Hidden Variable Theory of a Single World from Many-Worlds Quantum Mechanics

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We propose a method for finding an initial state vector which by ordinary Hamiltonian time evolution follows a single branch of many-worlds quantum mechanics. The resulting deterministic system appears to exhibit random behavior as a result of the successive emergence over time of information present in the initial state but not previously observed.

I. INTRODUCTION

Microscopic particles have wave functions spread over all possible positions. Macroscopic objects simply have positions, or at least center-of-mass positions. How to apply the mathematics of quantum mechanics to extract predictions registered in the macroscopic world of positions from experiments on microscopic systems having wave functions but not definite positions is well understood for all practical purposes. But less well understood, or at least not a subject on which there is a clear consensus, is how in principle the definite positions of the macroscopic world emerge from the microscopic matter of which it is composed, which has only wave functions but not definite positions. There is a long list of proposals. In the present article we will add yet one more entry to the list.

We begin in Section II with a brief reminder of the “problem of measurement” which arises for an experiment in which a microscopic system interacts with an external macroscopic measuring device with both systems assumed governed by quantum mechanics. We then review the many-worlds interpretation’s proposed resolution of this problem. The many-worlds interpretation, however, has well-known difficulties of its own. We next summarize how these problems are, to some degree, resolved by taking into account the process of environmentally-induced decoherence. But problems still remain. In Section III we introduce a hypothesis on the mathematical structure of environmentally-induced decoherence. Subject to this hypothesis, we propose a formulation of quantum mechanics which we believe resolves the difficulties in the initial combination of many-worlds and decoherence. The proposal incorporates purely deterministic time development by the usual unitary operator following, however, only a single branch of many-worlds quantum mechanics. The system’s initial state determines which branch is followed and is chosen from a particular random ensemble. In Section IV we show that the probability the random ensemble assigns each branch agrees with the probability that would be assigned to each branch by the Born rule. As a consequence the proposed formulation of quantum mechanics predicts the same observable results for experiments as would be obtained from the Born rule. In Section V we present a toy model which illustrates how the proposed deterministic theory works. In Section VI we apply the deterministic theory to a Bell’s theorem test and obtain, as required by the results of Section IV, the predictions expected of quantum mechanics and in violation of the constraints implied by Bell’s theorem for a certain class of hidden variable theories. We conclude in Section VII with several comments about the random ensemble from which initial states are drawn.

II. PROBLEMS

Let $S$ be a microscopic system to be measured, with corresponding state space $H_S$ for which a basis is $\{|s_i>\}, i > 0$. Let $M$ be a macroscopic measuring device with corresponding state space $H_M$ containing the set of vectors $\{|m_i>\}, i \geq 0$. For each different value of $i > 0$ the state $|m_i>$ is a macroscopically distinct meter reading. Let $|m_0>$ be an initial state showing no reading. In the combined system-meter product state space $H_S \otimes H_M$ a measurement of $S$ by $M$ over some time interval takes each possible initial state $|s_i> |m_0>$ into the corresponding final state $|s_i> |m_t>$ with the measuring device displaying the measured value of the microscopic system’s variable. By linearity of quantum mechanical time evolution, however, it then follows that a measurement with a linear superposition in the initial state will yield a final state also with a superposition

\[
\frac{1}{\sqrt{2}}|s_1> + \frac{1}{\sqrt{2}}|s_2> |m_0> \rightarrow \frac{1}{\sqrt{2}}|s_1> |m_1> + \frac{1}{\sqrt{2}}|s_2> |m_2> . \tag{1}
\]

In the measured final state, the meter no longer has a single value, but a combination of two values which cannot, by itself, be connected to a recognizable configuration of a macroscopic object. The absence of a recognizable configuration for the macroscopic device is the “problem of measurement”.

The resolution of this problem proposed by the many-worlds interpretation of quantum mechanics is that the states $|s_1> |m_1>$ and $|s_2> |m_2>$ actually represent two different worlds. In each world the meter has a definite position but with different positions in the two different worlds. Among the problems of this proposed
resolution is that
\[ \frac{1}{\sqrt{2}}|s_1 > |m_1 > + \frac{1}{\sqrt{2}}|s_2 > |m_2 > = \]
\[ \frac{1}{\sqrt{8}}(|s_1 > + |s_2 >)(|m_1 > + |m_2 >) + \]
\[ \frac{1}{\sqrt{8}}(|s_1 > - |s_2 >)(|m_1 > - |m_2 >). \] (2)
Thus again two worlds, but now with meter configuration of \(|m_1 > + |m_2 >\) in one and \(|m_1 > - |m_2 >\) in the other and thus a recognizable configuration for a macroscopic object in neither. According to which basis of \(\mathcal{H}_M\) should the two worlds be split? Without some reason for a choice, many-worlds quantum mechanics still has a problem, the “preferred basis problem”.

A resolution to this problem is proposed to occur through environmentally-induced decoherence. According to this proposal, in abbreviated summary, the system-meter combination should not be considered in isolation but instead an account is required of the rest of the macroscopic environment with which the meter can interact. When the value of a macroscopic meter is changed by recording the value of a microscopic coordinate, the meter’s new state rapidly becomes entangled with a large number of other degrees of freedom in the environment
\[ \frac{1}{\sqrt{2}}|s_1 > |m_1 > + \frac{1}{\sqrt{2}}|s_2 > |m_2 > \rightarrow \]
\[ \frac{1}{\sqrt{2}}|s'_1 > |m'_1 > |e_1 > + \frac{1}{\sqrt{2}}|s'_2 > |m'_2 > |e_2 >. \] (3)
For a particular choice of basis, determined by the system’s dynamics, entanglement of the meter with the environment is optimal in the sense that \(|s'_1 > |m'_1 >\) and \(|s'_2 > |m'_2 >\) are nearly equal to \(|s_1 > |m_1 >\) and \(|s_2 > |m_2 >\), respectively, \(|e_1 >\) and \(|e_2 >\) are nearly orthogonal, the process of the meter becoming entangled with the environment proceeds as quickly as possible, and \(|e_1 >\) and \(|e_2 >\) almost do not mix in the course of further time development becoming, as a result, permanent records of the measurement results. In addition, \(|e_1 >\) and \(|e_2 >\) include many redundant copies of the information in \(|s_1 > |m_1 >\) and \(|s_2 > |m_2 >\). Based on these various considerations it is argued that the coordinates with respect to the optimal basis behave essentially as classical degrees of freedom. Correspondingly, for many-worlds augmented with decoherence, the circumstance under which a system splits into distinct worlds is when a superposition has been produced mixing distinct values of one of these effectively classical degrees of freedom. Each distinct value of the coordinate in such a superposition goes off into a distinct world.

But a variety of problems remain for the combination of decoherence and many-worlds. In particular, environmentally-induced decoherence occurs over some finite time interval, no matter how short, occurs over some extended region of space, the entangled states \(|s'_1 > |m'_1 >\) and \(|s'_2 > |m'_2 >\) are not exactly equal to \(|s_1 > |m_1 >\) and \(|s_2 > |m_2 >\), respectively, \(|e_1 >\) and \(|e_2 >\) are not exactly orthogonal. When, exactly, over the time interval of decoherence, does the splitting of the world in two parts occur? And since the process extends over space, this timing will differ in different Lorentz frames. Which is the correct choice? In addition, how accurate do the approximate equalities and approximate orthogonality in the entanglement process have to be to drive a split? These questions may be of no practical consequence in treating the meter readings as classical degrees of freedom after entanglement and using the resulting values to formulate observable predictions. For any possible choice of the location of these boundaries, however, there will be at least some events in the vastness of space and time that either do or do not lead to new branches of the universe depending on essentially arbitrary choices of the parameters distinguishing acceptable from unacceptable entanglement events. These problems seem hard to square with the view that the state vector is the fundamental substance of reality and hard to accept as features of a process in which this fundamental substance splits into distinct copies. While it might be possible to argue that these elements of fuzziness in splitting events are necessary consequences of the quantum nature of the world, it still seems they would be better avoided if possible.

**III. DETERMINISTIC QUANTUM MECHANICS**

We now consider a modified version of many-worlds plus decoherence which avoids these three issues by pushing the process of world splitting infinitely far back into the past, or at least back before any potential world splitting events occur. In place of splitting, each possible world will be specified by a different initial state drawn from a random ensemble.

Choose some reasonable but otherwise arbitrary way of resolving the event time ambiguities. Let \(s_i\) be the set of possible outcomes of splitting event \(i\), occurring at time \(t_i\). We assume, for simplicity, only a finite sequence of \(N\) such splittings, the last occurring at time \(t_N\). Let \(h\) specify a sequence of splitting results \((r_0, r_1, \ldots, r_k)\), up to some time \(t_k\), with each \(r_i \in s_i\). Let \(S_k\) be the set of all such possible sequences for events up to and including time \(t_k\). Let \(\mathcal{H}\) be the space of states of the universe. Let \(|\Psi(t) > \in \mathcal{H}\) be the state of the universe at time \(t\)
\[ |\Psi(t) > = \exp(-iHt)|\Psi(0) > \] (4)
for system Hamiltonian \(H\).

We adopt from studies of decoherence the hypothesis that each macroscopic splitting event, after entanglement with the environment, leaves a permanent footprint in the degrees of freedom of \(\mathcal{H}\), consisting of multiple copies of the information which was recorded in
the event driving the split. Since each record is assumed permanent, records accumulate over time so that at any \( t \) shortly after event time \( t_k \), and before \( t_{k+1} \), the state vector can be expressed as a sum over branches of splitting events of the form

\[
|\Psi(t)\rangle = \sum_{h \in S_k} |\Psi(h, t)\rangle, \quad (5)
\]

where \( |\Psi(h, t)\rangle \) is a vector for which the environmental degrees of freedom carry a record of splitting history \( h \), the sum is over splitting histories up to time \( t_k \), and environmental record components of \( |\Psi(h, t)\rangle \) and \( |\Psi(h', t)\rangle \) for distinct \( h \) and \( h' \) are orthogonal, or nearly so. The decomposition in Eq. (5) will hold up until shortly after the next splitting time \( t_{k+1} \), at which time an updated expression with histories \( h \) drawn from \( S_{k+1} \) will take over.

From Eq. (5) applied at some time \( t \) after the last splitting event \( t_N \), for some history \( h \in S_N \), define from \( |\Psi(h, t)\rangle \) its evolution back to time 0

\[
|\Psi(h, 0)\rangle = \exp(iHt)|\Psi(h, t)\rangle. \quad (6)
\]

If \( |\Psi(h, 0)\rangle \) is now evolved forward again in time retracing its history up to \( t \), the result at each \( t_k \) has to follow the branch \( r_k \) specified in \( h \). If \( |\Psi(h, 0)\rangle \) is evolved forward again in time up to \( t \) the result will clearly have to return to \( |\Psi(h, t)\rangle \) exhibiting splitting history \( h \). But following any splitting event at some \( t_k \), the contribution \( r_k \) of that event to the history \( h \) is permanently recorded. Thus the only way \( |\Psi(h, t)\rangle \) can acquire a record of the full history \( h \) as of time \( t \) is if each \( r_k \) is correctly recorded at \( t_k \) and therefore the split at \( t_k \) occurs as specified by \( r_k \).

The system’s initial state we now assume is found by a single random choice at time 0 from an ensemble of \( |\Psi(h, 0)\rangle \) for \( h \) drawn from \( S_N \) with each having probability \( <\Psi(h, 0)|\Psi(h, 0)\rangle \). The result is that each history \( h \) will occur with the same probability that would be found if splitting events were treated as sequences of observations by some external observer and their probabilities then calculated by the Born rule. A proof of this statement will be given in Section [IV].

Each \( |\Psi(h, t)\rangle \) may be viewed as a different world. But splitting between the different worlds occurs only once, at the beginning of history.

One piece of the formulation of quantum mechanics we now propose remains approximate, but another has become exact. What has become exact is the time evolution by Hamiltonian \( H \) of the state vector \( |\Psi(h, 0)\rangle \) drawn from the initial random ensemble. This state vector we take as the underlying substance of the real world. What is approximate, on the other hand, is the extent to which this evolution follows splitting history \( h \). This tracking is approximate both because the time at which the entanglement equation Eq. (5) holds is approximate and because the orthogonality of the environmental record components of vectors \( |\Psi(h, t)\rangle \) and \( |\Psi(h', t)\rangle \) for distinct \( h \) and \( h' \) is approximate. Put differently, the underlying substance of the microscopic world follows an exact law, to which the macroscopic description is an approximation. By contrast, the combination of many-worlds and decoherence considered in Section [III] included only an approximate macroscopic world as the substance of reality.

The formulation we propose is a kind of hidden variable theory, with the hidden variables present in the initial \( |\Psi(h, 0)\rangle \). They emerge in macroscopic reality only over time through their influence on the sequence of splitting results \( (r_0, r_1, ..., r_N) \) forming the history \( h \).

### IV. BORN RULE

We will show that the probability weight \( <\Psi(h, 0)|\Psi(h, 0)\rangle \) assigned to \( |\Psi(h, 0)\rangle \) in the initial random ensemble is the same as the probability which the Born rule would assign to a series of observations of the system by an outside observer at splitting event times \( (t_0, t_1, ..., t_N) \) yielding the sequence of results \( (r_0, r_1, ..., r_N) \) of the history \( h \).

By the unitarity of time evolution, we can restore \( <\Psi(h, 0)|\Psi(h, 0)\rangle \) to a time \( t \) shortly after \( t_N \)

\[
<\Psi(h, 0)|\Psi(h, 0)\rangle = <\Psi(h, t)|\Psi(h, t)\rangle. \quad (7)
\]

But since \( h \) exhibits result \( r_N \) at \( t_N \) we have

\[
|\Psi(h, t)\rangle = P(r_N)|\Psi(h, t)\rangle \quad (8a)
\]

\[
P(r_N)\exp[-iH(t_N - t_{N-1})] \times |\Psi(h, t_{N-1})\rangle \quad (8b)
\]

where \( P(r_N) \) is the projection operator onto the subspace of \( \mathcal{H} \) for result \( r_N \).

Iterating the argument for Eqs. (8), we obtain

\[
|\Psi(h, t)\rangle = P(r_N)|\Psi(h, t)\rangle \exp[-iH(t_N - t_{N-1})] \times \ldots P(r_1)|\Psi(h, t_0)\rangle \quad (9a)
\]

\[
P(r_0)\exp[-iHt_0]|\Psi(h, 0)\rangle = Q(h)|\Psi(h, 0)\rangle, \quad (9d)
\]

where for notational convenience we define \( Q(h) \) to be the preceding product of projections and time evolution. For any \( h' \) which differs from \( h \) by at least one result \( r_i \), however, we have

\[
Q(h)|\Psi(h', 0)\rangle = 0, \quad (10)
\]

since if Eq. (9) is unfolded for \( |\Psi(h', 0)\rangle \), at step \( r_i \) the projection \( P(r_i) \) will annihilate the evolving image of \( |\Psi(h', 0)\rangle \).

From Eqs. (9d) (10), we have

\[
|\Psi(0)\rangle = \sum_{h \in S_N} |\Psi(h, 0)\rangle. \quad (11)
\]

Combining Eqs. (7, 11) gives

\[
<\Psi(h, 0)|\Psi(h, 0)\rangle = <\Psi(0)|Q(h)^\dagger Q(h)|\Psi(0)\rangle. \quad (12)
\]
The expression on the right hand side of Eq. (12), as advertised, is the probability which the Born rule would assign to a series of observations of the system by an outside observer at splitting event times \((t_0, t_1, \ldots t_N)\) yielding the sequence of results \((r_0, r_1, \ldots r_N)\) of the history \(h\).

A consequence of Eq. (12) is that the proposed formulation of quantum mechanics predicts the same observable results for experiments as would be obtained from the Born rule.

V. TOY MODEL

We now turn to a toy model. Consider a universe composed of a spin \(s\) taking two values \(|\uparrow\rangle\) and \(|\downarrow\rangle\) of \(z\)-direction spin which interacts with an environment composed of variables \(e_0, e_1, e_2,\) and \(e_3\) each taking values, \(0\) and \(1\). Except for transitions to be specified below at times \(t_1\) and \(t_2\), we assume a Hamiltonian diagonal in the basis \(|e_0\rangle > |e_1\rangle > |e_2\rangle > |e_3\rangle\) so that

\[
exp(-iHt)|e_0\rangle > |e_1\rangle > |e_2\rangle > |e_3\rangle = exp(-it \sum \omega_i e_i)|e_0\rangle > |e_1\rangle > |e_2\rangle > |e_3\rangle \equiv |e_0, t > |e_1, t > |e_2, t > |e_3, t > . \quad \text{(13)}
\]

The \(\omega_i\) we assume chosen in such a way that none of the states \(|e_0, t > |e_1, t > |e_2, t > |e_3, t >\) are degenerate. The result of these choices will be that the \(|e_0, t > |e_1, t > |e_2, t > |e_3, t >\) form an optimal environmentally selected memory basis for the events at \(t_1\) and \(t_2\).

Assume an initial state \(|\Psi(0)\rangle > \text{at time } 0\) of \(|\uparrow\rangle > |0, 0 > |0, 0 > |0, 0 > |0, 0 >\). From time 0 to some later \(t_1\) the system evolves according to Eq. (13). Then at \(t_1\) the \(z\)-direction spin of \(s\) is recorded by \(e_0\) and \(e_1\) according to the unitary time step \(U_1\)

\[
U_1 |\uparrow\rangle > |e_0, t > |0, t > |e_2, t > |e_3, t > = |\uparrow\rangle |e_0, t > |1, t > |e_2, t > |e_3, t > \quad \text{(14a)}
\]

\[
U_1 |\uparrow\rangle > |e_0, t > |1, t > |e_2, t > |e_3, t > = |\uparrow\rangle |e_0, t > |0, t > |e_2, t > |e_3, t > \quad \text{(14b)}
\]

\[
U_1 |\downarrow\rangle > |0, t > |e_1, t > |e_2, t > |e_3, t > = |\downarrow\rangle |1, t > |e_1, t > |e_2, t > |e_3, t > \quad \text{(14c)}
\]

\[
U_1 |\downarrow\rangle > |1, t > |e_1, t > |e_2, t > |e_3, t > = |\downarrow\rangle |0, t > |e_1, t > |e_2, t > |e_3, t > \quad \text{(14d)}
\]

which hold for any choice of \(e_0, \ldots, e_3\). Upon completion of this process, the system’s state becomes \(|\uparrow\rangle > |0, t_1 > |1, t_1 > |0, t_1 > |0, t_1 > \) The system again follows Eq. (13) until some later time \(t_2\) at which the \(x\)-direction spin of \(s\) is recorded on \(e_2\) and \(e_3\) according to the unitary time step \(U_2\) which is the same as \(U_1\) in Eqs. (14) but with \(e_0\) and \(e_1\) replaced, respectively, by \(e_2\) and \(e_3\) and with the spin states \(|\uparrow\rangle, |\downarrow\rangle\) of \(s\) replaced, respectively, by \(|\uparrow\rangle_x, |\downarrow\rangle_x\)

\[
|\uparrow\rangle_x = \frac{1}{\sqrt{2}} |\uparrow\rangle + \frac{1}{\sqrt{2}} |\downarrow\rangle, \quad \text{(15a)}
\]

\[
|\downarrow\rangle_x = \frac{1}{\sqrt{2}} |\uparrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\rangle . \quad \text{(15b)}
\]

Upon completing this step, the system’s state becomes

\[
\frac{1}{2} |\uparrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |1, t_2 > +
\]

\[
\frac{1}{2} |\downarrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |1, t_2 > +
\]

\[
\frac{1}{2} |\uparrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |0, t_2 > -
\]

\[
\frac{1}{2} |\downarrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |0, t_2 > . \quad \text{(16)}
\]

Histories can be labeled with the values of \((e_0, e_1, e_2, e_3)\). The two non-zero results left after \(t_2\) are \((0, 1, 0, 1)\) and \((0, 1, 1, 0)\). The random ensemble of states at time \(t_2\) thus consists of the two vectors

\[
\frac{1}{2} |\uparrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |1, t_2 > +
\]

\[
\frac{1}{2} |\downarrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |1, t_2 > , \quad \text{(17a)}
\]

\[
\frac{1}{2} |\uparrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |0, t_2 > -
\]

\[
\frac{1}{2} |\downarrow\rangle > |0, t_2 > |1, t_2 > |0, t_2 > |0, t_2 > , \quad \text{(17b)}
\]

each with probability of \(\frac{1}{2}\).

The ensemble of initial states \(|\Psi(h, 0)\rangle >\) can be recovered by applying the reversed time evolution operators \(U_2^\dagger, U_1^\dagger\) combined with the reverse of Eq. (13). The result is

\[
\frac{1}{\sqrt{2}} |\uparrow\rangle > |0, 0 > |0, 0 > |0, 0 > |0, 0 > +
\]

\[
\frac{1}{\sqrt{2}} |\downarrow\rangle > |1, 0 > |1, 0 > |0, 0 > |0, 0 > , \quad \text{(18a)}
\]

\[
\frac{1}{\sqrt{2}} |\uparrow\rangle > |0, 0 > |0, 0 > |0, 0 > |0, 0 > -
\]

\[
\frac{1}{\sqrt{2}} |\downarrow\rangle > |1, 0 > |1, 0 > |0, 0 > |0, 0 > . \quad \text{(18b)}
\]

A repeat of time evolution forward shows that the first of these states produces an \((e_0, e_1, e_2, e_3)\) history which is purely \((0, 1, 0, 1)\) and the second produces a history which is purely \((0, 1, 1, 0)\).
VI. BELL'S THEOREM

For a system $|\Psi> \times$ consisting of two spin $\frac{1}{2}$ particles in a total angular momentum 0 state, the standard formulation of quantum mechanics yields

$$<\Psi|(\sigma^1 \cdot \hat{u})(\sigma^2 \cdot \hat{v})|\Psi> = -\hat{u} \cdot \hat{v},$$

where $\sigma^i$ is the vector of sigma matrices acting on spin $i$ and $\hat{u}$ and $\hat{v}$ are two unit vectors. Bell's theorem is that the result of Eq. (19) cannot be realized in a particular class of hidden variable theories.

We now consider an experimental setup for deterministic quantum mechanics which measures the expectation value in Eq. (19). As expected from the proof in Section IV, the results predicted by the deterministic formulation of quantum mechanics will reproduce Eq. (19). For convenience, we will take $\hat{u}$ to be in the z-direction and $\hat{v}$ to be rotated from $\hat{u}$ by some angle $\theta$.

We assume a system composed of a large number of identical subsystems $W_1, ..., W_N$. Each subsystem includes two spins $s_0$ and $s_1$, each $s_i$ taking two values $|\uparrow> \text{ and } |\downarrow>$ of z-direction spin. Each subsystem includes also an environment composed of variables $e_0, e_1, e_2$, and $e_3$ each taking values, $|0> \text{ and } |1>$. As in Section V, except for transitions to be specified at a sequence of times $t_i$, we assume a Hamiltonian diagonal in the basis $|e_0 > |e_1 > |e_2 > |e_3 >$ according to Eq. (13), with corresponding vectors $|e_0, t> |e_1, t> |e_2, t> |e_3, t>$ forming an optimal environmentally selected memory basis for the event at each $t_i$.

Assume an initial state $|\Psi(0)>$ for subsystem $W_i$ at time 0 of

$$|\Psi(0)>_{1i} = \frac{1}{\sqrt{2}}(|\uparrow> \text{ and } |\downarrow> \text{ of } s_1 \text{ replaced by } |\uparrow> \text{ and } |\downarrow>$$

$$|0, 0, 0, 0 >|0, 0, 0, 0 > \text{.}$$

(20)

For each subsystem $W_i$ from time 0 to some later $t_i$, time evolution follows Eq. (13). Then at $t_i$ the z-direction spin of $s_0$ is recorded by $e_0$ and $e_1$ according to the unitary time step $U_1$ of Eqs. (14). In addition, at the same time, the spin of $s_1$ is recorded on $e_2$ and $e_3$ along a direction rotated by some angle $\theta$ from the z-direction toward the x-direction, according to the unitary time step $U_2$. Time step $U_2$ is the same as $U_1$ in Eqs. (14) but with $e_0$ and $e_1$ replaced, respectively, by $e_2$ and $e_3$ and with the spin states $|\uparrow> \text{ and } |\downarrow>$ of $s$ replaced, respectively, by $|\uparrow> \text{ and } |\downarrow>$

$$|\uparrow> = \cos(\frac{\theta}{2})|\uparrow> + \sin(\frac{\theta}{2})|\down>, \quad (21a)$$

$$|\down> = -\sin(\frac{\theta}{2})|\uparrow> + \cos(\frac{\theta}{2})|\down> \quad (21b)$$

Following $t_i$ the time evolution of $W_i$ again follows Eq. (13).

With these definitions, the observed value of the quantity in Eq. (19) recorded by each subsystem will be $(e_1 - e_0)(e_3 - e_2)$.

On the completion of all iterations after time $t_N$, each possible history $h$ can be expressed as a sequence of subhistories of the form

$$h = (e_0, e_1, e_2, e_3, ..., e_0N, e_1N, e_2N, e_3N).$$

(22)

The corresponding state vector $|\Psi(h, t_N)>$ has the form

$$|\Psi(h, t_N)> = |\Psi(e_0, e_1, e_2, e_3, t_N)>_1 \times ...|\Psi(e_0N, e_1N, e_2N, e_3N, t_N)>_N,$$

(23)

where each subsystem's states for its 4 possible subhistories are

$$|\Psi(0, 1, 0, 1, t_N)> = \frac{1}{\sqrt{2}} \sin(\frac{\theta}{2})|\uparrow> |\uparrow> \times$$

$$|0, t_N >|1, t_N >|0, t_N >|1, t_N > \text{,}$$

(24a)

$$|\Psi(0, 1, 1, 0, t_N)> = \frac{1}{\sqrt{2}} \cos(\frac{\theta}{2})|\uparrow> |\uparrow> \times$$

$$|0, t_N >|1, t_N >|0, t_N >|0, t_N > \text{,}$$

(24b)

$$|\Psi(1, 0, 0, 1, t_N)> = -\frac{1}{\sqrt{2}} \cos(\frac{\theta}{2})|\down> |\uparrow> \times$$

$$|1, t_N >|0, t_N >|0, t_N >|1, t_N > \text{,}$$

(24c)

$$|\Psi(1, 0, 1, 0, t_N)> = \frac{1}{\sqrt{2}} \sin(\frac{\theta}{2})|\down> |\uparrow> \times$$

$$|1, t_N >|0, t_N >|1, t_N >|0, t_N > \text{.}$$

(24d)

The weight of any history in the ensemble of initial states, equal by Eq. (17) to its weight in the ensemble of final states, becomes

$$<\Psi(h, t_N)|\Psi(h, t_N)> = \left[\frac{1}{2} \sin^2(\frac{\theta}{2})\right]^{p(h)} \left[\frac{1}{2} \cos^2(\frac{\theta}{2})\right]^{m(h)}.$$

(25)

Here $p(h)$ is the count of $i$ in $h$ for which $(e_{1i} - e_{0i})(e_{3i} - e_{2i})$ is 1, and $m(h)$ is the count of $i$ in $h$ for which $(e_{1i} - e_{0i})(e_{3i} - e_{2i})$ is -1. It follows that each choice of initial state according to Eq. (25) may be viewed as $N$ independent choices of the random variable $(e_{1} - e_{0})(e_{3} - e_{2})$ from an ensemble with probability $\sin^2(\theta)$ at value 1 and probability $\cos^2(\theta)$ at value -1. By the central limit theorem, we then find that for large $N$, the average over $N$ draws will nearly yield

$$<e_{1} - e_{0})(e_{3} - e_{2})> = \sin^2(\frac{\theta}{2}) - \cos^2(\frac{\theta}{2}).$$

(26a)

$$= -\cos(\theta),$$

(26b)

in agreement with Eq. (19).
VII. RANDOM STATE ENSEMBLES

A paradoxical feature of both states in the time 0 ensemble in Eqs. (18) is that they include $|\downarrow>$ components for the spin even though both show purely $|\uparrow>$ results for $e_1$ at the first event at time $t_1$. What this reflect, we believe, is that the system’s initial state contains hidden variables the values of which are not directly accessible. They are determinable only to the extent that they participate in events in which they become entangled with the degrees of freedom of the environment. The time 0 ensemble of Eq. (18) also includes linear combinations of distinct values of the environment variables, but which then assume single values at the time the entanglement events occur. What this implies, we believe, is that the environment variables do not become part of the macroscopic environmental record until the event at which they assume their permanently remembered values. Despite the peculiarities of the ensemble in Eq. (18), however, all that is required of the theory empirically is that it show macroscopic results at each event consistent with the Born rule. Which we have shown in Section IV the proposed theory does.

A further limitation restricting access to the initial state’s hidden variables is that the theory proposes to be a complete deterministic description of the universe and allows no outside observers. The macroscopic experiments which can occur are only whatever happens to have already been programmed into the initial state’s plan for history.

While the initial state ensembles constructed lead to time evolution without macroscopically ambiguous variables, the construction is by a roundabout path. A missing element is some criterion which can be applied directly to candidate initial state ensembles to determine which of these avoid macroscopically ambiguous time evolutions. An alternative view of the time evolution of the state vector, however, partially resolves this problem. Rather than taking the state vector as evolving forward in time from time 0, we could equally well propose it evolves backward from some distant future time $t$. The initial state for this reverse time evolution is then a random draw from an ensemble of $|\Psi(h, t)\rangle$ with corresponding probability weight $<\Psi(h, t)|\Psi(h, t)>$. Each of these vectors by construction in Eq. (5) will give rise to a macroscopically unambiguous time trajectory. The problem of finding an initial ensemble then becomes finding the decomposition in Eq. (5). Which task might be facilitated by the possibility that in the limit of large $t$, as the state $|\Psi(h, t)\rangle$ becomes fully entangled with the rest of the universe, the orthogonality of the environmental memory components of $|\Psi(h, t)\rangle$ and $|\Psi(h', t)\rangle$ might become exact for distinct $h$ and $h'$.

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