Abstract

The recently observed accelerated expansion of the universe has put a challenge for its theoretical understanding. As a possible explanation of this, it is considered that the most part of the present universe is filled with a form of energy that exerts a negative pressure called dark energy, which drives the acceleration. In the present work, we assume a dynamical dark energy model, where dark energy interacts with matter and grows at the expense of the latter. Using this model, we discuss the evolution of the universe within the context of loop quantum cosmology. Our work successfully explains the presently observed accelerated expansion of the universe, by predicting that the present universe is phantom dominated. We also found that in the past, the expansion of the universe was decelerated one and transition from deceleration to acceleration would occur at \( t_q = 0.688t_0 \), where \( t_0 \) is the present age of the universe. Again, our analysis predicted that at the transition time, the universe would be dominated with quintessence type dark energy.

Keywords Dark energy · Loop quantum cosmology · Deceleration parameter · Equation of state parameter

1 Introduction

In theoretical physics, classical cosmology, which is described by the Einstein’s general theory of relativity, solves many mysteries about the universe. General theory of relativity provides the idea about gravitational force and its implications, in cosmological and astronomical scales. But, it is unable to give a reasonable explanation on Big-Bang singularity in macroscopic scale. So we would like to orient our focus from classical aspect to the quantum one, which deals with the physics of matter in small scales. This general theory of relativity and quantum theory, are the two excellent proven theories of modern physics. However, both these individual theories cannot successfully explain the complications like Big-Bang singularity that arise in standard cosmology. A solution to that may be achieved by combining both the theories into a single theory, coined as quantum gravity theory. Loop Quantum Gravity (LQG) (Ashtekar and Lewandowski 2004; Rovelli 2004; Thiemann 2007; Han et al. 2007; Bodendorfer 2016; Chiou 2015; Ashtekar 2012) is one of the special alluring features of quantum gravity theories. LQG is completely non-perturbative, explicit background independent approach to quantum gravity. Generally, implication of LQG on cosmology for the survey of our universe, is called loop quantum cosmology (LQC) (Ashtekar et al. 2006a,b; Ashtekar and Singh 2011; Banerjee et al. 2012). The several outcome of LQC can be explained on the basis of two possible methods. The first kind of methods introduce modifications in the inverse scale factor below the critical scale factor. These methods also provide the idea about the replacement of classical Big-Bang by quantum bounce with some advantages of avoidance of singularities (Sami et al. 2006), inflation related problems (Zhang and Ling 2007; Bojowald et al. 2011) etc. But when the scale factor is very large, the modifications introduced by first kind of methods become ineffective, and then the second kind of methods come into picture (Chen et al. 2008; Jamil et al. 2011). The second kind of methods add a quantum correction term to the standard Friedmann equation (Ashtekar et al. 2006a,b; Singh 2006; Copeland et al. 2006; Artyomowski et al. 2009). This term is \(-\frac{\rho^2}{2\dot{\rho}}\), where \( \rho \) is the
density of mass-energy of the universe and \( \rho_c \) is the critical density at which bounce occurs. In a contracting universe, \( \dot{a} \) is less than zero. But when the density of mass-energy (\( \rho \)) is the same order as critical density \( \rho_c \), \( \dot{a} \) becomes zero, which indicates the end of contraction of the universe (Baggot 2018). Hence the universe moves from the classical Big-Bang to the quantum big bounce and oscillates forever. Again, this kind of modification shows the avoidance of Big-Bang singularity, Big Rip (Alonso-Serrano et al. 2018; Haro 2012) and several future singularities (Sami et al. 2006). In LQC, several features like quantum bounce (Zhu et al. 2017), super inflation (Xiao et al. 2013) and future singularity problem etc., attract cosmologists and mathematicians to a common platform to make out our own universe. Also, recently more researchers have paid attention for explaining presently observed accelerated expansion of the universe in LQC (Cognola et al. 2005; Fu et al. 2008; Wu and Zhang 2008; Xiao and Zhu 2010; Jamil 2010; Sadjadi and Jamil 2011; Oikonomou 2019).

On the other hand, observations (Adel et al. 2014, 2016a,b) demand that present universe is largely dominated by two components with unknown character and origin, named as dark energy and dark matter. In some models, it is assumed that perhaps the dark energy and dark matter are coupled to each other, so that they behave like a single dark fluid. Although this consideration sounds slightly phenomenological, but this possibility cannot be ruled out by any observations. So, one can of course think of some interaction between these two fields. The idea of coupling in the dark sectors was initiated by Wetterich (Wetterich 1995) and subsequently discussed by Amendola (Amendola 2000) and others. In the last couple of years, different kind of interacting dark energy models (Khurshudyan and Khurshudyan 2017; Xu and Zhang 2016; Zimdahl 2012; Majerotto et al. 2010; Yang et al. 2017, 2018a; Nayak 2020; Yang et al. 2018b; Bamba et al. 2012; Mishra et al. 2019a; Ray et al. 2019; Mishra et al. 2019b; Nayak and Singh 2009; Billyard and Coley 2000; Olives et al. 2005; del Campo et al. 2009; Väliivita et al. 2010; Pan and Chakraborty 2013; Yang and Xu 2014; Pan et al. 2015; Mukherjee and Banerjee 2017; Sahu and Nayak 2019; Odintso et al. 2017; Odintsov and Oikonomou 2018) have been studied by several researchers. But most of the studies on the dark energy models, are conducted in the light of classical Einstein gravity. However, the evolution of the universe seems plausible, if one considers quantum gravity. So the study on impact of interacting dark energy in the framework of quantum gravity will become a useful tool to understand the dynamical universe.

In this work, we focus on the evolution of the universe within the context of loop quantum cosmology by assuming a non-gravitational interaction between dark energy and dark matter. Here, we aim to discuss the variations of cosmic scale factor, matter-energy density and deceleration parameter for different cosmological eras on the basis of LQC.

In our study, we also try to find out the exact form of dark energy for the present universe. Again in this environment, we investigate the form of dark energy at transition period, where the expansion of the universe goes from decelerated to accelerated one. Comparing these two stages of evolution, we try to describe the nature of dark energy through its equation of state parameter, for whole evolution of the universe.

2 Basic framework

For a spatially flat FRW universe \( (k = 0) \) filled with dust and dark energy, the Friedmann equation, Raychaudhuri’s equation and energy conservation equation in LQC (Bojowald 2005; Ashtekar et al. 1998; Chen et al. 2008; Jamil et al. 2011; Li and Ma 2010) take the form

\[
H^2 = \frac{8\pi G}{3} (\rho_x + \rho_m) \left(1 - \frac{\rho_x + \rho_m}{\rho_c}\right),
\]

\[
\dot{H} = -4\pi G (\rho_x + \rho_m + p_x) \left(1 - \frac{2(\rho_x + \rho_m)}{\rho_c}\right),
\]

and

\[
(\dot{\rho}_x + \dot{\rho}_m) + 3H (\rho_x + \rho_m + p_x) = 0
\]

respectively. Where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter, \( \rho_x \) is dark energy density, \( \rho_m \) is matter density, \( p_x \) is pressure of the dark energy and \( \rho_c \) represents the critical value of energy density of the universe given by \( \rho_c = \frac{3}{16\pi} \rho_{pl} \) with \( \gamma = \frac{\ln 2}{\ln 3} \) is the dimensionless Barbero-Immirzi parameter (Ashtekar et al. 1998; Domagala and Lewandowski 2004; Meissner 2004) and \( \rho_{pl} \) is the energy density of the universe in Plank time.

In generalized interacting dark energy model, we assume that during evolution of the universe, dark energy and matter do not conserve separately but they interact with each other and one may grow at the expense of the other. Inserting this idea, energy conservation equation can be written as

\[
\dot{\rho}_m + 3H\rho_m = Q,
\]

\[
\dot{\rho}_x + 3H(1 + \omega)\rho_x = -Q,
\]

where \( Q = \Gamma \rho_x \) with \( \Gamma > 0 \), is the interaction rate (Sen and Pavón 2008), having dimension of Hubble parameter and \( \omega = \frac{p_x}{\rho_x} \) denotes the equation of state parameter for dark energy. But the observations (Adel et al. 2016a) demand that 68.3% of present universe is filled with dark energy and present age of the universe is \( 13.82 \times 10^9 \) years. Again, by considering that the early universe is completely filled with
dust, and dark energy appeared due to its decay with $\Gamma$ as the interaction rate, we can estimate $\Gamma$ as (Nayak 2020)

$$\Gamma \approx 4.942 \times 10^{-11} \text{(yr)}^{-1}.$$ (5)

Thus at any time $t$, the ratio of matter to dark energy density becomes

$$r = \frac{\rho_m}{\rho_x} = 1 - \frac{\Gamma t}{\Gamma t}.$$ (6)

### 3 Deceleration parameter

The cosmological parameter, which determines the nature of expansion of the universe, is known as deceleration parameter and mathematically, it can be written as $q = -1 - \frac{\dot{H}}{H^2}$. But from equations (1) and (2), we have

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \left(1 + \frac{1 + \rho}{1 + \rho} \right) \frac{1 - 2 \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}{1 - \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}.$$ (7)

So deceleration parameter $q$ becomes

$$q = -1 + \frac{3}{2} \left(1 + \frac{1 + \rho}{1 + \rho} \right) \frac{1 - 2 \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}{1 - \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}.$$ (8)

According to our model at any time $t$, the density of the universe is $\rho = \rho_m + \rho_x$ and in LQC, the expression for $\rho$ in radiation-dominated and matter-dominated eras can be written as (Dwivedee et al. 2014)

$$\rho (t)_{t < t_e} = \rho_0 \left[ \frac{\rho_0}{\rho_c} + \frac{\left(\frac{8\pi G}{3} \rho_0 \frac{1}{2} \left(t - t_e\right)\right)}{ \left(1 - \frac{\rho_0}{\rho_c}\right)^2 \left(1 + \frac{1 + \rho}{1 + \rho} \right) \left(1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)} \right],$$ (9)

and

$$\rho (t)_{t > t_e} = \rho_0 \left[ \frac{\rho_0}{\rho_c} + \frac{\left(\frac{8\pi G}{3} \rho_0 \frac{1}{2} \left(t - t_0\right)\right)}{ \left(1 - \frac{\rho_0}{\rho_c}\right)^2 \left(1 + \frac{1 + \rho}{1 + \rho} \right) \left(1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)} \right],$$ (10)

where $t_e$ is the time of radiation-matter equality and $t_0$ is the present age of the universe.

Now deceleration parameter can be written as

$$q = -1 + \frac{3}{2} \left(1 + \frac{1 + \rho}{1 + \rho} \right) \frac{1 - 2 \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}{1 - \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}.$$ (11)

At $\rho = \frac{\rho_c}{2}$, we get $q = -1$, which indicates the occurrence of inflation in the early universe (Guth 1981; Kofman et al. 1994). Because during inflation $a \propto e^{Ht}$, and hence $q = -1$.

Again for accelerated expansion, the second term of R.H.S. in equation (11) should be less than 1 i.e.

$$\frac{3}{2} \left(1 + \frac{1 + \rho}{1 + \rho} \right) \frac{1 - 2 \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)}{1 - \left(\frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)} < 1,$$

which gives

$$\omega < - (1 + r) + \frac{2}{3} (1 + r) \left[ \frac{1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x}}{1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x}} \right].$$ (12)

Thus $\omega$ is negative and its magnitude must be

$$|\omega| > (1 + r) - \frac{2}{3} (1 + r) \left[ \frac{1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x}}{1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x}} \right].$$ (13)

For present accelerated expansion

$$|\omega| > (1 + r_0) - \frac{2}{3} (1 + r_0) \left[ \frac{1 - \frac{\rho_0}{\rho_c}}{1 - \frac{\rho_0}{\rho_c}} \right].$$ (14)

Using the values of $r_0$, $\rho_0$ and $\rho_c$ in above equation (13), we get

$$|\omega| > 0.488.$$ (14)

In our calculation, we have taken $\rho_0 = 1.1 \times 10^{-29}$ (gm/cm$^3$), $\rho_c = 5.317 \times 10^{34}$ (gm/cm$^3$), $G = 6.673 \times 10^{-8}$ (dyne cm$^2$/gm$^2$), $t_0 = 4.36 \times 10^{17}$ (sec), $\Gamma \approx 4.942 \times 10^{-11}$ (yr$^{-1}$).

Now we construct Table 1 from equation (11), which shows the variation of deceleration parameter ($q$) with equation of state parameter ($\omega$) of dark energy, for present time ($t_0$).

Again, by comparing with present observational data (Santos et al. 2016), $q_0 \approx -0.55$, we can get $\omega_0 = -1.02449$. Here, subscript '0' refers to present value.

For the radiation-dominated era ($t < t_e$), the scale factor varies as (Dwivedee et al. 2014)

$$a (t)_{t < t_e} = \left[ \frac{\rho_0 a_0^3 a_e}{\rho_c} + \frac{\left(\frac{8\pi G}{3} \rho_0 \frac{1}{2} \left(t - t_e\right)\right)}{ \left(1 - \frac{\rho_0}{\rho_c}\right)^2 \left(1 + \frac{1 + \rho}{1 + \rho} \right) \left(1 - \frac{\rho_m + \rho_x}{\rho_m + \rho_x} \right)} \right],$$ (15)
Similarly for the matter-dominated era \((t > t_e)\), the scale factor varies as

\[
a(t)_{t > t_e} = \left[ \frac{\rho_0 a_0^3}{\rho_c} + \left\{ \frac{3}{2} \rho_0 \frac{1}{a_0^2} \sqrt{\frac{8\pi G}{3}} (t - t_0) \right\} \right] .
\]  

(16)

Thus, for matter-dominated era, redshift \((z)\) can be calculated as

\[
z = \left[ \frac{\rho_0}{\rho_c} + \left( a_0^3 \frac{\rho_0 a_0^3}{\rho_c} \right) \right]^{-\frac{1}{2}} \left( t/t_0 - 1 \right) .
\]

(17)

So the expression for \((t/t_0)\) can be written in terms of \(z\) as

\[
\frac{t}{t_0} = 1 + \left[ \left( 1 + z \right)^{-3 - \frac{1}{2}\frac{\rho_0}{\rho_c}} - \left[ 1 - \frac{1}{2}\frac{\rho_0}{\rho_c} \right]^{\frac{1}{2}} \right] .
\]

(18)

If we consider the present universe is dark energy dominated, then the dark energy domination would start from the time, at which transition from deceleration to acceleration would occur. At transition time, \(q = 0\) and thus equation (11) by the use of equation (6), gives

\[
\omega_{q=0} = \frac{2}{3t} \left( \frac{1 - \frac{\rho_0}{\rho_c}}{1 - 2\frac{\rho_0}{\rho_c}} \right) - \frac{1}{\Gamma t} .
\]

(19)

Using equations (17) and (19) we construct Table 2, which shows variations of transition redshift \((z_{q=0})\) and transition equation of state parameter of dark energy \((\omega_{q=0})\) with time.

| \(t/t_0\) | \(z_{q=0}\) | \(\omega_{q=0}\) |
|---|---|---|
| 1.0  | 0    | −0.4878 |
| 0.95 | 0.0580 | −0.5135 |
| 0.90 | 0.1252 | −0.5420 |
| 0.85 | 0.2042 | −0.5739 |
| 0.80 | 0.2987 | −0.6098 |
| 0.75 | 0.4143 | −0.6504 |
| 0.70 | 0.5596 | −0.6969 |
| 0.6880 | 0.6003 | −0.70908 |
| 0.65 | 0.7489 | −0.7505 |
| 0.60 | 1.0087 | −0.8130 |
| 0.55 | 1.3929 | −0.8870 |
| 0.50 | 2.0359 | −0.9757 |

### 4 Nature of dark energy and accelerated expansion

As we have seen from previous section, equation of state parameter of dark energy \(\omega\) varies with time in such a way that at \(t = 0.688t_0\), \(\omega = −0.70908\) and at \(t = t_0\), \(\omega = −1.02449\). Now for constructing the exact form of equation of state parameter of dark energy, we consider \(\omega\) in a general quadratic form as

\[
\omega = at^2 + bt
\]

where, \(a\) and \(b\) are the constant coefficients. Here, we have taken the quadratic form of \(\omega\). Because we have only two values of \(\omega\) for two different times and so we have the possibilities to take \(\omega\) as either a straight line or a quadratic function. But quadratic function contains more variation than a straight line and hence it is more appropriate for an unknown function, to be chosen as a quadratic function.

For the present era, \(\omega\) takes the form

\[
-1.02449 = at_0^2 + bt_0 .
\]

(20)
And similarly for the recent past, when the deceleration parameter is zero, $\omega$ takes the form

$$-0.70908 = a (0.688)^2 t_0^2 + b (0.688) t_0. \quad (21)$$

Solving Equations (20) and (21), we get

$$a = \frac{0.01971}{t_0^2} \quad \text{and} \quad b = -\frac{1.0442}{t_0}. \quad (22)$$

Putting the values of $a$ and $b$ in the expression of $\omega$, we get

$$\omega = 0.01971 \left(\frac{t}{t_0}\right)^2 - 1.0442 \left(\frac{t}{t_0}\right). \quad (22)$$

We plot Fig. 1 from equation (22) by using equation (18), which shows the variation of equation of state parameter of dark energy ($\omega$) with redshift ($z$). From Fig. 1, we found that $\omega < -1$ for present time ($z = 0$), which indicates that the present universe is phantom dominated.

The equation for deceleration parameter can be written by using equations (6), (10) and (22) in equation (11), as

$$q = -1 + \frac{3}{2} \left\{ 1 + \left(\frac{1 - \Gamma(t/t_0)\rho_c}{\Gamma(t/t_0)\rho_c}\right) + 0.01971 \left(\frac{t}{t_0}\right)^2 - 1.0442 \left(\frac{t}{t_0}\right) \right\} \left(\frac{1}{1 + \left(\frac{1 - \Gamma(t/t_0)\rho_c}{\Gamma(t/t_0)\rho_c}\right)}\right)^{-1}.$$ (23)

We plot Fig. 2 from equation (23) by using equation (18), which shows the variation of deceleration parameter ($q$) with redshift ($z$). From Fig. 2, we found that the present universe undergoes an accelerated phase of expansion and the transition from decelerated to accelerated expansion of the universe would be occurred at $z_q = 0 \approx 0.6$ (or $t_q = 0 \approx 0.688 t_0$).

5 Discussion and conclusion

We, here, have studied the evolution of the universe in LQC by using a dynamical interacting dark energy model, where dark energy interacts with matter and grows at the expense of latter. First we used the concept of interacting dark energy in evaluating the deceleration parameter for different cosmic eras. From our analysis, we found that for explaining the accelerated expansion, the present universe must be dominated by phantom type dark energy, since the value of equation of state parameter of dark energy is found to be $-1.0245$. Again we have picturised the nature of dark energy through evaluating its equation of state parameter for whole evolution of the universe. To get this, we first calculated the transition time and found that the transition from deceleration to acceleration would be occurred, when the age of the universe was nearly 0.688 times the present age. At that time, the value of equation of state parameter of dark energy would be $-0.709$, which shows that the universe was then dominated with quintessence type of dark energy. Comparing these two stages of evolution, one at present time and another at transition time, we construct an expression for equation of state parameter of dark energy for whole evolution of the universe. Using this, we have shown the variation of deceleration parameter with redshift for whole evolution of the universe and from our analysis, we concluded that
the universe will undergo an accelerated phase of expansion even in far future.

**Data Availability Statement** No new data were created or analyzed in this study. Data sharing is not applicable to this article.

**Declarations**

**Conflict of Interest** The authors declare that they have no conflicts of interest.

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