Microscopic nonlinear quantum theory of absorption of strong electromagnetic radiation in doped graphene

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Microscopic quantum theory of nonlinear stimulated scattering of 2D massless Dirac particles in doped graphene on Coulomb field of impurity ions at the presence of an external strong coherent electromagnetic radiation is developed. We consider high Fermi energies and low frequencies (actually terahertz radiation) to exclude the valence electrons excitations. The Liouville-von Neumann equation for the density matrix is solved analytically, taking into account the interaction of electrons with the scattering potential in the Born approximation. With the help of this solution, the nonlinear inverse-bremsstrahlung absorption rate for a grand canonical ensemble of 2D Dirac fermions is calculated. It is shown that one can achieve the efficient absorption coefficient by this mechanism.

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I. INTRODUCTION

Due to the known properties of graphene [1–6] the coefficient of interaction with the external electromagnetic (EM) radiation is very high in comparison with other systems, which opens a significant field of important applications of graphene in Nano-Opto-Electronics [7–17], as well as in low-energy physics, condensed matter physics, and quantum electrodynamics (QED).

High absorption coefficient [18] indicates that graphene strongly interacts with light, and, it is very important that, because of gapless structure of graphene such interaction can be efficiently realized with THz devices as a multitrion converter, as well as a protective material for nanodevices [19, 20]. The strong absorption of electromagnetic (EM) radiation in ultrasmall volumes (nanoscale) is a highly desirable property for shielding materials used in nanoelectronics, the aerospace industry, where strict requirements exist such as lightness and smallness or tightness.

Among the important processes induced by external radiation fields the multiphoton stimulated bremsstrahlung (SB) is a basic mechanism of energy exchange between the charged particles and plane monochromatic wave in plasma-like media to provide the energy-momentum conservation law for real absorption-emission processes that has been revealed immediately after the invention of lasers [21]. What concerns the electrons elastic scattering on impurity ions in graphene, there are many papers with consideration of this basic scattering effect which have been described mainly within the framework of perturbation theory by electrostatic potential (see, e.g., [22–30]). Regarding the SB process in graphene at moderate intensities of stimulated radiation, in case of its linear absorption by electrons (or holes), at the present time there are extensive investigations carried out in the scope of the linear theory, see, e.g. [31–35].

Taking into account the above mentioned unusual high EM nonlinearity of graphene, in this paper we have studied both analytically and numerically the nonlinear absorption process of external EM radiation in doped graphene. As a mechanism for the real absorption/emission of a plane-monochromatic wave by the charged particles (or plasma-like medium), we have assumed SB process of conductive electrons scattering on the charged impurities in doped graphene. We developed microscopic quantum theory of graphene nonlinear interaction with the coherent EM radiation of arbitrary intensity and frequency. With the help of the solution of Liouville-von Neumann equation for the density matrix, we calculated the nonlinear stimulated scattering of 2D Dirac particles in graphene on the Coulomb field of impurity ions at the presence of an external EM radiation field, taking into account the interaction of electrons with the scattering potential in the Born approximation. Here the selected frequency range of terahertz radiation excludes the valence electrons excitations at high Fermi energies.

In Sec. I the relativistic quantum dynamics of SB of conductive electrons in graphene is presented with analytical results for density matrix and inverse-bremsstrahlung absorption rate. In Sec. II the analytic formulas in case of screened Coulomb field of an impurity ion are considered numerically. Finally, conclusions are given in Sec. III.

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II. BASIC THEORY

Interaction of a free electron with the EM wave is described by the dimensionless relativistic invariant parameter of intensity $\xi = eE_0\lambda/(2\pi mc^2)$, which represents the wave electric field (with amplitude $E_0$) work on a wavelength $\lambda$ in the units of electron rest energy. Apart, for THz photons $h\omega \sim 0.01\text{eV}$, multiphoton effects take place at $\xi \sim 1$ that corresponds to intensities $I_\xi \sim 10^{14}\text{ Wcm}^{-2}$, while the massless electron-wave interaction in graphene is characterized by the dimensionless parameter $\chi = ev_F E_0/ (h\omega^2)$, which represents the work of the wave electric field on a period $1/\omega$ in the units of photon energy $h\omega$. Depending on the value of this parameter $\chi$, three regimes of the wave-particle interaction may be established: $\chi < 1$ – that corresponds to one-photon interaction regime [36–38], $\chi \gg 1$ – which is the static field limit of superpower fields in QED or Schwinger regime [39], and $\chi \approx 1$ – is the multiphoton interaction regime [8] with the corresponding intensity $I_\chi = \chi^2 \times 3.07 \times 10^{11}\text{ Wcm}^{-2}[h\omega/\text{eV}]^4$. Comparison of this intensity threshold with the analogous one for the free electrons or with the situation in common atoms shows the essential difference between the values of these thresholds: $I_\xi/I_\chi \sim 10^{11}$. Thus, for realization of multiphoton SB in graphene one can expect $10^{11}$ times smaller intensities than for SB in atoms [9], [10], [21, 40, 41]. In the presented work, the influence of multiphoton effects in SB absorption process with an external EM wave field of moderate intensities is considered. Note that the first nonrelativistic treatment of SB in the Born approximation has been carried out analytically in the work [21], and then this approach has been extended to the relativistic domain [40].

Let us consider the relativistic quantum theory of graphene nonlinear interaction with the arbitrary strong EM wave field by microscopic theory for the 2D Dirac fermions-ions interaction on the base of density matrix. We consider the interaction when the laser wave propagates in a perpendicular direction to the graphene plane ($XY$) to exclude the effect of magnetic field. This traveling wave for electrons in graphene becomes a homogeneous time-periodic electric field. It is directed along the $X$ axis with the form (constant phase connected with the position of the wave pulse maximum with respect to the graphene plane is set zero). We assume the EM wave to be quazimonochromatic and of the linear polarization with frequency $\omega$ and amplitude $E_0$:

$$\mathbf{E}(t) = \tilde{x}E_0 \cos \omega t.$$  \hspace{1cm} (1)

The corresponding wave vector potential $\mathbf{A}(t)$ will be:

$$\mathbf{A}(t) = -c \int_0^t \mathbf{E}(t') dt' = -\tilde{x}E_0 \omega \sin \omega t.$$  \hspace{1cm} (2)

To exclude the valence electrons excitations at high Fermi energies in graphene, we will assume for a EM wave actually a terahertz radiation. The impurity ions are assumed to be at rest and either randomly or nonrandomly distributed in the doped graphene, the arbitrary form electrostatic potential field of which is described by the scalar potential:

$$\varphi(\mathbf{r}) = \sum_i N_i \varphi_i (\mathbf{r} - \mathbf{R}_i).$$  \hspace{1cm} (3)

Here $\varphi_i$ is the potential of a single ion placed at the position $\mathbf{R}_i$, and $N_i$ is the number of impurity ions in the interaction region.

Let us consider the quantum kinetic equations for a single particle density matrix for SB process investigation, which can be derived from the second quantized formalism. As a basis for single particle wave functions we take the approximate solution of the massless Dirac equation in the strong EM wave field $\mathbf{A}(t)$, which may be presented in the form:

$$\Psi_p(\mathbf{r}, t) = \exp \left( \frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r} \right) f_p(t).$$  \hspace{1cm} (4)

Hear $\mathbf{r} = \{x, y\}$ is the 2D-radius vector. For determination of spinor wave function $f_p$ we will use the results of the paper [42] with the spinor wave function $f_p$ determined as follows:

$$f_p(t) = \frac{1}{\sqrt{2S}} \left( e^{i\varphi_p} e^{\frac{i}{\hbar} \frac{1}{2} \mathbf{A}(t)} \right) e^{-i\Omega(\mathbf{p}, t)}.$$  \hspace{1cm} (5)

where the temporal phase $\Omega(\mathbf{p}, t)$ (classical action in the field (2)) is defined as:

$$\Omega(\mathbf{p}, t) = \frac{v_F}{\hbar} \int \sqrt{(px + \frac{e}{c}Ax)^2 + p_y^2} dt.$$  \hspace{1cm} (6)

...
The function $\Theta(p)$ is the polar angle in momentum space and $S$ is the quantization area (graphene layer surface area). In terms of these parameters, the graphene linear dispersion law for quasiparticles energy-momentum $\mathcal{E}(p)$ defined by the characteristic Fermi velocity $v_F$, reads: $\mathcal{E}(p) = \pm v_F |p| = \pm v_F \sqrt{p_x^2 + p_y^2}$, where the upper sign corresponds to electrons and the lower sign - to holes. The states (1) are normalized by the condition

$$\int \Psi_p^+(r, t)\Psi_p(r, t)\,dr = \frac{(2\pi\hbar)^2}{S} \delta(p - p').$$

(7)

The Hamiltonian of the system in the second quantization formalism can be presented in the form:

$$\mathcal{H} = \int \hat{\Psi}^+ \hat{H}_0 \hat{\Psi} \,dr + \mathcal{H}_{sb},$$

(8)

where $\hat{\Psi}$ is the field operator for quasiparticles of the Fermi-Dirac sea in the graphene, $\hat{H}_0$ is the single-particle 2D Dirac Hamiltonian in the external field $A(t)$ (2), and the interaction Hamiltonian of SB process in the EM wave is

$$\mathcal{H}_{sb} = \frac{1}{c} \int \hat{\mathcal{J}}(\mathbf{r}) \,dr,$$

(9)

with the current density operator

$$\hat{\mathcal{J}} = -e v_F g_s g_v \int \hat{\Psi}^+ \hat{\sigma} \hat{\Psi} \,dr.$$  

(10)

Making Fourier transform of scalar potential

$$\varphi(\mathbf{r}) = \frac{1}{(2\pi)^2} \int V(q)e^{-iq\mathbf{r}} \,dq,$$

(11)

$$V(q) = \int \sum N_i c \varphi_i(\mathbf{r} - \mathbf{R}_i)e^{iq\mathbf{r}} \,dr,$$

the interaction Hamiltonian can be expressed in the following form

$$\mathcal{H}_{sb} = -\frac{g_s g_v v_F}{c(2\pi)^2} \int \hat{\Psi}^+ V(q)e^{-iq\mathbf{r}} \hat{\Psi} \,dq \,dr.$$  

(12)

In Eq. (10) $\hat{\sigma} = \{\sigma_x, \sigma_y\}$ - Pauli matrices, $g_s$ and $g_v$ are the spin and valley degeneracy factors, respectively.

Let us pass to Furry representation and present the Heisenberg field operator of the electron in the form of an expansion in the quasistationary Dirac states (4):

$$\hat{\Psi}(r, t) = \int d\Phi_p \hat{a}_p \Psi_p(r, t),$$

(13)

where $d\Phi_p = Sd^2p/\hbar(2\pi)^2$. We have excluded the hole operators in Eq. (13), since contribution of electron-holes intermediate states will be negligible for considered intensities and Fermi energies. The creation and annihilation operators $\hat{a}_p(t)$ and $\hat{a}_p^+(t)$, associated with positive energy solutions and satisfy the known anticommutation rules at equal times [11]:

$$\{\hat{a}_p^+(t)\hat{a}_p(t')\}_{t=t'} = \frac{(2\pi\hbar)^2}{S} \delta(p - p'),$$

(14)

$$\{\hat{a}_p^+(t)\hat{a}_p(t')\}_{t=t'} = \{\hat{a}_p(t)\hat{a}_p^+(t')\}_{t=t'} = 0.$$  

(15)

Taking into account anticommutation rules, Eqs. (13), (4)-(6), the second quantized Hamiltonian can be expressed in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{sb}(t),$$

(16)
where the first term is the Hamiltonian of the wave dressed two-dimensional massless Dirac fermion field:

\[ H_0 = v_F \int d\Phi_p p\hat{a}^+_p \hat{a}_p, \]  

(17)

and the second term

\[ H_{sb} = \int d\Phi_p \int d\Phi'_p C_{p'p}(t)\hat{a}^+_p \hat{a}_p \]  

(18)

is the Hamiltonian of interaction in SB process. Here \( P \) is the absolute value of the Dirac particle "quasimomentum" which is defined from kinematic momentum in EM field \( p \) as:

\[ P = \frac{\omega}{2\pi} \int_0^{2\pi} \sqrt{\left(p_x + \frac{e}{c} A_x(t)\right)^2 + p_y^2} \ dt. \]  

(19)

For the impurity potential of the arbitrary form electrostatic potential \( V(q) \) from the relation (13), (4)-(6) we have the following relations for the SB amplitudes:

\[ C_{p'p}(t) = -\frac{g_s g_v v_F}{2S} V(q) \left( 1 + e^{i[\Theta(p + \hat{z} A(t)) - \Theta(p' + \hat{z} A(t))] - \frac{i\hbar v_F}{2S} \int_0^t \beta^0 \left( \sqrt{(p'_x + \hat{z} A_x)^2 + p'_y^2} - \sqrt{(p_x + \hat{z} A_x)^2 + p_y^2} \right) dt \right), \]  

(20)

where \( q = \frac{p' - p}{\hbar} \) is the recoil momentum. In accordance to Eq. (20) the amplitude \( C_{p'p}(t) \) can be expressed in the following form:

\[ C_{p'p}(t) = \frac{1}{S} e^{-\frac{i\hbar v_F (p' - p) t}{2S}} B(t), \]  

(21)

where the time-depended function

\[ B(t) = -\frac{g_s g_v v_F}{2c} V(q) \left( 1 + e^{i[\Theta(p + \hat{z} A(t)) - \Theta(p' + \hat{z} A(t))] - \frac{i\hbar v_F}{2S} \int_0^t \beta^0 \left( \sqrt{(p'_x + \hat{z} A_x)^2 + p'_y^2} - \sqrt{(p_x + \hat{z} A_x)^2 + p_y^2} \right) dt \right). \]  

(22)

Making a Fourier transformation of the function \( B(t) \) (22) over \( t \), using the known relations

\[ B(t) = \sum_{n=-\infty}^{\infty} e^{-i\omega t} \tilde{B}^{(n)}_{p'p}, \]  

(23)

\[ \tilde{B}^{(n)}_{p'p} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} e^{i\omega t} B(t) dt, \]  

(24)

we can write the SB amplitude \( C_{p'p}(t) \) (20) as

\[ C_{p'p}(t) = \frac{1}{S} \sum_{n=-\infty}^{\infty} e^{-i\omega t} \tilde{B}^{(n)}_{p'p} e^{-\frac{i\hbar v_F (p' - p) t}. \]  

(25)

To present the microscopic relativistic quantum theory of the multiphoton inverse-bremsstrahlung absorption of external EM wave radiation in doped graphene we solve the Liouville-von Neumann equation for the density matrix \( \hat{\rho} \):

\[ \frac{\partial \hat{\rho}}{\partial t} = \frac{i}{\hbar} [\hat{\rho}, H_0 + H_{sb}(t)] \]  

(26)
with the initial condition

$$\hat{\rho}(0) = \hat{\rho}_g.$$  \hfill (27)

It is assumed that before the interaction with EM wave the system of graphene quasiparticles was an ideal Fermi gas in equilibrium (thermal and chemical) with a reservoir. Thus the density matrix $\hat{\rho}_g$ of grand canonical ensemble is:

$$\hat{\rho}_g = \exp \left[ \frac{1}{T_e} \left( W + \int d\Phi_p (\mu - v_F P) \hat{a}_p^+ \hat{a}_p \right) \right].$$  \hfill (28)

In Eq. (28) $T_e$ is the electrons temperature in energy units, $\mu$ is the chemical potential, $W$ is the grand potential.

The initial single-particle density matrix in momentum space will be a diagonal, and we will have the Fermi-Dirac distribution:

$$\rho(p_1, p_2, 0) = \text{Tr} \left( \hat{\rho}_g \hat{a}_p^+ \hat{a}_p \right) = f(P_1) \frac{(2\pi\hbar)^2}{S} \delta (p_1 - p_2),$$  \hfill (29)

where

$$f(P_1) = \frac{1}{\exp \left( \frac{v_F P_1 - \mu}{T_e} \right) + 1}.$$  \hfill (30)

Within the Born approximation, we consider SB interaction Hamiltonian $\mathcal{H}_{sb}(t)$ as a perturbation. So, we expand the density matrix as

$$\hat{\rho} = \hat{\rho}_g + \hat{\rho}_1,$$  \hfill (31)

taking into account the relations

$$\left[ \hat{a}_p^+ \hat{a}_p, \hat{\rho}_g \right] = \left( 1 - e^{\frac{v_F (P' - P)}{T_e}} \right) \hat{\rho}_g \hat{a}_p^+ \hat{a}_p,$$

$$\left[ \hat{\rho}_g, \mathcal{H}_0 \right] = 0,$$

for $\hat{\rho}_1$ we obtain:

$$\hat{\rho}_1 = -\frac{i}{\hbar} \int_0^t dt' \int d\Phi_p \int d\Phi_{p'} B(t') \times e^{\frac{v_F (P' - t) (P' - P)}{T_e}} \left( 1 - e^{\frac{v_F (P' - P)}{T_e}} \right) \hat{\rho}_g \hat{a}_p^+ \hat{a}_p.$$  \hfill (32)

The energy absorption rate of electrons due to inverse bremsstrahlung can be presented as

$$\frac{\partial E}{\partial t} = \text{Tr} \left( \hat{\rho}_1 \frac{\partial \mathcal{H}_{sb}(t)}{\partial t} \right).$$  \hfill (33)

It is convenient to represent the rate of inverse bremsstrahlung absorption via the mean number of absorbed photons per impurity ion, per unit time:

$$\frac{dN_{abs}}{dt} = \frac{1}{\hbar \omega N_i} \frac{\partial E}{\partial t},$$  \hfill (34)

where $N_i$ is the number of impurity ions in the interaction region.

Taking into account the decomposition relation:

$$\left( 1 - e^{\frac{v_F (P_1 - P_2)}{T_e}} \right) \text{Tr} \left( \hat{\rho}_g \hat{a}_p^+ \hat{a}_{p_2} \hat{a}_{p_3} \hat{a}_{p_4} \right) =$$
\[
\left(1 - e^{-\frac{x_F}{2}(p_1 - p_2)}\right) f_1 (1 - f_2),
\]
and making the some calculations using the relations (32)-(35), (18), (21), (22) for large \( t \) we obtain:
\[
\frac{dN_{abs}}{dt} = \sum_{n=1}^{\infty} \frac{dN_{abs}(n)}{dt},
\]
where the partial absorption rates have the following forms
\[
\frac{dN_{abs}(n)}{dt} = \frac{4\pi g_s g_v \nu n}{hN_c S^2} \int \int d\Phi_p d\Phi_{p'} |V(q)|^2 \left| \tilde{M}^{(n)}_{p,p'} \right|^2 
\times \delta(v_F P' - v_F P + n\hbar \omega) \left(1 - e^{-\frac{x_F}{2}(p' - p)}\right) 
\times f(P') (1 - f(P)),
\]
where
\[
\left| \tilde{M}^{(n)}_{p,p'} \right|^2 = \left[ \int_0^\infty \exp\left(\frac{t}{T}\right) \left(1 + e^{i[\Theta(p + x_A(t)) - \Theta(p' + x_A(t))]}\right) \exp(i\omega t) \right.
\times e^{-\frac{x_F}{2} f_r' \left[ \sqrt{(p_2 + x_A)^2 + p_0^2} - p' \right] - \left[ \sqrt{(p_0 + x_A)^2 + p_0^2} - p' \right]} dt' \right]^2.
\]

In Eq. (37) \( \delta(x) \) is the Dirac function \( \delta \) that expresses the energy conservation law of the SB process. The obtained formula for the absorption rate is true for grand canonical ensemble and positive only. With the help of (38) one can investigate the nonlinear inverse-bremsstrahlung absorption rate for degenerate quantum plasma state - the graphene electrons with the distribution function given by the Eq. (30).

III. NUMERICAL RESULTS FOR SB ABSORPTION COEFFICIENT FOR THE SCREENED COULOMB POTENTIAL OF IMPURITY IONS IN GRAPHENE

Now we utilize Eq. (38) in order to obtain the inverse-bremsstrahlung absorption coefficient in particular case of SB process on a screened Coulomb potential of impurity ions in graphene [25], [29], [42], [46]. In general, one should arise from the Linhard theory of screening, which is applicable for quantum plasmas [47]. For our calculations the Thomas-Fermi approximation is valid, and the potential of a single ion from the Linhard theory of screening, which is applicable for quantum plasmas [47]. For our calculations the Thomas-Fermi approximation is valid, and the potential of a single ion \( \phi \) varies slowly on the scale of Fermi wavelength \( \hbar / p_F \), where \( p_F \) is the Fermi momentum. In accordance with [25], [27], for randomly distributed charged impurity ions we have the Fourier transform of potential (11):
\[
|V(q)|^2 = N_c \frac{4\pi^2 e^4}{\kappa^2 q^2 \epsilon^2(q)},
\]
where \( \epsilon(q) (q = |q|) \) is the 2D static dielectric (screening) function in random phase approximation (RPA) appropriate for graphene [46], given by the formula
\[
\epsilon(q) = 1 + \frac{q_s}{q} \times \left\{ \begin{array}{ll}
1 - \frac{\sqrt{q^2 - k_F^2}}{q} & q \leq 2k_F \\
1 - \frac{\sqrt{q^2 - k_F^2}}{2q} - q \sin^{-1}\frac{2k_F}{4\kappa q} & q > 2k_F
\end{array} \right.
\]
(40)
Here \( k_F = \varepsilon_F / \hbar v_F \) is 2D Fermi wave vector, \( q_s = 4e^2 k_F / (\hbar \kappa v_F) \) is the effective graphene 2D Thomas-Fermi wave vector, and \( \kappa = \kappa (1 + \pi r_s / 2) \) is the effective dielectric constant of a substrate. The ratio of the potential to the
kinetic energy in an interacting quantum Coulomb system is measured by the dimensionless Wigner-Seitz radius
\[ r_s = \frac{e^2}{\kappa \hbar v_F}, \]
where \( \kappa \) is the background lattice dielectric constant of the system, \( e^2/\hbar v_F \approx 2.18 \) is "effective fine-
structure constant" in graphene (in the vacuum). Taking into account Eqs. (38)-(40), (30), and integrating in Eq. (37) over \( P' \), we obtain the following relation for the partial absorption rates \( dN_{abs}(n)/dt \) of SB process:

\[
\frac{dN_{abs}(n)}{dt} = \frac{g_s g_v n v_F}{\pi \hbar c} \left( \frac{r_s}{2 + \pi r_s} \right)^2 \int_{v_F P + n \hbar \omega} dP \int d\theta d\theta' \frac{|P|^2 |P'|^2 |\tilde{M}^{(n)}|}{(\hbar q)^2 \epsilon^2 (q)} \times \left( 1 - e^{-\frac{\hbar q}{\pi r_s}} \right) f (v_F P - n \hbar \omega) (1 - f (v_F P')).
\]

(41)

At the consideration of numerical results it is convenient to represent the differential cross-sections of SB on the charged impurities in the form of dimensionless quantities. For the dimensionless rates \( T dN_{abs}(n)/dt \) in the field of linearly polarized EM wave with the dimensionless vector potential \( \overrightarrow{A}(t) = -eX \sin(2\pi t) \) we have:

\[
T \frac{dN_{abs}(n)}{dt} = \frac{2 g_s g_v n}{300} \left( \frac{r_s}{2 + \pi r_s} \right)^2 \int_{\mathcal{P} + n} d\mathcal{P} \int d\theta d\theta' \frac{T (\mathcal{T} - n)}{\epsilon^2 (\mathcal{T})} \frac{|\tilde{M}^{(n)}|^2}{\epsilon^2 (\mathcal{T})}
\]

At the consideration of numerical results it is convenient to represent the differential cross-sections of SB on the charged impurities in the form of dimensionless quantities. For the dimensionless rates \( T dN_{abs}(n)/dt \) in the field of linearly polarized EM wave with the dimensionless vector potential \( \overrightarrow{A}(t) = -eX \sin(2\pi t) \) we have:
where

\[
|T_{p_{1}p_{1}}^{(n)}|^{2} = \left| \int_{0}^{1} d\tau \left( 1 + e^{i(\Theta_{\mathbf{p}_{1}\mathbf{p}_{1}} - \mathbf{x}_{\mathbf{p}_{1}} \sin(2\pi \tau) - \Theta_{\mathbf{p}_{1}\mathbf{p}_{1}} - \mathbf{x}_{\mathbf{p}_{1}} \sin(2\pi \tau))} \right) \times \exp \left\{ i2\pi n\tau - 2\pi i \int_{0}^{\tau} \left[ \left( (\mathbf{p}_{x} - \chi \sin(2\pi \tau'))^{2} + \mathbf{p}_{y}^{2} - \mathbf{P'} \right) \right] d\tau' \right\}^{2} \right|.
\]

In Eq. (42) the dimensionless momentum, energy, time, and relativistic invariant intensity parameter of EM wave introduced as follows:

\[
\mathbf{T}_{x,y} = \frac{v_{F}}{\hbar \omega} \mathbf{p}_{x,y}, \mathbf{P} = \frac{\mathcal{E}}{\hbar \omega}, \mu = \frac{\mu}{\hbar \omega}, \mathbf{T} = \frac{T_{e}}{\hbar \omega}, \mathbf{P} = \frac{v_{F}}{\hbar \omega} \mathbf{P},
\]

\[
\mathbf{T}_{F} = \frac{\mathcal{E}_{F}}{\hbar \omega}, d\tau = \frac{dt}{T}, \chi = \frac{e v_{F}}{\hbar \omega^{2}} E_{0}.
\]

The analytic integration over scattering angles \(d\theta_{x}, d\theta_{y}, dp_{x}, dp_{y} \) and momentum is impossible, so we make numerical integrations. For numerical analysis of SB cross sections in graphene we assume Fermi energy \(\epsilon_{F} \approx \mu = 20\hbar \omega \) (\(\epsilon_{F} \gg n\hbar \omega\)), electrons temperature \(T_{e} = 0.1\epsilon_{F}\), coherent EM linearly polarized radiation, dielectric environment constant \(\kappa = 2.5\) for an impurity strength in the presence of the SiO\(_{2}\) substrate \[40\]. Wigner-Seitz radius \(r_{s} = 0.87592\).

In the Fig. 1 and Fig. 2 the envelope of partial rate of inverse-bremsstrahlung absorption in graphene is shown for various wave intensities with energy of photons \(\epsilon \equiv \hbar \omega = 0.01\) eV (\(\lambda = 1.24 \times 10^{-2}\) cm) and \(\epsilon = 0.005\) eV (\(\lambda = 2.48 \times 10^{-2}\) cm), respectively. As seen from these figures, the multiphoton effects become essential with the increase of the wave intensity.

To show the dependence of the inverse bremsstrahlung absorption rate on the laser radiation intensity, the total SB rate \[50\] via the mean number of absorbed photons by per of impurity ion, per unit time in doped graphene versus the parameter \(\chi\) for various photon energies is shown in Fig. 3. To compare with the linear theory \[21\], in Fig. 4 we plot scaled absorption rate \(\chi^{-2} dN_{abs}/dt\) versus \(\chi\). As it is presented by the figure, for the large values of \(\chi\) the SB rate exhibits a slightly falling dependence on the wave intensity. To all other, in the scope of linear theory the scaled absorption rate does not depend on the wave intensity, while for the large values \(\chi\) it is suppressed with the increase of the wave intensity for the multiphoton SB process.

As was expected and as seen from these figures with the increasing of laser intensity the multiphoton effect becomes dominant compared to the one-photon scattering in linear theory \[21\]. For THz photons, the multiphoton interaction regime in graphene can be achieved already at the intensities \(I_{x} \sim 10^{7} W/cm^{2}\). Thus, for these intensities multiphoton SB process opens new channels for the wave absorption, and we can expect strong deviation of absorbance of a single layer doped graphene from linear one, which for frequencies smaller than Fermi energy is zero \[18\]. The latter opens up possibility for manipulating of electronic transport properties of the doped graphene by coherent radiation field.

**IV. CONCLUSION**

We have presented the microscopic relativistic quantum theory of multiphoton SB absorption in doped graphene in the presence of the coherent EM radiation of arbitrary intensity and frequency (actually terahertz radiation to exclude the valence electrons excitations at high Fermi energies). The Liouville-von Neumann equation for the density matrix has been solved analytically considering an external wave-field exactly, and the charged impurity ions arbitrary electrostatic potential in the Born approximation. These solutions for SB at the linear polarization of EM wave are used for derivation of a relatively compact formula for the nonlinear inverse-bremsstrahlung absorption rate when 2D Dirac fermions are represented by the grand canonical ensemble. The obtained relativistic analytical formulas have been analyzed numerically for screened Coulomb potential. The concluded results show that SB rate in graphene in the presence of strong terahertz radiation field being practically independent of the plasma Fermi energy, has an essentially nonlinear dependence on the increase of the wave intensity, and the multiphoton absorption/emission processes play a significant role already at moderate laser intensities. It is shown that one can achieve the efficient absorption coefficient by this mechanism for these intensities.
FIG. 3: Total rate of inverse bremsstrahlung in doped graphene vs the dimensionless parameter $\chi$ for setup of Fig. 1 at various photon energies $\varepsilon$.

FIG. 4: Total rates of the inverse bremsstrahlung absorption scaled to $\chi^2$ vs the parameter $\chi$.

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