**Coloured Scalar Mediated Nucleon Decays to Invisible Fermion**

Svjetlana Fajfer and David Susić

Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia and
Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

We investigate nucleon decays to light invisible fermion mediated by the coloured scalar $S_1 = (3, 1, -2/3)$ and compare them with the results coming from the mediation of $S_1 = (3, 1, 1/3)$. In the case of $S_1 = (3, 1, -2/3)$ up-like quarks couple to the invisible fermion, while in the case of $S_1 = (3, 1, 1/3)$ the down-like quarks couple to the invisible fermion. For the mass of invisible fermion smaller than the mass $m_p - m_K$, proton (neutron) can decay to $K$ and invisible fermion and the masses of $S_1$ and $S_1$ are in the region $\sim 10^{15}$ GeV. The decays of nucleons to pions and invisible fermion can occur at the tree-level, but in the case of $S_1$ they come from dimension-9 operator and are therefore suppressed by several orders of magnitude compared to the decays into kaons. For the invisible fermion mass in the range $(937.8 \text{ MeV}, 938.8 \text{ MeV})$, decay of neutron $n \to \chi \gamma$ induced by $S_1$ is possible at the loop level, while the proton remains stable. The branching ratio of such decay is $\lesssim 10^{-6}$, which does not explain neutron decay anomaly, but is in agreement with the Borexino experiment bound. We comment on low-energy processes with the nucleon-like mass of $\chi$ in the final state as $\Lambda \to \chi \gamma$ and heavy hadron decays to invisibles.

Many constraints on physics beyond the Standard Model (SM) at low-energies are already well established. Although, it seems that possibilities for New Physics (NP) at low energies are known and well studied, there are some chances that light neutral particles may have evaded experiments due to their long lifetime. Recently, the author of Ref. [1] suggested this possibility and investigated a number of scenarios with light fermions carrying baryon number. As already summarised by many authors [1-5], such interactions between quarks and right-handed fermions are mediated by coloured scalars. Obviously, coloured scalars can couple either to down-like quarks or to up-like quarks depending on their charge $-1/3$ or $2/3$. On the experimental side, the KamLAND Collaboration [7] has already searched for the invisible decays of neutrons, but assumed zero mass of the invisible state.

Leptoquarks mediate SM quark and lepton interactions. In the case where instead of a lepton there is a fermion with quantum numbers of a right handed neutrino, we name the mediator coloured scalar. Following the notation of [8], we present in Table 1 coloured scalars which have interactions with a such state as well as the di-quark interactions. The scalar $S_1$ couples to leptons $S_1(3, 1, 1/3)$, $\bar{S}_1(3, 1, -2/3)$ is a coloured-scalar (triplet of colour group, singlet of weak, with hypercharge and electric charge equal to $2/3$; here the weak hypercharge $Y$ is defined as $Q = I_3 + Y$). Due to its quantum numbers, $S_1$ might have interactions with SM doublets, quarks as leptons, while the coloured scalar $S_1 = (3, 1, -2/3)$ only has two type of interactions with right-handed fermions. One with up quarks and with neutral weak right-handed singlets and the second one is an interaction between different generations of the down-quarks [8].

In addition to the general study of Ref. [1], an interesting possibility was discussed in the literature with the main concern being stability of proton, while neutron or hydrogen atom are unstable [2-4, 10]. For example, the authors of [11] pointed out that there is a discrepancy between the neutron lifetime measured in beam and bottle experiments. This idea initiated new experimental studies which supported discrepancy between the two experimental results [12] on the level of $3.6\sigma$. The world average of the bottle experiment according to PDG [13] is $\tau_n^{\text{bottle}} = (880.2 \pm 1.0) \text{s}$ and $\tau_n^{\text{beam}} = (888.0 \pm 2.0) \text{s}$. In Ref. [11] this discrepancy was addressed by assuming that neutron can decay to dark matter (DM) and one photon, or two types of DM. In order to avoid proton destabilization, the authors of this proposal suggested that the dark fermion should have mass in the range $m_p - m_e \leq m_\chi \leq m_p + m_e$ (or $937.8 \text{ MeV} < m_\chi < 938.8 \text{ MeV}$) in the case of neutron decay to DM fermion and $\gamma$, while the photon energy is in the range $0.782 \text{ MeV} < E_\gamma < 1.664 \text{ MeV}$. The branching ratio for the decay $n \to \chi \gamma$ which explains the neutron lifetime anomaly should be $\sim 10^{-2}$. The selection of this narrow mass window enables the DM to remain stable. Unfortunately, the direct search for the $n \to \chi \gamma$ decay at the level required to explain the neutron lifetime anomaly was unsuccessful [14]. Another possibility for the DM presence in the nucleon dynamics was offered in [15] in which the neutron can convert into mirror neutron, its dark partner from parallel mirror sector.

Table 1. The coloured scalars $S_1$ and $S_1$ interactions with invisible fermions and two quarks. Here we use only right handed couplings of $S_1$. Indices $i, j$ refer to quark generations.

| Cloured Scalar | Invisible fermion | Di-quark |
|----------------|------------------|----------|
| $S_1 = (3, 1, 1/3)$ | $d_\chi^i \nu_d^i S_1$ | $u_\chi^i d_\chi^i S_1$ |
| $\bar{S}_1 = (3, 1, -2/3)$ | $u_\chi^i \nu_u^i \bar{S}_1$ | $d_\chi^i u_\chi^i \bar{S}_1$ |

* Electronic address: svjetlana.fajfer@ijs.si
† Electronic address: david.susic@ijs.si
The approach of [11] assumes that a state with quantum numbers of $S_1 = (3, 1, 1/3)$ mediates this interaction.

The fermionic dark matter in this approach is a colour weak singlet, neutral state $(1, 1, 0)$, which can couple to down-like quarks. Recently, the authors of [2] questioned a possibility that hydrogen atom is unstable, whereas proton remains stable. They considered a case where the photon is emitted with the energy smaller than the nuclear binding energy inside nucleus. They noticed that the results of the Borexino experiment [16] allow the possibility that hydrogen atom is unstable, whereas proton remains stable. They considered a case where the neutron mixes with an invisible fermion without the nucleon binding energy inside nucleus. They noticed that the neutron - anti-neutron oscillations do not occur. In such a way, we obtain a direct test of scenarios where the neutron mixes with an invisible fermion without the nuclear physics complications. The main message of this study is that Borexino data restrict the branching ratio of the $n \rightarrow \chi^\gamma$ to be smaller than $10^{-4}$. The existence of heavy neutron stars also gives the strong limits, since $n \rightarrow \chi^\gamma$ would allow neutron stars to reach masses below the observed ones [17].

The full Lagragian for $S_1$ and $S_1$ in Sec. I. INTERACTIONS OF $\bar{S}_1$ AND $S_1$

The Lagrangian describing $\bar{S}_1$ and $S_1$ in Sec. I. Then in Sec. II we consider decays of nucleons $p, n \rightarrow K \chi$ which can occur at tree-level, as well as $p, n \rightarrow \pi \chi$. We compare our results with results coming from the mediation of $S_1$. In Sec. III we discuss decay $n \rightarrow \pi \chi$ due to mediation of $S_1$. The Sec. III contains a discussion of consequences at low energies. In Sec. IV we summarise our results.

I. INTERACTIONS OF $\bar{S}_1$ AND $S_1$

The Lagrangian describing $\bar{S}_1 = (\bar{3}, 1, -2/3)$ interactions is

$$\mathcal{L}(\bar{S}_1) \supset + \frac{\bar{y}_{1ij}^R d_R^C i}{M_{\bar{S}_1}} \bar{S}_1^i \chi^j + \frac{\bar{y}_{1ij}^R d_R^C i}{M_{\bar{S}_1}} \bar{S}_1^i d_R^j + \text{h.c.}$$

(1)

This colour scalar does not couple to charged leptons and interacts only with two different down quarks. In principle, in this Lagrangian three species of invisible fermions $\chi^j \equiv (1, 1, 0)$ can exist with the quantum number of the right-handed neutrino $\nu_R$. In order to simplify the model, we assume that there is only one $\chi = \chi^1$ for $j = 1, 2, 3$ which can couple to the $u, c$ and $t$ quarks. In the matrix $y^R_{1ij}$ we then set $j = 1$. Strictly speaking, the Lagrangian refers to quarks and invisible fermions in the flavour basis. In order to get these fields in the mass basis, one has to perform appropriate rotations (see for details [20]). Since we consider Lagrangians with the right handed fields only, we treat our couplings in [1], as they are already in the mass basis. The colour indices are not presented in [1].

Note that $\bar{y}_{1ij}^R$ is an antisymmetric matrix in any flavour basis, as well as in colour indices (not specified here, but knowing that $\bar{y}_{ij}^R c_i d_R^j \rightarrow \epsilon_{\alpha\beta\gamma} d_{R,\alpha}^C d_{R,\beta}^C S_{1,\gamma}$ and $z_{1ij}^R = -z_{1ji}^R$).

In some proposals $\chi^j$ is considered to be a Majorana fermion whose mass can be introduced by the mass term $m_\chi \bar{\chi} \chi$. In such scenarios one can simply assign baryon number $B = 2/3$ to $\bar{S}_1$ and $B = +1$ to $\chi$ [23]. That means then that the interacting Lagrangian preserves baryon number, while only the Majorana mass term will be source of the baryon number violation.

The full Lagragian for $S_1$ is given in Eq. (9) of [8]. Here we give only two terms of it, which we use letter in our calculations

$$\mathcal{L}_{S_1} \supset \bar{y}_{1ij}^R d_R^C i \bar{S}_1^i \chi^j + \bar{z}_{1ij}^R a_R^C i \bar{S}_1^i d_R^j + \text{h.c.}$$

(2)

Note that the last term can come with the opposite chirality too, which is not the case with $\bar{S}_1$.

II. NUCLEON DECAYS TO PSEUDOSCALAR MESON AND INVISIBLE FERMION AT GUT SCALE

In reference [11] the author considers a number of cases with the invisible fermion having nonzero lepton or baryon number. The most general Lagrangian with $\chi$ having baryon number $B = 1$ can be written as

$$\mathcal{L}_\chi = \bar{\chi} (i \partial - m_\chi) \chi + \frac{u_d d_k \chi^i}{\Lambda^2_{ijk}} + \frac{Q_u Q_d d_k \chi^i}{\Lambda^2_{ijk}} + \text{h.c.}$$

(3)

Here $\Lambda$ and $\tilde{\Lambda}$ denote the scales of New Physics (NP). We use here notation introduced in [1] and only write the flavour indices, not indicating Lorentz, colour and isospin indices. Assuming baryon number conservation, neutron - anti-neutron oscillations do not occur.

Integrating out the leptoquark states, one can straight-forwardly write the effective Lagrangian for the $(w^i, d^k, d^j, \chi)$ interaction

$$\mathcal{L}_{\text{eff}} (\bar{S}_1) = \frac{\bar{y}_{1ij}^R z_{1jk}^R}{M_{\bar{S}_1}^2} \epsilon_{\alpha\beta\gamma} \left( \bar{\chi}^C P_R w^i_{\alpha} \right) \left( d^C_k P_R d^j_{\gamma} \right).$$

(4)
In the case of $S_1$ one finds

$$L_{\text{eff}}(S_1) = \frac{g_{\chi d} z^{RR}_{11 j} m^2_{S_1}}{M^2_{S_1}} \epsilon_{\alpha \beta \gamma} \left( \bar{\chi}^C P_R d_\alpha \right) \left( u^C_\beta P_R d_\gamma \right). \quad (5)$$

In eqs. (4) and (5) the dimension-6 operators are of the type $u_i d_j d_k \chi_{L}$ in Eq. (3). The last term in Eq. (3) can be generated only from the $S_1$ interactions with the left-handed quarks.

$$\Gamma(p \to K^+ \chi) = \frac{1}{32\pi} \left| \frac{g_{\chi d} z^{RR}_{11 j} m^2_{S_1}}{m^2_{S_1}} \right|^2 |W_0^{RR}|^2 \times \frac{m^2_{K^+} - m^2_{K^0} + m^2_{\chi}}{m^3_p} \lambda^{1/2}(m^2_{K^0}, m^2_{\pi}, m^2_{\chi}), \quad (8)$$

and the decay width for $n \to K^0 \chi$

$$\Gamma(n \to K^0 \chi) = \frac{1}{32\pi} \left( \frac{g_{\chi d} z^{RR}_{11 j} m^2_{S_1}}{m^2_{S_1}} \right)^2 |W_0^{RR}|^2 \times \frac{m^2_{n} - m^2_{K^0} + m^2_{\chi}}{m^3_n} \lambda^{1/2}(m^2_{K^0}, m^2_{n}, m^2_{\chi}), \quad (9)$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. We use the results $W_{\pi\pi}^{RR} = 0.122$ GeV, $W_{\pi\pi\pi}^{RR} = -W_{\pi K^+}^{RR} = -0.085$ GeV $^{27, 28}$. For the intermediate $S_1$ one can use above results by making the replacements $z^{RR}_{11 j} \to z^{RR}_{11 z}, m_{S_1} \to m_{S_1}$. Experimental results on nucleon decays to invisible fermions only exist for invisible fermion with the negligible mass. The bounds on the lifetimes are $\tau(p \to \pi^+ \nu) > (390 \times 10^{30})$ yr $^{27}$, $\tau(n \to \pi^0 \nu) > (1100 \times 10^{30})$ yr $^{27}$, $\tau(p \to e^+ \nu \nu) > (170 \times 10^{30})$ yr $^{29}$. As pointed out by the author of $^{1}$, these limits push the scale $m_{S_1}$ above $10^{15}$ GeV. For the nucleon decays to pion and invisible fermion induced by $S_1$ one can use $^{9}$, replacing $m_K \to m_\pi, z_{112} \to z_{111}$. However, the decay amplitude $N \to \pi \chi$ induced by $S_1$ can occur at loop-level or it can appear at tree-level, due to the operator of dimension-9, as explained in detail in $^{26}$. In

![Figure 1](image1.png) \text{The amplitude for } p \to \chi \pi^+, \text{ induced by } S_1.

$$< P(p) | \epsilon_{\alpha \beta \gamma} \left( u_\alpha^C P_T d_\beta \right) P_T s_J | N(P, s) > = \quad (6)$$

$$P_T \left( W_{0}^{TT}(q^2) - i g W_{1}^{TT}(q^2) \right) u_N(P, s),$$

with $W_i(q^2)$ being form-factors determined by lattice QCD. One can easily calculate $S_1$ mediated decay amplitudes for $p \to K^+ \chi$

$$- i M = - \frac{i g_{\chi d} z^{RR}_{11 j}}{m^2_{S_1}} W_{0}^{RR}(k^2_2) \bar{u}(k_2, s_2) P_R u_p(k_1, s_1) \quad (7)$$

In order to obtain matrix element of the operator between nucleon and pseudoscalar states one can use notation of Ref. $^{27}$

$$\Gamma(p \to \pi^+ \chi) \text{ induced by } S_1.$$
resulting in the amplitude
\[ M_{p \to \pi^+ \chi} = \frac{4G_F y_{11}^{RR} z_{12}^{RR}}{m^2_{S_1}} \frac{f_{\pi^+}}{m^2} \sum m \alpha_L u \bar{\chi} P_R u_p \]
and the decay width
\[ \Gamma(p \to \pi^+ \chi) = \frac{1}{4\pi} \alpha_L^2 \sum m \alpha_L \frac{m_{\pi^+}^2}{m^2} \frac{m^2}{m^2} \frac{m^2}{m^2} \frac{m^2}{m^2} \]

(11)

Here the parameter \( \alpha_L \) is defined as \( P_R u_p \alpha_L = e^{i \eta} \frac{m^2}{m^2} \frac{m^2}{m^2} \frac{m^2}{m^2} \frac{m^2}{m^2} \alpha_L \) with \( \alpha_L = 0.0100(12)(214) \) GeV \(^3\) obtained by the lattice calculation \( [23] \). \( f_{\pi^+} = 0.13 \) GeV. We do not discuss loop induced \( N \to \pi \chi \), due to the additional suppression by the loop factor \( 1/(16\pi^2) \) as explained in \( [23] \).

It is instructive to determine the suppression factor for the decay widths of \( p \to \pi^+ \chi \) and \( p \to K^+ \chi \) in the case of \( S_1 \) with \( m_\chi = 0.443 \) GeV
\[ \frac{\Gamma(p \to \pi^+ \chi)}{\Gamma(p \to K^+ \chi)} \bigg|_{S_1} \sim 10^{-10}, \]
(13)

and in the case of the same processes induced by \( S_1 \)
\[ \frac{\Gamma(p \to \pi^+ \chi)}{\Gamma(p \to K^+ \chi)} \bigg|_{S_1} \sim 10^{-1}. \]
(14)

In the case of \( S_1 \) one can derive bound
\[ \frac{y_{11}^{RR} z_{12}^{RR}}{M^2_{S_1}} \leq 2.83 \times 10^{-30} \text{ GeV}^{-2}. \]
(15)

III. NEUTRON DECAYS, WHILE PROTON IS STABLE

In the case where the mass of invisible fermion is in the range \( (937.8 \text{ MeV}, 938.8 \text{ MeV}) \) proton decay is avoided, but neutron transition to \( \chi \) is kinematically allowed. The lower bound on the mass of \( \chi \) comes from the request that none of the stable nuclei can decay to dark matter, whereas the upper bound is necessary for the stability of \( \chi \) \([2, 11, 17]\). In the case of experimental detection, the simplest way is to register photon of the energy \( 0.782 \text{ MeV} < E_{\gamma} < 1.646 \text{ MeV} \). In order to approach the \( n \to \gamma \chi \) decay amplitude according to \([11]\), one can assume the mixing of \( \chi \) and \( n \). Following \([11]\), the effective Lagrangian can be written as
\[ \mathcal{L}_{eff} = \bar{n}(i\gamma - m_n + \frac{g_n e}{2m_n} \sigma^{\alpha\beta} F_{\alpha\beta}) n + \bar{\chi}(i\gamma - m_\chi + \epsilon n) + \bar{\chi} \sigma^{\alpha\beta} F_{\alpha\beta} n. \]
(16)

In the case considered by \([11]\), the decay \( n \to \gamma \chi \) occurs at the tree-level with the mediation of the coloured scalar \((3, 1, 1/3)\). However, the \( S_1 \) coloured scalar can mediate such process only at the loop level. Actually, it has to be a box diagram with one \( S_1 \) and one \( W \) (see Fig. 1) for the \( n \to \chi \) transition. In principle, there is a possibility that in the case of \( u \chi \to s^* \bar{d} \) process, the \( s \bar{d} \) quark is transformed to \( d \) while the up-like quark and \( W \) are mediated in the loop. However, these contributions are suppressed by the mass of \( d \) quark and GIM mechanism and can therefore be neglected.

\[ \mathcal{L}_{eff} = \frac{g_n e}{2m_n} \frac{\epsilon}{m_\chi} \sigma^{\alpha\beta} F_{\alpha\beta} n. \]
(17)

In the case of kaons in the final state that is not possible due to \( m_{K^+} = 0.4937 \) GeV being smaller than \( m_{K^0} = 0.4976 \) GeV. In the pionic case \( m_{\pi^+} = 0.13957 \) GeV larger than \( m_{\pi^0} = 0.13497 \) GeV. One would think that mass of the invisible fermion should be larger than \( m_p - m_{\pi^+} \), which then kinematically forbids the decay \( p \to \pi^+ \chi \) and allows \( n \to \pi^0 \chi \). However, in both \( S_1 \) and \( S_1 \) cases, one can construct the dimension-9 operator which will allow decays \( p \to \chi e^+ \nu \) forcing \( S_1 \) (\( S_1 \)) to have mass of the order of a GUT scale. The same mechanism with mass of \( m_\chi < m_n - m_\eta \) will imply \( n \to \eta \chi \) can occur only at the GUT scale.

\[ \mathcal{L}_{eff} = \frac{g_n e}{2m_n} \frac{\epsilon}{m_\chi} \sigma^{\alpha\beta} F_{\alpha\beta} n. \]
(17)

In the case considered by \([11]\), the decay \( n \to \gamma \chi \) occurs at the tree-level with the mediation of the coloured scalar \((3, 1, 1/3)\). However, the \( S_1 \) coloured scalar can mediate such process only at the loop level. Actually, it has to be a box diagram with one \( S_1 \) and one \( W \) (see Fig. 1) for the \( n \to \chi \) transition. In principle, there is a possibility that in the case of \( u \chi \to s^* \bar{d} \) process, the \( s \bar{d} \) quark is transformed to \( d \) while the up-like quark and \( W \) are mediated in the loop. However, these contributions are suppressed by the mass of \( d \) quark and GIM mechanism and can therefore be neglected.

\[ \mathcal{L}_{eff} = \frac{g_n e}{2m_n} \frac{\epsilon}{m_\chi} \sigma^{\alpha\beta} F_{\alpha\beta} n. \]
(17)
with the integral

$$I(x_1, x_2, x_{S_1}) = \frac{1}{64m_W^2 \pi^2} \left[ \frac{(4 - x_1) x_1 \ln x_1}{(1 - x_1)(x_1 - x_{S_1})} - \frac{(4 + x_2) x_2 \ln x_1}{(1 - x_1)(x_1 - x_{S_1})} + \frac{(4 + x_{S_1}) x_{S_1} \ln x_{S_1}}{(1 - x_1)(x_1 - x_{S_1})(x_2 - x_{S_1})} \right].$$

(19)

In this expression $x_i = m_i^2/m_W^2$.

The dominant contribution from the box diagram comes from the $(c, b)$ ($t, b$) and $(c, s)$ quarks mediated in the box. In the box diagram in Fig. 5 the up quarks interact with the coloured scalar $S_1$. The down-like couplings to $S_1$ can be constrained using the oscillations of $K^0 - \bar{K}^0$, $B^0_{d,s} - \bar{B}^0_{d,s}$. In Appendix A, we present box diagram contributions to the transitions of $K^0 - \bar{K}^0$, $B^0_{d,s} - \bar{B}^0_{d,s}$ and determine bounds on the interactions of $S_1$ with the down quarks. Here we give bounds on the couplings we use in our calculation: $|z_{13,2}^{RR}| \leq 9.21 (M_{S_1}/\text{GeV})^{1/2} 10^{-4}$, $|z_{13}^{RR}| \leq 4.18 (M_{S_1}/\text{GeV})^{1/2} 10^{-3}$ and $|z_{12}^{RR}| \leq 0.028 (M_{S_1}/\text{GeV})^{1/2}$.

The neutron invisible decay width is given by [11]

$$\Delta \Gamma_{n \to \chi\gamma} = \frac{g^2 \epsilon^2}{8\pi} \frac{m_n \epsilon^2}{(m_n - m_\chi)^2} \left( 1 - \frac{m_\chi^2}{m_n^2} \right)^3.$$

(20)

According to [11], the branching fraction of neutron decay to invisible fermion and photon should be 1% to explain the neutron lifetime anomaly. In their case the parameter is $\epsilon = \beta y_{11j}^{\text{RR}} z_{11j}^{\text{RR}}/m_{S_1}^2$ ($S_1$ corresponds to $\phi$ in [11]). The parameter $\beta = 0.0144(3)(21)$ GeV$^3$ [2n] requires that the branching ratio for $n \to \chi\gamma$ is of the order 1%. They obtained that $y_{11j}^{\text{RR}}/m_{S_1}^2 \sim 8 \times 10^{-6}$ TeV$^{-2}$. Note that for $m_{S_1} \sim 1$ TeV the product of $y_{11j}^{\text{RR}}/m_{S_1}^2 \sim 10^{-6}$.

In Fig. 7 we present branching ratio dependence or the mass $m_\chi$ GeV and allow the couplings $y_{11j}^{\text{RR}} \simeq y_{11j}^{\text{RR}}$ to be in perturbative regime.

In Fig. 10 we present dependence of the branching ratio $Br(n \to \chi\gamma)$ on the mass of $\chi$ for a given $S_1$ mass. It is interesting that for the mass of $M_{S_1} = 1$ TeV the branching ratio is $2 \times 10^{-6}$, bellow the Borexino limit as discussed in [2]. The coloured scalar $S_1$ can have a mass within the TeV regime and is therefore appropriate for the LHC searches.

The authors of Ref. [2] explored the data with expectations of solar neutrinos and backgrounds from radioactivity to derive bounds on the neutron-mixing parameter $\epsilon/(m_n - m_\chi)$. They expressed the upper limits on the number of events as lower limits on the H lifetime are $10^{38}$ s, $10^{30}$ s and $10^{32}$ s. The green line is the 90% CL lower limit from their fit procedure to Borexino data.

The values of parameter $\epsilon/(m_n - m_\chi)$, coming from the calculation of $n - \chi$ oscillations, are allowed by the analysis of [2] and the mass of $S_1$ can be reached by LHC. In particular, the decay of $S_1$ to two jets and $S_1 \to c(t)\chi$ (monojet) studies were already done by the authors of [20] for larger masses of $\chi$, than the ones we use in this paper.

IV. POSSIBLE LOW-ENERGY SIGNATURES

The processes in which upper quarks couple to an invisible fermion $\chi$ might offer possible experimentally interesting signatures. Here we consider low-energy decays at the tree-level induced by $S_1$ with $\chi$ in the final state. These decays have invisible fermions in the final state with mass $m_\chi \approx 0.938$ GeV, allowed by the decay of neutron $n \to \chi\gamma$, leaving the proton stable. We comment on the loop-level decay $b \to s\chi\chi$. The coupling of top quark with $\chi$ and $S_1$ can be nonzero, making a search
for $t$ to two jets and invisible particle possible. However, it will be very difficult to distinguish such a signal from the decays of top to two jets at LHC.

A. $\Lambda \rightarrow \chi\gamma$

Assuming non-zero coupling of $\chi$ to $u$ quark ($\bar{y}_{111}^{RR} \neq 0$) one can generate oscillations of the $\Lambda$ baryon to $\chi$. By a simple replacement of $n$ by $\Lambda$ states and $g_n$ by $g_\Lambda$ in equation (20), one can write

$$L_{eff}(\Lambda) = \bar{\chi}(i\partial_\mu - m_\chi) + \epsilon_\Lambda(\bar{\lambda}_\chi + \bar{\chi}_\lambda), \quad (21)$$

leading to the decay width

$$\Delta \Gamma_{\Lambda \rightarrow \chi\gamma} = \frac{g_\Lambda^2 e^2}{8\pi} \left(\frac{m_\Lambda}{m_\chi}\right)^2 \left(1 - \frac{m_\chi^2}{m_\lambda^2}\right)^3 \cdot (22)$$

where $\bar{\epsilon}_\Lambda = \beta_\Lambda \left(\frac{\bar{y}_{111}^{RR} \bar{y}_{112}^{RR}}{M_{\tilde{S}_1}}\right)$. We use $g_\Lambda = -1.22$ as given in [30] and assume that the $SU(3)$ flavour symmetry holds. Then, the matrix element $0|\epsilon_{\rho\sigma\kappa} \bar{y}_{1\rho L}^{\rho\sigma} d_{R\sigma} s_{R\kappa}|\Lambda >$ is not very different from the matrix element for the neutron, $\beta_\Lambda \simeq \beta = 0.0144(3)(21)$ GeV$^3$ [27]. Current experimental limits on the rates for the baryon number violating processes $\Lambda \rightarrow \pi^+e^-$, $\Lambda \rightarrow \pi^+\mu^-$ are smaller than $6 \times 10^{-7}$ [30, 31] and for other searched channel the bounds are even weaker. Using Eq. (22), it is easy to calculate

$$Br(\Lambda \rightarrow \chi\gamma) \bigg|_{M_{\tilde{S}_1} = 5 \text{ TeV}} = \frac{|\bar{y}_{111}^{RR}|^2}{2.4 \times 10^{-5}}. \quad (23)$$

Obviously that such bound would require $\bar{y}_{111}^{RR} \ll 1$. It seems that the coupling of the $u$ quark to the invisible fermion should be very suppressed. From a number of cases studied in the literature (see e.g. [31, 32]), the couplings of the first quark generation to leptons and leptoquarks are very suppressed compared to the other two generations. Using the constraint from $D^0 - \bar{D}^0$ oscillations (see Appendix B) we notice, that the product is $|\bar{y}_{111}^{RR} \bar{y}_{121}^{RR}| < 1.1 \times 10^{-5} M_{\tilde{S}_1}/\text{GeV}$. Requirement that $\bar{y}_{111}^{RR}$ has to be small, leaves a possibility that the coupling $\bar{y}_{111}^{RR}$ can be of the order 1. This is exactly what is necessary for our analysis of $n \rightarrow \chi\gamma$. Obviously, if the $\Lambda \rightarrow \chi\gamma$ decay is forbidden, the coupling of $u$ quark should be set to zero.

![Figure 9. $B_s \rightarrow \chi\bar{\chi}$.](image)

B. Heavy hadron decays to invisibles

For the mass $m_\chi \simeq 0.938\text{ GeV}$, decays of charmed mesons to invisible fermions are not allowed kinematically. However, baryons containing one $c$ quark and two light quark, e.g. $\Lambda^+_c$ or $\Sigma^0_c$ can decay to invisible fermions. The processes as $\Lambda^+_c \rightarrow K^+\chi$ and $\Sigma^0_c \rightarrow \chi\gamma$ are allowed. Using Eq. (6), assuming that the matrix element of $< K^+|\epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'} C_P T_{1\alpha} d_{R\beta} s_{R\gamma'}|\Lambda^+_c >$ is not very different from the one in Eq. (6), using PGD data for the relevant parameters [30] we estimate that $Br(\Lambda^+_c \rightarrow K^+\chi) < 10^{-6}$ (for $\bar{y}_{121}^{RR}$ is about 1 and $M_{\tilde{S}_1} \sim 2$ GeV). The $\Sigma_c$ can decay to $\chi\gamma$. Taking the anomalous magnetic moment of $\Sigma^0_c$ to be $\simeq -2.7$, as calculated in [34], we obtain, by appropriate replacements in Eq. (20), that the rate for $\Sigma^0_c \rightarrow \chi\gamma$ is very suppressed, being in the order of $10^{-16}$, making it impossible to be seen.

Possible decays of heavy hadrons with baryon number violations were discussed in [34]. The decay $B^+ \rightarrow \Lambda_c\chi$ will be allowed within our approach, however very suppressed if the same assumptions as in [34] are used.
1. $J/\psi \to \chi \bar{\chi}$

The dominant contribution to $\Delta \Gamma(n \to \chi \gamma)$ induced by $S_1$ comes from the coupling of $c$ quark to $\chi$. One would immediately suggest that the $c\bar{c}$ bound state might decay to two invisible fermions. Only the lower bound $BR(J/\psi \to \text{visibles}) < 7 \times 10^{-4}$ is experimentally known.

The amplitude for decay $J/\psi \to \chi \bar{\chi}$ at the tree-level can be obtained using the effective Lagrangian approach as in \cite{36}.

$$\mathcal{L}_{eff} = \sqrt{2} G_F \frac{v^2}{2 M_{S_1}^2} |\bar{y}_{112}|^2 \left(\bar{c} \gamma_\mu P_R \epsilon (\chi \gamma^\mu P_R \chi)\right). \quad (24)$$

By introducing $< 0|\bar{c} \gamma_\mu c|J/\psi(\epsilon, P) > = f_{J/\psi} m_{J/\psi} \epsilon_\mu$ \cite{35}, the decay width is given by

$$\Gamma(J/\psi \to \chi \bar{\chi}) = \frac{f_{J/\psi}^2}{2 \pi m_{J/\psi}} (1 - 4 \bar{x}_c^2)^{1/2} (1 - x_\chi^2)|A|^2 \quad (25)$$

with $A \equiv \sqrt{2} G_F \frac{v^2}{2 M_{J/\psi}^2} |\bar{y}_{1211}|^2$ and $x_\chi = m_\chi / M_{J/\psi}$. The experimental bound is very week, allowing huge $\bar{y}_{112}$ coupling. For $m_\chi = 0.938$ GeV and $M_{S_1}$, given in TeV, branching ratio is

$$BR(J/\psi \to \chi \bar{\chi}) \leq \frac{|\bar{y}_{112}|^4}{M_{S_1}^2} \frac{1}{\text{TeV}^4} \times 10^{-7}. \quad (26)$$

This is three orders of magnitude smaller than the current experimental result in \cite{34}.

2. $b \to s \chi \bar{\chi}$

$$\begin{align*}
\mathcal{M}(b \to s \chi \bar{\chi}) &= \frac{8 G_F}{\sqrt{2}} \sum_{i,j=e,t} \bar{y}_{111} \tilde{y}_{i11} \tilde{y}_{j11} V_{ib} V_{js}^* \\
&\times m_{m_j} (\bar{s} \gamma^\mu P_L b) I(x_i, x_j, x_{S_1}). \quad (27)
\end{align*}$$

If we compare the appropriate Wilson coefficient for the $b \to s \chi \bar{\chi}$ and the numerical value for $M_{S_1} \sim 1$ TeV, we obtain that it is more than two orders of magnitude suppressed compared to the Wilson coefficient for the SM transition $b \to s \nu \bar{\nu}$ calculated in \cite{35}. This makes the invisible fermion search in the exclusive processes $B \to K^{(*)} \chi \bar{\chi}$ very difficult. The decays of $B \to K^{(*)} \chi \bar{\chi}$ were considered in Ref. \cite{37} for the mass of invisible fermions kinematically allowed.

V. SUMMARY

Invisible right-handed fermion can appear in different theoretical frameworks. Here we consider a model in which a coloured scalar $S_1 = (\bar{3}, 1, -2/3)$ couples either to up-like quarks and invisible right-handed fermion or two down-like quarks of different flavour species. In the case that both proton and neutron are unstable, decays of $N \to K \chi$ are possible with mass of $S_1$ at GUT scale. The neutron can decay to $n \to \pi^0 \chi$ for the mass of 0.7987 GeV $< m_\chi < 0.8045$ GeV, while decay $p \to \pi^- \chi$ is forbidden at tree-level by the dimension-9 operator. However, the dimension-9 operator might induce $p \to \chi l^+ \nu_l$ with $l = e, \mu$, forcing the mass of $S_1$ to be at GUT scale.

In the case when the neutron decays and the proton is stable, the mass of $\chi$ has a very narrow range. The $S_1$ can mediate $n \to \chi \gamma$ at loop-level with mass of coloured scalar $S_1$ of the order TeV scale, appropriate for the LHC searches. The contributions of $c$ and $t$ coupling to $\chi$ are largest in this case. The decay rate of $n \to \chi \gamma$ can reach $10^{-6}$, which is in agreement with the Borexino experiment bound. Further searches of such decays by Kam-Land and other experiments would help to distinguish between the models of invisible fermions. An interesting proposal to search for invisible fermions by their capture by atomic nuclei was done in Ref. \cite{36} suggesting that the large volume neutrino experiments can be used for such searches. This opens up new possibility for searches at DUNE, and at various xenon experiment as explained by the authors \cite{37}.

Further, we searched for possible signatures of the fermionic invisible particles, coupling to up-quarks via $S_1$ and found that at tree-level one can produce $\Lambda \to \chi \gamma$ decay. Obtaining the experimental bound on such decay rate would be very important for the model presented in this paper as well as for obtaining the constraint on the $u$ quark coupling to $\chi$. Search for $J/\psi \to \chi \bar{\chi}$ would shed more light on possible charm quark coupling to invisible fermions.

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VII. APPENDICES

A. Di-quark couplings

The contributions from the di-quark couplings in Lagrangian \( \mathcal{L}_{\Delta B = 2} \) appear in the oscillations of \( B_s - ar{B}_s \), \( B_d - ar{B}_d \) and \( K^0 - \bar{K}^0 \) mesons. In the case of \( B_s - ar{B}_s \), there are contributions from the two box diagrams with \( d \) quarks within the box. In the case of \( B_d - ar{B}_d \) (\( K^0 - \bar{K}^0 \)), internal \( s \) (\( b \)) quarks contribute. The couplings \( (\bar{z}_1)_{ij} \) are anti-symmetric \( (\bar{z}_1)_{ij} = -(\bar{z}_1)_{ji} \). The contributions of \( \bar{S}_1 \) box diagrams in the case of the \( B_s - ar{B}_s \) oscillation are

\[
\mathcal{L}_{\Delta B = 2}^{NP} = - \frac{1}{128\pi^2} \frac{\left(z_{12}^{RR}ight)^2 \left(z_{13}^{RR}ight)^2}{M_{\bar{S}_1}^2} \left(\bar{s}\gamma_\mu P_R b\right) \left(\bar{s}\gamma^\mu P_R b\right).
\]

\( (28) \)

Following their notation, one can write the modification of the SM contribution by the NP as in Ref. \([39] \)

\[
\frac{\Delta M_{SM + NP}^{S1}}{\Delta M_{SM}^{S1}} = 1 + \eta \frac{6}{23} \frac{C_{RR}^{SM}}{R_{SM}^{loop}}.
\]

\( (30) \)

They found that \( R_{SM}^{loop} = (1.31 \pm 0.010) \times 10^{-3} \) and \( \eta = \alpha_s(m_{NP})/\alpha_s(m_b) \). Relying on the Lattice QCD results of the two collaborations FNAL/MILC \([40] \), HPQCD \([41] \), the FLAG averaging group \([42] \) published following results, which we use in our calculations

\[
\Delta M_s^{FLAG2019} = (20.1^{+1.6}_{-1.0}) \text{ps}^{-1}, \quad \Delta M_d^{FLAG2019} = (582^{+0.049}_{-0.056}) \text{ps}^{-1},
\]

\( (31) \)

From these results, one can easily determine bound

\[
\frac{\left(z_{12}^{RR}\right)^2 \left(z_{13}^{RR}\right)^2}{M_{\bar{S}_1}^2} \leq 1.17 \times 10^{-4} \text{GeV}^{-2},
\]

\( (32) \)

while in the case of \( B_d - ar{B}_d \), following procedure of \([39] \), by appropriate replacements \( s \leftrightarrow d \), the constraint is

\[
\frac{\left(z_{12}^{RR}\right)^2 \left(z_{13}^{RR}\right)^2}{M_{\bar{S}_1}^2} \leq 2.58 \times 10^{-5} \text{GeV}^{-2}.
\]

\( (33) \)

Following work of \([22, 43, 44] \) for the treatment of \( K^0 - \bar{K}^0 \), we consider

\[
M_{K_{12}}^K = \frac{1}{2m_K} < \bar{K}^0|\hat{H}_{\Delta S = 2}|K^0 >. \quad (34)
\]

As discussed in \([43, 44] \) the short distance SM value for \( M_{K_{12}}^K \) is found to be

\[
M_{K_{12}}^{SM} = \frac{G_F}{12\pi} f_K^2 B_K m_K m_W F_0(x_c, x_t),
\]

\( (35) \)

with the function \( F_0(x_c, x_t) = \lambda_x^2 \eta_{c} S_0(x) + \lambda_y^2 \eta_{t} S_0(y) + 2\lambda_x \lambda_y \eta_{ct} S_0(x, y) \), \( B_K \) is a bag parameter and \( f_K \) is kaon decay constant. They are all introduced in \([43, 44] \). The effective Lagrangian can be straightforwardly derived by appropriate replacement in Eq. \((29) \).

Such Lagrangian gives the following contribution to \( M_{K_{12}}^{S1} \)

\[
M_{K_{12}}^{S1} = \frac{\left(z_{12}^{SR}\right)^2 \left(z_{13}^{SR}\right)^2}{M_{\bar{S}_1}^2} \frac{1}{192\pi^2} m_K^2 \hat{B}_K \eta^2. \quad (36)
\]

The values of \( \hat{B}_K = 0.727 \), \( m_K = 0.4976 \) and \( \eta = 0.58 \) as in \([18, 35] \). This leads to

\[
\frac{\left(z_{12}^{RR}\right)^2 \left(z_{13}^{RR}\right)^2}{M_{\bar{S}_1}^2} \leq 3.85 \times 10^{-6} \text{GeV}^{-2}. \quad (37)
\]

Using these constraints, one can find \( |z_{12}^{RR}| \leq 9.21 \times 10^{-4} \sqrt{M_{\bar{S}_1}/\text{GeV}}, |z_{13}^{RR}| \leq 4.18 \times 10^{-3} \sqrt{M_{\bar{S}_1}/\text{GeV}} \) and \( |z_{12}^{RR}| \leq 0.028 \sqrt{M_{\bar{S}_1}/\text{GeV}} \). \( \]

B. Constraints from \( D^0 - \bar{D}^0 \)

The effective Hamiltonian describing the \( D^0 - \bar{D}^0 \) oscillation is \( \mathcal{H} = C_6 (u_R \gamma_5 c_R) (\bar{u}_R \gamma_5 c_R) \). The effective Wilson coefficient in the case when two \( \chi \) and two \( \bar{S}_1 \) are exchanged within the box, one can easily calculate

\[
C_6(M_{\bar{S}_1}) = -\frac{\gamma_{11}^2 \gamma_{12}^2 {\bar{S}_1}^2}{64\pi^2 M_{\bar{S}_1}^2}.
\]

\( (38) \)

Usually, the hadronic matrix element \( <D^0| (u_R \gamma_5 c_R) (\bar{u}_R \gamma_5 c_R)|D^0> = \frac{1}{2} m_D F_D B \) with the
bag parameter $B_D(3\text{GeV}) = 0.757(27)(4)$, calculated in the MS scheme, has been computed on the lattice [14]. Due to large nonperturbative contributions, the SM contribution is not well known. Therefore, we can get the robust bound on the product of the couplings by requiring that the mixing frequency, in the absence of CP violation, should be smaller than the world average $x = 2|M_{12}|/\Gamma = (0.49^{+0.10}_{-0.11})\%$ as reported by HFLAV [15]. The bound can be obtained as in [16] from

$$|r C_6(M_{S_1})| \frac{2 m_D f_B^2 B_D}{3 \Gamma_D} < x,$$

where $r = 0.76$ is a renormalization factor due to running of $C_6$ from scale $M_{S_1} \approx 1.5$ TeV down to 3 GeV. One can easily get $|C_6| < 2.2 \times 10^{-13}$ or $y_{11}^{\text{LR}} y_{21}^{\text{LR}} < 1.1 \times 10^{-5} M_{S_1} / \text{GeV}$.

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