A domino model for geomagnetic field reversals

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Abstract

We solve the equations of motion of a one-dimensional planar Heisenberg (or Vaks-Larkin) model consisting of a system of interacting macro-spins aligned along a ring. Each spin has unit length and is described by its angle with respect to the rotational axis. The orientation of the spins can vary in time due to random forcing and spin-spin interaction. We statistically describe the behaviour of the sum of all spins for different parameters. The term “domino model” in the title refers to the interaction among the spins.

We compare the model results with geomagnetic field reversals and find strikingly similar behaviour. The aggregate of all spins keeps the same direction for a long time and, once in a while, begins flipping to change the orientation by almost 180 degrees (mimicking a geomagnetic reversal) or to move back to the original direction (mimicking an excursion). Most of the time the spins are aligned or anti-aligned and deviate only slightly with respect to the rotational axis (mimicking the secular variation of the geomagnetic pole with respect to the geographic pole). Reversals are fast compared to the times in between and they occur at random times, both in the model and in the case of the Earth’s magnetic field.

Keywords: Ising-Heisenberg model – geomagnetic field – reversals – statistical mechanics

1 Introduction

One of the most remarkable phenomena of geomagnetism is that the Earth has reversed the polarity of its almost dipolar magnetic field many times in the past at irregular intervals (e.g., Jacobs, 1994; Merrill et al., 1996). Similar reversals have also been observed in turbulent dynamo experiments (Berhanu et al., 2007) and in simulations of the geodynamo (Glatzmaier and Roberts, 1995), but the cause of the reversals has yet eluded a convincing explanation (Amit et al., 2010).

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The magnetic field of the Earth originates from dynamo action in the liquid outer core (e.g., Roberts, 2007). Numerical simulations of the geodynamo successfully reproduce many features of the magnetic field of the Earth including stochastic reversals (e.g., Christensen and Wicht, 2007). Depending on the importance of the inertial forces relative to the rotational forces, dynamos with either a dominant axial dipole or with a small-scale multipolar magnetic field are found (Kutzner and Christensen, 2002). The transition from dipolar to multipolar dynamos takes place at a local Rossby number of approximately 0.12 (Christensen and Aubert, 2006). The Earth lies close to the transition between both types (Olson and Christensen, 2006), which may explain why the dipole undergoes sporadic reversals.

Convection in fast rotating bodies like planetary liquid interiors organises itself in the form of convective columns that encircle the inner core tangent cylinder and are aligned in the $z$-direction parallel to the rotation axis. The flow becomes quasi two-dimensional (geostrophic), minimising any variation in $z$-direction (Proudman-Taylor theorem). Cyclonic and anticyclonic columns, rotating in the same and in the opposite direction as the planet, respectively, alternate in azimuthal direction (Busse, 1975). In strongly driven dynamos the number of columns increases. Secondary flows are directed away from the equatorial plane in anti-cyclonic columns, but towards the equatorial plane in cyclonic columns. The helicity, a key ingredient in the dynamo process, has therefore predominantly one sign in the northern hemisphere and the opposite sign in the southern hemisphere. The so-called $\alpha^2$-dynamo mechanism, described by Kageyama and Sato (1997) and Olson et al. (1999), can be thought of as a process where the helicity associated to each convective column produces its own magnetic field. The alignment with the rotation axis and the organised helicity guarantees that the sum of these individual contributions add up to form the dominant axial dipole field.

In the numerical simulations, Earth-like reversals are rare and only happen once the driving is large enough, whereby the fields are very complex. There have therefore only been a few attempts to unravel the reason for the polarity switches (Glatzmaier and Roberts, 1995; Kida and Kitauchi, 1998; Glatzmaier et al., 1999; Sarson and Jones, 1999; Kutzner and Christensen, 2004; Wicht and Olson, 2004; Wicht, 2005; Takahashi et al., 2005; Bouligand et al., 2005; Aubert et al., 2008; Wicht et al., 2009; Amit et al., 2010). Typically these simulations show that reversals go along with a breaking of the north-south (equatorial) symmetry in the flow of the aligned fluid columns. Aubert et al. (2008) identified flow upwellings as the cause of field reversals. These features rise from the inner-core boundary and produce inverse magnetic field. To some extend, the upwellings behave like tilted convective columns, at least what their role in the dynamo mechanism is concerned.

The reversal sequences in paleomagnetic data and in dynamo models have been analysed for their statistical properties. First estimates that geomagnetic reversals obey a Poissonian process where all reversals are independent of each other have been shown to fall too short. The statistical reversal rate has likely been changing over time due to the varying heat flux through the core-mantle boundary (Glatzmaier et al., 1999; Constable, 2000; Driscoll and Olson, 2009). The reversal sequence also suggests that the process may have a short and a long term memory, leading to changes in the statistical behaviour and the characterising distribution function of the times between reversals (Jonkers, 2003; Ryan and Sarson, 2007). Similar analysis for fully 3D numerical dynamo simulations are rare because it is very costly to compile a large number of reversals. The analysis by Wicht et al. (2009) and Driscoll and Olson (2009) indicate that the numerical simulations may follow a similar reversal statistics as the paleomagnetic record.
Simple parameterised models allow for a large number of reversals so that a statistical analysis becomes more meaningful. A famous example is the two-disk dynamo of Rikitake (1958), which exhibits sporadic reversals, but also a cyclic variation of the dipole moment during stable polarity periods. The extensions to $N$ coupled disks by Shimizu and Honkura (1985) and Ito (1988) improved on the latter weak point. Hoyng et al. (2001) considered a mean-field dynamo model with stochastic fluctuations of the induction effect; these lead to oscillations of the dipole field amplitude in a bistable potential with minima representing normal and reversed polarity and occasional jumps between them (Schmitt et al., 2001).

Here we study another class of simplified models, an Ising-Heisenberg model of interacting magnetic spins. Ising-like models have been used in molecular dynamics and statistical mechanics for describing, for instance, phase transitions in ferromagnetism, for modelling spin glasses and for pattern recognition in neural networks (e.g., Greiner et al., 1995). Coupled spin models of Ising type, where the individual spins can assume two scalar states $+1$ or $-1$ and interact with each other after certain rules, have also been suggested for describing geomagnetic polarity reversals and their statistics (Mazaud and Laj, 1989; Seki and Ito, 1993; Dias et al., 2008). We analyse a planar Heisenberg model consisting of a system of vectorial spins aligned along a ring. Each spin has unit length and is described by its angle with respect to the rotational axis, i.e., each spin has one degree of freedom. The orientation of the spins can vary in time due to spin-spin interaction and random forcing. The consecutive interaction of adjacent spins is described as “domino model”. We consider the time dependence of the average orientation of all spins, which exhibits a similar behaviour as the geomagnetic reversal record.

This sort of models are often classified according to spatial dimensionality and number of components of spin vectors. Our model is one-dimensional (i.e., spatial dimensionality of the lattice is one) and vectors (spins) are two-dimensional (i.e., they are contained within a plane). Thus, the domino model is a one-dimensional XY model, also referred to as the plane rotator model or the Vaks-Larkin model (Vaks and Larkin, 1966). Spin vectors in Ising and Heisenberg models are 1D and 3D, respectively. A clear classification can be found in the classical textbook by Stanley (1987).

The spins in our model might be associated with the convection columns, whose electromagnetic induction generates elementary dipoles. The tendency of the spins to be aligned with the rotation axis is a consequence of the Proudman-Taylor theorem, and the time variation of the spins is a measure of the vigor of convection and of the sporadic upwellings. The domino model may be thought of to mirror the full dynamo in a simplistic manner. One should, however, be careful not to take the analogies too far. The domino model is, after all, a toy model for a rather complex process.

The structure of the paper is as follows. In Sect. 2 the model is described. The results and the statistical analysis of our model are presented in Sect. 3. The influence of the various parameters as well as alternative model descriptions are given in Sect. 4. In Sect. 5 the results are compared with geomagnetic data, and in Sect. 6 we give our conclusions.

2 The domino model

We consider a system of $N$ macro-spins aligned along a ring and interacting pairwise like in a one-dimensional Vaks-Larkin model. The spins are embedded in a uniformly rotating medium and we take $\Omega = (0,1)$ as the unit vector along the rotational axis. Each spin $\mathbf{S}_i$, $i = 1, \ldots, N$ has unit length and is described by its angle $\theta_i$ with respect to the rotational axis, such that $\mathbf{S}_i = (\sin\theta_i, \cos\theta_i)$. The orientation of the spins can vary in time due to random forcing and spin-spin interaction (Fig. 1).
The kinetic and the potential energy $K(t)$ and $P(t)$ of the system are

$$K(t) = \frac{1}{2} \sum_{i=1}^{N} \dot{\theta}_i(t)^2,$$

(1)

$$P(t) = \gamma \sum_{i=1}^{N} (\Omega \cdot S_i)^2 + \lambda \sum_{i=1}^{N} (S_i \cdot S_{i+1}),$$

(2)

where $i + 1 = 1$ when $i = N$. Here $\gamma$ is a parameter characterising the tendency of the spins to be aligned with the rotation axis, while $\lambda$ is a parameter characterising the spin-spin interaction. The scalar product to the square in the $\gamma$-term ensures that there is no preferred polarity. The interaction is such that each spin interacts with the two neighbouring spins: spin 2 interacts with spins 1 and 3, spin 3 with spins 2 and 4 and so on. Spin $N$ interacts with spins $N - 1$ and 1, i.e. we are considering periodic boundary conditions.

The Lagragian for the system is $\mathcal{L} = K - P$. We set up a Langevin-type equation as follows

$$\frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} \right) = \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \kappa \dot{\theta}_i(t) + \frac{\epsilon \chi_i}{\sqrt{\tau}},$$

(3)

where the term $-\kappa \dot{\theta}_i(t)$ describes friction and the term $\epsilon \chi_i / \sqrt{\tau}$ is a random force acting on each spin. The parameters $\kappa$ and $\epsilon$ characterise the strengths of the friction and the random forcing, respectively. Finally, $\chi_i$ is a Gaussian-distributed random number with
zero mean and unit variance associated to each spin, which is updated each correlation time $\tau$.

Inserting the expressions for the kinetic and potential energy, (1) and (2), into equation (3) yields

$$
\ddot{\theta}_i - 2\gamma \cos \theta_i \sin \theta_i + \lambda \left[ \cos \theta_i (\sin \theta_{i-1} + \sin \theta_{i+1}) - \sin \theta_i (\cos \theta_{i-1} + \cos \theta_{i+1}) \right] \\
+ \kappa \dot{\theta}_i - \frac{\epsilon \chi_i}{\sqrt{\tau}} = 0, \quad i = 1, \ldots, N
$$

with $\theta_0 = \theta_N$ and $\theta_{N+1} = \theta_1$.

We integrate the equations of motion (4) forward in time with a 4th-order Runge-Kutta scheme starting from a random orientation of the spins between 0 and $2\pi$. A standard set of parameters $N$, $\gamma$, $\lambda$, $\kappa$, $\epsilon$ and $\tau$ is considered in Sect. 3, while the parameter dependence of the mean time between reversals is studied in Sect. 4.1. In Sect. 4.2 we also slightly alter the model using different alternatives of the $\gamma$-term, the $\lambda$-term and the forcing term.

As main output we consider the cumulative orientation of all spins and define

$$
M(t) = \frac{1}{N} \sum_{i=1}^{N} \Omega \cdot S_i(t) = \frac{1}{N} \sum_{i=1}^{N} \cos \theta_i(t)
$$

as the resulting total axial magnetic moment or “magnetisation”.

3 Results and statistical analysis of a typical model

The parameters of our standard model are $N = 8$, $\gamma = -1$, $\lambda = -2$, $\kappa = 0.1$, $\epsilon = 0.4$ and $\tau = 0.01$. The integration time step was $\Delta t = 0.01$, the total number of time steps was 30 000 000, and every 10th time step is outputted. The run comprises a total of 824 reversals, i.e. the mean time between reversals is 364. Identifying this time with the mean time between reversals in the case of the Earth, which is 300 kyr, the whole run with a time of 300 000 spans approximately 250 Myr. In Fig. 2 the first tenth of the full run is displayed.

![Figure 2: The magnetisation of the standard run as a function of time.](image)

The statistical analysis is done using the whole run. The power spectrum is shown in Fig. 3a. Over a large range comprising most of the reversals, the spectrum follows a power law with an exponent of about $-1.7$. The spectrum for small frequencies or long polarity
chrons (i.e., epochs of one polarity) is flatter, while the steeper decrease at high frequencies comes from the fast variations between reversals. The distribution of the magnetisation peaks near ±1 with a wide and deep valley between them (Fig. 3b). This reflects that the flipping or reversal times are short events compared to the average duration time between them and that the spins are most of the time closely aligned with the rotation axis.

The distribution of the duration of long chrons follows a power law with an exponent of approximately \(-1.5\) (Fig. 4a), while the short chrons are approximately exponentially distributed (Fig. 4b).

In Fig. 5 the details of one single reversal at \(t \approx 3450\) is shown. The reversal is triggered by a large fluctuation of spin 4 which is successively transferred to the neighbouring spins, which fluctuate until finally all spins reverse their polarity. We call our model “domino model” because of this consecutive interaction of neighbouring spins. The duration of the full reversal depends on how fast the original fluctuation is transferred to all other spins. The reversal here lasts about 10 time units, which corresponds to 8000 yr, roughly the time it takes for the geomagnetic field to perform a reversal. Shortly afterwards, at \(t \approx 3470\), some spins again show large variations and even reverse for a short time, but they fail to transfer this to all the spins. The total magnetisation shows an excursion.

Examples of a true reversal, an aborted reversal and two excursions are shown in Fig. 6.
Figure 5: The total magnetisation of all spins, $\sum_{i=1}^{N} \cos \theta_i / N$, (top panel) and the magnetisation of the individual spins, $\cos \theta_i$, before and after a reversal at $t \approx 3450$.

Figure 6: Examples of a true reversal (top panel), an aborted reversal (middle panel) and two excursions (bottom panel).

4 Parameter study

The model contains a number of free parameters. The main quantity which depends sensitively on their values is the frequency of reversals. The results, described in Sect. 4.1, are based on runs with at least three hundred reversals. The slope of the power spectrum depends only weakly on the model parameters. The distribution of the magnetisation is also similar to the model of Sect. 3. When there are very many reversals, the valley between the two stable states is less wide and deep.
4.1 Mean time between reversals

The dependence of the mean time between reversals on the number of spins is shown in Fig. 7. In the case of a few spins this time steeply increases with the number, but it seems to saturate for a large number of spins.

The parameter $\gamma$ measures the tendency of the spins to be aligned with the rotation axis. Large negative values stabilise the orientation and lead to fewer reversals (Fig. 8a).
A value of $\gamma = 0$ still behaves like a bistable oscillator, but with many reversals and an almost flooded valley between the stable states in the distribution function. Positive values of $\gamma$ do not lead to a stable magnetisation, but to oscillations around $M = 0$.

The influence of the spin-spin interaction parameter $\lambda$ is similar. Large negative values stabilise (Fig. 8b). For $\lambda \geq 0$ the system randomly oscillates around $M = 0$.

Increased friction, described by larger values of $\kappa$, quite naturally stabilises (Fig. 8c), while increased random forcing, described by larger values of $\epsilon$, destabilises the system, leading to shorter chrons and more frequent reversals (Fig. 8d).

### 4.2 Alternative model descriptions

To test the robustness of the domino model we also slightly altered its detailed description. Instead of the $\gamma$-term in Eq. (2) proportional to $\sum (\Omega \cdot S_i)^2$ we have considered a term proportional to $\sum |\Omega \cdot S_i|$. Since all $|\Omega \cdot S_i| \leq 1$ and thus $\sum |\Omega \cdot S_i| \geq \sum (\Omega \cdot S_i)^2$ very similar results were obtained for just somewhat smaller (absolute) values of $\gamma$. For a $\gamma$-term proportional to $\sum (\Omega \cdot S_i)$ no reversals were observed.

Furthermore, instead of additive forcing with Gaussian noise, described by the last term in Eq. (3), we also investigated white noise. This results in no much change, except for slightly less frequent reversals. We also studied multiplicative forcing proportional to $\cos \theta_i$; this results in much fewer reversals. Multiplicative forcing proportional to $\text{mod}(\theta_i, 2\pi)$ leads to random oscillations about 0.

As an alternative to the local interaction with neighbouring spins only, described by the $\lambda$-term in Eq. (2), we also considered a global or “mean-field” interaction with all other spins, described by $(2\lambda/N) \sum_{i<j} (S_i \cdot S_j)$. A normalisation factor of $2/N$ is included in order to compare with the standard interaction with just the adjacent neighbours. The mean-field model results in less frequent reversals, but shows otherwise qualitatively similar behaviour.

It is interesting to observe that in globally coupled models a qualitatively similar behaviour to the domino model described above is also found even without noise and without friction. The resulting system is conservative and only two parameters, $\gamma$ and $\lambda$, are left. We shall study this system in detail in a forthcoming paper (Nakamichi et al., 2011).

### 5 Comparison with geomagnetic data

We use the geomagnetic polarity time scale of Cande and Kent (1992, 1995) and Ogg (1995), which covers the past 166 Myr and comprises 332 reversals. Assigning a magnetisation of +1 for chrons of normal polarity and −1 for chrons of reversed polarity we derive Fig. 9, which is an analogue to Fig. 2. Reversals occurred at irregular intervals of $10^5$ to $10^7$ yr. The mean time between reversals is approximately 300 kyr, whereas reversals are fast events lasting a few kyr. The reversal frequency has considerably decreased towards and increased away from the Cretaceous superchron which lasted from 118 to 83 Myr BP (Constable, 2000).

The non-stationarity of the geomagnetic reversal record is not present in the domino model of Sect. 3, but could be easily accounted for by a gradual change of the model parameters with time (Sect. 4). It presents some difficulties in the statistical analysis of the record. The cumulative distribution of polarity chrons follows a power law with an exponent of −1.5 (Fig. 10a). Polarity intervals of a duration shorter than 1 Myr, which make up the vast majority of all intervals, follow an exponential or Poissonian distribution.
Figure 9: Geomagnetic reversal record from present to 166 Mio yr BP. The record comprises 332 reversals. A magnetisation of +1 is assigned for chronos with normal polarity and of −1 for chronos with reversed polarity.

with a mean of 300 kyr (Fig. 10b). Ryan and Sarson (2007) find that the full set of polarity intervals is better fitted by lognormal and loglogistic distributions rather than Poisson and gamma distributions (Constable, 2000). The power spectrum of the geomagnetic record follows power laws with an exponent of about −0.6 for chronos longer than about 3 Myr and an exponent of about −1.9 for chronos of shorter duration (Fig. 10c).

As a measure of the short-term variability of the geomagnetic field at times between reversals we analyse the virtual axial dipole moment (VADM) of the SINT-2000 data set (Fig. 11a) (Valet et al., 2005). The power spectrum of these fluctuations with a characteristic power index of −3 is displayed in Fig. 11b, while the distribution of the VADM is given in Fig. 11c. For a comparison of the distribution derived from the shorter SINT-800 data set of Guyodo and Valet (1999), see Hoyng et al. (2002).

When comparing the geomagnetic data with the behaviour of our reference model illustrated in Figs. 3 and 4 striking similarities become apparent. The fits to the chron durations suggest similar power law exponents of approximately −1.5 for the paleomagnetic sequence and for our simple model. In the latter we have disregarded the short chronos, which seem to represent brief statistical ventures into the other polarity because they follow a different behaviour. These may be identified with paleomagnetic excursions.

The power spectrum of the geomagnetic reversal record (Fig. 10c) only refers to polarity epochs and therefore does not contain the high frequency contributions in our model (Fig. 3c). The low-frequency range can be interpreted as the background variation in the reversal frequency. This leaves us with comparing the mid-frequency spectrum with a slope of −1.9 for the paleomagnetic data and −1.7 for our model. The high frequency part of the model can be compared with the SINT data analysis, which yield a slope of −3
Figure 10: Top left (a): Log-log plot of the number of reversals as a function of times between reversals of the geomagnetic reversal record shown in Fig. 9. Bottom left (b): Normalised distribution of the times between reversals for short duration chrons of less than 1 Myr. These make up 298 out of the total of 332 reversals. The bin size here is four times the interval size. The dashed line is the expected probability density function in the case of a Poissonian process with a mean polarity residence time of 300 kyr. Right (c): Power spectrum of the geomagnetic reversal record.

compared to $-6$ in the model. Not surprisingly, this discrepancy suggests that our model does a good job in replicating the reversal dynamics, but not in the details of the secular variation. A detailed discussion of the power spectrum of reversals as well as intensities is presented in Constable and Johnson (2005).

6 Discussion and conclusions

Our simple domino model of interacting magnetic spins reproduces the qualitative features of geomagnetic polarity reversals remarkably well. The orientation of the aggregate of all spins is most of the time nearly aligned or anti-aligned and deviates only slightly with respect to the rotational axis. Once in a while, at sporadic times, it starts flipping to ultimately change the orientation by almost 180 degrees or to move back to the original direction. The model thus mimics sporadic reversals of polarity, excursions and secular variation of the geomagnetic field. The power spectrum derived from the paleomagnetic reversal records as well as the distribution of the virtual axial dipole moment are qualitatively well represented in the model. Furthermore the statistics of the times between reversals is similar in the model and in the case of the Earth’s magnetic field. The mean time between reversals depends sensitively on the model parameters. Thus the drastic changes in the reversal frequency of the geomagnetic field could be explained by a
Figure 11: Top (a): Variability of the virtual axial dipole moment (VADM) during the past 2 Myr, the SINT-2000 data set. The absolute values of the VADM are given, disregarding the five reversals during this period. Bottom left (b): Power spectrum of the VADM time series. Bottom right (c): Distribution of the VADM.

Our model provides a convincing statistical representation of the geomagnetic field reversals process. One should be careful, however, when interpreting the model properties in terms of magnetohydrodynamics. Secular variation, which is mainly determined by the details of the convective flow dynamics, is certainly not captured correctly. The view that the convective columns to a certain degree represent building blocks of the full dynamo process seems to be strengthened by our results. A stable polarity can only established when the majority of these entities cooperate and produce field of the same polarity. Random forcing counteracts this and may sometimes be violent enough to cause a spin to flip significantly and leave the team. This may cause its neighbours to follow and ultimately lead to a reversal. The magnetic upwellings identified in full 3D dynamo simulations by Aubert et al. (2008) could be these events. When these upwellings last long enough or produce enough inverse field, they disrupt the normal dynamo process. The statistics of the complex interplay of many agents seems to be nicely describable by our domino model of Vaks-Larkin type of a set of interacting magnetic spins.

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