Many-body Effects in Landau Levels: Non-commutative Geometry and Squeezed Correlated States

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We discuss symmetry-driven squeezing and coherent states of few-particle systems in magnetic fields. An operator approach using canonical transformations and the SU(1,1) algebras is developed for considering Coulomb correlations in the lowest Landau levels.

Keywords: High Magnetic Fields; Coulomb Correlations; Coherent States.

I. INTRODUCTION

Coulomb interactions in strong magnetic fields are relevant in the various physical contexts such as, for example, the behavior of ions in atmospheres of neutron stars, electrons and holes in semiconductors, and charged quasiparticles in the Fractional Quantum Hall Effect regime. One of the general aspects of the problem are considerations of the relevant symmetries. In this work, we discuss an operator method that allows one to maintain both axial and magnetic translations geometric symmetries in two-dimensional (2D) systems in Landau Levels (LL). We also establish a connection with the dynamical SU(1,1) symmetry.

II. CHARGED e-h SYSTEMS

Let us consider a 2D system of two oppositely charged particles $-q_1 < 0$ and $q_2 > 0$, which we will denote as an “electron” and a “hole”, respectively. The total charge is negative, $-Q = q_2 - q_1 < 0$. The operator of magnetic translations (MT) is of the form $\tilde{K} = -i\hbar \nabla_1 + i\hbar \nabla_2 - B\times(q_1 r_1 - q_2 r_2)/2c$, where the symmetric gauge is used. The MT group is non-commutative, and the dimensionless MT operator $\mathbf{k} = K L_B / \hbar$ has canonically conjugate components, $[\hat{k}_x, \hat{k}_y] = i$, where $L_B \equiv \sqrt{\hbar c/Q B}$ is the effective magnetic length. This allows one to introduce a pair of Bose ladder operators for the whole system $\tilde{B}_c^\dagger = -i\hat{k}_- / \sqrt{2}$ and $\tilde{B}_e = i\hat{k}_+ / \sqrt{2}$ such that $[\tilde{B}_e, \tilde{B}_c^\dagger] = 1$, here $\hat{k}_\pm = \hat{k}_x \pm i\hat{k}_y$. In terms of intra-LL operators of individual particles, we have

$$
\tilde{B}_c^\dagger = \frac{-i\hat{k}_-}{\sqrt{2}} = u B_c^\dagger - v B_h , \quad (1)
$$

$$
\tilde{B}_h^\dagger = \frac{i\hat{k}_+}{\sqrt{2}} = \frac{q_1}{Q}, \quad v = \frac{q_2}{Q} . \quad (2)
$$

The second linearly independent annihilation operator is $\tilde{B}_h = u B_h - v B_c^\dagger$ so that we have two pairs of Bose ladder operators: $[\tilde{B}_h, \tilde{B}_c^\dagger] = [\tilde{B}_e, \tilde{B}_c^\dagger] = 1, [\tilde{B}_h, \tilde{B}_e^\dagger] = 0$, and $[\tilde{B}_h, \tilde{B}_e] = 0$. This is in fact Bogoliubov canonical transformation

$$
\left( \begin{array}{c} \tilde{B}_c^\dagger \\ \tilde{B}_h^\dagger \end{array} \right) = \left( \begin{array}{cc} \hat{S} B_h^\dagger & \hat{S} \hat{B}_h^\dagger \\ \hat{S} h B_h & \hat{S} h \hat{B}_h \end{array} \right) \left( \begin{array}{c} B_c^\dagger \\ B_h \end{array} \right) , \quad (3)
$$

$$
\hat{U} = \left( \begin{array}{cc} \cosh \Theta & -\sinh \Theta \\ -\sinh \Theta & \cosh \Theta \end{array} \right) \quad (4)
$$

performed by the unitary operator $\hat{S} = \exp(\Theta \hat{\mathcal{L}})$ with the generator $\hat{\mathcal{L}} = \hat{B}_h^\dagger \hat{B}_c - B_e B_h$: here $\Theta$ is the transformation parameter with $u = \cosh \Theta, v = \sinh \Theta$. This transformation introduces new quasiparticles with coordinates

$$
R_1 = \frac{q_1 r_1 - q_2 r_2}{Q} , \quad R_2 = \frac{\sqrt{q_1 q_2}}{Q} (r_2 - r_1) , \quad (5)
$$

in which the transformed operators assume the standard forms

$$
\hat{B}_c^\dagger = \frac{1}{\sqrt{2}} \left( \frac{Z_1^*}{2L_B} - \frac{2L_B}{\partial \partial Z_1} \right) , \quad (6)
$$

$$
\hat{B}_h^\dagger = \frac{1}{\sqrt{2}} \left( \frac{Z_2}{2L_B} - \frac{2L_B}{\partial \partial Z_2} \right) , \quad (7)
$$

where the 2D complex variables $Z_i = x_i + i y_i$ are used. Note that the interaction potential $U_{int} = U(r_1 - r_2)$ does not depend on $R_1$. The MT operator is diagonal in the new representation, $k^2 = 2\hat{B}_c^\dagger \hat{B}_c + 1$. It has the discrete spectrum $2k + 1$, where the oscillator quantum numbers $k = 0, 1, 2, \ldots$ determine the position of a guiding center of a charged system in $\mathbf{B}$. A complete basis of states in zero LL compatible with both axial and translational symmetries is given by

$$
\hat{\mathcal{B}}_c^k \hat{\mathcal{B}}_h^m \hat{0} \equiv | \tilde{m} \rangle , \quad (8)
$$

where $| \tilde{0} \rangle = \hat{\mathcal{S}} | 0 \rangle$ is the transformed vacuum and state $| \tilde{m} \rangle$ has total angular momentum projection $M_c = m - k$. The energy spectrum is degenerate with respect to $k$. Therefore, it is sufficient to consider only the states with $k = 0$ from $\mathbf{B}$; we denote such states as $| \tilde{m} \rangle$. The above procedure removes one degree of freedom and corresponds to a possible partial separation of variables in magnetic fields.
For, e.g., the Coulomb interaction \( U_{\text{int}} = -q_1 q_2 / |r_1 - r_2| \), the eigenvalues in the lowest LL can be calculated analytically as expectation values:

\[
U_m = \langle \hat{m} | U_{\text{int}} | \hat{m} \rangle = -E_0 \left( \frac{q_2}{q_1} \right)^{m+\frac{1}{2}} \sum_{k=0}^{m} C_k^m \sqrt{\Gamma(k + \frac{1}{2}) / \Gamma(k + 1)} \left( \frac{q_1 - q_2}{q_2} \right)^k ,
\]

where \( E_0 = \sqrt{\frac{q_1^2}{l_{B1}^2} + \frac{q_2^2}{l_{B2}^2}} \) are the magnetic lengths. Eigenenergies are shown in Fig. 1 for several values of parameter \( \epsilon = (q_2 / q_1)^{1/2} < 1 \). The spectra are completely discrete. However, in the limit \( q_2 \to q_1 \) (\( \epsilon \to 1^{-0} \)) the spectra become quasicontinuous and fill in the infinite neutral magnetoelectron band of width \( E_0 \).

In the terminology of quantum optics, the transformed vacuum \( |0\rangle = S |0\rangle \) is a two-mode squeezed state. For particles in a magnetic field squeezing has a direct geometrical meaning. Indeed, in the coordinate representation we have

\[
\langle r_1, r_2 | 0 \rangle = \frac{\sqrt{1 - \epsilon^2}}{2 \pi l_{B1} l_{B2}} \exp \left( -\frac{r_1^2}{4 l_{B1}^2} - \frac{r_2^2}{4 l_{B2}^2} + \frac{\epsilon z_1^* z_2}{2 l_{B1} l_{B2}} \right) .
\]

Using Eq. 10, the probability distribution can be presented in the following form

\[
|\langle r_1, r_2 | 0 \rangle|^2 \sim \frac{1}{\sigma_+ \sigma_-} \exp \left( -\frac{\rho_+^2}{\sigma_+^2} - \frac{\rho_-^2}{\sigma_-^2} \right) ,
\]

\[
\rho_\pm = \frac{r_1}{l_{B1}} \pm \frac{r_2}{l_{B2}} ,
\]

where \( \sigma_+^2 = 4 / (1 + \epsilon) \). This shows that in the new vacuum state \( |0\rangle \) the distribution probability for the difference coordinate \( \rho_- \) is squeezed at the expense of the sum coordinate \( \rho_+ \), see Fig. 2, for the \( x \)-components of coordinates \( \rho_+ \) and \( \rho_- \) (at \( y_+ = y_- = 0 \)) for several different values of the charge ratio \( q_2 / q_1 \). Note that in the limit \( q_2 \to q_1 \) (\( \epsilon \to 1^{-0} \)) the relative coordinate distribution becomes maximally squeezed while the center-of-charge coordinate becomes extended, \( \sigma_+ \to \infty \). This is because the components of \( \rho_1 \) and \( \rho_2 \) do not commute in the lowest LL approximation, see Sec. 3 below.

III. NEUTRAL e-h SYSTEMS

For a neutral system \( q_2 = -q_1 = q \) the MT operator is given by \( K = -i \hbar \nabla_1 - i \hbar \nabla_2 - q \mathbf{B} \times (r_1 - r_2) / 2c \). Its components commute [\( K_x, K_y \) = 0] so that the MT group is abelian. Therefore, the states of a neutral magnetoelectron (MX) can be labeled by the magnetic momentum \( K = (K_x, K_y) \).

The ground state in zero LL is a \( K = 0 \) state, which can be presented as a squeezed two-mode vacuum

\[
|K = 0 \rangle = S |0 \rangle , \quad S = \exp \left( B_1^\dag B_2^\dag \right) .
\]

The coordinate representation is given by

\[
\langle r_1, r_2 | K = 0 \rangle = \exp \left( -\frac{r_1^2 + r_2^2 - 2z_1^* z_2}{4 l_{B}^2} \right) .
\]

This is a coherent state of an infinite number of electron and hole states in zero LL. A state with a finite momentum \( K = (K_x, K_y) \) is given by

\[
|K \rangle = |K_x, K_y \rangle = S(K) |0 \rangle , \quad S(K) = \exp \left[ \left( B_1^\dag + \frac{i k_+}{\sqrt{2}} \right) \left( B_1^\dag + \frac{i k_-}{\sqrt{2}} \right) \right]
\]

and is a two-mode squeezed displaced vacuum state. Expectation value of the relative coordinate \( r = r_1 - r_2 \) in
state \( |15 \rangle \) is given by \( \langle K|v|K \rangle = \frac{2}{\hbar} \times K \hbar^{-1} |r \rangle \). Also, the zero-momentum state \( |14 \rangle \) can be considered to be a limiting case of a charged system state \( |10 \rangle \). Indeed, when \( q_2 - q_1 = 0 \), wavefunction \( |10 \rangle \) becomes extended (its norm tends to zero) and its coordinate dependence becomes identical to \( |14 \rangle \). Using Eq. \( |11 \rangle \) we deduce that the relative coordinate \( \mathbf{r} = r_1 - r_1 \) becomes maximally squeezed (to the magnetic length \( \lambda \)) less MT operator for a neutral magnetoexciton, \( \hat{K}_- \) becomes in this representation \( \hat{K}_- = e^{-B_1 B_1^\dagger} |K; M \rangle \),

\[
|K; M \rangle = \frac{1}{\sqrt{I_M(\kappa^2)}} \sum_{m=0}^{\infty} \left( \frac{ik}{\sqrt{2}} \right)^m \frac{e^{i\kappa L}}{\sqrt{(m + M)!m!}} |m + M, m \rangle
\]

where

\[
\kappa_0 = \frac{1}{2} \left( B_1^\dagger B_1 + B_1^\dagger B_1 + 1 \right), \quad \kappa_1 = B_1^\dagger B_1, \quad \kappa_2 = B_1^\dagger B_1
\]

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8. For a multiparticle \( e^2 \hbar \) system, charges \( -q_1, q_2 \) and coordinates \( r_1, r_2 \) correspond to the total charges and center-of-charge coordinates of the \( e^- \) and \( e^- \) subsystems.
9. Note that for, e.g., the electron operator, \( D_e(\alpha) B_1^\dagger D_1^\dagger(\alpha) = B_1^\dagger + \alpha \), where a well known displacement operator is given by \( D_e(\alpha) = \exp(\alpha B_e^\dagger - \alpha^* B_e) \).