Microconvection heat transfer in electrorheological fluids in rotating electric field

A A Mokeev, S A Gubarev, E V Korobko and N A Bedik

A.V. Luikov Heat and Mass Transfer Institute of NAS Belarus, 220072, 15 P. Brovka str., Minsk, Belarus

E-mail: evkorobko@gmail.com

Abstract. This work is dedicated to the study of heat transfer characteristics of electrorheological fluids in rotating electric fields. The problem of heat transfer in the electro-sensitive heat carrier considering the microconvection transport mechanism is formulated and numerically solved. It is shown on the example of a flat layer, that internal rotations of elementary particles intensify heat transfer in ERF.

1. Introduction

It is known that in a static electric field, a spontaneous rotation of insulators placed in a low conductive medium [1–5] occurs. When electrorheological fluids (ERF) are used as a medium the rotation of the insulator is enhanced by 1 – 2 orders of magnitude compared to the pure fluid [6]. This effect lies in the basis of dielectric motors [7, 8]. Recently papers studying the dynamics of electrorheological fluids in a rotating electric field [9] have appeared. In [10, 11] a principle of a centrifugal pump on that basis is proposed. In [12], based on differences in electrical properties of cell membranes and their size a method of their separation by applying a rotating electric field is suggested. The peculiarities of heat transfer in ERF when a rotating electric field is applied is not fully studied, in contrast to extensive studies in the field of microconvection heat and mass transfer in magnetic fluids under the influence of time-dependent magnetic fields [13–17]. This issue is particularly important in the aspect of controlled electric field devices for process intensification of heat exchange. This research investigates the features of the physical mechanism of microconvection heat transfer in electrorheological fluids and determines the influence of external alternating electric fields on heat transfer in them.

2. Physical and mathematical modeling of heat transfer in ERF in a rotating electric field

Let us consider the mechanism of heat transfer in ERF flowing in a flat slot channel under the influence of a rotating electric field. By analogy with the phenomenological approach proposed in [1], heat transfer, taking into account microconvection will be described using the effective thermal conductivity tensor, taking into consideration the energy transfer by two mechanisms – microconvection and conduction.

Electrorheological suspension is a quasi-homogeneous liquid dispersion in which the filler particles are evenly distributed throughout the volume. The electric field structures are formed, directed along the electric field vector.

1Corresponding author.
Macroscopic properties of ERF are expressed by values of physical quantities averaged over the local equilibrium subsystems $<F>$, located in a physically infinitesimal volume $dV(r,t)$, and containing a sufficiently large number of particles $dN(r,t)$. The values $<F>$ differ little from the average of the neighboring elementary volumes $dV(r+dr,t+dt)$ and are continuous functions of the coordinates of the centers of mass of subsystems. According to the mechanics of heterogeneous media [18], averaging over the local equilibrium systems is carried out separately for the liquid carrier and solid dispersed phase, and the results are summed up. The process of heat conduction in the absence of macroscopic convection of ERF according to the first law of thermodynamics:

$$
\rho_i \frac{d u_i}{d t} = \rho_i A_i + \rho_i Q_i ,
$$

where $\rho_i$ – density of $i$-th phase; $A_i$ – the work of internal forces; $u_i$ – mass density of the internal energy of the $i$-th phase; $\rho_i Q_i = -\text{div}(\vec{j}_{w}) + \sum Q_{ij}$ – the difference between incoming and outgoing heat fluxes due to heat $\vec{j}_{w}$ and contact heat fluxes between the phases $Q_{ij}$.

In the absence of phase transitions of the contact heat flow across the surface with a temperature difference $\Delta T_{ij}$:

$$
Q_{ij} = C_v dm \beta_{ij} \Delta T_{ij},
$$

where $C_v$ – heat capacity of perceiving the mass $dm$; $\beta_{ij}$ – the coefficient of contact heat transfer.

Solid particles in a stationary ERF under the influence of a constant external electric field strength $\vec{E}$ are in the form of chains connecting the channel wall in the direction of the vector $\vec{E}$. In particular, if the potential difference is applied to the walls of the flat channel, the chain will be built across the channel. This structure is constantly disturbed by Brownian motion and reduced again due to the shear flow due to the strength of the electric interaction.

Let us consider the case when in a rotating electric field with intensity of $\vec{E}(\Omega t)$ an elementary volume of fluid in steady motion rotates with the rotation frequency $\Omega$ in the plane of rotation of vector $\vec{E}$ maintaining chain structures. The interaction between the particles varies with the rate of relaxation. Mathematical modeling of heat and mass transfer in such a system based on the solution of the equations of hydrodynamics and heat for individual particles and the vortex flow of the dispersion medium between them is very complex. To resolve this problem in applications to magnetic fluids a phenomenological approach is proposed in [18], based on the use of the effective thermal conductivity tensor, taking into account the convective and conductive heat transfer mechanisms taking into consideration the anisotropy caused by the internal rotation of the particles.

We obtain similar expressions for the case of electrorheological fluids. For heat transfer in a stationary ERF it is possible to apply microscopic approach to the averaging physical quantities of the phase and volume [18]. In ERF a heterogeneous environment when using averaging microconvection flows of mass and heat is produced in cells that are released around each particle in the form of an almost spherical volume of radius $(b + a)$ (figure 1). These cells in the case of ERF with the particles form a chain, from wall to wall, and the averaging is performed on a pair of adjacent cells and carrier phase fluid surrounding the entire chain of cells.

By turning the electric field vector at an angle $\Theta = \Omega \Delta t$ with the angular velocity $\Omega$ chains tend to be reconstructed along a new direction, but in the stationary liquid particles remain in the chains of the former direction. Acting on the mass of liquid electrical forces only occurs in an inhomogeneous dielectric, i.e. on the surface of discontinuity of the dielectric constant and conductivity – on surfaces of the particles. Changing the direction of the electric field vector does not affect the mass transfer of the carrier liquid. Its movement arises only because of the rotating surface of the particle entrainment.
The moving surface of the particle carries away the surface layer of the liquid in rotation with the same frequency. Neighboring particles according to the dipole interaction draws the liquid in the opposite direction. In the separating layer of liquid (figure 2) almost planar shear flow appears in which the speed of the middle layer is equal to zero. The flow between the particles is approximately taken with a rectilinear Couette velocity profile. The physical mechanism of heat transfer is as follows.

Element of the fluid mass $dm$, followed by rotation of the moving particle with a linear velocity $\bar{v} = \frac{1}{2} \Omega \times \bar{b}$ in the time $\Delta t = \frac{T}{2} = \frac{\pi}{\Omega}$ receives from the heat:

$$\delta Q_1 = k \, dS \, \Delta T_1 = C_v \, dm \, \beta_i \, \Delta T_1,$$

where $\Delta T_1$ – temperature drop between the particle and fluid, °C; $\beta_i$ – a dimensionless coefficient of heat transfer.

The following half-period element of mass $dm$ is in contact with the corresponding element $dm^1$, moving along the surface of neighboring particles in the opposite direction and, in the case of steady-state heat transfer, it sends the same amount of heat. More precisely, different elements in series come in contact, but at the end of the half-period the element at the exit of the gap between the particles receives the amount of heat such as what it would have received under the continuous contact in the middle of the gap. Then the heat is transferred to the next link in the chain of particles, etc. to the opposite wall of the channel. On average, there is a closed stream.

In the steady state heat transfer process due to the symmetry of the system along the walls of the channel the amounts of heat transferred across the chains are mutually balanced. Heat is transferred only perpendicular to the walls. This average over the unit cell (particle and surrounding fluid) and the dispersion of phase (carrier fluid), the convective heat flux from the heated to the temperature $T_1$ to a wall cooled to a temperature $T_2$, the other wall, equal to half the width of the gap between the particles, depth of 2 $(a + b)$, covers the longitudinal chain of particles $N_c$ and 2 $(a + b) N_c$. In the first approximation, the velocity profile in the flow line $V(y) = ky$, so that
Given the differences between the actual shape of the particles from the cylindrical coefficient 2 in the last factor is omitted. Mass flow rate $Q_m$ and the mass flux density equal to:

$$Q_m = \frac{d}{dt} \int_{a+b}^{b+a} (a+b)V(y)dy = a \int_{a}^{b} aV(y)dy = a(a+b)^2 \Omega.$$  \hspace{1cm} (2.4)

Each element of mass $dm$ transfers heat quantity from the wall:

$$\delta Q = j_{qk} \cdot dS \cdot dt = C_v j_m dS \cdot dt \cdot \beta \Delta T_i,$$  \hspace{1cm} (2.7)

and the density of the convective heat flux $j_{qk}$ is:

$$j_{qk} = C_v j_m \beta \Delta T_i = C_v \rho \Omega a(a+b)^2 \beta \Delta T_i.$$  \hspace{1cm} (2.8)

We assume that the number of links in the chain is the same in the steady heat transfer process, so the total difference in temperature between the walls is the sum of temperature differences at each of the $N_c$ links in the chain of particles and the temperature gradient between the walls is equal to the gradient on the link:

$$\frac{\Delta T}{h} = \frac{\Delta T_i N_c}{(a+b)N_c} = \nabla T, \quad \Delta T_i = (a+b)\nabla T.$$  \hspace{1cm} (2.9)

Heat flux density along the chain is equal to the flux density through the link:

$$j_{qk} = C_v \rho \Omega(a+b) \beta \Delta T_i = C_v \rho \Omega(a+b)^2 \beta \nabla T = \lambda_q \nabla T.$$  \hspace{1cm} (2.10)

The total density of heat flow is the sum of convective and conductive components:

$$j_q = j_{qk} + j_{qm}.$$  \hspace{1cm} (2.11)

The molecular heat flux passes through a chain of particles with a density $j_{qp}$ intervals and in parallel along the liquid layer between the neighboring chains with a density $j_{qf}$:

$$j_{qm} = j_{qp} + j_{qf}.$$  \hspace{1cm} (2.12)

The flow along the chain in the same steady course through the particle and the layer of liquid between the particles: $j_{qp} = j_{qf} = \lambda_p T_p = \lambda_f \nabla T$, so

$$\nabla T_f = \frac{\lambda_p}{\lambda_f} \nabla T_p.$$  \hspace{1cm} (2.13)

On the other hand: $\nabla T_p = \frac{\Delta T_p}{2a}, \nabla T_f = \frac{\Delta T_f}{2b}$, so

$$\nabla T_f = \frac{\lambda_p}{\lambda_f} \frac{b}{a} \Delta T_p.$$  \hspace{1cm} (2.14)
Full temperature difference along the chain of particles $N_c$ is equal to

$$\Delta T = (\Delta T_p + \Delta T_f) N_p, \quad N_p = \frac{h}{2(a+b)}, \quad (2.15)$$

so

$$\Delta T_p + \Delta T_f = \frac{(a+b)\Delta T}{h} = (a+b)\nabla T, \quad \Delta T_p + \frac{\lambda_p}{\lambda_f} \frac{b}{a} \Delta T_p = 2(a+b)\nabla T,$$

$$1 + \frac{\lambda_p}{\lambda_f} \frac{b}{a} \Delta T_p = 2(a+b)\nabla T, \quad \Delta T_p = \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b} \nabla T, \quad j_{qc} = \lambda_p \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b} \nabla T$$

and the average coefficient of thermal conductivity of the molecular chains of particles is

$$\lambda_c = \lambda_p \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b}. \quad (2.16)$$

The molecular heat flux along the layer of fluid between adjacent chains of particles continuously from one side to the other channel is determined by the usual law of heat conduction $j_{qf} = \lambda_f \nabla T$, and added to the flow along the chains, so that the total molecular heat flux density is

$$j_{qm} = j_{qp} + j_{qf} = \left[ \lambda_c + \lambda_p \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b} \right] \nabla T \quad (2.17)$$

and the total coefficient of molecular thermal conductivity of ERF is

$$\lambda_m = \lambda_c + \lambda_p \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b}. \quad (2.18)$$

Full thermal conductivity across the layer of ERF is equal to the sum of convective and molecular components

$$\lambda_{ij} = \lambda_k + \lambda_m = C_\gamma \rho \frac{Q}{2(b+2a)} \beta_i + \lambda_c + \lambda_p \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b}. \quad (2.19)$$

Coefficient of thermal conductivity along the layer of ERF parallel to the walls of the channel due to the symmetry of stationary ERF has the same meaning. In the direction perpendicular to the plane of rotation of the electric field, the thermal conductivity is equal to the molecular

$$\lambda_{ij} = \lambda_m = \lambda_c + \lambda_p \frac{2(a+b)\lambda_f a}{\lambda_f a + \lambda_p b}. \quad (2.20)$$

Stationary ERF in a rotating homogeneous electric field is thermally homogeneous and isotropic, i. e., its coefficient of thermal conductivity in each element’s volume $dV(\rho)$ is the same in all directions perpendicular to the axis of rotation of $\vec{E}$.

Total heat flux

$$j_q = j_{qf} + j_{qf} = \partial_x \left( \lambda_{xx} \nabla_x T + \lambda_{xy} \nabla_y T + \lambda_{xz} \nabla_z T \right) + \partial_y \left( \lambda_{yx} \nabla_x T + \lambda_{yy} \nabla_y T + \lambda_{yz} \nabla_z T \right) + \partial_z \left( \lambda_{zx} \nabla_x T + \lambda_{zy} \nabla_y T + \lambda_{zz} \nabla_z T \right), \quad (2.21)$$

where $\lambda_{xx} = \hat{\lambda}_{ij}$, $\lambda_{xy} = \lambda_{yx}$, $\lambda_{xz} = \lambda_{zx}$, $\lambda_{yy} = \hat{\lambda}_{ij}$, $\lambda_{yz} = \lambda_{zy}$.

Thus, the thermal conductivity tensor has the following form
\[ \lambda_{\alpha\beta} = \begin{pmatrix} \lambda_{\perp} & 0 & 0 \\ 0 & \lambda_{\parallel} & 0 \\ 0 & 0 & \lambda_{\parallel} \end{pmatrix}. \]  

(2.22)

Substituting it into the equation of energy conservation

\[ \rho C_v \frac{\partial T}{\partial t} - \text{div} \left( \mathbf{j}_q \right) = f, \quad \mathbf{j}_q = \sum_{\alpha=1}^{3} (\hat{e}_\alpha \lambda_{\alpha\beta} \nabla \beta T) \]  

(2.23)

gives the equation of heat conduction

\[ \rho C_v \frac{\partial T}{\partial t} - \lambda_{\perp} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \lambda_{\parallel} \frac{\partial^2 T}{\partial z^2} = f, \]  

(2.24)

where \( f \) – the density of internal heat sources.

3. Convection heat transfer in ERF in a rotating electric field

With the shift of one of the electrodes in a layer of ERF Couette flow is created. It is divided into the physical motion of the infinitesimal volume elements \( dV \), within each of which the velocity profile \( V(x) \) can be considered linear in first approximation. ERF in each such microvolume flows as in a flat slot channel with the width \( h \) with a velocity profile \( V(x) = k x \). In the flowing ERF chains of particles are virtual, i.e. they are broken by a continuous unidirectional shift and re-formed from other elements downstream in the direction of rotation of the electric field vector. With a steady flow at a constant rate \([19]\) particles are fully entrained by a flow of carrying liquid.

With a steady flow – continuous unidirectional shift at a rate of the channel walls \( V \) the structural links of the particles in suspension are broken, the particles or small aggregates are almost independent.

If the fixed wall of the channel is heated to a temperature \( T_1 \), and moving to \( T_2 < T_1 \), then the channel creates a temperature field with a gradient \( \nabla x T \). Under the action of rotating with angular velocity in the plane with flow velocity \( \Omega \) of the electric field with intensity

\[ \vec{E}(\Omega \cdot t) = i E_x + j E_y, \quad E_x = E_0 \cdot \cos(\Omega \cdot t), \quad E_y = E_0 \cdot \sin(\Omega \cdot t), \]

a steady motion of a particle in orbit with a period \( T_\Omega = \frac{2\pi}{\Omega} \), with a frequency \( \Omega \) with a linear velocity of the surface points \( V_\Omega = \Omega \cdot b \) in the plane of rotation of intensity.

**Figure 3.** The location of the particles, rotating counterclockwise. Dotted line – the particle radius \( a \), the rectangle – a half of the gap \( b \).

Rotating particle carries the liquid carrier into the rotation at a speed \( V_\Omega = \Omega \cdot b \), and the next – at the same speed in the opposite direction. The velocity of the middle layer of liquid between them is
zero. Element of the fluid mass $dm$, adjacent to the particle surface, is entrained by it with this high speed. In the separating layer of liquid almost a planar shear flow appears in which the speed of the middle layer is equal to zero. The flow between the particles is approximately taken with a rectilinear Couette velocity profile. Elements of the fluid mass $dm$ with density $\rho$ with the specific heat $C_V$ entraining by the rotating surface of the particles are moving at an average linear velocity $b\Omega V_{rr} = \frac{1}{2}\Omega b$ and form microvortices, a compound of which creates a chain of micropumps, pumping the fluid, and with it the heat in the directions perpendicular and longitudinal to the flow.

Element of the fluid mass $dm$, isolated in the middle layer of fluid adjacent to the particle surface and carried along by it to meet the similar elements $dm$ of a neighboring particle, is in contact with them consistently. For a time equal to a half of the period of rotation of a particle $\Delta t = \frac{T}{2} = \frac{\pi}{\Omega}$ at the temperature difference of the wall and the element $\Delta T$ the amount of heat received from the wall of ERF of the mass $dm = \rho dV = \rho \Delta S a$, is determined by the density of heat flow $j_q$ through the contact area $\Delta S = [2(a + b)]^2$. Over time the amount of heat transferred to the contact:

$$\delta Q = j_q > \Delta S \Delta t = \beta \cdot C_V \cdot \rho \cdot dV \cdot \Delta T$$  \hspace{1cm} (3.1)

where $\beta$—dimensionless coefficient of heat transfer.

It is the product of the average density of heat flow through the average element of mass $dm$ in the interval between the particles and the contact area of $\Delta S$ for the contact time $\Delta t$. The average heat flux into the element $dm$ [5] is greater, the greater the heat element of mass $C_V$.

Heat flux density is equal to its amount in the volume of a single cylinder $\Delta S V_T \Delta t$ with a cross-sectional area $\Delta S$, perpendicular to the velocity of the carrier energy $V_T$, with the length of the side of $V_T \Delta t$, transmitted through $\Delta S$ at a time $\Delta t$, per unit area and unit of time:

$$j_q = \frac{\delta Q}{{dSdt}} = \beta \cdot C_V \cdot \rho \cdot V_T \cdot \Delta T.$$ \hspace{1cm} (3.2)

Heat transfer coefficient $\beta$ determines the velocity of propagation of thermal energy to a distance equal to the thickness of the layer of liquid between the particles.

The resulting amount of heat energy obtained by mass $dm$

$$\delta Q = j_q \cdot \Delta S \cdot \Delta t = \beta \cdot C_V \cdot \rho \cdot \Delta V \cdot \Delta T.$$ \hspace{1cm} (3.3)

is transferred by mass $dm$ during time $\Delta t$ along the axis OX with an average linear velocity $V_0 = \Omega (a + b)$ at a distance $\Delta x = V_0 \Delta t = \Omega (a + b) \Delta t$. After a time $\Delta t$ an element $dm$ is in contact with others, consistently successive elements of the mass $dm'$, each of which enjoys a neighboring chain in the structure of the particle temperature in the lower $\Delta T$. During the contacts on the area of $\Delta S = 4 (a + b)^2$, with a steady process of transfer the element $dm$ transfers to the elements $dm'$ the same amount of heat, as in (3.1).

At the same time this amount of heat is transferred along the axis OY to the next particle along the chain at a distance

$$\Delta X_v = \left( \frac{dV}{dx} \right) \cdot \Delta x = \left( \frac{dV}{dx} \right) \cdot (a + b).$$

The total length of heat transfer is defined as:

$$\Delta L = \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{(a + b) \cdot \Omega \cdot \Delta T \cdot \left[ 1 + \left( \frac{V(X)}{(a + b) \cdot \Omega} + \frac{1}{\Omega} \right) \right]}.$$ \hspace{1cm} (3.4)
and the rate of transfer:

\[
V_T = 2 \cdot (a + b) \cdot \Omega \cdot \sqrt{1 + \left[1 + \frac{V(X)}{(a + b)\Omega} + \frac{1}{\Omega} \frac{dV}{dX}\right]^2}.
\]  

(3.5)

Heat flux density at the temperature difference \(\Delta T = (a + b) \cdot \nabla x T\) is

\[
j_q = \frac{\delta Q}{dV} \cdot V_T = \beta \cdot C_v \cdot \rho \cdot (a + b)^2 \cdot \Omega \cdot \sqrt{1 + \left[1 + \frac{V(X)}{(a + b)\Omega} + \frac{1}{\Omega} \frac{dV}{dX}\right]^2} \cdot \nabla x T.
\]  

(3.6)

On the other hand

\[
j_q = \sqrt{(j_q)_x^2 + (j_q)_y^2},
\]  

(3.7)

and components of the heat flux density along the coordinate axes are equal in magnitude:

\[
(j_q)_x = \lambda_{xx} \cdot \nabla_x T,
\]  

(3.8)

\[
(j_q)_y = \lambda_{yy} \cdot \nabla_y T,
\]  

(3.9)

Where \(\lambda_{xx}\) and \(\lambda_{yy}\) – the thermal conductivity tensor components, determined by heat transfer during shear deformation of the structure:

\[
\lambda_{xx} = \beta C_v \cdot \rho \cdot (a + b)^2 \Omega,
\]  

(3.10)

\[
\lambda_{yy} = \lambda_{xx} \left[1 + \frac{V(X)}{(a + b)\Omega} + \frac{1}{\Omega} \frac{dV}{dX}\right],
\]  

(3.11)

\[
\lambda_{xz} = 0.
\]  

(3.12)

If the electric field vector rotates in a plane perpendicular to the flow, the amount of heat \(\delta Q\) transported is the same as at the rotation in the plane of flow and transport distance is equal to

\[
\Delta L = \sqrt{\Delta X^2 + \Delta Y^2} = (a + b)\Omega\Delta t \left[1 + \frac{V(X)}{(a + b)\Omega} + \frac{1}{\Omega} \frac{dV}{dX}\right]^{0.5},
\]  

(3.13)

then the components of the conductivity tensor in this case are:

\[
\lambda_{xx} = \beta C_v \cdot \rho \cdot (a + b)^2 \Omega,
\]  

(3.14)

\[
\lambda_{yy} = \lambda_{xx} \left[1 + \frac{V(X)}{(a + b)\Omega} + \frac{1}{\Omega} \frac{dV}{dX}\right],
\]  

(3.15)

\[
\lambda_{xz} = 0.
\]  

(3.16)

The difference between the thermal conductivity \(\lambda_{yy}\) from (3.11) reflects the absence of heat transfer in the direction of rotation of the flow velocity.

When temperature drops \(\nabla_x T\) along the velocity vector expressions are obtained for the components of the heat flux density along the coordinate axes:

\[
(j_q)_x = \lambda_{xx} \cdot \nabla_x T, \quad \lambda_{xx} = \beta \cdot C_v \cdot \rho \cdot (a + b)^2 \cdot \Omega,
\]  

(3.17)
\[
(j_q)_y = \lambda_{yy} \nabla_x T, \quad \lambda_{yy} = \beta \cdot C_v \cdot \rho \cdot (a+b)^2 \cdot \Omega \left[ 1 + \frac{V(X)}{(a+b) \cdot \Omega} + \frac{1}{\Omega} \frac{dV}{dX} \right], \quad (3.18)
\]

For an arbitrary direction of the temperature gradient in the plane xoy:

\[
\nabla T = i \nabla_x T + j \nabla_y T \quad (3.19)
\]

heat flux density is:

\[
\vec{j}_q = - \sum_{\alpha \beta} (\vec{e}_\alpha \cdot \lambda_{\alpha\beta} \cdot \nabla T) \quad (3.20)
\]

Effective thermal conductivity tensor of a plane flow of ERF is asymmetric and is the sum of symmetric and asymmetric tensors

\[
\lambda_{\alpha\beta} = \begin{pmatrix}
\lambda_{xx} & \lambda_{xy} & 0 \\
\lambda_{yx} & \lambda_{yy} & 0 \\
0 & 0 & \lambda_{zz}
\end{pmatrix} \quad (3.21)
\]

Substituting in the equation of energy conservation, that is, the equation of local first law of thermodynamics, the expression for the heat flux density, \( f \) – the density of heat sources

\[
\rho C_v \frac{\partial T}{\partial t} + \text{div} (j_q) = f,
\]

gives the equation of heat conduction

\[
\rho C_v \frac{\partial T}{\partial t} - \sum_{\alpha\beta} \frac{\partial}{\partial x_\alpha} \left( \lambda_{\alpha\beta} \frac{\partial T}{\partial x_\beta} \right) = f, \quad (3.22)
\]

Given the expressions of the tensor components of the effective thermal conductivity:

\[
\rho C_v \frac{\partial T}{\partial t} = \lambda_{xx} \frac{\partial^2 T}{\partial x^2} + \lambda_{yy} \frac{\partial^2 T}{\partial y^2} + \lambda_{zz} \frac{\partial^2 T}{\partial z^2} + \lambda_{xy} \frac{\partial^2 T}{\partial x \partial y} + \lambda_{yx} \frac{\partial^2 T}{\partial y \partial x} + f, \quad (3.23)
\]

If the temperature gradient is across the direction of flow, the equation (3.23) simplifies to:

\[
\rho C_v \frac{\partial T}{\partial t} - \beta \cdot C_v \cdot \rho \cdot (a+b)^2 \cdot \Omega \cdot \frac{\partial^2 T}{\partial x^2} = f. \quad (3.24)
\]

In the end channel at its edges \( \frac{\partial T}{\partial y} \neq 0 \), and the problem requires the solution of the previous equation with appropriate boundary conditions together with the equation of motion if the velocity profile \( V(x) \) is not specified.

4. The numerical solution

To assess the influence of the parameters of the rotating electric field on heat transfer in ERF let us solve the equation (3.24). Let us consider the one-dimensional case in the absence of internal heat sources, i.e. at \( f = 0 \). At one boundary the condition is given of the first kind (the known and constant temperature), the second – the condition of thermal equilibrium. At the initial time the temperature of ERF is assumed known and constant throughout the channel. Thus, we will solve the problem:
\[
\frac{\partial T}{\partial t} - \beta(a + b)^2 \Omega \frac{\partial^2 T}{\partial x^2} = 0, \quad (4.1)
\]

\[
T|_{x=0} = T_1; \quad \frac{\partial T}{\partial x}|_{x=h} = 0; \quad T(t = 0) = T_0. \quad (4.2)
\]

The problem (4.1) – (4.2) was solved numerically using the finite difference method. In the finite-difference form:

\[
\frac{1}{\Delta t} (T_i^k - T_{i-1}^k) = \beta(a + b)^2 \Omega \frac{1}{\Delta x^2} (T_{i+1}^k - 2T_i^k + T_{i-1}^k), \quad (4.3)
\]

\[
T_0^k = T_1; \quad T_N^k - T_{N-1}^k = 0; \quad T_i^0 = T_0. \quad (4.4)
\]

The problem (4.3) – (4.4) was solved by the sweep method [20]. The coefficients of the sweep:

\[
A_i = \beta(a + b)^2 \Omega \Delta t, \quad B_i = -\Delta x^2 - 2\beta(a + b)^2 \Omega \Delta t, \quad C_i = \beta(a + b)^2 \Omega \Delta t, \quad D_i = T_i^{k-1} \Delta x^2, \quad (4.5)
\]

\[
A_0 = 0, B_0 = 1, C_0 = 0, D_0 = -T_1; \quad A_N = 0, B_N = -1, C_N = 1, D_N = 0. \quad (4.6)
\]

Figure 4 shows the results of model calculations (4.1) – (4.2) at \(T_0 = 10^\circ\text{C}\) and \(T_1 = 50^\circ\text{C}\) after 1, 10, 30 and 60 s when applying the electric field. The thickness of the flat channel was assumed to be 1 cm. As electrosensitive carrier ERF prepared in the laboratory of rheophysics and macrokinetics of A.V. Luikov Heat and Mass Transfer Institute of National Academy of Sciences of Belarus was used, with the studied rheological, thermal and physical properties. The heating rate of a remote wall was considered at a given constant temperature of the wall heating.

**Figure 4.** The kinetics of heating ERF in a flat channel at different intensities of the internal rotation through: (a) 1 s, (b) 10 s, (c) 30 s and (d) 60 s.
As seen from figure 4 the presence of the internal rotation leads to a significant acceleration of heat transfer from one wall of a flat channel to another: after 1 min after the electric field at $\Omega = 3, 4$ and $5$ s$^{-1}$, channel wall temperatures were almost identical (up to 2 $^\circ$C).

Let us consider the case of cooling. Let the temperature of ERF flat channel walls to be known at the initial moment of time. We will study the process of cooling the system, setting the distal boundary conditions of heat transfer by natural convection and radiation. In this case we will vary the intensity of the internal rotations. The results of these calculations are shown in figure 4. The initial temperature was 50 $^\circ$C, the ambient temperature 10 $^\circ$C.

The coefficient of heat exchange with the environment $\alpha_{\text{env.}}$, in the absence of external convection is

$$\alpha_{\text{env.}} = \alpha_{\text{nat.conv.}} + \alpha_{\text{rad.}},$$

where $\alpha_{\text{nat.conv.}}$ – coefficient of heat transfer by natural convection of air from the outer surface of the absorber, W / (m$^2$K); $\alpha_{\text{rad.}}$ – coefficient of heat transfer by radiation from the outer surface of the absorber, W / (m$^2$K).

The value of $\alpha_{\text{nat.conv.}}$ was evaluated according to [21] for the horizontal plane: $\alpha_{\text{nat.conv.}} = 5$ W / (m$^2$K). The coefficient $\alpha_{\text{rad.}}$ was calculated by the Stefan–Boltzmann law: $\alpha_{\text{rad.}} = 5.7$ W / (m$^2$K).

The results of model calculations (2.25) – (2.26) for the described conditions are shown in figure 5. The time during which a plane boundary of the channel at $x = 0$ was cooled to within 1 $^\circ$C to ambient temperature was as follows: for $\Omega = 1$ s$^{-1}$ – the simulation time exceeded 1000 s, 2 s$^{-1} – 807$ s, 3 s$^{-1} – 538$ s, 4 – 404 s, 5 s$^{-1} – 323$ s.

Thus, the rotation of the particles under the influence of an alternating electric field has a significant influence on the rate of heat transfer in ERF. This effect can be used to intensify the process of heat transfer in channels of heat exchangers of different configurations.

![Figure 5](image-url)

**Figure 5.** The kinetics of cooling ERF in a plane channel at different intensities of the internal rotation through: (a) 1 s, (b) 10 s, (c) 30 s and (d) 60 s.
5. Conclusions
A study has been performed on physical and mathematical modeling of microconvection heat transfer in electrorheological fluid in an external alternating electric field varying with a given angular frequency, which takes into account the occurrence of microvortices carrying the energy and the mass of the dispersion medium as a result of particles' rotation of the dispersed medium.

The problem of heat transfer in a flat layer of electrically sensitive fluid in an alternating electric field with the microconvection is formulated and numerically solved. Numerical simulations showed that the presence of internal rotation significantly intensifies the heat exchange process in electrorheological fluids, which can be used to create heat exchangers, controlled by an external electric field.

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