Three Approaches for Personalization with Applications to Federated Learning

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Abstract

The standard objective in machine learning is to train a single model for all users. However, in many learning scenarios, such as cloud computing and federated learning, it is possible to learn one personalized model per user. In this work, we present a systematic learning-theoretic study of personalization. We propose and analyze three approaches: user clustering, data interpolation, and model interpolation. For all three approaches, we provide learning-theoretic guarantees and efficient algorithms for which we also demonstrate the performance empirically. All of our algorithms are model agnostic and work for any hypothesis class.

1. Introduction

A popular application of language models is virtual keyboard applications, where the goal is to predict the next word, given the previous words (Hard et al., 2018). For example, given “I live in the state of”, ideally, it should guess the state the user intended to type. Suppose we train a single model on all the user data and deploy it, then the model would predict the same state for all users and would not be a good model for most users. Similarly, in many practical applications, the distribution of data across clients is highly non-i.i.d. and training a single global model for all clients may not be optimal.

Thus, we study the problem of learning personalized models, where the goal is to train a model for each client, based on the client’s own dataset and the datasets of other clients. Such an approach would be useful in applications with the natural infrastructure to deploy a personalized model for each client, which is the case with large-scale learning scenarios such as federated learning (FL) (McMahan et al., 2017).

Before we proceed further, we highlight one of our use cases in FL. In FL, typically a centralized global model is trained based on data from a large number of clients, which may be mobile phones, other mobile devices, or sensors (McMahan et al., 2017; Konečný et al., 2016b,a) using a variant of stochastic gradient
descent called FedAvg. This global model benefits from having access to client data and can often perform better on several learning problems, including next word prediction (Hard et al., 2018; Yang et al., 2018) and predictive models in health (Brisimi et al., 2018). We refer to Appendix A for more details on FL.

Personalization in FL was first studied by Wang et al. (2019). They showed that federated models can be fine-tuned based on local data. They proposed methods to find the best hyper-parameters for fine-tuning and showed that it improves the next word prediction of language models in virtual keyboard applications. Recently, Jiang et al. (2019) drew interesting connections between FedAvg and first order model agnostic meta learning (Finn et al., 2017) and showed that FedAvg is already a meta-learning algorithm. Apart from these, personalization in the context of FL has not been well studied theoretically.

We provide a learning-theoretic framework, generalization guarantees, and computationally efficient algorithms for personalization. Since FL is one of the main frameworks where personalized models can be used, we propose efficient algorithms that take into account computation and communication bottlenecks.

2. Preliminaries

Before describing the mathematical details of personalization, we highlight two related models. The first one is the global model trained on data from all the clients. This can be trained using either standard empirical risk minimization (Vapnik, 1992) or other methods such as agnostic risk minimization (Mohri et al., 2019). The second baseline model is the purely local model trained only on the client’s data.

The global model is trained on large amounts of data and generalizes well; however it does not perform well for clients whose data distributions are very different from the global train data distribution. On the other hand, the train data distributions of local models match the ones at inference time, but they do not generalize well due to the scarcity of data.

Personalized models can be viewed as intermediate models between pure-local and global models. Thus, the hope is that they incorporate the generalization properties of the global model and the distribution matching property of the local model. Before we proceed further, we first introduce the notation used in the rest of the paper.

2.1. Notation

We start with some general notation and definitions used throughout the paper. Let $X$ denote the input space and $Y$ the output space. We will primarily discuss a multi-class classification problem where $Y$ is a finite set of classes, but much of our results can be extended straightforwardly to regression and other problems. The hypotheses we consider are of the form $h: X \rightarrow \Delta^Y$, where $\Delta^Y$ stands for the simplex over $Y$. Thus, $h(x)$ is a probability distribution over the classes that can be assigned to $x \in X$. We will denote by $\mathcal{H}$ a family of such hypotheses $h$. We also denote by $\ell$ a loss function defined over $\Delta^Y \times Y$ and taking non-negative values. The loss of $h \in \mathcal{H}$ for a labeled sample $(x, y) \in X \times Y$ is given by $\ell(h(x), y)$. Without loss of generality, we assume that the loss $\ell$ is bounded by one. We will denote by $\mathcal{L}_{\mathcal{D}}(h)$ the expected loss of a hypothesis $h$ with respect to a distribution $\mathcal{D}$ over $X \times Y$:

$$\mathcal{L}_{\mathcal{D}}(h) = \mathbb{E}_{(x, y) \sim \mathcal{D}}[\ell(h(x), y)],$$
and by $h_D$ its minimizer: $h_D = \arg\min_{h \in \mathcal{H}} \mathcal{L}_D(h)$. Let $\mathcal{R}_{\mathcal{D}, m}(\mathcal{H})$ denote the Rademacher complexity of class $\mathcal{H}$ over the distribution $\mathcal{D}$ with $m$ samples.

Let $p$ be the number of clients. The distribution of samples of client $k$ is denoted by $D_k$. Clients do not know the true distribution, but instead, have access to $m_k$ samples drawn i.i.d. from the distribution $D_k$. We will denote by $\widehat{D}_k$ the corresponding empirical distribution of samples and by $m = \sum_{k=1}^{p} m_k$ the total number of samples.

2.2. Local model

We first ask when it is beneficial for a client to participate in global model training. Consider a canonical user with distribution $D_1$. Suppose we train a purely local model based on the client’s data and obtain a model $\widehat{h}_{\widehat{D}_1}$. By standard learning-theoretic tools (Mohri et al., 2018a), the performance of this model can be bounded as follows: with probability at least $1 - \delta$, the minimizer of empirical risk $\mathcal{L}_{\widehat{D}_1}(h)$ satisfies

$$\mathcal{L}_{D_1}(\widehat{h}_{\widehat{D}_1}) \leq \mathcal{L}_{D_1}(h_{D_1}) + O\left(\sqrt{d \log 1/\delta} \sqrt{m_1}\right),$$

where $d$ is the VC-dimension of the hypothesis class $\mathcal{H}$. From (1), it is clear that local models perform well when the number of samples $m_1$ is large. However, this is often not the case. In many realistic settings, such as virtual keyboard models, the average number of samples per user is in the order of hundreds, whereas the VC-dimension of the hypothesis class is in millions (Hard et al., 2018). In such cases, the above bound becomes vacuous.

2.3. Uniform global model

The global model is trained by minimizing the empirical risk on the concatenation of all the samples. For $\lambda \in \Delta^p$, the weighted average distribution $D_\lambda$ is given by $\sum_k \lambda_k D_k$. The global model is trained on the concatenated samples from all the users and hence is equivalent to minimize the loss on the distribution $\widehat{U} = \sum_k \lambda'_k \widehat{D}_k$, where $\lambda'_k = m_k/m$. Since the global model is trained on data from all the clients, it may not match the actual underlying client distribution and thus may perform worse.

The divergence between distributions is often measured by a Bregman divergence such as KL-divergence or unnormalized relative entropy. However, such divergences do not consider the task at hand. To obtain better bounds, we use the notion of label-discrepancy between distributions (Mansour et al., 2009). For two distributions over features and labels, $D_1$ and $D_2$, and a class of distributions $\mathcal{H}$, discrepancy is given by

$$\text{disc}_{\mathcal{H}}(D_1, D_2) = \max_{h \in \mathcal{H}} |\mathcal{L}_{D_1}(h) - \mathcal{L}_{D_2}(h)|.$$

If the loss of all the hypotheses in the class is the same under both $D_1$ and $D_2$, then the discrepancy is zero and models trained on $D_1$ generalize well on $D_2$ and vice versa. Thus, discrepancy takes into account both the hypothesis set and the loss functions, and hence the structure of the learning problem.

With the above definitions, it can be shown that the uniform global model generalizes as follows: with probability at least $1 - \delta$, the minimizer of empirical risk on the uniform distribution satisfies

$$\mathcal{L}_{D_1}(h_{\widehat{U}}) \leq \mathcal{L}_{D_1}(h_{\widehat{D}_1}) + O\left(\sqrt{d + \log 1/\delta} \sqrt{\frac{m}{m_1}}\right) + \text{disc}_{\mathcal{H}}(D_1, U).$$

(2)
Since the global model is trained on the concatenation of all users’ data, it generalizes well. However, due to the distribution mismatch, the model may not perform well for a specific user. If $\mathcal{U} = \sum_k \lambda_k \mathcal{D}_k$, the difference between local and global models depends on the discrepancy between $\mathcal{D}_1$ and $\mathcal{U}$, $m_1$ the number of samples from the domain $\mathcal{D}_1$, and the total number of samples $m$. While in most practical applications $m_1$ is small and hence a global model usually performs better, this is not guaranteed. We provide the following simple example, which shows that global models can be a constant worse compared to the local model.

**Example 1** Let $\mathcal{X} = \mathbb{R}$ and $\mathcal{Y} = \{0, 1\}$. Suppose there are two clients with distributions $\mathcal{D}_1$ and $\mathcal{D}_2$ defined as follows. $\forall x, \mathcal{D}_1(x) = \mathcal{D}_2(x)$ and $\mathcal{D}_1(1|x) = 1$ if $x > 0$ and zero otherwise. Similarly, $\mathcal{D}_2(1|x) = 1$ only if $x < 0$ and zero otherwise. Let $\mathcal{H}$ be the class of threshold classifiers indexed by a threshold $t \in \mathbb{R}$ and sign $s \in \{-1, 1\}$ such that $h_{t,s} \in \mathcal{H}$ is given by $h_{t,s}(x) = 1_{(x-t)s>0}$. Further, suppose we are interested in zero-one loss and the number of samples from both domains is very large and equal. The optimal classifier for $\mathcal{D}_1$ is $h_{0,1}$ and the optimal classifier for $\mathcal{D}_2$ is $h_{0,-1}$, and they achieve zero error in their respective clients. Since the number of samples is the same from both clients, $\mathcal{U}$ is the uniform mixture of the two domains, $\mathcal{U} = 0.5 \mathcal{D}_1 + 0.5 \mathcal{D}_2$. Note that for all $h \in \mathcal{H}$, $\mathcal{L}_\mathcal{U}(h) = 0.5$ and hence the global objective cannot differentiate between any of the hypotheses in $\mathcal{H}$. Thus, with high probability, any globally trained model incurs a constant loss on both clients.

Since the uniform global model assigns weight $m_k/m$ to client $k$, clients with larger numbers of samples receive higher importance. This can adversely affect clients with small amounts of data. Furthermore, by (2), the model may not generalize well for clients whose distribution is different than the uniform distribution. Thus, (1) and (2) give some guidelines under which it is beneficial for clients to participate in global model training.

Instead of using uniform weighting of samples, one can use agnostic risk, which is less risk averse. We refer to Appendix B for details about the agnostic risk minimization.

### 2.4. Learning theory for personalization

We ask if personalization can be achieved by an intermediate model between the local and global models. This gives rise to three natural algorithms, which are orthogonal and can be used separately or together.

- Train a model for subsets of users: we can cluster users into groups and train a model for each group. We refer to this as user clustering, or more reﬁnely hypothesis-based clustering.
- Train a model on interpolated data: we can combine the local and global data and train a model on their combination. We refer to this as data interpolation.
- Combine local and global models: we can train a local and a global model and use their combination. We refer to this as model interpolation.

In the rest of the paper, we study each of the above methods.

### 3. User clustering

Instead of training a single global model, a natural approach is to cluster clients into groups and train a model for each group. This is an intermediate model between a purely local and global model and provides
a trade-off between generalization and distribution mismatch. If we have a clustering of users, then we can naturally find a model for each user using standard optimization techniques. In this section, we ask how to define clusters. Clustering is a classical problem with a broad literature and known algorithms (Jain, 2010). We argue that, since the subsequent application of our clustering is known, incorporating it into the clustering algorithm will be beneficial.

### 3.1. Baselines

If we have meta-features about the data samples and clients, such as location or type of device, we can use them to find clusters. This can be achieved by algorithms such as *k*-means or variants. This approach depends on the knowledge of the meta-features and their relationship to the set of hypotheses under consideration. While it may be reasonable in many circumstances, it may not be always feasible.

If there are no meta-features, a natural approach is to cluster using a Bregman divergence defined over the distributions $\mathcal{D}_k$ (Banerjee et al., 2005). However, it is likely that we would overfit as the generalization of the density estimation depends on the VC-dimension of the class $\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_p$, which in general can be much larger than that of the class of hypotheses $\mathcal{H}$. To overcome this, we propose an approach based on hypotheses under consideration which we discuss next.

### 3.2. Hypothesis-based clustering

Consider the scenario where we are interested in finding clusters of images for a facial recognition task. Suppose we are interested in finding clusters of users for each skin-color and find a good model for each cluster. If we naively use the Bregman divergence clustering, it may focus on clustering based on the image background e.g., outdoor or indoors to find clusters instead of skin color.

To overcome this, we propose to incorporate the task at hand to obtain better clusters. We refer to this approach as hypothesis-based clustering and show that it admits better generalization bounds than the Bregman divergence approach. We partition users into $q$ clusters and find the best hypothesis for each cluster. In particular, we use the following optimization:

$$
\min_{h_1, h_2, \ldots, h_q} \sum_{k=1}^{p} \lambda_k \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i),
$$

(3)

where $\lambda_k$ is the importance of client $k$. The above loss function trains $q$ best hypotheses and naturally divides $\mathcal{X} \times \mathcal{Y}$ into $q$ partitions, where each partition is associated with a particular hypothesis $h_k$.

In practice, we only have access to the empirical distributions $\widehat{\mathcal{D}}_k$. To simplify the analysis, we use the fraction of samples from each user $m_k/m$ as $\lambda_k$. An alternative approach is to use $\lambda_k = 1/p$ for all users, which assigns equal weight to all clients. The analysis is similar and we omit it to be concise. In particular, we propose to solve for

$$
\min_{h_1, h_2, \ldots, h_q} \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\widehat{\mathcal{D}}_k}(h_i).
$$

(4)
3.3. Generalization bounds

We now analyze the generalization properties of this technique. First we state a lemma about generalization bounds.

**Lemma 1 (Appendix C.1)** Let \( h_1^*, h_2^*, \ldots, h_q^* \) be the \( q \) models obtained by solving \( (3) \) and \( \hat{h}_1^*, \hat{h}_2^*, \ldots, \hat{h}_q^* \) be the \( q \) models obtained by solving \( (4) \). Then,

\[
\sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(\hat{h}_i^*) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) \
\leq 2 \max_{h_1, h_2, \ldots, h_q} \left| \sum_{k=1}^{p} \frac{m_k}{m} \cdot \left( \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i) - \min_{i \in [q]} \mathcal{L}_{\hat{\mathcal{D}}_k}(h_i) \right) \right|.
\]

Thus it suffices to provide bounds on this last term. Since we are bounding the maximum difference between true cluster based loss and empirical cluster based loss for all hypotheses, this bound holds for any clustering algorithm. Let \( C_1, C_2, \ldots, C_q \) be the clusters and \( m_{C_i} \) be the number of samples from cluster \( i \). Let \( \hat{C}_i \) and \( C_i \) be the empirical and true distributions of cluster \( C_i \). With these definitions, we now bound the generalization error of this technique.

**Theorem 2 (Appendix C.2)** With probability at least \( 1 - \delta \),

\[
\max_{h_1, h_2, \ldots, h_q} \left| \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\hat{\mathcal{D}}_k}(h_i) \right| 
\leq 2 \sqrt{\frac{p \log 2q}{m}} + \max_{C_1, C_2, \ldots, C_q} \frac{1}{m} \sum_{i=1}^{q} m_{C_i} \mathcal{R}_{C_i, m_{C_i}}(\mathcal{H}).
\]

**Corollary 3 (Appendix C.3)** Let \( d \) be the VC-dimension of \( \mathcal{H} \). Then with probability at least \( 1 - \delta \), the following holds:

\[
\max_{h_1, h_2, \ldots, h_q} \left| \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\hat{\mathcal{D}}_k}(h_i) \right| 
\leq 2 \sqrt{\frac{p \log 2q}{m}} + \frac{dq}{m} \log \frac{m}{d}.
\]

The above learning bound can be understood as follows. For good generalization, the average number of samples per user \( m/p \) should be larger than the logarithm of the number of clusters, and the average number of samples per cluster \( m/q \) should be larger than the VC-dimension of the overall model. Somewhat surprisingly, these results do not depend on the minimum number of samples per clients and instead depend only on the average statistics.

To make a comparison between the local performance \( (1) \) and the global model performance \( (2) \), observe that combining \( (5) \) and Corollary \( 3 \) together with the definition of discrepancy yields

\[
\sum_{k=1}^{p} \frac{m_k}{m} \cdot \mathcal{L}_{\mathcal{D}_k}(\overline{h}(f(k))) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{h \in H} \mathcal{L}_{\mathcal{D}_k}(h) 
\leq 2 \sqrt{\frac{p \log 2q}{m}} + \sqrt{\frac{dq}{m} \log \frac{m}{d}} + \sum_{k=1}^{p} \frac{m_k}{m} \cdot \text{disc}(\mathcal{D}_k, C_{f(k)}),
\]

where \( f: [p] \to [q] \) is the mapping from users to clusters. Thus, the generalization bound is in between that of the local and global model. For \( q = 1 \), it yields the global model, and for \( q = p \), it yields the local model.
Algorithm HYPCluster

**Initialize:** Randomly sample $P$ clients, train a model on them, and initialize $h^0_i$ for all $i \in [q]$ using them randomly. For $t = 1$ to $T$ do the following:

1. Randomly sample $P$ clients.
2. Recompute $f^t$ for clients in $P$ by assigning each client to the cluster that has lowest loss:
   \[ f^t(k) = \arg\min_i \mathcal{L}_{\hat{D}_k}(h^t_{i-1}). \] (6)
3. Run few steps of SGD for $h^t_{i-1}$ with data from clients $P \cap (f^t)^{-1}(i)$ to minimize
   \[ \sum_{k: P \cap (f^t)^{-1}(i)} m_k \mathcal{L}_{\hat{D}_k}(h^t_i), \]
   and obtain $h^t_i$.

Compute $f^{T+1}$ by using $h^T_1, h^T_2, \ldots, h^T_q$ via (6) and output it.

Figure 1: Pseudocode for HYPCluster algorithm.

As we increase $q$, the generalization decreases and the discrepancy term gets smaller. Allowing a general $q$ lets us choose the best clustering scheme and provides a smooth trade-off between the generalization and the distribution matching. In practice, we choose small values of $q > 1$. We further note that we are not restricted to using the same value of $q$ for all clients. We can find clusters for several values of $q$ and use the best one for each client separately using a hold-out set of samples.

3.4. Algorithm: HYPCluster

We provide an expectation-maximization (EM)-type algorithm for minimizing (4). A naive EM modification may require heavy computation and communication resources. To overcome this, we propose a stochastic EM algorithm in HYPCluster. In the algorithm, we denote clusters via a mapping $f: [p] \rightarrow [q]$, where $f(k)$ denotes the cluster of client $k$. Similar to $k$-means, HYPCluster is not guaranteed to converge to the true optimum, but, as stated in the beginning of the previous section, the generalization guarantee of Theorem 2 still holds here.

4. Data interpolation

From the point of view of client $k$, there is a small amount of data with distribution $\mathcal{D}_k$ and a large amount of data from the global or clustered distribution $\mathcal{C}$. How are we to use auxiliary data from $\mathcal{C}$ to improve the model accuracy on $\mathcal{D}_k$? This relates the problem of personalization to domain adaptation. In domain adaptation, there is a single source distribution, which is the global data or the cluster data, and a single target distribution, which is the local client data. As in domain adaptation with target labels (Blitzer et al., 2008), we have at our disposal a large amount of labeled data from the source (global data) and a small amount of labeled data from the target (personal data). We propose to minimize the loss on the concatenated data,

\[ \lambda \cdot \mathcal{D}_k + (1 - \lambda) \cdot \mathcal{C}, \] (8)
Algorithm DAPPER($h_c$)
For each client $k$ do the following:
1. Randomly sample $r \cdot m_k$ data points from $\hat{C}$. Let this dataset be $\hat{C}'$.
2. Let $\Lambda$ be a cover of $[0, 1]$. For each $\lambda \in \Lambda$, the client starts with $h_c$ and minimizes
   \[
   \lambda \mathcal{L}_{\hat{D}_k}(h) + (1 - \lambda) \mathcal{L}_{\hat{C}'}(h)
   \]
   using stochastic gradient descent for $r \cdot m_k$ steps, where at each step, it selects $\hat{D}_k$ with probability $\lambda$ and $\hat{C}'$ with probability $1 - \lambda$ and samples an element from the corresponding dataset to compute the stochastic gradient. Let $h_\lambda$ be the resulting model and $\mathcal{H}_\Lambda = \{h_\lambda : \lambda \in \Lambda\}$.
3. Output $\arg\min_{h \in \mathcal{H}_\Lambda} \mathcal{L}_{\hat{D}_k}(h)$.

Figure 2: Pseudocode for the DAPPER algorithm.

where $\lambda$ is a hyper-parameter and can be obtained by either cross validation or by using the generalization bounds of Blitzer et al. (2008). $\hat{C}$ can either be the uniform distribution $\mathcal{U}$ or one of the distributions obtained via clustering.

Personalization is different from most domain adaptation works as they assume they only have access to unlabeled target data, whereas in personalization we have access to labeled target data. Secondly, we have one target domain per client, which makes our problem computationally expensive, which we discuss next. Given the known learning-theoretic bounds, a natural question is if we can efficiently estimate the best hypothesis for a given $\lambda$. However, note that naive approaches suffer from the following drawbacks. If we optimize for each client separately, the time complexity of learning per client is $O(m)$ and the overall time complexity is $O(m \cdot p)$.

In addition to the computation time, the algorithm also admits a high communication cost in FL. This is because, to train the model with a $\lambda$-weighted mixture requires the client to admit access to the entire dataset $\hat{C}$, which incurs communication cost $O(m)$. One empirically popular approach to overcome this is the fine-tuning approach, where the central model is fine-tuned on the local data (Wang et al., 2019). However, to the best of our knowledge, there are no theoretical guarantees and the algorithm may be prone to catastrophic forgetting (Goodfellow et al., 2013).

We propose DAPPER, a theoretically motivated and efficient algorithm to overcome the above issues. The algorithm first trains a central model on the overall empirical distribution $\hat{C}$. Then for each client, it subsamples $\hat{C}$ to create a smaller dataset of size $\hat{C}'$ of size $r \cdot m_k$, where $r$ is a constant. It then minimizes the weighted combination of two datasets as in (7) for several values of $\lambda$. Finally, it chooses the best $\lambda$ using cross-validation. The algorithm is efficient both in terms of its communication complexity which is $r \cdot m_k$ and its computation time, which is at most $(r + 1) \cdot m_k$. Hence, the overall communication and computation time is $O(r \cdot m)$. 

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4.1. Convergence analysis

We analyze DAPPER when the loss function is strongly convex in the hypothesis parameters $h$ and show that the model minimizes the intended loss to the desired accuracy. To the best of our knowledge, this is the first fine-tuning algorithm with provable guarantees that ensures that there is no catastrophic forgetting.

To prove convergence guarantees, we need to ask what the desired convergence guarantee is. Usually, models are required to converge to the generalization guarantee and we use the same criterion. To this end, we first state a known generalization theorem. Let $h_{\lambda}^\wedge = \arg\min_\lambda \lambda L_{Dk}(h) + (1-\lambda)L_{C}(h)$ and $h_\lambda = \arg\min_\lambda \lambda L_{Dk}(h) + (1-\lambda)L_{C}(h)$.

Lemma 4 (Blitzer et al., 2008) If the VC-dimension of the hypothesis class is $d$, then with probability at least $1 - \delta$,

$$\lambda L_{Dk}(h_{\lambda}^\wedge) + (1-\lambda)L_{C}(h_{\lambda}^\wedge) - \lambda L_{Dk}(h_{\lambda}) - (1-\lambda)L_{C}(h_{\lambda}) = \mathcal{O}\left(\frac{\lambda^2}{m_k} + \frac{(1-\lambda)^2}{m_C} \cdot \sqrt{d \log \frac{1}{\delta}}\right).$$

Since the generalization bound scales as $\sqrt{\frac{\lambda^2}{m_k} + \frac{(1-\lambda)^2}{m_C}}$, the same accuracy in convergence is desired. Let $\epsilon_\lambda = \sqrt{\frac{\lambda^2}{m_k} + \frac{(1-\lambda)^2}{m_C}}$, denote the desired convergence guarantee. For strongly convex functions, we show that one can achieve this desired accuracy using DAPPER, furthermore the amount of additional data is a constant multiple of $m_k$, independent of $\lambda$ and $m$.

Theorem 5 (Appendix D) Assume that the loss function is $\mu$-strongly convex and assume that the gradients are $G$-smooth. Let $\mathcal{F}$ admit diameter at most $R$. Let $r \geq G^2 \left(\frac{G}{\mu} + 2R\right)^2$, a constant independent of $\lambda$. Let the learning rate $\eta = \frac{1}{G\sqrt{m_k}} \min\left(\frac{2G\lambda}{\mu(1-\lambda)}, R\right)$. Then after $rm_k$ steps of SGD, the output $h_A$ satisfies,

$$\mathbb{E}[\lambda L_{Dk}(h_A) + (1-\lambda)L_{C}(h_A)] \leq \mathbb{E}[\lambda L_{Dk}(h_{\lambda}^\wedge) + (1-\lambda)L_{C}(h_{\lambda}^\wedge)] + \epsilon_\lambda.$$

4.2. Practical considerations

While the above algorithm reduces the amount of data transfer and is computationally efficient, it may be vulnerable to privacy issues in applications such as FL. To overcome that, we propose several alternatives:

1. Sufficient statistics: in many scenarios, instead of the actual data, we only need some sufficient statistics. For example in regression with $\ell_2^2$ loss, we only need the covariance matrix of the dataset from $\hat{C}$.

2. Generative models: for problems such as density estimation and language modelling, we can use the centralized model to generate synthetic samples from $h_c$ and use that as an approximation to $\hat{C}$. For other applications, one can train a GAN and send the GAN to the clients and the clients can sample from the GAN to create the dataset $\hat{C}$ (Augenstein et al., 2019).

3. Proxy public data: if it is not feasible to send the actual user data, one could send proxy public data instead. While this may not be theoretically optimal, it will still avoid overfitting to the local data.
5. Model interpolation

The above approaches assume that the final inference model belongs to class \( \mathcal{H} \). In practice, this may not be the case. One can learn a central model \( h_c \) from a class \( \mathcal{H}_c \), and learn a local model \( h_l \) from \( \mathcal{H}_l \), and use their interpolated model

\[
\lambda \cdot h_l + (1 - \lambda) \cdot h_c.
\]
during inference. Such interpolated models are routinely used in applications such as virtual keyboards.

More formally, let \( h_c \) be the central or cluster model and let \( \tilde{h}_l = (h_{l,1}, h_{l,2}, \ldots , h_{l,p}) \), where \( h_{l,k} \) is the local model for client \( k \). Let \( \lambda_k \) be the interpolated weight for client \( k \) and let \( \lambda = \lambda_1, \lambda_2, \ldots , \lambda_p \). If one has access to the true distributions, then learning the best interpolated models can be formulated as the following optimization,

\[
\min_{h_c, h_l, \lambda_k} \sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{D_k}((1 - \lambda_k)h_c + \lambda_k h_{l,k}).
\]

Since, the learner does not have access to the true distributions, we propose the following optimization,

\[
\min_{h_c, h_l, \lambda_k} \sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{\tilde{D}_k}((1 - \lambda_k)h_c + \lambda_k h_{l,k}).
\]

5.1. Generalization bounds

We now show a generalization bound for the above optimization.

**Theorem 6 (Appendix E)** Assume that the loss \( \ell \) is \( L \) Lipschitz. Let \( \mathcal{H}_c \) be the hypotheses class for the central model and \( \mathcal{H}_l \) be the hypotheses class for the local models. Let \( h_c^*, \lambda^*, \tilde{h}_l^* \) be the optimal values and \( \bar{h}_c^*, \bar{\lambda}^*, \bar{\tilde{h}}_l^* \) be the optimal values for the empirical estimates. Then with probability at least \( 1 - \delta \),

\[
\sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{D_k}((1 - \bar{\lambda}_k^*)\bar{h}_c^* + \bar{\lambda}_k^* \bar{\tilde{h}}_l^*) - \sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{D_k}((1 - \lambda_k^*)h_c^* + \lambda_k^* h_{l,k}^*)
\]

\[
\leq 2L \left( \mathcal{R}_{l,m}(\mathcal{H}_c) + \sum_{k=1}^{p} \frac{m_k}{m} \mathcal{R}_{D_k,m_k}(\mathcal{H}_l) \right) + 2 \sqrt{\frac{\log \frac{1}{\delta}}{m}}.
\]

Standard upper bounds on Rademacher complexity by the VC-dimension, combined with Jensen’s inequality yields the following result.

**Corollary 7** Assume that \( \ell \) is \( L \) Lipschitz. Let \( h_c^*, \lambda^*, \tilde{h}_l^* \) be the optimal values and \( \bar{h}_c^*, \bar{\lambda}^*, \bar{\tilde{h}}_l^* \) be the optimal values for the empirical estimates. Then with probability at least \( 1 - \delta \), the following holds:

\[
\sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{D_k}((1 - \bar{\lambda}_k^*)\bar{h}_c^* + \bar{\lambda}_k^* \bar{\tilde{h}}_l^*) - \sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{D_k}((1 - \lambda_k^*)h_c^* + \lambda_k^* h_{l,k}^*)
\]

\[
\leq 2L \left( \sqrt{\frac{d_c}{m} \log \frac{em}{d_c}} + \sqrt{\frac{d_l p}{m} \log \frac{em}{d_l}} \right) + 2 \sqrt{\frac{\log \frac{1}{\delta}}{m}},
\]

where \( d_c \) is the VC-dimension of \( \mathcal{H}_c \) and \( d_l \) is the VC-dimension of \( \mathcal{H}_l \).
Algorithm MAPPER
Randomly initialize $h_0^c$ and for $t = 1$ to $T$, randomly select a client $k$ and do the following.
1. Let $\Lambda$ be a cover of $[0, 1]$. For each $\lambda \in \Lambda$, let
   
   $h_{l,k}(\lambda) = \arg\min_{h_{l,k}} \mathcal{L}_{D_k}(\lambda h_{l,k} + (1 - \lambda) h_c^{t-1}).$

   (9)

2. Find the best local model:
   
   $\lambda^* = \arg\min_{\lambda \in \Lambda} \mathcal{L}_{D_k}(\lambda h_{l,k}(\lambda) + (1 - \lambda) h_c^{t-1}).$

   (10)

3. Minimize the global model.
   
   $h_c^t = h_c^{t-1} - \eta \nabla \mathcal{L}_{D_k}(\lambda^* h_{l,k}(\lambda^*) + (1 - \lambda^*) h_c^{t-1}).$

   (11)

Let $h_c^T$ be the final global model. For each client $k$ rerun 1(a) and 1(b) to obtain the local model $h_{l,k}$ and the interpolation weight $\lambda_k$.

Figure 3: Pseudocode for the MAPPER algorithm.

Hence for models to generalize well, it is desirable to have $m \gg d_c$ and the average number of samples to be much greater than $d_l$, i.e., $m/p \gg d_l$. Similar to Corollary 3, this bound only depends on the average number of samples and not the minimum number of samples.

5.2. Algorithms

A common approach for model interpolation in practice is to first train the central model $h_c$ and then train the local model $h_l$ separately and find the best interpolation coefficients, i.e.,

$$\hat{h}_c = \arg\min_{h_c} \frac{1}{p} \sum_{k=1}^{p} \frac{m_k}{m} \mathcal{L}_{D_k}(h_c) \quad \text{and} \quad \hat{h}_{l,k} = \arg\min_{h_{l,k}} \mathcal{L}_{D_k}(h_{l,k}).$$

We first show that this method of independently finding the local models is sub-optimal with an example.

**Example 2** Consider the following discrete distribution estimation problem. Let $\mathcal{H}_c$ be the set of distributions over $d$ values and let $\mathcal{H}_l$ be the set of distributions with support size 1. For even $k$, let $\mathcal{D}_k(1) = \mathcal{D}_{even}(1) = 1.0$ and for odd $k$, let $\mathcal{D}_k(y) = \mathcal{D}_{odd}(y) = 1/d$ for all $1 \leq y \leq d$. Let the number of clients $p$ be very large and the number of samples per client a constant, say ten. Suppose we consider the log-loss.

The intuition behind this example is that since we have only one example per domain, we can only derive good estimates for the local model for even $k$ and we need to estimate the global model jointly from the odd clients. With this approach, the optimal solution is as follows. For even $k$, $h_{l,k} = \mathcal{D}_k$ and $\lambda_k = 1.0$. For odd $k$, $\lambda_k = 0.0$ and the optimal $h_c$ is given by, $h_c = \mathcal{D}_{odd}$. If we learn the models separately, observe that, for each client $\hat{h}_{l,k}$ be the empirical estimate and $\hat{h}_c$ would be $0.5 \cdot \mathcal{D}_{even} + 0.5 \cdot \mathcal{D}_{odd}$. Thus, for any $\lambda$, the algorithm would incur at least a constant loss more than optimal for any $\lambda_k$ for odd clients.
Table 1: Test loss of HYPCLUSTER as a function of number of clusters $q$.

| $q$ | 1  | 2  | 3  | 4  | 5  |
|-----|----|----|----|----|----|
| test loss | 3.6 | 3.2 | 3.0 | 2.8 | 2.8 |

Table 2: Test loss of various algorithms as a function of number of samples per user.

| $m_k$ | GLOBAL | HYPCLUSTER | DAPPER | MAPPER |
|-------|--------|------------|--------|--------|
| 10    | 3.7    | 3.0        | 3.6    | 2.4    |
| 100   | 3.6    | 2.8        | 3.0    | 2.1    |
| 1000  | 3.6    | 2.8        | 2.2    | 2.1    |

Since training models independently is sub-optimal in certain cases, we propose a joint-optimization algorithm. First observe that the optimization can be rewritten as

$$\min_{h_c, \lambda} \sum_{k=1}^{p} m_k \min_{h, \lambda} \mathcal{L}_{\mathcal{D}_k} ((1 - \lambda_k)h_c + \lambda_k h_{l,k}).$$

Notice that for a fixed $\lambda$ the function is convex in both $h^l$ and $h_c$. But with the minimization over $\lambda$, the function is no longer convex. We propose algorithm MAPPER for minimizing the interpolation models. At each round, the algorithm randomly selects a client. It then finds the best local model and interpolation weight for that client using the current value of the global model. It then updates the global model using the local model and the interpolation weight found in the previous step. In practice, dividing $\mathcal{D}_k$ into three parts: $\mathcal{D}_k'$, $\mathcal{D}_k''$, and $\mathcal{D}_k'''$ and using each of these separately for (9), (10), and (11) leads to better performance.

6. Experiments

6.1. Synthetic dataset

We first demonstrate the proposed algorithms on a synthetic dataset for density estimation. Let $X = \emptyset$, $Y = [d]$, and $d = 100$. Let $\ell$ be cross entropy loss and the number of users $p = 100$. We create client distributions as a mixture with a uniform component, a cluster component, and an individual component. The details of the distributions are in Appendix F.1.

We evaluate the algorithms as we vary the number of samples per user. The results are in Table 2. Note that MAPPER performs uniformly well across all values of $m_k$. However, the performance difference between HYPCLUSTER and DAPPER, depends on the number of samples per user.

In order to understand the effect of clustering, we evaluate various clustering algorithms as a function of $q$ when for all clients, $m_k = 100$, and the results are in Table 1. Since the clients are naturally divided into four clusters, as we increase $q$, the test loss steadily decreases till the number of clusters reaches 4 and then remains constant.

6.2. EMNIST dataset

We evaluate the proposed algorithms on the federated EMNIST-62 dataset (Caldas et al., 2018) provided by TensorFlow Federated (TFF). The dataset consists of 3400 users’ examples that are each labeled as one
Table 3: Test accuracy of seen and unseen clients (unshuffled).

| algorithm    | seen acc. | unseen acc. |
|--------------|-----------|-------------|
| BASELINE     | 85.2%     | 73.5%       |
| HYPCLUSTER   | 87.5%     | 81.0%       |
| FINETUNE     | 90.4%     | 88.7%       |
| DAPPER       | 90.6%     | 89.0%       |
| MAPPER       | 93.3%     | 91.2%       |

Table 4: Test accuracy of seen and unseen clients (shuffled).

| algorithm    | seen acc. | unseen acc. |
|--------------|-----------|-------------|
| BASELINE     | 83.1%     | 82.9%       |
| HYPCLUSTER   | 85.9%     | 85.7%       |
| FINETUNE     | 90.1%     | 90.0%       |
| DAPPER       | 90.2%     | 90.2%       |
| MAPPER       | 92.8%     | 92.9%       |

of 62 classes (lower and upper case letters and digits). The original TFF dataset is split only into train and test, so we further split train into train and eval such that all users have at least one example in each split. Additionally, within each split, we use the first 2500 users to train the global or clustered models and leave the remaining 900 as new unseen clients. The unseen clients do not participate in central model training and are reserved for evaluation only. The reported metrics are uniformly averaged across all clients similar to previous works (Jiang et al., 2019).

For model architecture, we use a two-layer convolutional neural net with hyper-parameters tuned using the eval dataset. We refer to Appendix F.2 for more details. We train the model for 1000 communication rounds with 20 clients per round and use server side momentum. One can use different optimizers as proposed by Jiang et al. (2019). Evaluating the combined effect of our approach and adaptive optimizers remains an interesting open direction. For FINETUNE and DAPPER, we use the best baseline model as the pretrained starting global model since its training is independent of client fine-tuning. For MAPPER we use the same model architecture for both local and global models and for the ease of training, at each optimization step, we initialize the local model using the parameters of the global model at that step.

We first split the seen and unseen clients using the original ordering. For this case, Table 3 reports the accuracy of each algorithm on the seen and unseen client test data averaged over 20 trials. However, there is a distinct difference in seen and unseen accuracy in BASELINE, which possibly indicates a natural ordering of clients. HYPCLUSTER further supports this as the best trials have two clusters and the unseen clients all map to only one. This experiment models the scenario where there is a distribution shift over the clients.

We then randomly shuffle the client before splitting into seen and unseen. The results for this case are in Table 4. After shuffling the clients, the BASELINE seen and unseen accuracy is much closer and the client cluster distribution in HYPCLUSTER is much more balanced.

We observe that in both the shuffled and unshuffled cases, MAPPER performs the best in seen and unseen accuracy, followed by DAPPER, FINETUNE, and HYPCLUSTER respectively. Additionally, all three novel approaches provide better and more balanced performance compared to BASELINE. This is especially pronounced in the unseen clients for the unshuffled scenarios.
7. Conclusion

We presented a systematic learning-theoretic study of personalization in learning and proposed and analyzed three algorithms: user clustering, data interpolation, and model interpolation. For all three approaches, we provided learning theoretic guarantees and efficient algorithms. Finally, we empirically demonstrated the usefulness of the proposed approaches on synthetic and EMNIST datasets.

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Appendix A. Federated learning

FL was introduced by McMahan et al. (2017) as an efficient method for training models in a distributed way. They proposed a new communication-efficient optimization algorithm called FedAvg. They also showed that the training procedure provides additional privacy benefits. The introduction of FL has given rise to several interesting research problems, including the design of more efficient communication strategies (Konečný et al., 2016b,a; Suresh et al., 2017; Stich, 2018; Karimireddy et al., 2019), the study of lower bounds for parallel stochastic optimization with a dependency graph (Woodworth et al., 2018), devising efficient distributed optimization methods benefiting from differential privacy guarantees (Agarwal et al., 2018), stochastic optimization solutions for the agnostic formulation (Mohri et al., 2019), and incorporating cryptographic techniques (Bonawitz et al., 2017), see (Li et al., 2019; Kairouz et al., 2019) for an in-depth survey of recent work in FL.

Federated learning often results in improved performance, as reported in several learning problems, including next word prediction (Hard et al., 2018; Yang et al., 2018), vocabulary estimation (Chen et al., 2019a), emoji prediction (Ramaswamy et al., 2019), decoder models (Chen et al., 2019b), low latency vehicle-to-vehicle communication (Samarakoon et al., 2018), and predictive models in health (Brisimi et al., 2018).

Appendix B. Agnostic global model

Instead of assigning weights proportional to the number of samples as in the uniform global model, we can weight them according to any \( \lambda \in \Delta^p \). For example, instead of uniform sample weights, we can weight clients uniformly corresponding to \( \lambda_k = \frac{1}{p} \), for all \( k \). Let \( \bar{D}_\lambda \) denote the \( \lambda \)-weighted empirical distribution and let \( h_{\bar{D}_\lambda} \) be the minimizer of loss over \( \bar{D}_\lambda \). Instead of the uniform global model described in the previous section, we can use the agnostic loss, where we minimize the maximum loss over a set of distributions. Let \( \Lambda \subseteq \Delta^p \). Agnostic loss is given by

\[
\max_{\lambda \in \Lambda} \mathcal{L}_{\bar{D}_\lambda}(h).
\]

Let \( h_{\bar{D}_\lambda} \) be the minimizer. Let \( s(\lambda, m) = \max_{\lambda \in \Lambda} s(\lambda, m) \). Let \( \Lambda_\epsilon \) be an \( \epsilon \)-cover of \( \Delta^p \). Let \( m \) denote the empirical distribution of samples \((m_1/m, m_2/m, \ldots, m_p/m)\). The skewness between the distributions \( \lambda \) and \( m \) is defined as \( s(\lambda, m) = \sum_{k=1}^{p} \frac{\lambda^2_k m_k}{m} \) where \( m_k = m_k/m \). With these definitions, the generalization guarantee of (Mohri et al., 2019, Theorem 2) for client one can be expressed as follows:

\[
\mathcal{L}_{D_1}(h_{\bar{D}_{\lambda^*}}) \leq \mathcal{L}_{D_1}(h_{D_1}) + O\left(\sqrt{s(\lambda^*\|m)} \cdot \frac{\sqrt{d + \log |\Delta^p|}}{\sqrt{m}} + \epsilon \right) + \text{disc}_H(D_1, D_{\lambda^*}),
\]

where \( \lambda^* = \arg\max_{\lambda} \mathcal{L}_{\bar{D}_{\lambda}}(h_{\bar{D}_{\lambda}}) \) is the mixture weight where the trained model \( h_{\bar{D}_{\lambda^*}} \) has the highest loss. Hence, this approach would personalize well for hard distributions and can be considered as a step towards ensuring that models work for all distributions. In this work, we show that training a different model for each client would significantly improve the model performance.
Appendix C. Proofs for user clustering

C.1. Proof of Lemma 1

\[
\sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{D_k}(\hat{h}_i^*) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) \\
= \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{D_k}(\hat{h}_i^*) + \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(\hat{h}_i^*) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) \\
+ \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) \\
\leq 2 \max_{h_1,h_2,...,h_q} \left| \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{D_k}(h_i) - \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i) \right|,
\]

where the inequality follows by observing that \( \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) \leq \sum_{k=1}^{p} \frac{m_k}{m} \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_i^*) \), by the definition of \( \hat{h}_i^* \).

C.2. Proof of Theorem 2

For any set of real numbers \( a_1, a_2, \ldots, a_q \) and \( b_1, b_2, \ldots, b_q \), observe that

\[
\min_i a_i - \min_i b_i = \min_i (a_i - b_i) - \min_i b_i \leq \min_i (a_i - b_i) - \min_i b_i = \max_i (a_i - b_i).
\]

We first prove the theorem for one side. Let \( f : [p] \rightarrow [q] \) be a mapping from clients to clusters. Applying the above result yields,

\[
\max_{h_1,h_2,...,h_q} \left( \sum_{k=1}^{p} m_k \cdot \min_{i \in [q]} \mathcal{L}_{D_k}(h_k) - \sum_{k=1}^{p} m_k \cdot \min_{i \in [q]} \mathcal{L}_{\mathcal{D}_k}(h_k) \right) \\
\leq \max_{h_1,h_2,...,h_q} \left( \sum_{k=1}^{p} m_k \cdot \max_{f \in [p]} (\mathcal{L}_{D_k}(h_k) - \mathcal{L}_{\mathcal{D}_k}(h_k)) \right) \\
= \max_{h_1,h_2,...,h_q} \left( \sum_{k=1}^{p} m_k \cdot \max_{f \in [p]} (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\mathcal{D}_k}(h_{f(k)})) \right) \\
= \max_{f} \max_{h_1,h_2,...,h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\mathcal{D}_k}(h_{f(k)})) \right).
\]

Since changing one sample changes the above function by at most 1, for a given \( f \), by the McDiarmid’s inequality, with probability at least \( 1 - \delta \), the following holds:

\[
\max_{h_1,h_2,...,h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\mathcal{D}_k}(h_{f(k)})) \right) \\
\leq \mathbb{E} \left[ \max_{h_1,h_2,...,h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\mathcal{D}_k}(h_{f(k)})) \right) \right] + 2 \sqrt{m \log \frac{1}{\delta}}.
\]

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The number of possible functions $f$ is $q^p$. Hence, by the union bound, for all $f$, with probability at least $1 - \delta$, the following holds:

$$
\max_{h_1, h_2, \ldots, h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\hat{D}_k}(h_{f(k)})) \right) \\
\leq \mathbb{E} \left[ \max_{h_1, h_2, \ldots, h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\hat{D}_k}(h_{f(k)})) \right) \right] + 2\sqrt{mp \log \frac{q}{\delta}}.
$$

For a given clustering $f$, by the sub-additivity of $\max$,

$$
\max_{h_1, h_2, \ldots, h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\hat{D}_k}(h_{f(k)})) \right) \\
= \max_{h_1, h_2, \ldots, h_q} \left( \sum_{k=1}^{p} \sum_{k: f(k) = i} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\hat{D}_k}(h_{f(k)})) \right) \\
\leq \sum_{i=1}^{q} \max_{h_1, h_2, \ldots, h_q} \left( \sum_{k: f(k) = i} m_k \cdot (\mathcal{L}_{D_k}(h_{i}) - \mathcal{L}_{\hat{D}_k}(h_{i})) \right) \\
= \sum_{i=1}^{q} \max_{h_i} \left( \sum_{k: f(k) = i} m_k \cdot (\mathcal{L}_{\hat{D}_k}(h_{i}) - \mathcal{L}_{\hat{C}_i}(h_{i})) \right) \\
= \sum_{i=1}^{q} \max_{h_i} \left( m_{C_i} \cdot (\mathcal{L}_{\hat{C}_i}(h_{i}) - \mathcal{L}_{\hat{C}_i}(h_{i})) \right),
$$

where $C_i$ is the cluster of clients such that $f(k) = i$ and $m_{C_i}$ is the number of samples in that cluster, and $\mathcal{C}_i$ is its distribution. Thus,

$$
\mathbb{E} \left[ \max_{h_1, h_2, \ldots, h_q} \left( \sum_{k=1}^{p} m_k \cdot (\mathcal{L}_{D_k}(h_{f(k)}) - \mathcal{L}_{\hat{D}_k}(h_{f(k)})) \right) \right] \leq \mathbb{E} \left[ \sum_{i=1}^{q} \max_{h_i} \left( m_{C_i} \cdot (\mathcal{L}_{\hat{C}_i}(h_{i}) - \mathcal{L}_{\hat{C}_i}(h_{i})) \right) \right] \\
\leq \sum_{i=1}^{q} \mathbb{R}_{e_{i}, m_{C_i}}(H)m_{C_i},
$$

where the last inequality follows from standard learning-theoretic guarantees and the definition of Rademacher complexity (Mohri et al., 2018b). The proof follows by combining the above equations, normalizing by $m$, and the union bound.

**C.3. Proof of Corollary 3**

We show that for any clustering,

$$
\sum_{i=1}^{q} \frac{m_{C_i}}{m} \mathbb{R}_{e_{i}, m_{C_i}}(H) \leq \sqrt{\frac{dp}{m} \log \frac{em}{d}}.
$$
The proof then follows from Theorem 2. To prove the above observation, observe that
\[
\sum_{i=1}^{q} \frac{mC_i}{m} g_{C_i, mC_i}(mC_i) \leq \sum_{i=1}^{q} \frac{mC_i}{m} \sqrt{\frac{dp}{mC_i} \log \frac{em}{d}} \leq \sum_{i=1}^{q} \frac{mC_i}{m} \sqrt{\frac{dp}{mC_i} \log \frac{em}{d}} \leq \sum_{i=1}^{q} \frac{1}{m} \sqrt{\frac{dpmC_i}{m} \log \frac{em}{d}} \leq \sqrt{\frac{dpmC_i}{m} \log \frac{em}{d}},
\]
where the last inequality follows from Jensen’s inequality.

Appendix D. Proof of Theorem 5

Let \( g(h) = \lambda \mathcal{L}_{\hat{D}_k}(h) + (1 - \lambda) \mathcal{L}_{\hat{C}}(h) \). Suppose we are interested in running \( T \) steps of SGD on \( g \), where at each step we independently sample \( \hat{D}_k \) with probability \( \lambda \) and \( \hat{C} \) with probability \( 1 - \lambda \) and choose a random sample from the selected empirical distribution to compute the gradient. This can be simulated by first sampling \( T \) elements from \( \hat{C} \), denoted by \( \hat{C}' \) and using \( \hat{C}' \) instead of \( \hat{C} \) during optimization. Hence to prove the theorem, suffices to show that \( T = rm_k \) steps of SGD on \( g \) using the above mentioned sampling procedure yields the desired bound.

We now ask how large should \( T \) be to obtain error of \( \epsilon \lambda \). By standard stochastic gradient descent guarantees, the output \( h_A \) satisfies
\[
\mathbb{E}[g(h_A)] \leq \mathbb{E}[g(h_{\hat{C}})] + \frac{\|h_c - h_{\hat{C}}\|^2}{2\eta} + \eta G^2 T.
\]
Since the loss is strongly convex and \( h_c \) is optimal for \( \mathcal{L}_{\hat{C}}(h_c) \),
\[
\mathcal{L}_{\hat{C}}(h_{\hat{C}}) - \mathcal{L}_{\hat{C}}(h_c) \geq \frac{\mu}{2} \|h_c - h_{\hat{C}}\|^2.
\]
Furthermore, since \( h_{\hat{C}} \) is optimal for a \( \lambda \)-mixture,
\[
\lambda \mathcal{L}_{\hat{D}_k}(h_{\hat{C}}) + (1 - \lambda) \mathcal{L}_{\hat{C}}(h_{\hat{C}}) \leq \lambda \mathcal{L}_{\hat{D}_k}(h_c) + (1 - \lambda) \mathcal{L}_{\hat{C}}(h_c).
\]
Hence,
\[
\frac{\mu}{2} \|h_c - h_{\hat{C}}\|^2 \leq \mathcal{L}_{\hat{C}}(h_{\hat{C}}) - \mathcal{L}_{\hat{C}}(h_c)
\]
\[
\leq \frac{\lambda}{1 - \lambda} \left( \mathcal{L}_{\hat{D}_k}(h_c) - \mathcal{L}_{\hat{D}_k}(h_{\hat{C}}) \right)
\]
\[
\leq \frac{G \lambda}{1 - \lambda} \|h_c - h_{\hat{C}}\|.
\]
Therefore,
\[
\|h_c - h_{\hat{C}}\| \leq \min \left( \frac{2G \lambda}{\mu(1 - \lambda)}, R \right)
\]
Combining the above equations, we get
\[
\mathbb{E}[g(h_A)] \leq \mathbb{E}[g(h_{\hat{C}})] + \frac{1}{2\eta} \min \left( \frac{2G \lambda}{\mu(1 - \lambda)}, R \right)^2 + \frac{\eta G^2 T}{2}.
\]
Substituting the learning rate and setting $T = rm_k$ yields

$$
\mathbb{E}[g(h_A)] \leq \mathbb{E}[g(h_\bar{\lambda})] + \frac{G}{\sqrt{rm_k}} \min \left( \frac{2G\lambda}{\mu(1 - \lambda)}, R \right).
$$

Hence if $r \geq G^2 \max_\lambda \min \left( \frac{2G}{\mu(1 - \lambda)}, \frac{R}{\lambda} \right)^2$, the above bound is at most $\sqrt{\frac{\lambda \mu}{m_k}} \leq \epsilon_\lambda$. Note that for any $\lambda$,

$$
\min \left( \frac{2G}{\mu(1 - \lambda)}, \frac{R}{\lambda} \right) \leq \frac{2G}{\mu(1 - \lambda)} 1_{\lambda < 1/2} + \frac{R}{\lambda} 1_{\lambda > 1/2} \leq \frac{4G}{\mu} + 2R,
$$

hence the theorem.

**Appendix E. Proof of Theorem 6**

Observe that

$$
\sum_{k=1}^p \frac{m_k}{m} \mathcal{L}_{D_k}((1 - \lambda_k) \bar{h}_c^* + \lambda_k^* h_{l,k}^*) - \sum_{k=1}^p \frac{m_k}{m} \mathcal{L}_{\bar{D}_k}((1 - \lambda_k) h_c^* + \lambda_k^* h_{l,k}^*)
\leq 2 \max_{h_c, \lambda, h_l} \left( \sum_{k=1}^p \frac{m_k}{m} \left( \mathcal{L}_{D_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) - \mathcal{L}_{\bar{D}_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) \right) \right).
$$

Changing one sample changes the above function by at most $1/m$. Thus by McDiarmid’s inequality, with probability at least $1 - \delta$,

$$
\max_{h_c, \lambda, h_l} \left( \sum_{k=1}^p \frac{m_k}{m} \left( \mathcal{L}_{D_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) - \mathcal{L}_{\bar{D}_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) \right) \right)
\leq \mathbb{E} \left[ \max_{h_c, \lambda, h_l} \left( \sum_{k=1}^p \frac{m_k}{m} \left( \mathcal{L}_{D_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) - \mathcal{L}_{\bar{D}_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) \right) \right) \right] + 2 \sqrt{\frac{\log \frac{1}{\delta}}{m}}.
$$

Let $\mathcal{H}_l$ be the Cartesian product of hypothesis classes where the $k^{th}$ hypothesis class is the hypothesis applied to $k^{th}$ client. Let $\mathcal{H} = \lambda \mathcal{H}_c + (1 - \lambda) \mathcal{H}_l$. Hence, by Talagrand’s construction lemma and the properties of Rademacher complexity,

$$
\mathbb{E} \left[ \max_{h_c, \lambda, h_l} \left( \sum_{k=1}^p \frac{m_k}{m} \left( \mathcal{L}_{D_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) - \mathcal{L}_{\bar{D}_k}((1 - \lambda_k) h_c + \lambda_k h_{l,k}) \right) \right) \right]
\leq \mathcal{R}_{\lambda + \bar{\lambda}, m}(\ell(\mathcal{H}))
\leq L \mathcal{R}_{\lambda + \bar{\lambda}, m}(\mathcal{H})
\leq L \mathcal{R}_{\lambda, m}(\mathcal{H}_c) + L \mathcal{R}_{\bar{\lambda}, m}(\mathcal{H}_l)
\leq L \left( \mathcal{R}_{\lambda, m}(\mathcal{H}_c) + \sum_{k=1}^p \frac{m_k}{m} \mathcal{R}_{\bar{\lambda}, m}(\mathcal{H}_l) \right),
$$

where the last inequality follows from the sub-additivity of Rademacher complexity.
Table 5: Test accuracy of seen and unseen clients for twice the number of layers (shuffled).

| algorithm | seen acc. | unseen acc. |
|-----------|-----------|-------------|
| BASELINE  | 83.5%     | 83.4%       |
| FINETUNE  | 90.6%     | 90.6%       |
| DAPPER    | 90.8%     | 90.8%       |

Table 6: Test accuracy of seen and unseen clients for twice the number of hidden units (shuffled).

| algorithm | seen acc. | unseen acc. |
|-----------|-----------|-------------|
| BASELINE  | 83.6%     | 83.4%       |
| FINETUNE  | 90.1%     | 90.0%       |
| DAPPER    | 90.2%     | 90.0%       |

Appendix F. Experiments

F.1. Synthetic dataset

Let $U$ be a uniform distribution over $\mathcal{Y}$. Let $P_k$ be a point mass distribution with $P_k(k) = 1.0$ and for all $y \neq k$, $P_k(y) = 0$. For a client $k$, let $D_k$ be a mixture given by

$$D_k = 0.5 \cdot P_k\%4 + 0.25 \cdot U + 0.25 \cdot P_k\%(d-4),$$

where $a \% b$ is $a$ modulo $b$. Roughly, $U$ is the uniform component and is same for all the clients, $P_k\%4$ is the cluster component and same for clients with same clusters, and $P_k\%(d-4)$ is the individual component per client.

F.2. EMNIST dataset

For the EMNIST experiments, we follow previous work for model architecture (Jiang et al., 2019). The model is a convolution neural network with two convolution layers with relu activation followed by a max pooling layer and two dense layers. For hyper-parameters, we perform a sweep over parameters and use the eval dataset to choose the best. The best hyperparameters after sweeping are as follows. We note that the best hyperparameters were the same for both the unshuffled and shuffled data splits.

- **BASELINE**: We train the baseline model using FedAvg for 1000 rounds with 20 clients participating per round, client learning rate 0.01, client batch size 20, client number of epochs 2, server learning rate 1.0, and server momentum 0.9.
- **HYPCLUSTER**: We initialize 2 clusters at the start of training and use the same hyper-parameters as baseline. We determined that 2 was the optimal number of clusters since for larger numbers of clusters, all clients essentially mapped to just 2 of them.
- **FINETUNE**: We use the best baseline model as the pre-trained starting global model and finetune for each client. We use the following finetune hyper-parameters: learning rate 0.005, batch size 10, and number of epochs 1.
- **DAPPER**: Similar to FINETUNE, we use the best baseline model as the pre-trained starting global model. For each client, we finetune the global model using a mixture of global and client data. Given $m_k$ client examples, we sample $5 \cdot m_k$ global examples and split the global and client examples into equal number of batches for training and use a learning rate of 0.01.
• MAPPER: We use the same global hyper-parameters as BASELINE with the following local hyper-parameters: learning rate 0.02, batch size 10, and number of epochs 5. As stated previously, we use the same architecture for both local and global models and at each optimization step, initialize the local model using the global parameters.

MAPPER interpolates two models and hence during inference it has twice the model size as the baseline. For a better comparison, we also evaluated the other approaches using neural architectures with twice the model size. Model size can be doubled either by increasing the number of layers or increasing the number of units and we evaluated both the approaches. Table 5 and Table 6 report the accuracy of each algorithm averaged over 20 trials when using twice the number of layers and twice the number of hidden units, respectively. Even though the algorithms with twice the model size perform better than the original BASELINE, DAPPER, and FINETUNE models, MAPPER with the original neural architecture remains the best performing approach.