Pion-photon exchange nucleon-nucleon potentials
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Abstract

We calculate in chiral perturbation theory the dominant next-to-leading order correction to the \(\pi\gamma\)-exchange NN-potential proportional to the large isovector magnetic moment \(\kappa_v = 4.7\) of the nucleon. The corresponding spin-spin and tensor potentials \(\tilde{V}_{S,T}(r)\) in coordinate space have a very simple analytical form. At long distances \(r \approx 2\) fm these potentials are of similar size (but opposite in sign) as the leading order \(\pi\gamma\)-exchange potentials. We consider also effects from virtual \(\Delta\)-isobar excitation as well as other isospin-breaking contributions to the \(2\pi\)-exchange NN-potential induced by additional one-photon exchange.

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Isospin violation in the nuclear force is a subject of current interest. Charge-independence breaking (i.e. the difference between the total isospin \(I = 1\) \(pn\)-scattering and \(nn\)- or \(pp\)-scattering) is large and well established. On the other hand, charge-symmetry breaking (i.e. the difference between \(nn\)- and \(pp\)-scattering after removal the long-range electromagnetic forces) is smaller and fairly well established. These isospin-violating contributions to the nuclear force play also an important role for explaining the 764 keV binding energy difference of \(^{3}\text{He}\) and triton [1, 2, 3]. The bulk of it, namely 648 keV, can already be understood in terms of the Coulomb interaction [1].

The isospin-violating NN-interaction of longest range is generated by the simultaneous exchange of a pion and a photon between the two nucleons. Since the photon is massless the \(\pi\gamma\)-exchange interaction is of nominal one-pion range, \(m_{\pi}^{-1} = 1.41\) fm. After earlier attempts made in refs.[4, 5] the complete leading order \(\pi\gamma\)-exchange NN-potential has been calculated in the systematic framework of chiral perturbation theory in ref.[6]. A crucial feature of that calculation has been to guarantee the gauge-invariance of the result by considering the full set of all 19 possible Feynman diagrams. The resulting expression for the complete leading order \(\pi\gamma\)-exchange potential in momentum space turned out to be surprisingly simple, see Eq.(1) below. However, due to its intrinsic smallness (a few permille of the \(1\pi\)-exchange interaction) the inclusion of this new isospin-breaking interaction had negligible effects on the \(^1S_0\) low-energy parameters and it lead only to a tiny improvement in the fits of the NN-scattering data [6]. Nevertheless it is important to know as accurately as possible the size of these well-defined long-range components before one introduces (adjustable) short-range isospin-violating terms.

The purpose of the present short paper is to calculate next-to-leading order corrections to the \(\pi\gamma\)-exchange NN-potential in the systematic framework of chiral perturbation theory. These corrections arise either as relativistic \(1/M\)-corrections (with \(M = 939\) MeV the average nucleon mass) to the static result of ref.[6] or they are generated by new interaction vertices from the next-to-leading order chiral Lagrangian \(\mathcal{L}_{\pi N}^{(2)}\) [7]. Experience with the isospin-conserving NN-potential has shown that the next-to-leading order corrections are dominated by the contributions proportional to the large low-energy constants, in that case \(c_{1,3,4}\) [8]. For the photon-nucleon coupling
which is of relevance in the present work, one readily identifies the isovector magnetic moment \( \kappa_v = 4.7 \) of the nucleon as an outstandingly large low-energy parameter. Therefore we will focus here on this particular contribution to the \( \pi\gamma \)-exchange NN-potential. Effects from virtual \( \Delta(1232) \)-isobar excitation which involve the equally strong \( \Delta \to N\gamma \) transition magnetic moment \( \kappa^* \approx 4.9 \) will also be considered.

Let us start with reanalyzing the leading order \( \pi\gamma \)-exchange NN-potential of ref.[6]. The corresponding T-matrix in momentum space reads:

\[
T^{(lo)}_{\pi\gamma} = \frac{\alpha g_A^2}{8\pi f_\pi^2}(\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^2\tau_2^3) \vec{\sigma}_1 \cdot \vec{q} \vec{q} \cdot \vec{q} \left\{ \frac{1}{q^2} - \frac{(m_\pi^2 - q^2)^2}{q^4(m_\pi^2 + q^2)} \ln \left(1 + \frac{q^2}{m_\pi^2}\right) \right\},
\]

with \( \alpha = 1/137.036 \) the fine structure constant, \( g_A = g_{\pi N}/f_\pi = 1.3 \) the nucleon axial vector coupling constant, \( f_\pi = 92.4 \text{ MeV} \) the pion decay constant, and \( m_\pi = 139.57 \text{ MeV} \) the charged pion mass. Furthermore, \( \vec{q} \) denotes the momentum transfer between both nucleons, and \( \vec{\sigma}, \vec{\tau}_1, \vec{\tau}_2 \) are the usual spin- and isospin operators of the two nucleons. The Fourier transformation, \(-2\pi)\cdot q \exp(\vec{i}\vec{q} \cdot \vec{r}) \ldots\), of Eq.(1) to coordinate space yields a local potential with spin-spin and tensor components:

\[
\left\{ \vec{V}_S(r) \cdot \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{V}_T(r) \cdot (3\vec{\sigma}_1 \cdot \vec{\tau} \vec{\sigma}_2 \cdot \vec{\tau} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \right\} \frac{1}{2}(\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^3\tau_2^3).
\]

The isospin factor \((\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^3\tau_2^3) / 2\) has been chosen such that it gives 1 for elastic \( pn \to np \) scattering. The leading order \( \pi\gamma \)-exchange spin-spin potential reads:

\[
\vec{V}_s^{(lo)}(r) = \frac{\alpha g_A^2 m_\pi^2 2e^{-x}}{(4\pi f_\pi^2)^2} \left\{ 2 \ln \frac{x}{2} + 2\gamma_E - \frac{1}{x} - \frac{1}{x^2} + \vec{E}(x) - 2\vec{E}(2x) \right\},
\]

with \( x = m_\pi r \) and \( \vec{E}(x) = \int_0^\infty d\zeta e^{-\zeta}(\zeta + x)^{-1} \) the modified exponential integral function. \( \gamma_E \approx 0.5772 \) is the Euler-Mascheroni number. The associated tensor potential has a similar form:

\[
\vec{V}_T^{(lo)}(r) = \frac{\alpha g_A^2}{(4\pi f_\pi^2)^2} \frac{e^{-x}}{3r^3} \left\{ x - 5 + 4(3 + 3x + x^2) \ln \frac{x}{2} + \gamma_E \right\} + (18 - x^2)\vec{E}(x) + 4(3x - 3 - x^2)\vec{E}(2x).
\]

In the first and second row of Table I we present some numerical values of these leading order \( \pi\gamma \)-exchange potentials for distances \( 1 \text{ fm} \leq r \leq 3 \text{ fm} \). Somewhat as a surprise, one observes a non-monotonic dependence on the nucleon distance \( r \). The spin-spin potential \( \vec{V}_s^{(lo)}(r) \) passes through zero at \( r = 2.53 \text{ fm} \) and it has a maximum at \( r = 3.34 \text{ fm} \). For the tensor potential \( \vec{V}_T^{(lo)}(r) \) these points are shifted inward and approximately reduced by a factor 1.9. It passes through zero at \( r = 1.31 \text{ fm} \) and it has its maximum value of about 8.8 keV at \( r = 1.78 \text{ fm} \).

In order to arrive at the analytical results Eqs.(1,3,4) one can actually circumvent the complete evaluation of all contributing one-loop diagrams. It is sufficient to calculate their spectral function or imaginary part using the Cutkosky cutting rule. The pertinent two-body phase space integral is most conveniently performed in the \( \pi\gamma \) center-of-mass frame where it becomes proportional to a simple angular integral:

\[
(\mu^2 - m^2)/\left(32\pi\mu^2\right) \int_{1}^{\infty} dz \ldots, \text{with } \mu \geq m_\pi \text{ the \( \pi\gamma \) invariant mass. Gauge invariance can be controlled by working with the \( (g_{\mu2} + \xi k_\mu k_\nu/k^2) / k^2 \) through the \( \xi \)-independence of the total spectral function. Using these techniques we obtain from the leading order one-loop \( \pi\gamma \)-exchange diagrams of ref.[6]:

\[
\text{Im} T^{(lo)}_{\pi\gamma} = \frac{\alpha g_A^2}{8\pi f_\pi^2}(\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^2\tau_2^3) \vec{\sigma}_1 \cdot \vec{q} \vec{q} \cdot \vec{q} \left( \frac{\mu^2 + m_\pi^2}{\mu^4(2m_\pi^2 - \mu^2)} \right).
\]


Table I: Pion-photon exchange $\sigma$-potentials in units of keV versus the nucleon distance $r$. The spin-spin and tensor potentials $\tilde{V}_{S,T}^{(lo)}$ correspond to the leading order in the chiral expansion. The next-to-leading order corrections $\tilde{V}_{S,T}^{(\kappa_{\nu})}$ are proportional to the large isovector magnetic moment $\kappa_{\nu} = 4.7$ of the nucleon, and $\tilde{V}_{S,T}^{(\kappa^{*})}$ arise from the magnetic $\Delta \rightarrow N\gamma$ transition.

The notation $\text{Im} T_{\pi\gamma}$ is meant here such that one is taking the imaginary part of the amplitude standing to the right of the spin- and isospin factors.

Fig. 1: Pion-photon exchange diagrams generating a non-vanishing imaginary part. The heavy dot symbolizes the magnetic coupling of the photon to the nucleon, or the magnetic $\Delta \rightarrow N\gamma$ transition. Diagrams for which the role of both nucleons is interchanged are not shown. These lead effectively to a doubling of the NN-potential.

Now we turn to the dominant next-to-leading correction to the $\pi\gamma$-exchange NN-potential proportional to the large isovector magnetic moment $\kappa_{\nu} = 4.7$. The relevant one-loop diagrams with a nucleon in the intermediate state are shown in Fig. 1. The pertinent Feynman rules can be found in appendix A of ref.[7]. From the calculated spectral function, $(\mu^2 - m_{\pi}^2)/\mu_5$ times a polynomial in $\mu^2$ and $m_{\pi}^2$, we can derive (via a once-subtracted dispersion relation) the following expression for the $T$-matrix in momentum space:

$$T_{\pi\gamma}^{(\kappa_{\nu})} = \frac{\alpha g_A^2 \kappa_{\nu}}{64 M f_{\pi}^2 q^3} \left\{ \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 - \tilde{\tau}_1^3 \tilde{\tau}_2^3 \right\} \left[ \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 \left( m_{\pi}^2 + q^2 \right) \left( 3m_{\pi}^2 - 5q^2 \right) \arctan \frac{q}{m_{\pi}} + m_{\pi} q \right]$$

As a side remark we note that the contribution proportional to the isoscalar magnetic moment $\kappa_s = 0.88$ vanishes identically. The reason for this feature are the vanishing angular integrals: $f_1 \, dz \, z^{-1} = 0 = f_1 \, dz \, z$. Fourier transformation of Eq.(6) to coordinate space yields spin-spin and tensor potentials of the following simple analytical form:

$$\tilde{V}_{S,T}^{(\kappa_{\nu})}(r) = \frac{\alpha g_A^2 \kappa_{\nu}}{48\pi M f_{\pi}^2} \frac{e^{-m_{\pi} r}}{r^4} \left( 1 + m_{\pi} r \right),$$
\[ V_T^{(\kappa_v)}(r) = -\frac{\alpha g_A^2 \kappa_v}{48\pi M f_\pi^2} \frac{e^{-m_{\pi}r}}{r^4} (5 + 2m_{\pi}r) . \] (8)

The more direct way to obtain these analytical expressions is to "Laplace transform" the spectral function \( \text{Im} T_{\pi\gamma}^{(\kappa_v)} \) (for details see Eqs.(3,4) in ref.[9]). In the third and fourth row of Table I we have collected some numerical values of these novel isospin-violating potentials \( V_T^{(\kappa_v)}(r) \). At long distances \( r \approx 2 \text{ fm} \) they are of the same size but opposite in sign as the leading order \( \pi\gamma \)-exchange potentials (compare with the first and second row in Table I). At shorter distances the tensor component \( V_T^{(\kappa_v)}(r) \) in Eq.(8) with its higher weight factor on the singular \( 1/r^4 \)-term becomes in fact dominant. Note also that there is some tendency for cancellation in the long-range tails of the tensor potentials.

Fig. 2: Isospin-breaking corrections to the two-pion exchange NN-interaction induced by one-photon exchange. The heavy dot in the right diagram symbolizes the magnetic coupling of the photon to the nucleon. The \( 2\pi \)-exchange diagrams for which the role of both nucleons is interchanged are not shown.

As argued in ref.[5], the \( \Delta(1232) \)-isobar with its relatively small excitation energy could also play a substantial role for the \( \pi\gamma \)-exchange interaction. The transition \( \Delta \to N\gamma \) is known to be predominantly of magnetic dipole type. In the effective chiral Lagrangian approach its strength is parameterized by a transition magnetic moment \( \kappa^* \). Using the empirical information [10] about the partial decay width:

\[ \Gamma(\Delta^+ \to p\gamma) = \frac{\alpha \kappa^{*2}(M_\Delta^2 - M^2)^3}{144M_\Delta^2 M^2}(3M_\Delta^2 + M^2) \simeq 0.68 \text{ MeV} , \] (9)

we can extract a value of \( \kappa^* \approx 4.9 \) for the \( \Delta \to N\gamma \) transition magnetic moment. Here, \( M_\Delta = 1232 \text{ MeV} \) denotes the mass of the delta-isobar. The possible one-loop diagrams with virtual excitation of a delta-resonance contributing to the \( \pi\gamma \)-exchange NN-interaction are shown in Fig. 1. The Feynman rules for the non-relativistic \( \pi N\Delta \)- and \( \gamma N\Delta \)-vertices read: \((g_{\pi N\Delta}/2M) \vec{S} \cdot \vec{k} T^a \) and \((\kappa^*/2M) \vec{S} \cdot (\vec{k} \times \vec{\varepsilon}) T^3 \), respectively, where \( \vec{k} \) denotes an ingoing pion or photon momentum. The \( 2 \times 4 \) spin- and isospin transition matrices \( S^i \) and \( T^a \) satisfy the relations \( S^i S^{ij} = (2\delta^{ij} - i e^{ijk} \sigma^k)/3 \) and \( T^a T^{ab} = (2\delta^{ab} - i e^{abc} \tau^c)/3 \). Using the empirically well-satisfied relation \( g_{\pi N\Delta} = 3g_{\pi N}/\sqrt{2} = 3g_A M/\sqrt{2} f_\pi \) for the \( \pi N\Delta \)-coupling constant we find the following result for their total spectral function:

\[
\text{Im} T_{\pi\gamma}^{(\kappa^*)} = \frac{\alpha g_A^2 \kappa^*}{96\sqrt{2} M f_\pi^2 \mu^3} \left( \vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^3 \tau_2^3 \right) \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 \left[ 2\mu_\Delta (\mu^2 - m_\pi^2) \\
+ (m_\pi^4 + 2m_\pi^2 \mu^2 - 3\mu^4 - 4\mu^2 \Delta^2) \arctan \frac{\mu_\Delta^2 - m_\pi^2}{2\mu_\Delta} \right] \right\},
\]
\begin{equation}
\begin{aligned}
&+ \bar{\sigma}_1 \cdot \bar{q} \cdot \bar{\sigma}_2 \cdot \bar{q} \left[ 2\Delta \left( \frac{m^2_\pi}{\mu} + 3\mu \right) \right. \\
&\left. + \left( 2m^2_\pi - 5\mu^2 + \frac{3m^4_\pi}{\mu^2} - \frac{4\Delta^2(3\mu^2 + m^2_\pi)}{\mu^2 - m^2_\pi} \right) \arctan \left( \frac{\mu^2 - m^2_\pi}{2\mu\Delta} \right) \right] \right). \tag{10}
\end{aligned}
\end{equation}

where \( \Delta = M_\Delta - M = 293 \text{ MeV} \) denotes the delta-nucleon mass difference. Note that the spectral function of the spin-spin term (proportional to \( \vec{\sigma}_1 \cdot \vec{\sigma}_2 \)) and the spectral function of the tensor term (proportional to \( \vec{\sigma}_1 \cdot \bar{q} \cdot \bar{\sigma}_2 \cdot \bar{q} \)) vanish both at threshold \( \mu = m_\pi \). From the mass-spectra given by Eq.(10) one can easily calculate the spin-spin and tensor potentials in coordinate space (following the decomposition in Eq.(2)) in the form of a continuous superposition of Yukawa functions [9]. The fifth and sixth row in Table I shows the corresponding numerical values for the \( \pi\gamma \)-exchange potentials \( V^{(x)}_{S,T}(r) \) for nucleon distances \( 1 \text{ fm} \leq r \leq 3 \text{ fm} \). One finds that the effects from virtual \( \Delta \)-excitation are about a factor 5 to 10 smaller than those generated by diagrams with only nucleon intermediate states. About such a suppression of the \( \Delta \)-isobar effects has already been speculated in the summary of ref.[5]. The present calculation provides now a quantitative answer to this question.

Fig. 2 shows another set of isospin-violating contributions to the \( 2\pi \)-exchange NN-interaction induced by an additional one-photon exchange. These effects could alternatively be interpreted as one-pion loop contributions to the electric and magnetic form factors of the nucleon which are introduced in order to describe the electromagnetic interaction between the extended (not point-like) nucleons. Irrespective of their classification the magnitude of such isospin-breaking effects should be quantified. The first two diagrams in Fig. 2 (allowing only for an intermediate \( n \)-state) with the photon coupling to the charge of the nucleon give rise to the following \( T \)-matrix:

\begin{equation}
T^{(lo)}_{\pi\gamma} = \frac{\alpha}{48\pi f^2_\pi} \tau_1 \tau_2 \left( \tau_1 + \tau_2 + 2\tau_1 \tau_2 \right) \left[ 1 + M^2_\pi + 4m^2_\pi \right] \frac{1}{\mu^2} \left( 1 + 2g^2_A \right) \left( \frac{q}{q + 4m^2_\pi + q^2} \right) \ln \left( \frac{q}{q + 4m^2_\pi + q^2} \right) \tag{11}
\end{equation}

\begin{equation}
V^{(lo)}_{C}(r) = \frac{\alpha}{3(8\pi f^2_\pi)^2} \tau_1 \tau_2 \left( \tau_1 + \tau_2 + 2\tau_1 \tau_2 \right) \frac{4m^2_\pi}{\mu^2} \left( 1 + 2g^2_A \right) \left( 1 - 5g^2_A \right) \tag{12}
\end{equation}

The corresponding central potential in coordinate space:

\begin{equation}
\text{Im} T^{(\Delta)}_{\pi\gamma} = \frac{\alpha g^2_A}{8f^2_\pi} \tau_1 \tau_2 \left( \tau_1 + \tau_2 + 2\tau_1 \tau_2 \right) \left( \frac{2m^2_\pi}{3} - \Delta^2 - \frac{5m^2_\pi}{12} \right) \frac{\sqrt{\mu^2 - 4m^2_\pi}}{\mu^2 - 4m^2_\pi} \left( \Delta^2 \mu^2 - m^2_\pi \right) \arctan \left( \frac{\mu^2 - m^2_\pi}{2\Delta} \right) \tag{13}
\end{equation}

The corresponding central potential \( V^{(\Delta)}_{C}(r) \) (see second row in Table II) comes out repulsive and it is approximately an order of magnitude smaller than the leading order one \( V^{(lo)}_{C}(r) \). The last
diagram in Fig. 2 involves the magnetic coupling of the photon to the nucleon. We are considering only the dominant contribution proportional to the isovector magnetic moment $\kappa_v = 4.7$ and find for the corresponding one-loop T-matrix:

$$T^{(N)}_{\pi\gamma} = \frac{\alpha g^2 \kappa_v}{32 M f^2_\pi} \tau_1^3 \tau_2^3 (\bar{\sigma}_1 \times \vec{q}) \cdot (\bar{\sigma}_2 \times \vec{q}) \left\{ \frac{2m_\pi}{q^2} - \frac{4m_\pi^2 + q^2}{q^2} \arctan \frac{q}{2m_\pi} \right\}. \quad (14)$$

When translated into coordinate space one obtains a spin-spin and tensor potential:

$$\bar{V}^{(N)}_S(r) = -\frac{\alpha g^2 \kappa_v}{96\pi M f^2_\pi} \tau_1^3 \tau_2^3 \frac{e^{-2m_\pi r}}{r^4} (1 + 2m_\pi r), \quad (15)$$

$$\bar{V}^{(N)}_T(r) = \frac{\alpha g^2 \kappa_v}{96\pi M f^2_\pi} \tau_1^3 \tau_2^3 \frac{e^{-2m_\pi r}}{r^4} (2 + m_\pi r), \quad (16)$$

of the typical two-pion range $(2m_\pi)^{-1} = 0.7$ fm. As the numbers in the third and fourth row of Table II indicate they differ from each other mainly in sign. The magnitude of $V^{(N)}_{S,T}(r)$ comes out substantially smaller than that of the central potential $V^{(C)}_{\pi\gamma}(r)$. This is to be expected since the magnetic interaction is a higher order relativistic $1/M$-correction. Finally, we include also a virtual $\Delta$-isobar in this two-pion exchange process followed by one-photon exchange. The corresponding spectral function:

$$\text{Im} T^{(\Delta)}_{\pi\gamma} = \frac{\alpha g^2 \kappa_v}{64M f^2_\pi \mu^3} \tau_1^3 \tau_2^3 \left( \bar{\sigma}_1 \cdot \bar{\sigma}_2 \mu^2 + \bar{\sigma}_1 \cdot \vec{q} \bar{\sigma}_2 \cdot \vec{q} \right) \times \left\{ -2\Delta \sqrt{\mu^2 - 4m_\pi^2} + (\mu^2 + 4\Delta^2 - 4m_\pi^2) \arctan \frac{\sqrt{\mu^2 - 4m_\pi^2}}{2\Delta} \right\}, \quad (17)$$

leads to the numerical values of the isospin violating spin-spin and tensor potentials $V^{(\Delta)}_{S,T}(r)$ presented in the fifth and sixth row of Table II. These potentials are of the same sign but a factor of 5 to 10 smaller than their counterparts $V^{(N)}_{S,T}(r)$ with pure nucleon intermediate states.

It is also instructive to compare our present results with previously calculated isospin-breaking $2\pi$-exchange potentials. Taking into account the mass difference between the charged and neutral pion, $m_{\pi^+} - m_{\pi^0} = 4.59$ MeV, in the pion-loops ref.[11] obtained the charge-independence breaking central potential $\delta V^{(cb)}_{2\pi}(r)$ (for an explicit expression, see Eq.(11) in ref.[11]). Moreover, the neutron-proton mass difference, $M_n - M_p = 1.29$ MeV, in intermediate nucleon states of the pion-loops leads to the charge-symmetry breaking potentials $\tilde{V}^{(cb)}_{C,S,T}(r)$ (proportional to $\tau_1^3 + \tau_2^3$) derived recently in ref.[12] (for details see Eq.(10) therein). As one can see from the numerical values in Table II the effects of these hadron mass splittings are substantially larger than the one-photon exchange corrections studies here. For more extensive recent work on isospin-violating NN-forces using the method of unitary transformations, see also ref.[13].

In summary we calculated in this work next-to-leading order corrections to the $\pi\gamma$-exchange NN-potential. The dominant contribution proportional to the large isovector magnetic moment $\kappa_v = 4.7$ turns out to be of similar size (but opposite in sign) as the leading order term. Effects from virtual $\Delta$-isobar excitation, involving the equally large $\Delta \to N\gamma$ transition magnetic moment $\kappa^* \approx 4.9$, are approximately one order of magnitude smaller. Furthermore, we have also evaluated several isospin-violating contributions to the $2\pi$-exchange NN-potential induced by additional one-photon exchange. In most cases these turned out to be smaller than the $\pi\gamma$-exchange terms and the effects from pion and nucleon mass splittings [11, 12]. The analytical expressions for the T-matrices and coordinate space potentials derived in this work are in a form that they can be easily implemented into NN-phase shift analyses or few-body calculations. Such numerical studies will reveal the role of the long-range isospin-violating NN-interaction generated by pion-photon exchange.
Table II: Isospin violating contributions to the two-pion exchange pp-potential in units of keV versus the distance $r$. The spin-spin and tensor potentials $V^{(N,\Delta)}_{S,T}$ are proportional to the large isovector magnetic moment $\kappa_v = 4.7$. The charge-independence breaking potential $\delta V_{2\pi}^{(cib)}$ proportional to the pion mass difference $m_{\pi^+} - m_{\pi^0} = 4.59$ MeV is taken from ref. [11]. The charge-symmetry breaking potentials $\tilde{V}_{C,S,T}^{(csb)}$ proportional to the neutron-proton mass difference $M_n - M_p = 1.29$ MeV are taken from ref. [12].

| $r$ [fm] | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $V_C^{(lo)}$ | -49.5 | -22.5 | -11.0 | -5.74 | -3.13 | -1.76 | -1.02 | -0.61 | -0.37 |
| $\tilde{V}_C^{(\Delta)}$ | 7.68 | 2.96 | 1.26 | 0.58 | 0.28 | 0.14 | 0.08 | 0.04 | 0.02 |
| $V_S^{(N)}$ | -21.3 | -8.66 | -3.89 | -1.88 | -0.96 | -0.51 | -0.28 | -0.16 | -0.09 |
| $\tilde{V}_S^{(N)}$ | 23.9 | 9.14 | 3.90 | 1.81 | 0.89 | 0.46 | 0.25 | 0.14 | 0.08 |
| $\tilde{V}_S^{(\Delta)}$ | -4.09 | -1.50 | -0.62 | -0.28 | -0.13 | -0.07 | -0.03 | -0.02 | -0.01 |
| $\tilde{V}_T^{(\Delta)}$ | 4.18 | 1.46 | 0.58 | 0.25 | 0.12 | 0.06 | 0.03 | 0.02 | 0.01 |

| $\delta V_{2\pi}^{(cib)}$ | 108 | 55.6 | 31.1 | 18.4 | 11.4 | 7.25 | 4.74 | 3.16 | 2.14 |
| $\tilde{V}_C^{(csb)}$ | -182 | -80.0 | -38.9 | -20.3 | -11.3 | -6.51 | -3.89 | -2.39 | -1.50 |
| $\tilde{V}_S^{(csb)}$ | 67.3 | 29.6 | 14.4 | 7.50 | 4.11 | 2.34 | 1.38 | 0.83 | 0.51 |
| $\tilde{V}_T^{(csb)}$ | -84.1 | -34.7 | -15.9 | -7.86 | -4.11 | -2.26 | -1.28 | -0.75 | -0.45 |

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