Gravitational Lensing Bound On The Average Redshift Of Gamma Ray Bursts In Models With Evolving Lenses

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Abstract

Identification of gravitationally lensed Gamma Ray Bursts (GRBs) in the BATSE 4B catalog can be used to constrain the average redshift \( < z > \) of the GRBs. In this paper we investigate the effect of evolving lenses on the \( < z > \) of GRBs in different cosmological models of universe. The cosmological parameters \( \Omega \) and \( \Lambda \) have an effect on the \( < z > \) of GRBs. The other factor which can change the \( < z > \) is the evolution of galaxies. We consider three evolutionary model of galaxies. In particular, we find that the upper limit on \( < z > \) of GRBs is higher in evolving model of galaxies as compared to non-evolving models of galaxies.

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1 Introduction

The use of gravitational lensing as a tool for the determination of cosmological parameters (e.g. $H_0$, $\Omega_0$, $\Lambda_0$) has frequently been discussed. To constrain these parameters either QSOs or galaxies have been used as sources. To use Gamma Ray Bursts (GRBs) as a source for gravitational lensing is not a new idea. As first discussed by Paczynski, if GRBs are cosmological then they should be gravitationally lensed just as quasars.

There is now overwhelming evidence that the majority of GRB sources lie at cosmological distances. The detection of GRB990123 which is believed to lie between redshift $1.6 \leq z < 2.14$, the identification of the host galaxy for GRB981214 at $z = 3.42$ and the detection of afterglows of many others GRBs in X-rays, optical and radio domain by BeppoSAX satellite are all suggestive of GRBs being at cosmological distances. Therefore it is expected that the GRBs must be gravitationally lensed as the probability of lensing increases with the source redshift. The primary effect of a gravitational lens on GRB would be to create more than one image of the burst. These images could not be angularly resolved with the present technology since BATSE’s spatial resolution is insufficient to resolve the images angularly but they could be temporally resolved. Several authors have presented detailed calculations of GRB lensing which can be used as a probe in the search for dark matter in the form of compact objects.

So far no lensed GRBs have been detected with BATSE which clearly implies an upper limit to the average redshift $<z>$ of
GRBs as first calculated by Holz, Miller & Quashnock\textsuperscript{15} [HMQ]. HMQ calculated an upper limit to \( < z > \) which is independent of the physical model of GRBs. They have calculated \( < z > \) for GRBs in different cosmological models.

Till date a substantial fraction of all GRBs observed occur at significant cosmological distances (\( z \sim 0.8 - 3.4 \)). Further the “no host galaxy problem” pushes the GRBs to be, either at very high redshift (\( z > 6 \)) or not to be in normal host galaxies.\textsuperscript{16} Several authors\textsuperscript{17, 18} propose that the GRBs rate should trace the star formation rate in the universe and consequently it places the very dim burst\textsuperscript{17} at \( z \geq 6 \). HMQ calculated the \( < z > \) of GRBs in a non-evolving model of galaxies (lenses) and the \( < z > \) at 95 \% confidence level (CL) comes out to be \( < 2.2, 2.8, 4.3 \) for different combinations of (\( \Omega, \Lambda \)) namely \( (0.3, 0.7), (0.5, 0.5) \) and \( (0.5, 0.0) \) respectively. These results get considerably modified when evolution in the properties of lensing galaxies (lenses) is considered. The modified results permit a larger value of \( < z > \) and are thus consistent to some extent with the high - z GRB scenarios. The purpose of this paper is to put the upper limit on \( < z > \) of GRBs in different models of evolving lensing galaxies.

We refine the analysis of HMQ by considering the evolution of galaxies. Normally the comoving number density of galaxies is assumed to be constant while calculating lensing probability. But it is an oversimplification to assume that galaxies are formed at a single epoch. Evolution tells us how the mass or number density of the lens varies with cosmic time scales. Merging between galaxies and the infall of surrounding mass into galaxies
are two possible processes that can change the comoving density of galaxies and/or their mass. The effect of galaxy merging or evolution has been studied by many authors. Most of them have focused on the statistical properties of gravitational lenses and the limits on the cosmological constant. This work is an attempt to check how galaxy evolution changes the upper limit on the average redshift of GRBs.

The structure of the paper is as follows. In section 2 we describe the evolutionary model of galaxies that we use. The lensing probability with evolving lenses is given in section 3. In section 4 we put the upper limits on the average redshift of GRBs, assuming that no lensing events are present in the BATSE 4B catalog. Results on $< z >$ of GRBs with evolving lensing galaxies in different cosmological models are described in section 5. Discussion is given in section 6.

2 Evolution Of Galaxies

Galaxy mergers and gravitational lensing are interlinked with each other as the merging of small galaxies gives rise to elliptical galaxies which further act as gravitational lenses. We consider three evolutionary models of galaxies

**Fast Merging**

This evolutionary model was proposed by Broadhurst, Ellis & Glazebrook (1992) [BEG] in order to resolve the faint galaxy population counts. The basic point of this model is to introduce strong number evolution or to assume that galaxy numbers are
not conserved with cosmic time but the total comoving mass remains conserved. This model assumes the comoving number density of the lenses to vary with cosmic time as:

\[ n(\delta t) = f(\delta t)n_0 \]  

where \( \delta t \) is the look-back time and the subscript ‘0’ indicates the present day values. The characteristic mass (or dispersion velocity) for self similar galaxy mass function of Singular Isothermal Sphere (SIS) lenses at \( \delta t \) is

\[ v(\delta t) = [f(\delta t)]^{-1}v_0 \]  

This model is better than the model proposed by Volmerange and Guiderdoni\textsuperscript{25} where the merging goes exponentially with \( (1 + z) \), because the merging rate doesn’t become high at early time as explained by BEG. According to the BEG model if we had \( n \) galaxies at look-back time \( \delta t \) each with velocity dispersion \( v \), they would by today have merged into one galaxy with a velocity dispersion \( [f(\delta t)]v \). The function \( f(\delta t) \) describes the time dependence and the strength of merging:

\[ f(\delta t) = \exp(QH_0\delta t) \]  

where \( H_0 \) is the Hubble constant at the present epoch and \( Q \) represents the merging rate. We take \( Q = 4 \) as suggested by BEG\textsuperscript{24}. The look back time \( \delta t \) is related to \( z_1 \) through

\[ H_0(\delta t) = \int_0^{z_1} \frac{(1 + y)^{-1}dy}{\sqrt{F(y)}} \]  

\[ \int \]
where $F(y) = \Omega_0(1 + y)^3 + (1 - \Omega_0 - \Lambda_0)(1 + y)^2 + \Lambda_0$

where $\Omega_0 = \frac{8\pi G \rho_0}{3H_0^2}$, $\rho_0$ is density of matter, $\Lambda_0 = \frac{8\pi G \rho_v0}{3H_0^2}$ and $\rho_v0$ is density of vacuum.

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**Slow Merging**

In this slow merging model\(^\text{26}\), the mass of an individual galaxy increases with cosmic time as $t^{2/3}$ while the comoving number density varies with cosmic time as $t^{-2/3}$, so the total mass of the galaxies within a given comoving volume is conserved. The cosmic time $t$ starts from the big bang. We also assume that the power law relation between the mass and velocity is $M \propto v^{3.3}$ for elliptical galaxies.\(^\text{27}\) Then

\[ n(\delta t) = n_0 \left[ 1 - \frac{\delta t}{t_0} \right]^{-2/3} \tag{5} \]

\[ v(\delta t) = v_0 \left[ 1 - \frac{\delta t}{t_0} \right]^{1/5} \tag{6} \]

where $t_0$ is present age of the universe.

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**Mass Accretion**

Mass accretion is the key factor for evolution of galaxies. In this model\(^\text{28}\), the comoving number density of the galaxies remains constant but the mass of galaxy increases with cosmic time as $t^{2/3}$ in the same way as in the slow merging model.
\[ n(\delta t) = n_0(\text{constant}) \]  \hspace{1cm} (7)

\[ v(\delta t) = v_0 \left[ 1 - \frac{\delta t}{t_0} \right]^{1/5} \]  \hspace{1cm} (8)

3 Basic Equations For Gravitational Lensing Statistics

The differential probability \( d\tau \) of a beam encountering a lens in traversing the path of \( dz_L \) is given by\textsuperscript{29, 30}

\[ d\tau = n_L(z) \sigma \frac{cdt}{dz_L} dz_L, \]  \hspace{1cm} (9)

where \( n_L(z) \) is the comoving number density.

The Singular Isothermal Sphere (SIS) provides us with a reasonable approximation to account for the lensing properties of a real galaxy. The lens model is characterized by the one dimensional velocity dispersion \( v \). The deflection angle for all impact parameters is given by \( \tilde{\alpha} = 4\pi v^2/c^2 \). This lens produces two images if the angular position of the source is less than the critical angle \( \beta_{cr} \), which is the deflection of a beam passing at any radius through a SIS:

\[ \beta_{cr} = \tilde{\alpha} D_{LS}/D_{OS}, \]  \hspace{1cm} (10)

Here we use the notation \( D_{OL} = d(0, z_L), D_{LS} = d(z_L, z_S), D_{OS} = d(0, z_S) \), where \( d(z_1, z_2) \) is the angular diameter distance between
the redshift $z_1$ and $z_2$. Then the critical impact parameter is defined by $a_{cr} = D_{OL} \beta_{cr}$ and the cross-section is given by

$$\sigma = \pi a_{cr}^2 = 16\pi^3 \left(\frac{v^*}{c}\right)^4 \left(\frac{D_{OL}D_{LS}}{D_{OS}}\right)^2,$$  \hspace{1cm} (11)

\textbf{The Evolutionary Model}

The differential probability $d\tau$ of a lensing event in an evolutionary model can be written as:

$$d\tau = \frac{16\pi^3}{cH_0^3} \phi_* v_*^4 \Gamma \left(\alpha + \frac{4}{\gamma} + 1\right) f(\delta t)^{(\frac{1-\gamma}{\gamma})} \times (1 + z_L)^3 \left(\frac{D_{OL}D_{LS}}{R_0D_{OS}}\right)^2 \frac{1}{R_0} \frac{cdt}{dz_L} dL$$  \hspace{1cm} (12)

$$d\tau = F(1 + z_L)^3 \left(\frac{D_{OL}D_{LS}}{R_0D_{OS}}\right)^2 f(\delta t)^{(\frac{1-\gamma}{\gamma})} \frac{1}{R_0} \frac{cdt}{dz_L} dL$$

where $F = \frac{16\pi^3}{cH_0^3} \phi_* v_*^4 \Gamma \left(\alpha + \frac{4}{\gamma} + 1\right)$. By substituting the values of $\phi_*, \alpha$ as given by Corray et al.\cite{31} and $v_*, \gamma$ as given by Nakamura & Suto\cite{27} we get $F = 0.035$. where $f(\delta t) = \exp(QH_0\delta t)$ for fast merging and $f(\delta t) = \left(1 - \frac{\delta t}{t_0}\right)^{-2/3}$ for slow merging. In case of mass accretion $f(\delta t) = \left(1 - \frac{\delta t}{t_0}\right)^{-2/3}$ but the exponent of $f(\delta t)$ for mass accretion model in eq. (12) becomes $(-1 - \frac{4}{\gamma})$ as the total mass in galaxies increases with time.

\textbf{The Non Evolutionary Model}
In the non merging model the optical depth is given by

\[ d\tau = \frac{16\pi^3}{cH_0^3} \frac{\phi_* v_*^4 \Gamma}{(\alpha + \frac{4}{\gamma} + 1)} (1 + z_L)^3 \]

\[ \times \left( \frac{D_{OL}D_{LS}}{R_0D_{OS}} \right)^2 \frac{1}{R_0} \frac{c dt}{dz_L} dz_L \]  

(13)

4 Bound On Average Redshift Of GRBs

Adopting the simplest matter distribution for the lensing galaxy as a Singular Isothermal Sphere (SIS), we calculate the probability \( \tau(z) \) of a beam from a source at redshift \( z \) is imaged by a lens in the filled beam approximation. Apart from the cosmological parameters \( \Omega \) and \( \Lambda \), the lensing probability depends not only on the model used for describing the evolution of galaxies but most importantly on the parameter \( F \). \( F \) parameterizes the distribution of galaxies as well as their effectiveness in the lensing process (defined in section 3). Thus the elliptical galaxies contribute strongly while spirals have a negligible effect. The Schechter distribution function often forms the basis of these estimates. However it is being increasingly realised that sub-distribution of specific types of galaxies have to be further taken into account. We take \( F = 0.035 \) which lies in the current estimate range \( 0.02 < F < 0.05 \). In the SIS model, the lensing event always consists of two images and the third central image is too faint to be observed. We take a constant BATSE efficiency \( \epsilon = 0.48 \) to see either image and \( \epsilon^2 = 0.23 \) to
see both images. So far no multiple images has been detected, which may due to the low instrumental efficiency.

In order to put the bound on the $<z>$ of GRBs, we follow the methodology of HMQ. As the distribution of GRBs in redshift is unknown, we use the simplifying assumption that all the sources are at the same redshift. We estimate the number of image pairs as

$$N_{<z>} = \left(\frac{N_{tot}}{\epsilon}\right) \epsilon^2 \tau(<z>) = N_{tot} \epsilon \tau(<z>) \quad (14)$$

where $N_{tot}$ be the total number of observed bursts in the BATSE 4B catalog (which are 1802) then $N_{tot}/\epsilon$ are actual burst sources above the BATSE threshold. The $<z>$ in the above equation tells the average redshift of GRBs, assuming that all the sources are at this redshift. If we define $\phi_{min}$ as the brightness threshold below which identification of lensed images from light-curve comparison is impossible, then the expected number gets modified to\[14,37\]

$$N_{<z>} = N_{tot} \epsilon \tau(<z>) \int_0^{\infty} dB(\phi) \frac{1}{[1 + \frac{\phi_{min}}{\phi(z)}]^2} \quad (15)$$

We plot this $N_{<z>}$ with $<z>$ in different cosmological models as shown in Fig. 1, Fig.2 and Fig.3 respectively. Here $B(\phi)$ is the the observed BATSE brightness distribution and the integrand is the conditional probability that the both images are above the brightness threshold. The value of this integral is equal to 0.57\[13\].
5 Results

We have taken three representative values of $(\Omega, \Lambda)$ as $(0.3, 0.7)$, $(0.5, 0.5)$ and $(0.2, 0)$. With these values we calculate the expected number of observable image pairs in the BATSE 4B as a function of average redshift for the mass accretion, fast merging, slow merging and no evolutionary models. We find that the best limit arises with a large cosmological constant where the lensing rate is quite high. At the 95% confidence level, we find an upper limit on $\langle z \rangle < 4.3, 7.8, 9.8$ in the non-evolutionary model for $(\Omega, \Lambda)$ values of $(0.3, 0.7), (0.5, 0.5)$ or $(0.2, 0)$ respectively. The upper limit on $\langle z \rangle$ at 95% CL in other evolutionary models are described in table 1. At 68% confidence level, the slow merging model gives upper limit on $\langle z \rangle < 2.3, 3.3$ or $3.9$ for $(\Omega, \Lambda)$ values of $(0.3, 0.7), (0.5, 0.5)$ or $(0.2, 0)$ respectively. Similarly fast merging model gives $\langle z \rangle < 2.9, 4.3$ or $5.2$ for $(\Omega, \Lambda)$ values of $(0.3, 0.7), (0.5, 0.5)$ or $(0.2, 0)$ respectively as shown in fig.1, fig.2, and fig.3 respectively. The mass accretion model at 68% CL gives an upper limit on $\langle z \rangle < 6.1$ only for $(\Omega = 0.3, \Lambda = 0.7)$. The result is described in Table 1 and Table 2.

6 Discussion

We notice that the evolution of lensing galaxies serves to increase the $\langle z \rangle$ of GRBs as compare to non- evolutionary model of galaxies. Throughout the paper, we have used the filled beam approximation (standard distance)\cite{29, 30} in which “smoothness parameter”
\( \alpha \) (which measures the degree of inhomogeneity of the universe) is equal to 1. On the other hand with \( \alpha = 0 \) one gets empty beam distance in which there is negligible intergalactic matter and the line of sight to a distant object doesn’t pass close to intervening galaxies.\(^{38, 39}\) At present we don’t know what value of smoothness parameter in the distance formula describes our universe best. Ehlers and Schneider\(^{40}\)(1986) argued that the direction to the source cannot be a random variable in the clumpy universe. At lower redshift the probability of lensing in different distance formulations remains approximately the same while at higher redshift the lensing rate become different in different distance approximations. So the question arises, which is the best distance approximation? But recently in an elegant paper, Holz & Wald (1998)\(^{41}\) have exhibited a new method for calculating gravitational lensing rates. This method is free from the ambiguities present in the distance formula. In this formulation, the optical depth at any redshift is greater than the optical depth calculated using the angular diameter distance in the filled beam approximation. Thus we expect that this formulation will increase the expected lensing rate, and hence decrease the upper limit on \(< z >\) of GRBs.

Another reason why our lensing rate could be an underestimate because of the assumption that lensing is only due to the SIS galaxies. But there are other lens models also like: point like mass as deflector, isothermal sphere with a softened core, asymmetric lens, spherical model (e.g. King model), cosmic strings and constant density sheet which in principle could also
contribute. Many authors\sup{13, 42–45} have explored the possibility of GRBs lensing by point masses. Grossman and Nowak\sup{37} also studied the lensing of GRBs by asymmetric and non singular isothermal spheres lenses. These additional lensing objects may change $< z >$.

The overall probability of strong lensing depends directly on the parameter $F$, which further depends upon four parameters $\alpha, \gamma, v_*, \phi_*$. There are many uncertainties associated with these parameters as mentioned in sec. 4 and discussed by several authors\sup{31, 34, 46, 47}. These uncertainties can change the value of $F$ by upto 30% and hence affect our results. HMQ used the value of $F = 0.1$ which is nearly three times higher than the value of $F$ used in this paper. Therefore the upper limit on the $< z >$ of GRBs in HMQ paper is less than our value of $< z >$.

The ability of BeppoSax satellite to detect GRBs opens up a new era in the studies of GRBs. Many counterparts of GRBs have been detected in X-ray, optical and radio domains from which important insight in this field has been gained. Nevertheless, the nature of the central engine that accelerates the relativistic flow is still not very clear. In order to calculate the amount of total energy in the burst, the precise measurement of redshift is required. For example, spectroscopic observations show that the host galaxy for GRB971214 is at a redshift $z = 3.418$. Given this high redshift and the known fluence of this
GRB the $\gamma$ - ray energy release of this burst is unexpectedly large, about $3 \times 10^{53}\text{erg}$, assuming isotropic emission. Energy released in other forms of radiation is not included in this energy calculation. The currently favored model for GRBs is coalescence of neutron stars. The coalescence model is expected to released about $10^{51}\text{erg}$ in the form of electromagnetic energy. So there is big gap between the model based calculation and the parameters based calculation. Similarly, with the combination of a redshift $\geq 1.6$ and fluence for GRB990123 imply that $\gamma$ ray energy release in this burst is $3.4 \times 10^{54}\text{erg}$, assuming the emission is isotropic. This would again put strain on any GRB model based on merging of neutrons star or black holes. At higher redshift this problem becomes more severe. We may be forced to consider even more energetic possibilities or to find ways of extracting more electromagnetic energy in coalescence models or we have to search for ways to reduce the estimated energy release from GRBs to resolve this problem.

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Figure Captions

**Figure 1.** Expected number of observable image pairs in the BATSE 4B ($N_{\langle z \rangle}$) as function of $< z >$, the effective average redshift of GRBs, in flat universe with $\Omega = 0.3$ and $\Lambda = 0.7$.

**Figure 2.** Expected number of observable image pairs in the BATSE 4B ($N_{\langle z \rangle}$) as function of $< z >$, the effective average redshift of GRBs, in flat universe with $\Omega = 0.5$ and $\Lambda = 0.5$.

**Figure 3.** Expected number of observable image pairs in the BATSE 4B ($N_{\langle z \rangle}$) as function of $< z >$, the effective average redshift of GRBs, in an open universe with $\Omega = 0.2$ and $\Lambda = 0.0$. 
Table 1. Limits On The Average Redshift \( \langle z \rangle \) Of GRBs In Different Models Of Galaxy Evolution At 95% Confidence Level with 1802 bursts

| \( \Omega + \Lambda \)   | No Evo. | Slow Merging | Fast Merging | Mass Accretion |
|-------------------------|---------|--------------|--------------|----------------|
| 0.3 + 0.7               | 4.3     | 5.0          | 7.6          | --             |
| 0.5 + 0.5               | 7.8     | 10.0         | --           | --             |
| 0.2 + 0.0               | 9.8     | --           | --           | --             |

Table 2. Limits On The Average Redshift \( \langle z \rangle \) Of GRBs In Different Models Of Galaxy Evolution At 68% Confidence Level with 1802 bursts

| \( \Omega + \Lambda \)   | No Evo. | Slow Merging | Fast Merging | Mass Accretion |
|-------------------------|---------|--------------|--------------|----------------|
| 0.3 + 0.7               | 2.2     | 2.3          | 2.9          | 6.1            |
| 0.5 + 0.5               | 3.0     | 3.3          | 4.3          | --             |
| 0.2 + 0.0               | 3.5     | 3.9          | 5.2          | --             |
Figure 1:
Figure 2:
Figure 3: