DARK MATTER AND BIG BANG NUCLEOSYNTHESIS

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ABSTRACT

Walker, Steigman, Schramm, Olive and Kang (WSSOK) have used the Standard Big Bang Nucleosynthesis (SBBN) theory and observed abundances of $^4$He, D+$^3$He and $^7$Li extrapolated to their primordial values, to argue that most of the baryons in the Universe are dark. But, we show here that the confidence level of the alleged agreement between the SBBN abundances and those inferred by WSSOK from observations is less than 0.02% for any baryon to photon ratio! If, however, the highly uncertain WSSOK essentially theoretical upper bound on the primordial abundance of D+$^3$He is ignored, then the predicted abundances of primordial $^4$He, D, $^3$He and $^7$Li are in agreement with the observations at a confidence level above 70%, provided that the cosmic baryon to photon ratio is $\eta = (1.60 \pm 0.1) \times 10^{-10}$. The predicted primordial D abundance is $(2.1 \pm 0.35) \times 10^{-4}$ and may be tested in the future by measuring absorption line systems in quasar spectra with the Hubble Space Telescope. The above baryon to photon ratio yields a baryon mass density, in critical density units, of $\Omega_B \approx 0.0059h^{-2}$, where $h$ is the Hubble parameter in units of $100$ km s$^{-1}$Mpc$^{-1}$. This baryon density is consistent with the observed mean density of luminous matter in the Universe (stars, X-Ray emitting gas, quasar light absorbing systems), $\Omega_{LUM} \approx 0.0060 \pm 0.0027$, in particular, if the most recent estimates, $0.75 \leq h \leq 1$, are correct. It does not provide reliable evidence that most of the baryons in the Universe are dark. Moreover, since the dynamics of clusters of galaxies and large scale structures indicate that $\Omega \gtrsim 0.15$, it does imply that most of the matter in the Universe is non baryonic dark matter.
I. INTRODUCTION
The quantitative agreement between the predictions of the Standard Big Bang 
Nucleosynthesis (SBBN) theory (Hayashi 1950; Alpher, Follin and Herman 1953; 
Peebles 1966; Wagoner, Fowler and Hoyle 1967; Wagoner 1969; Wagoner 1973; 
Yang et al. 1984; for reviews, see for instance Weinberg 1972; Schramm and Wag-
oner 1977; Bosegard and Steigman 1985; Kolb and Turner 1990; Olive 1991) and 
the observed abundances of $^4\text{He}$, $^3\text{He}$ and $^7\text{Li}$ extrapolated to their primordial 
values (see for instance Walker, Schramm, Steigman, Olive and Kang 1991, here-
after WSSOK) has been considered as one of the most convincing evidences for 
the validity of the Standard Hot Big Bang Model of the Universe (see for instance 
Weinberg 1972; Peebles et al. 1991). The SBBN predictions of the primordial 
abundances of the light elements depend essentially on well known nuclear reaction rates and on three additional parameters,
1) the number of generations of light neutrinos $N_\nu$,
2) the neutron lifetime $\tau_n$,
3) the ratio of baryons to photons in the Universe $\eta \equiv n_B/n_\gamma$.
Recently $N_\nu$, $\tau_n$ and $n_\gamma$ have also been measured quite accurately: The combined 
results of the LEP Collaborations (1991) at the Large Electron Positron Collider 
at CERN have shown that there are only three generations of light neutrinos, 
$N_\nu = 3.00 \pm 0.05$. Recent measurements of the neutron lifetime in a neutron bottle 
and in a Penning trap resulted in the precise values, $\tau_n = 887.3 \pm 3 \text{ sec}$ (Mampe 
et al. 1989) and $\tau_n = 893.6 \pm 5.3 \text{ sec}$ (Byrne et al. 1990), respectively, yielding 
a weighted “world average value” of (Hikasa et al. 1992) of $\tau_n = 889.1 \pm 2.1 \text{ sec}$ 
when combined with all previous measurements. Finally, the precise measurements 
of the spectrum of the cosmic microwave background radiation (MBR) by the 
Far Infrared Absolute Spectrometer (FIRAS) on the Cosmic Background Explorer 
(COBE) satellite (Mather et al. 1990) and by a helium cooled spectrometer carried 
by a high altitude rocket (Gush, Halpern and Wishnow 1990) gave a black body 
temperature of $T = 2.736 \pm 0.017 \text{ K}$, which yields a number density of relic photons 
from the Big Bang of $n_\gamma = 20.3 T^3 \approx 415 \pm 8 \text{ cm}^{-3}$. As a result, the SBBN 
predictions for the primordial abundances of the light elements depend essentially 
on a single unknown parameter, $n_B$, the mean baryon number density in the 
Universe. Hence, the primordial abundances of the light elements that are inferred 
from observations, if correct, can be used both to test the SBBN theory and to 
determine the mean baryon density in the Universe. Indeed, in recent papers Olive 
et al. (1990) and WSSOK have claimed to show that for baryon to photon ratio 
$2.8 \leq \eta_{10} \leq 4$, where $\eta_{10} \equiv \eta \times 10^{10}$, the predictions of the SBBN theory agree
with the observations and that this ratio implies (Schramm 1991) that most of the nucleons in the Universe are dark. However, in this paper we show that:

(a) The confidence level of the alleged agreement between the predictions of SBBN theory and the primordial abundances of the light elements that were inferred from observations by WSSOK is less than 0.02% for any value of $\eta$.

(b) If the highly uncertain upper bound on primordial D+$^3$He that was estimated by WSSOK is relaxed (see for instance Delbourgo-Salvador, Audouze and Vidal-Madjar 1987; Audouze 1987) and only the less restrictive, but more reliable, lower bounds on primordial D and $^3$He (their presolar abundances as inferred from meteorites) are trusted, then agreement between the SBBN theory and the primordial abundances of $^4$He, D, $^3$He and $^7$Li as inferred from observations is achieved with a high confidence level ($\gtrsim 70\%$) provided that $\eta_{10} \approx 1.60 \pm 0.10$.

(c) The above value for $\eta$ yields a mean baryon mass density in the Universe which is consistent with the best estimates of the total mass density of luminous matter (in the V, UV, IR, X and Radio bands) in the Universe. Thus, SBBN theory and the observed abundances of the light elements do not provide any reliable evidence that most of the baryons in the Universe are dark.

(d) Since there is considerable dynamical evidence from clusters of galaxies and large scale structures that the total mass-energy density in the Universe is greater than the mean density of luminous matter by more than an order of magnitude (see for instance Kolb and Turner 1990; Mushotzky 1991; Schramm 1991), it follows that most of the matter in the Universe is non baryonic dark matter.

II. THE SBBN PREDICTED ABUNDANCES

The primordial abundances of $^4$He, D, $^3$He and $^7$Li that are predicted by the SBBN theory with $N_\nu = 3$ and $\tau_n = 889.1$ s are displayed in Fig.1 for $1 \leq \eta_{10} \leq 10$. These are essentially the numerical results of WSSOK that were also obtained from the analytical calculations of Dar and Rudzsky (1992) using the same nuclear reactions cross sections. Over the range $2 \lesssim \eta_{10} \lesssim 8$, both the numerical results of WSSOK and the analytical calculations are well approximated (maximal deviation smaller than the uncertainty due to the uncertainties in the nuclear cross sections) by simple interpolating formulae:
Helium 4; The primordial mass fraction of $^4\text{He}$ is well described by

$$Y_p = 0.228 + 0.0105\ln\eta_{10} + 0.000208(\tau_n - 889.1 \text{ s}). \quad (1)$$

Deuterium; The primordial abundance of $^2\text{D}$, $y_{2p} \equiv N(\text{D})/N(\text{H})$, where $N(X)$ is the number density of atoms $X$, is well described by

$$y_{2p} = 4.6 \times 10^{-4} \eta_{10}^{-1.67}. \quad (2)$$

Helium 3; The primordial abundance of $^3\text{He}$, $y_{3p} \equiv N(^3\text{He})/N(\text{H})$, is well described by

$$y_{3p} = 3.0 \times 10^{-5} \eta_{10}^{-0.50}. \quad (3)$$

Lithium 7; The primordial abundance of $^7\text{Li}$, $y_{7p} \equiv N(^7\text{Li})/N(\text{H})$, is well described by

$$y_{7p} = 5.2 \times 10^{-10} \eta_{10}^{-2.43} + 6.3 \times 10^{-12} \eta_{10}^{2.43}. \quad (4)$$

The uncertainties in the absolute normalizations of the predicted abundances of $^4\text{He}$, $^2\text{D}$, $^3\text{He}$ and $^7\text{Li}$ due to uncertainties in nuclear cross sections are, $\pm 0.2\%$, $\pm 4\%$, $\pm 6\%$ and $\pm 20\%$, respectively.

### III. INFERRRED PRIMORDIAL ABUNDANCES

**Helium 4:** The primordial mass fraction $Y_p$ has been determined from observations of the most metal-poor extragalactic H II regions (see for instance Pagel 1990 and references therein) by a linear fit of $Y_p$ versus metalicity, using O, N and C as metalicity indicators. The extrapolations of these results to zero metalicity by WSSOK gave

$$O : \quad Y_p = 0.229 \pm 0.004; \quad N : \quad Y_p = 0.231 \pm 0.003; \quad C : \quad Y_p = 0.230 \pm 0.007,$$

where the errors are one standard deviations. Fuller, Boyd and Kalen (1991), however, have argued that the straight line fit to the $^4\text{He}$ data to obtain the zero metalicity intercept is inappropriate and places inordinate weight on high metalicity points. Using a linear regression procedure (Lyons 1986) for extrapolating
lower metalicity data to zero metalicity, and N and O as metalicity indicators, FBK found

\[ \text{O : } Y_p = 0.223 \pm 0.009; \quad \text{N : } Y_p = 0.220 \pm 0.010. \]  

(6)

**Deuterium:** Deuterium is easily destroyed already at relatively low temperatures. Consequently, its local abundance depends strongly on local chemical evolution and can vary markedly from site to site. Since the sun burned D into \(^3\text{He}\) during its pre-main sequence evolution, the difference between the largest observed \(^3\text{He}\) abundance in gas rich meteorites and the smallest observed \(^3\text{He}\) abundance in carbonaceous chondrites meteorites was used by WSSOK to estimate the presolar D abundance and to set a lower bound on primordial D due to its possible destruction during presolar galactic chemical evolution:

\[ y_{2p} \geq 1.8 \times 10^{-5}. \]  

(7)

The local chemical evolution that preceded the solar system is highly uncertain and precludes any reliable local determination of primordial D.

**Helium 3:** The smallest observed \(^3\text{He}\) abundance in carbonaceous chondrites meteorites was used by WSSOK as an estimate of its presolar abundance and as a lower bound on primordial \(^3\text{He}\) (due to possible destruction during presolar chemical evolution):

\[ y_{3p} \geq 1.3 \times 10^{-5}. \]  

(8)

WSSOK have also used a model dependent estimate for the survival of \(^3\text{He}\) during local presolar chemical evolution (primordial and that produced by burning away D) to bound (2\(\sigma\)) primordial D+\(^3\text{He}\):

\[ 3.3 \times 10^{-5} \leq y_{23p} \leq 1 \times 10^{-4}. \]  

(9)

**Lithium 7:** The primordial abundance of \(^7\text{Li}\), was determined from the most metal poor, Population II halo stars. Such stars, if sufficiently warm \((T \gtrsim 5500K)\), have apparently not depleted their surface Lithium and are expected to have nearly a constant Lithium abundance reflecting the Lithium abundance present at the early evolution of the Galaxy (Spite and Spite 1982a,b). WSSOK have fitted a
plateau value for 35 such stars with $T \geq 5500K$, which were selected from several observations, and concluded that

$$y_{7p} = (1.20 \pm 0.21) \times 10^{-10}. \quad (10)$$

Deliyannis and Demarque (1991), however, in their most recent paper derived from their detailed study of Population II halo stars with $5500K \lesssim T \lesssim 6400K$ a more conservative estimate for the early Galaxy Lithium abundance,

$$y_{7p} = (1.58 \pm 0.35) \times 10^{-10}. \quad (11)$$

IV. COMPARISON BETWEEN THEORY AND OBSERVATIONS

In order to test the agreement between the SBBN theory and observations we applied to them the standard $\chi^2$ test (we have used the standard CERN Computer Program MINUIT, James and Ross 1989), assuming the errors to be statistical. First the SBBN predictions for the primordial abundances of $^4$He, $D^3He$ and $^7$Li as obtained by WSSOK (Section II) were compared with the corresponding primordial abundances that were inferred by WSSOK from observations (Section III), for baryon to photon ratio in the range $1 \leq \eta_{10} \leq 10$. The confidence level of the “agreement” between theory and “observations” as a function of $\eta_{10}$ is displayed in Fig.1. Indeed, the best “agreement” is obtained for $\eta_{10} \approx 3.1$, but its confidence level is extremely poor, only 0.02% (errors were treated as purely statistical). In other words, the probability to be wrong in stating that the SBBN predictions and the primordial abundances of the light elements (with their quoted errors) as inferred from observations by WSSOK, disagree for any value of $\eta$, is less than 0.02%! Consequently, the confidence level of the value of the baryon mass density that was deduced by WSSOK, and of their conclusion that most of the baryons in the Universe are dark, is less than 0.02%!

The failure of the SBBN to reproduce the WSSOK primordial abundances suggests that something is wrong either with the SBBN theory or with the WSSOK abundances, or with both. However, the qualitative agreement between the SBBN predictions and the “observed” primordial abundances of the light elements that range over ten orders of magnitude, as well as other impressive successes of the Standard Big Bang Model (for a recent review see Peebles et al. 1991) suggest that the failure is probably due to the WSSOK primordial abundances. Perhaps not all the values inferred by WSSOK for the primordial abundances of the light
elements are correct. In fact, various authors (see for instance Delbourgo-Salvador, Audouze and Vidal-Madjar 1987; Audouze 1987 and references therein) have questioned the upper bound on the primordial abundance of D+3He that was inferred by WSSOK from observations because of large uncertainties in our knowledge of the local presolar chemical evolution. Indeed, even in the local interstellar medium (LISM) D abundances which were determined specroscopically from absorption spectra of the LISM (see for instance Murthy 1991) are very different for different directions. Thus, we have repeated the standard $\chi^2$ test for the SBBN predictions for primordial $^4$He, D, $^3$He and $^7$Li, ignoring the uncertain WSSOK upper bound on primordial D+3He. Satisfactory agreement (confidence level higher than 70\%) between theory and “observations” was obtained for

$$\eta_{10} \approx 1.60 \pm 0.1$$ (12)

when we used the WSSOK estimates of primordial $^4$He (Eq.5), the WSSOK lower bounds for primordial D and $^3$He (Eqs. 7,8) and the estimate by Deliyannis and Demarque (1991) of primordial $^7$Li (Eq. 11). This is demonstrated in Fig.2. Essentially the same values of $\eta$ and confidence level were obtained when we used the FBK estimates (Eq. 6) for the primordial abundance of $^4$He instead of the WSSOK estimates. The corresponding baryon number density is

$$n_B = \eta n_\gamma = (6.6 \pm 0.5) \times 10^{-8} \text{cm}^{-3}$$ (13)

and the baryon mass density is $\rho_B \approx n_B m_P = (1.11 \pm 0.08) \times 10^{-31} \text{g cm}^{-3}$. When expressed in critical density units, $\rho_C \equiv 3H_0^2/8\pi G \approx 1.88 \times 10^{-29} h^2 \text{g cm}^{-3}$, this baryon mass density is

$$\Omega_B \equiv \rho_B/\rho_C = (0.0059 \pm 0.0007) h^{-2}.$$ (14)

The observed light density in the Universe was estimated (e.g., Felten 1987) to be $(2.4 \pm 0.4) \times 10^8 L_\odot h \text{ Mpc}^{-3}$. The mass to light ratio which best reproduces rotation curves within the visible part of spiral galaxies (e.g., Rubin 1991) and X-ray luminosities from the visible part of elliptical galaxies is (Mushotzky 1991),
M/L \approx (7\pm 3)hM_{\odot}/L_{\odot}.\) Consequently, the mean mass density of luminous matter in the Universe is estimated to be,

\[ \Omega_{LUM} \approx 0.0060 \pm 0.0027. \] (15)

This mass density is not in disagreement with the mass density of baryonic matter obtained above from SBBN theory, in particular if one notes that the recent best estimates of the value of the Hubble parameter yield \(0.75 \leq h \leq 1.00\) (Jacoby et al. 1990, Fukugita and Hogan 1991, Tonry 1991). Conversely, if one constrains baryon mass density in SBBN to be equal to the observed density of luminous matter and \(h\) to the range \(0.75 \leq h \leq 1.00\), one obtains that \(\eta_{10} = 1.57 \pm 0.22\) and \(\Omega_B \approx \Omega_{LUM} \approx 0.59 \pm 0.15\) with a confidence level of 85%. For such baryonic mass density the SBBN theory predicts that the primordial abundances of D and \(^3\)He are \(y_{2p} = (2.10 \pm 0.35) \times 10^{-4}\) and \(y_{3p} = (2.37 \pm 0.18) \times 10^{-5}\), respectively.

V. CONCLUSIONS
The confidence level of the alleged agreement between the primordial abundances of the light elements that were inferred from observations by WSSOK and the predictions of the SBBN theory is less than 0.02% for any value of \(\eta\) (including \(2.8 \leq \eta_{10} \leq 4\)). Thus, SBBN theory and the WSSOK abundances do not provide any reliable evidence for the existence of cosmologically significant quantities of baryonic dark matter. Moreover, agreement between the SBBN theory and observations is achieved with a confidence level higher than 70% for baryon to photon ratio \(\eta_{10} = 1.60 \pm 0.22\), if the highly uncertain essentially theoretical upper bound for primordial D+\(^3\)He that was estimated by WSSOK is ignored. This range of \(\eta\) is essentially dictated by the present best estimates of primordial \(^4\)He and \(^7\)Li from observations and their uncertainties. It yields a mean baryon mass density of \(\Omega_B = (0.0058 \pm 0.0007)h^{-2}\), which is consistent with the best estimates of the mass density of luminous (baryonic) matter in the Universe, \(\Omega_{LUM} = 0.0060 \pm 0.0027\), provided that the most recent estimates \(0.75 \leq h \leq 1.00\) (Jacoby et al 1990, Fukugita and Hogan 1991, Tonry 1991) are correct. Since, there is dynamical evidence from clusters of galaxies and large scale structures that the total mass-energy density in the Universe satisfies (see for instance Kolb and Turner 1990 and references therein) \(\Omega \approx 0.15\), it implies that \(\Omega \gg \Omega_B\), i.e., that most of the gravitating matter in the Universe is non baryonic dark matter. Finally, SBBN with a baryon mass density similar to that of luminous matter predicts primordial abundances, \(y_{2p} = (2.10 \pm 0.35) \times 10^{-4}\) and \(y_{3p} = (2.37 \pm 0.18) \times 10^{-5}\), for D and
These predictions, in principle, can be tested by measuring high redshift absorption line systems in quasar spectra due to D and $^3$He in intergalactic space, perhaps by the Hubble Space Telescope.

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**FIGURE CAPTIONS**

**Figure 1.** (a) The predicted primordial mass fraction of $^4$He and the abundances (by number) of D, $^3$He, D+$^3$He and $^7$Li as a function of $\eta_{10}$. Also shown are the 95% confidence level bounds that were inferred from observations by WSSOK. The vertical band delimits the alleged range of $\eta_{10}$ where predictions agree with observations. (b) The values of $\chi^2$ (left scale) and the corresponding confidence level (right scale) of the alleged agreement between the predicted abundances and those inferred by WSSOK from observations, as function of $\eta_{10}$. Best “agreement” is obtained for $\eta_{10} \approx 3.1$, but its confidence level is only 0.02%!

**Figure 2.** (a) The predicted primordial mass fraction of $^4$He and the abundances (by number) of D, $^3$He, D+$^3$He and $^7$Li as a function of $\eta_{10}$. Also shown are the primordial mass fractions of $^4$He (1σ errors) that were inferred by WSSOK from observations of extragalactic HII regions by interpolating to zero metalicity, using O, N and C as metalicity indicators, the primordial abundance of $^7$Li (1σ error) that was inferred by Deliyannis and Demarque (1991) from observations of metal poor Population II halo stars and the lower bounds of WSSOK on primordial D and $^3$He from meteorites. The vertical line indicates the value of $\eta_{10}$ which is most consistent with the observations. (b) The values of $\chi^2$ (left scale) and the corresponding confidence level (right scale) of the agreement between the predicted abundances
and those inferred from observations, as function of $\eta_{10}$. Best agreement is obtained for $\eta_{10} \approx 1.60$ with a confidence level above 70%.