Reliability Range Through Upgraded Operation with Trapezoidal Fuzzy Number

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ABSTRACT
This work presents an application of upgraded arithmetic operations on trapezoidal fuzzy number to extend the reliability from point estimation to interval estimation. A realistic example of an accidental case has been taken to show the desired results. As we know that uncertainty is a disregardable phenomenon and in this situation crisp reliability may not be exact. To get a better hold on this situation, we introduce a method of trapezoidal fuzzy number and upgraded arithmetic operations. In this paper, the reliability of each factor is represented by the trapezoidal fuzzy number and the final reliability is obtained in the form of trapezoidal fuzzy number. Hence, the dejection of fuzzy number is preserved and the crisp region is increased to illustrate the highest membership grade as an interval. So it can help the decision maker to analyse the behaviour of the system and take effective decision accordingly to reduce the chance of mishapening.

1. Introduction

Reliable engineering is one of the most important tasks in designing and improving the technical system. If we calculate the reliability of any system, there are many factors responsible for it. We can say that reliability of any system depends on the number of subsystems arranged in series/parallel/mixed configuration. For calculating the terminal reliability, each subsystem is necessary to consider. However, if there is a crisp reliability at a single point in the system, then it estimates the full degree of satisfaction at only single point, but there exists some influence of uncertainty in estimation. Because the available data are uncertain due to the loss of information or human mistake in handling records and it can’t be ignored for any kind of system; otherwise, it leads to an illusion for the decision-maker. Hence the above situation can be acknowledged in a better way by trapezoidal fuzzy number, containing a specific interval with full membership grade. In this paper, we analyse the reliability range of the fuzzy system, using upgraded operation with Trapezoidal fuzzy number.

Many authors used different techniques to calculate the output of a system in terms of reliability. Abraham [1] used the Boolean function technique to find the probability of communication in a network system. It was concluded that this method was useful for large
networks. The calculation by this method was faster as compared to previous methods. Furuta and Shiraishi [2] attempted to simplify the calculation of structure function i.e. system success or system failure. Using fuzzy min/max operations, the importance of each basic event was examined. Kaufmann and Gupta [3] identified a number of different activities in the discipline of reliability engineering encompasses, in which reliability modelling vitality was the most important one. It is obvious that in real life generous knowledge is fuzzy rather than actual. In the last many years, enough effort in designing and evolution of reliable systems was made by engineers such that they remain in a working state. It is used to design broad systems for military application, power distribution, space exploration, aircrafts etc. However, fuzzy deals with an uncertainty in the data. Many researchers computed reliability with a simplified technique using fuzzy number arithmetic operations by expressing parameters with different fuzzy numbers. Singer [4] used AND, OR and NEG operators to construct a possibilistic fault tree. A real-life safety problem was taken to illustrate the example. He introduced a fuzzy set method for fault tree and reliability analysis, where the relative frequencies of the primary events were treated as fuzzy numbers. In this paper, parameters were defined with their tolerances to overcome the shortcomings of traditional fault tree and reliability approach. It motivates to extend the same real life safety problem with different kinds of fuzzy numbers for more realistic results. Cai et al. [5] did a considerable work for building an efficient theory of reliability established on the probability theory, where the probability of survival of the system was expressed in terms of the statistical information of its subsystems. Binary state assumption and Probability assumption, substitutive inferences in traditional reliability theory were presented. They identified different types of fuzzy reliability theories such as PROFUST and PROBST. Mishra [6] proposed the concepts of organisation and analysis of the data, fault-tree analysis reliability modelling and its assessment methods, under environmental conditions. Chen [7] analysed the behaviour of the fuzzy system using fuzzy arithmetic operations on triangular fuzzy number and it is very simplified technique to approach the same results by Singer [4] and Cheng and Mon [8] represented each factor by triangular fuzzy number. Cheng and Mon [8] gave the interval of confidence approach for fuzzy system reliability study. They proposed the use of fuzzy number over the interval of confidence in place of probability. They introduced the use of possibility theory for calculating the reliability in range value or interval value. However, extending the algebraic operations of fuzzy numbers techniques presented by Singer [4] and Cheng and Mon [8] demands a large amount of computation time due to the interval arithmetic operations of fuzzy numbers. Mon and Cheng [9] computed the fuzzy reliability of the system for its components with different membership grades. The authors used the fuzzy distribution functions in place of classical probability. Chen [10] calculated the reliability of weapon systems by fuzzy arithmetic operations and also used the defuzzification method to find the crisp value for the fuzzy result. Chang et al. [11] proposed the vague fault tree method to compute fault interval and vague reliability of the components of the automatic gun system. Zio [12] stated the role of the reliability in scientific era. It is rising in both qualitative and quantitative manner in this growing world. The main objective of this paper was to focus on the addition of reliability to the safety of the system and risk analysis. Zimmermann [13] introduced the various arithmetic operations on fuzzy numbers, which can be used for various applications in different fields. On the basis of Zadeh’s extension principle method, interval method and vertex method, Banerjee and Roy studied main arithmetic operations on gen-
eralised trapezoidal fuzzy numbers and the defuzzification method to convert fuzzy value to crisp value. Sharma and Sharma [14] focused on system reliability analysts along with managers/engineers/practitioners performing system RAM analysis using both the qualitative (RCA) and quantitative (Markov) approaches, which can help them model, evaluate and forecast the actions of production plants in a more practical and systematic way. Dhiman and Kumar [15] calculated the reliability of the thermal power plant and sensitivity analysis to find out the most sensitive component of the system. Dhiman and Kumar [16] analysed the performance of a repairable industrial system under genuine human errors. It showed that human (experienced or inexperienced) is also a major part of the structure. Deli [17] proposed a recent and efficient technique TOPSIS (Technique for order preference by similarity to ideal solution), of generalised trapezoidal hesitant fuzzy numbers (GTHF-numbers) used to select an appropriate robot among the alternative robots in an auto company. By using several distance measures, it becomes an easy task for the decision-makers. Deli and Keles [18] defined Intuitionary trapezoidal fuzzy multi-numbers (ITFM-numbers) for multiple criteria decision-making problems and some ITFM-number aggregation operators were developed. The ranking of the alternative was based on the alternative with regard to the positive ideal solution. The efficacy of the method was demonstrated by a numerical example. Deli and Karaaslan [19] used generalised hesitant trapezoidal fuzzy number to solve real-life decision-making problem in which membership degrees are expressed by different possible ways of trapezoidal fuzzy numbers. Deli [20] showed the use of generalised trapezoidal hesitant fuzzy numbers in the case of decision-making process. He applied the methodologies based on Bonferromi aggregation operators under generalised trapezoidal hesitant fuzzy surroundings. Then he compared these approaches with existing approaches. Deli and Karaaslan introduced the idea of generalised trapezoidal hesitant fuzzy number, in which membership grade is expressed by different ways of generalised trapezoidal fuzzy numbers. It was concluded that this method is more adequate to deal with real-life situations.

2. Preliminaries

Fuzzy Sets Theory was originated by Zadeh [21]. It is an extension/generalisation of classical/crisp sets. The major property of the fuzzy set is that it allows the partial membership. In the following section, we briefly review some definitions of fuzzy sets.

Definition 2.1 (Fuzzy Set): Let $X$ be the universe of discourse, $X = \{x_1, x_2, x_3, \ldots, x_n\}$, and a Fuzzy Set $\tilde{A}$ is a set of ordered pairs $\hat{A} = \{(x_1, f_{\tilde{A}}(x_1)), (x_2, f_{\tilde{A}}(x_2)), (x_3, f_{\tilde{A}}(x_3)), \ldots, (x_n, f_{\tilde{A}}(x_n))\}$, where $f_{\tilde{A}}$ is defined as $f_{\tilde{A}} : X \rightarrow [0, 1]$, is the membership function of $X$ in $\tilde{A}$, $f_{\tilde{A}}(x_i)$ indicates the grade of membership of $x_i$ in $\tilde{A}$.

Definition 2.2 (Convex Fuzzy Set): A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is Convex Fuzzy Set iff for all $x_1, x_2$ in $\tilde{A}$, $f_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \text{Min}(f_{\tilde{A}}(x_1), f_{\tilde{A}}(x_2))$ where $\lambda \in [0, 1]$.

Definition 2.3 (Normal Fuzzy Set): A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is Normal Fuzzy Set implying that there exists at least one $x_i \in X$ such that $f_{\tilde{A}}(x_i) = 1$.

Definition 2.4 (Fuzzy Number): A convex, normal membership function on real line $R$ is known as Fuzzy Number.
Definition 2.5 (Trapezoidal fuzzy number): A fuzzy number can be characterised by a trapezoidal distribution function specified by \((a, b, c, d)\), as shown in Figure 1. The membership function of trapezoidal fuzzy number \(\tilde{A}\) is defined as

\[
f_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x < b \\
1, & b \leq x < c \\
\frac{c-x}{c-b}, & c \leq x \leq d \\
0, & \text{otherwise}
\end{cases}
\]

where \(a > b > c > d\).

Definition 2.6 (Alpha-cut (\(\alpha\)-cut)): The \(\alpha\)-cut \((\tilde{A}_\alpha)\) of fuzzy set \(\tilde{A}\) in the universe of discourse \(X\) is defined by \(\tilde{A}_\alpha = [x_i | f_{\tilde{A}}(x_i) \geq \alpha, x_i \in X], \alpha \in [0, 1]\).

For example, Figure 2 shows a fuzzy number \(\tilde{A}\) of the universe of discourse \(X\) which is both convex and normal. Figure 2 shows \(\tilde{A}\) with \(\alpha\)-cuts, where

\[
\tilde{A}_\alpha = [A_\alpha(L), A_\alpha(R)] = [a + \alpha(b - a), d - \alpha(d - c)], \alpha \in [0, 1]
\]

Definition 2.7 (Probability assumption): It is defined as the nature of the system is completely in connection with probability measures.

3. Methodology

Let \(X\) be the Universe of discourse. Let \(\tilde{A}\) and \(\tilde{B}\) be the two fuzzy numbers and their membership functions are given as \(f_{\tilde{A}}\) and \(f_{\tilde{B}}\), respectively where \(f_{\tilde{A}}(x) : X \rightarrow [0, 1]\) and \(f_{\tilde{B}} : X \rightarrow [0, 1]\).
Let $x$ and $y$ be two real numbers in $\tilde{X}$ and $z$ is resultant of $x$ and $y$. The author defined arithmetic operations on fuzzy numbers [5] that are reviewed below:

- **Fuzzy number addition $\oplus$:**
  \[
  f_{A \oplus B}(z) = \bigvee_{z=x+y} (f_A(x) \wedge f_B(y))
  \]

- **Fuzzy number subtraction $\ominus$:**
  \[
  f_{A - B}(z) = \bigvee_{z=x-y} (f_A(x) \wedge f_B(y))
  \]

- **Fuzzy number multiplication $\otimes$:**
  \[
  f_{A \otimes B}(z) = \bigvee_{z=xy} (f_A(x) \wedge f_B(y))
  \]

- **Fuzzy number division $/:$**
  \[
  f_{A/B}(z) = \bigvee_{z=x/y} (f_A(x) \wedge f_B(y))
  \]

Let us consider two trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$. Fuzzy arithmetic operations of the trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are defined above, can be defined as follows:

- **Addition of two fuzzy numbers:**
  \[
  (a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2).
  \]

- **Subtraction of two fuzzy numbers:**
  \[
  (a_1, b_1, c_1, d_1) \ominus (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)
  \]
• Complement of a fuzzy number:
  \[ 1 - (a_1, b_1, c_1, d_1) = (1, 1, 1, 1) - (a, b_1, c_1, d_1) \]
  \[ = (1 - d_1, 1 - c_1, 1 - b_1, 1 - a_1) \]

• Multiplication of two fuzzy numbers:
  \[(a_1, b_1, c_1, d_1) \otimes (a_2, b_2, c_2, d_2) = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2)\]

• Division of two fuzzy numbers:
  \[(a_1, b_1, c_1, d_1) / (a_2, b_2, c_2, d_2) = (a_1 / a_2, b_1 / c_2, c_1 / b_2, d_1 / a_2)\]

4. A Technical Example

To show the application of trapezoidal fuzzy number, we consider a problem from the paper [2]. It is a system in which two grinding machines are working next to each other. In this, we have to find the probability that the people, who are coming in contact of these operating machines, are mainly injured by getting a chip into eye. The major reason to get injured is that the person obliged to wear safety glasses. Moreover, threatened persons are coming in contact of the working machines, entering without any reason and those persons who are bringing and carrying away the goods.

The main factors responsible for the injury of someone is described clearly with the help of fault tree in Figure 3.

The general factors contributing to the mishappening or injury are given in Table 1. It is clear that all the factors are mutually independent and reliability of the each basic factor
\[
\tilde{R}_i \text{ is characterised by trapezoidal fuzzy number } R_i. \text{ By keeping the uncertainty as an important measure, the reliability in this paper is characterised by trapezoidal fuzzy numbers symmetrised by } R_i = ((m_i - \alpha_i) - \delta_i, m_i - \alpha_i, m_i + \beta_i, (m_i + \beta_i) + \gamma_i). \text{ Here } m_i \text{ is the maximal value for which the value membership function is 1. Here } \alpha_i \text{ and } \beta_i \text{ are the left and right spread of } m_i, \text{ respectively. To extend the interval for } m_i, \text{ let us take } \alpha_i + \delta_i \text{ which is the left spread of } m_i - \alpha_i \text{ and } \beta_i + \gamma_i \text{ is the right spread of } m_i + \beta_i. \text{ According to this, trapezoidal fuzzy number is given by Figure 4. The values of spread vary according to the analyst demand/suggestion. Here the spread is taken 20\% and we can calculate the reliability of each factor accordingly as a trapezoidal fuzzy number using Table 1.}
\]

**Table 1.** The basic events contributing to the accident.

| Symbol | Basic event                                      | \( m_i \) | \( m_i - \alpha_i \) | \( m_i + \beta_i \) | \( (m_i - \alpha_i) - \delta_i \) | \( (m_i + \beta_i) + \gamma_i \) |
|--------|------------------------------------------------|-----------|---------------------|---------------------|---------------------------------|---------------------------------|
| A      | Operator 1 fails to wear safety glasses         | 0.02      | 0.016               | 0.024               | 0.0128                          | 0.0288                          |
| B      | Operator 2 fails to wear safety glasses         | 0.02      | 0.016               | 0.024               | 0.0128                          | 0.0288                          |
| C      | Machine 1 is operating                          | 0.80      | 0.640               | 0.960               | 0.5120                          | 1.1520                          |
| D      | Machine 2 is operating                          | 0.80      | 0.640               | 0.960               | 0.5120                          | 1.1520                          |
| E      | Persons entering the area without safety glasses| 1.00      | 0.800               | 1.200               | 0.6400                          | 1.4400                          |
| F      | Persons entering the endangered area            | 0.05      | 0.040               | 0.060               | 0.0320                          | 0.0720                          |
| G      | Persons entering the area carrying away made    | 0.05      | 0.040               | 0.060               | 0.0320                          | 0.0720                          |
|        | product                                         |           |                     |                     |                                 |                                 |
| H      | Persons entering the area for other reasons     | 0.01      | 0.008               | 0.012               | 0.0064                          | 0.0144                          |

5. **Study of Fuzzy System Reliability**

Consider a serial system in Figure 5. All the components are arranged in a series. This is called serial configuration. In this type, all the factors are major. If anyone fails, the complete system stops working. For getting output, all the factors need to perform their functions.
calculated as

\[
\alpha_fuzzy 
\]

Moreover, on the other hand, a parallel system is described in Figure 6. Here trapezoidal fuzzy number \( R_i \) indicates the reliability of each factor \( c_i \), where \( R_i = ((m_i - \alpha_i) - \delta_i, m_i - \alpha_i, m_i + \beta_i, (m_i + \beta_i) + \gamma_i) \) and \( 1 \leq i \leq n \). According to this, the membership function of trapezoidal fuzzy number is shown in Figure 4, which indicates \( \alpha_i, \delta_i \) and \( \beta_i, \gamma_i \) are the left and right spreads, respectively.

The reliability of the system in which factors are in series in Figure 5 can be calculated as

\[
\tilde{R}_1 \otimes \tilde{R}_2 \otimes \ldots \otimes \tilde{R}_n \\
= ((m_1 - \alpha_1) - \delta_1, m_1 - \alpha_1, m_1 + \beta_1, (m_1 + \beta_1) + \gamma_1) \\
\otimes ((m_2 - \alpha_2) - \delta_2, m_2 - \alpha_2, m_2 + \beta_2, (m_2 + \beta_2) + \gamma_2) \\
\otimes \ldots \otimes ((m_n - \alpha_n) - \delta_n, m_n - \alpha_n, m_n + \beta_n, (m_n + \beta_n) + \gamma_n) \\
= \left( \prod_{i=1}^{n} (m_i - \alpha_i) - \delta_i, \prod_{i=1}^{n} (m_i - \alpha_i), \prod_{i=1}^{n} (m_i - \alpha_i), \prod_{i=1}^{n} (m_i + \beta_i) + \gamma_i \right)
\]

Moreover, on the other hand, a parallel system is described in Figure 6. Here trapezoidal fuzzy number \( R_i \) indicates the reliability of each factor \( c_i \), where \( R_i = ((m_i - \alpha_i) - \delta_i, m_i - \alpha_i, m_i + \beta_i, (m_i + \beta_i) + \gamma_i) \) and \( 1 \leq i \leq n \). Then the reliability of the parallel system can be calculated as

\[
1 - \prod_{i=1}^{n} (1 - \tilde{R}_i) = 1 - \prod_{i=1}^{n} (1 - ((m_i - \alpha_i) - \delta_i, (m_i - \alpha_i), (m_i + \beta_i), (m_i + \beta_i) + \gamma_i)) \\
= 1 - \prod_{i=1}^{n} (1 - ((m_i + \beta_i) + \gamma_i), 1 - (m_i + \beta_i), 1 - (m_i - \alpha_i), 1 - ((m_i - \alpha_i) - \delta_i)) \\
= 1 - (1 - ((m_1 + \beta_1) + \gamma_1), 1 - (m_1 + \beta_1), 1 - (m_1 - \alpha_1), 1 - ((m_1 - \alpha_1) - \delta_1)) \\
\otimes (1 - ((m_2 + \beta_2) + \gamma_2), 1 - (m_2 + \beta_2), 1 - (m_2 - \alpha_2), 1 - ((m_2 - \alpha_2) - \delta_2)) \\
\otimes \ldots \otimes (1 - ((m_n + \beta_n) + \gamma_n), 1 - (m_n + \beta_n), 1 - (m_n - \alpha_n), 1 - ((m_n - \alpha_n) - \delta_n))
\]

### 6. Mathematical Computation

\[
\tilde{R}_A = (0.0128, 0.0160, 0.0240, 0.0288) \quad \tilde{R}_E = (0.6400, 0.8000, 1.2000, 1.4400) \\
\tilde{R}_B = (0.0128, 0.0160, 0.0240, 0.0288) \quad \tilde{R}_F = (0.0320, 0.0400, 0.0600, 0.0720) \\
\tilde{R}_C = (0.5120, 0.6400, 0.9600, 0.1520) \quad \tilde{R}_G = (0.0320, 0.0400, 0.0600, 0.0720) \\
\tilde{R}_D = (0.5120, 0.6400, 0.9600, 0.1520) \quad \tilde{R}_H = (0.0064, 0.0080, 0.0120, 0.0144)
\]
Figure 6. Components in a parallel system.

\[
\begin{align*}
U &= F \oplus G \oplus H \\
V &= C \oplus D \\
Z &= U \otimes V \otimes E \\
X &= A \oplus B \oplus Z
\end{align*}
\]

Then by applying the Reliability formula for AND-OR combination from section 5, we can get

\[
\hat{R}_U = 1 - ((1 - \hat{R}_F) \otimes (1 - \hat{R}_G) \otimes (1 - \hat{R}_H)) \\
= 1 - ((1 - (0.03200, 0.04000, 0.06000, 0.07200)) \otimes (1 - (0.032000, 0.04000, 0.06000, 0.07200))) \otimes (1 - (0.00640, 0.00800, 0.01200, 0.01440))) \\
= 1 - ((0.92800, 0.94000, 0.96000, 0.96800) \otimes (0.92800, 0.94000, 0.96000, 0.96800)) \otimes (0.98560, 0.98800, 0.99200, 0.99360)) \\
= 1 - (0.84878, 0.87300, 0.91423, 0.93103) \\
= (0.06897, 0.08577, 0.12700, 0.15122)
\]

\[
\hat{R}_V = 1 - ((1 - \hat{R}_C) \otimes (1 - \hat{R}_D)) \\
= 1 - ((1 - (0.51200, 0.64000, 0.96000, 1.15200)) \otimes (1 - (1 - (0.51200, 0.64000, 0.96000, 1.15200)))) \\
= 1 - ((-0.15200, 0.04000, 0.36000, 0.48800) \otimes (-0.15200, 0.04000, 0.36000, 0.48800)) \\
= 1 - (-0.02310, 0.00160, 0.12960, 0.23814) \\
= (0.76186, 0.87040, 0.99840, 1.02310)
\]

\[
\hat{R}_Z = \hat{R}_U \otimes \hat{R}_V \otimes \hat{R}_E \\
= (0.06897, 0.08577, 0.12700, 0.15122) \otimes (0.6400, 0.8000, 1.2000, 1.4400)
\]
Figure 7. Resultant reliability of mishappening.

\[ \otimes (0.6400, 0.8000, 1.2000, 1.4400) \]
\[ = (0.03363, 0.05972, 0.15216, 0.22279) \]

\[ \tilde{R}_X = 1 - (1 - (0.01280, 0.01600, 0.02400, 0.02880)) \]
\[ \otimes (1 - (0.03363, 0.05972, 0.15216, 0.22279))) \]
\[ = 1 - ((0.97120, 0.97600, 0.98400, 0.98720) \otimes (0.97120, 0.97600, 0.98400, 0.98720) \]
\[ \otimes (0.77721, 0.84784, 0.94028, 0.96637)) \]
\[ = 1 - (0.73309, 0.80763, 0.91043, 0.94179) \]
\[ = (0.05821, 0.08957, 0.19237, 0.26690) \]

7. Result Discussion

By using the proposed methodology, we computed the reliability of happening of an accident. Here we find the reliability of occurring of an accident as an interval for which the membership grade is 1. As a result of this, the dejection of fuzzy number is preserved and hence the crisp region increases to emphasise the highest membership grade at an interval \([0.08957, 0.19237]\). This is the required interval for which the membership grade of reliability is highest as given in Figure 7.

8. Conclusion

In this paper, we have shown the use of trapezoidal fuzzy number and their arithmetic operations in a real-life safety problem to extend the reliability from point estimation to interval estimation. As we stated that uncertainty, which is a disregardable phenomenon and the
crisp reliability may not be exact in a real-life situation. To overcome from this situation, we use upgraded arithmetic operations in which the reliability of each factor is processed by trapezoidal fuzzy number. By using fault tree, the arithmetic relation is obtained and hence arithmetic operations are applied. Hence, final reliability is calculated. We conclude that the dejection of fuzzy number is preserved and the crisp region is increased to illustrate the greatest value of membership function in a specific interval. It provides the resultant reliability of the happening of an accident in the range/interval. It describes that there are certain probability ranges $[0.08957, 0.19237]$ . So it can help out the analyst to analyse the reasons for accident and take effective decision to reduce the chance of mishappening.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

**Notes on Contributors**

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