Dynamic stability of viscoelastic rectangular plates with concentrated masses

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Abstract. Thin-walled constructions such as plates and shells, with installed units, devices and assemblies, are widely used in engineering and construction. In calculations, such attached elements are considered as concentrated at points and rigidly fixed elements. The influence of concentrated masses is taken into account in the equation of motion using the Dirac delta function. Recently, more and more attention has been paid to the nonlinear and inhomogeneous properties of a structure. Dynamic stability of viscoelastic orthotropic rectangular plates with concentrated masses in a geometrically nonlinear statement is considered in the paper. Using the Bubnov-Galerkin method, based on a polynomial approximation of deflections, the problem is reduced to solving a system of ordinary nonlinear integro-differential equations. The results of the problem are obtained by the proposed numerical method based on the use of quadrature formulas. Dynamic stability of viscoelastic rectangular plates with concentrated masses under various boundary conditions was studied over a wide range of changes in physico-mechanical and geometrical parameters of the plate.

1. Introduction

Plates and shells are widely used as the units of buildings, structures and various structure elements. Therefore, the analysis of the effect of additional masses on their dynamic behavior is of great importance. The role of such additional masses is often played by tie-plates, fastenings, stiffeners. In technical literature there are many publications devoted to the study of strength, vibrations and stability of plates, panels and shells carrying concentrated masses.

In [1], various methods for calculating eigenfrequencies of beams and plates with an arbitrary number of concentrated masses or springs were presented and compared.

The study in [2] was devoted to nonlinear forced vibrations of rectangular plates with a mass concentrated in the center.

Experimental and numerical studies of forced oscillations of rigidly fixed rectangular plates with concentrated masses were presented in [3].

The results of analytical and experimental studies of dynamic instability of hinged-supported rectangular plates with arbitrarily concentrated masses were given in [4]. The solution to the problem was obtained using the Bubnov-Galerkin method.
Based on the theory of shallow shells, the influence of a concentrated mass on shell frequency and mode of free vibrations was studied in [5]. To solve the problem, the Bubnov-Galerkin method was applied.

Designing and creating lightweight, durable and reliable structures made of composite materials requires solving more complex problems. It is known that most composite materials have pronounced viscoelastic properties [6,7]. Therefore, in engineering calculations, it becomes necessary to take into account the viscoelastic properties of the materials. But as a review of available literature shows, in the problems of viscoelastic systems behavior, either the Voigt differential model or the Boltzmann-Volterra integral model with a relaxation kernel in the form of an exponential kernel were used to solve the problems. But studies show that the Koltunov-Rzhanitsyn relaxation kernel describes the real processes occurring in the plates and shells at the initial moments of time [7].

An analysis of the free vibrations of composite cylindrical, spherical, and elliptical shells with cutouts and concentrated masses was presented in [8].

The development of computer technology has allowed the use of numerical methods to solve problems of hereditary theory of viscoelasticity and to expand the class of problems considered [9–14].

Earlier, the dynamic behavior of viscoelastic plates and cylindrical panels with concentrated masses was studied in [15–18].

But to obtain a more adequate pattern of the stress-strain state of thin-walled structural elements with concentrated masses, the studies must be carried out taking into account geometrical nonlinearity, viscoelastic and inhomogeneous properties of the material.

Dynamic stability of viscoelastic orthotropic rectangular plates with concentrated masses under rapidly growing compressive loads is studied in this paper.

2. Methods

Consider a viscoelastic orthotropic rectangular plate of constant thickness $h$ and sides $a$ and $b$ carrying concentrated masses $M_i$ at points with coordinates $(x_i, y_i)$, $i=1,2,...,I$. Let the plate undergo dynamic compression alongside a by force $P(t)=\nu t$ ($\nu$ - is the loading rate). Assume that the plate has initial deflections.

Under the accepted assumptions, taking into account the rapidly growing compressive force $P(t)\frac{\partial^2 w}{\partial x^2}$, the mathematical model of the problem with respect to displacements $u$, $v$ and deflection $w$ is described by the following system of integro-differential equations [18]

$$
\begin{align*}
B_{11} & \left(1 - R_{11}^*\right) \frac{\partial^2 u}{\partial x^2} + B_{12} \left(1 - R_{12}^*\right) \frac{\partial^2 v}{\partial y^2} + 2B \left(1 - R^*\right) \frac{\partial^2 w}{\partial x \partial y} - \\
& \left[ \rho + \frac{1}{h} \sum_{i=1}^{I} M_i \delta(x-x_i) \delta(y-y_i) \right] \frac{\partial^2 u}{\partial t^2} = 0, \\
B_{22} & \left(1 - R_{22}^*\right) \frac{\partial^2 v}{\partial y^2} + B_{21} \left(1 - R_{21}^*\right) \frac{\partial^2 u}{\partial x^2} + 2B \left(1 - R^*\right) \frac{\partial^2 w}{\partial x \partial y} - \\
& \left[ \rho + \frac{1}{h} \sum_{i=1}^{I} M_i \delta(x-x_i) \delta(y-y_i) \right] \frac{\partial^2 v}{\partial t^2} = 0,
\end{align*}
$$

$$
\frac{h^2}{12} \left[ B_{11} \left(1 - R_{11}^*\right) \frac{\partial^4 (w-w_0)}{\partial x^4} + 8B \left(1 - R^*\right) + B_{12} \left(1 - R_{12}^*\right) + B_{21} \left(1 - R_{21}^*\right) \frac{\partial^4 (w-w_0)}{\partial x^2 \partial y^2} + \\
B_{22} \left(1 - R_{22}^*\right) \frac{\partial^4 (w-w_0)}{\partial y^4} \right] - \frac{\partial}{\partial x} \left[ B_{11} \left(1 - R_{11}^*\right) \frac{\partial^2 w}{\partial x^2} + B_{12} \left(1 - R_{12}^*\right) \frac{\partial^2 w}{\partial y^2} + B_{21} \left(1 - R_{21}^*\right) \frac{\partial^2 w}{\partial x \partial y} + 2B \frac{\partial^2 w}{\partial y^2} \left(1 - R^*\right) \frac{\partial^2 w}{\partial x \partial y} \right] +
\end{align*}
$$
where $\Gamma_{11}^*, \Gamma_{12}^*, \Gamma_{21}^*, \Gamma_{22}^*, \Gamma^*$ - are the integral operators:

$$
\Gamma_{11}^* \phi(t) = \int_0^t \Gamma_{11}(t-\tau)\phi(\tau)d\tau, \quad \Gamma_{12}^* \phi(t) = \int_0^t \Gamma_{12}(t-\tau)\phi(\tau)d\tau, \\
\Gamma_{21}^* \phi(t) = \int_0^t \Gamma_{21}(t-\tau)\phi(\tau)d\tau, \quad \Gamma_{22}^* \phi(t) = \int_0^t \Gamma_{22}(t-\tau)\phi(\tau)d\tau, \\
\Gamma^* \phi(t) = \int_0^t \Gamma(t-\tau)\phi(\tau)d\tau,
$$

$\Gamma_{11}(t-\tau), \Gamma_{12}(t-\tau), \Gamma_{21}(t-\tau), \Gamma_{22}(t-\tau), \Gamma(t-\tau)$ – are the relaxation kernels in the case of uniaxial problem;

$$
B_{11} = \frac{E_1}{1-\mu_1\mu_2}, \quad B_{12} = -\frac{\mu_2E_1}{1-\mu_1\mu_2} = -\frac{\mu_1E_2}{1-\mu_1\mu_2}, \quad B_{22} = \frac{E_2}{1-\mu_1\mu_2}, \quad 2B = G,
$$

$E_1, E_2$ – are the linear elastic moduli, $G$ – is the shear modulus, $\mu_1, \mu_2$ are the Poisson’s ratios, $\rho$ is the density of the plate material; $q$ is the external static load.

Thus, a mathematical model of the problem in a geometrically nonlinear statement is described by integro-differential equations in partial derivatives of the form (1) under corresponding initial and boundary conditions.

The displacements $u(x,y,t), v(x,y,t)$ and the initial and total deflections $w(x,y,t)$ and $w_0(x,y)$ are sought in the form of an expansion in functions $\phi_{nm}(x,y), \varphi_{nm}(x,y), \psi_{nm}(x,y)$, satisfying corresponding boundary conditions

$$
uu(x,y,t) = \sum_{n=1}^N \sum_{m=1}^M u_{nm}(t)\phi_{nm}(x,y), \quad \nuv(x,y,t) = \sum_{n=1}^N \sum_{m=1}^M v_{nm}(t)\varphi_{nm}(x,y), \\
ww(x,y,t) = \sum_{n=1}^N \sum_{m=1}^M w_{nm}(t)\psi_{nm}(x,y),
$$

where $u_{nm} = u_{nm}(t), v_{nm} = v_{nm}(t), w_{nm} = w_{nm}(t)$ – are the unknown time functions; $\phi_{nm}(x,y), \varphi_{nm}(x,y), \psi_{nm}(x,y)$, $n = 1,2,\ldots,N$; $m = 1,2,\ldots,M$ are the coordinate functions satisfying the given boundary conditions of the problem.

Substituting (2) into system (1) and applying the Bubnov-Galerkin method, while introducing the following dimensionless quantities

$$
uu = \frac{mu}{h}, \quad \nuv = \frac{mv}{h}, \quad \ww = \frac{m}{h}, \quad \lambda = \frac{a}{b}, \quad \delta = \frac{b}{h}, \quad \iota = \frac{n}{h}, \quad P = \frac{p}{Pcr} = \frac{\rho h^2}{Pcr}, \quad P^* = \frac{P}{\sqrt{E_1E_2}}, \quad \bar{q} = \frac{q}{\sqrt{E_1E_2}},
$$

$$
S = Pcr^3 \left( \frac{\pi b}{\sqrt{E_1E_2}} \right)^2, \quad P_{cr}^* = \frac{P_{cr}}{\sqrt{E_1E_2}}, \quad \Gamma^* = \frac{\Gamma}{\sqrt{E_1E_2}}, \quad i,j = 1,2
$$

and maintaining the previous notation, to determine the unknowns $w_{nm} = w_{nm}(t), u_{nm} = u_{nm}(t), v_{nm} = v_{nm}(t)$, we obtain a system of nonlinear integro-differential equations.
Here $\omega = \sqrt{\frac{\pi^2}{4} \sqrt{\frac{E_1 E_2 h^2}{\rho c^4}}} \left(\rho b^4\right)$ is the frequency of the fundamental tone; $c = \sqrt{\frac{E_1 E_2}{\rho}}$ – is the speed of sound in the plate material.

In general, the boundary conditions can be of any type. If to assume that the plate is hinged-supported along the contour, then in the expansion of the Bubnov-Galerkin method (2), the approximating functions of deflections and displacements are selected in the form

$$\phi_{nm}(x, y) = \cos n \pi x \sin m \pi y, \quad \varphi_{nm}(x, y) = \sin n \pi x \cos m \pi y,$$

$$\psi_{nm}(x, y) = \sin n \pi x \sin m \pi y.$$  

Integration of equations obtained on the basis of monomial and polynomial approximation of deflection, taking into account various factors, was carried out using the numerical method based on the exclusion of singularity in the kernel [19].

As a criterion that determines the critical time and critical load, we accept the condition that the bending deflection should not exceed a value equal to the plate thickness [20]. To determine the dynamic critical load, we use a dynamic coefficient $C_D$ equal to the ratio of dynamic critical load to the upper static one. The results of calculations at various physical and geometrical parameters are reflected in the graphs shown in Figs.1-3. The weakly singular Koltunov-Rzhanitsyn kernels of the form given in [7] were used in calculations.

$$\Gamma(t) = Ae^{-\beta t} \cdot t^{\alpha-1}, \quad A > 0, \quad \beta > 0, \quad 0 < \alpha < 1.$$

3. Results and discussion

The effect of inhomogeneous material properties on the stability process of a plate was studied (Fig. 1). As can be seen from the figure, an increase in the parameter determining the degree of anisotropy $\Delta = \sqrt{\frac{E_1}{E_2}}$ (curve 1 - $\Delta = 1$; curve 2 - $\Delta = 1.5$ and curve 3 - $\Delta = 2$) leads to a later intensive increase in deflections, and accordingly, to an increase in critical value of $C_D$.

![Figure 1. Dependence of deflection on time at $\Delta=1$ (1); 1.5 (2); 2 (3)](image_url)

Next, the results of the study of concentrated mass effect in the center of the plate on its dynamic stability are presented (Fig. 2). At $M_1=0$; 0.2; 0.5 the magnitude of dynamic coefficient $C_D$ is 4.47, 4.61, 4.74, respectively.
Figure 2. Dependence of deflection on time at $M_1=0$ (1); 0.2 (2); 0.5 (3)

Fig. 3 reflects the relationship between the bending deflection and time at various values of the loading rate parameter $S$.

Figure 3. Dependence of deflection on time at $S=0.5$ (1); 2(2); 5 (3)

At these values of $S$ the coefficients $C_D$ are respectively 5.2; 3.8; 3.2. A decrease in dimensionless parameter of the loading rate $S$ leads to an increase in the critical load coefficient and time.

4. Conclusion
1. Dynamic stability of viscoelastic orthotropic plates with concentrated masses is estimated based on the polynomial approximation of deflections.
2. The effect of changes in physical, mechanical and geometrical parameters of the plate material on dynamic stability of plate is investigated.
3. It was established that the results of viscoelastic problem obtained using the exponential relaxation kernel almost coincide with the results of elastic problem, and when using the Koltunov-Rzhanitsyn kernel, the difference turns out to be significant and amounts to more than 40%.
4. It is shown that an increase in the value of loading rate $\psi$ in the viscoelastic case leads to an increase in critical load coefficient and time, and a rapid growth of deflections occurs at an earlier time than in the elastic case.
5. It was found that an increase in the parameter $\Delta$, which determines the degree of anisotropy, leads to a later intensive increase in deflections, and accordingly, to an increase in the critical value of $C_D$. 
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