HyperQB: A QBF-Based Bounded Model Checker for Hyperproperties

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Abstract. We present HyperQB, a push-button QBF-based bounded model checker for hyperproperties. HyperQB takes as input a NuSMV model and a formula expressed in the temporal logic HyperLTL. Our QBF-based technique allows HyperQB to seamlessly deal with quantifier alternations. Based on the selection of either bug hunting or synthesis, the instances of counterexamples (for negated formula) or witnesses (for synthesis of positive formulas) are returned. We report on successful and effective verification for a rich set of experiments on a variety of case studies, including information-flow security, concurrent data structures, path planning for robots, co-termination, deniability, intransitivity of non-interference, and secrecy-preserving refinement. We also rigorously compare and contrast HyperQB with existing tools for model checking hyperproperties.

1 Introduction

Hyperproperties [13] are system-wide properties (rather than the property of individual execution traces) that allow us to deal with important information-flow security policies (e.g., generalized non-interference (GNI) [21]), consistency models in concurrent computing [8] (e.g., linearizability [23]), and robustness conditions in cyber-physical systems [9,36]. The temporal logic HyperLTL [12] extends LTL with explicit and simultaneous quantification over execution traces, allowing to describe properties of multiple traces. For example, GNI as HyperLTL formula:

\[ \varphi_{\text{GNI}} = \forall \pi_A. \forall \pi_B. \exists \pi_C. \square (\text{high}_{\pi_A} \leftrightarrow \text{high}_{\pi_C}) \land \square (\text{low}_{\pi_B} \leftrightarrow \text{low}_{\pi_C}) \]

stipulates that for all traces \( \pi_A \) and \( \pi_B \), there must exists a \( \pi_C \), such that \( \pi_C \) agrees on high (i.e., high-security secret) with \( \pi_A \), and agrees on low (i.e., low-security observation) with \( \pi_B \). Satisfying \( \varphi_{\text{GNI}} \) implies that an attacker cannot infer the high-security value by speculating the observable parts of a program.

1.1 Related Model Checking Tools for HyperLTL

There has been a recent surge of model checking techniques for HyperLTL specifications [12,14,18,19]. These approaches employ various techniques (e.g., alternating automata, model counting, strategy synthesis, etc.) to verify hyperproperties. The tool MCHyper implements some of these ideas by computing the
self-composition of the input model and reducing the problem to LTL model checking on top of the model checker ABC [10,32]. However, these efforts generally fall short in proposing a general push-button method to deal with identifying bugs with respect to HyperLTL formulas involving arbitrary quantifier alternation. Indeed, quantifier alternation has been shown to generally elevate the complexity class of model checking HyperLTL specifications in different shapes of models [7,12].

A more recent model checker AutoHyper [3], written in F#, is an explicit-state tool and implements an automata-based verification approach. It supports full HyperLTL and is complete for properties with arbitrary quantifier alternations. However, AutoHyper can only verify or falsify a hyperproperty and cannot generate counterexamples in case of falsification. AutoHyperQ [4] (an extension of AutoHyper) is an automata-based model checker for HyperQPTL (i.e., HyperLTL with quantification over propositions). AutoHyperQ is capable of generating counterexamples but the performance may degrade significantly, as compared to AutoHyper.

1.2 The Bounded Model Checker HyperQB

In this paper, we introduce the tool HyperQB3, a fully automated bounded model checker (BMC) for hyperproperties based on the QBF-based technique introduced in [27]. In a nutshell, HyperQB works as follows:

- It takes as input a set of model(s) (up to one per trace quantifier) and a HyperLTL formula;
- The inputs are parsed into a quantified Boolean formula (QBF) encoding, which are unrolled together up-to a certain bound $k \geq 0$, and
- A query is generated to a solver that determines satisfiability of the QBF formula whose output (the satisfiability of the unrolled formulas) is interpreted to decide the outcome of the original model-checking problem.

Figure 1 shows the overall architecture of HyperQB. We use NuSMV [11] to express models and we use our home-grown grammar for HyperLTL formulas. First, these inputs are translated into a Boolean representation. For the models, our tool uses a home-grown parser (written in C++) for a subset of NuSMV and build transition relation. Next, this transition relation is handed over to another home-grown component named genqbf (written in OCaml), which unravels the transition relation along with the temporal part of the input HyperLTL formula. The unrolling depends on the bound and the choice of HyperLTL bounded semantics [27], provided by the user. It also adds quantification over variables to generate a QBF input instance. Finally, we run a QBF solver (currently QuAbS [33]) to check the satisfiability of the unrolled QBF formula and interpret the output. Our QBF encoding (introduced in [27]) is a natural generalization for HyperLTL of the classical BMC for LTL [5]. The solver will either output a

3 The tool and documentation are available at https://cse.msu.edu/tart/tools.
negative or affirmative verdict. If the original specification quantifier(s) are universal before negation, and the solver produces an affirmative answer, HyperQB generates a counterexample. It may also generate an inconclusive outcome, for example because a larger bound is needed. We note that the current version of HyperQB does not incorporate the loop conditions identified in [25]. This, of course, comes at the cost of incompleteness.

Following our results in [27], HyperQB allows to interpret a wide range of outcomes of the QBF solver and relate the outputs to the original model checking decision problem, based on the following bounded semantics for HyperLTL:

- **Pessimistic** semantics (like in LTL BMC [6]) under which pending eventualities are considered to be unfulfilled. This semantics works for *sometimes finitely satisfiable* (SFS) [22] temporal formulas and paves the way for bug hunting.

- **Optimistic** semantics considers the dual case, where pending eventualities are assumed to be fulfilled at the end of the trace. This semantics works for *sometimes finitely refutable* (SFR) [22] formulas, and allows us to interpret unsatisfiability of QBF as proof of correctness even with bounded traces.

- **Halting** variants of the optimistic and pessimistic semantics, which allow sound and complete decision on a verdict for terminating models.

We note that besides verification, HyperQB can also be used for synthesis through returning witnesses to existential quantifiers in the input HyperLTL formula. We will discuss such applications in Section 5.

### 1.3 Contributions

In summary, the contributions of this tool paper are:

1. The tool HyperQB that is able to perform BMC for HyperLTL. HyperQB has gone through a major update since the work in [27]. The main improvement
has been in QBF formula generation, where we use multi-gate constraints in QCIR generation rather than binary gates. This has resulted in significant performance gain (up to 20 times in some cases). Besides the inherent “bug hunting” feature of BMC, a key advantage of our approach—compared to state-of-the-art HyperLTL model-checkers such as MCHyper—is the ability of HyperQB to seamlessly handle formulas with quantifier alternation, which is a source of difficulty in model checking hyperproperties. HyperQB also has advantages compared to the explicit-state model checker AutoHyper and AutoHyperQ in of counterexample generation and also in terms of performance for the majority of case studies. We, of course do not claim a universal advantage.

2. We discuss comprehensive experimental evaluation, including those from [27], in addition to 15 new case studies. Our experimental evaluation includes a wide range of case studies, such as information-flow security, linearizability in concurrent data structures, path planning for robots, co-termination, deniability, intransitivity of non-interference, and secrecy-preserving mapping synthesis. Our evaluation shows that our technique is effective and efficient in identifying bugs in several prominent examples. All experiments evaluation in this paper are compared to our preliminary findings in [27] as well as with AutoHyper and AutoHyperQ.

Organization. The rest of the paper is structured as follows. We present the preliminary concepts in Section 2. Section 3 discusses the core algorithm implemented in HyperQB. Section 4 introduces the tool design and implementation in detail. Section 5 presents an empirical evaluation. Finally, Section 6 concludes. Detailed description of the new case studies appear in the appendix.

2 Background of HyperLTL Model Checking

Kripke Structures. We consider a model as the formal framework Kripke structure. Let AP be a finite set of atomic propositions and \( \Sigma = 2^{\text{AP}} \) be the alphabet. A Kripke structure is defined as \( K = (S, S_{\text{init}}, \delta, L) \), with a finite set of states \( S \), a set of initial states \( S_{\text{init}} \subseteq S \), a transition relation \( \delta \subseteq S \times S \), and a labeling function on \( S \). A path of \( K \) is an infinite sequence of states \( s(0)s(1)\cdots \in S^\omega \), such that \( s(0) \in S_{\text{init}} \), and for all \( i \geq 0 \), \( (s(i), s(i+1)) \in \delta \). A trace of \( K \) is a sequence \( t(0)t(1)t(2)\cdots \in \Sigma^\omega \), such that there exists a path \( s(0)s(1)\cdots \in S^\omega \) with \( t(i) = L(s(i)) \) for all \( i \geq 0 \). We write \( \text{Traces}(K) \) as a shorthand for the set of traces of \( K \) that start in all \( s \in S_{\text{init}} \).

The Temporal Logic HyperLTL. We consider hyperproperties as formulas in HyperLTL [12], which allows explicit quantification on traces. The syntax of HyperLTL formulas is defined by the following grammar:

\[
\varphi ::= \exists \pi. \varphi \mid \forall \pi. \varphi \mid \phi \\
\phi ::= \text{true} \mid a_\pi \mid \neg \phi \mid \phi \lor \phi \mid \phi \land \phi \mid \phi U \phi \mid \phi R \phi \mid \phi
\]

\[\varphi\]
The encoding of the formula of the form $\phi$ is first recap the core algorithm used in 3 The Algorithmic Backbone of HyperQB structure (see Appendix. A for detailed examples and explanation).

We say an interpretation $T$ satisfies $\varphi$, denoted by $T \models \varphi$, if $(T, \Pi_0, 0) \models \varphi$; and a family of Kripke structures $K$ satisfies $\varphi$, denoted by $K \models \varphi$, if $\langle \text{Traces}(K) \rangle_{\pi \in \text{Vars}(\varphi)} \models \varphi$. Fig. 2 (left) shows a simple program and its Kripke structure (see Appendix. A for detailed examples and explanation).

### 3 The Algorithmic Backbone of HyperQB

We first recap the core algorithm used in HyperQB from [27]. Let $\varphi$ be a HyperLTL formula of the form $\varphi = Q_A \pi_A Q_B \pi_B \ldots Q_Z \pi_Z \psi$ and $K = \langle K_A, K_B, \ldots, K_Z \rangle$, the encoding of the HyperLTL BMC problem in QBF is the following (for $\ast = \phi$):

where $a \in \text{AP}$ and trace variable $\pi$ from an infinite supply $V$. We also use other derived Boolean and temporal operators such as $\rightarrow$, and $\leftrightarrow$, eventually $\Diamond \varphi \equiv \text{true} \cup \varphi$ and globally $\Box \varphi \equiv \neg \Diamond \neg \varphi$. We write $T_\pi = \text{Traces}(K_\pi)$ to denote traces from $K_\pi$ that $\pi$ can range over. When traces come from multiple models, we write $K = \langle K_\pi \rangle_{\pi \in \text{Vars}(\varphi)}$ to denote a family of Kripke structures and their corresponding sets of traces (where $\text{Vars}(\varphi)$ is the set of trace variables in formula $\varphi$). An interpretation $T = \langle T_\pi \rangle_{\pi \in \text{Vars}(\varphi)}$ of $\varphi$ consists of a tuple of sets of traces (one set $T_\pi$ per $\pi$ in $\text{Vars}(\varphi)$). A trace assignment is a partial map (trace assignment) $\Pi : \text{Vars}(\varphi) \rightarrow \Sigma^\omega$ (then empty domain is denoted by $\Pi_0$).

We use pointed models, denoted by $\langle T, \Pi, i \rangle$, where $i \in \mathbb{Z}_{\geq 0}$ is a pointer that indicates the current evaluating position, and denote $\Pi[\pi \rightarrow i]$ the assignment of $\pi$ by $t$:

\[
\begin{align*}
(T, \Pi, 0) & \models \exists \pi. \psi & \text{iff} & \text{there is a } t \in T_\pi, \text{ such that } (T, \Pi[\pi \rightarrow t], 0) \models \psi, \\
(T, \Pi, 0) & \models \forall \pi. \psi & \text{iff} & \text{for all } t \in T_\pi, \text{ such that } (T, \Pi[\pi \rightarrow t], 0) \models \psi, \\
(T, \Pi, i) & \models \Diamond \psi & \text{iff} & (T, \Pi, i + 1) \models \psi, \\
(T, \Pi, i) & \models \psi_1 \cup \psi_2 & \text{iff} & \text{there is a } j \geq i \text{ for which } (T, \Pi, j) \models \psi_2 \text{ and for all } k \in [i, j), (T, \Pi, k) \models \psi_1, \\
(T, \Pi, i) & \models \psi_1 \Rightarrow \psi_2 & \text{iff} & \text{either for all } j \geq i, (T, \Pi, j) \models \psi_2, \text{ or, for some } j \geq i, (T, \Pi, j) \models \psi_1 \text{ and for all } k \in [i, j) : (T, \Pi, k) \models \psi_2.
\end{align*}
\]

We say an interpretation $T$ satisfies $\varphi$, denoted by $T \models \varphi$, if $(T, \Pi_0, 0) \models \varphi$; and a family of Kripke structures $K$ satisfies $\varphi$, denoted by $K \models \varphi$, if $\langle \text{Traces}(K) \rangle_{\pi \in \text{Vars}(\varphi)} \models \varphi$. Fig. 2 (left) shows a simple program and its Kripke structure (see Appendix. A for detailed examples and explanation).
Consider a closed HyperLTL of the form $\mathcal{Q}\pi_A\mathcal{Q}\pi_B\ldots\mathcal{Q}\pi_Z\varphi$, where $\mathcal{Q} \in \{\forall, \exists\}$. We assume that the formula has been converted into negation-normal form (NNF), so that the negation symbol only appears in front of atomic propositions, e.g., $\neg \alpha_{\pi_A}$. Without loss of generality and for the sake of clarity from other numerical indices, we use roman alphabet as indices of trace variables, we assume that $\text{Vars}(\varphi) \subseteq \{\pi_A, \pi_B, \ldots, \pi_Z\}$. Let $k \geq 0$ be the unrolling bound and let $\mathcal{T} = (T_A \ldots T_Z)$ be a tuple of sets of traces, one per trace variable. We start by defining a satisfaction relation between HyperLTL formulas for a bounded exploration $k$ and models $(\mathcal{T}, \Pi, i)$, where $\mathcal{T}$ is the tuple of set of traces, $\Pi$ is a trace assignment mapping, and $i \in \mathbb{Z}_{\geq 0}$ that points to the position of traces. as four different bounded semantics as presented in Table 1. Intuitively, bounded semantics are different strategies to predict the unseen future with finite observations. All these semantics coincide in the interpretation of quantifiers, Boolean connectives, and temporal operators up-to instant $k - 1$, but differ in their assumptions about unseen future events after the bound of observation $k$ (see [27] for more details).
A formula is declared \emph{false} unless it is witnessed to be true within the bound explored. If \((T, \Pi, 0) \models_{\text{w}} \varphi\), then \((T, \Pi, 0) \models \varphi\).

A formula is declared \emph{true} unless it is witnessed to be false within the bound explored. If \((T, \Pi, 0) \not\models_{\text{mt}} \varphi\), then \((T, \Pi, 0) \not\models \varphi\).

A formula is declared \emph{false} unless it is witnessed to be true before the program halts. If \((T, \Pi, 0) \models_{\text{hwa}} \varphi\), then \((T, \Pi, 0) \models \varphi\).

A formula is declared \emph{true} unless it is witnessed to be false before the program halts. If \((T, \Pi, 0) \not\models_{\text{hwa}} \varphi\), then \((T, \Pi, 0) \not\models \varphi\).

Table 1: Bounded Semantics for HyperLTL [27].

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The pessimistic semantics is aligned with the traditional BMC for LTL. In the pessimistic semantics a formula is declared false unless it is witnessed to be true within the bound explored. In other words, formulas can only get “truer” with more information obtained by a longer unrolling. Dually, the optimistic semantics considers a formula true unless there is evidence within the bounded exploration on the contrary. Therefore, formulas only get “falser” with further unrolling. For example, formula \(\Box p\) always evaluates to false in the pessimistic semantics. In the optimistic semantics, it evaluates to true up-to bound \(k\) if \(p\) holds in all states of the trace up-to and including \(k\). However, if the formula evaluates to false at some point before \(k\), then it evaluates to false for all \(j \geq k\).

In turn, the verdict obtained from the exploration up-to \(k\) can (in some cases) be used to infer the verdict of the model checking problem. As in classical BMC, if the pessimistic semantics find a model, then it is indeed a model. Dually, if our optimistic semantics fail to find a model, then there is no model.

### 3.2 QBF Encoding

Given a family of Kripke structures \(K\), a HyperLTL formula \(\varphi\), and bound \(k \geq 0\), we construct a QBF formula \([K, \varphi]_k\) whose satisfiability infers whether or not \(K \models \varphi\). We use our running example to describe the idea.

#### Encoding the family models.

We introduce variables \(n_0, n_1, \ldots\) to encode the state of the Kripke structure and use \(\text{AP}^* = \text{AP} \cup \{n_0, n_1, \ldots\}\) for the extended alphabet that includes these new variables. In this manner, the set of initial states of a Kripke structure is a Boolean formula over \(\text{AP}^*\), e.g., for the program in Alg. 1, the initial condition is as follows:

\[
I_A := (\neg n_0 \land \neg n_1 \land \neg n_2) \land \neg \text{high} \land \neg \text{low} \land \neg \text{halt} \land (\neg \text{PC}_0 \land \text{PC}_1)
\]
That is, \((-n_0 \land -n_1 \land -n_2)\) represents state \(s_0\), the value of \(high\) and \(low\) are both false initially, the program has not halted yet so \(halt\) is false, and the value of program counter \((-PC_0 \land PC_1)\) indicates initially an execution is at the first line of the program. All other states can be encoded in the same manner.

To encode the transition relation \(\delta\) into QBF, we populate all variables with a new copy of \(AP^*\) for each Kripke structure \(K_A\) and position in the unrolling. Then, we produce a Boolean formula that encodes the unrolling up-to \(k\). We use \(x_A^i\) for the set of fresh copies of the variables \(AP^*_A\) of \(K_A\) corresponding to position \(i \in [0, k]\). We use \(I_A(x)\) for the Boolean formula (using variables from \(x\)) that encodes the initial states, and \(\delta_A(x, x')\) (for two copies of the variables \(x\) and \(x'\)) for the Boolean formula whether \(x'\) encodes a successor states of \(x\). For example, for \(k = 3\), we unroll the transition relation up-to 3 as follows:

\[
\begin{align*}
[K_A]_3 = I_A(x_A^0) \land \delta_A(x_A^0, x_A^1) \land \delta_A(x_A^1, x_A^2) \land \delta_A(x_A^2, x_A^3)
\end{align*}
\]

which is the Boolean formula representing valid traces of length 4, using four copies of the variables \(AP^*_A\) that represent the Kripke structure \(K_A\).

**Encoding the inner LTL formula.** This is analogous to the standard BMC, except for the choice of different semantics described in Subsection 3.1. Consider the generalized non-interference formula \(\varphi_{\text{GNI}}\) we introduced in Section 1. With the negated formula (i.e., \(\neg \varphi_{\text{GNI}}\), as explained in Section 1) with bound \(k = 3\), we have:

\[
\begin{align*}
\llbracket (\text{high}_{\pi_A} \not\iff \text{high}_{\pi_C}) \lor (\text{low}_{\pi_B} \not\iff \text{low}_{\pi_C}) \rrbracket_3 := \\
((\text{high}^0_{\pi_A} \not\iff \text{high}^0_{\pi_C}) \lor (\text{high}^1_{\pi_A} \not\iff \text{high}^1_{\pi_C}) \lor (\text{high}^2_{\pi_A} \not\iff \text{high}^2_{\pi_C}) \lor (\text{high}^3_{\pi_A} \not\iff \text{high}^3_{\pi_C})) \lor \\
((\text{low}^0_{\pi_B} \not\iff \text{low}^0_{\pi_C}) \lor (\text{low}^1_{\pi_B} \not\iff \text{low}^1_{\pi_C}) \lor (\text{low}^2_{\pi_B} \not\iff \text{low}^2_{\pi_C}) \lor (\text{low}^3_{\pi_B} \not\iff \text{low}^3_{\pi_C}))
\end{align*}
\]

**Complete Formula.** Finally, to combine the model description with the encoding of the HyperLTL formula, we use two identical copies of the given Kripke structure to represent different paths \(\pi_A\) and \(\pi_B\) on the model, denoted as \(K_A\) and \(K_B\). The final resulting formula is:

\[
\llbracket K, \neg \varphi_{\text{GNI}} \rrbracket_k := \llbracket \exists x_A \exists x_B \exists x_C. \ (K_A)^k \land \llbracket K_B \rrbracket^k \land \llbracket (K_C)^k \rightarrow \llbracket \neg \varphi_{\text{GNI}} \rrbracket^k_{pes} \rrbracket \rrbracket,
\]

The satisfaction result shows that \(\llbracket K, \neg \varphi_{\text{GNI}} \rrbracket^k_{pes}\) is true, indicating finding a counterexample that consists of only one trace (i.e., witness to the existential quantifier in \(\neg \varphi_{\text{GNI}}\)). According to the pessimistic semantics, a successful detection of a counterexample allows to infer that \(K \not\models \varphi\) in the infinite semantics.

### 4 Implementation and Usage of HyperQB

HyperQB first translates the user inputs – model(s) and specification – into QBF, then unrolls the formulas based on the selected bounded semantics, and finally checks the satisfiability using a QBF-solver. In the following subsections, we elaborate on each step.
4.1 Model Description

The input modeling language of HyperQB is NuSMV [11]. We have developed a parser in C++ that translates a subset of NuSMV to a transition relation in Boolean this/next-state representation.

Our parser also analyzes the type of each defined variable to decide whether it needs to be bit-blasted (e.g., and integer). For each numerical variable $v_i$, the parser will automatically generate $\log(\text{max}(v_i))$ number of Boolean variables where $\text{max}(v_i)$ is a function that returns the maximum value of $v_i$ which is defined in the given NuSMV file. For example, consider the NuSMV model for Alg. 1 shown in Fig. 2 (right). This model contains three Boolean variables ($\text{low}$, $\text{high}$, $\text{halt}$) and one numerical variable ($PC$). Since $PC$ ranges from 1 to 3, our parser will first create two bits $PC_0$ and $PC_1$ to represent the value of $PC$.

4.2 HyperLTL Grammar

Our grammar to express HyperLTL formulas as input to HyperQB is shown on the right. This grammar supports specifying each trace using tid to their corresponding trace variable, where each can be universally or existentially quantified. Next, for the inner LTL formula, we support three different kinds of operations: (1) arithmetic comparisons (arith_comp), which must be applied to two arithmetic values; (2) binary temporal/propositional operators (binary_op), which can only be applied on two Boolean values; and, (3) unary temporal/propositional operators (unary_op), which must be followed by a single Boolean value. For example, the formula $\varphi_G^{NI}$ from Sec. 1 is written in our grammar as follows:

$$\forall A. \forall B. \exists C. \ X(\text{high}[A] \leftrightarrow \text{high}[C]) \land G(\text{low}[B] \leftrightarrow \text{low}[C])$$

In HyperQB, the syntax checking is performed by parser.cpp for both HyperLTL formulas and for NuSMV models. For example, the following are incorrect formulas and are rejected: ($PC[A] \land \text{halt}[B]$) (non-Boolean with binary_op), or ($PC[A] = \text{halt}[B]$) (treating Boolean variables as numerical). Furthermore, HyperQB also checks that (1) all the variables in the input formula are defined in the input NuSMV models, (2) all variables that appear in the input formula match the type definition in the given models, and (3) whether there are incorrect value assignments (i.e., value out of bound) in the formula. For example, if $\text{max}(PC) = 3$, then the expression ($PC[0] = 10$) is reported incorrect.
4.3 Unrolling of Model and HyperLTL Formula

The next step is to unroll the transition relation along with the input formula as described in Section 3.2. This unrolling mechanism is implemented in the component \texttt{genqbf} written in OCaml. It takes as input the Boolean representation of the NuSMV model (as a transition relation) and the input HyperLTL specification, as discussed in Sections 4.1 and 4.2. Then, \texttt{genqbf} creates multiple copies of the model to build the complete formula. This stage also implements user-specified features, including the bound of unrolling, the selected bounded semantics, and the decision on whether the input HyperLTL formula should be negated (i.e., to perform counterexample hunting), or not (i.e., to perform witness searching).

4.4 Tool Usage of HyperQB

The input arguments of HyperQB include:

- \texttt{<list of models>}, written in NuSMV format (as .smv files),
- \texttt{<formula>}, written in the grammar described in Sec. 4 (as a .hq file),
- \texttt{<k>}, a number \( \geq 0 \), specifying the unrolling bound,
- \texttt{<sem>}, the semantics, which can be -pes, -opt, -hpes or -hopt, and
- \texttt{<mode>}, to say performing classic BMC (i.e., negating the formula) or not, which can be -bughunt or -find (we use the former as default value).

Running HyperQB. HyperQB is running using shell script \texttt{hyperqb.sh} with the above-mentioned arguments in order. For example, the following invocation checks the symmetry property (\( \forall \exists \)) of the bakery algorithm using the pessimistic semantics with unrolling bound 10 (note that the list of models are the same since in this case, the sources of traces for both trace variables are the same):

\[
\texttt{./hyperqb.sh \ bakery.smv \ bakery.smv \ symmetry.hq \ 10 \ -pes \ -bughunt}
\]

For cases where each trace variable is pointing to a different model, for instance linearizability in SNARK, one should write as follows:

\[
\texttt{./hyperqb.sh \ SNARK_conc.smv \ SNARV_seq.smv \ lin.hq \ 18 \ -pes \ -bughunt}
\]

Tool Outputs. HyperQB returns either YES or NO as the final verdict of the HyperLTL BMC problem based on the SAT/UNSAT verdict by the QBF solver. In general, flags -bughunt and -find are duals, since one negates the formula and the other does not. If a counterexample or a witness is identified, HyperQB will return the path with

| \( \phi \) | SAT | UNSAT | \( \neg \phi \) | SAT | UNSAT |
|---|---|---|---|---|---|
| \( \exists^+ \phi \) | X | ✓ | \( \phi \) | ✓ | ✓ | \( \neg \phi \) | ✓ | X |
| \( \forall^+ \phi \) | X | ✓ | \( \phi \) | ✓ | ✓ | \( \neg \phi \) | ✓ | X |
| \( \exists^+ \forall^+ \phi \) | X | ✓ | \( \phi \) | ✓ | ✓ | \( \neg \phi \) | ✓ | X |

Table 2: Different output interpretations of HyperQB for BMC w.r.t. \( \phi \) with different forms of quantifiers, where: ✓: accepted; X: refuted; cex/witn: a counterexample/witness is found.
the value of each variable specified in each time step. For example, when performing -bughunt given a family of models $\mathcal{K}$ and a HyperLTL formula $\varphi$ of the form of $\forall \exists$, a SAT result indicates that $\mathcal{K} \not\models \varphi$, with a counterexample which is a trace from the first model of $\mathcal{K}$ that violates $\varphi$. Table 2 summarizes each possible outcome and its meanings. Note that in this work, we extended [27] to be able to handle more quantifiers (and alternations).

5 Empirical Evaluation and Discussions

We have evaluated HyperQB with a rich set of case studies (see Table 3). All experiments are run on an MacBook Pro with Apple M1 Max chip and 64 GB of memory.

5.1 Description of Case Studies

Benchmarks #0.1—#6.1 are from [27] and due to space limit, their detailed descriptions are in Appendix B. In this paper, we develop 15 new benchmarks for experimentation which we explain in this subsection.

**Co-termination.** This property asks whether two different programs agree on termination, which can simply be formulated as a $\forall \forall$ HyperLTL formula:

$$\forall \pi_A. \forall \pi_B. \Box (term_{\pi_A}) \leftrightarrow \Box (term_{\pi_B}).$$

We consider two simple programs from [34]. In this case, depends on their initial conditions, the programs might either diverge or agree on termination. Co-termination is a non-safety formula; however, our bounded semantics (in particular, opt), is able to give a meaningful verdict even though this is not a finitely-refutable property. This property is case #7.1 in Table 3.

**Deniability [30].** In a program, for every possible run $\pi_A$ (e.g., potentially being observed by an adversary), there must exist $2^N$ different runs, such that each agrees on $\pi_A$ on the observable parts, but differ on secret values. While deniability is usually an example of quantitative hyperproperties [20], here we demonstrate the case when the parameter is $N = 1$, that is, an $\forall \exists \exists$ formula:

$$\varphi_{\text{den}} = \forall \pi_A. \exists \pi_B. \exists \pi_C. \Box ((obs_{\pi_A} \leftrightarrow obs_{\pi_B}) \land (obs_{\pi_A} \leftrightarrow obs_{\pi_C}) \land (sec_{\pi_B} \not\leftrightarrow sec_{\pi_C})).$$

We evaluate this formula with an Wallet1 and Wallet2 models [1] (cases #8.1 and #8.2 in Table 3) with a possible attack, where the attacker can speculate the total amount of an account (sec) by repeatedly withdrawing a fix amount (obs). The UNSAT outcome for bug-hunting by HyperQB gives a positive verdict (i.e., $\mathcal{K} \models \varphi_{\text{den}}$).
Intransitive Non-interference [29]. Intransitivity down-grades non-interference (NI) in the cases that the systems secure correct information flow with a third-party. Formally, given three parties A, B and C, while the flow A to C is uncertain, intransitive NI permitted such flow if $A \rightarrow B \land B \rightarrow C$, then $A \rightarrow C$.

In this case we investigate a shared buffer model [37], which contains a secret (S) process, an unclassified (U) process, and a scheduler (sched). The main idea is to prevent $U$ from gaining secret information about $S$ by speculating $sched$, but imposing that this potential flow is allowable via identical $sched$. That is, if the two executions agree on $sched$, the flow from $S$ to $U$ is considered safe.

We apply this concept together with classic hyperproperties observational determinism (OD) and non-interference (NI), and wrote two variations that consider intransitivity:

$$\varphi_{OD_{intra}} = \forall \pi_A.\forall \pi_B. \Box (in^U_{\pi_A} \leftrightarrow in^U_{\pi_B}) \rightarrow ((sched_{\pi_A} \leftrightarrow sched_{\pi_B}) R (out^U_{\pi_A} \leftrightarrow out^U_{\pi_B}))$$

$$\varphi_{NI_{intra}} = \forall \pi_A.\exists \pi_B. \Box ((in^S_{\pi_A} = \epsilon) \land (out^U_{\pi_A} \leftrightarrow out^U_{\pi_B})) \lor \Box (sched_{\pi_A} \leftrightarrow sched_{\pi_B})$$

This property is investigated in case studies #9.1 – #9.3 in Table 3.

Termination-sensitive/-insensitive Non-interference. It is a classic definition [13] of whether leaking the information via termination channels is allowed, which derives two notions of non-interference (NI). For termination-insensitive, if one trace terminates, then there must exists another trace that either (1) terminates and obeys NI, or (2) not terminate. That is,

$$\varphi_{tini} = \forall \pi_A.\exists \pi_B. \Box (halt_{\pi_A}) \rightarrow \Box(halt_{\pi_B} \rightarrow ((high_{\pi_A} \neq high_{\pi_B}) \land (low_{\pi_A} = low_{\pi_B})))$$

Termination-sensitive strengthens the property by asking there must exists another trace that terminates and obeys NI. We verify a program from [34] with respect to termination sensitivity (cases #10.1, #10.2, and #11.1 in Table 3).

By using optimistic semantics, both return UNSAT, meaning no bugs can be found in the finite exploration. Hence, the program satisfies the properties.

Secrecy-preserving Refinement. Relating program at different levels (e.g., high vs. low, abstract vs. concrete) is often involved in system design. For example, secure compilation specifies that when the compiler transform the code (e.g., for optimization purpose), the compiled code should still satisfies the intended security property. We investigate secrecy-preserving refinement from [26] (cases #12.1 and #12.2 in Table 3). For instance, to preserve the classic $\forall \exists$ non-interference property during compilation an $\exists \forall \exists$ formula must be verified. That is, there exists a mapping $M$ that preserves NI from code $A$ to code $B$ (details in [26]), as follows:

$$\Phi_{NI-ABM} = \exists M. \forall \pi_{A_1}.\forall \pi_{B_1}.\exists \pi_{A_2}.\exists \pi_{B_2}.(\varphi_{map_1} \rightarrow (\varphi_{map_2} \land \psi_{NI})).$$
HyperQB is able to correctly synthesize correct mapping (i.e., the leading $\exists$) if one exists. Such formula with *multiple quantifier alternations* bumps up the complexity of model checking by one step in the polynomial hierarchy compared to the original non-interference formula. While our QBF-based approach in HyperQB does not suffer from it, the language-based approaches in AutoHyper seems to experience a complementation explosion. We provide more discussion in 5.2.

**LTL with Team Semantics.** TeamLTL [35] can be presented as HyperLTL formulas by avoiding explicit references to traces (details in [35]). Since our focus is on HyperLTL, we only borrow the example with team scenarios from [35].

Consider an unknown input that affects the system behavior. To specify that executions either agree on $a$ or $b$ depending on the input, one can write the following HyperLTL formula:

$$\varphi\text{team} = \exists \pi_A. \exists \pi_B. \forall \pi. (a_{\pi_A} \leftrightarrow a_{\pi}) \lor (b_{\pi_B} \leftrightarrow b_{\pi}).$$

Team scenarios as HyperQB is able to correctly verify and synthesize the two traces in the team (i.e., $\pi_A$ and $\pi_B$), correctly. This property is investigated in case studies #13.1 and #13.2 in Table 3.

**Nondeterministic Inputs/Transitions.** In order to investigate how non-deterministic choices affect the performance of model checking, we expand the running example of Fig. 2 in two ways. We first change the high and low as integers ranging $0 \ldots k$. Next, the model of #14.1 set the initial condition non-deterministically as a number from $0 \ldots k$. Another model in #14.2, instead, have high initially as 0, but on the next transition, have high set to a number $\leq k$. The formula is the classic $\forall \exists$ non-interference, but with arithmetic comparison instead of simply Boolean matching. Table 2 shows that this additional non-determinism do not create addition overhead for HyperQB despite the large state space and additional bit-blasting for HyperQB to encode the integers. However, it adds non-negligible overhead to AutoHyper for both input and transition non-determinism.

### 5.2 Analysis of Experimental Results

Table 3 summarizes our empirical results. Our case studies range over different fragments of HyperLTL. We break the running time of HyperQB in (1) parsing and translating the NuSMV model(s) and the HyperLTL formula using our parser; (2) generating a QBF query by genqbf; and (3) checking its satisfiability by QuAbS. In some cases, generating the QBF formula takes longer than checking its satisfiability, but sometimes is faster. The models in our experiments also have widely different sizes.
| Case | Model | $K$ | $\text{HLTL}$ | size | $k$ | $\text{parse}$ | $\text{genqbf}$ | $\text{QuAbS}$ | $\text{Total}$ | [27] | $\text{AH}$ | $\text{AHQ}$ |
|------|-------|-----|--------------|------|-----|-------------|-------------|-------------|-------------|-----|--------|--------|
| 0.1  | Bakery$_3$ | $\phi S1$ | 167 | 10 | 0.33 | 0.26 | 0.06 | $0.65 \times$ | $1.50$ | $0.48$ | $0.71$ |
| 0.2  | Bakery$_3$ | $\phi S2$ | 167 | 10 | 0.32 | 0.40 | 0.11 | $0.83 \times$ | $1.64$ | $0.56$ | $0.95$ |
| 0.3  | Bakery$_3$ | $\phi S3$ | 167 | 10 | 0.34 | 0.68 | 0.19 | $1.21 \times$ | $1.54$ | $19.64$ | $118.27$ |
| 1.1  | Bakery$_3$ | $\phi_{\text{sym1}}$ | 167 | 10 | 0.36 | 0.35 | 0.11 | $0.88 \times$ | $1.57$ | $4.05$ | $TO$ |
| 1.2  | Bakery$_3$ | $\phi_{\text{sym2}}$ | 167 | 10 | 0.53 | 0.37 | 0.06 | $1.24 \times$ | $1.84$ | $1.36$ | $TO$ |
| 1.3  | Bakery$_5$ | $\phi_{\text{sym1}}$ | 996 | 10 | 1.73 | 4.63 | 1.21 | $5.81 \times$ | $21.78$ | $250.28$ | $TO$ |
| 1.4  | Bakery$_5$ | $\phi_{\text{sym2}}$ | 996 | 10 | 1.52 | 4.62 | 1.17 | $5.79 \times$ | $21.58$ | $5.13$ | $TO$ |
| 2.1  | SNARK1 | $\phi_{\text{in}}$ | 18 | 49.13 | 31.20 | 22.55 | $102.88 \times$ | $598.19$ | $116.12$ | $TO$ |
| 2.2  | SNARK2 | $\phi_{\text{in}}$ | 30 | 50.57 | 99.34 | 45.48 | $195.39 \times$ | $785.13$ | $- -$ | $- -$ |
| 3.1  | 3T$_{\text{correct}}$ | $\phi_{\text{N}}$ | 368 | 50 | 0.50 | 3.81 | 2.24 | $6.55 \times$ | $14.58$ | $5.77$ | $787.36$ |
| 3.2  | 3T$_{\text{correct}}$ | $\phi_{\text{N}}$ | 64 | 50 | 0.24 | 0.70 | 0.19 | $1.13 \times$ | $2.37$ | $1.03$ | $665.41$ |
| 4.1  | NRP$_{\text{correct}}$ | $\phi_{\text{fair}}$ | 55 | 15 | 0.23 | 0.25 | 2.56 | $3.04 \times$ | $0.90$ | $0.45$ | $203.41$ |
| 4.2  | NRP$_{\text{correct}}$ | $\phi_{\text{fair}}$ | 54 | 15 | 0.24 | 0.23 | 1.64 | $2.11 \times$ | $1.14$ | $0.56$ | $207.51$ |

### 5.1 Planning (path synthesis, see Table 4)

| 6.1 Mutant | $\phi_{\text{mut}}$ | 32 | 10 | 0.20 | 0.08 | 0.03 | $0.31 \times$ | $0.46$ | $0.55$ | $2.54$ |

### Table 3: Performance of HyperQB, where column case denotes the artifact, ✓ denotes satisfaction, and ✗ denotes violation of the formula. AP$^*$ is the set of Boolean variables encoding $K$. size is the number of reachable states, $k$ is the BMC bound, parse, genqbf, and QuAbS denote run times for parsing, QBF generation, and QBF solving in HyperQB. AH stands for AutoHyper [3] and AHQ stands for AutoHyperQ [4]. TO means timeout (1200 seconds) and err means the third-party tool Spot [16] generates an error for AutoHyperQ. Finally '-' means the case was not implemented in the respective tool. All reported times are in seconds.
Comparison with [27]. The most complex case study is arguably the SNARK algorithm, where we identify both bugs in the algorithm in 102.88 and 195.39 seconds. Our implementation in [27] used to take 598.19 and 785.13 seconds, respectively. The same trend holds in the vast majority of case studies which clearly demonstrates significant performance improvement; in most cases, HyperQB outperforms our original early prototype in [27]. The cases that HyperQB is slower than [27] is most likely due to the structure of the temporal formula and the binary vs. multi-input gates in QCIR, which affects the solving time by QuAbS.

Comparison with AutoHyper and AutoHyperQ. We compare HyperQB with two explicit-state model checkers, AutoHyper [3] and AutoHyperQ [4]. First, recall that the intent of AutoHyper is only to verify or falsify a formula; i.e., AutoHyper cannot generate counterexamples. For the benchmarks #0.1—#6.1, the comparison presented in [3] shows that AutoHyper outperforms the implementation from [27] in most cases. However, due to the binary vs. multi-input gate optimization on QBF encoding and in QCIR generation in HyperQB, as mentioned in Section 1, HyperQB performs better in the vast majority of cases even on large model cases such as SNARK, compared to AutoHyper and although AutoHyper does not generate counterexamples. We now discuss in more detail.

Completeness vs. Finite Bound. Bounded model checking achieves fast verification/falsification by giving a finite bound $k$, which is typically much smaller than the diameter of the transition relation. HyperQB currently does not incorporate loop conditions and, hence, unlike AutoHyper, lacks completeness. Certain cases such as Bakery benefit from this bounded search in HyperQB. Since AutoHyper guarantees completeness, the times spent on either inclusion checking and/or product construction, are affected a lot by the type of models and the structure of formulas. For example, in cases #14.1 and #14.2, when a program contains many non-deterministic choices either on the initial states or on the transitions, the solving time of AutoHyper goes up due to language inclusion checking. In cases #8.1, #8.1, #12.1, and #12.2, where the number of quantifiers increases, HyperQB significantly outperforms AutoHyper due to the overhead of product construction in explicit state. In general, although the intent of the two tools are rather different, while AutoHyper ensures completeness, HyperQB seems to deal better with multiple quantifiers and their alternation, as QBF solvers benefit from quickly refuting or verifying a formula with small bound, which gives more opportunity to give out a verdict.

Counterexample Generation. Since the current implementation of AutoHyper does not generate counterexamples, we also compare our tool with AutoHyperQ, where obtaining concrete traces is possible. Since AutoHyperQ is not targeting efficient model checking; here we focus on cases that a concrete trace that serves as the certificate of a YES/NO answer is expected. Understandably, AutoHyperQ does not perform as well AutoHyper when attempting to obtain counterexamples
from AutoHyperQ. It also seems AutoHyperQ does not enjoy all the optimizations implemented in AutoHyper. Now, comparing AutoHyperQ and HyperQB, in all cases, HyperQB is able to synthesize the witness/counterexample more efficiently than AutoHyperQ. In fact, for many cases, AutoHyperQ either timeouts or the plugin tool Spot [16] to check language inclusion generates an error. For example, for Bakery with three processes and symmetry formula \( \phi_{sym} \), AutoHyperQ already times out after 20 minutes.

Finally, we elaborate more on scalability of the hyperproperty-based path planning for robots. Table 4 compares our approach for robustness in path planning for robots [38] for different grid sizes. First, one can observe that AutoHyper outperforms HyperQB since AutoHyper does not synthesize a path; i.e., it merely gives a YES/NO output meaning a path satisfying the specification does or does not exist without providing the witness. On the contrary, comparing the outcome of AutoHyper and AutoHyperQ, it shows that although automata-based approach is efficient in giving the correct YES/NO verdict, it is not yet efficient in witness/counterexample generation. In fact, HyperQB significantly outperforms AutoHyperQ as well as our implementation in [27].

**Summary of analysis.** Clearly, there is no silver bullet! Different tools show their strength in different contexts and there are several factors to be considered. This includes the intent of model checking, the need for completeness, structure of models, structure of formulas, depth of counterexamples, etc. More research is needed to better understand the role of each of these factors.

| grid | k | | parse | genqbf | QuAbS | Total | [27] | AH | AHQ |
|------|---|---|-------|-------|--------|-------|------|-----|-----|
| \( \varphi_{rb} \) | 10 | 20 | 266 | 0.44 | 1.09 | 1.13 | **2.22** | 5.38 | 0.40 | 4.03 |
| 20 | 40 | 572 | 0.62 | 6.35 | 5.39 | **11.74** | 26.55 | 0.73 | 32.54 |
| 40 | 80 | 1212 | 1.06 | 30.07 | 92.39 | **122.46** | 197.80 | 1.89 | 395.79 |
| 60 | 120 | 1852 | 1.63 | 75.05 | 303.49 | **378.54** | 1616.33 | 5.23 | 1926.88 |

Table 4: Path planning for robots and comparison to [27], AutoHyper, and AutoHyperQ. All cases use the halting pessimistic semantics and QBF solver returns SAT, meaning successful path synthesis.

6 Conclusion and Future Work

We introduced the tool HyperQB, a QBF-based bounded model checker for HyperLTL, which allows input models in the NuSMV language. HyperQB implements four different semantics that ensure the soundness of inferring the outcome of
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the model checking problem. To handle trace quantification in HyperLTL, we reduced the BMC problem to checking satisfiability of quantified Boolean formulas (QBF). This is analogous to the reduction of BMC for LTL to the simple Boolean SAT problem. Through a rich set of case studies, we demonstrated the effectiveness and efficiency of HyperQB in verification of information-flow properties, linearizability in concurrent data structures, path planning in robotics, and fairness in non-repudiation protocols.

We plan to extend HyperQB to incorporate loop conditions identified in [25] to gain completeness. We will also extend HyperQB to handle asynchronous hyperproperties, namely, the temporal logic A-HLTL [2, 24]. We will also add the following features: (1) choice of plug-ins for new QBF-solvers, (2) a graphical user interface, (3) a web-based interface to the tool, and (4) an API to provide the core functionality of the tool to other developers.

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A Detail Explanation of Preliminary using Examples

We provide here two example to explain (1) the concept of Kripke structures, and (2) the meaning of a HyperLTL formula.

Kripke structure Example. Consider the simple program in Fig. 2. There are two Boolean variables, high and low, representing high and low-security (i.e., secret and public) variables respectively. Initially, both high and low are set to false. In step 2 (i.e., program counter is 2), the value of high is nondeterministically set to either true or false, where the value of low stays as false. In step 3, low changes according to the conditional statement, and the whole program halts (i.e., halt = true). The Kripke structure of this program along with the NuSMV model are shown below it in Fig. 2 (respectively, left and right).

HyperLTL Formula Example. Consider the non-interference security policy specified by the HyperLTL formula: $\varphi_{NI} = \forall \pi_A. \exists \pi_B. (\text{high}_{\pi_A} \not\leftrightarrow \text{high}_{\pi_B}) \land [\text{low}_{\pi_A} \leftrightarrow \text{low}_{\pi_B}]$, which stipulates that for all traces $\pi_A$, there must exist a $\pi_B$ such that their high-security value high are different, but the low-security observation low always stays the same. Satisfying $\varphi_{NI}$ infers that an attacker cannot guess the secret value by observing the public information. It is straightforward to observe that the running example in Fig. 2 violates the formula, since the value of low variable depends on the value of high variable, a public observer can obtain the actual value of high by observing low: the path $s_0s_1s_2^\omega$ is a counterexample to $\varphi_{NI}$.

B Descriptions of Presented Case Studies from [27]

#1: Symmetry. Lamport’s Bakery algorithm is a mutual exclusion protocol for concurrent processes. The symmetry property states that no specific process is privileged in terms of a faster access to the critical section, which is a desirable property because it implies that concrete process ids are not relevant for faster accesses. Symmetry is a hyperproperty that can be expressed with different HyperLTL formulas (see Table 5 and [14]). In these formulas, each process $P_n$ has a program counter $pc(P_n)$; $select$ indicates which process is selected to process next; $pause$ if both processes are not selected; $sym\_break$ is which process is selected after a tie; and $sym(select_{\pi_A}, select_{\pi_B})$ indicates if two traces exchange the process ids of which processes proceeds. The basic Bakery algorithm does not satisfy symmetry (i.e. $\varphi_{sym}$), because when two or more processes are trying to enter the critical section with the same ticket number, the process with the smaller process ID has priority and process ID is statically fixed attribute. HyperQB returns SAT using the pessimistic semantics, indicating that there exists a counterexample to symmetry in the form of a falsifying witness to $\pi_A$ in formula $\varphi_{sym}$. The tool returns an observable witness within finite bound using the the pessimistic semantics. Therefore, we conclude that all future observations violate the property. Table 3 includes our result on other symmetry formulas presented in Table 5.
#2: Linearizability. The second study consists on verifying linearizability of the SNARK concurrent datatype [15]. SNARK implements a concurrent double-ended queue using double-compare-and-swap (DCAS) and a doubly linked-list. Linearizability [23] is a hyperproperty that requires that any history of execution of a concurrent data structure—where history is sequence of invocations and responses by different threads—matches some sequential order of invocations and responses. This is express as $\varphi_{\text{lin}}$ in Table 5. SNARK is known to have two linearizability bugs. With the use of pessimistic semantics, a witness of linearizability violation of length $k$ is enough to infer that the given system does not satisfy the linearizability property. HyperQB returns SAT identifying both bugs and producing two counterexamples. The bugs return are consistent with the ones reported in [15].

#3: Non-interference in multi-threaded programs. The hyperproperty of non-interference [21] states that low-security variables are independent from the high-security variables, thus preserving secure information flow. We consider the concurrent program example in [31], where PIN is high security input and Result is low security output. HyperQB returns SAT in the halting pessimistic semantics, indicating that there is a trace that we can spot the difference of high-variables by observing low variables, that is, violating non-interference. With HyperQB we also verified the correctness of a fix to this algorithm, proposed in [31] as well. In this case, HyperQB uses the UNSAT results from the solver (with halting optimistic semantics) to infer the absence of a violation.

#4: Fairness in non-repudiation protocols. A non-repudiation protocol ensures that a receiver obtains a receipt from the sender, called non-repudiation of origin (NRO), and the sender ends up having an evidence, named non-repudiation of receipt (NRR), through a trusted third party. A non-repudiation protocol is fair if both NRR and NRO are either both received or both not received by the parties. This is expressed as formula $\varphi_{\text{fair}}$ in Table 5. We studied two different protocols from [28], namely, $T_{\text{incorrect}}$ that chooses not to send out NRR after receiving NRO, and a correct implementation $T_{\text{correct}}$ which is fair. For $T_{\text{correct}}$, HyperQB returns UNSAT in the halting optimistic semantics which indicates that the protocol satisfies fairness. For $T_{\text{incorrect}}$, HyperQB returns SAT in the halting pessimistic semantics which implies that fairness is violated.

#5: Path planning for robots. In this case study we use HyperQB beyond verification, to synthesize strategies for robotic planning [38]. Here, we focus on producing a strategy that satisfies control requirements for a robot to reach a goal in a grid. First, the robot should take the shortest path, expressed as formula $\varphi_{\text{sp}}$ in Table 5. We also used HyperQB to solve the path robustness problem, meaning that starting from an arbitrary initial state, a robot reaches the goal by following a single strategy, expressed as formula $\varphi_{\text{rb}}$ in Table 5. HyperQB returns SAT for the grids of sizes up-to $60 \times 60$.

#6: Mutation testing. Another application of hyperproperties with quantifier alternation is the efficient generation of test suites for mutation testing. We
borrow a model from [17] and apply the original formula that describes a good test mutant together with the model, expressed as formula $\varphi_{\text{mut}}$ in Table 5. HyperQB returns SAT which implies the successful finding of a qualified mutant. We note that in [17] the authors were not able to generate test cases via $\varphi_{\text{mut}}$, as the model checker MCHyper is not able to handle quantifier alternation in push-button fashion.

| Property | Property in HyperLTL |
|----------|----------------------|
| Symmetry | $\varphi_S = \forall \pi_A, \forall \pi_B. (\neg \text{sym}(\text{select}_{\pi_A}, \text{select}_{\pi_B}) \lor \neg (\text{pause}_{\pi_A} = \text{pause}_{\pi_B})) \land ((\text{pc}(P_0)_{\pi_A} = \text{pc}(P_1)_{\pi_B}) \land (\text{pc}(P_1)_{\pi_A} = \text{pc}(P_0)_{\pi_B}))$ |
|          | $\varphi_{S2} = \forall \pi_A, \forall \pi_B. (\neg \text{sym}(\text{select}_{\pi_A}, \text{select}_{\pi_B}) \lor \neg (\text{pause}_{\pi_A} = \text{pause}_{\pi_B}) \lor \neg (\text{select}_{\pi_A} < 3) \lor \neg (\text{select}_{\pi_B} < 3) \land ((\text{pc}(P_0)_{\pi_A} = \text{pc}(P_1)_{\pi_B}) \land (\text{pc}(P_1)_{\pi_A} = \text{pc}(P_0)_{\pi_B}))$ |
|          | $\varphi_{S3} = \forall \pi_A, \forall \pi_B. (\neg \text{sym}(\text{select}_{\pi_A}, \text{select}_{\pi_B}) \lor \neg (\text{pause}_{\pi_A} = \text{pause}_{\pi_B}) \lor \neg (\text{select}_{\pi_A} < 3) \lor \neg (\text{select}_{\pi_B} < 3) \lor \neg \text{sym}(\text{sym\_break}_{\pi_A}, \text{sym\_break}_{\pi_B}) \land ((\text{pc}(P_0)_{\pi_A} = \text{pc}(P_1)_{\pi_B}) \land (\text{pc}(P_1)_{\pi_A} = \text{pc}(P_0)_{\pi_B}))$ |
|          | $\varphi_{\text{sym}_1} = \forall \pi_A, \exists \pi_B. (\square \text{sym}(\text{select}_{\pi_A}, \text{select}_{\pi_B}) \land (\text{pause}_{\pi_A} = \text{pause}_{\pi_B}) \land (\text{pc}(P_0)_{\pi_A} = \text{pc}(P_1)_{\pi_B}) \land (\text{pc}(P_1)_{\pi_A} = \text{pc}(P_0)_{\pi_B})$ |
|          | $\varphi_{\text{sym}_2} = \forall \pi_A, \exists \pi_B. (\square \text{sym}(\text{select}_{\pi_A}, \text{select}_{\pi_B}) \land (\text{pause}_{\pi_A} = \text{pause}_{\pi_B}) \land (\text{pc}(P_0)_{\pi_A} = \text{pc}(P_1)_{\pi_B}) \land (\text{pc}(P_1)_{\pi_A} = \text{pc}(P_0)_{\pi_B})$ |
| Linearizability | $\varphi_{\text{lin}} = \forall \pi_A, \exists \pi_B. (\Box \text{history}_{\pi_A} \rightarrow \text{history}_{\pi_B})$ |
| NI | $\varphi_{\text{NI}} = \forall \pi_A, \exists \pi_B. (\text{PIN}_{\pi_A} \neq \text{PIN}_{\pi_B}) \land ((\neg \text{halt}_{\pi_A} \lor \neg \text{halt}_{\pi_B}) \land ((\text{halt}_{\pi_A} \land \text{halt}_{\pi_B}) \land (\text{Result}_{\pi_A} = \text{Result}_{\pi_B}))$ |
| Fairness | $\varphi_{\text{fair}} = \exists \pi_A, \forall \pi_B. (\Box \text{mut}_{\pi_A} \land (\Diamond \text{NRO}_{\pi_A}) \land (\Diamond \text{NRO}_{\pi_A}) \land ((\Box \land \text{act}_{\pi_A} \rightarrow \text{act}_{\pi_A}) \rightarrow ((\Diamond \text{NRR}_{\pi_B} \rightarrow (\Diamond \text{NRO}_{\pi_A})) \land ((\Box \land \text{act}_{\pi_A} \rightarrow \text{act}_{\pi_A}) \rightarrow ((\Diamond \text{NRR}_{\pi_B} \rightarrow (\Diamond \text{NRO}_{\pi_A}))$ |
| Path Planning | $\varphi_{\text{path}} = \exists \pi_A, \forall \pi_B. (\neg \text{goal}_{\pi_B} \land \text{goal}_{\pi_A})$ |
|        | $\varphi_{\text{path}} = \exists \pi_A, \forall \pi_B. (\text{strategy}_{\pi_B} \rightarrow \text{strategy}_{\pi_A}) \land (\text{goal}_{\pi_A} \land \text{goal}_{\pi_B})$ |
| Mutant | $\varphi_{\text{mut}} = \exists \pi_A, \forall \pi_B. (\text{mut}_{\pi_A} \land \neg \text{mut}_{\pi_B}) \land ((\text{in}_{\pi_A} \rightarrow \text{in}_{\pi_B}) \land (\text{out}_{\pi_A} \neq \text{out}_{\pi_B})$ |

Table 5: Hyperproperties investigated in HyperQB case studies.