CP Violation in Neutrinoless Double Beta Decay

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Abstract

We argue three-flavour neutrino mixing. We consider the neutrinos as Majorana particles and see how the neutrinoless double beta decay constrains the neutrino mixing angles. Our formulation is widely valid and is applied to the neutrino oscillation experiment.

Key words: massive neutrino, neutrino oscillation.
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It is one of the most important problems in particle physics whether the neutrinos have masses or not. From the recent neutrino experiments \[1\] \[2\] \[3\] \[4\], it becomes very probable that the neutrinos have masses. However, if the neutrinos have masses, we must explain the reason why they are so tiny relative to the charged lepton masses. Seesaw mechanism is one of the most promising candidates for such an explanation. In this case, neutrinos become Majorana particles.

In this paper we consider the neutrinos as massive Majorana particles with three generations and see how this point of view constrains physics of lepton sector.

As is well known, the neutrino oscillation does not distinguish Majorana neutrinos from Dirac ones. So, let us first consider the neutrinoless double beta decay \((\beta\beta^0_0\nu)\) which occur only in the case of Majorana neutrinos.

\[\langle m_\nu \rangle \equiv \left| \sum_{j=1}^{3} U_{ej}^2 m_j \right|. \tag{1}\]

Here \(U_{\alpha j}\) is the left-handed neutrino mixing matrix which combines the weak eigenstate \((\alpha = e, \mu\) and \(\tau)\) to the mass eigenstate with mass \(m_j\) \((j=1,2\) and \(3)\). It takes the following form in the case of Majorana neutrinos,

\[
U = \begin{pmatrix}
    c_1 c_3 & s_1 c_2 e^{i\beta} & s_3 e^{i(\rho-\phi)} \\
    (-s_1 c_2 - c_1 s_2 s_3 e^{i\phi}) e^{-i\beta} & c_1 c_2 - s_1 s_2 s_3 e^{i\phi} & s_2 e^{i(\rho-\beta)} \\
    (s_1 s_2 - s_1 c_2 s_3 e^{i\phi}) e^{-i\rho} & (-c_1 s_2 - s_1 c_2 s_3 e^{i\phi}) e^{-i(\rho-\beta)} & c_2 c_3
\end{pmatrix}. \tag{2}\]

Here \(c_j = \cos \theta_j\), \(s_j = \sin \theta_j\) \((\theta_1 = \theta_{12}, \theta_2 = \theta_{23}, \theta_3 = \theta_{31})\), and beside \(\phi\), appear the two additional CP violating phases \(\beta\) and \(\rho\) for Majorana particle. So \(\langle m_\nu \rangle\) becomes

\[
\langle m_\nu \rangle = |m_1 c_1^2 c_3^2 - m_2 s_1^2 c_3^2 e^{-2i\beta'} - m_3 s_3^2 e^{-2i\rho'}|, \tag{3}\]

where we have introduced

\[
\beta' \equiv \frac{\pi}{2} - \beta, \quad \rho' \equiv \frac{\pi}{2} - (\rho - \phi). \tag{4}\]

CP violation occurs in the presence of imaginary part in \(\langle m_\nu \rangle\), though this process itself does not explicitly show CP violation.

We show that the neutrino mixing angles are constrained from the presence of CP violation. Here we follow the method given in \[3\].

From Eq.(3) it follows that

\[
\langle m_\nu \rangle^2 = (m_1 c_1^2 c_3^2 - m_2 s_1^2 c_3^2 \cos 2\beta' - m_3 s_3^2 \cos 2\rho')^2 + (m_2 s_1^2 c_3^2 \sin 2\beta' + m_3 s_3^2 \sin 2\rho')^2. \tag{5}\]
Rewriting $\cos 2\rho'$ and $\sin 2\rho'$ by $\tan \rho'$, we can consider Eq.(5) as an equation of $\tan \rho'$,

$$a_+ \tan^2 \rho' + b_\beta \tan \rho' + a_- = 0.$$  \hspace{1cm} (6)

Here $a_\pm$ and $b_\beta$ are defined by

$$a_\pm \equiv 4 \sin^2 \beta' m_2 s_1 t_3 (m_1 c_1^2 - m_2 s_1^2 + m_3 s_3^2) - \langle m_\nu \rangle^2,$$

$$b_\beta \equiv 4 m_2 s_1 t_3^2 \sin 2 \beta'.$$

So, the discriminant $D$ for Eq.(6) must satisfy the following inequality:

$$D \equiv b_\beta^2 - 4 a_+ a_-$$

$$= 4^3 (m_1 c_1^2 t_3^2) (m_2 s_1^2 t_3^2) (f_+ - \sin^2 \beta' (\sin^2 \beta' - f_-) \geq 0,$$

where

$$f_\pm \equiv \frac{(\langle m_\nu \rangle \pm m_3 s_3^2) - (m_1 c_1^2 - m_2 s_1^2 + m_3 s_3^2)}{4 m_1 m_2 c_1^2 s_1^2 t_3^2}.$$  \hspace{1cm} (7)

So we obtain

$$f_- \leq \sin^2 \beta' \leq f_+.$$  \hspace{1cm} (10)

It follows from Eq.(10) that

$$f_- \leq 1, \hspace{0.5cm} f_+ \geq 0.$$  \hspace{1cm} (11)

Quite analogously, rewriting $\cos 2\beta'$ and $\sin 2\beta'$ by $\tan \beta'$, and considering Eq.(5) as an equation of $\tan \beta'$, we obtain other inequalities,

$$g_- \leq \sin^2 \rho' \leq g_+.$$  \hspace{1cm} (12)

Here

$$g_\pm \equiv \frac{(\langle m_\nu \rangle \pm m_3 s_3^2) - (m_1 c_1^2 - m_2 s_1^2 + m_3 s_3^2)}{4 m_1 m_3 c_1^2 s_1^2 t_3^2}.$$  \hspace{1cm} (13)

So we get

$$g_- \leq 1, \hspace{0.5cm} g_+ \geq 0.$$  \hspace{1cm} (14)

The conditions (11) and (14) are consistency conditions. CP violating area is given by the more stringent condition

$$0 \leq f_- \leq \sin^2 \beta' \leq f_+ \leq 1 \hspace{0.5cm} or \hspace{0.5cm} 0 \leq g_- \leq \sin^2 \rho' \leq g_+ \leq 1.$$  \hspace{1cm} (15)

From the inequalities (11) and (14), we obtain the allowed region of the mixing angles in the $s_2^2$ versus $s_3^2$ plane once the neutrino masses $m_i$ and the ”averaged ” neutrino mass $\langle m_\nu \rangle$ are known. The magnitude of $\langle m_\nu \rangle$ is experimentally unknown at present. The neutrino masses may be safely ordered as $m_1 \leq m_2 \leq m_3$. So in the following discussions we consider the three cases:

a) $\langle m_\nu \rangle \leq m_1$,

b) $m_1 \leq \langle m_\nu \rangle \leq m_2$,  \hspace{1cm} (16)
and

c) $m_2 \leq \langle m_\nu \rangle \leq m_3$.

Note that the definition of $\langle m_\nu \rangle$ in Eq.(1) and the Schwartz inequality leads us to

$$\langle m_\nu \rangle \leq \sum_{j=1}^{3} m_j |U_{ej}^2| \leq m_3 \sum_{j=1}^{3} |U_{ej}| = m_3,$$

so $\langle m_\nu \rangle$ can not be larger than $m_3$. The allowed regions in the $s_1^2$ versus $s_3^2$ plane for each case (a), (b) and (c) are obtained from Eqs.(11) and (14), and are shown in Fig.2.

Fig.2

From Fig.2, we obtain the upper bound on $s_3^2$ as

$$s_3^2 \leq \frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2},$$

(17)

for any case. The CP violating areas in the $s_1^2$ versus $s_3^2$ plane given by Eq.(15) are also indicated by the oblique lines in Fig.2 for each case (a), (b) and (c). The above case (a) was considered also in [6] and [7]. In [6], the representation for the mixing matrix adopted by Cabibbo-Kobayashi-Maskawa was used. In [7], only the limiting case where all the neutrino masses are degenerate ($m_1 = m_2 = m_3$) was discussed. It should be noted that we consider the cases (b) and (c) in addition to (a) and that no condition on the neutrino masses has been imposed so far.

The above mentioned method is not restricted to Majorana particles but is widely applicable. Next, we consider the constraint from the neutrino oscillation experiment at CHORUS[8] and see how this method also gives the allowed region in $s_1^2$ versus $s_3^2$ plane. We assume here $\delta m_{31}^2 \equiv m_3^2 - m_1^2 \sim \delta m_{21}^2 \equiv m_3^2 - m_2^2 \gg \delta m_{32}^2 \equiv m_3^2 - m_2^2$.

In this case the approximate oscillation probability is given by

$$P(\nu_\mu \rightarrow \nu_\tau) = 4|U_{\mu 1}|^2|U_{\tau 1}|^2 \sin^2 \left(\frac{\delta m_{31}^2 L}{4E_\nu}\right).$$

(18)

Substituting the expression of Eq.(2) into Eq.(18), we obtain the following equation w.r.t. $\cos \phi$,

$$A \equiv \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4 \sin^2 \left(\frac{\delta m_{31}^2 L}{4E_\nu}\right)} = a_+ \cos^2 \phi - 2b \cos \phi + a_-$$

(19)

$$\equiv f(\cos \phi).$$

Here

$$a_+ \equiv - (2s_1 s_2 s_3 c_1 c_2)^2,$$

$$a_- \equiv (s_1^2 c_2^2 + c_1^2 s_2^2 s_3^2)(s_1^2 s_2^2 + c_1^2 s_2^2 s_3^2),$$

$$b \equiv s_1 s_2 s_3 c_1 c_2 (s_1^2 - c_1^2 s_3^2)(c_2^2 - s_2^2).$$

(20)
The oscillation process does not distinguish Majorana neutrino from Dirac one, and only $\phi$ phase takes place. Firstly the discriminant $D$ of Eq(19) leads to

$$D \equiv b^2 - a_+(a_- - A)$$
$$= (s_1s_2s_3c_1c_2)^2[(s_1^2 + c_1^2s_3^2)^2 - 4A] \geq 0. \quad (21)$$

Thus we obtain

$$\frac{(s_1^2 + c_1^2s_3^2)^2}{4} \geq A,$$ \quad (22)

which is irrelevant to $\theta_{23}$. Therefore the CHORUS data on $P(\nu_\mu \rightarrow \nu_\tau) \equiv P_{CHORUS}$ constrain the allowed region in the $s_1^2$ versus $s_3^2$ plane (Fig.3).

Unfortunately, we have only the upper bound on the $P_{CHORUS}$, $P_{CHORUS} < 2.5 \times 10^{-3}$ $[10]$. Setting $L = 600m$ (the midpoint of the maximum length, 800m and the minimum length, 400m), $E = 27GeV$ and $\delta m_{31}^2 \sim \delta m_{21}^2 = 6eV^2 \gg \delta m_{32}^2$, we have $A < 0.022$.

The broken line is the trajectory of

$$\frac{(s_1^2 + c_1^2s_3^2)^2}{4} = 0.022,$$ \quad (23)

and the allowed region is the upper part from the broken line. If the $P_{CHORUS}$ gives the lower value, the broken line moves downward to extend the allowed region. The shaded areas in Fig.3 are those of Fig.2(b) under the assumption that $\delta m_{31}^2 \sim \delta m_{21}^2 = 6eV^2 \gg \delta m_{32}^2$ and $m_1 \ll \langle m_\nu \rangle$ values. The more stringent constraints, though they depend on $\theta_{23}$, are also obtained from Eq.(19). $a_+$ is negative definite and $f(\pm 1)$ are positive definite. Therefore from the condition that $-1 \leq \cos \phi \leq 1$ we obtain the following inequalities:

**Case a-1:** \quad $0 \leq \frac{(s_1^2 - c_1^2)s_2^2}{4s_1s_2s_3c_1c_2} \leq 1$

$$0 \leq (s_1c_2 - c_1s_2s_3)^2(s_1s_2 + c_1c_2s_3)^2 \leq \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4\sin^2\left(\frac{\delta m_{31}^2}{4E_\nu}L\right)} \leq \frac{1}{4}\frac{(s_1^2 + c_1^2s_3^2)^2}{4} \quad (24)$$

**Case a-2:** \quad $1 \geq \frac{(s_1^2 - c_1^2)s_2^2}{4s_1s_2s_3c_1c_2}$

$$1 \geq \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4\sin^2\left(\frac{\delta m_{31}^2}{4E_\nu}L\right)} \leq \frac{(s_1c_2 + c_1s_2s_3)^2(s_1s_2 - c_1c_2s_3)^2}{(s_1c_2 - c_1s_2s_3)^2(s_1s_2 + c_1c_2s_3)^2} \leq \frac{(s_1c_2 + c_1s_2s_3)^2(s_1s_2 - c_1c_2s_3)^2}{(s_1c_2 - c_1s_2s_3)^2(s_1s_2 + c_1c_2s_3)^2} \quad (25)$$

**Case b-1:** \quad $-1 \leq \frac{(s_1^2 - c_1^2)s_2^2}{4s_1s_2s_3c_1c_2} \leq 0$

$$-1 \leq \frac{(s_1c_2 + c_1s_2s_3)^2(s_1s_2 - c_1c_2s_3)^2}{(s_1c_2 - c_1s_2s_3)^2(s_1s_2 + c_1c_2s_3)^2} \leq \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4\sin^2\left(\frac{\delta m_{31}^2}{4E_\nu}L\right)} \leq \frac{1}{4}\frac{(s_1^2 + c_1^2s_3^2)^2}{4} \quad (26)$$
Case b-2 : \[ \frac{(s_2^2-c_2^2)(s_1^2-c_1^2 s_3^2)}{4 s_1 s_2 s_3 c_1 c_2} < -1 \]

\( (s_1 c_2 + c_1 s_2 s_3)^2 (s_1 s_2 - c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4 \sin^2 \left( \frac{\delta m^2_{31}}{4E}\right)} \leq (s_1 c_2 - c_1 s_2 s_3)^2 (s_1 s_2 + c_1 c_2 s_3)^2 \)

(27)

As we have mentioned, we have experimentally only the upper bound of \( P(\nu_\mu \rightarrow \nu_\tau) \) at present. So the more meaningful inequalities than Eq.(22) comes from the lower bounds of Eqs.(24) \( \sim \) (27). Namely we have

Case a

\( (s_1 c_2 - c_1 s_2 s_3)^2 (s_1 s_2 + c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4 \sin^2 \left( \frac{\delta m^2_{31}}{4E}\right)} \) for \( (s_2^2 - c_2^2)(s_1^2 - c_1^2 s_3^2) \geq 0 \) (28)

Case b

\( (s_1 c_2 + c_1 s_2 s_3)^2 (s_1 s_2 - c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \rightarrow \nu_\tau)}{4 \sin^2 \left( \frac{\delta m^2_{31}}{4E}\right)} \) for \( (s_2^2 - c_2^2)(s_1^2 - c_1^2 s_3^2) \leq 0 \) (29)

From these inequalities (28) and (29), we obtain another allowed region in the \( s_1^2 \) versus \( s_2^2 \) plane for a fixed value of \( \theta_2 \). Using \( \delta m^2_{31} \sim \delta m^2_{21} = 6eV^2 \gg \delta m^2_{32} \) and \( A < 0.022 \), we show the allowed regions for \( \theta_2 = 0, \frac{\pi}{21}, \frac{\pi}{12}, \cdots, \frac{\pi}{2} \) in Fig.4.

Lastly we comment on another neutrino less process of \( \mu^- - e^+ \) conversion [11]. (Fig.5)

In this case the averaged neutrino mass which will be determined experimentally is given by

\[ \langle m_\nu \rangle_{\mu^- - e^+} = \sum_{i=1}^{3} m_i U^*_{ei} U_{\mu i} = \sum_{i=1}^{3} m_i U_{ei} U_{\mu i} \]  

(30)

Substituting the expression of Eq.(2) into Eq.(30), we obtain

\[ \langle m_\nu \rangle_{\mu^- - e^+}^2 = \sum_{i=1}^{3} m_i U^*_{ei} U_{\mu i}^2 = \sum_{i=1}^{3} m_i U_{ei} U_{\mu i}^2 \]

(31)
In contrast to the neutrinoless double beta decay, all the mixing angles and the phase parameters appear. So if we assume one of the phases and $\theta_{23}$, we can develop the same argument as that in the neutrinoless double beta decay.

In conclusion, we have proposed the new method to constrain the neutrino mixing angles from the observed data of $\langle m_\nu \rangle$. Our method, however, is widely valid and have been applied to the neutrino oscillation, having given the new constraints from the observed data of $P(\nu_\mu \rightarrow \nu_\tau)$. Our method will be applied to the other decay and oscillation processes.

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Figure Captions

**Fig.1:** Feynman diagram of neutrinoless double beta decay.

**Fig.2:** The allowed region on $\sin^2 \theta_{12}$ versus $\sin^2 \theta_{31}$ plane by the neutrinoless double beta decay is given by the shaded areas in the respective case:

(a) $\langle m_{\nu} \rangle \leq m_1$  \hspace{1cm} (b) $m_1 \leq \langle m_{\nu} \rangle \leq m_2$  \hspace{1cm} (c) $m_2 \leq \langle m_{\nu} \rangle \leq m_3$

In the allowed region, CP-violating area is specially indicated by the oblique lines.

**Fig.3:** Each allowed region by the neutrinoless double beta decay in the respective case:

$\langle m_{\nu} \rangle = 0.4, 0.8, 1.2, 1.6, 2.0, 2.4eV$

under the assumption that $\delta m_{31}^2 \sim \delta m_{21}^2 = 6eV^2 \gg \delta m_{32}^2$ and $m_1 \ll \langle m_{\nu} \rangle$. Broken line is given by Eq.(23). The upper part from the broken line is allowed by the neutrino oscillation experiment at CHORUS.

**Fig.4:** The allowed regions by the inequalities of Eqs.(28) and (29) under the condition that $P_{\text{CHORUS}} < 2.5 \times 10^{-3}$, $\delta m_{31}^2 \sim \delta m_{21}^2 = 6eV^2 \gg \delta m_{32}^2$ with given $\theta_2$.

(a) $\theta_2 = 0, \frac{\pi}{2}$  \hspace{1cm} (b) $\theta_2 = \frac{\pi}{21}, \frac{11\pi}{21}$  \hspace{1cm} (c) $\theta_2 = \frac{\pi}{12}, \frac{5\pi}{12}$  \hspace{1cm} (d) $\theta_2 = \frac{\pi}{8}, \frac{3\pi}{8}$

(e) $\theta_2 = \frac{\pi}{6}, \frac{\pi}{3}$  \hspace{1cm} (f) $\theta_2 = \frac{\pi}{24}, \frac{7\pi}{24}$  \hspace{1cm} (g) $\theta_2 = \frac{\pi}{4}$

**Fig.5:** Feynman diagram of the $\mu^- - e^+$ conversion.
Fig. 1
Fig. 2
Fig. 3

\[ \langle m_\nu \rangle = 2.4 \text{eV} \]
Fig. 4
Fig. 5