Soliton frequency shifts in subwavelength structures

Xiaohong Song, Ming Yan, Miaoli Wu, Zhihao Sheng, Zhizhen Hao, Chong Huang and Weifeng Yang

Department of Physics, College of Science, Shantou University, Shantou, Guangdong 515063, People’s Republic of China

E-mail: wfyang@stu.edu.cn

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Abstract
We investigate the soliton frequency shifts for few-cycle ultrashort laser pulses propagating through resonant media embedded within subwavelength structures, and we elucidate the underlying physics. Full-wave Maxwell–Bloch equations are solved numerically by using the finite-difference time-domain method. It is shown that both soliton blueshift and redshift can occur by changing the period of the structures. We found that the rereflected waves play an essential role in this process. When the pulse propagates through the periodic structures, the reflected waves can be rereflected back by the thin layers, which can further induce the controllable frequency shifts of the generated solitons. This suggests a way to tailor the light solitons over a large spectral range.

Keywords: full-wave Maxwell–Bloch equations, self-induced transparency soliton, few-cycle ultrashort laser pulses

1. Introduction

Optical solitons are self-localized pulses of light that occur in many natural and artificially created nonlinear systems such as Bragg reflectors [1–3], photonic crystal fibers [4], metamaterials [5–7], etc. They exist widely in media with resonant nonlinearity, Kerr nonlinearity, and quadratic nonlinearity [1, 8]. Usually, theoretical investigations of solitons are based on the slowly varying envelope approximation (SVEA). Recent technological progress in ultrafast optics has enabled the generation of few-cycle or even single-cycle light pulses [9–11]. For such pulses, the SVEA is no longer valid. As a result, several theoretical approaches are developed to model few-cycle solitons beyond the SVEA (see, e.g., the review [12]).

For resonant nonlinear systems, extremely short pulses can be described by directly solving the Maxwell–Bloch equations for two-level systems [1, 12–21]. It has been found that the $2\pi$ self-induced transparency soliton in resonant systems is essentially recovered for few-cycle ultrashort laser pulses, though some additional features were found as well [13]. When the Rabi frequency of few-cycle ultrashort laser pulses becomes comparable to the light frequency, the light-matter interaction enters the regime of extreme nonlinear optics [14], and several new phenomena occur, including the carrier-wave Rabi flopping [15, 16] and third-harmonic generation in the guise of second-harmonic generation [22]. In this regime, it has been demonstrated that the two-level system can still serve as a reference point [14–16]. Among other recent works about few-cycle resonant solitons, one can mention the validity of the area theorem for subcycle pulses [18], the carrier-envelope phase-dependent solitary pulse generation [19, 20], unipolar half-cycle pulse generation [21, 23, 24], femtosecond nanometer-sized optical solitons [25], etc.

Over the last decade, advances in nanofabrication techniques have led to a rich variety of different periodic nanostructures, providing an attractive way to control light propagation and light-matter interaction [1]. For example, photonic crystal fibers can be used to cancel the Raman soliton self-frequency shift (SSFS) due to their unique combination of dispersive and nonlinear properties [4]. SSFS is a well-known nonlinear phenomenon that involves, a short
soliton propagating in a Raman-active medium that is continuously redshifted due to the intrapulse Raman scattering [26, 27]. Recently, it has been predicted that the SSFS can be tailored in nonlinear metamaterials with controlled linear and nonlinear electromagnetic properties [7]. When few-cycle ultrashort laser pulses propagate in dense resonant nonlinear media, a soliton frequency shift can also occur [17]: a large blueshift in the transmitted solitary pulse and a large redshift in the reflected pulse. However, the physical mechanism is quite different than that of the Raman-induced SSFS. The frequency shift of the solitons in continuous resonant nonlinear media is caused by intrapulse four-wave mixing [17]. Recently, it was found that the frequency shift of the soliton can be suppressed if resonant media are embedded within periodic dielectric structures [28]. Here, we further show that the frequency shift of the solitons in resonant media can be controlled by the periodic structures. Both soliton redshift and blueshift can occur by manipulating the period of the structures. Moreover, the underlying physics of controllable soliton frequency shifts is clarified. We found that the rereflected Bragg waves participate in the four-wave mixing process, which further induces controllable frequency shifts of the generated solitary pulses.

2. Theoretical method

We consider the propagation of a few-cycle laser pulse along the z-axis through a one-dimensional periodic sequence of thin layers consisting of a two-level system (see figure 1). The electric field is linearly polarized along the x-axis. Initially, the few-cycle ultrashort laser pulse locates in the free space at \( z_0 = 10 \mu m \); then it partially penetrates into the subwavelength periodic structures at \( z_1 = 20 \mu m \). The penetrating part propagates through the medium and finally exits into the free space at \( z_2 = 150 \mu m \). The structure period is \( L = 130 \mu m \).

For the resonant propagation of laser pulses in the periodic structures, we numerically solve the full-wave Maxwell–Bloch equations beyond the SVEA and rotating wave approximation:

\[
\frac{\partial \mathbf{H}_i(z, t)}{\partial t} = -\frac{1}{\mu_0} \frac{\partial \mathbf{E}_i(z, t)}{\partial z},
\]

\[
\frac{\partial \mathbf{E}_i(z, t)}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_r} \frac{\partial \mathbf{H}_i(z, t)}{\partial z} - \frac{1}{\varepsilon_0 \varepsilon_r} \frac{\partial \mathbf{P}_i(z, t)}{\partial t},
\]

\[
\dot{u} = -\omega_0 v - \frac{\mu_1 \mu_2}{\hbar} E_i(z, t) w - \frac{1}{T_2} v,
\]

\[
\dot{w} = -\frac{\mu_1 \mu_2}{\hbar} E_i(z, t) v - \frac{1}{T_1} (w + 1).
\]

Here \( \mathbf{H}, \mathbf{P}, \mu_0 \), and \( \varepsilon_0 \) are the magnetic field, the macroscopic polarization, the permeability, and the permittivity of free space, respectively. \( \varepsilon_r = n_r^2 \) is the relative permittivity of the background material. \( \omega_0 \) is the resonant frequency. \( T_1 \) and \( T_2 \) are, respectively, the lifetime of the excited state and the dephasing time. The spatial modulation of \( \mathbf{P} \) is given by

\[
P(z, t) = \begin{cases} N \mu_1 \mu_2 u(z, t) & \text{if } z \in [2n\delta, (2n + 1)\delta], \\ 0 & \text{if } z \in [(2n + 1)\delta, (2n + 2)\delta]. \end{cases}
\]

where \( N \) is the medium density. The initial electric field is defined as \( \mathbf{E}_i(z, t = 0) = E_0 \sec h[1.76(z - z_0)/c\tau_p] \cos[\omega_0(z - z_0)/c] \), where \( \omega_0 \) is the central frequency and \( \tau_p \) is the full width at half maximum of the pulse intensity envelope. The envelope area of the pulse is determined by \( A = \mu_2 E_0 \tau_p \pi/1.76\hbar \). It has been demonstrated that under the extreme nonlinear regime, numerical solutions based on the well-known Maxwell–Bloch equations for two-level system have provided an adequate description of the experimental phenomena in solids [16, 29–32]. As a result, we take into account the practical parameters of GaAs for the resonant medium [16]. To avoid large absorption, we choose SiO\(_2\) for the background, which has small refractive index of \( n_b = \sqrt{\varepsilon_b} = 1.45 \). Our results can be scaled to various laser and material parameters. The parameters are as follows: \( \tau_p = 5 \) fs, \( \omega_0 = 2.16 \text{ fs}^{-1} \) (corresponding to \( \lambda_0 = 884 \text{ nm} \)), \( \mu_1 = 8 \times 10^{-20} \text{ As m}, T_1 = \infty, T_2 = 50 \) fs, the density of the resonant medium \( N = 3 \times 10^{19} \text{ cm}^{-3} \). The pulse area, \( A = 1.5\tau_p \), corresponds to a maximum electric-field amplitude, \( E_0 = 4.64 \times 10^6 \text{ V cm}^{-1} \), or an intensity of \( I = 1.33 \times 10^{12} E_0^2 = 2.86 \times 10^{10} \text{ W cm}^{-2} \). In simulation, we apply an input envelope area, \( A = 4\tau_p \). The full-wave Maxwell–Bloch equations are numerically solved by using the finite-difference-time-domain method. This method has been proven to be an accurate \textit{ab initio} tool to simulate the interactions between few-cycle ultrashort pulses and matters [14–17, 19–22].

3. Numerical results and discussion

We model few-cycle pulse propagation through various periodic structures with \( d = 0.14, 0.17, 0.21, 0.30, \) and \( 0.34 \mu m \),
respectively. The spectra and the corresponding temporal electric field profiles of the transmitted pulses detected outside the periodic structures are shown in figures 2 and 3, respectively. For the structure period, $d$ satisfies the Bragg condition, $d = \lambda_0/2n_b \approx 0.30 \mu m$ (see figure 2(d)), and for tighter periodic structures (see figures 2(a) and (b)), our simulations are essentially in agreement with the work presented in [28] (i.e., single-cycle gap solitons can be formed without any frequency shifts in these cases). However, we find that this rule does not hold for other structures with periods that slightly violate the Bragg condition. In figure 2(c), the spectrum of the transmitted pulse consists of two separated portions. The main part shows a redshift with a spectrum maximum position at approximately $0.85 \omega_0$, while the small part shows a blueshift. On the contrary, in figure 2(e), the main part of the transmitted spectrum exhibits a blueshift with a spectrum maximum position at $1.20 \omega_0$. From the time profiles of the transmitted electric fields, we can see that solitary pulses can also be formed in these two cases (see figures 3(c) and (e)). Nevertheless, in comparison with the solitary pulses in figures 3(a), (b), and (d), the pulse durations of the solitons in figures 3(c) and (e) are wider. By applying the Fourier transform to the generated solitary pulses, we found that the spectra of these generated few-cycle solitons correspond to the main parts of the total transmitted pulse spectra (see figure 2), which means that both soliton redshift and blueshift can occur. As a result, the soliton’s frequency shift can be controlled by the period of the periodic structures.

Figure 2. The transmitted pulse spectra for different periodic structures. (a) $d = 0.14 \mu m$; (b) $d = 0.17 \mu m$; (c) $d = 0.21 \mu m$; (d) $d = 0.30 \mu m$; (e) $d = 0.34 \mu m$. 

Figure 3. As in figure 2, but for the temporal electric field profiles.

Note that in addition to the soliton pulse, the transmitted electric field also contains a weak, long tailing part (see the right-hand side of figure 3). When the structure period satisfies the Bragg condition (figure 3(d)), the tailing part of the transmitted electric field is quite weak. It is strengthened and then weakened by further decreasing the structure period. The Fourier transforms of these tailing parts show that they contribute to the small parts of the transmitted pulse spectra, which are denoted by the arrows in figure 2. Such frequency components are quite weak in figures 2(a) and (b), but, they can still be observed on an enlarged scale (see insets).

To elucidate the underlying physics of the controllable frequency shifts of the soliton pulses and the origin of the tailing part of the transmitted electric fields, in figure 4 we further present the spectra of the reflected electric fields from the periodic structures. As one can see, the central frequency of the reflected pulse spectrum also depends on the period of the structures. By increasing the period of the structures, the frequency of the reflected spectrum decreases. We found that the spectral peaks in the reflected spectra locate at around $\omega = \pi c/n_b d$, which indicates that the peaks originate from the Bragg reflection. Note that the frequency of the reflected Bragg peaks can go beyond the bandwidth of the incident pulse. (The incident spectrum is shown in figure 4(b) with red line.) To investigate this, we present in figure 5 the spectrum of the reemitted field at the first subwavelength film. The reemitted field is proportional to the time derivative of the
In a continuous medium, it has been found that the transmitted pulse spectrum exhibits a blueshift, which was explained by intrapulse four-wave mixing [17]. In our case, we find that the rereflected waves also participate in the frequency-mixing processes, and play an essential role on the frequency shifts of the generated solitons. In figure 2(c), the rereflected Bragg peak shows a blueshift; note that the generated soliton exhibits a redshift. In figure 2(d), when the structure period d satisfies the Bragg condition, the rereflected Bragg peak is located around the input central frequency ω₀. Correspondingly, almost no frequency shift is observed in the transmitted pulse spectrum. In figure 2(e), the rereflected Bragg peak shows a redshift, and accordingly, the transmitted soliton exhibits a blueshift. In figures 2(a) and (b), the rereflected waves are very weak, and their influences on the transmitted field are negligible. Therefore, almost no frequency shift can be observed in these cases. Our simulations have demonstrated that the source for the controllable frequency shift of the generated soliton is due to the rereflected Bragg peak. To suppress the frequency shift of the generated soliton, one can use either a periodic structure that satisfies the Bragg condition or use tighter periodic structures to weaken the influence of the rereflected waves. On the contrary, to tailor a solitary pulse over a large spectral range, one can use denser media or higher intensities of the incident pulses to enhance the role of the rereflected waves.

4. Summary

In conclusion, we have investigated the physical mechanism of soliton frequency shifts in dense resonant two-level media with subwavelength structures. We found that the reflected electric field can be rereflected back by the thin layers and form a Bragg peak in the transmitted electric field, which further induces the frequency shifts of the generated solitary pulses. Since the frequency of the rereflected Bragg peak can be manipulated by the period of the media, both blueshifted and redshifted solitons can be obtained by changing the period of the structures. This offers opportunities for tailoring light solitons over a broad spectral range.

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