Top Quark Pair Production at $e^+e^-$ Colliders in the Topcolor-assisted Technicolor Model

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In the framework of topcolor-assisted technicolor model we calculate the contributions from the pseudo Goldstone bosons and new gauge bosons to $e^+e^-\rightarrow t\bar{t}$. We find that, for reasonable ranges of the parameters, the pseudo Goldstone bosons afford dominate contribution, the correction arising from new gauge bosons is negligibly small, the maximum of the relative corrections is $-10\%$ with the center-of-mass energy $\sqrt{s}=500 \text{ GeV}$; whereas in case of $\sqrt{s}=1500\text{ GeV}$, the relative corrections could be up to $16\%$. Thus large new physics might be observable at the experiments of next-generation linear colliders.

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I. INTRODUCTION

It is often argued that the standard model (SM) should be augmented by new physics at higher energy scales because of some unanswered fundamental questions. The recently reported 2.6 standard deviation of the muon anomalous magnetic moment over its SM prediction [1] may serve as the first evidence of existence of new physics at a scale not far above the weak scale. There are numerous speculations on the possible forms of new physics, among which supersymmetry and technicolor are the two typical different frameworks.

Technicolor—a strong interaction of fermions and gauge bosons at the scale $\Lambda_{TC}\sim 1\text{ TeV}$— is a scenario for the dynamical breakdown of electroweak symmetry to electromagnetism [2]. Based on the similar phenomenon of chiral symmetry breakdown in QCD, technicolor is explicitly defined and completely natural. To account for the mass of quarks, leptons, and Goldstone “technipions” in such a scheme, technicolor(TC), ordinary color, and flavor symmetry are embedded in a large gauge group, called extended technicolor (ETC) [3]. Because of the conflict between constraints on flavor-changing neutral currents (FCNC) and the magnitude of ETC-generated quark, lepton and technipion masses, classical technicolor was superseded by a “walking” technicolor and “multiscale technicolor” [4,5]. The incapability of explain the top quark’s large mass without afoul of either cherished notions of naturalness or experiments from the $\rho$ parameter and the $Z\rightarrow b\bar{b}$ decay rate by ETC led to the topcolor-assisted technicolor [7] and the technicolor with scalars model [8,9].

Up to now, top quark is the heaviest particle discovered in experiments. Its mass, $m_t=175\text{ GeV}$ [10], is of the order of the electroweak spontaneous breaking (EWSB) scale which means the top quark couples rather strongly to the EWSB sector. So effects from new physics would be more apparent in processes with the top quark than with any other light quarks. On the experimental side, it is possible to separately measure various production and decay form factors of the top quark at the level of a few percent [11]. Therefore, theoretical calculations of various corrections to the production and decay of the top quark are of much interest.

Top quark pair can be produced at various high energy colliders. This paper is devoted to examine the ability of the suggested future TeV energy $e^+e^-$ colliders in testing TC effects via $t\bar{t}$ production. It is organized as follows. In Sec. II, we present a brief review of the original topcolor-assisted technicolor (TOPCTC) model by C. T. Hill [13]. In Sec. III, we give out the corrections to the $e^+e^-\rightarrow t\bar{t}$ cross section at the center-of-mass (c.m.) energy $\sqrt{s}=0.5$, 1.0 and 1.5 TeV colliders in the TOPCTC model. Both the effects of pseudo Goldstone bosons (PGBs) and new gauge bosons are evaluated. Conclusions are included in Sec. IV. The analytic formulas for the form factors in the production amplitudes are presented in the Appendix.

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II. COLOR-ASSISTED TECHNICOLOR MODEL

The model assumes [12–14]: (i) electroweak interactions are broken by technicolor; (ii) the top quark mass is large because it is the combination of a dynamical condensate component \((1 - \varepsilon)m_t\), generated by a new strong dynamics, together with a small fundamental component \(\varepsilon m_t (\varepsilon \ll 1)\), generated by ETC; (iii) the new strong dynamics is assumed to be chiral critically strong but spontaneously broken by technicolor at the scale \(\sim 1\) TeV, and it generally couples preferentially to the third generation. This needs a new class of technicolor models incorporating “top-color” (TOPC). The dynamics at \(\sim 1\) TeV scale involves the gauge structure:

\[
SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \rightarrow SU(3)_{QCD} \times U(1)_{EM}
\]

where \(SU(3)_1 \times U(1)_{Y_1}\) \([SU(3)_2 \times U(1)_{Y_2}\)] generally couples preferentially to the third (first and second) generation, and is assumed to be strong enough to form chiral \(_t t\) but not \(_t b\) condensation by the \(U(1)_{Y_1}\) coupling. A residual global symmetry \(SU(3)_1 \times U(1)_{Y_1}\) implies the existence of a massive color-singlet heavy \(Z^0\) and an octet \(B^0\). A symmetry-breaking pattern outlined above will generally give rise to three top-pions, \(\pi_t\), near the top mass scale.

The new interactions by the topcolor for the process \(e^+e^- \rightarrow t\bar{t}\) give

\[
Z^\prime\mu e^+e^- : g_1 \tan \theta^\prime \gamma_{\mu} \left[ \frac{1}{2} L + R \right],
\]

\[
Z^\prime\mu t\bar{t} : g_1 \cot \theta^\prime \gamma_{\mu} \left[ \frac{1}{6} L + \frac{2}{3} R \right],
\]

where \(R(L) = (1 + \gamma_5)/2, g_1 = \alpha_{EM}/\cos \theta_W\) is the \(U(1)_Y\) coupling constant at the scale \(\sim 1\) TeV with \(\theta_W\) being the Weinberg angle. The SM \(U(1)_{Y}\) and the \(U(1)_{Y'}\) field \(Z^\prime\) are then defined by orthogonal rotation with mixing angle \(\theta^\prime\).

There exists the ETC gauge bosons including the sideways and diagonal gauge bosons \(Z^\prime\) in this model. The coupling of \(Z^\prime\) to the fermions and technifermions can be found in Ref. [16]. For the sake of simplicity, we assume that the mass of the sideways gauge boson is equal to the mass of the diagonal gauge boson, namely \(m_{Z^\prime}\), so the \(Z^\prime\mu e^+e^-\) and \(Z^\prime\mu t\bar{t}\) vertices by the ETC dynamics can be written as

\[
Z^\prime\mu e^+e^- : -\frac{\varepsilon m_t}{16\pi f_{\pi}} \frac{e}{s_W c_W} \gamma_{\mu} \left[ \frac{2N_C}{N_{TC} + 1} \xi_{t}^{-1} \left(1 - \xi_{e} \xi_{\nu} \right) \xi_{e} - \left(10\varepsilon \right)^{-2/3} \frac{2}{3} \xi_{e}^2 \right] L - \frac{2N_C}{N_{TC} + 1} \left(1 - \xi_{e} \xi_{\nu} \right) \xi_{e}^{-1} \xi_{\nu}^{-1} \frac{2}{3} R \right],
\]

\[
Z^\prime\mu t\bar{t} : -\frac{\varepsilon m_t}{16\pi f_{\pi}} \frac{e}{s_W c_W} \sqrt{\frac{N_{TC}}{N_C}} \gamma_{\mu} \left[ \frac{2N_C}{N_{TC} + 1} \xi_{t}^{-1} \left(1 + \xi_{t}^2 \right) L - \frac{2N_C}{N_{TC} + 1} \xi_{t}^{-2} R \right],
\]

where \(N_{TC}\) and \(N_C\) are the numbers of technicolors and ordinary colors, respectively; \(s_W = \sin \theta_W, c_W = \cos \theta_W; \xi_{t}, \xi_{e}\) and \(\xi_{\nu}\) are coupling coefficients and are ETC gauge-group dependent. Following Ref. [16], we take \(\xi_{t} = \xi_{e} = 1/\sqrt{2}, \xi_{\nu} = 0.1\xi_{e}^{-1}\).

In this TOPCTC model, there are 60 technipions in the ETC sector with decay constant \(f_{\pi} = 123\) GeV and three top pions \(\pi_t^0, \pi_t^\pm\) in the TOPC sector with decay constant \(f_{\pi_t} = 50\) GeV. The ETC sector is one generation technicolor model. The relevant technipions in this study are only the color-singlet \(\pi\) and color-octet \(\pi_8\). The color-singlets \(\pi\) include the isosinglet scalar \(\pi^0\), the isotriplet scalar \((p^\pm, p^0)\), while the color-octets \(\pi_8\) involve the isosinglet \(p^0\) and the iso-octet scalar \(p_3^\pm, p_3^0\). The color-singlet(octet) technipion-top(bottom) interactions are given by

\[
\frac{c_t \varepsilon m_t}{\sqrt{2} f_{\pi}} \left[ i\gamma_5 t\pi^0 + \frac{i}{\sqrt{2}} \lambda^a t\pi^a + \frac{1}{\sqrt{2}} (1 - \gamma_5) b\pi^+ + \frac{1}{\sqrt{2}} b(1 + \gamma_5) t\pi^- \right],
\]

\[
\frac{\sqrt{2} \varepsilon m_t}{f_{\pi}} \left[ i\gamma_5 a^0 t\pi^0 + i\gamma_5 a^a t\pi^a + \frac{1}{\sqrt{2}} (1 - \gamma_5) \lambda^a t\pi^a + \frac{1}{\sqrt{2}} b(1 + \gamma_5) \lambda^a t\pi^a \right],
\]

where the coefficient \(c_t = 1/\sqrt{6}\), and \(\lambda^a\) is a Gell-Mann matrix acting on ordinary color indices.

The interaction of the top pions with the top (bottom) quark has form

\[
\frac{(1 - \varepsilon)m_t}{\sqrt{2} f_{\pi_t}} \left[ i\gamma_5 t\pi^0_t + \frac{1}{\sqrt{2}} (1 - \gamma_5) b\pi^+_t + \frac{1}{\sqrt{2}} b(1 + \gamma_5) t\pi^-_t \right].
\]

More detail Feynman rules needed in the calculations can be found in Refs. [31,32].
III. THE $E^+E^- \rightarrow T\bar{T}$ CROSS SECTION

A. The PGBs contributions

The relevant Feynman diagrams arising from the PGBs contributing to the $e^+e^- \rightarrow t\bar{t}$ production amplitudes are shown in Fig. 1(a)-(g).

In our calculation, we use the dimensional regularization to regulate all the ultraviolet divergences in the virtual loop corrections, and adopt the Feynman gauge and on-mass-shell renormalization scheme [17]. The renormalized amplitude for $e^+e^- \rightarrow t\bar{t}$ contains

$$ M = M^{\text{tree}} + \delta M, $$

where the tree-level amplitude

$$ M^{\text{tree}} = \bar{u}(t)ie\gamma^\mu [v_t(1 + \gamma_5) + a_t(1 - \gamma_5)]v(\bar{t}) \frac{-ig_{\mu\nu}}{(P_t + P_{\bar{t}})^2 - m_Z^2} \bar{v}(e^+)ie\gamma^\nu [v_e(1 + \gamma_5)]u(e^-) + a_e(1 - \gamma_5)]u(e^-) + \bar{u}(t)\frac{2}{3}e\gamma^\mu v(\bar{t}) \frac{-ig_{\mu\nu}}{(P_t + P_{\bar{t}})^2} \bar{v}(e^+)(-ie)\gamma^\nu u(e^-).$$

Here $P_{t,\bar{t}}$ denote the momentum of the outgoing top quark pair. $\delta M$ represents the PGBs one-loop corrections which contains

$$ \delta M = \delta M_Z + \delta M_\gamma, $$

$$ \delta M_Z = \bar{u}(t)\Gamma_Z v(t) \frac{-ig_{\mu\nu}}{(P_t + P_{\bar{t}})^2 - m_Z^2} \bar{v}(e^+)ie\gamma^\nu [v_e(1 + \gamma_5) + a_e(1 - \gamma_5)]u(e^-), $$

$$ \delta M_\gamma = \delta M_Z |_{Z \rightarrow \gamma, m_Z=0, v_t=a_t=1/3, v_e=a_e=-1/2} $$
where \( v_t = \frac{(4/3)\sqrt{3}}{4\pi c_W}, \quad a_t = \frac{1-(4/3)\sqrt{3}}{4\pi c_W}, \quad v_c = \frac{2\sqrt{3}}{4\pi c_W}, \quad a_c = \frac{1-2\sqrt{3}}{4\pi c_W}. \) \( \Gamma_Z, \Gamma_\gamma \) denote the effect vertex \( Zt\bar{t} \) and \( \gamma t\bar{t} \) arising from PGBs corrections, respectively, and

\[
\Gamma_Z = i e [\gamma^\mu LF_{1Z} + \gamma^\mu RF_{2Z} + P_t^\mu LF_{3Z} + P_t^\mu RF_{4Z} + P_t^\mu LF_{5Z} + P_t^\mu RF_{6Z}],
\]

\[
\Gamma_\gamma = \Gamma |_{F_{(\gamma, t\bar{t})}}.
\]

The form factors \( \Gamma_{1Z}, \Gamma_{\gamma}, \) expressed in terms of two- and three-point scalar integrals are presented in the Appendix. It is easy to find that all the ultraviolet divergences cancel in the effective vertex.

Now we express the differential cross section of the process \( e^+e^- \rightarrow t\bar{t} \) as

\[
d\sigma = \frac{(2\pi)^4 \delta(\vec{P}_e + \vec{P}_e - \vec{P}_t - \vec{P}_\bar{t})}{4\sqrt{(\vec{P}_e + \vec{P}_e)^2 - (m_e + m_e)^2}} \sum |M|^2 \frac{d^3\vec{P}_e}{(2\pi)^3 2E_e} \frac{d^3\vec{P}_\bar{t}}{(2\pi)^3 2E_\bar{t}}.
\]

Integrating out phase space, we obtain the total cross section

\[
\sigma = \frac{\sqrt{s(s-4m_t^2)}}{32\pi s^2} \int_{-1}^{1} \sum |M|^2 d\cos \theta = \sigma_0 + \delta \sigma,
\]

where \( \sigma_0 \) is the cross section at the tree-level, and \( \delta \sigma \) denotes the Yukawa corrections arising from PGBs at the one-loop level.

Through the above equations, we calculate the relative corrections to the production rate \( \sigma \) arising from PGBs. Since the ETC sector of this model is one generation technicolor model, the masses of the PGBs are model dependent. In Ref. [13], the masses for \( \pi \) and \( \pi_8 \) are taken to be in the range \( 60 \text{ GeV} < m_\pi < 200 \text{ GeV}, \ 200 \text{ GeV} < m_{\pi_8} < 500 \text{ GeV}. \) In the TOPC sector, the mass of the top-pion, \( m_\pi \), a reasonable value of the parameter is around \( 200 \text{ GeV}. \) In the following calculation, we would rather take a slightly larger range, \( 150 \text{ GeV} < m_\pi < 450 \text{ GeV}, \) to see its effect, and shall take the masses of \( m_\pi, 150 \text{ GeV}, \) and \( m_{\pi_8}, 246 \text{ GeV}. \) Furthermore, the common mass \( m_\pi \) is assumed for color-singlets and \( m_{\pi_8} \) for color-octet scalars. The final numerical results are plotted in Figs. 2 and 3.

**FIG. 2.** The relative correction \( \delta \sigma/\sigma_0 \) curves as a function of \( m_\pi. \) The solid lines 1, 2 and 3 correspond to \( \varepsilon = 0.03, 0.07, 0.1 \) with \( \sqrt{s} = 500 \text{ GeV}, \) respectively, while the dashed ones 4, 5 and 6, with \( \sqrt{s} = 1500 \text{ GeV}. \)

Figure 2 shows \( \delta \sigma/\sigma_0 \) versus \( m_\pi. \) When \( \sqrt{s} = 500 \text{ GeV}, \) one can see that (i) in case of \( m_\pi > 250 \text{ GeV}, \) the size of the relative corrections \( \delta \sigma/\sigma_0 \) decrease near to zero with \( m_\pi, \) sensitively, (ii) the maximum of the relative corrections can reach \(-10\% \) for \( \varepsilon = 0.07, m_\pi = 150 \text{ GeV}. \) However, when \( \sqrt{s} = 1500 \text{ GeV}, \delta \sigma/\sigma_0 \) changes from positive to negative, and negative correction becomes larger when the top-pion is heavier. In this case, the relative corrections can be up to \( 16\% \) when \( m_\pi = 150 \text{ GeV}, \varepsilon = 0.03 \) or 0.1.

The dependence of the relative correction on \( \sqrt{s} \) is shown in Fig. 3. The negative contributions from PGBs to \( e^+e^- \rightarrow t\bar{t} \) are found at the outset stage, but at \( \sqrt{s} = 1150 \text{ GeV}, \) the effects become positive.

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FIG. 3. $\delta \sigma/\sigma_0$ as a function of $E = \sqrt{s}$ with $m_{\pi^t} = 220$ GeV, $\varepsilon = 0.07$.

B. The gauge bosons corrections

Now let’s consider the contributions from the new gauge boson to the $e^+e^- \rightarrow t\bar{t}$ cross section.

In the TOPCTC theory, there are two kinds of new gauge bosons: the ETC gauge bosons $Z^*$ including the sideways and diagonal gauge bosons, and the TOPC gauge bosons including the color-octet colorons $B_\mu$ and color-singlet $Z'$. The relevant gauge bosons to the process $e^+e^- \rightarrow t\bar{t}$ are only $Z^*$ in ETC sector and $Z'$ in TOPC sector, the Feynman diagram is shown in Fig. 1(h).

Using formula (2,3) we can obtain the effects of the gauge bosons. The contributions from ETC gauge boson to the $e^+e^- \rightarrow t\bar{t}$ cross section are found to increase quickly with $\varepsilon$ and $\sqrt{s}$, and decrease slowly with $m_{Z^*}$, and the maximum of the relative corrections $\delta \sigma_{Z^*}/\sigma_0$ is only the order of $10^{-12}$ whatever $\varepsilon$, $\sqrt{s}$ and $m_{Z^*}$ are taken in the above parameter ranges, therefore, can be neglected safely.

FIG. 4. The relative correction $\delta \sigma/\sigma_0$ of the gauge boson $Z'$ as a function $m_{Z'}$. The lines a, b and c correspond to $\sqrt{s} = 500$, 1000 and 1500 GeV, respectively.

As for the contributions from $Z'$, it is obvious the contributions to the $e^+e^- \rightarrow t\bar{t}$ cross section aren’t related to $\theta'$ (see Eq. (2)). In our calculation, we assume the mass of the gauge boson $Z'$ varying from 300 Gev to 1000 GeV.
to study the effects of $Z'$. The numerical results are plotted in Fig. 4. From this figure we find that except for the $Z'$ resonance region, the relative correction $\delta\sigma_{Z'}/\sigma_0$ is also quite small, and the corrections increase with $m_{Z'}$ at the outset stage and decrease at the back stage at $\sqrt{s} = 500$ and always increase in case of $\sqrt{s} = 1000, 1500$ GeV.

Now we would like extend this study to the top-color-assisted multiscale technicolor (TOPCMTC) model \[14,18\]. The main difference from the original TOPCTC model \[7\] is the ETC sector. Instead of the one generation technicolor and searching for the signs of technicolor.

We have studied the contributions from the pseudo Goldstone bosons and new gauge bosons in the original top-color-assisted technicolor model to $t\bar{t}$ production at the $\sqrt{s} = 0.5, 1.0,$ and $1.5$ TeV $e^+e^-$ colliders. We found for reasonable ranges of the parameters, the pseudo Goldstone bosons afford dominate contribution, the correction arising from new gauge bosons is negligibly small, the maximum of the relative corrections is $-10\%$ with the c.m. energy $\sqrt{s} = 500$ GeV; whereas in case of $\sqrt{s} = 1500$ GeV, the maximum of the relative corrections may reach $16\%$. We thus conclude that the $e^+e^- \rightarrow t\bar{t}$ experiments at the future colliders are really interesting in testing the standard model and searching for the signs of technicolor.

IV. THE DISCUSSIONS AND CONCLUSIONS

We have studied the contributions from the pseudo Goldstone bosons and new gauge bosons in the original top-color-assisted technicolor model to $t\bar{t}$ production at the $\sqrt{s} = 0.5, 1.0,$ and $1.5$ TeV $e^+e^-$ colliders. We found for reasonable ranges of the parameters, the pseudo Goldstone bosons afford dominate contribution, the correction arising from new gauge bosons is negligibly small, the maximum of the relative corrections is $-10\%$ with the c.m. energy $\sqrt{s} = 500$ GeV; whereas in case of $\sqrt{s} = 1500$ GeV, the maximum of the relative corrections may reach $16\%$. We thus conclude that the $e^+e^- \rightarrow t\bar{t}$ experiments at the future colliders are really interesting in testing the standard model and searching for the signs of technicolor.

APPENDIX: APPENDIX

The form factors $\Gamma_{12}, \Gamma_{13}$ of the effect vertex $Zt\bar{t}$ and $\gamma t\bar{t}$ arising from PGBs corrections are given out

$$F_{1Z} = \frac{1}{16\pi^2} \sum_{i=\pi,\eta_8,\varpi_8} \lambda_i \{ -2a_t(B_1(-P_t,m_b,m_i) + m_i^2B'_1(-P_t,m_b,m_i)) - 2kC^2_{24} $$

$$+ 2\nu_b(-B_0(\sqrt{s},m_b,m_b) - m_i^2C^1_0 + 2C^1_{24} - m_i^2C^1_{11}) \}

$$+ a_t[2am_i^2(2C^3_0 + C^3_{11}) + v_3(2C^3_{24} - B_0(-\sqrt{s},m_t,m_t) - B_1(-P_t,m_t,m_i)) $$

$$+ v_2m_t^2(-2C^3_{11} + 2C^3_{12} - C^3_0 - 2B'_1(-P_t,m_t,m_i)) \} \} \}

$$F_{2Z} = \frac{1}{16\pi^2} \sum_{i=\pi,\eta_8,\varpi_8} \lambda_i \{ 2m_i^2[v_tB'_1(-P_t,m_b,m_i) + v_b(C^1_0 + C^1_{11})] + a_t[2am_i^2(2C^3_0 + C^3_{11})$$

$$+ a_t(2C^3_{24} - B_0(-\sqrt{s},m_t,m_t) - B_1(-P_t,m_t,m_i)) + a_t m_i^2(-2C^3_{11} + C^3_{12} \}

$$- C^3_0 - 2B'_1(-P_t,m_t,m_i) - 2B'_0(-P_t,m_t,m_i)) \} \} \}

$$F_{3Z} = \frac{1}{16\pi^2} \sum_{i=\pi,\eta_8,\varpi_8} \lambda_i \{ [4v_b m_t(C^3_{12} + C^4_{22}) + km_t(-C^2_{11} + C^2_{12} - 2C^2_{21} - 2C^2_{22} + C^2_{23}) $$

$$+ a_t[2am_i^4(C^3_{22} - 2am_i^2(C^3_{22} - C^3_{23})] \} \}

$$F_{4Z} = \frac{1}{16\pi^2} \sum_{i=\pi,\eta_8,\varpi_8} \lambda_i \{ [4v_b m_t(-C^2_{22} + C^4_{23}) + km_t(-C^3_{12} + 2C^3_{22} - 2C^3_{23})]$$

$$+ a_t[2am_i^4(C^3_{22} - 2am_i^2(C^3_{22} - C^3_{23})] \} \}

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\[ F_{5Z} = \frac{1}{16\pi^2} \sum_{i=\pi,\pi_8,\pi_7} \lambda_i \{ [4v_b m_t (C_{22}^1 - C_{23}^1) + k m_t (C_{12}^1 - C_{11}^2 - 2C_{22}^2 + 2C_{23}^2)] \\
+ a_i [2a_i m_t (-2C_{11}^3 + 2C_{12}^3 - C_{21}^3 - C_{22}^3 + 2C_{23}^3) + 2v_t m_t (C_{32}^3 + C_{33}^3)] \} \]

\[ F_{6Z} = \frac{1}{16\pi^2} \sum_{i=\pi,\pi_8,\pi_7} \lambda_i \{ [4v_b m_t (-C_{11}^3 + C_{12}^1 - C_{21}^1 - C_{22}^1 + 2C_{23}^3) + k m_t (2C_{12}^2 + 2C_{22}^2)] \\
+ a_i [2v_t m_t (-2C_{11}^3 + 2C_{12}^3 - C_{21}^3 - C_{22}^3 + 2C_{23}^3) + 2a_i m_t (C_{32}^3 + C_{33}^3)] \} \]

where \( k = (1 - 2s_W^2)/(2s_W c_W), v_b = \frac{2}{3}s_W^2/(4s_W c_W), a_b = (-1 + 2s_W^2)/(4s_W c_W); B_i' = \frac{\partial}{\partial P_i} B_i |_{p_2 = m_t^2}; C^1 = C^1(P_t, -\sqrt{s}, m_t, m_b, m_\tau), C^2 = C^2(-P_t, \sqrt{s}, m_t, m_b, m_\tau), C^3 = C^3(-P_t, -\sqrt{s}, m_t, m_b, m_\tau). \) The basic two- and three-scalar integral functions can be found in Ref. [19], and

\[ F_{\gamma} = F_{IZ} |_{Z \rightarrow \gamma, m_Z = 0, v_3 = v_5 = 1/3, v_b = a_b = -1/6, v_\tau = a_\tau = -1/2, k = 1}. \]

For \( i = \pi, \)
\[ \lambda_\pi = \left( \frac{c_i m_t^4}{f_\pi} \right)^2, m_t^\prime = \varepsilon m_t, m_b = 0.1 m_t^\prime, a_i = 2. \]

For \( i = \pi_8, \)
\[ \lambda_\pi = \left( \frac{m_t^4 \lambda^a}{f_\pi} \right)^2, m_t^\prime = \varepsilon m_t, m_b = 0.1 m_t^\prime, a_i = 2. \]

And for \( i = \pi_7, \)
\[ \lambda_\pi = \left( \frac{(1 - \varepsilon) m_t^2}{f_\pi} \right)^2, m_b = 4.5 \text{GeV}, a_i = 1. \]

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