Improved Wolf Pack Algorithm for Solving a Class of Multi-Objective Route Optimization Problem

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Abstract. This paper considers a class of Multi-objective path planning and solving algorithms. The choice of step size is a very important factor that affects the iterative algorithm. This paper improves the classical Wolf Pack Algorithm (WPA) by designing an adaptive step size. This new improved wolf pack algorithm (IWPA) improves the classical wolf colony algorithm by designing an adaptive step size. It effectively solves the problem that the WPA may fall into a endless loop. The simulation results by MATLAB show that the IWPA has a good effectiveness than the WPA to solve multi-objective route optimization.

Keywords: Wolf pack algorithm; Multi-objective path optimization; Wolf pack algorithm; Adaptive step size; Simulation.

1. Introduction

Multi-objective programming is an important class of optimization model, which has many applications in science and technology. The optimization of one goal often has an impact on the optimization of other goals. The Wolf Pack Algorithm (WPA) is one of the bio-inspired of algorithms that can be employed in order to approximate solutions for various optimization problems. WPA is a useful intelligent swarm optimization algorithm [1] [2] for its good performance in global search and local development. WPA is a population-based meta-heuristic stirred by the social hunting behaviour of wolves. It consists essentially in making wolves hunt. Under the command of a chief wolf, the trace of prey can be found and captured. It is widely used in underwater robot hunting, TSP business travel problem, sensor optimal placement and so on [3] [4]. WPA has been widely used in optimization because of its characteristics, such as distributed, systematic, adaptive, diversity, freedom and feedback [5][6].

In this paper, the WPA is applied to solve the linear weighted multi-objective path planning. By establishing the mathematical model of Improving Wolf Pack Algorithm (IWPA) by given the relationship between the optimizing radius and iterative steps, the optimization results is obtained. Through the simulation experiment with MATLAB, the result is show that the iterative convergence of the improved Wolf pack algorithm is better than the classic WPA.

2. Multi-objective Path Planning

Multi-objective path planning is an important optimization problem. The traditional algorithm is very difficult to solve the multi-objective path optimal. So, it is usually to use Intelligent optimization algorithm to solve these kinds problems.
2.1. Mathematical Model of Linear Weighted Multi-objective Path Planning

Consider a kind of multi-objective optimization mathematical model which is widely used in solving practical problems. Each action function \( Y_i = f(X_i) \) has a weight \( w_i \). Choice the \( f_i \) to maximize

\[
Y = \sum_{i=1}^{m} w_i f(X_i)
\]

The mathematical model of linear weighted multi-objective path planning is given as follows:

\[
\max Y = \max \sum_{i=1}^{m} w_i f(X_i) \quad \text{subject to} \quad w_1 + w_2 + \cdots + w_m = 1
\]

(1)

2.2. Modeling of Multi-objective Problem and Wolf Pack Algorithm

Let the feasible search domain is a \( N \times D \) Euclidean space (Wolves’ hunting range), where \( N \) is the sum of wolves in the pack, and \( D \) is the number of variables to be optimized. The state of a following wolf \( i \) can be expressed as \( X_i = (x_{i1}, x_{i2}, \ldots, x_{id}) \), where \( x_{id} \) is the position of the following wolf \( i \) in the \( d \)-dimension variable space. The odor concentration of prey can be perceived by the following wolf is \( Y_i = f(X_i), i=(1, 2, \ldots, N) \), where \( Y_i \) is the corresponding objective function value. The distance between the following wolf \( i \) and the following wolf \( j \) is calculated by Manhattan distance as follows:

\[
L = \sum_{d=1}^{D} |x_{id} - x_{jd}|
\]

(2)

2.3. Multi-objective Wolf Pack Algorithm

The WPA has two mechanisms and three behaviours.

i. Leading wolf generation mechanism

The leading wolf changes with the capture process and iteration process. In each capture process, if the objective function value of an individual in the wolf pack is greater than the leading wolf in the iteration process, the leader wolf position will be replaced. During the iteration, the location of the first wolf is recorded \( X_{lead} \), and the objective function value of the leading wolf is recorded as \( Y_{lead} \).

![Figure 1](image)

Figure 1. Figure of optimization target

ii. Wolf Pack regeneration mechanism

The food distribution rules of wolf pack will lead to the starvation of some weak wolves. Therefore, in the WPA, the wolf individual with the worst objective function value should be removed. At the same time, a number of new wolf pack individuals should be added randomly. The wolf moves forward in the \( h \) direction called step \( step_a \). When the wolf moves forward in the \( p \) direction, the position of wolf \( i \) in the \( d \)-dimension space is as follows:

\[
x_{id}^p = x_{id} + \sin(2\pi \times p / h) \times step_a
\]

(3)

where \( h \) is the amount of search direction, \( step_a \) is the search step size.

iii. Seeking behavior
In the process of searching for prey, the wolves do not go out as a whole, but choose the best wolves except the first one to search for prey first. Set the number of wolves as $N = \alpha + 1$, $\alpha$ is the scale factor.

iv. Calling behavior. When the first wolf calls on the nearby fierce wolf to set the position of the first wolf in a step $step_b$, then the position of the $i$-th fierce wolf in the $d$-dimensional variable space in the $k+1$ iteration is as follows:

$$x^{k+1}_{id} = x^k_{id} + step^d_b \times (g^k_d - x^k_{id})/|g^k_d - x^k_{id}|$$  \(4\)

Where $g^k_d$ is the position of the $k$-th generation wolf in the $d$-dimensional space. When attacking the prey, if the wolf $i$ can smell the prey, $Y_i > Y_{lead}$, then $Y_{lead} = Y_i$, if $Y_i < Y_{lead}$, the wolf $i$ will continue to attack until it is in contact with the leader. $\left[\min_{d} , \max_{d}\right]$ is the optimization range of variable values.

Where $\omega$ is distance judgment factor. The different values $\omega$ will affect the convergence speed of the algorithm. Generally, if $\omega$ is increasing, the convergence speed of the algorithm will improve, but if $\omega$ is greater, it is difficult to make the follow wolf into the fine searching region.

v. Behaviour of calling and pursuit. When the fierce wolf is close to the prey, the fierce wolf cooperates with the detection wolf to round up the prey.

$$x^{k+1}_{id} = x^k_{id} + \lambda \cdot step^d_c \cdot |G^k_d - x^k_{id}|$$  \(6\)

Where $\lambda \in [-1, 1]$ is random numbers with uniform distribution, $G^k_d$ is the position of prey in the dimension $d$ space of $k$-th iteration, $step^d_c$ is the search step size.

In $d$-dimensional space, there are the following relations among the three kinds of steps of the Wolf Pack Algorithm. Here, $step^d_s$ is search step, $step^d_c$ is capture step, and $step^d_a$ is attack step. What the relationship between these three steps is:

$$step^d_a = step^d_b / 2 = 2 \cdot step^d_c = \left|\max_{d} - \min_{d}\right| / S$$

Where $S$ is the step factor which represents the fine degree of searching for the optimal value in solution space.

3. Improved Wolf Pack Algorithm (IWPA)

Here, the Wolf Pack Algorithm in the siege stage has been improved. The size of the step will greatly affect the efficiency of the capture. If it is not adjusted in time, it may fall into an infinite cycle. Therefore, the improvement is to replace the random step size $\lambda$ with an adaptive step size in the formula which changes linearly with the increase of the number of iterations $t$. The improved iterative formula is as follows:

$$x^{k+1}_{id} = x^k_{id} + \nu (1 - \varepsilon t / t_{max}) \times step^d_c \times |G^k_d - x^k_{id}|$$  \(7\)

where $\varepsilon$ is a random number with standard normal distribution from(0,1), $\nu$ is a random number with uniform distribution from [-1,1], $step^d_c$ is the search step size. The purpose of taking random numbers is to avoid limit to zero in the late iteration of the algorithm. If the “odor concentration of prey” perceived by the searching wolves after the siege is greater than the “odor concentration of prey”
perceived by the original position, the position of the head wolf will be updated. Otherwise, the position will be unchanged.

Figure 2. The Flow Chart of IWPA

4. Simulation Experiment

Here, simulation is given by software MATLAB. The maximum number of search set is 500 units. The step size is set to 10 units. The feasible region is a set of 100×100 square units. The simulation system consists of five “Wolfs” and one target. The capture radius is 150 units. The selection of step factor value is related to the radius of the seizing. The radius of this search region is 150 units. The siege is moving forward and returning, avoiding from missing the target and falling into the wireless cycle. Setting step factor is related to the radius of the siege. After many simulation experiments, the results are as follows: table 1 shows that the number of rounds iteration is smaller when the radius of the seizing is 15 times of the step size factor. The simulation experiments were carried out five times, the number of rounds was 1000 in one time, and the number of successful and failed rounds in the five tests was recorded in the table below. The success and failure ratio can be calculated.
Table 1. Set the length of Step

| Experiment times | Capture radius/Step size factor | Iteration times |
|------------------|--------------------------------|-----------------|
| 1                | 11                             | 480             |
| 2                | 13                             | 418             |
| 3                | 15                             | 379             |
| 4                | 17                             | 425             |
| 5                | 19                             | 469             |

Table 2 and table 3 show the experimental results of simulation of cooperative capture strategy based on basic Wolf Pack Algorithm and Improved Wolf Pack Algorithm respectively.

Table 2. The Results of Basic WPA for Static Target

| Experiment times | successful times | Failed times | Success rate |
|------------------|------------------|--------------|--------------|
| 1                | 609              | 391          | 1.56         |
| 2                | 623              | 377          | 1.65         |
| 3                | 611              | 389          | 1.57         |
| 4                | 591              | 409          | 1.45         |
| 5                | 587              | 413          | 1.42         |
| mean value       | 604.2            | 395.8        | 1.53         |

Table 3. The Results of IWPA for Static Target

| Experiment times | successful times | Failed times | Success rate |
|------------------|------------------|--------------|--------------|
| 1                | 679              | 321          | 2.12         |
| 2                | 663              | 337          | 1.97         |
| 3                | 661              | 339          | 1.95         |
| 4                | 691              | 309          | 2.24         |
| 5                | 627              | 373          | 1.68         |
| mean value       | 664.2            | 335.8        | 1.98         |

Compared with table 2 and table 3, the mean value of success rate of the IWPA has risen from 1.53 to 1.98. The simulation results show the effectiveness of IWPA.

5. Conclusion
In this paper, it is first to describe the classic Wolf Pack Algorithm. Based on the mathematical model, the design idea and key problem of the algorithm has been discussed. The optimization path has been given by designing an adaptive step size. Through simulation experiments, the feasibility of the cooperative siege strategy based on the classic WPA and the IWPA is verified. By comparing the two strategies, it is proved that the cooperative siege strategy is more suitable for the task of cooperative hunting than the classic WPA.

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