Flow resistance of ice slurry in bends and elbow pipes

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Abstract. The present paper covers the flow of ice slurry made of a 10.6% ethanol solution through small-radius bends and elbow pipes. The paper presents the results of experimental research on the flow resistances of Bingham-fluid ice slurry in bends and elbows. The research, performed for three pipe diameters and a relative bend radius of $1 \leq D/d_i \leq 2$, has made it possible to take into consideration the influence of friction resistances as well as the flow geometry on the total local resistance coefficients. The study attempts to make the local resistance coefficient dependent on the Dean number defined for a generalized Reynolds number according to Metzner-Reade

1. Introduction

Ice slurry belongs to a group of secondary refrigerants. It is a mixture of ice crystals and either water, or water containing a dissolved substance which lowers the freezing point (salt, glycol, alcohol). Ice slurry is a non-Newtonian fluid. The rheological models most frequently assigned to this fluid are the Ostwald-de Waele power model, the Bingham model and the Casson model [1-3].

Problems with determining flow resistances for Newtonian and non-Newtonian fluid flow through curved pipes can be divided into two groups. In the case of small-radius bends ($D/d_i<6$) [4] both friction- resistances and local resistances generated by a local change in flow geometry contribute to the drop in pressure. For $D/d_i<6$, Idielcik [4] suggests that the total resistance coefficient of elbows and bends be sum of the coefficient of local resistance of the bend and the friction coefficient. In this case, the local resistance coefficient is only a function of the geometrical parameters of the bend (angle, relative bend diameter). In [5] Ito proposes that a total local resistance coefficient be determined for the flow of non-Newtonian fluids through strongly curved pipes. For small Dean numbers (De<91), Ito makes the total local resistance coefficient directly dependent on the friction resistance coefficient and flow geometry. For Dean numbers of De>91, the local resistance coefficient is a function of the Reynolds number and the geometric parameters of the bend.

The other type of issues related to the flow of non-Newtonian fluids through curved pipes concerns their flow through spirally curved pipes and large-diameter bends. Ito [4], [6] considers the pressure drops in coil pipes and large-diameter bends $(D/d_i\geq6)$ to be dependent on the friction resistance coefficient, which is a function of the Dean number defined differently for laminar and turbulent flow. The basic works concerning the flow of non-Newtonian fluids through curved pipes include, among others, [7-9], which highlight the results of research on pipe coils. The paper by Mishra and Gupta [8] presents a generalization of the results of research on friction resistance coefficients for Ostwald-de

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Waele fluids. The results of research presented in [9] concern the flow resistances of a Bingham fluid in coil pipes. The generalized formula for the friction resistance coefficient takes on the form of the Ito relationship [6], corrected by implementing a correction function in the Dean number definition. [7] lists the results of research on flow resistances in curved circular and elliptical ducts. An analysis of the results of research covered in this paper indicates that various authors determine the friction resistance coefficient in curved pipes by means of generalizing the Ito formula. The modification of this relationship involves the implementation of correction functions both in the general form of the Ito formula and in the Dean number definition.

Table 1 presents a review of literature of slurry flow through fittings of [1], [3], [10-12]

| suspension | \(d_i\) (m) | type of bends | \(v_m\) (ms\(^{-1}\)) | \(x_s\) (%) | Literature, notes |
|------------|-------------|---------------|----------------|-----------|------------------|
| Ice slurry of 16\% propylene water glycol solution | 0.021; 0.027 | 90° bend | 0.5-1.5 | 0\(\leq x_s \leq 30\) | [10-11] The data included in the study have been presented in graphic form and have not been generalized. |
| Ice slurry of 10-12\% water solution of ethyl alcohol | 0.015 | 90° bend | 0.2-0.9 | 0\(\leq x_s \leq 25\) | [1] A lack of generalizations or information on bend radius makes it difficult to use the results. |
| 3.6-12.7\% laterite suspension | 0.025-0.5 | 45°, 90°, 180° bends \(D/d_i=0.25\) | \(0.5\rightarrow 16\) | 3.9-12.7 | [12] The relationships suggested in the aforementioned work are quite peculiar, as they present an individualized form of the formulas for the total local resistance coefficient for each \(D/d_i\) relation. |
| 10.7-30\% gypsum suspension | ~0.5-14 | | | 10.7-30 | |

The present paper concerns the flow of ice slurry made up of a 10.6\% ethanol solution through bends and elbow pipes. It contains the results of experimental research on flow resistances of the slurry through bends and elbow pipes in laminar and turbulent flow regime. The result of experimental research is a series of dimensionless relationships describing the local resistance coefficient for bend and elbow pipes and for ice slurry being a Bingham fluid, whose flow is treated as a generalized flow of a non-Newtonian fluid.

2. Measurement of ice slurry flow resistance in small-radius bends

The research programme concerning flow resistances in bends and elbow pipes included measurements for:

- Commercially available standard 90° bends and elbow pipes with diameters of \(d_i = 0.01; 0.016; 0.02\) (m), with the bend diameter-pipe diameter ratio of \(D/d_i = 2.0\) for bends and \(D/d_i = 1\) for elbow pipes.
- Mean flow velocities of \(0.1 \leq v_m \leq 4.5\) (ms\(^{-1}\)), with the corresponding Reynolds number values of \(200 \leq Re \leq 13\,700\),
- Mass fraction of ice in the slurry of \(0 \leq x_s \leq 30\%\).

The study of ice slurry flow encompassed a rheological identification of the ice slurry [13], measurements of pressure drops of the ice slurry in straight pipes [3], [13] as well as measurements of flow resistances in bends and elbow pipes. A detailed description of the measurement stand (figure 1A) has been provided in [2-3] and [13]. The measurement of ice slurry temperature is based on data
obtained from highly precise resistance sensors with large dimensions Pt100(7013) (HART Scientific, sensor mantle diameter – ca. 5 mm, active length – min. 0,02m). The flux of the ice slurry has been measured by means of a Danfoss MASSFLO 2100 mass flow-meter. Pressure drops have been measured using Fuji FKCV pressure-change transducers, with two measurement ranges: 0–32 kPa and 0–1 kPa (uncertainty: 0,07% of the measurement range).

The entire measurement system (figure 1B) used was composed of a straight-axis inlet and outlet sections (L_{in}, L_{out}), as well as a piece of fitting (bend, elbow). In order to determine the pressure drop in bends and elbows, the adopted method was similar to the one presented in [12]. Pressure drop measurements encompassed: the total pressure drop $\Delta p_T$ in a measurement element, as well as a friction-induced pressure drop in the straight-axis ($\Delta p_L$) measurement sections with a length of $L$. Pressure drops in a bend or an elbow were calculated using the following formula:

$$\Delta p_{ex} = \Delta p_T - \Delta p_L \cdot \frac{L_{ex} + L_{rest}}{L}$$

(1)

Figure 1. A) Schematic diagram of the test stand: 1-heated measuring segment, 2-elbow, 3- bend, 4-calorimetric measurement, 5-measurement of: density, volume change, ice and air content, 6-mass flow-meter, 7-heater, 8-wattmeter and autotransformer, 9-flow visualization, 10-air-escape, 11-ice generator, 12-accumulation container, 13-pump, 14-volume flow-meter; B) Outline of a measurement stand for investigating flow resistance in bends and elbows

Regardless of pipe diameter, the lengths of the inlet ($L_{in}$) and outlet ($L_{out}$) sections used in the measurement of pressure drops in bends and elbows were ten times the diameter of the pipe. It has to be noted that the entire measurement system was preceded and followed by straight pipes, which stabilized the flow of the agent.

Figure 2 presents the variation of pressure drops in the investigated bends ($D/d_i=2$) and elbows ($D/d_i=1$), for a pipe diameter of $d_i=0.02$ (m). The results of the measurements indicate that for $x_s \leq 10\%$, the influence of the mass fraction of ice on pressure drops both in bends and in elbows is negligible. For pipe flow of ice slurry with a mass fraction of ice of $x_s \geq 10\%$, there are velocities at which the flow resistance of the slurry is lower than the flow resistance of the carrying liquid [2-3], [13]. Ice slurry flow through bends and elbow pipes reduces the occurrence of this phenomenon in the examined fittings, leading to a situation in which, regardless of the type of movement, for the mass fraction of ice of $x_s \geq 10\%$, pressure drops in bends and elbows are always higher than the corresponding pressure drops of the carrying liquid. (Figure 2). For low values of ice content, the flow of the ice slurry is similar to the flow of a Newtonian fluid [2], [3], [13]. In the case of flow through elbow pipes, a greater pressure drop than in the case of a bend results from the fact that greater flow disturbances occur in the elbow than in the bend pipe. For mass fractions of ice of $x_s>15\%$ a
A significant change in the rheological properties of the fluid occurs. Ice crystals absorb part of the kinetic energy of the turbulences of the carrier liquid, muffling the turbulences occurring in the elbow pipe. This phenomenon results in the fact that the differences in pressure drops between the elbow and bend pipe for higher ice content values \((x_s > 15\%)\) are lower than for an ice content of \(x_s \leq 15\%\).

A)

![Diagram A]

**Figure 2.** Flow resistance \((\Delta p)\) as a function of the mean velocity of the slurry for A) a bend; \(d_i = 0.02 \text{ (m)};\ D/d_i = 2;\) B) an elbow; \(d_i = 0.02 \text{ (m)};\ D/d_i = 1.\)

Figure 3 presents the influence of pipe diameter, the mass fraction of ice and the Reynolds number on pressure drops (Figure 3a) and total flow resistance (Figure 3b) in elbows. An increase of flow resistances along with the increase of ice content results from the higher values of the dynamic coefficient of plastic viscosity \(\mu_p\) and increased values of yield shear stress \(\tau_p\) for the ice slurry. Similarly to Newtonian fluids, higher pressure drop values correspond to smaller flow cross-sections.

For a Newtonian fluid flow through curved pipes, Ito’s empirical formulas are the ones most frequently quoted: [4], [6-7], and [12]. Flow resistances in bends and elbows are caused both by friction resistance and resistance caused by flow disturbances as a result of the change of the direction of the flow of the medium. In this paper, the components of flow resistances are not determined separately. The calculated value of the resistance coefficient, similarly to paper [5] is the total local resistance coefficient of the medium flowing through a bend or elbow:

\[
\xi_{e,x} = \frac{2\Delta p_{e,x}}{\rho_s v_m^2},
\]  
(2)
This method of determining the total local resistance coefficient is prone to the smallest measurement error. For \( D/d_i < 6 \), the total local resistance coefficient is dependent on the Dean number \((\xi = f(De))\), which for laminar and turbulent flow is defined by formulas (3) and (4), respectively:

\[
De_L = Re \sqrt{\frac{d_i}{D}},
\]

\[
De_T = Re \left( \frac{d_i}{D} \right)^2.
\]

**Figure 3.** A) Influence of a generalised Reynolds number, pipe diameter and ice content on pressure drops in a elbows \((D/d_i = 1)\); B) Dependency total flow resistance on the generalised Dean number for flow through elbows

In this case it has been assumed that the Reynolds number \((Re)\) is a generalised Reynolds number according to Metzner-Reade:

\[
Re = \frac{\rho \sqrt{\nu L u} d_i}{8 K}.
\]
The generalised parameters \( n \) (characteristic flow behaviour index) and \( K \) (consistency index) depend on the rheological properties of the fluid, the mean shear stress at the wall, mean flow velocity, as well as on the dimensionless geometrical constants determined individually for various flow geometries of vertical flow cross-sections. For Bingham fluids and any cross-section geometry, these flows are calculated on the basis of the following formulas [14]:

\[
n = \frac{1/4 \left[ 1 - \epsilon_B^4 - 4/3 \epsilon_B \left( 1 - \epsilon_B^3 \right) \right]}{1 - 3/4 \epsilon_B \left[ 1 - \epsilon_B^4 - 4/3 \epsilon_B \left( 1 - \epsilon_B^3 \right) \right]}.
\]

\[
K = \left( \frac{1/4 \mu_p}{\tau_w} \right)^{1/n} \frac{1/3 \tau_w^2 \tau_p + 1/12 \tau_p^4}{1/4 \tau_w^4}.
\]

in which values of yield shear stress \( \tau_p \) and dynamic coefficient of plastic viscosity \( \mu_p \) for ice slurry made up of a 10.6% ethanol solution are calculated on the basic equations (8) and (9) [2-3], [13].

\[
\tau_p = 0.013 - 1.4284 \left( \frac{x}{100} \right)^2 + 73.453 \left( \frac{x}{100} \right)^3 - 394.64 \left( \frac{x}{100} \right)^4 + 835.82 \left( \frac{x}{100} \right)^5
\]

\[
\mu_p = 0.0035 + 0.0644 \left( \frac{x}{100} \right)^2 - 0.7394 \left( \frac{x}{100} \right)^3 + 5.6963 \left( \frac{x}{100} \right)^4 - 19.759 \left( \frac{x}{100} \right)^5 + 26.732 \left( \frac{x}{100} \right)^6
\]

The \( \tau_w \) and \( \epsilon_B \) in equations (6-7) correspond to the shear stress at pipe wall and the quotient of active shear stresses of a Bingham fluid \( \epsilon_B = \tau_p / \tau_w \), respectively.

3. Summary

Both the results of research presented in [12] and the authors’ own measurements of the flow resistance coefficient for ice slurry in bends and elbows (figure 3B), indicate that the substitute local resistance coefficient \( \xi \) depends on the Dean number and the relative bend diameter \( (D/d_i) \). When determining the formula for the local resistance coefficient for bends and elbows, it has been assumed that the required formula will correspond with the same relationship for bends and elbows \((1 \leq D/d_i \leq 2)\), which has the following form:

\[
\xi = \xi(De; d_i / D).
\]

Moreover, the research on flow resistances in bends and elbows concerned mainly the laminar flow range. Outside the laminar range, the flow occurred mainly in the transitional flow regime. Only for pipe diameters of \( d_i = 0.016 \) and \( 0.02 \) (m), part of the measurement points are located within the turbulent flow range. The following formula determines the relationship describing the local resistances coefficient in laminar flow regimes for a generalized flow of ice slurry through bends and elbows (11):

\[
\xi(De; d_i / D) = \frac{4.6 \left( \frac{d_i}{D} \right)^{0.33}}{[0.87 + 0.1 * \log(De_i)]^{0.1}}.
\]

In the turbulent flow range, the substitute local resistance coefficient for a generalised flow of ice slurry can be derived from the following formula:

\[
\xi(De; d_i / D) = \frac{\left( \frac{d_i}{D} \right)^{-0.306} \left[ 10.6 - 3.45 \left( \frac{d_i}{D} \right) \right]}{15.2 De_i^{2.204}}.
\]
A) $D_\text{t} = \text{Re}^{*}(d_i/D)^{0.5}$

B) $D_\text{t} = \text{Re}^{*}(d_i/D)^{0.5}$

C) $D_\text{t} = \text{Re}^{*}(d_i/D)^{2}$

D) $D_\text{t} = \text{Re}^{*}(d_i/D)^{2}$

**Figure 4.** Measured and best-fit curves of the total local resistance coefficient: A) laminar flow through bends, B) laminar flow through elbows, C) turbulent flow through bends, D) turbulent flow through elbows.

It needs to be noted that the superposition of the phenomena occurring within the inlet and outlet sections, as well as in the bent element itself, causes instability of the pressure signal, impeding the observation of, e.g. the nature of the changes in ice slurry movements. In this case, stabilization of the total local resistance coefficients for laminar flow has been assumed as the criterion of the movement change. It has been assumed that the Dean number, $D_e_\alpha = \text{Re}^{*}(\xi)^{\alpha} = 2500$, is in this case the criterion of the change in the character of the movement for ice slurry flow through bends and elbows.

Figures 4 A, B and 4 C, D present the measured and best-fit curves of total local resistance coefficients for ice slurry in bends and elbows for laminar and turbulent flow. With a 98% probability it can be assumed that the confidence intervals for formulae (11) and (12) equal 14% and 15%, respectively.

Figure 4 indicates that it might be interesting to attempt to improve the quantitative accuracy of formulae (11) and (12) with the results of experimental studies by making the constants in formulae (11) and (12) dependent on the inner pipe diameter. These problems may be further analysed by the authors in future studies.

4. Reference

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