In-medium Properties of the Low-lying Bottom Baryons in the Quark-Meson Coupling Model

Kazuo TSUSHIMA

1Laboratório de Física Teórica e Computacional-LFTC, Universidade Cruzeiro do Sul, 01506-000, São Paulo, SP, Brazil
E-mail: kazuo.tsushima@gmail.com

(Received January 15, 2019)

In-medium properties of the low-lying baryons are studied in the quark-meson coupling (QMC) model, focusing on the \( \Sigma \) and \( \Xi \) baryons. It is predicted that the Lorentz-scalar effective mass of \( \Sigma \) becomes smaller than that of \( \Xi \) at moderate nuclear matter density, and as the density increases, namely, \( m_{\Sigma} < m_{\Xi} \). We also study the effects of the repulsive Lorentz-vector potentials on the excitation energies of these bottom baryons.

KEYWORDS: bottom baryons, in-medium properties, quark-meson coupling model

1. Introduction

Studies of in-medium baryon properties, especially for the baryons which contain charm and/or bottom quarks, are very interesting, due to the emergence of heavy-quark symmetry. In particular, in-medium properties of heavy baryons which contain at least one light \( u \) or \( d \) quark, can provide us with important information on the dynamical chiral symmetry, and the roles of light quarks in partial restoration of chiral symmetry. To study the in-medium properties of heavy baryons, we use the quark-meson coupling (QMC) model [1], a quark-based model of nuclear matter, finite nuclei, and hadron properties in a nuclear medium [2, 3]. This report is based on the recent article [4]. (See Refs. [1–3] for details on the QMC model, and its successful applications.)

2. Finite (hyper)nucleus in the QMC model

Before discussing the in-medium baryon properties, we start with the case of finite (hyper)nucleus. Using the Born-Oppenheimer approximation, a relativistic Lagrangian density which gives the same mean-field equations of motion for a nucleus or a hypernucleus at the hadron level, may be given by the QMC model [4, 5]:

\[
\mathcal{L}_{\text{QMC}} = \mathcal{L}_{\text{QMC}}^N + \mathcal{L}_{\text{QMC}}^Y,
\]

\[
\mathcal{L}_{\text{QMC}}^N \equiv \bar{\psi}_N(\vec{r}) \left[ i \gamma \cdot \partial - m_N^e(\sigma(\vec{r})) - (g_{\sigma}\omega(\vec{r}) + g_{\rho}\frac{\tau^N_3}{2} b(\vec{r}) + \frac{\epsilon}{2}(1 + \tau^N_3)A(\vec{r})\gamma_0) \right] \psi_N(\vec{r}) - \frac{1}{2}[(\nabla \sigma(\vec{r}))^2 + m_N^e \sigma(\vec{r})^2] + \frac{1}{2}[(\nabla \omega(\vec{r}))^2 + m_N^e \omega(\vec{r})^2] + \frac{1}{2}[(\nabla b(\vec{r}))^2 + m_N^e b(\vec{r})^2] + \frac{1}{2}(\nabla A(\vec{r}))^2, \quad (1)
\]

\[
\mathcal{L}_{\text{QMC}}^Y \equiv \bar{\psi}_Y(\vec{r}) \left[ i \gamma \cdot \partial - m_Y^e(\sigma(\vec{r})) - (g_{\sigma}\omega(\vec{r}) + g_{\rho}^Y \frac{\tau^Y_3}{2} b(\vec{r}) + eQA(\vec{r})\gamma_0) \right] \psi_Y(\vec{r}), \quad (Y = \Lambda, \Sigma^0, \Xi^0, \Sigma^+, \Xi^+, \Sigma_c^{0,+,++}, \Xi_c^0, \Lambda_b, \Sigma_b^{0,+,+} \Xi_b^0, \Sigma_b^{0,+,+}). \quad (2)
\]

In the above \( \psi_N(\vec{r}) \) and \( \psi_Y(\vec{r}) \) are, respectively, the nucleon (N) and hyperon (Y) fields. The mean-meson fields represented by, \( \sigma, \omega \) and \( b \) which directly couple to the light quarks self-consistently,
are the Lorentz-scalar-isoscalar, Lorentz-vector-isoscalar and third component of Lorentz-vector-isovector fields, respectively, while \( A \) is the Coulomb field.

In an approximation that the \( \sigma \)-, \( \omega \)- and \( \rho \)-mean fields couple only to the \( u \) and \( d \) light quarks, the coupling constants for the hyperon appearing in Eq. (3) are obtained/identified as \( g^Y_\omega = (n_q/3)g_\omega \), and \( g^Y_\rho = g_\rho \), with \( n_q \) being the total number of valence light quarks in the hyperon \( Y \), where \( g_\omega \) and \( g_\rho \) are the \( \omega \)-\( N \) and \( \rho \)-\( N \) coupling constants. \( I^Y_3 \) and \( Q_Y \) are the third component of the hyperon isospin operator and its electric charge in units of the proton charge, \( e \), respectively. The approximation used in the QMC model, that the meson fields couple only to the light quarks, reflects the fact that the magnitudes of the light-quark condensates decrease faster than those of the strange and heavier quarks as the nuclear density increases. This is associated with partial restoration of chiral symmetry in a nuclear medium (dynamically symmetry breaking and its partial restoration).

The \( \sigma \)-field dependent \( \sigma \)-\( N \) \([g_\sigma(\sigma(\bar{\phi}))\] and \( \sigma \)-\( Y \) \([g^Y_\sigma(\sigma(\bar{\phi}))\) coupling strengths, associated with \( m^*_N(\sigma(\bar{\phi})) \) in Eq. (2) and \( m^*_Y(\sigma(\bar{\phi})) \) in Eq. (3), respectively, are defined by

\[
\begin{align*}
 m^*_N(\sigma(\bar{\phi})) &\equiv m_N - g_\sigma(\sigma(\bar{\phi})) \sigma(\bar{\phi}), \\
 m^*_Y(\sigma(\bar{\phi})) &\equiv m_Y - g^Y_\sigma(\sigma(\bar{\phi})) \sigma(\bar{\phi}), \quad (Y = \Lambda, \Sigma, \Xi, \Sigma^*, \Sigma, \Sigma^*, \Lambda, \Sigma, \Xi, \Lambda^*, \Xi^*).
\end{align*}
\]

where \( m_N \) (\( m_Y \)) is the free nucleon (hyperon) mass. The \( \sigma \)-dependence of these coupling strengths must be calculated self-consistently within the quark model [1, 2].

3. Baryon properties in symmetric nuclear matter

We consider the rest frame of symmetric nuclear matter, a spin and isospin saturated system with only strong interaction (Coulomb force is dropped) based on Eq. (1). Within the Hartree mean-field approximation, the nuclear (baryon) \( \rho_B \), and scalar \( \rho_s \), densities are, respectively, given by,

\[
\rho_B = \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |\vec{k}|) = \frac{2k^3_F}{3\pi^2}, \quad \rho_s = \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |\vec{k}|) \frac{m^*_N(\sigma)}{\sqrt{m^2_N(\sigma) + k^2}}.
\]

Here, \( m^*_N(\sigma) \) is the value (constant) of the Lorentz-scalar effective nucleon mass at a given nuclear density, and \( k_F \) the Fermi momentum.

The Dirac equations in the standard QMC model (modeled by the MIT bag) for the quarks and antiquarks in nuclear matter, in a bag of a hadron, \( h \) \((q = u \text{ or } d \text{ and } Q = s, c \text{ or } b \) hereafter\) neglecting the Coulomb force, are given by \([x = (t, \vec{x}) \text{ and for } |\vec{x}| \leq \text{bag radius}] \) [2],

\[
\begin{align*}
[i\gamma \cdot \partial_x - (m_q - V^p_{\sigma})] \psi_q(x) &= \gamma^0 \left(V^q_\omega + \frac{1}{2}V^q_\rho\right) \psi_q(x) = 0, \\
[i\gamma \cdot \partial_x - (m_q - V^p_{\sigma})] \psi_q(x) &= \gamma^0 \left(V^q_\omega - \frac{1}{2}V^q_\rho\right) \psi_q(x) = 0, \\
[i\gamma \cdot \partial_x - m_Q] \psi_Q(x) &= 0, \quad [i\gamma \cdot \partial_x - m_Q] \psi_Q^c(x) = 0,
\end{align*}
\]

where, (the constant) mean fields for a bag are defined by \( V^p_{\sigma} \equiv g^p_\sigma \sigma, V^q_\omega \equiv g^q_\omega \omega, \text{ and } V^p_\rho \equiv g^p_\rho b \), with \( g^p_\sigma, g^q_\omega \text{ and } g^p_\rho \) being the corresponding quark-meson coupling constants. The mass of the hadron \( h \) in a nuclear medium, \( m^*_h \), is calculated by [1, 2]

\[
m^*_h = \sum_{j=q,\bar{q},Q,\bar{Q}} \frac{n_j \Omega^j_q - z_h}{R^j_h} + \frac{4\pi R^3_h B_p}{3} \frac{\partial m^*_h}{\partial R_h} \bigg|_{R_h = R^*_h} = 0,
\]

where \( \Omega^j_q = \Omega^j_q = \left[x^2_q + (R^j_h m^*_q)^2\right]^{1/2} \), with \( m^*_q = m_q + g^q_\omega \omega = m_q - V^p_{\sigma} \), and \( \Omega^*_Q = \Omega^*_Q = \left[x^2_Q + (R^*_h m_Q)^2\right]^{1/2} \), and \( x_Q,\bar{Q} \) are the lowest mode bag eigenvalues.
In Table I we present the model inputs in vacuum, the quantities calculated in vacuum and at normal nuclear matter density, $\rho_0 = 0.15 \text{ fm}^{-3}$.

Table I. The parameters related with the zero-point energy $z_B$; baryon masses and the bag radii in free space [at normal nuclear matter density, $\rho_0 = 0.15 \text{ fm}^{-3}$] $m_B$(MeV), $R_B$(fm) [$m_B^*$, $R_B^*$]; and the lowest mode bag eigenvalues $x_1, x_2, x_3 [x_1^*, x_2^*, x_3^*]$ of baryon $B(q_1, q_2, q_3)$ with the corresponding valence quarks $q_1, q_2, q_3$ in the baryon $B$, where $z_B$'s are kept the same as those in vacuum, i.e., density independent. Free space mass values $m_B$ for the heavy baryons from Ref. [6], those for the strange hyperons from Ref. [2], and the nucleon bag radius $R_B = 0.8$ fm (and $m_q = 5$ MeV), are inputs. The light quarks are indicated by $q = u$ or $d$. Note that the baryons containing at least one light quark $q$ are modified in the medium in the QMC model, but $\Omega, \Omega_c$, and $\Omega_b$ are not modified in the QMC model. We recall that some inputs are updated from those in Refs. [2, 3] based on the data [6]. For the recent data for $\Sigma_b$, see Ref. [7], which gives the averaged mass of $m_{\Sigma_b} = 5813.1$ MeV, to be consistent with the value extracted from Ref. [6]. The entry with “NA” stands for “not applicable”.

| $B(q_1, q_2, q_3)$ | $z_B$ | $m_B$ | $R_B$ | $x_1$ | $x_2$ | $x_3$ | $m_{\Sigma}$ | $R_{\Sigma}$ | $x_{\Sigma}^*$ | $x_{\Sigma}^*$ | $x_{\Sigma}^*$ |
|---------------------|------|------|-------|-------|-------|-------|----------------|----------|-------------|-------------|-------------|
| $N(qqq)$            | 3.295 | 939.0 | 0.800 | 2.052 | 2.052 | 2.052 | 754.5 | 0.786 | 1.724 | 1.724 | 1.724 |
| $\Lambda(udss)$    | 3.131 | 1115.7 | 0.806 | 2.053 | 2.053 | 2.402 | 992.7 | 0.803 | 1.716 | 1.716 | 2.401 |
| $\Sigma(qqq)$      | 2.810 | 1193.1 | 0.827 | 2.053 | 2.053 | 2.409 | 1070.4 | 0.824 | 1.705 | 1.705 | 2.408 |
| $\Xi(qss)$         | 2.860 | 1318.1 | 0.820 | 2.053 | 2.053 | 2.406 | 1256.7 | 0.818 | 1.708 | 2.406 | 2.406 |
| $\Omega_{qss}$     | 1.930 | 1672.5 | 0.869 | 2.422 | 2.422 | 2.422 | NA   | NA   | NA   | NA   | NA   |
| $\Lambda_b(udb)$   | 1.642 | 2286.5 | 0.854 | 2.053 | 2.053 | 2.879 | 2164.2 | 0.851 | 1.691 | 1.691 | 2.878 |
| $\Sigma_b(qqb)$    | -0.622 | 5619.6 | 0.930 | 2.054 | 2.054 | 3.063 | 5498.5 | 0.927 | 1.651 | 1.651 | 3.063 |
| $\Xi_b(qsb)$       | -1.554 | 5813.4 | 0.968 | 2.054 | 2.054 | 3.066 | 5692.8 | 0.966 | 1.630 | 1.630 | 3.066 |
| $\Omega_b(ssb)$    | -0.785 | 5793.2 | 0.933 | 2.054 | 2.441 | 3.063 | 5732.7 | 0.931 | 1.649 | 2.440 | 3.063 |
| $\Omega_b(ssb)$    | -1.327 | 6046.1 | 0.951 | 2.446 | 2.446 | 3.065 | NA   | NA   | NA   | NA   | NA   |

We focus on the in-medium properties of $\Sigma_b$ and $\Xi_b$ baryons. In Fig. 1 we show the Lorentz-scalar effective masses, and excitation energies (Lorentz-scalar effective masses plus vector potentials) for two cases of vector potentials.

![Fig. 1. Lorentz-scalar effective masses ($m^*$), and excitation energies for two cases of vector potentials, with (left panel, denoted by $m^* + V_Q^V$) and without (right panel, denoted by $m^* + V_Q^V_{QMC}$) the phenomenologically introduced Pauli (vector) potentials [5] based on Pauli Principle at the quark level.](image-url)

First, one can notice in Fig. 1 that the Lorentz-scalar $\Sigma_b$ effective mass becomes smaller than...
that of $\Xi_b$, namely, $m_{\Sigma_b}^* < m_{\Xi_b}^*$, at baryon density range larger than about $0.3\rho_0$, although in vacuum, $m_{\Sigma_b} > m_{\Xi_b}$ (see Table I). This is one of the highlighted predictions in the present study.

Next, we discuss the excitation energy, which is the sum of the Lorentz-scalar effective masses plus Lorentz-vector potential. The left (right) panel is the case with (without) the phenomenologically introduced Pauli (vector) potentials based on the Pauli Principle at the quark level [5]. In the results shown in Fig. 1, the realistic case should be without the Pauli potentials (right panel) for the present case of bottom baryons. Interestingly, for the realistic case without the Pauli potentials, the excitation energies for $\Sigma_b$ and $\Xi_b$ are nearly degenerate in the nuclear matter density range $0.5\rho_0 < \rho_B < 1.5\rho_0$. Namely, $\Sigma_b$ and $\Xi_b$ can be produced at rest with nearly the same cost of energy. This may imply the emergence of many interesting phenomena, for example, in heavy ion collisions and reactions in the systems of dense nuclear medium, such as in the deep core of a neutron (compact) star.

The results shown in Fig. 1 also suggest that the two different types of the vector potentials can possibly be distinguished, and give important information on the dynamical symmetry breaking and partial restoration of chiral symmetry. For proving these suggestions, one needs to seek what kind of experiments can be made to get a clue. It might be very interesting to measure the valence quark (parton) distributions of $\Sigma_b$ and $\Xi_b$ in medium, since the supports of the parton distributions of these baryons reflect their excitation energies. Another possibility may be to measure the strangeness-changing semileptonic weak decay of $\Xi_b \rightarrow \Sigma_b$ in a medium, which reflects their excitation energy difference in a medium.

4. Summary and conclusion

We predict that the Lorentz-scalar effective mass of $\Sigma_b$ becomes smaller than that of $\Xi_b$ in the nuclear matter density range larger than $\approx 0.3\rho_0$ ($\rho_0 = 0.15$ fm$^{-3}$), while in vacuum the mass of $\Sigma_b$ is larger than that of $\Xi_b$.

We have also studied the effects of two different repulsive Lorentz-vector potentials on the excitation energies of $\Sigma_b$ and $\Xi_b$ baryons. In the case without the phenomenologically introduced Pauli potentials, which is expected to be more realistic, the excitation energies for $\Sigma_b$ and $\Xi_b$ are predicted to be nearly degenerate in the nuclear matter density range about $[0.3\rho_0, 1.5\rho_0]$. Thus, the production of $\Sigma_b$ and $\Xi_b$ cost nearly the same energy at rest in this density range.

To make possible connections of the findings for the Lorentz-scalar effective masses and/or excitation energies of $\Sigma_b$ and $\Xi_b$ baryons with experimental observables, one needs to seek relevant experimental methods and situations. It might be very interesting to measure the valence quark (parton) distributions of $\Sigma_b$ and $\Xi_b$ in medium, since the supports of the parton distributions of these baryons reflect their excitation energies. Another possibility may be to measure the strangeness-changing semileptonic weak decay of $\Xi_b \rightarrow \Sigma_b$ in a medium, which reflects their excitation energy difference in a medium.

To conclude, studies of the $\Sigma_b$ and $\Xi_b$ properties in a nuclear medium, can provide us with very interesting and important information on the dynamical symmetry and partial restoration of chiral symmetry, as well as the roles of the light quarks in a medium.

References

[1] P. A. M. Guichon, Phys. Lett. B 200 (1988) 235.
[2] For a review, K. Saito, K. Tsushima and A. W. Thomas, Prog. Part. Nucl. Phys. 58, 1 (2007).
[3] For a review, G. Krein, A. W. Thomas and K. Tsushima, Prog. Part. Nucl. Phys. 100, 161 (2018).
[4] K. Tsushima, Phys. Rev. D 99, 014026 (2019).
[5] K. Tsushima, K. Saito, J. Haidenbauer and A. W. Thomas, Nucl. Phys. A 630, 691 (1998).
[6] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.
[7] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122, 012001 (2019).