The thermomagnetic instability in superconducting films: Threshold magnetic field and temperature

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Abstract. The critical state of superconductors in transverse applied magnetic field is unstable with respect to a thermomagnetic instability, which gives rise to dendritic flux avalanches. We develop a quantitative theory for the threshold magnetic field and temperature for onset of the instability at low temperatures. The theory is confirmed by numerical simulations of the time-dependent distributions of magnetic flux and temperature in superconducting films taking into account the nonlinear and nonlocal electrodynamics, as well as its coupling to the flow of heat. The theory predicts that the thresholds are weakly sensitive to the variation rate of the applied magnetic field, and predicts the existence of undamped oscillations in temperature and electric field at the early stages of instability development.

PACS numbers: 74.25.Ha, 68.60.Dv, 74.78.-w
1. Introduction

The electromagnetic behavior of type-II superconductors exposed to magnetic fields is commonly explained by Bean’s critical state model [1]. The basic idea here is that the Lorentz force on flux lines from the current density, \( j \), is balanced by the pinning force from material defects. The pinning is characterized by the critical current density, \( j_c \), and the model assumes the existence of a metastable critical state with \( j = j_c \). However, the critical state is often showing strong intermittent behavior. In thin-film superconductors placed in a slowly increasing transverse field, large amounts of magnetic flux can suddenly rush in from seemingly random positions on the edges, forming complex branching structures. Such avalanches were observed in films of many materials, e.g., Nb compounds, MgB\(_2\) and YBa\(_2\)Cu\(_3\)O\(_x\) [2, 3, 4, 5, 6, 7].

In MgB\(_2\) – a most promising material for thin-film devices due to its high superconducting transition temperature, \( T_c = 39 \) K, and a large isotropic \( j_c \) [8, 9] – the dendritic flux avalanches are often present above a threshold field \( H_{th} \) and below a threshold temperature \( T_{th} \) [5]. The value of \( H_{th} \) increases with the sample size and substrate temperature [10]. The value \( T_{th} = 10 \) K is more universal, although avalanches nucleated by applied current pulses were observed up to 19 K [11]. At high magnetic fields the flux distribution can again become stable [12], depending on the field ramp rate [13]. At the same time, some samples, in particular films with smooth microstructure, tend to be more stable with respect to avalanches [14, 15, 16]. Suppression of avalanche activity can also be obtained by placing a normal-metal layer adjacent to the superconductor [17, 18, 19].

It is today widely accepted that the dendritic flux avalanches are caused by a thermomagnetic instability [20]. If \( j_c \) is reduced by a fluctuation, e.g., in the temperature, motion of magnetic flux is facilitated, easily leading to dissipation and further temperature increase. This positive feedback can cause a rapid rise in temperature and runaway of magnetic flux motion. Numerical simulations of the coupled Maxwell and heat transfer equations have reproduced both the nucleation and evolution of such avalanches, giving branched structures of striking similarity to those observed in experiments [21, 22, 23].

Linear stability analysis of the governing equations has predicted numerous experimentally observed features of the dendritic flux avalanches, such as the existence and quantitative values of \( H_{th} \) and \( T_{th} \) [24, 21, 10]. At the same time, the electric field, \( E \), has not yet been incorporated into the analytic modeling with the same success. Using \( E \) as an adjustable parameter, the best fit values often exceed the Bean model expectations by orders of magnitude [10]. The suggested explanation for the high \( E \) values is that they are caused by so-called micro-avalanches preceding the formation of dendrites [10]. Micro-avalanches are events of rapid displacement of small amounts of magnetic flux [25, 26, 27]; see [28] for a review. They are often observed in samples displaying also the dendritic flux avalanches [29]. At present time, there is no consensus on the origin of micro-avalanches, and consequently it is difficult to establish a quantitative relationship between the properties of micro-avalanches and the nucleation of the larger dendritic avalanches. Recently, an alternative explanation for the origin of dendritic flux avalanches was suggested based on Fourier space linear stability analysis [30]. The idea is that the Bean state is most unstable with respect to the spatially constant mode, accompanied by temporal oscillations in \( E \) and \( T \). As time goes, the spatially constant mode can – due to the nonlocal electrodynamics giving interaction between modes – develop into a dendritic flux.
avalanche. Consequently, it is possible that dendritic flux avalanches are nucleated directly by the low-\( E \) background from the Bean state, rather than micro-avalanches. This calls for a reexamination of the theory for onset of the thermomagnetic instability.

In this work, we consider the threshold magnetic field \( H_{th} \) and temperature \( T_{th} \) for onset of instability. Our goal is to develop a fully quantitative theory – i.e., a theory without fitting parameters – based on the Bean model expressions for the electric field and flux penetration depth. To verify the validity of the analytical theory, we compare it with numerical simulations solving the full nonlinear problem for the same parameters. Simulation runs at different substrate temperatures will be compared with theory. Also, we search for the temporal oscillations in temperature prior to avalanches predicted by theory.

2. Model

Consider a superconducting strip of thickness \( d \) and width \( 2w \), with \( d \ll w \), placed in an applied field \( H_a \) as shown in figure 1. The strip is in thermal contact with the substrate, which is kept at constant temperature \( T_0 \). The material law (\( E-J \) relation) of the superconductor is, due to flux pinning, taken as strongly nonlinear [31],

\[
\mathbf{E} = \rho \mathbf{J}, \quad \rho = \rho_n \begin{cases} (J/J_c)^{-n-1}, & J < J_c \text{ and } T < T_c, \\ 1, & \text{otherwise}. \end{cases}
\]

(1)

Here the sheet current \( J \) is defined from the current density by \( j = J \delta(z) \), where \( \delta(z) \) is the Dirac delta function. The critical sheet current is \( J_c = d_jc \). This form includes the effect of flux creep through a finite creep exponent \( n \). The Bean model corresponds to the limit \( n \to \infty \). The material law is combined with the Maxwell equations

\[
\nabla \times \mathbf{E} = -\mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = J \delta(z),
\]

(2)

where \( \mu_0 \mathbf{H} = \mathbf{B} \) and \( \nabla \cdot \mathbf{J} = 0 \).

The heat-flow in the strip is governed by the equation

\[
c \dot{T} = \kappa \nabla^2 T - \frac{h}{d}(T - T_0) + \frac{1}{d} \mathbf{J} \cdot \mathbf{E},
\]

(3)

where \( c \) is the specific heat, \( \kappa \) is the thermal conductivity, and \( h \) is the coefficient of heat transfer to the substrate.
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For a quantitative description of a specific material one must specify the temperature dependencies of the thermal and electromagnetic parameters. To represent MgB$_2$ films we use

$$c = c_0(T/T_0)^3, \quad \kappa = \kappa_0(T/T_0)^3, \quad h = h_0(T/T_0)^3, \quad j_c = j_{c0}(1 - T/T_c), \quad n = n_1T_c/T,$$

(4)

with $T_c = 39$ K, $j_{c0} = 10^{11}$ A/m$^2$, $c_0 = 35 \cdot 10^3$ J/m$^3$, $\kappa_0 = 160$ W/Km$^3$, $h_0 = 1.8 \cdot 10^4$ W/Km$^2$, and $n_1 = 50$. The values for $T_c$, $j_{c0}$, $c_0$, and $\kappa_0$ are comparable with the values used in previous theoretical works [10, 22], while $h_0$ is smaller, and $n_1$ is larger, as suggested by recent experiment [32]. As sample dimensions we use $w = 2$ mm and $d = 0.5 \mu$m, and the applied field is ramped from an initial zero-field-cooled state at the rate of $\mu_0\dot{H}_a = 600$ mT/s.

The state of the system is found by solving equations (2) and (3) by discrete time integration. Proper boundary conditions are implemented by an iterative real space-Fourier space hybrid method; see [22, 33] for details. The sample, taken periodic in the $y$ direction, is discretized on a $768 \times 512$ equidistant grid, with some free space in $x$ direction used to implement the boundary conditions.

The simulations serve as numerical experiments to be compared with the predictions from linear stability analysis. In particular, focus is set on (i) the threshold magnetic field, (ii) the threshold temperature, and (iii) possible traces of oscillations prior to avalanche events.

3. The instability thresholds

To derive expressions for the threshold magnetic field and temperature, we consider first the critical state from which the avalanches nucleate. As the applied magnetic field is increased, magnetic flux penetrates gradually from the edges. The flux-penetrated region has constant sheet current $J = J_c \equiv J_c(T_0)$. For a long strip of width of $2w$, the flux density is highest at the edge, $x = -w$, and falls to zero at the flux front at $x = -w + l$. The flux penetration length $l$ increases with $H_a$ as [34]

$$l = w - w / \cosh(\pi H_a/J_c).$$

(5)

The flux motion induces an electric field, which is zero at the flux front and has a maximum value at the edge given by [31]

$$E = \mu_0\dot{H}_aw \tanh(\pi H_a/J_c).$$

(6)

A thermomagnetic instability is nucleated when the electric field reaches the threshold value [30]

$$E_{th}^{(0)} = \frac{1}{n} \frac{\kappa(\pi/2l)^2 + h/d}{\kappa c/\mu_0 d j_c}, \quad \frac{1}{T^*} = \left| \frac{\partial \ln j_c}{\partial T} \right|, \quad (7)$$

This expression was derived using linear stability analysis under the assumption of formation of a spatially constant oscillating mode, for small flux penetration, $l \ll w$. The qualitative interpretation of (7) is straightforward: increased $c$, $\kappa$ or $h$ stabilize the superconductor while increased $n$ or $j_c/T^*$ destabilize it.

Let us consider the quantitative consequences of (7). In the Bean limit, $n \to \infty$, the threshold becomes independent of $\kappa$ and $h$, and the stability criterion is that the
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denominator of (7) should vanish. Using \( l = (w/2) (\pi H_a/d_jc)^2 \) from (5), valid at small \( H_a \), and solving for \( H_a \), one obtains the adiabatic threshold field (c.f. [29, 35])

\[
H_{th}^{\text{(adiab)}} = \sqrt{\frac{2}{\pi \mu_0} \frac{d}{w}}.
\]  

(8)

This field is lower than the adiabatic threshold field in bulks by a factor \( \sqrt{d/w} \). The adiabatic threshold field might be important for conventional superconductors at low temperatures, where flux creep is small.

With finite \( n \) the actual threshold is higher than the adiabatic one due to the lateral heat transport and heat removal to substrate. At sufficiently small \( l \), the lateral heat transport is most important. To isolate the effect, let us thus put \( c = h = 0 \) in (7), which yields

\[
E_{th}^{(0)} = \frac{T^* \kappa}{n j_c} \left( \frac{\pi}{2l} \right)^2.
\]  

(9)

For small flux penetration, both \( l \) and \( E \) are sensitive to \( H_a \). The Bean model expressions (5) and (6) give \( l = (w/2) (\pi H_a/d_jc)^2 \) and \( E = \mu_0 \dot{H}_a w (\pi H_a/d_jc) \). Inserting these formulas into (9) and isolating for \( H_a \) gives the threshold magnetic field as

\[
H_{th}^{(0)} = \frac{d_jc}{\pi} \left( \frac{2 \pi^2 \kappa T^*}{n w^3 j_c \mu_0 H_a} \right)^{1/5}.
\]  

(10)

The threshold magnetic field is a function of \( T_0 \) through \( \kappa, j_c, T^* \), and \( n \).

For deep flux penetration, the above calculated threshold field is not valid, since the most unstable mode has a finite wavelength. In this limit, the threshold penetration depth is [30]

\[
l_{th}^{(1)} = \frac{\pi}{2} \left( \frac{3 c n E}{\mu_0 d_jc} \right)^{1/2} \left( \frac{3 \kappa}{(n E j_c/T^* - h/d)^3} \right)^{1/4},
\]  

(11)

where \( E = \mu_0 \dot{H}_a w \). The threshold field \( H_{th}^{(1)} \) is determined by solving (5) with \( l = l_{th}^{(1)} \) for \( H_a \).

The threshold temperature \( T_{th} \) is defined as the temperature where \( l_{th}^{(1)} = w \), i.e., full penetration is reached without nucleation of instability. In this limit the dominant stabilizing factor is the heat removal to substrate, and an approximate criterion for the threshold temperature is when the denominator of (11) vanishes. We get

\[ n E j_c/T^* - h/d = 0. \]

By solving for \( T_0 \) with the temperature dependencies from (4) one gets the threshold temperature

\[
T_{th}^{(1)} = T_c \left( \frac{\mu_0 \dot{H}_a w d_jc n_l}{T_c h_0} \right)^{1/4}.
\]  

(12)

Equations (10) and (12) are the main results of this paper.
4. Analytical theory versus simulations

Let us conduct a numerical experiment to check the validity of the formulas for $H_{th}(T_0)$ and $T_{th}$, as derived in (10) and (12), respectively. We use the numerical simulation method described in section 2.

As the applied magnetic field is increased, the flux penetration is gradual from the edges, where a critical state is formed with $J = J_c$. When the first dendritic flux avalanche appears, there is a rapid increase in flux motion and temperature, and we identify the threshold field $H_{th}$ by the time $t_0$ when the maximum temperature in the sample reaches the superconductor transition temperature.

Figure 2 shows the $B_z$ distributions just after the first avalanche has ended, for $T_0 = 3 - 10$ K. From the figure it is clear that the depth of the background flux penetration, and the size of the branching structures, increase with $T_0$. For low $T_0$, the avalanches are small structures, with few main branches. With increasing $T_0$, the number of branches increases and in particular the 9 K structure is very large, covering most of the previously flux-free area. At 10 K the sample reaches full flux penetration without nucleation of the instability. This verifies that our set of...
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Figure 3. The threshold magnetic field as a function of the substrate temperature. The points are from simulations, the blue curve is $H_{th}^{(0)}$, and the black curve is $H_{th}^{(1)}$. The dashed red curve is the adiabatic threshold field $H_{th}^{(adiab)}$.

parameters is consistent with the notion of a threshold temperature. The numerical experiment of figure 2 is in excellent agreement with previous actual experiments on MgB$_2$ films [10].

Figure 3 shows the threshold field as a function of the substrate temperature. The points are the $H_{th}$ values extracted from the simulations. Below 7 K, $H_{th}$ increases slowly with $T_0$. Above 8 K it increases much faster and the highest recorded threshold field is 15.3 mT at 9 K. A run at $T_0 = 9.5$ K was stable, which means that threshold temperature is somewhere between 9 and 9.5 K. This is slightly lower than the $T_\text{th} = 10$ K expected from theory through (12). The solid blue line in figure 3 is a plot of $H_{th}^{(0)}$ from (10) while the almost vertical black curve is $H_{th}^{(1)}$ from (11). In combination, the two curves give the threshold field from the linearized theory. At low $T_0$, there is an excellent agreement between the calculated threshold field and the one from simulations. At higher $T_0$, the theoretical curve undershoots the threshold following from simulations. The dashed red curve in the bottom of figure 3 is the adiabatic threshold field calculated by (8). As expected, it is much lower than the other curves, and also much below the points from simulations. This means that the adiabatic criterion does not set the threshold.

The theory behind the thresholds considered in this work predicts that there will be oscillations in temperature and electric field as functions of time prior to avalanches [30], analogous to the oscillations in temperature prior to flux jumps in bulks [36, 37]. Such oscillations are important because they give information about the most unstable mode exactly at the onset of instability. Let us therefore search for oscillation in the simulation results.

Figure 4 shows the elevated temperature $T - T_0$ as function of $t - t_0$ prior to avalanche, for curves at $T_0 = 5$, 7, and 9 K. The temperature $T$ is extracted from the simulations as the maximum temperature in the quarter of the sample where the dendritic structure appears. Just as predicted by the theory, the curves shows pronounced oscillations prior to avalanche. For each curve, there seems to be one dominant frequency, which indicates that there is only one undamped mode. Just before $t_0$, the oscillations grow in amplitude, just as expected for unstable modes. The
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The temperature prior to avalanche as a function of time shows pronounced oscillations, for \( T_0 = 5, 7, \) and \( 9 \) K.

Presence of oscillations means that we can rule out the fingering modes considered in \[35, 10\] as the origin of instability.

In order to extract quantitative information about the onset of instability in the simulation, let us compare it with the theoretical expression for the oscillation frequency of the most unstable mode. By using the same approximations used for derivation of \[10\], the oscillation frequency becomes \[30\]

\[
\omega = \left( \frac{\pi}{2l} \right)^2 \sqrt{\frac{\kappa}{c \pi \mu_0 d_j c}}.
\]

Figure 4. The frequency of oscillations prior to avalanche, for \( T_0 = 4 \), \( 9 \). The points with error bars are extracted from the simulations, while the solid curve is \[13\].
With $E$ from (9) one gets

$$\omega = \mu_0 \dot{H}_a n \sqrt{\frac{2\pi w}{T^* \mu_0 d c}}.$$  \hspace{1cm} (13)

Figure 5 shows the oscillation frequency $\omega$ as a function of temperature $T_0$ in the range 4 to 9 K. The points with error bars are extracted from the simulations by collecting (for each temperature) the peaks of oscillations prior to avalanche, whereas the solid curve is a plot of (13). The points from simulations are in the range $(0.2 - 0.4) \cdot 10^6$ s$^{-1}$, showing a slight decrease with temperature. The analytical result is in rather good agreement, but it overestimates the frequency, in particular at low $T_0$. In any case, the presence of oscillations is a verification that the instability is nucleated by the spatially constant mode or another mode with long wavelength.

For comparison with experiments where $l$ is more accessible than $\dot{H}_a$, one can alternatively express the frequency as

$$\omega = \kappa \frac{j_c}{2l} \left( \frac{\pi}{2l} \right)^2 \sqrt{\frac{\pi T^*}{\mu_0 d c}}.$$  \hspace{1cm} (14)

5. Summary and discussion

The critical state in superconducting films is susceptible for intermittent flux penetration in form of dendritic flux avalanches. Such avalanches are potentially harmful for thin-film devices, e.g. using MgB$_2$ as a material, and it is thus crucial to find criteria for their appearance.

In this work we have considered the criteria for onset of the thermomagnetic instability, which gives rise to dendritic flux avalanches. The raw conditions, formulated by $E$, $l$, and $T$ from previously reported linear stability analysis, have been integrated with the Bean model for films in transverse applied field $H_a$, to yield quantitative formulas for $H_{th}(T_0, \dot{H}_a)$ and $T_{th}(\dot{H}_a)$. These quantities depend solely on the control variables of the experiment, $T_0$ and $\dot{H}_a$, and on the quantities characterizing the specimen. The derived thresholds are only weakly dependent on the rate of change of applied field, $H_{th} \propto \dot{H}_a^{-1/5}$, and $T_{th} \propto \dot{H}_a^{1/4}$.

The analytical formula for $H_{th}(T_0)$ was found to be in excellent agreement with numerical simulations of the full nonlinear and nonlocal dynamical problem using identical parameters. At the same time, the simulated $T_{th}$ was slightly below the analytical estimate. Hence we have verified that the simple threshold formulas from linear stability analysis are in agreement with the actual nonlinear and nonlocal equations governing the system. The simulations also verified the existence of temporal oscillations in temperature prior to avalanches, as predicted by the theory. The parameters used in the comparison and the results were in quantitative agreement with previous actual experiments on MgB$_2$ superconductors.

The results of this work suggest that additional experiments are needed to find the true triggering mechanism for dendritic flux avalanches in MgB$_2$ (and other materials). In such experiments, one should check the validity of (8), (10) and (12). If the experiments follow these predictions, it indicates that the thermomagnetic instability is nucleated directly from the critical state. If not, it indicates that the thermomagnetic instability is nucleated by micro-avalanches or some other event with high $E$. Also temporal oscillations in $T$, $E$, or $B_z$ on the sample edge prior to avalanches is an signature of a thermomagnetic instability nucleated directly from the critical state.
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Acknowledgments

This work was financially supported by the Research Council of Norway.

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