Derivation of the True Strain Rate Effect of Roller Compacted Concrete (RCC) from Impact-Induced Fragmentation

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Abstract

The true strain-rate effect for roller compacted concrete (RCC) removing the structural effect is a primary concern to achieve a reliable response prediction of RCC structures. In this study, a series of laboratory tests were carried out to study the dynamic compressive strength of RCC specimens with three different sizes subjected to various high-rate loadings. Based on the traditional decoupling method, the strength enhancements due to the inertial and end friction confinements were numerically evaluated and removed from the experimental DIF data. However, the mean values of material properties are generally employed in numerical simulation, which cannot reflect the high variability due to fast-successive constriction and mix proportion. This brings subjectivity and may result in incorrect evaluation of the structural effect and high variability in corrected DIF data. To solve this problem, the fractal dimension was introduced to quantify the fracture degree of impact-induced concrete fragments and describe the strain rate effect without significant variability. At last, a fragmentation-based approach was proposed to derive the true strain-rate effect of RCC and verified by comparison with the numerical and semi-theoretical methods.

1. Introduction

In recent decades, important infrastructures are facing huge threats of bomb attacks or accidental explosions from terrorism and local conflicts. Particularly, the safety of hydraulic dams subjected to such extreme loads has become a rising concern. The dam failure may lead to catastrophic consequences due to a sudden release of the reservoir water. Therefore, it is critical to understand the damage mechanism and failure modes of concrete dams suffering from all kinds of explosions, which has not been comprehensively considered in the design and construction of dams. As a special concrete material, roller compacted concrete (RCC) has a special mix design and construction technique, which has been widely used in the construction of high gravity dams or arch dams. Then, a reliable prediction of the dynamic response and antiknock performance of RCC dams requires an accurate evaluation of the strain-rate relation of RCC material.

It is well-accepted that concrete is a typical rate-dependent material. That is to say, the dynamic compressive strength of concrete material enhances with increasing strain rates. This compressive strength enhancement is generally characterized by the dynamic increase factor (DIF) (Ross et al. 1995; Zhou and Ge 2015; Hao et al. 2016), which is defined as the ratio of the compressive strength under dynamic and quasi-static loads. Based on various experiment results, many empirical DIF relations have been proposed to quantify the strength enhancement. These DIF relations can be applied simply for engineering design and structural analysis (Hao and Hao 2014; Hao et al. 2016). However, recent studies show that the strength enhancement obtained in experiments mainly attributes to not only the material strain-rate effect, but also structural effect due to lateral inertial and end friction confinements (Hao et al. 2016; Flores-Johnson and Li 2017). Then the DIF relations directly derived from experiment data may overestimate the dynamic response of concrete structures. Therefore, it is necessary to evaluate the true strain-rate effect exactly, removing the structural effect from experimental data.

Recently, efforts have been devoted to decoupling the lateral inertial effect and end friction effect to derive the true strain-rate effect. The lateral inertial effect in the split Hopkinson pressure bar (SHPB) tests has been widely investigated through numerical (Li and Meng 2003; Zheng et al. 2005; Li et al. 2009; Hao et al. 2010; Jin et al. 2019), experimental (Zhang et al. 2009, 2010), and analytical methods (Lee et al. 2018). Although most of them only confirm the existence and influence of lateral inertial confinement in SHPB tests, some studies also try to decouple the inertia-related strength enhancement from the experimental results. For example,
Hao and Hao (2011) suggested an approach to quantify the contribution from the inertial confinement to the DIF through numerical simulation. Based on a simple analytical model with the linear elastic theory for SHPB tests, Lee et al. (2018) proposed a new DIF formulation to describe the true strain-rate sensitivity of concrete, excluding the influence of the lateral inertial effect. On the other hand, the inevitable end friction confinement in experiments also results in an overestimation of the dynamic strength enhancement. An empirical relation was suggested to measure the end friction confinement with respect to the strain rate, friction coefficient, and aspect ratio (Hao et al. 2013a, 2013b). Consequently, based on the assumption that the true strain-rate effect, lateral inertial effect, and end friction effect are not related, the strength enhancement caused by the inertial and end friction confinement can be evaluated and removed from the experimental data. Finally, the true strain-rate effect can be obtained numerically and analytically. The above traditional decoupling method has been employed in the literature, the traditional decoupling method largely depends on the input parameters of numerical analysis, which is generally the mean value of material properties despite the variability in experiments. Thus, there is a certain degree of subjectivity in the traditional decoupling method.

As a special concrete material, the dynamic behavior of RCC has been confirmed in our previous studies that RCC shows an obvious strain-rate effect as well. Compared with normal concrete, the dynamic behaviors of RCC exhibit greater dispersion when subjected to higher strain rates. Moreover, more remarkable variability exists in the dynamic behaviors of RCC at high strain rates, mainly due to the distinct mixture (less water and fly ash mass fraction of fly ash in cementitious materials was 60%). A sand ratio of 31% was used to achieve a good mixture performance and an expected concrete strength. Limestone coarse aggregates, artificial medium sand with an apparent density of 2680 kg/m³, Portland cement with a strength grade of 42.5, and fly ash with fineness modulus of 19.60% were used in this study. Moreover, the high-performance superplasticizer and air-entraining agents were also employed to improve the performance of the RCC mixture. The vibrating-entrained (VC) value of the RCC mixture was tested to be 3.7 s. The maximum aggregate size was 19 mm to meet the requirement of the homogenization assumption. The fresh RCC mixture was firstly spread in a prefabricated trough with a thickness of 10 cm for each lift and 5 lifts in total. Each RCC lift experienced two static compactions at first to form a smooth working face, followed by eight vibration passes with a vibrating frequency of about 50 Hz and additional two static compactions. The compacted RCC was then cured in a standard condition for 90 days, and after curing RCC cores with diameters of 50, 75, and 100 mm were drilled. The hardened RCC was then cut and polished, ensuring the roughness of 0.5% of the thickness. As shown in Fig. 1, cylinders with different diameters but the same aspect ratio of 0.5 were prepared for further SHPB tests.

### 2. Size effect of RCC in SHPB tests

#### 2.1 Specimen preparation

The practical construction technique of RCC was simulated in the laboratory experiments, which has been described in detail in our previous studies (Wang et al. 2018a, 2018b). Table 1 shows the mixture proportion of RCC. The water/binder ratio (w/b) was 0.50, and the mass fraction of fly ash in cementitious materials was 60%. A sand ratio of 31% was used to achieve a good mixture performance and an expected concrete strength. Limestone coarse aggregates, artificial medium sand with an apparent density of 2680 kg/m³, Portland cement with a strength grade of 42.5, and fly ash with fineness modulus of 19.60% were used in this study. Moreover, the high-performance superplasticizer and air-entraining agents were also employed to improve the performance of the RCC mixture. The vibrating-entrained (VC) value of the RCC mixture was tested to be 3.7 s. The maximum aggregate size was 19 mm to meet the requirement of the homogenization assumption. The fresh RCC mixture was firstly spread in a prefabricated trough with a thickness of 10 cm for each lift and 5 lifts in total. Each RCC lift experienced two static compactions at first to form a smooth working face, followed by eight vibration passes with a vibrating frequency of about 50 Hz and additional two static compactions. The compacted RCC was then cured in a standard condition for 90 days, and after curing RCC cores with diameters of 50, 75, and 100 mm were drilled. The hardened RCC was then cut and polished, ensuring the roughness of 0.5% of the thickness. As shown in Fig. 1, cylinders with different diameters but the same aspect ratio of 0.5 were prepared for further SHPB tests.

#### 2.2 Experimental design

Dynamic compression tests were conducted by three SHPB sets with bar diameters of 50, 75, and 100 mm, adapting the specimens’ diameters. As listed in Table 2, four groups of gas pressures were adopted for each RCC category. The different experimental groups are denoted by the diameter of RCC specimens and the gas pressure in SHPB tests, for example, D50-0.30. To achieve the stress equilibrium and constant loading rate before specimen failure, the half-sine shock wave was achieved...
by the pulse-shaping techniques. The grease was daubed at the specimen-bar interface in order to minimize the influence of end friction. Besides, five 100 × 200 mm (diameter × height) cylindrical specimens were prepared to obtain the quasi-static compressive strength, and the strain rate used in the quasi-static tests was set to be 1e-5/s.

Since a major task of this study is to develop a fragmentation-based method to derive the true strain-rate effect of RCC, the accurate collection of the concrete fragments for further sieving analysis is extremely important herein. Therefore, the original mass of each RCC specimen was weighed before SHPB tests, which was used to calibrate the mass loss for the collected concrete fragments after experiments. The mass loss rate in this study could be less than 2% (Wang et al. 2018a). Although 10 specimens were prepared for each group, only 101 specimens successfully satisfied the assumptions of the SHPB test and the allowable mass loss rate.

### 2.3 Data processing and experimental validation

In SHPB experiments, the stress waves can be derived from the voltage signals, which were recorded by the strain gages mounted on the incident and transmitted bars. The smoothed stress waves for a typical test are shown in Fig. 2(a). Then, the stress \( \sigma(t) \) and strain rate \( \dot{\varepsilon}(t) \) can be obtained by Eqs. (1) and (2) (Lee et al. 2018). Based on the formula of the strain rate \( \dot{\varepsilon}(t) \), the strain history \( \varepsilon(t) \) can be calculated by integration via Eq. (3).

\[
\begin{align*}
\sigma(t) &= A_E [\varepsilon(t) + \varepsilon_i(t) + \varepsilon_r(t)]/(2A_s) \quad (1) \\
\dot{\varepsilon}(t) &= c_s [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon(t)]/H_s \quad (2) \\
\varepsilon(t) &= \varepsilon_i(t) - \varepsilon_r(t) - \int_0^t \varepsilon_i(t) - \varepsilon_r(t) - \varepsilon(t) dt / H_s \quad (3)
\end{align*}
\]

where \( A_s \) and \( A_e \) are the sectional areas of bars and specimen, respectively; \( c_s \) and \( E_s \) represent the wave propagation velocity and elastic modulus of the bars, respectively; \( H_s \) denotes the specimen length; \( \varepsilon_i(t) \), \( \varepsilon_r(t) \), and \( \varepsilon(t) \) denote the incident, reflected, and transmitted strain waves, respectively.

Figures 2(b) -2(d) show the typical stress-strain relations for RCC specimens with different sizes, as well as the strain rate varying with the strain. Meanwhile, the stress on the front surface of the specimens \( \sigma_f(t) \) is calculated by \( \sigma_f(t) = A_E [\varepsilon_f(t) + \varepsilon(t)]/A_s \), and the stress on back surface \( \sigma_b(t) \) can be expressed by \( \sigma_b(t) = A_E [\varepsilon_b(t)]/A_s \). When \( \varepsilon_i(t) + \varepsilon_r(t) = \varepsilon(t) \), then the stresses on both end surfaces of the specimen are equal, and the dynamic equilibrium state under impact loads is obtained. As illustrated in Fig. 2, it can be found that the stresses on the front and back surfaces of the specimens for each case are very close. Thus, the experimental tests in the present study are reliable, and the calculation of dynamic mechanical properties through Eqs. (1) - (3) can be simplified by the one-dimensional wave propagation theory. On the other hand, it is also

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**Table 2: Specimen size and shape for SHPB and quasi-static tests.**

| Test type          | Specimen dimensions (D×L: mm × mm) | L/D  | Group | Number | Gas pressure (MPa) |
|--------------------|------------------------------------|------|-------|--------|-------------------|
| Quasi-static tests | 100×200                            | 2.0  | /     | 5      | —                 |
|                    | 50×25                              | 0.5  | D50-0.30 | 8    | 0.30             |
|                    |                                    |      | D50-0.37 | 8    | 0.37             |
|                    |                                    |      | D50-0.45 | 8    | 0.45             |
|                    |                                    |      | D50-0.55 | 8    | 0.55             |
| SHPB tests         | 75×37.5                            | 0.5  | D75-0.40 | 7    | 0.40             |
|                    |                                    |      | D75-0.50 | 9    | 0.50             |
|                    |                                    |      | D75-0.60 | 8    | 0.60             |
|                    |                                    |      | D75-0.80 | 8    | 0.80             |
|                    | 100×50                             | 0.5  | D100-0.15 | 8    | 0.15             |
|                    |                                    |      | D100-0.22 | 10   | 0.22             |
|                    |                                    |      | D100-0.30 | 10   | 0.30             |
|                    |                                    |      | D100-0.37 | 9    | 0.37             |
found in Figs. 2(b) -2(d) that all these three specimens fail at a relatively stable phase of strain rate, achieved by the pulse-shaping techniques. Therefore, the constant strain-rate loading was realized in the experimental tests.

2.4 Size-dependent structural effect of dynamic compressive strength

It is widely accepted that the dynamic compressive strength of concrete is significantly influenced by the structural effect. More significant strain-rate sensitivity can be found in larger specimens, and the strength enhancement related to specimen size is nonlinear (Wang et al. 2018b). However, the size-dependent structural effect cannot reflect the material strain-rate effect. The experiment results of RCC specimens with different sizes at a similar loading rate (about 70 s⁻¹) are summarized in Table 3. The strain rate and the strain acceleration for the specified RCC specimens are determined by their averages until the peak strength is reached. It is obvious that the average compressive strength of RCC specimens increases with specimen size.

Figure 3(a) shows the size-dependence of dynamic compressive strength induced by the structural effect more intuitively, the standard deviation is used to illustrate the variability along with strain rate as well. The dynamic compressive strength of RCC shows obvious strain-rate sensitivity and size-dependence. Besides, its variability is also sensitive to the strain rate. Specifically, the variability of the dynamic compressive strength of RCC tends to increase with strain rates. Figure 3(b) shows the relationship between apparent DIF data and strain rate for specimens with different sizes. Mention that the apparent DIF data denotes the DIF including the structural effect, which is directly obtained from SHPB tests. The structural analysis based on the apparent DIF data may well lead to unconservative results by overestimating the dynamic compressive strength of RCC. Moreover, the significant variability in experiment re-

| Group | Indexes | Test 1 | Test 2 | Test 3 | Test 4 | Test 5 | Test 6 | Test 7 | Test 8 | Test 9 | Average |
|-------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| D50-A | Strain rate (s⁻¹) | 64.27 | 69.26 | 70.58 | 69.28 | 70.95 | 71.51 | 74.31 | 71.01 | — | 70.15 |
|       | Strength (MPa)    | 11.70 | 9.76  | 10.82 | 11.60 | 9.22  | 10.75 | 9.45  | 10.18 | — | 10.44  |
| D75-B | Strain rate (s⁻¹) | 69.73 | 72.00 | 74.66 | 74.83 | 78.35 | 79.67 | 80.17 | 80.21 | 80.68 | 76.70  |
|       | Strength (MPa)    | 12.99 | 21.74 | 12.66 | 20.79 | 20.59 | 28.79 | 26.38 | 18.67 | 14.57 | 19.69  |
| D100-D| Strain rate (s⁻¹) | 70.00 | 71.66 | 70.25 | 73.12 | 73.54 | 78.03 | 74.38 | 79.96 | 75.76 | 74.08  |
|       | Strength (MPa)    | 31.14 | 33.58 | 19.70 | 25.53 | 30.08 | 24.66 | 29.76 | 38.91 | 31.62 | 29.44  |

Table 3 Dynamic compressive strength for specimens with the strain rate around 70 s⁻¹.

Fig. 2 Experimental validation: (a) the smoothed stress waves for a specimen; (b) - (d) dynamic stresses and strain rates along with strain for different dimensional specimens.
3. Derivation of true strain-rate effect by traditional decoupling method

It is well-accepted that the lateral inertial effect, end friction effect, and true strain rate effect are coupled with each other (Li et al. 2009). If the DIF data obtained from SHPB tests is directly used, the corresponding dynamic compressive strength will be overestimated. In this section, it is assumed that the true strain rate effect, lateral inertial effect, and end friction effect are not related (Yu et al. 2013). Then the true strain rate effect can be defined as Eq. (4). The true strain-rate effect of RCC can be derived using the traditional decoupling method in the literature, by determining the contributions from the lateral inertial effect and end friction effect numerically.

\[
\text{DIF}_A = \frac{f_c}{f_c} = \left( f_c + \Delta f_i + \Delta f_f + \Delta f_i \right) / f_c
\]

where \( \text{DIF}_A \) denotes the apparent DIF directly obtained from SHPB tests; \( f_c \) and \( f_c \) are the dynamic compressive strength and that under quasi-static loads; \( \Delta f_i \) is the strength enhancement caused by the material strain-rate effect; \( \Delta f_i \) and \( \Delta f_f \) are the strength enhancements induced by the inertial and end friction confinements, respectively.

3.1 Numerical simulation method of SHPB test

Referencing previous work (Hao et al. 2010), a homogeneous numerical model was established for simplification to simulate the SHPB tests in LS-DYNA, as shown in Fig. 4(a). The three-dimensional (3D) Solid164 element was used to model the specimen, incident, and transmitted bars. On the balance of accuracy and computational economy, the mesh sizes of specimens and bars were designed to be 0.5 mm and 3.0 mm, respectively (Hao et al. 2010). Moreover, the keyword “CONTACT_SURFACE_TO_SURFACE” was used to model the contact between the specimen and rigid bars.

The KCC model (MAT_072R3) was selected to model the RCC specimen, because it can capture the key concrete damage behaviors. The parameters of KCC model were generated by the automation generation algorithm in LS-DYNA, besides the input of some necessary parameters obtained from quasi-static compressive tests (e.g., \( \rho_s = 2230 \text{ kg/m}^3 \), and \( f_c = 11.13 \text{ MPa} \)). The new relation for the strain-rate effect of concrete proposed in

![Fig. 3 SHPB test results for RCC specimens with different sizes: (a) variability of dynamic compressive strength under different loading rates; (b) the relationship between strain rate and apparent DIF.](image1)

![Fig. 4 Numerical model for calibration: (a) FEA model in LS-DYNA; (b) Comparison with the stress-strain curves.](image2)
the literature (Hao and Hao 2014) was used herein, which were regressed by the experimental data removing the structural effect. The incident and transmitted pressure bars were modeled by the isotropic elastic model (MAT_1). The parameters of different materials in the numerical simulation are listed in Table 4. Mention that the Young’s modulus and the density were determined by quasi-static tests, while the Poisson’s ratio took the general values of concrete materials.

The smoothed stress waves obtained from the SHPB tests were applied to the front surface of the incident bar. The keyword “MAT_ADD_EROSION” was utilized for concrete material in this study, and among all criteria for the deletion of the elements, the maximum principal strain (MXP) was employed herein. The above numerical model was verified by the stress-strain curve of a typical SHPB test. As shown in Fig. 4(b), the homogeneous numerical model cannot fully reproduce the heterogeneity of RCC specimens in SHPB tests, and there is a difference in the peak strain in stress-strain curves. However, the consistency of the peak stress indicates the reliability of the numerical simulation method to investigate the DIF relations of the compressive strength.

### 3.2 Decoupling analysis of experimental DIF

#### 3.2.1 Contribution from end friction confinement effect

It has been confirmed that the end friction at the specimen-bar interfaces can constrain the lateral deformation of the specimen, influencing the experimental results (Hao et al. 2013a, 2013b). Therefore, figuring out the strength enhancement induced by end friction confinement should be given careful consideration. This issue has been studied carefully through a mesoscale model considering the concrete components (Hao et al. 2013b). According to their findings, the strength enhancement caused by end friction confinement was coupled by aspect ratio (L/D), friction coefficient (μ), and strain rate (\(\dot{\varepsilon}\)). Moreover, an empirical relation in Eq. (5) was suggested to quantify the contribution from end friction confinement to dynamic compressive strength, whose accuracy was verified by experimental tests (Hao et al. 2013b).

For \(0.5 \leq L/D \leq 2.0\), \(0.0 \leq \mu \leq 0.5\) and \(10 / s^{-1} \leq \dot{\varepsilon} \leq 600 / s^{-1}\)

\[
\chi = \frac{DIF_{\mu > 0}}{DIF_{\mu = 0}} \approx e^{(1.0254 \times 10^{-3} (L/D) - 0.2844 (\mu - 0.1045) (\dot{\varepsilon} - 0.1563)) (5)}
\]

where \(DIF_{\mu > 0}\) represents the resulted DIF with end friction confinement in numerical simulation, while \(DIF_{\mu = 0}\) denotes the resulted DIF without end friction confinement. \(\chi\) is the ratio of \(DIF_{\mu > 0}\) to \(DIF_{\mu = 0}\).

In this study, the aspect ratio of all specimens remains 0.5. Then, the remaining variables that may significantly influence the end friction confinement are the friction coefficient and strain rate. Substituting all kinds of variables into Eq. (5), the contribution from end friction confinement can be illustrated by Fig. 5(a) under different parameter inputs. It is obvious that rational evaluation of the dynamic friction coefficient is the basis for measuring the contribution from the end friction confinement. However, it is difficult for us to measure the practical friction coefficient at the interface between RCC specimen and bars in SHPB tests. It is stated that the tested static friction coefficient (μ = 0.235) is usually smaller than the dynamic friction coefficient, and a good fitting result can be obtained when \(\mu = 0.10\) can be applied to correct the test data (Hao et al. 2013b). Thus, \(\mu = 0.10\) and \(L/D = 0.50\) are employed to furtherly evalu-

#### Table 4 Material models and parameters in numerical simulation.

| Material      | Density (kg/m³) | Young’s modulus (GPa) | Poisson’s ratio |
|---------------|-----------------|-----------------------|----------------|
| Pressure bar  | 7800            | 200                   | 0.3            |
| RCC           | 2230            | 11.3                  | 0.167          |

![Fig. 5](image) Strength enhancement from end friction confinement: (a) effect of friction coefficient on the end friction confinement; (b) strength enhancement due to the end friction confinement.
ate the strength enhancement from end friction confinement. Based on Eq. (6), the enhanced compressive strength of different size specimens under various strain rates can be calculated as shown in Fig. 5(b), and it can be used to eliminate the end-friction-related strength enhancement from the apparent DIF (DIF\textsubscript{a}).

3.2.2 Contribution from inertial confinement
To evaluate the true strain rate effect of RCC, it is important to exclude the inertia-induced strength enhancement from DIF\textsubscript{a}. For a strain-rate insensitive material, if defining DIF as unity in the material model in numerical simulation, the enhanced strength in measured DIF\textsubscript{a} is only induced by the inertial and end friction confinements (Hao and Hao 2011), as expressed by Eq. (7).

\[
\text{DIF}_{a} = \frac{f'_{c}}{f_{c}} = \left(1 + \Delta f_{i} + \Delta f_{s} \right)/f_{c}
\]  

(7)

Thus, the strength enhancement induced by the inertial confinement at different strain rates can be evaluated by Eq. (8) and then removed from experimental results.

\[
\Delta f_{i} = \left(\text{DIF}_{a} - 1\right)f_{c} - \Delta f_{s}
\]  

(8)

Based on the verified numerical model, the contribution of the lateral inertial effect can be determined. Contribution of inertial confinement under different strain rates, defined as \(\Delta f_{i}/(\text{DIF}_{a}f_{c})\), are plotted in Fig. 6(a). And the corresponding inertia-related strength enhancements in SHPB tests are also plotted in Fig. 6(b) for specimens with different sizes. As illustrated in Fig. 5, the inertial effect on dynamic compressive strength is both size-dependent and strain-rate sensitive. Higher strength enhancement due to the inertial confinement can be expected for a higher loading rate or a larger specimen.

3.3 True strain-rate effect through traditional decoupling method
The DIF data obtained from SHPB tests is caused by the interaction of the lateral inertia effect, end friction effect, and true strain rate effect. In the above sections, the strength enhancements caused by the end friction effect and lateral inertial effect have been determined, respectively. Then the true strain-rate effect can be determined, after removing them from DIF data obtained from SHPB tests, as shown in Eq. (9). Finally, typical empirical and theoretical analytical models to describe the true strain-rate effect of RCC material are proposed in this section.

\[
\text{DIF}_{\text{f}} = \left(\frac{f'_{d} - \Delta f_{i} - \Delta f_{s}}{f_{c}}\right)
\]  

(9)

The corrected DIF data reflecting the material strain-rate effect are shown in Fig. 7(a). Since Eq. (10) has been widely adopted to describe the strain-rate effect for quasi-brittle materials including concrete (Zhou and Hao 2008; Hao \textit{et al.} 2010), this study also uses it to fit the empirical relation between true DIF and strain rate.

\[
\text{DIF}_{\text{f}} = \begin{cases} \left(\dot{\varepsilon} / \dot{\varepsilon}_{0}\right)^{k_{1}} & \text{for } \dot{\varepsilon} < \dot{\varepsilon}_{c} \\ k_{0}\left(\dot{\varepsilon} / \dot{\varepsilon}_{0}\right)^{k_{1}} & \text{for } \dot{\varepsilon} \geq \dot{\varepsilon}_{c} \end{cases}
\]  

(10)

where \(\dot{\varepsilon}_{c}\) denotes the strain rate under quasi-static loading and equals to \(8.30 \times 10^{-5} \text{ s}^{-1}\); and \(\dot{\varepsilon}_{c}\) is the critical strain rate that equals to \(30 \text{ s}^{-1}\) (Zhou and Hao 2008; Hao \textit{et al.} 2010). \(k_{0}\) and \(k_{1}\) are two parameters to be fitted for describing the strain-rate effect.

By the least square fitting with the corrected DIF data in Fig. 7(a), an empirical model for Eq. (10) with parameters of \(k_{0} = 0.015\) and \(k_{1} = 0.017\) can be employed to determine the true strain-rate effect of RCC. Mention that \(k_{0}\) is determined from continuity requirements. Figure 7(a) also compares the proposed true DIF relations with other well-accepted classic models. Both the corrected DIF data and proposed true DIF relations fall into the range of classic models, despite the significant variability of corrected DIF data. Moreover, the proposed DIF relations of RCC extremely approach to the empirical relations suggested in the literature (\textit{fib} 2013; Hao and Hao 2014), especially for the strain-rate model in the \textit{fib} code.

![Fig. 6 Strength enhancement from inertial confinement: (a) size-dependence of the inertial confinement; (b) strength enhancement due to the inertial confinement.](image-url)
On the other hand, some theoretical analytical models were also introduced to investigate the inertial effect in SHPB tests (Forrestal et al. 2007; Lee et al. 2018). For example, based on the linear elastic theory, Lee et al. (2018) gave a detailed derivation for the strength enhancement induced by the inertial confinement, which depended on the strain rate, strain acceleration, and velocities at the specimen-bar interfaces. The key factors influencing the inertial confinement were the strain acceleration and specimen geometry. Thus, the resulted strength enhancement was simply expressed by Eq. (11), in which the strain acceleration \( \dot{\varepsilon}_s(t) \) can be obtained by the central difference method as illustrated by Eq. (12) and the average before the peak strength is used herein. The calculated experimental strain accelerations along with the strain rate are shown in Fig. 7(b), indicating a relatively stable relationship between strain rate and strain acceleration.

\[
\Delta f_s = k_1 \rho d_1^2 \dot{\varepsilon}_s + k_2 \rho d_2^2 \dot{\varepsilon}_s \tag{11}
\]

\[
\dot{\varepsilon}_s(t) = [\dot{\varepsilon}_s(t + \Delta t) - \dot{\varepsilon}_s(t - \Delta t)]/(2\Delta t) \tag{12}
\]

where \( k_1 \) and \( k_2 \) represent the rate-related parameters for the radial and axial inertial effects, respectively; \( t \) and \( \Delta t \) represent the instantaneous moment and the time interval in the derivation of strain acceleration.

Introducing Eq. (7) and Eq. (11) into Eq. (9), the rate-related parameters can be obtained by the least-square fitting method in Matlab platform with a non-negative boundary condition, i.e., \( k_1 = 0.0150 \), \( k_2 = 0.2633 \), \( k_3 = 0.0658 \). Moreover, \( k_0 = 5.177 \times 10^{-3} \) is used with due consideration of the continuity requirements for the strain-rate effect. Then, the true strain-rate effect of RCC derived from the semi-theoretical formulae can be obtained by removing strength enhancements induced by the inertial and end friction confinements.

A comparison of the two proposed empirical relations for the true strain-rate effect of RCC through the numerical method and semi-theoretical formulae is conducted in Fig. 7(a). It is obvious that the strain-rate effect expressed by the model from semi-theoretical formulae is a little lower than that from numerical derivation. But, the high strain-rate parts of them both change in the range between models proposed by fib code (fib 2013) and (Hao and Hao 2014).

### 4. Derivation of true strain-rate effect from RCC fragmentation

As illustrated in Chapter 3, the traditional decoupling method needs to evaluate the contributions from inertial and end friction confinements respectively. Then the above structural effect could be excluded from apparent DIF data to obtain the true strain-rate effect. Obviously, the traditional decoupling method largely depends on the input parameters in numerical simulation, where the mean values of material physical and mechanical properties are input. So, the traditional decoupling method cannot remove all the structural effects from the experimental data due to its high variability from fast-successive construction. As shown in Fig. 7(a), the variability of the true DIF data obtained by the traditional decoupling method becomes more obvious as the strain rate or specimen size increases. In this subsection, a new approach is suggested to determine the true DIF from impact-induced concrete fragmentation.

#### 4.1 Quantification of concrete fragmentation

In our previous studies, it has been proved that the fractal characteristics of impact-induced fragments could be used to explain the size-dependence and strain rate effect of RCC (Wang et al. 2018a). Thus, the objective of this section is to investigate the feasibility of deriving the true strain-rate effect via the fractal dimension from concrete fragmentation.

For this purpose, concrete fragments of each specimen after the SHPB test were carefully collected and sieved. It can be found in Fig. 8(a) that at a similar loading rate (about 70 s\(^{-1}\)), the content of coarse fragments decreases with specimen size, while the content of fine fragments exhibits a contrary rule. As shown in Fig. 8(b), the relationship between strain rate and strain acceleration.
the sieving results show the size-dependence of the failure pattern more obvious. Generally, more fine fragments are generated when the specimen becomes larger under the same loading rate, indicating a higher specific surface area of concrete fragments.

To quantitatively characterize the concrete fragments, Eq. (13) was introduced to illustrate the sieving curve with \( \frac{M(<d)}{M_T} \), which is determined by fractal dimension and the largest size of fragments. As shown in Table 5, all the concrete fragments can pass the sieving hole of 16 mm, but part of them cannot further pass the 10 mm hole. The largest size of fragments is then selected as 16 mm in this case, i.e., \( d_{\text{max}} = 16 \text{ mm} \). Moreover, according to Eq. (13), the fractal dimension can be calculated by the linearly fitting slope for the sieving results in the bi-logarithmic diagram of \( \frac{M(<d)}{M_T} \) and \( d \), as shown in Fig. 8(c). Thus, the fractal dimension is an ideal indicator to quantify the concrete fragmentation under the impact loads.

\[
\ln\left(\frac{M(<d)}{M_T}\right) = (3 - D_f)(\ln d - \ln d_{\text{max}})
\]

in which \( d \) is the nominal diameter of the concrete fragments; \( M(<d) \) and \( M_T \) represent the partial mass for fragments smaller than \( d \) and the total mass of the concrete fragments for a specimen, respectively; \( D_f \) is the fractal dimension and \( d_{\text{max}} \) is the largest size of the fragments.

### 4.2 Derivation of the true strain-rate effect from concrete fragmentation

As illustrated in Fig. 9(a), the relationship between apparent DIF and fractal dimension is size-independent, indicating that a higher dynamic compressive strength for a larger specimen is caused by the activation of more micro-cracks. As a result, more severe concrete fragmentation leads to a higher fractal dimension. The fractal dimension thus is a reasonable representation of apparent DIF without size-dependence, which can be explained theoretically by the specific surface area of concrete fragments in our previous work as described by Eq. (14) (Wang et al. 2018a). It can be found that the size of the largest fragment under high strain-rate loading significantly decreases and gets to be similar, no matter what the specimen size is (Wang et al. 2018a). In this case, the specific surface area is only related to the fractal dimension, and \( \partial A / \partial D_f > 0 \) always stands when \( 2 < D_f < 3 \) and \( d_{\text{max}} >> d_{\text{max}} \), indicating that a positive relationship between specific surface area of fragments and fractal dimension.

| Specimen | Percent of passing mass \( M(<d)/M_T \) (%) | \( M_T \) (g) | \( d_{\text{max}} \) (mm) | \( D_f \) |
|----------|--------------------------------|----------|----------------|-------|
| D50-74.31 s\(^1\) | 0.33 1.51 3.35 9.93 14.99 20.75 31.57 72.86 100 100 | 112.76 | 16 | 2.08 |
| D75-69.73 s\(^1\) | 0.90 2.04 4.11 12.17 17.93 22.62 33.67 80.77 100 100 | 412.04 | 16 | 2.18 |
| D100-71.66 s\(^1\) | 1.40 3.54 6.64 15.85 22.92 27.76 41.01 88.58 100 100 | 922.57 | 16 | 2.29 |

Fig. 8 Fractal characteristics of concrete fragments for RCC specimens with different sizes: (a) failure patterns at a similar loading rate (about 70 \( \text{s}^{-1} \)); (b) sieving results; (c) fractal dimension from concrete fragments.
where $A$ represents the specific surface area of concrete fragments; $A_s$ and $A_v$ are the shape coefficients for the fracture surface area and the estimated volume, respectively; $d_{\text{min}}$ represents the minimum size of the fragments and is small enough to ignore its effect on $A$.

On the other hand, as shown in Fig. 9(b), the fractal dimension is sensitive to the strain rate accompanied with the size-dependence. However, the variability of fractal dimension under high strain rates is much less significant than that of apparent DIF. The possible reason is that although mesoscopic defects can significantly influence the dynamic mechanical properties, the macroscopic concrete fragmentation only lies in the specimen size and loading rate and will exhibit the similar manner (i.e., the proportion of the coarse or fine fragments). Thus, the fractal dimension of impact-induced fragments is suggested to measure the strain-rate effect herein. Generally, the changing rule of the fractal dimension along with the strain rate can be defined by Eq. (15), in which the size effect is considered by the critical strain rate ($\dot{\varepsilon}_c$). The determination coefficient ($R$) is high enough to be 0.903. It is well-accepted that the critical strain rate decreases with the increase of specimen size (Qi et al. 2014, 2016), and it is 62 s$^{-1}$ for group D50, 41 s$^{-1}$ for group D75, and 30 s$^{-1}$ for group D100 in the experimental tests.

$$D_f = a_1 \ln(\dot{\varepsilon}/\dot{\varepsilon}_c - 1) + a_2 \quad (R = 0.903)$$

(15)

where $a_1 = 8.579 \times 10^{-2}$, $a_2 = 2.239$, $b_1 = 2.762$, $b_2 = -1.038$ based on the least-square method.

To describe the true strain-rate effect of RCC in terms of fractal dimension, the next object is to remove the inertia-induced increase of fractal dimension. Based on the above analysis, the structural effect in concrete fragmentation results in the size dependence of fractal dimension. Usually, when the specimen is small enough, the dynamic behaviors can be identified as the true material properties without structural effect. According to previous findings (Li et al. 2009; Zhang et al. 2009; Flores-Johnson and Li 2017), the structural effect will get to a relatively low level when the specimen size is smaller than 30 mm. Therefore, the apparent fractal dimension in this case can be identified as the true one to describe the true material fragmentation without structural effect when $d_s < 30$ mm, though the size of concrete specimen usually cannot decrease to this level for meeting the homogeneity assumption. Then, the true strain-rate effect from concrete fragmentation can be generally derived as follows: (1) calculate the fractal dimension from the impact-induced fragments for concrete specimens of different sizes; (2) obtain the model parameters in Eq. (15) by least-square method; (3) estimate the true fractal dimension by Eq. (15) at a certain strain rate but a small enough diameter; (4) estimate the true DIF by Eq. (16).

$$DIF = \left( \frac{D_f}{\dot{\varepsilon}_c} \right) = \left[ \frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_c} \right] + \exp \left( \frac{D_f - a_2}{a_1} \right)$$

(16)

in which $D_{10}$ and $\dot{\varepsilon}_{c0}$ denote the true fractal dimension and corresponding critical strain rate at a small enough specimen size.

However, the parameter $k_1$ in the empirical model of strain-rate effect varies in different sources, as shown in Table 6.

| Source | fib code | CEB code | Numerical method | Semi-theoretical formulae method | Fragmentation-based method |
|--------|----------|----------|-----------------|---------------------------------|---------------------------|
| Value  | 0.0120   | 0.0257   | 0.0170          | 0.0150                          | 0.0164                    |

Table 6 Estimation of the parameter $k_1$ for the strain-rate effect.

*Note: $\lg k_1 = 6.156\alpha - 2$, $\alpha = (5 + 9f_c/f_{oa})^{-1}$, $f_{oa} = 10$ MPa.

Fig. 9 Correlations among the parameters for strain-rate effect: (a) size-independence of apparent DIF under fractal dimension; (b) size-dependence of fractal dimension under strain rate.
In this study, RCC specimens of different sizes were carefully prepared and a series of SHPB tests were carried out. The resulted dynamic compressive strength verified the existence of the inertial and end friction confinements. In order to derive the true strain-rate effect of RCC, many attempts were made to decouple the structural effect from the apparent DIF data through numerical and theoretical methods. Furthermore, a new approach was proposed to determine the true strain-rate effect through impact-induced concrete fragmentation. The main conclusions of this study are listed as below:

1. The dynamic compressive strength in SHPB tests was significantly influenced by the structural effect caused by the inertial and end friction confinements. The obtained apparent DIF data were size-dependent and accompanied by high variability, especially at higher strain-rate loadings. The strength enhancements induced by the inertial and end friction confinements were decoupled from the apparent DIF data obtained from SHPB tests through numerical simulation and empirical relation, respectively.

2. Based on the traditional decoupling methods in the literature, the contributions of inertial and end friction confinements were determined, and then removed from the apparent DIF data. The true strain-rate effect of RCC was verified to be closely consistent with the models proposed by fib code and Hao. However, the evaluation of structural effect through traditional decoupling methods could not consider the change of material properties from fast-successive construction, leading to the high variability of the corrected DIF data.

3. To avoid the subjectivity of parameter input in traditional decoupling methods, the fractal dimension derived by impact-induced concrete fragments was suggested to achieve a more stable relation of the strain-rate effect. The fragmentation-based strain-rate effect could avoid the misestimation from the significant variability of DIF data, especially at high loading rates. The key parameter in the proposed model of the true strain-rate effect was discussed in detail and verified by comparison with the parameters estimated from other sources.

Table 6. In this study, the derived true DIF and true fractal dimensions are used to estimate the model parameter \( k_l \) whose strain rate is higher than the critical values (i.e., \( \dot{\varepsilon} > \dot{\varepsilon}^* \)). and \( k_l \) is fitted to be 0.0164 when \( d_z = 30 \text{mm} \) is used to be estimate the true fractal dimension. As shown in Fig. 10, it can be found that the parameter \( k_l \) obtained by the fractal dimension is approximate to those obtained by numerical method and semi-theoretical formulae method, indicating the validation of the suggested approach to determine the true strain-rate effect through impact-induced concrete fragmentation.

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