Perpendicular transmission of acoustic waves between two substrates connected by sub-wavelength pillars

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Abstract. We discuss theoretically the acoustic resonant transmission and zeros of transmission between two substrates connected by sub-wavelength pillars. The features of the transmission coefficient are explained in terms of the coupling of the incident waves with the Fabry–Perot oscillations inside the pillars and with the surface waves of both substrates. We discuss the dependence of the selective and zero transmission frequencies, in particular Fano resonances resulting from the proximity of a resonance to a zero of transmission, on the geometrical and physical parameters of the materials constituting the pillars and the substrate. These phenomena are studied in both one- and two-dimensional periodicities where the substrates are connected by a series of parallel plates or by a square lattice of cylindrical pillars, respectively. Finally, the calculation is extended to a periodic stacking of slabs and pillars that constitute a type of three-dimensional phononic crystal.

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1. Introduction

Since the original work of Ebbesen was published in 1998 [1], the extraordinary optical transmission through a metallic film patterned with periodical cylindrical holes has attracted a great deal of interest ([2] and references therein). The physical origin of this effect has been widely discussed in term of coupling between the surface plasmon polariton excitation with the Fabry–Perot resonances in the apertures. Over the last few years, the study of this phenomenon has been extended to acoustic waves that are incident on one-dimensional (1D) acoustic gratings with sub-wavelength slits [3–5] or on 2D panels periodically perforated with holes [6–8]. In those systems, a significant enhancement of the amplitude of the acoustic waves through very narrow apertures is seen. The origin of this phenomenon called extraordinary acoustic transmission [4, 5] or acoustic resonant transmission [3, 6–8] is because of the complex interplay of guided modes inside apertures and structurally induced waves on the surface of the gratings. Besides the acoustic resonant transmission, acoustic shielding can also be achieved, over a wide range of wavelengths [9–11].

Fundamental applications of such structures have been highlighted, as a tunable phononic crystal consisting of double crystal slabs. Varying the distance between the two plates permits us to tailor the bandwidth of a broadband sound blockage [12–14]. Sound collimation was also achieved by engineering a single slit through a perfect slab with corrugated surfaces [15–17]. Zhu et al [18] made the demonstration that a holey-structured metamaterial can act as a nearly perfect imaging device by reproducing the deep sub-wavelength information of an object.

It is worth noting that in all these previous works the embedding medium is a fluid and the acoustic resonant transmission or screening phenomena take place through holes or slits in a membrane. The aim of this paper is to consider the opposite situation where the transmission occurs between two solid substrates across a periodic array of sub-wavelength pillars. We discuss the existence of selective and rejective transmissions and the possibility of Fano resonances as a function of the geometrical and physical parameters of the constituting materials (a preliminary account of this work is presented in the conference papers [19]). In section 2, we present the model and the method of calculation, namely the finite difference time domain (FDTD) method, to obtain the transmission spectra. In section 3, we investigate the normal incidence acoustic transmission between two substrates connected by either 1D or 2D periodic arrays of pillars composed, respectively, of parallel plates or a square lattice of cylindrical pillars. Then we extend our calculation to the transmission across a 3D phononic...
Figure 1. (a) Schematic view of two substrates connected by a 1D periodic sub-wavelength grid with period $a$. The white arrows indicate the normal incidence of the incoming elastic wave. (b) Representation of the elementary unit cell composed of a rectangular plate of width $d$ and height $h$ sandwiched between two homogeneous silicon substrates.

The silicon is taken as a cubic material with elastic constants $C_{11} = 165.7$ GPa, $C_{12} = 63.9$ GPa and $C_{44} = 79.62$ GPa and mass density $\rho = 2331$ kg m$^{-3}$. For the purpose of the FDTD simulations, the hollow parts of the structure are filled with air of density $\rho_{\text{air}} = 1.4$ kg m$^{-3}$ and velocity $340$ m s$^{-1}$. Indeed, in these calculations, the continuity between the solid and air media replaces the use of vanishing normal stresses when the solid material is in contact with vacuum.

2. Model and method of calculation

Figure 1 is a schematic view of two silicon substrates connected by a 1D acoustic grid of period $a$ composed of rectangular silicon plates. The case of a 2D periodicity is sketched in figure 7. The silicon is taken as a cubic material with elastic constants $C_{11} = 165.7$ GPa, $C_{12} = 63.9$ GPa and $C_{44} = 79.62$ GPa and mass density $\rho = 2331$ kg m$^{-3}$. For the purpose of the FDTD simulations, the hollow parts of the structure are filled with air of density $\rho_{\text{air}} = 1.4$ kg m$^{-3}$ and velocity $340$ m s$^{-1}$. Indeed, in these calculations, the continuity between the solid and air media replaces the use of vanishing normal stresses when the solid material is in contact with vacuum. Figure 1(b) shows the elementary unit cell, which is composed of a rectangular plate of width $d$ and height $h$ sandwiched between two homogeneous media.

The transmission spectra presented in the following sections are performed with the help of a homemade FDTD code. In our calculations, the unit cell is discretized in two or three directions of the space, depending on whether the periodicity is in one or two directions. We use a mesh interval equal to $a/100$, which corresponds to the necessary spatial interval for a good definition of the solid–vacuum boundaries and the convergence of the results. The equations of motion are solved taking into account all the components of the displacement field, i.e. $U_x$, $U_y$ and $U_z$ in 3D structures with periodicity in two directions (see figure 7), or $U_x$ and $U_z$ in 2D simulations corresponding to a 1D periodicity (figure 1). The time integration step is defined by $\Delta t = \Delta x/(4c_l)$, where $c_l$ is the longitudinal velocity of sound in silicon, and the number of time steps is equal to $2^{21}$, which is the necessary time for good convergence of the numerical calculations.

As seen in figure 1, the $z$-axis corresponds to the direction of propagation, perpendicular to the grid, and the $x$-axis is chosen along the grid. Periodic conditions are applied on each side of the 2D box in the $x$-direction. As the box is finite along $z$, perfect matching layers are
Figure 2. Normal incidence transmission spectrum through the silicon grid embedded between two substrates, with an elementary rectangular bar of dimensions \( d = 0.2a \) and \( h = 1.4a \). The colored arrows represent three specific features which appear in the transmission spectrum: the maxima and minima of the Fabry–Perot oscillations in the height of the plates (in green); periodic zeros of transmission (in red) and Fano resonances (in blue).

applied on the top and bottom of the unit cell. A broadband wave packet is launched from the top, in the incident medium, in front of the periodic grid. This wave is a longitudinal pulse, with a polarization and Gaussian profile along the \( z \)-axis but uniform in the \( x \)-direction. The transmitted signal is recorded as a function of time in the outgoing medium, after the grid, and integrated along the \( x \)-axis over one unit cell for the component \( U_z \) of the displacement field. Finally, the signal is Fourier transformed and normalized by an equivalent signal propagating through a homogeneous silicon bulk to yield the transmission coefficient. In all the transmission curves presented in the paper the frequencies are given in the dimensionless unit \( \Omega = \omega a/2\pi c_t \), where \( c_t = 5844 \text{ m s}^{-1} \) is the transverse velocity of sound in silicon.

3. Transmission spectra in normal incidence

3.1. One-dimensional (1D) periodicity

Figure 2 represents a general view of the transmission spectrum obtained for a normal incident wave launched in front of the 1D periodic silicon plates (figure 1). The dimensions of the rectangular plates are \( d = 0.2a \) and \( h = 1.4a \) and, in particular, their thicknesses are taken to be small as compared to the period.

Three distinct features can be observed in the transmission spectrum.

The regular oscillations appearing in the low-frequency part of the spectrum can be associated with the excitation of Fabry–Perot resonances along the height of the plates. Figure 3 represents the evolution of these oscillations as a function of the geometrical parameters of the plates, i.e. \( (h/a) \) and \( (d/a) \). When we increase the height, the oscillations shift to lower frequencies, whereas decreasing the width of the plate enhances the amplitude of the oscillations. The frequencies of the peaks and their separation are closely related to the height \( h \) and, as a consequence, to the nature of the plates, but almost independent of the width of
the plate, the nature of the substrates and the period \(a\). The separation between these peaks would decrease if one increased the height of the plates or decreased their acoustic velocity. Two examples of the displacement component, corresponding respectively to a peak (point A) and a dip (point B) in the transmission coefficient, are presented in figures 4(a) and (b). For these two modes, mainly the longitudinal component \(U_z\) of the displacement field is different from zero. In both cases, the displacement field displays a strong enhancement inside the vertical plates, which results either in a full transmission (point A) or a significant rejection (point B).

A second remarkable feature of the spectrum of figure 2 is the existence of periodic zeros of transmission occurring at the reduced frequencies \(\Omega = 0.85, 1.7, 2.6 \text{ and } 3.4\). These frequencies are dependent upon the properties of the substrate but are not much affected by changing the properties of the vertical plates (provided they remain relatively thin as in figure 2). These zeros of transmission can be associated with the excitation of a surface mode at the boundary of the upper substrate as shown in the displacement field for both the longitudinal \((U_z)\) and transverse \((U_x)\) components of point C (figure 4(c)). One can note an enhancement of the field in the vicinity of the surface, whereas the wave does not penetrate inside the plates. Such an excitation can be explained by the fact that, at sufficiently high frequency, the normal incident wave with a wave vector \(k_{||}\) (parallel to the interfaces) equal to zero, can be coupled to surface modes having a wave vector equal to a reciprocal lattice vector, namely \(p(2\pi/a)\), where \(p\) is an integer. This explains the periodic occurrence of the zeros of transmission. In addition, one can check that their frequencies shift by changing the period \(a\) of the structure. These frequencies are only slightly dependent upon the properties of the vertical plates as far as the latter remain thin.
The third remarkable feature appears when a resonance of the structure becomes very close to a zero of transmission, yielding a very narrow and sharp Fano resonance corresponding to a high selective transmission, such as point D in figure 2. The displacement field of the latter mode, presented in figure 4(d) for the longitudinal ($U_z$) and transverse ($U_x$) components, shows an enhancement of the acoustic wave inside the plates and in the vicinity of the surfaces of both substrates. Therefore, it corresponds to a coherent coupling between the diffracted waves excited on both surfaces and the Fabry–Perot resonant modes inside the junctions. The quality factor of the Fano resonance is very sensitive to the proximity of the resonance with the zero of transmission. This is illustrated in figure 5 where the frequency of the resonance is shifted by changing slightly the height of the pillars. By increasing $h$ from $1.4a$ to $1.7a$, the Fano resonance is shifted to lower frequencies and traverses the zero of transmission, which remains independent of $h$.

Finally, above some threshold frequency, the transmission spectrum displays randomly fast oscillations. It is likely that this behavior happens when the plates can support transversely excited modes whose number increases when going to higher frequencies. It is also worth noting that in the case of full transmission, the ratio of transmission to unit area reaches 5 in the above calculations. Actually, this factor can even be increased further if we significantly decrease the area of the apertures. We will not discuss in detail the latter situations, which require a very fine mesh in the simulations and therefore increase the computation time too much.
Figure 5. Evolution of the normal transmission curves as a function of the height \( h/a \) of the bar of constant width \( d/a = 0.2 \). The Fano resonance shifts to low frequencies and crosses the zero of transmission (red dashed line) when \( h/a \) is increased.

To highlight the above trends, we present in figure 6 the transmission coefficients after the materials of either the substrates or plates are changed. Besides silicon, we use steel with the following elastic parameters: \( c_l = 5825 \text{ m s}^{-1}, c_t = 3227 \text{ m s}^{-1} \) and \( \rho = 7780 \text{ kg m}^{-3} \).

Figure 6(a) recalls the results already presented in figure 2 in the frequency range up to \( \Omega = 1 \). When the material of the plates is changed from silicon to steel (figure 6(b)), the zero of transmission is not shifted since its frequency is essentially related to the substrate material, but the Fabry–Perot resonances inside the plates become closer to each other. This is understandable owing to the lower values of acoustic velocities in steel than in silicon. This also produces a shift of the Fano resonance, which is now slightly more separated from the zero of transmission than in figure 6(a). In contrast, when the substrates are made of steel and the plates are made of silicon (figure 6(c)), one recovers the first Fabry–Perot oscillation as in figure 6(a) but the position of the zero of transmission is now shifted according to the acoustic velocities of the substrate (from \( \Omega = 0.85 \) to \( \Omega = 0.51 \)).

Finally, figure 6(d) illustrates the case of two different substrates. Now, the transmission spectrum displays two zeros occurring respectively at \( \Omega = 0.85 \) to \( \Omega = 0.51 \) that support the relationship between the zeros of transmission and localized waves at the surfaces of both
In this example there are no Fano resonances resulting from the proximity of a resonance mode with one of the transmission zeros. Let us also note that in this case the maxima of transmission do not reach unity because part of the incident wave is always reflected.

The general trends obtained from the above discussions are the following. The Fabry–Perot oscillations are strongly dependent upon the nature of the plates and their heights, but almost independent of the nature of the substrates. The zeros of transmission are linked to the nature of the substrates, and almost independent of the material constituting the plates as far as the plates remain thin as compared to the period. Finally, the Fano resonance exists only when the two substrates are similar and its frequency depends on both the nature of the substrates and of the plates.

3.2. 2D periodicity

In this section, we discuss the characteristic features of the transmission between two substrates across a 2D array of cylindrical pillars in the square lattice geometry (figure 7(a)). The unit cell
Figure 7. (a) Geometry of two substrates connected by a periodic subwavelength array of cylindrical pillars with period $a$. The white arrows indicate the normal incidence of the incoming elastic wave. (b) Representation of the elementary unit cell constituted of a pillar of diameter $d$ and height $h$ sandwiched between two homogeneous silicon substrates. (c) Normal incidence transmission spectrum with silicon pillars of dimensions $d/a = 0.4$ and $h/a = 1.4$. The colored arrows represent the three specific features appearing in the transmission spectrum: the maxima and minima of the Fabry–Perot oscillations in the pillars (in green); the zeros of transmission (in red) and the Fano resonances (in blue).

of the structure is shown in figure 7(b) where $d$ and $h$ are the diameter and the height of the pillars, respectively.

A typical transmission spectrum is shown in figure 7(c) with the geometrical parameters $h/a = 1.4$ and $d/a = 0.4$. The following remarkable features can be noted.

1. The low-frequency oscillations are associated with the Fabry–Perot resonances in the height of the pillars similarly to the 1D case.

2. Zeros of transmission, shown by red arrows in figure 7(c), can occur due to the coupling of the incident wave with the surface acoustic waves of the substrate. Since the incident wave has a wave vector $k_{||} = 0$, the excited surface wave should correspond to a $k_{||}$ equal to one reciprocal lattice vector. Since we are dealing with a 2D square lattice geometry, the selected wave vectors $k_{||}$ will be in units of $2\pi/a$: $1$, $\sqrt{2}$, $2$, $\sqrt{5}$, $\sqrt{8}$, $3$, $\sqrt{10}$, $\ldots$. These are associated with the frequencies $\Omega = 0.83$, $1.20$, $1.64$, $1.87$, $2.34$, $2.5$, and $2.71$, respectively, as shown by red arrows in figure 7(c).

3. Sharp Fano resonances, indicated by blue arrows in figure 7(c), can appear when a resonance falls in the vicinity of a zero of transmission. In this case, the incident wave
Figure 8. (a) Schematic representation of the stacking structure made up of a periodic repetition of slabs of thickness $e$ and arrays of pillars. (b) Transmission curves for the parameters $h/a = 1.4$, $d/a = 0.2$ and $e/a = 3.0$ and the number of layers of pillars is $N = 1–4$.

produces the excitation of the Fabry–Perot resonances in the height of the pillars together with the excitation of surface waves of both substrates. The quality factor of the Fano resonance can be tuned by changing the height of the pillars as in figure 5.

Let us also note that in the case of full transmission, the ratio of transmission to unit area reaches a factor of 8 in the above calculations, which can be actually increased by decreasing the area of the pillars.

3.3. 3D phononic crystal composed of a stacking of slabs and pillars

The previous effects can be enhanced and/or modulated if the space between the substrates contains a 3D phononic crystal composed of an alternating repetition of slabs and pillars along the vertical direction (figure 8(a)). For the sake of simplicity, we assume that the pillars are in the 1D geometry presented in section 3.1 (figure 8(a)). The thickness of each silicon plate is $e$ and the number of periods is denoted as $N$. The corresponding transmission spectra are presented.
Figure 9. Longitudinal component of the displacement fields of modes A, B and C in figure 8 for $N = 4$.

in figure 8(b) for the geometrical parameters $h/a = 1.4$, $d/a = 0.2$ and $e/a = 3.0$ and for a stacking structure containing one to four layers of pillars.

The case $N=1$ is the one already reported in section 3.1 (figure 2), displaying low-frequency Fabry–Perot oscillations, a zero of transmission at $\Omega = 0.85$ and the Fano resonance. By increasing the number of periods $N$, one can observe the formation of pass bands and band gaps as is usually the case in a phononic crystal, with the apparition of additional oscillations in the transmission coefficient inside each band due to the periodicity of the phononic crystal in the vertical direction. However, one set of pass bands (indicated by green arrows) originates from the initial Fabry–Perot oscillations in the pillars and is already present in the case $N = 1$. The second set, indicated by purple arrows, starts to form for $N > 1$ and is due to Fabry–Perot oscillations inside the silicon slabs of thickness $e$.

In figure 9, we show the displacement field associated with modes A and B in figure 8 (the case $N = 4$). For these two sets of peaks, only the longitudinal component is different from zero. It can be noted that mode A corresponds to an enhancement of the field inside the pillars, whereas mode B is associated with an enhanced vibration inside the silicon slabs. In addition, we have checked that the frequencies of the first set of bands is very sensitive to the physical parameters and heights of the pillars and almost independent of the thickness $e$ of the slabs, whereas the second set of bands can easily be shifted by changing the thickness $e$. The two sets of bands can be tuned independently by varying the parameters of either slabs or pillars, leading to the possibility of a separation, an interaction or a modification of the shape of the transmission bands. For instance, the band at $\Omega = 0.4$ remains well separated from its neighboring bands, whereas the band at $\Omega = 0.74$ results from a superposition of two bands coming from each set.

Finally, one can note that the zero of transmission (occurring at $\Omega = 0.85$) remains unchanged when $N$ increase from 1 to 4. The same holds for the Fano resonance which
becomes sharper as long as $N$ increases and gives rise to a very selective transmission peak. The longitudinal displacement field of the latter (mode C in figure 8) is displayed in figure 9(c) and shows an enhancement of the acoustic wave both inside the pillars and in the vicinity of the surfaces of all slabs as in the monolayer example (figure 4).

4. Conclusion

In conclusion, we have analyzed several fundamental properties of normal transmission occurring between two solid substrates, across a periodic array of sub-wavelength pillars. Two different pillars have been considered: rectangular plates and cylindrical pillars. In both cases, we have shown that it is possible to achieve low-frequency oscillations, zero of transmission and Fano resonance peak. The low-frequency oscillations correspond to the Fabry–Perot resonances in the pillar. The frequency and amplitude of the oscillations can be tuned by the variation of the height and width of the pillar, respectively. The second remarkable effect corresponds to zero of transmission, which has been explained as a coupling between the normal incident wave and the surface mode of the substrate. Finally, we found a high selective transmission, identified as a Fano resonance peak. The origin of this sharp peak corresponds to a coherent coupling between the diffracted waves excited on both the surfaces and the Fabry–Perot resonant modes inside the junctions. All these effects have been enhanced considering several periodic arrays of sub-wavelength pillars separated by silicon plates. Depending on the frequency domain, prospective applications of these mechanisms can be anticipated such as selective filters, sound blockers, sensing effect or the management of the thermal conductivity.

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