Smallness of Leptonic $\theta_{13}$
and Discrete Symmetry

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Abstract

The leptonic mixing angle $\theta_{13}$ is known to be small. If it is indeed tiny, the simplest explanation is that charged leptons mix only in the $\mu - \tau$ sector and neutrinos only in the 1-2 sector. We show that this pattern may be explained by the discrete symmetry $Z_2 \times Z_2$ of a complete Lagrangian, which has 2 Higgs doublets and 2 Higgs triplets (or 2 heavy right-handed singlet neutrinos). In the case of Higgs triplets, the Majorana neutrino masses are arbitrary, whereas in the case of heavy singlet neutrinos, an inverted hierarchy is predicted. Lepton-Flavor-Violation effects, present only in the $\mu - \tau$ sector, are analyzed in detail: the LFV $\tau$-decay rates are predicted below the present bounds by a few orders of magnitude, whereas LFV Higgs decays could allow for a direct test of the model.

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Recent experimental advances in measuring the neutrino oscillation parameters in atmospheric and solar data \[1\] have now fixed the $3 \times 3$ lepton mixing matrix $U$ to a large extent. Assuming that the neutrino mass matrix $M_\nu$ is Majorana and it is written in the basis where the charged-lepton mass matrix is diagonal, then for

$$U^T M_\nu U = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad (1)$$

with the convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2)$$

present data imply that $\theta_{23}$ is close to $\pi/4$, $\theta_{12}$ is large but far from $\pi/4$, and $\theta_{13}$ is small and consistent with zero ($\sin^2 \theta_{13} \leq 0.047$ at 3$\sigma$ C.L. \[2\]).

If data will significantly strengthen the upper bound on $\theta_{13}$, this will imply a very special pattern for the violation of lepton flavor, which begs for a theoretical rationale. In fact, it is possible to define quantitatively and experimentally when the $1-3$ mixing can be considered negligible: a value as tiny as $\sin^2 \theta_{13} \leq 10^{-4}$ can be generated by gravity effects alone \[3\] and neutrino factories could be sensitive to such small mixing \[4\].

The question of whether the origin of the lepton mixing $U = U_l^T U_\nu$ is in the neutrino or the charged-lepton sector has been discussed in many recent papers, e.g. \[5, 6, 7, 8, 9, 10\]. Just from the form of Eq. (2), it is apparent which is the most simple-minded realization of zero $1-3$ mixing: besides the diagonal contributions to the neutrino and charged-lepton mass matrices $M_\nu$ and $M_l$, one needs to generate off-diagonal entries only in the $1-2$ sector of $M_\nu$ and in the $\mu-\tau$ sector of $M_l$. In this case the atmospheric mixing originates in the charged-lepton sector and the solar mixing in the neutrino sector. In particular, this hybrid scenario has been shown to be generically associated with small values of $\theta_{13}$ \[11\].

We point out in this paper that the above-mentioned hybrid scenario with $\theta_{13} = 0$ is
realized by a discrete symmetry of the Lagrangian of a complete theory, with distinct experimentally verifiable predictions. Other models predicting $\theta_{13} = 0$ have also been proposed \[12, 13, 14, 15\].

Consider the discrete symmetry $Z_2 \times Z_2$, also known as the Klein group. There are 4 possible representations, i.e. $(+,-), (+, +), (-,-), (-, +)$. Suppose the 3 lepton families transform as follows:

$$(\nu_i, l_i), \ l^c_i \sim (+, -, (-, +), (-, -), (3)$$

with 2 Higgs doublets

$$(\phi_1^0, \phi_1^-) \sim (+, +), \ (\phi_2^0, \phi_2^-) \sim (+, -), (4)$$

and 2 Higgs triplets

$$(\xi_1^{++}, \xi_1^+, \xi_1^0) \sim (+, +), \ (\xi_2^{++}, \xi_2^+, \xi_2^0) \sim (-, -). (5)$$

Then the charged-lepton mass matrix linking $l_i$ to $l^c_j$ is given by

$$\mathcal{M}_l = \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & e & c \end{pmatrix}, (6)$$

where the diagonal entries $a, b, c$ are induced by $\langle \phi_1^0 \rangle$, and $d, e$ by $\langle \phi_2^0 \rangle$, and the Majorana neutrino mass matrix is given by

$$\mathcal{M}_\nu = \begin{pmatrix} A & D & 0 \\ D & B & 0 \\ 0 & 0 & C \end{pmatrix}, (7)$$

where $A, B, C$ come from $\langle \xi_1^0 \rangle$, and $D$ from $\langle \xi_2^0 \rangle$. The Higgs triplets are assumed to be very heavy ($\sim M_\xi$), so that they acquire naturally small vacuum expectation values ($\sim \langle \phi_1^0 \rangle^2 / M_\xi$) \[16\].
Then $\mathcal{M}_t$ is diagonalized by a rotation in the $2-3$ sector and $\mathcal{M}_\nu$ by a rotation in the $1-2$ sector. Hence $U$ is exactly of the form desired with $\theta_{13} = 0$ (models predicting Eqs. (6) and (7) by using different discrete symmetries can be found in [13]). In particular,

$$
\begin{bmatrix}
  a & 0 & 0 \\
  0 & b & d \\
  0 & e & c
\end{bmatrix}
= 
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & c_L & s_L \\
  0 & -s_L & c_L
\end{bmatrix}
\begin{bmatrix}
  m_e & 0 & 0 \\
  0 & m_\mu & 0 \\
  0 & 0 & m_\tau
\end{bmatrix}
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & c_R & -s_R \\
  0 & s_R & c_R
\end{bmatrix},
$$

with $s_L = s_{23}$, $c_L = c_{23}$. As for $\theta_{12}$, it is determined by Eq. (7) which also allows for arbitrary $m_{1,2,3}$. In other words, this model does not constrain any mass or mixing other than $\theta_{13} = 0$, but it identifies this particular limit as the result of a well-defined symmetry. Of course CP violation is not observable in oscillations, but it can appear in neutrinoless $2\beta$ decay, since $m_i$ are in general complex parameters.

One can ask the question if it is crucial for the above scenario to use Higgs triplets $\xi_i$ (type II seesaw) instead of right-handed neutrinos $N_i$ (type I seesaw). In this last case the predictions depend on the source of the $N_i$ Majorana masses. For definiteness, one can assume this source to be given by Higgs singlets $S_i$ which acquire super-heavy vacuum expectation values. In order to reproduce as closely as possible the above pattern, let us make the following assignments:

$$
N_i \sim (+,-), (-,+), (-,-), \quad S_1 \sim (+,+), \quad S_2 \sim (-,-).
$$

Then the neutrino mass matrix is given by

$$
\mathcal{M}_\nu = -\mathcal{M}_D\mathcal{M}_R^{-1}\mathcal{M}_D^T = 
\begin{bmatrix}
  a_\nu & 0 & 0 \\
  0 & b_\nu & d_\nu \\
  0 & e_\nu & c_\nu
\end{bmatrix}
\begin{bmatrix}
  A_R & D_R & 0 \\
  D_R & B_R & 0 \\
  0 & 0 & C_R
\end{bmatrix}
^{-1}
\begin{bmatrix}
  a_\nu & 0 & 0 \\
  0 & b_\nu & e_\nu \\
  0 & d_\nu & c_\nu
\end{bmatrix}
$$

(a general method to obtain texture zeros in type I seesaw matrices using flavor symmetries can be found in [17]). In this case the diagonalization of $\mathcal{M}_D$ requires also a right-handed
rotation, analogously to Eq. (8). Therefore a non-zero $\theta_{13}$ is in general induced. However, an interesting physical limit exists, such that $\theta_{13}$ is maintained to be zero, i.e. $M_3 \equiv C_R \to \infty$, so that the heaviest $N_3$ decouples, then it is easy to check that $\theta_{13} \to 0$ and, at the same time, $m_3 \to 0$. The smallness of $\theta_{13}$ is now related to the inverted hierarchy of the spectrum. Alternatively, we can simply eliminate $N_3$ from the beginning, i.e. keep only two right-handed neutrinos as in Ref. [18], which obtained the same result using a $U(1)$ flavor symmetry. In this scenario, since $\phi_2$ contributes both to $M_l$ and to $M_D$, the observable left-handed 2–3 mixing angle receives contributions from both $U_l$ and $U_\nu$.

There are only two other ways for $\theta_{13}$ to be zero in Eq. (10). If $b_\nu c_\nu - d_\nu e_\nu = 0$, then again $m_3 = 0$ as well as $\theta_{13} = 0$ (but without $M_3 \to \infty$: here one eigenvalue of $M_D$ vanishes instead). The third way is to have $b_\nu d_\nu + c_\nu e_\nu = 0$ (which allows $M_D$ of Eq. (10) to be diagonalized by a unitary transformation $U_L$ on the left and $U_R = 1$ on the right), then $m_3$ remains arbitrary as in the model with Higgs triplets. This form of the 2–3 submatrix of $M_D$ by itself is maintained for example by the discrete symmetry $S_3$ [19].

To test our model, we consider the details of the Higgs sector. Since the Higgs triplets are assumed to be very heavy, at the electroweak scale only the Higgs doublets are observable. Using Eq. (8), the Yukawa couplings of $\phi_1$ and $\phi_2$ are easily obtained as functions of $m_\mu$, $m_\tau$, $\theta_L$ and $\theta_R$, together with $v_1$ and $v_2$ subject to the constraint $\sqrt{v_1^2 + v_2^2} = 174$ GeV. The structure of Eq. (8) tells us that leptonic flavors only change between $\mu$ and $\tau$, apart from effects suppressed by the neutrino masses. In particular, the severe experimental constraints on $\mu \to e\gamma$ are automatically satisfied, as in the Standard Model. The couplings of $\phi^0_{1,2}$ are listed in Table 1. The physical charged Higgs boson is given by

$$h^- = \frac{v_2 \phi^-_1 - v_1 \phi^-_2}{\sqrt{v_1^2 + v_2^2}},$$

where $\phi^-_{1,2}$ couple to leptons as in Table 1, with $\mu$ replaced by $\nu_\mu$ and $\tau$ by $\nu_\tau$ respectively.

In the case of $M_\nu$ generated by Higgs triplets as in Eq. (7), we have $\theta_L = \theta_{23}$. In the
The neutral Higgs boson of the Standard Model (with the usual Yukawa couplings to leptons) is
\[ H^0 = \frac{\sqrt{2}(v_1 \text{Re} \phi_1^0 + v_2 \text{Re} \phi_2^0)}{\sqrt{v_1^2 + v_2^2}}, \]
but it is not in general a mass eigenstate in a two-Higgs-doublet model. It mixes with
\[ h^0 = \frac{\sqrt{2}(v_2 \text{Re} \phi_2^0 - v_1 \text{Re} \phi_2^0)}{\sqrt{v_1^2 + v_2^2}}, \]
which couples to leptons, in the same limit as in Eq. (12), according to
\[ \frac{m_\tau}{\sqrt{2} \sin 2\beta \sqrt{v_1^2 + v_2^2}} h^0 \left[ \sin 2\theta_R \mu + \cos 2\theta_R \tau \left( \frac{1 - \gamma_5}{2} \right) \mu - \cos 2\theta_R \tau \left( \frac{1 + \gamma_5}{2} \right) \tau \right]. \]
In general, $H^0$ may also mix with

$$A^0 = \frac{\sqrt{2}(v_2 \Im \phi^0_1 - v_1 \Im \phi^0_2)}{\sqrt{v_1^2 + v_2^2}}$$

(17)

which couples to leptons according to

$$-i m_\tau \frac{\sqrt{2}}{\sqrt{2} \sin 2\beta \sqrt{v_1^2 + v_2^2}} A^0 \left[ \sin 2\theta_R \bar{\mu} \gamma_5 \mu + \cos 2\theta_R \bar{\tau} \left( \frac{1 - \gamma_5}{2} \right) \mu - \cos 2\theta_R \bar{\mu} \left( \frac{1 + \gamma_5}{2} \right) \tau - \cos 2\beta \bar{\tau} \gamma_5 \tau \right].$$

(18)

If the Higgs potential has exact $Z_2 \times Z_2$ symmetry, then one can check that $CP$ is conserved and $A^0$ is a mass eigenstate (with odd $CP$) and does not mix with $h^0$ and $H^0$ which are even under $CP$. The decay of $A^0$ is thus another distinct signature of this model: the branching fractions of $A$ to $\tau^+ \tau^-$, $\tau^+ \mu^- + \mu^+ \tau^-$, and $\mu^+ \mu^-$ are proportional to $\cos^2 2\beta$, $\cos^2 2\theta_R$, and $\sin^2 2\theta_R$ respectively. If $Z_2 \times Z_2$ is allowed to be broken by soft terms of the Higgs potential, then $CP$ is violated and all 3 neutral Higgs bosons $A^0, h^0, H^0$ mix with one another. In the following we assume, for simplicity, that $A^0$ is a mass eigenstate and that

$$\begin{pmatrix} h^0_1 \\ h^0_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix},$$

(19)

where $h^0_{1,2}$ are the eigenstates with masses $m_{1,2}$.

Because of Eqs. (12), (16) and (18), the flavor-changing processes $\tau \rightarrow \mu \mu \mu$ and $\tau \rightarrow \mu \gamma$ are predicted, as well as an additional contribution to the muon anomalous magnetic moment.

Consider first $\tau \rightarrow 3\mu$. It proceeds through $A^0$ and $h^0$ exchange. Although $h^0$ mixes with $H^0$, the latter does not couple to $\bar{\mu} \tau$ and its coupling to $\bar{\mu} \mu$ is proportional to $m_\mu$. We obtain

$$\Gamma(\tau \rightarrow 3\mu) = \left[ \frac{m^2_\tau \sin 2\theta_R \cos 2\theta_R}{2 \sin^2 2\beta (v_1^2 + v_2^2)} \right] \frac{m^5_\tau}{4096 \pi^3} \left( \frac{1}{m^4_A} + \frac{1}{m^4_{h^0}} + \frac{2}{3m^2_A m^2_{h^0}} \right),$$

(20)

where $m_{h^0}$ is the effective contribution of $h^0$ exchange:

$$\frac{1}{m^2_{h^0}} \equiv \frac{\sin^2 \alpha}{m^2_A} + \frac{\cos^2 \alpha}{m^2_0}. \quad (21)$$
Numerically, for \( m_A = m_{h^0} = 100 \text{ GeV} \) and \( \sin 2\theta_R \cos 2\theta_R/\sin^2 2\beta = 1 \), this implies a branching fraction of \( 4.5 \times 10^{-9} \), well below the present experimental upper bound \([20]\) of \( 1.9 \times 10^{-6} \).

In the same approximation as above, the radiative decay rate of \( \tau \to \mu \gamma \) is given by

\[
\Gamma(\tau \to \mu \gamma) = \frac{\alpha_{em} m_\tau^5}{(64\pi^2)^2} (|A_L|^2 + |A_R|^2),
\]

(22)

where

\[
A_L = \frac{1}{3} \left[ \frac{m_\tau^2 \sin 2\theta_R \cos 2\theta_R}{2 \sin^2 2\beta (v_1^2 + v_2^2)} \right] \left[ \frac{1}{m_A^2} + \frac{1}{m_{h^0}^2} - \frac{1}{m_{h^-}^2} \right],
\]

(23)

and

\[
A_R = \left[ \frac{m_\tau^2 \cos 2\theta_R \cos 2\beta}{2 \sin^2 2\beta (v_1^2 + v_2^2)} \right] \left[ \sum_{i=1}^{2} \frac{k_i}{m_i^2} \left( \frac{8}{3} + 2 \ln \frac{m_\tau^2}{m_i^2} \right) - \frac{1}{m_A^2} \left( \frac{10}{3} + 2 \ln \frac{m_\tau^2}{m_A^2} \right) \right],
\]

(24)

with \( k_1 \equiv \sin^2 \alpha - \sin \alpha \cos \alpha \tan 2\beta \), \( k_2 \equiv \cos^2 \alpha + \sin \alpha \cos \alpha \tan 2\beta \). Numerically (using \( m_A = m_1 = m_2 = m_{h^-} = 100 \text{ GeV} \) and \( \cos 2\theta_R = \sin 2\theta_R = \sin 2\beta = 1/\sqrt{2} \)), this implies a branching fraction of \( 2.2 \times 10^{-12} \), again well below the experimental upper bound \([20]\) of \( 1.1 \times 10^{-6} \).

We computed also the contribution to the anomalous magnetic moment of the muon

\( a_\mu \equiv (g_\mu - 2)/2 \) from 1-loop diagrams mediated by \( h^-, h^0 \) and \( A^0 \):

\[
\delta a_\mu = \frac{m_\tau^2 m_\mu^2}{32\pi^2 (v_1^2 + v_2^2) \sin^2 2\beta} \left\{ \cos^2 2\theta_R \left[ \frac{1}{3m_{h^0}^2} + \frac{1}{3m_A^2} \right] + \right. \\
+ \left. \sin^2 2\theta_R \left[ \sum_{i=1}^{2} \frac{k_i}{m_i^2} \left( -\frac{7}{3} - 2 \log \frac{m_\mu^2}{m_i^2} \right) + \frac{1}{m_A^2} \left( \frac{11}{3} + 2 \log \frac{m_\mu^2}{m_A^2} \right) - \frac{1}{3m_{h^-}^2} \right] \right\},
\]

(25)

where \( k_1 \equiv \sin^2 \alpha \), \( k_2 \equiv \cos^2 \alpha \). Using the parameter values given above, \( \delta a_\mu \approx 6.2 \times 10^{-13} \), that is much smaller than the present uncertainty (\( \sim 10^{-9} \)) and therefore negligible as a possible explanation of the discrepancy (\( \sim 3 \times 10^{-9} \)) between the Standard Model prediction \([21]\) and the experimental value \([22]\).

In general, the leptonic Yukawa couplings of this model are at most of order \( m_\tau/M_W \) which is small enough to suppress all indirect Lepton-Flavor-Violation effects much below
the present experimental upper bounds, unless \( \tan \beta \) turns out to be very large. Thus the best hope of testing this model is through the direct production and decay of the extra Higgs bosons as already discussed.

Let us briefly review some features of the Majorana neutrino mass matrix with \( \theta_{13} = 0 \). In the basis where \( \mathcal{M}_l \) is diagonal, Eqs. (1) and (2) imply

\[
\mathcal{M}_\nu = \begin{pmatrix}
    c_{12}^2 m_1 + s_{12}^2 m_2 & s_{12} c_{12} c_{23} (m_1 - m_2) & s_{12} c_{12} s_{23} (m_1 - m_2) \\
    s_{12} c_{12} c_{23} (m_1 - m_2) & c_{23}^2 (s_{12}^2 m_1 + c_{12}^2 m_2) + s_{23}^2 m_3 & s_{23} c_{23} (s_{12}^2 m_1 + c_{12}^2 m_2 - m_3) \\
    s_{12} c_{12} s_{23} (m_1 - m_2) & s_{23} c_{23} (s_{12}^2 m_1 + c_{12}^2 m_2 - m_3) & s_{23}^2 (s_{12}^2 m_1 + c_{12}^2 m_2) + c_{23}^2 m_3
\end{pmatrix}.
\]

As shown in Ref. [15], this matrix by itself has a \( Z_2 \) symmetry. This may also be understood by its form invariance [23], i.e.

\[
U \mathcal{M}_\nu U^T = \mathcal{M}_\nu ,
\]

where

\[
U = \begin{pmatrix}
    1 & 0 & 0 \\
    0 & \cos 2\theta_{23} & \sin 2\theta_{23} \\
    0 & \sin 2\theta_{23} & -\cos 2\theta_{23}
\end{pmatrix}, \quad U^2 = 1 .
\]

The matrix of Eq. (26) was in fact obtained previously as the remnant of a complete \( D_4 \times Z_2 \) model [14]. Another model [24] based on the quaternion group \( Q_8 \) also obtains this structure (if one CP phase is put to zero) with the further restriction

\[
(\mathcal{M}_\nu)_{23} = 0 \quad \Leftrightarrow \quad s_{12}^2 m_1 + c_{12}^2 m_2 = m_3 .
\]

If \( c_{23} = s_{23} \) in Eq. (26), then \( \mathcal{M}_\nu \) has the \( Z_2 \) symmetry proposed in Ref. [25], which is realized in the \( A_4 \) model [26], with \( m_1 = m_2 = -m_3 \) (before radiative corrections). These examples and others in Ref. [13] show that our present proposal of \( Z_2 \times Z_2 \) is not unique for obtaining \( \theta_{13} = 0 \), but is rather the simplest scenario and it is also consistent with arbitrary charged-lepton and Majorana neutrino masses. It should also be noted that after the heavy Higgs triplets (or the right-handed neutrinos) are integrated away, the effective Lagrangian of this model (including the Higgs doublets) conserves \( L_e \) and \( L_\mu + L_\tau \) separately, broken only by the very small Majorana neutrino masses.
Quarks can be incorporated into this model, for example, assigning

\[ (u_i, d_i), \quad u_i^c, \quad d_i^c \sim (-, -), \quad (-, +), \quad (+, -). \quad (30) \]

In this way both up and down quark mass matrices contain only $1 - 2$ mixing, since the off-diagonal entries are induced by $\phi_2 \sim (+, -)$. Therefore the Cabibbo mixing can be reproduced while other mixing angles are suppressed. The three generations of fermions in Eqs. (3), (9) and (30) are associated with the nontrivial representations of $Z_2 \times Z_2$. They can be identified as the three components of the corresponding triplets of $SO(3)$, which breaks down to $Z_2 \times Z_2$ (i.e. a rectangle embedded inside a sphere). An alternative way to incorporate quarks in the model is to extend the discrete symmetry to $Z_2 \times Z_2 \times Z_2$, with leptons transforming trivially under the third $Z_2$ and quarks trivially under the second $Z_2$. The addition of $\Phi_3 \sim (-, +, -)$ and $\Phi_4 \sim (+, +, -)$ would then generate the complete quark mixing matrix, as in the $Q_8$ model [21].

Since left-handed and right-handed fermions transform in the same way under $Z_2 \times Z_2$, one could embed this model in a left-right symmetric theory. In particular, theories based on $SO(10)$ are a natural framework to provide both types of seesaw mechanism, since their particle spectrum may include both super-heavy right-handed neutrinos $N_i$ and scalar isotriplets $\xi_i$.

In conclusion, we pointed out that the absence of $1 - 3$ mixing in the lepton sector can be explained if the Standard Model Lagrangian is extended to include two Higgs doublets and an appropriate source for neutrino Majorana masses, in such a way to respect a $Z_2 \times Z_2$ family symmetry. In this scenario the atmospheric mixing angle originates in the $\mu - \tau$ sector of the charged lepton mass matrix and it relates with predictable Lepton-Flavor-Violation effects: the physical Higgs bosons have specific decay rates into muons and taus, while their indirect contributions to $\tau \rightarrow \mu \gamma$, $\tau \rightarrow 3\mu$ and $g_\mu - 2$ are in general negligible. The solar mixing angle originates in the neutrino mass matrix, that can be generated either by two
Higgs triplets or by two right-handed neutrinos.

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