Quantum Field Theory of the Laser Acceleration
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Abstract

After the historical background concerning the pressure of light, we derive the quantum field theory force of the laser radiation acting on electron. Numerically, we determine the velocity of an electron accelerated by laser beam, after acceleration time $\Delta t = 1\text{s}$.

1 Why laser acceleration?

The problem of acceleration of charged particles by the laser field is, at present time, one of the most prestigious problem in the accelerator physics. It is supposed that, in the future, the laser accelerator will play the same role in particle physics as the linear or circle accelerators working in today particle laboratories.

The acceleration effectiveness of the linear or circle accelerators is limited not only by geometrical size of them but also by the energy loss of accelerated particles which is caused by bremsstrahlung during the acceleration. The amount of radiation, following from the Larmor formula, emitted by accelerated charged particle is given generally as follows (Maier, 1991):

$$P = \frac{2r_0 m}{3c}(\gamma^6 a_{||}^2 + \gamma^4 a_{\perp}^2); \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}; \quad \beta = v/c$$

where $v$ is the velocity of a particle, $c$ is the velocity of light in vacuum, $a_{||}$ is parallel acceleration of a particle and $a_{\perp}$ is the perpendicular acceleration of a particle in the accelerator, $m$ is the rest mass of an electron. The quantity

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mc^2}$$

is the electron classical radius in SI units.

In terms of momenta

$$\hat{p}_{||} = \frac{\hat{E}}{\beta c}; \quad \hat{p}_{\perp} = m\gamma\hat{v}_{\perp},$$

the radiated power can be written as
\[ P = \frac{2}{3} r_0 c (\dot{\gamma}^2 + \gamma^2 p_\perp^2). \]  \hfill (4)

where \( E_0 = mc^2 \). Equation (4) shows that the same acceleration force produces a \( \gamma^2 \) times higher radiation power, if it is applied in perpendicular direction, compared to the parallel direction.

For a particle moving with a constant velocity in a circular machine with bending radius \( r \) the power radiated due to curvature of the orbit is

\[ P = \frac{2}{3} r_0 E_0 \frac{\beta^4 \gamma^4}{\gamma^2}. \]  \hfill (5)

So in the linear accelerator the energy loss caused by radiation is smaller than in circle accelerator and it means that to obtain high energy particles in linear accelerator is more easy than in the circle accelerator. In case of laser acceleration the situation radically changes. The classical idea of laser acceleration is to consider the laser light as the periodic electromagnetic field. The motion of electron in such a wave was firstly described by Volkov (Berestetzkii et al. 1989). However, it is possible to show that periodic electromagnetic wave does not accelerate electrons in classical and quantum theory, because the electric and magnetic components of the light field are mutually perpendicular and it means the motion caused by the classical periodic electromagnetic field is not linear but periodic (Landau et al., 1962).

The situation changes if we consider laser beam as a system of photons and the interaction of electron with laser light is via the Compton process

\[ \gamma + e \rightarrow \gamma + e. \]  \hfill (6)

We can see that the right side of equation (6) involves no bremsstrahlung photons and it means that there are no energy loss caused by emission of photons. It means also that laser acceleration is more effective than the acceleration in the standard linear and circle accelerators. It is evident that acceleration by laser can be adequately described only by quantum field theory. Such viewpoint gives us the motivation to investigate theoretically the effectiveness of acceleration of charged particles by laser beam.

2 Historical view on laser acceleration

The acceleration of charged particles by laser beam has been studied by many authors (Tajima and Dawson, 1979; Katsouleas and Dawson, 1983; Scully and Zubary, 1991; Baranova and Zel’dovich, 1994).

Many designs for such devices has been proposed. Some of these are not sufficiently developed to be readily intelligible, others seem to be fallacious and others are unlikely to be relevant to ultra high energies. Some designs were developed only to observe pressure of laser light on microparticles in liquids and gas (Ashkin, 1970; Ashkin, 1972).

The idea of laser acceleration follows historically the idea that light exerts pressure. The former idea was for the first time postulated by Johanness Kepler, the King astronomer in Prague, in 1619. He wrote that the pressure of Sun light is what causes the tails of comets to point away from the Sun. The easy explanation of that effect was
given by Newton in his corpuscular theory of light where this effect is evidently of the mechanical origin. Nevertheless, the numerical value of the light pressure was not known from time of Kepler to the formulation of the Maxwell theory of electromagnetism where Maxwell predicted in 1873 the magnitude of the light pressure.

The experimental terrestrial verification of the light pressure was given by the Russian physicist Lebedev and Nichols and Hull from USA (Nichols and Hull, 1903). The measurement consisted in determination of force acting on the torsion pendulum. At these experiments it was observed that the magnitude of the pressure of light confirmed the Maxwell prediction. It was confirmed that the pressure is very small and practically has no meaning if the weak terrestrial sources of light are used. Only after invention of lasers the situation changed because of the very strong intensity of the laser light which can cause the great pressure of the laser ray on the surface of the condensed matter. So, the problem of the determination of the light pressure is now physically meaningful because of the existence of high intensity lasers.

Here, we consider the acceleration of an electron by laser beam. We calculate the force due to Compton scattering of laser beam photons on electron using the methods of quantum field theory and quantum electrodynamics.

3 Quantum field theory of a laser beam acceleration

The dynamical equation of the relativistic particle with rest mass $m$ and the kinematical mass $m(v)$,

$$m(v) = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (7)$$

is as follows (Møller, 1972):

$$
\mathbf{F} = \frac{d(m(v)\mathbf{v})}{dt} = m(v) \frac{d\mathbf{v}}{dt} + \frac{dm(v)}{dt} \mathbf{v} = m(v) \frac{d\mathbf{v}}{dt} + \frac{1}{c^2} \frac{dW}{dt} \mathbf{v} =
$$

$$
m(v) \frac{d\mathbf{v}}{dt} + \left(\frac{\mathbf{F} \cdot \mathbf{v}}{c^2}\right) \mathbf{v}, \quad (8)
$$

or,

$$
m(v) \frac{d\mathbf{v}}{dt} = \mathbf{F} - \frac{\mathbf{v}}{c^2} (\mathbf{F} \cdot \mathbf{v}). \quad (9)
$$

If we consider a particle that is acted upon by a force $\mathbf{F}$ and which has an initial velocity in the direction of the force, then, according to Eq. (9) the particle will continue to move in the direction of force. Therefore the path of the particle will be a straight line, and we can choose this line as the $x$-axis. From Eq. (9) then follows for $\mathbf{v} \parallel \mathbf{F}$, and $F = |\mathbf{F}|$:

$$
\frac{d}{dt} \left\{ \frac{v}{(1 - \frac{v^2}{c^2})^{1/2}} \right\} = \frac{F}{m}, \quad (10)
$$

where force $F$, in case it is generated by the laser beam, contains also the velocity $v$ of particle as an integral part of the Doppler frequency.
Now, let us consider the laser acceleration of an electron by the monochromatic laser beam. The force of photons acting on an electron in a laser beam depends evidently on the density of photons in this beam. Using the definition of the cross section of the electron-photon interaction and with the energy loss \( \omega - \omega' \), it may be easy to define the force acting on electron by the laser beam, in the following way:

\[
F = n \int_{\omega_1}^{\omega_2} (\omega - \omega') \frac{d\sigma(\omega - \omega')}{d(\omega - \omega')} d(\omega - \omega'),
\]

where in the rest system of electron we have the following integral \( \omega' \)-limits (Sokolov et al., 1983):

\[
\omega_2 = \frac{\omega}{1 + \frac{\omega m}{m}} \leq \omega' \leq \omega = \omega_1.
\]

where \( \omega \) can be identified with the frequency of the impinging photon on the rest electron. Let us remark that Eq. (11) has the dimensionality of force if we correctly suppose that the dimensionality of \( d\sigma \) is \( m^2 \) and density of photons in laser beam is \( m^{-3} \). At the same time the combination \( \omega - \omega' \) in the cross section and differential can be replaced by \( \omega' \).

Since the dimensionality of the expression on the right side of the last equation is force, we at this moment connect \( m \) with \( c^2 \) in order to get \( mc^2 = E \). Quantity \( \omega \) will be later denoted as the frequency \( \omega_0 \) of the impinging photon. Using the expression for the differential Compton cross section (Berestetzkii et al., 1989), we get for the force:

\[
F = n \pi r_e^2 \frac{E}{\omega_0^2} \int_{\omega_1}^{\omega_2} d\omega' (\omega' - \omega) \left[ \frac{\omega}{\omega'} + \frac{\omega'}{\omega} + \left( \frac{m}{\omega} - \frac{m}{\omega'} \right)^2 - 2m \left( \frac{1}{\omega'} - \frac{1}{\omega} \right) \right],
\]

where \( r_e = e^2/mc^2 \) is the classical radius of an electron.

After \( \omega' \)-integration we obtain

\[
F = n \pi r_e^2 \frac{E}{\omega_0^2} \left\{ (3m^2 - \omega^2 + 2m\omega) \ln \frac{\omega_2}{\omega_1} + m^2 \omega \left( \frac{1}{\omega_2} - \frac{1}{\omega_1} \right) + \left( \omega - 4m - \frac{3m^2}{\omega} \right) (\omega_2 - \omega_1) + \frac{1}{3\omega} \left( \omega_2^3 - \omega_1^3 \right) + \left( \frac{m^2}{2\omega^2} + \frac{m}{\omega} - \frac{1}{2} \right) (\omega_2^2 - \omega_1^2) \right\},
\]

where the corresponding the \( \omega_i \)-combination are as follows:

\[
\frac{\omega_2}{\omega_1} = \frac{m}{m + 2\omega}; \quad \omega_2 - \omega_1 = -\frac{2\omega^2}{m + 2\omega}; \quad \frac{1}{\omega_2} - \frac{1}{\omega_1} = \frac{2}{m}
\]

and

\[
\omega_2^2 - \omega_1^2 = -\frac{4\omega^3(m + \omega)}{(m + 2\omega)^2}; \quad \omega_2^3 - \omega_1^3 = -\frac{2\omega^4}{(m + 2\omega)^3} (3m^2 + 6m\omega + 4\omega^2).
\]

Then, after insertion of the \( \omega_i \)-combinations and putting \( \omega \to \omega_0 \), we obtain for the accelerating force instead of equation Eq. (14) the following equation:

\[
F = n \pi r_e^2 \frac{E}{\omega_0^2} \times
\]
\[
\left\{(3m^2 - \omega_0^2 + 2m\omega_0) \ln \frac{m}{m + 2\omega_0} + 2m\omega_0 + \frac{2\omega_0(4m\omega_0 + 3m^2 - \omega_0^2)}{m + 2\omega_0} \right.

- \frac{2\omega_0^3(3m^2 + 6m\omega_0 + 4\omega_0^2)}{3(m + 2\omega_0)^3} + 2\omega_0(\omega_0^2 - m^2 - 2m\omega_0) \frac{(m + \omega_0)}{(m + 2\omega_0)^2} \right\}. \tag{17}
\]

Now, if we want to express the force \(F\) in the MKS system where its dimensionality is \(\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}\), we are forced to introduce the physical constants: velocity of light \(c\), Planck constant \(\hbar\) in the last formula. It is easy to see that the last formula expressed in the MKS system is as follows:

\[
F = n\pi r_e^2 \frac{E}{(\hbar\omega_0)^2} \times
\left\{(3m^2c^4 - \hbar^2\omega_0^2 + 2mc^2\hbar\omega_0) \ln \frac{mc^2}{mc^2 + 2\hbar\omega_0} + 2mc^2\hbar\omega_0 +
\frac{2\hbar\omega_0(4mc^2\hbar\omega_0 + 3m^2c^4 - \hbar^2\omega_0^2)}{mc^2 + 2\hbar\omega_0}
\right.

- \frac{2\hbar^3\omega_0^3(3m^2c^4 + 6mc^2\hbar\omega_0 + 4\hbar^2\omega_0^2)}{3(mc^2 + 2\hbar\omega_0)^3} +

2\hbar\omega_0(\hbar^2\omega_0^2 - m^2c^4 - 2mc^2\hbar\omega_0) \frac{(mc^2 + \hbar\omega_0)}{(mc^2 + 2\hbar\omega_0)^2} \right\}. \tag{18}
\]

The last formula is valid approximately only for nonrelativistic velocities because for laser photons the Doppler effect plays substantial role for moving electron in the laser field. The formula of the relativistic Doppler effect is as follows:

\[
\omega_0 \rightarrow \omega_0 \frac{1 - \frac{v}{c}}{(1 - \frac{v^2}{c^2})^{1/2}}, \tag{19}
\]

which means that for ultrarelativistic electron the frequency of photons accelerating the electron will be very small and that the acceleration will be also very small with regard to the electron moving with the relativistic velocities in the laser field. In order to obtain the exact description of the electron motion in the laser field, it is necessary to insert the Doppler frequency equation Eq. (19) in the formula Eq. (10).

Of course, it is not easy to obtain the general solution of Eq. (10) because it is strongly nonlinear. For \(v \ll c\) we obtain the solution \((E = mc^2, \hbar\omega_0 = \varepsilon)\):

\[
v = \frac{1}{m} \pi ntr_e^2 \times
\left\{(3E^2 - \varepsilon^2 + 2E\varepsilon) \ln \frac{E}{E + 2\varepsilon} + 2E\varepsilon +
\frac{2\varepsilon(4E\varepsilon + 3E^2 - \varepsilon^2)}{E + 2\varepsilon} - \frac{2\varepsilon^3(3E^2 + 6E\varepsilon + 4\varepsilon^2)}{3(E + 2\varepsilon)^3} \right\} +
\]

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\section*{4 Discussion}

We have seen, in this article, that the force accelerating an electron by a laser beam can be determined by means of the quantum field theory. The derived formula Eq. \eqref{eq:force} with Eq. \eqref{eq:compton} describes the force due to the Compton scattering of photons with electron moving in the laser monochromatic photon sea. We have used only the simple Compton process Eq. \eqref{eq:compton} and not the more complicated multiple Compton process defined by equation

\begin{equation}
  nγ + e \rightarrow γ + e, \quad (24)
\end{equation}
which follows for instance as a quantization of the Volkov equation (Berestetzkii et al., 1989).

The present article is the modification of the Pardy discussion on laser acceleration (Pardy, 1998), where the thermal statistical model of laser acceleration was proposed. The basic ansatz of that model was the energy loss formula

\[-\frac{dW}{dx} = \frac{1}{v} \int d\Gamma(\omega - \omega'),\]

(25)

where \(\Gamma\) is the differential reaction rate defined in different manner in quark quon plasma physics and in electrodynamical medium.

In that article, it was used the approach by Brown et al. (Brown and Steinke, 1997; Brown, 1992; Sokolov et al., 1983, Braaten and Thoma, 1991). At the same time we used the ideas of Blumenthal et al. (Blumenthal and Gould, 1970). Brown et al. (Brown and Steinke, 1997), applied the total electron scattering rate for determination of behaviour of electron in the Planckian photon sea inside of the pipe of the storage rings. While in the preceding article the thermal distribution function \(f(k)\) of photons was considered, here, we used the nonthermal density of photons \(n\).

The experimental perspective of the laser beam acceleration of elementary particles concerns not only the charged particles, however, also the neutral particles such as neutron, neutral \(\pi\)-meson, and so on. Also the system of particles with the opposite charges was considered to be simultaneously accelerated by the laser beam (Pardy, 1997). New experiments can be realized and new measurements performed by means of the laser accelerator, giving new results and discoveries. So, it is obvious that the acceleration of particles by the laser beam can form, in the near future, the integral part of the particle physics. In such laboratories as ESRF, CERN, DESY, SLAC and so on, there is no problem to install lasers with the sufficient power of the photon beam, giving opportunity to construct the laser accelerator.

To say the final words, we hope, the ideas of the present article open the way to laser accelerators and will be considered as the integral part of the today particle physics.

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