Research on Reservoir Water Quality Prediction Based on Grey Model

Yi Zhou\textsuperscript{1*}, Jiarui Hu\textsuperscript{2} and Mengfan Xie\textsuperscript{3}

\textsuperscript{1} School of Management, Tianjin University of Technology, Tianjin,300384,China
\textsuperscript{2} School of Management, Tianjin University of Technology, Tianjin,300384,China
\textsuperscript{3} School of Management, Tianjin University of Technology, Tianjin,300384,China

*Corresponding author’s e-mail: zhouyi111@stud.tjut.edu.cn

Abstract. Combining the gray model to predict the water quality of Qingshan Reservoir, the analysis results show that the grey model has good applicability for the prediction of reservoir water quality. The prediction errors of total nitrogen, total phosphorus, COD\textsubscript{5}, and ammonia nitrogen are all within 30%, meeting the accuracy requirements of water quality prediction. The research results have important method reference value for reservoir water quality prediction.

1. Introduction

At present, with the continuous increase of water environmental protection, water environmental protection planning has become a hot scientific issue of current research and attention\cite{1}. Water environmental protection planning needs to predict and analyze future water quality changes, and combine the results of predictive analysis to make a reasonable and effective water environmental protection plan\cite{2}. Reservoirs, especially reservoirs as water sources, their water environment status is a hot spot and focus of social and public attention\cite{3}, and there is an urgent need to predict their water environment status, combined with the prediction results, and formulate corresponding water environmental protection measures\cite{4}.

In recent years, with the deepening of water environment research, research on reservoir water quality prediction has been gradually carried out\cite{5}. Among them, the gray model through the establishment of variable regression equations has a better effect in reservoir water quality prediction\cite{6}, but the application effect of the model in different water areas is different. This paper takes Qingshan Reservoir as a research example, combines the gray model to predict the water quality of the reservoir, and combines the water quality sampling and analysis data to compare the applicability of the analysis model.

2. Research method

The main principle of the gray model is to calculate the value of the predictive variable by constructing the regression equation of the known variables and solving the equation. The known variables are mainly the analysis data series of each water quality index, and the regression equation is established to predict the corresponding index value. The calculation formula of the regression equation is:
\[
\frac{dx^{(i)}}{dt} + ax^{(i)} = u
\]

In the formula: \(a\) is expressed as the solution coefficient of the model; \(u\) is expressed as a predictor variable, in article is a predictive index of reservoir water quality; \(x^{(i)}\) is the predicted concentration of reservoir water quality index (mg/L); \(t\) is expressed as the model time step (h).

The calculation equation of the model solving coefficient is:
\[
\hat{a} = [a, u]^T
\]

In the formula: \(\hat{a}\) is the transposition coefficient of the solution of the model; \(T\) is the equation transposition symbol; the gray model combines the least square method to solve the equation (2), and the calculation equation is:
\[
\hat{a} = (B^TB)^{-1}B^TY_N
\]

In the equation, \(B\) and \(Y_N\) are the values of different solution variables of the model, and the calculation equations of each solution variable are:
\[
Y_N = [x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(N)]^T
\]

In the equation, \(x^{(0)}(2), x^{(0)}(3), x^{(0)}(N)\) are expressed as the water quality sample data series for sampling and analysis. The model water quality prediction equation can be converted into the following calculation formula:
\[
x^{(i)}(t+1) = \left(x^{(0)}(1) - \frac{u}{a}\right) e^{-at} + \frac{u}{a}
\]

3. Research results

3.1. Research area overview

This paper takes Qingshan Reservoir as the research object. The water quality indicators of the reservoir are total nitrogen, total phosphorus, \(\text{COD}_5\), and ammonia nitrogen. Based on the sample data of each water quality index of the reservoir for 30 years since 1990, the gray model is used to establish the regression equation of each water quality index, and the regression equation is combined to predict the water quality of the reservoir.

3.2. Model parameter settings

Based on the total nitrogen, total phosphorus, \(\text{COD}_5\), and ammonia nitrogen sample data series, the residual analysis method is used to set the model parameters and accuracy indicators. The setting results of each water quality indicator are shown in Table 1.

| Predictive index | Model parameters | Accuracy index |
|------------------|------------------|----------------|
| \(\text{COD}_5\)  | initial value: \(a=0.08352\), \(b=18.4835\), \(C=0.1352\), \(P=0.2734\) | \(\text{initial value:} a=0.08352, b=18.4835, C=0.1352, P=0.2734\) |
| \(\text{adjusted value:} a_1=0.09452, b_1=17.6353, C_1=0.4435, P_1=0.6742\) | \(\text{adjusted value:} a_1=0.09452, b_1=17.6353, C_1=0.4435, P_1=0.6742\) |
| \(\text{ammonia nitrogen} \) | initial value: \(a=0.04823\), \(b=0.5682\), \(C=0.2352\), \(P=0.2652\) | \(\text{initial value:} a=0.04823, b=0.5682, C=0.2352, P=0.2652\) |
| \(\text{adjusted value:} a_1=0.4653, b_1=0.0838, C_1=0.4652, P_1=0.5892\) | \(\text{adjusted value:} a_1=0.4653, b_1=0.0838, C_1=0.4652, P_1=0.5892\) |
| \(\text{total phosphorus} \) | initial value: \(a=-0.02356\), \(b=0.03263\), \(C=0.4238\), \(P=0.4782\) | \(\text{initial value:} a=-0.02356, b=0.03263, C=0.4238, P=0.4782\) |
| \(\text{adjusted value:} a_1=-0.1138, b_1=0.0215, C_1=0.2682, P_1=0.6975\) | \(\text{adjusted value:} a_1=-0.1138, b_1=0.0215, C_1=0.2682, P_1=0.6975\) |
From the first accuracy index of the COD₅ model, it can be seen that when the parameters are adjusted, the accuracy index coefficient is 0.1352, and the accuracy is low. After the model parameters are adjusted, the accuracy index can reach 0.4435. The adjusted model parameters can be used for COD₅ forecast.

3.3. Applicability analysis of COD₅ water quality index prediction

After the model parameters are set, the gray model is used to analyze the applicability of the COD₅ index in the model storage area. The applicability analysis results are shown in Table 2.

| Mouth | Monitoring value/(mg·L⁻¹) | Predictive value/(mg·L⁻¹) | Absolute error/(mg·L⁻¹) | Relative error/ (%) |
|-------|---------------------------|---------------------------|-------------------------|---------------------|
| 1     | 0.46                      | 0.40                      | 0.06                    | 12.44               |
| 2     | 0.46                      | 0.55                      | -0.09                   | -19.37              |
| 3     | 1.15                      | 1.01                      | 0.14                    | 11.66               |
| 4     | 0.61                      | 0.48                      | 0.14                    | 21.15               |
| 5     | 0.69                      | 0.55                      | 0.14                    | 19.43               |
| 6     | 0.46                      | 0.55                      | -0.09                   | -19.06              |
| 7     | 0.38                      | 0.19                      | 0.20                    | 50.09               |
| 8     | 0.92                      | 0.73                      | 0.20                    | 19.69               |
| 9     | 0.61                      | 0.48                      | 0.13                    | 20.09               |
| 10    | 1.22                      | 0.88                      | 0.35                    | 26.59               |
| 11    | 0.92                      | 0.75                      | 0.17                    | 17.33               |
| 12    | 0.77                      | 0.57                      | 0.20                    | 24.57               |

From the applicability analysis results, it can be seen that the absolute error between the COD₅ index calculated by the gray model and the sampled measured value is between 0.06 and 0.35, and the relative error is generally within 30%, which can basically meet the monthly water quality forecast Precision requirements.

3.4. Applicability analysis of Ammonia nitrogen water quality index prediction

The gray model is used to analyze the applicability of the ammonia nitrogen indicators in the storage area of the model, and the applicability analysis results are shown in Table 3.

| Mouth | Monitoring value/(mg·L⁻¹) | Predictive value/(mg·L⁻¹) | Absolute error/(mg·L⁻¹) | Relative error/ (%) |
|-------|---------------------------|---------------------------|-------------------------|---------------------|
| 1     | 0.24                      | 0.30                      | -0.06                   | -19.26              |
| 2     | 0.16                      | 0.18                      | -0.02                   | -8.00               |
| 3     | 0.27                      | 0.23                      | 0.04                    | 11.09               |
| 4     | 0.27                      | 0.24                      | 0.04                    | 8.67                |
| 5     | 0.26                      | 0.29                      | -0.03                   | -8.14               |
| 6     | 0.25                      | 0.20                      | 0.05                    | 13.69               |
| 7     | 0.17                      | 0.13                      | 0.04                    | 15.73               |
| 8     | 0.14                      | 0.17                      | -0.04                   | -17.28              |
| 9     | 0.30                      | 0.26                      | 0.04                    | 9.04                |
| 10    | 0.22                      | 0.27                      | -0.05                   | -15.87              |
| 11    | 0.28                      | 0.36                      | -0.08                   | -19.66              |
| 12    | 0.22                      | 0.16                      | 0.05                    | 17.66               |
From the analysis results, it can be seen that the gray model also has good accuracy in studying the ammonia nitrogen index prediction of the reservoir entrance. The error between the predicted value and the sampled analysis value is less than 20%. The absolute value between the predicted value and the sampled analysis value also can be controlled within 0.10 mg/L. It can be seen that the gray model also has good applicability in the prediction of ammonia nitrogen indicators in the research reservoir area. Ammonia nitrogen is an important indicator of reservoir water quality monitoring. Due to the continuity and consistency of sample data, the gray model has certain applicability in the prediction of ammonia nitrogen in the reservoir area. In addition, its prediction accuracy is generally better than the CODs index.

3.5. Applicability analysis of total nitrogen water quality index prediction
The gray model is used to analyze the applicability of the ammonia nitrogen indicators in the storage area of the model, and the applicability analysis results are shown in Table 4.

| Mouth | Monitoring value/(mg·L⁻¹) | Predictive value/(mg·L⁻¹) | Absolute error/(mg·L⁻¹) | Relative error/(%) |
|-------|---------------------------|---------------------------|-------------------------|-------------------|
| 1     | 4.40                      | 3.99                      | 0.41                    | 6.65              |
| 2     | 4.63                      | 5.19                      | -0.55                   | -8.72             |
| 3     | 3.80                      | 3.01                      | 0.79                    | 15.11             |
| 4     | 3.59                      | 2.57                      | 1.02                    | 20.74             |
| 5     | 3.13                      | 2.49                      | 0.64                    | 14.92             |
| 6     | 2.38                      | 1.77                      | 0.62                    | 18.73             |
| 7     | 4.30                      | 4.92                      | -0.62                   | -10.63            |
| 8     | 2.59                      | 1.92                      | 0.68                    | 19.04             |
| 9     | 2.57                      | 1.85                      | 0.72                    | 20.51             |
| 10    | 2.67                      | 1.99                      | 0.69                    | 18.67             |
| 11    | 2.82                      | 3.30                      | -0.48                   | -12.43            |
| 12    | 3.54                      | 4.34                      | -0.80                   | -16.50            |

It can be seen from the analysis results that the gray model also has good applicability in the prediction of total nitrogen in the reservoir area. The relative error between the predicted total nitrogen index and the actual measured total nitrogen index is between 6.65% and 20.51%. The absolute error is between 0.41~1.02 mg/L. This is mainly because the indicators of ammonia nitrogen and total nitrogen have a certain degree of correlation in the reservoir area, so the gray model can be used to predict the trend of total nitrogen at the reservoir entrance.

3.6. Analysis of applicability of total phosphorus water quality index prediction
The gray model is used to analyze the applicability of the ammonia nitrogen indicators in the model storage area. The applicability analysis results are shown in Table 5.

| Mouth | Monitoring value/(mg·L⁻¹) | Predictive value/(mg·L⁻¹) | Absolute error/(mg·L⁻¹) | Relative error/(%) |
|-------|---------------------------|---------------------------|-------------------------|-------------------|
| 1     | 0.03                      | 0.04                      | -0.01                   | -19.31            |
| 2     | 0.04                      | 0.04                      | -0.01                   | -15.44            |
| 3     | 0.04                      | 0.04                      | -0.01                   | -15.44            |
| 4     | 0.04                      | 0.04                      | -0.01                   | -15.44            |
| 5     | 0.04                      | 0.03                      | 0.01                    | 12.64             |
| 6     | 0.04                      | 0.03                      | 0.01                    | 12.64             |
| 7     | 0.04                      | 0.03                      | 0.01                    | 12.64             |
| 8     | 0.04                      | 0.03                      | 0.01                    | 12.64             |
| 9     | 0.04                      | 0.03                      | 0.01                    | 12.64             |
From the applicability analysis results of total phosphorus prediction, it can be seen that the gray model has good applicability in total phosphorus prediction in the reservoir area. The relative error between the predicted total nitrogen index and the actual measured total nitrogen index is 12.64%-19.31%, the absolute prediction error varies between 0.01 mg/L. This is mainly because the source of total phosphorus is mainly the consolidation of phosphorus in the air and agricultural non-point source pollution. In recent years, Qingshan Reservoir has increased its efforts to control agricultural non-point source pollution in the reservoir area, making the concentration of total phosphorus lower in the research reservoir.

4. Conclusions

1) Combined with the prediction accuracy index, the gray model can effectively adjust the model parameters predicted by the water quality index of the reservoir. After parameter adjustment, the gray model prediction accuracy is improved;

2) The gray model has good applicability in reservoir prediction in Qingshan reservoir. It is suggested that the model should be modified to predict the water quality of the reservoir in the water environment management and protection planning, so as to make a reasonable protection plan based on the trend prediction results.

References

[1] Niu F X, Ren D. (2019) Analysis of Fuzzy Mathematics and Single-factor Water Environment Evaluation Method. J. Groundwater, 41(05): 46-47.

[2] Georgia M, Kostas B, Panayiotis P. (2013) Operationalizing sustainability in urban coastal systems: A system dynamics analysis. J. Water Research, 47(20): 7235-7250.

[3] Selina M S. (2018) Rethinking marine resource governance for the united nations sustainable development goals. J. Current Opinion in Environmental Sustainability, 34: 54-61.

[4] Georgia M, Kostas B, Panayiotis P. (2013) Operationalizing sustainability in urban coastal systems: A system dynamics analysis. J. Water Research, 47(20): 7235-7250.

[5] Bu Z Y. (2016) Current water environment quality assessment methods and impact prediction—taking Dadu River as an example. J. Green Technology, 12: 109-110.

[6] Lu D. (2016) Research on Application of Improved Grey—Markov Model in Groundwater Quality Prediction. J. water conservation planning and design, 06: 86-89+99.