EVIDENCE FOR A NONUNIVERSAL STELLAR INITIAL MASS FUNCTION FROM THE INTEGRATED PROPERTIES OF SDSS GALAXIES

ERIK A. HOVERSTEN
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218

AND

KARL GLAZEBROOK
Centre for Astrophysics and Supercomputing, Swinburne University of Technology, Hawthorn, VIC 3122, Australia

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ABSTRACT

This paper revisits the classical Kennicutt method for inferring the stellar IMF from the integrated light properties of galaxies. The large-size, uniform high-quality data set from the SDSS DR4 is combined with more in-depth modeling and quantitative statistical analysis to search for systematic IMF variations as a function of galaxy luminosity. Galaxy Hα equivalent widths are compared to a broadband color index to constrain the IMF. This parameter space is useful for breaking degeneracies that are traditionally problematic. Age and dust corrections are largely orthogonal to IMF variations. In addition, the effects of metallicity and smooth SFH e-folding times are small compared to IMF variations. We find that for the sample as a whole the best-fitting IMF slope above 0.5 \( M_J \) is \( \Gamma = 1.4535 \), with a negligible random error of \( \pm0.0004 \) and a systematic error of \( \pm0.1 \). Galaxies brighter than around \( M_J < -20 \) (including galaxies like the Milky Way, which has \( M_J \sim -21 \)) are well fitted by a universal \( \Gamma \sim 1.4 \) IMF, similar to Salpeter, and smooth, exponential SFHs. Fainter galaxies prefer steeper IMFs, and the quality of the fits reveals that for these galaxies a universal IMF with smooth SFHs is actually a poor assumption. Several sources of sample bias are ruled out as the cause of these luminosity-dependent IMF variations. Analysis of bursting SFH models shows that an implausible coordination of burst times is required to fit a universal IMF to the \( M_J \sim -17 \) galaxies. This leads to the conclusions that the IMF in low-luminosity galaxies has fewer massive stars, by either steeper slope or lower upper mass cutoff, and is not universal.

Subject headings: galaxies: evolution — galaxies: stellar content — stars: luminosity function, mass function

1. INTRODUCTION

A precise measurement of the stellar initial mass function (IMF) and its functional dependence on environmental conditions would impact astronomy over a wide range of physical scales. It would be of great help to theorists in untangling the mysteries of star formation, and it is a key input in spectral synthesis models used to interpret the observed properties of galaxies both nearby and in the early universe.

The current question is whether the IMF is universal, i.e., the same regardless of time and environmental conditions. Kennicutt (1998b) concisely states the current understanding of IMF universality. It is difficult to believe that the IMF is universal given the diversity of galaxy types, environments, star formation rates, and populations within galaxies over the range of observable lookback times. On the other hand, while IMF measurements do vary, they are all consistent with a universal IMF within measurement errors and sampling statistics. The only way to proceed then is to strive for smaller measurement errors and improving sample sizes.

A definitive theoretical derivation of the IMF does not yet exist. Theoretical approaches to the IMF usually center around the Jeans mass, \( M_J \), the mass at which a homogeneous gas cloud becomes unstable. At first the collapse of a cloud is isothermal and the Jeans mass decreases, which leads to fragmentation of the cloud (Hoyle 1953). Both Rees (1976) and Low & Lynden-Bell (1976) suggested that at some point during the cloud collapse the cooling opacity becomes high enough that the collapse is no longer isothermal. At this point the Jeans mass increases and fragmentation stops. The minimum Jeans mass is the smallest fragment size at this point, and it provides a lower limit to the size of the stars formed.

Authors have calculated Jeans masses and minimum Jeans masses using a variety of methods. In the classical derivation of the Jeans mass \( M_J \propto T^{3/2}\mu^{-1/2} \), Low & Lynden-Bell (1976) find that \( M_{J,\text{min}} \propto m^{-16/7}\kappa^{-1/7} \), where \( m \) is the mass of gas atoms or molecules and \( \kappa \) is the opacity at the final fragmentation. More recently, turbulence in clouds has been studied. Padmanabhan et al. (1997) found \( M_{J,\text{min}} \propto n^{-1/2}T^{2}\sigma_e^{-1} \), where \( n \) is the number density and \( \sigma_e \) is the velocity dispersion of the gas. Other investigators have looked at the hierarchical fractal geometry of molecular clouds, thought to arise from turbulence, as a generator of the IMF (e.g., Elmegreen 1997). On a related note, Adams & Fatuzzo (1996) point out that molecular clouds exhibit structure on all resolvable spacial scales, suggesting that as no characteristic density exists for the clouds, neither does a single Jeans mass. They develop a semiempirical model for determining the final masses of stars from the initial conditions of molecular clouds without invoking Jeans mass arguments and use it to construct IMF models. The key components of their model are sound speed and rotation rate of cloud cores and the idea that stars help determine their final masses through winds and outflows.

But from the beginning the study of the IMF has been driven by measurements. In 1955 Salpeter was the first to make a measurement of the IMF, inferring it from his observed stellar luminosity function (Salpeter 1955). We parameterize the IMF by

\[
\frac{dn}{d\log m} \propto \begin{cases} 
0.5 & \text{if } 0.1 < m/M_\odot < 0.5, \\
\Gamma & \text{elsewhere}, 
\end{cases}
\]

(1)

following Baldry & Glazebrook (2003). Salpeter found that \( \Gamma = 1.35 \). It is often overlooked that his measurement only covered masses for which \( 0.4 < m/M_\odot \sim 10 \). Nonetheless, his original
measurement is surprisingly consistent with modern values over a wide range of masses. The IMF in equation (1) is similar to the Salpeter IMF for $\Gamma = 1.35$. The difference is that there are fewer stars with masses less than 0.5 $M_\odot$. We adopt a two-part power law as there is agreement among several authors that there is a change in the IMF slope near 0.5 $M_\odot$ (Kroupa 2001). The technique we use here is not sensitive to the IMF at low masses so we assume a constant value in that regime.

Salpeter’s idea continues to be used today in IMF measurements of resolved stellar populations. The technique can be applied to field stars as well as clusters. However, Salpeter’s method has several inherent limitations. The nature of stars presents a challenge. On the high-mass end stars are very luminous but live only a few million years, while on the low-mass end stars are faint but have lifetimes many times longer than the current age of the universe. There are very few star clusters that are both young and close enough to allow us access to the IMF over the full mass range. In addition, the main-sequence mass-luminosity relationship is a function of age, metallicity, and speed of rotation in addition to mass. It is not yet well known at the low- and high-mass extremes. Unresolved binaries can also affect the measured IMF (Kroupa 2001). The light from unresolved binaries is dominated by the more massive of the pair. As a result, the less massive star is typically not detected, which leads to a systematic undercounting of low-mass stars. A fraction of $42\% \pm 9\%$ of main-sequence M stars (Fischer & Marcy 1992) and 43% of main-sequence G stars (Duquennoy & Mayor 1991) are primary stars in multiple star systems. These are both lower limits as some companions may have eluded detection. As roughly half of stars are in multiple systems, it has a potentially large effect on the observed luminosity function.

Except for at the high-mass end, field stars in the solar neighborhood offer the best statistics for IMF measurements. However, the solar neighborhood IMF is found to be deficient in massive stars when compared to other galaxies. For example, the Miller & Scalo (1979) and Scalo (1986) solar neighborhood IMFs are rejected by integrated light approaches, i.e., Kennicutt (1983, hereafter K83), Kennicutt et al. (1994, hereafter KTC94), and Baldry & Glazebrook (2003).

Analysis of individual star clusters can be used to detect IMF variations. As methods and data quality can vary between authors, comparisons between individual clusters are difficult. However, Phelps & Janes (1993) studied eight young open clusters with the same technique. On the extremes they measured $\Gamma = 1.06 \pm 0.05$ for NGC 663 and $\Gamma = 1.78 \pm 0.05$ for NGC 581 over masses from around 1 to 12 $M_\odot$.

Kroupa (2001) considered the case in which one could have perfect knowledge of the masses of all stars in a cluster. Clusters have a finite size so even with no measurement errors uncertainty arises from sampling the underlying IMF. He shows that the observed scatter in IMF power-law values above 1 $M_\odot$ can be accounted for by sampling bias for clusters with between 102 and 103 members (the Phelps & Janes [1993] clusters have memberships in this range). Furthermore, dynamical evolution of clusters can affect measurements of the IMF. This can happen by preferentially expelling lower mass stars from the cluster and by breaking up binary systems within the cluster. In total Kroupa (2001) finds that for stars with $m \gtrsim 1 M_\odot$ the spread in the observed IMF power-law slopes in clusters is around 1 when both binary stars and sampling bias are considered even when the underlying IMFs are identical.

Stochastic processes can also influence the ability to detect systematic IMF variations. O and B stars produce ionizing photons and cosmic rays that affect the surrounding nebula. However, the probability of creating one of these massive stars is comparatively small. If one of these massive stars happens by chance to form first, it may drive up the nebular temperature and depress the formation of less massive stars compared to regions without massive stars (Robberto et al. 2005).

An alternative to IMF measurements based on the stellar luminosity functions of resolved stellar populations is to infer the characteristics of stellar populations from the integrated light of galaxies using spectral synthesis models. The advantage of using integrated light techniques is that many of the problems plaguing IMF investigations of resolved stellar populations are avoided. Stochastic effects are washed out over a whole galaxy. Unresolved star systems are irrelevant. The number of observable galaxies is large, and their environments span a much larger range than those of Milky Way clusters. Integrated light techniques can be applied to the high-redshift universe. This creates a strong motivation to develop IMF techniques and test them for galaxies in the local universe, which can later be used to probe earlier stages of galaxy formation. The assumption of a universal IMF has a huge influence on the interpretation of the high-redshift universe. The reionization of the universe and the Madau plot (the global star formation rate of the universe over cosmic history) are two areas where IMF variations could impact the current picture of galaxy formation and evolution.

On the downside, conclusions are dependent on the stellar evolution models used, which are not well constrained at high masses or with horizontal branch stars at low metallicities. The biggest problem is that changes in the IMF, star formation history (SFH), or age of a galaxy model can have similar effects in the resulting spectral energy distribution (SED). Any integrated light technique needs to address these degeneracies. It is also difficult to probe the IMF at subsolar masses using integrated light.

On the level of galaxies the concept of a universal IMF has recently become more complicated. A number of recent studies have investigated the effect of summing the IMF in individual clusters over a galaxy in the presence of power-law star cluster mass functions. Weidner & Kroupa (2005) argue that the integrated galaxial IMF will appear to vary as a function of galactic stellar mass even if the stellar IMF is universal. However, Elmegreen (2006) argues that the galaxy-wide IMF should not differ from the IMF in individual clusters based on analytical arguments and Monte Carlo (MC) simulations. Our approach can only measure the IMF averaged over whole galaxies and cannot address this distinction. Even so, systematic variations of any kind from the Salpeter slope have not been measured outside of some evidence for nonstandard IMFs in low surface brightness (LSB) galaxies and galaxies experiencing powerful bursts of star formation (Elmegreen 2006). Either way, observational evidence for systematic variations of the IMF in galaxies would be highly valuable.

The plan of this paper is as follows: In § 2 we explain our method for constraining the IMF. In § 3 we describe the SDSS data and our sample selection. Section 4 details our modeling scheme. In § 5 we discuss our statistical techniques. Section 6 reports our results. In § 7 we check our results against the Hα distribution of the data, and § 8 presents our conclusions.

2. METHODOLOGY

This paper revisits the “classic” method of K83 (and the subsequent extension KTC94) to constrain the IMF of integrated stellar populations. The method takes advantage of the sensitivity of the Hα equivalent width (EW) to the IMF. K83 showed that model IMF tracks can be differentiated in the $(B - V) - \log (H\alpha \text{ EW})$ plane.
The total flux of a galaxy at 6565 Å is the combination of the underlying continuum flux plus the flux contained in the Hα emission line. The Hα flux and the continuum flux have different physical origins, both of which can be used to gain physical insights into galaxies.

In the absence of active galactic nucleus (AGN) activity the Hα flux is predominantly caused by massive O and B stars that emit ionizing photons in the ultraviolet. O and B stars are young and found in the regions of neutral hydrogen in which they formed. In case B recombination, where it is assumed that these clouds and found in the regions of neutral hydrogen in which they formed. These photons experience smaller optical depths and can escape the cloud. The transition probabilities are weakly dependent on electron density and temperature and can be calculated. Through this process the measured Hα flux can be converted into the number of O and B stars currently burning in an integrated stellar population.

However, case B recombination is an idealized condition, and it is possible that ionizing photons can escape the cloud without this processing, a situation known as Lyman leakage. As such the Hα flux is a lower limit on the number of O and B stars present.

The continuum flux of a galaxy is due to the underlying stellar population. At 6565 Å the continuum is dominated by red giant stars in the 0.7–3 $M_\odot$ range, while the Hα flux comes from stars more massive than 10 $M_\odot$.

The EW is defined as the width in angstroms of an imaginary box with a height equal to the continuum flux level surrounding an emission or absorption line that contains an area equal to the area contained in the line. This is effectively the ratio of the strength of a emission or absorption line to the strength of the continuum at the same wavelength. Given the physical origins of the Hα flux and the continuum at 6565 Å, the Hα EW is the ratio of massive O and B stars to stars around a solar mass. Therefore, the Hα EW is sensitive to the IMF slope above around 1 $M_\odot$ and can be used to probe the IMF in galaxies.

As mentioned in the introduction, several degeneracies plague the study of the IMF from the integrated light properties of galaxies. Variations in the IMF, age, metallicity, and SFH of galaxy models can all yield similar effects in the resulting spectra. For example, increasing the fraction of massive stars, reducing the age of a galaxy, lowering the metallicity, and a recent increase in the star formation rate will all make a galaxy bluer.

Metallicity effects were not discussed in either K83 or KTC94, and galaxy ages were assumed: 15 Gyr for K83 and 10 Gyr for KTC94.

The SFH in K83 is addressed by calculating models with exponentially decreasing SFHs for a range of e-folding times, as well as a constant and a linearly increasing SFH. In the $(B - V) - \log$(Hα EW) plane the effect of varying the SFH e-folding time is orthogonal to IMF variations. However, this is only true for smoothly varying exponential and linear SFHs. Discontinuities, either increases (bursts) or decreases (gaps), in the star formation rate can affect the Hα EW relative to the color in ways similar to a change in the IMF. Along with the exponential SFHs KTC94 also uses models with instantaneous bursts on top of constant SFHs. However, this was done to access high EWs at an age of 10 Gyr rather than to fully flesh out the effects of SFH discontinuities.

The assumption of smoothly varying SFHs is a key assumption in our analysis. Most late-type galaxies are thought to form stars at a fairly steady rate over much of recent time, although bursts of star formation may play a significant role in low-mass galaxies (Kennicutt 1998a). For these galaxies smoothly varying SFHs are justified. However, there are other galaxies clearly in the midst of a strong burst of star formation (e.g., M82 and dwarf galaxies with complex SFHs, e.g., NGC 1569 (Angeretti et al. 2005), for which this assumption is a poor one. The effects of violations of our SFH assumptions are described in detail in § 6.3.

3. THE DATA

K83 cites four major sources of error, all of which are improved on or eliminated by the high, uniform quality of Sloan Digital Sky Survey (SDSS) spectroscopic and photometric data.

The Hα fluxes in his sample can be contaminated by nonthermal nuclear emission. In the updated investigation, KTC94, this problem is addressed by removing objects with known Seyfert or LINER activity and luminous AGNs. The SDSS spectra allow measurements of emission-line ratios that can be used to separate star-forming galaxies from AGNs (Baldwin et al. 1981).

The second problem is that the Hα emission flux will be underestimated if the underlying stellar absorption of Hα is not taken into consideration. The narrowband filter photometry of Kennicutt & Kent (1983) could not measure this effect for individual galaxies so a fixed ratio was assumed for all galaxies. The SDSS spectroscopic pipeline does not take this into account either. We use the Hα fluxes measured from the SDSS spectra by Tremonti et al. (2004), which fit the continua with stellar population models to more accurately measure the Hα emission. While the SDSS pipeline method is sufficient for strong emission lines, the Hα absorption EW can be as large as 5 Å, which is significant for weaker emission lines.

Third, their narrowband Hα imaging includes [N ii] emission, which is corrected for by assuming a constant [N ii]/Hα ratio. The Hα and [N ii] emission lines are resolved in the SDSS spectra so there is no need for a correction. This is a significant improvement. Kennicutt & Kent (1983) found from a literature survey that the mean value of the Hα/([N ii]) ratio is 0.75 ± 0.12 for spiral galaxies and 0.93 ± 0.05 in irregular galaxies. These mean corrections were applied uniformly to the K83 data. In KTC94 a uniform correction was applied using [N ii]/Hα = 0.5. For comparison in our sample the mean value of the Hα/([N ii] + [N ii]) ratio is a strikingly similar 0.752 and the mean [N ii]/Hα ratio is 0.340. However, [N ii]/Hα ranges from 0.0 to 0.6. If our Hα and [N ii] lines were blended, applying a fixed correction would introduce errors of as much as 25% in the Hα EWs of individual galaxies.

Finally, in both K83 and KTC94 extinction corrections were addressed by plotting data alongside models that either assumed an average value for the extinction or were unextincted. The Balmer decrement (Hα/Hβ) can be measured from SDSS spectra, which allows for extinction corrections for individual galaxies.

Another major advantage of this study is that the sample size is much larger than that of K83 and KTC94. The KTC94 sample contains 210 galaxies, whereas ours has $\sim$105. The large sample size allows us to investigate IMF trends as functions of galaxy luminosity, redshift, and aperture fraction with subsamples larger than the entire KTC94 sample.

There is a key disadvantage to this method as well. K83 and KTC94 were able to adjust the sizes of their photometric apertures to contain the entire disk of individual galaxies to a limiting isophote of 25$\mu$ given by the RC2 catalog (Kennicutt & Kent 1983). The advantage of narrowband measurements of Hα EW is that they can cover a much larger aperture and match the broadband measurements set to match the physical size of individual galaxies. The SDSS has fixed 3" spectroscopic apertures. This problem is partly offset by using matching 3" photometry apertures from the SDSS. However, this introduces aperture effects as observed.
galaxies have a wide range of angular sizes due to the range of physical sizes and distances present in the local universe. This is significant as radial metallicity gradients (e.g., Vila-Costas & Edmunds 1992) are observed in spiral galaxies that could incorrectly be interpreted as radial IMF gradients.

Even so, this method can constrain the IMF within the SDSS apertures. For our program galaxies 23% of the total light falls in the SDSS apertures. The fact that the more distant galaxies are more luminous and tend to be larger helps to balance out the larger physical scales of the fixed aperture size at greater distances. On average 17% of the light falls in the aperture for the faintest galaxies, while it is 25% for the brightest bin. In spite of their limited size, the SDSS apertures still contain a great diversity of stellar populations, which makes this data set an excellent test bed for IMF universality. We present extensive tests of aperture effects below.

3.1. Sample Selection

The sample is selected from SDSS data. The project goal of the SDSS is to image one-quarter of the sky in five optical bands with a dedicated 2.5 m telescope (York et al. 2000). From the imaging 10^7 galaxies and 10^5 quasars will be selected for spectroscopic follow-up. Photometry is done in the ugriz filter system described by Fukugita et al. (1996). Magnitudes are on the apecs system (Lupton et al., 1999), which approaches the AB system with increasing brightness. Spectra are taken with a multiobject fiber spectrograph with wavelength coverage from 3800 to 9200 Å and $R \sim 1800$ (Uomoto et al., 1999).

Our sample is a subsample of the Main Galaxy Sample (MGS) from the SDSS Fourth Data Release (DR4; Adelman-McCarthy et al. 2006). The MGS targets galaxies with $r \leq 17.77$ in Petrosian magnitudes (Stoughton et al. 2002). All galaxies in the MGS are strong detections so the differences between luptitudes and the AB system can be ignored. In order to avoid fiber cross talk in the camera, an upper brightness limit of $g = 15.0$, $r = 15.0$, and $i = 14.5$ is imposed. Targets are selected as galaxies from the imaging by comparing their point-spread function magnitudes to their de Vaucouleur’s and exponential profile magnitudes. Exposure times for spectroscopy are set so that the cumulative median signal-to-noise ratio satisfies $(S/N)^2 > 15$ at $g = 20.2$ and $i = 19.9$ fiber magnitudes. The time to achieve this depends on observing conditions but always involves at minimum three 15 minute exposures.

Due to the construction of the spectrograph, fibers cannot be placed closer than 55" to each other. This may be a source of bias in the sample. Cluster galaxies may be preferentially excluded from the sample. The SDSS collaboration has plans to quantify this effect in the near future (Adelman-McCarthy et al. 2006). LSB galaxies are excluded from the MGS with a surface brightness cut that may also bias the sample (Stoughton et al. 2002). There is some evidence that LSB galaxies may have IMFs that differ from a universal IMF. Lee et al. (2004) find that the comparatively high mass-to-light ratios of LSB galaxies can be explained with an IMF deficient in massive stars relative to normal galaxies.

Our sample begins with the SDSS DR4 (Adelman-McCarthy et al. 2006). DR4 covers 4783 deg^2 in spectroscopy for a total of 673,280 spectra, 567,486 of which are galaxy spectra. A total of 429,748 of these have flags set indicating that they are part of the MGS. DR4 also includes special spectroscopic observations of the Southern Stripe that were not selected by the standard algorithm but nonetheless have the TARGET_GALAXY flag set in primTarget, which usually indicates membership in the MGS. These objects are identified by comparing their spectroscopic plate number to the list of special plates in Adelman-McCarthy et al. (2006) and rejected. This leaves 423,285 spectra. The first round of cuts to our sample addresses general data quality. While the overall quality of SDSS data is high, there are a handful of objects with pathological values for one or more parameters. First, we require that all parameters of interest have reasonable, real values. This means that the Petrosian and fiber magnitudes must be between 0 and 25 and have errors smaller than 0.5 in all five ugriz bands. Line flux errors are capped at $10^{-12}$ erg cm$^{-2}$ s$^{-1}$ Å$^{-1}$ and equivalent widths at $10^3$ Å. For most of the parameters less than 1% of objects fail this loose requirement. However, 6.6% of the objects fail the Petrosian magnitude error requirement. This is most likely due to the photometric pipeline having a difficult time defining the Petrosian radius. As such this constraint is potentially biased against LSB galaxies or galaxies with unusual morphologies. In defense of this cut we later bin our data by luminosity and aperture fraction, both of which are determined in part by the Petrosian magnitudes and also by K-corrections determined from them. In addition, limiting flux errors to 50% is hardly unreasonable. Altogether 391,160 galaxies pass these combined requirements, which is 92.4% of the MGS.

Galaxies from photometry run 1659 are removed because of a known problem with the photometry. This excludes 4485 galaxies (1.1%) from a continuous strip on the sky and should not be a source of bias. We place a further constraint on the z-band fiber magnitude requiring that the error be less than 0.15. The z band generally suffers from the most noise, so this requirement ensures that the fiber magnitude quality is good enough to minimize the chance of erroneous K-corrections that could affect our colors. Only 2866 (0.7%) MGS galaxies fail this test.

Combining the general data quality requirements, the run 1659 rejection, and the fiber z-band error limit leaves 386,647 galaxies (91.3% of the MGS).

The next round of cuts to our sample, while necessary, has clear astrophysical implications. Many of the objects in the MGS have AGN components. As we are interested in studying only the underlying stellar populations of these objects, AGNs must be removed. This is done using the classical Baldwin et al. (1981) diagram comparing the logarithms of the [O III] $\lambda$5007/H$\beta$ and [N II] $\lambda$6584/H$\alpha$ emission line ratios. We used the criterion of Kauffmann et al. (2003) where objects for which

$$\log([\text{O III}]/H\beta) > \frac{0.61}{\log([\text{N II}]/H\alpha) - 0.05} + 1.3$$  \hspace{1cm} (2)$$

are classified as AGNs and rejected. Following Brinchmann et al. (2004), we require the S/N of the H$\alpha$, H$\beta$, [O III], and [N II] lines to be at least 3 to properly classify a galaxy as a star-forming one. A total of 131,807 galaxies (34.1% of MGS objects surviving our first round of cuts) survive this cut.

The above cut automatically rejects any galaxies with weak [O III] and [N II] lines. This excludes galaxies with weak star formation. To a lesser extent metal-poor galaxies that also have weak [N II] emission are rejected as well. Brinchmann et al. (2004) also define a low-S/N star-forming class of galaxies, which we identify and keep in our sample. These are the galaxies that have not already been classified as star-forming or AGNs by strong lines and equation (2), nor have they been identified as low-S/N AGNs by $[\text{N II}] \geq 0.6$ with $S/N > 3$ in both lines, yet still have H$\alpha$ with $S/N$ at least 2. A total of 79,548 (20.6%) of the objects in this sample fall into the low-S/N star-forming galaxy category. Combining the two classes, 211,355 (54.7%) of the galaxies survive the AGN cut.
The AGN cut also has a strong luminosity bias for two reasons. Galaxies with AGN components tend to be brighter. The bimodal distribution of galaxies in color-magnitude space (Baldry et al. 2004) also plays a role. The most luminous galaxies are predominantly red with minimal star formation and thus weak emission lines. Luminous galaxies are rejected both for having AGN components and for having low-S/N emission lines. Over 95% of galaxies fainter than $M_{r,0.1} = -19$ meet these criteria, but by $M_{r,0.1} = -24$ the fraction is only 38.9%.

A color bias is also introduced by the AGN cut. Over 95% of galaxies bluer than $(g - r)_{0.1} = 0.6$ pass, which drops to 24% by $(g - r)_{0.1} = 1.2$. This is mainly due to the S/N requirement for the emission lines. Redder galaxies tend to have weak emission lines and are rejected.

The Balmer decrement is used for the extinction correction so $H_\alpha$ and $H_\beta$ S/Ns are required to be at least 5 to reduce errors. A total of 214,912 galaxies (55.6%) have $H_\beta$ S/N > 5, which is the more restrictive of the two criteria. This cut has a clear luminosity bias. Roughly 85% of galaxies with $M_{r,0.1} > -20$ satisfy this requirement, but this fraction decreases with increasing luminosity until only 12.9% survive at $M_{r,0.1} = -24$. This is again due to the bimodal distribution of galaxies. The luminous red galaxies with weak emission lines are rejected.

There is also a color bias. Over 93% of the bluest galaxies blueward of $(g - r)_{0.1} = 0.8$ survive the cut, while only 23% of the reddest pass this requirement. As previously mentioned, by nature the reddest galaxies have weak Balmer lines as a result of their low SFRs and are preferentially rejected.

A redshift cut of 0.005 ≤ $z$ ≤ 0.25 is applied to ensure that peculiar velocities do not dominate at low redshift and to limit the range of galaxy ages. A total of 384,349 galaxies (99.4%) meet this criterion. Nearly all galaxies from $M_{r,0.1} = -17$ to $-23$ survive this cut. On the low-luminosity end only 27.4% of $M_{r,0.1} = -14$ galaxies are distant enough to pass, and on the high-luminosity end 93.1% of $M_{r,0.1} = -24$ galaxies are close enough to survive. Only 63% of the bluest galaxies pass. Many of the blue galaxies that fail are actually $H_{\alpha}$ regions of Local Group galaxies that are treated as their own objects by the SDSS pipeline, so removing them actually improves the integrity of our sample.

The stellar populations of galactic bulges can be significantly different from those in the spiral arms. The SDSS fibers have a fixed aperture of 3″ so over the large range of luminosities and distances in the MGS aperture affects can become very important. To remove outliers, we require that at least 10% of the light from a galaxy falls within the spectroscopic aperture. This is done by comparing the Petrosian magnitude to a fixed 3″ aperture fiber magnitude. Both of these quantities are calculated for all objects in the SDSS by the photometric pipeline. A total of 371,777 (96.2%) galaxies survive this cut. This cut rejects proportionally more faint galaxies; 99.2% pass at $M_{r,0.1} = -24$ compared to 50.9% at $M_{r,0.1} = -14$. The aperture cut has low sensitivity to color.

The intersection of the AGN, Balmer line S/N, redshift, and aperture fraction cuts leaves 140,598 galaxies: 36.4% of the high-quality MGS data defined by our first round of cuts and 33.2% of the MGS as a whole.

At this point three final cuts are applied to the sample. One galaxy is removed for surviving all criteria but having a negative H$\alpha$ EW.

The extinction of individual galaxies is estimated using the Balmer decrement ($H_\alpha/H_\beta$ emission flux ratio) for each galaxy. Given case B recombination, a gas temperature of 10,000 K, and density of 100 cm$^{-3}$, the Balmer decrement is predicted to be 2.86 (Osterbrock 1989). This ratio is weakly dependent on nebular temperature and density. Osterbrock (1989) lists values down to 2.74 for case B recombination in environments where both the temperature and electron density are high. A total of 3.2% of the galaxies suffer from the problem that the Balmer decrement is less than 2.86, and 2.1% have a Balmer decrement below 2.74. A total of 538 galaxies (0.4%) have Balmer decrements more than 3 $\sigma$ below 2.74, which is around 6 times more than expected. This does not suggest a problem with the case B assumption as the predicted Balmer decrements for case A recombination are nearly identical in each temperature regime.

To understand the reason behind this problem, around 100 of the offending spectra were inspected, revealing a few different causes for the problem. Around 80 galaxies are at redshifts where the telluric O $i$ $\lambda 5577$ line affects the measurement of H$\beta$. Many of these galaxies have very strong emission lines with extremely weak stellar absorption. Using the SDSS pipeline values instead of the Tremonti et al. (2004) values yields acceptable Balmer decrements. The rest are galaxies with low flux where the Balmer lines are in absorption. This shows that there are a few cases where attempting to fit the underlying stellar absorption lines fails and this failure is not reflected in the error values. These galaxies are rejected without any apparent introduction of bias.

All of these cuts combined leave 140,060 galaxies, which is 33% of the MGS and 36% of the high-quality MGS data. After removing duplicate observations, there are 130,602 galaxies in our sample. Of these objects, 1.7% overlap with the Luminous Red Galaxy Sample.

Overall the bulk of the galaxies are removed by AGN rejection and the H$\beta$ S/N requirement, with the rest of the cuts having little effect. Both of these cuts are necessary. AGNs must be removed to ensure that the H$\alpha$ emission represents the underlying stellar population and not an accretion disk. Our method requires that the galaxies have measurable Balmer emission lines. This, coupled with the need for accurate extinction corrections, justifies the Balmer line S/N requirement.

Our cuts bias our sample by preferentially excluding galaxies at both luminosity and both color extremes. The faintest galaxies are most affected by the redshift and aperture cuts, while the luminous galaxies succumb to the H$\beta$ S/N requirement. At the red extremes it is the H$\beta$ S/N and AGN requirements that play equally large roles, while the bluest objects are primarily rejected by the Hubble flow redshift requirement. Although our sample is biased by our cuts, each one is a necessary evil. We do not attempt to correct this bias, but we remind the reader that the following results are only representative of actively star-forming galaxies without any AGN activity.

The aim of this paper, however, is to test IMF universality. If the IMF is truly universal, it should be universal in any subsample of galaxies. The fact that our sample is slightly biased with respect to luminosity and color is not a significant barrier to achieving our goal.

### 3.2. Corrections

The SDSS includes a number of different calculated magnitudes. We use the fiber magnitudes, which are 3″ fixed aperture magnitudes. Although it was not the case in earlier versions of the photometric pipeline, fiber magnitudes are now seeing corrected (Abazajian et al. 2004). The fiber magnitudes were not originally intended for science purposes but rather to get an idea of how bright an object will appear in the spectrograph. We use the fiber magnitudes to reduce the aperture affects arising from comparing a 3″ spectroscopic aperture to Petrosian magnitudes. Originally the SDSS used “smear” exposures to correct spectra for light falling outside the 3″ aperture due to seeing, guiding errors, and
atmospheric refraction (Stoughton et al. 2002). The smear technique was later found to be an improvement only for high-S/N point sources, and its use was discontinued (Abazajian et al. 2004). The spectra here are not seeing corrected.

After paring the sample to its final size, a number of corrections must be made to both the photometric and spectroscopic data. Galactic reddening from the Milky Way must be corrected for. The SDSS database includes the Schlegel et al. (1998) dust map values for each photometry object.

The extinction of individual galaxies is estimated using the Balmer decrement for each galaxy. The data are corrected assuming that 2.86 is the true value of the Balmer decrement using the Milky Way dust models of Pei (1992). The assumption of Milky Way dust is not significant as models of the dust attenuation in the Milky Way, SMC, and LMC are nearly identical in the $g$ band and redward. As aforementioned, a few percent of our galaxies have Balmer decrements below 2.86. Our solution is to set the emission-line extinction to $A_{V,f} = 0.01$ mag for these galaxies.

Massive young stars and their surrounding ionized nebulae tend to be embedded in their star-forming regions more so than older, lower mass stars, which have had time to migrate from their birth regions. As such nebular emission lines will experience more extinction than the continuum. Calzetti et al. (1994) find that the ratio of emission to continuum line extinction is $f = 2.0 \pm 0.4$. We assume this value to be 2.0 and correct the continuum and emission lines separately. This is the same value used by K83 and KTC94 in their extinction-corrected models. We note that the spatial geometry of the dust can influence the extinction law, but this complication is beyond the scope of this paper.

Galaxy photometry is $K$-corrected to $z = 0.10$ using version 4.1.4 of the code of Blanton et al. (2003a). This redshift is roughly the median of the sample and is selected to minimize errors introduced by the $K$-corrections. The $(g-r)_{0.1}$ colors we use are the $g-r$ colors we would observe if the galaxies were all located at $z = 0.1$.

Stated explicitly, the $(g-r)_{0.1}$ color is
\[
c = (g-r)_{0.1} = (g - k_g - A_g - 1.153A_V/f) - (r - k_r - A_r - 0.834A_V/f),
\]
where $k_g$ and $k_r$ are $K$-corrections, $A_g$ and $A_r$ are Milky Way reddening values, and 1.153 and 0.834 relate the $V$-band extinction to the $g$ and $r$ bands assuming a Milky Way dust model. The corrected equivalent width is obtained as follows:
\[
w = w_0 \left[1 + z \right] \times 10^{-0.4(0.7754\lambda_{em})[(1-1/f)]^{-1}},
\]
where $w_0$ is the measured, uncorrected equivalent width. The $1 + z$ arises from the fact that the total flux in the $H\alpha$ line is not affected by cosmological expansion of the universe, but the flux per unit wavelength of the continuum is depressed by a factor of $1 + z$. The following term is the extinction correction. The 0.775 relates the $V$-band extinction to the $H\alpha$ line assuming a Milky Way dust model, and the $1 - 1/f$ is due to the fact that the emission line and continuum experience different amounts of extinction as previously explained.

Figure 1 shows the distribution of the galaxies in the color versus $H\alpha$ EW plane.

3.3. Errors

In order to conduct a likelihood analysis, we need error estimates that take into account both the errors induced by the photometry and spectroscopy and those by the aforementioned corrections. The error in the corrected color, $\sigma_c$, is given by
\[
\sigma_c = 0.03 + \left\{ \sigma_g^2 + \frac{1}{f} \left[ 0.319A/V/f \right]^2 \left( \frac{\sigma_{A/V}}{A/V} \right)^2 + \frac{1}{f} \left( \frac{1}{2} \right) \right\}^{1/2} + (0.0440A_g)^2 + \sigma_k^2,
\]
where $\sigma_g$ and $\sigma_f$ are the Poisson errors in the observed $g$- and $r$-band photometry, $f$ is the ratio of the emission line to continuum extinction, and $\sigma_k$ is the error introduced by the $K$-corrections. The terms inside the square root in equation (5) are obtained through a straight error propagation of equation (3). The 0.03 outside the square root is due to the systematic zero-point errors of the SDSS filter system. Following Calzetti et al. (1994), $f$ is fixed at 2.0 and $\sigma_f$ is set to 0.4. The error in emission-line $A_V$ is given by
\[
\sigma_{A_V}^2 = 9.440 \left( \frac{\sigma_{H\alpha}}{H\alpha} \right)^2 + \left( \frac{\sigma_{H\beta}}{H\beta} \right)^2.
\]
which is dependent on the fractional uncertainty of the Hα and Hβ emission line fluxes. Schlegel et al. (1998) report a 16% error in their Milky Way reddening values. Because a dust model is assumed, redenonings in the $g$ and $r$ bands are linearly related, so the error in $A_g - A_r$ is a function of $A_r$. This relationship, combined with the 16% error, yields the 0.0440 in equation (5). The median value of $g$-band reddening is 0.10 so the errors introduced by the Milky Way reddening correction are insignificant for the majority of objects. Errors in the redshift determination are negligible, typically 0.01%. The value of $\sigma_k$ is estimated to be 0.02 by visual inspection of a plot of $k_g - k_r$ as a function of redshift.

For a typical galaxy the term involving $\sigma_f$ is the largest contributor to the extinction-corrected color error. The median Poisson error from the photometry is 0.01 in both bands. The median value of $\sigma_e$ is 0.085.

The error in the corrected Hα equivalent width, $\sigma_w$, is given by

$$\sigma_w^2 = w^2 \left( \frac{\sigma_u}{w_0} \right)^2 + 0.5095 \left[ \left( \frac{\sigma_{A_w}}{1-f} \right)^2 + \left( \frac{\sigma_f A_w}{f} \right)^2 \right].$$

Equation (7) is the result of propagating the errors in equation (4), neglecting the insignificant redshift errors. Again, the term involving $\sigma_f$ is the largest contributor to the error for typical galaxies. The median error in the equivalent width is 17%. The median error bars for the sample are shown in Figure 1. For comparison, K83 reports equivalent width errors of around 10%, but the extinction is uncertain at the 20%–30% level.

4. MODELS

Model galaxy spectra were calculated using the publicly available PEGASE.2 spectral synthesis code (Fioc & Rocca-Volmerange 1997). Models are calculated for ages from 1 Myr to 13 Gyr. Twenty-five smoothly varying SFHs generated by analytic formulae are considered. The SFHs range from 19 exponentially decaying SFHs with time constants from 1.1 to 35 Gyr, a constant SFR, and four increasing SFHs that are proportional to $1 - \exp^{-\rho t}$, where $\rho$ is the time constant. The precise values of the time constants were selected to smoothly sample the Hα emission line fluxes. Schlegel et al. (1998) report a 16% error from the photometry is 0.01 in both bands. The median value of $\sigma_e$ is 0.02 by visual inspection of a plot of $k_g - k_r$ as a function of redshift.

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precise values for the IMF slope. In this limited space of observables the IMF models themselves are degenerate. What it does provide is a framework with which to detect variations in the IMF. Although we can construct similar tracks from different IMF models, we can still detect the differences between two groups of galaxies.

While we report our results as a function of $\Gamma$, it must always be kept in mind that it is degenerate with the upper mass cutoff and other fine structure in the IMF at high stellar masses.

The assumption of smoothly varying SFHs is of great consequence. In the event that a galaxy is experiencing or has recently experienced a burst, our SFH assumption can lead to measured $\Gamma$-values that are off by as much as 0.5. The effects of bursts are more closely examined in a later section.

Within the assumption of smoothly varying SFHs much can be said about the effects of the IMF, metallicity, age, and SFH in the color–H$\alpha$ EW plane. Figure 3 demonstrates these relationships. In both panels the ages of the models decrease from the upper left to lower right. The effects of the age of a stellar population are largely orthogonal to those of IMF variations. In Figure 3 the effects of changing the functional form of the smoothly varying SFH with fixed metallicity and $\Gamma$ are shown. SFH variation is degenerate with the IMF. However, the effect is relatively small over a wide range of SFHs. The solid lines are exponentially decreasing SFHs with $\tau = 1.1$ Gyr where the bulk of the star formation occurs early in the galaxy’s life. The dashed lines have SFHs that are increasing with time where most star formation occurs at late ages. The effect of variations in the form of smooth SFHs is larger at later ages and higher values of $\Gamma$ but does not dominate the effects of IMF variations. With all other parameters fixed the range of smooth SFHs causes systematic uncertainties at the level of ±0.1 in $\Gamma$.

In Figure 3 the effects of metallicity variations with fixed SFH and $\Gamma$ are shown. The metallicity variations are also degenerate with IMF variations. With all other parameters fixed between colors of 0.1 < $(g-r)_{0,1}$ < 0.4 the systematic uncertainty due to metallicity is less than 0.05 in $\Gamma$. This uncertainty increases to 0.35 at $(g-r)_{0,1} = -0.2$ and 0.7.

As aforementioned, the extinction correction is another potential problem. The arrows in Figure 3 show the length and direction of the extinction correction for typical galaxies in our sample. It is assumed that $f = 2$, but $f = 1$ and 4 are also plotted to show the potential effect of variations in $f$. The reddening vectors for $f = 2$ and 4 are fortuitously orthogonal to the IMF variations. Only when the continuum and emission extinctions are equal, when $f = 1$, do the extinction correction and variations in $f$ become a larger concern than metallicity and SFH. However, such low $f$ ratios are not observed in galaxies (Calzetti et al. 1994 and § 6.2.7).

Figure 4 shows all 18,480 model points with $\Gamma = 1.35$. Models are interpolated in SFH for fuller coverage of the color–H$\alpha$ EW plane. For each value of $\Gamma$ the models cover a stripe rather than a single line. It can be seen in the lower right of Figure 4 that the models become degenerate in $\Gamma$ for old, red galaxies with weak current star formation.

5. STATISTICAL TECHNIQUES

The data and models are compared using a “pseudo-$\chi^2$” minimization. For various reasons (detailed below) the classical $\chi^2$ estimator assumptions are violated so we cannot use traditional tables for error estimates, but we can still use the $\chi^2$ as a statistical estimator as long as the confidence regions are calibrated by MC techniques as we do. We proceed as follows: for each galaxy $i$ we have a measured $(g-r)_{0,1}$ color, $c_i$, and an H$\alpha$ EW, $w_i$, and measurement errors $\sigma_c$ and $\sigma_w$, given by equations (5) and (7). We also have model values $c(\Gamma, Z, \tau, \psi)$ and $w(\Gamma, Z, \tau, \psi)$ for a range of
IMF slopes $\Gamma$, metallicities $Z$, ages $t$, and SFHs $\psi$. We can then construct a $\chi^2$ value as

$$
\chi^2(\Gamma, Z, t, \psi) = \left[ \frac{c_i - c(\Gamma, Z, t, \psi)}{\sigma_{c_i}} \right]^2 + \left[ \frac{w_i - w(\Gamma, Z, t, \psi)}{\sigma_{w_i}} \right]^2,
$$

which is calculated by brute force. The goal of this paper, however, is to investigate the IMF with relatively simple measurements of the H$\alpha$ EW and a broadband color. While making crude measurements of the mean stellar metallicities of individual galaxies is possible, disentangling age and SFH effects on an individual basis is a daunting task. Assuming that it is possible to do, it does not scale up well to the high-redshift universe where observations will be of lower quality.

It does not make sense to minimize $\chi^2$ over all galaxies for a particular set of $(\Gamma, Z, t, \psi)$ because we have no a priori reason to think that all of the galaxies should have the same metallicity or SFH. In fact, we expect that they would not. The solution is to marginalize $\chi^2$ over metallicity, age, and SFH for each galaxy such that

$$
\chi^2(\Gamma) = \min \{ \chi^2_i(\Gamma, Z, t, \psi) \}_{Z,t,\psi}.
$$

This is somewhat unorthodox because for some galaxies the data points are overfitted, i.e., there will be a stripe in $(c, w)$ space corresponding to a given $\Gamma$ and we can get $\chi^2(\Gamma)$ values very close to zero (but not exactly because of the discrete nature of the model grid) for galaxies within the stripe. We note that we also have partial degeneracy between parameters such as age and metallicity: they both shift the tracks in similar directions largely orthogonal to $\Gamma$ (although not completely, which is why we have a stripe in parameter space and not a line). This makes it difficult to calculate the traditional “number of degrees of freedom.” Despite these limitations, it is clear that galaxies inconsistent with a particular $\Gamma$ will still have large values of $\chi^2(\Gamma)$: for example, a very blue galaxy with a low equivalent width in Figure 4. The complication is that the stripes for similar $\Gamma$-values overlap, and for red galaxies with low equivalent widths the stripes for vastly different $\Gamma$-values overlap. As such, the IMF for an individual galaxy is only broadly constrained. Measuring a precise best IMF for an individual galaxy boils down to random chance and the discrete nature of the models. However, by summing $\chi^2_i$ over many galaxies, the IMF is narrowly constrained for the sample being summed over as long as we are careful in our confidence region analysis.

Because of this overfitting and partial degeneracy, we cannot apply the textbook notions of the $\chi^2$ distribution, calculate degrees of freedom, and choose $\Delta \chi^2$ contours for different confidence regions. Furthermore, $c_i$ and $w_i$ are not truly independent variables. Both the colors and EWs are subject to the same extinction and reddening corrections, which tie the errors together. For galaxies with $z \leq 0.04$ the H$\alpha$ line is in the observed $r$ band, although this only affects a relatively small number of galaxies in the sample, almost all of which have $M_{r,0.1} > -20$. Also the direct statistical interpretation of $\chi^2$ is predicated on the assumption of normal errors. Equations (5), (6), and (7) reveal that our errors are complicated mixtures of individual measurements, which are most likely Poisson distributed. Thus, $\sigma_c$ and $\sigma_w$ are unlikely to be normally distributed. Bursty SFHs can potentially create outliers that are statistically significant due to the fact that neither $\sigma_c$ nor $\sigma_w$ includes a term for this difficult-to-quantify effect. The problem is even worse if the errors are nonsymmetric, which could potentially arise from the aforementioned bursty SFHs. In the case of nonsymmetric errors the best value of $\Gamma$ could erroneously be pulled away from the true value.

Given all this, we abandon the direct statistical interpretation of $\chi^2$ and regard it as an estimator of the goodness of fit whose confidence regions have to be calibrated empirically. We do this via MC simulations (as recommended by Press et al. 1992), where we simulate data points for a given $\Gamma$ with the correct error distributions and propagate everything through the analysis in the same way as for the actual data. For each of our MC simulations we add Poisson errors to the $ugriz$ fiber magnitudes, H$\alpha$ and H/$\beta$ fluxes, and the observed H$\alpha$ EW. We assume that these observed quantities have Poisson-dominated errors: as members of the MGS they are high-S/N measurements. The entire analysis described above is repeated, including a new extinction measurement and a recalculations of the $K$-corrections. For each value of $\Gamma$ we run 100 MC simulations to estimate the 95% confidence interval. Setting up the MC architecture in this way has the further advantage that we can use the same machinery to test the effect of systematic errors such as the violation of our smooth SFH assumptions, as we do later. The main downside of course is that this approach is computationally intense. Run times for the 100 MC simulations are typically 18 days on a 2 GHz desktop PC for the samples considered here.

In practice, it turns out that $\chi^2(\Gamma)$ is still a smooth well-behaved function with, not surprisingly, a quadratic minimum, which has the advantage that we can then interpolate it to increase the

![Figure 4](image-url)
resolution in the best-fitting \( \Gamma \) without incurring the additional computational expense. This arises of course from the fact that our estimator is similar to a traditional \( \chi^2 \) and is a good reason to stick with this similarity over some more exotic goodness-of-fit measure. An estimate of the systematic errors is discussed later.

Regardless of how poorly a sample is modeled by a universal IMF, the above method will still find a best-fitting \( \Gamma \) and corresponding confidence region. We still expect \( \chi^2 \) to be small for a model that is a good fit and large for one that is not. One nuance in comparing \( \chi^2 \) between different subsamples, as we do, is that the samples are often of considerably different sizes. Because of this, we choose instead to use the mean \( \chi^2 \), \( \overline{\chi^2} \), as a sample metric. This has the advantage that absolute \( \overline{\chi^2} \) values and confidence regions are more similar between the subsamples, although we note that the confidence regions on \( \overline{\chi^2} \) are still determined directly from our MC simulations.

6. MONTE CARLO RESULTS

Figure 5 shows the results of our analysis for the full sample of galaxies using just the observed data set. The cross marks the best-fitting IMF, where \( \Gamma = 1.4411 \) and \( \chi^2 = 60549.7 \) with \( \chi^2 = 0.46 \). At \( \Gamma = 1.00 \), \( \overline{\chi^2} = 3.61 \), while steeper IMFs are more heavily rejected with \( \overline{\chi^2} = 9.00 \) at \( \Gamma = 2.00 \).

For comparison several “classic” IMFs are also plotted in Figure 5 at their approximate equivalent values of \( \Gamma \). With \( \chi^2 = 7.73 \) the Miller \& Scalo (1979) solar neighborhood IMF is a particularly bad fit. Two more recent solar neighborhood IMFs, Scalo (1986) and Kroupa et al. (1993), yield \( \overline{\chi^2} = 2.38 \) and 1.98, respectively. These results reinforce the conclusions of K83, KTC94, and Baldry \& Glazebrook (2003) that the solar neighborhood is not representative of galaxies on the whole as far as the IMF is concerned.

On the other hand, the Scalo (1998) IMF, established from a review of star cluster IMF studies in the literature, is a better fit than the best \( \Gamma \)-value in our parameterization with \( \overline{\chi^2} = 0.43 \). This result highlights the degeneracy of the IMF models themselves in the color–H\( \alpha \) EW plane: two considerably different IMFs (one with one break and the other with two) fit nearly equally as well.

The results of the MC simulation show that the 95% confidence region is \( 1.4432 < \Gamma < 1.4443 \) for the data set as a whole. The MC simulation shows that additional data will not improve the overall results as the random errors are already small. Clearly and not surprisingly, systematic errors, which are discussed later, dominate.

The overall result of \( \Gamma = 1.4437 \pm 0.0005 \) is steeper than the original Salpeter value of \( \Gamma = 1.35 \). It is also steeper than the Baldry \& Glazebrook (2003) value of \( \Gamma = 1.15 \pm 0.2 \) derived from galaxy luminosity densities in the UV to near-infrared. It is, however, well within their 95% confidence limit of \( \Gamma < 1.7 \), as well as their measurement of \( \Gamma = 1.2 \pm 0.3 \) based on the H\( \alpha \) luminosity density. The difference between their two results suggests that the H\( \alpha \) and mid-UV to optical fluxes may have different sensitivities to massive stars. Scalo (1998) estimated the uncertainty in \( \Gamma \), due to either measurement uncertainties, real IMF variations, or both, in his star cluster IMF based on the spread of results in the literature. Our result is well within his range of uncertainty in both mass regimes: \( \Gamma = 1.7 \pm 0.5 \) for \( 1–10 M_\odot \) and \( \Gamma = 1.3 \pm 0.5 \) for \( 10–100 M_\odot \).

6.1. Luminosity Effects

The luminosity of a galaxy could potentially have an effect on the IMF within it. For one the ambient radiation field is likely higher in more luminous galaxies. Figure 6 shows the best-fitting
IMF and \( \chi^2 \) values as a function of \( M_{r,0.1} \) for all 130,602 galaxies. The galaxies have been binned in \( M_{r,0.1} \) such that there are 500 objects in each bin. The bin size was chosen to maximize coverage in \( M_{r,0.1} \) yet still keep the random errors in each bin small. The solid lines represent the lower and upper 95% confidence region determined from the MC simulation.

Figure 6 reveals a constant value of \( \Gamma \sim 1.4 \) for galaxies with \( M_{r,0.1} \) between \(-21 \) and \(-22 \) with linear increases in \( \Gamma \) for both brighter and fainter galaxies. There is also a sudden downturn in \( \Gamma \)-values for galaxies fainter than \( M_{r,0.1} = -16.5 \). Given the sizes of the random errors, the differences in \( \Gamma \) between \( M_{r,0.1} = -17 \) and \(-22 \) are substantial, from 1.59 to 1.41, and statistically significant. The agreement with the Salpeter slope is the best for galaxies between \(-21 \) and \(-22 \) in \( M_{r,0.1} \).

In many ways it is not surprising that previous investigators have not found this trend. The Milky Way is thought to have a luminosity of \( M_V = -20.9 \) (Delhaye 1965); the \( \Gamma \) and \( \gamma_0 \) filter curves cover roughly the same wavelengths. At comparable luminosities our results are similar to Salpeter. The galaxies in the K83 sample have a median \( M_B = -20.9 \) with only 16% (18 objects) fainter than \( M_B = -19.7 \) (Efstathiou et al. 1988). This is a significant bias toward more luminous galaxies where our results are in agreement with a universal IMF. By contrast, 30% of our sample is fainter than \( M_{r,0.1} = -20.44 \) (Blanton et al. 2003b). We have a sample of 39,350 galaxies fainter than \( L^* \).

The bottom panel of Figure 6 shows that the relative quality of the fits rapidly deteriorates as the luminosity of the galaxies decreases. For the brightest galaxies \( \chi^2 \) floats around 0.15, while in the faintest bin it is over 6. For comparison \( M_V = -18.5 \) for the Large Magellanic Cloud and \(-17.1 \) for the Small Magellanic Cloud (Courteau & van den Bergh 1999). This trend could indicate that a universal IMF is a good fit to the most luminous galaxies, but dwarf galaxies cannot be described by a universal IMF, even if a different universal slope is allowed. However, it could have a more mundane explanation. It could be that errors are over- or underestimated as a function of luminosity. It also could arise from deviations from our assumption of smoothly varying SFHs.

We cannot bin our data by stellar mass without assuming an IMF that is contrary to the goals of the project. We can repeat the analysis of Figure 6 using \( M_{r,0.1} \) in the place of \( M_{r,0.1} \). The quantity \( M_{r,0.1} \), being redder, is a better proxy to stellar mass. The resulting plot is nearly identical to Figure 6, which shows that the relationship persists across several wave bands.

Figure 6 reveals a clear, statistically significant trend in \( \Gamma \) and \( \chi^2 \) with respect to luminosity. The rest of this section focuses on whether this trend is a manifestation of true IMF variations or if it is the result of sample biases or poor assumptions.

### 6.2. Sources of Bias

If the IMF is truly universal and our method successfully probes the IMF, any subsample of galaxies that we could choose should yield the same \( \Gamma \)-value as any other in spite of any selection biases or aperture effects. Figure 6 clearly shows that the preceding statement is false. In this section we set aside the possibility of IMF variations and search for biases in our sample.

#### 6.2.1. Magnitude-limited Sample

Figure 6 shows that the overall result of \( \Gamma = 1.4437 \) is really a weighted average. The SDSS MGS is a magnitude-limited sample, one defined by flux limits, with both upper and lower limits. Table 1 gives the number of objects in each luminosity bin. There are 41,411 galaxies with \(-21.5 < M_{r,0.1} < -20.5 \), but only 28 for which \(-14.5 < M_{r,0.1} < -13.5 \). As such the overall result is heavily biased by more luminous galaxies.

Malmquist bias will affect any magnitude-limited sample. Because brighter objects can be seen at greater distances, a magnitude-limited sample contains bright objects from a greater volume of space than fainter objects. The result is that the ratio of objects by luminosity in a magnitude-limited sample differs from the true ratio in nature; brighter objects are overrepresented.

To eliminate Malmquist bias, volume-limited bins where subsamples are complete for a range of luminosities are constructed. Figure 7 details the construction of these bins. Given both the upper and lower flux limits of the MGS (15.0 < \( r < 17.77 \)), only a factor of 13 in luminosity falls in the sample at any given redshift. The redshift limits of each volume-limited bin are defined such that no galaxies within the magnitude limits of the bin are affected by the flux limits of the MGS. Within each box in Figure 7 the true ratio of galaxy luminosities is preserved and is thus free of Malmquist bias.

Figure 8 shows the results for volume-limited magnitude bins. Most error bars are smaller than the plotting symbols due to the
larger number of galaxies, 329 to 29,701 as given in Table 1, in each bin. Figures 6 and 8 show the exact same trends. The largest difference in $\Gamma$ between the whole and volume-limited samples is 0.0116 in the $M_{r,0.1}$ = $-17$ bin. The other notable difference is that the fainter galaxies have larger $\chi^2$ values in the volume-limited case. However, Malmquist bias across bins is not responsible for the luminosity trends in Figure 6.

6.2.2. Redshift

Another effect of magnitude-limited samples is that the faintest galaxies are much closer than the most luminous ones. The mean redshifts of the volume-limited magnitude bins range from $z = 0.013$ for $M_{r,0.1}$ = $-17$ to $z = 0.168$ for $M_{r,0.1}$ = $-23$. This corresponds to a difference in age of around 1.8 Gyr. As aforementioned, model tracks reveal that age is largely orthogonal to the IMF in our parameter space, but there could be other effects tied to age and distance. In addition, the IMF could evolve with time.

The large number of galaxies in our sample affords us the luxury of investigating the effects of luminosity and redshift simultaneously to obtain a better understanding of what role, if any, the redshift plays in our analysis. Figure 9 shows our fitted parameters for all 130,602 galaxies in bins that are 0.25 mag wide in luminosity and 0.005 wide in redshift. The top left panel shows the best-fitting $\Gamma$ for each two-dimensional (2D) bin. In the top right panel the width of the 95% confidence region in $\Gamma$ for each bin is shown. In the bottom left panel is the log $\chi^2$, and in the bottom right panel is the log of the number of galaxies in each bin. The white contour demarcates the region in which each bin contains at least 50 galaxies. The number 50 is arbitrary, but it shows the region where Poisson errors are expected to be small. The black areas are regions where there are no galaxies with the given parameters.

Using the plot of $\Gamma$ in the top left panel, we can look for potential redshift biases. This is complicated by the fact that at a fixed luminosity there is a limit to the range of redshifts in the sample due to the flux limits of the sample described earlier. Looking at vertical slices through the plot at any fixed luminosity, there is a trend toward larger values of $\Gamma$ with increasing redshift. However, for horizontal slices of fixed redshift the same relationship between $\Gamma$ and luminosity that is present for the whole sample is seen modulo a normalization factor.

The right panels of Figure 9 show a strong relationship between the number of galaxies per bin and the width of the 95% confidence region in $\Gamma$. This simply reflects the fact that larger samples are less affected by Poisson errors.

The bottom left panel of Figure 9 provides an excellent example of why our metric of fit quality, $\chi^2$, is so important. Bins with similar numbers of galaxies and $\delta \Gamma$ values can have vastly different values of $\chi^2$. It is worth reminding that the contours in $\chi^2$ are logarithmic. At fixed luminosity the galaxies are better fitted by a universal IMF at higher redshift. Similar to the sample as a whole, the quality of fit improves with luminosity.

While there does appear to be some weak trending of $\Gamma$ and $\chi^2$ with redshift, redshift effects are not driving the relationship seen between IMF and luminosity, as it persists at fixed redshifts.

6.2.3. Aperture Effects

One explanation for the trend in $\Gamma$ with redshift is aperture effects. Again, if the IMF is truly universal, aperture effects should not exist. The SDSS spectra have a fixed aperture of 3$''$ for all galaxies. Depending on the angular extent and distance to a galaxy, a different fraction of the total light of the galaxy will fall into the aperture. The problem is mitigated by the fact that the most distant galaxies are the most luminous and more likely to have a larger physical size. As the physical area contained in the aperture increases with distance, so too does the size of the galaxies being observed. However, the two effects do not exactly balance out. Table 1 shows that the mean aperture fraction for the $M_{r,0.1}$ = $-17$ bin is 0.20 and increases to 0.27 at $M_{r,0.1}$ = $-23$. On average 35% more of the most luminous galaxies fall within the aperture compared to the faintest.
Figure 10 shows the behavior of our fitted parameters for 2D bins of luminosity and aperture fraction in the same manner as Figure 9 did for luminosity and redshift. For fixed luminosities increasing aperture fraction leads to decreasing values of $C_0$. However, at fixed aperture fraction the qualitative IMF-luminosity trend remains. The $C_3$ values are a strong function of luminosity, but $C_0$ does increase with aperture fraction at each fixed luminosity.

The trend with aperture fraction is the exact opposite of what would be expected in the presence of a systematic effect operating given the redshift result in Figure 9. The nearest galaxies should have the smallest aperture fractions in a particular luminosity bin. The nearest galaxies in Figure 9 have the smallest values of $C_0$, while the smallest aperture fractions in Figure 10 have the largest values of $C_0$.

Figure 10 suggests that the measured IMF is more dependent on the aperture fraction than the redshift. There are several possible physical explanations for IMF trends with the aperture fraction, all of which are related to radial gradients in disk galaxies. Padoan et al. (1997) make the theory-based claim that the IMF should be a function of the original local temperature of the star-forming molecular clouds. Metallicity gradients are also known to exist in disk galaxies, including the Milky Way (Mayor 1976). Rolleston et al. (2000) measure a linear, radial light-metal (C, O, Mg, and Si) abundance gradient of $-0.07 \pm 0.01$ dex kpc$^{-1}$ in the disk of the Milky Way. Given the increased efficiency of cooling with metal lines, we would expect the most low-mass stars where metallicity is the highest, on average toward the center of galaxies. The trend in $C_0$ in Figure 10 is qualitatively consistent with this idea.

If there are radial IMF gradients in galaxies, one would expect the fits to decrease in quality with increasing aperture fraction. A blend of IMFs will not be fitted as well as a universal one given our technique. This idea is consistent with the results in Figure 10. However, in well-resolved stellar populations there is no evidence for a relationship between the IMF and metallicity, except perhaps at masses lower than those probed by our method (Kroupa 2002). If metallicity plays a role in determining the IMF, the effects are only being revealed as a global trend in our large sample of integrated stellar populations. For individual clusters metallicity must play a secondary role to stochastic effects.

6.2.4. Extinction Correction

The extinction correction is another potential source of bias. It is possible that there is a second-order correction that our fairly simple extinction correction fails to take into account. This could potentially lead to an erroneous IMF trend with extinction correction. This affects the luminosity results because more luminous galaxies tend to be dustier, as evidenced in Table 1. The problem is further complicated by the fact that dust is thought to play an
integral part in star formation so it is not unreasonable that an observed IMF trend with extinction may be real.

Figure 11, similar to Figures 9 and 10, shows the results of our analysis for 2D bins of luminosity and the extinction correction that was measured and applied. Vertical slices through the top left panel of Figure 11 show that $\Gamma$ does depend on $A_V$, trending toward lower $\Gamma$-values with increased extinction over the region where the Poisson error in $\Gamma$ is reasonable. Yet again, horizontal cuts of fixed extinction show the IMF-luminosity relationship. The decreasing $\Gamma$-values with increasing extinction are counterintuitive. Dustier regions tend to be more metal-rich. If metal cooling plays a significant role in the IMF, the dustiest regions should have the steepest IMFs.

At fixed luminosity $\chi^2$ increases with extinction. As aforementioned, our calculated errors in color and EW (eqs. [5] and [7]) have a functional dependence on the observed emission-line extinction. In both cases it is the term proportional to $\sigma_f A_V f$ that is on average the major contributor to the calculated error. Because luminous galaxies tend to be more heavily extincted, they will also be more likely to have larger errors. This is potentially problematic for our observed IMF trend with luminosity. If we are unknowingly underestimating the errors for faint galaxies with low extinction, the source of the poor fit qualities of these galaxies could be systematic instead of astrophysical. However, the bottom left panel of Figure 11 shows that the most extincted galaxies have the poorest fits where such a bias would suggest that they should fit the best due to the large accommodating errors.

6.2.5. Multiple Parameter Biases

It is also possible that biases in our $\Gamma$ measurements could depend on two parameters simultaneously. Figure 12 shows the measured value of $\Gamma$ as a function of both aperture fraction and measured emission-line extinction for six volume-limited luminosity bins.

When holding all other parameters fixed, increasing the aperture fraction leads to lower values of $\Gamma$ in all statistically significant areas of Figure 12. This is the same relationship found in the earlier section on aperture fraction. Decreasing values of $\Gamma$ are also seen for increasing extinction when all other parameters are constant. A notable exception to this is that galaxies with large extinction and small aperture fractions favor higher $\Gamma$-values.

Most importantly, when looking at a particular combination of aperture fraction and extinction, the IMF becomes shallower with increasing luminosity until the highest luminosities where it becomes steeper again. Even in the narrowest slices of the data set the same IMF-luminosity trend is seen, albeit with slightly different absolute values of $\Gamma$.

6.2.6. Star Formation Strength

As discussed previously, we have allowed two classes of star-forming galaxies into our sample. A total of 111,806 galaxies...
(86%) fall in the star-forming class, and the other 18,796 (14%) belong to the low-S/N star-forming class where the \([\text{O III}]\) or \([\text{N II}]\) lines are weak, but the H\(\alpha\) and H\(\beta\) lines still have S/N > 5. By comparing the results from these two subsamples, we can investigate a possible bias of the results with respect to the level of star formation.

Figure 13 shows the results for both classes as a function of luminosity. As it comprises 86% of the total sample, it is not surprising that the results for the star-forming class are similar to those of the sample as a whole in Figure 6.

The low-S/N class exhibits a similar qualitative behavior to the set as a whole with a few notable differences. The \(\Gamma\)-values are offset by at least 0.08 toward larger \(\Gamma\). The measured IMF turns toward steeper values at lower luminosities than for the sample as a whole. The \(\chi^2\)'s are several times lower as well.

While the galaxies in the low-S/N class meet the same requirement of S/N > 5 in the Balmer lines as the star-forming class, they are biased toward noisier H\(\alpha\) line measurements. This corresponds to lines that are either weak (low SFR) or weak compared to the continuum (low present SFR compared to the past), both of which lead to low EWs. Another issue at play is that the relationship between the H\(\alpha\) line flux and the SFR is dependent on the IMF. At a fixed metallicity and SFR, increasing \(\Gamma\) by 0.05 reduces the H\(\alpha\) flux by 20%. In fact, the H\(\alpha\) flux of a galaxy with \(\Gamma = 1.00\) will be 33 times larger than a galaxy with the same SFR and \(\Gamma = 2.00\). In the presence of real IMF variations at any fixed luminosity, the low-S/N class will be biased toward galaxies with steeper IMFs. Both low SFRs and steep IMFs potentially lead to low-S/N H\(\alpha\) flux. However, it is difficult to determine the level of influence of each effect.

As shown in Figure 1, low H\(\alpha\) EWs lead to larger values of \(\Gamma\) for any fixed color. As the low-S/N class tends toward noisier H\(\alpha\) fluxes and therefore EWs, it is easy to see from equations (5), (6), and (7) that the errors for this class will tend to be larger. This in turn leads to lower \(\chi^2\) values.

As the qualitative IMF-luminosity trend occurs in both star-forming classes, the strength of star formation is unlikely to be a significant bias on our results.

6.2.7. The \(f\) Ratio

As mentioned in § 3.2, the \(f\) ratio is the ratio of the extinction experienced by the nebular emission lines to that experienced by the stellar continuum. The assumption of a value for \(f\) could potentially bias our results. An alternative way of looking at the same problem is that our data in the color–H\(\alpha\) EW plane can be used to constrain the \(f\) ratio by assuming a universal Salpeter IMF.

Figure 14 gives the results of this analysis for the data set as a whole. A value of \(f = 2.0\) is found with \(\chi^2 = 1.068\). This is in good agreement with the Calzetti et al. (1994) value of \(f = 2.0^{+0.6}_{-0.4}\). The quality of the fit in the best case is worse than in Figure 5. Part of the reason for this is that the errors used were slightly smaller as the \(\sigma_{\gamma}\) terms in equations (5) and (7) are set equal to 0. The quality of the fit drops sharply below \(f = 2\) and...
more gradually for $f > 2$. Values of $f$ near 1 are heavily rejected. However, in this particular plot the results are dominated by luminous galaxies.

The values of $f$ as a function of luminosity are shown in Figure 15. For galaxies $M_{r,0.1} = -19$ and brighter the best value of $f$ is consistent with $f = 2$. The faintest two bins prefer an $f$ ratio closer to 2.5. However, the bottom panel shows that this new $f$-value does not translate to improved fit quality. In fact, the faintest galaxies have in general smaller measured extinctions and are therefore less susceptible to changes in $f$. The same qualitative trend of worsening fits with decreasing luminosity seen when allowing $\Gamma$ to float is seen with a varying $f$-value.

Together these two $f$-ratio plots provide a number of insights. For one it shows that our choice of the $f$ ratio is very sensible and provides an independent confirmation of other $f$ ratio measurements. The fact that our best-fitting $\Gamma$-values are at least 0.05 above the Salpeter value cannot be reconciled by changing the geometry of the dust screen. It provides further evidence that the relationship between $\Gamma$ and luminosity is not a function of extinction or a by-product of our extinction correction.

Fig. 12.—Best-fitting $\Gamma$-values as a function of aperture fraction and measured emission-line extinction for six volume-limited luminosity bins. The 2D bins extend 0.1 mag in extinction and 2% in aperture fraction. The volume-limited bins are defined as described in Fig. 8. The white contour shows the area where the 2D bins contain at least 50 galaxies. The shading levels are described by the gray-scale bars on the left.
In this section we have investigated several possible sources of bias to account for our observed trend between the IMF and luminosity. Relationships between the IMF and redshift, aperture fraction, extinction, and star formation strength have been uncovered. Two parameter biases were also found.

In all cases in narrow slices through the data where potential biases are held fixed the qualitative IMF-luminosity relationship appears. The parameters primarily act to offset the value of $\Gamma_0$ at a particular luminosity. The ratio of continuum to emission line extinction, $f$, was found to be a sensible choice, and the results are not sensitive to small changes in this value.

There are two possible interpretations of the relationships between the IMF and potential biases. One is that they are systematic effects due to some problem with our measurement of $\Gamma$. The second is that they are real physical effects. It is not clear from the data which of these statements is more correct.

6.3. Star Formation History

In the previous section several possible sources of bias were investigated, but none were able to account for our observed trend in $\Gamma$ with luminosity or the inability of a universal IMF to fit low-luminosity galaxies. Figure 16 shows the distribution in color-$\lambda_0$ EW space for the least and most luminous bins, $M_{\lambda_0} = -17$ and $-23$. From this figure it is apparent that the most luminous galaxies lie roughly parallel to the IMF tracks while the faintest galaxies are more perpendicular to the tracks. In the
low-luminosity bin there are galaxies that are simultaneously blue and have low EWs. These galaxies are not consistent with a universal IMF with $\Gamma = 1.35$ and, as mentioned before, are not consistent with a universal IMF with a different slope. In addition, the faintest galaxies have the lowest extinctions as they are the least sensitive to dust and $f$ ratio issues. Before concluding that this is evidence for IMF variations, we must first consider whether our model assumption of smooth SFHs is justified.

### 6.3.1. Effects of Star Formation Bursts

The SFH of individual galaxies is the most problematic aspect of the K83 analysis. A sudden burst on top of a smoothly varying background will immediately increase the H$\alpha$ EW. This is due to the formation of O and B stars, which indirectly increase the H$\alpha$ flux through processing of their ionizing photons. The new presence of O and B stars also makes the color of the galaxy bluer. Both of these effects are proportional to the size of the burst.

After the burst is over, the H$\alpha$ EW is smaller and the colors are redder than they would be if the burst had not occurred. The H$\alpha$ EW drops because there is no longer an excess of O and B stars and their ionizing photons, which reduces the H$\alpha$ flux to pre-burst levels. However, there is now an excess of red giants due to the less massive stars from the burst leaving the main sequence. This increases the continuum around the H$\alpha$ line, which further drops the EW in addition to making the galaxy colors redder. After enough time has elapsed after the burst, the galaxy returns to the same position in the color- H$\alpha$ EW plane it would have occupied had no burst occurred, although it will have taken longer to get there.

Figure 17 gives one example of this cycle. A solar metallicity galaxy with $\Gamma = 1.35$ and an exponentially decreasing SFH with $\tau = 2.15$ Gyr experiences a burst of star formation at an age of 4.113 Gyr, lasts 250 Myr, and forms 10% of the stellar mass. The black filled circles, spaced at 100 Myr intervals, show that comparatively more time is spent below the nominal track than above it. The peak H$\alpha$ EW is reached just 5 Myr after the start of the burst. If you happen to be observing the galaxy during the burst, a shallower IMF will be measured (assuming a burst-free SFH), after the burst for 1 Gyr a steeper IMF will be measured, and after that the effects of the burst largely disappear, although the galaxy will appear younger than it actually is.

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Figure 17 gives the best-fitting IMF slope from our analysis, which assumes no bursts, as a function of age for the galaxy in Figure 17. The jitter in the best-fit $\Gamma$-values is due to the discrete nature of our model grid and the fact that the model tracks for different IMFs run together at large ages. Within a 300 Myr period during and just after the burst the best-fitting IMF slope is anywhere from $\Gamma = 1.00$ to 1.95. This shows that even if a universal IMF exists, the SFH can mimic a huge range of IMF models. Roughly 1 Gyr after the burst the measured IMF is back to its true value. A galaxy with a bursty SFH viewed at a random time will be biased toward a steeper IMF than what is the true IMF. While one model is not an exhaustive study of the effects of the SFH on IMF measurements, it does give a good sense of what issues arise.

To eliminate this uncertainty, we investigated cutting the sample on SFH. In order to detect a relative lack or excess of present star formation, it is necessary to measure both the present and past star formation rates or at least be able to compare the two in some way. The problem lies in the fact that conversions of observables into star formation rates assume an IMF to do so. Our aim is to measure the IMF so we cannot make strong a priori assumptions about it. Instead of biasing our results from the start, we fit all
galaxies and then try to determine the effect of SFHs on our conclusions.

6.3.2. Single-Burst Models

The simplest burst model is that of a single burst at a random time on top of our smooth exponential SFHs. A grid of 1000 SFHs was constructed by first selecting one of the 24 smoothly varying SFHs at random. A burst lasting 200 Myr was superposed on the SFH at a time selected uniformly at random over a range of 12.5 Gyr. The strength of the burst was randomly selected up to 40% of the total stellar mass, with preference given to smaller bursts. The colors and EWs of these SFHs were calculated at 1 Myr intervals over 12.5 Gyr for the IMFs $\Gamma = 1.35$ and 1.80 and a fixed metallicity of $Z = 0.01$. This metallicity choice is based on the luminosity-metallicity relationship in Tremonti et al. (2004) for $M_r,0.1 = -17$ galaxies. Plotting these models in the color–$\text{H}$α EW plane shows that all observed data points are covered by either IMF.

To test whether the observed distribution of points in the $M_r,0.1 = -17$ bin can be explained by bursting SFHs, the MC techniques of §5 were used. The models of §4 were replaced with the grid of single-burst models. The 100 MC simulations were constructed as described earlier, but using only the 329 $M_r,0.1 = -17$ galaxies as a basis. The analysis yielded $\chi^2 = 0.027$ for $\Gamma = 1.35$ and $\chi^2 = 0.020$ for $\Gamma = 1.80$, both of which are over fits.

This shows than an individual galaxy can be fitted with an arbitrary IMF given the freedom to choose an SFH. However, our advantage is that we have many galaxies and the distribution of the properties of the best-fit models can be shown to be implausible.

Figure 19 demonstrates the problem with the single-burst model. In the left panels the distributions of the best-fitting burst strengths are plotted as a fraction of the total stellar mass formed. In the right panels is the distribution of the best-fitting burst ages relative to the burst onset. For example, a galaxy that best fits a model with a burst at 1.000 Gyr at an age of 1.211 Gyr has a time from burst onset of 211 Myr. This measure is used because in an investigation of the effects of bursts the age relative to the burst is more important than the age given that the bursts occur at different, random times across the models.

For both $\Gamma = 1.35$ and 1.80 the number of objects best fitted by a model prior to the burst is $11^2/3$, or 3%, and these are not plotted. In both cases the distributions of the best-fitting ages and ages at which the burst begins are roughly uniform. Given this fact, it is expected that half of the galaxies should be best fitted by a preburst model. Furthermore, the right panels show sharp, significant discontinuities in the distribution of best-fitting time from burst onset. Again, viewed at random times this distribution should be uniform but is highly peaked at 25 Myr at the start of the burst and the 25 Myr just after the burst ends. In both cases the errors bars show that the discontinuities are significant.

In the case of $\Gamma = 1.35$, 3.5 times as many galaxies are in the 200 Myr after the burst ends than the 200 Myr during it, and this 400 Myr accounts for 57% of all galaxies. Although our sample is $r$-band selected, the stars in the 0.7–3 $M_r$ range that dominate the red continuum in the red giant phase do not start to leave the main sequence for 300 Myr. The sharp increase in galaxies fitted at 200 Myr after the burst cannot be due to a selection effect.

Assuming a universal IMF, this points to a strong coordination of SFHs across a population of galaxies unrelated in space. These arguments show that while a single-burst model can fit the data extremely well, it does not do so in a physically self-consistent fashion.

6.3.3. Multiple-Burst Models

To find a physical motivation for SFH models for low-luminosity galaxies, we look toward the Local Group. There have been a number of recent studies of the SFHs of local dwarf galaxies that use the Hubble Space Telescope to get color-magnitude diagrams (CMDs) of resolved stellar populations. The SFH is determined by fitting isochrones to the CMD. The sample here is biased by Local Group membership and by what galaxies have been observed to date. The galaxies mentioned here give a point of reference rather than a well-defined distribution of SFHs.

The blue compact dwarf (BCD) UGCA 290 was found to quiescently form stars over the past Gyr up until a 10-fold increase in SFR from 15 to 10 Myr ago, which more recently has decreased to one-quarter of its peak value (Crone et al. 2002). The dwarf irregular IC 1613, which is relatively isolated and noninteracting, was found to have SFR enhanced by a factor of 3 from 3 to 6 Gyr ago without evidence of strong bursts (Skillman et al. 2003). The dwarf irregular NGC 6822, also relatively isolated, is found to have a roughly constant SFH (Wyder 2003). The BCD NGC 1705 is found to be gasping: an SFH marked by moderate activity punctuated by short periods of decreased star formation (Annibali et al. 2003). The authors also note that NGC 1705 is best fitted by an IMF with $\Gamma = 1.6$. NGC 1569 likely experienced three strong bursts in the last Gyr, as well as a quiescent phase from 150 to 300 Myr ago (Angeretti et al. 2005).

Inspired by the preceding Local Group SFHs, we constructed six SFH classes with multiple bursts. These SFH classes are described in Table 2. Each class starts with an underlying smooth SFH. SFHs 1 and 2 have no star formation, SFHs 3 and 4 have a constant SFH, and SFHs 5 and 6 have exponentially decreasing SFHs like those previously described. Star formation discontinuities are then superimposed on top of the smooth SFHs. These discontinuities are in the form of increased (bursts) or decreased (gasps) star formation for periods of 200 Myr. The time and spacing of the discontinuities are random, with the mean interval between bursts listed in Table 2.

For each SFH class described above we randomly generated 1000 SFHs. Colors and EW widths were calculated for each SFH using $\Gamma = 1.35$ and $Z = 0.01$. According to the SDSS mass-metallicity relationship (Tremonti et al. 2004), galaxies at $M_r,0.1 = -17$ will on average have $Z = 0.01$.

We then repeated our $\chi^2$ analysis with the 100 MC simulations of the $M_r,0.1 = -17$ galaxies in the same manner as for the single-burst models. The results of our analysis for each of the
six SFH classes are shown in Figure 20. SFH 5, the gasps on top of exponential SFHs, is the best fit with $\chi^2 = 0.09$. Extended periods of no star formation punctuated by bursts (SFHs 1 and 2) do not fit the data.

As was the case for the single-burst models, an unreasonable fraction of galaxies are best fitted by SFHs in the 20 Myr immediately following a burst or the first 20 Myr of a gasp. If the SFH models are reasonable, we should see roughly equal numbers of galaxies in each time bin. There is no reason why all of the low-luminosity galaxies across the large volume of space in the SDSS footprint should have experienced coordinated bursts. However, each panel of Figure 20 has at least 40% of the galaxies in one 20 Myr bin.

One explanation for this is that it is an artifact of our sample being selected in the $r$ band. However, spectral synthesis models show that for instantaneous bursts of star formation the $r_{0.1}$ magnitude is brightest at the burst time and decays smoothly for a range of $\Gamma$. If anything, it is more likely to catch galaxies during a burst rather than after one or after a gasp instead of during one.

Regardless of the SFH model, the presence of blue galaxies with low H$_\alpha$ EW requires a recent discontinuity in the SFR for $\Gamma = 1.35$. Based on the evolution of the $r_{0.1}$ band luminosity, we expect to see a similar number of galaxies with excess H$_\alpha$ EWs. The fact that these galaxies are missing shows that the discontinuous SFH models do not match our observations. Therefore, IMF variations are a more likely explanation for the observed distribution of $M_{r_{0.1}} = -17$ galaxies.

6.3.4. Recovering $\Gamma$ from Synthetic Data

As a last exercise, the best-fitting SFH models from the previous section can be run forward to see if the correct IMF can be recovered. For each SFH model grid 10,000 data points were chosen by selecting a random SFH and a uniformly distributed
age. Normal errors were added using the error characteristics of the $M_{r,0.1} = -17$ bin. These synthetic data were analyzed in the same fashion as the real data in the earlier sections. For the single-burst models the recovered IMF models for $\Gamma = 1.35$ and 1.80 were 1.34 and 1.79, respectively, with best-fitting $\chi^2 = 0.80$ and 0.65. For SFH 5 the recovered IMF was also 1.34 with $\chi^2 = 1.5$. In all three cases the correct IMFs were recovered although the fit was worsened by the burst activity. This reinforces the difficulty in producing enough blue, low Hα EW galaxies to match the observed data with simple SFH models.

7. Hα absorption

In the previous sections we have expanded on the K83 method and exploited the Hα and color information as much as possible. In § 6.2 we found that various possible biases do not fully explain either the increased values of $\Gamma$ or the poor fit to a single IMF in the lowest luminosity bins. In § 6.3 we found that an arbitrary $\Gamma$-value coupled with a plausible SFH with bursts or gasps can account for the position of any individual galaxy in the color–Hα EW plane. However, taking the population of $M_{r,0.1} = -17$ galaxies together necessitates an incredibly unlikely coordination of SFHs across the disparate group of objects. This points to the extraordinary conclusion that while the IMF may be universal across luminous galaxies, it is not in fact universal in low-luminosity galaxies. Such an extraordinary claim would ideally be backed by extraordinary evidence. In this section we take a look beyond the K83 method for some reinforcement of our result.

The Hα absorption feature can be used to gain additional insight into the nature of stellar populations. Hα absorption is due to absorption lines from stellar photospheres. The Balmer absorption lines are most prominent in A stars and weaken due to the Saha equation for both hotter and cooler stars. As such the Hα absorption is a proxy for the fraction of light of a stellar population being supplied by A stars, and to a lesser extent B- and F-type stars. In a stellar population of a uniform age the Hα absorption will peak after the O and B stars burn out, but before the A stars leave the

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**Fig. 20.**—Distributions of best-fitting times measured from the beginning of a burst or gasp for the synthetic $M_{r,0.1} = -17$ data for models with $Z = 0.01$, $\Gamma = 1.35$, and multiple-burst SFH models described in Table 2. The $\chi^2$ values for each family of models are also shown. The dotted lines indicate the 95% confidence regions determined by MC simulations.
main sequence. For this reason the strength of H$_\delta$ can be used to help determine the age of a population or to detect bursts of star formation that occurred around 1 Gyr in the past.

Worthey & Ottaviani (1997) describe two different methods for measuring H$_\delta$ absorption. The H$_\delta$ definition is tuned to most accurately measure the H$_\delta$ absorption from F stars. The H$_\delta$A definition has a wider central bandpass to match the line profiles for measuring H$_\delta$. The narrower H$_\delta$A definition is much more sensitive to population variations that determine the range of H$_\delta$A values. This is due to the fact that the steeper IMFs have fewer luminous massive stars to dilute the H$_\delta$ absorption features from the A star population. Second, the low-Î– models have H$_\delta$A values that start their initial increases at a later time. Finally, the low-Î– models reach their peak values later in time than those with fewer massive stars.

To compare the H$_\delta$A values for the $M_{*[0.1]} = -23$ and $-17$ bins to the models, the range of the middle 90% values from Figure 21 for each bin is overlaid on Figure 22. Once again the $M_{*[0.1]} = -23$ bin is in good agreement with our assumption of a universal IMF and smooth SFH. The range of H$_\delta$A values can be accomplished with a single Salpeter-like $\Gamma = 1.40$ IMF with only the proviso that most galaxies be either older than a few Gyr or younger than 300 Myr. However, the exact same statement can be made for $1.0 < \Gamma < 2.0$ so H$_\delta$A provides only a constraint on the age of the most luminous galaxies, but not the IMF.

For the $M_{*[0.1]} = -17$ galaxies the distribution of H$_\delta$A cannot be achieved with the shallowest IMFs investigated under the assumption of smooth SFHs. However, Salpeter and steeper IMFs can be accomplished. What changes is the range of ages over which the models have the correct H$_\delta$A. Steeper IMFs require that the galaxies be either older than a few Gyr or 200 Myr old to accommodate the observed H$_\delta$A values. Most troubling is that there does not appear to be any reason why H$_\delta$A should stack up at 6.1 for the $M_{*[0.1]} = -17$ galaxies.

H$_\delta$A values can also be calculated for the same SFH class models used earlier. Unfortunately, this does not provide any added constraints. H$_\delta$A values remain elevated for several hundred Myr after the start of a burst or the end of a gasp of SFH. The behavior of H$_\delta$A in the presence of SFH discontinuities provides no need for galaxies to stack up in the narrow 20 Myr intervals seen in §6.3.3, nor do these models suggest why the low-luminosity galaxies are consistent with a single value of H$_\delta$A.

There is no satisfactory model to account for the H$_\delta$A distribution of the low-luminosity galaxies. However, the SFH results from the previous section strongly suggest that the incredible coordination of discontinuities is highly unlikely. Steeper IMFs do allow for higher H$_\delta$A values for longer periods of time, thus relaxing the SFH coordination requirement. This agrees with our
earlier results for faint galaxies. The low-luminosity galaxies are most likely the result of a mix of IMFs that are on average steeper than Salpeter.

8. CONCLUSIONS

The goal of this paper was to revisit the K83 method for inferring the IMF from integrated stellar populations and to harness the richness of the SDSS data, improved spectral synthesis models, and greater computational power available today to make a state-of-the-art measurement of the IMF. The quality of the SDSS spectroscopy allowed us to address several of the limitations of K83 and KTC94: we resolve the [N ii] lines (a significant improvement in the accuracy of the Hδ/C11 EWs of individual galaxies), eliminate contamination from AGNs, make extinction corrections for individual galaxies, and fit underlying stellar absorption of Hδ/C11. We succeeded in achieving more accurate EWs for individual galaxies. The median total EW error for our sample is 17% compared to a 10% uncertainty in EWs combined with a 20–30% uncertainty in the extinction correction for K83.

We expanded the grid of models to allow for a range of ages and metallicities. We used χ² minimization to go beyond differentiating between two or three IMF models to actual fitting for the best IMF slope. The vast size of the SDSS sample allowed us both to drive down random errors and to cut the data into narrow parameter ranges that were still statistically viable.

The size of the DR4 sample yielded a ΔΓ = 0.0011 95% confidence region due to random error for the sample as a whole. Even the volume-limited M₀.₁ = −17 luminosity bin with only 329 objects has a random error of ΔΓ = 0.0086. Only in bins with fewer than 10 objects do the random errors become significant. Our IMF fitting is therefore dominated by systematics. Originally we believed that our systematics would be dominated by the effects of SFH discontinuities. However, we conducted several experiments where we selected populations of galaxies from models with bursting or gasping SFHs and gave them measurement errors consistent with those in the M₀.₁ = −17 luminosity bin. To our surprise, our χ² minimization revealed the true IMFs with ΔΓ = 0.01. The main effect was to reduce the quality of the fits. This is due to the fact that Hδ EWs return to nominal levels in a relatively short time after SFH discontinuities.

Another way to estimate the size of the systematic errors is to look at the trends of the M₀.₁ = −21 and −22 luminosity bins (because they have the largest membership) in Figures 9, 10, and 11. Assuming that the IMF is universal and that our method is perfect, we should get the same answer for any subset of the data we might choose. The largest ranges are ΔΓ ≈ 0.12 for redshift binning, ΔΓ ≈ 0.19 by aperture, and ΔΓ ≈ 0.19 by extinction. Conservatively then the systematic error is ±0.1.

There are two points to be kept in mind about this estimate of the systematic error. For one it is the systematic error in the exact

![Model values of the Hδ index as a function of age for four IMF models, Γ = 1.00 (top left), 1.40 (top right), 1.70 (bottom left), and 2.00 (bottom right). Each dot represents a model value for single IMF, metallicity, SFH (as described in § 4), and age. The four tracks identifiable toward the left of each panel are each of differing metallicities with the lowest metallicity models having the largest Hδ values at young ages. The gray area indicates the range of Hδ spanned by the middle 90% of the volume-limited M₀.₁ = −23 bin. The hashed area shows the middle 90% range for the volume-limited M₀.₁ = −17 bin.](image)
value of $\Gamma$. Even in Figure 12, where more narrow bands of measured extinction and aperture fraction are considered, the same trends with luminosity are seen as with the sample as a whole. The relative systematics between luminosity bins in these narrow slices is much smaller. The second thing to remember is that the way in which we empirically defined our systematic error discounts the possibility of IMF variations. What we have called systematics could actually be science. If galaxies have radial IMF gradients or if dust content plays a strong role in star formation, the systematic error could be much smaller. The main area in which we were unable to improve on the K83 and KTC94 studies is that they were able to match the aperture size to the galaxies, which avoids the issue of aperture effects.

In spite of a more quantitative approach, like K83 and KTC94 the results are mostly qualitative. However, there are four key results from our investigation.

First, for galaxies brighter than $M_{r,0.1} \approx -20$ the best-fitting IMFs are Salpeter-like ($\Gamma \sim 1.4$). In addition, the assumption of a universal IMF and smoothly varying SFHs is a good fit. This is reassuring as it follows the conventional wisdom and provides confidence that the method works.

Second, galaxies fainter than $M_{r,0.1} \approx -20$ are best fitted by steeper IMFs with larger fractions of low-mass stars. For these galaxies a universal IMF and smooth SFH are a poor assumption. This result is in qualitative agreement with evidence that LSB galaxies have bottom-heavy IMFs (Lee et al. 2004).

Third, while breaking the IMF-SFH degeneracy for individual galaxies using the Hα EW and color is hopeless, for a statistical sample of galaxies the degeneracy can be broken.

Finally, given our analysis of discontinuous SFHs, it appears that the IMF is not universal in low-luminosity galaxies and fewer massive stars are being created in these galaxies.

It is worth mentioning the main caveat of our $\Gamma$-values again. As illustrated in Figure 2, IMF parameterizations are themselves degenerate in our parameter space. Increasing the IMF slope has a similar effect to lowering the highest mass stars that are formed or increasing the fraction of intermediate-mass stars. This method cannot explicitly determine if two populations have the same underlying IMF. Figure 5 shows that for the sample as a whole the Scalo (1998) three-part power law yields nearly the same result as our two-part power law. However, our method is sensitive in many cases if the IMFs are different.

In terms of star-forming cloud temperatures the harsher ambient radiation and larger number of sources of cosmic rays present in more luminous galaxies agree qualitatively with our results. With the extra energy hitting the star-forming clouds, larger masses may be needed for contraction and fractionization may end sooner, suppressing the formation of less massive stars (Larson 1998). Cédrés et al. (2005) find that while the H α regions of the luminous grand-design spiral NGC 5457 (M31) can be reproduced by a single Salpeter IMF, for the low-luminosity flocculent galaxy NGC 4395 a blend of two IMFs is required. However, such trends are not seen in studies of well-resolved stellar populations (Kroupa 2002).

Another explanation for the absence of massive stars is that the massive stars are there but are not visible. Extinction to the center of star-forming regions, where massive stars preferentially exist, can reach $A_V \sim 20$ (Engelbracht et al. 1998). However, the low-luminosity galaxies have the lowest observed extinctions (see Table 1), which is the opposite of what would be expected given our IMF results.

It is also possible that the IMF is in fact universal, but the way in which it is sampled in embedded star clusters leads to an integrated galaxial IMF that varies from the true IMF. Weidner & Kroupa (2005) use a universal IMF with the assumption that stars are born in clusters where the maximum cluster mass is related to the star formation rate. For a range of models this leads to a narrow range for the apparent IMF in high-mass galaxies. For low-mass galaxies the IMF is steeper with a wider range of slopes. The results here are in qualitative agreement for some of the integrated galactic IMF scenarios in Weidner & Kroupa (2005) given that there should be a rough correlation between galaxy luminosity and mass. Once again, Elmegreen (2006) argues that the galaxy-wide IMF should be the same as the IMF in clusters regardless.

In light of the theory of Weidner & Kroupa (2005), whether the results of this paper speak to a relationship between environment and the formation of individual stars is open to interpretation. However, the impact on the modeling and interpretation of the properties of galaxies is clear. Köppen et al. (2007) note that the integrated galaxial IMF is the correct IMF to use when studying global properties of galaxies. Even if the IMF of stars is in truth universal, it may currently be misused in the modeling of galaxies. Furthermore, a varying integrated galaxial IMF could open the door to new insights in galaxy evolution. For instance, Köppen et al. (2007) suggest that the observed mass-metallicity relationship in galaxies naturally arises from a variable integrated galaxial IMF similar to the results of this paper.

Future work will expand in several directions. The success constraining the IMF with only two observed parameters (albeit carefully chosen to be orthogonal to systematic errors) motivates a more expansive analysis with more parameters. Information from wavelength regimes beyond the SDSS can be used. For instance, the absorption strength and P Cygni profile shape of the C iv $\lambda 1550$ line due to massive stars are sensitive to the IMF slope and upper mass cutoff (Leitherer et al. 1995). However, it could be contaminated by absorption from the interstellar medium and would need to be disentangled (Shapley et al. 2003). A full Markov Chain Monte Carlo analysis could be implemented fitting to multiple spectral features and marginalizing over a range of SFHs. In addition to luminosity, surface brightness and gas-phase metallicity can be tested for systematic IMF variations. While the results for luminous galaxies are already dominated by systematics, the continued progress of the SDSS can provide better statistics for a more detailed analysis of what physical processes are behind the IMF variations in faint galaxies.

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REFERENCES

Abazajian, K., et al. 2004, AJ, 128, 502
Adams, F. C., & Fatuzzo, M. 1996, ApJ, 464, 256
Adelman-McCarthy, J. K., et al. 2006, ApJS, 162, 38
Angeretti, L., et al. 2005, AJ, 129, 2203
Annibali, F., et al. 2003, AJ, 126, 2752
Baldry, I. K., & Glazebrook, K. 2003, ApJ, 593, 258
Baldry, I. K., et al. 2004, ApJ, 600, 681
Baldwin, J. A., Phillips, M. M., & Terlevich, R. 1981, PASP, 93, 5
Blanton, M. R., et al. 2003a, AJ, 125, 2348
———. 2003b, ApJ, 592, 819
Bonanos, A. Z., et al. 2004, ApJ, 611, L33
Brinchmann, J., et al. 2004, MNRAS, 351, 1151
Calzetti, D., Kinney, A. L., & Storchi-Bergmann, T. 1994, ApJ, 429, 582
Cedrés, B., Cepa, J., & Tomita, A. 2005, ApJ, 634, 1043
Courteau, S., & van den Bergh, S. 1999, AJ, 118, 337
Crone, M. M., et al. 2002, ApJ, 567, 258
Delhaye, J. 1965, in Galactic Structure, ed. A. Blaaun & M. Schmidt (Chicago: Univ. Chicago Press), 61
Duquennoy, A., & Mayor, M. 1991, A&A, 248, 485
Efstathiou, G., Ellis, R. S., & Peterson, B. A. 1988, MNRAS, 232, 431
Elmegreen, B. G. 1997, ApJ, 486, 944
———. 2000, ApJ, 548, 572
Engelbracht, C. W., Rieke, M. J., Rieke, G. H., Kelly, D. M., & Achtermann, J. M. 1998, ApJ, 505, 639
Figer, D. F. 2003, Nature, 434, 192
Fischer, D. A., & Marcy, G. W. 1992, ApJ, 396, 178
Fukugita, M., et al. 1996, AJ, 111, 1748
Hoyle, F. 1953, ApJ, 118, 513
Kauffmann, G., et al. 2003, MNRAS, 346, 1055
Kennicutt, R. C., Jr. 1983, ApJ, 272, 54 (K83)
———. 1998a, ARA&A, 36, 189
———. 1998b, in ASP Conf. Ser. 142, The Stellar Initial Mass Function, ed. G. Gilmore & D. Howell (San Francisco: ASP), 1
Kennicutt, R. C., Jr., & Kent, S. M. 1983, AJ, 88, 1094
Kennicutt, R. C., Jr., Tamblyn, P., & Congdon, C. W. 1994, ApJ, 435, 22 (KTC94)
Köppen, J., Weidner, C., & Kroupa, P. 2007, MNRAS, 375, 673
Kroupa, P. 2001, MNRAS, 322, 231
———. 2002, Science, 295, 82
Kroupa, P., Tout, C. A., & Gilmore, G. 1993, MNRAS, 262, 545
Larson, R. B. 1998, MNRAS, 301, 569
Le Borgne, D., et al. 2004, A&A, 425, 881
Lee, H., et al. 2004, MNRAS, 353, 113
Leitherer, C., Robert, C., & Heckman, T. M. 1995, ApJS, 99, 173
Low, C., & Lynden-Bell, D. 1976, MNRAS, 176, 367
Lupton, R. H., Gunn, J. E., & Szalay, A. S. 1999, AJ, 118, 1406
Mayor, M. 1976, A&A, 48, 301
Miller, G. E., & Scalo, J. M. 1979, ApJS, 41, 513
Osterbrock, D. E. 1989, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei (Sausalito: University Science Books)
Padoan, P., Nordlund, Å., & Jones, B. J. T. 1997, MNRAS, 288, 145
Pei, Y. C. 1992, ApJ, 395, 130
Phelps, R. L., & Janes, K. A. 1993, AJ, 106, 1870
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in C: The Art of Scientific Computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Rees, M. J. 1976, MNRAS, 176, 483
Robberto, M., et al. 2005, in IMF@50: The Initial Mass Function 50 Years Later, ed. E. Corbelli, F. Palla, & H. Zinnecker (Dordrecht: Kluwer), 455
Rolloston, W. R. J., Smartt, S. J., Dufton, P. L., & Ryans, R. S. I. 2000, A&A, 363, 537
Salpeter, E. E. 1955, ApJ, 121, 161
Scalo, J. M. 1986, Fundam. Cosm. Phys., 11, 1
———. 1998, in ASP Conf. Ser. 142, The Stellar Initial Mass Function, ed. G. Gilmore & D. Howell (San Francisco: ASP), 201
Schlegel, D. J., Finkbeiner, D. P., & Davis, M. 1998, ApJ, 500, 525
Shapley, A. E., Steidel, C. C., Pettini, M., & Adelberger, K. L. 2003, ApJ, 588, 65
Skillman, E. D., et al. 2003, ApJ, 596, 253
Stoughton, C., et al. 2002, AJ, 123, 485
Tremonti, C. A., et al. 2004, ApJ, 613, 898
Uomoto, A., et al. 1999, BAAS, 31, 1501
Vila-Costas, M. B., & Edmunds, M. G. 1992, MNRAS, 259, 121
Weidner, C., & Kroupa, K. 2004, MNRAS, 348, 187
———. 2005, ApJ, 625, 754
Worthey, G., & Ottaviani, D. L. 1997, ApJS, 111, 377
Wyder, T. K. 2003, AJ, 125, 3097
York, D. G., et al. 2000, AJ, 120, 1579