SEARCHING FOR HEAVY STERILE NEUTRINOS IN KAON DECAYS

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We present here a study of the impact of a heavy neutrino (or heavy neutral lepton) on leptonic and semileptonic kaon decays. We used a simplified model consisting of 3 light neutrinos responsible for neutrino oscillations and a heavy sterile neutrino. We found that it can lead to large deviations from the Standard Model predictions for leptonic decays, in conflict with experimental measurements of $\mathcal{K}_{\ell 2}$ and the lepton universality test $\mathcal{R}_K$. This allows to derive new constraints on the leptonic mixing for heavy sterile neutrinos. No tension was found when considering the semileptonic decays. Finally, we point out the potential of the decay $K_L \to \nu\nu$ as a clear signature of physics beyond the Standard Model.

1 Introduction

The observation of neutrino oscillations provides a clear evidence of the existence of physics beyond the Standard Model (SM). Among the many extensions of the SM that were introduced to explain this phenomenon and generate neutrino masses and mixing, one of the simplest is the addition of sterile neutrinos. Being fermionic gauge singlets, they can have a Majorana mass term whose value is not necessarily related to the typical mass scales of the SM like the QCD scale or the electroweak scale. Depending on their mass, sterile neutrinos can have very different yet observable effects. For example, eV-scale sterile neutrinos could solve neutrino oscillation anomalies\(^\text{1,2,3,4,5}\), while keV-scale sterile neutrinos can be dark matter candidates\(^\text{6,7}\). Above $10^9$ GeV, they could explain the observed baryonic asymmetry of the Universe through high-scale leptogenesis\(^\text{8,9}\). Sterile neutrinos in the range MeV–GeV have been introduced in minimal models like the $\nu$MSM\(^\text{6,10}\) and lead to visible effects in meson decays\(^\text{11,12,13,14}\).

We present here results of a study\(^\text{15}\) where we focused in particular on the impact of heavy sterile neutrinos on rare kaon leptonic and semileptonic decays with final state neutrinos. These charged kaon decays are searched for at NA62\(^\text{16,17}\) while neutral kaon decays are studied by KOTO\(^\text{18}\) and NA64\(^\text{19,20,21}\). Our study can be useful as well for the TREK/E36 experiment at J-PARC, where the data analysis is currently under way\(^\text{22}\). It will further test the lepton universality in kaon two-body decays ($K_{e2}$) and search for a heavy neutrino\(^\text{23}\).

2 A simplified model

In order to capture the generic effects due to the presence of heavy neutrinos, we use a simplified $3 + 1$ model with Majorana neutrinos. Three of them are light and responsible for neutrino oscillations while the fourth is heavier, its mass being a free parameter in our study. An immediate
Table 1: Input parameters in our random scan. Both samples were combined for this study.

| Parameter          | Sample 1                  | Sample 2                  |
|--------------------|---------------------------|---------------------------|
| Number of points   | 200000                    | 40000                     |
| $m_1$ (eV)         | $[10^{-21}; 1]$           | $[10^{-21}; 1]$           |
| $m_4$ (GeV)        | [0.1; 1]                  | [0.27; 0.35]              |
| $\theta_{14}, \theta_{24}$ | $[10^{-6}; 2\pi]$     | $[10^{-6}; 2\pi]$     |
| $\theta_{34}$      | $[10^{-6}; 2\pi]$         | [0.1; 2\pi]               |
| $\delta_{13}, \delta_{41}, \delta_{43}$ | [0; 2\pi]                | [0; 2\pi]                 |

The consequence is the modification of the charged and neutral currents as

$$\mathcal{L}_{W^\pm} = -\frac{g_2}{\sqrt{2}} W_\mu^- \ell_\alpha U_{\alpha i} \gamma^\mu P_L \nu_i,$$

$$\mathcal{L}_Z = -\frac{g_2}{4 \cos \theta_W} Z_\mu \bar{\nu}_i \gamma^\mu \left[ P_L (U^\dagger U)_{ij} - P_R (U^\dagger U)_{ij}^* \right] \nu_j,$$

where $g_2$ is the weak coupling constant, $\theta_W$ the weak mixing angle and $U$ is a $3 \times 4$ mixing matrix. Being rectangular, $U$ is obviously non-unitary but it nonetheless verifies $\sum_{i=1}^4 |U_{\alpha i}|^2 = 1$. It arises from the diagonalisation of the neutrino and charged lepton mass matrices and is defined as

$$U_{\alpha i} = \sum_{k=1}^3 V_{k\alpha}^* U_{\nu_k i},$$

with

$$\ell'_L = V \ell_L, \quad \nu'_L = U_\nu \nu_L.$$

where $\ell'$ and $\nu'$ are the weak gauge eigenstates while $\ell$ and $\nu$ are the mass eigenstates, $V$ and $U_\nu$ being unitary matrices. Thus, this modified lepton mixing matrix depends on the active-sterile mixing angles which are free parameters of our study as well.

### 3 Scan of the parameter space

Our study has 8 free parameters: the mass of the lightest neutrino $m_1$, the mass of the heavy neutrino $m_4$, the new active-sterile mixing angles $\theta_{14}$, $\theta_{24}$, $\theta_{34}$ used to build $U$ according to Eq. 4 of our study\textsuperscript{15} as well as the Dirac CP-violating phases $\delta_{13}$, $\delta_{41}$, $\delta_{43}$. We have explicitly checked that the three Majorana phases which are present as well do not significantly affect our result, either cancelling or giving a contribution suppressed by the light neutrino masses. The light neutrino mass differences and mixing are chosen according to the best fit point of a recent global fit to neutrino oscillation data\textsuperscript{24}. We performed three random scans of the parameter space, using flat priors on the Dirac CP phases and logarithmic priors on all other scan parameters, with the size of our samples and the ranges considered given in Table 1.

Focusing on the region where the heavy neutrino mass is between 0.1 and 1 GeV, we need to include experimental constraints on the active-sterile mixing. First, we consider the limits coming from the direct searches\textsuperscript{25} which give the strongest constraints for most of the parameter space. Then, we include constraints from both radiative lepton flavour violating decays and 3-body decays, comparing our predictions based on the formulas derived by Ilakovac et al.\textsuperscript{26} with the MEG\textsuperscript{27}, SINDRUM\textsuperscript{28}, Belle\textsuperscript{29} and BaBar\textsuperscript{30} results. We also compare our prediction of $W \to \ell \nu$, $Z \to \nu \nu$ and $\tau \to \ell \nu \nu (\ell = e, \mu)$, which agree with a previous independent calculation\textsuperscript{14}, with LHCb\textsuperscript{31} and LEP measurements\textsuperscript{32}. We include as well lepton universality tests in pion decays\textsuperscript{14}. Finally, we check that for the model used in this study and after applying all other constraints the Fermi constant $G_F$ is to an excellent approximation equal to $G_\mu$, its value extracted from muon decays. Points surviving constraints are presented in Fig. 1.
4 Numerical results

In this work, we focused on processes for which hadronic theoretical uncertainties are under control and which contain final state on-shell neutrinos, namely $K_{\ell 2}$, $K_{\ell 3}$, $K \to \pi \nu \nu$ and $K_L \to \nu \nu$. Detailed formulas including massive neutrinos can be found in our main article.\(^{15}\)

4.1 Leptonic decays, $K_{\ell 2}$

To better understand the impact of a heavy neutrino on these decays, we will first present analytical formulas for $\text{Br}(K \to \ell \nu)$ and highlight the difference with the SM. Shrock\(^{11,12}\) first pointed out the impact of sterile neutrinos on kaon and pion decays and their use to put bounds on neutrino masses and lepton mixing matrix elements. Since the heavy neutrino escapes the detector unobserved, one has to sum over all kinematically accessible neutrinos, whose number is denoted here by $N$, giving the expression

$$
\text{Br}(K \to \ell \nu) = \frac{G_F^2 \tau_K}{8 \pi m_K^2} |V_{us}|^2 f_K^2 \sum_{i=1}^N |U_{\ell i}|^2 \lambda^{1/2}(m_K^2, m_{\ell i}^2, m_{\nu i}^2) \left[ m_K^2 (m_{\ell i}^2 + m_{\nu i}^2) - (m_{\ell i}^2 - m_{\nu i}^2)^2 \right],
$$

where $\lambda$ is the Källén function, $\tau_K$ is the kaon lifetime, $f_K$ is the kaon decay constant and $V$ is the CKM matrix. First, the presence of a heavy neutrino modifies $U$ such that $\sum_{i=1}^3 |U_{\ell i}|^2 < 1$ which leads to non-unitarity effects for the light neutrino contributions, even when the heavy neutrino is not kinematically accessible. Second, the decay $K_{\ell 2}$ is helicity suppressed and the presence of a heavy neutrino in the final state would lift this helicity suppression, increasing the corresponding partial width. The above formula can be extended to other leptonic decays of pseudoscalar mesons by substituting $\tau_K$, $f_K$ and $V_{us}$ with the corresponding parameters.

Nowadays, lattice computations of $f_K$, and especially of $f_K/f_\pi$, have reached a sub-percent accuracy\(^{33}\), allowing to distinguish new physics effects at the percent level or smaller from the SM prediction. Our predictions are presented in Fig. 2 for the decay $K \to e \nu$, where we used the ratio

$$
R_O = \frac{O}{O_{\text{SM}}},
$$

of an observable $O$ to its SM value. We can see that a heavy neutrino with a mass between 400 MeV and $m_K$ can induce deviations at the percent-level while being agreement with all other experimental constraints. This demonstrates the potential of $K_{\ell 2}$ to provide additional constraints due to the partial lifting of the helicity suppression coming from an extra sterile neutrino. This was confirmed by investigating the $K_{\mu 2}$ decay, where the helicity suppression is weaker and the maximal deviation consequently smaller. However, the ratio $\Delta r_K = R_{K_{e2}} - R_{K_{\mu 2}} - 1$ allows to derive even stronger bounds as can be seen in Fig. 3 since it has smaller theoretical and experimental uncertainties. This will be even more relevant in the future because of the reduction of the experimental uncertainty by a factor of $\sim 2$ expected by the TREK experiment.\(^{23,22}\)
Figure 2 – $|R_{K_{e2}} - 1|$ as a function of the sterile neutrino mass $m_4$ (left) and of the leptonic mixing $U_{e4}$ (right). The red points agree with all constraints while the blue ones are in conflict with $R_{K_{e2}}^{\text{exp}}$ shown by the dashed line.

Figure 3 – $\Delta r_K$ as a function of the sterile neutrino mass $m_4$. The red points agree with all constraints while the blue ones are in conflict with $R_{K_{e2}}^{\text{exp}}$. The dashed blue line corresponds to the maximally allowed deviation from the experimental measurement.

4.2 Semileptonic decays, $K_{\ell 3}$

We focus here on the decays of $K_L$ which do not suffer from the uncertainties related to the isospin corrections that are present in the decays of charged kaons. These being 3-body decays, they are not helicity suppressed and, therefore, we do not expect a strong deviation from the SM prediction after experimental constraints are taken into account. While the non-unitarity effect and the modification of the phase-space due to the presence of a heavy sterile neutrino if it is kinematically accessible are always present, the stringent limits on leptonic mixing strongly limit the size of the allowed deviations. This is evident from Fig. 4 (left) where we present our predictions for the decays $K_L \rightarrow \pi^-e^+\nu$. We checked as well that no sizeable deviations are present in the lepton polarization asymmetry and in the forward-backward asymmetry.

4.3 Loop-induced weak decay $K \rightarrow \pi\nu\nu$

The semileptonic decays $K \rightarrow \pi\nu\nu$ is of particular interest since it is dominated by short-distance contributions which arise at the one-loop level. As a consequence, this process is especially
sensitive to the presence of new heavy particles in the loop. Thus a precise measurement of these decays would provide new constraints or could point towards the existence of new physics. It is worth noting that a control over the remaining long-distance hadronic contribution to the charged mode has recently been achieved, allowing for a reduced theoretical uncertainty. Both neutral and charged decays are also subjects of intense experimental searches at NA62 and KOTO. Our predictions for the branching ratio of $K_L \rightarrow \pi^+\nu\nu$ can be found in Fig. 4 (right).

While a heavy neutrino can induce percent-level deviations, the theoretical uncertainties on the SM predictions are at the 10% level. We obtained similar results for $K_L \rightarrow \pi^0\nu\nu$, which effectively makes these decays blind to the presence of an extra sterile neutrino.

4.4 “Invisible decay” $K_L \rightarrow \nu\nu$

The last process considered in this work is the decay $K_L \rightarrow \nu\nu$. In the SM, the branching ratio of this helicity-suppressed decay is exactly zero with massless neutrinos. When massive neutrinos are considered, we get the following result

$$\text{Br}(K_L \rightarrow \nu\nu) = \sum_{i,j=1}^{4} \left(1 - \frac{1}{2}\delta_{ij}\right) \frac{\alpha^2_{em} G_F^2 f_{K_L}^2}{8\pi^3 m_K^2 \sin^2 \theta_W} f_K^2 \lambda^{1/2}(m_K^2, m_{\nu_i}^2, m_{\nu_j}^2)$$

$$\times \left| \sum_{\ell \in \{e, \mu, \tau\}} \text{Re}(\lambda_c X_{\ell}^c + \lambda_t X_t) U_{\ell i}^* U_{\ell j} \right|^2 \left(m_{\nu_i}^2 + m_{\nu_j}^2 - (m_{\nu_i}^2 - m_{\nu_j}^2)^2\right)$$

$$+ 2 \sum_{\ell, \ell' \in \{e, \mu, \tau\}} \text{Re}(\lambda_c X_{\ell}^c + \lambda_t X_t) \text{Re}(\lambda_c X_{\ell'}^c + \lambda_t X_t) \text{Re}(U_{\ell i}^* U_{\ell j}^* U_{\ell' i} U_{\ell' j}) m_{\nu_i} m_{\nu_j} m_K^2$$

where, in addition to the quantities previously defined, we have $\lambda_c = V_{cs}^* V_{cd}$, $\lambda_t = V_{ts}^* V_{td}$ and the quark loop functions giving $X_t = 1.47(2)$ for the top contribution and $X_c = 6.5(6) \times 10^{-4}$ for the charm contributions. In the presence of only 3 light massive neutrinos with masses and mixing in agreement with oscillation data, we get an extremely suppressed prediction $\text{Br}(K_L \rightarrow \nu\nu) < 10^{-20}$. As such an observation of this decay with a branching ratio larger by a few orders of magnitude or more would be a clear signal of new physics. Our predictions are presented in Fig. 5 where we see that that a heavy neutrino could increase $\text{Br}(K_L \rightarrow \nu\nu)$ up to $1.2 \times 10^{-10}$. Unfortunately this falls short of the of NA64 expected sensitivity to this decay where this decay could be searched for by using $K^+$ produced from a $K^+$ beam hitting an active target.
sensitive to this decay or similar invisible decays. In any case, an experimental bound on this decay mode would be of great importance for studying physics beyond the SM.

5 Conclusion

In this work, we used a $3+1$ model to describe the effects of a heavy sterile neutrino on leptonic and semileptonic kaon decays, providing a simplified framework that should reproduce the effects of a more realistic model where the SM is extended to include one or more sterile neutrinos and reproduce the low-energy neutrino oscillation data. We have focused on a heavy neutrino with a mass close to the kaon mass and studied in particular the effect of a kinematically accessible sterile neutrino. We found that sizeable deviations from the SM prediction and experimental measurement are expected in $K_e2$ and lepton universality tests, allowing to derive new constraints on the mass and mixing of the sterile neutrino. We also derived the expressions for the semileptonic kaon decays $\text{Br}(K \to \pi \nu \nu)$ and $\text{Br}(K_L \to \nu \nu)$ in the case of massive neutrinos and provide generic results that can be used when studying new physics scenarios in which sterile heavy neutrinos present. Here, the deviations due to a heavy neutrino are much smaller than the experimental or theoretical uncertainties. Finally, we considered the decay $K_L \to \nu \nu$ which would be a smoking gun of new physics if observed due to the extremely suppressed SM prediction, smaller than $10^{-20}$. The presence of a heavy neutrino with a mass below $m_K$ could increase $\text{Br}(K_L \to \nu \nu)$ up to $O(10^{-10})$, a value much above the SM prediction and just a couple orders of magnitude below the expected sensitivity of NA64. This motivates the study of different experimental set-ups that could make use of the large number of kaons produced in fixed target experiments or in low-energy colliders like DAFNE.

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