EXAMPLES OF STRONGLY RIGID COUNTERABLE (SEMII)HAUSDORFF SPACES

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EXAMPLES OF STRONGLY RIGID COUNTABLE (SEMI)HAUSDORFF SPACES

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Abstract. A topological space $X$ is strongly rigid if each non-constant continuous map $f : X \to X$ is the identity map of $X$. A Hausdorff topological space $X$ is called Brown if for any nonempty open sets $U, V \subseteq X$ the intersection $U \cap V$ is infinite. We prove that every second-countable Brown Hausdorff space $X$ admits a stronger topology $T'$ such that $X' = (X, T')$ is a strongly rigid Brown space. This construction yields an example of a countable anticom pact Hausdorff space $X$ which is strongly rigid. By the same method, we construct a strongly rigid semi-Hausdorff $k$-metrizable space containing a non-closed compact subset.

1. Introduction

A topological space $X$ is called

- rigid if every homeomorphism $f : X \to X$ coincides with the identity map of $X$;
- strongly rigid if every non-identity continuous map $f : X \to X$ is constant.

Proposition 1.1. Every strongly rigid space $X$ is connected.

Proof. Assuming that $X$ is disconnected, we can write $X$ as the union $X = U \cup V$ of two disjoint nonempty open sets $U$ and $V$. Choose any points $u \in U$ and $v \in V$ and consider the continuous map $f : X \to \{u, v\}$ such that $f^{-1}(u) = V$ and $f^{-1}(v) = U$. This map witnesses that the space $X$ is not strongly rigid. □
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