Near-Field Noncircular Sources Localization Based on Fourth-Order Cumulant

XIAORAN LI*, QISHU GONG, SHUNAN ZHONG, AND SHIWEI REN†
School of Information and Electronics, Beijing Institute of Technology, Beijing 100081, China
Corresponding author: Shiwei Ren (renshiwei@bit.edu.cn)

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ABSTRACT

This paper presents a cumulant-based algorithm for near-field noncircular sources localization with a symmetric uniform linear array. It first constructs three extended matrices. Then, based on the rotational invariance property in extended cumulant-domain signal subspace, coarse direction-of-arrival (DOA) estimates are obtained. With the coarse results used as reference, the algorithm is able to disambiguate the cyclic phase ambiguities when the equivalent sub-array spacing exceeds a half wavelength. Thus, higher precision DOA estimates can be obtained. Finally, with the estimated DOAs, only one-dimensional searching is needed to obtain the range estimates via rank-reduction algorithm. The proposed algorithm avoids two-dimensional searching and parameters pairing. In addition, compared with the existing near-field noncircular sources localization algorithm, the significant characteristic of the proposed algorithm is that it exploits cumulant and the noncircularity of signal to achieve extended steering vector. Cumulant is insensitive to Gaussian (white or color) noise. Consequently, the proposed one provides DOA estimates with improved precision especially at low signal-to-noise ratios. Furthermore, it doubles the number of detectable sources. Total least square ESPRIT algorithm is exploited to yield search free estimates of near-field bearing parameters. Computer simulations are carried out to demonstrate the superiority of the proposed algorithm.

INDEX TERMS

Cumulant, near-field, noncircular sources, parameter estimation.

I. INTRODUCTION

Estimation of source localization has received significant attention in array signal processing fields such as radar, sonar, wireless communication, guiding systems, seismic detection, medical imaging and so on over the past decades [1]–[4]. Most of the classical algorithms, such as MUSIC [5], ESPRIT [6], and their derivations, assume far-field source localization. For far-field source, only the direction-of-arrival (DOA) estimate is required to describe its location while the corresponding wavefront can be considered as plane wave at the site of the sensor array. However, in many applications, when a source is located close to the array, its wavefront is spherical. The location of source should be parameterized by DOA and range [7]. Therefore, These traditional far-field source direction finding algorithms cannot be directly applied to near-field sources.

After more than 10 years of development, a large number of excellent algorithms and research results emerge for near-field sources localization, such as the maximum likelihood algorithm [8], the two-dimensional MUSIC algorithm [9], the path following algorithm [10] and its modified version [11], the polynomial rooting algorithm [12] and the second-order statistics (SOS) based algorithms [13], [14]. However, most of these algorithms require two-dimensional searching or suffer low resolution and reduced number of detectable sources for heavy aperture loss. To extend the virtual array aperture, various fourth-order cumulant (FOC) based algorithms have been proposed. For example, the classical ESPRIT-Like algorithm [15] deals with the near-field signals by constructing cumulant matrices with certain structure. The subspace-based algorithm [16] further avoids parameters pairing and aperture loss. Unlike the above algorithms, the quarter-wavelength element spacing constraint is not required in the cumulant-based algorithm [17]. The INF-MUSIC algorithm [18] improves the DOA estimation accuracy by exploiting physical property of spatial spectrum. The sparse array-based algorithm [19] and the extended-aperture ESPRIT algorithm [20] aim to achieve aperture extension respectively with a symmetric double...
nested array or a linear triple array. The algorithm [19] focuses on solving the under-determined cases based on symmetric double nested arrays, which is not considered in this paper. Such kind of methods increases the degree of freedom due to the specific array configurations. These algorithms are based on the complex circular source assumption. In this paper, we concentrate on performance improvement on the algorithmic level. Based on the isotropic uniform linear array (ULA), as the maximum number of detectable sources is less than the number of array elements, it is necessary to develop under-determined parameter estimation algorithm for near-field sources by exploiting the noncircularity of signals. The cases when noncircular signals incident on various array configuration such as sparse array [19] will be further studied in the future work.

In fact, noncircular signals, e.g., binary phase shift keying (BPSK), minimum shift keying (MSK) and offset quadrature phase shift keying (OQPSK), are widely used in modern wireless communication, radar, and satellite systems. By utilizing the noncircularity of the signals to extend the virtual array aperture, the DOA estimation accuracy and the maximum number of detectable sources can be improved [21]–[26]. The MUSIC-like algorithm for localization of noncircular sources and its performance bonds are presented in [21], [22]. The root-MUSIC-like algorithm is proposed to reduce computational complexity in [23]. References [24] and [25] present a noncircular sources localization method exploiting ESPRIT. Reference [26] presents a method via sparse representation. A more general case when impinging signals are mixture of circular and noncircular one has been studied for DOA estimation in [27]–[29]. Two MUSIC-like algorithms are proposed in [27] for coherently distributed sources localization in massive MIMO systems. A new polarization channel estimation algorithm [28] is proposed for base station (BS) equipped with a large polarization sensitive array. Reference [29] further proposes a high-resolution DOA estimation algorithm to track patients in environment with high patient density. These algorithms are all based on far-field sources assumption. There are very few studies on parameter estimations of noncircular sources in the near-field.

Reference [30] is an efficient method to resolve the above problem. Reference [31] further copes with a more complex case when mixed near-field and far-field noncircular sources incident on array. However, for a symmetric ULA with $2M + 1$ elements, the maximum number of detectable sources for [30], [31] is no more than $2M$. In other word, these methods still remain improvement to extend the array aperture. Furthermore, the parameter estimation performance of [30] is poor at relatively low signal-to-noise ratios (SNRs). In [32], an RARE-based near-field noncircular sources localization algorithm is proposed. However, it relies on a symmetric uniform linear array of cocentered orthogonal loop and dipole (COLD) antennas.

To get the near-field noncircular information more precisely, we present a fourth-order cumulant-based algorithm with scalar symmetric ULA in this paper. First, three special extended fourth-order cumulant matrices containing noncircular information of the source signals are constructed. These cumulants are formed by choosing array elements properly. Rotational matrix between these extended cumulant matrices is only characterized by the DOAs. Then, course DOA estimates are obtained based on the rotational invariance property of extended cumulant-domain signal subspace. These course results are further used as reference to disambiguate the cyclic phase ambiguities induced by the large equivalent spacing of sub-arrays which exceeds a half wavelength. Thus, higher precision DOA estimates can be obtained. Finally, with the DOA estimates, the ranges are generated through one-dimensional spectral searching via rank-reduction algorithm. The main contributions of the paper are listed as follows:

1. The proposed algorithm provides an improved localization accuracy especially at low SNRs due to the capability of cumulant being insensitive to Gaussian noise and the multi-resolution ESPRIT algorithm.

2. It also avoids the estimation failure problem and doubles the maximum number of detectable sources due to the extended steering vector based on the noncircularity of signals.

3. The resultant algorithm avoids two-dimensional searching and parameters pairing. In addition, total least square (TLS) ESPRIT is applied to yield search free estimates of near-field bearing parameters.

The rest of the paper is organized as follows. Section II develops the data model of noncircular source localization in the near-field. Section III presents the proposed parameter estimation algorithm. Analyses are provided in this section as well. The simulation results are presented in section IV to verify the effectiveness of the proposed method. Section V concludes the paper.

Notation: Upper(lower)bold symbols denote matrix (vector). $\cdot^*$, $(\cdot)^T,(\cdot)^H$, $(\cdot)^\dagger$ denote complex conjugation, transpose, conjugate transpose, and pseudo-inverse, respectively; $E \{\cdot\}$ represents the statistical expectation, $\text{cum} \{\cdot\}$ stands for the fourth-order cumulant, $\text{arg}(\cdot)$ means the argument of a complex number.

### II. SIGNAL MODEL

Suppose a symmetric uniform linear array is composed of $N = 2M + 1$ omnidirectional sensors with the inter-element spacing of $d$, as shown in Fig.1. The array is fully calibrated, with systematic errors and mutual coupling having been compensated.

$K$ narrowband, independent noncircular sources in the near-field impinge on this array. With the center of the array as the phase reference point, the signal received from the $m$th sensor can be modeled as [13]–[16]

$$x_m(t) = \sum_{k=1}^{K} s_k(t)e^{j\tau_{mk}} + n_m(t). \quad (1)$$
where $t = 1, 2, \ldots, T$ with $T$ being the snapshot number; $m$ denotes the sensor indices with $m = -M, -M + 1, \ldots, M - 1, M$; $s_k(t)$ denotes the $k$th source signal; and $n_m(t)$ represents the noise at the $m$th sensor. $\tau_{mk}$ stands for the propagation time delay of the $k$th source from the phase reference point to the $m$th sensor, which can be expressed as

$$\tau_{mk} = m\gamma_k + m^2\phi_k,$$

in which the so-called electric angles $\gamma_k$ and $\phi_k$ are described as

$$\gamma_k = -\frac{2\pi d}{\lambda} \sin \theta_k,$$

$$\phi_k = \frac{\pi d^2}{\lambda_R} \cos^2 \theta_k.$$  

$\gamma_k$ is only related to parameter $\theta_k$ whereas $\phi_k$ is related to both $\theta_k$ and $r_k$ where they are respectively denote the azimuth DOA and range parameter of the $k$th source signal. For source in the near-field, we have $0.62(D^3/\lambda)^{1/2} \leq r_k \leq 2D^2/\lambda$, where $D$ represents the aperture of the array and $\lambda$ represents the wavelength of the source signal.

In matrix form, the received data of the entire array can be written as

$$x(t) = A s(t) + n(t) = \sum_{k=1}^{K} a(\theta_k, r_k)s_k(t) + n(t),$$

where $x(t) = [x_{-M}(t), \ldots, x_0(t), \ldots, x_M(t)]^T$ is the array output vector; $s(t) = [s_1(t), \ldots, s_K(t)]^T$ is the signal vector of the noncircular sources; $n(t) = [n_{-M}(t), \ldots, n_0(t), \ldots, n_M(t)]^T$ is noise vector. $A$ represents the array steering matrix with the form of

$$A = [a(\theta_1, r_1), \ldots, a(\theta_k, r_k), \ldots, a(\theta_K, r_K)],$$

$$a(\theta_k, r_k) = [a_{-M}(\theta_k, r_k), a_{-M+1}(\theta_k, r_k), \ldots, a_{M-1}(\theta_k, r_k), a_M(\theta_k, r_k), \ldots, a_{M-1}(\theta_k, r_k), a_M(\theta_k, r_k), \ldots, a_{M-1}(\theta_k, r_k), a_M(\theta_k, r_k)]$$

$$= [e^{i(-M\gamma_k+(-M+1)\phi_k)}, e^{i(-M+1)\gamma_k+(M-1)^2\phi_k)}, \ldots, e^{i(M-1)\gamma_k+(M+1)^2\phi_k)}, e^{i(M\gamma_k+(M+1)^2\phi_k)}].$$

For noncircular sources $s(t)$, the corresponding pseudo-covariance matrix $E\{s(t)s^*(t)\}$ is not zero. Whereas for circular sources, it equals zero. Compared with the classical DOA estimation algorithm for circular sources, noncircular sources based algorithms [21]–[26] exploited both the covariance matrix $E\{s(t)s^*(t)\}$ and the pseudo-covariance matrix $E\{s(t)^2\}$ which can improve the information utilization rate of the received data, thus providing improved parameter estimation performance.

We consider the strictly noncircular source signals with maximum noncircular rate in this paper. The signal vector $s(t)$ can be written as

$$s(t) = \Psi_{0}^{1/2}z_0(t),$$

where $\Psi = diag\{e^{\psi_k}\}_{k=1}^{K}$ is a diagonal matrix with $\psi_k$ being noncircular phase of the signal $s_k(t)$; $s(t) = [s_0(t), s_0, 2(t), \ldots, s_0, K(t)]^T \in \mathbb{R}^{K \times 1}$, $s_0, k(t)$ is the zero-phase version of the $k$th source signal $s_k(t)$.

For the rest of this paper, the following assumptions are required:

1. The source signals are statistically independent, non-Gaussian, zero-mean, noncircular random process with non-zero kurtosis.
2. The sensor noise is zero-mean additive (white or color) Gaussian process and uncorrelated with the source signals.
3. The sensor array is a symmetric uniform linear array with element-spacing $d = \lambda/4$.
4. The source number $K$ is known or accurately estimated by the information theoretic criteria [33].

III. PROPOSED ALGORITHM

A. DOA ESTIMATION

The fourth-order cumulant of the array output $x_u, x_v, x_p, x_q$ where $u, v, p, q \in [-M, M]$ is defined as follows [34]

$$C_4(u, v, p, q) = cum(x_u, x_v, x_p, x_q)$$

$$= E[x_u, x_v, x_p, x_q] - E[x_{u}, x_{v}]E[x_{p}, x_{q}] - E[x_{u}, x_{p}]E[x_{v}, x_{q}] + E[x_{u}, x_{v}, x_{p}, x_{q}] - E[x_{u}, x_{v}]E[x_{p}, x_{q}] - E[x_{u}, x_{p}]E[x_{v}, x_{q}] + E[x_{u}, x_{v}, x_{p}, x_{q}] - E[x_{u}, x_{v}]E[x_{p}, x_{q}] - E[x_{u}, x_{p}]E[x_{v}, x_{q}] + E[x_{u}, x_{v}, x_{p}, x_{q})]$$

where $E[x_u, x_v, x_p, x_q]$ and $E[x_1, x_j](i, j \in \{u, v, p, q\}$ denote the fourth-order and the second-order moments, respectively. According to the definition (1) and the multi-linearity properties of the fourth-order cumulant, we have

$$cum(x_u, x_v, x_p, x_q)$$

$$= cum(\sum_{k=1}^{K} s_k e^{i\gamma_k+\phi_k}) - \sum_{k=1}^{K} cum(s_k e^{i\gamma_k+\phi_k})$$

$$= cum(\sum_{k=1}^{K} s_k e^{i\gamma_k+\phi_k}) - \sum_{k=1}^{K} cum(s_k e^{i\gamma_k+\phi_k})$$

$$= \sum_{k=1}^{K} cum(s_k, s_k, s_k, s_k).$$

Define $c_4(s_k, s_k, s_k, s_k)$ as the kurtosis of the $k$th source signal and $c_4(s_0, k, s_0, k, s_0, k, s_0, k)$ as the kurtosis of the zero-phase version of the source signal $s_0, k$.
Owing to the non-circularity of the sources, it can be concluded that \( c_{4n} = c_{40} A^4 \) and \( e^{i2\theta_i} \).

Cumulant can increase both the estimation accuracy and the number of detectable sources for array given number of sensors. Besides, it is insensitive to the Gaussian noise. With cumulant used, the proposed algorithm in this paper is expected to be able to alleviate aperture loss and localize near-field sources with high accuracy.

Denote \( k_1 = k_1 + M + 1 \) and \( k_2 = k_2 + M + 1 \). We construct three \( N \times N \) fourth-order cumulant matrices \( C_1, C_2, \) and \( C_3 \), whose \((k_1, k_2)\)th element is

\[
\begin{align*}
C_1(k_1, k_2) &= \text{cum}(x_{k_1}, x_{k_1}^*, x_{k_2}, x_{k_2}^*); \\
C_2(k_1, k_2) &= \text{cum}(x_{k_1}, x_{k_1}^*, x_{k_2}, x_{k_2}^*), \\
C_3(k_1, k_2) &= \text{cum}(x_{k_1}^*, x_{k_1}^*, x_{k_2}, x_{k_2}^*),
\end{align*}
\]

where \( k_1, k_2 = -M, -M + 1, \ldots, -1, 0, 1, \ldots, M - 1, M; \)
\( l \in [-M, M] \) is a determined constant. According to (10), we have

\[
\begin{align*}
C_1 &= \sum_{k=1}^{K} \alpha(\theta_k, r_k) s_k^* \phi_k, \\
C_2 &= \sum_{k=1}^{K} \alpha(\theta_k, r_k) s_k^* \phi_k, \\
C_3 &= \sum_{k=1}^{K} \alpha^*(\theta_k, r_k) s_k^* \phi_k,
\end{align*}
\]

where \( \text{cum}(s_k^*, s_k^*, s_k^*, s_k) = \text{cum}(s_k^*, s_k^*, s_k^*, s_k) = c_{40} \) and \( \text{cum}(s_k^*, s_k^*, s_k^*, s_k) = c_{40} \). Let \( C_{4S0} = \text{diag} \{ c_{40} \} \).

Then, \( C_1, C_2, \) and \( C_3 \) can be written as the compact matrix forms as

\[
\begin{align*}
C_1 &= A^T C_{4S0} A^H; \\
C_2 &= A^T C_{4S0} A^T; \\
C_3 &= A^T C_{4S0} A^T.
\end{align*}
\]

Apparently, \( C_2 \) contains noncircular information of the signal in \( \Psi \) whereas \( C_1 \) and \( C_3 \) do not. In order to make full use of the noncircular information, we construct a new matrix \( H_1 \) as

\[
H_1 = \begin{bmatrix}
C_1 & C_2 \\
C_2 & C_3
\end{bmatrix} = \begin{bmatrix}
A C_{4S0} A^H & A^\Psi C_{4S0} A^T \\
A^\Psi^H C_{4S0} A^H & A^* C_{4S0} A^T
\end{bmatrix} = \left[ A A^\Psi^* \right] C_{4S0} \left[ A A^\Psi^* \right]^H = BC_{4S0} B^H,
\]

where \( B = \begin{bmatrix}
A \\
A^\Psi^*
\end{bmatrix} \) is the extended virtual array steering matrix containing noncircular information. \( B = \{ b(\theta_1, r_1, \psi_1), \ldots, b(\theta_K, r_K, \psi_K) \} \in C^{2N \times K} \), and \( b(\theta_k, r_k, \psi_k) = \alpha^*(\theta_k, r_k) e^{-j\psi_k} \). It implies that the aperture of the array has been extended. Therefore, the maximum number of detectable sources can be increased.

Similarly, we construct the fourth-order cumulant matrices \( C_4, C_5, C_6, \) and \( C_7 \) with the \((k_1, k_2)\)th element as

\[
\begin{align*}
C_{4,1}(k_1, k_2) &= \text{cum}(x_{k_1}, x_{k_1}^*, x_{k_2}, x_{k_2}^*); \\
C_{5,1}(k_1, k_2) &= \text{cum}(x_{k_1}, x_{k_1}^*, x_{k_2}, x_{k_2}^*); \\
C_{6,1}(k_1, k_2) &= \text{cum}(x_{k_1}^*, x_{k_1}^*, x_{k_2}, x_{k_2}^*); \\
C_{7,1}(k_1, k_2) &= \text{cum}(x_{k_1}^*, x_{k_1}^*, x_{k_2}, x_{k_2}^*).
\end{align*}
\]

where \( k_1, k_2 = -M, -M + 1, \ldots, -1, 0, 1, \ldots, M - 1, M; \)
\( n \in [-M, -1] \cup [1, M] \) is a determined constant. Similarly, we have the following \( C_4, C_5, C_6, \) and \( C_7 \) as

\[
\begin{align*}
C_4 &= \sum_{k=1}^{K} \alpha(\theta_k, r_k) s_k^* \phi_k, \\
C_5 &= \sum_{k=1}^{K} \alpha(\theta_k, r_k) s_k^* \phi_k, \\
C_6 &= \sum_{k=1}^{K} \alpha^*(\theta_k, r_k) s_k^* \phi_k, \\
C_7 &= \sum_{k=1}^{K} \alpha^*(\theta_k, r_k) s_k^* \phi_k,
\end{align*}
\]

Noted that

\[
\alpha_n(\theta, r) = \alpha_{n-\theta} \alpha_{n-\theta}(\theta, r) = e^{i(\theta_2 \gamma + \theta_1 \gamma)} e^{i(-\gamma \theta_2 + \theta_1 \gamma)} \Rightarrow e^{i\gamma n \theta}.
\]

Let \( n = 1, C_4, C_5, C_6, \) and \( C_7 \) can be expressed as

\[
\begin{align*}
C_{4,1} &= A R_1 C_{4S0} A^H; \\
C_{5,1} &= A \Psi R_1 C_{4S0} A^T; \\
C_{6,1} &= A^\Psi^* R_1 C_{4S0} A^H; \\
C_{7,1} &= A^\Psi^* R_1 C_{4S0} A^T.
\end{align*}
\]

where \( R_1 = \text{diag} \{ e^{-j\gamma_1}, e^{-j\gamma_2}, \ldots, e^{-j\gamma_K} \} \) is a diagonal matrix. Combining \( C_{4,1}, C_{5,1}, C_{6,1}, \) and \( C_{7,1} \), we form a \( 2N \times 2N \) matrix \( H_2 \)

\[
H_2 = \begin{bmatrix}
C_{4,1} & C_{5,1} \\
C_{6,1} & C_{7,1}
\end{bmatrix} = \begin{bmatrix}
A & A^\Psi^* \\
A^\Psi & A^T
\end{bmatrix} C_{4S0} \begin{bmatrix}
A & A^\Psi^* \\
A^\Psi & A^T
\end{bmatrix}^H = BR_1 C_{4S0} B^H.
\]

Similarly, let \( n = M, C_4, C_5, C_6, \) and \( C_7 \) can be expressed as

\[
\begin{align*}
C_{4,M} &= A R_2 C_{4S0} A^H; \\
C_{5,M} &= A \Psi R_2 C_{4S0} A^T; \\
C_{6,M} &= A^\Psi^* R_2 C_{4S0} A^H; \\
C_{7,M} &= A^\Psi^* R_2 C_{4S0} A^T.
\end{align*}
\]

where \( R_2 = \text{diag} \{ e^{-j2M\gamma_1}, e^{-j2M\gamma_2}, \ldots, e^{-j2M\gamma_K} \} \). A \( 2N \times 2N \) matrix \( H_3 \) can be constructed by combining \( C_{4,M}, C_{5,M}, C_{6,M}, \) and \( C_{7,M} \) as

\[
H_3 = \begin{bmatrix}
C_{4,M} & C_{5,M} \\
C_{6,M} & C_{7,M}
\end{bmatrix} = \begin{bmatrix}
A & A^\Psi^* \\
A^\Psi & A^T
\end{bmatrix} R_2 C_{4S0} \begin{bmatrix}
A & A^\Psi^* \\
A^\Psi & A^T
\end{bmatrix}^H = BR_2 C_{4S0} B^H.
\]
From (14), (18), and (20), we can see that the differences between \( H_1, H_2, \) and \( H_3 \) lie on the diagonal matrix \( R_1 \) and \( R_2, R_1 \) and \( R_2 \) are parameterized by the incident angle of the sources signals only. Therefore, we can obtain DOAs as long as \( R_1 \) and \( R_2 \) are known. The matrix pencil \( \{ B, BR_1, BR_2 \} \) can be seen as equivalent steering matrix of sub-arrays with certain sub-array spacing. Denote \( \Delta_1 \) as the equivalent sub-array spacing between \( B \) and \( BR_1 \), and \( \Delta_2 \) as the equivalent sub-array spacing between \( B \) and \( BR_2 \). Under the assumption \( d = \lambda / 4 \), it can be deduced that \( \Delta_1 = \lambda / 2 \) and \( \Delta_2 = M \lambda / 2 \). That is to say, \( \{ B, BR_1, BR_2 \} \) is shift invariance matrix pair. Therefore, applying ESPRIT to the matrix pencil \( \{ B, BR_1, BR_2 \} \) would obtain the angle estimates of the near-field sources.

Combining \( H_1, H_2, \) and \( H_3 \) together, we have the 6\( N \times 2N \) matrix \( H \) as

\[
H = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \tilde{B}C_{4\lambda_0}B^H, \tag{21}
\]

where

\[
\tilde{B} = \begin{bmatrix} B \\ BR_1 \\ BR_2 \end{bmatrix} \in \mathbb{C}^{6N \times K}. \tag{22}
\]

A set of singular values and singular vectors can be obtained by performing singular value decomposition (SVD) on \( H \). Therefore,

\[
H = U_s\Lambda_sU_s^H + U_n\Lambda_nU_n^H, \tag{23}
\]

where \( \Lambda_s \in \mathbb{C}^{K \times K} \) and \( \Lambda_n \in \mathbb{C}^{(6N - K) \times (6N - K)} \) are the diagonal matrices containing the \( K \) largest and \((6N - K)\) smallest singular values of \( H \), respectively. \( U_s \) and \( U_n \) are the signal subspace and the noise subspace composed of the singular vectors of \( H \) corresponding to the \( K \) largest and \((6N - K)\) smallest singular values, respectively.

According to the subspace theory, \( U_s \) spans the same space of \( \tilde{B} \). That is

\[
\text{span} \{ U_s \} = \text{span} \{ \tilde{B} \}. \tag{24}
\]

There exists a \( K \times K \) non-singular matrix \( T \) such that

\[
U_s = \tilde{B}T. \tag{25}
\]

Dividing the matrix \( U_s \) into three sub-matrices \( U_{s1}, U_{s2}, \) and \( U_{s3} \) of the same size, we have

\[
U_{s1} = BT, \quad U_{s2} = BR_1T, \quad U_{s3} = BR_2T. \tag{26}
\]

(26) can be transformed as

\[
U_{s2} = U_{s1}\Omega_1, \quad U_{s3} = U_{s1}\Omega_2, \tag{27}
\]

where \( \Omega_1 = T^{-1}R_1T, \quad \Omega_2 = T^{-1}R_2T. \)

The TLS criterion [6] can be used to estimate \( \Omega_1 \) and \( \Omega_2 \). Performing SVD to obtain the right singular vector of a newly constructed \( 2N \times 2K \) matrix \( [U_{s1} U_{s2}] \). Let \( V \) be a \( 2K \times 2K \) matrix composed of these right singular vectors. We divide \( V \) into four sub-matrices of the same size as

\[
V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}. \]

Then, the estimation of \( \Omega_1 \) is

\[
\hat{\Omega}_1 = -V_{12}V_{22}^{-1}. \tag{28}
\]

Perform the eigenvalue decomposition (EVD) on \( \hat{\Omega}_1 \). Construct \( \hat{R}_1 = \text{diag}\{ \eta_k \}_{k=1}^K \) with \( \{ \eta_k \}_{k=1}^K \) being the eigenvalues of \( \hat{\Omega}_1 \). \( T_1 \) is the matrix of eigenvectors corresponding to \( \{ \eta_k \}_{k=1}^K \). Since \( \hat{R}_1 \) is characterized by DOAs only, the incident angle of \( k \)th near-field source can be calculated as

\[
\hat{\theta}_{\text{ref}}^{\text{cy}} = \sin^{-1}(\arg(\eta_k)\lambda / -2\pi \Delta_1). \tag{29}
\]

Similarly, we apply SVD to \( 2N \times 2K \) matrix \( [U_{s1}, U_{s3}] \). If the matrix \( U \) composed of its right singular vectors is partitioned into four \( K \times K \) sub-matrices \( U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \)

then the estimation of \( \Omega_2 \) is

\[
\hat{\Omega}_2 = -U_{12}U_{22}^{-1}. \tag{30}
\]

Similarly, perform EVD on \( \hat{\Omega}_2 \). Construct \( \hat{R}_2 = \text{diag}\{ \zeta_k \}_{k=1}^K \) with \( \{ \zeta_k \}_{k=1}^K \) being the eigenvalues of \( \hat{\Omega}_2 \). \( T_2 \) is composed of the eigenvectors corresponding to \( \{ \zeta_k \}_{k=1}^K \). The DOA of \( k \)th source can also be obtained through \( \hat{R}_2 \) by

\[
\hat{\theta}^{\text{cy}}_{\text{ref}} = \sin^{-1}(\arg(\zeta_k)\lambda / -2\pi \Delta_2). \tag{31}
\]

Two sets of DOA estimates can be obtained from the above discussions. Since \( \Delta_1 = \lambda / 2 \), according to Nyquist Sampling Theory, course DOA estimates \( \{ \hat{\theta}^{\text{ref}}_{\text{cy}} \}_{k=1}^K \) without aliasing can be obtained from (29). The second set of estimates \( \{ \hat{\theta}^{\text{cy}}_{\text{ref}} \}_{k=1}^K \) are based on a large equivalent sub-array spacing \( \Delta_2 = M\lambda / 2 \). Assuming \( M > 1 \), \( \Delta_2 \) is larger than a half wavelength, thus providing an aliased estimation but at a finer scale. The ratio of \( \Delta_2 \) to \( \Delta_1 \) is a measure of the gain on accuracy. Multi-resolution ESPRIT [35] can be adopted to get the unambiguous DOA estimates utilizing the above two sets of DOA estimates.

To resolve the cyclically ambiguous of \( \{ \hat{\theta}^{\text{cy}}_{\text{ref}} \}_{k=1}^K \), it is necessary to pair \( \{ \eta_k \}_{k=1}^K \) and \( \{ \zeta_k \}_{k=1}^K \) first. Since each is obtained via EVD, and the EVDs are accompanied by an unknown permutation matrix, the diagonal elements of \( \hat{R}_1 \) are not in one-to-one correspondence with those of \( \hat{R}_2 \).

We obtain the permutation matrix \( \hat{P}_{12} \) according to the similarity of columns of \( T_1 \) and \( T_2 \), i.e.,

\[
\hat{P}_{12} = T_1^{-1}T_2. \tag{32}
\]

If \( m \)th column of \( T_1 \) equals to the \( n \)th column of \( T_2 \), then the \((m, n)\)th element of \( \hat{P}_{12} \) is 1 while the other elements in the \( m \)th row are 0. In practice, the estimation of the permutation matrix \( \hat{P}_{12} \) will not be exactly 1 or 0. We set the element which is closest to 1 in each row as 1 and the others as 0. The pairing information is contained in permutation matrix \( \hat{P}_{12} \) and can be used to resolve the permutation ambiguity between \( \hat{R}_1 \) and \( \hat{R}_2 \with \)

\[
\hat{R}_2 = \hat{P}_{12} \hat{R}_2 \hat{P}_{12}^{-1}. \tag{33}
\]
Denote \( \{o_k\}_{k=1}^K \) as the diagonal elements of \( \tilde{R}_2 \). They are in one-to-one correspondence with those of \( \tilde{R}_1 \).

Let \( \{\mu_1^k\}_{k=1}^K \) and \( \{\mu_2^k\}_{k=1}^K \) be the phase of diagonal elements of \( \tilde{R}_1 \) and \( \tilde{R}_2 \), respectively. \( \mu_2^k \) is proportional to \( \mu_2^k \) plus an integer multiple of 2\( \pi \). According to
\[
\begin{align*}
\mu_1^k &= \arg(\eta_k) = -\frac{2\pi \sin \theta_k \Delta_1}{\lambda}, \\
\mu_2^k &= \arg(o_k) = -\frac{2\pi \sin \theta_k \Delta_2}{\lambda} - 2\pi n_k,
\end{align*}
\]
we have 
\[
-2\pi \sin \theta_k = \frac{\mu_2^k - 2\pi n_k}{\Delta_2} = \frac{\mu_2^k - 2\pi n_k}{\Delta_2},
\]
where the integer \( n_k \) is the winding coefficient. Therefore,
\[
n_k = \text{round}(\frac{\Delta_2 \mu_1^k}{\Delta_1} - \frac{\Delta_2^2}{\Delta_1^2})(2\pi).
\]

\( \text{round}(\cdot) \) represents the rounding operation. Then, substitute the estimated \( n_k \) into (34), the disambiguated DOA estimates \( \hat{\theta}_k \) with high-precision can be obtained
\[
\hat{\theta}_k = \sin^{-1}(\frac{\arg(o_k) + 2\pi n_k}{-2\pi \Delta_2}).
\]

Note that the ratio \( \Delta_2/\Delta_1 \) can be thought as the gain in resolution. The estimation of \( \theta_k \) based on \( o_k \) is a factor \( \Delta_2/\Delta_1 \) more accurate than that based on \( \eta_k \).

### B. RANGE ESTIMATION

Firstly, we construct the augmented data matrix as
\[
y(t) = \begin{bmatrix} x(t) \\ x^*(t) \end{bmatrix} = \begin{bmatrix} A \\ A^* \Psi^* \end{bmatrix} s(t) + \begin{bmatrix} n(t) \\ n^*(t) \end{bmatrix} = B s(t) + N(t).
\]

Then, performing EVD on the covariance matrix \( R = E[y(t)y^H(t)] \), we have
\[
R = E_s \sum_s E^H_s + E_n \sum_n E^H_n,
\]
where \( E_s \in \mathbb{C}^{K \times K} \) and \( E_n \in \mathbb{C}^{(2N-K) \times (2N-K)} \) are the diagonal matrices containing the \( K \) largest and \( (2N-K) \) smallest eigenvalues of \( R \). \( E_s \in \mathbb{C}^{K \times K} \) and \( E_n \in \mathbb{C}^{(2N-K) \times (2N-K)} \) are the corresponding eigenvector matrices.

Based on the conventional MUSIC algorithm, the noise subspace spanned by \( E_n \) is orthogonal to the signal subspace spanned by \( E_s \). Since the signal subspace can be also spanned by the extended steering vector \( B \), the following equation holds true
\[
b^H(\theta_k, r_k, \psi_k) \sum_n b(\theta_k, r_k, \psi_k) = 0, \quad k = 1, \cdots, K.
\]

The range estimates \( \{\hat{r}_k\}_{k=1}^K \) can be obtained by finding the maxima of the following spectral function \( f(\theta, r, \psi) = \{b^H(\theta_k, r_k, \psi_k) \sum_n b(\theta_k, r_k, \psi_k)\}^{-1} \). By substituting the DOA estimates \( \{\hat{\theta}_k\}_{k=1}^K \) into \( f(\theta, r, \psi) \), multidimensional search is required to find the local maxima which is computationally inefficient.

To reduce the complexity of computation, we need decouple the noncircular phase term from the DOA and range terms.

Notice that the extended steering vector \( b(\theta_k, r_k, \psi_k) \) can be rewritten as
\[
b(\theta_k, r_k, \psi_k) = b_1(\theta_k, r_k)b_2(\psi_k),
\]
where \( b_1(\theta_k, r_k) = \begin{bmatrix} \alpha(\theta_k, r_k) \\ 0 \end{bmatrix} \in \mathbb{C}^{2N \times 2} \), among which \( 0 \in \mathbb{C}^{2N \times 1} \) and \( b_2(\psi_k) = \begin{bmatrix} 1 \\ e^{-j\psi_k} \end{bmatrix} \). As such, (39) can be transformed as
\[
b_2^H(\psi_k)b_1^H(\theta_k, r_k) \sum_n \sum_n b_1(\theta_k, r_k) = 0, \quad k = 1, \cdots, K.
\]

We denote \( C(\theta, r) = b_1^H(\theta_k, r_k) \sum_n \sum_n b_1(\theta_k, r_k) \) for clarity. In fact, (41) implies that \( b_2(\psi_k) \) is the eigenvector corresponding to the smallest eigenvalue of Hermitian matrix \( C(\theta, r) \in \mathbb{C}^{2 \times 2} \). Therefore, once \( 2N - K \geq 2 \) (\( K \leq 2N - 2 \)) is satisfied, \( C(\theta, r) \) will be full rank. Since \( b_2(\psi_k) \neq 0 \) under the general assumption, (41) holds true only when \( C(\theta, r) \) drops rank at \( \theta = \theta_k \) and \( r = r_k \), \( k = 1, 2, \cdots, K \), or equivalently if the determinant
\[
\det \left[ b_2^H(\theta_k, r_k) \sum_n \sum_n b_1(\theta_k, r_k) \right] = 0.
\]

Now, by substituting the DOA estimates \( \hat{\theta}_k \) into \( b_1(\theta_k, r_k) \), the range estimates can be estimated by
\[
\hat{r}_k = \arg \max_r \frac{1}{\det \left[ b_1^H(\hat{\theta}_k, r_k) \sum_n \sum_n b_1(\hat{\theta}_k, r_k) \right]}.
\]

The searching range of \( r \) is \([0, 0.62(D/\lambda)^{1/2}], 2D/\lambda] \), where \( D = 2Md \). Note that \( \hat{\theta}_k \) and \( \hat{r}_k \) achieve automatic pairing without any extra processing.

### C. SUMMARY OF THE PROPOSED ALGORITHM

The proposed algorithm is summarized in TABLE 1:

| Input: \( T \) snapshots of the symmetric array output vector, \( \{x(t)\}_{t=1}^T \). |
| Output: DOA and range estimates of the near-field sources, \( \hat{\theta}_k \) and \( \hat{r}_k \). |

#### Step 1: Construct extended matrix \( H \) by (21).

#### Step 2: Perform SVD on \( H \) and obtain the corresponding signal subspace \( U_s \).

#### Step 3: Divide \( U_s \) into three parts of the same size \( U_{s1}, U_{s2}, \) and \( U_{s3} \).

#### Step 4: TLS criterion is applied to \( \{U_{s1}, U_{s2}\} \) and \( \{U_{s1}, U_{s3}\} \) to obtain the estimates of \( \hat{R}_1 \) and \( \hat{R}_2 \) via (28) and (30), respectively.

#### Step 5: Perform EVD on \( \hat{R}_1 \) to obtain the eigenvector matrix \( T_1 \) and matrix \( \hat{R}_2 \) which is composed of the eigenvalues \( \{\xi_{k1}\}_{k=1}^K \). Similarly, perform EVD on \( \hat{R}_2 \) to obtain the eigenvector matrix \( T_2 \) and matrix \( R_2 \) which is composed of the eigenvalues \( \{\xi_{k2}\}_{k=1}^K \).\]

#### Step 6: Calculate the permutation matrix \( F_1 \) by (32) and \( \tilde{F}_1 \) by (33).

#### Step 7: The disambiguated DOA estimates \( \hat{\theta}_k \) of the kth source with high-precision can be obtained from (36), where \( n_k \) can be calculated from (35).

#### Step 8: Construct augmented data matrix according to (37). Implement EVD on \( R \) via (38) and substitute the estimated \( \hat{\theta}_k \) into \( b_1(\hat{\theta}_k, r_k) \) to form the Hermitian matrix \( C(\theta, r) \).\]

The range estimates \( \hat{r}_k \) of the kth source can be calculated via (43).
D. DISCUSSION

In this section, we discuss the proposed algorithm from four aspects, i.e., source number limitation, estimation accuracy, parameter pairing, computational complexity. We assume all the compared algorithms are based on an array with \( N = 2M + 1 \) sensors except the classical ESPRIT-Like algorithm [15] with even number sensors \( N + 1 = 2M + 2 \).

1) Source Number Limitation: The classical ESPRIT-Like algorithm [15] can deal with at most \( M \) sources using \( N + 1 = 2M + 2 \) sensors due to the construction of several cumulant matrices with dimensions being \( M + 1 \). It does not perform well even with an extra array element used. Since the dimension of fourth-order cumulant matrices \( C_1 \) and \( C_2 \) is \( N \) in the subspace-based algorithm [16], its maximum detectable number is \( N - 1 = 2M \). The cumulant-based algorithm [17] can also deal with at most \( N - 1 = 2M \) sources since the dimension of the noise subspace in equation (25) of [17] should be no more than \( 2M + 1 \). The detectable number for the INF-MUSIC algorithm [18] is still \( N - 1 = 2M \) since the physical dimension of spatial spectrum used to improve estimation accuracy has no good for virtual aperture extension. The GESPRIT-based algorithm in [30] is proposed to resolve more sources by exploiting the noncircularity of the signals. It seems that this algorithm can resolve up to \( 2(N - 1) = 4M \) sources due to its extended subspace. However, as every column of (21) in [30] is linearly correlated to another column, the GESPRIT-based algorithm is only able to localize \( N - 1 = 2M \) sources with an \( N \)-sensor ULA, which cannot utilize all the potential of noncircular sources to extend the array aperture. As for the proposed algorithm in this paper, it can construct \((2N \times 2N)\) dimensional matrix \( H_1 \), \( H_2 \), and \( H_3 \). Therefore, it can localize up to \( 2(N - 1) = 4M \) sources. In other words, the proposed algorithm can double the number of detectable sources compared with the other algorithms [16]–[18], [30] for near-field noncircular sources.

2) Estimation Accuracy: Apart from the number of detectable sources, the proposed method also distinguishes itself from the GESPRIT-based algorithm [30] in estimation accuracy especially at relatively low SNRs. The GESPRIT-based algorithm performs extraordinary poor when the SNR is low. It means that it is more susceptible to the environmental influences. In comparison, as the fourth-order cumulant is insensitive to the Gaussian noise no matter it is a white or color one, the noise term can be excluded in DOA estimation. Thus, the proposed algorithm can achieve better performance at low SNRs. Moreover, compared with other fourth-order cumulant (FOC) based algorithms [15]–[18], the proposed algorithm increases the estimation accuracy not only owing to the extended array aperture by exploiting the noncircularity of signals, but also owing to the multi-resolution ESPRIT algorithm. The ratio of the longest equivalent sub-array baseline to the shortest one can be thought as the measure of the gain in accuracy provided by multi-resolution ESPRIT.

3) Parameter Pairing: The proposed algorithm estimates the DOAs \( \theta_k \), \( k = 1, 2, \cdots, K \) firstly. Then, \( \hat{\theta}_k \) is substituted into \( b_1(\hat{\theta}_k, r_k) \) to obtain the estimation of \( r_k \) by finding the local maxima of the function \( \{ \det(H^H_k(\theta_k, r_k) \sum_n \sum_n \hat{H}_1(n, n)(\hat{\theta}_k, r_k)) \}^{-1} \). The DOA estimates \( \hat{\theta}_k \) and the range estimates \( \hat{r}_k \) are one-to-one correspondence without any extra processing.

4) Computational Complexity: Define the search interval of DOA \( \theta \in \left[ -\pi/2, \pi/2 \right] \) as \( \Delta \theta \). The resulting multiplications required for the GESPRIT-based algorithm [30] are \( O((2(2M + 1))^2 \pi/2 + (2(2M + 1))^2 \pi/2 + 4/3(2(2M + 1))^3) \). Since second-order statistic is used in [30], its computational complexity is lower than that of those fourth-order cumulant based algorithms including the proposed one. The major computational load involved in the proposed algorithm contains the construction of the cumulant matrix \( H \) and the construction of the augmented covariance matrix \( R \), also theirs eigen-decompositions. Music spectrum search for DOA estimation is not required. Summing the above operations, we can determine the computational load as \( O(11 \times 9(2M + 1)^2 \pi/2 + 4/3(2(2M + 1))^3) \). It can be seen that the computational complexity of the proposed algorithm is higher than [30] due to the constructions of multiple cumulant matrices and the increase of matrix dimensions. It is the relatively high computational load that helps us double the maximum number of detectable sources and achieve the highest estimation accuracy.

IV. SIMULATIONS

In this section, several computational simulations are conducted to assess the performance of the proposed algorithm. For comparison, we simultaneously execute the GESPRIT-based algorithm [30] which is for noncircular sources localization in the near-field, as well as other fourth-order cumulant based algorithms for near-field situation with the noncircularity not considered. These fourth-order cumulant based algorithms include the classical ESPRIT-like algorithm [15], the subspace-based algorithm [16], the cumulant-based algorithm [17] and the INF-MUSIC algorithm [18]. The above algorithms are all based on ULA configuration. Other array configuration based algorithms such as [19] are not contained in the following simulations.

In the first example, we investigate the estimation capacity of the proposed algorithm. Consider an \( N = 3(M = 1) \) elements symmetric uniform linear array with sensor spacing \( d \) being \( \lambda/4 \). The corresponding near-field region of this array structure is \( (0.22\lambda, 0.5\lambda), \). It is assumed that the impinging sources are equi-power, statistically independent BPSK signals modeled as \( e^{\jmath \psi_{s,0,k}(t)} \). Suppose four BPSK signals impinge on the symmetric ULA with their location parameters being \((-40^\circ, 0.3\lambda), (-25^\circ, 0.25\lambda), (-10^\circ, 0.25\lambda), \) and \((40^\circ, 0.35\lambda)\), respectively. The additive noise is spatial white complex Gaussian process. To examine the estimation limitation and exclude the effect of irrelevant factor, the snapshot number and SNR are set relatively high which is equal to...
FIGURE 2. DOA and range estimation of the proposed algorithm. Four BPSK signals impinge on an N = 3(M = 1) elements symmetric ULA. SNR = 20 dB, and the snapshot number is 2000.

to say, the GESPRIT-based algorithm will suffer from the spurious peaks problem and thus may provide unreliable estimation.

In the third example, the estimation accuracy of the five algorithms versus SNR are studied. We consider a symmetric ULA composed of N = 7(M = 3) elements with element spacing d = λ/4 except one composed of N+1 = 8 elements for the classical ESPRIT-Like algorithm [15]. We assume that three uncorrelated narrow-band non-Gaussian BPSK signals with the location parameters being (−30°, 2λ), (−10°, 3λ), and (5°, 4λ) incident upon the array. The number of snapshots is set as 1000. In the following simulations, the performance is measured by the root mean-square error (RMSE) of 500 independent Monte Carlo runs. RMSEs are defined as

\[
RMSE_{\theta} = \sqrt{\frac{\sum_{k=1}^{K} \sum_{p=1}^{P} (\hat{\theta}_k(p) - \theta_k)^2}{KP}},
\]  

(44)
\[ \text{RMSE}_r = \sqrt{\frac{\sum_{k=1}^{K} \sum_{p=1}^{P} (\hat{r}_k(p) - r_k)^2}{KP}}, \quad (45) \]

where \( \hat{\theta}_k(p) \) and \( \hat{r}_k(p) \) are DOA and range estimate of the \( k \)th source from the \( p \)th Monte Carlo trial; \( P \) is the number of total Monte Carlo trials. Fig.4 and Fig.5 illustrate the RMSEs of DOA and range estimates of the three algorithms as a function of SNR varying from 0 dB to 10 dB. The Cramer-Rao bounds (CRBs) are also plotted for comparison. The azimuth DOA estimates are scaled in unit of degree whereas range estimates are scaled in unit of wavelength. From the two figures, we can conclude that the proposed algorithm has an estimation accuracy significantly higher than that of the other five algorithms. The reason why the proposed algorithm has better performance on DOA estimation than [30] is that the fourth-order cumulant we adopt is insensitive to the Gaussian components and thus can effectively suppress Gaussian noise. The proposed algorithm has higher estimation accuracy than the classical ESPRIT-Like algorithm [15], the subspace-based algorithm [16], the cumulant-based algorithm [17], and the INF-MUSIC algorithm [18] because these algorithms did not take the advantage of the noncircular information of the incident sources. However, we notice that the INF-MUSIC algorithm has extremely high accuracy at extremely low SNR. This is due to the use of the first derivative of spatial spectrum. As for the range estimation, the reason why the proposed algorithm performs better is that the range estimate depends on DOA estimate. The more accurate the DOA estimate is, the better range estimate would be. However, the RMSE of the range estimate of [30], which has the lowest precision DOA estimate, is the lowest compared with those of [16]–[18]. It is due to the different range estimation algorithms used in [30] and others. The GESPRIT-based algorithm in [30] takes the noncircular phase into consideration when estimating range. On the other hand, the range spectral searching algorithm adopted in [30] is better than range estimation directly obtained from the eigenvectors of a constructed matrix in [16].

In the last example, the RMSEs of the azimuth DOA and range estimates versus snapshot are shown in Fig.6 and Fig.7. The parameters are the same as that of the third example except that SNR is equal to 10 dB, and the number of snapshots varies from 100 to 1000 in the step of 100. From Fig.6 and Fig.7, we can see that RMSE decreases monotonically as the number of snapshots increase. In addition, the proposed algorithm is superior to the other five algorithms with its RMSE of DOA and range estimates closest to the CRB.
V. CONCLUSION

In this paper, for a symmetric ULA, a cumulant-based algorithm is presented for parameter estimation of noncircular sources in the near-field. By exploiting the rotational invariance property in extended cumulant-domain signal subspace, high precision DOA estimates can be obtained via multi-resolution ESPRIT. Then, one-dimensional search is used for range estimates with the estimated DOAs via rank-reduction algorithm. The proposed algorithm makes full use of the noncircular information of the signal. It also combines the capability of fourth-order cumulant being insensitive to Gaussian (white or color) noise and being able to extend array steering matrix. Compared with the GESPRIT-based algorithm [30], our investigation has shown capable to extend array steering matrix. Compared with the full use of the noncircular information of the signal. It also rank-reduction algorithm. The proposed algorithm makes via multi-resolution ESPRIT. Then, one-dimensional search

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XIAORAN LI received the B.Eng. degree in telecommunication engineering from the Beijing University of Posts and Telecommunications, Beijing, China, in 2011, and the M.E. and Ph.D. degrees in electronics science and technology from the Beijing Institute of Technology, Beijing, in 2013 and 2017, respectively. She was a Visiting Scholar with the Department of Electrical and Computer, Duke University, NC, USA, from 2015 to 2016. She is currently a Postdoctoral Research Fellow with the School of Information and Electronics, Beijing Institute of Technology. Her research interests include array signal processing and integrated circuits design.
QISHU GONG received the B.Eng. degree in electronics science and technology from the Beijing Institute of Technology, Beijing, China, in 2016, where she is currently pursuing the Ph.D. degree. Her research interests include the direction of arrival estimation and array signal processing.

SHUNAN ZHONG received the B.Eng. degree in applied physics, the M.E. degree in semiconductor physics and devices, and the Ph.D. degree in electromagnetic and microwave technology from the Beijing Institute of Technology, Beijing, China, in 1982, 1984, and 2002, respectively. He is currently a Professor and a Ph.D. Supervisor at the Department of Information, Beijing Institute of Technology. His research interests include array signal processing and sparse array design.

SHIWEI REN received the B.Eng. degree in information engineering from the Beijing Institute of Technology, in 2008, and the Ph.D. degree in signal and information processing from the University of Chinese Academy of Sciences, in 2013. In 2013, she joined the School of Information and Electronics, Beijing Institute of Technology, as a Faculty Member. Her research interests include the direction of arrival estimation and sparse array and signal processing.

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