Switch-Based Hybrid Beamforming Transceiver Design for Wideband Communications With Beam Squint

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Abstract—Hybrid beamforming (HBF) transceiver architectures based on frequency-independent phase shifters (PSs) are sensitive to phases and physical directions, resulting in limited capability to compensate for the detrimental effects of the beam squint. Motivated by the fact that switches are phase-independent and more power/cost efficient than PSs, we consider the switch-based HBF (SW-HBF) for wideband large-scale multiple-input multiple-output systems in this paper. We first derive a closed-form expression of the beam squint ratio unveiling that the severity of beam squint linearly increases with the number of antennas, the antenna spacing distance, and the fractional bandwidth. We then focus on the SW-HBF designs to maximize the spectral efficiency (SE) in both single-user (SU) and multi-user (MU) systems. The formulated problems in both cases exhibit intractable non-convex mixed-integer challenges. To address them, for the SU case, by combining the tabu search (TS) method and projected gradient descend (PGA), we propose an efficient heuristic PGA-TS algorithm to design analog beamformers while the digital ones admit closed-form solutions. For the MU case, we develop a two-step algorithm based on fractional programming and the PGA-TS method. Simulation results show that the proposed SW-HBF schemes are efficient and can outperform PS-based HBF architectures in terms of both SE and energy efficiency in wideband systems.

Index Terms—Beam squint, energy efficiency, hybrid beamforming, spectral efficiency, wideband systems.

I. INTRODUCTION

WIDEBAND communication systems are promising to meet the ever-increasing demand for ultra-high-speed data rates of future wireless networks [2]. Millimeter wave (mmWave) communications thereby have been considered essential for future wireless communication systems [3]. The short wavelength of mmWave signals allows compact deployment of large numbers of antenna elements, facilitating the implementation of large-scale or massive multiple-input multiple-output (MIMO) systems to compensate for severe path loss [4]. However, a large-scale MIMO system deploying the conventional fully digital beamforming (DBF) architectures requires excessively large numbers of power-hungry radio frequency (RF) chains causing prohibitive power consumption and hardware costs [5]. Such limitations pose significant challenges to the system. Therefore, hybrid beamforming (HBF) architectures have been proposed to divide the overall beamformer into a high-dimensional analog beamformer and a lower-dimensional digital beamformer [6]. Such architectures allow a low number of digital and RF branches while guaranteeing multiplexing gain, significantly reducing power consumption and implementation complexity [7].

A. Prior Works

Driven by the tradeoff between spectral efficiency (SE) and energy efficiency (EE), various HBF architectures have emerged based on the fact that the analog beamformer can be realized by a network of phase shifters (PSs) [8], [9], [10], [11], [12], switches (SWs) [13], [14], [15], or both [16], [17], [18], [19]. Specifically, it was shown in [13] that PS-based HBF (PS-HBF) architectures can reap more SE than SW-based HBF (SW-HBF) architectures, while the latter has more advantages of EE assuming the same connection style used in the analog beamformer. Inspired by this, the combination of PSs and SWs for HBF was investigated in [16], [17], [18], [19], which shows such HBF design can attain a better tradeoff between the SE and EE than the purely SW-HBF or PS-HBF schemes. The aforementioned works mainly focus on narrowband systems. In contrast, HBF design for wideband systems is more challenging because the frequency-flat analog beamformer must be shared across the whole signal bandwidth or all the subcarriers in the multicarrier case. To tackle the challenges, efficient PS-HBF algorithms have been proposed in [20], [21], [22], [23] by searching the eigenvector spaces and constructing the common analog beamformer across all frequencies. However, the detrimental beam squint effect [24], which can significantly degrade the performance of wideband systems, was ignored in those works. In conventional narrowband systems, the analog beamformer aligned with the physical direction at the central frequency can almost achieve the maximum array gain for signals at other frequencies. However, in wideband systems, due to frequency-independent PSs, the beam aligned with the central frequency squints to the incident angles of the incoming signals at other frequencies, resulting in non-negligible array gain loss, termed the beam squint effect [24]. Notably, this effect becomes more severe for systems with wider bandwidth and more antennas [25]. As such, the performance of the HBF strategies without considering the beam...
false. However, since the beam squint effect is essentially caused by the time delay spread over the antenna array [31], it cannot be eliminated solely relying on the frequency-independent phased array. Embedding true-time-delayers (TTDs) into the RF front-end can introduce frequency-dependent phase shifts, alleviating the beam squint. Therefore, TTD-aided HBF architectures have been proposed in [25], [26] wherein TTDs are inserted between the RF chain and the network of PSs to provide additional delay rather than replacing the PSs. While the approach has been shown to effectively eliminate the beam squint, the implementation can be a significant challenge in practice. Since the time delay over the antenna array increases with signal bandwidth and array size, more on-chip TTDs are required for compensation [30], which brings forth the scalability challenges regarding insertion losses, power consumption, chip area, and linearity [32]. For example, in a system operated at 300 GHz with a 256-antenna uniform linear array (ULA), the delay spread over the antennas could be up to 426 ps [25] and the required TTD could have a power consumption of 285 mW [33]. Furthermore, TTDs providing large variable delays can cause significant insertion loss at high frequencies, which can be greater than 40 dB for a delay of 400 ps at 20 GHz [34]. The above facts have motivated the deployment of uniform planar array (UPA) [28], [29], [35] to diminish the delays across the antenna elements, alleviating the demands for TTDs. Additionally, the intelligent transmission surface used in the HBF transmitter for beamforming [36] functions like a UPA and can also alleviate the beam squint. Furthermore, fixed-time TTDs (FTTDs) [37] can be used to further reduce power consumption. However, FTTDs still share the large variable delay and insertion losses as conventional TTDs do.

B. Contributions

In contrast to the TTD and PS-based analog architectures discussed above, a SW-based architecture boasts lower power consumption and insertion loss [13]. The exclusive use of switches simplifies the design of the RF chain and facilitates rapid adjustment to channel variations [14]. Specifically, the hardware implementation of the adjustable TTD typically involves numerous switches and other components [37], especially for large-scale systems. In contrast to the TTD-based architectures, SW-based ones obviate the necessity for additional power amplification to counteract insertion losses, thus, significantly reducing circuitual costs. Furthermore, switches designed for mmWave systems can operate at speeds on the order of nanoseconds or even sub-nanoseconds [38], which could be far less than the channel coherence time. For example, the coherence time for a carrier frequency of 110 GHz is approximately 0.2 ms [39]. Therefore, SW-based analog architectures are promising to meet the requirements of practical implementations. Moreover, SW-HBF architectures are less affected by the beam squint effect than PS-HBF ones. This characteristic contributes to improved SE and EE performance in wideband systems. However, to date, the advantages of SW-HBF architectures in wideband systems remain unexplored in the literature.

In this paper, our focus extends to wideband single-user and multiuser massive MIMO systems. Through an in-depth analysis of the beam squint effect and carefully designed SW-HBF, we show that SW-HBF architectures exhibit enhanced resilience to beam squint when compared to their PS-HBF counterparts. Given their cost effectiveness and low power consumption, SW-HBF architectures emerge as a compelling choice for wideband massive MIMO systems. Our specific contributions can be summarized as follows:

- We first obtain a closed-form expression of the beam squint ratio (BSR) that quantifies the severity of the beam squint effect in wideband MIMO systems [25]. The analytical results show that the severity of the beam squint effect increases linearly with the number of antennas, the antenna spacing distance, and the fractional bandwidth. This reveals that reducing the spacing of the antenna could be a promising approach to mitigate the beam squint. Moreover, our analysis shows that a SW-based beamformer may achieve a higher expected array gain than a PS-based one.
- We then focus on joint transmit precoding and receive combining in SW-HBF architectures for single-user (SU) MIMO communications, aiming to maximize the average SE. The formulated problems are challenging due to the non-convex mixed-integer nature. To overcome the challenge, we propose a projected gradient ascent-aided (PGA) tabu search (TS) algorithm, which finds an efficient solution to the analog beamformer, while the digital beamformers are obtained with closed-form solutions based on the solved analog ones.
- Furthermore, we study the hybrid precoding design in multiuser (MU) multiple-input single-output (MISO) downlink, aiming to maximize the average sum rate. To address the challenging non-convex mixed-integer problem, we propose an efficient two-step algorithm wherein the digital baseband beamformers and the relaxed analog one are first obtained with the fractional programming framework. Then an efficient analog beamformer is found by performing the PGA-TS algorithm based on the relaxed one.
- Extensive numerical simulations are provided to evaluate the performance of the proposed SW-HBF algorithms in both the considered SU and MU systems. The numerical results demonstrate that the proposed SW-HBF schemes perform close to the optimal exhaustive search (ES) scheme with significantly reduced complexity. Moreover, it is shown that under a severe beam squint effect, the SW-HBF scheme can outperform the TTD-based and PS-based HBF schemes.

To focus on the analysis and design of SW-HBF schemes, we assume the availability of full channel state information (CSI) at both the transmitter and the receiver [21], [37], [38], [40]. Thereby we gain a deep understanding of the fundamental performance limits of beam squint in wideband HBF architectures. Moreover, it is shown that CSI can be estimated with the compressed sensing method [13]. We note that this work is an extension of our previous conference paper [1], where we only considered the receiver design for SU-MIMO systems. We herein not only investigate the joint designs of both transmit precoding and receive combining for SU-MIMO but also study the MU-MISO downlink precoding design. Moreover, we also present a theoretical analysis of the beam squint effect, a detailed
analysis of the computational complexity of the proposed SW-HBF algorithms, and extensive numerical simulation results.

C. Organization and Notation

The remainder of this paper is structured as follows. We commence with the presentation of the signal and channel models in Section II. In Section III, we delve into an analysis of the beam squint effect. Subsequently, we introduce SW-HBF designs for SU-MIMO and MU-MISO systems in Section IV and Section V, respectively. Section VI provides our simulation results, and we conclude this paper in Section VII.

Throughout this paper, scalars, vectors, and matrices are denoted by the lower-case, boldface low-case, and boldface upper-case letters, respectively. \((\cdot)^T\), \((\cdot)^H\), and \((\cdot)^\dagger\) represent the transpose, conjugate transpose, and Moore-Penrose inverse operations. \(\|\cdot\|_p\) and \(\|\cdot\|_F\) denote the \(p\)-norm and Frobenius norm. The expectation operator is represented by \(\mathbb{E}(\cdot)\). Furthermore, we use \(A\) to denote a set and \(|A|\) to denote the cardinality of \(A\). In addition, \(|a|\) and \(|A|\) denote the absolute value of the scalar \(a\) and the determinant of the matrix \(A\). Moreover, complex and binary spaces of size \(m \times n\) are represented by \(\mathbb{C}^{m \times n}\) and \(\mathbb{B}^{m \times n}\), respectively. Finally, \(I_{m \times n}\) represents an identity matrix of \(m \times n\) of the channel at frequency \(f\). The expectation operator is represented by \(\mathbb{E}(\cdot)\). Furthermore, we use \(A\) to denote a set and \(|A|\) to denote the cardinality of \(A\). In addition, \(|a|\) and \(|A|\) denote the absolute value of the scalar \(a\) and the determinant of the matrix \(A\). Moreover, complex and binary spaces of size \(m \times n\) are represented by \(\mathbb{C}^{m \times n}\) and \(\mathbb{B}^{m \times n}\), respectively. Finally, \(I_{m \times n}\) represents an identity matrix of \(m \times n\).

II. SYSTEM MODEL

A. Signal Model

We consider a wideband SU-MIMO orthogonal frequency-division multiplexing (OFDM) system illustrated in Fig. 1 wherein the analog beamformers are realized by the fully connected SW-based networks. The transmitter (Tx) equipped with \(N_T\) antennas simultaneously transmits \(N_S\) data streams to the receiver (Rx) equipped with \(N_R\) antennas. For ease of exposition and without loss of generality, we assume that both the Tx and Rx are equipped with \(N_{RF}\) RF chains and \(N_S \leq N_{RF} \ll \min(N_T, N_R)\). Denote by \(K\) the number of subcarriers and by \(s_k \in \mathbb{C}^{N_S \times 1}\) the signal vector transmitted via the \(k\)-th subcarrier, \(E[s_k s_k^H] = I_{N_S}\), \(k = 1, \ldots, K\). At the Tx, \(s_k\) is first precoded by a digital precoder \(F_{BB}[k] \in \mathbb{C}^{N_{RF} \times N_S}\) in the frequency domain, which is then transformed into the time domain by the inverse fast Fourier transforms (IFFTs), and up-converted to the RF domain using the RF chains. The RF signals are then further processed by an analog precoder \(F_{RF} \in \mathbb{B}^{N_T \times N_{RF}}\). At the Rx, the received signal vector is first combined by an analog combiner \(W_{RF} \in \mathbb{B}^{N_T \times N_{RF}}\). Note that both the analog precoder \(F_{RF}\) and combiner \(W_{RF}\) are frequency-flat, i.e., they are used to process all the \(K\) subcarriers. After performing fast Fourier transforms (FFTs), the combined signal is further combined in the frequency domain by a baseband combiner \(W_{BB}[k] \in \mathbb{C}^{N_{RF} \times N_S}\) for each subcarrier. Let \(H_k \in \mathbb{C}^{N_T \times N_R}\) be the channel at the \(k\)-th subcarrier, and \(F_k = F_{RF} F_{BB}[k]\), \(\|F_k\|_F^2 \leq P_t, \forall k\), with \(P_t\) being the power budget for each subcarrier. The received signal at the \(k\)-th subcarrier is given as

\[
y_k = F_k H_k s_k + W_{RF} a_n, \quad (1)
\]

where \(W_{RF} = W_{RF} W_{BB}[k]\), and \(a_n \sim \mathcal{CN}(0, \sigma_n^2 I_{N_R})\) is the additive white Gaussian noise (AWGN) vector with \(\sigma_n^2\) being its variance.

B. Wideband Channel Model

We adopt a statistical tap-delay profile modeling the impulse response and multipath parameters for ultra-broadband channels [41]. Assuming that ULAs are employed, \(^1\) the \(d\)-th tap of the channel at frequency \(f\) can be modeled as [38], [42]

\[
H_f[d] = \zeta \sum_{l=1}^{L_p} \alpha_l \delta(dT_s - \tau_l) a_c(\theta_l^t, \theta_l^r) a_t^H(\theta_l^t, f), \quad (2)
\]

where \(\zeta \Delta \equiv \sqrt{N_T N_R L_p} / L_p\) with \(L_p\) being the number of distinct scattering paths, and \(\alpha_l \sim \mathcal{CN}(0, 1)\) and \(\tau_l\) are the complex channel gain and the delay of the \(l\)-th path, respectively; \(\theta_l^t\) and \(\theta_l^r\) represent the angles of arrival/departure (AoA/AoD) of the \(l\)-th path, respectively; \(T_s\) is the sampling period; \(p(\tau)\) denotes the pulse-shaping filter. In (2), \(a_c(\theta_l^t, \theta_l^r)\) and \(a_t(\theta_l^t, f)\) denote the transmit and receive array response vectors at frequency \(f\) for the \(l\)-th path with AoA \(\theta_l^t\) and AoD \(\theta_l^r\), respectively. The array response vector \(a(\theta, f) \in \{a_c(\theta, f), a_t(\theta, f)\}\) is given as

\[
a(\theta, f) = \frac{1}{\sqrt{N}} \left[1, e^{-j2\pi \Delta f \sin(\theta)}, \ldots, e^{-j2\pi (N-1) \Delta f \sin(\theta)} \right]^T, \quad (3)
\]

with \(N \in \{N_T, N_R\}\), \(\theta \in \{\theta_l^t, \theta_l^r\}\). Here, \(\Delta \equiv \lambda_{\text{deg}} / \lambda_c\) denotes the normalized antenna spacing where \(d_a\) and \(\lambda_c\) represent the physical antenna spacing and the wavelength of the central

\(^1\)In this paper, we consider the ULA for simplicity, but the analysis of the beam squint effect can be extended to the UPA [29]. Besides, the proposed SW-HBF schemes are applicable to antenna arrays of arbitrary geometry.
frequency $f_c$, respectively. The frequency-domain channel at the $k$-th subcarrier can be expressed as $H_k = \sum_{d=0}^{D-1} H_{fd}[d]e^{-j2\pi fd}$, where $D$ specifies the maximum delay spread. Here, $f_k = f_c + (k - \frac{K+1}{2})\frac{f}{R}$, $\forall k$ denotes the frequency of the $k$-th subcarrier with $B$ denoting the total system bandwidth.

### III. Analysis of the Beam Squint Effect

As shown in [25], the beam squint effect can cause significant performance degradation in wideband systems. The beam squint ratio is defined in [25] to quantify its severity so that a larger BSR implies a more severe effect. While it is a useful metric, the definition of the BSR involves integration, which is complex to compute and not straightforward to get insights into the factors causing the phenomenon. In this section, we first derive a closed-form expression of the BSR. We then analyze the expected array gains (EAGs) for both PS-based and SW-based beamformers to evaluate their corresponding relationship with the BSR.

#### A. Closed-Form BSR

The BSR is defined as the expectation of the ratio between the physical direction deviation and half of the beamwidth for all subcarrier frequencies and physical directions [23]. Specifically, let $N$ denote the number of antennas in a ULA, and let $\xi_k \triangleq \frac{k}{K}$ and $\vartheta \triangleq \sin \theta$, where $\theta$ denotes the AoA/AoD. Because the half-beamwidth is $\frac{\pi}{2N}$, and the physical direction deviation for the $k$-th subcarrier can be represented by $\frac{(1 - \xi_k)\vartheta}{1}$, the BSR is expressed as

$$\text{BSR} \triangleq 1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{K} \sum_{k=1}^{K} \frac{|(1 - \xi_k)\vartheta|}{\pi \Delta} d\vartheta. \quad (4)$$

Note that the BSR was originally defined in [25] for half-wavelength antenna spacing. We herein take the normalized antenna spacing $\Delta$ into consideration in (4). We present below a more insightful approximation of the BSR.

**Proposition 1:** The BSR can be approximately given as

$$\text{BSR} \approx \frac{N\Delta b}{8}, \quad (5)$$

where $N$, $b$, and $\Delta$ denote the number of antennas, fractional bandwidth, i.e., $b \triangleq \frac{B}{f_c}$, and the normalized antenna spacing, respectively. The approximation in (5) is tight when the number of subcarriers is sufficiently large.

*Proof:* Recalling that $f_k \triangleq f_c + (k - \frac{K+1}{2})\frac{f}{R}$, we obtain

$$\xi_k = \frac{f_k}{f_c} = 1 + \left(\frac{k - \frac{K+1}{2}}{2K}\right) b,$$

where $b \triangleq \frac{B}{f_c}$. Therefore, we can write

$$\text{BSR} = \frac{N\Delta b}{2K} \sum_{k=1}^{K} \left|\frac{k}{K} - \frac{K+1}{2K}\right| \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\vartheta| d\vartheta \approx \frac{N\Delta b}{2} \cdot \frac{1}{K} \sum_{k=1}^{K} \left|\frac{k}{K} - \frac{K+1}{2K}\right|$$

$$\approx \frac{N\Delta b}{2} \int_{0}^{1} |x - \frac{1}{2}| dx = \frac{N\Delta b}{8}, \quad (7)$$

where approximation (a) follows from the facts that $K \gg 1$ and $\int_{0}^{1} f(x) dx = \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} f\left(\frac{k}{K}\right)$ for a continuous real-valued function $f(\cdot)$ defined on the closed interval $[0, 1]$ [44].

Fig. 2 shows the proposed closed-form expression of the BSR (5) and its analytical definition (4). It can be observed that the closed form is sufficiently accurate for $K \geq 100$. Since the BSR concept targets systems with a large number of subcarriers, which is the typical case in wideband multichannel communications, the approximation in (5) can adequately approach the value given by (4). Proposition 1 implies that the severity of the beam squint effect linearly increases with the number of antennas $N$, fractional bandwidth $b$, and normalized antenna spacing $\Delta$, which was not observed in [25] and aligned with observations in [30]. Furthermore, it shows that the beam squint effect becomes more severe with a larger fractional bandwidth $b$ (rather than bandwidth) but can be mitigated by reducing the antenna spacing $\Delta$, e.g., via spatial oversampling. This is because an array with a reduced antenna spacing forms a larger beamwidth, enhancing the array gain of the marginal frequencies in the bandwidth. As shown in Fig. 2(a) and (b) on the top of the next page, when $\Delta$ decreases from 1/2 to 1/4, the beamwidth of the main lobes is broadened, and at the same time, the BSR is reduced by half. Fig. 2(c) shows the normalized array gains versus the subcarrier index. It is seen that the beam squint effect becomes negligible for BSR $\approx 0.1$. We will further demonstrate this through simulations in Section VI.

#### B. EAGs of SW- and PS-Based Beamformers

1) **PS-Based Beamformer:** The normalized array gain achieved by the PS-based analog beamforming vector $\mathbf{w} = [e^{j\pi a_1}, \ldots, e^{j\pi a_N}]^T$ for AoA $\theta$ at frequency $f_k$ can be given as [24]

$$g(\mathbf{w}, \theta, f_k) = \frac{1}{\sqrt{N}} |\mathbf{w}^H \mathbf{a}_k(\theta, f_k)|$$

$$= \frac{1}{N} \left| \sum_{n=1}^{N} e^{j\left[n\pi - 2\pi(n-\Delta f_c)]\sin \theta\right.} \right|. \quad (8)$$

For a multicarrier system, the beamformer $\mathbf{w}$ is conventionally designed for the central frequency, i.e., $f_c$. To obtain the highest array gain, the beamformer $\mathbf{w}$ should align with the AoA $\theta$ of the arriving signal [24]. Specifically, the $n$-th phase shift in $\mathbf{w}$ should be set as $\varphi_n = 2\pi(n-\Delta f_c)\sin \theta$, $n = 1, \ldots, N$. With this,
we can rewrite (8) as
\[
g(\theta, \xi_k) = \frac{1}{N} \sum_{n=1}^{N} e^{j2\pi(n-1)\Delta(1-\xi_k)\theta} = \frac{\sin (N\pi\Delta (1-\xi_k) \theta)}{N\sin (\pi\Delta (1-\xi_k) \theta)}. \tag{9}
\]
Then, the EAG over all possible physical directions is
\[
E_{\text{ps}}[g(\theta, \xi_k)] = \frac{1}{2\Delta} \sum_{k=1}^{K} \int_{-1}^{1} g(\theta, \xi_k) \, d\theta, \tag{10}
\]
which can be approximated by a closed-form expression as in the following proposition.

**Proposition 2:** The EAG of the PS-based array, i.e., \(E_{\text{ps}}[g(\theta, \xi_k)]\), can be approximated as
\[
E_{\text{ps}}[g(\theta, \xi_k)] \approx 2 \int_{0}^{\text{BSR}} |\text{sinc}(4x)| \, dx + \frac{1}{3}, \tag{11}
\]
where \(\text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x}\) and BSR is given as (5). The EAG \(E_{\text{ps}}[g(\theta, \xi_k)]\) monotonically decreases with BSR and is bounded by \(\frac{1}{4} \leq E_{\text{ps}}[g(\theta, \xi_k)] \leq 1\). The lower bound is attained as BSR \(\rightarrow \infty\), while the upper bound is achieved as BSR \(\rightarrow 0\).

**Proof:** See Appendix A.

**2) SW-Based Beamformer:** The normalized array gain achieved by the SW-based analog beamforming vector \(w = [w_1, \ldots, w_N]^T, w_n \in \{0, 1\}, n = 1, \ldots, N\) is given by
\[
g(w, \theta, \xi_k) = \frac{1}{\sqrt{||w||_1}} |w^H \alpha_r(\theta, f_k)| = \frac{1}{\sqrt{N||w||_1}} \left| \sum_{n=1}^{N} w_n e^{-j2\pi(n-1)\Delta \xi_k \theta} \right|. \tag{12}
\]
The EAG of the SW-based array can be written as
\[
E_{\text{sw}}[g(w, \theta)] = \frac{1}{2\Delta} \sum_{k=1}^{K} \int_{-1}^{1} g(w, \theta, \xi_k) \, d\theta, \tag{13}
\]
which is intractable for analysis due to the binary-valued vector \(w\). Instead, we obtain its approximated closed-form expression detailed in the following proposition.

**Proposition 3:** The EAG of the SW-based array, i.e., \(E_{\text{sw}}[g(w, \theta)]\), can be approximated as
\[
E_{\text{sw}}[g(w, \theta)] \approx \frac{2}{3} \sqrt{\frac{||w||_1}{N}} \leq 2 \frac{1}{3}. \tag{14}
\]

**Proof:** See Appendix B.

With Propositions 2 and 3, we can observe that the EAG of the PS-based beamformer can be smaller than that of the SW-based one when the beam squint effect is severe. For example, when BSR is sufficiently large, \(E_{\text{ps}}[g(\theta, \xi_k)] \rightarrow \frac{1}{3}\). In contrast, with \(||w||_1 \geq \frac{N}{4}\), we obtain \(E_{\text{sw}}[g(w, \theta)] > \frac{1}{4}\). This occurs because the PS-based beamformer is highly sensitive to the beam squint effect while the SW-based one is not. Consequently, the relative resilience of the SW-based beamformer to beam squint, compared to the PS-based one, may offer advantages for deploying the SW-HBF architecture over the PS-HBF one in wideband systems. We will further demonstrate this by the numerical results of Section VI.

### IV. SW-HBF Design for SU-MIMO

In this section, we focus on designing efficient SW-HBF schemes to maximize SE in wideband systems.

#### A. Problem Formulation

Considering the Gaussian signalling, based on signal model (1), the SE of the \(k\)-th subcarrier, denoted by \(R_k\), is expressed as \([11], [38]\)
\[
R_k = \log_2 \left| \text{I}_{N_S} + \frac{1}{\sigma_n^2} \text{F}_k \text{H}_k \text{F}_k^H \right|, \tag{15}
\]
where we recall that \(\text{F}_k = \text{F}_{RF} \text{F}_{BB}[k]\) and \(\text{W}_k = \text{W}_{RF} \text{W}_{BB}[k]\). We aim at a spectral-efficient design of the considered system by optimizing the HBF precoders and combiners to maximize the average SE of the system \([8], [21], [42]\). Let \(\{\text{F}_k\} \triangleq \{\text{F}_1, \ldots, \text{F}_K\}\) and \(\{\text{W}_k\} \triangleq \{\text{W}_1, \ldots, \text{W}_K\}\). The problem can be formulated as
\[
\max_{\{\text{F}_k\}, \{\text{W}_k\}} \frac{1}{K} \sum_{k=1}^{K} R_k \tag{16a}
\]
s.t. \(\|\text{F}_k\|_{F} \leq P_0, \forall k\), \(\text{F}_{RF} \in \mathcal{B}^{N_T \times N_{RF}}, \text{rank}(\text{F}_{RF}) \geq N_S\), \(\text{W}_{RF} \in \mathcal{B}^{N_H \times N_{RF}}, \text{rank}(\text{W}_{RF}) \geq N_S\). \(\tag{16c}\)

The joint optimization of the transmit and receive beamformers, i.e., \(\{\text{F}_k\}\) and \(\{\text{W}_k\}\), in problem (16) is challenging. It is a non-convex non-smooth mix-integer programming problem due to the binary and rank constraints of the analog beamformers. To overcome the challenges, we first decouple the problem into two subproblems: transmit and receive beamforming design, and then solve them successively. The procedure is elaborated below.
B. Transmit HBF Design

Assuming an optimal solution to \( W_k \) [9], [21], the analog and digital precoders, i.e., \( \mathbf{F}_{RF} \) and \( \{ \mathbf{F}_{BB}[k] \} \), can be obtained by solving the following problem

\[
\max_{\mathbf{F}_{RF}, \{ \mathbf{F}_{BB}[k] \}} \quad \frac{1}{K} \sum_{k=1}^{K} \tilde{R}_k
\]

s.t. \( (16c) \),

\[
\| \mathbf{F}_{RF} \mathbf{F}_{BB}[k] \|_F^2 \leq P_k, \forall k,
\]

where \( \tilde{R}_k \) denotes the achievable rate of the transmit precoder, i.e.,

\[
\tilde{R}_k = \log_2 \left( 1 + \frac{\gamma}{\sigma_n^2} \mathbf{H}_k^H \mathbf{F}_{RF} \mathbf{F}_{BB}[k] \mathbf{H}_k \mathbf{F}_{RF}^H \mathbf{H}_k \right).
\]

The binary and rank constraints in \( (16c) \) make problem \( (17) \) challenging non-convex mixed-integer, and it is difficult to jointly and optimally solve \( \{ \mathbf{F}_{RF}, \{ \mathbf{F}_{BB}[k] \} \} \). We consider their decoupled designs in the following.

1) Digital Precoding Design: Define \( \mathbf{H}_{eff}[k] \triangleq \mathbf{H}_k \mathbf{F}_{RF} \) \((\mathbf{F}_{RF}^H \mathbf{F}_{RF})^{-\frac{1}{2}} \). With a given analog precoder, the optimal digital precoder for the \( k \)-th subcarrier is given as [42]

\[
\mathbf{F}_{BB}[k] = (\mathbf{F}_{RF}^H \mathbf{F}_{RF})^{-\frac{1}{2}} \mathbf{V}_k \Gamma_k^{rac{1}{2}},
\]

where \( \mathbf{V}_k \in \mathbb{C}^{N_R \times N_S} \) is the matrix whose columns are the \( N_S \) right-singular vectors corresponding to the \( N_S \) largest singular values of \( \mathbf{H}_{eff}[k] \), and \( \Gamma_k \in \mathbb{C}^{N_S \times N_S} \) is the diagonal matrix whose diagonal elements are the power allocated to the \( N_S \) data streams at the \( k \)-th subcarrier, with \( \text{Tr}(\Gamma_k) = P_k \).

2) Analog Precoding Design: Inserting \( (19) \) into \( (18) \), we obtain an intractable function of \( \mathbf{F}_{RF} \). Alternatively, we can optimize its tight upper bound considering \( (21) \)

\[
\tilde{R}_k \leq \log_2 \left( 1 + \frac{\gamma}{\sigma_n^2} \mathbf{F}_{RF}^H \mathbf{H}_k \mathbf{F}_{RF} \right),
\]

where \( \gamma = P_k/N_S \) represents the power allocated to each data stream, and the relaxation is tight when \( N_S = N_R \). We note that for moderate and high signal-to-noise ratio (SNR) regimes, it is asymptotically optimal to adopt the equal power allocation for all streams in each subcarrier [21], [45]. By defining \( \mathbf{H}_k[k] \triangleq \mathbf{H}_k^H \mathbf{H}_k \), the problem of designing the analog precoder \( \mathbf{F}_{RF} \) is reformulated as

\[
\max_{\mathbf{F}_{RF}} \quad f(\mathbf{F}_{RF}) \triangleq \frac{1}{K} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{\gamma}{\sigma_n^2} \mathbf{F}_{RF}^H \mathbf{H}_k \mathbf{F}_{RF} \right)
\]

s.t. \( (16c) \).

Problem \( (21) \) remains challenging due to the binary and rank constraint of \( \mathbf{F}_{RF} \), and there is no existing solution to it in the literature. A relaxation of the binary variables may cause significant performance loss. Although the optimal solution can be obtained via the ES method, the resultant algorithm requires prohibitively high complexity. For such challenging mixed-integer programming, the tabu search (TS) approach [46], [47] can find a near-optimal solution with reduced complexity compared to the ES method. Specifically, the TS procedure explores candidates and their neighbors within the feasible region over iterations. In each iteration, the best non-tabu (i.e., unforbidden) neighbor is chosen as the candidate and stored in the tabu list to avoid cycling. As such, the TS method ensures convergence to a local optimum with a sufficiently large number of iterations [46].

In TS-based methods, the design of the neighbor set plays a pivotal role in balance performance and complexity. A larger neighbor set can generally enable better performance but result in a higher complexity. To efficiently tackle problem \( (21) \), we introduce a PGA-TS-aided analog precoding algorithm. This approach effectively diminishes the size of the neighbor set while maintaining satisfactory performance.

The idea of the PGA-TS scheme is to first apply the PGA method to find a relaxed but efficient solution in \([0, 1]\). Entries that are neither 0 nor 1 in the relaxed solution are considered erroneous and require further optimization with the TS procedure. By doing so, the search space of the TS procedure can be significantly diminished, facilitating faster convergence and lower complexity. Specifically, we first relax the rank and binary constraints in problem \( (21) \) and consider the following problem

\[
\max_{\mathbf{F}_{RF}} \quad f(\mathbf{F}_{RF}), \quad \text{s.t.} \quad \mathbf{F}_{RF} \in \mathcal{F},
\]

where \( \mathcal{F} \triangleq \{ \mathbf{F}_{RF} | \mathbf{F}_{RF}(i, j) \in [0, 1], \forall i, j \} \). Problem \( (22) \) can be efficiently solved using the PGA method, as summarized in Algorithm 1, where \( \nabla \mathbf{F}_{RF} f(\mathbf{F}_{RF}) \) denotes the gradient of \( f(\mathbf{F}_{RF}) \), given by [48]

\[
\nabla \mathbf{F}_{RF} f(\mathbf{F}_{RF}) = \frac{2}{K} \sum_{k=1}^{K} \mathbf{A}_k \mathbf{F}_{RF} \left( \mathbf{F}_{RF}^H \mathbf{A}_k \mathbf{F}_{RF} \right)^{-1} - 2 \mathbf{F}_{RF} \left( \mathbf{F}_{RF}^H \mathbf{F}_{RF} \right)^{-1}
\]

with \( \mathbf{A}_k \triangleq \mathbf{I} + \frac{\gamma}{\sigma_n^2} \mathbf{H}_k \). In step 4, the normalized gradient \( \nabla \mathbf{F}_{RF} \) is obtained and employed later in step 6, where the step size \( \mu \) is determined by the backtracking line search method [49] as in step 5, and \( \lceil . \rceil_x \) denotes the projection into \( \mathcal{F} \). The iterations continue until the procedure satisfies the stopping criteria. Since the objective function is non-decreasing during the iteration, the PGA algorithm is guaranteed to converge to a stationary point of the non-convex problem \( (22) \).

Observing that the non-integer entries of \( \mathbf{F}_{RF}^{pga} \) fall into \((0, 1)\), we further refine the solution before performing the TS procedure for efficiency. Specifically, define the following refinement function

\[
r(x) \triangleq \begin{cases} 
\delta & \text{if } |x - \delta| \leq \varepsilon, \\
\text{otherwise,}
\end{cases}
\]

where \( \delta \in \{0, 1\} \) and \( \varepsilon > 0 \) is a threshold rounding an entry to a binary value with an error upper bound \( \varepsilon \). Applying \( r(\cdot) \) to the entries of \( \mathbf{F}_{RF}^{pga} \), we obtain a refined solution \( \mathbf{F}_{RF}^{pga} \), i.e., \( \mathbf{F}_{RF}^{pga}(i, j) = r(\mathbf{F}_{RF}^{pga}(i, j)) \), \( \forall i, j \), in which the entries close to \( \delta \in \{0, 1\} \) are rounded to \( \delta \).
The PGA-aided TS (PGA-TS) algorithm is summarized in Algorithm 2. In step 1, the initial solution \( \mathbf{F}_{\text{RF}}^{\text{pga}} \) is obtained by applying the PGA algorithm and the refinement \( r(\cdot) \), as explained above. To facilitate the recovery of the analog precoder, the indices of non-integer entries of \( \mathbf{F}_{\text{RF}}^{\text{pga}} \) are stored in the set \( S_I \). Then, in the remaining steps, the TS procedure is performed in the reduced search space \( B^{S_I} \), where we recall that \( B = \{0, 1\} \). Specifically, let \( q(i) \in B^{S_I} \) be the candidate of the TS procedure in the \( i \)-th iteration. The neighbor set \( N(q(i)) \) is defined as the set consisting of \( |S_I| \) -dimensional vectors that have only one element different from \( q(i) \). Let \( N_{\text{nb}} \triangleq |N(q(i))| \) denote the size of the neighbor set satisfying \( N_{\text{nb}} \leq |S_I| \leq N_TN_R \). Note that because only the analog precoder entries with indices in \( S_I \) are updated in each iteration, the overall analog precoder needs to be recovered as in step 5. Specifically, based on \( S_I \), the entries of \( q(i) \) can be inserted back to the corresponding non-integer position in \( \mathbf{F}_{\text{RF}}^{\text{pga}} \) to recover the analog precoder. Here, an analog precoder is only valid if it satisfies the rank constraint (16c). Then in step 6, the neighbor set is examined to find the best neighbor \( q(i)^{*} \) whose corresponding analog precoder \( \mathbf{F}_{\text{RF}}^{q(i)^{*}} \) yields the largest objective value. Note that the best neighbor has to be in the tabu list. The best solution is updated if a larger objective value is found, as in steps 7–9. The best neighbor has to be not already in the tabu list. The best solution is updated if a larger objective value is found, as in steps 7–9. The best neighbor has to be not already in the tabu list. Finally, the candidate is updated for the next iteration, as in step 11. This iterative procedure can be terminated once the objective value converges or when the number of iterations exceeds a predefined threshold.

The convergence of Algorithm 2 is guaranteed because the objective values form a nondecreasing sequence over iterations. Furthermore, with the aid of PGA solution and refinement, the size of the neighbor set has decreased from \( N_TN_R \) to \( |S_I| \), which could result in \( 2^{N_TN_R} \) times reduction in the size of the search space based on our numerical simulations. Meanwhile, the PGA solution facilitates the search for a satisfactory local optimum, thereby significantly improving the efficiency of the iterative TS procedure, especially for the considered large-scale MIMO systems.

C. Receive HBF Design

For a given analog combiner \( \mathbf{W}_{\text{RF}} \), the optimal digital combiner of each subcarrier is the MMSE solution [8], [21]

\[
\mathbf{W}_{\text{BB}}[k] = (\mathbf{J}_k \mathbf{J}_k^H + \sigma_n^2 \mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{RF}})^{-1} \mathbf{J}_k,
\]

where \( \mathbf{J}_k \triangleq \mathbf{H}_k^H \mathbf{F}_k \). With (25), the problem of analog combiner design can be formulated as

\[
\max_{\mathbf{W}_{\text{RF}}} \frac{1}{K} \sum_{k=1}^{K} \log_2 \left| 1 + \frac{1}{\sigma_n^2} \mathbf{W}_{\text{RF}}^H \mathbf{T}_k \mathbf{W}_{\text{RF}} \right|
\]

s.t. (16d), (26)

where \( \mathbf{T}_k \triangleq \mathbf{H}_k \mathbf{F}_k \mathbf{F}_k^H \mathbf{H}_k^H \). It is observed that the problems of analog precoding and combining designs, i.e., problems (21) and (26), have a similar mathematical structure. Therefore, Algorithm 2 can be utilized to solve problem (26). We omit to apply the algorithm for the analog combiner design due to the limited space. Instead, we summarize the overall proposed SW-HBF design for both the Tx and Rx sides and the complexity analysis next.

Algorithm 2: PGA-TS for Designing Analog Precoder.

Output: \( \mathbf{F}_{\text{RF}}^{\text{pga}} \)

1. Perform the PGA algorithm to obtain \( \mathbf{F}_{\text{RF}}^{\text{pga}} \), and apply the refinement function \( r(\cdot) \) (where \( \epsilon = 0.1 \)) to obtain \( \mathbf{F}_{\text{RF}}^{\text{pga}} \).
2. Store the index of those non-integer entries of \( \mathbf{F}_{\text{RF}}^{\text{pga}} \) in \( S_I \).
3. Set \( i = 0 \) and generate the initial candidate \( q(0) \in B^{S_I} \), then recover analog precoder \( \mathbf{F}_R^{q(0)} \) based on \( q(0) \) and \( S_I \).
4. Set \( q(i) \leftarrow q(0) \), \( \mathbf{F}_{\text{RF}}^{q(i)} \leftarrow \mathbf{F}_{\text{RF}}^{q(0)} \), \( \mathbf{L}_{\text{ts}} \leftarrow \mathbf{L}_{\text{ts}} \cup q(0) \), and \( N_{\text{nb}} \).
5. while \( i \leq N_{\text{iter max}} \) and not converge do
   1. Find neighbor set \( N(q(i)) \) of the candidate \( q(i) \).
   2. Recover the analog precoder of each neighbor in \( N(q(i)) \) with the aid of \( S_I \), and cache the analog precoder into \( \hat{\mathbf{F}}_q(i) \) if it satisfies the rank constraint (16c).
   3. Find the best neighbor \( q(i)^{*} \) in \( N(q(i)) \) but not in the tabu list \( \mathbf{L}_{\text{ts}} \).
   4. The analog precoder of the best neighbor \( q(i)^{*} \), denoted as \( \mathbf{F}_{\text{RF}}^{q(i)^{*}} \), achieves the largest objective value \( f(\mathbf{F}_{\text{RF}}^{q(i)^{*}}) \) among \( \hat{\mathbf{F}}_q(i) \).
   5. if \( f(\mathbf{F}_{\text{RF}}^{q(i)^{*}}) > f(\mathbf{F}_{\text{RF}}^{q(i)}) \) then
      1. Update the best solution: \( \mathbf{F}_{\text{RF}} \leftarrow \mathbf{F}_{\text{RF}}^{q(i)^{*}} \).
   end
   6. Push \( q(i)^{*} \) to the tabu list: \( \mathbf{L}_{\text{ts}} \leftarrow \mathbf{L}_{\text{ts}} \cup q(i)^{*} \), and remove the first candidate in \( \mathbf{L}_{\text{ts}} \) if it is full.
   7. Set \( q(i+1) \) as the candidate for the next iteration, i.e., \( q(i+1) \leftarrow q(i)^{*} \).
   8. \( i \leftarrow i + 1 \).
end

Algorithm 3: Proposed SW-HBF Scheme.

Output: \( \mathbf{F}_{\text{RF}}, \mathbf{W}_{\text{RF}}, \mathbf{F}_{\text{BB}}[k], \mathbf{W}_{\text{BB}}[k], \forall k \)

1. Obtain \( \mathbf{F}_{\text{RF}} \) by solving problem (21) with Algorithm 2.
2. Obtain \( \mathbf{F}_{\text{BB}}[k], \forall k \) by (19).
3. Obtain \( \mathbf{W}_{\text{RF}} \) by solving problem (26) with modified Algorithm 2.
4. Obtain \( \mathbf{W}_{\text{BB}}[k], \forall k \) by (25).

D. Overall SW-HBF Design and Complexity Analysis

1) Overall SW-HBF Design: Our proposed SW-HBF scheme is summarized in Algorithm 3, in which, \( \mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}[k] \) and \( \mathbf{W}_{\text{RF}}, \mathbf{W}_{\text{BB}}[k] \) are solved sequentially. Here, while the analog beamforming matrices \( \mathbf{F}_{\text{RF}} \) and \( \mathbf{W}_{\text{RF}} \) are obtained with Algorithm 2, the digital ones \( \{\mathbf{F}_{\text{BB}}[k]\} \) and \( \{\mathbf{W}_{\text{BB}}[k]\} \) admit closed-form solutions, as shown in (19) and (25).

2) Complexity Analysis: We first evaluate the complexity of obtaining the analog beamformer with Algorithm 2. Considering \( N_{RF} \ll N_T \), the complexity to compute the objective function \( f(\cdot) \) is given as \( O(KN_{RF}(2N_T^2 + 4N_TN_{RF} + N_{RF}^2)) \approx O(2KN_{RF}N_T^2) \), which are mainly caused by Moore-Penrose inversion and matrix multiplications. In Algorithm 2, the complexity of the PGA algorithm is estimated as \( O(N_{pga}KN_{RF}(4N_T^2 + 4N_TN_{RF} + N_{RF}^2)) \approx O(4N_{pga}KN_{RF}N_T^2) \), where \( N_{pga} \) denotes the number of iterations. Considering that Algorithm 2 requires to compute the objective function \( N_{nb} \) times in each iteration, the complexity of Algorithm 2 is given by \( O(cKN_{RF}N_T^2) \), where \( c \triangleq 4N_{pga} + 2N_{iter max}N_{nb} \). Note that \( N_{nb} \) satisfying \( N_{nb} \leq |S_I| \leq N_T \) can be predefined to tradeoff the complexity and performance of Algorithm 2. By replacing \( N_T \) with \( N_R \), the complexity of obtaining the analog combiner \( \mathbf{W}_{\text{RF}} \)
can be derived similarly. In addition, the complexity of steps 2 and 4 of Algorithm 3 is approximate \(O(2KN_TN_k(N_T + N_R))\), which is mainly due to matrix multiplications. Therefore, the overall complexity of Algorithm 3 is estimated as
\[
O(cKNNRFN_k^2) + O(cKN_NRFN_k^2) + O(2KN_TN_k(N_T + N_R)) = O((cNRF + 2N_T)NKN_k^2 + (cNRF + 2N_R)NKN_k^2),
\]
where we assume that \(N_{RF}^k\), \(N_{RF}^{max}\), and \(N_{RF}^\text{step}\) in the precoder design are the same as those in the combiner design. In contrast, the complexity of an ES method is approximately \(2N_TN_kNKN_k^2 + 3N_TNNRFN_k^2\). Hence, the proposed method has a relatively very low complexity. We note that most of the complexity of the proposed method comes from the computations of the objective values \(f(\cdot)\) associated with the neighbor points to find the best one. These can be performed in parallel to reduce runtime. To further reduce the complexity, small \(N_{RF}^k\) can be used, and early termination can be employed during the neighbor search. However, these approaches cause performance loss. We will demonstrate this in Section VI-B.

V. HYBRID PRECODING DESIGN FOR MU-MISO

We now consider the SW-based hybrid precoding design in a MU-MISO OFDMA system where a base station (BS) with \(N_T\) antennas and \(N_{RF}\) RF chains serves \(N_U\) single-antenna users. The transmitted signal vector at the \(k\)-th subcarrier can be written as
\[
x[k] = \sum_{i=1}^{N_U} F_{RF} f_{BB,k}[i] s_i[k],
\]
where \(F_{RF} \in \mathbb{B}^{N_T \times N_{RF}}\) is the analog precoder, \(f_{BB,k}[i] \in \mathbb{C}^{N_{RF}}\) and \(s_i[k]\) are the digital precoder and data symbol intended for user \(i\) at subcarrier \(k\), respectively, with \(\mathbb{E}[|s_i[k]|^2] = 1, \forall i, k\).

Let \(h_m \in \mathbb{C}^{N_T}\) be the channel between the BS and the user \(m\). The \(m\)-th user receives \(y_m[k] = h_m^H x[k] + z_m[k]\), where \(z_m[k]\) represents the AWGN. The achievable rate of the user \(m\) at subcarrier \(k\) can be expressed as
\[
R_m[k] = \log_2 \left( 1 + \frac{|h_m^H[k]F_{RF}f_{BB,k}[i][k]|^2}{\sum_{i\neq m}|h_m^H[k]F_{RF}f_{BB,k}[i][k]|^2 + \sigma_n^2} \right),
\]
where \(\sum_{i=m}^{N_U}|h_m^H[k]F_{RF}f_{BB,k}[i][k]|^2\) represents the interference caused by the data intended for other users. Define \(\{F_{BB,k}\} \triangleq \{F_{BB,k}[k], k = 1, \ldots, K\}\) where \(F_{BB,k} \triangleq \{f_{BB,k}[1], \ldots, f_{BB,k}[K]\}\). The hybrid precoding design is formulated as the following average weighted sum rate maximization problem:
\[
\max_{F_{RF}, \{F_{BB,k}\}} \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{N_U} w_m R_m[k]
\]
subject to \(\|F_{RF}F_{BB,k}\|_F \leq P_k, \forall k\), \(F_{RF} \in \mathbb{B}^{N_T \times N_{RF}}, \text{rank}(F_{RF}) \geq N_U\).

This non-convex and mixed-integer nature of problem (29) renders it challenging to solve. Moreover, the interference term involves repeatedly computation for all \(N_U\) users over all \(K\) subcarriers, which further complicates the problem. To overcome these challenges, we propose a two-step method, where we first jointly design the baseband precoders and the relaxed analog beamforming matrix, and then obtain the analog beamforming matrix based on the relaxed one. Specifically, in the first step, we solve problem
\[
\max_{F_{RF}, \{F_{BB,k}\}} \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{N_U} w_m R_m[k]
\]
subject to \(\|F_{RF}F_{BB,k}\|_F \leq P_k, \forall k\), \(F_{RF} \in \mathbb{B}^{N_T \times N_{RF}}, \text{rank}(F_{RF}) \geq N_U\).

which is relaxed from (29). Here, we recall that \(\mathcal{F} \triangleq \{F_{RF}[i,j] \in [0,1], \forall i,j\}\). Then, in the second step, we leverage Algorithm 2 to obtain \(F_{RF}\) satisfying (29c) based on the solution to problem (30).

Despite the relaxation, problem (30) is still non-convex, and the analog and digital precoders are still highly coupled. We tackle this by resorting to the fractional programming (FP) theory [50] and transforming it into an equivalent but more tractable expression with the following proposition.

**Proposition 4:** The objective function is equivalent to the maximization of
\[
f_q(F_{RF}, \{F_{BB,k}[k], r[k], q[k]\})_{(k,m)} = \frac{G}{K \ln 2} + \frac{1}{K} \sum_{(k,m)} w_m \log_2 (1 + r_m[k]) - \frac{1}{K} \sum_{(k,m)} w_m r_m[k]
\]
with respect to \(\{r[k], q[k]\}_{(k,m)}\), where \(r[k] \in \mathbb{R}^{N_U}, \forall k, q[k] \in \mathbb{C}^{N_U}, \forall k\) are introduced auxiliary variables, and
\[
G \triangleq \sum_{(k,m)} 2 \sqrt{w_m (r_m[k] + 1)} |\mathcal{R}(q_m[k] h_m^H k F_{RF} F_{BB,k}[k])| - \sum_{(k,m)} |q_m[k]|^2 \left( \sum_{i} |h_m^H k F_{RF} F_{BB,k}[i][k]|^2 + \sigma_n^2 \right).
\]

Here, \(r_m[k]\) and \(q_m[k]\) are the \(m\)-th elements of \(r[k]\) and \(q[k]\), respectively. The optimal solution to \(r_m[k]\) and \(q_m[k]\) are given as
\[
r_m[k] = \frac{|h_m^H[F_{RF} F_{BB,k}[i][k]|^2}{\sum_{i \neq m}|h_m^H[F_{RF} F_{BB,k}[i][k]|^2 + \sigma_n^2},
\]
\[
q_m[k] = \frac{\sqrt{w_m (r_m[k] + 1)} h_m^H k F_{RF} F_{BB,k}[m] |^2}{\sum_{i} |h_m^H[F_{RF} F_{BB,k}[i][k]|^2 + \sigma_n^2}.
\]

**Proof:** See Appendix C.

Proposition 4 shows that the solutions of (30) can be obtained as
\[
\{F_{RF}^*, \{F_{BB,k}^*\}, r^*[k], q^*[k]\}_{(k,m)} = \arg \max_{F_{RF}, \{F_{BB,k}\}} \{F_{RF}, \{F_{BB,k}[k], r[k], q[k]\}_{(k,m)}\}
\]
subject to (29b) and (30b).

Define \(\{r[k]\} \triangleq \{r[k], k = 1, \ldots, K\}\) and \(\{q[k]\} \triangleq \{q[k], k = 1, \ldots, K\}\). It is observed that we can iteratively update \(F_{RF}, \{F_{BB,k}\}, \{r[k]\}\) and \(\{q[k]\}\) to solve problem (35). Since the optimal solutions to \(\{r[k]\}\) and \(\{q[k]\}\) are given in (33) and (34), respectively, we aim to develop the solution to \(F_{RF}\) and \(\{F_{BB,k}\}\) next.
A. Analog Precoder Design

With other variables fixed, the design of the analog precoder \( F_{RF} \) can be expressed as

\[
F_{RF} = \arg \max_g \ g(F_{RF}) \quad \text{s.t.} \quad (30b),
\]

with \( g(F_{RF}) \triangleq 2R\{\text{Tr}(EF_H^F)\} - \sum_{k=1}^K \text{Tr}(F_H^F Y_k F_{RF} Z_k) \),

where

\[
\Xi \triangleq \sum_{(k,m)}^\infty \sqrt{w_m(r_m[k]+1)} q_m[k] h_m[k] f_{BB,m}[k]^H, \tag{37}
\]

\[
Z_k \triangleq \sum_{i=1}^{N_U} f_{BB,i}[k] f_{BB,i}[k]^H, \quad \forall k, \tag{38}
\]

\[
Y_k \triangleq \sum_{i=1}^{N_U} |q_i[k]|^2 h_i[k] h_i^H[k], \quad \forall k. \tag{39}
\]

With the gradient of \( g(F_{RF}) \) given as \( \nabla_F g(F_{RF}) = 2\Xi - 2 \sum_k Y_k F_{RF} Z_k \), we can perform the PGA method similar to Algorithm 1 to efficiently find an optimal solution \( F_{RF}^* \) to the convex problem (36).

B. Digital Precoder Design

For given \( F_{RF} \) and \( \{r[k],q[k]\}_{k=1}^K \), the design of the digital precoder is formulated as

\[
\{F_{BB}^*[k]\}_{k=1}^K = \arg \max_{F_{BB}} \ G \quad \text{s.t.} \quad (29b), \tag{40}
\]

where \( G \) is given in (32). The optimal solution of problem (40) can be determined by the Lagrangian multiplier method. Specifically, the Lagrangian function is expressed as

\[
L_G = G + \sum_{k=1}^K \eta_k \left( P_b - \|F_{RF} F_{BB}^*[k]\|_F^2 \right),
\]

where \( \eta_k \geq 0, \forall k \) are introduced Lagrangian multipliers. With the first-order condition of optimality, we can derive the optimal digital precoder of subcarrier \( k \) as

\[
F_{BB}^*[k] = C_k^{-1} F_{RF}^* H[k] D_k, \quad \forall k, \tag{42}
\]

where

\[
C_k \triangleq F_{RF}^* (Y_k + \eta_k I) F_{RF}, \quad H[k] \triangleq [h_1[k], \ldots, h_{N_U}[k]], \quad D_k \triangleq \text{diag}(t_1[k], \ldots, t_{N_U}[k]) \quad \text{with} \quad t_i[k] \triangleq q_i[k] \sqrt{w_i(r_i[k]+1)}.
\]

The optimal Lagrangian multiplier \( \eta_k \) can be easily found by the bisection search method based on the complementary slackness, i.e., \( \eta_k (P_b - \|F_{RF} F_{BB}^*[k]\|_F^2) = 0, \forall k \).

The overall proposed FP-based algorithm for solving problem (30) is summarized in Algorithm 4, where the initial \( F_{RF} \) can be randomly set to satisfy (30b). Then, we can construct the effective baseband channel \( H[k] \triangleq F_{RF}^* H[k] H^H[k] F_{RF} \) by performing an SVD \( H[k] = U[k] S[k] V[k]^H \) for each subcarrier, the digital precoder is initialized as \( F_{BB}^*[k] = V[k]_{1:n_U} \), whose columns are the \( N_U \) right singular vectors of \( H[k] \). Finally, \( F_{BB}^*[k] \) is normalized to ensure the transmit power constraint, i.e., \( F_{BB}^*[k] = \frac{\sqrt{\|F_{RF} F_{BB}^*[k]\|_F^2}}{\|F_{RF} F_{BB}^*[k]\|_F} \). We note that Algorithm 4 converges to at least a stationary point since the objective function (31) is monotonically nondecreasing after each iteration [50].

With \( \{F_{BB}^*[k]\}_{k=1}^K \) and \( F_{RF}^* \in \mathcal{F} \) obtained by Algorithm 4, we can perform the TS procedure to find an efficient solution to the following problem

![Program 3](image-url)

Algorithm 4: FP-Based Algorithm for Problem (30).

Output: \( F_{RF}^*, \{F_{BB}^*[k]\}_{k=1}^K \)

1. Initialize \( F_{RF} \in \mathcal{F} \) and \( \{F_{BB}^*[k]\}_{k=1}^K \) satisfying (29b);
2. repeat
3. \( \text{Update} \quad (r^*[k])_{k=1}^K \) by (33);
4. \( \text{Update} \quad (q^*[k])_{k=1}^K \) by (34);
5. \( \text{Update} \quad F_{RF}^* \) as the solution to problem (36) by the PGA algorithm;
6. \( \text{Update} \quad \{F_{BB}^*[k]\}_{k=1}^K \) by (42);
7. until stopping criteria is satisfied.

Algorithm 5: Two-Step Algorithm for Problem (29).

Output: \( F_{RF}, \{F_{BB}^*[k]\}_{k=1}^K \)

1. Initialize \( F_{RF} \in \mathcal{F} \) and \( \{F_{BB}^*[k]\}_{k=1}^K \) satisfying (29b);
2. Obtain \( \{F_{BB}^*[k]\}_{k=1}^K \) and \( F_{RF} \in \mathcal{F} \) by Algorithm 4;
3. Obtain \( F_{RF}^* \) satisfying (29c) by the modified Algorithm 2 based on \( \{F_{BB}^*[k]\}_{k=1}^K \) and \( F_{RF} \in \mathcal{F} \).

\[
\max_{F_{RF}} \frac{1}{R} \sum_{k=1}^K \sum_{m=1}^{N_U} w_m R_m[k] \quad \text{s.t.} \quad (29c). \tag{43a}
\]

The above problem can be solved with Algorithm 2 by replacing its target objective function and PGA-enabled initialization with (43a) and \( F_{RF} \in \mathcal{F} \), respectively. We note that applying the PGA-TS algorithm here does not require performing the PGA initialization since \( F_{RF} \in \mathcal{F} \) is already given. We omit the detailed subsequent steps to avoid redundancy. Instead, we summarize the two-step algorithm for solving problem (29) in Algorithm 5 and present the complexity analysis next.

C. Complexity Analysis

We first evaluate the complexity of Algorithm 4 for the typical setting \( N_{RF}, N_U \ll N_T \) in downlink MU-MISO systems. The complexities of computing \( (r^*[k])_{k=1}^K \) and \( (q^*[k])_{k=1}^K \) are approximately the same, which is \( O(K N_U^2 N_T N_{RF}) \). To obtain \( F_{RF} \) in step 5, the matrices \( \Xi, \{Y_k\}_{k=1}^K, \) and \( \{Z_k\}_{k=1}^K \) are required but only computed once before the PGA procedure. The result complexity is approximate \( O(K N_U^2 N_T) \). On the other hand, the complexity of the PGA algorithm is approximately \( O(K N_{RF}^2 N_T^2) \), which is mainly due to computing the gradient \( \nabla_{F_{RF}} g(F_{RF}) \). Therefore, the overall complexity of step 6 is \( O(K (N_{PGA} N_{RF} + N_U) N_T^2) \), where \( N_{PGA} \) represents the number of iterations of the PGA algorithm. The complexity of step 6 is approximately \( O(K N_{RF}^2 N_T^2) \) due to matrix multiplications. Hence, the overall complexity of Algorithm 4 can be approximated as \( T_{FP} O(K (N_{PGA} N_{RF} + 2 N_U) N_T^2) \), where \( T_{FP} \) denotes the number of iterations of the PGA-TS algorithm. Therefore, the overall complexity of

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Algorithm 5 is approximately $O((c_{N_{RF}}K N_{RF}N_{RF}^2 + 2N_U)N_{max}^2 \Delta T_{SN}K N_{RF}N_{RF}^2 N_{SNR}^2)$. Finally, we summarize the complexities of proposed algorithms in Table I, where we recall $c \triangleq 4N_{pga} + 2N_{iter}^{max} N_{nb}$.

### VI. SIMULATION RESULTS

In this section, we provide numerical results to evaluate the performance of the proposed SW-HBF schemes. In the simulations, unless otherwise stated, we set $L_p=4$, $\Delta = \frac{1}{2}$, $f_c=300$ GHz, $K=128$, and $N_S = N_{RF} = 4$ for SU-MIMO and $N_U = 4$ for MU-MISO. The AoA/AoDs are uniformly distributed over $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and the pulse shaping filter is modeled by the raised cosine function [42] with a roll-off factor 1. The path delay is uniformly distributed in $[0, (\zeta - 1)T_s]$ where $\zeta$ is the cyclic prefix length, given by $\zeta = K/4$ according to 802.11ad specification [42]. Other parameters are specified as given in each figure. The SNR is defined as $\text{SNR} \triangleq \frac{P_{fb}}{\sigma_n^2}$. All reported results are averaged over $10^3$ channel realizations.

#### A. Convergence of the Proposed Algorithms

Fig. 4(a) shows the convergence of Algorithms 1, 2, and 4 with the normalized objective values. In simulations, we set the termination conditions for Algorithm 2 with $N_{nb} = |S_I|$, $N_{end} = 10$ and $N_{iter}^{max} = 200$. As a result, the PGA-TS algorithm is terminated if the objective value remains unchanged over $N_{end}$ consecutive iterations, or if the number of iterations exceeds $N_{iter}^{max}$. It can be observed that the PGA algorithm, PGA-TS algorithm, and the FP-based algorithm can converge after around 40, 70, and 70 iterations, respectively. In Fig. 4(b), we show the convergence of the PGA-TS algorithm with $N_{nb} \in \{8, 16, |S_I|\}$. The smaller neighbor set is obtained by randomly selecting $N_{nb}$ neighbor points from the full neighbor set. It can be observed that a smaller size of the neighbor set leads to a lower SE, though it reduces computational complexity. Additionally, examining fewer neighbor points lowers the likelihood of discovering a better solution, resulting in the observed staircase pattern in SE improvement over iterations. In subsequent figures, we set $N_{nb} = |S_I|$ unless specified otherwise.

#### B. Performance of SW-HBF in SU-MIMO Systems

In Fig. 5(a), we compare the SE attained by the proposed SW-HBF schemes with those achieved by the random and ES schemes with $N_S = N_{RF} = 2$ and various number of antennas ($N_T = N_R$). Specifically, in the random SW-HBF scheme, the connections between the RF chains and antennas are randomly generated, i.e., the entries of the analog beamformers are randomly set as 0 or 1. In contrast, the ES scheme examines all possible combinations of those connections to search for the best one, which requires a long run time and extremely high complexity. Therefore, we consider a system of moderate size with up to 12 antennas. It can be seen from Fig. 5(a) that the proposed PGA-TS algorithm performs close to the ES while requiring significantly lower complexity than the latter, verifying its efficiency. In the subsequent results, we will omit the ES scheme due to its extremely high complexity and focus on large-sized systems.

We next compare our proposed schemes to four state-of-the-art HBF designs, including the fully-connected PS-HBF [23], fully connected TTD-aided HBF (FC-TTD-HBF) [25], dynamically connected PS-HBF (Dyn-PS-HBF) [38], and dynamically connected FTTD HBF (Dyn-FTTD-HBF) [37]. Note that the FC-PS-HBF employs only PSs, while the FC-TTD-HBF uses both PSs and TTDs. In contrast, Dyn-PS-HBF utilizes a combination of SWs and PSs, and Dyn-FTTD-HBF uses a combination of SWs and FTTDs.

Because the works [25, 37] focus on only the precoder design, we modify the algorithms therein for combiner designs. Furthermore, since the FC-PS-HBF, Dyn-PS-HBF, and FC-TTD-HBF designs employ a large number of PSs, which can severely burden their power consumption, we adopt $\{1, 2, \infty\}$-bit PSs for the three schemes in the following simulations for a fair comparison, where $\infty$-bit PSs indicates the ideal infinite-precision PSs. In subsequent figures, we designate the HBF schemes with the infinite-precision PSs as the “Ideal” cases. Fig. 5(b) shows the SE versus SNR with $N_T = N_R = 256$ and $\beta = 30$ GHz, wherein FC-SW-HBF refers to the FC-TTD-HBF architecture studied in this work for clarity. The resultant BSR of the system according to Proposition 1 is 1.6, indicating the severe beam squint effect. We set the number of TTDs and FTTDs as one and two for each RF chain here and in the subsequent results, except Fig. 6(c) wherein the SE and EE of more TTDs and FTTDs will be demonstrated. We can observe from Fig. 5(b) that PS-based HBF schemes with ideal PSs achieve similar SE compared to those with 2-bit PSs. This occurs because the phase noise becomes a minor issue when the beam squint effect is relatively severe. Furthermore, it is observed that the proposed SW-HBF schemes perform better than the

### TABLE I

| Item | Computational complexity |
|------|-------------------------|
| Algorithm 1 | $O(4N_{pga}KN_{RF}N_{RF}^2)$ |
| Algorithm 2 | $O(cK N_{RF}N_{RF}^2)$ |
| Algorithm 3 | $O((c_{N_{RF}} + 2N_T)KN_{RF}^2 + c_{N_{RF}}N_{RF}^2)$ |
| Algorithm 4 | $O(I_{FF}K(N_{pga}N_{RF}^2 + 2N_U)N_{max}^2 + I_{TS}N_{RF}N_{RF}N_{SNR}^2)$ |
| Algorithm 5 | $O(I_{FF}K(N_{pga}N_{RF}^2 + 2N_U)N_{max}^2 + I_{TS}N_{RF}N_{RF}N_{SNR}^2)$ |

Fig. 4. Convergence of the proposed algorithms with $N_T = N_R = 64$, $N_S = N_{RF} = 4$ for SU-MIMO and $N_T = N_R = 64$, $N_U = 4$ for MU-MISO. In Fig. (a), the size of the neighbor set for the PGA-TS algorithm is set as $N_{nb} = |S_I|$. In Fig. (b), we show its convergence with $N_{nb} \in \{8, 16, |S_I|\}$. In subsequent figures, we set $N_{nb} = |S_I|$ unless specified otherwise.
Fig. 5. Achievable SE of SW-HBF schemes with $f_c = 300$ GHz, $B = 30$ GHz. Fig. (a) shows SE versus the number of antennas with $N_T = N_R$ and $N_S = N_{RF} = 2$. Fig. (b) shows SE versus SNR with $N_S = N_{RF} = 4$ and $N_T = N_R = 256$. Fig. (c) shows SE versus the number of data streams with $N_{RF} = N_S$ and SNR = 20 dB, $N_T = N_R = 256$.

Fig. 6. Average SE and average EE versus BSR = {0.1, 0.4, 0.7, 1.3, 1.6, 1.9, 2.2}, which are obtained with bandwidth $B = \{1.875, 7.5, 13.125, 18.75, 24.375, 30, 35.625, 41.25\}$ GHz and $f_c = 300$ GHz, $N_T = N_R = 256$, $N_S = N_{RF} = 4$, SNR = 20 dB. (a) Average SE versus BSR. (b) Average EE versus BSR. (c) Average SE versus average EE with $B = 30$ GHz.

| Benchmark Transceiver Architectures | Power consumption |
|-------------------------------------|-------------------|
| DBF                                 | $P_{\text{DBF}}$ = $(N_T + N_R)(P_{RF} + 2P_{ADC})$ |
| FC-PS-HBF                           | $P_{\text{FC-PS-HBF}}$ = $(N_T + N_R)N_{RF}P_{PS} + 2N_{RF}(P_{RF} + 2P_{ADC}) + (N_T + N_R)P_{SP}$ |
| Dyn-PS-HBF                          | $P_{\text{Dyn-PS-HBF}}$ = $(N_T + N_R)N_{RF}P_{PS} + 2N_{RF}(P_{RF} + 2P_{ADC}) + N_{RF}(N_{Rd} + N_{Ld})P_{TDD} + (N_T + N_R)P_{SP}$ |
| FC-TTD-HBF                          | $P_{\text{FC-TTD-HBF}}$ = $(N_T + N_R)N_{RF}P_{SW} + 2N_{RF}(P_{RF} + 2P_{ADC}) + N_{RF}N_{Rd}P_{FSS} + N_T + N_R + N_RP_{SP}$ |
| Dyn-FTTD-HBF                        | $P_{\text{Dyn-FTTD-HBF}}$ = $(N_T + N_R)N_{RF}P_{SW} + 2N_{RF}(P_{RF} + 2P_{ADC}) + N_{RF}N_{Rd}P_{FSS} + N_T + N_R + N_RP_{SP}$ |
| SW-HBF                              | $P_{\text{SW-HBF}}$ = $(N_T + N_R)N_{RF}P_{SW} + 2N_{RF}(P_{RF} + 2P_{ADC}) + (N_T + N_R)P_{SP}$ |

benchmarks schemes from medium to high SNRs. For example, at SNR = 20 dB, the PGA-TS SW-HBF scheme achieves SE improvements by 38%, 28%, and 12% compared to the FC-PS-HBF, FC-TTD-HBF, and Dyn-PS-HBF schemes with 2-bit PSs, respectively. Additionally, it achieves 30% higher SE than that of the Dyn-FTTD-HBF design. Particularly, setting $N_{nb} = \{8, 16\}$ only result in minor SE loss for the proposed SW-HBF design compared to $N_{nb} = |S|$. The former achieves 94% and 97% the SE of the latter at SNR = 20 dB and still outperforms the benchmark HBF designs. The superiority of SW-HBF over other schemes can also be seen in Fig. 5(c), which shows the SE versus the number of data streams with SNR = 20 dB and $N_{RF} = N_S$.

We next compare both the SE and EE of the considered schemes. The EE is defined as the ratio between the SE and the total power consumption of the transceiver [10], [25], [38]. The power consumption of the considered schemes is presented in Table II, where DBF represents the conventional digital beamforming scheme. Note that $N_{Td}$ and $N_{Ld}$ denote the number of TTDs (FTTDs) at each RF chain in the Tx and Rx, respectively. Furthermore, the power consumption of an RF
The chain is given as $P_{RF} = P_M + P_LO + P_{LPF} + P_{BBamp}$, where $P_M$, $P_LO$, $P_{LPF}$, and $P_{BBamp}$ are the power consumption of the mixer, the local oscillator, the low pass filter, and the baseband amplifier, respectively. The power consumptions of the low noise amplifier (LNA) at each receive antenna, power amplifier (PA) at each transmit antenna, and the analog-to-digital (digital-to-analog) converters at the Rx (Tx) are denoted as $P_{LNA}$, $P_{PA}$, and $P_{ADC}$, respectively. Furthermore, we denote $P_{common} = N_T P_{PA} + N_R P_{LNA}$ as the common power consumption for all architectures. Note that each RF chain requires two quantizers to quantize the in-phase and quadrature-phase signals separately [51]. In addition, the power consumption of the splitter, combiner, TTD, FTTD, PS, and SW are represented by $P_{SP}$, $P_{c}$, $P_{TTD}$, $P_{FTTD}$, $P_{PS}$ and $P_{SW}$, respectively. Finally, the power consumptions of all components are given in Table III.

In Fig. 6(a)-(b), we show the SE and EE of the considered schemes versus the BSR with $N_T = N_S = 256$, $N_3 = N_{RF} = 4$, SNR = 20 dB and various system bandwidth. We note the following observations. Firstly, at BSR = 0.1, the FC-PS-HBF scheme with 2-bit resolution achieves up to 97% of the SE of the optimal DBF. Although the system has a large bandwidth 1.875 GHz and a large number of antennas, the beam squint effect can be negligible, which confirms the findings in Section III. Secondly, it is seen for all the compared HBF schemes that the SE decreases as the BSR increases with the bandwidth. The FC-FTTD-HBF designs with 2-bit and ideal PSS outperform other schemes when BSR ≤ 0.7. However, it performs worse than the proposed FC-SW-HBF design for increased bandwidth due to the more pronounced beam squint effect. Notably, the SE of the FC-PS-HBF schemes decreases more rapidly with increasing BSR compared to the FC-SW-HBF scheme. This observation indicates that the FC-SW-HBF design is more resilient to the beam squint effect than the FC-PS-HBF scheme. Consequently, the former exhibits significantly better performance than the latter. For instance, at BSR = 1.6, the FC-SW-HBF design attains 38% SE improvement compared to the 2-bit FC-PS-HBF scheme. Furthermore, the higher power consumption of PSS makes the EE of the FC-SW-HBF scheme significantly lower than that achieved by the FC-SW-HBF scheme, as demonstrated in Fig. 6(b). Specifically, the FC-SW-HBF scheme attains 61% EE improvements compared to the FC-PS-HBF scheme with 2-bit PS at BSR = 1.6.

The Dyn-PS-HBF design outperforms the FC-PS-HBF scheme but still performs worse than the proposed FC-SW-HBF design in terms of both SE and EE. This is because the Dyn-PS-HBF is more sensitive to the beam squint and has a lower array gain than the FC-SW-HBF scheme. Moreover, the FC-SW-HBF scheme also achieves higher SE and EE than the FC-FTTD-HBF and Dyn-FTTD-HBF schemes for severe beam squint. Finally, we plot the SE–EE map of the considered schemes in Fig. 6(c) wherein $N_{td}$ ($N_{ftd}$) represents the number of TTDs (FTTDs) in each RF chain (assuming $N_{td}^r = N_{td}^l = N_{td}$ and $N_{ftd}^r = N_{ftd}^l = N_{ftd}$). It can be observed that deploying more TTDs and FTTDs leads to a better SE–EE tradeoff, consistent with the findings in [25] and [37]. However, for limited TTDs and FTTDs, the proposed FC-SW-HBF design demonstrates its superiority over the benchmarks.

Fig. 7 shows an example of the beam pattern of PS-based and SW-based analog beamformers with $f_c = 300$ GHz, $B = 30$ GHz, $N_T = N_R = 256$, $N_3 = N_{RF} = 1$. Fig. (a) and Fig. (b) display the beam pattern of the PS-based and SW-based analog beamformer, respectively.
and = 256 dB. In these simulations, we also = 4 and SNR = 9 with B = 300 GHz = and = 256 with = 20 dB. For example, has only a few non-integer entries that need to be = = N and after running step 2 of Algorithm =58= VI-B unless = 5 =. Consequently, the performance gap between Algorithm =5= and the PGA-random approach becomes = 20 dB. However, with more users or lower SNRs, the SW-HBF scheme marginally outperforms the proposed SW-HBF design. Nevertheless, the latter can achieve a higher EE than the former due to the substantially lower power consumption of switches compared to PSs.

Figs. 9 and 10 show the average weighted sum rate and EE versus system bandwidth, respectively, with = 300 GHz, = 256, = 4 and SNR = 20 dB. In these simulations, we also present the performance of the DBF scheme that employs the WMMSE algorithm [58]. It is observed that even though the performance of both SW-HBF and PS-HBF schemes deteriorate with increasing bandwidth due to the beam squint effect, the
former outperforms the latter in terms of both sum rate and EE. Moreover, although the DBF achieves the largest SE, it has the smallest EE due to the excessively high power consumption.

VII. CONCLUSION

We have studied the performance of SW-HBF in wideband mmWave communications. We first present a closed-form expression of the BSR, which reveals that the beam squint effect linearly increases with the number of antennas, fractional bandwidth, and normalized antenna spacing. Moreover, we propose a transceiver design for SW-HBF in SU-MIMO OFDM systems and a hybrid precoding algorithm for MU-MISO OFDM systems. The numerical results verify that in both SU and MU scenarios, the proposed SW-HBF schemes exhibit higher resilience to beam squint. Thus, they outperform the compared HBF architectures in terms of SE and EE, especially when the beam squint effect becomes severe. These new findings make the SW-HBF a compelling solution in wideband systems. Future work may concern the imperfect channel information and hardware components, such as ADCs/DACs and PAs. Further investigation into the impact of spatial oversampling, including the effects of grating lobes, and the intelligent transmission surface-based transceiver are important future research challenges.

APPENDIX A
PROOF OF PROPOSITION 2

As it is difficult to obtain the integration in (10), we utilize the Newton polynomial to fit it by three points \((-1, g(-1, \xi_k)), (0, g(0, \xi_k)),\) and \((1, g(1, \xi_k)),\) which leads to

\[
\int_{-1}^{1} g(\vartheta, \xi_k) \, d\vartheta \approx \int_{-1}^{1} \left[ g(-1, \xi_k) - g(0, \xi_k) \right] \vartheta^2 + g(0, \xi_k) \, d\vartheta
\]

\[
= \frac{4}{3} g(-1, \xi_k) + \frac{2}{3} g(0, \xi_k)
\]

Since the normalized sinc function \(\text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x}\) achieves the maximum value 1 at \(x = 0\), i.e., \(g(0, \xi_k) = 1\), the EAG of the PS-based array is expressed as \(E_{\text{ps}}[g(\vartheta, \xi_k)]\). With (6), we obtain

\[
E_{\text{ps}}[g(\vartheta, \xi_k)] = \frac{2}{3N} \sum_{k=1}^{K} \text{sinc}(N\Delta b c_k) + \frac{1}{3}
\]

\[
\approx \frac{2}{3N} \sum_{k=1}^{K} |\text{sinc}(N\Delta b c_k)| + \frac{1}{3}
\]

\[
\approx \frac{2}{3} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[ \text{sinc}(N\Delta b \left( x - \frac{1}{2} \right) ) \right] dx + \frac{1}{3}
\]

\[
= \frac{4}{3N\Delta b} \int_{0}^{\text{BSR}} \text{sinc}(x) dx + \frac{1}{3}, \quad (46a)
\]

where \(c_k \triangleq K - \frac{K+1}{2K}; (i)\) and \((ii)\) are due to \(|\Delta b c_k| \ll 1\) and \(K \gg 1\), respectively. Equation (46a) is obtained with the BSR in (5). Defining \(f(z) \triangleq \frac{1}{z} \int_{0}^{z} |\text{sinc}(4x)| \, dx\), we prove that \(f(z)\) monotonically decreases with \(z\) next. The first derivative of \(f(z)\) is

\[
f'(z) = \frac{1}{z} \left( |\text{sinc}(4z)| - \frac{1}{z} \int_{0}^{z} |\text{sinc}(4x)| \, dx \right)
\]

\[
\approx \frac{1}{z} \left( |\text{sinc}(4z)| - |\text{sinc}(4\varepsilon)| \right), \quad (47)
\]

where \((i)\) is due to the utilization of the Mean Value Theorem with \(\varepsilon \in (0, z)\).

For \(z \in \left( 0, \frac{1}{2} \right)\), the \(\text{sinc}(z)\) is non-negative and monotonically decreases, thus \(f'(z) < 0\).

For \(z \in \left( \frac{1}{2}, \infty \right)\), because of \(|\text{sinc}(4z)| = \left| \frac{\sin(4\pi z)}{4\pi z} \right| \leq \frac{1}{4\pi z} = \frac{1}{z} dx < 0\), with \(|\text{sinc}(4\pi z)| \leq 1\) and \(z > 0\), we obtain

\[
f'(z) \leq \frac{1}{z} \left( \frac{1}{4\pi z} - \frac{1}{z} \int_{0}^{\frac{z}{4\pi}} |\text{sinc}(4x)| \, dx \right)/4\pi
\]

\[
= \frac{1}{z^2} \left( 1 - \int_{0}^{\frac{z}{4\pi}} |\text{sinc}(4x)| \, dx - \int_{\frac{z}{4\pi}}^{z} |\text{sinc}(4x)| \, dx \right)
\]

Since \(\frac{1}{z^2} - \int_{0}^{\frac{z}{4\pi}} |\text{sinc}(4x)| \, dx < 0\) and \(f'(z) < 0\) for \(z \in \left( \frac{1}{2}, \infty \right)\).

In summary, the first derivative of \(f(z)\) is negative in \((0, \infty)\), indicating that \(f(z)\) monotonically decreases with \(z\) in \((0, \infty)\). Therefore, the EAG of the PS-based array monotonically decreases with BSR in (5). From (46a), we can obtain \(\frac{1}{2} \leq E_{\text{ps}}[g(\vartheta, \xi_k)] \leq 1\). The lower bound is attained as BSR \(\to \infty\), while the upper bound is achieved as BSR \(\to 0\). The proof is completed.

APPENDIX B
PROOF OF PROPOSITION 3

Define \(\Omega(\vartheta)\) as a \(K \times N\) matrix with the \(n\)-th column given by

\[
\frac{1}{K \sqrt{N}} \left( e^{-j2\Delta (n-1)\pi \xi_0, \vartheta, \ldots, e^{-j2\Delta (n-1)\pi \xi_K, \vartheta}} \right)^T.
\]

We have

\[
E_{\text{sw}}[g(w, \vartheta)] = \frac{1}{2K \sqrt{N \|w\|_1}} \int_{-1}^{1} \sum_{k=1}^{K} |\sum_{n=1}^{N} w_n e^{-j2\pi(n-1)\Delta \xi_k \vartheta}| \, d\vartheta
\]

\[
= \frac{1}{2} \int_{-1}^{1} \tilde{g}(w, \vartheta) \, d\vartheta, \quad (48)
\]

where \(\tilde{g}(w, \vartheta) \triangleq \frac{\|\Omega(\vartheta)w\|_1}{\sqrt{\|w\|_1}}\). By using the Newton polynomial fitting the integration in (48) at three points \((2, \tilde{g}(w, -1)), (0, \tilde{g}(w, 0))\) and \((1, \tilde{g}(w, 1))\), we obtain

\[
\tilde{g}(w, \vartheta) \approx \left( \tilde{g}(w, 1) + \tilde{g}(w, -1) - \tilde{g}(w, 0) \right) \vartheta^2
\]

\[
+ \tilde{g}(w, 1) - \tilde{g}(w, -1) \vartheta + \tilde{g}(w, 0), \quad (49)
\]

where \((i)\) is due to \(\tilde{g}(w, -\vartheta) = \tilde{g}(w, \vartheta)\). Therefore, we obtain

\[
E_{\text{sw}}[g(w, \vartheta)] \approx \frac{1}{2} \int_{-1}^{1} \left( (\tilde{g}(w, 1) - \tilde{g}(w, 0)) \vartheta^2 + \tilde{g}(w, 0) \right) \, d\vartheta
\]
\[
\frac{1}{3} \hat{g}(w, 1) + \frac{2}{3} \hat{g}(w, 0) \approx 2 \sqrt{\frac{\|w\|_1}{N}},
\]

where approximation (ii) follows.

\[
g(\hat{w}, 1) = \frac{1}{K} \ln \frac{\|w\|_1}{N},
\]

by the Law of Large Numbers as \(\|w\|_1 \gg 1\) in large-scale MIMO systems. Since \(\|w\|_1 \leq N\), we have \(E_{\hat{w}}[g(w, 0)] \leq \frac{2}{3}\), where the equality may hold when each RF chain is connected to all antennas.

\section*{Appendix C}

\subsection*{Proof of Proposition 4}

By introducing \(K\) auxiliary vectors \(r[k] = [r_1[k], \ldots, r_{N_C}[k]]^T\), \(\forall k\), the objective value of problem (29) can be achieved by the following problem

\[
\max_{\{r[k]\}_{k=1}^K} \quad \frac{1}{K} \sum_{(k,m)} w_m \log_2(1 + r_m[k])
\]

s.t. \(r_m[k] \leq \text{SINR}_m[k], \forall m, k\),

where \(\text{SINR}_m[k] \triangleq \frac{\|h[k]\|_2^2}{\sigma_n^2 + \sum_{t \neq m} \|h[k]\|_2^2} \). This is because the equality holds for (52b) at the optima of problem (52).

By introducing \(K\) multipliers \(\lambda[k] = [\lambda_1[k], \ldots, \lambda_{N_C}[k]]^T\), \(\forall k\), we can form a Lagrangian function as

\[
L\{\{r[k], \lambda[k]\}_{k=1}^K\} = \frac{1}{K} \sum_{(k,m)} w_m \log_2(1 + r_m[k])
\]

\[
- \sum_{(k,m)} \lambda_m[k] (r_m[k] - \text{SINR}_m[k]).
\]

With \(\partial L/\partial r_m[k] = 0\) and \(\partial L/\partial \lambda_m[k] = 0\), we obtain the optimal \(r_m[k]\) and \(\lambda_m[k]\) as

\[
r_m^*[k] = \text{SINR}_m[k], \forall k, m,
\]

\[
\lambda_m^*[k] = \frac{w_m}{K \ln 2 (1 + \text{SINR}_m[k])}, \forall k, m.
\]

Inserting (55) back to (53) and with some algebra, we obtain

\[
f_{r}(\mathbf{F}_{RF}, \{\mathbf{F}_{BB}[k], \mathbf{r}[k]\}_{k=1}^K) = \frac{1}{K} \sum_{(k,m)} w_m \log_2(1 + r_m[k]) - \frac{1}{K} \ln 2 \sum_{(k,m)} w_m r_m[k]
\]

\[
+ \frac{K \ln 2}{K} \sum_{(k,m)} \frac{w_m (r_m[k] + 1)}{K} \|h[k]\|_2^2 \|\mathbf{F}_{RF}\|_{\mathbf{F}_{BB}[k][\|\mathbf{r}[k]\|_2]}^2 + \sigma_n^2.
\]

In this case, setting \(\partial L/\partial \lambda_m[k] = 0\) yields (54), with which \(f_r(\mathbf{F}_{RF}, \{\mathbf{F}_{BB}[k], \mathbf{r}[k]\}_{k=1}^K)\) is same as the objective function of problem (29). It is observed that the last term of (56) has a multiple-ratio form, which is typical in FP problems. With the quadratic transform [50], we have

\[
\frac{\sum_{(k,m)} w_m (r_m[k] + 1) h_m^H[k] \mathbf{F}_{RF}\mathbf{F}_{BB}[k][\|\mathbf{r}[k]\|_2]}{\sum_i h_i^H[k] \mathbf{F}_{RF}\mathbf{F}_{BB}[k][\|\mathbf{r}[k]\|_2]^2 + \sigma_n^2} = \max_{\{q[k]\}_{k=1}^K} g(q[k]) \frac{K}{K-1}
\]

with

\[
q[k] = \sum_{(k,m)} 2 \sqrt{\frac{w_m (r_m[k] + 1)}{K}} \mathbf{F}_{RF}\mathbf{F}_{BB}[k][\|\mathbf{r}[k]\|_2].
\]

Therefore, with the optimal \(r_m[k]\) and \(q_m[k]\), given as (54) and (59), respectively, optimizing the objective function (56) is equivalent to maximizing the objective function (31) in the sense that they achieve the same optimal value and solutions. The proof is completed.

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