THE ORIGIN OF THE BLUE TILT IN EXTRAGALACTIC GLOBULAR CLUSTER SYSTEMS

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ABSTRACT

Some early-type galaxies show a correlation between color and integrated magnitude among the brighter metal-poor globular clusters (GCs). This phenomenon, known as the blue tilt, implies a mass–metallicity relationship among these clusters. In this paper, we show that self-enrichment in GCs can explain several aspects of the blue tilt and discuss predictions of this scenario.

Key words: galaxies: star clusters – globular clusters: general

1. INTRODUCTION

Several recent photometric studies of extragalactic globular cluster (GC) systems have discovered a novel feature. The blue, metal-poor population of GCs shows a color–magnitude relation at bright magnitudes: the more luminous GCs are redder. This “blue tilt” was first noticed in the GC systems of massive elliptical (E) galaxies (Harris et al. 2006; Strader et al. 2006), but has since been seen in the Sa/S0 NGC 4594 (Spitler et al. 2006), fainter early-type galaxies in Virgo (Mieske et al. 2006), and even in normal spirals (J. Strader et al. 2008, in preparation). Figure 1 shows the blue tilt in M87 (Strader et al. 2006; see also Mieske et al. 2006). The blue tilt is common but not ubiquitous—the massive E NGC 4472 does not have it, for example, and among galaxies that do show a tilt, the slope varies. There is no evidence for a tilt in the red, metal-rich subpopulation of GCs.

Many explanations for this phenomenon have been proposed. These include (1) self-enrichment, (2) “pollution” of the GC sequence with objects such as dwarf galaxy nuclei or merged star clusters (e.g., Bekki et al. 2007), and (3) gravitational capture of metal-rich field stars by GCs (Mieske et al. 2006). A number of other scenarios are discussed in Mieske et al. (2006) but are thought by them to be much less probable.

Option 2 is considered by both Mieske et al. (2006) and Strader et al. (2006), who conclude that it is unlikely. While the nuclei of Virgo dwarfs show their own color–magnitude relation (Lotz et al. 2001), the red mean colors of the luminous GCs are not due solely to the addition of bright red objects, but instead come from systematic shifts in the GC color distribution with magnitude. In addition, the number of nuclei needed is much larger than the handful of stripped nuclei that have been found in either the Virgo or Fornax clusters (see also discussion in Section 3).

Except for a few objects, the sizes of the blue-tilt GCs are generally small, inconsistent with the expectations of merged clusters. Merged clusters should be larger, as the orbital energy of the binary cluster is converted into kinetic energy in the merger product. The best candidate for a merged cluster, NGC 1846 in the Large Magellanic Cloud (Mackey & Nielsen 2007), has an unusually large projected half-light radius of \( \sim 15 \) pc (D. Mackey 2007, private communication). While it seems likely that such exotic objects contribute to the GC systems of massive galaxies at some level, especially at the brightest magnitudes (Harris et al. 2006; Strader et al. 2006), they are unlikely to be primarily responsible for the blue tilt.

Option 3 was investigated by Mieske & Baumgardt (2007) using collisional N-body simulations. They report that field star capture is inefficient even in optimistic circumstances and conclude that this is not the cause of the blue tilt.

Our working assumption is that self-enrichment remains the most likely explanation for the blue tilt. In this case, the basic explanation is that the color–magnitude relation reflects a physical mass–metallicity relation for metal-poor GCs. As discussed in Strader et al. (2006), there are a variety of GC self-enrichment models and simulations in the literature (e.g., Morgan & Lake 1989; Brown et al. 1991; Parmentier et al. 1999; Parmentier & Gilmore 2001; Parmentier 2004; Recchi & Danziger 2005). Here, we refer to models that focus on self-enrichment as the primary determinant of cluster metallicity, and not to the larger body of work on the origin of abundance anomalies in Galactic GCs that primarily involve light elements such as C, N, O, and Mg. Among GC self-enrichment models, there is no consensus on the dominant physical processes or expected level of enrichment as a function of cluster mass. The lack of a tilt in the red GCs might be expected if the self-enriched metals were a constant fraction of the stellar mass, since the overall metal content of red GCs is a factor of \( \sim 10 \) higher than the blue GCs (Strader et al. 2006; Mieske et al. 2006). We illustrate this point further in Section 3.

A possible relationship between GC self-enrichment and a mass–metallicity relation among GCs was discussed for the oldest halo Galactic GCs by Parmentier & Gilmore (2001), prior to the discovery of the blue tilt in external galaxies. However, Parmentier & Gilmore were attempting to explain an anticorrelation between GC metallicity and mass, the opposite of the blue tilt.

Here, we do not attempt detailed modeling of the self-enrichment process (see the studies cited in the previous paragraphs for such models). Our model differs from many of those previously cited in that we invoke a single generation of star formation; within the context of self-enrichment, we expect a spread in metallicity in individual star clusters. This is inconsistent with the chemical homogeneity observed in most Galactic GCs, but the Galaxy does not have a blue tilt (see additional discussion in Section 8).

Our approach is to use scaling relation assumptions about the putative enrichment to explore the conditions under which the...
observed GC mass–metallicity relationships may be reproduced. These conditions may then be used as additional constraints on GC formation.

2. DATA COMPILATION

The blue tilt is parameterized as $Z \sim M^\alpha$, where $Z$ is the GC metallicity, $M$ is the GC mass, and $\alpha$ is the slope of the tilt. Given their small spread in metallicity, the blue GCs are assumed to have a constant $M/L$ ratio. $Z$ must be calculated directly from the GC colors using a color–metallicity conversion. Unfortunately, nearly all of the individual studies have utilized different filters, so different (and possibly inconsistent) color–metallicity conversions must be used. There is no compelling evidence that a more complicated function than a single power law is favored.

The basic method is the same in all papers. The color–magnitude diagram is divided into magnitude bins, and within each bin Gaussian mixture modeling is performed to fit the mean color of each subpopulation within the bin. The slope estimates come from linear fits to the resulting mean colors of each subpopulation as a function of magnitude. The routine typically used for the mixture modeling is Kaye’s Mixture Model (KMM; Ashman et al. 1994), and as discussed in Mieske et al. (2006), KMM can introduce a systematic error into the derived peak locations if there is a large imbalance in the number of objects between the peaks in the usually bimodal color distribution. This can occur if the magnitude binning is done in a blue filter (for example, $B$ or $g$), since the large metallicity differences between the subpopulations lead to different cluster mass-to-light ratios at blue wavelengths. By contrast, red filters (e.g., $R$, $I$, or $z$) show little or no $M/L$ variations in the relevant metallicity range. All of the results cited below utilize a red filter for binning and so should not suffer from this systematic error. We note that mixture modeling with Nmix (Richardson & Green 1997), as in Strader et al. (2006) and Spitler et al. (2006), is more robust to this effect than KMM. Observations of Galactic GCs (e.g., Pryor & Meylan 1993; McLaughlin & van der Marel 2005) also show little or no correlation between metallicity and mass-to-light ratio.

Harris et al. (2006) studied the blue tilt in eight massive Es that are at the center of their respective group or cluster. The slope of the composite blue tilt is $\alpha = 0.55 \pm 0.06$, and the blue tilt appears at $M_I \lesssim -9.5$, corresponding roughly to cluster masses of $M \gtrsim 6 \times 10^9 M_\odot$. Below this “break” magnitude there is no evidence for a variation in the mean color. The diversity of the GC luminosity at which the tilt appears in their sample is worth noting—for NGC 3268 and NGC 3258 it is visible at $M_I \sim -8.5$ (near the GC luminosity function turnover at $2 \sim 3 \times 10^5 M_\odot$), while in NGC 4696 the tilt does not become obvious until $M_I \lesssim -10$ (nearly $10^6 M_\odot$). The photometry is relatively deep and these differences appear genuine. Due to the merging of the blue and red peaks at the brightest magnitudes, and the small number of very luminous GCs present, the precise behavior of the blue tilt at the bright end is unclear. Harris et al. (2006) estimate metallicities using the relation $[\text{Fe}/H] = 2.667 (B - I) - 5.757$, derived from a fit to Galactic GCs. Using this relation, the mean metallicity at the break mass of the blue tilt is $[\text{Fe}/H] \sim -1.4$, increasing to $[\text{Fe}/H] \sim -1.0$ for the most luminous GCs. To illustrate the systematic uncertainties in the color–metallicity conversion, Smith & Strader (2007) find $[\text{Fe}/H] = 1.898 (B - I) - 4.748$ from Galactic GCs with low reddening and $[\text{Fe}/H] < -0.5$; using this relation instead would give a slope $\alpha = 0.39 \pm 0.04$. Since there is some evidence for nonlinearity in color–metallicity relationships (e.g., Strader et al. 2007 and references therein), the latter slope may be more accurate, but this is uncertain at present.

More recently, using Gemini/GMOS imaging of NGC 3311 (the massive central galaxy in the Hydra Cluster), Wehner et al. (2008) found the blue tilt in $g - i$. The estimated slope is $\alpha = 0.6 \pm 0.2$, using a conversion between $g - i$ and metallicity derived from stellar population models: $(g - i) \propto 0.18 [\text{mH}]$. This paper is notable in demonstrating the blue tilt using ground-based imaging: the tilt is not a result of any artifact from Hubble Space Telescope observations (as claimed by Kundu 2008). The break mass is poorly determined but the tilt appears to extend to at least the globular cluster luminosity function (GCLF) turnover.

Spitler et al. (2006) find a slope of $\alpha = 0.29 \pm 0.04$ in NGC 4594 using $B - R$ photometry. In NGC 4594, the blue tilt extends at least to the GCLF turnover (at $\sim 2 \times 10^5 M_\odot$) and perhaps fainter. The exact magnitude at which the tilt begins is uncertain due to the larger photometric errors at these magnitudes. The color–metallicity relation used is $[\text{Fe}/H] = 3.06 (B - R) - 4.90$, derived from Galactic GCs with low reddening and a few NGC 4594 GCs with spectroscopic metallicities at the metal-rich end.

Using $g - z$ colors, Strader et al. (2006) detected the blue tilt in two massive Virgo Es (M87 and M60) with a firm nonde-tection in M49. These same galaxies were studied in the larger sample of Virgo cluster galaxies in Mieske et al. (2006). Since the measured blue-tilt slopes (in $g - z$) for M87 and M60 agree to within the errors for the two papers, we adopt the Mieske et al. (2006) values for consistency. They used the relation $[\text{Fe}/H] = 5.14 (g - z) - 6.21$ (from Peng et al. 2006), derived from Galactic GCs with low reddening and some M49 and M87 GCs with spectroscopic metallicities. This fit excludes objects with high metallicity. The resulting slopes for M87 and M60 are $\alpha = 0.54 \pm 0.19$ and 0.36 $\pm 0.11$. For these massive Es, the blue tilt clearly extends to at least the GCLF turnover, and perhaps even a magnitude fainter, near $10^5 M_\odot$.  

Figure 1. $z$ vs. $g - z$ color–magnitude diagram for GCs in M87, taken from Strader et al. (2006). Mixture modeling of GC colors in different bins in $z$ magnitude yields peak locations, and the overplotted linear fits to these points show the blue tilt (and lack of a corresponding red tilt).
We next consider the case, still assuming a self-gravitating cloud, in which star formation stops when the energy from supernovae is comparable to the initial binding energy of the gas cloud. Since the former is assumed proportional to the mass of stars formed ($M_s$), we have

$$M_s \propto M_g^2 / R$$  \hspace{1cm} (1)

at the cessation of star formation, where $M_g$ is the initial gas mass in the cloud, and $R$ is the radius. This equation assumes that the mass of the protocluster cloud is still dominated by gas at the time when star formation is turned off (i.e., $M_s \ll M_g$).

The mean metallicity $Z_m$ of the cloud and its subsequent GC after self-enrichment scales with $M_s/M_g$ (assuming zero initial metallicity), which leads to

$$Z_m \propto M_g^{(n-1)/(2n-1)}.$$  \hspace{1cm} (4)

A power-law index $n = 2$ corresponds to clouds in virial equilibrium, and $n = 3$ to equal density clouds. Combining relationships gives

$$M_s \propto M_g^{2-(1/n)}.$$  \hspace{1cm} (3)

The asymptotic behavior for large $n$ is $\alpha = 0.5$ (this corresponds to the case in which all clouds have the same radius); steeper slopes are not possible. If $n = 1$, there is no blue tilt.

For this model, we have not specified any value for the efficiency factor of GC formation $\epsilon = M_s/M_g$ (that is, the amount of mass in the protocluster cloud that ends up in the bound star cluster), although the above scenario implies that $\epsilon$ should scale with $M_s$ in an identical manner to the metallicity $Z_m$, i.e.,

$$\epsilon \propto M_s^{(n-1)/(2n-1)}.$$  \hspace{1cm} (5)

By neglecting the contribution of the star cluster to the binding energy of the protocluster gas, we have assumed that the mass of the cluster-formed $M_s$ is significantly smaller than the mass $M_g$ of the cluster protocloud, so the global efficiency $\epsilon$ is small. By contrast, it is generally thought that for an individual star-forming cloud core, the local efficiency must be high ($\sim 0.3$ or higher) for the resulting star cluster to remain bound (e.g., Boily & Kroupa 2003). Thus, our assumptions correspond to a scenario in which the bulk of a star cluster forms from a small core within a larger parent cloud. Within this core, the star-formation efficiency is high, while averaged over the entire cloud it is much lower. This is in accordance with the observations of molecular clouds and young star clusters summarized by Ashman & Zepf (2001). Energy from cluster evolution

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3. MODELS WITHOUT DARK MATTER

3.1. The Cluster Mass–Metallicity Relation

The simplest case we can consider is GC formation from an isolated, self-gravitating gas cloud with no mass loss. The mean metallicity $Z_m$ is determined by the simple closed-box model (e.g., Tinsley 1980), in which case $Z_m$ is equal to the assumed yield. There is no dependence on cluster mass, and thus $\alpha = 0$. In fact, this closed-box model would not predict any metallicity spread among GCs unless there was a range in the stellar yields.

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4 Relaxing this assumption makes the model only solvable numerically in most cases and obscures the relationship between the input parameters and results.
supernovae becomes dispersed throughout the parent cloud and eventually causes star formation to cease, and possibly the dispersal of the parent cloud. We again note here that we assume a single generation of star formation, and expect that self-enrichment will lead to metallicity inhomogeneity in individual GCs.

It would be possible to generalize our scenario to a twogeneration model in which a first generation of supernovae enrich a second generation of stars that forms at some later time, after the SN ejecta have been homogenized into the protocluster gas cloud. This would involve relating the star-formation efficiency of the first stellar generation to the second, and so would introduce more free parameters into the scenario without necessarily producing a chemically homogeneous cluster.

3.2. The Cluster Mass–Radius Relation

The scaling arguments above have not considered the radius $R_e$ of the star cluster that eventually results after gas loss from the parent cloud. We obtain a scaling relationship for $R_e$ by further assuming that the GC forms from a core within a much larger parent cloud in which there is an $r^{-2}$ density distribution, such that the mass within radius $r$ is $M(r) \propto r$. The star cluster forms from a central core of initial radius $R_c$ and mass $M_c$ within this cloud. The star-formation efficiency in the core is $\epsilon_c = M_c/M_s$, which is distinct from the efficiency $\epsilon$ averaged over the entire parent cloud of mass $M_g$. The assumed density distribution within the cloud leads to $M_c/M_g = R_c/R_s$, so that $R_c = (\epsilon/\epsilon_c)R_s$. With relations (2) and (5) we have

$$R_c \propto (1/\epsilon_c)M^*_{g}^{(2n-1)/(2n-1)}.$$  \hfill (6)

Suppose that the core expands adiabatically as gas is lost and the core mass drops from the original value of $M_c$ within a radius $R_c$ to the final mass of $M_s$ and radius $R_s$. If gas is lost slowly compared with a dynamical time then $M_s/R_s = M_c/R_c$ (e.g., Hills 1980; Ashman & Zepf 2001), which leads to

$$R_s \propto (1/\epsilon_c)^2M^*_s^{n/(2n-1)} \propto (1/\epsilon_c)^2M_g.$$  \hfill (7)

If $\epsilon_c$ within the protocluster core scales as $M_g^{1/2}$ then there is no cluster mass–radius relation. For a constant $\epsilon_c$, for all $n$ it is not possible to erase a GC mass–radius relation.

If we set $\epsilon = \epsilon_c$ such that $R_e = R$, i.e., the cluster forms throughout the entire cloud (although this seems less compatible with our assumption of $M_s \ll M_g$), then upon using relations (5) and (7) we get

$$R_s \propto M^*_g^{2-n)/(2n-1)}.$$  \hfill (8)

Thus in the case where a star cluster forms throughout the full volume of the parent cloud, a mass–radius relationship will be erased if $n = 2$. This is analogous to the situation considered by Ashman & Zepf (2001), who by assuming initially virialized gas clouds showed that the GC mass–radius relation can be eliminated if it is assumed that $\epsilon \propto M_g/R$, which in our notation reduces to $M_s \propto M^*_g/R$ (our Equation (1)). The point here is that rather than an arbitrary solution to the lack of a mass–radius relation for GCs, the Ashman & Zepf (2001) condition naturally emerges from the hypothesis that supernova feedback governs star formation in massive clusters. By contrast, Ashman & Zepf (2001) note that $M_c/R$ is the binding energy per unit mass and suggest that a different interpretation of the $\epsilon \propto M_c/R$ condition is that more tightly bound proto-GC clouds can convert gas into stars more efficiently. As shown above, such an assumption will lead to a blue-tilt relation between $Z_m$ and $M_s$ for the resulting GCs.

In our scenario of a GC forming within the central core of a much larger cloud, if we set $\epsilon_c \propto \epsilon$ then we can again retrieve a cluster mass–radius relation of $R_s \propto M$ where $R_s$ is the surface pressure of the cloud, gives a final mass–metallicity relation $Z_m \sim M^{1/3}P^{1/6}$. The dependence on the pressure is weak but could lead to some modification of the cluster metallicities in different environments.

In the above discussion, we have neglected the contributions of any pre-enrichment to the metal content of a GC protocluster. While this might be approximately true for blue GCs, it is unlikely to be the case for the red GC subpopulation of a galaxy. In fact, a self-enrichment origin for the blue tilt could be consistent with the lack of a metallicity–mass relation among the red subpopulations of GCs within galaxies. Suppose that the initial metallicity of a blue GC protocluster is $-1.5$ dex, equivalent to an initial $Z = 6 \times 10^{-5}$. This is the mean pre-enrichment metallicity of a blue GC. Further, suppose that self-enrichment leads to a metallicity for the final GC that is 0.4 dex higher (a typical value for the brightest blue GCs), by adding an extra 1.5 $Z_i$ metals per unit mass. In the case of a red GC, the initial pre-enrichment metallicity might be as much as 10$Z_i$. In this case, adding another 1.5$Z_i$ of metals per unit mass (assuming that the amount of enrichment does not scale as $Z_i$) gives an enhancement in metallicity of 0.06 dex. It is unlikely that such an effect could be readily detectable as a “red tilt.”

3.3. Globular Cluster and Cloud Mass Functions

An observable implication of this model is that the mass function of the GCs is, in most cases, flatter than that of the clouds themselves. This conclusion was reached by Ashman & Zepf (2001), whose results we recover as a specific case below.

First, we consider the case where star formation is dispersed throughout the cloud. Assume that the protocluster clouds and GCs have power-law mass functions over the mass range of relevance, with slopes $\beta_i$ and $\beta_f$, respectively. We then have

$$M^{-\beta_i}_s dM_s \propto M^{-\beta_f}_g dM_g dM_s.$$  \hfill (9)

Using Equation (3) and equating the exponents, we derive the general equation for $\beta_f$,

$$\beta_f = \frac{n(\beta_i + 1) - 1}{2n - 1}. \hfill (10)$$

For virialized clouds with $n = 2$, $\beta_f = 2(\beta_i + 1)/3$, recovering the result of Ashman & Zepf (2001). $\beta_f < \beta_i$ for all $n > 1$, and for large $n$ the equation approaches $\beta_f = (\beta_i + 1)/2$.

Rearranging the equation $\alpha = (n - 1)/(2n - 1)$, we can derive the predicted slope of the blue tilt $\alpha$ as a function of $\beta_i$ and $\beta_f$:

$$\alpha = (\beta_i - \beta_f)/(\beta_i - 1). \hfill (11)$$
This equation is remarkable in that it relates the predicted blue-tilt slope $\alpha$ to the quantities $\beta_f$ and $\beta_i$. For the case $\beta_f = 2$, this equation reduces to the admirably simple $\alpha = 2 - \beta_i$.

Next, we consider the case governed by Equation (7) in which the GC forms in the core of a star-forming cloud. We assume that the cloud core mass function follows a power law with slope $\gamma_i$. To make the problem tractable, we also assume the condition necessary to erase a GC mass–radius relationship, $\epsilon_c \propto M_c^{1/2}$. Combining this condition with Equation (3) gives an equation analogous to (10):

$$\beta_f = \frac{n}{2(n-1)} + \gamma_i \left[ 1 - \frac{n}{2(n-1)} \right]. \quad (12)$$

Though (12) looks different from (10), for the special case $n = 2$, it reduces to the equivalent result $\beta_f = 2(\gamma_i + 1)/3$. This is due to the self-similar nature of the cloud. For other $n$, the cloud and core results are generally similar but not identical. For example, if $n = 3$ in (10), then for $\beta_i = 2$, $\beta_f = 1.6$. In the core case, for $n = 3$ and $\gamma_i = 2$, $\beta_f = 1.7$.

Replacing $n$ with $\alpha$ gives

$$\alpha = \frac{2\beta_f - (\gamma_i + 1)}{\gamma_i - 1}. \quad (13)$$

At fixed $\alpha$, the difference between the cloud or core mass function slopes $\beta_i$ and $\gamma_i$ and the GC slope $\beta_f$ is smaller in the core case for the observed range of $\alpha$, $0.3 \lesssim \alpha \lesssim 0.5$. Recall that in this model $\alpha > 0.5$ cannot be produced.

One can imagine two different applications of these equations. For observed cloud (or core) and GC mass function slopes, $\alpha$ can be predicted. Alternatively, if one assumes that our self-enrichment model is correct, then one can use the observed values of $\alpha$ and $\beta_f$ in a galaxy to deduce the initial values of $\gamma_i$ or $\beta_i$.

### 4. DARK MATTER MODEL

Here, we investigate the case in which GCs form in individual dark matter halos, that is, they are “mini-galaxies.” In a proper cosmological setting, the clouds considered in Section 3 would also be hosted by dark matter halos. The crucial distinction is between a scenario in which the clouds are self-gravitating and that in which the potential well of the host halo dominates.

The basic model for the evolution of a GC in a dark matter halo is similar to that discussed in Section 3. Here, we follow the derivation in Dekel & Woo (2003). The mass of the proto-GC cloud is much smaller than that of the dark matter halo, and thus the formation of the cluster has essentially no effect on the halo. Star formation halts when $M_c \propto M_{\text{vir}} V_{\text{vir}}^2$, where $M$ and $V$ are the virial mass and velocity of the halo, respectively. These two quantities are related by $M_{\text{vir}} \propto a V_{\text{vir}}^3$ with $a = (1 + z)^{-1}$ being the expansion factor at virialization. Substitution yields $Z \sim M^{2/3} a^{-3/5}$. For low-mass halos the range in $a$ is expected to be small, so one simply reproduces the well-known result that $Z \propto M^{2/5}$. This is a good match to dwarf galaxies in the local universe, suggesting that their properties are consistent with supernova feedback in dominant dark halos. The slope is also consistent with a typical value for the blue tilt.

However, we must consider the other scaling relations expected in this model. Assuming a constant $a$, the radius of the cluster is given by $R_c \propto \lambda M_c^{1/5}$, where $\lambda$ is the dimensionless spin parameter of the halo. We would then require $\lambda \propto M_{\text{vir}}^{-1/5}$ to erase the mass–radius relationship (this same point is noted in Cen 2001). More crucial is the absolute value of $\lambda$; halos with typical values of $\lambda \sim 0.02$–0.03 have too much angular momentum to collapse to anything approaching the size of a GC. Even if one assumed that GCs came preferably from the low-$\lambda$ tail of the spin distribution—and the values would have to be quite small, naively a factor of $\sim 50$–100 lower than low-mass dwarf galaxies—one would still expect to see a much larger population of objects with sizes between those of GCs and dwarf galaxies that came from “intermediate” low-spin halos.

We can move a step further back and assume instead that we circumvent the angular momentum problem by forming the GC as the baryonic core of a more massive protogalaxy. The GC would be analogous to the nuclei of dwarf galaxies in the local universe. Such a scenario has been invoked for GCs such as $\omega$ Centauri (e.g., Zinnecker et al. 1988; Bekki & Freeman 2003) and more generally by Böker (2008). An upper limit to the expected effect of dark matter can be found by assuming a cusped Navarro–Frenk–White (NFW) profile for the dark matter; actual dwarfs may instead have cored dark matter profiles (e.g., Burkert 1995). NFW models of Milky Way dSphs, corresponding to virial masses of $\gtrsim 10^7 M_\odot$, have dark matter masses within the central $\sim 10$ pc of $\sim 10^4 M_\odot$, or slightly higher (e.g., Walker et al. 2007). This suggests that the dark matter would have little dynamical effect on a nuclear GC of mass $\sim 10^6 M_\odot$, and that the evolution would be similar to the self-gravitating cloud model of Section 3.

However, there are other issues with the stripped nuclei scenario. It is uncertain whether the halo and other associated baryonic material can effectively be stripped for hundreds of objects in a given galaxy, and whether the expected structural parameters of these nuclei can match that of the GCs. For example, Côté et al. (2006) find that the luminosity function of nuclei in Virgo cluster galaxies peaks $\sim 3.5$ mag brighter than the GCs; the nuclei also show a steep mass–radius relationship. Such observations are seemingly not in accordance with the GC–dwarf nuclei scenario, but perhaps biasing arguments could help address these issues.

### 5. SUBSEQUENT EVOLUTION OF THE BLUE TILT

The models described in Sections 3 and 4 strictly apply only to an initial mass–metallicity relationship for the metal-poor GCs. Subsequent mass loss from the GCs will modify the relation. This in fact would be the case for any scenario in which the blue tilt is a primordial phenomenon, whether caused by the internal self-enrichment of proto-GCs or by some other process related to the cluster environment at early times.

There are three principal sources of mass loss from star clusters: (1) prompt early gas expulsion, with an accompanying removal of stars, (2) mass loss due to stellar evolution, as stars eject gas via stellar winds and planetary nebula shells, and (3) slow mass loss from two-body relaxation and subsequent evaporation that lead to the loss of individual stars from a cluster. We consider these in turn.

The removal of gas from nascent GCs due to supernovae weakens the cluster potential and unbinds a fraction of the stellar mass of the cluster. How much mass is lost depends on the timescale of the gas removal (compared with the dynamical timescale of the GC) and on the efficiency of star formation. Numerical simulations indicate
that high efficiency ($\epsilon_c$ in Section 3) and slow gas removal lead to little mass loss, consistent with our assumptions in Sections 3.1 and 3.2 (Baumgardt & Kroupa 2007; Parmentier et al. 2008). For high enough efficiencies (say $\epsilon_c \gtrsim 0.6 - 0.7$), the timescale of the gas removal becomes less important. This is especially true if the tidal field of the GC is weak, as one might expect for metal-poor GCs forming in the proto-halos of their host galaxies.

We conclude that prompt mass loss from gas expulsion does not substantially alter the slope of the blue tilt under the assumptions of our model. However, for lower star-formation efficiencies or for very fast gas expulsion, our assumptions do not hold and the masses of the clusters may be changed substantially. This could act to induce scatter in the blue tilt, or erase it altogether. Therefore, the existence of a blue tilt may favor, or even require, high star-formation efficiency in the core of a proto-GC cloud.

The second source of mass loss is stellar evolution. All stars lose mass through winds, and more massive stars generally have higher mass loss rates. Models of the evolution of single stellar populations suggest that this form of mass loss is a constant fraction of the cluster mass for a given initial mass function (IMF)—see, e.g., the implementation in the models of Fall & Zhang (2001). Thus, mass loss from stellar evolution will change only the zero point of the blue tilt, not its slope, as long as the IMF of clusters does not vary. If we assumed that the IMF was a strong function of metallicity, then this could have a small effect on the slope of the blue tilt.

We finally consider mass loss from the slow dynamical evolution of star clusters. Two-body relaxation leads to energy equipartition and a quasi-Maxwellian distribution of velocities, and the stars in the tail of the distribution evaporate into the field. A number of authors have investigated this form of mass loss, from both theoretical and observational standpoints. In particular, Jordán et al. (2007) use data from the Advanced Camera for Surveys (ACS) Virgo Cluster Survey to argue that the GC mass functions of early-type galaxies in Virgo are best fit by a model in which the amount of mass lost is independent of the initial mass of the GC and is approximately equal to the classic “turnover” at $\sim 2 \times 10^5 M_\odot$.

There is no closed-form general expression to relate an initial blue-tilt slope to that after a constant amount of mass loss from all clusters. In particular, the power-law form is not retained, although the deviations from a power-law are small for typical parameters and would not be detectable in current data. A simple example demonstrates the basic features of the model. A typical observed blue tilt extends from $\sim 2 \times 10^5 M_\odot$ to $10^6 M_\odot$, with $\alpha = 0.4$. Adding in a constant mass loss of $2 \times 10^5 M_\odot$ and refitting over the entire mass range gives an initial slope of $\sim 0.59$. Thus, constant mass loss leads to shallower observed slopes. Less mass loss naturally produces a smaller change; for example, mass loss of $10^5 M_\odot$ would yield an initial slope of 0.50. Constant mass loss also has a smaller effect if only massive clusters are considered; for example, in some of the giant ellipticals in Harris et al. (2006), the break mass of the blue tilt is $\sim 10^6 M_\odot$. For these galaxies, constant mass loss of $2 \times 10^5 M_\odot$ would produce little change in the blue-tilt slope.

Overall, we conclude that mass loss subsequent to GC formation is likely to change both the slope and zero point of the observed blue tilt from its initial value. These changes should be taken into account in attempts to derive initial star-formation conditions from present-day blue tilts.

6. BLUE-TILT VARIATIONS

As described in Section 2, there are observed variations in both the slope of the blue tilt and the GC mass at which it becomes evident. While some of these variations may be due to modifications of GC masses subsequent to cluster formation (as discussed in Section 5), might there also be sources of intrinsic variation in any of the models discussed?

Within the context of a self-enrichment scenario, Recchi & Danziger (2005) found that dispersion in a GC mass–metallicity relation could be attributed to variations in the thermalization and mixing efficiencies of supernova ejecta, the stellar IMF, and external protocloud pressure. In the context of the scaling relations presented above, the slope of the blue tilt in the self-enrichment, self-gravitating model depends upon the initial mass–radius relation of the clouds. If the initial mass–radius relationship for clouds varied among galaxies, this would lead to differences in the blue-tilt slope. A relation between the surface pressure of the cloud and its mass might also change the expected tilt, although the dependence on pressure is weak. In addition, any scatter in the mass–radius relationship among GC parent clouds would act to erase a blue tilt among the resultant clusters. Thus, the presence or the absence of a blue tilt may reflect how well the parent clouds in a galaxy followed a well-behaved mass–radius law. In addition, the absence of a blue tilt in some galaxies could also be attributed to a protocloud mass–radius relation with $n = 1$.

The blue GC populations of present-day galaxies are expected to have assembled hierarchically through the merging of less massive galaxies. We may then speculate that an additional factor in blue-tilt variations among galaxies is the respective merging histories of the galaxies. If metal-poor GCs formed in low-mass halos at relatively high redshift (e.g., Cen 2001; Bromm & Clarke 2002; Rhode et al. 2005; Brodie & Strader 2006), then the blue tilt is expected to have been in place since then. The subsequent merging of halos with similar blue tilts would then produce a galaxy with that tilt; the merging of halos with different blue tilts (due, for example, to differing cloud mass–radius relationships as discussed above) could weaken or erase the progenitor tilts.

Nearly all of the blue tilts studied thus far appear to have a GC break mass at which the tilt appears. This mass varies among galaxies. We have assumed that GC self-enrichment is controlled by an energy balance between supernova feedback and the binding energy of the clouds. It is this balance that determines the ratio that results between parent cloud mass and the mass of cluster stars ultimately formed, which in turn sets the level of metallicity enrichment in the protocloud. This energy balance could be changed in several ways. For example, the fraction of supernovae energy that goes into cloud disruption might vary with cloud density. This could occur if the ratio between the radiative timescale of supernovae and the dynamical time of the cloud varied substantially with the cloud mass. If the radiative timescale had a strong inverse dependence on cloud density, then less dense clouds (equivalent to more massive clouds in virial equilibrium) might require more energy per unit gas mass to halt star formation. However, Dekel & Silk (1986) argue that this ratio is approximately constant for the relevant range of parameters, and it is not clear why it would vary among galaxies.

Another way to affect the energy balance is if the supernova energy in a single cloud does not dominate the overall energy input, for example, if the clouds are not isolated, but have energy
input from nearby star-forming regions. Consider a situation in which all of the proto-GC clouds are bathed with a field of uniform energy density. In the high-mass clouds this might be only a fraction of the energy from their own supernovae, and thus would have little effect. However, in clouds of lower mass, this additional energy input could serve to disrupt star formation before significant self-enrichment had occurred. For such clouds, the metallicity would then be set by the starting metallicity of the cloud. The initial cloud metallicity, set by pre-enrichment, should be independent of cloud mass. In this (speculative) scenario, the explanation for different blue-tilt break masses would be variations among galaxies in external energy sources during GC formation. However, it is not clear that the proposed mechanism for erasing the GC mass–radius relation would still be relevant if external energy input was important during cluster formation.

For the dark matter scenario, there seem to be fewer natural avenues for variations in the slope of the blue tilt, so if further studies strengthen the case for substantial differences in slope among galaxies, this could be evidence against this model. Our simple suggestion for variations in the break mass due to ambient radiation would naively seem to work equally well in the dark matter case.

7. THE BLUE TILT AND GALAXY–GLOBULAR CLUSTER CORRELATIONS

In the last few years, it has become clear that there are correlations between the mean color (and thus metallicity) of both old metal-poor and metal-rich GCs and the stellar mass of their parent galaxies (Larsen et al. 2001; Forbes & Forte 2001; Strader et al. 2004, 2006; Lotz et al. 2004; Peng et al. 2006). Here, we consider whether the blue tilt could have a significant effect on the relation between the mean color of the blue GC population and galaxy luminosity.

7.1. The Blue Tilt and Mean Globular Cluster Colors

The mean color of blue GCs is generally determined from the brighter clusters, due to incompleteness and larger photometric errors for fainter GCs. The existence of the blue tilt implies a dependence of the estimated mean color on the limiting magnitude of the study. Let us take M87 as an example. Assuming that the bright half of the blue GCLF is approximately lognormal, we use the parameters derived in Strader et al. (2006) for the $g$ band ($\mu_g = -8.2; \sigma_g = 0.92$). Mieske et al. (2006) and Strader et al. (2006) find very similar slopes for $g - z$ versus $z$: $0.042$ and $0.043$, respectively. Integrating from the GCLF turnover to 3 mag brighter gives an overall mean $g - z$ color 0.03 mag redder than the value at the turnover. In a “worst case” scenario in which the photometry extends only to 1 mag brighter than the turnover, the difference is large, with a mean color 0.06 mag redder than the turnover value.

The mean color derived from very deep photometry depends on the behavior of the blue tilt at faint magnitudes. If we assume that there is no tilt fainter than the turnover, then deep photometry can effectively dilute the blue tilt. For example, if the photometry extended to 1 mag beyond the turnover, the mean color would be less than 0.02 mag redder than the turnover color.

By influencing the mean color that is derived for a GC system, the blue tilt can conceivably contribute to the relation observed between mean GC color and parent galaxy luminosity. To assess whether this effect might be significant, let us consider the specific case of the relation obtained by Strader et al. (2006) between the mean color of the blue GCs and the parent galaxy luminosity. They found $(g - z) \propto -0.014 M_B$, implying a $(g - z)$ color difference of $\sim 0.07$ mag between the most luminous Es at $M_B \sim -21.5$ and the bulk of the dwarf ellipticals with $-17 \gtrsim M_B \gtrsim -16$. The largest expected effect of the blue tilt on the relationship between $M_B$ and $(g - z)$ for blue GC systems can be estimated by assuming that all of the massive Es have blue tilts similar to M87 and that all dwarf Es have no tilt. In reality, some massive Es (like NGC 4472) have no tilt, and there may be a weak tilt in the dwarfs (Mieske et al. 2006).

The $(g - z)$ colors in Strader et al. (2006) were estimated from photometry that reached typically 0.5–0.7 mag fainter in $z$ than the GCLF turnover. Using the model outlined above, then expect the massive Es to have mean colors $\sim 0.02$ mag redder ($\sim 0.1$ dex more metal rich) than the turnover color. Since the observed $(g - z)$ color of blue GCs is $\sim 0.07$ mag redder in giant Es than in dwarf Es, the blue tilt does not have a dominant effect on the slope of the blue GC mean color–galaxy luminosity relation, even using generous assumptions.

Even though galaxy-to-galaxy variations in the blue tilt do not appear likely to explain the relation between mean GC colors and parent galaxy properties in extragalactic GC systems, it is worth formalizing more precisely how the mean color of a GC system is to be defined. In the presence of a blue tilt, perhaps some fiducial magnitude range should be defined for the GCs from which the mean color is determined, or else the GC color at some reference magnitude such as the peak in the GCLF could be adopted. With sufficiently deep observations it would be valuable to determine if there is a correlation between parent galaxy magnitude and the mean color of those GCs that are fainter than the blue-tilt break magnitude.

7.2. Setting the Red Globular Cluster Metallicities

Regardless of these pragmatic issues, there is the deeper question of whether the origin of the blue tilt relates to the correlations that have been identified between galaxy absolute magnitude and the mean metallicities of both blue and red GC subpopulations. The blue tilt, as we have interpreted it in Section 3, is a product of the self-enrichment of GC protoclouds. By contrast, the mean GC color–galaxy luminosity relations refer to the average metallicity of ensembles of clusters. Given that the red GCs do show a correlation between mean color and parent galaxy luminosity, but do not show a red equivalent of the blue-tilt phenomenon, we would infer within the context of Section 3 that self-enrichment is not the underlying cause of the color–galaxy luminosity relation for red GCs. Rather this later relation would be attributed to the result of their parent galaxies influencing the pre-enrichment level of a red GC. In the case of the red GCs, this pre-enrichment level swamps any self-enrichment component, as discussed in Section 3.

If the red GCs trace the bulk chemical evolution of the galaxy spheroid then an “open-box model” with mass loss could apply. Such models were introduced for the GC system of the Milky Way by Hartwick (1976) and Searle & Zinn (1978) and have been applied more recently by VanDalsen & Harris (2004). In this case, the mean metallicity of the red GCs would depend
on two factors: (1) the ratio between the star-formation rate and the rate of mass loss from the spheroid component of which the red GCs are members and (2) the gas-to-star mass ratio at the end of the era of red GC formation (which might have been quite high). Systematic variations of either one of these factors could produce a red GC color–galaxy luminosity relation.

If the red GCs form as part of an early spheroidal system that behaves like a “closed box” (no mass loss from the spheroid), and they trace the chemical evolution of that larger halo (as opposed to being self-enriched), then the characteristic metallicity of the red GCs is $Z_{\text{red}} \sim Z_* + \gamma M/\dot{M}$, where $\gamma$ is the stellar yield, $M_*$ is the total mass within the galaxy that has been formed into stars at the time when red GC formation concludes, $\dot{M}$ is the gas mass of the galaxy, and $Z_*$ is the initial metallicity of the spheroid. (This approximation comes from the closed-box relation $Z = Z_* + \gamma \ln (\dot{M}_{\text{tot}}/\dot{M}_*)$ in the limit where $\dot{M}_* \ll \dot{M}$ for $M_{\text{tot}} = M_* + M_g$. Thus, there would be a relation between the average metallicity of red GCs and the efficiency of star formation during the early stages of galaxy chemical evolution wherein the red GCs are formed. In such a context, the observation that $Z_{\text{red}}$ correlates with the absolute magnitude of the parent galaxy indicates that in more massive galaxies the early star-formation efficiency was greater. This argument gets of the parent galaxy indicates that in more massive galaxies the halo (as opposed to being self-enriched), then the characteristic relation.

These factors could produce a red GC color–galaxy luminosity relation.

Most of the break masses are poorly determined, so deeper imaging of selected galaxies would be beneficial.

The assumption of a variable formation efficiency can be tested directly by measuring cloud and cluster masses in nearby systems of young massive clusters, as suggested by Ashman & Zepf (2001). With ALMA, accurate cloud masses and radii can be determined in such systems, testing both the efficiency of cluster formation and the initial mass–radius relationship for clouds as a function of external parameters such as pressure.

Self-enrichment has been separately proposed to explain a number of abundance anomalies in Galactic GCs, primarily involving light elements such as C, N, O, and Mg. The culprit is generally argued to be an unknown combination of massive main-sequence stars and intermediate-mass asymptotic giant branch (AGB) stars. A natural expectation of self-enrichment models for the blue tilt would be the appearance of abundance anomalies associated with blue-tilt GCs. The integrated abundances of C, N, and Mg can all be estimated in extragalactic GCs from low-resolution spectra. For example, Cenarro et al. (2007) find CN line strengths for blue-tilt GCs in NGC 1407 that are much larger than in Galactic GCs at comparable metallicities; unfortunately, less luminous GCs in NGC 1407 have not been observed. Studying the abundance patterns of GCs both above and below the blue-tilt break mass could constrain the timing and duration of the hypothesized self-enrichment events.

The self-enrichment model makes another prediction that was described in Sections 1 and 3. Unless star formation in the GC is oddly segregated by mass, such that the high-mass stars form, explode as supernovae, and enrich the intracluster gas before any low-mass stars form, then the blue-tilt GCs would be expected to have internal dispersions in not just the lighter elements such as C through Mg, but also in heavier elements produced by massive star supernovae. In the Milky Way, the only GC known to have a large star-to-star spread in the $\alpha$-elements heavier than Mg is $\omega$ Cen (e.g., Freeman & Rodgers 1975; Smith et al. 2000), a cluster that has been viewed as a test case of self-enrichment and chemical evolution (e.g., Carraro & Lia 2000; Ikuta & Arimoto 2000; Platais et al. 2003; Marcolini et al. 2007), and possibly M22 (Norris & Freeman 1983; Lehnert et al. 1991; Anthony-Twarog et al. 1995). The majority of Galactic GCs are homogeneous to a high degree in the $\alpha$-elements.

However, the Milky Way GC system is thought not to exhibit a blue tilt (although see Parmentier & Gilmore 2001), so there is no conflict with the self-enrichment scenario for this phenomenon. However, the challenge in other galaxies could be twofold: (1) if blue-tilt GCs are chemically homogeneous, then the self-enrichment picture may be less compelling; (2) if blue-tilt GCs are typically inhomogeneous in heavy elements, why is this not true in Galactic GCs of comparable mass? Regarding (1), it is intriguing that four of the most massive GCs in M31 have large internal metallicity spreads (Fuentes-Carrera et al. 2008). Forthcoming homogeneous photometric and spectroscopic data sets should allow the detection of a blue tilt in M31, if it exists. From a theoretical standpoint, self-enrichment models such as

8. DISCUSSION

We have examined a number of scenarios for the blue tilt, a mass–metallicity relationship for metal-poor GCs observed in many external galaxies. The model that seems to best reproduce current observations is self-enrichment in proto-GC clouds; in this model, star formation is governed by supernova feedback, and the efficiency is a function of the protocloud mass. Certain efficiency scalings produce no mass–radius relation for the resultant GC (in accordance with observations) and are equivalent to an energy balance condition for star formation. We have also speculated as to how the observed variations in the slope of the blue tilt and the break mass might be reproduced. Contrary to the comments in Strader et al. (2006), a dark matter halo is not necessarily required for effective self-enrichment, at least given the assumptions of the current model.
those of Brown et al. (1991) attempt to reconcile supernova-induced enrichment with chemical homogeneity. With regard to (2), we suggest reluctantly that it might be necessary to invoke fundamental differences between the parent clouds of Galactic GCs and those in blue-tilt galaxies.

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