In-plane Magnetoconductivity of Si-MOSFET’s:
A Quantitative Comparison between Theory and Experiment.

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For densities above \( n = 1.6 \times 10^{11} \text{ cm}^{-2} \) in the strongly interacting system of electrons in two-dimensional silicon inversion layers, excellent agreement between experiment and the theory of Zala, Narozhny and Aleiner is obtained for the response of the conductivity to a magnetic field applied parallel to the plane of the electrons. However, the Fermi liquid parameter \( F_0^\sigma (n) \) and the valley splitting \( \Delta_V(n) \) obtained from fits to the magnetoconductivity, although providing qualitatively correct behavior (including sign), do not yield quantitative agreement with the temperature dependence of the conductivity in zero magnetic field. Our results suggest the existence of additional scattering processes not included in the theory in its present form.

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Two-dimensional systems of electrons \([1, 2]\) and holes \([3, 4]\) have been the focus of a great deal of attention during the last few years \([5]\). Their resistance exhibits metallic temperature dependence above a critical electron density \( n_c \), and a strong dependence on in-plane magnetic field \([1, 2]\). Recent theory of Zala, Narozhny and Aleiner (ZNA) \([6, 7]\) attempts to account for the behavior of the conductivity of these strongly interacting electron systems using a Fermi liquid approach. Within this theory, the temperature and field dependence of the correction to the conductivity due to electron-electron interaction arises from the suppression of quantum interference between different electron trajectories (i.e. coherent scattering off Friedel oscillations). In this paper we present a detailed quantitative comparison of the ZNA theory \([6, 7]\) with the in-plane magnetoconductivity of the 2D electron system in Si-MOSFET’s measured at different temperatures and different electron densities.

Excellent agreement between theory and experiment is obtained for the magnetic field dependence, allowing the determination of the Fermi liquid constant \( F_0^\sigma \) and valley splitting energy \( \Delta_V \). However, the temperature dependence of the conductivity at zero magnetic field is considerably stronger than predicted theoretically (for the same values of \( F_0^\sigma \) and \( \Delta_V \)). Our results suggest the presence of additional scattering processes not included in the theory \([6, 7]\).

We begin with a very brief review of the theory. The conductivity of a disordered electron system (characterized by the large dimensionless conductance \( g = 2\hbar/e^2R_0 \gg 1; R_0 \) is the sheet resistance) can be expressed as follows \([3]\):

\[
\sigma = \sigma_D + \delta\sigma_{WL} + \delta\sigma_{ee} + O\left( \frac{1}{g} \right).
\]

Here the Drude conductivity is \( \sigma_D = ne^2\tau/m^* \), \( n \) is the electron density, \( \tau \) is the elastic mean free time and \( m^* \) is the effective mass. The weak localization correction \([3]\)

\[
\delta\sigma_{WL} = \frac{e^2}{2\pi^2\hbar} \ln \frac{\tau_c}{\tau},
\]

depends on external parameters through the dephasing time \( \tau_c \) (see Refs. \([3, 13]\) for details). In zero magnetic field the resulting temperature dependence is logarithmic, unlike the correction due to electron-electron interaction \( \delta\sigma_{ee} \), which is linear in temperature in the intermediate regime (see Ref. \([11]\) and below).

Based on Eq. \((3)\) and the available experimental data \([10]\), one can estimate the value of the weak localization correction \((3)\). For electron density \( n = 2.47 \times 10^{11} \text{ cm}^{-2} \) used in this work (see below), Eq. \((3)\) yields \( \delta\sigma_{WL} = -0.42 \times e^2/h \) at \( T = 0.25 \text{ K} \) and \( \delta\sigma_{WL} = -0.22 \times e^2/h \) at \( T = 1 \text{ K} \). These values are much smaller than typical variations of the conductivity with in-plane magnetic field \( H \) (see Fig. \(1\)). We therefore neglect the weak localization correction in the present paper.

The correction term due to interactions, \( \delta\sigma_{ee} \), was recently calculated by ZNA \([11]\) for all temperatures smaller than the Fermi energy. Physically, this correction arises due to coherent scattering off Friedel oscillations. For \( H = 0 \) the result of Ref. \([11]\) consists of the charge (singlet) channel contribution

\[
\delta\sigma_C = \frac{e^2}{\pi\hbar} \frac{T}{h} \left[ 1 - \frac{3}{8} f(T) \right] - \frac{e^2}{2\pi^2\hbar} \ln \frac{E_F}{T},
\]

and the triplet channel contribution (i.e. the contribution of spin-exchange processes)

\[
\delta\sigma_T = \frac{F_0^\sigma}{1 + F_0^\sigma} \frac{e^2}{\pi\hbar} \frac{T}{h} \left[ 1 - \frac{3}{8} f(T; F_0^\sigma) \right] - \left( 1 - \frac{1}{F_0^\sigma} \ln(1 + F_0^\sigma) \right) \frac{e^2}{2\pi^2\hbar} \ln \frac{E_F}{T}.
\]
Note that the latter must be multiplied by the number of channels (3 in the absence of valley degeneracy and the Zeeman splitting $E_z = \mu g_0 H$; the Lande $g$-factor $g_0 = 2$);

$$\delta\sigma_{ee} = \delta\sigma_C + 3\delta\sigma_T.$$ 

Magnetic field freezes those channels that correspond to non-zero total spin components, giving rise to a dependence of the conductivity on magnetic field \[12\]:

$$\delta\sigma(E_z) = \delta\sigma_{ee}(E_z) - \delta\sigma_{ee}(0) = \frac{e^2}{\pi}\left[\frac{2F_0^\sigma}{(1 + F_0^\sigma)}\right] T\tau \quad (3c)$$

$$\times K_b\left(\frac{E_z}{2T}, F_0^\sigma\right) + K_d\left(\frac{E_z}{2\pi T}, F_0^\sigma\right) + m(E_z\tau, T\tau; F_0^\sigma)$$.

Here the ballistic (i.e. dominant for $T\tau \gtrsim 1$) contribution to Eq. \([3d]\) is

$$K_b(x, F_0^\sigma) = x \coth x - 1 + K_2(x, F_0^\sigma), \quad (3d)$$

In the above expressions the dimensionless functions $f(x)$, $t(x, F_0^\sigma)$, and $m(y, x; F_0^\sigma)$ describe the crossover between the diffusive ($T\tau \ll 1$) and ballistic regimes. These functions decay smoothly from unity to zero and change the value of $\delta\sigma_{ee}$ by only few per cent outside the crossover region. The details regarding these functions, as well as explicit expressions for $K_2(x, F_0^\sigma)$ and the diffusive part of the magnetoconductivity $K_d(E_z/2\pi T, F_0^\sigma)$, can be found in Refs. \([11,12]\).

The purpose of this paper is to present a detailed quantitative comparison of the ZNA theory \([11,12]\) with the results of transport measurements in Si-MOSFET’s. Note that this theory (as described in Refs. \([1,12]\) and above) considers an idealized 2DEG without taking into account material-dependent details. For 2D electron systems in Si-MOSFET’s an important additional feature is the valley degeneracy \([17]\). If we neglect inter-valley scattering (in other words, assume that the valley index is a good quantum number) and assume that the two valleys are degenerate, then we observe that the valley index can be treated as a pseudo-spin. In this case electrons have not one but two “spin” indices, and the total number of scattering channels is thus 16 (as opposed to 4 in the usual case considered in Ref. \([12]\)). In reality, the valleys are split \([17]\) with an energy splitting $\Delta V$. This splitting is similar to the effect of the Zeeman field, since it only shifts energies of the electronic states without affecting the corresponding wave functions.

Taking into account the valley degeneracy, we can write the interaction correction to the conductivity in the following form:

$$\delta\sigma_{ee} = \delta\sigma_C + 15\delta\sigma_T + 2\delta\sigma(E_z)$$

$$+ 2\delta\sigma(\Delta V) + \delta\sigma(E_z + \Delta V) + \delta\sigma(E_z - \Delta V), \quad (4)$$

where all the terms in the second line are defined by Eq. \([4]\). Equation \([4]\) is the main theoretical result which we intend to test against the data. Our strategy is the following. The theoretical expressions Eqs. \([3,4]\) contain three phenomenological parameters: the Fermi liquid constant $F_0^\sigma$, the elastic mean-free time $\tau$, and the valley splitting $\Delta V$. For purposes of comparison with experiment, the parameters $F_0^\sigma$ and $\Delta V$ are free fitting parameters. Both will be determined below from the magnetic field dependence of the conductivity. On the other hand, one has less freedom in the determination of the parameter $\tau$. Since the theory calculates temperature corrections to the impurity scattering time, the value of $\tau$ in all theoretical expressions is the scattering time that the system would have at $T = 0$ in the absence of quantum corrections. Therefore $\tau$ must be determined from the temperature dependence of the conductivity: one has to extrapolate the linear temperature dependence from intermediate temperatures to $T = 0$ and use the y-intercept as the zero temperature value of the Drude conductivity (thus neglecting the logarithmic corrections; therefore this value does not coincide with the actual measured residual conductivity of the system).

The theory described above is valid \([1]\) when the dimensionless conductance of the system is large $g \gg 1$ and the system is not too close to the Stoner instability $T < (1 + F_0^\sigma)^2 E_F$. Also, the magnetic field dependence Eq. \([4]\) is valid for relatively weak fields, well below the full polarization of the electron system. The comparison between experiment and theory presented in this paper is restricted to a range of densities and temperatures well within these limits (see below and Figs. \([1,2]\)).

The remainder of the paper is devoted to a direct comparison of the data with Eq. \([4]\). Measurements were taken on two Si-MOSFET’s with mobility $\mu \approx 25,000 \text{ V}/(\mu\text{cm}^2\text{s})$ at 0.3 K. Data were obtained using standard four-terminal AC techniques on samples with split-gate geometry to 12 T at City College and in fields up to 20 T at the National High Magnetic Field Laboratory \([25]\). The dimensions of the measured portion of the 2DEG are $120 \times 50 \mu\text{m}^2$. Measurements were taken at temperatures down to 0.25 K in the linear regime using small currents of about $1 - 2 \text{ nA}$ to prevent overheating the electrons.

We first consider a comparison of the magnetoconductivity data with the ZNA theory. In Fig. \([1]\) we show the longitudinal conductivity $\sigma_{xx}$ as a function of magnetic field $H||$ applied parallel to the plane of the Si-MOSFET for electron density $n = 2.47 \times 10^{11} \text{ cm}^{-2}$ at different temperatures. In agreement with earlier results \([18]\), there is a substantial decrease of the conductivity with increasing magnetic field. The grey lines are obtained by direct evaluation of Eq. \([4]\) using $F_0^\sigma$ and $\Delta V$ as fitting parameters; the value of $\Delta V$ was constrained to be approximately independent of temperature, as expected within the theory. The theoretical results are shifted vertically to match the
The determination of the value of $\tau$ used in this calculation is more subtle. As pointed out earlier, the Drude conductivity was determined from the temperature dependence shown in Fig. 3 by extrapolating the linear part of the curves to zero temperature. In order to extract the value of $\tau$ one needs to know the effective mass of the electrons. One can measure the product $m^* g^*$ experimentally (by analyzing Shubnikov-de Haas data, for instance). The renormalized value of the Lande $g$-factor is related to the same Fermi liquid constant $g^* = g_0/(1 + F_0^\sigma)$. Using these relations self-consistently, we obtain the value of $\tau$ for each electron density (with $F_0^\sigma$ taken at $T = 0.25$ K). Once determined, $\tau$ was assumed to be independent of temperature. For the density $n = 2.47 \times 10^{11}$ we find $\tau = 5.28$ ps.

![Graph of conductivity vs. magnetic field](image)

**FIG. 1.** Conductivity of a Si-MOSFET versus in-plane magnetic field at $T = 0.25$, 0.5, 0.8, 1, 1.3, 1.6, 2.6, 3.6 (K) (from top). Symbols denote the experimental data; solid lines are calculated using Eqs. (3), (4). Inset: variation of $F_0^\sigma$ and $\Delta V$ with temperature obtained from fits to Eq. (4). The electron density is $n = 2.47 \times 10^{11}$ cm$^{-2}$. The scattering time used to evaluate Eq. (4) is $\tau = 5.28$ ps (see text for discussion).

The data in Fig. 1 demonstrate that there is very good agreement between experiment and theory. For electron density $n = 2.47 \times 10^{11}$ cm$^{-2}$ a fit to the theory Eq. (4) of the magnetoconductivity data taken at temperatures below 2 K yields $F_0^\sigma = -0.15$ and $\Delta V = 1$ K. At temperatures above 2 K we found an appreciable dependence of the parameter $F_0^\sigma$ on temperature; all other electron densities show similar behavior.

In Fig. 2 we show the in-plane magnetoconductivity for different densities at a fixed temperature $T = 1$ K. This temperature corresponds to the linear portion of the conductivity vs. temperature curve in zero magnetic field (see Fig. 3), i.e. to the ballistic regime $T \tau > 0.1$ of the theory [21]. The comparison was done for magnetic field below 6 T, where variations of the magnetoconductivity are relatively small (well within the range of applicability of the calculations). Again, the agreement between theory and experiment is impressive. In the inset to Fig. 2 we show the evolution of the fitting parameters $F_0^\sigma$ and $\Delta V$ with electron density. The Fermi liquid parameter $F_0^\sigma$ is nearly independent of density, while the valley splitting $\Delta V$ fluctuates between 0.5 and 1.2 K.

![Graph of conductivity vs. magnetic field](image)

**FIG. 2.** Conductivity of a Si-MOSFET versus in-plane magnetic field for densities $n = 3.3$, 3.0, 2.75, 2.47, 2.19, 1.92, 1.64 $\times 10^{11}$ cm$^{-2}$ (from top). Symbols denote the experimental data; the lines are calculated using Eq. (4) (for fields weaker than the field for full polarization). Inset: $F_0^\sigma$ and $\Delta V$ as a function of electron density. The temperature $T = 1$ K.

We now compare the measured temperature dependence of the conductivity in zero magnetic field with that predicted by the theory [11,12]. In Fig. 3 we show the data together with the predictions of the theory Eq. (4) obtained using the values of $F_0^\sigma$ and $\Delta V$ determined from the magnetic field dependence of the conductivity; see Fig. 2. The variations of the fitting parameters $F_0^\sigma$ and $\Delta V$ with temperature (see Fig. 1), which were not taken into account in this calculation, may change the high temperature tails of the calculated curves at $T > 2$ K in Fig. 3. However, it is clear from Fig. 3 that even for $T < 2$ K, the temperature dependence of the conductivity observed experimentally is considerably stronger than that calculated from the theory [11,12]. Thus, the parameters deduced from fitting the field dependence of the conduc-
tivity do not yield quantitative agreement with the observed temperature dependence.

![Graph](image)

FIG. 3. Temperature dependence of the conductivity of 2D electrons in a Si-MOSFET at $H = 0$ for densities $n = 3.3$, 3.0, 2.47, 2.19, 1.92, $1.64 \times 10^{11}$ cm$^{-2}$ (from top). The symbols indicate experimental data. The straight lines are calculated using the theory [1,2]. The straight line drawn for density $2.47 \times 10^{11}$ cm$^{-2}$ is an extrapolation to zero temperature of the (nearly) linear part of the conductivity used to determine the value of $\tau = 5.28$ ps (see text).

We have attempted to fit the temperature dependence and the magnetic field dependence simultaneously by allowing $F_0^0$ and $\Delta V$ to vary freely with temperature. Although a moderately good fit can be obtained, the resulting variation of $\Delta V$ with temperature, although similar for different densities, is unacceptably large (with variations up to a factor of 20). Another possible approach is to keep $\Delta V$ small (as in Fig. 2) to allow $F_0^0$ to assume the values needed to produce a good fit for the temperature dependence of the conductivity, and to compare these values with those in Fig. 2. Clearly, the steeper experimental curves in Fig. 2 correspond to larger absolute values of $F_0^0$, similar to those obtained in Ref. [23]. Such disagreement between the values of $F_0^0$ obtained from two separate measurements illustrates once more that the theory in its current form does not provide a consistent description of all measurements in Si-MOSFET’s. A possible explanation of the above discrepancy is the neglect of inter-valley scattering in the theory in its present form, or perhaps the presence of some other source of scattering that has not been identified.

In summary, for the strongly interacting two dimensional system of electrons in silicon MOSFET’s, excellent agreement between experiment and the theory of Zala, Narozhny and Aleiner [11,12] was obtained for the response of the conductivity to a magnetic field applied parallel to the plane of the electrons. The Fermi liquid parameter $F_0^0(n)$ and the valley splitting $\Delta V(n)$ obtained from fits to the magnetoconductivity, although providing qualitatively correct behavior (including sign), do not yield quantitative agreement with the temperature dependence of the conductivity in zero magnetic field. Our results suggest the existence of additional scattering processes not considered by the theory in its present form.

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[20] The fits are more sensitive to $F_0^0(n)$, which is determined with an accuracy of ±15%, than to the values chosen for $\Delta V$, which can be varied by 50%.
Note that the description of transport and thermodynamic properties of the 2DEG in terms of the single Fermi liquid parameter is an approximation. Strictly speaking, the constant that describes the susceptibility is the zero
\( \phi \)th harmonic of the spin part of the Landau function \( f(\theta) \), while the constant that describes the conductivity in the ballistic regime corresponds to backscattering, i.e. \( \theta = \pi \). The ZNA theory \[1\] assumes the Fermi liquid parameter to be independent of \( \theta \) and thus uses just one constant \( F_0^o \) to provide a phenomenological description of interaction in the triplet channel.