A microscopic model for detecting the surface states
photoelectrons in Topological Insulators

D. Schmeltzer

Physics Department,
City College of the City University of New York
New York, New York 10031

Abstract

We present a model for the photoelectrons emitted from the surface of a Topological insulator induced by a polarized laser source. The model is based on the tunneling of the surface electrons into the vacuum in the presence of a photon field. Using the Hamiltonian which describes the coupling of the photons to the surface electrons we compute the intensity and polarization of the photoelectrons.
Recently the use of an innovative spectrometer with a high laser-based light source have shown that the spin polarization of the photoelectrons emitted from the surface of the $Bi_2Se_3$ Topological Insulator (T.I.) \[7–10\] can be manipulated through the laser light polarization \[1\]. One finds that the photoelectrons polarization is completely different from the initial states and is controlled by the photon polarization. A number of theories have been proposed \[2–4\]. A correct theory for this effect is important since the photoemission experiments are used to investigate the nature of the surface states. The theory given by \[3\] suggest that the photoelectron current with the spin part in the $\hat{t}$ direction is given by $\vec{S} \cdot \hat{t}$ where $\vec{S}$ is the spin polarization of the photoelectrons. The state recorded by the photodetector at position $\vec{R}_d$ is given by $|\hat{t}, \vec{R}_d\rangle$. The photocurrent induced from the intial T.I. state $|i\rangle$ depends on the photoexcited states $|f\rangle$ of the Topological Surface States. (T.S.S.) is given in ref. \[3\] : $I_\hat{t} \propto |\sum_{E_f = E_i + \hbar \Omega} \langle \hat{t}, \vec{R}_d | f \rangle |^2$ where $H^{int}$ is the matrix element between the initial surface electron and final state and $\langle \hat{t}, \vec{R}_d | f \rangle$ is the projection of the final state $|f\rangle$ measured by the detector. This phenomenological description is based the Fermi Golden rule for the photoemission cross section. Recently it has been suggested \[13\] that interactions and hybridization between the bands in a T.I. might be responsible for the modification of the photoemission spectrum.

The purpose of this paper is to present a microscopic model which explains the photoemission experimental results. We present a model where the surface T.I. conduction band are coupled to the normal electrons of the vacuum states. Such a model emerges when we consider the bulk T.I. electrons which give rise to the surface states, the surface states have a non-zero amplitude to tunnel into the vacuum \[5, 10\]. The basic T.S.S. Hamiltonian in the presence of the vector potential $\vec{A}$ is given by the Weyl model \[11\]:

$$H = \hbar \nu \int d^2r \Psi^\dagger(\vec{r}) \left[ \sigma^1 \left( -i \partial_2 - \frac{e}{\hbar} A_2(\vec{r}, t) \right) - \sigma^2 \left( -i \partial_1 - \frac{e}{\hbar} A_1(\vec{r}, t) \right) \right] \Psi(\vec{r}),$$

(1)

The spinor operator $\Psi(\vec{r})$ is decomposed into the eigenvalues of the unperturbed Weyl Hamiltonian

$$\Psi(\vec{r}) = \sqrt{\nu} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{r}} [C(\vec{k}) u_+(\vec{k}) + B(\vec{k}) u_-(\vec{k})] \equiv \sqrt{\nu} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{r}} \Psi(\vec{k}).$$

(2)

where $|u_+(\vec{k})\rangle$ is the eigenvector for particles (positive energies) and $|u_-(\vec{k})\rangle$ is the spinor
for antiparticles (negative energies) :

\[
|u^{(-)}(\vec{k})\rangle = \frac{1}{\sqrt{2}} \left[ |\vec{k}\rangle \otimes |ie^{-i\chi(\vec{k})}, 1\rangle \right]^T, \\
|u^{(+)}(\vec{k})\rangle = \frac{1}{\sqrt{2}} \left[ |\vec{k}\rangle \otimes [1, -ie^{i\chi(\vec{k})}]^T \right].
\]

(3)

The eigenvectors are given in terms of the multivalued phase \( \chi(\vec{k}) \), \( \tan(\chi(\vec{k})) = \frac{k_y}{k_z} \). We choose for the particle operators \( C(\vec{k}), C^\dagger(\vec{k}) \) and for the anti-particles the operators \( B(\vec{k}), B^\dagger(\vec{k}) \). In order to construct the explicit model we will take the chemical potential \( \mu = 0.3 \) ev. We can consider only the particle band, electrons excitations above the Dirac Cone. The momentum \( \vec{k} \) is the momentum parallel to the surface located at \( z = 0 \). For simplicity we have ignored the hexagonal warping effect on the energy spectrum of the quasi-particles \[?] and consider only the effect on the spinors \( \Xi \).

Next we present the three parts of the model : A- the one band Hamiltonian and the coupling to the photon field ; B-The coupling of the surface electrons to the vacuum electrons ; C-The spin detection Hamiltonian:

**A- The one band Hamiltonian and the coupling to the external photon field is given by:**

\[
H^{(c)} = \sum_{\vec{k}} (\hbar v |\vec{k}| - \mu) C^\dagger(\vec{k}, z = 0) C(\vec{k}, z = 0)
\]

(4)

The Hamiltonian \( H^{(c)} \) with the chemical potential \( \mu \) describe the surface conduction electrons localized on the surface \( z = 0 \). In agreement with ref. [11] we only consider the conduction electrons and ignore the antiparticles excitations. The coupling to the photon field is given according to ref.[11] and depends explicitly on the matrix elements \( \langle u^{(+)}(\vec{k})|\sigma^1|u^{(+)}(\vec{p})\rangle \) and \( \langle u^{(+)}(\vec{k})| - \sigma^2|u^{(+)}(\vec{p})\rangle \).

\[
H^{ext}(t) = \int d^2r [J_1(\vec{r}, t)A_1(\vec{r}, t) + J_2(\vec{r}, t)A_2(\vec{r}, t)] = (-ev) \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2p}{(2\pi)^2} C^\dagger(\vec{k}, t)C(\vec{p}, t)[|u^{(+)}(\vec{k})| - \sigma^2|u^{(+)}(\vec{p})\rangle A_1(-\vec{k} - \vec{p}, t) + \langle u^{(+)}(\vec{k})|\sigma^1|u^{(+)}(\vec{p})\rangle A_2(-\vec{k} - \vec{p}, t)]
\]

\[
= (-ev) \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2p}{(2\pi)^2} \hat{C}^\dagger(\vec{k}, t)\hat{C}(\vec{p}, t) \left[ W_x(\vec{k}, \vec{p})A_x(-\vec{k} - \vec{p}, t) + W_y(\vec{k}, \vec{p})A_y(-\vec{k} - \vec{p}, t) \right]
\]

\[
W_x(\vec{k}, \vec{p}) = \cos[\frac{1}{2}(\chi(\vec{k}) + \chi(\vec{p}))]; W_y(\vec{k}, \vec{p}) = -\sin[\frac{1}{2}(\chi(\vec{k}) + \chi(\vec{p}))]
\]

(5)

The photon field \( \vec{A}(\vec{P}) \equiv \vec{P} - \vec{k}, p_z \) characterized by the coherent state \( |\Omega> \). The direction of the incoming photon with respect the surface located at \( z = 0 \) is given by the vector \( \frac{\vec{p}}{|\vec{p}|} \).

3
The two transversal polarization are given by the vectors \( \vec{e}_{s=1}(\vec{P}) \) and \( \vec{e}_{s=2}(\vec{P}) \) which are perpendicular to the photon propagation.

\[
A(\vec{P}, t) = \sum_{s=1,2} [c_s(\vec{P})a_s(\vec{P})e^{-i\omega t} + c^*_s(\vec{P})a_s^\dagger(\vec{P})e^{i\omega t}]
\]

\[
\vec{P} = (\sin[\theta]\cos[\phi], \sin[\theta]\sin[\phi], \cos[\theta])
\]

\[
c_{s=1}(\vec{P}) = (\cos[\theta]\cos[\phi], \cos[\theta]\sin[\phi], -\sin[\theta])
\]

\[
c_{s=2}(\vec{P}) = (-\sin[\phi], \cos[\phi], 0)
\]

(6)

The coherent state condition for the photon field with the frequency \( \Omega \) is given by: \( a_s(\vec{P})|\Omega> = \hat{a}_s(\vec{P})|\Omega> \). For the remaining part the photon momentum will be neglected in comparison with the electrons quasi-momentum.

**B-The hybridization between the surface electrons and emitted vacuum electrons.**

We will assume that the electrons in the vacuum are described by the creation and annihilation operators \( b_s^\dagger(\vec{k}, k_z) \) and \( b_s(\vec{k}, k_z) \) with the single particles energy, \( E_f(\vec{k}, k_z) = \frac{\hbar^2k^2}{2m} + V_0 \), where \( V_0 \) is the binding- work function for the T.S.S. surface. The tunneling Hamiltonian between the surface at \( z = 0 \) and the vacuum is given by:

\[
H_t^{z=0} = \int \frac{d^2k}{(2\pi)^2} \int \frac{dk_z}{2\pi} \int \frac{dk_z}{2\pi} [t(k_{z\perp}, k_{zz})C_{\alpha}^\dagger(\vec{k}, k_{z\perp})b_{\alpha}(\vec{k}, k_{zz}) + h.c.]
\]

\[
C_\alpha(\vec{k}, k_{z\perp}) \equiv C(\vec{k}, k_{z\perp})u_{\alpha}(+)(\vec{k}); u_{\alpha}(+)(\vec{k}) \equiv |\alpha><(\vec{k})|
\]

\[
H_0 = \sum_{\vec{k}, k_z} \left( \frac{\hbar^2k^2 + k_z^2}{2m} + V_0 - \mu \right)b_{\alpha}^\dagger(\vec{k}, k_z)b_{\alpha}(\vec{k}, k_z)
\]

(7)

t(k_{z\perp}, k_{zz}) is the tunneling matrix element between the surface and the vacuum. The momentum is not conserved in the \( z \) direction. The diagonal Hamiltonian is given in eq.(7):

\[
C_\alpha(\vec{K}) = \cos[\eta]\psi_\alpha(\vec{K}) + \sin[\eta]B_\alpha(\vec{K}); b_\alpha(\vec{K}) = \cos[\eta]B_\alpha(\vec{K}) - \sin[\eta]\psi_\alpha(\vec{K})
\]

\[
\tan[\eta] = -\frac{\sqrt{2}t(k_z, k_z)}{E_f(\vec{k}, k_z) - \hbar k}
\]

\[
H = \int \frac{d^2k}{(2\pi)^2} \int \frac{dk_z}{2\pi} \left( E_-(\vec{k}, k_z) - \mu \psi_\alpha^\dagger(\vec{k}, k_z)\psi_\alpha(\vec{k}, k_z) + (E_+(\vec{k}, k_z) - \mu)B_\alpha^\dagger(\vec{k}, k_z)B_\alpha(\vec{k}, k_z) \right)
\]

\[
E_{\pm}(\vec{k}, k_z) = \frac{1}{2}[\hbar v|\vec{k}| + \frac{\hbar^2(k_z^2 + k_z^2)}{2m} + V_0] \pm \Delta(k_z) \left[ 1 + \frac{t}{\sqrt{2}\Delta(k_z)^2} \right]
\]
\[ \Delta(\vec{k}, k_z) \equiv \frac{1}{2} \left[ \frac{\hbar^2 (k^2 + k_z^2)}{2m} \right] + V_0 - \hbar \nu |\vec{k}| \]

(8)

**C-The spin detection Hamiltonian.**

The detector is characterized by the vector \( \hat{t} \) which in the frame of the T.S.S. is given by the three vector components:

\[ \vec{t} = (\hat{t}_x, \hat{t}_y, \hat{t}_z) = (\sin[\theta_d]\cos[\phi_d], \sin[\theta_d]\sin[\phi_d], \cos[\theta_d]). \]

The momentum parallel to the surface of the T.I. is conserved therefore the angle \( \phi_d \) determines the phase \( \chi(\vec{k}) \) of the T.S.S. which is measured, \( \cos[\phi_d] = \cos[\chi(\vec{k})] \). The detector Hamiltonian measures the polarization energy as function of the local magnetization \( \vec{M} \).

**Computation of the Intensity of the polarized Photoelectrons**

The number of polarized photoelectrons measured in the \( \vec{t} \) direction is given by: \( I_t(\vec{Q}, \omega) = \sum_{r=x,y,z} \hat{t}_r < \sigma^r > W_r(q) e^{i\omega t} \). To first order in the laser field we find: \( I_t(\vec{Q}, \omega \to 0) \), this photon amplitude is a function of the coherent time and time duration of the laser pulse. For the coherent state \( |\Omega > \) we have: \( a_s(\vec{P})|\Omega > = \hat{a}_s(\vec{P})|\Omega > \) where the amplitude \( \hat{a}_s(\vec{P}) \) needs to be averaged over the coherency time or pulse duration. As a result this amplitude is reduced and \(< < \hat{a}_s(\vec{P}) >> 0 \) with the polarization \( P^{(1)}[\chi(\vec{k}), \theta, \phi, \phi_d] = \sum_{r=x,y,z} \hat{t}_r < \sigma^r > W_r(q) e^{i\omega t} \). In order to compute the number of polarized photoelectrons we substitute the operators \( b_\alpha(\vec{Q}, t) \) and \( C(\vec{Q}, t) \) in terms of the new operators \( \psi_\alpha(\vec{K}) B_\alpha(\vec{K}) \) given in eq.(8). Using Wick theorem \[12\] we perform the expectation values for the intensity operator \( I_t(\vec{Q}, \omega \to 0) \). We find:

\[
\begin{align*}
I_t^{(1)}(\vec{Q}, \omega) &= \int_{-\infty}^{\infty} dt e^{i\omega t} \langle [\hat{t}_r e^{i\omega t} b_\alpha^{\dagger}(\vec{Q}) b_\beta(\vec{Q})] |\sigma^{(r)}| \rangle \langle g |b_\alpha^{\dagger}(\vec{Q}) b_\beta(\vec{Q}) |g \rangle d^3Q = I_t^{(1)}(\vec{Q}, \omega) + I_t^{(2)}(\vec{Q}, \omega) \\
I_t^{(1)}(\vec{Q}, \omega) &= (-i) \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dt_1 \langle [\hat{t}_1 b_\alpha^{\dagger}(\vec{Q}, t) |\sigma^{(r)}| \rangle \langle t_1 b_\beta(\vec{Q}, t) ]H^{ext}(t_1) |0 \rangle d^3Q \\
I_t^{(2)}(\vec{Q}, \omega) &= \frac{(-i)^2}{2} \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \langle [\hat{t}_1 b_\alpha^{\dagger}(\vec{Q}, t) |\sigma^{(r)}| \rangle \langle t_1 b_\beta(\vec{Q}, t) ]H^{ext}(t_1) H^{ext}(t_2) |0 \rangle d^3Q
\end{align*}
\]

(9)
The photoelectron intensity $I_{t}^{(2)}(\vec{Q}, \omega \rightarrow 0)$ to second order in the photon field

The photoelectron intensity $I_{t}^{(2)}(\vec{Q}, \omega \rightarrow 0)$ is computed for two different photon polarizations $s = s' = 1,2$ as a function of the detector direction $\vec{t}$. The expectation values are obtained using the averaged Green’s function with the lifetime $\Gamma$ and the Fermi-Dirac occupation function.

\[
I_{t}^{(2)}(\vec{Q}, \omega \rightarrow 0) \propto (ev)^2 (\sin[2\eta] \cos[\eta])^2 (\hat{a}_s^\dagger \hat{a}_s + \frac{1}{2}) f_{F.D.}(E_{-}(\vec{q}, q_z) - \mu)[1 - f_{F.D.}(E_{+}(\vec{q}, q_z) - \mu)]^2
\]

\[
\frac{1}{(E_{+}(\vec{q}, q_z) - E_{-}(\vec{q}, q_z) - \hbar \Omega)^2 + (2\Gamma)^2}[1 - \frac{\hbar \Omega}{(E_{+}(\vec{q}, q_z) - E_{-}(\vec{q}, q_z))(1 + \frac{2\Gamma}{(E_{+}(\vec{q}, q_z) - E_{-}(\vec{q}, q_z))^2})}]
\]

\[
\frac{1}{(E_{+}(\vec{q}, q_z) - E_{-}(\vec{q}, q_z) + \hbar \Omega)^2 + (2\Gamma)^2}[1 + \frac{\hbar \Omega}{(E_{+}(\vec{q}, q_z) - E_{-}(\vec{q}, q_z))(1 + \frac{2\Gamma}{(E_{+}(\vec{q}, q_z) - E_{-}(\vec{q}, q_z))^2})}]
\]

\[
P^{(2)}[\chi(\vec{k}), \theta, \theta_d, \phi, \phi_d] d^3Q
\]

(10)

where $P^{(2)}[\chi(\vec{k}), \theta, \theta_d, \phi, \phi_d] \equiv P^{(2)}[\chi(\vec{k}), \vec{e}_s(\theta, \phi), \theta_d, \phi_d]$ is the polarization matrix of the photoelectrons. For the case considered $\vec{k} = \vec{q}$, the parallel component of the momentum which reaches the detector is equal to the surface momentum $\vec{k}$, therefore we have $\chi(\vec{k}) = \chi(\vec{q})$. The polarization is given by:

\[
P^{(2)}[\chi(\vec{k}), \vec{e}_s(\theta, \phi), \theta_d, \phi_d] = \left[\sum_{r=x,y} W_r(\vec{k}) \vec{e}_s^r(\theta, \phi)\right]^2 \sum_{r' = x,y,z} \hat{t}_r(\theta_d, \phi_d) < (\sigma^r)^* >
\]

(11)

We will consider first the scalar part of $I_{t}^{(2)}(\vec{Q}, \omega \rightarrow 0)$ and second we will study the explicit dependence of the polarization vector $P^{(2)}[\chi(\vec{k}), \theta, \theta_d, \phi, \phi_d]$.

a-The scalar amplitude of the emitted electrons

Using the formula $I_{t}^{(2)}(\vec{Q}, \omega \rightarrow 0)$ we plot the intensity as a function of the surface electron energy $\epsilon(\vec{k})$ for different values of the outgoing momentum which is determined by the angle $\theta_d$. We use the chemical potential $\mu = 0.3\text{ev}$, binding- work function $V_0 \approx 5\text{ev}$ and laser energy $\hbar \Omega \approx 5\text{ev}$. The energy of the emitted electrons can be obtained from the fact that the surface momentum $\vec{k} = q || = |\vec{Q}| \sin[\theta_d]$ and the $z$ component $q_z = |\vec{Q}| \cos[\theta_d]$. If $\epsilon$ is the surface electron energy the free electron energy is given by $E_f(q, q_z) = \frac{h^2 Q^2}{2m} + V_0 \equiv \frac{|\text{m} q_d |^2}{2m} + V_0$. The intensity $I_{t}^{(2)}(\vec{Q}, \omega \rightarrow 0)$ as a function of surface electrons energy $\epsilon$ for a fixed angle $\theta_d = \frac{\pi}{2}$ in figure 1. In figure 2 we tune laser frequency and the binding-work function to be $5\text{ev}$. The plots are for different detector angles $\theta_d$.

b-The polarization of the emitted electrons
FIG. 1: The intensity $I^{(2)}_t(Q, \omega \to 0)$ in arbitrary units as a function of surface electrons energy $\epsilon$ for a fixed angle $\theta_d = \pi/2$.

In order to evaluate the polarization of the photoelectrons we need to use the explicit form of the photon polarization vectors $\vec{e}_s(\theta, \phi)$, $s = 1, 2$. In addition we need the photon matrix elements $W^x(\vec{k})$, $W^y(\vec{k})$ and the polarization of the surface electrons $<\sigma^x>, <\sigma^y>$. 

\[ W^x(\vec{k}) = \cos[\chi(\vec{k})]; \quad W^y(\vec{k}) = -\sin[\chi(\vec{k})]; \quad <\sigma^x> = \sin[\chi(\vec{k})]; \quad <\sigma^y> = -\cos[\chi(\vec{k})]; \quad <\sigma^z> = 0 \]

(12)

Next we consider the case that the detector is oriented in the $\vec{t}$ direction. The polarization in the $\vec{t}$ direction is given by:

\[ \vec{t} \cdot <\vec{\sigma}> = \sin[\theta_d] \cos[\chi(\vec{k})] + \phi_d = \sin[\theta_d] \cos[2\chi(\vec{k})] = \cos[\theta_d] \cos[2\phi_d] \]

(13)

Therefore the surface polarization electrons will be given by: $P_{T.S.S.} = \vec{t} \cdot <\vec{\sigma}> = \sin[\theta_d] \cos[\chi(\vec{k})]$. For the incoming photons polarization $\vec{e}_{s=2}(\theta, \phi)$ we obtain the photo-electrons polarization $P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_{s=2}(\theta = 0, \phi), \theta_d, \phi_d]$: 

\[ P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_{s=2}(\theta = 0, \phi), \theta_d, \phi_d] = \sin[\theta_d] \cos[\chi(\vec{k}) + \phi_d] \sin^2[\phi + \chi(\vec{k})] = \]

7
FIG. 2: The intensity $I^{(2)}_t(\vec{Q}, \omega \to 0)$ in arbitrary units as a function of surface electrons energy $\epsilon$ for different detector angle $\theta_d$. Laser frequency and the binding-work function have been chosen to be 5 ev. We plot the intensity for $\theta_d = 0.87$ (the smallest peak) and $\theta_d = 0.27$ corresponds to the peak around $\epsilon = 0.2$. The values of $\theta_d$ considered are $\theta_d = 0.87; 0.77; 0.67; 0.47; 0.27$. The surface energies $\epsilon$ are normalized in units of 0.3 ev., the bottom of the surface conduction band is at $\epsilon = 0$. (1 corresponds to 0.3.)

$$\sin[\theta_d] \cos[2\chi(\vec{k})] \sin^2[\phi + \chi(\vec{k})]$$

$$P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_{s=2}(\theta = 0, \phi = 0), \theta_d, \phi_d] = \sin[\theta_d] \cos[2\chi(\vec{k})] \sin^2[\chi(\vec{k})]$$

(14)

The polarization operator $P^{(2)}$ for the photon polarization $\vec{e}_{s=1}(\theta, \phi)$ is given by:

$$P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_{s=1}(\theta, \phi), \theta_d, \phi_d] = \sin[\theta_d] \cos[\chi(\vec{k}) + \phi_d] \cos^2[\theta] \cos^2[\chi(\vec{k}) + \phi]$$

$$P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_{s=1}(\theta, \phi = 0), \theta_d, \phi_d] = \sin[\theta_d] \cos^2[\theta] \cos[2\chi(\vec{k})] \cos^2[\chi(\vec{k})]$$

(15)

We observe that for the surface polarization electrons given by $\cos[\chi(\vec{k})]$ the mea-
FIG. 3: The photoelectrons polarization $P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_{s=2}(\theta = 0, \phi = 0), \theta_d, \phi_d]$ for the photon polarization $\vec{e}_{s=2}(\theta = 0, \phi = 0)$ as a function the surface electron polar angle $\chi(\vec{k}) = \phi_d$.

Sured photoelectrons have a polarization proportional to $\sin^2[\chi(\vec{k})]$ for photons given by the polarization $\vec{e}_{s=2}(\theta = 0, \phi = 0)$ and $\cos^2[\chi(\vec{k})]$ for $\vec{e}_{s=2}(\theta, \phi = 0)$. The two polarizations are shown in figures 2 and 3.

To conclude a model for computing the photoelectrons intensity and polarization has been introduced. We show that the polarization of the photoelectrons depends on the laser polarization.
FIG. 4: The photoelectrons polarization $P^{(2)}[\chi(\vec{k}) = \phi_d, \vec{e}_s = 1(\theta, \phi = 0), \theta_d, \phi_d]$ for the polarization photon $\vec{e}_s = 1(\theta, \phi = 0)$ as a function the surface electron polar angle $\chi(\vec{k}) = \phi_d$

[1] Chris Jozwiak, Cheol-Hwan Park, Kenneth Gotlieb, Choongyu Hwang, Dung-Hai Lee, Steven G.Louie, Jonathan D. Denlinger, Costel R. Rotundu, Robert J. Birgeneau, Zahid Hussain and Ale Lanzara, Nature Physics 293, vol 9 May (2013).
[2] Oleg V.Yazyev, Joel Moore, and Steven G.Louie, Phys.Rev.Lett 105, 266806(2010).
[3] Phys.Rev.Lett 109, 097601(2012) Cheo-Hwan Park and Steven G.Louie, Phys.Rev.Lett 109, 097601(2012)
[4] Y.H.Wang, D.Hsieh D.Pilon, L.Fu, D.R. Gardner, Y.S. :ee and N.Gedik. Phys.Rev.Lett 107, 207602(2011)
[5] Fan Zhang, C.L. Kane, and E.J. Mele, Phys. Rev. B 86,081303(R)(2012)
[6] Liang Fu, Phys.Rev.Lett 103,266801 (2009).
[7] C.L.Kane and E.J. Mele Phys.Rev.Lett.75,146802(2005)
[8] B.A.Volkov and O.A.Pankratov Jetp Lett. 612015 (1988)
[9] Xiao-Liang Qi and Shou-Cheng-Zhang Rev.Of Modern Physics 831057(2011)
[10] D. Schmeltzer, Advances in Condensed Matter and Material Research Editors Hans Geelvinck and Sjaak Reyst volume 10 chapter 9, pages 379 – 403 (2011).

[11] D. Schmeltzer and Avadh Saxena, Phys. Rev. B 88 035140 (2013).

[12] S. Donaich and E. H. Sondheimer ”‘Green’s Functions for Solid State Physicists’” Imperial College Press (1988)

[13] Lin Miao, Z. F. Wang, Wenmei Ming, Meng-Yu Yao, Meixiano Wang, Fang Yang, Y. R. Song, Fengfeng Zhu, Alexei V. Fedorov, Z. Sun, C. L. Gao, Canhua Liu, Qi-Kun Xue, Chao-Xing Liu, Feng Liu, Dong Qian and Jin-Feng Jia, PNAS, 2758-2762, vol 110 no. 8 (2013).