The thermal waves induced by ultra-short laser pulses in $n$-dimensional space-time

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Abstract

In this paper the heat waves, induced by ultra-short laser pulses are considered. The hyperbolic heat transport in $n$-dimensional space-time is formulated and solved. It is shown that only for $n$—odd for heat waves the Huygens principle is fulfilled. The heat transport experiment for Cu$_3$Au alloy is considered.

Key words: Hyperbolic heat transport; Thermal waves; Huygens principle; Cu$_3$Au alloy.

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1 Introduction

The fact that we perceive the world to have three spatial dimensions is something so familiar to our experience of its structure that we seldom pause to consider the direct influence this special property has upon the laws of physics. Yet some have done so and there have been many intriguing attempts to deduce the expediency or inevitability of a three-dimensional world from the general structure of the physical law themselves.

As earlier as 1917 P. Ehrenfest [1] pointed out that neither classical atoms nor planetary orbits can be stable in a space with \( n > 3 \) and traditional quantum atoms cannot be stable either [2]. As far as \( n < 3 \) is concerned, it has been argued [3] that organism would face insurmountable topological problem if \( n = 2 \): for instance two nerves cannot across. In the following we will conjecture that since \( n = 2 \) offers vastly less complexity that \( n = 3 \), worlds with \( n < 3 \) are just too simple and barren to contain observers. Since our Universe appears governed by the propagation of classical and quantum waves it is interesting elucidate the nature of the connection the properties of the wave equation and the spatial dimensions.

In this paper we describe the partial differential equation (PDE) for the propagation of the thermal waves in \( n \)--dimensional space time. It is well known that for heat transport induced by ultra-short laser pulses (shorter than the relaxation time) the governing equation can be written as [4]

\[
\frac{1}{v^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{D} \frac{\partial T}{\partial t} + \frac{2Vm}{\hbar^2} T = \nabla^2 T, \tag{1}
\]

where \( T \) is the temperature, \( v \) denotes the thermal wave propagation, \( m \) is the mass of heat carriers and \( V \) is the potential.

In monograph [4] the solution of the equation for one-dimensional case, \( n = 1 \) was obtained. In this paper we develope and solve the analog of the equation for \( n \) = natural numbers \( n = 1, 2, \cdots \), separately for \( n \) = odd and \( n \) = even. The Huygens’ principle for thermal wave will be discussed. It will be shown that for thermal waves only in odd dimensional space the waves propagate at exactly a fixed space velocity \( v \) without “echoes” assuming the absence of walls (potentials) or inhomogeneities.

The three-dimensional heat transfer induced by ultra-short laser pulses in Cu$_3$Au alloy will be suggested.
2 The master equation for the thermal waves in $n$–dimensions

In the following we consider the $n$–dimensional heat transfer phenomena described by the equation \[4\]

$$\frac{1}{v^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{D} \frac{\partial T}{\partial t} + \frac{2Vm}{\hbar^2} T = \nabla^2 T$$ (2)

where temperature $T$ is the function in the $n$–dimensional space

$$T = T(x_1,\ldots,x_n,t)$$ (3)

We seek solution of equation (2) in the form:

$$T(x_1,x_2,\ldots,x_n) = e^{-\frac{t}{\tau}} u(x_1,\ldots,x_n,t)$$ (4)

After substitution of Eq. (4) to Eq. (2) one obtains

$$\frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} - \nabla^2 u + qu = 0$$ (5)

where

$$q = \frac{2Vm}{\hbar^2} - \left(\frac{mv}{2\hbar}\right)^2$$

for $D = \frac{\hbar}{m} \[4\]$.

We can define the distortionless thermal wave as the wave which preserves the shape in the field of the potential $V$. The condition for conserving the shape can be formulated as

$$q = \frac{2Vm}{\hbar^2} - \left(\frac{mv}{2\hbar}\right)^2 = 0$$ (6)

When Eq. (6) holds Eq. (5) has the form

$$\frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0$$ (7)

and condition (6) can be written as

$$V\tau \sim \hbar$$ (8)
We conclude that in the presence of the potential energy $V$ one can observe the undisturbed thermal wave only when the Heisenberg uncertainty relation (8) is fulfilled.

The solution of the Eq. (7) for the $n-$odd can be find in [5]. First of all let us change the variables in Eq. (7)

$$v : t \rightarrow t', \quad x \rightarrow x', \quad u \rightarrow u'$$

and obtain

$$\frac{\partial^2 u'}{\partial t'^2} - \nabla^2 u' = 0.$$  (9)

For

$$\lim_{x',t' \rightarrow (x^0,0)} u'(x',t') = g(x_0),$$  (10)

$$\lim_{x',t' \rightarrow (x^0,0)} \frac{\partial u(x',t')}{\partial t'} = h(x_0),$$

the solution have the form [5]

$$u'(x',t') = \frac{1}{\gamma_n} \left[ \left( \frac{\partial}{\partial t'} \right) \left( \frac{1}{t'} \frac{\partial}{\partial t'} \right) \frac{n-2}{2} \left( t'^n - \int_{\partial B(x',t')} g dS \right) + \left( \frac{1}{t'} \frac{\partial}{\partial t'} \right) \frac{n-2}{2} \left( t'^n - \int_{\partial B(x',t')} h dS \right) \right]$$  (11)

and $\gamma_n = 1 \cdot 3 \cdot 5 \cdots (n-2)$.

For $n-$ even the solution of equation [2] have the form [5].

$$u'(x',t') = \frac{1}{\gamma_n} \left[ \left( \frac{\partial}{\partial t'} \right) \left( \frac{1}{t'} \frac{\partial}{\partial t'} \right) \frac{n-2}{2} \left( t'^n - \int_{B(x',t')} \frac{g(y')dy'}{(t'^2 - |y' - x'|^2)^{1/2}} \right) + \left( \frac{1}{t'} \frac{\partial}{\partial t'} \right) \frac{n-2}{2} \left( t'^n - \int_{B(x',t')} \frac{h(y')dy'}{(t'^2 - |y' - x'|^2)^{1/2}} \right) \right]$$  (12)

and $\gamma_n = 2 \cdot 4 \cdots (n-2) \cdot n$. In formulae (11) and (12) $\int$ denotes integral over $n-$space.

Considering formulae (11) and (12) we conclude that for $n-$odd the solution (11) is dependent on the value of functions $h$ and $g$ only on the
hypersphere $\partial B(x', t')$. On the other hand for $n$—even the solution (12) is dependent on the values of the functions $h$ and $g$ on the full hyperball $B(x', t')$. In the other words for $n$—odd $n \geq 3$ the value of the initial functions $h$ and $g$ influence the solution (12) only on the surface of the cone $\{(y', t'), t' > 0, |x' - y'| = t'\}$. For $n = $ even the value of the functions $g$ and $h$ influences the solution on the full cone. It means that the thermal wave induced by the disturbance for $n = $ odd have the well defined front. For $n$—even the wave influences space after the transmission of the front. This means that Huygens’ principle is false for $n$—even. In conclusion: if we solve the wave equation in $n$—dimensions the signals propagate sharply (i.e. Huygens’ principle is valid) only for dimensions $n = 3, 5, 7, \ldots$. Thus three is the “best of all possible” dimensions, the smallest dimension in which signals propagate sharply.

3 Suggestions for experimentalists

The hyperbolic transport equation for heat transport (11)

$$\frac{1}{v_T^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{D_T} \frac{\partial T}{\partial t} + \frac{2Vm}{h^2} T = \nabla^2 T$$ (13)

or mass transport

$$\frac{1}{v_\rho^2} \frac{\partial^2 \rho}{\partial t^2} + \frac{1}{D_\rho} \frac{\partial \rho}{\partial t} + \frac{2Vm}{h^2} \rho = \nabla^2 \rho$$ (14)

are the damped wave equations. For very short time period $\Delta t \sim \tau$ both equations (13) and (14) can be written as

$$\frac{1}{v_{\rho,T}^2} \frac{\partial^2 u_{\rho,T}}{\partial t^2} - \nabla^2 u_{\rho,t} = 0$$ (15)

Eq. (15) is the generalization of equation (7). The solution of equation (15) in $n$—dimensional cases are described by formulae (11) and (12).

As was discussed in paragraph 2 only in 3—dimensional case the Huygens principle is fulfilled. It seems that in order to observe the thermal wave not disturbed by the ”echoes” and with sharp front the true three-dimensional experiment must be performed. Moreover the experiment must be performed in the relaxation regime, i.e. for materials with relatively long relaxation
time. The best candidates for “relaxation materials” is the Cu$_3$Au alloy [9]. As was shown in paper [6] the relaxation time is of the order of $10^4$ s in the temperature range 650–660 K. For $T > 660$ K the abrupt increasing, up to $1.5 \cdot 10^5$ s (due to order $\rightarrow$ disorder transition) was observed.
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