Spin 1 inversion: a Majorana tensor force for deuteron alpha scattering

S.G. Cooper†, V.I. Kukulin‡, R.S. Mackintosh† and V.N. Pomerantsev‡
†Physics Department, The Open University, Milton Keynes, MK7 6AA, U.K.
‡Institute of Nuclear Physics, Moscow State University, Moscow 119899, Russia.
(November 19, 2018)

Abstract

We demonstrate, for the first time, successful S-matrix to potential inversion for spin one projectiles with non-diagonal $S_{ll'}^j$ yielding a $T_R$ interaction. The method is a generalization of the iterative-perturbative, IP, method. We present a test case indicating the degree of uniqueness of the potential. The method is adapted, using established procedures, into direct observable to potential inversion, fitting $\sigma$, $iT_{11}$, $T_{20}$, $T_{21}$ and $T_{22}$ for d + alpha scattering over a range of energies near 10 MeV. The $T_R$ interaction which we find is very different from that proposed elsewhere, both real and imaginary parts being very different for odd and even parity channels.

PACS numbers: 21.30.-x, 13.75.Cs, 25.10.+s
The inverse scattering problem, i.e. the problem of determining a potential from the scattering matrix, has not until now been solved for the scattering of spin one projectiles. Spin one inversion is substantially different from that for spin zero or spin half because, even for a spin-zero target, the elastic scattering ‘S-matrix’ is no longer a complex number but a matrix. When spin-1 projectiles scatter from a spin-zero target, the conserved quantities are total angular momentum \( j \) and parity \( \pi \). When \( \pi = (-1)^{j+1} \), two values of orbital angular momentum, \( l \) and \( l' \), are, in general, coupled by a tensor interaction. Calculating the \( S \)-matrix \( S^l_{ll'} \) therefore requires a coupled channel calculation and inversion from \( S^l_{ll'} \) to a potential containing a tensor term requires a coupled channel inversion. Here we demonstrate a spin-one inverse scattering procedure. It is incorporated into a direct data-to-potential inversion procedure which is used to analyse \( ^2\text{H} - ^4\text{He} \) scattering data at around 10 MeV. We find evidence for a substantial parity dependent tensor interaction.

Coupled channel inversion is here carried out using a generalization of the iterative perturbative (IP) inversion procedure \([1,3]\), which is successful for \( S_{ll} \rightarrow V(r) + \mathbf{1} \cdot \mathbf{s} \psi_o(r) \) inversion for spin half projectiles. Features of the IP method (e.g. the ability to handle a range of energies simultaneously and to include Majorana terms for all potential components) all apply.

We seek a potential with many components, each component being labelled with index \( p \) identifying central, spin-orbit or tensor terms, each real or imaginary. The number of components further doubles where, as essential for light nuclei, parity dependence is permitted. We assume that the tensor force is of the form \( \hat{V} \), components further doubles where, as essential for light nuclei, parity dependence is permitted. The physical basis of the equations leading to \( \alpha_n \) is the the linear response of \( S_k \) to small perturbations in the potential \([1,3]\). This result applies to the non-diagonal as well as diagonal terms. For any given set of conserved quantum numbers, certain channels will be coupled by the nucleus-nucleus interaction and we use labels \( \kappa, \lambda, \mu, \nu \) for these channels. Thus the matrix element of the nucleus-nucleus interaction \( V \) between the wavefunctions for channels \( \kappa \) and \( \lambda \), corresponding to integrating over all coordinates but \( r \), will be written \( V_{\kappa\lambda}(r) \). The increment \( \Delta S_{\kappa\lambda} \) in the non-diagonal \( S \)-matrix which is due to a small perturbation \( \Delta V_{\kappa\lambda}(r) \), is

\[
\Delta S_{\kappa\lambda} = \frac{i\mu}{\hbar^2 k} \sum_{\mu\nu} \int_0^\infty \psi_{\kappa\mu}(r) \Delta V_{\mu\nu}(r) \psi_{\nu\lambda} dr
\]  

(1)

where \( \psi_{\kappa\nu} \) is the \( \nu \)th channel (first index) component of that coupled channel solution for the unperturbed non-diagonal potential for which there is in-going flux in channel \( \kappa \) (second index) only. The normalisation is \( \psi_{\kappa\nu} \rightarrow \delta_{\kappa\nu} I_{\kappa} - S_{\nu\kappa} O_{\kappa} \) where \( I_t \) and \( O_t \) are incoming
and outgoing Coulomb wavefunctions for orbital angular momentum $l$; there is no complex conjugation in the integral. Starting from Eq. 1, spin-one inversion becomes a straightforward generalisation of the procedure described in Refs. [3] and is now implemented in the code IMAGO [5].

**Testing spin-$1 \rightarrow V$ inversion.** We established that in forward (rather than inverse) mode, a given $T_R$ potential led to the same $S^l_{\nu'}$ and observables (including tensor analysing powers) as the standard deuteron scattering code DDTP [3]. A convenient property of the IP inversion method is the fact that when a sequence of iterations converges to small $\sigma^2$, as defined above, we can be sure that we have found a potential which reproduces the set of target $S^l_{\nu'}$ to a precision which corresponds to that value of $\sigma^2$. It follows that the forward-mode test verifies that we have found a correctly defined potential.

We have also tested the prescription for spin-$1 \rightarrow V$ inversion using a known potential to define the target $S^l_{\nu'}$. One set of results, for 20.2 MeV deuterons scattering from $^4$He is shown in Fig. 1. The target potential (solid line) has a complex central term and real $l \cdot s$ and $V(0)$ terms but no Majorana components. This potential reproduces the tensor analysing powers but with a $T_R$ interaction much stronger than folding models predict. The two inversion solutions shown in Fig. 1 both arise from a starting potential (SRP) with only one non-zero component, a real central Woods-Saxon potential 60 MeV deep. Although few partial waves contribute significantly to the inversion, only 3-4 iterations were necessary to give solution A (dotted) and solution B (dot-dashed), which reproduced all four volume integrals to better than 1 %, required not many more. Defects in the tensor and spin-orbit terms for $r \sim 0$ reflect the fact that both $l \cdot s$ and $T_R$ have zero diagonal matrix elements for $l = 0$. A test case involving parity dependence would necessitate a smaller inversion basis, and hence reproduce each component with less accuracy, since $S^l_{\nu'}$ for the same range of $j$ must determine twice as many components; this is relevant to the case described below.

**Direct data-to-potential inversion.** The preferred method of applying inversion methods to experimental scattering data is to convolute the phase shift fitting with the $S \rightarrow V$ inversion to form a direct data-to-potential inversion procedure. This can be coded in a single data fitting program. The two step procedure in which one first fits the data with $S^l_{\nu'}$ and then subsequently inverts $S^l_{\nu'}$ does not guarantee that the $S^l_{\nu'}$ for different energies are smoothly related like $S^l_{\nu'}$ derived from a single potential. The direct inversion formalism has been described and applied [3,4] to cases where there is no channel coupling and it carries over to the present case with no change in basic principle. It has been implemented in code IMAGO [5] and the convolution was thoroughly tested through explicit checking of all the derivative terms which are required [4, 5].

**Application to deuteron scattering.** We have analysed the scattering of tensor polarised deuterons from $^4$He over a series of energies near 10 MeV using direct data-to-potential inversion. This case is of particular interest both because it should become feasible to do quite realistic calculations using RGM or similar methods and also because the deuteron is the archetypal easily polarisable halo nucleus. From the data set for d – alpha scattering tabulated in Ref. [11], we have selected data of Jenny et al [12] which is of high quality and covers a wide angular range. We fit the following observables, $\sigma$, $iT_{11}$, $T_{20}$, $T_{21}$ and $T_{22}$, for the energies, 8, 9, 10, 11, 12 and 13 MeV. This range includes broad $2^+$ resonances and a region of strong mixing between the $1^+$ channels. Ref [13] applies direct data-to-potential IP inversion to data from the same source but without fitting $T_{2i}$.
We exploit a property of the IP method to fit all of the above data, for all the stated energies, with a single potential. The potential is of fixed radial form but the overall strength of each of the imaginary components is proportional to the energy above the inelastic threshold. To minimize the number of parameters the real components are taken to be energy independent.

It is well known that fits of $S$-matrix to data are inevitably subject to discrete and continuous ambiguities and direct data-to-$V$ inversion is not exempt. The uncertainties arising at the $S \rightarrow V$ stage are less important. Various strategies are possible, such as the inclusion of a priori information from theory \[14\]. Here we adopt the approach of using extremely small inversion bases (two or three Gaussian functions) so that, in effect, we find the smoothest, minimally energy-dependent, potential fitting the data at all six energies. We find that we can allow the spin orbit terms to be real and parity independent but the central and $T_R$ tensor terms must be complex and and parity dependent. Inversion studies involving fits to both empirical data and RGM theory \[15,16\] show that interactions between light nuclei are parity dependent for energies up to tens of MeV per nucleon. As in earlier work, we present the real central term in the form of Wigner and Majorana components: $V_W + (-1)^l V_M$. However, $\hat{V}(t)$ and the imaginary terms were treated somewhat differently: the data was reproduced most efficiently with inversion bases covering different radial ranges for the even and odd parity components. In Fig. 3 below we shall see that the odd parity $\hat{V}(t)$ terms cover a wider radial range. This is consistent with theoretical arguments \[17\] for the existence of tensor interactions of quite different strengths and shapes for odd and even parity. The SRP consisted only of real, Wigner, central and spin-orbit terms. Convergence was very rapid and typically about 3 iterations were required.

Figs 2 and 3 show two alternative potentials fitting the data for the six energies. The energy dependent imaginary parts were evaluated at 10 MeV and Fig. 4 shows the quality of fit to the 10 MeV data. The fit to the data at the ends of the energy range was poorer suggesting that a somewhat less simple energy dependence is required. The $\chi^2/N$ values of 8.59 and 8.08 are for the data at all six energies.\[1\]

Different components of the potential are determined to different degrees of uniqueness. The real central and spin-orbit Wigner potentials shown in Fig. 2 are well determined and consistent with predictions of RGM theory. The imaginary central component is highly parity dependent and is emissive near the origin. All potentials giving a good fit to the data over this energy range have such an emissive region near the nuclear centre. Such emissive regions are commonly present \[16\] in potentials found by inverting RGM $S$-matrix elements which manifestly do not break unitarity. Emissive regions often appear in local potentials which represent exchange and channel coupling non-locality. In the present case, the unitarity limit is only broken, to a small degree, for $L = 2$, $J = 2$ for the $\chi^2/N = 8.59$ and is not broken for the better fit.

The $V^{(t)}(r)$ interaction shown in Fig. 3 departs markedly from previous phenomenology. The even parity real and imaginary terms are very strong near $r = 1$. This is significant for

\[1\]For simplicity in this initial study, we made no selection, filtration or normalisation of input data as is usual in phase shift analyses.
$r >$ about 1.0 fm; for $r < 1$ the even-parity $V(t)(r)$ interaction is ill-determined since the diagonal matrix element of $T_R$ is zero for $l < 2$ so the comment concerning irregularities at the nuclear centre in Fig. 1 applies here. The combination of strong, short-ranged even-parity components and weaker, long ranged odd-parity components is precisely the pattern predicted to arise from a deuteron exchange mechanism discussed in Ref. [17]; see also Refs. [18,19] which also present a $\hat{V}(t)$ interaction strongly peaked inside $r = 2$ fm, but of the opposite sign.

The present analysis firmly establishes that most terms in the $d - ^4$He potential are substantially parity dependent, casting doubt on parity independent $d - ^4$He potentials, including the potential of Frick et al. [7] with its remarkably large $\hat{V}(t)$ interaction. RGM theory [14,16] predicts parity dependence for central components, but the strong parity dependence in $\hat{V}(t)$ suggests a new physical process.

In conclusion: we have demonstrated for the first time $S \rightarrow V$ inversion for spin-1 projectiles leading to a tensor interaction which couples channels. This has been incorporated into a direct data-to-potential inversion procedure yielding a remarkable new kind of $T_R$ interaction. The general procedure could be applied to other cases with channel spin one such as $p - ^3$H scattering. Spin-1 $S \rightarrow V$ inversion makes it possible to study theories for dynamic polarization and exchange contributions to central and non-central forces for spin-1 projectiles. The results can then be directly related to phenomenology since, as we have shown here, the inversion procedure can be made part of a powerful, rapidly converging, phenomenological method which, at low computational cost, fits multi-energy datasets with a single, energy dependent potential.

ACKNOWLEDGEMENTS

V.I.K. is grateful to Willi Gruebler for supplying the full tables of experimental data of the Zürich group. We are also grateful to the UK EPSRC for grant GR/L22843 supporting S.G. Cooper, the Russian Foundation for Basic Research (grant 97-02-17265) for financial assistance and to the Royal Society (UK) for supporting a visit by V.I. Kukulin to the UK. We thank Jeff Tostevin for sending us Goddard’s deuteron scattering code DDTP.
REFERENCES

[1] R.S. Mackintosh and A.M. Kobos, Phys. Lett. **116B**, 95 (1982); A.A. Ioannides and R.S. Mackintosh, Nucl. Phys. **A438**, 354 (1985).

[2] V.I. Kukulin, V.N. Pomerantsev and J. Horáček, Phys. Rev. **A 42**, 2719 (1990)

[3] S.G. Cooper and R.S. Mackintosh, Phys. Rev. **C 43**, 1001 (1991); Nucl. Phys. **A517**, 285 (1990); Nucl. Phys. **A576**, 308 (1994); Nucl. Phys. **A582**, 283 (1995); Phys. Rev. **C 54**, 3133 (1996)

[4] G.R. Satchler, Nucl. Phys. **21**, 116 (1960)

[5] S.G. Cooper, Program IMAGO, Open University Report

[6] R.P. Goddard, code DDTP, described and specified in University of Wisconsin Report, 1977. Our version was supplied by J.A. Tostevin.

[7] R. Frick, H. Clement, G. Graw, P. Schiemenz and N. Seichert, Phys. Rev. Lett. **44**, 14 (1980)

[8] P.W. Keaton, Jr. and D.D. Armstrong, Phys. Rev. **C 8**, 1692 (1973)

[9] V.I. Kukulin, V.N. Pomerantsev and S.B. Zuev, Yad. Fiz., **59**, 428 (1996); English translation: Physics of Atomic Nuclei, **59**, 403 (1996)

[10] S.G. Cooper, Nucl. Phys. **A 618**, 82 (1997)

[11] E.V. Kuznetsova and V.I. Kukulin, Yad. Fiz. **60**, 608 (1997); English translation: Physics of Atomic Nuclei, **60**, 528 (1997)

[12] B. Jenny, W. Gruebler, V. König, P.A. Schmelzbach, and C. Schwizer, Nucl. Phys. **A397**, 61 (1983)

[13] S.G. Cooper, V.I. Kukulin, R.S. Mackintosh and E.V. Kuznetsova, Phys. Rev. **C58**, R31 (1998)

[14] R.S. Mackintosh and S.G.Cooper, J. Phys. G. **24**, 1599 (1998)

[15] S.G. Cooper and R.S. Mackintosh, Phys. Rev. **C57**, 3133 (1996)

[16] R.S. Mackintosh and S.G.Cooper, Nucl. Phys. **A589**, 377 (1995); R.S. Mackintosh and S.G.Cooper, Nucl. Phys. **A625**, 651 (1997) S.G. Cooper, Nucl. Phys. **A626**, 715 (1997)

[17] V.I. Kukulin, V.N. Pomerantsev, S.G. Cooper and R.S. Mackintosh, Few Body Systems, in press.

[18] S.B. Dubovichenko, Physics of Atomic Nuclei, **61**, 162 (1988). Translated from Yad. Fiz. **61**, 210 (1988)

[19] V.I. Kukulin, V.N. Pomerantsev, S.G. Cooper, and S.B. Dubovichenko, Phys. Rev. **C 57**, 2462 (1998)
FIGURES

FIG. 1. For deuterons scattering from $^4$He at 20.2 MeV, the full line represents the potential of Frick et al, the long dashes represent the SRP (non-zero only for the real central term), the dots represent inversion solution A, and the dot-dashes represent solution B.

FIG. 2. From top: Real, Wigner and Majorana, central components; real, Wigner spin-orbit component; imaginary, central, even and odd parity components. All were evaluated at 10 MeV, and correspond to inverted potentials with $\chi^2/N = 8.59$ (solid line) and $\chi^2/N = 8.01$ (dashed line).

FIG. 3. Tensor components of the potentials of Figure 2, from top: Even and odd, real then even and odd imaginary. Again, solid line is for $\chi^2/N = 8.59$ and dashed line is for $\chi^2/N = 8.01$.

FIG. 4. For deuterons scattering from $^4$He at a laboratory energy of 10 MeV, $\sigma(\theta)$, $iT_{11}(\theta)$, $T_{20}(\theta)$, $T_{21}(\theta)$, $T_{22}(\theta)$ given by the potentials shown in Figures 2 and 3, with $\chi^2/N = 8.59$ (solid line) and $\chi^2/N = 8.01$ (dashed line), compared with the data of Jenny et al (solid points).
Fig. 1

![Graph showing various potentials and their solutions](image-url)

- **Target V**
- **SRP**
- **Solution A**
- **Solution B**

**Real Central**

**Imag. Central**

**Real Spin–orbit**

**Real Tensor**

- **V (MeV)**
- **r (fm)**
Fig. 2

Re. Cen Wigner

\[ \frac{\chi^2}{N} = 8.59 \]
\[ \frac{\chi^2}{N} = 8.08 \]

Re. Cen Majorana

Re. Spin–orbit

Im. Cen Even

Im. Cen Odd
Fig. 3

$\chi^2/N = 8.59$

$\chi^2/N = 8.08$

Re Tensor Even

Re Tensor Odd

Im Tensor Even

Im Tensor Odd

$V \text{ (MeV)}$

$r \text{ (fm)}$
Fig. 4

\[ \chi^2/N = 8.59 \]

Jeney et al.