Galactic and Accretion Disk Dynamos

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Dynamos in astrophysical disks are usually explained in terms of the standard alpha-omega mean field dynamo model where the local helicity generates a radial field component from an azimuthal field. The subsequent shearing of the radial field gives rise to exponentially growing dynamo modes. There are several problems with this model. The exponentiation time for the galactic dynamo is hard to calculate, but is probably uncomfortably long. Moreover, numerical simulations of magnetic fields in shearing flows indicate that the presence of a dynamo does not depend on a non-zero average helicity. However, these difficulties can be overcome by including a fluctuating helicity driven by hydrodynamic or magnetic instabilities. Unlike traditional disk dynamo models, this ‘incoherent’ dynamo does not depend on the presence of systematic fluid helicity or any kind of vertical symmetry breaking. It will depend on geometry, in the sense that the dynamo growth rate becomes smaller for very thin disks, in agreement with constraints taken from the study of X-ray novae. In this picture the galactic dynamo will operate efficiently, but the resulting field will have a radial coherence length which is a fraction of the galactic radius.

1. CONTEXT

The traditional focus of astrophysical dynamo theory has been on stars, where spherical symmetry is a reasonable first approximation, and the inward pull of gravity is balanced by the radial pressure gradient. In spite of the eponymous role of stars in astrophysics, this ignores the importance of magnetic fields in disks, where gravity is balanced by centrifugal forces. This traditional bias can be explained by the fact that we can observe the magnetic fields of at least one star in some detail, whereas the magnetic field of the Galactic disk presents itself as a bewildering mixture of structure on a range of scales. However, recent years have witnessed an accumulation of data concerning the structure of the magnetic field in our galaxy, and in external galaxies. In addition, it has become clear that magnetic fields play a critical dynamical role in accretion disks of all sizes, including some of the most luminous objects in the universe. Here I will summarize recent theoretical progress in understanding disk dynamos. In an unexpected twist, we will see that the role of global helicity in magnetic field generation may be small.
We start by considering the context of disk dynamos. Astrophysical disks can be divided into two general categories, galactic disks and accretion disks. The latter category includes disks around the supermassive black holes, as in active galactic nuclei (AGN), and stellar disks surrounding protostars or members of binary star systems. Although the physical conditions in these disks span an enormous range we will restrict ourselves in only two ways. First, we consider disks that are sufficiently ionized that ohmic dissipation is negligible on the scale of the disk thickness. This may exclude parts of protostellar disks, and some regions in the interstellar medium (ISM). Second, we restrict ourselves to disks that are primarily supported by rotation, and are consequently geometrically thin. There are cases where a large fraction of the disk support comes from radial pressure gradients or from the magnetic field of the accreting object. We ignore these cases because of the complicated physics involved, not because we believe them to be unimportant.

With this in mind, we can summarize the differences between galactic and accretion disks in the following manner. Galactic disks are confined vertically by the gravity of the disk and halo acting together. The typical galactic disk has an angular velocity $\Omega \propto r^{-1}$ over a broad range in radii. The gaseous disk itself is composed of a heterogeneous interstellar medium with a complicated history. It is typically marginally stable against local gravitational collapse. The number of dynamical time scales since the formation of the galaxy is limited, and it is unclear whether or not the magnetic field in the disk can be regarded as having reached a stationary state, or whether initial conditions might be important in understanding its structure. Finally, the orbital period is the longest natural time scale in these systems, followed by the dynamical time scale of local random motions within the disk, followed by the particle collision time in the gas. This is, in turn, greater than plasma time scales, such as the inverse of the ion cyclotron frequency or the inverse of the plasma frequency. Treating the gas in a galactic disk as a fluid is clearly a dangerous approximation, both because of its complicated substructure, and because the hydrodynamic approximation is unlikely to be accurate even within a relatively homogeneous volume of the ISM. Most of the volume of the gaseous disk is occupied by gas that is sufficiently ionized that ohmic dissipation is negligible on disk scales.

In contrast, vertical confinement in accretion disks is supplied by the vertical component of the gravity of the central object. This leads to an orbital frequency
Ω ∝ r^{-3/2} and, through the condition of hydrostatic equilibrium a thickness \( H \propto c_s/\Omega \), where \( c_s \) is the local sound speed. Accretion disks are relatively homogeneous, in the sense that the vertical sound crossing times are short and pressure equilibrium is a good approximation. In the absence of magnetic fields accretion disks are stable, although strongly unstable when they are present. Their age is greater than all other relevant time scales, so initial conditions can be ignored. The time it takes material to spiral inward to the central object is greater than the time scale for local thermal equilibrium, which is greater than an orbital period. Inverse plasma frequencies are typically much less than an orbital period, and usually much greater than the mean free collision time for a particle. Accretion disks are good fluids, although accretion disk coronae are not. Accretion disks are not always good conductors, but the exceptions are cold and difficult to observe.

2. CLUES AND CONSTRAINTS

Our knowledge of astrophysical magnetic fields is never as complete as we would like. For galactic magnetic fields we have a variety of diagnostics which tells us about the current state of the field. Direct observations of evolutionary effects are, of course, impossible. These diagnostics include the intensity and polarization of synchrotron radiation, the polarization of starlight, the polarization of infrared dust emission, Faraday rotation, and Zeeman splitting. (For a general review of galactic magnetic fields see Zweibel and Heiles 1997; Vallée 1997). It is important to note that each of these diagnostics involves other quantities, for example electron density or the physical properties of interstellar dust grains, for which we have only rough estimates. In addition, the direction of the magnetic field can be derived only from Faraday rotation, and only for the component along the line of sight.

Keeping these uncertainties in mind, we note that a rough concurrence among these methods allows us to conclude that the mean value of the magnetic field in the disk is approximately \( 10^{-5.5} \) Gauss, with comparable power in the large scale and ‘random’ (i.e. small scale) components. The large scale field is approximately aligned with the azimuthal direction, but tilted somewhat towards the direction of the local spiral arms. The number of large scale field reversals in the disk is unknown, but cannot be very large, since observations of Faraday rotation tend to give be consistent with a large scale field coherence length of at least several hundreds of parsecs.
Models of galactic magnetic field generation usually assign a rather large role to the galactic shear. We note that this is about $10^{-15}$ sec$^{-1}$ at our position in the Galaxy. Given a galactic disk age of $\sim 10^{10}$ years this gives a maximum growth of roughly 300 e-foldings. There are various suggestions for modifying fundamental physics in order to obtain a large scale primordial magnetic field, but these proposals are all highly speculative. Simply positing a primordial field as an initial condition poses severe problems for the successful standard cosmological model. If we restrict ourselves to magnetic fields generated by the stresses that accompany the formation of a galactic disk, then we obtain large scale seed fields in the range $10^{-18}$ to $10^{-19}$ Gauss (Lazarian 1992; Kulsrud, Cen, Ostriker and Ryu 1997) by invoking the Biermann battery in a realistic protogalaxy (Biermann 1950; for an exposition in English see Kemp 1982). This implies about 30 e-foldings of growth up to the present day, or a galactic dynamo growth rate which is no less than ten percent of the local shear rate. Since the current epoch in the history of our galaxy is unlikely to be special, in the sense that the magnetic field is unlikely to have just reached equipartition with the gas pressure, we would prefer a dynamo growth rate comfortably above this minimum.

For accretion disks we face a major observational difficulty. The magnetic field inside an accretion disk is completely unobservable. However, there are indirect constraints on the magnetic field strength. The luminosity of an accretion disk depends on the mass transport through the disk, and indirectly on the average radial velocity of the disk material. This is related to the dimensionless ‘viscosity’ $\alpha$ by

$$V_r \approx \alpha \frac{c_s^2}{r \Omega}$$

When a magnetic field is present, local instabilities in the field (see below) imply

$$\alpha \approx \frac{B_r B_\theta}{4\pi P} \propto \frac{V_A^2}{c_s^3}$$

In other words, the efficiency of radial mass transport is a measure of the ratio of magnetic pressure to gas pressure in an accretion disk.

For stationary systems this does not allow us to constrain the mean magnetic field, but the evolution of time varying systems is sensitive to the actual value of $\alpha$. In particular, a variety of systems, including dwarf novae and X-ray novae, undergo recurrent transitions between hot, ionized, luminous states and cold, mostly neutral quiescent states. The luminous outburst state is
marked by a relatively high mass flux through the disk while the quiescent state transfers little mass through the disk. Consequently, each system undergoes a thermal limit cycle, in which material accumulated near the outer edge of the disk during quiescence is spread through the disk, and onto the central object, during an outburst (for a review see Cannizzo 1993). Typical bright outbursts are marked by a fast rise and exponential decay. The rise marks appearance and spread of the hot state, typically starting far from the central object. The decay corresponds to the reappearance of the cold state, typically near the outer edge, and the subsequent progress of a cooling front to small radii. The duration of the outburst is sensitive to the rate at which a significant fraction of the total disk mass can be deposited on the central star, and therefore is a direct measure of \( \alpha_{\text{hot}} \), the average value of \( \alpha \) in the hot state. Conversely, the duration of a quiescent phase is a measure of how much mass can be accumulated without forcing the disk into outburst, and is therefore a measure of \( \alpha_{\text{cold}} \). Finally, the shape of the luminosity decay at the end of an outburst is a measure of how the cooling front velocity depends on radius. All of this data can be fit by taking

\[
\alpha \sim 35 \left( \frac{c_s}{r \Omega} \right)^{3/2},
\]

which also fits the difference in time scales between black hole candidate systems, with a central mass \( \sim 7M_\odot \), and white dwarf systems (Cannizzo, Chen, and Livio 1995; Vishniac and Wheeler 1996). The ratio \( c_s/(r \Omega) \) is not necessarily a sign that the orbital velocity of the disk material is directly connected to the dynamo rate. This is also the ratio of the disk height to radius and may have a purely geometric origin.

### 3. LOCAL MAGNETOHYDRODYNAMIC INSTABILITIES IN DISKS

In a purely hydrodynamic disk, i.e. when no magnetic field is present, there are no local instabilities aside from those induced by self-gravity or tidal effects from a companion. This encourages us to treat the evolution of a magnetic field in a smooth background. The dispersion relation is

\[
\left[ 1 - \frac{(x^2 + 3x_A^2)}{(x^2 - x_A^2)^2(1 + \kappa^2)} + \frac{9}{2} \frac{\kappa^2 x_A^2}{(x^2 - x_A^2)^2(1 + \kappa^2)} \right] u_r =
\]

\[
= \frac{9}{4} \frac{\kappa^2}{(1 + \kappa)} \partial_x^2 u_r + \frac{9}{2} \frac{\kappa^2}{(x^2 - x_A^2)} \partial_x u_r,
\]

for radial scales \( \ll r \) and ignoring the vertical structure of the disk (Vishniac and Diamond 1992; Matsumoto
and Tajima 1995). In this equation

\[ x \equiv \bar{\omega} = \frac{\omega}{\Omega} + k_\theta r, \]  

\[ x_A \equiv \frac{\omega_A}{\Omega} = \frac{k \cdot \vec{B}}{(4\pi \rho)^{1/2} \Omega}, \]  

\[ \kappa \equiv \frac{k_\theta}{k_z}, \]  

and \( u_r \) is the radial velocity perturbation. The frequency \( \bar{\omega} \) is the frequency measured by an observer rotating with the local fluid speed and \( \omega \) is the frequency measured by an external observer. Since \( \omega \) is a global quantity, while the dynamics of the perturbation are determined by \( \bar{\omega} \), which is a function of radius, the radial dependence cannot be generally assumed to be described by some radial wavenumber. Here we have taken advantage of the radial dependence of \( \bar{\omega} \) to use \( x \) as a radial coordinate.

In the axisymmetric limit this expression gives an instability. It is less obvious when \( k_\theta \neq 0 \) but this instability is generally present. It was first discovered by Velikhov (1959), and independently by Chandrasekhar (1961), and first applied to accretion disks by Balbus and Hawley (1991). Physically it is related to the famous tethered satellite experiment, except that it works. If magnetic field lines in the vertical or azimuthal direction are perturbed radially, then gas at smaller radii can transfer angular momentum outward to the slower moving gas on the same field line. This works whenever \( \Omega \) increases inward while specific angular momentum increases outward. In a accretion disk the large scale azimuthal field tends to dominate, so the non-axisymmetric case is the most important. One additional subtlety is that local nonaxisymmetric disturbances do not correspond to global linear modes, and only grow \( \sim k_z/k_\theta \) e-foldings before dissipating, but this is sufficient to ensure local instability in any practical sense of the phrase.

Our expectation, based on this linear dispersion relation, is that the dominant modes will have growth rates comparable to \( \Omega \), and wavelengths of roughly \( V_A/\Omega \) in all directions.

### 4. NUMERICAL SIMULATIONS

Linear theory gives us some understanding of the driving force behind the transition to turbulence, and consequently a set of dimensional estimates for the nature of the turbulent regime. However, any hope of obtaining a quantitative understanding of real systems
has to rest with numerical simulations. A number of groups have attempted simulations of the growth of the Balbus-Hawley instability in accretion disks (see, for example Brandenburg, Nordlund, Stein, and Torkelsson 1996; Hawley, Gammie, and Balbus 1996; Stone, Hawley, Gammie, and Balbus 1996). While these simulations have not completely overcome the technical difficulties involved in following MHD turbulence over a broad dynamical range, they do show some common results which we can take as a guide in considering real accretion disks. Since the nature of the simulations may play a large role in the results, we need to consider their common elements. First, in order to reduce the problem to a manageable size, the disk is idealized as a fluid in a shearing flow, with a scale height which is comparable to its radial extent. Rather than simulate an entire annulus the usual procedure is to make the box periodic in the azimuthal direction, with a total length which is typically about $2\pi$ vertical scale heights. (Although there have been simulations with azimuthal lengths up to four times longer.)

What do the results look like? First, naive expectations based on linear theory appear to be correct. There is a transition to turbulence, with the scales expected from the linear analysis. The resulting eddies are moderately anisotropic with $\lambda_\theta > \lambda_r > \lambda_z$.

Second, the evolution of the magnetic field typically has two phases. At first the magnetic field strength grows exponentially, with a rate $\sim \Omega$. However, this growth involves short wavelength components of the field. When this phase saturates, a slower growth appears, in which the large scale field components acquire a substantial fraction of the total magnetic energy. This latter phase frequently includes large scale field reversals, with a frequency which is roughly comparable to the growth rate of the large scale field.

Third, at saturation the field typically shows $\langle v^2 \rangle$ a fraction of $V_A^2$, which is in turn a large fraction of $c_s^2$. We expect $\alpha$ to scale with $(V_A/c_s)^2$, but in practice it remains small, typically less than a percent. However, the value of $\alpha$ varies from one simulation to another and appears to increase with increasing numerical resolution. It is plausible to suppose that for realistic Reynolds numbers $\alpha$ would reach reasonable values, although this involves a considerable amount of extrapolation.

Finally, one of the more striking features of this work is that the results are not qualitatively different for simulations which include vertical stratification and those that simply confine the fluid in a box with periodic vertical boundary conditions. In other words, vertical sym-
metry breaking does not play an important role in the dynamo present in these simulations.

5. DYNAMO THEORY

5.1. Conventional $\alpha - \Omega$ Dynamos

What generates the large scale field in the simulations, or, for that matter, in astrophysical disks? The usual answer is to appeal to mean field dynamo theory. In the context of strongly shearing astrophysical disks, the evolution equations for the large scale field can be written in a simplified form, i.e.

$$\partial_t B_r \approx -\partial_z (\alpha_{\theta \theta} B_\theta) + \partial_z D_T \partial_z B_r, \quad (8)$$

and

$$\partial_t B_\theta \approx -\frac{3}{2} \Omega B_r + \partial_z D_T \partial_z B_\theta, \quad (9)$$

where $D_T$ is the turbulent diffusivity and

$$\alpha_{\theta \theta} = \langle v_z \partial_\theta v_r - v_r \partial_\theta v_z \rangle \tau. \quad (10)$$

Here $\tau$ is the velocity correlation time. This formulation of mean field dynamo theory is referred to as the ‘$\alpha - \Omega$ dynamo, since the radial field is generated from the azimuthal field by helicity and the cycle is closed by the shearing of the radial field to create azimuthal field.

In order have a non-zero $\alpha_{\theta \theta}$ we need to have some systematic violation of symmetry with respect to the $\hat{z}$ direction. The same is also true for radial and azimuthal motions, but coriolis forces can be relied upon to generate correlations between motions and gradients in these two directions. Vertical symmetry breaking requires the presence of vertical stratification. However, as we saw in the last section, this does not play a crucial role in the simulations. Whatever dynamo is operating in them is indifferent to whether or not $\alpha_{\theta \theta} = 0$.

Notwithstanding this point, there have been several attempts to derive a dynamo theory for accretion disks using magnetic field buoyancy, or more specifically, the Parker instability (see, for example Tout and Pringle 1992). These models all face a basic theoretical problem. The growth rate for the Parker instability is of order $(V_A/c_s)\Omega$ with the fastest growing modes having azimuthal wavelengths similar to the pressure scale height, or $c_s/\Omega$. Shearing constraints imply that the corresponding radial wavelengths are of order $V_A/\Omega$, which is also the typical radial scale for the Balbus-Hawley instability. Consequently, rising and falling sections of the magnetic field are mixed at a rate $\sim \Omega$. Unless the magnetic field is already strong (i.e. $V_A \sim c_s$) this is much faster than the growth rate of the Parker
instability. In fact, numerical simulations with vertical stratification show little sign of the Parker instability, even when $V_A$ is large.

5.2. Incoherent and Chaotic Dynamos

What are the alternatives to the standard $\alpha - \Omega$ dynamo? One idea is that the magnetic field is sustained through a local, chaotic process, in which local field stretching amplifies the field up to equipartition with the ambient pressure. This picture was originally suggested by Batchelor (1950), although the first detailed treatment is due to Kazantsev (1967). It can be rigorously justified only in the limit of a weak magnetic field, which is never the case when the turbulence itself is driven by the field. In any case, if we accept this possibility in accretion disks then the large scale field would then be explained as the result of some sort of inverse cascade within a turbulent fluid. This model is not consistent with accretion disk phenomenology, in particular the thermal limit cycle and the decay from outburst of dwarf novae and soft X-ray transients mentioned above. It is also unclear why the very largest scales, with wavelengths equal to several eddy scales, always end up with a significant fraction of the total power.

An alternative explanation is that the large scale field is generated by an extension of the $\alpha - \Omega$ dynamo developed by Vishniac and Brandenburg (1997) called the ‘incoherent dynamo’. In the simplest version of this model the vertical symmetry is assumed to be unbroken, so that $\langle \alpha_{\theta\theta} \rangle = 0$. However, at any moment a magnetic domain containing $N$ eddies will have a helicity of

$$\alpha_{\theta\theta} \sim \frac{\alpha_{\theta\theta,E}}{N^{1/2}},$$

where $\alpha_{\theta\theta,E}$ is the helicity associated with a single eddy, which is comparable to the turbulent eddy velocity, $V_T$. In this case equation (8) can be written as a stochastic equation. It is also helpful to rewrite it in terms of the evolution of $\langle B_z^2 \rangle$ or

$$\partial_t \langle B_z^2 \rangle = 2\langle [\partial_z(\alpha_{\theta\theta} B_\theta)]^2 \rangle \tau - 2D_T \langle [\partial_z B_r]^2 \rangle.$$  \hspace{1cm} (12)

(Here the brackets denote only spatial averaging.) Combining equations (5) and (12) we can estimate the incoherent dynamo growth rate as

$$\gamma \approx \left( \frac{\langle \alpha_{\theta\theta,E}^2 \rangle \tau \Omega^2 L_z^2}{L_z^2 N} \right)^{1/3},$$

where $L_z$ is the vertical height of a magnetic domain. This growth is a combination of random walk in $B_r$,
driven by fluctuations in $B_\theta$, and the shearing of $B_r$. The fact that it gives exponential growth results from a tendency for the distribution of $B_r/B_\theta$ to be biased towards negative numbers. When this ratio becomes sufficiently positive the field undergoes a sudden reversal and $B_\theta$ switches sign. Typically non-axisymmetric domains are sheared out faster than they can grow, so the usual expression for $N$ in isotropic turbulence will be

$$N \approx \frac{L_z L_r 2\pi r}{\lambda_T^3}.$$  \hfill (14)

Incoherent dynamos are intrinsically noisy. In addition to the field fluctuations on eddy scales, the large scale field will undergo spontaneous field reversals with a frequency not far below the dynamo growth rate. Furthermore, the coupling between different domain scales implies that there is constant ‘crosstalk’ between different Fourier modes of the large scale magnetic field. Consequently, there are no well-defined linear eigenfunctions of this dynamo. Since individual annuli switch polarity on a regular basis, there seems little chance that the disk magnetic field will become uniform on radial scales larger than a few disk scale heights. Furthermore, this will reduce the strength of any large scale poloidal field produced via magnetic buoyancy. Different disk annuli will contribute randomly to any global field.

Finally, if we compare the growth rate $\gamma$ to the dissipation rate, $\sim V^2 \tau / L^2$, we see that the largest vertical scale domains will accumulate most of the energy. (This line of reasoning can’t be used to argue for larger radial scales since extending magnetic domains radially will lower the growth rate while leaving the dissipation rate unchanged.)

6. THE INCOHERENT DYNAMO IN ASTROPHYSICAL DISKS

6.1. Accretion Disks

If we wish to apply the incoherent dynamo to accretion disks then the obvious source of small scale turbulence is the Balbus-Hawley instability. Aside from the point that this is the only source of turbulence which is guaranteed to accompany a successful dynamo, only very strong convection is likely to survive the turbulent mixing caused by magnetic field instabilities. In this case we can write the dynamo growth rate as

$$\gamma \sim \left[ \left( \frac{V_A}{c_s} \right)^5 \frac{H}{r} G(\beta) \right]^{1/3},$$  \hfill (15)
where $G(\beta)$ describes the saturation of this mechanism as the ratio of magnetic to ambient pressure ($\beta^{-1}$) approaches unity. Here I have assumed that the magnetic domain is about as thick and wide as a disk vertical scale height. The dissipation rate is proportional to $\beta^{-1}\Omega$. This implies that the growth rate of the dynamo scales as the magnetic field strength to the $5/3$, while the dissipation rate scales as the magnetic field strength squared. Consequently, the saturated state will be sensitive to other aspects of the model, including numerical viscosity in the computer simulations.

We can get a sense of how this works for accretion disks by assuming

$$\alpha = \frac{1}{3} \left( \frac{V_A}{c_s} \right)^2, \quad (16)$$

and

$$G = 1 - \frac{V_A^2}{c_s^2}. \quad (17)$$

Both of these are meant to be illustrative rather than serious predictions, however they have roughly the properties we expect for the exact solution. The function $G$ should cut off sharply as $V_A \rightarrow c_s$, since in this limit the Balbus-Hawley instability disappears. Furthermore, for $V_A \ll c_s$ we expect $G$ to have a leading order correction term of order $\beta^{-1}$. The scaling of $\alpha$ is roughly consistent with the numerical simulations, but a bit on the high side, reflecting our expectation that current simulations tend to underestimate its value. Balancing dynamo growth and turbulent dissipation we find that

$$\alpha = \frac{C_0}{3} \left( \frac{c_s}{r\Omega} \right)^2 (1 - 3\alpha)^2. \quad (18)$$

The value of $C_0$ is difficult to estimate, and in any case is raised to such a high power that it has to be regarded as essentially a free parameter.

Applying equation (18) to real disks requires us to fit to phenomenological models of dwarf novae and soft X-ray transients. If we take $C_0 \sim 3$ then we can produce an acceptable fit to equation (18). In this case we find that for values of $c_s/(r\Omega)$ between 1 and 1/4 the predicted value of $\alpha$ drops from 0.32 to 0.29, i.e. negligibly. For values of $c_s/(r\Omega)$ more appropriate for dwarf novae systems, in the range 0.04 to 0.025, $\alpha$ drops to the range 0.15 to 0.1, with a slope with respect to $c_s/(r\Omega)$ of 0.75 to 1. Finally, if we take $c_s/(r\Omega)$ down to one percent, as expect for soft X-ray transients, we get $\alpha \approx 0.03$ with a slope of $\sim 5/3$. These values and slopes are consistent with models of these systems and with the results of computer simulations. The extremely weak response of $\alpha$ to changes in the disk height to radius ratio when
that ratio is not extremely small seems a bit odd. However, it is mostly the result of taking \( C_6/3 \) large, which is required by the thin disk models. A considerably smaller contribution to this effect comes from the sharp cutoff in \( G \) as \( V_A \to c_s \). Both of these effects are intrinsic to the incoherent dynamo model and would be expected in any phenomenologically acceptable version of the model.

6.2. Galactic Disks

Aside from the differences already noted between galactic and accretion disks, there is another point which is critical for any application of the incoherent dynamo to galactic disks. Since galactic magnetic fields start out weak, the scale of turbulence due to magnetic instabilities would have been small, and the incoherent dynamo would have been relatively ineffective. In order to have a strong dynamo from very early times we need to appeal to other sources of turbulent motion. In the case of a galactic disk, one plausible source would be local gravitational instabilities. Another might be violent outflows from star forming regions. In either case it is difficult to assign length scales and velocities from first principles.

Suppose we take the point of view that the kinds of motions present at early times were not very different from what we see today. If we take

\[ V_T \sim 10 \text{ km/sec}, \]  \hspace{1cm} (19)

\[ L_T \sim 300 \text{ parsecs}, \]  \hspace{1cm} (20)

and assume a magnetic field vertical scale of 1 kpc, then we get a growth rate of

\[ \gamma \sim 10^{-16} \text{ sec}^{-1}, \]  \hspace{1cm} (21)

with a slightly smaller dissipation rate. This estimate is just marginally fast enough, but ignores factors of order unity, which are bound to be important in this case. The only conclusion we can draw from this exercise is that it is possible that the incoherent dynamo is responsible for the growth of large scale galactic fields, but any real answer will require a firmer understanding of turbulence in the Galactic disk. On the other hand, the incoherent dynamo does make a testable prediction. Since the growth time is only slightly less than the reversal time, and since the typical magnetic domain has a radial extent comparable to the disk thickness, it follows that we should expect the large scale \( B_\theta \) to reverse over radial scales slightly larger than the disk thickness. This is consistent with current observations (see references in Zweibel and Heiles 1997), but the number of galaxies with observed reversals is still very small.
7. SUMMARY

We note several points in conclusion. First, disk dynamos do not require an average fluid helicity. They may require a mean square helicity, but this is a by-product of turbulence in general. Second, incoherent dynamo effects match phenomenological constraints on accretion systems. They are not inconsistent with numerical simulations, but are not yet clearly confirmed by such work. A clear signature of their presence would be a turn-down in the value of $\alpha$ in the limit of very long computational boxes. Third, the incoherent dynamo may be relevant for the rapid growth of galactic fields. However, models are sensitive to assumptions about the properties of turbulence in galactic disks. The only clear prediction is that large scale field reversals should be common on radial scales of a kiloparsec or more.

Fourth, at odds with the general theme of this conference, it is difficult to find a major role for either fluid or magnetic helicity in simulations of disk dynamos, or, perhaps, inside real astrophysical disks. The interaction of the disk field with its environment may present a mechanism for the generation of magnetic helicity by disks (cf. R. Matsumoto's contribution to this volume).

Acknowledgments. The work presented here was supported in part by part NAG5-2773 and NSF grant AST-9318185 (ETV). I am grateful for a number of helpful discussions with A. Brandenburg and E. Zweibel as well as the hospitality of MIT and the CfA for the 1997-98 academic year.

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