Cosmological model with movement in fifth dimension

W. B. Belayev *
Center for Relativity and Astrophysics,
185 Box , 194358, Sanct-Petersburg, Russia

March 24, 2022

Abstract

Presented cosmological model is 3D brane world sheet moved in extra dimension with variable scale factor. Analysis of the geodesic motion of the test particle gives settle explanation of the Pioneer effect. It is found that for considered metric the solution of the semi-classical Einstein equations with various parameters conforms to isotropic expanded and anisotropic stationary universe.

1 Introduction

Although conception of expanded universe is predominant, stationary model is also suggested [1], [2]. Godel [3] investigated a zero expansion model with global rotation. However, observations show that rotational effect can’t be main cause of the cosmological redshift. We consider metric for five-dimensional space-time, which produce model of flat space with movement along of additional coordinate. Kaluza- Klein theory proposes internal spaces with Plank size to form an extra dimension. Besides this approach, non-compactified interpretation of the fifth dimension is treated [4]. Machian interpretation of the Kaluza-Klein theory is in [5]. In framework of the brane world model the possibility that 3-brane worldsheet moves in higher dimensional space is discussed in [6]. Cosmology of the Randall-Sundrum (RS) model, considered as alternative of the compactification, with radion stabilization is analyzed in [7]. Charged Black Holes [8] and dynamical effects at the brane [9] are studied in generalized RS model. The fifth dimension was introduced for unification gravitational and electromagnetic fields [10], [4]. Geodesic deviation in multidimensional theories in presence of the Lorentz force is considered in [11]. On physical interpretation of the fifth dimension [12], [13] the rest mass $m$ changed with time is assumed to be $m \sim y$, where $y$ is fifth coordinate. Multidimensional theories yield appearance in four-dimensional space-time extra forces [14], [15]. Analysis of the bulk geodesic motion of the particle under influence of the extra non-gravitational force in brane world [16] admits the energy generation, caused by change of the proper mass.

2 5D field equations and its solution

Via a choice of units the speed of light $c$ and gravitational constant $G$ are rendered to $c = 1$ and $8\pi G = 1$. The field equations are taken to be 5D version [17] of the usual 4D
Einstein field equations, namely

\[ G^{ij} = T^{ij}, \]  

(1)

where \( G^{ij} \) is 5D analogy of the Einstein tensor, and \( T^{ij} \) is 5D extension of 4D energy-momentum tensor taken in general form \([13]\). With coordinates \( x^0 = t \) (time), \( x^{1,2,3} = \eta^{1,2,3} \) (spacelike coordinates) and \( x^4 = y \) (extra coordinate) the line element is given by

\[ ds^2 = dt^2 + 2B(t)dtdy - B(t)^{1/2}dy^2 + 2AB(t)^{1/2}dy + (1 - h^2)B(t)dy^2, \]  

(2)

where \( B(t) \) is function of \( t, h, q \) and \( A \) are constant parameters, for definiteness \( B(t) > 0 \). In order to avoid the existence of closed timelike curves is required \( h > 1 \). The case \( q \neq 0 \) correspond to the space with changed scale factor, with \( q = 0 \) 3D space is stationary.

With denotation \( F(t) \equiv (\partial B/\partial t)/B \) and \( Q(t) \equiv (\partial^2 B/\partial t^2)/B \) for \( q \) being equal 1 or 0 the non-vanishing field equations \([13]\) are following:

\[ \mu_g = [(1 + 47q)A^2 + (1 + 23q)A^4 - (1 + 47q)h^2A^2 + 24q(1 - 2h^2 + h^4)]F^2/[4(A^2 - h^2)^2], \]  

(3)

\[ S_1 = A[A^2(1 + 23q) + 24q(1 - h^2)]F^2/[4(A^2 - h^2)^2B^2], \]  

(4)

\[ S_4 = -[A^2(1 + 23q) + 24q(1 - h^2)]F^2/[4(A^2 - h^2)^2B], \]  

(5)

\[ \sigma_{11} = ([A^2h^2(1 + 11q) + 12q(A^2 + h^2 - h^4)])F^2 + 4(1 + 2q)[-A^2 + h^2 + A^2h^2 - h^4]Q/[4(A^2 - h^2)^2B^2], \]  

(6)

\[ \sigma_{22} = \sigma_{33} = ([A^2(1 - 13q) + 12q(-1 + h^2)]F^2 - 4(1 + 2q)[1 + A^2 - h^2]Q/[4(A^2 - h^2)^2B^2]), \]  

(7)

\[ \sigma_{14} = A([(-1 + 13q)A^2 + 2(1 - 7q)h^2 + 24q]F^2 + 2(1 + 5q)[A^2 - h^2]Q/[4(A^2 - h^2)^2B^1+q]), \]  

(8)

\[ \sigma_{44} = ([A^2(1 + 11q) + 24 - 12h^2]F^2 + 12q[A^2 - h^2]Q/[4(A^2 - h^2)^2B^2]), \]  

(9)

where \( \mu_g \) is mean energy density in the universe, \( S_i \) are components of vector of the energy flow, \( \sigma_{ij} \) is the 4D analogy of the 3D strain tensor. Energy conservation law \( \partial T^{ij}/\partial x^j = 0 \) yields relation

\[ \frac{\partial \mu_g}{\partial t} + \frac{\partial S_1}{\partial t} + \frac{\partial S_4}{\partial t} = 0. \]  

(10)

### 3 Geodesic motion

Denoting five-velocities as \( u^i = dx^i/ds \) let us find a solution of the geodesic line equations for particle with non-zero rest mass in form

\[ \frac{d}{ds}(g_{ij}u^i) - \frac{1}{2} \frac{\partial g_{mj}}{\partial x_i} u^m u^j = 0, \]  

(11)

where \( g_{ij} \) is metrical tensor. Spatial components are satisfied with comoving coordinates of the conventional type \( u^1 = u^2 = u^3 = 0 \). Then zero, first and fourth components of the
equations (11) yield
\[
\frac{d}{ds}(u^0 + Bu^4) - \frac{\partial B}{\partial t} u^0 u^4 - (1 - h^2) B \frac{\partial B}{\partial t} u^4 u^4 = 0, \\
\frac{d}{ds}(AB^{1+q} u^4) = 0, \\
\frac{d}{ds}[Bu^0 + (1 - h^2)B^2 u^4] = 0. 
\] (12)
A solution of these equations has to be compatible with the condition set by metric (2), which is
\[
(u^0 + Bu^4)^2 - h^2 B^2 (u^4)^2 = 1. 
\] (13)
In cases \(q = 0\) and \(q \neq 0\), \(A = 0\) equations (12) not constraining the signs of the velocities turn out to
\[
u^0 = \pm (h^2 - 1)^{0.5}/h, \quad u^4 = \pm 1/[Bh(h^2 - 1)^{0.5}]. 
\]
The ratio of these equations gives
\[
\frac{dy}{dt} = 1/[(h^2 - 1)B]. 
\] (14)

4 The Pioneer effect

In terms of the action of the system along of the world line, Lagrangian is written as
\[
L = -m[1 - B^{2q}\dot{\eta}^2 + 2B\dot{y} + 2AB^{1+q}\dot{\eta}^1 \dot{y} + (1 - h^2)B^2 \dot{y}^2]^{0.5}, 
\] (15)
where \(m\) is the rest mass of the particle, and overdot denotes derivative with respect to \(t\). Proposed model suggests explanation of the Pioneer effect [19], [20]. In comoving coordinate system with \(\dot{y} = 0\) for located at the distance \(r\) particle, moved slowly in weak gravitation field, Lagrangian (15) turn out
\[
L = -m[1 - \frac{1}{2}B^{2q}\dot{\eta}^2 + B\delta\dot{y} + AB^{1+q}\dot{\eta}^1 \delta\dot{y} + \frac{1}{2}(1 - h^2)B^2 \delta\dot{y}^2 + \varphi], 
\] (16)
where \(\varphi\) is gravity potential, \(\delta\dot{y}\) is difference of the velocity along of the fifth coordinate on the light cone and in locally scaled coordinates, which is assumed to be small. Taking into account relation (14) for geodesic line, for small variation \(B\), change of \(\dot{y}\) is given by
\[
\delta\dot{y} = F\delta t/[(h^2 - 1)B], 
\] (17)
where \(\delta t\) is time of the passing of the light of the distance from particle. The scale factor in case \(q = 1\) during time interval \(\delta t\) is assumed to be \(B \approx 1\). Then with \(|\delta\dot{y}| << |\dot{\eta}^1|\) equation (16) yields
\[
L = -m[1 - \frac{1}{2}\dot{\eta}^2 + F\delta t/(h^2 - 1) + \varphi]. 
\] (18)
The last two terms in (18) are considered as potential energy of the system
\[
U = m[Fr/(h^2 - 1) + \varphi], 
\] (19)
where \(r\) corresponds to the distance from Sun to the spacecraft. Besides the gravity, one determine the additional force, which yields for \(F > 0\) extra radial acceleration towards the Sun
\[
a_p = F/(h^2 - 1). 
\] (20)
This acceleration amounted \(7.5 \cdot 10^{-8}\) sm/s² [20] takes place without dependence on distance, direction of the radial movement and its and angular velocities that consists with the Pioneer effect.
5 Cosmological models

For expanding 3D space the Hubble parameter of the cosmological redshift is assumed to be

\[ H = F. \]  (21)

Then taking into account constrains for \( H \) equation (20) leads to \( h \approx \sqrt{2} \). With \( A = 0 \) equations (3)-(9) yield isotropic model of the universe with energy density \( \mu_g \approx \frac{3}{2} H^2 \).

The same field equations admit anisotropic stationary model with \( A \neq 0 \), however, in this case the cause of the cosmological redshift is not clear.

6 Conclusion

Presented five-dimensional cosmological model allows explanation of additional force produced Pioneer effect as a result of the movement of the 3D space along extra coordinate. This approach admits existence both isotropic expanded and anisotropic stationary universes, however, the first appears to be more probable.

Acknowledgements

This work was supported by MSO.

References

[1] J. Narlicar, H. Arp, Astrophus. J., 405, 51 (1993).
[2] D.F. Crawford, Astrophus. J., 440, 466 (1995).
[3] K. Godel, Rev. Mod. Phys., 21, 447 (1949).
[4] J.M. Overduin, P.S. Wesson, Phus. Rept., 283, 303 (1997).
[5] H. Liu, B. Mashhoun, Ann. Phys. (Leipzig), 4, 565 (1995).
[6] M. Pavsic, Phus. Lett. A, 116, 1 (1986).
[7] C. Csaki et al., Phys. Rev. D, 62, 045015 (2000).
[8] A. Chamblin et al., Phys. Rev. D, 63, 064015 (2001).
[9] R. Maartens, ”Geometry and dynamics of the brane-world”, invited talk at EREs2000, Spanish Relativity Meeting.
[10] V. Fock, Zeit. Phys. (Leipzig), 39, 226 (1926).
[11] R. Kerner et al., Phys. Rev. D, 63, 027502 (2001).
[12] P.S. Wesson, Astron. Astrophus., 119, 145 (1983).
[13] P.S. Wesson, ”Space-Time-Matter”, World Scientific, Singapore, 1999.
[14] Y.M. Cho, D.H. Park, Gen. Rel. Grav., 23, 741 (1991).
[15] P.S. Wesson et al., Phus. Lett. B, 456, 34 (1999).
[16] D. Youm, Phys. Rev. D, 62, 084002 (2000).
[17] M.J. Reboucas, A.F.F. Teixeira, *Int. J. Mod. Phys. A*, **13**, 3181 (1998).

[18] L.D. Landau, E.M. Lifshitz, "Teoriya Polja", Nauka, Moscow, 1973 (in Russian) pp. 105-109. [[L.D. Landau, E.M. Lifshitz, Classical Theory of Fields, Pergamon Press, Oxford, 1980 (in English).]]

[19] J.D. Anderson et al., *Phys. Rev. Lett.*, **81**, 2858 (1998).

[20] S.G. Turushev et al., "The Apparent Anomalous, Week, Long-Range Acceleration of Pioneer 10 and 11", XXXIV-th Rencontres de Morion Meeting on Gravitational Waves and Experimental Gravity 1999, Les Arcs, Savoi, France, 1999.

[21] A.H. Jaffe et al., *Phys. Rev. Lett.*, **86**, 3475 (2001).