Pure Luminosity Evolution Hypothesis for QSOs: From Luminosity Functions to Synthetic Catalogues

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Abstract. This paper describes the construction of realistic extragalactic Monte–Carlo catalogues, aimed at comparing the behaviour of cosmological tests versus input parameters. QSO catalogues are built on a luminosity function derived from data through a certain choice of any cosmological and evolutionary model, and through suitable computation of individual maximum volumes in complete (but magnitude– and redshift–limited) samples. Details of this computation are examined for the case of strong luminosity evolution. The values of the evolution parameter are derived for various cosmologies, corresponding to \( < V/V_{\text{max}} > = 1/2 \) in the sample of 400 Ultra–Violet Excess (UVX) QSOs (?). The various luminosity functions are compared, both for the whole sample and in redshift bins. A characteristic evolution time is defined and computed, depending strongly on the cosmology, but practically constant when expressed in terms of the age of the universe. Algorithms are given for producing catalogues based on the null hypothesis that the objects are uniformly distributed in volume but suffer pure luminosity evolution. The only input based on real data is the luminosity function, but this requires neither a redshift nor an apparent magnitude histogram. Simulated redshift and absolute magnitude histograms are compared to real data.

Key words: Cosmology, Quasars, Numerical simulations

1. Introduction

Various judicious tests have been developed to discriminate between different cosmological models, all requiring large and complete samples of extragalactic objects (quasars or galaxies). Unfortunately, the available samples are magnitude limited, selection effects induce poorly known systematic biases in magnitude and redshift, and extragalactic populations undergo strong evolution over a Hubble time. This paper describes the construction of extragalactic synthetic catalogues with various parent cosmologies, evolution laws and simulated observational biases, with the goal of applying them to the cosmological test described in Paper I (?). A preliminary study of the luminosity function of the real population and of its evolution in various cosmologies is necessary for such a construction; so it is also a by product of the present paper.

The two main extragalactic populations, quasars and galaxies, may be simulated: we focus on quasars in the following since they are more distant and a priori more suited to cosmological tests. A quasar being by definition stellar in appearance, the pair of apparent magnitude–redshift \((m, z)\) is sufficient to characterize it with respect to current cosmological tests. The result of the simulation is thus a list of apparent magnitudes and redshifts, \( m \approx z \) an image.

The observed population suffers strong evolution, as evinced by either the standard \( < V/V_{\text{max}} > \) analyse or direct derivation of the luminosity function in redshift bins. Pure Luminosity Evolution (PLE) seems to be the best model (?), but Pure Density Evolution (PDE) or mixed models are not rejected; and the question remains open for redshifts greater than 2.2 (?). Paper I describes a cosmological test which accounts for such evolution.

Quasars also suffer various biases in their redshift distribution and a strong variability in luminosity. Our initial aim was to construct synthetic quasar catalogues capable of quantifying the behaviour of the test as a function of the various parameters:

- sensitivity to the cosmological parameters;
- photometric uncertainties and strong quasar variability;
- redshift uncertainties and biasing in certain redshift ranges;
- catalogue size and redshift limitations, leading to a preferred observational strategy for the construction of quasar samples;
- this list is certainly not exhaustive.

Last, but not least, most cosmological tests using QSOs rely on some basic assumption concerning their evolution. As a consequence, their result in principle applies only within this framework. With Monte–Carlo catalogues, it is possible to test the uncertainty induced by any false hypothesis on evolution by making cross-tests between different assumed evolutions: this may concern the nature of the evolution, either Density or Luminosity Evolution (DE/LE), as well as its functional form, PoWer law or EXPonential (PWLE/LEXP). The present paper addresses only the case of PLE. PDE will be the subject of a forthcoming paper.

The paper is organized as follows: in Section 2 the basic method is exposed. After a brief review of the cosmological
for a given, actual, sample, gives a number \( V \)

volumetric and evolutionary model, define a QSO. Given the absolute magnitude are able to draw the two variables \( M \) variable

uniformly distributed in volume on Gpc scales, so that the

minosities are distributed according to the observed LF and

repeated until the desired catalogue size is reached.

two variables. For each cosmological model, the procedure i s

detailed in Section 3. In Section 4 the sample of Boyle is used

in sufficient number so that we may apply this average evolu-

tional parameter ensuring the condition

\[ \alpha \equiv 0, \quad \Omega_K \equiv 1 - \Omega_m - \Omega_L \]

The null hypotheses are: that PLE applies, that the lu-

minosity decreasing with increasing time, and that they a re

very important for the creation of synthetic catalogues because it

scales the distance and the absolute luminosity of the objects.

Nevertheless, \( H_0 \) has no influence on individual \( V/V'_a \)s, which

are scale free. The other known cosmological parameters are

related to these three parameters.

No arbitrary choice of a preferred cosmological model is

made. General formulae for the distance and the volume are

given in Caroll et al. (?). We define the conformal coordinate \( \chi(z) \) as:

\[ \chi(z) = 2|\Omega_K|^{1/2} \times \frac{dz'}{((1+z')^2(1+\Omega_m z') - z'(2+z')\Omega_K)^{1/2}} ; \]

The volume out to redshift \( z \) is written:

\[ V(z) = \left( \frac{c}{H_0^3} \right) \frac{1}{21K|\Omega_K|^{1/2}} \left[ \sinh(\chi(z)) - \chi(z) \right], \quad (1) \]

where \( \Omega_K = 1 - \Omega_m - \Omega_L \) is the curvature, \( \sinh \) is \( \sin \) if \( \Omega_L < 0 \) and \( \sinh \) if \( \Omega_L > 0 \). This equation is very useful for numerical

computation because it does not require inverse trigonometric

functions. Contrary to what happens in a closed universe, with

the expressions given, e.g., in Caroll et al. (?), Eq.(1) is single

valued and there is no discontinuity at the value \( \chi = \pi \), as

shown in Mathez et al. (?).

3.2. Evolution

The quasar population resides in an extremely young universe,

and it has evolved up until the present epoch. It is likely that

individual QSOs do not live for a long period of time, so we

speak rather about the evolution in a statistical sense. In any

case, assuming a PLE model for quasars amounts to assuming

that the fraction of active quasars is constant with epoch, their

luminosity decreasing with increasing time, and that they are

in sufficient number so that we may apply this average evolu-

tional law to individual quasars in order to derive maximum

redshifts. The absolute magnitude at redshift \( z \) is given by:

\[ M(z) = M_0 - 2.5\log_{10}(e(z)), \quad (2) \]

where \( M_0 \) is the absolute magnitude at \( z = 0 \), and \( e(z) \) is the

evolutionary law:

\[ e(z) \equiv (1 + z)^{k_L} \]

for a Power Law Luminosity Evolution (PWLE), and

\[ e(z) \equiv \exp(k_L t(z)/H_0) = \exp(t(z)/\tau) \]

(4)

for an Exponential Luminosity Evolution (LEXP); \( k_L \) is the

evolutionary parameter, \( t(z) \) the look-back time and \( \tau \) the char-

acteristic evolutionary time. Strictly speaking, as quoted by

Bigot and Triay (?), this evolution law has only a statistical

sense and does not apply to individual QSOs. However, to com-

pute each \( V/V'_a \) value, we fictitiously move each QSO toward

high redshifts, and it is necessary to apply this evolution law
to each QSO.

Assuming a power law spectrum \( f_\nu = \nu^{-\alpha} \) with a spec-
tral index \( \alpha = 0.5 \), the Mattig relation relates the observables

\((m, z)\) of a quasar to \( M_0 \), its absolute magnitude at \( z = 0 \):
Fig. 1. Four possible apparent magnitude – redshift plots: monotonically increasing, one maximum, both one maximum and one minimum, and monotonically decreasing. The evolutionary tracks are drawn according to the cosmology and exponential luminosity evolution as described by the top labels. The $m = m(z)$ curves (solid lines) correspond to bins of constant width in absolute magnitude. The horizontal dashed line represents a constant limiting magnitude $B = 21$; the dashed $m(z)$ line corresponds to the faint separator $S_2$ between magnitude-limited and redshift-limited quasars (see text). Depending on the parameters (cosmology and evolution), the same quasar may be either magnitude– or redshift–limited. In models where $m(z)$ shows a maximum (for example in Fig. 1b), for quasars lying between $S$ and $B = 21$ there exists a redshift slice where $m(z)$ is above $B = 21$. The corresponding volume must be subtracted from the available volume $V_2 - V_1$. Quasars of the complete Boyle et al. (1990) sample are superimposed.
here \( K(z) = (1+z)^{(a-1)} \) is the K-correction.

The evolutionary tracks \( m(z) \) on the Hubble diagram are shown in Fig. 1 for a given set of quasars in four different cosmologies and evolutionary histories. The curves appear to be quite different. In particular the quasars in Fig. 1b,c,e,f become brighter and brighter as the redshift increases, because the evolution dominates the cosmological dimming at large \( z \).

### 3.3. Distribution of the QSOs in the Hubble Plane

Available complete QSO samples usually have a well defined limiting magnitude, towards faint magnitudes. Due to saturation effects (in the photometry or in the spectroscopy), a bright limiting magnitude may also apply.

All UVX QSOs are limited to redshifts \( z \leq 2.2 \), precisely because of their selection as objects with a UV excess, a standard way to detect quasars in optical astronomy. As the redshift decreases (below \( z = 0.3 \)), there is another limitation because quasars become spatially resolved and can no longer be classified as stellar objects. These redshift limitations prevent the use of maximum (respectively minimum) redshifts outside the limiting redshifts of the sample. Assume that we have a QSO sample in the magnitude and redshift ranges \([m_1, m_2]\) and \([z_1, z_2]\), respectively. For UVX samples we would have \( z_1 = 0.3 \) and \( z_2 = 2.2 \).

We define \( \Phi(M, z) \) as the LF at redshift \( z \) and at absolute magnitude \( M \). However, for the case of PLE it can be written \( \Phi(M(z)) \); then all luminosity functions at different redshifts differ by an additive constant along the absolute magnitude axis. This allows us to work with the LF defined at a fixed epoch (\( z = 0 \)) where \( \Phi(M(z = 0)) = \phi(M_0) \), whatever the evolutionary law (?). The number of objects in the range of absolute luminosity \([M_0, M_0 + dM_0]\) and redshift \([z, z + dz]\) writes:

\[
d^2N = \phi(M_0) \, dM_0 \, H(m_2 - m) \, H(m_1 - m) \times \frac{dV}{dz} \, dz,
\]

with the following notations:

- \( dV \) is the comoving volume element of the shell of redshift \( z \) to \( z + dz \) in the solid angle \( \omega \) of the survey;
- \( \phi(M_0) \) is defined above;
- The four Heaviside functions are the selection function, ensuring that apparent magnitudes and redshifts remain inside 'box' \( B \), or \([m_1, m_2] \cap [z_1, z_2]\), the limitations due to the selection criteria.

We now define \( D \) as the domain in the Hubble plane where all quasars are redshift-limited.

Let \( z_{top} \) (respectively \( z_{bot} \)) be the redshift in the range \([z_1, z_2]\) at which the evolutionary tracks pass through the maximum (minimum) of the \( m(z) \) curve. \( z_{top} \) (\( z_{bot} \)) is the same for all quasars, since their \( m(z) \) curves are parallel, differing by an additive constant equal to the absolute magnitude. Consider a given absolute magnitude \( M_0 \), and let \( m_{M_0, top} \) (\( m_{M_0, bot} \)) be the corresponding faintest (brightest) apparent magnitude in box \( B \):

\[
m_{M_0, bot} = \inf_{z \in [z_1, z_2]} [m_{M_0}(z)] = m_{M_0}(z_{bot});
\]

\[
m_{M_0, top} = \sup_{z \in [z_1, z_2]} [m_{M_0}(z)] = m_{M_0}(z_{top});
\]

\[
m_{M_0, bot} = \inf_{z \in [z_1, z_2]} [m_{M_0}(z)] = m_{M_0}(z_{bot}).
\]

\( m_{M_0, bot} \) is the apparent magnitude corresponding to the absolute magnitude \( M_0 \) at redshift \( z \). \( z_{top} \) is equal to \( z_2 \) in the case of little or no luminosity evolution. Consider the \( m(z) \) curve corresponding to \( m_{top} = m_2 \): it is unique; let us call it the 'faint separator' \( S_2 \). There is also a 'bright separator' \( S_1 \) – the unique curve \( S_1 \) crossing \((z_{bot}, m_{bot} = m_1)\). \( S_2 \) is drawn on Fig. 1. Domain \( D \) is simply the domain located below \( S_2 \) and above \( S_1 \). Let \( M_0^2 \) be the two absolute magnitudes corresponding to the curves \( S_i \), \( i = 1, 2 \). The domain \( D \) simply consists of that part of the Hubble plane brighter than \( M_0^2 \) and fainter than \( M_0^1 \). We call \( D^* \) the complement of \( D \) with respect to the box \( B \). We call \( D \) and \( D^* \) quasars the quasars located in each of these domains.

### 3.4. The \( V/V_{max} \) Test

Since our method relies on \( V/V_0 \) values, it is necessary to review the \( V/V_0 \) computation, particularly in the case of strong evolution. Consider a quasar in the sample with \( (m, z) \) as its representative point in the Hubble diagram, and define the following quantities:

- \( M_0 \) – the quasar absolute magnitude at epoch \( z = 0 \);
- \( z_{max}(M_0, m_i) \) \( (i = 1, 2) \): the redshift (s) where the quasar has an apparent magnitude \( m_i \);
- \( V_0 \) – the available volume;
- \( V_i(1, 2) \): the comoving volumes enclosed by the two limiting redshifts \( z_i \).

The various possible shapes of the evolutionary track (Fig. 1) are crucial for understanding the \( V/V_0 \) test. The quasar may leave the sample in one of two ways: either its magnitude leaves the range \([m_1, m_2]\), or its redshift leaves the range \([z_1, z_2]\). The available volume \( V_0 \) does not reduce to \( V_{max}(M_0, m_2) \): it is the total volume \( V_0 = V_2 - V_1 \), minus \( \sum V_{out} \), the sum of the volumes of all slices of the universe where the evolutionary track is outside box \( B \), so that it prevents the quasar from remaining in the sample.

Assuming that volumes are randomly distributed over the range \([V_1, V_2]\), the variable uniformly distributed over \([0, 1]\) is the ratio \( x \) defined below. The number of quasars in the redshift range \([z_1, z_2]\) is:

\[
N = \int_{(M_0)_1}^{(M_0)_2} dM_0 \Phi(M_0) \int_{sup(z_2, z_{max}(M_0, m_2))}^{inf(z_2, z_{max}(M_0, m_2))} dz \, \frac{dV}{dz},
\]

\[
= \int_{(M_0)_1}^{(M_0)_2} dM_0 \Phi(M_0) V_0(M_0),
\]

where:

\[
V_0(M_0) = inf(V_2, V_{max}(M_0, m_2)) - sup(V_1, V_{max}(M_0, m_1)) - \sum V_{out}.
\]

Note that for \( D \) quasars \( inf(z_2, z_{max}) = z_2 \) and \( sup(z_1, z_{min}) = z_1 \) simultaneously.

It is straightforward to find the new variable \( x \):

\[
x = (V - V_{sup} - \sum V_{out})/(V_{inf} - V_{sup} - \sum V_{out}).
\]
Indeed, one shows that the mean of the ratio \( x \) over the sample is 1/2:

\[
< x > = \frac{1}{N} \int_{(M_0)}^{(M_0)2} dM_0 \Phi(M_0) \times \\
\int_{V_{in,f}}^{V_{in,p}} dV \left( \frac{V - V_{sup} - \sum V_{out}}{V_{in,f} - V_{sup} - \sum V_{out}} \right) \\
= 1/2, \quad \forall \Phi, \forall (M_0)_1, \forall (M_0)_2; \\
\text{moreover, since} \quad dV = V_a(M_0) dx, \quad \text{we have:} \\
\frac{dN}{dx} = \int_{(M_0)}^{(M_0)2} dM_0 \Phi(M_0) V_a(M_0) dx \\
= N dx,
\]

which characterizes a PDF of \( x \) which is uniform over \([0, 1]\). For a redshift–limited sample over \([z_1, z_2]\), taking \( V_{sup} = 0 \) overestimates \( V/V_a \) and taking \( V_{in,f} = V_a \) underestimates \( V/V_a \). In either case, < \( x > \) cannot be equal to 1/2 even in the correct cosmology and with the correct evolutionary law.

Varying the shape of the evolutionary track changes the expression of the variable \( x \). There may exist zero, one or several maxima of \( m(z) \), or the quasar may become fainter than \( m_2 \) at low redshift; in fact there may be several roots of the equation \( m(z) = m_2 \). Fig. 1 shows various curves \( m(z) \) for various cosmologies and evolutionary histories, in order to illustrate the existence of redshift ranges, lying inside the range \([z_1, z_2]\), but for which \( m(z) \) is not in the range \([m_1, m_2]\). Such ranges must be excluded from the volume computation.

4. Luminosity Functions in real samples

The LF is not a direct observable – on the contrary it depends on the choice of the cosmological model and of the evolutionary law. For our purpose, it is necessary to be realistic in reproducing the data; in particular, the artificial catalogues must have the same LF as the real sample whatever the choice of their parent cosmological model.

As discussed in Kassiola and Mathez, (7), given the cosmology and the evolution, there are two ways to derive the LF from data: firstly one can construct the Global Luminosity Function (GLF), by shifting the absolute magnitudes of the whole sample to the present epoch \( \epsilon = 0 \), according to the evolution law, and secondly one can compute the Restricted Luminosity Functions (RLF s), i.e. the LFs in different redshift bins. A necessary condition for PLE to apply is that the LF in all redshift bins differ by a simple shift along the absolute magnitude axis. To do this, one has to compare the RLFs. Assuming that PLE is correct, the reason for choosing the GLF to construct the catalogues is to avoid the noise created by binning the data in redshift. In Section 4.3, we test the PLE hypothesis by comparing of the RLF and the GLF.

4.1. Real data: the Boyle et al. sample

The complete sample of UVX quasars of Boyle et al. (1990) is used. It contains 383 quasars, if we exclude the Narrow Emission Lines (NL) and the redshifts outside the range \([0.3, 2.2]\).

4.2. The Global Luminosity Function and its derivation

We now consider the computation of the GLF for any redshift– and magnitude limited–QSO sample. The sample is binned in absolute magnitude \( M_0 \) with bin width \( \Delta M_0 \). Let \( N_i \) be the QSO number in the \( i^{th} \) absolute magnitude bin. The GLF in bin \( i \) is given by:

\[
\phi_{obs}(M_0) = \sum_{j=1}^{N_i} \left( \frac{1}{V'_{z'}} \right), \tag{10}
\]

\[
B_i = \left[ M_0 - \frac{\Delta M_0}{2}, M_0 + \frac{\Delta M_0}{2} \right], \tag{11}
\]

with an error on \( \phi \):

\[
\sigma'_{\phi} = \left( \sum_{j} V'_{z'}^{-2} \right)^{1/2}. \tag{12}
\]

For the case of the Boyle sample, none of the eight fields intersect; thus, the available volume follows directly from Eq. (8):

\[
V_a = \sum_{k} \omega_k \times \left[ \inf f(V^k_{max}, V_2) - \sup f(V^k_{min}, V_1) - \sum V_{out} \right] , \tag{13}
\]

where \( \omega_k \) is the solid angle of the \( k^{th} \) field.

The GLF looks similar for all of the cosmologies we tried. Special attention has been paid to some of the cosmological models (e.g. \( \Omega = 0; \Lambda = 0; \Omega = 1; \Lambda = 0; \Omega = 0; \Lambda = 0.8 \)), by various authors which found that the LF is well fitted by either a single (?) or a double (?) power–law. We adopted the double power–law model in all cosmologies and for the two functional forms of evolution:

\[
\phi_{model}(M_0) = \frac{\phi^*}{10^{\beta(M_0 - M_\star)\alpha + 1}} + \frac{\phi^*}{10^{\beta(M_0 - M_\star)\beta + 1}}. \tag{14}
\]

\( M_\star \) is the characteristic absolute magnitude of the power–law distribution, i.e. the knee of the distribution. Absolute magnitudes for the quasars are computed according to Eq. (3).

The evolution parameter \( k_L \) is computed by setting < \( x > = 1/2 \) for the whole sample. Such a procedure differs from current work (7; 9), in which \( k_L \) is fitted like the other parameters. The free parameters \( \phi^*, M_\star, \alpha, \beta \) are derived by a \( \chi^2 \) minimization of the binned GLF:

\[
\chi^2 = \sum_{i=1}^{N_{bin}} \left( \frac{\phi_{obs} - \phi_{model}}{\sigma'_{\phi_{obs}}} \right)^2 , \tag{15}
\]

where \( N_{bin} \) is the number of absolute magnitude bins. An important difference with other LF computations is that we have only four free parameters.

Table 1 and Figs. 2 and 3 show a panel of GLFs computed for four different cosmologies, scaled to \( H_0 = 50 \) \( \text{km s}^{-1} \text{Mpc}^{-1} \). The general behaviour of the GLF is identical for the exponential and power–law evolutions. The GLF is enlarged and QSOs are brighter for a \( \Lambda \)–dominated universe. This is due to the
Despite our different LF computation method, we find results quite similar to those of Malhotra et al. (7) and Boyle et al. (7).

4.3. The Restricted Luminosity Functions

Restricted LFs (RLFs) have been computed from the Boyle sample in three different redshift bins ([0.3, 1.0], [1.0, 1.7], [1.7, 2.2]). We are interested in checking whether or not the PLE hypothesis is confirmed. If this is the case, any RLF may be deduced from the GLF by a simple shift along the absolute magnitude axis according to the evolution law (Eq. 3). Since we have computed the GLF in the last Section, we are able to compare our RLF to the GLF. This is done by a $\chi^2$-procedure between the GLF and the three RLFs:

$$\chi^2 = \sum_{i=1}^{N_{bin}} \left( \frac{\phi_{GLF} - \phi_{RLF}}{\sigma_{\phi_{RLF}}} \right)^2.$$  

(16)

Here, $\phi_{RLF}$ and $\sigma_{\phi_{RLF}}$ refer to the RLF, computed from the data in the limited redshift bins, and $\phi_{GLF}$ is shifted to this redshift range.

The result is shown in Table 1 for four cosmologies and the two evolutionary models. The probabilities $P(> \chi^2)$ are high, and, except for one case (LEXP, $\Omega_{mat} = \Lambda = 1$), the test succeeds to better than the 15% confidence level. This result is in contradiction to those of a similar test done by Kassiola and Mathez (1991), which found strong discrepancies between the GLF and the RLF. However, they interpret these discrepancies as the lack of homogeneity of their composite catalogues. Since our results are consistent with PLE (favoured by Boyle et al., 1987), we use this hypothesis to construct our synthetic catalogues.

5. Evolution characteristic time in various cosmologies

In the exponential model, the characteristic evolution time $\tau$ enters explicitly in Eq. (3), the expression for $e(z)$. This is not the case for the power law model - but a characteristic time may be defined as the look-back time to which all luminosities were higher by a factor $e$:

$$\tau = t(z_L = exp(1/k_L) - 1).$$  

(17)

Table 2 (3) gives the evolutionary parameters $k_L$ for a grid of cosmological parameters in PWLE (LEXP). Table 4 (5) gives the corresponding PWLE (LEXP) characteristic times, computed according to Eqs. (4) and (5), respectively. These times are in units of the Hubble time $H_0^{-1}$. As is well known, the quasar evolution time is of the order of $H_0^{-1}/(7.7 \pm 3.5)$ for exponential evolution (7, 7, 7), strongly dependent on the cosmological model. For power-law evolution, the characteristic time is $H_0^{-1}/(3.7 \pm 0.6)$.

Tables 6 and 7 give the same characteristic times as Tables 4 and 5, but in units of the age of the universe in each cosmological model. With the exception of the $\Omega = 0$, $\Lambda = 1$ model, all ratios in Table 6 are close to 0.30±0.02, implying that the characteristic evolution time is about equal to $1/(3.2 \pm 0.2)$ of the age of the universe, whatever the cosmological model, in PWLE models. In LEXP models, this ratio is 1/[(6.7±0.9). The characteristic times in Tables 4 and 5, apparently quite dependent of the cosmology, are indeed surprisingly constant when expressed in terms of the age of the universe.
6. An algorithm for producing quasar catalogues

6.1. The algorithm

Fixing the evolutionary law, the cosmology (including the choice of \( H_0 \)) and the catalogue limits (i.e. \( m_1, m_2, z_1, z_2 \) which allow the box \( B \) and domains \( D \) and \( D^* \) to be defined), each QSO \((m, z)\) is fully determined by the other pair of variables \((V/V_\alpha, M_0)\). This is easy to understand from Fig. 1, where the Mattig functions appear. All of our previous discussion shows that under the PLE hypothesis (which was checked on the data), any sample is fully determined from its limiting magnitudes and redshifts, the GLF and a uniform distribution of the \( V/V_\alpha \). From the probability distribution function (PDF) of absolute magnitudes \( M_0 \), which is \( \phi(M_0) \times V_\alpha(M_0) \), and from the \( V/V_\alpha(M_0) \) ratios uniformly spread over \([0, 1]\), it is possible to draw Monte–Carlo catalogues.

The algorithm is shown in Fig. 4. Choosing the cosmology and a functional form for the evolution, the first part of the work is to compute the evolutionary parameter from the constraint \(< V/V_\alpha > = 0.5\), using the Boyle sample. At the same time, \( V_\alpha(M_0) \) is computed and used to obtain the GLF. In the second part, two series of random numbers, \( \epsilon_1 \) and \( \epsilon_2 \), are used to extract the \( V/V_\alpha \) and the absolute magnitude \( M_0 \) of each object in the synthetic catalogue from the relations:

\[
\begin{align*}
\epsilon_1 &= \frac{V/V_\alpha}{M} = \frac{V - V_{sup} - \sum V_{out}}{V_{inf} - V_{sup} - \sum V_{out}}, \\
\epsilon_2 &= \frac{\int_{M_0}^\infty dM \phi(M)V_\alpha(M)}{\int_{\infty}^\infty dM \phi(M)V_\alpha(M)}.
\end{align*}
\]

Obtaining the redshifts and apparent magnitudes is straightforward from a simple inversion procedure. Synthetic catalogues have been produced in the past, where both a LF plus a redshift histogram were jointly fitted. As already explained, this method suffers from at least two problems: Essentially, the two distributions may be incompatible in different cosmologies. Our method has the advantage of avoiding these problems since we use only one distribution, the GLF, and the other distributions (redshift, magnitude) are derived only after the cosmology and the evolution have been fixed. The algorithm shown in Fig. 4 ensures the coherence of these distributions, moreover it takes the evolution into account very accurately. This is not obviously true in previous methods where it is not clear how the evolution intervenes. In fact, we used the Cosmological Principle to replace the redshift distribution of the standard method by demanding the uniformity of \( V/V_\alpha \). Moreover, the limiting magnitude and the redshift range for the synthetic sample may be different from their values in the input sample.

6.2. Redshift and Magnitude Histogrammes

The input (7) and output (synthetic catalogue with the same observational biases) redshift histogrammes are compared in Fig. 5 for models \( b \) and \( f \) of Table 1. Similarly, the histogrammes of absolute magnitude are compared in Fig. 6. The fit is quite satisfactory for both models; this is true for all models we tried. It is possible to introduce various biases in the synthetic catalogues, depending on which sort of catalogue we want to simulate. For example, for UVX quasars it is well known that the color selection criteria introduce a deficiency of QSOs in the redshift range \([1.5, 1.8]\) (5). Moreover, QSOs are variable objects, which can be simulated by randomly selecting a magnitude variation in a pre–defined distribution function. Noise in the magnitude and redshift measurements may be simulated in the same way.

7. Conclusion

We have reviewed in detail the \( V/V_{\text{max}} \) computation in various cosmologies. The Mattig relation is used to understand how to compute the available volume \( V_\alpha \) of a QSO, even in complicated cases. Both the Pure Luminosity Evolution (PLE) assumption and the Cosmological Principle allow the variables \((M_0, V/V_\alpha)\) to be used instead of \((m, z)\) to construct synthetic QSO samples in a coherent way. Some of the previous catalogue constructions were shown to suffer problems. The Global Luminosity Function used for this purpose is derived in a slightly different way from previous calculations since we fixed the evolution parameter by the condition \(< V/V_\alpha > = 1/2\). This significantly changes the value of this parameter, and hence of the GLF.

A surprising result is that the QSO characteristic evolution times are constant when expressed in terms of the age of the universe, regardless of the cosmology. The PLE hypothesis was checked on the data, which confirms previous results (7). The advantage of such catalogues is that we tightly control all aspects of the sample: the parent cosmology; the strength and the functional form of the evolution, the luminosity function, the magnitude and redshift depths, and the effects of magnitude and redshift biases, all of which influence the results of the new cosmological test introduced in Paper I. In Paper I we apply the test to catalogues with different redshift limita-
tions, which leads us to propose a new observational strategy for the construction of future QSO samples. It is possible to test a catalogue with a wrong hypothesis, for example on the evolution, by assuming a power–law evolution and running the cosmological test with the exponential hypothesis. Moreover, our method is tractable with a Pure Density Evolution hypothesis, provided that density–weighted volume elements $\rho dV$ are substituted for all volume elements $dV$.

Table 2. Power Law Luminosity Evolution (PWLE) parameter in different cosmologies. The model $\Omega = 0$, $\Lambda = 1$, which is marked with an asterisk, is close to a universe without a Big–Bang, and not very reliable.

| $\Lambda$ | $\Omega$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|----------|----------|-----|-----|-----|-----|-----|-----|
| 0.0      | 2.94     | 3.16| 3.28| 3.44| 3.50| 3.60|
| 0.2      | 2.88     | 3.13| 3.28| 3.47| 3.50| 3.63|
| 0.4      | 2.78     | 3.10| 3.28| 3.47| 3.53| 3.66|
| 0.6      | 2.66     | 3.06| 3.28| 3.47| 3.60| 3.69|
| 0.8      | 2.47     | 3.00| 3.31| 3.47| 3.63| 3.75|
| 1.0      | 1.91(*)  | 2.94| 3.35| 3.50| 3.66| 3.81|

Table 3. Same as Table 2 for LEXP, Exponential Luminosity Evolution.

| $\Lambda$ | $\Omega$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|----------|----------|-----|-----|-----|-----|-----|-----|
| 0.0      | 6.65     | 8.02| 9.15| 10.10| 11.02| 11.77|
| 0.2      | 6.02     | 7.40| 8.65| 9.71 | 10.59| 11.40|
| 0.4      | 5.28     | 6.84| 8.15| 9.28 | 10.15| 11.03|
| 0.6      | 4.40     | 6.15| 7.53| 8.78 | 9.71 | 10.65|
| 0.8      | 3.40     | 5.40| 6.96| 8.21 | 9.28 | 10.15|
| 1.0      | 1.90(*)  | 4.59| 6.28| 7.65 | 8.78 | 9.71|
Table 1. The LF (Eq. (10)) in various redshift ranges for four arbitrary cosmologies and two evolution laws. Since we are interested in input parameters to construct catalogues, confidence intervals are not needed. Only the best fit parameters are given. We choose $H_0 = 50\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

| $\Omega \Lambda$ | $k_L$ | $\alpha$ | $\beta$ | $M_\star$ | $\phi^*$ | $P(>\chi^2)$ | $P(>\chi^2)$ | $P(>\chi^2)$ | $P(>\chi^2)$ |
|------------------|-------|----------|---------|-----------|---------|----------------|----------------|----------------|----------------|
|                  |       |          |         |           | $10^{-6}\text{Mpc}^{-3}$ | $z_1 = 0.3$ | $z_2 = 2.2$ | $P(>\chi^2)$ | $P(>\chi^2)$ |
| PWLE             |       |          |         |           |         | 0.97           | 0.99            | 0.89            | 0.99           |
| a                | 0.0   | 0.0      | -3.32   | -1.38     | -23.41  | 1.810^{-6}     | 0.97           | 0.20            | 0.78            | 0.80           |
| b                | 1.0   | 0.360    | -3.89   | -1.30     | -22.10  | 1.110^{-5}     | 0.99           | 0.99            | 0.69            | 0.92           |
| c                | 0.1   | 1.91     | -2.43   | -1.23     | -23.76  | 2.610^{-6}     | 0.89           | 0.15            | 0.88            | 0.58           |
| d                | 1.1   | 3.81     | -4.22   | -1.37     | -22.25  | 3.210^{-6}     | 0.99           | 0.92            | 0.66            | 0.81           |
| LEXP             |       |          |         |           |         | 0.99           | 0.99            | 0.69            | 0.92           |
| e                | 0.0   | 0.665    | -3.51   | -1.49     | -22.18  | 1.510^{-6}     | 0.99           | 0.58            | 0.78            | 0.40           |
| f                | 1.0   | 11.8     | -4.61   | -1.55     | -19.84  | 6.310^{-6}     | 0.97           | 0.88            | 0.57            | 0.24           |
| g                | 0.1   | 1.90     | -2.69   | -1.49     | -24.66  | 1.210^{-6}     | 0.76           | 0.16            | 0.23            | 0.58           |
| h                | 1.1   | 9.71     | -4.62   | -1.60     | -20.13  | 2.110^{-6}     | 0.99           | 0.64            | 0.57            | 0.01           |

Table 4. Evolution characteristic times: look–back time to which all luminosities were higher by a factor $e$, given the PWLE parameter in Table 2. Times are in units of $H_0^{-1}$, the mean is 0.27 and the dispersion is 0.04.

| $\Lambda \Omega$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|------------------|-----|-----|-----|-----|-----|-----|
| 0.0              | 0.29| 0.27| 0.26| 0.24| 0.23| 0.23|
| 0.2              | 0.30| 0.28| 0.26| 0.24| 0.23| 0.23|
| 0.4              | 0.32| 0.29| 0.27| 0.25| 0.24| 0.23|
| 0.6              | 0.35| 0.30| 0.28| 0.25| 0.24| 0.24|
| 0.8              | 0.39| 0.31| 0.28| 0.26| 0.25| 0.23|
| 1.0              | 0.52(*)| 0.33| 0.29| 0.27| 0.25| 0.24|

Table 5. Same as Table 4, but for LEXP. Mean characteristic time: 0.13; dispersion: 0.05.

| $\Lambda \Omega$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|------------------|-----|-----|-----|-----|-----|-----|
| 0.0              | 0.15| 0.12| 0.11| 0.10| 0.09| 0.08|
| 0.2              | 0.17| 0.13| 0.12| 0.10| 0.09| 0.09|
| 0.4              | 0.19| 0.15| 0.12| 0.11| 0.10| 0.09|
| 0.6              | 0.23| 0.16| 0.13| 0.11| 0.10| 0.09|
| 0.8              | 0.29| 0.18| 0.14| 0.12| 0.11| 0.10|
| 1.0              | 0.53(*)| 0.22| 0.16| 0.13| 0.11| 0.10|

Table 6. PWLE characteristic times (Table 4) in units of the age of the universe in the corresponding cosmology. Mean: 0.31; dispersion: 0.02. These ratios are far more independent of cosmology than the times in Table 4.

| $\Lambda \Omega$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|------------------|-----|-----|-----|-----|-----|-----|
| 0.0              | 0.29| 0.32| 0.33| 0.33| 0.34| 0.34|
| 0.2              | 0.28| 0.31| 0.32| 0.32| 0.33| 0.34|
| 0.4              | 0.28| 0.31| 0.32| 0.32| 0.33| 0.33|
| 0.6              | 0.27| 0.30| 0.31| 0.31| 0.32| 0.32|
| 0.8              | 0.25| 0.29| 0.30| 0.30| 0.31| 0.31|
| 1.0              | 0.13(*)| 0.28| 0.29| 0.30| 0.30| 0.31|

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Table 7. Same as Table 6, but for LEXP. Mean: 0.15; dispersion: 0.02. As for PWLE, the characteristic time depends far less on the age of the universe than on the Hubble time.

| Λ  | Ω  | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|----|----|-----|-----|-----|-----|-----|-----|
| 0.0| .15| .15 | .15 | .14 | .14 | .13 | .13 |
| 0.2| .16| .16 | .15 | .14 | .14 | .13 | .13 |
| 0.4| .16| .16 | .15 | .14 | .13 | .13 | .13 |
| 0.6| .17| .16 | .15 | .14 | .13 | .13 | .13 |
| 0.8| .19| .17 | .15 | .14 | .13 | .13 | .13 |
| 1.0| .13(*)| .18 | .16 | .14 | .14 | .13 | .13 |