QUARK-HADRON PHASE TRANSITION STUDY IN HADRON-NUCLEUS INTERACTIONS IN SELF-AFFINE SCALING SCENARIO

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Abstract: In the present work the phase transition and its dependence on target excitation has been studied in two dimensional (η-ϕ) self affine space using the experimental data of pions obtained from π⁻-AgBr interactions at 350 GeV/c. For studying target excitation dependence the data for produced pions are divided into three sets depending on the number of grey particles (ng). The different sets corresponds to the different degrees of target excitation. The Levy indices μ measured from the analysis fulfills the requirement of the levy stable region 0 ≤ μ ≤ 2. The Levy index μ ≥ 1 indicates that a thermal phase transition may exist in the π⁻-AgBr interactions at 350 GeV/c. Further the analysis indicates different degrees of multifractality for different target excitation. Moreover, the value of universal scaling exponent (ν) obtained from Ginzburg-Landau (GL) theory indicates that no evidence of second order phase transition has been found in the interaction.

Key Words: Hadron-nucleus interaction; Phase transition; Target excitation; Levy index; Ginzburg–Landau theory; self-affine scaling

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1. INTRODUCTION

It is predicted in quantum chromodynamics that a new matter state –quark-gluon plasma (QGP) – may be formed in high energy interactions. Then a phase transition occurs from the QGP to hadrons [1, 2]. The produced hadrons are expected to remember a part of the history of these interactions and are believed to be most informative about the collision dynamics, particle production mechanism etc. The study of the multiparticle production process may be very effective and useful for probing the formation of QGP.

The Levy stable law [3, 4] represents a useful tool to detect the existence of possible phase transition in multiparticle production process. This law is described by the Levy stability index μ. This parameter μ takes value in the range [0, 2] according to the requirement of Levy stability [3, 4]. Within the region of stability 0 ≤ μ ≤ 2, μ has a continuous spectrum. The index μ gives an idea about the estimation of the cascading rate [4]. The two bound axes of the Levy index indicate the degree of fluctuation in the particle production. μ=2 corresponds to the minimum fluctuation from self-similar branching processes. μ = 0 corresponds to the maximum fluctuation that characterizes the interacting system as monofractal [5, 6]. According to [4], when μ ≥ 1, there is a thermal phase transition. On the other hand, when μ ≤ 1, there is a non-thermal phase transition during the cascading process.

According to Van Hove [7] phase space in high-energy process is anisotropic. The fluctuation pattern is also expected to be anisotropic and the scaling behavior should also be different in different directions giving rise to self-affine scaling. In self-affine scenario, the phase space should be shrunk according to the inherent self-affine parameter --- Hurst exponent H. The Levy index μ obtained only in
this way is meaningful in characterizing the self-affine random cascading process. A very few Levy index analysis in self-affine space have been reported so far [8, 9].

The existence of second order phase transitions in multiparticle production process can be investigated by Ginzburg-Landau (GL) theory [6, 10-12]. Here, the anomalous fractal dimension \( d_q \) follows the relation

\[
\frac{d_q}{d_2} = (q - 1)^{-1}
\]

Where scaling exponent \( \nu = 1.304 \) [6, 10-12], a universal quantity that is valid for all systems describable by the GL theory. If the measured value of \( \nu \) is significantly different from the critical value, then the GL description is inappropriate and second order phase transition can most likely be ruled out. On the other hand, if it is close, then a second order quark-hadron phase transition can be expected.

The proper dynamics of the multiparticle production process can be understood by studying pions (shower particles). But the medium energy (30- 400 MeV) knocked out protons (grey particles), may also carry relevant information about the hadronization mechanism, since the time scale of emission of both the particles is of the same order.

The number of collisions in nuclei can be measured by the number of grey particles \( n_g \). Generally, \( n_g \) together with the number of pions are used as a measure of violence of the target fragmentation [13, 14]. To get more information about the inner dynamics of the particle production in high-energy interactions, the phase transition and its dependence on target excitation has to be studied thoroughly using the available tools. To do this, we have divided the data for produced pions for \( \pi^- - AgBr \) interactions at 350 GeV into three sets depending upon the number of grey tracks \( n_g \). The different data sets correspond to different degrees of target excitation.

In this present paper phase transition study of the produced pions is performed in two dimensional \((n - \phi)\) phase space under self-affine scenario imposing special emphasis on Levy stability analysis. Levy stable law has been used to determine the value of \( \mu \) for different target excitations (different values of \( n_g \)) in \( \pi^- - AgBr \) interactions at 350 GeV/c to assess the dependence of the phase transition on target excitation. Finally using the GL theory the value of \( \nu \) is determined to investigate the possibility of second order quark-hadron phase transition.

2. EXPERIMENTAL DETAILS

In this analysis hadron-nucleus interaction data of \( \pi^- - AgBr \) at 350 GeV/c has been used. A stack of G5 nuclear emulsion plate was exposed horizontally to a \( \pi^- \) beam at CERN with 350 GeV/c.

The emulsion plates were area scanned with a Leitz Metalloplan Microscope fitted with a semi-automatic scanning device, having a resolution along the X and Y axes of 1 \( \mu m \) while that along the Z axis is 0.5 \( \mu m \). A sample of 569 events of \( \pi^- - AgBr \) at 350 GeV/c was chosen, following the usual emulsion methodology for selection criteria of the events.

According to nuclear emulsion terminology [15], the particles emitted in high-energy interactions are classified as:

(a) Black particles: They are target fragments with ionization greater than or equal to 10 \( I_0 \), \( I_0 \) being the minimum ionization of a singly charged particle. Their ranges are less than 3 mm. Their velocity is less than 0.3 \( c \) and their energy is less than 30 MeV, where \( c \) is the velocity of light in free space.

(b) Grey particles: They are mainly fast target recoil protons with energy up to 400 MeV. The ionization power of gray particles lies between 1.4 \( I_0 \) to 10 \( I_0 \). Their ranges are greater than 3 mm and they have velocities between 0.3 \( c \) to 0.7 \( c \).

(c) Shower particles: They are mainly pions with ionization \( \leq 1.4 I_0 \). These particles are generally not confined within the emulsion pellicle.
3. METHODOLOGY

To analyse the fluctuation pattern of emitted particles in two-dimensional phase space the method of scaled factorial moment is used here. Denoting the two-phase space variables as \( x_1 \) and \( x_2 \), factorial moment of order \( q \) may be defined as [16]

\[
F_q(x_1, x_2) = \frac{1}{M} \sum_{m=1}^{M} \langle n_m(n_m - 1) \ldots (n_m - q + 1) \rangle / \langle n_m \rangle^q
\]  

where \( \delta x_1, \delta x_2 \) is the size of a two-dimensional cell. The brackets \( \langle \rangle \) denote the average over the whole ensemble of events. \( n_m \) is the multiplicity in the \( m \)th cell. \( M \) is the number of two-dimensional cells into which the considered phase space has been divided.

Let us fix a two-dimensional region \( \Delta x_1 \Delta x_2 \) and divide it in to sub cells of width \( \delta x_1 = \Delta x_1 / M_1 \) and \( \delta x_2 = \Delta x_2 / M_2 \). Here \( M_1 \) is the number of bins along \( x_1 \) direction and \( M_2 \) is the number of bins along \( x_2 \) direction. Cell size dependence of factorial moment is studied by shrinking the bin widths in both directions. There are two ways of doing it. Widths may be shrunked equally \((M_1 = M_2)\) or unequally \((M_1 \neq M_2)\) in the two dimensions. The shrinking ratios along \( x_1 \) and \( x_2 \) directions are characterised by a parameter \( H = \ln M_1 / \ln M_2 \) where \( H \) \((0 < H \leq 1)\) is called Hurst exponent [17, 18]. \( H = 1 \) signifies that the phase space is divided isotropically and consequently fluctuations are self-similar. When \( H < 1 \) it is clearly understood that the phase spaces along \( x_1 \) and \( x_2 \) directions are divided anisotropically consequently the fluctuations are self affine in nature.

The power law dependence of factorial moment on the cell size as cell size approaches zero is given by,

\[
\langle F_q \rangle \propto M^{\alpha_q}
\]  

The index \( \alpha_q \) is obtained from a linear fit of the form

\[
\ln \langle F_q \rangle = \alpha_q \ln M + a
\]  

where \( a \) is a constant.

According to the cascade model [3], the higher order scaled factorial moments are related to the second order scaled factorial moments by a modified power law

\[
F_q \propto F_2^{\beta_q}
\]  

which may provide some vital information about the underlying dynamics. It has been found that the slopes of the power law between higher order and second order SFM s are independent of phase space size and phase dimension [6, 10]. In other words the values of \( \beta_q \) summarize the scale invariance property on the global scale.

Now \( \beta_q \) is defined by the following relation

\[
\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{d_2}{d_1}(q-1)
\]  

\( \beta_q \) is related to Levy index \( (\mu) \) by the equation

\[
\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{q^\mu - q}{2^\mu - 2}
\]  

Here, \( \mu \), known as Levy index, is considered a measure of degree of multifractality [6].

Note that for \( \mu = 2 \), one expects minimum fluctuation in the self-similar branching process. On the other hand, for \( \mu = 0 \), \( d_q = d_2 \) i.e. \( d_q \) does not depend on \( q \), corresponds to monofractality and maximum fluctuation and might, therefore be a signal of QGP second order phase transition. When \( \mu > 0 \), \( d_q \neq d_2 \) i.e. \( d_q \) depends on \( q \), the condition for multifractality is satisfied.
According to GL theory for second order phase transition the anomalous fractal dimension follows the relation

\[
\frac{d_q}{dz} = (q - 1)^{-1}
\]  

(7)

or in terms of \( \beta_q \) the scaling behavior is represented by the following relation

\[
\beta_q = (q - 1)^{\nu}
\]  

(8)

With \( \nu = 1.304 \) as the critical exponent.

4. RESULT AND DISCUSSION

In this analysis the cumulative variables \( X_\eta \) and \( X_\phi \) are used instead of \( \eta \) and \( \phi \) [19, 20] to reduce the effect of non-flat average distribution. The new “cumulative” variable \( X_z \) is related to the original single- particle density distribution \( \rho(z) \) as,

\[
X_z = \int_{z_{\text{min}}}^{z_{\text{max}}} \rho(z') dz'/ \int_{z_{\text{min}}}^{z_{\text{max}}} \rho(z) dz'
\]  

(9)

where \( z_{\text{min}} \) and \( z_{\text{max}} \) are the two extreme points of the distribution. In the \( X_\eta - X_\phi \) space we divided the region \([0, 1]\) into \( M_\eta \) & \( M_\phi \) bins respectively. The partitioning was taken as \( M_\eta = M_\phi^H \). We choose the partition number along \( \phi \) direction as \( M_\phi = 2, 3, \ldots, 20 \). The \( (X_\eta - X_\phi) \) space is divided into \( M = M_\eta \times M_\phi \) cells and calculation is done in each bin independently.

To analyze the anisotropic nature of pions in the \( (X_\eta - X_\phi) \) phase space factorial moment of different orders for different Hurst exponents starting from 0.3 to 0.7 in steps of 0.1 and for \( H = 1 \) are calculated. The variation of average factorial moment \( \langle F_q \rangle \) against the number of the two-dimensional cells \( M \) in a log-log plot have been studied for different orders \( (q = 2, 3, 4 & 5) \) and for the considered \( H \) values. From the linear best fits intermittency exponents \( (\alpha_q) \) are extracted. \( \chi^2/\text{d.o.f.} \) values are calculated for each linear fits. We have also estimated the confidence level of fittings from the \( \chi^2 \) values. The minimum value of \( \chi^2 \) per degree of freedom indicates the best linear behavior. For pions the best linear fit occurs at \( H = 0.3 \) which shows that the anisotropic behavior is best revealed at \( H = 0.3 \). The values of \( \chi^2 \) per degree of freedom and the confidence level of fittings are tabulated in Table 1 for \( H = 0.3 \). The variation of \( \ln \langle F_q \rangle \) with \( \ln M \) for \( H = 0.3 \) is shown in Fig. 1(a). To compare the self- affine behaviour with the self-similar one the variation of \( \ln \langle F_q \rangle \) against \( \ln M \) corresponding to \( H = 1 \) is shown in Fig. 1(b) and the corresponding results are shown in Table 1. \( \chi^2 \) per degree of freedom values and confidence level of fittings at \( H = 0.3 \) are better than the corresponding values obtained at \( H = 1 \). So the dynamical fluctuation pattern of shower particles in \( \pi^- - \text{AgBr} \) interaction at 350 \( GeV/c \) is not self-similar but self-affine in nature.

The values of \( \beta_q \), calculated by using Eqn. (5), are listed in Table 2. The \( \beta_q \) versus \( q \) graph is shown in Fig. 2. It is observed that the parameter \( \beta_q \) increases with increasing order of moments indicating the fact that charged particle density distribution has multifractal structure. Therefore, we can infer that hadrons in the final state are produced due to self-similar cascade mechanism [3, 16].

Then Levy stability index \( \mu \) is calculated using Eqn. (6) and tabulated in Table 2. Here the Levy index obtained for the \( \eta - \phi \) space is \( \mu = 0.468 \pm 0.005 \)
Figure 1: Variation of $\ln \langle F_q \rangle$ as a function of $\ln M$ for full data set at (a) $H = 0.3$ (b) $H = 1$

| $H$  | $q$ | $\alpha_q$   | $\chi^2 / d.o.f.$ | Confidence level of fittings |
|------|-----|--------------|-------------------|-----------------------------|
| 0.3  | 2   | 0.56 ± 0.01  | 0.38              | 98.90 %                     |
|      | 3   | 1.22 ± 0.03  | 0.49              | 95.93 %                     |
|      | 4   | 1.92 ± 0.06  | 0.58              | 91.30 %                     |
|      | 5   | 2.63 ± 0.10  | 0.70              | 80.95 %                     |
| 1    | 2   | 0.59 ± 0.01  | 0.74              | 76.62 %                     |
|      | 3   | 1.29 ± 0.02  | 0.90              | 56.96 %                     |
|      | 4   | 2.02 ± 0.04  | 0.91              | 55.64 %                     |
|      | 5   | 2.72 ± 0.07  | 0.93              | 53.46 %                     |

Table 1: Values of intermittency exponent $\alpha_q$, $\chi^2 / d.o.f.$ and confidence level of fittings at $H=0.3$ and $H=1$ for full data set

| $H$  | $q$ | $\beta_q$ | $\mu$             | $\nu$             |
|------|-----|-----------|-------------------|-------------------|
| 0.3  | 2   | 1         | 0.468 ± 0.005     | 1.110 ± 0.002     |
|      | 3   | 2.16      |                   |                   |
|      | 4   | 3.39      |                   |                   |
|      | 5   | 4.65      |                   |                   |
Table 2: Values of different parameters ($\beta_q$, $\mu$ and $\nu$) for full data set

Figure 2: Variation of $\beta_q$ with $q$ for full data set for self-affine $H \# 1$ case

which is within the specified limit $0 \leq \mu \leq 2$. Here $\mu < 1$ would have indicated a thermal phase transition of second order.

Using Ginzburg – Landau (GL) theory it has been found that the scaling exponent $\nu = 1.110 \pm 0.002$. This value of $\nu$ (considering the errors) differs significantly from the critical value, 1.304. So it is evident that no second order QGP phase transition takes place in the hadronization process.

Fig. 3(a)  Fig. 3(b)
Figure 3: Variation of $\ln \langle F_q \rangle$ as a function of $\ln M$ for self-affine $H$ # 1 case for different $n_g$ intervals (a) $0 \leq n_g \leq 2$ (b) $3 \leq n_g \leq 5$ (c) $6 \leq n_g \leq 13$

For studying target excitation dependence the data set for pions is divided into three sets, $0 \leq n_g \leq 2$, $3 \leq n_g \leq 5$, $6 \leq n_g \leq 13$, depending on the number of grey tracks ($n_g$). The sets correspond to different degrees of target excitation. The division is made in such a way that each set contains a reasonable number of events. The self-affine analysis is repeated for the three data sets. The fluctuation pattern is self-affine in nature in all the three sets of $n_g$. The $\ln \langle F_q \rangle$ vs. $\ln M$ graphs for all $n_g$ intervals are shown in Fig. 3 and corresponding results are tabulated in Table 3. A low value of $H$ suggests that anisotropy is strong for $0 \leq n_g \leq 2$ and $6 \leq n_g \leq 13$ data sets.

| $n_g$       | $H$ | $q$  | $\alpha_q$ | $\chi^2 / d.o.f.$ | Confidence level of fittings |
|------------|-----|------|-------------|-------------------|-----------------------------|
| $0 \leq n_g \leq 2$ | 0.3 | 2    | 0.58 ± 0.01 | 0.13              | Almost 100 %                |
|            |     | 3    | 1.25 ± 0.05 | 0.51              | 94.89 %                     |
|            |     | 4    | 1.99 ± 0.10 | 0.54              | 93.67 %                     |
|            |     | 5    | 2.76 ± 0.18 | 0.70              | 80.67 %                     |
| $3 \leq n_g \leq 5$ | 0.7 | 2    | 0.55 ± 0.01 | 0.33              | Almost 100 %                |
|            |     | 3    | 1.20 ± 0.03 | 0.49              | 95.82 %                     |
|            |     | 4    | 1.88 ± 0.07 | 0.67              | 84.04 %                     |
|            |     | 5    | 2.56 ± 0.13 | 0.77              | 72.95 %                     |
| $6 \leq n_g \leq 13$ | 0.3 | 2    | 0.61 ± 0.02 | 0.24              | Almost 100 %                |
|            |     | 3    | 1.32 ± 0.04 | 0.56              | 92.07 %                     |
|            |     | 4    | 2.07 ± 0.07 | 0.57              | 91.55 %                     |
Table 3: Values of intermittency exponent $\alpha_q$, $\chi^2/d.o.f.$, and confidence level of fittings for different $n_g$ intervals (for self-affine $H$ # 1 case)

The Levy index analysis is repeated for the three target excitation data sets in the self-affine space. The $\beta_q$ versus $q$ graphs for the three data sets are shown in Fig. 4. It is observed that the parameter $\beta_q$ increases with increasing order of moments revealing multifractal pattern of produced pions in different $n_g$ intervals. The errors shown in the figures are standard errors. The values of $\mu$ are calculated following the same procedure as in the previous cases and listed in Table 4. We get $\mu < 1$ for three target excitation data sets indicating a second order thermal phase transition and thus probing for a possible QGP formation. Moreover, the values of Levy indices ($\mu$) vary consistently with degrees of target excitation.

Table 4

| $n_g$     | $H$ | $q$ | $\beta_q$ | $\mu$            | $\nu$          |
|-----------|-----|-----|------------|------------------|----------------|
| $0 \leq n_g \leq 2$ | 0.3 | 2   | 1          | 0.542 ± 0.002    | 1.131 ± 0.004 |
|           |     | 3   | 2.17       |                  |                |
|           |     | 4   | 3.46       |                  |                |
|           |     | 5   | 4.79       |                  |                |
| $3 \leq n_g \leq 5$ | 0.7 | 2   | 1          | 0.478 ± 0.012    | 1.112 ± 0.007 |
|           |     | 3   | 2.18       |                  |                |
|           |     | 4   | 3.42       |                  |                |
|           |     | 5   | 4.66       |                  |                |
| $6 \leq n_g \leq 13$ | 0.3 | 2   | 1          | 0.425 ± 0.012    | 1.098 ± 0.007 |
|           |     | 3   | 2.16       |                  |                |
|           |     | 4   | 3.37       |                  |                |
|           |     | 5   | 4.57       |                  |                |

Table 4: Values of different parameters ($\beta_q$, $\mu$ and $\nu$) for different $n_g$ intervals (for self-affine $H$ # 1 case)

Again according to the GL theory the values of $\nu$ for three $n_g$ intervals are calculated and are listed in Table 4. From the table it is observed that the values of $\nu$ are significantly different from the critical value of $\nu$ making the GL description inappropriate and second order phase transition can most likely be ruled out.
**5. CONCLUSIONS**

Quark-hadron phase transition and its dependence on target excitation have been studied for pions in self-affine $\eta - \phi$ phase space. The following interesting features are revealed from the present investigation:

1. The parameter $\beta_q$ increases with increasing order of moments $q$, which indicates that self-similar cascading to be the mechanism responsible for multiparticle production. From our analysis we find that the particle density distribution possesses multifractal structure and the degree of multifractality is different for different target excitations.
2. The values of the Levy stability index $\mu$ obtained in our study are consistent with the Levy stable region $0 \leq \mu \leq 2$.
3. We get $\mu \leq 1$ for full data set and as well as for three target excitation data sets indicating a second order thermal phase transition and thus may serve as a possible indication of QGP being formed.
4. The values of Levy indices ($\mu$) vary consistently with degrees of target excitation.
5. From the values of the critical exponent $\nu$ in our analysis, no evidence for the existence of second order phase transition has been found according to the GL theory.
6. The values of critical exponents ($\nu$) vary consistently with degrees of target excitation.
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