Higher twist effects in charmed-strange $\nu$DIS diffraction

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Abstract

The non-conservation of charmed-strange current in the neutrino deep inelastic scattering ($\nu$DIS) strongly affects the longitudinal structure function, $F_L$, at small values of Bjorken $x$. The corresponding correction to $F_L$ is a higher twist effect enhanced at small-$x$ by the rapidly growing gluon density factor. As a result, the component of $F_L$ induced by the charmed-strange current prevails over the light-quark component and dominates $F_L = F_{L}^{cs} + F_{L}^{ud}$ at $x \lesssim 0.01$ and $Q^2 \sim m_c^2$. The color dipole analysis clarifies the physics behind the phenomenon and provides a quantitative estimate of the effect.

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1 Introduction

Weak currents are not conserved. For the light flavor currents the hypothesis of the partial conservation of the axial-vector current (PCAC) [1] provides quantitative measure of the charged current non-conservation (CCNC) effect. [2]. The non-conservation of the charm and strangeness changing (cs) current is not constrained by PCAC. Here we focus on manifestations of the cs current non-conservation in small-x neutrino DIS. At small $x$ the color dipole (CD) approach to QCD [3, 4] proved to be very effective. Within this approach it is natural to quantify the effect of CCNC in terms of the light cone wave functions (LCWF)$^1$

$$\Psi \sim g\epsilon \nu j^\nu /\Delta E,$$

where $j^\nu = \bar{c}(k)\gamma^\nu(1 - \gamma_5)s(p)$, $\Delta E = E_q - E_p - E_k$ and $\epsilon^\nu$ is the four-vector of the so-called longitudinal polarization of the $W$-boson with the four-momentum $q$. Notice that $\epsilon^\nu \rightarrow q^\nu /Q$ for $Q^2 = -q^2 \rightarrow 0$.

The observable which is highly sensitive to the CCNC effects is the longitudinal structure function $F_L(x, Q^2)$ related, within the CD approach, to the quantum mechanical expectation value of the color dipole cross section,

$$F_L \sim Q^2 \langle \Psi |\sigma |\Psi \rangle.$$

Our finding is that the higher twist correction to $F_L$ arising from the cs current non-conservation appears to be enhanced at small $x$ by the BFKL [6] gluon density factor,

$$F_L^{cs} \sim \frac{m_c^2}{Q^2} \left(\frac{1}{x}\right)^\Delta.$$

The color dipole analysis reveals mechanism of enhancement: the ordering of dipole sizes

$$(m_c^2 + Q^2)^{-1} < r^2 < m_s^{-2}$$

typical of the Double Leading Log Approximation (DLLA) and the multiplication of log’s like

$$\alpha_s \log((m_c^2 + Q^2)/\mu_G^2) \log(1/x)$$

$^1$Preliminary results have been reported at the Diffraction’08 Workshop [5]
to higher orders of perturbative QCD. As a result, the component \( F_L^{cs} \) induced by the charmed-strange current
\[
F_L = F_L^{ud} + F_L^{cs}
\]
grows rapidly to small-\( x \) and dominates \( F_L \) at \( Q^2 \lesssim m_c^2 \) [7, 8].

## 2 CCNC in terms of LCWF

In the CD approach to small-\( x \) \( \nu \)DIS [9] the responsibility for the quark current non-conservation takes the light-cone wave function of the quark-antiquark Fock state of the longitudinal (\( L \)) electro-weak boson\(^2\). For Cabibbo-favored transitions the Fock state expansion reads
\[
|W_L^+⟩ = Ψ^{cs}|c\bar{s}⟩ + Ψ^{ud}|u\bar{d}⟩ + ...,
\]
where only \( u\bar{d}- \) and \( c\bar{s}-\)states (both vector and axial-vector) are retained.

In the current conserving eDIS the Fock state expansion of the longitudinal photon contains only \( S \)-wave \( q\bar{q} \) states and \( Ψ \) vanishes as \( Q^2 \to 0 \),
\[
Ψ(z, r) \sim 2δ_{L,-λ}Qz(1 - z) \log(1/εr).
\]
Here \( r \) is the \( q\bar{q} \)-dipole size and \( z \) stands for the Sudakov variable of the quark.

In \( \nu \)DIS the CCNC adds to Eq.(6) the \( S \)-wave mass term [11, 12]
\[
\sim δ_{L,-λ}Q^{-1} [(m ± μ)(1 - z)m ± zμ] \log(1/εr)
\]
and generates the \( P \)-wave component of \( Ψ(z, r) \),
\[
\sim iζ δ_{L,λ}e^{-i2λφ}Q^{-1}(m ± μ)r^{-1},
\]
where upper sign is for the axial-vector current, lower - for the vector one and \( ζ = 2λ \) - for the vector current and \( ζ = 1 \) - for the axial-vector one. Clearly seen are the built-in divergences of the vector and axial-vector currents \( \partial_νV^ν \sim m - μ \) and \( \partial_νA^ν \sim m + μ \). This LCWF describes the quark-antiquark state with quark of mass \( m \) and helicity \( λ = ±1/2 \) carrying fraction \( z \) of the \( W^+ \) light-cone momentum and antiquark having mass \( μ \), helicity \( \bar{λ} = ±1/2 \).
and momentum fraction $1 - z$. The distribution of dipole sizes is controlled by the attenuation parameter

$$\varepsilon^2 = Q^2 z (1 - z) + (1 - z) m^2 + z \mu^2$$

that introduces, in fact, the infrared cut-off, $r^2 \sim \varepsilon^{-2}$.

## 3 High $Q^2$: $z$-symmetric $c\bar{s}$-states

In the color dipole representation [3, 4] the longitudinal structure function $F_L(x, Q^2)$ in the vacuum exchange dominated region of $x \lesssim 0.01$ can be represented in a factorized form

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_W} \int dz d^2r |\Psi(z, r)|^2 \sigma(x, r) , \quad (9)$$

where $g$ is the weak charge, $\alpha_W = g^2/4\pi$ and $G_F/\sqrt{2} = g^2/m_W^2$. The light cone density of color dipole states $|\Psi|^2$ is the incoherent sum of the vector ($V$) and the axial-vector ($A$) terms,

$$|\Psi|^2 = |V|^2 + |A|^2 . \quad (10)$$

The Eq. (6) makes it obvious that for large enough virtualities of the probe, $Q^2 \gg m_c^2$, the $S$-wave components of both $F_L^{ud}$ and $F_L^{cs}$ in expansion (4) are dominated by the “non-

\footnote{for an alternative description of the $\nu$DIS structure functions see e.g. [10].}
partonic" configurations with \( z \sim 1/2 \) with characteristic dipole sizes \([13]\)

\[ r^2 \sim Q^{-2}. \]

In the CD approach the BFKL-\(\log(1/x)\) evolution \([6]\) of \(\sigma(x, r)\) in Eq.(9) is described by the CD BFKL equation of Ref.[14]. For qualitative estimates it suffices to use the DLLA (also known as DGLAP approximation \([15, 16]\)) Then, for small dipoles \([17]\)

\[ \sigma(x, r) \approx \frac{\pi^2 r^2}{N_c} \alpha_S(r^{-2}) G(x, r^{-2}), \]  

(11)

and from Eq.(9) it follows that

\[ F_{L}^{ud} \approx F_{L}^{cs} \approx \frac{2}{3\pi} \alpha_S(Q^2) G(x, Q^2), \]

(12)

where \( G(x, k^2) = x g(x, k^2) \) is the gluon structure function and \( \alpha_S(k^2) = 4\pi/\beta_0 \log(k^2/\Lambda^2) \) with \( \beta_0 = 11 - 2N_f/3. \)

The rhs of (12) is quite similar to \( F_{L}^{(e)} \) of eDIS \([15, 18]\) (see \([17]\) for discussion of corrections to DLLA-relationships between the gluon density \( G \) and \( F_{L}^{(e)} \)). Two \( S \)-wave terms in the expansion (4) that mimics the expansion (5) evaluated within the CD BFKL model of Ref.[19] are shown by dotted curves in Fig. 1. The full scale BFKL evolution of the \( \nu N \) structure function \( F_L(x, Q^2) \) with boundary condition at \( x_0 = 0.03 \) is shown in Fig. 2 of Ref.[20].

4 Moderate \( Q^2 \): asymmetric \( c\bar{s} \)-states and \( P \)-wave dominance

The \( S \)-wave term dominates \( F_L \) at high \( Q^2 \gg m_c^2 \). At moderate \( Q^2 \lesssim m_c^2 \) the \( P \)-wave component takes over (see Fig.1). To evaluate it we turn to Eq. (9). For \( m_c^2 \gg m_s^2 \) in Eq.(10),

\[ |V_L|^2 \sim |A_L|^2 \propto \left( \frac{m_c^2}{Q^2} \right) \varepsilon^2 K_1^2(\varepsilon r) \]

, where \( K_1(x) \) is the modified Bessel function and one can integrate in (9) over \( r^2 \) to see that the \( z \)-distribution, \( dF_{L}^{cs}/dz \), develops the parton model peaks at \( z \to 0 \) and \( z \to 1 \) \([7]\). To clarify the issue of relevant dipole sizes we integrate in (9) first over \( z \) near the endpoint \( z = 1 \).
For \( r^2 \) from the region

\[
(m_c^2 + Q^2)^{-1} \lesssim r^2 \ll m_s^2
\]

this yields [8]

\[
\int dz |\Psi^{cs}(z, r)|^2 \approx \frac{\alpha_W N_c m_c^2}{\pi^2 (m_c^2 + Q^2) Q^2 r^4}. \tag{13}
\]

This is the \( r \)-distribution for \( cs \)-dipoles with \( c \)-quark carrying a fraction \( z \sim 1 \) of the \( W^+ \)'s light-cone momentum. Thus, the singularity \( \sim r^{-4} \) in Eq.(13) together with the factorization relation (9) and \( \sigma(r) \sim r^2 \) give rise to nested logarithmic integrals over dipole sizes. Indeed, in the Born approximation the gluon density \( G \) in Eq.(11) is

\[
G(x, r^{-2}) \approx C_F N_c L(r^{-2}), \tag{14}
\]

where

\[
L(k^2) = \frac{4}{\beta_0} \log \frac{\alpha_S(\mu_G^2)}{\alpha_S(k^2)}. \tag{15}
\]

Notice, that perturbative gluons do not propagate to large distances and \( \mu_G \) in Eq.(15) stands for the inverse Debye screening radius, \( \mu_G = 1/R_c \). The lattice QCD data suggest \( R_c \approx 0.3 \text{ fm} \) [21]. Because \( R_c \) is small compared to the typical range of strong interactions, the dipole cross section evaluated with the decoupling of soft gluons, \( k^2 \ll \mu_G^2 \), would underestimate the interaction strength for large color dipoles. In Ref.[22, 19, 23] this missing strength was modeled by a non-perturbative, soft correction \( \sigma_{npt}(r) \) to the dipole cross section \( \sigma(r) = \sigma_{pt}(r) + \sigma_{npt}(r) \). Here we concentrate on the perturbative component, \( \sigma_{pt}(r) \), represented by Eqs.(11) and (14).

Then, for the charmed-strange \( P \)-wave component of \( F_L \) with fast \( c \)-quark \( (z \rightarrow 1) \) one gets

\[
F_{L}^{cs} \sim \frac{N_c C_F}{4} \frac{m_c^2}{(m_c^2 + Q^2)} \frac{1}{2!} L^2(m_c^2 + Q^2). \tag{16}
\]

There is also a contribution to \( F_{L}^{cs} \) from the region \( 0 < r^2 < (m_c^2 + Q^2)^{-1} \)

\[
F_{L}^{cs} \sim \frac{N_c C_F}{4} \frac{m_c^2}{(m_c^2 + Q^2)} \alpha_S(m_c^2 + Q^2) L(m_c^2 + Q^2) \tag{17}
\]

which is, however one \( L \) short. Thus, the CD analysis reveals the ordering of dipole sizes
Figure 2: The nucleon structure function $F_2$ at smallest available $x_{Bj}$ as measured in $\nu Fe$ CC DIS by the CCFR [24] (circles) and CDHSW Collaboration [25] (squares, $x_{Bj} = 0.015$). Triangles are the CHORUS Collaboration measurements [26] of $F_2$ in $\nu Pb$ CC DIS. Solid curves show the vacuum exchange contribution to $F_2$. Also shown are the charm-strange (dashed curves) and light flavor (dotted curves) components of $F_2$, dashed-dotted curves for the valence contribution to $F_2$.

$$(m_c^2 + Q^2)^{-1} \lesssim r^2 \ll m_s^{-2}$$

(18)

typical of the DGLAP approximation. The rise of $F_2^{cs}(x, Q^2)$ towards small $x$ is generated by interactions of the higher Fock states, $c\bar{s} + \text{gluons}$. The DLLA ordering of Sudakov variables and dipole sizes in the $n$-gluon state $|c\bar{s}g_1g_2...g_n\rangle$

$$x \ll z_n \ll ... \ll z_1 \ll z < 1$$

(19)

$$(m_c + Q^2)^{-1} \ll r^2 \ll \mu_1^2 \ll ... \ll \mu_n^2 \ll \mu_G^{-2}$$

(20)

results in the density $|\Phi_{n+1}|^2$ of multi-gluon states in the color dipole space [3]
\begin{equation}
\Phi_{n+1} = |\Psi(z, r)|^2 \frac{C_F \alpha_S(r^{-2})}{\pi^2} \cdot \frac{1}{z_1} \cdot r^2 \times \frac{C_F \alpha_S(\rho_1^{-2})}{\pi^2} \cdot \frac{1}{z_2} \cdot \frac{\rho_1^2}{\rho_1^4} \ldots \frac{C_F \alpha_S(\rho_{n-1}^{-2})}{\pi^2} \cdot \frac{1}{z_n} \cdot \frac{\rho_{n-1}^2}{\rho_n^4}.
\end{equation}

By virtue of (19,20) the $c\bar{s}g_1g_2...g_n$-state interacts like color singlet octet-octet state with the cross section $(C_A/C_F)\sigma(\rho_n)$. Then, making an explicite use of Eqs.(11,14) and (13) we arrive at the $P$-wave component of $F_L$ that rises rapidly to small $x$,

\begin{equation}
F_L^{cs} \approx \left( \frac{Q^2}{4\pi^2\alpha_W} \right) \pi^2 C_F \int dz d^2 r |\Psi(z, r)|^2 \times r^2 \alpha_S(r^{-2}) \sqrt{L(r^{-2})} I_1 \left( 2\sqrt{\xi(x, r^{-2})} \right)
\end{equation}

\begin{equation}
\approx \frac{N_c C_F}{4} \frac{m_c^2}{(m_c^2 + Q^2)} L(m_c^2 + Q^2) \eta^{-1} I_2 \left( 2\sqrt{\xi(x, m_c^2 + Q^2)} \right).
\end{equation}

In Eq.(22), which is the DGLAP-counterpart of Eq.(3),

\[ I_{1,2}(z) \approx \exp(z)/\sqrt{2\pi z} \]

is the Bessel function,

\[ \xi(x, k^2) = \eta L(k^2) \]

is the DGLAP expansion parameter with $\eta = C_A \log(x_0/x)$.

Additional contribution to $F_L^{cs}$ comes from the $P$-wave $c\bar{s}$-dipoles with “slow” $c$-quark, $z \to 0$. For low $Q^2 \ll m_c^2$ this contribution is rather small,

\begin{equation}
F_L^{cs} \approx \frac{N_c C_F}{4} \frac{(Q^2 + m_c^2)}{m_c^2} \left( \frac{\alpha_S^2}{\pi} \right)^2 \log(m_c^2/\mu_G^2).
\end{equation}

If, however, $Q^2$ is large enough, $Q^2 \gg m_c^2$, corresponding distribution of dipole sizes valid for

\[(m_c^2 + Q^2)^{-1} \lesssim r^2 \ll m_c^{-2}\]

is

\begin{equation}
\int dz |\Psi^{cs}(z, r)|^2 \approx \frac{\alpha_W N_c m_c^2}{Q^2} \frac{1}{Q^2 r^4}.
\end{equation}

The DLLA summation over the $s$-channel multi-gluon states, results in [5]

\begin{equation}
F_L^{cs} \approx \frac{N_c C_F}{4} \frac{m_c^2}{Q^2} L(Q^2) \eta^{-1} I_2 \left( 2\sqrt{\xi(x, Q^2)} \right).
\end{equation}

Therefore, at high $Q^2 \gg m_c^2$ both kinematical domains $z \to 1$ and $z \to 0$ (Eqs.(22) and (25), respectively) contribute (within the DLLA accuracy) equally to $F_L^{cs}$.
5 Low $Q^2$: light quark dipoles and Adler’s theorem.

The P-wave component of $F_{ud}^L$ is small because of small factor $m_q^2/Q^2$, where $m_q$ is the constituent $u, d$-quark mass. Here we deal with constituent quarks in the spirit of Weinberg [27]. This suppression factor, $m_q^2/Q^2$, comes from the light-cone wave function $\Psi_{ud} \sim m_q(Qr)^{-1}$ and is of purely perturbative nature.

In [20] we checked accuracy of the color dipole description of $F_L(x, Q^2)$ in the non-perturbative domain of low $Q^2$ making use of Adler’s theorem [2],

$$F_{ud}^L(x, 0) = \frac{f_\pi^2}{\pi} \sigma_{\pi},$$  \hspace{1cm} (26)

In (26) $f_\pi$ is the pion decay constant, $\sigma_{\pi}$ is the on-shell pion-nucleon total cross section.

Invoking the CD factorization, which is valid for soft as well as for hard diffractive interactions, we evaluated first the vacuum exchange contribution to both $\sigma_{\pi}$ and $F_L(x, 0)$. The parameter $f_\pi$ in Eq.(26) was evaluated within the CD LCWF technique [28, 29]. The approach successfully passed the consistency test: $\pi F_{ud}^L(x, 0)/(f_\pi^2\sigma_{\pi}) \approx 1$ to within 10%. The cross section $\sigma_{\pi}$ was found to be in agreement with data. However, the value of $f_\pi$ appeared to be underestimated. It was found that for $m_q = 150$ MeV, commonly used now in CD models successfully tested against DIS data, our $F_L$ at $Q^2 \rightarrow 0$ undershoots the empirical value of $f_\pi^2\sigma_{\pi}/\pi$ by about 40% [20], not quite bad for the model evaluation of non-perturbative parameters. One can think of improving accuracy at higher $Q^2 \sim m_c^2$ which we are interested in.

Notice, that Adler’s theorem allows only a slow rise of $F_{ud}^L(x, 0)$ to small $x$,

$$F_{ud}^L(x, 0) \propto \left(\frac{1}{x}\right)^{\Delta_{soft}},$$  \hspace{1cm} (27)

much slower than the rise of $F_{ud}^{\text{res}}$ following from our DLLA estimates. The value of the so-called soft pomeron intercept $\Delta_{soft} \simeq 0.08$ comes from the Regge parameterization of the total $\pi N$ cross section [30].
6 Comparison with experimental data.

We evaluate nuclear ($\nu A$) and nucleon ($\nu N$) structure functions within the color dipole BFKL approach [19] (for alternative approaches to nuclear shadowing in neutrino DIS see [31, 32, 33, 34, 35]).

The structure function $F_2$ for the $\nu Fe$ and $\nu Pb$ interactions are shown in Fig. 2. From comparison with experimental data [24], [25] and [26] we conclude that the excitation of charm contributes significantly to $F_2$ at $x \lesssim 0.01$ and dominates $F_2$ at $x \lesssim 0.001$ and $Q^2 \lesssim m_c^2$.

For comparison with data taken at moderately small-$x$ the valence component, $F_{2\text{val}}$, of the structure function $F_2$ should be taken into account. We resort to the parameterization of $F_{2\text{val}}(x, Q^2)$ suggested in [36]. This parameterization gives $F_{2\text{val}}(x, Q^2)$ vanishing as $Q^2 \to 0$ which is not quite satisfactory from the point of view of PCAC. The latter requires $F_{2\text{val}}(x, 0) = F_{2\text{val}}^{PCAC}(x, 0)$ with

$$F_{2\text{val}}^{PCAC}(x, 0) = \frac{f^2}{\pi} \sigma_{\pi}^{R}(W).$$

Here $x = m_a^2/W^2$ and $\sigma_{\pi}^{R}(W)$ stands for the secondary reggeon contribution to the total pion-nucleon cross section that diminishes at high cms collision energy as $\sigma_{\pi}^{R}(W) \sim (W^2)^{\alpha_R-1}$, where $\alpha_R \simeq 0.5$. However, at smallest values of $Q^2 \simeq 0.2 - 0.3$ GeV$^2$ accessible experimentally $F_{2\text{val}}(x, Q^2) \gg F_{2\text{val}}^{PCAC}(x, 0)$, remind, the characteristic mass scale in the axial channel is $m_a \sim 1$ GeV. Therefore, the accuracy of $F_{2\text{val}}(x, Q^2)$ of Ref.[36] is quite sufficient for our purposes. In Fig. 2 the valence contributions to $F_2$ are shown by dash-dotted curves. The agreement with data is quite reasonable.

One more remark is in order, the perturbative light-cone density of $ud$ states, $|\Psi^{ud}|^2 \sim r^{-2}$, apparently overestimates the role of short distances at low $Q^2$ (see Ch. 5) and gives the value of $F_L^{ud}(x, 0)$ which is smaller than the value dictated by Adler’s theorem [20]. This also may lead to underestimation of $F_2$ in the region of moderately small $x \gtrsim 0.01$ dominated by the light quark current.
7 Summary

Summarizing, it is shown that at small $x$ and moderate virtualities of the probe, $Q^2 \sim m_c^2$, the higher twist corrections brought about by the non-conservation of the charmed-strange current dramatically change the longitudinal structure function, $F_L$. The effect survives the limit $Q^2 \to 0$ and seems to be interesting from a point of view of feasible tests of Adler’s theorem [2] and the PCAC hypothesis.

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