Investigating the Consolidation of the Optimal Design of Distributed Energy Systems with Optimal Power Flow

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Abstract
Distributed energy systems have the potential to minimise costly network upgrades while increasing the proportion of renewable energy generation in the electrical grid. However, the integration of DES into existing distribution networks warrants further research, as poorly designed DES can upset the delicate balances in these networks. Although optimisation models have been used widely in the literature to design DES, most of these have excluded nonlinear and nonconvex constraints associated with AC power flow and the distribution network. A few studies have attempted to consolidate DES with a standalone class of models known as Optimal Power Flow (OPF), which contain detailed constraints of the distribution network but exclude DES. These studies have often overlooked the different types of distribution network configurations, and do not provide comparisons with DES-only formulations. This study aims to address this gap by presenting an open-source optimisation model for DES design combined with a nonlinear OPF framework employing four solution methods, including two new bi-level methods which can influence the DES design. A low-voltage distribution network is used to obtain results and compare the efficacy of these methods. Results suggest that under certain network configurations and scenarios, a combined DES-OPF framework is essential as typical mixed-integer and linear formulations do not provide solutions that are compatible with network constraints. Similarly, current violations that have been previously reported in literature are not seen under certain configurations, highlighting the importance of including such detail in combined DES-OPF models.

1. Introduction
Ageing power networks around the world are being transformed by the increasing integration of renewable and distributed energy resources (DERs). Previously unidirectional distribution networks have been converted to bidirectional active networks through collections of small-scale energy generation and storage technologies situated in proximity to the consumer. These are known as Distributed Energy Systems (DES) and consist of mostly renewable and low-carbon technologies. The term ‘DES’ has become synonymous with microgrids, energy hubs, distributed generation (DG), and smart local energy systems [1]. In most European countries, such as the United Kingdom, the technologies within a DES are often owned by the consumers themselves, who benefit from cost-reducing tariffs attached to low-carbon energy generation [2], [3].

When designed appropriately with operational and network-related constraints in mind, DES have the potential to lower energy losses and carbon emissions related to power generation, and the reduce the need for costly network reinforcements [4], [5]. However, if not designed and operated with care, these systems can potentially amplify voltage unbalance, increase power losses (such as via renewable energy curtailment), and subsequently lead to large economic losses [6]. Due to these implications, the design of a DES is often modelled as a mathematical optimisation problem. This can ensure the suitability of selected technologies over the system lifetime, with respect to relevant constraints and objective functions, such as the minimisation of annualised costs and/or carbon emissions [7]. Optimisation models account for operational aspects such as average demand and weather profiles, and to aid detailed analyses of how the system responds to dynamic conditions and requirements.

DES design is most commonly represented using mixed-integer formulations, where discrete decisions are used to choose the location of installation, the type of technology to be installed, and different states of operation. Continuous variables are used to describe energy flows, voltages, costs, etc. Some fundamental studies in this area, labelled “DES baseline models”, consider a wide selection of
renewable and low-carbon resources [8], heating pipeline networks [9], both environmental and economic objectives simultaneously [10], and the uncertainty present in inputs to models [11]. Although nonlinear formulations often more realistically depict the behaviour of certain technologies and underlying networks, most DES models, such as those mentioned above, have been formulated and solved as Mixed-Integer Linear Programming (MILP) problems [12]. These models either exclude nonlinear elements or use linear approximations, as nonlinear and nonconvex formulations can be challenging to formulate and solve, with no guarantee of global optimality and increased computational expense. However, without comparisons with high-fidelity models, globally optimal solutions from linearised models can be infeasible if implemented in reality. This highlights the need to achieve an accuracy-complexity balance when formulating and solving DES optimisation models [1].

1.1. The DES Design – OPF paradigm

In previous work [1], [13], we identified that nonlinearities associated with the underlying distribution network have often been oversimplified or excluded from DES design models, despite most DES operating in grid-connected mode. For example, recent studies that investigate the use of advanced features within design frameworks, such as bidirectional power flow in peer-to-peer trading [14] with blockchain protocols [15], utilising batteries to minimise stresses placed on the grid [16], and the integration of hydrogen storage [17], have all overlooked the inclusion of constraints related to the distribution network. Note that distribution networks are part of the larger electrical grid, which transmits and distributes alternating current (AC) power. It is utilised by a DES for active and reactive power flows, while the network also acts as both source and sink when demand within the DES does not match supply. Most DES design models only consider an active power balance, and exclude calculations of reactive power, voltages, and currents. A subset of studies have employed the DC approximation [18], which is widely considered as a simplified alternative to AC power flow. However, this does not accurately represent AC power flow as it calculates only the active power flow at each branch of the network, assuming that voltage magnitudes remain at nominal values and neglecting differences between voltage angles [19], [20].

A separate class of optimisation problems, known as Optimal Power Flow (OPF), has been used to study the allocation of distributed generators in transmission and distribution networks [21]. OPF models for AC networks contain detailed nonlinear and nonconvex constraints related to active and reactive power flows, voltages, and currents, but do not include constraints describing the operation of distributed generators and storage technologies. These models primarily use balanced power flow equations, as the powers, voltages, and currents are assumed to be balanced across all phases. Until recently, DES and OPF have largely been considered as two standalone problems in the literature, despite DERs installed within a DES having the potential to disrupt the normal functioning of the underlying distribution network. As exemplified above, DES models do not consider AC power flow constraints that are typically included in OPF models due to the complexity introduced by them. This has resulted in a disparity which has attracted increasing interest in recent times. Researchers have attempted to address this by combining the two formulations, which we label as the DES – OPF paradigm. While there are many studies focusing particularly on the operation of DES with the combined OPF framework [22], [23], few studies have paid attention to the importance of addressing this at a design level. Including OPF in the earlier stage of DES planning and implementation, such as in design models, has the potential to eliminate issues that may arise at the operational level. This would result in more robust DES that can operate safely within existing distribution networks.

Preliminary studies in this area, such as Moradi and Abedini [24] and Kaur et al. [25], have laid foundations for the DES design – OPF paradigm. These have focused on combining distributed generation planning with OPF, where greater attention has been paid to the latter. Compared to more recent work, these studies have excluded the presence of individual consumers and their demand profiles, detailed representations of the types of units that can be chosen, and weather profiles. Furthermore, metaheuristic approaches are used instead of mathematical programming approaches
due to the presence of nonconvexity. While metaheuristics can be easier to formulate and solve due to the absence of gradient information, mathematical programming approaches tend to provide superior solutions due to their speed and ability to guarantee local optimality at the minimum [26]. These studies have nevertheless shown that using combined and more detailed models can help evade network violations that may otherwise be present.

More recent studies in literature have specifically investigated the combined DES Design – OPF paradigm. Morvaj et al. [27] were the first to address this gap, followed by Mashayekh et al. [28]. Both have attempted to consolidate the previously separate problems by proposing linearised AC power flow equations that can be implemented within DES MILP formulations. A post-optimisation check using MATPOWER [29] (a popular power flow simulation tool for balanced AC networks) and a Genetic Algorithm (GA) design model have also been employed by Morvaj et al. [27]. The former is a key weakness of the framework, as the AC nonconvex network constraints have no influence on the DES design, thus merely serving as a numerical check. To remedy this, linearised power flow equations have been proposed in both studies. While these approximations seem promising as the percentage errors for bus voltages are low when compared with exact power flow solutions, the possibility and impacts of using higher fidelity AC OPF constraints within these models have not been explored. Neither use nonlinear formulations due to their inability to guarantee global optimality, and do not perform comparisons with such models to test for accuracy or ease of implementation. We argue that the feasibility of the solutions is more important than global optimality in this instance, as the solutions of these models should guarantee the synergistic operation of the DES within the existing distribution network. Morvaj et al. [27] also reports many current violations within the distribution network as a result of DES implementation, highlighting the need for further research in this area. In contrast, Sfikas et al. [30] and Jordehi et al. [31] have utilised nonlinear AC power flow equations as opposed to linear approximations. The former study analyses the suitability of the design in both grid-connected and islanded modes, but have excluded the use of binary decisions to solve the model using Sequential Quadratic Programming (SQP). This results in a more simplistic representation of the technologies and their operation. In comparison, the latter study [31] utilises a more detailed Mixed-Integer Nonlinear Programming (MINLP) framework, where results demonstrate how the placement of a battery swapping station can impact power losses and voltages in the network. While these results indicate the need for a DES-OPF framework, results of these DES Design – OPF models have not been compared with those of DES baseline models. Therefore, it is unclear whether the same designs and operational strategies can be obtained when network constraints are excluded in the latter.

The above studies have also utilised balanced AC power flow equations without further discussion or analysis on how the network and load configuration can affect the accuracy of the results. There are two main types of three-phase configurations found in the electrical transmission and distribution networks through which power is supplied and consumers or loads can be connected. These are wye or star (Y) and delta (Δ) [32]. The wye configuration is more widely used in the UK and other European distribution networks compared to the delta configuration [33]. This is because it allows both single-phase and three-phase connections, where single-phase connections are suitable for smaller residential loads. However, this configuration can lead to voltage unbalance due to unequal loads connected to each phase [34]. Full analysis of wye-connected radial networks requires more complex unbalanced power flow formulations which introduce additional nonconvexity [35] and are harder to formulate and solve, and hence have been approached using various approximations [36]. In contrast, the delta configuration is inherently balanced and allows the use of the balanced AC power flow formulations which are commonly used in classical AC OPF formulations [21]. It is used to connect three-phase loads such as motors, and in applications where a highly-reliable power supply is required, such as industrial plants [37]. While balanced formulations are useful for design and planning models, care must be taken to calculate the correct voltages and currents with respect to the overarching configuration. For example, branch currents in a delta system are generally three times higher compared to a wye system with the same line voltage and load. These differences are summarised in
Table 1. Excluding such information from DES – OPF models can result in practically feasible solutions being incorrectly discarded due to lack of detail within the model.

| Variable | Delta | Wye |
|----------|-------|-----|
| Line voltage ($V_L$) | $V_L = V_{ph}$ | $V_L = V_{ph} \times \sqrt{3}$ |
| Line current ($I_L$) | $I_L = I_{ph} \times \sqrt{3}$ | $I_L = I_{ph}$ |

With the increasingly multi-faceted nature of DES optimisation problems, the potential implications of a DES design not being compatible with existing infrastructure cannot be overlooked. Such designs can decrease efficiency while incurring additional costs to both consumers and network operators, straying from the true objectives of implementing a DES to reduce environmental impacts and minimise costs for all parties.

1.2. Contributions of this study

To address the research gaps identified above, this study incorporates the following novel aspects: i) a fully transparent DES Design model integrating OPF constraints is presented and solved using mathematical programming approaches, as opposed to metaheuristics; ii) two new bi-level MILP-NLP optimisation methods are introduced, where the OPF constraints can influence the DES model and can eliminate the need to solve a nonconvex MINLP model; iii) a modified formulation for integrating line current constraints for balanced networks with wye configuration is used; iv) results for a low voltage residential network using four modelling methods, which include high-fidelity DES Design – OPF models and a DES baseline model, are compared in detail for the first time; v) the impacts of considering network configuration are analysed, which has been overlooked in the literature thus far; vi) power flow formulations utilised are verified using Pandapower [38], a Python-based power flow simulation tool; vii) the limitations of using a balanced OPF formulation are recognised, particularly in networks with a wye configuration.

The aim of this study is to assess whether a combined DES-OPF framework is required when planning for the installation of DERs and DES within low voltage distribution networks. We also aim to provide guidance on which modelling methods are more suitable for achieving an accuracy-complexity balance in the presence of nonconvex constraints and a large number of binary variables.

2. Methodology

Four modelling methods are employed to enable the analysis and comparison of the combined DES Design – OPF framework with the baseline DES framework. These are:

- An MILP model (labelled MILP) representing a DES baseline model without any network constraints, as found in most DES literature
- A bi-level MILP-NLP model (labelled BL-1), where a nonlinear programming (NLP) OPF model under fixed design and binary variables is used to verify the feasibility of the MILP solution
- A bi-level MILP-NLP model (labelled BL-2) for the combined DES design – OPF formulation, where the binary decisions are determined by the MILP but the design and operational variables can be influenced by the OPF constraints
- An MINLP model (labelled MINLP) for the combined DES design – OPF formulation.

The overarching algorithm for these four methods is outlined in Figure 1. The three methods applicable to the combined DES-OPF framework, BL-1, BL-2, and MINLP, are further detailed in Section Error! Reference source not found.. The complete DES Design – OPF formulation can be broken down into two subsections, DES Design (Section 2.2) and OPF (Section 2.3), which are
subsequently linked by a number of linking constraints (Section 2.4) in the combined framework. Note that the objective function, applicable to both DES and DES-OPF models, is presented in Section 2.2.

Figure 1. The four methods employed in this study, where i) an MILP can be solved without any network constraints for comparison, ii) BL-1 allows the OPF constraints to influence the operational variables of the DES model while keeping binary and design variables fixed, iii) BL-2 allows the OPF constraints to influence both design and operational variables while keeping binary variables fixed, iv) an MINLP model where OPF constraints can influence all design and operational aspects of the DES model.

2.1. DES-OPF Modelling Methods

The modelling methods BL-1, BL-2, and MINLP employed in this study enable the integration of nonlinear and nonconvex OPF constraints for AC power flow within the DES design framework.

To allow the OPF constraints to both influence the DES model in varying degrees and check for the feasibility of the DES designs, we have proposed two new methods, BL-1 and BL-2. These are bi-level optimisation models, which first solve an MILP master problem to obtain all the binary or discrete decisions for a globally optimal DES design. These variables are then fixed and fed into a nonlinear (NLP) subproblem, which includes all the linear DES constraints and nonlinear OPF constraints. In addition to the binary variables, BL-1 requires the design variables, such as the capacities of the DERs installed, to be fixed as well. Therefore, the OPF constraints in BL-1 can only influence continuous operational variables in the DES-OPF model. Note that the operational variables can also be fixed in BL-1, should the modeller choose to perform a feasibility check of the MILP model. In contrast, BL-2 allows the OPF constraints in the nonlinear subproblem to influence both continuous design and operational variables, while keeping the binary variables initially determined by the MILP unchanged. This offers a larger feasible region to the nonlinear model compared to BL-1, and thus the ability to find better locally optimal solutions.

The MINLP framework offers the largest feasible region or greatest degrees of freedom, compared to both BL-1 and BL-2. It includes both the DES and OPF constraints, where the model can vary discrete...
and continuous variables. Although this is advantageous in terms of finding better feasible solutions, the presence of nonconvex constraints with a large number of binary variables makes MINLP models more computationally expensive to solve along with the possibility of finding only locally optimal solutions. Hence, this study compares the solutions of all three proposed methods to establish whether any of these methods can help modellers achieve an accuracy-complexity balance and obtain feasible solutions.

2.2. DES MILP Formulation

An MILP formulation is utilised as the DES baseline model, based on the works of Ren and Gao [8], Mehleri et al. [9], and Mauriad et al. [39]. Several modifications have been made to add flexibility and enable facile implementation when using the proposed DES – OPF methods, which are described in this section.

The base model has been formulated such that it can be solved independently as a seasonal model, if required, or across any number of seasons \( s \in S \). A common objective function to minimise the total annualised cost \( TAC \), which is applicable to both DES and DES-OPF frameworks, is used across all seasons:

\[
\min \ TAC = \sum_{G \in \text{DER}} C_{G}^{\text{INV}} + \sum_{s \in S} \left( C_{s}^{\text{grid}} + \left( \sum_{G \in \text{DER}} C_{s,G}^{\text{OM}} \right) - I_{s} \right) \tag{1}
\]

The total annualised cost is the sum of investment costs \( C_{G}^{\text{INV}} \) for DERs \( G \in \text{DER} \), seasonal costs for purchasing electricity from the grid \( C_{s}^{\text{grid}} \) and operation of DERs \( C_{s,G}^{\text{OM}} \), and seasonal income \( I_{s} \). Linking constraints across seasons are also utilised to ensure that the capacity of each DER \( \text{Cap}_{s,G}^{G} \) (kW) installed at each residence or building \( i \in I \) does not vary from season to season:

\[
\text{If } s > 1 \quad \text{Cap}_{s,G}^{G} = \text{Cap}_{s-1,G}^{G} \quad \forall i \in I \tag{2}
\]

Thus, the annualised investment costs for each DER can be calculated with respect to the selected capacity, capacity cost \( CC_{G}^{\text{INV}} \) (€/kW), and capital recovery factor \( CRF \) which takes the total lifetime of the DES into account:

\[
C_{G}^{\text{INV}} = \text{Cap}_{s,G}^{G} \cdot CC_{G}^{\text{INV}} \cdot CRF \quad \forall G \in \text{DER} \tag{3}
\]

Three of the most prevalent DERs implemented in residential settings have been included in this formulation, which are solar photovoltaics (PVs), batteries, and boilers. Note that additional technologies such as heating/cooling networks and other types of DERs have not been included in the formulation as the aim of this work is to study how electricity-generating technologies can impact the electricity distribution network. Constraints implemented in each seasonal model are further detailed in Appendix A – DES Seasonal Model.

2.3. OPF NLP Formulation

The bus injection model presented in Frank and Rebennack [21] is utilised for the OPF formulation, with an additional index representing time \( t \in T \). Complex voltage is denoted in polar form as \( V \angle \theta \), where the magnitude of voltage is denoted as \( V \) and the angle of the voltage is denoted as \( \theta \). This formulation assumes that the low voltage network is balanced, and therefore considers only one phase in the formulation. The per unit system is also utilised in the OPF formulation, such that voltages, powers, and currents are made dimensionless or ‘per unit’ using base values for voltage and apparent power. This has several advantages, including ease of formulation and improvement of numerical stability [21]. It can also be used for both delta and wye configurations, where the per-unit voltages are multiplied with the respective voltage base for each configuration. Figure 2 presents a line diagram of a small distribution network and introduces the set notations corresponding to the elements comprising the network. This notation is used in the OPF formulation below.
Considering a node \( n \in \mathbf{N} \) on branch or line \((n, m) \in \mathbf{L}\), the active power \( P_{n,t} \) and reactive power \( Q_{n,t} \) balances can be described as a function of polar voltage, as shown in the nonconvex equality constraints below:

\[
P_{n,t} = V_{n,t} \sum_{m=1}^{N} V_{m,t} ((G_{nm} \cos(\theta_{n,t} - \theta_{m,t})) + (B_{nm} \sin(\theta_{n,t} - \theta_{m,t})) ) \ \forall n \in \mathbf{N}, t \in \mathbf{T} \quad (4)
\]

\[
Q_{n,t} = V_{n,t} \sum_{m=1}^{N} V_{m,t} ((G_{nm} \sin(\theta_{n,t} - \theta_{m,t})) - (B_{nm} \cos(\theta_{n,t} - \theta_{m,t})) ) \ \forall n \in \mathbf{N}, t \in \mathbf{T} \quad (5)
\]

The parameters Conductance \( G_{nm} \) and Susceptance \( B_{nm} \) are the real and imaginary parts of the complex branch Admittance \( Y_{nm} \):

\[
Y_{nm} = Z_{nm}^{-1} = G_{nm} + jB_{nm} \quad (6)
\]

Note that branch admittance consists of both self-admittances (where \( Y_{nm}, n = m \)) and mutual admittances (where \( Y_{nm}, n \neq m \)). Computing branch admittance in complex form involves a series of steps. Firstly, it requires the calculation of branch series admittance \( y_{nm} \) using the positive sequence parameters Resistance \( R_{nm} \) and Reactance \( X_{nm} \). These are usually provided for each type of line or cable used within the network.

\[
y_{nm} = \frac{1}{R_{nm} + jX_{nm}} = \frac{R_{nm}}{R_{nm}^2 + X_{nm}^2} - j \frac{X_{nm}}{R_{nm}^2 + X_{nm}^2} \quad (7)
\]

Branch series admittance can then be used to calculate self and mutual branch admittances:

\[
Y_{nm} = \sum_{m:(n,m) \in \mathbf{L}} y_{nm} + \sum_{m:(m,n) \in \mathbf{L}} y_{nm} \quad (8)
\]

\[
Y_{nm} = -\sum_{m:(n,m) \in \mathbf{L}} y_{nm} - \sum_{m:(m,n) \in \mathbf{L}} y_{nm} \quad n \neq m \quad (9)
\]
Eq. (7) and (8) are simplified versions of the original equations presented in [21], by assuming that shunt susceptance and admittance are negligible, which is a reasonable assumption for short lines or cables, and nominal transformer turns ratio.

In grid-connected DES and microgrids, the main point of connection to the external grid is known as the slack bus, denoted by the subscript slack. Active and reactive power injections are not limited at this bus to purport the role of the external grid as both an unlimited source and sink for power, and to obtain feasible solutions for the scenarios tested. The voltage magnitude and angle are fixed at this bus, as shown below:

\[
V_{\text{slack},t} = 1, \quad \theta_{\text{slack},t} = 0 \quad \forall t \in T
\]  

(10)

The voltage magnitude at each node and timepoint must remain within pre-specified network bounds:

\[
V_{UB} \leq V_{n,t} \leq V_{LB} \quad \forall t \in T
\]  

(11)

where \( V_{UB} \) and \( V_{LB} \) are the per unit values of line voltage upper bound and lower bound, respectively.

The voltage angle is also constrained in a similar manner:

\[
-180^\circ \leq \theta_{n,t} \leq 180^\circ \quad \forall t \in T
\]  

(12)

In a typical OPF formulation, the upper and lower bounds of each generation and storage unit installed are also described. As these are included in the DES formulation, which ultimately gets combined with this OPF formulation, these constraints are not included here and can be found in Appendix A – DES Seasonal Model.

While the above constraints complete a typical OPF formulation, additional constraints, also known as side constraints, can be included to calculate variables such as the magnitude of current in each branch [21]:

\[
\left( V_{n,t} \cos \theta_{n,t} - V_{m,t} \cos \theta_{m,t} \right)^2 + \left( V_{n,t} \sin \theta_{n,t} - V_{m,t} \sin \theta_{m,t} \right)^2 \leq \frac{(I_{\text{max}})^2}{n_{nm}} \quad \forall (n, m) \in L
\]  

(13)

Here, \( y_{nm}^2 \) represents the square of the magnitude of the branch series admittance \( y_{nm} \), and \( I_{\text{max}} \) represents the maximum current that is allowed by the network or cables. While Eq. (13) is commonly used to calculate branch current in a balanced distribution network, it is only applicable to a network with a delta configuration at the load side. To derive the relationship between line current in a delta system \( I_L^\Delta \) and a wye system \( I_L^\gamma \), which have the same line current and resistive load \( R \), the relationships shown in Table 1 in Section 1.1 can be used:

\[
I_L^\Delta = I_{ph} \cdot \sqrt{3} = \frac{V_L}{R} \cdot \sqrt{3}
\]  

(14)

\[
I_L^\gamma = I_{ph} = \frac{V_L}{R\sqrt{3}}
\]  

(15)

Substituting for \( V_L \) in Eq. (14) with the relationship for \( I_L^\gamma \) in Eq. (15), we obtain:

\[
I_L^\Delta = 3 \cdot I_L^\gamma
\]  

(16)

Therefore, branch current within a wye configuration can be calculated from the line current constraint presented in Eq. (13) as shown below:

\[
\left( \left( V_{n,t} \cos \theta_{n,t} - V_{m,t} \cos \theta_{m,t} \right)^2 + \left( V_{n,t} \sin \theta_{n,t} - V_{m,t} \sin \theta_{m,t} \right)^2 \right) \leq \frac{(I_{\text{max}})^2}{y_{nm}^2} \quad \forall (n, m) \in L
\]  

(17)
Note that the square of branch current is calculated in Eqs. (13) and (17), and the delta branch current can be divided by 3 to obtain the wye branch current, as shown in Eq. (16).

The bus injection model relies on active and reactive power injections at each node. Nodes that neither consume nor generate power have net active and reactive power injections set to zero, as shown below:

\[
P_{n,t} = 0 \quad \forall n \in \mathbf{M}, t \in \mathbf{T}
\]

\[
Q_{n,t} = 0 \quad \forall n \in \mathbf{M}, t \in \mathbf{T}
\] (18) (19)

Note that \(n \in \mathbf{M} \subseteq \mathbf{N}\) represents the nodes that do generate and consume power, and the constraints for these are described in Section 2.4.

2.4. Linking constraints
The distribution network topology typically indicates which building \(i\) is located at which node \(n\). Nodes at which buildings are located, that act as either loads or generators, are described as load/generator nodes \(n \in \mathbf{M} \subseteq \mathbf{N}\) in this formulation. An indicator \(A_{n,i}\) is used to show the connection between such a node \(n\) and building \(i\):

\[
A_{n,i} \in \{0,1\} \quad \forall n \in \mathbf{M}, i \in \mathbf{I}
\] (20)

To connect the OPF formulation described in Section 2.3 with the DES formulation summarised in Section 2.2, load/generator nodes, which are potential sites for installing DERs, are assumed to be PQ nodes. The net active and reactive power injections at these nodes are typically expressed using the equations below:

\[
P_{n,t} = \sum_{i \in \mathbf{I}} A_{n,i}(P_{t,\text{Gen}} - P_{t,\text{Load}}) \quad \forall n \in \mathbf{M}, t \in \mathbf{T}
\] (21)

\[
Q_{n,t} = \sum_{i \in \mathbf{I}} A_{n,i}(Q_{t,\text{Gen}} - Q_{t,\text{Load}}) \quad \forall n \in \mathbf{M}, t \in \mathbf{T}
\] (22)

Where \(P_{t,\text{Gen}}\) and \(Q_{t,\text{Gen}}\) represent the total active and reactive power generated, while \(P_{t,\text{Load}}\) and \(Q_{t,\text{Load}}\) represent the total reactive power consumed. Eq. (23) is modified to align with the DES baseline and OPF formulations:

\[
P_{n,t} = \sum_{i \in \mathbf{I}} A_{n,i}\left(\frac{E_{t,\text{PV,sold}}^\text{PV,sold} - E_{t,\text{grid}}^\text{grid} - \sum_{c}E_{t,\text{grid,charge}}^\text{grid,charge}}{S_{\text{base}}}\right) \quad \forall n \in \mathbf{M}, t \in \mathbf{T}
\] (23)

\[
Q_{n,t} = \sum_{i \in \mathbf{I}} A_{n,i}\left(\frac{Q_{t,\text{Gen}} - Q_{t,\text{Load}}}{S_{\text{base}}}\right) \quad \forall n \in \mathbf{M}, t \in \mathbf{T}
\] (24)

where \(E_{t,\text{PV,sold}}^\text{PV,sold}\) is the excess electricity sold to the grid, \(E_{t,\text{grid}}^\text{grid}\) is the electricity purchased from the grid to satisfy consumer, and \(E_{t,\text{grid,charge}}^\text{grid,charge}\) is the total electricity purchased to charge the batteries \(c \in \mathbf{C}\) installed at each house. Note that the predefined apparent power base \(S_{\text{base}}\) (as mentioned in Section 2.3) is used to convert the powers to dimensionless quantities when fed into the OPF formulation.

This completes the linking of the two formulations, and the complete model can be solved using any of the methods described in Section 2.1.

3. Case Study
A European low voltage (0.4 kV) network supplying electricity to residential customers [40], also used by Morvaj et al. [27], is utilised to test the models and methods proposed in Section 2. A diagram of
the network including network parameters is presented in Figure 3. These parameters are used in the DES-OPF methods outlined in Section 0. It is assumed that all residential consumers are connected as three-phase loads under the respective configuration to maintain the assumption that the network is balanced. The power factor (PF) of solar panels is set to unity (i.e., 1.0) as inverters at residential buildings are typically set to provide active power only. The peak energy consumption and available space for DER installation at each residential building is recorded in Table 2. The models consider a discretised temporal horizon consisting of 24 hourly timepoints. Averaged daily profiles for electricity and heating demand are used for each season. The residential network is assumed to be located in the UK, therefore averaged solar irradiance profiles [41] (Figure 4) and the Feed-in-Tariff (FIT) scheme [2] applicable to the UK have been used. Other input parameters associated with the DERs used and pricing schemes are provided in Appendix B. Python/Pyomo-based models, all input data used, and results files have been made available via Github: https://github.com/Ishanki/DES-OPF-Design.

Table 2. Average daily demands for electricity and heating per day

| Building | Peak Electricity (kW) | Peak Heat (kW) | Area available (m²) | Volume available (m³) |
|----------|-----------------------|----------------|---------------------|-----------------------|
| A        | 3.8                   | 10.0           | 150                 | 5                     |
| B        | 18.4                  | 47.6           | 700                 | 5                     |
| C        | 14.1                  | 27.1           | 600                 | 5                     |
| D        | 3.8                   | 6.4            | 150                 | 5                     |
| E        | 12.0                  | 31.3           | 550                 | 5                     |
| **Total**| **52.2**              | **122.3**      | **2150**            | **25**                |

Figure 3. Line diagram of the low voltage distribution network with line parameters Resistance (R) and Reactance (X). The upper and lower bounds for voltage are indicated by UB and LB.

Figure 4. Averaged daily solar irradiance profiles for each season.
It is assumed that the installed DES will operate for 20 years in total. This also aligns with the FIT scheme, which promised fixed tariffs for small-scale renewable energy generation and export to the electricity grid, provided that the export occurs only if excess electricity is available after the demand of the consumer has been satisfied. In addition to these tariffs, the Economy 7 pricing strategy is considered [42], which provides different day and night electricity prices to consumers in the UK. As the day price is typically higher than the night price, this assists the study of battery installation and operation within the DES.

The models are tested under two scenarios to study the impact of the inclusion of electrical storage on the distribution network: i) including PVs and boilers only, i.e., without batteries providing electrical storage, and ii) including PVs, boilers, and batteries for electrical storage. The results of these scenarios using all four methods and under different network configurations (delta and wye) are presented in Section 4.

4. Results and Discussion

This section presents the results of all four methods under two different scenarios and network configurations. The MILP master problems have been solved using CPLEX [43], while the bi-level nonlinear subproblems have been solved using CONOPT [44]. The MINLP model has been solved using SBB [45]. The MILP master problem for the DES baseline model includes 4,473 continuous variables and 980 binary variables within 9,317 constraints. The complete DES Design – OPF formulation (used in the bi-level and MINLP methods) includes 8,217 continuous variables and 980 binary variables within 16,113 constraints. In the bi-level methods BL-1 and BL-2, binary variables have been solved within the MILP master problem and fixed prior to adding in the nonlinear constraints and associated variables.

4.1. DES Design with OPF Under Balanced Delta Configuration

Table 1 presents the results for Scenario 1, where electricity storage using batteries is excluded from the design. Significant percentage differences in the objectives and design costs can be observed between the DES-only (MILP) and DES-OPF models. Despite the MILP producing the lowest and globally optimal objective value for the proposed DES design, the solutions from both bi-level methods and the MINLP suggest that this design may not be practically feasible. The MILP has maximised income by installing a greater capacity of solar PVs, as evident from the high generation and export income reported in Table 1. Note that BL-1 has fixed capacities which are proposed by the MILP master problem. BL-2 and MINLP have opted for lower PV capacities (as recorded in Table 4) and therefore have a lower total income.

Table 3. Solutions obtained from all four models for Scenario 1 (no electricity storage) under the delta configuration.

| Breakdown                      | MILP  | BL-1 | BL-2 | MINLP |
|--------------------------------|-------|------|------|-------|
| Time taken (s)                 | 5.29  | 5.57 | 5.84 | 6.86  |
| Objective value (£)            | 43,793| 46,380| 46,158| 46,102|
| % Difference with MILP obj.    | -     | +5.9 | +5.4 | +5.3  |
| Relative optimality gap        | 0     | -    | -    | 0     |
| PV investment (£)              | 54,231| 54,231| 51,825| 50,693|
| Boiler investment (£)          | 480   | 480  | 480  | 480   |
| Battery investment (£)         | 0     | 0    | 0    | 0     |
| Grid electricity (£)           | 37,944| 37,944| 38,294| 38,448|
| PV operation (£)               | 5,446 | 5,382| 5,160| 5,055 |
| Boiler operation (£)           | 5,240 | 5,240| 5,240| 5,240 |
| Generation income (£)          | 50,976| 48,938| 47,310| 46,498|
| Export income (£)              | 8,572 | 7,959| 7,532| 7,316 |
Table 4. Capacities of PVs and boilers proposed by the four methods for Scenario 1 under the delta configuration.

| Building | PVs (kW_p) | Boilers (kW_th) |
|----------|------------|-----------------|
| MILP     | 307        | 122             |
| BL-1     | 307        | 122             |
| BL-2     | 294        | 122             |
| MINLP    | 287        | 122             |

To confirm whether the globally optimal solution of the MILP can be practically feasible with respect to the distribution network, BL-1 has been utilised to calculate the voltages, angles, and currents at each timepoint. The results from this model indicate that current constraints are violated when solar power generation is high, particularly in summer when solar irradiance levels are high. Currents at branch 1 (nodes 1 and 2) across all four models are presented in Figure 5, which has the highest branch current out of all branches within the distribution network. This confirms that the solution of the MILP model exceeds the maximum line current by nearly 40% at its peak. Furthermore, the solution of BL-1 indicates that excess solar power must be curtailed in order to avoid current violations, resulting in the highest objective value out of all four methods employed. No voltage violations exist in the solutions of all four models as voltage magnitude and angles remain within specified bounds.

![Figure 5. Currents in branch (1, 2) in summer, as calculated in all four models for Scenario 1. Note that 'l_max' is the maximum allowed line current.](image)

Results for Scenario 2, where the models are given the option of installing batteries to store electricity, are presented in Table 5 for comparison. Interestingly, the objectives and costs across all models are the same, and lower than that of Scenario 1. The total capacities installed are 307 kW_p for PVs, 322 kWh for batteries, and 122 kW_th for boilers. All models have opted to install the same PV capacity as the MILP solution in Scenario 1. However, instead of maximising export income as done in Scenario 1, a proportion of excess electricity has now been used to charge the batteries. The electricity stored during the peak solar generating hours in summer are discharged during the evening. These phenomena are illustrated in Figure 6, where the energy storage profile of Building A in summer is presented as a representative example in Figure 6a, and the total power sold in summer by the DES is given in Figure 6b. Notice that the operational schedules chosen by each of the models are different, warranting for further investigation. Once again, BL-1 with fixed operational variables for storage and selling electricity (in addition to design and binary variables) has been utilised to test the MILP solution. This model returns an infeasible solution, confirming that the MILP operational strategy cannot be realised in practice despite the design proposed being equivalent to the DES-OPF models. This also highlights that symmetrical globally optimal solutions exist within the MILP, some of which are practically feasible, while others are not. It is also evident that Scenario 2 costs 16% less annually for
this case study when compared with Scenario 1 (MILP solution), which does not include batteries. Thus, it demonstrates that the combination of PVs and batteries can provide greater flexibility to both the network and the consumer while lowering costs, provided that the batteries are scheduled with care and do not cycle unrealistically.

Table 5. Results for scenario 2 (with electricity storage) under the delta configuration

| Breakdown                | MILP  | BL-1 | BL-2 | MINLP |
|--------------------------|-------|------|------|-------|
| Time taken (s)           | 2.63  | 6.05 | 5.62 | 6.81  |
| Objective value (£)      | 37,566| 37,566| 37,566| 37,566|
| Relative optimality gap  | 0     | -    | -    | 0     |
| PV investment (£)        | 54,231| 54,231| 54,231| 54,231|
| Boiler investment (£)    | 480   | 480  | 480  | 480   |
| Battery investment (£)   | 8,534 | 8,534| 8,534| 8,534 |
| Grid electricity (£)     | 37,944| 37,944| 38,294| 38,448|
| PV operation (£)         | 5,446 | 5,382| 5,160| 5,055 |
| Boiler operation (£)     | 5,240 | 5,240| 5,240| 5,240 |
| Battery operation (£)    | 3,544 | 3,544| 3,544| 3,544 |
| Generation income (£)    | 50,976| 50,976| 50,976| 50,976|
| Export income (£)        | 5,235 | 5,235| 5,235| 5,235 |

Figure 6. Scenario 2 results showing a) energy storage levels over time for Building A in summer b) total solar-generated power sold in summer across all four models.

In Figure 6b, it appears that the power sold is capped at approximately 100 kW in the DES design-OPF models. Note that this is not a fixed cap enforced as a constraint within the models as the value taken by each of the models is between 102.8 kW – 103.1 kW, and is consequent of the nonconvex current constraints present in the OPF part of the models.

4.2. **DES Design with OPF Under Balanced Wye Configuration**

The models MILP and BL-1 have been run under this configuration with the modified current constraints applicable to a balanced wye system. As expected, the MILP solutions obtained in Section 4.1 for both scenarios under the delta system are obtained by the MILP here as well, which are summarised in Table 6. The design and operational strategies are verified using the modified BL-1, where operational variables are also fixed. This confirms that the MILP solution in this case is indeed globally optimal and can be realised in practice due to the absence of network current violations. Figure 7 illustrates the differences in current obtained across delta and wye systems in Scenario 1, where the lack of storage resulted in very high branch currents for the delta system.
Table 6. Results for MILP and BL-1 under a balanced wye system for both scenarios.

| Breakdown               | Scenario 1     | Scenario 2     |
|-------------------------|----------------|----------------|
| Objective value (£)     | 43,793         | 37,566         |
| Relative optimality gap | -              | 0              |
| PV investment (£)       | 54,231         | 54,231         |
| Boiler investment (£)   | 480            | 480            |
| Battery investment (£)  | 0              | 8,534          |
| Grid electricity (£)    | 37,944         | 37,944         |
| PV operation (£)        | 5,446          | 5,446          |
| Boiler operation (£)    | 5,240          | 5,240          |
| Battery operation (£)   | 0              | 3,544          |
| Generation income (£)   | 50,976         | 50,976         |
| Export income (£)       | 8,572          | 5,235          |

Figure 7. Peak currents in branch (1,2) in summer under delta and wye configurations for Scenario 1.

These results emphasise the importance of considering the overall network configuration when modelling the combined DES design and OPF problem, and especially when using balanced OPF formulations.

4.3. Verification of power flow formulations

To compare the accuracy of the nonlinear balanced AC power flow formulations presented in Section 2.3, the Newton-Raphson algorithm with timeseries power flow analysis on Pandapower [38] is used. BL-1 under delta configuration is used to simulate power flow analysis with the OPF formulation, without the capabilities of installing DES (i.e., power must be purchased from the external grid). Note that Pandapower reports current values as phase currents as opposed to line currents [46], therefore these values have been multiplied by $\sqrt{3}$ to obtain line currents for comparison. Percentage errors for voltage magnitude ($V_{ph}$) and angle ($\theta_{ph}$) at each bus, active ($P_{bus}$) and reactive ($Q_{bus}$) power at each bus, and branch current ($I_L$) are reported in Table 7. As all errors are within ±0.23%, the power flow formulation utilised is deemed sufficiently accurate.

Table 7. Percentage errors calculated between Pandapower solutions and power flow formulations used in this study.

| $V_{ph}$ | $\theta_{ph}$ | $P_{bus}$ | $Q_{bus}$ | $I_L$ |
|----------|---------------|-----------|-----------|-------|
| Min      | 0.000         | -0.029    | -0.211    | -0.036| -0.065|
| Max      | 0.000         | 0.018     | 0.221     | 0.029 | 0.063|
4.4. Discussion and Limitations

The results obtained suggest that a combined DES – OPF framework is essential when designing DES which are to be integrated into networks with a balanced delta configuration. DERs such as solar panels have the potential to increase branch currents to unreasonable levels, especially when exporting power during peak renewable generation times. However, this can be mitigated with the use of local electricity storage technologies such as batteries, which act as a buffer by storing excess electricity when the distribution network is overloaded. Computational times recorded for all four models in Table 3 show that mixed integer and/or nonlinear models can be solved within seconds using mathematical programming approaches, provided that the models receive sufficiently accurate initial conditions by solving an MILP master problem first (see Figure 1). However, if future studies choose to enlarge the DES formulation by including heating/cooling networks and additional DERs, bi-level models such as BL-2 may be more suitable than MINLP models due to their ability to strike a better accuracy-complexity balance. BL-2, in particular, has a comparatively lower computational expense and the ability to influence the DES design, while percentage differences between the objectives of BL-2 and the MINLP are also relatively small. Furthermore, the availability of well-tested and benchmarked open source MILP and NLP solvers, such as CBC [47] and IPOPT [48], make this method accessible to modellers and researchers.

On the other hand, it appears that DES-OPF models may not be essential when integrating DES into networks with a balanced wye configuration, as no network constraints have been violated by the MILP solution. This is consequent of lower branch currents in wye systems compared to delta systems, along with the absence of voltage violations. However, there are several limitations to this study, as a result of which we cannot rule out the use of a DES-OPF framework for wye systems. As mentioned in Section 1.1, wye systems are rarely balanced due to the presence of unequal single-phase loads, which may lead to the argument that balanced power flow formulations used in this work are inappropriate for studying such networks [49]. Unbalanced networks tend to have a different array of problems, such as increased power losses [50], voltage violations, and equipment overloading [49]. The impacts of combining DES design with AC OPF for unbalanced networks have not been studied to date but may be required in future studies. Furthermore, we do not use mixed delta- and wye-connected loads within the distribution network to maintain the assumption that the network is balanced. Should such a scenario be tested, it may render different results.

Overall, the results shed light on the synergies between DES and the underlying distribution network, and highlight the importance of considering network configuration within combined DES design and OPF frameworks.

5. Conclusions

This study presents a methodology for consolidating DES and OPF optimisation problems while paying attention to different types of network configurations that exist within electrical distribution networks. Four modelling methods are employed and compared, which include an MILP model to represent the DES baseline formulation, two bi-level methods where nonlinear OPF constraints can influence the linear DES model, and a combined MINLP model. Balanced nonlinear AC OPF formulations that are typically representative of a delta-configured network are utilised. To account for a balanced wye-configured network, a modified formulation for calculating branch currents is also presented. A low-voltage residential network is used to test all models under two scenarios, one including generation technologies only (excluding electricity storage), while the other includes both generation and storage technologies.

Results show that a combined DES design – OPF framework is essential for distribution networks under a delta configuration. Significant differences are seen in the resulting designs, particularly when electricity storage is unavailable. A bi-level model with detailed OPF constraints and fixed variables confirms that the proposed design from the MILP solution cannot be practically realised when storage
technologies are unavailable, despite being globally optimal. This is because network current constraints are violated through the excessive export of PV power in the distribution network. The second scenario highlights that the presence of electricity storage can ensure that the MILP design is feasible. However, combined DES-OPF models are required to determine practically feasible operational schedules, which the MILP model is incapable of achieving with respect to network constraints. Under these circumstances, locally optimal solutions from combined DES-OPF models are more useful and accurate than globally optimal solutions from less-detailed MILP models. On the other hand, under a wye configuration, the MILP design and operational solutions for both scenarios do not violate any network constraints and therefore may be practically feasible. This highlights the importance of considering network configuration within DES-OPF models, and suggests that most current violations reported in literature are only applicable to networks with delta configuration. Future work involves the use of unbalanced OPF formulations to more accurately represent wye-configured distribution networks, and analyse the impacts of integrating DES on network unbalance and power losses. Such efforts can further confirm the effectiveness of using more detailed optimisation models to design and operate robust DES within existing distribution networks.

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Appendix A – DES Seasonal Model
The constraints included in each seasonal model are described here. Note that the subscript $s \in S$, representing seasons, has now been removed from the formulation. Electrical power flows are denoted by $E$, while thermal power flows are denoted by $H$, with both calculated in kilowatts (kW).

Electricity demand $E_{i,t}^{load}$ at each building $i \in I$ and timepoint $t \in T$ can be satisfied using the equation below:

$$E_{i,t}^{load} = E_{i,t}^{grid} + E_{i,t}^{G} + \sum_{c} E_{i,t,c}^{disch} \quad \forall i \in I, t \in T$$

(25)

Where $E_{i,t}^{grid}$ is the purchased power, $E_{i,t}^{G}$ is the power generated by the electricity-generating distributed generators $G \in DER$, and $E_{i,t,c}^{disch}$ is the power discharged by the batteries $c \in C$ to satisfy consumer demand.

Heat demand $H_{i,t}^{load}$ is met using the thermal power generated by heat-generating DERs, $H_{i,t}^{G}$:

$$H_{i,t}^{load} = H_{i,t}^{G} \quad \forall i \in I, t \in T$$

(26)

The FIT tariffs require that power generated onsite must be used first to satisfy demand, allowing customers to sell excess power only, $E_{i,t}^{sold}$. This is ensured using the following constraints, where a binary variable $X_{i,t}$ is used to indicate when electricity is purchased by the consumer:

$$E_{i,t}^{grid} \leq E_{i,t}^{load} \cdot (1 - X_{i,t}) \quad \forall i \in I, t \in T$$

(27)

$$E_{i,t}^{sold} \leq M \cdot X_{i,t} \quad \forall i \in I, t \in T$$

(28)

Note that $M$ corresponds to large number within a big-M constraint, which is used here to avoid bilinear terms.

Each DER has a maximum rated capacity $Cap^{G}$, and any electrical or thermal power $P_{i,t}^{G}$ generated by this DER must be less than or equal to this:

$$P_{i,t}^{G} \leq Cap^{G} \quad \forall i \in I, t \in T, G \in DER$$

(29)
For solar PVs specifically, solar irradiance available at each timepoint \(Irr_i\) (kW/m\(^2\)), panel efficiency \(\eta_{\text{panel}}\), number of panels installed at each building \(N_{i}^{\text{Panel}}\), and area of each panel \(A_{\text{Panel}}\) (m\(^2\)) limits the amount of total power produced:

\[
E_{i,t}^{\text{PV,used}} + E_{i,t}^{\text{PV,sold}} + \sum_c E_{i,t,c}^{\text{PV,charge}} \leq N_{i}^{\text{Panel}} \times A_{\text{Panel}} \times Irr_i \times \eta_{\text{panel}} \quad \forall i \in I, t \in T
\]  

(30)

The total solar power generated is broken down to represent the power consumed by the building to meet demand \(E_{i,t}^{\text{PV,used}}\), the excess power sold \(E_{i,t}^{\text{PV,sold}}\), and power used to charge batteries \(E_{i,t,c}^{\text{PV,charge}}\).

The maximum available roof area \(A_{i}^{\text{roof}}\) (m\(^2\)) at each building limits the number of panels that can be installed:

\[
N_{i}^{\text{Panel}} \times A_{\text{Panel}} \leq A_{i}^{\text{roof}} \quad \forall i \in I
\]  

(31)

To calculate the total operational and maintenance costs of installing PVs \(C_{\text{OM, PV}}\) within the DES, the total power produced is multiplied by the variable operational cost \(C_{\text{OM,V, PV}}\) (\(£/\text{kWh}\)), number of days in each season \(N_{\text{days}}\), and the discretised time interval duration \(\Delta t\). Similarly, the fixed yearly operational cost \(C_{\text{OM,f, PV}}\) (\(£/\text{kW-year}\)) is accounted for with respect to the total capacity of panels installed:

\[
C_{\text{OM, PV}} = \sum_{i,t,c} \left( E_{i,t}^{\text{PV,used}} + E_{i,t}^{\text{PV,sold}} + \sum_c E_{i,t,c}^{\text{PV,charge}} \right) \times C_{\text{OM,V, PV}} \times N_{\text{days}} \times \Delta t
\]  

\[+ \sum_{i} N_{i}^{\text{Panel}} \times C_{\text{OM,f, PV}} \times \text{Cap}_{\text{Panel}} \times \frac{1}{365} \times N_{\text{days}}\]  

(32)

As boilers are utilised to generate heat \(H_{i,t}^{b}\), the maximum capacity required to satisfy the demand is determined by the maximum heat generated \(H_{i}^{b,\text{max}}\):

\[
H_{i}^{b,\text{max}} \geq H_{i,t}^{b} \quad \forall i \in I, t \in T
\]  

(33)

The total operational cost of boilers within the DES \(C_{\text{OM,b}}\) is a function of the fuel price \(C_{\text{gas}}\) (\(£/\text{kWh}\)) and thermal efficiency \(\eta^{b}\):

\[
C_{\text{OM,b}} = \sum_{i,t} H_{i,t}^{b} \times \Delta t \times C_{\text{gas}} / \eta^{b}
\]  

(34)

The capacity of the Lithium-ion batteries installed at each building \(\text{Cap}_{i,c}^{\text{batt}}\) (kWh) can be determined by the volume installed \(V_{i,c}\) (m\(^3\)) and the volumetric energy density of the battery \(VED_c\) (kWh/m\(^3\)) [39]:

\[
\text{Cap}_{i,c}^{\text{batt}} = V_{i,c} \times VED_c 
\]  

(35)

The volume of the battery installed cannot exceed the maximum available volume at each building \(VA_i\):

\[
\sum_c V_{i,c} \leq VA_i \quad \forall i \in I
\]  

(36)

If multiple battery-options are presented (such as Li-ion, Sodium-Sulphur, etc.), a binary variable \(W_{i,t}\) can be used to determine which type is installed (note that the options in this study are limited to Li-ion):

\[
\sum_c W_{i,c} \leq 1 \quad \forall i \in I
\]  

\[
\text{Cap}_{i,c}^{\text{batt}} \leq 100 \times W_{i,c}
\]  

(37)  

(38)
The maximum state of charge \( SoC_{c}^{\text{max}} \) and depth of discharge \( DoD_{c}^{\text{max}} \) allowed limits the amount of energy \( E_{t,c}^{\text{stored}} \) (kWh) a battery can store:

\[
E_{t,c}^{\text{stored}} \leq \text{Cap}_{i,c}^{\text{batt}} \times SoC_{c}^{\text{max}} \quad \forall i \in I, t \in T, c \in C
\]

\[
E_{t,c}^{\text{stored}} \geq \text{Cap}_{i,c}^{\text{batt}} \times (1 - DoD_{c}^{\text{max}}) \quad \forall i \in I, t \in T, c \in C
\]

The amount of energy stored in the battery is governed by the charging power \( E_{t,c}^{\text{ch}} \), discharging power \( E_{t,c}^{\text{disch}} \), and respective charging and discharging efficiencies, \( \eta_{c}^{\text{ch}} \) and \( \eta_{c}^{\text{disch}} \):

\[
\text{if } t = \text{start}: \quad E_{t,c}^{\text{stored}} = (E_{i,t,c}^{\text{ch}} \times \eta_{c}^{\text{ch}} \times \Delta t) - \frac{E_{i,t,c}^{\text{disch}} \times \Delta t}{\eta_{c}^{\text{disch}}} \quad \forall i \in I, t \in T, c \in C
\]

\[
\text{else}: \quad E_{t,c}^{\text{stored}} = E_{t,c}^{\text{stored}} + (E_{i,t,c}^{\text{ch}} \times \eta_{c}^{\text{ch}} \times \Delta t) - \frac{E_{i,t,c}^{\text{disch}} \times \Delta t}{\eta_{c}^{\text{disch}}} \quad \forall i \in I, t \in T, c \in C
\]

Note that at the first timepoint is denoted as \( t = \text{start} \). As no previous stored energy \( E_{t,c}^{\text{stored}} \) exists at this timepoint, an if-else statement is used, such that subsequent timepoints include the energy storage level at the previous timepoint.

To prevent the battery from discharging more energy than it already contains in the previous timepoint, a logical condition is presented:

\[
\text{if } t > 1 \quad \frac{E_{i,t,c}^{\text{disch}} \times \Delta t}{\eta_{c}^{\text{disch}}} \leq E_{t,c}^{\text{stored}} \quad \forall i \in I, t \in T, c \in C
\]

The storage levels at the beginning and end of the temporal horizon, denoted by \( t = \text{start} \) and \( t = \text{end} \), are also fixed, to ensure that there are no unaccounted differences in the storage between each day of the respective season:

\[
E_{t,c}^{\text{stored}} |_{t=\text{start},c} = E_{t,c}^{\text{stored}} |_{t=\text{end},c} \quad \forall i \in I, c \in C
\]

The batteries can be charged using either PV power \( E_{i,t,c}^{\text{ch,PV}} \) or power purchased from the grid \( E_{i,t,c}^{\text{ch,grid}} \):

\[
E_{i,t,c}^{\text{ch}} = E_{i,t,c}^{\text{ch,grid}} + E_{i,t,c}^{\text{ch,PV}} \quad \forall i \in I, t \in T, c \in C
\]

To prevent the battery from cycling unrealistically, scalar upper bounds \( S_1 \) and \( S_2 \) have been placed on charging and discharging at each timepoint:

\[
E_{t,c}^{\text{ch}} \times \Delta t \times \eta_{c}^{\text{ch}} \leq \text{Cap}_{i,c}^{\text{batt}} \times S_1 \quad \forall i \in I, t \in T, c \in C
\]

\[
E_{t,c}^{\text{disch}} \times \Delta t \times \eta_{c}^{\text{disch}} \leq \text{Cap}_{i,c}^{\text{batt}} \times S_2 \quad \forall i \in I, t \in T, c \in C
\]

Batteries are not allowed to charge and discharge at the same time, and therefore a binary variable \( Q_{i,t,c} \) is used to indicate when the battery is charging:

\[
E_{i,t,c}^{\text{ch}} \leq M \times Q_{i,t,c} \quad \forall i \in I, t \in T, c \in C
\]

\[
E_{i,t,c}^{\text{disch}} \leq M \times (1 - Q_{i,t,c}) \quad \forall i \in I, t \in T, c \in C
\]

Note that big-M constraints are once again utilised to prevent bilinear terms from introducing nonlinearity into the linear DES model.

The total operational cost of batteries installed within the DES \( C_{OM,Batt} \), is calculated using a fixed yearly operational cost \( P_{OM,batt} \)(€/kW-year):
\[ C_{OM,Batt} = \sum_{t} \frac{\sum C_{Cap_{l,c}}^{batt} \cdot P_{c}^{OM,batt}}{\Delta t} \cdot \frac{1}{365} \cdot N_{days} \] (50)

The total purchasing cost of electricity for the whole DES \( C_{grid} \) is calculated using the price of purchasing electricity \( p_{grid}^{\text{grid}} \) (\( £/\text{kWh} \)), which can vary for day and night under Economy 7 tariffs [42]:

\[ C_{grid} = \sum_{t} \left( E_{i,t}^{\text{grid}} + \sum_{c} E_{i,t,c}^{\text{ch,grid}} \right) \cdot \Delta t \cdot p_{grid}^{\text{grid}} \cdot N_{days} \] (51)

The total export income from selling excess power \( I_{\text{export}} \) can be calculated using the tariff prices offered by the FIT scheme \( p_{\text{tariff}} \) (\( £/\text{kWh} \)):

\[ I_{\text{export}} = \sum_{i,t} E_{i,t}^{PV,sold} \cdot \Delta t \cdot p_{\text{tariff}} \cdot N_{days} \] (52)

The FIT also rewards consumers for generating renewable energy. The total generation income can be calculated using the generation-specific tariff \( p_{\text{gen.tariff}} \):

\[ I_{gen} = \sum_{i,t} (E_{i,t}^{PV,used} + E_{i,t}^{PV,sold} + \sum_{c} E_{i,t,c}^{\text{charge}}) \cdot \Delta t \cdot p_{\text{gen.tariff}} \cdot N_{days} \] (53)

Appendix B – Input Parameters

Ap. B Table A. Technology- and pricing-related parameters used in the models

| Parameter                  | Value     | Units       | Reference          |
|----------------------------|-----------|-------------|--------------------|
| Interest rate              | 0.075     | -           | Assumed            |
| DES lifetime               | 20        | years       | Assumed            |
| Electricity purchasing price | 0.1389   | £/kWh       | Private correspondence\(^1\) |
| FIT export tariff          | 0.0477    | £/kWh       | [2]                |
| FIT generation tariff      | 0.1586    | £/kWh       | [2]                |
| **PVs**                    |           |             |                    |
| Investment cost            | 450       | £/panel     | [51]               |
| Efficiency                 | 0.135     |             | [39]               |
| Fixed operational cost     | 12.5      | £/kW-yr     | [39]               |
| Variable operational cost  | 0.005     | £/kWh       | Assumed            |
| Area                       | 1.75      | m\(^2\)/panel | [51]            |
| Rated capacity             | 0.25      | kW/panel    | [51]               |
| **Boilers**                |           |             |                    |
| Gas purchasing price       | 0.02514   | £/kWh       | Private correspondence\(^1\) |
| Boiler investment cost     | 40        | £/kW        | [52]               |
| Boiler efficiency          | 0.94      | -           | [53]               |
| **Li-ion Batteries**       |           |             |                    |
| Volumetric energy density  | 20        | kWh/m\(^3\) | [39]               |
| Max DoD                    | 0.85      |             | [39]               |
| Max SoC                    | 0.9       |             | [39]               |
| Investment cost            | 270       | £/kWh       | [39]               |
| Operational cost           | 11        | £/kWh-yr    | [39]               |
| Round trip efficiency (RTE)| 0.89      | -           | [39]               |
| Charge efficiency          | 0.94      | -           | [39]               |
| Discharge efficiency       | 0.94      | -           | Calculated using RTE |

\(^1\) The University of Surrey Estates and Facilities Department staff
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