Gravitational Waves from Rotating Proto-Neutron Stars

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Abstract. We study the effects of rotation on the quasi normal modes (QNMs) of a newly born proto neutron star (PNS) at different evolutionary stages, until it becomes a cold neutron star (NS). We use the Cowling approximation, neglecting spacetime perturbations, and consider different models of evolving PNS. The frequencies of the modes of a PNS are considerably lower than those of a cold NS, and are further lowered by rotation; consequently, if QNMs were excited in a sufficiently energetic process, they would radiate waves that could be more easily detectable by resonant-mass and interferometric detectors than those emitted by a cold NS. We find that for high rotation rates, some of the $g$-modes become unstable via the CFS instability; however, this instability is likely to be suppressed by competing mechanisms before emitting a significant amount of gravitational waves.

1. Introduction

It has recently been shown [1] that, during the first minute of life of a proto-neutron star (PNS) born in a gravitational collapse, the frequencies of its quasi normal modes (QNMs) change. Indeed, the star cools down and contracts and its internal structure is modified by neutrino diffusion and thermalisation processes. The evolutionary models used in [1], developed in [2, 3], describe the stellar evolution in terms of a sequence of equilibrium configurations; this quasi-stationary description has been shown to become appropriate after a few tenths of seconds from the stellar birth. By solving the equations of stellar perturbations it has been shown that the frequencies of all QNMs of a newly born PNS are much smaller than those of the cold NS which forms at the end of the evolution. In order to isolate the effects of the thermal and chemical processes from those induced by rotation, in [1] all stellar models were assumed to be non rotating. Subsequently, this study has been generalized to include rotation [4], and to explore the possibility that the modes become unstable due to Chandrasekhar–Friedman–Schutz (CFS) mechanism [5, 6, 7].

The CFS instability of the fundamental mode (f-mode) has been studied extensively in the literature [8, 9, 10], and it has been shown to act at very high stellar rotation rates, comparable to the break-up limit. Furthermore, unless the temperature is very low viscous dissipation mechanisms tend to stabilise the f-mode instability. The g-mode instability has also been extensively studied in recent years, after Andersson [11] discovered that it is generic for every rotating star. However, whether or not this
instability removes a considerable amount of rotational energy from the star is still matter of debate.

Since in the no rotation limit the \( g \)–modes have frequencies lower than that of the \( f \)–mode, they may become unstable for relatively small values of the angular velocity. This provides a good motivation for the study of CFS instability of \( g \)–modes, and indeed it has been studied in [18] for zero temperature stars using the Newtonian theory of stellar perturbations in the Cowling approximation. This study regarded a particular class of \( g \)–modes for which the buoyancy, which provides the restoring force for the modes, is due to the gradient of proton to neutron ratio in the interior of the star.

In [4] we studied the onset of the CFS instability of the lowest \( g \)–modes of a newly born, hot proto-neutron star, where the buoyancy is mainly due to the high entropy and composition gradients that dominate the stellar dynamics. We used the relativistic theory of stellar perturbations of a slowly rotating star in the Cowling approximation, which is known to reproduce with a good accuracy the \( g \)–mode frequencies, because such modes are associated with small gravitational perturbations.

We stress that today it is not definitely clear if CFS instability is relevant in newly born neutron stars. This and other similar mechanisms could explain why we do not observe pulsars spinning with periods of less than 5ms, which are predicted by most stellar models. [19]

2. Formulation of the problem

We consider a relativistic star in uniform rotation with an angular velocity \( \Omega \) so slow that the distortion of its figure from spherical symmetry is of order \( \Omega^2 \), and can be ignored. Following the approach of Hartle [20], we expand all equations with respect to the parameter \( \varepsilon = \Omega/\Omega_K \), where \( \Omega_K = \sqrt{\frac{M}{R^3}} \); we retain only first order terms \( O(\varepsilon) \).

On these assumptions, the metric can be written in the form

\[
 ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - 2\varepsilon \omega(r) \sin^2 \theta dt d\phi. 
\]

The star is assumed to be composed by a perfect fluid, whose energy momentum tensor is

\[
 T_{\mu\nu} = (p + \rho) u_{\mu} u_{\nu} + pg_{\mu\nu}, \quad (2)
\]

with pressure \( p \), energy density \( \rho \) and four-velocity components \( u^\mu = [e^{-\nu}, 0, 0, \Omega e^{-\nu}] \).

The metric functions \( \nu(r), \lambda(r) \) are found by solving the equations of hydrostatic equilibrium [20]. We use as a background the models of evolving proto-neutron stars developed in [2, 3], and study the non–radial, adiabatic perturbations of these models for selected values of the evolution time; we start from \( t = 0.5 \) s after the formation of the proto-neutron star, when the quasi–stationary description becomes appropriate to represent the stellar evolution.

The complete set of equations for the perturbations has been derived using the so–called BCL gauge [21] by Ruoff, Stavridis & Kokkotas [22]. The Cowling limit of these equations, which neglects the contribution of the gravitational perturbations, was studied in [23] for polytropic relativistic equations of state. In [4] we have used the Cowling approximation and, since we are interested in the evolution of the \( g \)– and \( f \)–modes, that have polar parity, we have neglected the coupling with axial parity perturbations. The results we show refer to the components with harmonic indices \( l = m = 2 \), which are the most relevant for gravitational wave emission.
3. Results

In order to find the frequencies of the quasi–normal modes, we have numerically integrated the perturbed equations both in the time and in the frequency domain for different values of the evolution time $t_{ev}$, and for selected values of the rotation parameter $\epsilon$. The rotation rate has been chosen to vary within $0 \leq \epsilon \leq 0.4$ because from preliminary calculations we find that for the models under consideration the mass shedding limit does not exceed $\epsilon = 0.4$. The results refer to the evolutionary model labelled as model A in [1]. We consider the first minute of life of the proto-neutron star, from $t_{ev} = 0.5$ s to $t_{ev} = 40$ s, during which the star significantly cools down and contracts, and processes related to neutrino diffusion and thermalization dominate the stellar evolution. The gravitational mass of the star, which is $M = 1.56 M_\odot$ at $t_{ev} = 0.2$ s, due to neutrino emission becomes $M = 1.46 M_\odot$ at $t_{ev} = 40$ s. The radius of the initial configuration is $R = 23.7$ km and reduces to $R = 12.8$ km at $t_{ev} = 40$ s.

The main results of our work are summarized in figures 1 and 2, where we plot the frequencies of the $f$-, $g_{1}$- and $g_{2}$- modes as a function of the rotation parameter $\epsilon = \Omega/\Omega_K$, for different values of the evolution time in the more interesting phases of the cooling process. We see that as the time elapsed from the birth of the PNS grows, the $f$-mode frequency increases and tends to that of the cold neutron star which forms at the end of the evolutionary process. Conversely, the frequency of the $g$-modes initially decreases, reaches a minimum for $t_{ev} \approx 12$ s, and subsequently smoothly increases. This behaviour is justified by the fact that the $g$-modes are associated to entropy and composition gradients, and whereas during the first 10-12 seconds the dynamical evolution of the star is dominated by strong entropy gradients that progressively smooth out, after $\approx 12$ s the entropy becomes nearly constant throughout the star and $g$-modes due to composition gradients take over.

The onset of the CFS instability is signaled by the vanishing of the mode frequency for some value of the angular velocity (neutral point). From figures 1 and 2 we see that while the $f$-mode does not become unstable during the first minute of the PNS life, both the $g_{1}$- and the $g_{2}$- modes do become unstable. The $g_{1}$- frequency remains positive during the first second, but at later times it vanishes for very low values of $\epsilon$. For instance, at $t_{ev} = 3$ s it crosses the zero axis for $\Omega = 0.17 \Omega_K$, even though its value for the corresponding nonrotating star is still quite high, $\nu_{g_{1}} = 486$ Hz. The behaviour of the $g_{2}$- mode is similar, but being the frequency lower the instability sets in at lower rotation rates.

It should be mentioned that in [1] we studied also a second model of evolving proto-neutron star, labelled as model B. The main difference between the two models is that model A has an equation of state softer than model B, and that at some point of the evolution a quark core forms in the interior of model B. We have integrated the perturbed equations also for model B, finding results entirely similar to those of model A; this indicates that the quark core that develops at some point of the evolution does not affect the overall properties of the modes in a relevant way.

A mode instability is physically significant if its growth time is sufficiently small with respect to the timescales typical of the stellar dynamics; in this case the instability has sufficient time to grow before other processes damp it out or the structure of the evolving star changes. In [4] we give an “order of magnitude” estimate of the growth time of the $g$–modes as follows: we compute the energy $E$ associated with a given mode in Newtonian approximation as in [18], and the gravitational luminosity $\frac{dE}{dt}$ using a multipole expansion as in [24]. The growth time associated to gravitational
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radiation reaction then is

\[ \frac{1}{\tau_{\text{gr}}} = -\frac{1}{2E} \frac{dE}{dt}. \] (3)

Since we are working in the Cowling approximation, we neglect the perturbation of the Newtonian potential. Furthermore, we neglect the contribution due to current multipoles, because they correspond to axial parity perturbations.

The growth times of the unstable $g_1$-modes shown in figure 2 are summarized in Table 1. The growth time appears to be orders of magnitude larger than the evolutionary timescale, which is of the order of tens of seconds. Although the estimate based on Newtonian expressions is a quite crude one, the growth time is so much larger than the evolutionary timescale that it is reasonable to conclude that the CFS instability of the lowest $g$-mode is unlikely to play a relevant role in the early evolution of proto-neutron stars. Similar conclusions can be drawn for the fundamental mode and for higher order $g$-modes.

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Table 1. Growth times for unstable $g_1$ mode of Model A for $t_{\text{ev}} = 3\, s, 12\, s, 40\, s$

| $\varepsilon$ | $\nu$ (Hz) | $\tau_{\text{gr}}$ (s) | $\varepsilon$ | $\nu$ (Hz) | $\tau_{\text{gr}}$ (s) | $\varepsilon$ | $\nu$ (Hz) | $\tau_{\text{gr}}$ (s) |
|---------------|-------------|---------------------|---------------|-------------|---------------------|---------------|-------------|---------------------|
| 0.1           | 200         | $\ldots$           | 0.1           | -160        | -4.6 $10^4$        | 0.1           | -70         | -3.7 $10^{10}$       |
| 0.2           | -90         | -1.5 $10^9$        | 0.2           | -475        | -1.7 $10^6$        | 0.2           | -430        | -1.3 $10^9$         |
| 0.3           | -683        | -2 $10^6$          | 0.3           | -760        | -3.4 $10^5$        | 0.3           | -900        | -3.6 $10^6$         |
| 0.4           | -789        | -2.2 $10^4$       | 0.4           | -1020       | -2.7 $10^5$        | 0.4           | -1250       | -1.3 $10^5$         |

Figure 1. The frequency of the fundamental mode of a newly born PNS is plotted as a function of the rotational parameter $\varepsilon = \Omega/\Omega_K$, for assigned values of the time elapsed from the stellar birth. As the time grows, the frequency increases and tends to that of the cold neutron star which forms at the end of the evolutionary process. The onset of the CFS instability occurs when the mode frequency becomes zero, and we see that the $f$-mode would become unstable only for extremely high values of the rotational parameter, as it is for cold stars.

Figure 2. The frequency of the $g_1$- and $g_2$- modes of a newly born PNS are plotted, as in figure 1, for the same stellar models. Unlike the $f$-mode, as the time grows the frequency of the $g$-modes decreases, reaches a minimum at about $t_{\text{ev}} = 12$ and then slightly increases (see text). For both modes the CFS instability sets in at values of the rotational parameter much lower than that needed for the $f$-mode.