Leptogenesis, \( CP \) violation and neutrino data: What can we learn?

G. C. Branco, R. González Felipe, F. R. Joaquim, M. N. Rebelo

Centro de Física das Interacções Fundamentais (CFIF), Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Abstract

A detailed analytic and numerical study of baryogenesis through leptogenesis is performed in the framework of the standard model of electroweak interactions extended by the addition of three right-handed neutrinos, leading to the seesaw mechanism. We analyze the connection between GUT-motivated relations for the quark and lepton mass matrices and the possibility of obtaining a viable leptogenesis scenario. In particular, we analyze whether the constraints imposed by SO(10) GUTs can be compatible with all the available solar, atmospheric and reactor neutrino data and, simultaneously, be capable of producing the required baryon asymmetry via the leptogenesis mechanism. It is found that the Just-So\(^2\) and SMA solar solutions lead to a viable leptogenesis even for the simplest SO(10) GUT, while the LMA, LOW and VO solar solutions would require a different hierarchy for the Dirac neutrino masses in order to generate the observed baryon asymmetry. Some implications on \( CP \) violation at low energies and on neutrinoless double beta decay are also considered.

1 Introduction

Baryogenesis via leptogenesis is one of the most appealing mechanisms for generating the observed baryon asymmetry of the Universe (BAU). The interest in leptogenesis has been reinforced after the strong evidence for neutrino oscillations reported by the Super-Kamiokande experiment [1] and recently confirmed by the results of the Sudbury Neutrino Observatory (SNO) [2], both pointing towards nonzero neutrino masses. From a theoretical point of view, the smallness of neutrino masses can be naturally explained through the

\footnote{E-mails: gbranco@cfif.ist.utl.pt (G.C. Branco), gonzalez@gtae3.ist.utl.pt (R. González Felipe), filipe@gtae3.ist.utl.pt (F.R. Joaquim), rebelo@alfa.ist.utl.pt (M.N. Rebelo)}
seesaw mechanism [3] which automatically follows, once right-handed neutrinos are added to the standard model (SM). Since the right-handed neutrino fields are singlets under the SU(3)$_c \times$ SU(2) $\times$ U(1) gauge symmetry of the SM, right-handed neutrino Majorana masses are not protected by the gauge symmetry so they can be much larger than the electroweak scale. These right-handed neutrino states appear naturally in Grand Unified Theories (GUT) such as SO(10) [4], where they are included in the same irreducible representation together with quarks and leptons.

At low energies, all the information about neutrino masses and mixing is contained in the effective neutrino mass matrix which can be expressed by the seesaw formula:

$$M_\nu \simeq -M_D M_R^{-1} M_D^T,$$

(1)

where $M_D$ and $M_R$ are the Dirac and right-handed neutrino mass matrices, respectively. Unfortunately, $M_D$ and $M_R$ are not fully determined by the low-energy experimental data. In this respect, GUTs can play an essential rôle since in their framework $M_D$ is closely related to the quark or charged lepton mass matrices. Another alternative to this approach is to impose some extra symmetries like horizontal or discrete symmetries [5].

In this paper we investigate the constraints on $M_D$ resulting from the requirement of a viable leptogenesis [6–8]. More specifically, we investigate whether leptogenesis can discriminate among the various solar neutrino solutions in the framework of some GUT-inspired patterns for $M_D$. We shall assume a special form for $M_D$ in the weak basis where $M_R$ and the charged leptonic mass matrix $M_\ell$ are diagonal. There is of course no loss of generality in choosing these two matrices real, positive and diagonal. Since our discussion does not rely on specific textures for these matrices it will cover a wide range of mass matrices. In this sense our analysis is quite general. Yet, as we shall see, the hierarchy of the masses resulting from $M_D$ plays a crucial rôle in the viability of the leptogenesis scenario. In fact, mass hierarchies such as the ones imposed by minimal SO(10) strongly constrain the allowed solar solutions. Nevertheless, it turns out that allowing for a more general choice of the mass spectrum of $M_D$, it is possible to reconcile the different solar solutions with the required cosmological BAU.

The connection between neutrino masses and mixing and the observed baryon asymmetry of the Universe becomes clear if one takes into account that the leptogenesis mechanism starts with the $CP$ asymmetry generated through the out-of-equilibrium $L$-violating decays of the heavy Majorana neutrinos [9,10,6], leading to a lepton asymmetry which is subsequently transformed into a baryon asymmetry by the $(B + L)$-violating sphaleron processes [11]. As it will be discussed later, this baryon asymmetry depends mainly on the heavy Majorana neutrino mass spectrum and the Dirac neutrino Yukawa coupling
matrix, in the basis where the mass matrix $M_R$ is diagonal. These matrices will be constrained by GUT-like relations and by the available information on neutrino mixing and mass squared differences measured in solar, reactor and atmospheric neutrino experiments. On the other hand, requiring the produced baryon-to-entropy ratio $Y_B \equiv n_B/s$ to be in the presently allowed range \cite{12},

$$1.7 \times 10^{-11} \lesssim Y_B \lesssim 8.1 \times 10^{-11},$$

(2)

can in principle constrain some mixing and mass parameters, or even discriminate the solar neutrino solutions which are compatible with the leptogenesis scenario. We shall see that the only parameters relevant to this analysis are the absolute value of the (1,3)-element of the leptonic mixing matrix, the Dirac-type phase and the two Majorana phases appearing in this matrix, as well as the mass of the lightest neutrino.

At this stage one may wonder whether there is any link between leptogenesis and leptonic $CP$ violation at low energies since they both arise from the phases appearing in the leptonic mixing matrix. Several authors have already addressed this question under different assumptions \cite{13,14}. Experimentally, leptonic $CP$-violating effects can be probed in neutrino oscillation experiments and they are only sensitive to the Dirac-type phase appearing in the leptonic mixing matrix. On the other hand, the size of double beta decays is affected by the presence of Majorana-type $CP$-violating phases.

The paper is organized as follows. In Section 2 we present the general framework and the strategy to follow. Special emphasis is given to the construction of the Dirac, Majorana and effective neutrino mass matrices as well as to the identification of the $CP$-violating phases relevant to leptogenesis. In Section 3 we present the simplest leptogenesis scenario, namely, the out-of-equilibrium decay of a single heavy Majorana neutrino. We briefly recall how to estimate the baryon asymmetry taking into account the washout effects due to inverse decays and lepton number violating scattering processes. This section is divided into two parts. First we develop a simple analytic approach which will allow us to obtain upper bounds on the baryon asymmetry for the different solar neutrino solutions and assuming different patterns for the light neutrino spectrum. We then perform a detailed numerical analysis in order to identify the regions of the parameter space where the leptogenesis mechanism is efficient enough to produce the required cosmological baryon asymmetry, taking into account the presently available low energy neutrino data. Section 4 is devoted to discuss some implications on leptonic $CP$ violation at low energies and neutrinoless double beta decay. Finally, our conclusions are presented in Section 5.
2 General framework

We shall work in the framework of a minimal extension of the SM with only one right-handed Majorana neutrino per generation. After spontaneous symmetry breakdown, the leptonic mass terms are of the form

\[ -\mathcal{L}_{\text{mass}} = \overline{\ell}_L^0 M_\ell \ell_R^0 + \overline{\nu}_L^0 M_D \nu_R^0 + \frac{1}{2} \nu_R^{0T} C \, M_R \nu_R^0 + \text{h.c.}, \]  

(3)

where $\ell_{L,R}^0$ and $\nu_{L,R}^0$ are the charged lepton and neutrino weak eigenstates, respectively. The charged lepton mass matrix $M_\ell$ and the Dirac neutrino mass matrix $M_D$ are in general $3 \times 3$ complex matrices whilst the heavy Majorana neutrino mass matrix $M_R$ is constrained to be symmetric. The right-handed Majorana mass term is SU(2)×U(1) invariant, as a result it is the only fermionic mass term present in the Lagrangian before the symmetry breaking. The Lagrangian in Eq. (3) can be rewritten as

\[ -\mathcal{L}_{\text{mass}} = \frac{1}{2} \, \overline{n}_L^T C \mathcal{M}^* n_L + \overline{\ell}_L^0 M_\ell \ell_R^0 + \text{h.c.}, \]  

(4)

with $n_L = (\nu_L^0, (\nu_R^0)^C)$ and

\[ \mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. \]  

(5)

The zero entry in $\mathcal{M}^*$ is due to the absence, at tree level, of a Majorana mass term for the left-handed neutrinos $\nu_L$. The diagonalization of $\mathcal{M}^*$ is performed through the transformation

\[ V^T \mathcal{M}^* V = \mathcal{D} = \text{diag}(m_1, m_2, m_3, M_1, M_2, M_3), \]  

(6)

where $m_i$ and $M_i$ denote the physical masses of the light and heavy Majorana neutrinos, respectively. It is useful to write $V$ and $\mathcal{D}$ in a block form as

\[ V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}, \quad \mathcal{D} = \begin{pmatrix} d_\nu & 0 \\ 0 & d_R \end{pmatrix}, \]  

(7)

with $d_\nu = \text{diag}(m_1, m_2, m_3), d_R = \text{diag}(M_1, M_2, M_3)$. From Eq. (6) one derives the approximate relations

\[ -K^\dagger M_D M_R^{-1} M_D^T K^* = K^\dagger M_\nu K^* = d_\nu, \]  

(8)

\[ S^\dagger = -K^\dagger M_D M_R^{-1}, \]  

(9)
together with the exact relation

\[ R = M_D T^* d_R^{-1} . \]  

(10)

The relation in Eq. (8) is the usual seesaw formula, which leads in a natural
way to small left-handed Majorana neutrino masses of order \( M_D M_R^{-1} \). The
matrix \( K \) is almost unitary, which is in agreement with the matrices \( S \) and \( R \)
being of order \( M_D M_R^{-1} \) as can be inferred from Eqs.(9) and (10).

The neutrino weak eigenstates \( \nu_{iL}^{0} \) are related to the mass eigenstates by

\[ \nu_{iL}^{0} = V_{i\alpha} \nu_{\alpha L} = (K, R) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix}, \quad i = 1, 2, 3, \quad \alpha = 1, 2, \ldots, 6, \]  

(11)

and thus the leptonic charged current interactions are given by

\[ -\frac{g}{\sqrt{2}} \left( \bar{\ell}_{L} \gamma_{\mu} K \nu_{L} + \bar{\ell}_{L} \gamma_{\mu} R N_{L} \right) W^{\mu} + h.c. , \]  

(12)

so that \( K \) and \( R \) give the charged current couplings of the charged leptons to
the light neutrinos \( \nu_{i} \) and the heavy ones \( N_{i} \), respectively. Without imposing
any restriction on the mass matrices, Eq. (3) can be rewritten in the weak
basis (WB) where \( M_{\ell} \) and \( M_{R} \) are real and diagonal in the form

\[ -\mathcal{L}_{\text{mass}} = \bar{\ell}_{L}^{0} d_{\ell} \ell_{R}^{0} + \bar{\nu}_{L}^{0} M_{D} \nu_{R}^{0} + \frac{1}{2} \nu_{R}^{0 T} C d_{R} \nu_{R}^{0} + h.c. , \]  

(13)

where \( d_{\ell} = \text{diag}(m_{e}, m_{\mu}, m_{\tau}) \). We have kept for simplicity the same notation
of Eq. (3) for the rotated fields and mass matrices in the Lagrangian (13). Obviously, in this WB all \( CP \)-violating phases appear in \( M_{D} \). Since for \( n \)
generations there are \( n(n-1) \) independent phases [14,15], the matrix \( M_{D} \) has
six \( CP \)-violating phases. In addition, the Lagrangian \( \mathcal{L}_{\text{mass}} \) has fifteen real
physical parameters: nine contained in \( M_{D} \) plus the six masses in \( d_{\ell} \) and \( d_{R} \).

It is easy to see how these phases appear in \( M_{D} \). Using the polar decom-
position, \( M_{D} \) can be written as the product of a unitary matrix \( U \) times a
Hermitian matrix \( H \). Factoring out all possible phases we can write

\[ M_{D} = U H = P_{\xi} U' P_{1} H' P_{2} , \]  

(14)

with \( P_{\xi} = \text{diag}(e^{i\xi_{1}}, e^{i\xi_{2}}, e^{i\xi_{3}}), P_{j} = \text{diag}(1, e^{i\varphi_{j}}, e^{i\varphi_{j}}) \) and the matrices \( U' \) and \( H' \) containing only one phase each. Since \( P_{\xi} \) can always be rotated away by
a simultaneous phase transformation of the left-handed charged lepton fields
and left-handed neutrino fields, only six phases are physically meaningful.

In the physical basis all \( CP \)-violating phases are shifted to the leptonic mixing
matrix and thus will appear in the matrices \( K \) and \( R \). Since to a very good
approximation $K$ is unitary and three of its phases can be rotated away, we end up with three physical phases in $K$, two of which are of Majorana character. These phases are in general complicated functions of the six phases of $M_D$ and can be obtained from Eq. (8) by replacing $M_R$ by $d_R$. The matrix $R$, on the other hand, verifies Eq. (10). In the WB where $M_R$ is real and diagonal, the matrix $T$ in (7) is close to the identity. As a result, from Eqs. (10) and (14) we obtain

$$R = U'P_1H'd_R^{-1}P_2.$$  (15)

Once again two of the phases appearing in $R$ are of Majorana character - the ones contained in $P_2$. As we shall see in the next section, leptogenesis is only sensitive to the phases appearing in $M_D^\dagger M_D$, or equivalently $R^\dagger R$. Therefore, from Eq. (15) we can conclude that the only CP-violating phases relevant for leptogenesis are the phase contained in $H'$ and the two Majorana phases of $P_2$.

In the exact decoupling limit, $R$ can be neglected and only $K$ is relevant. In this limit, the leptonic mixing matrix $K$ is usually referred as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [16,17], which from now on we shall denote as $U_{\nu}$. However, since we are interested in studying the connection between CP violation at low energies and leptogenesis we should keep both $K$ and $R$ or, alternatively, the Dirac neutrino mass matrix $M_D$ in the WB where $M_\ell$ and $M_R$ are diagonal.

Experimentally, only the charged lepton mass spectrum is very well known. In contrast, the experimental data on light neutrino masses and leptonic mixing define different allowed regions in parameter space, still leaving a lot of freedom in the choice of patterns for the matrices $U_{\nu}$ and $M_\nu$. The heavy neutrino masses and their mixing with the charged leptons are even less constrained.

From the theoretical point of view, grand unified theories such as SO(10) [4] are a suitable framework not only to analyze fermion masses but also to implement the seesaw mechanism. One of the attractive features of the SO(10) model is that its gauge group is left-right symmetric and, consequently, there exists a complete quark-lepton symmetry in the spectrum. In particular, the fact that all left-handed (right-handed) fermions of each family fit into the single irreducible spinor representation 16 ($\overline{16}$) of SO(10) and that the right-handed neutrino is precisely contained in this representation is remarkable. Several constraints on fermion masses are usually implied in these models [4]. For instance, if there is only one 10 Higgs multiplet responsible for the masses, then we have the relation $M_u = M_d = M_\ell = M_D$ (the indices $u$ and $d$ stand for the up and down quarks, respectively). Similarly, the existence of two 10 Higgs multiplets implies $M_\ell = M_d$ and $M_D = M_u$. On the other hand, if the fermion masses are generated by a VEV of the 126 of SO(10), then the SU(4) symmetry yields the relations $3 M_d = -M_\ell$ and $3 M_u = -M_D$. 

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Of course, equalities such as the ones arising purely from the 10-dimensional representation cannot be exact in realistic models, since they imply unphysical relations among the quark and charged lepton masses. Additional assumptions are therefore necessary in order to predict the correct fermion spectrum [18–20].

Without loss of generality, one can choose a WB where $M_\ell$ and $M_R$ are diagonal and real while $M_D$ is an arbitrary complex matrix which we write as:

$$M_D = V_L^\dagger d_D U_R ,$$

(16)

where $d_D = \text{diag}(m_{D1}, m_{D2}, m_{D3})$ and $V_L, U_R$ are unitary matrices. In a minimal SO(10) scenario it is expected a small misalignment between the Hermitian matrices $M_\ell M_\ell^\dagger$ and $M_D M_D^\dagger$, similar to that in the quark sector. The matrix $V_L$ is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the quark sector and under the above assumption of small misalignment, it should be close to the identity matrix. The Dirac neutrino mass spectrum will be constrained by SO(10) relations such as the ones discussed above. For definiteness, we will assume that $d_D$ is given by the up-quark spectrum, i.e

$$d_D = \text{diag}(m_u, m_c, m_t) .$$

(17)

A brief discussion of the most general case with arbitrary masses is presented at the end of Section 3.2.

In our analysis we choose $d_\nu$ and $U_\nu$ in agreement with the present experimental data. The corresponding effective neutrino matrix $M_\nu$ can be then computed from Eq. (8), i.e

$$M_\nu = U_\nu d_\nu U_\nu^T .$$

(18)

We shall consider different patterns for the PMNS matrix $U_\nu$ corresponding to the different solar solutions (LMA, SMA, LOW, VO, Just-So$^2$). For each solution we impose the corresponding mass squared differences $\Delta m^2_\odot \equiv \Delta m^2_{12} = |m_2^2 - m_1^2|$ (solar neutrinos) and $\Delta m^2_a \equiv \Delta m^2_{23} = |m_3^2 - m_2^2|$ (atmospheric neutrinos) to be in the presently allowed experimental range. We also let the mass of the lightest neutrino to vary in the appropriate range: its lower bound is typically fixed by requiring the heaviest Majorana mass to be below the Planck scale, while the upper bound is chosen so that neutrinoless double beta decays experiments are not violated.

An important feature of this approach is that once $M_\nu$ is chosen, and for a given$^2 V_L$, the masses of the heavy Majorana neutrinos are fixed together

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$^2$ Throughout our paper we either take $V_L$ equal to the identity or close to it.
with the matrix $U_R$, which appears in Eq. (16). Indeed, the relations

$$M_\nu = -M_D d_R^{-1} M_D^T = -V_L^\dagger d_D M d_D V_L^*,$$

$$M \equiv -d_D^{-1} V_L M_\nu V_L^T d_D^{-1} = U_R d_R^{-1} U_R^T,$$

(19)

imply that the matrix $M$ is fully determined. Obviously, we have $M M^\dagger = U_R d_R^{-2} U_R^\dagger$, corresponding to an eigenvalue equation. This equation determines the heavy Majorana spectrum $d_R$ and also the unitary matrix $U_R$ up to a phase ambiguity, which can be then fixed by going back to Eqs. (19).

Notice that Eqs. (19) clearly illustrate a remarkable feature of these kind of models [21], which is the fact the large mixing required for the atmospheric solution as well as some of the solar solutions can be obtained with small mixing in $U_R$, so that nearly maximal neutrino mixing is produced through the seesaw mechanism. In other words, starting from nearly diagonal mass matrices and as a result of the interplay between the small off-diagonal entries of $U_R$ and a strong hierarchy in $d_D$ and $d_R$, it is possible to obtain a large neutrino mixing via the seesaw mechanism.

### 3 Heavy Majorana neutrino decays, leptogenesis and neutrino oscillations

The crucial ingredient in leptogenesis scenarios is the $CP$ asymmetry generated through the interference between tree-level and one-loop heavy Majorana neutrino decay diagrams. In the simplest extension of the SM, such diagrams correspond to the decay of the Majorana neutrino into a lepton and a Higgs boson. Considering the decay of one heavy Majorana neutrino $N_i$, the $CP$ asymmetry in the SM is then given by

$$\epsilon_{N_i} = \frac{\Gamma (N_i \rightarrow l H) - \Gamma (N_i \rightarrow \bar{l} H^*)}{\Gamma (N_i \rightarrow l H) + \Gamma (N_i \rightarrow \bar{l} H^*)}.$$  

(20)

If the heavy Majorana neutrino masses are such that $M_1 < M_2 < M_3$ only the decay of the lightest Majorana neutrino $N_1$ is relevant for the lepton asymmetry$^3$ and one obtains [23,6]

$$\epsilon_{N_1} = \frac{1}{8 \pi v^2} \frac{1}{(M_D^\dagger M_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (M_D^\dagger M_D)_{ii}^2 \right] \left[ f \left( \frac{M_i^2}{M_1^2} \right) + g \left( \frac{M_i^2}{M_1^2} \right) \right],$$  

(21)

$^3$ This is a reasonable assumption if the interactions of the lightest Majorana neutrino $N_1$ are in thermal equilibrium at the time of the $N_{2,3}$ decays, so that the asymmetries produced by the heaviest neutrino decays are erased before the lightest one decays, or if $N_{2,3}$ are too heavy to be produced after inflation [22].
where $M_D$ is the Dirac neutrino mass matrix in the basis where $M_R$ is diagonal and $v = \langle H_0 \rangle / \sqrt{2} \simeq 174 \text{ GeV}$. The functions $f(x)$ and $g(x)$ denote the one-loop vertex and self-energy corrections, respectively, and are given by

$$f(x) = \sqrt{x} \left[ 1 + (1 + x) \ln \left( \frac{x}{1 + x} \right) \right], \quad g(x) = \frac{\sqrt{x}}{1 - x}. \quad (22)$$

In the limit $M_1 \ll M_2, M_3$ the $CP$ asymmetry (21) is approximately given by

$$\epsilon_{N_1} \simeq - \frac{3}{16 \pi v^2} \left( I_{12} \frac{M_1}{M_2} + I_{13} \frac{M_1}{M_3} \right), \quad (23)$$

where

$$I_{1i} \equiv \text{Im} \left[ (M_D^\dagger M_D)^2_{1i} \right] / (M_D^\dagger M_D)_{11}. \quad (24)$$

The lepton asymmetry $Y_L$ is related to the $CP$ asymmetry through the relation

$$Y_L = \frac{n_L - n_{\bar{L}}}{s} = d \frac{\epsilon_{N_1}}{g_*}, \quad (25)$$

where $g_*$ is the effective number of relativistic degrees of freedom contributing to the entropy and $d$ is the so-called dilution factor which accounts for the washout processes (inverse decay and lepton number violating scattering). In the SM case, $g_* = 106.75$.

The produced lepton asymmetry $Y_L$ is converted into a net baryon asymmetry $Y_B$ through the $(B + L)$-violating sphaleron processes. One finds the relation [24,6]

$$Y_B = \xi Y_{B-L} = \frac{\xi}{\xi - 1} Y_L, \quad \xi = \frac{8 N_f + 4 N_H}{22 N_f + 13 N_H}, \quad (26)$$

where $N_f$ and $N_H$ are the number of fermion families and complex Higgs doublets, respectively. Taking into account that $N_f = 3$ and $N_H = 1$ for the SM, we get $\xi \simeq 1/3$ and

$$Y_B \simeq - \frac{1}{2} Y_L. \quad (27)$$

The determination of the dilution factor involves the integration of the full set of Boltzmann equations. A simple approximated solution which has been
frequently used is given by [25]

\[
    d = \begin{cases} 
        \sqrt{0.1\kappa} \exp\left(-\frac{4}{3}\sqrt{0.1\kappa}\right), & \kappa \gtrsim 10^6 \\
        0.24(\kappa \ln \kappa)^{-3/5}, & 10 \lesssim \kappa \lesssim 10^6 \\
        1/(2\kappa), & 1 \lesssim \kappa \lesssim 10 \\
        1, & 0 \lesssim \kappa \lesssim 1
    \end{cases} 
\]  

(28)

where the parameter \( \kappa \), which measures the efficiency in producing the asymmetry, is defined as the ratio of the thermal average of the \( N_1 \) decay rate and the Hubble parameter at the temperature \( T = M_1 \),

\[
    \kappa = \frac{M_P}{1.7 \times 8\pi v^2 \sqrt{g_*}} \frac{(M_D^\dagger M_D)_{11}}{M_1},
\]

(29)

\( M_P \approx 1.22 \times 10^{19} \) GeV is the Planck mass\(^4\). A slightly modified approximation is used instead by the authors of Ref. [26]:

\[
    d = \begin{cases} 
        0.30(\kappa \ln \kappa)^{-3/5}, & 10 \lesssim \kappa \lesssim 10^6 \\
        1/(2\sqrt{\kappa^2 + 9}), & 0 \lesssim \kappa \lesssim 10
    \end{cases} 
\]

(30)

However, it has been recently pointed out [8] that in some cases the above approximations could seriously underestimate the suppression in the baryon asymmetry due to the washout effects. A more reliable result is obtained if one uses the empirical fit [8]

\[
    \log_{10} d = \log_{10}(1 - \xi) + \min\{d_1, d_2, d_3\},
\]

\[
    d_1 = 0.8 \log_{10}\kappa - 0.7 + 0.05 \log_{10} M_{10},
\]

\[
    d_2 = -1.2 - 0.05 \log_{10} M_{10},
\]

\[
    d_3 = -(3.8 + \log_{10} M_{10})(\log_{10} \kappa - 1) - (5.4 - 0.67 \log_{10}(M_1/\text{GeV}))^2 - 1.5,
\]

(31)

where \( \xi \) is defined in Eq. (26), \( M_{10} \equiv M_1/10^{10} \) GeV and the function \( \min \) evaluates to the smallest of the quantities \( d_i \). This fit reproduces considerably better the exact solution [6] of the Boltzmann equations for a wider range of \( \kappa \) and \( M_1 \). We notice however that for \( M_1 < 10^8 \) GeV and \( \kappa \gg 1 \) the approximation (30) tends to give asymptotically a better result than the empirical fit in Eq. (31) [27].

We shall therefore use a combined fit in our analysis in order to estimate the washout effects. If \( M_1 \gtrsim 10^8 \) GeV or \( M_1 < 10^8 \) GeV with \( \kappa \ll 1 \), we calculate

\[^4\text{Another variable commonly used in the literature [6,22,8] is the mass parameter } \tilde{m}_1 = (M_D^\dagger M_D)_{11}/M_1 \simeq 1.1 \times 10^{-3}\kappa \text{ eV.} \]
d using Eqs. (31). On the other hand, if \( M_1 < 10^8 \text{ GeV} \) and \( \kappa \gtrsim 1 \) we make use of the approximation (30). This will allow us to obtain a simple and reliable result for the solution of the Boltzmann equations without resorting to the full numerical solution of these equations.

3.1 Analytic approach

In this section we present a simple analytic approach that will allow us to obtain upper bounds for the baryon asymmetry in the present framework for the different solar neutrino solutions and assuming different patterns for the light neutrino spectrum. We shall divide our analysis in two parts. First we consider the case of large solar mixing, which includes four solar solutions: LMA, LOW, VO and Just-So\(^2\) solar solutions. Secondly, we discuss the case of small solar mixing, i.e. the SMA solar solution. We shall consider for each case two possible patterns for the light neutrino spectrum: hierarchical, i.e. \( m_1 \ll m_2 \ll m_3 \) and inverted-hierarchical, \( m_1 \sim m_2 \gg m_3 \), with the mass squared differences corresponding to the observed hierarchies \( \Delta m^2_\odot \gg \Delta m^2_\odot \). The analysis of another possible pattern, namely the case when the light neutrino masses are almost degenerate, \( m_1 \sim m_2 \sim m_3 \), is more subtle and crucially depends on the inclusion of the \( CP \)-violating phases in the leptonic mixing matrix. Therefore we shall not discuss it here. However, it turns out from our full numerical study of Section 3.2 that in the latter case the produced baryon asymmetry is highly suppressed for all the solar solutions.

The PMNS mixing matrix \( U_\nu \) can be parametrized in the standard form [12]

\[
U_\nu = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \cdot P,
\]

(32)

where \( c_{ij} \equiv \cos \theta_{ij} \), \( s_{ij} \equiv \sin \theta_{ij} \) and \( P = \text{diag} (1, e^{i\alpha}, e^{i\beta}) \); \( \delta \) is a Dirac-type phase (analogous to that of the quark sector) and \( \alpha, \beta \) are two physical phases associated with the Majorana character of neutrinos.

We shall assume for the Dirac neutrino mass spectrum the SO(10)-motivated hierarchy given in Eq. (17), i.e. \( M_D \sim M_u \). Moreover, since for the up-quark masses at GUT scale the hierarchy \( m_u : m_c : m_t \sim \epsilon^2 : \epsilon : 1 \) is verified (\( \epsilon \simeq 3 \times 10^{-3} \)), we shall write

\[
d_D = m_t \text{ diag}(\epsilon^2, \epsilon, 1).
\]

(33)

Finally, in order to simplify our analytical discussion, we neglect the possible
misalignment between the charged-lepton and Dirac neutrino mass matrices. In other words, we assume $V_L \simeq \mathbb{1}$ in Eqs. (16) and (19). As discussed in Section 2 such an approximation is reasonable in the context of SO(10), where one expects $V_L$ to be of the order of the CKM matrix. A more refined study which includes this effect will be given in the next section when we present our full numerical discussion.

Eqs. (32) and (33) together with the light neutrino mass spectrum are therefore the only input parameters necessary in our further analysis.

Case I: Large mixing (LMA, LOW, VO, Just-So$^2$)

We consider maximal mixing in the 2-3 sector of the leptonic mixing matrix, i.e. $\theta_{23} = \pi/4$, but keep the solar angle $\theta_{12}$ as a free parameter in order to account for deviations from maximal mixing in the 1-2 sector. Since our goal is to obtain an upper bound for the baryon asymmetry, for the moment we can neglect any $CP$-violating phase, i.e. assume $\delta = \alpha = \beta = 0$, and maximize the asymmetry by simply replacing the imaginary parts in the lepton asymmetry (21) with their corresponding absolute values. Finally, since the mixing angle $\theta_{13}$ is constrained by reactor neutrino experiments to be small, $U_{e3} \equiv |\sin \theta_{13}| \lesssim 0.2$ [28], the PMNS mixing matrix (32) can be approximately written in the form

$$U_\nu = \begin{pmatrix}
  c_\odot & s_\odot & U_{e3} \\
  -\frac{1}{\sqrt{2}}(s_\odot + c_\odot U_{e3}) & \frac{1}{\sqrt{2}}(c_\odot - s_\odot U_{e3}) & \frac{1}{\sqrt{2}} \\
  \frac{1}{\sqrt{2}}(s_\odot - c_\odot U_{e3}) & -\frac{1}{\sqrt{2}}(c_\odot + s_\odot U_{e3}) & \frac{1}{\sqrt{2}}
\end{pmatrix}, \quad (34)$$

where $s_\odot \equiv \sin \theta_{12}$, $c_\odot \equiv \cos \theta_{12}$. The effective neutrino mass matrix given in terms of the light neutrino masses is easily obtained from Eq. (18).

Now we introduce the symmetric matrix

$$M' = \frac{m_1^2 \epsilon^4}{\Delta} d_D^{-1} M_\nu d_D^{-1}, \quad \Delta = m_1 c_\odot^2 + m_2 s_\odot^2 + m_3 U_{e3}^2, \quad (35)$$

which can be expressed as

$$M' = \begin{pmatrix}
  1 & p \epsilon & r \epsilon^2 \\
  p \epsilon & q \epsilon^2 & s \epsilon^3 \\
  r \epsilon^2 & s \epsilon^3 & t \epsilon^4
\end{pmatrix}, \quad (36)$$

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with the coefficients
\[ p = \frac{1}{\sqrt{2\Delta}} \left[ -m_1(s_\odot + c_\odot U_{e3}) + m_2 s_\odot (c_\odot - s_\odot U_{e3}) + m_3 U_{e3} \right], \]
\[ q = \frac{1}{2\Delta} \left[ m_1(s_\odot + c_\odot U_{e3})^2 + m_2 (c_\odot - s_\odot U_{e3})^2 + m_3 \right], \]
\[ r = \frac{1}{\sqrt{2\Delta}} \left[ m_1 c_\odot (s_\odot - c_\odot U_{e3}) - m_2 s_\odot (c_\odot + s_\odot U_{e3}) + m_3 U_{e3} \right], \]
\[ s = \frac{1}{2\Delta} \left[ m_1 (c_{e3}^2 U_{e3}^2 - s_{e3}^2) - m_2 (c_{e3}^2 - s_{e3}^2 U_{e3}^2) + m_3 \right], \]
\[ t = \frac{1}{2\Delta} \left[ m_1 (s_\odot - c_\odot U_{e3})^2 + m_2 (c_\odot + s_\odot U_{e3})^2 + m_3 \right]. \quad (37) \]

The eigenvalues of \( M' \) are easily obtained in leading order of \( \epsilon \). We find
\[ \lambda_1 \simeq 1 + p^2 \epsilon^2, \quad \lambda_2 \simeq (q - p^2) \epsilon^2, \quad \lambda_3 \simeq \frac{s(2pr - s) + q(t - r^2) - p^2 t}{q - p^2} \epsilon^4. \quad (38) \]

According to Eqs. (19) and (35), these eigenvalues are nothing but the inverse of the right-handed Majorana masses with the proper normalization coefficient. Thus from Eqs. (35)-(38) we obtain
\[ M_i = \frac{m_i^2 \epsilon^4}{\Delta} \lambda_i^{-1}, \quad i = 1, 2, 3. \quad (39) \]

Next, we can find the unitary matrix \( U_R \) which diagonalizes \( M' \) and determines the structure of the Dirac neutrino mass matrix through the relations (16) and (19), i.e.
\[ M_D = d_D U_R. \quad (40) \]

In leading order of \( \epsilon \) we obtain
\[ U_R \simeq \begin{pmatrix} 1 - \frac{p^2}{2} \epsilon^2 & p \epsilon & \frac{ps - qr}{q - p'} \epsilon^2 \\ p \epsilon & -1 + \frac{p^2}{2} \epsilon^2 + \frac{(pr - s)^2}{2(q - p')} \epsilon^2 & \frac{ps - qr}{q - p'} \epsilon \\ r \epsilon^2 & \frac{ps - qr}{q - p'} \epsilon & 1 - \frac{(pr - s)^2}{2(q - p')} \epsilon^2 \end{pmatrix}. \quad (41) \]

It is now straightforward to estimate the baryon asymmetry. From Eqs. (23), (24), (33) and (39)-(41) we find
\[ Y_B = \frac{3}{32\pi g_*} \frac{d}{d} \frac{p^2(q - p^2 - 2r^2) + 2pr s + r^2(t - r^2) m_i^2}{1 + p^2 + r^2} \frac{\epsilon^4}{v^2}. \quad (42) \]
We can also write down the parameter $\kappa$ which together with the mass $M_1$ controls the washout effects. Using Eq. (29) we find

$$\kappa = \frac{M_P \Delta (1 + p^2 + r^2)}{1.7 \times 8\pi v^2 \sqrt{g_*}}.$$  \hspace{1cm} (43)

As mentioned before we are particularly interested in two limiting cases, namely, when the light neutrino spectrum is hierarchical or inverted hierarchical. Below we shall consider each one of these situations.

(i) Hierarchical spectrum

In this case $m_1 \ll m_2 \simeq \sqrt{\Delta m^2_\odot} \ll m_3 \simeq \sqrt{\Delta m^2_\odot}$ and the coefficients in the matrix (36) are approximately given by

$$p \simeq \frac{m_2 s_{2\odot} + 2m_3 U_{e3}}{2\sqrt{2}(m_2 s_{2\odot}^2 + m_3 U_{e3}^2)} , \quad q \simeq s \simeq t \simeq \frac{m_3}{2(m_2 s_{2\odot}^2 + m_3 U_{e3}^2)} ,$$

$$r \simeq -\frac{m_2 s_{2\odot} + 2m_3 U_{e3}}{2\sqrt{2}(m_2 s_{2\odot}^2 + m_3 U_{e3}^2)} ,$$  \hspace{1cm} (44)

with $s_{2\odot} \equiv \sin 2\theta_{12}$.

The right-handed Majorana neutrino masses read as

$$M_1 \simeq \frac{m_1^2 \epsilon^4}{m_2 s_{2\odot}^2 + m_3 U_{e3}^2} \simeq \frac{m_1^2}{s_{\odot}^2 \sqrt{\Delta m^2_{\odot}} + \sqrt{\Delta m^2_{a}} U_{e3}^2} ,$$

$$M_2 \simeq \frac{2m_1^2 \epsilon^2}{m_2 m_3 s_{2\odot}^2} \left( m_2 s_{2\odot}^2 + m_3 U_{e3}^2 \right) \simeq \frac{2m_1^2 \left( s_{2\odot}^2 \sqrt{\Delta m^2_{\odot}} + \sqrt{\Delta m^2_{a}} U_{e3}^2 \right)}{s_{2\odot}^2 \sqrt{\Delta m^2_{\odot}} \Delta m^2_{a}} ,$$

$$M_3 \simeq \frac{m_1^2 s_{2\odot}^2}{2m_1} .$$  \hspace{1cm} (45)

We notice that the requirement $M_3 \lesssim M_P$ implies a lower bound on the lightest neutrino mass:

$$m_1 \gtrsim \frac{m_1^2 s_{2\odot}^2}{2M_P} \simeq 2 \times 10^{-7} \text{ eV} ,$$  \hspace{1cm} (46)

for a typical value of $m_t \simeq 100 \text{ GeV}$ at GUT scale and a large solar mixing angle $\theta_{12} \simeq \pi/4$. Obviously, deviations from maximal solar mixing will slightly modify this bound.

We can also set an upper bound for the lightest right-handed Majorana mass
Indeed, from the first equation in (45) and taking $U_{e3} = 0$ we find

$$M_1 \lesssim \frac{m_u^2}{s_\odot^2 \sqrt{\Delta m_{\odot}^2}}.$$  \hspace{1cm} (47)

With $s_\odot$ and $\Delta m_{\odot}^2$ given by the lower bounds in the case of the Just-So$^2$ solar (cf. Table 1) we find $M_1 \lesssim 1.3 \times 10^9$ GeV. We notice that this value is consistent with the requirement $M_1 < T_{RH}$, where $T_{RH}$ is the reheating temperature after inflation, which in turn is constrained to be below $10^8 - 10^{10}$ GeV from considerations of the gravitino problem [22].

Moreover, for the mass ratios $M_1/M_i$, $i = 2, 3$ we have

$$\frac{M_1}{M_2} \simeq \frac{m_u^2 r_\odot s_\odot^2}{2m_e^2(r_\odot s_\odot^2 + U_{e3}^2)^2}, \quad \frac{M_1}{M_3} \simeq \frac{2m_e^2 m_1}{m_i^2 s_\odot^2 \sqrt{\Delta m_a^2(r_\odot s_\odot^2 + U_{e3}^2)}} \ll \frac{M_1}{M_2},$$  \hspace{1cm} (48)

where

$$r_\odot \equiv \left(\frac{\Delta m_{\odot}^2}{\Delta m_a^2}\right)^{1/2}.$$  \hspace{1cm} (49)

Let us also write down the approximate expression for the baryon asymmetry:

$$Y_B \simeq \frac{3}{32\pi} \frac{d}{g_*} \frac{m_2 m_3^3 s_\odot^2 U_{e3}^2}{(m_2 s_\odot^2 + m_3 U_{e3})^2} \frac{m_i^2}{v^2} \epsilon^4,$$  \hspace{1cm} (50)

i.e.,

$$Y_B \simeq \frac{3}{32\pi} \frac{d}{g_*} \frac{m_u^2 r_\odot s_\odot^2 U_{e3}^2}{v^2 (r_\odot s_\odot^2 + U_{e3}^2)^2 (r_\odot^2 s_\odot^2 + U_{e3}^2)}.$$  \hspace{1cm} (51)

As a function of $U_{e3}$, $Y_B$ reaches its maximum value when

$$U_{e3}^2 = \frac{s_\odot^2 r_\odot^3}{\sqrt{2}}.$$  \hspace{1cm} (52)

Substituting this value into Eq. (51) we obtain the upper bound

$$Y_B \lesssim \frac{3}{32\pi} \frac{d}{g_*} \frac{m_u^2}{v^2} \frac{1}{r_\odot s_\odot^2} \simeq 9.2 \times 10^{-15} \frac{d}{r_\odot s_\odot^2}.$$  \hspace{1cm} (53)

It is then clear that the larger the mass squared difference $\Delta m_{\odot}^2$ is, the higher is the suppression in the $CP$ asymmetry $\epsilon_N$. Thus, while the Just-So$^2$ vacuum oscillation solution is the most favoured in this framework, the LMA solar solution turns out to be highly disfavoured.
### Table 1

Constraints on neutrino masses and mixing angles coming from global analyses of the solar, atmospheric and reactor neutrino data [29,30]. The numbers in square brackets correspond to the best-fit values.

#### Atmospheric and reactor neutrinos

| \( \Delta m^2 \) (eV\(^2\))  | \( \tan^2 \theta_a \)   | \( U_{e3} \)    |
|-------------------------------|--------------------------|----------------|
| \((1.4 - 6.1) \times 10^{-3}\) | \((0.4 - 3.1) [1.4]\)    | <0.2           |

#### Solar neutrinos

| \( \Delta m^2 \) (eV\(^2\))  | \( \tan^2 \theta_\odot \)   | \( r_\odot = (\Delta m^2_\odot/\Delta m^2_a)^{1/2} \)          |
|-------------------------------|--------------------------|---------------------------------------------------------------|
| LMA                           | \((0.3 - 1) [0.4]\)       | \((0.6 - 4.6) [1.2]\) \times 10^{-1}                        |
| SMA                           | \((2 - 8) [4]\) \times 10^{-4} | \((2.6 - 8.4) [3.9]\) \times 10^{-2}                      |
| LOW                           | \((0.5 - 1.1) [0.7]\)    | \((0.2 - 1.2) [0.6]\) \times 10^{-2}                      |
| VO                            | \((1.5 - 4) [2.4]\)      | \((2.6 - 6.5) [3.8]\) \times 10^{-4}                      |
| Just-So\(^2\)                | \((0.5 - 2) [0.7]\)      | \((2.9 - 7.6) [4.2]\) \times 10^{-5}                      |

In Table 2 we give the upper bounds for the different solar solutions obtained using the present constraints on neutrino masses and mixing angles coming from global analyses of the solar, atmospheric and reactor neutrino data (cf. Table 1). We see that even if one neglects the washout effects (i.e. if one assumes \( d = 1 \)), the LOW and LMA solutions have an upper bound which is below the lower bound of the observed baryon asymmetry. On the other hand, the VO and Just-So\(^2\) solar solutions have an upper bound which lies inside the allowed range for the asymmetry.

A more realistic bound can be obtained by including the dilution effects. From Eq. (43) we find

\[
\kappa = \frac{1}{1.7 \times 8\pi v^2 \sqrt{g_*} m_2 s_\odot^2 + m_3 U_{e3}^2} \left[ \Delta m^2_\odot + \Delta m^2_a U_{e3}^2 \right] \frac{9.1 \times 10^2}{1 \text{eV}} s_\odot^2 \Delta m^2_\odot + \Delta m^2_a U_{e3}^2 \frac{1}{s_\odot \sqrt{\Delta m^2_\odot + \Delta m^2_a U_{e3}}^{1/2}}.
\]

Here we can distinguish two different regimes for the parameter \( \kappa \) depending...
Table 2
Upper bounds for the clean \((d = 1)\) and net \((d \neq 1)\) baryon asymmetries in the case of hierarchical neutrinos. The bounds are obtained from Eq. (53), using the solar, atmospheric and reactor neutrino data of Table 1.

| Solution | LMA | LOW | VO | Just-So\(^2\) | SMA |
|----------|-----|-----|----|--------------|-----|
| \(Y_B\) (clean) | 3.1 \times 10^{-13} | 8.7 \times 10^{-12} | 4.3 \times 10^{-11} | 3.2 \times 10^{-10} | 3.3 \times 10^{-10} |
| \(Y_B\) (net) | 1.4 \times 10^{-14} | 5.1 \times 10^{-13} | 2.5 \times 10^{-12} | 2.0 \times 10^{-11} | 1.7 \times 10^{-11} |

on the value of \(U_{e3}\):

\[
\kappa \simeq 9.1 \times 10^2 \frac{\Delta m^2_{\odot}}{1 \text{ eV}} \quad \text{for} \quad U_{e3}^2 \ll r_{\odot}^2 s_{\odot}^2 \ll r_{\odot}^2 s_{\odot}^2 ,
\]

\[
\kappa \simeq 9.1 \times 10^2 \sqrt{\frac{\Delta m^2_a}{\Delta m^2_{\odot}}} \quad \text{for} \quad U_{e3}^2 \gg r_{\odot}^2 s_{\odot}^2 \gg r_{\odot}^2 s_{\odot}^2 . \tag{55}
\]

Thus, for large values of \(U_{e3}\) the parameter \(\kappa\) is controlled by the mass squared difference of the atmospheric neutrinos and for all the large mixing solar solutions it lies in the range \(35 \lesssim \kappa \lesssim 70\). However, to establish an upper bound on the baryon asymmetry we use the value of \(U_{e3}\) in Eq. (52) which maximizes \(Y_B\). Substituting it into Eq. (54) we have

\[
\kappa \simeq 6.4 \times 10^2 \frac{(\Delta m^2_a \Delta m^2_{\odot})^{1/4}}{\text{eV}} , \tag{56}
\]

and according to Eq. (45), the mass of the lightest right-handed Majorana neutrino is approximately given by

\[
M_1 \simeq \frac{m^2_{u}}{s_{\odot}^2 \sqrt{\Delta m^2_{\odot}}} . \tag{57}
\]

Using these expressions to calculate the dilution factor \(d\) we obtain the upper bounds given in Table 2. These bounds are of course more stringent than the ones previously obtained neglecting the washout effects.

In Fig. 1 we plot the asymmetry \(Y_B\) as a function of \(U_{e3}\) for the mass of the lightest neutrino \(m_1 = 10^{-6} \text{ eV}\). Fig. 1a corresponds to the clean asymmetry, i.e. neglecting the dilution effects, while in Fig. 1b the net baryon asymmetry (after including the washout effects) is plotted. The parameter \(\kappa\) as defined by Eq. (29) is given in Fig. 1c. We notice that for very small values of \(U_{e3}\) this parameter is proportional to the mass squared difference of the solar neutrinos, \(\kappa \propto (\Delta m^2_{\odot})^{1/2}\), and as \(U_{e3}\) increases, it tends to a common value
Fig. 1. The baryon asymmetry $Y_B$ as a function of $U_{e3}$ for the different solar solutions. Fig. (a) corresponds to the clean asymmetry, while in Fig. (b) the net baryon asymmetry, which includes the washout effects, is given. The parameter $\kappa$ defined in Eq. (29) and the mass of the lightest right-handed Majorana neutrino $M_1$ are plotted in Figs. (c) and (d), respectively. The curves are given for the mass of the lightest neutrino $m_1 = 10^{-6}$ eV. The values for the neutrino oscillation parameters are taken from Table 1.

for all the solar solutions, which is determined by the mass squared difference of the atmospheric neutrinos, i.e. $\kappa \propto (\Delta m_{\odot}^2)^{1/2}$ (see Eqs. (55)). A similar behaviour is observed for the lightest Majorana neutrino mass $M_1$ as can be seen from Fig. 1d. In the latter case, $M_1 \propto (\Delta m_{\odot}^2)^{-1/2}$ for small values of $U_{e3}$ and $M_1 \propto (\Delta m_{a}^2)^{-1/2}$ for large values of $U_{e3}$ (see also Eqs. (45)).

At this point one may wonder whether the inclusion of a misalignment between the charged-lepton and Dirac neutrino mass matrices could change our conclusions. Since in SO(10)-motivated scenarios such a misalignment is expected to be proportional to the CKM quark mixing matrix, let us then assume for
the unitary matrix $V_L$ in Eq. (9) the following CKM-type real matrix

$$
V_L \simeq \begin{pmatrix}
1 - \lambda^2/2 & \lambda & 0 \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
0 & -A\lambda^2 & 1
\end{pmatrix}, \quad (58)
$$

with $A \simeq 1$ and $\lambda$ of the order of the Cabibbo angle, i.e. $\lambda \lesssim 0.2$.

Following the same analytic approach we can obtain upper bounds for the baryon asymmetry. Since the relevant formulae are more cumbersome in this case, it is more illustrative to present the results in a simple plot, similar to that of Fig. 1. The results are presented in Fig. 2. From this figure it is seen that the upper bounds previously found for $Y_B$ are essentially unaltered and that our previous conclusions remain valid in this case. To maximize the effect we have taken the Dirac phase $\delta$ in the PMNS neutrino mixing matrix to be $\pi$, as suggested from our full numerical study (see next section). The resonance behaviour in the curves at $U_{e3} \simeq 0.1$ is associated with our particular choice for the values of $\delta$ and $\lambda$.

(ii) Inverted-hierarchical spectrum
In this case \( m_3 \ll m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{\odot}} \) and the coefficients in the matrix (36) are approximately given by the expressions:

\[
\begin{align*}
p &\simeq \frac{(m_2 - m_1)s_{2\odot}}{2\sqrt{2}(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)} - \frac{U_{e3}}{\sqrt{2}}, \\
q &\simeq \frac{m_1 s_{\odot}^2 + m_2 c_{\odot}^2}{2(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)} - \frac{(m_2 - m_1)s_{2\odot}U_{e3}}{4(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)}, \\
r &\simeq -\frac{m_1 s_{\odot}^2 + m_2 c_{\odot}^2}{2(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)} - \frac{U_{e3}}{\sqrt{2}}, \\
s &\simeq -\frac{m_1 s_{\odot}^2 + m_2 c_{\odot}^2}{2(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)} + \frac{1}{2}U_{e3}^2, \\
t &\simeq \frac{m_1 s_{\odot}^2 + m_2 c_{\odot}^2}{2(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)} + \frac{(m_2 - m_1)s_{2\odot}U_{e3}}{4(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)},
\end{align*}
\]

with

\[
\frac{m_2 - m_1}{m_1 c_{\odot}^2 + m_2 s_{\odot}^2} \simeq \frac{1}{2} \frac{\Delta m^2_{\odot}}{\Delta m^2_{\odot}} = \frac{v^2_{\odot}}{2}.
\]

The right-handed Majorana neutrino masses are given in this limit by

\[
\begin{align*}
M_1 &\simeq \frac{m_1^2 e^4}{m_1 c_{\odot}^2 + m_2 s_{\odot}^2} \simeq \frac{m_a^2}{\sqrt{\Delta m^2_{\odot}}}, \\
M_2 &\simeq 2m_1^2 e^2 \left( \frac{c_{\odot}^2}{m_2} + \frac{s_{\odot}^2}{m_1} \right) \simeq \frac{2m_a^2}{\sqrt{\Delta m^2_{\odot}}}, \\
M_3 &\simeq \frac{m_1^2}{4m_3} \frac{m_1 + m_2}{m_1 c_{\odot}^2 + m_2 s_{\odot}^2} \simeq \frac{m_i^2}{2m_3}.
\end{align*}
\]

Once again requiring \( M_3 \ll M_P \) and taking \( m_i \simeq 100 \text{ GeV} \) at GUT scale, we find the following lower bound on the lightest neutrino \( m_3 \),

\[
m_3 \gtrsim \frac{m_i^2}{2M_P} \simeq 4 \times 10^{-7} \text{ eV}.
\]

The generated baryon asymmetry can be approximated in this case by the following expression

\[
Y_B \simeq \frac{3}{128\pi g_*} \frac{d}{m^2_{\odot} s_{2\odot}^2} \frac{m_1 m_2 (m_1 - m_2)^2 s_{2\odot}^2 + 2m_3 (m_1^3 + m_2^3) U_{e3}^2}{(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)(m_1 c_{\odot}^2 + m_2 s_{\odot}^2)^2} \frac{m_1^2 e^4}{v^2},
\]

or equivalently,

\[
Y_B \simeq \frac{3}{512\pi g_* v^2} \frac{d}{m^2_{\odot}} \left[ \frac{2}{s_{2\odot}^2} r_{\odot}^4 + \frac{16m_3 U_{e3}^2}{\sqrt{\Delta m^2_{\odot}}} \right].
\]
Taking for instance $m_2 \approx \sqrt{\Delta m_{32}^2} \ll m_{1,2} \approx \sqrt{\Delta m_{21}^2}$ and the maximum value of $U_{e3} \simeq 0.2$ it is easy to see that the dominant contribution to the asymmetry comes from the second term in the square brackets. Neglecting the washout effects ($d = 1$) we find the following upper bound:

$$Y_B \lesssim \frac{3}{32\pi} \frac{d}{g_* v^2} U_{e3} r_\odot \simeq 3.7 \times 10^{-16} r_\odot . \quad (65)$$

Thus we conclude that the baryon asymmetry is highly suppressed for all the large mixing solar solutions and for an inverted hierarchical neutrino spectrum, even without taking into account the washout effects. It is however worthwhile to have an idea of these effects. The parameter $\kappa$ reads as

$$\kappa = \frac{M_P}{1.7 \times 16\pi v^2 \sqrt{g_*}} \frac{m_1^2 + m_2^2}{m_1 e_\odot^2 + m_2 s_\odot^2} \simeq \frac{M_P \sqrt{\Delta m_{21}^2}}{1.7 \times 8\pi v^2 \sqrt{g_*}} . \quad (66)$$

Therefore,

$$\kappa \simeq 9.1 \times 10^2 \left[ \frac{\sqrt{\Delta m_{21}^2}}{\text{eV}} \right] \simeq 50 . \quad (67)$$

Since from Eq. (61) we have $M_1 \simeq 1.8 \times 10^4$ GeV, then from Eqs.(31) we find $d \simeq 10^{-2}$.

**Case II: Small mixing (SMA)**

Let us now consider the small mixing solar solution. Most of the relevant formulas can be easily obtained from the previously discussed large mixing case by just letting $c_\odot \to 1$ and assuming $s_\odot \ll 1$. We must however proceed with care since in this case there are two small parameters competing against each other, namely, the small solar mixing angle $\theta_{12}$ and $U_{e3}$. In this case the leptonic mixing matrix $U_\nu$ can be approximated as

$$U_\nu = \begin{pmatrix}
1 & s_\odot & U_{e3} \\
-\frac{1}{\sqrt{2}} (s_\odot + U_{e3}) & \frac{1}{\sqrt{2}} (1 - s_\odot U_{e3}) & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} (s_\odot - U_{e3}) & -\frac{1}{\sqrt{2}} (1 + s_\odot U_{e3}) & \frac{1}{\sqrt{2}}
\end{pmatrix} . \quad (68)$$

Next we follow the same steps performed in the large mixing case. We first
define the new matrix
\[ M' = \frac{m_t^2 \epsilon^4}{\Delta} d_D^{-1} M_d d_D^{-1}, \quad \Delta = m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2, \quad (69) \]
which can be written in the form (36) with the coefficients \( p, q, r, s, t \) as given in (37). The eigenvalues and eigenvectors of \( M' \) are given by the same expressions in Eqs. (38) and (41), respectively. The right-handed Majorana masses read as in Eq. (39). Finally, the baryon asymmetry will be given by Eq. (42) with the parameter \( \kappa \) defined in Eq. (43).

Let us now consider the limiting cases of hierarchical and inverted-hierarchical neutrino mass spectrum.

(i) Hierarchical spectrum

In the limit \( m_1 \ll m_2 \ll m_3 \) we find
\[
p \simeq \frac{m_2 s_\odot + m_3 U_{e3}}{\sqrt{2}(m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2)}, \quad q \simeq s \simeq t \simeq \frac{m_3}{2(m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2)},
\]

\[
r \simeq \frac{m_2 s_\odot}{\sqrt{2}(m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2)}. \quad (70)
\]
Moreover,
\[
M_1 \simeq \frac{m_t^2 \epsilon^4}{m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2} \simeq \frac{m_u^2}{m_1 + \sqrt{\Delta m_\odot^2 s_\odot^2 + \Delta m_\odot^2 U_{e3}^2}},
\]
\[
M_2 \simeq \frac{2m_t^2 \epsilon^2}{m_3} \frac{m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2}{m_1 + m_2 (s_\odot - U_{e3})^2} \simeq \frac{2m_c^2}{m_1 + \sqrt{\Delta m_\odot^2 s_\odot^2 + \Delta m_\odot^2 U_{e3}^2}} \frac{m_1 + \sqrt{\Delta m_\odot^2 s_\odot^2 + \Delta m_\odot^2 U_{e3}^2}}{m_1 + \sqrt{\Delta m_\odot^2 (s_\odot - U_{e3})^2}},
\]
\[
M_3 \simeq \frac{m_t^2}{2} \left( \frac{1}{m_2} + \frac{(s_\odot - U_{e3})^2}{m_1} \right) \simeq \frac{m_t^2}{2} \left( \frac{1}{\sqrt{\Delta m_\odot^2}} + \frac{(s_\odot - U_{e3})^2}{m_1} \right). \quad (71)
\]

From the requirement that \( M_3 \) be smaller than the Planck mass we find the following lower bound for the mass of the lightest neutrino:
\[
m_1 \gtrsim \frac{m_t^2 \sqrt{\Delta m_\odot^2 s_\odot (s_\odot - U_{e3})^2}}{2 M_P \sqrt{\Delta m_\odot^2 - m_t^2}} \simeq 4.1 \times 10^{-7} (s_\odot - U_{e3})^2 \text{ eV}. \quad (72)
\]

Using for instance the minimum value \( s_\odot = 0.014 \) and taking \( U_{e3} = 0 \) one has \( m_1 \gtrsim 8 \times 10^{-11} \text{ eV} \). The above bound is of course sensitive to the value of \( U_{e3} \) and it gets weaker as \( U_{e3} \) approaches \( s_\odot \).
Let us now estimate the baryon asymmetry. From Eqs. (42) and (70) we find

\[
Y_B \simeq \frac{3}{32 \pi g_* v^2} \frac{d \, m_1^2}{m_1^4 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2} \frac{m_1^3 (m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2)^2 (m_1^2 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2)}{(m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2)^2 (m_1^2 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2)} \epsilon^4
\]

\[
\simeq \frac{3}{32 \pi g_* v^2} \frac{d \, m_1^2}{m_1^4 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2} \frac{(\Delta m_a^2)^{3/2} (m_1 + s_\odot^2 \sqrt{\Delta m_a^2})^2 (m_1^2 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2)}{(m_1 + m_2 s_\odot^2 + m_3 U_{e3}^2)^2 (m_1^2 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2)}.
\]

(73)

We can also compute the parameter \( \kappa \) relevant for the dilution effects. From the definition (43) we find

\[
\kappa = \frac{M_P}{1.7 \times 8 \pi v^2 \sqrt{g_*}} \frac{m_1^4 + m_2^2 s_\odot^2 + m_3^2 U_{e3}^2}{m_1^4 + m_2 s_\odot^2 + m_3 U_{e3}^2}. \tag{74}
\]

As a function of \( U_{e3} \), the expression (73) reaches its maximum when

\[
U_{e3}^2 = \frac{(m_1 + s_\odot^2 \sqrt{\Delta m_a^2})^{1/2} (m_1^2 + m_2^2 s_\odot^2)^{1/2}}{\sqrt{2} (\Delta m_a^2)^{3/4}}. \tag{75}
\]

For \( m_1 \gg s_\odot \sqrt{\Delta m_a^2} \gg s_\odot^2 \sqrt{\Delta m_a^2} \) we have

\[
U_{e3}^2 \simeq \frac{1}{\sqrt{2}} \left( \frac{m_1}{\sqrt{\Delta m_a^2}} \right)^{3/2}, \tag{76}
\]

and therefore

\[
Y_B \lesssim \frac{3}{32 \pi g_* v^2} \frac{d \, m_1^2 \sqrt{\Delta m_a^2}}{m_1} \simeq 7.2 \times 10^{-16} \frac{d \, [eV]}{m_1}. \tag{77}
\]

For the parameter \( \kappa \) we have in turn

\[
\kappa \simeq \frac{6.4 \times 10^2}{1 \, \text{eV}} m_1^{1/2} (\Delta m_a^2)^{1/4} \simeq 1.8 \times 10^2 \sqrt{\frac{m_1}{\text{eV}}}. \tag{78}
\]

Finally, for the lightest right-handed Majorana mass we obtain

\[
M_1 \simeq m_1 \simeq 10^3 \text{GeV} \left[ \frac{\text{eV}}{m_1} \right]. \tag{79}
\]

It remains to estimate the dilution factor \( d \). Assuming e.g. \( m_1 = 10^{-4} \, \text{eV} \), so that the relation \( m_1 \gg s_\odot \sqrt{\Delta m_a^2} \) is satisfied, Eqs. (78) and (79) imply \( \kappa \simeq 1.8, M_1 = 10^7 \, \text{GeV} \), and therefore \( d \simeq 0.14 \). Thus, from Eq. (77) we obtain the upper bound

\[
Y_B \lesssim 10^{-12}, \tag{80}
\]
which is one order of magnitude below the required value for the observed asymmetry.

On the other hand, if \( m_1 \ll s_\odot^2 \sqrt{\Delta m_2^2} \ll s_\odot \sqrt{\Delta m_2^2} \) then the maximum value in Eq. (73) is obtained when

\[
U_{e3}^2 = \frac{s_\odot^2 r_\odot^{3/2}}{\sqrt{2}}.
\]  

(81)

In this case, the expression for the upper bound of the baryon asymmetry is the same as the one previously found for the large mixing solar solutions (see Eq. (53)):

\[
Y_B \lesssim \frac{3}{32\pi} \frac{d}{g_*} \frac{m_u^2}{v^2} \frac{1}{r_\odot s_\odot^2} \simeq 1.8 \times 10^{-9} d.
\]  

(82)

The dilution factor turns out to be crucial in obtaining a reliable estimate for \( Y_B \). In this limiting case, the parameter \( \kappa \) is also given by the same expression of the large mixing solutions, i.e. by Eq. (56). We find \( \kappa \simeq 8 \). For the lightest Majorana mass we have in turn,

\[
M_1 \simeq \frac{m_u^2}{s_\odot^2 \sqrt{\Delta m_2^2}} \simeq 2.5 \times 10^9 \text{ GeV},
\]  

(83)

and the mass ratios \( M_1/M_2 \) and \( M_1/M_3 \) are given by the approximate expressions:

\[
\frac{M_1}{M_2} \simeq \frac{1}{2} \left( \frac{m_u}{m_c} \right)^2 \frac{1}{r_\odot s_\odot^2}, \quad \frac{M_1}{M_3} \simeq 2 \left( \frac{m_u}{m_t} \right)^2 \frac{m_1}{s_\odot^4 \sqrt{\Delta m_2^2}}.
\]  

(84)

The above values for \( \kappa \) and \( M_1 \) imply then a dilution factor \( d \simeq 8 \times 10^{-3} \). Substituting this value into Eq. (82) we conclude that for the SMA solution

\[
Y_B \lesssim 1.4 \times 10^{-11},
\]  

(85)

a bound which is within the allowed range for the baryon asymmetry.

(\textbf{ii}) \textbf{Inverted-hierarchical spectrum}

Let us now consider an inverted-hierarchical neutrino spectrum, i.e. \( m_1 \simeq m_2 \simeq \sqrt{\Delta m_d^2} \gg m_3 \). In this case the coefficients of the matrix (36) can be
obtained from Eqs. (59) by setting \( c_\odot = 1 \) and assuming \( s_\odot \ll 1 \). We find
\[
p \simeq \frac{(m_2 - m_1)s_\odot}{\sqrt{2}m_1} - \frac{U_{e3}}{\sqrt{2}} \simeq \frac{s_\odot r_\odot^2}{2\sqrt{2}} - \frac{U_{e3}}{\sqrt{2}}, \quad q \simeq -s \simeq t \simeq \frac{m_2}{2m_1} \simeq \frac{1}{2},
\]
\[
r \simeq -\frac{(m_2 - m_1)s_\odot}{\sqrt{2}m_1} - \frac{U_{e3}}{\sqrt{2}} \simeq -\frac{s_\odot r_\odot^2}{2\sqrt{2}} - \frac{U_{e3}}{\sqrt{2}}.
\]

The right-handed Majorana neutrino masses will be given by the same expressions in Eqs. (61) and, in particular, the lower bound (62) for the lightest neutrino \( m_3 \) should be verified as well. Finally, the approximated expression for the baryon asymmetry is obtained from Eq. (64):
\[
Y_B \simeq \frac{3}{32\pi g_*} \frac{d}{v^2} \frac{m_3^2 U_{e3}^2}{\Delta m^2_a} \lesssim 3.7 \times 10^{-16} r_\odot \lesssim 3.1 \times 10^{-17},
\]
for \( d = 1 \), \( m_3 \simeq \sqrt{\Delta m^2_\odot} \) and \( U_{e3} = 0.2 \). Thus we conclude that for the SMA solution and an inverted-hierarchical neutrino spectrum the generated baryon asymmetry is highly suppressed.

### 3.2 Numerical Analysis

From the simple analysis performed in the last section we have concluded that in the SM framework with the simplest SO(10)-motivated hierarchy (17) for the Dirac neutrino mass spectrum, only the SMA and Just-So^2 solutions of the solar neutrino problem are compatible with the required value for the baryon asymmetry of the Universe. In this section we will perform a full numerical computation of the baryon asymmetry including not only all the \( CP \)-violating phases which appear in the PMNS leptonic mixing matrix \( U_\nu \) defined in Eq. (32), but also the possible misalignment which may result from the process of diagonalization of the charged-lepton and Dirac neutrino mass matrices. This misalignment is characterized by the matrix \( V_L \) in Eq. (16), for which we shall assume the CKM structure of Eq. (58).

We then proceed as follows. The PMNS mixing angles as well as the mass squared differences \( \Delta m^2_\odot \) and \( \Delta m^2_a \) are allowed to vary in the ranges indicated in Table 1. First we randomly fix a set of values for the PMNS mixing angles and the \( CP \)-violating phases \( \alpha \), \( \beta \) and \( \delta \). The latter are randomly chosen in the interval from 0 to 2\( \pi \). In order to compute the effective neutrino mass matrix we take as an input the mass of the lightest neutrino, \( m_1 \), which varies from \( 10^{-10} \) to 1 eV for the SMA solution and from \( 10^{-7} \) to 1 eV for all the large mixing solutions\(^5\). This means that unlike the analytic

\(^5\) The lower bounds for the mass of the lightest neutrino are taken in agreement
study, the full numerical analysis includes also the case of almost degenerate neutrinos. This is possible because we are taking into consideration all the phases that may be present and, consequently, possible cancellations. Assuming $m_1 < m_2 < m_3$, the values of $m_2$ and $m_3$ are determined from the expressions $m_2 = \sqrt{m_1^2 + \Delta m_{\odot}^2}$, $m_3 = \sqrt{m_1^2 + \Delta m_{\odot}^2 + \Delta m_{\odot}^2}$. The inverse hierarchical case is not discussed here since it leads to a very suppressed baryon asymmetry as already shown in the analytic discussion. The effective neutrino mass matrix $M_{\nu}$ in Eq. (18) is computed, and also the matrix $M$ in Eq. (19) with the up-quark masses given at the GUT scale: $m_u \simeq 1$ MeV, $m_c \simeq 0.3$ GeV and $m_t \simeq 100$ GeV [31] and the matrix $V_L$ in Eq.(58) with $\lambda$ randomly chosen in the interval $0 \leq \lambda \leq 0.22$. The matrix $M M^T$ is then diagonalized to obtain $U_R$ and the right-handed neutrino masses $M_i$. In the basis where $M_{\ell}$ and $M_R$ are diagonal the Dirac neutrino mass matrix $M_D$ is determined by Eq. (16). Finally we compute the baryon asymmetry from Eqs.(21), (22), (25) and (27), where the dilution factor $d$ is determined using a combined fit from the approximations given in Eqs. (30) and (31).

with our previous analytic estimates, so that the requirement $M_3 \lesssim M_P$ is always satisfied.
In Fig. (a), the baryon asymmetry as a function of $m_1$ in the case of the Just-So$^2$ solar solution. The black-dotted area corresponds to sets of input parameters which give an asymmetry within the allowed range (dashed lines) as given in Eq. (2). In Fig. (b), the right-handed Majorana neutrino masses are plotted as functions of $m_1$. From Fig. (c) we conclude that for relatively large values of $U_{e3}$ the Dirac phase $\delta$ should be close to $\pi$ to produce a baryon asymmetry in the allowed range. The allowed region in the plane $(\lambda, U_{e3})$ is shown in Fig. (d). The gray-dotted area is obtained by slightly relaxing the lower bound on the baryon asymmetry, $Y_B \gtrsim 10^{-11}$.

In Figs. 3 we present the results for the baryon asymmetry $Y_B$ as a function of $m_1$ for the LMA, SMA, LOW and VO solutions of the solar neutrino problem. The first immediate conclusion which can be drawn from these plots is that among these four solutions only the SMA solution is compatible with the experimental range for $Y_B$ with a mass for the lightest neutrino in the range from $2 \times 10^{-10}$ to $4 \times 10^{-6}$ eV. Moreover, the values of $U_{e3}$ should be small in this case, $|U_{e3}| \lesssim 10^{-2}$. By comparing the results plotted in Figs. 3 with the upper bounds for $Y_B$ obtained in the last section (see Table 2), we find the analytic bounds in very good agreement with the exact numerical ones.

Finally, in Figs. 4 we present the results for the only large mixing solar solution that produces the required $Y_B$, namely, the Just-So$^2$ vacuum oscillation
Fig. 5. The $CP$-violating Majorana phases $\alpha$ and $\beta$ in the cases of no Dirac-type $CP$ violation (Figs. (a) and (b)) and maximal Dirac-type $CP$ violation (Fig. (c)) for the Just-So$^2$ vacuum oscillation solar solution. The black-dotted area in the plots refers to the values which lead to an acceptable baryon asymmetry $Y_B$. 

The asymmetry as a function of the lightest neutrino mass $m_1$ is plotted in Fig. 4a. The black-dotted area corresponds to sets of input parameters which give an asymmetry within the allowed range delimited by the dashed lines (see also Eq. (2)). In Fig. 4b, the right-handed Majorana neutrino masses are plotted as functions of $m_1$. As before, the lower bound on $m_1$ is determined by requiring the mass of the heaviest Majorana neutrino $M_3$ to be below the Planck scale $M_P$. From Fig. 4c we conclude that for $|U_{e3}| \gtrsim 10^{-2}$ the Dirac phase $\delta$ should be close to $\pi$ to produce a baryon asymmetry in the allowed range. This in turn corresponds to larger values of the misalignment parameter $\lambda$ in the matrix (58) as can be seen from Fig. 4d.

From Figs. 3 and 4 it is also clear that an acceptable baryon asymmetry requires very small values of the mass of the lightest neutrino, $m_1 \ll 1$ eV. This in particular implies that, in the present framework, an almost degenerate neutrino spectrum is excluded.

To establish a relationship between $CP$ violation at high and low energies is not an easy task in the sense that a general analytic treatment with the full set of parameters would be difficult to perform. Nevertheless, one can investigate whether there is any correlation between the $CP$-violating phases $\alpha$, $\beta$ and $\delta$ in a plausible leptogenesis scenario. In Fig. 5 we present the results of a random analysis in the cases of no Dirac-type $CP$ violation ($\delta = 0, \pi$) and maximal Dirac-type $CP$ violation ($\delta = \pi/2$) for the Just-So$^2$ vacuum oscillation solar solution, which is the only large mixing solar solution compatible with leptogenesis in the simplest SO(10)-motivated framework considered here. The black-dotted area in the plots refers to the values of $\alpha$ and $\beta$ which lead to an acceptable baryon asymmetry $Y_B$. It is seen from Figs. 5a and 5b that one can obtain the right magnitude for $Y_B$ with a single nonvanishing Majorana phase.
This means that the possible absence of $CP$ violation effects in neutrino oscillations in future experiments may not $a$ $priori$ discard leptogenesis as the mechanism responsible for the generation of the BAU. In the case of maximal $CP$ violation in Fig. 5c, the phases $\alpha$ and $\beta$ are slightly less constrained than in the previous case.

We have seen that in the present framework and assuming a mass hierarchy for the Dirac neutrinos like the one of up-quarks (cf. Eq. (17)), it is not possible to reconcile the LMA, LOW and VO solar solutions with the leptogenesis scenario. We may then ask ourselves what type of hierarchies should the Dirac neutrino masses satisfy for these solutions to be realized and leptogenesis to be viable. To answer this question let us now relax our previous assumption (17) and write the most general form for the Dirac neutrino mass spectrum:

$$d_D = m_{D3} \text{diag}(\varepsilon_1, \varepsilon_2, 1),$$

with $\varepsilon_i \equiv m_{D_i}/m_{D3}$, $i = 1, 2$ and $0 < \varepsilon_1 \leq \varepsilon_2 \leq 1$.

For a given scale of the heaviest Dirac neutrino $m_{D3}$, we can vary the parameters $(\varepsilon_1, \varepsilon_2)$ in the above interval and look for the allowed regions where the cosmological baryon asymmetry is within the experimental range. To illustrate the results, we shall consider only the presently most favoured solar solution, i.e. the LMA solar solution. The LOW and VO solutions can of course be analyzed in a similar manner. The results are plotted in Fig. 6 for two typical scales, namely, $m_{D3}$ of the order of the bottom-quark mass $m_b \simeq 1$ GeV and around the top-quark mass $m_t \simeq 100$ GeV at GUT scale. It is then clear from the figure that one may obtain the correct baryon asymmetry, provided one uses a hierarchy for the eigenvalues of $M_D$ corresponding to $\varepsilon_i$ lying inside the allowed ranges indicated in Fig. 6. However, note that for $M_D \propto M_u$ or $M_D \propto M_d$ one is not able to have the LMA solar neutrino solution and yield the correct baryon asymmetry.

We emphasize that this analysis is quite general and does not rely on any specific texture for Dirac matrix $M_D$ or the Majorana matrix $M_R$, except for the fact that in the basis where the charged leptons and right-handed Majorana neutrinos are diagonal, we assume a CKM-type misalignment between $M_\ell$ and $M_D$.

4 Implications for low energy $CP$ violation and neutrinoless double beta decay

It is clear from the analysis presented in the previous sections that $CP$ violation is a crucial ingredient for the generation of the cosmological baryon asymmetry through leptogenesis. In our particular framework, the factor $M_D^\dagger M_D$
Fig. 6. Allowed region for the Dirac neutrino mass ratios $\varepsilon_i = m_{Di}/m_{D3}$ in the case of the LMA solar solution. We assume $m_1 = 10^{-3}$ eV and two different scales for the heaviest Dirac neutrino, $m_{D3} \sim m_b = 1$ GeV and $m_{D3} \sim m_t = 100$ GeV.

is of the form

$$M_D^\dagger M_D = (U_R^\dagger d_D V_L) (V_L^\dagger d_D U_R^* ) = U_R^\dagger d_D^2 U_R .$$

(89)

As a result, it does not depend directly on $V_L$, yet the presence of $V_L$ is felt in the determination of $U_R$ through Eqs. (19) and also of the heavy Majorana masses $M_i$ (i=1,2,3), which in turn appear in the lepton asymmetry $\epsilon_{N_i}$. In Eq. (89) with $U_R$ written as $P_\xi U_R' P_1$ (in the notation of Eq. (14)) it is obvious that the three phases in $P_\xi$ do not play any rôle for leptogenesis. Moreover, in the limit $V_L = \mathbb{1}$, we have

$$M_D = d_D U_R = P_\xi d_D U_R' P_1 ,$$

(90)

and in this case $P_\xi$ can be rotated away from $M_D$ so that only three physical $CP$-violating phases remain in $M_D$. In this case it is also possible to choose a WB where both $M_D$ and $M_\ell$ are diagonal and real, hence

$$M_R = U_R^* d_R U_R^\dagger ,$$

(91)

implying that all $CP$-violating effects could be generated at high energies with $CP$ softly broken.

One of the striking features of our numerical results is the fact that in the present framework leptogenesis favours a Dirac phase $\delta$ in the PMNS mixing matrix $U_\nu$, very close to $\pi$ for relatively large values of $|U_{e3}| \gtrsim 10^{-2}$. In the exact limit $\delta = \pi$ all $CP$-violating effects at low energies are due to the presence of the Majorana phases $\alpha$ and/or $\beta$. Nevertheless, the determination of the nature of the phases relevant to the lepton asymmetry is not trivial and would require a careful analysis of the matrix $R$ in Eq. (10). This is due to the fact that $U_R$ and $U_\nu$ are related in a complicated manner even in the limit $V_L = \mathbb{1}$ as can be seen from Eqs. (19). Another remarkable feature is
the fact that the Majorana phases $\alpha$ and $\beta$ can only give rise to the necessary baryon asymmetry in those regions of the parameter space represented by the bands in Figs. 5, where one can see that although all possible values of $\alpha$ and $\beta$ are allowed, they are not independent of each other. The type of pattern generated in the $(\alpha, \beta)$-plane remains very similar for different values of the Dirac phase $\delta$. It is also noticeable that leptogenesis is viable with a single nonzero $CP$-violating phase of Majorana character even for $\delta = 0, \pi$, whilst the Dirac phase alone is not sufficient.

The three phases $\delta, \alpha$ and $\beta$ contained in the mixing matrix $U_\nu$ (for 3 generations of light neutrinos) are precisely the ones related to $CP$-violating phenomena at low energies. It is well known that neutrino oscillations are only sensitive to Dirac-type phases since the combination of matrix elements of $U_\nu$ appearing in the oscillation probability is such that the Majorana phases cancel out. $CP$ violation in oscillations will manifest itself in the difference of $CP$-conjugated oscillation probabilities, e.g.

$$P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \propto J,$$

with $J$ a function of the PMNS matrix which can be expressed in terms of experimentally relevant quantities as

$$J = \frac{1}{4} \sin 2 \theta_\odot \sin 2 \theta_{a} (1 - |U_{e3}|^2) |U_{e3}| \sin \delta.$$ (93)

In the quark sector of the SM the parameter $J$ defined in terms of the $V_{CKM}$ elements is known to be $O(\lambda^6) \simeq 10^{-5}$. One might expect that for large solar mixing $J$ would be much larger, yet the constraints imposed by leptogenesis in the context of SO(10) with the Dirac neutrino masses fixed by the up quark spectrum imply an upper bound on $J$ of the order of $10^{-4}$, which is outside the experimental reach of the next generation of neutrino experiments. We notice however that within the LMA solution, which is at present the most favoured after the recent SNO results, and allowing for the most general form for the Dirac neutrino spectrum as in Eq. (88), one can reach values of $J$ of the order of $10^{-2}$, thus rendering $CP$-violating effects visible in the near future.

Neutrino oscillation experiments give no evidence about whether neutrinos are Dirac or Majorana particles. Processes which violate lepton number such as neutrinoless double beta decay ($(\beta\beta)_{0\nu}$-decay) of even-even nuclei, $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$, would imply that neutrinos have Majorana character. The probability amplitudes of these processes are proportional to the so-called “effective Majorana mass parameter”

$$|\langle m \rangle| = \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 \right|,$$ (94)
(with the possibility of complex $U_{ei}$), or equivalently,

$$|<m>| = |m_1 c_\beta^2 (1 - |U_{e3}|^2) + m_2 s_\beta^2 (1 - |U_{e3}|^2) e^{2i\alpha} + m_3 |U_{e3}|^2 e^{-2i\delta} e^{2i\beta}|.$$  

(95)

Presently, the most stringent constraint on the parameter $|<m>|$ comes from the $^{76}$Ge Heidelberg-Moscow experiment [32], indicating that $|<m>| < 0.35$ eV (90 % C.L.). This value is still too high to provide some information about the neutrino spectrum. In fact, it can only rule out some models with quasi-degenerate Majorana neutrinos. Nevertheless, higher sensitivity is planned to be reached in new generation experiments. The NEMO3 [33] and CUORE [34] experiments intend to achieve a sensitivity up to $|<m>| \simeq 0.1$ eV while for the EXO [35] and GENIUS [36] experiments this value is about one order of magnitude below.

A hierarchical spectrum with $m_1 \leq m_2 \simeq \sqrt{\Delta m_{12}^2} \ll m_3 \simeq \sqrt{\Delta m_{23}^2}$ together with the CHOOZ bound on $|U_{e3}|$ would imply $|<m>| \lesssim 10^{-3}$ eV, far below the present bound. On the other hand, for an almost degenerate spectrum with $m = m_1 \simeq m_2 \simeq m_3$, for $m > 0.35$ eV some cancellation between the terms in (95) is already required, while in the case of small solar mixing, the contribution comes mainly from the term proportional to $m_1$. Finally, for an inverted-hierarchical spectrum where $m_3 \leq m_1 \simeq m_2 \simeq \sqrt{\Delta m_{23}^2}$, the maximal possible value is $|<m>| \simeq \sqrt{\Delta m_{23}^2} \sim 7 \times 10^{-2}$ eV [37].

We conclude this section with the following remark. It has been recently claimed [38] based on a refined analysis of the data of the Heidelberg-Moscow experiment that there is evidence up to $3.1 \sigma$ for the observation of neutrino-less double beta decay with $|<m>| = (0.05 - 0.84)$ eV (at 95% CL) with a best-fit value of 0.39 eV. If confirmed, this result would imply an almost degenerate or inverted hierarchy for the light neutrino masses in the case of three generations [39]. If this is the case, baryogenesis via leptogenesis based on the simplest SO(10) GUT scenario would not be a viable mechanism, since it is not possible to reconcile any of the solar solutions with the required cosmological baryon asymmetry, as it follows from our analysis.

5 Conclusions

We have studied leptogenesis in the framework of a simple extension of the SM where the right-handed neutrinos are added to the standard spectrum, thus leading to the seesaw mechanism. In order to restrict the number of free parameters, we have made use of some GUT-inspired relations for the quark and lepton mass matrices. We have shown that the latter relations, together
with the constraints from the low energy neutrino data, imply important restrictions on the size of leptogenesis. In particular, we have pointed out that for the Just-So$^2$ and SMA solar solutions, one can generate sufficient BAU through leptogenesis even for the simplest SO(10) GUT. On the contrary, for the LMA, LOW and VO solar solutions, a different hierarchy for the Dirac neutrino masses is required in order to obtain a viable leptogenesis.

We expect our analysis and conclusions to remain valid also in the supersymmetric version of the present framework. Although in this case new decay channels will enhance the generated $CP$ asymmetry, these additional contributions tend to be compensated by the washout processes which are in general stronger than in the SM case [40].

A related and very important subject is the search for $CP$ violation in the leptonic sector, at low energies. This is at present one of the great challenges in particle physics and it has recently received a great deal of attention. Experiments with superbeams [41] and neutrino beams from muon storage rings (neutrino factories) [42] have the potential to measure directly the Dirac phase $\delta$ through $CP$ and $T$ asymmetries [43] or indirectly through oscillation probabilities which are themselves $CP$ conserving but also depend on $\delta$. An alternative method proposed and discussed recently [44] is to measure the area of unitarity triangles defined for the leptonic sector [45].

In this paper, we have investigated the possible connection between $CP$ violation at low energies and leptogenesis. In our SO(10) inspired framework, once masses and mixing for three light neutrinos are fixed, the baryon to entropy ratio $Y_B$ can be computed. It is well known that $CP$ violation at low energies depends on three physical phases appearing in the PMNS matrix. One of them, $\delta$, is a Dirac-type phase to which neutrino oscillations are sensitive, the other two are Majorana-type phases which are relevant to processes such as neutrinoless double beta decay. We have shown that the required cosmological baryon asymmetry can be produced with only one of the Majorana-type phases different from zero, while the same is not possible with only a nonvanishing Dirac-type phase. Concerning the important question of whether the strength of $CP$ violation at low energies will be sufficient to be measured through neutrino oscillations, we have verified that there are two possible scenarios: if one assumes minimal SO(10) then, the constraints imposed by leptogenesis render these effects too small to be seen in the next generation of neutrino experiments; on the other hand, if one allows for a more general spectrum of the Dirac neutrino mass matrix, then the strength of $CP$ violation can be sufficient to be visible at low energy neutrino oscillation experiments.

The leptogenesis scenario crucially depends not only on the mechanism that was responsible for populating the early Universe with right-handed neutrinos but also on the precise details of the reheating process after inflation. In this
paper we have only considered the conventional scenario, namely, the decays of right-handed Majorana neutrinos which are produced in thermal equilibrium processes. Other production mechanisms may be as well viable and even competitive [22]. Their realization will of course depend on the particular details of the inflationary epoch and the thermal evolution of our Universe.

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