Minimum Friction Losses in Planetary Stages of Wind Turbine Gearboxes

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Mechanical efficiency is not a critical design factor of wind turbine gearboxes since wind energy is abundant and costless. The design and sizing of the gears will be determined by the requirements of power density, gear ratio, fatigue safety, and allowable noise and vibration levels. However, reducing power losses results in lower lubricant heating, and therefore, a smaller cooling system is required with lower induced thermal stresses. Reducing friction losses will therefore be beneficial for the mechanical behavior of the gearbox. This paper provides a new method for the calculation of the optimal values of the outside radii and rack shift coefficients of planetary gear stages to minimize the friction power losses, maintaining unalterable geometrical and operating parameters that ensure the preestablished values of the aforementioned critical design factors.

1. Introduction

Wind turbine Gearboxes must be designed to work under severe operating conditions:

(i) High transmitted power at low input velocity, which means high gear ratio, high torque, and high stress levels
(ii) Complicated and expensive maintenance tasks due to difficult access to installations, which requires wide safety margins and high levels of reliability
(iii) Severe environmental and structural requirements related to noise and vibration levels

To meet such severe requirements, high levels of strength, durability, and reliability must be ensured in the final design of the gearbox [1], and therefore, mechanical efficiency is much less important. Since wind energy is overabundant, the energy extracted from the wind will be dependent on the obtained efficiency, but the electric output power supplied by the turbine will be the same (namely, the expected one) in both cases. Nevertheless, friction losses increase the temperature of the teeth surfaces and lubricant. High temperature on the teeth surfaces induces thermal stresses, which may produce cracking failures [2]. High lubricant temperature may result in several failure modes such as wear, scuffing, and pitting [3].

To avoid these problems, it is necessary to install a cooling system to ensure that the oil inlet temperature of the wind turbine gearbox does not exceed a maximum limit (usually 65–70°C). Reduced power losses result in lower lubricant heating, and hence, a smaller cooling system is required with subsequently lower induced thermal stresses. Consequently, although the mechanical efficiency is not a critical design factor of a wind turbine, reducing the power losses, and specifically the friction losses that directly affect the teeth surface temperatures, will be beneficial for the mechanical behavior of the gearbox, provided that such improvements are implemented in accordance with the required standards of safety, durability, and reliability.
For single-meshing gear stages, friction losses are at a minimum when the operating pitch point is located at the midpoint of the contact interval, due to the instantaneous power loss being zero at the pitch point and increasing as the contact point moves away from it. Nevertheless, in double-meshing gear stages, such as planetary gears, it is not possible to fix the pitch point at the midpoint of the contact interval of both sun-planet and planet-ring gear pairs.

Owing to the high gear ratio and power density required by a wind turbine, the gearbox is commonly based on planetary gear sets, primarily for the first stages [4]. Although power losses in planetary gears have recently become a topic of interest for some researchers [4–9], there is significantly more technical literature available related to power losses in parallel axis cylindrical gears [10–24]. The study of the various different components of the power losses in gearboxes [4, 11, 12, 25–27] and the determination of the friction coefficient [13, 24, 28–31] have also been of interest to researchers. However, due to the severe operating conditions of wind turbines, it is not feasible to improve the mechanical efficiency to the detriment of other, more critical design factors, such as tooth strength or load carrying capacity. Consequently, the results of the mentioned studies [4–24] maybe not entirely applicable to wind turbine gearboxes.

Recently, the authors have presented a methodology to optimize mechanical losses in planetary gearboxes [32], which keeps the critical design parameters unchanged in order to guarantee the desired operating conditions. The geometrical parameters of the gearbox (the module, teeth numbers, tooth height, pressure angle, helix angle, face width, and center distance) were not modified, allowing the size and weight to be maintained. Only the variation of the rack shift coefficients was considered in such a way that the corresponding variations of the outside radii provided the same values of the transverse contact ratios of both sun-planet and planet-ring gear pairs. Thus, the critical load points for bending and pitting do not change location with respect to the tooth root, resulting in insignificant variations in stress levels and load carrying capacities. In addition, noise emission, which is influenced by the gear ratios, is also unaffected. The rack shift coefficients were calculated according to two different criteria:

(i) Maintaining constant tooth height
(ii) Maintaining constant radial clearance

The second criteria may induce slight variations in the tooth height, which could subsequently affect the stress levels. However, the considered range of shift coefficients is small, and variations in the determinant stresses are not significant. The geometrical constraints of undercut, pointing, root interference, tooth tip interference, and pitch interference should be met in both sun-planet and planet-ring gearings, which significantly reduces the range of shift coefficients to be considered in the analysis.

A specific multi-MW wind turbine gearbox of Siemens-Gamesa, with two planetary gear stages, was studied. Optimal rack shift coefficients for sun, planets, and ring were obtained, and a small reduction in the friction power losses of around 1% in each stage was achieved [32]. However, the curves of friction power losses versus rack shift coefficient obtained from the analysis seemed to have a local minimum, which could not be reached due to some of the geometric constraints (teeth pointing or interferences) not being met.

The friction power losses do not depend directly on rack shift coefficients but on the outside radii and load sharing. There are various possible combinations of rack shift coefficient and tooth addendum to obtain the same outside radius, but all possible options result in the same friction losses. The differentiating factor is that some meet all geometric constraints whereas others do not. The influence on load sharing is small and neglectable.

In this paper, the power losses of a wind turbine planetary gear stage are studied and optimized as a function of the outside radii of the gears. The analysis is undertaken with the same operating conditions considered for the strength design of the gear stage, which includes the expected critical operating conditions, i.e., the highest transmitted torque, and the required safety margins. These design operating conditions are established according to Standards’ recommendations and Manufacturer’s experience. It is important to highlight that the goal of reducing friction losses is not to improve the efficiency but to reduce the size of the cooling system, which should be sized for the maximum instantaneous power losses. In addition, for any operating conditions, friction losses are proportional to the input load, and therefore, the load does not affect the optimum outside radii for minimum friction losses. In consequence, the result of the analysis will be valid for design, actual, or any other operating conditions.

The initial contact ratios of both sun-planet and planet-ring meshing will not be modified, which results in a single degree of freedom for the analysis. Accordingly, variations of the sun outside radius will be considered since the planets and ring outside radii will be given as a function of that of the sun. All the other geometrical parameters of the gears will be kept unaltered in order to maintain the initial size and weight of the gearbox. In addition, the input velocity, gear ratio, lubricant, and operating temperature will be maintained. Since the gear sizes, velocities, and lubricant viscosities will remain unchanged, all the load-independent power losses will not be affected, and therefore, the objective will be to minimize the friction power losses. Once the outside radii for minimum friction losses have been found, the optimal shift coefficients are calculated to meet the geometrical constraints and ensure the required stress level. The optimization method is applied to the multi-MW wind turbine gearbox of Siemens-Gamesa studied in [32], and additional improvements are discussed.

To compute the frictional losses, the load distribution model of minimum elastic potential energy for external [33–35] and internal [36, 37] gear pairs will be used.
2. Analysis of the Friction Power Losses

The energy lost through friction in a cylindrical helical gear (external or internal) throughout a meshing cycle $W_s$ can be expressed as

$$ W_s = F_T \frac{r_{b1}}{r_{b2}} (r_{b2} \pm r_{b1}) \frac{2\pi}{2} \xi_0 \int_0^{\xi_{max} + 1} \frac{1}{I_K (\xi_0)} \mu K_M (\xi) \frac{2\pi}{\xi} \xi - \tan \alpha_1 \frac{1}{d\xi} d\xi_0, $$

where $F_T$ is the transmitted load, $r_p$ is the base radius, $z$ is the number of teeth, $E_n$ is the integer part of the transverse contact ratio ($\varepsilon_0$), $\varepsilon$ is the axial contact ratio, $E_y$ is the integer part of the total contact ratio ($\varepsilon_y = \varepsilon_0 + \varepsilon$), $\mu$ is the instantaneous friction coefficient, $\alpha_1$ is the operating transverse pressure angle, subscript 1 denotes the pinion, subscript 2 denotes the gear, and the plus/minus signs correspond to external and internal gears, respectively. $\xi$ is the contact point parameter of each point of the line of contact, which is defined as follows:

$$ \xi = \frac{z_1}{2\pi r_{b1}} - 1, $$

$r_{b1}$ being the radius of the contact point on pinion. In (1), $\xi_{max}$ denotes the contact point parameter of the inner point of contact, and $\xi_0$ denotes the contact point parameter of the reference transverse section, which describes the contact position of the tooth pair. $K_M (\xi)$ is the single mesh stiffness of the contact pair in contact at point $\xi$ for unit face width $b$, (i.e., $b = 1$), and $I_K (\xi_0)$ is the integral of the single mesh stiffness along the complete line of contact, at contact point described by $\xi_0$.

According to [33–37], the single mesh stiffness can be accurately described by

$$ I_K (\xi_0) = \frac{K_{Mmax}}{b_0} \sum_{j=0}^{E_n} \left[ \sin \left( b_0 (\xi_{j-high} - \xi_{m}) \right) - \sin \left( b_0 (\xi_{j-low} - \xi_{m}) \right) \right], $$

where $b_0$ is the addendum coefficient, $C^*$ is the center distance, $r_p$ is the standard pitch radius, $m$ is the normal module, $x$ is the rack shift coefficient, and superscript * denotes the fictitious gear.

From (3), the integral of the single mesh stiffness can be expressed as [32, 33]

$$ K_M (\xi) = K_{Mmax} \cos \left( b_0 (\xi - \xi_{m}) \right), $$

where $K_{Mmax}$ is the maximum single mesh stiffness (whose value does not need to be calculated since both $K_M (\xi)$ and $I_K (\xi_0)$ are proportional to it), and parameters $b_0$ and $\xi_{m}$ are given by

$$ b_0 = \left[ \frac{\pi}{2} \left( \kappa_1 + \frac{\varepsilon_0}{2} \right)^2 - \kappa_2 \right]^{-1/2}, \xi_{m} = \xi_{inn} + \frac{\varepsilon_0}{2}, $$

Table 1: Values for coefficients $\kappa_1$ and $\kappa_2$ [35, 37].

|                  | $\kappa_1$ | $\kappa_2$ |
|------------------|------------|------------|
| External gears   | 1.11       | 1.17       |
| Internal gears   | 1.00       | 1.00       |

the relative curvature radius $\rho_C$, and the load $dF$ are all calculated at the pitch point, $\eta_{oil}$ is the

$$ \mu = 0.045 \left( \frac{\varepsilon_0}{bV_C \rho_C} \frac{dF}{d\xi} \right)^{0.2} \eta_{oil}^{-0.05} X_R X_L, $$

where the sum of velocities $V_2$, the relative curvature radius $\rho_C$, and the load $dF$ are all calculated at the pitch point, $\eta_{oil}$ is the
The instantaneous friction coefficient between gear teeth is not uniform along the path of contact [39, 40], but for evaluating the friction power losses an average friction coefficient as given by (7) is sufficient. The variation of the instantaneous friction losses along the path of contact cannot be properly defined with such a constant friction coefficient, but it is evident that these variations have no influence on the power of the cooling system, whose reduction is the objective of this analysis. Nevertheless, it is unable to predict the instantaneous behavior of the tooth pair at any point of the instantaneous line of contact.

The total friction losses $W_{s-T}$ will be the sum of those at the sun-planet meshing $W_{s-SP}$ and those at the planet-ring meshing $W_{s-PR}$, and multiplied by the number of planets $N_p$:

$$W_{s-T} = (W_{s-SP} + W_{s-PR})N_p,$$  

(9)

in which $W_{s-SP}$ and $W_{s-PR}$ are calculated separately, by using Eqsns. (1) to (8). The average power lost through friction will be equal to the total friction energy losses divided by the time corresponding to one meshing cycle:

$$P_{s-T} = \frac{z_o \omega_o}{2\pi} (W_{s-SP} + W_{s-PR})N_p,$$  

(10)

where $\omega$ is the angular velocity, and subscript S denotes the sun gear. Finally, the transmitted load $F_T$ can be expressed as

$$F_T = \frac{P_T}{N_p \omega_o r_{bs}},$$  

(11)

$P_T$ being the transmitted power.

According to this model of stiffness, load distribution, and friction coefficient, the friction losses are proportional to the transmitted load or input torque. Consequently, the optimum outside radii for minimum friction losses can be calculated irrespective of the operating conditions. Nevertheless, the cooling system power should be high enough to evacuate the heat generated under the most severe operating conditions. These determinant operating conditions correspond with maximum transmitted torque and are coincident with the design operating conditions considered for strength calculations. Consequently, the design operating conditions will be considered for the following analysis.

In wind turbine gearboxes, the bending safety factors are more critical than the losses or the efficiency. Consequently, the minimization of friction losses must be carried out in such a way that the values of the determinant tooth-root stresses remain at the same levels or lower. It can be assumed that the determinant stresses will remain approximately uniform if tooth height and transverse contact ratios are maintained. Thereby, the distance between the critical load points and the tooth roots is not modified, and tooth-root stresses will not vary significantly.

To maintain constant transverse contact ratios, the following conditions must be verified:

$$\xi_{oS} + \xi_{oP} = \frac{z_S + z_P}{2\pi} \tan \alpha_{o-SP} + \xi_{a-SP},$$

$$\xi_{oR} - \xi_{oP} = \frac{z_R - z_P}{2\pi} \tan \alpha_{o-PR} - \xi_{a-PR},$$

(12)

where $\xi_z$ is the profile parameter of the point at the outside radius, subscripts S, P, and R denote the sun, planets, and ring, respectively, and subscripts SP and PR denote the sun-planet gear and the planet-ring gear, respectively. This results in a single degree of freedom analysis since only one outside point parameter $\xi_o$ can be chosen, while the other two parameters will be given as a function of the first by (12). If the outside point parameter of the sun is chosen as an independent variable, the parameters of the planets and ring are given by

$$\xi_{oP} = \frac{z_s + z_p}{2\pi} \tan \alpha_{o-PR} + \xi_{a-PR} - \xi_{oS},$$

$$\xi_{oR} = \frac{z_r - z_p}{2\pi} \tan \alpha_{o-PR} + \xi_{a-PR} + \xi_{a-SP} - \xi_{oS}.$$  

(13)

The outside radii of the gears calculated according to (12) and (13) ensure constant contact ratios and, therefore, small variations in the determinant contact and tooth-root stresses. However, even such small variations in stress levels may be unacceptable if safety factors for bending and pitting are low—close to 1—and permitted stresses are exceeded. Accordingly, an evaluation of the variation of the nominal contact and tooth-root stresses will be included in the analysis.

From (1), the instantaneous friction power losses at a specific contact point described by $\xi$ at a specific meshing position described by $\xi_0$ are given by

$$dP_\xi = \omega_1 F_T r_{bs} \left( r_{b1} \pm r_{b0}\right) K_M \frac{\xi_0}{\xi} \frac{2\pi}{\varepsilon_1} \tan \alpha_{o-SP} \frac{1}{\xi_0} d\xi.$$  

(14)

Accordingly, the evolution of the instantaneous power losses at a specific transverse section (which will be described by $\xi = \xi_0 + \Delta$) throughout a complete meshing cycle (described by $\xi_{fin} \leq \xi \leq \xi_{fin} + \Delta$) is defined as

$$\frac{dP_\xi}{d\xi_0} = \omega_1 F_T r_{bs} \left( r_{b1} \pm r_{b0}\right) K_M \frac{\xi_0 + \Delta}{\xi_0} \frac{2\pi}{\varepsilon_1} \left( \xi_0 + \Delta \right) - \tan \alpha_{o-SP} \frac{1}{\xi_0} d\xi.$$  

(15)

Figure 1 presents the evolution of the instantaneous friction power losses of a specific transverse section for external (1(a) chart) and internal (1(b) chart) gear pairs. It can be observed that the instantaneous power losses are zero at the
operating pitch point (in which the relative sliding between surfaces is zero) and increase as the contact point moves away from the pitch point. In fact, as the distance from the contact point to the pitch point increases, the level of relative sliding grows. However, the mesh stiffness decreases with this distance, and therefore, the instantaneous power losses decrease slightly near the limits of the contact interval. From Figure 1, for minimum friction power losses, the pitch point should be located near the midpoint of the contact interval (the exact midpoint of the contact interval for constant tooth height on pinion and wheel, and standard center distance). Accordingly, the analysis of the power losses in the planetary gearbox should be extended to the intervals of the outside radii, which ensures that the pitch point is located inside the contact interval. For the sun-planet pair, this interval is described by

\[
\frac{z_{S}}{2\pi} \tan \alpha_{s-SP}^t \leq \xi_{oS} \leq \frac{z_{S}}{2\pi} \tan \alpha_{s-SP}^t + \epsilon_{a-SP},
\]

(16) which, according to (13), can be expressed in terms of \(\xi_{oS}\) as follows:

\[
\frac{z_{S} + z_{P}}{2\pi} \tan \alpha_{s-SP}^t - \frac{z_{P}}{2\pi} \tan \alpha_{r-PR}^t - \epsilon_{a-PR} + \epsilon_{a-SP} \leq \xi_{oS} \leq \frac{z_{S} + z_{P}}{2\pi} \tan \alpha_{s-SP}^t - \frac{z_{P}}{2\pi} \tan \alpha_{r-SP}^t + \epsilon_{a-SP}.
\]

(18)

The optimal \(\xi_{oS}\) for minimum friction power losses will most likely be located within the intersection of both intervals given by the (16) and (18). However, this intersection may be very small, or even nonexistent. For this reason, the analysis will be extended to the superposition of both intervals.

3. Analysis of Industrial Wind Turbine Planetary Stage

As an example, the first stage of an industrial large-scale wind-turbine gearbox [4] is studied. Geometrical and operating parameters of the planetary gear are given in Table 2. An ISO VG 320 synthetic polyalphaolefin lubricant was chosen. From this specific lubricant and the invariable

| Sun | Planets | Ring |
|----|--------|-----|
| Normal module/mm | 18.000 | 18.000 | 18.000 |
| Normal pressure angle/° | 20.000 | 20.000 | 20.000 |
| Addendum coefficient | 1.000 | 1.000 | 1.000 |
| Dedendum coefficient | 1.439 | 1.439 | 1.260 |
| Tool tip radius coefficient | 0.380 | 0.380 | 0.119 |
| Helix angle/° | 2.000 | 2.000 | 2.000 |
| Operating center distance/mm | 534.610 | 534.610 | 534.610 |
| Number of teeth | 24 | 34 | 92 |
| Rack shift coefficient | 0.4210 | 0.3181 | 1.0571 |
| Effective face width/mm | 568.000 | 568.000 | 568.000 |
| Outside radius/mm | 240.6985 | 328.9005 | 829.5330 |
| Output velocity/rpm | 57.000 | 57.000 | 57.000 |
| Number of planets | 4 | 4 | 4 |
| Transmitted power/MW | 3.000 | 3.000 | 3.000 |
| Outer point parameter (\(\xi_{oP}\)) | 2.4301 | 2.3615 | 2.3615 |
| Transverse contact ratio | 1.4407 | 1.4407 | 1.4407 |
| Total contact ratio | 1.7913 | 1.7913 | 1.7913 |

Figure 1: Instantaneous friction power losses of external (a) and internal (b) helical gear pair.
Figure 2: Scheme of the planetary gear of industrial large-scale wind turbine [4].

Figure 3: Average friction power losses of industrial wind turbine planetary stage: at sun-planet meshing (a), at planet-ring meshing (b), and at the planetary stage (c).
design parameters (such as contact ratios, velocities, curvature radii), the gear loss factor does not change, and the average friction coefficient can be assumed to be constant along the line of contact. A schematic of the planetary gear is shown in Figure 2. From (16) and (18), the intervals of $\xi_{oS}$ for the analysis are given by

$$1.6499 \leq \xi_{oS} \leq 3.0906,$$

$$1.4944 \leq \xi_{oS} \leq 3.0906,$$

and the superposition of both intervals is

$$1.4944 \leq \xi_{oS} \leq 3.0906. \quad (19)$$

Figure 3 presents the curves of average friction power losses at each sun-planet meshing (3(a) chart), at each planet-ring meshing (3(b) chart), and at the planetary gear stage (3(c) chart). The theoretical curves have been obtained from Eqns. (1) to (10), regardless of geometrical constraints of interferences or teeth pointing.

The following conclusions can be drawn from the charts in Figure 3.

(i) Curves of friction power losses at sun-planet meshing and planet-ring meshing have a theoretical minimum for a sun outer point parameter $\xi_{oS}$ close to the midpoint of the considered interval ($\xi_{oS} = 2.3975$ for sun-planet meshing and $\xi_{oS} = 2.2699$ for planet-ring meshing). It should be noted that the midpoint of the intervals ($\xi_{oS} = 2.3703$ for sun-planet meshing and $\xi_{oS} = 2.2925$ for planet-ring meshing) corresponds to the operating pitch point located at the midpoint of the contact interval.

(ii) Friction power losses at sun-planet meshing are significantly higher than those at the planet-ring. Therefor, the sun outer point parameter for minimum losses at the planetary stage ($\xi_{oS} = 2.3755$) is much closer to that for minimum losses at the sun-planet than those at the planet-ring.

(iii) The sun outer point parameter of the planetary stage ($\xi_{oS} = 2.4301$) is far from the optimum one ($\xi_{oS} = 2.3755$). From (1) and (9), the friction power losses could be reduced from 22.746 kW to 22.596 kW, which represents a reduction of 0.66%.

The outer point parameters of the planets $\xi_{oP}$ and ring $\xi_{oR}$ are given by (13) as a function of the sun outer point parameter $\xi_{oS}$. Consequently, the outside radii of the planets and ring are also given as a function of the sun outside radius. All these outside radii should be obtained from the combination of proper values of the respective tooth heights and rack shift coefficients. However, for a given value of the outside radius, there are infinite possible combinations of tooth height and rack shift coefficient. Two widely used criteria to select the rack shift coefficient are to maintain a constant tooth height and a constant radial clearance. For a given outside radius, the rack shift coefficient to maintain the tooth height is given by

$$x = \frac{r_o - r_p}{m} \pm (h_t - h_{ao}), \quad (21)$$

and the rack shift coefficient to maintain the radial clearance is

$$x = \left[\frac{C + r_o'}{m} - r_p \pm (h_{ao} - h_r)\right], \quad (22)$$

in which $r_p$ is the pitch radius, $mh_t$ is the tooth height, $h_{ao}$ is the dedendum coefficient, $r_o'$ is the outside radius of the mating gear, $h_r$ is the radial clearance at the root circle, the upper sign corresponds to external gears, and the lower sign corresponds to internal gears.

Not all rack shift coefficients obtained from (21) and (22) are possible due to the resulting combination of shift coefficient and tooth height being required to meet the restrictions on tooth pointing, root interference, tip interference, and pitch interference. Figure 4 shows the total friction power losses at the planetary stage for rack shift coefficients calculated with a constant tooth height (4(a)) and constant radial clearance (4(b)). The intervals of sun outer point parameter are displayed for both cases. Since the planet rack shift coefficient cannot ensure uniform radial clearance at the sun and ring tooth circles simultaneously, in the latter case, the ring shift coefficient is calculated to...
maintain a constant tooth height, while two additional coefficients are calculated to maintain the radial clearance at both root circles.

From Figure 4(b), the interval of possible values of the sun outer point parameter for maintaining constant radial clearance in sun-planet gearing is

\[ 2.4283 \leq \xi_{os} \leq 2.7429. \quad (23) \]

For smaller values of \( \xi_{os} \), pitch interference occurs at planet-ring contact, while for greater values, the sun teeth are pointed. It can be proved that the outside radii of the sun and planets in Table 2 have been chosen in such a way to maintain the radial clearance equal to the nominal clearance, namely \( h_p = h_{wp} - h_n \), and not to maintain the nominal tooth height, \( h_t = h_{wn} + h_n \). On the contrary, the ring outside radius maintains the nominal tooth height. If it is assumed that calculations were performed with this design criterion (nominal radial clearance for sun-planet meshing and nominal tooth height for ring gear), the analysis of the friction power losses corresponds to Figure 4(b), and the adopted solution (\( \xi_{os} = 2.4301 \)) is almost identical to the best possible, at the inner limit of the interval (\( \xi_{os} = 2.4283 \)). Nevertheless, the theoretical optimal solution cannot be reached from this design criterion.

Figure 4(a) presents the curve of friction power losses for outside radii computed to keep the tooth height. In this case, the interval of possible sun outer point parameters is

\[ 2.2952 \leq \xi_{os} \leq 2.7704. \quad (24) \]

For smaller values of \( \xi_{os} \), planet teeth are pointed, while for greater values, the same problem occurs at the sun teeth. The point of minimum friction power losses (\( \xi_{os} = 2.3755 \)) is contained in this interval, and therefore, the friction power losses can be optimized by selecting the following outside radii, computed from (2) and (13), (21)

\[
\begin{align*}
\xi_{os} &= 2.3755 \Rightarrow r_{os} = 239.152\text{mm} \Rightarrow x_S = 0.3351, \\
\xi_{op} &= 3.0525 \Rightarrow r_{op} = 330.317\text{mm} \Rightarrow x_P = 0.3968, \\
\xi_{or} &= 5.4435 \Rightarrow r_{or} = 830.540\text{mm} \Rightarrow x_P = 1.1131.
\end{align*}
\]

Since the tooth height, loads, and contact ratios are the same as those of the initial design, critical load points and bending moment arms will also be very similar, and therefore, the tooth-root stresses will be reasonably uniform, despite small variations in the tooth thickness at the root due to the variations in rack shift coefficient.

Figure 5 shows the relative variation of the nominal tooth-root stress of the sun \( \sigma_{F0} \) with respect to the initial value \( \sigma_{F0-D} \) according to ISO 6336–3:2019 [42]. It can be observed that the highest load increase, at the inner limit of the interval, is approximately 4.29%, while the increase for minimum friction power losses (\( \xi_{os} = 2.3755 \)) is 1.56%. Increases of nominal contact stress, computed according to ISO 6336–2: 2019 [43], are even smaller: 0.59% at the inner limit of the interval, and 0.83% at the minimum power loss point.

4. Analysis of the Planetary Stages of Multi-MW Wind Turbine Gearbox

A multi-MW wind turbine gearbox of Siemens-Gamesa was analyzed in [32]. It is made up of three stages: two planetary stages at the input and one helical stage at the output. The analysis of the planetary stages was undertaken by studying the influence of the rack shift coefficients on the friction power losses. The outside radii were calculated from the initial values of the transverse contact ratios, considering both the criteria of constant tooth height and constant radial clearance. The results of this analysis showed that the friction power losses could be reduced by 1% at the first stage and less than 0.3% at the second stage. A problem was that the analysis was restricted to the values of the outside radii corresponding to the rack shift coefficients that met all the geometrical restrictions. Consequently, there was no certainty that the point of minimum losses could be reached.

In this section, these specific input planetary stages will be analyzed using an alternative approach in which the tooth height can be modified slightly, allowing the point of minimum friction losses to be approached or reached. It will affect the contact and tooth-root determinant stresses, and therefore their variation will be studied.

The specific Siemens-Gamesa wind turbine is a recent design, currently available for new installations, so all the information, except the nominal power (5 MW) and the three-stage structure, is confidential. Nonetheless, the analysis illustrates the procedure and conveniently describes the application of the method. Numerical results are always given in relation to an initial design (or reference design) of the gearbox, allowing percentage variations of losses and stresses to be known.

4.1. Analysis of the First Planetary Stage. Figure 6 shows the theoretical curve of friction power losses for the first planetary stage, with indication of the interval of validity considering both criteria: constant tooth height (6(a)) and constant radial clearance (6(b)).
It can be observed that the theoretical point of minimum friction power losses corresponds to $\xi_{os} = 2.8606$, which is within the interval of validity for the constant radial clearance criterion. The optimal value of the average friction power losses at this point is 56.456 kW. The initial outside radius of the sun corresponds to $\xi_{os} = 2.7897$, for which the
calculated friction power losses are 57.152 kW. This indicates that the friction power losses can be reduced by 1.22% or 0.696 kW.

Figure 7 shows the relative variation of the nominal tooth-root stress and nominal contact stress of the sun, with respect to their initial values, considering the criteria of constant radial clearance for the rack shift coefficient calculation. It can be observed that, for an outside radius of the sun that is higher than the initial one, the tooth-root stress decreases, while the contact stress remains unaltered. Consequently, if considering the pitting and bending strength, there is no problem in enlarging the sun outside radius, and therefore, it is possible to reach the point of minimum friction power losses, which reduces the losses by 0.696 kW. As seen in Figure 7, for this outside radius, the tooth-root stress is reduced by 1.47%.

4.2. Analysis of the Second Planetary Stage. Figure 8 shows the theoretical curves of friction power losses for the second planetary stage. It can be observed that the theoretical point of minimum friction power losses corresponds to $\xi_{OS} = 2.2561$, and the calculated minimum power losses are 57.155 kW.

In this case, the initial outside radius of the sun corresponds to $\xi_{OS} = 2.2099$, which is outside the validity intervals of both calculation criteria. Calculated power losses are 57.441 kW. The points of the intervals closest to minimum power losses point are the corresponding upper limits, which are described by $\xi_{OS} = 2.2135$ for constant tooth height, and $\xi_{OS} = 2.2242$ for constant radial clearance. The calculated friction power losses are 57.392 kW and 57.270 kW, respectively. Reduction in power losses is 0.049 kW (0.08%) for the first case and 0.171 kW (0.29%) for the second.

This result may be significantly improved if the point of minimum friction power losses could be reached. The friction power losses would be reduced by 0.286 kW (0.50%). The intervals in the charts of Figure 8 show that it is not possible to reach this minimum-losses point by maintaining the tooth height or the radial clearance. However, small variations in the teeth height or radial clearance may be admissible if the determinant contact stress and tooth-root stress were not increased. Figure 9 shows the variation of the nominal contact stress and nominal tooth-root stress with the sun outer point parameter. Once again, the nominal contact stress remains uniform along the interval of validity, while the nominal tooth-root stress decreases slightly with the sun outside radius.

For the rack shift coefficient calculated for constant tooth height (9(a) chart in Figure 9), the sun outside radius for minimum friction power losses $\xi_{OS} = 2.2561$ produces pitch interference at sun-planet meshing and pointed teeth on the sun. For the rack shift coefficient calculated for constant radial clearance (9(b) chart in Figure 9), the sun outside radius for minimum friction power losses produces pointed teeth on the sun. The pitch interference can be avoided by reducing the sum of rack shift coefficients of sun and planets.

The condition of positive pitch clearance can be expressed as follows:

$$\frac{\tan \alpha_{t-Sp} - \alpha_i}{\tan \alpha_i - \alpha_i} - 2 \frac{x_S + x_P}{Z_S + Z_P} \tan \alpha_n \geq 0,$$

$$x_S + x_P \leq \frac{Z_S + Z_P}{2} \left[ (\tan \alpha_{t-Sp} - \alpha_{t-Sp}) - (\tan \alpha_i - \alpha_i) \right],$$

where $\alpha_i$ is the transverse pressure angle, and $\alpha_n$ is the normal pressure angle. The sun teeth pointing can be avoided by increasing the sun rack shift coefficient:

$$\frac{\pi}{Z_S} + 4 \frac{x_S}{Z_S} \tan \alpha_n + 2 \left( \frac{2}{r_{os}} - 1 \right) \tan \frac{2}{r_{os}} \frac{r_{os}}{r_{os}} \geq 0.3m,$$

$$x_S \geq \frac{Z_S}{4} \tan \alpha_n \left[ 0.3m + 2 \left( \frac{2}{r_{os}} - 1 \right) \tan \frac{2}{r_{os}} \right] - 2 (\tan \alpha_i - \alpha_i) - \frac{\pi}{Z_S}.$$

(27)
There is more than one combination of values of \( x_s \) and \( x_p \) that satisfy conditions (26) and (27). It is clear that a smaller sum of rack shift coefficients \((x_s + x_p)\) leads to greater pitch clearance, and therefore thinner mating teeth. Consequently, in order not to weaken the teeth, the value of \((x_s + x_p)\) should be maximized. The largest possible value of \((x_s + x_p)\) is one given by the equal sign in (26):

\[
(x_s + x_p) = \frac{Z_s + Z_p}{2} \tan \alpha_i \left[ (\tan \alpha_i - \alpha_i) - (\tan \alpha_i - \alpha_i) \right].
\]

(28)

The analysis will be undertaken by taking values of \( x_s \) according to (27) and the corresponding values of \( x_p \) according to (28). Figure 10 presents the relative variation of the sun nominal stresses for possible values of \( x_s \), for outside radii for minimum friction power losses, \( \xi_{os} = 2.2561 \). Abscissa axis represents the incremental values of the rack shift coefficient \( \Delta x_s \) with respect to the initial value. From (28), the incremental values of the planet rack shift coefficient are the opposite ones, \( \Delta x_p = -\Delta x_s \).

It can be observed that the sun nominal tooth-root stress decreases with the rack shift coefficient. However, this may not be a good result. As seen in Figures 5–9, the longer the outside radius, the lower the tooth-root stress. As the outside radius increases in length, the tooth-root stress decreases. Similarly, from Figure 10, as the rack shift coefficient increases, the tooth-root stress decreases. Nevertheless, according to (12) and (28), the greater the sun outside radius or rack shift coefficient, the smaller the planet outside radius or rack shift coefficient. Consequently, the lower the tooth-root stress at the sun teeth, the higher the tooth-root stress at the planet teeth.

Slightly lower stress values at the sun root will correspond to slightly higher stress values at the planet root. This can be considered a good result, since the safety factor for bending of the sun is usually lower than that of the planet. However, in the planetary gear stage analyzed in Figure 10, the reduction of the sun tooth-root stress is significant (up to 19%), which results in substantial increases on the planet tooth-root stress (up to 19.5%, in this case).

Since the sun tooth-root stress decreases with \( x_p \), along the entire interval of Figure 10, the optimal solution will be one providing the smallest increase of the planet tooth-root stress. This occurs for the sun shift coefficient corresponding to the inner limit of the interval in Figure 10, namely, \( \Delta x_p = 0.122 \), which results in a sun tooth-root stress 3.8% lower and a planet tooth-root stress 4.1% greater.

This result may be considered acceptable or not, depending on the safety factor for bending strength obtained in the initial design. The safety factor against pitting is frequently smaller (and therefore more critical) than that for bending strength, which is often not very close to the minimum permissible safety factor. This means that the increase of 4.1% in the planet tooth-root stress will be acceptable, and the point of minimum friction power losses can be reached.

Nevertheless, the analysis could be adjusted if this increase of 4.1% in the planet tooth-root stress was not admissible. In fact, for a sun outside radius greater than the initial one \( (\xi_{os} = 2.2099) \) but smaller than that for minimum friction power losses \( (\xi_{os} = 2.2561) \), the minimum \( x_s \) given by (27) will be smaller, which according to (28) will result in a greater \( x_p \) and therefore in a smaller value of the tooth-root stress.

Figure 11 presents the variation of the tooth-root stress on sun and planets with the sun outer point parameter, with adjusted rack shift coefficients.

![Figure 10](image-url)  
**Figure 10:** Relative variation of the nominal tooth-root stress and nominal contact stress with the sun rack shift coefficient, for outside radius for minimum friction power losses.

![Figure 11](image-url)  
**Figure 11:** Relative variation of the nominal tooth-root stress of sun and planets with the sun outer point parameter, with adjusted rack shift coefficients.

### 5. Conclusions

A methodology for the optimization of the friction power losses in the design stage of planetary gears of wind turbine gearboxes has been developed. The aim, better than improve the efficiency, is to reduce the heat generation and accordingly the cooling system size. All the geometrical...
parameters of the initial design are maintained to meet the severe requirements of power density, gear ratio, fatigue safety, and allowable noise and vibration levels for which they were calculated. Only variations of the outside radii have been considered, in such a way that the transverse contact ratios do not change, the critical load points do not move, and the stress levels and load carrying capacities remain practically unchanged. Despite this, relative variations of the determinant contact and tooth-root stresses have been calculated because even small modifications may be unacceptable if corresponding safety factors were close to 1. In addition, as the operating parameters (including velocity, torque, and lubricant) do not change, other power losses (such as windage and churning) also remain unchanged.

Calculations are made considering varying mesh stiffness along the meshing cycle and average value of the friction coefficient, which is considered constant along the instantaneous line of contact but variable along the meshing cycle.

The friction power losses are a function of the outside radii but not of the rack shift coefficients, because there is no single value of the shift coefficient for each specific value of the outside radius. Accordingly, the analysis has been undertaken by studying the influence of the outside radii on the friction power losses and subsequently determining the optimal shift coefficients to achieve the desired outside radii.

It has been found that for planetary gear stages, there are optimal values of the outside radii of sun, planets, and ring, for which the curve of average friction power losses reaches a minimum. If the rack shift coefficients are calculated to maintain constant initial tooth height or initial radial clearance, the optimal outside radii for minimum friction losses result in very small variations in the nominal contact stress (less than 1%) and tooth-root stress (approximately 2%). If the rack shift coefficients for constant tooth height or constant radial clearance do not meet any of the geometrical constraints (typically tooth pointing or pitch interference), equations for suitable values of the shift coefficients have been provided. In this case, the obtained nominal stress for minimum friction losses increases above 4%. If this increase is unacceptable, equations for the rack shift coefficients are provided to reach a compromise between increasing stress and reducing losses.

For three industrial planetary stages analyzed in this paper, reductions in friction power losses of 1.75%, 1.22%, and 0.50% were obtained, with increases in the nominal tooth-root stress of 2.22%, 1.5%, and 4.1%, respectively. If the last increase is unacceptable, it can be reduced to 2% smaller reduction on the power losses of 0.44%.

### Nomenclature

- $C$: Center distance, mm
- $F_T$: Transmitted load, N
- $h_a$: Addendum coefficient
- $h_{tao}$: Tool addendum coefficient
- $h_c$: Radial clearance coefficient
- $h_t$: Tooth height coefficient
- $I_K(\xi_0)$: Total mesh stiffness, N/mm
- $K_M(\xi)$: Single mesh stiffness, N/mm
- $m$: Normal module, mm
- $N_p$: Number of planets
- $P_{a-n}$: Average power losses, N-mm/s
- $P_T$: Transmitted power, N-mm/s
- $r_{bi}$: Base radius, mm
- $r_{ci}$: Radius of the contact point on pinion, mm
- $r_{pi}$: Standard pitch radius, mm
- $W_s$: Energy lost by friction, N-mm
- $x$: Rack shift coefficient
- $z$: Number of teeth
- $a_s$: Transverse pressure angle
- $a_{s+}$: Normal pressure angle
- $a_{s-}':$ Operating transverse pressure angle
- $\epsilon_a$: Transverse contact ratio
- $\epsilon_{g}$: Axial contact ratio
- $\epsilon_{n}$: Total contact ratio
- $\mu$: Friction coefficient
- $\sigma_{f0}$: Nominal tooth-root stress, MPa
- $\sigma_{fH}$: Nominal contact stress, MPa
- $\xi$: Contact point parameter
- $\xi_{inn}$: Inner point of contact parameter
- $\xi_{o}$: Outer point of contact parameter

### Data Availability

The data used to support the findings of this study come from two sources: reference [4] in the paper, and confidential information supplied by Gamesa Energy Transmission (GET).

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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