Circuit QED and sudden phase switching in a superconducting qubit array

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Superconducting qubits connected in an array can form quantum many-body systems such as the quantum Ising model. By coupling the qubits to a superconducting resonator, the combined system forms a circuit QED system. Here, we study the nonlinear behavior in the many-body state of the qubit array using a semiclassical approach. We show that sudden switchings as well as a bistable regime between the ferromagnetic phase and the paramagnetic phase can be observed in the qubit array. A superconducting circuit to implement this system is presented with realistic parameters.

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Circuit quantum electrodynamics (QED) has been intensively explored in the past few years. A number of quantum optical effects such as Lamb shift, nonlinear spectrum, and lasing have been demonstrated in experiments. Most recently, the Tavis-Cummings model has also been tested with multiple transmon qubits coupling to a resonator. The superconducting resonators hence not only provide a powerful tool to implement quantum information protocols, but can also facilitate the study of quantum coherence effects and many-body effects involving microwave photons.

When a quantum many-body system is coupled to a resonator cavity, novel many-body phenomena that have not been studied previously in condensed matter systems can be observed. In the atomic systems, it has been shown that atomic condensate can be coupled strongly to cavity photons to demonstrate interesting effects. In the superconducting circuits, given the diversity of the electromagnetic couplings and parameter regimes, a large variety of many-body Hamiltonians can be engineered that can not be emulated in atomic systems. In the past, critical behavior has been studied both theoretically and experimentally in Josephson junction arrays that can emulate spin models and quantum phase models. Recent progresses in superconducting quantum devices further expand such research to include fermionic Hamiltonians.

In this work, we study the nonlinear behavior in the many-body state of an array of superconducting qubits coupled to a superconducting resonator. Different from previous work on optical bistability, our work focuses on the behavior of the qubit array in this system. The qubits are arranged in a one-dimensional chain to form the quantum Ising model. In the bad cavity limit, the photon number of the resonator will depend nonlinearly on the average of the qubit operator. This gives rise to a sudden switching between different quantum many-body phases in the qubit array when the driving power on the resonator is varied, instead of the continuous quantum phase transition in a simple quantum Ising model. Our study also shows that a bistable regime exists where the qubit array can be in either the paramagnetic phase or the ferromagnetic phase. A circuit to implement this model can be constructed with realistic parameters, where the inductive coupling between the superconducting qubits and the resonator generates a shift of the resonator frequency that depends on the qubit operators. The nonlinear effects studied here can be experimentally demonstrated in a toy system with only a few qubits and a resonator. Furthermore, the resonator can act as a threshold detector to measure the phase switching in the bistable regime and act as a probe of the continuous quantum phase transition in the weak driving limit.

We consider a qubit array coupled to a resonator with the total Hamiltonian

$$H_t = H_{QI} + H_{int} + H_{c0} + H_\kappa + H_\gamma$$

where $H_{QI} = -J_x \sum_i \sigma_{xi} - \sum_i \sigma_{zi} \sigma_{zi+1}$ describes the quantum Ising model with a uniform transverse magnetic field $J_x$ and a ferromagnetic coupling between nearest neighbor qubits, $H_{c0} = h\omega_c a^\dagger a + h\epsilon(t)(a + a^\dagger)$ describes the Hamiltonian of the resonator and includes a driving term with the amplitude $\epsilon(t) = 2\epsilon \cos \omega t$, and $H_{int} = h g a^\dagger a \sum \sigma_{xi}$ is the coupling between the resonator and the qubits. Here, $\sigma_{xi}$ and $\sigma_{zi}$ are the Pauli matrices of the qubits and $a$ ($a^\dagger$) is the annihilation (creation) operator for the resonator. The magnitude of the ferromagnetic coupling is set to unity and is used as the energy unit in the following. The term $H_\kappa$ is the coupling between the resonator (qubit) and its environmental modes that causes damping (relaxation) of the resonator (qubit). Without the resonator, a second order quantum phase transition between the paramagnetic and the ferromagnetic phases occurs in the qubit array when the transverse field is varied.

The coupling $H_{int}$ shifts the frequency of the resonator by $g \sum \sigma_{xi}$. In the Heisenberg picture, we can derive the operator equation for the resonator in the rotating frame,

$$\dot{a} = i \Delta_c a - i \epsilon - i g a \sum_i (\sigma_{xi}) - \frac{\kappa}{2} a + \sqrt{\kappa} a_{in}$$

where $\Delta_c$ is the detuning, $\kappa$ is the damping rate of the resonator, and $a_{in}$ is the noise operator.
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The coupling constant \( g \) of the qubit has a small loop current in the qubits and the resonator, we consider a resonator with an inductance of the SQUID \( L \) be readily designed to satisfy this criterion. The frequency of the resonator becomes \( \frac{\omega_c}{\omega_r} \), which gives \( \Phi_c \) be the SQUID inductance at \( \Phi_c = \frac{\omega_c}{\omega_r} \). The total inductance of the resonator is \( L_{tot} = L_{sq} + L_r \), including the SQUID inductance and the loop inductance \( L_r \). Let \( L_{sq} \) be the SQUID inductance at \( \Psi_0 = 0 \) and \( \varphi_{ex} = \pi/4 \), the frequency of the resonator becomes

\[
\omega_c = \omega_{d0} + \omega_o \left( \frac{L_{sq}}{L_{sq} + L_r} \right)^2 \left( \frac{\pi \mu_0 R_0 I_{q2}}{2 \sqrt{2} \Phi_0} \right) \sum_i \sigma_{xi} \tag{9}
\]

with \( \omega_{d0} = \left( \frac{L_{sq} + L_r}{C_r} \right)^{-1/2} \). This expression includes the coupling between the qubits and the resonator in the form of \( H_{int} \) with

\[
g = \omega_o \left( \frac{L_{sq}}{L_{sq} + L_r} \right)^2 \left( \frac{\pi \mu_0 R_0 I_{q2}}{2 \sqrt{2} \Phi_0} \right) \tag{10}
\]

The coupling constant \( g \) depends on the qubit flux \( \Phi_c \), to the first order, which is a merit of this circuit that ensures a sizable coupling to demonstrate the nonlinear effect. We can choose the following parameters: \( L_r \sim 100 \mu \text{H} \), \( I_{q2} = 80 \text{nA} \), \( I_r = 1200 \text{nA} \) (i.e. \( L_{sq} = 100 \mu \text{H} \)), and the total capacitance of the resonator circuit \( C_r = 0.1 \mu \text{F} \), which give \( g = 2 \pi \times 1 \text{ MHz} \) and \( \omega_c = 2 \pi \times 29 \text{ GHz} \). With a ferromagnetic coupling of \( 2 \pi \times 2 \text{ GHz} \) chosen as the energy unit, we have \( g = 0.005 \) in the relative unit. Note that various other qubits such as the superconducting flux qubit and phase qubit can be used to construct the qubit array given the flexibility in the superconducting circuits [23, 24].

FIG. 2: (a) \( \tilde{J}_x \) vs. \( J_x \) before the sudden switching (solid curve) and after the switching (dashed curve) at \( \varepsilon_{z, 2} \). (b) the energy change at the switching points vs. \( J_y \). (c) many-body phases in the space of \( J_x \) and \( \varepsilon_z \), where ‘F’ (horizontal lines) indicates the ferromagnetic phase and ‘P’ (vertical lines) indicates the paramagnetic phase.

FIG. 3: Circuit of an array of superconducting qubits coupling to a resonator. The current in the qubit loops couples to the SQUID in the resonator via mutual inductance.

\[ B_{z_1}/B_{y_1} = C_1/C_0 < 1/2 \] In our system, \( C_1/C_0 \) can be readily designed to satisfy this criterion.

To generate a coupling \( H_{int} = \hbar g a^+ a \sum \sigma_{x i} \) between the qubits and the resonator, we consider a resonator that includes a SQUID in the circuit. In Fig. 3 each qubit has a small loop current \( I_{q2} = I_{q2} \sin(\delta_2/2) \sigma_{xi} \) that is controlled by the external flux \( \delta_2 \). This current is coupled to the SQUID via mutual inductance and modifies the flux in the SQUID loop by \( \Phi_r = -\mu_0 I_{q2} \sin(\delta_2/2) \sum_i \sigma_{xi} \) where \( R_0 \) is roughly the size of the qubit loop. The effective inductance of the SQUID can then be written as

\[
L_{tot} = L_{sq} + L_r \tag{8}
\]

where \( L_r \) is the critical current of the SQUID junctions and \( \varphi_{ex} \) is the external flux in the SQUID loop. The total inductance of the resonator is \( L_{tot} = L_{sq} + L_r \), including the SQUID inductance and the loop inductance \( L_r \). Let \( L_{sq} \) be the SQUID inductance at \( \Phi_c = 0 \) and \( \varphi_{ex} = \pi/4 \), the frequency of the resonator becomes

\[
\omega_c = \omega_{c0} + \omega_o \left( \frac{L_{sq}}{L_{sq} + L_r} \right) \left( \frac{\pi \mu_0 R_0 I_{q2}}{2 \sqrt{2} \Phi_0} \right) \sum_i \sigma_{xi} \tag{9}
\]

with \( \omega_{c0} = \left( \frac{L_{sq} + L_r}{C_r} \right)^{-1/2} \). This expression includes the coupling between the qubits and the resonator in the form of \( H_{int} \) with

\[
g = \omega_o \left( \frac{L_{sq}}{L_{sq} + L_r} \right)^2 \left( \frac{\pi \mu_0 R_0 I_{q2}}{2 \sqrt{2} \Phi_0} \right) \tag{10}
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\[
B_{z_1}/B_{y_1} = C_1/C_0 < 1/2 \] In our system, \( C_1/C_0 \) can be readily designed to satisfy this criterion.

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\[
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\]

with \( \omega_{c0} = \left( \frac{L_{sq} + L_r}{C_r} \right)^{-1/2} \). This expression includes the coupling between the qubits and the resonator in the form of \( H_{int} \) with

\[
g = \omega_o \left( \frac{L_{sq}}{L_{sq} + L_r} \right)^2 \left( \frac{\pi \mu_0 R_0 I_{q2}}{2 \sqrt{2} \Phi_0} \right) \tag{10}
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Decoherence plays an essential role in observing the nonlinear effect. In the above, we assume that the resonator is in the bad cavity limit where the damping of the resonator is stronger than the coupling constant and the decoherence of the qubits. This can be achieved either by using a noisy resonator or by connecting the resonator to a dissipative element which is shown as the resistor $R_\text{r}$ in Fig. 3. The decoherence of the qubits will relax the quantum many-body system to its ground state at the effective transverse field. Given the sub-megahertz decoherence rates that are recently measured in the superconducting qubits, the nonlinear effect can be studied in the above circuit with realistic parameters. Finite temperature may play a role in this system as well when the temperature $T$ becomes much smaller than the ferromagnetic coupling, where a low-lying excited state is nearly degenerate with the ground state. The final state can then be a mixture of the ground state and the excited state at a temperature of $T = 20 \text{ mK} \ (k_B T / \hbar = 2 \pi \times 0.4 \text{ GHz})$. In addition, the detection of the phase switchings in the qubit array can be achieved by measuring the photon number in the resonator. The jump of the photon number is directly associated with the phase switching when the driving is in the bistable regime.

To conclude, we studied the nonlinear effect in the quantum many-body state of a superconducting qubit array coupling to a superconducting resonator. We showed that sudden phase switching between the paramagnetic phase and the ferromagnetic phase can occur in the qubit array when the driving on the resonator is varied due to the strong back-action of the resonator on the qubits. The phase switching results in a finite energy jump in the qubit array, in analogy to a first order phase transition. These results showed that novel phenomena can be observed when a global resonator cavity is connected to a quantum many-body system. Many interesting effects await to be discovered in such systems.

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1. A. Blais et al., Phys. Rev. A 69, 062320 (2004); J. Q. You and F. Nori, Phys. Rev. B 68, 064509 (2003); F. Marquardt and C. Bruder, Phys. Rev. B 63, 054514 (2001).
2. A. Wallraff et al., Nature 431, 162 (2004); I. Chiorescu et al., Nature 431, 159 (2004); J. Johansson et al., Phys. Rev. Lett. 96, 127006 (2006).
3. D. I. Schuster et al., Nature 445, 515 (2007); A. Fragner et al., Science 322, 1357 (2008); J. M. Fink et al., Nature 454, 315 (2008); L. S. Bishop et al., Nat. Phys. 5, 105 (2009); O. Astafiev et al., Nature 449, 588 (2007); F. Deppe et al., Nat. Phys. 4, 686 (2008).
4. J. M. Fink et al., Phys. Rev. Lett. 103, 083601 (2009).
5. J. Clarke and F. K. Wilhelm, Nature 453, 1031 (2008); Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001); J. Q. You and F. Nori, Phys. Today 58, 42 (2005).
6. M. A. Sillanpää, J. I. Park, and R. W. Simmonds, Nature 449, 438 (2007); J. Majer et al., Nature 449, 443 (2007); M. Neeley et al., Nat. Phys. 4, 523 (2008); M. Hofheinz et al., Nature 459, 546 (2009).
7. L. Tian and R. W. Simmonds, Phys. Rev. Lett. 99, 137002 (2007); L. Tian and K. Jacobs, Phys. Rev. B 79, 144503 (2009).
8. M. J. Hartmann, F. G. S. L. Brandao, and M. B. Plenio, Nat. Phys. 2, 849 (2006); A. D. Greentree, C. Tahan, J. H. Cole, and L. C. L. Hollenberg, Nat. Phys. 2, 856 (2006); D. G. Angelakis, M. F. Santos, and S. Bose, Phys. Rev. A 76, 031805(R) (2007); N. Na, S. Utsumi, L. Tian, and Y. Yamamoto, Phys. Rev A 77, 031803(R) (2008).
9. N. Lambert, Y. N. Chen, R. Johansson, and F. Nori, Phys. Rev. B 80, 165308 (2009); P. Nataf and C. Ciuti, Phys. Rev. Lett. 104, 023601 (2010); G. Chen, Z. Chen, and J. Liang, Phys. Rev. A 76, 055803 (2007).
10. J.-M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. 73, 565 (2001); H. Mabuchi and A. Doherty, Science 298, 1372 (2002).
11. I. B. Mekhov, C. Maschler, and H. Ritsch, Nat. Phys. 3, 319 (2007); J. Larson, B. Danski, G. Morigi, and M. Lewenstein, Phys. Rev. Lett. 100, 050401 (2008); W. Chen et al., Phys. Rev. A 80, 011801(R) (2009); J.M. Zhang, F. C. Cui, D. L. Zhou, and W. M. Liu, Phys. Rev. A 79, 033401 (2009).
12. F. Brennecke et al., Nature 450, 268 (2007); K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, Nat. Phys. 4, 561 (2008).
13. R. Fazio and H. van der Zant, Phys. Rep. 355, 235 (2001); and references there in.
14. J. J. Garcia-Ripoll, E. Solano, and M. A. Martin-Delgado, Phys. Rev. B 77, 024522 (2008).
15. G. C. Garrison and R. Y. Chiao, Quantum Optics, Oxford University Press (2008); D. F. Walls and G. J. Milburn, Quantum Optics, 2nd ed., Springer-Verlag (2008).
16. R. Bonifacio and L. A. Lugiato, Phys. Rev. Lett. 40, 1023 (1978); R. Bonifacio and L. A. Lugiato, Phys. Rev. A 18, 1129 (1978).
17. I. Siddiqi et al., Phys. Rev. Lett. 93, 207002 (2004); I. Siddiqi et al., Phys. Rev. Lett. 94, 027005 (2005).
18. Y.-D. Wang, F. Xue, Z. Song, and C.-P. Sun, Phys Rev. B 76, 174519 (2007).
19. S. Sachdev, Quantum Phase Transitions, Cambridge University Press (1999).
20. P. G. Drazin, Nonlinear Systems, Cambridge University Press (1992).
21. P. R. Colares Guimaraes, J. A. Plascak, F. C. Sa Barreto, and J. Florescio, Phys Rev. B 66, 064413 (2002).
22. D. Vion et al., Science 296, 886 (2002).
23. L. S. Levitov, T. P. Orlando, J. B. Majer, and J. E. Mooij, cond-mat/0108266.
24. J. E. Mooij et al., Science 285, 1036 (1999); T. P. Orlando et al., Phys. Rev. B 60, 15398 (1999).
25. J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. 89, 117901 (2002); M. Steffen et al., Phys. Rev. Lett. 97, 050502 (2006).