Spin-torque shot noise in magnetic tunnel junctions

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Spin polarized current may transfer angular momentum to a ferromagnet, resulting in a spin-torque phenomenon. At the same time the shot noise, associated with the current, leads to a non-equilibrium stochastic force acting on the ferromagnet. We derive stochastic version of Landau-Lifshitz-Gilbert equation for a magnetization of a "free" ferromagnetic layer in contact with a "fixed" ferromagnet. We solve the corresponding Fokker-Planck equation and show that the non-equilibrium noise yields to a non-monotonous dependence of the precession spectrum linewidth on the current.

Magnetization dynamics of a ferromagnet under influence of a spin polarized current is a subject of intensive investigations (for recent reviews see Refs. 1 2). It was realized 2 4 that the spin current may transfer the angular momentum to the ferromagnet, resulting in a torque acting on its magnetization direction. In the case of a small ferromagnetic domain the torque may lead to a rotation of the magnetization as a whole, rather than to an excitation of spin waves. This phenomenon, allowing for an electronic manipulation of the magnetization, has a promise for a number of potential applications.

The effect has been recently observed 3 4 5 6 7 8 9 in a setup, where the spin-torque magnitude and direction are tuned to compensate exactly the dissipation force acting on the magnetization of the "free" ferromagnetic layer. This leads to an undamped precession which is detected through the induced microwave radiation. Both the spectral width and the generated power exhibit a strong dependence on the current flowing through the interface. This phenomenon may be accounted for by adding a fluctuating spin-torque term of Landau-Lifshitz-Gilbert (LLG) equation for the unit vector m → I s.

Interactive width and the generated power exhibit a strong dependence on the current flowing through the interface of the two ferromagnets. It was shown later 10 11 12 that the equilibrium thermal noise, first introduced in dynamics of micromagnets by F-L. Brown 12, may partially account for the observed linewidth.

On the other hand, since the experiments are performed under non-equilibrium conditions (spin current strong enough to balance the dissipation), one needs to address non-thermal sources of noise as well. The most essential of them is the spin shot noise associated with the discreteness of spin passing through the interface. This phenomenon may be accounted for by adding a fluctuating part to the spin current vector in the Slonczewski’s torque term of Landau-Lifshitz-Gilbert (LLG) equation \[ \mathbf{I}_s \rightarrow \mathbf{I}_s + \delta \mathbf{I}_s(t) \]. The resulting stochastic LLG equation for the unit vector \( \mathbf{m} = \mathbf{M}/M \) in the direction of the magnetization \( \mathbf{M} \) takes the form

\[
\frac{d\mathbf{m}}{dt} = -\gamma [\mathbf{m} \times \mathbf{H}_{\text{eff}}] + \alpha(\theta) \left[ \mathbf{m} \times \frac{d\mathbf{m}}{dt} \right] + \frac{\gamma}{MV} \left[ \mathbf{m} \times \left( \mathbf{I}_s + \delta \mathbf{I}_s \right) \times \mathbf{m} \right].
\]

(1)

Here \( \gamma \) is gyromagnetic ratio, \( \mathbf{H}_{\text{eff}} = -\partial F/\partial \mathbf{M} \) is the effective magnetic field, which includes both an external field and magnetic anisotropy, and \( V \) is a volume of the free ferromagnet. Gilbert damping \( \alpha(\theta) \) is renormalized by the coupling to the fixed ferromagnet 12 13 and is thus dependent on a relative orientation angle \( \theta \) of the fixed and free ferromagnets.

One could expect that the fluctuating part of the spin current vector \( \delta \mathbf{I}_s(t) \) is preferentially directed along the spin polarization of the incoming electron flux, i.e. along \( \mathbf{I}_s \). This is not the case, however, due to the quantum nature of the effect. Indeed, each spin-flip event transfers exactly one \( \hbar \) unit of the angular momentum to the free ferromagnet. Due to the uncertainty principle, direction of an ensuing magnetization rotation is completely random. As a result, the fluctuating part of the spin torque must have an isotropic correlator

\[
\langle \delta I_{s_x} \delta I_{s_y} \rangle = 2D(\theta) \delta_{ij} \delta(t - t'),
\]

(2)

where the variance \( D(\theta) \) does not depend on the cartesian indexes \( i, j = x, y, z \), but may depend on the mutual orientation of the two ferromagnets. Because of the isotropy of the stochastic torque, it can be equally well represented by a fluctuating magnetic field \( \mathbf{H}_{\text{eff}} \rightarrow \mathbf{H}_{\text{eff}} + \mathbf{h}(t) \), instead of the fluctuating spin flux \( \delta \mathbf{I}_s(t) \). In the latter case the correlator of the stochastic fields reads as

\[
\langle h_{ij}(t) h_{ij}(t') \rangle = 2D(\theta) V^2 \delta_{ij} \delta(t - t'),
\]

This type of non-equilibrium noise was considered in Ref. 13 in context of NFN structures.

Here we consider a model of a magnetic tunnel junction (MTJ) Ref. 14. Magnetization dynamics of the free ferromagnet is described using Holstein-Primakoff (HP) parametrization 15 of its total spin operator by deriving semiclassical equations of motions for HP bosons. We employ Keldysh formalism to allow for non-equilibrium conditions, i.e. voltage bias between the two ferromagnets 16 17 18. After integrating out the fermionic degrees of freedom in the second order in both tunneling and spin-flip processes, we obtain an effective action for HP bosons, which encapsulates deterministic forces (external magnetic field and spin-torque) along with the stochastic term. The latter contains an information about both equilibrium and non-equilibrium noise components.

The result of the program, outlined above, is the fol-
The non-equilibrium part of the noise is proportional to \( V \) and of Gilbert damping coefficient in LLG equation (1) context of NFN structures [19].

Both Eq. (11) and Eqs. (7), (8) should be interpreted in the sense of retarded regularization, or Ito calculus [20].

Let us first analyze deterministic dynamics described by Eqs. (7), (8) with \( D \to 0 \). For a strong enough (and negative for positive \( H_z \)) spin current the condition \( T_z(\theta) = 0 \) may be satisfied for a certain angle \( \theta \). Notice that it is the angular dependence of the enhanced Gilbert damping [2, 13], which is responsible for the angle selectivity. In such a case Eq. (8) describes a stable undamped precession with the frequency \( \bar{\omega}(\bar{\theta}) = \gamma H_z \). The intensity of the induced microwave radiation [3, 4, 7, 8, 4] is given by the square of the oscillating magnetic moment, i.e. proportional to \( \sin^2 \theta \).

To analyze effects of the noise we shall assume that \( \alpha(\theta) \ll 1 \), allowing for time scales separation. The fast variable is the azimuthal angle \( \phi(t) \), while the angle \( \theta(t) \) is the slow one. For a fixed slow variable \( \theta \) Eqs. (8), (9) lead to the Lorentzian shape of the emitted microwave power spectrum

\[
S(\omega, \theta) \propto \frac{2 \csc^2 \theta \dot{\Phi}(\theta)}{[\omega - \Omega(\theta)]^2 + \left[ \csc^2 \theta \dot{\Phi}(\theta) \right]^2}.
\]

To compare with the observed power spectrum this expression should be averaged over the stationary probability distribution \( P(\theta) \) of the slow degree of freedom \( S(\omega) = \int \sin \theta d\theta \int P(\theta) S(\omega, \theta) \). The distribution function \( P(\theta, t) \) obeys the Fokker-Planck equation which follows from Eqs. (7), (9) [12, 20]

\[
\dot{P} = \frac{1}{\sin \theta} \partial_\theta \left[ \sin^2 \theta T_z(\theta) P + \sin \theta \partial_\theta \left( \frac{\Phi(\theta)}{\gamma} \right) \right].
\]

The stationary solution of Eq. (11) is given by

\[
P(\theta) = \frac{1}{Z(\theta)} \exp \left\{ - \int_0^\theta \sin \theta' d\theta' T_z(\theta') \frac{\Phi(\theta')}{\gamma} \right\},
\]
where constant $Z$ is chosen to satisfy the normalization condition $\int_0^\pi \sin \theta \, d\theta \, P(\theta) = 1$. For a weak noise the stationary distribution function has a sharp maximum close to the angle $\bar{\theta}$, where the deterministic spin-torque compensates the dissipation.

Figure 1 shows the calculated spectral linewidth at the half maximum as a function of the applied voltage. The origin of the non-monotonous dependence may be understood by inspection of Eq. (11). The initial decline is due to the geometric factor $\csc^2 \theta$ in the width of the Lorentzian. As the voltage increases, so does the angle where the distribution function exhibits the maximum. Due to $\csc \theta$, coming from the noise term on the r.h.s. of Eq. (8), the amplitude of the phase noise decreases leading to a narrowing of the power spectrum. The initial decrease of the spectral width was discussed in Ref. 10, 11 in terms of the equilibrium thermal noise [12].

The subsequent increase of the spectral width at yet larger voltages was observed in a number of experiments [6, 7, 8] and, to the best of our knowledge, remained unexplained. It naturally comes about due to the non-equilibrium component of the noise. Indeed the noise correlator (3) grows with the applied voltage due to the growth of the spin-flip current. The dependence of the spin-flip current $I_{sf}(\theta)$ on the bias is faster than the linear because of the increase of the spin-flip conductance, $\Delta I_{sf}(\theta)$, on top of the over-all proportionality of $I_{sf}$ to the bias $V$. As a result, for $eV \gg 2T$ the noise variance $D$ grows rapidly, leading to the broadening of the power spectrum, cf. Eq. (11).

Another consequence of the non-equilibrium noise is the saturation of the spectral linewidth at small temperatures [9]. Since the devices are always operated at currents larger than the critical one, the noise intensity [13] is finite $D(\theta) = (\hbar/2)L_{sf}(\theta)$ even at $T \to 0$. Thus decreasing the temperature one should observe saturation of the linewidth at $T \sim eV$, provided the induced damping (the second term on the r.h.s. of Eq. (9)) is larger than the bare one.

In the remainder of the paper we outline derivation of Eqs. (11) – (13). The MTJ is modeled by the two itinerant ferromagnets, whose majority ($\sigma = +$) and minority ($\sigma = -$) bands are described by the operators $c_{k\sigma}^\dagger, c_{k\sigma}$ for the fixed ferromagnet and $d_{\sigma}^\dagger, d_{\sigma}$ for the free layer. We found convenient to work in the instantaneous reference frame, where the magnetization of the free layer points in the $z$-direction. The corresponding Hamiltonian takes the following form

$$H_0 = \sum_{k, \sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{l\sigma} (\epsilon_l - JSz\sigma) d_{l\sigma}^\dagger d_{l\sigma} - \gamma S \cdot H,$$

$$-J \left( S_+ s_- + S_- s_+ \right) + \sum_{kl, \sigma \sigma'} W_{kl}^{\sigma \sigma'} c_{k\sigma}^\dagger d_{l\sigma'} + h.c.$$

here the spin-dependent tunneling matrix elements are

$$W_{kl}^{\sigma \sigma'} = \langle \sigma | \sigma' \rangle W,$$

where the spin-transformation matrix is $\langle \sigma | \sigma' \rangle = e^{-i\sigma \phi / 2} \cos \theta / 2$ and $\langle \sigma | \sigma' \rangle = e^{i\sigma \phi / 2} \sin \theta / 2$; and $s = \frac{1}{2} \sum_{l\sigma} d_{l\sigma}^\dagger \sigma \sigma' d_{l\sigma'}$ is the spin of itinerant electrons, while $s_{\pm} = s_x \pm is_y$. We have explicitly accounted for the interactions of the itinerant electrons in the free layer with its total spin $S = M/\gamma$. To make the latter a dynamical variable we use HP parametrization [15]

$$S_+ = S - b^t b; \quad S_- = b^t \sqrt{2S - b^t b}; \quad S_{\mp} = \sqrt{2S - b^t b},$$

where $b^t, b$ are usual bosonic operators.

Next we write the corresponding action in terms of complex fermionic and bosonic fields $c_{l\sigma}(t), d_{l\sigma}(t), b(t)$, where the time variable runs along the closed Keldysh contour [10, 17, 18]. We then transform to two-component vector notations in terms of symmetric (classical "cl") and antisymmetric (quantum "q") combinations of the forward and backward propagating fields. One should keep in mind that the distribution functions of the $c$ and $d$ fermions have a relative shift of the chemical potentials by $eV$. The fermionic fields may be integrated out exactly and the remaining bosonic effective action expanded to the second order in the tunneling amplitude $W$ and to the first and second orders in the spin-flip processes $S_{\pm} s_{\pm}$. The corresponding processes are represented by the diagrams of Fig. 2. The approximations are justified by the weakness of tunneling and largeness of $S \gg h$.

The resulting action for the complex bosonic fields $b_{cl}(t), b_q(t)$ takes the form $S = S_0 + S_1 + S_2$ where the subscript indicates the order in spin-flips processes. Here
Those are nothing but modified to include a stochastic Langevin term which by a spin polarized current, the LLG equation should be regular momentum $\hbar W$. The contribution to the action is the deterministic spin-torque $[3, 4]$. The corresponding contribution to the action is $\S_1 = \frac{i}{\sqrt{2S}} \int dt \tilde{b}_q(t) I_\alpha \sin \theta e^{-i \phi} + c.c.$, where $I_\alpha$ is given by Eq. $[5]$ with $G_{\sigma \sigma'} = \frac{4\pi e^2}{\hbar} |W|^2 \nu_{\sigma'} \nu_{\sigma'}$ and $\nu_{c,d}$ are densities of states of the two ferromagnets in the $\sigma$ band. The second order processes in spin-flips are depicted by the diagram of Fig. $2b$. These are real (i.e. Golden rule) processes, which matrix elements include the spin-flips. They lead to dissipation as well as fluctuations. The corresponding action is $\S_2 = \int dt \left[ \alpha(\theta) (\tilde{b}_q \partial_t b_{c,d} - \tilde{b}_{c,d} \partial_t b_q) + \frac{2i}{S} D(\theta) \tilde{b}_q b_q \right]$, \hspace{1cm} \text{Eq. (15)}

where $D(\theta)$ and $\alpha(\theta)$ are given by Eqs. $[11, 12, 13]$ (without internal dissipation $\alpha_0$).

One then decouples the last term on the r.h.s. of Eq. (15) by means of the complex Hubbard-Stratonovich field $\delta A_{\pm}(t) = I_{sx} + i I_{sy}$. The remaining action is linear in $\tilde{b}_q(t)$ and $\tilde{b}_q(t)$. It constitutes thus resolution of functional $\delta$-functions of the first order Langevin equations on $b_{c,d}(t) = \sqrt{V/(2\gamma)} m_+ (t)$ and its complex conjugate. Those are nothing but $m_\pm$ components of Eq. (1), with the noise intensity given by Eqs. $[10, 11]$.\hspace{1cm} \text{Eq. (13)}

To conclude: in the presence of the spin-torque, caused by a spin polarized current, the LLG equation should be modified to include a stochastic Langevin term which accounts for the shot-noise, associated with the spin current. This term is different from the previously discussed thermal stochasticity in LLG equation $[12]$, because of its non-equilibrium origin. We have derived the corresponding noise correlator in the MTJ setup. We have argued that the non-equilibrium noise manifests itself in a non-monotonous voltage dependence and low-temperature saturation of the linewidth of the precession power spectrum.

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terms $\sim \partial_t b_{cl}$ along with those which contain only $b_q$. 