Checking Flavour Models at Neutrino Facilities

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Abstract

In the recent years, the industry of model building has been the subject of the intense activity, especially after the measurement of a relatively large values of the reactor angle. Special attention has been devoted to the use of non-abelian discrete symmetries, thanks to their ability of reproducing some of the relevant features of the neutrino mixing matrix. In this paper, we consider two special relations between the leptonic mixing angles, arising from models based on $S_4$ and $A_4$, and study whether, and to which extent, they can be distinguished at superbeam facilities, namely T2K, NOνA and T2HK.
1 Introduction

The recent measurement of a non-vanishing $\theta_{13}$ by Daya Bay \cite{1} and RENO \cite{2} has exerted some pressure on models for neutrino mixing based on the permutation groups (like $A_4$ and $S_4$, \cite{3}), as they are generically constructed to give at leading order very specific patterns in which $\theta_{13} = 0$ and the other angles are also completely fixed. Corrections from the charged sector or next-to-leading contributions to the neutrino mass matrix have to be invoked to correct such patterns and make the models compatible with the experimental data. The usual approach to model building is that of considering a Lagrangian invariant under a flavour group $G$ and to subsequently break $G$ into two different subgroups in the charged lepton and neutrino sector, is such a way to create two different rotations, responsible for a non-diagonal Pontecorvo-Maki-Nakagawa-Sakata ($U_{PMNS}$) mixing matrix. The structure of $G$ can also be reconstructed from the residual symmetries of the mass matrices after symmetry breaking; for example, using the criterion that a flavour group should be obtained from the neutrino mixing matrix without parameter tuning, it was shown in \cite{4} that the minimal group containing all the symmetries of the neutrino mass matrix and leading to the tri-bimaximal mixing (TBM \cite{5}) is $S_4$. The fact that the mixing angles are fixed to well defined values is the consequence of forcing all the symmetries of the mass matrix to belong to $G$. Moving from this consideration, in \cite{6} a different point of view was adopted: they assumed that the residual symmetries in both the charged lepton and neutrino sectors are one-generator groups. Indicating with $S_i$ and $T_\alpha$ ($\alpha = e, \mu, \tau$) the generators of the $Z_2$ and $Z_m$ discrete symmetries of the neutrino and charged leptons mass matrices, the previous condition implies that $\{S_i, T_\alpha\}$ form a set of generators for the flavor group $G$ for given $i$ and $\alpha$, with the meaning that all other symmetries appear accidentally. The structure of the generators is restricted by the additional requirements to be elements of $SU(3)$, for which $Det[S_i] = Det[T_\alpha] = 1$, so they can be written as:

\[
\begin{align*}
S_1 &= \text{diag}(1, -1, -1), \quad S_2 = \text{diag}(-1, 1, -1), \quad S_3 = \text{diag}(-1, -1, 1) \\
T_e &= \text{diag}(1, e^{2\pi i k/m}, e^{-2\pi i k/m}), \quad T_\mu = \text{diag}(e^{2\pi i k/m}, 1, e^{-2\pi i k/m}), \\
T_\tau &= \text{diag}(e^{2\pi i k/m}, e^{-2\pi i k/m}, 1).
\end{align*}
\]

The definition of $G$ requires a relation linking $S_i$ and $T_\alpha$, assumed to be

$(S_i T_\alpha)^p = (U_{PMNS} S_i U_{PMNS}^T T_\alpha)^p = I$. The lack of additional symmetry in $G$ has the direct consequence that the mixing angles are not all fixed (like in the TBM) but rather present some interesting correlations, or sum rules, that open the possibility to reconcile the predictions of the permutation groups with the experimental data already at leading order (see also \cite{7} for similar sum rules obtained in the context of $S_4$ and \cite{8} for sum-rules from residual $Z_2$ symmetries). The question we want to analyze in this paper is whether such correlations can be tested at neutrino facilities or, in other words, if model comparison and selection can be achieved at currently taking data or planned superbeams. It is clear that if two models live in completely different regions of the parameter space (given by the spanned values of all $\theta_{ij}$ and the leptonic CP phase) the measurement of the mixing parameters with huge precision will give the answer; however, we are still away from such an idealized situation, at least for what concerns the CP phase, and it is necessary to evaluate the performance of the neutrino facilities to face this problem. In this respect, we have selected two models from \cite{6}, called $1T$ and $2T$, which have been shown to be compatible with the current experimental data in the neutrino sector and with the hypothesis of TBM, and have used their different correlations to compute and compare (in a $\chi^2$ analysis) the expected event rates at T2K, NO$\nu$A and T2HK, with the aim of identifying the regions in the ($\theta_{13}, \delta$)-plane.
where the models can be distinguished at some confidence level. An interesting work along similar lines has been recently presented in [9], where the main focus was on the ability of next-generation of neutrino oscillation experiments to constraints correlations involving \( \theta_{23} \), \( \theta_{13} \) and \( \cos \delta \). We differ from [9] in that we consider different neutrino facilities, we use non-linear relations between the oscillation parameters and adopt a different statistical analysis with the purpose, given the lack of information on the CP phase, to present exclusion regions directly in the \((\theta_{13}, \delta)\) parameter space. It is important to stress again that such correlations are leading order predictions, in the sense that they are derived from group theoretical considerations and do not take into account possible higher order effects into the lepton mass matrices of new-physics effects [10], otherwise model-dependent features will appear with the main effect to spoil the sum rules and introduce additional indetermination of the parameter spaces where the models live. We do not take into account this possibility, as we are mainly interested to check whether the easiest case (validity of the sum rules) can be addressed at neutrino experiments. We revise the useful neutrino transition probabilities in Sect.2, where we also introduce the models 1\(T\) and 2\(T\) and discuss the parameter spaces allowed by the correlations; in Sect.3 we introduce the neutrino facilities used in our numerical simulation and discuss the results of the statistical analysis performed to distinguish the models. Our conclusions are drawn in Sect.4.

2 Setting the background

2.1 The relevant transition probabilities

Since we are interested in the performance of superbeam facilities, it is enough to consider the \( \nu_\mu \rightarrow \nu_e \) appearance and \( \nu_\mu \rightarrow \nu_\mu \) disappearance probabilities (and their CP-conjugate). Given the relatively large \( \theta_{13} \), we consider the probabilities up to first order in the small parameter \( r = \Delta m^2_{\mathrm{sol}} / \Delta m^2_{\mathrm{atm}} \sim 0.03 \) [11] while keeping their exact dependence on \( \theta_{13} \). In vacuum they read:

\[
P_{\mu e} = \sin^2 2\theta_{13} s^2_{23} \sin^2 \Delta - r \left[ \Delta s^2_{12} \sin^2 2\theta_{13} s^2_{23} \sin 2\Delta \right. \\
+ \Delta \sin 2\theta_{12}s_{13} c^2_{13} \sin 2\theta_{23}(-2 \sin \delta_{\mathrm{CP}} \sin^2 \Delta + \cos \delta_{\mathrm{CP}} \sin 2\Delta) \bigg], \tag{2}
\]

\[
P_{\mu\mu} = 1 - \sin^2 \Delta \left[ c^4_{13} \sin^2(2\theta_{23}) + s^2_{23} \sin^2(2\theta_{13}) \right] + r \left\{ \Delta \sin 2\Delta \left( c^2_{13} \left( \sin^2(2\theta_{23}) \left( c^2_{12} - s^2_{12}s^2_{13} \right) - 4s_{13} \cos \delta \sin 2\theta_{12} \right) \sin^3(\theta_{23}) \cos(\theta_{23}) \right) \\
+ s^2_{12}s^2_{23} \sin^2(2\theta_{13}) \right\}, \tag{3}
\]

where \( \Delta \equiv \frac{\Delta m^2_{\mathrm{sol}}}{4E_{\nu}} \), \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). Notice that:

\[
P_{\alpha\bar{\beta}} = P_{\alpha \beta}(\delta_{\mathrm{CP}} \rightarrow -\delta_{\mathrm{CP}}) \tag{4}
\]

\[
P_{\beta\alpha} = P_{\alpha \beta}(\delta_{\mathrm{CP}} \rightarrow -\delta_{\mathrm{CP}}), \quad \alpha, \beta = e, \mu, \tau. \tag{5}
\]

As it is well known, \( P_{\mu e} \) is mainly dependent of \( \theta_{13} \) and \( \delta \) whereas \( P_{\mu\mu} \) is recognized to be more sensitive to the atmospheric parameters; although the dependence on \( \delta \) is suppressed by the small \( r \), the approximation adopted shows that \( \theta_{13} \) already appears at leading order. We then expect that flavour models with different parameter spaces, that is with the mixing parameters living in different regions, are also characterized by different transition probabilities that, extracted from the experimental data, can help in distinguishing among them. In
our numerical computations we consider the mixing angles to vary within the $2\sigma$ intervals taken from [12]:

\begin{align*}
\sin^2 \theta_{23} &= 0.386^{+0.062}_{-0.038} \\
\sin^2 \theta_{13} &= 0.0241^{+0.0049}_{-0.0048} \\
\sin^2 \theta_{12} &= 0.307^{+0.035}_{-0.032},
\end{align*}

whereas the CP phase is left free to vary in the whole $[0, 2\pi)$ range. We consider the mass differences as constant quantities, $\Delta m_{31}^2 = 2.4 \times 10^{-3} \text{eV}^2$, $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{eV}^2$, since the models studied in this paper do not give any information on the neutrino masses.

2.2 A summary of the models 1$T$ and 2$T$

In this section we recall the main features of the correlations arising from two different models discussed in [6], of which we also follow the nomenclature. Both of them have $T_\alpha = T_e$. The first model, called 1$T$, uses the generator $S_1 = \text{Diag}(1, -1, -1)$ and the pair of values $(p, m) = (4, 3)$, which corresponds to the group $S_4$. The obtained relations among the mixing angles are:

\begin{align*}
\cos^2 \theta_{12} &= \frac{2}{3 \cos^2 \theta_{13}}, \\
\tan 2\theta_{23} &= \frac{1 - 5s_{13}^2}{2 \cos \delta s_{13} \sqrt{2(1 - 3s_{13}^2)}},
\end{align*}

also obtained in the explicit model of Ref. [13] and further studied in [14].

For any values of $\theta_{13}$, the first relation always gives an acceptable value of the solar angle, within the $2\sigma$ bounds quoted in eq.(6), so this relation does not set any restriction on the reactor angle. It has to be noted, however, that the dependence on the cosine function forces $\sin^2 \theta_{12}$ to be around $0.31 - 0.32$, very close to the current central value. On the other hand, eq.(8) imposes a constraint on the possible pairs of $(\theta_{13}, \delta)$ needed to fulfill the bounds for $\theta_{23}$ in eq.(6): in particular, the value of the CP phase can never be maximal in this model and, in order to have an atmospheric angle in the first octant, the relation $\cos \delta > \pi/2$ must hold. The exact bounds in the $(\theta_{13}, \delta)$-plane can be derived numerically from eq.(8) and are shown in Fig.(1) where, as expected, no restriction on the reactor angle is present and the CP phase is limited in a horizontal band above maximal violation.

The other model considered in our analysis is called 2$T$, which uses the generator $S_2 = \text{Diag}(-1, 1, -1)$ and the pair of values $(p, m) = (3, 3)$, which corresponds to the group $A_4$. In this model the mixing angles and the CP phase are related by the following relations:

\begin{align*}
\sin^2 \theta_{12} &= \frac{1}{3 \cos^2 \theta_{13}}, \\
\tan 2\theta_{23} &= \frac{1 - 2s_{13}^2}{\cos \delta s_{13} \sqrt{2 - 3s_{13}^2}}.
\end{align*}

The previous relations set important constraints on the reactor angle and the CP phase. In fact, given the bounds on $\theta_{12}$, the reactor angle is restricted to be $\sin^2 \theta_{12} \lesssim 0.025$, that is in a region where $\sin^2 \theta_{12} \sim 0.34$, still compatible with the range in eq.(6). In addition, the allowed range for the atmospheric angle restricts $\delta$ to be below the maximal value. The situation is illustrated again in Fig.(1), where we clearly see that the resulting parameter space in the $(\theta_{13}, \delta)$-plane for this model is quite different from that of the 1$T$ model.
Equipped with the correlations of eqs. (7)-(8) and (9)-(10), we can now eliminate the dependence on $\theta_{12}$ and $\theta_{23}$ in the transition probabilities and get the expressions for the various $P_{\alpha\beta}$ as predicted by the models, $P_{\alpha\beta}^{1T,2T}$. The resulting formulae are quite cumbersome, so we prefer not to show complicated analytical results that can hide the physical content of the probabilities. It is useful, instead, to study the differences:

$$\Delta P_{\alpha\beta} = |P_{\alpha\beta}^{1T} - P_{\alpha\beta}^{2T}|,$$

which give information on where to expect the largest differences among the two models. To make life easier, we assume the same $\theta_{12}$ for the two models, which is a good approximation implied by the two models, so a strong improvement in the measurement of the solar angle could be enough to distinguish among $1T$ and $2T$. On the other hand, there is a large overlap in the allowed $\sin^2 \theta_{23}$, due to the still relatively large uncertainty affecting the determination of this angle. In principle, a very precise measurement of $\sin^2 \theta_{23}$ with central value well below $\sim 0.39$ can tell the two models but this possibility seems at the moment quite disfavored.
because the intervals of $\theta_{12}^{1T}$ and $\theta_{12}^{2T}$ are contained in the 2σ uncertainties quoted in eq.(6), and to work in the intervals for $\theta_{13,23}$ where the models overlap; we take, however, different CP phases, called $\delta_1$ if referred to the model 1T and $\delta_2$ if referred to 2T. We get:

$$\Delta P_{\mu e} \sim \frac{8}{3} \sqrt{2} \rho \Delta \sin \theta_{13} \sin \Delta \left(\frac{\delta_1 - \delta_2}{2}\right) \sin \left[ \frac{1}{2} (2 \Delta + \delta_1 + \delta_2) \right]$$

$$\Delta P_{\mu\mu} \sim \frac{2}{3} \sqrt{2} \rho \Delta \sin \theta_{13} \sin 2\Delta \left(\cos \delta_2 - \cos \delta_1 \right).$$

A common feature of the $\Delta P_{\alpha\beta}$'s is the leading dependence on $\theta_{13}$; given that sin $\theta_{13}$ varies of about 17% in the range quoted in [6], we expect only a minor effects of $\theta_{13}$ in distinguishing the models, more pronounced for values close to the rightmost bounds of the regions in Fig.[1]. The dependence on the phases is, conversely, very significant. In the case of the $\nu_\mu \rightarrow \nu_\mu$ appearance channel, $\Delta P_{\mu e}$ is sensibly different from zero for $\delta_1 - \delta_2 \sim \pi$; this is not exactly the quantity separating the models under investigation since, as seen in Fig.[1], $\delta_1 - \delta_2 \in [0.6, 2.4]$; however, this range of values guarantees that $\Delta P_{\mu e} \neq 0$ and then that the two models can be, at least in principle, distinguished. For the $\nu_\mu$ disappearance channel we observe that, for $\delta_1 \sim \pi/2 + \delta_2$ (whose approximate validity can be appreciated again from Fig.[1]), we obtain $\cos \delta_2 - \cos \delta_1 = \cos \delta_2 + \sin \delta_2 > 0$, because $\delta_3 \lesssim \pi/4$; thus the phase dependence of $\Delta P_{\mu\mu}$ results in the addition of two positive contributions and can be relevant in distinguishing the models.

As a final remark, we want to stress that our considerations remain valid for neutrino facilities where matter effects are small, which is the case of interest in this paper.

3 Models at long baseline neutrino experiments

Having discussed the parameter space of the two models in an experimental-independent way, we now turn to the question on whether long baseline neutrino facilities will be able to tell model 1T from model 2T based on the measurement of $P_{\alpha\beta}$'s and the CP-conjugate transitions. Our previous considerations on probabilities are drawn from analytical expressions in vacuum. However, in studying the performance of a given experimental setup, one must take into account the experimental efficiencies to detect a given neutrino flavour. The expected number of events are computed according to:

$$N_{\beta} = N \int_{E_{\nu}} dE_\nu P_{\alpha\beta}(E_\nu) \sigma_\beta(E_\nu) \frac{d\phi_\alpha}{dE_\nu}(E_\nu) \varepsilon_\beta(E_\nu)$$

$$\tilde{N}_{\beta} = \tilde{N} \int_{E_{\nu}} dE_\nu P_{\bar{\alpha}\bar{\beta}}(E_\nu) \sigma_{\bar{\beta}}(E_\nu) \frac{d\phi_\bar{\alpha}}{dE_\nu}(E_\nu) \varepsilon_{\bar{\beta}}(E_\nu),$$

in which $\sigma_{\beta(\bar{\beta})}$ is the cross section for producing the lepton $\beta(\bar{\beta})$, $\varepsilon_{\beta(\bar{\beta})}$ the detector efficiency to reveal that lepton, $\phi_{\alpha(\bar{\alpha})}$ the initial neutrino flux at the source and $N, \tilde{N}$ are normalization factors containing the detector mass and the number of years of data taking. These events can also be grouped in neutrino energy bins, thus taking full advantage of different spectral information of $P_{\alpha\beta}$ or $P_{\bar{\alpha}\bar{\beta}}$. Since the probabilities we are interested in are $P_{\mu e}$ and $P_{\mu\mu}$, $\beta$ can be an electron or a muon (and their antiparticles).

In order to assess the capabilities of a given facility to tell 1T from 2T we adopt the following strategy:
• for any pair of the mixing parameters \((\bar{\theta}_{13}, \bar{\delta})\) in the regions allowed for the model \(2T\) we compute the expected number of events \(N_{a,i}^{2T}(\bar{\theta}_{13}, \bar{\delta})\) for a given final flavour \(\alpha\) and neutrino energy bin \(i\) \((\theta_{12}\) and \(\theta_{23}\) are then determined from eqs.\((9)-(10))\);

• we then compare \(N_{a,i}^{2T}(\bar{\theta}_{13}, \bar{\delta})\) to \(N_{a,i}^{1T}(\theta_{13}, \delta)\), where now the mixing parameters are those of the competing model \(1T\). In this procedure, we are implicitly assuming that the model \(2T\) and the pair \((\bar{\theta}_{13}, \bar{\delta})\) are the one chosen by Nature;

• in the next step, we minimize the following \(\chi^2\) variable over \(\theta_{13}\) and \(\delta\) in the \(1T\) allowed parameter space \([15]\):

\[
\chi^2 = \sum_{\alpha,i} \frac{\left[ N_{a,i}^{1T}(\theta_{13}, \delta) - N_{a,i}^{2T}(\bar{\theta}_{13}, \bar{\delta}) \right]^2}{\sigma_{a,i}^2},
\]

where the uncertainty is given by:

\[
\sigma_{a,i}^2 = N_{a,i}^{2T}(\bar{\theta}_{13}, \bar{\delta}) + B_{a,i} + (n_{a} N_{a,i}^{2T}(\bar{\theta}_{13}, \bar{\delta}))^2 + (b_{a} B_{a,i})^2,
\]

in which \(B_{a,i}\) is the background associated to \(N_{a,i}^{2T}(\bar{\theta}_{13}, \bar{\delta})\), \(n_{a}\) the overall systematic error related to the determination of \(N_{a,(\beta)}\), and \(b_{a}\) that of \(B_{a,i}\). For the sake of simplicity, \(n_{a}\) and \(b_{a}\) are constant in the whole energy range;

• if the obtained minimum is larger than some reference \(\chi^2\) value, \(\chi^2_{\text{min}} \geq \chi^2_{\text{cut}}\), then in the point \((\bar{\theta}_{13}, \bar{\delta})\) the two models can be distinguished at a given confidence level. The ensemble of such points identifies the wanted regions.

Obviously, the procedure can also be applied in the reverse order, that is considering \(1T\) as the true model and finding a minimum of the \(\chi^2\) function in the \(2T\) parameter space. The results will then be presented in the \(1T\) \((\theta_{13}, \delta)\)-plane.

In the following numerical simulations, we will proof our strategy at three different experimental setups: NO\(\nu\)A, T2K and T2HK. All events rates are computed using exact numerical probabilities in matter.

### 3.1 Results from NO\(\nu\)A \(\oplus\) T2K and T2HK

In this section, we briefly consider the experimental setups used in our numerical simulations.

• the NO\(\nu\)A detector \([16]\) is a 14 kt totally active scintillator detector (TASD), located at a distance of 810 km from Fermilab, with an off-axis angle of 0.8° from the NuMI beam. In the appearance mode \([17]\), the main backgrounds are due to the intrinsic beam \(\nu_e/\bar{\nu}_e\), mis-identified muons and single \(\pi^0\) events from neutral current interactions. In the disappearance mode \([18]\), we have to consider wrong-sign muon from \(\bar{\nu}_\mu\) \((\nu_\mu)\) contamination in \(\nu_\mu(\bar{\nu}_\mu)\) beam and neutral current events. Due to the relatively large \(\theta_{13}\), the collaboration has relaxed the cuts for the event selection criteria, allowing for more signal events along with more background events \([19]\). Our simulation is mainly based on the files provided by the GLoBES software \([20, 21]\), with migration matrices from \([22]\) and kindly provided by one of the authors of \([23]\). In this way, the signal and backgrounds events released by the NO\(\nu\)A Collaboration are reproduced \([16]\). For the sake of simplicity, we take all systematics effects at the level of 5%, that is \(n_{a} = b_{a} = 0.05\) for \(\alpha = e^-, e^+, \mu^-, \mu^+\).
For fixed $\alpha$ and $\beta$, the energy dependence of the probabilities $P^{1T}_{\alpha\beta}$ and $P^{2T}_{\alpha\beta}$ is quite different so, at least in principle, the facility could be efficient in discriminating the two models. In Fig. (3) we show both $P^{1T}_{\alpha\beta}$ (in light gray) and $P^{2T}_{\alpha\beta}$ (in dark gray) as a function of the neutrino energy, obtained varying all the mixing parameters in the respective allowed ranges, with $(\alpha, \beta) = (\mu, e)$ in the left panel and $(\alpha, \beta) = (\mu, \mu)$ in the right one. The solid line is a down-scaled version of the $\nu_\mu$ NO$\nu$A flux. Beside the large fluctuations of the probabilities below $E_\nu \lesssim 1$ GeV, which are less important due to the smallness of the neutrino flux, in both cases $P_{\mu e}$ and $P_{\mu \mu}$ show a different behavior close to the maximum of the flux, that is in an energy region where NO$\nu$A will collect the bulk of the events. In particular, in the appearance channel the spread we observe for the 1T model is mainly a consequence of a larger uncertainty on $\theta_{23}$, which reaches values smaller than $\sin^2 \theta_{23} \sim 0.39$ and then makes $P^{1T}_{\mu e} \lesssim P^{2T}_{\mu e}$ close to the pick. A smaller atmospheric angle also means a larger $\nu_\mu$ disappearance, so in the left plot we have $P^{1T}_{\mu e} \gtrsim P^{2T}_{\mu e}$ for energies above 1 GeV.

- for the T2K we consider the Super-Kamiokande water Cerenkov detector of fiducial mass of 22.5 kt, placed at a distance of 295 km from the source beam from J-PARC, at an off-axis angle of 2.5°. Our numerical simulation have been performed based to the information provided in the corresponding GLoBES files, described in [21, 24], to which we refer for details.

The appearance channels in T2K show an even increased capability to distinguish among $P^{1T}_{\mu e}$ and $P^{2T}_{\mu e}$: in fact, $P^{1T}_{\mu e}$ is generally smaller than $P^{2T}_{\mu e}$ for energies at and below the maximum of the T2K flux, thus making the prediction of the two models significantly different, see Fig. (4). On the other hand, for the disappearance channel we do not observe such a huge difference and we do not present the corresponding plot.

- for the T2HK setup we follow the proposal and the Letter of Intent presented in [25], with a WC detector with a fiducial mass of 560 kton, placed at a distance of 295 km from the source. We assume again $n_\alpha = b_\alpha = 0.05$.

It is clear that NO$\nu$A and T2K, taken individually, have the potential to make some sort of discrimination among the 1T and 2T models which, however, strongly depends on the assumed values of $n_\alpha$, so NO$\nu$A and T2K can say something relevant only in a limited portion of the parameter space. In particular, we have found that no distinction is possible.

![Figure 3: Range of values of $P^{1T,2T}$ (left panel) and $P^{1T,2T}_{\mu\mu}$ (right panel) as a function of the neutrino energy, for the NO$\nu$A setup. The solid line is a down-scaled version of the $\nu_\mu$ NO$\nu$A flux, in arbitrary units.](image-url)
Figure 4: Range of values of $P_{\mu e}^{1T,2T}$ as a function of the neutrino energy, for the T2K setup. The solid line is a down-scaled version of the $\nu_\mu$ T2K flux, in arbitrary units.

if we assume that $2T$ is the correct model, for any value of $n_\alpha$. On the other hand, under the assumption that $1T$ gives the values of the mixing parameters chosen by Nature and $n_\alpha = 0.05$, a limited discrimination is possible for those points in the $(\theta_{13}, \delta)$-plane with the largest possible values of the CP-phase, in agreement with our discussion below eq. (11). This can be seen in Fig. (5) where we show the results of our computation in the $(\theta_{13}, \delta)$-plane allowed for the $1T$ model, in the case on NO$\nu$A (left plot) and T2K (right plot) experimental setups. In both plots, the points above the solid lines, $\delta \gtrsim 2.06$, identify the region where the two models can be distinguished at the 90% of confidence level, using both appearance and disappearance channels. As expected, the capability of the considered facilities to distinguish

Figure 5: Regions in the $1T$ parameter space where the $1T$ and $2T$ models can be distinguished at 90% confidence level, using the appearance and disappearance channels. Left plot: for the NO$\nu$A setup. Right plot: for the T2K setup.

the two models is almost independent on the value of $\theta_{13}$, as emphasized in the previous section. For values of $n_\alpha$ as large as 10% no distinction is possible. The sensitivities are the results of a strong synergy among the appearance and disappearance channels; in fact, we have observed that:

- the appearance channel alone cannot give any useful information, since the sensitivity
lines lie above the maximum values of $\delta$ in the $1T$ parameter space;

• the $\nu_\mu \rightarrow \nu_\mu$ transition alone does not allow any discrimination among $1T$ and $2T$, given the mild dependence in $P_{\mu\mu}$ on $\theta_{13}$ and $\delta$, see eq. (3). However, when used in combination with the $\nu_\mu \rightarrow \nu_e$ channel, the disappearance transition sorts some effects, due to the ability of measuring $\theta_{23}$ whose allowed ranges are slightly different in the two models. Although we fixed the solar and mass differences to their best fit values quoted in [12], the inclusion of the uncertainty on $\Delta m^2_{31}$ (and, to a less extent, the one from $\Delta m^2_{21}$) does not change appreciably the regions where confusion is avoided, mainly due to the relatively small error on $\Delta m^2_{31}$ at the $2\sigma$ level, around 4-5% for both NO$\nu$A and T2K facilities.

A different situation arises if we combine the simulated data from both experiments. The most interesting feature is that a (reduced) region in the $2T$ parameter space appears where the two models can be distinguished. It involves values of $\delta$ no larger than 0.2, and only for values of the reactor angle close to its upper bound. In the $1T$ parameter space, we observe only a modest improvement with respect to the case of Fig. 5 due to the fact that the $\chi^2$ functions of the two setups are very similar in the portion of the parameter space considered, so that no powerful synergy is at work when combining the data. The different sensitivities observed in the $1T$ and $2T$ ($\theta_{13}, \delta$)-plane are easily understood in terms of intrinsic clones [26], that is in terms of points in the parameter space with the same number of expected events. Consider first the $2T$ model; the minimum of the $\chi^2$ in eq. (13) is expected to appear close to the points where the system of equations:

$$
N_{\mu,i}^{2T}(\bar{\theta}_{13}, \bar{\delta}) = N_{\mu,i}^{1T}(\theta_{13}, \delta)
$$

$$
N_{e,i}^{2T}(\bar{\theta}_{13}, \bar{\delta}) = N_{e,i}^{1T}(\theta_{13}, \delta)
$$

has a solution for $(\theta_{13}, \delta) \neq (\bar{\theta}_{13}, \bar{\delta})$. A numerical scan of the pairs $(\bar{\theta}_{13}, \bar{\delta})$ in the $2T$ parameter space, performed using the total event rates, has shown that many points with small $\bar{\theta}_{13}$ have a mirror in the $1T$ plane at values close to the smaller allowed $\delta$ and large $\theta_{13}$. Such $(\bar{\theta}_{13}, \bar{\delta})$
pairs are then not good to perform a discrimination. Changing $1T \leftrightarrow 2T$ into eq.\ref{eq:15} produces very similar results, in the sense that the region that was before the mirror region is now made of the $(\bar{\theta}_{13}, \bar{\delta})$ pairs in the $1T$ parameter space where discrimination is not possible (as they have clones located in the $2T$ space at small $\theta_{13}$). For the T2K setup, these regions (black areas) are presented in Fig.\ref{fig:7}. Taking into account that the solution of
\begin{equation}
\label{eq:15}
\end{equation}
only give an indicative position of the clone points and that NO\nu A has roughly the same $(L/E_\nu)$ as T2K, Fig.\ref{fig:7} shows many of the features of the allowed regions presented in Fig.\ref{fig:6} the distinction of the models in the $1T$ space happens at the largest possible values of $\delta$ and that in the $2T$ can happen only at large $\theta_{13}$.

For the T2HK setup, we get a much better capability of distinguishing the models, Fig.\ref{fig:7}; in fact, in both $1T$ and $2T$ parameter spaces the regions where confusion is possible (at 99% and 99.9% CL) are confined into thin stripes close to the lower ($1T$) and upper ($2T$) bounds, thus making this facility quite appropriate for model selection. The good performance with respect to the T2K setup has to be ascribed to the interplay between a larger detector mass and the use of the antineutrino modes. In particular, we have verified that the inclusion of the antineutrino mode into the analysis is crucial to get the sensitivities shown in Fig.\ref{fig:8} which, otherwise, would be a rescaled version of the T2K results shown in the right panel of Fig.\ref{fig:5} in the $1T$ parameter space, and a reduced sensitivity (for small $\delta$ and large $\theta_{13}$) in the $2T$ parameter space.

A summary of the previous considerations is presented in Tab.\ref{tab:1} where, for each of the facilities and combination analyzed above, we reported our estimates of the range of values of the CP phase where distinction is possible among the $1T$ and $2T$ models. These ranges are obviously modulated by $\theta_{13}$ (and in the table we use ”upper bound” to indicate the upper border of the $1T$ allowed parameter space), so they represent indicative intervals.

4 Conclusions

Starting from two different neutrino mixing sum rules we have studied if, and to which extent, NO\nu A, T2K and T2HK are able to falsify one of them in favor of the other. This is due to the fact that the two sum rules identify different set of values of the neutrino mixing
Figure 8: Regions in the $1T$ parameter space (left panel) and $2T$ parameter space (right panel) where the two models under investigation can be distinguished at 99% confidence level (solid line) and 99.9% confidence level (dashed line), in the case of the T2HK experimental setup.

|                  | NOνA                  | T2K                  | NOνA + T2K             | T2HK (99% CL)  |
|------------------|-----------------------|----------------------|------------------------|----------------|
| $1T$             | [2.06, upper bound]   | [2.06, upper bound]  | [2, upper bound]       | [1.83, upper bound] |
| $2T$             | -                     | -                    | [0, 0.1] for large $\theta_{13}$ | [0, 1]         |

Table 1: Estimates of range of values of $\delta$ where distinction is possible among the $1T$ and $2T$ models for the facilities analyzed in this paper. "Upper bound" refers the upper border of the allowed region for the $1T$ model. Dashes indicate that no discrimination is possible.

parameters, namely different regions in the CP phase $\delta$ and $\theta_{12}$ and partially overlapping regions for $\theta_{13}$ and $\theta_{23}$, all of them compatible with the experimental values at 2σ. Analytical considerations on the $\nu_{\mu} \rightarrow \nu_e$ and $\nu_{\mu} \rightarrow \nu_{\mu}$ transition probabilities revealed that distinguishing the two type of correlations is possible for large differences among the true values (chosen in one parameter space) and the fitted values (in the competing parameter space) of $\delta$. Our numerical simulations have shown that this is indeed the case; in particular, NOνA and T2K taken alone have the capabilities to tell the $1T$ model from the $2T$ model at 90% of confidence level, reducing the portion in the $(\theta_{13}, \delta)$-plane of the $1T$ model where confusion is possible. In the $2T$ parameter space we revealed a much worse performance, unless the combination of NOνA + T2K data is taken into account, and only in a very limited region at large $\theta_{13}$ and small $\delta$. On the other hand, the T2HK experimental facility, taking full advantage of a larger detector mass and of the use of the $\bar{\nu}_{\mu}$ flux compared to the T2K setup, has a much better performance in terms of model selection in both parameter spaces, leaving aside only a small portion of values of $\delta$ where confusion is still possible. These small regions disappear if we consider the setup of the NF10, thus making this facility useful to perform a selection of sum rules modified by the inclusion of various type of next-to-leading order effects.
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