Waking and Scrambling in Holographic Heating up

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ABSTRACT: We consider a holographic model of the heating up process. As a dual background we take a geometry describing thin shell accretion on a black brane. We find explicitly the time evolution of the mutual information during the non-equilibrium heating process from the initial temperature $T_i$ to the final temperature $T_f$ for the system of two intervals in the 1+1 dimensional case. We calculate widths and separation of two intervals for which the time dependence of the mutual information has the bell-like form, i.e. it starts from zero value at the wake up time, then reaches a maximal value and vanishes at the scrambling time. This form of the mutual information evolution was previously found in photosynthesis. The zone of the bell-like configurations exists for small distances $x < \log 2/2\pi T_i$ only for the particular interval sizes. For $x$ large enough, i.e. $x >> \log 2/2\pi T_i$, it exists only for large enough interval sizes and this zone becomes more narrow when $T_i$ increases and becomes larger with increasing of $T_f$.

KEYWORDS: AdS/CFT correspondence, holography, mutual information, entanglement entropy, scrambling, wake-up, heating up, thermalization, photosynthesis
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1 Introduction

The AdS/CFT correspondence [1–3] initially started as a duality between certain SYM theories and superstrings is a powerful tool for a description of general class of strongly-coupled quantum theories. Broad phenomena in QCD [4]-[6], condensed matter [7], cosmology [8] and even in biology [9] are described by the holographic duality.

One of the most challenging questions in traditional methods of quantum field theories is the thermalization process. In the AdS/CFT duality the temperature in the quantum field theory is associated with the black hole (or black brane) temperature in the dual background [10]-[13]. The thermalization is described in the dual language by a black hole formation process, see [4, 5] and references therein. One of the simplest descriptions of this process is given by the Vaidya deformation of a given background, see [14]-[22] and references therein.

In this paper we investigate the holographic instantaneous heating process. An instantaneous heating process can serve as an instant perturbation of open quantum systems (see for example [23]), in particular, systems considered in biology (photosynthesis) [9, 24–26]. The dual description of this process corresponds to a massless shell accretion on black hole and the corresponding metric describes the shell in the black hole background (we call it the BH-Vaidya metric). In the context of photosynthesis the time evolution of the quantum mutual information that has the bell-like form during heating process is of special interest. During this evolution the mutual information starts from zero value at the moment \( t_{\text{wup}} \), then increases up to the maximum value and then vanishes at the time \( t_{\text{scr}} \). This type of evolution of the mutual information has been found numerically in the holographic model of photosynthesis [9], where the thick shell BH-Vaidya metric has been considered for particular values of parameters, specifying the geometry of the system composed of two strips or intervals. Our goal is to perform a detailed study of the regions in the space of parameters, where the bell-like form of the time evolution of the mutual information takes place. Also we find the corresponding wake up \( t_{\text{wup}} \) and scrambling \( t_{\text{scr}} \) times. It is useful to have an explicit formula for the entanglement entropy evolution instead of numerical results for the thick shells [19, 20, 22, 27, 28] for this purpose. To this end we generalize the explicit results obtained for the thin AdS-Vadnya shell [14, 15, 17, 18] to the thin BH-Vaidya shell model. The explicit formulæ for the time evolution of the entanglement entropy for the thin BH-Vaidya shell are more cumbersome than ones for the thin AdS-Vadnya shell, however they allow us to simplify the detailed analysis of the mutual information evolution [28–30], [9], in particular, find the wake-up and scrambling times.

The bell-type behaviour of the time evolution of the mutual information takes place in a part of a larger region of parameters in which the finite scrambling time exists, but there is no the wake up regime. In other words, the bell zone of parameters
is a part of the scrambling zone. The scrambling zone in the space of geometrical parameters of our model depends on the final temperature only, meanwhile the bell zone depends on the initial and final temperature.

This paper is organized as following. In Sect.2 we describe the dual geometry of the heating process and present the explicit formulae for entanglement entropy evolution. Sect.3 is devoted to the study of the evolution of the mutual information during the heating process. Here we estimate the sizes of scrambling and bell zones in the parametric space as well as the scrambling and wake up time dependences on the final and initial temperatures.

2 BH-Vaidya geometry and geodesics

2.1 BH-Vaidya geometry

In this paper we consider black hole collapsing from an initial state defined by horizon position $z_H$ to the final state with horizon $z_h$ as a dual background. The BH-Vaidya metric is given by

$$v < 0 : \quad ds^2 = \frac{1}{z^2} \left(-f_H(z) dt^2 + \frac{dz^2}{f_H(z)} + dx^2\right), \quad v = t - z_H \arctanh \frac{z}{z_H},$$

$$v > 0 : \quad ds^2 = \frac{1}{z^2} \left(-f_h(z) dt^2 + \frac{dz^2}{f_h(z)} + dx^2\right), \quad v = t - z_h \arctanh \frac{z}{z_h},$$

and the functions $f_H$ and $f_h$ are defined as

$$f_H = 1 - \left(\frac{z}{z_H}\right)^2, \quad f_h = 1 - \left(\frac{z}{z_h}\right)^2, \quad z_h < z_H,$$ (2.3)

The initial and finite temperatures are

$$T_i = \frac{1}{2\pi z_H}, \quad T_f = \frac{1}{2\pi z_h}.$$ (2.4)

In more compact notations the metric is

$$ds^2 = \frac{1}{z^2} \left(-f(z, v) dt^2 + \frac{dz^2}{f(z, v)} + dx^2\right),$$ (2.5)

where

$$f(z, v) = \theta(v) f_h(z) + \theta(-v) f_H(z).$$ (2.6)

This metric describes the shell located at $v = 0$. Note, that the case $z_H < z_h$ corresponds to a model of cooling [16] and this model violates NEC condition [18].
2.2 The geodesics in the BH-Vaidya background.

The action for a geodesic connecting two points on the boundary $-\ell/2$ and $\ell/2$ at the boundary time moment $t$ in the background with metric (2.1) and parametrized as $z = z(x)$ is given by

$$S(\ell) = \int_{-\ell/2}^{\ell/2} dx \frac{\sqrt{Q}}{z}, \quad Q = 1 - 2v'z' - f(z,v)v'^2,$$  \hfill (2.7)

where

$$z(\pm \ell/2) = 0, \quad v(\pm \ell/2) = t.$$

(2.8)

The symmetry of the problem implies that $z'(0) = v'(0) = 0$ and we denote:

$$z(0) = z_*, \quad v(0) = v_*.$$

(2.9)

In the regions $v > 0$ and $v < 0$ the dynamical system (2.7) has two integrals of motion $E_\pm$ and $J_\pm$,

$$E_\pm = z' + f v', \quad J_\pm = z \sqrt{Q}. \hfill (2.10)$$

where we denote values of these integrals over the shell ($v > 0$) as $E_+$, $J_+$ and under the shell ($v < 0$) as $E_-$ and $J_-$. The integrals of motion $J_\pm$ are equal due to the $x$-independence of $Q$ and they are equal to the value of $z$ at the ”top” point $z_*$, the point where $z' = v' = 0$, i.e.

$$z_* = z_0 \sqrt{1 - 2v'_- z'_- - f_h(z_0)v'^2}_- = z_+ \sqrt{1 - 2v'_+ z'_+ - f_H(z_0)v'^2}_+.$$

(2.11)

Above the shell we have $E_- = 0$

$$z'_- + f_H(z)v'_- = 0,$$

(2.12)

and when crossing the shell $E$ is not conserved, $E_+ \neq E_-$. In the crossing point we can write down the equality for $E_+$

$$z'_{+c} + f_H(z_c)v'_{+c} = z'_+ + f_H(z)v'_+.$$

(2.13)

The geodesic passing through the shell $v = 0$ at a point $z_c$ has a discontinuity. Also, the integrals of motion $J_\pm$ can be matched in $z_c$

$$z_- \sqrt{1 - 2v'_- z'_- - f_H(z)_v'^2} = z_+ \sqrt{1 - 2v'_+ z'_+ - f_H(z)v'^2}_c,$$

$$z_+ \sqrt{1 - 2v'_+ z'_+ - f_H(z)v'^2}_c = z_+ \sqrt{1 - 2v'_+ z'_+ - f_H(z)v'^2}_c.$$  \hfill (2.14)
and from (2.12) we have the condition at the crossing point

\[ z_{-c}' + f_H(z_c)v_{-c}' = 0. \quad (2.15) \]

Integrating the equations of motions across the shell one can derive the continuity condition for function \( v \)

\[ v_{-c}' = v_{+c}'. \quad (2.16) \]

Solving system of equations (2.11)-(2.16) we can express the integral of motion \( E_+ \) and derivatives \( z_+', v_+ \) in terms of \( z_c \) and \( z_\ast \). For example, \( E_+ \) is

\[ E_+ = -\frac{z_c (z_H^2 - z_h^2)}{2z_H z_h^2} \sqrt{\frac{z_\ast^2 - z_c^2}{z_H^2 - z_c^2}}. \quad (2.17) \]

Let us introduce the following notations (compare with [14]):

\[ z_\ast = \frac{z_c}{s}, \quad z_c = \frac{z_h}{\rho}, \quad z_H = \frac{z_h}{\kappa}, \quad c = \sqrt{1 - s^2}, \quad \Delta = \sqrt{\rho^2 - \kappa^2}, \]

In terms of these variables the form of \( E_+ \) is

\[ E_+ = -\frac{c(1 - \kappa^2)}{2\Delta \rho s}. \quad (2.18) \]

The expression for the boundary time \( t \) in terms of \( E_+ \) is:

\[ t = \int_0^{z_c} \frac{dz}{f_h(z)} \left( \frac{E_+ z}{\sqrt{(z_\ast^2 - z^2)f_h(z) + E_+^2 z^2}} - 1 \right), \quad (2.19) \]

Evaluating integral in the RHS of (2.19) and substituting here \( E_+ \) from (2.18) we get

\[ \coth \frac{t}{z_h} = \frac{(-\kappa^2 + 2\rho^2 + 1)c + 2\rho \Delta}{2(\Delta + \rho c)}. \quad (2.20) \]

It is straightforward to obtain the expression for the extension \( \ell \) of geodesics along the boundary. It is the sum of two terms, corresponding to two parts of the geodesics, under and over the shell. Explicitly we have \( \ell/2 = \ell_- + \ell_+ \), where \( \ell_- \) and \( \ell_+ \) are

\[ \ell_-/2 = \int_{z_c}^{z_\ast} \frac{z^2 dz}{\sqrt{z^2 (z_\ast^2 - z^2)f_H(z)}}, \quad (2.21) \]

\[ \ell_+/2 = \int_0^{z_c} \frac{z^2 dz}{\sqrt{z^2 (z^2 - z_\ast^2)f_h(z) + E_+^2 z^4}}. \]
Introducing the notation $\gamma^2 = 1 - \kappa^2$ and integrating in (2.21) we get the explicit formulae

$$\ell_\pm = \frac{z_h}{2} \log \left( \frac{(c\kappa + \Delta s)^2}{\rho^2 s^2 - \kappa^2} \right)$$

(2.22)

The expressions for the length of each part of the geodesic

$$L_{\pm}/2 = z_s \int_{z_e}^{z_*} \frac{dz}{z^2 \sqrt{(1 - z^2/z_h^2) (z_e^2/z^2 - 1)}}$$

(2.23)

$$L_{+,\varepsilon}/2 = z_s \int_{\varepsilon}^{z_e} \frac{dz}{z^2 \sqrt{(1 - z^2/z_h^2) (z_e^2/z^2 - 1) + E_{\varepsilon}^2}}$$

(2.24)

where we introduced regularization $\varepsilon$ for divergent piece of the geodesic $L_{\varepsilon}$.

After integration and standard removal of the divergent part coming from $L_{+,\varepsilon}$ when $\varepsilon$ goes to zero, we get

$$L_{\pm}/2 = \frac{1}{2} \log \left( \frac{\rho (c^2 \rho + 2c \Delta + \rho) - \kappa^2}{\Delta^2 - c^2 \rho^2} \right),$$

(2.25)

$$L_{+,\varepsilon}/2 = \log \left( \frac{2\Delta z_h}{\sqrt{K_- K_+}} \right),$$

$$K_- = 2\Delta (\rho - 1) - c (\kappa^2 - 2\rho (\rho - 1) - 1),$$

$$K_+ = 2\Delta (\rho + 1) - c (\kappa^2 - 2\rho (\rho + 1) - 1).$$

Formulae (2.20), (2.22) and (2.25) in the limit $\kappa \to 0, \Delta \to \rho$ and $\gamma \to 1$ reproduce the corresponding formulae for thermalization from [14]. The holographic entanglement entropy for a single interval is given by the length of the geodesic that is anchored on this interval [31]. We can solve the equation (2.20) obtaining the time dependence analytically as some function $\rho = \rho(s, t)$ (we do not write out it explicitly due to its length). Substituting $\rho = \rho(s, t)$ in (2.18)-(2.25), we have

$$S(s, t) = 2 (L_-(s, t) + L_+(s, t)),$$

(2.26)

$$\ell(s, t) = 2 (\ell_-(s, t) + \ell_+(s, t)).$$

Next we invert numerically the functions of one variable (2.26) to get $S(\ell, t)$.

3 Evolution of the mutual information

3.1 Typical evolution of the mutual information

Now let us consider the mutual information evolution. The mutual information for two intervals system of lengths $\ell_1$ and $\ell_2$ divided by distance $x$ at the time moment
\( t \) is defined as
\[
I(\ell_1, \ell_2, x, t) = S(\ell_1, t) + S(\ell_2, t) - (S(\ell_1 + \ell_2 + x, t) + S(x, t)).
\] (3.1)

The behaviour of the mutual information thermalization has been investigated in many previous studies \([28, 29]\), where the AdS-Vaidya model has been considered. We study the general features of the mutual information behaviour in the BH-Vaidya model, i.e. when the initial state is already thermal. A quantum quench in the quantum theory starting from a thermal initial state has been studied in \([32]\).

We are looking for a very special mutual information evolution, namely the evolution that has the form of the bell. As has been noted in \([9]\), this type of behaviour of the holographical mutual information reproduces results of numerical calculations for a special non-equilibrium open quantum system describing the photosynthesis \([34]\).

![Figure 1](image)

**Figure 1.** Different regimes of the mutual information evolution in the heating process of two disjoint intervals. The blue curve starts from a non-zero value of the mutual information \( I_{\text{start}} \neq 0 \), then increases and reaches the maximum \( I_{\text{max}} \), and then starts to decrease up to a fixed positive value \( I_{\text{min}} \neq 0 \). The brown curve corresponds to the scrambling behaviour, \( I(t_{\text{start,scr}}) \neq 0, I(t_{\text{scr}}) = 0 \). The orange curve corresponds to the bell regime, \( I(t_{\text{wup}}) = I(t_{\text{scr,bell}}) = 0, t_{\text{wup}} < t_{\text{scr,bell}} \). The yellow line corresponds to mutual information vanishing during all process.

There are four different regimes of the mutual information time evolution during the heating process of two intervals:

- the regime where the mutual information starts from a non-zero value, then increases and reaches the maximum \( I_{\text{max}} \). After that it starts to decrease up to a fixed positive value \( I_{\text{min}} \neq 0 \) (the blue line in Fig.1 shows this type of the time evolution of the mutual information);

- the regime with a scrambling point, i.e. the regime, where at the time \( t_{\text{scr}} \) the mutual information vanishes (the brown line in Fig.1);
Figure 2. Contour plots of the mutual information for the system of two intervals for the fixed initial temperature at $z_h = 4$, while the final one varies. In the left plot: intervals of different lengths $\ell_1 = 4$, $\ell_2 = 6$ and $x = 1.68$; in the right plot intervals of equal length $\ell_1 = 5$, $\ell_2 = 5$ and $x = 1.68$. The light yellow domains correspond to the vanishing mutual information.

- the regime, where at the moment $t_{wup}$ the mutual information starts from zero value (the orange line in Fig.1);
- the regime with the vanishing mutual information (the yellow line in Fig.1).

These regimes are similar to the time evolution of the mutual information during the thermalization process [28–30] and physical explanations of these regimes are the same.

In Fig.2 the mutual information time evolution for different final temperatures for equal and different lengths of two intervals are presented. We see that the wake up time is smaller and scrambling time is bigger for the equal intervals.

In what follows we consider a special case $\ell_1 = \ell_2 = \ell$. In this case our parametric region is the 2-dimensional one $(x, \ell)$, $\ell > 0, x \geq 0$. It can be divided in four zones where one of the four typical regimes is realized, see Fig.3. It is obvious that for large enough $x$ and fixed $\ell$ the mutual information vanishes, as well as for large $\ell$ and small enough $x$ the mutual information behaves as shown by the blue line in Fig.1. The scrambling behavior occurs at the parameters for which the stationary mutual information of the whole system vanishes at the final temperature. The line in the $(x, \ell)$-plane, where the mutual information at the temperature $T_f$ vanishes, is given by the following equation

$$\ell(x, z_h) = 2z_h \text{arccoth} \left( \sqrt{2} \sinh \left( \frac{x}{2z_h} \right)^{-1} - \coth \left( \frac{x}{2z_h} \right) \right), \quad (3.2)$$

where $T_f$ is related with $z_h$ as usual, see eq.(2.4).

For small $x$ we have

$$\ell = (1 + \sqrt{2})x + \frac{24 + 17\sqrt{2}}{24z_h^3}x^3 + O(x^5). \quad (3.3)$$
and at large $\ell$ this curve approaches from the left to the vertical line

$$x \xrightarrow[\ell \to \infty]{} x_{scr} = z_h \ln 2,$$  

(3.4)

At $0 < t << t_{heat}$ the temperature of the system is defined by $z_H$ and the line separating the region of the bell shape mutual information from the scrambling regime is given by formula (3.2) with $z_H$ instead of $z_h$, and therefore at large $\ell$

$$x_{wup} = z_H \ln 2,$$  

(3.5)

see Fig.3, where the general structure of scrambling and bell regions is presented.

\[\begin{align*}
&z_H = 4, z_h = 2.5 & z_H = 4, z_h = 1.2 & z_H = 4, z_h = 1 \\
&z_H = 1.2, z_h = 1.1 & z_H = 1.2, z_h = 1 & z_H = 1.2, z_h = 0.9
\end{align*}\]

Figure 3. Zones of different regimes of the mutual information behaviour for different $\ell$ and separation $x$. The vertical axes correspond to $\ell$ and the horizontal axes to $x$. The black solid line corresponds to the critical value of $x_{scr} = z_h \log 2$, where the scrambling occurs and the dashed one corresponds to critical value of $x_{wup} = z_h \log 2$. The different colors correspond to the different regimes presented in Fig.1. In the up plots $z_H = 4$ and $z_h = 2.5, 1.2, 1$ (from the left to the right), in the bottom plots $z_H = 1.2$ and $z_h = 1.1, 1, 0.9$ (from the left to the right)

### 3.2 Configurations with the bell-type mutual information evolution

In this section we study in more details the region of parameters $\ell, x$, where the bell-type of the time evolution of the mutual information during the heating process
is realized. As has been mentioned above, this type of evolution of two disjoint intervals system (with parameters $\ell, x$) is such, that the mutual information is zero in the start of the heating process, then it becomes non-zero at the time moment $t_{wup}$, the so-called wake-up time, and then it vanishes again at the time moment $t_{scr}$, the scrambling time. For this purpose we study the scrambling and wake up times for different $\ell$ and $x$.

Figure 4. The dependence of the scrambling and wake up times on the intervals separation $x$. In the left panel the scrambling time (solid lines) and the wake up times (dashed lines) are shown for the different initial temperatures, $z_H = 1, 1.5, 2, 2.5, 3, 5$ (from the left to the right), and at the fixed final temperature, $z_h = 1$. In the central and right panels the scrambling time (solid lines) and the wake up times (dashed lines) are shown for the different final temperatures corresponding to $z_h = 1, 2$ and $3$ (from the left to the right), while $z_H = 4$. The intervals size is taken to be $\ell = 4$ for left and the central panels and $\ell = 6$ for the right panel. The dotted, dashed and solid black lines show the thermalization time of the intervals of the length $x, \ell$ and $x + \ell$, respectively.

In Fig.4 we plot the dependence of the wake up times (the dashed lines) and the scrambling times (the solid lines) on the intervals separation $x$ for fixed lengths of the intervals for different initial and final temperatures. In the left plot of Fig.4 the final temperature is fixed and we see that there is a critical $x_{cr,wup}(z_H)$ at which the wake up lines start at $t = 0$. The value of $x_{cr,wup}(z_H)$ does not depend on $z_h$ and along the wake up lines $x_{cr,wup}(z_H) < x_{wup}(t) < x_0(z_H, z_h)$ is satisfied. The wake up line disappears at $x_0(z_h, z_H)$, and the scrambling line starts at $x_0$. The scrambling lines satisfy $x_{cr,scr}(z_h) < x_{scr}(t) < x_0(z_H, z_h)$. We see that for larger $z_H$ we get a larger $t_{scr}$ and smaller $t_{wup}$. However, we cannot increase $z_H$ too much to get the less $t_{wup}$, since for fixed $x$ there is a critical $x_{cr,wup}(z_H)$. 

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Figure 5. The dependence of the scrambling and wake up times for the fixed initial temperature \(z_H = 4\) on \(x\), and A) equal lengths \(\ell_1 = \ell_2 = \ell\) and B) for non-equal lengths \(\ell_1 = 2\ell_2\). The final temperatures correspond to \(z_h = 3, 2, 1\) (green, pink and cyan surfaces, respectively)

We see, that for a small difference between the final and initial temperatures only in a very narrow interval \(\Delta x\) we can get the scrambling and wake up time simultaneously, i.e. the distance between two vertical lines in the left plot in Fig.4 decreases when the temperature difference is decreasing. For example, in Fig.4 \(\Delta x_1 < \Delta x_2\). Or in other words, a larger signal, i.e. a higher temperature difference, permits the information exchange for wider range set of parameters. We also see at the left plot of Fig.4, that \(x_{cr,scr}\) depends only on the \(z_h\) and does not depends on the \(z_H\). From the right and central plots of Fig.4 we see that \(x_{cr,wup}\) depends on the initial temperature and \(\ell\). Increasing \(\ell\) we increase \(x_{cr,wup}, x_0\) and \(x_0 - x_{cr,wup}\).

In Fig.5 we present the 3D picture of the scrambling and wake up times for the fixed initial and final temperatures and varying \(\ell_1, \ell_2\) and \(x\). To find the scrambling and wake up times corresponding to the given \(\ell_1, \ell_2\) and \(x\) we take a vertical line with fixed \(x\) and \(\ell\), and find its cross-sections with the surface. If there are two cross-sections then there are \(t_{wup}\) and \(t_{scr}\), as shown at Fig.5. One cross-section corresponds to \(t_{scr}\), and their absense means that we are in the blue zone on Fig.2. Comparing the left and right plots in Fig.5 we see that the bell zone is wide for equal intervals as compared to non equal ones, the same behavior on can see in Fig.2. Taking the different sections \(x = const\) of the surface depicted on Fig.5 we get dependences of the scrambling and wake up times on \(\ell\).

In Fig.6 we plot the dependence of the scrambling and wake times on the interval size, \(\ell_1 = \ell_2 = \ell\), for different final temperatures with the initial temperature being fixed, \(z_H = 4\). We see that the wake up time exists for \(\ell_0 < \ell < \ell_{cr,scr}\) and that increasing the distance between intervals \(x\) we increase \(\ell_0, \ell_{cr,scr}\) and \(\ell_{cr,scr} - \ell_0\).
Figure 6. The dependence of the scrambling (solid lines) and wake up (dashed lines) times on $\ell$ for the fixed initial temperature, $z_H = 4$ and the final one $z_h$ varying: $z_h = 1$ (the red curve), $z_h = 2$ (the green one), $z_h = 3$ (the blue one). In the left panel $x = 0.5, \ell = 1.4$, in the central panel $x = 1.4, \ell = 3.8$ and in the right one $x = 1.8, \ell = 5.3$. The mutual information time evolution is presented by the correspondently colored lines in the right parts of the panels.

Figure 7. In large plots the scrambling (solid lines) and the wake up (dashed lines) times as functions of the interval length $\ell$ for different initial temperatures parametrized by $z_H$ are presented. The scrambling time corresponds to solid color curves and the wake up time to dashed ones. The left big panel shows $z_H = 4.5, 2, 1.7, 1.54, 1.49$ and $z_H = 1.47$ (the green line) from the left to right. The right big panel shows $z_H = 1.47, 1.37, 1.3, 1.26, 1.23$ and $z_H = 1.21$ from the left to right. The green line is the same in the right and left panels. The solid black lines are the thermalization time for the interval size $2l + x$, the dashed ones are the same for the size $l$ and the dotted ones are for the size $x$. The intervals separation is taken to be $x = 1$ and the final temperature corresponds to $z_h = 1$. In the small plots the mutual information time evolution is shown at the same parameters (at by same color) as at large plots.
In Fig. 7 we plot the dependence of the scrambling and wake times on the interval size, \( \ell_1 = \ell_2 = \ell \), for different initial temperatures and the fixed final temperature, \( z_h = 1 \). We see that for \( z_H > z_{cr}(z_h) \), \( z_{cr}(z_h) = 1.47 \), there are critical \( \ell_{cr,wup} = \ell_{cr,wup}(z_H) \) and \( \ell_0(z_H) \), so that only for \( \ell_0 < \ell \leq \ell_{cr,wup} \) the wake up time exists. The scrambling time exists for \( \ell_0 < \ell \). Therefore, for \( z_H > z_{cr}(z_h) \) the bell form of the mutual information evolution takes place for the interval \( \ell_0 < \ell \leq \ell_{cr,wup} \). For \( z_H < z_{cr}(z_h) \), i.e. for high enough initial temperature, it let us remind again, that initial temperature has to be less than the final one, the bell form of the mutual information evolution always exists at large enough \( \ell \).

**Figure 8.** Zones of different regimes of the mutual information behaviour for different \( \ell \) and \( x \) for \( z_H = 1.2 \) and \( z_h = 1 \). In the left plot points \((x_1, \ell_0(x_1))\) and \((x_2, \ell_0(x_2))\) are on the border of the orange and yellow zones. In the right plot the points \( \ell = \ell_{wup}(x_1) \) is are on the border of the orange and yellow zones, meanwhile and \((x_2, \ell_0(x_2))\) is on the border of the brown and orange zones.

One can see that our plots Fig.4-Fig.7 confirm the zone structure presented in Fig.3. Indeed, let us consider Fig.8 that shows a small part of Fig.3. In the right plot let us set \( x_1 < z_H \log 2 \) fixed. We can always find \( \ell_0 = \ell_0(x_1) \), so that increasing \( \ell > \ell_0 \) we are in the bell zone till \( \ell < \ell_{wup}(x_1) \). However, if the distance \( x \) is taken \( x = x_2 > z_H \log 2 \) then the bell zone stars at \( \ell_0 = \ell_0(x_2) \) and exists till an arbitrary \( \ell > \ell_0 \). If we take \( \ell \), then we always can find \( x_1 \) and \( x_2 \), so that \( \ell = \ell_{wup}(x_1) \) and \( \ell = \ell_0(x_2) \), i.e. points are on the borders of the brown and orange zones and of the orange and yellow zones, respectively, and for any \( x, x_1 < x < x_2 \) the mutual information evolution has the bell-type form.

Fig.9 shows points that are on the border of the yellow and orange zones.
4 Conclusion

In this paper we have investigated different aspects of the holographic mutual information behaviour in the heating process using the AdS/CFT correspondence. The initial state is taken to be thermal, and as a dual background describing the heating process we take the BH-Vaidya metric in the thin shell approximation. We derive the explicit formulae for the geodesics in this 2+1 dimensional background and then describe the mutual information evolution for the system of two disjoint intervals. In the consequent work [33] using formulae for the evolution of holographic entanglement entropy we obtain the explicit description for different regimes of holographic heating up process.

We have paid the special attention to the bell form of the mutual evolution. This interest is due to a crucial role of this type of evolution in the photosynthesis [34]. The widest bell-like zone corresponds to the symmetric configurations and the larger temperature difference between the initial and final temperatures. This zone exists for small distances $x < z_H \log 2$, only for the particular interval sizes, $\ell_{\text{scr}}(x) < \ell < \ell_0(x)$ and for $x$ large enough $x > z_H \log 2$, exists only for large enough interval sizes, $\ell > \ell_{\text{scr}}(x) = \ell(x, z_H)$, where $\ell(x, z_H)$ is given by (3.2).

Notions of the quantum information help us in the understanding of the underlying quantum structure of the photosynthesis. The appearance of the bell-type evolution of the mutual information in the thermal states as a result of the global quench gives an opportunity of the information exchange between two subsystems even for non-zero temperature and in the context of the photosynthesis this can be interpreted as sending the signals from the antenna to the recreation center. The bell-type evolution under holographic smooth heating of higher dimensional two strips has been found numerically in [9]. It would be interesting to study the same problem for $3 \leq n \leq 7$ regions, especially of 3-dimensional ball forms as well as use more complicated holographic backgrounds [17, 35–37].

Figure 9. Points on the border of orange and yellow zones, here $z_H = 4, z_h = 1.2$. 
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