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Abstract

We consider some long multiplets describing bulk massive excitations of M-theory two-branes and IIB string three-branes which correspond to “non chiral” primary operators of the boundary $OSp(8/4)$ and $SU(2,2/4)$ superconformal field theories.

Examples of such multiplets are the “radial” modes on the branes, including up to spin 4 excitations, which may be then considered as prototypes of states which are not described by the K-K spectrum of the corresponding supergravity theories on $AdS_4 \times S^7$ and $AdS_5 \times S^5$ respectively.

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1 Introduction

The conjectured $AdS_{d+1}/CFT_d$ correspondence [1, 2, 3, 4] naturally associates the spectrum of the K–K excitations of the supergravity theory [5, 6, 7] describing the horizon geometry of M-theory and string theory branes [8, 9, 10] to the spectrum of “chiral” primary conformal operators of the boundary conformal field theory [11, 12, 3, 13, 14], related to the brane world-volume theory [15]. A detailed analysis of this correspondence has been carried out for all maximally supersymmetric theories [3, 13, 14], corresponding to 16 (Poincaré) supersymmetries on the world-volume theory, and also for many examples with reduced supersymmetry, corresponding to matter-coupled anti de Sitter supergravity in the bulk theory.

A particular property of these states is that their $AdS$ mass is quantized and this corresponds to the absence of anomalous dimensions of the corresponding “operators” describing the very same representations when the same superalgebra is realized as conformal field theory on the anti de Sitter boundary [3]. On the other hand the $CFT_d$ naturally contains operators which have the same quantum numbers of string or M-theory excitations not present in the K-K spectrum of the corresponding $AdS_{d+1}$ supergravity theory [3, 2, 16]. These states have “anomalous dimensions”, which in type IIB string compactified on $AdS_5 \times S_5$ grow as $(g^2 N)^{1/4}$, as a consequence of the relation between $AdS$ masses and conformal dimensions of the alluded correspondence [2]. An immediate consequence of this fact is that these excitations are described by “non chiral” primary superfields representations which, in $AdS_5$, describe “massive” string excitations in “long multiplets” of the $SU(2, 2/4)$ superalgebra [17].

In the present paper we describe the excitations corresponding to these multiplets and their degeneracy, which is a consequence of the underlying extended superalgebra.

Without loss of generality we will describe the simplest of these long multiplets in M-theory on $AdS_4 \times S_7$ and IIB string theory on $AdS_5 \times S_5$. This multiplet is associated to the radial mode of the two- and three-brane respectively, i.e. it is the supermultiplet whose (lowest dimensional) component is $Tr(X_T X_T)$, where $X_T$ are the “coordinates” transverse to the brane. Here the trace is taken over some Lie Algebra, present for the case of $N$ branes, which however does not play any role in our discussion. It turns out that this multiplet is actually the “simplest” long supermultiplet of the corresponding superalgebra and it contains $2^{16}$ states with spin range from 0 up to 4.
The paper is organized as follows: In section II we will review some properties of massive extended superfields. In section III we will describe the $d = 3$ case, corresponding to the M-theory 2-brane. This case is technically simpler even if the corresponding superconformal field theory exists only as the infrared limit of the gauge theory on the brane and is not known. In section IV we will then consider the case of IIB theory on AdS$_5 \times S_5$ and describe the simplest long multiplet of the SU$(2,2/4)$ superalgebra, sustaining “anomalous dimensions” for the primary conformal operators. This multiplet contains the same number of states as the $d = 3$ example even if the interpretation is different. We will give the quantum numbers and degeneracy of the bulk “excitations” corresponding to these multiplets, with regard to spherical harmonics on $S_5$.

The paper will end with some conclusions and outlooks.

2 Long multiplets in extended supersymmetry

In this section we review some properties of extended superfields in $d$-dimensional Minkowski space $M_d$, for the cases $d = 3$, $d = 4$.

In the case of N-extended supersymmetry, the $d = 3, 4$ algebras have a R-symmetry $O(N)$ and $U(N)$ respectively ($SU(4)$ for $N = 4$). The theories with maximal (conformal) supersymmetry correspond to $N = 8$ at $d = 3$ and $N = 4$ at $d = 4$, sustaining an algebra with 16 Poincaré supersymmetries. Poincaré superfields can be enlarged to conformal superfields, in the sense that a Poincaré superfield, through the method of induced representations, can induce a representation of the full superconformal algebra $[18, 19, 20, 21]$. If the superfield is unrestricted, it can carry “anomalous dimensions”. However, if the superfield is restricted, such as chiral superfields are, then the constraints are compatible with conformal invariance only if the “conformal dimension” is “quantized”. This is the origin of the quantized spectrum of “chiral primary superfields” which describe the K–K excitations of supergravity on AdS$_{d+1}$ in the CFT$_d$ correspondence. Examples of such superfields, with quantized conformal dimensions, are given in the $N = 1$, $d = 4$ theory by the chiral multiplets: $\mathcal{D}_{\dot{a}} S_{\{\alpha_1 \ldots \alpha_n\}} = 0$, where $S$ is a $(J,0)$ representation of SL$(2,C)$, or by the “current multiplets” $J_{a\dot{a}}, J$, such that $[13]: D^a J_{a\dot{a}} = \mathcal{D}^{\dot{a}} J_{a\dot{a}} = 0, D^2 J = \mathcal{D}^2 J = 0$. 

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These superfields describe respectively “hypermultiplets”, “gravity multiplet” and “vector multiplets” in the bulk \( N = 2 \) supergravity theory on \( AdS_5 \), with an underlying gauge algebra \( U(2,2/1) \times G_f \), where \( G_f \) is some flavour symmetry of the boundary SCFT\(_4\). The conformal dimension \( \ell \) of these superfields is \( \ell = q_J \) (\( q_J \) is the \( U(1) \) charge of the \( U(2,2/1) \) algebra for the lowest component \( S|_{\theta = 0} \)), \( \ell = 3 \) and \( \ell = 2 \) respectively \[22\].

Note that, for “short multiplets”, the number of components is always \( 2^2 r \), with \( r \) some spin representation (hypermultiplets have \( r = 2 \), the gravity multiplet \( r = 4 \), vector multiplets \( r = 2 \)).

In \( N = 1 \) an unconstrained superfield is a real scalar superfield which has \( 2^4 = 16 \) components, giving \( 4(0,0), 1(\frac{1}{2}, \frac{1}{2}), 2[(\frac{1}{2}, 0) + (0, \frac{1}{2})] \) states. These states are easily seen to describe a “massive” \( N = 2 \) long multiplet in the \( AdS_5 \) correspondence.

Since in the \( AdS/CFT \) correspondence the conformal dimension \( \ell \) and the \( SL(2,C) \) quantum numbers \( (J_1, J_2) \) of a given operator on the boundary are mapped into the quantum numbers \( (E_0, J_1, J_2) \) of the maximal compact subgroup \( O(2) \times SU(2) \times SU(2) \) of \( O(4,2) \) acting on states in the bulk, we will often interchange the notations.

Let us now consider a generic \( N \)-extended theory at \( d = 4 \). The R-symmetry is \( U(N) \) (\( SU(4) \) for \( N = 4 \)). If the conformal superfield is “short” (or chiral) then the superfield will contain \( 2N \theta \)’s (instead of \( 4N \)) and the \( \theta \) expansion will give \( 2^{2N} \) states. These representations have been described elsewhere and will not be repeated here \[23\]. Supersingletons correspond to “ultrashort representations” with multiplicity \( 2^N \) (up to CPT doubling when required). These representations have no particle interpretation on \( AdS_5 \) \[24, 25, 26, 7, 23\].

For long superfields the \( \theta \) expansion gives instead \( 2^{4N} \) states. These states can be thought as a basis for a representation of the Clifford algebra of the orthogonal group \( O(8N) \).

The left and right representations of \( O(8N) \) correspond to bosons and fermions, i.e. to even and odd powers of \( \theta \)’s \[27\].

To classify the states with respect to \( AdS_5 \) quantum numbers we have simply to decompose the rep. of the Clifford algebra, that is the spinor representation of \( O(8N) \) (long multiplets) with respect to the \( O(4) \) spin symmetry and \( U(N) \) R-symmetry.

In the case of \( O(8N) \) the decomposition is as follows \[28\]: \( 8N \rightarrow (4, 2N) \),
under $Sp(4) \times Sp(2N)$, with the further embedding:

$$Sp(4) \rightarrow O(4) \quad (4 \rightarrow (\frac{1}{2}, 0) + (0, \frac{1}{2})) \quad (1)$$
$$Sp(2N) \rightarrow U(N) \quad (2N \rightarrow N + \overline{N}). \quad (2)$$

It follows that each $O(4)$ rep. completes a $O(5)$ rep. and each $SU(4)$ rep. completes a $Sp(8)$ rep. States are therefore naturally classified under $O(5) \times Sp(8)$.

In $d = 3$ the story is very similar, but even simpler. In this case the spin subgroup of $AdS_4$ is simply $SU(2)$ and the R-symmetry of $SCFT_3$ is $O(N)$, with 2-component Majorana spinors. Maximal conformal supersymmetry corresponds to $N = 8$, this being related to $N = 8$ supergravity on $AdS_4$. Short representations of the $OSp(8/4)$ superalgebra both on $AdS_4$ and $\partial AdS_4$ have been described elsewhere. The simplest of them is the $N = 8$ graviton multiplet which is obtained tensoring the singleton representation $(\phi_s, \psi_c)$ where $\phi_s, \psi_c$ are a massless conformal scalar and spin-$\frac{1}{2}$ fields in the two 8 dimensional spinor representations $8_s, 8_c$ of $O(8)$. This multiplet contains $2^8$ states corresponding to the $N = 8$ supergravity multiplet [4].

Long supermultiplets correspond to massive supermultiplets in $AdS_4$ which do not correspond to “chiral” primary superfields on the boundary. The $\theta$ expansion will involve $16 \theta$’s and contains $2^{16}$ states with spin range $s = 0, \cdots, \frac{N}{2} = 4$.

To classify the degeneracy of these states with respect to the spin group $SU(2)$ and the R-symmetry $O(N)$ we decompose $O(4N)$ as follows: $4N \rightarrow (2, 2N)$, under $O(4N) \rightarrow SU(2) \times Sp(2N)$, with the further decomposition: $2N \rightarrow (2, N)$, under $Sp(2N) \rightarrow SU(2) \times O(N)$ . From the above we then note that massive long multiplets have a high degeneracy. For $N = 8$, states of a given spin are classified by $Sp(16)$ reps. Then $Sp(16)$ representations are naturally decomposed under the R-symmetry group $O(8)$ and an “internal” $SU_I(2)$ symmetry which gives the degeneracy of states in the same spin $SU(2)$ and R-symmetry $O(8)$ representation.
3 Non chiral primary operators in the $AdS_4/CFT_3$ correspondence: M-theory on $AdS_4 \times S_7$

In M-theory 2-branes the fundamental conformal multiplet can be taken to be the supersingleton representation of $OSp(8/4)$.

By a choice of triality basis, this multiplet contains a 3-d boson and a 3-d (Majorana) fermion in the $8_s$, $8_c$ spinor reps. of $O(8)$ respectively. This choice assigns the spinor charges to the vector rep. of $O(8)$ as in standard $N = 8$ supergravity in $AdS_4$.

In the dynamics of N M-branes, these multiplets are assumed to be Lie algebra valued in some group $G$ which depends on the N branes gauge dynamics [14].

The “chiral primary” operators are obtained by superfields whose lowest component is: $Tr(8_s \cdots 8_s) - \text{traces}$, i.e. gauge singlets in the “totally symmetric” irreducible multi spinor representation of $O(8)$.

The 2-branes coordinates and their superpartners are the supersingleton octets [8]. It is obvious that we can make bilinears (or multilinears) in singletons which do not give short representations with spin $s \leq 2$. In fact, by suitable multiplication, in the product of two (massless) supersingletons we may get arbitrarily high spin massless representations with the following structure [29]:

$$2s - 2, 2s - 3/2, 2s - 1, \cdots, 2s, \cdots, 2s + 1, 2s + 3/2, 2s + 2$$

Conserved currents of spin $s$ on the boundary of $AdS_4$ give two physical degrees of freedom (of helicity $\pm s$) in the bulk, as appropriate for massless particles [12]. Therefore the above multiplet contains $2 \times 2^8$ states (unless $s = 0$).

The multiplet (3), for $s = 0$, is the graviton multiplet with scalars in the $35^+, 35^-$ (in this case the degrees of freedom are $2^8$). This corresponds to a superfield starting with the $35$ of $8_s \times 8_s|_{S} = 1 + 35$.

The spin 4 multiplet ($s = 1$) corresponds instead to a massless superfield, starting with the singlet in $8_s \times 8_s|_1$. This is what we call the radial mode on the brane. In the next section we will see that a similar multiplet exists for the 3-brane of type IIB string on $AdS_5 \times S_5$.

We now consider the simplest superfield which corresponds to a long multiplet of the $OSp(8/4)$ algebra.
In a hypothetical 3-d interacting theory of singletons, we may think the multiplet to be obtained by a scalar unrestricted $N=8$ superfield whose lowest component is: $Tr(\phi_s\phi_s)$, where however now we do not assume $\phi_s$ (and $\psi_c$) to be massless.

As a result we expect to obtain a massive spin 4 multiplet with $2^{16}$ components classified in different representations of the spin $SU(2)$ and of the R-symmetry $O(8)$.

In this case the decomposition of the Clifford algebra of $O(32)$ with respect to $SU(2) \times Sp(16)$ is straightforward [27] because it is the same as the one which occurs for long Poincaré multiplets of $N=8$ Poincaré supergravity in $D=4$.

The result is that the representation of the Clifford algebra of $O(32)$ decomposes in a direct sum of irreducible (antisymmetric traceless) reps. of $Sp(16)$, where the $k$-fold antisymmetric component corresponds to spin $s = 4 - \frac{k}{2}$ ($k = 0, \cdots, 8$) each representation occurring with multiplicity one.

The R-symmetry content of each of these $Sp(16)$ reps. can be further analysed by decomposing $Sp(16)$ with respect to the maximal subgroup $SU_I(2) \times O(8)$ [30], where $SU_I(2)$ is an internal spin which gives the degeneracy of each $O(8)$ rep. for a given (space–time) spin $s$.

This multiplet is an obvious candidate for M-theory massive excitations obtained from the AdS$_4$/CFT$_3$ correspondence.

Note that since the spectrum of M-theory on $AdS_4 \times S_7$ is unknown, we cannot be sure that such a multiplet really occurs in the spectrum, but what we mean is that any long massive multiplet will have a similar structure. Indeed the multiplet considered before is the smallest long massive representation in $AdS_4$, as much as the same as the graviton multiplet is the smallest short rep. in $AdS_4$.

In tables 1, 2 we report the massive excitations contained in the long multiplet, corresponding to a scalar unrestricted $N=8$ superfield on $\partial AdS_4$, and the related R-symmetry representations.

Note that, for a given spin $s$, the internal spin $SU_I(2)$ classifies the conformal dimensions of operators in a given $O(8)$ representation according to the formula:

$$\ell_{J_3, J_s} = \ell + J_3 + 4; \quad -J \leq J_3 \leq +J; \quad 0 \leq J \leq 4 - s$$

(4)

where we denoted with $\ell$ the conformal dimension of the lowest component of the superfield.

From the spectrum we may just notice that the only $O(8)$ singlets are a spin
4 state (with conformal dimension $\ell + 4$), which is a scalar under $SU_I(2)$, and nine scalar states (with conformal dimensions $\ell, \ell + 1, \cdots, \ell + 8$), which have spin 4 under $SU_I(2)$.

In the free field theory limit ($\ell = 1$), where the singletons are massless, this multiplet shrinks to a massless representation of the $OSp(8/4)$ superalgebra and only two scalar states remain, in $Tr\phi_s\phi_s$ with $\ell = 1$ and in $Tr\psi_c\psi_c$ with $\ell = 2$.

Also the multiplicity of the higher spin states is just one for each anti-symmetric rep. of $O(N)$, according to (3).

4 “Non chiral” primary operators in the $AdS_5/CFT_4$ correspondence: IIB string theory on $AdS_5 \times S_5$

In $d = 4$, the $AdS/CFT$ correspondence is much more interesting because it relates 4-d superconformal invariant Yang–Mills theory to extended supergravity in $AdS_5$ with an underlying superalgebra $U(2,2/N)$.

The maximal case, corresponding to IIB string theory compactified on $AdS_5 \times S_5$, corresponds to $d = 4$, $N = 4$ Yang–Mills theories, which are conformal invariant for arbitrary values of the Yang–Mills coupling.

The K–K spectrum of ten dimensional IIB supergravity on $AdS_5 \times S_5$ has been considered in ref. [6, 7], and its correspondence with the “twisted chiral” primary superfields of the $N = 4$ superconformal algebra has been shown in detail in ref. [17, 21].

The “chiral” primary superfields correspond to short multiplets whose lowest component is $[20]: Tr(\phi_{\ell_1} \cdots \phi_{\ell_p}) - \text{traces}$, where $\phi_{\ell}$ is a $SU(N)$ Lie-algebra valued “transverse” coordinate and the $(0, p, 0) SU(4)$ rep. is singled out.

In ref. [2] Gubser, Klebanov and Polyakov have pointed out that a “Yang–Mills” composite operator which couples to string states exists such as: $Tr(F_{\mu_1\nu_1} \nabla_{\alpha_1} \cdots \nabla_{\alpha_n} F_{\mu_2\nu_2})$, where $F_{\mu\nu}$ is the Yang–Mills field strength. These operators, being related to string massive states, have anomalous dimensions $\delta$ which, through the $AdS_5/CFT_4$ correspondence, are predicted to grow, at strong coupling $g^2N$ large, as $\delta = (g^2N)^{1/4}$.

In the context of $N = 4$ superconformal symmetry, this necessarily implies that these operators are members of “long multiplets”, naturally associated to “excitations” in $AdS_5$ which do not correspond to “short reps” of the
SU(2, 2/4) algebra.

In this section we analyze in detail the simplest of these excitations, contained in the product of two singleton representations, namely the superfield whose lowest component is the “radial mode” on the brane: \( \Phi = Tr(\phi R^\ell) \). This composite scalar is expected to have anomalous dimension (growing as \((g^2N)^{1/4}\) in the strong coupling regime) and contains excitations on \( AdS_5 \times S_5 \) up to spin 4 [19].

This multiplet is the \( N = 4 \) version of the so-called Konishi multiplet [31], which in \( N = 1 \) notation is a real unrestricted superfield satisfying the relation: \( \mathcal{D}\mathcal{D}\Sigma = W \), where \( \Sigma = S i e^{g V S_i} \), \( W = g f_{\Lambda \Sigma \Delta} e^{ijk} S_i^\Lambda S_j^\Sigma S_k^\Delta \), \( i = 1, 2, 3 \), and \( \Lambda, \Sigma, \Delta \in \text{Adj}SU(N) \).

In the free field theory limit this multiplet corresponds to a short conserved current multiplet with \( \mathcal{D}\mathcal{D}\Sigma = D D \Sigma = 0 \) and \( \ell = 2 \). It corresponds to a massless vector multiplet in \( AdS_5 \) according to the discussion of section II. However in the non abelian gauge theory the \( \Sigma \) multiplet becomes “long” and corresponds to a “massive vector multiplet” in \( AdS_5 \). Some properties of this massive multiplet, containing \( 2^{16} \) states, were given in ref. [17]. Here we give the complete spectrum of the \( AdS_5 \) representations and their degeneracy.

Let us first notice that for an orthogonal group \( O(4NM) \) there is always a maximal subalgebra \( Sp(2N) \times Sp(2M) \), according to the embedding [28]: \( 4NM \rightarrow (2N, 2M) \) of the vector representation. A particular case of this decomposition (\( N = 1, M = 8 \)) was used in the previous section.

On the 4\( d \) boundary of \( AdS_5 \) the relevant decomposition is with respect to a group which contains the space–time spin \( O(4) \) and the R-symmetry \( SU(4) \) of the corresponding \( SU(2, 2/4) \) superalgebra. Then we see that the appropriate decomposition corresponds to \( N = 2, M = 4 \) i.e.: \( O(32) \rightarrow Sp(4) \times Sp(8), 32 \rightarrow (4, 8) \), with the further decomposition:

\[
4 \rightarrow (\frac{1}{2}, 0) + (0, \frac{1}{2}) \quad \text{under} \quad Sp(4) \rightarrow SU(2) \times SU(2) \\
8 \rightarrow 4 + 4 \quad \text{under} \quad Sp(8) \rightarrow SU(4) \times U(1) \quad (5)
\]

Having defined the embedding it is now straightforward to analyze the \( O(32) \) Clifford algebra in terms of \( Sp(4) \times Sp(8) \) representations. They are given by the Young tableaux exhibited in table 3 (\( U(1) \) charges have not been given here).

The further decomposition in \( SU(2) \times SU(2) \times SU(4) \) representations is given in tables 4, 5, 6.

As in \( d = 3 \) there is a sense to make the massless limit of this massive
representation. This corresponds to consider the product \( Tr(\phi_\ell \phi_\ell) \) where \( \phi_\ell \) is a free field \( [23] \). In this case one obtains a massless multiplet in \( AdS_5 \) up to spin 4, which contains \( 2^8(2(J_L + J_R) + 1) \) states with \( J_L + J_R = 2 \). The spectrum of the massless multiplets is given by Table 2 of ref. \([19]\) where all \((J_L, J_R)\) conformal fields, with \( J_L, J_R \geq \frac{1}{2} \), are conserved currents on the boundary (and have therefore \( E_0 = 2 + J_L + J_R \)). This is the same spectrum as obtained in Table 12 of ref. \([23]\).

As before we may easily identify states which are s-waves with respect to an hypothetical partial wave analysis on \( S_5 \).

The \( SU(4) \) singlets occur in the \( 1 (1), 36 (2), 308 (1), 825 (1) \) and \( 594 (3) \) of \( Sp(8) \). This gives rise to 7 scalar singlets, 4 \( (\frac{1}{2}, \frac{1}{2}) \) vectors, 5 \((1,1)\) tensors, 2 \( (\frac{3}{2}, \frac{3}{2})\) tensors, 2 \( ((2,0), (0,2), (\frac{3}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{3}{2}))\) tensors and 1 \((2,2)\) tensor. The \( SU(4) \) s-waves have the following value of the energy:

\[
\begin{align*}
(0,0) & \quad E_0 = \ell, \ell + 2, \ell + 4, \ell + 4, \ell + 6, \ell + 8 \\
\left(\frac{1}{2}, \frac{1}{2}\right) & \quad E_0 = \ell + 1, \ell + 3, \ell + 5, \ell + 7 \\
(1,1) & \quad E_0 = \ell + 2, \ell + 4, \ell + 4, \ell + 4, \ell + 6 \\
\left(\frac{3}{2}, \frac{3}{2}\right) & \quad E_0 = \ell + 3, \ell + 5 \\
(2,0), (0,2) & \quad E_0 = \ell + 2, \ell + 6 \\
\left(\frac{3}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{3}{2}\right) & \quad E_0 = \ell + 3, \ell + 5 \\
(2,2) & \quad E_0 = \ell + 4
\end{align*}
\] (6)

### 5 Concluding remarks

In this paper we have analyzed the structure of long multiplets, in the spirit of the \( AdS/CFT \) correspondence, for the maximally supersymmetric theories corresponding to \( d = 3, d = 4 \) SCFT’s with 16 (Poincaré) spinorial charges. As prototypes of these multiplets we considered the “radial mode” of the corresponding \((d-1)\)-brane, which contains up to spin 4 excitations and cannot therefore be described by supergravity on \( AdS_{d+1} \times S_{D-d-1} \).

Multiplets with these properties are expected to describe M-theory and string theory higher spin excitations. They can sustain anomalous dimensions and correspond to long massive multiplets on the \( AdS \) bulk. It is in principle possible to construct such kind of superconformal multiplets for arbitrarily high spin. The counterpart of this in a non interacting conformal
field theory is that it is possible to construct superconformal fields of arbitrary spin corresponding to conserved currents on the boundary, describing therefore massless fields in the bulk theory. It is likely that such operators for $d = 4, N = 4$ Yang–Mills theory give rise, in the corresponding theory on $AdS_5$, to stringy corrections $^{32, 33}$ in the scattering amplitudes for massless states. The latter can be computed from OPE techniques on the boundary conformal field theory $^{34, 17, 35}$.

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Tables
Table 1: Massive excitations on $AdS_4; O(32) \to SU(2) \times Sp(16) \to SU(2) \times SU(2)_I \times O(8)$ classification

| spin $\frac{1}{2}$ | $Sp(16)$ | $(SU(2)_I, O(8))$ |
|--------------------|----------|-------------------|
| 4                  | 1        | $(1, 1)$          |
| 3                  | 16       | $(2, 8)$          |
| 2                  | 119      | $(3, 28) + (1, 35)$ |
| 3                  | 544      | $(4, 56) + (2, 160)$ |
| 2                  | 1700     | $(5, 70) + (3, 350) + (1, 300)$ |
| 3                  | 3808     | $(6, 56) + (4, 448) + (2, 840)$ |
| 2                  | 6188     | $(7, 28) + (5, 350) + (3, 1134) + (1, 840')$ |
| 1                  | 7072     | $(8, 8) + (6, 160) + (4, 840) + (2, 1344)$ |
| $\frac{1}{2}$     | 4862     | $(9, 1) + (7, 35) + (5, 300) + (3, 840') + (1, 588)$ |
| rep | Young tableau | Dynkin label |
|-----|---------------|--------------|
| 1   | \(1\)         | \((0,0,0,0)\) |
| 8   | \(\square\)   | \((1,0,0,0)\) |
| 28  | \(\square\)   | \((0,1,0,0)\) |
| 35  | \(\square\)   | \((2,0,0,0)\) |
| 56  | \(\square\)   | \((0,0,1,1)\) |
| 160 | \(\square\)   | \((1,1,0,0)\) |
| 70  | \(\Box\)      | \((0,0,2)\) + \((0,0,2,0)\) |
| 350 | \(\Box\)      | \((1,0,1,1)\) |
| 300 | \(\Box\)      | \((0,2,0,0)\) |
| 448 | \(\Box\)      | \((1,0,2)\) + \((1,0,2,0)\) |
| 840 | \(\Box\)      | \((0,1,1,1)\) |
| 840' | \(\Box\)     | \((0,0,2,2)\) |
| 1134 | \(\Box\) | \((0,1,0,2)\) + \((0,1,2,0)\) |
| 1344 | \(\Box\) | \((0,0,1,3)\) + \((0,0,3,1)\) |
| 588 | \(\Box\) | \((0,0,0,4)\) + \((0,0,4,0)\) |
Table 3: Massive excitations on $AdS_5; Sp(4) \times Sp(8)$ classification and Young tableaux

| Spin | $Sp(4)$ dim | Sp(8) dim |
|------|-------------|-----------|
| 4    | 55          | 1         |
| $\frac{7}{2}$ | 80          | 8         |
| 3    | 81          | 27        |
|      | 30          | 36        |
| $\frac{5}{2}$ | 64          | 48        |
|      | 40          | 160       |
| 2    | 35          | 42        |
|      | 35'         | 315       |
|      | 14          | 308       |
| $\frac{3}{2}$ | 20          | 288       |
|      | 16          | 792       |
| 1    | 10          | 792'      |
|      | 5           | 825       |
| $\frac{1}{2}$ | 4           | 1056      |
| 0    | 1           | 594       |
| dim | $|\text{Sp}(4)|$ | $(\text{SU}(2),\text{SU}(2))$ rep |
|-----|----------------|---------------------------------|
| 1   | $(0, 0)$       |                                 |
| 4   | $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ |                               |
| 5   | $(\frac{1}{2}, \frac{1}{2}) + (0, 0)$ |                               |
| 10  | $(1, 0) + (\frac{1}{2}, \frac{1}{2}) + (0, 1)$ |                             |
| 16  | $(1, \frac{1}{2}) + (\frac{1}{2}, 1) + (\frac{1}{2}, 0) + (0, \frac{1}{2})$ |                           |
| 20  | $(\frac{3}{2}, 0) + (0, \frac{3}{2}) + (1, \frac{1}{2}) + (\frac{1}{2}, 1)$ |                         |
| 14  | $(1, 1) + (\frac{1}{2}, \frac{1}{2}) + (0, 0)$ |                       |
| 35' | $(\frac{3}{2}, \frac{1}{2}) + (\frac{1}{2}, \frac{3}{2}) + (1, 1) + (1, 0) + (0, 1) + (\frac{1}{2}, \frac{1}{2})$ |                     |
| 35  | $(2, 0) + (0, 2) + (\frac{3}{2}, \frac{1}{2}) + (\frac{1}{2}, \frac{3}{2}) + (1, 1)$ |                   |
| 40  | $(\frac{3}{2}, 1) + (1, \frac{3}{2}) + (1, \frac{1}{2}) + (\frac{1}{2}, 1) + (\frac{1}{2}, 0) + (0, \frac{1}{2})$ |                 |
| 64  | $(2, \frac{1}{2}) + (\frac{1}{2}, 2) + (\frac{3}{2}, 1) + (1, \frac{3}{2}) + (\frac{3}{2}, 0) + (0, \frac{3}{2}) + (1, \frac{1}{2}) + (\frac{1}{2}, 1)$ |               |
| 30  | $(\frac{3}{2}, \frac{3}{2}) + (1, 1) + (\frac{1}{2}, \frac{1}{2}) + (0, 0)$ |                 |
| 81  | $(2, 1) + (1, 2) + (\frac{3}{2}, \frac{3}{2}) + (\frac{3}{2}, \frac{1}{2}) + (\frac{1}{2}, \frac{3}{2}) + (1, 1) + (1, 0) + (0, 1) + (\frac{1}{2}, \frac{1}{2})$ |        |
| 80  | $(2, \frac{3}{2}) + (\frac{3}{2}, 2) + (\frac{3}{2}, 1) + (1, \frac{3}{2}) + (1, \frac{1}{2}) + (\frac{1}{2}, 1) + (\frac{1}{2}, 0) + (0, \frac{1}{2})$ |      |
| 55  | $(2, 2) + (\frac{3}{2}, \frac{3}{2}) + (1, 1) + (\frac{1}{2}, \frac{1}{2}) + (0, 0)$ |          |
### Table 5: $Sp(8) \rightarrow SU(4)$

| $\dim [Sp(8)]$ | $\dim [SU(4)]$ |
|----------------|----------------|
| 1              | 1              |
| 8              | $4 + \overline{14}$ |
| 27             | $6 + 6 + 15$   |
| 36             | $1 + 10 + \overline{10} + 15$ |
| 48             | $4 + \overline{14} + 20 + 2\overline{6}$ |
| 160            | $4 + \overline{14} + 20 + 2\overline{6} + 20 + 2\overline{6} + 36 + 3\overline{6}$ |
| 120            | $4 + \overline{14} + 20'' + 20'' + 36 + 3\overline{6}$ |
| 42             | $1 + 1 + 10 + \overline{10} + 20'$ |
| 315            | $6 + 6 + 10 + \overline{10} + 15 + 15 + 15 + 20' + 45 + 4\overline{15} + 64 + 64$ |
| 308            | $1 + 10 + \overline{10} + 15 + 20' + 20' + 20' + 64 + 64 + 84$ |
| 288            | $4 + \overline{14} + 4 + \overline{14} + 20 + 2\overline{6} + 20'' + 20'' + 36 + 3\overline{6} + 60 + 6\overline{6}$ |
| 792            | $4 + \overline{14} + 20 + 2\overline{6} + 20 + 2\overline{6} + 20 + 2\overline{6} + 36 + 3\overline{6} + 36 + 3\overline{6}$ |
|                | $+ 60 + 6\overline{6} + 60 + 6\overline{6} + 140 + 14\overline{6}$ |
| 792'           | $6 + 6 + 6 + 15 + 15 + 15 + 45 + 4\overline{15} + 45 + 4\overline{15} + 50 + 50$ |
|                | $+ 64 + 64 + 70 + 7\overline{6} + 175$ |
| 825            | $1 + 10 + \overline{10} + 10 + \overline{10} + 15 + 20' + 20' + 45 + 4\overline{15}$ |
|                | $+ 64 + 64 + 84 + 126 + 12\overline{6} + 175$ |
| 1056           | $4 + \overline{14} + 4 + \overline{14} + 20 + 2\overline{6} + 20 + 2\overline{6} + 20'' + 20'' + 36 + 3\overline{6}$ |
|                | $+ 60 + 6\overline{6} + 84' + 8\overline{6} + 140 + 14\overline{6} + 140' + 14\overline{6}$ |
| 594            | $1 + 1 + 1 + 10 + \overline{10} + 10 + \overline{10} + 20' + 20' + 35 + 3\overline{5}$ |
|                | $+ 84 + 105 + 126 + 12\overline{6}$ |
Table 6: $SU(4)$ reps of interest

| rep | Young tableau | Dynkin label |
|-----|---------------|--------------|
| 1   |               | (0,0,0)      |
| 4   |               | (1,0,0)      |
| 4   |               | (0,0,1)      |
| 6   |               | (0,1,0)      |
| 10  |               | (2,0,0)      |
| 20  |               | (1,1,0)      |
| 20' |               | (3,0,0)      |
| 15  |               | (1,0,1)      |
| 20' |               | (0,2,0)      |
| 45  |               | (2,1,0)      |
| 35  |               | (4,0,0)      |
| 36  |               | (2,0,1)      |
| 60  |               | (1,2,0)      |
| 84' |               | (3,1,0)      |
| 50  |               | (0,3,0)      |
| 126 |               | (2,2,0)      |
| 64  |               | (1,1,1)      |
| 140' |              | (1,3,0)      |
| 70  |               | (3,0,1)      |
| 140 |               | (2,1,1)      |
| 84  |               | (2,0,2)      |
| 105 |               | (0,4,0)      |
| 175 |               | (1,2,1)      |