Calculation of Iron Loss in Soft Ferromagnetic Materials using Magnetic Circuit Model Taking Magnetic Hysteresis into Consideration

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The establishment of a simple and practical method of calculating iron loss, including magnetic hysteresis behavior, in electric machines has recently been required in order to develop highly efficient machines quickly and at low cost. A novel and practical method of calculating hysteresis behavior based on a magnetic circuit model incorporated with the Landau-Lifshitz-Gilbert (LLG) equation was proposed in a previous paper. The proposed method enabled highly accurate calculation of the hysteresis loops of conventional non-oriented Si steel under sinusoidal and pulse width modulation (PWM) wave excitations. In this paper, we prove the usefulness and versatility of the proposed method by reporting our recent calculations of the hysteresis loops of two other types of soft ferromagnetic materials (low-loss Si steel with thickness of 0.2 mm and an amorphous core). The calculated results were compared with those obtained from an experiment.

Key words: magnetic circuit model, Landau-Lifshitz-Gilbert (LLG) equation, magnetic hysteresis

1. Introduction

In recent years, the establishment of a simple and practical method of calculating iron loss of electric machines including magnetic hysteresis behavior is required, in order to develop highly efficient electric machines quickly and at low cost. However, the hysteresis behavior is generally neglected in present general-purpose numerical analysis methods for the electric machines.

On the other hand, a lot of calculation methods for the hysteresis behavior have been proposed in previous papers. They can be classified into two types: physical and phenomenological models. The physical model (e.g., micromagnetic model, Stoner-Wohlfarth model) is used to understand details of magnetic phenomena, while the phenomenological model (Jiles-Atherton model, Preisach model), which can be defined by mathematical functions that fit experimental data, is generally more simple and practical than the physical model. Therefore, the phenomenological model is suitable for calculating the iron loss of the electric machines from a practical viewpoint.

In a previous paper, we have proposed a magnetic circuit model with look-up-table as one of the phenomenological models. It can be derived from dc hysteresis loops and iron loss curves, and coupled with an external electric circuit. This model is simple and able to calculate the iron loss including the hysteresis behavior under square wave excitation with various duty ratios. However, it can express only major loops, not minor ones.

To overcome the above problems, we have paid attention to Landau-Lifshitz-Gilbert (LLG) equation. The LLG equation represents the motion of magnetizations in a magnetic substance, and therefore it can express magnetic anisotropy, domain wall motion, etc., which is called “micromagnetic simulation,” namely, it should be classified as the physical model, and hardly adapted to a large-scale analytical object such as the electric machines in general. However, in a previous paper, we have proposed a practical magnetic circuit model incorporated with the LLG equation, which can be coupled with an external electric circuit and calculate the hysteresis behavior of a conventional non-oriented Si steel under sinusoidal and PWM wave excitation.

In this paper, to prove usefulness and versatility of the proposed method, the hysteresis behaviors of other two types of soft ferromagnetic materials (low-loss Si steel with thickness of 0.2 mm and amorphous core) are calculated and compared to experimental results.

2. Proposed Magnetic Circuit Model with LLG Equation

2.1 LLG equation used for the proposed model

The motion of magnetizations in a magnetic substance can be represented by the LLG equation. Fig. 1 shows a schematic diagram of the precession of a magnetization vector \( M_i \) given by the following LLG equation:

\[
\frac{dM_i}{dt} = -\gamma \left[ (M_i \times H_{eff}) + \frac{\alpha}{M_i} (M_i \times \frac{dM_i}{dt}) \right], \quad (i=1-N) \tag{1}
\]

where the effective field is \( H_{eff} \), the magnetization vector is \( M_i = M_s m_i \), the spontaneous magnetization is \( M_s \), the normalized magnetization vector is \( m_i \), the gyromagnetic ratio is \( \gamma \), and the damping constant is \( \alpha \), respectively. From (1), it is understood that the LLG equation represents the dynamic behavior of the magnetization.

The effective field \( H_{eff} \) in (1) is given by

\[
H_{eff} = H_{app} - \frac{\partial E_{anis}}{\partial M_i} - \frac{\partial E_{max}}{\partial M_i} - \frac{\partial E_{sud}}{\partial M_i}, \tag{2}
\]

where the applied field is \( H_{app} \), the magnetic anisotropy energy is \( E_{anis} \), the magnetostatic energy is \( E_{max} \), the
exchange energy is $E_{an}$, and the magnetoelastic energy is $E_{ela}$, respectively.

Equation (2) is simplified by the following assumption: each crystal grain $(0.1 \times 0.1 \times 0.1 \text{ mm}^3)$ has single domain particle, hence the magnetostatic energy $E_{mst}$ and exchange energy $E_{exc}$ in (2) can be neglected. Therefore, the domain wall motion is not expressed in the proposed model.

Since Si steel has a cubic crystal structure, the magnetic anisotropy energy $E_{an}$ is given by

$$E_{an} = \frac{h_{an}}{2} (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2),$$

where the direction cosines of magnetization vectors with respect to x, y, and z axes (easy axes) of each grains are $a_1$, $a_2$, and $a_3$, respectively.

The magnetoelastic energy $E_{ela}$ is assumed to be expressed by

$$E_{ela} = b_2 \frac{M_c^2}{M_s^2} + b_4 \frac{M_c^4}{M_s^4} + b_6 \frac{M_c^6}{M_s^6},$$

where the mean value of magnetization is $M_s$, and the parameters of Taylor expansion are $b_2$, $b_4$, and $b_6$, respectively. As shown in (4), the magnetoelastic energy is expressed as a function of the mean value of magnetization and does not depend on the location. Therefore, boundary conditions are not needed to be employed in the proposed model.

In this paper, to solve the LLG equation based on the Runge-Kutta method, equation (1) is rearranged so that $dM/dt$ in the right side of the equation is transported to the other side as follows:

$$\frac{dM_i}{dt} = -\frac{\gamma_i}{1+\alpha} \left[ (M_i \times H_{eff}) + \frac{\alpha}{M_s} (M_i \times (M_i \times H_{eff})) \right]$$

where the direction cosines of magnetization vectors with respect to $x$, $y$, and $z$ axes (easy axes) of each grains are $a_1$, $a_2$, and $a_3$, respectively.

2.2 DC hysteresis calculation by the LLG equation

Fig. 2 shows an analytical object and analytical space of the LLG equation. The core material is conventional non-oriented Si steel with cubic anisotropy. The number of winding turns is $N = 10$ and the winding resistance is $r = 0.021 \Omega$.

To simplify the LLG calculation, an analytical space, in which the distribution of mean magnetizations is calculated by the LLG equation, is limited in certain small-volume as shown in the figure, and it is assumed that the calculated distribution is uniformly observed in the whole magnetic core. In addition, although the calculated distribution has three-dimensional values, only the longitudinal component is taken into consideration as shown by

$$B = \mu_0 H_{app} + \bar{M}_c,$$

where the convergent mean value of magnetizations of the longitudinal ($z$-axis) component is $\bar{M}_c$, which is calculated by (1), and the vacuum permeability is $\mu_0$.

In the calculation of the LLG equation, only convergent-values of (5) are used for expressing the dc hysteresis. In more detail, when the applied field $H_{app}$ slightly changes at a certain calculation step, the effective field $H_{eff}$ and the magnetization vector $M_i$ are interacting with each other and converging to certain values based on (2) to (5). When the flux density $B$ is given by (6) using the above convergent-value of $M_i$, the relationship between the effective field $H_{eff}$ and the flux density $B$ is obtained at that step. Therefore, when the applied field $H_{app}$ changes for one cycle, the dc hysteresis can be obtained based on the above calculations.

The size of the analytical space is $L_x \times L_y \times L_z = 1.6 \times 1.6 \times 0.8 \text{ mm}^3$, which is divided into $n_x \times n_y \times n_z = 16 \times 16 \times 8$ elements. The spontaneous magnetization $M_s$ is 1.4 T. The magnetic anisotropy direction has three-dimensional random distribution and the constant $h_{an}$ has a normal distribution with a mean value of 120 A/m and a standard deviation of 30 A/m. The magnetoelastic energy coefficients $b_2$, $b_4$, and $b_6$ are 40 J, 30 J, and 6.7 J, respectively.

The above parameters can be determined by approximating the measured dc hysteresis loops as shown in Fig. 3. It is clear that the dc hysteresis loops in various maximum flux densities can be expressed by the LLG equation in which the same parameters described above are used.

2.3 Proposed magnetic circuit model

Reference 9 has reported that the iron loss of soft ferromagnetic materials can be approximately given by

$$W_i = A_f f B_m^2 + A_f f^2 B_m^2 + A_f f^{1.5} B_m^3,$$

where the frequency is $f$, and the maximum flux density is $B_m$. The last term is related to the domain wall motion-induced eddy current loss, which is called anomalous eddy current loss.

From (7), a relationship between $H$ and $B$ can be obtained as following equation 7;

$$H = H_{app} + \gamma_1 \frac{dB}{dt} \pm \gamma_2 \left( \frac{dB}{dt} \right)^{1/3},$$

In (8), the first term expresses the dc hysteresis, the second term indicates the classical eddy current loss, and the third term denotes the anomalous eddy current loss, respectively. The coefficients are $\gamma_1$ and $\gamma_2$, respectively.
Equation (8) can be rewritten by using the cross section \( S \) and the magnetic path length \( l \) as follows:

\[
Ni = H_{app}l + \gamma_1 \frac{d\phi}{dt} + \frac{\gamma_2}{S^0.5} \frac{d\phi}{dt}^1.5.
\]  

(9)

From (9), the proposed magnetic circuit model can be expressed in Fig. 4. In the proposed model, the dc hysteresis is calculated by the LLG equation as described in the previous section, while the classical and anomalous eddy current losses are taken into consideration by the inductance element and the dependent source of flux in the magnetic circuit.

To obtain the parameters \( \gamma_1 \) and \( \gamma_2 \) of (9), the iron loss equation is found from (8) as follows:

\[
W_i = \frac{1}{Tq} \int_{t=0}^{t=\tau}(\gamma_{i/2} \frac{d\phi}{dt} + \gamma_{i/4} \frac{d\phi}{dt}^{1.5})) dB + \frac{2\pi^2\gamma_i f^2 B_{i/2}^2}{q} + 8.763 \frac{\gamma_i B_{i/2}^{1.5}}{q} f^{1.5},
\]

(10)

where the period is \( T \), the mass density is \( q \), respectively. By dividing the each side of (10) by the frequency, the following equation is obtained:

\[
\frac{W_i}{f} = \frac{2\pi^2\gamma_i f^2 B_{i/2}^2}{q} + 8.763 \frac{\gamma_i B_{i/2}^{1.5}}{q} f^{1.5}.
\]

(11)

Therefore, the parameters \( \gamma_1 \) and \( \gamma_2 \) can be found by approximating \( W_i/f - f \) curves of core material with (11) based on least-square method. Fig. 5 shows the measured \( W_i/f - f \) curves of core material and their approximate lines. The parameters used in all the approximate lines are the same as \( \gamma_1 = 0.020 \) and \( \gamma_2 = 0.392 \), respectively.

Fig. 6 shows measured and calculated hysteresis loops of the conventional non-oriented Si steel under sinusoidal excitation\(^7\). The measured exciting voltage is used as the input voltage of the calculation. The figure reveals that the hysteresis loops in various maximum flux densities can be accurately obtained from the proposed model.

### 3. Simulation Results

In the previous paper\(^7\), it was demonstrated that the proposed model can express the hysteresis behavior with high accuracy as shown in Fig. 6. In this chapter, hysteresis loops of other two types of soft ferromagnetic materials (low-loss Si steel with thickness of 0.2 mm and amorphous core) are calculated and compared to experimental results. Table 1 shows properties of the materials. Table 2 indicates the parameters of the LLG equation and the magnetic circuit model used in the calculation.

#### 3.1 Low-loss Si steel with thickness of 0.2 mm

Fig. 7 shows measured and calculated hysteresis loops of low-loss Si steel with thickness of 0.2 mm under sinusoidal wave excitation. Fig. 8 shows comparison of measured and calculated iron losses. It can be seen that the proposed method can calculate the hysteresis loops and iron loss with high accuracy.

Figs. 9 and 10 show measured and calculated current...
waveforms under a binary PWM wave excitation. The measured exciting voltage is used as the input voltage of the calculation. The figures reveal that the proposed model can calculate current waveform under the PWM wave excitation with high accuracy. Fig. 11 shows measured and calculated hysteresis loops under the PWM wave excitation. In addition, Fig. 12 shows measured and calculated hysteresis loops obtained from the previous magnetic circuit model with look-up-table 5). The figures reveal that the proposed model can calculate hysteresis behavior under the PWM wave excitation including minor loops, while the previous model can express only major loops, not minor ones.

3.2 Amorphous core

Fig. 13 shows measured and calculated hysteresis loops of amorphous core under sinusoidal wave excitation. Fig. 14 shows comparison of measured and calculated iron losses. It is demonstrated that the proposed method can calculate the hysteresis loops and iron loss of amorphous core with high accuracy even though the shape of hysteresis loop is considerably different from Figs. 6 and 7.

![Fig. 6 Hysteresis loops under sinusoidal excitation (conventional non-oriented Si steel and \( f = 50 \text{ Hz} \).](image)

![Fig. 7 Hysteresis loops under sinusoidal excitation (low-loss Si steel with thickness of 0.2 mm and \( f = 200 \text{ Hz} \).](image)

![Fig. 8 Comparison of calculated and measured values of iron loss (low-loss Si steel with thickness of 0.2 mm).](image)

![Fig. 9 Exciting voltage and measured current waveforms (low-loss Si steel, binary PWM, and \( f = 200 \text{ Hz} \).](image)

![Fig. 10 Exciting voltage and calculated current waveforms (low-loss Si steel, binary PWM, and \( f = 200 \text{ Hz} \).](image)

| Table 1 | Properties of the materials. |
|---------|-----------------------------|
|         | Conventional Si steel | Low-loss Si steel | Amorphous Si steel |
| Thickness | 0.35 mm | 0.2 mm | 25 μm |
| \( q (\text{kg/m}^3) \) | 7700 | 7600 | 7180 |

| Table 2 | Parameters of the LLG equation and the magnetic circuit model. |
|---------|----------------------------------------------------------|
|         | Conventional Si steel | Low-loss Si steel | Amorphous Si steel |
| \( h_{\text{avg}} \) (A/m) | 120 | 108 | 26.5 |
| \( M_s \) (T) | 1.4 | 1.6 | 1.6 |
| \( h_2 \) (J) | 40 | 58 | 103 |
| \( h_4 \) (J) | 30 | 98 | 257 |
| \( h_6 \) (J) | 6.7 | 41 | 62 |
| \( \gamma_1 \) | 0.020 | 8.77×10^{-3} | 6.64×10^{-4} |
| \( \gamma_2 \) | 0.392 | 0.23 | 0.021 |
Fig. 11 Measured and calculated hysteresis loops under PWM excitation obtained from the proposed magnetic circuit model. (low-loss Si steel, binary mode, and $f = 200$ Hz)

Fig. 12 Measured and calculated hysteresis loops under PWM excitation obtained from the previous magnetic circuit model with look-up-table presented in Ref. 5. (low-loss Si steel, binary mode, and $f = 200$ Hz)

Figs. 15 and 16 show measured and calculated current waveforms under square wave excitation with a duty ratio of $D = 0.5$. The figures reveal that the measured and calculated results are in good agreement. Fig. 17 shows measured and calculated hysteresis loops under the square wave excitation. The figure reveals that the proposed method can calculate the hysteresis loops under square wave excitation with high accuracy.

4. Conclusion

This paper described the proposed magnetic circuit model incorporated with the LLG equation, and calculated the hysteresis behaviors of two types of soft ferromagnetic materials (low-loss Si steel with thickness of 0.2 mm and amorphous core).

As a result, it was demonstrated that the proposed model can express the hysteresis behavior under the sinusoidal, square, and PWM wave excitations with high accuracy including the minor loop. The usefulness and versatility of the proposed method was proved.

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Fig. 17 Measured and calculated hysteresis loops (amorphous core, square waveform, $D = 0.5$, and $f = 200$ Hz).

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