Two-stage pulse tube refrigerator with step displacer as phase shifter

Shaowei Zhu
Institute of Refrigeration and Cryogenics
School of Mechanical Engineering, Tongji University
4800, Cao’an Road, Shanghai, 201804, China
E-mail: swzhu2008@yahoo.com

Abstract. A two-stage pulse tube refrigerator with step displacer is discussed by numerical simulation. There is an optimum swept volume ratio of displacer over compressor and optimum phase angle difference between the displacer and the compressor. The volume of connecting tube between the pulse tube and the displacer can be used as a parameter to separate the mixing of the phase shifter effect and work distribution effect of the step displacer.

1. Introduction
In order to get cooling power at lower temperature with high efficiency, two-stage pulse tube refrigerator is necessary [1-4]. Till now, Stirling type two-stage pulse tube refrigerator is still not comparable with two-stage Stirling refrigerator. One of the problems is that the phase shifter is not strong enough, powerful phase shifter is needed. One of the choices is to use solid phase shifter, such as step displacer [5]. For the second stage, solid phase shifter is effective [6]. If the second stage uses a solid phase shifter, then to make it as a step type to let both stages use a solid phase shifter is a natural choice. If a step displacer is used, the phase shifter effect and work distribution effect are mixed together. In this paper, we will use the volume of the connecting tube to separate the phase shifter effect and work distribution effect.

2. Structure
Figure 1 is the schematic of the two-stage pulse tube refrigerator. An after cooler, a first stage regenerator, a second stage regenerator, a second stage cold head, a second stage pulse tube, a second stage gas distributor and a second stage connecting tube are connected serially. Between the first stage and the second stage regenerator, a first stage cold head, a first stage pulse tube, a first stage gas distributor and a first stage connecting tube are connected serially.

Compared to pre-cooling type two-stage [5], the first stage regenerator can work at desired condition, and there is no pre-cooling heat exchanger. The disadvantage is the difficult gas distribution problem in the regenerator if the regenerator is very big for high cooling power.

3. Mixing problem of phase shifter effect and work distribution effect
On the assumption of ideal pulse tube refrigerator, the swept volume for the first stage and second stage is related to the cooling power and refrigeration temperature, which depends on the mission and we cannot change. From the regenerator efficiency view point, the swept volume for the first stage and second stage have to be optimized to let the mass flow rate and pressure at the cold end of the pulse
tube have a suitable phase angle difference.

In general case, the swept volume for cooling power and for regenerator efficiency is difficult to be the same. This is the mixing problem of the phase shifter effect and work distribution effect of the step displacer.

In order to solve the problem, the swept volume ratio of the first stage displacer space and second stage displacer space is set for cooling power, while the swept volume of one displacer space is just fit for the pulse tube, the swept volume of another displacer space is bigger than the requirement and its over swept volume is balanced by the dead volume between the displacer head and the pulse tube. In this paper, connecting tube is used as the dead volume.

Step displacer is the simplest type. Though we can use different displacers for different stages, the structure is complex. For two-stage, two independent displacers may be acceptable. For three-stage, three independent displacers are too complex compared to one step displacer with three steps. Compared to mixing phase shifter of solid phase shifter and inertia tube, step displacer is also simple.

![Figure 1. Schematic of two-stage pulse tube refrigerator with step displacer.](image)

10. after cooler  11. first stage regenerator  12. first stage cold head  13. first stage pulse tube  14. first stage gas distributor  15. first stage connecting tube  16. first stage displacer space  21. second stage regenerator  22. second stage cold head  23. second stage pulse tube  24. second stage gas distributor  25. second stage connecting tube  26. second stage displacer space  31. compression space.

4. Numerical method
The numerical method in reference [7] is used for the simulation. It is the first numerical model of pulse tube refrigerator, and is further improved with momentum effect [8]. This numerical method has an essential contribution to the pulse tube refrigerator in the pulse tube refrigerator history. The basic working mechanism of the pulse tube refrigerator is explained with the numerical data of this numerical method, even the invention of the double inlet pulse tube refrigerator is mainly from the numerical results of this numerical method.

5. Swept volume ratio and phase angle difference
The basic data of the refrigerator is in table 1. The room temperature is 300K. The first stage temperature is 80K and the second sage temperature is 20K. Working medium is helium gas with 2MPa charge pressure. Operation frequency is 50Hz. The step ratio which is the swept volume of the first displacer space over the second displacer space is 4, which means that the input power for the first stage and second stage is the same at ideal condition. The swept volume of the compressor is 200cm$^3$. The swept volume ratio which is the total swept volume of the first displacer space and the second displacer space over the compression space and phase angle difference which is the phase angle delaying of the compression space over the displacer space are changed for finding the optimum swept volume ratio and phase angle difference.
Table 1. Main parameters.

| Parts                        | Parameters                                      |
|------------------------------|-------------------------------------------------|
| First stage regenerator      | Φ70mm×50mm, porosity70%, wire diameter 0.035mm   |
| First stage pulse tube       | Φ40mm×200mm                                     |
| First stage connecting tube  | Φ10mm×200mm                                     |
| Second stage regenerator     | Φ40×50, porosity62%, wire diameter 0.025mm       |
| Second stage pulse tube      | Φ20mm×200mm                                     |
| Second stage connecting tube | Φ5mm×200mm                                      |

Figure 2 shows the effect of swept volume ratio and the phase angle difference. There is an optimum swept volume ratio to let the first stage regenerator loss and the second stage regenerator loss become minimized. Near the optimum swept volume ratio, the first stage regenerator loss and the second stage regenerator loss increase with the increasing of phase angle difference. Other loss, which includes loss of the compressor and pulse tubes, increases with the increasing of swept volume ratio and the decreasing of phase angle difference. There is an optimum phase angle difference and swept volume ratio for efficiency. In the simulation range, the input power and cooling power of the second stage increase with the increasing of the swept volume ratio and phase angle difference. The cooling power of the first stage increases with the increasing of the phase angle difference. At phase angle difference 30° and 40°, the cooling power of the first stage increases with the increasing of the swept volume ratio. At phase angle difference 20°, the cooling power of the first stage has a peak point with the increasing of the swept volume ratio. The pressure ratio increases with the increasing of swept volume ratio and with the decreasing of the phase angle difference.

![Figure 2a. First stage regenerator loss.](image)

![Figure 2b. Second stage regenerator loss.](image)

![Figure 2c. Other loss.](image)

![Figure 2d. Efficiency.](image)
Figure 2e. Input power.

Figure 2f. First stage cooling power.

Figure 2g. Second stage cooling power.

Figure 2h. Pressure ratio.

Figure 3 shows the equivalent PV diagrams in the pulse tubes, mass flow rate and pressure wave at the cold end of the pulse tubes at the optimum point. There is a distance between the PV diagrams of the warm end and the cold end. The mass flow rate lags the pressure wave, indicating good condition for regenerator efficiency. In this case, the mass flow rate at the cold end of the first stage pulse tube and the second stage pulse tube are almost the same and the PV diagrams are very similar.

Figure 3a. Equivalent PV diagrams.

Figure 3b. Mass flow rate and pressure.
6. Connecting tube effect
At step ratio 8, the input power to the first stage is two times that of the second stage. In this case, if the swept volume of the second stage is optimized, the swept volume of the first stage would be too big. The dead volume of the connecting tube of the first stage should be increased so that the big first stage displacer swept volume can be balanced by the volume of the connecting tube.

Figure 4 shows the loss, cooling power of the first stage and second stage, efficiency, input power and pressure ratio vs. first stage connecting tube ratio, which is the volume of the first stage connecting tube over the volume of the first stage pulse tube. The loss of the first stage regenerator increases with the increasing of the first stage connecting tube ratio. The loss of the second stage and other loss decrease with the increasing of the first stage connecting tube ratio. There is an optimum first stage connecting tube ratio for efficiency and the cooling power of the second stage. The first stage cooling power, input power and pressure ratio decrease with the increasing of the first stage connecting tube ratio. The PV diagrams, mass flow rate and pressure at the cold end of the pulse tube with connecting tube ratio 0.0625 and 1 are shown in Figure 5 and Figure 6, respectively. It is shown that the first stage connecting tube can adjust the PV diagrams and mass flow rate.

![Figure 4a](image1.png)  ![Figure 4b](image2.png)

**Figure 4a.** Loss vs. first stage connecting tube ratio.

**Figure 4b.** Cooling power and efficiency vs. first stage connecting tube ratio.

At step ratio 2, the input power to the second stage is two times that of the first stage. In this case, if the swept volume of the first stage is optimized, the swept volume of the second stage would be too big. The volume of the connecting tube of the second stage should be increased so that the big second stage displacer swept volume can be balanced by the volume of the second stage connecting tube.
Figure 5a. PV diagrams at first stage connecting tube ratio 0.0625.

Figure 5b. Mass flow rate and pressure at first stage connecting tube ratio 0.0625.

Figure 6a. Diagrams at first stage connecting tube ratio 1.

Figure 6b. Mass flow rate and pressure at first stage connecting tube ratio 1.

Figure 7a. Loss vs. second stage connecting tube ratio.

Figure 7b. Cooling power and efficiency vs. second stage connecting tube ratio.

Figure 7 shows the loss, cooling power of the first stage and second stage, efficiency, input power and pressure ratio vs. second stage connecting tube ratio which is the volume of the second stage connecting tube over the volume of the second stage pulse tube. The loss of the first stage regenerator increases with the increasing of the second stage connecting tube ratio. The other loss decreases with the increasing of the second stage connecting tube ratio. There is an optimum second stage connecting tube ratio for efficiency, and second stage regenerator loss. The first stage cooling power, input power...
and pressure ratio decrease with the increasing of the second connecting tube ratio. There is an optimum second stage connecting tube ratio for the second stage cooling power.

The PV diagrams, mass flow rate and pressure at the cold end of pulse tubes with second stage connecting tube ratio 0.0625 and 1.75 are shown in Figure 8 and Figure 9, respectively. It is shown that the second stage connecting tube can adjust the PV diagrams and mass flow rate.

**Figure 7c.** Input power and pressure ratio vs. second stage connecting tube ratio.

**Figure 8a.** PV diagrams at second stage connecting tube ratio 0.0625.

**Figure 8b.** Mass flow rate and pressure at second stage connecting tube ratio 0.0625.

**Figure 9a.** PV diagrams at second stage connecting tube ratio 1.75.

**Figure 9b.** Mass flow rate and pressure at second stage connecting tube ratio 1.75.
7. Conclusion
There is an optimum swept volume ratio and phase angle difference for two stage pulse tube refrigerator with step displacer for efficiency, and the volume of the connecting tube can be used as an adjustable parameter to separate the phase shifter effect and power distribution effect to let the efficiency be optimized.

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