Pion in the Medium with a Light-Front Model

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Abstract. The pion properties in symmetric nuclear matter are investigated with the Quark-Meson Coupling (QMC) Model plus the light-front constituent quark model (LFCQM). The LFCQM has been quite successful in describing the properties of pseudoscalar mesons in vacuum, such as the electromagnetic elastic form factors, electromagnetic radii, and decay constants. We study the pion properties in symmetric nuclear matter with the in-medium input recalculated through the QMC model, which provides the in-medium modification of the LFCQM.

INTRODUCTION

A fundamental task in nuclear and particles physics is to understand the structure of hadronic systems in terms of the quarks and gluons, where their interactions are described by the strong interaction of quantum chromodynamics (QCD) [1, 2, 3]. Many experiments concerning the hadron properties are planned in some laboratories, among them, JLab (see Ref. [4] for details). However, another very important and interesting aspect with respect to hadronic physics is their properties in the nuclear medium. This includes the context of nuclear physics, i.e., NN interaction in a nucleus, neutron stars and particle properties in heavy ions collisions. The main questions here is, “How the hadron properties change in the dense nuclear medium?” and “What is the effect of the nuclear medium on the QCD structure of hadrons?”

To answer these questions, we study here the pion properties in symmetric nuclear matter. Our approach is to use the quark-meson-coupling (QMC) model [8, 9, 10] plus the light-front approach [15, 11]. Many studies of the pion properties in the nuclear medium exist in the literature and readers are asked to consult e.g. Ref. [16].

In 1949, Dirac proposed three possible forms of relativistic dynamics [5], namely, instant form, point form and front form, and the last one is used in this work.

The use of the light-front form, instead of the instant form, has some advantages as follows. Although the Fock state has infinite numbers of particles in general, only the valence component is necessary to calculate the electroweak properties of the hadronic systems [6]. However in the light-front approach, it is possible to take into account in the light-front wave function the higher Fock state components, which can be written in terms the lower ones [6, 7, 13]. Because of that, the light-front approach is an ideal framework to describe the hadronic bound states in terms of the valence component wave function, or in a picture of the constituent (quark) degrees of freedom. Thus, it can treat unambiguously the parton (quark) content of the meson and baryon wave functions. Another important advantages are that the vacuum for the free Hamiltonian is trivial, and the light-front Hamiltonian is Lorentz invariant [6, 7]. After the integration over the light-front energy \((k^- = k^0 - k^3)\) of a given Bethe-Salpeter amplitude, the hadronic valence wave function can be derived [14, 15].
The Model

For describing the nuclear matter, we use the QMC model developed in Ref. [8]. A similar approach using a confining potential was developed in Ref. [17]. QMC describes the nuclear matter based on the quark degrees of freedom. It has been successfully applied for studying the properties of finite nuclei [18], and the hadronic properties in dense nuclear medium (see Ref. [10], for more details).

The effective Lagrangian density for symmetric nuclear matter is given by [10],

\[
\mathcal{L} = \bar{\psi} \left[ i \gamma \cdot \partial - m_N^* \left( \partial \cdot \phi \right) - g_\omega \phi \gamma_\mu \gamma_5 \phi \right] \psi + \mathcal{L}_{\text{meson}},
\]

where, \( \psi, \phi, \partial, \) and \( \omega \) are respectively the nucleon, Lorentz scalar-isoscalar, and Lorentz vector-isoscalar field operators with

\[
m_N^* \left( \partial \cdot \phi \right) = m_N - g_\sigma \phi.
\]

The density \( (\sigma\text{-field}) \) dependent \( \sigma\text{-N coupling constant} \) in nuclear matter, \( g_\sigma \), is defined by Eq. (2), and \( g_\omega \) is the \( \omega\text{-N coupling constant} \). The meson Lagrangian density \( \mathcal{L}_{\text{meson}} \) in Eq. (2) is given by

\[
\mathcal{L}_{\text{meson}} = \frac{1}{2} \left[ \partial_\mu \partial_\nu \phi - m_\omega^2 \phi^2 \right] - \frac{1}{2} \left[ \partial_\mu \phi \partial_\nu \phi - \partial_\mu \phi \partial_\nu \phi \right] + \frac{1}{2} m_\omega^2 \phi \partial_\mu \phi \partial_\nu \phi.
\]

In the above the Lorentz vector-isovector dependence is ignored, because we considered the symmetric nuclear matter within the Hartree approximation [11].

We work in the nuclear matter rest frame hereafter. The Dirac equations for the up \((u)\) and down \((d)\) quarks are solved self-consistently with the same mean values of the \( \sigma \) and \( \omega \) fields, which also act on the nucleon and describe the properties of nuclear matter [18, 10]:

\[
\left[ i \gamma \cdot \partial - (m_q - V^{\sigma}_q) \mp \gamma \left( V^{\omega}_q + \frac{1}{2} V^{\rho}_q \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} = 0,
\]

\[
\left[ i \gamma \cdot \partial - (m_q - V^{\sigma}_q) \mp \gamma \left( V^{\omega}_q - \frac{1}{2} V^{\rho}_q \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} = 0,
\]

\( SU(2) \) symmetry is assumed in the above for the quarks \( u \) and \( d \). We define, \( m_u^* = m_q - V^{\sigma}_q = m_d^* = m_d \). Also, in symmetric nuclear matter, the \( \rho \)-meson mean field potential, \( V^{\rho}_q = 0 \), is dropped. The other mean-field potentials are defined by, \( V^{\sigma}_q \equiv g^{\sigma}_q \sigma = g^{d}_q \sigma < \sigma > \) and \( V^{\omega}_q \equiv g^{\omega}_q \omega = g^{d}_q \omega \), with \( g^{d}_q \) and \( g^{d}_q \) the corresponding quark-meson coupling constants.

The baryon density \( (\rho) \) via \( k_F \) the Fermi momentum, scalar density \( (\rho_s) \), and the total energy per nucleon \( (E^{\text{tot}}/A) \) are given by,

\[
\rho = \frac{4}{(2\pi)^2} \int d\vec{k} \, \theta(k_F - |\vec{k}|) = \frac{2k_F^3}{3\pi^2},
\]

\[
\rho_s = \frac{4}{(2\pi)^2} \int d\vec{k} \, \theta(k_F - |\vec{k}|) \frac{m_N^* (\sigma)}{\sqrt{m_N^* (\sigma) + k^2}},
\]

\[
E^{\text{tot}}/A = \frac{4}{(2\pi)^2} \rho \int d\vec{k} \, \theta(k_F - |\vec{k}|) \sqrt{m_N^* (\sigma) + k^2} + \frac{m_s^2 \sigma^2}{2\rho} + \frac{g^{2}_\omega \rho}{2m_\omega^2},
\]

where the nuclear matter saturation properties, namely, the binding energy per nucleon of 15.7 MeV at the saturation density \( \rho_0 = 0.15 \text{ fm}^{-3} \) are known, and from these the coupling constants \( g_\sigma \) and \( g_\omega \) are determined.

In order to calculate the pion properties in symmetric nuclear matter, we use the light-front model of Ref. [15]. That model reproduces quite well the pion experimental data, i.e., pion electromagnetic form factor, electromagnetic radius and also the weak decay constant. Then, in this work the light-front constituent quark model (LFQCM) and QMC, are combined to study the pion properties in symmetric nuclear matter.

The pion properties in symmetric nuclear matter is calculated with the effective Lagrangian density with a pseudoscalar coupling of the quarks to the pion [19], used before for the vacuum case [11],

\[
\mathcal{L}_t = -ig^* \Phi \cdot \gamma \bar{q} \bar{q} \Lambda^*.
\]
Here, $g^*$ is the coupling constant, and $\Lambda^*$ is the vertex function in the medium. The coupling constant is given by the Goldberger-Treiman relation, $g^* = m_q^*/f_{\pi}$, using the quantities in symmetric nuclear matter (with the asterisks).

The electromagnetic current in symmetric nuclear matter is obtained in the impulse approximation, represented by the Feynman triangle diagram [15],

$$\bar{j}^\mu = -i2e\frac{m^2}{f_{\pi}^2}N_c \int \frac{d^4k'}{(2\pi)^4} \text{Tr} \left[ S^*(k')\gamma^5 S^*(k' - P')\gamma^\rho S^*(k' - P)\gamma^5 \right] \Lambda^*(k', P') \Lambda^*(k', P),$$ \hspace{1cm} (8)

where the factor 2 comes from the isospin algebra.

The in-medium modifications can be implemented with the model of Ref. [15]. The in-medium quark propagators are given by,

$$S^*(p + V) = \frac{1}{p + V - m_q^* + i\epsilon}.$$ \hspace{1cm} (9)

In the symmetric nuclear matter, the quark properties are modified by the Lorentz-scalar-isoscalar and the Lorentz-vector-isoscalar mean field potentials. In mean field approximation, the modifications enter as the shift in the quark and anti-quark momentum, $p^\mu \rightarrow p^\mu + V^\mu = p^\mu + \delta^\mu_0 V_0^0$ and $p^\mu + V'^\mu = p^\mu + \delta^\mu_0 V'^0_0$, for the quark (+) and antiquark (-), in the case of the vector potential. For the Lorentz scalar part, we have a modification in the quark mass as $m_q \rightarrow m_q^* = m + V_0^0 (= m_q - V_0^0)$.

Also, the in-medium vertex function is given by [15, 11],

$$\Lambda^*(k + V, P) = \frac{C^*}{((k + V)^2 - m_q^* + i\epsilon)} + \frac{C^*}{((P - k - V)^2 - m_q^* + i\epsilon)}.$$ \hspace{1cm} (10)

The main motivation to choose the above symmetric regulator is that, this vertex function is symmetric by the exchange of the momentum between the two fermions, and we have a symmetric light-front valence wave function [11, 15].

In the present case the effect of the vector potentials in the loop integral associated with the Feynman diagram is cancel out, because of our choice of the pion vertex. Then, only the mass shift of the quarks is relevant for the loop integral. (See Ref. [11] for details on this point.) In the vertex function Eq. (10), the parameter $C^*$ is assumed to be the same as that in vacuum, because it is associated with the short-range scale of the wave function of the pion.

In the present work, we use the Breit-frame and the Drell-Yan condition ($q^2 = 0$), where the momentum transfer is given by $q^\mu = (0, 0, q/2, q^\perp) = (P' - P)^\mu$, $q^\perp = -q^\perp = 0$, $q_4 = -q/2$ and $q^2 = q^+q^- - (q^\perp)^2 = -Q^2$. The bound state mass is $m_B = \sqrt{P'^2 + q^2}/4$ with $P^+ = P'^+$. In order to calculate the covariant form factor, we use the following expression

$$\bar{j}^\mu = e(P^\mu + P'^\mu)F^*_\pi(q^2).$$ \hspace{1cm} (11)
The pion elastic electromagnetic form factor has two contributions in the light-front approach [14, 15, 37], i.e., the valence and the non-valence contributions (see the Fig. 2):

\[
F_{\pi}^{(1)}(q^2) = F_{\pi}^{(2)}(q^2) + F_{\pi}^{(3)}(q^2).
\]

(12)

The Bethe-Salpeter amplitude associated with the pion in the medium is given by [11, 15]

\[
\Psi^\prime(k + V, P) = \frac{k + N + m_k^*}{(k + V)^2 - m_k^*} + i\epsilon \gamma^5 A^\prime(k + V, P) \frac{k + N - P + m_k^*}{(k + V - P)^2 - m_k^*} + i\epsilon.
\]

(13)

In the expression above, the instantaneous terms are separated out in the quark propagators, and \( k^\mu + \delta^\mu_q V^0 \to k^\mu \) shift will be made for all the relevant places. Performing the light-front energy integration, \( k^- \), the valence pion wave function is obtained:

\[
\Phi^\prime(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp) = \frac{P^+}{m^2_\pi - M^2_0} \left[ \frac{N^+}{(1 - x)(m^2_\pi - M^2(m^2_{\pi 3}; m^2_{\pi 2}))} + \frac{N^+}{x(m^2_\pi - M^2(m^2_{\pi 3}; m^2_{\pi 2}))} \right].
\]

(14)

Here \( N^+ \) is a normalization factor, \( N^+ = C^m_{\pi \pi}(N_c)^{1/2} \), and \( x = k^+/P^+ \) with \( 0 \leq x \leq 1 \), \( M^2(m^2_{\pi 2}; m^2_{\pi 3}) = \frac{k^2 + m^2_i}{x} + \frac{(P^2 - k^2 + m^2_i)}{1 - x} - P^2_\perp \), and the square of the mass is \( M^2_0 = M^2(m^2_{\pi 2}; m^2_{\pi 3}) \). Note that the normalization factor is also affected in the medium, and the condition \( F_{\pi}^{(1)}(q^2 = 0) = 1 \) (the pion charge) is imposed to fix the normalization factor.

The final expression for the pion electromagnetic form factor in symmetric nuclear matter is given by:

\[
F_{\pi}^{(1)}(q^2) = \frac{1}{2\pi^3 P^+ + P^+} \int d^2k_\perp dk^+ \theta(k^+) \theta(P^+ - k^+) \Phi^\prime(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp, \frac{q_\perp}{2}) \times \left( k^+ P^+ P^+ - \frac{1}{2} \vec{k}_\perp \cdot \vec{q}_\perp (P^+ - P^+) - \frac{1}{4} k^+ q_\perp^2 \right) \Phi^\prime(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp, \frac{q_\perp}{2}).
\]

(15)

Next, we calculate the probability of the valence \( q\bar{q} \) state for the pion [11, 15],

\[
f^\prime(k_\perp) = \frac{1}{4\pi^2 m_\pi^2} \int_0^{2\pi} d\phi \int_0^{P^+} \frac{d^2k_\perp M^2_0}{k^+(P^+ - k^+)} \Phi^2(k^+, \vec{k}_\perp; m_\pi^*, \vec{0}),
\]

(16)

and the integration of \( f^\prime(k_\perp) \), gives

\[
\eta^* = \int_0^\infty dk_\perp f^\prime(k_\perp),
\]

(17)

which gives the probability of the valence component of the pion in symmetric nuclear matter.

In addition, we calculate the in-medium pion decay constant, \( f_\pi^* \), from the axial current:

\[
P_\mu \langle 0(\rho) | A^\mu_{\pi}(\pi^0) \rangle = im_\pi^2 f_\pi^* \delta_{ij} \approx im_\pi^2 f_\pi^* \delta_{ij},
\]

(18)

Using \( A^\mu_{\pi} = \bar{q} \gamma^\mu \gamma^5 \slash q \), the Lagrangian density [11], and after the \( k^- \) integration, the final expression for the in-medium decay constant is obtained as

\[
f_\pi^* = \frac{m_\pi^2(N_c)^{1/2}}{4\pi^3} \int d^2k_\perp dk^+ \frac{\Phi^\prime(k^+, \vec{k}_\perp; m_\pi^*, \vec{0})}{k^+(P^+ - k^+)},
\]

(19)

where the expression above is associated with the plus-component of the axial current. Thus, \( f_\pi^* \) cannot be separated into the time and space components as done in chiral perturbation theory [28, 29, 30, 31, 32, 33].
FIGURE 2. (Left) Feynman triangle diagram: (a) valence contribution, $F_I^\pi$, and (b), non-valence contributions, $F_{II}^\pi$. (Right) Electromagnetic form factor of the pion in symmetric nuclear matter, calculated for several nuclear densities, compared with the experimental data in the vacuum from Refs. [20, 21, 22, 23, 24, 25, 26].

TABLE 1. Summary of in-medium pion properties. $\eta^*$ is calculated via Eq. (16), the probability of the valence quark component in the pion.

| $\rho/\rho_0$ | $m_q^*$ [MeV] | $f_\pi^*$ [MeV] | $<r^2_\pi>^{1/2}$ [fm] | $\eta^*$ |
|----------------|----------------|-----------------|------------------------|---------|
| 0.00           | 220            | 93.1            | 0.73                   | 0.782   |
| 0.25           | 179.9          | 80.6            | 0.84                   | 0.812   |
| 0.50           | 143.2          | 68.0            | 1.00                   | 0.843   |
| 0.75           | 109.8          | 55.1            | 1.26                   | 0.878   |
| 1.00           | 79.5           | 40.2            | 1.96                   | 0.930   |

Results and discussions

The model presented here has two free parameters, i.e., the constituent quark mass $m_q = 0.220$ GeV used to describe the pion properties [34, 35, 36], and the regulator mass, $m_R = 0.600$ GeV, the same value used for the pion in the vacuum [15, 37]. The value of $m_R$ is obtained by the fit to the experimental value of the in-vacuum pion decay constant (see table 1), i.e., $f_\pi = 92.4$ MeV [38]. In the (cold) nuclear medium, the pion mass is approximately given by the in-vacuum value, $m_\pi^* = m_\pi = 140$ MeV (see Refs. [28, 30, 32] for discussion). The electromagnetic radius is calculated from the derivative of the electromagnetic form factor at a very low momentum, and with the parameters above, the radius obtained in the vacuum is $r_\pi = 0.74$ fm [11, 15], which is very close to the experimental value of $0.67 \pm 0.02$ fm [38].

In this work, we have studied the pion properties in symmetric nuclear matter with the light-front constituent quark model plus the QMC model. We have calculated the pion electromagnetic form factor, electromagnetic radius, valence quark component probability, up to the nuclear matter density with the plus component of the electromagnetic current in the light-front approach.

The results show that the electromagnetic form factor decreases with increasing the nuclear density as can be seen in Fig. (2b). Furthermore, the electromagnetic radius increases as the nuclear density increases (see Table I) (since it depends on the in-medium electromagnetic form factor). The in-medium pion decay constant decreases with increasing the nuclear density, and this agrees with the conclusion extracted from the pionic-atom experiments. Also the valence quark component probability in the medium, $\eta^*$, increases with increasing the nuclear density as shown in Table 1. This is because the decrease of the in-medium constituent quark mass makes it easier to excite the valence constituent quark, and yields to a larger valence quark distribution inside the pion.

In the near future we plan to explore the in-medium properties of kaons and D-mesons, as well as the vector
particles like $\rho$ and $\omega$ mesons. Such studies are under in progress.

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