1. INTRODUCTION

In the currently favored paradigm for the formation of structure in the universe, luminous objects, such as galaxies and galaxy clusters, are embedded within extended halos of cold dark matter (CDM). The nonlinear equilibrium structure of these halos has long been thought to contain clues as to the astrophysical nature of dark matter, a fact that has fueled strong interest in detailed theoretical predictions for the structure of dark matter halos, as well as in constraints placed by observations of disk galaxy rotation curves (Frenk et al. 1988; Flores et al. 1993; Flores & Primack 1994; Moore 1994; Moore et al. 1999b; McGaugh & de Blok 1998; van den Bosch et al. 2000; Swaters, Madore, & Trewhella 2000; van den Bosch & Swaters 2001), by gravitational lensing of galaxies and clusters (see, e.g., Tyson, Kochanski, & dell’Antonio 1998; Williams, Navarro, & Bartelmann 1999), and by detailed studies of the stellar dynamics of spheroidal galaxies (Carollo et al. 1995; Rix et al. 1997; Gerhard et al. 1998; Cretton, Rix, & de Zeeuw 2000; Kronawitter et al. 2000).

Unfortunately, observational constraints tend to be strongest near the center of dark matter halos, where most of the luminous material in galaxies resides but also where theoretical predictions are least robust. This is because of the difficulties inherent to simulating accurately through N-body methods regions where overdensities exceed ~10^6 and where particles may have completed thousands of orbits during a Hubble time. In spite of these difficulties, there has been steady progress in our theoretical understanding of the nonlinear structure of virialized dark matter halos, driven largely by direct numerical simulation. Following on pioneering numerical studies by Quinn, Salmon, & Zurek (1986), Frenk et al. (1988), Dubinski & Carlberg (1991), and Crone, Evrard, & Richstone (1994), Navarro, Frenk, & White (1996, 1997, hereafter NFW) found that, regardless of mass and of the value of the cosmological parameters, the density profiles of dark matter halos formed in various hierarchical clustering cosmogonies were strikingly similar. This “universal” structure can be characterized by a spherically averaged density profile that differs substantially from the simple power laws, ρ(r) ∝ r^−β, predicted by early theoretical studies (Gunn & Gott 1972; Fillmore & Goldreich 1984; Hoffmann & Shaham 1985; White & Zaritsky 1992). The profile steepens monotonically with radius, with logarithmic slopes shallower than isothermal (i.e., β < 2) near the center but steeper than isothermal (β > 2) near the virial radius of the system.

The NFW findings have been confirmed by a number of subsequent studies (Cole & Lacey 1996; Huss, Jain, & Steinmetz 1999; Fukushige & Makino 1997; Moore et al. 1998; Jing & Suto 2000), although there is some disagree-
ment among authors regarding the innermost value of the logarithmic slope. NFW argued that a fitting formula where \( \beta = (1 + 3y)/(1 + y) \) (where \( y = r/r_s \) is the radius in units of a suitably defined “scale radius” \( r_s \)) provides a good fit to the density profiles of simulated halos over two decades in radius. Moore et al. (1998), Ghigna et al. (2000), and Fukushige & Makino (1997, 2001) have argued that \( \beta \) converges to a value of \( \sim 1.5 \) near the center, rather than the \( \beta \approx 1 \) expected from the NFW fit. Kravtsov et al. (1998) initially obtained much shallower inner slopes (\( \beta \approx 0.7 \)) in their numerical simulations but have now revised their conclusions; these authors now argue that CDM halos have steeply divergent density profiles but, depending on evolutionary details, the slope of a galaxy-sized halo at the innermost resolved radius may vary between \(-1.0\) and \(-1.5\) (Klypin et al. 2001).

Steep inner slopes have been traditionally disfavored by rotation curve data, a fact that has often been used to rule out CDM as a viable cosmogony (Flores & Primack 1994; Moore 1994; McGaugh & de Blok 1998; Blais-Ouellette, Amram, & Carignan 2001; Côté, Carignan, & Freeman 2000; de Blok et al. 2001). However, recent reanalysis of the available data suggests that most rotation curves are broadly consistent with “cuspy” dark matter cores, provided that the innermost slope is shallower than \( \beta \approx 1.5 \) and that the “concentration” of the halos is low (Navarro 1998; van den Bosch et al. 2000; van den Bosch & Swaters 2001).

Although there may not be at present broad consensus regarding how steep the innermost slope is, or even whether there should be a well-defined asymptotic innermost slope, there is agreement that it will take extraordinary computational effort to reach a robust resolution of the controversy. What is required is a statistically significant sample of galaxy-sized halos with subkiloparsec resolution, an extremely onerous computational task that will stretch the capabilities of the most powerful massively parallel computers. Steps in this direction are currently being taken (e.g., Power et al. 2001, in preparation), but it will take some time until these efforts yield conclusive results.

From the theoretical point of view, a number of plausible arguments have been advanced to try to explain the innermost behavior of dark matter density profiles from stellar dynamical principles. These efforts, however, tend to yield nonunique results and have so far been unable to explain the remarkable similarity in the structure of dark matter halos of widely different mass formed in a variety of cosmogonies (Evans & Collett 1997; Syer & White 1998; Nusser & Sheth 1999; Lokas & Hoffman 2000). In this paper we investigate an empirical alternative to analytic efforts addressed at estimating the innermost slope of the density profile. Our proposal exploits the similarity between the phase-space density profiles of CDM halos and that of the self-similar solution for spherical collapse in an expanding universe found by Bertchinger (1985). It offers an attractive scenario for understanding the shape of halo density profiles, as well as a powerful tool for estimating their slope near the center.

The outline of this paper is as follows. In § 2, we present the phase-space density profiles for galaxy-sized CDM halos in several recent simulations and show that they are well approximated, over more than two decades in radius, by a power law. In § 3, we investigate equilibrium density profiles consistent with this constraint by solving the Jeans equation for spherically symmetric systems with isotropic velocity dispersion tensors. Section 4 compares these results with the results of numerical simulations. We discuss and summarize our findings in §§ 5 and 6.

2. THE PHASE-SPACE DENSITY PROFILE OF CDM HALOS

We have chosen for our analysis three high-resolution simulations of galaxy-sized CDM halos identified at \( z = 0 \). Each of these simulations has of order \( 10^7 \) particles within the virial radius of the halo and was run with PKDGRAV, a massively parallel \( N \)-body code developed by Joachim Stadel and Thomas Quinn at the University of Washington. These simulations are among the largest carried out so far for galaxy-sized objects and represent a major investment in computational resources.

Two of the halos were run in the former “standard” CDM cosmology (SCDM; \( \Omega_m = 1, \Lambda = 0, h = 0.5, \sigma_8 = 0.7 \)) and have circular velocities of \( \sim 180 \) and \( \sim 160 \) km s\(^{-1} \), respectively. These circular velocities are measured at the virial radius \( r_{200} \), where the mean inner density of the system is 200 times the critical value for closure. These two halos are part of the “Local Group” simulation reported in Moore et al. (1999a) (the larger of the two is referred to as M31, and the smaller one as MW, in Figs. 1 and 3). The third halo has a circular velocity of 200 km s\(^{-1} \). It was selected at random from a simulation of a 32.5 h\(^{-1}\) Mpc box in the popular LCDM cosmology (\( \Omega_m = 0.3, \Lambda = 0.7, h = 0.65, \sigma_8 = 0.9 \)) and was resimulated at higher resolution using the procedure described in Eke, Navarro, & Steinmetz (2001). All halos have thus been selected from large periodic boxes, and their formation and evolution preserve the full tidal field and cosmological context provided by structures on larger scales.

Gravitational softenings were chosen to be \( \sim 3 \times 10^{-3} r_{200} \) and \( 1.5 \times 10^{-2} r_{200} \) for the SCDM and LCDM runs, respectively. PKDGRAV uses a multisteping algorithm to integrate the equations of motion; particles with the smallest time bins may take up to 100,000 time steps to evolve from the initial redshift (\( z_i = 99 \) for SCDM and \( z_i = 49 \) for LCDM) to the present. The initial conditions for the SCDM runs were set up using the algorithms devised by the \( N \)-body shop at the University of Washington. Those for the LCDM run were set up in a completely independent way using the algorithms described by Navarro, Frenk & White (1996, 1997; see also Efstathiou et al. 1985). As we describe below, the results of these two independent runs are consistent with each other, which suggests that our conclusions are independent of both the cosmological model adopted and of the particular choice of initial conditions setup algorithms.

By analogy with the distribution function \( f(\mathbf{x}, \mathbf{v}, t) \) in kinetic theory, we can define a spherically averaged phase-space density that is inversely proportional to the local entropy at a given radius. This phase-space density is computed in spherical bins containing 2000 particles each and is defined as \( \rho/\sigma^2 \), where \( \rho \) is the mass density and \( \sigma^2/2 \) is the specific kinetic energy in the bin. The velocity dispersion tensor for all three halos is isotropic near the center and only mildly radially biased in the outer regions; \( \sigma_o \approx \sigma_r \approx 0.85 \sigma_8 \) near the virial radius. Thus, we will ignore the effects of anisotropy in this paper, but we plan to consider them in future work.

Figure 1 shows the phase-space density profile of the three CDM halos. Solid (dashed) lines are used for the SCDM (LCDM) halos. For ease of comparison, we have
chosen to normalize \( \rho/\sigma^3 \) so that all three curves coincide at 0.01\( r_{200} \). The important point illustrated by this figure is that, over more than two decades in radius, the phase-space density profile is very well approximated by a power law of slope \(-1.875 \) (thin straight line). This is quite remarkable, given that both the density profiles (shown in Fig. 2) and the velocity dispersion profiles of these halos deviate quite strongly from simple power laws, as described by NFW.

Also remarkable is that the slope of this power law coincides with that of the self-similar solution derived by Bertschinger (1985) for secondary infall onto a point mass perturber in an unperturbed Einstein–de Sitter universe: Bertschinger’s solution is plotted with circles in Figure 1. This solution corresponds to the self-similar equilibrium configuration of a \( \gamma = 5/3 \) gas formed by spherical accretion onto a point-mass perturber in an otherwise uniform Einstein–de Sitter universe. The quantity shown by the circles is equivalent to the phase-space density, \( \rho^{5/2}/\rho^{3/2} \), where \( P \) is the local (isotropic) pressure. As discussed by Bertschinger, this solution is the most appropriate to compare with our numerical results for CDM halos, given that the velocity dispersion tensor in this case is only mildly anisotropic. Radii are normalized assuming that \( r_{200} \) equals the shock radius of the self-similar solution, which corresponds to roughly one-third of the turnaround radius. The vertical normalization is arbitrary and has been chosen to match the \( N \)-body results at 0.01\( r_{200} \). Taking \( \rho^{5/2}/P^{3/2} \propto (T/\rho)^{-5/2} \) to be a measure of the local entropy of the system, Figure 1 shows that galaxy-sized CDM halos have the same radial entropy stratification as the simple spherical collapse solution. It is possible that this power-law stratification is a fundamental property that underlies the similarity of structure of cold dark matter halos.

### 3. DENSITY PROFILES

Density profiles consistent with the power-law phase-space density profile shown in Figure 1 can be obtained by assuming hydrostatic equilibrium. For an isotropic, spherically symmetric system of collisionless particles, the Jeans equation may be written as

\[
\frac{d(\rho \sigma^2)}{dr} = -\frac{d\Phi}{dr} = -\frac{GM(<r)}{r^2},
\]

where \( \Phi \) is the gravitational potential and \( M(<r) \) is the mass interior to \( r \) (Binney & Tremaine 1987, p. 198). Equation (1) is equivalent to the equation of hydrostatic equilibrium for a gas of pressure \( P = \rho \sigma^2 \). Dividing both sides of the equation by \(-G\rho/r\) and taking derivatives with respect to \( r \), we can rewrite this equation as

\[
\frac{d}{dr} \left[ -\frac{r^2}{G\rho} \frac{d(\rho \sigma^2)}{dr} \right] = \frac{d}{dr} M(<r) = 4\pi \rho r^2,
\]

where the last equivalence applies to a self-gravitating system. Assuming that the phase-space density is a power law of radius, i.e., that

\[
\rho/\sigma^3 (r) = \frac{\rho_0}{\sigma_0^3} \left( \frac{r}{r_0} \right)^{-x},
\]

where \( r_0 \) is an (arbitrary) reference radius, \( \sigma_0 = \sigma(r_0) \), and \( \rho_0 = \rho(r_0) \), and defining the dimensionless variables, \( x \equiv \)
r/r_o and y \equiv \rho/\rho_o$, equation (2) can then be written as

$$\frac{d}{dx} \left[ -\frac{x^3}{y} \frac{d}{dx} (y^{5/3} x^{2/3}) \right] = \kappa y x^2 ,$$

where

$$\kappa = 4\pi G \rho_o r_o^2/\sigma_o^2$$

is a dimensionless measure of the velocity dispersion at r_o.

A simple interpretation of the parameter \( \kappa \) can be found by choosing r_o to be r*, the radius where the circular velocity of the system peaks. In this case, and because \( \rho(r_p) = M(<r_p)/4\pi r_p^2 \), we can rewrite \( \kappa \) as

$$\kappa = \frac{4\pi G \rho_r r^2}{\sigma^2} = \frac{GM(<r_p)/r_p}{\sigma^2} = \frac{V^2_r}{\sigma^2}. \quad \text{(6)}$$

In other words, \( \kappa \) measures the local velocity dispersion of the system in units of its circular velocity at the radius where the circular velocity peaks.

Equation (4) has the form

$$y'' + f(y', y, x) = 0 \quad \text{(7)}$$

(where \( y'' \equiv dy/dx \), \( y' \equiv d^2y/dx^2 \)), so, given initial conditions \( y(1) = \rho(r_o)/\rho_o = 1 \) and \( y'(1) = y' \) at \( x = 1 \) (\( r = r_o \)), we can solve it numerically for \( r > r_o \) or \( r < r_o \) by integrating outward or inward. The resulting solutions are parameterized by \( x, y', \) and \( \kappa \), so for \( x = 1.875 \), we obtain a two-parameter family of solutions. Clearly, integrating equation (7) with respect to some different variable \( z \equiv tx \), where \( t \) is a constant, will produce an identical solution, provided that we scale \( y' \) by \( t \) and \( \kappa \) by \( t^{2/3} \), so in fact the solutions are degenerate, and there is effectively a single free parameter (which we can take to be \( \kappa \)) that characterizes the full solution set.\(^2\)

Equation (7) admits power-law solutions. Solving for \( y = x^{-\beta} \) yields the following conditions. For \( \kappa = 0 \), we have

$$\beta = 2\alpha/5 \quad \text{or} \quad \beta = \alpha + 3/2 , \quad \text{(8)}$$

while for \( \kappa \neq 0 \) the conditions are

$$\beta = 6 - 2\alpha \quad \text{and} \quad \kappa = 8(\alpha - 3/2)(5/2 - \alpha) . \quad \text{(9)}$$

For example, the singular isothermal sphere is recovered for \( \kappa = x = \beta = 2 \).

Thus, assuming \( \kappa \neq 0 \), that is, a finite velocity dispersion at \( r_o \), there is a single power-law solution to equation (7) with slope \( \beta = 6 - 2\alpha = 2.25 \) for \( \alpha = 1.875 \).\(^3\) It is convenient to set \( y'_c = 2a - 6 = -2.25 \) and to generate the family of solutions simply by varying \( \kappa \). Figure 2 shows a few of these solutions (for \( \alpha = 1.875 \)), labeled by the particular value of \( \kappa \) beside each curve. With the exception of the power law, the shape of the equilibrium density profiles is complex, with an outer cutoff, one or more inflection points, and an inner cusp. For \( \kappa \) larger than some critical value \( \kappa_{\text{crit}}(\alpha) \), the solutions become nonmonotonic and the density vanishes at a finite radius. All solutions with \( 1.875 < \kappa < \kappa_{\text{crit}} \) seem to have a steep inner cusp with asymptotic slope close to \( \beta = 6 - 2\alpha \), but the critical solution approaches \( \beta = 2\alpha/5 = 0.75 \) asymptotically as \( r \) tends to 0.

4. COMPARISON WITH NUMERICAL DENSITY PROFILES

As discussed in the previous section, the isotropic Jeans equation admits a family of solutions for the density profile under the constraint \( \rho/\sigma^2 \propto r^{-\beta} \). The family includes a power law, \( \rho \propto r^{-\beta} \), with \( \beta = 6 - 2\alpha = 2.25 \) (for \( \alpha = 15/8 = 1.875 \)), which corresponds to Bertschinger's spherical infall solution. This is shown with circles in Figure 3, where we also show the density profiles corresponding to the CDM halos in the N-body simulations. Radii have been scaled here, as in Figure 2, to the radius where the slope of the density profile is \(-2.25 \) and densities to the critical density. Clearly, the power-law solution is a poor fit to the results of the N-body simulations, which become noticeably shallower than \( r^{-2.25} \) near the center.

The N-body results can be reasonably well fitted by the profile proposed by NFW (Navarro, Frenk, & White 1996, 1997), which is shown by the triangles in Figure 3. Interestingly, over approximately three decades in radius, \( \text{the shape of the NFW profile is very similar to the critical solution} \) alluded to above. Discrepancies occur only at large radii, where substructure leads to systematic deviations from the power-law behavior for the phase-space density. The meaning of the critical solution may be illustrated by considering the phase-space density distribution corresponding to the different solutions. This is shown in Figure 4 for the solutions illustrated in Figure 2, for systems normalized to have the same total mass, energy, and mean

\(^2\) This assumption breaks down if we choose \( y'_c \) to be positive or to be negative but very small. In that case, \( \rho(r) \) increases with \( r \) and the solutions are unstable and lack direct physical interpretation.

\(^3\) Curiously, besides the singular isothermal sphere, this is the only case where the power-law solution has \( \kappa = \alpha \).
that each has the same total mass, energy, and mean density. The straight line corresponds to the power-law solution, $\kappa = 1.875$. As $\kappa$ increases, the distribution becomes more sharply peaked. Dashed lines correspond to the hollow (nonphysical) density profiles. The critical solution, $\kappa = \kappa_{\text{crit}} = 2.678$, corresponds to the most sharply peaked phase-space density distribution or, equivalently, to the case where phase-space density is most uniformly distributed across the system. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 4.—Phase-space distribution function of the five solutions shown in Fig. 2. Distributions were normalized by choosing a cutoff radius such that each has the same total mass, energy, and mean density. The straight line corresponds to the power-law solution, $\kappa = 1.875$. As $\kappa$ increases, the distribution becomes more sharply peaked. Dashed lines correspond to the hollow (nonphysical) density profiles. The critical solution, $\kappa = \kappa_{\text{crit}} = 2.678$, corresponds to the most sharply peaked phase-space density distribution or, equivalently, to the case where phase-space density is most uniformly distributed across the system. [See the electronic edition of the Journal for a color version of this figure.]

density inside some fiducial radius. Clearly, the phase-space distribution function is broadest for the power-law solution ($\kappa = \alpha = 1.875$) and gets progressively narrower as $\kappa$ increases; the critical solution ($\kappa = \kappa_{\text{crit}}$) corresponds, then, to the most sharply peaked phase-space distribution compatible with a monotonically decreasing (nonhollow) density profile and with the entropy stratification constraint. The critical solution may thus be interpreted as a maximally “mixed” configuration, where the phase-space density is as uniform as possible across the system.

This leads to the following interpretation for the origin of the NFW profile. Gravitational assembly of CDM halos leads to a simple power-law radial stratification of the phase-space density. If spherical symmetry is imposed, as in the case treated by Bertschinger (1985), the collapsing radial mass shells generate progressively larger entropies (lower phase-space densities) as they pass through the shock and settle into hydrostatic equilibrium, leading to steeply cusped power-law profiles with slope $\beta = 6 - 2\alpha$. On the other hand, when the assumption of spherical symmetry is released and the collapse proceeds through many stages of hierarchical merging, mass shells are continuously mixed and the profiles tend to the critical solution: that corresponding to the most uniform entropy distribution compatible with a nonhollow density profile and with the power-law entropy stratification constraint. This density profile resembles the NFW profile over a large dynamic range in radius.

Finally, it is important to note that, despite the similarity shown in Figure 3, there are important differences between the NFW profile and the critical solution, the most notable being that the latter tends to an asymptotic central slope of $-\beta = -2\alpha/5 = -0.75$ rather than to $-1$, as in NFW’s fitting formula. A simple approximation to the radial dependence of the slope of the critical density profile is given by

$$
\frac{d \ln \rho}{d \ln x} = \frac{0.75 + 2.625x^{1/2}}{1 + 0.5x^{1/2}},
$$

which is accurate to 3% for $x = r/r_0 < 4$. Here $r_0$ is the radius where the logarithmic slope of the density profile equals $-2.25$ and $r_0 = (5/3)r_s$, in terms of the NFW scale radius $r_s$.

5. DISCUSSION

The power-law phase-space density profile offers a natural way to describe the structure of dark matter halos at radii where simulations become increasingly difficult and expensive, but also where observational constraints are strongest. Provided that the velocity dispersion tensor remains nearly isotropic, the critical solution provides a clear prediction as to the behavior of the logarithmic slope of the density profile: it should become progressively shallower toward the center, converging asymptotically to a value of $-2\alpha/5 = -0.75$. This is interesting because, as mentioned in § 1, slopes shallower than $-1.5$ seem to be consistent with the recent reanalysis of the rotation curve data set by van den Bosch et al. (2000) and van den Bosch & Swaters (2001). It is important to stress, however, that a shallow central slope does not guarantee consistency with observations, which constrain the detailed radial dependence of the density profile slope as well. Our results do, however, offer a clear prediction for extrapolating the mass profiles to regions that are very difficult to probe numerically. Is the extrapolation of the power-law behavior to very small radii warranted? This question ultimately will have to be answered by direct numerical simulation, although there is no obvious a priori reason why a power-law scaling that is valid over more than two decades in radius should break down nearer the center.

One important point to note is that the critical solution is clearly at odds with the proposal of Moore et al. (1998) and Ghigna et al. (2000) that the innermost slopes of CDM halos converge to a value not shallower than about $-1.5$. However, it should be emphasized that their conclusion was based on the simulation of a single halo in a standard CDM universe, on a significantly different mass regime (galaxy clusters) than probed here. Thus, the possibility remains that this particular system may not be representative of the general population or that the density profiles of clusters are steeper than those of galaxy-sized halos. Because these authors use the same N-body code as in the present work—and, indeed, we use their own results for galaxy-sized halos in this paper—it is unlikely that the discrepancy is due to subtle errors associated with the numerical setup of the simulations. Confirming which of these possibilities holds will require a statistically significant sample of halos simulated with resolution comparable to that of the systems used here. Finally, it is also possible that the cluster simulated by Moore et al. (1998) and Ghigna et al. (2000) differs from the galaxy-sized halos we present here in other, more subtle ways. For example, it may be significantly more triaxial than the systems analyzed here, or perhaps its velocity dispersion tensor is very anisotropic, in conflict with the...
assumptions of this work. Again, a detailed reanalysis of the discrepant system, extended to a statistically meaningful sample, seems necessary in order to explain this discrepancy conclusively. We are currently working on this issue and plan to report our findings soon (Stadel et al. 2001, in preparation).

6. SUMMARY

We examine the spherically averaged phase-space density profiles of N-body simulations of galaxy-sized CDM halos in the SCDM and LCDM cosmologies and find that they are very well approximated by a power law \((\rho / \sigma^2 \propto r^{-1.875})\) over more than two decades in radius. The slope of this power law is consistent with that of the self-similar solution for spherical secondary infall derived by Bertschinger (1985). Assuming that the phase-space density profile is a power law and assuming isotropy, hydrostatic equilibrium is satisfied by a family of density profiles controlled by a single parameter. This parameter is determined by the ratio of the velocity dispersion to the “natural” velocity dispersion of the system at some fiducial radius \(r_0\), \(\kappa = 4\pi G \rho(r_0) r_0^2 / \sigma^2(r_0)\). The parameter \(\kappa\) can also be expressed in terms of the ratio of the circular velocity to the one-dimensional velocity dispersion at \(r_0\), the radius where the circular velocity peaks \([\kappa = V_c^2(r_0)/\sigma^2(r_0)]\). For \(\kappa = 1.875\), the density profile is a power law, which agrees with Bertschinger’s solution. As \(\kappa\) increases, the density profiles become increasingly curved, although they still approach the steep power-law divergent behavior near the center. For \(\kappa\) greater than some critical value \(\kappa_{\text{crit}} \approx 2.678\), the density profiles become unphysical, vanishing at some finite radius near the center.

The critical density profile \((\kappa = \kappa_{\text{crit}})\) corresponds to the maximum value of \(\kappa\) consistent with a nonvanishing density profile at the center and follows closely the profile proposed by Navarro, Frenk, & White (1996, 1997) over more than two decades in radius. This solution corresponds to the most uniform (or most sharply peaked) phase-space distribution compatible with a monotonically decreasing density profile and with the entropy stratification constraint. This suggests that the structure of CDM halos is determined by a radial phase-space density stratification process similar to that established through collapse onto a point-mass perturbation in an unperturbed expanding universe and by the mixing of phase-space density that occurs, presumably, as a result of the many merger and satellite accretion events that characterize the assembly of a CDM halo. This identification leaves a couple of important questions unanswered, however: (1) why should the phase-space density be a power law of radius? and (2) why is the exponent the same as in Bertschinger’s self-similar solution? Although we have no clear answer to these questions at this point, our results suggest that explaining the origin of the structural similarity of CDM halos pointed out by Navarro, Frenk, & White may entail unraveling why the radial stratification of phase-space density in CDM halos is the same power law generated by the simple spherical collapse model. Discovering a mechanism that achieves this may provide a simple explanation for the universal structure of cold dark matter halos.

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