Improving the Sensitivity of Planar Fabry–Pérot Cavities via Adaptive Optics and Mode Filtering

Jakub Czuchnowski and Robert Prevedel*

Fabry–Pérot (FP) cavities are fundamental and ubiquitous optical elements frequently used in various sensing applications. Here, the authors introduce a general theoretical framework to study arbitrary light–cavity mode interactions for planar FP and show how optical aberrations, intrinsic to the interrogating beam or due to imperfect cavities, reduce optical sensitivity by exciting higher-order spatial modes in the cavity. It is found that particular Zernike aberrations play a dominant role in sensitivity degradation, and that the general loss of sensitivity can be significantly recovered by appropriate wavefront correction or mode filtering. The authors then demonstrate their theoretical findings also experimentally and show that in practice the sensitivity of realistic planar FP micro-cavity sensors can be improved up to threefold by a synergistic combination of adaptive optics and passive mode filtering.

1. Introduction

Fabry–Pérot (FP) cavities are omnipresent in many important devices used in optics, such as in laser cavities or as narrowband wavelength filters in spectrometers. Their potential for ultra-high sensitivity measurements was recently demonstrated by LIGO with the detection of gravitational waves.[1] While most FP cavities come in a confocal configuration which makes them in general more stable, some recently emerging optical systems require planar FP configurations. Among them are FP micro-cavity based pressure sensors employed in, for example, high-sensitivity photo-acoustic imaging,[2] or so-called virtually imaged phased arrays (VIPAs), which enable high-resolution spectroscopy applications such as Brillouin microscopy.[3,4] As all-optical devices, the performance of FP cavities depends not only on FP intrinsic parameters such as mirror reflectivities, surface homogeneity, etc., but also on extrinsic properties such as the incident light beams that are used to interrogate them. Here in particular, wavefront distortion of the incident light beam due to imperfect optics and/or their alignment, or due to air turbulence have to be considered.

The effects of such wavefront distortions on FP sensitivity were previously studied for confocal FP cavities, inside which the electro-magnetic field is quasi-stationary, that is, not changing between individual reflections and round trips. In case of gravitational wave detectors, Bond et al.[5] have studied the role of mirror distortions, described using Zernike polynomials, and showed that they redistribute power among Laguerre–Gaussian modes, an effect that was later experimentally demonstrated for several aberrations by Gatto et al.[6] In shorter cavities, Mah and Talghader[7] as well as Takeno et al.[8] have explored the use of FP cavities for aberration sensing by relying on spectral properties of the transmitted beam and intensity of the higher order spatial modes reflected away from the cavity, respectively, while Liu and Talghader[9] examined the effects of imperfections in tunable micromirror cavities on Gaussian beams.

However, the effects of optical aberrations on planar FP cavities, that is, composed of flat mirrors, are generally much less studied, and in particular for micro-cavities interrogated by focused light (Figure 1a). Such a configuration has recently been shown to exhibit promising performance as highly sensitive pressure sensors for use in photo-acoustic imaging modalities.[2,9] Here, large efforts have been devoted to improving techniques for their manufacture,[11,12] however, a general theoretical framework to study the interaction of arbitrary light modes with their cavity counterparts is currently lacking. While recent work by Marques et al.[13] provides an accurate theoretical model for calculating reflectivity spectra for ideal FP micro-cavity illuminated with focused beams based on interference of plane waves, their model does not allow to draw general conclusions on the effects of aberrations on FP sensitivity.
In our work, we introduce an alternative general framework to study arbitrary light–cavity interactions that allow us to gain a broader understanding of the mechanisms by which optical aberrations degrade FP sensitivity. Our framework is based on extending the “unfolded cavity approach”\cite{14,15} in order to account for beam aberrations by combining it with Gaussian beam mode analysis (GBMA).\cite{16} GBMA is based on Laguerre–Gaussian mode decomposition and beam propagation, which enables us to numerically investigate the coupling of arbitrarily aberrated Gaussian beams, expressed in terms of Zernike-modes, to Laguerre–Gaussian cavity modes (Figure 1c). This framework allows us to investigate how particular beam and cavity aberrations affect the FP’s overall sensitivity. Our simulations show that the loss of sensitivity is generally caused by coupling and exciting higher-order cavity modes (Figure 1d), and that optimal sensitivity can be restored by optical mode filtering and active aberration correction.

**Figure 1.** a) A schematic conceptualizing the effects of beam and cavity aberrations on the wavefront of the beam, where: $R_1, R_2$ are the reflectivities of the two mirrors, $\omega_0$ is the beam waist radius, $l_0$ is the cavity thickness, $n$ is the refractive index of the elastic material inside the cavity. b) Phase profile of the first 15 Zernike aberrations. c) Intensity profile of some of the low order $(l, p)$ Laguerre–Gaussian modes. d) Decomposition of an arbitrarily aberrated beam into Laguerre–Gaussian modes. e,f) Effect of beam size on FPI transfer functions for two LG modes. e) The fundamental LG00 displays little dependence on increasing divergence of the beam (decreasing the spot size). Here, we also show schematically how changes in cavity thickness are optically amplified at the bias wavelength. f) In contrast, a higher order LG15 mode displays pronounced TF distortions as a function of beam divergence. Additionally simulations for large spot sizes ($\omega_0 = 250\,\mu \text{m}$) show little difference between the two modes confirming the calculations for an ideal FPI (Appendix). For the simulations, we choose the following cavity parameters: $l_0 = 20\,\mu \text{m}$, $R_{1/2} = 0.95$, which are commonly used design parameters for photoacoustic tomography systems.\cite{2}
correction techniques. We further demonstrate these effects experimentally and show that in practice the sensitivity of realistic planar FP micro-cavity sensors can be improved up to three-fold by a combination of adaptive optics and passive mode filtering.

2. Fabry–Pérot Micro-Cavities for (Photo-)Acoustic Sensing

Since the main motivation of our work is connected to FP micro-cavity based pressure sensors and their use in photo-acoustic imaging, we start by shortly introducing the concept and working principle of the technique. Photoacoustic tomography is a non-invasive deep-tissue imaging modality that uses light-induced acoustic waves to combine optical contrast with high-resolution ultrasound detection.[7] To overcome limitations of classical ultrasound detectors, several optical methods for detection of photoacoustic waves were developed over the years (see ref. [18] for review). Here, the use of planar FP micro-cavities has been particularly promising, as it combines high sensitivity with the ability to measure acoustic waves at well-defined spatial locations (given by the interrogating beam size on the sensor), which is important for high-resolution tomographic image reconstruction. In this approach, an elastic FP micro-cavity is formed by sandwiching a layer of elastomere (e.g., Parylene C) between two dichroic mirrors. The cavity can then deform elastically upon incidence of a pressure wave, thus modulating the position of the μFP interferometer’s transfer function (ITF) which depends on the (optical) thickness of the cavity. By tuning the interrogation laser wavelength to the point of maximum slope on the ITF (so-called bias wavelength) one obtains maximum sensor sensitivity, that is, the incident acoustic wave is maximally amplified (so-called bias wavelength) one obtains maximum sensor sensitivity.

This definition of $S_{o}$ allows normalization for both laser relative intensity noise, as well as shot noise which are the dominant sources of noise in typical, realistic μFP systems and therefore is directly proportional to the SNR. Estimation of $S_{o}$ requires calculation of $E(r, \phi, z, \lambda)$ for different $z$ planes (Equation (4)). This cannot be done analytically for aberrated Gaussian beams in general and thus requires a new theoretical framework. A number of approaches exist including numerically solving the Fresnel integral,[9] or the use of extended Zernike–Njober theory.[20]

Here, we chose to use Gaussian beam mode analysis, an established method based on decomposing fields into Gaussian mode bases (such as the Laguerre–Gauss base), as an efficient way of performing diffraction calculations,[16] also for beam propagation of aberrated fields.[21] This approach was also recently used for analysis of beam aberrations in confocal Fabry–Pérot cavities.[7] Our choice was motivated by the fact that Laguerre-Gaussian beams (Figure 1c) are natural modes side, respectively.

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We start by defining the general LG mode with indices $l, p$ as:

$$LG_{l p}(r, \phi, z, \lambda) = C^{l p}_{G} \left[ \frac{r \sqrt{2}}{w(z)} \right] L_{l}^{p} \left[ \frac{2 r^{2}}{w(z)^{2}} \right] \exp(-i l \phi) G(r, z, \lambda)$$

(5)

where $C^{l p}_{G}$ is a normalization constant, $L_{l}^{p}(x)$ is the Laguerre polynomial, and $G(r, z, \lambda)$ denotes a general Gaussian beam (see Section S1, Supporting Information, for full definition):

$$G(r, z, \lambda) = \frac{w_{0}}{w(z)} \exp \left[ - \frac{r^{2}}{w(z)^{2}} \right] \exp \left[ -i \left( \frac{2 \pi}{\lambda} \frac{z}{z} + \frac{\pi r^{2}}{\lambda R(z)} - \psi(z) \right) \right]$$

(6)

We now consider an aberrated Gaussian beam of the form:

$$G_{a}(r, \phi, z, \lambda) = G(r, z, \lambda) \exp \left[ 2 \pi i \sum_{j} a_{j} Z_{j} \right]$$

(7)

where $a_{j}$ is the amplitude coefficient of Zernike aberrations expressed in waves and $Z_{j}$ is Zernike polynomial indexed.
using the OSA/ANSI standard indices \( (Z_j = Z_n^m, \text{ where } j = \frac{1}{2}(n(n+2)+m)) \).

\[
Z_n^m(r, \phi) = \begin{cases} 
A_n^m R_n^m(r) \cos(m \phi) & \text{for } m \geq 0 \\
A_n^m R_n^m(r) \sin(m \phi) & \text{for } m < 0 
\end{cases}
\] (8)

with

\[
R_n^m(r) = \sum_{s=0}^{[n=m]/2} \frac{(-1)^s (n-s)!}{s!((n+m)/2-s)!((n-m)/2-s)!} r^{x-2s} 
\] (9)

and \( A_n^m \) being a normalization factor chosen so that

\[
\max_{r \in [0,1]} Z_n^m(r, \phi) - \min_{r \in [0,1]} Z_n^m(r, \phi) = 1 
\] (10)

This normalization allows better direct comparison with experimental systems using deformable mirrors or spatial light modulators since their dynamic range is limited by the maximum amplitude of the mode they can display and thus are often calibrated in mode amplitude units.

We now wish to calculate the coupling or overlap of aberrated Gaussian beam expressed in Zernike modes with the LG modes that propagate inside the FP cavity. For this we seek the electric field \( E(r, \phi, z, \lambda) \) of the cavity,

\[
E(r, \phi, z, \lambda) = \sum_{j=0}^{\infty} \sum_{\alpha} c_{\alpha j} LG_{\alpha}(r, \phi, z, \lambda) 
\] (11)

where \( |c_{\alpha j}|^2 \) denotes the fraction of optical power coupled into a particular LG_{\alpha} mode. These decomposition coefficients can be obtained from

\[
c_{\alpha j} = \iint_A LG_{\alpha}(r, \phi, z_0, \lambda) G_n(r, \phi, z_0, \lambda) dA
\] (12)

where \( A \) is the aperture over which the field is measured. This now allows us to numerically simulate the Fabry-Pérot interfering field (Equation 4) by calculating the aberrated electric fields for different reflections in the Fabry-Pérot interferometer.

4. Effects of Beam Aberrations on FPI Sensitivity

Our theoretical framework allows us to explore the effects of optical aberrations on the sensitivity of the FP cavity. We start by exploring the properties of an ideal Gaussian beam and note that for a non-aberrated beam all power is confined in the fundamental cavity mode (LG_{00}). However, in the presence of aberrations, we start to see significant coupling into higher order LG-modes of the cavity (Figure 1d). This effect was also observed in ref. [7] for confocal cavities, although it does not negatively affect the sensitivity as the ITFs of different LG-modes are fully separated spectrally in this case.

Next, we consider an ideal FP micro-cavity with flat mirrors and show that for such a cavity illuminated with a perfectly collimated beam aberrations have no effect on sensitivity (Appendix). In essence, all LG modes show the same transfer function (Figure 1e,f) if the beam diameter is sufficiently large compared to the cavity thickness. This extreme case serves as a check of our analytical result (see Appendix). However, in realistic experimental conditions the interrogation light is focused on the FP micro-cavity with spot size \( w_0 \leq 50 \mu m \) which will significantly reduce the Rayleigh range of the beam, effectively leading to spectrally shifted and distorted transfer functions, especially for higher order LG-modes (Figure 1f). This in turn, will have serious consequences for the robustness of the cavity as coupling into higher order modes will cause broadening of the ITF and loss of sensitivity.

With this general observation in mind, we now start investigating the effects of single Zernike modes on the sensitivity of the μFPI. The conceptual procedure of our simulations is given in Figure 2a and is based on numerical assessment of the equations outlined in Section 3 over a \( (r, \phi) \) grid. We constrained the range of Z-modes to be analyzed in this study to Z3–Z14 due to several factors. First, we excluded Z0 (piston) and Z1–Z2 (tip-tilt) because they are known to only shift the phase of the beam and move the laser spot focus laterally, respectively, and thus are not expected to affect the optical sensitivity of the FPI. Second, we did not focus on higher modes for the sake of clarity and because our experimental setup is not capable of correcting modes higher than Z14.

We observe that while FP sensitivity (S_{\lambda}) generally declines with increasing amplitude for all aberration modes, the magnitude of their effect is heterogeneous (selected examples in Figure 2b; for all Z-modes see Figure S1, Supporting Information). Only the positive values of \( \alpha_j \) coefficients are plotted as the effect of isolated aberrations is symmetric and independent of the sign of the aberration. To directly compare all selected individual Zernike modes, we calculate the Zernike amplitude (\( \alpha_j \)) that reduces sensitivity to 50% (Figure 2c). This characteristic point \( S_{\lambda,50} \) is important from a practical perspective as it characterizes the effective strength of different modes in degrading the optical sensitivity. Interestingly, we find that Zernike mode I2, that is, primary spherical aberration (Z12) has the strongest negative effect on sensitivity, followed by Z4 (defocus) and Z11/ Z13 (secondary astigmatism). These modes couple most strongly to higher LG-modes, presumably because of a combination of factors.

First, Zernike polynomials, due to differences in shape, have a variation of volume under the polynomial (VuP) for a constrained maximum amplitude resulting in differences in the overall phase aberration introduced in the beam (Figure S2, Supporting Information). Z12 and Z4 show the largest VuP which might explain their strong effect on sensitivity of the μFPI. This observation does not, however, fully account for the differences between individual Zernike modes, like in the case of Z11/Z13. Additionally, more subtle properties such as the ring-shaped phase of Z12 (Figure 1b) matches well the profiles of higher order LG-modes (Figure 1c), therefore facilitating an efficient coupling.

In practice, aberrations are never present in isolation, but rather occur as a mixture with varying weights. Hence, we decided to further explore the interactions between different Zernike modes. For this, we pursued a Monte-Carlo approach to analyze these interactions in a high-throughput manner (Figure 2a). In order to constrain optical aberrations in groups, we decided to keep the total aberration magnitude constant for each group (\( Z_{\text{total}} = \sum \alpha_j = \text{const} \)). We observed that the mean
sensitivity decreases as the total aberration magnitude increases but also the variance in sensitivity increases for stronger aberrations (Figure 2d). We analyzed the source of this variation by calculating the correlation between the sensitivity and aberration magnitude for each of the modes ($\alpha_j$) within a group where $Z_{\text{tot}} = \text{const}$. For $Z_{\text{tot}} = 2$, we observe a strong negative correlation.
with mode Z12 (spherical aberration) (Figure 2e), which is in line with the results for single modes in which Z12 has a much stronger impact on the sensitivity than other modes (Figure 2c).

The most prominent finding of our investigations of beam aberrations is that a single parameter showed a strong linear correlation with the simulated optical sensitivity $S_o^M$ (Figure 2f). This parameter is the power fraction conserved in the fundamental LG00 mode, which seems to suggest that the principal mechanism behind loss of sensitivity is the loss of power in the fundamental mode induced by aberrations.

5. Effects of Cavity Aberrations on FPI Sensitivity

In the previous section, we discussed the effect of beam aberrations on the transfer function of the μFPI. However, there is another special class of optical aberrations that needs to be treated separately in our framework, namely, the optical aberrations that are accumulated while the beam is propagating inside the cavity. Since the beam makes several round trips (~35 – 45 as reported in ref. [2]) inside the cavity, any phase delay will add up at each reflection. These spatially varying phase delays can, for example, be induced by mirror imperfections (Figure 3a) and require the following theoretical treatment.

Here, the main insight is that we can describe the cavity mirror shape again by a combination of Zernike polynomials[5]

$$\mathcal{M}(\phi, \phi, \lambda) = \sum_{j} \gamma(\lambda) Z_{j}(\phi, \phi) \quad (13)$$

where, $\gamma(\lambda)$ is the magnitude of the phase delay introduced to the beam by the mirror deformation expressed in waves. Because this phase delay is introduced at each reflection, it requires a modification of the approach from previous sections. By combining Equation (11) with Equation (4), we can express the interfering electric field $E_{\text{FP}}(\phi, \phi, \lambda)$ in terms of Laguerre–Gaussian modes:

$$E_{\text{FP}}(\phi, \phi, \lambda) = \sum_{r=0}^{\infty} \sum_{p=0}^{\infty} c_{r}^{p} LG_{r}(\phi, \phi, \lambda) \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \beta_{l} LG_{l}(\phi, \phi, \lambda')$$

$$(14)$$

where $s = (|| + p)^2 + l + || + p$ and $l, p$ are LG indices, but because of cavity aberrations, the decomposition of beam into LG modes of weight $c_{r}$ changes at each reflection $k$. Therefore, the interfering electric field in an aberrated cavity takes the form

$$E_{\text{FP}}(\phi, \phi, \lambda) = \sum_{r=0}^{\infty} \sum_{p=0}^{\infty} c_{r}^{p} LG_{r}(\phi, \phi, \lambda) \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \beta_{l} LG_{l}(\phi, \phi, \lambda')$$

$$(15)$$

where $|c_{r}^{p}|^2$ denotes the fraction of power coupled into mode LG at reflection $k$. Because the beam decomposition into LG modes is changing in the cavity at each reflection, the coefficients $c_{r}^{p}$ need to be calculated in an iterative fashion:

$$c_{r}^{p} = \sum_{k} c_{r}^{p-1} \int \int_\lambda LG_{r}(\phi, \phi, \lambda') LG_{l}(\phi, \phi, \lambda') \delta(\phi, \phi, \lambda) dA \quad (16)$$

This approach is computationally expensive in practice, but can be greatly simplified by assuming that the increase of beam diameter inside the cavity during propagation is small. In our particular case where beam diameter $w_0 = 50\mu m$ and cavity thickness $l_0 = 20\mu m$, the beam will only increase in diameter $\approx 10\%$ for an optical path length of 100 reflections. This is negligible for the qualitative conclusions we aim to draw (Figure S3, Supporting Information). Because the electric field $E_{k}$ for reflection $k$ can always be expressed as

$$E_{k}(\phi, \phi, \lambda) = \sum_{s} c_{s} LG_{s}(\phi, \phi, \lambda) \quad (17)$$

where $s = (|| + p)^2 + l + || + p$, we can consider the electric field $E_{k}$ as a vector in a vector space with LG-modes as an orthonormal base $\{e_{s} = LG_{s}(\phi, \phi, \lambda)\}$ and $c_{s}$ as coefficients:

$$E_{k} = \sum_{s} c_{s} e_{s} \quad (18)$$

This allows us to define an algebraic operator ($\mathcal{M}$) that describes the mode evolution of the beam inside the cavity (see Section S2, Supporting Information, for derivation):

$$E_{k+1} = \mathcal{M}E_{k} \quad (19)$$

or, in terms of coefficients:

$$c_{s}^{k+1} = \sum_{s} \mathcal{M}_{s-k} c_{s-k} \quad (20)$$

This considerably simplifies the calculation of the amplitude coefficients and power fractions $|c_{s}^{k}|^2$ coupled into mode LG, and the interfering electric field $E_{\text{FP}}(\phi, \phi, \lambda)$ can then be calculated according to Equation (15).

Cavity aberrations have important differences from beam aberrations when considering aberration correction using active wavefront control (adaptive optics, AO). Because experimental AO methods only allow wavefront determination and control at a single chosen plane along the optical axis at a time, it is not possible to fully correct cavity aberrations as they evolve through each reflection (Figure 3a). Thus, we aim to explore which extent cavity aberrations can be corrected. In particular, we perform in silico AO experiments for various cavity deformations by establishing an appropriate simulation pipeline (Figure 3b). Here, we simulate the AO correction by applying Z-modes with amplitudes $c_{s} \in [-1, 1]$ to the illuminating beam. Conversely to the case of isolated aberrations from Section 4, in the case of cavity aberrations, there exists a clear asymmetry where the sign of the Z-mode amplitude will determine whether the aberration is compensated or enhanced and thus the range of $c_{s}$ need to be expanded to also encompass negative values. Utilizing our simulation routine, we first investigated and characterized the capability of individual Z-modes to correct a given cavity deformation (Figure 3c; Figure S4, Supporting Information).

Here, we found that more than a single Z-mode can interact with the cavity, even when the cavity is deformed using only a single Zernike polynomial. This led to the observation that, for example, Z2 can be corrected by Z4, Z12 as well as Z24 (Figure 3d; Figure S4, Supporting Information). We explored this in a more rigorous fashion and found that Z-modes within the same “family” have the ability to partially compensate for each other (Figure 3e). Furthermore, when combined together, they can even further improve FP sensitivity (Figure 3d).
Figure 3. a) A schematic conceptualising the limitations of adaptive optics in fully compensating the effects of cavity aberrations. b) Flowchart of the simulation procedure. c) AO correction of a $\gamma_{12} = 0.02$ aberrated cavity using mode Z12. d) Quantification of sensitivity for correcting $\gamma_{12}$ using different Z-modes as well as their combination, which achieves the highest sensitivity. Here, the cavity deformation was chosen as $\gamma_{12} = 0.02$. e) Z-mode interactions in different deformed cavities. For all cavities: $\gamma = 0.02$. 
6. Active and Passive Aberration Correction in FPI Systems

6.1. Beam Aberrations

After discussing the effects of both beam and cavity aberrations in the previous sections, we now proceed to explore potential approaches for correcting them, and thus recovering the loss of sensitivity. Typically, beam aberrations in microscopy setups are addressed with the use of deformable mirrors (DMs) or spatial light modulators (SLMs) which can apply spatially varying, controlled phase delays. However, based on our simulations, we also hypothesized that a much simpler approach could be effective. Since the loss of sensitivity is caused mainly by leakage of power to higher-order cavity modes (Figure 2f), we speculated that sensitivity could be improved by mode filtering, for example, by using a passive element such as a single mode fibre. This serves to reject all the reflected light that propagates outside the fundamental LG_{00} (i.e., Gaussian) mode from reaching the detector.

We tested our hypothesis both in simulations as well as experimentally. For our simulations we calculated the coupling of light power from the interfering field $E_{\text{FPI}}$ into the fundamental Gaussian mode:

$$I_{\text{SM}}(\lambda) = \int A E_{\text{FPI}}(r, \phi, \lambda) G(r, \phi, 0, \lambda) dA$$ (21)

We observed that this mode filtering approach has the same effect as active AO correction in recovering the ideal transfer function (Figure 4a), while being much simpler to implement experimentally. The only disadvantage is that for large aberration the power loss through filtering becomes significant (Figure 4b).

6.2. Cavity Aberrations

In the previous section, we treated the effects of beam aberrations and described potential ways to tackle them using both active and passive aberration correction. In this section, we will discuss the case of cavity aberrations and how both active and passive correction could synergize.

We start by stating that similarly to active correction, passive mode filtering will also not be fully effective in tackling sensitivity loss due to cavity aberrations, since the power distribution between the modes will change in-between reflections, thus distorting the Gaussian mode interference pattern (Figure 4c).

This highlights the importance of manufacturing cavities with high uniformity of thickness as cavity deformations cannot be fully corrected and thus lead to an irreversible loss of sensitivity, unless techniques to actively alter the cavity structure locally are implemented. As some of such techniques are being actively developed, it remains to be seen if they prove to be effective in tackling cavity induced loss of sensitivity.

Our simulations show that both AO and single-mode filtering (SM) individually do improve sensitivity over the aberrated case (Figure 4d,e). Contrary to beam aberrations, for cavity aberrations, AO and SM have different correction mechanisms which can in fact complement each other. Interestingly if AO is performed while the beam is being mode filtered, we can achieve even higher sensitivity than by mode filtering the AO corrected beam (AO(SM) > SM(AO)). The reason for this lies in the fact that for AO(SM) only the fundamental mode is optimized for amplitude and phase distribution between reflections. This yields a different overall correction compared to optimizing reflections for all LG modes in case of AO. However, combining passive and active aberration correction can reject significant amounts of light, which might prove prohibitive for some practical applications (Figure 4f).

6.3. Experimental Validation

To validate our theoretical findings, we also performed experiments using an all-optical photoacoustic tomography setup based on a Fabry–Pérot micro-cavity sensor that conceptually follows ref. [2] but was modified by adding an adaptive optics module (Figure S5a, Supporting Information) consisting of a deformable mirror conjugated to the back focal plane of the scan lens. First, the output of the interrogation laser is collimated and its size matched to the diameter of the active aperture (~10 mm) of the deformable mirror (DM, DMP40/M-P01, Thorlabs). Two relays (L2-L3 and L4-L5) then reduce the beam diameter by 0.6x and 0.625x, respectively, to match the required NA for the scan lens (TSL-1550-15-80, Wavelength Opto-Electronic) to achieve a ~50 μm spot radius on the Fabry–Pérot interferometer (FPI). The back reflected light is redirected by a quarter-waveplate ($\lambda$/$4$) and polarizing beamsplitter (PBS) to the detection path and then either directly focused, or fiber coupled into a single-mode fibre, before detection by a photodiode (PD, PDA05CF2, Thorlabs). The DM was factory-precalibrated to display Zernike modes 3 to 15. The tuneable interrogation laser, DM, and data acquisition (NI-6259, National Instruments) are controlled by a custom written LabView software. The μFP interferometer transfer function (ITF – Equation (1); Figure S5b, Supporting Information) is acquired by first setting a particular Zernike mode pattern on the DM and then tuning the wavelength of the laser in a stepwise manner to avoid spectra deformations connected to continuous wavelength sweeping.

To accurately estimate the μFP sensitivity, the ITF data is fitted numerically, considering the function of a Gaussian beam propagating in an ideal cavity and varying the reflectivity of the two mirrors ($R_1$, $R_2$, Figure S5b, Supporting Information). This fitting approach showed good performance and can be computed efficiently.

It is important to note here that in our experiments both the exact beam ($\alpha$) as well as cavity ($\gamma$) aberrations remain in principle unknown, therefore preventing precise modeling of the experimental situation. Nevertheless, qualitative comparisons can be made to gain intuitive insights into the system. In our experiments, we observed that mode filtering indeed increases the sensitivity compared to performing adaptive optics only which stands in agreement with our simulations (Figure 4g). Furthermore, the quantification of the characteristic points also shows similarities with the relative sensitivity improvements mostly conserved AO(SM) > SM(AO) > AO/SM(G) > G (Figure 4h). Differently to the simulation AO < SM(G), however, this may be due to contributions from other cavity and beam aberrations which are not experimentally characterized, as mentioned above. Finally, we...
quantified the power loss for various AO approaches, which also shows qualitative agreements with our simulations (i.e., AO(SM) < SM(AO) < G, see Figure 4i). Here, we note that SM(AO) and AO(SM) are normalized to the fiber coupled power at $\alpha_{12} = 0.0$ (SM(G)) to disentangle the experimental power loss due to fiber coupling and therefore SM(G) is not shown in Figure 4i.
7. Discussion

We have shown theoretically as well as experimentally that active and passive techniques to correct aberrations have the capability to increase optical sensitivity of FP micro-cavity sensors. One unexpected finding of our investigation is the fact that passive mode filtering can achieve significant gains in optical sensitivity. We expect this to have impact in practical realizations of FPIs, such as in photoacoustic imaging, because of the simplicity and ease of its experimental implementation. The only disadvantage and limitation lies in the fact that, depending on the amplitude of the aberration, a considerable part of the interrogating laser light might be rejected. We further found that, in more realistic cases when both cavity and beam aberrations are present, combining active and passive techniques yields the overall best improvements in sensitivity (Figure 4e,h). As again power loss of the passive filtering might be limiting, the optimal solution may thus not only depend on the effective increase in sensitivity, but also on the effects due to reduced power and signal-to-noise ratio on the detector side. This might therefore require a more complex optimization metric which takes into account these additional considerations concerning signal and detector noise sources.

In our experiments, we chose the Z12 mode for validating our theoretical predictions. This choice was motivated by the fact that our simulations predicted the most detrimental effects of Z12 on sensitivity, while other modes either showed less strong effects (e.g., Z6 and Z9) or are difficult to correct with our deformable mirror in practice. The latter is due to its limited resolution, which effectively induces an undesirable focal shift of the beam on the surface of the interferometer for particular classes of Zernike modes (e.g., Z7/Z8 or Z11/Z13).

An important insight of our work with much broader applicability and potential impact is our theoretical observation that higher order aberrations can actually be partially corrected by lower order modes (e.g., Z4 can correct Z6). This has important practical implications since it might allow to use active optical elements such as deformable mirror with lesser degrees of freedom. This would greatly reduce both the cost as well as technical complexity of experimental AO correction, and thus might lead to a more widely uptake in the field. Furthermore, we note that our theoretical approach can also be used to investigate other metrics of the FP transfer function, such as visibility or linewidth, which are important for other FP-based sensing applications.

Moreover, recent work has shown that the use of non-Gaussian beams can in principle further increase FP measurement sensitivity, for example, by utilizing LG_{33} modes in LIGO detectors,[23] or Bessel beams in FP micro-cavity-based pressure sensing.[24] Our model could therefore be used to further evaluate and explore the robustness of these and other non-Gaussian beams against beam and cavity aberrations in FPI interrogation. Finally, we expect that our theoretical framework will find application beyond photoacoustics-based pressure sensing. For example, our general finding might be also further explored for other imaging modalities such as multi-photon microscopy where AO is utilized to correct more complex aberrations induced by living tissue,[25] or to increase the spectral resolution in VIPA-based, non-confocal Brillouin spectrometers.[26]

A. Appendix

Limit Case for Ideal FP Cavity Illuminated with a Non-Diverging Beam

The properties of a planar Fabry–Pérot cavity illuminated with non-diverging beams can be analyzed analytically to evaluate the effect of optical aberrations on the ITF. We aim to calculate the transfer function \( I_{\text{FP}}^{\text{ND}}(\lambda) \) where we use the superscript \( \text{ND} \) to denote non-divergence of the beam. From Equation (3) we know that

\[
I_{\text{FP}}^{\text{ND}}(\lambda) = \int_\lambda E_{\text{FP}}^{\text{ND}}(r, \phi, \lambda) \, dA
\]  

(A1)

and by combining Equation (11) with Equation (4), we can express the interfering electric field \( E_{\text{FP}}^{\text{ND}} \) in terms of Laguerre–Gaussian modes:

\[
E_{\text{FP}}^{\text{ND}}(r, \phi, \lambda) = \sum_{i=0}^{\infty} \sum_{p=0}^{\infty} \sum_{\lambda_p} \beta_i \mathcal{L}_i^{\lambda_p}(r, \phi, z, \lambda)
\]

(A2)

where

\[
\beta_i = \begin{cases} r_i & \text{for } i = 0 \\ 2(t_i)^i(r_i^*)^{i-1}(r_i^*) & \text{for } i > 0 \end{cases}
\]

(A3)

Now we need to explore the properties of non-diverging LG modes. We start by defining a non-diverging LG mode as the limit when the Rayleigh range of the beam \( z_R \) approaches infinity

\[
\mathcal{L}_i^{\lambda_p}(r, \phi, z, \lambda) = \lim_{z_R \to \infty} \mathcal{L}_i^{\lambda_p}(r, \phi, z, \lambda) = \lim_{z_R \to \infty} \mathcal{C}_i^{\xi_p}
\]

\[
\times \exp(-i\phi) \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w(z)^2}\right) \exp\left(-i\frac{2\pi}{\lambda} z + \frac{\pi r^2}{\lambda R(z)} - \psi(z)\right)
\]

(A4)

We will now explore the limits of all the parts dependent on \( z_R \) separately:

\[
\lim_{z_R \to \infty} w(z) = \lim_{z_R \to \infty} \sqrt{1 + \left(\frac{z}{z_R}\right)^2} = w_0
\]

(A5)

\[
\lim_{z_R \to \infty} \psi(z) = \lim_{z_R \to \infty} \text{arctan}\left(\frac{z}{z_R}\right) = 0
\]

(A6)

The value of \( r \) is also indirectly dependent on \( z_R \) because \( r = w_0 \) for proper beam sampling and from Equation (S5), Supporting Information, we know that \( w_0^2 = z_R \) so \( r^2 \approx z_R \):

\[
\lim_{z_R \to \infty} \frac{z_R}{R_i(z)} = \lim_{z_R \to \infty} \frac{\sqrt{z_R^2 + z_R^2}}{0} = 0
\]

(A7)
With these we come back to Equation (A4):

\[
\begin{align*}
\text{LG}^\text{ND}(r, \phi, z, \lambda) &= \lim_{r, \phi, z, \lambda \to \infty} \text{LG}^\text{ND}(r, \phi, z, \lambda) = \lim_{r, \phi, z, \lambda \to \infty} \text{c}_{LP}^G \left( \frac{r \sqrt{2}}{w(z)} \right) \text{L}_p^G \left( \frac{2r^2}{w^2(z)} \right) \\
&\times \exp\left( -i \phi \right) \left( \frac{w(z)}{w(z)} \right) \exp\left( -\frac{r^2}{w^2(z)} \right) \exp\left( -i \frac{2\pi}{\lambda} \right) (z - \lambda) \\
&= \text{c}_{LP}^G \left( \frac{r \sqrt{2}}{w(z)} \right) \text{L}_p^G \left( \frac{2r^2}{w^2(z)} \right) \exp\left( -i \phi \right) \left( \frac{w(z)}{w(z)} \right) \exp\left( -\frac{r^2}{w^2(z)} \right) \exp\left( -i \frac{2\pi}{\lambda} \right) z
\end{align*}
\]

(A8)

where we reach our first conclusion by observing that

\[
\text{LG}^\text{ND}(r, \phi, z, \lambda) \text{LG}(r, \phi, z, \lambda)
\]

which leads to the first property of non-diverging LG modes:

\[
\text{LG}^\text{ND}(r, \phi, z, \lambda) = \text{LG}^\text{ND}(r, \phi, z, \lambda) \exp\left( -i \frac{2\pi}{\lambda} (z - \lambda) \right)
\]

(A10)

The second property flows directly from orthonormality of LG-modes:

\[
\delta_{\phi \delta_{LP}} = \int \text{LG}^\text{ND}(r, \phi, z, \lambda) \text{LG}^\text{ND}(r, \phi, z, \lambda) \, dA
\]

(A11)

We combine these to achieve

\[
\delta_{\phi \delta_{LP}} \exp\left( -i \frac{2\pi}{\lambda} (z - \lambda) \right) = \int \text{LG}^\text{ND}(r, \phi, z, \lambda) \text{LG}^\text{ND}(r, \phi, z, \lambda) \, dA
\]

(A12)

Now we return to Equation (A1) and proceed to calculate the transfer function of an ideal FP cavity

\[
I_{FP}^\text{ND}(\lambda) = \int \text{E}_{FP}^\text{ND}(r, \phi, \lambda) \text{E}_{FP}^\text{ND}(r, \phi, \lambda) \, dA
\]

(A12)

\[
\begin{align*}
&= \left[ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \beta_i \beta_j \text{LG}^\text{ND}(r, \phi, z, \lambda) \right] \left[ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \beta_p \beta_q \text{LG}^\text{ND}(r, \phi, z, \lambda) \right] \, dA \\
&= \left[ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \beta_i \beta_j \beta_p \beta_q \text{LG}^\text{ND}(r, \phi, z, \lambda)^2 \right] \, dA \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \beta_i \beta_j \beta_p \beta_q \delta_{\omega \delta_{LP}} \exp\left( -i \frac{2\pi}{\lambda} (z - \lambda) \right) \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \beta_i \beta_j \beta_p \beta_q \exp\left( -i \frac{2\pi}{\lambda} (z - \lambda) \right)
\end{align*}
\]

(A13)

We conclude

\[
I_{FP}^\text{ND}(\lambda) = E_{FP}^\text{ND}(\lambda) E_{FP}^\text{ND}(\lambda)^* = I_{FP}^\text{ND}(\lambda)
\]

(A16)

This results show that if the beam is non-diverging it will create an ideal airy interference pattern inside a Fabry–Pérot cavity regardless of its decomposition into LG-modes. As any beam can be represented as a linear combination of LG-modes this shows that an Fabry-Pérot cavity illuminated with a non-diverging beam is inherently resistant to beam aberrations. Importantly, however, this conclusion does not hold for confocal cavities because interference patterns of different LG-modes experience a spectral shift due to the curvature of the mirrors.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

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