Double-Lepton Polarization Asymmetries in $B \rightarrow K_1 l^+ l^-$
Decay in Universal Extra Dimension Model

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Abstract

Double-lepton polarization asymmetries for the exclusive decay $B \rightarrow K_1 l^+ l^-$ in the Universal Extra Dimension (UED) Model is studied. It is obtained that double-lepton polarization asymmetries are very sensitive to the UED model parameters. Experimental measurements of double lepton polarizations can give valuable information on the physics beyond the Standard Model (SM).

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1 Introduction

The rare B-meson decays pointed out by the flavor-changing neutral currents (FCNC) have been significant channels for acquiring knowledge on the SM parameter and analyzing the new physics predictions. Rare B meson decays are not allowed at the tree level in the SM and seem at loop level. By rare B decays, one generally comprehend Cabibbo-suppressed $b \to u$ transitions or flavour-changing neutral currents (FCNC) $b \to s$ or $b \to d$. So rare decays are significant testing basic of the SM and take an important part in the search for new physics. The examinations of different FCNC processes can be used to determine different fundamental parameters of SM like elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, various decay constants etc. Between testing SM the FCNC processes can be very important for discovering indirect effects of possible TeV scale extensions of SM. Therefore, we examine $b \to q (q = d, s)$ transitions in terms of an effective Hamiltonian. For observing to the new physics in these decays, there are two different ways. First of all, the differences in the Wilson coefficients form the ones existing in the SM. And the second one the new operator in the effective Hamiltonian which are absent in the SM. All decay channels of B meson include many physically quantities which are very useful testing for the SM and investigating for new physics beyond the SM. Exclusive processes such as $B \to K(K^*)l^+l^-$ and $B \to \gamma l^+l^-$ decays [4, 5, 6, 7, 8] have been studied extensive in literature. Colangelo et al. have studied $B \to K(K^*)l^+l^-$ decays in framework of one Universal Extra Dimension model (ACD), proposed in the Ref. [16] and analyzed the branching ratio and forward-backward asymmetry. In meanwhile, in the Ref. [14] the single lepton polarizations is studied for $\mu$ for the $B \to K_1 l^+l^-$ decay in UED model. The Branching ratios (BR) of the Semileptonic decays $B(B \to K^* l^+l^-) = 7.8 \pm 1.2 \times 10^{-7}$ [2] and $B(B \to K l^+l^-) = 5.5 \pm 0.02 \times 10^{-7}$ [11] have been measured by BELLE [11] and BaBar [2] collaborations. It is noted that the measurement of the polarization of the $b \to s$ decay can provide important information about more observables. Some of the single lepton polarization asymmetries can be too small to be observed. Since it might not provide number of observables for control the structure of the effective Hamiltonian, we calculate to double lepton polarization for more observables [9]. Among the different models of physics beyond the SM, extra dimensions is very interesting models. Since the extra dimension model contain of gravity, they give to clue on the hierarchy problem and a connection with string theory. The model of Appelquist, Cheng and Dobrescu (ACD) [10, 11, 20] with one universal extra dimension (UED), where all the SM particles can propagate in the extra dimension. Compactification of the extra dimension leads to Kaluza-Klein model in the four-dimension. In the extra dimension model, we have extra free parameter is $1/R$, which is inverse of the compactification radius. With the aid of $1/R$, we can determined all the masses of the KK particles and their interactions with SM particles. In the meanwhile, If we have not tree level contribution of KK states to the low energy processes, KK parity is conservation in ACD model at scale $\mu \ll 1/R$.

In this work, we study the double-lepton polarization asymmetries for the $B \to K_1 l^+l^-$ decay in the UED model. In section 2, we shortly examine ACD model. In section 3, we obtain matrix element for the $B \to K_1 l^+l^-$ decay. In section 4, Double lepton polarization for the $B \to K_1 l^+l^-$ decay are calculated. Section 5 is devoted to the numerical analysis and discussion of our results.
\section{\(B \to K_1 l^+l^-\) Decay in ACD Model}

Before calculation of the double lepton polarizations few words about the ACD model. This model is the minimal extension of the SM to the \(4 + \delta\) dimensions. We consider simple case which is \(\delta = 1\). In the universe, we have 3 space + 1 time dimensions and one possibility is the propagation of gravity in whole ordinary plus extra dimensional universe. The five-dimensional ACD model with a single UED uses orbifold compactification, the fifth dimension \(y\) that is compactified in a circle of radius \(R\), with points \(y = 0\) and \(y = \pi R\) that are fixed points of the orbifolds \([11,12,13,14]\). The Lagrangian in ACD model can be written as:

\[
\mathcal{L} = \int d^4x dy \{ \mathcal{L}_A + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y \}
\]

where

\[
\begin{align*}
\mathcal{L}_A &= -\frac{1}{4} W^{MN \alpha} W_{MN}^\alpha - \frac{1}{4} B^{MN} B_{MN} \\
\mathcal{L}_H &= (D^M \phi) \phi^\dagger - V(\phi) \\
\mathcal{L}_F &= \bar{Q}(i \Gamma^M D_M) Q + \bar{u}(i \Gamma^M D_M) u + \bar{D}(i \Gamma^M D_M) D \\
\mathcal{L}_Y &= -\bar{Q} \tilde{Y}_\alpha \phi u - \bar{Q} \tilde{Y}_\alpha \phi d + h.c.
\end{align*}
\]

where \(M\) and \(N\) are the five-dimensional Lorentz indices which can run from 0, 1, 2, 3, 5. \(W_{MN}^\alpha = \partial_M W_N^\alpha - \partial_N W_M^\alpha + \tilde{g} \varepsilon^{abc} W_M^b W_N^c\) are the field strength tensor for the \(SU(2)_L\) electroweak group, \(B_{MN} = \partial_M B_N - \partial_N B_M\) are that of the \(U(1)\) group. \(D_M = \partial_M - i \tilde{g} W_M^a T^a - i \tilde{g}' B_M Y\) is the covariant derivative, where \(\tilde{g}\) and \(\tilde{g}'\) are the five-dimensional gauge couplings for the \(SU(2)_L\) and \(U(1)\) groups. \(\Gamma^M\) are five-dimensional matrices which is \(\Gamma^\mu = \gamma^\mu, \mu = 0, 1, 2, 3\) and \(\Gamma^5 = \gamma^5\). \(F(x_t, y)\) is the periodic function of \(y\) which is \(1/R\). It can be written as follow:

\[
F(x_t, y) = F_0(x_t) + \sum_{n=1}^{+\infty} F_n(x_t, x_n)
\]

where \(x_t = m_t^2/m_w^2\), \(x_n = m_n^2/m_w^2\) and \(m_n = n/R\). These function can be found in [10,15].

\section{Effective Hamiltonian for \(B \to K_1 l^+l^-\) Decay}

At quark level, the exclusive \(B \to K_1 l^+l^-\) decay is described by \(b \to sl^+l^-\) transition governed by effective Hamiltonian:

\[
\mathcal{H}_{\text{eff}} = - \frac{4}{\sqrt{2}} G_F V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)
\]

where \(O_i\)'s are local quark operators and \(C_i\)'s are Wilson coefficients. \(G_F\) is the Fermi constant and \(V_{ij}\) are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element for \(B \to
$K_1l^+l^-$ decay is obtained by $b \rightarrow sl^+l^-$ sandwiching transition amplitude between initial and final meson states. Using effective Hamiltonian the matrix element of the $B \rightarrow K_1l^+l^-$ decay which can be written as follows:

$$
\mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ -2m_b C_7^{e\mu} \bar{s}_l \sigma_{\mu\nu} q'' (1 + \gamma_5) b l \gamma^\mu l \\
+ C_9^{e\mu} \bar{s}_l \gamma^\mu (1 - \gamma_5) b l \gamma^\mu l + C_{10} \bar{s}_l \gamma^\mu (1 - \gamma_5) b l \gamma^\mu \gamma_5 l \right\} 
$$

(2)

where $s = q^2$, $q$ is the momentum transfer, $q = p_1 + p_2 = p_B - p_K$. Here, $p_1$, $p_2$, $p_B$ and $p_K$ are the four-momenta of the leptons, $B$ meson and $K_1$ meson respectively. Already the free quark decay amplitude $\mathcal{M}$ contains certain long-distance effects which usually are absorbed into a redefinition of the Wilson coefficient. These coefficients in UED are calculated by Ref.[11] and [12] which can be written as follows,

$$
C_7^0(\mu_w) = -\frac{1}{2} D'(x_t, 1/R), \\
C_9(\mu) = P_0^{NDR} + \frac{Y(x_t, 1/R)}{\sin^2 \theta_w} - 4Z(x_t, 1/R) + P_1 E(x_t, 1/R), \\
C_{10} = -\frac{Y(x_t, 1/R)}{\sin^2 \theta_w}
$$

(3)

where $P_0^{NDR} = 2.60 \pm 0.25$ and referring to leading log approximation. Explicit expression the functions of the detail $D'(x_t, 1/R), Y(x_t, 1/R)$ and $Z(x_t, 1/R)$ are calculated in Ref.[11][12][16]. From Eq.(2) it follows that, for obtaining matrix element for the $B \rightarrow K_1l^+l^-$ decay we need to know following matrix elements $\langle K_1(k, \varepsilon) \mid \bar{s}_l \gamma_5 (\gamma_5) b \mid B(p) \rangle$ and $\langle K_1(k, \varepsilon) \mid \bar{s}_l \sigma_{\mu\nu} q'' b \mid B(p) \rangle$. These matrix elements in terms of form factors are parametrized as

$$
\langle K_1(k, \varepsilon) \mid \bar{s}_l \gamma_5 b \mid B(p) \rangle = i \varepsilon^*_\mu (m_B + m_{K_1}) V_1(s) - (p + k) \mu \varepsilon^* q \frac{V_2(s)}{m_B + m_{K_1}},
$$

$$
\langle K_1(k, \varepsilon) \mid \bar{s}_l \gamma_5 \gamma_5 b \mid B(p) \rangle = \frac{2i \varepsilon_{\mu\nu\alpha\beta}}{m_B + m_{K_1}} \varepsilon^* \nu \rho^\alpha k^\beta A(s)
$$

(4)

(5)

$$
\langle K_1(k, \varepsilon) \mid \bar{s}_l \sigma_{\mu\nu} q'' b \mid B(p) \rangle = \left[ (m_B^2 - m_{K_1}^2) \varepsilon_{\mu} - (\varepsilon.q)(p + k)_\mu \right] F_2(s) \\
+ \left( \varepsilon^* . q \right) \left[ q_{\mu} - \frac{s}{m_B^2 - m_{K_1}^2} (p + k)_\mu \right] F_3(s)
$$

(6)

$$
\langle K_1(k, \varepsilon) \mid \bar{s}_l \sigma_{\mu\nu} q'' \gamma_5 b \mid B(p) \rangle = -i \varepsilon_{\mu\nu\alpha\beta} \varepsilon^* \nu k^\beta F_1(s)
$$

(7)

where $\varepsilon$ is the polarization vector of the $K_1$ meson. The form factors entering Eq.(4) and (5) are estimated in [18][19].
\[ V_1(s) = \frac{V_1(0)}{(1 - s/m_{B_A}^2)(1 - s/m_{B_A}^2)} \left( 1 - \frac{s}{m_B^2 - m_{K_1}^2} \right) \]  
\[ V_2(s) = \frac{\tilde{V}_2(0)}{(1 - s/m_{B_A}^2)(1 - s/m_{B_A}^2)} - \frac{2m_{K_1}}{m_B - m_{K_1}} \frac{V_0(0)}{(1 - s/m_{B}^2)(1 - s/m_B^2)} \]  
\[ V_3(s) = \frac{m_B + m_{K_1}}{2m_{K_1}} V_1(s) - \frac{m_B - m_{K_1}}{2m_{K_1}} V_2(s) \]  
\[ A(s) = \frac{A(0)}{(1 - s/m_B^2)(1 - s/m_B^2)} \]  

We can also define to the other matrix elements of the \( B \to K_1 l^+ l^- \) decay in terms of penguin form factors. Using the Ward identities following relationship between form factors, we get

\[ F_1(s) = -\frac{(m_b - m_s)}{(m_B + m_{K_1})} 2A(s) \]  
\[ F_2(s) = -\frac{(m_b + m_s)}{(m_B - m_{K_1})} V_1(s) \]  
\[ F_3(s) = -\frac{V_1(s)}{s} [V_3(s) - V_0(s)] \]  

In order to avoid the kinematical singularity in the matrix element at \( s = 0 \) we demand \( F_1(0) = 2F_2(0) \). The corresponding values at \( s = 0 \) are given by \([14][17][18][19]\).

\[ A(0) = -(0.52 \pm 0.05) \]  
\[ V_1(0) = -(0.24 \pm 0.02) \]  
\[ \tilde{V}_1(0) = -(0.39 \pm 0.03) \]  
\[ V_0(0) = -(0.29 \pm 0.04) \]  
\[ A_1(0) = (0.23 \pm 0.02) \]  
\[ \tilde{A}_2(0) = (0.33 \pm 0.05) \]  

Using Eq.(4),(5),(6) and (7) for the matrix element of the \( B \to K_1 l^+ l^- \) decay we set,

\[ \mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts} m_B \left\{ A(\hat{s}) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu\nu} p_{B}^\alpha p_{K_1}^\beta - iB(\hat{s}) \epsilon_\mu^* + iC(\hat{s}) (\epsilon^* \cdot p_B)(p_B + p_{K_1})_\mu \\
+ iD(\hat{s}) (\epsilon^* \cdot p_B) q_\mu (\bar{l}\gamma^\mu l) + \left[ E(\hat{s}) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_{B}^\alpha p_{K_1}^\beta - iF(\hat{s}) \epsilon^* \\
+ iG(\hat{s}) (\epsilon^* \cdot p_B)(p_B + p_{K_1})_\mu + iH(\hat{s}) (\epsilon^* \cdot p_B) q_\mu (\bar{l}\gamma^\mu \gamma^5 l) \right] \right\} \]  

where
Having the explicit expression for the matrix element for the $B \to K_1 l^+ l^-$ decay, the next task is the calculation its differential decay rate. In the center of mass frame (CM) of the dileptons $l^+ l^-$, where we take $z = \cos \theta$ and $\theta$ is the angle between the momentum of the $B$ meson and that of $l^-$, differential decay width is found to be like follows,

$$\frac{d\Gamma}{d\hat{s}}(B \to K_1 l^+ l^-) = \frac{G^2_F \alpha^2 |V_{tb}V_{ts}^{\ast}|^2}{8m_B^4 \pi^2} \Delta \quad (18)$$

where $\lambda = r^2 + (-1 + \hat{s})^2 - 2r(1 + \hat{s})$ with $\hat{s} = q^2/m_B^2$ and $r = m_l^2/m_B^2$ and $m_l = m_l/m_B$. $s = q^2$ is the dilepton invariant mass. The function $\Delta$ is defined as follows:

$$\Delta = \frac{2}{3} m_B^2 \left( 2 m_B^4 (2 \hat{m}_l^2 + \hat{s}) \lambda A^2 + \frac{1}{r \hat{s}} (2 \hat{m}_l^2 + \hat{s}) (r^2 + (-1 + \hat{s})^2 + 2r(-1 + 5\hat{s})) |B|^2 \
+ \frac{1}{r \hat{s}} m_B^4 (2 \hat{m}_l^2 + \hat{s}) \lambda^2 |C|^2 - 2 m_B^4 (4 \hat{m}_l^2 - \hat{s}) \lambda |E|^2 \
+ \frac{1}{r \hat{s}} \left[ \hat{s} (r^2 + (-1 + \hat{s})^2 + 2r(-1 + 5\hat{s})) + 2 \hat{m}_l^2 \left( r^2 + (-1 + \hat{s})^2 - 2r(1 + 13\hat{s}) \right) \right] |F|^2 \
+ \frac{1}{r \hat{s}} \left[ m_B^4 \hat{s} \lambda^2 + 2 \hat{m}_l^2 (1 + r^2 + 4\hat{s} - 2\hat{s}^2 + r(-2 + 4\hat{s})) \right] |G|^2 + \frac{1}{r} 6 m_B^4 \hat{m}_l^2 \hat{s} \lambda |H|^2 \
+ \frac{1}{r \hat{s}} 2 m_B^2 (2 \hat{m}_l^2 + \hat{s}) \left( r^3 + (-1 + \hat{s})^3 - r^2 (3 + \hat{s}) - r(-3 + 2\hat{s} + \hat{s}^2) \right) \text{Re}(B^{\ast}C) \
+ \frac{1}{r \hat{s}} \left[ 2 m_B^2 \lambda \left( 2 \hat{m}_l^2 (-1 + r - 2\hat{s}) + \hat{s}(-1 + r + \hat{s}) \right) \right] \text{Re}(F^{\ast}G) \right)$$
\[
- \frac{1}{r} 12 m_B^2 \hat{m}_t^2 \lambda Re(F^* H) - \frac{1}{r} 12 m_B^2 \hat{m}_t^2 \lambda (-1 + r) Re(G^* H) \right) \}
\]

(19)

4 Lepton Polarization Asymmetries

Now, we would like to discuss the lepton polarizations in the \( B \rightarrow K_1 l^+ l^- \) decays. For calculation of the double lepton polarization asymmetries, in the rest frame of \( l^+ l^- \), unit vectors \( s_i^\pm \) \( (i = L, T, N) \) are defined as [8, 13]

\[
s_L^- = (0, \vec{e}_L^-) = \left( 0, \frac{\vec{p}_-}{|\vec{p}_-|} \right),
\]

\[
s_T^- = (0, \vec{e}_T^-) = \left( 0, \vec{e}_L \times \vec{e}_N^- \right),
\]

\[
s_N^- = (0, \vec{e}_N^-) = \left( 0, \frac{\vec{p}_{K_1} \times \vec{p}_-}{|\vec{p}_{K_1} \times \vec{p}_-|} \right),
\]

\[
s_L^+ = (0, \vec{e}_L^+) = \left( 0, \frac{\vec{p}_+}{|\vec{p}_+|} \right),
\]

\[
s_T^+ = (0, \vec{e}_T^+) = \left( 0, \vec{e}_L^+ \times \vec{e}_N^+ \right),
\]

\[
s_N^+ = (0, \vec{e}_N^+) = \left( 0, \frac{\vec{p}_{K_1} \times \vec{p}_+}{|\vec{p}_{K_1} \times \vec{p}_+|} \right).
\]

(20)

where \( \vec{p}_\pm \) and \( \vec{p}_{K_1} \) are the three-momenta of the leptons \( l^+ l^- \) and \( K_1 \) meson in the center of mass frame (CM) of \( l^+ l^- \) system, respectively. The longitudinal unit vector \( S_L \) is boosted to the CM frame \( l^+ l^- \) under the Lorentz transformation:

\[
(s_L^\pm)_{CM} = \left( \frac{|\vec{p}_\pm|}{m_l}, \frac{E_l \vec{p}_\pm}{m_l |\vec{p}_\pm|} \right),
\]

(21)

where \( \vec{p}_\pm = -\vec{p}_- \), \( E_l \) and \( m_l \) are the energy and mass of leptons in the CM frame, respectively. The transversal and normal unit vectors \( s_T^\pm \), \( s_N^\pm \) are not changed under the Lorentz boost. The double lepton polarization asymmetries are defined as:

\[
P_i^\pm(s) = \frac{\frac{d\Gamma}{ds}(\vec{n}_i^\pm = \vec{e}_i^\pm) - \frac{d\Gamma}{ds}(\vec{n}_i^\pm = -\vec{e}_i^\pm)}{\frac{d\Gamma}{ds}(\vec{n}_i^\pm = \vec{e}_i^\pm) + \frac{d\Gamma}{ds}(\vec{n}_i^\pm = -\vec{e}_i^\pm)}
\]

(22)

where \( \vec{n}_i^\pm \) is the unit vectors in the rest frame of the lepton. The next step, we calculated double-lepton polarization asymmetries which is define as \( P_{ij} \):

\[
P_{LL} = \frac{1}{3} m_B^2 \left\{ 2 m_B^4 (2 \hat{m}_t^2 - \hat{s}) \lambda |A|^2 + \frac{1}{r \hat{s}} (2 \hat{m}_t^2 - \hat{s}) (r^2 + (-1 + \hat{s})^2 + 2 r (-1 + 5 \hat{s})) |B|^2 + \frac{1}{r \hat{s}} m_B^4 (2 \hat{m}_t^2 - \hat{s}) \lambda^2 |C|^2 + 2 m_B^4 (4 \hat{m}_t^2 - \hat{s}) \lambda |E|^2 \right\}
\]

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\begin{align}
- \frac{1}{rs} & \left[ \hat{s}(r^2 + (-1 + \hat{s})^2 + 2r(-1 + 5\hat{s})) - 2\hat{m}_l^2 \left( 5r^2 + 5(-1 + \hat{s})^2 + 2r(-5 + 7\hat{s}) \right) \right] |F|^2 \\
- \frac{1}{rs} & \left[ m_B^2 \hat{s}\lambda^2 - 2\hat{m}_l^2 (5 + 5r^2 - 4\hat{s} + 2\hat{s}^2 - 2r(5 + 2\hat{s})) \right] |G|^2 + \frac{1}{r} 6m_B^4 \hat{m}_l^2 \hat{s}\lambda |H|^2 \\
+ \frac{1}{rs} & 2m_B^2 (2\hat{m}_l^2 - \hat{s}) \left( r^3 + (-1 + \hat{s})^3 - r^2(3 + \hat{s}) - r(-3 + 2\hat{s} + \hat{s}^2) \right) \text{Re}(B^*C) \\
+ \frac{1}{rs} & 2m_B^2 \lambda \left( 2\hat{m}_l^2(-5 + 5r + 2\hat{s}) - \hat{s}(-1 + r + \hat{s}) \right) \text{Re}(F^*G) \\
- \frac{1}{r} & 12m_B^2 \hat{m}_l^2 \lambda \text{Re}(F^*H) - \frac{1}{r} 12m_B^4 \hat{m}_l^2 \lambda (-1 + r) \text{Re}(G^*H) \right \} \\
(23) \\

P_{NN} & = \frac{1}{\Delta} \frac{2}{3} m_B^2 \left \{ m_B^4 (-4\hat{m}_l^2 + \hat{s})\lambda |A|^2 + \frac{1}{rs} \left[ \hat{s}\lambda + 2\hat{m}_l^2(r^2 + (-1 + \hat{s})^2 + 2r(-1 + 5\hat{s})) \right] |B|^2 \\
- \frac{1}{rs} & m_B^4 (2\hat{m}_l^2 + \hat{s})\lambda^2 |C|^2 + m_B^4 (4\hat{m}_l^2 - \hat{s})\lambda |E|^2 + \frac{1}{rs} (2\hat{m}_l^2 + \hat{s})\lambda |F|^2 \\
+ \frac{1}{rs} & m_B^4 \hat{s}\lambda^2 + 2\hat{m}_l^2 (1 + r^2 + 4\hat{s} - 2\hat{s}^2 + 2r(-2 + 4\hat{s})) \left | G \right |^2 + \frac{1}{r} 6m_B^4 \hat{m}_l^2 \hat{s}\lambda |H|^2 \\
- \frac{1}{rs} & 2m_B^2 (2\hat{m}_l^2 + \hat{s}) \left( r^3 + (-1 + \hat{s})^3 - r^2(3 + \hat{s}) - r(-3 + 2\hat{s} + \hat{s}^2) \right) \text{Re}(B^*C) \\
+ \frac{1}{rs} & 2m_B^2 \lambda \left( 2\hat{m}_l^2(-1 + r - 2\hat{s}) + \hat{s}(-1 + r + \hat{s}) \right) \text{Re}(F^*G) \\
- \frac{1}{r} & 12m_B^2 \hat{m}_l^2 \lambda \text{Re}(F^*H) - \frac{1}{r} 12m_B^4 \hat{m}_l^2 \lambda (-1 + r) \text{Re}(G^*H) \right \} \\
(24) \\

P_{TT} & = \frac{1}{\Delta} \frac{2}{3} m_B^2 \left \{ m_B^4 (4\hat{m}_l^2 + \hat{s})\lambda |A|^2 + \frac{1}{rs} \left[ -\hat{s}\lambda + 2\hat{m}_l^2(r^2 + (-1 + \hat{s})^2 + 2r(-1 + 5\hat{s})) \right] |B|^2 \\
+ \frac{1}{rs} & m_B^4 (2\hat{m}_l^2 - \hat{s})\lambda^2 |C|^2 + m_B^4 (4\hat{m}_l^2 - \hat{s})\lambda |E|^2 \\
+ \frac{1}{rs} & \lambda(-10\hat{m}_l^2 + \hat{s}) |F|^2 \\
+ \frac{1}{rs} & m_B^4 \hat{s}\lambda^2 - 2\hat{m}_l^2 (5 + 5r^2 - 4\hat{s} + 2\hat{s}^2 - 2r(5 + 2\hat{s})) \left | G \right |^2 - \frac{1}{r} 6m_B^4 \hat{m}_l^2 \hat{s}\lambda |H|^2 \\
+ \frac{1}{rs} & 2m_B^2 (2\hat{m}_l^2 - \hat{s}) \left( r^3 + (-1 + \hat{s})^3 - r^2(3 + \hat{s}) - r(-3 + 2\hat{s} + \hat{s}^2) \right) \text{Re}(B^*C) \\
+ \frac{1}{rs} & 2m_B^2 \lambda \left( 2\hat{m}_l^2(-5 + 5r + 2\hat{s}) + \hat{s}(-1 + r + \hat{s}) \right) \text{Re}(F^*G) \\
+ \frac{1}{r} & 12m_B^2 \hat{m}_l^2 \lambda \text{Re}(F^*H) + \frac{1}{r} 12m_B^4 \hat{m}_l^2 \lambda (-1 + r) \text{Re}(G^*H) \right \} \\
(25) \\

P_{LN} & = \frac{1}{\Delta} \frac{1}{r\sqrt{s}} m_B^2 \hat{m}_l \pi \sqrt{\lambda} \left[ (-1 + r + \hat{s}) \text{Im}(B^*F) + m_B^2 \lambda \text{Im}(C^*F) ight] \\
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\[ P_{LT} = \frac{1}{\Delta} \frac{1}{\sqrt{s}} m_B^2 \hat{m}_\pi \lambda \sqrt{1 - \frac{4\hat{m}_\pi^2}{s}} \left( -\frac{1}{r}(-1 + r + \hat{s})|F|^2 - \frac{1}{r}m_B^4(-1 + r)\lambda|G|^2 + 2m_B^2\hat{s}Re(B^*E) \right) \]

\[ + \frac{1}{r}m_B^4\hat{s}\lambda Re(G^*H) \]  

\[ P_{TL} = \frac{1}{\Delta} \frac{1}{\sqrt{s}} m_B^2 \hat{m}_\pi \lambda \sqrt{1 - \frac{4\hat{m}_\pi^2}{s}} \left( -\frac{1}{r}(-1 + r + \hat{s})|F|^2 - \frac{1}{r}m_B^4(-1 + r)\lambda|G|^2 - 2m_B^2\hat{s}Re(B^*E) \right) \]

\[ - \frac{1}{r}m_B^4\hat{s}\lambda Re(A^*F) - \frac{1}{r}m_B^2(2 + 2r^2 + \hat{s}^2 - r(4 + \hat{s}))Re(F^*G) + \frac{1}{r}m_B^2\hat{s}(-1 + r + \hat{s})Re(F^*H) + \frac{1}{r}m_B^4\hat{s}\lambda Re(G^*H) \]  

\[ P_{TN} = \frac{1}{\Delta} \frac{1}{\sqrt{s}} m_B^2 \hat{m}_\pi \lambda \sqrt{1 - \frac{4\hat{m}_\pi^2}{s}} \left[ -m_B^4r\hat{s}Im(A^*E) + Im(B^*F) + m_B^2\left( -1 + r + \hat{s}\right)Im(C^*F) \right] \]

\[ + \left( -1 + r + \hat{s}\right)Im(B^*G) + m_B^2\lambda Im(C^*G) \]  

\[ P_{NL} = \frac{1}{\Delta} \frac{1}{\sqrt{s}} m_B^2 \hat{m}_\pi \lambda \left[ -\left( -1 + r + \hat{s}\right)Im(B^*F) - m_B^2\lambda Im(C^*F) \right] \]

\[ - m_B^2(-1 + r)(-1 + r + \hat{s})Im(B^*G) - m_B^4(-1 + r)\lambda Im(C^*G) \]

\[ + m_B^2\hat{s}(-1 + r + \hat{s})Im(B^*H) + m_B^2\hat{s}\lambda Im(C^*H) \]  

\[ P_{NT} = \frac{1}{\Delta} \frac{1}{\sqrt{s}} m_B^2 \hat{m}_\pi \lambda \sqrt{1 - \frac{4\hat{m}_\pi^2}{s}} \left[ -m_B^4r\hat{s}Im(A^*E) + Im(B^*F) + m_B^2\left( -1 + r + \hat{s}\right)Im(C^*F) \right] \]

\[ + \left( -1 + r + \hat{s}\right)Im(B^*G) + m_B^2\lambda Im(C^*G) \]  

## 5 Numerical analysis and discussion

In this section, we present our numerical results on the double lepton polarization asymmetries for the \( B \rightarrow K_1^+ l^- \) decays. First, we present the values of input parameters are:

\[ m_B = 5.28 \text{ GeV}, \ m_{B^*} = 5.32 \text{ GeV}, \ m_{K_1} = 1.402 \text{ GeV}, \]

\[ m_b = 4.8 \text{ GeV}, \ m_a = 0.13 \text{ GeV}, \ m_s = 0.105 \text{ GeV}, \ m_s = 1.77 \text{ GeV}, \]

\[ |V_{tb}V_{ts}^*| = 0.04, \ \alpha^{-1} = 137, \ G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}, \ \tau_B = 1.53 \times 10^{-12} \text{ s}. \]  

The \( B \rightarrow K \) transition form factors are the main input parameters in performing the numerical analysis, which are embedded into the expressions of the double-lepton polarization asymmetries.
For them we have used their expression given by Eq. (8-15). The differential decay rate for $B \rightarrow K_1 l^+ l^-$ can be defined in terms of integration on $\hat{s}$, which is determined to the range of the $4m_t^2 \leq s \leq (m_B - m_{K_1})^2$.

In Fig.1, we present the dependence of the $P_{LL}$ for the $B \rightarrow K_1 \mu^+ \mu^-$ decay as a function of $s/m_B^2$. We see that, $P_{LL}$ in UED compatible with the SM result. Increasing $\hat{s}$, $P_{LL}$ is moderate for the low of $\hat{s}$. The effect of KK contribution in the Wilson coefficient are consistent for $1/R = 200 GeV$ at low value of $\hat{s}$. $1/R = 200 GeV$ value is greater than $1/R = 400 GeV$. In Fig.2, Double lepton longitudinal polarization asymmetries for the $B \rightarrow K_1 \tau^+ \tau^-$ decay is presented. From this figure is follows, UED model prediction coincide with the SM result. One can see that the value of the longitudinal polarization is different in the low of $\hat{s}$ for the $B \rightarrow K_1 \tau^+ \tau^-$ decay. While $1/R = 200 GeV$ value is max in the UED model, The SM result is approximately two times lower than this value. In Fig.3, For the $B \rightarrow K_1 \mu^+ \mu^-$ decay, we analysis to the normal polarizations. We obtained good result at the $1/R = 200 GeV$ in UED model. We can see that the effect of extra dimension are very noticeable at the small value of $\hat{s}$. When the value of $\hat{s}$ close to 0.2, all the value of normal polarization is coincide with each other. In $\hat{s} = 0.36$, the value of $1/R = 200 GeV$ is five times bigger than SM result. But in Fig.4, for the $B \rightarrow K_1 \tau^+ \tau^-$ decay, it is similar to the $P_{LL}$ result. In Fig.5, We examine to the transversal polarization for the $B \rightarrow K_1 \mu^+ \mu^-$ decay. At the $1/R = 200 GeV$ value, we compared to that of the SM prediction $P_{TT}$ is larger from SM. Again, the effects of extra dimension are distinguished at the small value of momentum transfer $\hat{s}$ where $P_{TT}$ is minimum. For the $\hat{s} = 0.53$ value, all polarization values are decreases. In Fig.6, We analysis to transversal polarization as a function of the $\hat{s}$ for the $B \rightarrow K_1 \tau^+ \tau^-$ decay. We observe a little contributions from UED model, especially in the $1/R = 400 GeV$ value. But UED model is better than SM in this figure. All model values come together with the SM result in the $\hat{s} = 0.53$ value. In Fig.7, we investigate $P_{LT}$ polarization. We see that increasing $\hat{s}$, $P_{LT}$ increase until $\hat{s} = 0.5 GeV^2$. After this value of $\hat{s}$ two models are decrease until $\hat{s} = 0.55 GeV^2$. $(P_{LT})_{UED} = 2(P_{LT})_{SM}$ at $1/R = 200 GeV$. So it is also very useful for establishing new physics. In Fig.8, We show our predictions for the $P_{TL}$ for $B \rightarrow K_1 \tau^+ \tau^-$ decay. We get $|(P_{TL})_{UED}| > |(P_{TL})_{SM}|$. This result can serve as a good test for discrimination of two models. The other polarizations for the $B \rightarrow K_1 l^+ l^-$ decay, we have imaginary part and therefore there is no interference terms between SM and UED model contributions.

In conclusion, we have studied the double-lepton polarization asymmetries in the UED model. We obtain different double-lepton polarization asymmetries which is very sensitive to the UED model. It has been shown that all these physical observebles are very sensitive to the existence of new physics beyond SM and their experimental measurements can give valuable information on it.

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Figure 1: The dependence of the Longitudinal polarization, for $B \rightarrow K_1 \mu^+ \mu^-$ decay, as a function of the $\hat{s}$.

Figure 2: The dependence of the Longitudinal polarization, for $B \rightarrow K_1 \tau^+ \tau^-$ decay, as a function of the $\hat{s}$.
Figure 3: The dependence of the Normal polarization, for $B \rightarrow K\mu^+\mu^-$ decay, as a function of the $\hat{s}$.

Figure 4: The dependence of the Normal polarization, for $B \rightarrow K\tau^+\tau^-$ decay, as a function of the $\hat{s}$.
Figure 5: The dependence of the Transversal polarization, for $B \rightarrow K_{1}\mu^{+}\mu^{-}$ decay, as a function of the $\hat{s}$.

Figure 6: The dependence of the Transversal polarization, for $B \rightarrow K_{1}\tau^{+}\tau^{-}$ decay, as a function of the $\hat{s}$.
Figure 7: The dependence of the $P_{LT}$ polarization, for $B \to K_1\mu^+\mu^-$ decay, as a function of the $\hat{s}$.

Figure 8: The dependence of the $P_{TL}$ polarization, for $B \to K_1\tau^+\tau^-$ decay, as a function of the $\hat{s}$. 

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$P_{LT}(B\rightarrow K^*\tau\bar{\tau})$

$s/m_B^2$

$1/R=200$

$1/R=250$

$1/R=300$

$1/R=400$
\( P_{TL} (B \rightarrow K \mu \mu) \)

\( s/m_B^2 \)

- SM
- 1/R=200
- 1/R=250
- 1/R=300
- 1/R=400