Cosmology and Local Physics

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Abstract

This article is dedicated to the memory of Dennis Sciama. It revisits a series of issues to which he devoted much time and effort, regarding the relationship between local physics and the large scale structure of the universe - in particular, Olber’s paradox, Mach’s principle, and the various arrows of time. Thus the focus is various ways in which local physics is influenced by the universe itself.

1 Introduction

The major thrust of present day scientific cosmology is that of examining the effect of local physical laws on the large-scale structure of the cosmos. This started with Einstein’s application of the local law of gravitation, expressed via the Einstein field equations, to determine space-time structure in the large [1, 2]. This then led to the Friedmann-Lemaître-Eddington demonstration of how these equations imply an evolution of the universe [3], followed by the prediction, on the basis of classic gravitational theory, of a beginning to the universe at a space-time singularity [1, 2, 3, 4]. This thrust attained new power through the understanding of how nuclear physics processes in the hot early expansion phase of the universe would lead to synthesis of light elements out of primordial constituents, providing an explanation both for the origin of the light elements and the existence of the 3K blackbody Cosmic Blackbody Radiation (‘CBR’) [5] - a theme that Dennis Sciama found most exciting, and contributed to in his inimitable way [5, 6]. And it has reached its climax in the understanding, following Alan Guth’s innovative realisation of the power of the idea of an inflationary era of expansion in the early universe caused by energy-condition violating quantum fields [10], of how particle physics processes in the very early universe could be of major importance in determining the large-scale structure of the universe today [11, 12].

However there has also from the earliest times been a counter theme: the study of the way that global properties of the universe can influence its local properties. This was particularly embodied in Mach’s principle, and was always dear to Dennis’ heart, inspired particularly by his discussions with Fred Hoyle, Tommy Gold, and Herman Bondi when they were together in Cambridge in the 1950’s. He devoted much thought to this topic, resulting in his famous vector
gravitational model \[13\] and various popular books \[14, 15\]. In his writings on these topics, he consistently emphasized the interconnectedness of the universe \[14\]: each part interacting with each other part, and with very distant parts being as important as local regions in these interactions, a prime example being Olber’s paradox \[16\]. Because of this interconnection, in principle one can obtain some understanding of the whole from any part; indeed if one was clever enough and understood enough physics, one could in principle completely deduce the nature of the whole from a sufficient study of its parts. An example of this line of argument is the suggestion that one could in principle deduce the expansion of the universe, and even the approximate value of the Hubble constant, from the existence of bus tickets.

A major further theme is the uniqueness of the universe, which is what gives cosmology its specific unique nature as a science \[14, 17\]. Because of this uniqueness, one runs into major problems in distinguishing boundary conditions from physical laws. What appears to be an inviolable physical law may just be a consequence of the particular boundary conditions that happen to hold in this particular universe. This might even be true for some of the apparently most fundamental of laws at the macroscopic scale, such as the second law of thermodynamics and its associated arrows of time; but we cannot test this supposition, for we cannot re-run the universe, nor can we investigate any other universe. Conversely, one can propose that if the universe were different, the laws of physics would be different. This leads to the idea that as the universe is changing, maybe the laws of nature are changing with it, or at least this might be true for the values of some of the ‘fundamental constants’ of physics, for example the gravitational constant \[18\]. From this line of argument follows the need to consider seriously alternative physics as well as alternative boundary conditions when we consider the relation between local physics and cosmology.

Many years ago, Dennis and I jointly wrote a survey article on these global-to-local relations \[19\]. The purpose of the present paper is to revisit these topics, and comment on what progress has been made in the intervening years. I deal in turn with Olber’s paradox; Mach’s principle; the arrow of time; the framework for relating local to global physics; and the relation between initial conditions and local physics. This paper is dedicated to the memory of Dennis, who was an inspiring teacher, enthusiastic colleague, and good friend.

2 Olber’s Calculation

The basic point of Olber’s paradox (‘Why is the sky dark at night?’), actually developed by Halley, is the need for integration over all very distant sources in determining the intensity of integrated radiation from astronomical sources. These distant sources cannot be neglected in flat space-time, because their number goes up as \(r^2\) and this compensates for the decrease in flux from each source, which goes down like \(1/r^2\), where \(r\) is the standard radial distance in Minkowski

\[1\] See Roger Penrose’s article, pages 314-315 in \[20\], for a nice account both of Dennis’ infectious passion for physics, and his belief in the importance of non-local effects.
space. Doing the calculation in Robertson-Walker universes with $r$ chosen as the area distance gives the same result, and the reciprocity theorem \[21\] shows this will in fact be true in any curved space-time, for it is equivalent to the result that total intensity $I$ of radiation received from a source (that is, flux per unit solid angle) is independent of the source’s area distance; it will vary as

$$I = \frac{1}{(1+z)^4} I_G$$

where $I_G$ its bolometric surface brightness, and $z$ its redshift, while the pointwise specific intensity $I_\nu$ of radiation received (the intensity per unit frequency range) in direction $\theta, \phi$ is

$$I_\nu(\theta, \phi) = \frac{I(\nu(1+z))}{(1+z)^3} I_G(\theta, \phi)$$

where $I(\nu)$ is the source spectrum and $I_G(\theta, \phi)$ its surface brightness in the direction of observation. Hence in an expanding universe, very distant sources at high $z$ appear much fainter than nearby ones, and so the sky can appear dark even though every line of sight eventually intersects a source of some kind.

However, as emphasized by Ted Harrison \[22\], that is not the whole story: in the end, the dark night sky is the result of the fact that there is not enough radiation in the universe to generate a bright night sky. Most of the lines of sight from the earth effectively end up on the surface of last scattering of radiation after the hot big bang, because although there are numerous intervening galaxies, they are mainly transparent (this is clear from both lensing studies and QSO Lyman forest observations).

### 2.1 Discrete Sources and Background radiation

Thus the present day astronomical version of the Olber’s studies consists of two parts. The first is the study of the integrated radiation from all individually unresolvable sources, resulting in a predicted spectrum of background radiation at all wavelengths coming from these sources, in particular, optical, radio, and X-ray backgrounds. The detailed theory of this background then resides in detailed astrophysical speculations about these sources and their evolution \[23\]; observations are incomplete, because of galactic and inter-galactic absorption.

The second part is a dominant theme of present day theoretical and observational cosmology: namely prediction and observation of the CBR spectrum and anisotropies, resulting from us seeing the point by point brightness of the surface of last scattering of this radiation \[13\]. Observations provide an angular power spectrum, and theory relates that to the formation of structure in the expanding universe; comparison of theoretical predictions with the observations then enable us to get tight limits on cosmological parameters \[24\]. Thus this can legitimately be regarded as the culmination of Olber’s investigations into the integrated radiation emitted by all sources in the universe. The source here is the uniformly distributed matter at the surface of last scattering, which at
later times will either be incorporated into galaxies or will reside in intergalactic space. It is the furthest matter we can see by any kind of electromagnetic radiation, because the universe is opaque at earlier times, and hence this matter comprises the visual horizon \[25\]. This is the proper endpoint of the integral in the Olber’s calculation, for we cannot receive radiation from more distant matter. The temperature of this surface is about \(T_e = 3000 \text{K}\), and it emits black body radiation at that temperature because the matter and radiation are very close to equilibrium at that epoch; by Eqn. \(1\) that radiation is received as blackbody radiation at a temperature \(T_o = T_e(1 + z)^{-1}\) where \(z\) is the redshift of emission. Because of the redshift factor of about 1000 for that matter, we receive black body radiation at about 3000\(K/1000 = 3\)\(K\) from that surface. There are small variations to this temperature over the sky due to gravitational fields and velocity effects, and study of these anisotropies is a major part of present day cosmology \[12, 24\].

Thus in the real universe, the Olber’s integral does not extend to infinity, and 3\(K\) is the temperature of the dominant radiation today, and hence is the temperature we measure for most of the night sky (and for the day sky as well, away from the sun). That temperature is not due to discrete sources, but rather is due to the primeval radiation from the hot big bang.

The further new point is the realisation that this calculation is of considerable importance, apart from its use in helping us determine cosmological parameters. It implies that the earth is in contact with a thermal reservoir at a temperature of 3\(K\), which is the sink into which we dispose of our excess entropy, generated by all the processes of life. That sink is essential to the thermodynamic functioning of the biosphere, and hence to the existence of life on earth, for the biosphere functions by receiving the black body radiation from the sun, using it to run biological and atmospheric processes, and then radiating the waste heat away to the sky \[26\]. If the temperature of the sky were much higher, and certainly if it were above 300\(K\), life like ours on earth would not be possible. Thus from this viewpoint, the reason we observe the night sky to be dark is that if that were not true, we would not be here to see it! This is of course part of the ‘anthropic universe’ argument, briefly touched on below.

2.2 The Gravitational Version

The same issue of course arises in Newtonian gravitational theory, for that too is an inverse square law. The present-day form of this problem arises in studies of the large-scale motions of matter induced by the gravitational field of large-scale inhomogeneities in the universe, such as the ‘Great Attractor’ (see for example the POTENT series of studies \[27\]). In any particular case, the question here is, has the integral for the gravitational potential causing the acceleration and hence inducing these velocities, converged or not? Do we need to consider further-out matter in our calculation of these integrals?

There are however some major differences from the Olber’s calculation. Firstly, the integral here is carried out in spacelike surfaces rather than on the past light cone, for it corresponds to the Newtonian limit of the relativistic
equations, where the constraint equations are equivalent to relations that must be satisfied at constant time (i.e. at each instant) in the expanding universe. How this is compatible with the relativistic nature of the field equations was discussed in [19]. In essence: these initial conditions have to satisfy constraint equations, such as the Hamiltonian constraints, which are usually expressed as ‘instantaneous’ conditions. This requirement has to be built into the initial conditions for the universe, which raises significant conceptual problems in some cases. Once they are satisfied, they will remain satisfied because of the consistency of the time evolution equations with the constraint equations (see Maartens [28] and Friedrich and Nagy [29] for explicit demonstration of this consistency in particular cases).

Secondly, in contrast to the Olber’s case just discussed, there is no natural cut-off to the integral in this case - one must in principle extend it to spatial infinity, if the universe is indeed spatially homogeneous - and then it diverges! (this is indeed the reason that Newtonian cosmology was not arrived at until some decades after relativistic cosmological models were available). But this is compensated for by the third point: although the gravitational field in principle diverges, its local effect is of a vector nature (causing a vectorial acceleration of matter), and hence the effects from opposite directions cancel.

Thus in effect one renormalizes by omitting the divergent integral due to a uniform background, and only calculates the remnant effect due to inhomogeneities superimposed on this uniform background (this is implied in the rescaling of variables relative to the background expansion that is used in the standard calculations of peculiar velocities, see for example Peebles [30]). Thus in this context, ‘convergence’ means continuing the integral until the effects of all more distant matter are isotropic and cancel each other out when one does the required vector addition of resulting gravitational forces. It is assumed here that the further out matter is indeed isotropically distributed about us on the largest scales; and this supposition receives some support from the high degree of CBR isotropy we measure. An issue that could perhaps still be reflected on is to what degree the success of the resulting calculations of large scale velocities can be taken as providing evidence for isotropy of the universe outside our visual horizon, and perhaps even outside our particle horizon. This question, alluded to in [19], remains open. The problem is that locally we can only measure the total integrated effect on the motion of matter, and it does not seem that one can separate out the contributions to the effect from shells of matter around us at different distances.

3 Mach’s Principle

This discussion leads naturally on to Mach’s famous conjecture, largely motivated by his position in the philosophical debates about relative versus absolute motion, that the origin of inertia is interactions with very distant matter [15, 14, 16]. This issue remains as open today as ever.

On the one hand, there are a variety of ‘Anti-Machian’ solutions of Einstein’s
Field Equations (EFE) - non-singular solutions of the vacuum equations \[31\] - suggesting to many that these equations do not by themselves incorporate Mach’s principle, despite Einstein’s hope that this would be so \[32\]. This kind of view is supported by cosmological analyses showing that distant galaxies are at rest in the local inertial rest-frame iff the cosmological vorticity is very close to zero \[33, 5\] - and there clearly are both Newtonian and relativistic cosmological models in which this is not true. The implication is that Machian solutions are a subset of all solutions of the EFE, characterized by some suitable kind of boundary conditions, for examples Raine’s isotropy conditions \[34\] resulting from an analysis of solutions of the remarkable Sciama-Waylen-Gilman (SWG) integral formulation of the EFE \[35\]. This then comes close to supporting Penrose’s proposal \[26, 36\] of the necessity of isotropic singularities at the Big Bang in order that ordinary thermodynamic properties can hold, solutions allowing such singularities being studied in some detail by Tod and Newman \[37\].

On the other hand, some people - notably Barbour \[38\] - maintain that Einstein’s equations already incorporate the full meaning of what Mach intended, and there is no need for further conditions on the solutions in order to have a Machian character. This view is imbedded in a much larger philosophical position supporting the relativity of all measurements, and suggesting that the passage of time is an illusion - a position that has received little support from other quarters.

A recent account of the debate on Mach’s Principle is given in \[38\]. This idea has of course been of enormous importance in the evolution of the theory of general relativity. Three related ideas are worth mentioning as being of some practical interest.

Firstly, there is the idea of ‘dragging of inertial frames’ occurring in rotating solutions of the EFE, for example the Kerr rotating black hole solutions (see section 5 of \[38\]). This then ties Machian ideas in to the physics of accretion disks in rotating black holes, albeit in a rather weak way. Secondly, there have been a series of accurate experiments relating to Mach’s principle (see section 6 of \[38\]), based on links between Machian ideas and testable features of local physics. And thirdly, there is the important distinction made in studies of cosmological perturbations \[39\] between scalar, vector, and tensor gravitational modes, and their relation to large scale structure formation and associated velocities (scalar modes), vorticity (vector modes), and gravitational waves (tensor modes). Machian issues arise in each case: the degree to which each is affected by very distant as opposed to local matter. And here an important point arises: tensor modes are determined by distant matter, because the corresponding characteristic velocity is the speed of light (gravitational waves propagate at speed \(c\)), but in the case of pressure-free matter (the recent universe) the characteristic velocities of the scalar and vector modes are zero \[40, 41\] - so they are only directly affected by nearby matter. However as mentioned in the previous section, this result holds only once one has factored out the background geometry and dynamics - and it is precisely that full effect (incorporating integrals over all matter) that is the concern of Mach’s principle, not just the perturbation effects
around the background geometry. That is of course both the fatal attraction and flaw in the basic idea - it is fundamental in its nature (determining the nature of all inertial effects) but beyond experimental testing in its full extent (precisely because of that nature). Overall, the idea remains a source of both inspiration, and irritation, because it is so hard to tie it down satisfactorily - see page 530 of [38] for 21 different formulations of the idea!

4 Arrows of Time

One of the most intractable and fundamental problems in physics is the relation between reversible fundamental (micro) physics laws, and irreversible macro-physics effects and associated phenomenological laws, with the various arrows of time (radiation, thermodynamical, quantum mechanical, biological for example) dominant in real physical applications [12]. It seems to be agreed by most that in one way or another this major discrepancy must reside in the difference between initial conditions and final conditions in the universe, which effectively disallow half of the solutions that are in principle allowed by the time-reversible microphysical equations (see e.g. [19, 26]). There is however an alternative view that maybe we are mistaken about the fundamental laws of physics - maybe they should be formulated in a fundamentally time-irreversible way [43]. This view has not attracted as much a following, despite the fact that the irreversibility of physics is clearly already evident in the quantum measurement process [26], and so is not solely a macro-physical phenomenon.

It is clear that the arrow of time is closely related to the definition and evolution of entropy. The relation of a phenomenological (macro) definition to microscopic properties is obtained via a process of coarse graining [44], through which the macro-description (given only in terms of macroscopic variables) explicitly loses information that is available in the detailed micro description (given in terms of microscopic variables); and then the basic quantitative question is how many different micro-states correspond to the same macro-state. A macro-state is more probable if it corresponds to a greater number of different micro states, and time evolution will tend to go from a less probable to a more probable state, defined in this way [26]. So far so good: this can be made precise in terms of Boltzmann’s H-theorem, proving the second law of thermodynamics for suitably defined entropy (defined as an integral over microstate occupancies) on the basis of microscopic physical laws (see [1] for a beautiful derivation of this result in the case of relativistic kinetic theory). But the problem is that this argument applies equally in both directions of time: it completely fails to determine which is the forward direction of time. It predicts the entropy will increase in both directions of time! The only plausible basis so far for making a choice, is that the local direction of time is determined by boundary conditions on the physical equations at the beginning (and perhaps also at the end) of the evolution of the universe [14, 28]. This seems to be the logical explanation - but how this master arrow of time uniquely determines the directions of each of the separate physical and biological arrows still needs convincing explication. This
is one of the most important unsolved problems in present day physics, for it represents a major gulf between the macro-physics that dominates everyday life, and the microphysics that (on a reductionist viewpoint) is supposed to explain that macrophysics. The small amount of attention paid to it is presumably due to the intractability of the problem.

The important application of this idea is in terms of the questions raised by Roger Penrose regarding the idea of an inflationary universe and the initial conditions for physical fields required in order to obtain a macroscopic second law of thermodynamics that agrees with the direction of the inflationary expansion. He argues that this will work only if the initial state is very special relative to a random state, where many black holes might occur with huge initial entropies. Hence the universe must start off from a highly special state corresponding to small Weyl tensor magnitudes and an initially isotropic expansion, if the arrow of time is to work out in terms of the gravitational field - in contrast to the random initial conditions suggested by many theories of inflation, for example chaotic inflation. It is something of a mystery that this argument does not seem to be given the attention it deserves. This may have more to do with the sociology of science than the merits of the theory. The strength of his remarks is strengthened by an appreciation of the trans-Planckian problem of some inflationary cosmology models: namely that the perturbations that are supposed to lead, on the inflationary view, to some large scale astronomical structures, can lie deep within the Planck era, where the linearised gravitational theory that is normally used simply does not apply; and indeed we might expect some kind of space-time foam description to be accurate. Amplifying such inhomogeneity through inflationary expansion to large scales would lead to anything but a smooth structure. This issue remains unresolved.

4.1 Gravitation and Entropy

A crucial element that is missing from our understanding of the growth of structure in the expanding universe, is a suitable definition of the entropy of an arbitrary gravitational field. Of course a huge amount has been written on the concept of entropy of a black hole; but that does not deal with the fundamentally important question of what entropy is associated with inhomogeneities of the gravitational field when no black holes are involved.

The importance of this topic is that it underlies the growth of astronomical structure (galaxies, stars, and planets) in the universe, which in turn allows our own existence. If you take the statements about entropy in almost every elementary textbook, and indeed most advanced ones, they are contradicted when the gravitational field is turned on and is significant. For example, in the famous case of the gas container split into two halves by a barrier, with all the gas initially on one side, the standard statement is that the gas then spreads out to uniformly fill the whole container when the barrier is removed, with the entropy correspondingly increasing. But when gravitation is turned on, the final state is with all the matter clumped in a blob somewhere in the
container, rather than being uniformly spread out. This is related to the issue of the negative specific-heat behaviour of gravitational systems, and the associated ‘gravithermal catastrophe’ \[50\]. The question then is whether there is a definition of entropy for the gravitational field itself (as distinct from the matter filling space-time), and if so if the second law of thermodynamics applies to the system when this gravitational entropy is taken into account. The answers are far from obvious. Dyson \[51\] claims there is no such entropy, while Penrose \[36, 26\] claims there is, and that it is related to some integral of the Weyl tensor on spacelike surfaces. A flurry of recent work has given local definitions of gravitational entropy in relation to the idea of ‘holography’ in the expanding universe \[52\], but that literature has not shown how this entropy behaves as structure is formed. Tavakol and Ellis (unpublished) have conjectured that gravitational entropy is related to the spatial divergence of the electric part of the Weyl tensor measured by an observer moving with 4-velocity \(u^a\), which certainly seems to have some of the desired properties, because of the Bianchi identity

\[ \nabla^a E_{ab} = \frac{1}{3} \nabla_b \rho \]

relating that divergence to the spatial gradient of the matter density \(\rho\) (in linearised gravitational theory), where \(\nabla\) is the covariant derivative operator orthogonal to \(u\). However that work is incomplete - we have not succeeded in showing some suitable function of this quantity has all the desirable properties for a gravitational entropy. Of course the problem is related to the well-known difficulties in obtaining local definitions of the mass of an isolated system in general relativity. Recent work by Ashtekar \[53\] may open the way here, and that in turn might provide a new approach to the entropy issue, because of the well-known relations between entropy and energy on the one hand, and between mass and energy on the other.

This issue remains one of the most significant unsolved problems in classical gravitational theory, for as explained above, even though this is not usually made explicit, it underlies the spontaneous formation of structure in the universe - the ability of the universe to act as a ‘self-organizing’ system where more complex structures evolve by natural processes, starting off with structure formed by the action of the gravitational field. If solved in a generic way, it would also presumably lead to another view of the nature of black hole entropy, for it would certainly have to tie in to that concept in a congruent way. It would presumably also relate to the idea of the coarse graining of the gravitational field, and hence to the whole difficult averaging problem in general relativity theory \[54\], which may not be approachable in a covariant and gauge invariant manner (cf. \[55, 56\]).

Given a suitable definition of gravitational entropy and a proof that it has the required local properties, the further issue that will remain is how this entropy can tie in to the master arrow of time given by the expansion of the universe. At one level, implicit answers to this are given by the standard theory of structure formation in the expanding universe \[11, 12\]; however, firstly this has not been
explicitly related to a general theory of gravitational entropy, and secondly, as
reported above, Penrose has raised major questions about whether the usual
explanation in terms of an inflationary model will in fact work, when one looks
at its implications for entropy [45].

5  Finite Infinity and Local Physics

In looking at the evolution of ‘isolated systems’, such as the solar system, our
Galaxy, or the local group of galaxies, it is common to use the idea of asymptotic
flatness as the setting for the local system, and to put boundary conditions on
local physical fields ‘at infinity’. This has been taken to a very high level of
sophistication [57], particularly by use of Penrose’s concept of conformal infinity
[58, 2]. But that description makes it very difficult to look at the relation
between such ‘isolated systems’ and the universe in which they are imbedded,
precisely because it ignores the structure of that universe. In the real universe,
there will probably be no asymptotically flat region at or ‘near’ infinity, for the
real universe is almost certainly not asymptotically Minkowskian at very large
distances, and indeed it may not be spatially infinite.

This led me some years ago to ask the question: ‘How far away is an effec-
tive ‘infinity’ to use in discussing boundary conditions for local physical systems of
this kind?’ Since that was written, answers have been given for the solar system
[59] (between 1.5 and 3 light years) and for the local group of galaxies [60]
(about 1.2 Mpc). So the obvious proposal [54] is that we should put boundary
conditions on all fields at that distance, rather than at infinity itself, leading
to the concept of a ‘finite infinity’ $F$: a smooth timelike surface at a large but
finite distance $r_*$ from the centre of the system considered, separating it from
the surrounding universe, and lying in an almost-flat space-time region at that
distance. Then incoming and outgoing radiation conditions can be imposed on
that surface $F$, rather than at infinity or conformal infinity $I$ as is usual [57].

One can then examine the physics of the interaction between the interior and
exterior regions by relating each to energy, momentum, matter, and any fields
that cross this surface. This timelike surface will, in conformally flat coordinates,
be an extremely long, thin tube (the radius of the tube for the solar system will
be about one light year, but we will want to consider incoming and outgoing
radiation for many hundreds of years when we look for example at the stability
of the solar system).

This idea has not been fully developed yet, except to some degree in the
case of the ‘swiss-cheese’ models of spherical inhomogeneities of the gravita-
tional field [61], but raises many interesting issues. One needs a clear definition
of when the bounding surface lies in an ‘asymptotically flat’ region around the
isolated system but at a finite distance from it, which can be defined by requir-
ing (i) existence of suitable local almost-flat coordinates in a neighbourhood of
the surface, and (ii) limits on the amount of matter and radiation present there.
In the cosmological context, (iii) the velocity of the surface must not be greatly
different from that of the cosmic background radiation, as otherwise that radi-
ation will be experienced as high intensity radiation as it crosses $F$, and so the system will no longer be ‘isolated’. As regards (i), we might for example require the existence of an atlas including a set of local coordinates $x^i = (t, r, \theta, \phi)$ in a region $U = I_1 \times I_2 \times S_2$ where $I_1 = (t_1, t_2)$, $I_2 = (r_1, r_2)$ are finite intervals with $r_* \in I_2$, such that (a) in $U$, the metric takes the almost-flat form

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + h_{ab}(x^i)dx^a dx^b, \quad |h_{ab}| << |g_{ab}| \quad (2)$$

(b) $U$ surrounds the finite system (either the coordinates $x^i$ in fact cover a larger region of space-time, including the isolated system and it centre, but is only asymptotically flat in the region $U$; or other coordinates cover the interior of $U$ and include the system), and (c) the exterior universe lies outside $U$ (usually this will require a separate coordinate system).

Such a surface will not be uniquely defined; if one such surface exists there will be an allowed family of such surfaces, related to each other by a rather large group of transformations - from (i), they must not be too far out, nor too far in; from (iii), their velocities must not be too different from each other; and they should be suitably smooth; apart from this, they can be chosen arbitrarily. One might allow a density discontinuity across this surface, as in the case of the Swiss-cheese models, in order to match a fluid-filled exterior to a vacuum interior; but that will need to be handled with caution (and would require a modification of the formulation of condition (i) given above). In that case, the exterior boundary must be chosen as comoving. The obvious choice of velocity for this family of surfaces will be that velocity field which sets the microwave background anisotropy dipole to zero; this will be comoving with the matter to a high degree of accuracy, and the degree to which this is not true will be an important statement about the nature of the interaction of the chosen system with the universe.

To develop the dynamical implications of this idea, one needs to develop the boundary value problem for all the fields involved, generalizing the usual proofs to a surface with both spacelike and timelike segments. In a major step forward, Helmut Friedrich and G Nagy [29] have developed this boundary value problem for the case of the vacuum gravitational field. They did not give a physical interpretation to the mathematical variables defined in their theorems, but such an interpretation should be forthcoming in terms of a preferred family of observers associated with the allowed family of surfaces $F$, particularly when the outer region is defined in a cosmological context. They also did not consider the implications when this surface lies in an almost flat region, as for example specified by Eqn. (2) above. The further challenge then will be to use a suitable definition of mass defined from data on this surface (cf. e.g. [22]) and then generalize to this setting the results by Bondi et al, Sachs, and Newman and Penrose relating incoming and outgoing radiation at infinity to mass loss and the ‘news function’ [33]. Furthermore the famous positive mass theorems [24] should also be generalized to this case. These will be more complex than the existing proofs with the limit taken at infinity, but in my view the added complication
will not be gratuitous, but will be essential to the physics of the situation. Additionally, the impact of the constraints on the allowed incoming and outgoing radiation needs careful analysis. This will then provide a rigorous setting for discussing the real physics of the interaction of the interior and the exterior regions, with physical estimates of the magnitude of incoming and outgoing radiation that crosses this surface, and limits on these quantities in order that the system can indeed be regarded as physically isolated.

This will then be the proper setting in which to look at the relation of local physics to cosmology, and hence to look at the other questions discussed in this paper - the Mach and Arrow of Time issues, for example (this kind of description is already implicit in the Olber’s type of calculations that are carried out). A useful step in this regard is a paper by Hogan and Ellis determining the radiation part of the electromagnetic field at such a surface at a finite distance from the source, in a flat background spacetime (and hence, by conformal transformation, in FLRW geometries). This is in contrast to usual definitions of the radiation part of the electromagnetic field, which use a limit at infinity. It would be useful to carry out similar calculations for the gravitational field, probably best based on the SWG integral formalism mentioned above.

This may also be the best setting for numerical calculations for ‘isolated systems’, which often talk about ‘integrating to infinity’, but in most cases do nothing of the sort. As in the rest of theoretical physics, it would be advantageous to have a theoretical framework that corresponds more closely to actual calculations - namely an integration to a surface at a finite distance from the centre of coordinates. It is usual to make that surface a null surface; the suggestion here is that it would be better to make it timelike, corresponding to the region in the real universe where the exterior is physically separated from the local system. A null surface does not work well in this context, hence the importance of Friderich’s and Nagy’s work on the mixed initial value problem. It might be useful seeing if a conformal version of their theory would be suitable for numerical work.

Overall, the point is that no system can be completely isolated; the context suggested here allows one to monitor the degree to which any local system is indeed isolated, and to examine the nature of its interaction with the external world - that is, with the rest of the universe. The criterion for ‘isolation’ will be a real physical one in terms of limits on incoming and outgoing effects (matter, radiation of all kinds, and tidal forces) across the separating surface \( F \), rather than statements on limits at an unattainable infinity, as has been customary up to now (for example in studies of gravitational radiation and of the Hawking effect). In my view this will make the analysis much more useful, and genuinely physical based in terms of relating to real estimates of the magnitude of these effects. The difference corresponds to the transition in mathematics between calculus based on infinite limits, and analysis based on \( \varepsilon \) and \( \delta \) bounds. I believe it will have the same kind of beneficial effects.

\[ \text{But see H"ubner for a conformal method that integrates to null infinity.} \]
5.1 Possibility of Newtonian Physics

A particular interesting point then, is what kinds of conditions on a surface $\mathcal{F}$ surrounding a local physical system will be required in order that that system can validly be described in terms of Newtonian physics (cf. [67] for an examination of the associated consistency conditions from another point of view). The point here is that too much interference from the outside will prevent a good Newtonian limit existing, for example a local system imbedded in a universe where high-intensity gravitational waves abound will not have a good Newtonian description. Thus there will be limits on the particles and gravitational waves crossing any bounding surface like $\mathcal{F}$, in order that such a description be possible.

It seems likely that existence of a surface $\mathcal{F}$ in an almost flat region at a finite distance, such as suggested above, may be sufficient to show the possibility of Newtonian-like behaviour (it will not guarantee it, since black holes may form inside such a surface). It is possible that such existence can only occur if the exterior universe is reasonably similar to a standard Friedmann-Lemaître-Robertson-Walker (‘FLRW’) model, and with almost co-moving boundaries chosen in the exterior region. That is, it seems likely that if the universe is not suitably close to a FLRW model, then no suitable surface $\mathcal{F}$ will exist and a Newtonian limit will not be attainable locally in such a universe. This is an idea that needs to be checked. Whether this is the case or not, the condition here is a significant one that needs investigation: when does a universe allow Newtonian-like behaviour in local regions? This is clearly an important aspect of how global structure affects the nature of local physics. This corresponds to the dual micro-question: when does a quantum system behave in an almost classical way, and what kind of cosmologies allow the emergence of classical regimes from the early quantum domain?

6 Initial Conditions and Local Physics

The setting just described enables one to distinguish matter and radiation that crosses the surface $\mathcal{F}$, in either an ingoing or outgoing direction, from matter which does not do so, and hence is associated with the isolated system on a long-term basis. Such matter relates the ‘isolated system’ to local initial conditions at the same position (as defined by comoving with matter) but at very early times, rather than to exterior fields at later times. This also applies to scalar and vector gravitational modes, for the crucial point was made above: the characteristics for these modes are timelike curves rather than null surfaces, and how they behave is determined by conditions near our world line at very early times. Thus the true domain of dependence of such modes is not the usual domain of dependence bounded by null curves, but rather a much smaller region close to the matter world lines.

The practical issue is that many local conditions at the present time are determined by conditions near our world line in the very early universe [21].
and this applies particularly to element abundances (based on nucleosynthesis) and baryon abundances (based on baryosynthesis). Each are relics of non-equilibrium phases in the early universe’s history. A specific set of such relics results from specific initial conditions, and determines the nature of what can exist locally at the present time (stars, planets, and living beings have to be constructed out of whatever matter is present locally). The power of the assumption of equilibrium physics in the early universe is that the nature of these relics is very largely independent of what existed at very early times, for whatever one might feed in will get transformed into an equilibrium mixture (this is the cosmological version of the famous statement that the nature of black holes is independent of what is put into them). Only conserved quantities will survive unchanged; and there are very few of those in the extreme conditions of the very early universe.

7 The Existence of Life

A key issue for the future is to clarify in more detail the relation of the nature of the universe to the existence of life, both in terms of initial conditions, and of the nature of the laws of physics. This is the highly contested terrain of the Anthropic Principle. Strangely, Dennis wrote rather little on this, but it has been a very active area and certainly is a legitimate concern within the broad terrain under discussion.

This concept has been regarded with considerable suspicion by many because of some rather unguarded or ill-thought out statements regarding the nature and application of the anthropic principle. There is a clear distinction between the Weak Anthropic Principle (‘WAP’) from the Strong Anthropic Principle (‘SAP’) [68, 69]. The former is an unobjectionable selection principle (‘we can only view the universe from space-time regions that allow our existence’), while the latter is a highly disputable philosophical claim (‘the universe must allow the existence of life’), argued on a number of different grounds, for example the need for observers to exist in order that quantum theory can make sense. The problem then is that firstly, some papers seem to argue the SAP case by an inversion of normal logic, for example the statement by Collins and Hawking at the end of an important study of Bianchi cosmologies that ‘the universe must exist because we are here’ [70]; and secondly there have been some attempts to stretch the concept into quite undefensible territory [71], in particular Barrow and Tipler’s Final Anthropic Principle (‘FAP’): ‘life not only must exist, but once it has come into existence must continue to exist until the end of the universe’ [8]. This dubious proposition led Gardner in a famous review to refer to the Completely Ridiculous Anthropic Principle (‘CRAP’).

These extremes are unfortunate, because they have obscured important arguments regarding the nature of the laws of physics and boundary conditions in the universe necessary to the existence of life. If one leaves aside the contentious claims, such as those mentioned above, one is left with an important selection principle that may indeed be essential both in terms of trying to explain the
value of the cosmological constant\footnote{72} and in relating concepts such as chaotic inflation to observations\footnote{46}. There is also an intrinsic interest in charting out what variations in physical laws will allow life to survive\footnote{73}. Thus in my view this is indeed an important part of the range of issues discussed in this article, and the need is to strictly separate out the unexceptional range of concerns that can be classed as WAP issues, and their use for example in terms of relating observations to chaotic inflation, from the whole range of controversial concerns raised under the SAP banner. These relate to fundamental metaphysical issues that are of a different nature than the issues pursued here; they are of course important, but are of a different character and need a different kind of discussion of a philosophical and metaphysical character\footnote{74} rather than relating to the strictly physical issues discussed in this paper.

The WAP issues are very much part of the theme of this paper. As they relate the rest of the arguments to questions concerning human life and existence, they are of major importance and interest to us as human beings as well as scientists.

8 Conclusion

As well as the major way that microphysics affects macrophysics in a ‘bottom-up’ way, there are many themes whereby there is a ‘top-down’ action of the cosmos as a whole on local physical systems. Some unsolved problems of physics may be related to this theme, in particular the ‘arrow of time’ issue which is still a major puzzle for theoretical physics. Many of the themes discussed here have practical applications in terms of being related to tests of cosmological theories, precisely because if the universe has an influence on local systems, then observing local systems tells us something about the universe. This range of themes remains of interest today; there is still interesting work to be done on them. These issues were amongst the driving forces of Dennis’ career, and he presented them with force on many occasions.

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