Students Watching Stars Evolve

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Abstract We describe a study of period changes in 59 RR Lyrae stars, using times of maximum brightness from the GEOS database. The work was carried out by outstanding senior high school students in the University of Toronto Mentorship Program. This paper is written in such a way that high school or undergraduate physics and astronomy students could use it as a guide and template for carrying out original research, by studying period changes in these and other types of pulsating variable stars.

Introduction

The sun is a star, one of hundreds of billions in our Milky Way galaxy, which is one of tens of billions in the universe. Our sun makes life possible on Earth, and other stars may nourish life on their own planets. As stars radiate energy, however, they gradually exhaust their fuel, leading to change and eventual death.

A star is an immense sphere of gas, about 3/4 hydrogen, 1/4 helium, and 2% heavier elements, typically a million times larger and more massive than Earth. Throughout most of its life, it generates energy in its hot, dense core by thermonuclear fusion of hydrogen into helium, currently at the rate of $4 \times 10^{26}$ W, in the case of the sun. Gradually the hydrogen in the core is depleted. The structure and properties of the star change. It expands into a red giant star. The aging of the star takes billions of years, but it has direct consequences that can be observed, measured, and analyzed by students!

The Physics of Stars

The structure of a star is governed by simple laws of physics. Deeper in the star, the pressure of the gas is greater, because of the greater weight of the overlying layers. Since the gas generally obeys the perfect gas law, the temperature and density also increase inward. The outward thermal gas pressure gradient balances the inward pull of gravity on the gas.

Heat always flows from hot to less hot, so energy must flow outwards, either by radiation or convection, depending on the transparency of the gas to radiation. If the star is to remain stable, the outward-flowing energy must then be replenished by energy generation in the core; otherwise the core will cool, pressure will decrease, and the star will shrink. These principles, along with conservation of mass and energy, can be expressed as four equations. These, plus the input mass and composition, the gas law, and information about the transparency of the gas, and about the thermonuclear energy generation, can be solved on a computer to produce a "model" of the star – its physical properties from center to surface. The changes in these properties due to thermonuclear fusion of hydrogen into helium can then be incorporated into the calculations to produce an "evolutionary sequence of models". These predict how the star will age with time. They can be tested in many ways.

Pulsating Stars

At various stages in a star’s life, its properties make it unstable to pulsation – in-and-out (radial) vibration being the simplest kind. The outflowing radiation interacts with the gas in the outer layers of the star, converting radiant energy into mechanical energy. The period $P$, the time for one full vibration, decreases as the radius $R$ increases: $P \propto R^n$, where $n$ is the radial order of the mode. For the first mode, $n = 0$, the period decreases with radius $P \propto R^{-2}$. A red giant star may pulsate with several modes observable as well as the simplest, the fundamental radial mode.
Figure 1: Light curves of an RRab (top) and an RRc (bottom) star – apparent magnitude (brightness increasing vertically) versus time, in units of cycles measured from maximum brightness. RRab stars are pulsating in the fundamental mode, and have sharp maxima. RRc stars are pulsating in the first overtone mode, and have rounded maxima.

A graph of the brightness of the star versus time is called a light curve (Fig. 1). It plots the "apparent magnitude", which is proportional to -2.5log(brightness), versus time (usually the "Julian Date") or the phase, which is the fraction of a cycle elapsed since maximum brightness.

Period Changes in Pulsating Stars

As a pulsating star ages, it initially expands slowly, so the period lengthens and the separation of the times of maximum brightness gradually increases. [Later in its life, the star may contract, in which case the period would decrease.] Specifically: \(\frac{dP}{dt}/P \sim 1.5 \left(\frac{dR}{dt}\right)/R\). This process of period change, though slow, is observable because its effects are cumulative. Consider the analogy of a perfect clock A, and a less perfect clock B that runs one second more slowly every day, starting with the day when it runs at the correct rate. The cumulative error in clock B, after 1, 2, 3, 4 and 5 days, relative to clock A, is: 0, 1, 1+2=3, 3+3=6, 6+4=10 seconds etc. The cumulative error increases as the square of the elapsed time. The possibility of watching stars slowly age by measuring the change in their pulsation period was recognized almost a century ago.

Cepheids, named after the star delta Cephei, are an important class of pulsating stars. Their periods, which range from 1 to 100 days, are correlated with their average power, so their distance can be estimated from the inverse-square law of brightness by comparing their average apparent brightness with the power deduced from their pulsation periods. Polaris (the North Star) is the best-known star in the northern sky. It is a Cepheid with a period of 3.96925... days, which is increasing by 4.5 seconds per year as the star ages.

Educational Context
Over the decades, co-author Percy’s research program on variable stars and stellar evolution has been enriched by the participation of undergraduates, and of outstanding senior high school students through the University of Toronto Mentorship Program (UTMP). These students develop and integrate their science, math, and computing skills, motivated by doing real science, with real data. Co-authors MacNeil, Meema-Coleman, and Morenz were participants in the UTMP in 2011. They met with Percy regularly; they produced a poster paper, on this project, for the end-of-year Research Fair and Reception. MacNeil and Meema-Coleman also developed and gave an engaging astronomy outreach program for 70 grade six students at a local inner-city public library.

This Project

In 2011, for the UTMP, we decided to use pulsation period changes to study the evolution of RR Lyrae stars (the nomenclature of variable stars, leading to names such as RR Lyrae, is explained on an excellent “stars” website maintained by Professor Jim Kaler). They are old sunlike stars, with pulsation periods of about half a day, which have exhausted the hydrogen fuel in their core, swelled into red giants, lost about half their mass, and are now fusing helium into carbon. Since helium fusion is less efficient than hydrogen fusion, and because the star is now emitting about 100 times the power of the sun, it fuels the star for only a few million years, before the star again exhausts its fuel and again swells into a red giant.

There are complications to be expected. Some RR Lyrae stars (designated RRab) pulsate in the fundamental (simplest) mode, others (designated RRc) in the first overtone, and a few (designated RRd) in both. Some RR Lyrae stars also show the Blazhko effect, a long-term (weeks) variation in the amplitude and shape of the light curve which may be due to interference between the in-and-out pulsation mode, and a more complicated “non-radial” mode of pulsation.

Data
cially by skilled amateur astronomers. The American Association of Variable Star Observers is the world leader in coordinating such measurements; it celebrated its centennial in 2011. Amateur astronomers and students can make these measurements with a small telescope and the unaided eye, though most of the measurements are now made with CCD (charge-coupled device) digital cameras. We used times of maximum brightness in a database maintained by the Groupe Européen d’Observation Stellaire (GEOS), including times from previous publications, from the AAVSO and other such organizations, and from robotic telescopes. The database includes about 50,000 times of maximum of over 3,000 stars, covering up to a century. There were 59 stars with data that were long and continuous enough for straightforward analysis.

**Analysis**

For any star, the observed times of maximum brightness $t(i)$ are compared with the predicted times $t(0) + NP$ where $t(0)$ is an initial time of maximum, $P$ is the assumed period in days, and $N$ is an integer – the number of cycles between $t(0)$ and $t(i)$. $t(i) - (t(0) + NP)$ is called $(O-C)$, observed minus calculated. A graph of $(O-C)$ versus $t$ – an $(O-C)$ diagram or OCD – will be a straight line if the period $P$ is constant; its slope will be positive, zero, or negative, depending on whether $P$ is greater than, equal to, or less than the true period. This slope can therefore be used to correct or refine the existing period of the star. If $P$ is changing linearly (i.e. $P = P(0) + kt$ where $k$ is a constant), then the OCD will be a parabola, opening upward or downward, depending on whether $k$ is positive or negative. A parabola can be fitted to the OCD using least-squares software, such as the fitting function in spreadsheet software such as Excel.

A complication occurs if there are long gaps between the $t(i)$: the number of cycles in the gap is uncertain. If the gaps are not too long, and if the measurements before and after the gaps are reasonably continuous, it should be possible to infer the number of cycles in the gap by extrapolation of the existing measurements.

The database was scanned for stars having a sufficient number and continuity of $t(i)$. Periods and $t(i)$s were entered into a spreadsheet; values of $C$ and of $(O-C)$ were determined; OCDs were plotted. If their slopes were non-zero, the existing periods were modified.

We initially classified the OCDs as approximately (i) parabolic (P); (ii) linear (L); (iii) wavy (W); (iv) otherwise peculiar (?) – not parabolic or linear; or (v) too scattered (S) to classify. We later decided that parabolic-linear was a continuum in the sense that “apparently linear” corresponded to a parabola with very small curvature, and that “wavy” was a special case of “peculiar”.

The accuracy of the $t(i)$ is greater for the RRab stars, because they have sharp-peaked light curves, whereas the RRc have more sinusoidal ones (Fig. 1). The accuracy is generally also greater if the $t(i)$ has been measured with a CCD camera, rather than with the unaided eye or photography. The accuracies are typically a few minutes.

A characteristic time scale, for period change and therefore for change in the size of the star, is the period divided by the rate of change of the period $P/(dP/dt)$, in appropriate units, i.e. it is the time required for the period to change by $P$. We designate the time scale by $\tau$ (tau). This is approximately the length of time that the RR Lyrae phase of evolution would last, though that time more correctly depends on how long the star’s properties cause it to be unstable to pulsation. Alternatively, the time scale could be defined as the time required for the period to change by a factor of 2, or $e$. These time scales apply to the few decades which are covered by our data, though evolutionary models predict that the rate of period change should be uniform over millions of years.
Results

Tables 1 and 2 list the stars analyzed: their name, period (day), (O-C) diagram type, RR Lyrae subtype, \([\text{Fe}/\text{H}]\), \(dP/dt\) in seconds per 100 years, and the characteristic evolution time \(\tau = P/dP/dt\) in millions of years. \([\text{Fe}/\text{H}]\) is the logarithm of the ratio of a star’s iron abundance to its hydrogen abundance, compared with that of the sun; it is used as a measure of the abundance of elements heavier than helium.

1. Of the 59 stars that were analyzed, 10 were too scattered (S) to classify, 27 were parabolic (P), 14 were linear (L), \(P + L = 41\) in total; 3 were wavy (W) and 5 were otherwise peculiar (?), \(W + ? = 8\) in total.

2. The two RRd stars were both among the stars with too much scatter for analysis; their period changes must be studied in more sophisticated ways.

3. The Blazhko-effect stars showed no particular tendency for scatter, though 5 of the 18 had peculiar OCDs.

4. Of the RRab stars whose (O-C) diagrams could be fit with parabolas, 15 had increasing periods (median \(\tau\) of 10 million years) and 14 had decreasing periods (median \(\tau\) of 6.7 million years) i.e. there were approximately the same number increasing as decreasing, and the rates of evolution were similar.

5. The RRc stars have sinusoidal light curves. The times of maximum are more difficult to measure than for the RRab stars, whose light curves have sharp maxima, so they were not studied by Le Borgne et al.\(^{11}\). Of the 14 RRc stars in our sample, 4 had scattered OCDs, 2 had peculiar OCDs, 5 showed increasing periods (median \(\tau\) of about 20 million years), and 3 showed decreasing periods with a large scatter in the rate. The median \(\tau\) suggests rather slow evolution, compared with the RRab stars. The sample size, however, is small.

Two examples of OCDs are shown in Figs. 3 and 4. RR Leo has a parabolic OCD, corresponding to a period increase of 2.7 seconds per century, and RR Gem has a peculiar one, neither parabolic nor linear. A few OCDs seem to show abrupt pulsation period changes, which are difficult to explain theoretically. Various explanations have been proposed for these peculiar OCDs, including “burps” of convection\(^{12}\), deep in the star, or magnetic cycles in their outer layers.

Discussion

It is not easy to compare our results in detail with the predictions of evolutionary models\(^{13}\) because the sign and magnitude of the predicted changes depends strongly on the exact mass and composition of the star, especially the abundance of the elements heavier than helium (denoted \(Z\)). The value of \([\text{Fe}/\text{H}]\) (see Tables 1 and 2) is used as a proxy for \(Z\). Indeed, the stars in our sample have a wide range of values of \([\text{Fe}/\text{H}]\), and probably of masses (though most are probably between 0.4 and 0.6 times the sun’s). The magnitudes of the observed period changes, and the values of \(\tau\) are reasonably consistent with the models, though the models predict many more period increases than decreases. See references 11-13 for more detailed comparisons between observations of period changes, and predictions of evolutionary models.

Pulsation period changes have been studied in other kinds of variable stars, with a variety
pulsate with periods of several hundred days. In these stars, the pulsation drives off the outer layers of the star into space, leaving a white dwarf corpse, a million times denser than water. The pulsation of these stars is complicated by random cycle-to-cycle period fluctuations of a few percent, leading to wave-like (O-C) diagrams. Times of maximum brightness are available for analysis on the AAVSO website.

Eclipsing binary stars are pairs of stars in mutual orbit in which one or both stars periodically eclipses the other star, resulting in a brief decrease in brightness. (O-C) analysis of the times of minimum brightness, also available on-line, provides information on mass transfer or loss in the system.

The period changes in Cepheid pulsating stars are particularly important for testing evolutionary models, and have been extensively studied by Leonid Berdnikov, David Turner, and their colleagues. High school (and university) students can observe the variability of Cepheids using the unaided eye or a camera, and verify the slow period changes in these stars. Students’ observations of these or other variable stars can be entered into the AAVSO International Database, which now contains over 20 million observations, for other researchers to use.

The detectability of linear period changes, due to the aging of the star, increases as the square of the timespan of the data, so it is worthwhile for observers to continue their measurements, and for analysts to repeat the (O-C) analysis as longer datasets become available. For instance: the period changes in some of the RRab stars in our sample were studied a few years ago but the analysis should be repeated periodically. We believe that this type of analysis provides an engaging education experience for undergraduates or for senior high school students; the physical and mathematical concepts are simple, data is available on-line and, with it, students can actually observe the effects of the slow aging of the stars.

Conclusions

Students can use existing data on the times of maximum brightness of pulsating stars to

Figure 3: The parabolic OCD for RR Leo, corresponding to a linear period increase of 2.7 seconds per century.
Figure 4: The non-parabolic, non-linear OCD for RR Gem. The OCD is not explainable as due solely to evolution, but must be due to one or more other processes.

observe and measure the evolutionary changes in the stars, even though the characteristic times, for the changes, are millions to billions of years. In this way, they can make meaningful use of their science and math skills, motivated by doing real science, with real data.

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| Name     | Period day | OCD | Type | [Fe/H] | dP/dt s/100Y | τ MY |
|----------|------------|-----|------|--------|--------------|------|
| DX Del   | 0.47261807 | P   | ab   | -0.11  | 0.412        | 9.60 |
| EZ Cep   | 0.37899919 | P   | c    |        | 0.404        | 7.85 |
| EZ Lyr   | 0.52526415 | L:  | ab   | -1.30  | 0.047        | 93.5 |
| GI Gem   | 0.43326648 | L:  | ab   |        | -0.87        | -4.17|
| IO Lyr   | 0.57712192 | L:  | ab   | -1.14  | -0.22        | -21.9|
| OV And   | 0.47058076 | L:  | ab   |        | -0.116       | -33.9|
| RR Cet   | 0.55302795 | P   | ab   | -1.60  | 0.260        | 17.8 |
| RR Gem   | 0.39731060 | ?   | ab   | -0.29  |             |      |
| RR Leo   | 0.45238969 | P   | ab   | -1.60  | 2.660        | 1.42 |
| RR Lyr   | 0.56683295 | L:  | ab   | -1.14  | -0.676       | -7.01|
| RS Boo   | 0.37733834 | P   | ab   | -1.36  | 0.381        | 8.29 |
| RU CVn   | 0.57324490 | W   | ab   |        |             |      |
| RU Psc   | 0.39038500 | S   | c    | -1.65  |             |      |
| RV Cap   | 0.44774566 | P   | ab   | -1.61  | -1.238       | -3.03|
| RV CrB   | 0.33156500 | S   | c    | -1.69  |             |      |
| RV Oct   | 0.57116250 | S   | ab   | -1.71  |             |      |
| RV UMa   | 0.46806000 | W   | ab   | -1.04  |             |      |
| RW Cnc   | 0.54720569 | P:  | ab   | -1.67  | 1.910        | 2.40 |
| RW Dra   | 0.44291700 | ?   | ab   | -1.55  |             |      |
| RX Cet   | 0.57371095 | L:  | ab   | -1.28  | 1.108        | 4.33 |
| RZ Cep   | 0.30868530 | S   | c    | -1.77  |             |      |
| RZ Lyr   | 0.51124591 | P   | ab   | -1.69  | -1.578       | -2.71|
| S Com    | 0.58658810 | P   | ab   | -1.91  | -0.892       | -5.50|
| SS Cnc   | 0.36733833 | P   | ab   | -0.24  | 0.185        | 16.6 |
| SS Tau   | 0.36991119 | L   | ab   |        | -1.270       | -2.44|
| ST Boo   | 0.62229368 | L   | ab   | -1.72  | 0.348        | 15.0 |
| ST Leo   | 0.47798421 | L   | ab   | -1.17  | -0.416       | -9.61|
| SU Dra   | 0.66042067 | P   | ab   | -1.39  | 0.534        | 10.3 |
| SW And   | 0.44226806 | P   | ab   | -0.24  | -1.446       | -2.56|
| SW Aqr   | 0.45930290 | L   | ab   | -1.63  | 0.067        | 57.4 |

Table 1: Rates of period change, and characteristic evolution times for RR Lyrae stars.
| Name      | Period   | OCD | Type | [Fe/H] | dP/dt s/100Y | τ MY |
|-----------|----------|-----|------|--------|--------------|------|
| SW Dra    | 0.56967136 | P   | ab   | -1.12  | 0.353        | 13.5 |
| SX Aqr    | 0.53571243 | P   | ab   | -1.87  | -0.686       | -6.53|
| V394 Her  | 0.43605526 | P:  | ab   | -       | -0.894       | -4.08|
| SZ Hya    | 0.53724022 | ?   | d    | -       | -            | -    |
| TU UMa    | 0.55765749 | L:  | ab   | -1.05  | -0.160       | -29.1|
| SX UMa    | 0.30711780 | ?   | c    | -1.82  | -            | -    |
| AA Aql    | 0.36178731 | P:  | ab   | -0.20  | 0.032        | 94.6 |
| AH Cam    | 0.36872857 | B   | ab   | -       | -4.100       | -0.75|
| AQ Cnc    | 0.54851950 | P:  | ab   | -       | -0.353       | -13.0|
| AR Per    | 0.42554959 | P   | ab   | -0.17  | 0.250        | 14.2 |
| AV Peg    | 0.39037583 | P   | ab   | 0.02   | 1.537        | 2.13 |
| AA CMi    | 0.47632365 | P   | ab   | -0.15  | 0.499        | 7.99 |
| AC And    | 0.71123760 | S   | d    | -1.16  | -            | -    |
| AN Ser    | 0.52207130 | S   | ab   | -0.07  | -            | -    |
| AQ Lyr    | 0.35714240 | S:  | ab   | -       | -            | -    |
| AR Her    | 0.47002800 | ?:  | ab   | -1.30  | -            | -    |
| AT And    | 0.61691220 | S   | ab   | -1.18  | -            | -    |
| AR Ser    | 0.57514160 | S   | ab   | -0.85  | -            | -    |
| BC Dra    | 0.71958926 | L:  | ab   | -2.00  | -0.777       | -7.75|
| BD Dra    | 0.58906655 | P:  | ab   | -       | 4.240        | 1.16 |
| TV Boo    | 0.31256050 | P   | c    | -       | 0.087        | 30.1 |
| AE Boo    | 0.31489430 | L:  | c    | -       | 0.248        | 10.6 |
| ST CVn    | 0.32904500 | W   | c    | -       | -            | -    |
| BB CMi    | 0.39642400 | S   | c    | -       | -            | -    |
| VZ Dra    | 0.32102829 | P   | c    | -       | -1.441       | -1.93|
| LS Her    | 0.23080777 | P:  | c    | -       | 0.052        | 42.4 |
| TV Lyn    | 0.2405125 | P:  | c    | -       | 0.091        | 22.9 |
| DH Peg    | 0.2551056 | L:  | c    | -       | -0.037       | -60.3|
| SS Psc    | 0.28779248 | P   | c    | -       | -0.207       | -10.2|

Table 2: Rates of period change, and characteristic evolution times for RR Lyrae stars (continued).