A Unifying Framework for Combining Complementary Strengths of Humans and ML toward Better Predictive Decision-Making

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Abstract
Hybrid human-ML systems are increasingly in charge of consequential decisions in a wide range of domains. A growing body of empirical and theoretical work has advanced our understanding of these systems. However, existing empirical results are mixed, and theoretical proposals are often mutually incompatible. In this work, we propose a unifying framework for understanding conditions under which combining the complementary strengths of humans and ML leads to higher quality decisions than those produced by each of them individually—a state which we refer to as human-ML complementarity. We focus specifically on the context of human-ML predictive decision-making and investigate optimal ways of combining human and ML predictive decisions, accounting for the underlying sources of variation in their judgments. Within this scope, we present two crucial contributions. First, taking a computational perspective of decision-making and drawing upon prior literature in psychology, machine learning, and human-computer interaction, we introduce a taxonomy characterizing a wide range of criteria across which human and machine decision-making differ. Second, formalizing our taxonomy allows us to study how human and ML predictive decisions should be aggregated optimally. We show that our proposed framework encompasses several existing models of human-ML complementarity as special cases. Last but not least, an initial exploratory analysis of our framework presents a critical insight for future work in human-ML complementarity: the mechanism by which we combine human and ML judgments should be informed by the underlying causes of divergence in their decisions.

1 Introduction
Recent years have witnessed a rapid growth in the use of Machine Learning (ML) models to assist decision-making across a wide range of high-stakes domains, including healthcare [Patel et al., 2019, Rajpurkar et al., 2020, Tschandl et al., 2020, Bien et al., 2018], credit lending [Bussmann et al., 2021, Kruppa et al., 2013], criminal justice [Angwin et al., 2016, Kleinberg et al., 2017], and employment [Raghavan et al., 2020, Hoffman et al., 2017]. For example, in the criminal justice system, algorithmic recidivism risk scores inform pre-trial bail decisions for defendants [Angwin et al., 2016]. In credit lending, lenders routinely use credit-scoring models to assess the risk of default by applicants [Kruppa et al., 2013]. The excitement around modern ML systems facilitating high-stakes decisions is fueled by the promise of these technologies to tap into large datasets, mine the relevant statistical patterns within them, and utilize those patterns to make more accurate predictions at a lower cost and without suffering from the same cognitive biases and limitation as human decision-makers. Growing evidence, however, suggests that ML models are vulnerable to biases of

*indicates equal contribution. The ordering was decided by a coin toss.
their own [Angwin et al., 2016], instability [Finlayson et al., 2018], and opaqueness [Burrell, 2016]. Additionally, they may lack crucial human strengths such as contextual knowledge and commonsense reasoning [Holstein and Aleven, 2021, Lake et al., 2017, Miller, 2019]. These observations have led to calls for effective human involvement in high-stakes decision-making—with the hope of combining and amplifying the respective strengths of human experts and ML models through carefully designed hybrid decision-making systems. Such systems consist of ML models and human experts jointly making decisions, and they are common in practice—including in the domains mentioned above.

Researchers have proposed and tested various hybrid human-ML designs, ranging from human-in-the-loop [Russakovsky et al., 2015] to algorithm-in-the-loop [De-Arteaga et al., 2020, Saxena et al., 2020, Brown et al., 2019, Green and Chen, 2019] arrangements. However, empirical findings regarding the success and effectiveness of these proposals are mixed [Lai et al., 2021, and references therein]. Simultaneously, a growing body of theoretical work has attempted to formalize these hybrid designs [Gao et al., 2021, Bordt and von Luxburg, 2020] and study optimal ways of aggregating human and ML decisions within them [Madras et al., 2018, Mozannar and Sontag, 2020, Wilder et al., 2020, Keswani et al., 2021, Raghu et al., 2019, Okati et al., 2021, Steyvers et al., 2022]. Additionally, prior work has studied human-ML complementarity under a variety of conditions [Donahue et al., 2022]. Existing theories, however, are hard to navigate and make sense of as a whole due to (often implicit) idiosyncratic assumptions within different formalizations—making it challenging to compare existing proposals and foresee the conditions under which one would outperform another. Even within the same conceptual framework, empirical results are inconclusive and sensitive to the context, human expertise, and other situational factors [Lai et al., 2021].

While prior work has attempted to combine human and machine decisions optimally under various stylized settings, it has not systematically addressed the underlying causes of human-ML complementarity, the condition in which an aggregation mechanism can take the human and ML decisions as inputs and output decisions that outperform both human- and ML decisions in isolation. We argue, therefore, that there is a clear need to form a deeper, more fine-grained understanding of what types of human-ML systems lead to complementarity and under what conditions. In this work, we respond to this gap in the literature by building and formalizing a taxonomy around sources of human-ML complementarity. This taxonomy aims to provide a shared understanding of the causes and conditions of complementarity so that researchers and practitioners can design more effective hybrid systems—by investigating and enumerating the distinguishing characteristics of human vs. ML-based decision-making within their context of interest.

**Scope of inquiry.** We focus on a specific paradigm of hybrid human-ML decision-making, which we refer to as combining predictive decisions in static environments. We deliberately narrow down the scope of inquiry as follows:

- **Static environments:** We set aside decision-making contexts that are inherently sequential in nature. In particular, we rule out settings where (1) the decision at hand is only meaningful if viewed as one piece in a sequence of decisions made for the corresponding unit; (2) the decision for the unit at hand heavily influences decisions for other units.

- **Predictive decisions:** The decision at stake is solely based on predicting some outcome of interest [Mitchell et al., 2018]. Henceforth, we use the terms ‘prediction’ and ‘decision’ interchangeably. Some examples of predictive decisions are diagnosis of diabetic retinopathy [Gulshan et al., 2016] and consumer credit risk prediction [Bussmann et al., 2021, Krupp

1Note that “outperforming” does not always refer to accuracy gains. We will revisit this point in Section 2.
• **Combining human and ML decisions:** We consider hybrid designs in which an independent third party combines human and ML predictions into a joint decision. Such designs are only sensible in domains where human and ML decisions are comparable and equally credible. This holds true for application domains such as image classification [Beck et al., 2018, Russakovsky et al., 2015, Kerrigan et al., 2021], crowdsourcing [Kamar et al., 2012], and clinical radiology [Patel et al., 2019, Rajpurkar et al., 2020, Bien et al., 2018].

We emphasize at the outset that the above scope does not capture a large number of practical decision-making scenarios, but its stylized nature facilitates more rigorous investigations.

**Computational approach.** Taking a computational perspective toward decision-making and drawing upon prior literature in human psychology, machine learning, and human-computer interaction, we introduce a taxonomy characterizing a wide variety of criteria across which human and machine decision-making differ. Importantly, we follow traditions from cognitive science and computational social science [Lake et al., 2017, Marr and Poggio, 1977], and contrast human decision-making with the machine through a computational lens. As with any modeling approach, adopting this computational view involves making several simplifying assumptions in exchange for useful abstractions. As we will show in this work, it proves to be a powerful perspective for deepening our understanding of human-ML complementarity.

**Contributions.** To build our taxonomy of human-ML complementarity, we reviewed literature on human psychology, machine learning, AI, and human-computer interaction to understand the essential factors across which human and ML decision-making processes differ. Our taxonomy provides a detailed list and characterization of the advantages and disadvantages of human and ML predictions in a static predictive decision-making regime (Section 2.2). We formalize and capture these factors within a unifying mathematical framework developed to analyze optimal ways of combining human and ML predictions according to their distinctive strengths (Section 2.3). In this manner, we provide a generalized recipe (with the appropriate tools) to combine human and ML predictions optimally in a variety of contexts. We also demonstrate how our framework encapsulates many recent human-ML combination schemes, notably those proposed by Mozannar and Sontag [2020] and Raghu et al. [2019] (Section 3). Finally, as a proof of concept, we analyze a special case of our framework (Section 4). In particular, we derive the optimal linear aggregation schemes under three distinct complementarity conditions: (1) human and ML model have access to different features, (2) human decisions are inconsistent, and (3) ML models are trained under biased target labels. Contrasting optimal aggregation under these special cases provides us with a critical insight: the mechanism by which we combine human-ML judgments should be informed by the unique strengths and weaknesses that each one brings to the table. We hope that our contributions provide a common language and an organizational structure to inform future research in the increasingly essential space of human-AI complementarity.

### 2 A Framework to Understand Human-ML Complementarity

In this section, we propose a framework for hybrid decision-making that captures different sources of human-ML complementarity. Our framework draws on existing syntheses in human psychology, machine learning, AI, and human-computer interaction to capture the distinguishing characteristics of human decision-makers and ML in the context of predictive decision-making [Holstein et al., 2020, Lai et al., 2021, Lake et al., 2017]. Our framework views human decision-making through a
computational lens. While computational models of human behavior are inherently reductive, they are often useful in making sense of complex phenomena for this very reason. Specifically for our work, they allow us to cast several existing models of human-ML complementarity as special cases (Section 3), and show that the choice of the appropriate aggregation scheme must be informed by the underlying causes of divergence in human vs. ML predictions (Section 4)—an insight that we believe will be fundamental for future work in the area.

2.1 A Computational Model of Predictive Decision-making

With a computational perspective toward decision-making by an agent, we consider an account of a task the agent performs, the inputs that the agent takes in, the ways in which the agent perceives and processes these inputs, and the kinds of outputs that the agent produces. The agent can be either a human expert or an ML model. Accordingly, we organize sources of human-ML complementarity into four broad categories, characterizing their differences across (1) task definition, (2) input, (3) internal processing, and (4) output of decision-making.

Formally, the agent’s decision-making setting is specified by a covariate space \( X \), an action space \( A \), and the space of observed outcomes, \( O \). At a high-level, the agent perceives an instance \( x \in X \), chooses to take an action \( a \in A \) based on its relevant prior knowledge and experiences, and observes an outcome \( o \in O \) as a result. We model this as follows. The agent’s perception of an instance \( x \in X \) is denoted by \( s(x) \), where \( s : X \to X \). With this perception \( s(x) \) in mind, the agent chooses to take an action \( a \in A \), utilizing its relevant prior knowledge and relevant experiences encompassed in a set \( D \). Similar to the input instance, the agent’s past experiences are subject to distortions. To model such distortions, we assume the existence of a correspondence \( \delta \) mapping the set of all relevant prior experiences to another set, \( \delta(D) \). After choosing an action, the agent observes the outcome \( o \in O \) of the instance as the result. To emphasize that the outcome, \( o \), is influenced by \( x \) and \( a \), we slightly abuse the notation and use \( o(x,a) \) to denote the outcome as the result of taking action \( a \) for instance \( x \).

As a concrete running example, consider a treatment prescription setting where the decision of interest is choosing a treatment to prescribe to a patient [Hammer et al., 1996]. In this example, \( x \) describes the patient’s medical records; \( s(x) \) is the decision-maker’s perception of medical records (e.g., the patient’s blood test results may be perceived by the doctor with limited precision); \( D \) specifies information about how patients have reacted to various treatments in the past; \( \delta(D) \) may emphasize certain salient cases and down-weight others; \( a \) is the prescribed treatment (e.g., Zidovudine); and \( o \) is the patient’s T cell count once the treatment has been prescribed.

The goal of decision-making agent is to choose a policy, \( \pi : \mathcal{X} \to \mathcal{A} \), in the space of feasible policies \( \Pi \), such that \( \pi \) leads to favorable overall outcome quality, measured by an evaluation function \( F \). We assume \( F \) takes in a policy and a dataset that the policy is evaluated upon and outputs a real number. Note that a policy maps the perceived case to an action and the dataset that the policy is evaluated upon is the agent’s perceived relevant experience \( \delta(D) \). An example for the evaluation function is the expected outcome of the policy \( F(\pi, D) := \mathbb{E}_P[o(x, \pi(s(x)))] \), where the expectation is over \( P \) denoting the population and \( \delta \) is the identity function. The agent chooses a policy

\[
\pi^* \in \text{OPT}_{\pi \in \Pi} F(\pi, \delta(D)),
\]

We expect, and in fact invite, future additions and modifications to our taxonomy, and hope that it serves as a stepping stone toward a unified understanding of human-ML complementarity under much broader conditions.

Throughout the paper, we use lower cases to refer to both a random variable and a sample of it.
where OPT denotes the process the agent uses for finding policies with favorable $F$ values.

In summary, an agent’s decision-making can be characterized by the following components:

- **Task definition** specifies the observable outcomes, $O$, and the goals of decision-making, $F$.
- **Input** specifies the information the agent receives about the current instance, $x \in X$, and the prior knowledge and experience they utilize to come up with a decision for the current instance, $D = \{(x_k, a_k, o_k)\}_{k=1}^K$.
- **Internal processing** captures how the agent perceives the current instance ($s(x)$), utilizes past experiences ($\delta(D)$), and picks a decision rule $\pi \in \Pi$ using process OPT.
- **Output** determines the action space that the agent can choose from ($A$), and the way the agent can communicate about their choice of actions.

### 2.2 A Taxonomy of Human vs. ML Strengths & Weaknesses in Predictive Decision-making

The decision-making agent can be either a human expert, denoted by $H$, or a machine learning model, denoted by $M$. We use $X_H \subseteq X$ and $X_M \subseteq X$ to denote the covariate space for $H$ and $M$, respectively. Similarly, we use $A_H \subseteq A$ and $A_M \subseteq A$ to denote the set of actions available to $H$ and $M$, respectively. Finally, we use $O_H$ and $O_M$ to specify the space of outcomes $H$ and $M$ may observe.

Each decision-maker $i \in \{H, M\}$ perceives an input through a function $s_i : X \rightarrow X_i$. It then applies a policy $\pi_i : X_i \rightarrow A_i$ from a set of feasible policies $\Pi_i$. A policy $\pi_i$ maps an instance $x$ to an action $a$. Note that the mapping may be non-deterministic. For instance, suppose the action of interest is continuous (e.g., choosing the dosage of a drug). To capture the noise in choice of action according to policy, we may assume $\forall x \in X$, $a = \pi_i(s_i(x)) + \epsilon_i(s_i(x))$, where $\pi_i(s_i(x))$ denotes the prescribed action for instance $x$ and $\epsilon_i(s_i(x))$ is an instance-dependent mean-zero noise.

Each decision-maker $i \in \{H, M\}$ utilizes parts of their relevant prior experiences and knowledge to choose a policy. In particular, we denote by $D_i$ prior knowledge and experiences that $i$ can utilize to choose their policy. We assume there exists a correspondence $\delta_i$ which maps $D_i$ to a distorted version of it. Decision-maker $i \in \{H, M\}$ chooses their policy by solving

$$\pi^*_i \in \text{OPT}_i \quad F_i(\pi, \delta_i(D_i)),$$

where $F_i$ is $i$’s subjective measure of quality for outcomes and OPT$_i$ is the process $i$ uses to find a policy with favorable evaluations.

Next, we utilize the above computational framework to provide a taxonomy for sources of human-ML complementarity as investigated in prior work.

**Task definition**

- **Misaligned construct of interest** ($O_H \neq O_M$). Human decision-making often relies on inferences about unobservable theoretical constructs (e.g., patient’s medical needs) as outcomes that cannot be directly measured [Jacobs and Wallach, 2021]. ML models are often based on simplistic operationalizations of these constructs, which may diverge substantially from the ways humans infer and reason about these constructs [Kawakami et al., 2022]. For example, Obermeyer et al. [2019] traced the racial disparities in risk scores for healthcare provision to the proxy target variable (cost of healthcare) meant to capture medical need.
• **Objectives** \((F_H \neq F_M)\). Most machine learning models optimize a single mathematically tractable notion of performance (e.g., \(F_M\) can be the 0-1 loss in supervised learning). While there has been growing research on building models with respect to a more diverse set of objectives [Leqi et al., 2019, Khim et al., 2020, Chouldechova and Roth, 2020, Lipton, 2018, Miller, 2019], these objectives are often less complex than the objectives considered by expert decision-makers [Kleinberg et al., 2017]. For example, a doctor may care not only about the patient’s chance of survival as a result of the treatment, but also account for the cost and suffering each treatment regimen may impose on the patient and their family.

**Input**

• **Access to different information about the current instance** \((X_H \neq X_M)\). Human decision-makers may rely on information that are not amenable to codification for the machine. For example, a doctor can see the physical presentation of a patient. Similarly, a judge may learn about the predisposition of the defendant to future crime by observing their real-time behavior in the courtroom [Kleinberg et al., 2018].

• **Nature of relevant prior experiences and expertise** \((D_H \neq D_M)\). The nature of human experience and expertise—often amassed over a long period of time—differs substantially from the training datasets used by ML systems [Klein, 2008]. For example, ML models are often trained using a large number of prior instances of a very specific predictive task. Humans make their decisions with reference to a lifetime of experiences across a range of domains, and it is often infeasible to explicitly enumerate the prior experiences they take into account in reaching a decision [Cioffi, 2001].

**Internal processing**

• **Models of the world** \((\Pi_H \neq \Pi_M)\). Machine learning algorithms often rely on mathematically tractable hypothesis classes (e.g., \(\Pi_M\) can be the set of all neural networks of a certain depth and width) to capture statistical patterns in data. In contrast to ML systems, human experts’ mental models tend to be compositional and causal [Lake et al., 2017]. These strong prior beliefs about the world can enable humans to learn rapidly in comparison to ML systems, and to make impressive inferential leaps based on very limited data [Gopnik and Wellman, 2012, Lake et al., 2017, Tenenbaum et al., 2011].

• **Input processing and perception** \((s_H \neq s_M \text{ and } \delta_H \neq \delta_M)\). The ways decision-makers perceive inputs (both information about the current instance \(x_i\) and relevant prior experiences \(D_i\)) may differ substantially [Gentner and Stevens, 2014, Holstein et al., 2020]. Following research in human cognition and ML, we highlight two important sources of distortion in input perception: (1) differences in mental capacity, and (2) differences in human versus machine biases. For instance, compared with ML systems, humans demonstrate less capacity to perceive small differences in numerical values [Amitay et al., 2013, Findling and Wyart, 2021]. Furthermore, both humans and ML systems can bring in both adaptive and maladaptive biases, based on their experiences and models of the world, which in turn shape the ways they process and perceive new situations [Fitzgerald and Hurst, 2017, Wistrich and Rachlinski, 2017, Kleinberg et al., 2018, Gentner and Stevens, 2014].

• **Choosing among models of the world** \((OPT_H \neq OPT_M)\). Given the task definition, a class of models of the world, and a set of data, modern ML models such as neural networks are commonly trained using first-order methods to optimize the objective in the model class and may require a huge amount of computational resource [Bottou, 2010]. (Note that while training
can be time-consuming in ML systems, once a model has been trained, it can generate larger volumes of decisions in less time than human decision-makers.) On the other hand, humans may employ heuristics, such as satisficing, that can be executed in a relatively short amount of time to choose a decision-making model [Simon, 1979]. These simple strategies may have advantages over more complex models under conditions such as high task uncertainty. For a comprehensive review on when and how heuristics may be more preferable, we refer readers to Kozyreva and Hertwig [2021].

Output

- **Available actions** ($A_H \neq A_M$) ML models often have to choose their action from a restricted set of well-defined alternatives. Human experts, on the other hand, may have a larger set of alternatives at their disposal [Kawakami et al., 2022]. Relatedly, humans and ML have differing abilities in communicating the reasoning behind their decisions and their level of confidence in the quality of those decisions. For instance, Miller [2019] argues that human explanations are contrastive, selected in a biased manner, and most importantly they are social and contextual. In contrast, ML predictions are traceable, in that we can trace the precise computational steps that led to a specific prediction for a given instance [Hu et al., 2019].

- **Consistency** ($\varepsilon_H \neq \varepsilon_M$) We define a given decision-maker to have a consistent output when they always produce the same decision for the same input. Research in human behavior and psychology has shown that human judgments show inconsistency [Kahneman et al., 2016]. More specifically, it is possible for a given human decision-maker to change their decisions given the exact same instance at two different times. We call this within-person inconsistency, which does not include inconsistency across more than one human. Within-person inconsistency in human judgments has been observed across many domains, including medicine [Koran, 1975, Kirwan et al., 1983], clinical psychology [Little, 1961], finance and management [Kahneman et al., 2016]. This form of inconsistency is not exhibited by standard machine learning models.\(^4\)

2.3 Aggregation Mechanisms for Complementarity

We study the combination of H and M’s policies to obtain a joint policy $\pi: \mathcal{X} \rightarrow \mathcal{A}$ that improves the overall quality of outcomes. We assume that the joint policy is obtained by combing $\pi_H$ and $\pi_M$ through an aggregation function $G: \mathcal{X} \times \mathcal{A} \times \mathcal{A} \rightarrow \mathcal{A}$. In particular, given the actions recommended by $\pi_H \in \Pi_H$ and $\pi_M \in \Pi_M$, we assume the joint policy $\pi \in \Pi$ to be defined as follows:

$$\forall x \in \mathcal{X}, \quad \pi(x) = G(x, \pi_H(s_H(x)), \pi_M(s_M(x))).$$

In this work, we use a weighted aggregation function of the following semi-parametric form:

$$\pi(x) = w_H(x)\pi_H(s_H(x)) + w_M(x)\pi_M(s_M(x)),$$

(2)

where $\forall x \in \mathcal{X}, w_H(x), w_M(x) \in [0, 1]$, and $w_H(x) + w_M(x) = 1$. Observe that the joint decision is a convex combination of H and M’s actions. Several existing works have studied such weighted aggregation functions (e.g., [Mozannar and Sontag, 2020, Gao et al., 2021, Raghu et al., 2019, Donahue et al., 2022]). In Section 3, we show how our framework captures these prior works on combining human- and ML decision-making.

To understand conditions for complementarity, we first provide a formal definition as follows:

\(^4\)We don’t consider randomized ML models as inconsistent, since they can be directly mapped to deterministic models by fixing the seed of randomization or using deterministic thresholds.
**Definition 1.** The joint policy \( \pi \) defined in (2) exhibits complementarity if and only if

\[
F(\pi, \mathbb{P}) > \max\{F(\pi_H, \mathbb{P}), F(\pi_M, \mathbb{P})\}.
\]

We denote the the optimal joint policy that maximizes \( F(\pi, \mathbb{P}) \) by \( \pi^* \) and its corresponding aggregation function and weighting functions by \( G^* \), \( w^*_M \) and \( w^*_H \).

**Quantitative notions of complementarity** To capture the *extent* of complementarity, we introduce two measures: the across-instance and within-instance complementarity. In (2), for a given instance \( x \), either one of the human or ML makes the final decision \( w_M(x) = 0 \) or \( w_H(x) = 0 \), or both decision-makers contribute to the final decision partially \( w_M(x) \neq 0 \) and \( w_H(x) \neq 0 \). In the first type, since there is no complementarity within a single instance we call it across-instance complementarity. In the second type, since both the human and the ML model contribute to the same instance \( x \), we call it within-instance complementarity.

Formally, consider a joint policy \( \pi \), obtained via the aggregation function \( G \) with weight functions \( w_H \) and \( w_M \). We define a factor \( c(\pi) = F(\pi, \mathbb{P}) - \max\{F(\pi_H, \mathbb{P}), F(\pi_M, \mathbb{P})\} \) that quantifies the total amount of complementarity exhibited in \( \pi \). When \( c(\pi) < 0 \), there is no complementarity in \( \pi \). The across- and within-instance optimality are defined as follows:

- **Across-instance complementarity** quantifies the *variability* of \( w_H \) and \( w_M \) across different \( x \in X \):

\[
d_{\text{across}}(w_H, w_M) = \mathbb{E}_x \left[ (w_H(x) - \mathbb{E}[w_H(x)])^2 \right] = \mathbb{E}_x \left[ (w_M(x) - \mathbb{E}[w_M(x)])^2 \right],
\]

where the equality follows from the constraint \( \forall x \in X, w_M(x) + w_H(x) = 1 \). In case of no variability across instances, that is if for \( i \in \{H, M\} \), \( w_i(x) \) is a constant for all \( x \in X \), then \( d_{\text{across}}(w_M, w_H) = 0 \). The type of complementarity studied in Mozannar and Sontag [2020], Madras et al. [2018] is across-instance.

- **Within-instance complementarity** quantifies how much on average \( \pi_H(x) \) and \( \pi_M(x) \) contributes to each instance, expressed as

\[
d_{\text{within}}(w_H, w_M) = \mathbb{E}_x \left[ 1 - (w_H(x) - w_M(x))^2 \right].
\]

Within-instance complementarity is maximized when each decision-maker contributes equally to a problem instance, that is, \( \forall x \in X, w_H(x) = w_M(x) = 0.5 \). \( d_{\text{within}} \) is minimized when there is no contribution from one of the two decision-makers, i.e., \( \forall x \in X, w_H(x) \in \{0, 1\} \). Within-instance complementarity is demonstrated in certain Human-ML teams [Patel et al., 2019, Tschandl et al., 2020].

Finally, when comparing the across- and within-instance complementarity among different policies, we multiply (3) and (4) by \( c(\pi) \). For more discussions on these complementarity notions, we refer the readers to Appendix B.

**3 Prior Work as Special Cases of Our Framework**

Prior work in hybrid decision-making has considered two general collaborative strategies among others, one wherein the machine policy and the aggregation policy are optimized simultaneously Mozannar and Sontag [2020], and the other wherein the aggregation policy and the machine policy are optimized one after the other [Raghu et al., 2019]. In this section, using Example 1 and Example 2,
we illustrate that these two different strategies fit neatly into our framework (1). In the binary classification setting considered in Mozannar and Sontag [2020], for any given instance \( x \in \mathcal{X} \), the machine makes a choice to either defer the decision to the human expert or to make the decision itself. In either case the decision is the predicted binary label \( A = \{0, 1\} \). In other words, under this setting, the predictions made by the decision-makers are the actions/decisions taken by them. The outcome for a given instance is determined by the loss incurred by the decisions. Formally, a \( \pi \) is learned to minimize the expected loss (i.e., the machine’s expected outcome and a property of the aggregation function (i.e., whether the final decision comes from the human decision-maker):

\[
F(\pi, P) := -E[\ell(\pi(x), y) + w_H(x) \cdot c(x, y, \pi_H(x))], \tag{5}
\]

where \( w_H(X) \cdot c(x, y, \pi(x)) \) specifies the cost incurred by deferring the final decision to a human expert. The goal is to find the joint policy \( \pi \) that minimizes \( F(\pi, P) \). Finally, these examples consider a special case of the aggregation function (2) by restricting \( w_H \) and \( w_M \) to be binary and they differ at the machine’s policy class \( \Pi_M \).

**Example 1** (Mozannar and Sontag [2020]). Under the above specification, We can equivalently write the optimization problem (1) as

\[
\min_{w_M, w_H: \mathcal{X} \to \{0,1\}, \pi_M \in \Pi_M} E[\ell(\pi(x), y) + w_H(x) \cdot c(x, y, \pi_H(x))]
\]

s.t. \( \forall x \in \mathcal{X}, \pi(x) = w_H(x)\pi_H(x) + w_M(x)\pi_M(x), w_H(x) + w_M(x) = 1 \).

As shown in Mozannar and Sontag [2020], the learned joint policy outperforms the individual expert and the machine policy by ensuring the machine to be trained upon cases where deferring to the human expert incurs a high cost.

In settings such as the one studied by Raghu et al. [2019], the machine model \( \pi_M \) is learned first and the aggregation function is optimized at a second stage. The next example shows how to instantiate our framework in such settings as a bi-level optimization program.\(^5\)

**Example 2.** The machine policy \( \pi_M^* \) is learned to minimize the expected loss (i.e., the machine’s evaluation function is \( F_M(\pi_M, P) = E[\ell(\pi_M(x), y)] \), under a model class \( \Pi_M \). We rewrite (1) as

\[
\min_{w_M, w_H: \mathcal{X} \to \{0,1\}} E[\ell(\pi(x), y) + w_H(x) \cdot c(x, y, \pi_H(x))]
\]

s.t. \( \pi_M^* \in \arg\min_{\pi_M \in \Pi_M} E[\ell(\pi_M(x), y)], \forall x \in \mathcal{X}, \pi(x) = w_H(x)\pi_H(x) + w_M(x)\pi_M^*(x), w_H(x) + w_M(x) = 1 \).

In Example 1 and 2, the human-ML complementarity are across-instance not within-instance, since an instance is either assigned to a human expert or a machine to decide. This is also captured

\(^5\)One may consider Example 2 to be a special case of Example 1.
in our proposed complementarity notions, i.e., \( d_{\text{across}}(w^\text{H}_i, w^\text{M}_i) > 0 \) and \( d_{\text{within}}(w^\text{H}_i, w^\text{M}_i) = 0 \) where \( d_{\text{across}} \) and \( d_{\text{within}} \) are defined in (3) and (4) respectively.

While these works provide ways of obtaining complementary decisions by allocating decisions to either the human expert or the ML model, they do not provide conditions under which one should look for such types of complementarity. In the following section, we showcase settings where knowing how the human and machine decision-making differs helps guide the optimal aggregation mechanism (and, hence, the collaboration strategy) for the joint decision-making policy.

4 An Illustration of Complementarity Analysis

We analyze the optimal aggregation function in a special case of binary classification. Through this analysis, we demonstrate how the optimal aggregation function can vary depending on the source of complementarity (Section 2.2). All proofs for this section are deferred to Appendix C.

We first set up the binary classification task where both the human and the machine have a finite covariate space. Each decision maker \( i \in \{H, M\} \), given an instance \( x \in X \), perceives it as \( s_i(x) \in X_i \) (this includes settings where the human and machine have access to different features), makes a decision \( a \in [0, 1] \) indicating the predicted probability of the covariate being labelled as positive. This decision incurs an outcome measured by the squared error loss \( o = (a - y)^2 \) where \( y \in \{0, 1\} \) is the true label of the instance. This implies that, for both \( H \) and \( M \), the action space is \( A = [0, 1] \) and the outcome space is \( O = \mathbb{R}_+ \). In the following complementarity analysis, the joint policy is learned by minimizing the expected outcome \( F(\pi, \mathbb{P}) = \mathbb{E}[(\pi(x) - y)^2] \).

In our special case, we deal with settings where human decisions are inconsistent and ML models are trained on biased target labels. To model this, we first introduce a notion of best-fitting model. A decision-maker’s best-fitting model is defined as follows:

\[
\pi^\text{b}_i \in \arg\min_{\pi_i \in \Pi_i} F(\pi_i, \mathbb{P}) \quad \forall i \in \{H, M\},
\]

where \( \Pi_i \) contains all measurable functions mapping from \( X_i \) to \( A \).

**Lemma 1.** For any decision-maker \( i \in \{H, M\} \), its best-fitting model is given by: \( \forall x \in X \), \( \pi^\text{b}_i(s_i(x)) = \mathbb{E}[y|s_i(x) = s_i(x)] \).

To account for the **inconsistency in human decisions** and **target label bias in machines**, with the help of our taxonomy in Section 2.2, we model the human and machine decision-makers \( \pi_H, \pi_M \), using their best-fitting models as follows:

- **Inconsistency in human decisions:** Human decision-makers can be inconsistent in their decisions while most machines are consistent. In this case, the human policy is given by \( \pi_H(s_H(x)) = \pi^\text{b}_H(s_H(x)) + \varepsilon(s_H(x)) \) where \( \varepsilon(s_H(x)) \) are independent mean-zero noises with bounded variance \( \sigma^2(s_H(x)) \).

- **Target label bias for machines:** We consider that the machine has access to biased labels and the label bias \( B \) is constant and independent of label \( y \). However, \( \beta \) can potentially depend on the covariates \( x \). Under this case, the machine policy is the best-fitting model under the biased data \( (x, y + \beta) \) instead of \( (x, y) \). That is, \( \pi_M(s_M(x)) = \pi^\text{b}_M(s_M(x)) + \beta(s_M(x)) \), where \( \beta(s_M(x)) = \mathbb{E}[\beta|s_M(x)] \) (Appendix C).

Next, we present the optimal aggregation mechanism—the corresponding weighting functions that maximize \( F(\pi, \mathbb{P}) \)—in this setup.
Proposition 1. Define
\[ \tilde{w}(x) = \frac{(\pi^b_H(x) - \pi^b_M(x) - \beta(s_M(x)))(E[y|x] - \pi^b_M(x) - \beta(s_M(x)))}{(\pi^b_H(x) - \pi^b_M(x) - \beta(s_M(x)))^2 + \sigma^2(s_H(x))}. \]

The optimal weighting function \( w^*_H(x) \) and \( w^*_M(x) = 1 - w^*_H(x) \) are given as follows: for all \( x \in X \),
- If \( \pi^b_H(x) - \pi^b_M(x) - \beta(s_M(x)) = 0 \) and \( \sigma(s_M(x)) = 0 \), then \( w^*_H(x) \) can be any value in \([0, 1]\).
- Otherwise, \( w^*_H(x) \) is unique and is defined as \( w^*_H(x) = \min\{1, \max\{0, \tilde{w}(x)\}\} \).

Implications In settings where the human and machine policies are the best-fitting model \( \pi^b_H \) and \( \pi^b_M \) respectively (i.e., the human decisions are consistent and the machine policy is learned under no label bias), the optimal weight \( w^*_H(x) \) has a simpler form: When \( \pi^b_H(x) = \pi^b_M(x) \), \( w^*_H(x) \) can by any value in \([0, 1]\). When \( \pi^b_H(x) \neq \pi^b_M(x) \), we have
\[ w^*_H(x) = \frac{E[y|x = x] - E[y|s_M(x) = s_M(x)]}{E[y|s_H(x) = s_H(x)] - E[y|s_M(x) = s_M(x)]}. \] (6)

In such a case, we have some direct observations of the optimal weights: the joint policy relies on the human (or machine) fully, i.e., \( w^*_H(x) = 1 \) (or \( w^*_M(x) = 1 \)) if and only if its decision \( \pi^b_H(x) \) (or \( \pi^b_M(x) \)) is the same as \( E[y|x = x] \). For example, if a decision-maker has access to all features \( s_i(x) = x \), then the joint policy will always follow that decision-maker \( (\forall x \in X, w^*_i(x) = 1) \). In settings where the human and machine both have access to a subset of the features, the optimal weight depends on how “relevant” their features are to the decision. For example, as \( E[y|s_H(x) = s_H(x)] \) gets closer to \( E[y|x = x] \), the weight \( w^*_H(x) \) gets closer to 1. When the noise variance and the machine bias are nonzero, the optimal weight depends on their values. For example, when \( w^*_H(x) = \tilde{w}(x) \), as the variance of the noise gets larger, \( w^*_H(x) \) gets closer to 0, suggesting that the human decisions should contribute less to the final decision.

Thus, we demonstrate the use of our framework to investigate human-ML complementarity in this classification setting, further illustrated in Figure 1. Using Proposition 1, we show that complementarity exists, i.e., the optimal joint policy outperforms both the individual policies, in terms of the evaluation function. Specifically, for some \( x \in X \), we see that \( w^*_H(x) \) (or \( w^*_M(x) \)) lies in \((0, 1)\), thus the optimal joint decision does not rely on one decision-maker alone, and depending on the source of complementarity (e.g., feature access, inconsistency, biased target labels), the optimal weights and hence the form of complementarity are different. For further analysis, we compute across-instance (3) and within-instance complementarity (4) for the two cases considered in Figure 1. For case (1), we get \( d_{\text{across}}(w^*_H, w^*_M) = 0.18 \) and \( d_{\text{within}}(w^*_H, w^*_M) = 0.048 \), while for case (2) we get \( d_{\text{across}}(w^*_H, w^*_M) = 0.16 \) and \( d_{\text{within}}(w^*_H, w^*_M) = 0.075 \). We observe that as we go from case (1) where the human and machine have access to different features to case (2) where human has access to all features but is inconsistent, the within-instance complementarity increases and the across-instance complementarity decreases. The increase in within-instance complementarity may be attributed to the inconsistency in human decisions, since while the human expert has access to all features to inform their decisions, their decisions can be inconsistent and they will benefit from within-instance collaboration. Thus, by measuring the within-instance and across-instance variability of the optimal weights assigned to the humans or ML, we quantify the extent of complementarity for this setting.
Figure 1: We provide a toy example to illustrate results in Section 4. In this toy example (a), each grid point represents a distinct covariate value in \( \mathcal{X} \subset \mathbb{R}^2 \). The number on top of the grid point gives \( \mathbb{E}[y|x = x] \). For instance, \( \mathbb{E}[y|x = (-1, -1)] = 0.2 \). All covariate values are equally probable. We consider two different cases in Figures (b) and (c), and for each of these two cases, we derive the optimal weighting function for human \( w^*_H \) according to Proposition 1 and plot that as a heatmap, wherein \( w^*_H(x) = 1 \) (black) indicates the entire weight for covariate \( x \) is given to the human. The two cases are as follows: (1) Human has access to \( x_1 \) while machine has access to \( x_2 \), and both use their best-fitting model (Lemma 1). (2) Human has access to both \( x_1 \) and \( x_2 \), while machine has access to only \( x_2 \). The human decision-maker is inconsistent, with variance \( \forall x \in \mathcal{X}, \sigma^2_H(x) = 0.04 \). In case (2), even though the human has the true \( \mathbb{E}[y|x = x] \), due to their inconsistency, the optimal weights lie between (0, 1), exhibiting both across- and within-instance complementarity.

5 Discussion and Future Work

Our work offered an initial yet critical contribution toward a more comprehensive understanding of complementarity in human-ML decision-making. We aimed to bring much-needed organization to the growing line of work on hybrid human-ML decision-making. Towards this broader goal, we deliberately scoped our inquiry to “combining predictive decisions in static environments”. Within this scope, we presented a taxonomy characterizing differences between human and machine decision-making as well as a general optimization-based framework for optimal aggregation of human and machine decisions. Our proposed framework unified several existing approaches to combining human-ML decisions. Critically, the analysis of our framework suggested that the mechanism by which human-ML judgments are combined should be informed by the relative advantages and disadvantages they exhibit in the specific setting at hand. Our optimization framework can be additionally utilized to generate hypotheses about the optimal aggregation schemes in practical settings. Testing such hypotheses both in theory and practice are of great interest.

The scoping considered in this work, while rich, is limited. In particular, it does not apply to settings where a human decision-maker makes the final call or to cases where predictions do not translate to decisions in a straightforward manner. Our framework is based on a relatively narrow decision-making paradigm. As such, it does not capture a wide range of alternative human-ML collaboration paradigms, including those that focus on sequential decisions under resource constraints. These limitations point to critical directions of future research that we hope the research community takes on toward a more comprehensive science of hybrid human-ML decision-making systems.
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Appendix

A  Prior Work through the Lens of Our Taxonomy

As mentioned in the introduction, we believe the empirical and theoretical findings in existing human-ML collaboration literature can be hard to navigate as a whole, in part due to a wide range of (often implicit) assumptions made in different research articles. To elaborate, in this section, we attempt to contextualize prior literature on human-ML collaborative decision-making through the lens of our taxonomy. With this exercise we highlight ambiguities in existing literature about sources of human-AI complementarity through the lens of our taxonomy.

Much prior work has studied settings where the AI alone outperforms either the human alone or the human-AI team. These studies have often focused on tasks where there are no reasons to expect upfront that the human and the AI system will have complementary strengths [Bansal et al., 2021a, Holstein and Aleven, 2021, Lurie and Mulligan, 2020]. For example, some research has studied human-AI collaboration on tasks where AI systems have clear advantages over humans under our taxonomy. Other studies have examined untrained crowdworkers’ performance on decision-making tasks that require strong domain expertise [Fogliato et al., 2021, Lurie and Mulligan, 2020] (see Section 2.2). In the context of these study designs, there does not seem to be room for complementarity.

To systematically investigate HAI complementarity, Bansal et al. [2021b] artificially constrained the AI’s accuracy to be comparable to that of human crowdworkers in their study, in order to simulate conditions that can emerge in real deployment settings. Through experiments, the authors found that human accuracy was high on samples the AI got incorrect and vice versa. These disparities in accuracy, coupled with comparable overall accuracy, yielded complementary performance of the HAI team. In terms of our taxonomy, this work considers a setting where there is overall performance complementarity between the human and the AI, but the mechanisms behind this complementary performance are not well understood. In another recent work, Rastogi et al. [2022] attempt to achieve complementary performance between the human and the machine by providing each with access to different task-relevant information. They also showed how training the machine on different features led to complementarity in the accuracy of the human alone versus the AI alone. Despite this, the overall HAI team accuracy was similar to that of either the human or the AI alone, indicating that complementarity in input processing and perception was not sufficient to yield overall performance complementarity in this case.

Human-AI collaboration has been implemented and studied in real-world scenarios as well. For instance, in healthcare, Tschandl et al. [2020] and Patel et al. [2019] demonstrate better HAI team performance in skin cancer recognition and chest radiograph diagnosis respectively. Here, the ML model has access to a larger training dataset than an individual human decision-maker, including more training examples and access to finer-grained pixel-level data. Tschandl et al. [2020] note that the benefits of complementary performance increase, as the human collaborators’ expertise increases. In our taxonomy, this suggests that human decision-makers’ models of the world encode task relevant knowledge that the AI is missing, for example knowledge of causal mechanisms. Similarly, in the context of child maltreatment screening, Cheng et al. [2022] found that human workers were able to mitigate the impacts of racial disparities in recommendations from an ML-based decision support tool by systematically overriding the ML model’s recommendations in certain cases. Through both quantitative analyses of historical decision data and qualitative analyses of
field data from a child welfare agency, the authors found that workers were able to do so by taking advantage of **complementary information** to which they had access but the model did not, and by **reasoning about causal mechanisms** that might underlie their observations, not just the statistical patterns that the ML model took into account.

**B  Definition and Measures of Complementarity**

Definition 1 provides a general definition of complementarity for human-ML teams. We propose further quantification of complementarity by introducing two different measures of complementarity. In human-ML combined decision-making (2), we see that for a given instance \(x\) we either have only one of human or ML making the final decision (\(w_M(x) = 0\) or \(w_H(x) = 0\)), or both decision-makers contributing to the final decision partially (\(w_M(x) \neq 0\) and \(w_H(x) \neq 0\)). In the first type, since there is no complementarity within a single instance we call this **across-instance complementarity**. In the second type, since both human and ML model contribute to the same instance \(x\), we call this **within-instance complementarity**. Importantly, measuring these two types of complementarity separately helps distinguish between different instance allocation strategies analysed in human-ML collaboration research.

The two measures of complementarity rely on properties of the optimal aggregation function \(G^*\) and, in particular, on the optimal weighting functions \(w^*_H, w^*_M\) as defined earlier in Section 2. Formally, we define the two measures as follows.

- **Across-instance complementarity** quantifies the *variability* of \(G^*(x, \pi_H(x), \pi_M(x))\) across different \(x \in \mathcal{X}\), expressed as
  \[
  d_{\text{across}}(w^*_M, w^*_H) = E_x \left[ (w^*_M(x) - E[w^*_M(x)])^2 \right] = E_x \left[ (w^*_H(x) - E[w^*_H(x)])^2 \right],
  \]
  where the equality follows from the constraint \(w_M + w_H = 1\). In case of no variability across instances, that is if for \(d \in \{H, M\}\), we have \(w^*_d(x)\) is a constant for all \(x \in \mathcal{X}\), then \(d_{\text{across}}(w^*_M, w^*_H) = 0\). In Mozannar and Sontag [2020], Madras et al. [2018], the type of complementarity studied is across-instance.

- **Within-instance complementarity** quantifies how much *on average* \(\pi_H(x)\) and \(\pi_M(x)\) contributes to each instance, and is expressed as
  \[
  d_{\text{within}}(w^*_H, w^*_M) = E_x \left[ 1 - (w^*_H(x) - w^*_M(x))^2 \right].
  \]
  Importantly, within-instance complementarity is maximized when each decision-maker contributes equally and maximally to a problem instance, that is, \(w_H = w_M = 0.5\). The measure is minimized when there is no contribution from one of the two decision-makers \(w_H, w_M \in \{0,1\}\). Further, it increases monotonically as \(w^*_H(x)\) and \(w^*_M(x)\) get closer to each other in value, that is the two decision-makers’ contributions to the final decision get closer. This type of complementarity is demonstrated in Human-ML teams in Patel et al. [2019], Tschandl et al. [2020].

**Examples.** We provide some illustrative toy examples to explain the two complementarity metrics introduced. Consider \(\mathcal{X} = \{x_1, x_2, x_3, x_4\}\) where each covariate is equally likely.

1. If \(w^*_H(x_1) = w^*_H(x_2) = w^*_H(x_3) = w^*_H(x_4) = 0\), then \(d_{\text{within}}(w^*_H, w^*_M) = 0\), and \(d_{\text{across}}(w^*_H, w^*_M) = 0\).
2. If \( w_\text{H}(x_1) = w_\text{H}(x_2) = 0, w_\text{H}(x_3) = w_\text{H}(x_4) = 1 \), then \( d_{\text{within}}(w_\text{H}^*, w_\text{M}^*) = 0 \), and \( d_{\text{across}}(w_\text{H}^*, w_\text{M}^*) = 0.75 \).

3. If \( w_\text{H}(x_1) = w_\text{H}(x_2) = w_\text{H}(x_3) = w_\text{H}(x_4) = 0.3 \), then \( d_{\text{within}}(w_\text{H}^*, w_\text{M}^*) = 0.84 \), and \( d_{\text{across}}(w_\text{H}^*, w_\text{M}^*) = 0 \).

We note that in the second example, we have \( d_{\text{within}}(w_\text{H}^*, w_\text{M}^*) = 0 \) and \( d_{\text{across}}(w_\text{H}^*, w_\text{M}^*) > 0 \), which is opposite to the third example. Both examples demonstrate complementarity in some form. This shows that across-instance complementarity captures aspects of human-ML complementarity, that are not captured by within-instance complementarity and vice versa.

C Proofs

Proof of Lemma 1. Recall that for \( d \in \{ \text{H}, \text{M} \} \), the best-fitting model is defined to be

\[
\pi_d^b \in \arg \min_{\pi_d \in \Pi_d} \mathbb{E}[(\pi_d(x) - y)^2].
\]

We denote \( \mathcal{X}_d = \{ s_d(x) \mid x \in \mathcal{X} \} \), the set of unique covariates accessible by \( d \). For a given covariate \( x \), we use \( x_d \) to denote the features that are accessible by \( d \), i.e., \( x_d = s_d(x) \) and \( x_{\neg d} \) to denote the features that are not accessible by \( d \). For example, \( x = (x_\text{H}, x_{\neg \text{H}}) \). Rewriting the objective function, we have that

\[
\min_{\pi_d \in \Pi_d} \mathbb{E}[(\pi_d(X) - y)^2] = \min_{\pi_d \in \Pi_d} \sum_{x \in \mathcal{X}} P(x = x) \mathbb{E}_y[(\pi_d(x) - y)^2 \mid X = x]
\]

\[
= \min_{\pi_d \in \Pi_d} \sum_{x_d \in \mathcal{X}_d} P(X_d = x_d) \mathbb{E}_{x_{\neg d}, y}[(\pi_d(x_d) - y)^2 \mid X_d = x_d]
\]

\[
= \sum_{x_d \in \mathcal{X}_d} P(X_d = x_d) \min_{\pi_d(x_d) \in [0,1]} \mathbb{E}_{x_{\neg d}, y}[(\pi_d(x_d) - y)^2 \mid X_d = x_d],
\]

where the last line is true due to the definition of \( \Pi_d \), i.e., the decision for \( x \) and \( x' \) is the same if \( s_d(x) = s_d(x') = x_d \). The proof completes by realizing that

\[
\mathbb{E}_{x_{\neg d}, y}[(\pi_d(x_d) - y)^2 \mid X_d = x_d] = \mathbb{E}_{x_{\neg d}, y}[(\pi_d(x_d) - \mathbb{E}[y|x_d = x_d])^2 \mid X_d = x_d] + \mathbb{E}_{x_{\neg d}, y}[(\mathbb{E}[y|x_d = x_d] - y)^2 \mid X_d = x_d].
\]

Thus, we have that \( \pi_d^b(x) = \pi_d^b(x_d) = \mathbb{E}[y|x_d = x_d] = \mathbb{E}[y|s_d(x) = s_d(x)] \), which completes the proof.

Target label bias for machines: In our discussion, we have claimed that when having access to data \((x, y + \beta)\) (where \( \beta \) is independent of \( y \) but dependent on \( x \)) instead of \((x, y)\), the machine’s best-fitting model is \( \pi_M(x) = \pi_M^b(x) + \mathbb{E}[\beta|s_M(x) = s_M(x)] \). This is true because by replacing the label \( y \) by \( y + \beta \) in Lemma 1, we have its best-fitting model to be

\[
\mathbb{E}[y + \beta | s_M(x) = s_M(x)] = \pi_M^b(x) + \mathbb{E}[\beta|s_M(x) = s_M(x)].
\]

Proof of Proposition 1. Recall that for all \( x \in \mathcal{X} \), we have that \( w_\text{H}(x), w_\text{M}(x) \in [0,1] \) and \( w_\text{H}(x) + w_\text{M}(x) = 1 \). Thus, we can rewrite our objective (1) to

\[
\min_{w_\text{H}} \mathbb{E}[(w_\text{H}(x) \pi_\text{H}(x) + (1 - w_\text{H}(x)) \pi_\text{M}(x) - y)^2] = \min_{w_\text{H}} \mathbb{E}[(w_\text{H}(x) (\pi_\text{H}(x) - \pi_\text{M}(x)) + \pi_\text{M}(x) - y)^2]
\]

\[
= \sum_{x \in \mathcal{X}} \mathbb{P}(x = x) \min_{w_\text{H}(x) \in [0,1]} \mathbb{E}_{y,z|x}[(w_\text{H}(x) (\pi_\text{H}(x) - \pi_\text{M}(x)) + \pi_\text{M}(x) - y)^2 | x = x].
\]
Thus, our goal can be reduced to
\[
\min_{w_H(x) \in [0,1]} \mathbb{E}_{y,\varepsilon|x}[(w_H(x)(\pi_H(x) - \pi_M(x)) + \pi_M(x) - y)^2|x = x]
\]
\[
= \min_{w_H(x) \in [0,1]} \mathbb{E}_{y,\varepsilon|x}[(w_H(x)(\pi_H(x) + \varepsilon(s_H(x))) - \pi_M(x) - \beta(s_M(x))) + \pi_M(x) + \beta(s_M(x)) - y)^2|x = x]
\]
\[
= \min_{w_H(x) \in [0,1]} w_H^2(x)(\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x)))^2 + w_H^2(x)\sigma^2(s_H(x)) + \mathbb{E}_Y[(\pi_M^b(x) + \beta(s_M(x)) - Y)^2]
\]
\[
+ 2 \left( w_H(x)(\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x))) \right) (\pi_M^b(x) + \beta(s_M(x)) - \mathbb{E}[Y|x = x]).
\]

First, we note that the above objective is convex in $w_H(x)$, suggesting that it is sufficient to use the first-order condition to find a global optimum. Observe that when $(\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x)))^2 = 0$ and $\sigma^2(s_H(x)) = 0$, the optimal weighting function can be any value in $[0,1]$. When $(\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x)))^2 + \sigma^2(s_H(x)) \neq 0$, the objective function is quadratic in $w_H(x)$ and the optimum is unique. We obtain the following first-order condition:

\[
w_H^*(x) = (\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x)))(\mathbb{E}[y|x = x] - \pi_M^b(x) - \beta(s_M(x))).
\]

We denote
\[
\bar{w}(x) = \frac{(\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x)))(\mathbb{E}[y|x = x] - \pi_M^b(x) - \beta(s_M(x)))}{(\pi_H^b(x) - \pi_M^b(x) - \beta(s_M(x)))^2 + \sigma^2(s_H(x))}
\]

Given the geometry of our objective function, we have that the optimal weighting function $w_H^*$ to be: for all $x \in \mathcal{X}$,

- If $\bar{w}(x) \in [0,1]$, then $w_H^*(x) = \bar{w}(x)$.
- If $\bar{w}(x) < 0$, $w_H^*(x) = 0$.
- If $\bar{w}(x) > 1$, $w_H^*(x) = 1$.