A Strong Formulation for Stochastic Multiple Constrained Resources Air Traffic Flow Management with Reroutes

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This paper addresses the air traffic flow management research problem of determining reroute, ground delay and air delay for flights using stochastic weather forecast information. The overall goal is to minimize system-wide reroute and delay costs. This problem is a primary concern in United States and especially in its northeastern region, and is also the key in enhancing the performance of the new FAA Traffic Management Initiative called Collaborative Trajectory Options Program (CTOP). In this work we present two stochastic integer programming models, including a two-stage static model and a multistage dynamic model, which are both based on a notable strong deterministic flight-by-flight level air traffic flow formulation. Our preliminary numerical results show that completely integer solutions can be achieved from linear relaxation, for both models and for both no-route and reroute cases.

Key words: air traffic flow management; collaborative trajectory options program; stochastic model; strong formulation

History:

Nomenclature

\( R \) \hspace{1cm} \text{Set of resources, including departure airports and PCAs} \\
\( C \) \hspace{1cm} \text{Set of ordered pairs of resources.} \quad (r, r') \in C \text{ iff } r \text{ is connected to } r' \text{ in the directed graph of departure airports and PCAs} \\
\( \Delta_{r,r'} \) \hspace{1cm} \text{Number of time periods to travel from resource } r \text{ to } r', \text{ defined for all pairs } (r, r') \in C \\
\( Q \) \hspace{1cm} \text{Set of scenarios, } q = 1, \cdots, |Q| \\
\( F \) \hspace{1cm} \text{Set of flights, } i = 1, \cdots, |F| \\
\( p_q \) \hspace{1cm} \text{Probability that scenario } q \text{ occurs} \\
\( M_{i,q}^r \) \hspace{1cm} \text{Real capacity of PCA } r \text{ in time period } t \text{ under scenario } q \\
\( w_{ij,t}^r \) \hspace{1cm} \text{Whether flight } i \text{ taking route } j \text{ departs from/arrives at airport/PCA } r \text{ by time } t \\
\( w_{ij,t}^{rq} \) \hspace{1cm} \text{Whether flight } i \text{ taking route } j \text{ departs from/arrives at airport/PCA } r \text{ by time } t \text{ under scenario } q
1. Introduction

The goal of air traffic flow management is to alleviate projected demand-capacity imbalances at airports and in en route airspace. As a new tool in the Federal Aviation Administration (FAA) NextGen portfolio, Collaborative Trajectory Options Program (CTOP) enables air traffic managers to control traffic through multiple congested airspace regions with a single program, which allows traffic to be managed in an integrated way. CTOP also allows airline flight operators to submit a set of reroute options (called a Trajectory Options Set or TOS), which provides great flexibility and efficiency to airspace users.

This paper aims to answer the following research question: given reroute options and probabilistic weather forecast information, what is the theoretical best system performance we can achieve in terms of total route and delay costs? This research question is important in designing CTOP program and in analyzing CTOP performance. It is also rather general and fundamental, and can be meaningful for researchers in other countries. In this work, we will tackle this research question in the stochastic programming framework, in which probabilistic weather forecast will be translated into scenario-based capacity data. We present two stochastic integer programming models to find the optimal delay and reroute policy, each with varying degree to which traffic managers can modify or revise flights controlled departure times and reroute.

\[
T^r_{ij} \quad \text{Set of allowed time periods for flight } i \text{ taking route } j \text{ to departs from/arrives at airport/PCA } r
\]

\[
\bar{T}^r_{ij} \quad \text{First time period in the set } T^r_{ij}
\]

\[
\underline{T}^r_{ij} \quad \text{Last time period in the set } T^r_{ij}
\]

\[
\delta_{qtij} \quad \text{Binary indicator whether flight } i \text{ will depart in time period } t \text{ and take route } j \text{ under scenario } q
\]

\[
\tilde{\delta}_{ij} \quad \text{Binary indicator whether flight } i \text{ will take route } j
\]

\[
\tilde{\delta}_{qij} \quad \text{Binary indicator whether flight } i \text{ will take route } j \text{ under scenario } q
\]

\[
\text{Dep}_i \quad \text{Original scheduled departure time of flight } i
\]

\[
t^r_{ij} \quad \text{Time period in which flight } i \text{ taking route } j \text{ is scheduled to cross PCA } r
\]

\[
\Omega_{ij} \quad \text{Ordered set of indices of the airport/PCAs which flight } i \text{ passes if taking route } j
\]

\[
\Omega^k_{ij} \quad \text{The } k\text{-th resource along route } j \text{ of flight } i
\]

\[
N_{ij} \quad \text{Number of PCAs along route } j \text{ of flight } i
\]

\[
c_{ij}, c_g, c_a \quad \text{Cost for flight } i \text{ taking route } j, \text{ cost for unit ground delay and air delay}
\]

\[
B \quad \text{Set of branches in the scenario tree, } b = 1, \ldots, |B|
\]

\[
N_b \quad \text{Number of scenarios corresponding to branch } b
\]

\[
o_b, \mu_b \quad \text{Start and end nodes of branch } b
\]
Flight-by-flight level air traffic management models tend to be $\mathcal{NP}$-hard, even in the deterministic case as shown in Bertsimas and Patterson (1998). Considering the uncertainty in capacity can only compound the problem. Thus, having a good formulation is crucial to solve realistic size problem instance in real time. These two stochastic models presented in this work are based on a famous strong air traffic flow formulation, proposed in Bertsimas and Patterson (1998) and Bertsimas, Lulli, and Odoni (2011). To differentiate this work with a previous work (Zhu et al. (2019)), we will call models in this work binary models, because all decision variables are binary. We will call models in Zhu et al. (2019) as integer models, since some of the key decision variables are integers.

2. Preliminary Concepts

2.1. Potential Constrained Area and Capacity Scenarios

In this paper, we will model a constrained airspace resource as a Potentially Constrained Area (PCA), in which air traffic demand may exceed capacity and whose future capacity realization is represented by a finite set of scenarios arranged in a scenario tree. A related concept is the airport-PCA network, which refers to a directed graph that links the airports and PCAs, and models the potential movement of traffic between them. Figure 1 shows an example of PCA network, which includes three en route PCAs and one constrained airport EWR. Figure 2 shows the scenario tree used in this paper. In multi-resource air traffic management problem, the change of operating condition at any PCA will result in a branch point in the scenario tree. Therefore, this scenario tree models the evolution of the future capacities of all four PCAs in Figure 1.

2.2. Resources along a Route

In this study, we assume each flight will choose a route from its TOS set, depart from its origin airport, traverse one or more constrained en route PCAs, and land at destination airport (if it is
also constrained) or directly exit the PCA system (grey arrow in Figure 1). We define for each flight a list of elements it passes, denoted as $\Omega_{ij}$. By assumption, $\Omega^0_{ij}$ is the departure airport of flight $i$ and $\Omega^{k\geq 1}_{ij}$ will include all the PCAs along the route.

3. Two-stage Static Model

In this section, we introduce the two-stage aggregate stochastic model. In this two-stage model, the first stage decisions are the reroute decision and ground delay assignment, and the second stage decisions are the air delays flights need to take in response to the actual weather scenarios.

The primary decision variable in this work is $w^r_{ijt}$, which is a binary variable indicating whether flight $i$ will take $j$ and departs from/arrives at airport/PCA $r$ by time $t$. To be more clear, when $r$ is an airport ($r = \Omega^0_{ij}$), if route $j$ is chosen for flight $i$, $w^r_{ijt} = 0$ implies that flight is still on the ground. The first time period $w^r_{ijt} = 1$ is when this flight is released for departure. When $r$ represents a PCA and $j$ is chosen, $w^r_{ijt} = 0$ means flight $i$ is still on its way to PCA $r$, and $w^r_{ijt}$ first becomes 1 when it is admitted to PCA $r$. In two-stage stochastic model, the first stage decisions are made while a flight is still on the ground and are the same for all scenarios, hence we can drop $q$ for $w^r_{ijT_{ijr}}$ when $r = \Omega^0_{ij}$.

In the first set of constraints we ensure that one and only route is chosen for each flight:

$$w^r_{ijT_{ijr}} = \delta_{ij} \quad \forall i \in F, j \in F_i, r = \Omega^0_{ij}$$

$$\sum_{j \in F_i} \delta_{ij} = 1 \quad \forall i \in F$$

(1)

If $j$ is indeed selected for flight $i$, then this flight must depart by the last allowed departure time period $T_{ijr}$. Here $\delta_{ij}$ is only an ancillary variable.
There are two types of connectivity constraints in this problem: connectivity in time and connectivity between resources. Connectivity between time ensures that if a flight has arrived at a resource by time \( t \), then \( w_{i,j,t}^r - w_{i,j,t-1}^r \geq 0 \) for all later time periods \( t' > t \).

\[
\begin{align*}
\text{Connectivity between time ensures that if a flight has arrived at a resource by time } t, \text{ then } w_{i,j,t}^r - w_{i,j,t-1}^r &\geq 0 \quad \forall i \in F, j \in F_i, r \in \Omega_{ij}^0, t \in T_{ij}^r, q \in Q \\
\text{Connectivity between resources impose that if a flight arrives at resource } r' \text{ by } t + \Delta_{r,r'}, \text{ it must has arrived at } r \text{ which is the upstream resource on route } j \text{ by } t.
\end{align*}
\] (2)

\[
\begin{align*}
\text{Ground delay for flight } i \text{ is:}
\quad g_i = \sum_{j \in F_i} \left[ \sum_{t \in T_{ij}^r, t = \Omega_{ij}^0} t (w_{i,j,t}^r - w_{i,j,t-1}^r) - \delta_{ij} \text{Dep}_i \right]
\end{align*}
\] (8)

\[
\begin{align*}
\text{Air delay for flight } i \text{ under scenario } q \text{ is:}
\quad a_{iq} = \sum_{j \in F_i} \left[ \sum_{t \in T_{ij}^r, t = \Omega_{ij}^{N_{ij}}} \left( t (w_{i,j,t}^r - w_{i,j,t-1}^r) - \delta_{ij} \text{Dep}_i \right) \right] - g_i q
\end{align*}
\] (9)

The boundary conditions are:

\[
\begin{align*}
\text{Ground delay for flight } i \text{ is:}
\quad g_i = \sum_{j \in F_i} \left[ \sum_{t \in T_{ij}^r, t = \Omega_{ij}^0} t (w_{i,j,t}^r - w_{i,j,t-1}^r) - \delta_{ij} \text{Dep}_i \right]
\end{align*}
\] (5)

\[
\begin{align*}
\text{Air delay for flight } i \text{ under scenario } q \text{ is:}
\quad a_{iq} = \sum_{j \in F_i} \left[ \sum_{t \in T_{ij}^r, t = \Omega_{ij}^{N_{ij}}} \left( t (w_{i,j,t}^r - w_{i,j,t-1}^r) - \delta_{ij} \text{Dep}_i \right) \right] - g_i q
\end{align*}
\] (9)

In this work, since we assume flight cannot depart before scheduled time and cannot speed up, therefore \( \text{Dep}_i = T_{ij}^0 \), \( t^r = T_{ij}^{N_{ij}} \).

The objective function minimizes the total reroute, ground delay, and expected air delay costs. Arranging the terms in the following formula

\[
\begin{align*}
\text{min } \sum_{i \in F} \left( c_g g_i + \sum_{q \in Q} c_q a_{iq} + \sum_{j \in F_i} c_{ij} \delta_{ij} \right)
\end{align*}
\]

we obtain

\[
\begin{align*}
\text{min } \sum_{i \in F} \sum_{j \in F_i} \left[ c_{ij} \delta_{ij} + (c_g - c_a) \sum_{t \in T_{ij}^r, t = \Omega_{ij}^0} \left( t (w_{i,j,t}^r - w_{i,j,t-1}^r) - \delta_{ij} t^r \right) \right] + c_a \sum_{t \in T_{ij}^r, t = \Omega_{ij}^{N_{ij}}} \left( t (w_{i,j,t}^r - w_{i,j,t-1}^r) - \delta_{ij} t^r \right)
\end{align*}
\] (10)
4. Multistage Dynamic Model

In this section, we introduce the multistage stochastic model which can dynamically adjust flight release time and reroute choice before actual departure. The formulation is listed as follows:

\[
\min \sum_{q \in Q} p_q \sum_{i \in F} \sum_{j \in F_i} \left[ \left( c_{ij} - (c_g - c_a) T_{ij}^{r=\Omega_{ij}^0} - c_a N_{ij}^{\Omega_{ij}^N} \right) \tilde{q}_{ij} + (c_g - c_a) \sum_{t \in T_{ij}^{r=\Omega_{ij}^0}} t(w_{ij,t}^{r,q} - w_{ij,t-1}^{r,q}) + c_a \sum_{t \in T_{ij}^{r=\Omega_{ij}^0}} t(w_{ij,t}^{r,q} - w_{ij,t-1}^{r,q}) \right]
\]

\[\delta_{qij} = w_{ij,t}^{r,q} - w_{ij,t-1}^{r,q} \quad \forall i \in F, j \in F_i, r = \Omega_{ij}^0, q \in Q\]  

\[\delta_{qij} = \sum_{t \in T_{ij}^{r=\Omega_{ij}^0}} \delta_{qij} \quad \forall i \in F, j \in F_i, q \in Q\]  

\[\sum_{j \in F_i} \delta_{qij} = 1 \quad \forall i \in F, q \in Q\]  

\[w_{ij,t}^{r,q} - w_{ij,t-1}^{r,q} \geq 0 \quad \forall i \in F, j \in F_i, r \in \Omega_{ij}, t \in T_{ij}^{r}, q \in Q\]  

\[w_{ij,t}^{r,q} - w_{ij,t+\Delta r'}^{r',q} \leq 0 \quad \forall i \in F, j \in F_i, r, r' \in \Omega_{ij}, t \in T_{ij}^{r}, q \in Q\]  

\[\sum_{(i,j) \in F_i} w_{ij,t}^{r,q} - w_{ij,t-1}^{r,q} \leq M_{ij}^{r} \quad \forall r \in \Omega_{ij}^{h \geq 1}, t \in T, q \in Q\]  

\[\delta_{qN_{ij}}^{b} = \cdots = \delta_{qN_{ij}}^{b} \quad \forall i \in F, j \in F_i, t \geq \text{Dep}_b = T_{ij}^{\Omega_{ij}^B}, b \in B, \mu_b \geq t \geq \alpha_b\]  

\[w_{ij,T_{ij}} = 0 \quad \forall i \in F, j \in F_i, r \in \Omega_{ij}, q \in Q\]  

\[w_{ij,T_{ij}} = \delta_{qij} \quad \forall i \in F, j \in F_i, q \in Q, r = \Omega_{ij}^{N_{ij}}\]  

The first three set of constraints make sure one and only route will be chosen for each flight. \(\delta_{qij}\) is an ancillary binary variable indicating whether flight will take route \(j\) and depart in time period \(t\). \(\tilde{\delta}_{qij}\) is another ancillary variable which shows whether flight will choose route \(j\) under scenario \(q\). [15] and [16] are connectivity in time constraint and connectivity between resources constraint. [17] is the capacity constraint, which has exactly the same expression as in two-stage model. In multistage model, we will also have a set of nonanticipativity constraints [18], which ensures that decisions are made solely based on the information available at that time. [19] and [20] are boundary conditions.

5. Experimental Results

To demonstrate the performance of the proposed models, we created an operational use case based on actual events from July 15, 2016. This use case primarily addresses convective weather activity in southern Washington Center (ZDC) and EWR airport. Figure 3 shows the pattern of convective weather activity for that day. There is a four-hour capacity reduction in ZDC/EWR from 2000z to 2359z. By analyzing the traffic trajectory (Figure 4) and weather data, we can build the airport-PCA network, shown in Figure 1.
5.1. Capacity Profiles and Traffic Demand

For comparison purposes, we use the same capacity data as in [Zhu et al. (2019)]. The detailed capacity information is listed in Table 1. The three scenarios correspond to optimistic, average, and pessimistic weather forecast. We can see that in scenario 1 at 2100Z PCA1’s 15-minute capacity changes from 44 to 50, the EWR’s capacity changes from 8 to 10; in scenario 2 at 2230Z, the capacities of PCA1 and EWR return to the nominal values. These two changes correspond to the two branch points in the scenario tree shown in Figure 2.

In GDP optimization, we usually add one extra time period to make sure all flights will land at the end of the planning horizon. Because CTOP has multiple constrained resources, we need to add more than one time period depending on the topology of the FCA-PCA network. In this case, we add eight extra time periods, because the longest travel time between the three en route PCAs and EWR among all TOS options is around 2 hours (8 time periods). For any time periods outside the CTOP start-end time, e.g. the eight extra time periods in Table 1, nominal capacities are used.

We use flight trajectory data from System Wide Information Management (SWIM) and Coded Departure Route (CDR) database for traffic demand modeling. In total 1098 flights are captured by this CTOP, among them 890 flights that traverse the PCAs in their active periods. And there are in total 1368 TOS options for 890 flights, on average 1.54 options per flight.

5.2. Model Comparisons

The optimization models are solved using Gurobi 8.1 on a laptop with 3.6 GHz processors and 32 GB RAM. The main results are listed in Table 2 to 4. In this test example, integer solutions can be directly obtained from linear programming relaxation, for both two-stage and dynamic models and for both no-route and reroute cases.
6. Conclusions

This preliminary result shows that the new binary stochastic programming model seems to be a better formulation compared with previous work. We are currently doing more numerical test and theoretical analysis.

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