Hydrodynamical description of Galactic Dark Matter *

Luis G. Cabral-Rosetti†, Darío Núñez‡, and Roberto A. Sussman§
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México (ICN-UNAM), Apartado Postal 70-543, 9510 México, D.F., México.

Tonatiuh Matos¶
Departamento de Física, Centro de Investigación y Estudios Avanzados del IPN, Apartado Postal 14-740, México D. F., México.

We consider simple hydrodynamical models of galactic dark matter in which the galactic halo is a self-gravitating and self-interacting gas that dominates the dynamics of the galaxy. Modeling this halo as a spherically symmetric and static perfect fluid satisfying the field equations of General Relativity, visible baryonic matter can be treated as “test particles” in the geometry of this field.

We show that the assumption of an empirical “universal rotation curve” that fits a wide variety of galaxies is compatible, under suitable approximations, with state variables characteristic of a non-relativistic Maxwell-Boltzmann gas that becomes an isothermal sphere in the Newtonian limit. Consistency criteria lead to a minimal bound for particle masses in the range $30 \text{eV} \lesssim m \lesssim 60 \text{eV}$ and to a constraint between the central temperature and the particle mass. The allowed mass range includes popular supersymmetric particle candidates, such as the neutralino, axino and gravitino, as well as lighter particles ($m \sim \text{keV}$) proposed by numerical N-body simulations associated with self-interacting “cold” and “warm” dark matter structure formation theories.

PACS numbers: 12.60.-i, 51.30.+1, 95.35.+d, 95.35.Gi, 95.30.Lz

I. INTRODUCTION

The presence of large amounts of dark matter at the galactic lengthscale is already an established fact. It is currently thought that this dark matter is made of relic self-gravitating gases which are labeled as “cold” (CDM) or “hot” (HDM), depending on the relativistic or non-relativistic nature of the particles energetic spectrum at their decoupling from the cosmic mixture [1]-[3]. HDM scenarios are not favoured, as they seem to be in-compatibility with current theories of structure formation [2]. CDM, usually examined within a newtonian framework, can be considered as non-interactive (collisionless particles) or self-interactive [4]. N-body numerical simulations are often used for modeling CDM gases [5]-[8]. However, in recent numerical simulations (see [9]) non-interactive CDM models present the following discrepancies with observations at the galactic scale [10],[11]: (a) the “substructure problem” related to excess clustering on sub-galactic scales, (b) the “cusp problem” characterized by a monotonic increase of density towards the center of halos, leading to excessively concentrated cores. In order to deal with these problems, the possibility of self-interactive dark matter has been considered, so that nonzero pressure or thermal effects can emerge, thus leading to self-interactive models of CDM (i.e. SCDM) [12],[13],[14],[15] and “warm” dark matter (WDM) models [16]-[22] that challenges the duality CDM vs. HDM. Other proposed dark matter sources consist replacing the gas of particles approach by scalar fields [23],[24] and even more “exotic” sources [25].

Whether based on SCDM or WDM, current theories of structure formation point towards dark matter characterized by particles having a mass of the order of at least keV’s (see [12]-[22]), thus suggesting that massive but light particles, such as electron neutrinos and axions (see Table 1), should be eliminated as primary dark matter candidates (though there is no reason to assume that these particles would be absent in galactic halos). Of all possible weakly interactive massive particles (WIMPS), complying with the required mass value of relic gases, only the massive Neutrinos (the muon or tau neutrinos), have been detected, whereas other WIMPS (neutralino, gravitino, photino, sterile neutrino, axino, etc.) are speculative. See [26],[27],[28] and Table 1 for a list of candidate particles and appropriate references.

In this paper we develop an alternative description of galactic DM. Since the dark matter halo constitutes about 90% of the galactic mass, we consider the galactic gravitational field as a spacetime whose sole, self-gravitating, source is this halo, described as a perfect fluid. Assuming this galactic spacetime to be static and...
spherically symmetric, the barionic dark matter can be thought of as test particles following stable and circular geodesics of this spacetime curvature. Since the tangential velocity, \( v \), along these geodesics can be calculated for such a spacetime, we can express the field equations in terms of \( v \). However, the velocity profile \( v = v(r) \) has been observed for a wide range of galaxies, leading to “Universal Rotation Curves” (URC’s) that provide an empiric fit to these rotation velocities\(^\text{[31]}\). Therefore, by inserting the empiric formula for the URC derived by Persic and Salucci\(^\text{[30]}\) into the field equations (given in terms of \( v(r) \)), we obtain a constraint on one of the metric coefficients. Once this constraint is solved, we can obtain the state variables characterizing the galactic dark fluid associated with this URC. We solve this constraint assuming that the velocities are non-relativistic, thus expanding around \( v_0/c \ll 1 \), where \( v_0 \) is the terminal velocity associated with the flattening of the URC.

### II. FIELD EQUATIONS

Considering the line element of a static spherically symmetric space time

\[
d s^2 = -A^2(r) c^2 dt^2 + \frac{d r^2}{1 - 2M(r)/r} + r^2 (d \theta^2 + \sin^2 \theta d \varphi^2),
\]

the tangential velocity of test particles along stable circular geodesic orbits can be expressed in terms of the metric coefficients as

\[
\frac{V^2}{c^2} = v^2(r) = \frac{r A'}{A}.
\]

Becomes a dynamical variable replacing \( A(r) \). Assuming as source of \( (1) \) a perfect fluid momentum energy tensor: \( T^{ab} = (\rho + p) u^a u^b + p g^{ab} \), with \( u^a = A^{-1} \delta^a_\chi \), the following field equations in terms of \( (2) \) become

\[
M' + \frac{(-3 - 5v^2 + 4vv' + r + 2v^2)}{1 + v^2} M - \frac{v(-2v + 2v' + r + v^2)}{1 + v^2} = 0.
\]

(3)

\[
\kappa p = \frac{2M - 2Mv^2 + v^2r}{r^3},
\]

(4)

\[
\kappa p = \frac{[-8vv' - 2(2v^2 + 1)(v^2 - 3)] M}{r^3(1 + v^2)} + \frac{4v^2v' + 2v^2(2 - v^2)}{r^3(1 + v^2)} r
\]

(5)

where \( \kappa = 8\pi G/c^4 \) and a prime denotes derivative with respect to \( r \). Writing the field equations in terms of the orbital velocity, \( v \), provides a useful insight into how an (in principle) observable quantity relates to spacetime curvature and with physical quantities (state variables) which characterize the source of spacetime.

### III. THERMODYNAMICS

If we assume that the self gravitating ideal “dark” gas exists in physical conditions far from those in which the quantum properties of the gas particles are relevant, we would be demanding that these particles comply with Maxwell-Boltzmann (MB) statistics. Following\(^\text{[31]}\), the condition that justifies an MB distribution is given by

\[
\frac{n \hbar^3}{(m k_B T)^{3/2}} \ll 1,
\]

(6)

where \( n, T, \hbar \) and \( k_B \) are, respectively, the particle number density, absolute temperature, Planck’s and Boltzmann’s constants. If the constraint \((1)\) holds and we further assume thermodynamical equilibrium and non-relativistic conditions, the ideal dark gas must satisfy the equation of state of a non-relativistic monatomic ideal gas

\[
\rho = mc^2 n + \frac{3}{2} nk_B T, \quad p = nk_B T,
\]

(7)

whose macroscopic state variables can be obtained from a MB distribution function under an equilibrium Kinetic theory approach (the non-relativistic and non-degenerate limit of the Jüttner distribution)\(^\text{[32]}\). An equilibrium MB distribution restricts the geometry of spacetime\(^\text{[33]}\), resulting in the existence of a timelike Killing vector field \( \beta^a = \beta u^a \), where \( \beta = mc^2/k_B T \), as well as the following relation (Tolman’s law) between the 4-acceleration and the temperature gradient

\[
\dot{u}_a + h^b_a \left( \ln T \right)_b = 0, \quad h^b_a = u_a u^b + \delta^b_a,
\]

(8)

leading to

\[
\frac{A'}{A} + \frac{T'}{T} = 0 \quad \Rightarrow \quad T \propto A^{-1}.
\]

(9)

The particle number density \( n \) trivially satisfies the conservation law \( J^a_{;a} = 0 \) where \( J^a = n u^a \), thus the number of dark particles is conserved. Notice that given \((1)\) and \((6)\), the equation of state \((8)\) and the temperature from the Tolman law \((9)\), we have two different expressions for \( n \)

\[
n = \frac{p}{k_B T} \propto p A,
\]

(10)
velocity law given by Persic and Salucci \[30\], \[31\], and (11) yield the same expression for (4) with
\[n = \frac{1}{mc^2} \left[ p - \frac{3}{2} p \right] \]
\[= \frac{-8 \nu v r + (v^2 + 9) (2 v^2 + 1)}{mc^2 r^3 (1 + v^2)} M + \frac{+4 \nu v r^2 - v^2 (7 + v^2) r}{mc^2 r^3 (1 + v^2)} .\] 

The quantity $mc^2 n$ in (11) follows directly from equations (5) and (6), while $n$ in (10) also follows from $p$ in (4), with $A \propto \exp \int (v^2/r)dr$. Consistency requires that (4) and (11) yield the same expression for $n$.

IV. DARK FLUID HYDRODYNAMICS

We shall assume for $v^2$ the empiric dark halo rotation velocity law given by Persic and Salucci \[30\], \[31\]
\[v^2 = \frac{v_0^2 x^2}{a^2 + x^2} , \quad x = \frac{r}{r_{opt}} \] (12)
where $r_{opt}$ is the “optical radius” containing 83% of the galactic luminosity, whereas the empiric parameters $a$ and $v_0$, respectively, the ratio of “halo core radius” to $r_{opt}$ and the “terminal” rotation velocity, depend on the galactic luminosity. For spiral galaxies we have: $v_0^2 = v_0^2(1-\beta)(1+a^2)$, where $v_{opt} = v(r_{opt})$ and the best fit to rotation curves is obtained for: $a = 1.5(L/L_\odot)^{1/5}$ and $\beta = 0.72 + 0.44 \log_{10}(L/L_\odot)$, where $L_\odot = 10^{10.4}L_\odot$. The range of these parameters for spiral galaxies is $125 \text{ km/sec} < v_0 < 250 \text{ km/sec}$ and $0.6 < a < 2.3$.

Inserting (12) into (2) and (3) we obtain
\[A = [1 + x^2]^{-v_0^2/2} \Rightarrow T = T_c [1 + x^2]^{-v_0^2/2} , \] (13)
\[M = \frac{(v_0^2 - 2)(a^2 + x^2)^{v_0^2 - v_0^2} v_0^4 x^3}{[a^2 + (1 + v_0^2)x^2]^{v_0^2/2(1+v_0^2)}} \times r_{opt} \int [a^2 + (1 + v_0^2)x^2]^{(1-v_0^2)/(1+v_0^2)} \frac{x dx}{(a^2 + x^2)^{3-v_0^2}} . \] (14)

where $T_c = T(0)$ and we have set an integration constant to zero in order to comply with the consistency requirement that $v_0 = 0$ implies flat spacetime ($A = 1, M = 0$). Since the velocities of rotation curves are Newtonian, $v_0 \ll c$ (typical values are $v_0/c \approx 0.5 \times 10^{-3}$), instead of evaluating (14) we will expand this quadrature around $v_0/c$ (in order to keep the notation simple, we write $v_0$ instead of $v_0/c$). This yields
\[\kappa \rho r_{opt}^2 = \frac{2(a^2 + x^2)}{(a^2 + x^2)^2} v_0^2 \]
\[- \frac{5x^4 + 23a^2 x^2 + 6a^4}{(a^2 + x^2)^3} v_0 + \mathcal{O}(v_0^8) , \] (16)
\[\kappa \rho r_{opt}^2 = \frac{2a^2 + x^2}{(a^2 + x^2)^2} v_0^4 \]
\[- \frac{2x^4 + 7a^2 x^2 + 3a^4}{(a^2 + x^2)^3} v_0^6 + \mathcal{O}(v_0^8) , \] (17)
while the expanded form for $T$ follows from (13).
As a function of $v$, the minimal mass for which the Maxwell-Boltzmann distribution is applicable. This graph displays $m$ (in keV's) as a function of $v_0/c$ and $b = a x$, respectively, the terminal velocity and 'halo core radius' associated with the URC given in (22). Assuming typical ranges for spiral galaxies: $125 \text{ km/sec} \leq v_0 \leq 250 \text{ km/sec}$ and $1 \text{ kpc} \leq b \leq 5 \text{ kpc}$, we obtain masses in the range of $30 \text{ eV} \leq m \leq 60 \text{ eV}$ that follow from the right hand side of the relation (23), providing the criterion for applicability of the Maxwell-Boltzmann distribution. Dark matter particle candidates complying with an MB distribution must have much larger mass than the plotted values.

![Graph](image)

**FIG. 2:** Minimal mass for which the Maxwell-Boltzmann distribution is applicable. This graph displays $m$ (in keV’s) as a function of $v_0/c$ and $b = a x$, respectively, the terminal velocity and 'halo core radius' associated with the URC given in (22). Assuming typical ranges for spiral galaxies: $125 \text{ km/sec} \leq v_0 \leq 250 \text{ km/sec}$ and $1 \text{ kpc} \leq b \leq 5 \text{ kpc}$, we obtain masses in the range of $30 \text{ eV} \leq m \leq 60 \text{ eV}$ that follow from the right hand side of the relation (23), providing the criterion for applicability of the Maxwell-Boltzmann distribution. Dark matter particle candidates complying with an MB distribution must have much larger mass than the plotted values.

In order to compare $n$ obtained from (13) and (14), we substitute (12) and (14) into (11) and expand around $v_0$, leading to

\[ T = T_c \left[ 1 - \frac{1}{2} \ln \left( 1 + x^2 \right), v_0^2 \right. \]

\[ + \left. \frac{1}{8} \ln^2 \left( 1 + x^2 \right) v_0^4 - O(v_0^6) \right]. \]  

(18)

In order to compare $n$ obtained from (13) and (14), we substitute (12) and (14) into (11) and expand around $v_0$, leading to

\[ n_{r_{\text{opt}}}^2 = \frac{1}{\kappa m c^2} + \frac{2(3a^2 + x^2)(a^2 + x^2)^2}{2(a^2 + x^2)^3} v_0^2 \]

\[ - \frac{18a^4 + 55a^2x^2 + 13a^4}{2(a^2 + x^2)^3} v_0^4 + O(v_0^6) \].

(19)

while $n$ in (10) follows by substituting (14) into (17), using $T$ from (13) and then expanding around $v_0$. This yields

\[ n_{r_{\text{opt}}}^2 = \frac{1}{\kappa m c^2} + \frac{2(3a^2 + x^2)(a^2 + x^2)^2}{2(a^2 + x^2)^3} v_0^2 \]

\[ + \frac{2(2a^2 + x^2)(a^2 + x^2) \ln (1 + x^2)}{2(a^2 + x^2)^3} v_0^6 \]

\[ - \frac{2(2a^2 + x^2)(3a^2 + x^2)}{2(a^2 + x^2)^3} v_0^6 + O(v_0^8) \].

(20)

Since $v_0/c \ll 1$, a reasonable approximation is obtained if the leading terms of $n$ from (13) and (20) coincide. By looking at these equations, it is evident that this consistency requirement implies

\[ \frac{1}{2} m v_0^2 = \frac{3}{2} k_B T_c. \]

(21)

where $v_0$ denotes a velocity (cm/sec) and not the adimensional ratio $v_0/c$. Since higher order terms in $v_0/c$ have a minor contribution, the two forms of $n$ are approximately equal. This is shown in Figure 1 displaying the adimensional quantity $\kappa mc^2 n_{r_{\text{opt}}}^2$ from (13) and (20) as functions of $x$ for typical values $v_0/c = 0.0006, a = 1$ and eliminating $T_c$ with (21). Equation (14) shows how “flattened” rotation curves, as obtained from the empiric form (12), lead to $M \propto r^3$ for $r \approx 0$ and $M \propto r$ for large $r$. Equations (13) to (21) represent a relativistic generalization of the “isothermal sphere” that follows as the newtonian limit of an ideal Maxwell-Boltzmann gas characterized by $\rho \approx mc^2 n, p \ll \rho$ and $T \approx T_c$. In fact, using newtonian hydrodynamics we would have obtained only the leading terms of equations (13) to (21). It is still interesting to find out that the isothermal sphere can be obtained from General Relativity in the limit $v_0/c \ll 1$ by demanding that rotation curves have a form like (12). The total mass of the galactic halo, usually given as $M$ evaluated at the radius $r = r_{200}$ (the radius at which $\rho$ is 200 times the mean cosmic density). Assuming this density to be $\approx 10^{-29} \text{ gm/cm}^3$ together with typical values $v_0 = 200 \text{ km/sec}$ and $a = 1$ yields $r_{200} \approx 150 \text{ kpc}$. Evaluating $M$ at this values yields about $10^{17} M_\odot$, while $M$ evaluated at a typical “optical radius” $r = 15 \text{ kpc}$ leads to about $10^{12} M_\odot$, an order of magnitude larger than the galactic mass due to visible matter.

**V. DISCUSSION.**

So far we have found a reasonable approximation for galactic dark matter to be described by a self gravitating Maxwell-Boltzmann gas, under the assumption of the empiric rotation velocity law (12). The following consistency relations emerge from equations (19), (21) and (22)
TABLE I: Particle candidates for a MB Dark Matter gas.

| SCDM/WDM | mass in keV | References |
|----------|------------|------------|
| Light Candidates |           |            |
| Light Gravitino | ~ 0.5 | [24] |
| " | ~ 0.75 – 1.5 | [24] |
| Sterile Neutrino | ~ 2.6 – 5 | [24] |
| " | < 40 | [24] |
| " | 1 – 100 | [24] |
| Standard Neutrinos | ~ 1 | [11] |
| Light Dilaton | ~ 0.5 | [11] |
| Light Axino | ~ 100 | [11] |
| Majoron | ~ 1 | [18] |
| Mirror Neutrinos | ~ 1 | [18] |
| CDM | mass in GeV | References |
| Neutralino | > 32.3 | [51] |
| " | > 46 | [51] |
| Axino | ~ 10 | [16] |
| Gravitino | ~ 100 | [50] |

hence, bearing in mind that \( n \leq n_c \) and \( T \approx T_c \), the condition (18) for the validity of the MB distribution together with (22) yields the condition
\[
m \gg \left[ \frac{3^{5/2} \hbar^3}{4\pi G a^2 r_{opt} v_0} \right]^{1/4}, \tag{23}
\]
a criteria of applicability of the MB distribution that is entirely given in terms of \( m \), the fundamental constants \( G, \hbar \) and the empiric parameters \( v_0 \) and \( a r_{opt} \) (the “terminal” rotation velocity and the “core radius”) [29]. For dark matter dominated galaxies (spiral and low surface brightness (LSB)) [30] these parameters have a small variation range: \( r_{opt} \approx 15 \text{kpc}, 0.6 \leq a \leq 2.3 \) and \( 125 \text{km/sec} \leq v_0 \leq 300 \text{km/sec} \), the constraint (23) does provide a tight estimate of the minimal value for the mass of the particles under the assumption that these particles form a self gravitating ideal dark gas complying with MB statistics. As shown in Figure 2, this minimal value lies between 30 and 60 eV, thus implying that appropriate particle candidates must have a much larger mass than this figure [29]. This minimal bound excludes, for instance, light mass thermal particles such as the electron neutrino \( (m_{e\nu} < 2.2 \text{ eV}) \). The axion is also very light \( (m_A \approx 10^{-5} \text{ eV}) \) but it is not a thermal relic and so we cannot study it under the present framework. The currently accepted estimations of cosmological bounds on the sum of masses for the three active neutrino species is about 24 eV, a value that would apparently rule out all neutrino flavours. However, recent estimations of these cosmological bounds have raised this sum to about 1 keV [40], hence more massive neutrinos could also be accommodated as dark matter particle candidates. Estimates of masses of various particle candidates are displayed in Table 1.

FIG. 3: Relation between particle mass and central temperature. This graph displays the relation between \( \log_{10}(T_c) \) (in K) and \( \log_{10}(m) \) (in keV’s) that follows from equation (24) for a terminal velocity \( v_0 = 200 \text{ km/sec} \). Almost identical plots are obtained for other velocities in the observed range 125 km/sec \( \leq v_0 \leq 250 \text{ km/sec} \). The circle and box symbols respectively denote the proton and electron mass yielding central temperatures of the order \( T_c \approx 10^6, 10^5 \text{ K} \). The central temperature for light particles in the range \( 0.5 \text{ keV} \leq m \leq 100 \text{ keV} \) is less than 300 K (rectangle in the left), while for massive supersymmetric particles in the range \( 1 \text{ GeV} \leq m \leq 100 \text{ GeV} \), we have \( T_c \) as large as \( 10^9 \text{ K} \) (rectangle on the right). However, such high temperatures cannot rule out these weakly interactive particles as components of the dark matter MB gas.

Since \( T \approx T_c \), the consistency condition [21] provides the following constraint on the temperature and particles mass of the dark gas
\[
\frac{m}{T_c} = \frac{3 k_B}{v_0^2} \approx 0.4 \times 10^3 \text{ eV/K}, \tag{24}
\]
where we have taken \( v_0 = 300 \text{ km/sec} \). Considering in [24] the minimal mass range that follows from (23), we would obtain gas temperatures consistent with the assumed typical temperatures of relic gases: \( T_c \approx 2 \to 4 \text{ K} \). However, since we have no way of inferring a value for the temperature of the ideal dark gas, we have no clear cut criterion for the estimate of a maximal bound for this mass. If we assume that the ideal dark gas is made of electrons or barions, so that \( m = m_e \) or \( m = m_B \), then condition (23) for applicability of the MB distribution is certainly satisfied and (24) implies a temperature of the order of \( T_c \approx 10^3 \text{ K} \) for electrons and \( T_c \approx 10^9 \text{ K} \) for barions. Obviously, barions or electrons at such a high temperatures would radiate and certainly not remain unobservably “dark”. However, as long as the interaction
is weak and the particles are not charged, we cannot rule out any other particle candidate only on the basis of the gas temperature, even if this temperature is very high (see Figure 3) as in the case of massive supersymmetric particles. As shown in Table 1, a wide range of weakly interactive particles can be considered as possible main components of a MB dark gas, including popular supersymmetric particles (the neutralino), as well as hypothetical light particles predicted by current literature based on WDM models of structure formation [2], [3]. The main novelty of the present paper is the fact that it is based on a general relativistic hydrodynamics, as opposed to numerical simulations [1], [11], Newtonian or Kinetic Theory perturbative approaches (see [12]-[22]).

Finally, the fact that we have obtained a minimal mass on the range $30 – 60$ eV, that seems to discriminate against very light thermal particles like the electron neutrino, coincides with the fact that these HDM particle candidates tend to be ruled out because of their inability to produce sufficient matter clustering [2], [3]. In spite of these arguments, a self gravitating gas of this type of particles accounting for a galactic halo, would have to be modeled, either as a relativistic MB gas (very light particles can be relativistic even at low temperatures) and/or in terms of a distribution that takes into account Fermi-Dirac or Bose-Einstein statistics. These studies will be undertaken in future papers [29].

Acknowledgments

We thank Professor Rabindra N. Mohapatra for calling our attention to the important papers of Ref. [13] and [14] and N. Fornengo for useful discussions. R. A. S. is partly supported by the DGAPA-UNAM, under grant (Project No. IN122498), T. M. is partly supported by CoNaCyT México, under grant (Project No. 34407-E) and L. G. C. R. has been supported in part by the DGAPA-UNAM under grant (Project No. IN109001) and in part by the CoNaCyT under grant (Project No. I37307-E).
[37] S. D. Burns, e-Print Archive: astro-ph/9711304.
[38] S. H. Hansen, J. Lesgourgues, S. Pastor and J. Silk, e-Print Archive: astro-ph/0106108.
[39] A. D. Dolgov and S. H. Hansen, e-Print Archive: hep-ph/0009083.
[40] K. Abazajian, G. M. Fuller and M. Patel, Phys. Rev. D64, 023501 (2001).
[41] Chun Liu and J. Song, Phys. Lett. B512, 247 (2001).
[42] G. F. Giudice, E. W. Kolb, A. Riotto, D. V. Semikoz and I. I. Tkachev, Phys. Rev. D64, 043512 (2001).
[43] Y. M. Cho and Y. Y. Keum, Mod. Phys. Lett. A13, 109 (1998).
[44] S. A. Bonometto, F. Gabbiani and A. Masiero, Phys. Rev. D49, 3918 (1994).
[45] V. Berezinsky and J. W. F. Valle, Phys. Lett. B318, 360 (1993).
[46] K. S. Babu, I. Z. Rothstein and D. Seckel, Nucl. Phys. B403, 725 (1993).
[47] A. Dolgov, S. Pastor and J. W. F. Valle, e-Print Archive: hep-ph/0102323.
[48] Z. G. Berezhiani and R. N. Mohapatra, Phys. Rev. D52, 6607 (1995).
[49] Z. G. Berezhiani, Acta Phys. Polon. B27, 1503 (1996).
[50] P. Abreu et al. Phys. Lett. B 489, 38-35 (2000).
[51] J. Ellis, T. Falk, G. Ganis and K. A. Olive Phys. Rev. D 62, 075010 (2000).
[52] L. Covi, H. B. Kim, J. E. Kim and L. Roszkowski, JHEP 0105, 033 (2001).
[53] Leszek Roszkowski, invited plenary talk at the 4th International Workshop on Particle Physics and the Early Universe (COSMO-2000), Cheju, Cheju Island, Korea, September 4-8, 2000 and the 3rd International Workshop on the Identification of Dark Matter (IDM-2000), York, England, 18-22 September 2000. e-Print Archive: hep-ph/0102323.
[54] M. Kawasaki and T. Moroi Prog. Theor. Phys. 93, 879-900 (1995).