THE LARGE-SCALE CLUSTERING OF MASSIVE DARK MATTER HALOES

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The statistics of peaks of the initial, Gaussian density field can be used to interpret the abundance and clustering of massive dark matter haloes. I discuss some recent theoretical results related to their clustering and its redshift evolution. Predictions from the peak model are qualitatively consistent with measurements of the linear bias of high mass haloes, which also show some evidence for a dependence on the halo mass \( M \) at fixed peak height \( \nu \). The peak approach also predicts a distinctive scale-dependence in the bias of haloes across the baryon acoustic feature, a measurement of which would provide strong support for its validity. For 2\( \sigma \) density peaks collapsing at \( z = 0.3 \), this residual scale-dependent bias is at the 5-10\% level and should thus be within reach of very large simulations of structure formation.

1 Peaks in Gaussian random field

The peak model introduced by \(^1\) assumes that dark matter haloes are associated with peaks of the initial (Gaussian) density field. Although dark matter haloes are the local maxima of the evolved mass distribution, there is a clear correspondence with initial density maxima for massive objects only. In the following, I will focus on the large-scale clustering properties of initial density peaks and show there is nontrivial scale-dependence both in the linear spatial and velocity bias. I will discuss some implications of these results.

2 First order biasing of initial density peaks

Following \(^1\), one usually smooths the initial density fluctuations at redshift \( z_i \gg 1 \) with a filter of characteristic mass scale \( M \) before identifying local maxima of height \( \nu \). Even though density peaks form a well-behaved point process, the large-scale asymptotics of the 2-point correlation and pairwise velocity can be though of as arising from the continuous bias relation \(^6\) \(^\text{[68]}\):

\[
\delta n_{pk}(x) = b_{\nu} \delta_M(x) - b_{\zeta} \Delta \delta_M(x), \\
v_{pk}(x) = v_M(x) - \frac{\sigma_0^2}{\sigma_1^2} \nabla \delta_M(x),
\]

where \( \delta n_{pk} \) and \( v_{pk} \) are the peak count-in-cell density and velocity, \( \delta_M \) and \( v_M \) are the initial mass density and velocity field smoothed on scale \( M \), and the (Lagrangian) bias parameters \( b_{\nu} \) and \( b_{\zeta} \) are

\[
b_{\nu}(\nu, \gamma_1) = \frac{1}{\sigma_0} \left( \frac{\nu - \gamma_1 \bar{u}}{1 - \gamma_1^2} \right), \quad b_{\zeta}(\nu, \gamma_1) = \frac{1}{\sigma_2} \left( \frac{\bar{u} - \gamma_1 \nu}{1 - \gamma_1^2} \right).
\]
Here, $\tilde{u} \equiv \tilde{u}(\nu)$ denotes the mean curvature of peaks of height $\nu$, $\gamma_1(M) = \frac{\sigma_1^2}{\sigma_0 \sigma_2}$ and $\sigma_0$, $\sigma_1$ and $\sigma_2$ are spectral moments which depend upon the shape of the linear mass power spectrum. Note that $b_\zeta$ is strictly positive, whereas $b_\nu$ can be positive or negative. In Fourier space, wavemodes of the peak number density $\delta_{pk}(k)$ can be obtained by multiplying $\delta_M(k)$ with (here and henceforth, I will omit the dependence on $\nu$ and $\gamma_1$ for brevity)

$$b_{pk}(k) = b_\nu + b_\zeta k^2 .$$

This defines the spatial peak bias at the first order. In practice, the peak-background split approach, which is based on count-in-cells statistics, can also be used to estimate $b_{pk}$.

The peak velocity $v_{pk}(\mathbf{x})$ as defined in Eq.(3) is consistent with the assumption that initial density peaks move locally with the dark matter. However, the 3-dimensional velocity dispersion of peaks is smaller than that of the mass $\sigma_1$, i.e. $\sigma_{v_{pk}}^2 = \sigma_1^2 (1 - \gamma_0^2)$ with $\gamma_0 = \sigma_0^2 / \sigma_1 \sigma_2$, because large-scale flows are more likely to be directed towards peaks than to be oriented randomly. As shown in [8], this leads to a $k$-dependence of the peak velocities as can be seen upon taking the divergence of $v_{pk}(\mathbf{x})$ and Fourier transforming it,

$$\theta_{pk}(\mathbf{k}) = \left( 1 - \frac{\sigma_0^2}{\sigma_1^2} k^2 \right) W(k, M) \theta(\mathbf{k}) \equiv b_{vel}(k) \theta_M(\mathbf{k}) ,$$

where $\theta \equiv \nabla \cdot v$ is the mass velocity divergence and $W(k, M)$ is the Fourier transform of the filter. This defines the statistical velocity bias $b_{vel}(k)$. Note that $b_{vel}(k)$ does not depend on $\nu$ and, for the highest peaks, remains scale-dependent even though the spatial bias $b_{pk}(k)$ has no $k$-dependence in this limit.

### 3 Redshift evolution of the peak correlation

Pairwise motions induced by gravitational instabilities will distort the primeval peak correlation. The gravitational evolution of the correlation of initial density peaks can be addressed with the Zel’dovich ansatz [14], assuming they behave like test particles moving with the dark matter. In this first order approximation, the gravitationally-evolved peak correlation $\xi_{pk}(r, z)$ is the Fourier transform of the peak power spectrum [9]

$$P_{pk}(k, z) = G^2(k, z) [b_{vel}(k) + b_{pk}(k, z)]^2 P_M(k, 0) ,$$

where $b_{pk}(k, z) = D(z_i) / D(z) b_{pk}(k)$ and the function

$$G^2(k, z) = \left( \frac{D(z)}{D(0)} \right)^2 e^{-\frac{1}{2} k^2 \sigma_{v_{pk}}^2(z)}$$

is a damping term induced by velocity diffusion. It is similar to the propagator $G_\delta(k, z)$ introduced in [11], although the latter involves the matter velocity dispersion $\sigma_{z-1}$. The first term in the square bracket reflects the fact that peaks stream towards (or move apart from) each other in high (low) density environments, but this effect is $k$-dependent owing to the statistical velocity bias. Therefore, the Eulerian and Lagrangian linear bias parameters are related according to

$$b_\nu^E(z) \equiv 1 + \frac{D(z_i)}{D(z)} b_\nu(z_i), \quad b_\nu^L(z) \equiv \frac{D(z_i)}{D(z)} b_\nu(z_i) - \frac{\sigma_0^2}{\sigma_1^2} .$$

The first relation is the usual formula for the Eulerian, linear scale-independent bias [11]. The second relation shows that $b_\nu^E$ approaches $-\sigma_0^2 / \sigma_1^2$ with time.
4 The large-scale bias of dark matter haloes

The large-scale bias contains important information on the abundance and clustering of biased tracers of the density field. To compare theoretical expectations with measurements of dark matter halo bias, I will assume that peaks of height $\nu = \delta_c/\sigma_0(R,z)$ identified in the initial, smoothed density field $\delta_M$ are associated with objects of mass $M$ collapsing at redshift $z$.

The peak model predicts that, for moderate peak height, $b_E$ is significantly smaller than the value $1+\nu^2/\delta_c$ derived for thresholded regions due to the correlation between the peak height and the peak curvature. However, in the limit $\nu \gg 1$, $b_E(\nu) \approx 1 + (\nu^2 - 3)/\delta_c$ which shows that the evolved linear bias of initial density peaks of height $\nu$ indeed converges towards the prediction of $10$. This should be compared to well-known expressions derived from the extended Press-Schechter formalism which, in the same limit, evaluate to $b_{MW}^E(\nu) = 1 + (\nu^2 - 1)/\delta_c$ and $b_{ST}^E(\nu) \approx 1 + (a\nu^2 - 1)/\delta_c$. In the latter case, $a = 0.75$ follows from normalising the Sheth-Tormen mass function to N-body simulations. Note that, whereas $b_{MW}^E$ and $b_{ST}^E$ depend only upon the peak height, $b_E^E$ is a function of both $\nu$ and $M$ (through $\gamma_1(M)$).

In Fig. 1, these various predictions are compared with measurements of the linear bias of massive haloes extracted from numerical simulations of structure formation. Error bars show the scatter among various realisations. The measured halo bias appears to depart from the Sheth-Tormen scaling at large $\nu$, in agreement with recent measurements of the halo bias. Furthermore, the data shows evidence for a dependence on $M$, but the exact magnitude of the effect is sensitive to the halo finder. Because the best choice of filter is a matter of debate, I treat $\gamma_1$ as a free parameter and show $b_E^E(\nu, \gamma_1)$ for $\gamma_1 = 0.4$ and $0.5$ (a Gaussian filter yields $\gamma_1 \approx 0.65$ for the mass range considered), which provide a reasonably good fit to the bias of $>2\sigma$ haloes. Note that the peak expression $b_E^E$ is also found to match the bias of massive haloes in scale-free cosmologies rather well.

5 Peak biasing and the baryon acoustic oscillation

Having checked that the peak model predicts a large-scale halo bias $b_E(\nu)$ consistent with simulations, I consider now the impact of the scale-dependent piece $b_E^E(\nu)k^2$. The presence of such a term amplifies the contrast of the baryon acoustic oscillation (BAO) in the correlation of initial
density peaks relative to that in the linear theory correlation. Eq. (6) can be used to estimate how much of this effect survives at virialization redshift (a more realistic calculation should include the mode-coupling power).

To emphasise the effect of $b_E(z)k^2$, Fig. 2 compares the redshift evolution of the large-scale, 2-point correlation $\xi_{\text{pk}}$ of initial density peaks (left) with that of “linear tracers”, $\xi_{\text{lt}}$, for which $P_{\text{lt}}(k, z) \equiv G^{2}(k, z) |b_E(z)|^2 P_M(k, 0)$ (middle). The right panel displays the ratio between the two correlations. Results are shown for $2\sigma$ density peaks collapsing at $z_c = 0.3$ and identified on a mass scale $5 \times 10^{13} \, M_\odot/h$ with a Gaussian filter. The relative amplification of the BAO contrast in $\xi_{\text{pk}}(r, z_i)$ induces a scale-dependence in the bias that decays with time owing to the smearing from velocity dispersion. At the collapse redshift however, the model predicts residual scale-dependence across the BAO feature at the 5-10% level (right), a measurement of which in numerical simulations would provide strong support for the validity of the peak approach.

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