Valve closure based on pump runaway characteristics in long distance pressurized systems

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ABSTRACT

In order to guarantee the safety of pumps, valves are installed at the outlet of each pump in long-distance pressurized water supply systems. However, water hammer pressure caused by improper valve closure can tremendously exceed the standard of the pipeline. In this paper, the effects of valve closure on speed of pump and pressure along the pipeline were investigated. Valve closure formula based on pump runaway characteristics were proposed and verified using numerical simulation. In addition, when valves were closed under the formula proposed in this paper and other closure laws, the minimum speed, minimum and maximum pressure along the pipeline were compared. The results showed that formulas agree well with the numerical results. In the high lift supply systems, compared with the other closure laws, the minimum speed and minimum pressure along the pipeline under valve closure formula were the largest, and the maximum pressure along the pipeline was the smallest. Moreover, in the low lift supply systems, the minimum speed under valve closure formula did not exceed the rated speed. Compared with the other closure laws, the minimum pressure along the pipeline was the largest and the maximum pressure along the pipeline was the smallest.

Key words | long distance pressurized water supply system, pump runaway characteristics, valve closure, water hammer

HIGHLIGHTS

- Valve closure formula based on pump runaway characteristics were proposed.
- In high lift systems, the minimum speed and minimum pressure along the pipeline under valve closure formula were the largest, and the maximum pressure along the pipeline was the smallest.
- In the low lift systems, the minimum speed under valve closure formula did not exceed the rated speed.
- Minimum or maximum pressures were the safest.
GRAPHICAL ABSTRACT

INTRODUCTION

Long-distance pressurized water supply system is the most effective approach to solve uneven distribution of water resources (Wang et al. 2019a, 2019b; Ni et al. 2020). It is pressurized by pumps and then piped to transfer water resources from low-altitude areas to high-altitude areas. Therefore, cross-regional long-distance pressurized water supply systems can be seen all over the world (Sun et al. 2016; Miao et al. 2017). However, due to large flow and multi-objective water supply of the systems, the safety of pumps and along pipelines is a difficult problem. In order to regulate the flow and ensure the safety of pumps, valves are installed at the outlet of each pump. When pump trip happens, to prevent reverse speed of the pump from exceeding the standard, valves should be closed. Generally, the faster the valves are closed, the easier the reverse speed of the pump meets the requirements, but water hammer pressure caused by improper closure of valves can tremendously exceed pressure standard. If the pressure exceeds the pressure standard of the pipeline, it may cause enormous damage to the system (Adamkowski & Lewandowski 2015; Guo et al. 2020). Therefore, it is necessary to investigate how to effectively prevent the valves from being improperly closed of a long-distance pressurized water supply system.

When pump trip happens in the long-distance pressurized water supply system, it is generally accepted that valves installed at the outlet of the pump are components to control the reverse speed of the pump by effectively regulating the flow (Wan et al. 2014; Essaidi & Triki 2020). Due to the fast and convenient regulating flow, the valves have been widely used in water supply systems all over the world (Bettaieb & Taieb 2020). In previous research, there are a large number of studies about the valve (Kim & Yoon 2015). According to the closing rate of the valve, the valves are divided into slow closing valves and fast closing valves. Slow closing valves include ball valves (Ferreira et al. 2018) and butterfly valves. The most common type of fast closing valve is the axial flow check valve (Yu & Yu 2015). There are many published papers regarding valve closure law (Li et al. 2018). According to the different curves of valve closure, the valve closure laws can be divided into one straight-line closure, two straight-line closure, and three straight-line closure. However, the above studies have focused on discussing the influence of valve closure curve on the water hammer pressure of long-distance water supply systems.

The long-distance water supply systems are mainly divided into pressurized and gravitational water supply systems. The gravitational system relies on topographical drop to supply water, and there is no pump in gravitational systems (Wang et al. 2019a, 2019b). Then, only the effect of different valve closure curves on the water hammer pressure of the system needs to be considered. However, compared with the gravitational system, the pressurized system relies on being pressurized by pumps to supply water. The pressure drop behind the pump is influenced by the characteristics of the pump. Hence, it is insufficient to only consider the effect of valve closure curves on water hammer pressure in long-distance pressurized water supply systems. Therefore, motivated by the above discussions, this paper focuses on the influence of valve closure law considering pump runaway characteristics on water hammer pressure in the long-distance pressurized water supply system.
MATHEMATICAL MODEL

For low Mach number fluids, the continuity and momentum equations of unsteady pipe liquid flow can be expressed as follows (Wylie et al. 1995; Chaudhry 2014):

\[
\frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} = 0
\]  
(1)

\[
\frac{\partial H}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + f \frac{|V|}{2gD} = 0
\]  
(2)

where, \(H\) is piezometric head (m), \(V\) is velocity of flow (m³/s), \(D\) is pipe internal diameter (m), \(A\) is cross-sectional area (m²), \(g\) is gravitational acceleration (m/s²), \(a\) is wave speed (m/s), \(t\) is computational time (s), \(x\) is axial distance (m), \(f\) is friction coefficient.

Equations (1) and (2) are converted to ordinary differential equations along the characteristic lines. The characteristic lines are shown in Figure 1. The characteristic equations can be obtained by integrating the ordinary differential equations.

The negative characteristic equation \(C^-\)

\[ H_{Pi} = C_M + B_M Q_P \]  
(3)

The positive characteristic equation \(C^+\)

\[ H_{Pi} = C_P - B_P Q_P \]  
(4)

where, \(H_{Pi}\) is unknown piezometric head of section \(i\) at time \(t\). \(Q_P\) is unknown discharge of section \(i\) at time \(t\).

\(C_M = H_{i+1} - BQ_{i+1}, B_M = B + R|Q_{i+1}|, C_P = H_{i-1} + BQ_{i-1}, B_P = B + R|Q_{i-1}|, B = a/gA, R = f\Delta x/2gDA^2.\)

Valve model

The relationship between the head and discharge may be written as:

\[ Q_P = C_d A_r \sqrt{2g\Delta H_P} \]  
(5)

where, \(C_d\) is coefficient of discharge, \(C_d = 1/\sqrt{\epsilon}\), \(\epsilon\) is loss coefficient of the valve, \(\epsilon = 2gA^2\Delta H_P Q_P^2\), \(A_r\) is area of the valve opening (m²), \(\Delta H_P\) is losses at the valve (m).

Through Equations (3) and (4), it can be obtained that:

\[ Q_P = \frac{C_P - C_M}{B_P + B_M + \frac{\epsilon(Q_P)}{2gA^2}} \]  
(6)

As the \(\Delta t\) is small, the \(Q_P\) at the right side of Equation (6) can be replaced by the instantaneous discharge rate \(Q_{P0}\) at the time \(t_0 = t - \Delta t\). Therefore, the discharge at the left can be directly obtained, and the values of \(H_{P1}\) and \(H_{P2}\) are determined.

Pump model

In long-distance pressurized water supply systems, pumps are important components. The Suter characteristic curves of a centrifugal pump are shown in Figure 2. \(WH(x)\) and \(WB(x)\) are separately the head and torque characteristics of the pump under different discharge and rotational speed conditions.

\[
\begin{align*}
WH(x) &= \frac{h}{q^2 + n^2} \\
WB(x) &= \frac{m}{q^2 + n^2} \\
x &= \pi + \tan^{-1} \frac{g}{n}
\end{align*}
\]  
(7)

![Figure 1](characteristic-grid.png)  
**Figure 1** | Characteristic grid.

![Figure 2](suter-characteristics.png)  
**Figure 2** | Suter characteristic curves of a centrifugal pump.
where, $h = H/H_r$, $H$ is head of pump (m), and $H_r$ is the rated head of pump (m). $n = N/N_r$, $N$ is speed of pump (r/min), and $N_r$ is the rated speed of pump. $q = Qp/Q_{pr}$. $Q_p$ is the discharge of pump (m$^3$/s), $Q_{pr}$ is the rated discharge of pump (m$^3$/s). $m = M/M_r$, $M_r$ is the torque of pump (KN.m), $M_r$ is the rated torque of pump (kN.m).

The equation of speed for the pump

$$J \frac{dn}{dt} = M - M_p$$

(8)

where, $J$ is moment of inertia (t·m$^2$), $\omega = 2\pi N/60 = n\omega_r$ is angular velocity (rad/s), $M_g$ is the motor torque (KN.m). $M = mM_g = mP_r/\omega_r$. For $M = mM_g = mP_r/\omega_r$, $T_a = \omega_r^2/P_r$, $P_r$ is the rated output of pump (kW). Then, Equation (8) can be written as

$$T_a \frac{dn}{dt} = -m$$

(9)

The integral of Equation (9) is obtained:

$$n = n_0 - \frac{1}{T_a} \int_{t_0}^{t} m dt$$

(10)

Using the Taylor expansion with Equation (10):

$$n = n_0 - \frac{\Delta t}{T_a} (1.5m_0 - 0.5m_{00})$$

(11)

where, $m_{00}$ is the relative value of torque at $t_0 - \Delta t$. $m_0$ is the relative value of torque at $t_0$. $m_{00}$ and $m_0$ can be solved by Suter characteristic curves, respectively.

Therefore, the relative speed $n$ at $t$ can be obtained directly by Equation (11).

The head equation of the pump:

$$H = H_{P1} - H_{P2}$$

(12)

where, $H_{P1}$, $H_{P2}$ are piezometric head for the inlet and outlet of the pump, respectively (m).

According to Equation (7), it can be obtained that:

$$H = hH_r = H_r(q^2 + n^2)WH(x)$$

(13)

Through Equations (3), (4), (12), and (13), we can get

$$H_r(q^2 + n^2)WH(x) - C_M + C_p - (B_M + B_P)Q_{pr} q = 0$$

(14)

Relative discharge $q$ can be calculated by Equation (14) using the Newton-Raphson method. According to $q = Q_p/Q_{pr}$, $Q_p$ can be obtained. Then, according to Equations (3) and (4), $H_{P1}$ and $H_{P2}$ can be obtained.

### VALVE CLOSURE BASED ON PUMP RUNAWAY CHARACTERISTICS

#### Theoretical analysis of pump runaway characteristic

A long-distance pressurized water supply system is shown in Figure 3. $P$ and $B$ are the locations of pump and downstream reservoir, respectively. The compatibility equation of the MOC (method of characteristics) can be integrated from B to P.

$$H_{P2} - H_{B1} - \frac{a}{gA} (Q_{P2} - Q_{B1}) + \int_{B1}^{P2} (Qdx)$

(15)

where, subscripts 0, 1, and 2 indicate the time after pump failures, which are time at time $t = 0$, $t = L/a$, $t = 2L/a$, respectively.

By simplifying Equation (15), we can get

$$\Delta H = \frac{aQ_0}{gA} + \frac{fLQ_0^2}{2gDA^2} - \left( \frac{a}{gA} + \frac{fLQ_0}{2gDA^2} \right) Q_{P2}$$

(16)

where, $\Delta H = H_{P0} - H_{P2}$, $\Delta H$ is the pressure drop at $P$ when $t = 2L/a$ (m).

![Figure 3](image-url)
Generally, the water level of the upstream reservoir can be constant to $H_U$. Therefore, it is approximated that the change of pump head is equal to the change of piezometric head behind the pump. According to the above analyses, it can be obtained that $h = H/H_U = H_0 - \Delta H/H_U$ and $q = Q_p/Q_{pr} = Q_p/0 - \Delta Q_p/Q_{pr}$. By substituting $h$ and $q$ into Equation (7), the following expression can be obtained:

$$H_0 - \Delta H = WH(x) \left(1 + \frac{1}{\tan^2 x}\right) \left(\frac{Q_p}{Q_{pr}}\right)^2$$

(17)

When pump trip happens, the pump is runaway at time $t = 2L/a$. According to Equation (7), $WB(x) = 0$ at that time. Then, according to the Suter characteristic curves of a centrifugal pump, the pump must be at point a or point b at time $t = 2L/a$. Based on the above analyses, when $t = 2L/a$, the values of $x$ and $WH(x)$ can be known.

By combining Equations (16) and (17), we can get

$$Q_p = \frac{IC_1 \pm \sqrt{(IC_1)^2 + 4C_2(H_0 - C_1Q_0)}}{2C_2}$$

(18)

where, $i$ is the number of the same parallel pumps in the system. $C_1 = a/gA + fLQ_0/2gDA^2. C_2 = WH(x) \left(1 + 1/\tan^2 x\right)H_U/Q_{pr}^2.$

Based on the above analyses, when pump trip happens, the pump is in the runaway condition at time $t = 2L/a$, and the pump must be at point a or point b.

If the pump is at point a, then $x = x_a$. The pump has reverse speed and reverse flow. $Q_p < 0$, $WH(x_a) > 0$, $C_2 > 0$. So $IC_1 \pm \sqrt{(IC_1)^2 + 4C_2(H_0 - C_1Q_0)} < 0$. Because $C_1 > 0$, it can be obtained that $IC_1 - \sqrt{(IC_1)^2 + 4C_2(H_0 - C_1Q_0)} < 0$. Then, $C_2(H_0 - C_1Q_0) > 0$. Therefore, $H_0 - C_1Q_0 = H_B - H_U - aQ_0/gA < 0$.

In summary, when the long-distance pressurized water supply system is high lift, then $H_B - H_U - aQ_0/gA > 0$, the pump must be at point a at time $t = 2L/a$, and the pump has reverse speed and reverse flow. On the contrary, when the long-distance pressurized water supply system is low lifted, then $H_B - H_U - aQ_0/gA < 0$, the pump must be at point b at time $t = 2L/a$, and the pump has positive speed and positive flow.

**Valve closure formulas**

In long-distance pressurized water supply systems where friction of the pipeline is constant, the value of pressure drop behind the pump is only influenced by the characteristics of the pump. When the pump outlet is not protected and the valve behind the pump does not move, variation of the pressure behind the pump is shown in Figure 4.

In Figure 4, $T_d$ is the time it takes for the pressure behind the pump to drop to $\Delta H$ (s). $\Delta H$ is maximum value of the pressure drop behind the pump before $2L/a$ (m). $T_s$ is the time it takes for the pressure behind the pump to drop to the minimum (s).

The backflow can cause the pump trip in a long-distance pressurized water supply system. In order to prevent the pump having reverse speed exceeding the standard, the valve at the outlet of the pump should be closed. Based on the above analyses, the value of pressure drop behind the

![](https://i.imgur.com/3G5JQ5G.png)

**Figure 4** | Variation of the pressure behind the pump.
pump is influenced by the characteristics of pump and the valve closing law. If the closing time of the valve is \( T_g \), and \( T_g < 2L/a \), then, direct pump-stopping water hammer behind the valve can be obtained.

\[
\Delta H = \frac{a}{g} \Delta V \tag{19}
\]

where, \( \Delta V = (V_0 - 0) \), \( V_0 \) is the flow velocity throughout the pipeline at time \( t = 0 \) (m/s).

\[
\Delta H_s = \frac{a}{g} \Delta V_s \tag{20}
\]

where, \( \Delta V_s = (V_0 - V_2) \), \( V_2 \) is the flow velocity throughout the pipeline at time \( t = 2L/a \) (m/s).

According to the analyses, when pump trip for the system with \( H_B - H_U - aQ_0/gA > 0 \) happens, the four quadrant of the zones of pump operation is the reversed speed dissipation at time \( t = 2L/a \). And the pump has reverse speed and reverse flow. Thus, \( V_2 < 0 \) and \( \Delta V_s > \Delta V \). If \( T_g < T_d \), according to Equations (19) and (20), it can be obtained that \( \Delta H_P = \Delta H < \Delta H_s \). Based on the above analyses, it can be believed that, when \( H_B - H_U - aQ_0/gA < 0 \), slow closure of valves is beneficial to the safety of long-distance pressurized water supply systems.

Based on the above analyses and discussions, the valve closure formulas considering pump runaway characteristics can be obtained.

\[
f(t) = T_g = T_d, \quad H_B - H_U - \frac{a}{gA}Q_0 > 0 \tag{21}
\]

\[
f(t) = T_g > T_s, \quad H_B - H_U - \frac{a}{gA}Q_0 < 0 \tag{22}
\]

where, \( f(t) \) is the valve closure formula.

**CASE STUDY AND ANALYSIS**

The pump runaway characteristics are theoretically analyzed in long-distance pressurized water supply systems. According to the theoretical analyses, valve closure formulas considering pump runaway characteristics are proposed. In this section, the formulas are verified using numerical simulation based on practical projects. Layouts of the long-distance pressurized water supply systems are shown in Figure 5.

![Figure 5](http://iwaponline.com/aqua/article-pdf/70/4/493/899081/jws0700493.pdf)
Case 1: Three identical centrifugal pumps are operated in parallel. The rated discharge of each pump is 0.55 m$^3$/s, and the pump head is 300.0 m. The length and diameter of pipe are 75.0 km and 1.4 m, respectively, and roughness coefficient is 0.0115. The wave speed is 1,000 m/s. A valve is installed at the outlet of the pump. Upstream and downstream water levels are 1,365.5 m and 1,602.5 m. The elevation and piezometric head along the pipeline are shown in Figure 5(a), and the parameters of the system are listed in Table 1.

Case 2: Three identical centrifugal pumps are operated in parallel. The rated discharge of each pump is 0.27 m$^3$/s, and the pump head is 82.0 m. The length and diameter of pipe are 18.0 km and 1.0 m, respectively, and roughness coefficient is 0.0120. The wave speed is 1,000 m/s. A valve is installed at the outlet of the pump. Upstream and downstream water levels are 515.0 m and 578.0 m. The elevation and piezometric head along the pipeline are shown in Figure 5(b), and the parameters of the system are listed in Table 1.

Valve closure formula for high lift supply systems

Verification of theoretical analyses

The elevation and piezometric head along the pipeline of Case 1 are shown in Figure 6(a). In order to verify the accuracy and rationality of the valve closure, Equation (21) is proposed in this paper. Based on the information in the section ‘Mathematical model’, the transition process of the long-distance pressurized water supply system is calculated and analyzed through MOC. Figure 6 presents the variations of the pressure behind the pump, discharge of pump and speed of pump, when the pump outlet is not protected and the valve at the outlet of the pump does not move. The length of the system for Case 1 is 75.0 km, $t = 2L/a = 150$ s. $H_B = 1, 602.5$ m, $H_U = 1, 365.5$ m, and $aQ_0/gA = 99.33$ m. Therefore, the system of Case 1 is $H_B - H_U - aQ_0/gA = 137.67 > 0$. From Figure 6(a) and 6(b), when the pump outlet is not protected and the valve at the outlet of the pump does not move, the initial pressure behind the pump at time $t = 0$ is 296.6 m, and the pressure decreases from the initial pressure to 101.7 m at the time $t = 2L/a = 150$ s. Thus, $\Delta H_s = 194.9$ m. According to Table 1, the initial flow velocity is 0.974 m/s. Based on Equation (19), it can be obtained that $\Delta H = a/g\Delta V = a/g(V_0 - 0) = 99.3 < \Delta H_s = 194.9$. It shows that the mathematical model and calculation method established in this paper are reasonable.

The variations of discharge and speed of the pump are shown in Figure 6(c) and 6(d). At the time $t = 2L/a = 150$ s, the discharge of the pump at the time is $-0.2$ m$^3$/s, and the speed is $-954.3$ t/min. Based on the above analyses, it can be believed that, when the system is $H_B - H_U - aQ_0/gA > 0$, the pump must be at point a at time $t = 2L/a$, the four quadrants of the zones of pump operation have reversed speed dissipation at the time. The pump has reverse speed and reverse flow at time $t = 2L/a$. The results are consistent with the theoretical analyses. It shows that the theoretical analyses are reasonable.

Sensitivity analyses

According to Equation (21), the valve should be completely closed when the pressure behind the pump drops to 197.3 m, in which $197.3 = H_0 - \Delta H_s = 296.6 - 99.3$. From Figure 6(b), $T_b = T_d = 1.0$ s. In order to verify the accuracy and rationality of the valve closure Equation (21), the sensitivity of valve closing time $T_b$ is analyzed. The analysis schemes are shown in Table 2. Under these analysis schemes, variations of pressure behind the pump, discharge of pump, speed of pump, maximum and minimum pressure curves along the pipeline are shown in Figure 7.

In the system of Case 1, $H_B - H_U - aQ_0/gA = 137.67 > 0$. From Figure 7(a) and 7(b), when the valve

| Systems | Rated speed (r/min) | Motor power (kW) | Moment of inertia (kg·m$^2$) | Discharge (m$^3$/s) | Velocity (m/s) | Roughness coefficient |
|---------|--------------------|-----------------|-----------------------------|-------------------|---------------|----------------------|
| Case 1  | 1,480.0            | 2,800.0         | 753.04                      | 1.50              | 0.974         | 0.0115               |
| Case 2  | 1,480.0            | 350.0           | 72.84                       | 0.81              | 1.03          | 0.0120               |
The pressure behind the pump drops 101.6 m, 101.5 m, 101.9 m, 149.1 m, and 181.9 m in 6 seconds, respectively. Under these schemes, when $T_g/C_20 > T_d = 1.0$ s, the pressure drops behind the pump are basically equal. However, when $T_g/C_20 = T_d = 1.0$ s, with the increase of valve closing time, the pressure drop increases gradually. The reason for this is that, according to the variations of the discharge and speed as shown in Figure 7(c) and 7(d), under $T_g/C_20 = T_d = 1.0$ s, the pump has no reverse speed and no reverse flow. Under $T_g > T_d = 1.0$ s, the pump has no reverse speed and no reverse flow. And the longer the valve closing time, the greater the reverse speed and reverse flow of the pump. Then, pressure drop behind the pump is greater.

The speed of the pump, the maximum and minimum pressure curves along the pipeline are shown in Figure 7(d)–7(f), respectively. When the valve closing times are 0.4 s, 0.7 s, 1.0 s, 10.0 s, and 15.0 s, the minimum speeds are $-12.08$ r/min, $-12.10$ r/min, $-12.11$ r/min, $-677.5$ r/min, and $-1,037.12$ r/min, respectively. Minimum pressures along the pipeline are $-53.8$ m, $-38.1$ m, $-24.5$ m, $-41.8$ m, $-60.6$ m, and $-65.7$ m, and maximum pressures along the pipeline are 319.9 m, 318.8 m, 318.7 m, 336.0 m, 361.2 m, and 363.4 m. Compared with the closing time of

| Table 2 | The sensitivity analysis schemes of valve closing |
|---------|-----------------------------------------------|
| Schemes | Scheme 1 | Scheme 2 | Scheme 3 | Scheme 4 | Scheme 5 |
| Valve closing time (s) | 0.4 | 0.7 | 1.0 | 10.0 | 15.0 |

Figure 6 | (a) and (b) Variations of the pressure behind the pump, (c) variations of the discharge of the pump, and (d) variations of the speed of the pump.
valves less than $T_d$ and greater than $T_d$, these results show that, when $T_{g} = T_d = 1.0$ s, the minimum speed is the largest, and the minimum pressure along the pipeline is the largest, and the maximum pressure is the smallest. Based on the above analyses, it can be believed that, for the system with $H_B - H_U - a Q_0 / g A > 0$, both the pump and
the pressure along the pipeline are the safest under the valve closing Equation (21).

**Valve closure formula for low lift supply systems**

**Verification of theoretical analysis**

The elevation and piezometric head along the pipeline of Case 2 are shown in Figure 5(b). In order to verify the valve closure Equation (22) is proposed. Based on the section ‘Mathematical model’, the transition process of the long-distance pressurized water supply system is calculated and analyzed through MOC. Figure 8 presents the variations of the pressure behind the pump, discharge of pump and speed of pump, when the pump outlet is not protected and the valve does not move.

![Figure 8](image-url)

**Figure 8** | (a) Variations of the pressure behind the pump, (b) variations of the discharge of the pump, and (c) variations of the speed of the pump.

The length of the system of Case 2 is 18.0 km, $t = 2L/a = 36$ s. $H_B = 578.0$ m, $H_U = 515.0$ m, and $aQ_0/gA = 108.15$ m. Therefore, the system of Case 2 is $H_B - H_U - aQ_0/gA = -45.13 < 0$. From Figure 8(a), when the pump outlet is not protected and the valve does not move, the initial pressure behind the pump at time $t = 0$ is 100.6 m, and the pressure decreases from the initial pressure to 17.4 m at the time $t = 2L/a = 36$ s. Thus, $ΔH_s = 83.2$ m. According to Table 1, the initial flow velocity is 1.03 m/s. Based on Equation (19), it can be obtained that $ΔH = a/gΔV = a/g(V_0 - 0) = 105.0 > ΔH_s = 83.2$. It shows that the mathematical model and calculation method established in this paper are reasonable.

The variations of discharge and speed of pump are shown in Figure 8(b) and 8(c). At time $t = 2L/a = 36$ s, the discharge of the pump at the time is 0.09 m$^3$/s, and the speed is 160.6 r/min. Based on above analyses, it can be
believed that, for the system of Case 2, which is $H_B - H_U - aQ_0/gA < 0$, the pump must be at point b at time $t = 2L/a$, and the pump is of positive speed and of positive flow at time $t = 2L/a$. The results are also consistent with the theoretical analyses. It shows that the theoretical analyses are reasonable.

**Sensitivity analysis**

According to Equation (22), the closing time of the valve should be longer than $T_s$. From Figure 8(a), $T_g > T_s = 36$ s. In order to verify the valve closure Equation (22), the sensitivity of valve closing $T_s$ is analyzed. The analysis schemes are shown in Table 3. Under these analysis schemes, variations of pressure behind the pump, discharge of pump, speed of pump, maximum and minimum pressure curves along the pipeline are shown in Figure 9.

In the system of Case 2, $H_B - H_U - aQ_0/gA = -45.13 < 0$. From Figure 9(a), when the valve closing times are 1.0 s, 18.0 s, 36.0 s, 54.0 s, and 90.0 s, the pressure behind the pump drop is 120.4 m, 119.9 m, 119.3 m, 89.6 m, and 83.6 m in 36 seconds, respectively. Under these schemes, when $T_g \leq T_s = 36$ s, the pressure drop behind the pump is basically equal. However, when $T_g > T_s = 36$ s, with the increase of valve closing time, the pressure drop decreases greatly. The reason for this is that, according to the variations of the discharge and speed as shown in Figure 9(b) and 9(c), under $T_g \leq T_s = 36$ s, the pump has reverse speed and reverse flow. On the contrary, under $T_g > T_s = 36$ s, the pump has positive speed and positive flow. The longer the valve closing time, the greater the positive speed and positive flow of the pump. Then, pressure drop behind the pump is smaller.

The speed of the pump, the maximum and minimum pressure curves along the pipeline are shown in Figure 9(c)–9(e), respectively. When the valve closing times are 1.0 s, 18.0 s, 36.0 s, 54.0 s, and 90.0 s, the minimum speeds are $-6.72$ r/min, $-10.01$ r/min, $-11.58$ r/min, $-778.68$ r/min, and $-1241.77$ r/min. Minimum pressures along the pipeline are $-70.6$ m, $-28.6$ m, $-28.0$ m, $-28.0$ m, and maximum pressures along the pipeline are 172.0 m, 172.3 m, 171.6 m, 150.9 m, and 136.8 m. Compared with the closing time of valves less than $T_s$ and greater than $T_s$, respectively, these results show that, when $T_g \leq T_s = 36$ s, with the increase of valve closing time, the minimum pressure along the pipeline increases sharply, but the pump speed and the maximum pressure along the pipeline hardly change. However, when $T_g > T_s = 36$ s, with the increase of valve closing time, the minimum pressure along the pipeline is almost equal to $-28.0$ m. Moreover, the maximum pressure along the pipeline decreases, and the reverse speed of the pump increases, but the reverse speed does not exceed the rated speed of the pump. Based on the above analyses, it can be believed that, for the system with $H_B - H_U - aQ_0/gA < 0$, the pump is safe, and the pressure along the pipeline is the safest under the valve closing Equation (22).

**DISCUSSION**

In this paper, in order to reduce the water hammer pressure caused by pump trip in long-distance pressurized water supply system, the valve closure formulas considering pump runaway characteristics are derived. Then, the formulas are verified using numerical simulation based on practical projects. In this section, the effects of valve closure formulas on water hammer pressure are discussed.

1. The effects of the characteristics of long-distance pressurized water supply system on the pump runaway are compared. When the pump trip occurs, and the system is high lift, $H_B - H_U - aQ_0/gA > 0$, the variations of discharge and speed of the pump are shown in Figure 6(c) and 6(d). At the time $t = 2L/a = 150$ s, the discharge and speed are $-0.2$ m$^3$/s and $-954.3$ r/min. It can be obtained that the pump must be at point a at time $t = 2L/a$, and the four quadrants of the zones of pump operation are the reversed speed dissipation. The pump...
is reverse speed and reverse flow at time $t = 2L/a$. In addition, when the system is low lift, $H_B - H_U - aQ_0/gA < 0$, and the variations of discharge and speed are shown in Figure 8(b) and 8(c). From Figure 8(b) and 8(c), the discharge and speed are 0.09 m$^3$/s and 160.6 r/min at the time $t = 2L/a = 56$ s.
It can be obtained that the pump must be at point b at time $t = 2L/a$, and the pump has positive speed and positive flow at time $t = 2L/a$. The above results are consistent with the theoretical analyses. It can be proved that the theoretical analyses in this paper are reasonable.

2. The effects of valve closure on speed of pump and pressure along the pipeline for the high lift system are compared. Under different valve closure laws, variations of speed, maximum and minimum pressure along the pipeline are shown in Figure 7. Figure 7 indicates that, when the valve closure Equation (21) is adopted, and $T_s = T_d = 1.0$ s, the minimum speed is the largest, and the minimum pressure along the pipeline is the largest, and the maximum pressure is the smallest. They are $-12.11 \text{ r/min}$, $-24.5 \text{ m}$, and $318.7 \text{ m}$, respectively. It can be believed that, for the high lift system, both the pump and the pressure along the pipeline are the safest under the valve closing Equation (21).

3. The effects of valve closure on speed of pump and pressure along the pipeline for the low lift system are compared. Under different valve closure laws, variations of speed, maximum and minimum pressure along the pipeline are shown in Figure 9. From Figure 9, when the closing time is $90.0$ s, compared with the other closure laws, the minimum speed is $-1,241.77 \text{ r/min}$, which does not exceed the rated speed. Moreover, minimum pressure along the pipeline is the largest, which is $-28.0 \text{ m}$, and the maximum pressure along the pipeline is the smallest, which is $136.8 \text{ m}$. Therefore, it can be believed that, for the low lift system, the pump is safe, and the pressure along the pipeline is the safest under the valve closing Equation (22).

4. The effects of the characteristics of pump runaway on the valve type selection are compared. For the high lift system, when the pump trip occurs, the pump has reverse speed and reverse flow at time $t = 2L/a$. From Figure 7, in order to satisfy the safety of pumping and pipelines, the closing time of valves is very short. Therefore, if the system is high lift, a fast closing valve should be selected, such as axial flow check valve. In addition, for the low lift system, the pump has positive speed and positive flow at time $t = 2L/a$. As shown in Figure 9, if the valve closing time is longer than $36.0$ s, the fluctuation of pressure along the pipeline can be reduced. It can be obtained that, if the system is low lift, a slow closing valve should be selected, such as ball valves, butter-fly valves, piston valves.

CONCLUSIONS

For long-distance pressurized water supply systems, in this paper, the valve closure formulas considering the pump runaway characteristics were proposed. Then, the formulas were verified using numerical simulation based on practical projects. The results showed that the formulas agree well with the numerical results. In addition, the pressure along the pipeline, speed of the pump and pressure behind the pump were compared under different valve closure laws. Based on the results, the main conclusions are drawn as follows.

When the system is high lift, and $H_B - H_U - aQ_0/gA > 0$, the four quadrants of the zones of pump operation have reversed speed dissipation at time $t = 2L/a$, and the pump has reverse speed and reverse flow at time $t = 2L/a$. Compared with the other closure laws, the minimum speed and minimum pressure along the pipeline under valve closure Equation (21) are the largest, and the maximum pressure along the pipeline is the smallest. In addition, when the system is low lift, and $H_B - H_U - aQ_0/gA < 0$, the pump has positive speed and positive flow at time $t = 2L/a$. Compared with the other closure laws, when the valve closure Equation (22) is adopted, the minimum speed does not exceed the rated speed, minimum pressures along the pipeline is the largest, and the maximum pressure along the pipeline is the smallest.

This study provides theoretical support for the valve closure law of the long-distance pressurized water supply system. As well, it must be pointed out that valve closure laws in long-distance pressurized water supply systems are only investigated for pump runaway characteristics in this paper. Other factors are not considered, such as, moment of inertia of pump and water hammer protection devices. Therefore, future research is needed regarding the impact of other factors on the valve closure law.
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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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