Spectrum bandwidth narrowing of Thomson scattering X-rays with energy chirped electron beams from laser wakefield acceleration

Tong Xu, Min Chen, Fei-Yu Li, Lu-Le Yu, Zheng-Ming Sheng, and Jie Zhang

1Key Laboratory for Laser Plasmas (Ministry of Education), Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
2SUPA, Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK
3Beijing National Laboratory of Condensed Matter Physics, Institute of Physics, CAS, Beijing 100190, China

We study incoherent Thomson scattering between an ultrashort laser pulse and an electron beam accelerated from a laser wakefield. The energy chirp effects of the accelerated electron beam on the final radiation spectrum bandwidth are investigated. It is found that the scattered X-ray radiation has the minimum spectrum width and highest intensity as electrons are accelerated up to around the dephasing point. Furthermore, it is proposed the electron acceleration process inside the wakefield can be studied by use of 90° Thomson scattering. The dephasing position and beam energy chirp can be deduced from the intensity and bandwidth of the scattered radiation.

PACS numbers: 52.38.Kd, 41.75.Jv, 41.60.-m, 41.60.Ap

Nowadays high bright X-ray sources based on intense laser pulses have attracted a lot of attentions due to their wide applications, relative low cost, easy operation, and unique characters [1]. Compared with traditional accelerator based synchrotron radiation sources these new sources have shorter duration and easier synchronization with lasers, which makes them more flexible for pump-probe technique. Many mechanisms are proposed to generate X-ray radiations. For incoherent sources, radiation based on inner shell electron excitation through laser-solid interaction are extensively studied [2–4]; electron betatron radiation inside a laser wakefield or radiation from laser Thomson scattering through laser gas interactions are also studied [5–8]. For coherent sources, radiation based on high harmonics generation in laser-solid interaction or coherent Thomson scattering from laser nanometer electron sheet interaction are studied [9–11]. The latter has attracted more and more interests recently due to the feasibility of electron sheet generation resulting from laser plasma interactions [12–14].

In this paper, we focus on incoherent radiation from laser electron Thomson scattering. We use the electron beam accelerated from a laser wakefield accelerator [15, 16], which may allow easy synchronization between the electron beams and laser pulses. We show that, due to the energy chirp of the electrons accelerated inside a wakefield, the final spectrum of the scattered radiation can be narrowed compared with a normal un-chirped electron beam. In the meanwhile, we propose to deduce the acceleration process inside a wakefield by diagnosing the radiation spectrum. This provides a new possible approach to detect the wakefield.

For simplicity, here we use a one-dimensional (1D) model for the electron beam accelerated in a wakefield. This gives a proof-of-principle description for Thomson scattering with chirped electron beams. As one knows for the electrons accelerated along the same phase space trajectory in a wakefield whose normalized phase velocity is \( \beta_p = v_p/c \), the relation between its longitudinal momentum \( (p_z) \) and phase position \( \psi \) satisfies: \( p_z = \beta_p \gamma_p^2 [H + \phi(\psi)] \pm \gamma_p [\gamma_p^2 (H + \phi(\psi)^2) - \gamma_p^2]^{1/2} \), where \( H \) is the Hamiltonian along the specific trajectory in the phase space set by the wakefield, \( \phi \) is the potential of the wake and \( \gamma_p = (1 - \beta_p^2)^{-1/2} \). Usually electrons injected at different instant time can get different acceleration length which contributes to the final energy spread of the accelerated electron bunch. Besides this, electrons injected at the same physical position may be put into different trajectories in the phase space, which also results in energy spread to the final beam [17]. Thus the whole beam usually shows energy chirp before accelerating to the dephasing length where the fastest electrons begin to decelerate. To get small energy spread one usually let the electrons be accelerated further a little bit over the dephasing position where both acceleration and deceleration happen to the fast and slow electrons, respectively. At this point, the electrons have relative high energy in the beam center and low energy both in front and at end. Prior to and after this stage, beams show monotonic energy chirp. As shown in the following, the Thomson scattering from these electrons is quite different and the spectrum actually can be used as a diagnostic method to find the dephasing position.

On the other hand, laser Thomson scattering is a well-known process in which electrons oscillate inside a laser field and radiate new electromagnetic field [18]. Normally classical electrodynamics is enough to describe such process once the emitted photon’s energy in the electron rest frame is far less than the electron energy, i.e. quantum recoil effect can be neglected. The radiation spectrum shows synchrotron radiation characters. Depends on the laser intensity the radiation may include high harmonic components or show single peak characters. The normalized laser vector potential \( a = eA_L/m_e c^2 \) acts as the strength parameter of an undulator or wiggler inside a synchrotron facility. The resonant frequency is \( \omega_{rl} = 4\gamma_{eL}^2 n a_2/(1 + a^2/2 + \gamma_{eL}^2) \). Here \( \omega_{rl} \) is the laser frequency and \( \theta \) is the radiation angle related to the longitudinal motion direction of the electron. For values of \( a < 1 \), the laser pulse acts as an undulator and emitted radiation by a single electron will be narrowly peaked about the fundamental angle peak character. The normalized laser vector potential \( \gamma_{eL}^2 = A_L/m_e c^2 \) acts as the strength parameter of an undulator or wiggler inside a synchrotron facility. The resonant frequency is \( \omega_{rl} = 4\gamma_{eL}^2 n a_2/(1 + a^2/2 + \gamma_{eL}^2) \). Here \( \omega_{rl} \) is the laser frequency and \( \theta \) is the radiation angle related to the longitudinal motion direction of the electron. For values of \( a < 1 \), the laser pulse acts as an undulator and emitted radiation by a single electron will be narrowly peaked about the fundamental

\[ eA = eA_L/m_e c^2 \]
resonant frequency $\omega_\ell (n=1)$. As $\ell$ increases, the pulse is more like a wiggle and the emitted radiation will appear at harmonics of the resonant frequency as well ($\omega_{n\ell}$). The final spectrum consists of many closely spaced harmonics. For electrons with different energy ($\gamma_{n\ell}$) inside a bunch these spectrum incoherently superimposed and the final spectrum appears broadband. A continuum of radiation is generated which extends out to a critical frequency, $\omega_c$, beyond which the radiation intensity goes down.

The resonant frequency along the axis ($\theta = 0$) is $\omega_1 = 4\gamma_{n\ell}^2 \omega_{n\ell} / (1 + \alpha^2 / 2)$. Even for a monoenergetic electron bunch (with same $\gamma_{n\ell}$) the radiation spectrum will be broadened due to the different laser intensities ($\alpha$) along the interaction path. Usually for a normal temporally Gaussian pulse [$\alpha \propto \exp(-z^2/\ell^2T^2)$] with $z = z - ct$, the maximum intensity is in the center of the pulse. If one use an electron beam with special energy chirp to make $\gamma_{n\ell}^2(z)[1 + a^2(z)] = \text{constant}$, the emitted radiation will be narrowed. We call such a beam a matched beam with the laser pulse. As discussed above, for the electrons accelerated to the dephasing position inside the laser wakefield, they can automatically match with a normal laser pulse interaction geometry, in which the emitted resonant frequency on axis will be the same during the scattering process with a 180° laser-beam interaction geometry, in which a monoenergetic beam is assumed. These two schemes (electron energy matching or laser frequency matching) share similar ideas. The main difference is that in our scheme a normal laser pulse and a natural accelerated electron beam accelerated from a wake field can be used and the 180° interaction geometry is not necessary.

We use the VDSR code to simulate the electron laser Thomson scattering process. The classical radiation calculation model is used and end-points effects are considered. In the simulation we fix the laser pulse and vary the electron beam properties. The spatial and spectrum distribution of the far field incoherent radiation is recorded by a virtual detector inside the code. The incoherent radiation is calculated by summing the radiation intensity contributed by each single electron.

We first study the 90° Thomson scattering from a laser pulse interaction with a normal un-chirped electron beam. A linearly polarized laser pulse with normalized electric field of $a = eE / m_0 c \propto a_0 \exp(-r^2/W^2 - r^2/T^2)$ propagates along the $-x$ direction, where $a_0 = 0.5$. The pulse is set to be focused at $(x,y,z)=(0,0,0)$. The pulse center is initially at $(60,0,0)$ with the normalization length of $\lambda_0$. Here $T = 20T_0$, $W = 20\lambda_0$, $\omega_0 = 2\pi/T_0$ and $T_0 = 2.67fs$, $\lambda_0 = 0.8\mu m$ are the period and wavelength of the laser, respectively. An electron beam with charge of 10 pC is launched from (0,0,-80). The beam has a cylindrical shape with transverse radius of 0.1$\lambda_0$ and longitudinal size of 40$\lambda_0$. The initial longitudinal momenta of the electrons are sampled as $p_z = \langle p_z \rangle (1 + \alpha \cdot \delta p_z)$ with $\alpha$ a random number uniformly distributed between $[-1,1]$, central momentum $\langle p_z \rangle = 98m_ec$, and momentum spread $\delta p_z = 0.02$. It corresponds to a beam with central energy of 50 MeV and rms energy spread of 1.1%. For simplicity the beam initially has zero transverse emittance ($\delta p_x = 0$). From above parameters we know the centers of the electron beam and laser beam will collide at zero point. Due to Doppler effect, the main radiation from the electron beam will be focused within a cone with half open angle of $1/\gamma$ rad which is around 0.58°. In our code the detector records the radiation intensity $d^2I/d\omega d\Omega$ which is a function of $\omega$, $\theta$, and $\varphi$. According to the scattering parameters, we set the collection angle as: $0° < \theta < 0.8°$ with totally 80 bins and $0 < \omega < 5.0 \times 10^4 \omega_0$ with totally 5000 bins. We fix $\varphi = 0°$ here. The setup of the laser beam interaction is shown in Fig. 1.

FIG. 1: Schematic view of Thomson scattering. The laser pulse is focused at zero point and propagates along the $-x$ direction. The electron beam propagates along $z$ direction.

The final angular and energy distributions of the radiation for this unmatched beam is shown in the left part of Fig. 2. The on axis ($\theta = 0°$) radiation spectrum for this case is also shown in this figure by the right dashed curve. As we see the radiation shows harmonics characters and the first resonant radiation frequency is around $2\gamma^2(1 + a_0^2/2)\omega_0 \approx 1.7 \times 10^4 \omega_0$. It corresponds to a photon energy of 26.46 keV which is close
the spatial coordinates of the electrons in the wake rest frame. Here $v_p$ is the phase speed of the wake, which is close to the group velocity of the driver pulse $v_g \approx c \sqrt{1 - n_e/n_i}$ inside a plasma with density of $n_e/n_i = m_e\omega_0^2/4\pi e^2$ is the critical density for the driver pulse. Electrons between the two vertical lines and along the trajectory are used for the Thomson scattering calculation. Here the slice energy spread of the beam is set to be 0 in the simulations. (a-c) correspond to beams with central position relative to the beam center. The center of the beam is the phase space of the electrons with coordinate of $[0, 100]$. It is easy to see the electrons with coordinate of $[100, 100]$ will feel the laser field as $a = a_0 \exp\left[-(\xi_0 + ct)^2/W^2 - t^2/T^2\right]$. To make a matched condition, one should make the variation of $\gamma^2(\xi_0)/(1 + a^2/2)$ as small as possible for all the electrons with different initial positions ($\xi_0$). To satisfy this in a second simulation the beam’s longitudinal momentum is sampled as $p_z = p_{max} \sqrt{[1 + a^2 \exp(-2\xi_0^2/W^2)/2]/(1 + a_0^2/2)}$ with $p_{max} = 100 m_ec$. The laser and electron parameters except this longitudinal momentum distribution are the same as the previous simulation (shown in Fig. 2 left part). In this case, it corresponds to a beam with maximum energy of 51.1 MeV and rms energy spread of 1.1%. The final radiation spectrum is shown in the right part of Fig. 2. The on-axis radiation spectrum is shown by the solid curve. It shows the radiation peak locates at photon energy of 28.33 keV and the FWHM spread of the radiation spectrum is 1.19 keV. As we can see the radiation spectrum has been narrowed to 38% of the previous spectrum width and on axis radiation intensity has been increased more than 4.1 times. These simple calculations show the importance of a matched beam on the final X-ray radiations.

It should be pointed out that the above optimized scheme needs a very precise synchronization between the laser pulse and the electron beam with an approximately tolerable delay deviation as large as the beam length. For separated laser pulse and beam this is obviously quite difficult. However for electron beam accelerated by laser plasma wakefield it is not a big issue since the accelerated beam is usually just behind the driver laser with a delay length of the wake wavelength. The scattered laser pulse can be split from the driver pulse or easily synchronized with such pulses. On the other hand as we mentioned before, for the electron beam inside a wakefield, the energy chirp always exists. In the following we show the Thomson scattering of a laser wakefield accelerated electron beam with different energy chirps.

To compared with the above simulation results, in the simulations the same laser parameters are used and a trajectory with central position $-5\lambda_0$, 0, and $5\lambda_0$ away from the dephasing position, respectively. (d) shows the on-axis radiation spectra from the electron beams with different central position deviations from the dephasing position.

FIG. 3: Electron beam distribution in the phase space. $\xi = z - v_p t$ is the spatial coordinates of the electrons in the wake rest frame. Here $v_p$ is the phase speed of the wake, which is close to the group velocity of the driver pulse $v_g \approx c \sqrt{1 - n_e/n_i}$ inside a plasma with density of $n_e/n_i = m_e\omega_0^2/4\pi e^2$ is the critical density for the driver pulse. Electrons between the two vertical lines and along the trajectory are used for the Thomson scattering calculation. Here the slice energy spread of the beam is set to be 0 in the simulations. (a-c) correspond to beams with central position relative to the beam center. The center of the beam is the phase space of the electrons with coordinate of $[0, 100]$. It is easy to see the electrons with coordinate of $[100, 100]$ will feel the laser field as $a = a_0 \exp\left[-(\xi_0 + ct)^2/W^2 - t^2/T^2\right]$. To make a matched condition, one should make the variation of $\gamma^2(\xi_0)/(1 + a^2/2)$ as small as possible for all the electrons with different initial positions ($\xi_0$). To satisfy this in a second simulation the beam’s longitudinal momentum is sampled as $p_z = p_{max} \sqrt{[1 + a^2 \exp(-2\xi_0^2/W^2)/2]/(1 + a_0^2/2)}$ with $p_{max} = 100 m_ec$. The laser and electron parameters except this longitudinal momentum distribution are the same as the previous simulation (shown in Fig. 2 left part). In this case, it corresponds to a beam with maximum energy of 51.1 MeV and rms energy spread of 1.1%. The final radiation spectrum is shown in the right part of Fig. 2. The on-axis radiation spectrum is shown by the solid curve. It shows the radiation peak locates at photon energy of 28.33 keV and the FWHM spread of the radiation spectrum is 1.19 keV. As we can see the radiation spectrum has been narrowed to 38% of the previous spectrum width and on axis radiation intensity has been increased more than 4.1 times. These simple calculations show the importance of a matched beam on the final X-ray radiations.

It should be pointed out that the above optimized scheme needs a very precise synchronization between the laser pulse and the electron beam with an approximately tolerable delay deviation as large as the beam length. For separated laser pulse and beam this is obviously quite difficult. However for electron beam accelerated by laser plasma wakefield it is not a big issue since the accelerated beam is usually just behind the driver laser with a delay length of the wake wavelength. The scattered laser pulse can be split from the driver pulse or easily synchronized with such pulses. On the other hand as we mentioned before, for the electron beam inside a wakefield, the energy chirp always exists. In the following we show the Thomson scattering of a laser wakefield accelerated electron beam with different energy chirps.

To compared with the above simulation results, in the simulations the same laser parameters are used and a trajectory with central position $-5\lambda_0$, 0, and $5\lambda_0$ away from the dephasing position, respectively. (d) shows the on-axis radiation spectra from the electron beams with different central position deviations from the dephasing position.

FIG. 3: Electron beam distribution in the phase space. $\xi = z - v_p t$ is the spatial coordinates of the electrons in the wake rest frame. Here $v_p$ is the phase speed of the wake, which is close to the group velocity of the driver pulse $v_g \approx c \sqrt{1 - n_e/n_i}$ inside a plasma with density of $n_e/n_i = m_e\omega_0^2/4\pi e^2$ is the critical density for the driver pulse. Electrons between the two vertical lines and along the trajectory are used for the Thomson scattering calculation. Here the slice energy spread of the beam is set to be 0 in the simulations. (a-c) correspond to beams with central position relative to the beam center. The center of the beam is the phase space of the electrons with coordinate of $[0, 100]$. It is easy to see the electrons with coordinate of $[100, 100]$ will feel the laser field as $a = a_0 \exp\left[-(\xi_0 + ct)^2/W^2 - t^2/T^2\right]$. To make a matched condition, one should make the variation of $\gamma^2(\xi_0)/(1 + a^2/2)$ as small as possible for all the electrons with different initial positions ($\xi_0$). To satisfy this in a second simulation the beam’s longitudinal momentum is sampled as $p_z = p_{max} \sqrt{[1 + a^2 \exp(-2\xi_0^2/W^2)/2]/(1 + a_0^2/2)}$ with $p_{max} = 100 m_ec$. The laser and electron parameters except this longitudinal momentum distribution are the same as the previous simulation (shown in Fig. 2 left part). In this case, it corresponds to a beam with maximum energy of 51.1 MeV and rms energy spread of 1.1%. The final radiation spectrum is shown in the right part of Fig. 2. The on-axis radiation spectrum is shown by the solid curve. It shows the radiation peak locates at photon energy of 28.33 keV and the FWHM spread of the radiation spectrum is 1.19 keV. As we can see the radiation spectrum has been narrowed to 38% of the previous spectrum width and on axis radiation intensity has been increased more than 4.1 times. These simple calculations show the importance of a matched beam on the final X-ray radiations.

It should be pointed out that the above optimized scheme needs a very precise synchronization between the laser pulse and the electron beam with an approximately tolerable delay deviation as large as the beam length. For separated laser pulse and beam this is obviously quite difficult. However for electron beam accelerated by laser plasma wakefield it is not a big issue since the accelerated beam is usually just behind the driver laser with a delay length of the wake wavelength. The scattered laser pulse can be split from the driver pulse or easily synchronized with such pulses. On the other hand as we mentioned before, for the electron beam inside a wakefield, the energy chirp always exists. In the following we show the Thomson scattering of a laser wakefield accelerated electron beam with different energy chirps.

To compared with the above simulation results, in the simulations the same laser parameters are used and a trajectory with central position $-5\lambda_0$, 0, and $5\lambda_0$ away from the dephasing position, respectively. (d) shows the on-axis radiation spectra from the electron beams with different central position deviations from the dephasing position.
The radiation angular distributions for beams with deviation length of 1.5λ₀ and 3.0λ₀ are shown in Fig. 4. Again one can see the radiation is also more focused in space and narrowed in spectra for a matched electron beam. The more deviation from the dephasing point the wider the radiation spectrum.

In summary we studied the Thomson scattering from laser interaction with energy chirped electron beams accelerated from laser wakefield acceleration. By matching the beam energy distribution with the laser pulse intensity, a narrowed spectrum of radiation is obtained. The maximum radiation intensity is about 5 times larger than an unmatched beam. Our scheme maybe used to optimize the Thomson/Compton scattering spectrum in future experiments for laser plasma based X-ray sources [8]. By using 90° Thomson scattering, this method can also be used to detect the electron dephasing position in the LWFA. Radiation from the electrons through betatron radiation inside the wake or scattering from other magnetic or laser undulator has already been used to characterize the electron beams [21, 22]. For other scattering angles (other than 90°), to narrow the radiation spectrum one can use an electron beam with transverse spatial energy chirp or use a laser pulse with transverse spatial chirp. Such laser pulse has already been used to improve the HHG from laser gas interaction [23]. We should mention that the slice energy spread for electron beams inside the phase space has been neglected in our model. This energy spread is due to the injection of the electrons into different trajectories inside the wakefield. Numerous computational studies have proved that usually this spread is far less than the one due to different acceleration length, which corresponds to the energy spread included here. In our simulation we also found once the slice energy spread contribution is less than 10 percent of total energy spread, the spectrum variation results from the mechanism described here is obvious.

This work is supported in part by the National Basic Research Program of China (Grant No. 2013CBA01504), the National Science Foundation of China (Grant No. 11205101, 11121504, 11374209, and 11374210), and the MOST international collaboration project 082013GR0050. MC appreciates supports from Shanghai Science and Technology Commission (Grant No. 13PJ1403600) and National 1000 Youth Talent Project of China. We also appreciate helpful discussions with Dr. Feng Liu. The simulations were carried out on the Π supercomputer in Shanghai Jiao Tong University.

[1] S. Corde, K. Ta Phuoc, G. Lambert, R. Fitour, V. Malka and A. Rousse, Rev. Mod. Phys. 85, 1 (2013) and reference therein.
[2] L.M. Chen, M. Kando, M.H. Xu, et al., Phys. Rev. Lett. 100, 045004 (2008).
[3] A. Levy, F. Dorchies, P. Audebert, et al., Appl. Phys. Lett. 96, 151114 (2010).
[4] N.L. Kugland, C.G. Constantin, P. Neumayer, et al., Appl. Phys. Lett. 92, 241504 (2008).
[5] S. Kneip, et al., Nature Phys. 6, 980 (2010).
[6] S. Cipiccia, M.R. Islam, B. Ersfeld, et al., Nat. Phys. 7, 867 (2011).
[7] M. Schnell, A. Saevert, I. Uschmann, et al., Nat. Commun. 4, 2421 (2012).
[8] S. Chen, N.D. Powers, I. Ghebregziabher, et al. Phys. Rev. Lett. 110, 155003 (2013).
[9] B.H. Shaw, J. van Tilborg, T. Sokollik, et al., J. Appl. Phys. 114, 043106 (2013).
[10] T. Baeva, S. Gordienko, A. Pukhov, Phys. Rev. E 74, 046404.
(2006); R. Lichters, J. Meyer-ter-Vehn, A. Pukhov, Phys. Plasmas 3, 3425 (1996).
[11] H.C. Wu, J. Meyer-ter-Vehn, B.M. Hegelich, Phys. Rev. ST Accel. Beams 14, 070702 (2011).
[12] F.Y. Li, Z.M. Sheng, Y. Liu, J. Meyer-ter-Vehn, et al., Phys. Rev. Lett. 110, 135002 (2013).
[13] H.C. Wu, J. Meyer-ter-Vehn, J. Fernandez, and B.M. Hegelich, Phys. Rev. Lett. 104, 234801 (2010).
[14] Y. Liu, F.Y. Li, M. Zeng, et al., Laser Part. Beams 31, 233 (2013).
[15] T. Tajima and J.M. Dawson, Phys. Rev. Lett. 43, 267 (1979).
[16] E. Esarey, C.B. Schroeder, and W.P. Leemans, Rev. Mod. Phys. 81, 1229 (2009).
[17] M. Chen, E. Esarey, C.B. Schroeder, C.G.R. Geddes, and W.P. Leemans, Phys. Plasmas 19, 033101 (2012); M. Chen, Z.M. Sheng, Y.Y. Ma, J. Zhang, J. Appl. Phy. 99, 056109,(2006).
[18] E. Esarey, B.A. Shadwick, P. Catravas, and W.P. Leemans, Phys. Rev. E 65, 056505 (2002).
[19] I. Ghebregziabher, B.A. Shadwick, and D. Umstadter, Phys. Rev. ST Accel. Beams 16, 030705 (2013).
[20] M. Chen, E. Esarey, C.G.R. Geddes, et al., Phys. Rev. Spec. Topics-Acc. Beams 13, 030701 (2013).
[21] G.R. Plateau, C.G.R. Geddes, D.B. Thorn, et al., Phys. Rev. Lett. 109, 064802 (2012).
[22] S. Corde, C. Thaury, K.T. Phuoc, et al., Plasma Phys. Contr. Fusion 54, 124023 (2012).
[23] K. T. Kim, C. Zhang, T. Ruchon, et al, Nat. Photon. 7,651(2013).