Optical Fibre Angle Sensor Used in MEMS

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Abstract. There is a need for displacement and angle measurements in many movable MEMS structures. The use of fibre optical sensors helps to measure micrometre displacements and small rotation angles. Advantages of this type of transducers are their simple design, high precision of processing, low costs and ability of a non-contact measurement. The study shows an analysis of a fibre-optic intensity sensor used for MEMS movable structure rotation angle measurement. An intensity of the light in the photodetector is basically dependent on a distance between a reflecting surface and a head surface of the fibre transmitting arm, and the deflection angle. Experimental tests were made for PMMA 980/1000 plastic fibres, \( \theta_{NA} = 33^\circ \). The study shows both analytical and practical results. It proves that calculated and experimental characteristics for the analysed transducers are similar.

1. Introduction

There is a need of a displacement and angle measurement in many movable MEMS structures. The use of fibre optical sensors helps to measure micrometre displacements and small rotation angles [1,2,3].

The study shows an analysis of a fibre-optic intensity sensor used for MEMS movable structure rotation angle measurement. Figure 1 shows a basic structure of the transducer. The transducer consists of two arms made of bunches of fibres or a single fibre. An optical source lightens an inspected surface (a mirror surface, e.g. of a silicon cantilever). Reflected light is collected by a receiving fibre and it is transmitted to a photodetector. An intensity of the light in the photodetector is basically dependent on a distance between the reflecting surface and a head surface of the fibre transmitting arm, and the deflection angle.

![Figure 1. Construction of an optical fibre transducer (where: T – transmitting fibre, Rn – receiving fibres).](image-url)
2. Analytical model of the sensor

A case of the reflecting surface (reflection coefficient \( R_m = 1 \) ) turned through an angle \( \alpha \) in the axis of the fibres was analysed in the study (figure 2). For step-index fibres, a \( \Theta_{NA} \) angle depends on an aperture \( NA \) of the fibre, a refractive index \( n_0 \), and it is described by an equation \( \Theta_{NA} = \arcsin (NA/n_0) \).

![Figure 2. Principle of operation of the fibre optic transducer.](image)

A power \( P_i \) coming out from the transmitting fibre is described by a formula \( P_i = F_i(n_0)P_t \), where \( P_t \) is overall power in the transmitting fibre, \( F_i(n_0) \) is the Fresnel’s transmission coefficient. The power emitted from the transmitting fibre can be described by the dependence of light intensity \( I(r,h) \)\[4\]:

\[
P_i = \int_0^{2\pi} \int_0^h I(r,h) r \, dr \, d\phi
\]

where: \( r \) – distance from the axis of the transmitting fibre in a plane perpendicular to the axis, \( \phi \) – azimuth angle, \( R \) – radius of the light cone in the distance of \( 2h \) given by a formula:

\[
R = a + 2h \cdot \tan(\Theta_{NA})
\]

where: \( a \) – radius of the fibre.

A distribution of the light intensity \( I(r,h) \) can be described by the equation [4]:

\[
I(r,h) = \frac{2 \cdot P_t}{\pi \cdot R^2(h)} \left(1 - \frac{r^2}{R^2(h)}\right)
\]

An optical power \( P_o \) received by the photodetector can be described by the equation [2,5]:

\[
P_o(r, h, n_0, \alpha) = 2P_t \int_{R_0}^{R_2} \int_0^{\phi} R_m(\lambda, \alpha) \cdot T_i(n_0) T_i(r, h, n_0, \lambda) \cdot \frac{2}{\pi \cdot R^2(h)} \left(1 - \frac{r^2}{R^2(h)}\right) r \, dr \, d\phi
\]

where: \( \phi = \arccos((3a^2 + r^2)/2a^2) \), \( R_m \) – refractive index of the mirror surface.

For the case when the reflecting surface rotates in an axis of the fibres, the light emitted by the transmitting fibres has a shape of a cone. Cross-sections of the arms of the transducer are elliptical. The light intensity of the cone cross-section is described by the equation:

\[
I(h, \alpha) = \frac{P_i}{\pi \cdot R(h) \cdot R(h, \alpha)}
\]
where $P_i$ is a power on the transmitting arm output, radius $R(h)$ is described by the equation (2) and $R'(h, \alpha)$ denotes a radius of the ellipse described by the equation:

$$R'(h, \alpha) = \frac{h \cdot \sin \Theta_{NA}}{2} \left( \frac{1}{\cos(\alpha - \Theta_{NA})} + \frac{1}{\cos(\alpha + \Theta_{NA})} \right)$$

(6)

For the analysed case the mirror was placed in long-range field ($h > z_a$). Optical power for the receiving fibre’s surface, for $h > z_a$, equals:

$$P_o(h, \alpha) = \frac{P_i \cdot a^2 \cdot R_m}{R(h) \cdot R'(h, \alpha)}$$

(7)

3. Analytical and practical results

To calculate an output signal for a multi-fibre intensity transducer signal, the superposition method was used. The intensity of the light for the branch is a sum of intensities for every single fibre.

Figure 3 shows a dependency of the received optical power as a function of the mirror rotation angle for the step-index fibres – one transmitting and one receiving ($\Theta_{NA} = 33^0$, $a = 0.49$ mm) for the power $P_i = 170 \, \mu W$, $R_m = 0.91$ for aluminium mirror and the distance $h = 1$ mm. The distance between the axes of the transmitting and receiving fibres is 1 mm. Figure 4 shows a characteristics for two receiving fibres. The characteristic for one receiving fibre has only a rising slope (figure 3) in difference to the other constructions.

![Figure 3](image_url)

**Figure 3.** Calculated characteristic of received optical power as a function of a rotation angle, for one receiving fibre.

A practical research was made for PMMA 980/1000 plastic fibres, $\Theta_{NA} = 33^0$, with the aid of a test-stand containing a light source (AFL OLS1 – LED, -10dBm output optical power into a 980µm fibre, wave-length 660nm) and a rotating stand equipped with glass mirror and an optical power meter Thorlabs PM 100D with a Si photodiode sensor (400-1100nm).
In further research an optical head with one central transmitting fibre and four receiving fibres was used (see figure 5). As in the previous case, PMMA fibres were used. The measuring-head was positioned in the distance $h = 1$ mm from the mirror.

![Figure 4](image)

**Figure 4.** Experimental and modelling results of received optical power versus rotation angle, for two receiving fibres.

Figure 6 shows analytical results for optical head containing one central transmitting fibre and four receiving fibres (as shown in figure 5). Collected optical power was over 200% higher than for the construction with two receiving fibres. It reached 7.5 µW, while for the two receiving fibres about 2 µW of optical power was observed. The construction was analysed for rotation angles from -10° to +10°. The sensitiveness of this transducer’s construction increased in comparison to the construction with two receiving fibres.

![Figure 5](image)

**Figure 5.** Construction of the optical transducer consisting of one central transmitting fibre and four receiving fibres.
4. Conclusion

The study shows the analysis of the experimental and numerical results covering different structures of the measuring head (the number of receiving fibres and their placement) and different parameters of the mirrors. The study proves that calculated and measured characteristics for the analysed transducers are similar. Constructions were tested in the range of the rotation angles from \(-10^\circ\) to \(10^\circ\). Received optical powers for the real constructions are slightly lower due to attenuation not considered in the equations used for the calculations. The research shows that maximum optical power for the transducer containing four receiving fibres is over 200\% higher than for the construction with two receiving fibres. The surface of the receiving fibres is bigger, so it collects more optical signal. The maximum optical power reached 7.5 µW. The sensitiveness of the transducer also increases in the case of four receiving fibres.

Advantages of this type of the transducers are their simple design, high precision of processing, low costs and ability of a non-contact measurement.

5. References

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Figure 6. Calculated characteristics for the construction of the transducers containing four receiving fibres.