Theoretical studies of filtration movement of moisture in capillaries of woody plants

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Abstract. The article presents theoretical studies of filtration transport of moisture through capillaries of woody plants that is determined by humidity gradients. The emerging humidity fields and the magnitude of the gradients significantly affect the growth and development of woody plants biomass. The aim of theoretical research was to use mathematical tools of operational calculus that provide an opportunity to consider the filtration of moisture through the capillaries of wood materials using a third-kind boundary condition, when the substance is exchanged with the environment at the considered boundary. As a result of the use of Laplace transform a mathematical model for studying the movement of moisture in the capillaries of woody vegetation through filtration transport has been obtained. Modelling of a moisture transport process in wood materials is an important tool in theoretical and experimental studies. This is particularly important for conducting long-term experimental studies of the site quality of stand of trees from the technogenic impact of vehicles on roadside ecological systems.

1. Introduction

The growth and development of forest tree species along highways significantly depends on the roadside environmental conditions created by the technogenic impact of vehicles. Wood belongs to capillary-porous colloidal substances. Thanks to such a structure, moisture and nutrients are filtered through the wood capillaries. The main reason for the movement of moisture through the capillaries of woody material is its uneven distribution by volume in the direction of low humidity. This type of movement is called hydraulic conductivity of wood. The intensity or density of the moisture flow, moving inside the material, is proportional to the moisture concentration gradient, that is the coefficient of filtration transfer.

Theoretical studies of the filtration movement of moisture in the capillaries of woody vegetation, that is its hydraulic conductivity, are similar to the problems of the theory of thermal conductivity, in which solid fundamental theoretical research has been conducted so far. An academician A V Lykov [1, 2], as well as other scientists [3-10], has made the main contribution to these studies. These scientific papers examined: the use of environmental indicators for intensive and extensive description of an ecosystem at the various stages of its development [4], an engineering approach to the construction, identification and the use of mathematical models of heat and mass transfer for the selection of optimal heating modes for capillary-porous bodies [5], thermodynamic optimization [6, 7], the method for determining the coefficient of thermal conductivity and humidity of wood [9], basics of computational fluid dynamics [3] and thermodynamics [8, 10]. However, these studies are
insufficient and they do not reflect the ongoing biological processes of filtration transport of moisture in the capillaries of woody plants and, therefore, they have not been sufficiently reviewed in domestic and foreign literature. Filtration transport of moisture in wood capillaries and bast vessels has been mainly studied within biological science. Nowadays, as research shows, the filtration process has to be considered from a physical point of view according to the existing cycle of energy and substances (mass exchange) in the biosphere.

The study of the filtration movement of moisture in the capillaries of woody vegetation was conducted, as in the theory of thermal conductivity, with the use of the operational calculus while applying the Laplace transform \([11-13]\). During the research, the authors of the article observed the negative technogenic impact of vehicles on roadside ecosystems connected with decrease in the movement of moisture in wood materials \([14, 15]\). This is reflected in the site quality of stand of trees depending on the distance from the highway. That is why the purpose of the research is theoretical studies of the technogenic impact of vehicles on the filtration transport of moisture in the capillaries of woody vegetation and reduction of the site quality of roadside forest plantation, which is important both for the state of ecosystems and for the science in general.

2. Materials and methods

Theoretical studies of the filtration transport of moisture in the capillaries of woody vegetation have been examined, which allow us to determine the impact of technogenic pollution from vehicles on the state of forest plantation located along highways. Using mathematical tools in research enables us to significantly reduce the time factor in unlimited variation in the parameters of hydraulic conductivity of wood in various environmental conditions created by vehicles along highways. The obtained mathematical models enable us to enter various parameters of humidity and its gradients that affect the growth and development of woody plants biomass and determine the overall changes in the state of the environment.

Mathematical methods of operational calculus and the theory of heat and mass transfer were used in the studies of the filtration transport of moisture in the capillaries of woody plants \([1, 13-15]\).

3. Results and discussion

Mass transfer of moisture and nutrients in capillary-porous vascular systems of woody-shrub plants can be described by the Fourier-Kirchhoff differential equation \([2]\):

\[
\frac{\partial u}{\partial t} = a \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = a \nabla^2 u
\]  

(1)

where \(u\) – the concentration of moisture and nutrients; \(t\) – the time; \(x, y, z\) – the coordinates; \(a\) – the diffusion coefficient of moisture and nutrients; \(\nabla^2\) – the Laplace operator.

Since the bast sieve tubes and xylem vessels are capillary-porous anisotropic material, the problem of mass transfer in vascular systems can be reduced to a "one-dimensional" problem with non-stationary movement of nutrients and moisture in the capillaries, that is

\[
\frac{\partial u(x,t)}{\partial t} = a \frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t)
\]

(2)

Differential equation (2) defines the field of concentration of moisture and nutrients, expresses the relationship between their measurement \(u\) in time \(t\) and the propagation of the value \(u'\) along the capillary at the \(x\) coordinate. We assume that in the area of the capillaries under consideration there are no ongoing reactions characterized by the reaction function \(f(x,t)\). Then equation (2) will take the form

\[
\frac{\partial u(x,t)}{\partial t} = a \frac{\partial^2 u(x,t)}{\partial x^2}
\]

(3)

Equation (3) provides a mathematical description of the transfer of moisture and nutrients in the capillaries of wood materials. The left part of the equation describes \(\frac{\partial u(x,t)}{\partial t}\) the rate of change of
moisture and nutrients at a certain point of the capillary with time \( t \), the right-the spatial distribution \( u(x,t) \) near this point. Indeed, the partial derivative \( \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial u(x,t)}{\partial x} \right) \) determines the intensity of the gradient change \( u(x,t) \) in the direction of the \( x \) axis.

In order to find the field \( u(x,t) \) in capillaries of wood materials at any time, that is, to solve the differential equation (3), it is necessary to determine \( u(x,t) \) at the initial time (initial conditions – \( u(x,0) \)) and the law of propagation \( u(x,\tau) \) in capillaries of wood materials – boundary conditions.

We assume in the solution of equation (3) boundary conditions of the third kind, when a substance is exchanged with the external environment at the considered boundary of the region.

The solution of equation (3) of the movement of nutrients in the capillaries of wood materials using the initial and boundary conditions will finally take the form:

\[
a \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial x} = 0 \quad (4)
\]

starting conditions

\[
t = 0; \quad u(x,0) = U_2 \quad (5)
\]

boundary condition

\[
\begin{cases}
x = 0; \quad u(0,t) = u_1 = u_2 + bt \\
x = \ell; \quad \frac{\partial u(\ell,t)}{\partial x} = -h[u(\ell,t) - u_2]
\end{cases} \quad (6)
\]

where \( \ell \) – the considered area of capillary length; \( h \) – the coefficient of nutrient filtration; \( u_2 \) – the humidity at the initial time in the wood material.

To solve equation (4) under the conditions (5) and (6), we apply the Laplace transform, relative to one of the variables, for example, time \( t \) [13]:

\[
L_1[u(x,t)] = \int_0^\infty u(x,t) \cdot e^{-st} dt = T(x,g) \quad (7)
\]

Then the image of the function of the first term in equation (4) will be

\[
L_1 \left[ \frac{\partial^2 u(x,t)}{\partial x^2} \right] = \frac{\partial^2 u(x,g)}{\partial x^2} = T''(x,g) \quad (8)
\]

The image of the second term in equation (4) will be determined using the Laplace transform for the image of the derivative according to the formula:

\[
L_1[f^{(n)}(t)] = g^n \cdot F(g) - g^{(n-1)} \cdot f(0) - g^{(n-2)} \cdot f'(0) - ... - f^{(n-1)}(0),
\]

in this case we have

\[
L_1[u(x,t)] = gT(x,g) - u(x,0) = gT(x,g) - u_2 \quad (9)
\]

where \( u(x,0) = u_2 \) – from the initial conditions (5).

Finally get

\[
T''(x,g) - \frac{g}{a} T(x,g) + \frac{u_2}{a} = 0 \quad (10)
\]

or

\[
T''(x,g) - \frac{g}{a} \left[ T(x,g) - \frac{u_2}{g} \right] = 0 \quad (11)
\]

In equation (12), we introduce a replacement

\[
T(x,g) - \frac{u_2}{g} = F(x,g) \quad (12)
\]

In this case, the second derivative of the replacement (13) corresponds to \( F''(x,g) = T''(x,g) \),
assuming that \( u_2 \) and \( g \) are constant values. As a result of the substitution in equation (12), we get:

\[
F''(x, g) - \frac{g}{a} F(x, g) = 0 .
\]  

(14)

In this equation (14), using the Laplace transform with respect to the \( x \)-coordinate variable, we obtain:

\[
S^2\Phi(s, g) - SF(0, g) - F'(0, g) - \frac{g}{a} \Phi(s, g) = 0 .
\]

(15)

Solve (15) as a simple algebraic equation

\[
\Phi(s, g) = \frac{SF(0, g) + F'(0, g)}{S^2 - \frac{g}{a}}.
\]

(16)

To do this, enter another replacement:

\[
A = F(0, g) \quad \text{and} \quad B = F'(0, g) \cdot \sqrt{\frac{a}{g}}.
\]

(17)

Then equation (16) can be written

\[
\Phi(s, g) = A \cdot \sqrt{\frac{g}{a}} + B \cdot \sqrt{\frac{g}{a}}.
\]

(18)

Applying the inverse Laplace transform to equation (18) in tables [13], we obtain:

\[
L_x^{-1}\{ \Phi(s, g) \} = F(x, g) = A \cdot ch \left( \sqrt{\frac{g}{a}} x \right) + B \cdot sh \left( \sqrt{\frac{g}{a}} x \right),
\]

if you consider that

\[
ch \left( \sqrt{\frac{g}{a}} x \right) = \frac{e^{\sqrt{\frac{g}{a}} x} + e^{-\sqrt{\frac{g}{a}} x}}{2} ; \quad sh \left( \sqrt{\frac{g}{a}} x \right) = \frac{e^{\sqrt{\frac{g}{a}} x} - e^{-\sqrt{\frac{g}{a}} x}}{2},
\]

(20)

then, taking into account (20), we will write

\[
F(x, g) = A \cdot ch \left( \sqrt{\frac{g}{a}} x \right) + B \cdot sh \left( \sqrt{\frac{g}{a}} x \right) = A_1 \cdot e^{\sqrt{\frac{g}{a}} x} + B_1 \cdot e^{-\sqrt{\frac{g}{a}} x},
\]

(21)

where \( A_1 = \frac{A + B}{2} ; \quad B_1 = \frac{A - B}{2} \).

To solve \( T(x, g) \), substitute the value from (13) in equation (21):

\[
T(x, g) - \frac{u_2}{g} = A \cdot ch \left( \sqrt{\frac{g}{a}} x \right) + B \cdot sh \left( \sqrt{\frac{g}{a}} x \right).
\]

(22)

We define the constants \( A \) and \( B \) after the Laplace transform from the boundary conditions:

\[
x = 0 : \quad T(0, g) = \frac{u_1}{g} = \frac{u_2}{g} + \frac{b}{g^2} \]

\[
x = \ell : \quad T' (\ell, g) = -h \left[ T(\ell, g) - \frac{u_2}{g} \right].
\]

(23)

For \( x = 0 \), from equations (22) and (23), we have the following:

\[
T(0, g) = \frac{u_2}{g} + A \cdot 1 = \frac{u_2}{g} + \frac{b}{g^2}.
\]
From here

\[ A = \frac{b}{g^2}. \]  

(24)

When \( x = \ell \) we get:

\[ A \cdot \sqrt[4]{\frac{g}{a}} \left[ \sqrt{\frac{g}{a}} \ell + B \cdot \sqrt[4]{\frac{g}{a}} \ell h \right] = -h \left[ A \cdot \sqrt[4]{\frac{g}{a}} \ell + B \cdot \sqrt[4]{\frac{g}{a}} \ell h + \sqrt[4]{\frac{g}{a}} \ell - \frac{u_2}{g} \right]. \]  

(25)

From (25) we define \( B \):

\[ B = \frac{b}{g^2} \left[ \sqrt[4]{\frac{g}{a}} \ell + h \sqrt[4]{\frac{g}{a}} \ell h \right]. \]  

(26)

Thus, the solution for image (22) will look like this:

\[ T(x, g) = \frac{u_2}{g} - \frac{b}{g^2} \sqrt[4]{\frac{g}{a}} x \left[ \sqrt[4]{\frac{g}{a}} x - \frac{b \cdot \sqrt[4]{\frac{g}{a}} x^2}{g^2 \left( \sqrt[4]{\frac{g}{a}} x + h \sqrt[4]{\frac{g}{a}} x h \right)} \right]. \]  

(27)

In order to get the solution of equation (27) in the original, consider each of the three components of the right side of the equation separately. In this case, for the first term, write:

\[ \sqrt[4]{\frac{g}{a}} x = \sqrt[4]{\frac{g}{a}} K [14]: \]

\[ \frac{1}{\sqrt{\left( 1 + \frac{1}{4} x \right)^n}} e^{\frac{1}{4} x} = L \left[ (4t)^{1/2} \cdot I^n \cdot e^{-\frac{x}{2\sqrt{at}}} \right], \]  

(29)

where \( I^n \cdot e^{\zeta} = \int_{-\infty}^{\infty} e^{-\zeta} d\zeta \) — integral of function \( e^{\zeta} \) at \( z = \frac{x}{2\sqrt{at}} \); \( \text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{z} e^{-\zeta^2} d\zeta \) — the Gauss error function [15].

In this case, for the first term on the right side of equation (27), we get:

\[ L^{-1} \left[ \frac{b}{g^2} \sqrt[4]{\frac{g}{a}} x \right] = \frac{2b}{2} \left[ 4t \cdot I^2 \cdot e^{\frac{x}{2\sqrt{at}}} \right]. \]  

(30)

Taking into account that [13]:

\[ 0.5 = \left( \frac{\sqrt[4]{\frac{g}{a}} \ell}{\sqrt{\frac{g}{a}} \ell h} \right)^2 \]
Let's define the roots of the polynomial standing in the denominator does not contain a free term, that is, the condition of the generalized decomposition theorem is met and it can be used to move the image (33) to the solution for the original:

\[ L^{-1} \left[ \varphi_1(g) \right] = \sum_{n=1}^{n} \varphi_1(g_n) \cdot e^{g_n t}, \]  

where \( g_n \) – roots of the polynomial \( \psi_1(g) \).

Let's define the roots of \( \psi_1(g) \) to do this, equate the denominator polynomial (33) to zero:
\[ \psi_1(g) = g \left( \sqrt[3]{\frac{g}{a}} \cosh \left( \sqrt[3]{\frac{g}{a}} \ell + h \right) \right) = g \left( \frac{\mu}{\ell} \cosh \mu + h \right) = 0. \]  

(35)

Receive:
1) \( g_0 = 0 \) – simple root;
2) \( g_n = -\frac{a}{\ell^2} \mu_n^2 \) – innumerable roots defined from the equation \((35)\), where \( \mu = i \sqrt[3]{\frac{g}{a}} \).

From equation \((35)\) we get:
\[ \text{ctg} \mu = -\frac{B_i}{\mu}, \]

(36)

where \( B_i = h \cdot \ell \).

Applying the inverse Laplace transform using the decomposition theorem \((34)\), we define \( \psi_1'(g) \):
\[ \psi_1'(g) = \frac{3}{2} \left( \sqrt[3]{\frac{g}{a}} \cosh \left( \sqrt[3]{\frac{g}{a}} \ell + h \right) \right) = \frac{1}{2} \frac{g \ell}{\sin^2 \mu} - \frac{1}{2} h. \]

(37)

In equation \((37)\), the expression in parentheses is zero based on equality \((35)\). In this regard, we have:
\[ \lim_{g \to 0} \psi_1'(g) = -\frac{1}{2} h; \]

and
\[ \lim_{g \to g_n} \psi_1'(g) = -\frac{1}{2} \left( \frac{g_n \cdot \ell}{\sin^2 \mu_n} - h \right) = \frac{1}{2} \frac{\mu_n^2}{\ell^2 \sin^2 \mu_n} - h = -\frac{\mu_n^2 + B_i \sin^2 \mu_n}{2 \ell^2 \sin^2 \mu_n}; \]

Define \( \varphi_1(g) \) for different roots:
\[ \varphi_1(0) = \frac{b}{a} x; \quad \varphi_1(g_n) = \frac{b}{a} \frac{\sinh \sqrt[3]{\frac{g_n}{a}}}{\sqrt[3]{\frac{g_n}{a}}} = \frac{b \cdot \ell}{a} \frac{\sin \mu_n}{\mu_n}. \]

Finally, for the second expression \((27)\) and \((33)\) we have:
\[ L^{-1}_{\psi_1'(g)} \left[ \frac{\varphi_1(g)}{\psi_1(g)} \right] = -\frac{b x}{a h} - 2 \frac{b \cdot \ell^2}{a} \sum_{n=1}^{\infty} \frac{\sin \mu_n}{\mu_n^3 + \mu_n \cdot B_i \sin^2 \mu_n} \cdot e^{-\frac{\mu_n^2 a t}{\ell^2}} = -\frac{b x}{a h} - 2 \frac{b \cdot \ell^2}{a} \sum_{n=1}^{\infty} \frac{\sin \mu_n}{\mu_n (B_i + B_i^2 + \mu_n^2)} \cdot \exp \left( -\frac{\mu_n^2 a t}{\ell^2} \right). \]

(38)

The third term of the right part of the equation \((27)\) is defined similarly to the second term \((33)\):
Using the decomposition theorem (40) for the expression,”
\[ g^2 \left( \frac{1}{h} \sqrt{\frac{g}{a}} + th \frac{g}{a} \right) = g^2 \left( \frac{1}{h} \sqrt{\frac{g}{a}} \right) \frac{a}{h} + th \frac{g}{a} = g^2 \left( \frac{1}{h} \sqrt{\frac{g}{a}} \right). \]  

(39)

From equation (41) we get:

\[ f(t) = \frac{1}{(k-1)!} \lim_{g \to \infty} \left[ \frac{\varphi_2(g)(g-a)^k}{\psi_2(g)} e^{igt} \right]. \]

(40)

Let us define the roots by equating the denominator of the expression (39) to zero:

\[ g^2 \left( \frac{1}{h} \sqrt{\frac{g}{a}} + th \frac{g}{a} \right) = 0; \quad \frac{1}{h} \sqrt{\frac{g}{a}} + th \frac{g}{a} = \mu \cdot ctg \mu + B_i = 0. \]

(41)

Receive:

1) \( g_0 = 0 \) – double root;

2) \( g_n = - \frac{a}{\ell^2} \mu_n^2 \) – innumerable roots defined from the equation (41), where \( \mu = i \sqrt{\frac{g}{a}} \).

From equation (41) we get:

\[ ctg \mu = - \frac{B_i}{\mu}, \]

(42)

where \( B_i = h \cdot \ell \).

Using the decomposition theorem (40) for a two-fold zero root, we obtain:

\[ \lim_{g \to 0} \left[ d \left( \frac{\varphi_2(g)}{\psi_2(g)} e^{igt} \right) \right] = \lim_{g \to 0} \left( t \cdot e^{igt} \frac{\varphi_2(g)}{\psi_2(g)} + e^{igt} \frac{\varphi_2'(g)}{\psi_2(g)} - e^{igt} \frac{\varphi_2(g) - \psi_2'(g)}{\psi_2(g)} \right) = t \cdot h \cdot b \cdot x. \]

(43)

because \( \varphi_2(0) = bx; \varphi_2'(0) = 0; \varphi_2(0) = \frac{1}{h}; \varphi_2'(0) = 0. \)

For the other roots, we use the decomposition theorem (34). For this purpose, we define \( \psi_2'(g_n) \) and \( \varphi_2(g_n) \):
The analytical equation that determines the distribution of moisture and nutrients in the capillaries of wood materials according to the linear filtration law and after conversion will finally have the following form:

\[ u(x,t) - u_2 = b \left[ R(1 - x) + \frac{x}{a} \left( 1 + \frac{2}{h} \right) \right] + \sum_{n=1}^{\infty} A_n \left( \frac{\ell}{a \cdot h \mu_n} + \frac{a \cdot h \cdot \mu_n}{\ell} \right) \cdot \sin \left( \frac{\mu_n x}{\ell} \right) \cdot \exp \left( -\mu_n^2 \frac{at}{\ell^2} \right). \]  

(46)

where \( A_n = \frac{2B_i}{\mu_n \left( B_i^2 + \mu_n^2 \right)} \cdot (-1)^{n+1} \) – the initial amplitude of filtration of moisture and nutrients when considered in the initial time period \( (t = 0) \) and when it changes linearly at the capillary boundaries; \( B_i = h \cdot \ell \) – the Bio criterion (characterizes the mass exchange of humidity with the environment); \( F_0 = \frac{a \cdot t}{\ell^2} \) – the Fourier criterion (characterizes the process of pure filtration of moisture and nutrients along the length of capillaries); \( \mu_n \) – the root of the characteristic equation; \( \ell \) – the considered length of capillaries of wood materials; \( t \) – the time; \( u \) – moisture in the capillaries of the wood material, which moves under the action of the created motion gradients; \( b \) – constant coefficient.

From the analysis of equation (46), it follows that the last term of the sum is a converging series, that is, an algebraic sum with a gradually fading amplitude \( A_n \cdot \exp(-F_0 \mu_n^2) \), which decreases both with increasing \( \mu_n \) and over time (more precisely, the criterion \( F_0 = \frac{at}{\ell^2} \)). Therefore, moisture and nutrients along the length of the capillaries of wood materials \( \ell \) are a function of time and obey a linear law according to the accepted boundary conditions (6). In the case of negligible values of the
series (the last term of the mathematical model (46)), a quasi-stationary mode will be observed for the field of moisture gradients and nutrients in the capillaries of wood materials.

4. Conclusion
So, as a result of the theoretical research, a mathematical model (46) has been obtained, allowing us to determine the field of moisture distribution and nutrients \( u(x,t) - u_2 \) due to their filtration transport in the capillaries of woody materials at any time of the research.

The determination of filtration mass transfer in woody materials could be an important factor in the growth and development of woody materials, as well as a factor for monitoring the environmental condition from the technogenic impact of vehicles on forest plantation located along highways. The conducted theoretical research is of scientific interest for forest science and forest ecological systems.

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