Sum rules for strong decays of 10 and 27 dibaryon multiplets in broken SU(3) symmetry

G. Sánchez-Colón* and E. N. Polanco-Euán

Departamento de Física Aplicada.
Centro de Investigación y de Estudios Avanzados del IPN.
Unidad Mérida.
A.P. 73, Cordemex.
Mérida, Yucatán, 97310. MEXICO.

C. E. López-Fortín
Facultad de Ingeniería.
Universidad Autónoma de Yucatán.
A.P. 150, Cordemex.
Mérida, Yucatán. MEXICO.

(Dated: February 20, 2024)

Abstract

10 and 27 SU(3) multiplets of hexaquarks states with baryon number $B = 2$, known as dibaryons, are considered. Previous analyses are complemented and sum rules for strong decay amplitudes with SU(3) symmetry violation up to the first order are determined for the 10 and 27 dibaryon multiplets into a baryon octet plus a baryon decuplet and for the dibaryon 27 into two baryon octets.

PACS numbers: 14.20.Pt, 13.30.-a

Keywords: dibaryon, sextaquarks, sum rule

*gabriel.sanchez@cinvestav.mx
I. INTRODUCTION

A dibaryon denotes a sextaquark state with baryon number $B = 2$. The first known dibaryon has been the deuteron discovered in 1932 [1]. In 1963, Oakes [2] examined the role of deuteron in the “eightfold way” [3, 4] and found that it belongs to the irreducible representation $10^*$ resulting from the SU(3) decomposition of two baryon octets, $8 \otimes 8$.

Interest in the study of dibaryons rose in 1977 when Jaffe [5] predicted the so-called H-dibaryon, a bound $\Lambda \Lambda$ system containing two strange quarks. Diverse amount of predicted states triggered a worldwide rush of experimental dibaryon searches [6–8]. Recently, two groups announced that lattice QCD calculations [9–11] provide evidence for a bound H-dibaryon, as predicted. Despite the historical difficulties in the experimental search for multi-quarks states, tetraquarks ($q_1 \bar{q}_2 q_3 \bar{q}_4$) and pentaquarks ($q_1 q_2 q_3 q_4 \bar{q}_5$) systems have been recently observed [12, 13]. Moreover, WASA at COSY collaboration have finally confirmed solid evidence for the existence of a dibaryon [14, 15], denoted $d^*(2380)$. Theoretical models and analysis of experimental data have also suggested possible existence of dibaryon states belonging to $8$ [16] and $27$ [17, 18] SU(3) representations.

It is in this context that the study of sum rules for decay amplitudes turns out to be helpful in the hunting for possible multi-quark states.

A dibaryon has a total of six quarks, for it to be a color singlet it must be a baryon-baryon state and its SU(3) properties depend on the SU(3) properties of the two baryon it is made of. If both baryons are octets, then the SU(3) transformation properties of the dibaryon can be $1$, $8$, $10$, $10^*$, or $27$. If one of the baryons is an octet and the other one is a decuplet then the dibaryon can transform as $8$, $10$, $27$, or $35$.

Flavor SU(3) symmetry breaking effects up to the first order are taken into account in Refs. [19, 20] and sum rules are calculated for strong decays of dibaryon octet $8$ (denoted $D_8$) into two baryon octets (denoted $B_8$), $D_8 \rightarrow B_8 + B_8$, and into a baryon octet plus a baryon decuplet (baryon resonances, denoted $R_{10}$), $D_8 \rightarrow B_8 + R_{10}$. Corresponding sum rules for decays of dibaryon SU(3) decuplets, $10^*$ and $10$ (denoted $D_{10^*,10}$, respectively), into two baryon octets, $D_{10^*,10} \rightarrow B_8 + B_8$, are reported in Ref. [21].

In this paper, analyses of Refs. [19–21] are complemented. Sum rules for strong decays amplitudes with first order breaking of the SU(3) symmetry are determined for dibaryon multiplets $10$ ($D_{10}$) and $27$ (denoted $D_{27}$) into a baryon octet plus a baryon decuplet,
$D_{10,27} \rightarrow B_8 + R_{10}$.

Additionally, sum rules in broken SU(3) symmetry up to first order are determined for strong decays of $D_{27}$ into two baryon octets, $D_{27} \rightarrow B_8 + B_8$.

Using the traditional method, the SU(3) symmetry breaking interaction term transforms as the eight component of the unitary spin, that is, like the hypercharge [3, 4].

In Sec. II, to establish notation and for practical use, the general formula for the two body decay amplitude including SU(3) first order breaking is determined. Secs. III and IV include the analysis of $D_{10,27} \rightarrow B_8 + R_{10}$ decays, respectively. $D_{27} \rightarrow B_8 + B_8$ decays are studied in Sec. V. Finally, a summary of results obtained and concluding remarks are presented in Sec. VI.

II. DECAY AMPLITUDE IN BROKEN SU(3) GENERAL FORMULA

Consider a two body decay, $a \rightarrow b + c$, of an initial state $|a\rangle \equiv |\mu_a, n_a\rangle$ into $|b\rangle \equiv |\mu_b, n_b\rangle$ plus $|c\rangle \equiv |\mu_c, n_c\rangle$, where $\mu$ is the belonging representation of the corresponding state and $n = (Y, I, I_3)$ collectively denotes its quantum numbers of hypercharge, isospin, and the third component of isospin, respectively. The decay amplitude is given by the matrix element

$$G[a \rightarrow bc] \equiv (\langle c|b\rangle) H_{\text{int}} |a\rangle = (\langle c|b\rangle \langle \mu_b, n_b | H_{\text{int}} |\mu_a, n_a\rangle$$

$$= \sum_{\mu, \gamma, n} \left( \begin{array}{c} \mu_b \mu_c \\ n_b \\ n_c \end{array} \right) \left( \begin{array}{c} \mu_{\gamma} \\ n \end{array} \right) \langle \mu_{\gamma}, n | H_{\text{int}} |\mu_a, n_a\rangle,$$  

(1)

where $H_{\text{int}}$ is the interaction Hamiltonian. Final composite state has been expanded by using SU(3) Clebsch-Gordan coefficients following notation of Ref. [22]. Representation $\mu_{\gamma}$ goes through all representations appearing in the $\mu_b \otimes \mu_c$ decomposition and quantum numbers $n$ are obtained from $n_b$ and $n_c$ by $Y = Y_b + Y_c$, $I = |I_b - I_c|, \ldots, I_b + I_c$, and $I_3 = I_{3b} + I_{3c}$.

In the flavor SU(3) symmetry limit, that is, with $H_{\text{int}} = H_{\text{st}}$, the matrix element appearing in second line of general expression (1) satisfies

$$\langle \mu_{\gamma}, n | H_{\text{st}} |\mu_a, n_a\rangle = g^0_{\mu_{\gamma}} \delta_{\mu_{\gamma}, \mu_a} \delta_{n, n_a},$$  

(2)

where $g^0_{\mu_{\gamma}}$ is a coupling constant parameter.
To apply the octet “spurion” formalism in this case, define a fictitious state as the $Y = 0, I = 0$ component of the SU(3) octet, $|S_8\rangle \equiv |8, n_S\rangle$, with $n_S = (0,0,0)$. Then, up to first order, SU(3) symmetry breaking effects are absorbed into the interaction of initial state with $|S_8\rangle$ and the original decay, $a \xrightarrow{H_{ms}} b + c$, can then be treated as an SU(3) invariant “dispersion”, $S_8 + a \xrightarrow{H_{st}} b + c$.

With $H_{int} = H_{ms}$, the matrix element in second line of (1) is then given by:

$$
\langle \mu, n | H_{ms} | \mu_a, n_a \rangle = \langle \mu, n | H_{st} (|S_8\rangle | \mu_a, n_a \rangle)
$$

$$
= \sum_{\nu, n'} \left( 8 \begin{array}{c} \mu_a \\
S \begin{array}{c} \mu \\
n \begin{array}{c} n_a \\
\nu \\
n' \\n\end{array} \end{array} \end{array} \right) \langle \mu, n | H_{st} | \nu, n' \rangle
$$

$$
= \sum_{\nu} \left( 8 \begin{array}{c} \mu_a \\
S \begin{array}{c} \mu \\
n \begin{array}{c} a \\
\nu \\
\end{array} \end{array} \end{array} \right) g_{\nu, \delta_{\mu, \nu_a}}
$$

Now, it is the initial composite state, $|S_8\rangle | \mu_a, n_a \rangle$, that has been expanded by using Clebsch-Gordan coefficients. $\nu$ goes through all representations in $8 \otimes \mu_a$, quantum numbers $n' = n_a$ due to $n_S = (0,0,0)$, and $g_{\nu}$ is a coupling constant parameter.

From the previous analysis, the general expression of the decay amplitude that includes violations up to first order of the SU(3) flavor symmetry is obtained from Eqs. (1) to (3) with $H_{int} = H_{st} + H_{ms}$:

$$
G[a \rightarrow b c] = \sum_{\mu, \nu} \left( \begin{array}{c} \mu_b \\
\mu \begin{array}{c} \mu \end{array} \end{array} \right) \left( \begin{array}{c} \mu_c \\
_\nu \begin{array}{c} \nu \\
n \begin{array}{c} a \\
\nu \end{array} \end{array} \end{array} \right) g_{\mu, \delta_{\mu, \mu_a}}
$$

$$
+ \sum_{\mu, \nu} \left( \begin{array}{c} \mu_b \\
\mu \begin{array}{c} \mu \end{array} \end{array} \right) \left( \begin{array}{c} \mu_c \\
_\nu \begin{array}{c} \nu \\
n \begin{array}{c} a \\
\nu \end{array} \end{array} \end{array} \right) \sum_{\nu, n'} \left( 8 \begin{array}{c} \mu_a \\
S \begin{array}{c} \mu \\
n \begin{array}{c} a \\
\nu \\
n' \begin{array}{c} \nu \\
\end{array} \end{array} \end{array} \end{array} \right) g_{\nu, \delta_{\mu, \nu_a}}
$$

first line in the right hand side corresponds to the SU(3) symmetry limit and second line to SU(3) first order breaking. Representation $\mu, \gamma$ goes through all representations in $\mu_b \otimes \mu_c$ and $\gamma$ distinguishes repeated appearances of a representation in this expansion. Similarly, $\nu$ goes through all representations in $8 \otimes \mu_a$ and $\tau$ distinguishes repeated appearances of a representation in $8 \otimes \mu_a$. 

4
III.  \( D_{10} \rightarrow B_8 + R_{10} \)

Strong decays of dibaryons in the decuplet into an octet plus a decuplet of baryons, \( D_{10} \rightarrow B_8 + R_{10} \), are analyzed.

Decay amplitudes in first order SU(3) breaking will be obtained from Eq. (4). In this case, \( |a\rangle = |10, n_D\rangle \equiv |D_{10}^n\rangle \), \( |b\rangle = |8, n_B\rangle \equiv |B\rangle \), and \( |c\rangle = |10, n_R\rangle \equiv |R\rangle \), so that, \( \mu_a = 10 \), \( \mu_b = 8 \), and \( \mu_c = 10 \). Therefore, decompositions of composite final state \( (\mu_b \otimes \mu_c) \) and spurion-initial particle \( (8 \otimes \mu_a) \) product representations are the same:

\[
\mu_b \otimes \mu_c = 8 \otimes \mu_a = 8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35,
\]

no repeated representations are present in these expansions so that indexes \( \gamma \) and \( \tau \) are not necessary here.

General formula (4) reduces in this case to:

\[
G \left[ D_{nD}^{10} \rightarrow B R \right] = \left( \begin{array}{c|c} 8 & 10 \\ \hline n_B & n_D \end{array} \right) g_{10}^0 + \sum_\mu \left( \begin{array}{c|c} 8 & 10 \\ \hline n_B & n_R \end{array} \right) \left( \begin{array}{c|c} \mu & \mu \\ \hline n_S & n_D \end{array} \right) g_\mu,
\]

with \( \mu = 8, 10, 27, 35 \). In exact SU(3), only one parameter, \( g_{10}^0 \), determines all the strong decay amplitudes of the dibaryon decuplet \( D_{10} \) into \( B_8 + R_{10} \). Due to SU(3) breaking, there are four extra parameters \( g_8, g_{10}, g_{27}, \) and \( g_{35} \).

A.  \( D_{10} \rightarrow B_8 + R_{10} \) sum rules

In this section, sum rules in broken SU(3) for the strong decays \( D_{10} \rightarrow B_8 + R_{10} \) are determined. Quantum numbers of components states, \( D_{nD}^{10} \), \( B \), and \( R \), of corresponding dibaryon decuplet \( D_{10} \), \( J^P = (1/2)^+ \) baryon octet \( B_8 \), and \( J^P = (3/2)^+ \) baryon decuplet \( R_{10} \) are shown in Figs. 1, 2, and 3, respectively.

By the use of Eq. (6) and following sign conventions for SU(3) Clebsch-Gordan coefficients from Ref. [22], it is found that there are 13 linearly independent decay amplitudes, they are taken as:

\[
G \left[ D_{(1,3/2,+3/2)}^{10} \rightarrow p \Sigma^{++} \right] \equiv X_1, \quad G \left[ D_{(1,3/2,+3/2)}^{10} \rightarrow \Sigma^+ \Delta^+ \right] \equiv X_2,
\]

(7)
Relationships for the remaining non-zero dependent amplitudes in terms of the independent ones are given in Sec. A 1 of Appendix A.

The 13 independent decay amplitudes are given in terms of the 5 parameters $g_{10}^0, g_8, g_{10}, g_{27},$ and $g_{35},$ consequently, 8 sum rules in broken SU(3) can be established:

\[ X_1 - \sqrt{3}X_4 + X_8 = 0, \]  
\[ -2X_2 + \sqrt{3}X_4 + \sqrt{6}X_5 - 2X_8 = 0, \]  
\[ -4X_3 - 3\sqrt{6}X_4 + 6X_6 + 6\sqrt{2}X_8 - 2X_{12} = 0, \]  
\[ -X_2 + \sqrt{2}X_3 + 2\sqrt{3}X_7 + 2X_8 - \sqrt{2}X_{12} = 0, \]  
\[ -X_2 + \sqrt{3}X_4 - 2X_8 + \sqrt{3}X_9 = 0, \]  
\[ -\sqrt{2}X_3 - 3\sqrt{3}X_4 + 6X_8 + 3\sqrt{2}X_{10} - 2\sqrt{2}X_{12} = 0, \]
\[-\sqrt{2}X_2 + 2X_3 + \sqrt{6}X_4 + 2\sqrt{3}X_{11} - 2X_{12} = 0, \quad (20)\]

\[-X_2 + \sqrt{2}X_3 + 2\sqrt{3}X_4 - 2X_8 - \sqrt{2}X_{12} + 2X_{13} = 0. \quad (21)\]

IV. $D_{27} \rightarrow B_8 + R_{10}$

Strong decays of dibaryons in the $27$-multiplet into an octet plus a decuplet of baryons, $D_{27} \rightarrow B_8 + R_{10}$, are analyzed. SU(3) first order breaking decay amplitudes are given by Eq. (4). In this case, $|a\rangle = |27, n_D\rangle \equiv |D^{27}_{n_D}\rangle$, $|b\rangle = |8, n_B\rangle \equiv |B\rangle$, and $|c\rangle = |10, n_R\rangle \equiv |R\rangle$, so that, $\mu_a = 27$, $\mu_b = 8$, and $\mu_c = 10$. Therefore,

$$\mu_b \otimes \mu_c = 8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35, \quad (22)$$

in this case, index $\gamma$ is not necessary because there are no repeated representations in this expansion. Also,

$$8 \otimes \mu_a = 8 \otimes 27 = 8 \oplus 10 \oplus 10^* \oplus 27 \oplus 27' \oplus 35 \oplus 35^* \oplus 64, \quad (23)$$

so that, index $\tau$ is necessary due to the double appearance of the $27$.

General expression (4) reduces to

$$G\left[D^{27}_{n_D} \rightarrow B R\right] = \begin{pmatrix} 8 & 10 \\ n_B & n_R \end{pmatrix} \begin{pmatrix} 27 \\ n_D \end{pmatrix} g^0_{27}$$

$$+ \sum_{\mu} \begin{pmatrix} 8 & 10 \\ n_B & n_R \end{pmatrix} \begin{pmatrix} \mu \\ n_D \end{pmatrix} \sum_{\nu} \begin{pmatrix} 8 & 27 \\ n_s & n_D \end{pmatrix} \begin{pmatrix} \nu \tau \\ n_D \end{pmatrix} g_{\nu, \delta_{\mu,\nu}}, \quad (24)$$

with $\mu = 8, 10, 27, 35$, and $\nu_{\tau} = 8, 10, 10^*, 27, 27', 35, 35^*, 64$. In exact SU(3), one parameter, $g^0_{27}$, determines all the strong decay amplitudes of $D_{27}$ into $B_8 + R_{10}$. Due to SU(3) breaking, there are five extra parameters $g_8, g_{10}, g_{27}, g_{27'}$, and $g_{35}$.
A. \( D_{27} \to B_8 + R_{10} \) sum rules

Sum rules for strong decay amplitudes in broken SU(3) of dibaryon 27-plet, \( D_{27} \) (quantum numbers of the components states, \( D_{27}^{R_D} \), are shown in Fig. 4), into a final state of an ordinary baryon octet (Fig. 2) plus a baryon decuplet (Fig. 3), \( D_{27} \to B_8 + R_{10} \), are determined.

From Eq. (24) and Ref. [22], there are 22 linearly independent decay amplitudes that can be taken as:

\[
G[D_{27}^{27(2,1,+1)} \to p\Delta^+] \equiv Y_1, \quad G[D_{27}^{27(1,3/2,+3/2)} \to p\Sigma^{+*}] \equiv Y_2, \quad (25)
\]

\[
G[D_{27}^{27(1,3/2,+3/2)} \to \Sigma^+\Delta^+] \equiv Y_3, \quad G[D_{27}^{27(1,3/2,+3/2)} \to \Lambda\Delta^{++}] \equiv Y_4, \quad (26)
\]

\[
G[D_{27}^{27(1,1/2,+1/2)} \to p\Sigma^0\pi^+] \equiv Y_5, \quad G[D_{27}^{27(1,1/2,+1/2)} \to \Sigma^+\Delta^0] \equiv Y_6, \quad (27)
\]

\[
G[D_{27}^{27(0,2,+2)} \to \Sigma^+\Sigma^{++}] \equiv Y_7, \quad G[D_{27}^{27(0,2,+2)} \to \Xi^0\Delta^{++}] \equiv Y_8, \quad (28)
\]

\[
G[D_{27}^{27(0,1,+1)} \to p\Xi^0\pi^+] \equiv Y_9, \quad G[D_{27}^{27(0,1,+1)} \to \Sigma^+\Sigma^0\pi^+] \equiv Y_{10}, \quad (29)
\]

\[
G[D_{27}^{27(0,1,+1)} \to \Lambda\Sigma^{++}] \equiv Y_{11}, \quad G[D_{27}^{27(0,1,+1)} \to \Xi^0\Sigma^+] \equiv Y_{12}, \quad (30)
\]

\[
G[D_{27}^{27(0,0,0)} \to p\Xi^-\pi^+] \equiv Y_{13}, \quad G[D_{27}^{27(0,0,0)} \to \Sigma^+\Sigma^-\pi^+] \equiv Y_{14}, \quad (31)
\]

\[
G[D_{27}^{27(-1,3/2,+3/2)} \to \Sigma^+\Xi^0\pi^+] \equiv Y_{15}, \quad G[D_{27}^{27(-1,3/2,+3/2)} \to \Xi^0\Sigma^{++}] \equiv Y_{16}, \quad (32)
\]

\[
G[D_{27}^{27(-1,1/2,+1/2)} \to p\Omega^-] \equiv Y_{17}, \quad G[D_{27}^{27(-1,1/2,+1/2)} \to \Sigma^+\Xi^-\pi^+] \equiv Y_{18}, \quad (33)
\]

\[
G[D_{27}^{27(-1,1/2,+1/2)} \to \Lambda\Xi^0\pi^+] \equiv Y_{19}, \quad G[D_{27}^{27(-1,1/2,+1/2)} \to \Xi^0\Sigma^0\pi^+] \equiv Y_{20}, \quad (34)
\]

\[
G[D_{27}^{27(-2,1,+1)} \to \Sigma^+\Omega^-] \equiv Y_{21}, \quad G[D_{27}^{27(-2,1,+1)} \to \Xi^0\Xi^0\pi^+] \equiv Y_{22}. \quad (35)
\]
Relationships for the remaining non-zero dependent amplitudes in terms of the independent ones can be found in Sec. A 2 of Appendix A.

The 22 independent amplitudes are described by 6 parameters, so, 16 sum rules can be established:

\[ Y_1 + \sqrt{2}Y_2 - \sqrt{2}Y_3 - Y_7 = 0, \]  
(36)

\[ \sqrt{6}Y_1 - \sqrt{3}Y_3 - \sqrt{6}Y_4 + \sqrt{2}Y_8 = 0, \]  
(37)

\[-6\sqrt{3}Y_1 - 4\sqrt{6}Y_2 + 6\sqrt{5}Y_5 + 3\sqrt{15}Y_9 = 0, \]  
(38)

\[ 9\sqrt{3}Y_1 + \sqrt{6}Y_2 - 5\sqrt{6}Y_3 - 6\sqrt{5}Y_5 + 6\sqrt{10}Y_6 + 3\sqrt{30}Y_{10} = 0, \]  
(39)

\[ 3\sqrt{2}Y_1 - 2Y_2 - 2\sqrt{2}Y_4 - \sqrt{30}Y_5 + \sqrt{15}Y_{11} = 0, \]  
(40)

\[-3\sqrt{6}Y_1 - 5\sqrt{3}Y_3 + 3\sqrt{6}Y_4 - 12\sqrt{5}Y_6 + 6\sqrt{30}Y_{12} = 0, \]  
(41)

\[ 2\sqrt{3}Y_1 - 3\sqrt{5}Y_5 + \sqrt{10}Y_{13} = 0, \]  
(42)

\[-4Y_1 + \sqrt{15}Y_5 - 2\sqrt{30}Y_6 + \sqrt{30}Y_{14} = 0, \]  
(43)

\[-6\sqrt{2}Y_1 - 4Y_2 + 4Y_3 + \sqrt{30}Y_6 - \sqrt{15}Y_7 + 3Y_{15} = 0, \]  
(44)

\[ 6\sqrt{2}Y_1 - 2Y_2 - Y_3 - 3\sqrt{2}Y_4 - \sqrt{30}Y_5 + \sqrt{15}Y_6 + 3Y_{16} = 0, \]  
(45)

\[-3\sqrt{2}Y_1 - Y_2 + \sqrt{30}Y_5 + \sqrt{5}Y_{17} = 0, \]  
(46)

\[ 21\sqrt{2}Y_1 + 2Y_2 - 5Y_3 - 5\sqrt{30}Y_5 + 8\sqrt{15}Y_6 + 3\sqrt{15}Y_{18} = 0, \]  
(47)
\[ 3Y_1 - \sqrt{2}Y_2 - Y_4 - \sqrt{15}Y_5 + \sqrt{5}Y_{19} = 0, \quad (48) \]

\[ -6\sqrt{2}Y_1 + 2\sqrt{2}Y_2 - 5Y_3 + 3\sqrt{2}Y_4 + \sqrt{30}Y_5 - 4\sqrt{15}Y_6 + 3\sqrt{30}Y_{20} = 0, \quad (49) \]

\[ -9Y_1 - \sqrt{2}Y_2 + \sqrt{2}Y_3 + 2\sqrt{15}Y_5 - \sqrt{30}Y_6 + \sqrt{3}Y_{21} = 0, \quad (50) \]

\[ 9\sqrt{6}Y_1 - 4\sqrt{3}Y_2 + \sqrt{3}Y_3 - 3\sqrt{6}Y_4 - 6\sqrt{10}Y_5 + 6\sqrt{5}Y_6 + 3\sqrt{6}Y_{22} = 0. \quad (51) \]

V. \( D_{27} \rightarrow B_8 + B_8 \)

Strong decays of dibaryons in the 27-multiplet into two baryon octets, \( D_{27} \rightarrow B_8 + B_8 \), are analyzed.

Decay amplitudes with first order SU(3) symmetry breaking are obtained from Eq. (4). In this case, \(|a\rangle = |27, n_D\rangle \equiv |D_{27}^{n_D}\rangle, |b\rangle = |8, n_B\rangle \equiv |B\rangle, \) and \(|c\rangle = |8, n_{B'}\rangle \equiv |B'\rangle\), so that, \( \mu_a = 27, \mu_b = \mu_c = 8 \). For sake of clarity, a prime is placed on the baryon in second octet, but it is understood that \( B \) and \( B' \) are identical baryons. Here,

\[ \mu_b \otimes \mu_c = 8 \otimes 8 = 1 \oplus 8 \oplus 8' \oplus 10 \oplus 10' \oplus 27, \quad (52) \]

index \( \gamma \) is necessary because the octet appears twice in this expansion. Also,

\[ 8 \otimes \mu_a = 8 \otimes 27 = 8 \oplus 10 \oplus 10' \oplus 27 \oplus 27' \oplus 35 \oplus 35' \oplus 64 \quad (53) \]

and index \( \tau \) is also necessary due to the double appearance of the 27.

General expression (4) reduces to

\[
G \left[ D_{n_D}^{27} \rightarrow BB' \right] = \left( \begin{array}{c|c} 8 & 8 \\ n_B & n_{B'} \end{array} \right) \begin{array}{c} 27 \\ n_D \end{array} g_{27}^0 + \sum_{\mu_\gamma} \left( \begin{array}{c|c} 8 & 8 \\ n_B & n_{B'} \end{array} \right) \sum_{\nu_\tau} \left( \begin{array}{c|c} 8 & 27 \\ n_S & n_{D} \end{array} \right) \left( \begin{array}{c|c} \nu_T \\ n_D' \end{array} \right) g_{\nu_\tau} \delta_{\mu_{\gamma}, \nu_{\tau}}, \quad (54)
\]

with \( \mu_{\gamma} = 1, 8, 8', 10, 10', 27 \), and \( \nu_{\tau} = 8, 10, 10', 27, 27', 35, 35', 64 \).
In the final state decomposition, Eq. (52), $8'$ octet and $10$ and $10^*$ decuplets are antisymmetric under exchange of the two octet baryon states. Singlet $1$, octet $8$, and $27$-plet are symmetric. There are seven parameters in Eq. (54), three of them (all coming from SU(3) breaking) correspond to couplings with flavor-antisymmetric final state representations $\langle 8'|8 \rangle$, $\langle 10|10 \rangle$, and $\langle 10^*|10^* \rangle$, denoted $g_{8'}$, $g_{10}$, and $g_{10^*}$, respectively. Couplings with flavor-symmetric final state representations $\langle 8|8 \rangle$, $\langle 27|27 \rangle$, and $\langle 27|27' \rangle$, introduce four additional parameters, $g_{27}^0$ (from exact SU(3)) and $g_{8}$, $g_{27}$, and $g_{27'}$ (from SU(3) first order breaking).

A. $D_{27} \rightarrow B_8 + B_8$ sum rules

Due to the presence of two identical fermions in the final state of $D_{27} \rightarrow B_8 + B_8$ decays, it is important to consider symmetry properties under interchange of the two final baryons. Both cases, antisymmetric and symmetric final state, will be analyzed separately.

1. Flavor-antisymmetric final state

Sum rules for the strong decays of the $27$-plet of dibaryons (Fig. 4) into a flavor-antisymmetric final state of two ordinary $J^P = (1/2)^+$ baryon octets (Fig. 2) are determined. As mentioned above, in this case, the corresponding decay amplitudes will be described by 3 parameters, $g_{8'}$, $g_{10}$, and $g_{10^*}$, all of them from first order SU(3) symmetry breaking.

From Eq. (54) and Ref. [22], there are 10 linearly independent decay amplitudes, they can be chosen as:

\[ G[D_{(1,3/2,+3/2)}^{27} \rightarrow p\Sigma^+] \equiv Z_1^A, \quad G[D_{(1,1/2,+1/2)}^{27} \rightarrow p\Sigma^0] \equiv Z_2^A, \]

\[ G[D_{(1,1/2,+1/2)}^{27} \rightarrow n\Sigma^+] \equiv Z_3^A, \quad G[D_{(0,1/2,+1/2)}^{27} \rightarrow p\Xi^0] \equiv Z_4^A, \]

\[ G[D_{(0,1,+1)}^{27} \rightarrow \Sigma^+\Sigma^0] \equiv Z_5^A, \quad G[D_{(0,1,+1)}^{27} \rightarrow \Sigma^+\Xi^0] \equiv Z_6^A, \]

\[ G[D_{(0,0,0)}^{27} \rightarrow p\Xi^{-}] \equiv Z_7^A, \quad G[D_{(-1,1/2,+1/2)}^{27} \rightarrow \Sigma^+\Xi^{-}] \equiv Z_8^A, \]

\[ G[D_{(1,1/2,+1/2)}^{27} \rightarrow n\Xi^+] \equiv Z_9^A, \quad G[D_{(-1,1/2,+1/2)}^{27} \rightarrow n\Xi^0] \equiv Z_{10}^A. \]
\[ G[D_{(-1,1/2,1/2)}^{27} \to \Lambda \Xi^0] \equiv Z_5^A, \quad G[D_{(-1,3/2,3/2)}^{27} \to \Sigma^+ \Xi^0] \equiv Z_{10}^A. \tag{59} \]

Relationships for the remaining non-zero dependent amplitudes in terms of the independent ones are given in Sec. A 3 (a) of Appendix A.

Since the 10 independent decay amplitudes are described by three parameters, seven SU(3) broken sum rules may be established:

\[ 4Z_1^A - \sqrt{30}Z_2^A + 3\sqrt{10}Z_3^A - 3\sqrt{10}Z_4^A = 0, \tag{60} \]

\[ 2Z_1^A - 2\sqrt{30}Z_2^A + 3\sqrt{5}Z_5^A = 0, \tag{61} \]

\[ 2Z_1^A + \sqrt{30}Z_2^A - \sqrt{10}Z_3^A + \sqrt{15}Z_6^A = 0, \tag{62} \]

\[ 3Z_2^A + \sqrt{3}Z_3^A - 2\sqrt{2}Z_7^A = 0, \tag{63} \]

\[ \sqrt{2}Z_1^A - \sqrt{15}Z_2^A - \sqrt{5}Z_3^A + \sqrt{30}Z_8^A = 0, \tag{64} \]

\[ 2Z_1^A + \sqrt{30}Z_2^A + \sqrt{10}Z_3^A - 2\sqrt{10}Z_9^A = 0, \tag{65} \]

\[ \sqrt{15}Z_2^A - \sqrt{5}Z_3^A + \sqrt{2}Z_{10}^A = 0. \tag{66} \]

2. Flavor-symmetric final state

Now, sum rules for SU(3) symmetry breaking decays of dibaryons in the 27 (Fig. 4) into a flavor-symmetric final state of two baryon octets (Fig. 2) are calculated. As described at the beginning of this section, four coupling constants are involved in this case: \( g_0^{27} \) from exact SU(3), and \( g_8, g_{27}, \) and \( g_{27}^\prime, \) from SU(3) breaking.

From Eq. (54) and Ref. [22], there exist 14 linearly independent decay amplitudes:

\[ G[D_{(2,1,+1)}^{27} \to pp^\prime] \equiv Z_1^S, \quad G[D_{(1,3/2,3/2)}^{27} \to p\Sigma^+] \equiv Z_2^S. \tag{67} \]
\[ G[D_{(1,1/2,+1/2)}^{27} \rightarrow p \Sigma^0] \equiv Z_3^S, \quad G[D_{(1,1/2,+1/2)}^{27} \rightarrow p \Lambda'] \equiv Z_4^S, \] (68)

\[ G[D_{(0,2,+2)}^{27} \rightarrow \Sigma^+ \Sigma^+] \equiv Z_5^S, \quad G[D_{(0,1,+1)}^{27} \rightarrow p \Xi^0] \equiv Z_6^S, \] (69)

\[ G[D_{(0,1,+1)}^{27} \rightarrow \Sigma^+ \Lambda'] \equiv Z_7^S, \quad G[D_{(0,0,0)}^{27} \rightarrow p \Xi^{-}] \equiv Z_8^S, \] (70)

\[ G[D_{(0,0,0)}^{27} \rightarrow \Sigma^+ \Sigma^{-}] \equiv Z_9^S, \quad G[D_{(0,0,0)}^{27} \rightarrow \Lambda \Lambda'] \equiv Z_{10}^S, \] (71)

\[ G[D_{(-1,3/2,+3/2)}^{27} \rightarrow \Sigma^+ \Xi^0] \equiv Z_{11}^S, \quad G[D_{(-1,1/2,+1/2)}^{27} \rightarrow \Sigma^+ \Xi^{-}] \equiv Z_{12}^S, \] (72)

\[ G[D_{(-1,1/2,+1/2)}^{27} \rightarrow \Lambda \Xi^0] \equiv Z_{13}^S, \quad G[D_{(-2,1,+1)}^{27} \rightarrow \Xi^0 \Xi^0] \equiv Z_{14}^S, \] (73)

Relationships for the remaining non-zero dependent amplitudes in terms of the independent ones are given in Sec. A 3 (b) of Appendix A.

The 14 independent amplitudes are described by four parameters and ten SU(3) broken sum rules may be deduced:

\[ Z_1^S - 2\sqrt{2} Z_2^S + Z_5^S = 0, \] (74)

\[ 3\sqrt{3} Z_1^S - 2\sqrt{6} Z_2^S + 3\sqrt{5} Z_3^S - 3\sqrt{15} Z_4^S + 3\sqrt{15} Z_6^S = 0, \] (75)

\[ 3 Z_1^S - 2\sqrt{2} Z_2^S - 2\sqrt{15} Z_3^S - 2\sqrt{5} Z_4^S + \sqrt{30} Z_7^S = 0, \] (76)

\[ \sqrt{3} Z_1^S - 3\sqrt{5} Z_3^S - \sqrt{15} Z_4^S + 2\sqrt{10} Z_8^S = 0, \] (77)

\[ Z_1^S - 4\sqrt{15} Z_3^S - 2\sqrt{30} Z_9^S = 0, \] (78)

\[ 3\sqrt{3} Z_1^S - 4\sqrt{15} Z_4^S + 2\sqrt{10} Z_{10}^S = 0, \] (79)
\[ 6Z_1^s - 4\sqrt{2}Z_2^s - \sqrt{15}Z_3^s - 3\sqrt{5}Z_4^s + 3\sqrt{2}Z_{11}^s = 0, \quad (80) \]

\[ 6Z_1^s - 2\sqrt{2}Z_2^s - 7\sqrt{15}Z_3^s - 3\sqrt{5}Z_4^s + 3\sqrt{30}Z_{12}^s = 0, \quad (81) \]

\[ 6Z_1^s - \sqrt{2}Z_2^s - 5\sqrt{15}Z_3^s - 5\sqrt{5}Z_4^s + 2\sqrt{5}Z_{13}^s = 0, \quad (82) \]

\[ 9\sqrt{3}Z_1^s - 2\sqrt{6}Z_2^s - 6\sqrt{5}Z_3^s - 6\sqrt{15}Z_4^s + 3\sqrt{3}Z_{14}^s = 0. \quad (83) \]

VI. SUMMARY AND CONCLUDING REMARKS

If a dibaryon is made of two baryon octets, its SU(3) transformation properties can be \( 1, 8, 10, 10^* \), or \( 27 \). If a dibaryon is made of one baryon octet and one baryon decuplet, then it can transform as \( 8, 10, 27 \), or \( 35 \). In Refs. [19–21], sum rules for strong decays in broken SU(3) have been reported for \( D_{8,10,10^*} \rightarrow B_8 + B_8 \) and for \( D_8 \rightarrow B_8 + R_{10} \) decays.

In this work, previous studies are complemented and sum rules of strong decay amplitudes in first order SU(3) symmetry breaking are determined for dibaryon decays \( D_{10,27} \rightarrow B_8 + R_{10} \) and \( D_{27} \rightarrow B_8 + B_8 \). Summary of results obtained and references to corresponding equations are given in Table I.

Experimentally, in each decay the width \( \Gamma \) determines the decay amplitude \( G \) since \( \Gamma \propto |M|^2 \) (with \( M \) the Feynman invariant amplitude) and \( M \propto G \). Notice that although dibaryons in the \( 10 \) and \( 27 \) multiplets are very unlikely to be stable, some of the decay modes may be kinematically forbidden and the corresponding decay amplitude would be zero, which would simplify sum rules where it appears.

It is worth noting that with appropriate changes, sum rules determined here and in previous works [19–21, 23] apply to decays of other multiquark states like tetraquarks and pentaquarks. This subject will be studied separately.

Acknowledgments

G. Sánchez-Colón and E. N. Polanco-Euán would like to thank CONACyT (México) for partial support.
Appendix A  DEPENDENT DECAY AMPLITUDES

Expressions for linearly dependent non-zero decay amplitudes in terms of the chosen independent ones are given. These relationships were determined by following the sign convention for SU(3) Clebsch-Gordan coefficients from Ref. [22].

1  $D_{10} \rightarrow B_8 + R_{10}$

\[ G[D_{(1,3/2,+,3/2)}^{10} \rightarrow \Sigma^0 \Delta^{++}] = -\sqrt{\frac{3}{2}} X_2, \] (84)

\[ G[D_{(1,3/2,-,3/2)}^{10} \rightarrow p\Sigma^0] = \sqrt{\frac{2}{3}} X_1, \quad G[D_{(1,3/2,+,3/2)}^{10} \rightarrow n\Sigma^+] = \frac{1}{\sqrt{3}} X_1, \] (85)

\[ G[D_{(1,3/2,+1/2)}^{10} \rightarrow \Sigma^+ \Delta^0] = \frac{2}{\sqrt{3}} X_2, \quad G[D_{(1,3/2,+1/2)}^{10} \rightarrow \Sigma^0 \Delta^+] = -\frac{1}{\sqrt{6}} X_2, \] (86)

\[ G[D_{(1,3/2,+1/2)}^{10} \rightarrow \Sigma^- \Delta^{++}] = -X_2, \quad G[D_{(1,3/2,+1/2)}^{10} \rightarrow \Lambda\Delta^+] = X_3, \] (87)

\[ G[D_{(1,3/2,-1/2)}^{10} \rightarrow \Sigma^- \Delta^+] = \frac{1}{\sqrt{3}} X_1, \quad G[D_{(1,3/2,-1/2)}^{10} \rightarrow n\Sigma^0] = \sqrt{\frac{2}{3}} X_1, \] (88)

\[ G[D_{(1,3/2,-1/2)}^{10} \rightarrow \Sigma^+ \Xi^-] = X_2, \quad G[D_{(1,3/2,-1/2)}^{10} \rightarrow \Sigma^0 \Delta^0] = \frac{1}{\sqrt{6}} X_2, \] (89)

\[ G[D_{(1,3/2,-1/2)}^{10} \rightarrow \Sigma^- \Delta^+ ] = -\frac{2}{\sqrt{3}} X_2, \quad G[D_{(1,3/2,-1/2)}^{10} \rightarrow \Lambda\Delta^0 ] = X_3, \] (90)

\[ G[D_{(1,3/2,-3/2)}^{10} \rightarrow n\Sigma^- ] = X_1, \quad G[D_{(1,3/2,-3/2)}^{10} \rightarrow \Sigma^0 \Delta^- ] = \sqrt{\frac{3}{2}} X_2, \] (91)

\[ G[D_{(1,3/2,-3/2)}^{10} \rightarrow \Sigma^- \Delta^0 ] = -X_2, \quad G[D_{(1,3/2,-3/2)}^{10} \rightarrow \Lambda\Delta^- ] = X_3, \] (92)

\[ G[D_{(0,1,+1)}^{10} \rightarrow \Sigma^0 \Xi^+] = -X_5, \quad G[D_{(0,1,+1)}^{10} \rightarrow \Xi^- \Delta^{++} ] = -\sqrt{3} X_7, \] (93)

\[ G[D_{(0,1,0)}^{10} \rightarrow p\Xi^- ] = \frac{1}{\sqrt{2}} X_4, \quad G[D_{(0,1,0)}^{10} \rightarrow n\Xi^0 ] = \frac{1}{\sqrt{2}} X_4, \] (94)
\[ G[D^{10}_{(0,1,0)} \to \Sigma^+ \Sigma^-] = X_5, \quad G[D^{10}_{(0,1,0)} \to \Sigma^- \Sigma^+] = -X_5, \quad (95) \]

\[ G[D^{10}_{(0,1,0)} \to \Lambda \Sigma^0] = X_6, \quad G[D^{10}_{(0,1,0)} \to \Xi^0 \Delta^0] = \sqrt{2}X_7, \quad (96) \]

\[ G[D^{10}_{(0,1,0)} \to \Xi^- \Delta^+] = -\sqrt{2}X_7, \quad (97) \]

\[ G[D^{10}_{(0,1,1)} \to n \Xi^-] = X_4, \quad G[D^{10}_{(0,1,1)} \to \Sigma^0 \Sigma^-] = X_5, \quad (98) \]

\[ G[D^{10}_{(0,1,-1)} \to \Sigma^- \Sigma^0] = -X_5, \quad G[D^{10}_{(0,1,-1)} \to \Lambda \Sigma^-] = X_6, \quad (99) \]

\[ G[D^{10}_{(0,1,-1)} \to \Xi^0 \Delta^-] = \sqrt{3}X_7, \quad G[D^{10}_{(0,1,-1)} \to \Xi^- \Delta^0] = -X_7, \quad (100) \]

\[ G[D^{10}_{(-1,1/2,+1/2)} \to \Sigma^0 \Xi^0] = -\frac{1}{\sqrt{2}}X_9, \quad G[D^{10}_{(-1,1/2,+1/2)} \to \Xi^- \Sigma^+] = -\sqrt{2}X_{11}, \quad (101) \]

\[ G[D^{10}_{(-1,1/2,-1/2)} \to n \Omega^-] = X_8, \quad G[D^{10}_{(-1,1/2,-1/2)} \to \Sigma^0 \Xi^-] = \frac{1}{\sqrt{2}}X_9, \quad (102) \]

\[ G[D^{10}_{(-1,1/2,-1/2)} \to \Sigma^- \Xi^0] = -X_9, \quad G[D^{10}_{(-1,1/2,-1/2)} \to \Lambda \Xi^-] = X_{10}, \quad (103) \]

\[ G[D^{10}_{(-1,1/2,-1/2)} \to \Xi^0 \Sigma^-] = \sqrt{2}X_{11}, \quad G[D^{10}_{(-1,1/2,-1/2)} \to \Xi^- \Sigma^0] = -X_{11}, \quad (104) \]

\[ G[D^{10}_{(-1,2/0,0)} \to \Xi^- \Xi^0] = -X_{13}, \quad (105) \]

\[ \text{2} \quad D_{27} \to B_8 + R_{10} \]

\[ G[D^{27}_{(2,1,+1)} \to n \Delta^+] = -\sqrt{3}Y_1, \quad G[D^{27}_{(2,1,0)} \to p \Delta^0] = \sqrt{2}Y_1, \quad (106) \]
\begin{align}
G[D_{(2,1,0)}^{27} \to n\Delta^+] &= -\sqrt{2}Y_1, \quad G[D_{(2,1,-1)}^{27} \to p\Delta^-] = \sqrt{3}Y_1, \quad (107)\\
G[D_{(2,1,-1)}^{27} \to n\Delta^0] &= -Y_1, \quad (108)\\
G[D_{(1,3/2,+,3/2)}^{27} \to \Sigma^0\Delta^{++}] &= -\sqrt{\frac{3}{2}}Y_2, \quad G[D_{(1,3/2,+,1/2)}^{27} \to p\Sigma^{0*}] = \frac{\sqrt{2}}{3}Y_2, \quad (109)\\
G[D_{(1,3/2,+,1/2)}^{27} \to n\Sigma^{++}] &= \frac{1}{\sqrt{3}}Y_2, \quad G[D_{(1,3/2,+,1/2)}^{27} \to \Sigma^+\Delta^0] = \frac{2}{\sqrt{3}}Y_3, \quad (110)\\
G[D_{(1,3/2,+,1/2)}^{27} \to \Sigma^0\Delta^+] &= -\frac{1}{\sqrt{6}}Y_3, \quad G[D_{(1,3/2,+,1/2)}^{27} \to \Sigma^-\Delta^{++}] = -Y_3, \quad (111)\\
G[D_{(1,3/2,+,1/2)}^{27} \to \Lambda^0\Delta^+] &= Y_4, \quad G[D_{(1,3/2,+,1/2)}^{27} \to p\Sigma^{-*}] = \frac{1}{\sqrt{3}}Y_2, \quad (112)\\
G[D_{(1,3/2,+,1/2)}^{27} \to n\Sigma^0*] &= \sqrt{\frac{2}{3}}Y_2, \quad G[D_{(1,3/2,+,1/2)}^{27} \to \Sigma^+\Lambda^-] = Y_3, \quad (113)\\
G[D_{(1,3/2,+,1/2)}^{27} \to \Sigma^0\Lambda^0] &= \frac{1}{\sqrt{6}}Y_2, \quad G[D_{(1,3/2,+,1/2)}^{27} \to \Sigma^-\Delta^+] = -\frac{2}{\sqrt{3}}Y_3, \quad (114)\\
G[D_{(1,3/2,+,1/2)}^{27} \to \Lambda\Lambda^0] &= Y_4, \quad G[D_{(1,3/2,+,3/2)}^{27} \to n\Sigma^{-*}] = Y_2, \quad (115)\\
G[D_{(1,3/2,+,3/2)}^{27} \to \Sigma^0\Delta^-] &= \sqrt{\frac{3}{2}}Y_3, \quad G[D_{(1,3/2,+,3/2)}^{27} \to \Sigma^-\Delta^0] = -Y_3, \quad (116)\\
G[D_{(1,3/2,+,3/2)}^{27} \to \Lambda\Lambda^-] &= Y_4, \quad (117)\\
G[D_{(1,3/2,+,3/2)}^{27} \to n\Sigma^{++}] &= -\sqrt{2}Y_5, \quad G[D_{(1,1/2,+,1/2)}^{27} \to \Sigma^0\Delta^+] = -\sqrt{2}Y_6, \quad (118)\\
G[D_{(1,1/2,+,1/2)}^{27} \to \Sigma^-\Delta^{++}] &= \sqrt{3}Y_6, \quad G[D_{(1,1/2,+,1/2)}^{27} \to p\Sigma^{-*}] = \sqrt{2}Y_5, \quad (119)\end{align}
\[ G[D_{(1,1/2\rightarrow 1/2)}^{27} \rightarrow n\Sigma^{0*}] = -Y_5, \quad G[D_{(1,1/2\rightarrow 1/2)}^{27} \rightarrow \Sigma^{+} \Delta^{-}] = \sqrt{3}Y_6, \quad (120) \]

\[ G[D_{(1,1/2\rightarrow 1/2)}^{27} \rightarrow \Sigma^{0} \Delta^{0}] = -\sqrt{2}Y_6, \quad G[D_{(1,1/2\rightarrow 1/2)}^{27} \rightarrow \Sigma^{-} \Delta^{+}] = Y_6, \quad (121) \]

\[ G[D_{(0,2,1)}^{27} \rightarrow \Sigma^{+} \Sigma^{0*}] = \frac{1}{\sqrt{2}}Y_7, \quad G[D_{(0,2,1)}^{27} \rightarrow \Sigma^{0} \Sigma^{0*}] = \frac{1}{\sqrt{2}}Y_7, \quad (122) \]

\[ G[D_{(0,2,1)}^{27} \rightarrow \Xi^{0} \Delta^{+}] = \frac{\sqrt{3}}{2}Y_8, \quad G[D_{(0,2,1)}^{27} \rightarrow \Xi^{-} \Delta^{++}] = \frac{1}{2}Y_8, \quad (123) \]

\[ G[D_{(0,2,0)}^{27} \rightarrow \Sigma^{+} \Sigma^{-*}] = \frac{1}{\sqrt{6}}Y_7, \quad G[D_{(0,2,0)}^{27} \rightarrow \Sigma^{0} \Sigma^{0*}] = \frac{\sqrt{2}}{\sqrt{3}}Y_7, \quad (124) \]

\[ G[D_{(0,2,0)}^{27} \rightarrow \Sigma^{-} \Sigma^{0*}] = \frac{1}{\sqrt{6}}Y_7, \quad G[D_{(0,2,0)}^{27} \rightarrow \Xi^{0} \Delta^{0}] = \frac{1}{\sqrt{2}}Y_8, \quad (125) \]

\[ G[D_{(0,2,0)}^{27} \rightarrow \Xi^{-} \Delta^{+}] = \frac{1}{\sqrt{2}}Y_8, \quad G[D_{(0,2,0)}^{27} \rightarrow \Xi^{-} \Sigma^{0*}] = \frac{1}{\sqrt{2}}Y_7, \quad (126) \]

\[ G[D_{(0,2,0)}^{27} \rightarrow \Sigma^{-} \Sigma^{0*}] = \frac{1}{\sqrt{2}}Y_7, \quad G[D_{(0,2,0)}^{27} \rightarrow \Xi^{0} \Delta^{-}] = \frac{1}{2}Y_8, \quad (127) \]

\[ G[D_{(0,2,0)}^{27} \rightarrow \Xi^{-} \Delta^{0}] = \frac{\sqrt{3}}{2}Y_8, \quad G[D_{(0,2,0)}^{27} \rightarrow \Xi^{-} \Sigma^{-*}] = Y_7, \quad (128) \]

\[ G[D_{(0,2,1)}^{27} \rightarrow \Xi^{-} \Delta^{-}] = Y_8, \quad (129) \]

\[ G[D_{(0,1,1)}^{27} \rightarrow \Sigma^{0} \Sigma^{+*}] = -Y_{10}, \quad G[D_{(0,1,1)}^{27} \rightarrow \Xi^{-} \Delta^{++}] = -\sqrt{3}Y_{12}, \quad (130) \]

\[ G[D_{(0,1,0)}^{27} \rightarrow p\Xi^{-*}] = \frac{1}{\sqrt{2}}Y_9, \quad G[D_{(0,1,0)}^{27} \rightarrow n\Xi^{0*}] = \frac{1}{\sqrt{2}}Y_9, \quad (131) \]

\[ G[D_{(0,1,0)}^{27} \rightarrow \Sigma^{0} \Sigma^{-*}] = Y_{10}, \quad G[D_{(0,1,0)}^{27} \rightarrow \Sigma^{-} \Sigma^{+*}] = -Y_{10}, \quad (132) \]

\[ G[D_{(0,1,0)}^{27} \rightarrow \Lambda \Sigma^{0*}] = Y_{11}, \quad G[D_{(0,1,0)}^{27} \rightarrow \Xi^{0} \Delta^{0}] = \sqrt{2}Y_{12}, \quad (133) \]
\[G[D^{27}_{(0,1,0)} \rightarrow \Xi^- \Delta^+] = -\sqrt{2}Y_{12}, \quad G[D^{27}_{(0,1,-1)} \rightarrow n\Xi^-] = Y_9, \quad (134)\]

\[G[D^{27}_{(0,1,-1)} \rightarrow \Sigma^0 \Sigma^-] = Y_{10}, \quad G[D^{27}_{(0,1,-1)} \rightarrow \Sigma^- \Sigma^0] = -Y_{10}, \quad (135)\]

\[G[D^{27}_{(0,1,-1)} \rightarrow \Lambda \Sigma^-] = Y_{11}, \quad G[D^{27}_{(0,1,-1)} \rightarrow \Xi^0 \Delta^-] = \sqrt{3}Y_{12}, \quad (136)\]

\[G[D^{27}_{(0,1,-1)} \rightarrow \Xi^- \Delta^0] = -Y_{12}, \quad (137)\]

\[G[D^{27}_{(0,0,0)} \rightarrow n\Xi^{0*}] = -Y_{13}, \quad G[D^{27}_{(0,0,0)} \rightarrow \Sigma^0 \Sigma^0] = -Y_{14}, \quad (138)\]

\[G[D^{27}_{(0,0,0)} \rightarrow \Sigma^- \Sigma^+] = Y_{14}, \quad (139)\]

\[G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Sigma^+ \Xi^-] = \frac{1}{\sqrt{3}}Y_{15}, \quad G[D^{27}_{(-1,3/2,+1/2)} \rightarrow \Sigma^0 \Xi^{0*}] = \sqrt{\frac{2}{3}}Y_{15}, \quad (140)\]

\[G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Xi^0 \Sigma^0] = \sqrt{\frac{2}{3}}Y_{16}, \quad G[D^{27}_{(-1,3/2,+1/2)} \rightarrow \Xi^- \Sigma^+] = \frac{1}{\sqrt{3}}Y_{16}, \quad (141)\]

\[G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Sigma^0 \Xi^-] = \sqrt{\frac{2}{3}}Y_{15}, \quad G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Sigma^- \Xi^0] = \frac{1}{\sqrt{3}}Y_{15}, \quad (142)\]

\[G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Xi^0 \Sigma^-] = \frac{1}{\sqrt{3}}Y_{16}, \quad G[D^{27}_{(-1,3/2,+1/2)} \rightarrow \Xi^- \Sigma^0] = \sqrt{\frac{2}{3}}Y_{16}, \quad (143)\]

\[G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Sigma^- \Xi^-] = Y_{15}, \quad G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Xi^- \Sigma^-] = Y_{16}, \quad (144)\]

\[G[D^{27}_{(-1,1/2,+1/2)} \rightarrow \Sigma^- \Xi^-] = -\frac{1}{\sqrt{2}}Y_{18}, \quad G[D^{27}_{(-1,1/2,+1/2)} \rightarrow \Sigma^- \Xi^-] = -\sqrt{2}Y_{20}, \quad (145)\]
\[ G[D_{(-1,1/2,-1/2)}^{27}] \rightarrow n\Omega^- = Y_{17}, \quad G[D_{(-1,1/2,-1/2)}^{27}] \rightarrow \Sigma^0\Xi^- = \frac{1}{\sqrt{2}} Y_{18}, \quad (146) \]

\[ G[D_{(-1,1/2,-1/2)}^{27}] \rightarrow \Sigma^-\Xi^0* = -Y_{18}, \quad G[D_{(-1,1/2,-1/2)}^{27}] \rightarrow \Lambda\Xi^-* = Y_{19}, \quad (147) \]

\[ G[D_{(-1,1/2,-1/2)}^{27}] \rightarrow \Xi^0\Sigma^-* = \sqrt{2} Y_{20}, \quad G[D_{(-1,1/2,-1/2)}^{27}] \rightarrow \Xi^-\Sigma^0* = -Y_{20}, \quad (148) \]

\[ G[D_{(-2,1,0)}^{27}] \rightarrow \Sigma^0\Omega^- = Y_{21}, \quad G[D_{(-2,1,0)}^{27}] \rightarrow \Xi^0\Xi^-* = \frac{1}{\sqrt{2}} Y_{22}, \quad (149) \]

\[ G[D_{(-2,1,0)}^{27}] \rightarrow \Xi^-\Xi^0* = \frac{1}{\sqrt{2}} Y_{22}, \quad G[D_{(-2,1,-1)}^{27}] \rightarrow \Sigma^-\Omega^- = Y_{21}, \quad (150) \]

\[ G[D_{(-2,1,-1)}^{27}] \rightarrow \Xi^-\Xi^-* = Y_{22}. \quad (151) \]

3 \[ D_{27} \rightarrow B_8 + B_8 \]

(a) \text{ Antisymmetric final state}

\[ G[D_{(1,3/2,+1/2)}^{27}] \rightarrow p\Sigma^0\ = \sqrt{\frac{2}{3}} Z_1^A, \quad G[D_{(1,3/2,+1/2)}^{27}] \rightarrow n\Sigma^+\ = \frac{1}{\sqrt{3}} Z_1^A, \quad (152) \]

\[ G[D_{(1,3/2,-1/2)}^{27}] \rightarrow p\Sigma^-\ = \frac{1}{\sqrt{3}} Z_1^A, \quad G[D_{(1,3/2,-1/2)}^{27}] \rightarrow n\Sigma^0\ = \sqrt{\frac{2}{3}} Z_1^A, \quad (153) \]

\[ G[D_{(1,3/2,-3/2)}^{27}] \rightarrow n\Sigma^-\ = Z_1^A, \quad (154) \]

\[ G[D_{(1,1/2,+1/2)}^{27}] \rightarrow n\Sigma^+\ = -\sqrt{2} Z_2^A, \quad G[D_{(1,1/2,-1/2)}^{27}] \rightarrow p\Sigma^-\ = \sqrt{2} Z_2^A, \quad (155) \]

\[ G[D_{(1,1/2,-1/2)}^{27}] \rightarrow n\Sigma^0\ = -Z_2^A, \quad G[D_{(1,1/2,-1/2)}^{27}] \rightarrow n\Lambda\ = Z_3^A, \quad (156) \]

\[ G[D_{(0,1,0)}^{27}] \rightarrow p\Xi^-\ = \frac{1}{\sqrt{2}} Z_4^A, \quad G[D_{(0,1,0)}^{27}] \rightarrow n\Xi^0\ = \frac{1}{\sqrt{2}} Z_4^A, \quad (157) \]
\[ G[D_{(0,1,0)}^{27} \rightarrow \Sigma^+\Sigma^-' = Z_5^A, \quad G[D_{(0,1,0)}^{27} \rightarrow \Sigma^0\Lambda' = Z_6^A, \quad (158) \]

\[ G[D_{(0,1,-1)}^{27} \rightarrow n\Xi^- = Z_4^A, \quad G[D_{(0,1,-1)}^{27} \rightarrow \Sigma^-\Sigma^0' = -Z_5^A, \quad (159) \]

\[ G[D_{(0,1,-1)}^{27} \rightarrow \Sigma^-\Lambda' = Z_6^A, \quad (160) \]

\[ G[D_{(0,0,0)}^{27} \rightarrow n\Xi^0 = -Z_7^A, \quad (161) \]

\[ G[D_{(-1,1/2,+1/2)}^{27} \rightarrow \Sigma^0\Xi^0' = -\frac{1}{\sqrt{2}}Z_8^A, \quad G[D_{(-1,1/2,-1/2)}^{27} \rightarrow \Sigma^0\Xi^- = \frac{1}{\sqrt{2}}Z_8^A, \quad (162) \]

\[ G[D_{(-1,1/2,-1/2)}^{27} \rightarrow \Sigma^-\Xi^0' = -Z_8^A, \quad G[D_{(-1,1/2,-1/2)}^{27} \rightarrow \Lambda\Xi^- = Z_9^A, \quad (163) \]

\[ G[D_{(-1,3/2,+1/2)}^{27} \rightarrow \Sigma^+\Xi^- = \frac{1}{\sqrt{3}}Z_{10}^A, \quad G[D_{(-1,3/2,+1/2)}^{27} \rightarrow \Sigma^0\Xi^- = \sqrt{\frac{2}{3}}Z_{10}^A, \quad (164) \]

\[ G[D_{(-1,3/2,-1/2)}^{27} \rightarrow \Sigma^0\Xi^- = \sqrt{\frac{2}{3}}Z_{10}^A, \quad G[D_{(-1,3/2,-1/2)}^{27} \rightarrow \Sigma^-\Xi^0' = \frac{1}{\sqrt{3}}Z_{10}^A, \quad (165) \]

\[ G[D_{(-1,3/2,-3/2)}^{27} \rightarrow \Sigma^-\Xi^- = Z_{10}^A, \quad (166) \]

with identical relationships for \( G[D_{(Y,I,I_3)}^{27} \rightarrow B'B] = -G[D_{(Y,I,I_3)}^{27} \rightarrow BB']. \)

(b) Symmetric final state

\[ G[D_{(2,1,0)}^{27} \rightarrow pn' = \frac{1}{\sqrt{2}}Z_1^S, \quad G[D_{(2,1,-1)}^{27} \rightarrow nn' = Z_1^S, \quad (167) \]

\[ G[D_{(1,3/2,+1/2)}^{27} \rightarrow p\Sigma^0' = \frac{\sqrt{2}}{3}Z_2^S, \quad G[D_{(1,3/2,+1/2)}^{27} \rightarrow n\Sigma^+ = \frac{1}{\sqrt{3}}Z_2^S, \quad (168) \]
\[G[D^{27}_{(1,3/2,-1/2)} \rightarrow p \Sigma^-] = \frac{1}{\sqrt{3}} Z_2^S, \quad G[D^{27}_{(1,3/2,-1/2)} \rightarrow n \Sigma^0'] = \sqrt{\frac{2}{3}} Z_2^S, \quad (169)\]

\[G[D^{27}_{(1,3/2,-3/2)} \rightarrow n \Sigma^-] = Z_2^S, \quad (170)\]

\[G[D^{27}_{(1,1/2,+1/2)} \rightarrow n \Sigma^+] = -\sqrt{2} Z_3^S, \quad G[D^{27}_{(1,1/2,-1/2)} \rightarrow p \Sigma^-] = \sqrt{2} Z_3^S, \quad (171)\]

\[G[D^{27}_{(1,1/2,-1/2)} \rightarrow n \Sigma^0] = -Z_3^S, \quad G[D^{27}_{(1,1/2,-1/2)} \rightarrow n \Lambda'] = Z_4^S, \quad (172)\]

\[G[D^{27}_{(0,2,+1)} \rightarrow \Sigma^+ \Sigma^0] = \frac{1}{\sqrt{2}} Z_5^S, \quad G[D^{27}_{(0,2,0)} \rightarrow \Sigma^+ \Sigma^-] = \frac{1}{\sqrt{6}} Z_5^S, \quad (173)\]

\[G[D^{27}_{(0,2,0)} \rightarrow \Sigma^0 \Sigma^0'] = \sqrt{\frac{2}{3}} Z_5^S, \quad G[D^{27}_{(0,2,-1)} \rightarrow \Sigma^0 \Sigma^-] = \frac{1}{\sqrt{2}} Z_5^S, \quad (174)\]

\[G[D^{27}_{(0,2,-2)} \rightarrow \Sigma^- \Sigma^-] = Z_5^S, \quad (175)\]

\[G[D^{27}_{(0,1,0)} \rightarrow p \Xi^-] = \frac{1}{\sqrt{2}} Z_6^S, \quad G[D^{27}_{(0,1,0)} \rightarrow n \Xi^0] = \frac{1}{\sqrt{2}} Z_6^S, \quad (176)\]

\[G[D^{27}_{(0,1,0)} \rightarrow \Sigma^0 \Lambda'] = Z_7^S, \quad G[D^{27}_{(0,1,-1)} \rightarrow n \Xi^-] = Z_6^S, \quad (177)\]

\[G[D^{27}_{(0,1,-1)} \rightarrow \Sigma^- \Lambda'] = Z_7^S, \quad (178)\]

\[G[D^{27}_{(0,0,0)} \rightarrow n \Xi^0] = -Z_8^S, \quad G[D^{27}_{(0,0,0)} \rightarrow \Sigma^0 \Sigma^0'] = -Z_9^S, \quad (179)\]

\[G[D^{27}_{(-1,3/2,+1/2)} \rightarrow \Sigma^+ \Xi^-] = \frac{1}{\sqrt{5}} Z_{11}^S, \quad G[D^{27}_{(-1,3/2,+1/2)} \rightarrow \Sigma^0 \Xi^0'] = \sqrt{\frac{2}{3}} Z_{11}^S, \quad (180)\]

\[G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Sigma^0 \Xi^-] = \sqrt{\frac{2}{3}} Z_{11}^S, \quad G[D^{27}_{(-1,3/2,-1/2)} \rightarrow \Sigma^- \Xi^0'] = \frac{1}{\sqrt{3}} Z_{11}^S, \quad (181)\]
\begin{equation}
G[D_{(-1,1/2,+,1/2)}^{27} \rightarrow \Sigma^0 \Xi^0] = -\frac{1}{\sqrt{2}} Z^S_{12}, \quad G[D_{(-1,1/2,-,1/2)}^{27} \rightarrow \Sigma^0 \Xi^-] = \frac{1}{\sqrt{2}} Z^S_{12}, \quad (182)
\end{equation}

\begin{equation}
G[D_{(-1,1/2,-,1/2)}^{27} \rightarrow \Sigma^- \Xi^0] = -Z^S_{12}, \quad G[D_{(-1,1/2,-,1/2)}^{27} \rightarrow \Lambda \Xi] = Z^S_{13}, \quad (183)
\end{equation}

\begin{equation}
G[D_{(-1,3/2,-,3/2)}^{27} \rightarrow \Sigma^- \Xi^-] = Z^S_{11}, \quad (184)
\end{equation}

\begin{equation}
G[D_{(-2,1,0)}^{27} \rightarrow \Xi^0 \Xi^-] = \frac{1}{\sqrt{2}} Z^S_{14}, \quad G[D_{(-2,1,-,1)}^{27} \rightarrow \Xi^- \Xi^-] = Z^S_{14}, \quad (185)
\end{equation}

with identical relationships for $G \left[ D_{(Y,I,I_3)}^{27} \rightarrow B'B \right] = G \left[ D_{(Y,I,I_3)}^{27} \rightarrow BB' \right]$. 

[1] H. C. Urey, F. G. Brickwedde, and G. M. Murphy, Phys. Rev. 39, 164 (1932).
[2] R. J. Oakes, Phys. Rev. 131, 2239 (1963).
[3] M. Gell-Mann, “The Eightfold Way: A Theory of Strong Interaction Symmetry”, California Institute of Technology Synchrotron Laboratory Report CTSL-20 (1961), unpublished.
[4] S. Okubo, Prog. Theor. Phys. 27, 949 (1962).
[5] R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977); Erratum Phys. Rev. Lett. 38, 617 (1977).
[6] A. Yokosawa, Phys. Rep. 64, 47 (1980).
[7] M. P. Locher, M. E. Sainio, and A. Svarc, Adv. Nucl. Phys. 17, 47 (1986).
[8] I. I. Strakovsky, Sov. J. Part. Nucl. Phys. 17, 296 (1991).
[9] S.R. Beane, et al., Phys. Rev. Lett. 106, 162001 (2011).
[10] S.R. Beane, et al., Phys. Rev. D 87, 034506 (2013).
[11] T. Inoue, et al., Phys. Rev. Lett. 106, 162002 (2011).
[12] S.-K. Choi, et al., Belle Collaborations, Phys. Rev. Lett. 91, 262001 (2003).
[13] R. Aaij, et al., LHCb Collaborations, Phys. Rev. Lett. 115, 072001 (2015).
[14] M. Bashkanov et al., NSTAR2015 Proceedings (2015).
[15] H. Huang et al., NSTAR2015 Proceedings (2015).
[16] M. Oka, Phys. Rev. D 38, 298 (1988).
[17] S. Xie, Q. Zhang, Phys. Let. B 143, 441 (1984).
[18] S. Xie, *Journal of Phys. G* 15, 287 (1989).

[19] E. N. Polanco-Euán, V. Gupta, and G. Sánchez-Colón, *Mod. Phys. Lett. A* 32, 1750041 (2016).

[20] E. N. Polanco-Euán, V. Gupta, G. Sánchez-Colón, and B. A. Bambah, *J. Phys. Conf. Ser.* 761, 012086 (2016).

[21] V. Gupta and G. Sánchez-Colón, *Mod. Phys. Lett. A* 30, 1550010 (2015).

[22] P. McNamee S. J. and F. Chilton, *Rev. Mod. Phys.* 36, 1005 (1964).

[23] V. Gupta and V. Singh, *Phys. Rev.* 135, B1442 (1964).
FIG. 1: \((Y, I, I_3)\) states of the dibaryon 10 multiplet \(D_{10}\).

FIG. 2: \((Y, I, I_3)\) states of the baryon octet \(B_8\).
FIG. 3: \((Y, I, I_3)\) states of the baryon decuplet \(R_{10}\).

FIG. 4: \((Y, I, I_3)\) states of the dibaryon 27 multiplet \(D_{27}\).
TABLE I: Summary of studied decays and results found in this article. References to the corresponding sections and equations are shown.

| Decay          | Independent amplitudes | Sum rules      |
|----------------|------------------------|----------------|
| $D_{10} \rightarrow B_8 + R_{10}$ | 13                     | 8              |
| Sec. III       | Eqs. (7) - (13)        | Eqs. (14) - (21) |
| $D_{27} \rightarrow B_8 + R_{10}$ | 22                     | 16             |
| Sec. IV        | Eqs. (25) - (35)       | Eqs. (36) - (51) |
| $D_{27} \rightarrow B_8 + B_8$   |                        |                |
| Antisymmetric final state | 10                     | 7              |
| Sec. V A 1     | Eqs. (55) - (59)       | Eqs. (60) - (66) |
| $D_{27} \rightarrow B_8 + B_8$   |                        |                |
| Symmetric final state | 14                     | 10             |
| Sec. V A 2     | Eqs. (67) - (73)       | Eqs. (74) - (83) |