Isotropy equilibrium of the double woven fabric with cotton face and wool reverse fibrous compositions

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Abstract. The equilibrium of the masses and the mechanical properties between the warp and the weft is a determining factor for the quality of the woven fabrics. When the fabric has a multi-layered structure and is designed for protective clothing, the uniform distribution of the elasical resistance acquires a paramount importance for the consumer properties. Isotropy in the sense of absolute equalising of the properties between the base and the weft evaluates the achieved optimum cohesion between the weaving threads and directs the weaving cycle settings. The possible variation of the ratio between the elastic modules of the warp and the weft, depending on the weft spacing and the warp tension, is the basic idea of this article.

1. Introduction
Designing of a woven fabric goes through a variety of procedures and usually takes a long time, which is inevitable in the mass production of textiles. However, when the woven fabric has a complex weave with a multi-layered structure and a varied fibrous composition of the threads, the requirements for it are related to the general equilibrium between the warp and the weft. If the fabric is intended for protective garment, operational expectations add new requirements to its quality and properties.

The strength of the fabric is paramount in determining its quality. From this point of view, the fabric project focuses on the useful equalising of the mechanical resistance in the direction of the warp and the weft threads. Well-known studies of the multi-layer fabrics as well as the technological experience have identified two main factors for variation the structure and the properties of fabrics during the weaving cycle: weft spacing and stretching of the warp.

The subject of this article is the variation of ratio between the elastic module on warp and on weft for woven double fabric with a cotton face layer and a wool reverse layer. The aim of the work is to establish the relationship between the fabric structure and its possible plain isotropy. Performance tasks include: literary research, conducting a planned experiment, processing the results with analysis and discussion.

2. Theory assumptions
In spite of their spatial structure, the textiles are regarded as lamellas. One of the reasons is that the thickness of the fabric is many times smaller than its length and width. The other reason is that the establishment of the basic quality of the fabric, i.e. its strength is achieved by applying forces and deformations along its plain surface.
For two-dimensional continuous mediums, the theory of the elasticity [3] considers the stress-strain process as a homogenous in the thickness \( z \) of the fabric. The deformation tensor \( \varepsilon \) is a function of the surface coordinates only, such as \( x \) and \( y \), and does not depend on \( z \). From the boundary conditions of the stresses \( \sigma \) on the surface of the fabric subjected to longitudinal forces [3], the stresses on the normal vector \( (z) \) are negligible:

\[
\sigma_l \cdot n_k = 0 \quad \Rightarrow \quad \sigma_x = \sigma_y = \sigma_z = 0
\]  
(1)

The isotropy of homogeneous materials is subject to study in three research areas. First of all, the theory of elasticity [3] and the elasticity in a continuous medium [5] define the isotropy as the equation of the components of the material tensor \( \lambda = c_{ij} \leftrightarrow \lambda_{ij} \), regardless of their location in the matrix, i.e. regardless of target \( \sigma_x \).

\[
\lambda_{ij} = \lambda_{kl} = \lambda_{lk} \quad \text{(2)}
\]

Also, for the isotropic materials the external load is expressed as a function of the stress tensor \( \sigma \) by the following way:

- Either as a symmetrical function of the fundamental stress \( \sigma_x, \sigma_y \text{ and } \sigma_z \);
- Either as a function of the stress tensor invariants \( \sigma_i \).

Secondly, the theory of plasticity [6] and [5] describes the elastic behaviour of the isotropic materials by only one modulus of elasticity - \( E_Y \), one Poisson’s coefficient \( \nu \) and one bonding modulus - \( G \). In this case, the orthogonal symmetry of the stress-strain process does not depend on the orientation of the coordinate system. Moreover, the elastic isotropy can be achieved for a body with a crystalline structure, which is made up of oriented anisotropic elements. These are the textile fibres.

The third is the rheological equation of the state of the bodies or fluids [7] - \( \varepsilon = f(\tau) \).

In the absence of shear deformations \( \varepsilon_l \), then the normal stress \( \tau_l \) does not depend on the orientation of the surface under consideration:

\[
\forall \nu, j \rightarrow \varepsilon_l = 0 \rightarrow \tau_x = \tau_y = \tau_z \quad \text{(3)}
\]

In this case, the stress-strain process treats the material as homogeneous and isotropic.

The fabric, however is complicated the weave, or the fibrous composition of the threads is varied, has clearly defined parameters of the plain orthotropy. On the one hand, the fabric can be examined as a two-dimensional body. On the other hand, the perpendicular interlacing of the warp threads and the weft threads implicitly determines the orientation of the two-dimensional coordinate system.

Given the particularities of the weaving technics and the contexture of the woven fabrics, the isotropy can not be expected as the quality of the textile product in the theoretical sense of the term. The logical distinction between the elastic behaviour of the fabric on warp and on weft is subject to description by examining the corresponding elastic modules: \( E_Y \) and \( E_W \).

The ratio between the two property indicators is an universal evaluation of the variation of the fabric plain orthotropy, depending on its contexture. In this case, interest is the way by which the elasticity modulus \( E_Y \), or of Young, can be calculated from the experimental rheograms. The different sources [1], [8] and [9] determine the approximate limits of elasticity as the visible end of the linear part from the experimental rheogram. From this point of view, the coordinates of the boundary elastic elongation \( [\varepsilon] \) and the boundary elastic force of the reaction \( [F] \), \( N \) can be indicated with sufficient precision, and then the modulus of elasticity \( N \) is available for calculation:

\[
E_Y = \frac{[F]}{[\varepsilon]} \quad \text{(4)}
\]

3. Experimental work

The experimental work is carried out in industrial conditions and covers all stages of fabrication of the weaving threads up to fabrication of the fabric and the metrological testing of the various variants.
3.1. Experimental sets

3.1.1. Fibrous composition and structure of double woven fabric

The weaving threads for the double fabric with cotton face and wool reverse sides are the result of a preliminary experimental work whose results are consistently published [12, 13 and 14].

The distribution of the yarns and the man-made twisted silks towards the warp and the weft is as follows:

- Warp threads for the face fabric: $T_f = 20 \times 2, 100\% C$;
- Warp/weft threads for the face/reverse fabric: $T_f = 16.2 (8.4 V - R + 7.6 P - S)$;
- Weft thread for the reverse fabric: $T_f = 40 \times 1, 100\% W$.

The weave of the double fabric is a result of an autonomous development, which is protected by the patents [11]. The contexture of the double woven fabric with multilayer structure is shown in Figure 1.

![Figure 1. Warp direction contexture of the double woven fabric](image)

Double woven fabric for protective garment includes face and reverse fabrics. The face fabric is made up of four twill weaves with a warp effect of interlacing and diagonal furrows from bottom to top and from left to right with four warp threads and four weft threads. The reverse fabric is made up of four twill weaves with a warp effect of interlacing and diagonal furrows from bottom to top and from right to left with three warp threads and three weft threads. The obtained common double fabric contains fourteen warp threads and fourteen weft threads.

The bonding of the two fabrics is accomplished by interlacing of the fourth warp thread $4_c$ with the third weft thread $3_t$ and the eleventh warp thread $11_c$ with the tenth weft thread $10_t$. The two linkages between the face and the reverse fabric combine the face and the reverse sublayers into an intermediate layer of chemical fibres.

The three-layered structure of the double fabric is due to the particular combination of three types of threads. The warp threads of the fabric are from twisted cotton yarn. The weft threads of the reverse fabric are made from single wool (worsted) yarn. The weft threads of the face fabric and the warp threads of the reverse fabric are from twisted viscose rayon with synthetic silk (polyamide or polyester) and are at least twice as thin as the cotton or the wool yarns. These threads form respectively the face and the reverse sublayer of chemical fibres in the double fabric. The connections between the two fabrics are due to the additional interlacing between the silk threads. In this way, three separate layers are formed in the relatively independent two fabrics: face and reverse. The outer, face layer of the fabric is composed predominantly of cotton fibres with high density. The inner, reverse layer of the fabric is composed predominantly of wool fibres with reduced density and
increased porosity and fluffiness. The intermediate, bonding layer is located between the face and the reverse layers and remains hidden inside the fabric. This layer is made up by the face and reverse sublayers of the twisted chemical silks.

3.1.2. Technological and metrological equipment
The experimental samples are made on the weaving loom with flexible rapiers - Vamatex Leonardo P1001es. Dynamometric tests are performed on a Titan - Universal Tester Model 510 dynamometer. Tests comply with strip method EN ISO 13934-1 or ASTM D 5035, [8]. The microscopic photos of the facial and packing surfaces of the fabric are made using a Leica MS microscope.

3.1.3. Experimental planning
The experimental work was carried out on the basis of a central composite rotatory experiment (D-plans) with two factors. In this case the first factor is the weft spacing - \( L_d \) \([p / d]\), and the second factor is the stretching of the warp - \( P_w \) \([k]\). These types of plans by regression analysis give the possibility of compiling second-order optimization polynomials:

\[
Y_k(X_1, X_2) = b_0 + \sum b_i X_i + b_1 X_1 X_2 + \sum b_i X_i^2
\]  

(5)

3.2. Run the experiment
The order of the experimental samples is determined by the random principle. After weaving the given variants, the fabric was left to relax for 48 hours. The test specimens for the dynamometer were then prepared in accordance with the standard requirements.

3.3. Experimental results and statistical treatment
Figure 2 shows the rheograms of the dynamometric tests on the warp direction, and Figure 4 – the weft direction.

![Figure 2](image)

Figure 2, Dynamometer rheograms of the warp direction resistance

Also, the method for practical determination of the boundary elastic reaction \([P]_e\) and the boundary elastic elongation \([\varepsilon]_e\) is shown.
Figure 3, Dynamometer rheograms of the weft direction resistance

The calculated values of the Young’s modulus - $E_y[N]$ from the tested samples on warp and weft are filled in Table 1. It gives the primary results of the experimental variants through which the regression coefficients can be calculated.

Table 1, Experimental planning of the variants levels and laboratory results

| Variants | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9x5 vars |
|----------|---|---|---|---|---|---|---|---|----------|
| Factor 1 | - | + | - | + | -1,4142 | 1,4142 | 0 | 0 | 0        |
| Factor 2 | - | - | + | + | 0 | 0 | -1,4142 | 1,4142 | 0        |
| F1-weft spacing | 447 | 553 | 447 | 553 | 425 | 575 | 500 | 500 | 500      |
| F2-warp tension | 7,00 | 7,00 | 9,10 | 9,10 | 8,00 | 8,00 | 6,50 | 9,50 | 8,00     |

Table 2 gives the regression coefficients of the optimization polynomial for the elastic modulus on warp - $E_w$, the elastic weft module - $E_y$, as well as the $R_E$ ratio between them.

Table 2, Coefficient of regression of the optimising polynom

| Coefficients of regression | b0 | b1 | b2 | b12 | b11 | b22 | f | Fr |
|----------------------------|----|----|----|-----|-----|-----|---|----|
| $E_w$ [N]                  | 6740,1 | 211,0 | 56,6 | -34,8 | -260,1 | -2,7 | 9 | 2,16 |
| $E_y$ [N]                  | 3070,1 | -190,7 | -96,1 | -151,3 | -5,4 | -87,9 | 8 | 3,40 |
| $R_w/w$                    | 2,18 | 0,21 | 0,10 | 0,12 | -0,07 | 0,08 | 7 | 0,90 |
4. Analyse and discussion
The Variation of the elastic modulus depending on the experimental factors is given in Figure 4.

![Figure 4](image-url)

**Figure 4.** Response surfaces of the Young’s modules of warp and weft

The first impression from the response surface of Figure 4a refers to the significant difference in the influence of both factors. It is obvious that the stress of the warp threads in the weaving cycle slightly alters the elastic resistance of the fabric on the warp direction. The variation in the aggregate stretching on the whip roll of $P_w = 6,5 \div 9,5 k$, as well as the centre of the experimental area at $P_{thr} = 8,0 k$ as a mean load over approximately 11,000 warp threads is reduced to $P_{thr} \approx 2,5 C / t_i$ for one warp thread. Thus the average stress level approaches the boundary elastic tension of the twisted cotton yarn and the twisted chemical silks in the weaving warp. The high stress state of the warp threads determines the relatively constant level of the elastic module of the warp in two directions. The warp threads are stressed enough and the variations in the stretching of the warp no longer affect their behaviour. This deduction is confirmed by the uniform, smooth shape of the dynamometrical rheograms on warp where the specific mechanical characteristics of the two types of warp threads are not profiled.

The influence of the weft spacing on the elastic resistance of the fabric on warp is subject to major changes. The presence of an extreme area at the maximum weft spacing around $D_{u} \approx 575 p / d$ is a testimony of the boundary saturation of the fabric and the weave by weft threads. In essence, this maximum serves as a benchmark for optimizing of the weaving. Increasing the elastic resistance of the fabric on warp at increasing the weft density is an indicator of the augmented cohesion between the warp and the weft. At the same time, the free length of the warp threads between two wefts decreases and this is another reason to increase the strength of the fabric on warp.

The decrease in elastically resistance, expressed by the reduction of the Young’s modulus after the boundary weft spacing of about $D_{u} \approx 530 p / d$ is due to the depletion of the elastic capacity of the warp threads. The increased density of the weft threads causes further waveform deformations of the warp threads during the interlacing and the warp run-in. Their respective additional elongation passes the boundary elastic elongation of the twisted cotton yarns. The decreasing elastically resistance, i.e. the drop in the Young’s modulus is a testimony to exhausted elastic capacity. In this sense, the maximum values of the Young’s modulus can serve as an optimization criterion.

The influence of the weft spacing and the stretching of the weaving warp on the elastic resistance of the fabric on weft is shown in Figure 4b through the Young’s modulus response surface. Already in
the experimental rheograms of the dynamometric tests, figure 2, the specific character of both the wool single yarns and the twisted chemical silks appears.

The main tendency in the influence of the experimental factors is the reduction of the elastic modulus with the simultaneous increase of the weft density and the warp stretching. This is mainly due to the highly stressed warp threads, which cause increased waveform deformations of the weft threads. The reduced resistive elasticity, i.e. respectively, increased elongation is a logical consequence.

In the surface of the response, three tendencies in the influence of the weft density on the elastic resistance of the fabric on weft stand out. With a low-stretched warp, \( R_w \approx 6.5 \, k \), the weft density does not affect the Young’s modulus, which maintains constant values around \( E_w \approx 3.05 \, k \). Bigger quantity of the weft threads logically does not alter the elastic features. At average stretching of the weaving warp, which coincides with the centre of the experimental factor, the increase in weft density results in a reduction in the elastic modulus \( E_w \approx 3.2 \, k \). The simultaneous impact of the two experimental factors is due to the additional stress that occurs in the weaving warp due to the increased number of wefts. In turn, the more heavily stressed warp threads cause greater run-in, or else larger waveform deformations of the weft threads. The clear drop of the elastic modulus, depending on the weft spacing at maximum stretching of the warp \( R_w \approx 9.5 \, k \), confirms the physical explanation of the cohesion and the deformation of the warp and weft threads.

The isotropy of the woven fabric has a practical meaning when viewed in plain state. Also, the basic property whose behaviour in the various directions on the fabric has operational significance is the elasticity, which is represented in the case by the Young’s modulus. Absolute orthogonal symmetry, i.e. \( R_E = E_w / E_w = 1.0 \), cannot be achieved with a woven fabric as a technological product. From this point of view, the relationship between the two boundary elasticities: on warp and on weft can only produce a relative assessment of the achieved isotropy with practical significance.

**Figure 5**, Response surface of the ratio between the warp and weft modules

Firstly, the maximum value of the ratio \( R_E \approx 2.9 \) is at the maximum value of the experimental factors. The maximum weft density and maximum stretching of the warp are technological conditions in which the woven fabric acquires maximum mass (fibrous) density and homogeneity. It is obvious
that the homogeneous contexture of the fabric is not a condition for plain isotropy. In this case, the
different fibrous composition and the mechanical properties of the weaving threads: yarns and twisted
chemical silk influence. Second is the role of the average by value stretching of the warp \( P_w \cong 8,0 \, k \), which coincides with the central field of the experimental factor. A smooth zone of stability of the modulus \( R_E \) is plotted on the groove in the response surface, which slightly increases with increasing the weft density. Apparently, under this maintained stress of the warp threads, the mutual
deformation of the warp and the weft has approximately the same parameters in increasing the
weft density. The minimal plain isotropy, expressed by the ratio \( R_E \cong 1,75 \), is due to the minimum
weft density at \( E_d \cong 425 \, p \, /d \) and to the relatively increased stretching of the warp with \( P_w \cong 8,5 \, k \). At the same experimental factor value, the Young’s modulus on warp shows a
minimum value, whereas the weft modulus shows maximum values. From the point of view of the
simultaneous satisfaction of the technological and operational requirements for the fabric, its plain
isotropicity is of a compromising character and can compose one of the optimization parameters of the
weaving cycle.

5. Conclusion
The increase in the weft density results in an increase in the homogeneous distribution of the mass in
its multilayer volume, but does not increase the isotropicity between the directions of the warp and the
weft. The isotropy of the double fabric in the sense of orthogonal symmetry of the elastic resistance
can be achieved by some extent as a compromise between the warp stretching and the weft spacing.
The next development of this study should be accompanied by structural analysis of the fabric and
determination of the geometric parameters of the warp threads and the wefts.

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