Black composite M-branes

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Abstract

We generalise all the known supersymmetric composite M-branes to the corresponding black configurations. Thermodynamical formulae is written by using the simple rules to construct these black branes.
1 Introduction

In the past year string theory has addressed a longstanding problem in black hole physics, the statistical origin of black hole entropy. In particular, by interpreting certain BPS saturated black holes carrying R-R charges as Dirichlet branes [1], it has been possible to provide a microscopic explanation of the Bekenstein-Hawking entropy [2–6]. The existence of supersymmetric black holes with finite horizon area in four [7, 8, 9] and five [10, 11] dimensions has been crucial in these calculations. The entropy and Hawking temperature of near extremal black holes have also been successfully calculated [3, 11, 12].

During the same period M-theory has been a subject of intensive research. The effective field theory limit of this conjectured theory is 11D supergravity. Remarkably, it seems that the different superstring theories and corresponding duality symmetries have an eleven-dimensional origin [13–16]. A related issue is the eleven-dimensional origin of the supersymmetric brane solutions of type IIA superstring theory [17]. Therefore it is important to study the supersymmetric brane solutions of 11D supergravity, not only to try to understand what M-theory may or may not be, but also to find new supersymmetric brane solutions of superstring theories. The first step in this direction was given by Townsend and Papadopoulos [18] who reinterpreted some of the brane solutions found by Güven [19] as orthogonal intersections of basic membranes [20]. They also found the corresponding magnetic duals interpreted as orthogonal intersections of the basic 5-branes [19]. Tseytlin [21] (see also [22, 23]) has extended this work and formulated a general rule, the harmonic function rule, to construct orthogonal intersecting branes with the basic 2 and 5 branes as its constituents. Some configurations can be boosted along a common string to all branes [21] and/or superposed with a Kaluža-Klein (KK) monopole [21] yielding, upon dimensional reduction to $D = 10$, brane solutions with charges arising from the R-R 2-form field strength of type IIA superstring theory. Intersecting configurations with the $\frac{1}{2}$ supersymmetric $(2 \subset 5)$-brane [25] among its constituents were found in [24].

The black hole solutions of lower dimensional theories may be interpreted as reductions of extreme and non-extreme composite M-branes. One of the challenges of M-theory is to provide the statistical counting of the Bekenstein-Hawking entropy of these black holes from an eleven dimensional perspective [22, 26–28]. It is therefore important to have a complete classification of the black composite M-branes and corresponding black hole solutions of lower dimensional theories [29, 30]. The aim of this paper is to carry on such a program by considering the non-extreme configurations associated with the new extreme solutions found in [24].

This paper is organised as follows. In section two we start by presenting the black $(2 \subset 5)$-brane solution and by reviewing the composite M-brane rules. We then state the rules to construct the black composite M-branes. In section three we will give the eleven-dimensional interpretation of black hole solutions in $D$-
dimensional spacetime for $4 \leq D \leq 10$. Finally, in section four we discuss the thermodynamics of these black hole solutions.

2 Black M-branes from extreme M-branes

Through this paper we will be considering bosonic solutions of 11D supergravity whose action is

$$I_{11} = \frac{1}{2} \int d^{11}x \sqrt{-g} \left[ R - \frac{1}{24!} F^2 \right] + \frac{1}{12} \int F \wedge F \wedge A,$$

(2.1)

where $F = dA$ and $A$ is a 3-form field. In this section we will describe the method to construct the most general black composite M-branes from the corresponding extreme configurations. We will start by presenting the non-extreme generalisation of the $(2 \subset 5)$-brane solution. The composite M-brane rules needed for our purposes will then be reviewed. We will follow the notation of [24].

2.1 Black $(2 \subset 5)$-brane

Nearly all of the new supersymmetric configurations found in [24] had $(2 \subset 5)$-branes among its constituents. Even though the corresponding Chern-Simons term in the action (2.1) is non-vanishing those configurations solve the equations of motion that follow from this action. It turns out that there is also a generalisation of the $\frac{1}{2}$ supersymmetric $(2 \subset 5)$-brane to the corresponding non-extreme configuration. This anisotropic black $p$-brane is described by ($5 \leq p \leq 7$)

$$ds^2 = \left( \bar{H} \bar{H} \right)^{\frac{3}{2}} \left[ H^{-1} (-f dt^2 + dy_1^2 + dy_2^2) + \bar{H}^{-1} (dy_3^2 + dy_4^2 + dy_5^2) + dy_6^2 + ... + dy_p^2 + f^{-1} dr^2 + r^2 d\Omega_{9-p}^2 \right],$$

$$\mathcal{F}_{(2\subset 5)} = \mathcal{F}_{(2)} + \mathcal{F}_{(5)} - \tan \zeta \ d \left( \bar{H}^{-1} - 1 \right) \wedge \xi,$$

(2.2)

where $\ast \mathcal{F}_{(2)} = Q(\epsilon_{9-p} \wedge \eta)$, $\mathcal{F}_{(5)} = P(\mu \wedge \epsilon_{9-p})$, $f = 1 - \frac{\mu}{r^{p-1}}$, $H = 1 + \frac{\eta}{r^{9-p}}$ and $\bar{H} = \sin^2 \zeta + \cos^2 \zeta \ H$. The dual operation in the expression for $\mathcal{F}_{(2)}$ is defined for the metric with $\cos \zeta = 0$. $\epsilon_{9-p}$ is the unit $(9-p)$-sphere volume form. $\mu = dy_6 \wedge ... \wedge dy_p$ and $\eta = dy_3 \wedge dy_4 \wedge dy_5 \wedge \mu$ are the volume forms on the relative transverse spaces of the 5-brane ($\mathcal{M}^{p-5}_{(5)}$) and 2-brane ($\mathcal{M}^{p-2}_{(2)}$), respectively. $\xi = dy_3 \wedge dy_4 \wedge dy_5$ is the volume form on the space $\mathcal{M}^3_{(5/2)}$ spanned by the vectors that are tangent to the 5-brane’s worldvolume but are orthogonal to the 2-brane’s worldvolume. If $\cos \zeta = 0$ we obtain the black 2-brane solution (with $p - 2$ extra flat directions) and if $\sin \zeta = 0$ the black 5-brane solution (with $p - 5$ extra flat
directions). If \( \mu = 0 \) we obtain the extreme supersymmetric configuration. The electric charge is defined by \( Q = \frac{1}{V_{(2)}} \int_{\Sigma} \ast F \), where \( V_{(2)} \) is the volume of \( M_{(2)}^{p-2} \) and \( \Sigma = S^{9-p} \times M_{(2)}^{p-2} \) is an asymptotic spacelike hypersurface. The magnetic charge is defined by \( P = \frac{1}{V_{(5)}^{p-5}} \int_{\Sigma} F \) with \( \Sigma = M_{(5)}^{p-5} \times S^{9-p} \) and \( V_{(5)}^{p-5} \) the volume of \( M_{(5)}^{p-5} \). These charges are given by

\[
\left( \frac{Q}{(8-p) A_{9-p}} \right)^2 = \alpha (\mu + \alpha) \sin^2 \zeta ,
\]

\[
\left( \frac{P}{(8-p) A_{9-p}} \right)^2 = \alpha (\mu + \alpha) \cos^2 \zeta .
\]

where \( A_{9-p} \) is the volume of the unit \((9-p)\)-sphere.

Two remarks are in order. Firstly, this solution has a non-vanishing Chern-Simons term. This fact slightly complicates the corresponding field equations. It also shows that the black \( p \)-brane solutions found in \cite{29} are not the most general solutions. In fact, in \cite{29} (and in the related work \cite{31}) a \( D \)-dimensional Lagrangian derived by reduction of the 11\( D \) supergravity bosonic Lagrangian was used \cite{32}. The Chern-Simons term in the action (2.1), as well as the Chern-Simons type modifications to the field strengths due to the dimensional reduction process were then assumed to vanish. Secondly, the existence of this solution may be argued as a consequence of the \( SL(2, \mathbb{R}) \) electromagnetic duality of \( D = 8, N = 2 \) supergravity. This can be done by starting with the (electrically or magnetically charged) black membrane solution of the latter theory and by following the same steps that led to the construction of the extreme \((2 \subset 5)\)-brane solution \cite{25}.

### 2.2 Extreme M-brane rules

As mentioned at the end of the previous subsection, black \( p \)-brane solutions can be obtained by performing a consistent truncation of the 11\( D \) supergravity bosonic Lagrangian to \( D \)-dimensions \cite{29}. Not only this approach as the disadvantage of enforcing the constraint that the Chern-Simons terms have to vanish, but also it is much more complicated due to the presence of a much larger number of fields. The eleven-dimensional approach is simpler, does not impose any constraint and provides an immediate interpretation of the solutions in terms of compositions of M-branes. Because the non-extreme composite M-branes can be easily obtained

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1We use the same Greek letter to denote the volume form on the space \( M_{(5)}^{p-5} \) and the non-extremality parameter. It should be obvious from the context which one we mean.

2We remark that from the 4-form field equation, the conserved electric charge is in fact given by \( Q = \frac{1}{V_{(2)}} \int_{\Sigma} (\ast F + \frac{1}{2} A \wedge F) \). In the cases where the Chern-Simons term in this integral is non-vanishing as it is the case for this solution, we will always choose a gauge such that it vanishes at spatial infinity and therefore the charge is still given by the expression in the text.
from the corresponding one-centered extreme solutions we will briefly review the composite M-brane rules.

The rules to construct configurations of $N \frac{1}{2}$ supersymmetric M-branes are:

(i) To each basic $q_i$-brane we assign an harmonic function $H_i$ on the overall transverse space. If the coordinate $y$ belongs to several constituents $q_i$-branes ($q_1, ..., q_n$) then its contribution to the metric written in the conformal frame where the overall transverse space is 'free' is $H_i^{-1} ... H_n^{-1} dy^2$. The contribution to the conformal factor of the $i$-th $q_i$-brane is $H_i^{-1}$. The 4-form field strength is given by

$$F = \sum_{i=1}^{N} F_{(q,i)},$$

where

$$\ast F_{(2,i)} = Q_i (\epsilon_{9-p} \wedge \eta_i), \quad F_{(5,i)} = P_i (\mu_i \wedge \epsilon_{9-p}),$$

whether the $i$-th brane is a 2 or 5-brane, respectively. In the former case, $\eta_i$ is the volume form on the i-th 2-brane relative transverse space $M_{p-2}^{(2,i)}$ and the electric charge is defined by $Q_i = \frac{1}{V_{p-2}^{(2,i)}} \int_{\Sigma} \ast F_{(2,i)}$ with $\Sigma = S^{9-p} \times M_{p-2}^{(2,i)}$ and $V_{p-2}^{(2,i)}$ the volume of $M_{p-2}^{(2,i)}$. In the latter case, $\mu_i$ is the volume form on $M_{p-5}^{(5,i)}$ and the magnetic charge is defined by $P_i = \frac{1}{V_{p-5}^{(5,i)}} \int_{\Sigma} F_{(5,i)}$ with $\Sigma = M_{p-5}^{(5,i)} \times S^{9-p}$ and $V_{p-5}^{(5,i)}$ the volume of $M_{p-5}^{(5,i)}$. Both the electric and magnetic charges are given by

$$\frac{Q_i}{(8-p) A_{9-p}}, \quad \frac{P_i}{(8-p) A_{9-p}} = \pm \alpha_i.$$  

(ii) $q$-branes of the same type can intersect orthogonally over $(q-2)$-branes. A 2-brane can intersect orthogonally a 5-brane over a string.

(iii) If there are $(2 \subset 5)$-branes among the $N$ intersecting constituents then in the cases where these branes reduce to the basic 2 or 5-branes the resulting configuration should be compatible with the previous rules (i) and (ii).

The 4-form field strength is given by

$$F = \sum_{i=1}^{N} F_{(q,i)},$$

where $F_{(q,i)}$ is given either by one of the expressions in (2.5) or by

$$F_{(2 \subset 5,i)} = F_{(2,i)} + F_{(5,i)} - \tan \zeta_i \ H_i^{-1} - 1 \wedge \xi_i.$$  

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3There is an extra minus sign if $y$ is the time coordinate $t$.
4The sign of the third term in (2.8) depends on how we distribute the constituent M-branes. We will always make the minus sign choices.
The dual operations are defined for \( \cos \zeta_i = 0 \) and \( \xi_i \) is the volume form on the space \( \mathcal{M}^3_{(5/2,i)} \). The electromagnetic charges are defined as before. They are given by

\[
\frac{Q_i}{(8-p)A_{9-p}} = \alpha_i \sin \zeta_i, \quad \frac{P_i}{(8-p)A_{9-p}} = \alpha_i \cos \zeta_i, \quad (2.9)
\]

where \( \cos \zeta_i = 0 \) or \( \sin \zeta_i = 0 \) if the i-th brane is a basic 2 or 5-brane, respectively.

Some configurations can be boosted along a common string to all branes \([21]\) and/or superposed with a KK monopole \([24]\). For black configurations we will consider a variation of this fourth rule. The last rule is concerned with the amount of preserved supersymmetry and it will not be used here.

### 2.3 Black composite M-brane rules

One-center supersymmetric compositions of M-branes can be used to construct the corresponding non-extreme configurations. These configurations may be seen as anisotropic black \( p \)-branes and can be built according to the following rules:

**(i)** Perform the following replacements in the metric \([29, 30]\)

\[
dt^2 \to f dt^2 \\
dx^i dx_i \to f^{-1} dr^2 + r^2 d\Omega_{9-p}^2,
\]

where \( i = 1, ..., 10 - p \) and \( f = 1 - \frac{\mu}{r^{8-p}} \) with \( r^2 = x^i x_i \).

**(ii)** The 4-form field strength ansatz remains the same with the charge redefinitions

\[
\left( \frac{Q_i}{(8-p)A_{9-p}} \right)^2 = \alpha_i^2 \sin^2 \zeta_i \to \alpha_i (\mu + \alpha_i) \sin^2 \zeta_i, \\
\left( \frac{P_i}{(8-p)A_{9-p}} \right)^2 = \alpha_i^2 \cos^2 \zeta_i \to \alpha_i (\mu + \alpha_i) \cos^2 \zeta_i,
\]

where \( \cos \zeta_i = 0 \) or \( \sin \zeta_i = 0 \) when the corresponding i-th brane is a 2 or 5-brane, respectively.

Notice that rule (ii) above has been given in \([29]\) for the cases where \( \cos \zeta_i = 0 \) or \( \sin \zeta_i = 0 \). As explained before the more general cases considered here allow configurations with non-vanishing Chern-Simons terms.

In some configurations we can add KK charges yielding, upon dimensional reduction to \( D = 10 \), branes with charges arising from the R-R 2-form field strength of type IIA superstring theory. Correspondingly, the Kaluza-Klein reduction of
11D supergravity yields $N = 2A, D = 10$ supergravity. The reduction of the bosonic fields is performed by writing

$$
\begin{align*}
 ds^2 &= g_{MN} dx^M dx^N = e^{-\frac{2}{3} \phi} g_{mn} dx^m dx^n + e^{-\frac{4}{3} \phi} (dx^{10} - A_m dx^m)^2, \\
 A_{MNP} &= A_{mnp}, \quad A_{MN10} = B_{mn}, \quad \text{with} \quad M, N, P = 0, \ldots, 9.
\end{align*}
$$

(2.12)

Unless stated capital letters range from 0 to 10 and in the lower case from 0 to 9. The rule to generate black branes with KK charges is:

(iii) If the corresponding extreme configuration has a common string to all constituent M-branes, say along $y$, then a KK electric charge can be added by applying the boost transformation \[30\]

$$
\begin{align*}
 t &\rightarrow \left(1 + \frac{\alpha}{\mu}\right)^{\frac{1}{2}} t \mp \left(\frac{\alpha}{\mu}\right)^{\frac{1}{2}} y, \\
y &\rightarrow \mp \left(\frac{\alpha}{\mu}\right)^{\frac{1}{2}} t + \left(1 + \frac{\alpha}{\mu}\right)^{\frac{1}{2}} y,
\end{align*}
$$

(2.13)

where the $\mp$ sign choice will correspond to positive or negative KK charge, respectively. The $g_{tt}$ and $g_{yy}$ elements of the black $p$-brane are then transformed to

$$
-f dt^2 + dy \rightarrow -H^{-1} ft^2 + H \left(dy \mp \sqrt{\frac{\alpha (\mu + \alpha)}{r^{8-p} + \alpha}} dt\right)^2,
$$

(2.14)

where $H = 1 + \frac{\alpha}{r^{8-p}}$. Compactifying along the $y$ direction we have a black composite brane with a 0-brane among its constituents.

If the overall transverse space has dimension bigger than three, a magnetic monopole can be added by making all the harmonic functions $^5$ to depend only on three of this space coordinates and performing the substitution (we start with an anisotropic black $p$-brane)

$$
\begin{align*}
 f^{-1} dr^2 + r^2 d\Omega_{9-p}^2 &\rightarrow dy_{p+1}^2 + \ldots + dy_6^2 \\
 + H^{-1} \left(dy_7 \pm \sqrt{\alpha (\mu + \alpha)} \cos \theta d\phi\right)^2 + H (f^{-1} dr^2 + r^2 d\Omega_2^2),
\end{align*}
$$

(2.15)

where $H = 1 + \frac{\alpha}{r^p}$, $f$ is transformed to $f = 1 - \frac{\mu}{r}$ and the $\pm$ sign choice will correspond to positive or negative KK charge, respectively. Compactifying along the $y_7$ direction we have a black composite brane with a 6-brane among its constituents.

There are cases where we can add both electric and magnetic KK charges and a further compactification is required.

\[5\text{We refer to } H_i \text{ as harmonic functions even though they are no longer harmonic functions of the non-extreme metrics.}\]
We remark that this rule, for both electric and magnetic KK charges, may be recast in the corresponding dimensionally reduced theories in a form similar to the rule (ii) above (for \( \cos \zeta_i = 0 \) or \( \sin \zeta_i = 0 \)).

### 3 Black holes in lower dimensions

In Table I we present the most general black M-brane configurations that reduce to black holes in lower dimensions after dimensional reduction. Consider first black holes in four dimensions with \( n = 4 \). The black \( 2 \perp 2 \perp 5 \perp 5 \) and \( 5 \perp 5 \perp 5 + \text{boost} \) branes were presented in [30] and the other two follow from the KK monopole rule stated in section 2.3. All these solutions have been obtained from a consistent reduction to four dimensions of the 11D theory [29]. Here we present the corresponding eleven dimensional interpretation. The most general \( D = 4 \) configurations with \( n = 3 \) are also presented. In order to avoid an excessive lengthy exposition and since it is straightforward to construct these solutions by using the rules of the previous section we will just present the black

| \( D \) | \( n \) | Black composite M-brane |
|-------|-----|------------------------|
| 4     | 4   | \( 2 \perp 2 \perp 5 \perp 5 \) \( 5 \perp 5 \perp 5 + \text{boost} \) \( 2 \perp 2 \perp 2 + \text{KK monopole} \) \( 2 \perp 5 + \text{boost} + \text{KK monopole} \) |
| 5     | 3   | \( (2 \subset 5) \perp (2 \subset 5) \perp (2 \subset 5) \) \( (2 \subset 5) \perp 5 \perp \text{boost} \) \( (2 \subset 5) \perp 2 + \text{KK monopole} \) \( (2 \subset 5) + \text{boost} + \text{KK monopole} \) |
| 5     | 2   | \( 2 \perp 2 \perp 2 \) \( 2 \perp 5 + \text{boost} \) \( (2 \subset 5) \perp 2 \) |
| 6     | 2   | \( (2 \subset 5) + \text{boost} \) \( 2 \perp 2 \) |
| 7     | 2   | \( 2 \perp 2 \) \( 2 + \text{boost} \) |
| 8, 9  | 2   | \( 2 + \text{boost} \) |
| 10    | 1   | \( \text{boost} \) |
\[(2 \subset 5) \perp (2 \subset 5) \perp (2 \subset 5)\) brane solution. It is described by

\[
\begin{align*}
    ds^2 &= \left( \prod_{i=1}^{3} (H_i \tilde{H}_i)^4 \right) \left[ - (H_1 H_2 H_3)^{-1} f dt^2 + (H_1 \tilde{H}_3)^{-1} dy_1^2 \right. \\
    & \quad \left. + (H_1 \tilde{H}_2)^{-1} dy_2^2 + (H_2 \tilde{H}_1)^{-1} dy_3^2 + (H_2 \tilde{H}_3)^{-1} dy_4^2 + (H_3 \tilde{H}_2)^{-1} dy_5^2 \right. \\
    & \quad \left. + (H_3 \tilde{H}_1)^{-1} dy_6^2 + (\tilde{H}_1 \tilde{H}_2 \tilde{H}_3)^{-1} dy_7^2 + f^{-1} dr^2 + r^2 d\Omega_2^2 \right],
\end{align*}
\]

\[
\mathcal{F} = \sum_{i=1}^{3} \left( \mathcal{F}_{(2,i)} + \mathcal{F}_{(5,i)} \right) - \tan \zeta_i \left( \tilde{H}_i^{-1} - 1 \right) \wedge \xi_i, \tag{3.1}
\]

where \(*\mathcal{F}_{(2,i)} = Q_i (\epsilon_2 \wedge \eta_i)\) and \(\mathcal{F}_{(5,i)} = P_i (\mu_i \wedge \epsilon_2)\). The electric and magnetic charges are given by

\[
\begin{align*}
    \left( \frac{Q_i}{\Delta_2} \right)^2 &= \alpha_i (\mu + \alpha_i) \sin^2 \zeta_i, \tag{3.2} \\
    \left( \frac{P_i}{\Delta_2} \right)^2 &= \alpha_i (\mu + \alpha_i) \cos^2 \zeta_i,
\end{align*}
\]

with \(i = 1, 2, 3\). Four-dimensional solutions with \(n \leq 2\) can be obtained by allowing some of the charges of the corresponding configurations in table I to vanish.

The eleven-dimensional interpretation of black hole solutions in \(D\) spacetime dimensions for \(5 \leq D \leq 10\) is also described in table I. There are solutions that have the same eleven-dimensional interpretation but reduce to black hole solutions in different spacetime dimensions. The reason is that the corresponding harmonic functions have a different dependence on the overall transverse space. Solutions with \(n = 1\) can be obtained by allowing all charges but one to vanish.

### 4 Thermodynamics

Remarkably, toroidal compactification of all black \(p\)-branes of M-theory along the branes spatial directions yields the following \((11 - p)\)-dimensional Einstein’s frame metric

\[
\begin{align*}
    ds^2 &= -\lambda^{3-D}(r) f dt^2 + \lambda(r) \left[ f^{-1} dr^2 + r^2 d\Omega_{D-2}^2 \right], \tag{4.1} \\
    \lambda(r) &= \left( \prod_{i=1}^{n} H_i(r) \right)^{\frac{1}{12}}.
\end{align*}
\]

Notice that for configurations involving the eleven-dimensional \((2 \subset 5)\)-brane the metric still reduces to this simple form. However, the dilaton field as well as the field strengths will be much more complicated than in the other cases.
The ADM mass and the electric and magnetic charges of this black hole solutions are given by

\[
\frac{2M}{A_{D-2}} = (D - 2)\mu + (D - 3)\sum_{i=1}^{n} \alpha_i, \\
\left(\frac{Q_i}{(D-3)A_{D-2}}\right)^2 = \alpha_i (\mu + \alpha_i) \sin^2 \zeta_i, \\
\left(\frac{P_i}{(D-3)A_{D-2}}\right)^2 = \alpha_i (\mu + \alpha_i) \cos^2 \zeta_i,
\]

where \(\cos \zeta_i = 0\) or \(\sin \zeta_i = 0\) when the corresponding electric or magnetic charge does not originate from the eleven-dimensional \((2 \subset 5)\)-brane. Labelling all the charges by \(Q_i\) (electric, magnetic and KK in origin) and defining for the originally \((2 \subset 5)\)-brane case an electromagnetic charge \(Q_i\) by

\[
Q_i^2 = P_{2\subset5,i}^2 + P_{2\subset5,i} \mu + (D-3) \sum_{i=1}^{n} \alpha_i,
\]

the ADM mass can be written as

\[
\frac{2M}{(D-3)A_{D-2}} = \lambda \mu + \sum_{i=1}^{n} \left(\frac{\mu}{2}\right)^2 + \left(\frac{Q_i}{(D-3)A_{D-2}}\right)^2,
\]

where \(\lambda = \frac{D-2}{D-3} - \frac{n}{2}\).

We now write thermodynamical formulae. Let us start by calculating the Hawking temperature. Write the \(g_{tt}\) and \(g_{rr}\) metric elements as \(g_{tt} = -M(r)f\) and \(g_{rr} = L(r)f^{-1}\). We then have \(\frac{L(r)}{M(r)} = \prod_{i=1}^{n} H_i(r)\). Performing the coordinate transformation \(\rho^2 = f(r)\), analytically continuing to Euclidean spacetime and examining the behaviour of the metric (4.1) in the vicinity of the horizon \(r = \mu^{\frac{1}{D-3}}\), this temperature is seen to be

\[
T_H = \frac{D-3}{4\pi} \mu^{-\frac{1}{D-3} + \frac{n}{2} \prod_{i=1}^{n} (\mu + \alpha_i)^{-\frac{1}{2}}}. \\
\]

We can write the Hawking temperature as a function of the physical charges \(Q_i\)

\[
T_H = \frac{D-3}{4\pi} \mu^{-\frac{1}{D-3} + \frac{n}{2} \prod_{i=1}^{n} \left(\mu + \alpha_i\right)^{-\frac{1}{2}} + \left|Q_i\right|}.
\]

In the extremal limit \(\mu \to 0\) we have

\[
T_H \to \frac{D-3}{4\pi} \mu^{-\frac{1}{D-3} + \frac{n}{2} \prod_{i=1}^{n} \left(\frac{|Q_i|}{(D-3)A_{D-2}}\right)^{-\frac{1}{2}}}.
\]
For \( n = 1 \) and \( D = 4 \) the temperature diverges as \( \mu \to 0 \). For \( n = 2 \) and \( D = 4 \), or \( n = 1 \) and \( D = 5 \) we have \( T_H \to \frac{D-3}{4\pi} \prod_{i=1}^{\frac{n}{2}} \left( \frac{|Q_i|}{(D-3)A_{D-2}} \right)^{-\frac{1}{2}} \). For \( n > 2 \) and \( D = 4 \), \( n > 1 \) and \( D = 5 \), or \( 6 \leq D \leq 10 \) the temperature converges to zero.

The Bekenstein-Hawking entropy of the black hole solutions described by (4.1) is given by \( \frac{A_H}{4G_D} \), where \( A_H \) is the horizon area, \( G_D \) the \( D \)-dimensional Newton’s constant and in our units \( 4G_D = (2\pi)^{-1} \). This entropy is given by

\[
S_{BH} = 2\pi A_{D-2} \mu^\lambda \prod_{i=1}^{\frac{n}{2}} (\mu + \alpha_i)^\frac{1}{2}.
\]

(4.7)

In terms of the physical charges it is given by

\[
S_{BH} = 2\pi A_{D-2} \mu^\lambda \prod_{i=1}^{\frac{n}{2}} \left( \frac{\mu}{2} + \left( \frac{\mu}{2} \right)^2 + \left( \frac{Q_i}{(D-3)A_{D-2}} \right)^2 \right)^{\frac{1}{2}}.
\]

(4.8)

In the extremal limit we have

\[
S_{BH} \to 2\pi A_{D-2} \mu^\lambda \prod_{i=1}^{\frac{n}{2}} \left( \frac{|Q_i|}{(D-3)A_{D-2}} \right)^\frac{1}{2}.
\]

(4.9)

Extremal black holes with finite entropy are those for which \( \lambda = 0 \), i.e. \( D = 4 \) and \( n = 4 \), \( D = 5 \) and \( n = 3 \).

We now write the mass formula for our black hole solutions. Defining \( \Phi_H^i = \frac{1}{2(\text{D-3})A_{\text{D-2}}} \frac{1}{\mu + \alpha_i} \) to be the horizon electromagnetic potential due to the \( i \)-th brane (or the originally \( i \)-th \((2 \subset 5)\)-brane), we have

\[
M = \frac{D - 2}{D - 3} T_H S_{BH} + \sum_{i=1}^{\frac{n}{2}} \Phi_H^i Q_i.
\]

(4.10)

The differential form of this equation gives the first law of thermodynamics

\[
dM = T_H dS_{BH} + \sum_{i=1}^{\frac{n}{2}} \Phi_H^i dQ_i.
\]

(4.11)

In the cases where the charge \( Q_i \) arises from the \((2 \subset 5)\)-brane we can have \( dQ_i = 0 \) (\( dM = 0 \) and \( dS = 0 \)) while changing the composite branes electric and magnetic charges. It corresponds to a rotation of the electromagnetic charges.

\(^6\)The \( Q - M, T - Q \) and \( T - M \) plots for the case \( n = 1 \) were given in \[33\] in the context of KK electrically charged solutions. The case \( n = 2 \) and \( D = 4 \) exhibits the same features as the the case \( n = 1 \) and \( D = 5 \). All the other cases are similar to the cases with \( n = 1 \) and \( 6 \leq D \leq 10 \) (see \[33\] for some 3D plots).

\(^7\)The Newton’s constant in 11 and \( D \) dimensions are related by \( G_{11} = G_D L^p \), where \( L \) is the length of each of the \( p \) compact directions. We have taken \( 4G_{11} = (2\pi)^{-1} \) and \( L = 1 \).
To conclude, we have generalised all known supersymmetric M-branes to the corresponding black configurations. We also wrote the thermodynamical formulae that follows from the simple rules to construct the black composite M-branes. This way we have extended the previous work of Duff, Lü and Pope [29] and of Cvetič and Tseytlin [30].

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