Research on "concentric drum" project based on discrete model

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Abstract. The "concentric drum" project tests team members' ability of cooperation, and the team members can hit the ball more times by working together. It is a typical dynamic problem. In this paper, the complete elastic collision model and the drum surface segmentation model are established to complete the discrete model. Through the optimization of the system, the good solution results of the problem are obtained gradually. The "concentric drum" strategy based on the discrete model has a certain reference for the solution of dynamic problems.

1. Problem description

The "concentric drum" is a highly collaborative project. As shown in Figure 1, the fixed points of the rope tied on the concentric drum are evenly distributed along the circumference. Each team member pulls a rope and throws the ball to keep the concentric drum surface level. According to the rules of the game, the sports ball must rise to a certain height, and the players can only grasp the end of the rope and cannot touch other positions.

Figure 1. Demo picture of concentric drum project.

The concentric drum problem is a kind of qualitative dynamic problem. In an ideal state, players can precisely control the direction, timing and strength of force, so that a certain strategy can go on indefinitely. However, the obtained cooperation strategy is not realistic, so it is necessary to consider the nonideal situations such as non-uniform force, non-simultaneous force, and drum surface tilt.

In this paper, an ideal cooperation strategy is established firstly. Then, on this basis, through the idea of discretization model, the complete elastic collision model and drum surface segmentation model are established, and the good solution results of the problem are obtained gradually.
2. Problem analysis

Under ideal conditions, the problem is in good agreement with the physical definition of a completely elastic collision [1, 2], based on the kinetic energy theorem and the momentum theorem [3, 4]. At the same time, considering the proportion of waist length and height, the objective equation is established, and the best value of force and time is obtained [5, 6].

When there are nine relatively simple error cases in Table 1, the analysis of them can provide help for the establishment of subsequent models. By building a drum surface segmentation model, we divide the concentric drum into eight parts, and analyse them discretely. The height of each discrete point is calculated respectively, and the drum inclination can be obtained by using the inverse function of the ratio of the difference between the high and low points and the drum diameter [4].

| No. | Parameter | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|-----------|---|---|---|---|---|---|---|---|
|     | Time(s)   | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1   | Force(N)  | 90| 80| 80| 80| 80| 80| 80| 80|
|     | Time(s)   | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2   | Force(N)  | 90| 90| 80| 80| 80| 80| 80| 80|
|     | Time(s)   | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3   | Force(N)  | 90| 80| 80| 90| 80| 80| 80| 80|
|     | Time(s)   | -0.1| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4   | Force(N)  | 80| 80| 80| 80| 80| 80| 80| 80|
|     | Time(s)   | -0.1| -0.1| 0 | 0 | 0 | 0 | 0 | 0 |
| 5   | Force(N)  | 80| 80| 80| 80| 80| 80| 80| 80|
|     | Time(s)   | -0.1| 0 | 0 | -0.1| 0 | 0 | 0 | 0 |
| 6   | Force(N)  | 80| 80| 80| 80| 80| 80| 80| 80|
|     | Time(s)   | -0.1| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7   | Force(N)  | 90| 80| 80| 80| 80| 80| 80| 80|
|     | Time(s)   | 0 | -0.1| 0 | 0 | -0.1| 0 | 0 | 0 |
| 8   | Force(N)  | 90| 80| 80| 90| 80| 80| 80| 80|
|     | Time(s)   | 0 | 0 | 0 | 0 | -0.1| 0 | 0 | -0.1|
| 9   | Force(N)  | 90| 80| 80| 90| 80| 80| 80| 80|

In order to adapt to the more general situation, we analyse the above nine special cases. We decompose the problem into two parts, one is to consider the factors of excessive force and the other is the factors of pre-emptive force, and then analyse the problem discretely. In order to make the speed of the eight discrete points to be the same, apply the opposite force with the same time, so that the
angle of inclination and the distance of horizontal movement will not change any more, and the number of throws will become more.

Considering the direction of inclination and the magnitude of force. Make the drum surface level, try to reduce the number of players who change the force and change the force.

3. Establishment and solution of discretization model

3.1. The strategy of hitting the ball under ideal condition

Under the ideal condition, it is more reasonable to use the arm strength rope between the waist and shoulder, and the distance between the players is a straight line, assuming that the height of the players is the same [7, 8, 9]. In order to make the number of throws as many as possible, the height of the throw is set as the minimum height of 40 cm when the ball is bumped.

The collision between the ball and the concentric drum is a complete elastic collision, during which there is no energy loss. According to the conservation of momentum and conservation of energy, as well as the motion state of the ball and the concentric drum before and after the collision, the analysis is carried out [1, 2].

Let the mass of the ball be \( m_1 \) (kg), the velocity of the ball at the moment before the collision be \( v_1 (m/s) \), the displacement of the ball at the time of collision with the concentric drum be \( x(m) \), the mass of the concentric drum be \( m_2 \) (kg), the acceleration of the concentric drum upward be \( a \), the velocity of the concentric drum at the moment before the collision be \( v_2 (m/s) \), after the collision, the velocity of the volleyball be \( v_3 \), the velocity of the concentric drum be \( v_4 \). The acceleration of the drum is \( a (m/s^2) \), the acceleration of gravity is \( g = 10 (m/s^2) \) and the displacement before the collision of ball and drum is \( x(m) \). The detailed motion diagram, as shown in Figure 2.

![Figure 2](image)

Through the above analysis, the following equations are listed for solution:

\[
\begin{align*}
\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 &= \frac{1}{2} m_1 v_3^2 + \frac{1}{2} m_2 v_4^2 \\
-m_1 v_1 + m_2 v_2 &= -m_1 v_3 + m_2 v_4 \\
m_1 g x &= \frac{1}{2} m_1 v_1^2 \\
m_2 a \times (0.4 - x) &= \frac{1}{2} m_2 v_2^2 \\
\frac{1}{2} at^2 + x &= 0.4
\end{align*}
\]

(1)
Using LINGO to solve the equation: the shortest rope length is 54.23 cm, the angle between the pull direction and the horizontal plane is 64.1379°, each team member uses 5.58371 N, and the time of applying force to the concentric drum and the time of ball falling is 0.2623 s.

### 3.2. Analysis of the inclination angle of drum surface after discretization

Firstly, the whole drum surface is divided into eight discrete parts according to the number of players [4, 10] by the method of discretization to determine the spatial position relationship between the rope and the drum. According to the data given in Table 1, analyse the influence of the force size and timing of individual players on the inclination angle and direction of the drum surface one by one, and obtain the physical quantities of the drum surface and the ball in different motion states, the most important of which is the position of the highest and lowest discrete points.

When the number of players with mis-operation increases, the highest point of drum is not in the position with the largest rope tension, but in the position between the players. Then we analyzed 9 possible scenarios, as shown in Figure 3.

![Figure 3. Positions of the highest and lowest discrete points in 9 cases.](image)

After getting the position and height of the highest point is $s_{max}$, the other point where it intersects the center of the drum is the lowest point and its height is $s_{min}$. In order to find the final included angle, the difference between the highest and the lowest rising displacement is set as $d$, which includes:

$$d = s_{max} - s_{min}$$

When the diameter of drum surface is known as $D$, the final inclination angle $\theta$ of drum surface can be obtained as follows:
\[
\sin \theta = \frac{d}{D \cos \beta} \frac{\beta}{2}
\] (4)

Where \( \beta \) is the angle between the line between the two highest (lowest) discrete points of the drum and the center of drum.

For some cases where it is impossible to simply solve the highest and lowest discrete points, the least square method can be used to fit the position when the height difference is the largest [11, 12]. Using MATLAB to solve the problem, the angle of the drum surface is 0.0180 °, 0.0146 °, 0.0117 °, 0.3947 °, 0.3174 °, 0.2539 °, 0.4739 °, 0.2499 ° and 0.2422 °.

3.3. Strategy change under non ideal condition

According to the above analysis, it is found that it is impossible for team members to achieve accurate control in reality, so the previous strategies must be adjusted due to the influence of human uncertainties. The goal is still to get as many consecutive throws as possible, and the player can control drum easily.

Because the uncertain factors are mainly the influence of time and force, we think that players can improve the position of drum surface by increasing the upward force, and then increase the speed of ball upward movement after collision; if the force is not increased in place, there is error, and the ball can hit the target by translating the drum.

3.3.1. Height of the ball. The inconsistent operation of the players leads to the tilt collision, the ball no longer continues to move upward along the vertical direction, resulting in a certain amount of energy loss. In order to make the number of consecutive throws as much as possible, the players need to increase their strength and make up for the loss in the time of lifting the drum. In order to make the height of the ball thrown away from the drum surface more than 40 cm, set the force increased by each player in the vertical direction as \( \Delta F \), use the model of 3.1, and solve it through LINGO, and get the results, as shown in Table 2:

| No. | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| \( \Delta F \) (N) | 0.0022 | 0.0014 | 0.0009 | 1.0754 | 0.6890 | 0.4385 | 1.5715 | 0.4244 | 0.3987 |

3.3.2. Displacement of drum center. As the drum surface inclines, if no force in horizontal direction is applied to change the current situation in this process, the incline angle will be larger and larger, which will have a negative impact on the subsequent adjustment. Therefore, the adjustment plan is: the relative direction of the team members, with 0.1s time to complete the same operation as the fault team members, so that the final drum surface angle remains unchanged, and the horizontal speed is 0 m/s. Using MATLAB to solve, the horizontal displacement of the concentric drum is \( s \); the mechanism to measure the displacement direction is established the angle around the drum is represented by the number 8, and the displacement direction is the lowest point of the angle of the drum turning relative to the original position relative to the number 8. The results are shown in Table 3:

| No. | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| \( \Delta S \) (m) | 0.0277 | 0.0104 | 0.0043 | 0.0898 | 0.1659 | 0.0687 | 0.1010 | 0.0689 | 0.0644 |
| Direction | 6     | 6.5   | 7.5   | 6     | 6.5   | 7.5   | 6     | 8.36  | 1.5   |
| Angle | 0.0180 | 0.0146 | 0.0117 | 0.3947 | 0.3174 | 0.2539 | 0.4739 | 0.2499 | 0.2422 |
3.4. The ball throwing strategy under rough situation
Considering that there is a certain angle of ball rebound under different parameters, it is found that the position of the highest point of ball vertical bounce and the position of player's pull point are no longer evenly distributed.

In order to make the ball move in the vertical direction after collision, it is necessary to adjust the tilt angle of the drum surface. The process of adjustment should be the opposite of the original process of generating tilt angle. The idea of discretization is adopted for consideration, and at the same time, the adjustment is carried out with as few players as possible (and everyone's force is the same as far as possible), so only the two action points near the lowest point are considered to change the tension. Then analyse the vertical movement and the horizontal force offset to find the final force.

Investigate the number of people, rope length, rebound height of the ball and other parameters. Based on the previous model, according to the tilt direction of ball, the two players closest to this direction adjust the force size, so that the tilt angle is 0, that is, try to reduce the number of players who change the force and the size of force, so that the force time is the same. The displacement difference between the high and low points is 0.006981 m from the chord length formula, and the flight time of the ball in the air is 0.2898120 s from the rebound height. In order to reduce the influence of the additional force, the new applied force is 21.41631 N and 36.14447 N from the quadrilateral law.

4. Model optimization
We can adjust the force size and direction according to the individual difference, and build a systematic model to simulate the motion state of the object through force analysis and kinematics law. Through the simulation experiment, we can more intuitively see the difference between the model and the actual phenomenon, so as to modify. Of course, it is also a scheme that is hard to be perfect in a short time.

For the discrete mathematical and physical model, we simply analyse the players and drum surface according to the number of force points, that is, the number of players. Although this is the most efficient way, it also brings some errors. According to the original discrete principle, more discrete points can be established. For example, 8 people beat drums, 16 discrete points, 24 discrete points or 8n discrete points are established. Through simulation verification, the results can be seen that the discrete points reach a certain degree, and the results are closer to the target requirements.

In the process of calculating the inclination angle, there is a case that the highest point of No. 8 is no longer at the discrete point. In order to find the position of the highest point and the lowest point, we use the method of linear fitting to simulate the distance between the two closest discrete points of the highest distance, and there is a certain error [6]. It can also be used to increase the number of discrete points to approximate the exact position of the highest point. Through the analysis of serial number 8, the result is 3.33. It can be seen that when the number of discrete points is 24, the position of the highest discrete point can be accurately obtained.

5. Evaluation of the model
In the process of gradual refinement, we can find that for the study of this kind of physical model and control strategy, we can divide the complex model by the number of force points to achieve simplification.

The model uses the isolation method to build the discrete model of the object according to the force, and abstracts some more complex irrelevant factors, which makes the model more simple. Among them, the complete elastic collision model, free falling model and force analysis are all used in the case of neglecting some secondary factors, so that the solution process is more intuitive and the solution speed is greatly improved.

At the same time, the discrete model has universality and good generalization value. The same approach applies to more complex changes. For example, the discretization model and the force synthesis method used in the model can be used to analyse the force magnitude and direction of various objects (such as bridge and house design); the complete elastic collision, kinetic energy
theorem and momentum theorem can be extended to vehicle collision test and speed analysis; the cooperation strategy used in the model can be extended to cargo handling and umbrella structure design, etc. It is of practical significance to solve the above engineering problems.

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