Structural scheme for the synthesis of stable motion of a walking drive using a self-oscillation control system

A S Polyanina

Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

E-mail: A.S.Churzina@mail.ru

Abstract. Walking robots are one of leading directions of scientific and technical developments. An important problem is the creation of control systems that ensure the stable motion of a walking drive along various types of trajectories. In work the structural scheme of a system that implements the controlled motion of the foot point of a walking robot using a self-oscillation generator is given. Modeling of periodic motion generators is based on the use of the second Lyapunov method.

1. Introduction

The study of the dynamic modes of walking robots motion using mathematical modeling methods is the subject of many fundamental works. The creation of methods of controlling walking robots will solve important technical problems. In this work a control algorithm synthesis method which allows receiving cyclic motions of foot points of a walking robot is considered. For multidimensional spatial robotic systems, two directions for solving this problem can be proposed. These are theoretical methods and methods based on the use of human as a control link. The second direction is developing intensively in medicine when creating exoskeletons. The main theoretical method for the synthesis of controlled motion of mechanical systems is the inverse dynamic method, which allows one to determine the program motions of the robot and its drives that implement the given law of motion in the general case.

The main element of walking robot control systems is the gait generator, which calculates the interrelated coordinates of the motion of all links of the robot. Basically, existing gait generators use piecewise interpolation of program trajectories by trigonometric functions, resulting in acceleration surges at site boundaries. Another approach is to use different self-oscillation systems as gait generators.

There are various methods for constructing control algorithms for nonlinear dynamic systems. Allocate into an individual class of the system, in the state space of which the condition for the continuity of control functions is broken. The sliding mode method proposed by Utkin V.I. [1] deals with discontinuous controls and considers variable structure systems (VSS). To synthesize VSS, first of all, switching control laws are entered, which corresponds to the construction in the state space of a system of switching surfaces on which there are discontinuity points of control functions. It is said that in VSS on these surfaces sliding modes of motion arise [1]. Varying the parameters of the obtained control law, the sliding mode in a VSS will acquire asymptotic stability. As soon as the number of switching increases rapidly, the image point of the system stabilizes in the equilibrium position.
For the development of adaptive control algorithms the speed gradient method (MSG) proposed in works of Fradkov A.L. is used [2].

The purpose of the control is to reduce the value of some smooth objective function. MSG requires the involvement of the second Lyapunov method. At the end of the implementation of the speed gradient algorithm, it remains to replace the Lyapunov function with the objective function.

Motion in robotic systems is described by differential-algebraic equations [3], [4]. This form of equations makes it difficult to find control functions when implementing the sliding mode method or the speed gradient method.

Kolesnikov A.A. describes the synergetic method for the synthesis of controlled motion of nonlinear systems in his works. According to the synergetic approach stable areas of attraction are formed in the state space of the system. Such behavior is ensured by the corresponding control laws, which change the right-hand sides of the differential equations of the system and thereby become the means of its target self-organization [6].

The synergetic conception can be identified in most existing methods of control synthesis; the difference will be in the type of hyper-surface. In terms of the synergetic approach, there are two parts of the mathematical model of the controlled object. It is a control object and a control contour model that provides the specified properties of the entire object. Self-oscillation systems are often used as models of control contours [6]. The main method of synergetic control theory is the method of analytical design of aggregated regulators (ADAR), which allows to obtain the desired attractor. In this case, attracting sets are introduced into the system using macro substitutions, which include functions determining the shape of limit cycles.

To find the trajectories of motion of walking robot drive, it is necessary to set control functions that describe the program motion of individual points of the system. Such functions are generally piecewise continuous, which impairs the quality of control. In this regard, methods are being developed in which the function that defines the program motion must be analytic, asymptotically stable, and differ little from the program trajectory [3], [5]. The method for solving the problem is based on the construction of the Lyapunov function. One of the level surfaces of Lyapunov function is compact and invariant for a control system. The stability of motion in the vicinity of the surface is determined by the conditions for control parameters of the system. Trajectories of the system cannot cross the surface, this leads to stabilization of motions of the system in its vicinity. Stable limit cycles are formed in the phase space of the system.

It is required to ensure the stability of motion of both an individual link and the entire control system as a whole.

The method is well designed for equations of motion written in Cauchy form. However, the use of equations in the Cauchy form allows one to consider a fairly limited class of mechanical systems.

2. Generator of controlled motion trajectories walking robot

The gait generation unit consists of three parts - a generator of foot point’s trajectories, a generator of mass center trajectories and a generator of additional links.

The solution to the problem of synthesis of controlled motion of a walking robot is obtained from the solution of system of differentially-algebraic equations [3], [4]. To realize control on the basis of such a system, the link equation contains functions that determine the programmed motion of points of the robot body, functions that specify the motion of foot points of walking drives, and functions that ensure the stability of the position of the robot by moving some auxiliary points that are part of the additional linking equations [4].

Trajectories of foot points are well interpolated by self-oscillation systems [5], [7]. For the case of a walking machine such closed trajectories consist of almost straight sections.

2.1. Problem formulation.

Consider a system of self-oscillations [8]. A mathematical model of this type can be represented as follows.
\begin{equation}
\begin{cases}
\dot{x}_{2i-1} = \pm \alpha_{2i} x_{2i}^{2m-1} + U_{2i-1}(X), \\
\dot{x}_{2i} = \mp \alpha_{2i} x_{2i}^{2m-1} + U_{2i}(X),
\end{cases}
\end{equation}

where \( i = 1, 2; \; m \in \mathbb{N}. \)

The constructed control function of \( U = (U_1(X), U_2(X), U_3(X), U_4(X))^T \) should provide stabilization of the system along four channels in the vicinity of the \( \Omega \) manifold defined by the equation of \( \sum_{i=1}^{4} x_{2i}^{2m} = 1. \)

The required feedback controls are sought in the form:

for channels of the first subsystem it is

\[
\begin{align*}
U_1(X) &= \beta_1 x_1 + \sum_{j=1}^{2} (\beta_{2j-1,1} x_{2j-1}^{2m} x_1 + \beta_{2j,1} x_{2j}^{2m} x_1), \\
U_2(X) &= \beta_2 x_2 + \sum_{j=1}^{2} (\beta_{2j-1,2} x_{2j-1}^{2m} x_2 + \beta_{2j,2} x_{2j}^{2m} x_2);
\end{align*}
\]

for channels of the second subsystem it is

\[
\begin{align*}
U_3(X) &= \beta_3 x_3 + \sum_{j=1}^{2} (\beta_{2j-1,3} x_{2j-1}^{2m} x_3 + \beta_{2j,3} x_{2j}^{2m} x_3), \\
U_4(X) &= \beta_4 x_4 + \sum_{j=1}^{2} (\beta_{2j-1,4} x_{2j-1}^{2m} x_4 + \beta_{2j,4} x_{2j}^{2m} x_4);
\end{align*}
\]

Further, the problem is reduced to finding control parameters.

2.2. Solution method

Using the output scheme [4], the control coefficients were obtained:

for the first subsystem it is

\[
\begin{align*}
\alpha_1 &= 2m \alpha_1^{2m}, \quad \alpha_2 = 2m \alpha_2^{2m}, \quad \beta_1 = \pm 1, \quad \beta_2 = \pm 1, \\
\beta_{2j-1,1} &= \mp \alpha_{2j-1}^{2m}, \quad \beta_{2j-1,2} = \mp \alpha_{2j-1}^{2m}, \quad \beta_{2j,1} = \mp \alpha_{2j}^{2m}, \\
\beta_{2j,2} &= \mp \alpha_{2j}^{2m};
\end{align*}
\]

for the second subsystem it is

\[
\begin{align*}
\alpha_3 &= 2m \alpha_3^{2m}, \quad \alpha_4 = 2m \alpha_4^{2m}, \quad \beta_3 = \pm 1, \quad \beta_4 = \pm 1, \\
\beta_{2j-1,3} &= \mp \alpha_{2j-1}^{2m}, \quad \beta_{2j-1,4} = \mp \alpha_{2j-1}^{2m}, \quad \beta_{2j,3} = \mp \alpha_{2j}^{2m}, \\
\beta_{2j,4} &= \mp \alpha_{2j}^{2m},
\end{align*}
\]

where \( j = 1, 2. \)

For this, the following invariance condition of the \( \Omega \) manifold was used:

\[
\sum_{i=1}^{2} \left( \pm \alpha_{2i} x_{2i}^{2m-1} + U_{2i-1}^\tau(X) \frac{\partial F(X)}{\partial x_{2i-1}} + (\mp \alpha_{2i-1} x_{2i-1}^{2m-1} + U_{2i}^\tau(X)) \frac{\partial F(X)}{\partial x_{2i}} \right) = \pm 2m \sum_{i=1}^{2} \left( x_{2i-1}^{2m} + x_{2i}^{2m} \right) \left( 1 - \sum_{i=1}^{2m} \frac{x_{2i}^{2m}}{a_{2i}^{2m}} \right),
\]

where minus or plus signs are taken in front of the coefficients of the control functions \( U_{2i-1}^\tau(X), \) \( U_{2i}^\tau(X) \) depending on whether "+" or "−" are in front of the first degree members \( x_{2i-1}, x_{2i}, \)
\( i = 1, 2 \), of control.

Thus when relations (2), (3) are satisfied for controlled parameters, the surface \( \Omega \) will be an invariant manifold of system (1).

In particular, control functions

\[
\begin{align*}
U_{2j-1}(X) &= x_{2j-1} + \sum_{j=1}^{2} (\beta_{2j-1,2j-1} x_{2j-1}^2 x_{2j-1}^2 + \beta_{2j,2j} x_{2j}^2 x_{2j}^2), \\
U_{2j}(X) &= x_{2j} + \sum_{j=1}^{2} (\beta_{2j-1,2j} x_{2j-1}^2 x_{2j}^2 + \beta_{2j,2j} x_{2j}^2 x_{2j}^2),
\end{align*}
\]

where \( i = 1, 2 \), will stabilize the trajectories of motion in the vicinity of the \( \Omega \) manifold.

Therefore the \( \Omega \) surface is an invariant asymptotically stable manifold of control system (1).

The proposed method for the synthesis of nonlinear self-oscillation generators (1) is used to form control of walking robot foot points.

2.3. Numerical modeling of stable controlled motion

System (1) is a first order autonomous system. Possible motions are convenient to analyse in subspaces of system states. The results of this analysis are shown in fig. 1, 2.

![Figure 1. Vertical motion of the robot foot](image1)

![Figure 2. Self-oscillations in the second subsystem](image2)

The existence of a simultaneous exit of processes to self-oscillating modes in the corresponding subspaces of the system is shown. The trajectory obtained by the proposed method contains sections that are close to straight, which is necessary for work of a walking robot. This ensures resistance to disturbances and changes in system parameters.

3. Structural scheme of the control system

In the work the inverse dynamic method scheme is used as the structure of the object control scheme [9]. Control element is generator of program motions of walking robot providing stability of drive trajectory. The structural scheme of a system that implements the motion of a walking drive using a self-oscillation generator is shown in fig. 3. The equations of controlled object dynamics are numerically integrated together with the equations of self-oscillation generator [3], [4]. For the formation and numerical integration of dynamics equations, an invariant modeling system is used. The measured coordinates \((x_{\text{meas}}, y_{\text{meas}})\) of the point on the foot of the walking drive are transmitted to the numerical model, which, at each sampling step, are used as initial conditions for the corresponding coordinates of the equations of the self-oscillation generators.

At solution of equations values of kinematic parameters of motion of the whole robot and each drive, in particular, relative stroke and speed of the drive are determined. By their inverse dynamic method there are control forces in drives. Values of control forces from computer are provided to control block in which signals are generated to drives [9].

A significant difference between the proposed method for the synthesis of a given closed trajectory using asymptotically stable self-oscillation generators from, for example, deviation control is the
following. In the proposed scheme the measured controlled coordinates are used only as initial conditions for some variable of generator equations and stability is ensured by the properties of the self-oscillation generator.

\[ U_j, \ldots, U_n \]

\[ F_j, \ldots, F_n \]

Amplifier generates controls to drives

Walking robot drive

Detectors of foot position

Solver of equations of a controlled object model connected with self-oscillation generator

Block for calculation of forces in drives by inverse dynamic method

\[ \lambda_1, \ldots, \lambda_n \]

\[ x_{\text{meas}}, y_{\text{meas}} \]

\[ V_{\text{program}} \]

**Figure 3.** Structural scheme

4. Conclusion
Stabilizing controls are obtained to solve problems of control of nonlinear systems. The results of the research can be used in the projection of control systems for robotic complexes [7], [9].

At present, despite the relatively high level of mechanization and automation of production processes, many more operations have to be performed by a person. Soon, walking robots will become active helpers of human. The use of walking robots capable of carrying out emergency, assembly, building and repair works will make it possible to take another important step forward in scientific and technical progress.

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