Status and prospects for $CPT$ and Lorentz invariance violation searches in neutral meson mixing

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Abstract

An overview of current experimental bounds on $CPT$ violation in neutral meson mixing is given. New values for the $CPT$ asymmetry in the $B^0$ and $B^0_s$ systems are deduced from published BaBar, Belle and LHCb results. With dedicated analyses, LHCb will be able to further improve the bounds on $CPT$ violation in the $D^0$, $B^0$ and $B^0_s$ systems. Since $CPT$ violation implies violation of Lorentz invariance in an interacting local quantum field theory, the observed $CPT$ asymmetry will exhibit sidereal- and boost-dependent variations. Such $CPT$-violating and Lorentz-violating effects are accommodated in the framework of the Standard-Model Extension (SME). The large boost of the neutral mesons produced at LHCb results in a high sensitivity to the corresponding SME coefficients. For the $B^0$ and $B^0_s$ systems, using existing LHCb results, we determine with high precision the SME coefficients that are not varying with sidereal time. With a full sidereal analysis, LHCb will be able to improve the existing SME bounds in the $D^0$, $B^0$ and $B^0_s$ systems by up to two orders of magnitude.
1 Introduction

In the weak interaction of the Standard Model, the symmetries under transformations of charge conjugation ($C$), parity ($P$), and time reversal ($T$) are broken. Nevertheless, the combined $CPT$ transformation is observed to be an exact fundamental symmetry of nature. From a theoretical perspective, $CPT$ symmetry is required by any Lorentz-invariant, local quantum field theory. Many experimental searches for $CPT$ violation have been performed over the last decades. Interferometry in the particle-antiparticle mixing of neutral mesons is a natural and very sensitive method to search for deviations from $CPT$ invariance. Since most $CPT$ tests have been performed with neutral kaons, progress can still be made in the $D^0$, $B^0$, and $B^0_s$ systems.

As $CPT$ violation implies Lorentz violation in an interacting local quantum field theory [1], any $CPT$-violating observable must also break Lorentz invariance. In the framework of the Standard Model Extension [2,3] (SME), spontaneous $CPT$ violation and Lorentz invariance violation appear in a low-energy effective field theory. In this sense, small $CPT$-violating effects at low energies provide a window to the quantum gravity scale [4]. Such effects are expected to be suppressed by $m^2/M_{Pl}$, with $M_{Pl} \approx 10^{19}$ GeV the Planck mass and $m$ the relevant low-energy mass, which depends on the underlying unified theory and possibly ranges from the mass of the neutral meson to the electroweak mass [5]. The Lorentz violation introduces a boost- and direction-dependent variation in the $CPT$-violating parameters. From an experimental point of view, the direction dependence results in a modulation with the sidereal phase. Such modulations would provide an unambiguous signature of $CPT$ violation.

We will show that a high sensitivity to these effects can be obtained by exploiting the large sample of heavy flavour decays obtained at the LHCb experiment, in particular taking advantage of the forward boost of the neutral mesons. Using published LHCb results, corresponding to a luminosity of 1 fb$^{-1}$, we can already deduce improved constraints on the SME parameters. Based on naive extrapolations, further improvements are possible with dedicated analyses on the existing 3 fb$^{-1}$ data set.

2 Formalism

The time evolution of a neutral meson system, $P^0 - \bar{P}^0$, is governed by an effective $2 \times 2$ Hamiltonian $H = M - i \Gamma/2$. Following the notation in Ref. [6], we write the light and heavy mass eigenstates, with eigenvalues $m_{L,H} - i \Gamma_{L,H}/2$, as

$$|P_L\rangle = p\sqrt{1-z}|P^0\rangle + q\sqrt{1+z}|\bar{P}^0\rangle$$

$$|P_H\rangle = p\sqrt{1+z}|P^0\rangle - q\sqrt{1-z}|\bar{P}^0\rangle,$$

(1)

where $p$ and $q$ spawn the eigenvectors under $CPT$ symmetry and $z$ is the complex, $CPT$-violating parameter. The definition of $z$ is independent of phase convention [7]. The mixing parameters are defined as $\Delta m \equiv m_H - m_L$ and $\Delta \Gamma \equiv \Gamma_H - \Gamma_L$, and the average mass and decay rate as $m \equiv (M_{11} + M_{22})/2$ and $\Gamma \equiv (\Gamma_{11} + \Gamma_{22})/2$. This definition implies that
\( \Delta \Gamma < 0 \) for the \( K^0, B^0 \) and \( B^0_s \) systems, and \( \Delta \Gamma > 0 \) for the \( D^0 \) system in the Standard Model. The CPT-violating parameter in \( P^0, \bar{P}^0 \) mixing can be written as
\[
z = \frac{\delta m - i \delta \Gamma / 2}{\Delta m - i \Delta \Gamma / 2},
\]
where \( \delta m \equiv (M_{11} - M_{22}) \) and \( \delta \Gamma \equiv (\Gamma_{11} - \Gamma_{22}) \) are the differences of the diagonal mass and decay rate matrix elements of \( H \). This equation makes clear that \( z \) is sensitive to small values of \( \delta m \) or \( \delta \Gamma \) due to the smallness of \( \Delta m \) and \( \Delta \Gamma \) in neutral meson systems.

By measuring the time-dependent decay rates of an initial \( P^0 \) or \( \bar{P}^0 \) state to a final state \( f \) or \( \bar{f} \), information on \( z \) can be obtained. For simplicity, we only consider CPT violation in \( P^0, \bar{P}^0 \) mixing. Direct CPT violation is experimentally difficult to separate from direct CP-violating effects. In both cases, it causes a difference in the instantaneous decay amplitudes, i.e., \( A_f \neq \bar{A}_f \), where \( A_{f, \bar{f}} (\bar{A}_{f, \bar{f}}) \) is the direct decay amplitude of a \( P^0 (\bar{P}^0) \) meson to a final state \( f \) or \( \bar{f} \). In the following, any direct CP-violating term implicitly includes possible direct CPT violation. For a complete expression of the decay rates we refer to Ref. [6]. Although those equations apply to the more general case of coherent production of \( B^0, \bar{B}^0 \) pairs, we will ignore this additional complication here and assume incoherent production by setting the decay amplitude of the tagging particle to either zero or one.

It is instructive to construct an observable CPT asymmetry
\[
A_{CPT}(t) = \frac{P_f(t) - \bar{P}_f(t)}{P_f(t) + \bar{P}_f(t)},
\]
where \( P_f (\bar{P}_f) \) is the time-dependent decay probability of an initial \( P^0 \) (\( \bar{P}^0 \)) meson to a final state \( f \) (\( \bar{f} \)). For decays to pure flavour-specific final states (i.e., \( A_f = \bar{A}_f = 0 \)), this asymmetry can be written as
\[
A_{CPT}(t) = A_{dir} + \frac{2 \text{Re}(z) \sinh \Delta \Gamma t / 2 - 2 \text{Im}(z) \sin \Delta m t}{(1 + |z|^2) \cosh \Delta \Gamma t / 2 + (1 - |z|^2) \cos \Delta m t},
\]
where the direct CP asymmetry \( A_{dir} \equiv (|\bar{A}_f|^2 - |A_f|^2) / (|\bar{A}_f|^2 + |A_f|^2) \) is assumed to be small. On the other hand, the CP asymmetry, defined as
\[
A_{CP}(t) = \frac{\bar{P}_f(t) - P_f(t)}{\bar{P}_f(t) + P_f(t)},
\]
and the CPT asymmetry become equivalent for decays to CP eigenstates \( f = \bar{f} \), and their effects become automatically connected. The CPT or CP asymmetry can be written as
\[
A_{CPT,CP}(t) = \left[ A_{mix}^2 / 2 + D_f \text{Re}(z) \right] \cosh \Delta \Gamma t / 2 - \left[ C_f + D_f \text{Re}(z) \right] \cos \Delta m t + \left[ D_f A_{mix}^2 / 2 + \text{Re}(z) \right] \sinh \Delta \Gamma t / 2 + \left[ S_f - \text{Im}(z) \right] \sin \Delta m t,
\]
where \(D_f = 2 \text{Re}(\lambda_f)/(1 + |\lambda_f|^2)\), \(C_f = (1 - |\lambda_f|^2)/(1 + |\lambda_f|^2)\) and \(S_f = 2 \text{Im}(\lambda_f)/(1 + |\lambda_f|^2)\). The parameter \(\lambda_f = (q/p)(\bar{A}_f/A_f)\) is introduced for convenience, and \(A_f^\text{mix} = (1 - |q/p|^4)/(1 + |q/p|^4)\) describes CP violation in mixing only. In the absence of CP violation in mixing (i.e., \(|q/p| = 1\)), \(C_f\) is equivalent to \(A_f^\text{dir}\). Only leading-order terms in \(\lambda_f\) and \(z\) are retained in Eq. 6. Comparing Eqs. 4 and 6, it becomes apparent that flavour-specific final states and CP eigenstates have different, complementary sensitivities to \(\text{Re}(z)\) and \(\text{Im}(z)\). We will come back to this point later.

Up to now we have assumed that \(z\) is a constant of nature for each of the four neutral meson systems. We will refer to this assumption as the classical approach. In the SME Lagrangian, CP-violating and Lorentz-violating terms are introduced for the fermions with coupling coefficients \(a_\mu\). The observable effect is determined by the contributions from the two valence quarks, \(q_1\) and \(\bar{q}_2\), in a meson as \(\Delta a_\mu \simeq a_\mu^q - a_\mu^\bar{q}\), hereby ignoring small effects from binding and normalization. In the SME approach, the equations above remain valid, but now \(z\) depends on the four-velocity \(\beta^\mu = \gamma(1, \vec{\beta})\) of the neutral meson as

\[
z \simeq \frac{\beta^\mu \Delta a_\mu}{\Delta m - i \Delta \Gamma/2}.
\]

An overview of experimental bounds on \(\Delta a_\mu\) and other SME parameters is given in Ref. [9]. In the SME, \(\Delta a_\mu\) is required to be real [10], which implies \(\delta \Gamma = 0\). The real and imaginary parts of \(z\) then become connected through

\[
\text{Re}(z) \Delta \Gamma = 2 \text{Im}(z) \Delta m.
\]

As we will see later, this constraint has implications for CP violation searches within the SME framework.

In such a search, the four-velocity of the neutral mesons at any time needs to be determined with respect to fixed stars. A useful reference frame is the Sun-centred frame defined in Ref. [10]. In this frame, the \(Z\)-axis is directed North, following the rotation axis of Earth, the \(X\)-axis points away from the Sun at the vernal equinox and the \(Y\)-axis complements the right-handed coordinate system. For an experiment where the neutral mesons are produced in a horizontal direction, fixed with respect to the Earth’s coordinate system, the dependence on the four-velocity can be written as

\[
\beta^\mu \Delta a_\mu = \gamma[\Delta a_0 + \beta \Delta a_Z \cos \chi + \beta \sin \chi(\Delta a_Y \sin \Omega \hat{t} + \Delta a_X \cos \Omega \hat{t})],
\]

where \(\Omega\) is the sidereal frequency and \(\cos \chi = \cos \theta \cos \lambda\) with \(\theta\) the azimuth of the neutral mesons and \(\lambda\) the latitude. The time coordinate \(\hat{t}\) is chosen such that the boost direction aligns with the \(X\)-axis at \(\hat{t} = 0\) in the \(XY\) projection. We have used the same convention as in Ref. [10], where the spatial coordinates of the \(\Delta a_\mu\) field are defined such that \(\Delta a^{X,Y,Z} = -\Delta a_{X,Y,Z}\). Equation [9] makes clear that \(z\) not only depends on the size of the boost, but also that it has a constant component, independent of the sidereal phase, and a component that exhibits a sidereal modulation. The sidereal variation is largest when the experiment is oriented east-west or when it is close to the North Pole. For the LHCb experiment, we determine the latitude \(\lambda = 46.24^\circ\) and azimuth \(\theta = 236.3^\circ\) east of north, which gives \(\cos \chi = -0.38\) and \(\sin \chi = 0.92\). This means that the constant component scales with \((\Delta a_0 - 0.38 \Delta a_Z)\) and that the sidereal variation at LHCb is close to maximal.
3 Experimental results and potential measurements

In the following, we present an overview of experimental searches for CPT violation in the four neutral meson systems. We interpret published results that are sensitive to CPT violation. These new values are summarized in Table 1 and discussed in the following. We also include prospects for analyses that can be conducted with current data from the LHCb experiment. The expected sensitivities on the CPT-violating parameters with the existing 3 fb\(^{-1}\) data set are given in Table 2.

Table 1: Overview of new values derived in this paper from published results, compared to existing CPT violation results. The new values for \(\Delta a_0 - 0.38\Delta a_Z\) in the \(B^0\) and \(B^0_s\) systems should be regarded as crude estimates as they are based on an estimate for the average \(B\) momentum.

| System | Parameter | Current best value | New value |
|--------|-----------|--------------------|-----------|
| \(B^0\) | \(\text{Re}(z)\) | (1.9 ± 4.0)% | \(0.7 ± 2.4)%\(^\dagger\) |
| \(N_B(\Delta a_0 - 0.30\Delta a_Z)\) | \((-3.0 ± 2.4) \times 10^{-15}\text{GeV}\) | \((0.9 ± 2.8) \times 10^{-15}\text{GeV}\) |
| \(\Delta a_0 - 0.38\Delta a_Z\) | | | |
| \(B^0_s\) | \(\text{Re}(z)\) | \(-\) | \((6 ± 4)\)% |
| \(\Delta a_T\) | \((3.7 ± 3.8) \times 10^{-12}\text{GeV}\) | \((5 ± 3) \times 10^{-14}\text{GeV}\) |
| \(\Delta a_0 - 0.38\Delta a_Z\) | | | |

\(^\dagger\) \(N_B \equiv \Delta \Gamma_d/\Delta m_d\), which is about 1/190 in the Standard Model.

\(^\dagger\dagger\) \(\Delta a_T\) is the constant component of \(\Delta a_{\mu}\) that depends here on the orientation of \(D^0\).

Table 2: Expected statistical sensitivities (one standard deviation) on CPT parameters with the existing 3 fb\(^{-1}\) data set from LHCb using the listed decay modes, compared to current experimental limits. The uncertainties are expected to be dominated by the statistical uncertainty as in the current measurements.

| System | Parameter | Current best limit | LHCb 3 fb\(^{-1}\) | Decay mode |
|--------|-----------|--------------------|-----------------|------------|
| \(D^0\) | \(|\text{Re}(z)y - \text{Im}(z)x|\) | (0.83 ± 0.77)% \(^\dagger\) | 0.02% \(^\dagger\) | \(D^0 \to K^\mp\pi^\mp\) |
| \(\Delta a_{\mu}\) | \(~ 3 \times 10^{-13}\text{GeV}\) | \(~ 1 \times 10^{-14}\text{GeV}\) | \(D^0 \to K^-\pi^+\) |
| \(B^0\) | \(\text{Im}(z)\) | \((-0.8 ± 0.4)\)% | 0.1% | \(B^0 \to D^{(*)\mp}\mu^\mp\nu_{\mu}\) |
| \(\text{Re}(z)\) | \((1.9 ± 4.0)\)% | 7% | \(B^0 \to J/\psi K_{s}^0\) |
| \(\Delta a_{\mu}\) | \(\mathcal{O}(10^{-13})\text{GeV}\) | \(~ 1 \times 10^{-15}\text{GeV}\) | \(B^0 \to J/\psi K_s^0\) |
| \(B_{s}^0\) | \(\text{Im}(z)\) | \(~ 0.4\)% | \(B_{s}^0 \to D_{s}^\mp\pi^\mp\) |
| \(\text{Re}(z)\) | \(~ 2\)% | \(B_{s}^0 \to J/\psi\phi\) |
| \(\Delta a_{\mu}\) | \(\mathcal{O}(10^{-12})\text{GeV}\) | \(~ 1 \times 10^{-15}\text{GeV}\) | \(B_{s}^0 \to J/\psi\phi\) |

\(^\dagger\) Assuming that \(x \approx y \approx 0.5\%).
3.1 Neutral kaons

In the neutral kaon system, there are many experimental searches for CPT violation. Most of them have been performed within the classical framework, i.e., assuming $z$ to be constant. In the PDG review [18], combining results from the KLOE, KTeV, CPLEAR and NA48 experiments, average values of $\text{Re}(\delta) = (2.4 \pm 2.3) \times 10^{-4}$ and $\text{Im}(\delta) = (-0.7 \pm 1.4) \times 10^{-5}$ are reported, where $\delta \approx -z/2$. An experimental limit on direct CPT violation is also included in this review.

A search for sidereal variations in the SME framework has been performed at the KLOE experiment [19]. The kaons are produced from the $\phi$ resonance, which has a small boost of $\beta\gamma \approx 0.015$, and detected in the $\pi^+\pi^-$ final state. Limits on all four SME parameters are reported with uncertainties on $\Delta a_{\mu}$ of about $2 \times 10^{-18}$ GeV.

Another search for sidereal variations using KTeV data is presented in Ref. [20], which has not been published in a peer-reviewed journal. Due to the high boost of the uncorrelated kaons ($\beta\gamma \approx 100$), strong limits on the sidereal-phase-dependent SME parameters have been set to $\Delta a_{X,Y} < 9.2 \times 10^{-22}$ GeV at 90% confidence level (CL). Kaons produced at the E773 experiment are also highly boosted ($\beta\gamma \approx 100$). Using E773 results, a bound on the constant SME parameters has been determined in Refs. [8,16] to $|\Delta a_0 - 0.6 \Delta a_Z| \lesssim 5 \times 10^{-21}$ GeV. Even though cross sections for kaon and $\phi$ production are high at the LHC, it will be difficult for LHCb to compete with the dedicated kaon experiments due to the limited decay time acceptance (roughly up to one $K^0_S$ lifetime), lower boost and larger backgrounds.

3.2 Neutral charm

Only the FOCUS collaboration has reported limits on CPT violation in $D^0$ mixing [17]. About 35k Cabibbo-favoured $D^0 \rightarrow K^-\pi^+$ decays have been analysed, both in the classical and SME approach. This final state is not a pure flavour-specific eigenstate, since there is also a small contribution from doubly Cabibbo-suppressed $D^0 \rightarrow K^+\pi^-$ decays. Due to the small mixing in the $D^0$ system [12], the CPT asymmetry can be approximated to first order as

$$A_{\text{CPT}}(t) = A_{\text{dir}} - \sqrt{R_D} \sin \phi (x \cos \delta + y \sin \delta) \Gamma t + (\text{Re}(z) y - \text{Im}(z) x) \Gamma t ,$$

where $x \equiv \Delta m/\Gamma$, $y \equiv \Delta\Gamma/2\Gamma$, $R_D = (0.349 \pm 0.004)\%$ [12] is the decay rate ratio of doubly Cabibbo-suppressed over Cabibbo-favoured decays and $\phi$ and $\delta$ are the corresponding weak and strong phases. The second term, the contribution from CP violation, is maximally of $\mathcal{O}(10^{-4})$ [12] and is neglected in the FOCUS analysis. In their classical analysis, a value of $\text{Re}(z) y - \text{Im}(z) x = (0.83 \pm 0.77)\%$ is reported. Assuming $x \approx y \approx 0.5\%$, this measurement provides only a weak bound on $\text{Re}(z) - \text{Im}(z)$ of $\mathcal{O}(1)$.

At LHCb, many more $D^0 \rightarrow K^-\pi^+$ decays are available. In the current $3\text{ fb}^{-1}$ data sample, more than 50M Cabibbo-favoured decays have been observed [21], which means a possible improvement of the FOCUS measurement by a factor of about 40 and a precision

\footnote{Natural units are used with $c = 1$.}

\footnote{The inclusion of charge-conjugated decay modes is implicit.}
on \( \text{Re}(z)y - \text{Im}(z)x \) of 0.02\%. At this precision, the \( CP \)-violating term cannot be ignored anymore and needs to be taken into account in the analysis. In addition, the observed \( CP \) asymmetry will include effects from production and detection asymmetries. Fortunately, the latter two effects are expected to be independent of the \( D^0 \) decay time, adding only to the constant contribution from direct \( CP \) violation, \( A_{\text{dir}} \).

The same FOCUS paper \[17\] also presents a full sidereal analysis in the SME framework. The average boost of the \( D^0 \) mesons is \( \langle \beta \gamma \rangle \approx 39 \). Due to the SME constraint, the \( CP \)-violating term in Eq. \[10\] is zero and a further expansion in \( x \) and \( y \) is required, which reduces the sensitivity to \( \text{Re}(z) \). The expansion to second and third order in decay time gives

\[
A_{\text{dir}}(t) = \frac{\text{Re}(z)(x^2 + y^2)(\Gamma t)^2}{2x} \times \left[ \frac{xy}{3} \Gamma t + \sqrt{R_D}(x \cos \delta + y \sin \delta) \right],
\]

where \( A_{\text{dir}} \) and all \( CP \)-violating terms are omitted. Assuming again \( x \approx y \approx 0.5\% \), the uncertainties on the \( \Delta a_\mu \) parameters are found to be about \( 3 \times 10^{-13} \text{ GeV} \). At LHCb, with their large sample of \( D^0 \rightarrow K^-\pi^+ \) decays and assuming a comparable boost factor, it should be possible to improve the FOCUS bounds by a factor 40. Note, however, that it will not be possible to constrain \( \text{Re}(z) \) to be smaller than one, since \( \text{Re}(z) \) is suppressed in Eq. \[11\] by \( \mathcal{O}(10^{-6}) \). Nevertheless, no assumptions on the smallness of \( |z| \) have been made so far. Extrapolating to the statistically larger sample, LHCb should be able to reach a sensitivity on the \( \Delta a_\mu \) parameters of about \( 1 \times 10^{-14} \text{ GeV} \).

### 3.3 \( B^0 \) mesons

Due to the small value of \( \Delta \Gamma_d \) in the \( B^0 \) system, decays to flavour-specific final states are sensitive to \( \text{Im}(z) \), while decays to \( CP \) eigenstates are sensitive to \( \text{Re}(z) \) (cf. Eqs. \[4\] and \[5\]). This is a key point that is used below for \( B^0 \) decays, but it is also valid for \( B^0_s \) decays. Using only dilepton (i.e., flavour-specific) final states, the BaBar collaboration published \( \text{Im}(z) = (-1.39 \pm 0.80)\% \) \[22\]. Similarly, the Belle collaboration reported \( \text{Im}(z) = (-0.57 \pm 0.47)\% \), mainly using flavour-specific final states \[11\]. The average value of both results is \( \text{Im}(z) = (-0.8 \pm 0.4)\% \) \[18\]. Using the same dilepton final states, the BaBar collaboration also reported a measurement of \( \text{Re}(z)\Delta \Gamma_d = (-7.1 \pm 4.4) \times 10^{-3} \text{ ps}^{-1} \) \[22\]. When inserting the theoretical expectation of \( \Delta \Gamma_d \approx -(2.7 \pm 0.7) \times 10^{-3} \text{ ps}^{-1} \) \[23\], this measurement gives only a weak constraint on \( \text{Re}(z) \) of \( \mathcal{O}(2) \). Since \( |z|^2 \) terms have been ignored in this analysis, this means that a higher sensitivity to \( \text{Re}(z) \) could have been achieved when including \( |z|^2 \) terms in the fits to the decay rates.

Due to the relatively low tagging performance in a hadron-collider environment, the untagged asymmetry for flavour-specific decays, defined as

\[
A_{\text{untagged}}(t) \equiv \frac{[P_f(t) + \overline{P}_f(t)] - [P_f(t) + \overline{P}_f(t)]}{[P_f(t) + \overline{P}_f(t)] + [P_f(t) + \overline{P}_f(t)]},
\]

gives a higher sensitivity to \( \text{Im}(z) \) than the tagged asymmetry as defined in Eq. \[3\]. Including experimental effects from a possible detection asymmetry \( A_D \) and from a production
asymmetry $A_P$, the observed asymmetry becomes

$$A_{\text{untagged}}^{\text{observed}}(t) = A_D + A_{\text{mix}}^{\text{obs}}/2 - \left( A_{\text{mix}}^{\text{obs}}/2 - A_P \right) \cos \Delta m_d t + \text{Im}(z) \sin \Delta m_d t , \quad (13)$$

whereby $|z|^2$ terms have been ignored and $\Delta \Gamma_d$ is approximated to be zero. Compared to Eq. (4), the sensitivity to $\text{Im}(z)$ is only reduced by a factor 2, rather than a reduction by a factor 20 - 30, which is the typical loss due to the flavour tagging in a hadronic environment. In Eq. (13), $A_{\text{mix}}^{\text{obs}}$ is the flavour-specific CP asymmetry in $B^0$ mixing. At LHCb, using inclusive $B^0 \to D^{(*)-}\mu^+\nu_\mu$ decays, a high-precision measurement of $\text{Im}(z)$ is possible, since the dilution of the amplitude of the oscillation due to the partial reconstruction is small [24]. We estimate about 3 million inclusive $B^0 \to D^{(*)-}\mu^+\nu_\mu$ decays in the 3 fb$^{-1}$ data set, using the observed yields in $B^0_s \to D^- \mu^+\nu_\mu$ decays [25] and the production ratio of $B^0$ and $B^0_s$ mesons [26]. Hence, a statistical precision on $\text{Im}(z)$ of 0.1% is in reach.

In $B^0$ decays to CP final states, $Re(z)$ appears in the cosine term of the time-dependent CP asymmetry. Neglecting CP violation in mixing (i.e., $A_{\text{mix}}^{\text{mix}} = 0$), and setting $\Delta \Gamma_d = 0$ and $\text{Im}(z) = 0$, the time-dependent (tagged) asymmetry from Eq. (6) becomes

$$A_{\text{CPT,CP}}(t) = D_f Re(z) - [C_f + D_f Re(z)] \cos \Delta m_d t + S_f \sin \Delta m_d t . \quad (14)$$

Effects from $\text{Im}(z)$ are expected to be negligible and this assumption can be tested with experimental input from flavour-specific decay modes as described above. Similarly, $A_{\text{mix}}^{\text{mix}}$ is also negligible at the current experimental precision [12]. Direct CP violation ($C_f$) and $Re(z)$ both contribute to the cosine term. In principle, the time-independent offset is also sensitive to $Re(z)$, however, this offset is additionally affected by production, detection and tagging asymmetries. Hence, in practice most information on $Re(z)$ will come from the oscillating term.

For $B^0$ decays to the CP final state $J/\psi K^0$, we can identify $C_f = 0$, $D_f = \cos 2\beta$ and $S_f = \sin 2\beta$, where $\beta$ is the usual CKM parameter. We ignored for simplicity small effects coming from CP violation in kaon and $B^0$ mixing and direct CP violation due to the penguin contributions. The contribution from direct CP violation gives the dominant uncertainty on $C_f$ and therefore on the determination of $Re(z)$. Theoretically, it is estimated to be at most a few times $10^{-3}$ [27]. Experimentally, the direct CP asymmetry in $B^+ \to J/\psi K^+$ decays is $(0.3 \pm 0.6)\%$ [18], which is expected to be largely equal to that in $B^0 \to J/\psi K^0_s$ decays using isospin symmetry [28]. Another experimental constraint comes from the $B^0 \to J/\psi \pi^0$ decay, which can be used to determine the direct CP violation in $B^0 \to J/\psi K^0_s$ to be $(1 \pm 1)\%$ [29].

The Belle collaboration has measured $Re(z) = (1.9 \pm 5.0)\%$, where the sensitivity mainly comes from $B^0 \to J/\psi K^0_{s\pi}$ decays [11]. Similarly, the BaBar collaboration has measured with a small fraction of the data $Re(z)Re(\lambda)/|\lambda| = (1.4 \pm 4.9)\%$ [6]. We can remove the factor $Re(\lambda)/|\lambda| = D_f = \cos(2\beta) = 0.722^{+0.016}_{-0.020}$, where we used the measured value of the CKM angle $\beta$ from Ref. [30]. Then, this measurement translates to $Re(z) = (1.9 \pm 6.8)\%$, where the uncertainty from the factor $Re(\lambda)/|\lambda|$ is negligible. This result was left unnoticed in the PDG world average of $Re(z)$ [18]. Averaging here both numbers, we find $Re(z) = (1.9 \pm 4.0)\%$. Both results neglect the possible contribution
from direct CP violation. A more recent and accurate value on Re(z) can actually be obtained using the world average on the cosine coefficient of (0.5 ± 1.7)% \cite{12}. With Re(\lambda)/|\lambda| \approx 0.72 and setting C_f = 0, this results in Re(z) = (0.7 ± 2.4)%.

Finally, we briefly mention the BaBar analysis \cite{31} where the CP, CPT and T asymmetries are tested separately. For instance, CPT asymmetries for \( B^0 \) mixing are constructed by simultaneously interchanging the time ordering of initial \( B^0 \) and \( \bar{B}^0 \) decays and substituting \( K^0_L \) and \( K^0_S \) states. Although this method is statistically not competitive, it does allow to cleanly separate effects from CP, T and CPT violation. Unfortunately, such tests are only possible at the \( \Upsilon(4S) \) experiments, and not at hadron collider experiments where the \( B^0 \) mesons are produced incoherently and the reconstruction of \( K^0_L \) mesons is much more challenging.

In the SME framework, due to the constraint Re(z)\( \Delta \Gamma_d \approx 2\text{Im}(z)\Delta m_d \), the real part of z is about 380 times larger than the imaginary part. Without loss of generality, we used here the theoretical expectation value of \( \Delta \Gamma_d \) from Ref. \cite{23}; experimentally 2\( \Delta m_d/\Delta \Gamma_d \) is already bounded to be larger than 77 at 95% CL \cite{18}. Therefore, \( B^0 \) decays to CP eigenmodes are more sensitive to the \( \Delta a_\mu \) variables than \( B^0 \) decays to flavour-specific modes. The BaBar collaboration published a paper \cite{13} where the \( \Delta a_\mu \) parameters are determined in a full sidereal analysis. The boost of the \( B^0 \) mesons is \( \beta \gamma = 0.55 \). They used only dilepton events, rather than CP eigenmodes. Using the expected \( \Delta \Gamma_d \) value \cite{23}, the uncertainties on the \( \Delta a_\mu \) parameters are \( \sim (5 - 25) \times 10^{-13} \text{GeV} \), corresponding to uncertainties on Re(z) of order one. Just as in their classical analysis \cite{22}, this means that a higher sensitivity to \( \Delta a_\mu \) would have been possible in case \( |z|^2 \) terms are not ignored.

When using the location and orientation of the BaBar and Belle experiments and their measurements of Re(z), stronger constraints on the constant \( \Delta a_\mu \) term can be set, however, at this point we focus on LHCb where an even higher precision can be reached due to the larger boost of the \( B^0 \) mesons. The average momentum of b hadrons at LHCb is \( \langle p \rangle \approx 80 \text{GeV} \) \cite{32}, corresponding to a relativistic boost of \( \langle \beta \gamma \rangle \approx 15 \). The LHCb collaboration reported a value of \( C_f = (3 \pm 9)\% \) using \( B^0 \to J/\psi K^0_S \) decays in the 1 fb\(^{-1} \) data set \cite{14}. This corresponds to Re(z) = (4 ± 12)%. Using the LHCb beam direction, a measurement of the constant combination of SME parameters of \( (\Delta a_0 - 0.38 \Delta a_Z) = (0.9 \pm 2.8) \times 10^{-15} \text{GeV} \) is obtained. Although this number is only a crude estimate, mainly due to the uncertainty on the average \( B^0 \) momentum, it improves the current best value \cite{13} by two orders of magnitude. By making use of the \( B^0 \) momentum in each event and with a full sidereal analysis on the 3 fb\(^{-1} \) data set, LHCb should be able to reach a sensitivity on \( \Delta a_\mu \) of about \( 1 \times 10^{-15} \text{GeV} \).

### 3.4 \( B^0_s \) mesons

The discussion for the \( B^0_s \) system is very similar to that for the \( B^0 \) system. In this system \( \Delta \Gamma_s \) is not anymore negligible, but still small enough such that flavour-specific final states primarily give access to Im(z), while CP eigenmodes give access to Re(z). No dedicated CPT measurements have been done with \( B^0_s \) mesons to date. In the classical approach, LHCb would be able to measure Im(z) using the flavour-specific \( B^0_s \to D^-_s \pi^+ \)
decays. In the 3 fb\(^{-1}\) data set, \(N = 100k\) untagged signal decays can be expected\(^{33}\). Following Eq.\(^{13}\), this corresponds to a statistical uncertainty on \(\text{Im}(z)\) of \(\sqrt{2/N} = 0.4\%\) (see Table\(^2\)). Alternatively, also the more abundant inclusive \(B^0_s \rightarrow D^\pm_s \mu^\mp \nu_\mu\) decays can be used to measure \(\text{Im}(z)\). Due to the partial reconstruction, however, the worse time resolution washes out the oscillations already after a 1 ps (see Ref.\(^{24}\)), reducing the sensitivity.

Constraints on \(\text{Re}(z)\) can be made using \(B^0_s\) decays to the \(CP\) eigenstate \(J/\psi \phi\). This decay mode is the \(B^0_s\) equivalent of \(B^0 \rightarrow J/\psi K^0\). Equation\(^6\) gives the observable asymmetry. The phase \(\arg(\lambda_f) = \phi_s\) is expected\(^{30}\) and experimentally measured\(^{12}\) to be small, leading to \(D_f \approx 1\) and \(S_f \approx 0\). Any effect from \(A^\text{mix}\) can be ignored at the current level of precision\(^{12}\). The LHCb collaboration has published a value of \(|\lambda_f| = 0.94 \pm 0.04\) using the 1 fb\(^{-1}\) data set\(^{15}\). Ignoring again direct \(CP\) violation, a first evaluation of \(\text{Re}(z) \approx (1 - |\lambda_f|^2)/2 = (6 \pm 4)\%\) can be made in the \(B^0_s\) system.

In the \(B^0\) system, the SME constraint \(\text{Re}(z) \Delta \Gamma_s \approx 2 \Delta m_s \text{Im}(z)\) leads to a \(\text{Re}(z)\) that is a factor 450 larger than \(\text{Im}(z)\). Even more than for the \(B^0\) system, this means that one should focus on decays to \(CP\) final states, such as \(B^0_s \rightarrow J/\psi \phi\). An interesting relation between the \(K^0, B^0\) and \(B^0_s\) systems is pointed out in Ref.\(^{16}\). As the expectation value of \(\Delta a_\mu\) is dominated by the valence quarks, a sum rule relating these three neutral meson systems can be written as

\[
\Delta a^K_\mu - \Delta a^B_\mu + \Delta a^{B^0}_\mu \approx 0.\]  

(15)

Since the constraints on \(\Delta a^K_\mu\) are most strong and compatible with zero, this sum rule implies that possible \(CPT\)-violating effects in the \(B^0\) and \(B^0_s\) system should be of the same order. In that sense, the \(B^\lambda\) system is more interesting, since the production rate of \(B^\lambda\) mesons is higher and the mass difference \(\Delta m_d\) is smaller (c.f. Eq.\(^7\)). Ideally, the mass difference should be such that one could just measure one period of oscillation, which is the case for \(B^0\) oscillations. Nevertheless, it remains important to measure possible \(CPT\) violation in all possible systems to verify this sum rule.

Using the like-sign dimuon asymmetry measured in the D0 data, a value for \(\Delta a_\mu\) has been derived in Ref.\(^{16}\). Assuming that the only source of \(CPT\) violation comes from \(B^0_s\) decays (like-sign dimuons originate from both \(B^0\) and \(B^0_s\) mixing) and using the average boost of \(\langle \beta \gamma \rangle = 4.1\), the constant \(\Delta a_\mu\) term becomes \((3.7 \pm 3.8) \times 10^{-12}\) GeV. This corresponds to \(\text{Re}(z) = 1.0 \pm 0.8\). Stronger limits on \(\text{Re}(z)\) can be set with the \(CP\) eigenmode decay \(B^0_s \rightarrow J/\psi \phi\). Using again \(\text{Re}(z) = (6 \pm 4)\%\), derived from LHCb results\(^{15}\), and taking as average boost \(\langle \beta \gamma \rangle \approx 15\), we find as a crude estimate \((\Delta a_0 - 0.38 \Delta a_Z) = (5 \pm 3) \times 10^{-14}\) GeV, which is an improvement by two orders of magnitude. With the existing LHCb data set and a dedicated sidereal analysis, it should be possible to reach a sensitivity of about \(1 \times 10^{-14}\) GeV or below.

4 Conclusion

We have presented new results on \(CPT\) violation in \(B^0\) and \(B^0_s\) mixing in both the classical and SME approach, derived from published BaBar, Belle and LHCb results. The new
results in the SME approach should be regarded a crude estimates, as a precise estimate of the average $B$ momentum is missing. In both approaches there is a significant improvement over previous results (see Table 1). LHCb should be able to further improve these numbers in the $B^0$ and $B_s^0$ systems, as well as in the $D^0$ system, with dedicated analyses on the existing 3 fb$^{-1}$ data set (see Table 2). In most cases these possible LHCb measurements would improve the current best values by orders of magnitude and the corresponding precision on $\Delta a_\mu$ is approaching the interesting region of $m^2/M_{Pl}$. Further improvements can be expected with the LHCb data from run II, starting in 2015. On the longer time scale, much stronger limits can be expected from Belle II and the LHCb upgrade.

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References

[1] O. Greenberg, CPT violation implies violation of Lorentz invariance, Phys. Rev. Lett. 89 (2002) 231602, arXiv:hep-ph/0201258.

[2] D. Colladay and V. A. Kostelecky, CPT violation and the standard model, Phys. Rev. D55 (1997) 6760, arXiv:hep-ph/9703464.

[3] D. Colladay and V. A. Kostelecky, Lorentz violating extension of the standard model, Phys. Rev. D58 (1998) 116002, arXiv:hep-ph/9809521.

[4] S. Liberati, Tests of Lorentz invariance: a 2013 update, Class. Quant. Grav. 30 (2013) 133001, arXiv:1304.5795.

[5] V. A. Kostelecky and R. Potting, CPT and strings, Nucl. Phys. B359 (1991) 545; V. A. Kostelecky and R. Potting, CPT, strings, and meson factories, Phys. Rev. D51 (1995) 3923, arXiv:hep-ph/9501341.

[6] BaBar collaboration, B. Aubert et al., Limits on the decay-rate difference of neutral-$B$ mesons and on CP, T, and CPT violation in $B^0$-$\bar{B}^0$ oscillations, Phys. Rev. D70 (2004) 012007, arXiv:hep-ex/0403002.

[7] V. A. Kostelecky, CPT, T, and Lorentz violation in neutral-meson oscillations, Phys. Rev. D64 (2001) 076001, arXiv:hep-ph/0104120.
[8] V. A. Kostelecky, *Sensitivity of CPT tests with neutral mesons*, Phys. Rev. Lett. **80** (1998) 1818, arXiv:hep-ph/9809572.

[9] V. A. Kostelecky and N. Russell, *Data tables for Lorentz and CPT violation*, Rev. Mod. Phys. **83** (2011) 11, arXiv:0801.0287.

[10] V. A. Kostelecky, *Signals for CPT and Lorentz violation in neutral meson oscillations*, Phys. Rev. **D61** (2000) 016002, arXiv:hep-ph/9909554.

[11] Belle collaboration, T. Higuchi et al., *Search for time-dependent CPT violation in hadronic and semileptonic B decays*, Phys. Rev. **D85** (2012) 071105, arXiv:1203.0930.

[12] Heavy Flavor Averaging Group, Y. Amhis et al., *Averages of b-hadron, c-hadron, and τ-lepton properties as of early 2012*, arXiv:1207.1158, updated results and plots available at [http://www.slac.stanford.edu/xorg/hfag/](http://www.slac.stanford.edu/xorg/hfag/).

[13] BaBar collaboration, B. Aubert et al., *Search for CPT and Lorentz violation in $B^0$-$\bar{B}^0$ oscillations with dilepton events*, Phys. Rev. Lett. **100** (2008) 131802, arXiv:0711.2713.

[14] LHCb collaboration, R. Aaij et al., *Measurement of the time-dependent CP asymmetry in $B^0 \to J/\psi K_S^0$ decays*, Phys. Lett. **B721** (2013) 24, arXiv:1211.6093.

[15] LHCb collaboration, R. Aaij et al., *Measurement of CP-violation and the $B^0_s$-meson decay width difference with $B^0 \to J/\psi K^+K^-$ and $B^0 \to J/\psi \pi^+\pi^-$ decays*, Phys. Rev. **D87** (2013) 112010, arXiv:1304.2600.

[16] A. Kostelecky and R. Van Kooten, *CPT violation and B-meson oscillations*, Phys. Rev. **D82** (2010) 101702, arXiv:1007.5312.

[17] FOCUS collaboration, J. Link et al., *Charm system tests of CPT and Lorentz invariance with FOCUS*, Phys. Lett. **B556** (2003) 7, arXiv:hep-ex/0208034.

[18] Particle Data Group, K. Olive et al., *Review of Particle Physics*, Chin. Phys. **C38** (2014) 090001.

[19] KLOE-2 collaboration, D. Babusci et al., *Test of CPT and Lorentz symmetry in entangled neutral kaons with the KLOE experiment*, Phys. Lett. **B730** (2014) 89, arXiv:1312.6818.

[20] H. Nguyen, *CPT results from KTeV*, in *CPT and Lorentz symmetry II* (V. A. Kostelecky, ed.), World Scientific, Singapore, 2002, arXiv:hep-ex/0112046.

[21] LHCb collaboration, R. Aaij et al., *Measurement of $D^0-\bar{D}^0$ mixing parameters and search for CP violation using $D^0 \to K^+\pi^-$ decays*, Phys. Rev. Lett. **111** (2013) 251801, arXiv:1309.6534.
[22] BaBar collaboration, B. Aubert et al., Search for $T$, $CP$ and $CPT$ violation in $B^0 - \bar{B}^0$ mixing with inclusive dilepton events, Phys. Rev. Lett. 96 (2006) 251802, arXiv:hep-ex/0603053.

[23] A. Lenz and U. Nierste, Theoretical update of $B_s - \bar{B}_s$ mixing, JHEP 06 (2007) 072, arXiv:hep-ph/0612167.

[24] LHCb collaboration, R. Aaij et al., Observation of $B^0_s$ mixing and measurement of mixing frequencies using semileptonic $B$ decays, Eur. Phys. J. C73 (2013) 2655, arXiv:1308.1302.

[25] LHCb collaboration, R. Aaij et al., Measurement of the flavour-specific $CP$-violating asymmetry $a_{s1}$ in $B^0_s$ decays, Phys. Lett. B728 (2014) 607, arXiv:1308.1048.

[26] LHCb collaboration, R. Aaij et al., Measurement of the fragmentation fraction ratio $f_s/f_d$ and its dependence on $B$ meson kinematics, JHEP 04 (2013) 001, arXiv:1301.5286.

[27] Y. Grossman, A. L. Kagan, and Z. Ligeti, Can the $CP$ asymmetries in $B \to \psi K^0_S$ and $B \to \psi K^0_L$ differ?, Phys. Lett. B538 (2002) 327, arXiv:hep-ph/0204212.

[28] R. Fleischer and T. Mannel, General analysis of new physics in $B \to J/\psi K$, Phys. Lett. B506 (2001) 311, arXiv:hep-ph/0101276.

[29] S. Faller, M. Jung, R. Fleischer, and T. Mannel, The golden modes $B^0 \to J/\psi K^0_S$ in the era of precision flavour physics, Phys. Rev. D79 (2009) 014030, arXiv:0809.0842.

[30] CKMfitter Group, J. Charles et al., $CP$ violation and the $CKM$ matrix: Assessing the impact of the asymmetric $B$ factories, Eur. Phys. J. C41 (2005) 1, arXiv:hep-ph/0406184, updated results and plots available at http://ckmfitter.in2p3.fr.

[31] BaBar collaboration, J. Lees et al., Observation of time reversal violation in the $B^0$ meson system, Phys. Rev. Lett. 109 (2012) 211801, arXiv:1207.5832.

[32] M. P. Altarelli and F. Teubert, $B$ physics at LHCb, Int. J. Mod. Phys. A23 (2008) 5117, arXiv:0802.1901.

[33] LHCb collaboration, R. Aaij et al., Precision measurement of the $B^0_s - \bar{B}^0_s$ oscillation frequency $\Delta m_s$ in the decay $B^0_s \to D_{s}^+ \pi^-$, New J. Phys. 15 (2013) 053021, arXiv:1304.4741.