Sakharov Curvature in Rowlands Duality Spacetime: Do vacuum ‘spacetime forces’ curve matter?

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Abstract. Space and time, inertia and gravity are unseen yet they are the most fundamental concepts needed to understand physics classically, relativistically, and quantumly. Non-Euclidean geometries and relativity theory have yielded new insights in treating the four interrelated metaphysical concepts. Minkowski linked time and space. Einstein linked Newton’s first and second laws of motion. Attempts to link electromagnetism / gauge forces to theories of gravity have been somewhat of a challenge. Could discrete inertia be the unrecognized link between quantum mechanics and relativity theory? Newtonian mechanics and Maxwell’s equations are based on the mathematics of continuous functions and relativistic effects count Newtonian laws as approximations. The classical period of physics, based on ideas of continuity came to an end when discreteness and quantum considerations, like the Planck black-body spectrum, became experimentally and theoretically relevant. General relativity and quantum mechanics have reshaped our way of thinking about classical geometric concepts of space and time as well as inertia and gravity. Unsolvable mathematical frameworks have not provided the inertial / gravitational and space / time superstructures needed to address the unification problem. The first attempts to link gravity to the behavior of subatomic particles originated in papers written by two Russian physicists. Sakharov proposed that Newtonian gravity could be interpreted as a dipolar van der Waals force that emerges from the action of quantum fluctuations of the empty vacuum. Zeldovich gave a new meaning to Einstein’s cosmological constant, as a measure of the curvature of space, allowing for new insights into cosmological inflation and dark energy. It will be argued in this paper that zero-point fluctuations which produce inertial reaction forces also produce the elasticity / curvature of spacetime suggested by Sakharov / Zeldovich. Is it possible that dynamic spacetime gives matter its curvature? We will attempt to show the feasibility of this idea by appealing to Rowlands quaternion representation of spacetime and gauge / gravity duality hypothesis. We will then argue that Sakharov’s induced continuous gravitational field emerges which balances the discrete inertial spacetime forces satisfying a zero-totality condition.

1. Introduction
The “origin and nature of inertia” was described by Vigier [1] as an “unsolved mystery in modern physics” even though it is known that inertia instantaneously opposes acceleration. Newton viewed inertia as an internal property of matter. Mach, on the other hand, thought that inertia was an external property collectively linked to all matter in the universe. General relativity (GR) links spacetime curvature to gravitational fields and inertia is accommodated by Einstein’s equivalence principle (EEP). GR cannot, however, accommodate Machian-type inertia and attempts to explain gravity as the source of inertial forces in the geometrical interpretation fails. Any appeal to the equivalence principle leads, rather, to a circular argument. Solutions to Einstein’s field equations have been formulated for
empty de Sitter spaces and rotating Godel universes. Vigier’s investigations into inertia from a relativistic perspective revealed other conflicts when Mach’s principle is considered and he attempted to reconcile GR and Mach’s principle proposing that non-Machian forces operated in “local interactions of a Dirac subquantum aether model stemming from Einstein-de Broglie-Bohm causal stochastic quantum mechanics”. Also called the Dirac Sea, a theoretical model of the vacuum with negative energy, Vigier’s aether model combines matter and antimatter, particles and antiparticles. The aether’s density is extremely large but cannot be detected because the observable effects cancel each other out. Fluctuations in the vacuum can be understood as the production of a weak dipolar fermion / antifermion pair, a harmonic oscillator creation-annihilation type of mechanism. The same fluctuations are also responsible for the Casimir effect and the Van der Waals force corresponding to the potential for a fluctuating dipole-dipole interaction.

Another model for inertia that uses the electromagnetic part of the Unruh radiation has been suggested by Rueda and Haisch [2-3]. They substitute electromagnetic fields of the quantum vacuum for Mach’s principle as the “external causative agent of inertia” providing a “locally-originating reaction force.” The “quantum inertia vacuum hypothesis” that inertia originates in interactions between quarks and electrons and the zero-point field of the vacuum was inspired by Sakharov’s 1967 conjecture [4]. Cosmological spacetime, surprisingly, can be considered as a phenomenon associated with the physics of particles and fields. Spacetime structure is not directly observed nor measured but is deduced indirectly by looking at the propagation of radiation and matter and has been assimilated into a theory called quantum field theory (QFT). Inertial properties of neutrons and protons arise in the zero-point fluctuations (ZPF) scattering at the level of individual quarks. ZPF, zero-point energy (ZPE), zero-point field, and vacuum energy considerations all appeal to theories in particle physics. The density of the ZPE is linked to Newton’s gravitational constant G and the particles may originate in the cosmic microwave background (CMB) which has been experimentally validated. The quantum vacuum stress between conducting plates caused by modifications to the electromagnetic ZPF was predicted by Casimir in 1948. This effect has also been confirmed by experiment. The Zeeman, Aharon-Bohm, Sagnac, and Lamb effects, which are considered as evidence of the Dirac polarized vacuum, are based on classical physical conditions but cannot be replicated in laboratory experiments. The continuity of the filled Dirac vacuum of mass-energy, the Higgs field, and the need for instantaneous correlation of inertial reaction on discrete fermionic or bosonic states requiring an instantaneous and continuous gravitational force to balance it seem to satisfy Rowlands’ spacetime zero-totality condition. The following sections will investigate these ideas.

2. Induced gravity

Einstein’s theory about gravity appeared in 1915. It is also theory about space-time, a geometry which is not fixed but governed by a set of field equations. Gravity, rather than being described as a Newtonian force on a fixed space-time manifold, is embedded in the geometry of space-time. Unknown origin of Newtonian-type forces are no longer relevant in this picture of gravity. Sakharov, on the other hand, proposed that vacuum fluctuations, forces, were relevant in gravitational theory. He suggested that gravity was “analogous to the discussion of quantum electrodynamics (QED).” Sakharov’s hypothesis “identified the action (in Einstein’s theory of gravity) as being the change in quantum fluctuations of vacuum resulting in a distortion of space.” Sakharov obtained an expression for Newton’s gravitational constant G which is essential in Einstein’s field equation. The expression

\[ G = \frac{c^4L^2}{\hbar} = \frac{hc^3}{p_0^2} \]

connects Newton’s gravitational constant to the elementary Planck length \( L \), the cutoff length of Einstein’s action, and a limiting momentum \( p_0 = \frac{\hbar}{L} \). If the evolution of the universe in the big bang theory is followed backwards in time it can be shown that the entropy is a consequence of baryon asymmetry. In 1967 Sakharov proposed a general model for the non-zero baryon number in which hypothetical heavy bosons mediated interactions between quarks
and leptons. The following year he linked gravitation to spacetime and vacuum emergence treating gravity, not as a fundamental interaction but, as a secondary effect induced by non-gravitational forces. This raises the question about spacetime. Is spacetime really non-Euclidean? Could the measurements of spacetime curvature be nothing more than light propagation through a quantum polarized vacuum? Is it possible that the spacetime vacuum gives curvature to matter? A view supported by the helical molecular structure at the DNA scale in which bipolar van der Waals forces operate. Van der Waal forces are a fundamental aspect of the nature of the weak vacuum which plays a significant role in all material phase transitions. These transitions occur with the creation of nuclear interbaryonic matter through the strong forces between quarks which is observed as a Bose-Einstein condensation.

Today Planck-size forces are collectively called gauge fields and include the electric, weak, and strong interactions. Sakharov’s view of gravity as “a metric elasticity of space” arises from particle interactions. He identified the action term of Einstein’s geometrodynamics with the “change in the action of quantum fluctuations of the vacuum… and the dependence of the action of the quantum fluctuations on the curvature of space.” Sakharov viewed Einstein’s action

$$S(R) = -\frac{1}{16\pi G} \int (dx) \sqrt{-g} R$$ \hspace{1cm} (1)

of spacetime as a function of curvature. The invariant Ricci curvature tensor, $R$, determines how matter diverges or converges in time. This leads to Sakharov’s idea of an expanding and contracting universe. Sakharov postulated that the “presence of the action (1) leads to a ‘metrical elasticity’ of space, i.e., to generalized forces which oppose the curving of space.” Opposing the curving of space implies that a gravitational field is present. It may also imply, if we appeal to Einstein’s equivalence principle, that an inertial field is present. And, at the Planck scale where gauge forces reside, it would also imply a quantized inertia. Sakharov expands the Lagrange density function in powers of the curvature

$$L(R) = L(0) + A \int kdk \cdot R + B \int \frac{dk}{k} R^2 + ...$$ \hspace{1cm} (2)

where the first term corresponds to Einstein’s cosmological constant and the second term, corresponds to the action (1)

$$G = -\frac{1}{16\pi A} \int kdk$$, where $A \sim 1$. \hspace{1cm} (3)

Integrating (3) and solving for $k$ yields $k_0 \sim 10^{38}$ eV $\sim 10^{33}$ cm$^{-1}$ which corresponds to the inverse of the Planck length. The third term in (2) “leads to corrections, non-linear in $R$, to Einstein’s equations.” Sakharov argues that the “quantity $k_0$ determines the mass of the heaviest particles existing in nature” and “determines the limit of applicability of present-day notions of space and causality.” He argues that the “density of the vacuum Lagrange function in a simplified ‘model’ of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and ‘virtual’ particles (i.e., hypothetical particles which give an opposite contribution to that of the real particles to the $R$ dependent action) a finite change of action arises that is proportional to $M^2 R$ and which we identify with $R/G$. ” The opposite contribution proportional to $R$ is significant as it correlates with a repulsive force. For a modern perspective on Sakharov’s induced gravity see [5]. The finite change of action identified by Sakharov that is proportional to $M^2 R$ can be correlated with an anti-gravitational Hooke-like repulsive term. Newton’s second law originally accommodated a discrete Hooke-like term but was rejected several times as not being a natural physical law. Freeing gravity from being a nebulous force does not resolve the problem of continuity and discreteness. A unified picture of the continuous wave / discrete particle nature of light revealed in slit experiments has yet to be resolved. A mathematical attempt at a unified treatment of the discrete point / continuous manifolds has been resolved. The mathematical solution found by Grothendieck is the topos which combines topological spaces and the combinatorial structures found in arithmetic. When applied to
quantum formalism it provides a solution for the coexistence of discrete and continuous variables. Combining continuous variables with discrete ones appears to be a necessary condition for all physical theories to have universal validity, whether within a physical or mathematical framework. For a full account with historical details see [6-7].

Einstein introduced a cosmological constant to produce a static universe. Recent data collected by the Supernova Cosmology Project and the High-Z Supernova Team provide irrefutable evidence that the universe is expanding. Zeldovich [8] characterized the cosmological constant as the “curvature of empty space.” A positive cosmological constant $\Lambda \approx 10^{-52}$ cm$^{-2}$ ostensibly gives the required repulsive force to cause the expansion.

3. Virtual particles

Sakharov’s virtual particles produce a repulsive force proportional to $R$, the second term in (2). Zeldovich regarded the cosmological constant, the first term in (2), as curvature. It seems reasonable to combine these two ideas. The introduction of the cosmological constant into Einstein’s field equations yields the following equation

$$ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}. $$

(4)

Equation (4) leads to the Friedmann equation where $\rho_m$ represents matter density and $\rho_\Lambda = \frac{A}{8\pi G}$ is the vacuum energy density

$$ R'^2 = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda) R^2 - kc^2. $$

(5)

Differentiating (5) and simplifying gives

$$ R'' = \left[ \frac{8\pi G}{3} (\rho_m + \rho_\Lambda) \frac{A}{3} \right] R. $$

(6)

It follows from (6) that a universe where $R > 0$ is accelerating. The general spherically symmetric vacuum solution of (4) results in the Schwarzschild-de Sitter metric having an asymptotic curvature induced by vacuum energy corresponding to $\Lambda$. The weak field limit leads to the potential

$$ \phi = \frac{GM}{r} + \frac{Ac^2}{6r^2} $$

(7)

from which the full-force Newton second law can be derived

$$ a = \frac{F}{m} = \nabla \phi = -\frac{GM}{r^2} + \frac{Ac^2}{3r^2}. $$

(8)

Newton’s second law of motion (N2L) is clearly visible as the first term in (8). A Hooke-like repulsive second term, identified by Sakharov, also makes an appearance. The repulsive Hooke-like term, originally found by Newton, is derived from the Einstein action, regarded as a repulsive factor by Zeldovich, and examined in astronomical satellite-based studies advanced. Treating $F/m$ as acceleration due to gravity we can set $a = 0$ we can solve for $r$. We find the distance $r = \sqrt{\frac{3GM}{Ac^2}}$ at which a galaxy of mass $M$, or a star like our Sun, no longer has attractive influence. For a historical overview of the three century rejection of the repulsive Hooke-like term in (8) see [11]. An important thing to note in the two terms in (8) is that continuous gravitational attraction is linked to the discrete inertial repulsion. This combination is very significant. Gravity and inertia are now seen to be linked by a Machian principle. The Planck mass can be understood to represent an inertial quantum, and an event horizon for the time-delayed inertial force, through which inertial mass appears to be generated. For a fuller discussion of this and the significance of discreteness and continuity problem see Chapters 7, 15, and 16 in Rowlands [9].
4. Singularities

Einstein’s theory of gravity leads to a QFT which cannot be normalized by standard quantization methods. An increasing number of singularities are encountered when spacetime is curved. Sakharov’s hypothesis is that gravitational phenomenon is induced by particle interactions producing a “metrical elasticity of space” which opposes the curving of spacetime. It is known that the quantum vacuum makes a contribution to inertial mass. Part of the inertial force of opposition to acceleration, the inertial reaction force, originates in the zero-point fluctuations (ZPF) of the quantum vacuum. ZPF, zero-point field, vacuum energy, and zero-point energy describe an energy-momentum flux as experienced in a Rindler constant acceleration frame and the Unruh-Davies radiation. Both emerge from event-horizon effects in accelerating reference frames. The force of radiation pressure produced by the Rindler flux is proportional to the acceleration of the reference frame. The hypothesis that the inertia of an object is due to the individual and collective interaction of its quarks and electrons with the energy momentum flux allows for a new interpretation of Sakharov’s conjecture about the elasticity of spacetime. If we regard the inertial reaction forces as being emergent from vacuum fluctuations then the elasticity of spacetime, i.e. the resistance to gravity, may be considered as the origin of inertial mass while simultaneously giving rise to curvature in quantum spacetime. Vacuum energy, according to Sakharov, cannot generate a gravitational field but it can generate an inertial field. These considerations naturally lead to the following conclusions (a) Planck length spacetime inertial reaction forces are created by vacuum fluctuations, (b) the inertial forces are responsible for curvature, and (c) the gauge forces are brought into balance by a continuous gravitational field. The third conclusion listed as (c) is an application of Rowlands’ zero-totality condition [10] and conjecture that it is discrete inertial forces that are amenable to quantization which result in the emergence of a continuous gravitational field that balances gauge forces. The continuous / discrete tradeoff, similar to the action-reaction of continuous kinetic and discrete potential energy conservation, can also be seen in the two terms (8). The interchange between a continuous attractive gravitational force and a discrete repulsive Hooke-like force can produce a Sakharov contracting / expanding universe. Rowlands’ discrete gauge / continuous gravity duality is a conservation law analogous to the kinetic-potential energy conservation law. In classical physics we assume that space is a vacuum; there is nothing in it; it is a complete void. Quantum theory on the other hand postulates that space is full of activity, virtual because it is unobserved, with elementary particles being created and annihilated simultaneously and instantaneously, resulting in a virtual void called zero-point field (ZPF). The random and complex radiation field is considered to be made up of zero-point energy (ZPE) and is still present even at absolute zero. Sakharov suggested that spacetime emerges curved due to the vacuum fluctuations and that gravity corresponds to the “generalized forces which oppose the curving of space.” Inertia is a force which opposes gravity. This suggests that spacetime curvature is linked to inertia, not gravity. Einstein’s equivalence of gravitational and inertial mass and Rowlands zero-totality condition can then be applied to argue that Sakharov elasticity and emergent curvature, an inertial action force, are intrinsic properties of mass and inherited by matter. Gravity, a dual reaction force to gauge forces, emerges as an inverse equilibrium force satisfying Rowlands’ universal zero-totality (ZT) condition. In an earlier paper it was shown how ZT can be regarded as a generalization of Newton’s third law [11]. Combining Rowlands’ extension of the relativistic Dirac equation to an order 64 Dirac algebra space / anti-space of four fundamental parameters (space, time, mass, charge) and Sakharov’s theory of gravitation yields additional insights into the connections between discrete inertial forces and continuous gravitational fields.

5. The search continues

Einstein’s search for an extension of the special theory of relativity was to express fundamental physical laws in arbitrary coordinate systems. A satisfactory general theory about gravity was formulated when EEP showed that there is no essential difference between inertia and gravitation. Einstein’s thought experiment demonstrated that a person in gravitational free-fall does not feel their own weight. It is as if though the gravitational field did not exist at all. This physical interpretation of
the influence of gravity on a human is similar to Galileo’s realization, the original principle of relativity, that for a person in uniform motion it seems as if motion does not exist. In any inertial frame of reference, motion is not detected and physical laws are invariant. It is remarkable that a person who does not feel the effects of a first derivative, i.e. uniform motion, is like a person in free fall who does not feel effects of the second derivative, i.e. acceleration. Newton’s first law of motion (N1L) is a codification of Galileo’s principle of relativity. Although Einstein argued that the general theory of relativity “rests exclusively on this [equivalence] principle” many have challenged its validity. Some argue that accelerometers of arbitrarily small size can detect tidal variations in a non-homogeneous gravitational field. Others argue that small droplets of liquid falling freely in the gravitational field of a spherical body will not be perfectly spherical and tidal effects cause them to be slightly ellipsoidal. Others insist that the equivalence principle is only valid for infinitesimal regions of spacetime and that this restriction renders it meaningless.

A purely geometrical interpretation of gravity would be impossible if gravitational and inertial acceleration were not identical. Gravitation, according to GR, is not something that exists in spacetime, but is considered to be an attribute of spacetime itself; coordinates that curve are gravitation. The ontological phenomena called gravity and acceleration are analogous to electric and magnetic fields in the context of special relativity; there are two ways at looking at electric and magnetic phenomena in terms of different coordinate systems. Similarly, in quantum mechanics we have two ways of describing wave-particle duality. We can use special relativity and Heisenberg’s quantum mechanics or Lorentzian relativity and Schrödinger’s wave mechanics. Special relativity has successfully unified the electric and magnetic fields in free space without trying to describe the nature of the elementary charged particles. Likewise, GR unifies gravity, acceleration, and non-linear coordinates in free space. The question of how gravity is ultimately produced and why the elementary massive particles have the masses they have are still open questions. The purely geometrical effects of non-inertial coordinates is associated with the physical phenomenon called gravitation. It is argued that the equivalence principle considers only the passive response of inertial mass points to a gravitational field. A complete account of gravity surely must include the active source of each mass point in the production of the gravitationally curved field. Einstein himself emphasized that both electromagnetism and gravitational theories were incomplete, precisely because they did not include the “source” of the interactions. He said, “Maxwell’s theory of the electric field remained a torso, because it was unable to set up laws for the behavior of electric density, without which there can, of course, be no such thing as an electro-magnetic field. Analogously the general theory of relativity furnished a field theory of gravitation, but no theory of the field-creating masses.” Einstein agreed that GR correlates spacetime curvature with gravity but provides no insights nor answers questions related to field-creating masses or inertial forces. There are various terms which describe spacetime deformations. The terms curvature, elasticity, expansion, contraction, and compactification are used interchangeably to describe gravitational phenomena, including gravitational fields, gravitational potentials, time and space dilations, gravitational waves, or simply gravity itself. The geometry of GR is not a suitable framework for describing behaviors where high energies, quantum considerations, and where canonical quantization of the electromagnetic field uses Coulomb gauge not subject to Lorentz invariance. Quantization implies discontinuity; GR equations and its classical Newtonian approximation based on ordinary and partial differential equations are governed by continuous variables. Newton’s laws allow for physical interpretations given to the solutions of the ordinary differential equations that describe terrestrial and planetary motion. The focus of special relativity has been on the physics of objects that move at or near the speed of light; it is the speed at which field-creating masses move. Quantum effects have undermined the ability to produce valid geometric descriptions of spacetime. Quantum theoretical considerations impose fundamental limitations on the accuracy with which spacetime measurements can be made. It is not possible, for example, to approximate with arbitrary precision free test-point particles in which any spacetime structure is measured according to GR. Time considerations of quantum mechanical clocks also reveal fundamental limitations to measurements of duration as well as distance. The physical meaning given
to spacetime points and manifolds has been seriously questioned. The continuum of spacetime required by GR fails at the quantum mechanical Planck length \( l_p = \frac{\hbar}{c} \approx 10^{-33} \text{ cm} \) where black holes are theoretically studied. Einstein resisted ideas about origins of the universe, such as the big bang, as being produced from any singularity. In 1939 he published a paper in the Annals of Mathematics proving that black holes could not be formed by the gravitational collapse of a star. In that same year it was clearly shown how spacetime curvature can be strong enough to prevent even light from escaping. According to general relativistic constructs it would appear to outside observers as pitch black. The bending effect inside a black hole is so extreme that not only is spacetime ripped apart but a singularity is formed as curvature becomes infinite. The physics modeled by GR appears not to apply on small scales and requires a new theory which accommodates a so-called vacuum.

Historical studies reveal that it was as early as 1916 that Nernst [12] proposed that the vacuum was not empty. Pauli [13], in the 1920s, also became concerned with the gravitational effects of zero-point energy. Dirac, in 1933, speculated about a huge vacuum energy. The ZPF-inertial mass connection agrees with classical representations of the ZPF which have been used to derive a variety of quantum results. These include van der Waals binding force, the ground-state behavior of the quantum-mechanical harmonic oscillator, the Casimir and Davies-Unruh effects, and the spectrum of blackbodies as well. It was Zeldovich, however, who was the first to suggest that vacuum fluctuations contributed to the cosmological constant leading him to conclude that the zero-point energy needed to be considered even when gravity is taken into account. Puthoff [14], like Sakharov, proposed that a cut-off at the Planck mass in the zero-point field could generate a long-range van der Waals type force with an inverse-square property of gravity. Rueda and Haisch [15], expanding on Puthoff’s idea, showed that the Davies-Unruh anisotropy of vacuum fluctuations is equivalent to a non-vanishing Poynting vector. This leads to a resistance to acceleration which may be interpreted in the context of Newton’s second law (N2L) as inertia or inertial mass, \( F = ma \). Haisch and Rueda, like Sakharov, require a Planck length zero-point cut-off. Their use of a Poynting vector within the discrete event horizon is likely equivalent to a gravomagnetic inertial field which generates the inertial mass. Their argument depends on the linearity of the gravitational field being determined by the zero-point cut-off which applies only to the discrete inertial repulsion, and not to the continuous gravitational attraction. The assertion that inertia is time-delayed, while gravity is instantaneous, reinforces the claim that inertia is amenable to quantization and not gravity as Rowlands and McCulloch have shown theoretically and experimentally.

6. Conclusion

Einstein’s equivalence principle allows us to write Rowlands gauge / gravity duality [10] as a gauge / inertia duality which the author has written in the form of a generalized Newton third law (N3L) [11]

\[
gauge \text{ forces (+) gravitational force} = Z, \tag{9}\]

where \( Z \) represents a generalized zero-totality condition. In this particular case, \( Z = 0 \). We now rewrite (9) in the form

\[
gauge \text{ forces (+) inertial force} = 0 \tag{10}\]

agreeing with Sakharov / Zeldovich that spacetime is produced in the vacuum by particle interactions. These forces, which we might call spacetime forces, give us a of view spacetime as being dynamic, producing discrete inertial forces, and simultaneously creating the curvature properties which are inherited by all matter. In equation (10) the inverse force, a gravitational field, balances the gauge forces satisfying Rowlands’ zero-totality condition. Spacetime curves matter. This idea also appears to reverse Mach’s principle about inertia and that it is matter in the universe that contributes to gravitational field rather than inertia. The work of another British researcher, McCulloch [16-17], proposes such a model in which inertia, rather than gravity, is quantized. McCulloch also explains the connection between an object’s inertial mass and the emission of photons in the form of Unruh radiation. Inertial mass is generated by the object’s acceleration with respect to surrounding matter; a Modification of Inertia resulting from a Hubble-scale Casimir effect (MiHsC), or Quantised Inertia for
short, follows. The hypothesis of Sakharov and Zeldovich about the effect of vacuum energy on spacetime, viz. its elasticity and curvature, the inertial effects of the vacuum suggested by Haisch and Rueda, the gauge / gravity duality and quantized inertial models of Rowlands, and “quantised inertia” of McCulloch all point to a dynamic spacetime that curves matter while simultaneously producing a reverse Machian ether-type cosmological gravitational field.

Finally, it is important to note that zero-point energy, anticommutativity, and electron spin derive from the harmonic oscillator and can be related to the $\hbar / 2$ in the Heisenberg uncertainty relation. Assuming a state is represented by a Poynting vector $\psi$ the expectation value of the variable $p$, becomes $\langle p^2 \rangle = \psi P\psi^*$ and the mean squared variance $(\Delta p)^2 = \psi \{ P - \langle p \rangle I \} \psi^* = \psi P'\psi^*$ if $P' = P - \langle p \rangle I$ and $I$ is a unit matrix. For operator $Q$, $(\Delta q)^2 = \psi \{ Q - \langle q \rangle I \} \psi^* = \psi Q'\psi^*$. Since $P'\psi$ and $Q'\psi$ are vectors,

$$(\Delta p)^2 (\Delta q)^2 = (\psi P'\psi^*) (\psi Q'\psi^*) \geq (\psi P Q'\psi^*) (\psi Q P'\psi^*) \geq \frac{1}{4} \left\{ (\psi P'Q'\psi^* - \psi Q'P'\psi^*) \right\}^2 \geq \frac{1}{4} [P, Q]^2.$$

Therefore $(\Delta p)(\Delta q) \geq \frac{1}{2} [P, Q] \geq \frac{\hbar}{2}$ if $P$ and $Q$ do not commute. The significance of this proof is seen in the factor 2. The expression $\hbar / 2$ comes from the non-commutation of the P operator. Non-commutativity occurs in the quaternion relationship since $ij + ji = 0$. We can think of $i$ and $j$ as space-operators which are anti-commutative. They also satisfy a zero-totality condition which we may view as N3L action-reaction. We see the same result when applying two rotations in succession to vectors. Reversing the order of the differentiation yields two different results

$$T_{\mu
u} - T_{\nu\mu} = R_{\mu
u}^\sigma T_\sigma, \quad (11)$$

where $R_{\mu
u}^\sigma$ is the Riemann curvature tensor. The vanishing of (11) is a necessary and sufficient condition for the manifold to be metrically flat which means it is free of intrinsic curvature. This tensor can be regarded as a measure of the degree of non-commutativity of covariant derivative operators in the manifold and can be linked to the Heisenberg uncertainty principle. This idea will be explored in a follow up study.

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