Outlier Detection of Space Trilateration Localization Based on Geometric Inequalities

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Abstract. The three-dimensional positioning problem remains as a challenge in modern commercial communication networks. Yet conventional algorithms, such as the trilateration localization algorithm, are vulnerable to abnormal distance input values which are common in praxis. To address this problem, we introduced an outlier detection approach based on geometric reasoning prior to localization. This process detects and discards abnormal distance values and thus increase the robustness of conventional localization algorithms. Our main concern is the validity of geometry, for example, whether triangle or tetrahedron can be formed is valued in the spatial trilateration localization algorithm. This method is generally used to locate target on the ground, and the problem of ambiguity can be avoided. To this end, geometric reasoning based algorithms such as the triangle inequality and tetrahedral inequality are applied for detection. In this paper, we have mathematically validated the correctness of the proposed method. We have also demonstrated that the proposed method outperforms conventional methods using \textit{in silico} simulation.

1. Introduction
Positioning of handheld terminal devices is mainly computed based on measurements from the base stations, where three-dimensional (3D) positioning is still considered as a technical challenge \cite{1} in modern commercial communication networks.

High-precision 3D positioning is also expected to provide more value to customers \cite{2}, such as basic technologies in intelligent warehousing, intelligent factories, fixed asset tracking and other vertical industries sensitive to 3D coordinate information, as well as business push services based on accurate 3D geographic location information \cite{3, 4}.

Current state-of-the-art two-dimensional (2D) positioning algorithms are mostly based on the time of arrival (TOA) \cite{5, 6}, which offers more accurate positioning as well as low-cost real-time performance. However, these methods have not been implemented in 3D space.

In this paper, we consider the positioning problem of the unmanned aerial vehicle (UAV) on a ground target, therefore, the traditional 3-point TOA positioning algorithm can be extended to the three-dimensional problem without the blurring situation. Since the propagation path of the signal in three-dimensional space is more complicated, the abnormal measurement distance is more likely to occur than the two-dimensional space. Traditional trilateration localization method is vulnerable to
abnormal distance input, in which small distance errors would incur significant positioning offsets. To solve this problem, we introduce an outlier detection technique based on geometric reasoning [7]. Triangle inequality and tetrahedral inequality are used to detect abnormal measurement distance groups, and improve positioning accuracy.

The workflow of the following section can be summarized as: The problem is described in Section 2. Section 3 introduces improved algorithms. Section 4 shows performance of proposed algorithm and comparison with conventional algorithm. Section 5 concludes the paper.

2. Problem Description

Suppose there are three UAVs in the air receiving signals from ground target, and the position of target can be estimated by trilateration localization algorithm without blurring situation. As shown in Figure 1, the coordinates of the UAV A, B, and C are \((x_A, y_A, z_A), (x_B, y_B, z_B), (x_C, y_C, z_C)\), the three UAVs should not be collinear, the ground target coordinate is \((x_D, y_D, 0)\), and the distances measured by UAV A, B, and C are given by:

\[
\begin{align*}
    a &= \left[ (x_D - x_A)^2 + (y_D - y_A)^2 + (0 - z_A)^2 \right]^{1/2} \\
    b &= \left[ (x_D - x_B)^2 + (y_D - y_B)^2 + (0 - z_B)^2 \right]^{1/2} \\
    c &= \left[ (x_D - x_C)^2 + (y_D - y_C)^2 + (0 - z_C)^2 \right]^{1/2}
\end{align*}
\]

(1)

The position estimation of target can be obtained by solving the linear system of equations:

\[
\begin{bmatrix}
    x_D \\
    y_D
\end{bmatrix} = \begin{bmatrix}
    2(x_A - x_C) & 2(y_A - y_C) \\
    2(x_B - x_C) & 2(y_B - y_C)
\end{bmatrix}^{-1} \begin{bmatrix}
    c^2 - a^2 - x_C^2 + x_A^2 - y_A^2 - z_A^2 + z_C^2 + z_A^2 \\
    c^2 - b^2 - x_C^2 + x_B^2 - y_B^2 - z_B^2 + z_C^2 + z_B^2
\end{bmatrix}
\]

(2)

Figure 1. UAV positioning on the ground target

Once the measured distance \(a, b, c\) appear an abnormal value, the positioning result has a large error. How to use geometric reasoning to detect the existence of abnormal distance values is the problem that will be discussed in the next section.
3. Location algorithm

3.1 Triangle inequality algorithm
A triangle with side lengths $l_1, l_2, l_3$ must satisfy the following inequality:

$$|l_1 - l_2| < l_3 < l_1 + l_2$$

(3)

As can be seen from Figure 1, the triangle inequality can be used for trilateration localization by judging whether the three measurement distances $a, b, c$ satisfy the following inequalities:

$$|a - b| < c' < a + b$$

$$|b - c| < a' < b + c$$

$$|c - a| < b' < c + a$$

(4)

If not, then this set of measurement distances cannot be used for localization.

3.2 Tetrahedral inequality algorithm
Let the three sets of opposite edges of the tetrahedral ABCD (Figure 2) be $a, a'; b, b'; c, c'$. Making the following equations:

$$p_1 = \left( a'^2 + c'^2 - b'^2 \right) \left( b^2 + a'^2 - c^2 \right)$$

(5)

$$p_2 = \left( 2a'^2 c'^2 + 2a'^2 b'^2 + 2b'^2 c'^2 - a'^4 - b'^4 - c'^4 \right)^{1/2}$$

(6)

$$p_3 = \left( 2a'^2 b'^2 + 2b'^2 c'^2 + 2c'^2 a'^2 - a'^4 - b'^4 - c'^4 \right)^{1/2}$$

(7)

The following equation can be obtained by spreading the $\triangle BCD$ and $\triangle ABC$ on the plane (Figure 3):

$$AD^2 = b^2 + c^2 - \left[ \frac{p_1}{2a'^2} - \frac{p_2}{2a'} \right]$$

(8)

And the following equation is obtained by stacking side BCD and ABC on the plane (Figure 4):

$$A'D^2 = b'^2 + c'^2 - \left[ \frac{p_1}{2a'^2} + \frac{p_2}{2a'} \right]$$

(9)

Figure 2. The tetrahedron ABCD.

Figure 3. Spread the $\triangle BCD$ and $\triangle ABC$ on the plane.
Thus the necessary and sufficient condition for the line segment $a, a', b, b', c, c'$ to be the three sets of the opposite sides of the tetrahedron is:

$$A'D' < a < AD$$  \hspace{1cm} (10)

And it is equal to

$$A'D'^2 < a^2 < AD^2$$  \hspace{1cm} (11)

The following inequality can be obtained by substituting equations (8) and (9) into (11):

$$|2a'^2(a^2 - b^2 - c'^2) + p_j| < p_2 \cdot p_3$$  \hspace{1cm} (12)

That is to say, if the line segments $a, a', b, b', c, c'$ can be three sets of opposite edges of the tetrahedron, they must satisfy the inequality (12).

As can be seen from Figure 1, we can detect whether the three measured distances satisfy the inequality (12). If not, the set of distances will be discarded.

4. Simulation

In comparison with the traditional spatial trilateration localization algorithm, this section analyses the performance of the proposed algorithm. For simplicity, the algorithm uses the triangle inequality to detect outlier before the traditional algorithm is called Triangle location, the algorithm uses the tetrahedral inequality is called Tetrahedral location. And the traditional algorithm is called Traditional location.

4.1 Scenario

Assuming there are three UAVs A, B, C in the air, as shown in Figure 5, the positions are respectively $(1000, 0, h)$ (m), $(0, 0, h)$ (m), $(0, 1000, h)$ (m), $h$ is the altitude of the UAV from the ground, the target D is on the ground, the horizontal and vertical coordinates vary from 0 to 1000(m), and the distance between AB is $c'$ (m), the AC is $b'$ (m) and the BC is $a'$ (m). The distance between the UAV A and the target D is $a$ (m), the distance between B and D is $b$ (m), and the distance between C and D is $c$ (m).
4.2 Result

In the simulation, the measurement noise is Gauss white noise with zero mean and the variation range is 15%. Change the height $h$ of the UAVs, and perform 1000 experiments for each $h$. The horizontal and vertical coordinates of point D are randomly selected in each experiment within the range, and the root mean square error (RMSE) of the positioning results after using Triangle location, Tetrahedral location and Traditional location algorithm are calculated at different height $h$.

| UAV height $h$ (m) | RMSE of Traditional location (m) | RMSE of Triangle location (m) | RMSE of Tetrahedral location (m) |
|------------------|---------------------------------|-------------------------------|----------------------------------|
| 150              | 235.46                          | 224.41                        | 200.13                           |
| 250              | 270.02                          | 253.25                        | 216.94                           |
| 350              | 288.32                          | 254.10                        | 218.39                           |
| 450              | 337.81                          | 283.83                        | 252.47                           |
| 550              | 341.38                          | 318.59                        | 276.32                           |
| 650              | 383.98                          | 340.62                        | 299.17                           |
| 750              | 446.72                          | 381.24                        | 340.99                           |
| 850              | 482.66                          | 435.53                        | 394.94                           |
| 950              | 561.31                          | 473.96                        | 441.81                           |
| 1000             | 661.31                          | 634.51                        | 518.85                           |
| 2000             | 1772.02                         | 1459.41                       | 1292.07                          |
| 3000             | 3910.77                         | 2584.84                       | 2176.22                          |
| 4000             | 6790.70                         | 3586.13                       | 3014.82                          |
| 5000             | 11129.02                        | 4624.80                       | 3918.47                          |
| 6000             | 15474.88                        | 5447.31                       | 4710.25                          |

From the table 1, we can see that at the same height, the RMSE of Triangle location and Tetrahedral location decrease compared to the RMSE of Traditional location, and the RMSE of
Tetrahedral location decreases more. The higher the height, the better the performance of Triangle location and Tetrahedral location.

5. Conclusion
In this paper, we present a novel technique that applies detection before space trilateration localization. This technique analyses the collected distance values and correlates the values that form impractical triangle and tetrahedron. We have demonstrated the improvement of the proposed method comparing to direct trilateration localization based on in silico simulation in realistic scenario.

Acknowledgement
This work was supported in part by the National Natural Science Foundation of China under Grant U1533125, and Grant 61771108, in part by the National Science and Technology Major Project under Grant 2016ZX03001022, in part by the Fundamental Research Funds for the Central Universities under Grant ZYGX2015Z011 and Sichuan science and technology planning project (key R & D project 18ZDYF0990).

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