Abstract

In supersymmetry, there are gauge invariant dimension 5 proton decay operators which must be suppressed by a mass scale much larger than the Planck mass. It is natural to expect that this suppression should be explained by a mechanism that explains the hierarchical structure of the fermion mass matrices. We apply this argument to the case where wave functions of chiral multiplets are localized under a kink background along an extra spatial dimension and the Yukawa couplings as well as the coefficients of the proton decay operators are determined by the overlap of the relevant wave functions. A configuration is found in the context of SU(5) supersymmetric grand unified theory that yields realistic values of quark masses, mixing angles, $CP$ phase and charged lepton masses and sufficiently small genuine dimension 5 proton decay operators. Inclusion of SU(5) breaking effects is essential in order to obtain non-vanishing $CP$ phase as well as correct lepton masses. The resulting mass matrix has a texture structure in which texture zeros are a consequence of extremely small overlap of the wave functions. Our approach requires explicit breaking of supersymmetry in the extra dimension, which can be realized in (de)constructing extra dimension.
I. INTRODUCTION

One of the puzzles of modern particle physics is the origin of the structure of masses and mixing in quarks and leptons. There have been many approaches to explain the Yukawa couplings mostly based on some flavor symmetry.

In the supersymmetric (SUSY) extension of the Standard Model, besides the renormalizable Yukawa interaction, one can write non-renormalizable dimension 5 operators in the superpotential, which violate baryon and lepton numbers. After dressed by superparticles, these operators induce nucleon decay [1]. The present experimental limit on the nucleon lifetime gives very stringent bounds on these operators, whose coefficients must be much smaller than the inverse of the Planck scale for weak-scale SUSY breaking. It is then natural to expect that the mechanism which explains the fermion mass structure will also explain the smallness of the genuine dimension 5 proton decay.

Some time ago, Arkani-Hamed and Schmaltz [2] proposed a new approach to these issues based on a fermion localization mechanism in extra spatial dimensions. It is known that chiral fermions are localized in solitonic backgrounds [4]. In Ref. [2], the quarks and leptons have Gaussian wave functions along one extra dimension under a kink background, and the overlap of the wave functions of the two quarks (leptons) determines the relevant Yukawa coupling. The small Yukawa couplings are then attributed to the small overlap of the wave functions. Realistic configurations were given in Ref. [5]. In the same way, the proton decay will be suppressed if the overlap of the wave functions of three quarks and one lepton is small.

The purpose of this paper is to investigate whether this idea indeed works in the context of supersymmetric grand unified theories (GUTs). Issues of the fermion masses in SUSY GUTs using the idea of the fermion localization were studied in Ref. [9]. In addition to the quark and lepton chiral multiplets, the Higgs supermultiplets are also assumed to have the Gaussian wave functions along the extra dimension and realistic masses and mixing angles of quarks are obtained. Here we extend their approach to include the proton decay operators in the analysis. We will show that one has to split the wave functions of the quarks in the same SU(5) multiplet in order to obtain sufficient $CP$ phase. The splitting is realized by the mechanism proposed in Ref. [10] (see also [11]). Note that the same mechanism can realize the triplet-doublet Higgs mass splitting and the suppression of the proton decay mediated by the triplet higgsino exchange.

We shall obtain a configuration of the chiral multiplets in the minimal supersymmetric standard model (MSSM), which is consistent with the SU(5) GUT and explains the quark masses, the mixing angles and the $CP$ phase, the charged lepton masses, and the suppression of the genuine dimension 5 proton decay. The resulting mass matrices of the quarks and leptons have a variant of Fritzsch-type texture. In our approach, texture zeroes are attributed to very small overlap of the wave functions.

\[\text{\textsuperscript{1}A similar proposal but with exponential Higgs configuration was given by Ref. [3].}\]

\[\text{\textsuperscript{2}Issues of the fermion localization and the fermion masses were discussed in the warped extra dimension in Refs. [6–8].}\]
The organization of the paper is as follows. In section 2, we review the mechanism to localize the wave functions along the fifth dimension. In section 3, we explain the basic ideas of our approaches in the simple case where the wave functions have SU(5) invariant form. It turns out that CP violation is too small in this case. Then in section 4, we split the wave functions to obtain sufficiently large CP phase. Here we need to contrive to suppress dimension 5 proton decay operators. A workable configuration of the wave functions is given there. In section 5, we argue various energy scales of our model and discuss some subtle issues in this approach. The final section is devoted to conclusions and discussion.

II. A BRIEF REVIEW OF LOCALIZATION MECHANISM

We begin with a brief review of the localization mechanism of fields in the fifth dimension, which is based on the formalism of [9,12]. Throughout this paper, we consider 5D SU(5) SUSY GUT with the fifth dimension y compactified. 5D Planck scale is denoted by $M_\star$ and the compactification scale by $M_c$.

We shall assume that 5D super-Poincare invariance is broken in some way and only $N = 1$ supersymmetry (in 4D sense) is respected. This allows us to write Yukawa interactions in the bulk in the form of superpotential, as we will see shortly. An appropriate formalism will be provided by utilizing the idea of (de)constructing extra dimensions [13] which does not necessarily have 5D $N = 1$ SUSY in the presence of matter interactions [14,15].

Let us thus consider the Lagrangian

$$\mathcal{L} = \int dy \left\{ \int d^4 \theta \left( \Phi(y)\Phi(y) + \Phi^C(y)\Phi^C(y) \right) + \int d^2 \theta \Phi^C(y) [\partial_y + M(y)] \Phi(y) + h.c. \right\}$$

of 5D spacetime. Here, $\Phi$ is a 4D $N = 1$ SUSY chiral superfield and $\Phi^C$ is a charge conjugated chiral superfield, both of which combine to produce a 5D $N = 1$ SUSY hypermultiplet. $M(y)$ is decomposed as

$$M(y) = \Xi(y) + M,$$

where $\Xi(y)$ is the vacuum expectation value (vev) of a scalar component of a background chiral superfield and $M$ is a mass parameter. As we mentioned above, the interactions in the superpotential (1) break 5D $N = 1$ SUSY (or $N = 2$ in 4D SUSY).

Equations of motion for zero modes of scalar component $\phi$ and spinor one $\psi$ of $\Phi$ are obtained as

$$\left( \partial_y + M(y) \right) \phi(y) = 0, \quad \left( \partial_y + M(y) \right) \psi(y) = 0,$$

and for ones of $\Phi^C(y)$ as

$$\left( \partial_y - M(y) \right) \phi^C(y) = 0, \quad \left( \partial_y - M(y) \right) \psi^C(y) = 0.$$

We assume that $\Xi$ has a kink configuration along the fifth dimension and approximate it as

$$\Xi(y) = 2\mu^2 y, \quad \mu^2 > 0$$

near the origin. Thus, by appropriate boundary conditions we obtain the zero mode wave functions with Gaussian profile as
\[ \phi(y) = \psi(y) = \left( \frac{2\mu^2}{\pi} \right)^{1/4} \exp \left[ -\mu^2 (y - l)^2 \right], \]

localized around \( l \equiv -M/2\mu^2 \).

If the extra dimension were non-compact, an anti-chiral zero mode would not exist under the kink background because its wave function would not be normalizable. However in the compactified extra dimension, the wave function will generically become normalizable, and one has to elaborate to forbid the anti-chiral zero mode. Here zero modes for the anti-chiral fields, \( \phi(y)^C \) and \( \psi(y)^C \), are assumed to be absent. A possibility is that the anti-chiral zero modes will be projected out by an orbifold boundary condition. Another possibility to realize such a situation is to consider a \( y \) dependent \( Z \) (wave-function) factor in the Lagrangian (1), where the \( Z \) factor vanishes at the boundaries of the extra dimension. This makes the anti-chiral zero mode having non-normalizable wave function, and thus it does not survive. A detail on this realization will be given in Appendix A.

There are also massive modes called Kaluza-Klein (KK) modes on top of these zero modes (massless modes). For the time being, we will focus on the zero modes with Gaussian profile (6).

### III. SU(5) INVARIANT CONFIGURATION

In this and the next sections we try to produce the hierarchy of the fermion masses and very small proton decay operator coefficients, using the localization mechanism described above. The important point is that smallness of these constants is a consequence of small overlaps of wave functions. The main purpose in the two sections is to illustrate our procedure to determine localization points of fields to reproduce fermion mass matrices and survive proton decay constraints. We will obtain an almost realistic model except that it has no \( CP \) violating phase in the mass matrix. This model nicely illustrates our procedure because the structure is rather simple and also it suggests a hint how we should extend it to obtain non-vanishing \( CP \) violation. We will give in section 4 a more realistic model with \( CP \) violation.

First, we consider the case that the MSSM fields which belong to the same multiplet in SU(5) have the same wave functions localized at the same point along the extra dimension. We assign the MSSM matter fields to the SU(5) multiplets as usual:

\[
\Psi(10) = \frac{1}{\sqrt{2}} \begin{pmatrix} U^C & Q \\ -Q & E^C \end{pmatrix}, \\
\Phi(5^*) = (D^C, L),
\]

(7)

where \( Q \) is the left-handed quark doublet, \( U^C \) and \( D^C \) are the charge conjugated right-handed up- and down-type quarks, \( L \) is the left-handed lepton doublet, and \( E^C \) is the charge conjugated right-handed charged lepton. In the minimal case, Higgs multiplets are

\[
H(5) = (H_T, H_u) \\
\bar{H}(5^*) = (H_T, H_d),
\]

(8)
where $H_u$ and $H_d$ are the MSSM Higgses and $H_T$ and $H_T$ are their color triplet partners. Thus, the 5D Yukawa couplings which leads to fermion masses are
\[
\mathcal{L} = \int dy d^2 \theta \left\{ \frac{1}{4} f_{Uij}^2 \Psi_i \Psi_j H + \sqrt{2} f_{Dij}^2 \Psi_i \Phi_j \bar{H} \right\},
\]
where $f_{U,D}^ij$ are dimensionless coefficients and $i, j$ are family indices. In non-minimal cases, Yukawa couplings take other form. Any way, we assume that after SU(5) breaking these couplings lead to
\[
\int dy d^2 \theta \left\{ f_{ij}^U \sqrt{M_*} Q_i U^C U_j H_u + f_{ij}^D \sqrt{M_*} Q_i D^C D_j H_d + f_{ij}^L \sqrt{M_*} E_i L^C L_j H_d \right\}.
\]
For the moment, we concentrate on the quark sectors and discard the wrong relations between down-type quark masses and charged lepton ones.

Substituting zero mode wave functions of the form (6) into eq.(10) and integrating over the extra dimension, we obtain 4D Yukawa couplings. Here we denote Gaussian widths of the matters (quarks and leptons), the up-type Higgs and the down-type Higgs by $\mu_u, \mu_d, \mu_H_u$ and $\mu_H_d$, respectively, and define their ratios as $r_u = \mu_{H_u}/\mu, r_d = \mu_{H_d}/\mu$. We assume that all matters have the same width for simplicity. Relaxing this will not drastically change our main conclusions. Without loss of generality, we set the location of $H_u$ at the origin. Resulting 4D up-type Yukawa coupling constants are
\[
y_{ij}^U = F_{ij}^U \sqrt{r_u} \frac{2^{3/4}}{2 + r_u^{3/4}} \exp \left\{ -\frac{\mu^2}{2 + r_u^2} \left[ (1 + r_u^2)(l_i^2 + l_j^2) - 2l_i l_j \right] \right\}, \quad F_{ij}^U \equiv f_{ij}^U \sqrt{\frac{\mu}{M_*}}. \]
Exponentially small coupling constants are a result of small overlap of the wave functions, which can be generated by at most $O(10)$ distance among fields relative to typical magnitude of width.

Similarly, down-type Yukawa coupling constants are written as
\[
y_{ij}^D = F_{ij}^D \sqrt{r_d} \frac{2^{3/4}}{2 + r_d^{3/4}} \exp \left\{ -\frac{\mu^2}{2 + r_d^2} \left[ (1 + r_d^2)(\tilde{l}_i^2 + \tilde{l}_j^2) - 2\tilde{l}_i \tilde{l}_j \right] \right\}, \quad F_{ij}^D \equiv f_{ij}^D \sqrt{\frac{\mu}{M_*}}. \]
where $\tilde{l}_i \equiv l_i - l_{H_d}, \tilde{k}_i \equiv k_i - l_{H_d}$, and $k_i$ represents the location of $\Phi(5^*)_i$.

To obtain physical masses and mixings, we must translate fields from flavor basis into mass basis through unitary matrices as
\[
U^T_Q y_U U_U = \hat{y}_U \equiv \text{diag}(y_u, y_c, y_t)
\]
\[
U^T_Q y_D U_D = \hat{y}_D \equiv \text{diag}(y_d, y_s, y_b)
\]
\[
U^T_L y_L U_L = \hat{y}_L \equiv \text{diag}(y_e, y_\mu, y_\tau),
\]
from which we obtain the CKM matrix as
\[
V_{KM} = U^T_Q U_Q.
\]
A. Up-type Masses

Next, we determine the locations of the fields using experimental data. 5D Yukawa coupling constants $f^{ij}$ run from $M_s$ to $M_c$, and then are matched to 4D Yukawa coupling constants $y^{ij}$, and run again to low energy scale. However we ignore the running between $M_s$ and $M_c$, and rescale Yukawa coupling constants by using one-loop MSSM renormalization group equations (RGE) from the usual GUT scale $M_{GUT} = 2 \times 10^{16}$ GeV to the EW scale, by identifying that $M_s = M_{GUT}$. This simplification is justified when we consider uncertainties of $f^{ij}_U, f^{ij}_D$ in the 5D Lagrangian.

We adopt as experimental bounds the running quark masses evaluated at the $Z$-boson mass scale [16,17]

$$
m_u = 2.33^{+0.42}_{-0.45} \text{ MeV}, \quad m_c = 677^{+56}_{-61} \text{ MeV}, \quad m_t = 175 \pm 6 \text{ GeV}$$

$$
m_d = 4.69^{+0.60}_{-0.66} \text{ MeV}, \quad m_s = 93.4^{+11.8}_{-13.0} \text{ MeV}, \quad m_b = 3.00 \pm 0.11 \text{ GeV}$$

$$
m_e = 0.487 \text{ MeV}, \quad m_\mu = 102.7 \text{ MeV}, \quad m_\tau = 1.747 \text{ GeV}, \quad (15)$$

the magnitude of the CKM mixing matrix [17]

$$
\begin{pmatrix}
0.9742 \sim 0.9757 & 0.219 \sim 0.226 & 0.002 \sim 0.005 \\
0.219 \sim 0.225 & 0.9734 \sim 0.9749 & 0.037 \sim 0.043 \\
0.004 \sim 0.014 & 0.035 \sim 0.043 & 0.9990 \sim 0.9993
\end{pmatrix}, \quad (16)
$$

and the Jarlskog invariant [18]

$$
J \equiv V_{us}V_{cb}V_{ub}^*V_{cs} = (1.8 \sim 3.1) \times 10^{-5}, \quad (17)
$$

which represents $CP$ violation.

First we seek the locations of $\Psi_i$ from the observed up-type quark masses. Since diagonal parts of $y_U$ contain the same field, we expect that $y_U$ is nearly diagonal. Therefore in this case we can approximate locations of $\Psi_i$ as

$$
|\mu_{l_i}| \sim \sqrt{\frac{2 + r_u^2}{2r_u^2}} \log \frac{F^{ii}_{U}v \sin \beta R_u}{\sqrt{2m_{U_i}}}, \quad R_u \equiv \sqrt{\frac{r_u}{2 + r_u^2}} \frac{2^{3/4}}{\pi^{1/4}}, \quad (18)
$$

where $v \sim 246 \text{ GeV}$ is the vacuum expectation value (vev) of the standard model Higgs and $\beta$ parameterizes the ratio of the MSSM Higgses' vevs as $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$. Since $\sin \beta \sim 1$ for $\tan \beta \gtrsim 2$, $|\mu_{l_i}|$ are sensitive to $r_u$.

There are two types of $\Psi_i$'s configuration which can lead to realistic up-type quark masses. One is (i) $\mu_3 \sim 0 < \mu_2 < \mu_1$ type as in [9], and the other is (ii) $\mu_1 < \mu_3 \sim 0 < \mu_2$ type, up to sign. Recall that the wave function of $H_u$ is localized at the origin, and thus the order one top Yukawa coupling requires $\mu_3 \sim 0$.

For example, in the case that $F^{ii}_{U} = 1, \tan \beta = 20$ and $r_u = 0.5$, we obtain for the case (i), $\mu_1 = 7.30, \mu_2 = 5.10, \mu_3 = 0.00,

$$
m_u = 0.74 \text{ MeV}, \quad m_c = 321 \text{ MeV}, \quad m_t = 104 \text{ GeV}, \quad (19)
$$

and for the case (ii), $\mu_1 = -7.20, \mu_2 = 5.20, \mu_3 = 0.60,$
Here we have used \( U_0^u = U_U \sim 1 \) and the running quark masses are those at energy scale \( M_{\text{GUT}} \). After RGE running to lower energy, these values can reside in the range given in eq.(15).

**B. Constraints From Proton Decay**

Although both of the two cases, (i) and (ii), succeed in obtaining up-type quark masses, they must also explain the null result of proton decay experiments. We impose \( R \)-parity to forbid dimension 4 proton decay throughout this paper, and concentrate on genuine dimension 5 proton decay operators suppressed by 5D Planck scale. Ordinary dimension 5 operators which arise from triplet Higgs exchange can be adequately suppressed by the mechanism described in [10]. Namely using the splitting of the wave functions of the doublet and the triplet, we can realize the configuration that the triplet Higgs fields are located far away from the quarks and leptons, suppressing the Yukawa couplings of the triplet to them and thus the proton decay through the triplet Higgs exchange.

What we are concerned with are the genuine dimension 5 proton decay operators which are induced from the following interactions:

\[
\mathcal{L}_5 = \int dy \, d^2 \theta \frac{\sqrt{2}}{4} \frac{d^{ijkl}}{M^*_s} (\Psi_i \Psi_j) (\Psi_k \Phi_l),
\]

where the brackets are contracted to the fundamental and the anti-fundamental representations, and \( d^{ijkl} \) are constants which satisfy \( d^{ijkl} = d^{jikl} \). Since the gauge symmetry as well as the \( R \)-parity conservation does not forbid this type of interactions, we expect that they have unsuppressed coefficients \( d^{ijkl} \sim O(1) \). By integrating over the fifth dimension, the above operators lead to the genuine dimension 5 proton decay operators

\[
\mathcal{L}_5 = \int d^2 \theta \left( \frac{1}{2} C^{ijkl}_{L} Q_i Q_j Q_k L_l + C^{ijkl}_{R} E^C_i U^C_j U^C_k D^C_l \right),
\]

in terms of 4D fields. Here \( C^{ijkl} \) are constants proportional to \( d^{ijkl} \):

\[
C^{ijkl}_{L,R} = \frac{D^{ijkl}}{\sqrt{\pi}} \exp \left[ -\frac{\mu^2}{4} (3p_j^2 + 3p_k^2 + 3q_l^2 - 2p_j p_k - 2p_k q_l - 2p_j q_l) \right], \quad D^{ijkl} \equiv d^{ijkl} \frac{\mu}{M^*_s},
\]

where \( p_j \) and \( q_l \) denote \( \Psi_i \)'s and \( \Phi_i \)'s relative locations from \( \Psi_i \), namely \( p_j \equiv l_j - l_i, q_l \equiv k_l - l_i \). Note that \( Q_i Q_j Q_k L_l \) and \( E^C_i U^C_j U^C_k D^C_l \) identically vanish because of the Bose statistics. What we hope is that the coefficients \( C^{ijkl}_{L,R} \) become sufficiently small due to small overlaps of the wave functions, which we will study from now on.

In the above equation \( C^{ijkl}_{L,R} \) are written in flavor basis, which are related to ones in mass basis as

\[
C^{ijkl}_{Lm} = (U^u_Q)^i_m (U^n_Q)^j_n (U^u_U)^k_p (U^T_L)^l_q C^{mnpq}_{Lf},
\]

\[
C^{ijkl}_{Rm} = (U^T_E)^i_m (U^n_U)^j_n (U^u_U)^k_p (U^T_D)^l_q C^{mnpq}_{Rf}.
\]
The decay mode $p \rightarrow K^+\bar{\nu}$ gives the most severe constraints on these constants. The partial life time for this mode is larger than $1.9 \times 10^{33}\text{yr}$ [19], which implies [20] for the first term of eq.(22) (LLLL operators)

$$
\sum_k \sqrt{|C_{Lmk}^{iijk}|^2} \lesssim C_c^{iijk} \left(\frac{m_{\text{SUSY}}}{1 \text{ TeV}}\right) \left(\frac{0.03 \text{ GeV}^3}{|\beta_p|}\right) \left(\frac{10}{A_L}\right)
$$

$$
C_c^{112k} = 3.6 \times 10^{-11}, \quad C_c^{221k} = 1.3 \times 10^{-10}
$$

$$
C_c^{113k} = 9.7 \times 10^{-10}, \quad C_c^{331k} = 8.5 \times 10^{-8}
$$

(25)

if cancellation among coefficients in the proton decay amplitudes does not occur. These values are those given at the GUT scale. Here, we have assumed a common mass $m_{\text{SUSY}}$ to all superparticles in the MSSM for simplicity, and $\beta_p$ represents the hadronic matrix element parameter, which is evaluated as

$$
|\beta_p| = 0.003 \sim 0.03 \text{ GeV}^3
$$

(26)

by various methods [21]. Hereafter we take $m_{\text{SUSY}} = 1 \text{ TeV}$, and the largest value $\beta_p = 0.03 \text{ GeV}^3$. $A_L$ represents one loop renormalization effect due to gauge interaction from $M_{\text{GUT}}$ to 1 GeV. Here we neglect effect due to Yukawa interaction since it does not change our conclusions.

Similarly, for the second term of eq.(22) (RRRR operators) we have the following constraints:

$$
|C_{Rmk}^{ijkl}| \lesssim C_c^{ijkl} \left(\frac{\sin 2\beta}{0.10}\right) \left(\frac{m_{\text{SUSY}}}{1 \text{ TeV}}\right) \left(\frac{0.03 \text{ GeV}^3}{|\alpha_p|}\right) \left(\frac{6.5}{A_R}\right)
$$

$$
C_c^{3311} = 4.7 \times 10^{-9}, \quad C_c^{3211} = 4.5 \times 10^{-8}
$$

(27)

at the GUT scale, where $\alpha_p$ is also the hadronic parameter which satisfies $|\alpha_p| = |\beta_p|$ and $A_R$ is renormalization effect. The appearance of $\tan \beta$ is due to the fact that the proton decay diagram is generated by Higgsino exchange, which inevitably depends on the Yukawa couplings.

First we decide the location of $\Psi_i$ by considering constraints on the LLLL operators. If $U_Q^u = 1$, we find

$$
\sum_i |C_{Lmk}^{iijk}|^2 = \sum_l |C_{Lmk}^{iijk}|^2.
$$

(28)

When $\Psi_i$ and $\Psi_j$ are localized close to each other, the overlap of their wave functions becomes large. To suppress the proton decay, the localization point of the other one $\Phi_k$ must be far away from them. Numerically we find

$$
q_k < q_-, \quad \text{or} \quad q_+ < q_k,
$$

(29)

where

$$
\mu q_{\pm} \equiv \frac{1}{3} \left\{ \mu p_j \pm \sqrt{12 \log \frac{D^{iijk}}{C_c^{iijk} \sqrt{\pi}} - 8\mu^2 p_j^2} \right\}.
$$

(30)
On the other hand, if $\Psi_i$ is localized far from $\Psi_j$, or more explicitly

$$\mu|l_j - l_i| = \mu|p_j| > \sqrt{\frac{3}{2}} \log \frac{D^i_{ijk}}{C_i_{ijk}^c} \sqrt{\sigma},$$

(31)

then we do not have any constraints on the position of $\Phi_k$, and thus arbitrary $q_k$, namely $k_k$, is allowed. Using the bounds (25) and taking $|D_{ijk}| = 1$, we find if

$$\mu|l_1 - l_2| \gtrsim 5.9, \quad \mu|l_1 - l_3| \gtrsim 5.5,$$

(32)

then the proton decay induced by the LLLL operators is always suppressed to an experimentally allowed level as far as $U_{QQ}^U = 1$.

In the case (i), $\mu l_1 = 7.30, \mu l_2 = 5.10, \mu l_3 = 0.00$ do not satisfy Eq. (32). Thus the locations of the $\Phi_k$ are very constrained. For instance, when $|D_{ijk}| = 1$, the region $1.2 < k_k < 11.4$ is prohibited even for $|\beta_p| = 0.003$ GeV$^2$. Even if we take $|D_{ijk}| = 10^{-2}$, the corresponding excluded region is $1.9 < k_k < 10.7$. Because of such large excluded regions, it is difficult to obtain realistic down-type quark masses and CKM parameters within at most $O(10)$ relative distance. Thus we will not consider the case (i) any more.

On the other hand, in the case (ii), $\mu l_1 = -7.20, \mu l_2 = 5.20, \mu l_3 = 0.60$ (with $|D_{ijk}| = 1$) satisfy the conditions (32), and thus we do not have any constraints on the locations of $\Phi_k$.

Next we consider the RRRR operators. If $U_U = 1$,

$$C_{ijkl} = (U_E^T)^i_m (U_D^T)^j_l C_{Rm}^{mjkl}.$$

(33)

Even for large $\tan \beta$ constraints on these are weaker than ones from the LLLL operators, since matter sector has SU(5) symmetry. Therefore, we expect that the case (ii) survives the proton decay constraint.

In the above consideration we have taken $U_Q^u = U_U = 1$. Thus the suppression of the proton decay rates only relies on the fact that $\Psi_1$ lives far from the other fields. However, if $U_Q^d$ or $U_U$ are different from the unit matrix, very rapid proton decay may be induced through off-diagonal elements, in particular from $(U_Q^U)_{1i}$ and $(U_U)_{1i}$. We will come to this point later on.

C. Texture

Based on the aforementioned observations, we now seek locations of $\Phi_i$ which generate realistic down-type quark masses and CKM parameters.

The proton decay constraint prefers the case (ii) where $\Psi_1$ lives on the opposite side of $\Psi_2$. It makes the structure of the mass matrix very interesting as we will see soon. Let us suppose that the $\Phi(5^*)$ are placed in the following order along the extra dimension:

$$\Psi_1 - \Phi_2 - \Psi_3 - \Phi_3 - \Psi_2 - \Phi_1$$

(34)

with Higgses localized around the third generation. This can lead to a variant of Fritzsch type texture [22] as follows [5]
Approximate zeros arise in the matrix when the overlaps of the wave functions between \( \Phi_i \) and \( \Psi_j \) are extremely suppressed.

\( y_D \) is diagonalized as eq.(13). Since we have \( U_Q^u \simeq 1 \), we can approximate \( V_{\text{CKM}} \simeq U_Q^d \). Therefore,

\[
y_d \sim \frac{aa'd}{bd'}, \quad y_s \sim \frac{bd'}{\sqrt{d'^2 + d^2}}, \quad y_b \sim \sqrt{d'^2 + d^2}
\]

\(|V_{us}| \sim \frac{ad}{bd'}, \quad |V_{ub}| \sim \frac{ad'}{d^2 + d'^2}, \quad |V_{cb}| \sim \frac{bd}{d^2 + d'^2}.
\]

This leads to a prediction

\[
\frac{m_s^2}{m_b^2} \sim \frac{|V_{ub}V_{cb}|}{|V_{us}|}
\]

at \( M_{\text{GUT}} \). Observed values satisfy this relation very well.

### D. Quark Masses and Mixings

We are now at the position to describe the configuration of the wave function locations which generate realistic fermion masses and mixings.

For simplicity, we set \( F_{ij}^U = F_{ij}^D = 1 \) and \( \tan \beta = 20 \). A choice of the parameter set (see Fig. 1)

\[
r_u = 0.66, \quad r_d = 0.22, \quad \mu l_{H_d} = -0.55
\]

\[
\mu l_1 = -5.7, \quad \mu l_2 = 4.1, \quad \mu l_3 = -0.35
\]

\[
\mu k_1 = 7.1, \quad \mu k_2 = -2.3, \quad \mu k_3 = 1.3
\]

yields Yukawa coupling matrices

\[
y_U = \begin{pmatrix} 5.90 \times 10^{-6} & 7.31 \times 10^{-22} & 1.52 \times 10^{-8} \\ 7.31 \times 10^{-22} & 1.61 \times 10^{-3} & 9.37 \times 10^{-6} \\ 1.52 \times 10^{-8} & 9.37 \times 10^{-6} & 0.629 \end{pmatrix}
\]

\[
y_D = \begin{pmatrix} 1.02 \times 10^{-36} & 7.29 \times 10^{-4} & 8.33 \times 10^{-12} \\ 7.70 \times 10^{-4} & 4.78 \times 10^{-10} & 4.99 \times 10^{-3} \\ 1.77 \times 10^{-13} & 6.01 \times 10^{-2} & 0.101 \end{pmatrix}
\]

These values give quark masses

\[
m_u = 1.03 \text{ MeV}, \quad m_c = 280 \text{ MeV}, \quad m_t = 110 \text{ GeV}
\]

\[
m_d = 1.54 \text{ MeV}, \quad m_s = 23.8 \text{ MeV}, \quad m_b = 1.02 \text{ GeV}
\]

and the magnitude of the mixing matrix
at $M_{\text{GUT}}$. After the RGE evolution, we obtain

$$m_u = 2.50 \text{ MeV}, \quad m_c = 679 \text{ MeV}, \quad m_t = 176 \text{ GeV}$$

$$m_d = 5.26 \text{ MeV}, \quad m_s = 81.3 \text{ MeV}, \quad m_b = 2.98 \text{ GeV}$$

(42)

and

$$\begin{pmatrix}
0.975 & 0.221 & 0.004 \\
0.220 & 0.975 & 0.042 \\
0.013 & 0.040 & 0.999
\end{pmatrix}$$

(43)

at the $Z$-boson mass scale. These are consistent with observed values.

As for proton decay, including contribution from off-diagonal elements of unitary matrices we obtain

$$\sum_k \sqrt{|C_{112k}|^2} \sim 6.1 \times 10^{-15}, \quad \sum_k \sqrt{|C_{221k}|^2} \sim 6.8 \times 10^{-15}$$

$$\sum_k \sqrt{|C_{113k}|^2} \sim 4.1 \times 10^{-10}, \quad \sum_k \sqrt{|C_{331k}|^2} \sim 2.9 \times 10^{-9}$$

$$C_{Rm}^{331} \sim 6.7 \times 10^{-10}, \quad C_{Rm}^{321} \sim 1.0 \times 10^{-14},$$

(44)

which survives the current experimental bounds (25). Thus this mechanism succeeds to explain not only quark masses and mixings but also the suppression of the proton decay which arises from the genuine dimension 5 operators. To get small numbers, we used only at most $O(10)$ parameters, i.e. some of $\mu_l$, and $\mu_k$; become $\sim 10$. This is due to the Gaussian profiles of the wave functions of the fields.

A similar argument can apply for $SO(10)$ GUT. However, it seems difficult to explain proton decay and mass matrices simultaneously as far as all matters in the same generation share a common wave function.

One might worry that the configurations of Fig. 1 would generate $\mu$ parameter (the supersymmetric mass of the doublet Higgses) of order $M_*$. This problem may be solved by introducing a singlet field $S$ localizing far from $SU(2)_L$ doublet Higgses and considering the Yukawa interactions $S H_u H_d$ to yield a small $\mu$ parameter [10].

E. Problems

Although we can obtain the realistic quark masses and mixings, the above texture in which up-type Yukawa coupling matrix is diagonal cannot generate sufficient $CP$ violation [23] in the CKM matrix.

Since up-type Yukawa coupling matrix is diagonal,

$$y_D y_D^\dagger = V_{KM} y_D y_D^T V_{KM}^T$$

(45)
On the other hand, eq.(35) implies
\[ y_D y_D^\dagger = \begin{pmatrix} x & 0 & x \\ 0 & x & x \\ x & x & x \end{pmatrix}, \] (46)
where \( x \)s represent non-zero values. Thus, we find
\[ y_d^2 V_{ud} V_{cd}^* + y_s^2 V_{us} V_{cs}^* + y_b^2 V_{ub} V_{cb}^* = 0. \] (47)
Using this relation and the unitarity of the CKM matrix, we obtain a relation
\[ \frac{y_s^2 - y_d^2}{y_b^2 - y_d^2} = -\frac{V_{ub} V_{cb}^*}{V_{us} V_{cs}^*}. \] (48)
This is nothing but the prediction of eq.(37) and leads to
\[ J = Im(V_{us} V_{cb}^* V_{ub} V_{cs}^*) = 0, \] (49)
which means there is no \( CP \) violation in the quark sector. In fact, the Yukawa coupling matrices of eq.(39) with arbitrary phases, give extremely small \( CP \) violation, numerically \( J \lesssim 1 \times 10^{-8} \).

Another problem is that this texture predicts equality of down-type masses and charged lepton masses at \( M_{\text{GUT}} \). This is inevitable as far as GUT breaking Higgs does not couple with matters.

IV. SU(5) BREAKING CONFIGURATION

In this section, we construct a model which explains the fermion masses, mixings, \( CP \) violation and the null results from proton decay experiments at the same time. As we saw in the previous section, our approach gave almost realistic mass matrices. The two difficulties mentioned above are originated from the fact that the configurations of the wave functions of the quarks and leptons have SU(5) invariant form, namely the fields belonging to the same multiplet have the same wave function along the extra dimension. Thus we will consider how SU(5) breaking is incorporated to improve these points. Here, an adjoint Higgs field \( \Sigma \), which spontaneously breaks the SU(5) gauge symmetry, plays a crucial role in splitting the wave functions of the quarks from ones of the leptons which belong to the same multiplets [10], so that we obtain the sufficient \( CP \) violation and the mass difference between down-type quarks and charged leptons.

A. Wave function Splitting Mechanism

Let us here briefly review how the wave functions in a SU(5) multiplet can be split after SU(5) breaking. Besides the usual matters, \( \Psi \) and \( \Phi \), the model has their conjugate fields, \( \Psi^C \) and \( \Phi^C \). Thus we can write Yukawa interactions of matters with the adjoint Higgs \( \Sigma \),
\[ \mathcal{L} = \int d^2 \theta \left\{ \Psi^C_i \left[ \partial_y + M_i(y) - \frac{2}{3} g_i \Sigma \right] \Psi_i + \Phi^C_i \left[ \partial_y + M_i(y) + \frac{1}{3} g_i \Sigma \right] \Phi_i \right\}. \] (50)
Here, $i$ is a generation index, $g, \bar{g}$ are Yukawa coupling constants, and $M(y), \bar{M}(y)$ are position dependent 5D masses including the $\Xi(y)$. We assume that the adjoint Higgs $\Sigma$ develops the constant vev along the extra dimension

$$\langle \Sigma \rangle = V \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & -3 \\ -3 & -3 & 2 \end{pmatrix},$$

(51)

by which the SU(5) group is spontaneously broken. Therefore, after the GUT breaking, the above Lagrangian becomes of the form

$$\mathcal{L} = \int d^2 \theta F_i^C (\partial_y + M_i + g_i V Y) F_i,$$

(52)

where $F$’s are the MSSM fields, $Q, U^C, D^C, L$ and $E^C$, and $F^C$’s are their conjugated fields, and $Y$ represents the hypercharge of $F$ as

$$
\begin{array}{c|cccc}
F & Q & U^C & D^C & L \\
Y & 1/3 & -4/3 & 2 & 2/3 & -1,
\end{array}
$$

(53)

and $M$ and $g$ include $\bar{M}$ and $\bar{g}$. In this setting, the locations of zero mode wave functions of the MSSM fields are split as

$$l_i = -\frac{M_i + g_i Y V}{2\mu^2}, \quad k_i = -\frac{\bar{M}_i + \bar{g}_i Y V}{2\mu^2},$$

(54)

with

$$l(E^C) = 2l(Q) - l(U^C).$$

(55)

Since we no longer have SU(5) symmetry in the matter sector, this mechanism can explain the difference between down-type masses and charged lepton masses without using Georgi-Jarlskog mechanism [24]. However, the unification of $m_b$ and $m_\tau$ is accidental.

**B. CP Violation**

The difficulty in generating sufficient CP violation in previous SU(5) invariant case comes from the fact that the up-type Yukawa matrix is diagonal. Thus we will consider a slight deviation from the diagonal matrix for $y_U$ as follows:

$$y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & x \\ 0 & 0 & y_t \end{pmatrix}, \quad y_D = \begin{pmatrix} 0 & a & 0 \\ a' & 0 & b e^{i\alpha} \\ 0 & d' & d \end{pmatrix},$$

(56)

where complex phase only appears through the phase $\alpha$. Since the above matrices have several zero entries, it is sufficient to consider only one phase. We keep elements $(y_U)^{11}$ and $(y_U)^{11}$ zero in order to avoid rapid proton decay. Otherwise, proton decay coefficients in
mass basis $C_{l,r}^{ijkl}$ may become too large by rotating unsuppressed coefficients e.g. $C_{l,r}^{223k}$ by $(U_Q^{u})^3$. The unitary matrices which contribute to the CKM matrix are decomposed in orthogonal matrices $O$ and a phase matrix $P = \text{diag}(1, e^{-i\alpha}, 1)$ as

$$U_Q^u = O_Q^u, \quad U_Q^d = PO_Q^d. \quad (57)$$

These give the CKM matrix,

$$V_{KM} = O_Q^{uT}PO_Q^d. \quad (58)$$

For $O_Q^u = 1$ like in the previous SU(5) invariant case, obviously $J = 0$. On the other hand, for $O_Q^u \neq 1$ as in eq.(56) CP violation compatible with experimental results can arise.

In the texture of eq.(56), following a similar argument given in section III E, we obtain

$$J \sim \frac{x}{y_t} m_b^2 - m_s^2 |V_{ub}|^2 |V_{tb}| |V_{cb}| \sin \alpha. \quad (59)$$

In order to produce $J \sim 10^{-5}$, at least $x/y_t \gtrsim 10^{-2}$ is required.

C. Realistic Fermion masses, Mixings and CP Violation

Here we numerically describe a parameter set of models consistent with experimental results. In searching parameter space, for simplicity, we take $|F^U_{ij}| = 1$ as

$$F_U = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad F_D = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & e^{i\alpha} \\ 1 & 1 & 1 \end{pmatrix}, \quad (60)$$

and fix $\tan \beta = 20$. In order to produce the texture eq.(56) and not to destroy the preferred features in the SU(5) invariant case, we simply split $\Psi_2$ and keep $\Psi_{1,3}$ unbroken.

An interesting parameter set we found by numerical survey is (see Fig. 2.)

$$r_u = 0.66, \quad r_d = 0.34, \quad \mu l(H_d) = -3.65$$

$$\mu l(Q_1) = -5.75, \quad \mu l(Q_2) = 2.15, \quad \mu l(Q_3) = -0.35$$

$$\mu l(U^C_1) = -5.75, \quad \mu l(U^C_2) = 4.30, \quad \mu l(U^C_3) = -0.35$$

$$\mu l(E^C_1) = -5.75, \quad \mu l(E^C_2) = 0.00, \quad \mu l(E^C_3) = -0.35$$

$$\mu k(D^C_1) = 4.00, \quad \mu k(D^C_2) = -2.15, \quad \mu k(D^C_3) = 0.24$$

$$\mu k(L_1) = -9.67, \quad \mu k(L_2) = 1.76, \quad \mu k(L_3) = -1.45, \quad (61)$$

which are derived from the following fundamental parameters,

$$g_1 V/\mu = 0, \quad g_2 V/\mu = 2.58, \quad g_3 V/\mu = 0,$$

$$\bar{g}_1 V/\mu = -16.404, \quad \bar{g}_2 V/\mu = 4.692, \quad \bar{g}_3 V/\mu = -2.028,$$

$$M_1/\mu = 11.5, \quad M_2/\mu = -5.16, \quad M_3/\mu = 0.7,$$

$$\bar{M}_1/\mu = 2.936, \quad \bar{M}_2/\mu = 1.172, \quad \bar{M}_3/\mu = 0.872 \quad (62)$$
when $H_u$ is not shifted after GUT breakdown. From these values, we obtain the Yukawa coupling matrices as

$$
y_U = \begin{pmatrix}
4.81 \times 10^{-6} & 6.37 \times 10^{-23} & 1.10 \times 10^{-8} \\
5.79 \times 10^{-15} & 1.58 \times 10^{-3} & 2.16 \times 10^{-2} \\
1.10 \times 10^{-8} & 3.29 \times 10^{-6} & 0.629
\end{pmatrix},
$$

$$
y_D = \begin{pmatrix}
4.97 \times 10^{-22} & 7.69 \times 10^{-4} & 7.50 \times 10^{-9} \\
6.53 \times 10^{-4} & 1.14 \times 10^{-5} & 6.28 \times 10^{-3} e^{i\alpha} \\
1.49 \times 10^{-6} & 5.34 \times 10^{-2} & 0.104
\end{pmatrix},
$$

$$
y_L = \begin{pmatrix}
3.84 \times 10^{-5} & 2.13 \times 10^{-13} & 4.89 \times 10^{-5} \\
2.15 \times 10^{-21} & 1.14 \times 10^{-2} & 6.95 \times 10^{-2} e^{i\alpha} \\
5.69 \times 10^{-20} & 6.88 \times 10^{-3} & 0.121
\end{pmatrix}
$$

(63)

For $\alpha = 50^\circ$, the above parameter set predicts

$$
m_u = 0.838 \text{ MeV}, \quad m_c = 275 \text{ MeV}, \quad m_t = 110 \text{ GeV}
$$

$$
m_d = 1.29 \text{ MeV}, \quad m_s = 26.3 \text{ MeV}, \quad m_b = 1.02 \text{ GeV}
$$

$$
m_e = 3.35 \text{ MeV}, \quad m_\mu = 70.7 \text{ MeV}, \quad m_\tau = 1.22 \text{ GeV},
$$

(64)

$$
\begin{pmatrix}
0.975 & 0.222 & 0.003 \\
0.221 & 0.974 & 0.037 \\
0.010 & 0.035 & 0.999
\end{pmatrix}
$$

(65)

and

$$
J = 1.7 \times 10^{-5}
$$

(66)

at the $M_{\text{GUT}}$ scale. After the RGE evolution,

$$
m_u = 2.03 \text{ MeV}, \quad m_c = 666 \text{ MeV}, \quad m_t = 176 \text{ GeV}
$$

$$
m_d = 4.40 \text{ MeV}, \quad m_s = 89.8 \text{ MeV}, \quad m_b = 2.96 \text{ GeV}
$$

$$
m_e = 0.491 \text{ MeV}, \quad m_\mu = 104 \text{ MeV}, \quad m_\tau = 1.75 \text{ GeV},
$$

(67)

$$
\begin{pmatrix}
0.975 & 0.222 & 0.003 \\
0.221 & 0.974 & 0.043 \\
0.012 & 0.041 & 0.999
\end{pmatrix}
$$

(68)

and

$$
J = 2.3 \times 10^{-5}
$$

(69)

at the $Z$-boson mass scale. Furthermore, from eq.(23) we obtain

$$
\sum_k \sqrt{|C_{Lm}^{112k}|^2} \sim 6.4 \times 10^{-13}, \quad \sum_k \sqrt{|C_{Lm}^{221k}|^2} \sim 1.3 \times 10^{-10}
$$

$$
\sum_k \sqrt{|C_{Lm}^{113k}|^2} \sim 1.9 \times 10^{-11}, \quad \sum_k \sqrt{|C_{Lm}^{331k}|^2} \sim 1.4 \times 10^{-9}
$$

$$
C_{Rm}^{331} \sim 3.3 \times 10^{-10}, \quad C_{Rm}^{321} \sim 3.0 \times 10^{-14}.
$$

(70)
It turns out that the proton decay is suppressed to a level consistent with experimental limit.

As for the lepton sector, the locations of $E_i^C$ are fixed by eq.(55), and those of $L_i$ are adjusted to reproduce the observed charged lepton masses.

V. SCALES OF MODEL

Next we would like to clarify several points on the scales of this model.

So far, we have used the approximate form (eq.(5)) for $\Xi(y)$, which is valid near the origin. In this approximation, zero modes have Gaussian profiles (eq.(6)), and adding mass parameter $M$ to eq.(5) is merely to shift wave functions along the fifth dimension. However, this configuration will not hold in the entire extra dimension, but rather it will behave for instance like

$$\langle \Xi \rangle = \begin{cases} 
\xi & (y \geq L) \\
\mu^2 y & (-L < y < L) \\
-\xi & (y \leq -L)
\end{cases}$$

(71)

with $\xi \equiv \mu^2 L$ for some $L$. A natural expectation is $\xi \sim M_*$. In order that all our previous arguments are justified, $L$ must be sufficiently large: otherwise we would have different configurations for wave functions. To obtain the fermion mass hierarchy we need that $\mu L$ is at least order 10. Combined it with the expectation $\xi \sim M_*$, we find

$$\mu L \sim \frac{M_*}{\mu} \gtrsim 10.$$ 

(72)

Another point we wish to clarify is behavior of KK modes. Wave functions for the KK modes obey the following eigenvalue equations

$$\left[ -\partial^2_y + M^2(y) - \partial_y M(y) \right] L_n = m^2_n L_n$$

$$\left[ -\partial^2_y + M^2(y) + \partial_y M(y) \right] R_n = m^2_n R_n$$

(73)

where $L$ and $R$ represent components of chiral superfields and anti-chiral super fields respectively, and $n$ labels the excitation level of a KK mode and $m_n$ is its mass. For $m_n \ll \xi$, eq.(73) become

$$\left[ -\partial^2_y + (2\mu^2 y)^2 - 2\mu^2 \right] L_n = m^2_n L_n$$

$$\left[ -\partial^2_y + (2\mu^2 y)^2 - 2\mu^2 \right] R_n = m^2_n R_n,$$

(74)

and the wave functions are written in terms of Hermite polynomials. Thus a KK mode with mass $m_n \ll \xi$ is localized with narrow width. A higher KK mode has a more spread wave function, but is still bounded. On the contrary, when $m_n > \xi$, since ‘energy’ $m^2_n$ is always larger than ‘potential’ $M(y)^2 \pm \partial_y M(y)$, KK modes can freely propagate all over the extra dimension. Such KK modes, if exist in the color triplet Higgses would be very dangerous, as
they would mediate unacceptably fast proton decay. To be safe, we should introduce cut-off below $M_*$ to eliminate such freely propagating KK modes.\footnote{Summation over the whole KK tower would, however, reproduce the 5D picture in which the contribution to the proton decay from the heavy triplet is exponentially suppressed. We thank Martin Schmaltz for pointing it out.}

The (de)constructing extra dimension naturally provides such a cut-off. In this scenario, a (dynamical) scale $\Lambda$ is implemented in the theory. Below the energy scale $\Lambda$, the theory looks 5 dimensional with one extra dimension. Above this scale, the extra dimension is resolved and the theory looks 4 dimensional with gauge group $SU(5)^N$ with $N$ being some large integer. In this setup, it is very natural to expect that there is no KK mode above $\Lambda$, which provides the cut-off we want to have.

Furthermore in the (de)constructing extra dimension, gravity does not propagate this extra dimension, so that we can identify the 5D Planck scale $M_*$ with the 4D (reduced) Planck scale $2.4 \times 10^{18}$ GeV.

Summarizing these arguments, we find that a set of the parameters as

\[
M_* \sim 10^{18} \text{ GeV} \\
\mu \sim 10^{17} \text{ GeV} \\
L^{-1} \sim 10^{16} \text{ GeV}
\]  

provide an example of the setting in which our mechanism works.

Perturbativity of 5D gauge interactions requires

\[
N_{KK} \frac{g^2}{16\pi^2} \lesssim 1,
\]

where $g$ is a gauge coupling constant and $N_{KK}$ is the number of the KK modes below the cut-off $\Lambda$. We expect that

\[
N_{KK} \sim \Lambda L \lesssim M_* L \sim 100,
\]

and thus with the gauge coupling of order unity, eq. (76) is satisfied.

**VI. CONCLUSIONS AND DISCUSSION**

In this paper, we have pursued a very natural expectation that the mechanism which explains the masses and mixing of the quarks and leptons should also explain the smallness of the dangerous genuine dimension 5 proton decay operators in supersymmetry. This philosophy provides a test of models of flavor and here we applied to the mechanism of localizing fermions under kink background along the extra dimension. We showed that the localization mechanism can provide a successful configuration of the wave functions of the chiral multiplets which is consistent with the SUSY $SU(5)$ and yields the realistic fermion mass structure and suppresses the dimension 5 proton decay.

Here we would like to summarize our ingredients on the localization mechanism.
• The mechanism advocated by Arkani-Hamed and Schmaltz is used. In this approach, there exists a singlet filed $\Xi$ which has a kink configuration along the extra dimension. Bulk fermions are assumed to have Yukawa interactions with the singlet in the bulk, which is a necessary ingredient of this localization mechanism. The width and the localization of the fermion wave function are controlled by the Yukawa coupling and the invariant mass of the fermion. In the context of SUSY, $D = 5$ full SUSY would prevent such Yukawa interactions in the superpotential. We argue that in the deconstructing extra dimension $D = 5$ SUSY can be explicitly broken to $D = 4 \ N = 1$ SUSY, allowing the desired superpotential.

• In order to obtain a chiral theory, one needs to elaborate to kill anti-chiral zero modes. One way would be to consider a $Z_2$ orbifold compactification, in which appropriate boundary conditions would allow only chiral zero modes. An alternative way is to consider a non-trivial $Z$ factor for the fermion kinetic terms, in which $Z$ vanishes at the boundaries. In this case, the anti-chiral zero modes have non-normalizable wave functions and thus they do not exist.

To obtain a realistic configuration, one further requires

• Splitting of wave functions in a single SU(5) multiplet is required to have a non-vanishing CKM phase.

• Appropriate couplings and masses should be chosen to reproduce the fermion masses and mixing and to suppress the proton decay to an experimentally acceptable level.

In this paper, we extensively discussed the SU(5) GUT. It is interesting to consider the case of SO(10) and larger groups. Here we briefly mention the SO(10) case. As is well-known, all quarks and leptons as well as a right-handed neutrino can be embedded into a single 16 dimensional representation in SO(10). These fields in the same multiplet are localized in different points after the SO(10) breaking. To see this point closely, let us consider the following symmetry breaking chain $SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. In the first symmetry breaking, 10, $\bar{5}$, and 1 (right-handed neutrino) in SU(5) terminology are split. Thus our arguments given for the SU(5) case can apply to this case. A crucial difference is that the positions and widths of the right-handed neutrinos are not freely chosen, but are related to those of 10 and $\bar{5}$ because the splitting is attributed to the expectation value of the Higgs responsible for the breaking $SO(10) \rightarrow SU(5)$. It is interesting to see whether the realistic neutrino masses and mixing are realized in this case, which is however beyond the scope of this paper.

To conclude, the idea of the fermion localization along the extra dimension can pass the phenomenological test of models of flavor. Our explicit construction shows that this idea can indeed work. Of course, ours is just one possibility among divergent approaches to the extra dimensions. Further investigation along this line should be encouraged.

After completion of this work, we received a preprint [25] which deals with a similar subject.
ACKNOWLEDGMENT

This work was supported in part by the Grant-in-aid from the Ministry of Education, Culture, Sports, Science and Technology, Japan, priority area (#707) “Supersymmetry and unified theory of elementary particles,” and in part by the Grants-in-aid No.11640246 and No.12047201.

APPENDIX A: A MECHANISM TO OBTAIN CHIRAL ZERO MODES

In this appendix, we describe a mechanism to obtain only the zero modes of a chiral superfield $\Phi$ while forbidding those of a charge conjugated chiral superfield $\Phi^C$ in five dimensions with the extra dimension compactified on the orbifold $S_1/Z_2$. The key point for this mechanism to work is introduction of a $Z$ factor with a non-trivial profile along the extra dimension. Here, we assume that the $Z$ factor is nearly constant in the bulk but vanishes at the boundaries of the extra dimension. Here we do not specify the origin of the $Z$ factor.

Instead of eq.(1), we thus suppose the following five-dimensional action:

$$S = \int d^5x \left\{ \int d^4\theta Z(y)(\Phi^+\Phi + \Phi^C+\Phi^C) \ight. \\
+ \int d^2\theta \left( \frac{1}{2}Z(y)(\Phi^C\partial_y \Phi) + M(y)\Phi^C\Phi + h.c. \right) \right\}, \quad (A1)$$

where $y$-derivative does not act on $Z(y)$.

We expand the fields into their KK modes,

$$\Phi = \frac{1}{\sqrt{Z}} \sum_{n=0}^{\infty} f_L^{(n)}(y)\Psi^{(n)}(x), \quad \Phi^C = \frac{1}{\sqrt{Z}} \sum_{n=0}^{\infty} f_R^{(n)}(y)\Psi^{(n)}(x), \quad (A2)$$

where $\{f_L^{(n)}\}$ and $\{f_R^{(n)}\}$ span complete orthonormal bases as follows:

$$\int dy f_L^{(m)*}f_L^{(n)} = \delta_{mn}, \quad \int dy f_R^{(m)*}f_R^{(n)} = \delta_{mn}. \quad (A3)$$

Integrating over the extra dimension, we obtain

$$S = \sum_{n=0}^{\infty} \int d^4x \left\{ \int d^4\theta \left( \Psi^{(n)*}\Psi^{(n)} + \Psi^{(n)*}\Psi^{(n)} \right) + \int d^2\theta \left( m_n\Psi^{(n)}\Psi^{(n)} + h.c. \right) \right\}, \quad (A4)$$

if the wave functions $f_{L,R}^{(n)}$ satisfy the following eigenvalue equations:

$$\left( \partial_y + \frac{M(y)}{Z(y)} \right) f_L^{(n)} = m_n f_L^{(n)}, \quad \left( -\partial_y + \frac{M(y)}{Z(y)} \right) f_R^{(n)} = m_n f_R^{(n)}. \quad (A5)$$

Notice that the mass term $M(y)$ is practically replaced with $M(y)/Z(y)$. In particular, the equations for the zero modes are written

$$\left( \partial_y + \frac{M(y)}{Z(y)} \right) f_L^{(0)} = 0, \quad \left( -\partial_y + \frac{M(y)}{Z(y)} \right) f_R^{(0)} = 0, \quad (A6)$$
whose general solutions are easily obtained as

\[
  f_L^{(0)}(y) = N_L \exp \left( -\int_0^y dy' \frac{M(y')}{Z(y')} \right), \quad f_R^{(0)}(y) = N_R \exp \left( \int_0^y dy' \frac{M(y')}{Z(y')} \right),
\]

(A7)

where \( N_{L,R} \) are normalization constants. Since \( M(y)/Z(y) \) diverges at the boundaries of the extra dimension, one of the zero-mode wave functions, say \( f_R^{(0)} \), is not normalizable. Thus there exists only the chiral zero mode for \( \Phi \). Notice that the shape of the zero mode \( f_L^{(0)} \) is not changed drastically compared with the case \( Z = 1 \).
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FIG. 1. Profiles of fields which not only produce realistic quark masses and mixing angles but also suppress genuine dimension 5 proton decay. The resulting texture provides no $CP$ violation phase in the CKM matrix.
FIG. 2. Profiles of fields which can produce realistic fermion masses, CKM parameters and suppress genuine dimension 5 proton decay. Non-vanishing $CP$ phase in the CKM matrix is obtained in this case.