An Atom Laser Based on Raman Transitions

G.M. Moy, J.J. Hope and C.M. Savage

Department of Physics and Theoretical Physics, The Australian National University,

Australian Capital Territory 0200, Australia.

(March 21, 2022)

Abstract

In this paper we present an atom laser scheme using a Raman transition for the output coupling of atoms. A beam of thermal atoms (bosons) in a metastable atomic state is pumped into a multimode atomic cavity. This cavity is coupled through spontaneous emission to another cavity for the atomic ground state. Above a certain threshold pumping rate a large number of atoms build up in the lowest energy state of the second cavity, while the higher energy states remain unpopulated. Atoms are then coupled to the outside of the cavity with a Raman transition. This changes the internal level of the atom and imparts a momentum kick, allowing the atoms to leave the system. We propose an implementation of our scheme using hollow optical fiber atom waveguides.

42.50.Vk,03.75.Be,42.50.Ct,03.75.Fi
I. INTRODUCTION

There has been much recent interest in the design of a device that would emit a coherent beam of bosonic atoms - an atom laser. An atom laser would have many applications in atom optics including atom lithography and nanofabrication, as well as fundamental tests of quantum mechanics such as those involving atom interferometry. A number of theoretical atom laser schemes have already been proposed [1–6]. These have involved some method of cooling atoms in an atomic cavity, and a schematic model for coupling the atoms to the external freely traveling atomic modes. Our approach differs from previous work since we model a particular atom laser scheme emphasizing the importance of output coupling. Our model is quantitative and based on rate equations. There is a lasing threshold analogous to that of the optical laser. We consider a method for output coupling based on changing the internal atomic state of the atoms using a Raman transition in a spatially localized region.

We propose an implementation of our atom laser scheme using hollow optical fibers. This has several advantages, including providing a directed output beam, and minimization of the reabsorption of spontaneously emitted photons.

In recent experiments which have produced a Bose-Einstein condensate [7–10], a macroscopic number of atoms have been condensed in the ground state of an atomic trap. This has generated increased interest in atom lasers, which also involve the production of large occupation numbers in a single quantum mechanical state. In the reported theoretical atom laser models the ground state of an atomic trap or cavity (the lasing mode) is filled with a large number of atoms by using the higher energy modes of the trap as a continuously pumped atomic source. The coupling between higher energy modes and the laser mode is achieved through cooling. In reference [2] the mechanism is dark state cooling, and atoms are transferred from the source to the lasing mode irreversibly by spontaneous emission. Spreeuw et al. [5] use precooled atoms and a pump followed by spontaneous emission to transfer atoms into the lasing mode. Holland et al. [1] and Guzman et al. [4] use inelastic binary collisions to transfer atoms from the source mode to the lasing mode. In reference
two atoms collide to produce one atom in the lasing mode, and another in a higher energy mode. This process is made irreversible by using evaporative cooling to rapidly remove the higher energy atom from the system. A conceptually similar method is used in \cite{4}. The number of atoms in the lasing mode depends on the pumping and loss rates. Above threshold the number of atoms in the lasing mode saturates. This produces a reasonably well defined number of atoms in a single cavity mode.

The atom laser proposals mentioned above have been schematic in nature and the only proposed methods of output coupling have involved either quantum mechanical tunneling \cite{2} or periodically turning off the cavity mirrors \cite{4}. Wiseman et al. \cite{2} find that any physically realizable optical potential barrier confining the atoms leads to an extremely small tunneling rate. Turning off the cavity mirrors, while effective for output coupling, will not provide a continuous beam. We present an atom laser scheme using two atomic cavities - one for the source atoms in atomic level $|1\rangle$ and a second for the lasing atoms in level $|2\rangle$. This second cavity has only one significantly populated mode - the ground state mode. Raman transitions are used to change the state of the atoms in this mode to a non-trapped state, to allow the output coupling of the atoms from the lasing mode.

In Sec. II we give an overview of our basic atom laser scheme. In Sec. III we investigate the input of bosons into the lasing mode. We discuss the rate of transfer of the atoms from the pump cavity to the ground state of the lasing cavity, considering both the atomic transition rate due to the spontaneous emission of photons and the overlap between the wavefunctions of the two cavities. In Sec. IV we discuss our output coupling method. This method could in principle be used for other atom laser schemes. In Sec. V we derive rate equations and compare our atom laser model with the standard optical laser and similar rate equations for the atom laser described by Spreeuw et al. \cite{5}. We also investigate the threshold condition. In Sec. VI we present an outline of a possible implementation of our basic scheme using hollow optical fibers. Finally, in Sec. VII we summarize our conclusions and discuss some of the limitations of our work.
II. ATOM LASER SCHEME

In this section we present an overview of our atom laser scheme. The details are discussed in the following sections. The model consists of atoms with four energy levels, as outlined in Fig. 1. Level $|1\rangle$ is the input pump level, level $|2\rangle$ is the lasing level and level $|4\rangle$ is the output level. Level $|3\rangle$ mediates the output coupling Raman transition. There are two atomic cavities for confining atoms in states $|1\rangle$ and $|2\rangle$. One of these cavities, the lasing cavity, traps atoms in the level $|2\rangle$. Only a single mode of this cavity becomes populated as we show in Sec. V. We wish to build up a large number of atoms in this ground state mode, in an analogous way to the standard optical cavity in a laser. The other cavity, the pump cavity, traps a large number of atoms in the internal metastable level $|1\rangle$. The two cavities are spatially overlapping, see Fig. 2.

Initially the atoms are prepared in level $|1\rangle$, cooled and injected into the pump cavity. Injection could be achieved by a number of methods. For example, a partially reflecting atomic cavity mirror could be employed. Such atomic mirrors can be produced using the repulsive potentials created by blue detuned laser beams [11]. The transmittivity of such a mirror may be very small for a practical cavity [4], however extremely large input fluxes of atoms are possible allowing useful numbers of atoms into the cavity. Another possible input mechanism would be to inject the atoms into the cavity in a non-trapped atomic state and then employ a change of atomic state (for example using a Raman transition) to the trapped state. This reverses our output coupling method.

Atoms change from the pump level $|1\rangle$ to the lasing level, $|2\rangle$ at a rate $r_{12}$ in the absence of Bose-enhancement. Atoms which make this change of level, however, do not necessarily become trapped in the ground state of the lasing cavity - the lasing mode. However the wavefunction overlap with this ground state is larger than the overlap with other higher energy states. Tunneling losses out of this state are also lower. Atoms that do transfer to the ground state of the lasing cavity are trapped, and so further transitions to this state will be enhanced by a factor of $(N_{21} + 1)$ where $N_{21}$ is the number of atoms in atomic level $|2\rangle$. 
and the ground state of the cavity. With suitable pumping of atoms into the system, a large number of atoms will build up in this single quantum mechanical state of the combined atomic and cavity system. For the parameters considered in this paper (Sec. VI) other higher energy modes of the cavity state are not significantly populated.

A Raman transition couples atoms out of the system. Two lasers transfer the atoms from level \( |2\rangle \) to a final atomic level \( |4\rangle \). The lasers are confined to the lasing cavity, and are shone diagonally across the cavity so that they are counter-propagating in one direction (the longitudinal direction) and co-propagating in the transverse direction, Fig. 2. Thus a momentum kick is imparted on atoms in the longitudinal direction, but no net momentum kick occurs in the transverse direction. The longitudinal momentum kick pushes the atoms out of the interaction region. If the rate at which atoms leave the system due to this kick is much larger than the Raman transfer rate then we have an effective irreversible transfer of atoms from the lasing mode out of the system.

In Sec. VI we present an implementation of the scheme using a hollow optical fiber to create the transverse confinement for the atoms, and focussed laser light to form the longitudinal atomic mirrors. For a fiber with a hole of diameter \( \approx 2\mu m \) it is appropriate to model the confining potential in the transverse direction as a harmonic oscillator potential. In the longitudinal direction we model the confining lasers for the pump cavity as a square well, and those of the lasing cavity as a harmonic oscillator. One possible problem with coupling the pump and lasing cavity by spontaneous emission is photon reabsorption. A simple argument suggests that reabsorption might reduce the number of atoms in the ground trap state. However, Cirac and Lewenstein \[12\] have shown that under certain conditions reabsorption may increase the number of atoms in the ground state. Moreover, reabsorption can be circumvented by ensuring that the lasing cavity is smaller than the mean free path of a photon, as suggested by Spreeuw et al. \[5\]. Hollow optical fibers with hole diameters comparable to the photon wavelength achieve this.
III. POPULATION OF THE LASING MODE

Atoms enter the system from a cooled thermal source. The initial cavity is produced using atomic mirrors for atoms in the metastable level $|1\rangle$. We consider an initial rate of atoms entering this cavity which we call the pump rate, $r_1$. The coupling between the pump and laser cavity occurs through spontaneous emission. The rate of transfer of atoms from the pump cavity to the lasing cavity depends both on this atomic transition rate, $r_{12}$, and on the average wavefunction overlap between the modes of the pump cavity and the lasing cavity. The average overlap between the pump cavity and the $j$th mode of the lasing cavity, $g_j$, is equal to the sum over all the pump cavity states of the probability of finding an atom in that state, multiplied by the overlap between it and the $j$th energy state of the lasing cavity,

$$g_j = \sum_{i=0}^{\infty} n_i g_{ij}/N_1,$$

(1)

where $N_1$ is the number of atoms in level $|1\rangle$ in any pump cavity state. $g_{ij}$ is the value of the wavefunction overlap between the $i$th energy state of the pump cavity and the $j$th energy state of the lasing cavity, as outlined in the appendix. This overlap includes the effect of the random momentum kick from the spontaneously emitted photon. $n_i$ is the number of atoms in the $i$th pump cavity state. The exact nature of the pump cavity distribution does not affect the qualitative behaviour of the atom laser scheme, so for calculational purposes we will assume that the population of states is a Bose-Einstein distribution,

$$n_i = \frac{z \exp[-E_i/k_b T]}{1 - z \exp[-E_i/k_b T]},$$

(2)

where $E_i$ is the energy of the $i$th cavity state, $k_b$ is the Boltzmann constant and $T$ the temperature. $z$ is the fugacity, which is related to the chemical potential, and can be found by solving the equation $\sum_{i=0}^{\infty} n_i = N_1$. The total rate $R_{12j}$ at which atoms transfer from the pump cavity to the $j$th mode of the lasing cavity is given by

$$R_{12j} = (N_{2j} + 1)N_1 r_{12} g_j,$$

(3)
where \((N_{2j} + 1)\) is the Bose-enhancement factor due to the presence of \(N_{2j}\) bosons in level \(|2\rangle\) and the \(j\)th state of the lasing cavity. \(g_j\) is given by Eq. (1).

### IV. OUTPUT COUPLING

Other schemes for atom lasers \([1–5]\) have not given a description of a practical output coupling mechanism. We present here a method of switching the atomic state of the atoms, using a Raman transition, to allow output coupling from the system. A pair of lasers at frequencies \(\omega_1\) and \(\omega_2\) induce a Raman transition from level \(|2\rangle\) to the output level \(|4\rangle\) of the atom, see Fig. 1. This output level is not trapped.

The lasers are oriented as discussed in Sec. [4] so as to impart a net momentum kick to the atoms of magnitude \(2\hbar k_x\) in a particular direction, \(x\). For the hollow fiber situation discussed in Sec. [7], the \(x\) direction is the longitudinal fiber direction, so that the atoms are pushed out of the fiber. The lasers are on resonance with the Raman \(|2\rangle \leftrightarrow |4\rangle\) transition. They are far detuned from the \(|2\rangle \rightarrow |3\rangle\) and \(|4\rangle \rightarrow |3\rangle\) resonances to reduce excitation to level \(|3\rangle\). The Raman transition from level \(|2\rangle\) to level \(|4\rangle\) has a transition rate that depends on the Rabi frequencies, \(\Omega_1\) and \(\Omega_2\), of the two lasers for their respective transitions \(|2\rangle \rightarrow |3\rangle\) and \(|4\rangle \rightarrow |3\rangle\). This rate also depends on the detuning of the two lasers from level \(|3\rangle\). This transition rate can be found from scattering theory \([13]\)

\[
r_{24} = \frac{\Omega_1^2 \Omega_2^2}{8\Delta^2 (1/t_0)},
\]

where \(\Delta = \omega_{23} - \omega_1\) is the detuning and \(t_0\) is the time scale on which atoms are irreversibly lost from the system due to the momentum kick. \(1/t_0\) is then the single atom loss rate from the system. The atoms that are lost from the system due to this momentum kick are the atom laser output. We assume that the total momentum kick is purely in the longitudinal direction, \(x\). We are also assuming that \(1/t_0 > \gamma_4\), where \(\gamma_4\) is the linewidth of level \(|4\rangle\), so that the output level \(|4\rangle\) has a long lifetime. Here, \(t_0\) is estimated by

\[
t_0 = \frac{l_x}{2\hbar k_x/m},
\]
where $m$ is the mass of the atom, $l_x$ is the length of the light-atom cavity interaction region in the $x$ direction and $2\hbar k_x$ is the size of the momentum kick. To transfer the population out of the system, we consider a regime where the loss rate out of the interaction region, $1/t_0$ is larger than the Raman transition rate from level $|4\rangle$ back to $|2\rangle$. In this regime effectively all atoms that transfer to level $|4\rangle$ will exit the system. This condition is given by

$$(N_{21} + 1) r_{24} < \frac{1}{t_0},$$

where $r_{24}$ is the Raman transition rate given in Eq. (4). The factor $(N_{21} + 1)$ is a Bose-enhancement factor for the backward process, due to the lasing mode having an occupation of $N_{21}$ bosons. Here we have ignored the transitions into other higher energy modes of the lasing cavity, as in practice we find that only the ground state ($j = 1$) mode has a significant non-zero population.

We require a coherent transfer of the population from level $|2\rangle$ to level $|4\rangle$ for our atom laser output. To ensure that the transfer is unitary we require negligible spontaneous emission from level $|3\rangle$. More specifically, we require that the rate at which atoms populate and then spontaneously emit from level $|3\rangle$ is much less than the rate at which atoms leave the system coherently, $N_{21} r_{24}$. This constraint is given by

$$\frac{\Omega_1^2 \Gamma_3}{\Omega_1^2 + \Delta^2} \frac{N_{21} \Gamma_3}{2} \ll N_{21} r_{24}.$$  

We have only considered in this expression the population of $|3\rangle$ due to atoms in level $|2\rangle$. The population due to level $|4\rangle$ atoms is negligible compared to this, as the number of atoms in state $|4\rangle$ is always small, (see Sec. [V]). In the physical implementation discussed in Sec. [V] we evaluate Eq. (4) for $r_{24}$ using values that satisfy constraints (6) and (7).

In the discussion above we assumed that the lasers impart a fixed momentum kick of magnitude $2\hbar k_x$ to the atoms. This assumption corresponds to modelling the lasers as plane waves. Atoms undergo a Raman transition, absorbing a photon from one of the beams and emitting into the other. In a plane wave model of the lasers, the final state of the atoms in the longitudinal direction will correspond to the initial wavefunction in the ground state.
of the lasing cavity combined with the momentum kick $2\hbar k_x$. In the transverse direction, the output state of the atoms remains the ground state mode of the hollow optical fibre. This is achieved using output coupling lasers with a sufficiently narrow linewidth compared with the separation of the lasing cavity transverse energy levels. By suitably tuning such Raman lasers to the output atomic level it is impossible for the atoms to excite into a higher transverse mode of the fiber, as the energy required to change internal atomic levels and excite the atoms to a higher transverse mode is higher than the available energy from the Raman photons. For a fiber approximately $2\mu$m in diameter this requires the Raman lasers to have a linewidth of only a few kHz, which can be achieved by active stabilization.

In the longitudinal direction, the output atoms in state $|4\rangle$ are no longer trapped. The atoms couple to a continuum of momentum eigenstates. Since the initial wavefunction is an energy eigenstate of the lasing cavity the atoms do not have a definite momentum, due to the position-momentum uncertainty principle. Hence, when the atoms leave the trap they have a momentum distribution with some width $\hbar \Delta k$. For our parameters this width is smaller than the longitudinal momentum imparted to the atoms, $2\hbar k_x$. Based on the time-energy uncertainty relation we expect the output linewidth to be narrower for slower output coupling. We can obtain an upper bound for the output width by considering fast output coupling. By fast we mean that the internal state of the atom is changed without time for the spatial wavefunction to evolve. Then the spread in momentum due to the presence of a momentum distribution in the cavity is identical to the momentum spread one would obtain from a pulsed atom beam created by removing the walls of the cavity as suggested by Guzman et al. [4].

For the parameters considered in Sec. VI, the longitudinal kick is of the order of $2k_x \approx 1.3 \times 10^7$ m$^{-1}$, in comparison to the momentum spread of the atoms in the lasing mode of length $2\mu$m of $\Delta k \approx 10^6$ m$^{-1}$. A further factor which contributes to the final momentum spread of the atomic beam is the shape of the Raman laser beams. We can estimate the size of this spread by considering a Gaussian laser beam focussed down to a waist of size $w_0 = 2\mu$m. This corresponds to focussing onto an interaction region of the size of the laser
cavity that we consider in Sec. VI. The gaussian transverse wavevector spread has standard deviation $\sigma = 5 \times 10^5 \text{m}^{-1}$. Thus the lasers impart a range of kicks which is somewhat smaller than the spread in the momentum in the cavity, $\Delta k$. We can further decrease this spread by increasing the beam waist. This, however, increases the size of the interaction region, eventually invalidating inequality (6). Nevertheless, even when it is invalid, some atoms will continue to couple out of the system.

V. THE ATOM LASER RATE EQUATIONS

The atom laser discussed here contains analogous elements to those found in the optical laser. One of the differences is in the pumping process. Here our pumping consists of the loading of the pump cavity. In an optical laser, pumping involves exciting atoms which then emit photons. This difference is a consequence of the inability to create atoms in a manner analogous to creation of photons. Instead, atoms must be transferred from different states to the lasing state. Another difference is that the output coupling of our atom laser also involves changing the state of the atoms.

One important characteristic of the optical laser that is observed in our atom laser model is the presence of a threshold condition. This threshold condition occurs in an optical laser when the net amplification between the mirrors for a single photon circulating the cavity equals the loss at the mirrors. Similarly for the atom laser threshold, we consider atoms injected into an otherwise empty system ($N_{2j} = 0$). The threshold condition occurs when the single atom input rate into the lasing cavity, $g_1r_{12}$, is just sufficient to dominate the loss rate, $r_{24}$. We thus expect a threshold when $r_{12} = r_{24}/g_1$. The threshold condition can be expressed in terms of the injection rate into the pump cavity, $r_1$, using the fact that for the empty system considered for the onset of threshold, $r_1 \approx r_{12}$ at steady state. This gives the threshold condition in terms of the rate of input into the pump cavity as $r_1 \approx r_{24}/g_1$.

We now present rate equations for this atom laser scheme. Similar equations have been investigated independently by Spreeuw et al. in the context of another atom laser scheme.
In the regime where all the atoms that transfer to level $|4\rangle$ are lost from the system, we find that our rate equations reduce to a form equivalent to those of Spreeuw et al., though the overlap factors $g_j$, and rates, $r_{12}$ and $r_1$ in our equations are different due to physical differences between the schemes. Spreeuw et al. do not consider a separate output atomic level for their scheme so they have no corresponding equation to our Eq. (8c) for $N_4$. These rate equations allow us to investigate the number of atoms in the modes of the lasing cavity as a function of the pumping rate and of time, and to verify the threshold condition. We are interested in the population of each of the combined atom and cavity states using realistic parameters for our laser model. Using the notation presented earlier, rate equations for each of the atom laser levels are given by

\[
\frac{dN_1}{dt} = r_1 - \sum_j g_j(N_{2j} + 1)r_{12}N_1 - (1 - \sum_j g_j)r_{12}N_1, \tag{8a}
\]

\[
\frac{dN_{2j}}{dt} = g_jr_{12}N_1(N_{2j} + 1) - r_{24}N_{2j} + G_j(N_{2j} + 1)N_4r_{24}, \tag{8b}
\]

\[
\frac{dN_4}{dt} = \sum_j N_{2j}r_{24} - \sum_j G_j(N_{2j} + 1)N_4r_{24} - N_4 \frac{1}{t_0}. \tag{8c}
\]

Eq. (8a) describes the pump level. The first term gives the input rate into the pump level. The second term corresponds to the transfer of atoms from the pump cavity to the lasing modes as described in Eq. (3). This term includes the bosonic enhancement of transitions into lasing states due to the presence of $N_{2j}$ bosons. The final term gives the loss from level $|1\rangle$ into states which are not in the laser cavity. Eq. (8b) describes the population of the various modes of the lasing cavity. The first term corresponds to the Bose enhanced input into the levels of the lasing cavity from the pump. The second term describes the loss from the lasing states into the output level, $|4\rangle$. The final term describes a coupling of atoms from level $|4\rangle$ back into the lasing state. Finally, Eq. (8c) describes the population of the output level, $|4\rangle$. $N_4$ is the number of atoms in level $|4\rangle$ which are still confined to the system. The
first term describes the gain in level $|4\rangle$ due to atoms transferring from the lasing states. The last two terms describe losses out of level $|4\rangle$ due to coupling back to the lasing cavity and out of the system respectively.

Level $|4\rangle$ actually consists of a manifold of states $|4,j\rangle$, where state $|4,j\rangle$ corresponds to an atom that has made a Raman transition from the $j$th mode of the lasing cavity to the electronic level $|4\rangle$. Since the transformation is unitary each of these $|4,j\rangle$ states only couples back to its corresponding $|2,j\rangle$ state, where $|2,j\rangle$ describes an atom in the $j$th mode of the lasing cavity. For simplicity of notation, we have not considered separate equations for the states $|4,j\rangle$. Instead, we define a coupling constant, $G_j$, for transitions between $|4\rangle$ and $|2,j\rangle$. $G_j = N_{4j}/N_4$ is the probability that an atom with an electronic level $|4\rangle$ is in the state $|4,j\rangle$. We approximate $G_j \approx N_{2j}/N_2$ in the numerical calculations discussed below, however it is found that the results are insensitive to the value of $G_j$ as this back-coupling term is small in the regime defined by inequality (6). This approximation for $G_j$ is based on the fact that the lasing atoms transfer to level $|4\rangle$ at a rate which is independent of the lasing mode from which they come. Since the lasing atoms are the only source for $|4\rangle$, the number of atoms in each of the $|4,j\rangle$ states depends only on the number of atoms in the corresponding $|2,j\rangle$ state of the lasing cavity.

From Eqs. (8a-8c), the steady state population of the lasing mode $N_{21}$ can be found. The solution is somewhat complicated, though a simplified solution can be found for the case where we assume that all atoms that enter level $|4\rangle$ are rapidly lost from the system, as required by inequality (6). In this regime the steady state is given by

$$N_{2j} = \frac{1}{2g_j} \left[ \left( R_j - (1 + \sum_{j' \neq j} g_{j'j} N_{2j'}) \right) + \left( R_j - (1 + \sum_{j' \neq j} g_{j'j} N_{2j'}) \right)^2 + 4R_j g_j \right]$$

(9)

where the $R_j$ are dimensionless pumping rate parameters, given by

$$R_j = \frac{r_1 g_j}{r_{24}}$$

(10)
The time dependent solutions of Eqs. (8a)-(8c) can be found numerically. Fig. 3 shows a logarithmic plot of the number of atoms in the lasing mode as a function of time, for an input pumping rate of \( r_1 = 1000 \, \text{s}^{-1} \). For the parameters of Sec. VI it takes a time on the order of 10 seconds for a large number (\( \gg 1 \)) of atoms to build up in the lasing mode. After this time the number of atoms reaches a steady state value and remains constant. The number of atoms populating the next highest energy state of the lasing cavity is also plotted in Fig. 3. In this plot we see that due to gain competition and the Bose-enhancement of transitions into the highly populated ground state mode the steady state population of the next highest mode is negligible. All higher modes are also found to have negligible population. In the regime where the populations of all but the ground state mode are negligible, the steady state equations given in Eq. (9) reduce to the form

\[
N_{21} = \frac{1}{2g_1} \left[ (R_1 - 1) + \sqrt{(R_1 - 1)^2 + 4R_1 g_1} \right].
\]  

(11)

This result is analogous to the standard laser population equation [14] and equivalent to results of Spreeuw et al. [5]. In the limit of strong pumping \( r_1 \to \infty \) the number of atoms in the lasing mode, \( N_{21} \), increases linearly with the pumping rate \( R_1 \), with a slope of \( 1/g_1 \). We assume numerical values for the transition rates, \( r_{12} = 0.1 \, \text{s}^{-1} \), \( r_{24} = 0.125 \, \text{s}^{-1} \) and for the wavefunction overlap, \( g_1 = 0.00571 \). These parameters are justified in Sec. VI and correspond to a particular implementation of our scheme using hollow optical fibers.

A logarithmic plot of the number of atoms at steady state in the lasing cavity, \( N_{21} \), as a function of the dimensionless pumping rate \( R_1 \) is given in Fig. 4. The threshold pumping rate is \( R_1 = 1 \), which corresponds to an input pumping rate, \( r_1 \approx 21.9 \, \text{s}^{-1} \).

VI. IMPLEMENTATION

We propose here a possible implementation of this atom-laser model using hollow optical fibers. A schematic diagram of this is shown in Fig. 2. Single mode hollow optical fibers, with holes of about 1.5\( \mu \text{m} \) have been proposed for guiding atoms [15,16] and multimode fibers
have already been demonstrated experimentally to guide atoms [17–19]. The hollow fibre acts as a waveguide for atoms. However in contrast with the optical case, the longitudinal atomic motion along the fiber decouples from the transverse motion, so there is a continuum of longitudinal plane wave modes which atoms can couple into. A detailed development of the theory of hollow optical fiber waveguides is given by Marksteiner et al. [15]. Laser guiding of atoms using blue-detuned light has been observed by Ito et al. [19]. An enhancement of 20 times the ballistic flux was observed, indicating the guiding of atoms down a fiber. The main mechanism which destroys the coherence in hollow-fiber atom optics experiments is spontaneous emission by atoms in the confining light field. This is particularly a problem in those experiments that use red-detuned light as the atoms are continuously in the light field. However, for the blue detuned case the atoms only interact with the light field when they approach the inner wall of the hollow fiber. The effect of spontaneous emission while the atoms are in this light field can be reduced by increasing the detuning of the light field from the atomic resonance. We assume confining light with blue detuning of $\Delta_{\text{max}} = 2\pi \times 50$THz. We assume a typical linewidth of $\gamma = 2\pi \times 6$MHz for an (unspecified) upper level $|e\rangle$. The spontaneous emission rate is given by the usual formula, $\Gamma_{\text{se}} = \gamma \Omega^2 / 4\Delta^2$, where the Rabi frequency is determined by the required potential height. As discussed by Hope and Savage [20] we assume that the minimum excited state population is limited by the maximum possible detuning, rather than by the available laser power. Then the minimum possible spontaneous emission rate is given by Eq. (14) of Hope and Savage [20]

$$\Gamma_{\text{se,min}} = \frac{\gamma (k_b T)}{\hbar \Delta_{\text{max}}}. \quad (12)$$

The required potential height has been expressed in terms of Boltzmann’s constant $k_b$ and the atom temperature $T$. With a temperature of $T = 200 \text{nK}$ these parameters give a spontaneous emission rate of approximately $0.003 \text{s}^{-1}$. Thus a typical atom must spend a time of order 300s inside the confining light fields before a spontaneous emission event is likely. Note that this time is an upper bound since we have assumed that the atom always experiences the maximum field. Nevertheless it is possible to further reduce this spontaneous
emission rate by a factor of almost 100, to $6.0 \times 10^{-5}\text{s}^{-1}$, by using the Raman scheme of Hope and Savage \[20\] to create the potential.

The pump cavity is formed using light induced potentials from blue-detuned lasers shone transversely across the optical fiber. Because the pump cavity is long and narrow, the transverse mode energy level spacings are much larger than the longitudinal mode spacings.

The lasing cavity is also produced in the fiber by using two blue detuned lasers. These are much closer together than in the pump cavity, producing a much larger energy level spacing. Due to the large overlap of the lowest pump cavity modes with the ground state of the lasing cavity only it becomes significantly populated. To maximize this overlap the input atoms must be pre-cooled to a few hundred nanokelvin so as to populate the lower energy states of the pump cavity. Such temperatures can be achieved by evaporative cooling.

We calculate the average overlap for a pump cavity modeled as discussed in Sec. II. The pump cavity is modeled as a square well with sides of length $100\mu\text{m}$ in the longitudinal direction. The lasing cavity is modeled as a three dimensional harmonic oscillator. The oscillator frequencies $\omega_i$ are specified in terms of the ground state width, $d_i$

$$d_i = 2\sqrt{\frac{\hbar}{2m\omega_i}}. \quad (13)$$

In the longitudinal direction the width of the lasing cavity is $d_i = 2\mu\text{m}$. Transverse confinement for both cavities is modeled by oscillators in the transverse directions with widths of $d_i = 1.5\mu\text{m}$. The lasing cavity is displaced a distance $\bar{x}$ from the center of the pump cavity. For definiteness, we consider a spontaneous emission kick from the $|1\rangle \rightarrow |2\rangle$ transition of magnitude $k_0 = 10^6 \text{ m}^{-1}$ which corresponds to an infrared transition. Modeling a lasing cavity placed at the edge of the pump cavity, $\bar{x} = 48\mu\text{m}$, we calculate the average overlap for a spontaneous emission kick of magnitude $10^6 \text{ m}^{-1}$ to be $g_1 = 0.00571$. The overlaps, $g_j$ with the higher exited states ($j > 1$) are smaller, with the next greatest overlap, $g_2 = 0.00362$ occurring with the state $n_x = 1, n_y = n_z = 0$. In the analysis leading to Fig. 3 we use these values, and consider the six lowest energy states of the lasing cavity. All of these apart from the first are found to have negligible population in steady state because of their weaker
overlap with the pump cavity modes, and due to the Bose-enhancement of transitions into already populated states. In the rate equations we also use an atomic transition rate, $r_{12}$, given by the inverse lifetime of the initial atomic level, $|1\rangle$. For definiteness we assume that $r_{12} = 0.1 \, \text{s}^{-1}$, corresponding to $|1\rangle$ being metastable.

The output coupling is achieved by shining two lasers diagonally across the lasing cavity. This has the dual purpose of localizing the interaction region, as well as providing a total momentum kick, $2\hbar k_x$, directed along the longitudinal axis of the optical fiber (the transverse components cancel). The atoms move out of the interaction region due to the momentum kick, thus forming the atom laser beam. For definiteness we arrange a value of $2k_x = 1.31 \times 10^7 \, \text{m}^{-1}$. This value could be achieved using lasers with wavelengths $\lambda = 480 \, \text{nm}$ oriented at $60^0$ to the long axis of the fiber. It is possible to produce light of this wavelength with a frequency doubled titanium-sapphire laser. Assuming a typical atomic mass $m = 10^{-26}$ kg and size of the interaction region, $l_x = 2 \times 10^{-6} \, \text{m}$, we find that the timescale on which atoms leave the system due to the momentum kick is given by Eq. (5) to be $t_0 = 1.5 \times 10^{-5} \, \text{s}$. Using this value for $t_0$, we find that constraint (6) can be fulfilled for lasing mode populations of $N_{21}$ up to $N_{21} \approx 10^5$ with large detunings, and relatively small Rabi frequencies. If state $|3\rangle$ has a linewidth of $\Gamma_3 = 2\pi \times 1.6 \, \text{MHz}$ then both constraints (3) and (4) can be satisfied with the output lasers detuned by an amount, $\Delta = 2\pi \times 1.6 \, \text{GHz}$ and Rabi frequencies, $\Omega_1 = 2\pi \times 50 \, \text{kHz}$ and $\Omega_2 = 2\pi \times 1.6 \, \text{MHz}$. With these values the single atom rate constant, $r_{24}$ is given by Eq. (4) to be $r_{24} = 0.125 \, \text{s}^{-1}$. The total rate at which atoms leave the system as the atom-laser beam is then given by $N_{21}r_{24}$.

We do not propose a particular atom in which our level scheme might be implemented. One possibility is to use a metastable, rather than a hyperfine, level for the output state $|4\rangle$. Atoms with two accessible metastable levels, for the pump state $|1\rangle$ and the output state $|4\rangle$, include manganese.
VII. CONCLUSION

In this work we have analyzed an atom laser model and calculated values for the lasing population using an implementation of the scheme based on hollow optical fibers. We have proceeded by analogy with the photon laser and have described a device that will produce a large number of atoms in a single atomic state and a method of coherently coupling a beam of atoms out of this device. We found that the output energy spread is bounded by the momentum spread of the atoms in the lasing cavity. Since the spread in momentum introduced by the Raman lasers is smaller than the lasing cavity spread, the Raman output coupling gives an output spectral density similar to that produced by dropping the walls of the cavity. This is because an atom making the Raman transition changes to a non-trapped state where the atom is no longer affected by the confining potential. An important advantage of the Raman scheme over switching the cavity walls is that atoms can be let out continuously. Moreover, the Raman transition gives the atoms a longitudinal momentum kick.

Other atom laser schemes \cite{2,3} have found large output linewidths due to collisions. This may increase the linewidth of our atom laser, though as our linewidth is already broad we do not expect this to be a limiting factor and have not considered this in our model. A criticism that has recently been made of a number of atom laser schemes \cite{1} is the lack of consideration given to inelastic collisions. Such collisions are used explicitly in a number of models \cite{1,3,4} to provide the transitions to the lasing mode. We assume large blue detuning to minimize the excited state population due to the confining light. This minimizes the interatomic interactions in the lasing cavity. The Raman transition scheme of Hope and Savage \cite{20} for generating mechanical potentials provides a method for further reducing spontaneous emission.

We have presented an atom laser model, discussing input and output coupling mechanisms as well as a possible implementation of our scheme in hollow optical fibers. We found that for reasonable parameters we get a large build up of atoms in the lasing mode, above
a threshold pumping rate, in a manner analogous to the threshold found in optical lasers.

ACKNOWLEDGEMENT

The authors would like to thank Dr. J. Eschner for his many helpful discussions.

APPENDIX

The overlap, $g_{ij}$ between an atom in the $i$th state of the pump cavity and the $j$th state of the lasing cavity is given by

$$g_{ij} = \int_0^\pi d\phi \frac{d\theta}{4\pi} \sin(\phi) \times$$

$$\left| \int d\mathbf{r} \phi_i^*(\mathbf{r})\psi_j(\mathbf{r}) \exp \left[i\mathbf{k} \cdot \mathbf{r}\right] \right|^2$$

(14)

where the domain of the $k$ space integrals is a spherical shell of radius $k_0$. This accounts for the spontaneous emission of a photon with wavevector $\mathbf{k}$ of magnitude $k_0$. For simplicity we assume that spontaneous emission is isotropic. Other emission distributions are found to give similar overlap results for a particular $k_0$. $\phi_i$ and $\psi_j$ are the wavefunctions of atoms in the $i$th and $j$th energy states of the pump and lasing cavity respectively. The lasing states are modelled in this paper as the eigenstates of three dimensional harmonic oscillators. The transverse mode structure of the pump cavity is likewise modelled as harmonic oscillators, while the longitudinal states are eigenstates of a square well.
REFERENCES

* Email address: Glenn.Moy@anu.edu.au

[1] M. Holland, K. Burnett, C. Gardiner, J.I. Cirac and P.Zoller, Phys. Rev. A 54, R1757 (1996).

[2] H.M. Wiseman and M.J.Collett, Physics Lett. A 202,246 (1995).

[3] H.M. Wiseman, A. Martin and D.F. Walls, Quantum Semiclass. Opt. 8, 737 (1996).

[4] A.M. Guzman, M. Moore and P.Meystre, Phys. Rev. A 53, 977 (1996).

[5] R.J.C. Spreeuw, T. Pfau, U. Janicke and M. Wilkens, Europhysics Letters 32, 469 (1995).

[6] M. Olshanii, Y. Castin and J. Dalibard, Proc. of the 12th Int. Conference on Laser Spectroscopy, edited by M. Inguscio, M. Allegrini and A. Sasso. (1995).

[7] M.H. Anderson et al., Science 269, 198 (1995).

[8] C.C Bradley et al., Phys. Rev. Lett. 75, 1687 (1995).

[9] K.B. Davis et al., Phys. Rev. Lett. 75, 3969 (1995).

[10] M.O. Mewes et al., Phys. Rev. Lett. 77, 416 (1996).

[11] N. Davidson et al., Phys. Rev. Lett. 74, 1311 (1995).

[12] J.I. Cirac and M. Lewenstein, Phys. Rev. A 53, 2466 (1996).

[13] R. Loudon, The Quantum Theory of Light (Clarendon Press, Oxford 1983).

[14] A. E. Siegman, Lasers (University Science Books, California 1986).

[15] S. Marksteiner et al., Phys. Rev. A 50, 2680 (1994).

[16] C. Savage et al., Fundamentals of Quantum Optics III, edited by F. Ehlotzky (Springer-Verlag, Berlin,1993).
[17] H. Ito et al., Optics Comm. 115, 57 (1995).

[18] M. Renn et al., Phys. Rev. A 53, R648 (1996).

[19] H. Ito et al., Phys. Rev. Lett. 76, 4500 (1996).

[20] J.J. Hope and C.M. Savage, Phys. Rev. A 53, 1697 (1996).
FIGURES

FIG. 1. Schematic diagram of atomic states and output coupling lasers.

FIG. 2. Schematic diagram of a possible implementation of our atom laser model using a hollow optical fiber. The lasing cavity is confined to a region of size 2$\mu$m. The pump cavity is approximately 100$\mu$m long. The input coupling is by a partially transmitting atomic mirror for atoms in state $|1\rangle$, indicated by the laser on the left of the figure. Lasers are also used for the other end mirror of the pump cavity, and for the lasing cavity. The two lasers $\omega_1$ and $\omega_2$ are the output coupling lasers. They are localized to the lasing cavity, and provide a momentum kick along the longitudinal axis of the fiber as shown.

FIG. 3. Plot of number of atoms in lasing mode $N_{21}$ (solid line) and in the next highest energy mode $N_{22}$ (dashed line) of the lasing cavity as a function of time, $t$ for an input pumping rate, $r_1 = 1000$ s$^{-1}$, average overlap, $g_1 = 0.00571$ and $g_2 = 0.00362$ and single atom output rate, $r_{24} = 0.125$ s$^{-1}$. The dimensionless pumping rate, $R_1 = 45.7$.

FIG. 4. Plot of the steady state number of atoms in the lasing mode, $N_{21}$ as a function of the dimensionless pumping rate, $R_1$. Average overlap, $g_1 = 0.00571$. Threshold occurs at $R_1 = 1$. 

21
Pump level \[ |1\rangle \]
Lasing level \[ |2\rangle \]
Output level \[ |4\rangle \]

\[ \omega_1 \]
\[ \omega_2 \]

\[ \Delta \]

MOY - AR5628 Fig1
Lasing Cavity ≈ 2 µm

Pump Cavity ≈ 100 µm

Source

ω1

ω2

Output beam.

Lasing Cavity ≈ 2 µm

MOY - AR5628 Fig2
