Some Remarks on Lower Bounds for Queue Machines
(Preliminary Report)

Holger Petersen*
Reinsburgstr. 75
70197 Stuttgart

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Abstract

We first give an improved lower bound for the deterministic online-simulation of tapes or pushdown stores by queues. Then we inspect some proofs in a classical work on queue machines in the area of Formal Languages and outline why a main argument in the proofs is incomplete.

1 Introduction

A classical result states that a storage organized as a queue can be the basis of universal computations. Implicitly this has been shown by Post in [Pos43], who proved the universality of normal productions. A normal production \( u \rightarrow v \) transforms a string \( ux \) into \( xv \). In order to obtain the equivalence with general formal systems nonterminal symbols are required.

This idea leading back to Post’s work is used in [Man74], where a Post machine is defined as an automaton with a finite control and queue storage. Some articles are devoted to similar models in the area of Formal Languages. Vollmar [Vol70] considers automata with queue storage. He focusses on automata working in real-time (the number of steps executed is equal to the length of the input) accepting with empty storage. Separation of deterministic and non-deterministic variants of this model are shown, the language classes are compared to the Chomsky Hierarchy, closure properties and decidability are considered. We will discuss some lower bound proofs from this article in Section 4. Another work on machines with queue storage in the area of Formal Languages is [Bra80].

While the universality of queue storages was known for a long time, results on the complexity of computations on such models were obtained much later. In [LLV92] Li, Longrée and Vitanyi mainly lower bounds for the mutual simulation of machines with a varying number of queues, pushdowns, and tapes are shown. The simulation of a pushdown by a queue requires \( \Omega(n^{4/3}/\log n) \) steps in the deterministic and nondeterministic case (here \( n \) denotes the number of

*Part of the research was done while the author was with the Universität Stuttgart.
steps simulated). An optimal quadratic lower bound holds for the deterministic simulation of a queue by a tape \cite{LV88}, while a nondeterministic simulation is possible in time $O(n^{3/2}\log^{1/2} n)$ \cite{Li88}. The lower bound for this simulation is $\Omega(n^{4/3}/\log^{2/3} n)$ \cite{LV88}.

For linear tapes it is known that many storages can be simulated efficiently on two storages. In the deterministic case the bound $O(n \log n)$ is due to Hennie and Stearns \cite{HS66}, while a real-time solution exists for nondeterministic machines \cite{BG70}. The latter result can be transferred to queue storages \cite[Theorem 4.5]{LLV92} and \cite[Theorem 4.2]{Li88}. Li, Longpré and Vitanyi left open the question whether a subquadratic solution is possible for queue storages in the deterministic case. Hühne \cite{Huh93} could show that machines with several linear storages can be simulated by $k$ queues in time $O(n^{1+1/k})$. For online simulations this solution is almost optimal since a lower bound $\Omega(n^{1+1/k}/\text{polylog } n)$ holds.

Below we give an overview of lower and upper bounds for simulations by queue machines.

### Deterministic simulations by one queue:

| simulation | lower bound | upper bound |
|------------|-------------|-------------|
| one one-turn pushdown | $\Omega(n^{4/3}/\log n)$ \cite[Theorem 3.2]{LLV92} | $O(n^{3/2})$ \cite[Theorem 3.2]{PR06} |
| one or two pushdowns one tape | $\Omega(n^{4/3}/\log n)$ \cite[Theorem 3.2]{LLV92} | $O(n^2)$ \cite[Theorem 3.2]{PR06} |
| two queues | $\Omega(n^2)$ \cite[Theorem 4.13]{LLV92} | $O(n^2)$ |

### Deterministic simulations by $k$ queues:

| simulation | lower bound (online) | upper bound |
|------------|----------------------|-------------|
| $k + 1$ queues | $\Omega(n^{1+1/k}/\log^{1/k} n)$ \cite[Theorem 4.2]{Huh93} | $O(n^{1+1/k})$ \cite[Theorem 3.2]{Huh93} |
| three pushdowns two tapes | $\Omega(n^{1+1/k}/\log^{1/k} n)$ \cite[Theorem 4.11]{LLV92} | $O(n^{1+1/k})$ \cite[Theorem 3.2]{Huh93} |

### Nondeterministic simulations by one queue:

| simulation | lower bound | upper bound |
|------------|-------------|-------------|
| two queues | $\Omega(n^2/\log^2 n \log \log n)$ \cite[Theorem 4.5]{LLV92} | $O(n^2)$ |

### Nondeterministic simulations by two queues:

| simulation | lower bound |
|------------|-------------|
| queues | $O(n)$ \cite[Theorem 4.2]{LLV92} |
| one pushdown | $O(n \log n)$ \cite{Ros98} |
| multi-dimensional tapes | $O(n \log^2 n)$ \cite{PR06} |

| upper bound |
|-------------|
| $O(n)$ \cite[Theorem 4.2]{LLV92} |
| $O(n \log n)$ \cite{Ros98} |
| $O(n \log^2 n)$ \cite{PR06} |
2 Preliminaries

Queue machines are usually defined in the same way as Turing machines, the storage consisting of one or several queues instead of tapes [Vol70, LV88, LLV92, Huh93]. The machines have a separate one-way input tape and (determined by a finite control) can do the following in one step:

- Read zero or one symbol from the input.
- Pop zero or one symbol from the queue.
- Push zero or one symbol onto the queue.
- Change state.

Acceptance is indicated in different ways: By writing a 0 or a 1 [LV88] or by empty storage [Vol70, LLV92, Huh93]. A simple buffer automaton in the sense of [Vol70] is a queue machine accepting with empty storage and working in real-time (one input symbol is read in each step). Notice that in [Vol70] the empty string is by definition not accepted.

The Post machine from [Man74] is more restricted and has no separate input tape. The input is initially stored on the queue instead. This model is surprisingly powerful and can accept languages like \{a^n b^n | n \geq 0\} in linear time (the algorithm in [Man74, Example 1-9] is quadratic). Lower bounds carry over to this model, since the contents of the input tape can be transferred to the queue and then the Post machine can be simulated in real-time.

3 A Lower Bound for Simulating Several Storages

Hühne has shown in [Huh93, Corollary 4.4], that the online-simulation of 2(k+1) tapes or pushdown stores by k queues requires \(\Omega(n^{1+1/k}/\log^{1/k} n)\) steps and that the analogous simulation of two tapes or three pushdown stores requires \(\Omega(n^{1+1/k}/\log^{1+2/k} n)\) time [Huh93, Corollary 4.5]. The proof is based on lower bounds for the computation of certain functions by machines with k queues. Our proof uses the same functions as the proof of Hühne, the difference being that we give specific algorithms for computing the functions on the models which are hard to simulate by machines with k queues. In contrast, Hühne’s proof uses general simulation results causing an additional overhead.

Function \(F_k\) is defined via a machine with k queues running in linear time. The relevant subset of possible input strings – on other inputs the output is arbitrary – is defined as

\[
L_k = \{x_{1,1} \cdots x_{1,f_1} \# \cdots \# x_{i,1} \cdots x_{i,f_i} \# \cdots \# x_{k,1} \cdots x_{k,f_k} | \\
x_{1,f_1+1} \cdots x_{i,f_i+1} \cdots x_{k,f_k+1} \# \cdots \# \\
x_{1,f_1+j} \cdots x_{i,f_i+j} \cdots x_{k,f_k+j} \# \cdots \# \\
x_{1,f_1+m} \cdots x_{i,f_i+m} \cdots x_{k,f_k+m} \# | \\
\forall 1 \leq i \leq k, 1 \leq j \leq f_i : x_{i,j} \in \{0,1\}\}
\]

On an input from the set \(L_k\) the k queue machine \(M_k\) fills its i-th queue with \(x_{i,1} \cdots x_{i,f_i}\) from the section of the input before the first \# (in this phase
the output is irrelevant). After the first symbol $ has been read, \( M_k \) works in \( m \) rounds. In round \( j \) machine \( M_k \) reads the symbols of the \( j \)-th string between $-symbols and appends sequentially the \( x_{i,f_i+j} \) to queue \( i \), while the first symbols are deleted and output. Every $ is output directly. Consequently the lengths of the queues are kept constant after the first $ has been read, and the output is \( x_{1,1} \cdots x_{k,1} \cdots x_{1,j} \cdots x_{k,j} \cdots x_{1,m} \cdots x_{k,m} \). Machine \( M_k \) works in linear time and computes \( F_k \).

**Lemma 1** For every \( k \geq 1 \) the function \( F_k \) defined above can be computed by a deterministic Turing machine with one tape and one pushdown store in linear time.

Proof. Turing machine \( T_k \) computing \( F_k \) uses \( k \) tracks on the work-tape in order to store the \( k \) strings on the queues of \( M_k \). After \( x_{i,1} \cdots x_{i,f_i} \) has been read and copied onto the \( i \)-th track, \( T_k \) returns to the first cell of the portion of the tape storing this string. After reading the first $, machine \( T_k \) starts to output the stored symbols and replaces them with symbols just read. Additionally they are marked as new. When the string on track \( i \) has been output completely, \( T_k \) returns to the first new symbol while copying the symbols read onto the pushdown store, then moves its head to the position where reading had been suspended, and writes the contents of the pushdown store onto the following cells of track \( i \). The section read is marked as old and reading resumes at the original position.

The initial phase in which the string \( x_{i,1} \cdots x_{i,f_i} \) is read can be completed in a number of steps proportional to the length of the string before the first $.

Theorem 1 Every online-simulation of one tape and one pushdown store, two tapes, or three pushdown stores by deterministic machines with \( k \) queues requires \( \Omega(n^{1+1/k}/\log^{1/k} n) \) steps.

Proof. The two other models can simulate one tape and one pushdown store in linear time. Therefore it is sufficient to show the lower bound for the latter model.

According to Lemma 1 the function \( F_{k+1} \) can be computed in linear time by one tape and one pushdown store. The proof of Theorem 4.2 in [Hüh93] shows, that every machine with \( k \) queue computing \( F_{k+1} \) requires \( \Omega(n^{1+1/k}/\log^{1/k} n) \) steps for input length \( n \).

4 An Analysis of Lower Bound Proofs for Queue Machines

In [Vol70] several separation results are presented related to the Chomsky Hierarchy. At the heart of the proofs is an argument claiming a quadratic lower bound on the time required to accept a certain language. We will review the proofs of these results.

**Theorem 2** (Satz 3.4 (b) of [Vol70]) There are deterministic context-sensitive, non-contextfree languages that are not accepted by deterministic simple buffer automata.
Approximately 20 years later, Li, Longpré, and Vitányi obtained a stronger result by showing that the following deterministic context-sensitive, non-context-free language in Section 4.2 of [LLV92] requires almost quadratic time on non-deterministic queue machines:

\[
Q = \{ a \& b_0 b_1 \cdot \cdot \cdot b_k \# b_0 b_1 b_2 b_3 \cdot \cdot \cdot b_{2i} b_i b_{2i+1} \cdot \cdot \cdot b_{k-1} b_{(k-1)/2} b_k \\
\cdot \cdot \cdot b_{2i \mod (k+1)} b_i b_{(2i+1) \mod (k+1)} \cdot \cdot \cdot b_{k-i} b_k a | b_i \in \{0,1\}^* \text{ for } 0 \leq i \leq k, \ k \text{ is odd}, a \in \{0,1\}^* \}
\]

(the language is renamed in order to disambiguate it from \( L \) below).

The earlier claim is nevertheless interesting, since it is based on the simple witness-language

\[
L = \{ wvcew | v \in \{0,1\}^+, w \in \{a,b\}^+ \}
\]

with a much shorter proof. We outline the main steps in the proof from [Vol70]:

1. A deterministic queue machine \( B \) is assumed to accept \( L \).
2. For every first portion \( wvc \) of the input, \( B \) has to enter a unique configuration.
3. Using a counting argument, the length of the queue has to be proportional to \(|wv|\) when \( wvc \) has been read.
4. By choosing \( w \) sufficiently long, not all of the queue contents can be removed while reading \( vcv \) and \( v \) can be arbitrarily long.
5. After having read the queue contents caused by \( w \) it is necessary to compare the queue contents encoding the two copies of \( v \).
6. For the comparison the two copies have to be moved to the front of the queue, which implies a quadratic running time, contradicting the real-time restriction of \( B \).

The first arguments are convincing, but from item 5 the proof is incomplete.

We are not able to give a more efficient algorithm for \( L \) than suggested by the proof in [Vol70]. We will define a language \( L' \) instead that satisfies the first items listed above, but can be accepted by a deterministic queue machine in a way contradicting item 5 above.

Let

\[
L' = \{ wvcw \pi(w) | v \in \{0,1\}^*, w \in \{a,b\}^*, |w| = 2^k \}
\]

where \( \pi \) is a permutation of the symbols of a word of length \(|w| = 2^k \) for \( k \geq 0 \) defined by

\[
\pi (w) = x_1 x_3 \cdot \cdot \cdot x_{2k-1} x_2 x_6 \cdot \cdot \cdot x_{2k-2} \cdot \cdot \cdot x_{2m} \cdot \cdot \cdot x_{2k-2m} \cdot \cdot \cdot x_{2k}
\]

where \( x_i \in \{a,b\} \) for \( 1 \leq i \leq 2^k \).

Permutation \( \pi \) can be defined arbitrarily on strings not having a length of a power of two.
Observation 1  
Language $L'$ can be accepted by a deterministic queue machine $M$ that works in real-time on the prefix $wvcv$ of an accepted input and in linear overall time.

Proof. We make use of the fact that computations making a bounded number of steps between reading consecutive input symbols can be converted into real-time computations (by compressing the storage).

The queue machine $M$ accepting $L'$ first stores a prefix of the input from $\{a, b\}^* \{0, 1\}^* c \{0, 1\}^*$ on the queue. When reading the next symbol from $\{a, b\}$, the machine starts to read one symbol $x$ from the queue, compares it to the current input symbol, and rejects if the symbols are different. The next input symbol is stored on the queue. Continuing this process $M$ removes one out of two symbols from the queue until a symbol from $\{0, 1\}$ is at the head of the queue. $M$ remembers this symbol in its finite control and drops it while keeping all symbols from $\{0, 1\}$ until $c$ is read from the queue. The next symbol from $\{0, 1\}$ after $c$ is compared with the symbol stored in the finite control rejecting the input if the symbols are different. The following string from $\{0, 1\}^*$ is cycled to the rear of the queue. If all stored symbols of $w$ and $v$ can be removed in this way, $M$ accepts its input.

The symbols of a prefix $w \in \{a, b\}^*$ of the input will be permuted according to $\pi$ on the queue under the assumption that the length constraints are satisfied. If the input has the form $wvcv'w'$ with $v, v' \in \{0, 1\}^+$, $|v| = |v'|$, $w, w' \in \{a, b\}^+$, and $2^{|v|} = |w| = |w'|$ then $M$ accepts if $v = v'$ and $\pi(w) = w'$. If the length constraints are not satisfied, $M$ rejects.

For the time bound notice that the steps of $M$ after having read $wvcv$ are determined by the size of the queue. If we divide $M$’s computation into cycles that start with a queue storing a string from $\{a, b\}^* \{0, 1\}^* c \{0, 1\}^*$, then the length of the queue at the start of cycle $i$ is $2^k - i + 1 + 2(k - i + 1) + 1$, where $k = |v|$.

The total running time after having read $wvcv$ can be bounded by

$$2 + \sum_{i=1}^{k+1} 2^{k-i+1} + 2(k - i + 1) + 1 = 2 + 2^{k+1} - 1 + k^2 + 2k + 1 = O(n).$$

Since the first part of the computation is real-time we obtain the claimed time bound. □

Another result the proof of which refers to the incomplete argument of Satz 3.4 (b) is:

Theorem 3 (Satz 3.5 (b) of [Vol70]) There are deterministic context-free languages that are not accepted by deterministic simple buffer automata.

The witness language is:

$$\{ wvcv^R w^R \mid v \in \{0, 1\}^+, w \in \{a, b\}^+ \}.$$

Here the lower time bound $\Omega(n^{4/3} / \log n)$ on palindrome recognition by non-deterministic one-queue machines from [LLV92] applies to the witness language.

Finally the proof of the following non-closure property is affected:

Claim 1 (Satz 4.1 (a) of [Vol70]) The class of languages accepted by deterministic simple buffer automata is not closed under intersection.

This claim does not seem to follow easily from known results.
5 Discussion

For the online simulation of two tapes or three pushdown stores by \( k \) queues we could improve the lower bound \( \Omega(n^{1+1/k} / \log^{1+2/k} n) \) to \( \Omega(n^{1+1/k} / \log^{1/k} n) \).

Then we investigated proofs in a classical paper on queue machines. It turned out that the main argument in one of the proofs is incomplete, although some results relying on the argument are true as can be concluded from complexity results published much later.

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