Calculation method of creep under decreasing stress considering variation of natural environmental factors

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Abstract. The actual structure is always in the state of natural changing environment and stress decreasing in the section. At present, there is little method for calculating the creep under decreasing stress which takes environmental variation into account. As excessive deflection becomes a serious problem for long span continuous rigid frame bridges during the operation period, in this paper, a creep calculation method is proposed for the continuous decrease of stress in most girder sections, and considers the environmental factors comprehensively. Firstly, the multifactor (temperature, relative humidity and reinforcement ratio) correction was made to creep model of the current code JTG D62. And five creep recovery models were compared and analyzed simultaneously. Based on the elastic creep theory and the superposition principle, the improved creep calculation method was proposed. Then theoretical calculation was carried out for the assumed structure. The results showed that the calculated value obtained by the proposed method were obviously different from those obtained by the code, and were significantly different from those obtained without considering creep recovery, which proves the superiority of the proposed method under the action of multiple environmental factors and decreasing stress.

1. Introduction

Long-span continuous rigid frame bridges have suffered from different degrees of cracking and excessive deflection in the operation stage, which poses a threat to the safety and durability of bridges. Concrete creep is one of the most important factors causing such problems[1]. At present, researchers have not formed a consistent understanding on the calculation of concrete creep resulting of plenty of factors affecting concrete creep[2].

The main girder sections of long-span continuous rigid frame bridges mostly present the phenomenon of stress decreasing, and concrete members exhibit creep recovery behavior after unloading[3]. Through experimental research, Neville found that the ratio of creep recovery to creep was positively correlated with mortar strength[4]. Narintsoa carried out creep tests on high-performance concrete cylinder and prism specimens under conditions of compression, tension and bending, and gave creep recovery curves of concrete under each condition[5]. Based on five sets of tests, Lin analyzed concrete creep deformation and its recoverability, and established a mathematical formulation describing concrete creep and creep recovery. The feasibility of the formulation is verified by the results of concrete creep tests under sustained constant loading, completely unloading after loading, repeated loading and step-up-step unloading[6].
Since the continuous rigid frame bridges are in natural environment, the development of concrete creep will be affected by various natural environmental factors\(^7\). Based on the basic creep coefficients under constant temperature and relative humidity, Yang introduced the correction factors of temperature and humidity to establish a compositive creep model for predicting concrete creep under actual environmental conditions. It is found that the maximum error ratio between the predicted result and the measure result is 6%, which is 7% lower than that calculated by the creep model in the specification\(^8\). Cao introduced the reinforcement coefficient to predict the creep of reinforced concrete on the basis of the age-adjusted effective modulus method. The results show that the reinforcement has obvious inhibition effect on concrete creep, and the relative effect is strongest with small reinforcement ratio\(^9\).

Considering the above two aspects, this paper presents an improved calculation method for concrete creep under decreasing stress, which describes the creep using creep coefficient modified by environmental factors and the hysteretic recovery using a separate creep recovery function.

2. Creep model and creep recovery model

2.1. Creep model

2.1.1. Single-factor correction for creep model.

Although the creep model in Specifications for Design of Highway Reinforced Concrete and Prestressed Concrete Bridges and Culverts (JTG 3362-2018)\(^10\) takes into account the influence of component size, concrete strength and relative humidity, it cannot reflect the influence of ambient temperature and reinforcement ratio on creep development. In practical bridge construction, the ambient temperature and relative humidity are constantly changing with time, and there is a certain amount of reinforcement. Therefore, it is necessary to modify the above aspects of the model and carry out single-factor sensitivity analysis.

In terms of temperature, the nominal creep coefficient and creep development coefficient were modified.

the nominal creep coefficient: \(\phi_{\text{RT,T}} = \phi_T + (\phi_{\text{RH}-1}) \phi_T^2 \), \(\phi_T = e^{0.015(T/T_e-20)}\)

the creep development coefficient: \(\beta_{\text{RT}} = \beta_h \beta_T \), \(\beta_T = e^{1000/(273+T_e-15)}\)

Reinforcement can restrain the development of concrete creep. At present, some research results have been obtained on the influence of reinforcement on concrete creep\(^11-15\), CEB-FIP (1970) method\(^12\) is applied in this paper, which is as follows:

\[ k_p = \frac{100}{100 + n \rho}, \rho = 100A_p / A_c \]

The modified creep coefficient is shown as follows:

\[ \phi_p = \phi \cdot k_p \]

In order to investigate the influence of single factor on concrete creep, a C50 concrete prism specimen with loading age of 7 days and theoretical thickness of 50cm was analyzed:

a. When the ambient temperature is 20°C and the relative humidity is 60%, 70%, 80% and 90% respectively, the creep coefficient of concrete specimen is shown in figure 1.

b. When the relative humidity is 60% and the ambient temperature is 10, 15, 20, 25 and 30°C respectively, the creep coefficient of concrete specimen is shown in figure 2.

c. When the ambient temperature is 20°C, the relative humidity is 60%, and the reinforcement ratio is 0.5%, 1.0%, 1.5% and 2.0% respectively, the creep coefficient is shown in figure 3.

As can be seen from figure 1-3, creep coefficient is positively correlated with ambient temperature and negatively correlated with ambient relative humidity and reinforcement ratio. When the ambient temperature increases from 10 to 30°C, the creep coefficient increases by about 73%. When the relative humidity increases from 60% to 90%, the creep coefficient decreases by about 53%. When the
reinforcement ratio increases from 0.5% to 2.0%, the creep coefficient decreases by about 12%. The above results sufficiently show that relative humidity, ambient temperature and reinforcement ratio have great influence on concrete creep development.

Fig. 1 Creep coefficient with different relative humidity

Fig. 2 Creep coefficient with different ambient temperature

Fig. 3 Creep coefficient with different reinforcement ratio

2.1.2. Multiple-factor correction for creep model.

In order to calculate the concrete creep effect in natural environment more accurately, it is necessary to modify the creep model considering comprehensively variable and multiple factors. Based on the superposition principle proposed by Boltzmann in 1878[16], the creep coefficient can be calculated by the following formulation:

$$\phi(t_n, t_0) = \sum_{i=1}^{n} \left[ \phi(t_{i-1}, t_0), T, RH, h, \rho_i \right] - \phi(t_{i-1}, t_0), T, RH, h, \rho_i \right]$$

In order to apply the modified creep coefficient to calculation, a synthetic function expression should be simulated. According to the concrete creep behavior, the Dirichlet series is constructed with exponential function to fit the creep coefficient, the fitting formulation is as follow:

$$\phi(t, t_0) = \sum_{i=1}^{n} C_i(t_0) \left[ 1 - e^{-\alpha(t-t_0)} \right]$$

2.2. Creep recovery model

Yue and Taerwe[17] investigated the creep and creep recovery behavior through creep tests with various loading age and load duration time, and presented a creep recovery model as follow:

$$\phi(t, t_0) = \left( \frac{0.35}{\alpha^{0.22}} \right) \times \left( \frac{t - \tau}{t - \tau + 300 \alpha} \right)^{0.24}$$
Where, \( \alpha = 1 - \exp\left\{-0.1[t_0 + 0.05(t-t_0)]\right\} \)

Wang [18] proposed a prediction model for creep recovery of normal strength concrete with considering the influence of stress level and load history:

\[
\phi_i(t,t_0,\tau) = \phi_{\infty}K(t_0,\tau)\beta_i(t,t_0,\tau)
\]

\[
K(t_0,\tau) = 1 + \exp\left[-at_0 - b(t-t_0)\right]
\]

\[
\beta_i(t,t_0,\tau) = \frac{(t-\tau)^{\alpha}}{(t-\tau)^{\alpha} + \beta(t_0,\tau)}
\]

\[
\beta(t_0,\tau) = c - d\cdot K(t_0,\tau)
\]

Where, \( \phi_{\infty} = 0.35, a = 0.12, b = 0.0055, c = 7, d = 3, \alpha = 0.5 \)

The CEB Mode Code of 1978 (MC78) [19] considers the creep under changing stress as the sum of recoverable strain and non-recoverable strain. The creep recovery model is shown in the following equation.

\[
\phi_i(t,t_0,\tau) = 0.4 \times \left(\frac{t-\tau}{t-\tau+328}\right)^{1/4.2}
\]

The creep recovery in improved CEB Model Code (IMC78) [19] is calculated as follows:

\[
\phi_i(t,t_0,\tau) = 0.35 \times \left\{1 - \exp\left[-0.53(t-t_0)\right]\right\}^{0.28}
\]

The Revised Summation Model (RSM) [19] is as follows:

\[
\phi_i(t,t_0,\tau) = \frac{0.9}{f_{\infty}} \times \left[\beta_d(t_0,\tau) - \left[\beta_d(t,t_0) - \beta_d(t,\tau)\right]\right]
\]

In which, \( \beta_d(t,\tau) = 0.25 + \frac{46}{60 + t^{0.5}} \times \left(0.24 + \sum_{i=1}^{n} a_i \left[1 - \exp\left[-b_i(t^{0.35} - \tau^{0.35})\right]\right]\right) \)

In order to study the development of recoverable deformation in each model, the creep recovery of C50 concrete with loading age of 7 days and unloading age of 35 days was calculated. The creep recovery curve is shown in figure 4.

![Fig.4 Comparison of creep recovery curves](image)

It can be seen from figure 4 that the creep recovery of the five models has basically the same trend with time, which shows steeper growth in the early days and finally tends to be stable. The curves of Yue’s Model and Wang’s Model are generally consistent, and the final value of creep recovery coefficient is larger, about 0.42. The curves of MC78 and IMC78 are close, and the final value of creep recovery coefficient is relatively small, about 0.32. From the perspective of strain development rate, the creep recovery deformation calculated by the first four models developed slowly—when the unloading duration reached 300-400 days, the deformation gradually approached the final value.
However, the creep recovery of RSM develops more rapidly—about 100 days after unloading, the deformation tends to be stable.

3. The proposed calculation method

For concrete members under axial pressure, when the stress is stepwise decreasing stress, as shown in figure 5, the sum of elastic strain and creep strain is:

$$\varepsilon(t) = \sigma_n \left[ \frac{1}{E(t_0)} + \frac{\phi(t_0, t_0)}{E_{28}} \right] + \sum_{i=1}^{n} \Delta \sigma_n \left[ \frac{1}{E(t_i)} + \frac{\phi(t_i, t_i)}{E_{28}} \right] - \sum_{i=1}^{n} \Delta \sigma_n \left[ \frac{1}{E(t)} + \frac{\phi(t, t_i, t)}{E_{28}} \right]$$

The key point of the proposed creep calculation method is that the creep deformation caused by loading and the creep recovery deformation caused by unloading were calculated by different development functions, i.e. the creep strain under loading is calculated by the creep model with multiple-factor correction, while the creep recovery strain when unloading is calculated by the creep recovery model. The concrete creep under continuous decreasing stress in real structure can be calculated by step-by-step method and successive approximation.

4. Application

4.1. The application conditions

The specimen is a hollow prism exposed to an axial compression with a height of 70cm, concrete strength rank of C50, and steel grade of Φ6. The cross section of the specimen is shown in the figure6.

| month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|
| temperature (℃) | -2.5 | 2.9 | 8.1 | 14.7 | 19.8 | 24.8 | 26.6 | 25.3 | 19.7 | 16.5 | 7.9 | 2.3 |
4.2. Result and analysis

According to the multi-factor modified creep calculation method under decreasing stress proposed in this paper, Wang’s model, MC78 and RSM were adopted as the creep recovery model in the calculation process. The creep strain calculated by the proposed method and the specification method is shown in figure 7 and figure 8.

![Fig.7 creep strain calculated by specification method](image1)

![Fig.8 creep strain calculated by the proposed method](image2)

It can be seen from figure 7 and 8 that the creep strain of concrete increases consistently when taking no account of creep recovery model. When creep recovery is taken into account, the calculated deformation still shows an overall rising trend. However, after the stress decreases, the strain curve carries out a tendency of first decreasing and then gradually increasing. The creep rate in the early period of unloading is less than the creep recovery rate, which leads to the short-term decline of strain. While the creep recovery development rate decreasing to less than the creep rate, the creep strain will gradually increase. It can be seen from the strain obtained by three different creep recovery models that there is a difference in the results, but the difference is small. Compared with the creep effect calculated by specification method and the proposed method, the difference is larger, and the former is generally smaller than the latter.

5. Conclusions

In this paper, taking reinforced concrete structure under decreasing stress in natural environment as the study object, relevant theoretical research on creep calculation method is carried out, and the following conclusions are obtained:

a. Creep strain is positively correlated with ambient temperature and negatively correlated with relative humidity and reinforcement ratio.

b. After single-factor correction of ambient temperature and reinforcement ratio, the creep model can be comprehensively modified with various factors by using Boltzmann superposition principle.

c. The creep and creep recovery are expressed by different functions, which is more consistent with the development and essence of creep under decreasing stress.

d. The results obtained by the proposed method are obviously different from those obtained by the specification, and are obviously smaller than those obtained without considering creep recovery model.
Therefore, for the calculation of concrete creep under natural conditions and decreasing stress, the creep recovery and the environment factors must be considered.

c. The proposed method can be applied to complex stress condition.

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