Quark Antiscreening at Strong Magnetic Field and Inverse Magnetic Catalysis

E. J. Ferrer\(^1\), V. de la Incera\(^1\), and X. J. Wen\(^{1,2}\)

\(^1\) Department of Physics, University of Texas at El Paso, 500 W. University Ave., El Paso, TX 79968, USA

\(^2\) Institute of Theoretical Physics, Shanxi University, Taiyuan 030006, China

The dependence of the QCD coupling constant with a strong magnetic field and the implications for the critical temperature of the chiral phase transition are investigated. It is found that the coupling constant becomes anisotropic in a strong magnetic field, and that the quarks, confined by the field to the LLL, produce an antiscreening effect. These results lead to inverse magnetic catalysis, providing a natural explanation for the behavior of the critical temperature in the strong field region.

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Introduction. The study of the QCD phase diagram in the temperature \((T)\) - density \((\mu)\) plane has attracted much attention for many years \([1]\). The more recent possibility to experimentally reach the high-energy regions where the quark-gluon plasma is realized has activated even more this research field. On the other hand, effects of strong magnetic fields in quark matter have also been an active research area for a long time \([2,12]\). At present, such studies have been reactivated by the possibility to reach magnetic field values in heavy ion collisions in an energy range which is beyond the intrinsic QCD scale \(\Lambda_{QCD} \sim 200\) MeV. There are both theoretical and experimental indications that the colliding charged ions can generate very strong magnetic fields, estimated to be of order \(eB \sim 2m_{\pi}^2(\sim 10^{18}G)\) for the top collision, \(\sqrt{s_{NN}} \sim 200\) GeV, in non-central Au-Au impacts at RHIC, or even larger, \(eB \sim 15m_{\pi}^2(\sim 10^{19}G)\), at future LHC experiments \([8]\).

As is known, a magnetic field can affect the QCD chiral phase transition. In fact, due to the dimensional reduction of the lowest Landau level (LLL) dynamics, a magnetic field can catalyze the chiral symmetry breaking even if the attractive interaction between fermions is weak. This phenomenon, known as magnetic catalysis of chiral symmetry breaking (MC\(\chi\)SB), consists of the dynamical generation of a fermion/anti-fermion (chiral) condensate that modifies the vacuum properties and induces dynamical parameters that depend on the applied field. This effect has been actively investigated during the last two decades \([1,2]\).

In the original studies of MC\(\chi\)SB \([1]\), the phenomenon was assumed to lead only to the conventional, scalar chiral condensate, and to the generation of a single dynamical parameter, the fermion mass. In recent years, however, it has been found in both QED \([5]\) and QCD \([6]\) that the same mechanism of MC\(\chi\)SB is also responsible for the formation of a magnetic moment condensate and the dynamical generation of an anomalous magnetic moment (AMM) for the quasiparticles. In the case of quarks, this result was obtained in a Nambu-Jona-Lasinio (NJL) model with interaction channels consistent with the symmetries of QCD in the presence of a magnetic field \([4]\). There, the generation of the AMM condensate increases the critical temperature of the chiral phase transition.

So far, all the studies of the MC\(\chi\)SB phenomenon in QCD have led to an increase of both the dynamical mass, (for a review see \([7]\) and references therein), and the AMM \([8]\) with the magnetic field. Consequently, the critical temperature \(T_c\) for the restoration of the chiral symmetry is found to increase with the magnetic field. This result is in sharp contrast, however, with recent QCD-lattice calculations showing a decrease of the critical temperature with the magnetic field \([8]\), a phenomenon that has been termed as inverse magnetic catalysis. Several attempts to address this disagreement already exist in the literature \([9,12]\). In \([9]\), the authors argued that the inverse catalysis is already embedded in the NJL approach because the neutral meson effects would suppress the chiral condensate at fields stronger than the scale of the hadron structure. Other authors \([10,12]\) have focused on the renormalization effects of the strong coupling in a magnetic field. The analysis in \([11]\) is based on the proposition that while in the strong field region the chiral condensate grows linearly with the field \([8,12]\), the dynamical quark mass should be however field-independent in this region. Other efforts \([11]\) introduced the effects of confinement in NJL through the Polyakov loop, using the so-called PNJL model, and argued that one could reproduce the inverse catalysis if the lattice data were fitted by making the critical temperature \(T_c\) a parameter of the PNJL model. In \([12]\), the inverse magnetic catalysis was connected to the running of the coupling with the magnetic field. The main point of \([12]\) was to propose an ansatz for the NJL coupling \(G\) that assumed a logarithmic dependence with \(B\).

In the present paper, we also adopt the point of view that the origin of the inverse magnetic catalysis lies on the effects of a strong field in the running of the strong coupling. The two new key elements of our findings are however the proof that in the strong field region \((qB \gg \Lambda_{QCD}^2)\), where the infrared dynamics is relevant, the QCD running coupling becomes anisotropic, and the quarks, confined by the field to the LLL, produce antiscreening because they are all paired with antiquarks forming color-electric dipoles. These results naturally lead to inverse catalysis and also allow us to identify the
physical origin of the $T_{C_x}$ behavior in the strong field region.

**Anisotropic Coupling Constant.** One limitation of the effective low-energy NJL-like models where the MCXSB has been found in QCD, is that the role of the gluons is reduced to multi-fermion contact interactions. The gluon dynamics, responsible for confinement, is then absent in the NJL approach. It is then natural to expect a connection of the inverse magnetic catalysis found in lattice QCD with the running of the coupling in the presence of a strong magnetic field. This running should in turn be driven by the region of momenta on which the MCXSB mechanism is operative. On the other hand, the MCXSB mechanism is effective as long as the fermions remain mostly confined to the LLL. This implies that the average momenta exchanged by particles should be much smaller than $\sqrt{qB}$. To explore the running of the coupling in such an infrared region, one needs to incorporate non-perturbative effects in the perturbative calculation of the running, as done in [13].

Furthermore, the magnetic field affects the coupling constant through quark loops with gluon external legs. In the strong-field region ($qB \gg \Lambda_{QCD}^2$), the dimensional reduction of the quarks in the LLL leads to a significant anisotropy in the gluon self-energy and consequently, in the coupling. This can be revealed by calculating the color Coulomb potential energy in the presence of a strong magnetic field [16],

\[
V(k^2) = -\frac{16\pi}{3} \frac{\alpha_s^0(\mu)}{C_\perp k^2 + C_\parallel k_\parallel^2} \tag{1}
\]

where $k^2 = k_\perp^2 + k_\parallel^2$. The coefficient

\[
C_\perp = 1 + \frac{11\alpha_s^0(\mu)N_c}{12\pi} \ln \left( \frac{k^2 + M_B^2}{\mu^2} \right) \tag{2}
\]

has only contributions from gluons and ghosts, while $C_\parallel$ also gets contributions from the quarks in the LLL. Each quark flavor gives a term similar to QED [17]. Hence,

\[
C_\parallel = C_\perp + \frac{\alpha_s^0(\mu)}{3\pi} \sum_{i=1}^{N_f} \frac{|q_iB|}{\tau} \exp \left( -\frac{k_\perp^2}{2|q_iB|} \right). \tag{3}
\]

In (1)-(3), $\alpha_s^0(\mu) = 12\pi/(11N_c \ln (\mu^2 + M_B^2)/\Lambda^2)$ is the running coupling constant at scale $\mu = 1.1$ GeV for $\Lambda_V = 0.385$ GeV, which, as in [10], has taken into account the nonperturbative QCD vacuum corrections calculated in [15] with $M_B \approx 1$ GeV and string tension $\tau = 0.18$ GeV$^2$. It should be pointed out that a complete formulation of the infrared behavior of $\alpha_s(k)$ is one of the most challenging problems in QCD. In this sense, the approach of Ref. [13], followed here, has the advantage of incorporating vacuum nonperturbative configurations that ensure confinement in the infrared region. Notice that contrary to the case of an applied electric field [18], a magnetic field does not change the string tension for a neutral string, which links a quark with an antiquark.

![FIG. 1. Coupling constant $\alpha_\parallel^0$, as a function of the magnetic field for a typical momentum $|k_\parallel| = \Lambda_{QCD}$](image)

From (1), a splitting of the couplings for momenta parallel and transverse to the field follows. They are given respectively by

\[
\alpha_\parallel^0(k_\parallel) = \frac{\alpha_0^0(\mu)}{C_\parallel(k_\perp = 0)}, \quad \alpha_\perp^0(k_\perp) = \frac{\alpha_0^0(\mu)}{C_\perp(k_\parallel = 0)}. \tag{4}
\]

From (1)-(4) it is evident that $\alpha_\perp^0(k_\perp)$ only gets contributions from gluons loops, thus it does not depend on the magnetic field. In contrast, as gathered from the plot in Fig.1, $\alpha_\parallel^0(k_\parallel)$ decreases with the magnetic field for momenta $k_\parallel \lesssim \Lambda_{QCD}$. Looking at Eqs. (2)-(4), it is worthy noticing, as already pointed out in [20], that the LLL quarks do not contribute to the running of the coupling. This is a consequence of the dimensional reduction in the LLL that produces finite quark loop contributions.

**Behavior of $T_c$ with $B$.** As shown in Ref. [6], the breaking of the rotational symmetry by a uniform magnetic field $B$ induces a separation between longitudinal and transverse fermion modes. This separation leads to the effective splitting of the couplings in the one-gluon exchange interactions on which the NJL models are based. This splitting is therefore reflected in the four-fermion couplings of a QCD-inspired NJL model in a magnetic field, and one can use the Fierz identities in a magnetic field, to show that the NJL Lagrangian in this case should be of the form

\[
\mathcal{L} = \bar{\psi}i\gamma^\mu D_\mu \psi + \frac{G}{2} [(\bar{\psi}\gamma^3\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2] + \frac{G'}{2} [\bar{\psi}(-A_\mu + A_\mu^{\text{ext}}) + \bar{\psi}i\gamma^5 A_\mu^{\text{ext}}] \tag{5}
\]

with $D_\mu = \partial_\mu + igA_\mu^{\text{ext}}$, $A_\mu^{\text{ext}} = (0, Bx_1, 0)$, for a constant and homogeneous magnetic field $B$ in the $x_3$-direction, and $\Sigma^3 = \frac{i}{2}[\gamma^1, \gamma^2]$ (see [6] for details). The couplings $G$ and $G'$ are related to the split gluon-quark vertex couplings $g_\parallel$ and $g_\perp$ through $G = (g_\parallel^2 + g_\perp^2)/2\Lambda^2$. 
\[ G' = (g_0^2 - q_s^2) / 2\Lambda^2, \] with \( \Lambda \) the energy scale of the effective NJL theory. We can define \( G' = \eta G \) with \( 0 < \eta < 1 \).

In the strong-field region, \( \epsilon B / \Lambda^2 \sim 1 \), and then \( \eta \approx 1 \). In this region, all the fermions are confined to the LLL and the only modes contributing to the coupling are the longitudinal ones. This is the origin of the anisotropy in the strong-coupling vertex in the presence of a magnetic field.

In the mean-field approximation, one can show that the theory has two separate chiral condensates, \( \langle \bar{\psi} \gamma^5 \psi \rangle \) and \( \langle \bar{\psi} \gamma^1 \gamma^2 \psi \rangle \), that minimize the free-energy and give rise to dynamical mass and dynamical anomalous magnetic moment. In the strong-field region, their solutions are respectively

\[ \sigma = \left( \frac{2G\Lambda}{G + G'} \right) \exp \left[ \frac{-2\pi^2}{(G + G')N_c q_B} \right] \] (6)

and \( \bar{\sigma} = \frac{G}{G + G'} \sigma \).

Chiral symmetry restoration in NJL models with MC\(\chi\)SB typically occurs as a second order transition at a critical temperature \( T_{C_{\chi}} \approx m_q \ll \sqrt{qB} \) with \( m_d \) the dynamical mass of the model considered. In agreement with this behavior, the critical temperature for the single flavor case considered in [6] was found to be

\[ T_{C_{\chi}} = 1.16 \sqrt{qB} \exp \left[ \frac{-2\pi^2}{(G + G')N_c q_B} \right]. \] (7)

In vacuum, the four-fermion interaction coupling constant \( G \) that enters in the dynamical mass found in NJL is connected via one-gluon exchange interactions with the dimensionless QCD coupling as \( G = 4\pi \alpha_s / \Lambda^2 \). In the presence of a strong magnetic field, the effective entering in the dynamical mass is actually \( G + G' \), and the corresponding relation becomes

\[ G + G' = \frac{g_0^2}{\Lambda^2} = \frac{4\pi \alpha_s}{q_B} \] (8)

where, in agreement with the strong-field regime, we have assumed that all the quarks are in the LLL, so that \( \eta = 1 \), \( \Lambda = \sqrt{qB} \), and the momentum transfer between quarks and gluons is effectively driven by the longitudinal modes (this is why in [5] we write \( \alpha_s^{\parallel} \)). In terms of \( \alpha_s^{\parallel} \), the critical temperature [6] can be written as

\[ T_{C_{\chi}} = 1.16 \sqrt{qB} \exp \left[ \frac{-\pi}{2\alpha_s^{\parallel}} \right]. \] (9)

We can infer from [6] and the behavior of \( \alpha_s^{\parallel} \) with the field shown in Fig. 1 that in the strong-field region, where \( |k| \ll \sqrt{qB} \), the critical temperature for chiral restoration decreases with the magnetic field, displaying an inverse catalysis behavior in qualitative agreement with the results of lattice QCD [5]. This becomes apparent in the plotting of Fig. 2, where we took \( \alpha_s^{\parallel} \) from Eq. 4 for \( N_c = 3 \) and energy scale \( k_3 = \Lambda_{QCD} \ll \sqrt{qB} \).

The strong-field approximation of course is valid for \( T \ll \sqrt{qB} \), what allows to neglect any T-dependence in \( \alpha_s^{\parallel} \). Notice that \( T_{C_{\chi}} \) is consistent with this approximation.

**Discussion.** We have found that in the strong-field limit the coupling of quarks and gluons becomes anisotropic with respect to the directions parallel and transverse to the magnetic field. The transverse coupling only gets contributions from gluons and does not change with the magnetic field, while the parallel coupling gets contributions from the quarks in the LLL and decreases with the field.

From the running of the coupling constant in vacuum

\[ \alpha_s(k) = \frac{\alpha_s^0(\mu)}{1 + \frac{\beta}{4\pi} \ln(k^2 / \mu)} \] (10)

where \( \beta = \frac{4}{\pi} N_c - \frac{2}{3} N_f \), it is clear that the quark contribution to \( \beta \) (\( \sim N_f \)) enters with a negative sign and thus produces screening. The gluon part (\( \sim N_c \)) however enters with a positive sign, so it gives rise to the antiscreening responsible for asymptotic freedom.

In a strong magnetic field, the quarks are confined to their LLL where they pair with antiquark via the MC\(\chi\)SB mechanism. In contrast to the vacuum quark contribution, the LLL quarks contribute to \( \alpha_s^{\parallel} \) with a positive sign (see Eqs. 2-4). Because of this, they produce antiscreening as the gluons. The physical mechanism behind this radical change of the quarks’ effect on a test color charge can be understood as follows. The pairs of LLL quarks and antiquarks form color-electric dipoles that would polarize in the presence of a test color charge. The dipole polarization leaves at long distances an effective color charge, equal in sign to the original test charge. The variation of the coupling constant with the field is then produced because the larger the magnetic field, the larger the density of states of the LLL, so more dipoles...
From the dependence on $\alpha_\parallel$ of the critical temperature in Eq. (9), we see that only if $\alpha_\parallel$ decreases with the magnetic field, the critical temperature can decrease with $B$, to be in agreement with lattice calculations. On the other hand, the magnetic field can only enter in the running coupling through the quark loops contribution, since the gluons are electrically neutral. Therefore, the inverse magnetic catalysis must be directly linked to the antiscreening effect of the LLL contribution.

Finally, we point out that our approach is essentially different from the one considered in Ref. [12], where the relevance of the LLL and the infrared dynamics in the MC/CSB was ignored, so the coupling was considered in the region of relative weak fields ($eB << A_{QCD}^2$), on which the authors proposed an ansatz for the running of the coupling and used momenta of the order of the magnetic field scale that equally affected the gluon and the quark contributions.

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1. D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. 53, 43 (1981); B. Svetitsky, Phys. Rept. 132, 1 (1986); H. Kogut-Ortmanns, Rev. Mod. Phys. 63, 473 (1996); K. Fukushima, T. Hatsuda, Rept. Prog. Phys. 74, 014001 (2011).
2. S. P. Klevansky and R. H. Lemmer, Phys. Rev. D 39, 3478 (1989); D. Ebert and M. K. Volkov, Phys. Lett. B 272, 86 (1991); S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992); I. V. Krive and S. A. Naftulin, Phys. Rev. D 46, 2737 (1992); K. G. Klimenko, Z. Phys. C 54, 323 (1992); V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Phys. Lett. B 349, 477 (1995); I. Shushpanov and A. V. Smilga, Phys. Lett. B 402, 351 (1997); A. Yu. Babansky, E. V. Gorbar, and G. V. Shchepanyuk, Phys. Lett. B 419, 272 (1998); D. Ebert, K. G. Klimenko, M. A. Vdovichenko, and A. S. Vshivtsev, Phys. Rev. D 61, 025005 (1999); N. O. Agasian and I. A. Shushpanov, Phys. Lett. B 472, 143 (2000); V. C. Zhubovskii, K. G. Klimenko, V. V. Khudyakov, D. Ebert, JETP Lett. 74, 523 (2001); D. Kabat, K. Lee, and E. Weinberg, Phys. Rev. D 66, 014004 (2002); E. S. Fraga and A. J. Mizher, Phys. Rev. D 78, 025016 (2008); A. J. Mizher, M. N. Chernodub, E. S. Fraga, Phys.Rev. D 82, 105016 (2010); P. Watson and H. Reinhardt, Phys. Rev. D 89, 045005 (2014).
3. V.V. Shokov, A. Yu. Illarionov and V.D. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009).
4. K. G. Klimenko, Z. Phys. C 54, 323 (1992); Teor. Mat. Fiz. 90, 3 (1992); V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Phys. Rev. Lett. 73 (1994) 3499; Nucl. Phys. B 563, 361 (1999); C. N. Leung, Y. J. Ng and A. W. Achle, Phys. Rev. D 54, 4181 (1996); D.-S. Lee, C.N. Leung and Y.J. Ng, Phys. Rev. D 55, 6504 (1997); E. J. Ferrer and V. de la Incera, Phys. Rev. D 58, 065008 (1998); E. J. Ferrer and V. de la Incera, Phys. Lett. B 481, 287 (2000); Yu. I. Shilnov, and V.V. Chitov, Phys. Atom. Nucl. 64 (2001) 2051 [Yad. Fiz. 64 (2001) 2138]; C.N. Leung and S.-Y. Wang, Nucl. Phys. B 747, 266 (2006); N. Sadooghi, A. S. Jallili, Phys. Rev. D 76, 065013 (2007); E. Rojas, A. Ayala, A. Bashir, and A. Raya, Phys. Rev. D 77, 093004 (2008); A. Raya and E. Reyes, Phys. Rev. D 82 016004 (2010).
5. E. J. Ferrer and V. de la Incera, Phys. Rev. Lett. 102, 052002 (2009); Nucl. Phys. B 824, 217 (2010).
6. E. J. Ferrer, V. de la Incera, I. Portillo and M. Quiroz, Phys. Rev. D 89, 054034 (2014).
7. R. Gatto, M. Ruggieri, Lett. Notes Phys. 871, 87 (2013).
8. G. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, S. Krieg, A. Schäfer, and K. K. Szabó, JHEP 1202, 044 (2012); G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, and A. Schäfer, Phys. Rev. D 86, 071502 (2012).
9. K. Fukushima and Y. Hidaka, Phys. Rev. Lett. 110, 031601 (2013).
10. K. Kojo and N. Su, Phys. Lett. B 720, 192 (2013); Phys. Lett. B 726 839 (2013).
11. F. Bruckmann, G. Endrodi and T. G. Kovacs, JHEP 1304, 112 (2013); M. Ferreira, P. Costa, O. Lourenço, T. Frederico, and C. Providência, Phys.Rev. D 89, (2014) 116011.
12. R. L. Farias, K. P. Gomes, G. Krein and M. B. Pinto, arXiv:1404.3931 [hep-ph].
13. P.V. Buividovich, M.N. Chernodub, E.V. Luschevskaya and M. I. Polikarpov, Phys. Lett. B 682, 484 (2010).
14. V. A. Miransky and I. A. Shovkovy, Phys. Rev. D 66, 045006 (2002).
15. Yu. A. Simonov, Phys. At. Nucl. 74, 1223 (2011); 58, 107 (1995).
16. M. A. Andreichikov, V. D. Orlovsky and Yu. A. Simonov, Phys. Rev. Lett. 110, 162002 (2013).
17. I. A. Batalin and A. E. Shabad, Zh. Eksp. Teor. Fiz. 58 (1970) 1223 (1971); A. E. Shabad, Ann. Phys. (N.Y.) 90, 166 (1975), A. E. Shabad, Lettere al Nuovo Cim. 2, 457 (1972).
18. E. J. Ferrer, E. S. Fradkin and V. de la Incera, Phys. Lett. B 248, 281 (1990).
19. S. Ferrari and M. Porrati, Mod. Phys. Lett. A 8, 2497 (1993); E. J. Ferrer and V. de la Incera, Phys. Rev. D 49, 2926 (1994); Int. J. Mod. Phys. A 11, 3875 (1996).
20. D. K. Hong, Phys. Lett. B 445, 36 (1998).