

$$g^{\perp q}_L(x, k_\perp) = -h^{\perp q}_1(x, k_\perp),$$

$$g^{\perp q}_{1T}(x, k_\perp) = -h^{\perp q}_{1T}(x, k_\perp),$$

and the relations $f^{0q}(x, k_\perp) = \frac{1}{2} f^q_t(x, k_\perp) + \frac{1}{2} f^{\perp q}(x, k_\perp), g^{0q}_{1T}(x, k_\perp) = \frac{1}{2} g^{\perp q}_{1T}(x, k_\perp) + \frac{1}{2} g^{\perp q}_L(x, k_\perp), g^{0q}_L(x, k_\perp) = \frac{1}{2} g^{\perp q}_L(x, k_\perp) + \frac{1}{2} g^{\perp q}_1(x, k_\perp)$ are to be considered as linearly independent. This implies 9 linear equations among the 14 TMDs.
However, there are also non-linear relations, for example,

\[ h_1^q(x, k_\perp) h_{1T}^\perp(x, k_\perp) = - \frac{1}{2} \left[ h_{1L}^q(x, k_\perp) \right]^2, \quad (10) \]

\[ g_T^q(x, k_\perp) g_T^\perp(x, k_\perp) = \frac{1}{2} \left[ g_{1T}^q(x, k_\perp) \right]^2 - g_T^q(x, k_\perp) g_T^\perp(x, k_\perp). \quad (11) \]

The Eqs. (10, 11) are independent in the sense that it is impossible to convert one into the other upon use of the linear relations (1-9). With the 9 linear relations (1-9) and the 2 non-linear relations (10, 11) we find altogether 11 relations among 14 TMDs in the bag model.

The deeper reason, why in the bag model relations among TMDs appear, is ultimately related to Melosh rotations which connect longitudinal and transverse nucleon and quark polarization states in a Lorentz-invariant way [3]. This was elucidated in detail in Ref. [4].

3.1 Comparison to other quark models

An important issue, when observing relations among TMDs in a model, concerns their presumed validity beyond that particular model framework. For that it is instructive to compare to other models.

- Eq. (1): its \( k_\perp \)-integrated version was discussed in bag model in [5] and [6, 7] and in light-cone constituent models in [5]. The unintegrated version was discussed in bag and light-cone constituent models [9,10].

- Eq. (2): its integrated version was observed in the bag model previously in [6].

- Eq. (4): was first observed in the spectator model of [11] and later also in light-cone constituent models [9] and the covariant parton model of Ref. [12].

- Eq. (6): was found in the spectator model of Ref. [11].

- Eq. (7): was first observed in the bag [10]. It is valid also in the spectator [11], light-cone constituent [9], and covariant parton [12] models.

- Eqs. (3, 5, 8, 9): are new in the sense of not having been mentioned previously in literature. But the latter 3 are satisfied by the spectator model results from [11].

- The non-linear relation (10) connecting all T-even, chiral-odd leading-twist TMDs was found in the covariant parton model approach [12]. Eq. (11) was not discussed prior to [2].

The detailed comparison, in which models these relations hold and in which they are violated, gives some insight into the question to which extent these relations are model-dependent.

Let us discuss first Eqs. (1-3), which connect polarized and unpolarized TMDs. For these relations SU(6)-spin-flavor symmetry is necessary, but not sufficient. For example, the spectator model of [11] is SU(6) symmetric. But it does not support (1-3) which are spoiled by the different masses of the (scalar and axial-vector) spectator diquark systems. Also (1) are not supported in the covariant parton model approach of [12]. However, in that approach it is possible to ‘restore’ these relations by introducing additional, restrictive assumptions, see [12] for a detailed discussion. We conclude that the relations (1-3) require strong model assumptions. It is difficult to estimate to which extent such relations could be useful approximations in nature, though they could hold in the valence-x region with an accuracy of (20–30)\% (see [13]).

From the point of view of model dependence, it is ‘safer’ [10] to compare relations which include only polarized TMDs such as Eqs. (4, 9). (Or, only unpolarized TMDs,
It is gratifying to observe that these relations are satisfied not only in the bag model, but also in the spectator model version of Ref. [11]. The relations among the leading twist TMDs, Eqs. (4, 7), hold also in light-cone constituent [9] and covariant parton [12] models.

Of course, quark model relations among TMDs have limitations, even in quark models. In [14] various versions of spectator models were used, and in some versions the relations were not supported (4, 7). Also the quark-target model [15] does not support (4, 7). Finally, in QCD none of such relations is valid, and all TMDs are independent structures. It would be interesting to ‘test’ the quark model relations in other models, lattice QCD, and in experiment.

3.2 A relation among PDFs

It is worth to discuss in some more detail one particularly interesting relation, which includes only functions known from the collinear case. By eliminating the transverse moment of the pretzelosity distribution from Eqs. (7, 8), and integrating over transverse momenta, we obtain (this relation holds in its unintegrated form)

\[ g_q^T(x) - h_q^L(x) = g_q^T(x) - h_q^L(x). \] (12)

There are several reasons, why this relation is interesting.

First, it involves only collinear PDFs, which is the only relation of such type in bag model. The QCD evolution equation for all these functions are different, which shows the limitation of this relation: even if for some reason (12) was valid in QCD at a certain renormalization scale \( \mu_0 \), it would break down at any other scale \( \mu \neq \mu_0 \).

Second, for the first Mellin moment this relation is valid model-independently presuming the validity of the Burkardt-Cottingham sum rule and an analog sum rule for \( h_q^L(x) \) and \( h_q^T(x) \). In QCD there are doubts especially concerning the validity of the Burkardt-Cottingham sum rule. However, it is valid in many models such as bag [5] or chiral quark soliton model [16].

Third, it would be interesting to learn whether (12) is satisfied in nature approximately. Also this relation can be tested on the lattice, especially for low Mellin moments and in the flavour non-singlet case. Lattice QCD calculations for Mellin moments of \( g_q^T(x) \) were reported in [17].

Forth, the relation (12) can be tested in models where collinear PDFs were studied. Some results can be found in literature. For example, calculations in the bag [5,6] and spectator [11] model support this relation. Also one counter-example is known: the chiral quark-soliton model does not support this relation [16,18]. The models supporting (12) have only the components in the nucleon wave-function with the quark orbital angular momenta up to \( L = 0, 1, 2 \). The chiral quark soliton model, which does not support (12), contains all quark angular momenta.

Fifth, an important aspect of model relations is that they inspire interpretations. The relation (12) means that the difference between \( g_q^T \) and \( h_q^L \) is to the same extent a ‘measure of relativistic effects in the nucleon’ as the difference between helicity and transversity. Both these differences are related to the transverse moment of pretzelosity, see Eqs. (7, 8) and (10).

4 Pretzelosity and quark orbital angular momentum

In quark models, in contrast to gauge theories, one may unambiguously define the quark orbital angular momentum operator as \( \hat{L}_q = \hat{\psi}_q e^{ikx^j} \hat{p}^j \hat{\psi}_q \) (for clarity the
'hat' indicates a quantum operator. In the absence of gauge fields this definition follows uniquely from identifying that part of the generator of rotations not associated with the intrinsic quark spin. We introduce a ‘non-local version’ of this operator $\hat{L}_i^q(0,z) = \bar{\psi}_q(0)\varepsilon^{ikl}\hat{r}^k\hat{p}^l\psi_q(z)$ with $\hat{r}^k = i\frac{\partial}{\partial p^k}$ and $\hat{p}^l = p^l$ in momentum space. Next let us define the quantity

$$L_i^q(x,p_T) = \int \frac{dz^+ d^2z_T}{(2\pi)^3} e^{ipz} \langle N(P,S^3)|\bar{\psi}_q(0)\varepsilon^{ikl}\hat{r}^k\hat{p}^l\psi_q(z)|N(P,S^3)\rangle \bigg|_{z^+=0, p^+=xP, p^+ = xP}.$$  

(13)

We consider a longitudinally polarized nucleon with the polarization vector $S = (0, 0, 1)$, and focus on the $j = 3$ component in (13). Evaluating (13) in the bag model we obtain as in [19, 20]

$$L_3^q(x, p_T) = (-1)h^{(1)}_{1T}^{(1)}q(x, p_T).$$  

(14)

Thus, $L_3^q = \int dx \int d^2p_T L_3^q(x, p_T) = (-1) \int dx h^{(1)}_{1T}^{(1)}q(x)$ is the contribution to the nucleon spin from the quark orbital angular momentum of the flavour $q$. Adding up the contribution of the intrinsic quark spin, $S_3^q = \frac{1}{2} \int dx g_{1T}^q(x)$, we obtain $2J_3^q = 2S_3^q + 2L_3^q = P_q$ which is the consistent result for the contribution of flavor $q$ the nucleon spin in SU(6). We stress that the relation of pretzelosity and orbital angular momentum, Eq. (14), is at the level of matrix elements of operators, and there is no a priori operator identity which would make such a connection.

5 Conclusions

We presented a study of a complete set relations among T-even twist-2 and twist-3 TMDs in the MIT bag model, and discussed to what extent these relations are supported in other quark models. Special attention was paid to the relation of the difference of $g_q^1$ and $h_q^1$ to the (1)-moment of pretzelosity, and the relation of the latter to quark orbital angular momentum. It is interesting to ask, whether a quark model relation of the type (14) may inspire a way to establish a rigorous connection between TMDs and OAM in QCD? We hope our results will stimulate further studies in quark models.

Sorrry space limitations did not allow us to address other interesting questions like Lorentz invariance relations [21], inequalities [22], Wandzura-Wilczek-type approximations [23], and numerical results which support the Gauss Ansatz [24]. All these issues can be found in Ref. [2].

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