Scale Invariant Density Perturbations from Cyclic Cosmology

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Abstract

It is shown how quantum fluctuations of the radiation during the contraction era of a CBE (Comes Back Empty) cyclic cosmology can provide density fluctuations which re-enter the horizon during the subsequent expansion era and at lowest order are scale invariant, in a Harrison-Zel’dovich-Peebles sense, as necessary to be consistent with observations of large scale structure.
1 Introduction

In a cyclic cosmology based on the CBE (Comes back empty) assumption [1,2] it has been shown that flatness ($\Omega \approx 1$) is achieved with high precision without an inflationary era. It arises from two reasons, firstly that during the contraction phase $\Omega = 1$ is approached naturally by the Friedmann equation and secondly the significant reduction in size of the contracting introverse relative to the expanding extroverse makes the flatness even more precise.

The CBE cyclic universe is motivated by satisfying the second law of thermodynamics where the propensity of entropy to increase provides a well-known stumbling block [3]. By the fact that general relativity is valid for 99% of the cycle, and joining the radiation-dominated contraction and early expansion eras, the duration of the expansion and contraction eras were estimated [2] as 1.3 trillion years, close to a hundred times the present age.

Another requirement of a cosmological theory is to explain the observed density fluctuations which set up the initial conditions for structure formation. The necessary properties can be deduced from the anisotropy of the Cosmic Microwave Background (CMB), also from the distribution of matter as revealed by galaxy redshift surveys.

The density fluctuation $\delta(x)$ is defined by

$$\delta(x) \equiv \frac{\rho(x)}{\bar{\rho}} - 1 = \int dk \delta_k \exp(ik.x)$$

where $\bar{\rho}$ is the average density and $k$ is the wave number of the fluctuation.

The isotropic power spectrum $P(k)$ with $k \equiv |k|$ is defined by the two-point function

$$< \delta_k \delta_{k'} > = \frac{2\pi^2}{k^3} \delta(k - k')P(k)$$

and the behavior of $P(k)$ is characterized by

$$P(k) \propto k^{n_s-1}$$

where $n_s$ is the scalar spectral index. $n_s = 1$ corresponds to scale invariance [4,6]. The latest determination of $n_s$ from Planck [7,8] gives

$$n_s = 0.9655 \pm 0.0062$$

at 68% confidence level so there is approximate scale invariance. The fact that $n_s < 1$ implies a spectral tilt towards the red because lower frequencies are enhanced relative to the exactly scale-invariant $n_s = 1$ case. In the present article, we shall be content to derive
scale-invariant fluctuations with $n_s = 1$ and leave the study of the higher order corrections which redden the spectrum to $n_s \simeq 0.96$ for future research.

The plan of this paper is that in Section 2 the CBE cyclic cosmological model is discussed, in Section 3 density perturbations in inflationary cosmology are reviewed as a preparation, in Section 4 the density perturbations in CBE cyclic cosmology are derived, and finally in Section 5 there is discussion.
2 CBE Model

The CBE cosmological model makes use of the superluminal expansion first observed in 1998 [9,10]. It separates the universe into the visible universe, or particle horizon, in CBE called the introverse, and the remaining part of spacetime outside the introverse in CBE called the extroverse.

The motivation for the CBE assumption is consistency with the second law of thermodynamics and explanation of the vanishing entropy at the beginning of expansion.

The idea first introduced in 2007 [11] is that only the introverse be considered because everything outside is unobservable, and further that the introverse be chosen, as is true for almost every choice, to be empty of matter and to include only dark energy together with tiny amounts of radiation and curvature.

One striking property of the observed geometry of the visible universe is its proximity to flatness $\Omega_{TOTAL} = 1$.

In [2], the turnaround time was established as $t_T = 1.3\,\text{Ty}$ at which time the radii of the introverse(IV) and extroverse(EV) are respectively $R_{IV}(t_T) = 58\,\text{Gly}$ and $R_{EV}(t_T) = 4.4 \times 10^{42}\,\text{Gly}$. This implies in the notation of [1] that

$$f(t_T) = \frac{R_{IV}(t_T)}{R_{EV}(t_T)} = 1.1 \times 10^{-41}$$

which can be substituted in

$$|\Omega(t_B) - 1| = f(t_T)^4 C_\omega t_B$$

in which $C_\omega = 3.9 \times 10^{-16}\,\text{s}^{-1}$ and $t_B$ is the bounce time given by

$$t_B = 10^{-44}\,\text{s} \left( \frac{10^{19}\,\text{GeV}}{T_B} \right)^2$$

where $T_B$ is the bounce temperature in GeV. Taking typical temperature values $T_B = 10^6, 10^{11}, 10^{16}$ GeV the bounce times are $t_B = 10^{-18}, 10^{-28}, 10^{-38}$ s.

However, because $f(t_T)$ in Eq.(5) is so extremely small, when we calculate the value of the present total density for any of these $t_B$, the result for $|\Omega_{TOTAL}(t_0) - 1|$ is infinitesimal, well below an inverse googol, and its exact result become academic:

$$|\Omega_{TOTAL}(t_0) - 1| \ll 10^{-100}$$

which is interesting.
This implies that any departure from $\Omega_{TOTAL}(t_0) = 1$ can falsify the CBE model. The expected limit to the observational accuracy in the measurement is comparable to the size of the observed perturbations $\sim 10^{-5}$. Therefore falsification of CBE would follow from

$$|\Omega_{TOTAL}(t_0) - 1| > 10^{-5}$$

(9)

On the other hand, the smaller this quantity becomes, the more it will favor CBE over inflation which predicts $|\Omega(t_0) - 1|$ to be small but not identically zero.

The contraction period is radiation dominated and begins at the turnaround $t_T \sim 1.3$ Ty. The introverse at that time has a scale factor $\hat{a}(t_T) = 1.11$ and subsequently contracts as $\hat{a}(\hat{t}) \sim \hat{t}^{1/2}$ where for convenience during contraction we define a displaced time $\hat{t}$ by $\hat{t} \equiv (t_B - t)$ with $t_B$ the time of the bounce, which is $t_B \sim 2.6$ Ty.

In terms of $\hat{t}$ the contraction scale factor $\hat{a}(\hat{t})$ therefore shrinks according to

$$\hat{a}(\hat{t}) = \hat{a}(t_T) \left( \frac{\hat{t}}{t_T} \right)^{1/2}$$

(10)

for $0 < \hat{t} < t_T$ and must be matched on to the expansion scale factor $a(t)$ at the onset its matter domination $t = t_m$ so that

$$\hat{a}(\hat{t} = t_m) = a(t_m) = 2.1 \times 10^{-4}$$

(11)

and from then to and from the bounce the expansion $a(t)$ and the contraction $\hat{a}(t)$ are equal.

So far the discussion is classical without fluctuations. Density fluctuations are expected to arise from quantum effects so first we shall discuss how this happens in inflationary models in the next section, then show how the CBE model can produce scale-invariant density perturbations.
3 Density perturbations with inflation

In the inflationary scenario, the superluminal accelerated expansion during inflation makes quantum fluctuations of the inflaton enlarge to macroscopic size and freeze in after leaving the horizon. They later re-enter the horizon as classical density perturbations which provide the initial conditions necessary for structure formation.

Let us flesh out some of the mathematical details, extracted from [12]. This is not original but will be useful in addressing the perturbation issue for the CBE model in the subsequent section.

We take a single inflaton field $\phi(x,t)$ in a locally flat spacetime as in [13–15] whereupon its classical field equation is

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}\nabla^2\phi + V' = 0$$

with $V' = dV/d\phi$.

At lowest order, a perturbation $\delta\phi_k$ for wave number $k$ therefore satisfies

$$\ddot{\delta\phi_k} + 3H\dot{\delta\phi_k} + \left(\frac{k}{a}\right)^2\delta\phi_k + V''\delta\phi_k = 0$$

with $V'' = d^2V/d\phi^2$.

For a light field $V \ll H$ and $V \ll (k/a)^2$, so that

$$\ddot{\delta\phi_k} + 3H\dot{\delta\phi_k} + \left(\frac{k}{a}\right)^2\delta\phi_k = 0$$

We are concerned only with a few Hubble times $H^{-1}$ around the exit time during which we may take the slowly varying $H$ to be constant at $H_k$. Defining conformal time $\eta$ by $\eta = -1/aH$, we find an oscillator equation

$$\frac{d^2\phi_k(\eta)}{d\eta^2} + \omega_k^2(\eta)\phi_k(\eta) = 0$$

where

$$\omega_k^2 = k^2 - \left(\frac{2}{\eta^2}\right) = k^2 - 2(aH_k)^2$$

Before horizon exit there is constant wave number $k$. In a small spacetime region with $k^{-1} \ll \Delta\eta \ll (aH_k)^{-1}$ there are still many oscillations but the spacetime curvature is negligible. During the interval $\Delta\eta$, $k$ is the physical wave number because the second term in Eq. (16) is negligible.
We can decompose the Fourier component $\hat{\phi}_k(\eta)$ as

$$(2\pi)^3 \hat{\phi}_k(\eta) = \phi_k(\eta)\hat{a}(k) + \phi_k^*(\eta)\hat{a}^\dagger(-k)$$  \hspace{1cm} (17)$$

The required solution is

$$\phi_k(\eta) = \frac{e^{-ik\eta}(k\eta - i)}{\sqrt{2k}k\eta}$$ \hspace{1cm} (18)$$

which long after the horizon exit approaches

$$\dot{\phi}_k(\eta) = -\frac{i}{\sqrt{2k}k\eta}$$ \hspace{1cm} (19)$$

It is assumed that the state corresponds to vacuum with no particles and $<\phi_k>=0$. We have a gaussian random field whose ensemble average is the vacuum expectation value. The power spectrum is defined by the two-point function

$$<\phi_k\phi_{k^\prime}> = \frac{2\pi^2}{k^3} \mathcal{P}_\phi(k) \delta^3(k + k^\prime)$$ \hspace{1cm} (20)$$

Using the earlier expressions, we now find the scale invariant result

$$\mathcal{P}_\phi(k) = \left(\frac{H_k}{2\pi}\right)^2$$ \hspace{1cm} (21)$$

where $H_k$ is a constant, $H_k = H$.

The mean square perturbation of the field is given by

$$<\delta\phi^2(x, t)> = \left(\frac{H}{2\pi}\right)^2N(t)$$ \hspace{1cm} (22)$$

where $N(t)$ is the number of e-folding of inflation after leaving the horizon.

Finally, long after the fluctuation leaves the horizon the k-mode in Eq.(17) becomes purely imaginary with a definite constant value for measurements and hence can be regarded as classical. These density perturbations re-enter the horizon later in the expansion.
4 Density perturbations in CBE model

Because the photon is massless, the CBE contraction era is classically scale invariant. There is no scalar inflaton and the quantum fluctuations of relevance are in the electromagnetic field $A_\mu(x, t)$.

In inflation, quantum fluctuations of the inflaton freeze-out, exit the horizon during expansion then later in the expansion re-enter the horizon as classical density perturbations.

In the CBE cyclic scenario quantum fluctuations of the radiation field freeze in after leaving the horizon during contraction and later re-enter the horizon as classical density perturbations after the bounce when the universe is expanding, thereby providing the initial conditions necessary for structure formation.

The classical field equation for a perturbation $\delta(A_\mu)_k$ may be cast into the simple harmonic oscillator equation

$$\frac{d^2(A_\mu)_k(\eta)}{d^2\eta^2} + \omega^2_k(\eta)(A_\mu)_k(\eta) = 0 \quad (23)$$

where

$$\omega^2_k = k^2 - \left(\frac{2}{\eta^2}\right) = k^2 - 2(aH_k)^2 \quad (24)$$

Quantum k-modes are then defined by

$$(2\pi)^3(\hat{A}_\mu)_k(\eta) = (A_\mu)_k(\eta)\hat{a}(k) + (A_\mu)_k^*(\eta)\hat{a}^\dagger(-k) \quad (25)$$

Subject to the initial condition

$$(A_\mu)(\eta) = \frac{1}{\sqrt{2k}}\epsilon_\mu(k)e^{-ik\eta}, \quad (26)$$

the appropriate solution is then

$$(A_\mu)_k(\eta) = \epsilon_\mu(k)\frac{e^{-ik\eta}(k\eta - i)}{\sqrt{2k} \cdot k\eta} \quad (27)$$

Well after exiting the horizon, this solution becomes purely imaginary and freezes in with time-independent eigenvalues just as if classical, namely

$$(A_\mu)_k(\eta) = -\epsilon_\mu(k)\frac{i}{\sqrt{2k} \cdot k\eta} \quad (28)$$
The two point function may be defined in the 't Hooft-Feynman gauge by

\[ <(A_\mu)_k(A_\nu)_{k'}>=\frac{2\pi^2}{k^3}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\mathcal{P}_A(k)\delta^3(k+k') \] (29)

Proceeding with the parallel steps as before one arrives at the scale-invariant power law, at this lowest order

\[ \mathcal{P}_A(k) = \left(\frac{Hk}{2\pi}\right)^2. \] (30)

The mean square perturbation is given by considering a comoving box of side \((aL)\) by

\[ <|\delta A_\mu(x,t)|^2>=\left(\frac{H}{2\pi}\right)^2\int_{L^{-1}}^{aH}d\frac{dk}{k}=\left(\frac{H}{2\pi}\right)^2\ln(LHa) \] (31)

These are the scale invariant density perturbations in the CBE model. They do not re-enter the horizon during the contraction era but only after the bounce. That is when they enter as frozen-in classical density perturbations and seed large-scale structure formation.

There is no superluminal accelerated expansion in the early universe, only starting at \(t \simeq 9.8\text{Gy}\) when dark energy begins to dominate over matter very much later in the expansion era.

In the CBE cyclic cosmological model, the density perturbations arise quite differently from in inflationary cosmology because (i) they originate not during expansion but during contraction and (ii) they arise from quantum fluctuations not of an inflaton field but of the electromagnetic field.

This keeps alive the beautiful idea that the large scale structure in the present universe originates from quantum fluctuations in the very early universe.
5 Discussion

Previously it was shown how a cyclic universe with the CBE assumption, that the contracting universe is an introverse empty of matter, can explain the observed flatness of the present universe without an inflationary era. It also led to an estimate of the time until the turnaround from expansion to contraction of 1.3 Ty, or about one hundred times the present age.

In the present article we have analyzed the appearance of density perturbations in the CBE model. Also, in Eq. (9), there was an observational method to falsify CBE.

So where do we stand on the key question of whether the present expansion era began with a big bang or a bounce at a time \( t_0 \) in the past? At least, we know \( t_0 = 13.80 \pm 0.04 \) Gy. The two alternatives coincide after the first picosecond, \( 10^{-12} \) s, but differ completely in the earlier universe.

It is necessary to be clear on what "big bang" theory means. We take it to mean that time starts \( t_0 \) ago and that at that time the density and temperature were both extremely large. They are not necessarily infinite, as suggested by the classical Friedmann-LeMaître equation [16, 17] because, for times \( t < t_{\text{Planck}} = 10^{-44} \) s, time itself becomes ill-defined due to quantum fluctuations of spacetime. In the distant future, the big-bang model is normally taken to imply expansion for an infinite time.

In big-bang theory, to explain the flatness and horizon properties a brief period of superluminal accelerates expansion helps during the extremely early universe. Even with such inflation, however, there still remains a mystery about the initial conditions, especially why the entropy is so singularly low [18].

By "bounce" which is our clear preference, it is meant that at the bounce time \( t_B \) satisfying

\[
10^{-38} \text{s} < t < 10^{-18} \text{s}
\]

(32)

the present expansion era began immediately preceded by contraction. There was no inflation era. Nevertheless, the flatness property is predicted and, as we have shown in this article, so are scale invariant density perturbations at lowest order. One expects the reddening from spectral index \( n_s = 1 \) to the observed \( n_s \simeq 0.96 \) to be calculable in higher orders, similarly to what happens in inflation.

It should be added that the CBE assumption first introduced in [11], and considerable refined here, is very speculative but, to our knowledge, there is no alternative resolution of the Tolman conundrum [3].

One outstanding issue is how the turnaround and bounce occur dynamically. If we may close with a speculation, the correct theory of quantum gravity could have a classical limit agreeing with general relativity everywhere, except in the close vicinity of the turnaround or bounce.
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