Warping and Supersymmetry Breaking

Michael R. Douglas\textsuperscript{1,\&}, Jessie Shelton\textsuperscript{1} and Gonzalo Torroba\textsuperscript{1}

\textsuperscript{1}NHETC and Department of Physics and Astronomy
Rutgers University
Piscataway, NJ 08855–0849, USA

\&I.H.E.S., Le Bois-Marie, Bures-sur-Yvette, 91440 France
mrd, jshelton, torrobag@physics.rutgers.edu

Abstract

We analyze supersymmetry breaking by anti-self-dual flux in the deformed conifold. This theory has been argued to be a dual realization of susy breaking by antibranes. As such, one might expect it to lead to a hierarchically small breaking scale, but only if the warp factor is taken into account. We verify this by explicitly computing the warp-modified moduli space metric. This leads to a new term, with a power-like divergence at the conifold point, which lowers the breaking scale. We finally point out various puzzles regarding the gauge theory interpretation of these results.
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## 1 Introduction

Over the last few years there has been much progress in understanding flux compactifications of string theory \[1\]. Supersymmetric configurations have been well studied \[2, 3, 4\], and as a general rule one can understand their physics using either supergravity, or a dual gauge theory picture in which fluxes are replaced by branes. As a basic example, consider pure super Yang-Mills theory. Its vacuum structure can be usefully described by the Veneziano-Yankielowicz effective superpotential, which leads to an exponentially small gaugino condensate. The same physics can be described by a dual theory with fluxes near a conifold singularity, and an analysis based on the flux superpotential \[2\].

The situation for supersymmetry breaking configurations is much less clear. While there are many known examples of gauge theories in which the dynamical scale sets the scale of supersymmetry breaking, these always
seem to involve special matter content or tuning in the superpotential, and until fairly recently this evidence seemed to suggest that in a “generic” set of theories (such as would come out of an ensemble of string vacua), very few would break supersymmetry.

More recently, this belief has been significantly revised in light of new work incorporating ingredients such as metastability, “retrofitting,” obstructions in the dual geometry, and many others [5, 9, 6, 7, 11, 12], suggesting that a far broader and more generic class of gauge theories will break supersymmetry.

It should be said however that, so far, the constructions which produce low scales still require either tuning of parameters or discrete symmetries, so the question of what distribution of supersymmetry breaking scales we expect to come out of string theory remains open. To illustrate this, consider the problem of finding string theory realizations of the “retrofitting” construction of [9], in which a small parameter in the superpotential (for example the small quark mass required in [5]) is obtained as the dynamical scale of a second gauge group. While one can easily find superpotential couplings such as \((\text{Tr} W^2\alpha)(\tilde{Q})^2\) which do this, the harder problem is to find a mechanism which suppresses an order one bare quark mass term \((\tilde{Q})^2\). The only obvious candidate is a discrete R symmetry [14], which appears to be even less natural in flux vacua than the tuning we are hoping to explain [15].

One might imagine that a truly generic construction would work both at weak coupling (gauge theory) and strong coupling (the flux dual). On the flux side, nonsupersymmetric flux compactifications also exist and appear to be generic [13, 3]. However, both statistical analyses [8] and more intuitive arguments [16, 17] suggest that these favor a high scale of supersymmetry breaking. Although flux superpotentials contain the ingredients needed to obtain low scales, in the work so far these do not lead to a low supersymmetry breaking scale without tuning.

In exploring these issues, we tried to find the simplest possible flux compactifications with susy breaking. One simple and popular idea is to break supersymmetry by combining branes and antibranes, which preserve incompatible \(\mathcal{N} = 1\) subalgebras of an underlying \(\mathcal{N} = 2\) supersymmetry algebra of type II Calabi-Yau compactification. This includes the anti D3-brane in a conifold throat worked out in [19], a controlled construction using wrapped D5-branes to get D-term breaking in [20], a suspended brane construction in [11], and many others.

In [7], a system of D5 and anti-D5 branes wrapped on a pair of resolved conifolds was studied, and argued to have a simple flux dual involving imaginary anti-self-dual flux. On the other hand, as we will review, an \(\mathcal{N} = 1\) effective field theory analysis of this theory along the lines of [7] leads to high
scale breaking. While this may at first appear paradoxical, actually it is to be expected, as the scale of suzy breaking in this model is the anti-D-brane tension, of order the string scale.

Of course, as has been much discussed in the string compactification literature, starting with [18], supersymmetry breaking by antibranes can lead to low scale breaking, but only if the antibranes are localized in a region of large warp factor. Now in the limit considered by [7], in which \( \alpha' \to 0 \) before considering other effects, the warp factor is not present, since it is sourced by fluxes which are quantized in units of \( \alpha' \). Thus their results appear self-consistent, but this suggests that we need to incorporate the warp factor to see low scale susy breaking.

In this work, we will do just this. By a detailed analysis of warping effects on the metric on complex structure moduli space, we show the existence of a term with a power-like divergence close to the conifold point. This contribution will be the one responsible for lowering the scale of the breaking.

Having explained our basic results, the order of the discussion will be reversed, for clarity. We begin in section 2 by reviewing the anti-D5 brane theory and its flux dual. In section 3 we develop the general theory of warped Calabi-Yau compactification, along lines initiated by Giddings and collaborators [22, 23]. In particular, we analyze general properties of the warped metric \( G_{\alpha \bar{\beta}} \) for complex deformations. We show that throats contribute new zero modes to the warp factor, that are responsible for enhancing \( G_{\alpha \bar{\beta}} \) near conifold points in moduli space. Since the general analysis leaves numerical factors undetermined, in section 1 we compute explicitly \( G_{\alpha \bar{\beta}} \) for the deformed conifold. This analysis reveals a new \( |S|^{-4/3} \) contribution.

In subsection 4.3 we use this to understand the supergravity behavior near the tip of the warped deformed conifold. We also show how a negative \( F_3 \) flux through the 3-cycle of the conifold, and a far away \( O7 \) plane preserving a misaligned supersymmetry, can give parametrically small supersymmetry breaking. This still requires potentially non-generic ingredients, depending on choices made in the bulk, so the question of whether this mechanism generically leads to low scale breaking remains open.

Finally, section 5 comments on the dual gauge theory description, in which anti \( D5s \) wrap the resolved cycle. While the new term \( |S|^{-4/3} \) actually matches with old expectations for the Kähler potential for the gaugino condensate, we are left with numerous unanswered questions here.
2 Supersymmetry breaking without warping

We begin by reviewing the suggestion of [7] that the dual of an anti-D5-brane wrapped on the resolved conifold is an anti-self-dual flux in the deformed conifold geometry, leading to supersymmetry breaking. However, using the unwarped moduli space metric, we find that this is high scale breaking.

2.1 Flux compactification of IIB string

We start with $\mathcal{N} = 2$ compactification of type IIB string theory, in which the complex structure moduli live in vector multiplets. We choose a symplectic basis $(A_\alpha, B^\alpha)$ of $H_3(X, \mathbb{Z})$, and take as coordinates on moduli space the A cycle periods $\{ (S^\alpha) \}$. The B cycle periods can then be integrated to define a prepotential $\mathcal{F}$, so that

$$ S^\alpha = \int_{A_\alpha} \Omega, \quad \frac{\partial \mathcal{F}}{\partial S^\alpha} = \int_{B^\alpha} \Omega. \tag{2.1} $$

The metric on moduli space can then be determined either from Calabi-Yau geometry, or from the prepotential, as

$$ G_{\alpha\beta} = \text{Im} \frac{\partial\partial F}{\partial \bar{\partial} F} = -\frac{\int \chi_\alpha \wedge \chi_{\bar{\beta}}}{\int \Omega \wedge \bar{\Omega}} \tag{2.2} $$

where $\chi_\alpha$, $\alpha = 1, \ldots, h^{2,1}$ are a basis of $H^{2,1}(X, \mathbb{C})$.

We next consider a compactification with three-form fluxes $G_3 := F_3 - \tau H_3$. We define the flux parameters $(N^\alpha_R, N^\alpha_{NS}, \beta^R_\alpha, \beta^{NS}_\alpha)$ by

$$ 4\pi^2 \alpha' \int_{A_\alpha} G_3 = N^\alpha_R - \tau N^\alpha_{NS}, \quad -4\pi^2 \alpha' \int_{B^\alpha} G_3 = \beta^R_\alpha - \tau \beta^{NS}_\alpha. \tag{2.3} $$

These are quantized in units of $4\pi^2 \alpha'$, which we generally set to one in the following. We follow the notation in [24].

As discussed in [2], the vacuum energy of the fluxes depends on the complex structure moduli and dilaton, leading to moduli stabilization. This can be analyzed by minimizing a scalar potential derived from the Gukov-Taylor-Vafa-Witten superpotential

$$ W = \int G_3 \wedge \Omega, \tag{2.4} $$

using the standard $\mathcal{N} = 1$ supergravity formalism.

We should say from the start that this $\mathcal{N} = 1$ effective supergravity description in terms of the Calabi-Yau moduli fields is in general only a very
partial description of the physics. For example, there might be other light modes, from KK modes in string compactification, other degrees of freedom in the gauge theory dual, and the like. This will be even more true once we take warping into account, as discussed in [21]. We do not intend to study this question in detail here, but rather will limit ourselves to finding an effective potential which properly describes the vacuum energy, moduli stabilization and supersymmetry breaking. At least on the strongly coupled (supergravity) side, we believe our arguments (based on explicit $d = 10$ solutions) suffice to demonstrate this.

2.2 Geometric properties

For completeness we include a quick review of the basic properties of the conifold. The algebraic variety describing the deformed conifold is

$$u^2 + v^2 + y^2 - x^2 + S = 0.$$  \hfill (2.5)

Its holomorphic properties are very simple. There are only two nontrivial 3-cycles, $(A, B)$, $A \cap B = 1$; for $S \to 0$, $A \to 0$ and $B$ is noncompact. The $A$ cycle is an $S^2$ fibration ($u, v \in \mathbb{R}$) over the cut $x \in (-\sqrt{S}, +\sqrt{S})$ of the hyperelliptic curve

$$F(x, y) = y^2 - x^2 + S = 0.$$  \hfill (2.6)

The noncompact $B$-cycle extends between $y = \pm \infty$ and runs through the previous cut. We introduce a geometrical cutoff $\Lambda_0$ such that the points at infinity become $\pm \Lambda_0$. From the usual monodromy arguments, the periods are

$$\int_A \Omega = S, \quad \int_B \Omega = \frac{\partial F}{\partial S} = \Pi_0 + \frac{1}{2\pi i} S \log \frac{\Lambda_0^3}{S} + \ldots$$  \hfill (2.7)

where $\ldots$ are analytic subleading contributions and $F$ is the prepotential of the geometry.

2.3 SUGRA on the warped deformed conifold

A simple embedding of the conifold in a compact Calabi-Yau orientifold was constructed in [2]. However the details of the embedding do not matter for our discussion, so we consider the following simplified model. We ‘zoom in’ to a local neighborhood of a compact CY $X$, containing the deformed conifold (2.5). The only information that we keep from the rest of $X$ is that there is an orientifold projection, breaking $\mathcal{N} = 2 \to \mathcal{N} = 1$, and preserving the chiral supermultiplet with scalar component $S$. While a complete discussion requires solving the D3 tadpole condition, which usually requires adding
wandering D3 branes, we keep these away from the conifold, so that they don’t enter these results.

In the presence of the 4-form $C_4$, compactifying on the conifold contributes an $\mathcal{N} = 2$ 4d vector multiplet $\mathcal{A} = (S, \psi, \lambda, A_\mu)$, where $S$ is the complex modulus of the conifold

$$\frac{\partial g_{ij}}{\partial S} = -\frac{1}{\|\Omega\|^2} S(x) \bar{\Omega}^k_i \chi_S k\bar{j}.$$

and $A_\mu$ is the $U(1)$ gauge field from $C_4 = A_\mu(x) dx^\mu \wedge \chi_S$. $\psi$ and $\lambda$ are the fermion superpartners.

The orientifold action is [30]

$$\mathcal{O} = (-)^{F_L} \Omega_p \sigma^*, \ \sigma^* \Omega = -\Omega;$$

$\Omega_p$ is the worldsheet parity, $F_L$ is the left moving 4d fermion number and $\sigma^*$ is the holomorphic involution (acting on forms). This will produce $O3/O7$ planes. Orientifolding splits the $\mathcal{N} = 2$ vector multiplet into an $\mathcal{N} = 1$ chiral multiplet $(S, \psi)$ and a vector multiplet $(\lambda, A_\mu)$. Since we want to keep $S$ as a low energy 4d field, we take the action of the involution to be $\sigma^* \chi_S = -\chi_S$. In this way the vector multiplet is projected out and we are left with only $(S, \psi)$.

As the next step we turn on the following quantized fluxes:

$$\int_A F_3 = N, \ -\int_B F_3 = \beta^R, \ -\int_B H_3 = \beta^{NS}. \ \ \ \text{(2.8)}$$

Because of the monodromy $B \rightarrow B + nA$, $\beta^R$ is defined mod $N$, and will play the role of a discrete $\theta$-angle. In an $\mathcal{N} = 2$ formalism, fluxes may be seen as FI terms for the auxiliary components of the superfield $\mathcal{A}$. Based on this identification, it was noted in [7] that for $N > 0$ (we always take $\beta^{NS} > 0$), the supersymmetry variations are

$$\delta \psi = 0, \ \delta \lambda = i \epsilon \frac{1}{\text{Im} \partial^2 S \mathcal{F}} \left( \frac{i}{g_s} \beta^{NS} + N \bar{g}^2 S \mathcal{F} \right).$$

Therefore positive flux respects the same supersymmetry as the orientifold; we still have an $\mathcal{N} = 1$ theory because $\lambda$ is projected out from the spectrum, so $\delta \lambda \neq 0$ is not seen.

Using (2.7) and (2.8), the GVW superpotential (2.4) for the conifold reads

$$W = \frac{N}{2\pi i} S (\log \frac{\Lambda_0}{S} + 1) - (\beta^R - \frac{i}{g_s} \beta^{NS}) S. \ \ \ \text{(2.9)}$$
For $N > 0$, $\beta^{NS} > 0$, solving $\partial_s W = 0$,

$$S = e^{-2\pi i \beta^R/N} e^{-2\pi \beta^{NS}/g_s N} \Lambda_0^3.$$  

(2.10)

There are $N$ degenerate vacua coming from $\beta^R = 0, \ldots, N-1$; to simplify our results, we will in general set $\beta^R = 0$ and remember this degeneracy. The supergravity background with fluxes corresponds to the warped deformed conifold, appropriately glued into the compact CY, as discussed in section 4.2.

This type of flux dual was used in the discussion of supersymmetry breaking by anti-D3 branes in [19].

2.4 Supersymmetry breaking

We now consider the effect of misaligning the supersymmetry preserved by the $O7$ and the one preserved in the conifold, by turning on negative flux $N < 0$.

In the case $N < 0$, Aganagic et al [7] showed that $\delta \lambda = 0$ but

$$\delta \psi = i \epsilon \left( \frac{1}{\text{Im} \partial_s^2 \mathcal{F}} \left( \frac{i}{g_s} \beta^{NS} + N \partial_s^2 \mathcal{F} \right) \right).$$  

(2.11)

and hence this flux configuration breaks $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ spontaneously.

For $N < 0$, $\beta^{NS} > 0$, (2.10) would give a result $S \gg \Lambda_0^3$! It was argued in [7] that the physical vacuum is instead the minimum of the scalar potential

$$V = e^K \left( G^{ij} \partial_i W \partial_j W^* \right),$$

obtained from Eq. (2.9) and Eq. (2.2). This is located at

$$S_{N<0} = e^{-2\pi i \beta^R/N} e^{-2\pi \beta^{NS}/g_s N} \Lambda_0^3.$$  

(2.12)

On this vacuum,

$$\partial_s W_{N<0} = 2i \frac{\beta^{NS}}{g_s} \neq 0$$  

(2.13)

and hence supersymmetry is broken by an explicit non-zero F-term $\partial_s W$.

2.5 Scale of supersymmetry breaking

In principle there are various ways one could define this scale, and in [7] definitions involving the mass splittings among supermultiplets were studied.
However, in a spontaneously broken $\mathcal{N} = 1$ supergravity theory, the standard definition is the norm of the $F$ terms, or equivalently the scale determined by the $F$ term contribution to the scalar potential

$$M_{\text{susy}}^4 = V = e^K \left( G^{ii} D_i W D_i W^* \right).$$

In a realistic compactification with near-zero cosmological constant, this scale will also determine the gravitino mass, as $m_{3/2} = M_{\text{susy}}^2 / \sqrt{3} M_{\text{Planck}}$. How exactly it enters into observable susy breaking depends on the mediation mechanism, but very generally one expects soft terms of order $M_{\text{susy}}^2 / M_{\text{Planck}}$ from gravitational couplings and gravitino loop effects. Thus one generally requires $M_{\text{susy}} < 10^{11}\text{GeV}$ (the intermediate scale) for a model which naturally solves the hierarchy problem, and this is the operational definition of a low scale of susy breaking.

Besides the superpotential Eq. (2.4) and the Kähler potential $K$, one also needs the dependence on the Kähler moduli of $X$ to get the full scalar potential. Before taking into account stringy and quantum corrections, this leads to “no scale” structure. This suffices to define $M_{\text{susy}}$ as above, and we will discuss the nonperturbative effects later.

The factor $e^K$ also includes normalization factors determined by doing the dimensional reduction, leading to

$$V = \frac{1}{2\kappa_4^2} \frac{1}{V_W \text{Im} \tau (\text{Im} \rho)^3} \frac{1}{|| \Omega ||^2 V_W} G^{S\bar{S}} |\partial_S W|^2. \quad (2.14)$$

Using Eq. (2.7) and Eq. (2.2), we find

$$G_{S\bar{S}} \sim c \log \frac{\Lambda_0^3}{|S|},$$

and thus

$$V = \frac{1}{2\kappa_4^2} \frac{g_s}{(\text{Im} \rho)^3} \left[ c \log \frac{\Lambda_0^3}{|S|} \right]^{-1} \left| \frac{N}{2\pi i} \log \frac{\Lambda_0^3}{S} + \frac{\beta^{NS}}{g_s} \right|^2 \sim N \frac{\beta^{NS}}{g_s}. \quad (2.15)$$

Since we are working in conventions in which $\alpha'$ is order one, the upshot is that $\mathcal{N} = 1$ supersymmetry is broken at a high scale. This can be confirmed by a $d = 10$ computation of the mixing between the gravitino and the goldstino, here the fermionic component of $S$. The essential content of this computation is already present in the supersymmetry variation Eq. (2.11).

Since the energy Eq. (2.15) is the expected tension of $N$ anti D5-branes, in retrospect this result should not be very surprising. However it raises the question of whether and how it would be changed by including the warp factor.
3 Warped Compactifications

We start by reviewing the basic features of warped compactifications, and then we describe the warp effects on the geometry of the complex moduli space. We will concentrate on the complex moduli stabilization.

3.1 Warping and fluxes

We mainly follow DeWolfe and Giddings [22]. Starting from an underlying CY $X$ with metric $g_{mn}(y)$, turning on fluxes produces a warped metric

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}(y)dy^m dy^n$$

(3.1)

$m, n = 1, \ldots, 6$. To avoid confusion, we want to stress that we are not using the usual GKP notation $\tilde{g}_{mn}$ for the CY metric, because we want to avoid tildes appearing in all our formulas. Throughout the work we will rise and lower indices only with respect to $g_{mn}$, so that the dependence on the warp factor is always explicit.

A complete and consistent dimensional reduction is very involved, containing subtle issues from KK modes, compensators and warping contributions [23]. However, the results that we will discuss in this work refer to the vacuum structure of the theory, arising purely from the effective potential. Fortunately, this is free of many of these issues.

In [22], a Kaluza-Klein reduction is done from $d = 10$ IIB supergravity to a $d = 4$, $\mathcal{N} = 1$ effective supergravity, taking into account the warp factor. To get a self-consistent $\alpha' \rightarrow 0$ limit in which the warp factor remains, one simultaneously increases the flux $G_3$, keeping the flux parameters $(N_\alpha^R, N_\alpha^NS, \beta_\alpha^R, \beta_\alpha^NS)$ fixed. The other $\alpha'$ and quantum corrections in the full string theory are dropped.

The result is that, in the presence of warping, the superpotential still takes the form Eq. (2.4), but the metric on complex structure moduli space is deformed to

$$G_{\alpha\beta} = \frac{\int e^{-4A}x_\alpha \wedge \chi_\beta}{\int e^{-4A}\chi \wedge \Omega},$$

(3.2)

where the warp factor $e^{-4A}$ is determined by a supergravity equation of motion. In a supersymmetric background, this is [2]

$$-\nabla^2(e^{-4A}) = \pm \frac{G_{mn}G_{mnp}}{12\text{Im}\tau} \pm 2\kappa_{10}^2 T_3\rho^f$$

(3.3)

where

$$\nabla^2 := \frac{1}{\sqrt{g}}\partial_m \sqrt{g} g^{mn} \partial_n.$$

(3.4)
Here the plus (minus) sign corresponds to ISD (IASD) fluxes and D3 (anti D3) brane charge, with the consequence that the r.h.s. of Eq. (3.3) is always positive, as it should in order to get a positive definite metric. These signs are easy to understand: denoting, as in [2]

$$\tilde{F}_5 = (1 + \ast)[d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3];$$

$\alpha$ will of course depend on the sign of its sources (3-fluxes or 3-branes). But the BPS-like condition of GKP, extended to IASD fluxes also, is $e^{4A} = \pm \alpha$, with the result that the warp factor only depends on the absolute value of the sources.

In this limit, the scalar potential takes the no-scale form, cancelling $|W|^2$ against $|D_\rho W|^2$, where $\rho$ are the Kähler moduli. The scalar potential for the complex moduli and dilaton then takes the standard $\mathcal{N} = 1$ form

$$V = \frac{G^{\alpha\beta} D_\alpha W D_\beta \overline{W}}{\int e^{-4A} \Omega \wedge \overline{\Omega}}. \quad (3.5)$$

3.2 ‘Warped’ geometry of the moduli space

Implicit in the previous results is the claim that the warp-deformed moduli space metric (3.2) is Kähler. In [22] it was suggested that this metric may be derived from the Kähler potential

$$K(S, \bar{S}) = -\log\left( -i \int e^{-4A} \Omega \wedge \overline{\Omega} \right), \quad (3.6)$$

in other words that $\partial_\alpha \partial_\beta K = G_{\alpha\beta}$.

Seeing this requires properly treating the zero mode of the warp factor, which will be important when we match the noncompact conifold solution onto a compact bulk manifold. We start from

$$\frac{\partial \Omega}{\partial S^\alpha} = k_\alpha \Omega + \chi_\alpha, \quad k_\alpha = \frac{\int e^{-4A} \partial_\alpha \Omega \wedge \overline{\Omega}}{\int e^{-4A} \Omega \wedge \overline{\Omega}}. \quad (3.7)$$

One can check that this expression is valid with or without the warp factor $e^{-4A}$ as after integration it cancels between numerator and denominator. Therefore,

$$\partial_\alpha K = -k_\alpha - \frac{\int (\partial_\alpha e^{-4A}) \Omega \wedge \overline{\Omega}}{\int e^{-4A} \Omega \wedge \overline{\Omega}}. \quad (3.8)$$
Note that the usual special geometry relation $\partial_\alpha K = -k_\alpha$ is not yet apparent. After some algebra,

$$\partial_\alpha \partial_\beta K = G_{\alpha\beta} + \left( \frac{\int \Omega \wedge \bar{\Omega} }{\sqrt{\left| \Omega \right|^2} } \right) \left( f \Omega \wedge \bar{\Omega} e^{-4A} \right) \left( f \Omega \wedge \bar{\Omega} e^{-4A(y)} \right) \left( f \Omega \wedge \bar{\Omega} e^{-4A(y)} \right),$$

(3.9)

The consistency of $\mathcal{N} = 1$ supergravity requires that $\partial_\alpha \partial_\beta K = G_{\alpha\beta}$, in other words that the second and third terms on the right hand side vanish. The easiest way for this to happen is if

$$0 = \int \Omega \wedge \bar{\Omega} \partial_\alpha e^{-4A}, \quad (3.10)$$

i.e. the zero mode $c$ of the warp factor, defined by writing

$$e^{-4A(y)} = c + e^{-4A_0(y)}, \quad (3.11)$$

is independent of variations of the complex structure. This is sensible because in a true compactification, the equation of motion Eq. (3.3) does not determine the zero mode of $e^{-4A}$. Rather, the zero mode enters into the overall volume as a Kähler modulus (this is before bringing in nonperturbative effects depending on Kähler moduli).

Another way to define the zero mode is to use the relation

$$\sqrt{\text{det} g} = \frac{1}{3!} (g_{ij} dy^i dy^j)^3 = \frac{\Omega \wedge \bar{\Omega} }{||\Omega||^2} = \frac{\Omega \wedge \bar{\Omega} }{\frac{1}{3} g^{ij} g^{jj} g^{kk} \Omega_{ijk} \Omega_{i j k}}, \quad (3.12)$$

One can rewrite the Ricci flatness condition on $g$ as the condition that the denominator $||\Omega||^2$ in this expression is constant on the Calabi-Yau, and thus Eq. (3.10) is equivalent to the condition that

$$0 = \int \sqrt{\text{det} g} \partial_\alpha e^{-4A}.$$  

Given an initial choice of the zero mode, this condition determines a unique solution of Eq. (3.3)

Note that it will not in general be the case that $\partial_\alpha e^{-4A} = 0$ pointwise, only under the integral. Similarly, even though the unwarped volume $\int \sqrt{\text{det} g}$ can be defined to be independent of complex structure, the functional form of the volume element $\sqrt{\text{det} g}$ can vary. Thus the actual warped volume, defined as

$$V_w = \int \sqrt{\text{det} g} e^{-4A},$$

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could in general depend on the complex structure moduli.

To summarize: in the absence of fluxes, compactifying on \( X \) gives \( \mathcal{N} = 2 \) supersymmetry and correspondingly the complex moduli space \( \mathcal{M} \) is a special Kahler manifold. When we turn on fluxes, \( X \) gets a warp contribution and we break \( \mathcal{N} = 2 \to \mathcal{N} = 1 \). \( \mathcal{M} \) is still a Kahler manifold but no longer special. The prepotential \( \mathcal{F} \) is not enough to specify all the geometrical data of the conformal Calabi-Yau, and the information on the fluxes comes in through the factor \( e^{-4A} \).

### 3.3 Zero modes of the warp factor

Evidently the \( e^{-4A} \) warp factor plays a crucial role. While in ten-dimensional terms, it is determined by Eq. [3.3], as yet there is no simple four-dimensional ansatz for this factor. Thus we will shortly need to look at the details of the ten-dimensional solution. However there is a simple argument which suggests what we are looking for.

As we discussed earlier, the general solution for the warp factor is

\[
e^{-4A(y)} = c + e^{-4A_0(y)},
\]

where \( c \) is a free parameter related to the total warped volume. For example, in the large volume limit we can take \( c \to \infty \), which allows one to identify \[23\]

\[V_{\text{CY}} \sim c^{3/2}.\]

In this limit, the warp factor becomes irrelevant. On the other hand, for any fixed \( c \), the factor \( e^{-4A_0(y)} \) might become so large at some \( y \) that it cannot be neglected. In this sense, the neglect of the warp factor at large volume is an order of limits problem.

The large variation of the warp factor is due to the source terms in Eq. [3.3]. As an example, around a D3-brane, which is a delta function source \( c_2 \delta^6(r) \) at \( r = 0 \), one has \( e^{-4A} \sim c_2/r^4 \). A non-zero flux, of either sign, while not localized, has a similar effect.

While the details depend on \( d = 10 \) physics, the general behavior is governed by a ‘localized’ (rapidly decaying) but non-constant zero mode of the source-free equation, in other words the Laplacian on \( e^{-4A} \). For simplicity, let us consider this in a single throat located (in some local coordinate system) at \( r = 0 \), embedded in a compact Calabi-Yau. The discussion extends without changes to any number of throats. Then, in a conifold geometry, the Laplacian takes the form

\[
\frac{1}{r^5} \frac{\partial}{\partial r} \left( r^5 \frac{\partial}{\partial r} G \right) + \frac{1}{r^2} \nabla^2 G = 0,
\]

(3.15)
where $r$ is the conical distance from the singularity $r = 0$, and $\Psi$ denotes the five angular variables. Hence the zero mode with trivial angular dependence on the throat is

$$G(r) = c + \frac{c_2}{r^4}. \quad (3.16)$$

The general solution (3.13) becomes

$$e^{-4A(y)} = c + \frac{c_2}{r^4} + e^{-4A_0(y)}. \quad (3.17)$$

The zero mode term $c_2/r^4$ will then dominate other effects in the throat. Expanding the Kahler potential in the large volume limit,

$$K(S, \bar{S}) = -\log \left( -i \int_{\text{bulk}} e^{-4A} \Omega \wedge \bar{\Omega} - i \int_{\text{conif}} e^{-4A} \Omega \wedge \bar{\Omega} \right) \approx K_0 + ie^{K_0} \int_{\text{conif}} (c + \frac{c_2}{r^4}) \Omega \wedge \bar{\Omega}, \quad (3.18)$$

with $e^{-K_0} := -i \int_{\text{bulk}} c \Omega \wedge \bar{\Omega}$. This is essentially the limit from local special geometry to rigid special geometry, valid in the neighborhood of the conifold point in moduli space [24].

This shows that there can be a warp enhancement of the Kahler potential at the tip of the throat. Moreover, the metric computed from here will be

$$G_{SS} \approx ie^{K_0} \int_{\text{conif}} (c + \frac{c_2}{r^4}) \chi_S \wedge \chi_S, \quad (3.19)$$

which agrees with the general result (3.2). We will see below that, at least for the deformed conifold, the $(2, 1)$ forms $\chi_\alpha$ are also localized at $r = 0$, contributing to the enhancement effect.

The coefficient $c_2$ is fixed by a standard Stokes theorem argument. Given a region $R \subset \mathcal{M}$ with boundary $\partial R$, we have [4]

$$\int_{\partial R} \sqrt{g_5} \partial_\alpha e^{-4A} = \int_R \sqrt{g_6} \nabla^2 e^{-4A} = \pm \int_R \sqrt{g_6} \frac{G_{mnp} \tilde{G}^{mnp}}{12 \text{Im} \tau} \pm 2 \kappa_{10}^2 T_3$$

so the normal derivative of $e^{-4A}$ integrated over the boundary is proportional to the total source contained in the region. At large $r$, the boundary integral is dominated by the contribution of the zero mode $c_2/r^4$.

Thus, D3 charge near the conifold, including that induced by fluxes, leads to warping. Of course this charge will be compensated by negative charge elsewhere, say from O3 planes.

---

1 Recall that, from Eq. [3.3], a plus (minus) sign must be used in the e.o.m. of the warp factor for ISD (IASD) fluxes and positive (negative) D3 charge.
4 Explicit analysis of the deformed conifold

Let us see how this shows up in an explicit treatment. Thus, following Klebanov and Strassler [25] we discuss the CY metric corresponding to (2.5). We warm up with the singular conifold, with \( S = 0 \). It has

\[
ds_6^2 = dr^2 + r^2 ds_{T,1,1}^2 , \quad ds_{T,1,1}^2 = \frac{1}{9} (g^5)^2 + \frac{1}{6} \sum_{i=1}^{4} (g^i)^2 .
\]  

(4.1)

The basis of one forms \( g^i \) was introduced in [26, 25]; they arise from the angular variables of the base \( S^2 \times S^3 \). In terms of

\[
\omega_2 := \frac{1}{2} (g^1 \wedge g^2 + g^3 \wedge g^4) , \quad \omega_3 := \frac{1}{2} g^5 \wedge (g^1 \wedge g^2 + g^3 \wedge g^4) ,
\]

(4.2)

the \( (2,1) \) form reads

\[
\chi_S = \frac{1}{8\pi^2} \left( \omega_3 - 3i \frac{dr}{r} \land \omega_2 \right).
\]

(4.3)

Notice that this form is ‘localized’ at small \( r \). The normalization chosen is related to the fact that \( \int_{S^3} \omega_3 = 8\pi^2 \), although the final results are independent of this.

The actual solution of interest is the deformed conifold. In the basis \((\tau, g^i)\) of [25] the metric is diagonal

\[
ds_6^2 = \frac{1}{2} |S|^{2/3} K(\tau) \left[ \frac{d\tau^2 + (g^5)^2}{3 K^3(\tau)} + \cosh^2 \frac{\tau}{2} \left( (g^3)^2 + (g^4)^2 \right) + \sinh^2 \frac{\tau}{2} \left( (g^1)^2 + (g^2)^2 \right) \right]
\]

(4.4)

where

\[
K(\tau) := \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh \tau}.
\]

Note that all the moduli dependence is contained in the single prefactor \(|S|^{2/3}\). For large \( \tau \), the relation with the conical radius is

\[
r^2 = \frac{3}{2^{5/3}} |S|^{2/3} e^{2\tau/3}.
\]

(4.5)

The \( (2,1) \) form is now more complicated:

\[
\chi_S = g^5 \wedge g^3 \wedge g^4 + d \left[ F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4) \right] - i d \left[ f(\tau) g^1 \wedge g^2 + k(\tau) g^3 \wedge g^4 \right],
\]

(4.6)

where the functions \( F, f \) and \( k \) were computed in [25]:

\[
F(\tau) = \frac{\sinh \tau - \tau}{2 \sinh \tau} , \quad f(\tau) = \frac{\tau \coth \tau - 1}{2 \sinh \tau} (\cosh \tau - 1) ,
\]

\[
k(\tau) = \frac{\tau \coth \tau - 1}{2 \sinh \tau} (\cosh \tau + 1).
\]

(4.7)
4.1 Computation of the moduli space metric

Next we turn on $N$ units of $F_3$ flux through the deformed 3-cycle $A$ of the conifold; as explained in [25] this generates an $H_3$ flux through the dual $B$ cycle. In the noncompact model it doesn’t matter whether $N$ is positive or negative, and, indeed we will see that the formulas depend only on $N^2$. The difference will appear when we embed the configuration in a compact CY; bulk interactions between the ‘misaligned’ flux $N < 0$ and the orientifold will break supersymmetry. This will be addressed in the next section.

The configuration might be seen either as the gravity side of the Dijkgraaf-Vafa duality, or as the end of the duality cascade of Klebanov-Strassler. In any case, the Calabi-Yau is warped as in (3.1); $e^{-4A}$ may be written as [25]

$$e^{-4A(\tau)} = 2^{2/3} \frac{(g_s N\alpha')^2}{|S|^{4/3}} I(\tau)$$

(4.8)

where $I(\tau)$ is an integral expression defined in [25]. It is not known how to evaluate it in terms of elementary functions, but fortunately we will only need its derivative:

$$\frac{d}{d\tau} e^{-4A(\tau)} = -4 \times 2^{2/3} \frac{(g_s N\alpha')^2}{|S|^{4/3}} \frac{f + F(k - f)}{(\sinh 2\tau - 2\tau)^{2/3}}.$$  

(4.9)

Although the form of $\chi_S$ is complicated, surprisingly $\chi_S \wedge \chi_S$ is a total $\tau$-derivative: from (4.6),

$$\chi_S \wedge \chi_S = -\frac{2i}{64\pi^4} d\tau \wedge \left( \prod_i g^i \right) \frac{d}{d\tau} \left[ f + F(k - f) \right].$$

(4.10)

Integrating by parts, the warped metric reads

$$G_{\bar{S}S} = -\frac{2i}{\|\Omega\|^2 64\pi^4 V_W} \left( \int \prod_i g^i \right) \left[ \int_{\tau}^{\tau_A} d\tau \frac{d}{d\tau} \left( e^{-4A(\tau)}(f + F(k - f)) \right) + \right.$$

$$\left. - \frac{d e^{-4A}}{d\tau} (f + F(k - f)) \right].$$

As explained in section 2.2, the noncompact model is regularized at $\Lambda_0$, which determines $\tau_A$ through (4.5),

$$\tau_A = \frac{3}{2} \log \frac{2^{5/3}}{3} + \log \frac{\Lambda_0^3}{|S|}.$$  

(4.11)

We only have to integrate the last term in $G_{\bar{S}S}$. Fortunately the integrand decays rapidly; its maximum is 0.03 at $\tau \approx 2.5$ and already at $\tau \approx 10$, its
value is $5 \times 10^{-5}$. Since $\tau_\Lambda \gg 1$, with negligible error we may extend the integral to infinity, giving

$$
\int_0^\infty \! d\tau \frac{d e^{-4A}}{d\tau} (f + F(k - f)) \approx 0.093 \times (-4 \times 2^{2/3}) \frac{(g_s N \alpha')^2}{|S|^{4/3}}.
$$

Putting together these results, we finally obtain the explicit expression for the metric:

$$
G_{\bar{S}S} = -\frac{i}{||\Omega||^2 \pi V_W} \left[ c \log \frac{\Lambda_0^3}{|S|} + (8 \times 2^{2/3} \times 0.093) \frac{(g_s N \alpha')^2}{|S|^{4/3}} \right]. \tag{4.12}
$$

The volume of the base $T^{1,1}$ contributes a factor $64 \pi^3$ and, as before, $c$ is the universal Kahler modulus.

The first term in (4.12) is the usual one, determined by special geometry and interpreted as integrating out BPS $D3$ branes wrapping the $A$ cycle. The second term is the new contribution; such a term could not appear in $N = 2$ compactification, both on mathematical grounds [28] and because loop effects of massless particles cannot lead to this type of power-like divergence. However it is a natural consequence of warping in $N = 1$, and also has a suggestive interpretation in the dual gauge theory, as we discuss later.

Notice that at small enough $|S|$, the second term will dominate. Since it is singular at $S = 0$, one should ask whether it is valid in this regime. We will examine the consistency condition in supergravity in subsection 4.3, concluding that for $g_s N >> 1$ (the standard supergravity regime) this is valid all the way down to $S = 0$.

### 4.2 Gluing the conifold to a compact CY

The advantage of using formula (3.2) to compute $G_{\alpha\bar{\beta}}$ close to the conifold point ($S \rightarrow 0$) is that it is insensitive to how we patch the noncompact conifold into the Calabi-Yau. Indeed, in the previous two subsections we saw that the form $\chi_S$ is localized on the vanishing cycle, so the dominant contribution to the metric will come from this region. However, since we are trying to compute a small effective potential and supersymmetry breaking scale, we might worry that small corrections coming from the bulk or the patching prescription might qualitatively change the results.

The basic argument that these do not matter, is that the new term $|S|^{-4/3}$ in Eq. (4.12) grows more quickly as $S \rightarrow 0$ than any possible bulk term. At this point we cannot show this for all possible bulk solutions. But we do know it for the original unwarped bulk solutions described by special geometry – as argued in [28, 29], these moduli space metrics can have at most logarithmic
divergences, as in the $c$ term in Eq. (4.12). This comes from the integral down to small $r$ and will not get contributions from elsewhere in the bulk. Then, if we can argue that the enhancement due to warping is always of the general form described by Eq. (3.19), we will know that possible bulk contributions will be subleading to the conifold contribution we computed.

Let us see how Eq. (3.19) works for our explicit example. Volume integrals of the warp factor in the deformed conifold are hard to make so, for simplicity, we discuss the case of a singular conifold, which gives the right intuitive picture. We should point out that even without warping, introducing a cutoff $\Lambda_0$ will make the conifold volume and the bulk volume depend on $\Lambda_0$ and $S$:

$$V_{CY} = V_{bulk}(\Lambda_0, |S|) + \text{vol}(T^{1,1}) \int_{|S|^{1/3}}^{\Lambda_0} dr r^5 \quad (4.13)$$

$$= V_{bulk}(\Lambda_0, |S|) + \frac{16\pi^3}{27} \frac{1}{6} (\Lambda_0^6 - |S|^2) \quad (4.14)$$

As we discussed earlier, it is natural to define the total unwarped volume to be independent of the complex structure, in which case the bulk volume should have a dependence $\sim |S|^2$ to cancel the conifold contribution.

In the singular conifold, the warp factor generated by $N$ fractional $D3$ branes is

$$e^{-4A(r)} = c + \frac{81}{8} (g_s N_3')^2 \left( \log \frac{r}{|S|^{1/3}} \right) \frac{\log (r/|S|^{1/3})}{r^4}. \quad (4.15)$$

From the gauge theory point of view, the constant zero mode $c$ corresponds to a dimension 8 operator, which arises naturally as a correction in the Born-Infeld action. It depends on the amount of flux turned on, but we can neglect it for large cutoff. On the other hand, $c_2$ is just the ‘bare’ contribution to the warp factor, in agreement with the general discussion of subsection 3.3.

Now, replacing the value of $c_2$ and (4.3) in (3.19),

$$G_{S\bar{S}} = -\frac{i64\pi^3}{||\Omega||^2 V_W} \left[ c \log \frac{\Lambda_0^3}{|S|} + \frac{81}{32} (g_s N_3')^2 \right]. \quad (4.16)$$

Comparing to (4.12), we see that both results have exactly the same dependence, so our gluing prescription reproduces the right metric. The log$|S|$ coefficients match exactly because they come from the asymptotic behavior of the conifold, which is correctly described by the singular conifold. The ones of $|S|^{4/3}$ have the same order of magnitude, but we don’t expect them to agree. Indeed, this term comes from the contribution of the deformed 3-cycle, which is described only qualitatively by the simplified approach of using the singular conifold and introducing a cutoff at $r = |S|^{1/3}$.
The expression for the vacuum energy from dimensional reduction is [22]

\[
V = \frac{1}{2\kappa^2} \frac{1}{V_W \text{Im} \tau (\text{Im} \rho)^3} \frac{1}{\| \Omega \|^2} V_W G^{SS} |\partial_S W|^2 .
\]

(4.17)

Using the known expression for \( G_{SS} \), we obtain

\[
V = \frac{1}{2\kappa^2_{10}} \frac{g_s}{(\text{Im} \rho)^3} \left[ c \log \frac{\Lambda_0^3}{|S|} + c' \frac{(\alpha' g_s N)^2}{|S|^{4/3}} \right]^{-1} \left| N \frac{\Lambda_0^3}{S} + i \frac{\beta_{NS}}{g_S} \right|^2 .
\]

(4.18)

To avoid cluttering, we have absorbed order one numerical factors into the constants \( c \) and \( c' \), although \( c \) still denotes the universal Kahler modulus \( c \sim V_W^{2/3} \).

### 4.3 A closer look into warping effects

Near the mouth of the throat, where warping is small, the usual intuition from special geometry and the deformed conifold is valid. In particular, in the limit \( S \to 0 \), the \( S^3 \) collapses, its radius being controlled by the factor \( |S|^{2/3} \) in the metric (4.14).

We may, however, tune the fluxes to get strong warping \( e^{-4A} \gg c \). As we now discuss, this changes radically the picture, even before considering particular models for supersymmetry breaking. In this subsection we analyze the supersymmetric \( N > 0 \) case and, in the next one we break supersymmetry by setting \( N < 0 \).

To begin with, consider a sample potential with and without warping, plotted in Figure 1. We have taken order one parameters so that the various regimes are easily visible on the same plot.

The dashed curve is the potential without the \( S^{-4/3} \) correction to the metric. It has a minimum given by (2.10), where it vanishes, while it goes to infinity at \( |S| \to 0 \) and at \( |S| \to \Lambda_0^3 \). One might have expected that for \( S \) small, the system should become unstable and undergo a geometric transition.

On the other hand, the behavior of the potential (Eq. (4.18)) including the warped metric is quite different. At

\[
c \log \frac{\Lambda_0^3}{|S|} \sim \frac{(\alpha' g_s N)^2}{|S|^{4/3}}
\]

the \( S^{-4/3} \) starts to dominate; a maximum value is attained and after that the system starts to roll down to \( S = 0! \)

The reason why this effect was not detected before is that fluxes break \( N = 2 \) softly, so at string tree level the Kahler metric is still given by special
Figure 1: Behavior of the potential (4.18) for the supersymmetric $N > 0$ case, with (full line) and without (dashed line) warping effects. The point $S = 1$ is the supersymmetric vacuum.

geometry. However, if we want to analyze the geometric transition in more detail, we have to consider what happens for $S \rightarrow 0$. In this case, the $g_s$ correction of (4.18) is important, showing that the system becomes unstable.

Clearly, the supergravity solution is singular at $S = 0$. For which range of small (but finite) $S$ can we trust the supergravity analysis? To answer this we need to study the curvature of the background. We consider the ‘near horizon’ limit $\tau \rightarrow 0$, where the largest curvatures may be generated; strong warping implies the boundary condition $e^{-4A(\tau)} \rightarrow 0$ as $\tau \rightarrow \infty$, which is exactly the KS end of the cascade. In this case, the metric for the warped-deformed conifold

$$ds_{10}^2 = e^{2A(\tau)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(\tau)} ds_6^2$$

with $ds_6^2$ given in (4.1), becomes

$$ds_{10}^2 \approx \frac{1}{2^{1/3} a_0^{1/2}} \frac{|S|^{2/3}}{\alpha' g_s N} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{a_0^{1/2}}{6^{1/3}} \alpha' g_s N \left[ \frac{d\tau^2}{2} + d\Omega_2^2 + d\Omega_3^2 \right]. \quad (4.19)$$

Here we used the fact that for $\tau \rightarrow 0$, the function $I(\tau)$ introduced in (4.8) behaves as $I(\tau \rightarrow 0) \rightarrow a_0 \sim 0.7180$ [25]. Furthermore, we included explicitly the $S^2$ and $S^3$ at the base of the cone:

$$d\Omega_2^2 = \frac{\tau^2}{2} ((g^1)^2 + (g^2)^2),\ d\Omega_3^2 = \frac{1}{2} (g^5)^2 + (g^3)^2 + (g^4)^2. \quad (4.20)$$
The $S^3$ has finite radius, while the $S^2$ collapses, as expected.

The fact that the $S$ dependence cancels out in $e^{-2A(r)} \, ds_6^2$ is quite striking; this was already derived in [25], but we would like to point out some of its consequences. In the strong warping limit, we see that the volume of the $S^3$ is not proportional to $S$; in particular this 3-cycle does not vanish when $S \to 0$! The order of limits matters and we cannot recover the (strongly) warped deformed conifold by taking $S \to 0$ in the deformed conifold and then introducing the warp factor for the singular conifold. The modulus $S$ no longer parametrizes the size of a cycle in the warped deformed geometry. Note, however, that not all the dependence on $S$ of the six-dimensional geometry has disappeared. Indeed, unlike $r$, $\tau$ is a dimensionless coordinate; cutting off the conifold at some finite $\tau_\Lambda$ requires both scales $\Lambda_0$ and $S$, as showed in (4.11). Hence, as $S \to 0$, the throat becomes infinite (even at fixed $\Lambda_0$). Of course, once $e^{-4A}$ is small enough, the bulk effects become relevant, cutting off the geometry; but still, this behavior is very different to the deformed case without warping.

The analysis of the curvature tensor of (4.19) is straightforward. A crucial point is that the only dependence on $S$ is through $\eta_{\mu\nu} \, dx^\mu \, dx^\nu$; since the curvature does not depend on $x^\mu$, defining orthonormal Minkowski coordinates

$$\tilde{x}^\mu := \left( \frac{1}{\sqrt{a_0} \, a^1/2 \, g_s N} \right)^{1/2} \frac{|S|^{2/3}}{\sqrt{\alpha' \, g_s N}} x^\mu,$$

(4.21)

none of the components of $R_{MNR}^S$ will depend on $S$. An explicit computation to order $\tau^2$ gives the scalar curvature

$$R = -\frac{6^{1/3}}{5 \sqrt{a_0}} \frac{1}{\alpha' g_s N} \left[ 3(1 + 20k) - (6 + 9k + 880k^2) \tau^2 \right] + O(\tau^3) \quad (4.22)$$

where $I(\tau) \sim a_0(1+k\tau^2)$, $k$ being an order one constant. Therefore, unlike the unwarped case, we can trust the supergravity approach as long as $g_s N \beta^{NS} \gg 1$, even if $S \to 0$ (but finite). Incidentally, (4.21) implies that the time $x^0$ necessary to roll down to $S = 0$ tends to infinity, at fixed orthonormal time $\tilde{x}^0$.

If the modulus $S$ doesn’t have now a geometric interpretation, what is its meaning? As explained by KS, (4.19) is the ‘supergravity version’ of confinement. Since the prefactor multiplying $\eta_{\mu\nu} \, dx^\mu \, dx^\nu$ is finite for $\tau = 0$, Wilson loops will have an area law. Furthermore, we see that the theory generates dynamically a confinement scale

$$M^2_{\text{conf}} \sim \frac{|S|^{2/3}}{\alpha' g_s N},$$

(4.23)
controlled by $S$. From this point of view, the previous noncommutativity of limits is expected: the warped singular conifold cannot reproduce these nonperturbative effects.

### 4.4 Breaking supersymmetry at strong warping

We have finished assembling the necessary tools to understand how warp effects influence the nonsupersymmetric $N < 0$ case. The unwarped case was considered in [7] and summarized in subsection 2.3.

To evaluate the scale of this breaking we need $G_{S\bar{S}}$, which was computed explicitly in subsection 4.1. Integrating (4.12), the Kahler potential for the conifold, in rigid special geometry, reads

$$K(S, S) = -\frac{i64\pi^3}{\|\Omega\|^2V_W} [c|S|^2 \log \frac{A_0^3}{|S|} + (72 \times 2^{2/3} \times 0.093)\alpha'g_sN|S|^{2/3}]. \quad (4.24)$$

We are interested in the regime where the new warp correction dominates, namely

$$c \log \frac{A_0^3}{|S|} \ll \frac{(\alpha'g_sN)^2}{|S|^{4/3}}. \quad (4.25)$$

From (2.12) this may be attained by an adequate choice of fluxes $\beta^{NS} \gg g_sN$.

Replacing in (2.14) the values of $S_{N<0}, \partial S W_{N<0}$ and $G_{S\bar{S}}$ that we found,

$$M_{susy}^4 := V_{N<0} = \frac{k}{\kappa^2} \frac{1}{V_W(\text{Im} \rho)^3} g_s \left| \frac{\beta^{NS}}{g_sN} \right|^2 \exp\left( -\frac{8\pi}{3} \frac{\beta^{NS}}{g_s|N|} \right) A_0^4. \quad (4.26)$$

This has the desired exponential suppression in the semiclassical limit $\beta^{NS}/g_sN \gg 1$. Note that in the limit $V_W \to \infty$, the orientifold will be far away from the throat and $V_{N<0} \to 0$, which agrees with the idea that the system is locally supersymmetric.

From the point of view of the potential (4.18), the prescription of [7] for the physical vacuum (2.12) puts us in an unstable point, rolling directly to $S = 0$! One option would be that the present description, in terms of a single field $S$ is not valid in strongly warped regimes. Indeed, in the known holographic descriptions of confined pure SYM [25, 35, 36], the masses of KK modes (from dimensional reduction on the conifold) are comparable to the glueball mass. Including these fields is not a simple task, requiring, in particular, a better understanding of the Green’s functions on the deformed conifold.

Here we briefly discuss another option, namely that the breaking of the no-scale structure (due to the absence of supersymmetry) may stabilize the vacuum. Indeed, as the analysis of [5] suggests, metastable vacua in general
require two scales, one generated by the gauge theory, and another coming from UV effects (in their case, the small mass \( m \) for quarks). Our discussion so far has no analog of this second scale.

To begin a full discussion, one would have to incorporate the various ingredients of moduli stabilization discussed in \[18\], including stabilization of the dilaton and the complex structure moduli other than \( S \), and breaking of no-scale structure and stabilization of Kähler moduli due to stringy and quantum corrections which depend on the these moduli. We now assume that this has been done in some way which does not affect the physics in the throat, and discuss the remaining physics in the throat after integrating these modes out, using the supergravity potential

\[
V = \kappa_3^2 e^K \left[ G^{S\bar{S}} |D_S W|^2 - 3 |W|^2 \right] \tag{4.27}
\]

Actually this expression would only be exact in a limit in which the other moduli were infinitely massive; otherwise it will receive corrections from cross-coupling between the other moduli and \( S \). However, one can easily state conditions under which these effects will not qualitatively affect the results, so we neglect this.

Now, taking the anti-self-dual flux configuration, an important point is that the dual period \( \partial \mathcal{F} / \partial S \) does not vanish in the limit \( S \to 0 \). Writing

\[
\int_B \Omega = \frac{S}{2\pi i} \log \frac{\Lambda_0^3}{S} + \Pi_0 ,
\]

then

\[
W = \frac{N}{2\pi i} S \left( \log \frac{\Lambda_0^3}{S} + 1 \right) + N\Pi_0 + \frac{i}{g_s} \beta^{NS} S \tag{4.28}
\]

and the condition for having a minimum at small \( S \) is

\[
S^{1/3} \log \frac{\Lambda_0}{S^{1/3}} \approx \frac{2\pi e' \Pi_0}{\int e^{-4A\Omega \wedge \overline{\Omega}}} (\alpha' g_s N)^2 . \tag{4.29}
\]

Here we are using the notation of Eq. \[4.18\].

Typically,

\[
\frac{2\pi \Pi_0}{\int e^{-4A\Omega \wedge \overline{\Omega}}} \sim V_W^{-1/2}
\]

so by choosing a bulk volume \( V_W^{1/2} \gg \alpha'^2 g_s N / \beta^{NS} \), the modulus is stabilized at a parametrically small (though no longer exponentially small) scale. The vacuum energy here is of the order

\[
V_{\text{min}} \approx \frac{1}{2\kappa_{10}^2} \frac{g_s}{\Im \rho} \frac{|\Pi_0|^2}{\int e^{-4A\Omega \wedge \overline{\Omega}}} .
\]

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Since the natural scale of the potential away from the \( S \to 0 \) limit is set by \( \beta^{NS}/g_s N \), the height and breadth of the barrier separating this minimum from the true vacuum scale in the same way. It’s worth mentioning at this point that in the GKP conifold setup the parameter choices leading to a controllable hierarchy are \( 1 << g_s N \), so that the supergravity approximation is reliable at the tip, and \( 1 << \beta^{NS}/g_s N \), which in the supersymmetric GKP setup sets the scale of the hierarchy. These are precisely the same relations which yield a reliable metastable vacuum here.

Finally, once one has found a stable vacuum from the point of view of the \( \mathcal{N} = 1 \) effective Lagrangian, one needs to ask whether other effects could destabilize it, in particular whether a KK mode which was dropped in deriving the Lagrangian could go tachyonic. The basic answer to this question is that, since there is a limit in which the throat solution would have been \( \mathcal{N} = 1 \) supersymmetric had not that supersymmetry been projected out by the orientifolding, it will satisfy the constraints of this \( \mathcal{N} = 1 \) supersymmetry, up to small corrections. Thus, one can restrict attention to the light modes in the \( \mathcal{N} = 1 \) effective Lagrangian, and see whether the new couplings introduced at this point destabilize any of them; massive KK modes will be stable since the original KS solution was stable.

5 The dual gauge theory

The discussion we just gave should be valid for \( g_s N >> 1 \). For small \( g_s N \), we would expect a description in terms of the gauge theory on the wrapped anti-D5 branes to be more appropriate. We do not know how such a description would work in detail, but we can make the following comments on the problem.

Let us start by considering the embedding of the conifold with anti-self-dual flux into an \( \mathcal{N} = 2 \) compactification. There, the gauge theory under discussion is the same as the gauge theory usually invoked in this duality, namely the \( U(N_1) \times U(N_2) \) supersymmetric gauge theory of \cite{25} in the UV, undergoing a “cascade” down to pure \( U(N) \) super Yang-Mills theory. This theory has \( N \) supersymmetric vacua, and we recover the standard discussion, with the sole change being the sign of the RR fluxes and the identification of the unbroken \( \mathcal{N} = 1 \) subalgebra in \( \mathcal{N} = 2 \).

As we saw in the supergravity analysis, it seems very plausible that the essential phenomenon is a misalignment of the \( \mathcal{N} = 1 \) supersymmetries preserved by the bulk and by the antibrane. To describe this in gauge theory terms, we might try to identify the action of bulk \( \mathcal{N} = 2 \) supersymmetry on the gauge theory, and the \( \mathcal{N} = 2 \) stress tensor multiplet, which would couple
to the $d = 4, \mathcal{N} = 2$ supergravity obtained by KK reduction. The difference between the D5 and anti-D5 theories then arises when we do the orientifold projection, obtaining a $d = 4, \mathcal{N} = 1$ supergravity. Whereas for the D5 theory, we couple the $\mathcal{N} = 1$ stress tensor multiplet to $\mathcal{N} = 1$ supergravity, for the anti-D5-brane we would instead couple to the broken $\mathcal{N} = 1$ subalgebra of $\mathcal{N} = 2$.

This idea is simple to realize in the case of branes embedded in flat space. Consider for example the world-volume theory of $N$ D3-branes; it is $\mathcal{N} = 4$ super Yang-Mills with 16 linearly realized supersymmetries. It also has 16 nonlinearly realized supersymmetries, the constant shifts of the diagonal components of the gauginos. An analog of the theory under discussion is obtained by truncating this to a linearly realized $\mathcal{N} = 1$ and a nonlinearly realized $\mathcal{N} = 1$. Thus, the antibrane couples to the $\mathcal{N} = 1$ gravitino, not through the standard supercurrent, but through the gaugino.

This leads to spontaneous supersymmetry breaking, at a scale controlled by the antibrane tension. However, it is not obvious how strong coupling effects could lower this scale. Naively, since the supersymmetry breaking is all in the coupling to the $U(1)$ sector, the nonabelian Yang-Mills sector does not seem to play any role. However, the sectors could be coupled by higher dimension operators, so this conclusion is probably too quick.

According to the usual discussions of the AdS/CFT correspondence, the $\mathcal{N} = 2$ supersymmetry of the underlying string background, is reflected in the $\mathcal{N} = 1$ superconformal symmetry of the gauge theory. Thus, the idea would be to couple the gravitino of $\mathcal{N} = 1$ supergravity, not to the standard supercurrent, but to the superconformal current of the gauge theory.

Unfortunately, this idea is not consistent as it stands, as the superconformal symmetry in these gauge theories is explicitly broken by quantum effects (the beta function is non-zero) and we cannot gauge an explicitly broken symmetry. Nevertheless it might be correct if a suitable compensator field is present in the bulk theory.

## 5.1 Effective potential

Granting that there is a microscopic definition of the theory as a gauge theory coupled to $\mathcal{N} = 1$ supergravity, we next ask whether the effective potential we have derived and justified at $g_s N >> 1$, should be expected to give a good qualitative description for small $g_s N$. As we commented earlier, even in the supersymmetric vacua the precise interpretation of this type of effective action is not entirely clear, so we limit ourselves to questions about vacuum energy, supersymmetry breaking and stability.

We begin with the $|S|^{2/3}$ term in the Kähler potential. It is amusing and
perhaps significant that such a term was already suggested in the pioneering work of Veneziano and Yankielowicz on pure $SU(N)$ SYM [34]. The argument there was that the gaugino bilinear $S$, being a composite field, does not have the canonical dimension of a scalar field. At weak coupling, its dimension should be close to that of a fermion bilinear in free field theory, namely $[S] = 2[\lambda] = 3$. On the other hand, a $d^4\theta$ kinetic term should have dimension 2. If we are not allowed any dimensionful constants, this forces

$$K(S\bar{S}) = \alpha(\bar{S}S)^{1/3}$$

(5.1)

for some numerical constant $\alpha$. This precisely matches the new term coming from warping in (4.24)!

Unfortunately, for $S \to 0$ the gauge theory is strongly coupled, and it is not known how to compute the Kähler potential in this regime. On general grounds one would expect corrections controlled by the dynamical scale $\Lambda$. While it is true that $\Lambda$ does not appear explicitly in the superpotential, only emerging upon solving for the vacuum, there is no obvious reason that the Kähler potential should work the same way. Thus at this point we can not say we have strong evidence for such a term at weak coupling, although it is certainly a very suggestive coincidence.

In any case, if we accept that the theory breaks supersymmetry at the dynamical scale, the claim that the metric $G_{S\bar{S}} \sim |S|^{-\alpha}$ for some $\alpha > 0$ would seem to be a very natural way to describe this in an $\mathcal{N} = 1$ effective Lagrangian. It might not be inevitable, as one can also imagine inverse powers of $\Lambda$ playing this role. However, this would violate the general principle that nonperturbative effects should vanish in the weak coupling limit $\Lambda \to 0$, so it seems a reasonable hypothesis that such effects are not present.

But, as we saw in our explicit example, any structure in which the vacuum energy is warped down by a power of $S$, leads directly to a potential with a zero energy minimum at $S = 0$. We discussed how in a string theory compactification this might be prevented by bulk effects. But in the gauge theory limit, such effects would presumably be absent, so the result would be a theory which rolls down to $S = 0$.

Could there be another supersymmetric vacuum at $S = 0$? A suggestion that super Yang-Mills theory has additional vacua at $S = 0$ was made in [37], however at present this is not believed to be the case.

One straightforward way to reconcile these claims is if the effects we are discussing, in particular the correction to the Kähler potential and the corresponding lowering of the supersymmetry breaking scale, are not present at small $g_sN$. Now some brane-antibrane realizations of supersymmetry breaking, for example [10, 11], lead to a non-trivial phase structure, and it might
be the case here. However, in these realizations, the supersymmetry breaking vacuum exists at weak coupling, and disappears at strong coupling, so the opposite claim might be surprising.

It also seems possible to us that while this effective field theory is qualitatively valid, the configuration rolling down to $S = 0$ is not a conventional vacuum. This is true in the supergravity limit, as the value of $S$ controls the warp factor in $d=4$, so that $S = 0$ cannot be realized. One can still imagine solutions in which $S$ rolls to zero, but these are essentially dynamical. In particular, since the warp factor multiplies the $g_{00}$ component of the metric, the time evolution is very different than the flat space evolution in such a potential. As we explained in the discussion below Eq. (4.22), this suggests that the minimum $S = 0$ is not reached in finite physical time. We intend to study the physics of this more carefully in a future work.

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