Positive and Negative Energy Symmetry and the Cosmological Constant Problem

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Abstract

The action for gravity and the standard model includes, as well as the positive energy fermion and boson fields, negative energy fields. The Hamiltonian for the action leads through a positive and negative energy symmetry of the vacuum to a cancellation of the zero-point vacuum energy and a vanishing cosmological constant in the presence of a gravitational field solving the cosmological constant problem. To guarantee the quasi-stability of the vacuum, we postulate a positive energy sector and a negative energy sector in the universe which are identical copies of the standard model. They interact only weakly through gravity. As in the case of antimatter, the negative energy matter is not found naturally on Earth or in the universe. A positive energy spectrum and a consistent unitary field theory for a pseudo-Hermitian Hamiltonian is obtained by demanding that the pseudo-Hamiltonian is $\mathcal{PT}$ symmetric. The quadratic divergences in the two-point vacuum fluctuations and the self-energy of a scalar field are removed. The finite scalar field self-energy can avoid the Higgs hierarchy problem in the standard model.

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1 Introduction

The cosmological constant problem is considered to be one of the major problems of modern physics [1]. The particle physics origin of the problem arises because of the quartic divergence of the zero-point vacuum energy in the presence of a gravitational field. The constant zero-point energy cannot be shifted to zero in the action due to the universal coupling of gravity to energy including vacuum energy. It is hoped that a natural symmetry exists that explains why the cosmological constant is zero or small. However, the obvious candidates, supersymmetry and conformal invariance,
cannot supply this solution, for they are badly broken in Nature. We shall introduce a positive and negative energy symmetry of particles that removes the generic infinity of the zero-point vacuum energy in the presence of a gravitational field.

Kaplan and Sundrum \[2\] have introduced a discrete parity symmetry which transforms positive energy into negative energy through a projection operator $P$. As in the case of Linde’s \[3, 4\] two–universe proposal, the authors postulate that to avoid a breakdown of the vacuum due to the negative energy particles, the two copies of the standard model matter fields, corresponding to positive and negative energy particles, interact only weakly through gravity. To prevent excessively rapid decay of the vacuum, it is also postulated that gravitational Lorentz invariance breaks down at short distances. ’t Hooft and Nobbenhuis \[5\] have studied a symmetry argument exploiting a complex space with both positive and negative energy particles.

A local quantum field theory with positive and negative energy particle symmetry was recently developed \[6\], following a method of quantizing fields introduced by Dirac in 1942 \[7\] and investigated in detail by Pauli \[8\] and Sudarshan and collaborators \[9, 10, 11\]. We investigate a method of field quantization motivated by Bender and collaborators \[12, 13, 14, 15, 16, 17\] that can implement a real energy spectrum for a pseudo-Hermitian Hamiltonian, involving an indefinite metric in Hilbert space, and a unitary $S$-matrix.

The stability of the vacuum is assured by postulating a positive energy and a negative energy sector, which are identical copies of the standard model of particles. The positive energy and negative energy particles called positons and negatons, respectively, only interact weakly through the gravitational field. Moreover, negative energy matter in the form of negaton particles does not occur naturally in the universe. A negative energy “shadow” universe could exist separated from a positive energy universe without being annihilated.

The positive and negative energy symmetry leads to a cancellation of the zero-point vacuum energy in the presence of a gravitational field and to the vanishing of the cosmological constant. Possible corrections to the vanishing of the cosmological constant due to interaction with the gravitational field can be small for a finite quantum gravity.

A calculation of positon and negaton propagators leads to the cancellation of quadratic divergences in the Higgs self-energy and in two-point vacuum fluctuations.

\section{Indefinite Metric in Hilbert Space}

Dirac \[7, 8\] generalized field quantization by introducing an indefinite metric in the Hilbert space of the state vectors. The normalization of a state vector $\Psi$ is normally defined by

$$\mathcal{N}_+ = \int dq \Psi^* \Psi,$$

\begin{equation}
\end{equation}
where $\Psi^*$ is the complex conjugate of $\Psi$. The scalar product of two complex state vectors $\Phi$ and $\Psi$ is given by
\[ B_+ = \int dq \Phi^* \Psi. \] (2)

Instead, we consider the more general bilinear form
\[ B = \int dq \Phi^* \eta \Psi, \] (3)
in which the operator $\eta$ is an Hermitian operator to guarantee real normalization values.

The expectation value of an observable $O$ described by a linear operator is now defined by
\[ \langle O \rangle = \int dq \Psi^* \eta O \Psi. \] (4)

The generalization of the standard Hermitian conjugate operator $O = O^\dagger$ is given by the adjoint operator
\[ \tilde{O} = \eta^{-1} O^\dagger \eta^\dagger = \eta^{-1} O^\dagger \eta. \] (5)

All physical observables have to be self-adjoint, $\tilde{O} = O$, to guarantee that their expectation values are real. In particular, the Hamiltonian operator $H$ has to be self-adjoint, $\tilde{H} = H$, which has the consequence that
\[ \frac{d}{dt} \int dq \Psi^* \eta \Psi = i \Psi^* \eta (\tilde{H} - H) \Psi = 0, \] (6)
guaranteeing the conservation of the normalization with time.

A transformation of the Hermitian matrix $\eta$ to normal, diagonal form can have the values 1 or $-1$. The positive definite form yields a unit matrix and positive probabilities. However, in general positive eigenvalues can have negative probabilities, i.e. one can introduce negative probabilities that certain positive eigenvalues of an observable are realized. We shall discuss the possibility of obtaining a physical interpretation of quantum field theory in which the $S$-matrix is unitary and only a positive energy spectrum is observed when the pseudo-Hermitian Hamiltonian is quantized, and is invariant under parity $P$ and time reversal invariance $T$.

### 3 The Action and Field Quantization

The action takes the form ($c = \hbar = 1$):
\[ S = S_{\text{Grav}} + S_M(\phi_+) + S_M(\phi_-), \] (7)

where
\[ S_{\text{Grav}} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ (R - 2\Lambda_0) - (\overline{R} - 2\overline{\Lambda}_0) \right]. \] (8)

The $R$ denotes the normal Ricci scalar associated with positon gravitons, while $\overline{R}$ is associated with negaton gravitons. Moreover, $\Lambda_0$ and $\overline{\Lambda}_0$ denote the “bare”
cosmological constants corresponding to positon and negaton gravitons, respectively. The $\phi_+$ and $\phi_-$ fields denote positon and negaton matter fields, respectively.

We define an effective cosmological constant

$$\Lambda_{\text{eff}} = \Lambda_{0\text{eff}} + \Lambda_{\text{vac}},$$

where $\Lambda_{0\text{eff}} = \Lambda_0 - \Lambda_0$, $\Lambda_{\text{vac}} = 8\pi G \rho_{\text{vac}}$ and $\rho_{\text{vac}}$ denotes the vacuum density.

We expand the metric tensor $g_{\mu\nu}$ about Minkowski flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2),$$

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. We will study the lowest weak field approximation for which $\sqrt{-g} = 1$.

Let us consider as a first simple case a real scalar field $\phi(x)$ in the absence of interactions. The action is

$$S_{\phi} = \frac{1}{2} \int d^4x \left[ \partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2 \right],$$

and $\phi$ satisfies the wave equation

$$(\partial^\mu \partial_\mu + \mu^2)\phi = 0.$$ (12)

As is well known, this equation has both positive and negative energy solutions, as is the case with the Dirac equation [18, 19]. We have

$$k_0 \equiv \omega(k) = \pm \sqrt{|k|^2 + \mu^2}.$$ (13)

The equal time commutation relations for the field operator $\phi$ are

$$[\phi(x), \phi(x')] = [\pi(x), \pi(x')] = 0,$$

$$[\phi(x), \pi(x')] = i\delta^3(x - x'),$$

where $\pi = \dot{\phi}$ is the conjugate momentum operator.

We shall retain both positive and negative energy solutions and following Dirac [7] and Pauli [8], decompose $\phi$ into positive and negative energy parts:

$$\phi(x) = A(x) + \bar{A}(x).$$ (16)

The quantization of $A(x)$ with $-k_0x_0$ in the phase factor occurs in the usual way, corresponding to positive energy particles, while the other part with $+k_0x_0$ in the phase factor is quantized such that it leads to negative energy particles. We have for $t = 0$:

$$A(x) = \int \frac{d^3k}{((2\pi)^32k_0)^{1/2}} \{A_+(k) \exp[i(k \cdot x)] + A_-(k) \exp[i(-k \cdot x)] \},$$ (17)
The equal time commutation relations for the \( A \) operators assume that all the \( A_+(k), \tilde{A}_+ \) operators commute with the \( A_-(k), \tilde{A}_- \) and the \( A_+(k) \) operators with the \( A_+(k) \) and the \( A_-(k) \) with the \( A_-(k) \). Moreover, we have

\[
[A_+(k), \tilde{A}_+(k')] = \delta^3(k - k'), \quad [A_-(k), \tilde{A}_-(k') ] = -\delta^3(k - k').
\]

The equal time commutation relations (14) and (15) follow from the commutation relations for the \( A_\pm \) operators.

The Hamiltonian takes the form

\[
H(x) = \frac{1}{2} \int d^3 x \left[ (\nabla \phi(x))^2 + (\partial_0 \phi(x))^2 + \mu^2 \phi^2(x) \right].
\]

Substituting (19) and (20) into \( H(x) \), we get

\[
H(x) = \frac{1}{(2\pi)^3} \int d^3 k d^3 k' \exp[i(k + k') \cdot x] \left\{ -\frac{\sqrt{\omega(k)\omega(k')}}{4} B(k) + \frac{-k \cdot k' + \mu^2}{4\sqrt{\omega(k)\omega(k')}} C(k) \right\},
\]

where

\[
B(k) = [A_+(k) + \tilde{A}_-(k) - A_-(k) - \tilde{A}_+(k)]
\times[A_+(k') + \tilde{A}_-(k') - A_-(k') - \tilde{A}_+(k')],
\]

\[
C(k) = [A_+(k) + \tilde{A}_-(k) + A_-(k) + \tilde{A}_+(k)]
\times[A_+(k') + \tilde{A}_-(k') + A_-(k') + \tilde{A}_+(k')].
\]

By integrating \( H(x) \) we obtain

\[
H \equiv \int d^3 x H(x) = \frac{1}{(2\pi)^3} \int d^3 k \omega(k) [N_+(k) - N_-(k)],
\]

where

\[
N_+(k) = \tilde{A}_+(k) A_+(k), \quad N_-(k) = -\tilde{A}_-(k) A_-(k),
\]
denote the occupation numbers of the positons and negatons, respectively. The field momentum is given by

\[ P = \frac{1}{(2\pi)^3} \int d^3k [N_k^+ - N_k^-]. \]  

(28)

We see that the zero-point vacuum energy contributions corresponding to the infinite c-number \( \delta(0) \) have cancelled. The Fock vacuum is the state which satisfies

\[ A_k^+(|0\rangle) = 0, \quad A_k^-(|0\rangle) = 0, \]  

(29)
corresponding to the eigenvalue \( E_{\text{vac}} = 0 \).

In standard second quantized quantum field theory with only positon scalar fields, we have

\[ E = \sum_k k_0 \left[ \frac{1}{2} + N_k^+ \right]. \]  

(30)

For the vacuum (ground state), \( N_{+k} = 0 \), and we obtain the zero-point vacuum energy

\[ E_{\text{vac}} \equiv E_0 = \frac{1}{2} \sum_k k_0 = \frac{1}{2(2\pi)^3} \int d^3k \sqrt{|k|^2 + \mu^2}. \]  

(31)

The zero-point vacuum energy \( E_0 \) diverges quartically and is the root of the cosmological constant problem in the presence of a gravitational field, since graviton loops can couple to the vacuum energy “bubble” graphs which cannot be time-ordered away, i.e. we cannot simply shift the infinite constant vacuum energy, \( E_0 \), such that only \( E' = E - E_0 \) is observed.

An alternative quantization procedure consists of defining besides the field \( \phi(x) \) another scalar field \( \chi(x) \), the adjoint of which is \( \bar{\chi}(x) = -\chi(x) \) [7, 8]. Then we have

\[ \chi(x) = \frac{1}{\sqrt{2}} [A(x) - \tilde{A}(x)], \]  

(32)

with the Fourier decomposition

\[ \chi(x) = V^{-1/2} \sum_k (2k_0)^{-1/2} \{ \bar{\chi}(k) \exp[i(k \cdot x - k_0 x_0)] - \chi(k) \exp[i(-k \cdot x + k_0 x_0)] \}. \]  

(33)

The \( \chi \) field is quantized according to

\[ [\chi(k), \bar{\chi}(k)] = -1, \]  

(34)

which gives

\[ \bar{\chi}(k)\chi(k) = -N_{\chi}(k). \]  

(35)

The Hamiltonian is now given by

\[ H = \frac{1}{2} \int d^3x \left[ (\nabla \phi)^2 + (\partial_0 \phi)^2 + \mu^2 \phi^2 - (\nabla \chi)^2 - (\partial_0 \chi)^2 - \mu^2 \chi^2 \right]. \]  

(36)
This leads to the energy
\[ E = \sum_k k_0 [N_\phi - N_\chi]. \]  
(37)

As before, the positive and negative energy symmetry of the vacuum state leads to the cancellation of the zero-point vacuum energy, \( E_0 \). We have
\[ \phi(k) = \frac{1}{\sqrt{2}}[A_+(k) + \tilde{A}_-(k)], \quad \tilde{\phi}(k) = \frac{1}{\sqrt{2}}[\tilde{A}_+(k) + A_-(k)], \]  
(38)
\[ \chi(k) = \frac{1}{\sqrt{2}}[\tilde{A}_+(k) - A_-(k)], \quad \tilde{\chi}(k) = \frac{1}{\sqrt{2}}[A_+(k) - \tilde{A}_-(k)]. \]  
(39)

A spinor field \( \psi \) satisfies in the presence of an electromagnetic interaction the Dirac equation
\[ [\gamma^\mu(p_\mu - eA_\mu) - m] \psi = 0, \]  
(40)
and its charge conjugate equation
\[ [\gamma^\mu(p_\mu + eA_{c\mu}) - m] \psi_c = 0. \]  
(41)
These equations yield both positive and negative energy solutions of the Dirac equation. The two spinor fields \( \psi \) and \( \psi_c \) and the two photon fields \( A_\mu \) and \( A_{c\mu} \) are associated with positive and negative energy fermions and neutral gauge fields, respectively. We introduce the positive and negative energy spinor field
\[ \psi(x) = \Psi(x) + \tilde{\Psi}(x), \]  
(42)
and for the photon field we define
\[ A_\mu(x) = U_\mu(x) + \tilde{U}_\mu(x). \]  
(43)
For the charge conjugation transformation we have
\[ \psi_c = C\gamma^0 \psi^* = C\overline{\psi}^T, \]  
(44)
where \( C \) is the charge conjugation matrix which satisfies
\[ C^{-1} \gamma^\mu C = -\gamma^{\mu T}, \]  
(45)
and \( \gamma^{\mu T} \) denotes the transpose of the Dirac \( \gamma \) matrix. A similar transformation exists for the gauge field \( A_\mu \) which transforms it into the anti-particle gauge field.

We have the spinor wave expansion
\[ \psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \sqrt{\frac{m}{E_p}} \left\{ b_{+,r}(p)u_{+,r}(p) \exp(-ip \cdot x) + \tilde{d}_{+,r}(p)v_{+,r}(p) \exp(ip \cdot x) 
+ b_{-,r}(p)u_{-,r}(p) \exp(-ip \cdot x) + d_{-,r}(p)v_{-,r}(p) \exp(-ip \cdot x) \right\}, \]  
(46)
where $E_p = \sqrt{|p|^2 + m^2}$. The conjugate momentum is $\pi(x) = i\bar{\psi}(x)$ and the $\psi(x)$ satisfy the anti-commutation relation

$$\{\psi(x, t), \bar{\psi}(x', t)\} = \delta^3(x - x').$$  

(47)

The Hamiltonian is given by

$$H = \int d^3x \bigl\{\bar{\psi}(x)(-i(\alpha \cdot \Delta + \beta m)\psi) = i \int d^3x \bar{\psi}(x)\partial_\mu \psi(x),$$  

(48)

where $\alpha$ and $\beta$ are Dirac matrices. The anti-commutation relations for the $b$ and $d$ operators are of the form

$$\{b_{+,r}(p), \bar{b}_{+,r}(p)\} = \delta_{rr}\delta^3(p - p), \quad \{d_{+,r}(p), \bar{d}_{+,r}(p)\} = \delta_{rr}\delta^3(p - p),$$  

$$\{b_{-,r}(p), \bar{b}_{-,r}(p)\} = -\delta_{rr}\delta^3(p - p), \quad \{d_{-,r}(p), \bar{d}_{-,r}(p)\} = -\delta_{rr}\delta^3(p - p).$$  

(49)

All the other anti-commutation relations of the $b_+, d_+, b_-, d_-$ vanish. We obtain for the energy:

$$E = \int d^3p E_p [N_{+}^{(+)}(p) - N_{+}^{(-)}(p) - N_{-}^{(+)}(p) + N_{-}^{(-)}(p)].$$  

(50)

The occupation numbers are defined by

$$N_{+}^{(+)}(p) = \bar{b}_{+}(p)b_{+}(p), \quad N_{+}^{(-)}(p) = \bar{d}_{+}(p)d_{+}(p),$$  

$$N_{-}^{(+)}(p) = -\bar{b}_{-}(p)b_{-}(p), \quad N_{-}^{(-)}(p) = -\bar{d}_{-}(p)d_{-}(p).$$  

(51)

The vacuum state is

$$N_{+}^{+}(p)|0\rangle = 0, \quad N_{-}^{+}(p)|0\rangle = 0, \quad N_{+}^{(-)}(p)|0\rangle = 0, \quad N_{-}^{(-)}(p)|0\rangle = 0.$$  

(52)

We see from (50) that the zero-point vacuum energy for fermions has cancelled. In the standard treatment of the quantization of Dirac spinors, a normal ordering of the operators results in the cancellation of the infinite zero-point vacuum energy. However, this entails subtracting a quartically divergent (infinite) constant from the actual energy. The cancellation of the vacuum energy, due to the positive and negative energy symmetry of the vacuum, does not invoke unphysical infinite constants. Moreover, in the presence of a gravitational field the normal ordering of the fermion operators is not valid.

The normalization of the state vector $\Psi$ is determined by $[8]$:

$$\mathcal{N} = \sum_{N_{+}(k), N_{-}(k)} (-1)^{\sum N_{-}(k)} \Psi^\ast(...N_{+}(k)\ldots, \ldots N_{-}(k)\ldots)$$

$$\times \Psi(...N_{+}(k)\ldots, \ldots N_{-}(k)\ldots) = \text{const.}$$  

(53)

This demonstrates that “negative probability” states will exist with an odd number of particles in states with negative energy. We shall see in the following section, how
we can quantize the field in the presence of interactions and avoid a catastrophic instability due to these negative probabilities and negative energy particles. In Section 5, we shall investigate how we can formulate the quantum field theory, so that we obtain a real and positive energy spectrum and a unitary $S$-matrix.

It can be shown that an equivalent quantization procedure for complex charged scalar fields, neutral vector gauge fields $A_\mu$, spin-2 graviton fields, can be derived that leads to the cancellation of the zero-point vacuum energy in the presence of a gravitational field. This is again due to the positive-negative energy symmetry of the vacuum state $|0\rangle$ for these fields.

4 Positive and Negative Energy Mirror Sectors

In standard second quantized field theory and in the current interpretation of the standard model, the creation and annihilation operators for a negative energy particle are interpreted as the annihilation and creation of a positive energy particle with the opposite charge $-|e|$ corresponding to an antiparticle. The negative energy particle vacuum is empty as is the vacuum of positive energy particles.

We must now assure that the visible positons are stable against decay into negatons and that the annihilation of positons and negatons does not destabilize the vacuum. The problem of the stability of matter when negative energy particles are included has been investigated [21, 22]. When an ordinary positive energy positon collides with a negaton, the positon energy can increase due to the compensating negative energy of the negaton. Since there is an infinite amount of phase space available, the rate of decay is infinite when arbitrarily high momenta are included; the negaton particles can have arbitrarily large negative energies.

We postulate the existence of a visible positon matter sector and a negative energy negaton matter “shadow” sector, which are identical copies of the standard model of particles. These two sectors only couple weakly through gravity.

Let us denote positon particles with positive energy by $\phi_+$ and negatons with negative energy by $\phi_-$. Consider the Feynman tree-graph decay of a positon particle $\phi_+$ into a positon $\phi_+$ plus a negaton $\phi_-:

$$\phi_+ \rightarrow \phi_+ + \phi_-.$$  

(54)

Neither this reaction nor the reaction

$$\phi_+ + \phi_- \rightarrow \phi_+,$$  

(55)

In previous work [6], we postulated that the vacuum was fully occupied by negative energy fermions and bosons. The negative energy bosons were required to satisfy parastatistics in an attempt to obtain a Pauli exclusion principle for the sea of negative energy bosons. However, any number of parabosons can occupy an antisymmetric quantum state, excluding the possible existence of a paraboson exclusion principle [20].

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can occur through standard model couplings. Similarly, the process

\[ e_+^{(-)} \rightarrow \mu_+^{(-)} + \nu_{e+} + \bar{\nu}_{\mu+} + \phi^{(0)}, \] (56)

is not allowed. Here, \( e_+^{(-)}, e_{-}^{(+)}, \mu_+^{(-)}, \bar{\nu}_{\mu+}, \phi^{(0)} \) denote the positive energy electron, positron, muon, and neutral negaton, respectively. Moreover, the energy conserving annihilation process

\[ e_+^{(-)} + e_+^{(+)} \rightarrow \gamma_+ + \gamma_-, \] (57)

where \( \gamma_- \) denotes the negaton photon. Similar reactions of positon quarks into negaton gluons, for example, are also forbidden.

We also postulate that negative energy matter is not found naturally on Earth and in the universe, except in vanishingly small quantities. This is similar to positive energy antimatter, which is also not found naturally in the universe and very briefly in vanishingly small amounts as the results of radioactive decay or cosmic rays. This avoids the catastrophic annihilation of positon and negaton matter and guarantees the meta-stability of the vacuum. The matter-antimatter symmetry was broken in the very early universe. In a similar way, the positon-negaton symmetry was also broken in the early universe, avoiding the instability of Minkowski spacetime. As in the breaking of matter-antimatter symmetry in the early universe, the breaking of positive-negative energy symmetry requires an explanation.

We must consider the possible decay of negative energy negaton particles. Consider the tree-graph decays of negatons:

\[ \phi_- \rightarrow \phi_{1+} + \phi_{2+}. \] (58)

This is forbidden since

\[ |0\rangle \rightarrow \phi_- + \phi_{1+} + \phi_{2+} \] (59)

is not allowed. But the decay into one negaton and one positon is allowed

\[ \phi_{1-} \rightarrow \phi_{2-} + \phi_+, \] (60)

provided that

\[ \phi_{2-} \rightarrow \phi_{1-} + \phi_. \] (61)

This reaction requires that \( m_2 > m_1 + m_\phi \). To avoid any catastrophic instability of negaton reactions, we postulate that the interaction of negaton particles with one another is weak, \( \alpha_{\text{neg}} \ll \alpha_w \), where \( \alpha_w \sim 10^{-6} \) denotes the electroweak standard model coupling constant.

Positon and negaton gravitons can interact with the visible positon matter and the shadow negaton matter. The negaton decay channel involving the smallest number of positons and a positon graviton is

\[ \phi_- \rightarrow g_+ + \phi_{1-} + \phi_{2-}, \] (62)
where $g$ denotes a graviton. An effective field theory with a cutoff $\Lambda_{\text{cut}}$ can lead to a sufficiently stable vacuum, provided that higher derivative couplings are suppressed by the cutoff \cite{21,22}.

These postulates lead to a meta-stable vacuum that has a life-time greater than the age of the universe, i.e., a lifetime greater than the Hubble time, $H_0^{-1} \sim 10^{60} M_P^{-1}$, where $H_0$ and $M_P$ denote the Hubble constant and the Planck mass, respectively.

In contrast to supersymmetry, we do not postulate new species of particles such as supersymmetry partners with differing spins and satisfying opposite particle statistics. The dual energy symmetry holds for all known species of particles and corresponds to a doubling of the degrees of freedom of known standard model particles.

## 5 Pseudo-Hermitian Hamiltonian and Unitarity

In standard quantum field theory the Hamiltonian is Hermitian, $H^\dagger = H$, and we are guaranteed that the energy spectrum is real and positive and that the time evolution of the operator $U = \exp(itH)$ is unitary and probabilities are positive and preserved for particle transitions. However, in recent years there has been a growth of activity in studying quantum theories with pseudo-Hermitian Hamiltonians, which satisfy the generalized property of adjointness, $\hat{H} = \eta^{-1} H^\dagger \eta = H$, associated with an indefinite metric in Hilbert space \cite{12,13,14,15,16,17}.

Spectral reality and unitarity can in special circumstances follow from a symmetry property of the Hamiltonian in terms of the symmetry under the operation of $\mathcal{PT}$, where $\mathcal{P}$ is a linear operator represented by parity reflection, while $\mathcal{T}$ is an anti-linear operator represented by time reversal. If a Hamiltonian has an unbroken $\mathcal{PT}$ symmetry, then the energy levels can in special cases be real and the theory can be unitary and free of “ghosts”. The operation of $\mathcal{P}$ leads to $x \to -x$, while the anti-linear operation of $\mathcal{T}$ leads to $i \to -i$. It follows that under the operation of $\mathcal{PT}$ the Hamiltonian $H$ in (23) is invariant under the $\mathcal{PT}$ transformation, which is necessary but not sufficient to assure the reality of the energy eigenvalues.

The proof of unitarity follows from the construction of a linear operator $\mathcal{C}$. This operator is used to define the inner product of state vectors in Hilbert space:

$$\langle \Psi | \Phi \rangle = \Psi^{\mathcal{CPT}} \cdot \Phi.$$  \hspace{1cm} (63)

Under general conditions, it can be shown that a necessary and sufficient condition for the existence of the inner product (63) is the reality of the energy spectrum of $H$. With respect to this inner product, the time evolution of the quantum theory is unitary. In quantum mechanics and in quantum field theory, the operator $\mathcal{C}$ has the general form

$$\mathcal{C} = \exp(Q) \mathcal{P},$$  \hspace{1cm} (64)

where $Q$ is a function of the dynamical field theory variables. The form of $\mathcal{C}$ must be determined by solving for the function $Q$ in terms of chosen field variables and field equations. The form of $\mathcal{C}$ has been calculated for several simple field theories, e.g.
\( \phi^3 \) theory and also in massless quantum electrodynamics with a pseudo-Hermitian Hamiltonian. The solution for \( C \) satisfies

\[
C^2 = 1, \quad [C, \mathcal{P}T] = 0, \quad [C, H] = 0. \tag{65}
\]

We shall not attempt to determine a specific generalized charge conjugation operator \( C \) in the present work.

It has also been shown that a special form of \( P \) leads to a Lorentz invariant scalar expression for the operator \( C \) \[10\].

### 6 Propagators and Vacuum Fluctuations

We can evaluate the field commutator for \( \phi(x) \):

\[
[\phi(x), \phi(x')] = \frac{1}{(2\pi)^3} \int \frac{d^3k d^3k'}{\sqrt{2\omega_k 2\omega_{k'}}} \left\{ [A_+(k), \tilde{A}_+(k')] \exp[-ik \cdot x + ik' \cdot x'] + [\tilde{A}_+(k), A_+(k')] \exp[i k \cdot x - i k' \cdot x'] + [A_-(k), \tilde{A}_-(k')] \exp[i k \cdot x + i k' \cdot x'] + [\tilde{A}_-(k), A_-(k')] \exp[-ik \cdot x - ik' \cdot x'] \right\}.
\]

We obtain the result

\[
\frac{1}{2} [\phi(x), \phi(x')] = i \Delta(x - x'). \tag{66}
\]

The equal time commutator of the \( \phi \) fields vanishes for space-like separation \( (x - x')^2 < 0 \) in agreement with Lorentz invariance and microscopic causality.

Let us define for charged scalar particles

\[
\phi(x) = \frac{1}{2} [\phi_1(x) + i \phi_2(x)], \tag{68}
\]

where

\[
(\partial_\mu \partial^\mu + \mu^2) \phi_1 = 0, \quad (\partial_\mu \partial^\mu + \mu^2) \phi_2 = 0. \tag{69}
\]

Then, the commutator for the charged scalar fields is

\[
\frac{1}{2} [\phi_i(x), \phi_j(x)] = i \delta_{ij} \Delta(x - x'). \tag{70}
\]

The commutators of \( \phi_i \) with \( \phi_j \) and \( \phi^*_i \) with \( \phi^*_j \) vanish.

We define the Feynman propagator for the positons and negatons:

\[
i \Delta_F(x' - x) = \langle 0 | \phi(x') \phi^*(x) | 0 \rangle \theta(t' - t) + \langle 0 | \phi^*(x) \phi(x') | 0 \rangle \theta(t - t'). \tag{71}
\]
We obtain the modified Feynman propagator:

\[ i \Delta_F(x' - x) = \frac{i}{(2\pi)^4} \int d^4k \left\{ \exp[-ik \cdot (x - x')] \left[ \frac{1}{k^2 - \mu^2 + i\epsilon} - \frac{1}{k^2 - \mu^2 - i\epsilon} \right] \right\}. \quad (72) \]

We have the result that

\[ (\partial_\mu \partial_\mu + \mu^2) \Delta_+ F(x' - x) = \delta^4(x' - x), \quad (73) \]
\[ (\partial_\mu \partial_\mu + \mu^2) \Delta_- F(x' - x) = \delta^4(x' - x), \quad (74) \]

where

\[ \Delta_+(x' - x) = \frac{1}{2} [\Delta_+(x' - x) + \Delta_-(x' - x)], \quad (75) \]
\[ \Delta_+(x' - x) = \frac{i}{2(2\pi)^4} \int d^4k \frac{\exp[-ik \cdot (x - x')]}{k^2 - \mu^2 + i\epsilon}, \quad (76) \]
\[ \Delta_-(x' - x) = -\frac{i}{2(2\pi)^4} \int d^4k \frac{\exp[-ik \cdot (x - x')]}{k^2 - \mu^2 - i\epsilon}. \quad (77) \]

We also find that

\[ i \Delta_F(0) = \frac{i}{2(2\pi)^3} \int d^4k \left[ \frac{1}{k^2 - \mu^2 + i\epsilon} - \frac{1}{k^2 - \mu^2 - i\epsilon} \right] = \frac{1}{16\pi^3} \int d^4k \delta(k^2 - \mu^2), \quad (78) \]

where we have used the identity

\[ \frac{1}{k^2 - \mu^2 + i\epsilon} - \frac{1}{k^2 - \mu^2 - i\epsilon} = -2i\pi \delta(k^2 - \mu^2). \quad (79) \]

We observe that \( i \Delta_F(0) \) has a finite value in contrast to the usual result for positons that \( i \Delta_F(0) \) diverges quadratically.

A calculation of the two-point vacuum fluctuations yields

\[ \langle 0 | \phi(x) \phi(x') | 0 \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2k_0} \left( \exp[-ik \cdot (x - x')] - \exp[ik \cdot (x - x')] \right). \quad (80) \]

We find that for \( x = x' \):

\[ \langle 0 | \phi^2(0) | 0 \rangle = \Delta(0) = 0. \quad (81) \]

Thus, due to the positive and negative energy symmetry, we avoid the quadratic divergence of the two-point vacuum fluctuations in standard second quantization for positons.
7 Resolution of the Cosmological Constant and Higgs Hierarchy Problems

We shall postulate that

$$\Lambda_{\text{eff}} = \Lambda_0 - \bar{\Lambda}_0 = 0.$$  \hspace{1cm} (82)

The vanishing of the zero-point vacuum energy in our quantum field theory, including higher-order graviton tree-graph couplings and loops, then assures that (82) is protected against all higher order radiative vacuum corrections.

If we assume that a spontaneous symmetry breaking of the positive and negative energy symmetry of the vacuum occurs, then this will create a small “observed”, effective cosmological constant $\Lambda_{\text{eff}}/8\pi G \sim (2 \times 10^{-3} \text{eV})^4$, needed to provide a cosmological constant explanation of the accelerating expansion of the universe [23, 24, 25]. However, it is possible that the accelerating expansion of the universe can be explained by a late-time inhomogeneous cosmological model [26, 27, 28], in which the cosmological constant $\Lambda_{\text{eff}} = 0$ and there is no need for a negative pressure “dark energy”. Moreover, we also have to account for small higher-order gravitational corrections.

Let us consider the scalar field theory with the interaction term:

$$S_I = \frac{\lambda}{4!} \int d^4x \phi^4(x).$$  \hspace{1cm} (83)

We shall use the propagator (72) in momentum space to calculate the self-energy of the scalar field:

$$\Sigma(p^2) \equiv \Sigma(0) = \frac{\lambda}{2(2\pi)^4} \int d^4k \left( \frac{1}{k^2 - m^2 + i\epsilon} - \frac{1}{k^2 - m^2 - i\epsilon} \right).$$  \hspace{1cm} (84)

We now obtain the result by using (79):

$$\Sigma(0) = -\frac{i\pi\lambda}{(2\pi)^4} \int d^4k \delta(k^2 - m^2).$$  \hspace{1cm} (85)

Transforming to Euclidean momentum space we have: $k^0 = ik^0$, $k^2 = k^2 + k^0 k^0$, $d^4k = id^4k'$ and $\int d^4k' = \pi^2 \int dk^2 k^2$ which yields

$$\Sigma(0) = \frac{\lambda m^2}{16\pi}.$$  \hspace{1cm} (86)

We see that $\Sigma(0)$ is finite and we avoid the quadratic divergence of the scalar field self-energy in standard QFT. This resolve the Higgs hierarchy problem in the standard model.

A Casimir vacuum energy has been experimentally observed. In our quantum field theory, the vanishing of the zero-point vacuum energy is only valid in the absence of material boundary conditions as are necessary for the Casimir effect [29].
When material boundary conditions such as the parallel metal plates required to perform the Casimir experiments are imposed, then we can no longer demand that the positive and negative energy symmetry of the vacuum state is preserved; the breaking of this symmetry of the vacuum will produce a non-vanishing zero-point energy effect.

Jaffe [29] points out that the Casimir effect gives no more or less evidence for the “reality” of the vacuum fluctuation energy of quantum fields than any other one-loop effect in quantum electrodynamics, e.g. the vacuum polarization effect associated with charges and currents in atomic physics. Like all other observable effects in quantum electrodynamics, the Casimir effect vanishes as the fine structure constant $\alpha$ goes to zero.

8 Conclusions

We have formulated a quantum field theory based on an indefinite metric in Hilbert space with a generalization of the Hermitian Hamiltonian operator $H = \hat{H}^\dagger$ to an adjoint operator $\tilde{H} = \eta^{-1}H^\dagger\eta$ and we have $\tilde{H} = H$. The quantization of fields in the presence of gravity is performed with a positive and negative energy particle interpretation, which leads to the cancellation of the zero-point vacuum energy due to the positive and negative dual energy symmetry of the vacuum. We have

$$H = H_+ + H_-,$$

where

$$H_+|0\rangle = E_{\text{vac}}|0\rangle, \quad H_-|0\rangle = -E_{\text{vac}}|0\rangle,$$

and

$$\langle 0|H|0\rangle = 0.$$

We postulate that the effective, classical “bare” cosmological constant $\Lambda_{\text{eff}} = 0$. The condition (89) leads to a protection of the vanishing of the effective cosmological constant, $\Lambda_{\text{eff}}$, from gravitational and external field quantum corrections. Any possible higher-order gravitational corrections will be expected to be small in a finite quantum gravity theory. This can be interpreted as a resolution of the cosmological constant problem. The scalar field self-interaction is finite and can resolve the standard model Higgs hierarchy problem.

The stability of the vacuum and positive energy standard model particles is assured by postulating two identical standard model particle sectors, a visible positon matter sector and a shadow negaton matter sector, which interact only weakly through gravitational interactions or the curved geometry of spacetime. A similar scenario has been proposed by Kaplan and Sundrum [2]. Possible observational consequences of this gravitational interaction between the two sectors for the early universe will be considered in a future publication.

The indefinite Hilbert space state vector metric can generate negative probabilities for the transitions of particles and violate the unitarity of the $S$-matrix. To
guarantee positive probabilities and the unitarity of transition and scattering am-
pitudes, we incorporate the $\mathcal{PT}$ operation on field operators and the action. The
generalized charge conjugation operator $\mathcal{C}$, introduced by Bender and collabora-
tors [12, 13, 14, 15, 17] is invoked to affirm that the energy spectrum for gravity
and the standard model particle theory is positive, and assure that probabilities are
positive and conserved and that the $S$-matrix is unitary. Further investigation of
the properties of the operator $\mathcal{C}$ that will guarantee a physical energy spectrum and
unitary $S$-matrix for the standard model and gravity will be presented elsewhere.

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