Analysis of Output Parameters of a Three-Stage Asymmetric Vibration Device

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Abstract. There is a long-felt need for a wider use of asymmetric vibration devices in technological processes of construction and road-building operations. The methodology of engineering and design of multi-stage asymmetric vibration devices is in the conceptual phase and requires additional analytical studies, design studies and industrial tests. Asymmetric vibration devices may consist of two or more stages of unbalanced directional vibrators, speeds of rotation of the drive shafts whereof are divisible. This article considers the effect of the interrelation of static moments of unbalance of a three-stage asymmetric vibration device on the system's dynamic amplification factor. The article describes a method to determine the rational interrelation of static moments of unbalance using a three-stage asymmetric vibration device as an example. The article will be interesting to specialists in the sphere of production of vibrating production machines and equipment.

1. Introduction
There is a whole range of industrial sectors, e.g. construction and road building, where technologies employing mechanical vibration are widely used [1, 2, 3, 4]. Depending on the industrial workflow type, generators with circular, elliptical, directional, or asymmetric vibration may be used (Figure 1).

The key to figure 1: $m_0$ – unbalance mass, $\omega$ – angular velocity of rotation of unbalance, $r$ – radius of the circle described by the center of mass of unbalance, $m_0 \cdot r$ – static moment of unbalance, $F$ – impact force, $A_x$, $A_y$ - amplitude of vibration along coordinate axes x and y, respectively, $A_1$, $A_2$ - amplitude of vibration along axes of the ellipse, $F_1$ and $F_2$ – components of the impact force of directional or asymmetric vibration acting in opposite directions, $k_d$ – dynamic amplification factor of an asymmetric vibration system.

Asymmetric vibration may be obtained by superposing several mechanical vibrations [5, 6, 7, 8]

$$F = m_1 \cdot r_1 \cdot \omega_1^2 + m_2 \cdot r_2 \cdot \omega_2^2 + m_3 \cdot r_3 \cdot \omega_3^2 \ldots$$

where $1, 2, 3 \ldots n$ - an ordinal number of an asymmetric vibration system's stage.

Study methods. Articles [9, 10] provide results of a study of a two-stage asymmetric vibration system, the highest possible dynamic amplification factor $k_d$ for which requires a specific rational interrelation of static moments of unbalance $(m_1 \cdot r_1):(m_2 \cdot r_2)$.

It is known [11] that the dynamic amplification factor for a three-stage asymmetric vibration system tends to three, i.e. $k_{d,3} = 3$. This means that the impact force component in the "working" direction of operation is three times stronger than the component acting in the opposite ("return stroke") direction.
However, it is necessary to take into account the average instantaneous dynamic amplification factor observed at the "return stroke" to design and develop a three-stage asymmetric vibration system.

Thus, for a three-stage asymmetric vibration system (table 1) the parameters are as follows: \( \omega_1: \omega_2: \omega_3 = 500:1,000:1,500 \) rpm; \( m_1: m_2: m_3 = 10:1.65:0.36 \) kg; \( r_1 = r_2 = r_3 = 1.0 \) cm, respectively.

![Diagram](image)

*Figure 1.* Schemes of vibration modes of machine implements depending on design peculiarities of the vibration assembly and its attachment to the technological equipment: a - scheme of an unbalanced vibrator, b - circular vibration, c - elliptical vibration, d - directional vibration, e - asymmetric vibration.

### Table 1. Input data for a calculation of the impact force of a three-stage asymmetric vibration system.

| Vibrator No. | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|--------------|----|----|----|----|----|----|----|
| Weight (kg)  | 10 | 1.65 | 0.36 | 0  | 0  | 0  | 0  |
| Radius (cm)  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| Initial phase (deg) | 0 | 0 | 0 | 2,000 | 2,500 | 3,000 | 3,500 |
| Velocity (rpm) | 500 | 1,000 | 1,500 | 2,000 | 2,500 | 3,000 | 3,500 |

See results of the calculation of the impact force in table 2.

The period of vibration of the system \( T = 0.12 \) s is divided into 20 intervals \( \Delta t = 0.006 \) s. Impact forces of each stage \( F_1, F_2, F_3 \) constitute an aggregate resulting impact force \( F \). Negative values of the force are on the "return stroke" side, whereas positive values are on the "working stroke side"
See the graph of behavior of impact forces (Yr) of the first and the second stages, as well as of the total impact force in Fig. 2. Impact forces are given in kN. As the program allows making calculations for up to seven stages, the graph in Fig. 2 demonstrates impact forces of the first and the second stages, as well as the total impact force.

**Table 2.** Results of a calculation of the impact force of an asymmetric vibration device within one period of vibration.

| No. | T   | F₁  | F₂  | F₃  | F₄, aggregate |
|-----|-----|-----|-----|-----|----------------|
| 0   | 0.000 | 0.55 | 0.36 | 0.18 | 1.09           |
| 1   | 0.006 | 0.52 | 0.29 | 0.10 | 0.92           |
| 2   | 0.012 | 0.44 | 0.11 | -0.05 | 0.50           |
| 3   | 0.018 | 0.32 | -0.11 | -0.17 | 0.04           |
| 4   | 0.024 | 0.17 | -0.29 | -0.14 | -0.27          |
| 5   | 0.030 | 0.00 | -0.36 | 0.00 | -0.36          |
| 6   | 0.036 | -0.17 | -0.29 | 0.14 | -0.32          |
| 7   | 0.042 | -0.32 | -0.11 | 0.17 | -0.27          |
| 8   | 0.048 | -0.44 | 0.11 | 0.05 | -0.28          |
| 9   | 0.054 | -0.52 | 0.29 | -0.10 | -0.33          |
| 10  | 0.060 | -0.55 | 0.36 | -0.18 | -0.36          |
| 11  | 0.066 | -0.52 | 0.29 | -0.10 | -0.33          |
| 12  | 0.072 | -0.44 | 0.11 | 0.05 | -0.28          |
| 13  | 0.078 | -0.32 | -0.11 | 0.17 | -0.27          |
| 14  | 0.084 | -0.17 | -0.29 | 0.14 | -0.32          |
| 15  | 0.090 | 0.00 | -0.36 | 0.00 | -0.36          |
| 16  | 0.096 | 0.17 | -0.29 | -0.14 | -0.27          |
| 17  | 0.102 | 0.32 | -0.11 | -0.17 | 0.04           |
| 18  | 0.108 | 0.44 | 0.11 | -0.05 | 0.50           |
| 19  | 0.114 | 0.52 | 0.29 | 0.10 | 0.92           |
| 20  | 0.120 | 0.55 | 0.36 | 0.18 | 1.09           |

**Figure 2.** Graph of behavior of impact forces of unbalance of the first (1) and the second (2) stages, as well as of the aggregate impact force $F_{agg}$ of an asymmetric vibration system.
2. Main part
The graph in Fig. 2 demonstrates that the duration of action of the impact force in the "working stroke" ("effective output") direction constitutes 30% of the period of vibration, as it takes place at 0-0.018 s and at 0.102-0.12 s. The "return stroke" duration constitutes 70% of the period, respectively, and takes place at 0.018-0.102 s. The strongest negative aggregate impact force in the "return stroke" part of the graph at $t = 0.03$ s is 0.36 kN. Therefore, the dynamic amplification factor at this point is

$$k_{d,0.030} = \frac{F_{agg,0.030}}{-F_{agg,0.030}} = \frac{1.09}{0.36} = 3.03$$

The dynamic amplification factor (could be called the "instantaneous" dynamic amplification factor of an asymmetric vibration system) for following points is as follows:

$$k_{d,0.036} = 4.04; k_{d,0.042} = 3.89; k_{d,0.054} = 3.30; k_{d,0.060} = 3.03$$

Thus, the interval from 0.030 to 0.06 s is characterized as follows:
- dynamic amplification factor of an asymmetric vibration system $k_{d,0.030} = 3.03$;
- sequence of "instantaneous" dynamic amplification factors of an asymmetric vibration system: 3.03; 4.04; 3.89; 3.03; 3.03;
- average dynamic amplification factor $k_{d,av} = 3.404$.

It ought to be noted that the dynamic amplification factor at $t = 0.024$ s cannot characterize the process, as it is close to $Y = 0$, so interrelation $k_d \to \infty$ becomes possible.

For practical calculations, the following interrelation of static moments of unbalance of a three-stage asymmetric vibration device is recommended [5]:

$$(m_1 \cdot r_1):(m_2 \cdot r_2):(m_3 \cdot r_3) = 100:16.64:3.68$$

See the analysis of trends of dynamic amplification factor of an asymmetric vibration system $k_{d,0.030}$ and system's average dynamic amplification factor $k_{d,av}$ in table 3.

Table 3. Change of the dynamic amplification factor and the average dynamic amplification factor of the system

| Interrelation of static moments of unbalance $(m_1 \cdot r_1):(m_2 \cdot r_2):(m_3 \cdot r_3)$ | System's dynamic amplification factor $k_d$ | Average dynamic amplification factor $k_{d,av}$ |
|-------------------------------------------|---------------------------------|-------------------|
| $m_1 \cdot r_1$ | $m_2 \cdot r_2$ | $m_3 \cdot r_3$ | $k_d$ | $k_{d,av}$ |
| 100 | 16.64 | 2 | 2.63 | 3.15 |
| 100 | 16.64 | 2.5 | 2.84 | 3.22 |
| 100 | 16.64 | 3 | 2.91 | 3.39 |
| 100 | 16.64 | 3.3 | 2.95 | 3.39 |
| 100 | 16.64 | 3.5 | 2.98 | 3.43 |
| 100 | 16.64 | 3.68 | 3.0 | 3.48 |
| 100 | 16.64 | 4 | 2.92 | 3.58 |
| 100 | 16.64 | 4.1 | 2.89 | 3.6 |
| 100 | 16.64 | 5 | 2.7 | 3.95 |

Calculation results in table 2 demonstrate that deviation from a certain rational interrelation of static moments of unbalance of the given three stages of vibrators results in a lower dynamic amplification factor of an asymmetric vibration device.

The rational interrelation of static moments of unbalance of a three-stage asymmetric vibration device results in the following interrelation of the forces constituting the aggregate impact force:

$$F_{agg} = 0.505F_1 + 0.3333F_2 + 0.1662F_3$$

(2)

The following method ought to be used at the stage of design calculation of operational parameters of a three-stage asymmetric vibration device.

The following ought to be considered the initial parameters:
- aggregate impact force in the "effective output" direction - $F_{AGG}$;
- dynamic amplification factor of a three-stage asymmetric vibration device $k_d = 3.0$.

Component forces $F_1$, $F_2$ and $F_3$ are determined using equation (2).

The scope of use and application of a three-stage asymmetric vibration device in known conditions helps to make a structural choice of the unbalance radius of each stage ($r_1$, $r_2$, $r_3$) and calculate weight of each unbalance ($m_1$, $m_2$, $m_3$).

Example. Design calculation of operational parameters of a three-stage asymmetric vibration device.

Set aggregate impact force $F_{AGG} = 20$ kN.

Solution.

According to (2), $F_1$, $F_2$, $F_3$ are as follows:

$$20 = 10.1 + 6.67 + 3.33 \text{ (kN)}$$

Let us assume the following for design purposes: $r_1 = 10$ cm, $r_2 = 2$ cm, $r_3 = 2$ cm.

See initial data in table 4.

| Vibrator No. | 1      | 2      | 3      |
|--------------|--------|--------|--------|
| Weight (kg)  | 18.4   | 15.2   | 3.3    |
| Radius (cm)  | 10     | 2      | 2      |
| Initial phase (deg) | 0     | 0      | 0      |
| Velocity (rpm)| 500   | 1,000  | 1,500  |

See calculation results in table 4.

| No. | $t$ | $F_1$ | $F_2$ | $F_3$ | $F$, aggregate |
|-----|-----|-------|-------|-------|----------------|
| 0   | 0.000 | 10.09 | 6.67  | 3.26  | 20.01          |
| 1   | 0.006 | 9.60  | 5.39  | 1.91  | 16.90          |
| 2   | 0.012 | 8.16  | 2.06  | -1.01 | 9.22           |
| 3   | 0.018 | 5.93  | -2.06 | -3.10 | 0.77           |
| 4   | 0.024 | 3.12  | -5.39 | -2.63 | -4.91          |
| 5   | 0.030 | 0.00  | -6.67 | 0.00  | -6.67          |
| 6   | 0.036 | -3.12 | -5.39 | 2.63  | -5.88          |
| 7   | 0.042 | -5.93 | -2.06 | 3.10  | -4.89          |
| 8   | 0.048 | -8.16 | 2.06  | 1.01  | -5.10          |
| 9   | 0.054 | -9.60 | 5.39  | -1.91 | -6.12          |
| 10  | 0.060 | -10.09| 6.67  | -3.26 | -6.68          |
| 11  | 0.066 | -9.60 | 5.39  | -1.91 | -6.12          |
| 12  | 0.072 | -8.16 | 2.06  | 1.01  | -5.10          |
| 13  | 0.078 | -5.93 | -2.06 | 3.10  | -4.89          |
| 14  | 0.084 | -3.12 | -5.39 | 2.63  | -5.88          |
| 15  | 0.090 | 0.00  | -6.67 | 0.00  | -6.67          |
| 16  | 0.096 | 3.12  | -5.39 | -2.64 | -4.91          |
| 17  | 0.102 | 5.93  | -2.06 | -3.10 | 0.77           |
| 18  | 0.108 | 8.16  | 2.06  | -1.01 | 9.22           |
| 19  | 0.114 | 9.60  | 5.39  | 1.91  | 16.90          |
| 20  | 0.120 | 10.09 | 6.67  | 3.26  | 20.01          |

System's dynamic amplification factor: $k_d = 20.1 : 6.67 = 3.0$. 

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**Table 4. Initial data for a sample calculation**

| Vibrator No. | 1      | 2      | 3      |
|--------------|--------|--------|--------|
| Weight (kg)  | 18.4   | 15.2   | 3.3    |
| Radius (cm)  | 10     | 2      | 2      |
| Initial phase (deg) | 0     | 0      | 0      |
| Velocity (rpm)| 500   | 1,000  | 1,500  |

**Table 5. Sample calculation results**

| No. | $t$ | $F_1$ | $F_2$ | $F_3$ | $F$, aggregate |
|-----|-----|-------|-------|-------|----------------|
| 0   | 0.000 | 10.09 | 6.67  | 3.26  | 20.01          |
| 1   | 0.006 | 9.60  | 5.39  | 1.91  | 16.90          |
| 2   | 0.012 | 8.16  | 2.06  | -1.01 | 9.22           |
| 3   | 0.018 | 5.93  | -2.06 | -3.10 | 0.77           |
| 4   | 0.024 | 3.12  | -5.39 | -2.63 | -4.91          |
| 5   | 0.030 | 0.00  | -6.67 | 0.00  | -6.67          |
| 6   | 0.036 | -3.12 | -5.39 | 2.63  | -5.88          |
| 7   | 0.042 | -5.93 | -2.06 | 3.10  | -4.89          |
| 8   | 0.048 | -8.16 | 2.06  | 1.01  | -5.10          |
| 9   | 0.054 | -9.60 | 5.39  | -1.91 | -6.12          |
| 10  | 0.060 | -10.09| 6.67  | -3.26 | -6.68          |
| 11  | 0.066 | -9.60 | 5.39  | -1.91 | -6.12          |
| 12  | 0.072 | -8.16 | 2.06  | 1.01  | -5.10          |
| 13  | 0.078 | -5.93 | -2.06 | 3.10  | -4.89          |
| 14  | 0.084 | -3.12 | -5.39 | 2.63  | -5.88          |
| 15  | 0.090 | 0.00  | -6.67 | 0.00  | -6.67          |
| 16  | 0.096 | 3.12  | -5.39 | -2.64 | -4.91          |
| 17  | 0.102 | 5.93  | -2.06 | -3.10 | 0.77           |
| 18  | 0.108 | 8.16  | 2.06  | -1.01 | 9.22           |
| 19  | 0.114 | 9.60  | 5.39  | 1.91  | 16.90          |
| 20  | 0.120 | 10.09 | 6.67  | 3.26  | 20.01          |
3. Conclusion.
The author analyzed operational parameters of a three-stage asymmetric vibration device and substantiated the need in such an analysis. The author demonstrated that the highest dynamic amplification factor of three, i.e. $k_d = 3.0$ is achieved when the rational interrelation of static moments of unbalance of a three-stage asymmetric vibration device. The author gave an example of operational parameters of a three-stage asymmetric vibration device to confirm results of theoretical studies.

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