Study on the low-frequency forbidden transmission properties of one-dimensional ternary solid-fluid sonic crystals by FEM

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Abstract. Based on the finite element method (FEM), an acoustic transmission model of one-dimensional ternary solid-fluid sonic crystals under the condition of oblique incidence is established. Transmission spectra of acoustic waves with different incident angles and frequencies in sonic crystals are calculated. The results show that two broadband forbidden band gaps exist in the low frequency range within two separated incident angle ranges for the ternary structure, which has one more band gap than the binary solid-fluid structure. By adjusting the solid filling fraction, period and lattice constant of the ternary structure, the angle widths of these two band gaps can be effectively regulated. This work will be of great significance to the design of underwater low-frequency acoustic filters.

1. Introduction

Since the concept of sonic crystal or phononic crystal was proposed, its novel properties including band gaps, negative refraction, defect states, etc., have attracted widespread attention [1-6]. It has been demonstrated that the band gap properties of sonic crystals have significant application potential in acoustic insulator, vibration damper and other artificial acoustic devices. So far, a large amount of outstanding research work are mainly focused on the band gap engineering in two or three dimensional systems based on Bragg scattering and/or local resonance mechanisms [7-10]. As is well known, Bragg scattering sonic crystals are used to have a structure with a lattice constant comparable to the blocking wavelength, resulting in major obstacles in acoustic manipulation in the low frequency domain.

In recent years, it has been suggested that the one-dimensional solid-fluid sonic crystal also exists a low-frequency forbidden band gap (LFB) originated from the transmission zeros [11], whose frequency range is much lower than the first Bragg band gap. Since the non-Bragg scattering mechanism of LFB, the high-efficient blocking effect can be achieved by a one-dimensional solid-fluid sonic crystal with less periods [12]. Therefore, LFB-based sonic crystals can have diverse potential applications in the field of low-frequency acoustic manipulation and the design of sub-wavelength devices. Although the frequency of LFB is broadband, its angle range is relatively narrow. Previous studies have shown that one-dimensional binary solid-fluid sonic crystals can produce LFB only in one particular angle range [13-14]. More recently, two different finite one-dimensional solid-
fluid sonic crystals were cascaded to realize the LFB with double or ultra-wide angle ranges [15]. However, the research on the LFB properties of one-dimensional polynary sonic crystal is still unthorough.

In this work, a finite element model of acoustic wave propagation in one-dimensional ternary solid-fluid sonic crystals under the condition of oblique incidence is established. On this basis, the transmission characteristics of LFBs in the ternary structure are revealed. In addition, the influence of structural parameters on the LFBs is analyzed by the commercial software COMSOL Multiphysics.

2. Model and method
In order to study the low-frequency band gap properties of one-dimensional ternary solid-fluid sonic crystals, the model is established by using the acoustic-solid interaction module of COMSOL Multiphysics. Due to the whole structure extends infinitely in the direction perpendicular to the cross-section and the forces exert on the cross-section is identical, based on the assumption of plane strain, the three-dimensional structure can be simplified into a two-dimensional model. The schematic diagram of this structure is shown in Figure 1, the infinite solid I (epoxy) and solid II (PMMA) layers are periodically arranged in fluid background (water). The one-layered epoxy and PMMA, together with the two-layered water constitute one unit cell of this structure, and the total period is \( N (N=3) \). The thickness of the solid and fluid layers in a unit cell are \( ds \) and \( df \), respectively. Thus, the lattice constant is \( D = 2(ds + df) \), and the total thickness of this structure is \( L = ND \). The material parameters adopted are as follows: the longitudinal wave velocity, transverse wave velocity and mass density for epoxy are 2400 m/s, 1100 m/s, and 1100 kg/m\(^3\), respectively; the acoustic velocity and mass density for water are 1500 m/s and 1000 kg/m\(^3\), respectively; and the longitudinal wave velocity, transverse wave velocity and mass density for PMMA are 2700 m/s, 1300 m/s and 1180 kg/m\(^3\), respectively.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Schematic diagram of one-dimensional ternary solid-fluid sonic crystal (\( N=3 \)).

The wave equation of acoustic waves in fluid medium can be written as:

\[
\nabla \cdot \left( -\frac{1}{\rho} \nabla p \right) - \frac{\omega^2}{c^2} p = 0
\]

where \( \rho \) is the mass density of medium, \( c \) is the acoustic velocity in medium, \( p \) is acoustic pressure and \( \omega \) is circular frequency. The equation of acoustic waves in solid medium is as follows:

\[
\begin{cases}
- \rho \omega^2 U = (\lambda + 2\mu) \frac{\partial^2 U}{\partial x^2} + \mu \frac{\partial^2 U}{\partial y^2} + (\lambda + \mu) \frac{\partial^2 V}{\partial x \partial y} \\
- \rho \omega^2 V = (\lambda + 2\mu) \frac{\partial^2 V}{\partial y^2} + \mu \frac{\partial^2 V}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 U}{\partial x \partial y}
\end{cases}
\]

where \( \lambda \) and \( \mu \) are Lamé constants, \( U \) and \( V \) represent the horizontal and vertical displacements, respectively. Due to the coupling effect between solid and fluid media, the acoustic-structure
boundaries are adopted at the solid-fluid interfaces. In order to avoid the boundary reflection, the left and right sides are set as plane wave radiation boundaries. Considering that the model extends infinitely along the \( y \)-axis and the acoustic wave is obliquely incidence, the Floquet periodic boundary condition is adopted at the upper and lower boundaries. The vector \( \mathbf{k} \) of the Floquet period can be decomposed into \( \mathbf{k}_x \) and \( \mathbf{k}_y \), which are expressed as follows:

\[
\begin{align*}
\mathbf{k}_x &= k_0 \cos \theta \\
\mathbf{k}_y &= -k_0 \sin \theta
\end{align*}
\]  

(3)

where \( k_0 \) is the plane wave vector at any point in space, and \( \theta \) is the incident angle of acoustic wave.

Then, the field distribution of incident acoustic pressure satisfies

\[
p = p_0 \exp[-i(\mathbf{k}_x x + \mathbf{k}_y y)]
\]

(4)

where \( p_0 \) is the amplitude of incident acoustic pressure. In the calculation, the value of \( p_0 \) is set as 1 Pa.

For the incident plane wave with a given incident angle and frequency, the amplitude of transmitted acoustic pressure \( p_t \) on the right side of the structure can be calculated, and then the acoustic energy transmittance \( T = \frac{p_t^2}{p_0^2} \) can be obtained.

3. Results and discussion

For the physical model shown in Figure 1, the period of the whole structure is 3, the thickness of the solid and fluid layers are both 1.5 mm, thus the lattice constant \( D \) is 6 mm. The frequency of the incident wave is 20 kHz, whose wavelength \( \lambda \) is about 12.5 times of \( D \). The second-order triangular elements are commonly used in simulations to discretize the physical model. For obtaining effective and reliable solutions, the mesh size is set at least 1/10 of the wavelength in the frequency range studied. Figure 2 shows the spatial distribution of solid total displacement and fluid acoustic pressure when acoustic waves incident from the left side to the structure at different incident angles. We can find that when the incident angles are 10°[see Figure 2(a)] and 30°[see Figure 2(b)], waves can pass through the ternary solid-fluid sonic crystal with very high transmittance. However, under the condition that incident angles are 41°[see Figure 2(c)] and 50°[see Figure 2(d)], the acoustic pressures are almost zeros on the right side of the sonic crystals. Regular point pressure distribution on the left side of the sonic crystals originates from the superimpose of incident and reflected waves. The effectively blocking effects for low-frequency underwater acoustic waves at incident angles of 41° and 50° show potential applications in low-frequency filters and sub-wavelength devices.

![Figure 2. The spatial distribution of solid total displacement field and fluid acoustic pressure field of the ternary structure though FEM simulation.](image-url)
The transmittance of acoustic waves passing through the above sonic crystal ($N=3$) as a function of incident angle is also shown in Figure 3 with blue lines. We can find that there are two separated LFBs within angle ranges around 41° and 50°, respectively. Besides, we also calculate the transmittance for sonic crystals with different $N$, which are also shown in Figure 3. With the increase of $N$, the width of the band gap become wider. It should be mentioned that, even in the case of $N=1$, two distinct LFBs can still be found in the transmission curve due to the non-Bragg scattering mechanism, whose lowest value can reach $10^{-5}$.

The frequency responses of the ternary solid-fluid sonic crystals in Figure 3 are shown in Figure 4. When the incident angle is 41° [see Figure 4(a)], we can find that the transmittance keep a low value in a broad bandwidth. Besides, the larger the periodic number $N$, the lower the transmittance. Same results can also be found for the case of 50°[see Figure 4(b)]. When $N>3$, curves with unsmooth shape are due to the error accumulation of finite element calculation.

In order to research the influence of structural parameters on the angle widths of LFBs, we keep the lattice constant $D$ unchanged, and only change the solid filling fraction, which is defined as $f_c = d_s/(d_s + d_f)$. Figure 5 shows the effect of $f_c$ on the widths of LFBs at 20 kHz. When $f_c=0.2$, the angle widths of LFBs are very narrow, and the angle ranges of transmittance below $10^{-3}$ are 41.08°-41.18° and 49.88°-50.12°, respectively. By increasing the value of $f_c$ up to 0.8, their ranges separately expand to 40.74°-41.2° and 49.2°-50.22°. Obviously, a larger solid filling fraction can widen the angle range of LFBs.

Furthermore, the influence of lattice constant $D$ on the angle widths of LFBs is also investigated, as shown in Figure 6, where $f_c$ is 0.5 and $N$ is 3. When $D=6$ mm, the two LFBs are both relatively narrow. With the increase of $D$, the upper and lower angle of LFBs expand to both sides, which leads to broad LFBs. Thus, the lattice constant $D$ can also play an important role in controlling LFBs.
Figure 4. Transmittance as a function of frequency for incident waves with angles of 41° and 50°.
4. Conclusions

In this paper, a FEM model of acoustic waves obliquely incident upon one-dimensional ternary solid-fluid sonic crystal is established. We found that two low-frequency forbidden band gaps can be produced in one-dimensional ternary solid-fluid sonic crystals. The influence of structural parameters on the angle widths of LFBs is further studied. The results show that increasing the solid filling fraction and lattice constant can effectively broaden their angle widths. This study can be of great significance to the application of low-frequency forbidden band gap in underwater acoustic energy flow manipulation, such as sub-wavelength acoustic filter for multi-angle ranges. The theoretical investigation based on the transfer matrix method is currently under consideration and will be discussed elsewhere.

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