Radiation drag effects in black hole outflows from super-critical accretion disks via special relativistic radiation magnetohydrodynamics simulations

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Abstract

By performing 2.5-dimensional special relativistic radiation magnetohydrodynamics simulations, we study super-critical accretion disks and the outflows launched via the radiation force. We find that the outflow is accelerated by the radiation flux force, but the radiation drag force prevents the outflow velocity from increasing. The outflow velocity saturates around 30%–40% of the light speed around the rotation axis, since then the flux force balances with the drag force. Our simulations show that the outflow velocity is kept nearly constant in the region of $\dot{M}_{\text{BH}} \sim 10^{2-3} L_{\text{Edd}}/c^2$, where $\dot{M}_{\text{BH}}$ is the mass accretion rate, $L_{\text{Edd}}$ is the Eddington luminosity, and $c$ is the light speed. Such a faster outflow is surrounded by a slower outflow of $\sim 0.1c$. This velocity is also determined by the balance between the radiation flux force and the radiation drag. The radiation drag works to collimate the slower outflow in cooperation with the Lorentz force, although the faster outflow is mainly collimated by the Lorentz force. The kinetic energy is carried by the slower outflow rather than by the faster outflow. The total kinetic luminosity of the outflow as well as the photon luminosity is $\sim L_{\text{Edd}}$, almost independent of the mass accretion rate.

Key words: accretion, accretion disks—black hole physics—magnetohydrodynamics (MHD)—radiation: dynamics

1 Introduction

A black hole accretion disk system is one of the most energetic phenomena in the universe. A mass accretion on to a black hole results in effective release of gravitational energy. According to the mass accretion rate, three accretion modes, i.e., the standard disk model, the radiatively inefficient accretion flow model, and the slim disk model, have been proposed by Shakura and Sunyaev (1973), Ichimaru (1977), Abramowicz et al. (1988), and Narayan and Yi (1994) (see also Kato et al. 1998 for a review).
Whereas the above three models are established by a one-dimensional approach, multi-dimensional hydrodynamic and magnetohydrodynamic (MHD) numerical simulations of the accretion disks have also been performed. In MHD simulations, the phenomenological viscosity model is not used, since the disk viscosity is magnetic in origin, i.e., magnetorotational instability exists (Hawley & Balbus 1991). However, the radiation transfer should be solved in order to investigate the luminous accretion modes.

By performing 2.5-dimensional (2.5D) radiation magnetohydrodynamics (RMHD) simulations, Ohsuga et al. (2009) succeeded for the first time in reproducing three accretion modes by one numerical code (see also Ohsuga & Mineshige 2011). They revealed that the super-critical accretion, of which the mass accretion rate is greater than the critical rate ($L_{E}/c^{2}$), shines at the super-Eddington luminosity, where $L_{E}$ is the Eddington luminosity and $c$ is the light speed. In this case, since the huge amount of photons is mainly released towards the rotation axis of the disks, the radiation force does not prevent the mass accretion along the disk plane (Ohsuga & Mineshige 2007). From the surface of the super-critical accretion disks, powerful jets or outflows are launched by the strong radiation force. Takeuchi, Ohsuga, and Mineshige (2010) showed that the jets are accelerated by the radiation force and collimated by the Lorentz force. This type of jet seems to explain the mildly relativistic, powerful jet from the micro-quasar SS 433. However, their simulations do not fully take account of the relativistic effect, though the maximum velocity of the jet is several 10% of the light speed.

The highly relativistic jets, of which the velocity is close to the light speed, are thought to be associated with the black hole accretion flows; e.g., microquasar GRS 1915+105 (Mirabel & Rodríguez 1994; Fender et al. 1999), active galactic nuclei (Biretta et al. 1995; Giroletti et al. 2012), and gamma-ray bursts (Abdo et al. 2009; Rykoff et al. 2009). The relativistic effects should play important roles in such highly relativistic flows. For instance, the radiation drag force deaccelerates the outflows in contrast to the acceleration via the radiation flux force. Thus, for a non-relativistic approach, the outflow velocity should be overestimated. The relativistic RMHD simulations are required to study the radiatively-driven high-velocity outflows. Recently, special relativistic (SR) (Takahashi et al. 2013; Takahashi & Ohsuga 2013) and general relativistic (GR) (Farrar et al. 2008; Zanotti et al. 2011; Roedig et al. 2012; Sadowski et al. 2013) RMHD codes have been developed, and GR-RMHD simulation of the super-critical disks have been initiated (McKinney et al. 2014; Sadowski et al. 2014, 2015).

In this paper, we perform 2.5D SR-RMHD simulations of the super-critical accretion disks and launching outflows. For the outflows, we investigate the deceleration via the radiation drag as well as the acceleration via the radiation flux force. The terminal velocity is determined by the balance between the above two forces. This paper is organized as follows. In section 2, we introduce basic equations, and describe initial and boundary conditions. We show global inflow-outflow structure and detailed analysis of acceleration/deceleration of the outflow in section 3. Lastly, section 4 is devoted to conclusions and discussion.

2 Basic equations, initial, and boundary conditions

We solve a full set of SR-RMHD equations. Here, the Greek suffixes $\mu$ and $\nu$ take values of 0, 1, 2, and 3, while the Latin suffixes of $i,j,k$ take values of 1, 2, and 3. By taking light speed $c$ as unity hereafter, the basic equations of ideal magnetofluids consist of the mass conservation equation

$$\partial_{\mu}(\rho u^{\mu}) = 0, \quad (1)$$

the energy-momentum conservation,

$$\partial_{\mu} \left[ \left( w_{\mu} + \frac{b_{\mu}^{2}}{4\pi} \right) u^{\nu} u^{\nu} - \frac{b_{\mu} b^{\nu}}{4\pi} + \left( p_{\mu} + \frac{b_{\mu}^{2}}{8\pi} \right) \eta^{\mu\nu} \right] = C_{\tau\nu}^{\mu} + f_{\text{grav}}^{\mu}, \quad (2)$$

and the induction equation,

$$\partial_{\mu}(u^{\mu} b^{\nu} - u^{\nu} b^{\mu}) = 0, \quad (3)$$

where $\rho$ is the proper mass density, $p_{\mu}$ is the gas pressure, $u^{\mu} = [\gamma (1, v^{i})]$ is four velocity, $\gamma = 1/\sqrt{1 - v^{i} v^{i}}$ is the Lorentz factor, and $\eta^{\mu\nu}$ is the Minkowski metric (of which the signature is $[-, +, +, +]$ in the present paper). By supposing a simple $\Gamma$-law polytropic equation of state, the gas enthalpy, $w_{3}$, is given by

$$w_{3} = \rho + \frac{\Gamma}{\Gamma - 1} p_{\mu}, \quad (4)$$

with $\Gamma$ being assumed to be 5/3 throughout the present study. A covariant magnetic field $b^{\mu}$ is related to the magnetic field in laboratory frame, $B^{i}$, as

$$b^{\mu} = \left[ u_{i} B^{i}, B^{j} + (u_{i} B^{j}) u^{i} \right], \quad (5)$$

and an external force is described as $f_{\text{grav}}^{\mu} = -\gamma^{-2} w_{3} (u^{\mu} \mathbf{\nabla} \psi, \mathbf{\nabla} \psi)$, where $\psi = -GM_{\text{BH}}/(r - r_{s})$ is the pseudo-Newtonian potential (Paczynsky & Wiita 1980). Here, $M_{\text{BH}}$ is the black hole mass, $r$ is the distance from the central black hole, and $r_{s}(= 2GM_{\text{BH}})$ is the Schwarzschild radius. In the present paper, we set $M_{\text{BH}}$ to be $10 M_{\odot}$.
The radiation four force, \( G^0_{\text{rad}} \), is given by

\[
G^0_{\text{rad}} = -\rho \kappa \left( 4\pi \gamma B - \gamma E_{\text{rad}} + u_i F^i_{\text{rad}} \right) - \rho \kappa \left[ \gamma (\gamma^2 - 1) E_{\text{rad}} + \gamma u_i u_k P^k_{\text{rad}} - (2\gamma^2 - 1) u_i F^i_{\text{rad}} \right],
\]

and

\[
G^j_{\text{rad}} = -4\pi \rho \kappa a B u^j + \rho (\kappa_a + \kappa_s) (\gamma F^j_{\text{rad}} - u_k P^j_{\text{rad}}) - \rho \kappa_s u^j (\gamma^2 E_{\text{rad}} - 2\gamma u_k F^k_{\text{rad}} + u_i u_j P^l_{\text{rad}}),
\]

where \( \kappa_s = 6.4 \times 10^{22} \rho T_g^{-3.5} \text{ cm}^2 \text{ g}^{-1} \) and \( \kappa_s = 0.4 \text{ cm}^2 \text{ g}^{-1} \) are the Rosseland mean free–free absorption coefficient and the electron scattering coefficient measured in the comoving frame, \( E_{\text{rad}} \) is the radiation energy density, \( F^j_{\text{rad}} \) is the radiation flux, \( P^j_{\text{rad}} \) is the radiation stress tensor, \( B \) is the blackbody intensity, and \( T_g \) is the gas temperature.

The radiation energy density and the radiation flux are solved using the zeroth and first-order moment equations of

\[
\partial_t E_{\text{rad}} + \partial_j F^j_{\text{rad}} = -G^0_{\text{rad}},
\]

and

\[
\partial_t F^j_{\text{rad}} + \partial_i P^{ij}_{\text{rad}} = -G^j_{\text{rad}}.
\]

The blackbody intensity \( B \) is described by gas temperature \( T_g \) as

\[
B = \frac{a_{\text{rad}} T_g^4}{4\pi},
\]

where \( a_{\text{rad}} \) is the radiation constant, and the gas temperature is determined by the Boyle–Charles’s law:

\[
\rho_B = \frac{\rho_B T_g}{\mu m},
\]

where \( \rho_B \) is the Boltzmann constant, \( m_p \) is the proton mass, and \( \mu (= 0.5) \) is a mean molecular weight. As a closure, we adopt M-1 closure in which the Eddington tensor \( D^{ij} \), expressed as

\[
D^{ij} = \frac{1}{2} \chi \delta^{ik} + \frac{3\chi - 1}{2} n' n^k,
\]

where

\[
\chi = \frac{3 + 4|f|^2}{3 + 2\sqrt{4 - 3|f|^2}},
\]

\[
f^j = \frac{F^j_{\text{rad}}}{F_{\text{rad}}},
\]

and

\[
n' = \frac{F^j_{\text{rad}}}{|F_{\text{rad}}|}.
\]

We assume the axisymmetric (\( \partial_\phi = 0 \)) and reflecting boundary at \( \theta = 0, \pi \). At \( \theta = 0 \) and \( \pi \), \( \rho, \rho_B, v', B', E_{\text{rad}}, \) and \( F_{\text{rad}} \) are symmetric, while others are antisymmetric. For the inner (\( r = 2 r_s \)) and outer (\( r = 534 r_s \)) boundaries, we use free boundary conditions and allow for matter and the radiation to go out but not to come in. If the radial components of the velocity and the radiative flux are positive (negative) at the innermost (outermost) grid, they are set to be zero. For the magnetic fields, we also employ the free boundary condition for the tangential components, \( B^\theta \) and \( B^\phi \), while the normal component, \( B^r \), is determined to satisfy \( \nabla \cdot B = 0 \).

We start the simulation with a low-density, non-rotating, and non-magnetized corona surrounding the black hole. The coronal density is given by

\[
\rho = \rho_c \exp \left\{ -\frac{\mu m_p}{k_B T_c} \left[ \psi(r) - \psi(r = 100 r_s) \right] \right\}.
\]

Here \( T_c = 10^{12} \text{ K} \) is the coronal temperature and \( \rho_c = 10^{-3} \text{ g cm}^{-3} \). The radiation temperature, \( T_{\text{rad}} \equiv (E_{\text{rad}}/\rho g)^{1/4} \), is uniform as \( T_{\text{rad}} = 10^5 \text{ K} \).

Following Igumenshchev, Narayan, and Abramowicz (2003) we continuously inject gas inside a torus, which is located on the equatorial plane and surrounds the black hole. The curvature radius of the torus and the radius of the torus tube are \( R_{\text{torus}} = 80 r_s \) and \( r_{\text{torus}} = 15 r_s \), respectively. In the torus, an increment of the gas density per unit time is \( \dot{\rho}_{\text{inj}} = M_{\text{inj}} / (2\pi r_{\text{torus}}^2 R_{\text{torus}}) \), where \( M_{\text{inj}} \) is the mass injection rate and set to be \( 10^7 L_e \). The temperature and the angular momentum of the injected matter are set to be \( 10^{10} \text{ K} \) and be equal to the Keplerian angular momentum at \( r = R_{\text{Kep}} = 60 r_s \). In the torus, we also inject the poloidal magnetic field at each time step, which is given by the increment of the azimuthal component of the vector potential as

\[
\Delta A^\phi = \sqrt{\frac{8\pi \Delta \rho c^2_{s,\text{inj}} r_{\text{torus}} R_{\text{torus}}}{\beta_{\text{inj}}} r \sin \theta} \exp \left[ -\frac{8(r^2 + R_{\text{torus}}^2 + 2r R_{\text{torus}} \sin \theta)}{r_{\text{torus}}^2} \right].
\]

where \( \Delta \rho = \dot{\rho}_{\text{inj}} \Delta t \) and \( c_{s,\text{inj}} \) are the increase of the mass density inside the torus within the time step, \( \Delta t \), and the sound velocity of the injected gas, respectively. We set \( \beta_{\text{inj}} = 100 \). We compute the plasma-\( \beta \) inside the torus at each time step and suspend injection of the magnetic field.
when plasma-$\beta$ is smaller than 30, for numerical stability (Igumenshchev 2008).

We have to note that our simulation has been performed in 2.5-dimension by assuming axisymmetry, so that the anti-dynamo effect works and the magnetic field cannot be maintained (Cowling 1933). So as to avoid this problem, we inject gas which has a poloidal magnetic field, realizing a steady accretion in the present work (see subsection 3.1). Our procedure would not be unnatural, since the magnetized matter is thought to be transported from the outer region to the disks in reality. Also, the phenomenological dynamo model has been employed by other researchers (Bucciantini & Del Zanna 2013; Bugli et al. 2014; Sądowski et al. 2015). 3D simulations, in which the anti-dynamo does not work, are performed by McKinney et al. (2014). The impact of our boundary condition on the resulting accretion flow structure should be verified by comparing results of these different models.

3 Results

3.1 Overview

After simulation starts, a gas is injected in the torus. Since the injected gas is not in dynamical equilibrium and since the gas pressure is larger in the torus than in the corona, the injected gas expands by the gas pressure gradient force at once. This gas falls back towards the equatorial plane and accumulates around $r = R_{Kep}$. Such expansion behavior is remarkable only for the elapse time of $t \lesssim 1$ s. After a few seconds, MHD turbulence develops and gas starts to accrete on to the black hole. The mass accretion rate on to the black hole rapidly increases at $t = 2.4$ s. After that, the gas is continuously swallowed by the black hole at the super-critical rate, $\gg L_E$. Then, the radiation energy is enhanced in the accretion disks since a part of the internal energy of the gas is converted through the free–free emission. The strong radiation force drives the outflows from the disks. At that time, since the torus is embedded in the dense matter, the expansion behavior does not occur. Hence, the incipient expansion does not affect the accretion motion or the launching outflows.

In the left-hand panel of figure 1, we show the distribution of the radiation energy density (color), and the radiation flux normalized by the radiation energy density is denoted by arrows in the $R$-$z$ plane. Here $R(= \sin \theta)$ and $z(= r \cos \theta)$ are horizontal and vertical distances. The right-hand panel indicates the density distribution (color) overlaid with the velocity vectors. Solid and dashed curves in the right-hand panel show $\tau_{\text{tot}} = 1$ and $\tau_{\text{eff}} = 1$, respectively.

We find in the right-hand panel that a geometrically thick disk forms (red). This disk is supported by the strong radiation pressure. Indeed, it is found that the radiation energy density is enhanced in the disk. Since the disk is very optically thick for the electron scattering, most of the radiation energy is trapped inside the accretion disks.

In contrast to $|F_{\text{rad}}| \ll E_{\text{rad}}$ in the disk, the radiation energy is effectively transported above the disk ($z/R \gtrsim 2$) due to small density (small optical depth). Especially, we find $|F_{\text{rad}}| \sim E_{\text{rad}}$ around the rotation axis, meaning that the photons freely move in the vertical direction. Then, the radiation flux force effectively accelerates the gas, leading to the outflows. Such an outflowing motion is clearly shown in the right-hand panel (see velocity vectors above the disk).

We plot in figure 2 a time evolution of the mass accretion rate on to the black hole (thin black curve),

$$\dot{M}_{\text{BH}} \equiv -2\pi (2r_S)^2 \int_{-\pi}^\pi \gamma \rho v \sin \theta \, d\theta,$$  \hspace{1cm} (18)

and the mass outflow rate (thick black curve),

$$\dot{M}_{\text{out}} \equiv 2\pi \int_0^{150r_S} \gamma \rho v^2 H(v' - v_{\text{esc}}) RdR \bigg|_{z=100r_S} - 2\pi \int_0^{150r_S} \gamma \rho v^2 H(v' - v_{\text{esc}}) RdR \bigg|_{z=-300r_S}. \hspace{1cm} (19)$$

Fig. 1. Global structure of radiation-dominated accretion disks and launching outflows at $t = 10$ s. Color shows $E_{\text{rad}}$ and $\rho$, and arrows indicate $F_{\text{rad}}/E_{\text{rad}}$ and $\theta$ for left- and right-hand panels, respectively. Solid and dashed curves in the right-hand panel show $\tau_{\text{tot}} = 1$ and $\tau_{\text{eff}} = 1$, respectively. (Color online)
$H$ is the Heaviside function,

$$H(x) = \begin{cases} 0 & (x < 0) \\ 1 & (x > 0) \end{cases}$$ (20)

and $v_{\text{esc}} = \sqrt{\frac{GM}{r}}$ is the escape velocity. We ignored relativistic effects here since the outflow velocity is only mildly relativistic (see subsection 3.2) and the rest mass is the dominant energy density. The photon luminosity, $L_{\text{ph}}$ (thick red), the kinetic power, $L_{\text{kin}}$ (thick blue), the photon luminosity swallowed by the black hole, $L_{\text{ph,BH}}$ (thin red), and the kinetic power swallowed by the black hole, $L_{\text{kin,BH}}$ (thin blue) are also represented in this figure. These values are evaluated as

$$L_{\text{ph}} \equiv 2\pi \int_0^{150r_s} F_{\text{rad}}^z R dR \bigg|_{z=300r_s} - 2\pi \int_0^{150r_s} F_{\text{rad}}^z R dR \bigg|_{z=-300r_s},$$ (21)

$$L_{\text{kin}} \equiv 2\pi \int_0^{150r_s} \gamma (\gamma w_c - \rho) v^2 H(v' - v_{\text{esc}}) R dR \bigg|_{z=300r_s} - 2\pi \int_0^{150r_s} \gamma (\gamma w_c - \rho) v^2 H(v' - v_{\text{esc}}) R dR \bigg|_{z=-300r_s},$$ (22)

After $t \simeq 3$ s, we find that the mass accretion rate on to the black hole is about $M_{\text{BH}} \simeq 1000 L_E$ and slowly increases with time. Due to the release of the gravitational energy via the mass accretion, the radiation energy is enhanced, driving the outflow from the accretion disk. The outflow carries a large amount of gas at the rate of $\sim 10\%$ of the mass accretion rate, which exceeds the critical rate ($L_E$). The kinetic power and the photon luminosity are almost comparable to the critical rate, i.e., $L_{\text{kin}} \sim L_{\text{ph}} \sim L_E$. In addition, we find $L_{\text{kin}}$ and $L_{\text{ph}}$ are much smaller than $L_{\text{kin,BH}}$ and $L_{\text{ph,BH}}$, respectively. This means that most of kinetic and radiation energies are swallowed by the black hole. Since the huge amount of photons is swallowed by the black hole with accreting gas (photon trapping), the photon luminosity is not sensitive to the mass accretion rate (Begelman 1978; Ohsuga et al. 2002, 2003). This is clearly shown in figure 3, where $L_{\text{ph}}$ (red open circle) as well as $L_{\text{ph,BH}}$ (red filled circle) is plotted as a function of the accretion rate, $M_{\text{BH}}$. In contrast with $L_{\text{ph}}$, we find $L_{\text{ph,BH}}$ increases with an increment of $M_{\text{BH}}$. Such effective photon trapping is a characteristic feature in super-critical accretion disks (Ohsuga et al. 2005; Ohsuga & Mineshige 2007, 2011).
gas is injected at a constant rate. The total mass in the simulation box and the mass accretion rate on to the black hole increases with time. However, the disk is actually in steady state, which is understood from figure 4. In this figure, we plot the mass inflow (\(M_i\), dashed curves) and outflow (\(M_o\), dotted curves) rates as a function of \(r\),

\[
M_i(r) = -2\pi \int \gamma \rho v H(-v') r^2 \sin \theta d\theta, \tag{25}
\]

and

\[
M_o(r) = 2\pi \int \gamma \rho v H(v') r^2 \sin \theta d\theta. \tag{26}
\]

Note that here \(M_i\) is the same as \(M_{BH}\) when we take \(r = 2 r_s\). A total mass flow rate (\(M_i - M_o\)) is also plotted, by solid lines. We can see that the inflow rate is larger than the outflow rate, and the total mass flow rate is almost constant in the region of \(r = [2–30] r_s\). This indicates that the inflow equilibrium is attained, while the mass flow rate slowly increases with time. Thus, the results obtained by our simulation are not transient behavior but the quasi-steady structure for various mass accretion rates, \(M_{BH}\).

Also, Sadowski et al. (2015) showed that the mechanical energy weakly increases with \(M_{BH}\), but it is almost independent of it in our results. Such a discrepancy might be due to the general relativistic effects and/or \(M_{BH}\). In Sadowski et al. (2015), the general relativistic effects are taken into consideration and the mass accretion rate is around \(\leq 100 M_*\). The present simulations are special relativistic versions, and the mass accretion exceeds \(100 M_*\). Detailed study of such a difference is left as an area for important future work.

### 3.2 Outflow properties

Next, we focus on the outflow structure. We hereafter show time-averaged values over \(t = 10 – 11\) s. Figure 5 shows vertical profiles of the outflow velocity \(v^z\) for \(R = 10 r_s\) (black solid line), \(30 r_s\) (black dashed line), and \(60 r_s\) (black dotted line). We can see that the gas is accelerated at the small altitudes, and its velocity finally saturates at the outer region due to the radiation drag (we will discuss this in the next subsection). In particular, the outflow velocity for \(R = 10 r_s\) is proportional to the altitude at \(z \lesssim 80 r_s\), and is kept constant in the region of \(z \gtrsim 80 r_s\). For \(R = 30 r_s\) and \(60 r_s\), the gas is accelerated up to \(z \sim 240 r_s\) and \(\sim 280 r_s\). The terminal outflow velocity is larger near the rotation axis, where the gas is blown away at the speed of \(\sim 0.3\). The velocity for \(R = 60 r_s\) is slightly over \(0.2\) at a maximum.

Such \(R\)-dependence of the outflow velocity at \(z = 300 r_s\) is clearly shown in figure 6 (black solid line). As noted before, the faster outflow \((v^z \gtrsim 0.3)\) is concentrated near the rotation axis (\(R < 30 r_s\)), while slower outflow extends up to \(R \sim 100 r_s\). Its speed is typically \(\sim 0.1–0.2\), which
exceeds the escape velocity (dashed line). This structure is similar to spine-sheath structure (Sol et al. 1989; Meier 2003), in which a faster outflow/jet is surrounded by a slower outflow.

In this figure, we also plot the \( R \)-dependent kinematic power \( (\Delta L_{\text{kin}}, \text{blue}) \) and photon luminosity \( (\Delta L_{\text{ph}}, \text{red}) \). They are assessed as

\[
\Delta L_{\text{kin}} = 2\pi \int_{R-\Delta R/2}^{R+\Delta R/2} \gamma'(\gamma w_g - \rho) v^2 H(v' - v_{\text{esc}}) RdR |_{z=300r_s} 
- 2\pi \int_{R-\Delta R/2}^{R+\Delta R/2} \gamma' w_g v^2 H \times (v' - v_{\text{esc}}) RdR |_{z=300r_s},
\]

and

\[
\Delta L_{\text{ph}} = 2\pi \int_{R-\Delta R/2}^{R+\Delta R/2} F^i_{\text{rad}} RdR |_{z=300r_s} 
- 2\pi \int_{R-\Delta R/2}^{R+\Delta R/2} F^i_{\text{rad}} RdR |_{z=300r_s},
\]

where \( \Delta R = 2 r_s \). We find that the kinetic energy is mainly carried by the slower outflow. The relation of \( \Delta L_{\text{kin}} \propto R \) implies that the kinetic energy flux (the kinetic energy transported per unit surface and per unit time) is almost constant within \( R \sim 40 r_s \). This figure also shows that such a kinetic energy flux is larger near the rotation axis \( (R \lesssim 40 r_s) \) than in the region of \( R \gtrsim 40 r_s \). The profile of \( \Delta L_{\text{ph}} \) is similar to that of \( \Delta L_{\text{kin}} \). Thus, the vertical radiation flux is mildly collimated within \( R \sim 40 r_s \). However, due to a larger opening angle, the kinetic power of the slower outflow is dominant over that of the faster outflow, and the radiation energy is mainly released from the region of \( R \sim 40-100 r_s \).

### 3.3 Radiation drag force

In this subsection, we show effects of the radiation drag in the outflow. Since the flow is quasi-steady, here, we introduce the steady-state equation of motion,

\[
\gamma^2 w_g (v \cdot \nabla) v^j = f^i_{\text{gas}} + f^i_{\text{mag}} + f^i_{\text{grav}} + f^i_{\text{rad}},
\]

where

\[
f^i_{\text{gas}} = -\partial_j p_g,
\]

\[
f^i_{\text{mag}} = \frac{1}{4\pi} [(v \times B)^j \partial_j (v \times B)^i] + \frac{1}{4\pi} [(\nabla \times B) \times B]^i,\]

\[
f^i_{\text{rad}} = f^i_{\text{rad--flux}} + f^i_{\text{rad--drag}} + f^i_{\text{rad--corr}},
\]

are the gas pressure gradient, Lorentz, and radiation forces. The radiation force consists of the following three components: the radiation flux force,

\[
f^i_{\text{rad--flux}} = \gamma \rho (\kappa_g + \kappa_i) F^i_{\text{rad}},
\]

the radiation drag,

\[
f^i_{\text{rad--drag}} = -\gamma \rho (\kappa_g + \kappa_i) \left( E_{\text{rad}} v^j + v_k b^k_{\text{rad}} \right),
\]

and the relativistic correction,

\[
f^i_{\text{rad--corr}} = \gamma \rho (\kappa_g + \kappa_i) (v_j F^j_{\text{rad}}) v^i.
\]

The radiation drag, which is of the order of \( v \), works to slow down the relativistic flow. The relativistic correction is \( O(v^2) \) so that it plays an important role only for highly relativistic flow.

In the top panel of figure 7, a vertical component of the radiation force (orange) and gravitational force (black) are plotted along the vertical lines of \( R = 10 r_s \) (solid) and \( 30 r_s \) (dashed). We can see that the radiation force tends to be weaker than the gravity at small altitude (within the disk), but exceeds the gravity above the disk. The turn-around altitude is around \( 20 r_s \) for \( R = 10 r_s \) and \( 110 r_s \) for \( R = 30 r_s \). Although the gas pressure-gradient force and the Lorentz force are not plotted in this panel, they are much
smaller than the gravity. Therefore, we conclude that the outflows are accelerated by the radiation force. The gas is mainly accelerated just above the turn-around altitude since $f_{\text{rad}}^z + f_{\text{grav}}^z$ peaks there, and the radiative acceleration becomes ineffective at large altitude. Note that the deviation between the radiation force and the gravity is significant near the rotation axis. This is the reason why the faster outflow/jet forms around the axis (see figures 1 and 5).

Why does the radiative acceleration decrease at large altitude? This is due to the radiation drag. The bottom panel of figure 7 shows the vertical profile of $f_{\text{rad}}^z$ (orange), $f_{\text{rad}}^z - f_{\text{drag}}^z$ (red), $-f_{\text{drag}}^z$ (blue) and $f_{\text{corr}}^z$ (green) at $R = 10\,r_s$. It is found that the $f_{\text{rad}}^z$ is much larger than the gravity at large altitude. This implies that the radiation flux force continues to accelerate the gas. However, the radiation drag, which is the downward force, is comparable to the radiation flux force at $z \gtrsim 80\,r_s$. Thus, the radiative acceleration becomes inefficient at the large altitude.

In contrast, the radiation flux force is much larger than the radiation drag force, $f_{\text{rad}}^z - f_{\text{drag}}^z \gg f_{\text{drag}}^z$ just above the turn-around altitude ($z \sim 20\,r_s$), leading to the effective acceleration. Here we note that the outflow velocity is mildly relativistic so that $f_{\text{corr}}^z$ is negligible.

The force balance of $f_{\text{rad}}^z - f_{\text{drag}}^z \sim f_{\text{drag}}^z$ is nearly equivalent to $F_{\text{rad}}^z \sim 0$, where $F_{\text{rad}}^z$ is the radiation flux measured in the comoving frame. This is because that $F_{\text{rad}}^z$ is related to the radiation fields in the laboratory frame as $F_{\text{rad}}^z = F - F_{\text{rad}} u^z - u_j P_{\text{rad}}^{\phi^j}$, and it is rewritten as $(f_{\text{rad}}^z - f_{\text{drag}}^z)(\rho \kappa_i + \kappa_i)$. From this fact, we can obtain the saturation velocity from $F_{\text{rad}}^z = 0$ as

$$v_{\text{sat}} = \frac{F_{\text{rad}}^z}{E_{\text{rad}} + P_{\text{rad}}^{zz}}. \tag{36}$$

Here we assume $|u_j P_{\text{rad}}^{\phi^j} | \gg |u_j P_{\text{rad}}^{zz}|, |v^z P_{\text{rad}}^{zz}|$. In figure 5, we plot the saturation velocity using orange lines. This figure clearly shows $v^z < v_{\text{sat}}$ at $z \lesssim 220$ for $R = 10\,r_s$. In this region, the vertical component of the radiation flux in the comoving frame is positive and the gas is pushed in the vertical direction. We find $v^z \sim v_{\text{sat}}$ above that region, implying that the radiation flux is almost zero in the comoving frame. The radiation force thus cannot accelerate the gas further. In this figure, we can also see that $v^z$ is slightly less than $v_{\text{sat}}$ at $z \lesssim 200\,r_s$ for $30\,r_s$, leading to the weak acceleration of the gas.

This radiation drag plays an important role when $M_{\text{BH}} \gg L_E$. In figure 3, we show a maximum outflow velocity, $v_{\text{max}}^z$ (blue), which is computed at a surface of $|z| = 300\,r_s$ and $R = [0, 150\,r_s]$. We also plot a ratio of saturation velocity and outflow velocity averaged over the same surface for maximum outflow velocity, $v_{\text{sat}} / v_{\text{sat}}^z$ (green). This figure shows that the maximum velocity is about 0.1–0.4, and it is not so sensitive to the mass accretion rate. We also find that $v^z / v_{\text{sat}}$ is very close to unity independently of the mass accretion rate, although we find $v^z / v_{\text{sat}} \lesssim 1$ in the range of $M_{\text{BH}} \lesssim 10^{-2.5}L_E$. It means that the outflow velocity is determined by the balance between the radiation drag and the radiation flux force. The maximum velocity of the radiatively driven outflow/jet from the super-critical accretion disks cannot largely exceed a few 10% of the light velocity.

The saturation velocity given in equation (36) is obtained by assuming that $|u_j P_{\text{rad}}^{\phi^j} | \gg |u_j P_{\text{rad}}^{zz}|, |v^z P_{\text{rad}}^{zz}|$. This assumption comes from the fact that the diagonal terms of the radiation pressure tensor dominates over the off-diagonal terms. This result does not change even if we apply the Eddington approximation instead of M-1 closure. We calculated the off-diagonal terms using the Eddington approximation, and confirmed that the diagonal terms are larger than off-diagonal terms. The difference of the radiation dragging force is less than 1%, implying that the saturation velocity does not change so much. It is, however, noted that the radiation pressure tensor in the Eddington approximation is computed based on $E_{\text{rad}}, F_{\text{rad}}^i$, and $u^j$ obtained from the present simulation. If the numerical simulations are performed with the Eddington approximation, we may obtain different results since the Eddington approximation gives a different radiation flux from the M-1 method (Takahashi & Ohsuga 2013; see also section 4).

Here it seems that the terminal velocity approaches 0.5 in the free streaming limit (e.g., $F_{\text{rad}}^z = E_{\text{rad}}, P_{\text{rad}}^{zz} = E_{\text{rad}}$), according to equation (36). This is because we ignored the relativistic correction, $f_{\text{corr}}^z$. If we take this term into account correctly, and if we assume $F_{\text{rad}} = E_{\text{rad}}$ (and $P_{\text{rad}}^{zz} = E_{\text{rad}}$), the terminal velocity approaches the light speed. Although we have $F_{\text{rad}} < E_{\text{rad}}$ and $v \ll 1$ in the present simulations since the radiation comes from the funnel-shaped photosphere of the disk, $E_{\text{rad}}$ does approach $E_{\text{rad}}$ at very large altitude. But then, the radiation is attenuated and the gas would not be effectively accelerated.

In cooperation with the Lorentz force, the radiation drag also plays an important role for the collimation of the outflow. The top panel of figure 8 shows the horizontal component of the force densities at $z = 300\,r_s$. Black, orange, red, magenta, and blue curves denote the force densities due to the gravitational force, the radiation force, the gas pressure gradient force, the centrifugal force, and the Lorentz force, respectively. In the region of $R \lesssim 100\,r_s$, the gravitational and the radiation and the Lorentz forces are negative (i.e., $-R$ direction), while the gas pressure gradient and centrifugal forces are positive. The sum of $F_{\text{gas}}^R$ and $F_{\text{cent}}^R$ approximately balances with the Lorentz force. This implies that the gas pressure gradient and centrifugal forces work to expand the outflow in the $R$-direction, in contrast to how
found that the outflow is accelerated by the radiation flux force and is subjected to the radiation drag force, which prevents the outflow from speeding up. The outflow velocity is determined by the balance between the above two forces and becomes 30%-40% of the light speed near the rotation axis. Such a velocity does not change very much in the super-critical accretion region of \(M_{\text{BH}} \sim 10^{-2}-10^{3}L_{\text{Edd}}\).

Such a faster outflow is surrounded by a slower outflow, of which the velocity is \(\sim 0.1\). It is similar to so-called spine-sheath structure. It is found that the radiation drag force works to collimate the slower outflow in cooperation with the Lorentz force, though the faster outflow is mainly collimated by the Lorentz force. Most of the kinetic energy is carried by the slower outflow, and the radiation energy is larger in the slower outflow than in the faster outflow. Although the mass accretion rate on to the black hole largely exceeds the critical rate \(L_{c}\), the total photon and kinematic luminosities are comparable to \(L_{c}\), since a huge amount of the radiation energy is trapped and swallowed by the black hole with accreting matter. The swallowed photon luminosity is roughly proportional to the mass accretion rate.

The resulting outflow velocity of \(\sim 0.3-0.4\) nicely agrees with the jets (\(\sim 0.26\)) observed in SS 433 (Marshall et al. 2002). However, it is inconsistent with the highly relativistic jets in GRS 1915+195 (Mirabel & Rodriguez 1994; Fender et al. 1999), or some active galactic nuclei (Biretta et al. 1995; Giroletti et al. 2012). The outflow velocity might increase if the slower outflow is optically thick. This is because the outflow is self-shielded and the spine of the outflow can avoid suffering from the radiation drag. In addition, if the photon bubble instability occurs near the innermost regions of the accretion disks, the outward radiation flux increases, and thus the outflow velocity is larger due to the enhanced radiation flux force (Begelman 1978; Turner et al. 2003). On the other hand, the outflow velocity does not increase even if the pair-plasma appears in the outflow, since the radiation drag acts on the positron as well as the electron.

We included only opacities due to the electron scattering and free–free absorption. Recently, Kawashima et al. (2009) performed non-relativistic radiation hydrodynamic simulations including thermal Comptonization. They showed that although the Compton effect does not impact on the disk structure very much, the gas temperature of the outflow drastically decreases due to the Compton up-scattering. A decrease in the gas temperature leads to softening of the X-ray spectrum (Kawashima et al. 2012). We need to include the Comptonization in relativistic radiation MHD simulation (Sadowski et al. 2015) to explain the spectral properties observed in microquasars.

4 Conclusions and discussion

We performed 2.5D special relativistic radiation magnetohydrodynamics simulations to study the super-critical accretion disks and the radiatively driven outflows. We
Although we assume the axisymmetry in the present study, three-dimensional study is required in order to estimate the outflow velocity more precisely. If the density of the disk fluctuates in the azimuthal direction, the photon-trapping effect might degrade since the photon can escape from the less dense region. Then, the radiation fields change and the outflow velocity alters. The dynamo would change the evolution of the magnetic fields, leading to the change of the disk structure. The saturation velocity in such a case might differ from that of the present study. The 3D simulation has been initiated in the Newtonian limit (Jiang et al. 2014a) and in general relativity (McKinney et al. 2014). Jiang, Stone, and Davis (2014a) performed a 3D simulation of slim disks by solving a full radiation transfer equation while keeping terms of \(O(v)\). They found that the outflow velocity is about 0.3, which is consistent with our results. Comparing results in detail between these different models is also very important future work to understand the validity of the closure relation.

The light bending should decrease the photon luminosity, since more photons are swallowed by the black hole. Then, the kinetic luminosity of the outflow would go down. In contrast, the spin of the black hole is thought to enhance the outflow (e.g., Blandford & Znajek 1977). Recently, GR simulations have been revealed that indicate that the strong outflow is generated from the super-critical accretion disks around a Kerr black hole (Sadowski et al. 2014; McKinney et al. 2014).

Finally, we discuss the difference between our simulations and previous non-relativistic simulations. Takeuchi, Ohsuga, and Mineshige (2010) performed a non-relativistic radiation MHD simulation with the flux-limited diffusion approximation (FLD: Levermore & Pomraning 1981), reporting that the outflow velocity is about 0.6–0.7, which is faster than our results (∼0.4). Such a difference would be caused by the radiation drag effect, which is not taken into account in Takeuchi, Ohsuga, and Mineshige (2010). Thus, we stress that it is important to include the radiation drag effects to study acceleration mechanisms of outflows from black hole accretion disks.

The different algorithms make differences especially in the outflow regions, since the approximate radiative transfer algorithms (e.g., FLD approximation, the Eddington approximation, and the M-1 method) are known to be problematic in the optically thin region and, in contrast, give accurate radiation fields in the optically thick diffusion limit. The FLD approximation cannot be properly applied to the relativistic fluid since it violates a causality (e.g., the radiation energy equation becomes parabolic). On the other hand, the Eddington approximation is utilisable in relativistic simulations. In the relativistic one-dimensional test problems, Takahashi and Ohsuga (2013) showed that \(F_{\text{rad}}/F'_{\text{rad}}\) becomes larger for the M-1 treatment than for the Eddington approximation. Thus, it is expected that the terminal velocity becomes slower if the Eddington approximation is adopted. However, the performing simulation with the Eddington approximation is time-consuming since \(6 \times 6\) matrix inversion at each grid point is needed to compute the Eddington tensor. Hence, the detailed comparison between Eddington approximation and M-1 closure is beyond the scope of the present paper and is left as an important future work. In addition, recently, a more accurate method, which solves radiative transfer equation, has been proposed by Jiang, Stone, and Davis (2014b). Comparing results between these different models is also very important future work to understand the validity of the closure relation.

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References

Abdo, A. A., et al. 2009, Science, 323, 1688
Abramowicz, M. A., Czerny, B., Lasota, J. P., & Szuszkiewicz, E. 1988, ApJ, 332, 646
Begelman, M. C. 1978, MNRAS, 184, 53
Biretta, J. A., Zhou, F., & Owen, F. N. 1995, ApJ, 447, 582
Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433
Bucciantini, N., & Del Zanna, L. 2013, MNRAS, 428, 71
Bugli, M., Del Zanna, L., & Bucciantini, N. 2014, MNRAS, 440, L41
Cowling, T. G. 1933, MNRAS, 94, 39
Farris, B. D., Li, T. K., Liu, Y. T., & Shapiro, S. L. 2008, Phys. Rev. D, 78, 024023
Fender, R. P., GArrington, S. T., McKay, D. J., Muxlow, T. W. B., Pooley, G. G., Spencer, R. E., Stirling, A. M., & Waltman, E. B. 1999, MNRAS, 304, 865
Giroletti, M., et al. 2012, A&A, 538, L10
Hayw, J. F., & Balbus, S. A. 1991, ApJ, 376, 223
Ichimaru, S. 1977, ApJ, 214, 840
Igumenshchev, I. V. 2008, ApJ, 677, 317
Igumenshchev, I. V., Narayan, R., & Abramowicz, M. A. 2003, ApJ, 592, 1042
Jiang, Y.-F., Stone, J. M., & Davis, S. W. 2014a, ApJ, 796, 106
Jiang, Y.-F., Stone, J. M., & Davis, S. W. 2014b, ApJS, 213, 7
