THE $b - u$ SKEWED PARTON DISTRIBUTIONS

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THE $b-u$ SKewed Parton DISTRIBUTIONS

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The $b-u$ skewed parton distributions (SPDs) are discussed. The SPDs allow an unambiguous superposition of overlap and resonance contributions and their zeroth order moments represent the $B \to \pi$ transition form factors. The values of the form factors at maximum recoil are found to be $F_+(0) = F_0(0) = 0.22 \pm 0.05$ in agreement with measurements of the $B \to \pi\pi$ branching ratio. The branching ratios for the semi-leptonic $B \to \pi$ decays are evaluated too.

1. INTRODUCTION

A good theoretical understanding of the $B \to \pi$ transition form factors are of utmost interest. Accurate predictions of these form factors would permit a determination of the less well-known Cabbibo-Kobayashi-Maskawa matrix element $V_{ub}$ from experimental rates of semi-leptonic $B \to \pi$ transitions. The $B \to \pi$ form factors also form an important ingredient of the calculation of $B \to \pi\pi$ decay rates; the factorization hypothesis relates the form factors at maximum recoil to the decay rates. Thus, not surprisingly, the $B \to \pi$ and other heavy-to-light form factors, attracted the attention of many theoreticians. In several of these approaches two distinct dynamical mechanism are considered which build up the $B \to \pi$ form factors: The $B\pi$ resonances which control the form factors at small recoil and the overlap of the meson wave functions that dominates at large recoil. Other mechanisms, like the perturbative one, provide only small and often negligible corrections. The crucial questions are how to match these two prominent contributions at intermediate recoil and how to avoid double counting. In a recent article [1], on which I am going to report here, we proposed to start from skewed parton distributions [2]. The SPDs allow an unambiguous superposition of $B\pi$ resonances and the overlap contribution.

2. $b-u$ SKewed Parton DISTRIBUTIONS

To be specific let us consider the semi-leptonic decay $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$. Instead of the usual form factors, $F_+$ and $F_0$, (see e.g. [3,4]) it is more appropriate here to use the definition

$$\langle \pi^+: p' | \bar{u}(0) \gamma_\mu b(0) | \bar{B}^0; p \rangle = F^{(1)}(q^2) p'_\mu + F^{(2)}(q^2) \left( q_\mu - \frac{q^2}{M_B^2} p_\mu \right),$$

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where \( q = p - p' \) and \( M_B (M_\pi) \) is the \( B (\pi) \) mass. A convenient frame of reference is a generalized Breit frame in which the mesons move collinearly. In this frame the momentum transfer is given by

\[
q^2 = \zeta M_B^2 \left( 1 - \frac{M_\pi^2}{M_B^2 (1 - \zeta)} \right),
\]

where the so-called skewedness parameter, \( \zeta \), is defined by the ratio of light-cone plus components

\[
\zeta = \frac{q^+}{p^+} = 1 - \frac{p'^+}{p^+}.
\]

The skewedness parameter \( \zeta \) covers the interval \([0, 1 - M_\pi/M_B]\) in parallel with the variation of the momentum transfer from zero (the lepton mass is neglected) to \( q^2_{\text{max}} = (M_B - M_\pi)^2 \) in the physical region of the \( B \to \pi \) transitions. The pion mass is neglected in the calculation of SPDs and form factors. The advantage of the generalized Breit frame is that the \( B \to \pi \) matrix element of the current’s plus component is related to the form factor \( F^{(1)} \) solely while that of the minus component is related to \( F^{(2)} \). The matrix elements of the transverse currents are zero.

The flavour non-diagonal b-u SPDs \( \tilde{F}_\zeta^{(i)} \), \( i = 1, 2 \) are defined by the \( B - \pi \) matrix elements of bilocal products of quark field operators, e.g.

\[
\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle \pi^+; p' | \bar{u}(0) \gamma^+ b(z^-) | \bar{B}^0; p \rangle = (1 - \zeta) \tilde{F}_\zeta^{(1)}(x, q^2).
\]

The second SPD, \( \tilde{F}_\zeta^{(2)} \), is analogously defined with \( \gamma^+ \) being replaced by \( \gamma^- \). The variable \( q^2 \) is redundant in the generalized Breit frame, see Eq. (2). Integration of (4) over \( x \) reduces the bilocal operator product to the local one that defines the form factors, see Eq. (1). Hence, one has the reduction formula

\[
F^{(i)}(q^2) = \int_{-1+\zeta}^{1} dx \tilde{F}_\zeta^{(i)}(x)
\]

for \( i = 1, 2 \).

Depending on the value of \( x \), the SPDs describe different physical situations:

i) For \( 1 \geq x \geq \zeta \) a b-quark with momentum fraction \( x \) is emitted from the \( B \)-meson and a u-quark carrying a momentum fraction \( x' = k^+/p^+ \) is absorbed, turning the \( B \)-meson into a pion. This part of the SPDs can be modelled as overlaps of \( B \) and \( \pi \) light-cone wave functions. For the valence Fock states, for instance, the overlap reads

\[
\tilde{F}_\zeta^{(1)}(x) = \frac{2}{1 - \zeta} \int \frac{d^2 k_\perp}{16\pi^3} \Psi_\pi(x' = \frac{x - \zeta}{1 - \zeta}, k_\perp) \Psi_B(x, k_\perp),
\]

where \( k_\perp \) is the intrinsic transverse momentum of the b (u) quark with respect to the \( B (\pi) \)-meson momentum.

ii) For \( 0 \leq x < \zeta \) the fraction \( x' \) is negative. Interpreting a parton with a negative momentum fraction as an antiparton with a positive fraction, one sees that the physical
situation is now the emission of a bπ pair from B-meson and the formation of the pion from the remaining partons. This contribution can be described by overlaps for N+2 \rightarrow N parton processes; it is found to be very small numerically. Bπ resonances contribute to the SPDs in that region as well \[3\]. The contribution of the most important resonance, the B*- vector meson, reads

\[
\tilde{F}^{(1)}_{\xi \text{res}}(x) = \frac{f_{B^*} g_{BB^*\pi}}{M_{B^*}} \left( M_{B^*}^2 - \frac{1}{2} \zeta M_B^2 \right) \frac{\phi_{B^*}(x/\zeta)}{M_{B^*}^2 - \zeta M_B^2}, \tag{7}
\]

\[
\tilde{F}^{(2)}_{\xi \text{res}}(x) = -\frac{1}{2} M_B^2 \frac{f_{B^*} g_{BB^*\pi}}{M_{B^*}} \frac{\phi_{B^*}(x/\zeta)}{M_{B^*}^2 - \zeta M_B^2},
\]

where \(\phi_{B^*}(y)\) is the valence Fock state distribution amplitude of the B*-meson. The argument of the \(B^*\) distribution amplitude, \(x/\zeta\), equals the momentum fraction, \(k_+/q_+\), which the b-quarks carries w.r.t. the B*-meson.

iii) For \(-1 + \zeta \leq x < 0\) where \(x'\) is negative too, the SPDs describe the emission of a \(\overline{B}\)-quark and the absorption of a \(\pi\) one. Since the probability of finding a b\(\overline{B}\) sea-quark pair in the B-meson is practically zero, \(\tilde{F}^{(i)}_{\xi}(x) \approx 0\) in this region.

Combining all these contributions, one finds for the b – u SPDs the superposition

\[
\tilde{F}^{(i)}_{\xi}(x) = \theta(x - \zeta) \tilde{F}^{(i)}_{\xi \text{ann}}(x) + \theta(\zeta - x) \theta(x) \left( \tilde{F}^{(i)}_{\xi \text{ann}}(x) + \tilde{F}^{(i)}_{\xi \text{res}}(x) \right). \tag{8}
\]

The relative importance of the overlap contribution to the SPDs on the one side and the sum of annihilation and resonance one on the other side, change with the momentum transfer as a consequence of the relation \(2\). At large recoil, \(q^2 \simeq 0\), the annihilation and resonance parts do not contribute while they dominate at small recoil, \(q^2 \simeq q_{\text{max}}^2\). The superposition \(\tilde{F}^{(i)}\) is controlled by the skewedness parameter \(\zeta\) in an unambiguous way, i.e. there is no danger of double counting.

For the numerical estimate of the overlap contribution a simple Gaussian wave function is used to describe the pion’s valence Fock state

\[
\Psi_{\pi}(x, k_\perp) = \frac{\sqrt{6}}{f_{\pi}} \exp \left[ -\frac{1}{8\pi^2 f_{\pi}^2} \frac{k_\perp^2}{x (1-x)} \right]. \tag{9}
\]

\(f_{\pi} (= 132\, \text{MeV})\) is the usual pion decay constant. The wave function \(\Psi_{\pi}\) has been tested against experiment and found to work satisfactorily in many exclusive reactions involving pions (e.g. \[9\]). It is also supported by theoretical studies, e.g. \[8\].

For the bd wave function of the B meson a slightly modified version of the Bauer-Stech-Wirbel (BSW) function \[3\] is used

\[
\Psi_B(x, k_\perp) \propto f_B x (1-x) \exp \left[ -a_B^2 M_B^2 (x-x_0)^2 + k_\perp^2 \right]. \tag{10}
\]

\(m_b\) is taken to be 4.8 GeV, \(a_B = 1.51\, \text{GeV}^{-1}\), \(f_B = 180\, \text{MeV}\) and the wave function is normalized to unity. The distribution amplitude exhibits a pronounced peak, its position is approximately at \(x \simeq x_0 = m_b/M_B\).

In principle, the overlap parts of the SPDs receive contributions from all Fock states. The generalization of the overlap representation \(\tilde{F}^{(i)}\) to higher Fock states is a straightforward application of the methods outlined in \[3\]. Using suitably generalized N-particle...
wave functions, one can show that the higher Fock state contributions to the SPD $\tilde{F}_\zeta^{(1)}$ are very small and can be neglected; they represent power corrections $(\bar{\Lambda}/M_B)^n(N)$.

In order to estimate the resonance contribution the same ansatz as for the $B$-meson is employed for the $B^*$-meson distribution amplitude. Its explicit form is however irrelevant for the transition form factors. The product of the $B^*$ decay constant, $f_{B^*}$ and the $BB^*\pi$ coupling constant is taken to be $20f_B$. The numerical results for the b-u SPD $\tilde{F}_\zeta^{(1)}$ are shown in Fig. 1. Both the contributions exhibit characteristic bumps which are generated by the pronounced peaks in the $B$ and $B^*$ distribution amplitudes.

3. $B-\pi$ FORM FACTORS

The $B-\pi$ form factors $F^{(i)}$ can be evaluated form the SPDs through the reduction formula (5). Since the form factors $F_+$ and $F_0$, being linearly related to the $F^{(i)}$, are more suitable in applications to decay processes, I only present results for them. In addition to the overlap and resonance contributions the form factors also receive contributions from perturbative QCD where a hard gluon, with a virtuality of the order of $M_B^2$, is exchanged between the struck and the spectator quark. In Ref. [10] the perturbative contributions have been evaluated at large recoil within the modified perturbative approach and one can make use of these results. At small recoil the perturbative contributions cease to be reliable because of the small virtualities some of the internal off-shell quarks and gluons acquire in this region.

Numerical results for the form factors are displayed in Fig. 2. In the case of the form factor $F_+$ one sees the dominance of the overlap contribution at large recoil while the resonance contribution takes the lead at small recoil (cf. Eq. (6)). The perturbative contribution provides a correction to $F_+$ of the order of 10% at large recoil and can be neglected at small recoil. The sum of the three contributions to $F_+$ is in fair agreement with lattice QCD results [11]. Due to the absence of the $B^*$ pole the form factor $F_0$ behaves differently; it is rather flat over the entire range of momentum transfer. The perturbative contribution makes up a substantial fraction of the total result for $F_0$ at intermediate momentum transfer. Since, as is mentioned above, it becomes unreliable for $q^2 \gtrsim 18 \text{GeV}^2$ $F_0$ cannot reliably be predicted at large $q^2$. A calculation of $F_0$ in that
Figure 2. The form factors $F_+(q^2)$ and $F_0(q^2)$ vs. momentum transfer. Our predictions (solid lines) for the form factors are decomposed into resonance, overlap and perturbative contributions. The lattice QCD data, taken from Ref. [11], are shown for comparison. CT indicates the Callan-Treiman value, $f_B/f_\pi$.

region would also require a detailed investigation of the scalar $B\pi$ resonances of which not much is known at present. Despite of this drawback the results for this form factor are also in fair agreement with the lattice QCD results [11] and, in tendency, seem to extrapolate to the $B$-sector analogue of the Callan-Treiman value.

An assessment of the theoretical uncertainties of the predictions for the $B \to \pi$ transition form factors leads to an uncertainty of about $20 - 25\%$. It includes estimates of: Sudakov suppressions in the end-point region ($x \to 1$), contributions from two-particle twist-three wave functions, deviations from the asymptotic form of the pion distribution amplitude, uncertainties of the input parameters ($g_{BB^*\pi}, f_B, f_{B^*}, m_b$) and from the order $\bar{\Lambda}/M_B$ corrections. In particular for the form factors at maximum recoil, $F_+(0) = F_0(0)$ a value of $0.22 \pm 0.05$ is obtained.

With the form factors at hand one can evaluate the semi-leptonic decay rates $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$. For the branching ratio of the light-lepton modes one finds

$$B[\bar{B}^0 \to \pi^+ e\bar{\nu}_e] \approx B[\bar{B}^0 \to \pi^+ \mu\bar{\nu}_\mu] = 1.9 \cdot 10^{-4} \cdot \left( \frac{|V_{ub}|}{0.0035} \right)^2.$$ (11)

The theoretical uncertainty of this prediction, dominated by that of the overlap contribution, amounts to about $30\%$. This result is to be compared with the CLEO measurement [12]: $(1.8 \pm 0.4 \pm 0.3 \pm 0.2) \cdot 10^{-4}$. For the $\tau$ channel one obtains a value of $1.5 \cdot 10^{-4}$ for the branching ratio ($|V_{ub}| = 0.0035$).

The exclusive $B$-decays into pairs of pions are usually calculated on the basis of a factorization hypothesis according to which the decay amplitudes can be written as a product of two weak current matrix elements

$$M = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub} \langle \pi^- ; q J_W^{\mu} | 0 \rangle \langle \pi^+ ; p' J_W^{\mu} | \bar{B}^0 ; p \rangle.$$ (12)

The first matrix element defines the usual pion decay constant ($\propto f_\pi q^\mu$) while the second
one defines the $B \to \pi$ transition form factors \cite{1}. The factorizing contribution alone leads to the following branching ratio

$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = 10.5 \cdot 10^{-6} \left( \frac{|V_{ub}|}{0.0035} \right)^2 \left| \frac{F_+(0)}{0.33} \right|^2. \tag{13}$$

Ignoring the short-distance corrections which seem to amount to about $10 - 20\%$ \cite{13}, and choosing $|V_{ub}| = 0.0035$, one finds agreement between the prediction for $F_+(0)$ from the SPD approach and the recent CLEO measurement \cite{14} for the $\bar{B}^0 \to \pi^+\pi^-$ branching ratio of $(4.3^{+1.6}_{-1.4}\pm0.5) \cdot 10^{-6}$. The experimental value is much smaller than expected (based on $F_+(0) \simeq 0.3 - 0.33$) and a revision of the theoretical analysis of exclusive $B$-decays seems to be required.

4. CONCLUSIONS

The $b-u$ SPDs are calculated from light-cone wave function overlaps and a contribution from the $B^*$ resonance. The chief advantage of the SPD approach is that the skewedness parameter clearly separates the overlap from the resonance contribution and both the contributions can be added unambiguously. The $B \to \pi$ transition form factors are calculated from the $b-u$ SPDs by means of reduction formulas. $F_+$ is obtained in the entire range of momentum transfer and $F_0$ up to about 18 GeV$^2$. In particular, a value of $0.22 \pm 0.05$ is found for the form factors at maximum recoil. This value appears to be in agreement with the recent CLEO measurement \cite{12} of $B \to \pi\pi$ decays (if the latter process is analysed on the basis of the factorization hypothesis). The prediction for the total decay for the process $\bar{B}^0 \to \pi^+e^-\bar{\nu}_e$ is also in agreement with a CLEO measurement \cite{12}. In both the cases, the $\pi\pi$ and the semi-leptonic decay, a value of 0.0035 is used for the CKM matrix element $V_{ub}$.

REFERENCES

1. T. Feldmann and P. Kroll, Eur. Phys. J. C12, 99 (2000).
2. D. Müller et al., Fortschr. Physik 42, 101 (1994), hep-ph/9812448 X. Ji, Phys. Rev. Lett. 78, 610 (1997); A.V. Radyushkin, Phys. Rev. D56, 5524 (1997).
3. M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C29, 637 (1985).
4. A. Khodjamirian and R. Rückl (1998), in: “Heavy Flavours II”, World Scientific.
5. A.V. Radyushkin, Phys. Lett. B449, 81 (1999).
6. P. Kroll and M. Raulfs, Phys. Lett. B387, 848 (1996).
7. R. Jakob, P. Kroll and M. Raulfs, J. Phys. G G22, 45 (1996).
8. R. Akhoury, A. Sinkovics and M.G. Sotiropoulos, Phys. Rev. D58, 013011 (1998); B. Chibisov and A. R. Zhitnitsky, Phys. Rev. D52, 5273 (1995).
9. M. Diehl, T. Feldmann, R. Jakob, and P. Kroll, Eur. Phys. J. C8, 409 (1999).
10. M. Dahm, R. Jakob, and P. Kroll, Z. Phys. C68, 595 (1995).
11. J.M. Flynn, hep-lat/9611016.
12. J.P. Alexander et al., CLEO Collaboration, Phys. Rev. Lett. 77, 5000 (1996).
13. A. Ali, G. Kramer and C.-D. Lü, Phys. Rev. D58, 094009 (1998).
14. D. Cronin-Hennessy et al., CLEO collaboration, hep-ex/0001010.