Abstract
Australian schools, like schools elsewhere, have been through a period of closure. The closure creates both threats and opportunities for teachers and students. In the context of a project exploring approaches to teaching in early years, we outline some considerations and offer advice to teachers and educators on strategies for welcoming students back to school.

Keywords Mathematics · Remote learning · Threats

Introduction
There is ongoing debate about optimal ways to teach mathematics. On one hand, there are those who argue that mathematics teaching should mainly include clear explanation of procedures followed by practice and correction (Kirschner et al. 2006; Przychodzin et al. 2004). This approach is relatively easy to adapt to online environments in that there are many short videos that offer demonstrations and explanations and practice exercises or games that can be easily sourced online or created by teachers. There are a number of threats if this is the only experience students have. Learning this way may be less interesting to students working by themselves. It is hard for the teacher to provide explanations and targeted practice that meets students’ needs. But most of all, students are not thinking about mathematics, they are not doing mathematics, when working
online, and they are likely to form the impression that mathematics is something that is done to them rather than knowledge and connections that they create.

On the other side of this debate are those who argue that mathematics learning is best when students solve mathematics tasks for themselves, when they have opportunities to explain and justify their reasoning, and when they connect different aspects of mathematics (Kapur 2014; Schwartz and Martin 2004; Sullivan et al. 2020). This approach is more difficult to arrange in remote and online environments.

Of course a balanced mathematical program would have elements of both. But it is possible that students might return to school after the early 2020 COVID-19 shutdowns having had substantial experience with the former and not much of the latter.

This article describes our experience of a project focusing on elements of the latter approach that commenced with teachers and their Foundation to Year 2 students in schools not long before the lockdown. It summarises aspects of our experience during this time, lists some of the considerations for learning due to the lockdown and offers some principles for teachers to consider when students return to school.

**Our project**

We are working with two school systems on the latter approach to teaching mathematics, described as student centred structured guided inquiry, which can be argued to be an optimal way to engage students in learning (Alfieri et al. 2011). We encourage teachers to pose challenging learning experiences that activate cognition (Organisation for Economic Co-operation Development 2014), and we offer strategies for differentiating tasks with which students engage (Sullivan et al. 2009).

The pedagogies that align with this approach include the following: tasks are posed without instructing students on solution methods; students are allowed time to engage with tasks initially by themselves, perhaps later in small groups; actions are taken by teachers to differentiate tasks for students who might require additional support and those who finish quickly; and responses to the tasks are observed and selected by teachers during lessons to orchestrate classroom dialogue between students, emphasising their explorations and mathematical thinking (see Sullivan et al. 2016). Critical to our current thinking are ways of consolidating learning activated by the initial learning experiences (Dooley 2012; Dreyfus and Tsamir 2004).

We structure our pedagogical advice around five practices recommended by Smith and Stein (2011) including:

- Anticipating students’ responses
- Monitoring student work (which connects to formative assessment)
- Selecting examples for students to present
- Sequencing presentation of those examples
- Connecting students’ responses to the mathematical purpose of the experience

To these five practices, we add encouraging students to listen to others and using efficient technologies to allow students to project and present work samples to the class to facilitate whole class discussion on learning (e.g. use of document cameras).
In particular, we have developed and distributed to research partners some hypothetical learning sequences, described by Simon (1995) as trajectories. The intention of the sequences is to allow teachers to focus their planning on pedagogies rather than developing resources. Nevertheless, we intend that the experience of working with trialled sequences provides teachers with models from which they can create their own resources subsequently.

**Locking down Australian schools**

Australia is a federation of states which, along with the national government, have overlapping and sometimes conflicting responsibilities. In late March 2020, schools were closed to all but children of essential workers who could not work from home. The expectation was that teachers would prepare learning resources, many of which incorporated aspects of communication technologies, to be delivered remotely for their students. Generally restrictions imposed have had the effect of reducing the spread of the virus, and deaths per millions of population to date are among the lowest in the world (see https://www.worldometers.info/coronavirus/ for up-to-date figures).

One of the school education systems in which we are working anticipates returning to full operation in mid-May, while the other system is working towards return in mid-July.

We note that not all students will return to school when invited to do so. This might be due to some of them having compromised immune systems or to an increased appetite for home schooling either by parents or students or both or to residual fears of a second wave of contagion. Some online learning will continue so ways to overcome difficulties identified will presumably be helpful.

**Project response**

Initially we anticipated that support mechanisms for professional learning, either by system or project personal, could be replicated in some way to support students’ remote learning experiences in mathematics. This consisted of a range of approaches including annotated slide shows, session running sheets, talking head videos, or special purpose video conferences.

We are interested in the nature of remote learning experiences and media that incorporate our project principles and also the challenges students and teachers will face after students return to the classroom.

We have identified some experiences that are ideal for home learning. One example was when early years students were asked to work out which person in a family would be tallest if measured by lengths of their shoe. In this example, students explored a range of measurement ideas including the notion of iteration, the concept of a zero, the importance of the unit, especially if the unit varies, and keeping track of counting. It is noteworthy that in at least some instances, the experiences sparked additional student-initiated inquiries around the same idea. One student decided to compare all her dolls with shoes to her family members, discovering that most dolls had far smaller feet in comparison with their heights than people. This response sparked a conversation about
the unrealistic body dimensions of dolls, emphasising how richly contextualised mathematical inquiries can connect to important social issues, such as conversations about body image and identity. Rather than using mathematics to illustrate a point about gender stereotypes (e.g. Norton et al. 1996), the issue of body image “fell out” of an authentic mathematical inquiry, something arguably all the more powerful because it arose in the child’s home environment outside of the mathematics classroom.

We have also found a session structure to be productive in which an easily posed task requiring minimal explanation but a considered response is followed by video conference discussion, after which further similar tasks are posed. When posing online learning experiences, we have found that three questions are enough for primary students and that this structure maximises chances of success on the second and third tasks. For example, one task asked students to draw on squared paper some rectangles using 12 square counters, after which there was a discussion, followed by which students were asked to draw some rectangles or squares using 16 square counters. The learning from the first task informed not only the engagement with the second task but also success. We have established that at least some students are willing to persist for longer periods at home than might be expected in classrooms.

Nevertheless, we have found that the creation of online learning experiences of the type we value is difficult to design for remote learning. We have substantial experience in creating mathematics lessons for classrooms, but adaptation of those skills to remote learning took many iterations and discussions. A further challenge for us was to become aware of the many wonderful tools for online learning already available.

**Considerations on return to school and how to address them**

At the time of writing, we anticipate that there will be new pressures on teachers once students return to classrooms, many of which are idiosyncratic to this lockdown-then-relaxation situation. The following describes some of the considerations followed by some advice on principles for teaching.

No doubt, student progress will have fallen behind the previously developed teaching program, whether it was designed topic-by-topic or something else. Even where topics have been addressed remotely, teachers might assume that at least some students have learned little over the lockdown period. A possible response is for teachers to rush quickly over missing content, feeling it is better to touch on everything lightly rather than doing fewer aspects in depth. There may be a tendency to emphasise procedural rules and dependent approaches that can be expected to foster short-term success without the understanding that is likely to be robust and adaptable. An associated consideration is that teachers, who otherwise see importance of meaningful experiences, might resort to teacher-centred approaches, minimising opportunities for problem-solving and reasoning, arguing that those proficiencies can be addressed in subsequent years.

A further consideration, especially if most examples and tasks have been short during remote learning, or repetitive practice exercises, is that students might come to see such experiences as mathematics and maybe have even enjoyed it. The remote experience might create a reluctance to engage with more difficult ideas that take time, both for the student and their teachers when preparing lessons. The effect could be that
students with a positive growth mindset have moved to having a fixed mindset (Dweck 2000).

As if differences in style and rates of learning between class members were not already sufficient to create barriers to ongoing learning, the shutdown is likely to have exacerbated those differences. An obvious factor is access to required technology. It can be assumed that students who have been able to access both planned and incidental online opportunities would benefit more than those who have not had such opportunities. Students who already were, or became, self-directed, persistent, resilient learners may have even been advantaged by opportunities to work with limited supervision. Those who were not so motivated, especially if their parents did not encourage engagement, are likely to have gained less from time away from the classroom.

All classrooms have students who are more anxious than others. While the time closeted with the family might have moderated the anxiety, it is likely to re-emerge once students return to school, especially if they feel they have missed out on learning opportunities that other students have had.

A further consideration is that parents may have become more interested in their children’s education and seek to play a more active role in students’ learning. This will be especially true if parents fostered learning experiences that are extra curricula. It is also possible that parents had more relaxed and enjoyable conversations with their children. Maybe even some of the parental anxieties were reduced when they observe the learning their children were doing at home.

Some principles to inform return to school

Of course, the key is anticipating challenges when students return to school and finding ways to address them. Some principles that might inform strategies for welcoming students back to mathematics classrooms are as follows.

Build the relationship again

This probably applies more to foundation level and students in their first year of secondary school than to others. It is possible that students missed their friends more than they did teachers, and there will be a need to induct students again into the routines and expectations of classrooms, more so if routines are different from the remote learning experience. This common sense stance is supported by research that consistently links the quality of peer relationships and sense of belongingness to school to academic achievement (Osterman 2000; Wentzel 2005).

Avoid the temptation to talk fast, loud and long

Generally teachers are conscious that students should be talking at least as much as them if not more and that slowing the pace of teacher explanations is important. For example, there is research to suggest that when given opportunities to reflect on their own practice, teachers view “talking too fast” as undermining student agency and opportunities to learn (Yow 2012). When face-to-face schooling resumes, there is a very real risk that teachers might seek to cover content quickly and do this by offering
longer and faster explanations. This is unlikely to be productive and might even have negative effects on long-term learning.

**Spend as much time on a given topic as normally**

Even under normal circumstances, we know at least some teachers are reluctant to employ inquiry-based approaches in part because they worry about covering a crowded curriculum (Russo and Hopkins 2019). We might expect perceived pressures to “cover all the content” to be exacerbated when schools return. However, speeding up will not help. Most mathematics curriculums revisit important ideas over a number of years. For example, teachers cover aspects of fractions every year from year 3 to year 9 (at least). Approaches that emphasise understanding are likely to be better for learning than ruled-based approaches (Clarke and Roche 2009). It is preferable to engage students in thinking deeply about fewer topics over the rest of this year rather than superficially about many topics.

**Give students time to learn, to engage with challenges and to think**

Have a deliberate strategy to assist students to engage with substantial thinking by minimising routine practice experiences. It can be anticipated that some students have become used to easy successes; tasks that require extended thinking might seem confrontational. This has been noted in some of our previous research when students are confronted with challenging, non-routine tasks, possibly for the first time in their mathematical learning. For example, Russo et al. (2019) documented how a small number of mathematically capable students, who were used to having success, chose to “act out (e.g. throw objects around the room, have tantrums) when they perceived a task or concept as too difficult” (p. 19). Teachers perhaps could lead discussions specifically intended to create awareness among students including how school and home learning are different.

**Use previously successful experiences again**

Especially if teachers offered experiences early in the year that gave primacy to student thinking, there would be value in repeating those same experiences. For example, if the experiences were based on open-ended tasks, perhaps with low floors and high ceilings, students can engage this second time in different and new ways. Indeed, in a previous iteration of our project, teachers have observed that there is value in revisiting rich tasks throughout the school year: “some of these tasks can be repeated over and over again… [children] need multiple exposures of the same concepts” (Russo et al. 2019, p. 14). In any case, using familiar inquiries can be a useful strategy for reducing anxiety and reconnecting students with a learning culture.

**Match the style of assessment with the assessment goals**

If most of the student experiences during the closure were intended to practice previously learned skills, then a conventional test of fluency and understanding will work. But if teachers value the importance of engaging students in thinking more
deeply about fewer and inquiry experiences, open-ended challenges are more likely to reveal useful formative assessment information (Downton and Wright 2016). In any case, formative assessment focusing on problem-solving and reasoning is important for planning future learning (Davidson 2019).

Prepare to engage more with parents

It seems that many parents have taken their roles as home tutors seriously. Where parents spent time engaging with the ideas, listening to their children, and encouraging effort, the time will have been productive. We know from the literature that parent engagement is associated with high levels of aspiration for children's learning, as well as learning outcomes (Phillipson et al. 2017). Even if parents have become focussed on fluency with number facts, this is not likely to have done any harm. Finding ways to broaden the nature of home learning experiences will be an important challenge for some time.

Conclusion

As argued above, it is possible that learning from home might be productive especially if teachers developed mathematical learning experiences that challenged student thinking. But it is also possible that time away from school might have exacerbated differences in learning opportunities. Finding ways to reengage students with school while also addressing those differences productively will be an ongoing challenge for the rest of 2020 and beyond.

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