Viewing Angle Effects in Quasar Application to Cosmology

Raj Prince\(^1\), Bożena Czerny\(^1\), and Agnieszka Pollo\(^2\)

\(^1\) Center for Theoretical Physics, Polish Academy of Sciences, Al. Lotników 32/46, 02-668, Warsaw, Poland
\(^2\) National Centre for Nuclear Research, ul. Pasteura 7, 02-093 Warsaw, Poland

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Abstract

The symmetry axes of active galactic nuclei (AGNs) are randomly distributed in space, but highly inclined sources are heavily obscured and are not seen as quasars with broad emission lines. The obscuring torus geometry determines the average viewing angle, and if the torus geometry changes with the redshift, this average viewing angle will also change. Thus, the ratio between the isotropic luminosity and observed luminosity may change systematically with redshift. Therefore, if we use quasars to measure the luminosity distance by evaluating the isotropic absolute luminosity and measuring the observed flux, we can have a redshift-dependent bias that can propagate to cosmological parameters. We propose a toy model for testing the effect of viewing angle uncertainty on the measurement of the luminosity distance. The model is based on analytical description of the obscuring torus applied to one-parameter observational data. It illustrates the possible change of the torus covering factor between the two chosen redshift ranges. We have estimated the possible errors in specific cosmological parameters \((H_0, \Omega_m)\) for the flat Lambda cold dark matter cosmology if a method is calibrated at low redshift and applied to the higher redshift. The errors in the cosmological parameters due to potential dependence of the viewing angle on redshift are found to be potentially significant, and the effect will have to be accommodated in the future in all quasar-based cosmological methods. A careful systematic study of AGNs means that a viewing angle across the redshift is necessary, with the use of appropriate samples and models that uniquely determine the inclination of each source.

Unified Astronomy Thesaurus concepts: Cosmology (343); Galaxies (573); Quasars (1319)

1. Introduction

Quantifying the accelerated expansion of the universe is one of the key issues of cosmology (for a recent reviews, see, e.g., Matarrese et al. 2011; Rubin & Hayden 2016; Czerny et al. 2018). Various probes are used for this purpose, like observations of the cosmic microwave background (Planck Collaboration et al. 2020), Supernovae Ia (SN Ia; Riess et al. 1998; Perlmutter et al. 1999), baryon acoustic oscillations (Bassett & Hlozek 2010), gravitational lensing (Wong et al. 2020a), and gamma-ray bursts (Schafer 2003). Quasars, or more generally, active galactic nuclei (AGNs) also joined the class of sources with cosmological applications, and several specific methods to use these objects have been proposed: continuum time delays (Collier et al. 1999; Cackett et al. 2007), emission line time delays (Watson et al. 2011; Haas et al. 2011; Czerny et al. 2013), broadband spectral shape (nonlinear dependence between the UV and X-ray flux; Risaliti & Lusso 2015; Lusso et al. 2019), virial broadening estimator studies (Franca et al. 2014; Marziani & Sulentic 2014; Marziani et al. 2020), statistical properties of the quasar distribution reflecting the large scale structure (e.g., de Carvalho et al. 2020), and quasar strong lensing (Wong et al. 2020b).

Several recent measurements based on different methods imply the tension between the Hubble constant \(H_0\) determination based on an early universe and the value coming from the relatively local measurements (see, e.g., Riess 2019; Verde et al. 2019, for recent reviews). However, high accuracy of measurements is necessary, and the control of the systematic errors is critical, since the systematic error of 0.2 mag in the luminosity distance is enough to eliminate the \(H_0\) tension.

The use of quasars for cosmology has important advantages. Quasars are numerous, and they nicely cover the broad range of redshifts, starting from relatively nearby AGNs to redshifts of order of 7 (Mortlock et al. 2011; Bañados et al. 2018). They also do not show strong extinction effects, since bright quasars efficiently clear the line of sight through the host galaxy, and in addition the color selection allows for removing the highly reddened minority from the sample. They also do not seem to show evolutionary effects related to the metallicity, since even high redshift quasars are metal-rich, with solar or even supersolar abundance.

Quasar-based constraints of the cosmological parameters have already been very successful. Combining the quasar method based on the broadband spectral shape with several other cosmological probes (Lusso et al. 2019) showed that there is a 4\(\sigma\) tension with the standard Lambda cold dark matter (ΛCDM) cosmology. Quasar-based results will flourish in the next years, with more AGNs being a subject of systematic reverberation monitoring (Du et al. 2015; Grier et al. 2017; Du et al. 2018; Grier et al. 2019; Homayouni et al. 2020; Cackett et al. 2020), and with the future surveys approaching, like the Large Synoptic Survey Telescope (Ivezić et al. 2019).

Since the number of sources is growing, the cosmological constraints will be determined with increasingly smaller statistical errors. However, the systematic errors inherent in the methods will not be eliminated with the rise of the statistical samples. In the case of AGNs, one of such potentially important issues is the effect of the viewing angle of an active nucleus. AGNs are not pointlike sources; their appearance also joined the classification (see e.g., Antonucci 1993; Urry & Padovani 1995; Netzer 2015, for a review). This by itself does not pose a problem as long as, statistically, there is no trend in the average viewing angle with the redshift. However, if such a trend is likely to exist, it can be expected to
2. Method

An AGN is believed to have a supermassive black hole at the center, which accretes the matter from its surroundings. In vigorously accreting systems this matter forms cold Keplerian accretion disks. Close to the disk, the broad-line region (BLR) forms. This central part of an AGN is usually surrounded by a dusty/molecular torus. Thus, the viewing angle plays an important role in how the AGN appears, and consequently, in how it is classified. The disk is flat, so the observed disk flux decreases with the increase of the viewing angle. In addition, the observer looking at a very high viewing angle (measured from the symmetry axis) will not be able to see the direct disk emission, and instead, will register only a processed emission from the molecular torus (plus, eventually, a scattered emission). At intermediate viewing angles, both the disk anisotropy and the partial obscuration are important. Therefore, the observed radiation from any AGN mostly depends on two important parameters, e.g., viewing angle (i) and the optical depth (τ) along the line of sight. Here we consider our study as a toy model, illustrating the potential dangers of neglecting viewing angle issues in quasar cosmology.

2.1. Description of the Geometry

The AGN geometry to be considered in this work is similar to that given by Elitzur (2008), and is shown in Figure 1, where the torus is considered to have a smooth-density distribution of dust in the angular direction. The inclination angle measured from the symmetry axis is denoted as i.

We assume that the radiation produced in the disk is modified by the torus before it reaches the observer, and the effect depends on the viewing angle. Let us assume that the τ is the optical depth of molecular torus and it is defined as

\[ \tau = \alpha e^{-\beta/\sigma^2}, \]

where “A” is the normalization constant, \( \beta = (\pi/2 - i) \) is the angle measured from the equatorial plane, and \( \sigma \) is the characteristic width of the torus. We thus just use the projection of the torus onto the sky plane of the central source and neglect possible variations in the radial distribution of the dusty material. Therefore, constants A and \( \sigma \) are the only two free parameters of the torus model itself. The third parameter is the viewing angle.

2.2. Disk/torus System

Here we consider a toy model that allows us to see how the change in the CF measured by Gu (2013) can relate to the torus parameters and the viewing angle, and further, how it can affect the measured luminosity in the case of high redshift AGN (2.0 \( \leq z \leq 2.4 \)) if we fix the geometry by analyzing the low redshift sample (0.7 \( \leq z \leq 1.1 \)). We do not aim here at actual measurement of the observational trend, as this will have to be done in the future with a better torus model and a large sample of objects over a redshift range with broadband spectra, which will allow for uniquely measuring the model parameters, including the viewing angle, from the data.

The disk luminosity seen by the observer is anisotropic, since the AGN standard accretion disk is geometrically thin, so the disk flux is proportional to the cosine of the viewing angle (i), i.e., the intrinsic disk flux \( L_{disk} \) seen by observer in the absence of the dusty torus is

\[ L_{disk}(i) = 2L_{disk} \cos i, \]  

where \( L_{disk} \) is the intrinsic average disk luminosity. When the observer is looking along the symmetry axis, where \( i = 0 \), the observed disk luminosity, \( L_{disk} \), would be \( 2L_{disk} \) (see, e.g., Frank et al. 2002). This anisotropy effect is neglected in Elitzur (2008) as well as in the Gu (2013) analysis. In principle, an additional factor related to the limb brightening could also have been included here, but for the sake of simplicity, we neglect it in the current approach.

Finally, the disk intrinsic anisotropic emission is modified in a viewing-angle-dependent way when the light is passing through the dusty torus, which gives us the final expression for the observed disk flux:

\[ L_{disk}(i) = 2L_{disk} \cos i e^{-\tau}, \]

where \( \tau \) is the optical depth of dusty torus. The IR radiation produced in the molecular torus comes from the nuclear emission, which is intercepted by the torus. In order to calculate it from the model, assuming that the intrinsic disk emission and torus parameters are known, we perform an integral over all

![Figure 1. Illustration of a dusty torus with half opening angle \( \sigma \) and with decreasing density away from the equatorial plane. The viewing angle is defined as \( i \) measured from the symmetry axis.](image-url)
viewing angles:

\[ L_{\text{IR}} = 2L_{\text{disk}}^\text{aver} \int_0^{\pi/2} \sin{\beta} \cos{\beta}(1 - e^{-Ae^{-\beta} \sin{\beta}})^2 d\beta. \]  

(4)

Here, we use the angle \( \beta \) measured from the equatorial plane.

We further assume that the emission of the torus in the IR band is isotropic. This is an oversimplification in compared to the approach of Elitzur (2008) based on radiative transfer, but we do not aim to study the spectral features in the IR band. This assumption of isotropy is relatively correct if the torus is optically thin and/or we observe the emission at relatively longer wavelengths. Here we concentrate on a simple approach to the overall radiation budget.

Thus, we define the CF of the AGN as the ratio of observed IR to the intrinsic disk luminosity:

\[ \text{CF} = \frac{L_{\text{IR}}}{L_{\text{disk}}^\text{aver}} = \frac{L_{\text{IR}}}{L_{\text{disk}}^*} \frac{L_{\text{disk}}^*}{L_{\text{disk}}^\text{aver}}. \]  

(5)

On the other hand, if we assume that the IR emission is isotropic but the disk emission is highly anisotropic, then what we measure directly from the data is instead the ratio

\[ \frac{L_{\text{IR}}}{L_{\text{disk}}} = \frac{L_{\text{IR}}}{L_{\text{disk}}^\text{aver}} \frac{L_{\text{disk}}^\text{aver}}{L_{\text{disk}}^\text{aver}}. \]  

(6)

The numerator in the above equation is estimated through the numerical integration from Equation (4). The denominator is the rewritten form of Equation (3),

\[ L_{\text{disk}}^\text{aver} = 2 \cos{\beta} e^{-\beta}, \]

(7)

and after substituting the value \( i = \pi/2 - \beta \), Equation (7) is transformed to

\[ L_{\text{disk}}^\text{aver} = 2 \sin{\beta} e^{-Ae^{-\beta} \cos{\beta}^2}. \]  

(8)

Although the viewing angle has such importance for the appearance of an AGN, we rarely measure it for individual sources. A range of viewing angles would lead to a scatter in the ratio of \( L_{\text{IR}}/L_{\text{disk}}^\text{aver} \). This scatter is around some mean value, unknown a priori, and it needs to be determined observationally. This value is sensitive to the properties of the torus.

In reverberation mapping, we use only type 1 AGN, with emission lines well visible. The torus shields the BLR from the observer for high inclinations for which the torus becomes optically thick. The range of the viewing angles shielding the nucleus depends on the dust spatial distribution, i.e., dust properties. In addition, the transition from the unshielded to shielded case is continuous for most torus models. Here we assume that the line of sight with the value \( \tau = 1 \) separates the type 1 and type 2 AGN. We mark the corresponding value of the angle \( \beta \) as \( \beta_0 \), and we use Equation (1) to estimate its value knowing the torus parameters. Our torus model gives

\[ \beta_0 = \sigma \sqrt{\ln{(A)}}, \]  

(9)

and hence, \( \sin{\beta_0} = \sin{\sigma \sqrt{\ln{(A)}}} \).

Thus, type 1 sources are those with a viewing angle between \( \beta = 0 \) and \( \beta = \beta_0 \), and statistically the mean (effective) viewing angle is given by \( \sin{\beta_{\text{eff}}} = 1/2(1 + \sin{\beta_0}) \), which by substituting the value of \( \beta_0 \) reduces to \( 1/2(1+ \sin{\sigma \sqrt{\ln{(A)}}}) \).

Therefore, if we consider a sample of randomly oriented type 1 sources, we can substitute \( \beta \) with \( \beta_{\text{eff}} \) in Equation (8), and use Equation (6) to get the average value for the normalized disk luminosity,

\[ \frac{L_{\text{IR}}}{L_{\text{disk}}} = \frac{L_{\text{IR}}/L_{\text{disk}}^\text{aver}}{(1 + \sin{\sigma \sqrt{\ln{(A)}}})^* e^{-Ae^{-\beta_{\text{eff}}(\sigma \sqrt{\ln{(A)}}^2)}}}. \]  

(10)

The values of \( A \) and \( \sigma \) are model parameters, and \( \sigma \) can vary between 0° and 90°, but \( A \) must be larger than 1, otherwise we would have no type 2 sources and the definition of \( \beta_0 \) in Equation (9) would not apply. If the ratio of \( L_{\text{IR}}/L_{\text{disk}} \) is measured observationally, we can constrain the geometry of the torus, and in particular, the average viewing angle and intrinsic extinction.

2.3. Observational Data

Gu (2013) studied the sample of quasars from the DR9Q (Pâris et al. 2012) and DR7Q (Schneider et al. 2010) catalogs by combining the photometric data from the Sloan Digital Sky Survey (SDSS) with the data from other observatories, including the Wide-field Infrared Survey Explorer (WISE; Wright et al. 2010), the UKIRT Infrared Deep Sky Survey (UKIDSS; Lawrence et al. 2007), and the Galaxy Evolution Explorer (GALEX; Martin et al. 2005).

The distribution of quasar samples in DR9Q and DR7Q peak in two different redshift ranges. The distribution with redshift can be seen in Pâris et al. (2012) and Schneider et al. (2010), where a large number of sources in DR9Q found in the redshift range of 2.0–2.4 with median 2.2 and in DR7Q the source distribution with redshift shows two peaks, one at 0.8 and another at 1.6. Now to consider the significantly different redshift ranges for low and high redshift sources, Gu (2013) focused on the peak at 0.8 and selected the sources within the redshift range of 0.7–1.1. The multiwavelength data collected from other telescopes like WISE, GALEX, and UKIDSS along with SDSS are used to constrain the sample size and finally the multiwavelength spectral energy distribution (SED) was produced. The SED was used to estimate the IR and bolometric luminosity for all the sources, and further, their ratio is defined as the CF by Gu (2013). The distribution of sources for low and high redshift with CF are shown in Figure 5 of Gu (2013), and their corresponding median values differ significantly, suggesting a clear evolution with redshift. Gu (2013) performed a Kolmogorov–Smirnov statistic test to confirm the difference between low and high redshift CF distribution. Both sub-samples showed a wide distribution of values, but there is an overall strong shift in the reported values toward higher dust covering at large redshift despite no clear effect in the distribution of black hole mass and Eddington ratio.

It can always be argued that the difference seen in the torus geometry at two different redshift ranges possibly results from the biases present in the sample selection. But it can only be verified in future by having more observed sources over a larger redshift range, with more precise observations.

For use in the present paper we concentrate only on the median values of the IR to bolometric luminosity (CF), which is 0.478 for the low redshift subsample and 0.871 for the high redshift subsample. Gu (2013) directly reported the ratio of the IR to the bolometric luminosity and considered that as a measurement of the CF, assuming a spherically symmetric (isotropic) emission of the central source and the even simpler torus model without stratification along the \( \beta \) angle. In our approach, Gu’s measurement actually corresponds to the ratio defined by Equation (10).
The change in this factor between the low and the high redshift subsamples implies a systematic change in the effective viewing angle in quasars with redshift. If this effect is ignored in studies using quasars for cosmology, it will lead to a clear systematic apparent drift in the cosmological parameters. In the next section, we describe how neglecting this information affects the results. We summarize the parameters adopted from Gu (2013) in Table 1.

Since Gu (2013) provided a single value for each source, we cannot uniquely derive our two torus parameters and the viewing angle from Gu’s measurements. The assumption that we concentrate on the mean viewing angle relates the viewing angle to the torus parameters, but still two parameters remain to be determined from a single measurement. Therefore, we cannot expect to determine the actual trend with the redshift, ready for cosmological applications, but nevertheless we obtain a very interesting illustration of what may happen if the issue is neglected.

2.4. Application to Cosmology

We first provide simplified analytical estimates of how the lack of knowledge of the viewing angle of a quasar leads to an incorrect determination of the Hubble constant from the reverberation mapping of a single quasar. Next, we use the data from Table 1 to show how large errors in estimating the cosmological parameters can result if the systematic change in the torus parameter between the lower redshift and higher redshift subsamples is ignored.

2.4.1. Simple Example of the Role of the Viewing Angle Applicable at Very Low Redshift

In a standard cosmological approach to use quasar reverberation measurements, we ignore the issue of the viewing angles, since we do not measure them routinely for the reverberation measured sources. We then measure the time delay for a given source, \( \tau_d \), and we measure the observed monochromatic flux, for example, at 5100 Å, \( F_{\text{obs}} \). We use the theoretical formula, ignoring the issue of the viewing angle, to define the observed monochromatic luminosity,

\[
\log L_{44,5100} = -2.61 + 2 \log(\tau_d),
\]

where the constant was calculated assuming the dust temperature \( T_{\text{dust}} = 1000 \text{ K} \), and effective viewing angle, \( i_{\text{eff}} = 39.2^\circ \) in Equations (2)–(4) of Czerny & Hryniewicz (2011). Here, the time delay \( \tau_d \) of the H\( \beta \) line with respect to the continuum at 5100 Å is measured in days and the monochromatic luminosity at 5100 Å in units of \( 10^{44} \text{ erg s}^{-1} \text{ cm}^{-2} \). Thus, the measurement of the time delay gives us the absolute luminosity of the source. Knowing the absolute luminosity, one can estimate the source luminosity distance by following the relation

\[
D_L^2 = \frac{L_{5100}}{4 \pi F_{5100}^3},
\]

where \( F_{5100} \) is the observed flux measured at 5100 Å, and easily available for the studied quasar. Derived \( D_L \) can be compared to the cosmological model through the prescription for the cosmological distance \( D_L(z, H_0, \Omega_m, \Omega_k) \) if we know the redshift, \( z \) (also easily measured for the quasar from the position of the emission line). Thus, measuring \( \tau_d, z \), and \( F_{5100} \) we can do the cosmology. In particular, if the redshift is much smaller than 1, we can use a linear part of the Hubble flow to determine the Hubble constant:

\[
H_0 = \text{const} \frac{F_{5100}^{1/2}}{\tau_d}. \tag{13}
\]

The problem appears if the viewing angle is not consistent with the value \( i_{\text{eff}} = 39.2^\circ \) adopted by Czerny & Hryniewicz (2011). So one can ask the question, what happens if we use a single object to obtain \( H_0 \) but the source is seen at an angle \( i \) different from \( i_{\text{eff}} \)?

Now we ignore the effect of the viewing angle in a time delay geometrical setup (see Equation (3) of Czerny & Hryniewicz 2011), since this effect is highly dependent on the BLR 3D geometry. In that case, the time delay measurement gives us the true viewing-angle independent absolute luminosity (and the product of the black hole mass and accretion rate), where the constant was adjusted to the viewing angle of 39.2\(^\circ\). However, the measured flux, \( F_{5100} \) would still depend on the actual viewing angle, so the luminosity distance would contain the term \( \left( \frac{1}{\cos i} \right)^{-1/2} \), and the measured Hubble constant would actually be

\[
H_0 = \text{const} \left( \frac{\cos i}{\cos i_{\text{eff}}} \right)^{1/2} \frac{F_{5100}^{1/2}}{\tau_d}. \tag{14}
\]

This means we will be making an error by a factor of \( (\cos i / \cos i_{\text{eff}})^{1/2} \) in \( H_0 \) if this factor is ignored, and that is mostly the case, since we did not measure the viewing angle to the source. This is the problem in any quasar reverberation mapping, either based on emission line delay, or continuum time delay (see Equation (15) of Cackett et al. 2007). In addition, in this simplified approach we also neglected the effect of the extinction due to the dusty torus.

Through this article, we have tried to shed some light on the issue of possible systematic errors due to disk emission anisotropy and the dust properties, which might affect the application of a quasar to cosmology.

2.4.2. Estimated Errors Based on the Data of Gu (2013)

Now we use the full model described in Section 2.2, and we include the disk anisotropy as well as the extinction caused by the torus, and we do not limit ourselves to very low redshift, so the approach is not analytical until the very end, as in Section 2.4.1. However, our approach still remains relatively simple.

We now assume that all the parameters for the low redshift \( (0.7 \lesssim z \lesssim 1.1) \) subsample are properly adjusted to the correct cosmological model, and we study the errors that would be made if the cosmological parameters are estimated from the
large redshift ($2.0 \leq z \leq 2.4$) subsample but the change in the torus properties visible in the data of Gu (2013) is neglected.

When we constrain the cosmology using the high redshift quasar sample, we measure the luminosity distance and we use the geometry appropriate for high redshift sources. Considering all the appropriate effects, the observed flux can be defined as

$$F_{5100} \propto \left( \frac{\tau_d \cos i \ e^{-\tau}}{D_L^2} \right)$$

where $\tau$ is the optical depth measured along the line of sight and $\tau_d$ is the time delay. The above equation is similar to the equation defined by Cackett et al. (2007) in Equation (14), though they did not consider the effect of optical depth. The luminosity distance from the above equation can be defined as

$$D_L^2 \propto \left( \frac{\tau_d \cos i \ e^{-\tau}}{F_{5100}} \right)$$

or

$$D_L \propto \frac{\tau_d \cos i)^{1/2} \ e^{-\tau/2}}{(F_{5100})^{1/2}}.$$  \hspace{1cm} (17)

As we noted before, we aim at using a statistical sample so we do not need to know the mean, or effective viewing angle in each redshift bin. Thus, if one wants to estimate the correct luminosity distance at the high redshift bin, one should use the effective viewing angle at high redshift ($\cos i_{\text{eff}}(\text{high}-z)$). If the effective viewing angle appropriate for low redshift bin is used, then the luminosity distance at the high redshift bin measured by using the low redshift geometry would be incorrect. The correct and incorrect luminosity distance at high redshift bin can be defined as

$$D_L^{\text{correct}} \propto \left( \frac{\tau_d \cos i_{\text{eff}}(\text{high}-z))^{1/2} \ e^{-\tau_{\text{high}}/2}}{(F_{5100})^{1/2}} \right)$$

and

$$D_L^{\text{incorrect}} \propto \left( \frac{\tau_d \cos i_{\text{eff}}(\text{low}-z))^{1/2} \ e^{-\tau_{\text{low}}/2}}{(F_{5100})^{1/2}} \right).$$ \hspace{1cm} (19)

The relative error in measuring the luminosity distance caused by the incorrect use of the low redshift geometry to the high redshift sample can be estimated as by taking the ratio of Equation (19) to Equation (18),

$$\frac{D_L^{\text{incorrect}}}{D_L^{\text{correct}}} \propto \left( \frac{\cos i_{\text{eff}}(\text{low}-z)}{\cos i_{\text{eff}}(\text{high}-z)} \right)^{1/2} \frac{e^{-\tau_{\text{low}}/2}}{e^{-\tau_{\text{high}}/2}}.$$ \hspace{1cm} (20)

where the time delay $\tau_d$ does not appear in the final Equation (20).

In Figure 2, we show the variation of luminosity distance for high redshift objects (assuming at lower redshift all the measurements are correct) from Equation (20). The luminosity distance at lower redshift is estimated for standard cosmology (Planck Collaboration et al. 2020), and the effective viewing angle, $i_{\text{eff}}$. is set at $39.2^\circ$. In addition, the optical depth is assumed to be constant for the purpose of this particular plot, and the viewing angle is chosen within the allowed range.

The error in the luminosity distance can be propagated to an error in the Hubble constant, $H_0$, and can be represented by the following relation, if we assume that we know the other cosmological parameters,

$$D_L^{\text{incorrect}} = D_L(H_0^{\text{incorrect}}, \Omega_m, \Omega_\Lambda).$$ \hspace{1cm} (21)

To reduce the number of free parameters, we consider a flat cosmology, where $\Omega_m + \Omega_\Lambda = 1$, and Equation (21) is finally modified to

$$D_L^{\text{incorrect}} = D_L(H_0^{\text{correct}}, \Omega_m^{\text{correct}}).$$ \hspace{1cm} (22)

where $\Omega_m^{\text{correct}}$ is the value taken from Planck Collaboration et al. (2020). Equation (22) can only be solved numerically. We used the correct value of $H_0$ and other cosmological parameters from Planck Collaboration et al. (2020) to estimate the correct luminosity distance. Using the correct luminosity distance in Equation (20), we estimated the incorrect luminosity distance at higher redshift caused by the geometry mismatch. And again we used Equation (22) to estimate the $H_0^{\text{incorrect}}$ for the incorrect luminosity distance at fixed values of the other cosmological parameters.

Similarly, we can assume that the Hubble constant is known from other measurements but the aim is to obtain the $\Omega_m$. In this case, we have to solve the equation

$$D_L^{\text{incorrect}} = D_L(H_0^{\text{correct}}, \Omega_m^{\text{incorrect}})$$ \hspace{1cm} (23)

in order to see the error caused by an inappropriate assumption about the torus geometry for the high redshift sample.

To perform this exercise, we need the observational constraints for the mean viewing angles and optical depths at the low and high redshift samples.

3. Results

3.1. Constraints for the Torus Evolution with Redshift and the Corresponding Change in the Viewing Angle from the Gu (2013) Sample

In Section 2, we discussed our simple torus model, which we use to estimate the CF analytically. Equation (10) is the final
equation that gives the CF for different values of \( A \) and \( \sigma \). By comparing our analytical value of CF to the observational values from Gu (2013), we formulate constraints for the two torus parameters. Since at a given redshift, we measure only one parameter (see Table 1) while the model has two parameters, the results are not unique.

The disk luminosity significantly depends both on the optical depth along the equatorial plane (\( A \)), and on the width or the opening (\( \sigma \)) of the torus. Thus, with a single constraint from the data at a given redshift, we obtain a relation between the two model parameters, separately for each redshift. In Figure 3, we considered the entire possible range of \( \sigma \) between 0° (at the equatorial plane) and 90° (at the symmetry axis). The red curve shows the range of \( \sigma \) for low redshift sources (Table 1 for \( CF = 0.478 \); Gu 2013) and the green curve covers the \( \sigma \) for high redshift sources (Table 1, \( CF = 0.871 \)). For the low redshift sources the maximum \( \sigma \) allowed is 45°, shown as the upper dotted blue horizontal line in Figure 3. This upper limit comes from the fact that the average viewing angle for type 1 sources is \( \sim 40° \) (Czerny & Hryniewicz 2011), which allows the maximum \( \sigma \) to go close to 50°. Hence, to be on safe side, we constrained our analysis to \( \sigma \) equals to 45° as an upper limit. The lower dotted blue horizontal line is constrained by the minimum allowed value of \( \sigma \) for high redshift sources in our sample. More details about constraining the \( \tau_{\text{equatorial}} \) are discussed in Section 3.2. The two lines never cross, which means that we do not see any possibility of having the same torus mean parameters for the low and high redshift sources. However, we have a region where the \( \sigma \) is common for both the low and high redshift sources, which means that on average, we can have a similar opening angle of dusty torus in the SDSS DR9Q and DR7Q catalogs (Gu 2013), i.e., at redshifts \( \sim 0.8 \) and \( \sim 2.2 \). However, even in this case the mean effective viewing angle is not identical, since it also depends on the extinction, which is set by the parameter \( A \) equal to \( \tau_{\text{equatorial}} \).

We illustrate it in Figure 4. We choose to keep the maximum value of \( \tau_{\text{equatorial}} \sim 100 \), which has also been suggested by Nenkova et al. (2008) for a single cloud in their clumpy torus model. Of course, the observational appearance of the source is also strongly affected, since the ratio of the IR emission to the optical/UV emission also rises steeply with the torus optical depth (see Figure 5). The CF increases very sharply below \( A = 10 \), and after that the rise slows down and CF finally almost saturates for higher value of \( A \). The parameter \( A \) can have a large range of values starting from 1 (constrained by Equation (10)) to any other higher value. We discussed the possibility of constraining the upper limit of \( A \) in Section 3.2.

### 3.2. Available Parameter Space

To constrain the upper limit of \( A \), we consider the case of Compton thick type AGN. The maximum value of the torus optical depth is not easy to measure, Burlon et al. (2011) gives the constraint for the column density, \( N_H > 10^{25} \text{ cm}^{-2} \). Now to calculate the optical depth along the equatorial plane (\( A \) or \( \tau_{\text{equatorial}} \)), we assume \( N_H = 5 \times 10^{25} \text{ cm}^{-2} \), and hence \( \tau_{\text{equatorial}} \sim \sigma T^2 N_H \) gives \( \tau_{\text{equatorial}} \sim 33 \), where \( \sigma T \) is the Thompson scattering cross section.

We adopt this value as the upper limit, and therefore Figure 3 shows a plot for \( \tau_{\text{equatorial}} \) and \( \sigma \) for low (\( z \sim 0.8 \)) and high (\( z \sim 2.2 \)) redshift sources, where \( A \) is constrained from 1–33, and \( \sigma \) from 25°–45° and 38°—85° for the low and high redshift sources, respectively.

![Figure 3](image-url) **Figure 3.** The relation between the torus parameters for low and high redshift sources based on constraints listed in the data in Table 1.

![Figure 4](image-url) **Figure 4.** The effective viewing angle of the torus with \( \sigma = 40° \) as a function of torus optical depth.

![Figure 5](image-url) **Figure 5.** Illustration of the IR emission dominance with the increase of the torus optical depth for three values of the torus opening angles.
3.3. Effect on the Luminosity Distance

Having constrained the $\sigma$ for the low and high redshift sources from Gu (2013), we can now estimate the potential error in the determination of the luminosity distance, if the effect of the change in the geometry between the low and high redshift is neglected, making use of Equation (20).

We pick up the torus parameters from the two lines shown in Figure 3, which forms a 2D plot of the possible errors in the luminosity distance, parameterized by the values of $\sigma$ on the low redshift branch and at the high redshift branch. The possible errors in luminosity distances at higher redshift due to the viewing angle and the torus extinction are plotted in Figure 6. The color bar of the contour plot shows the relative error of the luminosity distance, and the $X$- and $Y$-axes represent the half opening angle at lower and higher redshifts, respectively. At the higher redshift, if the half opening angle of the torus is less than 45° (i.e., $\sigma < 45^\circ$), we underestimate the luminosity distance. However, if $\sigma > 45^\circ$, we overestimate the luminosity distance. The error in the luminosity distance for the maximum possible $\sigma$ would be $\sim 40\%$, but this would happen only if the true torus opening angle at high redshift is as high as 80°. If the used parameter space is too broad, and in reality the torus at high redshift has an opening angle of about 40°, then the maximum possible error is smaller, below 20%. This error is just an approximation, and it can be modified in the future by considering a more appropriate model of torus geometry and measuring the viewing angle of the individual objects, which will constrain the parameter space much better.

3.4. Potential Errors in the Cosmological Parameters

Since the luminosity distance is used to measure the Hubble constant in a local universe and to determine other parameters of the cosmological model, we see that the potential of neglecting the systematic trend in the viewing angle of the torus may lead to considerable bias in determining these parameters from quasar reverberation studies. To see how large are the potential errors in quasar-based method are we translate the error in the determination of the luminosity distance to an error in the cosmological parameters. With the two-redshift approach based on the data in Gu (2013) and our model, we can only provide a simple illustration.

First, we assume that the standard $\Lambda$CDM model correctly describes the expansion of the universe and that we know the values of the parameters $\Omega_{\Lambda}$ and $\Omega_m$, but we aimed at determining the Hubble constant. Using Equation (22), we thus determine the error in $H_0$ resulting from the incorrect representation of the quasar geometry at a high redshift of 2.2. The result is shown in Figure 7. The contour plot has a similar shape to the error in luminosity distance but relative errors are much smaller. The errors in the whole parameter space are up to 18%, but even if the opening angle of the torus at high redshift is actually of order of 40°, the error could reach 16%. Since the error can be both positive and negative, with luck we will have no errors. However, if the aim of the quasar-based method is to meet the limits of 1%, achieved with SN Ia (Riess et al. 2020), then the issue of the viewing angle trend in AGNs cannot be ignored. From our plot we can estimate that in order to have 1% accuracy in $H_0$ we would need to determine (statistically) the values of $\sigma$ with 0.5° accuracy.

Next we assume that we know the Hubble constant, but we want to use quasars to estimate the other cosmological parameters, for example $\Omega_m$, assuming that the universe is flat. Now using Equation (23) we derive the contour error map for the $\Omega_m$ (see Figure 8). We see that both negative and positive errors are expected, and if a broad parameter range used for the
plot is allowed the error is huge. If the realistic values of \( \sigma_{\text{high} - z} \) are rather close to 40°, the error is smaller, but achieving an error of order of 1%, required for precision cosmology, will clearly require taking the systematic errors connected with the viewing angle into account.

We do not claim here that the systematic errors in the published papers are actually as large as illustrated in Figures 6—8, since here we only use one-parameter data input and two-parameter models so some ranges of the parameter space are unlikely. However, we expect that the plots well argue that the effect is important and requires further studies to validate cosmological applications. In principle, we could use exactly the approach of Gu (2013) and interpret directly the measured ratio as the viewing angle, thus assuming that the torus can be modeled as a solid body, without extinction or disk anisotropy included. In that case, we would obtain a unique result from Gu’s measurements: the viewing angle in the lower redshift sources would be 40.5° and at the high redshift sample 55.6°. Under these strong assumptions, the corresponding error in the luminosity distance would then be 16%.

4. Discussion

The structure of an AGN can be divided in two parts. The central part (BH + disk + BLR) of a bright AGN is dominated by the optical and UV emission from the flat optically thick disk. At larger distances the central parts are partially surrounded by the dusty torus, which emits mostly the reprocessed emission in the IR band. The dusty torus can be optically thin or optically thick. The emission from the central part of the AGN is anisotropic because the flat disk emission is anisotropic, and the optically thick dusty torus further modifies the radiation in an angle-dependent way. Therefore, the observed bolometric luminosity of the AGN depends on the viewing angle. The issue is well known; Netzer (1987) discussed how the central radiation source in AGN was responsible for the anisotropic emission, but it was not taken into account in the context of quasar applications to cosmology.

In this paper we used the observational data from Gu (2013) and a very simple torus model to evaluate implications of this effect when quasars are used for cosmology. Our toy model of torus is continuous, with the optical depth dependent on the angular distance from the equatorial plane, we assumed that the disk emission (also dependent on the viewing angle) is filtered by such a torus, and we obtained constraints for the torus structure at two values of redshift corresponding to the mean properties of the sample from Gu (2013). Later we assumed that the cosmological method is calibrated at lower redshift and we checked to see what happens if the change in geometry is ignored when the study of the high redshift sample is done. We specifically had in mind the reverberation method when the absolute luminosity of a quasar is determined either from the time delay of the BLR lines with respect to the continuum, or from the relative delays of two continuum bands. Such time delay depends mostly on the actual size of the source and only weakly on the source inclination, while the observed flux does depend on inclination and obscuration.

Considering the effect of the viewing angle and the torus optical depth, we estimated the bolometric luminosities and the luminosity distance for high redshift AGNs, applying either the correct geometry, or the geometry appropriate for the low redshift quasar sample. We then calculated the error that one would make in the measurement of luminosity distance if one does not consider the effect of the change in the viewing angle and torus optical depth (Figure 6). The result shows that the errors are quite significant, and they cannot be ignored. Since the luminosity distance can be used to do the cosmology, the error in the cosmological model can also be estimated. The errors in \( H_0 \) and \( \Omega_m \) are shown in Figures 7 and 8.

For a single source, as can be seen from Figure 7, the 4% error in \( H_0 \) is equivalent to the difference in the \( \sigma \) by \( \sim 2° \). We know that \( \sigma \) is directly related to the viewing angle \( i \) and hence the change in \( i \) would be order of 2°. As has been seen in many studies (Riess et al. 2020) the 1% error in \( H_0 \) is significant enough to formally claim the tension in \( H_0 \). Hence, from a single object to claim the error of 1% in \( H_0 \) we need an accuracy of 0.5° in the effective viewing angle. The viewing angle estimated in a single source with such accuracy would be rather unrealistic, but for cosmological applications we need only to know the trend in the statistical sense. Thus, if in a single source we can estimate the viewing angle with 10° accuracy, we would need 400 sources in a redshift bin to have the statistically requested accuracy.

4.1. The Accretion Disk Model

In this paper we used the simplest possible representation of the accretion disk (see Equation (2)) without explicitly using the disk spectral shape. The important fact we needed was the strict proportionality of the observed flux to \( \cos i \), which is correct for the standard Shakura–Sunyaev disk model (Shakura & Sunyaev 1973). This simple model applies quite well to bright quasars in the optical and near-UV range. As an illustration, in Figure 9 we show such a model fitted to the recent quasar composite spectrum based on XSHOOTER data (Selsing et al. 2016). The model represents well the optical data, although at the longest wavelengths traces of the host galaxy contribution start to be seen. However, at the far-UV and in the X-ray band, the standard model does not apply, and the soft and hard X-ray emission come from the innermost part of the disk, from the warm and hot corona. In the IR, the torus emission and the contribution from the host galaxy dominate the broadband spectrum (for an observational discussion of

![Figure 9. The composite spectrum from Selsing et al. (2016) (black line) compared to the standard Shakura & Sunyaev (1973) accretion disk model (red line) for the parameters of the black hole mass \( 6 \times 10^8 M_\odot \) and the Eddington ratio \( L/L_{\text{edd}} = 0.3 \).](image-url)
broadband quasar spectra see Richards et al. 2006). Modeling this broadband emission is usually done using some parametric models, usually aimed at a specific energy band. Modeling IR emission requires a model of the dusty torus, and modeling the X-ray spectra requires a model of the corona/coronas and the model of the effects of reprocessing. For just the broadband spectrum, a convenient model with two coronas was proposed by Kubota & Done (2018). The theoretical modeling of the innermost part of the disk is complex (see, e.g., Czerny & Naddaf 2018 for a recent review), particularly if the important role of the magnetic field is postulated. At very high accretion rates, the advection has to be included (Abramowicz et al. 1988). At much lower Eddington rates than typically seen in quasars an inner hot flow replaces the cold disk (e.g., Ichimaru 1977; Narayan & Yi 1994; Melia & Falcke 2001). These processes do not affect the optical emission considerably. However, if the broadband spectrum is used, then some representation of these processes is required.

4.2. Advanced Torus Models

There are various methods to estimate the viewing angles of quasars. First of all, more advanced torus models combined with the good SED data can be used for that purpose. In the past, various models have been proposed in the context of dusty torus based on X-ray observations. The models MYtorus (Murphy & Yaqoob 2009) and BNTorus (Brightman & Nandra 2011) have been used extensively for spectroscopic studies of obscured AGNs. However, they have a very limited number of parameters to describe the physical scenarios of a torus. In the MYtorus model, the covering factor is fixed to 50%. In the BNTorus model, the opening angle of the torus is considered to be a free parameter, but the column density along the line of sight is assumed to be equal to the torus column density (which is the case for Compton thick AGN). The torus model used in our study is more like the BNTorus model where the opening angle varies from 0°–90° (along the equatorial plane—symmetric axis), and the column density we chose is more like Compton thick AGN ($N_H \sim 10^{25} \text{ cm}^{-2}$). A recent dusty torus model (borus02) was proposed by Baloković et al. (2018), where they assumed a toroidal geometry similar to the BNTorus model, but the new model is more flexible in terms of available parameter space, and it allows for a line-of-sight column density of the torus that is different from the total torus column density. Also, the covering factor is a free parameter, and it varies with the torus opening angle ($\cos \theta_{\text{torus}}$) measured from the symmetry axis. They also assumed that the smooth-density torus scenario that reasonably represents the physical reality of the torus. Along with the smooth-density torus, there are studies that show the possibility of a clumpy torus also. A recent study by Buchner et al. (2019) discussed the clumpy torus model based on X-ray studies. The clumpiness in the torus has been observationally confirmed through the X-ray eclipse events as reported by Risaliti et al. (2002) and Markowitz et al. (2014). The XCLUMPY model of Buchner et al. (2019) explains such events.

A number of other interesting studies have also been done addressing the aspect of the quasar radiation anisotropy. Venanzi et al. (2020) studied the impact of anisotropic radiation from the central AGN on the emergence of dusty winds. Their simulation shows that the anisotropic radiation from the central source changes the outflow opening angle by making the cone wider with respect to isotropic radiation. Our model also assumed that anisotropic AGN radiation where more radiation is expected along the symmetric axis than toward the dusty torus. In their studies, Yamada et al. (2020) included not only the effect of the torus but also the dusty polar outflows in their study of anisotropy. The effect of the orientation of an individual AGN on the dispersion on their luminosity and line width was included by Negrete et al. (2018) in their study based on the virial broadening estimator, where they also included the limb darkening effect of the disk emission, neglected in our approach. However, if even a small fraction of the central flux is back scattered toward the disk, the limb darkening changes to a limb-brightening effect (see, e.g., Zyk & Czerny 1994, and references therein), and any enhancement of the dissipation profile within the disk close to the surface will also result in the same effect (see, e.g., Różańska et al. 2015; Petrucci et al. 2020).

A recent study by Bisogni et al. (2019) showed that the low EW[O III] sources have a face-on view and have a flatter IR/ optical SED compared to the high EW[O III] sources, which have an edge-on view with respect to the observer. They also claim that their finding suggests that the torus is clumpy.

Although the clumpy torus model is the best we have currently, a more advanced torus model that can measure the torus parameters and the viewing angle independently is needed in the future.

4.3. Viewing Angle as a Solution to the Quasar-based Claims of Tension in Cosmology

In our paper we mostly focused on future corrections to quasar-based cosmological methods using time delay measurements. However, the issue of the potential problem also applies to the most developed quasar method proposed by Risaliti & Lusso (2015). They tested the ΛCDM cosmological model by using the nonlinear relation between the UV and X-ray luminosities of quasars. In a recent paper by Risaliti & Lusso (2019), they considered the quasars in the redshift range of 0.5–5.5 and plotted the Hubble diagram. Their results show that the value of the Hubble parameter $H_0$ matches that of
Suzuki et al. (2012) at lower redshift (0–1.4). However, at higher redshift, it deviates from the ΛCDM model at a statistical significance of 4σ (Risaliti & Lusso 2019). Considering the effect of viewing angle in their Equation (2) (Risaliti & Lusso 2015), we derive the luminosity distance dependence on the viewing angle, i.e., \( D_L \sim (\cos i)^{0.75} \) for \( \gamma \sim 0.6 \), which is even steeper than \( D_L \sim (\cos i)^{0.5} \) characteristic for reverberation-based methods.

Assuming that the extinction is unimportant, we illustrate how the viewing angle can mimic the departure from the standard (ΛCDM) model. We assume that, statistically, the effective viewing angle \( i \) of an AGN may vary as \( \cos i = \cos \theta_0 (1 + z)^2 \). We plot the distance modulus for the standard ΛCDM model in Figure 10. We also plot the best fit to the quasar data provided by Risaliti & Lusso (2019) in the form of a cosmographic approach, taking their values of the best-fit parameters. We should note here that the use of this expansion has been questioned by Banerjee et al. (2020), but it represents the observational results from Figure 2 of Risaliti & Lusso (2019). If we fix the slope at \( \beta = 0.3 \), our points roughly reproduce the observed departure. The requested change in the viewing angle (values provided in the plot) is considerable, but plausible. If the extinction plays a role like in our torus model (see Section 2.2), the requested change in the viewing angle might be even lower.

Our example does not imply that only the standard ΛCDM is correct, and the tension must be attributed to the viewing angle issues. The tension can be real, particularly if the claim is based on probes other than quasars. The need for a revision of the standard model has now been proposed in numerous papers (Wong et al. 2020a; Riess et al. 2019; Lusso et al. 2019), and there are already some studies that have discussed, e.g., the possibility of an \( R_0 = ct \) universe instead of the ΛCDM model to explain the Hubble tension based on quasar data (Melia 2019; López-Corredoira et al. 2016). It is also possible that the claim of the departure from the standard model in Risaliti & Lusso (2019) is not justified; Melia (2019) pointed out that the justification for introducing more free parameters (1 for standard ΛCDM and 2 for more general expansion) have to be done properly, and some aspects of the analysis itself have been questioned by Yang et al. (2020). However, in this paper, we only concentrate on demonstrating that if quasars are used for cosmology, and the method involves the use of the quasar observed flux in the optical/UV band, the viewing angle issue needs to be addressed.

Thus, future quasar-based studies aimed to do precision cosmology should carefully address the issue of the potential statistical evolution of the mean torus properties with the redshift. In their paper, Khadka & Ratra (2020) considered a combination of different cosmological probes and concluded that the observed disagreement with the ΛCDM model at redshifts from 2–5 is most likely caused by problems with quasi-stellar object data. This seems like an important warning.

5. Summary

Our work is based on the results of Gu (2013), which show the redshift evolution of the CF between two samples of quasars at two values of redshift: ~0.8 and ~2.2. The higher covering factor at higher redshift suggests that the structure and density distribution of a dusty torus could be different than at the low redshift quasars. This change would statistically affect the viewing angles of the sources and their observed luminosity. This work is a pilot study to show how the viewing angle can play an important role in measuring the luminosity distance and further the cosmological parameters. We have developed an analytical method to estimate the CF by considering the viewing angle and smooth-density distribution of a dusty torus. Using our method, we estimated the possible error in the luminosity distance that we could make if we did not include the viewing angle. Our results shown in Figure 6 show the error in luminosity distance might go up to 40% for high CF sources (with a high torus opening angle). This effect should be verified by measuring a large sample of quasars, with much better modeling of dusty torus reprocessing and much better data coverage. Our study shows that to have a 1% error in \( H_0 \), one needs to have a statistical accuracy of 0.5° in the effective viewing angle in the quasar sample, and for that, one would need nearly 400 sources in a single redshift bin.

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ORCID iDs
Raj Prince © https://orcid.org/0000-0002-1173-7310
Bożena Czerny © https://orcid.org/0000-0001-5848-4333

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