Slow nonequilibrium dynamics: parallels between classical and quantum glasses and gently driven systems

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Abstract

We review an scenario for the non-equilibrium dynamics of glassy systems that has been motivated by the exact solution of simple models. This approach allows one to set on firmer grounds well-known phenomenological theories. The old ideas of entropy crisis, fictive temperatures, free-volume... have clear definitions within these models. Aging effects in the glass phase are also captured. One of the salient features of the analytic solution, the breakdown of the fluctuation-dissipation relations, provides a definition of a bonafide effective temperature that is measurable by a thermometer, controls heat flows, partial equilibrations, and the reaction to the external injection of heat. The effective temperature is an extremely robust concept that appears in non-equilibrium systems in the limit of small entropy production as, for instance, sheared fluids, glasses at low temperatures when quantum fluctuations are relevant, tapped or vibrated granular matter, etc. The emerging scenario is one of partial equilibrations, in which glassy systems arrange their internal degrees of freedom so that the slow ones select their own effective temperatures. It has been proven to be consistent within any perturbative resummation scheme (mode coupling, etc) and it can be challenged by experimental and numerical tests, some of which it has already passed.

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1 Introduction

The theoretical understanding of the non-equilibrium dynamics of glassy systems is now being reached. Setting the accent on the dynamics, and leaving aside subtle and possibly undecidable questions as whether there exists a thermodynamic glass transition, allows for a unified analytical treatment of many glassy phenomena. Indeed, in spite of important differences in the microscopic constituents, the interactions among them, and the nature of the microscopic dynamics, many of the macroscopic dynamic features of glassy materials are similar and can be captured by a generic approach. It is the aim of this article to review the scenario for the out of equilibrium dynamics of glassy systems that emerges from the study of simple test models \[1\]. The outcome of its detailed analysis can be, and to a certain extent has been, put to test by comparing it to the numerical analysis of more realistic models and to experiments in real systems.

Glassy states of matter are ubiquitous in nature. When a liquid is annealed at a sufficiently rapid rate, it reaches a temperature region in which its dynamics can no longer follow the pace imposed by the thermal environment. The system enters the glassy phase where it cannot equilibrate. The relaxation at all lower temperatures occurs out of equilibrium. One of its hallmarks are aging effects, or the impossibility of reaching stationarity \[2\]. Typical examples of such glassy systems are polymer glasses, spin glasses, structural glasses, orientational glasses, vortex glasses, etc. In all these systems the crossover temperature is sufficiently high as to make quantum effects totally irrelevant.

Quantum glassy phases, where quantum fluctuations are at least as important as thermal ones, have been identified in a number of materials. Two such examples are the spin glass compound and the amorphous insulator studied in \[3\] and \[4\], respectively. Another interesting realization is the so-called Coulomb glass in which localized electrons interact via Coulomb two-body potentials and hop between localization centers \[5\]. In both systems the dynamics is extremely slow and strong history dependences as well as other glassy features have been observed \[3, 4, 5\].

The above examples concern systems that are not able to reach equilibrium with its environment in measurable times but that, let evolve on astronomical time-scales, will eventually equilibrate. Other ways of establishing non-equilibrium states with slow dynamics are also possible. For instance, a dense liquid can be driven to a slow out of equilibrium stationary regime by a weak shear. This perturbation accelerates the dynamics in such a way that the structural relaxation time (equivalently the viscosity) decreases with increasing shear strength (shear thinning) \[6\]. Non-potential forces also have a strong effect on aging systems: they introduce a time scale beyond which aging is interrupted.

The driven dynamics of granular matter is now receiving renewed attention \[7\]. Since the potential energy needed to displace a macroscopic grain by a distance equal to its diameter is much larger than the characteristic thermal energy, \(k_B T\), thermal activation is totally irrelevant for granular systems. Therefore, granular systems are blocked in metastable states in the absence of external driving. Instead, when energy is pumped in in the form of shearing, vibration or tapping, transitions
between the otherwise stable states occur and granular matter slowly relaxes toward configurations with higher densities. Glassy features such as hysteresis as a function of the amount of energy injected, slow dynamics, and non-stationary correlations have been exhibited.

Many of the recent developments in the understanding of the similarities in the behavior of a priori so different systems are based on the analysis of simple mean-field-like models. In a series of seminal papers Kirkpatrick, Thirumalai and Wolynes (KTW) \[8\] showed that disordered spin models such as a fully connected Ising model with random \( p \) spin interactions \((p \geq 3)\), or the Potts glass, capture many of the expected properties of the glass dynamic transition. Consequently, it realizes exactly some of the older phenomenological descriptions. Neither the fact that the dynamic variables are spins nor the presence of disorder are essential; they are used only as a calculational tool. This was signaled by these authors and later made explicit in a series of articles where non-disordered models with similar phenomenology have been proposed and studied \[1\].

In short, KTW showed the following. In the liquid phase the \textit{exact} dynamic equation for the auto-correlation function coincides with the schematic mode-coupling equation \[9\]. Hence, the dynamics of these models in the high temperature phase captures the same physics as the schematic mode-coupling theory and, in particular, the dynamic transition at \( T_d \) \((\equiv T_{\text{mct}})\) toward a glassy phase. The dynamic transition is \textit{discontinuous}, in the sense that a plateau in the decay to zero of the correlations already appears in the (equilibrium) liquid phase when \( T_d \) is approached from above; it is of second order in the usual sense since thermodynamic quantities as, for instance, the energy density are continuous when going across \( T_d \). The dynamic transition is due to the proliferation of metastable states as proved by the analysis of the Thouless-Anderson-Palmer (TAP) free energy density.

Since the model is defined by a simple Hamiltonian, the static properties are also easily accessible. A static transition between a liquid and a glassy thermodynamic phase occurs at a lower temperature \( T_s \) \((< T_d)\) where the configurational entropy vanishes. This phenomenon is related to the Kauzmann paradox; \( T_s \) is then associated to \( T_K \). The static transition is also discontinuous since the order parameter, \( q_{EA} \), jumps at \( T_s \) but it is of second order thermodynamically.

The relaxational dynamics below \( T_d \) was solved in \[10\]. The model does not reach equilibrium with the thermal bath unless exponentially diverging times with \( N \), the number of degrees of freedom, are considered. The non-equilibrium solution is then relevant for most cases of practical interest, since such diverging times are unrealistic. One of the main features of the analytic solution is that aging effects, much as those observed experimentally, are captured by this model. The reason why they subsist asymptotically can be grasped from the analysis of the metastable states, again using TAP’s approach. Extensions of the \( p \) spin-glass model to other mean-field models with internal dimension lead to predictions about spatio-temporal scalings \[11\]. Above \( T_d \) these are equivalent to the extensions of the mode-coupling approach to super-cooled liquids \[9\]; below \( T_d \) they are related to mode-coupling approximations of the realistic (hard to treat) models in which the assumption of equilibration is not done \[12, 13\].
It is important to stress that, even if the $p$ spin model has quenched disorder in its interactions, it is not a spin-glass model. It is rather well established that spin-glasses (in the absence of an external field) have second order phase transitions with no discontinuity of any order parameter. Moreover $T_s = T_d$ for a spin-glass.

In the models and approximations mentioned above, the dynamic and static transitions are sharp. In real glasses, $T_d$ is a crossover while $T_s$ might not exist. In spite of this and other defects, the interest of these solutions is that they have a great power of prediction of so far unknown effects. The rather accurate comparison of these to numerical simulations [14] and, to the extent of their availability, experiments [15,16,17], supports the proposal that the mechanism in these models is similar to the one responsible for the glass transition, and the glassy dynamics, in real materials. Moreover, some of the ingredients missing in the full analytic solution of the mean-field models that will render their description of real materials more accurate, have been identified (preasymptotic corrections when the thermodynamic limit has been taken, analysis of the dynamics in time-scales that diverge with $N$, the rôle played by fluctuations, etc.). For the moment, their analytical treatment has proven too difficult.

Another important consequence of the study of these solvable models is that it has allowed us to identify an effective temperature, $T_{\text{eff}}$, that can be measured experimentally in a direct manner [18]. This quantity has all the expected properties of a temperature: it controls heat transfer, partial equilibrations, and the reaction to external heat transfers. More importantly, different observables that evolve in the same time-scale and interact should have the same effective temperature. The solution of the models and approximations with space and time [11,13] do indeed respect this property. Recent numerical simulations in Lennard-Jones mixtures have also shown the validity of this important property [19]. This concept lead to rather straightforward extensions of the dynamic scenario to other non-equilibrium systems with small entropy production as quantum glasses [20,21], vibrated glasses [23,24] and weakly perturbed liquids and glasses [23,25,26]. In our opinion, the mean-field models are explicit and solvable realizations of a general scenario [27] based on the generation of effective temperatures in generic non-equilibrium systems with slow dynamics.

In this respect, it is interesting to compare the effective temperature to the proposal of Tool, Narayanasamy, Moynihan and collaborators [28] to introduce phenomenological fictive temperatures in order to characterize the departure from equilibrium of glasses. It is important to stress that, even if similar in spirit, the effective temperature is more than a phenomenological definition. In particular, the fictive temperature is defined with respect to a given physical property and it can vary from one property to another. The effective temperature instead has to be the same for all physical properties (that interact) when studied in the same time-scales.

The definition of a bonafide effective temperature has rekindled the search for an extension of thermodynamics to describe the quasi-static properties of the glassy non-equilibrium dynamics. In fact, Moynihan et al had studied the modification of several thermodynamic relations, e.g. the Ehrenfest relations and the Prigogine-Defay ratio, when a (time-dependent) fictive temperature is added as an “internal”
parameter \[29\]. More recently, these questions have been revisited \[30\] using the effective temperature as a supplementary parameter in the thermodynamic potentials. This was also motivated by the solution to \(p\) spin-like models for which “mixed” thermodynamic potentials are relevant.

Another interesting problem is the explicit search of a relation between the effective temperature and the structure of metastable states in the free-energy landscape. Again, the exact results for the mean-field models acted as a guideline of immense help. Indeed, for \(p\) spin like models \[31, 23\] the value of \(T_{\text{eff}}\) coincides with a quasi-static quantity defined as \(\partial S_{\text{TAP}}(f,T)/\partial f |_{f_{\text{th}}}\) with \(f\) the TAP free-energy density, \(S_{\text{TAP}}\) the logarithm of the number of stationary points of the TAP free-energy density in the interval \([f, f + df]\), and \(f_{\text{th}}\) a particular value with a dynamical meaning (it is the free-energy of the threshold level, the one related to the energy density reached asymptotically \([11, 32]\)). This result has been subsequently analysed in more general terms in \[33\].

The relation between the effective temperature and the TAP entropy makes contact with the proposal of Edwards \[34\] that an ensemble of equiprobable blocked configurations describes the statistical properties of gently driven granular matter in the steady state. (It is important to stress that the definition of blocked configuration actually depends on the dynamics, e.g. in a spin system blocked may mean stable with respect to single spin flips.) Edwards’ entropy, \(S_E(\mathcal{E})\), is defined as the logarithm of the number of blocked configurations with energy density in \([\mathcal{E}, \mathcal{E} + d\mathcal{E}]\), and Edwards’ temperature is then \(T_E^{-1} = \partial S_E(\mathcal{E})/\partial \mathcal{E}\). It has been recently checked that this definition coincides with the effective temperature in a number of glassy models at zero temperature \[35, 36\]; some cases where it does not work have also been exhibited, showing that the connection cannot be generically valid. An early experimental attempt to test the assumption of a flat distribution in a granular system has been performed by Nowak \textit{et al} \[37\].

Similar ideas have been explored \[38, 39, 40\] in the context of the inherent structure construction of Stillinger and Weber \[41\], and its extensions. Mainly, the idea is to relate the aging dynamics of, e.g., a finite size mean-field model \[39\] or a Lennard-Jones glass former \[40, 38\] to the evolution in the multidimensional potential energy landscape once partitioned in basins of minima and saddles. Thermal activation is totally irrelevant in granular matter and the use of energy densities, instead of free-energy densities, is justified in this case. At finite temperature potential energy densities and free-energy densities are not equivalent. Hence, the connection between the approaches based on the study of the potential energy landscape and the ones based on mean-field models at finite temperatures have to be taken with care \[42\].

### 2 Slow relaxation in systems out of equilibrium

After having very briefly described the main ingredients of the behaviour of freely relaxing mean-field like models, let us now compare in more detail the main features of the dynamic relaxation in free evolving classical and quantum glassy systems, and
externally driven complex liquids and glasses.

We have argued that the experimentally relevant sizes and times are such that the thermodynamic limit has to be taken first and the asymptotic limit can be taken only afterwards. When a transient following preparation has elapsed, in all the systems and experimental setups described in the introduction (with the proviso that energy has to be weakly injected in the case of driven systems) the macroscopic relaxation occurs in several time scales. The separation of time scales is most clearly shown by the evolution of the self-correlation, in the case of spin models, or by the decay of the incoherent scattering function in the case of particle systems. We denote both by $C_k(t + t_w, t_w)$, with $k$ a chosen wave-vector (usually, $k = 0$ for the spin model), $t_w$ the waiting-time measured from the moment in which the experimental glass transition $T_g$ is crossed and $t + t_w \geq t_w$ the measuring time.

Above the glass transition, super-cooled liquids have a stationary relaxation for all waiting times that are longer than the finite equilibration time. $C_k(t)$ has a rich behavior with a rapid decay toward a plateau (visible in logarithmic scale) and a second slow decay toward zero that occurs in the $\alpha$ relaxation time $t_\alpha(T)$. Below the glass transition the decay after a waiting-time also occurs in two steps. There is a first stationary decay toward a plateau whose value depends on the external parameters ($T$, field, etc.) and $k$, and a second slow decay toward zero that occurs in a waiting-time dependent scale $t_\alpha(t_w, T)$, with $t_\alpha(t_w, T)$ a growing function of $t_w$: the older the system the slower the decay. The top-left panel in Fig. 1 displays the decay of the self-correlation $C(t + t_w, t_w)$, for instance $C(t + t_w, t_w) = N^{-1} \sum_i \langle s_i(t + t_w) s_i(t_w) \rangle$, with $s_i$ the dynamic variables and the brackets representing an average over different thermal histories, in the spin system).

The two step relaxation can be simply visualized in a number of examples that provide different interpretations for the glassy dynamics. Phenomenological descriptions of glassy dynamics based on the motion of a point, that represents the full system, in a roughed free-energy landscape are popular (more precisely, an average over subsystems is needed to obtain continuous results). However, since phase space is infinite dimensional ($2^N$ dimensional in a spin system) the intuition based on the knowledge of diffusion processes in finite dimensional spaces are misleading and can lead to misconceptions when applied to the glassy problem. The free-energy landscape (TAP free-energy), the non-equilibrium dynamics, and the relation between them, are completely known for solvable glassy models like the $p$ spin model. In the asymptotic limit, the point that represents the system in phase space approaches a path made of flat directions, the threshold, that is higher in free-energy density than the equilibrium level [10]. The rapid first step relaxation is related to the rapid motion “transverse” to the flat directions while the slow $\alpha$-relaxation has to do with the slow diffusion along the path and it is not an activated process over barriers. Since phase space is infinite dimensional, times that diverge with $N$ are needed to penetrate below the threshold. These processes are now of a different type, being due to activation from one well in the TAP free energy to another. When contact is made with real materials such long times might be unrealistic, unless close to $T_g$, where transitions between metastable states are certainly easier and can be responsible for the observed cooling rate dependences. (With a slower annealing the system spends
longer periods at higher temperatures where transitions are more favorable and it
can then penetrate more deeply below the threshold. All (one-time) quantities such
as the internal energy density will consequently approach lower values and be closer
to the “optimum”.) The relevance of saddles in the free-energy landscape is not
peculiar to the non equilibrium dynamics of mean-field models [43].

The two step relaxation of the self-correlation toward and away from a plateau
captures the well-know cage effect. In a system of interacting particles, any particle
is surrounded by a small number of neighbors that form a cage around it. The short
time-difference dynamics corresponds to the rapid rattling of particles in their cages
and it resembles the dynamics of a system in equilibrium in that it is stationary.
The second step is instead due to the destruction of the cages, each particle escapes
its own cage and there is a structural rearrangement. How much can the particles
decorrelate within the cages depends on temperature: the height of the plateau in
the correlation, that is related to the “size” of the cage, decreases with increasing
temperature. The time needed to destroy the cages, $t_w(t_w, T)$, depends on the
waiting-time (and $T$). Even if in these models there is no notion of interacting
particles, the models capture a cage effect and they predict a form for the decay
of $C_k(t + t_w, t_w)$ that has been later observed numerically in more realistic glassy
models [14].

Certainly, such a two-step process is also captured by coarsening models, the
simplest examples being spin models undergoing domain growth. In these, after
the quench from high temperatures, two ordered phases grow with none of them
conquering the sample if the thermodynamic limit has been taken at the outset.
However, it has remained an open question if any such growing order exists in most
glassy systems. It is also worth mentioning that models with an infinite hierarchy
of relaxation times exist [44, 11].

The two-step decay is robust with respect to changes in the microscopic dy-
namics. On the one hand, the same kind of two-step relaxation is observed using
molecular dynamics or Montecarlo to simulate the dynamics of a given model. On
the other hand, more important changes in the driving dynamics slightly modify
this picture, as shown below.

The simplest effect of quantum fluctuations is to introduce oscillations in the first
step of relaxation. These disappear at long enough time-differences and they are to-
tally suppressed from the second decay, that superficially looks classical [20, 21]. (A
more dramatic effect of quantum mechanics is related to the very strong role played
by the quantum environment on the dynamics of a quantum system. Indeed, the
location of the transition line strongly depends on the type of quantum bath con-
sidered and on the strength of the coupling between system and environment [45].)
The top-right panel in Fig. 1 shows the decay of the symmetrical correlation for the
quantum extension of a $p$ spin model in its glassy phase.

A weak shear has a spectacular effect on the relaxation of macroscopic correla-
tions. It introduces a shear-dependent time scale $t_{sh}$ and, for waiting-times that are
longer than $t_{sh}$, aging is interrupted. The effective age is a decreasing function of the
pumped energy. This effect has been known for long experimentally [3, 47] and it is
related to the shear-thinning behavior in Ref. [3]. It has been found and explored
recently within the theoretical framework that we review [23, 26]. Under a weak shear the decay still occurs in two steps though it is fully stationary (for waiting times that are longer than $t_{\text{st}}$), as shown in in the bottom-left panel in Fig. 1.

The effect of an ac-field on a glassy system is complex and it is somehow similar to the action of a quantum bath on a quantum glass. When the test glassy model is externally perturbed by an oscillatory field, $H \sin(\omega t)$, the correlation oscillates with a frequency that is determined by the one of the external drive. If one explores the dynamics by using stroboscopic time, that is to say by using a single point for each cycle, the relaxation becomes monotonic and smooth with a clearcut time-separation. Aging is not interrupted by the external perturbation but the location of the liquid-to-glass transition in the $(T, H, \omega)$ parameter space depends on $\omega$. The effect of an oscillatory field on a spin system is weaker than the one of a dc field with the same amplitude: the projection of the glassy phase in the $(T, H, \omega)$ parameter space on to the $(T, H)$ plane, is larger for an ac-field than for a dc one [24]. The bottom-right panel in Fig. 1 displays the decay of the self-correlation for different waiting times when an ac field is applied. Temperature is zero and there is no first decay since the plateau is at $C = 1$. Waiting-time dependences are apparent.

The threshold level in the TAP free-energy still plays an important role in all these modified problems. A quantum TAP free-energy density can be defined and, for $p$ spin-like models, it is similar to the classical one with a threshold level, many minima below it, etc [22]. In systems perturbed by non-potential forces, the energy injection serves to keep the system in a steady state above the threshold level, the higher the stronger the drive.

3 Responses and effective temperatures

The solution to dynamics of glassy models relates the spontaneous fluctuations $C_k(t + t_w, t_w)$ and the induced ones, $\chi_k(t + t_w, t_w)$, defined as the integrated linear response over a finite time interval $[t_w, t + t_w]$. It was shown in [18] that the slope of the plot of $\chi_k(t + t_w, t_w)$ against $C_k(t + t_w, t_w)$ for $t_w$ fixed and using $t$ as a parameter, defines a measurable effective temperature.

In classical thermal systems without external forces the effective temperature takes different values in different time-scales [10]. In the rapid time-scale, where $C_k$ decays from 1 to the plateau $q_k$, the equilibrium fluctuation-dissipation theorem (FDT) is satisfied and the effective temperature is that of the environment. The parametric construction yields a straight line of slope $-1/T$. This result is natural. For a particle system, this regime corresponds to the motion of particles within their cages that strongly resembles equilibrium. Consequently, the kinetic energy in a glass satisfies equipartition with the temperature of the environment. In a coarsening spin system it corresponds to the thermal spin flips within the domains that is also very reminiscent of an equilibrium problem. During the $\alpha$ structural relaxation instead the effective temperature takes a different value, that is close (but not identical) to the critical temperature $T_d$ (see the top-left panel in Fig. 2). In $p$ spin like models there is only one independent correlation and response. In the
Figure 1: The auto-correlation function $C_{k=0}(t + t_w, t_w)$ for several choices of the waiting times $t_w$. Clockwise from the top-left panel: the relaxing spherical $p$ spin model at $T = 0.2$, its quantum extension at $T = 0$, the vibrated case at $T = 0$ and the “sheared” one at $T = 0.2$ (aging is interrupted in this case).

extensions based on models of finite dimensional manifolds in random potentials all the FD relations that one can construct using different $k$ lead to the same value of the effective temperature, $T_{\text{EFF}}(k) = T_{\text{EFF}}$ \[11, 13\]. The reason for this property is that all observables evolving in the same time-scale, and interacting, equilibrate and hence acquire the same value of the effective temperature \[18\]. This property has also been checked numerically in Lennard Jones mixtures \[19\]. (In models with a hierarchy of time-scales, each time-scale has its own effective temperature \[44\].)

The quantum FDT is a retarded non-linear relation between the spontaneous symmetrical correlations and the linear response. Quantum glassy systems have (at least) two time-regimes \[20, 21\]. In the rapid regime the quantum FDT holds. This is the reason why the plot on the right-top panel in Fig. 2 has a strange form and does not show a straight line when the correlation varies from 1 to $q_k$. Instead, in the second time-regime the parametric construction yields a linear relation between $\chi_k$ and $C_k$. The implications of this result are very interesting. The classical FD relation develops, with a slope given by a non-trivial $T_{\text{EFF}}$ (even at zero external
Figure 2: The parametric plot of the integrated response, $\chi$, against the auto-
correlation, $C = C_{k=0}(t + t_w, t_w)$, for several values of the waiting time. Clockwise:
the relaxing spherical $p$ spin model at $T = 0.2$, its quantum extension at $T = 0$, the
vibrated case at $T = 0$ and the “sheared” one at $T = 0.2$.

temperature). The dynamics looks classical and a decoherent effect appeared.

In systems perturbed by non-potential forces, like shear, a non-equilibrium steady
state is reached that is characterized by two time-scales with different effective tem-
peratures. The $p$ spin models with asymmetric couplings \cite{25} is the mean-field
realization of a super-cooled liquid or a glass perturbed in this way \cite{26} (see the
bottom-left panel in Fig. 2). Binary Lennard-Jones mixtures show the same phe-
nomenology \cite{46}.

In a granular system the microscopic dynamics is not thermal; the dynamics
is driven by the energy pump and dissipation is not proportional to the velocity.
If two time-regimes develop, the fast regime should not be characterized by an
effective temperature. Instead, if the structural relaxation is slow and similar to
that of a glassy system, there should be an effective temperature controlling this
slow time-scale, in the same way as in the quantum system we recovered a classical
effective temperature in this regime. This scenario has been checked using a $p$ spin
model perturbed by an ac force (where the bath temperature is still relevant, right-bottom panel in Fig. 2) [24] and more realistic models of granular matter externally sheared [36]. In the vibrated case, the existence of an effective temperature is better visualized in stroboscopic time, when one divides away the oscillations directly related to the external drive.

4 Conclusions

We have exhibited several features of the dynamics of simple models that have also appeared in more realistic ones and real systems. Without wanting to stretch the domain of applicability of these models too far (some of their limitations are obvious!), we believe that the scenario they suggest is very close to the one of the dynamics of generic systems in the limit of small entropy production (glassy, gently driven).

The number of models showing relaxations as in Fig. 1 and the existence of an effective temperature is now very large (spin systems with and without disorder, models of interacting particles and polymers, kinetically constrained models, realistic models of granular matter, etc.). Moreover, in Figs. 1 and 2 we have shown the effect of three rather different ways of modifying the microscopic dynamics that still leave untouched the separation of time scales and the existence of a non-trivial effective temperature for the slow one (apart from slightly changing its precise value). One may wonder why the dynamic features presented here are so robust.

One reason for this is technical. For any dynamical problem one can write Schwinger-Dyson equations for the evolution of the two-point correlations and the corresponding linear responses. These equations involve vertices and self-energies that depend on the same two-point functions but that, in general, cannot be calculated explicitly. Two ways of approximating these equations are [12]:

– to change the departing Hamiltonian to a mean-field like one for which explicit Schwinger-Dyson equations are exactly derived.
– to approximate the vertices and self-energy of the original theory with a self-consistent procedure (MCT [13], SCSA, etc) in order to render them explicit. The structure of the approximate equations is very constrained by the symmetries in the problem (including the rather abstract supersymmetry of stochastic processes) that act as a guideline to derive them.

As explained in [12], there is a one-to-one correspondence between one method and the other. It is then no surprise that the behavior of mean-field models, approximation schemes and real systems are indeed very similar.

The general Schwinger-Dyson equations have a structure such that, when a fast time-scale can be separated from the slow one(s), the existence of a non-trivial effective temperature corresponds to a symmetry breaking [27]. The structure we have shown here, even if initially derived in the mean-field p spin model, is then consistent even for finite dimensional problems (it does not include non-perturbative effects though). This argument explains why it is easier to change the time-dependences in the relaxation (e.g. from aging to stationary in the sheared case) than the organiza-
tion of effective temperatures [18]. However, whether one given system will realize this many temperature structure or not cannot be easily guessed from looking at the microscopic Hamiltonian (much as it is difficult to guess the order of a static phase transition for a model with complicated interactions).

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Having given these reasons to justify the “universality” in the thermal properties of these systems, it is not clear to us why two ways of injecting energy into the system, \textit{viz} with a force that does not derive from a potential, or with a time-dependent perturbation, have such different effect on the time scales for relaxation.