Abstract

We show that a novel decay mode $Z\ell h$ of the bound state of stop-anti-stop pair in the ground state $^1S_0(\tilde{t}_1\tilde{t}_1^\ast)$ may have a significant branching ratio if the $CP$ violating mixing appears in the stop sector, even after we apply the stringent constraint from the measurement of the electric dipole moment (EDM) of the electron. We show that the branching ratio can be as large as 10% in some parameter space that it may be detectable at the LHC.
I. INTRODUCTION

So far the Higgs boson discovered in 2012 is the only fundamental particle of scalar in nature [1]. On the other hand, colored scalar bosons are definitely signs of physics beyond the standard model (SM), which often appears in many new physics models. One outstanding example is the scalar-top (stop) quark – superpartner of the top quark – in the Minimal Supersymmetric Standard Model (MSSM). Their strong interaction allows them to be produced abundantly at hadron colliders if kinematically allowed. The current search for the stop at LHC has pushed its mass above about 500 GeV [2]. To escape its detection, the mass of the lightest stop state $\tilde{t}_1$ is compressed just above the lightest neutralino mass so that there is not much missing momentum for tagging the event at the LHC. In such a scenario, the stop state is rather long lived in comparison to the time scale of QCD hadronization. Therefore, the stop-anti-stop pair can form the bound state, called the stoponium [3], which is produced through the gluon-gluon fusion [3–5] as in the squarkonium [6] production. The ground state $\eta \equiv \ 1S_0(\tilde{t}_1\tilde{t}_1^*)$ of the stoponium can then be identified by its distinctive decay modes, such as $hh, WW, ZZ, \gamma\gamma$, etc. Among them, the channel $hh$ stands out [3] for its significant decay rate with clean detection signature. Recent studies of the stoponium at LHC can be found in [7–12]. There are also efforts in studying the QCD corrections [13, 14], the lattice calculation [15], the mixing between the Higgs boson and the stoponium [16], and the role of the stoponium [17] in the dark matter co-annihilation.

Surprisingly in all studies about the stoponium decay, the channel $Zh$ is not given. In fact, the process is forbidden by the underlying assumption of the $CP$ conservation, which implies the cancellation of amplitudes à la the Furry theorem. However, there is no strong argument against $CP$ violation in the stop sector. We are going to show in this article that $\eta \to hZ$ can have a significant branching ratio when $CP$ violating parameters are chosen yet within the experimental constraint due to the electron electric dipole moment (eEDM) measurement.

If the mass of the stoponium is close to the mass $m_A$ of the pseudoscalar Higgs boson, substantial enhancement of the $Zh$ decay mode happens due to the resonance effect. Nevertheless, for a stoponium mass around $1.2$ TeV $\sim m_A$ the $e$EDM places a very stringent constraint on the choice of the $CP$-violating parameter such that the $B(\eta \to Zh) \sim 10^{-3}$. On the other hand, if the mass of the second stop is not too far from the lightest stop,
substantial cancellation between the stop contributions to the eEDM can happen, such that the \( CP \)-violating parameter can be chosen to be much larger and the branching ratio 
\[ B(\tilde{\eta} \to Zh) \sim 10^{-1} \]. In the extreme case that the \( m_A \to \infty \) when the eEDM is not effective, the branching ratio can reach a large value, 
\[ B(\tilde{\eta} \to Zh) \sim O(0.5) \]. This is the major result of our work. Furthermore, due to the heavy stoponium decay the \( Z \) and \( h \) bosons are very boosted, in which both bosons can be identified as boosted objects with advanced boost techniques to suppress backgrounds. Such rather straightforward detection of the \( Z \) and \( h \) bosons makes the mode \( Zh \) a wonderful place to look for the new particle as well as \( CP \) violation.

The organization is as follows. In the next section, we give details about the mixing in the stop sector, as well as the \( CP \)-violating couplings to the Higgs boson and \( Z \) boson. In Sec. III, we analyze the decay mode \( Zh \) together with the eEDM constraint. In Sec. IV, we estimate the observability of the \( Zh \) mode at the LHC. We summarize in Sec. V.

II. \( CP \)-VIOLATION IN THE STOP SECTOR

Let us start with the \( Z \) boson couplings to the stops \( t_i (i = 1, 2) \). The convective current among stop states is

\[
J_{ij}^\mu = i \tilde{t}_i^\dagger \partial \tilde{t}_j \quad \text{where} \quad \partial \equiv \partial - \tilde{\partial} .
\]

Our convention for the Feynman vertex amplitude is

\[
(\tilde{t}_i(p_i)|J_{ij}^\mu|\tilde{t}_j(p_j)) = (p_j + p_i)^\mu ,
\]

for the incoming \( p_j \) and the outgoing \( p_i \). Under the charge conjugation \( C \), \( \tilde{t}_i \leftrightarrow t_i \). So \( J_{ij}^\mu \leftrightarrow -J_{ji}^\mu \). The negative sign in the transformation of \( J \) comes from that in \( \partial \).

Consequently, we need to make the \( C \)-odd transformation for the \( Z \) gauge boson, \( Z^\mu \leftrightarrow -Z^\mu \). The hermiticity of the unitary interaction \( \mathcal{L} \supset \sum_{ij} g_{ij}^Z J_{ij}\mu Z^\mu \) requires \( g_{ij}^Z = g_{ji}^Z \). If the charge conjugation is a good symmetry, we have \( g_{ij}^Z = g_{ji}^Z \). From this, we know that a complex \( g_{ij}^Z \) (for \( i \neq j \)) if its phase is not removable implies \( C \)-parity violation.

In general, if the states \( \tilde{t}_{L,R} \) mix with each other by the complex \( 2 \times 2 \) matrix into the mass eigenstates \( \tilde{t}_{1,2} \), we expect the complex off-diagonal \( g_{12}^Z \) coupling to the \( Z \) boson. However, we can set \( g_{12}^Z \) real by redefining the relative phase between the two stop fields \( \tilde{t}_1, \tilde{t}_2 \). Indeed
in the next section, we adopt such a choice in our convention. To have a genuine $C$-parity non-conservation, we need additional complex coupling coefficient $y$, which appears in the Higgs vertex of $yh(\tilde{t}_2^*\tilde{t}_1^*)$. Then there is no more freedom to remove its phase.

For the renormalizable interaction of the pure bosonic sector, operators of dim 4 or less do not involve the $P$-odd Levi-Civita $\epsilon$-symbol. Therefore, the $P$-parity is conserved in the $Z$ vertex. Consequently, the $C$-parity violation is the $CP$-violation. Our example is the decay of the ground state of the stoponium in $^1S_0(\tilde{t}_1^*\tilde{t}_1^*)$ into $Zh$. The exchange of $\tilde{t}_2$ can appear in the $t$-channel and in the $u$-channel, as shown in the first two diagrams in Fig. 1. The phase of $g_{ij}^Z$ is tied with another vertex $yh\tilde{t}_1^*\tilde{t}_2$, and thus overall unremovable. The two amplitudes of the $u$ and $t$ channels cancel if the coupling factor is real, but add up if imaginary. The production of $Zh$ from such a decay is a sign of $CP$-violation.

**FIG. 1.** Feynman diagrams for the stoponium decaying into $Zh$ via the $t,u,s$ channels from the left to the right.

Furthermore, there exists the direct coupling of the pseudoscalar $A^0$ to the stops, $A^0(\tilde{t}_1^*\tilde{t}_1^* - \tilde{t}_2^*\tilde{t}_2)$, which is $CP$-violating. The ground state of the stoponium $\tilde{\eta}$ can annihilate into the virtual $A^0$ in the $s$-channel as in the third diagram in Fig. 1 and then become $Zh$ via the $ZA^0h$ gauge vertex. If the mass of the stoponium is close to the mass $m_A$ of the pseudoscalar Higgs boson, substantial enhancement of the $Zh$ decay mode happens, indeed the $Zh$ mode is significant in such a scenario. Nevertheless, it is restricted by the eEDM especially when the mass eignestates of the stop sector is widely separated and $m_A$ is moderate. When $m_A$ is chosen to be very heavy, then the constraint of eEDM disappears and the $CP$ parameter can be chosen very large and the branching ratio into $Zh$ can be as large as $O(0.5)$. 


A. Complex mixing in the Stop sector

Input parameters in the calculation of the $\tilde{\eta} \to Zh$ decay mode include masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, mixing parameters $\theta_\tilde{t}$, $\delta_u$, Re$[\mu^*e^{-i\delta_u}]$, Im$[\mu^*e^{-i\delta_u}]$, and $\tan \beta$ is the ratio of the VEV of the two Higgs doublets.

The relative phase between the $\mu$ parameter and the trilinear $A_t$ parameter can be established in the following $\tilde{t}_L\tilde{t}_R^*$ term in the Lagrangian:

$$\mathcal{L} \supset -y_tA_t\tilde{t}_L\tilde{t}_R^*H_u^0 + y_t\mu^*\tilde{t}_L\tilde{t}_R^*H_d^0 + H.c. + ....,$$

(1)

where $y_t = \frac{\sqrt{2}m_t}{v \sin \beta}$, $v = 246$ GeV, and

$$H_u^0 = \frac{1}{\sqrt{2}} \left[ vs_\beta + (c_\alpha h + s_\alpha H) + i(A^0 c_\beta - G^0 s_\beta) \right],$$

$$H_d^0 = \frac{1}{\sqrt{2}} \left[ vc_\beta + (-s_\alpha h + c_\alpha H) + i(A^0 s_\beta + G^0 c_\beta) \right],$$

(2)

where $c_\beta, s_\beta$ are shorthand notation for $\cos \beta$ and $\sin \beta$, $c_\alpha, s_\alpha$ are for $\cos \alpha$ and $\sin \alpha$, respectively, $\tan \beta \equiv u_d/v_u$ is the ratio of the VEV of the two Higgs doublet, and $\alpha$ is the mixing angle between the two neutral components of the Higgs doublets.

The stop mass matrix can be expressed as

$$\begin{pmatrix}
    (\tilde{t}_L, \tilde{t}_R^*) \\
    \phantom{(}\tilde{t}_L, \tilde{t}_R^*)
\end{pmatrix}
\begin{pmatrix}
    m_t^2 + M_Q^2 + M_Z^2(\frac{1}{2} - \frac{2}{3}x_W) \cos(2\beta) & m_t(A_t^* - \mu \cot \beta) \\
    m_t(A_t - \mu^* \cot \beta) & m_t^2 + M_Q^2 + M_Z^2(\frac{2}{3}x_W) \cos(2\beta)
\end{pmatrix}
\begin{pmatrix}
    \tilde{t}_L \\
    \tilde{t}_R
\end{pmatrix}.$$ 

We can define a phase $\delta_u$ by

$$A_t - \mu^* \cot \beta = |A_t - \mu^* \cot \beta|e^{i\delta_u},$$

(3)

then the mass matrix can be diagonalized by an orthogonal transformation with an angle $\theta_\tilde{t}$ into mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$:

$$\begin{pmatrix}
    \tilde{t}_L \\
    \tilde{t}_R
\end{pmatrix}
= \begin{pmatrix}
    1 & 0 \\
    0 & e^{i\delta_u}
\end{pmatrix}
\begin{pmatrix}
    \cos \theta_\tilde{t} & -\sin \theta_\tilde{t} \\
    \sin \theta_\tilde{t} & \cos \theta_\tilde{t}
\end{pmatrix}
\begin{pmatrix}
    \tilde{t}_1 \\
    \tilde{t}_2
\end{pmatrix}.$$ 

The stop mass matrix can be re-expressed in terms of $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_\tilde{t}$, and $\delta_u$ as

$$\begin{pmatrix}
    (\tilde{t}_L, \tilde{t}_R^*) \\
    \phantom{(}\tilde{t}_L, \tilde{t}_R^*)
\end{pmatrix}
\begin{pmatrix}
    m_{\tilde{t}_1}^2 \cos^2 \theta_\tilde{t} + m_{\tilde{t}_2}^2 \sin^2 \theta_\tilde{t} & e^{-i\delta_u}(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) \sin \theta_\tilde{t} \cos \theta_\tilde{t} \\
    e^{i\delta_u}(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) \sin \theta_\tilde{t} \cos \theta_\tilde{t} & m_{\tilde{t}_1}^2 \sin^2 \theta_\tilde{t} + m_{\tilde{t}_2}^2 \cos^2 \theta_\tilde{t}
\end{pmatrix}
\begin{pmatrix}
    \tilde{t}_L \\
    \tilde{t}_R
\end{pmatrix}.$$ 

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By comparing the off-diagonal elements of the above two stop mass matrix, we can express $A_t$ in terms of $\text{Re}[\mu^*e^{-i\delta_u}]$, and $\text{Im}[\mu^*e^{-i\delta_u}]$:

$$\text{Re}[A_t e^{-i\delta_u}] = \text{Re}[\mu^*e^{-i\delta_u}] \cot \beta + \left( \frac{m_t^2 - m_{t_1}^2}{m_t} \right) \sin \theta_t \cos \theta_t,$$

$$\text{Im}[A_t e^{-i\delta_u}] = \text{Im}[\mu^*e^{-i\delta_u}] \cot \beta.$$ (4)

**B. Relevant Couplings for $Zh$ decay mode**

The interaction between $h$ and $\tilde{t}_{L,R}$ is

$$\mathcal{L} \subset h(t_{L}^{\tilde{t}}_L, t_{R}^{\tilde{t}}_R) \left( \begin{array}{c}
V_{LL} & V_{LR}^* \\
V_{LR} & V_{RR}
\end{array} \right) \left( \begin{array}{c}
t_L^{\tilde{t}} \\
t_R^{\tilde{t}}
\end{array} \right),$$

$$= h(t_{L}^{\tilde{t}}_L, t_{R}^{\tilde{t}}_R) \left( \begin{array}{c}
-\frac{g m_t^2 c_{\alpha}}{m_W s_{\beta}} + \frac{g m_t^2}{\sqrt{1-x_W}} \left( \frac{1}{2} - \frac{2}{3} x_W \right) s_{\alpha + \beta} \\
-\frac{1}{2} \frac{g m_t^2}{m_W s_{\beta}} (A_t c_{\alpha} + \mu^* s_{\alpha}) + \frac{g m_t^2 c_{\alpha}}{m_W s_{\beta}} + \frac{g m_t^2}{\sqrt{1-x_W}} \left( \frac{2}{3} x_W \right) s_{\alpha + \beta}
\end{array} \right) \left( \begin{array}{c}
t_L^{\tilde{t}} \\
t_R^{\tilde{t}}
\end{array} \right),$$

$$\equiv h(t_{L}^{\tilde{t}}_L, t_{R}^{\tilde{t}}_R) \left( \begin{array}{c}
y_{t_1 t_1}^{h} c_{\alpha} + y_{t_1 t_2}^{h} c_{\alpha} \\
y_{t_2 t_1}^{h} + y_{t_2 t_2}^{h}
\end{array} \right) \left( \begin{array}{c}
t_1^{\tilde{t}} \\
t_2^{\tilde{t}}
\end{array} \right),$$ (5)

where $m_W = \sqrt{g^2 + g^2 v}$, $m_Z = m_W/\sqrt{1-x_W}$, and

$$y_{t_1 t_1}^{h} = V_{LL} c_{\theta_t}^2 + V_{RR} s_{\theta_t}^2 + 2 s_{\theta_t} c_{\theta_t} \text{Re}[V_{LR} e^{-i\delta_u}]$$

$$y_{t_2 t_2}^{h} = V_{LL} s_{\theta_t}^2 + V_{RR} c_{\theta_t}^2 - 2 s_{\theta_t} c_{\theta_t} \text{Re}[V_{LR} e^{-i\delta_u}]$$

$$y_{t_1 t_2}^{h} = s_{\theta_t} c_{\theta_t} (V_{RR} - V_{LL}) + (c_{\theta_t}^2 - s_{\theta_t}^2) \text{Re}[V_{LR} e^{-i\delta_u}] + i \text{Im}[V_{LR} e^{-i\delta_u}]$$

and

$$\text{Re}[V_{LR} e^{-i\delta_u}] = -\frac{1}{2} \frac{g m_t}{m_W} \left\{ \frac{c_{\beta - \alpha}}{s_{\beta}^2} \text{Re}[\mu^* e^{-i\delta_u}] + \frac{c_{\alpha}}{s_{\beta}} \left( \frac{m_t^2 - m_{t_1}^2}{m_t} \right) \sin \theta_t \cos \theta_t \right\}.$$ (6)

$$\text{Im}[V_{LR} e^{-i\delta_u}] = -\frac{1}{2} \frac{g m_t}{m_W} \left\{ \frac{c_{\beta - \alpha}}{s_{\beta}^2} \text{Im}[\mu^* e^{-i\delta_u}] \right\}.$$ (7)

For the interaction between the heavy Higgs $H$ and stops $\tilde{t}_{1,2}$, we need to change the above $h, \tilde{t}_{1,2}$ interactions by substitutions

$$h \rightarrow H, \ c_\alpha \rightarrow s_\alpha, \ -s_\alpha \rightarrow c_\alpha.$$ (8)
On the other hand, the interaction between $A^0$ and $\tilde{t}_{L,R}$ is

$$
\mathcal{L} \supset - \frac{im_t}{v \sin \beta} A^0(\tilde{t}_L, \tilde{t}_R) \begin{pmatrix} 0 & -(A_t^* c_\beta + \mu s_\beta) \\ A_t c_\beta + \mu^* s_\beta & 0 \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = \frac{m_t}{v \sin \beta} A^0(\tilde{t}_1, \tilde{t}_2) \begin{pmatrix} 2s_\theta_t c_\theta_t \text{Im}[\hat{A}_t] & i(c_\theta_t^2 \hat{A}_t^* + s_\theta_t^2 \hat{A}_t) \\ -i(c_\theta_t^2 \hat{A}_t + s_\theta_t^2 \hat{A}_t^*) & -2s_\theta_t c_\theta_t \text{Im}[\hat{A}_t] \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}
$$

$$
\equiv A^0(\tilde{t}_1, \tilde{t}_2) \begin{pmatrix} y_{1\tilde{t}_1} \tilde{t}_1 & y_{1\tilde{t}_2} \tilde{t}_2 \\ y_{2\tilde{t}_1} \tilde{t}_2 & y_{2\tilde{t}_1} \tilde{t}_2 \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}
$$

(9)

where $\hat{A}_t = (A_t c_\beta + \mu^* s_\beta) e^{-i\delta_a}$, and $\text{Im}[\hat{A}_t] = \text{Im}[\mu^* e^{-i\delta_a}] / \sin \beta$. Also, $y_{1\tilde{t}_1} = -y_{2\tilde{t}_2}$.

The interaction between $Z$ boson and $\tilde{t}_{L,R}$ is

$$
\mathcal{L} \supset \frac{g}{\sqrt{1-x_W}} Z^\mu(\tilde{t}_L, \tilde{t}_R) i \partial_\mu \begin{pmatrix} -\frac{1}{2} + Q_i x_W & 0 \\ 0 & Q_i x_W \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}
$$

$$
= \frac{g}{\sqrt{1-x_W}} Z^\mu (\tilde{t}_1, \tilde{t}_2) i \partial_\mu \begin{pmatrix} -\frac{1}{2} c_\theta_t + Q_i x_W & \frac{1}{2} s_\theta_t c_\theta_t \\ \frac{1}{2} s_\theta_t c_\theta_t & -\frac{1}{2} s_\theta_t - Q_i x_W \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}
$$

$$
\equiv Z^\mu (\tilde{t}_1, \tilde{t}_2) i \partial_\mu \begin{pmatrix} g_{\tilde{t}_1\tilde{t}_1} Z & g_{\tilde{t}_1\tilde{t}_2} Z \\ g_{\tilde{t}_2\tilde{t}_1} Z & g_{\tilde{t}_2\tilde{t}_2} Z \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}
$$

(10)

where the two-way derivative $i \partial_\mu$ applies only to the stop fields, and picks up $(p - p')_\mu$ of the stop momenta $p, p'$ flowing into the vertex in the Feynman diagram.

The process $\tilde{t}_1 \tilde{t}_1^* \rightarrow hZ$ involve the $s$-channel diagram going by the $A^0$ exchange, as well as the $t$-channel and the conjugated $u$-channel by the $\tilde{t}_2$ exchange, as shown in Fig. 1.

In the non-relativistic approximation, the overall amplitude is

$$
\mathcal{M}(\tilde{t}_1 \tilde{t}_1^* \rightarrow hZ) = - \left[ \frac{4i\text{Im}(g^Z_{\tilde{t}_1\tilde{t}_2} y_{h\tilde{t}_1\tilde{t}_2}^h)}{m_h^2 + m_Z^2 - 2(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)} + \frac{2g_{A\tilde{t}_1\tilde{t}_2}^Z g_{A\tilde{t}_1\tilde{t}_2}^Z}{4m_{\tilde{t}_1}^2 - m_A^2} \right] (P \cdot \varepsilon_Z),
$$

(11)

where $g_{A\tilde{t}_1\tilde{t}_2}^Z = \frac{g}{2\sqrt{1-x_W}} \cos(\beta - \alpha)$. The overall transition rate requires the polarization sum,

$$
\sum_{\varepsilon_Z} (P \cdot \varepsilon_Z)^2 = P^\mu \left( -g_{\mu\nu} + \frac{p_{Z\mu} p_{Z\nu}}{m_Z^2} \right) P^\nu = \lambda(s, m_h^2, m_Z^2) \frac{\lambda(s, m_h^2, m_Z^2)}{4m_Z^2}.
$$

Here we use $2P \cdot p_Z = s + m_Z^2 - m_h^2$ and $s = m_h^2 \simeq 4m_{\tilde{t}_1}^2$. The kinematic function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. Note that all amplitudes are suppressed by the non-alignment factor $\cos(\beta - \alpha)$, which appears in both $g_{A\tilde{t}_1\tilde{t}_2}^Z$ and $\text{Im}[y_{h\tilde{t}_1\tilde{t}_2}^h] = \text{Im}[V_{LR} e^{-i\delta_a}]$. 

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The partial decay width in the non-relativistic approximation is

\[ \Gamma(\tilde{t}_1\tilde{t}_1^* \to hZ) = \frac{1}{(2\pi)^2} \sum_{\epsilon_Z} |\mathcal{M}(\tilde{t}_1\tilde{t}_1^* \to hZ)|^2 |\psi(0)|^2 \frac{3}{8\pi} \lambda^2 (1, m_h^2/s, m_Z^2/s) , \]

where the bound state wave function at the origin is estimated by the Coulomb type expression,

\[ |\psi(0)|^2 = \frac{1}{27\pi} (\alpha_s m_{\tilde{t}_1})^3 . \]

In comparison, we show the partial decay width of the gluon-gluon mode .

\[ \Gamma(\tilde{t}_1\tilde{t}_1^* \to gg) = \frac{4\pi \alpha_s^2}{3m_{\tilde{t}_1}^2} |\psi(0)|^2 . \]

### C. Contributions to the Electron EDM

The most recent eEDM gives a very stringent constraint\[18\]

\[ |d_e| < 8.7 \times 10^{-29} \text{ e} \cdot \text{cm} , \text{ at } 90\% \text{ C.L.} \] (12)

In MSSM, the relevant contribution to the eEDM based on the \( CP \) violating parameters in the stop sector \( \tilde{t}_{1,2} \) arises via the two-loop Barr-Zee diagrams \[19\].

\[ \left( \frac{d_e}{e} \right)_{2-\text{loop}} = 2Q_e Q_{\tilde{t}} \frac{3\alpha_{\text{em}} m_e}{64\pi^3 m_A^2} \left( \frac{\sin 2\tilde{t} m_i \text{Im}[\mu^* e^{-i\delta_i}]}{v^2 \sin \beta \cos \beta} \right) \left[ F\left( \frac{m_{\tilde{t}_i}^2}{m_A^2} \right) - F\left( \frac{m_{\tilde{t}_2}^2}{m_A^2} \right) \right] , \] (13)

where \( \alpha_{\text{em}} = e^2/(4\pi) \), \( v \simeq 246 \text{ GeV} \), and \( F(z) \) is a two-loop function given by

\[ F(z) = \int_0^1 dx \frac{x(1-x)}{z-x(1-x)} \ln \left[ \frac{x(1-x)}{z} \right] . \] (14)

In fact the eEDM contribution vanishes in two different limits, first when \( A^0 \) becomes heavy and decoupled, and second when \( m_{\tilde{t}_1} \simeq m_{\tilde{t}_2} \) so that their effects cancel each other. Our numerical results show that even in general cases, an ample parameter space satisfies the eEDM constraint, but still gives significant branching ratio mode of \( hZ \).

Although the one-loop contributions to the eEDM 1-loop also exist in the neutralino-selectron diagram, and the chargino-sneutrino diagram, they involve totally different \( CP \) violating parameters and can be tuned to give tiny eEDM \[20\]. Therefore, we ignore their one-loop effect in eEDM. In another approach \[21\] \[22\], one can allow the sole contribution of one type of diagrams to exceed the current experimental limit, where one can expect that there might be other types of diagrams that would cancel one another.
III. ANALYSIS

The input parameters that are relevant for the stoponium decay into \( Zh \) are: \( m_{\tilde{t}_1}, m_{\tilde{t}_2}, \) 
\[ \text{Re}[\mu^* e^{-i\delta_u}], \text{Im}[\mu^* e^{-i\delta_u}], \theta_T, \tan \beta, \text{and } m_A. \] In the computation of the branching ratios of the stoponium, it also involves the gluino mass \( m_{\tilde{g}} \) and \( \cos(\beta - \alpha) \).

Since we expect the pseudoscalar resonance can enhance the decay rate when \( m_{\tilde{t}} \) is around the heavy pseudoscalar \( A^0 \) mass, we study the following cases,

1. Near and below the pole, \( m_{\tilde{t}} < m_A \) by setting \( 2m_{\tilde{t}_1} = 1200 \text{ GeV} \) and \( m_A = 1.5 \text{ TeV} \).
2. Well below the pole, \( m_{\tilde{t}} \ll m_A \) by setting \( 2m_{\tilde{t}_1} = 1200 \text{ GeV} \) and \( m_A = 2.5 \text{ TeV} \).
3. Far from the pole for an extremely heavy \( m_A \). We set \( 2m_{\tilde{t}_1} = 1200 \text{ GeV} \ll m_A \). In this case, we simply remove the \( s \)-pole contribution. Note that in this limiting case, the two-loop contribution of the pseudoscalar boson \( A^0 \) to the eEDM vanishes as well.

Note that we do not choose \( m_A \) very close to \( m_{\tilde{t}} \) in case (1), because for such a low \( m_A \) the contribution to the eEDM would be large. In Fig. 2, we show the branching ratios of the stoponium in upper panels with the corresponding predictions for the eEDM in the lower panels, where we have chosen the heavier stop mass \( m_{\tilde{t}_2} \) to be 1 TeV and \( m_A = 1.5 \text{ TeV} \). For simplicity we have also chosen \( \text{Re}[\mu^* e^{-i\delta_u}] = 0 \). We note that the partial width into \( Zh \) depends on \( \text{Im}[\mu^* e^{-i\delta_u}] \), as indicated in Eq. (7), and the eEDM is also proportional to \( \text{Im}[\mu^* e^{-i\delta_u}] \), as shown in Eq. (13). Therefore, we cannot choose the parameter \( \text{Im}[\mu^* e^{-i\delta_u}] \) arbitrarily large. It is clear from the lower panels in Fig. 2 that \( \text{Im}[\mu^* e^{-i\delta_u}] = 200 \text{ GeV} \) is the largest allowed value without violating the constraint of eEDM under the set of other input parameters shown in the figure caption. The branching ratio in \( Zh \) is also small, and of order \( 10^{-3} \) only.

An interesting observation can be found in Eq. (13) that when the heavier stop mass is indeed close to the lightest stop mass, a significant cancellation between these two contributions is possible. In Fig. 3, we show the branching ratios of the stoponium and the corresponding predictions for the eEDM with \( m_{\tilde{t}_2} = 650 \text{ GeV} \) and heavier \( m_A = 2.5 \text{ TeV} \) (case 2). The parameter \( \text{Im}[\mu^* e^{-i\delta_u}] \) can be chosen as large as 2000 GeV without violating the constraint of eEDM. With such a large \( \text{Im}[\mu^* e^{-i\delta_u}] \) the branching ratio into \( Zh \) can be as large as 10%. With such a large branching ratio, the stoponium decay into \( Zh \) now becomes very interesting and detectable.
FIG. 2. Upper panels show the branching ratios of the stoponium with the corresponding predictions for the predicted eEDM from the two-loop Barr-Zee diagrams shown in the lower panels. We set $\delta_u = 0$, and $\mu$ purely imaginary, i.e. $\text{Re}[\mu^* e^{-i\delta_u}] = 0$. In the left (right) panel, we choose $\text{Im}[\mu^* e^{-i\delta_u}] = 100 (200)$ GeV. For all panels we fix $m_{\tilde{t}_1} = 600$ GeV, $m_{\tilde{t}_2} = 1$ TeV, $m_{\tilde{g}} = 2$ TeV, $\tan \beta = 10$, $\cos(\beta - \alpha) = 0.1$, $m_h = 125$ GeV, $m_{H,A} = 1.5$ TeV, and vary $\theta_t \subseteq [0, \pi/2]$. We include the binding energy effect in the stoponium mass, $m_{\tilde{\eta}} = 1195$ GeV.

In the extreme case of case (3), the mass of the pseudoscalar $A^0$ is set to be very heavy. Practically, we ignore the term involving the $A^0$ exchange. We show in Fig. 4 the branching ratios for the stoponium with $m_{\tilde{t}_1} = 600$ GeV and $m_{\tilde{t}_2} = 1000$ GeV, except for the lower-right panel where $m_{\tilde{t}_2} = 650$ GeV. Since there are no more $A^0$ contribution to the eEDM, we can set the parameter $\text{Im}[\mu^* e^{-i\delta_u}]$ large enough to achieve a dominant branching ratio for the $Zh$ mode. We have chosen $\text{Im}[\mu^* e^{-i\delta_u}] = 100, 200, 5000,$ and $5000$ GeV, respectively. Note that increasing $\text{Im}[\mu^* e^{-i\delta_u}]$ will also increase the $hh$ mode, because the partial width $\Gamma(\tilde{\eta} \to hh) \propto |y_{ht,\tilde{t}_1\tilde{t}_2}^h|^2$, and $\Gamma(\text{stoponium} \to hZ) \propto \text{Im}[y_{ht,\tilde{t}_1\tilde{t}_2}^h]$. In the most favorable case, the branching ratio into $Zh$ can be of order $O(0.5)$, as indicated in the lower-right panel.
FIG. 3. Upper panels show the branching ratios of the stoponium with the corresponding predictions for the predicted eEDM from the two-loop Barr-Zee diagrams shown in the lower panels. We set $\delta_u = 0$, and $\mu$ purely imaginary, i.e. $\text{Re}[\mu^* e^{-i\delta_u}] = 0$. In contrast to Fig. 2, here we set $m_{H,A} = 2.5$ TeV, $m_{\tilde{t}_2} = 650$ GeV, and $\text{Im}[\mu^* e^{-i\delta_u}] = 1000$ (2000) GeV for the left (right) panels. The other input parameters are the same as Fig. 2.

IV. OBSERVABILITY AT THE LHC

The leading order (LO) production process for $\tilde{\eta}$ at LHC is through the gluon-gluon fusion, $gg \rightarrow \tilde{t}_1 \tilde{t}_1^*$. The cross section can be expressed in term of its gluonic decay width as

$$\sigma(pp \rightarrow \tilde{\eta}) = \frac{\pi^2}{8m_{\tilde{\eta}}^2} \Gamma(\tilde{t}_1 \tilde{t}_1^* \rightarrow gg) \int_{\tau}^{1} dx \frac{\tau}{x} g(x,Q) g(\tau/x,Q),$$

(15)

where $g(x,Q)$ is the gluon parton distribution function, and $\tau \equiv m_{\tilde{\eta}}^2/s$ with the center of mass energy of $pp$ collision $\sqrt{s}$. For the parton distribution function, we used CTEQ6 [23] with the factorization scale $Q = m_{\tilde{\eta}}$. The K-factor, which is the ratio between the next leading order (NLO) and the LO cross sections, we take a reasonable value about 1.4. For
In this extreme case of $m_A \to \infty$ the contribution to eEDM vanishes. Here we show the branching ratios of the stoponium for $m_{\tilde{t}_1} = 600$ GeV. The other relevant parameters are $m_{\tilde{t}_2} = 1$ TeV in the upper-left, upper-right, and lower-left panels, while $m_{\tilde{t}_2} = 650$ GeV in the lower-right panel. The $\text{Im}[\mu^\ast e^{-i\delta_u}] = 100, 200, 5000$, and $5000$ GeV, respectively. The other parameters are the same as in Fig. 2.

For a more detailed NLO calculation, we refer to Ref. [14]. At NLO, we obtain the production cross section for $m_{\tilde{t}} \simeq 1.2$ TeV at the LHC of $\sqrt{s} = 13$ TeV.

$$\sigma(pp \to \tilde{t}) \simeq 1 \text{ [fb]}.$$ \hfill (16)

The $Zh$ decay mode of the stoponium can be searched for via $h \to b\bar{b}$ and $Z \to \ell^+\ell^-$ or $Z \to jj$. At the LHC, such searches have been performed [24–27], in which hadronic or leptonic modes of the $Z$ boson and $b\bar{b}$ mode of the Higgs boson have been used. It is clear that the leptonic mode of the $Z$ boson is clean but suffers from a small branching ratio. The hadronic mode of $Z$ boson was believed to be suffered from large QCD background. Nevertheless, with the advance of various boosted-jet techniques the hadronic decays of the
Z boson and $h$ can be performed with reasonable success. Since the stoponium is rather heavy $\sim 1.2 - 1.5$ TeV here, the Z boson and the Higgs boson are very boosted with $p_T \sim 0.6 - 0.75$ TeV. The opening angle between the decay products of the Z or the Higgs boson is $\sim 2M/p_T \sim 0.3 - 0.5$. This is in the right ballpark for excellent detectability of boosted jets in contrast to the conventional QCD background.

The recent search for $pp \rightarrow X \rightarrow Zh \rightarrow jjb\bar{b}$ performed by ATLAS\cite{24} at the LHC gave an upper limit on $\sigma(pp \rightarrow X \rightarrow Zh) \times B(h \rightarrow b\bar{b} + c\bar{c}) < 20 - 30$ fb around the resonance mass $1.2 - 1.5$ TeV. On the other hand, the search $pp \rightarrow X \rightarrow Zh \rightarrow \ell^+\ell^-b\bar{b}$ was also performed \cite{26}. The upper limit on $\sigma(pp \rightarrow X \rightarrow Zh) \times B(h \rightarrow b\bar{b} + c\bar{c}) < 10$ fb. Note that these searches was designated for vector resonances. In the same paper, they also gave $\sigma(pp \rightarrow A \rightarrow Zh) \times B(h \rightarrow b\bar{b}) < 10$ fb for $m_A \approx 1.2$ TeV. Therefore, the production cross section of the stoponium times the branching ratio into $Zh$ is well below the current limits at the LHC.

With a project luminosity of 300 fb$^{-1}$ at the end of Run II, we can expect about 15 events for $\tilde{\eta} \rightarrow Zh \rightarrow (jj, \ell\ell) + b\bar{b}$ for an optimistic branching ratio $B(\tilde{\eta} \rightarrow Zh) \sim 10\%$. We emphasize again that in $CP$-conserving case the stoponium would not decay into $Zh$, yet a small branching ratio into $Zh$ would signal a violation of $CP$ symmetry.

V. CONCLUSIONS

We have demonstrated that the decay mode of the ground state of the stoponium, $\tilde{\eta} \rightarrow Zh$, can have a dominant or significant branching ratio if we choose suitable $CP$ violating mixing in the stop sector, which is still allowed by the eEDM measurement. Observation of such a decay mode of the stoponium is clean signal of $CP$ violation. The detailed phenomenology will be investigated in a separate analysis.

Our framework for the decay mode $Zh$ from the scalar pair in the ground state can be extended to other models that have fundamental colored scalar bosons, such as the technipion\cite{28} or the colored octet Higgs\cite{29}.

We offer a few comments before closing.

1. Both the partial width of $\tilde{\eta} \rightarrow Zh$ and eEDM increase with increase in the parameter $\text{Im}[\mu^*e^{-ib_u}]$. Therefore, we cannot make it arbitrarily large. When $m_A = 1.5$ TeV and $m_{\tilde{t}_1} = 600$ GeV, $\text{Im}[\mu^*e^{-ib_u}]$ can only be $100 - 200$ GeV.
2. The $A^0$ contribution would be suppressed with increases in $m_A$. Further suppression can be achieved with a smaller mass difference between $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$. For $m_{\tilde{t}_1} = 600$ GeV and $m_{\tilde{t}_2} = 650$ GeV, and $m_A = 2.5$ TeV, the parameter $\text{Im}[\mu^*e^{-i\delta_u}]$ can be as large as 2000 GeV. The branching ratio for $Zh$ can be enhanced to about 0.1.

3. In the extreme case of very heavy $m_A$, the $A^0$ contribution to eEDM vanished. Thus, we can choose a very large $\text{Im}[\mu^*e^{-i\delta_u}]$ such that the branching ratio into $Zh$ can be of order $O(0.5)$.

4. There are other contributions to the eEDM from 1-loop diagrams in supersymmetric models, such as chargino-selectron loop and neutralino-sneutrino loop, and other 2-loop diagrams such as Barr-Zee diagrams with chargino, neutralino, stau, etc. Here in this work we only focus on a particular contribution from $A^0$. In principle, we can allow some level of cancellation from other contributions, such that the sole contribution from $A^0$ may be over the current constraint. In such a case, the parameter $\text{Im}[\mu^*e^{-i\delta_u}]$ could be chosen a larger value and the branching ration into $Zh$ could increase.

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