Probabilistic topic modeling is a popular choice as the first step of crosslingual tasks to enable knowledge transfer and extract multilingual features. While many multilingual topic models have been developed, their assumptions on the training corpus are quite varied, and it is not clear how well the models can be applied under various training conditions. In this paper, we systematically study the knowledge transfer mechanisms behind different multilingual topic models, and through a broad set of experiments with four models on ten languages, we provide empirical insights that can inform the selection and future development of multilingual topic models.

1. Introduction

Popularized by Latent Dirichlet Allocation (Blei, Ng, and Jordan 2003), probabilistic topic models have been an important tool for analyzing large collections of texts (Blei 2012, 2018). Their simplicity and interpretability make topic models popular for many natural language processing tasks, such as discovering document networks (Chen et al. 2013; Chang and Blei 2009) and authorship attribution (Seroussi, Zukerman, and Bohnert 2014).

Topic models take a corpus $D$ as input, where each document $d \in D$ is usually represented as a sparse vector in a vocabulary space, and project these documents to a lower-dimensional topic space. In this sense, topic models are often used as a dimensionality reduction technique to extract representative and human-interpretable features.

Text collections, however, are often not in a single language, and thus there has been a need to generalize topic models from monolingual to multilingual settings. Given a corpus $D^{(1,\ldots,L)}$ in languages $\ell \in \{1,\ldots,L\}$, multilingual topic models learn topics in each of the languages. From a human’s view, each topic should be related to the same theme, even if the words are not in the same language (Figure 1b). From a machine’s view, the word probabilities within a topic should be similar across languages, such that the low-dimensional representation of documents is not dependent on the language. In other words, the topic space in multilingual topic models is language agnostic (Figure 1a).

This paper presents two major contributions to multilingual topic models. We first provide an alternative view of multilingual topic models by explicitly formulating a crosslingual knowledge transfer process during posterior inference (Section 3). Based on this analysis, we unify different multilingual topic models by defining a function called the transfer operation. This function provides an abstracted view of the knowledge transfer mechanism behind these models, while enabling further generalizations and
Multilingual topic models project language-specific and high-dimensional features from the vocabulary space to a language-agnostic and low-dimensional topic space. This figure shows a t-SNE (Maaten and Hinton 2008) representation of a real dataset.

Figure 1: Overview of multilingual topic models.

improvements. Using this formulation, we analyze several existing multilingual topic models (Section 4).

Second, in our experiments we compare four representative models under different training conditions (Section 5). The models are trained and evaluated on ten languages from various language families to increase language diversity in the experiments. In particular, we include five languages with relatively high resources and the others with low resources. To quantitatively evaluate the models, we focus on topic quality in Section 5.5.1, and performance of downstream tasks using crosslingual document classification in Section 5.5.2. We investigate how models are sensitive to different language resources (i.e., parallel/comparable corpus and dictionaries), and analyze what factors cause this difference (Section 6).

2. Background

We first review monolingual topic models, focusing on Latent Dirichlet Allocation, and then describe two families of multilingual extensions. Based on the types of supervision added to multilingual topic models, we separate the two model families into document-level and word-level supervision.

Topic models provide a high-level view of latent thematic structures in a corpus. Two main branches for topic models are non-probabilistic approaches such as Latent Semantic Analysis (LSA, Deerwester et al. (1990)) and Non-Negative Matrix Factorization (NMF, Xu, Liu, and Gong (2003)), and probabilistic ones such as Latent Dirichlet Allocation (LDA, Blei, Ng, and Jordan (2003)) and probabilistic LSA (PLSA, Hofmann (1999)). All these models were developed for monolingual data and further adapted to multilingual
situations. Though there are works of adaptations on non-probabilistic models based on “pseudo-bilingual” corpora approaches (Littman, Dumais, and Landauer 1998), most multilingual topic models that are trained on corpora in different languages are based on probabilistic models, especially LDA. Therefore, our work is mainly focused on the probabilistic topic models in multilingual settings, and in the following section, we start with LDA.

2.1 Monolingual Topic Models

The most popular topic model is Latent Dirichlet Allocation (LDA), introduced by Blei, Ng, and Jordan (2003). This model assumes each document $d$ is represented by a multinomial distribution $\theta(d)$ over topics, while each “topic” $k$ is a multinomial distribution $\phi(k)$ over the vocabulary $V$. In the generative process, each $\theta$ and $\phi$ are generated from Dirichlet distributions parameterized by $\alpha$ and $\beta$, respectively. The hyperparameters for Dirichlet distributions can be asymmetric (Wallach, Mimno, and McCallum 2009), though in this work we use symmetric priors. Figure 2 shows the plate notation of LDA.

2.2 Multilingual Topic Models

We now describe a variety of multilingual topic models, organized into two families based on the type of supervision they use. Later (Section 4), we focus on a subset of the models described here for deeper analysis using our knowledge transfer formulation, selecting the most general and representative models.

2.2.1 Document Level. The first model proposed to process multilingual corpora using LDA is the Polylingual Topic Model (PLTM, Mimno et al. (2009); Ni et al. (2009)). This model extracts language-consistent topics from parallel or highly comparable multilingual corpora (for example, Wikipedia articles aligned across languages), assuming that document translations share the same topic distributions. This model has been extensively used and adapted in various ways for different crosslingual tasks (Krstovski and Smith 2016; Liu, Duh, and Matsumoto 2015; Vulic and Moens 2014; Moens and Vulic 2013; Krstovski and Smith 2011).

In the generative process, PLTM first generates language-specific topic-word distributions $\phi(k,\ell) \sim \text{Dir}(\beta)$ for topics $k = 1, \ldots, K$ and languages $\ell = 1, \ldots, L$. Then, for each document tuple $d = (d^{(1)}, \ldots, d^{(L)})$, it generates a tuple-topic distribution $\theta(d) \sim \text{Dir}(\alpha)$. Every topic in this document tuple is generated from $\theta(d)$, and the word tokens in this document tuple are then generated from language-specific word distributions $\phi(k,\ell)$ for each language. To apply PLTM, the corpus must be parallel or closely comparable to provide document-level supervision. We refer to this as the document links model (DOCLINK).

Models that transfer knowledge on the document level have many variants, including SOFTLINK (Hao and Paul 2018), comparable bilingual latent Dirichlet allocation (C-
BiLDA, Heyman, Vulic, and Moens (2016)), the partially-connected multilingual topic model (pcMLTM, Liu, Duh, and Matsumoto (2015)), and multi-level hyperprior polylingual topic model (mlhPLTM, Krstovski, Smith, and Kurtz (2016)). SOFTLINK generalizes DOCLINK by using a dictionary, so that documents can be linked based on overlap in their vocabulary, even if the corpus is not parallel or comparable. C-BiLDA is a direct extension of DOCLINK which also models language-specific distributions to distinguish topics that are shared across languages from language-specific topics. pcMLTM adds an additional observed variable to indicate the absence of a language in a document tuple. mlhPLTM uses a hierarchy of hyperparameters to generate section-topic distributions. This model was motivated by applications to scientific research articles, where each section \( s \) has its own topic distribution \( \theta(s) \) shared by both languages.

### 2.2.2 Word Level.

Instead of document-level connections between languages, Boyd-Graber and Blei (2009) and Jagarlamudi and Daumé III (2010) proposed to model connections between languages through words using a multilingual dictionary and apply hyper-Dirichlet Type-I distributions (Andrzejewski, Zhu, and Craven 2009; Dennis III 1991). We refer to these approaches as the vocabulary links model (voclink). See Figure 3 for an illustration.

Specifically, VOCLINK uses a dictionary to create a tree structure where each internal node contains word translations, and words that are not translated are attached directly to the root of the tree \( r \) as leaves. In the generative process, for each language \( \ell \), VOCLINK first generates \( K \) multinomial distributions over all internal nodes and word types that are not translated, \( \phi^{(k,r)} \sim \text{Dir}(\beta^{(r\rightarrow i)}) \), where \( \beta^{(r\rightarrow i)} \) is the Dirichlet prior from root \( r \) to an internal node \( i \) or an untranslated word \( w \). Then under each internal node \( i \), for each language \( \ell \), VOCLINK generates a multinomial \( \phi^{(k,\ell,i)} \sim \text{Dir}(\beta^{(i\rightarrow w)}) \) over word types in language \( \ell \) under the node \( i \). Thus, to draw a word in language \( \ell \) is equivalent to generating a path from the root to leaf nodes, i.e., \((r \rightarrow i, i \rightarrow w^{(\ell)})\) or \((r \rightarrow w^{(\ell)})\):

\[
\Pr(r \rightarrow i, i \rightarrow w^{(\ell)}|k) = \Pr(i|k) \cdot \Pr(w^{(\ell)}|k, i), \tag{1}
\]

\[
\Pr(r \rightarrow w^{(\ell)}|k) = \Pr(w^{(\ell)}|k). \tag{2}
\]

Document-topic distributions \( \theta^{(d)} \) are generated in the same way as monolingual LDA, since no document translation is required.

The use of dictionaries to model similarities across topic-word distributions has been formulated in other ways as well. ProbBiLDA (Ma and Nasukawa 2017) uses inverted indexing (Søgaard et al. 2015) to encode assumptions that word translations are generated from same distributions. ProbBiLDA does not use tree structures in the parameters as in VOCLINK, but the general idea of sharing distributions among word translations is similar. Gutiérrez et al. (2016) use part-of-speech taggers to separate topic words (nouns) and perspective words (adjectives and verbs), developed for the application of detecting cultural differences, i.e., how different languages have different perspectives on the same topic. Topic words are modeled in the same way as in VOCLINK, while perspective words are modeled in a monolingual fashion.

### 3. Crosslingual Knowledge Transfer in Probabilistic Topic Models

In this section, we define the general forms of crosslingual topic models used as the building blocks for our analysis. We distinguish the joint generative model, in which
both languages are generated jointly, from the **conditional generative model**, in which the two languages are generated sequentially, with one conditioned on the other. Our analysis focuses on bilingual models that generate data in two languages.

Multilingual topic models usually assume that the observations of all the languages are generated at the same time, sharing certain distributions among document or word translation pairs. We refer this process as the joint generative model.

**Definition 1** (Joint generative model)
A joint generative model $G^{(\ell_1, \ell_2)}$ is a probabilistic graphical model that generates all the observations of languages $\ell_1$ and $\ell_2$ at the same time.

The simultaneity in joint generative models masks the important process of knowledge transfer during model training; in this paper, we want to understand in detail how knowledge from one language is **transferred** to another language in such models, i.e., how the sampled parameters from one language influence the sampling of parameters in another language. To make this process explicit, we define conditional generative models.

**Definition 2** (Conditional generative model)
A conditional generative model $G^{(\ell_1, \ell_2)}_{\ell_1}$ is a probabilistic graphical model that generates observations of language $\ell_1$ first from $G^{(\ell_1)}$, and that of language $\ell_2$ conditioned on $\ell_1$ from $G^{(\ell_2|\ell_1)}$.

Adding conditionality to conditional generative models make them no longer be in a conventional form of generative models, because the conditionality can go both ways (e.g., $\ell_2$ could be generated first, with $\ell_1$ conditioned on $\ell_2$). In this section, we will show two conditional generative models, $G^{(\ell_1, \ell_2)}_{\ell_1}$ and $G^{(\ell_1, \ell_2)}_{\ell_2}$, can be combined as an instance of pseudo-likelihood (Besag 1975), where the joint likelihood of two documents is

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1 In this definition, we put a subscript under the graph notation to indicate that $\ell_1$ generates observations first. We will use the same convention for probability notation in the rest of the paper.
approximated as the product of each document’s conditional likelihood given the other, i.e., \( \Pr\left( d_{\ell_1}, d_{\ell_2} \right) \approx \Pr\left( d_{\ell_1} | d_{\ell_2} \right) \cdot \Pr\left( d_{\ell_2} | d_{\ell_1} \right) \).

**Theorem 1** (Pseudo-likelihood approximation)
The maximum *a posteriori* (MAP) estimate of a joint generative model is equivalent to the MAP estimate of the product of the two corresponding conditional generative models:

\[
\arg \max_{\Theta, \Phi} \Pr\left( G_{\ell_1, \ell_2} \right) = \arg \max_{\Theta, \Phi} \Pr\left( G_{\ell_1 \mid \ell_2} \right) \cdot \Pr\left( G_{\ell_2 \mid \ell_1} \right),
\]

(3)

The pseudo-likelihood approximation is not a numerically accurate approximation to the likelihood of \( G_{\ell_1, \ell_2} \), but the optimal parameters for the pseudo-likelihood function are the same as for the original joint likelihood function (Besag 1975; Koller and Friedman 2009; Leppä-aho et al. 2017). See Hao and Paul (2018) for a proof of this property that is specific to multilingual topic models. This property thus provides a convenient objective to learn consistent estimates of parameters, and importantly, this formulation provides an explicit way to analyze how model parameters are transferred from one language to another.

### 3.1 Transfer Operation

**Theorem 1** enables us to now formulate the knowledge transfer process in detail.

**Definition 3** (Transfer operation)
A transfer operation on a distribution \( \Omega \) from language \( \ell_1 \) to \( \ell_2 \) is defined as a function \( h_\Omega \left( z_{\ell_1}, \xi, S_{\ell_1, \ell_2} \right) \) where \( z_{\ell_1} \) is the observations of topics from \( \ell_1 \), \( \xi \) is the prior knowledge, and \( S_{\ell_1, \ell_2} \) is any bilingual supervision needed in this knowledge transfer process.

A transfer operation \( h_\Omega \left( z_{\ell_1}, \xi, S_{\ell_1, \ell_2} \right) \) is used as the prior for distribution \( \Omega \) to generate observations in \( \ell_2 \). If an observation \( z \) in language \( \ell_2 \) is generated by a multinomial \( \Omega \) from a Dirichlet, we can impose the transfer operation by:

\[
\begin{align*}
\Omega & \sim \text{Dirichlet} \left( h_\Omega \left( z_{\ell_1}, \xi, S_{\ell_1, \ell_2} \right) \right), \\
z & \sim \text{Multinomial}(\Omega).
\end{align*}
\]

See Figure 4 for an illustration.

Given this definition, for a topic model that transfers knowledge at the document level, we can impose transfer operations on the document-topic distribution \( \theta \):

\[
\Pr\left( G_{\ell_2 \mid \ell_1} \right) = \Pr\left( w_{\ell_2}, z_{\ell_2}, \theta, \phi \mid h_\theta \left( z_{\ell_1}, \alpha, S_{\ell_1, \ell_2} \right), \beta \right) = \Pr\left( z_{\ell_2}, \theta \mid h_\theta \left( z_{\ell_1}, \alpha, S_{\ell_1, \ell_2} \right) \right) \cdot \Pr\left( w_{\ell_2}, \phi \mid z_{\ell_2}, \beta \right),
\]

(4)

(5)

where \( S_{\ell_1, \ell_2} \) could be document-level translation information. Note that the distribution \( \Omega \) in the definition of transfer operation could be any distribution to be modeled, not only \( \theta \) as we show here. We provide more examples of transfer operations in Section 4.
Figure 4: An illustration of a transfer operation on a 3-dimensional Dirichlet distribution. Using observations in $\ell_1$, $z^{(\ell_1)}$, prior knowledge $\xi$, and a supervision $S^{(\ell_1,\ell_2)}$, the transfer operation provides priors for the Dirichlet distribution from which a multinomial $\Omega$ in $\ell_2$ is drawn.

Figure 5: The Gibbs sampling procedures for multilingual topic models can be treated as sampling a joint graphic model (left), or equivalently, two conditional graphic models iteratively.

3.2 Collapsed Gibbs Sampling

General approaches to infer posterior distributions over topic model parameters include collapsed Gibbs sampling, variational inference, or hybrid approaches (Hu et al. 2014; Kim, Voelker, and Saul 2013). We focus on collapsed Gibbs sampling (Griffiths and Steyvers 2004). Figure 5 illustrates an example of Gibbs sampling procedures in multilingual topic models.

Typically, in each iteration of Gibbs sampling (a “sweep” of samples), the sampler goes through each token in each document sequentially, which can be viewed as sampling from the full posterior of a joint generative model (Figure 5 left). Alternatively, we can separate each sweep into two subprocedures, one for each language. In Figure 5 (right), when the sampler goes through English (EN) tokens, the latent topic assignments for Swedish (SV) tokens are fixed, and therefore it is sampling from the posterior of the conditional generative model of $G^{(EN,SV)}$. When Swedish tokens are being sampled, it is sampling from the posterior of $G^{(EN,SV)}$. Importantly, the different languages share the same calculation for the posterior sampling distribution (Hao and Paul 2018).

4. Representative Models

In this section, we describe four representative multilingual topic models in terms of the transfer operation formulation. These are also the models we will experiment on in
Section 5. The plate notations of these models are shown in Figure 6. Note that we use the original formulation of these models without explicitly showing transfer operations, so only SOFTLINK uses the transfer distributions $\delta$.

4.1 DOCLINK

The document links model (DOCLINK) uses parallel/comparable datasets, so we use an indicator matrix $\delta \in \mathbb{N}_+^{D^{(\ell_2)} \times D^{(\ell_1)}}$ to indicate if a document $d^{(\ell_1)} \in D^{(\ell_1)}$ is a translation to a document $d^{(\ell_2)} \in D^{(\ell_2)}$,

$$
\delta_{d^{(\ell_2)},d^{(\ell_1)}} = \mathbf{1} \left\{ d^{(\ell_2)} \text{ and } d^{(\ell_1)} \text{ are translations} \right\}.
$$

Thus, the transfer operation at the document level for each document $d^{(\ell_2)}$ can be defined as

$$
h_{\theta_{d^{(\ell_2)}}} \left( z^{(\ell_1)}, \alpha, S^{(\ell_1,\ell_2)} \right) = h_{\theta_{d^{(\ell_2)}}} \left( z^{(\ell_1)}, \alpha, \delta \right) = \left( \delta \cdot N^{(\ell_1)} \right)_{d^{(\ell_2)}} \oplus \alpha,
$$

Figure 6: Plate notations of DOCLINK, C-BiLDA, SOFTLINK, and VOCLINK (from left to right). We use red lines to make the knowledge transfer component clear.
where \( z^{(\ell_1)} \) is re-shaped into a matrix \( N^{(\ell_1)} \in \mathbb{N}^{[D^{(\ell_1)}] \times K} \), \( K \) is the number of topics, and \( \oplus \) denotes element-wise addition.

Since no dictionary is required, the transfer operation at the word level in DOCLINK is straightforward. For every topic \( k = 1, \ldots, K \) and each word type \( w \) regardless of its language, \( h_{\theta(k,t)} (z^{(\ell_1)}, \beta, S^{(\ell_1,\ell_2)}) = \beta \), where \( \beta \) is the Dirichlet prior for the topic-vocabulary distributions \( \phi^{(k,\ell)} \).

### 4.2 C-BiLDA

As a variation of DOCLINK, C-BiLDA has all of the components of DOCLINK, so this model transfers knowledge on document level as well. C-BiLDA additionally models topic-language distributions \( \eta \).\(^2\) Thus, in terms of transfer operations \( h_\theta \) and \( h_\phi \), C-BiLDA uses the same formulations as in Equation (7), and we will now focus on the topic-language distributions \( \eta \).

In C-BiLDA, for each document pair \( d \) and each topic \( k \), a bivariate Bernoulli distribution over the two languages \( \eta^{(k,d)} \) is drawn from a Beta distribution:

\[
\eta^{(k,d)} \sim \text{Beta} \left( \chi^{(d,\ell_2)}, \chi^{(d,\ell_1)} \right), \tag{8}
\]

\[
\ell^{(k,m)} \sim \text{Bernoulli} \left( \eta^{(k,d)} \right), \tag{9}
\]

where \( \ell^{(k,m)} \) is the language of the \( m \)-th token which is assigned to topic \( k \) in this document pair \( d \).

To define a transfer operation on the topic-language distribution, we use the same supervision \( \delta \) as defined in Equation (6), and similarly, we reshape the observations \( z^{(\ell_1)} \) from language \( \ell_1 \) into a matrix \( N^{(\ell_1)} \in \mathbb{N}^{[D^{(\ell_1)}] \times K} \). Thus, the transfer operation on the topic-language distribution is:

\[
h_{\eta(k,d,\ell_2)} \left( z^{(\ell_1)}, \chi^{(d,\ell_1)}, \delta \right) = \left( \delta \cdot N^{(\ell_1)} \right)_{d(\ell_2),k} + \chi^{(d,\ell_1)}, \tag{10}
\]

where \( \left( \delta \cdot N^{(\ell_1)} \right)_{d(\ell_2),k} \) is the number of topics \( k \) in document \( d^{(\ell_1)} \) that are linked to document \( d^{(\ell_2)} \).

The transfer process for C-BiLDA can be formed as follows. As usual, the model \( G^{(\ell_1)} \) first generates documents in \( \ell_1 \) using monolingual LDA. When generating documents in \( \ell_2 \), the conditional model \( G^{(\ell_2|\ell_1)} \) generates topics \( z \) for each token using the document-level transfer operation as defined in Equation (7). Instead of directly drawing a new word type according to \( z \), C-BiLDA additionally uses the transfer operation \( h_{\phi(k,\ell_1,\ell_2)} \) to draw a language for this token \( \ell' \). Since the current token is known to be in language \( \ell_2 \), if \( \ell' \neq \ell_2 \), we skip this token, and draw the next topic \( z \); otherwise, we use \( \phi^{(z,\ell_2)} \) to draw a word type and attach it to the current document. Conceptually, C-BiLDA adds an additional “selector” in the generative process, to decide if a topic should appear more in \( \ell_2 \) based on documents in \( \ell_1 \). We use Figure 7 as an illustration to show the difference between

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\(^2\) The original notation for topic-language distribution is \( \delta \) (Heyman, Vulic, and Moens 2016). To avoid confusion in Equation (6), we change to \( \eta \). We also follow the original paper where the model is for bilingual case.
DOCLINK and C-BiLDA. As we will discuss in Section 6.5.1, this selector mechanism is useful when the paired documents have mismatched document lengths.

4.3 SOFTLINK

In the two document-level models above, δ is constructed using comparable datasets as supervision to enable knowledge transfer. SOFTLINK defines δ differently, constructing “soft” document links but without requiring a comparable corpus, motivated by the fact that this requirement is too demanding for low-resource languages (Hao and Paul 2018). This model extends the same concept of document-level supervision from DOCLINK, but generalizes δ to be a probabilistic matrix rather than a one-to-one parallel document mapping as in DOCLINK. See Figure 8 for an illustration.

Specifically, in SOFTLINK, each row in \( \delta \in \mathbb{R}^{|D^{(e_2)}| \times |D^{(e_1)}|} \) is a transfer distribution over all documents in \( D^{(e_1)} \) for a document \( d^{(e_2)} \). In Hao and Paul (2018), the transfer distribution is defined with the pairwise document Jaccard index according to a dictionary \( \mathcal{L}^{(e_1,e_2)} \):

\[
\delta_{d^{(e_2)},d^{(e_1)}} \propto \frac{| \{ w^{(e_1)} \} \cap \{ w^{(e_2)} \} |}{| \{ w^{(e_1)} \} \cup \{ w^{(e_2)} \} |}, \quad \forall \ w^{(e_1)} \in d^{(e_1)} \text{ and } w^{(e_2)} \in d^{(e_2)},
\]

where \( \{ w^{(e)} \} \) contains all the word types that appear in document \( d^{(e)} \), and \( \{ w^{(e_1)} \} \cap \{ w^{(e_2)} \} \) indicates all word pairs \( (w^{(e_1)}, w^{(e_2)}) \) that can be found in a dictionary \( \mathcal{L}^{(e_1,e_2)} \) as translations.

Hao and Paul (2018) show that instead of a dense distribution, it is more efficient to make the transfer distributions sparse by thresholding:

\[
\tilde{\delta}_{d^{(e_2)},d^{(e_1)}} \propto 1 \left\{ \delta_{d^{(e_2)},d^{(e_1)}} > \pi \cdot \max(\delta) \right\} \cdot \delta_{d^{(e_2)},d^{(e_1)}},
\]
Figure 8: An example of how topic knowledge is transferred through DOCLINK and SOFTLINK. To generate observations in $\ell_2$, both models use topics in $\ell_1$ as a prior knowledge to shape the Dirichlet distribution in $\ell_2$. This transfer happens in DOCLINK through document translations from a parallel corpus, while transfer happens in SOFTLINK through the generalized transfer distribution $\delta$.

where $\pi$ is a fixed threshold parameter. When the predicate of max is the entire matrix $\delta$, this is referred to as a corpus-wise selection scope; when it is one row $\delta_{d(\ell)}$ with respect to a document $d(\ell)$, this is called a document-wise selection scope. Regardless of the selection scope, $\tilde{\delta}_{d(\ell)}$ is re-normalized for each row, i.e., each document $d(\ell)$.

SOFTLINK is a generalization of DOCLINK but uses dictionary instead of parallel documents as supervision. Therefore, the transfer operations are defined in the same way as in DOCLINK, with a different calculation method of matrix $\delta$.

4.4 VOCLINK

As defined by Jagarlamudi and Daumé III (2010) and Boyd-Graber and Blei (2009), the original VOCLINK only transfers knowledges on word level. Thus, the transfer operation at the document level for every document $d(\ell)$ is $h(\alpha, S(\ell_1, \ell_2)) = \alpha$.

VOCLINK uses tree-structured priors over the word distributions (Figure 3). Each word is associated with at least one path, denoted as $\lambda_w$. If a word is translated, the path is $\lambda_w(\ell) = (r \rightarrow i, i \rightarrow w(\ell))$ where $r$ is the root and $i$ an internal node. Along this path, two multinomial distributions are modeled: from root node $r$ to internal nodes $i$, $\phi(k, \ell, r)$, and from internal nodes to a word type $w(\ell)$ in language $\ell$, $\phi(k, \ell, i)$. On the other hand, if a word is untranslated, it is directly attached to the root $r$, and the path is $\lambda_w(\ell) = (r \rightarrow w(\ell))$, and only $\phi(k, \ell, r)$ is modeled.

Note that $\phi(k, \ell, r)$ is a distribution over all internal nodes and untranslated word types in language $\ell$. Therefore, different languages in fact model different $\phi(k, \ell, r)$ due to different vocabularies of untranslated words. However, knowledge transfer can still happen for internal nodes where both languages share sub-distributions.

Suppose $\ell_2$ has a vocabulary of size $|V(\ell_2)|$, and there are $I$ internal nodes according to a dictionary, excluding the root. We create an indicator matrix $\delta \in \mathbb{N}^{\nu(\ell_2) \times I}$,

$$
\delta_{w(\ell_2), i} = \mathbf{1}\left\{ w(\ell_2) \text{ is under internal node } i \right\}.
$$

3 While some models as in Hu et al. (2014) transfers knowledge on both document and word levels, in this analysis, we only focus on word level assuming no transfer happens on document level. The generalization, however, is straightforward, by modifying transfer operation on $\delta$. 

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Table 1: Summary of transfer operations defined in the compared models. We assume the direction of transfer is from $\ell_1$ to $\ell_2$.

| Model   | Document level | Word level | Parameters of $h$ |
|---------|----------------|------------|-------------------|
| DoCLink | $\left[ \delta \cdot N^{(\ell_1)} \right]_{d(t_2)} \oplus \beta$ | $\beta$ | $\delta$: indicator matrix; $N^{(\ell_1)}$: doc-by-topic matrix; $S$: comparable documents; |
| C-BiLDA | $\left[ \delta \cdot N^{(\ell_1)} \right]_{d(t_2)} \oplus \beta$, $\delta \cdot N^{(\ell_2)}_{d(t_1)} + \chi(d,t_1)$ | $\beta$ | $\delta$: transfer matrix; $N^{(\ell_1)}$: doc-by-topic matrix; $S$: dictionary; |
| SoftLink | $\left[ \delta \cdot N^{(\ell_1)} \right]_{d(t_2)} \oplus \beta$ | $\beta$ | $\delta$: indicator matrix; $N^{(\ell_1)}$: node-by-topic matrix; $S$: dictionary; |
| VocLink | $\alpha$ | $(\delta \cdot N^{(\ell_1)})_{k} \oplus \beta^{(i \rightarrow j)}$, $\beta^{(i \rightarrow w(t))}$, $\beta^{(r \rightarrow w(t))}$ | $\delta$: indicator matrix; $N^{(\ell_1)}$: node-by-topic matrix; $S$: dictionary; |

Thus, for a word type $w^{(\ell_2)}$ whose path is $\lambda^{(\ell_2)} = \{(i \rightarrow j)\}$, the transfer operation of each edge $i \rightarrow j$ along the path is:

$$h_{\phi(k,\ell,i)} \left(z^{(\ell_1)}, \beta^{(i \rightarrow j)}, \delta\right) = \begin{cases} \left(\delta \cdot N^{(\ell_1)}\right)_k \oplus \beta^{(i \rightarrow j)} & \text{if } i = r, \\ \beta^{(i \rightarrow j)} & \text{otherwise,} \end{cases}$$

(14)

where $z^{(\ell_1)}$ is re-shaped into a matrix $N^{(\ell_1)} \in \mathbb{R}^{I \times K}$, discarding the statistics of untranslated word types in language $\ell_1$, and $\beta^{(i \rightarrow j)}$ the Dirichlet prior on the edge of $i \rightarrow j$.

### 4.5 Summary

For clarity, we summarize the definitions of transfer operation in different models in Table 1. Other models can also be analyzed in this way, since the observations from $z^{(\ell_1)}$ help shape the multinomial distributions from which observations in $\ell_2$ are drawn by using different transfer operations. In this paper, we will experiment on the four models in Sections 4.1 through 4.4, since these models are the most generalizable.

### 5. Experiment Settings

Multilingual topic models can be applied to any tokenizable language with sufficient language resources (document translations or dictionaries) available. With the huge gap of available resources among different languages, some natural questions to consider are, which models perform best for which languages, and which models are more generalizable regardless of availability of multilingual resources?

To answer these questions, we train the four models described in the previous section in ten languages. Considering the resources available, we separate the ten languages into two groups: high-resource languages (HighLAN) and low-resource languages (LowLAN). For HighLAN, we have relatively abundant resources such as dictionary entries and document translations. We additionally use these languages to simulate the settings of LowLAN by training multilingual topic models with different amount of resources. For LowLAN, we use all resources available to verify experiment results and conclusions from HighLAN.
We evaluate the models using crosslingual topic coherence as an intrinsic task, and crosslingual document classification as an extrinsic task.

5.1 Language Groups and Preprocessing

We separate the ten languages we experiment with into two groups, i.e., HIGHLAN and LOWLAN. In this section, we describe the preprocessing details of these languages.

5.1.1 HIGHLAN. Languages in this group have relatively large amount of resources, and have been widely experimented on in multilingual studies. Considering language diversity, we select representative languages from five different families: Arabic (AR, Semitic), German (DE, Germanic), Spanish (ES, Romance), Russian (RU, Slavic), and Chinese (ZH, Sinitic). We follow standard preprocessing procedures: we first use stemmers to process both documents and dictionaries (segmenter for Chinese), then we remove stopwords based on a fixed list and the most 100 frequent word types in the training corpus. The tools for preprocessing are listed in Table 2.

5.1.2 LOWLAN. Languages in this group have much fewer resources than those in HIGHLAN, considered as low-resource languages. We also select five languages from different families: Amharic (AM, Afro-Asiatic), Aymara (AY, Aymaran), Central Khmer (KM, Austroasiatic), Burmese (MY, Sino-Tibetan), and Swahili (SW, Niger-Congo). Preprocessing in this language group needs more consideration. Because they represent low-resource languages that most natural language processing tools are not available for, we do not use a fixed stopword list but we do remove the most 100 frequent word types. Stemmers are also not available for these languages, so we do not apply stemming. However, KM and MY need segmentation, so we collect a word list for both languages, and use FOMA to segment the texts, following Hulden (2009).

5.2 Training Sets

There are many resources available for multilingual research, such as the European Parliament Proceedings parallel corpus (EuroParl, Koehn (2005)), The Bible, and Wikipedia. EuroParl provides a perfectly parallel corpus with precise translations, but

Table 2: List of source of stemmers and stopwords used in experiments for HIGHLAN.

| Language | Family     | Stemmer                      | Stopwords |
|----------|------------|------------------------------|-----------|
| EN       | Germanic   | SnowBallStemmer 4            | NLTK      |
| DE       | Germanic   | SnowBallStemmer             | NLTK      |
| ES       | Romance    | SnowBallStemmer             | NLTK      |
| RU       | Slavic     | SnowBallStemmer             | NLTK      |
| AR       | Semitic    | Assem’s Arabic Light Stemmer | GitHub 5  |
| ZH       | Sinitic    | Jieba 6                     | GitHub 7  |
it only contains 21 European languages, which limits its generalizability to most of the languages. The Bible, on the other hand, is also perfectly parallel and is widely available in 2,530 languages.\(^8\) Its disadvantages, however, are that the contents are archaic (mostly about family and religion), the dataset size is small (1,189 chapters), and many languages do not have digital format (Christodoulopoulos and Steedman 2015).

Compared to EUROPARL and The Bible, Wikipedia provides comparable documents in many languages with a large range of content, making it a very popular choice for many multilingual studies. In our experiments, we create ten bilingual Wikipedia corpora, each containing documents in one of the languages in either HIGHLAN or LOWLAN, paired with documents in English (en). Though most multilingual topic models are not restricted to training bilingual corpora paired with English, this is a helpful way to focus our experiments and analysis.

### 5.3 Test Sets

For extrinsic evaluation, we also require held-out test data. One of the most common applications of topic models is to use the topic-word distributions trained from training sets to infer document-topic distributions on unseen documents, or test sets. In our experiments, we construct test sets from two sources: TED Talks 2013 (ted) and Global Voices (gv). TED contains parallel documents in all languages in HIGHLAN. Thus, for a language pair (EN, ℓ), the topic-word distributions \(\phi^{(en)}\) and \(\phi^{(ℓ)}\) can be used to infer topics on documents in both English and another language. The second test set, Global Voices (gv) contains all languages from both HIGHLAN and LOWLAN.

Using the two multilingual sources, we create two types of test sets for each language pair (EN, ℓ)—**TED+TED** and **TED+GV**. In **TED+TED**, we infer document-topic distributions on documents from TED in English and the paired language. This only applies to HIGHLAN, since TED do not have documents in LOWLAN. In **TED+GV**, we infer topics on English documents from TED, and infer topics on documents from GV in the paired language. This test set includes all the languages. The two types of test sets also represent different application situations. TED+TED implies that the test documents in both languages are parallel and come from the same source, while TED+GV represents how the topic model performs when the two languages have different data sources.

### 5.4 Model Specifications

The goal of this study is to bring current multilingual topic models together, studying their corresponding strengths and limitations. To keep the experiments as comparable as possible, we use constant hyperparameters that are consistent across the models.

For all models, we set number of topics \(K = 20\) and the Dirichlet hyperparameter \(\alpha = 0.1\). We run 1,000 Gibbs sampling iterations on the training set and 200 iterations on the test set. For specific model configurations, see Table 3.

### 5.5 Evaluation

We evaluate all models using both intrinsic and extrinsic metrics. Intrinsic evaluation is used to measure the topic quality or coherence learned from the training set, while extrinsic evaluation measures performance after applying the trained distributions to

---

\(^8\) Reported by the United Bible Societies: [https://www.unitedbiblesocieties.org/](https://www.unitedbiblesocieties.org/)
We set $\beta = 0.01$ for all word types of all the languages, and use the MALLET implementation for training (McCallum 2002). To enable consistent comparison, we disable hyperparameter optimization provided in the package.

Following the experiment results from Heyman, Vulic, and Moens (2016), we set $\chi = 2$ to make the results more competitive to doclink. We also set $\beta = 0.01$ for all word types.

We use the document-wise thresholding approach for calculating the transfer distributions. The focus threshold is set to 0.8. We also set $\beta = 0.01$ for all word types.

We set the hyperparameter from the root to internal nodes or leaves $\beta = 0.01$. For those from internal nodes to leaves, we set $\beta^{(i\rightarrow w)} = \beta' = 100$, following the settings in Hu et al. (2014).

Table 3: Model specifications.

downstream crosslingual applications. For all the following experiments and tasks, we start by analyzing languages in HighLan. Then we apply the analyzed results to LowLan.

We choose topic coherence (Hao, Boyd-Graber, and Paul 2018) and crosslingual document classification (Smet, Tang, and Moens 2011) as intrinsic and extrinsic evaluation tasks, respectively. The reason for choosing these two tasks is that they examine the models from different angles: topic coherence looks at topic-word distributions, while classification focuses on document-topic distributions. Other evaluation tasks, such as word translation detection and crosslingual information retrieval, also utilize the trained distributions, so we choose the most straightforward tasks to analyze different models.

5.5.1 Intrinsic Evaluation: Topic Quality. We first compare models using an intrinsic evaluation developed specifically for crosslingual topic models, called Crosslingual Normalized Pointwise Mutual Information (CNPMI, Hao, Boyd-Graber, and Paul (2018)). In contrast to held-out likelihood which is negatively correlated with human judgments of topics (Chang et al. 2009) and is not established for multilingual settings, CNPMI extends the widely-used standard topic model evaluation, Normalized Pointwise Mutual Information (NPMI, Lau, Newman, and Baldwin (2014)) to multilingual settings, and correlates well with bilingual speakers’ quality judgments as well as with predictive performance in downstream applications.

**Definition 4 (CNPMI)**
Given a bilingual topic $k^{(\ell_1,\ell_2)}$ in languages $\ell_1$ and $\ell_2$ and a parallel reference corpus $R^{(\ell_1,\ell_2)}$, the CNPMI of topic $k$ is calculated as:

$$\text{CNPMI}(\ell_1, \ell_2, k) = \frac{1}{C^2} \sum_{i,j} \log \frac{1}{\Pr \left( w^{(\ell_1)}_i, w^{(\ell_2)}_j \right)} \cdot \log \frac{\Pr \left( w^{(\ell_1)}_i, w^{(\ell_2)}_j \right)}{\Pr \left( w^{(\ell_1)}_i \right) \Pr \left( w^{(\ell_2)}_j \right)},$$

$$\Pr \left( w^{(\ell_1)}_i, w^{(\ell_2)}_j \right) \triangleq \frac{|\left\{ d : w^{(\ell_1)}_i \in d^{(\ell_1)}, w^{(\ell_2)}_j \in d^{(\ell_2)} \right\}|}{|R^{(\ell_1,\ell_2)}|},$$

where $C$ is the total number of word types in both languages and $\ell_1$ and $\ell_2$ represent the languages.
Infer topics on unseen documents
Train multilingual topic models on
corpus
corpus
test corpus
Classifier
test corpus
Labels for
test corpus

\[ D(\ell_1) \]
\[ D(\ell_2) \]

Train multilingual topic models on \( D(\ell_1, \ell_2) \)

Infer topics on unseen documents

\[ \hat{\theta}_{d(\ell_1)} \]
\[ \hat{\theta}_{d(\ell_2)} \]

In multilingual topic models, document-topic distributions \( \theta \) can be used as features for classification, where the \( \theta^{(d, \ell_1)} \) vectors in language \( \ell_1 \) train a classifier tested by the \( \theta^{(d, \ell_2)} \) vectors in language \( \ell_2 \). A better classification performance indicates more consistent features across languages. See Figure 9 for an illustration.

In our experiments, we use a linear support vector machine (SVM) to train multilabel classifiers with five-fold cross-validation. Then, we use micro-averaged F-1 scores to evaluate and compare performance across different models.

6. Experiments

Two important factors that can affect the performance of multilingual topic models are:

1. **Comparability**: how comparable is the training corpora (e.g., parallel, comparable, partially comparable, and incomparable).
2. **Dictionary**: how many dictionary entries are available.
Both factors are considered as different types of supervision. Therefore, in the following experiments we vary these two factors and train different models to compare the topic qualities across these factors.

6.1 Sensitivity to Training Corpus

The first factor is the comparability of training corpora. Although only DOCLINK and C-BiLDA rely on this information to transfer document-level knowledge across languages, all models are potentially affected by the training set.

For each language pair \((en, \ell)\), we construct a random subsample of 2,000 documents from Wikipedia in each language (4,000 in total). To vary the comparability, we vary the proportion \(r\) of linked Wikipedia articles between the two languages, from 0, 0.01, 0.05, 0.1, 0.2, 0.4, 0.8, to 1. When \(r = 0\), the bilingual corpus is entirely incomparable—no document-level translations can be found in another language, and DOCLINK and C-BiLDA degrade into monolingual models. The indicator matrix introduced in Section 4.1 is a zero matrix \(\delta = 0\). When \(r = 1\), meaning all the documents from the two languages are linked, the corpus is considered fully comparable, and \(\delta\) is an identity matrix \(I\). Any number between 0 and 1 makes the corpus partially comparable to different degrees.

We show \(cnpmi\) and crosslingual classification results in Figures 10a and 10b, respectively. As expected, models transferring knowledge at the document level (DOCLINK and C-BiLDA) are very sensitive to the training corpus: the more aligned the corpus is, the better topics the model learns. On the other hand, VOCLINK roughly stays at the same performance level, which is also expected, since these models do not use linked documents as supervision. It is quite interesting to see SOFTLINK, a document-level model, also insensitive to the training corpus, and outperforms other models most of the time. This informs us that SOFTLINK can deal with incomparable or partially comparable corpus much better than other models.

We observe a similar pattern as \(cnpmi\) for classification performance, where SOFTLINK and VOCLINK are much less sensitive to the training corpus than DOCLINK and C-BiLDA. We also notice that when the test set contains different document sources across the two languages (TED+GV), the performance differences among models are less obvious.

6.2 Sensitivity to Dictionaries

The second factor is the number of dictionary entries used, especially in word-level models. Therefore, along this dimension, we run experiments on VOCLINK and SOFTLINK. While SOFTLINK does not transfer knowledge at the word level, it uses dictionary information to enable document-level transfer, so we include this model as well.

Similar to the previous section, we vary the proportion of the dictionary available to these models, from 0.01, 0.05, 0.1, 0.2, 0.4, 0.8, to 1, called the dictionary coverage. When dictionary coverage is 0, trivially, no word translations are added in the model, making VOCLINK degrade into two monolingual LDA models, one for each language, so we exclude this case. When dictionary coverage is 1, we add all available word translations.

We present \(cnpmi\) scores in Figure 11a, where the results are averaged over all five languages in HIGHLAN. It is clear that SOFTLINK outperforms VOCLINK, regardless of training corpus and the size of the dictionary. This implies that SOFTLINK could potentially learn coherent multilingual topics even when the training conditions are unfavorable, i.e., the training corpus is incomparable and there is only a small number of
Figure 10: Both SOFTLINK and VOCLINK stay at a stable performance level of either CNPMI or F-1 scores, while DOCLINK and C-BiLDA expectedly have better performance as there are more linked Wikipedia articles.

6.3 Results on LOWLAN

In this section, we change the language set from HIGHLAN to LOWLAN. For SOFTLINK and VOCLINK, we use all dictionary entries to train languages in LOWLAN, since the sizes of dictionaries in these languages are very small. We again use a subsample of 2,000 Wikipedia document pairs with English to make the comparison with HIGHLAN meaningful. In Table 4a, we also present results of models for HIGHLAN using fully comparable training corpora and full dictionaries for direct comparison of the effect of language resources.
Hao and Paul  

Crosslingual Transfer in Topic Modeling

Test sets:

(a) Average cnpmi scores on multilingual topic coherence.

| Dictionary Coverage | SOFTLINK | VOCLINK |
|---------------------|----------|---------|
| 1.0                 | .348     | .322    |
| 0.8                 | .349     | .321    |
| 0.6                 | .334     | .281    |
| 0.4                 | .324     | .246    |
| 0.2                 | .319     | .231    |
| 0.1                 | .301     | .232    |
| 0.05                | .286     | .235    |
| 0.01                | .281     | .236    |

(b) Multilabel crosslingual document classification F-1 scores in HighLan.

| Dictionary Coverage | SOFTLINK | VOCLINK |
|---------------------|----------|---------|
| 1.0                 | .536     | .536    |
| 0.8                 | .534     | .544    |
| 0.6                 | .538     | .558    |
| 0.4                 | .522     | .511    |
| 0.2                 | .487     | .513    |
| 0.1                 | .500     | .521    |
| 0.05                | .510     | .473    |
| 0.01                | .539     | .439    |

Test sets: TED+GV Test sets: TED+TED

Figure 11: Adding more dictionary entries has higher impact on word-level model SOFTLINK learns better quality topics than VOCLINK. SOFTLINK also generally performs better on classification, while VOCLINK has competitive performance when the test set is TED+GV.

Similar to the results in HighLan, both document-level models, DOCLINK and C-BiLDA, can learn high quality topics, since their cnpmi scores are very close. In comparison, we notice that SOFTLINK and VOCLINK perform relatively well on HighLan, while their performance is much lower than DOCLINK and C-BiLDA in LowLan.

Note that VOCLINK depends on the number of dictionary entries available, and we show in Figure 11a that it can achieve a relatively good performance only when the dictionary is large enough. For LowLan, the number of dictionary entries is extremely low, so it is not surprising that the topic qualities of VOCLINK are the lowest.
On the other hand, although softlink does not model vocabularies directly as in voclink, transferring knowledge at the document level with a limited dictionary still yields competitive cnpmi scores. Therefore, in this experiment on LowLAN, we arrive at the conclusion again that transferring knowledge at the document level is the most efficient way.

We also present a comparison of F-1 scores between HighLAN and LowLAN in Table 4b. The test set used for this comparison is TED + GV, since TED does not have articles available in LowLAN. Also, languages such as Amharic (AM) and Khmer (KM) have fewer than 50 GV articles available, which is an extremely small number for training a robust classifier, so in these experiments, we only train classifiers on English (TED articles) and test them on languages in HighLAN and LowLAN (GV articles).

In contrast to the topic quality comparison in Table 4a where C-BiLDA and doclink consistently learn high-quality topics, we notice that the best classification performance is much variable among different languages, and the performance gap for a certain language among different models is also higher. For LowLAN, we emphasize that the results have large variations because the test sets are so small. For example, KM has only 32 documents in GV and an even smaller portion of documents have labels. Therefore, we will mainly focus on HighLAN in later sections, since it is difficult to reach clear conclusions for LowLAN.

6.4 Limitations of Word-Level Transfer

From the results so far, it is empirically clear that transferring knowledge at the word level tends to be less efficient than at the document level. This is arguably counter-intuitive. Coherence scores such as CNPMI examine co-occurrence of word pairs with highest probabilities in each topic. In other words, the higher CNPMI, the better the topic-word distributions are modeled. The word-level model voclink adds supervision on word translations and models transfer in the topic-word distributions directly, yet its CNPMI scores are lower.
In this section, we try to explain this apparent contradiction. We first analyze the dictionary usage of VOCLINK (Section 6.4.1), and then lead our discussion on the transfer strength comparisons between document and word levels for all models (Sections 6.4.2 and 6.4.3).

6.4.1 Dictionary Usage. In practice, the assumption of VOCLINK is also often weakened by another important factor—the presence of word translations in the training corpus. Given a word pair \((w^{(ℓ_1)}, w^{(ℓ_2)})\), the assumption of VOCLINK is valid only when both words appear in the training corpus in their respective languages. If \(w^{(ℓ_2)}\) is not in \(D^{(ℓ_2)}\), \(w^{(ℓ_1)}\) will be treated as an untranslated word instead. In Table 5, we present the available dictionary entries for each language, and the actual number of entries VOCLINK present in the corpus, which is very low. Note that these statistics are based on the fully comparable Wikipedia datasets from our experiments. We conclude from Figure 11a that adding more dictionary entries will slowly improve VOCLINK, but even when we use the complete dictionary to create the tree structure in VOCLINK, only less than 50% of entries are actually used. Thus, the full potential of VOCLINK is very difficult to achieve due to the properties of the training corpus.

A possible solution is to first extract word alignments from parallel corpora, and then create a tree structure using those word alignments, as experimented in Hu et al. (2014). However, when parallel corpora are available, we have shown that document-level models such as DOCLINK work much better anyway, and the accuracy of word aligners is another possible limitation to consider.

6.4.2 Topic Analysis. While we show the statistics of dictionaries above, we further look into the actual topics trained from SOFTLINK and VOCLINK in this section. The general idea is to look into the same topic output from SOFTLINK and VOCLINK, and see what topic words they have in common (denoted as \(W^+\)), and what words they have exclusively (denoted as \(W^{-,\text{SOFT}}\) and \(W^{-,\text{NOCL}}\) for SOFTLINK and VOCLINK respectively). The words in \(W^{-,\text{NOCL}}\) are those with lower topic coherence and are thus the key to understanding the suboptimal performance of VOCLINK.

To this end, we first define the corpus distribution of a word \(w\) as \(\psi^{(w)}_d = \Pr (d|w) \propto \frac{n_{w|d}}{n_d}\) where \(n_d\) is the total number of tokens in document \(d\), and \(n_{w|d}\) is the count of word type \(w\) in \(d\). The corpus entropy for a word \(w\) is then the entropy over the corpus distribution. For every aligned topic pair \((k_i, k_j)\) where \(k_i\) is from SOFTLINK and \(k_j\) from VOCLINK, we calculate the average corpus entropy over all words in \(W^+, W^{-,\text{SOFT}}\) and \(W^{-,\text{NOCL}}\) respectively, using training corpora. The results are shown in Figure 12.

We observe that the average corpus entropies over words in \(W^{-,\text{NOCL}}\) are consistently lower in all languages, while those in \(W^+\) are higher. This implies that VOCLINK tends to gather words that only appear in a few documents into the same topic. In other words, VOCLINK gives high probabilities to words that only appear in specific contexts, such as...
name entities. Thus, when evaluating topics using a reference corpus, the co-occurrence of such words with other words is relatively low due to lack of that specific context in the reference corpus.

We show an example of an aligned topic in Figure 13. In this example, we see that although both VOCLINK and SOFTLINK can discover semantically coherent words (in $W^+$), VOCLINK focuses more on words that only appear in specific contexts: there are many words (mostly named entities) in $W^{−,VOC}$ with zero corpus entropy, meaning they only appear in one document. Due to lack of this very specific context in the reference corpora, the co-occurrence of these words with other more general words is very likely to be zero, thus resulting in lower CNPMI.

6.4.3 Comparing Transfer Strength. Both VOCLINK and SOFTLINK use the same dictionary resource as crosslingual supervision, but VOCLINK is less efficient than SOFTLINK. The difference of knowledge transfer manners with same resource leads to a suspicion that document level has a “stronger” transfer power.

Generally, the conditional distribution for a token, denoted as $P$, can be factorized to two individual conditionals, i.e., document conditional likelihood $P_\theta$ and word conditional
Table 6: The ratio of similarities $r = \frac{\cos(P_\theta, P)}{\cos(P_\phi, P)}$ averaged over all tokens. Although VOCLINK transfers knowledge at the word level, the context term still dominates the conditional distributions.

|       | HIGHLAN |       | LOWLAN |
|-------|---------|-------|--------|
|       | AR      | DE    | ES     | RU     | ZH     | AM    | AY    | KM    | MY    | SW    |
| DOCLINK | 1.369  | 1.386 | 1.298  | 1.406  | 1.375  | 1.591 | 1.599 | 1.689 | 1.529 | 1.495 |
| C-BiLDA | 1.296  | 1.318 | 1.296  | 1.279  | 1.371  | 1.825 | 1.445 | 1.629 | 1.110 | 1.110 |
| SOFTLINK | 0.830  | 1.027 | 0.918  | 0.866  | 1.104  | 0.868 | 1.108 | 1.094 | 1.271 | 0.905 |
| VOCLINK | 1.955  | 2.472 | 2.352  | 1.643  | 1.759  | 1.787 | 1.428 | 1.614 | 1.252 | 1.252 |

The consistently high ratios from VOCLINK verifies our analysis that even if VOCLINK transfers knowledge at the word level, the transfer strength is low and the topic assignment is more likely to be determined by $P_\theta$. This also explains why, in the previous section, VOCLINK tends to find topic words with low corpus entropy, i.e., words usually appearing in only a few documents.

In the previous section, we explored the characteristics of word-level models that lead to inefficiencies in transferring knowledge. In this section, we now turn our attention to document-level models, examining their knowledge transfer characteristics empirically.

We start by considering characteristics of the training corpus, since document-levels are most sensitive to training corpora (Section 6.1). Recall that document-level models define transfer operations on $\theta^{(d)}$ with document connections. Thus, for DOCLINK, C-BiLDA, and SOFTLINK, the sampling equation for conditional document likelihood looks
very similar,

$$\Pr \left( z_{d^{(t_2)},m} = k; \mathbf{z}_{d^{(t_2)}}^{(t_1)}, \alpha, h_{\theta_d^{(t_2)}} \left( \mathbf{z}^{(t_1)}, \alpha, \delta \right) \right) \propto n_{k;\theta_d^{(t_2)}} + \delta \cdot \mathbf{N}^{(t_1)} + \alpha,$$

(19)

where \(\delta\) is either an indicator matrix (in DOCLINK and C-BiLDA) or a matrix of transfer distributions in SOFTLINK. The supervision matrix \(\delta\) is one factor that may cause performance differences among models, but we also consider how \(\mathbf{N}^{(t_1)}\), the observation matrix of \(\ell_1\) might also be a factor.

Document-level models use \(\delta\) to select one or multiple source distributions \(\theta^{d,\ell_1}\) to guide the sampling of topics in \(d^{(t_2)}\) by biasing the samples toward an ideal target distribution \(\theta^{d,\ell_2}\). In practice, however, one can only sample \(\hat{\theta}^{d,\ell_1}\) using the sufficient statistics, i.e., \(\mathbf{N}^{(\ell_1)}\), which are less informative when the linked document has a very small number of tokens. When the document lengths are imbalanced across linked documents, the knowledge transfer will not be balanced.

To verify this hypothesis, we measure the balance of document lengths by calculating the Jaccard distance between the two languages of each document pair,

$$J\text{-}dist \left( d^{(\ell_1)}, d^{(\ell_2)} \right) = \frac{\left| n_{d^{(\ell_1)}} - n_{d^{(\ell_2)}} \right|}{n_{d^{(\ell_1)}} + n_{d^{(\ell_2)}}},$$

(20)

and show the statistics in Figure 14. It is not surprising that Wikipedia articles in LOWLAN have very large differences in document lengths between EN-linked pairs.

In Table 7, we calculate the Pearson correlation between the average Jaccard distance of a training corpus and the F-1 scores on the two test sets for HIGHLAN. The strong negative correlations of DOCLINK and SOFTLINK imply that the higher the Jaccard distance is, the lower the classification performance is. In other words, these models required balanced documents in order to transfer knowledge effectively.

C-BiLDA, on the other hand, is almost invariant to differences in the Jaccard distances. The most surprising result is VOCLINK—if the test sets are from the same source and parallel (TED+TED), it has negative correlations between Jaccard distances and F-1; but if the test sets are from different sources (TED+GV), it has positive correlations. We further investigate the effect of length balancing on C-BiLDA and VOCLINK in the next two subsections.
In terms of classification performance, both DOCLINK and SOFTLINK are very sensitive to the Jaccard distances between document pairs in the training corpus.

### 6.5.1 Language Selector in C-BiLDA

We first study the factors that make DOCLINK sensitive to Jaccard distances while its simple extension, C-BiLDA, is mostly insensitive. Recall that C-BiLDA explicitly models the language (see Section 4.2). By setting $\chi = 2$, according to Heyman, Vulic, and Moens (2016), the Beta distribution prior becomes more sensitive to Jaccard distances while its simple extension, the $m$-th token in document $d(\ell_2)$, the posterior estimation of $\eta$ becomes:

\[
\Pr \left( \ell_{d(\ell_2)} | z_{d(\ell_1), m} = k, z_{d(\ell_2)}, w_{d(\ell_2)} ; h_{n_d(\ell_1)} \left( z_{d(\ell_1)}, \chi_{d(\ell_1)} ; \delta \right) \right) = \frac{n_{k,d(\ell_2)} + n_{d(\ell_2)}}{n_{k,d(\ell_1)} + n_{d(\ell_1)} + n_{k,d(\ell_2)} + n_{d(\ell_2)}}.
\]

(21)

The original C-BiLDA paper motivated this model by discussing the benefit that it can “distinguish shared from non-shared topics,” which relaxes the assumption that the topic content is perfectly comparable across languages. However, we notice an additional benefit of this model: it relaxes the assumption that the length of documents is comparable across languages. We now show how the language distribution adjusts for the length balance between document pairs.\(^9\)

When $n_{d(\ell_1)} \approx n_{d(\ell_2)}$, the DOCLINK part of the equation transfers enough knowledge so that $\hat{\theta}(d,\ell_1)$ and $\hat{\theta}(d,\ell_2)$ are sufficiently similar. Thus, Equation (22) is roughly around $\frac{1}{2}$ which does not affect the distribution.

When there is instead a large token difference, we have the following approximation,

\[
\Pr (\ell | z = k, \alpha, \beta) \approx \begin{cases} 1, & n_{d(\ell_2)} \ll n_{d(\ell_1)} \quad \text{for} \quad n_{d(\ell_2)} \gg n_{d(\ell_1)} \\ \frac{1}{n_{k,d(\ell_2)} + n_{d(\ell_2)}}, & \end{cases}
\]

(23)

The two cases in Equation (23) are two extreme approximations, where the first one is getting closer to DOCLINK while the second one is getting away from it. Without loss of generality, we assume $n_{d(\ell_2)} \ll n_{d(\ell_1)}$ and approximate the posterior for a topic:

\[
\Pr \left( z_{d(\ell_1), m} = k | z_{d(\ell_1)}, \alpha, \beta, h_{n_d(\ell_1)} \left( z_{d(\ell_1)}, \alpha, \delta \right), h_{n_d(\ell_1)} \left( z_{d(\ell_2)}, \chi_{d(\ell_1)} ; \delta \right) \right)
\]

(24)

\[^9\text{Note that the language of document } d(\ell_2) \text{ is already observed as } \ell_2. \text{ Therefore, in actual sampling, it is not functioning as selecting a language; rather, it serves the purpose of learning parameters based on more realistic assumptions about how the corpora differ across languages.}\]
\[ \alpha \left( n_{k|d(\ell_1)} + \alpha \right) \left( \frac{n_{w|k} + \beta}{n_{|k} + V(\ell_1) \beta} \right) \]
\[ \Pr \left( z_{d(e_2), m} = k \middle| z_{-d(e_2)}, w_{-d(e_2)}^{(\ell_2)}, z^{(\ell_1)}, h_{d(e_2)}^{(\ell_1)} \right) \]
\[ \propto \left( \frac{n_{k|d(\ell_1)} + \alpha}{n_{k|d(\ell_1)} + n_{d(\ell_1)}} \right) \left( \frac{n_{w|k} + \beta}{n_{|k} + V(\ell_2) \beta} \right) \]

Note that in the approximation of \( \ell_2 \) tokens, the conditional document likelihood is fixed because it is only related to \( \ell_1 \) tokens. Therefore, in fact, we can treat the sampling process in this way: when sampling \( \ell_1 \) tokens, the process is very similar to monolingual LDA; when sampling the paired language \( \ell_2 \), we only model the conditional word probability with a scalar defined by documents in \( \ell_1 \). Thus, C-BiLDA actually ignores the context information from documents in \( \ell_2 \), or borrows the context from the paired documents in \( \ell_1 \), and only models topic-word distributions so that they are closer to \( \ell_1 \).

Recall the disparity between the generative process and the real situation for Voclinc. In fact, Doclinck has a similar fallacy because it assumes every token generated will be attached to the document. C-BiLDA, however, uses this additional topic-language distribution to better avoid this problem. This better explains the difference we discussed in Figure 7.

6.5.2 Voclinc. As mentioned above regarding Table 7, the correlation between Jaccard distance and classification F-1 scores for Voclinc is quite interesting. When inferred on the parallel corpus (TED+TED), the correlation is negative, while on the incomparable corpus (TED+GV) it is positive. Neither correlation is perhaps strong enough for a conclusive result, but it is surprising enough to warrant further investigation.

Recall that in Table 6, we concluded that Voclinc is more influenced by conditional document likelihood \( P_\theta \) than by conditional word likelihood \( P_\phi \). If the training set has lower Jaccard distance, the context terms for both languages are more comparable, even though no transfer is happening. This means when we use the \( \phi^{(k,\ell)} \) in a similar context to infer topics (e.g., TED+TED in our experiments), they are more likely to provide consistent features. Therefore, the lower the Jaccard distance, the higher the performance on TED+TED.

However, a lower Jaccard distance means that during sampling, a token’s context is usually ignored if that token is in a language with a much shorter document length (say \( \ell_2 \)). In this case, knowledge transfer entirely depends on word translations in another language (say \( \ell_1 \)). Without document context, topic-word distributions \( \phi^{(k,\ell_2)} \) can be loosely interpreted as learned from \( \phi^{(k,\ell_1)} \). Therefore, \( \phi^{(k,\ell_2)} \) can be more adaptive to situations like TED+GV where different languages have different contexts.

Similar analysis applies to correlations between CNPMI and Jaccard distances, where Doclinck and Voclinc have positive correlations. The larger the Jaccard distances are, the more dependent on topic-word distributions Voclinc is, and the more dependent on another language’s contexts Doclinck is. Introducing more contexts from another language (Softlink) or using a language selector (C-BiLDA) removes the effects of token differences between two languages and thus gives almost zero correlation.
7. Remarks and Conclusions

Multilingual topic models use corpora in multiple languages as input with additional language resources as supervision. These traits of the models inevitably lead to a wide variety of training scenarios, especially when a language’s resources are scarce, while most previous studies on multilingual topic models have not analyzed in depth the appropriate of different models for different training situations and resource availability. For example, experiments are most often done in European languages, with models that are typically trained on parallel or comparable corpora.

The contributions of our study are to provide a unifying framework of these different models, and to systematically analyze their efficacy in different training situations. We conclude by summarizing our findings along two dimensions—training corpora characteristics and dictionary characteristics, since these are the necessary components to enable crosslingual knowledge transfer.

7.1 Model Selection

Document-level models are shown to work best when the corpus is parallel or at least comparable. In terms of learning high-quality topics, DOC LINK and C-BiLDA yield very similar results. However, since C-BiLDA has a “language selector” mechanism in the generative process, it is slightly more efficient for training Wikipedia articles in low-resource languages, where the document lengths have large gaps compared to English. SOFT LINK, on the other hand, only needs a small dictionary to enable document-level transfer, and yields very competitive results. This is especially useful for low-resource languages when the dictionary size is small and only a small number of comparable document pairs are available for training.

Word-level models are harder to achieve full potential of transfer, due to limits in the dictionary size and training sets, and unrealistic assumptions of the generative process. The representative model, VOCLINK, has similarly good performance on document classification as other models, but the topic qualities according to coherence-based metrics are lower. Comparing to SOFT LINK, which also requires a dictionary as resource, directly modeling word translations in VOCLINK turns out to be a less efficient way of transferring dictionary knowledge. Therefore, when a dictionary is available, we recommend SOFT LINK over VOCLINK.

7.2 Relations to Other Crosslingual Representations

As an alternative method to learning crosslingual representations, crosslingual embeddings have been gaining attention (Ruder, Vulić, and Søgaard 2017; Upadhyay et al. 2016). Similar to the topic space in multilingual topic models, crosslingual embeddings learn semantically consistent features in a shared embedding space for all languages. A very common strategy for learning crosslingual embeddings is to use a projection matrix as supervision (Vulić and Korhonen 2016; Faruqui and Dyer 2014) or sub-objective (Tsvetkov and Dyer 2016; Dinu and Baroni 2014) to learn a common embedding space for separately trained monolingual embeddings.

In multilingual topic models, the supervision matrix $\delta$ plays the role of a projection matrix between languages. For example, in DOC LINK, $\delta_{d^{(x)},d^{(t)}}$ projects document $d^{(x)}$ to the document space of $l_1$ (Equation (6)). SOFT LINK provides a simple extension by forming $\delta$ to a matrix of transfer distributions based on word-level document similarities. VOCLINK applies projections in the form of word translations.
Thus, we can see that the formation of projection matrices in multilingual topic models is still static, i.e., used as supervision, and restricted to an identity matrix or a simple pre-calculated matrix. An generalization would be to add learning the projection matrix itself as an objective into multilingual topic models. This could be a way for improving VOCLINK by extending word associations to polysemy across languages, and making it less dependent on context.

7.3 Future Directions

Our study inspires future work in two directions. The first direction is to increase the efficiency of word-level knowledge transfer. For example, it is possible to use co-location information of translated words to transfer knowledge, though cautiously, to untranslated words. It has been shown that word-level models can help find new word translations, for example by using the existing dictionary as “seed”, and gradually adding more internal nodes to the tree structure using trained topic-word distributions. Additionally, our analysis showed the benefits of using a “language selector” in C-BiLDA to make the generative process of DOCLINK more realistic, and one could also implement a similar mechanism in VOCLINK to make the conditional distributions for tokens less dependent on specific context.

The second direction is more general. By systematically synthesizing various models and abstracting the knowledge transfer mechanism through an explicit transfer operation, we can construct models that shape the probabilistic distributions of a target language using that of a source language. By defining different transfer operations, more complex and robust models can be developed, and this transfer formulation may provide new ways of constructing models than with a traditional joint formulation. For example, SOFTLINK is generalization DOCLINK based on transfer operations that does not have an equivalent joint formulation. This framework for thinking about multilingual topic models may lead to new ideas for other models.
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