A hybrid decision-making framework under complex spherical fuzzy prioritized weighted aggregation operators

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Abstract
Complex spherical fuzzy set, an extended version of spherical fuzzy set, is a very powerful tool to capture fourfold information (typically yes, no, abstain and refusal), in which the range of degrees occurs in the complex plane with unit disk. Through this prominent feature, complex spherical fuzzy sets outperform earlier concepts of fuzzy sets and their extensions. This research article utilizes complex spherical fuzzy sets and prioritized weighted aggregation operators to construct the complex spherical fuzzy prioritized weighted averaging/geometric operators. We present their most noticeable properties. Further, we establish a decision-making approach that takes full advantage of the aforesaid operators. To explore their superiority and applicability in decision making, we apply our algorithm to a numerical example. Finally, we compare this decision-making approach with prevailing methods in this context.

KEYWORDS
boundedness, complex spherical fuzzy numbers, decision matrix, prioritized weighted aggregation operators, score function

1 | INTRODUCTION

Fuzzy set (FS) theory, originated with Zadeh (1965), is applicable in a variety of areas, including computer science, engineering, management sciences, decision theory and many others. Nevertheless, there exist some drawbacks in FS theory. One of them is the fact that Zadeh considered only the membership degree (MD) of an object in a FS. Motivated by this downside, Atanassov (1986) put forward the remarkable notion of intuitionistic fuzzy set (IFS). This convenient tool captures two independent degrees, namely, membership degree or MD ($\mu$) with the same interpretation as above, and non-membership degree or NMD ($\nu$); they are asked to satisfy the condition that at any point, their sum should be smaller than or equal to one. After the introduction of IFSs, various models insisted on expanding their scope by either relaxing their requirements or adding new features. Thus for example, Alcantud, Khameneh, and Kilicman (2020) explored temporal IFSs. Yager (2013 2014) presented the idea of Pythagorean fuzzy set (PyFS), which generalized them by relaxing the condition on MD and NMD to $\mu^2 + \nu^2 \leq 1$. This model received considerable attention. Wang and Li (2020) introduced Pythagorean fuzzy interaction power Bonferroni mean AOs, and Wang and Garg (2020) first adopted the rules of Archimedean t-conorm and t-norm for the production of Pythagorean fuzzy interaction aggregation operators. Relatedly, Cuong (2013a 2013b)
presented the concept of picture fuzzy set (PFS), which is also more general than IFS. To this purpose he introduced three degrees of an element in a PFS, namely, MD ($\mu$), abstention degree (AD) or neutral degree ($\gamma$), and NMD ($\nu$); they satisfy the joint condition $0 \leq \mu + \gamma + \nu \leq 1$ at any point.

PFSs are capable of dealing with the human opinions of type: yes, abstain, no and refusal. Gündoğdu and Kahraman (2019) originated a new extension of PFS named spherical FS (SFS). This model broadened the space of MD ($\mu$), AD ($\gamma$), and NMD ($\nu$) in the interval $[0, 1]$ with the condition $0 \leq \mu^2 + \gamma^2 + \nu^2 \leq 1$. For further study on SFSs and their applications, the readers are referred to Gündoğdu and Kahraman (2020), Kahraman, Gündoğdu, Onar, and Oztaysi (2019), and Yang, Li, Garg, and Qi (2020). Li et al. established a new extension, named q-rung PFS with the condition $0 \leq \mu^q + \gamma^q + \nu^q \leq 1$, $q \geq 1$ (Li, Zhang, Wang, Shang, & Bai 2018). Akram et al. studied the notion of SFS by providing the solution of a decision-making (DM) problem and Akram (2020) presented a DM method based on spherical fuzzy graphs (Akram, Saleem, & Al-Hawary, 2020c).

Despite the introduction of different sets and their extensions, the aforementioned models do not suffice to manage periodic information or two-dimensional phenomena. To overcome this difficulty, Ramot et al. produced the concept of complex FS (CFS) by extending the range of membership from $[0, 1]$ to the unit circle in the complex plane (Ramot et al., 2003; Ramot, Milo, Friedman, & Kandel 2002). After the emergence of CFS, researchers investigated and worked on CFS and its extensions (Akram, Bashir, & Garg, 2020a; Akram, Khan, & Karaaslan, 2020; Alkouri & Salleh 2012; Alkouri & Salleh 2013; Luqmán, Akram, Al-Kenani, & Allocutd, 2019; Luqmán, Akram, & Smarandache, 2019; Ullah, Mahmood, Ali, & Jan 2020).

This paper explores the scope of application of the concept that combines the benefits of SFSs and CFS, namely, complex spherical fuzzy sets (CSFSs). This model has already been combined with the well-known VIKOR methodology since Akram, Kahraman, and Zahid (2021) developed the complex spherical fuzzy VIKOR method. In a CSFS, there are three complex valued membership degrees (MD, AD and NMD) which are represented in polar coordinates. Therefore, CSFSs emerge as a combination of the CFS and SFS theories, and it incorporates their respective benefits. Hence, keeping the advantages of CSFSs, we introduce basic operational laws between two complex spherical fuzzy numbers (CSFNs). These items capture the assessment on belongingness of one particular element to the CSFS. Moreover, we introduce two aggregation operators (AOs) on CSFSs, respectively called the CSFPWA and CSFPWG operators.

AOs are a helpful tool for the aggregation of information. They have been applied in decision-making (DM) and other fields of research (Garg & Kumar 2019; Rani & Garg 2018; Tang, Meng, Xu, & Yuan 2020). DM attempts to identify the most beneficial choice from a set of feasible alternatives. In particular, multi-criteria decision-making (MCDM) provides the most suitable solution after examining the alternatives over multiple criteria. Applications of MCDM problems abound in different fields. A main issue in these problems is the right combination of information from various sources. Researchers made numerous attempts to produce adequate aggregations. From different perspectives, quantitative AOs were presented in order to convert the overall data into a single value. For further study on AOs, the readers are referred to Shahzadi, Akram, and Al-Kenani (2020), and Waseem, Akram, and Alcantud (2019).

In relation with the context that we investigate in this paper, Ashraf and Abdullah (2019) presented spherical AOs with an application to DM. (Bi, Dai, & Hu 2018) and (Bi, Dai, Hu, & Li 2019) proposed geometric and arithmetic AOs in a complex fuzzy environment. Akram and Bashir (2020) developed complex fuzzy ordered weighted quadratic averaging operators and applied them to a numerical example. Akram and Khan (2020) presented complex Pythagorean Dombi fuzzy arithmetic and geometric AOs.

In order to allow for the prioritization of the opinions, some researchers presented certain prioritized AOs. They are designed to account for the fact that the priority level of the criteria may be different. Yu (2013a) presented prioritized AOs under the IF environment with their applications. Yu (2013b) also developed IF generalized prioritized weighted averaging and geometric AOs and their application. For further study on prioritized AOs, one may refer to Arora and Garg (2018), Gao (2018), Khan, Abdullah, Ali, and Amin (2019), Verma and Sharma (2015), Yager (2008), and Akram, Khan, and Karaslan (2020).

The main objectives of the present article are:

- We develop prioritized weighted AOs within the CSF environment. Spherical fuzzy prioritized weighted AOs exist and they are quite efficient to deal with data having a prioritization relationship. But these prioritized AOs for the SF case deal with one-dimensional information only. Prioritized weighted AOs have not yet been applied to CSFSs. Therefore, we have presented our work within the CSF environment in order to provide a platform which is able to use CSF data with the assistance of prioritized AOs.
- Two AOs, namely, the CSFPWA and CSFPWG operators, are successfully established to initiate the idea of prioritized AOs under the CSF framework.
- A new score function to rank the alternatives is put forward.
- An MCGDM algorithm, based on complex spherical fuzzy prioritized weighted AOs, is introduced to solve practical problems. Furthermore, a fully developed numerical example along with a validity test and a comparison analysis are presented in order to guarantee the applicability and reliability of the proposal.

The following points have motivated us to conduct this research:

- CSFSs are very effective to represent the two dimensions of certain objects, as the phase term of a CFS is used to capture their second dimension. In addition, SFSs are quite remarkable as a general model for fuzzy information, which becomes indispensable when neutral opinions occur.
The CSFS theory merges the features of CFS theory and SFS theory. The potential of a CSFS to represent the three degrees of an object makes it outperform earlier notions of FSs and its extensions. The salient characteristics of a CSFS motivated us to present our work in a CSF environment.

In many decision-making problems, chances exist that the priority levels are not the same for criteria and decision makers. Put differently, imposing the same priority levels for the criteria and decision makers may be inconvenient in certain circumstances. This prominent feature of prioritized AOs – that can circumvent this inconvenience – motivated us to apply them on CSFSs.

The contributions of this article are summarized as follows:

- Operational laws play a key role during the aggregation process. We have presented some fundamental operations on CSFNs in order to be better equipped at that stage. We have also given a new score function that enables us to compare CSFNs.
- We develop two AOs under CSF environment, namely, the CSFPWA and CSFPWG operators. Some of their basic properties are discussed, inclusive of idempotency, monotonicity and boundedness.
- We have stated an algorithm and solved a decision-making problem by the resort to these AOs. Furthermore, a validity test is performed and a comparative analysis with existing methods is presented. This indicates the reliability and superiority of the proposed work.

The remainder of the paper is structured as: Section 2 recalls some basic concepts. Section 3 introduces some operational laws, score function and the complex spherical fuzzy prioritized weighted AOs. These include the CSFPWA and CSFPWG operators with their fundamental properties. Section 4 presents an algorithm that takes advantage of the application of our proposed operators. Section 5 discusses a comparison of the proposed technique with existing methods. Section 6 reveals the advantages of our presented method, and finally Section 7 summarizes our findings. To facilitate the reading of this article, Table 1 summarizes the meaning of the symbols that are used.

### 2 | PRELIMINARIES

The model we use to capture information is an improvement of the next definition:

**Definition 1** (Gündoğdu & Kahraman, 2019) An SFS \( \mathcal{S} \) on a universe \( Y \) is represented as

\[
\mathcal{S} = \{(g, \mathcal{N}_g(g), h_g(g), \rho_g(g))| g \in Y\},
\]

where \( \mathcal{N}_g(g), h_g(g), \rho_g(g) \in [0,1], 0 \leq \mathcal{N}_g^2(g) + h_g^2(g) + \rho_g^2(g) \leq 1 \) for all \( g \in Y \). We consider the triplet \((\mathcal{N}_g(g), h_g(g), \rho_g(g))\) as spherical fuzzy numbers (SFN) and denote it by \( S = (\mathcal{N}_g, h_g, \rho_g) \). Note that \( \mathcal{N}_g \), \( h_g \) and \( \rho_g \) are the membership degree (MD), abstinence degree (AD) and non-membership degree (NMD) of \( S \), respectively. Further \( \pi_g = \sqrt{1 - (\mathcal{N}_g^2(g) + h_g^2(g) + \rho_g^2(g))} \) is the hesitancy degree of \( g \) in \( S \).

**Table 1** A mathematical description of the symbols

| Symbols | Mathematical description |
|---------|-------------------------|
| \( \mathcal{S} \) | Spherical fuzzy set |
| \( \mathcal{N} \) | Membership degree of amplitude term |
| \( h \) | Neutral or abstinence degree of amplitude term |
| \( \rho \) | Non-membership degree of amplitude term |
| \( \pi \) | Hesitancy degree |
| \( \delta \) | Complex spherical fuzzy set |
| \( \sigma \) | Membership degree of phase term |
| \( \varsigma \) | Neutral or abstinence degree of phase term |
| \( \xi \) | Non-membership degree of amplitude term |
| \( S \) | Score function of SFNs |
| \( S^* \) | Score function of CSFNs |
| \( Q \) | Collection of proper alternatives |
| \( \mathcal{C} \) | Set of criteria or attributes |
| \( \mathcal{D} \) | Set of decision makers |
| \( \mathcal{E}^p_{\mathcal{D}} \) | CSFNs assigned by the decision makers |

Abbreviation: CSFNs, complex spherical fuzzy numbers.
The seminal Gündoğdu and Kahraman (2019) gave a geometrical representation of SFSs, as well as the intuitive meaning of the distance between two SFSs.

**Definition 2** (Akram, Khan, and Karaaslan, 2020) Suppose \( \delta = (\xi, \eta, \rho) \) is an SFN. The new score function to score the alternatives is defined as

\[
S(\delta) = \frac{1}{3} \left( \left( k^2 - h^2 - \rho^2 \right) \right) \text{where } S(\delta) \in [0, 2].
\]

The inclusion of a periodical component produces a more general model, which is developed in the following section.

### 3  COMPLEX SPHERICAL FUZZY PRIORITIZED WEIGHTED AGGREGATION OPERATORS

**Definition 3.** A complex spherical fuzzy set \( \delta \) on a set \( Y \) is represented as

\[
\delta = \left\{ \left( g, N_\xi(g), N_\eta(g), N_\rho(g) \right) \mid g \in Y \right\},
\]

where \( N_\xi(g) \), \( N_\eta(g) \), and \( N_\rho(g) \) represent the membership degree, neutral degree and non-membership degree, respectively, characterized by a mapping \( N_\xi(g), N_\eta(g), N_\rho(g) : Y \to [0, 1] \). The amplitude terms of the membership degree, neutral degree and non-membership degree, represented by \( \sigma, \tau, \gamma \), where \( 0 \leq \sigma + \tau + \gamma \leq 1 \) and belong to the unit interval \([0,1]\). The phase terms of the membership degree, neutral degree and non-membership degree, represented by \( \sigma_\delta, \tau_\delta \) and \( \gamma_\delta \) belong to the closed interval \([0,2\pi]\) and \( \iota = \sqrt{-1} \).

Moreover,

\[
\sigma_\delta(g) = \sqrt{1 - N_\tau^2(g) - N_\rho^2(g) - N_\xi^2(g)} \text{ and } \tau_\delta(g) = \sqrt{1 - N_\sigma^2(g) - N_\rho^2(g) - N_\xi^2(g)}
\]

represents the refusal degree of \( \delta \) for all \( g \in Y \). We consider each triplet \( (N_\xi(g), N_\eta(g), N_\rho(g)) \) as a CSFN and denote it by \( \delta = (N_\xi, N_\eta, N_\rho) \).

In this section, we procure some aggregation operators that act on the components of CSFSs, i.e., on CSFNs. We propose the CSFPWA and CSFPWG operators. Moreover, we discuss some of their fundamental properties. First of all we define some basic operations between two CSFNs, as well as a new score function to score the alternatives defined by CSFNs.

**Definition 4.** Suppose \( \delta_1 = (\xi_1, \eta_1, \rho_1) \) and \( \delta_2 = (\xi_2, \eta_2, \rho_2) \) are two CSFNs on a universe of alternatives \( Y \). Then some basic operations between \( \delta_1 \) and \( \delta_2 \) are given as follows:

1. \( \delta_1 \oplus \delta_2 = \left( \sqrt{N_{\xi_1}^2 + N_{\xi_2}^2 - N_{\xi_1}N_{\xi_2}} e^{i \pi \left( \frac{1}{2} - \frac{N_{\xi_1}^2}{N_{\xi_2}^2} \right)} \right), \)

2. \( \delta_1 \otimes \delta_2 = \left( \sqrt{N_{\xi_1}N_{\xi_2} e^{i \pi \left( \frac{1}{2} - \frac{1}{2} \frac{N_{\xi_1}^2}{N_{\xi_2}^2} \right)}} \right), \)

3. \( \rho \delta = \left( \frac{1 - (1 - N_{\xi_2})}{e^{i \pi \left( \frac{1}{2} - \frac{1}{2} \frac{N_{\xi_1}^2}{N_{\xi_2}^2} \right)}} \right), (\rho_1)^e, (\rho_2)^e, \) where \( e \geq 0 \)
Definition 5. The new score function to score the alternatives is defined as: when \( \delta = (N_k \epsilon^{x_k}, h_k \epsilon^{x_k}, \rho_k \epsilon^{x_k}) \) is a CSFN,

\[
S^*(\delta) = \frac{1}{3} \left( (2 + k^2 - h^2) + (2 + x^2 - \rho^2) \right).
\]

Observe that similarly to the case of Definition 2, \( S^*(\delta) \in [0, 2] \) for each CSFN.

### 3.1 Complex spherical fuzzy prioritized weighted averaging operator

Now we are ready to define our first aggregation operator on CSFNs:

**Definition 6.** The complex spherical fuzzy prioritized weighted averaging (CSFPWA) operator is defined by the mapping \( \text{CSFPWA} : \delta' \rightarrow \delta \), such that when \( \delta = (N_k \epsilon^{x_k}, h_k \epsilon^{x_k}, \rho_k \epsilon^{x_k}) \), \( s = 1, 2, \ldots, v \), is a collection of CSFNs,

\[
\text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) = \bigotimes_{v=1}^{v} \left( \frac{A_s}{\sum_{s=1}^{A_s} \delta_s} \right) = \left( \frac{A_1}{\sum_{s=1}^{A_s} \delta_s} \oplus \frac{A_2}{\sum_{s=1}^{A_s} \delta_s} \oplus \cdots \oplus \frac{A_v}{\sum_{s=1}^{A_s} \delta_s} \right).
\]

In this formula \( A_s = \prod_{s=1}^{v} S^*(\delta_s) \), \( s = 1, 2, \ldots, v \), with \( A_1 = 1 \), and \( S^*(\delta) \) denotes the score of \( \delta_s \).

Our next Theorem proves that the CSFPWA operator is well defined:

**Theorem 1.** If \( \delta = (N_k \epsilon^{x_k}, h_k \epsilon^{x_k}, \rho_k \epsilon^{x_k}) \), \( s = 1, 2, \ldots, v \), is a collection of CSFNs, then their aggregate value by CSFPWA is a CSFN. This aggregate value can be obtained by the following formula:

\[
\text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) = \bigotimes_{v=1}^{v} \left( \frac{A_s}{\sum_{s=1}^{A_s} \delta_s} \right) = \left( \frac{1 - \prod_{s=1}^{v} (1 - h_s^k) \sum_{s=1}^{A_s} \delta_s}{1 - \prod_{s=1}^{v} (1 - h_s^k) \sum_{s=1}^{A_s} \delta_s} \right)^{\frac{2\alpha}{\sum_{s=1}^{V} h_s^k}} \left( \frac{1}{\prod_{s=1}^{v} (1 - h_s^k) \sum_{s=1}^{A_s} \delta_s} \right)^{\frac{2\alpha}{\sum_{s=1}^{V} h_s^k}} \left( \frac{1}{\prod_{s=1}^{v} (1 - h_s^k) \sum_{s=1}^{A_s} \delta_s} \right)^{\frac{2\alpha}{\sum_{s=1}^{V} h_s^k}} \left( \frac{1}{\prod_{s=1}^{v} (1 - h_s^k) \sum_{s=1}^{A_s} \delta_s} \right)^{\frac{2\alpha}{\sum_{s=1}^{V} h_s^k}} \right)\]
In this formula $A_s = \prod_{j=1}^{s-1} S'(\delta_j)$, $s = 1, 2, ..., v$, with $A_1 = 1$, and $S'(\delta)$ denotes the score of $\delta$.

Proof. Theorem 1 can be easily proved by induction. We first prove that Equation (3) holds for $v = 2$:

\[
A_1 - \delta_1 = \sum_{i=1}^{A_1} \delta_i = \left( \prod_{i=1}^{A_1} \delta_i \right) \sum_{i=1}^{A_1} \delta_i e^{2x} \left( \sum_{i=1}^{A_1} \delta_i e^{2x} \right) \left( \sum_{i=1}^{A_1} \delta_i e^{2x} \right),
\]

\[
A_2 - \delta_2 = \sum_{i=1}^{A_2} \delta_i = \left( \prod_{i=1}^{A_2} \delta_i \right) \sum_{i=1}^{A_2} \delta_i e^{2x} \left( \sum_{i=1}^{A_2} \delta_i e^{2x} \right) \left( \sum_{i=1}^{A_2} \delta_i e^{2x} \right),
\]

Thus

\[
CSFPWA(\delta_1, \delta_2) = \frac{A_1}{\sum_{i=1}^{A_1}} \delta_1 \bigoplus \frac{A_2}{\sum_{i=1}^{A_2}} \delta_2
\]

\[
= \left( \prod_{i=1}^{A_1} \delta_i \right) \sum_{i=1}^{A_1} \delta_i e^{2x} \left( \sum_{i=1}^{A_1} \delta_i e^{2x} \right) \left( \sum_{i=1}^{A_1} \delta_i e^{2x} \right)
\]

\[
\bigoplus \left( \prod_{i=1}^{A_2} \delta_i \right) \sum_{i=1}^{A_2} \delta_i e^{2x} \left( \sum_{i=1}^{A_2} \delta_i e^{2x} \right) \left( \sum_{i=1}^{A_2} \delta_i e^{2x} \right)
\]

\[
= \left( \prod_{i=1}^{A_1} \delta_i \right) \sum_{i=1}^{A_1} \delta_i e^{2x} \left( \sum_{i=1}^{A_1} \delta_i e^{2x} \right) \left( \sum_{i=1}^{A_1} \delta_i e^{2x} \right)
\]

So, Equation (3) is true for $v = 2$.

For $v = 1$, suppose that the induction hypothesis is true, that is, that Equation (3) holds.
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Suppose

With the avail of Theorem 1, we proceed to prove some properties (idempotency, monotonicity, and boundedness) satisfied by the CSFPWA

Therefore Equation (3) is true for

Now we prove that Equation (3) holds true for

With the avail of Theorem 1, we proceed to prove some properties (idempotency, monotonicity, and boundedness) satisfied by the CSFPWA operator.

Theorem 2. (Idempotency) Suppose \( \delta_i = (N_i e^{\rho_i}, h_i e^{\rho_i}, \rho_i e^{\rho_i}) \) \( \forall i = 1, 2, ..., \nu \) is a collection of CSFNs with the condition \( \delta_i = \delta \) (for all \( s \)). Then

\[
\text{CSFPWA}(\delta_1, \delta_2, ..., \delta_\nu) = \delta.
\]
(Monotonicity) Suppose $\delta_s = \delta$ for all $s$, we have

$$\text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) = \left( \sum_{i=1}^{v} \frac{A_i}{\sum_{i=1}^{v} A_i} \delta_i \right) = \left( \frac{A_1}{\sum_{i=1}^{v} A_i} \delta_1 + \frac{A_2}{\sum_{i=1}^{v} A_i} \delta_2 + \cdots + \frac{A_v}{\sum_{i=1}^{v} A_i} \delta_v \right)$$

$$= \left( \sqrt{1 - \prod_{i=1}^{v} (1 - N_i^2) } e^{\frac{1}{2} \sum_{i=1}^{v} (h_i) e^{2\alpha_i}} \right) \left( \prod_{i=1}^{v} (h_i) e^{2\alpha_i} \right),$$

where $N_i = \frac{\sum_{i=1}^{v} A_i}{\sum_{i=1}^{v} A_i}, h_i = r_i \delta_i, \alpha_i = \sigma_i \delta_i, \beta_i = \xi_i \delta_i,$ and $N_i, h_i, \alpha_i, \beta_i$ are non-negative for all $s$. Then

$$\text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) \leq \text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v).$$

Theorem 3. (Monotonicity) Suppose $\delta_s = \left( N_s e^{\alpha_s}, h_s, r_s e^{\beta_s} \right)$ and $\delta_s = \left( N_s e^{\alpha_s}, h_s, r_s e^{\beta_s} \right)$ are two collections of CSFNs, with $s = 1, 2, \ldots, v$, $N_s \leq N_s, h_s \geq h_s, r_s \geq r_s, \alpha_s \leq \alpha_s, \beta_s \geq \beta_s, \text{ and } \alpha_s \geq \alpha_s,$ for all $s$. Then

$$\text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) \leq \text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v).$$
Theorem 4. Suppose that the collections of CSFNs \( \delta'_s \) and \( \delta_s \) have the following prioritized weight vectors:

\[
\begin{align*}
A'_{\delta'} = & \left( \frac{A'_{\delta'_1}}{\sum_{i=1}^{v} A'_{\delta'_i}}, \frac{A'_{\delta'_2}}{\sum_{i=1}^{v} A'_{\delta'_i}}, \ldots, \frac{A'_{\delta'_v}}{\sum_{i=1}^{v} A'_{\delta'_i}} \right), \\
A_{\delta} = & \left( \frac{A_{\delta_1}}{\sum_{i=1}^{v} A_{\delta_i}}, \frac{A_{\delta_2}}{\sum_{i=1}^{v} A_{\delta_i}}, \ldots, \frac{A_{\delta_v}}{\sum_{i=1}^{v} A_{\delta_i}} \right),
\end{align*}
\]

respectively, where \( \frac{A'_{\delta'_i}}{\sum_{i=1}^{v} A'_{\delta'_i}}, \frac{A_{\delta_i}}{\sum_{i=1}^{v} A_{\delta_i}} \in [0, 1] \) meet the conditions \( \sum_{i=1}^{v} \frac{A'_{\delta'_i}}{\sum_{i=1}^{v} A'_{\delta'_i}} = \sum_{i=1}^{v} \frac{A_{\delta_i}}{\sum_{i=1}^{v} A_{\delta_i}} = 1 \). Consider

\[
\begin{align*}
\text{CSFPWA}(\delta'_1, \delta'_2, \ldots, \delta'_v) &= (\kappa e^{i\epsilon^r}, h e^{i\epsilon^c}, \rho e^{i\epsilon^l}), \\
\text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) &= (\kappa e^{i\epsilon_r}, h e^{i\epsilon_c}, \rho e^{i\epsilon_l}).
\end{align*}
\]

Since \( N'_v \leq N_v \), therefore \( \sqrt{\left(N'_v\right)^2} \leq \sqrt{\left(N_v\right)^2} \). We first show that \( N' \leq N \). So, we have

\[
\sqrt{1 - \left(N'_v\right)^2} \geq \sqrt{1 - \left(N_v\right)^2}.
\]

Thus, \( N' \leq N \). Moreover, \( h' \geq h \). Then,

\[
\prod_{i=1}^{v} \left(h'_iight)^{1-\frac{A'_{\delta'_i}}{\sum_{i=1}^{v} A'_{\delta'_i}}} \geq \prod_{i=1}^{v} \left(h_iight)^{1-\frac{A_{\delta_i}}{\sum_{i=1}^{v} A_{\delta_i}}}.
\]

Similarly, we can show that \( \rho' \geq \rho, \sigma' \leq \sigma, \xi' \geq \xi, \zeta' \geq \zeta \). Thus, the theorem is proved.

\[
\square
\]

Theorem 4. (Boundedness) Suppose \( \delta_s = (N_s e^{i\epsilon_s}, h_s e^{i\epsilon_c}, \rho_s e^{i\epsilon_l}) \) (s = 1, 2, ..., v) is a collection of CSFNs with \( \delta_{\min} = \min(\delta_1, \delta_2, \ldots, \delta_v) \) and \( \delta_{\max} = \max(\delta_1, \delta_2, \ldots, \delta_v) \). Then

\[
\delta_{\min} \leq \text{CSFPWA}(\delta_1, \delta_2, \ldots, \delta_v) \leq \delta_{\max}.
\]

Proof. Suppose that \( \delta_{\min} = \min(\delta_1, \delta_2, \ldots, \delta_v) = (N^- e^{i\epsilon^r}, h^- e^{i\epsilon^c}, \rho^- e^{i\epsilon^l}) \) and \( \delta_{\max} = \max(\delta_1, \delta_2, \ldots, \delta_v) = (N^+ e^{i\epsilon^r}, h^+ e^{i\epsilon^c}, \rho^+ e^{i\epsilon^l}) \). Therefore,

\[
N^- = \min(N_s), h^- = \min(h_s), \rho^- = \min(\rho_s), N^+ = \max(N_s), h^+ = \max(h_s), \rho^+ = \max(\rho_s).
\]

\[
\sigma^- = \min(\sigma_s), \xi^- = \min(\xi_s), \sigma^+ = \max(\sigma_s), \xi^+ = \max(\xi_s).
\]

The inequality for amplitude term of membership grade is given as follows:

\[
\sqrt{1 - \prod_{i=1}^{v} \left(1 - (N_s)^2\right)^{\frac{A_{\delta_i}}{\sum_{i=1}^{v} A_{\delta_i}}}} \leq \sqrt{1 - \prod_{i=1}^{v} \left(1 - (N_s)^2\right)^{\frac{A'_{\delta'_i}}{\sum_{i=1}^{v} A'_{\delta'_i}}}} \leq \sqrt{1 - \prod_{i=1}^{v} \left(1 - (N_s)^2\right)^{\frac{A_{\delta_i}}{\sum_{i=1}^{v} A_{\delta_i}}}.
\]

Similarly, the inequality for phase term of membership grade is given as follows:
The complex spherical fuzzy prioritized weighted geometric (CSFPWG) operator is defined by the mapping

$$\text{CSFPWG}(\delta_1, \delta_2, ..., \delta_v) = \prod_{s=1}^{v} \left( \frac{\sum_{j}^{n_s} \sigma_{s,j}^{(s)}}{\delta_{s,j}^{(s)}} \right).$$

This section provides an alternative aggregation operator on CSFNs and studies some of its properties. Its formulation is as follows:

**Definition 7.** The complex spherical fuzzy prioritized weighted geometric (CSFPWG) operator is defined by the mapping \(\text{CSFPWG}: \delta^{(s)} \rightarrow \delta\), such that when \(\delta = (\mathbf{N}_s, \mathbf{h}_s, \mathbf{\gamma}_s, \rho_s, \mathbf{e}_s)\), \(s = 1, 2, ..., v\), is a collection of CSFNs,

$$\text{CSFPWG}(\delta_1, \delta_2, ..., \delta_v) = \prod_{s=1}^{v} \left( \frac{\sum_{j}^{n_s} \sigma_{s,j}^{(s)}}{\delta_{s,j}^{(s)}} \right).$$

In this formula \(\lambda_s = \prod_{j=1}^{n_s} S(\delta_j)\), with \(\lambda_1 = 1\) and \(S(\delta_j)\) denotes the score of \(\delta_j\).

Our next Theorem proves that this operator is well defined:

**Theorem 5.** If \(\delta = (\mathbf{N}_s, \mathbf{h}_s, \mathbf{\gamma}_s, \rho_s, \mathbf{e}_s)\) \((s = 1, 2, ..., v)\) is a collection of CSFNs, then their aggregate value by CSFPWG is a CSFN. This aggregate value can be obtained by the following formula:

$$\text{CSFPWG}(\delta_1, \delta_2, ..., \delta_v) = \prod_{s=1}^{v} \left( \frac{\sum_{j}^{n_s} \sigma_{s,j}^{(s)}}{\delta_{s,j}^{(s)}} \right).$$

**Proof.** Similar to the proof of Theorem 1.

Below we demonstrate the properties of idempotency, monotonicity, and boundedness for CSFPWG operator.

**Theorem 6.** (Idempotency) Suppose \(\delta_s = (\mathbf{N}_s, \mathbf{h}_s, \mathbf{\gamma}_s, \rho_s, \mathbf{e}_s)\) \((s = 1, 2, ..., v)\) is a collection of CSFNs with the condition \(\delta_s = \delta\) (for all \(s\)). Then
Proof. Using Equation (5), we have

\[
\text{CSFPWG}(\delta_1, \delta_2, \ldots, \delta_v) = \bigotimes_{s=1}^{v} \left( \sum_{i_1}^{\delta_{1s}} e^{i_1} \prod_{i_2}^{(\bar{\delta}_{1s}^*)} e^{i_2} \right) = \left( \sum_{i_1}^{\delta_{1s}} e^{i_1} \prod_{i_2}^{(\bar{\delta}_{1s}^*)} e^{i_2} \right),
\]

\[
\prod_{i_1}^{(N)} e^{i_1} \prod_{i_2}^{(\bar{\delta}_{1s}^*)} e^{i_2}, \prod_{i_3}^{(h)} e^{i_3} \prod_{i_4}^{(\bar{\delta}_{1s}^*)} e^{i_4}, \ldots, \prod_{i_v}^{(h)} e^{i_v} \prod_{i_v}^{(\bar{\delta}_{1s}^*)} e^{i_v},
\]

\[
\left( 1 - \prod_{i_1}^{(1 - \rho_1^2)} e^{i_1} \prod_{i_2}^{(1 - (\bar{\delta}_{1s}^*)^2)} e^{i_2}, 1 - \prod_{i_3}^{(1 - \rho_2^2)} e^{i_3} \prod_{i_4}^{(1 - (\bar{\delta}_{1s}^*)^2)} e^{i_4}, \ldots, 1 - \prod_{i_v}^{(1 - \rho_v^2)} e^{i_v} \prod_{i_v}^{(1 - (\bar{\delta}_{1s}^*)^2)} e^{i_v} \right)
\]

\[
= \left( (N)e^{(i_1^2 - \rho_1^2)} e^{(i_2^2 - \rho_2^2)} e^{(i_3^2 - \rho_3^2)} \ldots e^{(i_v^2 - \rho_v^2)} \right) = \text{CSFPWG}(\delta_1, \delta_2, \ldots, \delta_v).
\]

Theorem 7. (Monotonicity) Suppose \( \delta'_s = (N_{s1} e^{i_1}, h_{s1} e^{i_2}, \rho_{s1} e^{i_3}) \) and \( \delta_s = (N_{s2} e^{i_1}, h_{s2} e^{i_2}, \rho_{s2} e^{i_3}) \) are two collections of CSFNs, with \( s = 1, 2, \ldots, v \), \( N_{s1} \leq N_{s2} \), \( h_{s1} \geq h_{s2} \), \( \rho_{s1} \geq \rho_{s2} \), \( \sigma_{s1} \leq \sigma_{s2} \), \( \zeta_{s1} \leq \zeta_{s2} \), and \( \xi_{s1} \geq \xi_{s2} \), for all \( s \). Then

\[\text{CSFPWG}(\delta'_1, \delta'_2, \ldots, \delta'_v) \preceq \text{CSFPWG}(\delta_1, \delta_2, \ldots, \delta_v)\].

Proof. Suppose \( \delta'_s = (N_{s1} e^{i_1}, h_{s1} e^{i_2}, \rho_{s1} e^{i_3}) \) and \( \delta_s = (N_{s2} e^{i_1}, h_{s2} e^{i_2}, \rho_{s2} e^{i_3}) \) are two collections of CSFNs with the following prioritized weight vectors,
Theorem 8. (Boundedness) Suppose \( \sigma_\lambda \) is a collection of CSFNs with \( \delta_1, \delta_2, \ldots, \delta_v \) \((s = 1, 2, \ldots, v)\) is a collection of CSFNs with \( \delta_{\min} = \min(\delta_1, \delta_2, \ldots, \delta_v) \) and \( \delta_{\max} = \max(\delta_1, \delta_2, \ldots, \delta_v) \). Then

\[
\delta_{\min} \leq \text{CSFPWG}(\delta_1, \delta_2, \ldots, \delta_v) \leq \delta_{\max}.
\]

Proof. Suppose that \( \delta_{\min} = \min(\delta_1, \delta_2, \ldots, \delta_v) = (\mathcal{N}^+ e^{\mathcal{N}^+}, \mathcal{N}^- e^{\mathcal{N}^-}, \rho^+ e^{\mathcal{N}^+}) \) and \( \delta_{\max} = \max(\delta_1, \delta_2, \ldots, \delta_v) = (\mathcal{N}^+ e^{\mathcal{N}^+}, \mathcal{N}^- e^{\mathcal{N}^-}, \rho^+ e^{\mathcal{N}^+}) \). Therefore,

\[
\mathcal{N}^+ = \min[\mathcal{N}], \quad h^- = \min[h], \quad \rho^- = \min[\rho], \quad \mathcal{N}^+ = \max[\mathcal{N}], \quad h^+ = \max[h], \quad \rho^+ = \max[\rho].
\]

\[
\sigma^- = \min[\sigma], \quad \zeta^- = \min[\zeta], \quad \rho^+ = \max[\rho], \quad \xi^- = \min[\xi], \quad \sigma^+ = \max[\sigma], \quad \zeta^+ = \max[\xi].
\]

The inequality for amplitude term of membership grade is given as follows:

\[
\prod_{i=1}^{v} \left( \mathcal{N}^+ \right)^{\frac{\delta^-_i}{\delta^+_i}} \leq \prod_{i=1}^{v} \left( \mathcal{N}^- \right)^{\frac{\delta^-_i}{\delta^-_i}} \leq \prod_{i=1}^{v} \left( \mathcal{N}^+ \right)^{\frac{\delta^+_i}{\delta^+_i}}.
\]
Similarly, the inequality for phase term of membership grade is given as follows:

\[
\prod_{i=1}^{v} \left( \frac{\sigma_{\text{ei}}}{2\pi} \right)^{\alpha_{i}} \leq \prod_{i=1}^{v} \left( \frac{\sigma_{\text{ei}}}{2\pi} \right)^{\beta_{i}} \leq \prod_{i=1}^{v} \left( \frac{\sigma_{\text{ei}}}{2\pi} \right)^{\delta_{i}} .
\]

The inequality for amplitude term of non-membership grade is given as follows:

\[
\sqrt{1 - \prod_{i=1}^{v} \left( 1 - (\rho_{i})^{2} \right)^{\frac{2\pi}{\alpha_{i}}} } \leq \sqrt{1 - \prod_{i=1}^{v} \left( 1 - (\rho_{i})^{2} \right)^{\frac{2\pi}{\beta_{i}}} } \leq \sqrt{1 - \prod_{i=1}^{v} \left( 1 - (\rho_{i})^{2} \right)^{\frac{2\pi}{\delta_{i}}} } .
\]

We can prove the other inequalities in a similar manner.

4 | MCGDM TECHNIQUE BASED ON CSF INFORMATION

In this section, we propose a novel MCGDM technique that benefits from either the CSFPWA or the CSFPWG operator. Our approach assumes that the information comes in the form of CSFNs. Furthermore, we present an algorithm for this technique. Finally, we provide a numerical example to demonstrate the validity and implementability of the new operators.

4.1 | Mathematical description of the MCGDM problem and its solution

Let \( Q = \{ Q_1, Q_2, \ldots, Q_n \} \) be the collection of feasible alternatives and \( \mathcal{C} = \{ \mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_c \} \) be the set of criteria or attributes. There is prioritization among the criteria which can be expressed by the ordering \( \mathcal{C}_1 > \mathcal{C}_2 > \cdots > \mathcal{C}_c \), where the criterion \( \mathcal{C}_p \) is preferred over the criterion \( \mathcal{C}_q \), \( p < q \). Let \( \mathcal{D} = \{ D_1, D_2, \ldots, D_l \} \) be the set of decision makers and the prioritization among the decision makers can be expressed by the ordering \( D_1 > D_2 > \cdots > D_l \), which shows that the decision maker \( D_1 \) is preferred over the decision maker \( D_2 \), \( \delta < \beta \). Also, let \( \mathcal{D}^{(a)} = \{ E^{\alpha}_{pa} \}^{m \times n} = (N^{(a)}_{pq} e^{\alpha_{pq}}, h^{(a)}_{pq} e^{\alpha_{pq}}, \eta_{pq} e^{\alpha_{pq}}) \) be the complex spherical fuzzy decision matrix (CSFDM) and \( \mathcal{D}^{(a)} = (N^{(a)}_{pq} e^{\alpha_{pq}}, h^{(a)}_{pq} e^{\alpha_{pq}}, \eta_{pq} e^{\alpha_{pq}}) \) be the CSFNs assigned by the decision makers. Here, \( N^{(a)}_{pq}, h^{(a)}_{pq} \) and \( \eta^{(a)}_{pq} \) represents the membership grade, neutral grade and non-membership grade for amplitude terms and \( \sigma_{\text{ei}}, \sigma_{\text{ni}} \) and \( \varsigma_{\text{ei}} \) represents the membership grade, neutral grade and non-membership grade for phase terms of the alternatives with respect to the criteria, satisfying the condition \( \left( N^{(a)}_{pq} \right)^{2} + \left( h^{(a)}_{pq} \right)^{2} + \left( \eta^{(a)}_{pq} \right)^{2} = 1 \), \( \sigma_{\text{ei}}, \sigma_{\text{ni}} \in [0, 2\pi] \) for all \( p = 1, 2, \ldots, m \) and \( q = 1, 2, \ldots, n \).

Basically, there are two types of criteria: benefit type (larger value is better) and cost type (smaller value is better). So, we have to convert these criteria into the same type. Therefore, we transform the CSFDM \( \mathcal{D}^{(a)} = (E^{\alpha}_{pa})^{m \times n} \) into the normalized CSFDM \( \mathcal{D}^{(a)} = (M^{\alpha}_{pq})^{m \times n} \), where

\[
M^{\alpha}_{pq} = \begin{cases} 
E^{\alpha}_{pq} & \text{for benefit type criteria}, \\
\left( E^{\alpha}_{pq} \right)^{c} & \text{for cost type criteria},
\end{cases}
\]

where \( \left( E^{\alpha}_{pq} \right)^{c} \) denotes the complement of \( E^{\alpha}_{pq} \), such that \( \left( E^{\alpha}_{pq} \right)^{c} = (\sigma_{\text{ei}}, \sigma_{\text{ni}}, \varsigma_{\text{ei}}, N^{(a)}_{pq} e^{\alpha_{pq}}, h^{(a)}_{pq} e^{\alpha_{pq}}, \eta^{(a)}_{pq} e^{\alpha_{pq}}) \), for all \( p = 1, 2, \ldots, m \) and \( q = 1, 2, \ldots, n \). The decision makers provide the information for the MCGDM problem in the following matrix form:

\[
\mathcal{D} = \\
= \begin{bmatrix}
(N_{11} e^{\alpha_{11}}, h_{11} e^{\alpha_{11}}, \rho_{11} e^{\alpha_{11}}, h_{11} e^{\alpha_{11}}, \rho_{11} e^{\alpha_{11}}) & \cdots & \begin{bmatrix}
(N_{12} e^{\alpha_{12}}, h_{12} e^{\alpha_{12}}, \rho_{12} e^{\alpha_{12}}) \\
(N_{12} e^{\alpha_{12}}, h_{12} e^{\alpha_{12}}, \rho_{12} e^{\alpha_{12}}) \\
\vdots \\
(N_{1m} e^{\alpha_{1m}}, h_{1m} e^{\alpha_{1m}}, \rho_{1m} e^{\alpha_{1m}})
\end{bmatrix} \\
\vdots & \ddots & \vdots \\
(N_{m1} e^{\alpha_{m1}}, h_{m1} e^{\alpha_{m1}}, \rho_{m1} e^{\alpha_{m1}}) & \cdots & \begin{bmatrix}
(N_{m2} e^{\alpha_{m2}}, h_{m2} e^{\alpha_{m2}}, \rho_{m2} e^{\alpha_{m2}}) \\
(N_{m2} e^{\alpha_{m2}}, h_{m2} e^{\alpha_{m2}}, \rho_{m2} e^{\alpha_{m2}}) \\
\vdots \\
(N_{mn} e^{\alpha_{mn}}, h_{mn} e^{\alpha_{mn}}, \rho_{mn} e^{\alpha_{mn}})
\end{bmatrix}
\end{bmatrix}

Algorithm 1 explains the various steps of our solution to this problem. This algorithm is applied to a numerical example in the next section. The flow chart to find the best alternative is shown in Figure 1.
Algorithm 1

Steps to solve MCGDM problem by using CSFPW aggregation operators

Step 1. Organize the rating values of the alternatives, assigned by the experts corresponding to the criteria in the form of CSFDM as represented in matrix $\mathbb{D}$.

Step 2. Find the values of $A_{pq}^{a}$ ($a = 1, 2, \ldots, l$) as follows:

$$A_{pq}^{a} = \prod_{i=1}^{l} S^{i} \left( M_{pq}^{a} \right), \quad (a = 2, 3, \ldots, l), \text{ such that } A_{pq} = 1.$$ 

Step 3. Aggregate the CSF decision matrices $\mathbb{D}^{a \leftarrow (a)} = \left( M_{pq}^{a} \right)_{p \times q}, \quad (a = 1, 2, \ldots, l)$ into the combined CSFDM $\mathbb{D} = (M_{pq})_{m \times n}, \quad (p = 1, 2, \ldots, m) \quad (q = 1, 2, \ldots, n)$ by using CSFPWA or CSFPWG operator as follows:

$$M_{pq} = \left( N_{pq} e^{\psi_{pq}}, A_{pq} e^{\phi_{pq}}, \rho_{pq} e^{\psi_{pq}} \right) = \text{SFPWA} \left( M_{pq}^{1}, M_{pq}^{2}, \ldots, M_{pq}^{l} \right)$$

or by utilizing SFPWG operator

$$M_{pq} = \left( N_{pq} e^{\psi_{pq}}, A_{pq} e^{\phi_{pq}}, \rho_{pq} e^{\psi_{pq}} \right) = \text{CSFPWG} \left( M_{pq}^{1}, M_{pq}^{2}, \ldots, M_{pq}^{l} \right)$$

Step 4. Find the values of $A_{pq} (p = 1, 2, \ldots, m)(q = 1, 2, \ldots, n)$ as follows

$$A_{pq} = \prod_{a=1}^{l} S^{a} \left( M_{pq} \right), \quad (p = 1, 2, \ldots, m) \quad (q = 1, 2, \ldots, n), \text{ such that } A_{pq} = 1.$$ 

Step 5. For each alternative $Q_{pq}$ aggregate the CSFN $M_{pq}$ by using the presented CSFPWA or CSFPWG operator.

$$M_{pq} = \left( N_{pq} e^{\psi_{pq}}, A_{pq} e^{\phi_{pq}}, \rho_{pq} e^{\psi_{pq}} \right) = \text{CSFPWA} \left( M_{pq}^{1}, M_{pq}^{2}, \ldots, M_{pq}^{l} \right)$$
3. Personal skills, which include analytical skills and adaptability.

2. Competence, which includes academic background and awareness.

1. Communication skills, which include verbal skills and written skills.

A textile company wants to hire a Marketing Manager for a vacant seat. For this purpose, the board of the company decides to appoint an interview panel formed by three decision makers, namely $D_1$: Owner of the textile industry, $D_2$: General Manager, and $D_3$: Marketing Executive. Four candidates, namely, $Q_1$, $Q_2$, $Q_3$, and $Q_4$, are considered for interview after preliminary screening. The prioritization among the decision makers is $D_1 > D_2 > D_3$, which indicates that the decision maker $D_1$ is at a higher priority level than the other two, and that the decision maker $D_2$ is at a higher priority level than $D_3$. The appointment is totally unbiased, that is, it is free from political or any other kind of influence. The interview panel made their evaluations among the four candidates for the position of Marketing Manager on the basis of the following three criteria:

1. $\hat{C}_1$: Communication skills.
2. $\hat{C}_2$: Competence.
3. $\hat{C}_3$: Personal skills.

For the CSFNs, the above criteria can be divided into the following categories:

1. Communication skills, which include verbal skills and written skills.
2. Competence, which includes academic background and awareness.
3. Personal skills, which include analytical skills and adaptability.

The example is related to the hiring of a Marketing Manager for an open position at a textile company. It aims at illustrating the prioritization phenomenon among the decision makers and among the criteria. Within a relevant panel, each decision maker has a different priority level. Similarly, each criterion has its own priority within the set of criteria. Thus, the structure of the formulation conforms to the characteristics of our techniques. We shall solve the example by the resort to the CSFPWA operator and then with the alternative CSFPWG operator for comparison.

The actual description of the problem is as follows.

A textile company wants to hire a Marketing Manager for a vacant seat. For this purpose, the board of the company decides to appoint an interview panel formed by three decision makers, namely $D_1$: Owner of the textile industry, $D_2$: General Manager, and $D_3$: Marketing Executive. Four candidates, namely, $Q_1$, $Q_2$, $Q_3$, and $Q_4$, are considered for interview after preliminary screening. The prioritization among the decision makers is $D_1 > D_2 > D_3$, which indicates that the decision maker $D_1$ is at a higher priority level than the other two, and that the decision maker $D_2$ is at a higher priority level than $D_3$. The appointment is totally unbiased, that is, it is free from political or any other kind of influence. The interview panel made their evaluations among the four candidates for the position of Marketing Manager on the basis of the following three criteria:

1. $\hat{C}_1$: Communication skills.
2. $\hat{C}_2$: Competence.
3. $\hat{C}_3$: Personal skills.

For the CSFNs, the above criteria can be divided into the following categories:

1. Communication skills, which include verbal skills and written skills.
2. Competence, which includes academic background and awareness.
3. Personal skills, which include analytical skills and adaptability.

$$\begin{align*}
\text{Step 6. Calculate the score values by using Equation (2).} \\
\text{Step 7. Choose the alternative having highest score value.}
\end{align*}$$
The criterion \( \tilde{C}_1 \) is at higher priority level than the other criteria and \( \tilde{C}_3 \) has the lowest priority. Therefore, the prioritization among the criteria is \( \tilde{C}_1 > \tilde{C}_2 > \tilde{C}_3 \). The decision makers submit the information in the form of CSFNs. As all the criteria under consideration are of benefit type, normalization is not needed.

Now we proceed to reproduce the steps of Algorithm 1.

**Step 1.** The CSF decision matrices \( \mathbb{D}^a = (E^a)_{4 \times 3} \) \((a = 1,2,3)\) are represented in Tables 2–4.

**Step 2.** Calculate the values of \( A_{pq}^a \) \((a = 1,2,3)\).
Table 2: CSFDM from \( D^1 = (E^1_{pq})_{4 \times 3} \)

| \( C_1 \) | \( C_2 \) | \( C_3 \) |
|---|---|---|
| \( Q_1 \) | (0.8e^{2(0.8)}, 0.3e^{2(0.4)}, 0.5e^{2(0.2)}) | (0.3e^{2(0.3)}, 0.4e^{2(0.3)}, 0.7e^{2(0.3)}) | (0.6e^{2(0.9)}, 0.5e^{2(0.1)}, 0.5e^{2(0.2)}) |
| \( Q_2 \) | (0.3e^{2(0.3)}, 0.7e^{2(0.4)}, 0.4e^{2(0.3)}) | (0.8e^{2(0.8)}, 0.3e^{2(0.4)}, 0.5e^{2(0.3)}) | (0.5e^{2(0.8)}, 0.4e^{2(0.3)}, 0.3e^{2(0.2)}) |
| \( Q_3 \) | (0.8e^{2(0.7)}, 0.5e^{2(0.4)}, 0.3e^{2(0.3)}) | (0.7e^{2(0.7)}, 0.5e^{2(0.2)}, 0.2e^{2(0.1)}) | (0.7e^{2(0.4)}, 0.5e^{2(0.3)}, 0.4e^{2(0.3)}) |
| \( Q_4 \) | (0.6e^{2(0.7)}, 0.6e^{2(0.4)}, 0.3e^{2(0.3)}) | (0.9e^{2(0.6)}, 0.3e^{2(0.7)}, 0.2e^{2(0.1)}) | (0.1e^{2(0.7)}, 0.6e^{2(0.5)}, 0.4e^{2(0.2)}) |

Abbreviation: CSFDM, complex spherical fuzzy decision matrix.

Table 3: CSFDM from \( D^2 = (E^2_{pq})_{4 \times 3} \)

| \( C_1 \) | \( C_2 \) | \( C_3 \) |
|---|---|---|
| \( Q_1 \) | (0.5e^{2(0.7)}, 0.7e^{2(0.3)}, 0.3e^{2(0.2)}) | (0.6e^{2(0.3)}, 0.3e^{2(0.4)}, 0.4e^{2(0.7)}) | (0.9e^{2(0.5)}, 0.3e^{2(0.3)}, 0.1e^{2(0.3)}) |
| \( Q_2 \) | (0.9e^{2(0.6)}, 0.2e^{2(0.4)}, 0.1e^{2(0.3)}) | (0.3e^{2(0.1)}, 0.5e^{2(0.4)}, 0.6e^{2(0.5)}) | (0.8e^{2(0.7)}, 0.5e^{2(0.3)}, 0.3e^{2(0.2)}) |
| \( Q_3 \) | (0.3e^{2(0.1)}, 0.8e^{2(0.8)}, 0.3e^{2(0.4)}) | (0.7e^{2(0.6)}, 0.4e^{2(0.3)}, 0.3e^{2(0.1)}) | (0.7e^{2(0.8)}, 0.4e^{2(0.2)}, 0.2e^{2(0.1)}) |
| \( Q_4 \) | (0.5e^{2(0.5)}, 0.4e^{2(0.3)}, 0.7e^{2(0.4)}) | (0.5e^{2(0.6)}, 0.4e^{2(0.3)}, 0.2e^{2(0.2)}) | (0.6e^{2(0.7)}, 0.4e^{2(0.3)}, 0.3e^{2(0.2)}) |

Abbreviation: CSFDM, complex spherical fuzzy decision matrix.

Table 4: CSFDM from \( D^3 = (E^3_{pq})_{4 \times 3} \)

| \( C_1 \) | \( C_2 \) | \( C_3 \) |
|---|---|---|
| \( Q_1 \) | (0.7e^{2(0.7)}, 0.4e^{2(0.1)}, 0.4e^{2(0.3)}) | (0.5e^{2(0.1)}, 0.3e^{2(0.3)}, 0.3e^{2(0.4)}) | (0.5e^{2(0.8)}, 0.6e^{2(0.2)}, 0.6e^{2(0.3)}) |
| \( Q_2 \) | (0.2e^{2(0.5)}, 0.6e^{2(0.4)}, 0.5e^{2(0.3)}) | (0.8e^{2(0.1)}, 0.3e^{2(0.4)}, 0.5e^{2(0.3)}) | (0.5e^{2(0.7)}, 0.4e^{2(0.3)}, 0.3e^{2(0.1)}) |
| \( Q_3 \) | (0.7e^{2(0.8)}, 0.6e^{2(0.3)}, 0.3e^{2(0.5)}) | (0.7e^{2(0.7)}, 0.5e^{2(0.2)}, 0.3e^{2(0.3)}) | (0.7e^{2(0.3)}, 0.4e^{2(0.2)}, 0.5e^{2(0.3)}) |
| \( Q_4 \) | (0.6e^{2(0.8)}, 0.6e^{2(0.4)}, 0.3e^{2(0.3)}) | (0.9e^{2(0.6)}, 0.2e^{2(0.5)}, 0.3e^{2(0.2)}) | (0.2e^{2(0.7)}, 0.5e^{2(0.4)}, 0.4e^{2(0.1)}) |

Abbreviation: CSFDM, complex spherical fuzzy decision matrix.

Step 3. Apply the CSFPWA operator to combine all the matrices into an aggregate matrix (see Table 5).

Step 4. Calculate the values of \( \Lambda_{pq}^q \) (\( q = 1, 2, \ldots, n \)), \( q = 1, 2, \ldots, n \).

\[
\Lambda_{mn}^{(1)} = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}, \\
\Lambda_{mn}^{(2)} = \begin{bmatrix}
1.62 & 1.0766 & 1.54 \\
0.9467 & 1.62 & 1.5033 \\
1.38 & 1.4167 & 1.3533 \\
1.4233 & 1.5133 & 1.23
\end{bmatrix}, \\
\Lambda_{mn}^{(3)} = \begin{bmatrix}
2.0898 & 1.2739 & 2.4537 \\
1.5084 & 1.7658 & 2.3351 \\
1.0902 & 2.0495 & 2.2014 \\
1.7079 & 2.1590 & 1.8532
\end{bmatrix}.
\]

Step 5. Apply the CSFPWA operator to combine the values \( E_{pq}^q \) (\( q = 1, 2, 3, 4 \)) of \( D^1 \) in Table 5 to get \( E_p \).

\[
Q_1 = (0.6519e^{2(0.6760)}, 0.4106e^{2(0.2448)}, 0.3731e^{2(0.3469)}) , \\
Q_2 = (0.6564e^{2(0.7048)}, 0.4139e^{2(0.3393)}, 0.3604e^{2(0.2255)}) , \\
Q_3 = (0.6882e^{2(0.5384)}, 0.4803e^{2(0.2479)}, 0.3175e^{2(0.2721)}) , \\
Q_4 = (0.6418e^{2(0.6742)}, 0.4147e^{2(0.3227)}, 0.3289e^{2(0.1767)}) .
\]
### Table 5

Aggregated CSFD $\mathbf{D} = (E_p)_{4 \times 3}$

| $C_1$ | $E_{p1}$ | $E_{p2}$ | $E_{p3}$ | $E_{p4}$ |
|------|----------|----------|----------|----------|
| $Q_1$ | $(0.6776 e^{-0.7658}, 0.4561 e^{-0.2335}, 0.3798 e^{-0.4032})$ | $(0.6273 e^{-0.5744}, 0.4643 e^{-0.2610}, 0.3016 e^{-0.3834})$ | $(0.6475 e^{-0.4726}, 0.6383 e^{-0.2843}, 0.3000 e^{-0.5243})$ | $(0.5692 e^{-0.7040}, 0.5218 e^{-0.1727}, 0.4017 e^{-0.2801})$ |
| $Q_2$ | $(0.4949 e^{-0.2127}, 0.3269 e^{-0.3290}, 0.4237 e^{-0.4768})$ | $(0.7021 e^{-0.7436}, 0.3623 e^{-0.4000}, 0.5348 e^{-0.3000})$ | $(0.7000 e^{-0.5355}, 0.4658 e^{-0.2678}, 0.2739 e^{-0.2258})$ | $(0.8388 e^{-0.6000}, 0.2730 e^{-0.4556}, 0.2412 e^{-0.1724})$ |
| $Q_3$ | $(0.7240 e^{-0.7764}, 0.4671 e^{-0.1978}, 0.3329 e^{-0.2766})$ | $(0.6348 e^{-0.7249}, 0.4287 e^{-0.3447}, 0.3000 e^{-0.1431})$ | $(0.7000 e^{-0.5667}, 0.4201 e^{-0.2186}, 0.3626 e^{-0.2164})$ | $(0.3794 e^{-0.7000}, 0.4888 e^{-0.3428}, 0.3668 e^{-0.1460})$ |
| $Q_4$ | $(0.6463 e^{-0.4036}, 0.4846 e^{-0.2491}, 0.3514 e^{-0.4156})$ | $(0.4981 e^{-0.5991}, 0.4144 e^{-0.3366}, 0.4234 e^{-0.3213})$ | $(0.4083 e^{-0.6584}, 0.4145 e^{-0.3205}, 0.3806 e^{-0.2111})$ |

Abbreviation: CSFD, complex spherical fuzzy decision matrix.

**Step 6.** Calculate the score values of all CSFNs obtained in Step 5.

$$S'(Q_1) = 1.4646, \quad S'(Q_2) = 1.4868, \quad S'(Q_3) = 1.4314, \quad S'(Q_4) = 1.4836.$$  

Therefore,

$$S'(Q_2) > S'(Q_3) > S'(Q_1) > S'(Q_4).$$

**Step 7.** The best alternative is $Q_2$.

Now, we solve the same problem by utilizing CSFPWG operator.

**Step 1.** This Step is same as that of Step 1.

**Step 2.** This Step is same as that of Step 2.

**Step 3.** Apply the CSFPWG operator to combine all the individual matrices into an aggregate single matrix (Table 6).

**Step 4.** Calculate the values of $A_{pq}$ ($p = 1, 2, \ldots, m$, $q = 1, 2, \ldots, n$).

$$A_{pq} = \begin{bmatrix} 1 & 1.3837 & 1.6072 \\ 1.2538 & 1.6211 \\ 1.1235 & 1.6183 \\ 1.3667 & 2.0657 \end{bmatrix}.$$

**Step 5.** Apply the CSFPWG operator to combine the values $E_{pq}(p = 1, 2, 3, 4)$ of $\mathbf{D}$ in Table 6 to get $E_p$.

$$Q_1 = (0.5623 e^{-0.4408}, 0.4103 e^{-0.2457}, 0.4761 e^{-0.4878})$$

$$Q_2 = (0.4981 e^{-0.5991}, 0.4144 e^{-0.3366}, 0.4234 e^{-0.3213})$$

$$Q_3 = (0.6463 e^{-0.4036}, 0.4846 e^{-0.2491}, 0.3514 e^{-0.4156})$$

$$Q_4 = (0.4083 e^{-0.6584}, 0.4145 e^{-0.3205}, 0.3806 e^{-0.2111})$$

**Step 6.** Calculate the score values of all CSFNs obtained in Step 5.
\[ S/C_3 Q_1(\pi) = 1.2724, \quad S/C_3 Q_2(\pi) = 1.3465, \quad S/C_3 Q_3(\pi) = 1.3292, \quad S/C_3 Q_4(\pi) = 1.3787. \]

Therefore,

\[ S/C_3 Q_4(\pi) \succ S/C_3 Q_2(\pi) \succ S/C_3 Q_3(\pi) \succ S/C_3 Q_1(\pi). \]

**Step 7.** The best alternative is \( Q_4 \).

Table 7 represents the score values and ranking order of alternatives. Thus, by applying the CSFPWA operator the best alternative is \( Q_2 \), whereas the resort to the CSFPWG operator recommends \( Q_4 \) as the best alternative.

### 4.3 Validity test

Wang and Triantaphyllou (2008) developed a testing criteria to examine the validity and effectiveness of MCDM techniques. It consists of the following tests:

- **Test criterion 1**: The outcome of an effective MCDM technique should remain unaltered if we replace the decision values of a non-optimal alternative with those of a worse alternative, without changing the relative importance of each criteria.
- **Test criterion 2**: An effective MCDM technique should obey the transitive property.
- **Test criterion 3**: The ranking of alternatives should not alter on splitting the problem into smaller sub-problems and then applying the same MCDM technique on these subproblems.

Let us check the validity of our MCDM approach in terms of the proposed operators, by testing these criteria:

1. **Validity test by criterion 1**: If we change the decision values of a non-optimal alternative \( Q_3 \) with those of a worse alternative \( Q_4 \) in the decision matrices given by the experts, then the decision values are presented in Table 8

   Now the score values of the alternatives by utilizing the proposed technique for CSFPWA operator are \( S'(Q_1) = 1.4551, S'(Q_2) = 1.4868, S'(Q_3) = 1.4599, \) and \( S'(Q_4) = 1.4836. \) From the score values, the ranking order of alternatives is \( Q_2 \succ Q_4 \succ Q_3 \succ Q_1 \), and the best alternative is \( Q_2 \). Hence, the optimal alternative is the same as that of the original ranking. Thus, Algorithm 1 is effective under the test criterion 1.
The application of the operators studied in Section 4 shows that the best alternatives are $Q_2$ and $Q_4$, whereas the best alternative is $Q_3$ when we apply the existing operators. Notice that the existing operators can only deal with one dimension. Their inability to deal with CSF data is the cause of a severe loss of information. Thus, the operators presented here are more general, as they can capture two dimensions and are capable of representing more complete information.

The CSFPWA and CSFPWG operators recommend different top alternatives. The reason is that the CSFPWA operator considers the role of overall data, whereas the CSFPWG operator highlights the role of individual data (Table 12).

The graphical representation of the score values of alternatives using both novel and existing operators is shown in Figure 2.

Table 13 summarizes the distinctive features of different models inclusive of our benchmark model, for the purpose of comparison.
| TABLE 11 | Final ranking |
|-----------|---------------|
| Operators | Final ranking |
| SFPWA operator<sup>a</sup> | \( S(Q_3) > S(Q_2) > S(Q_1) \) |
| SFPWG operator<sup>a</sup> | \( S(Q_3) > S(Q_2) > S(Q_4) \) |
| CSFPWA operator proposed here | \( S'(Q_3) > S'(Q_2) > S'(Q_1) \) |
| CSFPWG operator proposed here | \( S'(Q_3) > S'(Q_2) > S'(Q_4) \) |

<sup>a</sup>Akram, Khan, and Karaaslan (2020).

| TABLE 12 | Comparative study |
|-----------|-------------------|
| Operators | Best alternative |
| SFPWA operator<sup>a</sup> | \( Q_3 \) |
| SFPWG operator<sup>a</sup> | \( Q_3 \) |
| CSFPWA operator proposed here | \( Q_2 \) |
| CSFPWG operator proposed here | \( Q_4 \) |

<sup>a</sup>Akram, Khan, and Karaaslan (2020).

| FIGURE 2 | Graphical representation of the results of four operators |

| TABLE 13 | Comparison of the CSFS model with related models |
|-----------|--------------------------------------------------|
| Model     | Membership degree | Non-membership degree | Neutral degree | Hesitancy degree | Represents two dimensions |
| Fuzzy set | ✓ | ✗ | ✓ | ✗ | ✗ |
| q-rung orthopair fuzzy set (\( q \geq 1 \)) | ✓ | ✓ | ✗ | ✗ | ✗ |
| Complex fuzzy set | ✓ | ✗ | ✗ | ✓ | ✗ |
| Complex q-rung orthopair fuzzy set | ✓ | ✓ | ✗ | ✓ | ✓ |
| Spherical fuzzy set | ✓ | ✓ | ✓ | ✓ | ✗ |
| Complex spherical fuzzy set | ✓ | ✓ | ✓ | ✓ | ✓ |

6 | ADVANTAGES OF THE PROPOSED METHOD AND JUSTIFICATION OF THE PROPOSED RESULTS

A list of advantages of the proposed methodology follows.

- The most influential feature of CSFSs is its ability to represent two-dimensional phenomena that require the allocation of membership, non-membership and neutral membership grades. This quality makes CSFSs more dominant to express the desired information that get over imperfections of the existing theories such as CPFS and SFS.
Hence the CSFS model is a generalization of existing theories that deals with both imprecision and periodicity simultaneously.

In many real life situations, the criteria and decision makers have different priority levels. The most noticeable feature of prioritized AOs is to capture the prioritization phenomenon among the aggregated arguments. By keeping the benefits of prioritized AOs, we have applied them on CSFSs.

Our proposed MCGDM strategy is not only applicable to two-dimensional data, but it can also be adapted to spherical fuzzy data by associating them with a null phase term. Comparisons can be made easily as this model is embedded into this one (cf., Section 5).

Thus our proposed method is more flexible and convenient for approaching very general MCGDM problems.

Our comparative study justifies the need for the benchmark setting. If we make full use of all the information that our numerical problem embodies, then the best alternative is either $Q_2$ (if we use the CSFPWA operator) or $Q_4$ (if we use the CSFPWG operator). However when we only use the spherical fuzzy formulation of the problem, i.e., we force the phase term to be equal to zero, then neither of these alternatives is justified (as the recommendation is $Q_3$).

This phenomenon also justifies the introduction of operators that extend the performance of the SFPWA and SFPWG operators when phase terms are non-trivial.

7 | CONCLUSIONS AND FUTURE DIRECTIONS

The CSFS theory possesses the joint characteristics of the CFS and SFS theories. CSFSs represent three degrees in polar coordinates form. They capture membership, non-membership, abstinence or falsity, in such way that the range of the degrees is extended from $[0, 1]$ to the unit disk in a complex plane. In this research article, we have been motivated by the idea that prioritized AOs for CSFNs would enable us to solve a new class of decision-making problems. We have successfully designed complex spherical fuzzy prioritized AOs by employing the prioritized AOs on CSFNs. Then we have shown their applicability and conducted numerical and qualitative analyses of this novel aspect.

First, we have presented some basic operational laws and a new score function to rank the CSFNs. Inspired by the idea of prioritized AOs and CSFNs, we have presented prioritized weighted aggregation operators for a collection of CSFNs, including CSFPWA operator and CSFPWG operator, and investigated their substantial properties in detail. In addition, we successfully form an MCGDM method by utilizing prioritized AOs under CSF environment, having prioritization relationship over the criteria. We have provided the mathematical description of the MCGDM problem and then an algorithm and a flow chart is initiated for the MCGDM problem. Further, we have applied the presented MCGDM approach to a numerical example and provided the solution of the problem. We have also tested the effectiveness and validity of our proposed technique through a validity test. Utilizing the proposed operators, we have provided a comparative analysis with existing operators available in the literature to show the superiority of proposed operators. From the arguments above, it is concluded that the present work provides more adequate ways to manage the complex spherical fuzzy information to solve practical problems.

Although the AOs established in this paper are effective to aggregate the imprecise information, they still have some limitations. The aggregation theory here presented does not apply when the sum of squares of the triplets (membership, non-membership and neutral degrees) for the amplitude and phase terms exceeds 1. Consequently, the subsequent decision-making theory fails to apply in this case too. We are therefore planning to extend our work to: (a) complex $q$-rung picture fuzzy prioritized AOs, (b) complex spherical fuzzy prioritized Dombi AOs, and (c) complex spherical fuzzy prioritized Yager AOs. Extensions of the Pythagorean fuzzy framework is another promising area of application of these techniques (Peng & Selvachandran 2019).

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CONFLICT OF INTEREST

The authors declare there is no conflict of interest.

AUTHOR CONTRIBUTIONS

All authors contributed equally to this work.

DATA AVAILABILITY STATEMENT

Datasets of the numerical example at Sections 4.2 and 5 are available from the public repository https://github.com/gsantosgarcia/CSFPWA- and- CSFPWG-operators. Its supporting information files are licensed under the GNU General Public License v3.0 and can be distributed under GNU General Public License v3.0.
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