Interactions between zero-point radiation and electrons

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Abstract

Knowing the magnitude of the energy flow inherent to zero-point radiation allows us to approach the question of its possible interaction with particles of matter. Its photons are not different from the rest, and must in principle be subject to the Compton effect and the Klein-Nishima-Tann formula for its cross section. On this assumption, it is shown here that zero-point radiation may be powerful enough to explain Poincaré’s tensions and to supply an efficient cause for gravitation. This could be only the case if the classic radius of the electron measures $8.143375 \times 10^{20} q_{\lambda}$, where $q_{\lambda}$ is the minimum wavelength for electromagnetic radiation, and if the wavelength of the most energetic photon in the actual zero-point radiation is $5.275601 \times 10^{27} q_{\lambda}$. To the first of these numbers there corresponds the energy $3.5829 \times 10^{23}$ MeV for the photon whose wavelength is $1 q_{\lambda}$. This gives also the relation $q_{\lambda} = (2\pi\alpha)^{1/2} L_P$, where $L_P$ is the Planck Length. Finally the relation between the force of gravity and the electrostatic force is explained by the equations obtained in this paper.
I. PRELIMINARY CONSIDERATIONS ON ZERO-POINT RADIATION

Sparnaay’s 1958 experiments [1] exposed the existence of zero-point radiation, which Nernst had considered as a possibility in 1916. However, Sparnaay was not looking for it, since he was only trying to check Casimir’s hypothesis about the mutual attraction of two uncharged conductor plates placed very close together [2]. This attraction should have disappeared when the temperature approached absolute zero, but Sparnaay found that at near that temperature there was still some attraction not accounted for by Casimir’s hypothesis, being independent of temperature and obeying a very simple law; it is directly proportional to the surface of the plates, and inversely proportional to the 4^{th} power of the distance \(d\) between them. Sparnaay observed this force when the plates were placed in a very complete vacuum at near zero-absolute temperature. For a distance of \(5 \times 10^{-5}\) cm between the plates, he was able to measure a force of 0.196 gcms\(^{-2}\), and deduced the formula

\[
f = \frac{k_s}{d^4}, \quad \text{where } k_s = 1.3 \cdot 10^{-18} \text{ ergcm}
\]

In a near-perfect vacuum and at a temperature very near absolute zero, which implies the absence of any “photon gas”, the phenomenon observed by Sparnaay could only be produced by a radiation inherent to space. This could be only the case if its spectrum is relativistically invariant, which could only happen if its spectral distribution is inversely proportional to the cubes of the wavelengths—in other words, if the number of photons of wavelength \(\lambda\) which strike a given area within a given time is inversely proportional to \((\lambda)^3\).

A function of spectral distribution which is inversely proportional to the cubes of the wavelengths, implies a distribution of energies which is inversely proportional to the 4^{th} power of the wavelengths, because the energies of the photons are inversely proportional to the wavelengths. In 1969, Timothy H. Boyer [4] showed that the spectral density function of zero-point radiation is

\[
f_{\varphi}(\lambda) = \frac{1}{2\pi^2} \frac{1}{(\lambda_*)^3}, \tag{1}
\]

where \(\lambda_*\) is the number giving the measurement of wavelength \(\lambda\).

This function produces the next, for the corresponding energies

\[
E_{\varphi}(\lambda) = \frac{1}{2\pi^2} \frac{hc}{\lambda} \frac{1}{(\lambda_*)^3}
\]

for \(\lambda \rightarrow 0\), \(E_{\varphi}(\lambda) \rightarrow \infty\). There must therefore be a threshold for \(\lambda\), which will hereafter be designated by the symbol \(q_\lambda\).
In the Sparnaay effect, the presence of a force which is inversely proportional to the 4\textsuperscript{th} power of the distance $d$ between the plates, while the distribution of energies is also inversely proportional to the 4\textsuperscript{th} power of the wavelengths, leads us to infer that the cause of the apparent attraction must lie in some factor which varies inversely with the wavelength, and which causes a behaviour different from the photons of wavelengths equal to or greater than $d$, since:

$$\sum_{n=d}^{\infty} Kn^{-5} \rightarrow \frac{K}{4d^4}$$

The factor in question turns out to be the proportion of photons which are reflected, not absorbed; that is the coefficient of reflection $\rho$, while the different behaviour is caused by the obstacle which the presence of each one of the two plates offers to the reflection from the inner face of the other one, of photons of wavelength equal or greater than “$d$”. As we know, the energy which is transferred by a reflected photon is double that transferred by one which is absorbed.

To simplify the following arguments, it is convenient to use the $(e, m_e, c)$ system of measurements in which the basic magnitudes are the quantum of electric charge, the mass of the electron and the speed of light. In this system, the units of length and time are respectively $l_e = c^2/m_e c^2$ and $t_e = c^2/m_e c^3$.

The results of Sparnaay’s experiments provide a link with reality which may be enough to show the intensity of the energy flow belonging to zero-point radiation. A recent paper [5] has shown that this flow is one which corresponds to the incidence in an area $(q\lambda)^2$ of one photon of wavelength $q\lambda$ every $q\tau$, plus one photon of wavelength $2q\lambda$ every $2^2 q\tau$, etc. up to one photon of wavelength $nq\lambda$ every $n^3 q\tau$, where $q\tau = q\lambda/c$ [6]. This radiation implies the energy flow per $(q\lambda)^2$ and $q\tau$ which is given by

$$W_0 = \frac{hc}{q\lambda q\tau} \left\{1 + \frac{1}{2^4} + \cdots + \frac{1}{n^4}\right\} = \frac{\pi^5}{45\alpha}(k\lambda)^2 \frac{m_e c^2}{t_e},$$

where $k\lambda = l_e/q\lambda = t_e/q\tau$ [7].

This energy flow produces a force per $(l_e)^2$ which is given by

$$F_{(e,m_e,c)} = \frac{\pi^5}{45\alpha}(k\lambda)^4 \frac{m_e c}{t_e}$$

which when expressed in the c.g.s. system and per cm$^2$ is given by

$$F_{(c.g.s.)} = 3.409628 \times 10^{34}(k\lambda)^4 \frac{\text{gcm}}{\text{s}^2}.$$
In cosmic rays, photons have been observed with energies of the order of more than $10^{19}$ eV, [8] which implies that the value of $k_\lambda$ must be greater than $2.273 \times 10^{10}$. The value of $F_{(c.g.s.)}$ is therefore immense. In consequence the analysis of the possible interactions of zero-point radiation with the elementary particles assumes enormous interest.

II. ZERO-POINT RADIATION AND POINCARÉ TENSIONS

“If we have a charged sphere, all the electrical forces repel, and the electron will tend to fly apart... The charge must be maintained over the sphere by something which stops it from flying off. Poincaré was the first to point out that this ‘something’ must be allowed for in the calculation of energy and momentum”.

The Feynman Lectures in Physics, Vol. II, 28-1 to 28-14.

If the electron’s mass and charge were distributed according to some spatial configuration, the repulsion of the charge against itself would tend to cause the particle to fly apart. Seeing this, Poincaré suggested that something must exist which can counteract this repulsion and this ‘something’ has therefore been named ‘Poincaré’s tensions’. The possible configuration must be one which has no favoured direction, which implies that it must have spherical symmetry. Electrostatic repulsion would appear in the form of a centrifugal force. To meet this, Poincaré’s tensions would have to be arranged as centripetal forces able to counteract it. When submitted to these fields of opposing forces, the charge of the electron would tend to distribute itself equally over a spherical surface. Zero-point radiation, which arrives equally from all directions of space, provides centripetal energy flows towards every imaginable spherical surface, and from these flows there could be derived equally centripetal flows, to fill the role of Poincaré’s tensions. Sparnaay’s experiments have allowed us to measure the intensity of the energy flows inherent to zero-point radiation; Compton’s experiments discovered the laws which govern the phenomena of dispersal and energy transfer produced by the interaction of photons with free particles and matter. Finally the differential cross section for these phenomena follows the Klein-Nishima-Tann formula. With this knowledge we can attempt to calculate the centripetal force which could be produced by the zero-point radiation, on an electron with its charge distributed equally over a spherical surface of radius $r_x$. 
The collision of a photon with a free particle of matter, shown in Fig. 1, produces the Compton effect. The colliding photon of energy $E$ and wavelength $\lambda$, loses part of its energy to the particle, and is diverted at an angle $\theta$ to its previous trajectory. When losing energy, it increases its wavelength by an amount which is given by Compton’s equation

$$\Delta \lambda = \lambda_c (1 - \cos \theta),$$

where $\lambda_c = \frac{h}{mc}$, where $m$ is the said particle mass. For the electron, Compton’s wavelength $\Delta \lambda = \lambda_{ce} = \frac{2\pi}{\alpha l_e}$.

In the case of zero-point radiation, the free particle of matter, which is subjected to equal forces from all directions, does not move, but suffers compression towards its centre.

The encounter of a photon with an electron does not always produce the Compton effect; the differential cross section for this $d\sigma$ is given by the Klein-Nishima-Tann formula for dispersal with electrons, which is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} (r_e)^2 \left( \frac{\omega'}{\omega} \right)^2 \left( \frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2 \theta \right),$$

where $\theta$ is the angle of dispersal, $\omega$ the frequency of the arriving photon, $\omega'$ that of the dispersed photon, $r_e = l_e = e^2 m_e^{-1} c^{-2}$ the classical radius of the electron, $d\sigma$ is the differential cross section for the Compton effect, and $d\Omega = 2\pi \sin \theta d\theta$. By bringing in this relation and remembering that $\frac{\omega'}{\omega} = \frac{\lambda}{\lambda'}$, we can write

$$d\sigma = \pi (r_e)^2 \left( \frac{\lambda}{\lambda'} \right)^2 \left[ \left( \frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} \right) \sin \theta - \sin^3 \theta \right] d\theta$$

(4)

Taking all the foregoing into consideration, we also know that:

- The number of photons of zero-point radiation with wavelength $nq_\lambda$, which converge from all directions in an area $(q_\lambda)^2$ is $1/n^3$ every $q_r$.
The energy transferred to the electron by a photon of wavelength \( nq\lambda = \lambda \), which after interacting is diverted at an angle \( \theta \) from its trajectory, is

\[
E = hc \left[ \frac{1}{\lambda} - \frac{1}{\lambda'} \right] = hc \left[ \frac{1}{nq\lambda} - \frac{1}{nq\lambda + \frac{2\pi}{\alpha}l_e (1 - \cos \theta)} \right].
\]

Therefore, the energy transferred every \( q\tau \) by these photons of point-zero radiation converging on an area \( (q\lambda)^2 \) is given by

\[
E_{Tn} = \frac{\pi (r_e)^2}{n^3} hc \int_0^{\pi/2} \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) \left( \frac{\lambda}{\lambda'} + \frac{\lambda'}{\lambda} - \sin^2 \theta \right) \sin \theta \, d\theta, \tag{5}
\]

where

\[
\lambda = nq\lambda,
\]
\[
\lambda' = nq\lambda \left[ 1 + 2\pi k_\lambda (1 - \cos \theta)/\alpha n \right],
\]
\[
k_\lambda = l_e/q\lambda,
\]

whence we derive the following definite integrals:

\[
\int_0^{\pi/2} \frac{\lambda^2}{\lambda'^2} \sin \theta \, d\theta = \frac{1}{nq\lambda} \frac{1}{2} \left[ \frac{1}{1 + A} + \frac{1}{(1 + A)^2} \right],
\]

\[
\frac{1}{nq\lambda} \int_0^{\pi/2} \frac{\sin \theta \, d\theta}{\lambda'/\lambda^2} = \frac{1}{nq\lambda} \frac{1}{A} \ln(1 + A),
\]

\[
- \frac{1}{nq\lambda} \int_0^{\pi/2} \frac{\lambda}{\lambda'^2} \sin^2 \theta \, d\theta = \frac{1}{nq\lambda} \frac{1}{A} \left[ \frac{1}{1 + A} + \frac{2A}{(1 + A)^2} \right]
\]

\[
- \frac{1}{nq\lambda} \int_0^{\pi/2} \frac{\lambda^3}{(\lambda')^2} \sin \theta \, d\theta = -\frac{1}{3nq\lambda} \left[ \frac{1}{1 + A} + \frac{1}{(1 + A)^2} + \frac{1}{(1 + A)^3} \right],
\]

\[
- \frac{1}{nq\lambda} \int_0^{\pi/2} \frac{\lambda}{(\lambda')^2} \sin^3 \theta \, d\theta = \frac{1}{nq\lambda} \frac{-1}{nq\lambda} \left[ \frac{1}{1 + A} \right] \left( \frac{1}{1 + A} \right),
\]

\[
+ \int_0^{\pi/2} \frac{\lambda^2 \sin^3 \theta \, d\theta}{(\lambda')^3} = \frac{1}{nq\lambda} \frac{1}{A} \left[ \frac{1}{A} - \frac{1}{2(1 + A)^2} - \frac{1}{A^2} \ln(1 + A) \right],
\]

where \( A = \frac{2\pi k_\lambda}{\alpha n} \). The addition of this integrals gives

\[
\frac{1}{nq\lambda} \left[ \frac{l(1 + A)}{A} - \frac{5}{6(1 + A)} + \frac{1}{6(1 + A)^2} - \frac{1}{3(1 + A)^3} + \frac{3}{A^2} + \frac{1}{A(1 + A)} - \frac{1}{2A(1 + A)^2} - \frac{(3 + 2A)}{A^3} l(1 + A) \right] \tag{6}
\]
Therefore we have: \( E_{T_n} = \pi (r_e)^2 \frac{hc}{nq_\lambda} [A] \); where

\[
[A] = \left[ \frac{l(1 + A)}{A} - \frac{5}{6(1 + A)} + \frac{1}{6(1 + A)^2} + \cdots \right]
\]
as in (6). Therefore \( \frac{E_{T_n}}{E_n} = \pi (r_e)^2 [A] \); obviously \( E_n = \frac{hc}{nq_\lambda} \) and \( \frac{E_{T_n}}{E_n} < 1 \).

Since in the \((e, m_e, c)\) system \( r_e = 1 \), this factor can be ignored and we can write

\[
\frac{E_{T_n}}{E_n} = \pi [A], \tag{7}
\]
and

\[
E_{T_n} = \frac{hc}{nq_\lambda} \pi [A]. \tag{8}
\]

A first attempt to solve the problem showed us that the energy flow (3) would be excessive. Sparnaay's measurements allow us to deduce that, in the case of photons with wavelength equal to or greater than \( 5 \times 10^{-10} \) cm, the energy flow of the zero-point radiation agrees with the energy flow corresponding to the incidence, in an area \((q_\lambda)^2\), of the photon of wavelength \( q_\lambda \) every \( q_\tau \), plus another of wavelength \( 2q_\lambda \) every \( 2^3 q_\tau \), up to a photon of wavelength \( nq_\lambda \) every \( n^3 q_\tau \). However this does not mean that, at present, the photon with the shortest wavelength in the zero-point radiation, has to be the photon of wavelength \( 1q_\lambda \). If we suppose that its wavelength is \( xq_\lambda \), the flow of energy to be taken into account is:

\[
W_x = \frac{hc}{q_\lambda q_\tau} \sum_{x}^{\infty} \frac{1}{n^4} = \frac{2\pi}{3\alpha} \frac{(k_\lambda)^2 m_e c^2}{x^3 t_e}, \quad \text{per} \quad (q_\lambda)^2
\]

instead of

\[
W_0 = \frac{\pi^5 (k_\lambda)^2 m_e c^2}{45\alpha t_e},
\]
as given by (3).

The condition \( \frac{E_{T_n}}{E_n} < 1 \) implies

\[
\pi \left[ \frac{l(1 + A)}{A} - \frac{5}{6(1 + A)} + \frac{1}{6(1 + A)^2} - \frac{1}{3(1 + A)^3} + \frac{3}{A^2} + \cdots \right] < 1
\]

It is reasonable to suppose that the wavelength \( xq_\lambda \) of the most energetic photon in the actual zero-point radiation is much greater than the wavelength of the photon whose energy is \( m_e c^2 \), which is \( \frac{2\pi}{\alpha} l_e = \frac{2\pi}{\alpha} k_\lambda q_\lambda \). If this assumption does not cause any future contradiction
and leads to a value of \( k_\lambda > 2.273 \times 10^{10} \), which means that the energy of the photon with wavelength 1\( q_\lambda \) is greater than 10\(^{19} \) eV, it must be considered as reasonable. Accordingly \( A = \frac{2\pi k_\lambda}{\alpha x} < 1 \), and we can write:

\[
l(1 + A) = A - \frac{1}{2}A^2 + \frac{1}{3}A^3 + \cdots + (-1)^{m+1}\frac{A^m}{m}
\]

Besides, we have for any value of \( A \):

\[
(1 + A)^{-1} = 1 - A + A^2 - A^3 + \cdots + (-1)^m A^m, \\
(1 + A)^{-2} = 1 - 2A + 3A^2 - 4A^3 + \cdots + (-1)^m(m + 1)A^m, \\
(1 + A)^{-3} = 1 - 3A + 6A^2 - 10A^3 + \cdots + (-1)^m\frac{(m + 1)(m + 2)}{2}A^m.
\]

By introducing these series into \([A]\), as in (6), we obtain:

\[
[A] = \frac{7}{12}A - \frac{11}{10}A^2 + \frac{37}{20}A^3 + \cdots + \frac{(-1)^m}{m+1} + \frac{2}{m+2} - \frac{3}{m+3} - 1 - \frac{m(m-1)}{6} \frac{A^m}{m}
\]

(10)

To obtain the total energy transferred each \( q_\tau \) to an area \((q_\lambda)^2\), we can write:

\[
E_{T_x} = \sum_x E_{T_n} \frac{1}{n^3}.
\]

From (8) we have

\[
E_{T_n} = \frac{hc}{nq_\lambda} \pi [A] = \frac{2\pi^2 k_\lambda}{\alpha m} [A] m_e c^2
\]

whence

\[
E_{T_x} = \frac{2\pi^2}{\alpha} k_\lambda \left( \sum_x \frac{[A]}{n^4} \right) m_e c^2
\]

each \( q_\tau \).

Therefore the energy flow per \((q_\lambda)^2\) each \( t_e \) is:

\[
W_{T_x} = \frac{m_e c^2 2\pi^2}{t_e \alpha} (k_\lambda)^2 \sum_x \frac{1}{n^4} \left\{ \frac{7}{12}A - \frac{11}{10}A^2 + \cdots + \frac{(-1)^m}{m+1} + \frac{2}{m+2} - \frac{3}{m+3} - 1 - \frac{m(m-1)}{6} \right\} A^m \right\}
\]

(11)
to an area \((q\lambda)^2\). Since \(A = \frac{2\pi k\lambda}{\alpha n}\) we can write:

\[
W_{Tz} = \frac{m_e c^2 2\pi^2}{t_e \alpha} (k\lambda)^2 \left[ \frac{7}{12} \frac{2\pi k\lambda}{\alpha} \sum_{x}^{\infty} \frac{1}{n^5} - \frac{11}{10} \left( \frac{2\pi k\lambda}{\alpha} \right)^2 \sum_{x}^{\infty} \frac{1}{n^6} + \cdots + \right. \\
\left. + (-1)^{m-1} \left\{ \frac{1}{m+1} + \frac{2}{m+2} - \frac{3}{m+3} - 1 - \frac{m(m-1)}{6} \right\} \left( \frac{2\pi k\lambda}{\alpha} \right)^m \sum_{x}^{\infty} \frac{1}{n^{m+4}} \right] 
\]

(12)

to an area \((q\lambda)^2\), whence

\[
W_{Tz} = \frac{m_e c^2 2\pi^2}{t_e \alpha} (k\lambda)^2 \left[ \frac{7}{48} \frac{2\pi k\lambda}{\alpha} \frac{1}{x^4} - \frac{11}{50} \left( \frac{2\pi k\lambda}{\alpha} \right)^2 \frac{1}{x^5} \cdots + \right. \\
\left. + T_m \left( \frac{2\pi k\lambda}{\alpha} \right)^m \frac{1}{x^{m+3}} \right]
\]

to an area \((q\lambda)^2\); where

\[
T_m = (-1)^{m-1} \left[ \frac{1}{m+1} + \frac{2}{m+2} - \frac{3}{m+3} - 1 - \frac{m(m-1)}{6} \right] \frac{1}{m+3}
\]

By introducing \(B = \frac{2\pi}{\alpha} \left( \frac{k\lambda}{x} \right)\) we can write:

\[
W_{Tz} = \frac{m_e c^2 2\pi^2}{t_e \alpha} x^3 \left( \frac{7}{48} B - \frac{11}{50} B^2 + \cdots + T_m B^m \right) 
\]

(13)

to an area \((q\lambda)^2\), in such a way that

\[
\frac{W_{Tz}}{W_x} = 3\pi [B]_m, 
\]

(14)

where we used

\[
\{B\}_m = \left( \frac{7}{48} B - \frac{11}{50} B^2 + \cdots + T_m B^m \right).
\]

Fig. 2 shows an electron configured as a spherical surface as a result of the equilibrium between centrifugal forces of electrostatical repulsion and centripetal forces derived from the interaction with zero-point radiation. If this interaction happens with zero-point radiation falling on an area \((q\lambda)^2\) situated on \(N\) on the surface of the electron, and coming from all directions of the half space defined by the tangent plan at \(N\) and opposite to \(O\), the total energy flow over the said area will be \(W_x/2\), and the total energy flow transferred to the electron will be \(\frac{W_{Tz}}{2}\).
This energy flow produces a force \( F = \frac{W_T}{2c} \). If we imagine a half sphere whose center is at \( N \) and whose radius has a measure equal to \( F \) the components according the direction \( \overrightarrow{NO} \) of the forces \( F \) coming from the referred half space have the same measurements as the distances of the points of the surface of the said sphere to the tangent plane at \( N \) and the number which expresses their sum is that which measures the volume of the said half sphere; i.e. \( \frac{2\pi}{3} F^3 \). Now, in the present case we must divide this number by the area of the said half sphere, because the total force \( \overrightarrow{F_{NO}} \) is the sum of those which come from all directions. To each point of this area there corresponds one of those directions. Therefore the sum of the components according the radius at the point of arrival of the forces which strike in an area \((q\lambda)^2\) is given by

\[
F_{NO} = \frac{(2/3)\pi F^3}{2\pi F^2} = \frac{F}{3} = \frac{W_T}{6c},
\]

which gives

\[
F_{NO} = \frac{\pi^2}{3\alpha} \frac{(k\lambda)^2}{x^3} \frac{m_e c}{t_e} \tag{15}
\]

The electrostatic repulsion for the fraction \( e/4\pi(r_x)^2(k\lambda)^2 \) of the charge \( e \) which corresponds to a fraction \((q\lambda)^2\) of a sphere with radius \( r_x k\lambda(q\lambda) \) is

\[
F_e = \frac{e}{4\pi(r_x)^2(k\lambda)^2} \frac{1}{(r_x)^2(l_e)^2} = \frac{1}{4\pi(r_x)^4(k\lambda)^2} \frac{m_e c}{t_e} \tag{16}
\]

By equalising (15) and (16) we obtain

\[
x^3 = \frac{4\pi^3}{3\alpha} (k\lambda)^4 (r_x)^4 [B] m
\]

\[
\tag{17}
\]
III. BEHAVIOUR OF TWO ELECTRONS IMMERSED IN ZERO-POINT RADIATION

Figure 3 shows two electrons whose centers are $dl_e$ apart. A fraction of the zero-point radiation flow which reaches the electron whose center is at $O_1$ is spent to equalize the Poincaré’s tensions at its surface. According to (15) this fraction implies the force.

$$F_P = 4\pi (r_x)^2 \frac{\pi^2 (k\lambda)^2}{3\alpha (x)^3} [B]_m \frac{m_e c}{t_e}. $$

Therefore, the zero-point flows which emerge after interacting with this particle imply forces towards $O_1$ whose intensities at the distance $dle$ from this point would be equal to $F_P/3\pi d^2$. The same occurs with the zero-point radiation which interacts with the electron at $O_1$ after having interacted with the electron at $O_2$. Both reductions produce an apparent attraction given by

$$F_d = \frac{2\pi^2 (r_x)^2 (k\lambda)^2}{3\alpha d^2 (x)^3} [B]_m \frac{m_e c}{t_e} \quad (18)$$

The expression of $G$ in the $(e, m, c)$ system is $G = G_e \left( \frac{e}{m_e} \right)^2$, where $G_e = 2.399998 \times 10^{-43}$.

The attraction between two electrons which are $dl_e$ distant is:

$$F_G = \frac{G_e m_e l_e}{d^2} \quad (19)$$

By equalising (18) and (19) we obtain

$$x^3 = \frac{2\pi^2 (k\lambda)^2 (r_x)^2 [B]_m}{3\alpha G_e} \quad (20)$$

From (17) and (20) we obtain

$$k\lambda r_x = \left[ \frac{1}{2\pi G_e} \right]^{1/2}.$$
where \( k_\lambda r_x \) is the length of the radius of the electron and \( k_\lambda = \frac{l_e}{q_\lambda} \), the number of \( q_\lambda \) by one \( l_e \). The quantum character of \( \left( \frac{e^2}{c^2} \right)^2 = m_e l_e \) requires that if \( m_e = 1 \), \( r_x = N l_e \), where \( N \) is a digit.

For \( N = 1 \) we have

\[
k_\lambda = \left( \frac{1}{2\pi G_e} \right)^{1/2}.
\]

(21)

The Planck length \( L_P = \left( \frac{\hbar G}{c^3} \right)^{1/2} \), expressed in the \((e, m_e, c)\) system is \( L_P = \left( \frac{G_e}{\alpha} \right)^{1/2} l_e \). Therefore \( q_\lambda = l_e / k_\lambda = (2\pi\alpha)^{1/2} L_P = 0.214276 L_P \).

\( L_P \) is bigger than \( q_\lambda \), on the same way that \( h = \frac{2\pi e^2}{\alpha c} \) is bigger than \( \frac{e^2}{c} \). Notwithstanding the real wavelength of the photon of maximum energy is \( q_\lambda \) and not \( L_P \). The quantum of the magnitude \( ML^2 \) is bigger than the easy combination \( e^2 / c \). The quantum \( q_\lambda \) can be expressed as \( q_\lambda = \left( \frac{2\pi G e^2}{c^3} \right)^{1/2} \), which is minor than the easy combination \( \left( \frac{G e^3}{\alpha c^3} \right)^{1/2} \).

IV. CONCLUSIONS

The minimum value of \( r_x \) is 1, and for it: \( q_\lambda = (2\pi\alpha)^{1/2} L_P \) and

\[
k_\lambda = \left( \frac{1}{2\pi G_e} \right)^{1/2} = 8.143375 \times 10^{20}
\]

\[
x = 5.257601 \times 10^{27}
\]

\[
z = \frac{k_\lambda}{x} = 1.548877 \times 10^{-7}
\]

The wavelengths of the most energetic photons observed in cosmic rays are more than \( 10^5 q_\lambda \).

The relation between the force of gravitation on the electrostatic force is:

\[
\frac{G}{e^2} = \frac{1}{2\pi (k_\lambda)^2} = 2.4000001 \times 10^{-43}
\]

References

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[6] Note that Sparnaay’s experiments do not imply that $q_\lambda$ must be the wavelength of the most energetic photon in zero-point radiation.

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