Probabilistic quadratic programming model for Sewon-Bantul facultative ponds optimization

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Abstract. This article is addressed to show the result of a probabilistic mathematical model implementation in Sewon, Bantul facultative wastewater treatment pond to analyse the performance of the pond. The model was developed in a probabilistic quadratic optimization model and was solved by using probabilistic programming by using deterministic equivalent approach. LINGO 18.0 optimization software package was employed to do the computation. The optimization result was giving the optimal decision value for the wastewater volume that should be processed in the pond in order to gain the best performance and the storing time of the wastewater for each facultative pond. For each facultative pond, the load volume of the wastewater should be 1199.5 kg where the storing time is 13.2 day for the first pond, 13.24 day for the second pond, 25.59 day for the third pond and 30.30 day for the fourth pond. This optimal decision, then, can be used by the decision maker (WWTP operator) in order to optimize the performance of the treatment ponds.

1. Introduction
Wastewater treatment plants (WWTP) work by reducing the pollutant through natural processes in bacteria, algae, and zooplankton [1]. There are many treatment parts in a WWTP where one them is facultative stabilization pond. The facultative stabilization pond works by storing the wastewater and then processing the wastewater along some storing time [2]. To observe and analyse the performance of a wastewater facultative stabilization pond, many researches were conducted by using various approaches like quantitative method approach to analyse the pollutant degradation coefficient [3], a linear optimization model approach [4], analysing the effectiveness and purification for biological oxygen demand concentration [5], optimizing the construction cost [6], and natural adsorbent model approach [7]. Some more advance analysis approaches were also conducted in order to develop the previous approach like treatment of sewage [8], removal mechanism analysis of nitrogen [9], a model for optimization on maturation pond [10], for energy saving and mitigation purposes [11]. Furthermore, some advanced optimization models were formulated to optimize the performance of a specific methods and purposes, e.g. optimization of stabilization pond using statistical analysis [12], an optimization model for distillation column of wastewater pond [13], analysis of phosphorus content in wastewater pond [14], analysis of wastewater pond using integrated model of hydraulic, physio-chemical & micro-biological model [15], a model of the combination between oxygen electrode & biological approach [16], organism & organic matter optimization model [17], utilization of residual wastewater for bricks [18], wastewater recycle [19], and a quadratic programming approach for wastewater treatment optimization [20]. The analysis and optimization approaches above were use certain parameters hence the model was deterministic. For advance problem, if some parameters are uncertain, then a
mathematical model that can handle uncertain parameters is needed to develop. Generally, an uncertain parameter is approached by using a probabilistic parameter where a mathematical optimization model which contains some probabilistic parameters will be a probabilistic programming. Some researches in many fields can be found that showing us the application of a probabilistic programming, e.g. fiber reinforced polymer (FRP) strengthening planning [21], optimization in operation of energy hub [22], and optimization of dose coverage in radiotherapy [23].

In this paper, we develop a probabilistic mathematical optimization model to optimize the performance of Sewon-Bantul wastewater facultative stabilization pond where some parameters are uncertain which is approached by a random variable with some probability distribution function formulated from observation data. The decision variable that we determine is the wastewater volume that should be processed in pond and the storing time that should be used that the value of pond’s efficiency is optimal.

2. Method
First, we employ some assumptions as follows: (1) all observation data were taken in Sewon, Bantul WWTP located in Yogyakarta, Indonesia (2) The efficiency index value of a facultative pond is the percentage of the BOD degradation rate times the storing time divided by one plus BOD degradation rate times storing time (3) the quality standard requirement of the wastewater is taken from the local government’s policy (4) The model only covers the wastewater processing in the facultative pond. Next, we identify the probabilistic parameter in the problem, where in our case the probabilistic parameter included in the model is the inflow rate of wastewater entering each facultative pond which may differ day by day. From the data collected in the pond, we formulate the probability distribution function for this probabilistic parameter. The remain parameters are certain and were taken in Sewon, Bantul WWTP as reported in our previous articles. After that, we formulate the objective function which is the total amount of the wastewater processed in all four facultative ponds and the tracking reference for the efficiency index value where this objective function allows the decision maker to optimize the efficiency index value by bringing the efficiency index value to some desired value. Finally, we construct all the constraints that should be held by the model which are representing the conditions occurred in the WWTP. Then, we solve the optimization problem by inputting the model first into LINGO programming language and then using the optimization algorithm to calculating the optimal decision variable value. Finally, the optimal decision is determined from the optimization result.

Figure 1. Sewon Bantul wastewater treatment plant [4]
3. Mathematical Model

Sewon-Bantul wastewater treatment plant is consisting of inlet valve, four facultative ponds, two maturation ponds, and outlet valve which is illustrated by Figure 1. The wastewater is entering the treatment plant via inlet valve and it is distributed into four facultative ponds and stored in these ponds for some storing time. For mathematical model formulation, these are the notations to be used:

Decision variables:
\[ L^e_i \quad : \quad \text{Load volume of the wastewater processed in facultative pond } i \text{ (kg/day)} \]

Deterministic parameters:
\[ L_0 \quad : \quad \text{Wastewater load entering the treatment plant through inlet valve (kg/day)}; \]
\[ L_i \quad : \quad \text{Load volume of the wastewater prior to facultative pond } i \text{ (kg/day)}; \]
\[ C_i \quad : \quad \text{Biological oxygen demand (BOD) concentration at pond } i \text{ (mg/L)}; \]
\[ L \quad : \quad \text{Total of wastewater load volume in all ponds (kg/day)}; \]
\[ E_i \quad : \quad \text{Efficiency index value of the wastewater treatment at pond } i \text{ (in percentage)}; \]
\[ BM \quad : \quad \text{Quality standard of wastewater decided by the government.} \]

Probabilistic parameter:
\[ Q^e_i \quad : \quad \text{Inflow rate of wastewater entering facultative pond } i \text{ (m}^3/\text{day).} \]

We have formulated the mathematical model as follows. The objective function is formulated as the total load volume of the wastewater that will be processed in the facultative ponds. The aim of the optimization is maximising the total load volume of the wastewater and minimizing the difference between efficiency value to a reference point, which can be modelled as maximizing the following objective function
\[
\max Z = L^e_1 + L^e_2 + L^e_3 + L^e_4 - \sum_{i=1}^{4} \left( E_i - E^r_i \right)^2 .
\]  

The constraints of the model are formulated as follows.

1. The efficiency (percentage of the BOD concentration) in each facultative pond must be less or equal to the quality standard:
\[
E_i C_i \leq BM, i = 1, 2, 3, 4; \]

2. The load volume of the wastewater should be less or equal to the load volume of the wastewater prior to the pond:
\[
L^e_i \leq L_i, i = 1, 2, 3, 4; \]

3. The load volume of the wastewater entering to these facultative ponds is the flow rate of the wastewater including the organic matter i.e. Biochemical Oxygen Demand (BOD):
\[
L^e_i = \left( \frac{Q^e_i C_i}{1000} \right), i = 1, 2, 3, 4; \]

4. The efficiency of each facultative pond \( i \) is formulated as:
\[
E_i = \frac{k_t}{1 + k_t}, i = 1, 2, 3, 4; \]

where \( k \) is the BOD degradation rate per day and \( t \) is the storing time (in day) of the wastewater in the pond.

Hence, the complete mathematical model can be rewritten as follows:
\[
\max Z = \sum_{i=1}^{4} L_i^e + \sum_{i=1}^{4} \left( E_i - E_i^f \right)^2 
\]

subject to:

\[
E_i \cdot \left( \frac{1000 \cdot L_i}{Q_i} \right) \leq BM, i = 1, 2, 3, 4;
\]

\[
L_i^e \leq L_i;
\]

\[
L_i^e \leq L_i - L_i^c;
\]

\[
L_i^e \leq L_i - L_i^c;
\]

\[
L_i^e \leq L_i - L_i^c.
\]

4. Results and Discussions

To determine the optimal decision for Sewon, Bantul WWTP, we have used the observation data from our previous works shown in Table 1 where the wastewater load in pond I and II is a half of the inflow of \( L_0 = 4.799.6 \) kg/day and the used BOD degradation coefficient is 1.1\% \[4\]. The quality standards of the BOD is 50 mg/L following the Yogyakarta’s province regulation \[24\]. The probability distribution function of the inflow rate of wastewater entering facultative pond \( i \) (m\(^3\)/day) was fitted using observation data in our previous work \[3\] using normally distribution fitting and we have derived that the mean is 11238 m\(^3\)/day with variance of 2506 m\(^3\)/day. Let the decision maker wants to have the efficiency index to be as closed as possible to 0.15 for pond I and II and 0.25 for pond III and pond IV. We have solved (5) using these value of parameters by using LINGO 18.0 optimization tool in common computer with operating system of Windows 10 x64, processor of Dual Core 3.0 GHz and memory of 4 GB.

| Pond (\( p \)) | \( L_i^e \) | \( E_i \) | \( E_i^f \) | \( L_i \) | \( k_i \) | \( S_i \) |
|---------------|------------|------------|------------|--------|--------|--------|
| 1             | 1199.5     | 0.126617845 | 0.15       | 2399   | 1.1    | 13.17946918 |
| 2             | 1199.5     | 0.127197695 | 0.15       | 2399   | 1.1    | 13.24862084 |
| 3             | 1199.5     | 0.219679765 | 0.25       | 1199.5 | 1.1    | 25.59319479 |
| 4             | 1199.5     | 0.250000124 | 0.25       | 1199.5 | 1.1    | 30.30305468 |

The calculation results shown in Table 1 conclude that that the optimal wastewater volume in each pond should be 1199.5 kg per day where the treatment efficiency of the pond I and II is 12.7\% and the treatment efficiency value of pond III is 21.97\% and pond IV is 25.00\%. The corresponding storing time in pond I is 13.18 days, pond II is 13.25 days, pond III is 25.59 days and pond IV is 30.30 days.

As a fact that the longer the storing time then the less the pollutant concentration in the wastewater, the derived storing time shown in Table 1 is just our recommendation achieved from our proposed mathematical optimization model. As a consequence, the decision maker can store the wastewater longer or shorter. Furthermore, the since the condition of the ponds is full of uncertainty, then the treatment efficiency value may vary from the value shown in Table 1. Furthermore, the other variables may also be varying at the different processing time, for instant, sunny or rainy season. The BOD degradation rate used in this paper was measured in sunny season to be 1.1. It means that while the environment of the ponds is changed, the performance of the pond may also change. Finally, the decision achieved in this research can be used by the decision maker with suitable condition and while the condition is changed then the model can be recalculated using the suitable parameter values.
5. Concluding Remarks

In this paper, a probabilistic optimization model was considered to optimize the wastewater processing of Sewon, Bantul WWTP. From the formulated optimization model, the optimal wastewater volume for each pond and the storing time for each pond in order to achieve the optimal performance of the ponds. In our future works, we will calculate the optimal decision using larger observation data so the probabilistic parameter will be closer to the real data. Furthermore, we also will develop the model so it covers the wastewater processing in maturation pond.

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References

[1] Sunarsih S, Purwanto P, and Budi WS. *J Urban Environ Eng*. 2013 7 293

[2] Sunarsih S, Purwanto, P, and Setia Budi W, *Int. J of Tech*. 2015 6 689

[3] Huang BS, Hong CH, Du HH, Qiu J, Liang X, Tan C, and Liu D, *J Hydrodyn*. 2017 29 118

[4] Sunarsih, Widowati, Kartono, and Sutrisno E3S Web Conf. 2018 31 1

[5] Kogo BK, Biamah EK, Langat PK. *J Geosci Environ Prot*. 2017 5 14

[6] Olukanni DO, Ducoste *J Ecol Eng*. 2011 37 1878

[7] Gopakumar A, Narayan R, Nagath SA, P N, Mohammed. S R, and Chandran SS *Mater. Today Proc*. 2018 5 17699

[8] Recio-Garrido D, Kleiner Y, Colombo A, Tartakovsky B. *Water Res*. 2018 144 444

[9] Mayo AW, Abbas M *Phys Chem Earth*. 2014 72 77

[10] Cortés Martínez F, Treviño Cansino A, Sáenz López A, González Barrios JL, and De La Cruz Acosta F*J Appl Res Technol*. 2016 14 93

[11] Borzooei S, Campo G, Cerutti A, Meucci L, Panepinto D, Ravina M, Riggio V, Ruffino B, Scibilia G, Zanetti M, *Sci Total Environ*. 2019 691 1182

[12] Halters F, Zondervan E, and Haan AD *J Hazard Mater*. 2010 179 480

[13] Ferella F *J Environ Chem Eng*. 2018 6 377

[14] Sells MD, Brown N, Shilton AN *Water Res*. 2018 132 301

[15] Ho LT, Alvarado A, Larriva J, Pompeu C, and Goethals P *Water Res*. 2019 151 170

[16] Novak M, and Horvat P 2012 36 3813

[17] Martínez FC, Cansino AT, Aracelia M, García A, Kalashnikov V, and Rojas RL *Math Probl Eng*. 2014 2014 1

[18] Rodrigues LP, Nilson J, and Holanda F *Procedia Mater Sci*. 2015 8 197

[19] Raghuvanshi S, Bhakar V, Sowmya C, and Sangwan KS *Procedia CIRP*. 2017 61 761

[20] Sunarsih, Sasongko DP, and Sutrisno *Mat MIJAM*. 2019 35 171

[21] Yang DY, Frangopol DM, and Teng JG *Eng Struct*. 2019 188 340

[22] Rakipour D, and Barati H *Energy*. 2019 173 384

[23] Tilly D, Holm Å, Grussell E, and Ahnesjö A *Phys Imaging Radiat Oncol*. 2019 10 1

[24] Gubernur DI Yogyakarta (Governor of Special Region Yogyakarta). *Surat Keputusan Gubernur Kepala Daerah Istimewa Yogyakarta (Decree of Special Region Yogyakarta Governor) No. 214/KPTS/1991*.; 1991