Plane model of particle separation in disintegrators with internal circulation of the grinding material

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Abstract. The paper offers a method for evaluating the separation process of fine particles in the design of a classifier of a disintegrator with internal circulation of the dispersed material. Disintegrator type mills are fairly common in the construction materials industry and related industries. However, there are a number of technological problems with this type of equipment, which hinder their wider use in the production of dispersed powders with a high specific surface area. One of these problems is the presence of under-ground (coarse) particles in the volume of the finished product of a given fraction. The quality of classification is evaluated taking into account the velocity parameters of fine particles in the classifier device built into the disintegrator body. A new design of a gravity-type classifying device is considered, which contributes to the efficient return of large substandard particles to the finishes grinding in the disintegrator grinding chamber. The publication provides a method for determining the speed parameters of particle dynamics in the chamber of the classifier device of a disintegrator mill, and also presents a model for numerical calculation of the efficiency of the classifier device. This theoretical method of assessing the quality of separation of particles in the disintegrator is to define a measure of the degree of separation, to assess the effectiveness of recycle coarse fractions of grinding material powder: degree of operational separation shows the ability of disintegrator to operate in various process modes, and sets the granulometric boundaries for obtaining a particular fraction after grinding in the disintegrator.

1. Introduction

Along with the development of building materials science, it is necessary to solve the problem of obtaining dispersed materials of a narrow granulometric composition without the presence of particles with a diameter greater than the critical one in the finished product. Developed by a team of authors, the disintegrator [1] with internal circulation of the flow of grinding materials allows obtaining dispersed powders of a narrow granulometric composition with the absence of particles in their volume whose diameter is greater than the critical one.

Let us consider the design features of a gravitational type classifying device, which can be equipped with a disintegrator, or a rotary mill, of any design (figure 1).
Figure 1. Scheme for calculating particle classification parameters in a disintegrator mill: a) scheme for calculating the speed mode of air-material flow in a conical multi-chamber classifier: 1 - core of the gushing layer; 2 - central part of the fountain; 3 - descending part of the fountain; 4 - area of bulk circular layer;

b) – scheme for determining the trajectories of movement and speeds of particles in the classifier.

To establish analytical expressions for determining the parameters of the process of dynamics, deposition and possible separation of particles from the conical surface of the classifier, we make the following assumptions: we assume that the particles move singly, have a spherical shape, the movement occurs in a limited volume of the classifier device, and the air (two-phase) flow is constant in speed and direction.

2. Materials and methods
In the weighed layer of a two-phase flow a fixed particle in the gushing layer is affected by the total force that can be formed from:

\[ \sum F = G + F_a + F_{pod} + F_M. \]

Based on these components of the total force (1), we will keep in mind the model of particle classification in the disintegrator (figure 1b) in the self-similar region of the flow of a spherical particle. This means that the horizontal part of the particle velocity in the classifier in the projection on the X1 axis will have the form:

\[ m_a \frac{du_p}{dt} = m_p a - 0.05 \mu \cdot u_p^2 \cdot \rho_p \cdot \rho_p g + m_p g + V_p \cdot \rho_p g. \]

(2)

where \( a \) - particle acceleration in the air stream, m/s^2; \( u_p \) - the horizontal speed of the particle, m/s; \( \rho_p \) - density of particle material, kg/m^3; \( m_p \) - the density of the material particle, kg/m^3; \( m \) - mass of the material particle, kg; \( \mu \) - dynamic viscosity of two-phase flow, Pa\cdot s; \( V_p \) - volume of a particle as a spherical whole, m^3; \( g \) - acceleration of gravity, m/s^2; \( d_{kp} \) - the critical diameter of the particle, defined from the expression [2]:

\[ d_{kp} \geq \frac{9 R_{ck} S_{ck} \ln \mu}{\pi (\rho_\rho - \rho) \sin^2 \psi_{ck} R_{ck}^2 H v_{ck}^2 - \nu_{ck}^2}. \]

(3)

here \( S_{ck} \) - the area of the hole for entering the two-phase flow into the classifier, m^2; \( R_{ck} \) - radius of the hole for entering the two-phase flow into the classifier, m; \( R_{ck} \) - radius of the classifier exit hole, m; \( H \) - the chamber height of the classifier, m; \( v_{ck} \) - the speed of air and material flow at the entrance to the classifier, m/s; \( \psi_{ck} \) - the angle between the axis of the hole and the direction of movement of the air-material flow, °; \( \rho_\rho \) - air density, kg/m^3; \( j_\omega \) - the ratio of windage of particle.

Let us represent a nonlinear quadratic expression (2) in letter form:

\[ \frac{du_p}{dt} = A_1^2 - B_2^2 \cdot u_p^2. \]

(4)
where parameters can be defined like this

\[ A_1 = a \cdot (1 - \bar{\rho}); \quad B_1 = \frac{0.05\pi \mu d_{wp}^2}{\rho_p}, \]

(5)

where \( P \) - full static pressure, Pa; \( \bar{\rho} \) - the ratio of the particle density to the flow density of the carrier medium-air.

Let us integrate expression (4) under the initial condition \( u_p (0) = 0 \):

\[ \int_0^u \frac{du_p}{A_1 - B_1 u_p} = \int_0^t dt. \]

(6)

Therefore, the equation for horizontal velocity will have the form:

\[ u_p = \frac{A_1}{B_1} \frac{\exp(2A_1 B_1 t - 1)}{\exp(2A_1 B_1 t + 1)}. \]

(7)

Substituting the expression (5) in (7), we obtain an equation for determining the horizontal velocity of the particle movement in the classifier chamber:

\[ u_p = \frac{m_p a \cdot (1 - \bar{\rho})}{0.05\pi \mu d_{wp}^2}, \]

(8)

We determine the movement of the particle from the central part of the two-phase flow to the periphery of the chamber along the X-axis - expression (8) is re-integrated. We will have with substitutions (5) for the initial condition \( X_1 (t) = 0 \) for \( t = 0 \) the following expression for \( X_1 \):

\[ X_1 = \frac{m_p a \cdot (1 - \bar{\rho})}{0.05\pi \mu d_{wp}^2} \ln \left[ 1 + \exp(2d_{wp} \cdot t \sqrt{a \cdot (1 - \bar{\rho})} \frac{0.05\pi \mu d_{wp}^2}{\rho_p}) \right] - d_{wp} \cdot t \sqrt{a \cdot (1 - \bar{\rho})} \frac{0.05\pi \mu d_{wp}^2}{\rho_p}. \]

(9)

Thus, an equation is obtained that allows the particles to move in the volume of the gushing layer at beginning of coordinates (figure 1 b). Along this trajectory, a heavy particle just needs to pass through and leave the active zone of the gushing (weighed) layer to settle on the surface of the conical classifier and return to re-grinding.

The obtained analytical dependences consider a flow evenly dusted with particles with a single value of their diameter and a certain concentration of \( C_{vx} \). In this case, we use the condition that if large, heavy particles under the action of gravity \( G \) reach the lower cone surface of the classifier, we will assume that they differentiated, lagged behind the two-phase flow. When the two-phase flow exits the lower chamber of the conical classifier with a height \( H_1 = f (L(y)) \) and enters another chamber located above the first one, we observe that the dispersed two-phase flow with a thickness \( Y_1 \) leaves the solid particles of the crushed material. Therefore, it is necessary to install with the mass characteristics of a two-phase flow. Then the total mass flow rate of the particles separated in the classifier per unit height of each of the chambers will be determined by the expression:

\[ M_V = C_{vx} \cdot V_V \cdot L, \]

(10)

where \( V_V \) - the speed of the particle along the axis of the hole of the conical classifier, which depends on the speed of the particle hovering \( V_h, m/s \). However, it should be noted that the hovering speed \( V_h \) can be calculated using the work formula [3]:

\[ V_h = \sqrt{\frac{4(\rho_p - \rho) g d_{st}}{3 \rho_c c_s}}, \]

(11)

where \( d_{st} \) - average diameter of the grinding material particles, m.

It should be noted that the dependence of the velocity of vertical movement of the particle along the axis of the hole of the conical classifier is experimentally confirmed, taking into account the gushing two-phase flow [4], which can be represented as:

\[ V_V = V_p \sqrt{\frac{V_M^V}{V_M^V + \frac{S_{int}}{S_M^V}}}, \]

(12)

where \( V_M^V \) and \( V_M^K \) - the volume of material in the air flow at the entrance and exit of their classifier chambers, m³; \( S_M^V \) and \( S_M^K \) - the area of the interphase boundary at the entrance and exit of the classifier chambers, m².
To calculate the vertical movement of a particle in a two-phase flow that is formed in the classifier along the Y axis, we integrate the expression (12) by time t. For initial conditions $Y_1(t) = 0$ for $t = 0$, we get:

$$L(y) = t \cdot \sqrt{\frac{4(\rho_1 - \rho_2)g \Delta_1}{3p c_p}} \cdot \frac{V_V}{V_{en}} \cdot \frac{S_{se}}{S_{er}}.$$  
(13)

Expressions for $u_p$ and $X_1$, as well as $V_V$ and $L(y)$ define the main parameters of particle dynamics in the conical classifier and help control the separation process in the device [5]. Using the obtained analytical equations it is possible to determine the degree of separation of particles crushed in a conical classifier of the disintegrator according to the method proposed below:

We determine the total mass flow rate of particles at the entrance to the cylindrical channel of the cone classifier:

$$\Sigma M_{vx} = 2C_{vx} \cdot V_V \cdot R_{vx},$$  
(14)

We calculate the degree of separation of particles with a critical diameter from a two-phase flow using the empirical expression:

$$\eta_d = \frac{M_V}{\Sigma M_{vx}} = \frac{H}{2R_{vx}} = \frac{H}{d_{vx}}.$$  
(15)

We find the value of the degree of separation of particles in a two-phase flow entering the conical classifier, taking into account the substitution (6) and the time of movement in the channel $t = \frac{H(L(y))}{V_V}$:

$$\eta_d = K_{sep} \left[ \ln \left( \frac{1 + \exp \left(2S_p \cdot K_{sep} \right)}{S_p \cdot K_{sep}} \right) - 1 \right].$$  
(16)

Here $S_p$ - the flow parameter that affects the particle, determined from the works [6-7]:

$$S_p = \frac{0.05\pi d_{vp}^2 \cdot \rho_{vp}}{m_p}.$$  
(17)

$K_{sep}$ - separation coefficient for the quadratic law of particle flow, calculated by the results of the work [3]:

$$K_{sep} = \sqrt{\frac{(1-\eta) \cdot \sigma^2}{2S_p \cdot R_{sep} \cdot V_V}}.$$  
(18)

The distribution function (pass) can be expressed as follows, based on the fact that a two-phase flow in the volume of a conical classifier contains particles of different sizes:

$$\Phi(d_p) = \frac{M_{d_p}}{M},$$  
(19)

where $M_{d_p}$ - mass flow rate of separated particles, kg, determined as $2r_p = d_p$; $M$ - mass flow rate of all particles, kg.

The total mass flow rate of particles with a diameter $d_p$ within the deviation $\Delta d_p$ at a certain degree of separation $\eta_d$ can be determined by their equations:

$$\Sigma M_{S} = M \cdot \eta_d \frac{d\Phi(d_p)}{dd_p} \Delta d_p.$$  
(20)

The mass flow rate of separated particles $\Sigma M_{S}$, after integration, will be represented by the degree of separation of particles from the two-phase flow in the chambers of the conical classifier, which will look like this:

$$\eta = \frac{M_{S}}{M} = \int_{d_p}^{\infty} \eta_d \frac{d\Phi(d_p)}{dd_p} \, dd_p.$$  
(21)

In the infinite set of volumes of dispersed dusts the logarithmic law of particle distribution is valid:

$$\Phi(d_p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left( -\frac{1}{2} \cdot \frac{d_p}{\sigma} \right) \, dd_p,$$  
(22)

where $\tau_1 = \frac{\ln(d_{50})}{\ln \sigma}$ - argument of the distribution function, and $\sigma$ - variance that characterizes the standard deviation, $d_{50}$ - the median diameter of the distribution at which the mass of particles with $d_p < d_{50}$ is equal to the mass of particles $d_p > d_{50}$.

Finally, the equation for the degree of separation of particles taking into account (21) will take the form:
where $\tau_{kr} = \tau_1$.

3. Results
The use of probabilistic expression (22), which is confirmed by experimental data, allowed obtaining theoretically equation (23), which establishes the dependence of the degree of separation of particles in the conical classifier of the disintegrator on the distribution parameters in the volume of the classifier chambers. Expression (23) includes all parameters that characterize this process. It should be noted that the obtained expression (23) is sufficiently correlated with experimental data obtained by electronic sieve analysis of particles after grinding and classification of various materials in the disintegrator.

Equations (16) and (23) allow determining the degree of separation under the influence of forces of different nature: gravitational, inertial, centrifugal, etc. in the working volume of the mill classifier of the developed design.

4. Summary
As a result of mathematical and experimental research, analytical dependences were obtained that allow determining the velocity parameters of particles in the working volume of the disintegrator chambers, which are based on the design and technological parameters of the conical classifier and the disintegrator separately. Taking into account the influence of processes occurring in the disintegrator, the effect of the disintegrator operation modes on the particle separation process in the classifier is established, and the relationship between the speed parameter of particle movement in the classifier and the degree of separation of fine particles in the two-phase circulation flows of the disintegrator mill is obtained.

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