The Modified Probability Hypothesis Density Filter With Adaptive Birth Intensity Estimation for Multi-Target Tracking in Low Detection Probability

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ABSTRACT The existing Probability Hypothesis Density (PHD) filters with birth intensity estimation only operate on single or two consecutive scan data for multi-target tracking. However, for those targets with low detection probability, it is hard to achieve a satisfactory level of track initiation and maintenance. To overcome the weakness above, we propose a modified PHD filter with adaptive birth intensity estimation. The core of the proposed filter is to define two state sets as the formal set and the temporary set. In the framework of measurement driven estimation, we classify the measurements into three categories depending on whether it is in the neighborhood of the state in above two sets. And the birth states of the formal set and the temporary set are generated by the classified measurements respectively. In addition, if there is no matching measurement for the state in the formal set, duplicate the corresponding state as the birth state of the temporary set. For each state in temporary set, we introduce a forgetting factor and a dynamic detection probability in filter to cope with the rapid decrease of its intensity due to the absence of measurement. If its forgetting factor is over the dead threshold, the state will be deleted from the set. Based on the principles above, we derive the Gaussian-mixture (GM) implementation of the PHD filter proposed in this paper. Experiment results show that, in low detection probability scenario, the modified PHD filter outperforms other PHD filters with birth intensity estimation.

INDEX TERMS Probability hypothesis density filter, multi-target tracking, birth intensity estimation, low detection probability.

I. INTRODUCTION

With the rapid development of sensors, multi-target track has become a research hotspot in recent years [1]–[12]. Generally, there are two branches in multi-target track method. One branch is the track methods with the help of the traditional data association techniques to find the exact relationship between the sensor measurement and the target state. Typical methods in this branch are nearest neighbor (NN) algorithm [3], joint probabilistic data association (JPDA) algorithm [4], [5], multiple hypothesis tracking (MHT) [6], [7] and etc. The other branch is the track method in the framework of the random finite set (RFS). As the RFS theory is a systematic and direct procedure to estimate the numbers and states of the multiple targets without dealing with the complex association between target states and measurements, it draws considerable attentions in multi-target track area [8], [9]. The Probability Hypothesis Density (PHD) filter is the most popular and typical filter among those track methods based on RFS [10]–[12]. It is the first order statistical moment of the RFS, which is an approximation solution to the optimal multi-target Bayes filter.

The traditional PHD filter assumes the birth intensity as prior information. However, in many real cases, multiple targets occur at random in the surveillance field. And the distribution of new born target is unknown. The tracking performance will degrade significantly without target birth prior information. In existing literature, the PHD filter with birth intensity estimation is based on the principle of measurement driven. Ristic et al. designed an adaptive target birth intensity estimation strategy for PHD and CPHD filter [13], [14]. The target birth intensity is approximated by the newborn particles drawn from each of the measurement with same weight. And this idea is also applied into the Cardinality
Balanced Multi-Target Multi-Bernoulli (CBMeMBer) filter [14–16] only presents the SMC (Sequential Monte Carlo) implementations of the filter proposed. In [17], Houssineau and Laneuville deduced its Gaussian mixture GM-PHD version under the assumption that the received measurements can entirely embody new born targets. Inspired by [13] and [17], Zhu et al. [18] proposed an improved version of the algorithm. To make it closer to the truth, the proposed algorithm directly estimates the unmeasured component, velocity, with the help of the two-point measurement difference in consecutive scan [19]. To cope with strong clutter situation, [20] presents a dual threshold particle PHD filter with unknown target birth intensity. Different from above solutions based on direct measurement-driven estimation, [21] and [22] give another way to deal with the target birth intensity. Before the PHD update operator, both introduce extra steps to distinguish the newborn targets originated measurements and other measurements. In [21], the birth intensity is estimated by two steps. The first step is based on the criterion of maximum a posteriori (MAP) relating to the entropy distribution of the intensity weight. The second step is based on the coverage rate. In [22], the birth intensity is estimated based on iterative random sample consensus (RANSAC) algorithm within a sliding window.

All the above PHD filters with birth intensity estimation only operate on the current or the current and previous consecutive scan data for multi-target tracking, except for [22]. But in the iterative RANSAC based filter, in essence, the line model is constructed by using the measurements of the current scan and the previous scan. Measurements of other scans only play a role of verification. Thus, for those targets with low detection probability, it is hard to maintain the accuracy and time cost of track initiation to a satisfactory level when continuously missing measurements. In recent years, passive radar has attracted great interests due to its low-cost and feasibility of various illuminators [23]. One of its main challenges is the low detection probability due to the difficulties in accurate synchronization [24]. It is necessary to explore an effective track filter under the low detection probability situation. On the basis of the existing algorithms, we propose a PHD filter with modified adaptive birth intensity estimation in this paper. The core of the proposed filter is to define two state sets as the formal set and the temporary set. In the framework of measurement driven estimation, we classify the measurements into three categories depending on whether it is in the neighborhood of the state in the above two sets. And the birth states of the formal set and the temporary set are generated by the classified measurements respectively. In addition, if there is no matching measurement for the state in the formal set, duplicate the state as the birth state of the temporary set. For each state in temporary set, we introduce a forgetting factor and a dynamic detection probability in filter to cope with the rapid decrease of its intensity due to the absence of measurement. If its forgetting factor is over the dead threshold, the state will be deleted from the set. Based on the principles above, we derive the Gaussian-mixture (GM) implementation of the PHD filter proposed in this paper. Experiment results show that, in low detection probability scenario, the modified PHD filter outperforms other PHD filters with birth intensity estimation.

The remaining part is organized as follows. Section II introduces the background on the multi-target tracking algorithm under RFS. Section III is the modified PHD filter with adaptive birth intensity estimation. Section IV introduces the Gaussian-mixture implementation of the proposed PHD filter. Section V is the simulation and result. Section VI is conclusion.

II. MULTI-TARGET TRACKING ALGORITHM UNDER RFS

Multi-target Bayes filter under RFS framework has been a popular systematic approach to multi-target tracking. For multi-target track, the main goal is to track and estimate the target states correctly based on sensor measurements.

A. THE RANDOM FINITE SET MODEL FOR MULTI-TARGET TRACKING

Both target states and measurements can be modeled in RFS. Suppose $\mathbf{x}_k$ and $\mathbf{z}_k$ represent the RFS of target states and measurements at time step $k$ respectively.

$$
\mathbf{x}_k = \{x_k^{(1)}, \ldots, x_k^{(\text{card} (\mathbf{x}_k))}\}, \quad \mathbf{z}_k = \{z_k^{(1)}, \ldots, z_k^{(\text{card} (\mathbf{z}_k))}\}
$$

(1)

where $\text{card} (\cdot)$ denotes the cardinality of the set. (2) and (3) describe the composition of $\mathbf{x}_k$ and $\mathbf{z}_k$ respectively.

$$
\mathbf{x}_k = \mathbf{x}^P_{k|k-1} \cup \mathbf{x}^E_{k|k-1} \cup \mathbf{x}^B_{k|k-1}
$$

(2)

The target states set $\mathbf{x}_k$ is the union of the surviving target states $\mathbf{x}^P_{k|k-1}$ from $\mathbf{x}_{k-1}$, the extended target states $\mathbf{x}^E_{k|k-1}$ from $\mathbf{x}_{k-1}$ and the new born target states $\mathbf{x}^B_{k|k-1}$.

$$
\mathbf{z}_k = \xi_k \cup \mathbf{c}_k
$$

(3)

The measurements set $\mathbf{z}_k$ is the union of the measurements set $\xi_k$ generated by targets and the false alarm measurements set $\mathbf{c}_k$.

B. BAYESIAN TRACKING THEORY

In the Bayesian track theory, there are two main procedures, that is prediction and updating.

For prediction,

$$
P_{k|k-1}(\mathbf{x}_k | \mathbf{z}_{1:k-1}) = \int f_{k|k-1}(\mathbf{x}_k | \mathbf{x}) p_{k-1}(\mathbf{x} | \mathbf{z}_{1:k-1}) d\mathbf{x}
$$

(4)

For updating,

$$
P_{k|k}(\mathbf{x}_k | \mathbf{z}_{1:k}) = \frac{g_k(\mathbf{z}_k | \mathbf{x}_k) P_{k|k-1}(\mathbf{x}_k | \mathbf{z}_{1:k-1})}{\int g_k(\mathbf{z}_k | \mathbf{x}) P_{k|k-1}(\mathbf{x} | \mathbf{z}_{1:k-1}) d\mathbf{x}}
$$

(5)

where $P_{k|k-1}(\mathbf{x}_k | \mathbf{z}_{1:k-1})$ is the prediction probability of the multi-targets. $P_{k|k}(\mathbf{x}_k | \mathbf{z}_{1:k})$ is the updated probability of the
multi-targets. $f_{k|k-1}(X_k | X)$ denotes the multiple target transition density and $g_k(Z_k | X_k)$ is the likelihood function of the measurement $Z_k$.

**C. THE STANDARD PHD FILTER**

Due to the set integral issue in the Bayes formula, it becomes intractable for the RFS to involve in the Bayesian track theory. To reduce the complexity, Mahler adopted the first order statistical moment $D(x)$ of the RFS to estimate the multi-target states in each iteration and deduced the standard PHD filter [11]. In the framework of the RFS and the Bayesian theory, the recurrence form of the PHD filter is given by (6) and (7).

For prediction,

$$D_{k|k-1}(x) = y_k(x) + \int p_{s,k}(\tau)f_{k|k-1}(x | \tau)D_{k-1|k-1}(\tau) d\tau + \int \beta_{k|k-1}(x | \tau)D_{k-1|k-1}(\tau) d\tau$$

(6)

For updating,

$$D_{k|k}(x) = (1 - p_{D,k}(x))D_{k|k-1}(x) + \sum_{z \in Z_k} p_{D,k}(x) g_k(z | x) D_{k|k-1}(x)$$

$$+ \int p_{D,k}(\tau) g_k(z | \tau) D_{k|k-1}(\tau) d\tau$$

(7)

where $D_{k|k-1}(x)$ denotes the predicted state intensity from time step k-1 to time step k. $D_{k|k}(x)$ denotes the updated state intensity at time step k. $y_k(x)$ is the state intensity function of new born targets. $p_{s,k}(x)$ is the survival probability. $\beta_{k|k-1}(x | x')$ is the intensity function of extended targets. $p_{D,k}(x)$ is the detection probability. $\kappa_k(z)$ is the intensity function of clutter.

**III. THE MODIFIED PHD FILTER WITH ADAPTIVE BIRTH INTENSITY ESTIMATION**

Low detection probability leads to the absence of lots of measurements generated by targets, which can degrade the tracking performance significantly, especially in track initiation. To ensure the performance of the PHD filter, we propose a modified PHD filter with adaptive birth intensity estimation in this section. In the standard PHD filter, the state intensity will drop rapidly when there is no measurement in the neighborhood. To cope with the absence of measurements, the strategy of the proposed algorithm is to keep the states generated by the unconfirmed measurements as long as possible for future verification, while considering the algorithm efficiency at the same time. Fig. 1 shows the flowchart of the proposed algorithm.

There are three processing modules in the modified PHD filter. First of all, a preprocessor in the measurement set is designed before the PHD predictor. In the framework of measurement driven estimation, we classify the measurements $Z_k$ into three categories ($Z^P_k$, $Z^B_k$ and $Z^C_k$) depending on whether
it is in the neighborhood of the state in the formal set and the temporary set \(X_{k|k-1}^{FM}\) and \(X_{k|k-1}^{TP}\). Then, we define the state set generated by \(Z_{k0}^{B2}\) and \(Z_{k}^{B}\) above as the formal set and the temporary set respectively. The left module describes the procedure in the formal set. The right module describes the procedure in the temporary set. There is a transfer state process from the right to the left. The processing progress in the formal state set \(X_{k}^{FM}\) from the right to the left. The processing progress in the formal set.

The classification of measurements.

| NAME | FORMULA |
|------|---------|
| \(Z_{k}^{P0}\) | \(|x_{k}^{(i)}| \left( x_{k}^{(i)} - h(x_{k|k-1}^{TP}) \right)^{T} \left( x_{k}^{(i)} - h(x_{k|k-1}^{TP}) \right) < U_{1}; x_{k} \in Z_{k}^{P}, x_{k|k-1}^{TP} \in X_{k|k-1}^{TP}\) \(i = 1, \ldots, \text{card}(X_{k|k-1}^{TP}), j = 1, \ldots, \text{card}(Z_{k})\) (11) |
| \(Z_{k}^{B2}\) | \(|x_{k}^{(i)}| \left( x_{k}^{(i)} - h(x_{k|k-1}^{TP}) \right)^{T} S(x_{k|k-1}^{TP} - x_{k}^{(i)})^{-1} (z_{k}^{(i)} - h(x_{k|k-1}^{TP})) < U_{1}; z_{k} \in Z_{k}^{B2}, x_{k|k-1}^{TP} \in X_{k|k-1}^{TP}\) \(i = 1, \ldots, \text{card}(X_{k|k-1}^{TP}), j = 1, \ldots, \text{card}(Z_{k})\) (12) |
| \(Z_{k}^{B}\) | \(|x_{k}^{(i)}| - Z_{k}^{P} - Z_{k}^{B2} = Z_{k}^{B}\) (13) |

Note:
1. In (11), \(U_{1}\) is a threshold, obtained through inverse of the chi-square cumulative distribution function. \(X_{k|k-1}^{FM}\) is the predicted state set of formal set \(X_{k|k-1}^{FM}\).
2. In (12), \(Z_{k}^{P0}\) denotes the complement set of \(Z_{k|k-1}^{PO}\) in the \(Z_{k}\). The element in \(Z_{k}^{P0}\) is represented as \(z_{k}, x_{k|k-1}^{TP}\) is the predicted state set of the temporary set \(X_{k|k-1}^{TP}\). \(\lambda_{k}\) is the forgetting factor corresponding to the \(x_{k|k-1}^{TP}\) \(z_{k}\) \(S(x_{k|k-1}^{TP} - x_{k})^{-1} (z_{k} - h(x_{k|k-1}^{TP}))\) matrix. \(\lambda_{k}\) is a correlation matrix about the state \(x_{k|k-1}^{TP}\). \(\lambda_{k}\) is the forgetting factor \(\lambda_{k}\). Since the states in temporary set have two sources. One is generated by measurements in \(Z_{k}^{P}\). The other is directly duplicated by the partial previous states in formal set. We distinguish the source of the state according to its velocity vector. The form of \(S\) depends on the source of the state. Thus, \(S\) can be derived into (14). For the specific derivation and analysis process, see appendix A. In (14), \(U_{v}\) is the threshold of velocity. \(||*||\) denotes the modulus of vectors. \(*_{p1}\) denotes the velocity vector of \(*. T\) is the interval of the time step.
3. \(Z_{k}^{B}\) is the residual set of \(Z_{k|k-1}^{PO}\) in the \(Z_{k}\).

The target measuring model is assumed as
\[
z(x) = h(x) + w
\]
(10)
where \(h(\ast)\) is the measuring function. \(w\) is the measuring Gaussian noise with covariance matrix \(R\).

In the proposed PHD filter, we define two state sets as the formal state set \(X_{k}^{FM}\) and the temporary state set \(X_{k}^{TP}\). The former is the state set, in which each target state has been confirmed by the association with previous states. So, the source of \(X_{k|k-1}^{FM}\) is the states generated by \(Z_{k}^{B2}\). The latter is the state set, in which each target state has not been confirmed. So, the main source of \(X_{k|k-1}^{TP}\) is from the states generated by \(Z_{k}^{B}\). Moreover, in low detection probability scenario, not all states in \(X_{k|k-1}^{TP}\) have matching measurements in \(Z_{k|k-1}^{PO}\). To maintain the track, we duplicate those states as the birth states of \(X_{k|k-1}^{TP}\), which is another source of \(X_{k|k-1}^{TP}\).

Next is going to introduce the primary classification method for \(Z_{k}\). Based on the definitions above, \(Z_{k|k-1}^{PO}\), \(Z_{k}^{B2}\) and \(Z_{k}^{B}\) can be calculated by formula (11), (12) and (13) in Table. 1 respectively.

\[
S(x, \lambda) = \left[ \begin{array}{ccc}
R \\
\frac{v_{max}^2}{3} & \frac{\lambda T^{2}}{2} & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \frac{v_{max}^2}{3} & \frac{\lambda T^{2}}{2} \\
\vert x_{[i]} \vert & \cdots & \cdots & \vert x_{[i]} \vert > U_{V}
\end{array} \right] + R
\]
(14)
TABLE 2. The birth intensity estimation.

| NAME       | FORMULA                                                                 |
|------------|-------------------------------------------------------------------------|
| \( \gamma_{k,1}(x,\lambda) \) | \[ \sum_{i=1}^{\text{card}(\tilde{Z}^B_k)} w_{b1,k} N\left( x; \tilde{x}^{(i)}_{k,1}, \tilde{m}^{(i)}_{b1,k-1}, \tilde{p}^{(i)}_{b1,k-1} \right) \] (16) |
| \( \gamma_{k,2}(x) \) | \[ \sum_{i=1}^{\text{card}(\tilde{Z}^B_k)} w_{b2,k} N\left( x; \tilde{m}^{(i)}_{b2,k-1}, \tilde{p}^{(i)}_{b2,k-1} \right) \] (18) |

Note:
1. \( w_{b1} \) and \( w_{b2} \) are the preset weight value for the suspected new born target and the confirmed new born target respectively.
2. \( x_{[p]} \) denotes the position vectors in state \( x \). \( h^{-1}(\cdot) \) is the function of the transition from \( \tilde{z}^B_k \) to \( x_{[p]} \). \( T \) is the interval time of two adjacent time step. \( v_{\text{max}} \) is the preset maximum velocity of the target.
3. In the temporary set, we introduce a forgetting factor \( \lambda \) corresponding to each Gaussian component of state. The initiation of the \( \lambda \) is set as 0. Thus, the parameter composition of \( \gamma_{k,1}(x) \) is a bit different from \( \gamma_{k,2}(x) \).

Besides, there is an attachment parameter set \( \beta_{k,2} \), with the \( \tilde{Z}^B_{k} \), as (15) shown. \( \beta_{k,2} \) denotes the association index of element in \( X^T_{k|k-1} \). For example, if \( \beta_{k,2} = p \), that means the \( p \)-th element in \( X^T_{k|k-1} \) is most closely related to the \( j \)-th element in \( \tilde{Z}^B_{k} \).

\[
\beta_{k,2} = [i] \left( \tilde{z}^{B,0}_{k,2} - h\left(x_{T,k|k-1}^{(i)}\right) \right)^T U_1 \left( \tilde{x}^{(i)}_{T,k|k-1}, \lambda_{k-1}^{(i)} \right)^{-1} < U_2 \leq \beta_{k,2} \in Z^B_k, x_{T,k|k-1}^{(i)} \in X^T_{k|k-1}, i = 1, \ldots, \text{card}(X^T_{k|k-1}), j = 1, \ldots, \text{card}(Z^B_k) \] (19)

B. MEASUREMENT-DRIVEN BIRTH INTENSITY ESTIMATION

In this sub-section, we design the method of birth intensity estimation based on the measurement categories above. Assume the target state \( x \) consists of the position vector \( p \) and the kinematic vector \( u \). The former can be calculated through the measurement set directly. The latter requires further calculations relating to the whole kinematic procedure. Inspired by E-PHD [18] and two-point track initiation algorithm [19], we consider the kinematic vector during the birth intensity estimation.

In the measurement driven framework, \( Z^B_k \) generates the confirmed new born intensity \( \gamma_{k,2}(x) \) in the formal set. \( Z^B_k \) generates the suspected new born intensity \( \gamma_{k,1}(x) \) in the temporary set. Table 2 gives the estimation methods of both \( \gamma_{k,1}(x) \) and \( \gamma_{k,2}(x) \).

C. DEDUCTION OF THE MODIFIED PHD FILTER

As the proposed birth intensity estimation method has introduced in the above sub-section, we deduce the modified PHD filter in this sub-section. There are two state sets, that is the formal set and the temporary set, in the modified PHD filter. Suppose the target state intensity at time step \( k \) is \( D_k(x) \). Therefore, \( D_k(x) \) can be represented by formula (20).

\[
D_k(x) = D^T_{k}(x,\lambda) + D^FM_{k}(x) \quad (20)
\]

where \( D^T_{k}(x,\lambda) \) and \( D^FM_{k}(x) \) are the intensity of the target state in the temporary set and the formal set respectively.

The standard PHD filter has two steps, that is prediction and updating. We deduce the proposed modified PHD filter into (23) and (30), as shown at the bottom of page 7.

In general, researches on the PHD filter with birth intensity estimation omit the extended target states for facilitation of deduction [15–22].

Assume the intensity of the replica is \( D^0_k(x,\lambda) \). The corresponding forgetting factor \( \lambda \) is set as 0. The prediction equation of the modified PHD filter is given by (21).

\[
D_{k|k-1}(x) = \gamma_{k,2}(x) + \gamma_{k,1}(x,\lambda) + \int p_{s,k}(\tau)f_{k|k-1}(x|\tau)D^0_0(\tau)\, d\tau + \int p_{s,k}(\tau)f_{k|k-1}(x|\tau) \times \left( D^T_{k-1|k-1}(x,\lambda) + D^FM_{k-1|k-1}(x,\lambda) \right)\, d\tau \quad (21)
\]

For facilitation of following derivation, combine \( D^0_k(x,\lambda) \) and \( D^T_{k-1|k-1}(x,\lambda) \) together before the prediction. Therefore, we introduce an updating step in temporary set before the prediction, given by (22).

\[
D^T_{k-1|k-1}(x,\lambda) = D^0_k(x,\lambda) + D^T_{k-1|k-1}(x,\lambda) \quad (22)
\]

So, the prediction equation can be rewritten as:

\[
D_{k|k-1}(x) = \left( \gamma_{k,2}(x) + D^FM_{k-1|k-1}(x,\lambda) \right) + \left( \gamma_{k,1}(x,\lambda) + D^T_{k-1|k-1}(x,\lambda) \right) \quad (23)
\]

Assumption 1: To simplify the model, we assume the detection probability is supposed to be a constant value \( p_D \). Since the birth intensity is totally generated by the measurements at time step \( k \), the detection probability for the new born targets is equal to 100\%.
Besides, in low detection probability situation, the absence of measurements leads to great difficulties in track initiation. The state intensity for new born targets will drop significantly, if there is no associated measurement at next time step. So, the aim of the proposed algorithm is trying to make full use of the historical unassociated measurements and keeping the survival life of the states generated by them as long as possible. From (7), we can find that the intensity updating equation of PHD filter has two parts. One is the previous state intensity under the assumption that target is not detected. The state intensity for new born targets will drop significantly, if there is no associated measurement at next time step. So, the proposed solution is to update the state intensity with the help of historical unassociated states. Thus, the proposed solution is to estimate birth intensity with the help of historical unassociated states. So, it only requires for the contribution of the likelihood function about \( Z^B_{k0} \cup Z^B_{k2} \) to update \( D^{TP}_{k|k-1}(x) \). Based on the analysis above, the calculating strategy of the likelihood function is given by (26) ~ (29).

\[
g_k(z|x) |_{x \in X^P_{k|k-1}} = \begin{cases} g_k(z|x) & \text{if } z \in Z^P_{k} \\ 0 & \text{if } z \notin Z^P_{k} \end{cases} \tag{26}
\]

\[
g_k(z|x) |_{x \in X^P_{k}} = \begin{cases} g_k(z|x) & \text{if } z \in Z^B_{k} \\ 0 & \text{if } z \notin Z^B_{k} \end{cases} \tag{27}
\]

\[
g_k(z|x) |_{x \in X^P_{k|k-1}} = \begin{cases} g_k(z|x) & \text{if } z \in (Z^P_{k} \cup Z^P_{k2}) \\ 0 & \text{if } z \notin (Z^P_{k} \cup Z^P_{k2}) \end{cases} \tag{28}
\]

\[
g_k(z|x) |_{x \in X^P_{k}} = \begin{cases} g_k(z|x) & \text{if } z \in Z^B_{k} \\ 0 & \text{if } z \notin Z^B_{k} \end{cases} \tag{29}
\]

Take (23) ~ (29) into (7), and obtain the updating equation of the modified PHD filter as (30). For the specific derivation and analysis process, see appendix B.

where \( L_1(z) \), \( L_2(z) \) and \( L_3(z) \) are given by (31)-(33).

\[
L_1(z) = \mathbb{L}(z) \mid_{z \in Z^P_{k}} = k_k(z) + g_0 w_{b1,k} \nonumber \\
+ \int \rho d \tau \frac{1}{\pi \lambda^2} g_k(z|\tau) D^{TP}_{k|k-1}(\tau,\lambda) d \tau \tag{31}
\]

\[
L_2(z) = \mathbb{L}(z) \mid_{z \in Z^B_{k}} = k_k(z) + g_0 w_{b2,k} \nonumber \\
+ \int \rho d \tau \frac{1}{\pi \lambda^2} g_k(z|\tau) D^{TP}_{k|k-1}(\tau,\lambda) d \tau \tag{32}
\]

\[
L_3(z) = \mathbb{L}(z) \mid_{z \in Z^P_{k} \cup Z^P_{k2}} = k_k(z) + \int \rho d \tau \frac{1}{\pi \lambda^2} g_k(z|\tau) D^{TP}_{k|k-1}(\tau,\lambda) d \tau \tag{33}
\]

At the meantime, update the forgetting factor of the state in temporary set, as (34) shown.

\[
\lambda_{k+1}(x) = \lambda_k(x) + 1(x \in X^P_{k|k}) \tag{34}
\]

And delete those states in the temporary set whose forgetting factor exceeds a preset threshold.

After each loop of prediction and updating, extract the target state whose intensity is over a preset threshold. The target extraction has three steps. Firstly, the extraction of target state performs on the formal set and the temporary set respectively.
Then, merge the extracted states and half down the intensity.
Finally, extract the target from the merging result, if its intensity is over the preset threshold.

IV. GAUSSIAN MIXTURE IMPLEMENTATION OF THE PROPOSED FILTER

In this section, we derive the Gaussian mixture implementation method of the proposed filter.

Assume the survival probabilities is the constant value \( p_s \).
And the state transition function and the measurement likelihood function are both linear Gaussian model, as formula (35), (36) shown [12].

\[
f_{k|k-1}(x_k | x_{k-1}) \sim \mathcal{N}(x_k; F_{k-1} x_{k-1}, Q_{k-1}) \quad (35)
\]

\[
g_k(z_k | x_k) \sim \mathcal{N}(z_k; H_k x_k, R_k) \quad (36)
\]

Suppose the posterior intensity at time k-1 is a Gaussian mixture form:

\[
D_{k-1}(x) = D_{k-1}^{FM}(x) + D_{k-1}^{TP}(x, \lambda)
\]

\[
= \sum_{i=1}^{\text{card}(X_{k|k-1}^{FM})} w_{F,k-1}^{(i)} \mathcal{N}(x; m_{F,k-1}^{(i)}, P_{F,k-1}^{(i)})
+ \sum_{i=1}^{\text{card}(X_{k|k-1}^{TP})} w_{T,k-1}^{(i)} \mathcal{N}(x, \lambda^{(i)}; m_{T,k-1}^{(i)}, P_{T,k-1}^{(i)})
\]

(37)

Update the temporary set by (22) and obtain the new

\[
D_{k|k}^{TP}(x, \lambda).
\]

For prediction,

\[
D_{k|k-1}(x) = y_{k,1}(x) + y_{k,2}(x) + D_{k-1}^{FM}(x, \lambda)
+ D_{k-1}^{TP}(x, \lambda)
\]

(38)

\[
D_{k|k}(x) = D_{k|k}^{FM}(x) + D_{k|k}^{TP}(x, \lambda)
= \left( \sum_{j=1}^{\text{card}(Z_{k}^{G})} g_{b_{1,k}} N(x; m_{b_{1,k|k-1}}^{(i)}, P_{b_{1,k|k-1}}^{(i)}) + pD e^{-\frac{1}{\mu_{i}}} g_k(z | x) D_{k|k-1}(x) \right) \frac{L_2(z^{(i)})}{L_2(z^{(i)})} + (1 - pD) D_{k|k-1}^{FM}(x, \lambda)
+ \sum_{j=1}^{\text{card}(Z_{k}^{D})} \frac{pD g_k(z^{(i)} | x) D_{k|k-1}^{FM}(x, \lambda)}{L_3(z^{(i)})}
+ \left( \sum_{j=1}^{\text{card}(Z_{k}^{L})} g_{b_{2,k}} N(x, \lambda^{(j)}; m_{b_{2,k|k-1}}^{(j)}, P_{b_{2,k|k-1}}^{(j)}) + pD e^{-\frac{1}{\mu_{i}}} g_k(z | x) D_{k|k-1}(x, \lambda) \right) \frac{L_1(z^{(j)})}{L_1(z^{(j)})} + (1 - pD) D_{k|k-1}^{TP}(x, \lambda)
\]

(39)

The weights and the Gaussian components are given by (40).

\[
\begin{align*}
D_{k|k}^{FM}(x) & = p_{s} e^{w_{F,k-1}^{(i)}}; \\
D_{k|k}^{TP}(x) & = p_{s} e^{w_{T,k-1}^{(i)}}; \\
D_{k|k}^{FM}(x, \lambda) & = F_{k-1} m_{F,k-1}^{(i)}; \\
D_{k|k}^{TP}(x, \lambda) & = F_{k-1} m_{T,k-1}^{(i)}; \\
P_{F,k-1}^{(i)} & = Q_{k-1} + F_{k-1} P_{F,k-1}^{(i)} F_{k-1}^T; \\
P_{T,k-1}^{(i)} & = Q_{k-1} + F_{k-1} P_{T,k-1}^{(i)} F_{k-1}^T
\end{align*}
\]

(40)

For updating,

\[
D_{k|k}(x) = D_{k|k}^{FM}(x) + D_{k|k}^{TP}(x)
= \left( D_{k|k}^{FM}(x) + D_{k|k}^{TP}(x) \right)
+ \left( D_{k|k}^{FM}(x, \lambda) + D_{k|k}^{TP}(x, \lambda) \right)
\]

(41)
where

\[
\begin{align*}
L_1(z^{(i)}) &= L_2(z^{(i)}) + \frac{\sigma}{\sigma_t} w^{(i)}_{T,k-1} N(z^{(i)}; \eta_{T,k}, \psi_{T,k}), \\
L_2(z^{(i)}) &= L_3(z^{(i)}) + \frac{\sigma}{\sigma_t} w^{(i)}_{T,k-1} N(z^{(i)}; \eta_{T,k}, \psi_{T,k}), \\
L_3(z^{(i)}) &= \kappa_k(z^{(i)}) + \frac{\sigma}{\sigma_t} w^{(i)}_{T,k-1} N(z^{(i)}; \eta_{T,k}, \psi_{T,k}), \\
\end{align*}
\]

(43)

The target state extraction has three steps:

a. Respectively extract the states in the \(X_{i,k}^{FM}\) and \(X_{i,k}^{TP}\) by (44).

\[
\begin{align*}
\hat{X}_{i,k}^{FM} &= \{m_{FM,k}^{(i)}; P_{FM,k}^{(i)}\} w^{(i)}_{F,k} > Th; \\
i = 1 \ldots \text{card}(X_{i,k}^{FM}) \\
\hat{X}_{i,k}^{TP} &= \{m_{TP,k}^{(i)}; P_{TP,k}^{(i)}\} w^{(i)}_{T,k} > Th; \\
i = 1 \ldots \text{card}(X_{i,k}^{TP})
\end{align*}
\]

(44)

where \(Th\) is a preset weight threshold for extraction. \(X_{i,k}^{FM}\) and \(X_{i,k}^{TP}\) are the updated formal state set and the updated temporary set at time step k.

b. Merge the \(\hat{X}_{kF}^{FM}\) and \(\hat{X}_{kT}^{TP}\). Obtain the merged state set \(\hat{X}_{kM}\) and its corresponding merged weights. Reduce the weight by half.

c. Extract the target states in the merged set \(\hat{X}_{kM}\) by (45). \(\hat{X}_k\) and \(N_k\) are the extracted state set and the cardinality of the set at time step k.

\[
\begin{align*}
\hat{X}_k &= \{(m_{M,k}^{(i)}; P_{M,k}^{(i)}) w_{M,k} > Th; \\
i = 1 \ldots \text{card}(X_{kM}) \} \\
N_k &= \text{card}(\hat{X}_k)
\end{align*}
\]

(45)

In this end, we provide the detailed pseudocode of the GM-implementation of the proposed filter in Appendix C for readers.

V. SIMULATION AND RESULTS

In this section, we conduct experiments to test the performance of the proposed filter. To make it an intuitive performance comparison, we choose three representative PHD filters based on adaptive birth intensity estimation. The information about those comparative filters will be given in detail in sub-section VA. The experiments focus on the issue of multi-targets tracking in the low detection probability.

A. PARAMETERS SETTING

In this section, we designs a scenario in which an unknown number of targets appear and die at different time steps. The dimension of the state is set as four. The targets move in the Cartesian coordinates. The state of a target at time t can be denoted by \(x_t = [x_t, y_t, v_x, v_y]^T\). The state transition model is a constant velocity model with (46) and (47).

\[
F = \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(46)

\[
Q = \frac{T^4}{4} \sigma_v^2 \begin{bmatrix}
T^3 & \frac{T^3}{2} & \sigma_v^2 & 0 \\
\frac{T^3}{2} & \sigma_v^2 & 0 & 0 \\
0 & 0 & T^4 & \frac{T^3}{2} \sigma_v^2 \\
0 & 0 & \frac{T^3}{2} \sigma_v^2 & T^2 \sigma_v^2
\end{bmatrix}
\]

(47)

where \(\sigma_v = 5\) is the standard deviation of the process noise.

The observation model is linear with (48) and (49).

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(48)

\[
R = \begin{bmatrix}
\sigma_v^2 & 0 & 0 \\
0 & \sigma_v^2 & 0
\end{bmatrix}
\]

(49)
where $\sigma_x = 10$ and $\sigma_y = 10$ are the standard deviation of the observation noise. The clutter points are uniformly distributed in the surveillance scope $[-1000 \times 1000] \times [-1000 \times 1000]$. The number of clutter points obeys a poison distribution with an average rate $\rho_c$ per scan. The survival probability $p_s$ of the state is set as 0.99. The maximum number of the gaussian components in both the formal set and the temporary set are set as 100. The surveillance time length is 100s. We set five targets in simulation scenario. Table 3 shows the parameter setting of these five tracks.

**TABLE 3. Parameter setting of tracks.**

| No. | Initial position | Velocity | Appearing time | Disappearing time |
|-----|------------------|----------|----------------|-------------------|
| 1   | (0, 0)           | (0, -10) | 1              | 80                |
| 2   | (-600, -600)     | (8, 4)   | 10             | 90                |
| 3   | (500, -600)      | (-8, 1)  | 20             | 100               |
| 4   | (-400, 400)      | (11, 0)  | 25             | 100               |
| 5   | (700, 100)       | (-3, 8)  | 30             | 100               |

For the proposed filter, we set $w_{b1}$ and $w_{b2}$ as $10^{-2}$ and $10^{-2}$ respectively. The setting of the maximum forgetting factor $\lambda_{\text{max}}$ is assumed to be related to the detection probability $p_D$. Because if $\lambda_{\text{max}}$ is larger, the false alarm rate is getting greater in low $p_D$. To make a balance, (50) gives the setting principle of $\lambda_{\text{max}}$.

$$
\lambda_{\text{max}} = \lfloor 3 + 12p_D \rfloor
$$

(50)

where $\lfloor \ast \rfloor$ denotes round down. The constant coefficient $\mu$ in (24) is also related to $p_D$, given by (51).

$$
\mu = \max(1, 10^{(2.5 - 5p_D)})
$$

(51)

We choose three representative PHD filters based on adaptive birth intensity estimation for comparison. They are the extended PHD filter (E-PHD) [18], the PHD filter based on entropy distribution (ED-PHD) [21] and the PHD filter based on iterative RANSAC (IR-PHD) [22]. The E-PHD combines the single-point and two-point difference track initialization with the adaptive birth intensity estimation. In [21], the ED-PHD is the PHD filter with entropy distribution and coverage rate-based birth intensity estimation. And it mainly applies in the area of multi-target visual tracking. However, our research focus on the point-target track. The coverage rate makes no contribution in this simulation. Thus, the ED-PHD in simulation only considers the entropy distribution-based birth intensity updating by the criterion of the maximum a posterior (MAP). The IR-PHD adopts iterative random sample consensus (I-RANSAC) with a sliding window to estimate the birth intensity. The unique parameter settings of three comparative filters in simulation are given in Table 4.

The common parameter settings in simulation are given in Table 5.

**TABLE 4. Parameter settings of comparative filters.**

| Filter       | Weight for one-point initiation | Weight for two-point termination | Length of the sliding window |
|--------------|---------------------------------|---------------------------------|-------------------------------|
| E-PHD        | $10^{-2}$                        | $10^{-2}$                       | 6                             |
| ED-PHD       |                                 | $10^{-2}$                       |                               |
| IR-PHD       |                                 |                                 | Distance threshold in RANSAC  |
|              |                                 |                                 | Minimum ratio of points to    |
|              |                                 |                                 | sliding window                |

**TABLE 5. Common parameter settings.**

| NAME                       | VALUE |
|---------------------------|-------|
| Maximum velocity          | 30    |
| Prune threshold           | $10^{-5}$ |
| Merge threshold           | 4     |
| State extraction threshold| 0.8   |

**B. SIMULATION RESULTS**

1) **MULTI-TARGET TRACKING RESULTS IN LOW DETECTION PROBABILITY SCENARIO**

In this subsection, we conduct the simulation with the detection probability $p_D = 0.3$. And the average rate $\rho_c$ of poison distribution for clutters is set as 2. Fig. 3 illustrates all measurements of five tracks in this scenario. Blue ‘x’ denotes the location of measurements.
FIGURE 4. Tracking results of four filters in scenario \( p_{D} = 0.3 \). (a)(b) are the tracking results of the proposed PHD filter. (c)(d) are the tracking results of E-PHD. (e)(f) are the tracking results of ED-PHD. (g)(h) are the tracking results of IR-PHD. (a)(c)(e)(g) illustrate the tracking results in Cartesian coordinates. Red line denotes the true track. Red circle ‘o’ denotes the start position of the track. Green ‘∗’ denotes measurements generated by true target. Blue ‘x’ denotes the tracking results. (b)(d)(f)(h) illustrate the tracking results in x coordinate and y coordinate. Dark line denotes the true track. Dark point denotes the estimated coordinate. Light ‘x’ denotes the measurement.
The tracking results of four filters are illustrated in Fig. 4. Fig. 4(a)(b) are the tracking results of the proposed PHD filter. Fig. 4(c)(d) are the tracking results of E-PHD. Fig. 4(e)(f) are the tracking results of ED-PHD. Fig. 4(g)(h) are the tracking results of IR-PHD. Fig. 4(a)(c)(e)(g) illustrate the tracking results in Cartesian coordinates. At meantime, to make a reference in visual, we also plot the true track and the measurements generated by true targets. Red line denotes the true track. Red circle ‘o’ denotes the start position of the track. Green ‘∗’ denotes measurements generated by true target. Blue ‘x’ denotes the tracking results. Fig. 4(b)(d)(f)(h) illustrate the tracking results in x coordinate and y coordinate. Dark line denotes the true track. Dark point denotes the estimated coordinate. Light ‘x’ denotes the measurement.

**TABLE 6. Number of scans for correct track initiation.**

| Track NO. | Proposed PHD | E-PHD | ED-PHD | IR-PHD |
|-----------|--------------|-------|--------|--------|
| Track 1   | 6            | 7     | 6      | 13     |
| Track 2   | 10           | 24    | 24     | 63     |
| Track 3   | 5            | 32    | 9      | 32     |
| Track 4   | 6            | 17    | 16     | 26     |
| Track 5   | 6            | 5     | 10     | 45     |

The tracking results of four filters are illustrated in Fig. 4. From Fig. 4, it is obvious that the proposed filter has the best performance on track formation and maintenance in low detection probability. The tracking performance of E-PHD and ED-PHD are at same level, while IR-PHD performs worst. Besides, the number of false track points in the result of proposed PHD and IR-PHD are less than the one in the results of E-PHD and ED-PHD.

To evolve the tracking error, we choose the optimal sub-pattern assignment (OSPA) distance [25], which captures the cardinality error, as well as the distance of individual elements, of two finite sets. The OSPA distance of two finite sets $X = \{x_1, \ldots, x_m\}$ and $Y = \{y_1, \ldots, y_n\}$ is given by (52)

$$d_{OSPA}^{c, p}(X, Y) \triangleq \begin{cases} \left[ \frac{1}{n} \min_{\pi \in \Pi_n} \sum_{i=1}^m d^{(c)}(x_i, y_{\pi(i)})^p + c^p (n - m) \right]^{\frac{1}{p}} & m \leq n \\ d_{OSPA}^{c, p}(Y, X), & m > n \end{cases}$$

(52)

where $\Pi_k$ is the set of permutations on $\{1, 2, \ldots, k\}, k > 0$, and the distance $d^{(c)}(x, y)^p = \min(c, \|x - y\|)$. $p \geq 1$ is the order of the distance, and $c$ is the cut-off parameter.
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FIGURE 5. Tracking error of different filter in scenario $p_D = 0.3$. The error indicators are the OSPA distance, the OSPA location and the OSPA cardinality, displayed from top to bottom. Different color to distinguish different filter. Red line denotes the tracking error of the proposed PHD filter. Black line denotes the tracking error of the E-PHD filter. Blue line denotes the tracking error of the ED-PHD filter. Green line denotes the tracking error of the IR-PHD filter.

Set the cut-off parameter $c$ as 100 and the order $p$ as 1. The OSPA distance of tracking results, as well as the OSPA location and the OSPA cardinality, are illustrated in Fig. 5. Use different colors to distinguish the filter. Red line denotes the tracking error of the proposed PHD filter. Black line denotes the tracking error of the E-PHD filter. Blue line denotes the tracking error of the ED-PHD filter. Green line denotes the tracking error of the IR-PHD filter.

Thus, the tracking performance of the proposed filter is superior over other comparative filters in low detection probability scenario.
The error indicators are the OSPA distance, the OSPA location and the OSPA cardinality, displayed from top to bottom. Red line denotes the tracking error of the proposed PHD filter. Black line denotes the tracking error of the E-PHD filter. Blue line denotes the tracking error of the ED-PHD filter. Green line denotes the tracing error of the IR-PHD filter.

2) MONTE-CARLO SIMULATION IN DIFFERENT DETECTION PROBABILITY

The computer configuration for simulation: Intel(R) Core(TM) i7-4790 CPU@3.6GHz 3.6GHz. RAM:16.0 GB. Matlab R2018b.

In this sub-section, the simulation scenario is same as the above subsection. The only variable parameter is the detection probability. Change the detection probability \(p_D\) from 0.25 to 0.8 with the step length of 0.01. Do Monte-Carlo experiment for 100 times at each \(p_D\). Use three indexes (A. Number of scans for track initiation \(N_{\text{initial}}\). B. Average OSPA distance \(d_{\text{OSPA(\text{av})}}\). C. Time consuming \(t\)) to describe the performance of the tracking result. The definition of \(N_{\text{initial}}\) is the average number of consumed scans of five tracks for correct track initiation. The definition of \(d_{\text{OSPA(\text{av})}}\) is the average OSPA distance over the whole surveillance time period. The definition of \(t\) is the time consuming on Matlab to complete tracking in the scenario of specific detection probability.

Fig. 9 illustrates the Monte-Carlo experiment results of time-consuming. Red line denotes the time-consuming of the proposed PHD filter. Black line denotes the time-consuming of the E-PHD filter. Blue line denotes the time-consuming of the ED-PHD filter. Green line denotes the time-consuming of the IR-PHD filter.

Monte-Carlo results, it is obvious that the proposed filter requires the least number of scans for correct track initiation, especially in low detection probability situation.

Fig. 10 illustrates the Monte-Carlo experiment results of the average number of consumed scans for track initiation \(N_{\text{initial}}\). Red line denotes the \(N_{\text{initial}}\) of the proposed PHD filter. Black line denotes the \(N_{\text{initial}}\) of the E-PHD filter. Blue line denotes the \(N_{\text{initial}}\) of the ED-PHD filter. Green line denotes the \(N_{\text{initial}}\) of the IR-PHD filter.
FIGURE 11. The Monte-Carlo experiment results of average OSPA distance $d_{\text{OSPA}}^{(av)}$. Red line denotes the $d_{\text{OSPA}}^{(av)}$ of the proposed PHD filter. Black line denotes the $d_{\text{OSPA}}^{(av)}$ of the E-PHD filter. Blue line denotes the $d_{\text{OSPA}}^{(av)}$ of the ED-PHD filter. Green line denotes the $d_{\text{OSPA}}^{(av)}$ of the IR-PHD filter.

C. TEST BY THE FIELD DATA

In this part, we use the field data to test the proposed PHD filter, as well as other three comparative filters. The field data is the radar data from a passive radar exploring uncooperative radar illuminator. As such radar is hard to achieve the synchronization accuracy to a satisfactory level, the detection probability is relatively low. Fig. 12 illustrates a piece of passive radar data with the time length of 4 minutes around.

FIGURE 12. The passive radar data with the time length of 4 minutes. There are two true tracks pointed out in the surveillance scope.

FIGURE 13. Tracking results of the field passive radar data: (a) the proposed PHD filter. (b) the E-PHD filter. (c) the ED-PHD filter. (d) the IR-PHD filter. The original measurements are plot by green ‘•’. The tracking results are plot by blue ‘x’.
The time step is 2.5 s. And the data consists of 100 scans. According to the data from the active radar, we confirm two tracks of true targets in the surveillance scope, which is pointed out in Fig. 12. The detection probability in this scenario is about 0.5.

Besides, there are a few scans with no measurements. And the distribution of the clutters is not uniform. In addition, dense clutters and interferences are distributed in the right upper region of the scope. In the field experiment, the baseline of the illuminator and the receiver is relative short to the detecting range. And both face in the similar direction for surveillance. So, the observation model is approximately linear with (48) and (49). The target state transition model is approximately CV model with (46) and (47), where \( T = 2.5 \) and \( \sigma_v = 100 \). The maximum velocity is set as 900 m/s. The tracking results of the four filters are shown in Fig. 13. The original measurements are plotted by green ‘*’. The tracking results are plotted by blue ‘x’. Fig. 13(a)-(d) are the tracking results of the proposed algorithm, E-PHD, ED-PHD and IR-PHD respectively. The tracking result shows the proposed PHD filter is superior among four filters. The proposed PHD filter can successfully track the two confirmed targets. The E-PHD filter can partially track these targets, but there are too many false-alarm clutters in the result. Also, the ED-PHD filter can partially track these targets. Its track maintenance ability, is relatively weak under the real low detection probability scenario, especially no measurements and clutters in several consecutive scans. The IR-PHD filter is fail to track in this field experiment. The reason we analysis is that the birth state estimation in IR-PHD filter requires two true measurements in two consecutive scans for the construction of line model, while the absence of measurement in several scans leads to the difficulty of track initiation of IR-PHD.

D. CALCULATIONAL COMPLEXITY ANALYSIS

Suppose the cardinality of the formal set and the temporary set at time step k-1 is \( N_{f,k-1} \) and \( N_{t,k-1} \) respectively. The cardinality of the measurement set at time step k is \( L_k \). \( L_{P0,k} \) denotes the cardinality of \( Z_k^{P0} \). \( L_{B2,k} \) denotes the cardinality of \( Z_k^{B2} \). Assume the detection probability is \( p_D \). The cardinality of the replica \( D_k^* \) can be approximately regarded as \((1−p_D)N_{f,k-1}\). The proposed algorithm has two main steps: prediction and updating. Prediction:

(a) Predict the states of previous time step. The calculational complexity: \( O(N_{f,k-1} + N_{t,k-1}) \).

(b) Classify the measurement set. The calculational complexity: \( O(N_{f,k-1}L_k + N_{t,k-1}(L_k - L_{P0,k})) \).

(c) Calculate the intensity of new born target. The calculational complexity: \( O(L_k - L_{P0,k}) \).

Updating: (a) Update the formal set. The calculational complexity: \( O(N_{f,k-1}L_{B2,k} + (N_{t,k-1}+(1−p_D)N_{f,k-1})L_{B2,k} + N_{f,k-1}L_{P0,k}) \).

(b) Update the temporary set. The calculational complexity: \( O((N_{f,k-1} + (1−p_D)N_{f,k-1})L_{B2,k} + L_{P0,k} - L_{B2,k} + (N_{t,k-1}+(1−p_D)N_{f,k-1})(L_k - L_{P0,k} - L_{B2,k})) \).

In summary, the calculational complexity of the proposed PHD filter is \( O((N_{f,k-1} + N_{t,k-1})(L_kC1)) \).

VI. CONCLUSION

In this paper, we proposed a modified PHD filter with adaptive birth intensity estimation to improve the tracking performance in the low detection probability scenario. The core of the proposed filter is to define two state sets as the formal set and the temporary set. And introduce a forgetting factor into the temporary set. The measurement-driven birth intensity estimation is modified based on the proposed mechanism. We derive the Gaussian mixture implementation of the proposed PHD filter in this paper. The experiment results show the proposed PHD filter outperforms other filters in the low detection probability scenario, especially in OSPA distance and the track initiation. However, due to the introduction of the temporary set, the time-consuming of the proposed filter is increased. For better application in engineering, it is meaningful to explore the improvements on the efficiency of the propose filter in the future work.

APPENDIXES

APPENDIX A

THE ANALYSIS ON S AND THE DETAILED DERIVATION OF (14)

The primary source of the state in temporary set is distinguished by its velocity vector. If the velocity is over a threshold, we assume the source of the state is the formal set. Otherwise, the source is the measurements in \( Z_k^B \). According to different situation, we derive the correlation matrix \( S \) respectively.

For those states originated from the formal set, its velocity information is meaningful and has acted on the previous Kalman filtering loop. So, the classification principle of \( Z_k^{B2} \) follows (10). As \((x−h(x))^T R^{-1}(x−h(x))\) obeys the Chi-squared distribution, \( S \) equals to \( R \). And given the maximum probability value, \( U_1 \) can be obtained through inverse of the chi-square cumulative distribution function.

For those states originated from the measurements set \( Z_k^B \), its velocity is usually very low, since there are no measurements in the neighborhood in previous filtering steps. So the classification principle of \( Z_k^{B2} \) should consider the forgetting factor \( \lambda \) of the state. And the measurements model is constructed by (55).

\[
\begin{align*}
\bar{z} &= h(x + \lambda T \bar{a}) + w
\end{align*}
\]

\( T \) is the interval time step. \( \bar{a} \) is the scremen vector of the state in each time step. \( \bar{a} \) consists of two parts, that is position vector \( a_{[p]} \) and velocity vector \( a_{[v]} \). \( \bar{a}_{[p]} \) denotes the position vector of \( \bar{z} \). \( \bar{a}_{[v]} \) denotes the velocity vector of \( \bar{z} \). To simplify the analysis, we suppose the states in temporary set follow CV (constant velocity) model and the measuring function \( h(\bar{z}) \) is partial linearization. So (55) can be rewritten as (56).

\[
\begin{align*}
z &= h(x) + \lambda T a_{[p]} + w
\end{align*}
\]
As the velocity is unknown, we assume $\alpha_\ell$ obeys the gaussian distribution $\mathcal{N}(0, C)$, where $C$ is
\[
\begin{bmatrix}
\frac{v_{max}^2}{3} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \frac{v_{max}^2}{3}
\end{bmatrix}_{N \times N}
\]
is the maximum velocity value. $N$ is the dimension of the position of the state. Suppose $\gamma(x) = z - h(x) = \lambda T \alpha_\ell + w$. So $\gamma(x)$ obeys the mixture gaussian distribution, that is $\gamma(x) \sim \mathcal{N}(0, C_T)$. Where $C_T = \begin{bmatrix} \frac{v_{max}^2}{3} \lambda^2 T^2 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \frac{v_{max}^2}{3} \lambda^2 T^2
\end{bmatrix}_{N \times N}$
+ $R$. Thus, $\gamma(x)\mathcal{T} C_T^{-1} \gamma'(x)$ obeys the Chi-squared distribution. And $S$ equals to $C_T$.

Based on the analysis above, $S(x, \lambda)$ can be calculated by (57).

\[
S(x, \lambda) = \begin{bmatrix}
\frac{R}{v_{max}^2} |x_{[v]}| < U_V \\
\vdots \\
\frac{R}{v_{max}^2} |x_{[v]}| \geq U_V 
\end{bmatrix} + R
\]

where $U_V$ is the threshold of velocity. $|\cdot|$ denotes the modulus of vectors.

**APPENDIX B**

**THE DETAILED DERIVATION OF (30)**

The updating equation $D_{k|k} (x)$ can be derived into (58), as shown at the bottom of this page.

Assume
\[
\mathcal{L} (z) = \kappa_k(z) + \int p_{D,k} (\tau) g_k (z | \tau) \mathcal{Y}_{k,2} (\tau) d\tau
\times (\mathcal{Y}_{k,2} (\tau) + D_{k|k-1}^{FM} (\tau) + \mathcal{Y}_{k,1} (\tau) + D_{k|k-1}^{TP} (\tau, \lambda)) d\tau
\]

(59)

So, $D_{k|k} (x)$ can be further derived into (60), as shown at the bottom of this page. The further derivation of $D_{k|k}^{FM} (x, \lambda)$ and $D_{k|k}^{TP} (x, \lambda)$ are given by (61) and (62), respectively, as shown at the bottom of the next page.

Now analysis the $\mathcal{L} (z^{(0)})$:

\[
\mathcal{L} (z^{(0)}) \overset{(24)}{=} \kappa_k(z^{(0)}) + \int g_k (z^{(0)} | \tau) \mathcal{Y}_{k,2} (\tau) d\tau
+ \int p_{D,k} (\tau) g_k (z^{(0)} | \tau) \mathcal{Y}_{k,1} (\tau, \lambda) d\tau
+ \int p_{D,k} \left( \frac{1}{\mathcal{L} (z^{(0)})} \right) D_{k|k-1}^{FM} (\tau) d\tau
+ \int p_{D,k} \left( \frac{1}{\mathcal{L} (z^{(0)})} \right) D_{k|k-1}^{TP} (\tau, \lambda) d\tau
\]

(63)

\[
D_{k|k} (x) = (1 - p_{D,k} (x)) D_{k|k-1} (x) + \sum_{z \in Z_k} p_{D,k} (x) g_k (z | x) D_{k|k-1} (x)
\overset{(21)}{=} (1 - p_{D,k} (x)) \left( \mathcal{Y}_{k,2} (x) + D_{k|k-1}^{FM} (x) + \mathcal{Y}_{k,1} (x, \lambda) + D_{k|k-1}^{TP} (x, \lambda) \right)
+ \sum_{z \in Z_k} \kappa_k(z) + \int p_{D,k} (\tau) g_k (z | \tau) \mathcal{Y}_{k,2} (\tau) + D_{k|k-1}^{FM} (\tau) + \mathcal{Y}_{k,1} (\tau, \lambda) + D_{k|k-1}^{TP} (\tau, \lambda)) d\tau
\]

(58)

\[
D_{k|k} (x) \overset{(24)}{=} \left( \sum_{z \in Z_k} g_k (z | x) \mathcal{Y}_{k,2} (z) \mathcal{L} (z) + (1 - p_D) D_{k|k-1}^{FM} (x) + \sum_{z \in Z_k} p_{DG} (z | x) D_{k|k-1}^{FM} (x) \mathcal{L} (z) \right)
+ \left( \sum_{z \in Z_k} g_k (z | x) \mathcal{Y}_{k,1} (\lambda) \mathcal{L} (z) + (1 - p_D) D_{k|k-1}^{TP} (x, \lambda) \mathcal{L} (z) \right)
+ \left( \sum_{z \in Z_k} p_{DG} (z | x) D_{k|k-1}^{TP} (x, \lambda) \mathcal{L} (z) \right)
\]

(60)
Assume
\[
\mathcal{L}_1 (z^{(i)})
= \mathcal{L} (z^{(i)}) \bigg|_{z^{(i)} \in \mathbb{Z}_k^p} = \kappa_k (z^{(i)}) + g_{0w^b, k} + \int p_D e^{-\frac{1}{\mu_l}} g_k (z^{(i)} | \tau) D_{TP}^{FM} (\tau, \lambda) d \tau
\]

\[
\mathcal{L}_2 (z^{(i)})
= \mathcal{L} (z^{(i)}) \bigg|_{z^{(i)} \in \mathbb{Z}_k^p} = \kappa_k (z^{(i)}) + g_{0w^b, k} + \int p_D e^{-\frac{1}{\mu_l}} g_k (z^{(i)} | \tau) D_{TP}^{TP} (\tau, \lambda) d \tau
\]

\[
\mathcal{L}_3 (z^{(i)})
= \mathcal{L} (z^{(i)}) \bigg|_{z^{(i)} \in \mathbb{Z}_k^p} = \kappa_k (z^{(i)}) + \int p_D g_k (z^{(i)} | \tau) D_{TP}^{FM} (\tau, \lambda) d \tau
\]

So, \( D_{FM}^{F^*} (x) \) and \( D_{TP}^{F^*} (x, \lambda) \) can be derived into (67) and (68), as shown at the bottom of this page, respectively.

\[
D_{FP}^{F^*} (x) = \sum_{j=1}^{\text{card}(Z_k^p)} g_{0w^b, k} N(x; \mu_{b,k|k-1}^{(j)}, \Sigma_{b,k|k-1}^{(j)}) \frac{1}{\mathcal{L}_2 (z^{(i)})} + (1 - p_D) D_{TP}^{F^*} (x)
\]

Regard the last part of the \( D_{TP}^{F^*} (x, \lambda) \) as the transferred state intensity from the temporary set to the formal set, that is \( D_{TP}^{F^*} (x) \).

\[
D_{TP}^{F^*} (x) = \sum_{j=1}^{\text{card}(Z_k^p)} p_D e^{-\frac{1}{\mu_l}} g_k (z | x) D_{TP}^{FM} (\tau, \lambda) \frac{1}{\mathcal{L}_2 (z^{(i)})}
\]

So, the final updating formula of both sets are given by (70) and (71), respectively, as shown at the top of page 20.

And \( D_{k|k} (x, \lambda) = D_{FM}^{F^*} (x) + D_{TP}^{F^*} (x, \lambda) \).
APPENDIX C
THE PSEUDOCODE OF THE PROPOSED ALGORITHM

Input parameters:
The model parameter: $H, R, P, Q$
The measurement set at time step $Z_k$,
$w_{b1}, w_{b2}, w_{max}, \lambda_{max}, \theta, M_{gth}$ (merge threshold), $P_{rth}$ (prune threshold), $P_g$ (gate size of the distance in percentage), $U_P, T$

Initial parameters:
The weights and Gaussian component set in the formal set
$\left\{w_{F,0},m_{F,0},p_{F,0}\right\}$
The weights and Gaussian component set in the temporary set
$\left\{w_{T,0},m_{T,0},p_{T,0}\right\}$
Given $P_g$, and use the inverse of the chi-square cumulative distribution function to calculate $U_P$.

STEP 1. PREDICTION

a. Predict the updated states of previous time step.
Predict the states both in the formal set and in the temporary set by using (40).
And obtain $\left\{w_{F,k|k-1},m_{F,k|k-1},p_{F,k|k-1}\right\}$
and $\left\{w_{T,k|k-1},m_{T,k|k-1},p_{T,k|k-1}\right\}$.

b. Classify the measurement set into 3 categories.
for $i = 1 \ldots \text{card}(X_{k|k-1}^{FM})$
\[d^{(i)} = \left( z_k^{(i)} - h(m_{T,k|k-1}) \right)^T R^{-1} \left( z_k^{(i)} - h(m_{T,k|k-1}) \right); j = 1 \ldots \text{card}(Z_k)\]
Find the index set
$Q = \{q|d^{(q)} < U_1, q = 1 \ldots \text{card}(Z_k)\}$
If $Q$ is empty
Add $i$ into $Q_{r,m,k}$
Else
Add $i$ into $Q_{z,k}$
End
Remove the repeated elements from $Q_{z,k}$.
\[Z_k^{\beta} = \left\{z_k^{(i)} \mid q_{z,k} \in Q_{z,k}, z_k \in Z_k, m = 1 \ldots \text{card}(Q_{z,k})\right\}\]
Find the complement set of $Z_k^{\beta}$ in $Z_k$ as $Z_k^{\alpha}$.

For $i = 1 \ldots \text{card}(X_{k|k-1}^{TP})$
Calculate the modulus of the velocity vector of $m_{T,k|k-1}$ as $v^{(i)}$.
If $v^{(i)} < U_V$
\[S^{(i)} = \begin{bmatrix} \frac{w_{max}}{3} \lambda^{(i)} T^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{w_{max}}{3} \lambda^{(i)} T^2 \end{bmatrix}_{N \times N} + R\]
End
End
for $j = 1 \ldots \text{card}(Z_k)$
for $i = 1 \ldots \text{card}(X_{k|k-1}^{TP})$
\[d^{(i)} = \left( z_k^{(i)} - h(m_{T,k|k-1}) \right)^T S^{(i)} \left( z_k^{(i)} - h(m_{T,k|k-1}) \right); i = 1 \ldots \text{card}(X_{k|k-1}^{TP})\]
Find the index $p$, where $d^{(p)}$ is the minimum value among $\{d^{(i)}|i = 1 \ldots \text{card}(X_{k|k-1}^{TP})\}$.
\[\beta^{(p)}_{k,b} = p\]
End

Step 2. UPDATING
\[X_0 = \begin{bmatrix} w_{F,0} & m_{F,0} & p_{F,0} \\ w_{T,0} & m_{T,0} & p_{T,0} \end{bmatrix}_{Q_{r,m,k} \in Q_{r,m,k}} = 1 \ldots \text{card}(Q_{r,m,k})\]
Duplicate $X_0$ into the temporary set.
Given $Z_k^{\alpha}$ and $X_{k|k-1}^{FM}$, adopt Kalman-updating algorithm by (42) and obtain $\{r_{F}^{(i)}, m_{F,k|k-1}^{(i)}, p_{F,k|k-1}^{(i)}\}; i = 1 \ldots \text{card}(Z_k^{\beta})$.
Given $Z_k^{\beta}$ and $X_{k|k-1}^{TP}$, adopt Kalman-updating algorithm by (42) and obtain $\{r_{T2}^{(i)}, m_{T2,k|k-1}^{(i)}, p_{T2,k|k-1}^{(i)}\}; i = 1 \ldots \text{card}(Z_k^{\beta})$.
\[ i = 1 \ldots \text{card}(Z_k^{B2}), j = 1 \ldots \text{card}(Z_k^{P2}) \]

\[ \text{card}(x_k^{PP}) \text{ and } x_k^{PP}(i) \text{ denote the likelihood function between } z_k \text{ and } x_k^{PP}(i). \]
\[ r_{T2}^{(i)} \text{ denotes the likelihood function between } z_k \text{ and } x_k^{PP}(i). \]

**Calculate the weights of states in both two sets:**

**For** \( i = 1 \ldots \text{card}(Z_k^{B2}) \)

\[ L_1(Z_k^{B2}) = \]
\[ \kappa_k(z_k^{B2}) + g_0 w_{B2,1} + \]
\[ \sum_{j=1}^{\text{card}(x_k^{PP})} p_D e^{-\frac{1}{\kappa_{T1}^2}} w_{T,k|k-1}^{(j)} r_{T1}^{(i,j)} \]
\[ w_{k,k}^{1,(i,j)} = \frac{g_0 w_{B2,k}}{L_1(z_k^{B2})} \]

\[ \text{For } j = 1 \ldots \text{card}(x_k^{PP}) \]
\[ w_{k,k}^{2,(i,j)} = \frac{p_D e^{-\frac{1}{\kappa_{T1}^2}} w_{T,k|k-1}^{(j)} r_{T1}^{(i,j)}}{L_1(z_k^{B2})} \]
\[ \lambda_{k,B}^{(i)} = \lambda_{k-1}^{(i)} + 1; \]

End

**For** \( i = 1 \ldots \text{card}(Z_k^{B0}) \)

\[ L_2(z_k^{B0}) = \]
\[ \kappa_k(z_k^{B0}) + g_0 w_{B2,1} + \]
\[ \sum_{j=1}^{\text{card}(x_k^{PP})} p_D e^{-\frac{1}{\kappa_{T2}^2}} w_{T,k|k-1}^{(j)} r_{T2}^{(i,j)} \]
\[ w_{k,k}^{1,(i,j)} = \frac{g_0 w_{B2,k}}{L_2(z_k^{B0})} \]

\[ \text{For } j = 1 \ldots \text{card}(x_k^{PP}) \]
\[ w_{k,k}^{2,(i,j)} = \frac{p_D e^{-\frac{1}{\kappa_{T2}^2}} w_{T,k|k-1}^{(j)} r_{T2}^{(i,j)}}{L_2(z_k^{B0})} \]

End

**For** \( i = 1 \ldots \text{card}(Z_k^{P0}) \)

\[ L_3(z_k^{P0}) = \]
\[ \kappa_k(z_k^{P0}) + \sum_{j=1}^{\text{card}(x_k^{FM})} p_D w_{F,k|k-1}^{(j)} r_{F}^{(i,j)} \]

\[ \text{For } j = 1 \ldots \text{card}(x_k^{FM}) \]
\[ w_{k,k}^{3,(i,j)} = \frac{p_D w_{F,k|k-1}^{(j)} r_{F}^{(i,j)}}{L_3(z_k^{P0})} \]

End

**For** \( j = 1 \ldots \text{card}(x_k^{FM}) \)

\[ w_k^{(i,j)} = (1 - p_D) w_{F,k|k-1}^{(i,j)} \]

End

**For** \( j = 1 \ldots \text{card}(x_k^{PP}) \)

\[ w_k^{(i,j)} = \left(1 - p_D e^{-\frac{1}{\kappa_{T1}^2}}\right) w_{F,k|k-1}^{(i,j)} \]

End

**Obtain the updated formal set** \( \hat{X}^{FM}_{k|k} \):

\[ \{w_{k,k}^{(i,j)}, w_{p,k}^{(i,j)}, w_{y,k}^{(i,j)}, w_{y,k}^{(i,j)}\} \]
\[ \{m_{F,k|k-1}^{(i,j)}, m_{b1,k|k-1}^{(i,j)}, m_{T2,k|k}^{(i,j)}, m_{T1,k|k}^{(i,j)}\} \]
\[ \{p_{F,k|k-1}^{(i,j)}, p_{b1,k|k-1}^{(i,j)}, p_{T2,k|k}^{(i,j)}, p_{T1,k|k}^{(i,j)}\} \]

**For** \( i = 1 \ldots \text{card}(x_k^{FM}) \), \( j = 1 \ldots \text{card}(Z_k^{B2}) \)

**Obtain the updated temporary set** \( \hat{X}_{k|k}^{TP} \):

\[ \{w_{k,k}^{(i,j)}, w_{b,k}^{(i,j)}, w_{b,k}^{(i,j)}, w_{b,k}^{(i,j)}\} \]

**STEP 3. STATE EXTRACTION**

**For** \( i = 1 \ldots \text{card}(x_k^{TP}) \)

\[ \lambda_{k,B}^{(i)} > \lambda_{\text{max}} \]

Delete the \( i \)-th state from \( \hat{X}_{k|k}^{TP} \).

End

Prune the state whose weight is less than \( Pr_{th} \) off both \( \hat{X}_{k|k}^{TP} \) and \( \hat{X}_{k|k}^{FM} \).

Delete the state whose \( \lambda \) is over \( \lambda_{\text{max}} \) from \( \hat{X}_{k|k}^{TP} \).

Given the merge threshold \( Mg_{th} \), merge the states in the \( \hat{X}_{k|k}^{FM} \).

Given the merge threshold \( Mg_{th} \), merge the states in the \( \hat{X}_{k|k}^{TP} \).

**For** \( i = 1 \ldots \text{card}(x_k^{FM}) \)

\[ w_{k,k}^{(i,j)} > T_h \]

Add the \( i \)-th state into \( \hat{X}_{k|F} \).

End

**For** \( i = 1 \ldots \text{card}(x_k^{TP}) \)

\[ w_{k,k}^{(i,j)} > T_h \]

Add the \( i \)-th state into \( \hat{X}_{k|F} \).

End

Given the merge threshold \( Mg_{th} \), merge the \( \hat{X}_{k|F} \) and \( \hat{X}_{k|F} \) together. Obtain the merged state set \( \hat{X}_{k|F} \).

**For** \( i = 1 \ldots \text{card}(x_k^{FM}) \)

\[ \hat{w}_{k|F}^{(i)} = \frac{1}{2} \hat{w}_{k|F}^{(i)} \]

If \( \hat{w}_{k|F}^{(i)} > T_h \)

Add the \( i \)-th state into \( \hat{X}_{k|F} \).

End
\[
D_{k|k}^{FM}(x) = D_{k|k}^{FM*}(x) + D_{k|k}^{FT}(x, \lambda) = \frac{\text{card}(Z_k^p)}{\sum_{j=1}^{\text{card}(Z_k^p)} g_0 w_{2, k} N \left( m_{b, k|k-1}, p_{b, k|k-1} \right)} + p_D e^{-\frac{1}{2 \sigma^2}} g_k (z|x) D_{k|k-1}^{TP}(x, \lambda)
\]
\[
L_2 (z^{(t)})
\]
\[
D_{k|k}^{TP}(x, \lambda) = D_{k|k}^{TPw}(x, \lambda) - D_{k|k}^{FT}(x, \lambda) = \frac{\text{card}(Z_k^p)}{\sum_{j=1}^{\text{card}(Z_k^p)} g_0 w_{1, k} N \left( m_{b, k|k-1}, p_{b, k|k-1} \right)} + p_D e^{-\frac{1}{2 \sigma^2}} g_k (z|x) D_{k|k-1}^{TP}(x, \lambda)
\]
\[
L_1 (z^{(t)})
\]

\[
Q. Zhu
\]

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