Modeling and Analysis of Millimeter Wave 5G Cellular Networks Based on 3-D Spatial Model

Xu Teng, Pan Ziyu*, Zhang Haolin, Zou Qiqi and Bao Can
Department of Information and Communication Engineering, Nanjing Institute of Technology, Nanjing, 211167, China
*panziyu@njit.edu.cn

Abstract. Wireless cellular networks are usually modeled and analysed in two-dimensional (2-D) space. The 2-D model is suitable for the analysis of cellular networks in suburb area but not for the dense millimeter cellular networks in the urban environments. In this work, a three-dimensional (3-D) model based on stochastic geometry is proposed, in which the distribution of base stations (BSs) are modelled as a 3-D Poisson point process (PPP), the blockage is modelled as line of sight (LOS) ball, the shadowing of wireless channel is modelled as Nakagami-m fading, and both the transmitters and receivers obtain maximum gain of beamforming by a large array of antennas. Based on the model, the distribution of the distance between the target user and the nearest the BS is given, and then the average coverage probability and transmission rate of the networks are derived. We analyse the impact of parameters such as path loss, cell radius on average coverage and the relationship between BS density and average rate through Monte Carlo simulation. The simulation results show that in the dense urban environment, the performance of 3-D PPP model of the millimeter wave cellular network analysis is more precise.

1. Introduction
With the explosive growth of mobile communication data, the current spectrum resources have been unable to meet the needs of future wireless communication. Therefore, the 5G cellular communication system uses millimeter wave to obtain more spectrum resources[1]. Millimeter wave frequency can be used to add the current saturated 700 MHz to 2.6 GHz radio spectrum band to wireless communication. Using millimeter wave spectrum can make service providers significantly expand the channel bandwidth, far exceeding the 20 MHz channel bandwidth currently used by 4G customers, greatly increasing data capacity and greatly reducing the waiting time of digital traffic[2].

The location distribution of each base station (BS) in millimeter wave cellular network is random relative to users. Therefore, the performance of millimeter-wave cellular networks is generally analysed by random geometry analysis method [3]. In this method, the spatial distribution of BSs is modelled as a point process such as Poisson point process (PPP), Poisson cluster processes (PCP), matérn hard core point process (MHCP) and etc..

In [4], PPP is used to model the distribution of BSs in millimeter-wave cellular networks in two-dimensional (2-D) space, and analyses the average coverage probability and rate performance index. The model based on two-dimensional space is easy to analyse, but the 2-D space model is not accurate in urban areas with dense base stations, especially in communication hotspots.
In [5], the 3-D PPP model is first used to analysis the coverage in small BS cellular networks. In [6], the authors analyse the 3-D spatial model for millimeter-wave cellular network, which assumes that the fading channel is Rayleigh fading. However, the Rayleigh fading channel is suitable for Sub-6GHz traditional microwave communication, but not for millimeter-wave communication [7]. In addition, it is studied in [8] that in millimeter-wave communication, 5G base stations are equipped with large-scale antenna arrays for transmitting and receiving, which can provide a lot of beamforming gains. Therefore, the performance analysis of millimeter-wave cellular networks must consider the influence of antenna gains on system performance.

In this paper, 3-D PPP is studied and proposed to simulate the actual distribution of BSs more accurately. We assume that each link in millimeter wave communication experiences an independent Nakagami-m fading channel. Simulation results show that our model is more suitable for millimeter wave cellular networks in dense city scene.

2. System Model
The system model of millimeter wave cellular networks mainly includes the spatial distribution model of BSs, the blockage model, the antenna model and the channel model.

2.1. Spatial distribution model
In the 3-D millimeter wave cellular networks, the spatial distributions of BSs and users are assumed to be two independent 3-D independent PPP distributions, as shown in figure 1. Moreover, the signal interference noise ratio (SINR) of the target user and other users have the same distribution independent of the BS distribution. Therefore, the performance of other users can be represented by studying the performance of the target user [3]. In figure 1, we assume that the target user is located at the center of the 3-D space. According to slivnyak theory [9], this assumption does not affect the distribution of the entire PPP. In this paper, we consider the most serious interference situation, that is, all BSs use full frequency multiplexing mode and transmit signals at the same time, will bring interference to the target users. The spatial distribution of Los base station is modeled as a 3-D PPP $\Phi$, with its density $\lambda_c = \frac{\rho}{4/3 \pi R^3}$, which is distributed in a sphere with radius $R$, where $\rho$ is the number of BSs.

![Figure 1. Three-dimensional Poisson point process diagram](image)

2.2. Blockage model
In this paper, we consider the line of sight (LOS) spherical obstruction model, as shown in figure 2. The study in [4] has proved that this method is accurate for dense millimeter wave cellular networks. In this model, the LOS radius is defined as $R$, in which the probability of the link in the LOS link is 1; while 0 outside the radius. Obstruction not only increases the randomness of average path loss, but also changes the path loss parameters of LOS link and non line of sight (NLOS) link. In the case of
dense millimeter wave base stations, the interference of NLOS link can be ignored [4]. In this paper, only the interference of LOS link is considered.

### Figure 2. LOS ball blockage model

#### 2.3. Antenna model

In millimeter wave communication, both transmitter and receiver are equipped with beam forming. In this paper, we use a simple analogy beamforming antenna structure, assuming that the target user and its serving base station have perfect channel state information. The beamforming gain from base station $X_k$ to target user is defined as $G_k$. The probabilities of BS and user in main lobe gain are $C_{BS} = \frac{\theta_{BS}}{2\pi}$ and $C_{MS} = \frac{\theta_{MS}}{2\pi}$, respectively. The main lobe gain of the user is $M_s$, the side lobe gain is $m_s$ and the main lobe beam width is $\theta_s$, $s \in \{MS, BS\}$. We assume that the interference link has in the beam gain $G_k = a_k$, $a_k = M_s m_s$ with probability $b_k \neq 0$, $k \in \{1,2,3,4\}$. The signal link of the target user has perfect beam gain $G_0 = a_1$, where the probability mass function of $a_k$ and $b_k$ is shown in Table 1.

#### Table 1. Probability mass function of $a_k$ and $b_k$.

| $k$ | $a_k$ | $b_k$ |
|-----|-------|-------|
| 1   | $M_{MS} M_{BS}$ | $c_{MS} c_{BS}$ |
| 2   | $M_{MS} m_{BS}$ | $c_{MS} (1 - c_{BS})$ |
| 3   | $m_{MS} M_{BS}$ | $(1 - c_{MS}) c_{BS}$ |
| 4   | $m_{MS} m_{BS}$ | $(1 - c_{MS})(1 - c_{BS})$ |

#### 2.4. Channel model

In this paper, the channel model is consisted of two parts: pass loss and fading. The path loss of LOS link is $l(r) = \beta r^{-\alpha}$, $\alpha, \beta$ are parameters. The fading of each link in millimeter wave communication is independent Nakagami-m fading, which is more general and easy to handle. Its probability distribution function is

$$p(x) = \begin{cases} \frac{2^m}{\Gamma(m)} \left( \frac{m}{\Omega} \right)^m x^{2m-1} e^{\frac{-ms^2}{\Omega}}, & x > 0 \\ 0, & \text{other} \end{cases}$$ (1)
where $\Omega=E[X^2]$, $\Gamma(m)$ is the gamma random variable, $m$ is the fading index. In this paper, $m=2$.

3. performance analysis

3.1. Average coverage probability and probability density function of distance

The average coverage probability of 3-D millimeter wave cellular environment is defined as $P_c(T) = P\{\text{SINR} \geq T\}$, and $T$ is the SINR threshold.

Based on the system model given in Section 1, the SINR of the target user is defined as

$$\text{SINR} = \frac{\sum_{b \in \Phi \setminus \Phi_0} \frac{h_b G_b r^{-\alpha}}{\sigma^2} + \sum_{b \geq 0, x \notin \Phi_0} \frac{h_b G_b r^{-\alpha}}{\sigma_n^2 + I}}{\sum_{b \geq 0, x \notin \Phi_0} \frac{h_b G_b r^{-\alpha}}{\sigma_n^2 + I}}. \quad (2)$$

In formula (2) $\sigma^2$ is the normalized noise power, $h_b$ is the Nakagami-m channel gain, $G_b$ is the antenna gain, $l(r_b)$ is the path loss, $I = \sum_{b \geq 0, x \notin \Phi_0} \frac{h_b G_b r^{-\alpha}}{\sigma_n^2 + I}$, where $\Phi_0$ denotes the elements other than $x_0$ in set $\Phi$, and $x_0$ represents the location of the BS communicating with the target user.

According to the 3-D PPP, the probability density function (PDF) will be

$$f(r) = \frac{dF(r)}{dr} = 4\pi r^2 \exp(-\frac{4}{3} \pi \lambda r^3), \quad r \geq 0 \quad (3)$$

3.2. Average coverage probability

First, we use the Alzer's lemma[9] to deal with gamma random variables, then get the Laplace transform of interference $L(s)$, at last calculate the covering probability $P_c(T)$.

Lemma 1 (Alzer's lemma) Let $h$ as a normalized gamma random variable with parameter $v$. When the constant $\gamma > 0$, the conclusion is as follows

$$P(h > \gamma) \geq 1 - [1 - e^{-\eta \gamma}]^v = \sum_{n=1}^{v} (-1)^{n+1} \binom{v}{n} e^{-\eta \gamma} \quad (4)$$

where $\eta = v(\nu!)^{-1/\nu}$, the equal sign holds when $\nu = 1$.

According to the definition, the average coverage probability can be expressed as:

$$P_c(T) = E[P\{\text{SINR} \geq T \mid r\}] = \int_0^\infty P\{h_b \geq T(\sigma_n^2 + I)r^\alpha\}f(r)dr \quad (5)$$

In formula (5), $P\{h_b \geq T(\sigma_n^2 + I)r^\alpha\}$ can be derived as

$$P\{h_b \geq T(\sigma_n^2 + I)r^\alpha\} \geq 1 - E[(1 - \exp(-\varepsilon T(\sigma_n^2 + I)r^\alpha))^m]$$

$$= \sum_{n=1}^{m} (-1)^{n+1} \binom{m}{n} \exp(-\varepsilon T \sigma_n^2 r^\alpha) L_{\varepsilon}(\varepsilon n Tr^\alpha) \quad (6)$$

where $\varepsilon = m(m!)^{-1/m}$.

The Laplace transform of interference $L_{\varepsilon}[s]$ is

$$L_{\varepsilon}(s) = E[e^{-st}] = E_{\Phi_0} \left[ \exp(-s \sum_{i \in \Phi_0 \setminus x_0} G_i h_i x^{-\alpha}) \right] = E_{\Phi_0} \left[ \prod_{i \in \Phi_0 \setminus x_0} E_{h_i G_i} \exp(-s G_i h_i x^{-\alpha}) \right] \quad (7)$$
According to the probability generating function (PFG) of the 3-D PPP, $E[\prod_{x \in \Phi} f(x)] = \exp(-\lambda \int_{\mathbb{R}^3} (1 - f(x))dx)$, formula (7) can be derived as
\[
L'_t(s) = \exp(-\lambda \int_{\Phi/\alpha_0} (1 - E_{(h_i, a_i)} \exp(-s G_i x^{-\alpha}))d\nu)
\] (8)
The point process is transformed from rectangular coordinates to polar coordinates in $\mathbb{R}^3$.
\[
L'_t(s) = \exp(-\lambda \int_{0}^{2\pi} \int_{0}^{\pi} \int_{r}^{\mathcal{R}} (1 - E_{(h_i, a_i)} \exp(-s G_i x^{-\alpha}))x^2 \sin \phi dx d\phi d\theta)
\] = $\exp(-4\pi \lambda \int_{r}^{\mathcal{R}} (1 - E_{(h_i, a_i)} \exp(-s G_i x^{-\alpha}))x^2 dx)$ (9)

Because of the LOS model of the base station $x \in (r, \mathcal{R})$
\[
L'_t(s) = \exp(-4\pi \lambda \int_{r}^{\mathcal{R}} E_{(h_i, a_i)} [1 - \exp(-s G_i x^{-\alpha})]x^2 dx)
\] (10)

Then the mean value of antenna gain is calculated as
\[
L'_t(s) = \exp(-4\pi \lambda \int_{r}^{\mathcal{R}} (1 - \sum_{k=1}^{\lambda} b_k (1 + sa_k^{-\alpha}))x^2 dx)
\] (11)

Calculate the matrix function in the gamma variable about $h_i$, we get
\[
L'_t(s) = \exp(-4\pi \lambda \int_{r}^{\mathcal{R}} (1 - \sum_{k=1}^{\lambda} b_k (1 + e\lambda Tr a_k^{-\alpha})^{-m})x^2 dx)
\] (12)

Let $s = \epsilon n Tr^a$, and substitute it to (12), we get
\[
L'_t(\epsilon n Tr^a) = \exp(-4\pi \lambda \int_{r}^{\mathcal{R}} (1 - \sum_{k=1}^{\lambda} b_k (1 + \epsilon n Tr^a a_k^{-\alpha})^{-m})x^2 dx)
\] (13)

The coverage probability can be obtained by substituting formula (3), (10), (13) into equation (7).

3.3. Downlink average rate
The downlink average rate of the target user in millimeter wave 5G cellular networks is derived as
\[
\tau = E[\log_2(1 + SINR) | r] = \int_{0}^{\mathcal{R}} \exp(-4\pi r^3)E[\log_2(1 + \frac{h_\alpha}{\sigma + 1})]4\pi r^2 dr
\] (14)

By transforming the form, we get
\[
\tau = \int_{0}^{\mathcal{R}} \exp(-4\pi r^3) \int_{0}^{\infty} P\{h_\alpha > r^a (\sigma^2 + I_\infty)(2^\epsilon - 1)\}dt 4\pi r^2 dr
\] (15)

To the formula $P\{h_\alpha > r^a (\sigma^2 + I_\infty)(2^\epsilon - 1)\}$, we can also use Alzer’s lemma and get
\[
P\{h_\alpha > r^a (\sigma^2 + I)(2^\epsilon - 1)\} \geq 1 - E[1 - \exp(-\epsilon(2^\epsilon - 1)(\sigma^2 + I)r^a)]
\] = $\sum_{n=1}^{m} (-1)^{n+1} \binom{m}{n} \exp(-\epsilon(2^\epsilon - 1)\sigma^2 r^a)L(\epsilon(2^\epsilon - 1)n r^a)$ (16)

According to (12) and let $s = \epsilon(2^\epsilon - 1)n r^a$, we can get
\[
L'_t(\epsilon(2^\epsilon - 1)n r^a) = \exp(-4\pi \lambda \int_{r}^{\mathcal{R}} (1 - \sum_{k=1}^{\lambda} b_k (1 + \epsilon(2^\epsilon - 1)n r^a a_k^{-\alpha})^{-m})x^2 dx)
\] (17)

Substituting formula (17) and (16) into equation (15), we can get the downlink average rate of the target user in millimeter wave 5G cellular networks.
4. Simulation results and analysis
In this section, the mathematical expressions derived in the previous section were simulated and analysed. Matlab is the basic tool of simulation, and theoretical values are calculated in Matlab. The basic parameters of simulation are listed in Table 2.

| Parameters | Value     | Parameters | Value     |
|------------|-----------|------------|-----------|
| $\lambda$  | $10^{-6}/m^3$ | $R$       | 200m      |
| $P_t$      | 1W        | $\beta$   | -60dB     |
| $\alpha$   | 4         | $M_{BS} \cdot M_{MS}$ | 10dB, 10dB |
| $\sigma_0^2$ | -145dB     | $m_{BS} \cdot m_{MS}$ | -10dB, -10dB |
| $m$        | 2         | $\theta_{BS} \cdot \theta_{MS}$ | 30°, 30° |

Figure 3 analyses the influence on coverage probability when $\alpha$ is taken as 3, 4 and 5 respectively, and other experimental parameters do not change into the data in Table 2. We find that the theoretical curve is in good agreement with the simulation curve, which confirm the correctness of the derivation in this paper. It can also be seen that the larger the $\alpha$ is, the larger the average coverage probability will be, which indicates that the influence on average coverage probability will increase when the path loss parameter fluctuates too much.

![Figure 3. Coverage probability in different path loss parameter](image1)

![Figure 4. Coverage probability in different radius $R$.](image2)

Figure 4 analyzes the influence of different radius $R$ of LOS horizon sphere, that is, different number of BSs in LOS horizon sphere, on average coverage probability. It can be concluded that when $R$ is larger, that is, the number of base stations is larger, the average coverage probability will be lower, which indicates that the larger $R$ is, the more serious the received interference will be. Therefore, how to allocate the number of base stations is very important for improving the performance of millimeter-wave cellular networks.

Figure 5 shows the relationship curve between BS density and the average rate in unit Hz under the. It can be concluded that when the BS is in $0.4 \times 10^5$, there will be a peak, which indicates that increasing the number of BSs excessively and increasing the number of BSs too little in a region will lead to the decrease of the downlink rate of the network. This provides a direction for our next research on energy efficiency optimization.
5. Conclusions
This paper presents a performance analysis method of dense millimeter wave cellular network in the real city environment based on 3-D PPP model. The theoretical results of average coverage probability and transmission rate are obtained by this method. The simulation results show that: ① The 3-D PPP model can well describe the performance of millimeter wave cellular networks. ② When the SINR threshold is constant, the larger the parameter $\alpha$ is, the larger the average coverage probability will be. ③ The larger the radius $R$ of LOS sphere, the lower the average coverage probability. ④ Under normal conditions, the BS density will lead to a peak in average rate, so it is of great significance to control the base station density for its spectral efficiency.

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