Factorization breaking for higher moments of harmonic flow

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We study correlations between harmonic flow vectors squared measured at different transverse momenta. One of the flow harmonics squared is taken at a fixed transverse momentum and correlated to the momentum averaged harmonic flow squared of the same order. Such four particle correlators, dependent on transverse momentum, have been recently measured experimentally. Factorization coefficients based on the ratio of such four-particle correlators allow the independent measurement of the flow vector and flow magnitude factorization breaking coefficient. Moreover, the correlation of the angles of flow harmonics as a function of transverse momentum can be extracted. Results are compared to preliminary data of the ALICE Collaboration. We also present predictions for the momentum dependent factorization breaking coefficient between mixed flow harmonics. The correlators with squares of mixed harmonics can serve as a way to independently measure the flow vector, flow magnitude, and flow angle correlations, and could be used to gain additional information on the fluctuating initial state and the dynamics in heavy-ion collisions.

I. INTRODUCTION

One of the main goals of the experimental program in high-energy nuclear collisions is to study the properties of the dense matter created in the collision. The dense matter created in the interaction region of the collision expands and a collective flow of matter appears\cite{1,2,3}. One of the essential characteristics of high-energy collisions is the presence of event-by-event fluctuations in the initial state\cite{1,2,3}. Initial state fluctuations manifest themselves in event by event fluctuations of the collective flow generated in the collisions.

One of the aspects of collective flow fluctuations is the decorrelation of the harmonic flow vectors measured at different momenta. In the longitudinal direction, it means that the correlation coefficient for the harmonic flow vectors measured at two different pseudorapidity is smaller than one\cite{11,12,13,14,15,16,17}. In the transverse momentum it manifests itself as a deviation from one of the correlation coefficient (called in this context the factorization breaking coefficient) for flow vectors measured at two different transverse momenta\cite{18,19,20,21,22,23,24,25,26,27}.

The decorrelation between two flow vectors is due to the decorrelation of the flow vector magnitudes and of the flow vector angles\cite{22,23,24,25,26,27}. The two effects can be separated when four-particle correlators in pseudorapidity are measured\cite{25,26}. For the correlation of flow vectors in transverse momentum a similar procedure would involve the measurement of four-particle correlators for harmonic flow vectors at different transverse momenta\cite{22,23,24,25,26,27}. Such four-particle correlators have been measured experimentally only for the case when two harmonic flow vectors are defined at fixed transverse momentum and two are momentum averaged\cite{27}.

We present calculations for the flow vectors squared and flow magnitudes squared factorization breaking coefficients and compare them to the preliminary results of the ALICE Collaboration. We check in the model that the estimation of the flow angle decorrelation used by the ALICE Collaboration is a good measure of the event by event flow angle decorrelation between harmonic flow vectors. The flow angle decorrelation between first moments of flow vectors cannot be measured, but can be approximated as the square root of the flow angle decorrelation between two flow vectors squared. Finally, we present prediction for the momentum dependent correlation between mixed harmonics. Also in that case, using squares of flow vectors (or flow magnitudes) makes possible the separate measurement of the flow vector, flow vector magnitude, and flow angle correlations between mixed harmonics.

II. FACTORIZATION BREAKING FOR HARMONIC FLOW VECTORS AND FLOW MAGNITUDES

The harmonic flow coefficients measure the azimuthal asymmetry in the distribution of particles emitted in a collision. The particle distribution is written as

\begin{equation}
\frac{dN}{dpd\phi} = \frac{dN}{2\pi dp} \left( 1 + 2 \sum_{n=1}^{\infty} V_n(p)e^{in\phi} \right) . \quad (1)
\end{equation}

The complex vector $V_n(p) = v_n(p)e^{i\Psi_n(p)}$ denotes the harmonic flow vector of order $n$ for particles emitted at the transverse momentum $p$; $v_n$ and $\Psi_n$ are the corresponding flow magnitude and angle. The harmonic flow averaged over the whole range of transverse momentum is $V_n$. The average over a sample of events in a given centrality bin is denoted by $\langle \ldots \rangle$. The two-particle cumulant formula for the harmonic flow coefficient takes the form

\begin{equation}
v_n\{2\} = \langle V_nV_n^* \rangle \quad (2)
\end{equation}
for the momentum averaged flow and

\[ v_n\{2\}(p) = \frac{\langle V_n V_n^* (p) \rangle}{\sqrt{\langle V_n^2 \rangle}} \]  

(3)

for the momentum dependent flow.

In this paper we study the harmonic flow as a function of transverse momentum. All results presented are obtained using a boost invariant version of the hydrodynamic model MUSIC [8, 28, 29] for Pb+Pb collisions at 5.02 TeV. The initial entropy distributions for the hydrodynamic evolution are taken from two models; a two-component Glauber Monte Carlo model [30] and the TRENTO model [31]. Unless otherwise stated, we use a constant shear viscosity to entropy density ratio \( \eta/s = 0.08 \).

Event by event fluctuations in the initial conditions cause fluctuations in the final harmonic flow. One of the effects of these fluctuations is the decorrelation between the harmonic flow vectors in two different kinematic regions [11, 18]. The correlation between flow vectors at two different transverse momenta can be measured using the factorization breaking coefficient (correlation coefficient)

\[ r_n(p_1,p_2) = \frac{\langle V_n(p_1)V_n^*(p_2) \rangle}{\sqrt{\langle v_n^2(p_1) \rangle} \sqrt{\langle v_n^2(p_2) \rangle}} . \]  

(4)

The factorization breaking coefficient \( r_n(p_1,p_2) \) has been measured in experiment [15, 32, 33] and calculated in models [18, 23].

The factorization breaking coefficient (4) is defined as the correlation coefficient between two flow vectors (as complex numbers). In the following, we study similar factorization breaking coefficients between flow vectors squared and flow vector magnitudes squared, as well as for mixed flow harmonics. In this context, the decorrelation is understood as the deviation of these factorization breaking coefficient from one. The quantity defined as the correlation for the flow angles between two flow vectors involves the cosine of the angle difference, which is not the Pearson correlation coefficient. It is the circular correlation coefficient weighted with flow vector magnitudes. In that case we use the names correlation between flow vector angles, and flow angle decorrelation.

The decorrelation between harmonic flow vectors at two different momenta involves both the decorrelation of the flow magnitudes and of the flow angles [12, 24]. The flow magnitude or the flow angle decorrelation cannot be measured experimentally with two-particle correlators. The flow angle or flow magnitude decorrelation in pseudorapidity can be estimated independently of the flow vector decorrelation using four-particle correlators [25].

Previous experimental results of the ATLAS Collaboration have shown that the decorrelation of harmonic flow vectors in pseudorapidity is composed in roughly equal strength from flow magnitude and flow angledecorrelations [29].

For the correlation of harmonic flow vectors at two different transverse momenta a similar procedure can be used [22]. Four-particle correlators, measuring the factorization breaking between two flow vectors squared, can be defined as,

\[ r_{n,2}(p_1,p_2) = \frac{\langle V_n^2(p_1)V_n^*(p_2)^2 \rangle}{\sqrt{\langle v_n^2(p_1) \rangle} \sqrt{\langle v_n^2(p_2) \rangle}} \]  

(5)

and for the flow magnitudes squared, it is constructed as,

\[ r_n^2(p_1,p_2) = \frac{\langle |V_n(p_1)|^2|V_n(p_2)|^2 \rangle}{\sqrt{\langle v_n^2(p_1) \rangle} \sqrt{\langle v_n^2(p_2) \rangle}} \]  

(6)

In practice, the above formulae are difficult to use in experiment due to low statistics in high momentum bins.

\section{RESULTS FOR Pb+Pb COLLISIONS AT \( \sqrt{s_{NN}} = 5.02 \) TeV}

The ALICE Collaboration has presented measurements of the harmonic flow factorization breaking using only one bin in transverse momentum [27, 34]. That way the limitations from low multiplicity in bins at high transverse momentum can be partly overcome. The factorization breaking coefficient is defined as

\[ r_n(p) = \frac{\langle v_n v_n^*(p) \rangle}{\sqrt{\langle v_n^2(p) \rangle} \sqrt{\langle v_n^2(p) \rangle}} \]  

(7)
and it measures the correlation of the harmonic flow vectors between the momentum averaged flow and the flow at the transverse momentum $p$. The factorization breaking coefficient is also written as

$$r_n(p) = \frac{v_n[2](p)}{v_n[2](p)}$$  \hspace{1cm} (8)

and is a measure of the difference between two definitions of the differential harmonic flow coefficient $v_n[2](p)$ (Eq. 3) and

$$v_n[2](p) = \sqrt{\langle V_n^2(p) V_n^*(p) \rangle}$$  \hspace{1cm} (9)

The factorization breaking coefficient for the harmonic flow vector squared is

$$r_{n:2}(p) = \frac{\langle V_n^2 V_n^*(p) \rangle^2}{\sqrt{\langle v_n^2 \rangle \langle v_n^4 \rangle}}.$$  \hspace{1cm} (10)

Note that the factorization breaking coefficient (10) is defined as the correlation coefficient between $V_n^2$ and $v_n^2$.

$V_n^2(p)$. In Figs. 1, 2, and 3 the results obtained in the hydrodynamic model are shown. The decorrelation of the flow vectors is the largest for centralities where fluctuations dominate, i.e. for the elliptic flow in central collisions and for the triangular flow at any centrality (the results for the triangular flow at other centralities are qualitatively similar as for the centrality 30–40%, shown in Fig. 3). The model results are similar to the data for central collisions (Fig. 1). For semi-central events we find a much stronger decorrelation in the model for the flow vectors squared than in the data. The main difference shows up for $p > 2.0$ GeV. The model calculations do not include any contribution from non-flow correlations, while the experimental data have non-flow effects reduced using rapidity gaps. We cannot discriminate quantitatively in this study how much of the difference is due to remaining non-flow effects and how much can be attributed to genuine difference in flow fluctuation between the model and experiment.

With a four-particle correlator one can measure the factorization breaking coefficient for the square of the magnitude of the harmonic flow:

$$r_{n:2}^v(p) = \frac{\langle v_n^2 v_n^2(p) \rangle}{\sqrt{\langle v_n^4 \rangle \langle v_n^4 \rangle}}.$$  \hspace{1cm} (11)

In Figs. 4 and 5 are shown the results obtained for the elliptic flow in the hydrodynamic model. The results for the triangular flow are qualitatively similar (not shown). We observe that the magnitude decorrelation accounts for roughly one half of the flow vector decorrelation

$$[1 - r_{n:2}(p)] \simeq 2 \left[ 1 - r_{n:2}^v(p) \right].$$  \hspace{1cm} (12)
This means that the remaining angle decorrelation accounts roughly for the other half of the total vector decorrelation. While in general, any proportion between angle and magnitude decorrelation is possible, we note that previous experimental measurements \[20\] and model calculations \[19\] have observed an approximately equal strength of angle and magnitude decorrelation for flow correlations in rapidity. Similar to the flow vector factorization breaking coefficients, also for the magnitude factorization breaking coefficient, the data lie significantly above the simulations at high transverse momentum, which may be partly due to the contribution of non-flow correlations. This discrepancy is particularly pronounced for the 30–40% centrality. In fact, some of the data points lie above one, suggesting the dominance of non-flow correlations.

A comment is in order on how the flow vectors squared and flow magnitudes squared factorization breaking coefficients are extracted from the experimental data of the ALICE Collaboration. The experiment does not measure directly the flow vectors and the flow magnitudes factorization breaking coefficients (Eqs. \[10\] and \[11\] \[27\]). The difficulty lies in the extraction of the four-particle correlator in the denominator of the correlation formulae \(\langle v_n^2(p)\rangle\), with all four particles in a narrow transverse momentum bin. On the other hand the fourth moment \(\langle v_n^4\rangle\) can be measured. The ALICE Collaboration presents the results for the correlators with different scaling, e.g., for flow magnitude squared correlations,

\[
\frac{\langle v_n^2v_n^2(p)\rangle}{\langle v_n^2\rangle\langle v_n^2(p)\rangle}.
\]  

(13)

By dividing this scaled correlator by \(\langle v_n^2\rangle^2\) \[27\] an estimate of the flow vector squared correlation

\[
r_{n;2}(p) \approx \frac{\langle V_n^2V_n^*(p)\rangle^2}{\langle V_n^2\rangle\langle v_n^2(p)\rangle}. \tag{14}
\]

or the flow magnitude squared correlation

\[
r_{n}^{v_n^2}(p) \approx \frac{\langle v_n^2v_n^2(p)\rangle\langle v_n^2\rangle}{\langle v_n^2\rangle\langle v_n^2(p)\rangle}. \tag{15}
\]

can be obtained. The difference between Eqs. \[14\] and \[15\] and the factorization breaking coefficients defined in Eqs. \[10\] and \[11\] is a factor

\[
\sqrt{\frac{\langle v_n^2(p)\rangle\langle v_n^2\rangle}{\langle v_n^2\rangle^2}}.
\]  

(16)

We have checked that in the hydrodynamic model the deviation of this factor from 1 is less than \(6 \times 10^{-3}\) for 0.5 GeV < \(p\) < 3.0 GeV. The experimental data for the flow vector and magnitude correlation shown in the Figures are obtained using the formulae \[14\] and \[15\].

## IV. FLOW ANGLE DECORRELATION

An experimental observable directly measuring the flow angle correlation cannot be defined. An estimate of the flow angle correlation can be obtained as the ratio of a four-particle flow vector correlator and a flow magnitude correlator \[25\] \[27\]. The correlator measure used by the ALICE Collaboration for the correlation of the flow angle in transverse momentum is

\[
F_n(p) = \frac{\langle V_n^2V_n^*(p)\rangle^2}{\langle v_n^2\rangle\langle v_n^2(p)\rangle}. \tag{17}
\]

It is the ratio of the flow vector squared \[10\] and flow magnitude squared \[11\] factorization breaking coefficients. In Figs. \[6\] \[7\] and \[8\] are shown the results of the hydrodynamic simulations compared to the ALICE Collaboration data. The model simulations predict a noticeable flow angle decorrelation between the global flow \(V_n\) and the differential, momentum dependent flow \(V_n(p)\), both for the elliptic and triangular flows. The preliminary data of ALICE Collaboration are consistent with the simulation for the elliptic flow in central collisions. For the centrality 30–40\% (Fig. \[7\]), the simulation and the experiment show a smaller angle decorrelation than in central collisions. Non-flow correlations are expected to be relatively more important in that case. The difference in the strength of the decorrelation between central and semi-central collisions comes from the global correlation of the elliptic flow due to the initial geometry in non-central collisions.

The formula in Eq. \[17\] is a way to estimate the flow angle decorrelation in experiment. It measures

\[
\frac{\langle v_n^2v_n(p)^2\cos[2n(\Psi_n(p) - \Psi_n)]\rangle}{\langle v_n^2\rangle\langle v_n^2(p)^2\rangle}. \tag{18}
\]

Its use as an estimate of the flow angle decorrelation

\[
\frac{\langle v_n^2\cos[2n(\Psi_n(p) - \Psi_n)]\rangle}{\langle v_n^2\rangle} , \tag{19}
\]
that the two formulae give similar results, suggesting that the experimental measure \([17]\) could be used to estimate the weighted flow angle decorrelation. Please note that the angle correlation is weighted by the fourth power of flow magnitude. Only the \(v^4\) weighted flow angle decorrelation is measured. We have checked in our simulation, that using a simple average
\[
\langle \cos [2n (\Psi_n(p) - \Psi_n)] \rangle
\]
instead, would give a very different result. This results are not unexpected. In events with large flow (large \(|V_n|\) and large \(|V_n(p)|\) the random decorrelation between the two vector is relatively smaller, and the angle decorrelation is small. For a detailed discussion of the effect we refer the reader to Ref. \([16]\). Please note, that the simple angle correlation \([20]\) cannot be measured experimentally.

In the experiment, only the angle decorrelation between the flow vectors \textit{squared} can be measured. In the model one can check how it is related to the angle correlation between first moments of flow vectors
\[
\frac{\langle v^2_n \cos [n (\Psi_n(p) - \Psi_n)] \rangle}{\langle v^4_n \rangle}
\]
We find that the flow angle correlation between the first moment can be approximated (Fig. \(\ref{fig:30-40}\)) as the squared root of the angle decorrelation for the flow vectors squared
\[
\frac{\langle v^2_n \cos [n (\Psi_n(p) - \Psi_n)] \rangle}{\langle v^4_n \rangle} \simeq \sqrt{F_n(p)}.
\]
A similar relation was found for correlators of higher moment of flow vectors in pseudorapidity \([26]\). In Fig. \(\ref{fig:30-40}\) the upper limit for the flow angle correlation \(\sqrt{(1 + F_n(p))/2}\), proposed by the ALICE Collaboration \([27]\), is also shown.
V. MIXED HARMONICS

Correlations between event planes for mixed flow harmonics are interesting as a measure of nonlinearities in the hydrodynamic expansion or of correlations in the initial state \[35,46\]. Quantities discussed in these studies are based on correlators of harmonic flow vectors of different orders. Typically these moments involve the momentum averaged harmonic flow. The momentum dependence of the nonlinear response \[47\] or the covariance \[18\] between flow harmonics of different orders have been discussed in models, using moments of mixed flow harmonics in bins of transverse momentum. Such studies of many-particle correlators in small bins cannot be easily performed experimentally due to limited statistics.

A correlation coefficient (in this context the name correlation coefficient is used in the literature instead of the factorization breaking coefficient) for mixed flow harmonics can be defined with only one of the flow harmonics restricted to a transverse momentum bin

\[
\frac{\langle V_m^*(p) V_k V_n \rangle}{\sqrt{\langle V_m^2(p) \rangle \langle V_k^2 V_n^2 \rangle}},
\]

with \(m = k + n\). In Fig. 10 are shown the results for the correlation coefficient measuring the nonlinear coupling between \(V_4(p)\) and \(V_2^2\) as a function of the transverse momentum. We present the correlation coefficient for semi-central collisions only, where the nonlinear component in \(V_4\) is dominant. The dependence on transverse momentum gives an additional constraint on the initial state and on the hydrodynamic evolution. The correlation is the largest for small transverse momenta. In Fig. 11 is shown the analogous result for the coupling between \(V_5\) and \(V_3 V_2\). The results are qualitatively similar as for the \(V_4-V_2^2\) coupling.

In order to extract the flow angle correlation for mixed flow harmonic separately from the magnitude decorrelation, correlators with higher powers of flow harmonics must be considered. Taking as an example the nonlinear coupling between \(V_4\) and \(V_2\), the correlation coefficient for the flow vectors squared is

\[
\frac{\langle V_4^2 V_2^*(p)^2 \rangle}{\sqrt{\langle V_4^2 \rangle \langle V_2^4(p) \rangle}},
\]

the factorization breaking coefficient for the flow magni-
The measure (26) of event by event flow angle correlation is given by:

\[
\frac{\langle v_4^2 v_2(p)^2 \rangle}{\sqrt{\langle v_2^4 \rangle}} = \frac{\langle v_4^2 v_2(p)^2 \rangle}{\sqrt{\langle v_2^4 \rangle}}, 
\]

and the flow angle correlation can be estimated from the ratio of the above as:

\[
\frac{\langle V_2^2 V_4^*(p)^2 \rangle}{\langle v_2^2 v_4(p)^2 \rangle}.
\]

The correlation coefficient for higher powers of the flow vectors (Fig. 12) is smaller than for the lower powers of the respective flow vectors (Fig. 10). In semi-central collisions, the flow magnitude decorrelation (Fig. 13) accounts for about one half of the flow vector decorrelation shown in Fig. 12.

The measure (26) of event by event flow angle correlation for mixed harmonics involves six-particles correla-
We notice that the flow angle correlation \[^{26}\text{r}_{n,2}(p)\] defined as the ratio of the correlation coefficient for flow vectors and of the factorization breaking coefficient of the flow magnitudes is a good approximation of flow angle correlation weighted with powers of flow magnitudes
\[
\frac{\langle v_n^2 v_2^2 \cos [8 (\Psi_4(p) - \Psi_2)] \rangle}{\langle v_n^2 v_2^2 \rangle}.
\]
(27)

For completeness, in Fig. 15 we show the centrality dependence of flow angle decorrelation between momentum averaged flow vectors \(V_4^2\) and \(V_2^2\). Together with the momentum dependent flow angle correlation shown in Fig. 14 it can serve as an additional experimental observable, sensitive to the model of the initial conditions.

VI. SUMMARY AND OUTLOOK

We study observables defined as correlators of several flow harmonics where one or two of them are measured in a fixed bin of transverse momentum. The separate measurement of flow magnitude and flow factorization breaking requires the calculation of such four-particle correlators. The ALICE Collaboration has measured four-particle correlators, from which the factorization breaking coefficient, or the flow vector magnitudes factor acceptance can be used. The separate measurement of flow harmonics and in pseudorapidity provide additional information sensitive to the initial state and the dynamics in the hydrodynamic evolution. Correlators for the squares of the harmonic flow vectors (e.g. between \(V_4^2(p)\) and \(V_2^2\) or between \(V_5^2(p)\) and \(V_2 V_3\)) allow to measure separately the flow vector correlation coefficient, the flow vector magnitudes factorization breaking coefficient, or the flow angle correlation.

The simulation results for centrality 30–40% do not reproduce the preliminary experimental data of the ALICE Collaboration. Generally, we find a stronger decorrelation, while in the experiment harmonic flow correlation is sometimes larger than one. This may indicate that a large contribution of non-flow correlations is present. This issue requires further study.

An estimate of correlation measures involving four harmonic flow vectors can be constructed using separate bins in pseudorapidity, in order to reduce the non-flow correlations. Using four bins in pseudorapidity located at \(-\eta_F, -\eta, \eta, \eta_F\), the factorization coefficient for flow vectors squared can be estimated as

\[
r_{n,2}(p) \simeq \frac{\langle V_n(-\eta_F)V_n^*(-\eta,p)V_n(\eta,p)V_n(\eta_F)\rangle}{\langle V_n(-\eta_F)V_n^*(-\eta,p)V_n(\eta,F)\rangle}{\langle V_n^*(-\eta,p)V_n(p)\rangle},
\]
(28)

the factorization breaking coefficient between flow vector magnitudes can be defined as

\[
r_{n,2}(p) \simeq \frac{\langle V_n(-\eta_F)V_n(-\eta,p)V_n(\eta,p)V_n^*(\eta,F)\rangle}{\langle V_n(-\eta_F)V_n(-\eta,\eta_V^*(\eta,F))\rangle}{\langle V_n^*(-\eta,p)V_n(p)\rangle},
\]
(29)

and the flow angle correlation is estimated from the ratio the two quantities above

\[
F_n(p) \simeq \frac{\langle V_n(-\eta_F)V_n^*(-\eta,p)V_n(\eta,p)V_n(\eta_F)\rangle}{\langle V_n(-\eta_F)V_n(-\eta,p)V_n(\eta,F)\rangle}{\langle V_n(-\eta_F)V_n^*(-\eta,\eta_V(\eta,F))\rangle}{\langle V_n(\eta,F)V_n^*(\eta)\rangle}.
\]
(30)

The four-particle correlators in the above formulae involve flow vectors at different pseudorapidity and transverse momentum. Therefore, flow decorrelation in transverse momentum and in pseudorapidity combine in the result. However, the decorrelation in the longitudinal direction cancels between the numerator and the denominator, assuming it factorizes from the decorrelation in transverse momentum. In those formulae we also use the approximation

\[
\sqrt{\frac{\langle v_n^2(p)\rangle}{\langle v_n^2(p)\rangle}} \simeq \sqrt{\frac{\langle v_n^2\rangle}{\langle v_n^2\rangle}}.
\]
(31)

as discussed in sect. 111 It is used in order to avoid a four-particle correlator where all flow vectors are defined in a small transverse momentum bins. Note, that the flow vectors at the forward and backward rapidities \(\pm \eta_F\) do not require the measurement of particle transverse momenta and forward/backward calorimeters could used to measure them. Only the flow vectors \(V_n(\pm \eta, p)\) require the measurement of the transverse momenta of individual particles. For that purpose two bins well separated in pseudorapidity in the central rapidity region of the detector acceptance can be used. The separate measurement
of the flow magnitudes and the flow angles decorrelation in transverse momentum would provide a sensitive probe to the initial state fluctuations in the heavy-ion dynamics and could further constrain the initial state models.

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[1] J.-Y. Ollitrault, J. Phys. Conf. Ser. 312, 012002 (2011)
[2] U. Heinz and R. Snellings, Ann.Rev.Nucl.Part.Sci. 63, 123 (2013)
[3] C. Gale, S. Jeon, and B. Schenke, Int.J.Mod.Phys. A28, 1340011 (2013)
[4] C. E. Aguiar, Y. Hama, T. Kodama, and T. Osada, Nucl. Phys. A698, 639 (2002)
[5] J. Takahashi, B. M. Tavares, W. L. Qian, R. Andrade, F. Grassi, Y. Hama, T. Kodama, and N. Xu, Phys. Rev. Lett. 103, 242301 (2009)
[6] B. Alver et al. (PHOBOS), Phys. Rev. Lett. 98, 242302 (2007)
[7] B. Alver and G. Roland, Phys. Rev. C81, 054905 (2010)
[8] B. Schenke, S. Jeon, and C. Gale, Phys. Rev. Lett. 106, 042301 (2011)
[9] B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012)
[10] D. Teaney and L. Yan, Phys. Rev. C83, 064904 (2011)
[11] P. Bozek, W. Broniowski, and J. Moreira, Phys. Rev. C83, 034911 (2011)
[12] J. Jia and P. Huo, Phys. Rev. C90, 034915 (2014)
[13] L.-G. Pang, G.-Y. Qin, V. Roy, X.-N. Wang, and G.-L. Ma, Phys. Rev. C91, 044904 (2015)
[14] L.-G. Pang, H. Petersen, G.-Y. Qin, V. Roy, and X.-N. Wang, Eur. Phys. J. A52, 97 (2016)
[15] V. Khachatryan et al. (CMS), Phys. Rev. C92, 034911 (2015)
[16] P. Bozek and W. Broniowski, Phys. Rev. C97, 034913 (2018)
[17] J. Cimerman, I. Karpenko, B. Tomášik, and B. A. Trzeciak, Phys. Rev. C104, 014904 (2021)
[18] F. G. Gardim, F. Grassi, M. Luzum, and J.-Y. Ollitrault, Phys.Rev. C87, 031901 (2013)
[19] T. Kozlov, M. Luzum, G. Denicol, S. Jeon, and C. Gale(2014), arXiv:1405.3976 [nucl-th]
[20] F. G. Gardim, F. Grassi, P. Ishida, M. Luzum, P. S. Magalhães, and J. Noronha-Hostler, Phys. Rev. C 97, 064919 (2018)
[21] W. Zhao, H.-j. Xu, and H. Song, Eur. Phys. J. C 77, 645 (2017)
[22] P. Bozek, Phys. Rev. C98, 064906 (2018)
[23] L. Barbosa, F. G. Gardim, F. Grassi, P. Ishida, M. Luzum, M. V. Machado, and J. Noronha-Hostler(5 2021), [arXiv:2105.12792 [nucl-th]]