Modeling nonlinear in Bowman’s paradox: the case of Pakistan

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Abstract
Prospect theory frequently explains the empirical results of Bowman’s paradox (negative relationship between risk and return). However, the empirical econometric model of these researches is misspecified. This study used a data-driven approach to improve the econometric model. Empirical results based on the improved econometric model are also reinforced by data visualization to be illustrated in depth. For this purpose, we used the data of 622 listed firms on the Pakistan Stock Exchange from 2000 to 2019. Our results contradict the literature on prospect theory based on the improved econometric model.

Keywords Bowman’s paradox · Prospect theory · Modeling nonlinear relationship

JEL Classification G32 · C58

1 Introduction
The seminal work of Bowman (1980) observed that risk–return relationship is negatively associated across firms within industries. Several researchers empirically

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verified it and proposed different explanations. Following the categorization of Andersen et al. (2007) and Henkel (2000), these explanations can roughly be categorized into those based on prospect theory (Bowman 1982; Fiegenbaum and Thomas 1988, 1990; Fiegenbaum 1990; Jegers 1991; Johnson 1992; Sinha 1994; Gooding et al. 1996; Chou et al. 2009; Santacruz 2019; Nuir and Asri 2019), strategic and organizational factors (Bowman 1980; Bettis and Mahajan 1985; Jemison 1987; Andersen et al. 2007; Henkel 2009; Greve 2011) and model misspecifications (Ruefli 1990; Wiseman and Bromiley 1991; Oviatt and Bauerschmidt 1991; Gooding et al. 1996; Lehner 2000).

A few detailed reviews are also available on this topic, which discusses Bowman’s paradox in several dimensions (Ruefli et al. 1999; Bromiley et al. 2001; Nickel and Rodriguez 2002; Brockett et al. 2003).

The conditional decision-making framework is based on the level of risk preference; therefore, the risk–return relationship could change depending on the choice context. The condition is derived while comparing the firm’s performance with a particular threshold value (Denrell 2004, 2008; Andersen et al. 2007). Prospect theory states that if an individual’s forecast suggests the likelihood of an event is positive in the future, the individual’s attitude will be risk-averse. Therefore, high expected results would lead to a risk-averse attitude and, consequently, a positive relationship between risk and return and vice versa. Hence, the individual’s decision relies entirely on her information and the position around the reference point. Furthermore, it posits that the negative relationship is steeper than the positive one (Kahneman and Tversky 1979). Most of the empirical research is consistent with prospect theory by using Compustat data (Fiegenbaum and Thomas 1988; Chou et al. 2009; Kliger and Tsur 2011; Santacruz 2019), while Jegers (1991) used Belgian data, Sinha (1994) employed Australian data, and Nuir and Asri (2019) used Indonesian data set.

The literature used three hypotheses to validate the existence of the Bowman paradox: (1) negative relationship between risk and return below the reference point, (2) above the reference point, the relationship is positive, and (3) the relationship below the reference point is steeper than after the reference point (Chou et al. 2009; Nuir and Asri 2019).

We have found the following issues in the literature. (1) In general, the literature on Bowman’s paradox used US financial data, and very few employed the other data sets; hence, there is a need to validate Bowman’s paradox with different economic environments (Nickel and Rodriguez 2002; Brockett et al. 2003). Therefore, we used the data of listed companies on the Pakistan Stock Exchange to compare the empirical results with the literature. (2) The literature estimated a simple linear regression model to examine the relationship between risk and return. However, in the case of the risk–return relationship, the data suffer from several issues, such as outliers at the extreme position that adversely affect the line’s slope (See Sect. 2). Moreover, in heteroscedasticity, the estimated parameters’ standard error is negatively affected, and hence t statistics, confidence intervals and all inference. (3) Furthermore, some researchers divided the data into two halves, i.e., before and after the middle value, and then set up a separate linear regression for each half. Estimating linear regression can raise issues, such as (a) the loss of degrees of freedom, and hence inconsistent estimates, and (b) choice of middle value of the data based on central tendency measure is itself an issue. Therefore, to overcome this issue, we suggest linear regression.
with the quadratic term, which is the best fit for the risk, returns data and avoids the abovementioned issues. (4) Moreover, in the present study, we replace the ordinary least-squares method with the generalized least-squares method to overcome the problem of heteroscedasticity.

This research aims to contribute to the empirical analysis of Bowman’s paradox with the modified econometric model to estimate the risk–return relationship more accurately. Furthermore, it will help researchers reconsider the risk–return relationship for estimation and analyze it based on the improved econometric model and data visualization to understand the relationship between risk and return in detail, respectively.

Section 1 has introduced the topic. Then, Sect. 2 presents the data and methodology while explaining the issues with the econometric model in detail. Finally, Sect. 3 concludes the study (Table 1).

2 The data and methodology

2.1 The data set

Since the study’s primary purpose is to test the statistical relationship between risk and return, we have collected the data on these variables from the Pakistan Stock Exchange of listed companies. We have collected the data from 2001 to 2019 and do not include the information on 2020 and 2021 for empirical analysis because of the massive amplification in financial vulnerability due to COVID-19. From 2001 to 2019, 952 firms registered on the Pakistan Stock Exchange, out of which 275 firms’ have been operating for less than five years and therefore are deleted from the selected sample. We have 677 firms that have been working for more than five years and remain listed on the Pakistan Stock Exchange. These 677 firms fall into 27 industries, as per the criteria of the Pakistan Stock Exchange. We have deleted those industries with less than ten firms and merged some related industries, such as textile spinning, textile weaving and textile composite. Finally, there are eight industries1 with 622 firms.2

The data set that we have in our hands is time series. The firm’s return is calculated as a median of the ROA series, and risk is calculated as a standard deviation of the ROA series, so we have a single observation against each firm (Fiegenbaum and Thomas 1988; Fiegenbaum 1990; Sinha 1994). We discarded (trimmed) those firms from the data set whose risk or return falls outside the three standard deviations to minimize the level of bias between estimated and true parameters (Chou et al. 2009).

1 Chemical and pharmaceutical, construction industry, engineering, financial institutions, food industry, fuel and energy, textile industry, and transport and communication.

2 Our data set might have some level of supervisor bias. (For details, see Chou et al. 2009.) However, if we strictly follow the rules to overcome the problem of supervisor bias, which leads to quite low number of firms in hands, then the analysis would not be interesting. Furthermore, it is not the interest of the present study.
| Researchers               | Measures          | Return | Risk              | Reference point | Sample source                      | Firms | Source            | Period       |
|--------------------------|-------------------|--------|-------------------|-----------------|------------------------------------|-------|-------------------|--------------|
| Brick et al. (2015)      | ROE               |        | Variance          | Mean            | US firms                           | 2135  | Compustat         | 1971–2009    |
| Nguyen et al. (2021)     | ROA               |        | Variance of ROA   | Median          | China, Japan and Vietnam           | 10,623| Compustat         | 2012–2017    |
| Chari et al. (2019)      | ROA               |        | Standard deviation| Mean            | US firms                           | 1500  | S&P 500           | 1995–2016    |
| Phuang (2022)            | ROA, ROE          |        | Standard deviation| Mean, Median    | Vietnamese stock market            | 727   | Vietstock.vn      | 2017–2020    |
| Becerra and Markarian (2021) | ROA               |        | Standard deviation| Mean, Median    | US firms                           | 2681  | S&P, Compustat, CRSP | 1980–2014    |
| Khan et al. (2022)       | ROA, ROE          |        | Market-based risk | CAPM Model      | Asian emerging countries           | 4609  | Asian emerging countries | 2013–2017    |
| Marques et al. (2021)    | ROE               |        | Coefficient of variation | Median       | Brazilian companies                | 292   | Brazilian data    | 2008–2018    |
| Maurer (2008)            | ROA, ROE          |        | Standard deviation| Mean            | French firms                       | 38    | French CAC 40     | 1988–2002    |
| Patel et al. (2018)      | ROA               |        | Standard deviation| Mean            | Asian, European and South African countries | 12,235 | Compustat         | 1998–2012    |
2.2 Some methodological issues

2.2.1 Measure the quality of fit

Most of the literature used a simple linear regression model: \( risk_i = \alpha + \beta \times return_i + \epsilon_i \) for the estimation of the statistical relationship between risk and return. \( \alpha \) is an intercept, \( \beta \) is the rate of change in risk due to change in return, \( return_i \) is the average of return on assets and/or return on equity for each firm in the industry, \( risk_i \) is the standard deviation of the return on assets and/or return on equity, and \( \epsilon_i \) is the error term of the model.

This simple linear regression model has some limitations. For example, neither the estimated parameters would hold the best linear and unbiased estimator (BLUE) property, nor does the fitted line has a minimum standard error of the regression, including others. We construed the scatter plots to understand these issues, which occur with the simple regression model, and possible solutions. Figure 1 represents the risk–return relationship for the transportation and communication industry. Each point is a combination of risk (standard deviation) and returns (average of return on assets) for each firm, separately, at a given time. The solid line represents the fitted line by using the simple regression model: \( risk_i = \alpha + \beta \times return_i + \epsilon_i \), while the dashed line represents the fitted line of a simple regression model with the quadratic term, i.e., \( risk_i = \alpha + \beta \times return_i + \gamma \times return_i^2 + \epsilon_i \), where \( \beta < 0 \) and \( \gamma > 0 \).

To evaluate the performance of simple linear regression vs linear regression with quadratic terms on a given data set, we need some way to measure how well its predictions match with the observed data. We need to quantify the extent to which the predicted response value of a given observation is close to the true response value for

![Risk and Return Relationship](image)

**Fig. 1** Transportation and communication industry data were collected from the Pakistan Stock Exchange. The X axis represents the return level, and the Y axis represents the risk level for each firm at a certain point in time. The solid line represents the simple linear regression fitted line. The dashed line represents the linear regression fitted line with the quadratic term of risk.
the observations (James et al. 2013). In the regression setting, the most commonly used measure is the mean square error (MSE), given by 
\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (\hat{risk}_i - risk_i)^2. \]

The MSE will be zero if the predicted value \((\hat{risk}_i)\) is equal to the actual value \((risk_i)\). Hence, if the distance between the expected value is close to the actual value, then the MSE will be small, and vice versa. Practically, we found that the MSE of the simple linear regression model is 50% higher than the MSE of the linear regression with the quadratic term. Furthermore, as the distance between predicted and actual values decreases, the explained part of the regression analysis increases and hence the value of \(R^2\); the value of \(R^2\) for the linear regression model with the quadratic is higher than for the simple regression model. Therefore, we will add the quadratic term to the empirical regression model to estimate the statistical relationship between risk and return for other industries.

2.2.2 Data visualization

This research explores possible misspecification that has not received attention in the literature. Understanding what data are and where they come from is essential, which helps draw reliable inferences by linking with the empirical results. The risk and return are the single values against each firm, calculated by taking the median and standard deviation of the ROA (and/or ROE) series. It means that we have discarded the time series properties of the data, and now, we have observations at a single point in time. For example, in the transportation and communication industry, in Fig. 1, a single point represents each firm, and the complete figure represents the picture of the industry at a particular point in time. At a single point in time, some firms are below the reference point (for example, zero), which implies that the specific firm was consistently making a loss because each point is derived from a time series, and vice versa (Henkel 2000, 2009).

Furthermore, any firm at the time of decision faces different kinds of risk, such as risk within a firm, an industry and the economy (national and international). By contrast, in the case of an individual, she neither afford losses for a long time nor faces a different kind of risk. Therefore, it is not plausible to explain the Bowman paradox by prospect theory.

2.2.3 Outlier and simple linear regression model

At a single point in time (property of cross-sectional data), firms in a single industry are heterogeneous in many aspects: attitude toward risk (add something to it). Concerning the performance level, the risk lover manager of the firm goes for the risky investment, whereas the risk-averse manager gives less weight to return. Hence, at a single point in time, some firms could be in risk-averse positions, and some could be at risk lover positions and maybe lie in extreme positions. In the language of statistics, these extreme

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3 The linear regression model \(risk_i = \alpha + \beta \ast return_i + \epsilon_i\) has two coefficient parameters, while the linear regression with the quadratic term \(risk_i = \alpha + \beta \ast return_i + \gamma \ast return_i^2 + \epsilon_i\) has one more extra parameter. Usually, the value of \(R^2\) increases with the addition of variables.
positions are known as outliers. These outliers significantly impact the simple linear regression models and are represented in Fig. 2. Using the same data set, shown in Fig. 1, we deleted two extreme values, one on the top right and one on the top left. Deleting two extreme values significantly impacts the regression analysis, as shown in Fig. 2—on the estimated parameters and statistical inferences.

The left panel represents the impact of deleting one extreme value from the left side. If we compare the simple linear regression with Fig. 1, it implies that the rate of change has increased by approximately 12%, and the value of $R^2$ has increased by 9%. Similarly, in the case of the right panel, the situation is worst, where the positive slope becomes negative. The value of the slope parameter decreased by approximately 20%, and the value of $R^2$ has increased by 10%. These results are consistent with Sinha (1994). Contrary, in the case of linear regression with the quadratic term, approximately, there is no change in estimated parameter values. This is the second reason why linear regression with the quadratic term is more appropriate.

2.2.4 Heteroscedasticity

Most of the literature used a simple linear regression model to test the statistical relationship between risk and return and estimate this model with the ordinary least-squares (OLS) method. Moreover, the estimates of this regression are valid only if all the assumptions are fulfilled. In the case of cross-sectional data, the most relevant assumption is constant variance. Therefore, the standard errors, confidence intervals and hypothesis testing associated with the linear model only rely upon this assumption. Unfortunately, in the case of a risk–return relationship, it is often violated.

Similarly, in our case, we reject the null hypothesis of homoscedasticity even at 1%. Contrary, the linear regression with quadratic terms does not suffer from this problem. However, the quadratic term might not always solve this problem; therefore, it is suggested to replace the OLS method with the generalized least-squares method.

2.2.5 Degrees of freedom and law of large numbers

Figure 1 consists of the transportation and communication industry based on 19 Firms. If this sample is divided into two halves, as most of the literature did, one sample will contain nine observations, and the other will have ten observations. Running separate
regressions for 9 or 10 observations will not only yield high standard errors but also could have biased estimated parameters. In other words, we have fewer observations to interpret a parameter accurately in each regression. We estimate that a linear regression with the quadratic term will have 19 observations and three estimated parameters, and then, the degrees of freedom will equal 16. If we run a simple regression as suggested by the literature, the degrees of freedom will be 7 and 8, respectively, relatively low. Furthermore, according to the law of large numbers—the estimated parameter approach to the true parameter as we increase the number of observations. Hence, the linear regression with the quadratic term will also resolve this issue.

2.3 The methodology

Most previous research used a linear regression model to test the empirical relationship between risk and return by splitting the data set into two groups. However, splitting the data into groups and estimating two separate regressions would harm the degrees of freedom. There is a pronounced relationship between risk and return, but this relationship is nonlinear: The data suggest a curved relationship (Fig. 1). A simple approach for incorporating nonlinear association is a linear model to include a transformed version of the predictions in the model, i.e., by adding the quadratic term in the linear model form. Therefore, the present study added the quadratic term to estimate the risk–return relationship for each industry without dividing the industry into two groups. Hence, we have the following empirical model for estimating the risk–return relationship.

\[
\text{risk}_{i} = \alpha + \beta \times \text{return}_{i} + \gamma \times \text{return}_{i}^2 + \varepsilon_{i}
\]  

(1)

where \( \alpha \), \( \beta \) and \( \gamma \) are the parameters and \( \varepsilon_{i} \) is the error term. These empirical results will only be valid for explaining Bowman’s paradox if \( \alpha > 0, \beta < 0 \) and \( \gamma > 0 \) and must be significant. We then apply the F statistic for testing the null hypothesis that \( \beta = \gamma = 0 \) are jointly equal to zero. A rejection of the null indicates that no quadratic relationship exists; hence, we can conclude that prospect theory cannot be used to explain Bowman’s paradox.

Furthermore, this linear risk–return relationship is estimated using OLS method in the literature. It works well if all the classical linear regression method assumptions are fulfilled. However, due to heteroscedasticity, the standard error of the estimated parameters is incorrect. Therefore, in the present study, we used the generalized least-squares method to obtain the estimated parameter with the minimum standard error and inferences.

2.3.1 Empirical results

Table 2 represents the empirical results of the linear regression model with the quadratic term for eight industries. The number of firms varies from 19 in transportation & communication industry to 189 in the textile industry.\(^4\) We have separately estimated

\(^4\) The number of the firms mentioned in “The data set” section is not comparable here, because we applied trimming method to minimize the impact of outlines on the estimated parameters.
Table 2 Regression results linear regression model

| Industry                  | No. of firms | $\alpha$ | $\beta$ | $\gamma$ | $R^2$ | Summary of issues               |
|---------------------------|--------------|----------|---------|----------|-------|---------------------------------|
| 1 Chemical and pharmaceutical | 39           | 0.09***  | -0.07   | 0.22     | >0.00 | $\beta = 0, \gamma = 0$        |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (0.937)                         |
|                           | ROA          |          |         |          |       |                                 |
|                           | ROE          | 0.26***  | -0.82***| 1.24***  | 0.44  | Valid model                     |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (13.89 (> 0.000))***            |
| 2 Construction industry   | 38           | 0.04***  | 0.03    | 1.36**   | 0.22  | $\beta > 0, \beta = 0$          |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (1.32 (0.278))                  |
|                           | ROA          |          |         |          |       |                                 |
|                           | ROE          | 0.12***  | -0.03   | 0.93     | 0.03  | $\beta = 0, \gamma = 0$        |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (0.55 (0.579))                  |
| 3 Engineering             | 36           | 0.12***  | -0.20*  | 0.79**   | 0.14  | Valid model                     |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (2.65 (0.085))                  |
|                           | ROA          |          |         |          |       |                                 |
|                           | ROE          | 0.12***  | -0.34** | 1.28**   | 0.23  | Valid model                     |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (4.89 (0.014))                  |
| 4 Financial institutions  | 177          | 0.10***  | -0.52***| 1.62***  | 0.50  | Valid model                     |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (86.44 (> 0.000))***            |
|                           | ROE          | 0.20***  | -0.46***| 2.94***  | 0.55  | Valid model                     |
|                           |              |          |         |          |       | $H_0: \beta = \gamma = 0$       |
|                           |              |          |         |          |       | (105 (> 0.000))***              |
| 5 Food industry           | 55           | 0.09     | -0.07   | 0.14     | >0.00 | $\alpha = 0, \beta = 0, \gamma = 0$ |
| Industry                  | No. of firms | $\alpha$   | $\beta$   | $\gamma$   | $R^2$  | Summary of issues                        |
|--------------------------|--------------|------------|-----------|------------|-------|------------------------------------------|
| Fuel and energy          | 34           |            |           |            |       | $H_0 : \beta = \gamma = 0 \ 0.11 \ 0.1583$ (>) $0.000$*** $0.11^{**}$ $-0.21$ $1.02^{***}$ $0.38$ $\beta = 0$ |
| Textile industry         | 189          |            |           |            |       | $H_0 : \beta = \gamma = 0 \ 0.1205$ (>) $0.000$*** $0.06^{***}$ $-0.22^{**}$ $0.84^{***}$ $0.27$ $H_0 : \beta = \gamma = 0$ |
| Transportation and       | 19           |            |           |            |       | $H_0 : \beta = \gamma = 0 \ 0.1062$ (>) $0.001$*** $0.09^{***}$ $-0.09^{**}$ $1.44^{***}$ $0.57$ $H_0 : \beta = \gamma = 0 \ 0.3893$ (>) $0.000$*** $0.13$ $2.26^{**}$ $0.33$ $0.86$ $\alpha(0, \alpha = 0, \beta 0, \gamma = 0$ |

***, ** and * represents the statistical significance of at 1%, 5% and 10%, respectively. The dependent variable is a risk, the standard deviation of the ROA or ROE. We have estimated a linear regression model with quadratic terms for most industries. A generalized least-squares method was used to estimate these models. The value of the $R^2$ lies between 0 and 1, and in our case, this is close to 0; therefore, we added the sign of greater (>)
the risk–return relationship for each industry based on ROA and ROE. The last column of Table 2 contains the summary of issues found in the empirical, respectively.

In the case of the construction industry and transportation and communication industry, the sign of the estimated $\beta$ is positive, which is inconsistent with our hypothesis (ROA model and ROE model, respectively).

Furthermore, $\beta$ is statistically insignificant construction industry and food industry. Similarly, $\beta$ is also statistically insignificant in the case of chemical and pharmaceutical (ROA) and fuel and energy (ROE). $\alpha$ and $\gamma$ are also insignificant in some cases. The signs of all other estimated parameters are as per our expectations and statistically significant. These models imply that each model’s estimated quadratic line decreases from some positive value (due to positive intercept) toward the minimum value (reference point); after that, it increases at an increasing rate.

The joint null hypothesis ($H_0 : \beta = \gamma = 0$) cannot be rejected in the case of four industries, which implies that there is no relationship between $risk_i$ and $return_i$, and $risk_i$ and $return_i^2$; for example, in the case of the chemical and pharmaceutical industry ROA model. Furthermore, the value of $R^2$ is close to zero in the case of the chemical and pharmaceutical and food industry ROA model. Moreover, the estimated parameters are insignificant, implying that there is no relationship between risk and return in these industries; hence, they are independent of each other. The possible reason could be that these industries contain the necessities commodities.

Our empirical results contradict the hypothesis of prospect theory. To illustrate these contradictory results, we construed a scatter plot. Figure 3 represents the scatter plot with the quadratic fitted line. At first, the quadratic line fits more accurately in the data. Suppose we combine the empirical results of Table 2 with Fig. 3. In that case, we can conclude that our hypothesized econometric linear model with the quadratic term is appropriate for estimating prospect theory compared to linear regression.

The first hypothesis of prospect theory states that there is a negative relationship between risk and return before the reference point. In contrast, the chemical and pharmaceutical industry, construction industry, engineering and food industry in case of ROA, and construction industry, engineering and food industry in case of ROE have either a straight line or have an insignificant negative relationship between risk and return. Transportation and communication industry is a particular case where the negative region also presents a positive relationship. The second hypothesis of prospect theory states that above the reference point, risk and return are positively related, which is confirmed in almost all cases. However, in some cases, it is insignificant.

The third hypothesis of prospect theory is risk aversion, which is the central hypothesis, i.e., the line is steeper before the reference point than after the reference point. However, our empirical results are also contradictory to this hypothesis. Construction, engineering, food, textile and transportation and communication industries violate the risk aversion hypothesis. So, we can say that we did not find any significant evidence of prospect theory for the firms listed on the Pakistan Stock Exchange. Hence, this research found mixed empirical results, which vary within the industry and across industries; therefore, it raises the questions of the generalizability of prospect theory.

Figure 3 represents that some firms are below the reference point (minimum value of the quadratic line), and some are above it, which firmly explains Bowman’s paradox.
Fig. 3 Scatter plot of each industry along with a linear regression model with a quadratic term fitted line. The X axis represents the level of risk, and the Y axis represents the level of return. Each point represents the risk and returns for a particular firm in that industry. The estimated parameters of each plot are of the generalized least-squares method.
in the industries of Pakistan. Some firms are performing well at a specific time, and some are unwell concerning the reference point. Any firm can make future decisions by estimating the gap between the firm’s expected performance and aspired performance. Those firms lie below the reference point representing that they are not working well and making losses. To overcome these losses and aspire to future growth, firms will ultimately take risky decisions by changing their operational level. On the other hand,
there is no need to change the operations of the firms already doing well and making a profit.

Evidence from the literature The empirical results of the literature are consistent with the present study, but the literature did not use data visualization to analyze the compatibility between the properties of the data in hand with the properties of the econometric model. Fitting an econometric model without understanding the properties of the data could lead to spurious regressions. Subsequently, the literature estimates the relationship between risk and return with the linear regression, which was not true in most cases; instead, the true choice of econometric model is a linear regression with the quadratic term.

For example, the values of $\beta$ under the heading of the above median in Table 1 of Sinha (1994), industry 01 and 15 are negative. However, theoretically, they must be positive to remain consistent with prospect theory and Bowman’s paradox; subsequently, if we compare the value of $\beta$ below and above the median, in many cases, the value $\beta$ after the median is higher, such as in industries 01, 02 and 06. Similarly, the value of $\beta$ in Table 2 of Chou et al. (2009) is higher after the median than before the reference point. Table A.1 of Fiegenbaum (1990) has that value of $\beta$ above the target value is negative for mobile homes, paperboard mills, perfume preparations, bolts, motors and generators. Similarly, the value of $\beta$ for bituminous coal, plastic materials, construction machinery, special industrial machinery, radio and TV sets and electronics is higher above the target level compared to below the target level. Table 1 of Gooding et al. (1996) is another example that does not follow Bowman’s paradox.

3 Conclusion

The primary objective of the present study is to revalidate the prospect theory hypothesis and assess the empirical validity of Bowman’s paradox. To achieve the goal, we gathered the data of 622 listed firms on the Pakistan Stock Exchange from 2000 to 2019. These firms are categorized into eight industries. We tested the statistical relationship between risk and return using linear regression with a quadratic term and estimated it through the generalized least-squares method. We found that the suggested hypothesis for prospect theory in the literature holds neither in the present study nor in the literature, but the literature remains silent about it. Furthermore, we have also used data visualization to illustrate it in depth, confirming that the prospect theory’s hypotheses do not hold. Therefore, prospect theory cannot be used to back the empirical results of Bowman’s paradox.

We found significant evidence of Bowman’s paradox across Pakistan’s industries. Some firms in each industry are located below the reference point, and some are found above the reference point, implying that some firms are making a profit and some are making losses simultaneously. Those firms that are not doing well must decide to improve the organization’s health by changing their operations.

In reality, we are never sure about the true functional form or the specification of the econometrics model. Therefore, searching for a valid econometric model based on data visualization will give a more accurate clue, minimizes the chance of spurious regression and helps understand the relationship between the variables. Hence, the
selected empirical model will be more powerful. Subsequently, this study used data visualization to improve the empirical model of prospect theory.

The choice of the ROA and/or ROE as a proxy for the return is also questionable, because the construction of ROA entirely depends on the assets that a firm holds. Similarly, ROE is constructed by accumulating the equity of the firms. Thus, the foundations of both ROA and ROE are pretty different from each other. Therefore, future research needs to explore the valid proxy of return.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any authors.

References

Andersen TJ, Denrell J, Bettis RA (2007) Strategic responsiveness and Bowman’s risk-return paradox. Strateg Manag J 28(4):407–429
Becerra M, Markarian G (2021) Why are firms with lower performance more volatile and unpredictable? A vulnerability explanation of the bowman paradox. Organ Sci 32(5):1327–1345
Bettis RA, Mahajan V (1985) Risk/return performance of diversified firms. Manag Sci 31(7):785–799
Bowman EH (1980) A risk/return paradox for strategic management. Sloan Manag Rev 21:17–31
Bowman EH (1982) Risk seeking by troubled firms. Sloan Manag Rev (pre-1986) 23(4):33
Brick IE, Palmon O, Venezia I (2015) On the relationship between accounting risk and return: is there a (Bowman) paradox? Eur Manag Rev 12(2):99–111
Brockett PL, Cooper WW, Kwon KH, Rueli TW (2003) Commentary on “a review of research on the negative accounting relationship between risk and return: Bowman’s paradox by MN Nickel and MC Rodriguez.” Omega 31(5):409–412
Bromiley P, Miller K, Rau D (2001) Risk in strategic management research. In: Hitt M, Freeman RE, Harrison JS (eds) The Blackwell handbook of strategic management. Blackwell, Malden, pp 259–288
Chari MD, David P, Duru A, Zhao Y (2019) Bowman’s risk-return paradox: an agency theory perspective. J Bus Res 95:357–375
Chou PH, Chou RK, Ko KC (2009) Prospect theory and the risk-return paradox: some recent evidence. Rev Quant Finance Acc 33(3):193–208
Denrell J (2008) Organizational risk taking: adaptation versus variable risk preferences. Ind Corp Change 17(3):427–466
Denrell J (2004) Risk taking and aspiration levels: two alternative null-models. In: Academy of management proceedings, vol 2004. Academy of Management, Briarcliff Manor, no 1, pp J1–J6
Fiegenbaum A (1990) Prospect theory and the risk-return association: an empirical examination in 85 industries. J Econ Behav Organ 14(2):187–203
Fiegenbaum A, Thomas H (1988) Attitudes toward risk and the risk-return paradox: prospect theory explanations. Acad Manag J 31(1):85–106
Fiegenbaum A, Thomas H (1990) Strategic groups and performance: the US insurance industry, 1970–84. Strateg Manag J 11(3):197–215
Gooding RZ, Goel S, Wiseman RM (1996) Fixed versus variable reference points in the risk-return relationship. J Econ Behav Organ 29(2):331–350
Greve HR (2011) Positional rigidity: low performance and resource acquisition in large and small firms. Strateg Manag J 32(1):103–114

Henkel J (2000) The risk-return fallacy. Schmalenbach Bus Rev 52(4):363–373

Henkel J (2009) The risk-return paradox for strategic management: disentangling true and spurious effects. Strateg Manag J 30(3):287–303

James G, Witten D, Hastie T, Tibshirani R (2013) An introduction to statistical learning, vol 112. Springer, New York, p 18

Jegers M (1991) Prospect theory and the risk-return relation: some Belgian evidence. Acad Manag J 34(1):215–225

Jemison DB (1987) Risk and the relationship among strategy, organizational processes, and performance. Manag Sci 33(9):1087–1101

Johnson HJ (1992) The relationship between variability, distance from target, and firm size: a test of prospect theory in the commercial banking industry. J Socio-Econ 21(2):153–171

Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. Econometrica 47(2):363–391

Khan A, Khan MI, ur Rehman Z, Khan S (2022) Does prospect theory explain Bowman’s paradox in Asian emerging markets? Manag Finance 48(7):1029–1046

Kliger D, Tsur I (2011) Prospect theory and risk-seeking behavior by troubled firms. J Behav Finance 12(1):29–40

Lehner JM (2000) Shifts of reference points for framing of strategic decisions and changing risk-return associations. Manag Sci 46(1):63–76

Marques VA, Costa BMLF, Lopes MB, Ramos SW (2021) The paradox fallacy: an analysis of the risk-return binomial in the Brazilian capital market. Adv Sci Appl Account 14(2):109–122

Maurer F (2008) Risk and return: new insights for theory, measurement and management. J Appl Bus Res 24(4):51–64

Nguyen NC, Wei NING, Alikaj A (2021) Bowman’s risk-return paradox: evidence from eastern countries. Stud Bus Econ 16(1):244–254

Nickel MN, Rodriguez MC (2002) A review of research on the negative accounting relationship between risk and return: Bowman’s paradox. Omega 30(1):1–18

Nuir RS, Asri M (2019) Bowman’s paradox: prospect-theory-based risk-return relationship (some recent evidence in Indonesia). Indones Cap Mark Rev 11(1):1

Oviatt BM, Bauerschmidt AD (1991) Business risk and return: a test of simultaneous relationships. Manag Sci 37(11):1405–1423

Patel PC, Li M, Park HD (2018) A replication of Bowman’s paradox across 28 countries. J Innov Knowl 3(3):128–142

Phuong LCM (2022) Bowman’s risk-return relationship: empirical evidence in a Frontier market. Invest Manag Financ Innov 19(2):191–200

Rueff TW (1990) Mean-variance approaches to risk-return relationships in strategy: paradox lost. Manag Sci 36(3):368–380

Rueff TW, Collins JM, Lacugna JR (1999) Risk measures in strategic management research: Auld Lang Syne? Strateg Manag J 20(2):167–194

Santacruz L (2019) Measures of firm risk-taking: revisiting Bowman’s paradox. Manag Finance 46:421–434

Sinha T (1994) Prospect theory and the risk-return association: another look. J Econ Behav Organ 24(2):225–231

Wiseman RM, Bromiley P (1991) Risk-return associations: paradox or artifact? An empirically tested explanation. Strateg Manag J 12(3):231–241

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