An Arena for Model Building
in the Cohen-Glashow Very Special Relativity

M.M. Sheikh-Jabbari\textsuperscript{1} and A. Tureanu\textsuperscript{2}

\textsuperscript{1}School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O.Box 19395-5531, Tehran, Iran\textsuperscript{*}

\textsuperscript{2}Department of Physics, University of Helsinki and Helsinki Institute of Physics, P.O.Box 64, FIN-00014 Helsinki, Finland\textsuperscript{†}

The Cohen-Glashow Very Special Relativity (VSR) algebra \cite{1} is defined as the part of the Lorentz algebra which upon addition of $CP$ or $T$ invariance enhances to the full Lorentz group, plus the space-time translations. We show that noncommutative space-time, in particular noncommutative Moyal plane, with \textit{light-like noncommutativity} provides a robust mathematical setting for quantum field theories which are VSR invariant and hence set the stage for building VSR invariant particle physics models. In our setting the VSR invariant theories are specified with a single deformation parameter, the noncommutativity scale $\Lambda_{NC}$. Preliminary analysis with the available data leads to $\Lambda_{NC} \gtrsim 1 - 10$ TeV. This note is prepared for the Proceedings of the G27 Mathematical Physics Conference, Yerevan 2008, and is based on \cite{2}.

I. INTRODUCTION AND MOTIVATION

According to Special Relativity (SR) physical theories and observables are invariant under the \textit{Poincaré group}, that is the set of Lorentz transformations plus space-time translations. Mathematically, the Poincaré algebra is the isometry of the $3 + 1$ dimensional Minkowski space. One may then consider possible extensions or restrictions of the Poincaré group and study the theories which are invariant under specific extensions or restrictions. The

\textsuperscript{*}Electronic address: jabbari@theory.ipm.ac.ir
\textsuperscript{†}Electronic address: anca.tureanu@helsinki.fi
maximal extension of the Poincaré algebra is the conformal group $so(4,2)$ which cannot be a symmetry of the particle physics models even at classical tree level due to the presence of massive particles. Besides the conformal group, one can extend Poincaré group (or algebra) by the addition of the discrete symmetries of space and time inversion $P$, $T$.

The discrete symmetries, $P$, $T$ and charge conjugation $C$, at low energy (where QED+QCD is at work) are individually good symmetries of nature. However, at higher energies, as is built in the particle physics standard model (SM), the weak interactions do not respect the parity invariance. Moreover, experiments and observations confirm that the charge conjugation times parity $CP$, and hence $T$, are also violated in the strange and $b$-meson systems. The $CP$ violating parameters in the SM are encoded in the generation mixing matrices. Theoretically, there is a generally held belief that the observed $CP$ violation can be traced back to physics beyond the electroweak scale (beyond SM). On the other hand, although in the experiments and observations so far we do not have a decisive signal of Lorentz symmetry violation, it is conceivable that Lorentz symmetry is not an exact symmetry at energies above the electroweak scale. Cohen and Glashow in their idea of very special relativity (VSR) [1] were in fact seeking a connection between the two usually thought to be unrelated phenomena, the $CP$ and the possible Lorentz symmetry violation.

The Cohen-Glashow Very Special Relativity (VSR) [1] is defined as symmetry under certain subgroups of Poincaré group, containing space-time translations and a proper subgroup of Lorentz group $SO(3,1)$ with the property that when supplemented with parity, $CP$ or time-reversal $T$ it enlarges to the full Lorentz group. In other words, a theory with VSR symmetry is not strictly Lorentz invariant and also not parity or time-reversal invariant. As it is seen from the definition, and will be made explicit in our setup for the realization of VSR invariant theories, the Lorentz violation and $CP$ violation are linked together.

Although currently we do not have any observation or experiment signaling departure from Lorentz symmetry, with the advance of technologies we will be able to trace such deviations with ever increasing precision. With the prospect of upcoming experiments various possible deviations from Lorentz invariance at high energies have been studied, both theoretically and phenomenologically (for an incomplete list see e.g. [3, 4]). The problem open in [1] is whether Lorentz and nearly $CP$ invariant theories, like the Standard Model, could emerge as effective theories from a more fundamental scheme, perhaps operative at the Planck scale.
One of the very crucial implications of the Lorentz symmetry is in building physical models based on Lorentz invariant quantum field theories: in field theories the fields and/or the states in their Fock space are labeled by representations of the Lorentz algebra and in particular particle states are specified by their mass and spin and they obey the spin-statistics relation. In the formulation of any theory in which the Lorentz invariance is relaxed, like the VSR invariant theories, one should re-examine whether one can use the outcomes of the Lorentz symmetry about the representations of the matter content of the theory. In the particular case of VSR subgroups of the Lorentz group, as it will become clear momentarily, they only admit one-dimensional representations. Being proper subgroups, the representations of the VSR subgroups of Lorentz are automatically representations of the Lorentz group, but the reciprocal is not true. As a result, if we construct a VSR invariant quantum field theory based on the one-dimensional representations of the VSR subgroups, when requiring also $P$, $T$ or $CP$ invariance, although the theory becomes invariant under the whole Lorentz group, the fact about the one-dimensional representations of VSR does not change and hence the effective theory would be doomed by its very poor representation content. This is what we call “representation problem” in the VSR invariant theories.

In view of the above argument and recalling the fact that the observed elementary particles are neatly classified and described by the representations of Lorentz group, one possibility for resolving the “representation problem” is that the Lorentz violating terms (the terms which reduce the symmetry of the Lagrangian to VSR) are added as perturbations to ordinary Lorentz invariant Lagrangians and hence the theories have the usual matter content allowed by Lorentz invariance. However, such a realization of VSR may not be thought of as a “fundamental” or “master” theory which leads to Lorentz invariant theories at low energies; this approach does not provide a firm theoretical setting for building the VSR invariant theories.

Here we present an alternative way for resolving the “representation problem”, paving the way for formulating VSR invariant quantum field theories. This could be achieved noting that one can include in the picture of symmetries not only the commutation relations defining an algebra, but also the action of the generators of the symmetry on the tensor product of their representation spaces (the so-called co-product). In more mathematical terms, the reasoning in terms of Lie groups/algebras can be extended to considering (deformed) Hopf algebras. In the framework of Hopf algebras, there are deformations which leave the
commutation relations and structure constants of the algebra untouched, but affect other properties of the Hopf algebra, i.e. the co-algebra structure [5]. Since the commutation relations of generators are not deformed, it follows automatically that the Casimir operators are the same and the representation content of the deformed Hopf algebra is identical to the one of the undeformed algebra. On the other hand, the deformation of the co-algebra structure reduces the symmetry of the scheme. Such deformations are the twists introduced in Ref. [6], and are hence called “Drinfeld twist”. Since the twisting reduces the symmetry, one may try to use the same concept to reduce the Lorentz symmetry to its VSR subgroups. This is indeed the idea we are putting forward here to give a robust mathematical framework for constructing the VSR invariant theories.

The paper is organized as follows. In section II, we review the Cohen-Glashow VSR subgroups of Lorentz. In section III, we recall some aspects about noncommutative space-times and in particular the Moyal space. In section IV, we show that there are specific noncommutative space-times, with light-like noncommutativity, that are invariant under the $T(2)$, $E(2)$ or $SIM(2)$ subgroups of the Lorentz group. In section V, we present a general framework for writing $T(2)$ VSR invariant quantum field theory.

II. THE COHEN-GLASHOW VSR: A BRIEF REVIEW

Energy-momentum conservation, and hence invariance under rigid space-time translations, should be preserved in VSR invariant theories. The minimal version of the VSR algebra contains, besides the generators of translations $P_\mu$, the subgroup $T(2)$ of the Lorentz group, which is generated by

$$T_1 = K_x + J_y \quad \text{and} \quad T_2 = K_y - J_x,$$

where $J_i$ and $K_i$, $i = x, y, z$ are respectively generators of rotations and boosts. It is then immediate to check that $[T_1, T_2] = 0$ and hence $T(2)$ is an Abelian subalgebra of Lorentz algebra $so(1,3)$. Moreover, upon action of parity $P$,

$$T_1 \rightarrow T_1^P = -K_x + J_y , \quad T_2 \rightarrow T_2^P = -K_y - J_x ,$$

and similarly under $T$. It is straightforward to see that the algebra obtained from $T_1$, $T_2$, $T_1^P$ and $T_2^P$ closes on the whole Lorentz group and therefore, $T_1, T_2, P_\mu$ form (the smallest possible) VSR algebra.
The group \( T(2) \) can be identified with the translation group on a two dimensional plane. The other larger versions of VSR are obtained by adding one or two Lorentz generators to \( T(2) \), which have geometric realizations on the two dimensional plane:

- **\( E(2) \)**, the 3-parametric group of two dimensional Euclidean motion, generated by \( T_1, T_2 \) and \( J_z \), with the structure:

  \[
  [T_1, T_2] = 0, 
  [J_z, T_1] = -iT_2, 
  [J_z, T_2] = iT_1; 
  \] (3)

- **\( HOM(2) \)**, the group of orientation-preserving similarity transformations, or homotheties, generated by \( T_1, T_2 \) and \( K_z \), with the structure

  \[
  [T_1, T_2] = 0, 
  [T_1, K_z] = iT_1, 
  [T_2, K_z] = iT_2; 
  \] (4)

- **\( SIM(2) \)**, the group isomorphic to the four-parametric similitude group, generated by \( T_1, T_2, J_z \) and \( K_z \).

Some comments about the above VSR subgroups are in order:

- It is obvious that once we add parity or time-reversal conjugates of the generators to either of the above three VSR cases, similarly to the \( T(2) \) case, they enhance to the full Lorentz algebra.

- The \( T(2) \) VSR has an invariant vector \( n_\mu = (1, 0, 0, 1) \) as well as an invariant two form [1], to which we shall return later.

- \( n_\mu = (1, 0, 0, 1) \) is also an invariant vector of the \( E(2) \) VSR [1]. \( E(2) \) does not have any invariant two form.

- The \( HOM(2) \) and \( SIM(2) \) do not admit any invariant vector or tensors.

- The VSR subgroups only admit one dimensional representations. While all the representations of VSR are also representations of the Lorentz group, the converse is not true.

One way to realize \( T(2) \) or \( E(2) \) VSR is to recall that they admit invariant vector or tensor and use the idea of *inverse* spontaneous Lorentz symmetry breaking and give VEVs to a vector or a tensor in such a way that in low energies the VEV goes away, or become
negligible and we recover the full Lorentz symmetry. This was indeed the idea put forward by Cohen and Glashow [1, 3] and some other authors. However, this may spoil the nice features of Lorentz invariant theories and in principle is a phenomenological approach which introduces many parameters in the theory.

$HOM(2)$ and $SIM(2)$ do not admit invariant tensors and their formulation should be done in some other ways. The $SIM(2)$ case as the largest VSR has been studied more (see e.g. [10]).

### III. A BRIEF REVIEW ON NONCOMMUTATIVE SPACES AND THE TWISTED POINCARÉ ALGEBRAS

As mentioned above, the Poincaré algebra is the isometry of the Minkowski space. The idea we will follow here is whether there are $3+1$ dimensional space-times whose “isometry” group is either of the VSR subgroups. Noncommutative spaces which are defined through the commutation relations

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

among their coordinates, where $\theta^{\mu\nu}$ is in general a function of coordinates (of course, with the condition that it satisfies the Jacobi identity), provide a setup to address this issue.

The commutation relations (5) usually spoil the Lorentz invariance (and the translational invariance if $\theta$ has space-time dependence). Nonetheless, depending on $\theta$, specific subgroups of the Poincaré group under which the commutation relation (5) is preserved still provide a symmetry (or “isometry”) of the noncommutative space-time.

The essential element for our discussion is that for specific choices of $\theta$ the commutation relations can be obtained from the associative star-products coming from introducing twisted co-product for the Poincaré algebra [8, 9] (see also [11, 12]). The advantage of using the twisted Poincaré language for constructing physical theories is that, in spite of the lack of full Lorentz symmetry, the fields carry representations of the full Lorentz group [13, 14] and the spin-statistics theorem is still valid; the deformation then appears in the product of the fields (interaction terms) and therefore, we have a way of overcoming the “representation problem”.

Depending on the structure of the r.h.s. of (5), there exist three types of noncommutative deformations of the space-time which can be realized through twists of the Poincaré algebra
[8, 9, 11]:

- **Constant $\theta^{\mu\nu}$:** the Heisenberg-type commutation relations, defining the Moyal space:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu},$$

where $\theta^{\mu\nu}$ is a constant antisymmetric matrix. This is the most studied case and various aspects of QFTs on the Moyal space have been analyzed. Here we briefly review them. Since $\theta^{\mu\nu}$ is an anti-symmetric two tensor, noncommutative spaces can be classified based on the values of the two Lorentz invariants

$$\Lambda^4 \equiv \theta_{\mu\nu}\theta^{\mu\nu}, \quad L^4 \equiv \epsilon^{\alpha\beta\mu\nu}\theta_{\mu\nu}\theta_{\alpha\beta}.$$  

(7)

$\Lambda^4$ is related to the noncommutativity scale, the scale where noncommutativity effects will become important, while $L^4$ is related to the smallest (space-time) volume that one can measure in a noncommutative theory.

Depending on whether $L^4$ and $\Lambda^4$ are positive, zero or negative one can recognize nine cases. The $L^4 \neq 0$ cases cannot be obtained as a decoupling (low energy) limit of open string theory. (However, the $\Lambda^4 = 0, L^4 \neq 0$ case is the famous Doplicher-Fredenhagen-Roberts [15] noncommutative space.)

For $L^4 = 0$, depending on the value of $\Lambda^4$, there are three types of noncommutative spaces:

1) $\Lambda^4 > 0$ – space-like (space-space) noncommutativity;

ii) $\Lambda^4 < 0$ – time-like (time-space) noncommutativity;

iii) $\Lambda^4 = 0$ – light-like noncommutativity.

When $\Lambda$ is constant, for the case ii), it has been shown that there is no well-defined decoupled field theory limit for the corresponding open string theory [16]. In the field theory language this shows itself as instability of the vacuum state $^1$ and non-unitarity of the field theory on time-like noncommutative space [17]. For the space-like case i) and light-like case iii), noncommutative field theory limits are well-defined and the corresponding field theories are perturbatively unitary.

$^1$ This instability is similar to the instability caused by background electric fields due to pair creation if the theory has massless charged particles.
• **Linear** $\theta^{\mu\nu}$, with the Lie-algebra type commutators:

$$[x^\mu, x^\nu] = iC^\rho_{\mu\nu} x^\rho .$$

(8)

This case describes an (associative but) noncommutative space if $C^\rho_{\mu\nu}$ are structure constants of an associative Lie algebra.

• **Quadratic noncommutativity**, with the quantum group type of commutation relations:

$$[x^\mu, x^\nu] = \frac{1}{q} R^\rho_{\mu\nu} x^\rho x^\sigma .$$

(9)

All the above-mentioned cases of noncommutative space-time have originally been studied in [18] with respect to the formulation of NC QFTs on those spaces.

IV. **NONCOMMUTATIVE SPACE-TIMES INVARIANT UNDER THE VSR**

SUBGROUPS OF LORENTZ

As already pointed out, the VSR is defined through a proper subgroup of the Lorentz group $SO(3,1)$, which could be either of $T(2), E(2), HOM(2)$ or $SIM(2)$, plus space-time translations generated by $P_\mu$. Among the three cases of NC space-time discussed in the previous section only the constant $\theta^{\mu\nu}$ case preserves the space-time translational invariance in all directions. In the cases of linear and quadratic noncommutativity, translational invariance along some or all of the space-time directions is lost. Therefore, the Moyal case is the one relevant to the VSR theory. For completeness, however, we also discuss the relevance of the linear and quadratic noncommutative cases to the VSR subgroups of Lorentz.

A. **$T(2)$ symmetry implies light-like noncommutativity**

Motivated by the above arguments, we set about finding a configuration of the antisymmetric matrix $\theta^{\mu\nu}$. Since $T(2)$ is the only VSR which admits an invariant two tensor [1], we focus on this case. If we denote the elements of the $T(2)$ subgroup by

$$\Lambda_1 = e^{i\alpha T_1} \quad \text{and} \quad \Lambda_2 = e^{i\beta T_2},$$

(10)

the invariance condition for the tensor $\theta^{\mu\nu}$ is written as:

$$\Lambda_i^{\mu} \Lambda_i^{\nu} \theta^{\alpha\beta} = \theta^{\mu\nu}, \quad i = 1, 2,$$

(11)
and infinitesimally:
\[ T_1^\mu \theta^{\alpha \nu} + T_2^\nu \theta^{\mu \beta} = 0, \quad i = 1, 2. \]  \hspace{1cm} (12)

The matrix realizations of the generators \( T_1 \) and \( T_2 \) are (see, e.g., [19]):
\[ T_1 = \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \]  \hspace{1cm} (13)

and
\[ T_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ -i & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}. \]  \hspace{1cm} (14)

Plugging these values into (12) we find the solution
\[ \theta^{0i} = -\theta^{3i}, \quad i = 1, 2, \]  \hspace{1cm} (15)
all the other components of the antisymmetric matrix \( \theta^{\mu \nu} \) being zero. Note that to obtain the above result we did not assume any special form for the \( x \)-dependence of \( \theta^{\mu \nu} \) and hence this holds for either of the three constant, linear and quadratic cases. With the above condition on \( \theta^{\mu \nu} \), we see that \( \Lambda^4 = L^4 = 0 \), that is

\textit{Regardless of its space-time dependence, a light-like} \( \theta^{\mu \nu} \) \textit{is invariant under} \( T(2) \).

One may use the light-cone frame coordinates
\[ x^\pm = (t \pm x^3)/2, \quad x^i, \ i = 1, 2. \]  \hspace{1cm} (16)

In the above coordinate system the only non-zero components of the light-like noncommutativity (15) is \( \theta^{-i} = \theta^{0i} = -\theta^{3i} \) (and \( \theta^{+-} = \theta^{+i} = \theta^{ij} = 0 \)). In the light-cone coordinates (or light-cone gauge) one can take \( x^+ \) to be the light-cone time and \( x^- \) the light-cone space direction. In this frame, (light-cone) time commutes with the space coordinates. In the light-cone \((+,-,1,2)\) basis
\[ \theta^{\mu \nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \theta & \theta' \\ 0 & -\theta & 0 & 0 \\ 0 & -\theta' & 0 & 0 \end{pmatrix}. \]  \hspace{1cm} (17)
B. \( E(2) \) and \( SIM(2) \) invariant NC spaces

A constant \( \theta^{-i} \) breaks rotational invariance in the \((x^1, x^2)\)-plane and hence larger VSR subgroups are not possible in the Moyal NC space case. The \( E(2) \) invariant case can be realized in the linear, Lie-algebra type noncommutative spaces and \( SIM(2) \) can be realized by quadratic noncommutativity.

1. The \( E(2) \) case

\( E(2) \) is made up of \( T_1, T_2, J_z \). \( x^\pm \) are invariant under \( J_z \). \( \delta_{ij} \) and \( \epsilon_{ij} \) are the (only) two invariant tensors under \( J_z \) while \( x^i \) transform as vector under \( J_z \). Therefore, \( \theta^{-i} = \ell \epsilon_{ij} x^j \) and \( \theta^{-i} = \ell x^i \) lead to \( E(2) \) invariant spaces, namely

\[
[x^-, x^i] = \ell \epsilon^{ij} x^j, \tag{18a}
\]

or

\[
[x^-, x^i] = i \ell x^i. \tag{18b}
\]

With the above choices, it is evident that the translational symmetry along \( x^\pm \) is preserved while along \( x^i \) it is lost.

Instead of \( x^i \) coordinates we may work with the cylindrical coordinates on \( x^-, x^1, x^2 \) space. If we denote the radial and angular coordinate on the \((x^1, x^2)\)-plane by \( \rho \) and \( \phi \),

\[
\rho e^{\pm i\phi} = x^1 \pm ix^2, \tag{19}
\]

the case (18a) is then described by:

\[
[x^-, \rho] = 0, \quad [\rho, e^{\pm i\phi}] = 0, \quad [x^-, e^{\pm i\phi}] = \pm \lambda e^{\pm i\phi}, \quad \lambda = 2\ell. \tag{20}
\]

Since \( \rho \) commutes with both \( x^- \) and \( \phi \) we can treat it as a number (rather than an operator). The above space is then a collection of NC cylinders of various radii and the axes of the cylinders is along \( x^- \). Demanding the wave-functions to be single valued under \( \phi \to \phi + 2\pi \), leads to the discreteness of the spectrum of \( x^- \) in units of \( \lambda \).

The (18b) case in the cylindrical coordinates takes the form

\[
[x^-, e^{\pm i\phi}] = 0, \quad [\rho, e^{\pm i\phi}] = 0, \quad [x^-, \rho] = i \ell \rho. \tag{21}
\]
Here we may treat $\phi$ just as a number and work in a basis where $\rho$ is diagonal. In this basis $x^- = i\ell\rho \frac{\partial}{\partial \rho}$.

There is a twisted Poincaré algebra which provides the symmetry for the case of (18a) while the other case cannot be generated by a twist [25]. In the above, $\ell$ and $\lambda$ are deformation parameters of dimension length.

2. The SIM(2) case

Since $K_z$ acts on $x^\pm$ as scaling (scaling $x^-$ by, say, $\kappa$ and $x^+$ by $\kappa^{-1}$) while keeping $x^i$ intact, it is readily seen that it is impossible to find $\theta^{-i}$ linear in the coordinates which is invariant under $HOM(2)$. It is, however, possible to realize $SIM(2)$ (and hence $HOM(2)$, too) with quadratic $\theta^{-i}$. To have both the $K_z$ and $J_z$ invariant noncommutative structures, from the above discussions we deduce that we should take $\theta^{-i}$ which is linear in both $x^-$ and $x^i$, therefore the two possibilities are

$$[x^-, x^i] = i\frac{\xi - 1}{\xi + 1} \epsilon^{ij} \{x^-, x^j\}, \quad \xi \in \mathbb{R}$$  \hspace{1cm} (22a)

or

$$[x^-, x^i] = i \tan \chi \{x^-, x^i\},$$  \hspace{1cm} (22b)

preserving translational symmetry only along $x^+$ (where $\xi$ and $\chi$ are dimensionless deformation parameters). For neither of the above cases there is any twisted Poincaré of the form discussed in [11] to provide these commutators [25]. The case (22b) in the above mentioned cylindrical coordinates $x^-, \rho, \phi$ takes the familiar form of a quantum (Manin) plane [20] with $x^-$ and $\rho$ being the coordinates on the Manin plane.

As mentioned above, the Cohen-Glashow VSR requires translational invariance, which is only realized in the constant $\theta^{\mu\nu}$ case, therefore we continue with the discussion of QFTs on the light-like Moyal plane, as the VSR-invariant theories. Further analysis of the linear and quadratic noncommutativity cases will be discussed in a future work [25].

V. NC QFT ON LIGHT-LIKE MOYAL PLANE AS VSR INARIANT THEORY

So far we have shown that a Moyal plane with light-like noncommutativity is invariant under the $T(2)$ VSR. We now provide a prescription for writing VSR invariant QFT for any
given ordinary Lorentz invariant QFT. Our prescription is:

For any given QFT on commutative Minkowski space its VSR invariant counterpart is a noncommutative QFT, NCQFT, which is obtained by replacing the usual product of functions (fields) with the nonlocal Moyal $\ast$-product (for a review on NC QFTs see [24])

$$(\phi \ast \psi)(x) = \phi(x) e^{\frac{i}{2} \theta_{\mu\nu} \partial_\mu \partial_\nu} \psi(x),$$

(23)

where $\theta^{\mu\nu}$ is the constant light-like noncommutativity matrix given in (15) or (17). Without loss of generality one may use the freedom in choosing the direction of the axes in the $(x^1, x^2)$-plane such that $\theta^i = 0$ and our VSR invariant theory is specified with a single deformation parameter $\theta$.

Due to twisted Poincaré symmetry, the fields carry representations of the full Lorentz group, but the theory is only invariant under transformations in the stability group of $\theta^{\mu\nu}$, $T(2)$ [13, 14]. Consequently, the NC QFT constructed on this space possess also the same symmetry [7], as well as twisted Poincaré symmetry [8, 9].

VI. DISCUSSION AND OUTLOOK

We have given a framework for constructing VSR invariant quantum field theories. In analogy with the Poincaré algebra which has the geometric interpretation of the isometry group of the Minkowski space, our realization of the VSR subgroups, among other things, provides a geometric interpretation for these groups, as the isometry groups of specific “noncommutative” space-times, with light-like noncommutativity. In particular, demanding invariance under space-time translations restricts us to light-like noncommutative Moyal plane which is specified by a single deformation (noncommutativity) parameter. This case realizes the $T(2)$ invariant Cohen-Glashow VSR. Our realization of VSR theory naturally resolves the “representation problem” that, in spite of the lack of full Lorentz symmetry, one can still label fields by the Lorentz representations in a consistent manner. For the NC QFTs we can rely on the basic notions of fermions and bosons, spin-statistics relation and CPT theorem [21–23]. However, as shown in [21] for NC QED, $C$, $P$ and $T$ symmetries are not individually preserved and these symmetries, along with the full Poincaré symmetry may be recovered only in the $\theta^{\mu\nu} \rightarrow 0$ limit (or at energies much below the noncommutativity scale).
Through the parameter \( \theta \) of the NC QFT realization of \( T(2) \) VSR which has dimension length-square we define the noncommutativity scale \( \Lambda_{NC} = 1/\sqrt{\theta} \). To find bounds on \( \Lambda_{NC} \) we need to compare results based on the NC models to the existing observations and data. These data can range from atomic spectroscopy and Lamb-shift (see, e.g., [26]) to particle physics bounds on the electric-dipole moments of elementary particles. The preliminary analysis leads to \( \Lambda \gtrsim 1 - 10 \) TeV. A thorough analysis of obtaining bounds on \( \Lambda_{NC} \) is postponed to a future work [25]. First steps in this direction should involve constructing VSR invariant Standard Model, which could be done along the lines of [27, 28].

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