Statistical control of forming process

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Abstract. A qualitative product must be obtained through a stable and repeatable process. In order to control a defining characteristic of the quality of a specific product, various specific methodologies have been developed in time. One of the most known methods of this kind is the statistical control of the process which provides users with a powerful and useful range of tools designed to ensure process stability and improve capacity by reducing variability. These tools have the following great advantages: they are simple to use; are effective and can be applied to all processes. This paper presents a software application designed to conduct a forming process by the statistical control method. The program analyzes a database of measured values of the traceability characteristics and identifies whether these values have a statistically normal distribution. In the affirmative case, the main statistical indicators of the considered sample are calculated: maximum and minimum values, amplitude, average value, median, dispersion, process control limits and process capability index. These statistics are displayed both numerically and graphically. By using the software, it is possible to track the process and, if exists the tendency to overcome the control limits, to change the process parameters that caused the change in the quality characteristics.

1. Introduction
It is knowing that a qualitative product must be obtained through a stable and repeatable process. If this condition is fulfilled, the process is able to assure values of product characteristics which vary very little around the nominal values.

In order to check a defining characteristic of a product, a suite of specific methods was developed [1-4].

One of these methods is represented by the process statistical process control, which provides user a suite of powerful and useful tools, assuring the process stability and improving its capabilities by reducing the variability of the tracked quality characteristic [2, 3].

These tools have some advantages as: easy to use; efficiency and generality in application. Generally, it is considered that a technological process is controllable from statistic point of view if the values of tracked characteristics have a normal distribution, process take place only under influence of random causes, the quality characteristic values deviation is constant over time and the average value of samples are roughly constant over time [2].

In this paper is proposed applying statistical process control to a cold forming process.
In case of cold forming, is expected that the product dimensional characteristic variation has a larger spreading that in case of others manufacturing processes due to the material’s anisotropy increased effect, manifesting in particular on elastic spring back.

However, can be assumed that, in this case too, the tracked dimensional characteristics comply the same criteria which determine whether the process is statistically controllable.

2. Forming process control
A forming process for strip blanks was monitored. From these strips by bending were obtained the pieces presented in figure 1.

![Figure 1. Pieces obtained by forming.](image)

The manufacturing was performed on a die with forms and dimensions presented in figure 2.

![Figure 2. Die used for forming:](image)

(a) Photo of the die.  
(b) Dimensions of the die.

The blank’s material is OL 42, with thickness of 0.8 mm and the pressure was 6 MPa. The traced quality characteristic is the angle marked onto the figure 3.

![Figure 3. Quality characteristic.](image)
The measurement of this dimension was made onto a Sinow on 3D video measuring machine. Fifteen samples were measures, each sample with unique observation. Each sample was measured for both sides and the average value of the angle was taken into consideration.

3. The off-line control software description
The measured data were registered into a “.csv” file. This file can be analysed with in-house software made for this and programmed in Java language. The graphic user interface of this program is presented in figure 4.

![Graphic user interface of the software.](image)

The button 1 allows selecting the file where the numerical data are. Once this file open, pressing the button 2 it is made the statistical processing of the data.

3.1. Checking for existence of aberrant values
After loading the data and the calculus starting, first of all, the existence of aberrant values is checked. This is made using the Chauvenet test [1].

According this test, the $X_i$ value is regarded as aberrant if it is accomplished the Chauvenett criterion, equation (1):

$$|X_i - \bar{X}| > z \cdot \sigma$$

where $\sigma$ is the standard deviation, see equation (2)

$$\sigma = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \bar{X})^2}$$

$X_i$ being the value with index $i$.

In equation (1), the $z$ value is given by:

$$z = \frac{0.435 - 0.862 \cdot a}{1 - 3.604 \cdot a + 3.213 \cdot a^2}; \quad a = \frac{2 \cdot n - 1}{4 \cdot n}.$$
The software displays possible aberrant values and the user can choose to delete these values, using the buttons 3 and 4.

After calculating and eventually eliminating the aberrant values, the statistical parameters are displayed: number of values; maximum and minimum value; average; the median and the module.

3.2. The checking of normal distribution
Pressing the 5 button is possible to check if the values have normal distribution. This check is made based on the $\chi^2$ test.

This test assumes to follow the next algorithm [1-3, 5]:

3.2.1. Sorting values in ascending order. The string of values is sort in ascending order and is divided in $k$ class, $k$ given by equation (4):

$$k = 1 + 3.322 \cdot \ln n$$  

(4)

The range of each class is calculated with equation (5):

$$a = \frac{R}{k}$$  

(5)

$a$ being the range of class, $k$ number of classes and $R$ the range of the values string ($R = X_{\text{max}} - X_{\text{min}}$).

3.2.2. Determining the upper limits for each class. The upper limits of each class are determined with equation (6):

$$X_{\text{max} \_ k} = X_{\text{max} \_ k-1} + a$$  

(6)

3.2.3. Determining the frequency in each class. The number of data values in each class is determined. If in a class are less than five values, this class merges with the next class. The merging stage is resumed until in each class are more than five values.

3.2.4. Calculating $t_i$. For each class the $t_i$ values is calculated using the equation (7):

$$t_i = \frac{X_i - \bar{X}}{\sigma}, \quad i = 1 + \nu,$$  

(7)

$\nu$ is the freedom degree of the data string, $\nu = k' - 1$, $k'$ is the number of new classes after merging.

3.2.5. The calculus of probability intervals. For each class is calculated the probability interval using equations (8) and (9):

$$p_i = \Phi(t_i) - \Phi(t_{i-1}), \quad i = 1 + (\nu + 1)$$  

(8)

$$\Phi(t) = \frac{1}{2} - \frac{e^{-\frac{t^2}{2}}}{\sqrt{2 \cdot \pi}} \left(0.4362 \cdot a - 0.1202 \cdot a^2 + 0.9373 \cdot a^3\right), \quad a = \frac{1}{1 + 0.3326 \cdot t}.$$  

(9)
3.2.6. Calculating the $\chi^2$ value, using equation (10):

$$\chi^2 = \sum_{i=1}^{v+1} \left( \frac{n_i - n \cdot p_i}{n \cdot p_i} \right)^2,$$

(10)

where $n_i$ is the number of values in class $i$.

3.2.7. Calculating $\chi^2_{cr}$, equation (11):

$$\chi^2_{cr} = a + b \cdot v + c \cdot v^2 + d \cdot v^3,$$

(11)

$$a = \frac{0.2046 - 0.2032 \cdot \alpha}{1 - 1.9562 \cdot \alpha + 0.9563 \cdot \alpha^2}; \quad b = \frac{0.685 - 0.6819 \cdot \alpha}{1 - 1.6662 \cdot \alpha + 0.6673 \cdot \alpha^2};$$

(12)

$$c = \frac{-0.1507 + 0.1508 \cdot \alpha}{10 - 17.1239 \cdot \alpha + 7.122 \cdot \alpha^2}; \quad d = \frac{0.07773 - 0.07928 \cdot \alpha}{100 - 165.0049 \cdot \alpha + 641821 \cdot \alpha^2};$$

In equations (11) and (12) $\alpha$ is the confidence level. For this case, the confidence level was considered 99.5%.

3.2.8. Applying the $\chi^2$ test. The test check if the inequality (13):

$$\chi^2 > \chi^2_{cr},$$

(13)

is accomplished. If yes, the spreading of values is regarded as randomly.

3.3. Check the process static stability.

The process static stability is performed in order to determine if the process take place only under random causes.

For checking the total number of iterations is used [2].

The ordered values string splits in values of type $a$ or $b$ regarding the magnitude of each value compared with the string median value. The median is calculated with equation (14) depending if the value number is even or odd:

$$m_e = \frac{X_{n/2} + X_{n+1/2}}{2}; \quad n = 2 \cdot k; \quad m_e = X_{n/2}; \quad n = 2 \cdot k + 1.$$

(14)

If a value is bigger than the median, is considered as a type value, if is smaller is considered as $b$ type value.

The transition from an $a$ value to a $b$ value and inverse constitutes an iteration.

The iterations which emerge in the initial string of values (before sorting in ascending order) are counted and, this number is compared with the $R_a$ reference value given by equation (15).

$$R_a = \frac{1}{2} \left( n + 1 - z_a \cdot \sqrt{n - 1} \right); \quad a = 5\% = 0.05; \quad \alpha = 1 - 2 \cdot \Phi(z_a),$$

(15)

$\Phi(z_a) = (1 - a)/2$, is the frequency function.

For the calculated value of the frequency function, from literature, [3], is considered the $z_a$ value, equation (16):
4. Tracing of control charts

Among the statistical tools for process control, the control charts are especially useful, being designed to notice if the process trend to become out-control.

In this control software as control chart was choosing the Shewhart chart [4].

In order to trace the Shewhart chart, in manufacturing time \( n \) samples from the analysed part are random collected. Each sample contains \( m \) specimens.

The average value of each sample is calculated with equation (17):

\[
\bar{X}_i = \frac{\sum_{j=1}^{m} X_j}{m}
\]

the sample range, equation (18):

\[
R_i = X_{i,\text{max}} - X_{i,\text{min}}
\]

and the deviation of values in each sample, equation (19):

\[
s_i = \sqrt{\frac{1}{m-1} \cdot \sum_{j=1}^{m} (X_{ij} - \bar{X}_i)^2}
\]

Further, the average of the string’s values is calculated as so as the average deviation, see equation (20):

\[
\bar{X} = \frac{\sum_{i=1}^{n} \bar{X}_i}{n}, \quad \bar{s} = \frac{\sum_{i=1}^{n} s_i}{n}.
\]

The process performances can be assessed comparing the analysed characteristic values with the control limits calculated based on established formula [4].

- upper control limit for \( X \) value:

\[
LCSX = \bar{X} + \frac{Z}{d_3(m) \cdot \sqrt{m}} \cdot \bar{s};
\]

- lower control limit for \( X \) value:

\[
LCIX = \bar{X} - \frac{Z}{d_3(m) \cdot \sqrt{m}} \cdot \bar{s}.
\]

In equations (21) and (22), \( Z \) and \( d_3(m) \), are calculated with [4]:
The process is in-control if $\overline{X}_j$ is between the LCSX and LCIX limits.

The process variability can be assessed using as guide the $s_i$ deviation in each of analysed sample. If the $s_i$ values are between the control limits given by equation (24):

$$LCSS = \overline{s} + Z \cdot \frac{1-d_3^2(m)}{d_3(m)} \cdot \overline{s}, \quad LCIS = \overline{s} - Z \cdot \frac{1-d_3^2(m)}{d_3(m)} \cdot \overline{s};$$

then, the process is in-control.

**NOTE**: Although the software was designed in this way that the values string can be analysed for subgroups with more than one specimen, the default option refers to $m=1$. This was considered as optimal taking into consideration the reduced number of specimens (15 exemplary) and the fact that the time between two specimens manufacturing is short.

In this case, the $\overline{X}, R$ charts are applied for monitoring individual observation. For individual observation ($m=1$), the $MR_i$ values are calculated using equation (25), ($MR$ is the range between two consecutive values):

$$MR_i = X_{i+1} - X_i; \quad i = 1 \div n-1; \quad \overline{MR} = \frac{\sum_{i=1}^{n-1} MR_i}{n-1}. \quad (25)$$

In this case, the control limits will be:

$$LCSX = \overline{X} + 2.33 \cdot \overline{MR}; \quad LCIX = \overline{X} - 2.33 \cdot \overline{MR}; \quad LCSS = 3.27 \cdot \overline{MR}; \quad LCIS=0. \quad (26)$$

**5. Results**

Measuring the 15 manufactured specimens, the following results were obtained: maximum value 143.785°; minimum value 139.637°; average 141.969°; median 142.115°; range 4.148°; standard deviation 1.181°. For the considered string, the module cannot be calculated due to the fact that each value emerges once. The Shewhart charts are presented in figures 5 and 6.

The control limits calculated according to the equations (21), (22) and (26) results: $LCSX=145.153°$; $LCIX=138.786°$; $LCSS=4.468°$; $LCIS=0°$.

![Figure 5. Chart R.](image-url)
6. Conclusions

According [4] the process behaviour can be analysed using a set of rules for charts interpretation. Western Electric Handbook (1956) [4] suggests some aspects which have to be traced for detecting non-random patterns in control charts:
1. One or more points outside of the control limits.
2. Four or five consecutive points beyond the 1 sigma limits.
3. A run of eight consecutive points on one side of the centre line.
4. Six points in a row steadily increasing or decreasing.
5. Fifteen points in a row in 1 sigma zone.
6. Fourteen points in a row alternating up and down.
7. Eight points in a row on both sides of the centre line with none in 1 sigma zone.
8. An unusual or non-random pattern in the data.

According to the chart presented in figure 5 is obviously that none of these aspects does occur in the measured values string.

Analysing the results, we can state that the process is in-control and is possible to control it from the statistic point of view.

7. References

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Acknowledgments

This work was supported by a grant of the Romanian National Authority for Scientific Research and Innovation, CNCS-UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0446 - TFI PMAIAA, within PNCDI III.