Bose-Einstein condensates (BECs) have become a cornerstone of current developments in quantum simulation [1], and one would expect a complete description of their properties to be available. However, in spite of considerable theoretical effort, the atom number fluctuations between the thermal and condensed component of an interacting Bose gas at relevant atom densities are still not fully understood. In principle, such a quantum system can be described through all moments of its probability distribution, which have indeed not been obtained for large interacting BECs [2–6]. To date, experiments have been limited to the first moment corresponding to the condensate fraction [7–9] as well as the critical temperature indicating the onset of condensation [10, 11]. Only recently BEC atom number fluctuations corresponding to the second moment have become experimentally accessible [12].

These fluctuations pose a subtle problem with surprising challenges and rich physics [6]. Historically, a description of the non-interacting Bose-gas was developed within the grand canonical ensemble, which shows unphysically large fluctuations below the critical temperature [13]. This is referred to as the grand canonical catastrophe [14, 15], and is one of the few examples where the predictions of different statistical ensembles differ dramatically. Thus, any description of the fluctuations must be based on canonical or microcanonical approaches.

In the canonical ensemble, the asymptotic behavior of the condensate atom number variance in the non-interacting Bose gas was first obtained by Politzer [2]. Later, exact numerical recursion relations for this non-interacting case were found [16], which recently allowed for an initial comparison with experimental results [12]. However, the canonical ensemble assumes existence of an external reservoir exchanging energy with the system under investigation. This assumption is not met in the case of an ultracold gas isolated from the environment. Therefore, the microcanonical ensemble needs to be invoked, which is numerically demanding and currently does not allow for a full comparison at typical experimental parameters.

In the microcanonical ensemble the key quantity is the partition function $\Gamma(E, N)$, i.e. the number of ways to distribute the energy $E$ between $N$ atoms. Calculation of $\Gamma(E, N)$ for

\[ \Delta N^2 \propto E^{1.34} \]

FIG. 1. Atom number fluctuations of a BEC. (a) Calculated variance of BEC atom number as a function of relative temperature in the canonical (red diamonds) and microcanonical (blue circles) ensembles, without (filled symbols) and with perturbatively included interaction (empty symbols) for $N = 1000$ $^{87}$Rb atoms at an aspect ratio of $\lambda = 10$. (b) Experimental variance of BEC atom number as a function of temperature (blue points). The vertical error bars are statistical uncertainties of the variance. The dashed line is a fit (see text), where the blue shaded band represents the uncertainty of the fitted model. The gray area indicates the fitted offset due to technical fluctuations. The data was acquired at an aspect ratio of $\lambda = 7.28(2)$ and atom number $N_p = 1.49(6) \times 10^8$. The results are plotted as a function of the temperature rescaled with the temperature corresponding to the peak fluctuations. The solid black lines in both panels are exact canonical calculations for a non-interacting gas [16].
a non-interacting gas in a harmonic trap is closely related to the classic mathematical problem of partitions. This was studied extensively inspired by discussions between Leibnitz and Bernoulli in 1674 [17], with breakthroughs achieved a century ago by Ramanujan and Hardy [18]. In particular it was shown that the canonical and microcanonical fluctuations agree for large atom number in the non-interacting 1D gas [19].

The results for the 3D case in the microcanonical ensemble were not known until 1997 [3], when a fourth statistical ensemble, called the Maxwell’s Demon ensemble, was proposed. In this ensemble the system exchanges particles with the reservoir, without changing its energy. It is applied to the thermal cloud at sufficiently low temperatures, where the BEC comprises the reservoir. The ensemble yields an asymptotic expression for the microcanonical fluctuations in the non-interacting gas [20]

$$\Delta N_0^2 = \left( \frac{\zeta(2)}{\zeta(3)} - \frac{3\zeta(3)}{4\zeta(4)} \right) N \left( \frac{T}{T_c} \right)^3.$$  (1)

Here $T_c^0$ is the critical temperature of a non-interacting gas in a harmonic trap and $\zeta(x)$ is the Riemann zeta-function. In particular, the second term corresponds to the reduction in the number fluctuations in the microcanonical ensemble with respect to the canonical result given by the first term. Thus, the ratio between the canonical and microcanonical fluctuations in the 3D isotropic case is 0.39, corresponding to a reduction by 61%.

Importantly, interactions are expected to modify the fluctuations of the condensate atom number $\Delta N_0^2$. In the homogeneous case, initial results showed that interactions suppress the fluctuations by a factor of two at very low temperatures due to a strong pair-correlation of the atoms reducing the number of degrees of freedom [4, 5, 21]. Moreover, interactions have been included using numerous methods such as the aforementioned Maxwell’s Demon ensemble [22], number-conserving quasi-particle methods [23, 24], master-equation and hybrid quasi-particle approaches [25, 26], and a correlated many-body approach [27]. However, only some of these methods are based on a microcanonical ensemble and none of them allow for the analysis of microcanonically trapped interacting BECs at typical experimental atom numbers. Moreover, most approaches are only valid at low temperature and do not capture the peak fluctuations near $T_c^0$.

Additionally, the fluctuations are expected to scale anomalously with atom number: $\Delta N_0^2 \propto N^{1+\gamma}$, with $\gamma \neq 0$ [4, 5]. This is in stark contrast to most classical systems where the central limit theorem ensures normal scaling of the fluctuations $\Delta N^2 \propto N$ due to the existence of a finite microscopic coherence length scale [21]. The characterization of BEC atom number fluctuations is thus an important outstanding challenge that has been hindered by technical fluctuations of BEC experiments until recently. This technical limitation was mitigated in our recent experiments, enabling the current experimental characterization of BEC fluctuations.

In this letter, we characterize the atom number fluctuations in a weakly interacting BEC as a function of atom number and trapping geometry. To avoid technical fluctuations, BECs with a well-controlled atom number and temperature are prepared through a combination of non-destructive detection and active stabilization of the cooling sequence. We quantitatively analyze the fluctuations in two ways. The fluctuations are compared to an exact canonical calculation for the non-interacting Bose gas, which shows fluctuations reduced by 27% with respect to the canonical expectation, indicating the microcanonical nature of the system. Secondly, the fluctuations are fitted with a phenomenological model showing anomalous atom number scaling with an exponent $\gamma = 0.134(5)$, which we compare to the expectation in the thermodynamic limit.

In a first step, we theoretically estimate the importance of using an interacting microcanonical ensemble with respect to the more readily available canonical ensemble calculations. This is achieved for ensembles of up to $10^4$ atoms using a new computational method. In brief, we apply the usual Metropolis algorithm but perform a random walk in the space of Fock states, accounting for Bose enhancement with perturbatively included interactions. The random walk results in the set of visited Fock states obeying canonical ensemble statistics. By post-selecting samples according to their energy, we obtain representative “microstates” of the microcanonical ensemble. Figure 1(a) show the results for $N = 1000$ atoms for trap aspect ratio $\lambda = 10$. Importantly, the microcanonical result lies clearly below the canonical expectation as indicated by Eq. (1). In the elongated case shown here, the ratio, $S$, between the peak variance in the microcanonical and canonical ensembles is $S = 0.72$, corresponding to a reduction by 28% for this atom number. The effect of interactions is compara-
tively small in this case and leads to a minor increase of the variance in both ensembles.

Given the small effect of interactions, we calculate $S$ for a range of aspect ratios and atom numbers for the non-interacting case as shown in Fig. 2. For low aspect ratios $\lambda$ and large atom numbers the ratio $S$ tends towards the limiting value 0.39 given by Eq. (1). In highly elongated traps with large aspect ratios, $S$ tends towards 1, since microcanonical and canonical fluctuations are identical in the non-interacting 1D case. The dependence of $S$ on the atom number is shown in Fig. 2 (inset) for $\lambda = 6$. This indicates that the microcanonical and canonical results scale differently with atom number. In particular we observe a roughly logarithmic dependence of $S$ on $N$ for atom numbers $10^2 < N < 10^4$. These results show that a clear reduction of the fluctuations below the canonical result is expected under realistic conditions and motivates the following experimental investigation.

The experimental apparatus used to produce BECs was previously described in detail in [28]. Briefly, around $10^9 ^{87}$Rb atoms are first captured and cooled in a magneto-optical trap. The cloud is then optically pumped to the $|F = 2, m_F = 2 \rangle$ state and transported into a cigar-shaped quadrupole-Ioffe-configuration magnetic trap, where it is cooled by radio-frequency (RF) forced evaporation.

The sequence outlined above is typically subject to technical fluctuations that prevent the observation of atom number fluctuations. To overcome this challenge, a stabilization procedure is initiated when the cloud contains $\sim 4 \times 10^6$ atoms at a temperature $14 \mu$K. The cloud is probed using minimally destructive Faraday imaging, and the atom number is corrected using a weak RF pulse of controllable duration. The pulse removes excess atoms, and the outcome of the stabilization procedure is verified by a second Faraday measurement. This allows us to prepare a well-controlled number of atoms with a relative stability at the $10^{-4}$ level [28, 29].

For efficient cooling towards BEC, a tight magnetic trap and consequently large collision rate is desirable. In its most compressed configuration, our trap has an aspect ratio $\lambda = 17$. This can lead to significant phase fluctuations across the spatial extent of the cloud [30], which evolve into density modulations during time-of-flight (TOF) and thus hinder the precise determination of atom number and temperature. We therefore decompress the magnetic trap for our measurements, limiting the aspect ratio to $4.5 < \lambda < 10$ where the phase coherence length is larger than the condensate length in the long direction.

In the final step of the experimental sequence, BECs are produced by forced evaporation ending at an RF-frequency $\omega$ tuned towards the desired BEC temperature. To ensure proper thermal equilibrium, the BEC is first held at the final RF frequency for 800 ms and a further 400 ms without RF-radiation before the trap is turned off.

The clouds are probed after 35 ms TOF expansion using resonant absorption imaging on the $F = 2 \rightarrow F' = 3$ cycling transition. Fringes in the processed image due to vibrations of optical components are mitigated by minimizing the time-

![FIG. 3. Peak fluctuations of a BEC. (Data points) Observed peak variance of the BEC atom number as a function of peak atom number $N_p$ and trap aspect ratio $\lambda$. (Surface) Theoretical expectation for a non-interaction canonical ensemble scaled according to Eq. (6).](image-url)

delay between the absorption and reference image. Optical pumping is applied between these images to transfer the atoms to the transparent $F = 1$ state. Thus, the atom and beam images can be taken only 340 μs apart, limited only by the camera shift speed.

For each experimental configuration corresponding to a chosen aspect ratio and atom number, the fluctuations were measured as a function of the temperature $T$. At every temperature (corresponding to an RF end frequency) a set of 60 measurements according to the experimental sequence outlined above was taken [31].

To determine the BEC atom number in each image, the wings of the cloud are fit with a Bose-enhanced thermal distribution from which the temperature is obtained. The fitted distribution is subtracted from the image, and the BEC atom number $N_0$ is obtained by integration of the remaining density. However, the variance cannot be determined directly from $N_0$, since small remaining drifts of the magnetic offset field lead to minor temperature variations with a median standard deviation of $\sim 3 \text{nK}$. We eliminate this drift by subtracting a linear fit of the condensate number as a function of the total atom number [12] and determine the residuals $\eta_i$, where $i$ indicates the order in time. The BEC atom number variance is then given by a two-sample variance of the residuals

$$\Delta N_0^2 = \frac{1}{2} \left( \langle \eta_{i+1} - \eta_i \rangle \right)^2 .$$

Thus, this two-sample variance contains the BEC fluctuations and detection noise, but excludes slow technical drifts. Figure 1(b) shows this variance of the measurements as a function of $T$ for a given configuration of the experiment.

The peak fluctuations are determined by a fit to $\Delta N_0^2$ also illustrated in Fig. 1(b). The fit model is inspired by the asymptotic behaviour of the fluctuations in a non-interacting gas

$$\Delta N_0^2 = \xi(2) \left( \frac{\mu}{m} \right)^3 T^3 ,$$

where $\xi$ is the geometric mean of the trapping frequencies [2]. Moreover, the fluctuations decay in near step-like manner close to the critical temperature,
which we model with a Heaviside step function $\Theta(T_p - T)$, where $T_p$ is the temperature at the peak fluctuations. To account for small temperature drifts the expression is furthermore convolved with a normal distribution $\mathcal{N}(T, \sigma_T)$ centered on the temperature $T$ with a standard deviation $\sigma_T$ given by the median of the measured temperature variation. Thus, we fit the data with the model

$$\Delta N^2_0(T) = (f * g)(T) + \mathcal{O}$$ \tag{3}$$

where $f$ and $g$ are given by

$$f(T) = \Delta N^2_{0,p} \left( \frac{T}{T_p} \right)^3 \Theta(T_p - T),$$ \tag{4}$$

and

$$g(T) = \mathcal{N}(T, \sigma_T).$$ \tag{5}$$

The three fit parameters are the peak atom number variance $\Delta N^2_{0,p}$, $T_p$ and an offset $\mathcal{O}$ which accounts for experimental noise [12]. The atom number $N_p$ at $T_p$ is subsequently obtained from a spline interpolation of the atom number as a function of the temperature for all measurements in a given configuration. Figure 3 (data points) shows the measured peak atom number variance for all 13 experimental configurations of aspect ratio and atom number.

We quantitatively evaluate the peak fluctuations using two approaches. First, we compare the measured fluctuations to the expected fluctuations in a canonical non-interacting gas, which can be calculated under conditions comparable to the experiment. Secondly, the fluctuations are investigated using a phenomenological model to probe their dependence on atom number and trapping geometry.

To compare experiment and theory the average ratio $S$ between the observed fluctuations and the exact theoretical result for the non-interacting canonical ensemble $\Delta N^2_{0,p,can}$ is evaluated. We obtain $S$ from a fit of

$$\Delta N^2_{0,p} = S(1 + a \delta \lambda + b \delta \ln(N))\Delta N^2_{0,p,can}.$$ \tag{6}$$

inspired by the scalings with atom number and aspect ratio in Fig. 2. The coefficients $a$ and $b$ allow for a linear scaling with the trap aspect ratio and the logarithm of the atom number, where $\delta x = x - \langle x \rangle$ denotes that the mean has been subtracted. This yields an average ratio $S = 0.73(5)$ corresponding to a $27\%$ reduction of the fluctuations and clearly reveals the microcanonical nature of our system with reduced fluctuations below the canonical ensemble expectation. The coefficients $a = 0.04(4)$ and $b = 0.24(26)$ represent a correction to $S$ which are barely resolved within our data range, supporting the interpretation that the microcanonical nature of the system constitutes the most important deviation from the canonical result.

We use a complementary approach to analyze the scaling of the fluctuations with atom number and aspect ratio $\lambda$. Inspired by Eq. (1), we use the phenomenological model

$$\Delta N^2_{0,p}/N = a + b \lambda N^\gamma,$$ \tag{7}$$

where $a$, $b$ and $\gamma$ are fit coefficients allowing for linear aspect ratio and a non-linear number dependence. In particular, the exponent $\gamma$ quantifies the degree of anomalous atom number scaling.

We first fit the exact canonical result $\Delta N^2_{0,p,can}$ with this model which yields $a = 1.327(14)$, $b = 1.5(5)$ and $\gamma = -0.27(3)$. Thus, the coefficient $a$ is close to its limiting value $\zeta(2)/\zeta(3) = 1.37$ corresponding to the first term in Eq. 1. More significantly, the negative sign of $\gamma$ combined with the relatively large value of $b$ show that even the exact theoretical result scales anomalously with $N$. This can be attributed to the transition from a system with reduced dimensionality at low atom numbers to the limiting 3D case [12]. Hence, we do not expect our experiment to reflect the thermodynamic limit, where the scaling has been investigated theoretically [4, 5, 22].

Figure 4 shows the experimental data and the fit according to Eq. (7). Contrary to the non-interacting theoretical case, we find anomalous scaling with $\gamma = 0.134(5) > 0$. We interpret our result as being due to the interplay between the dimensionality effects in a microcanonical system and interaction effects, which are predicted to yield $\gamma = 1/3$ in the thermodynamic limit [4, 5]. In particular, Fig. 2 shows that a microcanonical system generally scales different from a canonical one in both $\lambda$ and $N$. Since $\gamma > 0$, Eq. (7) diverges and the coefficient $a = 0.55(4)$ cannot be interpreted as a limiting value. Moreover, $b = 0.020(12)$ is very small due to the large value of $N^\gamma$. This result thus provides a simple analytical model as a benchmark for future theoretical investigations.

In conclusion, we have measured the peak fluctuations of a weakly interacting Bose-Einstein condensate as a function of atom number and trap aspect ratio. The significant differences between a canonical and microcanonical ensemble description of these fluctuations were investigated numerically by post-processing ensembles generated via the quantum Metropolis algorithm. Experimentally the fluctuations were found to be reduced by $27\%$ relative to the expectation for a canonical non-interacting gas, in qualitative agreement with our
theoretical prediction. This clearly demonstrates the microcanononical nature of the system and thus guides future theoretical work. In addition the peak fluctuations were found to follow a weakly anomalous scaling with the total atom number $\Delta N^2_{dp} \propto N^{1.134}$. We interpret the scaling as an interplay between dimensional effects and the limiting case in the thermodynamic limit with $\Delta N^2_{dp} \propto N^{4/3}$.

Future experiments will aim to observe the slight increase of the fluctuations due to interactions, predicted by our theoretical calculation. To date, this effect is shrouded by measurement uncertainties and the dependence of the fluctuations on trapping geometry. Moreover, the use of larger atomic clouds will provide experimental insight in the fluctuations in the thermodynamic limit [4, 5]. Finally, technical improvements leading to shorter experimental cycles will allow for larger data sample sizes and enable the investigation of the higher moments of the atom number distribution [6].

M.B.C., T.V., A.J.H., M.A.K., and J.J.A. acknowledge support by the Villum Foundation, the Carlsberg Foundation, and the Danish National Research Foundation through the Center of Excellence “CCQ” (Grant agreement no.: DNRF156). K.R., M.K. and D.H. acknowledge support from the (Polish) National Science Center Grant 2018/29/B/ST2/01308. K.P. acknowledge support from the (Polish) National Science Center Grant 2019/34/E/ST2/00289. This research was supported by the Villum Foundation, the Carlsberg Foundation, and the Danish National Research Foundation through the Center of Excellence “CCQ” (Grant agreement no.: DNRF156).

M.B.C., T.V., A.J.H., M.A.K., and J.J.A. acknowledge support by the Villum Foundation, the Carlsberg Foundation, and the Danish National Research Foundation through the Center of Excellence “CCQ” (Grant agreement no.: DNRF156). K.R., M.K. and D.H. acknowledge support from the (Polish) National Science Center Grant 2018/29/B/ST2/01308. K.P. acknowledge support from the (Polish) National Science Center Grant 2019/34/E/ST2/00289. This research was supported in part by PLGrid Infrastructure. Center for Theoretical Physics of the Polish Academy of Sciences is a member of the National Laboratory of Atomic, Molecular and Optical Physics (KL FAMO).

[1] Immanuel Bloch, Jean Dalibard, and Syvälnascimbène, “Quantum simulations with ultracold quantum gases,” Nat. Phys. 8, 267–276 (2012).
[2] H. D. Politzer, “Condensate fluctuations of a trapped, ideal Bose gas,” Phys. Rev. A 54, 5048–5054 (1996).
[3] Patrick Navez, Dmitri Bitouk, Mariusz Gajda, Zbigniew Idziaszek, and Kazimierz Rzążewski, “Fourth statistical ensemble for the Bose-Einstein condensate,” Phys. Rev. Lett. 79, 1789–1792 (1997).
[4] S. Giorgini, L. P. Pitaevskii, and S. Stringari, “Anomalous fluctuations of the condensate in interacting Bose gases,” Phys. Rev. Lett. 80, 5040–5043 (1998).
[5] F. Meier and W. Zwerger, “Anomalous condensate fluctuations in strongly interacting superfluids,” Phys. Rev. A 60, 5133–5135 (1999).
[6] Vittal V. Kocharovsky, Vladimir V. Kocharovsky, Martin Holthaus, C. H. Raymond Ooi, Anatoly Svidzinsky, Wolfgang Ketterle, and Marlan O. Scully, “Fluctuations in ideal and interacting Bose-Einstein condensates: From the laser phase transition analogy to squeezed states and Bogoliubov quasiparticles,” Adv. At., Mol., Opt. Phys. 53, 291–411 (2006).
[7] F. Gerbier, J. H. Thywissen, S. Richard, M. Hubert, P. Bouyer, and A. Aspect, “Experimental study of the thermodynamics of an interacting trapped Bose-Einstein condensed gas,” Phys. Rev. A 70, 013607 (2004).
[8] R. Meppelink, R. A. Rozendaal, S. B. Koller, J. M. Vogels, and P. Van Der Straten, “Thermodynamics of Bose-Einstein-condensed clouds using phase-contrast imaging,” Phys. Rev. A 81, 053632 (2010).
[9] Naaman Tammuz, Robert P. Smith, Robert L. D. Campbell, Scott Beattie, Stuart Moulder, Jean Dalibard, and Zoran Hadzibabic, “Can a Bose gas be saturated,” Phys. Rev. Lett. 106, 230401 (2011).
[10] F. Gerbier, J. H. Thywissen, S. Richard, M. Hubert, P. Bouyer, and A. Aspect, “Critical temperature of a trapped, weakly interacting Bose gas,” Phys. Rev. Lett. 92, 030405 (2004).
[11] Robert P. Smith, Robert L. D. Campbell, Naaman Tammuz, and Zoran Hadzibabic, “Effects of interactions on the critical temperature of a trapped Bose gas,” Phys. Rev. Lett. 106, 250403 (2011).
[12] M. A. Kristensen, M. B. Christensen, M. Gajdacz, M. Iglicki, Krzysztof Pawłowski, C. Klemp, J. P. Sherson, K. Rzążewski, Kazimierz, A. J. Hilliard, and J. J. Airt, “Observation of atom number fluctuations in a Bose-Einstein condensate,” Phys. Rev. Lett. 122, 163601 (2019).
[13] Robert M. Ziff, George E. Uhlenbeck, and Mark Kac, “The ideal Bose-Einstein gas, revisited,” Phys. Rep. 32, 169–248 (1977).
[14] Siegfried Grossmann and Martin Holthaus, “Microcanonical fluctuations of a Bose system’s ground state occupation number,” Phys. Rev. E 54, 3495–3498 (1996).
[15] Martin Holthaus, Eva Kalinowski, and Klaus Kirsten, “Condensate fluctuations in trapped Bose gases: Canonical vs. microcanonical ensemble,” Ann. Phys. 270, 198–230 (1998).
[16] C. Weiss and M. Wilkens, “Particle number counting statistics in ideal Bose gases,” Opt. Express 1, 272–83 (1997).
[17] L. E. Dickson, History of the Theory of Numbers: Diophantine Analysis (Chelsea Publishing Co., New York, 1971).
[18] G. H. Hardy and S. Ramanujan, “Asymptotic formulae in combinatorial analysis,” Proc. London Math. Soc. 17, 75–115 (1918).
[19] Siegfried Grossmann and Martin Holthaus, “Maxwell’s demon at work: Two types of Bose condensate fluctuations in power-law traps,” Opt. Express 1, 262–271 (1997).
[20] The Maxwell’s Demon ensemble yields the variance of the number of atoms in the thermal cloud. Due to conservation of the total number of atoms, this also exactly corresponds to the variance of the number of atoms in the condensate.
[21] W. Zwerger, “Anomalous fluctuations in phases with a broken continuous symmetry,” Phys. Rev. Lett. 92, 027203 (2004).
[22] Zbigniew Idziaszek, Mariusz Gajda, Patrick Navez, Martin Wilkens, and Kazimierz Rzążewski, “Fluctuations of the weakly interacting Bose-Einstein condensate,” Phys. Rev. Lett. 82, 4376–4379 (1999).
[23] V. V. Kocharovsky, Vl. V. Kocharovsky, and Marlan O. Scully, “Condensate statistics in interacting and ideal dilute Bose gases,” Phys. Rev. Lett. 84, 2306–2309 (2000).
[24] V. V. Kocharovsky, Vl. V. Kocharovsky, and Marlan O. Scully, “Condensation of N bosons. III. analytical results for all higher moments of condensate fluctuations in interacting and ideal dilute Bose gases via the canonical ensemble quasiparticle formulation,” Phys. Rev. A 61, 053606 (2000).
[25] Anatoly A. Svidzinsky and Marlan O. Scully, “Condensation of N interacting bosons: A hybrid approach to condensate fluctuations,” Phys. Rev. Lett. 97, 190402 (2006).
[26] Anatoly A. Svidzinsky and Marlan O. Scully, “Condensation of N bosons: Microscopic approach to fluctuations in an interacting Bose gas,” Phys. Rev. A 82, 063630 (2010).
[27] Satadal Bhattacharyya and Barnali Chakrabarti, “Condensate fluctuation and thermodynamics of mesoscopic Bose-Einstein condensates: A correlated many-body approach,” Phys. Rev. A
[28] Mick Kristensen, M. Gajdacz, P. L. Pedersen, Carsten Klempt, J. F. Sherson, J. J. Arlt, and A. J. Hilliard, “Sub-atom shot noise Faraday imaging of ultracold atom clouds,” J. Phys. B: At., Mol. Opt. Phys. 50, 034004 (2017).

[29] M. Gajdacz, A. J. Hilliard, M. A. Kristensen, P. L. Pedersen, C. Klempt, J. J. Arlt, and J. F. Sherson, “Preparation of ultracold atom clouds at the shot noise level,” Phys. Rev. Lett. 117, 073604 (2016).

[30] D. Hellweg, L. Cacciapuoti, M. Kottke, T. Schulte, K. Sengstock, W. Ertmer, and J. J. Arlt, “Measurement of the spatial correlation function of phase fluctuating Bose-Einstein condensates,” Phys. Rev. Lett. 91, 010406 (2003).

[31] Measurements for which the stabilization process failed were excluded, such that each set includes at least 45 measurements.