Biordered superconductivity and strong pseudogap state

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Interrelation between the two-particle and mean-field problems is used to describe the strong pseudogap and superconducting states in cuprates. We present strong pseudogap state as off-diagonal short-range order (ODSRO) originating from quasi-stationary states of the pair of repulsing particles with large total momentum ($K$ - pair). Phase transition from the ODSRO state into the off-diagonal long-range ordered (ODLRO) superconducting state is associated with Bose-Einstein condensation of the $K$ - pairs. A checkerboard spatial order observable in the superconducting state in the cuprates is explained by a rise of the $K$ - pair density wave. A competition between the ODSRO and ODLRO states leads to the phase diagram typical of the cuprates. Biordered superconducting state of coexisting condensates of Cooper pairs with zero momentum and $K$ - pairs explains some properties of the cuprates observed below $T_c$: Drude optical conductivity, unconventional isotope effect and two-gap quasiparticle spectrum with essentially different energy scales.

PACS numbers: 74.20.-z, 74.20.De, 74.72.-h

I. INTRODUCTION

The high-$T_c$ cuprate superconductor can be considered a doped two-dimensional (2D) antiferromagnetic (AF) insulator with strong Coulomb repulsion. Most commonly, one accepts that the physics of the cuprates can be understood within the simplest one-band models of strongly correlated systems such as the Hubbard and $t-J$ models. It seems impossible to obtain analytic solutions for the ground states of these 2D models. Therefore, a variational approach based on a choice of an appropriate trial wave function can be considered as a natural way to solve the cuprate problem.

The wave function of the resonating valence bond (RVB) ground state, derived from Gutzwiller projected d - wave ground state of the Bardeen-Cooper-Schrieffer (BCS) model eliminates a possibility of double occupancy of a site (no double-occupancy constraint) and corresponds to extremely strong on-site correlations. A great many of important results, obtained within the RVB approach, can be considered as a ground of the physics of the cuprates.

However, the problem is complicated by the fact that, in a strong correlated system, ground state energies of different ordered states may be close to each other d - wave superconductor (dSC), staggered flux phase, spin and charge density waves (SDW and CDW, respectively) and some others. Account of the competition and coexistence of such ordered states within extremely simplified models by numerical tools leads to a wide variety of phase diagrams. It is clear that, among them, one can always find the diagrams reminding those typical of the cuprates.

Various approximations within the on-site Coulomb repulsion models often lead to antipodal conclusions concerning a possibility of the superconducting (SC) state itself. The complete suppression of the double occupancy under Gutzwiller’s projection promotes a rise of an insulating rather than SC order. In this connection, Laughlin has proposed an alternative approach to clarify the problem of the superconductivity of the cuprates. Instead of numerical study of a highly simplified Hamiltonian, he suggested to select a reasonable ground state in order to determine the Hamiltonian leading to such a state.

To take into account a realistic on-site repulsion, the ground state of Laughlin’s gossamer superconductor is chosen in the form of an incomplete projected BCS $d$ - wave state with partially suppressed double occupancy. Hamiltonian with such an exact forethought ground state, along with strong on-site repulsion, manifests an attractive term which can lead to dSC state.

Gossamer superconductivity of the underdoped compound can be associated with a band of states with relatively low spectral weight inside a pronounced insulating forbidden band so that the chemical potential turns out to be pinned near the middle of this band.

The repulsion-induced dSC state is highly sensitive to electron dispersion. The simplest tight-binding approximation taking into account only the nearest neighbors ($t$ - model) seems to be insufficient, therefore, it is necessary to consider more complicated dispersion with the next nearest neighbor terms ($t-t'$ - model). Numerical study shows that the stable dSC state corresponds to hopping integral ratio $t'/t$ within a narrow range near $t'/t = -0.3$. Just the same value of $t'/t$ is consistent with available angle resolved photoemission spectroscopy (ARPES) data.

As follows from the $SU(2)$ approach to the RVB problem, it is necessary to consider doublets of fermions and bosons to realize the spin-charge separation correctly. Two minima of the $SU(2)$ boson dispersion are relative to the points (0, 0) and $(\pi, \pi)$ in the 2D Brillouin zone, therefore, the SC pairing channel corresponding to large pair momentum should be taken into account along with the Cooper channel corresponding to zero pair momentum.

Geshkenbein et al. have assumed that an enhancement of the scattering between the saddle points of electron dispersion results in the fact that the electron-
electron interaction with large momentum transfer can be “less repulsive” with respect to small transfer. Therefore, in the vicinities of the saddle points, fermions may be paired into bosons. Such noncoherent preformed pairs arising near the antinodal arcs of the Fermi contour (FC) might exist in the pseudogap state of underdoped cuprates as a normal Bose liquid.\textsuperscript{11}

The FC outside of the arcs corresponds to unpaired fermions coexisting with the preformed pairs. Bose-Einstein condensation (BEC) of the preformed pairs with large momentum due to their interaction with unpaired particles results in the SC gap on the whole of the FC. The SC state that arises in such a way describes reasonably rather wide (intermediate with respect to BCS and BEC limiting cases) fluctuation region above $T_c$.

Instabilities in 2D strong correlated electron system were investigated within the $t - t'$ – model at small $t'$ by renormalization-group (RG) methods\textsuperscript{12,13} using a discretization of the FC into a finite number of patches. The singularity in the Cooper channel exhibits a squared logarithmic divergence at low energies. For insulating Peierls channel with electron-hole pair momentum $Q = (\pi, \pi)$, the singularity also exhibits a squared logarithm in the particular case $t' = 0$ when nested FC has the form of a square coinciding with the boundary of the magnetic Brillouin zone of the parent compound. At $t' \neq 0$ and low doping, that is in the case of a deviation of the FC from the perfect nesting, the divergence is found more weak with logarithmic enhancement of the order of $\ln |t/t'|$ under the condition that $|t'| < |t|$. The singularities in the insulating and SC channels corresponding to zero and $Q = (\pi, \pi)$ pair momenta, respectively, are found to be logarithmic in the case of small but nonzero $|t'/t|$. Such a case corresponds to approximately nested FC disposed close to saddle-point van Hove singularity. RG approach, involving the nesting effects\textsuperscript{14} gives a possibility to select singular contributions into pairing channels but remains corresponding pre-exponential factors to be undetermined.

General symmetry consideration, based on Zhang’s SO(5) theory\textsuperscript{15} or SU(4) theory by Guidry et al.\textsuperscript{16} shows that one should take into account a closed set of competing ordered states to describe key features of correlated electron system. In this sense, singlet SC pairing channel with large momentum incorporating singlet orbital insulating long-range (possibly, hidden\textsuperscript{17,18} or short-range (fluctuating between dSC and staggered flux states)\textsuperscript{19} order may be naturally connected with the Cooper channel. Thus, there should be two SC gap parameters related to large and zero pair momentum, respectively.

The SC gap, which determines $T_c$ and corresponds to a rise of the coherence in the system of electron pairs, can be directly extracted from experiments on Andreev reflection\textsuperscript{20} or Josephson tunneling.\textsuperscript{21} The observation of two SC gaps of about 10 meV and 50 meV, respectively, in tunnel experiment\textsuperscript{22} in Bi2212 (in particular, a suppression of the lesser gap in high magnetic field at temperatures $30 – 50 \, mK$) may be considered as an indirect evidence in favour of two SC energy scales in the cuprates.

One more energy scale, observed in ARPES and tunnel spectra of underdoped cuprates,\textsuperscript{22,23} can be associated with the strong pseudogap state.\textsuperscript{24} To describe this state one can start from a reasonably chosen one-particle Green function. Recently, Yang et al.\textsuperscript{25} developed RVB phenomenology of the pseudogap state based on the assumption that this state can be viewed as a liquid formed by an array of weakly coupled two-leg Hubbard ladders. The coherent part of the Green function obtained within the random phase approximation is consistent with the Luttinger theorem\textsuperscript{26} and describes evolution of the FC (from small pockets to closed contour) with doping. Similar results follows from both the spin-charge separation approach\textsuperscript{22} and phenomenological account of short range insulating order above $T^\ast$.\textsuperscript{27}

In this paper, we develop the concept of Coulomb pairing\textsuperscript{28} leading to the biorered state originating from two SC pairing channels, with large and zero pair moments. The all-sufficient conditions of repulsion-induced superconductivity in these two channels are discussed in Sec. II. Sec. III deals with the strong pseudogap state arising from incoherent quasi-stationary states of pairs with large momenta that are inherent in the screened Coulomb pairing potential. In Sec. IV, we consider the symmetry and two-gap spectrum of the biorered state. Finally, some possible manifestations of the strong pseudogap and biorered SC states are discussed in Sec. V.

## II. COMPETING PAIRING CHANNELS

Screening of Coulomb repulsion in three-dimensional isotropic degenerate electron gas results in momentum dependent interaction energy of two electrons,

$$U(k) = 4\pi e^2/|k^2\epsilon(k)|,$$

(1)

where static permittivity has the form\textsuperscript{30}

$$\epsilon(k) = 1 + 1 + x^2 2k^2 4x ln(1 + x)/(1 - x).$$

(2)

Here, $x = k/2k_F$, $k_F$ and $k_F^{-1} = (4\pi e^2 n g)^{1/2}$ are Fermi momentum and screening length, respectively, $n$ and $g$ are electron concentration and density of states on the Fermi level.

Kohn singularity at $k = 2k_F$ leads to the interaction energy with damped Friedel oscillation in the real space. At a distance $r \gg k_F^{-1}$, this potential can be written as\textsuperscript{30}

$$U(r) \approx e^2 \cos 2k_F r / 2\pi r^3.$$  

(3)

Kohn and Luttinger\textsuperscript{31} have argued that attractive contribution into screened Coulomb repulsion originating from Friedel oscillation is sufficient to ensure Cooper pairing with non-zero angular momentum. Because of the weakness of the Kohn singularity, corresponding SC transition temperature turns out to be very low.\textsuperscript{32}
In the case of nested FC, the Kohn singularity transforms into the Peierls one with strong anisotropy of $\epsilon(k)$. Therefore, effective pairing interaction can be enhanced both in particle-hole and particle-particle channels. In particular, this can give rise to CDW or SDW in singlet or triplet insulating pairing channels, respectively.

Peierls singularity in a particle-hole channel originates from the fact that momentum transfer turns out to be equal to nesting vector $Q$ for any particle on the FC (Fig. 1a). For the sake of simplicity, in Fig. 1 the FC is presented as a square corresponding to the $t$ - model at half-filling. In the case of low doping and under the condition that $|t'| \ll t$, one can expect that the FC may be found close to this square.

In the Cooper channel, Peierls enhancement of the Kohn singularity emerges as appreciably more weak because momenta before and after scattering ($p$ and $p'$ in Fig. 1b, respectively) giving rise to this enhancement should be related as $p - p' \approx Q$. Integration over momentum transfer $p' - p$ along the whole of nested FC smoothes down the Peierls singularity from the Cooper channel. Logarithmic singularity ($\ln(|\varepsilon_0/\varepsilon|)$, where $\varepsilon_0$ is a cut-off energy) in the Cooper channel is ensured by a general feature of electron dispersion, $\varepsilon(-p) = \varepsilon(p)$, that holds for any momentum $p$ (statistical weight of the Cooper pair is proportional to the length of the entire FC).

Density of states of 2D system manifests logarithmic van Hove singularities originating from saddle-point vicinities with hyperbolic metric. Due to close proximity of the FC and the isoline connecting saddle points ($\pm \pi, \pm \pi$), effective coupling constant $w$ turns out to be logarithmically enhanced, $w \to w \cdot \ln(2k^2/|\varepsilon - \varepsilon_0|)$ Here, $\varepsilon_0$ is saddle point energy (within the $t$ - model, $\varepsilon_0 = 0$) and $k_0$ has meaning of a scale of the part of 2D Brillouin zone with hyperbolic metric.

In the case of SC pairing with large total momentum $K$ ($K$ - channel), the momenta of the particles composing a pair with given momentum ($K$ - pair), both being either inside or outside the FC at $T = 0$, should belong to only a part of the Brillouin zone (domain of kinematic constraint) rather than the whole one. In a general case, kinetic energy of the pair with relative motion momentum $k$,

$$2\varepsilon_K(k) = \varepsilon(K/2 + k) + \varepsilon(K/2 - k) - 2\mu,$$

vanishes only at some points of the FC inside this domain ($\mu$ is the chemical potential). Therefore, in contrast with the Cooper pairing when $\varepsilon_0(k) = 0$ on the whole of the FC, integration over $k$ eliminates the logarithmic singularity in the $K$ - channel. However, if kinetic energies, $\varepsilon(K/2 + k)$ and $\varepsilon(K/2 - k)$ coincide on finite pieces of the FC (“pair” Fermi contour, PFC), logarithmic singularity $\ln(|\varepsilon_0/\varepsilon|)$ survives and the $K$ - channel can result in the SC order.

Mirror nesting condition,

$$\varepsilon(K/2 + k) = \varepsilon(K/2 - k),$$

determines the locus in the momentum space that logarithmically contributes to the $K$ - channel. Statistical weight of $K$ - pair, proportional to the length of the PFC, can be less in comparison with the Cooper channel.

Mirror nesting is a necessary (not all-sufficient) condition of the SC pairing with large momentum. Indeed, this condition is perfectly satisfied in the case of nested FC when $K = Q_\pi$ (Fig. 1c). However, it is obvious that, in such a case, domain of kinematic constraint degenerates into a line resulting in zero statistical weight of the paired state. To obtain finite statistical weight, one can choose incommensurate pair momentum $K \neq Q_\pi$ This results in the domain of kinematic constraint in the form of relatively narrow strip containing the PFC as shown in Fig. 1d. The coefficient of the logarithmic contribution to the $K$ - channel should be proportional to the length of the PFC. Considering the PFC as two patches connected by nesting vector $Q_\pi$ (Fig. 1d), one can conclude that this coefficient has to be logarithmically enhanced.

**FIG. 1:** Nested Fermi contour (bold line) in the form of a square coinciding with 2D magnetic Brillouin zone of parent compound with half-filled conduction band (within tight-binding model with nearest-neighbor interactions). a: electron-hole pairing with momenta $p$ and $p' = p - Q$ ($Q$ is nesting momentum); b: Cooper pairing with zero total momentum ($p$ and $p'$ are momenta before and after scattering, respectively); c: SC pairing with nesting momentum ($p_+ = Q/2 \pm k$, where $k$ is momentum of the relative motion of the pair). Domain of kinematic constraint is degenerated into a line coinciding with one of the sides of the square; d: SC pairing with an incommensurate total momentum $K$ ($p_+ = K/2 \pm k$). Domain of kinematic constraint is bounded by the line 1-2-3-4-5-6. Parts 2-6 and 3-5 of this line are nested pieces of the FC resulting in singular contribution into this pairing channel.
by umklapp scattering inherent in the Peierls channel.\cite{footnote12} Patch approximation\cite{footnote14} appears to be relatively good just in the case of short PFC. The reason is that integration over momentum transfer $p - p' \approx Q_x$ cannot completely eliminate enhancement of the pairing interaction due to logarithmic singularities of the permittivity.

Another necessary condition of SC pairing under repulsion is connected with the existence of oscillating attractive contribution into the pairing potential. It should be noted that an oscillation itself cannot ensure a rise of a bound state. For example, simple step-wise repulsive potential $U(k) = U_0 > 0$ defined in a finite domain of the momentum space oscillates in the real space. However, by analogy with the problem of a bound state in one-dimensional asymmetric potential well,\cite{footnote14} such a potential cannot result in a bound state even under mirror nesting.

One can consider screened Coulomb potential $U(k - k')$ as a kernel of Hermite integral operator with complete orthonormal system of eigenfunctions defined within domain of kinematic constraint $\Xi$,

$$\varphi_s(k) = \lambda_s \sum_{k' \in \Xi} U(k - k') \varphi_s(k').$$

Here, a set of $\lambda_s$ represents the spectrum of a pairing operator which can be written in the form of the Hilbert-Schmidt expansion,

$$w(k, k') = \sum_s \frac{\varphi_s(k) \varphi_s^*(k')}{\lambda_s}.$$ 

The necessary (and sufficient, under mirror nesting) condition of the SC pairing under repulsion is the existence of at least one negative eigenvalue of the pairing operator.\cite{footnote20}

In the case of comparatively small domain of kinematic constraint, one can replace the screened Coulomb potential by its expansion in powers of momentum transfer, $\kappa = k - k'$, up to the term of the second order,

$$w(\kappa) = U_0 r_0^2 (1 - \kappa^2 r_0^2/2)$$

where $U_0$ and $r_0$ have meaning of an on-site repulsive energy and screening length, respectively. The simplest repulsive kernel,\cite{footnote3} defined inside $\Xi$, has two even and two odd (with respect to inversion $k \rightarrow -k$) eigenfunctions. Singlet SC order parameter should be determined by only even eigenfunctions belonging to eigenvalues $\lambda_1$ and $\lambda_2$ of opposite sign.

Scattering between nested pieces of the FC leads to strong anisotropy of the permittivity. Therefore, expansion of $w(k, k')$ in powers of momentum transfer close to nesting momentum $Q$ appears to be anisotropic as well. Resulting pairing interaction kernel, analogous to Eq.\cite{footnote3}, preserves its eigenvalue feature $\lambda_1 \lambda_2 < 0$. It should be noted that nesting momentum $Q \neq Q_x$, therefore, $Q$ has the meaning of a new nesting momentum, inherent in the real FC, which can result in the Peierls enhancement of the SC pairing.

FIG. 2: Repulsive pairing potential, $w(r)$, and bound state, $|\psi|^2$ (dotted line), distributions in the real-space (schematically). Energies $E_b$ and $E_q$ correspond to bound and quasi-stationary states, respectively. Barrier height $E_b$ corresponds to a break of the pair without tunnelling through the barrier.

Matrix of pairing operator Eq.\cite{Eq.17} between its eigenfunctions is diagonal, $w_{ss'} = \lambda_s^{-1} \delta_{ss'}$. The necessary condition of the existence of nontrivial solution to the self-consistency equation with kernel\cite{footnote3} has the form $\lambda_1 \lambda_2 < 0$. Written in arbitrary basis, it takes the form of the Suhl inequality,

$$w_{11}w_{22} - w_{12}w_{21} < 0,$$

introduced as a necessary condition of superconductivity within two-band model.\cite{footnote33}

The effective pairing potential oscillates in the real space and, in agreement with Laughlin’s proposal,\cite{footnote2} manifests repulsive core at small distance (corresponding to incomplete double-occupancy constraint) and attractive contribution outside of the core. Thus, there is a possibility of a rise not only of a bound (with negative energy, $E < 0$) but also of quasi-stationary (with $E > 0$) paired state with large momentum (Fig. 2).

The singular contribution into SC order parameter is determined by relatively small vicinity of the PFC with an energy scale $\varepsilon_0$. In this respect, repulsion-induced $K$-pairing seems to be similar to phonon-mediated pairing arising from attraction with negative coupling constant $V$. Rough estimation\cite{footnote4} of Coulomb repulsion within the phonon-mediated mechanism of superconductivity leads to the fact that, to ensure SC pairing, $|V|$ must exceed a threshold value,

$$|V| > \frac{U_c}{1 + gU_c \ln (E_F/\varepsilon_D)}.$$ 

Here, $E_F$ and $U_c$ are Fermi and average Coulomb energies, respectively. Phonon-mediated attraction is defined inside a narrow layer (domain of dynamic constraint with energy scale of the order of Debye energy $\varepsilon_D$) enveloping the FC.

One can expect that a deviation from perfect mirror nesting condition\cite{footnote3} outside of the PFC does not eliminate the logarithmic singularity in the $K$-channel. Typical of high-$T_c$ cuprates, nearly nested FC in the form...
III. STRONG PSEUDOGAP STATE

Taking into account the ground state instability due to a rise of pairs, the mean-field approach to the problem of superconductivity excludes fluctuations of paired states from consideration. Within the mean-field theory, the SC gap is directly relative to the binding energy of a pair resulting from the two-particle Cooper problem. In the case of the \( K \)-pairing, such a problem may admit more complicated solution as compared to the attraction-induced pairing.

Integrable equation which determines a wave function of the relative motion of two interacting particles (holes) above (below) the FC can be written as

\[
\psi(k) = G^{(0)}(\omega; k) \sum_{k' \in \Xi} w(k, k') \psi(k').
\]

Here, \( \omega \) and \( k \) \((k')\) are an energy and momentum of the relative motion before (after) scattering, respectively, and

\[
G^{(0)}(\omega; k) = [\omega - \xi(k) + i \gamma]^{-1}
\]

is one-particle Green function corresponding to free relative motion of the \( K \)-pair, \( \gamma \to +0 \). In contrast with one-particle Landau Fermi liquid Green function, the condition that \( [G^{(0)}(0, k)]^{-1} = 0 \) does not determine a closed FC. Indeed, a locus in the momentum space resulting from this condition written in equivalent form \( \xi_\lambda(k) = 0 \), is either some isolated points or finite pieces of mirror nested FC.

In the case of mirror nested FC, one can separate a singular contribution to the Green function originating from relatively small (with energy scale \( \varepsilon_0 \)) part \( \Xi_\delta \) of the domain of kinematic constraint \( \Xi \). The rest of the domain, including an energy range from \( \varepsilon_0 \) up to a cut-off value of about \( E_p \), results in a regular contribution into \( G^{(0)} \). One can consider this contribution in a way similar to the account for the Coulomb repulsion within phonon-mediated pairing attraction scenario and renormalize kernel Eq. \( \Xi \) to a kernel, defined inside \( \Xi_\delta \), with the same spectrum.

One-particle Green function \( G(\omega; k) \) corresponding to relative motion of \( K \)-pair of particles (holes) excited above (below) the FC can be represented in the basis formed by the eigenfunctions of the renormalized pairing operator \( w(k, k') \),

\[
G_{ss'}(\omega) = \sum_{k \in \Xi} \varphi_s^*(k) G(\omega; k) \varphi_{s'}(k).
\]

Matrix elements \( G_{ss'}^{(0)}(\omega) \) are the solutions to Dyson equation,

\[
\sum_{s''} \{ \delta_{ss''} - \lambda_{ss'}^{-1} G_{ss''}^{(0)}(\omega) \} G_{s''s'}(\omega) = G_{ss'}^{(0)}(\omega),
\]

in which matrix elements \( G_{ss'}^{(0)}(\omega) \) of free Green function are defined similar to Eq. \( \Xi \).

Pairing operator with two even eigenfunctions results in \( 2 \times 2 \) matrix \( \Xi \). One can resolve Eq. \( \Xi \) with respect to \( G_{ss'}(\omega) \) and then obtain \( G(\omega; k) \) in the form

\[
G(\omega; k) = D^{-1}(\omega) [G^{(0)}(\omega; k) - B(\omega; k)],
\]

where

\[
B(\omega; k) = \lambda_1^{-1} \lambda_2^{-1} B(\omega) \sum_{s=1}^2 \lambda_s |\varphi_s(k)|^2,
\]

and

\[
D(\omega) = 1 - \frac{G_{11}^{(0)}(\omega)}{\lambda_1} - \frac{G_{22}^{(0)}(\omega)}{\lambda_2} + B(\omega) \frac{\lambda_1 \lambda_2}{\lambda_1^2 + \lambda_2^2}.
\]

In the case of mirror nested FC, Green function manifests a pole resulting in a bound state with negative energy \( \omega = E_i \) determined from equation \( D(\omega) = 0 \) in which all functions \( G_{ss'}^{(0)}(\omega) \) are real. This pole is related to instability of the ground state with respect to a rise of pairs. Within a small vicinity of the pole, Green function can be represented as

\[
G(\omega; k) = \frac{[G^{(0)}(\omega; k) - B(\omega; k)]}{D'(E_i)} \frac{1}{\omega - E_i}
\]

where \( D' = dD/d\omega \).

At \( \omega > 0 \), Green functions are complex and equation \( D(\omega) = 0 \) can lead to complex solution \( \omega = E_q - i\Gamma \) where \( E_q \) and \( \Gamma \) have meanings of energy and decay of quasi-stationary state (QSS) of the relative motion of \( K \)-pair, respectively. Near this complex pole, Green function has the form of Eq. \( \Xi \) where \( E_i \) should be replaced by \( E_q - i\Gamma \).

Wave functions of the relative motion of \( K \)-pair corresponding to both bound state and QSS, are localized, in main, in a wide region of the real space outside the repulsive core as shown schematically in Fig. 2.
The $K$ - pairs can exist above $T_c$ as long-living QSS due to considerable increase of density of states in a narrow vicinity of $E_g$. To overcome the potential barrier before tunnel decay, such a non-coherent pair should accumulate an energy exceeding barrier height $E_g$. Thus, the energy $E_g - E_i$ is sufficient to destroy SC coherence whereas corresponding pair-break energy should exceed $E_b - E_i$. A temperature range between the SC transition temperature $T_c \sim E_g - E_i$ and a crossover one, $T_{str} \sim E_b - E_i$, can be interpreted as a strong pseudogap state observable above $T_c$ in underdoped cuprates. If density-of-states peak at $\omega \approx E_i$ turns out to be smoothed due to $\Gamma$ being large enough, the strong pseudogap state becomes unobservable. In such a case, the SC transition from coherent into non-coherent state should be assisted with a break of pairs at energies $\approx E_b - E_i$ similar to that in the BCS theory.

By analogy with the interrelation between the Cooper two-particle problem and BCS theory, the pair-break energy $E_b - E_i$ due to direct excitation of particles from bound state into continuous spectrum should be transformed into momentum-dependent energy gap $\Delta(k)$ in the quasiparticle spectrum. In the strong pseudogap state, this gap, due to a non-coherence of QSS, can be presented as $\Delta = \sqrt{\Delta^2_p + \Delta^2_p}$. Here, $\Delta^c \sim E_g - E_i$ corresponds to transition from the coherent into non-coherent QSS and $\Delta^p \sim E_b - E_i$ can be related to a break of $K$ - pair as a result of transition between two non-coherent states.

Microscopically, SC gap $\Delta_p$ and strong pseudogap $\Delta_p$ emerge with random phases. Therefore, mean-field value $\Delta_p$ vanishes at any temperature whereas $\Delta^c$ becomes nonzero below $T_c$ due to Bose condensation of $K$ - pairs from QSS into the bound state and vanishes only above $T_c$. However, nonzero mean square strong pseudogap, $\langle \Delta_p \rangle^2 \neq 0$, may become apparent well above $T_c$. In this sense, pseudogap parameter $\Delta^p$, corresponding to decay of QSS of $K$ - pairs, reminds RVB spin liquid pseudogap introduced by Yang et al. However, it has different physical meaning.

Green function $G^{(0)}(0; k)$ changes sign on the PFC from positive to negative through an infinity. Therefore, Green function $G^{(0)}(0; k)$ at $\omega = 0$ manifests the same feature. It should be noted that, in the case of Cooper pairing, the Green function changes sign on the whole of the FC. In addition, Green function $G^{(0)}(0; k)$ changes sign on a zero line determined by equation $G^{(0)}(0; k) = B(0; k)$. This line does not coincide with the PFC.

Green function of the two-particle problem has a pole corresponding to a bound state of the relative motion of the pair. Therefore, one can suppose, in line with Yang et al., a phenomenological BCS-like form of the coherent contribution to the normal (diagonal) Gor’kov Green function of the mean-field problem

$$G(\omega; k) = z_k \left[ \frac{u^2_z(k)}{\omega - E(k) + i\Gamma} + \frac{u^2_z(k)}{\omega + E(k) - i\Gamma} \right].$$  \hspace{0.5cm} (20)

where $E = \sqrt{\xi^2_K + |\Delta|^2}$ and $2u^2_z = 1 \pm \xi_K/E$ are quasiparticle energy and coherence factors, respectively. In accordance with (19), one should suppose that momentum-dependent quasiparticle weight $z_k$ vanishes on the line of zeroes and corresponds to a finite value $z$ ($0 < z < 1$) on the PFC. Two terms can be referred to pairs above and below the FC.

Diagonal Green function describes a non-superconducting state with off-diagonal short-range (ODSR) corresponding to the existence of non-coherent $K$ - pairs above $T_c$. Below $T_c$, the ODSRO transforms into the off-diagonal long-range order (ODLO) introduced by Yang.

Excitation with a transition from the bound paired state into long-living QSS corresponds to quite small but finite decay $\Gamma = \Gamma(\omega; k)$. The transitions into stationary states above barrier energy $E_b$ should be associated with an infinitesimal decay, $\gamma \to +0$, leading to conventional Fermi-liquid behavior of diagonal Gor’kov function above $T_{str}$. Thus, a rise of QSS results in a non-Fermi-liquid behavior of diagonal Green function that can be manifested in rather wide temperature range $T_c < T < T_{str}$ relating to strong pseudogap state. This range corresponds to transitions between boson-like bound and quasi-stationary states. Therefore, Eq. (20) can be considered as a bridge between the BCS and BEC approaches to the problem of superconductivity, in accordance with the assumption by Geshkenbein et al.

IV. SUPERCONDUCTING STATE WITH LARGE MOMENTUM

One can believe that the nearly nested FC of underdoped (up to optimum doping) cuprate compound in the form of a square with rounded corners, shown schematically in Fig. 3, results in the fact that the $K$ - channel corresponding to an incommensurate momentum $K$ dominates the Cooper channel. In such a case, it is a rise of coherence in the system of $K$ - pairs that determines SC transition temperature $T_c$. Therefore, there is a temperature range well below $T_c$ in which mean-field SC order parameter $\Delta_c(k)$, relating to the $K$ - channel, can be approximately considered as governed by the only self-consistency equation,

$$\Delta_c(k) = -\frac{1}{2} \sum_k \frac{\omega(k, k')\Delta_c(k')}{E(k')} \tanh \left( \frac{E(k')}{2T} \right).$$ \hspace{0.5cm} (21)

Quasiparticle energy,

$$E(k) = \sqrt{\xi^2_K(k) + |\Delta_c(k)|^2 + |\Delta_p(k)|^2},$$ \hspace{0.5cm} (22)

aside from $\Delta_c(k)$, includes strong pseudogap parameter, $\Delta_p(k)$, associated with QSS that can exist above $T_c$. Thus, Eq. (21) reflects the fact that SC order arises from the state other than normal Fermi liquid. Therefore, Eq. (21) differs from the conventional BCS self-consistency equation. It is reasonable to assume that
\( \Delta_c(k) \) is small above optimum doping and gradually increases with underdoping. This leads to smoothing of the singularity in Eq. (21) and, as a result, to a gradual decrease in \( T_c \) with underdoping.

One can reduce summation in Eq. (21) to small part \( \Xi_s \) of the domain of kinematic constraint similar to the two-particle problem considered above. Under repulsive interaction, a non-trivial solution to Eq. (21) can arise due to a competition of positive and negative contributions to the right-hand side of this equation. Thus, such a solution should have a line of zeroes (nodal line, NL) inside \( \Xi_s \).

In the case of phonon-mediated pairing with account of Coulomb repulsion\(^{22}\), the NL of the order parameter coincides with the boundary enclosing the domain of dynamic constraint. In this domain, attraction dominates logarithmically weakened repulsion in accordance with Eq. (11). This NL is disposed everywhere outside of the FC, therefore, mirror nesting of the FC can be considered as the only condition of the \( K \) - pairing under attraction.

Peierls enhancement of the pairing interaction results in a strong anisotropy of the NL disposed close to the FC inside \( \Xi_s \). This corresponds to an increase in the dominant part of \( \Xi_s \) that mainly contributes into the logarithmic singularity in the self-consistency equation and thus increases a magnitude of \( \Delta_c \) (it is clear, that \( \mid \Delta_c \mid \) should be much lesser than \( \delta_0 \)).

Since the SC order parameter arising in the \( K \) - channel is essentially momentum-dependent, there are three characteristic lines of zeroes: 1) the PFC on which kinetic energy of the pair equals zero, \( 2 \varepsilon_k(k) = 0 \); 2) the NL of the order parameter determined by \( \Delta_c(k) = 0 \); 3) the curve on which quasiparticle group velocity changes sign, \( \nabla_k E(k) = 0 \).

These three lines may have common points of intersection inside \( \Xi_s \), therefore, the NL can be disposed both above and below the PFC. This results in qualitatively different non-monotonic momentum dependence of coherence factors \( u^2_k(k) \) for two kinds of directions in the \( k \) - space intersecting at first the PFC and then the NL and vice versa. Under pairing repulsion, the scattering across the NL turns out to be dominating in comparison with scattering inside or outside the NL in accordance with Suhl inequality Eq. (9).

Due to the fact that \( \mid \Delta_p \mid \neq 0 \) in the strong pseudogap state, coherence factors in diagonal Gor’kov function (20) may overlap each other near the PFC even above \( T_c \). On the contrary, the BCS coherence factors are step-wise functions without an overlap in the normal Fermi liquid state.

The SC state that arises below \( T_c \) should be described by both diagonal and off-diagonal (anomalous) Gor’kov functions. Taking into account the fact that mean-field (averaged over random phases) pseudogap parameter \( \Delta_p \) vanishes whereas mean-field SC condensate parameter \( \Delta_c \) \neq 0 below \( T_c \), one can introduce off-diagonal Gor’kov function \( F^+(\omega; k) \) in a phenomenological way similar to that we use to obtain diagonal Gor’kov function (20). This function describes the ODLRO state and can be written as

\[
F^+(\omega; k) = -\frac{z_k \Delta_c}{(\omega - E(k) + i\Gamma)(\omega + E(k) - i\Gamma)}.
\]  

Factor \( z_k \) is defined inside each of the crystal equivalent domains of kinematic constraint \( \Xi_j \) where \( j = 1,2,3,4 \) in the case of tetragonal symmetry of cuprate planes. Paired states with large total momenta \( K_j \), both coherent and non-coherent, arise exactly inside these domains. Parameters \( \Delta_{cj} \), \( \Delta_{pj} \) and \( \Gamma_j \) are identical for any of \( \Xi_j \) differing only by the domain of definition of \( k \).

In the whole of the Brillouin zone, the SC order parameter in the mixed representation, \( \Delta_{cj}(R,k) \), can be presented as a superposition,

\[
\Delta_c(R,k) = \sum_{j=1}^{4} \gamma_j(k) e^{iK_j R} \Delta_{cj}(k),
\]  

where \( R \) is center-of-mass radius-vector, coefficients \( \gamma_j(k) \) should be chosen in accordance with the symmetry of the order parameter.

As a function of \( R \), order parameter (24) arising in the \( K \) - channel turns out to be spatially modulated similar to Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state. Such a modulation with relatively short wavelength reflects a variation of \( K \) - pair density and can be associated with a pair density wave with a checkerboard order in cuprate planes. Thus, the \( K \) - pairing leads to a microscopic ground of the pair density wave concept introduced phenomenologically by Zhang.\(^{25} \)
The SC order parameter can be expanded over the eigenfunctions of pairing operator \( \varphi_s(k) \)

\[
\Delta_c(R, k) = \sum_s \Delta_{cs}(R) \varphi_s(k). \quad (25)
\]

In the simplest case, repulsion-induced SC pairing can be described by two-component order parameter, \( \Delta_{cs} \), in the vicinities of the PFC and entire FC, relative to the magnitude of \( \Delta_0 \)

to the whole of the FC should be induced by \( \Delta_0 \)

and, as a result, in the superfluid density which becomes proportional to the whole of the FC length at \( T \lesssim T'_c \).

In the vicinities of the PFC, two branches \((m = 1, 2)\) of strong anisotropic quasiparticle spectrum of the biordered superconductor can be written in the form

\[
E_m(k) = \sqrt{\xi_0^2(k) + |\Delta_p(k)|^2 + |\Delta_s(k)|^2}. \quad (26)
\]

Here, we take into account the fact that kinetic energy of \( K \) - pair is equal to kinetic energy of Cooper pair,

\[
2\xi_0(k) = \varepsilon(K/2 + k) + \varepsilon(-K/2 - k) - 2\mu. \quad (27)
\]

Two-gap spectrum \((24)\), with the lesser gap \( |\Delta_c - \Delta_0| \)

observable at excitation energies up to the greater gap \( |\Delta_c + \Delta_0| \), should be apparent at \( T < T'_c \).

Above the greater gap the spectral weight transfers from the low to high-energy branch of quasiparticle spectrum.

Diagonal and off-diagonal Gor’kov functions of the biordered SC state preserve their form, Eqs.\((24)\) and \((25)\), respectively, with the exception of the fact that SC order parameter \( \Delta_s \) has to take into account both SC pairing channels. Thus, these two channels result in two coexisting ODLRO states.

Most likely, Cooper channel, including both Coulomb and phonon-mediated pairing, cannot result in a rise of QSS. Therefore, SC gap parameter turns out to be BCS-like everywhere on the FC with the exception of the PFC on which the unconventional \( K \) - channel is opened.

Symmetry of the biordered SC state is determined by Eqs.\((24)\) and \((25)\) where \( \Delta_s \) should be considered as the components of momentum dependent order parameter arising as a result of both Cooper and \( K \) - pairing. One can approximately represent the gap parameter in the conventional form

\[
\Delta(k) = D(k)(\cos k_x \pm \cos k_y) \quad (28)
\]

where momentum dependent magnitude \( D(k) \) reflects mainly strongly anisotropic contribution of the \( K \) - pairing. Upper (lower) sign in Eq.\((28)\) corresponds to extended \( s(d) \) - wave symmetry of the order parameter.

It should be noted that, in the case of Peierls enhanced \( K \) - pairing, nodal line of order parameter \( \Delta(k) \) may pass through the center of the corresponding domain of kinematic constraint. This results in the fact that order parameter \( \Delta(k) \) differs in sign on the opposite parts of the PFC.

VI. CONCLUSION

We believe that biordered superconductivity may be generic for such superconductors as doped cuprate compounds. Unconventional features of these compounds,
especially, universality of their phase diagram (Fig.4), can be associated with evolution of the FC and pairing interaction with doping.

It is clear that singular contribution to Coulomb pairing interaction is sensitive to doping dependent form of the FC. Therefore, one can suppose that oscillating real-space pairing potential $\omega(r)$ varies with doping in such a way that only noncoherent QSS of SC pairs arise under extremely low doping. This corresponds to strong pseudogap penetrating into insulating region of doping below the onset of superconductivity at $x = x_*$.

In overdoped regime $x_ < x < x_{opt}$, along with QSS, there is a bound state. Both bound state energy $|E_b|$ and QSS decay $\Gamma$ increase with doping so that, near optimal doping $x_{opt}$, pair-break energy approximately coincides with the energy corresponding to the loss of phase coherence. Thus, in overdoped regime $x_{opt} < x < x^*$, pairing interaction can result in only bound state.

As doping increases, the $K$-channel may be dominated by the Cooper channel and overdoped SC state can manifest properties inherent in conventional BCS state. When doping exceeds $x_{opt}$, a decrease in $T_c$ down to $T_c = 0$ at $x = x^*$ can be also associated with doping dependence of the pairing interaction. This interaction becomes more repulsive at $x > x_{opt}$ as the FC gradually leaves the vicinity of the extended van Hove saddle point.

A suppression of the phonon-mediated component of the SC pairing may have the same origin. Effective increase in repulsion can result in the fact that inequality $\Delta_{p}^{0}$ can be reversed because of not too large ratio $E_F/\varepsilon_D$ typical of cuprates.

There is rather strong evidence that, in underdoped cuprates, dimensionless ratio $2\Delta(0)/T_c$ considerably exceeds universal BCS value $3.52$. Here, $\Delta(0)$ has meaning of the SC energy gap extrapolated down to $T = 0$. In underdoped biordered superconductor, this parameter should be determined by both Cooper and $K$-pairing, therefore, one can assume that $\Delta(0) = \sqrt{\Delta^2 + \Delta_0^2 + \Delta_p^2}$.

Taking into account the fact that $T_c$ is determined by $K$-pairing only and $\Delta^2$, $\Delta_0^2$ and $\Delta_p^2$ are, generally speaking, of the same order, one can easily conclude that ratio $2\Delta(0)/T_c$ may considerably exceed $3.52$ (observed in Ref. $^{23}$ values $2\Delta(0)/T_c \geq 10$). In overdoped regime, strong pseudogap parameter $\Delta_p \to 0$ and the Cooper channel dominates $K$-pairing. Therefore, $2\Delta(0)/T_c$ should be close to $3.52$ in accordance with the BCS theory.

Superfluid density $\rho_s$ should be determined by condensation of $K$-pairs within a broad temperature range below $T_c$, down to the onset of the Cooper channel. Below $T'_c$, superfluid density increases considerably; conversely, off-condensate particle density decreases. Drude-like behavior of the coherent contribution into optical conductivity $\sigma_1(\omega) \sim \omega^{-2}$, observed below $T_c$, can be connected with rather high off-condensate density (experimental data available$^{24}$ show that spectral weight of the off-condensate particles may exceed spectral weight of the SC condensate below $T_c$). At $T \lesssim T'_c$, Drude component of the optical conductivity of biordered superconductor, $\sigma_1(\omega)$, should be suppressed due to shedding of the off-condensate particles into the condensate of Cooper pairs.

Two-gap excitation spectrum of biordered superconductor should be consistent with tunnel conductance measurements$^{22}$ and quasi-linear temperature dependence of heat capacity, $c_V = \gamma(T)/T^{4^2}$. All homologous cuprate series investigated demonstrate universal dependence of $T_c$ on the number of CuO$_2$ layers in the unitary cell, $T_c(n)$, with maximum at $n = 3$.$^{43}$ Strong initial increase in $T_c(n)$ cannot be associated with local real-space pairing interaction. Weak interlayer tunnelling can explain this feature qualitatively by rather small effective enhancement of the coupling constant.$^{49}$ Coulomb pairing with finite screening length ensures strong correlation between electrons in the nearest-neighbor layers and results in a quantitative explanation of $T_c(n)$ leading to almost triple increase of the coupling constant.$^{50}$

Doping dependent isotope effect, observed in cuprate superconductors$^{24}$ is highly sensitive to sample quality and reflects the contribution of phonon-mediated component to the SC pairing interaction. Depending on the interrelation between Coulomb and phonon-mediated contributions$^{25}$ the exponent of the isotope effect on $T_c$ can be close to both zero, in the case of dominating repulsion, and BCS limit in the opposite case of dominating phonon-mediated attraction. Relative isotope shift, negligible above optimum doping, increases with underdoping. We believe that this can be considered as an indirect evidence in behalf of the fact that, in the case of low
doping, Coulomb correlation effects dominate phonon-mediated contribution to the $K$ - channel which determines $T_c$ in b iodoped superconductor.

The isotope effect on the London penetration depth $\lambda_L$, absent within the BCS theory, also turns out to be enhanced with underdoping. The penetration length is weakly sensitive to isotope substitution in a wide temperature range well below $T_c$. Then, starting from $T \approx T_c$, the isotope shift on $\lambda_L$ increases gradually at $T \to 0$. As $\lambda_L^2 \sim \rho_s$, such a behavior of isotope effect on $\lambda_L$ can be associated with temperature and doping dependence of superfluid density inherent in the biordered SC state.

Acknowledgments

We thank A.V. Chubukov, V.F. Efetov, V.L. Ginzburg, L.V. Keldysh, and S.I. Vedeneev for very useful discussions. This work was supported in part by the Russian Foundation for Basic Research (project nos. 05-02-17077, 06-02-17186).

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