Geometric Measurement of Topological Susceptibility on Large Lattices

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The topological susceptibility of the quenched QCD vacuum is measured on large lattices for three $\beta$ values from 6.0 to 6.4. Charges possibly induced by $O(a)$ dislocations are identified and shown to have little effect on the measured susceptibility. As $\beta$ increases, fewer such questionable charges are found. Scaling is checked by examining the ratios of the susceptibility to three physical observables computed in Refs. [1, 2, 3, 4] and $\Lambda_{latt}$.

The topological susceptibility of the QCD vacuum is defined by

$$\chi_t = \langle Q^2 \rangle / V, \quad Q = \frac{-1}{16\pi^2} \int_V d^4x F(x) \tilde{F}(x).$$

(1)

The computation of $\chi_t$ on a lattice is not straightforward. The simplest method, replacing $F \tilde{F}$ by a product of plaquettes, suffers mixing with $F^2$ and a constant, and a large perturbatively determined renormalization factor, but cooling and smearing algorithms have been proposed to refine this method and is not susceptible to multiplicative renormalization but contains additive divergences from compact dislocations so that

$$\chi_t^{measured} = \chi_t + \int_{p_0}^\infty dp C(p) a^p.$$

(3)

The dislocation described by Göckeler et al. produces a power $p_0 < 0$ in (3) and thus a power law divergence in the measured susceptibility as $a \to 0$. The factor $C(p)$ for $p < 0$ is crucial to determining the onset of divergent behavior in the measured susceptibility for finite $a$, and we investigate whether divergent terms affect the result at realistic values of $a$ used in our calculation.

We are examining several aspects of topology on the lattice. First, we compute $\chi_t$ using the geometric method and examine the systematic effect of dislocations in the range $\beta = 6.0$ to $\beta = 6.4$. We plan to study the various methods of measuring $\chi_t$, including cooling, and use the geometric topological charge density to examine the spin asymmetry in deep inelastic scattering off a proton. At present we have completed the first stage and report the results here.

Our lattice ensembles are listed in Table 1. We evaluate the integrals derived by Göckeler et al. on the surfaces of each hypercube on the lattice, obtaining a local integer for each hypercube, and add the local integers in each configuration to obtain the topological charge $Q$. The integration requires a two-stage process. First, all hypercubes are integrated using two different coarse ($\sim 5^3$) cubic meshes. Hypercubes which show no definite evidence of convergence toward zero are integrated more accurately, with $\sim 10^3$ meshes on subcubes. Most hypercubes are brought within $10^{-4}$ to $10^{-5}$ of an integer with the finer mesh, but a few difficult hypercubes require further refinement. The number of difficult hypercubes per unit topological charge decreases as $\beta$ increases.

We attempt to separate spurious charges, caused by low-action dislocations, from physical charges. These dislocations in $SU(2)$ typically contain a central plaquette near $-1$ and have a radius of about 1 lattice spacing. Since the topological structure of $SU(3)$ is determined by

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Figure 1. Normalized nearest neighbor correlations from charged hypercubes to nearest plaquette with $RDC < 0.15$ (Eq. 3), versus $r_{\text{min}}^4$ in lattice units. Highlighted correlations at $r_{\text{min}} < 2a$ indicate possible compact dislocations.

Figure 2. $\chi_1^{1/4}/\sqrt{\sigma}$ (circles), $\chi_t^{1/4}/(100\Lambda_{\text{latt}})$ (crosses), $\chi_t^{1/4}/m_\rho$ (diamonds), and $\chi_t^{1/4}/(10f_\pi)$ (squares) versus $\beta$. Scaling is indicated when the ratios are independent of $\beta$. The actual values of these ratios are unimportant, since we consider only their dependence on $\beta$. The $16^3 \times 40$ lattice results are shown for $\beta = 6.0$.

embedded $SU(2)$ windings, we search for correlations between hypercubes of nonzero charge and plaquettes near the $SU(3)$ cut locus, the analog to the $-1$ group element in $SU(2)$. Plaquettes whose radial distance to cut locus ($RDC$) is less than 0.15 are selected for presentation here. In this definition (4) the $\theta_i$ are the three eigenangles of the $SU(3)$ matrix in ascending order, so that their sum is zero. For each charged hypercube in the ensemble the distance $r_{\text{min}}$ from the center of the hypercube to the center of the nearest selected plaquette is found. The number of occurrences at each lattice distance $r_{\text{min}}$, $N(r_{\text{min}})$ is divided by the weight factor $g(r_{\text{min}})$, the number of available plaquettes at distance $r_{\text{min}}$, to obtain a normalized nearest neighbor correlation $n(r_{\text{min}})$. If the charges and nearly critical plaquettes are decoupled, $n(r_{\text{min}})$ will fall exponentially as a function of volume spanned by $r_{\text{min}}$, $n(r_{\text{min}}) \sim e^{-ar_{\text{min}}^4}$.

As we investigate possible dislocations, we find some encouraging results. Our plot (Fig. 1) shows $n(r_{\text{min}})$ decaying exponentially as a function of $r_{\text{min}}^4$, except for a dramatic rise for $r_{\text{min}} < 2a$, indicating that questionable charges within $2a$ of a nearly critical plaquette may indeed be dominated by compact dislocations which are lattice artifacts. Where $N_q$ is the number of questionable charges in a configuration, we find that $\langle N_q \rangle/\langle Q^2 \rangle = 0.21$ at $\beta = 6.0$, and 0.05 at $\beta = 6.4$. This decrease suggests that for the $\beta$ values considered here the additive term in Eq. 3 is dominated by the $p > 0$ part of the integral, and
the divergent part \( p < 0 \) gives at most a negligible contribution to our measured \( \chi_t \). Also, as \( \beta \) increases, the physical charge is spread over a larger lattice volume and the short range peak of \( \chi_t \) becomes more pronounced relative to the background exponential, allowing a clearer filtering of dislocation-induced charges. Upon removing these charges we find that \( \chi_t \) changes by about 1/3 of the statistical uncertainty for \( \beta = 6.0 \) and only about 1/10 of the uncertainty for \( \beta = 6.4 \). We take this as evidence that low-action dislocation effects are negligible and control finite size effects. We conclude that the geometric method yields \( \chi_t \) about a factor of four higher than both the Witten-Veneziano prediction and the \( \chi_t \approx (180 \text{ MeV})^4 \) obtained by cooling\[ 3 \] and that the superficially divergent part of Eq. (3) is unlikely to cause this discrepancy.

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### Table 1
Lattice Ensembles and Results

| \( \beta \) | Lattice Size | Sample Size | \( \chi_t^{1/4} \) \((\sqrt{\sigma})\) | \( \chi_t^{1/4} \) \((m_\rho)\) |
|---|---|---|---|---|
| 6.0 | \( 16^4 \times 40 \) | 34 | 228(15) | 264(19) |
| 6.0 | \( 24^4 \times 40 \) | 23 | 220(20) | 255(34) |
| 6.2 | \( 32^4 \times 48 \) | 22 | 251(16) | 251(28) |
| 6.4 | \( 32^4 \times 48 \) | 21 | 268(18) | 264(20) |

### Table 2
Lattice Observables (Lattice Units)

| \( \beta \) | \( \sigma^{1/4} a \) | \( m_\rho a \) | \( f_\pi a \) |
|---|---|---|---|
| 6.0 | 0.220(2) | 0.333(8) | 0.056(15) |
| 6.2 | 0.158(1) | 0.277(25) | 0.044(7) |
| 6.4 | 0.119(2) | 0.211(7) | 0.040(8) |