Exotica in rotating compact stars

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Abstract. The determination of mass and radius of a single neutron star EXO 0748-676 has been reported recently. Also, the estimate of radius from the measurement of moment of inertia of pulsar A in double pulsar system PSR J0737-3039 would be possible in near future. Here we construct models of static and uniformly rotating neutron stars involving exotic matter and compare our theoretical calculations with the recent findings from observations to probe dense matter in neutron stars.

1. Introduction
Neutron stars are ideal astrophysical laboratories to study properties of cold and dense matter [1, 2]. The investigation of dense and cold matter in neutron star core has entered into a new phase. Observatories such as Chandra X-ray observatory, X-ray Multi Mirror (XMM)-Newton, Rossi X-ray Timing Explorer (RXTE) are pouring in many new and interesting data on compact stars. It is now possible to estimate mass and radius of a single neutron star from observations. These informations may shed light on the composition and equation of state (EoS) of matter at low temperature and high baryon chemical potential.

A lot of interest has been generated in thermonuclear x-ray bursts from the low mass x-ray binary EXO 0748-676. Already Cottam et al. [3] estimated the gravitational redshift \( z = 0.35 \) of three absorption lines in x-ray bursts from EXO 0748-676 with XMM-Newton. They also reported the narrow widths of those lines. Mass to radius ratio is immediately known from the measurement of \( z = \left( 1 - \frac{2GM}{c^2R} \right)^{-1/2} - 1 \), where \( M \) and \( R \) are neutron star mass and radius. On the other hand, the widths of absorption lines provide information about the surface rotational velocity \( (v_{\text{rot}}) \) which is related to radius and spin frequency \( v_{\text{rot}} \propto \nu_{\text{spin}}R \). Villarreal and Strohmayer [4] reported the neutron star spin frequency \( \nu_{\text{spin}} \) of 45 Hz in the x-ray burst oscillations of EXO 0748-676 with RXTE. These observations lead to a radius of 9.5-15 km and mass 1.5-2.3M\( _{\odot} \) for EXO 0748-676 [4]. For the first time, it has been possible to determine mass and radius of a single neutron star.

Recently discovered double pulsar system PSR J0737-3039 [5] provides another opportunity for the determination of mass and radius of same neutron star. Two pulsars in this double pulsar system are denoted by pulsar A and pulsar B. Pulsar A has a spin period of \( \sim 23 \) ms and mass 1.34M\( _{\odot} \) whereas those of pulsar B are 2.8 s and 1.25M\( _{\odot} \). Double neutron star binaries provide unique laboratories for testing relativistic gravity. Soon it will be possible to measure spin-orbit coupling of pulsar A from pulsar timing data [6, 7]. The moment of inertia for pulsar A could be determined from the measurement of spin-orbit coupling. Moment of inertia is dimensionally proportional to mass times radius squared. Already, the mass of pulsar A is known. Therefore, the measurement of moment of inertia will provide an estimate of radius of the compact object and put important constrain on the properties of neutron star matter.
Now the observed mass-radius (M-R) relationship can be directly compared with that of theoretical calculation. Consequently, the composition and EoS of dense matter in neutron stars may be probed. Here we discuss exotic forms of matter and their influence on various properties of static and rotating compact stars. In section 2, we describe the models of static and uniformly rotating compact stars and EoS adopted for this calculation. Results are discussed in section 3. And section 4 is devoted to summary and conclusions.

2. Model
Models of static and rotating compact stars are constructed by solving Einstein’s field equations. The equilibrium structures of non-rotating stars are obtained from Tolman-Oppenheimer-Volkoff (TOV) equations. For rotating neutron stars, we consider stationary and axisymmetric equilibrium configurations. In this case, the metric is taken in the form \[ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2d\theta^2) + e^{\gamma-\rho}r^2\sin^2\theta(d\phi - \omega dt)^2,\] (1)
where \(\omega\) is the angular velocity of the local inertial frame. The metric functions in the line element depend on the radial coordinate \(r\) and the polar angle \(\theta\).

We model the neutron star matter as a perfect fluid and the stress-energy tensor is given by \[T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - P g^{\mu\nu}\] (2)
where \(\varepsilon\) and \(P\) are the total energy density and pressure of the matter respectively and \(u^{\mu}\) is the fluid’s four velocity. The moment of inertia of the star is defined by \(I = J/\Omega\); angular momentum \((J)\) is,
\[J = \int d^3x\sqrt{-g}(\varepsilon + P)u^{t}(g_{\phi\phi}u^{\phi} + g_{\phi t}u^{t})\] (3)
where, \(u^{\phi}\) and \(u^{t}\) are components of fluid four velocity and \(\Omega\) is angular velocity of the star. For uniformly rotating neutron stars, we adopt a numerical code based on Komatsu-Eriguchi-Hachisu [9, 10] method.

Composition and structure of neutron stars depend on the nature of strong interaction. Matter density could exceed by a few times normal nuclear matter density in a neutron star interior. Consequently, the baryon and lepton chemical potential increase rapidly with baryon density. This might lead to the formation of exotic matter with a large strangeness fraction such as hyperon matter, Bose-Einstein condensate of \(K^-\) mesons and quark matter. Here we discuss equations of state with various compositions of dense matter in neutron star cores.

We construct hadronic EoS using relativistic field theoretical models [11, 12]. Here baryon-baryon interaction is mediated by exchange of mesons. The hadronic phase is composed of all members of the baryon octet, electrons and muons. The hadronic phase is to satisfy beta-equilibrium and charge neutrality conditions. In compact stars interior, the generalised \(\beta\)-decay processes may be written in the form \(B_1 \rightarrow B_2 + l + \bar{\nu}_l\) and \(B_2 + l \rightarrow B_1 + \nu_l\) where \(B_1\) and \(B_2\) are baryons and \(l\) represents leptons. The generic equation for chemical equilibrium condition is
\[\mu_i = b_i\mu_n - q_i\mu_e,\] (4)
where \(\mu_n\), \(\mu_e\) and \(\mu_l\) are respectively the chemical potentials of neutrons, electrons and i-th baryon and \(b_i\) and \(q_i\) are baryon number and electric charge of i-th baryon respectively. The charge neutrality in hadronic phase is given by \(Q^h = \sum_B q_B n_B^h - n_e - n_\mu = 0\), where \(n_B^h\) is the number density of baryon \(B\) in pure hadronic phase and \(n_e\) and \(n_\mu\) are number densities of electrons and muons respectively.

Next we consider Bose-Einstein condensate of \(K^-\) mesons. Experimental results from heavy ion collisions suggest that the in-medium kaon-nucleon interaction is repulsive whereas it is
where $\epsilon$ is the volume fraction in phase II. The total energy density in the mixed phase is given by $\chi K \epsilon$.

In this calculation, we also consider a first order phase transition from hadronic to quark matter. The pure quark matter is composed of members of the baryon octet embedded in the condensate, electrons and muons and maintains charge neutrality and beta-equilibrium. The charge neutrality condition is $Q_i = 0$, where $Q_i$ is the number of members of the baryon octet $i$. The above condition determines the onset of $K^-$ condensation as a first order phase transition. Here we adopt a relativistic field theoretical model to describe (anti)kaon-baryon interaction [11, 12, 17]. In this case, (anti)kaon-baryon interaction is treated in the same footing as baryon-baryon interaction. The pure $K^-$ condensed phase is composed of members of the baryon octet embedded in the condensate, electrons and muons and maintains charge neutrality and beta-equilibrium conditions. The charge neutrality condition is $Q^K = \sum_B q B n^B_K - n_{K^-} - n_e - n_\mu = 0$, where $n^B_K$ is the number density of baryon $B$ in pure antikaon condensed phase and $n_{K^-}$ is the number density of antikaons.

Strangeness changing processes such as, $n \rightarrow p + K^-$ and $e^- \rightarrow K^- + \nu_e$, in a neutron star interior are responsible for the onset of $K^-$ condensation. The requirement of chemical equilibrium yields

\[ \mu_n - \mu_p = \mu_{K^-} = \mu_e , \]

where $\mu_{K^-}$ is the chemical potentials of $K^-$ mesons. The above condition determines the onset of $K^-$ condensation.

The deconfinement phase transition from hadronic to quark matter is a possibility in a neutron star interior. In this calculation, we also consider a first order phase transition from hadronic to quark phase. The pure quark matter is composed of $u$, $d$ and $s$ quarks. The EoS of pure quark matter is described in the MIT bag model [18].

As we are considering first order phase transitions, the mixed phase of two pure phases is governed by Gibbs phase rules and global conservation laws [1]. The Gibbs phase rules read, $P^I = P^{II}$ and $\mu^B_I = \mu^B_{II}$, where $P^I$, $P^{II}$ and $\mu^B_I$, $\mu^B_{II}$ are pressure and chemical potentials of baryon $B$ in phase I and II respectively. The conditions for global charge neutrality and baryon number conservation are respectively $(1 - \chi)Q^I + \chi Q^{II} = 0$ and $n_B = (1 - \chi)n^I_B + \chi n^{II}_B$ where $\chi$ is the volume fraction in phase II. The total energy density in the mixed phase is given by $\epsilon = (1 - \chi)\epsilon^I + \chi \epsilon^{II}$.

3. Results & Discussions

We adopt GM1 parameter set [11, 12, 19] for this calculation where nucleon-meson coupling constants are determined from the nuclear matter saturation properties. The vector meson coupling constants for (anti)kaons and hyperons are determined from the quark model [20] whereas the scalar meson coupling constants for hyperons and antikaons are obtained from the potential depths of hyperons and antikaons in normal nuclear matter [12, 20]. As the phenomenological fit to the $K^-$ atomic data yielded a very strong real part of antikaon potential $U_{K^-$ at normal nuclear matter density, we perform this calculation with an antikaon optical potential of -160 MeV at normal nuclear matter density ($n_0 = 0.153 fm^{-3}$). The coupling constants for strange mesons with hyperons and (anti)kaons are taken from Ref.[12, 20]. Also, we consider a bag constant $B^{1/4} = 200$ MeV and strange quark mass $m_s = 150$ MeV for quark matter EoS.
Figure 1. Pressure \( P \) (MeV fm\(^{-3}\)) as a function of \( \mu_n \) (MeV) and \( \mu_e \) (MeV) for pure hadronic phase (wired surface) and \( K^- \) condensed phase (shaded surface). The solid line is the charge neutrality line in pure hadronic phase, the mixed phase and pure \( K^- \) condensed phase.

We compute equations of state for four different compositions of neutron star matter. The EoS with \( n, p, e \) and \( \mu \) is denoted by "np". With further inclusion of hyperons, it becomes "npH". The EoS undergoing a first order phase transition from nuclear to quark matter is represented by "npQ". In the last case, the EoS involves first order phase transitions from nuclear to \( K^- \) condensed matter and then from \( K^- \) condensed matter to quark matter and is referred to as "npKQ" [21].

It is evident from Eq. (4) that neutron and electron chemical potentials are two independent quantities. One can express energy density and pressure of neutron star matter using \( \mu_n \) and \( \mu_e \). As an example, we demonstrate here the construction of EoS involving a first order phase transition from nuclear to \( K^- \) condensed matter. In Figure 1, pressure is plotted as a function of neutron and electron chemical potentials for pure hadronic and \( K^- \) condensed phase. Here shaded surface denotes the antikaon condensed phase whereas the wired surface represents the hadronic phase. It is noted that two surfaces intersect along a common line. This implies that two phases having same pressure along the intersection line could co-exist. In this figure, the solid line represents the EoS of charge neutral and beta-equilibrated matter. It has three parts. Below \( P=42 \) MeV fm\(^{-3}\), the pressure in pure hadronic phase is higher than that of the antikaon condensed phase making the hadronic phase physically preferred one. Therefore, the solid line below pressure 42 MeV fm\(^{-3}\) denotes the charge neutrality line in pure hadronic phase. This phase is mainly composed of neutrons, protons, electrons and muons. Two surfaces first meet at \( 2.23n_0 \) and \( P=42 \) MeV fm\(^{-3}\) and this is the beginning of the mixed phase. After the onset of the phase transition, the mixed phase represented by the solid line follows the intersection part of two surfaces. The Gibbs conditions of phase equilibrium and global charge neutrality are satisfied along the intersection line of two surfaces. We find that the electron chemical potential \( \mu_e \) decreases in the mixed phase. This is attributed to the fact that with the formation of \( K^- \) condensate, electrons are replaced by \( K^- \) mesons. The phase transition ends at \( 3.51n_0 \) and pressure 70 MeV fm\(^{-3}\). The solid line above pressure 70 MeV fm\(^{-3}\) represents the charge
Figure 2. Mass-radius relationship of non-rotating stars calculated with different equations of state.

neutral line in pure antikaon condensed phase.

In Figure 2, the mass-radius relationship of non-rotating neutron stars is displayed for different EoS. For np case, the maximum neutron star mass is 2.39 $M_\odot$ and the corresponding radius is 11.94 km. With the inclusion of softening components such as hyperons, Bose-Einstein condensate of $K^-$ mesons or quarks, the maximum neutron star masses are reduced. For npH case, the maximum mass and the corresponding radius are 1.74$M_\odot$ and 12.41 km respectively. Maximum masses and the corresponding radii are 1.83 $M_\odot$ and 12.88 km for npQ case and 1.75$M_\odot$ and 12.10 km for npKQ case, respectively. Among all cases considered here, the maximum neutron star mass for np case is the largest because the EoS in this case is the stiffest. Now we compare our results with the recent findings from EXO 0748-676. Villarreal and Strohmayer [4] have predicted that the best fit values of mass and radius for EXO 0748-676 are $M \sim 1.8\ M_\odot$ and $R \sim 11.5\ km$ respectively. It is found that theoretical M-R relationships shown in Figure 2 are consistent with the observed values. It shows that there might be room for exotic matter in neutron star interior.

Now we focus on the estimate of radius from the calculation of moment of inertia of pulsar A in the double pulsar binary PSR J0737-3039. In this calculation, we adopt some of EoS used for explaining EXO 0748-676 data. Also, we use the inputs gravitational mass $M=1.337M_\odot$ and angular velocity $\Omega = 276.8\ s^{-1}$ corresponding to a spin period $P=2\pi/\Omega=22.7\ ms$ to calculate the moment of inertia of the uniformly rotating star. It follows from the calculation that for np, npH and npQ cases the central energy density corresponding to pulsar A is $\sim 5.4\times10^{14}\ g\ cm^{-3}$. We also note that the moment of inertia and radius of pulsar A are $I \sim 4.90\times10^{45}\ g\ cm^2$ and $R \sim 14\ km$ for all these cases studied here. It exhibits that hyperons or quark matter does not change the moment of inertia and radius appreciably.

In Figure 3, we have shown the moment of inertia in dimensionless unit as a function of compactness parameter ($M/R$) of the neutron star for different EoS. Already, it has been noted that these EoS give rise to maximum neutron star masses $> 1.7\ M_\odot$. Therefore, $I/MR^2$ versus
Figure 3. Moment of inertia ($I$) is plotted with compactness parameter ($M/R$) for three different EoS.

$M/R$ curves shown in Figure 3 can be approximated by [7]

$$I \approx (0.237 \pm 0.008) \, MR^2 \left[ 1 + 4.2 \, \frac{M \, km}{M_\odot \, R} + 90 \left( \frac{M \, km}{M_\odot \, R} \right)^4 \right]. \quad (7)$$

The values of moment of inertia that we have obtained from the rotating neutron star model for fixed angular frequency are comparable with the values calculated using Eq. (7). Thus from the measurements of moment of inertia and mass of pulsar A, the radius could be determined by inversion of Eq. (7).

It has been shown that the moment of inertia of a neutron star depends quite strongly on the EoS. The predictions for neutron star moment of inertia and radius are quite different in non-relativistic and relativistic models [6]. Therefore, a measurement of moment of inertia with 10 percent uncertainty could impose new constraints on the EoS and could rule out some classes of equations of state that currently exist.

4. Summary and conclusions
We have computed various properties of static and uniformly rotating compact stars using EoS with first order nuclear to $K^-\text{condensed matter}$ phase transition and also $K^-\text{condensed matter}$ to quark matter phase transition. Mass and radius of EXO 0748-676 have been measured. These informations put stringent constraints on EoS. Our results are consistent with the mass and radius of EXO 0748-676. This, in turn, shows that bizarre matter might exist in this star. We have also calculated the moment of inertia for pulsar A of double pulsar binary PSR J0737-3039. It is found that the values of moment of inertia and radius obtained with relativistic EoS are quite different from that of the nonrelativistic model of Akmal and Pandharipande [6]. Soon the measurement of moment of inertia for pulsar A would be possible. This would lead to an estimate of the radius of pulsar A. Therefore, it would be possible to rule out some EoS with the help of the measured value of moment of inertia and mass of pulsar A. Lattimer and
Prakash showed that there was an empirical correlation between the radius and matter pressure in the density region $n_0$ to $2n_0$ [22]. Therefore, the determination of radius of pulsar A having a mass $M=1.337M_\odot$ could give an estimate of matter pressure around saturation density. It is interesting to note that we have informations about mass and radius of a single neutron star. More events like this would be available in future. Therefore, it would be worth investigating the dense matter EoS from the knowledge of masses and radii of neutron stars and using a numerical inversion of neutron star structure equation [23].

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