Mathematical modeling of deposition process of charged polymeric particles on moving surface

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Abstract. The deposition of charged polymeric particles in the gravitational and electrostatic fields on a moving surface is considered. A mathematical model is proposed, as a result of a number of simplifications analytical solutions are found. The rational design parameters of the installation and the technological modes of its operation are determined.

1. Introduction
Let us consider a gas suspension – a gaseous medium containing small polymeric particles [1, 2]. Since the shape and sizes of the particles are different we will turn our attention to the simplest case, when it is assumed that the particles of a gas suspension have a spherical shape with an equivalent diameter

\[ d_p = \frac{6 \beta_p V}{\pi \cdot n_p} \]

Here \( V \) is the control volume of the gas suspension, \( \beta_p \) is the volume concentration of powder particles, \( n_p \) is the total number of particles in the control volume.

If the total mass \( M_p \) of \( n_p \) particles is known then the equivalent diameter

\[ d_p = \frac{6 M_p}{\pi \cdot \rho_p \cdot n_p} \]

where \( \rho_p \) – the density of the material of the particles.

2. Governing equations
Further, we assume that the spherical particles moving along a flat surface in the initial vertical section of the gas suspension have a spherical shape with an equivalent diameter.
Since the particles deposition area is long and flat, the medium is low viscous, we can assume, that when it flows along the material surface, the vertical component \(v_g\) will be small \((v_g \approx 0.0)\) compared to the horizontal velocity \(u_g\).

Taking into account the above assumptions, we write the following equation of motion for the reference particle \(K\) which at the initial time \(\tau = 0\) is in the initial section at the height \(Y_{k0}\):

\[
m_p \frac{d\tilde{v}_p}{d\tau} = \tilde{P}_p + \tilde{F}_a + \tilde{F}_e.
\]

Here \(\tau\) – time, \(m_p = 4\pi \rho_p \cdot r_p^3/3\) – the average particles mass, \(r_p = 0.5d_p\) – the equivalent radius, \(\tilde{v}_p = \bar{v}_p(\tau)\) – the current velocity vector of the reference particle, \(P_p = m_p \bar{g}\) – the gravity force, \(\bar{g}\) – free-fall acceleration vector, \(\tilde{F}_a\) – aerodynamic drag force vector of the particle, \(\tilde{F}_e = q_p \bar{E}\) – force caused by the action of the electric field with intensity \(\bar{E}\) on the reference particle with the charge \(q_p\).

It should be noted that in this case the center-of-gravity motion of the particle is considered, its possible rotation, mechanical or electrostatic interaction with the neighboring particles, as well as the non-uniformity of the spatial distribution of intensity are not taken into account.

Assuming that the gravity force acts perpendicular to the processed surface for the \(\tilde{P}_p\) vector components we get:

\[
P_{px} = 0, \quad P_{py} = -m_p g.
\]

\((g\) – free-fall acceleration).

Since the transverse sizes of the particles are small (~ 100 \(\mu m\)) their speed of movement relative to carrier gas medium (air) is small, in the first approximation, we use the Stokes formula to calculate the aerodynamic drag force:

\[
\tilde{F}_a = -6\pi \mu_g r_p \bar{w}_p.
\]

Here \(\mu_g\) – the carrier medium viscosity (for air under normal conditions \(\mu_g = 18.37 \cdot 10^{-6} \text{ kg/(m \cdot s)}\)), \(\bar{w}_p = \bar{v}_p - \bar{v}_g\) – relative particle velocity.

In the simplest case, when the charged particle is small in size its positive charge is considered as a point charge, the intensity in the working space is constant, the equipotential lines (surfaces) are parallel to the \(OX\)-axis, and accordingly, the force lines are parallel to the \(OY\)-axis, the intensity \(E = -dU/dl = \text{const}\),

where \(dU\) – the change in the potential difference of the electrostatic field on the length \(dl\) of the force line.

Hence, the components of the force acting on the single charged particle will be [3-5]:

\[
F_{ex} = 0, \quad F_{ey} = q_p E = -q_p \frac{dU}{dl}.
\]

Then, going over to the dimensionless variables in the equations of motion for the particle and integrating them with the corresponding initial conditions, we find relationships describing this motion:

\[
\zeta = x_p/r_p = \tau - \Re\left(1 - \exp(-\tau/\Re)\right),
\]

\[
\eta = y_p/r_p = G_p \left(-\tau + \Re\left(1 - \exp(-\tau/\Re)\right)\right) + \eta_0.
\]

Here \(\tau = \tau/\tau_p, \quad \tau_p = r_p/u_g, \quad \eta_0 = Y_{kn}/r_p, \quad \Re = (1/6\pi k_{cf})(m_p u_g/\mu_g r_p)\) – the analogue of the Reynolds number \((k_{cf} – \text{ correction factor})\), the parameter \(G_p = (\bar{g} + \bar{q}_p \bar{E})/\Re\) characterizes the collective effect of gravitational and electrostatic forces \(\bar{g} = g r_p/u_g^2, \quad \bar{q}_p \bar{E} = (q_p E) r_p/(m_p u_g^2)\) on the particle.

According to the obtained solutions the dimensionless flight time of the reference particle \(K\) to the processed surface
\[ \tau_f \approx \Re + \eta_0 / G_p , \]
dimensional time
\[ \tau_f \approx \bar{\tau}_f \left( r_p / u_g \right) = \left( 1/6 \pi k_{ef} \right) \left( m_p / \mu_r u_g \right) + Y_{0\theta} / \left( u_g G_p \right) . \]

It can be seen that for all other parameters unchanged the flight time \( \tau_f \) increases with increasing the distance \( Y_{0\theta} \) and with decreasing the gravitational and electrostatic influence.

For a time \( \tau_f \) the reference particle will move in the horizontal direction approximately at a distance:
\[ x_f \approx r_p \left( \bar{\tau}_f - \bar{\Re} \right) = Y_{0\theta} / G_p , \]
where relative to the dimensional values
\[ G_p = \frac{1}{6 \pi k_{ef}} \left( m_p / \mu_r u_g \right) \left( g + q_p E \right) / m_p . \]

Taking into account these relationships, we estimate the deposited particles distribution in the case when the surface being processed moves with the velocity \( u_m \) in the flow direction of the carrier medium for that we select the reference region with the length \( L_0 \) and time \( \tau_n = L_0 / u_m \). It is easy to show that during this time \( N_0 \) particles deposit uniformly on the region \( L_0 \):
\[ N_0 = \tau_n \sum_{K=1}^{K_n} \left( 1 / \left( \kappa_p + l_p K \left( u_g G_p \right) \right) \right) , \]
where \( K_n = H_0 / l_p , H_0 = G_p L_0 \), \( \kappa_p = \left( 1/6 \pi k_{ef} \right) \left( m_p / \mu_r u_g \right) \). We also note that the values \( l_p , H_0 , L_0 \) are essentially the design parameters of the installation for applying the particles.

If the number of particles \( N_0 \leq L_0 / d_p \) then they shall be placed on the material in one layer and for a larger quantity – in several layers.

Provided that the deposited particles are located on the moving surface being processed in one layer we determine the required number of powder particles on the control length \( L_0 \):
\[ N^*_p = L_0 / S_p = 4 \alpha_{\rho} \delta_m L_0 / \pi d_p^2 . \]
Here \( S_p = S_p / \left( \alpha_{\rho0} \delta_m \right) = 0.25 \pi d_p^2 / \left( \alpha_{\rho0} \delta_m \right) \) – the average distance between neighbouring sprayed particles, \( \alpha_{\rho0} \) – the specified parameter characterizing the particles density, \( \delta_m \) – average fabric thickness.

Equating the values of \( N_0 \) and \( N^*_p \) we obtain the formula for estimating time \( \tau_n \):
\[ \tau_n = \alpha_{\rho0} \delta_m L_0 / \left( \pi d_p^2 \sum_{K=1}^{K_n} \left( 1 \left( \kappa_p + l_p K \left( u_g G_p \right) \right) \right) \right) . \]

Assuming that the term \( l_p K / \left( u_g G_p \right) < \kappa_p \), we approximately write:
\[ \sum_{K=1}^{K_n} \frac{1}{\kappa_p + l_p K / \left( u_g G_p \right)} \approx \frac{1}{\kappa_p} \sum_{K=1}^{K_n} \left( 1 - \frac{l_p K}{\kappa_p u_g G_p} \right) = \frac{H_0}{\kappa_p l_p} \left( 1 - 0.5 H_0 / \left( \kappa_p u_g G_p \right) \right) . \]

Hence
\[ \tau_n \approx \alpha_{\rho0} \kappa_p l_p \delta_m L_0 / \left( \pi d_p^2 H_0 \left( 1 - \gamma_p \right) \right) , \]
where
\[ \gamma_p = \gamma_p \left( \kappa_p , u_g , G_p , H_0 \right) = 0.5 H_0 / \left( \kappa_p u_g G_p \right) = 177.5 k_{ef} H_0 \left( \mu_r u_g / m_p \right)^2 \left( g + q_p E / m_p \right) . \]
Therefore, when other parameters are specified the regulated density of particles packing on the processed surface is provided in the case when

\[ u_t = \pi d_p^2 H_0 \left( 1 - \gamma_p \right) / \left( \alpha_p \kappa_p \rho_p \delta_m \right). \]

On the other hand, if the velocity \( u_t \), values of \( \delta_m, H_0 \), and the parameters \( d_p, \rho_p, \kappa_p \) characterizing the particles are given, then:

\[ \gamma_p = 1 - \left( \alpha_p \kappa_p \rho_p \delta_m u_t / \left( \pi d_p^2 \right) \right), \]

\[ \frac{q_p E}{m_p} = -g + 177.5k_{ij}^2 H_0 \left( \frac{u_x r_p}{m_p} \right)^2 / \left( \left( 1 - \left( \alpha_p \kappa_p \rho_p \delta_m u_t / \left( \pi d_p^2 \right) \right) \right) \right). \]

At small \( \alpha_p^* = \alpha_p \kappa_p \rho_p \delta_m u_t / \left( \pi d_p^2 \right) \) the value of \( q_p^* = q_p E / m_p \) describing the electrostatic force acting on the charged particle is approximately equal to

\[ q_p^* = q_p E / m_p \approx 177.5k_{ij}^2 H_0 \left( \frac{u_x r_p}{m_p} \right)^2 - g. \]

In particular, for \( k_{ij} = 1.0, \alpha_p = 0.1, l_p = 5 \cdot 10^{-4} \text{ m}, \delta_m = 2 \cdot 10^{-4} \text{ m}, u_t = 0.1 \text{ m/s}, H_0 = 0.5 \text{ m} \) and the above values of \( u_x, r_p \), the average mass of particles \( m_p = 3 \cdot 10^{-8} \text{ kg}, q_p^* \approx 4.4 \text{ m/s}^2. \)

3. Conclusions
Thus, we obtained dependencies for evaluating the design parameters of the installation for electrostatic application of polymeric particles on the surface being processed, relationships for calculating rational technological regimes of its operation taking into account the existing requirements for the operating characteristics of the deposited layer.

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