Resolution of the Proton Spin Problem

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Abstract

A number of lines of investigation into the structure of the nucleon have converged to the point where we believe that one has a consistent explanation of the well known proton spin crisis.
There is no more fundamental challenge for strong interaction physics than mapping the distribution of energy, momentum, spin and angular momentum onto the quarks and gluons that compose the nucleon. For the past two decades there has been a tremendous level of activity associated with the latter two, sparked by the discovery, almost 20 years ago, by the European Muon Collaboration (EMC) of a proton “spin crisis” [1]. Much of the early theoretical effort was focused on the important task of understanding the role of polarized gluons and the axial anomaly in resolving this crisis. Impressive experimental work at CERN, DESY, JLab, RHIC and SLAC has established a number of important pieces of the information needed to understand the puzzle.

According to EMC [1] the experimental indication was that the quark spin was near zero: $14 \pm 9 \pm 21\%$. This led to the exciting possibility [2, 3, 4, 5] that the proton might contain a substantial quantity of polarized glue which could contribute to reducing the quark spin through the famous U(1) axial anomaly. It has taken almost 20 years to investigate this fascinating possibility experimentally and there are still important measurements underway. Nevertheless, it is already clear that the gluon spin is nowhere near as large as would be required to explain the spin problem. For example, the most recent measurements of inclusive $\pi^0$ jets at RHIC are best fit with $\Delta G = 0$ [6, 7] and Bianchi [8, 9] reported $\Delta G/G \sim 0.08$ at Pacific-SPIN07.

As the accuracy of experimental investigation of the spin of the proton has increased, the fraction of the spin carried by quarks has moved significantly far towards the top of the range quoted by EMC. We now know that the sum of the helicities of the quarks in the proton corresponds to about a third its total spin [10, 11],

$$\Sigma_{\text{inv}} = 0.33 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.}),$$  \hspace{1cm} (1)

considerably higher than the initial EMC suggestion. Nevertheless, the modern value is still astonishingly small.

The apparent failure of polarized glue as an explanation for the spin problem leads us to focus again on suggestions made soon after the EMC announcement [12, 13, 14], which were based on physics that is more familiar to those modeling non-perturbative QCD. As we shall explain, these ideas have important implications for experimental efforts in this area. In particular, they suggest that most of the missing spin of the proton must be carried as orbital angular momentum by the valence quarks, which in turn makes the
study of Generalized Parton Distributions (GPDs) after the 12 GeV Upgrade at JLab extremely interesting.

We begin our discussion by summarizing the key physics leading to the observed quark spin, $\Sigma_{\text{inv}}$, before explaining each term in more detail. There are three factors which, in our view, are needed in order to understand the data:

- the relativistic motion of the valence quarks
- the virtual excitation of anti-quarks in low-lying p-states through the one-gluon-exchange hyperfine interaction – in nuclear physics terms this would be termed an exchange current correction
- the pion cloud of the nucleon.

These three pieces of physics, tested in many independent ways, all have the effect of converting quark spin to orbital angular momentum. The first reduces the spin by about one third, the second yields a reduction by an amount of order 0.15 and the third gives a multiplicative reduction by a factor of order 0.80 – the details and estimates of uncertainties are given below. Altogether, these effects reduce the fraction of the proton spin carried by its quarks to about one third, in very good agreement with the data.

We now present some details of these three major reduction factors, which lead to the small value of $\Sigma_{\text{inv}}$.

1. Relativistic Valence Quark Motion

This effect was well understood even at the time of the EMC discovery. A spin-up, light quark in an s-state, moving in a confining potential, has a lower Dirac component in which the quark is in p-wave. The angular momentum coupling is such that for this component the spin is preferably down and reduces the “spin content” of the valence quarks. In the bag model, for example, where the massless quark’s ground state energy equals $\Omega/R \simeq 2.043/R$, the reduction factor $B = \Omega/3(\Omega - 1) \simeq 0.65$. The same factor reduces the value of $g_A$ from 5/3 to $\simeq 1.09$ in a bag model and this value changes little if one uses typical light quark current masses. Even in more modern relativistic models, where quark confinement is simulated by forbidding on-shell propagation through proper-time regularization, the reduction factor is very similar – e.g., in Ref. $[15]$ $\Delta u + \Delta d$ is 0.67. In terms of following where the nucleon spin has gone, the relativistic motion transfers
Figure 1: One gluon exchange graphs contributing to the proton spin.

roughly 35% of the nucleon spin from quark spin to valence quark orbital angular momentum.

2. The One-Gluon-Exchange Hyperfine Interaction
It is well established that the spin-spin interaction between quarks in a baryon, arising from the exchange of a single gluon, explains a major part of the mass difference between the octet and decuplet baryons – e.g., the nucleon-Δ mass difference \[16, 17\]. This spin-spin interaction must therefore also play a role when an external probe interacts with the three-quark baryon state. That is, the probe not only senses a single quark current but a two-quark current as well. The latter has an intermediate quark propagator connecting the probe and the spin-spin interaction vertices, and is similar to the exchange-current corrections which are well known in nuclear physics. In the context of spin sum rules the probe couples to the various axial currents in the nucleon. In the case of the two-quark current, first investigated in detail in Ref. \[18\], using the MIT bag model, the quark propagator was written as a sum over quark eigenmodes and the dominant contributions were found to come from the intermediate p-wave anti-quark states. The primary focus of Ref. \[18\] was actually the one-gluon-exchange corrections to the magnetic

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moments and semi-leptonic decays of the baryon octet. For example, this exchange current correction is vital to understand the unusual strength of the decay $\Sigma^- \to n + e^- + \bar{\nu}_e$.

Myhrer and Thomas [12] realized the importance of this correction to the flavor singlet axial charge and hence to the proton spin, finding that it reduced the fraction of the spin of the nucleon carried by quarks, calculated in the naive bag model by 0.15, i.e., $\Sigma \to \Sigma - 3G$ [12]. The correction term, $G$, is proportional to $\alpha_s$ times certain bag model matrix elements [18], where $\alpha_s$ is determined by the “bare” nucleon-$\Delta$ mass difference. Again, the spin lost by the quarks is compensated by orbital angular momentum of the quarks and anti-quarks (predominantly $\bar{u}$ in the p-wave).

3. The pion cloud

We know that many static baryon observables, such as the baryon magnetic moments and charge distributions, acquire important contributions from their pion cloud [19]. This pion cloud is an effective description of the quark-antiquark excitations which are required by the chiral symmetry of QCD. In fact, describing a physical nucleon as having a pion cloud which interacts with the valence quarks of the quark core (the “bare” nucleon), in a manner dictated by the requirements of chiral symmetry, has been very successful in describing the properties of the nucleon [20, 21, 22]. The cloudy bag model (CBM) [20, 21] reflects this description of the nucleon and in this model the nucleon consists of a bare nucleon, $|N>$, with a probability $Z \sim 1 - P_{N\pi} - P_{\Delta\pi} \sim 0.7$, in addition to being described as a nucleon (N) and a pion and a $\Delta$ and a pion, with probabilities $P_{N\pi} \sim 0.20 - 0.25$ and $P_{\Delta\pi} \sim 0.05 - 0.10$, respectively. The phenomenological constraints on these probabilities were discussed, for example, in Refs. [23, 24]. One of the most famous of these constraints is associated with the excess of $\bar{d}$ over $\bar{u}$ quarks in the proton, predicted on the basis of the CBM [25]. Indeed, to first order the integral of $\bar{d}(x) - \bar{u}(x)$ is $2/3 \ P_{N\pi}$, which is experimentally consistent with the range just quoted [26].

The pion cloud effect was investigated early by Schreiber and Thomas, who wrote the corrections to the spin sum-rules for the proton and neutron explicitly in terms of the probabilities set out above [13]. For our purposes it is helpful to summarize the results of Ref. [13] for the proton and neutron. The pion cloud correction to the flavor singlet combination modifies
the proton spin in the following manner:

\[ \Sigma \rightarrow \left( Z - \frac{1}{3} P_{N\pi} + \frac{5}{3} P_{\Delta\pi} \right) \Sigma. \]  

(2)

From the point of view of the spin problem, the critical feature of the pion cloud is that the coupling the spin of the nucleon and the orbital angular momentum of the pion in the \( N\pi \) Fock state favors a spin down nucleon and a pion with +1 unit of orbital angular momentum. This too has the effect of replacing quark spin by quark and anti-quark orbital angular momentum. Note that in the \( \Delta\pi \) Fock component the spin of the baryon tends to point up (and the pion angular momentum down), thus enhancing the quark spin. Nevertheless, the wave function renormalization factor, \( Z \), dominates, yielding a reduction by a factor between 0.7 and 0.8 for the range of probabilities quoted above.

4. Discussion

The corrections described here, which arise from either the pion cloud or gluon exchange, lead to a significant movement from the theoretically expected value of \( \Sigma \approx 0.65 \) (because of relativistic motion of the quarks) towards the experimental value. The one-gluon-exchange correction moves \( \Sigma \) down to 0.50, while the pion cloud correction reduces \( \Sigma \) to between 0.46 and 0.52. At the time these corrections were first discussed, neither the one-gluon-exchange correction, nor the pion cloud, seemed to yield a correction large enough to be relevant to resolving the crisis. Furthermore, we were reticent to combine the one-gluon-exchange and pion cloud corrections as it was expected that the latter might contribute a substantial fraction of the observed splitting between the \( N \) and \( \Delta \), which would in turn reduce the strength of the one-gluon-exchange term. However, progress in the analysis of lattice QCD calculations, especially in the last few years, changes the situation. In particular, the chiral analysis of quenched and full QCD data for the \( N \) and \( \Delta \) masses as a function of quark mass [27, 28], has led to the conclusion that pion effects likely contribute 50 MeV or less of the observed 300 MeV mass difference. As a result we no longer need to worry about significant double counting and can therefore combine the one-gluon-exchange and pion cloud corrections to the quark spin sum.

In fact, it is apparent that if we combine the one-gluon-exchange and pion cloud corrections, which we have just summarized, one finds a value for \( \Sigma \) between 0.35 (\( P_{N\pi} = 0.25, P_{\Delta\pi} = 0.05 \)) and 0.40 (\( P_{N\pi} = 0.20, P_{\Delta\pi} = 0.10 \)) in excellent agreement with the modern data.
Until now we have used the symbol $\Sigma$ to denote the quark spin evaluated within a model. This has variously been identified in the literature with either $\Sigma_{\text{inv}}$ or with the value measured at some relatively low scale, $\mu^2 < 1 \text{ GeV}^2$. The difficulty arises because the flavor singlet spin operator has a divergence in QCD involving an operator of dimension 4, so that its matrix element is not renormalization group invariant. Fortunately, at the current level of precision, this issue is of little practical importance for us. In the former case, the model result is $\Sigma \in (0.35, 0.40)$, which agrees very well with the experimental value $\Sigma_{\text{inv}}$ -- c.f. Eq. (1). In the latter case, which is motivated by the observation that a valence dominated quark model can only match experiment for parton distribution functions at a relatively low scale [29, 30, 31], the calculated value of the quark spin would need to be multiplied by a non-perturbative factor involving the QCD $\beta$–function and the anomalous dimension, $\gamma$, of the flavor singlet axial charge, which has been calculated to three loops by Larin and Vermaseren [32, 33]. As this factor is truly non-perturbative, its evaluation through even three-loop perturbation theory is at best semi-quantitative [34]. Nevertheless, it is rigorously less than unity and an evaluation at three-loops gave a value of order 0.6–0.8 [35]. Multiplying the quark spin obtained above by this factor yields a value for $\Sigma \in (0.21, 0.32)$, which is also in excellent agreement with the current experimental value.

In conclusion, the tremendous experimental progress aimed at resolving the spin problem has established that the quarks carry about one third of the spin of the nucleon and that the polarization of the gluons is most likely too small to account for the difference. Instead, well known aspects of hadron structure involving its pion cloud and the hyperfine interaction mediated by one-gluon exchange, in combination with the relativistic motion of the confined quarks, are able to explain the modern data very satisfactorily. As a consequence of these new insights, we expect that the missing spin should be accounted for by the orbital angular momentum of the quarks and antiquarks – the latter associated with the pion cloud of the nucleon and the p-wave anti-quarks excited by the one-gluon-exchange hyperfine interaction. Finally, we note that the exploration of the angular momentum carried by the quarks and anti-quarks is a major aim of the scientific program associated with the 12 GeV Upgrade at Jefferson Lab.

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