New Concept of Semi-precast Concrete Slab on Pre-tensioned Boards

Rafal Szydlowski 1, Magdalena Szreniawa 2

1 Cracow University of Technology, Warszawska Street 24, 31-155 Cracow
2 TCE Structural Design & Consulting, Domikanow Street 14, 31-409 Cracow

rszydlowski@pk.edu.pl

Abstract. There are many fully or half precast slab systems in construction industry. Each of them has its advantages and disadvantages. One of them is better here, another elsewhere. The authors of this paper decided to develop a new solution. It tries to reconcile the advantages of different systems. The concrete slab consisted of pre-tensioned concrete boards as a bottom formwork and reinforcement as well as concrete topping is proposed. The tight concrete boards with important bending capacity allows to obtain the solid concrete slab of little thickness with avoiding the traditional formwork and supports. It has a great importance for acoustic slab properties. A small slab thickness makes them chipper in transport. The paper presents some structural details as well as the results of calculation of the slab with span of 8.0m and the thickness of 200mm.

1. Introduction

Many precast or semi-precast concrete slab systems based on pre-tensioned concrete elements are available on the Polish market. The first group are beam-and-block systems consisted of beams spaced at a maximum of 600mm and filling bricks made of light concrete, ceramic or even wood waste. The second group are systems consisting of large sized elements (hollow core slabs or TT plates). Each of these systems has its advantages and disadvantages. The first group do not require tight formwork or heavy equipment, but they are time-consuming in assembly, are not rigid enough for long span [1], are of pretty big height, and transfer noise with their low mass. The large-sized slabs, although being fast in assembly, do not require large amounts of wet concrete, but they are quite expensive in transport and assembly. The authors of the study decided to try to develop a new type of slab composed of concrete formwork in the form of pre-tensioned boards and a wet-cast concrete layer. The proposed solution may prove to be more effective than the existing systems in some solutions. It will not require tight formwork, it will be a solid cross-section slab (desired due to acoustic reasons). Because only a thin layer of slab it will be transported from the prefabrication plant to the construction site, it will allow for a significant reduce of transportation cost of a slab square meter. The low mass of composite elements will not require engaging heavy cranes, which has high significance in the case of small constructions.

The use of pre-tensioned boards in construction as a lower reinforcement in bended elements was proposed by W. Grzegorzewski in the fifties of the last century [2]. After a period of laboratory tests and experiments on some natural scale elements, more extensive research has been carried out and a number of facilities have been implemented, e.g. about 20 road bridges with a span up to 10m and
Figure 1. The slab constructed with pre-tensioned elements proposed by W. Grzegorzewski [2]: a) the slab with tight bottom reinforcement (1 – pre-tensioned boards, 2 – pre-tensioned bars, 3 – concrete topping); b) the slab with openwork bottom reinforcement (1 – bars of bottom reinforcement layer, bars of second reinforcement layer, 3- concrete topping)

prestressed liquid tanks. In the context of using pre-tensioned boards for concrete floors, the author of the work [2] proposed two solutions. The first of them was based on laying the boards edge to edge in the direction of transfer (figure 1a), at the same time giving them the function of tight formwork reinforcement. In this variant, it was necessary to use transverse strips to reduce the faulting. The second application of pre-tensioned boards (with a small cross section) was dedicated to two-way reinforced slabs and their role was limited only to reinforcement.

Due to the difficulty in the boards production and their lack of standardization, the development of such structures was abandoned in those days. Nowadays, the increased use of precast concrete in floors in the last decades may be the reason for undertaking attempts to reactivate pre-tensioned boards.

2. The concept of a new slab with pre-tensioned boards
The composite slab planned to be developed will consist of pre-tensioned concrete boards and a concrete layer cast over them. The boards will both constitute bottom formwork and the lower tensioned zone of the bending slab (figure 2). At the time of concreting, the boards will be supported by linear supports with a spacing of 2÷3m. According to the regulations in force, the horizontal barriers in buildings should be designed to provide 60 minutes of work in fire conditions (REI 60). An analysis of the currently applicable fire protection standards of the structure showed that this resistance would be achieved with a concrete cover of not less than 18mm. Taking into account the biphasic work of the board and various statics of its work in different phases, it was found that axial prestressing would be the most beneficial. The above assumptions indicate a minimum board thickness amounting to 50mm. The optimum seems to be a board with a cross section from 150×50mm to 500×50mm. Boards with a smaller width can be used in single family housing due to their relative ease of transportation and assembly.

Figure 2. The cross-sections of proposed slab: perpendicular to the boards (a), longitudinal to the boards (b)
Wider elements can be used in the case of slabs with much larger span, i.e. in multi-family housing or public utility places. On boards laid side by side (without additional joining), a layer of monolithic concrete will cast. The rigidity of the slab in the transverse direction to the boards will be provided by small, steel cores hidden in the concrete topping with a spacing of approx. 1.0m. These cores will be formed by four 10mm diameter bars, joined by stirrups $\phi$ 4.5 mm every 400mm. The cores formed in such a way will only have to be protected against sliding through their mutual connection with $\phi$6mm bars every 1.0m.

This solution is dedicated to the construction of slabs in buildings, with the span of up to 10.0m. Preliminary design analyzes have indicated that the total thickness of such a slab should be: 150mm for a span of 6.0m; 200mm for 8.0m and 250mm for 10.0m.

The proposed solution has the following advantages:

- obtaining a monolithic concrete slab without the need for using formwork and conventional reinforcement. The acoustical comfort of using areas in buildings (sound insulation and no vibration transmission) can only be provided by slabs of appropriate mass,

- low transportation cost per square meter of precast elements in relation to the existing solutions (only a 50mm thick slab layer is precast without additional protruding elements, which, with standard 24tonne trucks, will allow for a one-off transportation of 190m$^2$ of precast elements). Concreting in situ mixture is much more available everywhere and then cheaper in transport,

- reduction in consumption of steel (approximately 7÷8kg/m$^2$),

- ease and safety of assembly of equipment and installations under the slab (lack of prestressing concentration in beams, to which installations and suspensions in buildings are always assembled in the case of beam-and-block floor systems).

3. Calculation

Computational analysis was applied to a slab of 8.0m span consisting of boards with a cross section of 50×300mm and concrete topping with a thickness of 150mm (figure 3). The board is pre-tensioned with two seven-wire strands with a diameter of 11.3mm (for two strands $A_p = 186$mm$^2$) made with a steel of strength equals 1860MPa. The boards will be made of C45/55 concrete. It was assumed that in the assembly phase the boards will be supported in a span of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ to form a 4-span continuous beam (figures 3 and 4a). The overlay concrete class of C25/30 prepared on a natural aggregate will be used.

![Figure 3. Arrangement of analyzed slab](image-url)
Three phases of construction work were considered:

- **prestressing** - an axially compressed board resting on the ground,
- **assembly phase** - board in the assembly phase resting on intermediate supports, loaded with own weight and weight of wet concrete,
- **usable phase** - composite slab loaded with own weight, weight of finishing layers and service load.

### 3.1. Fire resistant analysis

The computational analysis of required concrete cover with regard to fire protection standards has been conducted according to 5.2(7) [3]. The basic value of cover „a“ according to table 5.8 [3], for 60 minutes of work in fire conditions (REI 60) is equal to 20mm. This value should be increased by a component Δa, which for simplified calculations and prestressing steel is equal to 15mm. Carrying out a detailed computational analysis allows for decreasing the component Δa through the reduction of stresses in the prestressing steel used in a fire conditions. The maximum recommended value of stresses in the prestressing steel during the fire has been calculated in accordance with: 

\[
\sigma_{p,fi} = \eta_{fi} \cdot \frac{f_{pk(20^\circ C)}}{\gamma_p} \cdot \frac{A_{p,req}}{A_{p,prov}} \tag{1}
\]

where:

- \( \eta_{fi} \) – reduction coefficient of load level design in a fire conditions, equal to 0.618, according to:

\[
\eta_{fi} = \frac{g^+\psi_{21}q_k}{\gamma_g \gamma_y \gamma_q q_k} \tag{2}
\]

- \( f_{pk(20^\circ C)} \) – yield strength of prestressing steel equal to 1860MPa,
- \( \gamma_p \) – safety coefficient of prestressing steel, in a fire conditions equal to 1.0,
- \( \frac{A_{p,req}}{A_{p,prov}} \) – required to provided area of prestressing steel ratio equal to 0.52, which corresponds to the ratio of maximum stresses of prestressing steel in design situation \( \sigma_p \) and characteristic prestressing steel strength \( f_{pk} \).

The value of maximum stresses in prestressing steel \( \sigma_{p,fi} \) was obtained 597.9MPa.

Based on the value of \( \frac{\sigma_{p,fi}}{f_{pk(20^\circ C)}} \) the critical temperature \( \Theta_{cr} \) was assumed 452 °C. This assumption allows to determine the value of Δa accoring to the equation:

\[
\Delta a = 0.1 \cdot (500 - \Theta_{cr}) = 4.8 \text{mm} \tag{3}
\]

As a result of detailed computational analysis the value of „a“ has been reduced to 25mm, hence the thickness of pre-tensioned board with axial prestressing can be assumed 50mm \( (h = 2a) \). The saving of the thickness of board was able to get because of reduction of prestressing steel effort. Taking into account the immediate and time-dependent losses and the increase of stresses in prestressing steel caused by external loads (dead loads and live loads), it was necessary to reduce the initial steel stresses to 0.64\( f_{pk} \), despite the recommended value of 0.80\( f_{pk} \) [4].

### 3.2. Influence of concrete shrinkage difference

In the analysis of the effect of the shrinkage difference of pre-tensioned concrete boards and concrete topping, the following assumptions were made in:

- parameters of the board concrete: class C45/55, fast hardening cement R;
- overlay concrete parameters: class C25/30, normal cement N,
- age of boards at the moment of combination: 30 days, ambient humidity 50%.

Due to the assumptions, the following shrinkage values were obtained:

- in the boards: \( 12.1 \times 10^{-5} \) at the time of combination, \( 40.2 \times 10^{-5} \) as the final value of shrinkage,
- in the concrete overlay: the shrinkage final value is \( 37.2 \times 10^{-5} \).
The calculated shrinkage induces a bending moment in the cross section with a value of 0.84kNm. The stresses created as a result of shrinkage difference are respectively 0.12 and 0.35MPa in the lower and upper fibers of the board and -0.37 and 0.20MPa in the lower and upper fibers of the concrete overlay.

Table 1. The load value assumed in each of the two phases of work.

| Phase of work  | Kind of load        | $q$ [kPa] |
|----------------|---------------------|-----------|
| Assembly phase | Boards self-weight  | 1.25      |
|                | Wet concrete        | 3.75      |
| Usable phase   | Slab self-weight    | 5.0       |
|                | Finishing layers    | 1.5       |
|                | Service load        | 3.0       |

These are insignificant stress values. These stresses were added to the normal stresses in the cross-section after slab composition (shown in figure 5b).

3.3. Static analysis and normal cross-section stresses

Table 1 presents the load values assumed in the calculation analysis in individual situations. In the assembly phase, the weight of the board resting on the mounting supports and the weight of the wet concrete was assumed. In the usable phase, additional load with finishing layers as well as service load was applied. This time, these loads (including the slab weight) operate on a single-span beam scheme with a theoretical span of 8.2m (figure 4b).

![Figure 4.](image)

**Figure 4.** The distribution of bending moments: in assembly phase (a), after composite (b)

The distribution of bending moments in a single board is shown in figure 4 (4a for the assembly situation, 4b for the situation after the composition). This figure also presents characteristic cross sections (A-A, B-B, C-C and D-D) selected for the verification of stresses.

Figure 5 presents the development of stresses in the board and in the composite slab in four cross sections A-A, B-B, C-C and D-D, indicated in figure 4a. The initial stresses in the pre-tensioned boards resting on the substrate is 12.7MPa. The distribution and values of bending moments in the assembly phase are shown in figure 4a. In this situation, the board in all sections is compressed (figure 5a). The highest compressive stresses were obtained at the bottom of the B-B cross-section (above the mounting support) and has a value of 15.9MPa. The smallest compression occurs at the top of the
same cross section; the stresses have a value of 5.7MPa. Figure 5b shows the increase of stress in the composite slab caused by the bending moment shown in figure 4b and the difference in shrinkage of the boards and overlay concrete.

![Stress Diagram](image)

**Figure 5.** Normal stresses in composite cross-section: in assembly phase (a), increase of stresses after composite (b), final stresses (c).

The final stress in a design situation (figure 5c) is a superposition of the stresses from two usable phases (figures 5a and 5b).

Table 2 shows the values of the prestressing force in particular stages of board operation in the most strained slab cross-section of D-D. The initial prestress force of the board amounts to $P_0 = 221.4$kN. The force after immediate losses is $P_{\text{m0}} = 200.7$kN, whereas force after rheological losses has a value of $P_{\text{mr}} = 170.3$kN. The final prestressing steel stress value after time-dependent losses is $\sigma_{\text{pmt}} = 930.4$ MPa. Using the stress diagram in the cross section of the board, stress growth in prestressing steel in cross-section D-D was determined. The average concrete cross-section stress growth at board height is 5.0MPa (figure 5b). Taking into account the ratio of elastic modulus of prestressing steel and concrete $E_p/E_{cem} = 6.5$ stress growth in steel was calculated: $\Delta \sigma_p = 5.0 \times 6.5 = 32.5$MPa. As a result, the total stresses in prestressing steel are equal:

$$\sigma_p = \sigma_{\text{pmt}} + \Delta \sigma_p = 930.4 + 32.5 = 962.9 \text{MPa}$$
Table 2. Prestressing force value in concrete board as well as prestressing steel stress

| Design situation         | $P$ [kN] | $\sigma_p$ [MPa] |
|--------------------------|----------|------------------|
| Tensioning               | $P_0 = 221.4$ | $\sigma_{p0} = 1270$ |
| After immediate losses   | $P_{m0} = 200.7$ | $\sigma_{pm0} = 1079$ |
| After time-dependent losses | $P_{mt} = 170.3$ | $\sigma_{pmt} = 930$ |
| Final operational situation | $P_{ef} = 179.1$ | $\sigma_p = 963$ |

The use of prestressing steel in a final usable situation is therefore $\frac{\sigma_p}{\sigma_{pk}} = 0.52$. The assumed level of steel stress was not exceeded.

Analysing the presented stress diagrams, it can be observed that in all analysed cross-sections there was no total reduction in the initial compressive stresses in the board. It is easy to observe the tensile stresses in the overlay concrete at the place of contact with the board (max -2.0MPa in cross-section D-D). Cracking of the overlay concrete near the surface of composite undoubtedly weakens the shear capacity of contact surface. These are, however, values below the cracking values. Any unforeseen cracks would significantly reduce the shear capacity of contact surface. These stresses, however, reach the biggest value in the middle of the span and will disappear as they approach the supports, where the delaminating forces are highest. It can be considered that this tension is not dangerous for the point of work, even its underestimation will not worsen the working conditions of the slab. The first assumption of the absence of connecting reinforcement supports its validity.

3.4. Deflection control

Deflection of the slab as a simply supported element with a span of 8.2 m was calculated on the basis of known structure mechanics dependency. The analysis takes the following assumptions:

- non-cracked cross-section,
- moment of cross-section inertia $I_{cs} = 0.00036 m^4$, brought to the properties of overlay concrete with creep coefficient $\phi = 2.7$,
- effective concrete elastic modulus $E_{c,eff} = 7.9 MPa$ (for $\phi = 2.7$),
- bending moment assuming 30% of the service load as a constant and growth due to the difference in shrinkage in both concretes: $M = 19.5 kNm$.

A permanent long-time deflection was obtained amounting to 48 mm. The $L/250$ permitted value is 32 mm. The obtained values show that the problem of deflection in the assumed solution of the slab is important. This is due to the fact that in the case of typical pre-tensioned slab, the prestressing is introduced into a finished plate with a full cross-section resulting in camber (figure 6a). In the proposed solution, the prestressing is only applied to the lower layer of the slab and the remaining slab height is overlaid on a straight-lined bottom element (figure 6b). There is therefore lack of negative deflection (camber). However, the camber can be simply set by lifting the mounting supports. In the analysed example, the problem of excessive deflection will be solved by initial negative deflection of the slab by 20mm, which constitutes 1/410 of the span. Final deflection will then be 48 - 20 = 28mm, so it will be below the permitted value.
3.5. Connecting surface shear capacity

One of the decisive factors regarding the correctness of the adopted solution is the shear capacity of contact surface between the precast concrete and the overlay concrete. The project has an assumption that the boards will be devoid of any staple reinforcement. Its presence would severely worsen the economic benefits resulting from transport and storage of boards. It is also desirable for the prefabricated surface to be natural without additional adhesion treatments.

\[ u_{Edi} = \beta \cdot \frac{V_{Ed}}{z_b} \]  

where:
- \( \beta \) – the ratio of the longitudinal force acting on the cross-section of the overlay concrete to the whole longitudinal force acting in the compression zone of considered cross-section; a value equal to 1 is assumed,
- \( V_{Ed} \) – shear force acting on a one meter of slab width equal to 17.64kN,
- \( z \) – an arm of internal forces in a composite cross-section equal to 0.125m,
- \( b_i \) – width of the contact surface equal to 1.0 m.

The design shear strength in the connecting surface \( u_{Rdi} \) is determined the basis of a 6.25 formula template [4], which, in the absence of connecting reinforcement and bypassing the influence of normal stresses on the contact surface, is reduced to \( u_{Rdi} = c \cdot f_{ctd} \). The surface roughness coefficient \( c \) is assumed to be the same for smooth surfaces, equal to 0.4, and the tensile concrete strength \( f_{ctd} \) was assumed as 1.0MPa. Computational analysis has shown that the shear stresses at the contact surface above the supports will be 118kPa. Meanwhile the shear resistant is 200kPa (assuming a smooth surface). The results show with a large reserve, that the goal is achievable.

4. Conclusions

The work presents an alternative solution of semi-prefabricated concrete slabs to the ones existing on the market. This solution is the result of the creative activity of the authors and continues to evolve. The innovative idea is supported by theoretical calculations for selected specific conditions of use. However, it should be noted that the analyses carried out were based on a number of simplified assumptions. The obtained results of the calculations allow to conclude the validity of the proposed solution. However, it is important to keep in mind the randomness of the behaviour of concrete components, especially of prestressed concrete. This slab will be the subject of laboratory research.
conducted on elements on a natural scale that will certainly solve many doubts. The obtained results will be reported in the future.

References

[1] W. Derkowski, S. Kańka, “Strunobetonowe stropy gęstożebrowe – badania w skali rzeczywistej (Pretensioned beam-and-block floor systems – real scale tests),” Technical-Transactions, 3-B/2012, pp. 35–49.

[2] W. Grzegorzewski W, “Deski strunobetonowe w budownictwie (Pre-tensioned concrete boards in building construction),” Arkady, p. 165.

[3] PN-EN 1992-1-2 Eurocode 2: Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design, European Committee of Standardization, May 2008.

[4] EN 1992-1-1: Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings, European Committee of Standardization, December 2004.