Approximating symmetrized estimators of scatter via balanced incomplete \( U \)-statistics

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Abstract
We derive limiting distributions of symmetrized estimators of scatter. Instead of considering all \( n(n - 1)/2 \) pairs of the \( n \) observations, we only use \( nd \) suitably chosen pairs, where \( d \geq 1 \) is substantially smaller than \( n \). It turns out that the resulting estimators are asymptotically equivalent to the original one whenever \( d = d(n) \to \infty \) at arbitrarily slow speed. We also investigate the asymptotic properties for arbitrary fixed \( d \). These considerations and numerical examples indicate that for practical purposes, moderate fixed values of \( d \) between 10 and 20 yield already estimators which are computationally feasible and rather close to the original ones.

Keywords  Asymptotic normality · Incomplete \( U \)-statistic · Independent component analysis · Linear expansion · \( U \)-statistic