Imaging the Phase of an Evolving Bose-Einstein Condensate Wavefunction

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We demonstrate a spatially resolved autocorrelation measurement with a Bose-Einstein condensate (BEC) and measure the evolution of the spatial profile of its quantum mechanical phase. Upon release of the BEC from the magnetic trap, its phase develops a form that we measure to be quadratic in the spatial coordinate. Our experiments also reveal the effects of the repulsive interaction between two overlapping BEC wavepackets and we measure the small momentum they impart to each other.

A trapped Bose-Einstein condensate has unique value as a source for atom lasers and matter-wave interferometry because its atoms occupy the same quantum state, with uniform spatial phase. However, when released from the trapping potential, a BEC with repulsive atom-atom interactions expands, developing a non-uniform phase profile. Understanding this phase evolution will be important for applications of coherent matter waves. We have developed a new interferometric technique using spatially resolved autocorrelation to measure the functional form and time evolution of the phase of a BEC wavepacket expanding under the influence of its mean field repulsion.

In 1997, the coherence of weakly interacting BECs was demonstrated by releasing two spatially separated condensates and observing their interference. Subsequent experiments have further investigated condensate coherence properties. One used velocity-resolved Bragg diffraction to probe the momentum spectrum of trapped and released BECs. A complementary experiment that used matter-wave interferometry can be interpreted as a measurement of the spatial correlation function, whose Fourier transform is the momentum spectrum. These experiments showed that a trapped condensate has a uniform phase, and a released condensate develops a non-uniform phase profile. (Recently the influence of non-zero temperature on coherence properties was also investigated). The experiments reported in this Letter combine spatial resolution and interferometry to measure the functional form of the time-dependent phase profile of a released condensate. We also make the first measurement of the velocity imparted to two equal BEC wavepackets from their mutual mean-field repulsion.

We perform our experiments with a condensate of 1.8(4) × 10⁶ sodium atoms in the 3S¹/₂, F = 1, m_F = −1 state. The sample has no discernable non-condensed (i.e. thermal) component. The condensate is prepared following the method of Ref. and is held in a magnetic trap with trapping frequencies ω_x = ω_y = 2ω_z = 2π×27 Hz. Using a scattering length of a = 2.8 nm, the calculated Thomas-Fermi diameters are 47 µm, 66 µm, and 94 µm, respectively.

We release the BEC from the magnetic trap and it expands, driven mostly by the mean-field repulsion of the atoms. This expansion implies the development of a nonuniform spatial phase profile (recall that the velocity field is proportional to the gradient of the quantum phase). After an expansion time T₀, we probe the phase profile with matter-wave Bragg interferometry. Our interferometer splits the BEC into two wavepackets and recombines them with a chosen overlap, producing interference fringes, which we measure with absorption imaging. From the dependence of the fringe spacing on the overlap, we extract the phase profile of the wavepackets.

Our atom interferometer consists of three optically induced Bragg-diffraction pulses form the interferometer. The condensate is released for a time T₀ before the first Bragg pulse. The centers of ψ_A and ψ_B are separated by δx at the time of the third Bragg pulse, which splits them into ψ_A1, ψ_B1, and ψ_A2, ψ_B2. Before imaging the atoms, we allow the output ports to separate for a time T₃ ≈ 2 ms. The image shows the output ports when T₀ = 3 ms, T₁ = 1 ms, and T₂ = 1.3 ms.

Our atom interferometer consists of three optically-induced Bragg-diffraction pulses applied successively in time (Fig. 1). Each pulse consists of two counter-propagating laser beams whose frequencies differ by 100 kHz. They are detuned by about −2 GHz from atomic resonance (λ = 2π/k = 589 nm) so that spontaneous emission is negligible. The first pulse has a duration of 6 µs and intensity sufficient to provide a π/2 pulse.
which coherently splits the BEC into two wavepackets, \(\psi_A\) and \(\psi_B\). The wavepackets have about the same number of atoms and only differ in their momenta: \(p = 0\) and \(p = 2\hbar k\). At a time \(T_1 = 1\) ms after the first Bragg pulse, the two wavepackets are completely separated and a second Bragg pulse \((a \pi\) pulse) of 12 \(\mu\)s duration transfers \(\psi_B\) to a state with \(p \approx 0\) and \(\psi_A\) to \(p \approx 2\hbar k\). After a variable time \(T_2\) the wavepackets partially overlap again and we apply a third pulse, of 6 \(\mu\)s duration \((\pi/2\) pulse). This last pulse splits each wavepacket into the two momentum states. The interference of the overlapping wavepackets in each of the two momentum states allows the determination of the local phase difference between \(\psi_A\) and \(\psi_B\) at the time of the final Bragg pulse. The set of data at different \(\delta x\) constitutes a new type of spatial autocorrelation measurement that is similar to the “FROG” technique \([17]\) used to measure the complete field of ultrafast laser pulses. From these measurements we obtain the phase profile of the wavepackets in the \(x\) direction.

Figure 2a-e shows one interferometer output port for different \(\delta x\) \((\text{different } T_2)\) after an expansion time \(T_0 = 4\) ms. In general, we observe straight, evenly spaced fringes (although for small \(T_0\) and \(T_2\) the fringes may be somewhat curved). There is a value of \(\delta x = x_0 \neq 0\) where we observe no fringes (Fig. 2c) and the fringe spacing decreases as \(|\delta x - x_0|\) increases. Figure 2f, a cut through Fig. 2b, shows the high-contrast fringes \([18]\). Our data analysis uses the average fringe period \(d\), obtained from plots like Fig. 2d.

The fringes come from two different effects: the interference of two wavepackets with quadratic phase profile, and a relative velocity between the wavepackets’ centers. The data can be understood by calculating the fringe spacing along \(x\) at output port 1 \([19]\). We assume that the phase \(\phi\) of the wavefunction \(e^{i\phi}\) can be written as \(\phi = \frac{\pi}{2} x^2 + \beta x\). The equal spacing of the fringes implies, as predicted in the Thomas-Fermi limit \([20]\), that \(\phi\) has no significant higher-order terms \([21]\). The curvature coefficient \(\alpha\) describes the mean-field expansion of the wavepackets and \(\beta\) describes a relative repulsion velocity. The velocity arises because the wavepackets experience a repulsive push as they first separate and again as they recombine. The density at port 1 \((\text{see Fig. 1})\) just after the final interferometer pulse is the interference pattern \(|\psi_{A1} + \psi_{B1}|^2\) of the wavepackets \(\psi_{A1}\) and \(\psi_{B1}\):

\[
|f(x - \delta x)e^{i\left(\frac{\pi}{2}(x - \delta x)^2 - \beta(x - \delta x)\right)} + f(x)e^{i\left(\frac{\pi}{2}x^2 + \beta x\right)}|^2,
\]

where we assume that the amplitudes and curvatures of the wavepackets are equal and their velocities have equal magnitude and opposite direction. The cross term of \([1]\) is

\[
2f(x - \delta x)f(x)\cos\left(\frac{\alpha \delta x + M \delta v}{\hbar}\right)x + C,
\]

where \(M\) is the sodium mass, \(M \delta v/\hbar \equiv 2\beta\), and \(C\) is independent of \(x\) \([22]\). \(\delta v = v_B - v_A\) is the relative repulsion velocity between the wavepackets \(\psi_{A1}\) and \(\psi_{B1}\). Expression \([2]\) predicts fringes with spatial frequency,

\[
\kappa = \alpha \delta x + M \delta v/\hbar
\]

where \(|\kappa| = 2\pi/d\). When there are no fringes, \(\kappa = 0\) and the wavepacket separation \(\delta x = x_0 \equiv -M \delta v/\alpha h\).

Figure 3 plots the measured \(\kappa\) vs. \(\delta x\) \([23]\) for \(T_0 = 1\) and 4 ms. The data are well fit by a straight line as expected from Eq. \((3)\) in the approximation that \(\alpha\) and \(\delta v\) are independent of \(\delta x\). The slopes of the lines are the phase curvatures \(\alpha\), and the \(\kappa\) intercepts give the relative velocities \(\delta v\).

We checked the validity of the data analysis procedure by analyzing data simulated with a 1-D Gross-Pitaevskii (GP) treatment. Despite variations of \(\delta v\) and \(\alpha\) with \(\delta x\) (due to their continued evolution during the variable time \(T_2\)), we find that \(\kappa\) is still linear in \(\delta x\). The slopes and intercepts in general are averages over the range of \(\delta x\) used in the experiment.

The interference fringes used to determine \(\alpha\) and \(\delta v\) are created at the time of the final interferometer pulse. Because the two outputs overlap at that moment, we wait a time \(T_3\) for them to separate before imaging. During this time, the wavepackets continue to expand.
1-D simulations show that the fringe spacings and the wavepackets expand in the same proportion. We correct \( \kappa \) (by typically 15\%) for this, using the calculated expansion from a 3-D solution of the GP equation described below.

A trapped condensate has \( \frac{1}{2} \mu \) average total energy per particle, where \( \mu \) is the chemical potential \([11]\). After release from the trap, it has \( \frac{1}{2} \mu \) average mean-field energy per particle. Applying a \( \pi/2 \) Bragg pulse to the BEC causes a density corrugation, which increases the mean-field energy to \( \frac{1}{2} \mu \) per particle. In the approximation that the wavepackets do not deform as they separate and recombine, one can show that 1/3 of the total mean-field energy goes into expansion of the wavepackets, and 2/3 is available for kinetic energy of center-of-mass motion. Therefore \( \frac{1}{2} \mu \) of mean-field energy per particle is available for repulsion. The corresponding repulsion velocity is only about \( 10^{-2} \) of a photon recoil velocity. The repulsion energy and \( \delta v \) decrease for larger \( T_0 \) because both are inversely proportional to the condensate volume, which we calculate with the LVM. The two curves shown in Fig. 2b are the calculated \( \delta v \) when \( \delta x = 0 \) (solid curve) and \( \delta v \) averaged over the different \( \delta x \) used in the experiment (dashed curve). The 1-D GP simulations suggest that for small \( T_0 \), the results of the experiment should be closer to the solid curve; and for large \( T_0 \), closer to the dashed curve. The data is consistent with this trend.

\[ T_0 \text{ [ms]} \]
\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]
\[ 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \]
\[ \delta v \text{ [mm/s]} \]
\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \]

**FIG. 4.** (a) Plot of the phase curvature \( \alpha \) versus the initial expansion time \( T_0 \) showing the phase evolution from mean-field expansion. The solid line is a calculation using the Lagrangian Variational Method (LVM). (b) Plot of the relative repulsion velocity \( \delta v \) versus \( T_0 \). The solid curve is the calculated maximum repulsion velocity (when \( \delta x = 0 \)) and the dashed curve is the repulsion velocity averaged over the range of \( \delta x \) used in the experiment.

The different slopes and intercepts of the two lines in Fig. 3 show that the curvature \( \alpha \) and relative velocity \( \delta v \) of the wavepackets depend on the release time \( T_0 \) before the first interferometer pulse. Figure 3 plots the dependence of \( \alpha \) and \( \delta v \) on various release times \( T_0 \). The condensate initially has a uniform phase so that immediately after its release from the trap \( \alpha = 0 \). We nevertheless measure a nonzero \( \alpha \) for \( T_0 = 0 \) ms because the BEC expands during \( T_1 \) and \( T_2 \). As a function of time, \( \alpha \) behaves as \( \dot{D}/D \) where \( D \) is the wavepacket diameter and \( \dot{D} \) is its rate of change \([20]\). At early times when the mean-field energy is being converted to kinetic energy, \( \dot{D} \) increases rapidly, increasing \( \alpha \). At late times, after the mean-field energy has been converted, \( D \) increases while \( \dot{D} \) is nearly constant, decreasing \( \alpha \).

We predict the time evolution of \( \alpha \) using the Lagrangian Variational Method (LVM) \([24]\). The LVM uses trial wavefunctions with time dependent parameters to provide approximate solutions of the 3-D time-dependent GP equation. In the model, the effect of the interferometer pulses is to replace the original wavepacket with a superposition of wavepackets having different momenta; e.g., the action of our first interferometer pulse is

\[ \psi_0 \rightarrow (\psi_0 + e^{i2k_\perp x} \psi_0) / \sqrt{2} \]

We use Gaussian trial wavefunctions in the LVM and, for simplicity, neglect the interaction between the wavepackets, to calculate the phase curvature \( \alpha \) at the time of the last interferometer pulse. This result, with \( T_1 = T_2 \), is the solid line of Fig. 4a.

We use energy conservation to calculate the relative repulsion velocity \( \delta v \) between \( \psi_{A1} \) and \( \psi_{B1} \) because we neglect wavepacket interactions in the LVM. In the Thomas-Fermi approximation, we can calculate the amount of energy available for repulsion when \( T_0 = 0 \).

\[ \dot{v} \quad \text{Time} \]
\[ v \text{ [mm/s]} \]
\[ 1 \quad 0 \quad -1 \quad -2 \]
\[ T \text{ [ms]} \]
\[ 0 \quad 0.4 \quad 0.8 \quad 1.2 \quad 1.6 \]

**FIG. 5.** (a) Schematic representation of the interferometer in the trap, with the principle difference from Fig. 1 being the curved arrows indicating the acceleration of the wavepackets. (b) The relative velocity \( v \) between the two trapped wavepackets versus the interferometer time \( T \). The solid line is a fit.
the first Bragg pulse, $\psi_A$ remains at the minimum of the magnetic potential while $\psi_B$ is displaced. Wavepacket $\psi_B$ therefore spends more time away from the center of the trap and experiences more acceleration than $\psi_A$. Following the last Bragg pulse, $\psi_{A1}$ and $\psi_{B1}$ have a velocity difference which for our parameters can be approximated by $v \approx -\frac{\hbar k}{2\mu} \sin^2(\omega x T_A) + \delta v$. Figure $\ref{fig:velocity}$ plots $v$ versus $T_A$ and the curve is a fit to the above expression. We obtain the trap frequency $\omega_x/2\pi = 26.7(15) \text{ Hz}$, in excellent agreement with an independent measurement. We also obtain the relative velocity from the mean-field energy shift $\delta v = 0.49(12) \text{ mm/s}$, which we expect to be somewhat larger than for the released measurements because the wavepackets contract, producing a larger mean field.

In conclusion, we demonstrate an autocorrelating matter-wave interferometer and use it to study the evolution of a BEC phase profile by analyzing spatial images of interference patterns. We study how the phase curvature of the condensate develops in time and measure the repulsion velocity between two BEC wavepackets. Our interferometric method should be useful for characterizing other interesting condensate phase profiles. For example, it can be applied to detect excitations of a BEC with characteristic phase patterns, such as vortices and solitons. The method should be useful for further studies of the interaction of coherent wavepackets and to study the coherence of atom lasers.

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[22] In practice, C includes a random phase from mirror vibrations.
[23] We use $\delta x = \frac{2\hbar k}{\mu} (T_2 - T_1) - x_s$ where $\frac{2\hbar k}{\mu} = 5.9 \text{ cm/s}$ is the two photon recoil velocity of sodium and $x_s$ is a small correction of the order $\delta v T_1$ due to the repulsion of the wavepackets. We include the correction in our data analysis in a self-consistent manner. The correction modifies $\alpha$ insignificantly, but increases the final values of $\delta v$ by ≈ 0.05 mm/s.
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