A comparative analysis of the volatility nature of cryptocurrency and JSE market

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Abstract

Despite the rapid growth of developing markets, aided by globalization, comparative studies of cryptocurrency and stock market volatility have focused on the developed markets and neglected developing ones. In this regard, this study compares cryptocurrency volatility with that of the Johannesburg Stock Exchange (JSE), a developing market. GARCH-type models are applied to daily log returns of Bitcoin, Ethereum, and the FTSE/JSE 40 in two ways. Firstly, the models are applied directly; secondly, structural breaks are tested and accounted for in the models. The sample period was from September 18, 2017, to May 27, 2021. The results show higher volatility and higher volatility persistence in cryptocurrency than in the JSE market. They also show that persistence is overestimated for cryptocurrencies when structural breaks are not accounted for. The opposite was true for the JSE.

Moreover, the two cryptocurrencies were found to have close to identical volatility plots that differ from that of the JSE. High volatility periods of cryptocurrency also did not coincide with that of JSE and those of JSE did not coincide with the cryptocurrency ones. There is also evidence of an inverse leverage effect in cryptocurrency, which opposes the normal leverage effect of the JSE market.

INTRODUCTION

Volatility is the most crucial of all the stylized facts that come with financial data. This is because it determines the riskiness of an asset. Higher volatility signifies more risk and a higher likelihood of making losses.

Nakamoto (2008) suggested the first idea of cryptocurrency. He introduced Bitcoin, a new form of currency based on blockchain computer technology and free from institutional control. Over a decade later, Bitcoin and other cryptocurrencies created afterward are considered alternative investment options. While the initial purpose was to create an alternative currency, cryptocurrencies are now considered an asset. Baur et al. (2018) and Glaser et al. (2014) highlight the classification of cryptocurrency as an asset. This revelation is not surprising given the rapid increase of Bitcoin from being given for free on launch, reaching 10 cents in 2010, and attaining an all-time high of over $60,000 in 2021. However, despite the rapid growth in the value of cryptocurrencies, the growth has been marred by high volatility. There is, therefore, a need to study this volatility so investors can better understand how to invest amid the turmoil.
One way of mitigating risks is by investing into different assets. These assets must ideally have different volatility levels and reactions to market shifts. For example, given a market shock, the market’s reaction speed differs for each asset. This difference can be noted in the reactions of cryptocurrency and the stock market when there is a market shock. For example, interest rate changes can quickly affect stock prices but not the cryptocurrency market. As a result, investors can shift some of their funds away from troubled stocks and into cryptocurrency until stability returns.

The primary investment area for investors has long been the stock markets. Hence, the stock market has been extensively studied. With the coming of cryptocurrency, there have been numerous comparative studies on its volatility compared to that of the stock market. This is to give investors the knowledge to make informed about investment decisions, particularly in portfolio creation, where investors must know the asset volatilities and use them as a hedge against risk. However, current studies have focused on comparative studies using the developed stock markets, such as the US stock market, while neglecting the developing markets. This means that the results used in developing markets are the generalization of those made in developed market studies. This generalization may carry some bias as developing markets are at different growth phases than developed markets. It is on this basis that this study seeks to compare cryptocurrency volatility with that of the JSE market, a developing market.

1. LITERATURE REVIEW

The high volatility of cryptocurrency has been the major talk in the financial markets, with many investors voicing their worries about the level of volatility. Zhang et al. (2018), Katsiampa et al. (2019), and Hu et al. (2019) have reiterated the highly volatile nature of cryptocurrency. High volatility has been attributed to speculation, lack of regulation, and bad news, amongst others. The explanation for the unusually high volatility found in cryptocurrencies is not as simple as it seems. Hence, various reasons have been brought forward. Dowd (2014) and Cheah and Fry (2015) attribute the volatility to speculation. This reason contradicts Blau (2017), who found no evidence of speculation as a cause for volatility. Corbet et al. (2018) found that volatility increased after the advent of Bitcoin futures trading. The study also revealed that using Bitcoin futures as a hedging tool was ineffective and that the futures were insignificant in the asset classification of Bitcoin. In agreement with this study, Klein et al. (2018) studied Bitcoin and Gold returns. The study revealed that Bitcoin was not a reliable hedging alternative. Another cryptocurrency study was conducted by Akyildirim et al. (2020). They used the DCC GARCH models to examine the dependencies between cryptocurrency and financial markets and found evidence of increased volatility when investor fear was high in the market.

Similarly, the stock market volatility, which Kaseke et al. (2021), Gil-Alana et al. (2020), Aloosh and Ouzan (2020), and Liang et al. (2019) have shown to be below that of cryptocurrency, has different causes. However, for stocks, the leading causes are policies made by governments. These policies tend to alter the investors’ perspectives and their expected profit margins. Despite the policies being made with market stabilization in mind, Smith Jr. (1988) argues that the markets would be better off with less institutional interference. This point of self-regulatory is the main reason for the creation of cryptocurrency. Based on the observations so far, such decentralization has not lessened volatility. This argument is based on the fact that cryptocurrency remains more volatile than stock markets. Another reason for the high volatility and the leverage effect can be derived from Duffee (2002), who argued that balance-sheet effects are a potential source of asymmetric volatility. The study shows that the financial leverage of stocks increases when the prices of stocks decrease. When this happens, the stocks get more volatile. In contrast, an increase in price causes the opposite. Cryptocurrency has shown in its entire existence that the value tends to drop suddenly; hence, following the reason given by Duffee (2002), this increases the financial leverage, fueling the volatility.

The GARCH-type models have been the go-to for determining the half-life and volatility. The most common models are the GARCH, GJR-
GARCH, and EGARCH. For most cases, these models use the student $t$, skewed student $t$, and the general error distributions. Examples of such studies include those of Muguto and Muzindutsi (2022), who used the GARCH, GJR-GARCH, and EGARCH to quantify the half-life of volatility in the BRICS markets and the G7 markets. The same models were used by John et al. (2019), who studied the half-life of Bitcoin. The models helped determine the half-life in these studies, but they also show that a slight change in persistence significantly affects the half-life value. In a similar study, Ghoddusi et al. (2020) used the same GARCH models but accounted for structural breaks in the data. Dummy variables accounted for the structural breaks. The results showed that the presence of structural breaks leads to overestimated persistence and hence an underestimated half-life. This finding is coherent with the study of Lamoureux and Lastrapes (1990), where the GARCH model was applied to stock data, and the results revealed persistence measures that were almost 1. This high persistence, they discovered, was attributed to structural breaks in the unconditional variance. The same conclusion is reached by Chatzikonstanti (2017) in the stock market using the US stock market indices. Besides using dummy variables, the study split the data at breakpoints, and the change in persistence was recorded per segment. In cryptocurrency, the presence of structural breaks is identified by Mensi et al. (2019), Charles and Darné (2019), and Latif et al. (2017). Overall, these studies show that accounting for structural breaks is crucial to avoid bias in results.

With regards to mean reversion and momentum (persistence), Zaremba et al. (2021) and Pavlov (2022) explain that for traders, mean reversion and persistence, if notable, can be exploited for profit-making. These traders believe in buying the upward momentum and selling the downward momentum. However, for this to occur, the momentum and mean reversion should be known. In particular, Zaremba et al. (2021) found a powerful one-day reversal in cryptocurrencies. High return cryptocurrencies on the previous day were on the next day outperformed by those that had not performed well on the previous day. The study also shows that 2% of the biggest coins exhibit momentum rather than reversal, while the remaining 98% exhibit mean reversion (Zaremba et al., 2021, p. 13). The study argues this difference is due primarily to the effects of liquidity, whereby the less liquid small cryptocurrencies are affected by shocks of supply and demand that they cannot absorb. Catania and Grassi (2021) also found significant long-term memory concerning stocks. Finally, Cubbin et al. (2006) studied the mean reversion in the JSE market. They found evidence of mean reversion, with high price-to-earnings ratio portfolios outperforming those with lower price-to-earnings ratios.

When it comes to price discovery in cryptocurrency, using Bitcoin in their study, Corbet et al. (2018) discovered that uninformed investors in the spot market contributed to the cryptocurrency pricing. This conclusion was based on the information leadership share, which states that 97% of the information affecting Bitcoin prices was reflected in the spot market. The remaining 3% was reflected in the futures market (Corbet et al., 2018, p. 26). This finding contradicts the widely known fact that futures markets tend to influence price discovery.

Kwon (2020) compared the tail behavior of cryptocurrency against the US dollar and stock index. The study revealed similar tail behaviors across the assets concerning contemporaneous correlation. Furthermore, the correlation was negative, suggesting using Bitcoin as a portfolio hedge for the dollar and the stock market.

Bouri et al. (2017a) studied the changes in Bitcoin volatility and its return prices. The study used GARCH models to reveal an inverse volatility asymmetry in Bitcoin. They discovered that the reverse property vanished after the crash period. The takeaway was that before the crash, Bitcoin could have been used as a safe haven, but not after. The same study used S&P500 returns for the same periods. The results showed normal volatility asymmetry, which differed from the Bitcoin market results. Furthermore, the study compared a portfolio of 50% Bitcoin and 50% S&P500 to a portfolio with 100% S&P500. The demonstration results showed that adding Bitcoin reduced the portfolio risk for both periods, but mainly before the crash when the inverse asymmetry was found. In a similar study, López-Cabarcos et al. (2021) used the GARCH and EGARCH models to examine the effects of Bitcoin volatility on S&P500 VIX returns and investor sentiment. The results

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showed evidence of higher volatility in bitcoin as compared to S&P500. The volatility of Bitcoin was unstable during speculative periods. Another study by Dasman (2021), which used t-tests and F tests to compare the mean returns and variance of Bitcoin, the Indonesian Composite Index, and gold, found higher volatility in Bitcoin than in gold and the stock market. However, the average return of Bitcoin was significantly higher, while the stock returns and gold were not significantly different.

From the reviewed literature, it is clear that most studies look at stock and cryptocurrency volatilities separately. Of those that studied both simultaneously, the focus was on the developed markets, or they focused on the spillover between cryptocurrency and stocks. The results obtained from these studies are then generalized for the developing markets. Studies show that volatility is higher in cryptocurrencies than in stocks. However, considering that the state of the developing markets is more dynamic, generally underdeveloped, and most likely inefficient compared to the developed markets, it is justified that the findings may be different. Furthermore, developing markets are still a go-to investment area for investors seeking higher returns due to the inefficiency they exhibit. This consistent attempt to profit from inefficiency may result in unusual volatility in developing markets.

The information presented by the reviewed literature may not apply to all markets. This gap is explored and filled in this study. The common feature of this study with others is that it will use cryptocurrency returns and GARCH-type models. However, it will differ in the sample period, as more information was available at the time of this analysis compared to the reviewed studies. These additional data also set the study apart because it presents an opportunity to learn more about cryptocurrency properties that may not have been previously revealed.

2. METHODOLOGY

Three main GARCH models are used. These are the GARCH model of Bollerslev (1986), the Exponential GARCH (EGARCH) model of Nelson (1991), and the GJR-GARCH of Glosten et al. (1993).

The GARCH model was developed to improve on the shortcomings of the ARCH model by Engle (1982), which required many parameters to capture the volatility process adequately. The GARCH model, however, reduces the number of parameters required to the point where GARCH (1,1) is sufficient in most cases. The GARCH is parsimonious because it models the conditional variance to depend on the past squared residuals and past conditional variance. Despite the modifications made to the GARCH model, it still did not capture some features found in other time series, such as the leverage effects. All the GARCH models follow a similar framework, which consists of two equations; the first is the mean model, which captures the conditional mean of the process. The second is the conditional variance equation. The mean equation is given by:

\[ r_t = \mu + \alpha_t, \]

\[ \mu_t = \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_i a_{t-j}, \]  \hspace{1cm} (1)

\[ y_t = r_t - \phi_0 - \sum_{j=1}^{p} \beta_j x_{j-t}. \]

The mean equation for all GARCH models is the same, but they differ in how the conditional variance \( \sigma^2 \) evolves over time. The conditional variances for the three models to be used are:

\[ \sigma^2_t = \omega + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j}, \]  \hspace{1cm} (2)

\[ \ln(\sigma^2_t) = \omega + \sum_{i=1}^{q} \left( a_{t-i} \right) + \frac{\gamma_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j \ln(\sigma^2_{t-j}), \]  \hspace{1cm} (3)

\[ \sigma^2_t = \omega + \sum_{i=1}^{q} \left( \alpha^2 + \gamma_i S_{t-i} \right) e_{t-i}^2 + \beta \sigma^2_{t-i}, \]  \hspace{1cm} (4)

The GARCH, EGARCH, and GJR-GARCH conditional variance equations are represented by equations 2, 3, and 4. \( \omega \) is a constant, \( \beta \) captures the past conditional variance effects on the current volatility, \( \gamma \) captures the past shock effects on the current conditional variance. Model adequacy tests are performed to get the final lags required per model, which means modeling lower lags until insignificant higher lags are obtained. The GARCH model assumes the absence of serial autocorrelations in the data. This assumption means that, if present, the autocorrelations must first be removed. The re-
moval is achieved by fitting an AR(p) model with the lag p determined by the significant lags from an ACF plot.

For the standard GARCH model, it must be that $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and that $\alpha + \beta < 1$ for positivity and conditional variance stationarity to hold. The persistence of volatility is measured by $\alpha + \beta$. To ensure covariance stationarity, then $\alpha + \beta < 1$. Despite its success, the standard GARCH model has its limitations. While it captures stylized facts such as persistence and mean-reversion, it retains the weakness of the ARCH model of responding equally to both positive and negative shocks.

The GJR-GARCH improves the GARCH model by allowing the asymmetric effect to be modeled. This asymmetry is captured by an additional parameter, $\gamma$, which indicates the presence or absence of the leverage effect. If $\gamma = 0$, then there is no leverage effect. However, if $\gamma > 0$, negative shocks will have a bigger impact on the volatility as compared to positive shocks. An indicator function $S_{t-1}$ is used to capture the asymmetry. This indicator has a value of one in the case of a negative shock and zero in the case of a positive shock.

The GJR-GARCH also improves the GARCH model by considering the asymmetry and leverage effects. It differs from the GJR-GARCH in that there is no need for additional constraints to avoid violating the none negativity conditions since the conditional variance is modeled using the natural log, making the variance positive by construction. However, $\omega > 0$ and $\alpha + \beta < 1$ should still hold. $\gamma$ is the leverage effect parameter of $a_{t-1}$. If $\gamma$ is 0, then there is no leverage effect. Otherwise, if $\gamma$ is negative, negative shocks will have a bigger impact on the volatility as compared to positive shocks. The opposite is true for a positive $\gamma$. The persistence of the EGARCH model is given by $\sum_{i=1}^{p} \beta_i$. If the persistence parameter is less than 1, the return series will exhibit mean reversion. However, if the persistence parameter is equal to 1, then the series follows the random walk (Gbenro & Moussa, 2019, p. 4).

Another concept closely related to persistence is the mean reversion. The mean reversion of volatility is defined as the average level of volatility to which volatility will eventually return (Engle & Patton, 2007, p. 239). This means that, despite any wild swings, the volatility will remain at its average level. In other words, this means that current volatility will not affect future volatility in the long run. For any unit of measurement, investors may want to know the average periods it takes for volatility to return to its average level; this can be found using the half-life by Engle and Patton (2007). Engle and Patton (2007, p. 239) defined the half-life "as the time it takes the volatility to move halfway back towards its unconditional mean." The formula is:

$$\ell = \frac{\ln 0.5}{\ln \text{(Volatility persistence)}},$$

where $\ell$ is the half-life and the Volatility persistence is the volatility persistence of the model used. In the case of the GARCH,

$$\ell = \frac{\ln 0.5}{\ln (\alpha + \beta)},$$

for the EGARCH,

$$\ell = \frac{\ln 0.5}{\ln \left(\sum_{i=1}^{p} \beta_i\right)},$$

(Mert & Demireli, 2020), and for the GJR-GARCH,

$$\ell = \frac{\ln 0.5}{\ln \left(\alpha + \beta + \frac{1}{2} \gamma\right)}.$$
ected using the Pruned Exact Linear Time (PELT) method from the Changepoint package in R developed by Killick and Eckley (2014). These change points are then included in the GARCH type models in the mean and variance equations as dummy variables. The results are then compared to those of models without the structural breaks.

The three GARCH models were fitted to each asset under three error distributions, the student t, the skewed student t, and the generalized error distribution, to find the best fit model for each asset. The normal distribution was not included since the preliminary analysis of the data showed that it was leptokurtic and diverted from the normal distribution. Hence, heavy-tailed distributions were preferred to capture the tails. These heavy-tailed distributions are ideal as they can capture rare financial events, such as random crashes and booms that yield extreme values. In the literature, the choice of the error distribution has varied for stock return data, but the most used are student t, skewed student t, and the general error distribution. For cryptocurrency, the same distributions have proven to be efficient in capturing the tails. The choice of the error term distribution usually does not change the results when using the three distributions. However, since new patterns may emerge with new data, all three distributions are used. The best model will be chosen using two information criteria: the AIC and the BIC.

3. DATA AND METHODS

This study uses the Bitcoin, Ethereum, and FTSE/JSE 40 index daily log returns. Bitcoin and Ethereum were chosen as the representations of cryptocurrencies because they are the two biggest by market cap. The Johannesburg Stock Exchange (JSE) market is represented by the FTSE/JSE 40 because it takes the top 40 companies listed on the JSE, making it a fair representation of the entire market. All the data was retrieved as daily closing prices from Investing (n.d.). The daily returns are calculated from the daily closing prices. The sample period runs from September 18, 2017, to May 27, 2021, with 1,348 observations per asset. This period is chosen to allow for uniformity in the data series based on the availability of the data. The daily prices \( P_t \) were converted to returns using the formula:

\[
R_t = \frac{P_{t+1} - P_{t-1}}{P_{t-1}}.
\]  

The simple returns \( R_t \) are converted to log returns \( r_t \). The reason for the conversion is that log returns have statistical properties that are more tractable. They can be used with many statistical theories, such as the need for normalization (Quigley & Ramsey, 2008). The log-returns are obtained by:

\[
r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \times 100.
\]

The log-returns are multiplied by 100 to work with percentage returns rather than raw returns, which will have many decimal places.

| Statistic       | Bitcoin | Ethereum | JSE  |
|-----------------|---------|----------|------|
| nobs            | 1348    | 1348     | 1348 |
| Minimum         | −39.181611 | −44.54723 | −9.922903 |
| Maximum         | 22.551201  | 25.957246 | 9.479792  |
| Mean            | 0.267411   | 0.325399  | 0.030626  |
| Median          | 0.162155   | 0.325399  | 0.030626  |
| Variance        | 17.995898  | 28.817646 | 1.521986  |
| Stdev           | 4.242157   | 5.368207  | 1.233688  |
| Skewness        | −0.277674  | −0.288557 | −0.30232  |
| Kurtosis        | 7.843084   | 6.18759   | 10.132279 |

As seen in Table 1, the average daily returns were highest in cryptocurrency, with 0.27% for Bitcoin and 0.33% for Ethereum, while the JSE market had 0.03%, which is nine times less than the daily return of Bitcoin. The standard deviations indicate higher volatility in cryptocurrency, with Bitcoin having 4.2 and 5.4 for Ethereum, compared to 1.2 for the JSE. These results are not surprising as they confirm the findings of many studies, such as those by Bouri et al. (2017b), which showed that cryptocurrency has higher returns and a higher standard deviation than stocks. In addition, all three assets exhibited negative skewness, indicating heavier left tails than the upper right tail. This result suggests that the lower returns are more probable than the higher returns. Negative skewness in the Bitcoin and Ethereum return series is also observed by Baur and Dimpfl (2018) and Kim et al. (2021).

Another observation is that the highest and lowest returns of the JSE are almost symmetrical, i.e., a maximum of 9.48% and a minimum of −9.92%.
This observation contradicts the two cryptocurrencies with higher absolute values on the negative side, as seen by the maximums of 22.55%, 25.96%, and minimums of –39.18% and –44.55% for Bitcoin and Ethereum, respectively. An important observation compared with other studies is that the skewness and other descriptive statistics are sample period dependent. Periods concentrating on the bull period and those with more of the bear period will have different results. For example, Brauneis and Mestel (2018) and Uzonwanne (2021) used sample periods with negative skewness. Stationarity was checked using the Augmented Dicky Fuller test. This test is essential because stationarity is a model assumption for the models used. Stationary data means consistency in series properties, making the model results more reliable, while non-stationary data results in non-consistent and biased results. The results in Table 2 reject the null hypothesis of lack of stationarity, concluding that the data is stationary.

### Table 2. Stationarity tests for the returns

| Asset  | Test statistic | 1%  | 5%  | 10% | Conclusion |
|--------|----------------|-----|-----|-----|------------|
| Bitcoin| –24.8279       | –2.58| –1.95 | –1.62 | Stationary |
| Ethereum| –24.5003      | –2.58| –1.95 | –1.62 | Stationary |
| JSE    | –25.1035       | –2.58| –1.95 | –1.62 | Stationary |

Apart from the statistics, the log returns’ visual plots, which are presented in Figure 1, are also considered. These plots show the presence of heteroscedasticity and volatility clustering for all the return series. For example, the returns of Bitcoin and Ethereum fluctuated around –10 and 10, while those of the JSE fluctuated around –5 and 5. This shows that the two cryptocurrencies have higher returns and losses than the JSE. Periods of random extreme shocks are also visible. As expected, these extreme shocks are similar to those in cryptocurrency. For example, there was a significant negative shock for Bitcoin and Ethereum around July 2019, but none on the JSE.

Similarly, there was a substantial negative shock in the JSE market at the end of 2019, followed by a substantial positive shock. These shocks do not coincide with any of the shocks in the cryptocurrency plots. According to López-Cabarcos et al. (2019), this could indicate that investors were fleeing the stock market during the turmoil.

In general, for financial data, serial dependence in a stock return series $r_t$ tends to be weak or does not exist (Tsay, 2014). This is the case with the ACF plots shown in Figure 2, where the ACF plots generally have weak low-order correlations. The presence of correlations is also tested using the Portmanteau test, and the results confirm that the low lag correlations are significant. LeBaron (1992) mentions that serial autocorrelations can be introduced into the series during the final stock price index and return calculation. However, this
is more common in high-frequency data. The low order autocorrelations also indicate that the markets are not efficient according to the efficient market hypothesis of Fama (1998). The squared residuals will have significant autocorrelations. However, these are more pronounced in JSE returns and less in cryptocurrency returns. These significant squared returns show ARCH effects present in the data. This conclusion is cemented by performing the ARCH effects test using the ARCH LM test. The results all had a p-value of less than 0.05 for all three assets. Therefore, the null hypothesis of no ARCH effects is rejected.

The QQ plots in Figure 3 show that for all three assets, the normal assumption is not a good fit for the data, as evident by the data deviating from the line at the ends. This deviation indicates tails that are heavier than the normal distribution, i.e., leptokurtic. To affirm the findings from the QQ plots,
the Jarque-Bera (JB) test for normality is carried out. The results of the test had p values < 0.05. Hence, the normality for log return is rejected.

4. RESULTS AND DISCUSSIONS

Three models were fit for each of the assets. The best model was selected using two information criteria, the AIC and BIC values, as shown in Table 3. Based on the information criterion, the best models were the EGARCH(1,1) with Student t errors for Bitcoin, the GARCH (1,1) with GED for Ethereum, and the GJR-GARCH(1,1) with GED for JSE.

The resulting model parameters of the three final models are discussed. The two cryptocurrencies had positive and significant AR terms, showing autocorrelation in the log returns. The presence is a sign that future returns of the cryptocurrencies can be explained by their past values and that there is a general mean reversion. For the JSE, the AR terms were not significant. The AR terms in the JSE were only added in the model to allow a more balanced model comparison between the cryptocurrency and the JSE market. Another implication of significant serial autocorrelations is that this is evidence against the weak form of market efficiency. This inefficiency enables investors who can employ technical strategies to profit by setting up positions based on the information.

The effect of past shocks on returns, as shown by the parameter α, was positive for both Bitcoin and Ethereum. However, it was only significant for Bitcoin, indicating that for Bitcoin, positive past shocks could affect the future returns and volatility of Bitcoin. This finding also points to Bitcoin being an inefficient market, which agrees with the suggestions of significant autocorrelations. For JSE, α was rounded off to zero in the model. The β was significant for all three assets, suggesting that the past volatilities can explain the future volatility. This finding is not surprising as it supports the volatility clustering identified on the return plots and is also a known stylized fact of financial data.

For Bitcoin, the EGARCH model had a positive γ, indicating an inverse leverage effect; this means that positive returns increased volatility more than negative ones. This finding agrees with Huang et al. (2022), who also find an inverse leverage effect in both Bitcoin and Ethereum. In another study, Zhang et al. (2021) discovered that the inverse leverage effect exists in the short run; however, but not in the long run. In addition, the JSE model had a positive and significant asymmetry parameter. Therefore, negative shocks have a larger impact on volatility than positive shocks. This effect shows that the market is more leveraged and therefore more risky, so the volatility should increase (Bauwens et al., 2012).

Table 4. Parameter estimates for the selected models

| Asset   | Bitcoin | Ethereum | JSE    |
|---------|---------|----------|--------|
| Model   | AR(2) + | AR(2) +  | AR(2) + |
|         | EGARCH  | GARCH    | GJR–GARCH |
| µ       | 0.140885** | 0.083601* | 0.030258 |
| AR1     | −0.076549*** | −0.109732*** | 0.008896 |
| AR2     | 0.054814*** | 0.046798*** | −0.016794 |
| ω       | 0.008961** | 1.233171** | 0.032637*** |
| α       | 0.012456  | 0.084114*** | 0 |
| β       | 0.997064*** | 0.875849*** | 0.898653*** |
| γ       | 0.202255*** | 0.144779*** | 0 |
| η       | 2.790695*** | 0.96962*** | 1.590065*** |
| Persistence | 1.00952 | 0.959963 | 0.898653 |
| Half-Life | 235.7388085 | 16.96373158 | 23.58843566 |

Note: *, **, and *** mean statistical significance at the 0.05, 0.01, and 0.001 critical level, respectively.

Table 3. Model selection

| CRITERION | ERROR | BITCOIN | ETHEREUM | JSE |
|-----------|-------|---------|----------|-----|
|           |       | GARCH   | EGARCH   | GJR-GARCH |
| AIC STD   | 5.3575 | 5.3452  | 5.3580   | 5.9407 |
| BIC       | 5.3564 | 5.3464  | 5.3589   | 5.9398 |
| AIC STD   | 5.3512 | 5.3936  | 5.9709   | 5.9738 |
| BIC STD   | 5.3614 | 5.3580  | 5.3628   | 5.9316 |
| AIC STD   | 5.3885 | 5.3889  | 5.3937   | 5.9587 |

Note: All models fit with an AR(2) component, which is not shown due to space constraints.
Model diagnostics are performed to check the model's adequacy in capturing volatility. The first diagnostic is to examine the models' standardized residuals. The data fit the distribution assumptions reasonably using the QQ plots in Figure 4. The plots show that the heavy-tailed error distributions fit the data reasonably well. The heavier tails of the std error used for Bitcoin and the GED error used for Ethereum and JSE are an improvement from the normal error, which failed to capture the heavy tails. At the ends, a few points deviate from the line, indicating the presence of some rather extreme events. This result is not surprising as financial data produces extreme data points due to random and extreme market shocks. The ACF plots of the residuals are also considered to check if the autocorrelations are removed to give white noise residuals. The ACF plots in Figure 4 show one significant low autocorrelation for all three assets. Despite the minor lag, the models describe the conditional mean adequately. Different GARCH and error combinations were tried, but the lower-order autocorrelation remained. This suggests the need for other models or a change in the error type to be able to remove the significant autocorrelation. A suggestion would be to try extreme value theory distributions.

The conditional volatility plots are presented in Figure 5. It is visible that the volatility is higher in the two cryptocurrencies as compared to the JSE. The similarities in the volatility plots for cryptocurrencies are striking. The scale range is similar between the two plots, with a maximum value of just over 10, and periods of high volatility are the same. For example, the volatility is slightly higher around January/February 2018 for both, coinciding with the 2018 crypto-crush after the 2017 boom. Another spike is witnessed in the cryptocurrencies around February/March 2020, which is not observed in the JSE market. This period coincides with the beginning of the COVID-19 pandemic. In June and July 2020, there was high volatility in the JSE market, but the levels in cryptocurrency were normal. This observation is consistent with other studies using developed markets, such as one by Mariana et al. (2021). Using Bitcoin and Ethereum, they observed concurrent high volatility in the cryptocurrency, which was not observed in the stocks. The same occurred when volatility was high in the stocks. It was not the same in the two cryptocurrencies. Another explanation of the difference in the volatility magnitude is the safe-haven effect explained by Mariana et al. (2021), where investors move funds to cryptocurrency in times of turmoil in the stock market. Therefore, the differing volatility periods suggest using cryptocurrency in a portfolio with JSE as a hedge.

![QQ plots and ACF plots for the standardized residuals](image-url)

*Figure 4. QQ plots and ACF plots for the standardized residuals*
4.1. Structural breaks

Table 5 reports the results of the breakpoints identified from the change point package using the PELT method and a penalty of 45. For Bitcoin and Ethereum, 4 breakpoints each were identified. As expected, the occurrence of these breakpoints were in a similar position (not exact) for three of the four identified breakpoints. This is not surprising as both are in the same market and have a tendency to face similar market shocks. For the JSE, 2 breakpoints were identified, and these did not coincide with the cryptocurrency ones.

The structural breaks are shown visually in Figure 6.

From Table 6, the structural breaks are only significant in the mean equation of the Ethereum

Table 5. Breakpoints identified in the return series

| Asset | Number of breaks | Position of breaks | Breakpoint dates          |
|-------|------------------|--------------------|---------------------------|
| Bitcoin | 4                | 207,906,918,1185  | 2018/04/12, 2020/03/11, 2020/03/23, 2020/12/15 |
| Ethereum | 4                | 485,902,914,1202  | 2019/01/15, 2020/03/07, 2020/03/19, 2021/01/01 |
| JSE    | 2                | 1033,1069          | 2020/07/16, 2020/08/21 |

Table 6. Parameter estimates for models with structural breaks

| Asset | Bitcoin | Ethereum | JSE |
|-------|---------|----------|-----|
| Model | AR(2) + GARCH | AR(2) + GARCH | AR(2)+ GJR–GARCH |
| **µ** | 0.137542** | 0.083347*** | 0.031314 |
| AR1   | −0.077026*** | −0.108463*** | 0.006387 |
| AR2   | 0.054056** | 0.048026*** | −0.016128 |
| S.Break | 2.678021 | −2.489631*** | −1.003498*** |
| **ω** | 0.01207*** | 1.245745** | 0.031556*** |
| α     | 0.011052 | 0.08497*** | 0 |
| β     | 0.995729*** | 0.874469*** | 0.902421*** |
| γ     | 0.201342*** | 0.138688*** | 0 |
| S.Break | 0.321027 | 0 | 0 |
| **η** | 2.824784*** | 0.966816*** | 1.570265*** |
| Persistence | 0.995729 | 0.959439 | 0.971765 |
| Half–Life | 161.9447223 | 16.74004094 | 24.20099041 |

Figure 5. Volatility plots with Bitcoin atop followed by Ethereum and lastly JSE
and the JSE model. For the variance equation, the breakpoint parameter is estimated as 0 for Ethereum and JSE returns but is a positive value for Bitcoin which is however statistically insignificant. A surprising result is a difference in results between Bitcoin and Ethereum. This bizarre result is further interrogated by running the Bitcoin under GJR-GARCH and GARCH model, and the results had a significant break under the mean equation. This revelation shows that the significance of the structural break variable is sensitive to the model used. Therefore, the focus is on the implication of the persistence rather than the significance of the parameter itself.

Regarding the model’s parameters, the same conclusions are attained from the model with structural breaks. For the cryptocurrency, the AR parameters remained significant, while for the JSE, the non-significance is retained. The mean and the alpha parameter of the JSE are insignificant, which agrees with the results of Muguto and Muzindutsi (2022), who also used the structural breaks, which did not change these parameters. Their study revealed a significant structural break for the variance equation. This structural break parameter was 0.0003, close to the zero estimates. Critical for the study is the effect of the structural break on the persistence of volatility and, therefore, the half-life. There is an abundance of research that shows that ignoring the presence of structural breaks causes persistence overestimation in most cases. Using a Monte Carlo simulation experiment, Lamoureux and Lastrapes (1990) showed that structural changes affect persistence as measured by GARCH models. This study’s results show that for cryptocurrency, this is true, in particular for Bitcoin, whose half-life went from 235 to 162 days. Surprisingly for Ethereum, there was a slight increase in persistence, which did not change the half-life. For JSE, the persistence also increased and led to a 1-day increase in the half-life.

The results of this study are of significant importance to investors, especially those investing in the JSE market, who need to make informed decisions on how to fully utilize cryptocurrency in their portfolios. Firstly, the findings reveal how the volatility in the cryptocurrency market is always higher than that of the JSE market. This finding warns risk-averse investors to be wary of investing in cryptocurrency. However, risk-tolerant investors can invest with the knowledge of the persistence patterns and hence can limit potential losses by exiting positions based on the persistence and half-life.
Furthermore, persistence is lower in the JSE market than in the cryptocurrency market, a phenomenon also observed in the developed markets. This observation implies that investors in the JSE can use this persistence to their advantage in the Bitcoin market to have risk-adjusted portfolio positions. However, it should be noted that the same does not hold with Ethereum, as it had lower persistence than that observed in Bitcoin. This Ethereum result is surprising as, generally, Ethereum moves in tandem with Bitcoin. Such a revelation also warns investors not to assume that all cryptocurrencies behave like Bitcoin.

With these results, portfolio allocation can be made with the know-how of the behavior of cryptocurrency. Secondly, given the effect of structural breaks on the persistence and, subsequently, the half-life, there is a need to use models that cater for structural breaks whenever breaks are detected.

Thirdly, there is evidence of an inverse leverage effect in Bitcoin, unlike the leverage effect observed in the JSE. Running the EGARCH for Ethereum also shows that there is inverse leverage. Therefore, investors must be wary of the effect of the positive shocks on increasing volatility rather than the negative shocks. Lastly, based on market inefficiency in cryptocurrencies, especially Bitcoin, some investors who use technical analysis can take advantage of the inefficiency to make abnormal profits. For example, investors, once they identify the structure of the correlations and the level of persistence, can use this to improve their forecasts and, consequently, the potential profitability of their trading strategy. Such findings may indicate the need for some form of cryptocurrency market regulation. The inefficiency also means that traditional asset pricing models such as the CAPM and APT will be inadequate to model returns in these markets (Muguto & Muzindutsi, 2022, p. 21).

Figure 7. QQ plots and ACF plots for residuals for models with structural breaks
Another important takeaway is that the high volatility periods of cryptocurrency and the JSE market do not coincide. This finding means that investors in the JSE market can look into moving funds to cryptocurrency during the turmoil in the JSE market. In essence, Bitcoin and Ethereum act as a safe haven. Such conclusions were also reached by researchers in European and Asian markets, such as Bouri et al. (2017a), Selmi et al. (2018), and Mariana et al. (2021). However, according to Bouri et al. (2017b), the properties of safe havens differ across markets. Other researchers also share such sentiments. Moreover, it has been shown that Bitcoin’s volatility behaves differently across time. López-Cabarcos et al. (2019, p. 5) put it this way, ‘when stock markets are volatile, Bitcoin can be used as a safe haven; when stock markets are stable, Bitcoin becomes appealing to speculative investors.’

CONCLUSION

Because different markets have different behaviors, any findings in one market may not apply to the others. Currently, literature has studies that compare cryptocurrency volatility to stock market volatility, but these focus on developed economies, leaving African economies with generalized conclusions. Such sweeping generalizations may prove fatal for local investors. This study fills the gap in two ways: comparing a developing market and accounting for structural breaks in the comparisons.

The study used GARCH-type models in the absence and presence of structural breaks. The results showed that cryptocurrency possesses most features associated with financial data, such as fat tails, volatility clustering, asymmetry, and persistency. However, it is the magnitude of these features that differs. Similar to developed markets, cryptocurrency volatility is higher than that of the JSE stock market. For the risk-averse investor, they are better off investing in stocks than in cryptocurrency. The presence of structural breaks caused an overestimation of persistence. However, the significance was affected by the model used. A slight difference in persistence has a considerable impact on the half-life. This observation itself casts doubt on the reliability of the half-life measures from GARCH models, as changing models may considerably affect the half-life.

Overall, having an understanding of the nature of the volatility of different assets is essential for investors. In addition, this information can assist African investors in having information calculated from local markets and not relying on generalizations made from developed markets.

A limitation of this paper is that only the JSE is considered as a representative of developing markets, yet there are many other developing markets.

AUTHOR CONTRIBUTIONS

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