Optimization of the mean-absolute deviation portfolio investment in some mining stocks using the singular covariance matrix method

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Abstract. Investing in mining stocks, investors often face risk problems. Usually to minimize risk, it is done by forming an investment portfolio. This paper aims to discuss the optimization of the investment portfolio. The data analyzed are several mining stocks traded on the capital market in Indonesia. Optimization is done using the mean-absolute deviation model with the singular covariance matrix method and the non-singular covariance matrix method to determine the optimal weight of the two existing methods. Based on the results of the optimization, we can obtain a weight allocation composition that provides an optimal portfolio. In addition, we also estimate the amount of return on expectations and risks in the optimal portfolio formed. So that the composition of this optimal weight can be used as a consideration for investors in investing their capital in several analyzed mining stocks.

1. Introduction
Investment in the mining sector is quite interesting for investors in Indonesia and abroad. The return on investment returns is a major concern for every investor [1, 2]. Risk tolerance limits for each investor vary based on preference [3]. The general strategy that investors make in investing is by forming a portfolio [4-6]. Portfolio formation, aims to provide maximum total return with certain risks or minimum total risk with a certain return [7-9]. The problem that is commonly faced by investors is determining the weight (proportion) of funds to be invested in each share to obtain an optimal portfolio [10]. The stock investment portfolio optimization process can be done by forming a covariance matrix from the stock data obtained, then determining the inverse of the covariance matrix [1]. A matrix has an inverse if the matrix is nonsingular, or in other words the determinant of the matrix is not zero [11, 12]. However, to determine the inverse of a singular matrix can be done with Pseudo Inversion [13, 14]. Pseudo inverse is used to find a solution to the inverse approach of a singular matrix [15].

Mean-variance model portfolio optimization was first put forward by Markowitz [16] which is the basis of modern financial development. Along with developments, researchers have developed many portfolio optimization models. For example Tayaki and Toluena [17] use the non-negative dimension reduction method, with the aim of increasing the efficiency of portfolio optimization using the mean-variance model. Whereas in the study of Zhifeng and Wang [18], the portfolio optimization process of the mean-variance model was improved to reduce unwanted impacts due to uncertainty in parameters and estimation errors so that a better portfolio optimization model was obtained. Li [19] conducted portfolio optimization by using robust asymmetry mean absolute deviation (ARMAD) model on historical small capitalized stock data to distinguish stocks with high returns in the near term. In the
study of Qin [20] used the mean-absolute deviation model in the portfolio optimization process, using Fuzzy random variables as a measure of risk to describe uncertainty in the selection of investments. Similar discussions can also be found at [21-25] to determine optimal portfolio. Other portfolio optimization models have been formulated by previous researchers to determine the optimal portfolio. As in Zhu [26] discusses the selection of optimal investment portfolios in risk assets, with the limitation of Maximum Value-at Risk (MVaR) using regime-switching hidden markovian models. Whereas in Sukono et.al. [5] discusses the optimization of the Mean-VaR portfolio model with risk tolerance, using quadratic utility functions assuming that asset returns have a certain distribution.

Based on the description of the above problems, we are interested in conducting a mean-absolute deviation model portfolio optimization analysis with risk tolerance using the singular covariance matrix method. The objects analyzed are several mining stocks traded on the capital market in Indonesia. The goal is to obtain the optimum portfolio weight allocation, so that optimum investment return and investment portfolio risk can be obtained.

2. Material and Method
In this section, we provide the material and methods used in the study, as follows:

2.1. Material
The data we use in this study is mining stock data in Indonesia. The historical data used is daily data obtained from https://finance.yahoo.com for the period of 2017/04/05 to 2019/04/05. We use a portfolio of 5 (five) mining stocks, namely TINS, PGAS, MDKA, CITA and BSSR.

2.2. Method
In this section, we discuss the methods used in the optimization of mining stock portfolios, using the singular covariance matrix. Discussions include: stock returns, singular covariance matrix, and mean-absolute deviation models.

3. Mathematical models
In this section, we discuss mathematical models namely pseudo inverse, calculation of stock returns, portfolio formation, portfolio optimization of mean-absolute deviation models.

3.1. Pseudo Invers
The inverse matrix concept in general, if a matrix is \( n \times n \) and non-singular then the matrix has an inverse. Whereas for \( m \times n \) or \( n \times n \) matrices the singular has no inverse. However, it can be done by generalizing inverse matrices for \( m \times n \) or \( n \times n \) singular sized matrices [12, 15]. The inverse of the matrix is usually called pseudo inverse.

**Definition 1** [11]
If \( A \) matrix is \( n \times m \) a real or complex number, there is a unique \( m \times n \) matrix \( A^\dagger \), then \( A^\dagger \) is the inverse pseudo of the matrix \( A \) if it satisfies:
1) \( AA^\dagger A = A \)
2) \( A^\dagger AA^\dagger = A^\dagger \)
3) \( (AA^\dagger)^T = AA^\dagger \)
4) \( (A^\dagger A)^T = A^\dagger A \)

3.2. Return
Return of an asset is the result obtained as a result of an asset investment activity that has been carried out, usually expressed as a percentage of the initial investment price [27]. Mathematically the return of an asset is formulated as follows:

\[
R_t = \frac{P_t - P_{t-1}}{P_{t-1}}
\]  

(1)

with \( R_t \) is asset return in period \( t \), \( P_t \) is asset price in period \( t \), and \( P_{t-1} \) is asset price in period \( t-1 \). Furthermore, the expected return of an asset is formulated as follows:

\[
\mu_t = E[R_t] = \int_{-\infty}^{\infty} R_t f(R_t) dR_t
\]

(2)
Expectation of returns shows how much investors gain profits in the future. In order for this to be achieved, investors need to determine whether the probability value of all possible returns.

Probabilities will be calculated based on historical performance or similar investments that are modified from investors’ expectations in the future [28].

Calculating portfolio risk can be done by determining the variance and standard deviation in each stock. Portfolio variances denoted by $\sigma^2_i$ are mathematically formulated as follows:

$$\sigma^2_i = E[(r_i - \mu_i)^2] = \int_{-\infty}^{\infty} (r_i - \mu_i)^2 f(r_i)dr_i$$

Where $\sigma_{ij}$ states the covariance between shares $i$ and $j$, is defined as follows:

$$\sigma_{ij} = E[(r_i - \mu_i)(r_j - \mu_j)] = \rho_{ij}\sigma_i\sigma_j$$

3.3. Portfolio formation
From the expectations of each stock, an average vector will be formed as follows:

$$\mu^T = (\mu_1, \mu_2, \ldots, \mu_N).$$

If $N$ states the number of shares in the portfolio, then the unit vector $e$ element consists of number 1 as much as $N$ can be stated as follows:

$$e^T = (1, 1, \ldots, 1)$$

From the variance and covariance of each stock, arranged in the form of a variance-covariance matrix which is defined as follows:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{pmatrix}$$

with a weight vector

$$w^T = (w_1, w_2, \ldots, w_N)$$

If $w_i$ states the proportion (weight) of funds invested in assets $i$, and $w$ is the portfolio weighting vector return, then the ratio of portfolio $R_p$ is given as follows:

$$R_p = \sum_{i=1}^{N} w_i R_i \quad (3)$$

Based on equation (3), the average (return expectation) of the portfolio is given as follows:

$$E[R_p] = E\left[ \sum_{i=1}^{N} w_i R_i \right] \quad \text{or} \quad \mu_p = \sum_{i=1}^{N} w_i E[R_i] = \sum_{i=1}^{N} w_i \mu_i = w^T \mu \quad (4)$$

3.4. Mean-absolute deviation model portfolio optimization
Portfolio optimization using the mean-absolute deviation model can be solved by solving two existing problems, namely for cases $\sigma_p > 0$ and $\sigma_p < 0$. The optimization problem of the mean-absolute deviation portfolio model is formed by completing:

For $\sigma_p > 0$ problems

$$\max \left\{ 2\tau \mu_p - \sigma_p \right\}$$

Or it can be formed in a vector-matrix equation

$$\max \left\{ 2\tau w^T \mu - (w^T \Sigma w)^{1/2} \right\}$$
Obstacles $\sum_{i=1}^{N} w_i = 1$

With $\tau$ is the risk tolerance owned by investors.

Of the above problems are arranged in the Lagrange multiplier function, given as follows:

$$L = 2\tau w^T \mu - (w^T \Sigma w)^{1/2} + \lambda (w^T e - 1)$$

$$\frac{\partial L}{\partial w} = 2\mu - \frac{1}{2} \frac{\Sigma w}{(w^T \Sigma w)^{1/2}} + \lambda e = 0$$

(5)

$$\frac{\partial L}{\partial \lambda} = w^T e - 1 = 0$$

(6)

From equation (5) is obtained

$$\frac{\Sigma w}{(w^T \Sigma w)^{1/2}} = 2\mu + \lambda e$$

(7)

from equation (7) the two segments multiplied by $\Sigma^{-1}$, are obtained:

$$\frac{w}{(w^T \Sigma w)^{1/2}} = 2\tau \Sigma^{-1} \mu + \lambda \Sigma^{-1} e$$

(8)

for equation (8) the two segments multiplied by $e^T$, are obtained:

$$\frac{w e^T}{(w^T \Sigma w)^{1/2}} = 2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e$$

$$\frac{1}{(w^T \Sigma w)^{1/2}} = 2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e$$

(9)

Then from equation (9) substitution to equation (8), so that we get the weighting vector as follows:

$$\frac{w}{(w^T \Sigma w)^{1/2}} \times \frac{(w^T \Sigma w)^{1/2} \times \frac{w}{(w^T \Sigma w)^{1/2}}}{1} = \frac{2\tau \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e}$$

$$w = \frac{2\tau \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e}$$

(10)

To determine $\lambda$, from equation (7) multiplied by $w^T$, is obtained:

$$\frac{w^T \Sigma w}{(w^T \Sigma w)^{1/2}} = 2\tau w^T \mu + \lambda w^T e$$

$$\frac{(w^T \Sigma w)^{1/2}}{w^T \Sigma w} = 2\tau w^T \mu + \lambda$$

(11)

Furthermore, from Equations (9) and (10) substitution to equation (11), is obtained:

$$\frac{1}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e} = 2\mu^T w + \lambda$$

$$\frac{1}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e} = 2\mu^T \frac{2\tau \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e} + \lambda$$

$$\frac{1}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e} = 2\mu^T \frac{2\tau \Sigma^{-1} \mu + \lambda \Sigma^{-1} e + \lambda (2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e)}{2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e}$$

$$1 = 2\mu^T (2\tau \Sigma^{-1} \mu + \lambda \Sigma^{-1} e) + \lambda (2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e)$$

$$1 = 4\tau \mu^T \Sigma^{-1} \mu + 2\mu^T \Sigma^{-1} e + \lambda (2\tau e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e)$$

$$4\tau \mu^T \Sigma^{-1} \mu - 1 + \lambda (2\mu^T \Sigma^{-1} e + 2\mu^T \Sigma^{-1} \mu) + \lambda e^T \Sigma^{-1} e = 0$$

(12)
\[ \lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{for } \lambda > 0 \]  

(12)

with

\[ a = e^T \Sigma^{-1} e, \quad b = (2 \mu^T \Sigma^{-1} e + 2 \sigma^T \Sigma^{-1} \mu) \]  

and

\[ c = 4 \sigma^2 \mu^T \Sigma^{-1} \mu - 1 \]

For \( \sigma_p < 0 \) problems

\[ \max \left\{ 2 \tau \mu_p + \sigma_p \right\} \]

Or it can be formed in a vector-matrix equation

\[ \max \left\{ 2 \tau w^T \mu + (w^T \Sigma w)^{1/2} \right\} \]

Obstacles \[ \sum_{i=1}^{N} w_i = 1 \]

With \( \tau \) is the risk tolerance owned by investors.

Of the above problems are arranged in the Lagrange multiplier function, given as follows:

\[ \hat{L} = 2 \tau w^T \mu + (w^T \Sigma w)^{1/2} + \lambda (w^T e - 1) \]

\[ \frac{\partial \hat{L}}{\partial w} = 2 \tau \mu + \frac{1}{2} 2 \frac{\Sigma w}{(w^T \Sigma w)^{1/2}} + \lambda e = 0 \]

\[ \frac{\partial \hat{L}}{\partial \lambda} = w^T e - 1 = 0 \]

(13)

(14)

In the same way from equations (5) and (6), the weighting vector is obtained as follows:

\[ w = \frac{2 \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{2 \sigma^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e} \]

\[ \lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{for } \lambda > 0 \]  

(15)

(16)

With

\[ a = e^T \Sigma^{-1} e, \quad b = (2 \mu^T \Sigma^{-1} e + 2 \sigma^T \Sigma^{-1} \mu) \]  

and

\[ c = 4 \sigma^2 \mu^T \Sigma^{-1} \mu - 1 \]

From the completion of the two constraints above \( \sigma_p > 0 \) and \( \sigma_p < 0 \) we obtain the same weighting vector and \( \lambda \) coefficient, which are equations (10), (12) and equations (15), (16). So that the mean-absolute deviation model is obtained to determine the weight (proportion) of funds to be invested, i.e.

\[ w = \frac{2 \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{2 \sigma^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e} \]

With

\[ \lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{for } \lambda > 0 \]

With

\[ a = e^T \Sigma^{-1} e, \quad b = (2 \mu^T \Sigma^{-1} e + 2 \sigma^T \Sigma^{-1} \mu) \]  

and

\[ c = 4 \sigma^2 \mu^T \Sigma^{-1} \mu - 1 \]

\( \Sigma^{-1} \) is the inverse of the matrix \( \Sigma \).
4. Numerical Simulation

In this section, numerical simulations are carried out to describe the results of the analysis of stock data on the formulated model. Numerical simulations are carried out using $e^T$ unit vectors and $\mu^T$ average vectors, used to calculate the investment portfolio optimization process. For average vectors obtained $\mu^T = \begin{bmatrix} 0.00074 & -0.00011 & 0.0007 & 0.00096 & 0.0004 \end{bmatrix}$ and the portfolio to be analyzed is five stock data, unit vectors are obtained $e^T = [1 \ 1 \ 1 \ 1 \ 1]$.

While the covariance value of the five shares given in Covariance Matrix $\Sigma$ is obtained as follows:

$$
\Sigma = \begin{bmatrix}
0.00075 & -0.00085 & -0.000689 & -0.001054 & 0.000958 \\
-0.00085 & 0.000963 & 0.000781 & 0.001195 & -0.001085 \\
-0.000689 & 0.000781 & 0.000633 & 0.000969 & -0.00088 \\
-0.001054 & 0.001195 & 0.000969 & 0.001482 & -0.001347 \\
0.000958 & -0.001086 & -0.00088 & -0.0001347 & 0.001224
\end{bmatrix}.
$$

Because the determinant of the covariance matrix is $3.33 \times 10^{-28}$ or close to zero, it is assumed that the covariance matrix is singular. So based on these assumptions, the portfolio optimization process will be carried out a comparative analysis of two methods, namely comparing the non-singular covariance matrix method ($\det(\Sigma) \neq 0$) and the singular covariance matrix method ($\det(\Sigma) = 0$). Consecutive singular and non-singular covariance matrix inversions are given as follows:

$$
\Sigma^+ = \begin{bmatrix}
2.7259 \times 10^6 & 0 & -0.4112 \times 10^6 & 1.3909 \times 10^6 & 0.8985 \times 10^6 \\
0 & 0 & 0 & 0 & 0 \\
-0.4112 \times 10^6 & 0 & 0.8833 \times 10^6 & 1.0863 \times 10^6 & 2.1523 \times 10^6 \\
1.3909 \times 10^6 & 0 & 1.0863 \times 10^6 & -0.0203 \times 10^6 & -0.3299 \times 10^6 \\
0.8985 \times 10^6 & 0 & 2.1523 \times 10^6 & -0.3299 \times 10^6 & 1.8883 \times 10^6
\end{bmatrix}, \text{ and}
$$

$$
\Sigma^{-1} = \begin{bmatrix}
1.8769 \times 10^6 & -0.7087 \times 10^6 & 1.1501 \times 10^6 & 1.3153 \times 10^6 & 0.1772 \times 10^6 \\
-0.7087 \times 10^6 & -0.5916 \times 10^6 & 1.3033 \times 10^6 & -0.0631 \times 10^6 & 0.8979 \times 10^6 \\
1.1501 \times 10^6 & 1.3033 \times 10^6 & -1.988 \times 10^6 & 1.2252 \times 10^6 & 0.1742 \times 10^6 \\
1.3153 \times 10^6 & -0.0631 \times 10^6 & 1.2252 \times 10^6 & -0.027 \times 10^6 & -0.2342 \times 10^6 \\
0.1772 \times 10^6 & 0.8979 \times 10^6 & 0.1742 \times 10^6 & -0.2342 \times 10^6 & 0.5255 \times 10^6
\end{bmatrix}
$$

In the next process, the results of the determination of the average vector, unit vector, inverse matrix of non-singular covariance and singular covariance inverse matrix are used in the process of portfolio optimization with the mean-absolute deviation model. For the mean-absolute deviation model investment portfolio optimization process with risk tolerance $0 \leq \tau \leq 26$. The results of the mean-absolute deviation model portfolio optimization process using the non-singular covariance matrix method and the singular covariance matrix method from the results of numerical analysis are given in Table 1 and Table 2. The increase in risk tolerance value from the initial value of 0.00 has increased by 0.02, up to the risk tolerance value increased to 0.26.
Based on the results of the portfolio optimization calculation process in Tables-1 and Table-2, mean values and portfolio standard deviations change with increasing risk tolerance $\tau$. The results of the non-singular covariance matrix method numerical analysis from Table-1 obtained the optimum portfolio at the time of risk tolerance $\tau = 0.24$ with a portfolio ratio $R_p = 2.1642$ for the portfolio mean value $\mu_p = 0.0006925$ and portfolio standard deviation $\sigma_p = 0.0003200$. While the results of numerical analysis of singular Covariance Matrix Method from Table-2 obtained the optimum portfolio when risk tolerance $\tau = 0.22$ with portfolio ratio $R_p = 2.3745$ for portfolio mean value $\mu_p = 0.0007208$ and portfolio standard deviation $\sigma_p = 0.0003353$. The results of numerical analysis from Table 1 and Table 2 are presented in the form of efficient portfolio graphs of mean-absolute deviation presented in Figure 1 and Figure 2.
In Figure 1, the minimum portfolio value occurs when the value of risk tolerance 0.00 with the mean portfolio \( \mu_p = 0.0006599 \) and portfolio standard deviation \( \sigma_p = 0.000312 \). Minimum portfolio results for weighting combinations of five stocks compiled as \( w_{\text{Min}} = (0.371053, 0.081579, 0.181579, 0.215789, 0.15) \). While the maximum portfolio value occurs when the magnitude of the risk tolerance value is 0.26 with the mean portfolio \( \mu_p = 0.0006953 \) and portfolio standard deviation \( \sigma_p = 0.0003214 \). Maximum portfolio results for weighting combinations of five stocks compiled as \( w_{\text{Max}} = (0.553372, 0.114787, 0.06985, 0.258554, 0.003438) \).

In Figure 2, the minimum portfolio value occurs when the value of risk tolerance 0.00 with the mean portfolio \( \mu_p = 0.0006844 \) and portfolio standard deviation \( \sigma_p = 0.0002954 \). Minimum portfolio results for weighting combinations of five stocks compiled as \( w_{\text{Min}} = (0.245016, 0.000000, 0.323881, 0.185645, 0.245459) \). While the maximum portfolio value occurs when the magnitude of the risk tolerance value is 0.26 with the mean portfolio \( \mu_p = 0.0007279 \) and portfolio standard deviation \( \sigma_p = 0.0003069 \). Maximum portfolio results for weighting combinations of five stocks compiled as \( w_{\text{Max}} = (0.370137, 0, 0.272498, 0.214767, 0.142598) \).

5. Conclusion
The optimal solution obtained from the mean-absolute deviation portfolio model by considering risk tolerance is expressed in the form of a weighting vector. The results of non-singular covariance matrix method numerical analysis obtained optimum portfolio when risk tolerance \( \tau = 0.24 \) with portfolio ratio \( R_p = 2.1642 \) for portfolio mean value \( \mu_p = 0.0006925 \) and portfolio standard deviation \( \sigma_p = 0.0003200 \). The resulting global optimum weight combination is \( w_{\text{Glob}} = (0.538594, 0.112096, 0.078906, 0.255088, 0.015317) \). Whereas the results of the singular covariance matrix method numerical analysis obtained the optimum portfolio when risk tolerance \( \tau = 0.22 \) with portfolio ratio \( R_p = 2.3745 \) for portfolio mean value \( \mu_p = 0.0007208 \) and portfolio standard deviation \( \sigma_p = 0.0003035 \). The resulting global optimum weight combination is \( w_{\text{Glob}} = (0.538594, 0.112096, 0.078906, 0.255088, 0.015317) \). Based on the results of a comparative analysis of the two methods namely Non-Singular Covariance Matrix Method and Singular Covariance Matrix Method of the five stocks that have been compiled. The Singular Covariance Matrix Method provides more optimal results compared to the Non-Singular Covariance Matrix Method. This is because the Singular Covariance Matrix Method has a global optimal portfolio with the ratio between the mean to standard deviation being the largest.
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7. References
[1] Soeryana E, Halim N B A, Sukono, Rusyaman E and Supian S 2016 Mean-Variance Portfolio Optimization on Some Stocks by Using Non Constant Mean and Volatility Models Approaches Proceedings of the International Conference on Industrial Engineering and Operations Management, Vol. 8-10 March 2016, pp. 3124-3131.
[2] Soeryana E, Halim N B A, Sukono, Rusyaman E and Supian S 2017 Mean-Variance Portfolio Optimization by Using Non Constant Mean and Volatility Based on the Negative Exponential Utility Function AIP Conference Proceedings 1827 020042.
[3] Elton E J and Gruber M J 1991 Modern Portfolio Theory and Investment Analysis. Fourth Edition (New York: John Wiley & Sons, Inc).
[4] Soeryana E, Halim N B A, Sukono, Rusyaman E and Supian S 2017 Mean-Variance Portfolio Optimization by Using Time Series Approaches Based on Logarithmic Utility Function IOP Conf. Series: Materials Science and Engineering 166 012003.
[5] Sukono, Lesmana E, Susanti D and Napitupulu H 2017 Estimating the Value-at-Risk for Some Stocks at the Capital Market in Indonesia Based on ARMA-FIGARCH Models IOP Conf. Series: Journal of Physics: Conf. Series 909 012040.
[6] Golafshani Z Y and Emamipoor S 2015 Portfolio optimization using two methods of mean-variance analysis and mean risk in Tehran Stock Exchange Technical Journal of Engineering and Applied Sciences.
[7] Gambrah P S N and Pirvu T A 2014 Risk Measures and Portfolio Optimization Journal Risk Financial Management 7 113.
[8] Hasbullah E S, Suyudi M, Halim N B A, Sukono, Gustaf F and Putra A S 2018 A Comparative Study of Three Pillars System and Banking Methods in Accounting Long-Term Purposes of Retiree in Indonesian Saving Account IOP Conf. Series: Materials Science and Engineering 332 012017.
[9] Sukono, Hidayat Y, Lesmana E, Putra A S, Napitupulu, H and Supian S 2018 Portfolio Optimization by Using Linear Programming Models Based on Genetic Algorithm IOP Conf. Series: Materials Science and Engineering, 300 012001.
[10] Sukono, Susanti D, Najmia M, Lesmana E, Napitupulu H, Putra A S and Supian S 2018 Analysis of Stock Investment Selection Based on CAPM Using Covariance and Genetic Algorithm Approach IOP Conf. Series: Materials Science and Engineering 332 012046.
[11] Boullion T L and Odell P L 1971 Generalized Inverse Matrices (New York: John Wiley & Sons, Inc.)
[12] Penrose R 1955 A generalized inverse for matrices Proc. Cambridge Philos. Soc. 51 406.
[13] Carnia E, Ernawati and Supriatna A K 2018 A Pseudo-Inverse Method as an Alternative in Forecasting Geothermal Energy Consumption and Palm Fruit Production AIP Conference Proceedings 2043 020006.
[14] Ma J, Gao F and Li Y 2019 An efficient method to compute different types of generalized inverses based on linear transformation Applied Mathematics and Computation 349 367.
[15] Rao C R and Mitra S K (1971) Generalized Inverse of Matrices and Its Applications. in: Probability and Statistics Series (London: Wiley).
[16] Markowitz H M 1952 Portfolio selection J. Finance 7 77.
[17] Tayali H A and Tolun S 2018 Dimension reduction in mean-variance portfolio optimization Expert Systems With Applications 92 161.
[18] Zhifeng Dai Z and Wang F 2019 Sparse and robust mean–variance portfolio optimization problems Physica A 523 1371.
[19] Li P, Han Y and Xia Y 2016 Portfolio optimization using asymmetry robust mean absolute deviation model Finance Research Letters 18 353.
[20] Qin Z 2017 Random fuzzy mean-absolute deviation models for portfolio optimization problem
with hybrid uncertainty *Applied Soft Computing* **56** 597.

[21] Qin Z 2015 Mean-variance model for portfolio optimization problem in the simultaneous presence of random and uncertain returns *European Journal of Operational Research* **245** 480.

[22] Stempień J P, Chan S H 2017 Addressing energy trilemma via the modified markowitz Mean-Variance Portfolio Optimization theory *Applied Energy* **202** 228.

[23] Santos-Alamillos F J, Thomaidis N S, Usaola-García J, Ruiz-Arias J A, Pozo-Vázquez D 2017 Exploring the mean-variance portfolio optimization approach for planning wind repowering actions in Spain *Renewable Energy* **106** 335e342.

[24] Grechuk B and Zabarankin M 2014 Inverse portfolio problem with mean-deviation model *European Journal of Operational Research* **234** 481.

[25] Sukono, Sidi P, Bon A T, and Supian S 2017 Modeling of Mean-VaR Portfolio Optimization by Risk Tolerance When the Utility Function is Quadratic *Statistics and its Applications. AIP Conf. Proc.* **1827** 020035.

[26] Zhu D M, Xie Y, Ching W K and Siu T K 2016 Optimal portfolios with maximum Value-at-Risk constraint under a hidden Markovian regime-switching model *Automatica* **74** 194.

[27] Schulmerich M, Leporcher Y M and Eu C H 2015 *Applied Asset and Risk Management* (London: Springer).

[28] Reilly F K and Brown K C 2012 *Investment Analysis & Portfolio Management* (Ohio: South Western a division of Thomson Learning Ohio)