DARK MATTER AND SOLAR NEUTRINOS IN SUSY†

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ABSTRACT

In the Supersymmetric extension of the Standard Model with minimal particle content the three neutrinos can have non trivial masses and mixings, generated at 1 loop due to renormalizable lepton number violating interactions. We show that the resulting mass matrix can provide simultaneously a significant amount of the Dark Matter of the Universe and solve the solar neutrino problem, if the free parameters of the model are fixed to values which are consistent with all the present accelerator and cosmological constraints. The theory also predicts new effects in future experiments looking for neutrino oscillations.

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Cosmological and astrophysical considerations strongly suggest that neutrinos are massive particles, in spite of the present lack of any accelerator evidence.

First, neutrinos are the only known particles which can contribute significantly to the Dark Matter (DM) of the Universe if the sum of their masses is in the $\sim 10$ eV range. Recent indications\(^1\) suggest that hot DM, such as neutrino DM, should compose the $\sim 30\%$ of the Universe, corresponding to $\sum \nu m_\nu \simeq 7$ eV.

Second, the experimental measurements of the flux of the neutrinos from the sun imply\(^2\) non trivial mass matrices\(^3\) and/or nonstandard interactions\(^4\) for the neutrinos. The simplest and most appealing solution to this “solar neutrino problem” is provided by the Mikheyev–Smirnov–Wolfenstein (MSW)\(^3\) scenario, which describes all the observations assuming that the electron neutrino $\nu_e$ is resonantly converted into another state such as $\nu_\mu$ or $\nu_\tau$, resulting in a reduction of the observed flux in the earth. The allowed region for the squared mass difference and mixing angle between the two states is

$$\Delta m^2 = (0.5 - 1.2) \times 10^{-5} \text{eV}^2, \quad \sin^2 2\theta = (0.3 - 1.0) \times 10^{-2},$$

(1)

in the case of small vacuum mixing\(^5\).

In the Standard Model (SM) of Particle Physics, neutrinos are strictly massless as a consequence of the gauge symmetry and particle content of the theory. In extensions of the SM, neutrinos can acquire a mass due to the presence of sterile neutrinos, and in this case the smallness of the mass of the known neutrino states can be elegantly explained by the see-saw mechanism\(^6\). The resulting mass matrix among the light states can then account both for the hot Dark Matter (e.g. with $m_\nu \sim 10$ eV) and for the MSW solution of the solar neutrino problem (e.g. by $\nu_e - \nu_\mu$ oscillations)\(^7\).

In some very popular extensions of the SM, such as the Supersymmetric SM (SSM) with minimal particle content and the simplest SUSY-SU(5) theory, no nonstandard neutral fermion is present, so that the see-saw mechanism cannot be implemented. However, in the supersymmetric case the SU(2)$\times$U(1) gauge symmetry and the minimal particle content allow for renormalizable lepton number violating operators\(^8\), in contrast to the non supersymmetric case. The three left handed neutrino flavours acquire then a non trivial mass matrix, and the heaviest state can be naturally in the $\sim 10$ eV range\(^9\). We will study here the possibility that this happens while the two lighter components have masses and mixing such as to account also for the MSW mechanism, in the framework of the SSM with minimal particle content, making just a minimum number of naturalness assumptions in order to minimize the number of free parameters. The result is that it is possible
to solve simultaneously the DM and solar neutrino problems just by supersymmetrizing the SM and allowing for the most general superpotential, which involve lepton number violating interactions, without contradicting any accelerator or cosmological constraint, and this solution imply predictions which will be tested in the future experiments looking for neutrino oscillations.

In the Supersymmetric Standard Model (SSM) with minimal particle content, lepton number violation can arise in the superpotential through renormalizable terms, involving either two lepton and an antilepton or quark–antiquark–lepton, chiral (“left–handed”) superfields. Fig. 1 shows the generic one loop diagrams that contribute to the mass entry \( m_{\nu_i\nu_j} \) \((i, j, k = 1, \ldots, 3\) are the flavour indices). The fermionic \( f_k \) and scalar \( \tilde{f}_k \) internal lines can be either charged ±1 lepton–slepton pairs, or charged ±1/3 quark–squark pairs. Diagrams (a) and (b) must be considered together, and summed, because they are proportional to the same product of lepton number violating coupling constants; for the diagonal matrix elements \((i = j)\), if the two flavors running in the loop are the same \((k = k')\), the two diagrams coincide, so that just one should be considered.

In the diagram of fig. 1 an helicity flip on the internal fermion line is necessary. As explicitly indicated, this also requires a mixing of the scalars \( \tilde{f}_k \) and \( \tilde{f}'_k \), described by the “mass insertions” \( \Delta_k \). For instance, in the usual “Minimal” SSM (MSSM), with soft supersymmetry breaking arising from low energy supergravity, every contribution to the mixing \( \Delta_k \) is proportional to the mass \( m_k \) of the fermionic superpartner \( f_k \) of \( \tilde{f}_k \),

\[
\Delta_k = \tilde{m} m_k, \tag{2}
\]

where \( \tilde{m} \) is a typical supersymmetry–breaking mass parameter. For simplicity, we will assume in the following that eq. (2) holds.

The generic single diagram of fig. 1 contributes to \( m_{\nu_i\nu_j} \) as

\[
\delta m_{\nu_i\nu_j} \simeq N_c \frac{\lambda_{i'kk'} \lambda_{j'kk'} m_{k'}}{16\pi^2} \frac{\Delta_k}{m_k^2} \tag{3}
\]

where \( N_c \) is a colour factor \((N_c = 3 \text{ for the diagrams with quark flavors, } N_c = 1 \text{ otherwise})\), \( m_{k'} \) is the mass of the internal charged fermion \( f_{k'} \), and \( \tilde{m}_k^2 \) is the average mass of the two mass eigenstates resulting from the mixing between \( f_k \) and \( \tilde{f}_k \). Of course \( m_{k'}/\tilde{m}_k \ll 1 \) and we are also making an expansion in \( \Delta_k/\tilde{m}_k^2 \). The lepton number breaking coupling constants \( \lambda_{ikh} \) for the purely leptonic vertices, \( \lambda_{ikh} = \lambda'_{ikh} \), are antisymmetric in the indices \( h, k \), and they differ in general from the couplings for the lepton–quark–squark vertices, \( \lambda_{ikh} = \lambda''_{ikh} \). The sets of the \( \lambda' \) and \( \lambda'' \) consist then of 18 and 27 free parameters, respectively. We stress again that there is no compelling a priori reason to put all these coupling constants to zero, assuming the conservation of a symmetry generalizing lepton number in the supersymmetric case,
called R-parity, since this is no longer a result of the gauge symmetry, unlike in the non supersymmetric SM. We will make the reasonable assumption that any hierarchy amongst the coupling constants \( \lambda_{ihk} \) can be justified by an (approximate) flavour symmetry. In particular, we assume that all the coupling constants \( \lambda \) that violate only a single lepton flavour number, say \( L_i = L_e \), are of the same order, their value measuring the amount of breaking of the symmetry \( L_i \). The remaining coupling constants, \( \lambda'_{123}, \lambda'_{132} \) and \( \lambda'_{231} \), arise when all the lepton numbers are broken, so that they are expected to be not larger than any of the others. This implies that for any mass entry \( m_{\nu_{\nu'}} \), there is just one dominating contribution from fig. 1, corresponding to the largest product \( m_k \Delta_k = m_k \tilde{m} m_k \), that is to the exchange of bottom quarks and squarks. The full 3 \times 3 neutrino mass matrix depends then just on three parameters, \( \lambda_e = \lambda'_{133}, \lambda_\mu = \lambda'_{233} \) and \( \lambda_\tau = \lambda'_{133} \), as

\[
\mathbf{m} = \frac{N_c}{16\pi^2} \frac{m^2 \tilde{m}}{\tilde{m}_b^2} \begin{pmatrix}
\lambda_e^2 & 2\lambda_e \lambda_\mu & 2\lambda_e \lambda_\tau \\
2\lambda_e \lambda_\mu & \lambda_\mu^2 & 2\lambda_\mu \lambda_\tau \\
2\lambda_e \lambda_\tau & 2\lambda_\mu \lambda_\tau & \lambda_\tau^2
\end{pmatrix}.
\] (4)

Let us assume that one neutrino state constitutes the hot DM with a mass \( \sim 7 \) eV, and that the remaining two states have a mass difference and mixing angle given by eq. (1) corresponding to the MSW solution of the solar neutrino problem. These properties should result from the diagonalization of the mass matrix \( m \) in eq. (4), so that all the three independent parameters \( \lambda_e \tilde{m}/\tilde{m}_b^2, \lambda_\mu \tilde{m}/\tilde{m}_b^2 \) and \( \lambda_\tau \tilde{m}/\tilde{m}_b^2 \) will be determined. The non trivial fact is that the solution will correspond to experimentally allowed values for the \( \lambda \)'s. Furthermore, once the parameters \( \lambda \)'s are fixed, all the entries of the mass matrix (4) will be determined, and we will have unambiguous predictions for the mixing angles in the two channels that do not correspond to the MSW mixing.

Let us consider the case \( \lambda_\tau > \lambda_\mu > \lambda_e \), that is \( L_\tau \) is violated first, providing a DM neutrino \( \nu_{DM} \sim \nu_\tau \) of mass \( m_{\nu_{DM}} \approx 7 \) eV. Then the breaking of \( L_\mu \) provides the MSW mass scale, and finally the smaller violation of \( L_e \) generates also the MSW mixing of eq. (1). From the diagonalization of the matrix \( m \) in eq. (4) we get then

\[
\begin{align*}
\lambda_\tau \sqrt{\frac{\tilde{m}}{100\text{GeV}}} \left( \frac{100\text{GeV}}{\tilde{m}_b} \right) & \approx 10^{-3}, \\
\lambda_\mu \sqrt{\frac{\tilde{m}}{100\text{GeV}}} \left( \frac{100\text{GeV}}{\tilde{m}_b} \right) & \approx 10^{-5}, \\
\lambda_e \sqrt{\frac{\tilde{m}}{100\text{GeV}}} \left( \frac{100\text{GeV}}{\tilde{m}_b} \right) & \approx 5 \times 10^{-7}.
\end{align*}
\] (5)

These values are consistent with the present accelerator and astrophysical limits \(^{11}\). Furthermore, by diagonalizing the matrix (4) corresponding to these values for the \( \lambda \)'s one gets automatically \( \nu_\mu - \nu_\tau \) oscillations with \( \sin^2 2\theta \approx \ldots \).
$1.3 \times 10^{-3}$, 3 times below the present limit \(^\text{12}\). The predicted $\nu_e - \nu_\tau$ mixing is negligibly small, $\sin^2 2\theta \simeq 10^{-6}$.

Similar conclusions hold if for some reason the vertices $\lambda^h$ are suppressed with respect to the purely leptonic couplings $\lambda^l$. In this case, the largest diagrams involve $\tau - \tilde{\tau}$ exchange, and the heavy DM should be mostly $\nu_\mu^9$, since the entry $m_{\nu_e,\nu_\tau}$ of the mass matrix involve the exchange of muonic flavours in the loop due to the antisymmetric character of the $\lambda^l$s.

We see that, in order to account for both the hot DM and the MSW mechanism, a three order of magnitude hierarchy is needed in eq. (5) between the $L_\tau$ violating couplings, $\lambda_\tau$, and the $L_e$ violating couplings $\lambda_e$. This is comparable to the hierarchy between the charged lepton masses $m_\tau$ and $m_e$. We notice also that the values in eq. (5) do not satisfy the stringent cosmological bound $|\lambda| \lesssim 2 \times 10^{-7}(\tilde{m}_b/100\text{GeV})^{1/2}$, obtained from the requirement that the primordial Baryon Asymmetry of the Universe (BAU) be not washed out \(^\text{13}\). However, this bound depends on the assumption that the BAU was generated at the GUT scale, which may not be the case. In particular, it has been shown that the BAU could have been produced just below the electroweak scale due to the (possible) lepton number violating interactions themselves, and the required values \(^\text{14}\) for the $\lambda$s can agree with eq. (5) for some special choice of the supersymmetric mass parameters.

We conclude noticing that in this framework it is impossible to account also for the atmospheric neutrino data \(^\text{5}\), if the indication of non vanishing mixing is confirmed. In general, it is very hard to reconcile hot DM, MSW and atmospheric neutrino mixing with just the three standard left handed neutrinos. If all these phenomena were taken seriously, this could be an hint for some more light neutral state, such as a singlet neutrino \(^\text{15}\).

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FIGURE CAPTION

Figure 1. Contributions to the neutrino mass element $m_{\nu_i,\nu_j}$ in the SSM with broken R–parity.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9402202v1