Supplementary Materials

Ultrasound Localization of Nitinol Wire of Sub-Wavelength Dimension

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I. INTRODUCTION

The main lobe of ultrasound transmission is approximately a cylinder in the near-field, with the width of the cylinder the same as the width of the transmitting aperture, and the depth of the near-field is approximately the square of the radius of the transmitting aperture \(a\) divided by the wavelength \(\lambda\) of the ultrasound signal. Shorter wavelengths (higher frequencies) thus would indicate higher resolution and deeper penetration depth, but increased attenuation at higher frequencies reduces penetration depth. Beyond the near field the lateral beam width spreads by the angle of divergence which is approximately \(\sin^{-1} 0.61 \frac{\lambda}{a}\), further reducing the effective resolution.

Reflection and transmission of waves happens at the interface of materials. When the wave is coming in normal (incident) to the surface (90º), then the intensity reflection coefficient \([S1]\) is given by

\[ R = \frac{Z_2 - Z_1}{Z_2 + Z_1}^2, \]

where \(Z\) is the characteristic impedance of the material and the subscripts denote the material the wave starts in (denoted by 1) and the material the wave is heading toward (denoted by 2). The intensity transmission coefficient \([S1]\) is given by

\[ T = \frac{4Z_1Z_2}{(Z_2 + Z_1)^2} \]

For reference, metals have a \(Z\) value that is 10–20 times that of soft tissue, and 2.5–5 times that of bone \([S2]\). Metals therefore have a very bright reflection. For reflected waves the angle of reflection is the same as the angle of incidence, though the transmitted angle is given by Snell’s Law:

\[ \sin \theta_t = \frac{c_1}{\sin \theta_i} \]

where \(\theta_i\) and \(\theta_t\) are the angles of incidence (transmission) and the speed of sound in the incident/transmitted media.

When the ultrasound beam encounters objects or inhomogeneities the size of the ultrasound wave or smaller, then the ultrasound wave is scattered in many directions. For very small objects compared to the wavelength, the scattering cross section is given by:

\[ \sigma_S = \frac{64\pi^5}{3\lambda^6} \left[ \frac{\kappa - \kappa_S}{\kappa} \right]^2 + \frac{1}{3} \left( \frac{\rho_S - \rho}{\rho_S + \rho} \right)^2 \]

where \(\kappa\) is the adiabatic compressibility and \(\rho\) is the density, with the subscript \(s\) denoting the scattering material and the non-subscripted variables the surrounding material (tissue). \(\lambda\) is the wavelength and \(r\) is the small object’s radius \([S1]\). In the case of very small objects like red blood cells (≈ 7 µm), most of the radiation reflects back to the source, making the red blood cell act like a larger object and serve as the underlying idea of Doppler ultrasound \([S1]\).

In the case of a thin Nitinol wire (≈ 20 µm) this cross-section equation is a good approximation. The adiabatic compressibility is inversely proportional to bulk modulus, which is a standardly tabulated value for both soft tissue and metal, so roughly \(\kappa_S \approx 0.1\kappa\) for Nitinol to soft tissue. In terms of density, Nitinol is about 6.5 times denser than soft tissue. Using these numbers, the small Nitinol wire appears about the same effective size as an object seven (7) wavelengths across. Doubling the size starts to break down the effect of returning most of the energy the scattering cross section relies on; however, since the refractive index is still large (10 times that of soft tissue), the wire will still generate a noticeable speckle pattern.
II. MATERIALS AND METHODS

Machine learning is a subset of artificial intelligence. It uses statistics to enable computers to learn from experience by developing computational algorithms or models that can automatically infer hidden patterns from data. Deep learning is a subset of machine learning that uses multilayer neural networks to perform desired tasks by using trained models. Neural networks are nonlinear mapping systems whose structure and function are loosely based on the idea of the nervous systems in humans and animals. A neural network consists of a large number of processors (similar to neurons in the nervous system) linked by weighted connections. The power of the system is from combining many processing units in a network. These unit transform inputs linearly from other nodes and generate output as a single scalar by applying a nonlinear function [S3]. The output is distributed to and acts as an input to the next layer in the network. The weights and biases used in the linear transformation are learned and updated through backpropagation. Backpropagation adjusts the network weights so the network produces the desired output in response to every input pattern in a predetermined set of training patterns [S3]. The derivative chain rule is applied to calculate the derivatives of the network training error with respect to the weights. Thus the parameters are updated based on the gradients of the cost function in relation to the previous layer.

A convolutional layer transforms the input by convolving it with a kernel and applying a non-linearity to the output, known as a feature map. A pooling layer aims to reduce the size of the feature map by taking the average or the maximum of small regions in the input. A fully connected layer is placed before the output (last) layer to tune the weights and biases to create a stochastic likelihood representation of each class based on the activation maps created by the previous layers.

VGG achieved good results in both image classification and localization at the 2014-ILSVRC (Imaginet) competition. VGG-16 is made up of thirteen convolutional layers and three fully connected layers. All convolutional and fully connected layers have a ReLU activation function applied to their output except for the last layer which has a softmax activation function applied to it. The smallest size of 3 by 3 filters is included in the model to help with learning more complex features by increasing the depth of the network [S3]. The main limitation of VGG-16 is the number of 138 million parameters which can become a burden to computational resources.

III. RESULTS AND DISCUSSION

Images are shown in Fig. S2 at 7.5 MHz frequency as a 50-μm-diameter wire was advanced from position L2 to position U2. The ultrasound probe was fixed at position C0. Images were taken when the tip of the wire was at each of the labeled positions. All images shown are for 0.25-in-diameter tubing. This data set parallels Fig. 2 in the manuscript, while successfully applying the ultrasound localization technique to a wire of even smaller diameter.

A summary of results from the VGG-16 CNN model is displayed as a confusion matrix in Fig. S3 for both raw images (Fig. S3(a)) and difference images relative to the image at position L2 (Fig. S3(b)). In each matrix, the number (percentage) is reported in each quadrant. True positive is shown in the upper left quadrant, with false positive in the upper right, false negative in the lower left, and true negative in the lower right.

In seven instances, the CNN model applied to raw images yielded false negative results. The images corresponding to those cases are shown in Fig. S4. Below each image are the imaging frequency, wire diameter, and wire tip position for the raw image. There were five false negatives for the CNN analysis of difference images. The corresponding raw images are shown in all five cases in Fig. S5. Note that these are the raw images before the L2 image was subtracted. Note that no false negatives occurred for 10 MHz images.
Fig. S3. Confusion matrices obtained from (a) raw images and (b) difference images relative to the image at position L2.

Fig. S4. All false negative ultrasound images for the regular image CNN analysis.
Fig. S5. All false negative ultrasound images for the difference image CNN analysis.

REFERENCES

[S1] A. G. Webb, *Introduction to Biomedical Imaging*, Hoboken, NJ: Wiley-IEEE Press, 2002.
[S2] J.L. Prince and J. M. Links, *Medical Imaging Signals and Systems*, Upper Saddle River, NJ: Pearson, 2014.
[S3] R. D. Reed and R. J. Marks, *Neural Smithing: Supervised Learning in Feedforward Artificial Neural Networks*, Cambridge, MA: The MIT Press, 1999.