A proposed method for calculating dimension of irrigation channel section

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Abstract. In general, the cross section of irrigation channels in Indonesia is planned to be trapezoidal. The calculation of the dimensions of the trapezoidal section uses the quadratic equation when calculating the hydraulic depth in the channel. Calculations using quadratic equations can produce imaginary numbers if the discriminant value is negative, resulting in a failed calculation of channel dimension. This study proposes a method for calculating the dimensions of a trapezoidal cross-sectional irrigation channel, in which the discriminant value of quadratic equation is not negative. The analysis is carried out on all formulas that produce the independent variable for the discriminant value in the quadratic equation. The independent variable is the mean flow velocity in the channel or the channel base slope. This study produces a formula for calculating the mean flow velocity in a channel as well as a formula for calculating the channel base slope which results do not result in a negative discriminant value. Thus, the calculation of the hydraulic depth in the channel has not failed and the calculation of other channel dimensions can be continued.

1. Introduction

Irrigation channel planning requires that the dimensions of the planned channel must have the capacity to drain a water discharge amount equal to or greater than the discharge of irrigation water needs [1, 2]. The channel dimensions that determine the channel capacity or channel discharge are the cross-sectional dimensions and the longitudinal sectional dimensions. From these two sectional dimensions, the area of the channel section and the mean flow velocity in the channel can be calculated. The multiplication of the area of channel section with the mean flow velocity gives the channel discharge value [3, 4].

Generally, in Indonesia, the cross section of irrigation channels that is planned is a trapezoidal section [5]. The calculation of the dimensions of the trapezoidal section uses the quadratic equation when calculating the hydraulic depth in the channel [3]. Calculations using this quadratic equation require conditioning of the discriminant value not to be negative [6], because the negative discriminant value produces imaginary numbers and results in failure in the planning of irrigation channel dimensions. Therefore, it is necessary to conduct a study to produce a method of calculating the dimensions of the irrigation channel that discriminant value is not negative when quadratic equation is used to calculate hydraulic depth.

In this study, an analysis of all independent variables in the formula was carried out to calculate the discriminant value. From this analysis, the formula for calculating the independent variable is obtained which results in a non-negative discriminant value. Then proposed a method for calculating the
dimensions of a trapezoidal cross-sectional irrigation channel in the form of a calculation sequence and a calculation flow chart.

2. Literature review

2.1 Discharge of channel

For any flow, the discharge $Q$ at a channel section is expressed by formula below [3, 4].

$$Q = AV$$

Where $Q$ is discharge at a channel section, m$^3$/sec; $A$ is the flow cross-sectional area normal to the direction flow, m$^2$; and $V$ is the mean velocity, m/sec.

The mean velocity of flow in channel, $V$ can be calculated using Manning Formula as below [3,4,7]

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

Where $n$ is roughness coefficient of Manning; $R$ is hydraulic radius, m; and $S$ is base slope of channel.

2.2 Geometric elements of channel section

Artificial channel is usually designed with section of regular geometric shapes, such as trapezoid and rectangle. The formulas of geometric element of channel section, especially trapezoid section is shown in Figure 1 [3].

![Figure 1. Trapezoid section of channel [3]](image)

- $A = (B + zY)Y'$
- $P = B + 2Y\sqrt{1 + z^2}$
- $R = \frac{A}{P}$

Where $A$ is area of channel section; $B$ is width of channel; $Y$ is hydraulic depth; $P$ is wetted perimeter; $R$ is hydraulic radius; and $z$ is side slope.

3. Study methodology

In the planning of the dimensions of the irrigation channel, the amount of water that is flowed by the channel or channel discharge, $Q$ must be greater than or equal to the amount of irrigation discharge, $Q_{ir}$ so that irrigation water needs are met. This relationship is shown below [1, 2].

$$Q \geq Q_{ir}$$

The discharge of irrigation needs, $Q_{ir}$ is obtained from calculating the need for irrigation water which has been known in advance. Based on formula 3.1, in calculating the dimensions of the channel, the amount of channel discharge, $Q$ is equal to the amount of irrigation discharge, $Q_{ir}$.

Calculation of channel dimensions begins with analyzing the all formulas in the literature review section above. The analysis carried out is described in the following

1. From formula 4, formula 6 is obtained.
2. Then, from the substitution of formula 6 into formula 3, formula 7 is obtained.

\[ A = (B + zY)Y \]
\[ A = \left( P - \left( 2\sqrt{1 + z^2} \right) Y \right) Y + zY^2 \]
\[ A = PY - \left( 2\sqrt{1 + z^2} \right) Y^2 + zY^2 \]  

(7)

Formula 7 above is a quadratic equation which is then simplified to formula 8.

\[ \left( 2\sqrt{1 + z^2} - z \right) Y^2 - PY + A = 0 \]  

(8)

3. As it is known that the solving of quadratic equations is done with equations as shown in Formulas 9 and 10 [6].

\begin{align*}
Y_1 &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\
Y_2 &= \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\end{align*}

(9)

(10)

The values \(a\), \(b\), and \(c\) in formulas 9 and 10 are obtained from formula 8, namely:

\begin{align*}
a &= 2\sqrt{1 + z^2} - z \\
b &= -P \\
c &= A
\end{align*}

(11)

(12)

(13)

4. The requirement that the results of the quadratic equation calculation of formulas 9 and 10 above do not produce imaginary values is discriminant value, \(D\) cannot be negative [6].

\[ D = b^2 - 4ac \geq 0 \]  

or

\[ b^2 \geq 4ac \]

(14)

5. To meet the requirements of \(D\) value cannot be negative, it is necessary to take steps to solve \(b^2\), starting from formula 5 to be changed to formula 15

\[ R = \frac{A}{P} \]
\[ P = \frac{A}{R} \]

(15)

Then, by substituting formula 15 into formula 12, then formula 16 is obtained.

\[ b = -P \]
\[ b = -\frac{A}{R} \]

(16)

From formula 1, formula 17 is obtained.

\[ Q = AV \]
\[ A = \frac{Q}{V} \]

(17)

Then from formula 2, formula 18 is obtained.

\[ V = \frac{1}{n} R^{2/3} S^{1/2} \]
Furthermore, by substituting formula 17 and formula 18 into Formula 16, then the Formulas 19 and 20 are obtained.

\[ b = -\frac{A}{R} \]
\[ b = -\left(\frac{Q}{V}\right) \frac{V}{\left(V n \right)}^{1/2} \]
\[ b = -\left(\frac{Q S^{3/4}}{V^{5/2} n^{1/2}}\right) \]  
(19)
\[ b^2 = \frac{Q^2 S^{3/2}}{V^5 n^3} \]  
(20)

The Formula 20 above is the solution for \( b^2 \) in the Formula 14.

6. Next, the solution for \( 4ac \) is in Formula 14, starting by substituting formula 17 into Formula 13, then Formula 21 is obtained.

\[ c = A \]
\[ c = \frac{Q}{V} \]  
(21)

Then based on the value of \( a \) in Formula 11 and the value of \( c \) in Formula 21, the solution for \( 4ac \) is obtained as shown in Formula 22.

\[ 4ac = 4\left(2\sqrt{1+z^2}-z\right)\frac{Q}{V} \]  
(22)

The formula 21 above is the solution for \( 4ac \) in the formula 10.

7. Then \( b^2 \) in formula 20 and \( 4ac \) in formula 21 are substituted into formula 14, then formula 23 is obtained.

\[ b^2 \geq 4ac \]
\[ \frac{Q^2 S^{3/2}}{V^5 n^3} \geq 4\left(2\sqrt{1+z^2}-z\right)\frac{Q}{V} \]
\[ V^4 \leq \frac{Q S^{3/2}}{4\left(2\sqrt{1+z^2}-z\right)n^3} \]  
if \( \alpha = 4\left(2\sqrt{1+z^2}-z\right) \)
\[ V \leq \left(\frac{Q S^{3/2}}{\alpha n^3}\right)^{1/4} \]  
(23)

The formula 23 above is the formula for calculating the mean flow velocity in the channel, \( V \) with the condition of the channel base slope, \( S \) is determined. If it turns out that the value of \( V \) is less than the minimum velocity allowed or greater than the maximum velocity allowed, then the calculation is carried out to find the channel base slope, \( S \) with the value of mean flow velocity in the channel, \( V \) is determined. The formula is as follows.
4. Result and discussion

The methodology section above shows that the calculation of the dimensions of the irrigation canal must go through the quadratic equation, which requires discriminant value, \( D \) must not be negative in order to avoid imaginary numbers. Based on non-negative discriminant conditions and after analyzing the formulas in the literature review section, a formula is obtained to calculate the mean flow velocity in the channel, \( V \) with the channel base slope, \( S \) is determined as well as the formula is obtained to calculate the channel base slope, \( S \) with the mean flow velocity at channel, \( V \) is determined.

In calculating the dimensions of the irrigation channel, several variables have been determined in advance. The variable of channel discharge, \( Q \) is equal to irrigation discharge, \( Q_{ir} \) which is calculated based on irrigation water needs. Then the channel side slope, \( z \) and Manning’s roughness coefficient, \( n \) values are both determined based on the type of channel material. Whereas the variable mean flow velocity in the channel, \( V \) and the channel base slope, \( S \) is selected one of their values.

Based on the results of the analysis that has been carried out, the method of calculating the dimensions of the irrigation channel with a trapezoidal cross-section is obtained, namely the calculation of the hydraulic depth, \( Y \) and width of channel, \( B \) in the order described below.

1. Calculate the value of \( \alpha \) with the side slope value of the channel, \( z \) has been determined as planned.

\[
\alpha = 4 \left( 2 \sqrt{1 + z^2} - z \right)
\]

2. Calculate the value of the mean flow velocity in the channel, \( V \) or the base slope the channel, \( S \) with the Manning roughness coefficient, \( n \) and the channel discharge, \( Q \) has been predetermined as planned.

\[
V = \left( \frac{Q S^{3/2}}{\alpha n^3} \right)^{1/4} \quad \text{or} \quad S = \left( \frac{V^4 \alpha n^3}{Q} \right)^{2/3}
\]

3. Calculate the value of \( a \)

\[
a = 2 \sqrt{1 + z^2} - z
\]

4. Calculate the value of \( b \)

\[
b = - \left( \frac{Q S^{3/4}}{V^{5/2} n^{3/2}} \right)
\]

5. Calculate the value of \( c \)

\[
c = \frac{Q}{V}
\]

6. Calculate the value of the hydraulic depth in channel, \( Y_1 \) and \( Y_2 \)

\[
Y_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]

\[
Y_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\]

7. Determine the \( P \) value.

\[
P = -b
\]

8. Calculate width of channel, \( B_1 \) and \( B_2 \)

\[
B_1 = P - \left( 2 \sqrt{1 + z^2} \right) Y_1
\]
\[ B_2 = P - \left(2\sqrt{1 + z^2}\right)Y_2 \]

9. Select a channel dimension pair \(Y_1\) and \(B_1\) or \(Y_2\) and \(B_2\)

10. Perform the control calculations in the following order.
\[
A = (B + zY)Y
\]
\[
P = B + 2Y\sqrt{1 + z^2}
\]
\[
R = \frac{A}{P}
\]
\[
V = \frac{i}{n}R^{2/3}S^{1/2}
\]
\[
Q = AV \geq Q_o
\]

The flow chart of the method for calculating the dimensions of a trapezoidal cross-sectional irrigation channel is illustrated by Figure 2.

\[ \text{Figure 2. The flow chart of the method for calculating the dimensions of a trapezoidal cross-sectional irrigation channel.} \]
As an example, the proposed method is applied to calculate the dimensions of the primary canal in the Krueng Jreu irrigation area located in Aceh Besar District, Aceh Province. According to [8] Krueng Jreu Irrigation has an area of 3,175 ha, the length of the primary channel is 1,839.80 m, the maximum need for irrigation water is 1.7 ltr/sec/ha, so that the irrigation discharge that is flowed by the primary channel is, 
\[ Q = \frac{5.4}{m^3/sec}. \]
The primary channel is planned to be made of concrete, has a trapezoidal cross-section, and the slope of the base of the channel in the direction of the flow is the same as the slope of the irrigation area, 
\[ S = 0.0016. \]
According to [5] for concrete channels: roughness coefficient of Manning, 
\[ n = 0.015, \]
the side slope can be planned, 
\[ z = 1.5 \]
and the maximum permissible velocity, 
\[ V_{\text{max}} = 3 \text{ m/sec}. \]

At first, the calculation was tried by following the procedure for calculating the dimensions of the channel according to [3], whose calculation steps are as follows.

1. \[ V = \frac{1}{n} R^{2/3} S^{1/2} \]
\[ R = \left( \frac{n V}{S^{1/2}} \right)^{3/2} = \left( \frac{0.015 \times 3}{0.0016^{1/2}} \right)^{3/2} = 1.2 \text{ m} \]

2. \[ Q = AV \]
\[ A = \frac{Q}{V} = \frac{5.4}{3} = 1.8 \text{ m}^2 \]

3. \[ P = \frac{A}{R} = \frac{1.8}{1.2} = 1.5 \text{ m} \]

4. \[ A = (B + z Y) Y = B Y + z Y^2 \]
\[ 1.8 = B Y + 1.5 Y^2 \]

5. \[ P = B + 2 Y \sqrt{1 + z^2} \]
\[ B = P - 2 Y \sqrt{1 + z^2} = 1.5 - 2 Y \sqrt{1 + 1.5^2} = 1.5 - 3.61 Y \]

6. Substitute 5 into 4, then the quadratic equation is obtained.
\[ 1.8 = (1.5 - 3.61 Y) Y + 1.5 Y^2 = 1.5 Y - 2.1 Y^2 \]
\[ 2.1 Y^2 - 1.5 Y + 1.8 = 0 \]
The quadratic equation above has \( a = 2.1; b = -1.5; \) and \( c = 1.8 \)

7. Solving the quadratic equation to calculate the value of hydraulic depth, \( Y \).
\[ Y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ Y_{1,2} = \frac{1.5 \pm \sqrt{1.5^2 - 4 \times 2.1 \times 1.8}}{2 \times 1.8} = \frac{1.5 \pm \sqrt{-12.87}}{4.2} \]

It turns out that the value of flow depth, \( Y \) cannot be obtained from the above calculation procedure because the discriminant value, \( D = b^2 - 4ac \) is negative, resulting in an imaginary number.

In order for the discriminant value not to be negative, it is calculated using the proposed method which is the sequence of calculations as follows.

1. \[ \alpha = 4 \left( 2 \sqrt{1 + z^2} - z \right) = 4 \left( 2 \sqrt{1 + 1.5^2} - 1.5 \right) = 8.4 \]
2. \[ V = \left( \frac{Q S^{3/2}}{\alpha n^3} \right)^{1/4} = \left( \frac{5.4 \times 0.0016^{3/2}}{8.4 \times 0.015^3} \right)^{1/4} = 1.8 \text{ m/sec} \]

\[ V < V_{\text{max}} \quad \text{and} \quad V > V_{\text{min}} = 0.76 \text{ m/sec} \ [3]. \]

3. \[ a = 2\sqrt{1 + z^2} - z = 2\sqrt{1 + 1.5^2} - 1.5 = 2.1 \]

4. \[ b = -\left( \frac{Q S^{3/4}}{V^{5/2} n^{3/2}} \right) = -\left( \frac{5.4 \times 0.0016^{3/4}}{1.8^{5/2} \times 0.015^{3/2}} \right) = -5.4 \]

5. \[ c = \frac{Q}{V} = 5.4 \]

6. \[ Y_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{5.4 + \sqrt{5.4^2 - 4 \times 2.1 \times 3}}{2 \times 2.1} = 1.760 \text{ m} \]

\[ Y_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{5.4 - \sqrt{5.4^2 - 4 \times 2.1 \times 3}}{2 \times 2.1} = 0.812 \text{ m} \]

7. \[ P = -b = 5.4 \]

8. \[ B_1 = P - \left( 2\sqrt{1 + z^2} \right) Y_1 = 5.4 - \left( 2\sqrt{1 + 1.5^2} \right) 1.8 = -0.9 \text{ m} \]

\[ B_2 = P - \left( 2\sqrt{1 + z^2} \right) Y_2 = 5.4 - \left( 2\sqrt{1 + 1.5^2} \right) 0.8 = 2.5 \text{ m} \]

Selected: width of channel, \( B = 2.5 \text{ m} \) and hydraulic depth, \( Y = 0.812 \text{ m} \), as displayed in Figure 3.

![Figure 3. Dimensions of the cross section of the trapezoidal channel](image)

Control:

\[ A = (B + z)Y = BY + zY^2 = 2.5 \times 0.812 + 1.5 \times 0.812^2 = 3.019 \text{ m}^2 \]

\[ P = B + 2Y\sqrt{1 + z^2} = 2.5 + 2 \times 0.812 \times \sqrt{1 + 1.5^2} = 5.428 \text{ m} \]

\[ R = \frac{A}{P} = \frac{3.019}{5.428} = 0.556 \text{ m} \]

\[ V = \frac{1}{n} R^{2/3} S^{1/2} = \frac{1}{0.015} \times 0.556^{2/3} \times 0.0016^{1/2} = 1.804 \text{ m/sec} \]

\[ Q = AV = 3.019 \times 1.804 = 5.446 \text{ m}^3/\text{sec} > Q = 5.4 \text{ m}^3/\text{sec} \] (fulfill the requirements).

5. Conclusion

The hydraulic depth in the irrigation channel with a trapezoidal section calculated by the quadratic equation, the discriminant value in the quadratic equation must not be negative so that the calculation results are not imaginary. The negative value can be avoided by analyzing the independent variable,
namely the mean flow velocity or the channel base slope. Based on the non-negative discriminant value, it are obtained the formula for calculating the mean flow velocity with the channel base bottom slope is determined as well as the formula for calculating the channel base slope with the mean flow velocity is determined. The proposed method for calculating the dimensions of the irrigation channel with a trapezoidal cross-section is in the form of a calculation sequence and a calculation flow chart which, if followed, will avoid calculation failure due to imaginary numbers appearing in the calculation of the hydraulic depth in the channel calculated by the quadratic equation.

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