Supplementary data for “Gauge matters: Observing the vortex-nucleation transition in a Bose condensate”

L J LeBlanc, K Jiménez-García, R A Williams, M C Beeler, W D Phillips, I B Spielman
E-mail: lindsay.leblanc@ualberta.ca, ian.spielman@nist.gov

1. Experimental conditions

Effects of modulation: This manuscript and Ref. [S1] share the same underlying data set, put to very different purpose. In this data set, the harmonic potential along $e_x$ was periodically modulated, but sufficiently weakly that the system remained in the linear response regime. In the data presented here, release always occurred at the same phase in the modulation cycle. (Modulation was used to determine transport coefficients, which evidenced the superfluid Hall effect reported in Ref. [S1].) For the present analysis, we chose those modulation times for which the shear is entirely due to the induced circulation due to the applied $B$ and the vector-potential turn-off. We confirmed with our GPE simulations that the modulation had no effect on the shapes of the clouds in situ or in TOF.

Effects of Raman dressing: In addition to producing an artificial magnetic field, the combination of Raman dressing and real magnetic field gradient used in these experiments contributed a scalar potential that locally depended on both Raman coupling strength and detuning from Raman resonance. (The spatially-dependent, lowest-energy eigenstates of the system with Raman coupling plus magnetic field and field gradient along $e_y$ experienced an effective harmonic antitrapping potential.) This variation was measured experimentally by recording the frequency of dipole oscillation in the harmonic trap, confirming the weakened trapping potential along $e_y$. We fit the resulting trap frequency to a phenomenological second-order polynomial as a function of cyclotron frequency and included this modification of the trapping potential in the superfluid hydrodynamic calculations: $\omega_y/2\pi = [c - a(\Omega_C/2\pi + b)^2]$, where $a = 0.038(3)$ (Hz)$^{-1}$ and $b = 2.1(9)$ Hz and $c = 47.5(4)$ Hz. We also fully accounted for the Raman-induced anistropic effective mass in this system [S1, S2] in the superfluid hydrodynamics calculations. In contrast, both the modified trapping potential and the effective mass are automatically accounted for in the Raman-system GPE calculations, where the full dispersion relationship was used, and no corrections are applied (see Notes on GPE.
Atom number: Our measurements of shear are relatively insensitive to atom-number differences between experimental realizations. For $B < B_{cr}$, the shear $a_{xy}$ is depends only weakly on $N$. The atom number, $N = 1.4(3) \times 10^5$ was determined from the TOF Thomas-Fermi radii $R_y$. The uncertainty is the standard deviation of the measurements, and represents shot-to-shot fluctuations.

2. Trap and $B$ turn-off

The optical trapping potential was turned off at $t = 0$ in less than 1 $\mu$s. Concurrently, the artificial magnetic field was removed by adiabatically transforming the Raman-dressed superposition into a single Zeeman state, $|f = 1, m_F = 1\rangle$, by simultaneously ramping the Raman intensity to zero and sweeping the bias magnetic field away from Raman resonance. This process was complete over the first 2 ms of TOF. Under this process, the $1/e$ time of turn-off of the vector potential was 130 $\mu$s and during this time, the cloud expanded only slightly while artificial field effects were significant. Our GPE numerical calculations ignore the effects of this artificial field during the initial mean-field expansion and assume that all expansion occurred without the Raman beams present. The agreement between our experimental and numerical results validate this assumption, as expected for an impulse that is short compared to typical timescales for expansion. In the main text, we simplify the turn-off and consider “$t = 0^+$” to be the time when all Raman coupling effects are removed.

Due to the large final Raman detuning, the final vector potential $\mathbf{A}_f = A_f \mathbf{e}_x$ is non-zero, but uniform. This turn-off can be thought of as a two-step process. After the first step, $\mathbf{A} \rightarrow 0$ and the mechanical momentum $p'_m(t_{0+}) = p_m(t_{0-}) - B y e_x$ [equal to the initial canonical momentum $p(t_0)$] was changed by an amount proportional to $\mathbf{A}$ in the Landau gauge. Physically, this change in mechanical momentum resulted from the electric field induced as $\mathbf{A} \rightarrow 0$. In the second step $\mathbf{A} = 0 \rightarrow \mathbf{A} = \mathbf{A}_f$, giving the final mechanical momentum $p_m(t_{0+}) = p'_m(t_{0+}) - \mathbf{A}_f$, shifted by a constant $\mathbf{A}_f$ from the intermediate momentum $p'_m(t_{0+})$, because neither $\mathbf{A} = 0$ nor $\mathbf{A} = \mathbf{A}_f$ have any spatial variation. As it is the spatially dependence that is of interest here, we consider in the main text the quantity $p'_m(t_{0+})$. Effectively, the spatial variations of $p'_m(t_{0+})$ and $p_m(t_{0+})$ are identical and independent of $\mathbf{A}_f$, allowing us to consider only the $\mathbf{A}_f = 0$ case in the main text, where we use $p_m(t_{0+})$.

3. Expressions for Landau-gauge canonical momentum of trapped BEC subject to artificial magnetic field

Due to the single-valuedness of the BEC order parameter, the canonical momentum is necessarily irrotational. We find expressions for this (gauge-dependent) canonical momentum that are relevant in our experiment. In the vortex-free case (ii), we follow
the procedures outlined in Ref. [S3] and seek a solution for the canonical momentum of the spatially-continuous form \( p(\mathbf{r}) = \alpha \nabla(xy) \), where \( \alpha \) is a constant. Next, we substitute this solution into the superfluid hydrodynamic equations for a harmonically trapped system, together with the expression for the vector potential in the Landau gauge \( \mathbf{A} = -B_y \mathbf{e}_x \), as outlined in the supplementary materials of Ref. [S1]. In doing this, we find that this is a valid solution, when we use a constant \( \alpha \) such that

\[
p_{\text{ii}}(\mathbf{r}) = -B \left( \frac{\bar{\epsilon} + 1}{2} \right) \nabla(xy),
\]

(1)

where \( \bar{\epsilon} \) is an anisotropy parameter that is determined, implicitly, given the harmonic trapping parameters for this system. When \( B = 0 \), we recover case (i) in which \( p(\mathbf{r}) = 0 \).

For case (iii), in which vortices are included in the system, the canonical momentum is modified, but only near the location of the vortex core. We explore, as an example, the simple case in which a single vortex is located at the centre of the system. We assume the vortex has a phase \( \phi(\mathbf{r}) = \tan^{-1}(x/y) \) and the canonical momentum associated with this vortex is \( p_V(\mathbf{r}) = \hbar \nabla[\tan^{-1}(x/y)] \). The total canonical momentum contains contributions both from the vortex and the variations due to the field, as in case (ii), resulting in an overall canonical momentum of the approximate form

\[
p_{\text{iii}}(\mathbf{r}) = \hbar \nabla \left[ \tan^{-1} \left( \frac{x}{y} \right) \right] - B \left( \frac{\bar{\epsilon} + 1}{2} \right) \nabla(xy).
\]

(2)

Note again that these expressions for canonical momentum are gauge-dependent, and assume the Landau gauge.

4. Notes on GPE calculations

We modelled our system using a 2+1 dimensional simulation, wherein we assumed a Thomas-Fermi profile along \( \mathbf{e}_z \), and solved the resulting 2D GPE in the \( \mathbf{e}_x - \mathbf{e}_y \) plane, appropriately modified to account for the real 3D profile by including a position-dependent interaction factor [S4]. We used imaginary-time propagation [S5] to determine the initial wavefunctions, and evolved the system using a split-time spectral method [S6]. For the Raman-coupled GPE calculations, we assumed that the atoms remained in the lowest Raman dressed beam and used the energy versus momentum dispersion relation from the exact 3-level Raman-coupling Hamiltonian [S4] to describe the GPE’s kinetic energy (which in this case is only modified along \( \mathbf{e}_x \)). In the presence of a detuning gradient along \( \mathbf{e}_y \), this dispersion depended upon \( y \). This simulation accounts for both the non-uniformity of \( B \) and all contributions to the potential energy. The calculation is valid for all three cases described in the main text, and for example correctly predicts the low-field shear, \( \Omega_C \) for the onset of vortex nucleation, and the shear with vortices.

The regular structure in the simulated TOF distribution we see in case (iii) [Fig. 2(i)] results from the regularity of vortex positions \textit{in situ}. If we start the GPE calculation with vortices seeded at random positions and allow the system to equilibrate, the vortices will eventually form a single row along the long axis, as seen
5. Vortex nucleation

In the main text, we approximate the canonical momentum in case (iii) under the assumption of a single vortex at the centre of the cloud, and make the assumption that the healing length, i.e., the vortex core size, is much smaller than the Thomas-Fermi radius.

Measurements of spatial irregularity: The spatial irregularity plotted in Fig. 3(a) in the regime of case (iii) is non-monotonic as a function of $\Omega_C$ due to the details of the imaging process: though we see an initial increase as $B$ increases beyond $B_{cr}$, it does not continue to increase. As more vortices enter the system, the TOF density variations increase in their spatial frequency, and due to the limited resolution of our imaging, this measure of spatial irregularity decreases for higher spatial frequencies.

References

[S1] LeBlanc L J, Jiménez-García K, Williams R A, Beeler M C, Perry A R, Phillips W D and Spielman I B 2012 Proc. Nat. Acad. Sci. (USA) 109 10811–10814

[S2] Lin Y J, Compton R L, Jimenez-Garcia K, Phillips W D, Porto J V and Spielman I B 2011 Nature Phys. 7 531

[S3] Recati A, Zambelli F and Stringari S 2001 Phys. Rev. Lett. 86 377–380

[S4] Spielman I B 2009 Phys. Rev. A 79 063613 (pages 7)
REFERENCES

[S5] Dalfovo F and Stringari S 1996 Phys. Rev. A 53 2477–2485
[S6] Bao W, Jin S and Markowich P A 2003 SIAM J. Sci. Comput. 25 27–64
[S7] Fetter A L 2009 Rev. Mod. Phys. 81 647–691