Estimation of johnson-cook constitutive model constants from sheppard and jockson model

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Abstract. This work introduces a novel approach for determining the material constants of the Johnson-Cook constitutive model, which is used to estimate flow stress when a material is open to higher strain rates, stresses, and temperatures. In this study, the material characteristics/constants of the Johnson-Cook constitutive model were derived by using another flow stress model which was introduced by Sheppard and Jackson. The input data was collected from published hot compression test results. This method can potentially reduce the experimental efforts required for evaluating the data of the Johnson-Cook model. In this work, aluminum alloys 7075-T651 and Ti-6Al-4V have been considered for analysis. These materials have their mechanical properties at high temperatures and are of excellent significance for new manufacturing procedures that use deformation to produce a bonding in the solid state, such as friction stir welding and magnetic impulse welding. The flow stress variation of the two models and similar characteristics have been observed at some points over the temperature range. The strain rate constant (c) and temperature softening coefficient (m) for each material have been determined and results were compared with experimental data which were found in the literature. By adopting this method, it is possible to eliminate torsion test to know the strain rate effect on a material. These data could also be used to simulate possibly the best processes as forging, rolling at higher temperatures, and creep.

Keywords. Johnson-Cook constitutive model, Zener-Hollomon parameter, Sheppard and Jackson model, Aluminum alloys, material constants, friction stir welding

1. Introduction
A typical model consisting of the plastic behavior in the current generation manufacturing processes, such as friction stir welding (FSW), magnetic impulse welding and other well-recognized processes, such as hot rolling, forging to estimate reliable material properties, strains, and strain rates at high temperatures are needed. There is always a need for the newer techniques as the current techniques have lack of experimental measurements at high temperatures, high strain rates, and high strains. The problems involved in measuring mechanical properties at high temperatures are also high.
In the case of friction stir welding (FSW), reproducing the conditions anticipated in the shear layer experimentally is exceptionally hard, and the community agrees that one of the most difficult challenges to bypass the creation of effective and reliable FSW simulations is a shortage of available mechanical properties data. Atomistic simulations can be used to obtain mechanical behaviour of monolithic metals and alloys, however simulations of alloys with grain sizes higher than 1 µm are extremely difficult.

In the absence of real measurements at high temperatures, strain rates, and strains, realistic extrapolations based on trustworthy prepared models are the next best choice. For hot deformation of materials, the Johnson-Cook material model and the constitutive model presented by Sheppard and Jackson are the most extensively used.

For materials subjected to huge strains, high strain rates, and high temperatures, Johnson and Cook (1983) established a constitutive model. This model requires data from torsion testing over a wide range of strain rates, static tensile tests, and dynamic Hopkinson bar tests at elevated temperatures for the material constants. Maheswari et al., (2013) established a dimensionless constitutive modelling containing the relationship between stress, strain rate and temperature is the prominent input for assessing thermo mechanical deformation processing results. The proposed model uses the results of hot compression test. Huang et.al. (2012) proposed a paradigm to determine material parameters Johnson-Cook constitutive model. The direct method and optimization method identifying constitutive model parameters are proposed.

Tello et al. (2010) presented newer constants for the Sellars and Tegart constitutive model for hot metalworking with various metals, which had previously been unpublished. Dorogoy et al. (2009) used the shear compression specimen (SCS) to determine the Johnson Cook material parameters, which included the recognition of the thermal softening effect in quasi-static and dynamic loading, including the strain rate hardening effect in dynamic loading. Slais et al. (2012) proposed a method for determining five parameters of Johnson-Cook constitutive equation for the Ti-6Al-4V titanium alloy.

Mousavi et al. (2010) studied the hot compression behaviour of a medium carbon low alloy steel, analyzing the impact of Zener-Hollomon parameters, strain, and strain rate on the flow stress and estimating the material’s deformation activation energy. Yang Hui et al. (2006) used hot compression experiments to investigate the warm-hot deformation behaviour of 20CrMnTi steel. The influence of the Zener–Hollomon parameter on microstructures and mechanical characteristics in pure Cu were explored by Li et al. (2008) using a method including varied plastic deformation with multiple strain rates and at different temperatures. On a Gleeble-1500 mechanical testing machine, Ou, L., Nie, Y., and Zheng, Z. (2014) investigated isothermal hot compressive deformation in the temperature range of 350-500 °C at strain rates ranging from 0.01-10/s up to a real strain of 0.9. The flow stress rapidly climbed to a maximum value. With increase in deformation temperature and decreasing strain rate, the peak stress was decreased. A Zener-Hollomon parameter with an Arrhenius term can be used to model the effects of strain rate and temperature on hot deformation behaviour. Ning et al. (2018) [11] adapted the Johnson–Cook constitutive model to define the deformation behavior of the material when it was exposed to high temperatures and strain rates. Ballistic impact testing was used to verify the correctness of the improved model. The hot deformation behaviour of aluminum matrix composites was investigated by Neelima et al., (2010) [13] developing constitutive equations using Johnson–Cook (JC), modified JC (m-JC), Arrhenius, and modified Zerilli and Armstrong (m-ZA) models, and the capacity of these models was estimated and compared using average absolute error.

In this work a comparison of both Johnson-Cook constitutive model and Sheppard-Jackson model was carried out by using MATLAB software for two materials Al7075-T651 and Ti-4Al- 6V. With these observations, the material constants strain rate constant (c) and temperature softening coefficient (m) of Johnson-Cook constitutive model have been determined and the results were compared with known values which were found in the literature.
1.1. Johnson-Cook Constitutive Model

Gordon R. Johnson and William H. Cook proposed this model in 1983 for materials subjected to significant strains, high strain rates, and high temperatures. The model’s basic form is as shown:

\[
\sigma(\varepsilon_p, \varepsilon_p, T) = [A + B\varepsilon_p^n] \left[1 + c\varepsilon_p^m \left[1 - \left(\frac{T - T_{room}}{T_{melt} - T_{room}}\right)^{n_2}\right] \right]
\]

(1)

Where

- \(\sigma\) is the flow stress
- \(\varepsilon_p\) is the equivalent plastic strain
- \(\dot{\varepsilon}_p\) is the dimensionless plastic strain rate for \(\dot{\varepsilon}_0 = 1.0s^{-1}\)
- \(T\) is the homologous temperature.

A, B, n, C, m are the material constants.

In this model the data for the material constants is collected from torsion tests over a broad spectrum of strain rates, static tensile tests, dynamic Hopkinson bar tensile tests and Hopkinson bar tests at elevated temperatures. The following sections provide detailed information about these tests.

1.1.1. Torsion Test

Torsion tests across a wide variety of strain rates are the primary source of Johnson-Cook material data. The state of the test specimen is well defined, high strains may be reached without forming geometric instabilities, and small to big range of strain can be obtained with the same testing technique, are some important features.

1.1.2. Dynamic Hopkinson Bar Tensile Test

This is also a important test in getting the data of the Johnson-Cook material model. This is done over a range of temperatures. The higher temperatures can be achieved by surrounding the in-place test specimen in an oven such that the temperatures are gradually given for several minutes before to testing. It is also possible to test materials to larger strains; the Hopkinson bar tensile test cannot be accurately evaluated after necking begins to occur in the tensile specimens. In addition, the effects of adiabatic heating at significant strains might make the results even more complicated. The materials are softening as a result of the higher temperatures.

1.1.3. Tensile Test

Static tensile tests should also be used in order to obtain test data of the Johnson-Cook model. The stress for the tension test data is based on the neck current area, and the strain is referred as \(\ln(A_0/A)\), where \(A_0\) and \(A\) represent the initial and areas of the neck.

Static tensile testing should also be used to get test results for the Johnson-Cook model. Because of the presence of hydrostatic tension, this is the case. The Bridgman correction factor is used to approximate the equivalent tensile flow stress obtained from the tension measurements. Usually both tensile and torsion tests have to be conducted such that the disagreement between two modes of deformation can be recognized and rewarded.

1.2 Sheppard-Jackson Constitutive Model

Constitutive equation introduced by sheppard and Jackson
Where $Z$ is the Zener-Hollomon parameter:

$$Z(T, \dot{\varepsilon}) = \exp \left( \frac{Q}{RT} \right)$$  \hspace{1cm} (3)

Where

- $T$ is temperature
- $\dot{\varepsilon}$ is the strain rate
- $Q$ is the activation energy
- $R$ is gas constant
- $A,n, \eta$ are the material constants.

The Zener-Hollomon parameter is used to relate the flow stress to the deformation strain-rate and temperature. It can be used to describe the material properties during solid state joining, such as friction stir welding, cold spray and magnetic impulse welding.

2. Study of Constitutive Models

2.1. Material Constants

Material constants are used to describe the constitutive model for a particular material. Experimental work has been done by many researchers to derive the data of the constitutive models. Material constants for few materials was given in Table 1.

| Model                  | Material          | $A$ \text{ s}^{-1} | $N$   | $Q$ \text{ KJ mol}^{-1} | $\eta$ \text{ Pa}^{-1} | $R$   | Melting point, K |
|------------------------|-------------------|-------------------|------|----------------------|------------------------|-------|------------------|
| Sheppard and Jackson   | Ti-6Al-4V         | 3.92x10^9         | 3.6  | 176                  | 2.13x10^{-5}           | 8.314 | 1877             |
| Sheppard and Jackson   | Al7075-T651       | 1.03x10^9         | 5.41 | 129                  | 1.41x10^{-5}           | 8.314 | 750              |
| Johnson and Cook       | Ti-6Al-4V         | 802               | 995  | 0.01                 | 0.5                    | 0.6   | 1877             |
| Johnson and Cook       | Al7075-T651       | 527               | 575  | 0.017                | 0.72                   | 1.61  | 750              |

2.2. Flow Stress Variation

When a material is subjected to large strains, high strain rates and high temperatures its flow stress varies. Many models were proposed for this flow stress variation with temperature and strain rates. Johnson-Cook constitutive model and Sheppard-Jackson model were the most useful in estimating this flow at particular temperature and strain rate. The flow stress variation for Al7075-T651 using both models at different strain rates is given in Figure 1.
The flow stress variation for Ti-4Al-6V using Johnson-Cook model and Sheppard-Jackson model at different strain rates are given in Figure 2.

2.3 Comparison of Flow Stress

From the above graphs (Figure 3.) it can be say that the value flow stress is increasing with strain rate at a temperature in both models. We have increased strain rate value from 1s^{-1} to 100s^{-1} and the corresponding increment in flow stress in both models has been drawn in flowing figures.

Constitutive equation given by sheppard and Jackson is

\[
\sigma(T,\varepsilon) = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \ln \left( \frac{T}{T_0} \right) + 1 + \left( \frac{T}{T_0} \right)^{\frac{2}{m}} + \left[ 1 + \left( \frac{T}{T_0} \right)^{\frac{2}{m}} \right]^{1/2}
\]  

(4)

First derivative of flow stress with respect to temperature is given as

\[
\frac{\partial \sigma(T,\varepsilon)}{\partial T} = \frac{1}{\eta} \left( \frac{1}{\dot{\varepsilon}_0} \right) + \frac{2}{\dot{\varepsilon}_0} \left( \frac{T}{T_0} \right)^{\frac{2}{m}} + \frac{1}{\dot{\varepsilon}_0} \left( \frac{T}{T_0} \right)^{\frac{2}{m}} \left( \frac{T}{T_0} \right)^{\frac{2}{m}}
\]  

(5)
Figure 3. Comparison of Flow stress vs Temperature for Ti-6Al-4V material for both the models at different strain rates

Figure 4. First derivative of Flow stress vs Temperature for Ti-6Al-4V material by Johnson-Cook model

Johnson-Cook Constitutive model is given by

\[ \sigma_{JC}(d, \varepsilon_p, T) = [A + B\varepsilon_p^n][1 + c \ln \varepsilon_p][1 - \left(\frac{T - T_{room}}{T_{melt} - T_{room}}\right)^m] \] (6)
By taking the partial derivative of first order with respect to temperature

$$\frac{\partial \sigma}{\partial T} = -m \left(A + B \varepsilon\right) \left(1 + C \ln \frac{\varepsilon}{\varepsilon_0}\right) \left(\frac{T - T_{ref}}{T_{ref} - T_{m}}\right)^{-1} \frac{1}{\varepsilon_0}$$  \hspace{1cm} (7)

The variation of its first derivative over temperature is given in the following Figure 5

![Figure 5](image-url)

**Figure 5.** First derivative of Flow stress vs Temperature for Ti-6Al-4V material by Sheppard-Jackson model

From the above two figures (Figure 4 & Figure 5) it can be said that in Johnson-Cook model the flow stress is decreasing very rapidly with temperature and in Sheppard-Jackson model flow stress is decreasing gradually over the temperature.
The change in flow stress value with increasing strain rate from $1 \text{s}^{-1}$ to $100 \text{s}^{-1}$ in both models was shown in above Figures (Figure 6 (a-f)). By using these observations the detailed procedure to determine $C$ and $m$ was explained in the following section.

### 3. Determination of Material Constants

By observing above graphs, at a particular temperature, as we increase the strain rate, flow stress is also increasing. And this increment in flow stress is proportionally equal. This characteristic of these two models can be used to predict the value of strain rate constant of the Johnson-Cook constitutive model by using data of the sheppard-Jackson model.

#### 3.1 Determination of Strain Rate Constant $C$

At a particular temperature, as we increase the strain rate, flow stress is also increasing. And this increment in flow stress is proportionally equal in both the models from Fig. 3.6, with these observations the increment in flow stress at room temperature considered equal for both models and the material constants strain rate constant $C$ and temperature softening coefficient of Johnson-Cook constitutive model have been determined as follows.

For Ti-4Al-6V using Sheppard-Jackson model

At $\dot{\varepsilon}_p = 1 \text{s}^{-1}$, $T=T_{\text{room}}$

Flow stress value $= 7.7479 \times 10^8 \text{ pa}$

Johnson-Cook constitutive model becomes

$$\sigma_{JC}(\dot{\varepsilon}_p, \varepsilon_p, T) = (A + B\varepsilon_p^m)$$

(8)

And at $\dot{\varepsilon}_p = 100 \text{s}^{-1}$, $T=T_{\text{room}}$

Flow stress value $= 8.3285 \times 10^8 \text{ pa}$

$$\sigma_{JC}(\dot{\varepsilon}_p, \varepsilon_p, T) = (A + B\varepsilon_p^m)(1 + C \ln 100)$$

(9)

From above graphs for Ti-4Al-6V subtractive equation 1 from 2

$$(A + B\varepsilon_p^m)(C \ln 100) = (8.3285-7.7479) \times 10^8$$

(10)
From simple quasi-static tensile test
A=802MPa, B=9.95MPa, n=0.5, equivalent plastic strain =0.4
By substituting above values in equation 10
We get C=0.0091
For Al7075-T651 using Sheppard-Jackson model

\[ \sigma_f(\varepsilon_p, \varepsilon_p, T) = (A + B\varepsilon_p^n) \]

And at \( \dot{\varepsilon}_p = 100s^{-1}, T=T_{room} \)

\[ \sigma_f(\varepsilon_p, \varepsilon_p, T) = (A + B\varepsilon_p^n)(1 + C \ln(100)) \]  

(11)

(12)

From above graphs for Al7075-T651 subtractive equation 12 from 13

\[ (A + B\varepsilon_p^n)(C \ln(100)) = (5.3549-4.7512) \times 10^8 \]  

(13)

From simple quasi-static tensile test
A=527MPa, B=575MPa, n=0.72, equivalent plastic strain =0.03
By substituting above values in equation 13
We get C=0.021

4. Results

5. The change in value of strain rate constant C effects flow stress variation with temperature at a strain rate. The flow stress variation and error in flow stress from experimental value is shown in the below Figure 7.

![Figure 7](image)

**Figure 7.** Flow stress variation with C=0.017 and error in flow stress for Al7075-T651

![Figure 8](image)

**Figure 8.** Flow stress variation with C=0.0091 and error in flow stress for Ti-4Al-6V
The above observations and calculations were summarized in the following Table 2.

| Material       | Experimental | Using sheppard Jackson model | Error in value of C (%) | Maximum Error in flow stress(MPa) |
|----------------|--------------|------------------------------|-------------------------|----------------------------------|
| Ti-6Al-4V      | 0.01         | 0.0091                       | 9                       | 6                                |
| Al7075-T651    | 0.017        | 0.021                        | 23.5                    | 4                                |

5 Conclusions
From the above results and observations it is concluded that
- The Johnson-Cook constitutive model always gives more flow stress than Sheppard-Jackson model at a particular temperature and strain rate for any material over temperature range. From the graphs of first derivative of the flow stress.
- Flow stress of the material using Johnson-Cook model is decreasing with temperature rapidly. But it was decreasing gradually if use Sheppard and Jackson model. Flow stress is increasing with strain rate at a particular temperature in both models and it was observed that the increment in flow stress is proportionally equal in the both models.
- The values of C and m for the materials have been calculated from the Sheppard and Jackson model. The results were well approximated with the know values which were found in the literature. The value of C for Ti-4Al-6V is found to be 0.0091 and for Al7075-T651 is 0.021 and the error from the experimental value is found to be low.

Generally to determine material constants of Johnson-Cook constitutive model three tests namely torsion test, Split Hopkinson bar test and quasi-static tensile test have to be conducted. This work can eliminate torsion test which is used to find strain rate constant “C”. If we are able to find thermal softening coefficient “m” in future by any method with doing experiment, it is also possible to avoid split Hopkinson bar test. These results can also be used to model the plastic behavior of the material in FSW joining processes and also in well established process such as Creep, hot rolling, and forging.

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