A Study on the Detection Method of Dark Matter using Gravitational Waves

Zheng Li1, Chenyu Yang2, Xinen Zhou3, *  
1Chongqing No.1 International Studies School, Chongqing, China  
2Beijing National Day School, Beijing, China  
3Department of Physics & Astronomy, University of California, Irvine, California, United States  

*Corresponding author: xinenz@uci.edu

Abstract. Dark matter is a type of invisible matter that analytically exists in the universe. Nowadays, scholars have yet detected it and confirmed its presence experimentally. Einstein predicted gravitational waves based on his general theory of relativity. In 2015, the Laser Interferometer Gravitational-Wave Observatory (LIGO) first detected the gravitational wave. This paper reviews the background of dark matter and gravitational waves and introduces the method of detecting dark matter with gravitational waves. Moreover, the feasibility of the scenario has been verified based on information retrieval and theoretical analysis. These results shed light on the future detection schemes of dark matter detection.

Keywords: Dark matter, gravitational waves, LIGO, Hubble constant

1. Introduction

Dark matter is a hypothetical form of matter that accounts for about 85% of the universe today. The detection of this unknown matter is one of the most popular topics in science today. Among all the models, the ΛCDM model is used the most since it can capture the features of the large universe. However, there're still many discrepancies in this model. In the paper, we introduce some of the existing problems in this model. Many methods have been used to detect dark matter, including the particle collision in the Large Hadron Collider (LHC), the space telescope detection, etc.

In 2015, the Laser Interferometer Gravitational-Wave Observatory (LIGO) made the first direct observation of the gravitational wave produced by the disturbance in the curvature of spacetime and generated by accelerated mass. Einstein first predicted it based on his general theory of relativity, published in 1915 [1]. General relativity interprets the gravitational field as the curvature of spacetime. The basic formula of general relativity is Einstein's field equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$  (1)
where $R_{\mu\nu}$ is Ricci tensor, $R$ is curvature scalar, $g_{\mu\nu}$ is metric tensor, $T_{\mu\nu}$ is an energy-momentum tensor, $c$ is the speed of light, and $G$ is the gravitational constant. In addition, gravitational waves can be a way of finding dark matter.

In this article, we will review some of the ongoing problems with the dark matter and the approach of detection. Subsequently, the basic descriptions of gravitational waves and the issues facing nowadays will be described. Then some basic knowledge about how gravitational waves will be introduced, finding dark matter with contemporary technology. Afterward, the detection of dark matter based on the gravitational wave will be discussed and evaluated. All the methods can be categorized into two types: direct detection and indirect detection. For direct detection, people believe that weakly interacting massive particles (WIMPs) are consisting dark matter. There're many methods in indirect detection, including gravitational lensing, cosmological simulations, etc. In this article, we will introduce the method of gravitational waves.

2. Basic descriptions of dark matter

One of the early propositions of Dark Matter was in the 1930s by the Dutch astronomer Jan Oort [2]. Based on observing the nearby galaxies and calculating their stars' moving velocity, he found that the galaxy was in clusters rather than apart. It is concluded that there must be mass in the galaxy to keep the nearby stars from escaping. However, the matter was unknown. Later in the 1970s, the Galactic Rotation Curves further proved the existence of Dark Matter. At that time, scientists compared the velocity and the orbital radius of the stars when they observed other galaxies. Surprisingly, as the orbital radius increases, the linear velocity of the planets does not drop at all. Instead, it becomes a flat line indicating that the velocities of the planets remain constant, which is clearly against Newton's law of gravity. The only explanation is that there is an "extra" mass around the galaxy. The "extra" mass we cannot detect by light or any electromagnetic waves. Therefore, the 1980s was the time most scientists accept the existence of dark matter.

Moreover, gravitational lensing also indicates the existence of dark matter. In 1997, the Hubble Space Telescope captured an image that had an unexpected bend of light by the cluster in the image's foreground. The mass of the cluster could be 250 times the visible part.

![Figure 1](image-url)

**Figure 1.** The rotation curve is flat rather than decreasing toward the righthand side [2]

As for the composition of dark matter, there are different forms of dark matter being predicted today, e.g., hot dark matter, cold dark matter, warm dark matter, and self-interacting dark matter. Hot dark matter consists of particles with ultra-relativistic velocity. In contrast, cold dark matter is composed of particles with slow speed. Self-interacting dark matter is an alternative to cold dark matter. Some researchers also proposed warm dark matter, whose properties are intermediate between hot dark matter and cold dark matter.
In general, the form that's been studied the most is cold dark matter. Since contemporary studies of dark matter are mainly based on the Lambda cold dark matter model (ΛCDM). The ΛCDM model is probably the best model to describe the large structure of the universe and big bang cosmology. This is attributed to it simply contains cold dark matter and baryonic matter. Therefore, after being proposed in 2000, the ΛCDM model is considered the standard cosmological model. However, since the 1960s, there have been four discrepancies on cold dark matter.

The first is the core-cusp problem. Nowadays, scientists have had different simulations about Cold Dark Matter. According to simulations, the density of cold dark matter should be increasing as it is closer to the center. However, the observation of disk galaxies shows that the density almost stays the same throughout the cold dark matter halos. The other discrepancy is the Missing Satellites Problem. Compared to the Local Group, the number of stars in cold dark matter halos is tiny. For example, numerical predicts that there should be hundreds of dark matter subhalos in the Milky Way. However, there exist only 11 dwarf satellites galaxies. Therefore, scientists think that satellites are missing around cold dark matter halos.

The other problem is Too-Big-to-Fail Problem. Scientists expected those observed satellites to form the largest subhalos given the Missing Satellites’ small number of milky way satellites. However, the central density of the large subhalos is too dense to hold those satellites. To solve the core-cusp problem and the missing satellites problem above, Spergel & Steinhardt proposed an alternative of cold dark matter, called self-interacting dark matter. Compared to cold dark matter, its advantage includes reduced central density, which directly aims at the core-cusp problem. Generally, self-interaction dark matter is considered an alternative since it has the large-scale structure formation of ΛCDM but also affecting structure at late times and only on small scales in the dense inner regions of halos [2].

Detection of dark matter is also a popular topic. There are several ways of detection, such as observation by telescopes and particle detection in the Large Hadron Collider (LHC). Because dark matter is also frequently related to supersymmetry, finding supersymmetric particles becomes an interest of scientists in the LHC. The use of telescopes is another significant way of finding dark matter. The detection of gamma rays and gravitational lensing are crucial to the detection of dark matter as well. Moreover, since gravitational waves were directly detected in 2015, it has been considered a good way of finding the activities of cosmological objects. Therefore, in this paper, gravitational waves are being discussed as a way to detect dark matter.

Figure 2. Subhalos within the Milky Way and Virgo cluster in ΛCDM simulations are much less than the observed Milky Way satellites and galaxies in the Virgo cluster [3].
3. The gravitational wave sources and detection schemes

The gravitational sources are divided into four categories which are compact binary coalescence, continuous, burst, and stochastic sources. In the case of the gravitational source, the wave equation should include the source term:

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

The formal solution is:

$$h_{\mu\nu} = \frac{4G}{c^4} \int T_{\mu\nu}(t - \frac{|x - x'|}{c}, x') d^3x' \approx \frac{4G}{rc^4} \int T_{\mu\nu}(t - \frac{|x - x'|}{c}, x') d^3x'$$

where \( r \) is the distance of the observer from the source, which is greater than the size of the source. A covariant derivative can be written as:

$$\partial \tau^{tt} + \partial \tau^{kt} \partial x^k = 0,$$

because the conservation equation equals zero. This can be differentiated another time with respect to time to get the equation:

$$\partial^2 \tau^{tt} \partial t^2 = \partial^2 \tau^{kl} \partial x^k \partial x^l,$$

where \( \partial_k T^{tt} + \partial_i T^{lk} = 0 \) for \( \nu = k \) is used in this equation. It is shown below that the strain amplitude can be expressed by multiplying \( x'/x^k \) by the above equation and integrating with respect to the spatial volume:

$$\bar{h}_{jk} = \frac{2G}{rc^4} \frac{d^2 Q_{jk}(t-r)}{dt^2}.$$  

We calculate the second time derivative of the quadrupole moment (i.e., quadrupole kinetic energy):

$$Q_{jk} \sim \frac{(mass)}{(size)^2} \frac{(transit time)^2}{(transit time)^2} = \epsilon M c^2,$$

\( \epsilon \) is much smaller than 1 in most situations. Therefore, the gravitational waves are weak. However, it could become almost 0.1 in some cases. It can be estimated for sources with mass \( M \) at distance \( r \) and amplitude \( h \):

$$h_{jk} \sim 10^{-22} \left( \frac{\epsilon}{M/M_\odot} \frac{M}{M_\odot} \frac{100Mpc}{r} \right).$$

This equation can show that the best sources of gravitational waves that can be detected are black holes and neutron stars. The reason is that these objects are very compact to make \( \epsilon \) be 0.1. Furthermore, only a binary system of black holes and neutron stars can produce the gravitational waves that are detectable now [4].
In the 1960s, Joseph Weber, a physicist at the University of Maryland, tried to detect gravitational waves using resonance. He built a resonance-type gravitational wave detector. The bar detector consists of multiple layers of aluminum tubes, one meter in diameter and two meters in length, with a mass of about 1,000 kilograms, and is suspended by a filament. When gravitational waves pass through the cylinder, it resonates, which can be detected by piezoelectric sensors attached to the cylinder. Weber once placed the same bar detectors at two locations 1,000 kilometers apart. Only when the detectors detected the same signal at the same time was it recorded. In 1969, Weber announced that he had detected gravitational waves [6]. This is causing an immediate stir in the scientific community, but subsequent repeated experiments have found nothing.

In the 1970s, Rainer Weiss, a physicist at MIT, began thinking about using laser interferometry to detect gravitational waves. But detecting gravitational waves requires a very sensitive instrument that can sense a change of 10^{-18} meters at a distance of 1,000 meters, equivalent to one thousandth of the diameter of a proton. It was not until the 1990s that the technical conditions required for such high sensitivity gradually matured. In 1991, THE Massachusetts Institute of Technology (MIT) and the California Institute of Technology (Caltech) began joint construction of the Laser Interferometer Gravitational-wave Observatory (LIGO) with funding from the National Science Foundation (NSF). The advanced LIGO installation was completed in 2014. The gravitational waves were first detected by Laser Interferometer Gravitational-wave Observatory (LIGO) on September 14, 2015.
The basic principle of a Laser interferometer is to compare the time it takes for light to pass between its two perpendicular multiple pass arms. When a gravitational wave is an incident in the plane perpendicular to the interferometer, it distorts space in the form of a quadrupole moment because of its polarization. That is, stretching space in one direction at the frequency of the gravitational wave, squeezing space in the direction perpendicular to it, and vice versa. In the case of a laser interferometer, gravitational waves naturally stretch or compress the part of space where the interferometer's perpendicular arms are. The effects of gravitational waves can be detected by comparing the time it takes for light to pass between perpendicular arms.

4. Detection of Dark matter based on gravitational wave

Dark matter detection can be divided into direct detection, indirect detection, and accelerator detection. Direct detection involves weakly interacting massive particles (WIMP), the particles now widely accepted for consisting of dark matter. When WIMP collides with nuclei in the detector and scatters them, the heat and light emitted during interaction can be detected.

Nevertheless, the problem occurs where the ratio is not fixed so that we cannot get the strength of the weak interaction [8]. Direct observation, though simple in theory, is troublesome in practice. First, the interactions between WIMPs and ordinary matter are so weak that they have little chance of being captured. Second, the cost of locating deep underground laboratories to exclude the effect from cosmic microwave background radiation, such as the Sasso National Laboratory in Italy, has increased. The lack of conclusive evidence for the existence of dark matter particles limits their mass and the intensity of their interactions.

Figure 5. Evolution of GW detectors in Japan.

We compare the measured sensitivity curves obtained from TAMA300(2018) and CLIO (2010) with the design sensitivity of bKAGRA, which will be accomplished in 2022 in the current plan. The sensitivity curves of LIGO during the science run in 2009-2010 and of Advanced LIGO (aLIGO)/Advanced Virgo (AdV) designs are also shown [9].

Then, more feasible schemes would be indirect detection, in which dark matter can be detected by interfering with the light and gravity of nearby objects or by the signals produced by their annihilation decay, such as a gravitational wave. In the universe, sources of gravitational waves large enough to be detected are usually the following types, interacting compact binary star systems, rapidly rotating compact objects, random backgrounds of gravitational waves, supernovae, and gamma-ray bursts, and massive black holes. As mentioned above, the best sources for gravitational wave detection are black
holes and neutron stars that emit strong gravitational fluctuations due to their high density, so gravitational waves from dark matter are relatively difficult to detect.

So far, KAGRA, a new GW interferometer detector that effectively helps pinpoint gravitational wave sources and provides high precision parameters, has made the appeal scheme more practical. The design of KAGRA is more advanced than that of LIGO and Virgo, which firstly detected gravitational waves. It will use the sapphire mirror at low temperature, an important property that helps improve accuracy to about 100Hz, and is currently being built into the 2.5 generation of gravitational wave detectors. It can be seen from Fig. 5 that the sensitivity curve of KAGRA is significantly improved, which makes the location of GW more accurate. In 2019, with the formation of the space GW detection network by the Taiji probe launched by China along with LISA, it became possible to obtain the Hubble Constant accurate to the thousandth place within ten years.

In obtaining the Hubble constant with the LISA-Taiji network, the distribution density of a counterpart galaxy can be expressed as:

\[
\frac{dN}{dRd\Omega} = R \exp \left[ \left( \frac{R}{R_*} \right)^{-4} \right]
\]  

(9)

with R as the co-moving distance and \( R_* \) as the Hubble distance. If we only consider a small redshift such as \( z < 1 \), the exponential part of Eq. (9) will always be \( \sim 1 \). The projected number density \( dD/d\Omega \) is \( \sim 300 \) galaxies given by the Hubble Deep Field. We can normalize Eq. (9) by integrating it into the projected number density, which should be \( < 300 \) galaxies. Then, we have:

\[
\frac{dN}{dRd\Omega} < \frac{600R}{R_0^2} \text{galaxies}
\]

(10)

where \( R_0 \) is the distance at \( z = 1 \) [10]

5. Conclusion

In summary, the paper mainly summarizes recent progress on detecting dark matter via gravitational waves. We first introduce the basic information about dark matter and gravitational waves. Subsequently, this paper introduces a method to detect dark matter using gravitational waves. According to our analysis, it is a feasible path to detect dark matter. If gravitational waves from dark matter can be detected, it will be possible to better understand dark matter’s nature. This result offers a guideline for detecting dark matter and pave a roadmap to exploring the universe.

Reference

[1] Einstein A. Zur allgemeinen Relativitaetstheorie. Sitzungsber K Preuss Akad Wiss, 1915, 1: 778 786, 7
[2] M. White: https://w.astro.berkeley.edu/~mwhite/darkmatter/rotcurve.html
[3] S. Tulin and H. Yu, arXiv1705.02358
[4] Lee, H. M. . (2018). Long journey toward the detection of gravitational waves and new era of gravitational wave astrophysics. Journal of the Korean Physical Society, 73(6), 684-700.
[5] https://cdn.mos.cms.futurecdn.net/g9S9gKUKAjRDcqdVj2jPUL-1024-80.jpg.webp
[6] Weber, & J. (1969). Evidence for discovery of gravitational radiation. Physical Review Letters, 22(24), 1320-1324.
[7] Muehliner, D. J. , & Weiss, R. . (1972). Gravitation Research.
[8] https://www.elsevier.com/open-access/userlicense/1.0/
[9] https://www.researchgate.net/publication/330480470
[10] The LIGO Scientific Collaboration. Advanced LIGO. Class. Quantum Gravity 32.074001(2015)