The Pioneer anomaly and the holographic scenario

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Abstract In this paper we discuss the recently obtained relation between the Verlinde’s holographic model and the first phenomenological Modified Newtonian dynamics. This gives also a promising possible explanation to the Pioneer anomaly.

Keywords Modified Newtonian Dynamics, Cosmology, Pioneer anomaly

1 The phenomenological version of MOND

The Modified Newtonian dynamics theory (MOND) was introduced by Milgrom to solve the galaxies rotation curves problem as an alternative to the dark matter. The MOND can be implemented by a modification of the Newton’s second law or the Newton’s law of gravity.

In particular Milgrom (in the formulation where the Newton’s second law is modified) allowed for an inertia term not to be proportional to the acceleration of the object but rather to be a more general function of it. More precisely, it has the form

\[ m_i \frac{\mu}{a} a = F, \]

where \( \mu(x) \approx 1 \), and \( \mu(x) \approx x \) and \( a = |a| \), replacing the classical form \( m_i a = F \). Here \( m_i \) is also the inertial mass of a body moving in an arbitrary static force field \( F \) with acceleration \( a \), see Milgrom (1983). For accelerations much larger than the acceleration constant \( a_0 \), we have \( \mu \approx 1 \), and Newtonian dynamics is restored. However for small accelerations \( a \ll a_0 \) we have that \( \mu = a/a_0 \). In this case if \( F \) is the gravitational force of a central mass \( M \), then the modulus of the acceleration is \( a = \sqrt{a_0 GM/r} \). This acceleration gives a constant velocity \( v = \sqrt{GMa_0} \) in a circular orbit and the correct value of the galactic rotational curves. However, it has been shown that Milgrom theory, while solving a few difficulties, gives rises to other fresh problems, see for instance Felter (1984); Sanders (2006). The fundamental objection to a modification of the inertia is that it violates the equivalence principle, tested to an accuracy of \( 10^{-13} \) \( Kg \), see Baeßler et al. (1998), and the energy conservation. The version of MOND presented is not a consistent theory and it is only a phenomenological approach. To solve these problems Bekenstein and Milgrom proposed in Bekenstein & Milgrom (1984) a nonrelativistic potential theory for gravity which differs from the Newtonian one. In Famaey & Binney (2005) simple analytical forms of \( \mu(x) \) are analyzed and satisfactory fits to the observationally determined terminal velocity curve are obtained. A theoretical argument that supports a certain form of \( \mu(x) \) against other is still not known. In Giné (2009) we made a first approximation to the problem and deduced the following form of \( \mu(x) \), in the context of the Mach’s principle

\[ m_i \left( \frac{a}{a + a_0} \right) a = F, \]

(1)

This simple form of \( \mu(x) \) yields very good results in fitting the terminal velocity curve of Milky Way and others, see Famaey & Binney (2005). Moreover, in Giné (2010) a new form for the \( \mu(x) \) appearing in the Milgrom formula was obtained:

\[ m_i \left( \frac{|a|}{a + a_0} \right) a = F, \]

(2)

where \( a_e \) is an effective acceleration given by \( a_e = a_0(1 - R_{obs}/R_U)R_U \), \( R_{obs} \) is the distance to the object and \( R_U \) is the radius of the causal connected universe.
For local objects we have \( a_e \sim a_0 \) and for far away objects \( a_e \sim 0 \). Equation (2) contains \( \mu(x) \) as a particular case. The form of \( \mu(x) \) presented in Gine (2010) is a modification of the inertia following the ideas developed by Milgrom in Milgrom (1999, 2005), using the relativity principle of motion and assuming the proven fact of the accelerated expansion of the universe. In the formula (2) we have a vectorial sum of accelerations and depending if the vectors are quasi-collinear or are perpendicular the vectorial sum gives different values. In Gine (2010) it is also established a relation between the MOND and the deceleration parameter of the expansion of the universe.

2 Verlinde holographic scenario

Verlinde propose a model where the second Newton law and Newton’s law of gravitation arise from basic thermodynamic mechanisms. In the context of Verlinde’s holographic model, the response of a body to the force may be understood in terms of the first law of thermodynamics. We consider a holographic screen in the plane \( yz \) that intersects de \( x \) axis at \( x + \Delta x \), where \( \Delta x \) is a small increment distance. As the body approaches the screen, its descriptive information becomes encoded holographically on the screen. The entropy of the screen increase by some amount \( \Delta S \). In a similar way in which a particle approaching the event horizon of a Schwarzschild black hole increases the entropy of the horizon, in Verlinde (2011) is proposed that

\[
\Delta S = 2\pi k_B \frac{m c^2}{a} \Delta x. \tag{3}
\]

When the body traverse the distance \( \Delta x \), its energy changes by an amount \( \Delta E = F \Delta x \), which is the incremental work done by the force \( F \). Using the first law of thermodynamics, the model sets that

\[
F \Delta x = T \Delta S. \tag{4}
\]

An observer in an accelerated frame experiences the associated Unruh temperature

\[
T = \frac{1}{2\pi} \frac{\hbar a}{k_B c}. \tag{5}
\]

The second law of Newton \( F = ma \) follows from substituting in (4) equations (3) and (5). Now is supposed that the boundary is a closed surface, it is assumed that is an sphere. Assuming that the holographic principle holds, the maximal storage space, or the total number of bits, is proportional to the area of the boundary

\[
N = \frac{Ac^2}{G} = \frac{4\pi R^2c^3}{G}. \tag{6}
\]

where a new constant \( G \) is introduced. The total energy is given by the equipartition rule

\[
E = \frac{1}{2} N k_B T. \tag{7}
\]

Now we consider the total energy enclosed by the screen is given by a mass \( M \) i.e. is satisfied \( E = Mc^2 \). Now equating this equation with equation (7) and substituting equations (6) and (8) we obtain the Newton’s law of gravitation

\[
F = G \frac{m M}{R^2}, \tag{8}
\]

and the constant \( G \) is the universal gravitational constant. From this arguments it is stated in Verlinde (2011) the entropic origin of gravity because the acceleration is related with an entropy gradient. More precisely, gravity is explained as an entropic force caused by changes in the information associated with the positions of material bodies. The consequences of this general theory are being analyzed and discussed. The cosmological acceleration can be explained using the entropic force, see Easson et al. (2011). Other important consequence related to this work is that the Verlinde’s holographic model in an asymptotically de Sitter space leads to a new form of the second law of motion which is the required by the MOND theory proposed by Milgrom, see Funkhouser (2010). Therefore the phenomenological Milgrom formulation is supported by Verlinde’s theory. In Funkhouser (2010) it is demonstrate that, in a universe endowed by a positive cosmological constant \( \Lambda \), the holographic model described by Verlinde leads naturally to a modification of the second Newton’s law of the form

\[
m[(a^2 + k^2)^{1/2} - k] = F, \tag{9}
\]

where \( k = \sqrt{\Lambda/3} \). Moreover equation (9) is identical to the specific formulation of MOND suggested by Milgrom in Milgrom (1999). In the limit \( a/k \) arbitrarily large (9) becomes identical to the Newton second law and for \( a/k \ll 1 \) we have

\[
m\frac{a^2}{2k} = F,
\]

where \( 2k \) plays the role of the constant acceleration \( a_0 \). In fact, if we assume that the present evolution of the universe is dominated by the cosmological constant \( \Lambda \), as corroborated by observation Tegmark et al. (2001), we can set \( cH_0 \sim \Lambda^{1/2} \) which implies that \( k \sim a_0 \) in orders of magnitude. The relation between \( a_0 \) and the cosmological constant it is also discussed in Gine (2011) in the context of the scaling laws that suggest a fractal universe.
3 The Pioneer anomaly in this context

The Pioneer anomaly [Anderson et al. 1998, 2002] consists of unexpected, almost constant and uniform acceleration directed approximately towards the Sun $8.74 \pm 1.33 \times 10^{-10} \text{m s}^{-2}$ first detected in the analyzed data of the Pioneer probes after they passed the threshold of 20 Astronomical units. However, the recent new data of the Pioneer anomaly suggest that it is variable and environment dependent rather than a fixed value and still is not clear its direction with the possibility that be Earth directed, see Tuyyshev & Toth (2009, 2010). The effects of the Pioneer anomaly are non-detected on the major bodies of the solar system and in several papers is studied its gravitational origin, see Iorio (2010c); Tangen (2007) and references therein. Meanwhile there exits other works where it is studied and in several papers is studied its gravitational origin, see Bertolami et al. (2008); Rievers et al. (2009).

The Pioneer anomaly is similar to the galaxy rotation problem which also involves an unexplained acceleration. Milgrom realized that MOND could explain the Pioneer anomaly, see Milgrom (2001). The modified-inertia approaches to solve the Pioneer anomaly have been also considered under Unruh radiation by McCulloch, see McCulloch (2007). This proposal acquires its meaning in the wake of the holographic scenario established in the work of Verlinde (2011). In McCulloch (2007) it is found that the acceleration of the Pioneer craft is given by

$$a = \frac{GM_\odot}{r^2} + \beta \pi^2 c^2 \frac{\Theta}{r^2},$$

where $M_\odot$ is the sun mass, $\beta$ appear in the Wien’s constant and has the value $\beta = 0.2$, and $\Theta$ is the Hubble diameter $\Theta = 2c/H_0 = 2R_U$. The second term can be rearranged to give

$$a = \frac{GM_\odot}{r^2} + 2 \beta \pi^2 c H_0 \approx \frac{GM_\odot}{r^2} + 0.99 \times c H_0. \quad (10)$$

We are going to see that we obtain equation (10) in the context of phenomenological formulation of MOND. We use equation (2) for the Pioneer craft, with the approximation $a_e \sim a_0$ because we are dealing with a local object and taking into account that the accelerations are quasi-collinear because the Pioneer craft performs an orbit away from us (hence we can use equation (1)). In a strong Newtonian regime, we can develop the term

$$\frac{a}{a + a_0} = \frac{1}{1 + a_0/a} \approx 1 - \frac{a_0}{a}, \quad (11)$$

in the case $a_0/a \ll 1$ i.e. $a_0 \ll a$. Now substituting in (11) (taking the modulus) we have

$$m_i \left(1 - \frac{a_0}{a}\right) a = F = \frac{GM_\odot m_g}{r^2}.$$  

We can rearrange this equation to obtain

$$m_i a = \frac{GM_\odot m_g}{r^2} + m_i a_0. \quad (12)$$

From the equivalence principle we have $m_i = m_g$ and (12) becomes

$$a = \frac{GM_\odot}{r^2} + a_0. \quad (13)$$

In Giné (2009) and Giné (2011a) it is justified by different arguments that $a_0 \sim c H_0$, where $H_0$ is the actual value of the Hubble constant, see also Giné (2010). Therefore we have obtained equation (10). The arguments to obtain $a_0 \sim c H_0$ are the following. In Giné (2009) using the equivalence principle, which implies the equality between inertial mass $m_i$ and gravitational mass $m_g$, it is obtained the relation $GM_U = c^2 R_U$ where $M_U$ and $R_U$ is the mass and the radius of the universe respectively. Then substituting this expression in the definition of $a_0$ in the sense of the Mach’s principle the result follows. In Giné (2011a) the relation $a_0 \sim c H_0$ is obtained through the scale factor of the universe $R(t)$ and the Hubble law of expansion of the universe. Hence, the Pioneer anomaly is given by $a_0$ that taking into account that $H_0 = 2.3 \pm 0.9 \times 10^{-18} \text{s}^{-1}$ we obtain that $a_0 = 6.9 \pm 3.5 \times 10^{-10} \text{m s}^{-2}$, which is in agreement with the observed value anomaly $8.74 \pm 1.33 \times 10^{-10} \text{m s}^{-2}$. The 40% ($\pm 3.5$) uncertainty arises because of uncertainties in the Hubble constant.

4 Final comments

The value of $a_0 = 6.9 \pm 3.5 \times 10^{-10} \text{m s}^{-2}$ is about six times larger than the acceleration constant $1.2 \times 10^{-10} \text{m s}^{-2}$ required for MOND of Milgrom for fitting galaxy velocity curves. However, the constant acceleration $a_0$ is also present in the inner solar system where it is dramatically inconsistent with the motion of the inner planets if we use equation (1) or similar versions of $\mu(x)$. In fact, it fails completely in the strong gravity regime where $a \gg a_0$, and thus cannot be valid in the Solar system. For instance, the upper limit on an additional constant acceleration imposed by the observed precession of the orbit of Mercury is more than a factor of 10 smaller than $a_0$. Similar constrains result from the observed motion of Icarus. In general, such peculiar acceleration is constrained by observations to be
about one or two orders of magnitude lower than $a_0$ in the inner Solar system, see Sanders (2006). Hence, one must argue that the MOND acts in a very different way for local bound objects like planets. This has already pointed out by Milgrom in Milgrom (2001). Sanders in Sanders (2006) concludes that if the effects of the MONDian modification of gravity are not observed in the motion of the outer planets in the solar system, the acceleration cannot be due to MOND. Solar system constraints on multifield theories of modified dynamics. The $\mu(x)$ function (2) used in this paper also present these problems. Nevertheless, the result obtained for the Pioneer anomaly reinforces that it must be Earth directed and variable and environment dependent, because it depends on the relative position of the Pioneer craft and the Earth in its own movement along its orbit around the Sun.

A simple modification of the $\mu(x)$ function does not save MOND from its inherent problems. In a recent review of the Pioneer anomaly is said that the Pioneer anomaly has nothing to do with MOND, see Turyshev & Toth (2010). In this survey it is also said that “the exact form of $\mu(x)$ remains unspecified in both MOND and the relativistic version of TeVeS proposed by Bekenstein Bekenstein (2004). It is conceivable that an appropriately chosen $\mu(x)$ might reproduce the Pioneer anomaly even as the theory’s main result, its ability to account for galaxy rotation curves, is not affected”. This also happens with the new expression of $\mu(x)$ presented in this paper. It is still open to find the form of $\mu(x)$ consistent with the observational data which establish differences between the unbounded orbits (like the Pioneer craft) and the bounded orbits (like the planets). In the framework of MOND, the internal dynamics of a gravitating system $S$ embedded in a larger one $S'$ is affected by the external background field $E$ of $S$ even if it is constant and uniform, thus implying a violation of the strong equivalence principle: it is the so-called External Field Effect (EFE). Milgrom Milgrom (1983) originally introduced EFE in order to explain that the observed mass in certain open star clusters in the galactic neighborhood of the solar system was very low, although their internal accelerations were 5 or 10 times smaller than $a_0$. The galactic acceleration felt by such open clusters is just of the order of $a_0$. The first, preliminary attempts to look at EFE in the Oort cloud were made by Milgrom in Milgrom (1983, 1986). More detailed analysis on EFE in the Oort cloud is made by Iorio in Iorio (2010a). EFE was adapted to the planetary regions of the Solar System, where the field is strong, see Milgrom (2009). Some implications were discussed in Blanchet & Novak (2011); Iorio (2010b, 2011). Finally it should be mentioned that several studies of MOND were performed in the solar system, see Bekenstein & Magueijo (2000); Blanchet & Novak (2011); Iorio (2008, 2009, 2010a, b, 2011); Milgrom (1999); Sanders (2006); Sereno & Jetzer (2006); Talmadge et al. (1988). Anyway, the correct version of MOND to be constructed in the future must be derived from the new holographic scenario. In Hajdukovic (2010, 2011), based on the hypothesis of the gravitational repulsion between matter and antimatter, what allows considering, the virtual particle–antiparticle pairs in the physical vacuum, as gravitational dipoles, it is argued that the Pioneer Anomaly and the MOND is related to the quantum vacuum fluctuations. Two speculative but exciting papers which may help provide insight into the nature of the dark energy of the Universe.

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References

Anderson, J.D., Liang, P.A., Lau, E.L., Liu, A.S. Nieto, M.M., & Turyshev S.G. 1998, Phys. Rev. Lett., 81, 2858

Anderson, J.D., Liang, P.A., Lau, E.L., Liu, A.S., Nieto M.N., & Turyshev, S.G. 2002, Phys. Rev. D, 65, 082004

Baessler, S, Heckel, B.R., Adelberger E.G., Gundlach, J.H. Schmidt, U, & Swanson H.E. 1998, Phys. Rev. Lett., 83, 3585

Bekenstein, J.D. 2004, Phys. Rev. D, 70, 083509

Bekenstein, J, & Maguirejo, J. 2004, Phys. Rev. D, 73, 103513

Bekenstein, J. & Milgrom, M. 1984, Astrophys. J., 286, 7

Bertolami, O., Francisco, F., Gil, P.J.S., & Páramos J. 2008, Phys. Rev. D, 78, 103001

Blanchet, L, & Novak, J. 2011, Mon. Not. R. Astron. Soc., 412, 2530

Bessaon, D.A., Frampton, P.H., & Smoot G.F. 2011, Phys. Lett. B, 696, 273

Famaey, B., & Binney, J. 2005, Mon. Not. R. Astron. Soc., 363, 603

Felten, J.E. 1984, Astrophys. J., 286, 3

Funkhouser, S. 2010, preprint arXiv: 1009.5126

Giné, J. 2009, Chaos Solitons Fractals, 41, 1651

Giné, J. 2010, preprint, Udl, http://web.udl.es/usuario /4088/454/ssl/Prepublicaciones/PS/mon.pdf

Giné, J. 2011a, Internat. J. Theoret. Phys, 50, 607

Giné, J. 2011b, preprint, Udl, http://web.udl.es/usuario /4088/454/ssl/Prepublicaciones/PS/fractal.pdf

Hajdukovic, D.S. 2010, Astrophys. Space Sci. 330, 207

Hajdukovic, D.S. 2011, Astrophyics. Space Sci. 334, 215

Iorio, L. 2008, J. Gravitational Physics, 2, 26

Iorio, L. 2009, Astrophys. Space Sci., 323, 215

Iorio, L. 2010a, The Open Astronomy Journal, 3, 156

Iorio, L. 2010b, The Open Astronomy Journal, 3, 1

Iorio, L. 2010c, Mon. Not. R. Astron. Soc., 405, 2615

Iorio, L. 2011, preprint arXiv:1101.2634

McCulloch, M.E. 2007, Mon. Not. R. Astron. Soc., 376, 338

Milgrom, M. 1983, Astrophys. J., 270, 365

Milgrom, M. 1986, Astrophys. J., 302, 617

Milgrom, M. 1994, Ann. Phys., 229, 384

Milgrom, M. 1999, Phys. Lett. A, 253, 273

Milgrom, M. 2001, Acta Phys. Polon. B, 32, 3613

Milgrom, M. 2005, EAS Publications Series, 9

Milgrom, M. 2009, Mon. Not. R. Astron. Soc., 399, 474

Rievers, B., Lämmerzahl, C., List, M., Bremer, S. & Dittus, H. 2009, New Journal Physics, 11, 113032

Sanders, R.H. 2006, Mon. Not. R. Astron. Soc., 370, 1519

Sereno, M. & Jetzer, Ph. 2006, Mon. Not. R. Astron. Soc., 371, 626

Talmadge, C., Berthias, J.P., Hellings, R.W. & Standish, E.M. 1988, Phys. Rev. Lett., 61, 1159

Tangen, K. 2007, Phys. Rev. D, 76, 042005

Tegmark, M., Zaldarriaga, M., & Hamilton, A.J.S. 2001, Phys. Rev. D, 63, 043007

Turyshev, S.G., & Toth, V.T. 2009, Space Sci. Rev., 148, 149

Turyshev, S.G., & Toth, V.T. 2010, Living Reviews Relativity, 13, 4

Unruh W.G. 1976, Phys. Rev. D, 14, 870

Verlinde, E. 2011, Journal High Energy Physics, 4, 29

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