Is radio jet power linearly proportional to the product of central black hole mass and Eddington ratio in AGN?

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\textbf{Abstract} A model for the relation between radio jet power and the product of central black hole (BH) mass and Eddington ratio of AGN is proposed, and the model is examined with data from the literature. We find that radio jet power positively correlates but not linearly with the product of BH mass ($m$ in solar mass) and Eddington ratio ($\lambda$), and the power law indices ($\mu$) are significantly less than unity for relatively low accretion ($\lambda < 0.1$) AGN, $P_j \propto (\lambda m)^\mu$, in the radio galaxies and the Seyfert galaxies. This leads to a negative correlation between radio loudness and $\lambda m$ for the low luminosity AGN, i.e. $R \propto (\lambda m)^\rho$ with $\rho = (7/6)\mu - 1 < 0$, which may be attributed to a contribution of BH spin to total jet power assuming that the spin induced jet is gradually suppressed as the accretion rate increases. Whereas, for the high-z quasars which often show the slope $\mu \geq 1$, a positive correlation between the radio loudness and disc luminosity is predicted. We discuss that the jet powers of the high-z FRII quasars are likely dominated by the accretion disc rather than by the BH spin.

\textbf{Keywords} black hole physics • galaxies: jets • quasars: general • accretion, accretion disks

1 Introduction

It is generally accepted that active galactic nuclei (AGN) harbor massive black holes (BHs). The black holes have only three physical parameters: mass, spin and net charges, the net charges are often considered to be zero. The mass (and its time derivative) and spin (and its time derivative) of BHs are crucial for understanding the AGN phenomena; most models and simulations suggest that it is the mass accretion and/or spin of black holes that produce the jets of AGN, e.g. see Blandford & Znajek (1977) model for the BH spin induced jet, Blandford & Payne (1982) model for the disc accretion induced jet, and see recent reviews for the simulations of accretion disc (Fragile 2013) and for hot accretion flows (Yuan & Narayan 2014). The co-evolution of central BH and its host AGN makes it possible for measuring the BH mass via various ways, and the mass accretion rate can be estimated through the measurements of Eddington ratio. While BH spins are still difficult to measure, although there is an explosion of BH spin measurements in recent years, only some 19 AGN may have BH spin measurements (Reynolds 2013).

About 10\% of quasars are radio loud (Kalfountzou et al. 2012), and many more are in radio weak/quiet state, while a higher radio loud fraction was found in low luminosity AGN (Ho 2002). It is not well understood how a radio jet is related to the central BH mass, spin, and accretion rate (or Eddington ratio) in AGN. Observationally, radio jet powers are not well correlated with BH masses (Ho 2002), whereas there are several findings that radio luminosity correlates with the narrow emission line luminosity or the bolometric luminosity which assumed to be proportional to disc accretion rate (Rawlings & Saunders 1991, Wilott et al. 1999, Cao & Rawlings 2004, van Velzen & Falcke 2013). In recent development of the Blandford & Znajek (1977) mechanism, Tchekhovskoy et al. (2011) and McKinney et al. (2012) demonstrate that the magnetically arrested flows can extract BH spin energy efficiently. A jet-spin correlation has been suggested in X-ray binaries.
2 Relation of radio jet power to BH mass and Eddington ratio

In the Newtonian approximation – this is suitable to a distance beyond a few Schwarzschild radii of BH (Meier 2012), where a jet may be formed from the disc accretion of AGN if not to consider a contribution from the BH spin, the binding energy of unit mass in a Keplerian orbit is $GM/2r$, where $G$ is the gravitational constant. For an accreting mass of $\Delta M$ in time of $\Delta t$, the binding energy $E$ per unit time is

$$E = \Delta M/\Delta t \times (GM/2r) = G\dot{M}/2r. \quad (1)$$

Where $\dot{M}$ is the accretion rate defined by accreted mass $\Delta M$ per unit time. For the radiative efficiency of $\varepsilon$ in an accretion disc, the disc luminosity:

$$L_{\text{disc}} = \varepsilon G\dot{M}M/2r. \quad (2)$$

Similarly assuming radio jet power is also proportional to the binding energy with a jet efficiency $\eta$, we will have the radio jet power $P_j$:

$$P_j = \eta G\dot{M}M/2r. \quad (3)$$

Considering the ratio of disc luminosity to the Eddington luminosity:

$$\frac{L_{\text{disc}}}{L_{\text{Edd}}} = \frac{\varepsilon G\dot{M}M/2r}{4\pi G M m_p c/\sigma_T} \quad (4)$$

We have

$$L_{\text{disc}}/L_{\text{Edd}} = 5.28 \times 10^{-12} \varepsilon \dot{M}/r. \quad (5)$$

Taking this relation into equation (3), $r$ is cancelled, and we have

$$P_j = 1.26 \times 10^{38}(\eta/\varepsilon)\left[\frac{L_{\text{disc}}}{L_{\text{Edd}}}M/M_\odot\right](\text{erg/s}). \quad (6)$$

This relation shows that the jet power is the disc accretion dominated, which is expected to be linearly proportional to the product of the Eddington ratio ($\lambda = L_{\text{disc}}/L_{\text{Edd}}$) and the BH mass ($M$) of AGN, with the coefficient of $\eta/\varepsilon$. The jet power depends not only on the Eddington ratio but also BH mass in this relation. It is equivalent to the relation (from equations 2 and 3):

$$P_j = (\eta/\varepsilon)L_{\text{disc}}. \quad (7)$$

3 Statistical analysis of the jet power, BH mass and Eddington ratio

To test the formula (6), we searched for suitable AGN samples which have information on radio jet luminosity, BH mass and Eddington ratio (or disc luminosity). Firstly, we study from the Sikora et al. (2013) sample, the sample consists of 404 narrow line radio galaxies (NLRGs) consisting of FRIs and FRIIs, double-double radio lobes, X-shaped lobes, and one side lobes (for details, see Sikora et al. 2013), and the sample is limited to redshift $<0.4$. We use only the data of FRIs and FRIIs, they are the majority of the sample. The 1.4 GHz radio luminosity computed from 1.4 GHz fluxes in the NVSS catalog, BH mass and $H_\alpha$ (also $[O_{III}]$) line luminosity are available in the Sikora sample, where the black hole masses were estimated from the observed stellar velocity dispersion given in the SDSS using the relation by Tremaine et al. (2002) with typical error of log$M$ less than 0.3 dex. The radio jet power $P_j$ can be estimated from low frequency (151 MHz) radio luminosity with the minimal energy argument for synchrotron emission (Willott et al. 1999), assuming that the jet output results in energy stored in radio source lobes together with associated work done on the source environment. We convert the jet power formula of Willott et al. (1999) from 151 MHz to 1.4 GHz assuming source spectral indices $S_\nu \propto \nu^{-0.8}$, and we have

$$P_j = 2.32 \times 10^{20}(f/3)^{3/2}(P_{1.4})^{6/7} [W/H\bar{z}](\text{erg/s}) \quad (8)$$

The $f$ (in range 1-20) represents several uncertainties associated with estimating $P_j$ from 151 MHz luminosity (Willott et al. 1999). We here use the median value $f = 10$ (Blundell & Rawlings 2000) and use
Sikora et al. (2013) used a simplified formula \( P_{1.4} \) than that of equation (8). Where the \( P_{1.4} \) is the 1.4 GHz luminosity, the radio jet power is generally less than the bolometric disc luminosity in our case. The bolometric luminosity \( L_{\text{bol}} \) estimated with the \( H\alpha \) line luminosity, the same as in Sikora et al. (2013), see also Netzer (2009):

\[
L_{\text{disc}} = L_{\text{bol}} \approx 2000 L_{\text{line}}(\text{erg/s})
\]

(9)

The Eddington ratio is computed from \( L_{\text{disc}}/L_{\text{Edd}} \), and the Eddington luminosity \( L_{\text{Edd}} \) depends only on BH mass.

The radio loudness parameter \( R \) is defined by the 1.4 GHz radio luminosity over the \( H\alpha \) line luminosity, i.e. \( R = P_{1.4}/L_{H\alpha} \).

To test and fit the equations (6)-(7), we rewrite the equation (6)-(7) as a power law form \( P_j = (\eta'/\varepsilon')L_{\text{disc}}^m \), i.e.

\[
\log(P_j) = b + \mu \log(\lambda M/M_\odot),
\]

(10)

where \( b = \log[1.26 \times 10^{38}(\eta'/\varepsilon')] \), the ratio \( \eta'/\varepsilon' \) is a coefficient for the power law form and it returns to \( \eta/\varepsilon \) when \( \mu = 1 \).

For the anti-correlations between the radio loudness and disc luminosity (or Eddington ratio) found by Greene & Ho (2002), Sikora et al. (2007, 2013), it is reasonable to assume a power law relation, i.e: \( R = \xi L_{\text{disc}}^p \), and rewrite it with the form \( (m \equiv M/M_\odot) \):

\[
\log(R) = c + \rho \log(\lambda m)
\]

(11)

Then, we plot the radio power \( P_j \) versus the production of BH mass and Eddington ratio for the sample, as well as the radio loudness \( R \) versus \( \lambda m \), and fit the data with the equation (10),(11) respectively, as shown in Fig. 1 and Fig. 2. The linear regression fitted parameters are listed in Table I as well as the correlation coefficient and the null hypothesis probability in Table 2.

The result shows that the radio power of the NL-RGs (FRIs+FRIIs) positively correlates with the \( \lambda m \), with the power law index of 0.52±0.06 with high correlation coefficient and confidence level in Table 2 it is not a linear proportionality as one might expect from equations (6)-(7). The radio loudness is anti-correlated with the \( \lambda m \) with the power law index of -0.40±0.08. It shows that the jet powers have no correlation with the BH masses (Fig. 3). If we fit separately for FRIIs and FRIs in the sample, the slope \( \mu \) is 0.52±0.08 and 0.49±0.18 for the FRIIs and FRIs respectively.

We also study the Sikora et al. (2007) sample, that consists of five sub-samples including broad-line radio galaxies (BLRGs), radio-loud quasars (RLQs), FRI radio galaxies, optically selected quasars (PG quasars), and Seyfert galaxies plus LINERS (only 3), all the sources are at redshift \( z < 0.5 \). The 5 GHz radio luminosity, BH mass and Eddington ratio are available in the Sikora et al. (2007) sample. We searched total 1.4 GHz flux densities available from the literature (Condon et al. 1998) and data from the NED and references therein) for the sample, for that the lower frequency radio luminosity would reflect the isotropic properties of emission in the lobes and extended jets.

The BH masses in Sikora et al. (2007), were estimated using a broad-line region size-luminosity relation, assuming virial velocities of the gas, which produces broad \( H\alpha \) lines (Greene & Ho 2003), or from the appropriate references. The uncertainties of BH mass from Greene & Ho (2003), typically ~20% in the virial mass formula that depends on the \( H\alpha \) line alone. The Eddington ratio is \( \lambda=L_{\text{bol}}/L_{\text{Edd}} \), and the bolometric luminosity \( L_{\text{bol}} \) is assumed to be 10 times optical B-band nuclear luminosity (at 4400Å), i.e. \( L_{\text{bol}} = 10L_B \) (see, e.g., Richards et al. 2006), where \( L_B = \nu L_{\nu} \) which is from directly measured apparent magnitudes of the nuclear regions or also estimated from the \( H\alpha \) line, the errors of \( H\alpha \) flux can be \( \leq 30\% \) (Greene & Ho 2005), see Sikora et al. (2007) for more details on the estimation of BH mass and nuclear disc luminosity. Here we use the equation (8) to estimate the jet power by using \( f = 10 \) and \( P_{1.4} \).

As each subsample of the Sikora et al. (2007) sample is relatively small, to compare with the sample in Fig. 1 which are narrow line FRI/FRII galaxies, we combine the FRI galaxies and BLRGs into radio galaxies (RG) in the Sikora et al. (2007) sample, also because the FRI and FRII galaxies may have similar accretion mode (Cao & Rawlings 2004). The BLRGs and RLQs which divided by absolute magnitude \( M_V > -23 \) and \( M_V < -23 \), are almost all the FRIIs (Sikora et al. 2007). We plot the jet power \( P_j \) derived from 1.4 GHz luminosity versus \( \lambda m \) in Fig. 3 as well as the radio loudness versus \( \lambda m \) in Fig. 5. A few outliers are excluded in our analysis (e.g., the NGC1275 from Seyfert galaxies, which is actually hosted by a giant elliptical galaxy, and 5 PGQs which are in the RLQ area), and those data with only upper/lower limits are excluded. The radio loudness is computed with \( R = 1 \times 10^3 L_{1.4}/L_B \) which converted from the formula \( R = 1.36 \times 10^3 L_{5.0}/L_B \) at 5 GHz in Sikora et al. (2007), assuming source spectral
index $S_\nu \propto \nu^{-0.8}$. Linear regression fittings to the RGs, Seyferts+LINERS, and PG quasars are shown in Fig. 3 with the fitted parameters in Table 1 as well as the correlation coefficient and null probability in Table 2.

The quality of a linear regression can be measured by the coefficient of determination (COD), a value from 0 to 1. If it is close to 0 the relationship between X and Y will be regarded as very poor, the COD theoretically equals to the square of the Pearson coefficient of linear correlation. We try to do linear fitting with slope=1 (dash line in Fig. 1), the resulted COD is 0.07, which is much lower than the COD=0.52 of the best linear fit for the FRIs+FRIIs. This is also true for the subsamples in Fig. 4, except the PG quasars which is close to slope=1 but with larger errors in Table 1. The correlation coefficient and confidence level are quite high for the radio galaxies, Seyferts+LINERS in the two samples in Table 2. We did not fit for the radio loud quasars (RLQs), because they are clustering in a small area making it difficult to fit them properly.

In the results, we show that the jet powers at 1.4 GHz positively correlate with the $\lambda m$, but not linearly, i.e. the power law indices ($\mu$) are significantly less than unity for the radio galaxies and Seyferts+LINERS. The radio loudness is anti-correlated with the $\lambda m$ for the radio galaxies and Seyferts+LINERS. For PG quasars, there appears to be a linear proportionality between the jet power and disc luminosity but with larger errors, that leads to no correlation between radio loudness and disc luminosity.

To further study the anti-correlation between the radio loudness and the $\lambda m$ we found, considering $P_j = \eta'/\varepsilon' L_{\text{disc}}^\rho$ and equation (8)-(9), we have radio loudness:

$$R = P_{\lambda4}/L_{\text{line}} \propto P_j^{7/6} / L_{\text{disc}}^{7/6} \mu^{-1} \propto L_{\text{disc}}^\rho$$ (12)

The ratio $\eta'/\varepsilon'$ can be estimated from the b of the linear fit parameters in Table 1. From equation (12) we find a relation $\rho = (7/6)\mu - 1$. This relation accords with the fitting results in Table 1, e.g. for $\mu = 0.52$, the $\rho = (7/6)\mu - 1 = -0.39$ is close to the measured value $\rho = -0.40 \pm 0.08$ in Table 1 for the NLRGs. So the anti-correlation is apparently explained, that is due to the $\mu < 6/7$, so that $\rho = (7/6)\mu - 1 < 0$.

The physics for $\mu < 6/7 (0.86)$ needs to be explored further in the $P_j = (\eta'/\varepsilon') L_d^{\rho}$, it implies that the jet power increases less efficiently than the disc luminosity increases. For $\mu < 1$, the $\eta$ and $\varepsilon$ may be not constants but vary with the accretion rate. Sikora et al. (2013) used the relation of $\varepsilon \propto \lambda^{2/5}$ for the BH magnetosphere of a truncated disc, in which the jet power is dominated by a BH spin. With $R = P_{\lambda4}/L_{\text{line}} \propto \eta'/\varepsilon$ approximately, they explain that the anti-correlation in...
Fig. 3  Log[jet power] vs. log[m] for the narrow line radio galaxies (FRIs+FRIIs), m is black hole mass in solar mass unit.

Fig. 4  Log[jet power] vs. log[λm] for the radio galaxies (RG), Seyfert galaxies and LINERS (Sey+L), and PG quasars (PGQ), with the best linear fit to each subsample, the dash line is fitted with the fixed slope=1.

Fig. 5  Log[radio loudness] vs. log[λm] for the sub-samples (same as in Fig.4), with the best linear fit to each subsample.

Table 1  The linear fit parameters of $\log[P_j] = b + \mu \times \log[\lambda m]$ along with the derived value of $\eta' / \varepsilon'$ in the upper part, and the linear fit parameters of $\log[R] = c + \rho \times \log[\lambda m]$ in the lower part, for narrow line radio galaxies (NLRG), radio galaxies (RG), Seyfert galaxies and LINERS (Sey+L), and PG quasars (PGQ), respectively. The number in the brackets of second column is the coefficient of determination (COD) which ranges from 0-1, it basically equals to the square of the Pearson coefficient of linear correlation. The error is 2σ (standard deviation from the linear fit program).

| Sample  | subsample | $\mu$  | $b$    | $\eta' / \varepsilon'$ |
|---------|-----------|--------|--------|------------------------|
| Sikora13| NLRG(0.52)| 0.52±0.06 | 40.10±0.34 | 100.0                  |
| Sikora07| RG(0.51)  | 0.35±0.10 | 41.06±0.60 | 912.0                  |
|         | Sey+L(0.47)| 0.32±0.12 | 38.32±0.58 | 1.7                    |
|         | PGQ(0.47) | 0.92±0.44 | 34.08±3.42 | 9.5E-5                 |
| Sikora13| NLRG(0.32)| -0.40±0.08 | 3.46±0.40  |                        |
| Sikora07| RG(0.69)  | -0.59±0.12 | 7.22±0.70  |                        |
|         | Sey+L(0.73)| -0.63±0.14 | 4.02±0.66  |                        |
|         | PGQ(0.00) | 0.07±0.52 | -0.89±3.96 |                        |
radio loudness and Eddington ratio could be due to the $\varepsilon \propto \lambda^{3/5}$, i.e. $R \propto \lambda^{-0.4}$.

van Velzen & Falcke (2013) find that from 763 FRII quasars with the median redshift of 1.16, a linear correlation between 1.4 GHz luminosity and bolometric disc luminosity with $P_{j,1.4} \propto L_{bol}$. If we use the equation (8), the jet power will be $P_j \propto \rho^{6/7} \propto L_{bol}^{0.86}$. They claim that, for this nearly linear proportionality, the power output from the inner part of the accretion disc dominates over the power extracted from the black hole by the Blandford-Znajek mechanism (BZ-jet, hereafter).

Kalfountzou et al. (2012) investigated the $[O_{III}]$ emission line properties of 18508 quasars at $z < 1.6$ drawn from the Sloan Digital Sky Survey (SDSS) quasar sample. The quasar sample has 1692 radio-loud (RLQs) and 16816 radio-quiet quasars (RQQs), according to the traditional radio-loud/quiet division of the radio-to-optical flux ratio of 10, and the radio loudness computed using the radio flux density at 1.4 GHz and the optical (7480Å) flux density from the SDSS. They found a strong correlation between 1.4 GHz radio luminosity and narrow emission-line luminosity, with a power law index of $\mu \sim 1.6$ and $\sim 1.2$ for the RLQs and RQQs respectively. For radio jet power, using the equation (8), the slope reduces to $\mu \sim 1.4$ and $\sim 1.0$ for the RLQs and RQQs respectively. The optical line luminosity is linearly proportional to the disc luminosity in Kalfountzou et al. (2012).

4 Discussion

From the results above, in general there is a positive power law correlation between radio jet power and disc luminosity $P_j = \eta / \varepsilon L_{disc}^{\mu}$. The power law indices are significantly less than unity for the Seyfert galaxies, FRI and FRII galaxies at $z < 0.5$ and the Eddington ratio $\lambda < 0.1$ (or typically $\lambda m < 7$ in Fig. 1 and Fig. 3). Second, there is an anti-correlation between radio loudness and disc luminosity for the samples which have the $\mu < 6/7$. We find that the negative correlation is caused by $(7/6)\mu < 1$ with the relation $\rho = (7/6)\mu - 1 < 0$. On the other hand, in high-$z$ quasars from the literature, we have $\mu \sim 0.86$ for the FRII quasars in van Velzen & Falcke (2013) and $\mu > 1$ for the quasars in Kalfountzou et al. (2012). It will lead to a positive correlation between radio loudness and disc luminosity for $\mu > 6/7$ and no correlation for $\mu \sim 6/7$ in the high-$z$ samples, and we will investigate these in future.

The radio loudness problem appears to be resolved with our interpretation, the underlying physics is still not clearly identified. Are there different accretion discs or jet forming mechanisms for the power-law correlation index $\mu \ll 1$ and $\mu \geq 1$? In the BH accretion models, there are cold and hot accretion flows. Cold accretion flow consists of optically thick and geometrically thin gas, e.g. the standard thin disc (Shakura & Sunyaev 1973), which occurs at a fraction of the Eddington mass accretion rate, and the slim disc at super-Eddington rates (Abramowicz et al. 1988). Hot accretion flows, however, are virially hot and optically thin, they occur at lower mass accretion rates, and are described by models such as the advection-dominated accretion flow (ADAF; Narayan & Yi 1994) and luminous hot accretion flow. Observations show that hot accretion flows are often associated with jets, and they are present in low-luminosity AGN (LLAGN) (see Yuan & Narayan 2014 for a review on hot accretion flows). Ho & Peng (2001) and Ho (2002) find that most of the LLAGN in nearby galaxies are radio-loud, and the radio loudness is inversely correlated with the disc luminosity. Sikora et al. (2013) and Sikora & Begelman (2013) find that the anti-correlation of radio loudness and disc luminosity could be explained with the $\varepsilon \propto \lambda^{3/5}$ in the BH spin paradigm, implying that the BH spin plays an important role in the jet powers. Therefore the Seyfert galaxies and radio galaxies in our samples are likely powered by both the inner part of accretion disc and the BH spin. If the BZ-jet power is gradually suppressed when the accretion rate increases, the jet power could be less proportional to the disc luminosity and this may lead to an anti-correlation of radio loudness and disc luminosity. This is some similar to the transition from hard/low (with low accretion rate and radio jet) to soft/high (with high accretion rate and no jet) state in the X-ray binaries, in which a contribution of jet power from black hole spin is possible (Fender et al, 2004). However, this explanation needs a high fraction of rapid spin BHs in the low-z AGN.

Martínez-Sansigre & Rawlings (2011a,b) analyzed the ratio of jet power to disc luminosity, they find that at $z<0.5$ the low-excitation galaxies have low accretion rates and bimodal spin distributions, with approximately half of the population having maximal spins, while high exciting galaxies are explained as high-accretion rate but very low spin objects at higher redshifts ($z\sim1$) and only a small population of nearly maximally spinning high accretion rate objects is possible. This may be supported by the findings that much higher fraction of radio loud galaxies at low accretion rates (Ho & Peng 2001; Ho 2002) than the fraction ($\sim10\%$) of radio loud quasars at higher redshifts (Kalfountzou et al. 2012). And it has been suggested that low redshift FRIs have rapid spinning BHs (Wu et al 2011).
Radio jet power, black hole mass and Eddington ratio

In high-z quasars, a steep slope $\mu \approx 1$ or $>1$ between jet power and disc luminosity is often observed, e.g. van Velzen & Falcke (2013), Willott et al. (1999), Fernandes et al. (2011), Kalfountzou et al. (2012). The quasars are at high accretion state ($\lambda > 0.1$) and hosted by elliptical galaxies. The massive ellipticals are most likely formed via major merger events, and their nuclei may have different nuclear environments than disc galaxies. Falder et al. (2010) find evidence for the environmental source density to increase with the radio luminosity of AGN at around $z=1$. The jets of FRII quasars are probably launched through the Bondi accretion of hot interstellar gas (Bondi 1952; Allen et al. 2006; Gaspari et al. 2013; Sikora et al. 2014), with efficient jet powers (Werner et al. 2012). Allen et al. (2006) find a tight positive correlation between the Bondi accretion rate and the radio power required to inflate cavities observed in the surrounding X-ray emitting gas, suggests that the Bondi formulae provide a reasonable description of the accretion process for powerful jets. Furthermore, the powerful jet may live in a short time for quasars, given that the majority of quasars are radio quiet.

Meier (2001) proposed a hybrid model combined both the disc accretion and the BH spin/magnetosphere effects, it can produce powerful jets if the BH spin $a_* > 0.9$ ($a_*$ is dimensionless spin parameter ranging from 0-1, see also Nemmen et al. 2007). Therefore, both the hybrid model and the magnetically arrested disc (Tchekhovskoy et al. 2011; McKinney et al. 2012) needs maximally spinning BHs for producing FRII jets, it is suggested that only a small population of nearly maximally spinning BHs in high accretion rate objects, and spins of BHs are generally low at around $z=1$ (Martínez-Sansigre & Rawlings 2011a,b). As noted by van Velzen & Falcke (2013) the powerful jets of FRII quasars are mainly controlled by the inner part of accretion flows rather than by the power extracted from the BH spin (also see Livio et al. 1999), in contrast to that the BZ-jet may control the negative correlation of radio loudness and disc luminosity in the low luminosity AGN as we discussed above.

Fernandes et al. (2011) find from a radio selected sample of 27 radio galaxies at a narrow redshift range $z=0.9-1.1$ that is unbiased to evolutionary effects, that there is a tight positive correlation between the radio luminosity and the mid-IR (also the $[O_\text{II}]$) line luminosity with power law indices $\mu > 1$. They added optical selected quasars (OSQs) into the analysis, the correlation for the brightest radio sources appears to become an upper envelope. However, Kalfountzou et al. (2012) investigated both radio loud and quiet quasars at $z < 1.6$ drawn from the SDSS quasar sample. They find a strong correlation between 1.4 GHz radio luminosity and narrow emission-line luminosity, for both RLQs and RQQs, with power law indices $\mu > 1$. This sample is much larger than the Fernandes et al. (2011) sample, so it could be statistically more significant. Our studies concentrate mainly on radio loud sources, and it is needed to include radio quiet ones in larger and complete samples into analysis in future.

Finally, we note that the different slopes of correlation between radio jet power and disc luminosity might be affected by selection effects, e.g. sample sizes etc., however, we have high confidence for that in high-z quasars the slopes are much steeper than those of low redshift galaxies in our analysis. There are no correlation found between radio luminosities and source sizes in Willott et al. (1999) and Sikora et al. (2013) for FRIs and FRIIs, however, caution must be taken if there is a wide range of source sizes in a relative small sample (Shabala & Godfrey 2013). The Doppler boosting effect is not considered here, since our samples are mainly the FRIs, FRIIs and Seyfert galaxies, and the blazars are not included. Furthermore, as noted by Singal & Rajpurohit (2014), the radio power and/or the disc luminosity may be also related to the redshift, a redshift – luminosity correlation (Malmquist bias) in a flux-limited sample. We checked our data, find there are weak positive correlations for the radio galaxies while stronger correlation for the PG quasars between the radio jet power (or disc luminosity) and redshift. In fact, the correlation is stronger between the radio power and the disc luminosity than that between the radio (or disc) luminosity and redshift in the radio galaxies of our samples, these type of source are mainly studied in this paper, but the Malmquist bias might still have some effects which we were currently not able to remove from our data. As analyzed by Singal & Rajpurohit (2014), a flux limited sample with a wide range of radio luminosities within narrow redshift ranges (and vice versa) will be needed to disentangle the effects in future.

5 Summary

A model for the relation between radio jet power and the product of central BH mass and Eddington ratio of AGN is proposed. The model is examined with data from the literature.

We find that radio jet power positively correlates but not linearly with the product of BH mass and Eddington ratio, and the power law indices are significantly less than unity for relatively low accretion ($\lambda < 0.1$) AGN, $P_j \propto (\lambda m)^\mu$, in the radio galaxies and the Seyfert galaxies. This leads to a negative correlation between
radio loudness and $\lambda m$ for the low luminosity AGN, i.e.
$R \propto (\lambda m)^\rho$ with $\rho = (7/6)\mu - 1 < 0$, which may be attributed to a contribution of BZ-jet to total jet power assuming that the BZ-jet power is gradually suppressed as the accretion rate increases.

On the contrary, for the high-z quasars which often show the slope $\mu \geq 1$, a positive correlation between the radio loudness and disc luminosity is predicted. We discuss that the jet powers of the high-z FRII quasars are likely dominated by the accretion disc rather than by the BH spin.

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| Subsample | Pearson | Spearman | Kendall |
|-----------|---------|----------|---------|
| NLRG      | 0.72(0) | 0.71(0)  | 0.53(0) |
| logR      | -0.57(0) | -0.52(0) | -0.36(1.6E-18) |
| RG        | 0.71(5.7E-9) | 0.64(6.4E-7) | 0.45(5.7E-6) |
| logR      | -0.83(1.1E-13) | -0.81(1.1E-12) | -0.62(2.5E-10) |
| Sey+L     | 0.68(4.2E-6) | 0.68(5.2E-6) | 0.51(1.5E-5) |
| logR      | -0.85(3.6E-11) | -0.86(2.4E-11) | -0.70(2.2E-9) |
| PGQ       | 0.69(5.5E-4) | 0.64(0.002) | 0.57(3.9E-4) |
| logR      | 0.06(0.78) | 0.17(0.45) | 0.17(0.29) |

Table 2 The Pearson, Spearman and Kendall linear correlation coefficient between $\log[P_j]$ and $\log[\lambda m]$, and that between $\log$[radio loudness] and $\log[\lambda m]$ marked ‘logR’ in second line, with null hypothesis probability (non correlation probability) in brackets.
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