Discussion on the Issues of Rotor-Stator Interference in Pump Turbine

Hongquan Xu¹, Zhonghua Gui², Tieyou Li¹, Cuilin Liao¹,
Long Meng¹ and Lice Zhao¹

¹ China Institute of Water Resources and Hydropower Research, Beijing, China
² Technology Center State Grid Xinyuan Company LTD., Beijing, China

E-mail: xuhq@iwhr.com

Abstract. There are serious vibration issues of powerhouse in some pumped storage power plants and it affects severely the safe and stable operation of power plant. In order to settle the issues which have troubled the hydropower technology for several decades, extensive researches have been made. The rotor-stationary interference between rotating runner and stationary guide vanes are one of the researched issues concerned most and has the biggest influences. In this paper, the different opinions are presented. Firstly, the characteristics of pressure variations before and after the guide vanes are discussed and it indicates that, the pressure distribution is not uniform due to the turbulent flow around guide vanes but there is no pressure fluctuation with the dominant frequency of rotational frequency multiplying guide vane number. The interference precondition of pressure waves is there are two pressure waves with the same frequency, thus the new opinion, there is no rotor-stationary interference in the vaneless area, is presented. Secondly, equations of rotor-stationary interference presented by Hiroshi Tanaka and its derivation process are introduced. The equation foundation, physical meanings of parameters and equation dimensions are analyzed, and it is concluded that it is wrong to combine the pressure waves from rotating part and stationary part by multiplying the two pressure waves. And also the following mistakes and unreasonable issues are described. The studied objects are not consistent with that in practical application, the pump condition is not considered, the steady operation is taken as unsteady, and the definition of nodes is mistaken. The correct combination method of rotor-stationary interaction are presented.

1. Introduction
In order to adapt the new variations and requirements of energy development brought about by climate change and environmental pollution, especially to adapt to the new challenges from large-scale application of new energy to the safe operation of power grid, Chinese government made “13th Five-Year Plan” for energy development. It is planned to speed up the construction progress of pumped storage power plants during this period. New pumped storage power plants with the capacity of 60000000 kW are started, and the total installed capacity reaches 40000000 kW by the year 2020 and 90000000 kW by year 2025. The pumped storage power plants will provide support and protection for the high-efficiency application of new energy power and safe operation of power grid.

Chinese pumped storage power plants were constructed and developed rapidly and the total capacity reached 26690MW at the end of year 2016. Great technical breakthrough has been got, and most power plants operate steady and they play a huge role in the safety and stability of power grid. However,
vibration problems of powerhouses and units exist in some large pumped storage power plants [1-3] and vibration accelerations of some powerhouses even exceed 1g (g=9.8 m/s²) [3]. Because of the local vibration of powerhouse in Zangheawan pumped storage power plant, penetrative cracks appeared in the stair pillars, and local decorated layer of generator floor peeled off from the ground. Due to the severe vibration of the local powerhouse in Guangzhou pumped storage power plant, concentrated structure cracks appeared on the floor slab, beam, pillar near the construction joints on the generator floor, middle floor and pump turbine floor.

Reversible Francis pump turbine are adopted mostly in modern large pumped storage power plant for generating and pumping. It is different from conventional centrifugal pump and equipped with guide vanes and stay vanes. It is believed by most researchers, severe vibrations of head cover, bracket, turbine-generator unit and other equipment are caused by the pressure fluctuations with high frequencies and large amplitudes due to the interference between the rotatory runner blades and static guide vanes. In order to fix the issues, lots of researches are made at home and abroad. It is put forward that most powerhouse vibrations are caused by the hydraulic sources, and the main vibration frequency is generally the same as that of head cover and lower bracket, which is multiple of the blade-passage frequency (it is the product of rotational frequency and blade number), 2 or 3 times the blade-passage frequency generally.

In the 1990s, Mr. Hiroshi Tanaka made researches on the interference between runner blades and guide vanes, the concept about rotor-stator interference were presented, and the equations for rotor-stator interference are derived from the superposition of two pressure waves induced by rotational runner blades and stationary guide vanes [4]. It caused great repercussions [5]. Since then, the pressure fluctuations in vaneless area of pump turbine, namely the region between guide vanes and runner blades, are more widely concerned. Lots of researches are made through numerical simulations and model tests, and the pressure fluctuations in vaneless area are mostly considered as a result of rotor-stator interference based on the thoughts of Mr. Hiroshi Tanaka [6-17]. Rotor-stator interference between runner blades and guide vanes is regarded as the main vibration source of powerhouses and pump-turbine units in pumped storage power plants. The equations of rotor-stator interference presented by Mr. Hiroshi Tanaka are used by some enterprises for optimizing the number of runner blades and guide vanes. However, actually the methods to reduce and avoid rotor-stator interference are not achieved through more than twenty years of research and study, and the effect of mitigating vibration is not obvious after numbers of runner blades and guide vanes are optimized and matched using equations of rotor-stator interference.

An in-depth investigation is made on the steady operation of pumped storage power plants, particularly on the generating sources and propagating laws of pressure fluctuations in vaneless area. Then it is found that there are errors in the concept of rotor-stator interference, and the number combinations of runner blades and guide vanes in some power plants, in which the powerhouses vibrate seriously, don’t conform to the equations of rotor-stator interference presented by Mr. Hiroshi Tanaka. And then, it is found that there are mistakes in definition and derivation of the equations of rotor-stator interference.

2. Discussion on the Concept of Rotor-Stator Interference

2.1. Wave Interference

Wave interference is a physical phenomenon. Superposition of two waves with the same frequency will make the vibration strengthen in some areas, while weaken in some areas. The strengthening areas and weakening areas occur alternately and this phenomenon is called interference.

Wave interference is a special form of superposition of waves, and any two waves could be superimposed, whereas there are necessary requirements for two waves to form steady interference. The necessary requirements are, two waves have the same frequency and vibration direction as well as unchanged phase difference. In case the frequencies are different or the phase difference is variable for two waves, the amplitudes of different points on the wave superposition vary with time and
strengthening areas or weakening areas are variable. Hence the steady interference could not be formed and there are no interference waves.

2.2. Moving and Stationary Coordinate Systems and Relative Movements

To discuss the issues on rotor-stator interference, firstly it is necessary to figure out the original conditions of the pressures in moving and stationary coordinate systems, then the mutual effects of pressure fluctuations in two coordinate systems.

The velocity and pressure distributions are uneven in the vaneless areas, which are before and after the guide vanes, due to the turbulent flow around guide vanes. The uneven pressure distribution at turbine inlet may be formed under turbine condition as shown in Figure 1, but it is not pressure fluctuation wave because around the circle it is location coordinate and not time coordinate in the figure. The pressure at any point is only a function of location rather than a function of time without regard to pressure fluctuations due to turbulent flow, Karman vortices and other positions, hence there is no pressure fluctuations. That is to say, in stationary coordinates, there is only pressure fluctuation with blade-passage frequency caused by runner rotation.

Actually, no pressure fluctuation, the dominant frequency of which is multiplying rotation frequency with guide vane number, is found during pressure measurements in vaneless areas. For example, for a model pump turbine, runner blade number \( Z_r = 9 \), guide vane number \( Z_g = 20 \), the dominant frequency of pressure fluctuations in vaneless areas under turbine condition is \( 9f_n \) as shown in Figure 2, and no pressure fluctuation with dominant frequency of \( 20f_n \) is found.

Conversely, in the rotating coordinate of runner, the velocity and pressure distribution at inlet of runner blades is the function of location rather than time, hence there is no pressure fluctuation with blade-passage frequency. Whereas, affected by relative movements, the pressure distribution at the outlet of guide vanes in stationary coordinate is rotating in an opposite direction of runner relative to the runner in the rotating coordinate. Then pressure fluctuation, the dominant frequency of which is product of rotation frequency \( f_n \) and guide vane number \( Z_g \), is generated. As shown in Figure 3, for a Francis turbine with 24 guide vanes and 15 runner blades, pressure fluctuations on suction sides of blades with the dominant frequency of \( 24f_n \) rather than \( 15f_n \) are measured.

![Figure 1. Schematic diagram of the pressure distribution after guide vanes around the circumference.](image1)

![Figure 2. Frequency spectra of pressure fluctuations in vaneless area of pump turbine.](image2)

![Figure 3. Frequency spectra of pressure fluctuations on runner blade of Francis turbine.](image3)
2.3. **Rotor-stator Interference and Wave Interference**

Rotor-stator interference and wave interference in literatures [5-6] are compared and great differences are found.

1. Wave interference is the phenomenon which occurs in the propagation process of two waves and the basic feature is propagation. Whereas, rotor-stator interference involves the generation and formation of the pressure waves as well as mutual interactions. The basic feature is generation of pressure waves.

2. Wave interference results in stationary wave and it is superposition of two waves. Whereas rotor-stator interference results in traveling waves and the waves propagate constantly.

3. The waves involved in wave interference have the same frequency, while the waves involved in rotor-stator interference have different frequencies. One frequency is the product of rotational frequency and runner blade number, $Z_r f_n$, and the other frequency is product of rotational frequency and guide vane number, $Z_g f_n$.

4. The waves involved in wave interference are in the same coordinate, while the waves involved in the rotor-stator interference are in the different coordinates, namely stationary coordinate and rotating coordinate respectively.

5. Wave interference involves two kinds of waves. While rotor-stator interference only involves one kind of pressure wave either in rotating coordinate or stationary coordinate and it is mutual effects of pressure waves and uneven pressure fields.

Obviously, there are great differences between rotor-stator interference and wave interference. The use and spread of rotor-stator interference easily lead to confusion in the concept and it is recommended to stop using it for the following reasons.

1. Interference is a special term in the technical field, which refers to a special physical phenomenon. Whereas the physical phenomenon, which is referred to by rotor-stator interference, is not interference.

2. It is easy to make people misunderstand that rotor-stator interference really induces stationary waves in which strengthening areas and weakening areas occur alternately. But when interference of vibration waves occurs between two same units, it may be difficult to distinguish the real wave interference.

### 3. Equations of Rotor-Stator Interference

The rotor-stator interference of rotating runner blades and guide vanes in pump turbine is analyzed in detail in literature [4] and equation of interference between rotating runner blades and guide vanes is given as follows.

$$n \cdot Z_g \pm k = m \cdot Z_r$$

where, $Z_g$: number of guide vanes  
$Z_r$: number of runner blades  
$k$: number of diametrical nodes between guide vanes and runner blades, an integer  
$n$: a positive integer  
$m$: a positive integer

Equations are derived as the follows [4].

The frequency of the $n$-th harmonic of hydraulic excitation exerted on runner due to the interference is given as follows.

$$f_r = n \cdot Z_g \cdot N$$  \hspace{1cm} (1)

where, $N$ is rotational speed of runner (rps).

If we assume that the vibration with $k$ diametrical nodes by $X_k$, the vibration of runner with the frequency of $f_r$ is expressed by following equation as a sum of the modes with various numbers of diametrical nodes.
\[ X = \sum_{k=0}^{\infty} X_k \quad (2) \]

The vibration of runner consisting of \( k \) diametrical nodes which is induced by excitation acting on blade \( R(1) \) is expressed as follows.

\[ X_k = A \cdot \cos(k \cdot \varphi) \cdot \sin(2\pi \cdot f_s \cdot t) \quad (3) \]

In the same way, the vibration of runner excited by the \( i \)-th blade, \( R(i) \), is expressed as follows, considering the time lag necessary for the blade \( R(i) \) to reach the \( j \)-th guide vanes, \( S(j) \).

\[ X_{ij} = A \cdot \cos[k \cdot (\varphi - \varphi_i)] \cdot \sin[2\pi \cdot f_s \cdot (t - (\theta_j - \varphi_j) / (2\pi \cdot N))] \quad (4) \]

where, \( \varphi_i \) is the angular coordinate of location of the blade \( R(i) \) in the coordinates fixed to runner and written as follows.

\[ \varphi_i = 2\pi(i - 1) / Z_r \quad (5) \]

Also, \( \theta_j \) represents the location of the guide vane \( S(j) \) on the stationary coordinates, which is written as follows.

\[ \theta_j = 2\pi(j - 1) / Z_g \quad (6) \]

By substituting equations (1) and (6) into equation (4),

\[ X_{ij} = A \cdot \cos[k \cdot (\varphi - \varphi_i)] \cdot \sin[2\pi \cdot f_s \cdot (t + \varphi_j / (2\pi \cdot N))] \quad (7) \]

Therefore, the vibration of \( k \) diametrical nodes excited on a runner having \( Z_r \) blades is given as follows;

\[ X_k = \sum_{i=1}^{Z_r} X_{ij} = \sum_{i=1}^{Z_r} \{A \cdot \cos[k \cdot (\varphi - \varphi_i)] \cdot \sin[2\pi \cdot f_s \cdot (t + \varphi_j / (2\pi \cdot N))]} \]

By substituting equation (5) into equation (8),

\[ X_k = (A/2) \sum_{i=1}^{Z_r} \{ \sin[(2\pi \cdot f_s \cdot t - k \cdot \varphi) + 2\pi(i - 1)(n \cdot Z_g + k) / Z_r] \\
+ \sin[(2\pi \cdot f_s \cdot t + k \cdot \varphi) + 2\pi(i - 1)(n \cdot Z_g - k) / Z_r] \} \quad (9) \]

This equation represents the superposition of many waves with different phase. By carrying out the summation, we obtain,

\[ X_k = (A/2) \{ C_1 \cdot \sin[(2\pi \cdot f_s \cdot t - k \cdot \varphi) + \pi(Z_r - 1)(n \cdot Z_g + k) / Z_r] \\
+ C_2 \cdot \sin[(2\pi \cdot f_s \cdot t + k \cdot \varphi) + \pi(n \cdot Z_g - k) / Z_r] \} \quad (10) \]

where,

\[ C_1 = \sin[\pi(n \cdot Z_g + k)] / \sin[\pi(n \cdot Z_g + k) / Z_r] \quad (11) \]

\[ C_2 = \sin[\pi(n \cdot Z_g - k)] / \sin[\pi(n \cdot Z_g - k) / Z_r] \quad (12) \]

The numerators of constants, \( C_1 \) and \( C_2 \), in the terms of vibration amplitudes is always zero since \( n \cdot Z_g \pm k \) is integer. If the denominators of \( C_1 \) and \( C_2 \), which are \( \sin[\pi(n - Z_g \pm k) / Z_r] \), are not zero, the value of \( X_k \) given by equation (10) becomes zero and the vibration \( k \) diametrical nodes is not excited on the runner. On the contrary, the vibration with \( k \) diametrical nodes can be excited only in case the denominators of \( C_1 \) and \( C_2 \) are zero. This is given by the following condition.

\[ n \cdot Z_g \pm k = m \cdot Z_r \quad (13) \]

Under this condition (13), the values of \( C_1 \) and \( C_2 \) are indeterminate. They are obtained by computing the limit value of \( \sin[\pi(n \cdot Z_g - x)] / \sin[\pi \cdot x] \) when \( x \) converges to \( m \). The value of \( C_1 \) and \( C_2 \), thus obtained, is \( \pm Z_r \), where plus sign for even numbers of \( m(Z_r - 1) \) and minus sign is for odd numbers of \( m(Z_r - 1) \).

Therefore, when equation (13) is satisfied, equation (10) becomes as follows.

\[ X_k = (A/2) \cdot Z_r \cdot \{ \sin[2\pi \cdot f_s \cdot t - k \cdot \varphi] + \sin[2\pi \cdot f_s \cdot t + k \cdot \varphi] \} \quad (14) \]
If the above vibration is viewed from the stationary coordinates, it is expressed as follows by substituting \( \Phi = \theta - 2\pi N \cdot t \).

\[
X_k = (A / 2) \cdot Z_r \cdot \{ \sin[2\pi \cdot (f_r + k \cdot N) \cdot t - k \cdot \theta] + \sin[2\pi \cdot (f_r - k \cdot N) \cdot t + k \cdot \theta] \}
\]

To simply it as follows.

\[
X_k = (A / 2) \cdot Z_r \cdot \{ \sin(2\pi \cdot f_r \cdot t - k \cdot \theta) + \sin(2\pi \cdot f_r \cdot t + k \cdot \theta) \}
\] (15)

The angular velocity of the modes is \( \pm (f_r / k) \text{Hz} \) against the rotating coordinate and \( \pm (f_r / k) \text{Hz} \) against stationary coordinate. The plus sign indicates the mode turning in the same direction and the minus sign the opposite.

Note: The derivation formulas in this chapter are quoted from literature [4].

4. Discussion on Equations of Rotor-Stator Interference

Equation (13) is widely used. It reminds us of the threat caused by mutual interference between rotating runner and stationary guide vanes, paying attention to the design and selection of numbers of runner blades and guide vanes, avoiding the rotor-stator interference due to mismatching parameters. However, there are some issues on the equations and it will be discussed from the basic concepts, formula derivation and practical application as below.

4.1. Combination Method of Rotor-Stator Interference

In the formula derivation, equations (3) and (4) are the basic formulas and the other equations are derived from the two equations. The concepts are discussed firstly.

The equations (3) and (4) are multiplying sine function by cosine function and the physical meanings of two functions are not defined. As shown in equations (1) and (4), sine function in equation (4) includes the number of guide vanes \( Z_g \) and it reflects the static effect from guide vanes. As shown in equation (5), cosine function in equation (4) includes the number of runner blades \( Z_r \) and it reflects the rotating effect from runner blades. Obviously, in literature [4], it is believed that vibration excitation acting on the runner blades is the product of vibrations from rotating excitation and static excitation and this is known as interference between rotating runner blades and guide vanes, namely rotor-stator interference.

Similarly, the views are presented in literature [6]. The pressure fluctuations in stationary coordinate and rotating coordinate, \( P_s \) and \( P_r \), are expressed as formulas below.

\[
P_s(\theta_s, t) = \sum_{n=1}^{\infty} A_n \cos(n \cdot Z_g \cdot \theta_s + \varphi_n)
\] (16)

\[
P_r(\theta_r, t) = \sum_{m=1}^{\infty} A_m \cos(m \cdot Z_r \cdot \theta_r + \varphi_m)
\] (17)

The results of interference between rotating and stationary parts are the product of the two excitations [7] as shown in equations (16) and (17), which are two kinds pressure fields in literature [6].

Whether or not there are stationary or rotating pressure fluctuations, no matter the interference exists or not, the combination results of the interference should be the sum of two pressure fluctuations, rather than the product. The reasons are analyzed as follows.

1. The basic principle is analyzed. The combination method is determined by the physical meanings and relationship of the physical quantities. For example, the power \( P \) should be the product of head \( H \) and flowrate \( Q \). The addition should be used to combine two pressure fields (or pressure fluctuations) because the combined terms and the combined result are all pressure fields (or pressure fluctuations). For the definition of interference of pressure waves, it is superposition of two pressure waves with the same frequency and phase difference, and addition should be used, rather than multiplication. Even in the literature [6], which presents equations (16) and (17), it states that, the effects from guide vanes or runner on velocity field are obtained by superposition and addition, and also flow field affected by runner pressure field or guide vane wake as well as both of them are shown in Figure 4. Known from the Bernoulli equation, pressure filed affected by both runner blades and guide vanes should be combined by superposition and addition.
(2) Physical meanings and dimensions are analyzed. The dimension of equations (16) and (17) is that of pressure, and the dimension of the combined result, which is the product of equations (16) and (17), is that of square of pressure. The physical meaning of the combined result is not clear, but it certainly is not the pressure. If it is combined by superposition and addition, physical meaning of combine result is still pressure and its dimension is that of pressure. Similarly, the vibrations caused by guide vanes and runner blades are expressed by sine function and cosine function in equations (3) and (4) respectively. The physical unit of combined result, which is the product of equations (3) and (4), is that of square of vibration. The physical meaning of the combined result is also not clear. If the vibration is a value of displacement, the dimension of the displacement vibration is length. The dimension of combined result through multiplication is area and it is obviously unreasonable.

(3) Practical application is analysed. For example, it is assumed that the pressure caused by the runner at a certain moment and a certain location is a negative value, the pressure caused by the guide vanes is also a negative value, and the product of the two is a positive value, which is obviously very unreasonable.

Flow field distortion due to runner pressure field
Flow field distortion due to guide vane wake
Combination of both effects

Figure 4. Schematic diagram of combined velocity field at runner inlet.

4.2. Issues on Basic Concepts
Issues on the foundation and basic concepts of equations of rotor-stator interference exists as below.

(1) Equation (1) is derived for the runner and it is in the rotating coordinate, while powerhouse vibrations and pressure fluctuations in vaneless zone is in stationary coordinate. Even if the equation is right, it is not applicable to powerhouse vibrations, pressure fluctuations in vaneless zone and others in stationary coordinate.

(2) The formula derivation does not specify whether it is for pump conditions or turbine conditions. From the actual analysis, it refers to turbine conditions, obviously it is not suitable for pump conditions, because at pump conditions the water flows from runner into guide vanes and the effects from guide vanes to runner is indirect, and the formulas (3) and (4) are not established.

(3) For the analysis of the runner, rotating coordinate is adopted, in which the runner is relatively stationary, and the flow and pressure distribution in the runner are not uniform (as shown in Figure 4(a)). However, when the influence of the relative motion of the stationary part such as the guide vane is not considered, flow in runner is steady. In Figure 4, horizontal coordinate is the location, not the time. As runner rotates and the time changes, the velocity and pressure at each point in Figure 4 remain unchanged. It is impossible for the runner flow to generate pressure fluctuations itself, and it is even less likely that the flow in one channel causes an excitation to the flow in other channel and generate pressure fluctuations.

(4) The object of the formulas is the runner, and the main influencing factor is from guide vanes. But equation (8) does not accumulate the vibrations of guide vanes, but accumulates the vibrations of runner blades which almost have no influence on each other.

(5) The formula doesn’t specify clearly whether its object is a whole runner or a single blade channel. For a whole runner, the number of guide vanes is mostly even, the pressure on both symmetric sides of the runner is equal, the action direction is opposite, and the overall action is offset. Only for a single blade channel, the uneven pressure distribution caused by guide vanes will arouse pressure fluctuations.
4.3. Issues on Formula Derivation

There are also some problems in the derivation process of equation (13) in the literature [4]. The most obvious problem occurs in the conversion process from equation (4) to equation (7).

The literature [4] clearly states that the equations (1) and (6) are put into the equation (4) to obtain equation (7). However, comparing the equations (7) and (4), the only difference between them is that, term $\theta_j$ is deleted in equation (7).

According to the derivation, after correctly substituting equations (1) and (6) into equation (4), equation (7) should be as below.

$$X_{ki} = A \cdot \cos[k \cdot (\varphi - \varphi_i)] \cdot \sin\left\{2\pi \cdot f_r \cdot \left[t + \varphi_i / (2\pi \cdot N)\right] - 2\pi \cdot f_r \cdot (j-1) / (Z_g \cdot f_n)\right\}$$

$$= A \cdot \cos[k \cdot (\varphi - \varphi_i)] \cdot \sin\left\{2\pi \cdot f_r \cdot \left[t + \varphi_i / (2\pi \cdot f_n)\right] - 2\pi \cdot n \cdot (j-1)\right\}$$

Although different simplifications will result in different equations, the result of substitution is deleting $\theta_j$, which is undoubtedly unreasonable. Therefore it indicates that, whether the derivation after equation (7) is correct or not, the final result, referring to equation (13), is not the correct derivation of basic formula, referring to equation (4), and the final derived result can not be ensured to be correct.

In addition, the term $\theta_j$ is deleted in equation (7), which is equivalent to ignoring the influence of guide vanes. When vibrations of runner blades are superimposed in equation (8), the influences from different guide vanes on runner blades are not considered and the original intention of designing the term $\theta_j$ in equation (4) is violated.

4.4. Discussion on Number of Diametrical Nodes $k$

In equation (13), the number of diametrical nodes $k$ is involved. How to define it involves both conceptual and derivation problems, and also has great influence on the application of equation (13).

4.4.1. Confusion in the definition of diametrical nodes. The concept of diametrical nodes is applied in many fields, and it usually refers to junctions. In mechanical engineering, a diametrical node is a tangent point of two pitch circles on a pair of meshing gears.

In the literature [4], the explicit definition of the diametrical node is not given, but the number of diametrical node $k$ is described in the schematic diagram (as shown in Figures. 5 and 6). In Figure 5, 20 guide vanes are evenly arranged around the outer circle of the runner, and 6 runner blades are evenly arranged around the inner circle of the runner. And blade rotation angle and node rotation angle are defined as $\theta_r$ and $\theta_h$. Obviously, the pinion in the figure is a diametrical node, which is the point at which the rotating blade and the stationary vane coincide at the same angle $\theta_h$. In the figure, there are 2 pinions, and the number of diametrical node $k$ is 2. That is to say, when one blade rotates to coincide with one guide vane in the radial direction, the other $k-1$ blades coincide with the other guide vanes at the same time. There are $k$ coincident points, and $k$ is the number of diametrical node. Mathematically, the number of diametrical node $k$ is the greatest common divisor of $Z_g$ and $Z_r$.

![Figure 5. Schematic diagram of diametrical nodes.](image-url)
Figure 6. Schematic diagram of diametrical nodes and vibrations in literature [4].

However, the number of diametrical node \( k \) in literature [4] is applied confusedly. The definition of diametrical node in Figures 5 and 6 is completely different from the meanings in equation (2). In equation (2), \( X_k \) is superposed by \( k=0 \) to \( k=\infty \), and the physical meaning of \( k \) is obviously not the number of diametrical node, because the number of diametrical node \( k \) is unique, it is impossible to be 0 or infinity. For example, if \( Z_g=20, Z_r=7 \), then \( k=1 \) and the superposition in equation (2) has no physical meaning.

Obviously, \( k \) in equation (2) is the sequence number of intersection points of the guide vanes and runner blades, rather than the number of diametrical node. Runner blades and guide vanes meet \( Z_r \cdot Z_g \) times each rotation of the runner and there are \( Z_r \cdot Z_g \) intersection points which appear in sequence. If the runner continues to rotate, \( k \) can reach infinity. Whether it is parameter \( k \) in the formulas for \( X_k \) or it is subscript \( k \) in symbol \( X_k \), \( k \) is sequence number of intersection points, not the number of diametrical node. Obviously, it is inconsistent with the definition of the number of diametrical node \( k \) in literature [4].

4.4.2 Effects of the number of diametrical node on formula derivation. In fact, even if the parameter \( k \) is defined as the sequence number of intersection points of the guide vanes and the runner blades, equation (2) is also incorrect. Since in equation (3) \( X_k \) has been defined as a periodic function, the infinite superposition of \( X_k \) is bound to superimpose the amplitudes and make the amplitudes infinite.

Considering rotor-stator interference, the number of diametrical node can also be understood as the impact point. The influence on the runner caused by that runner rotates from the second impact point to the third one, as well as the influence on the runner caused by that runner rotates from the first impact point to the second one, should be consistent. The periodic function from the first impact point to the second one should not be superimposed from 0 to infinity. The only correct treatment way is to modify equation (2) as below.

\[
X = \sum_{i=1}^{k} X_i \quad (2')
\]

That is to say, in the case the runner and the guide vanes have \( k \) impact points, the vibration of runner should be superimposed by the runner modes with various numbers of diametrical nodes from 1 to \( k \).

In summary, there are mistakes of different degrees in the definitions, concepts and derivations which are involved in equations (2), (3), (4), (5), (7) and (8) proposed or derived by literature [4], hence it can be confirmed that equation (13) is groundless and it is not a necessary condition for judging whether or not rotor-stator interference occurs.

4.4.3 Confusion in practical applications of parameter \( k \). According to the above definition, the number of runner blades \( Z_r \), guide vanes \( Z_g \) and diametrical nodes \( k \) in some pumped storage power plant in China have been shown in Table 1. Obviously, the number of diametrical nodes \( k \) of most pump turbines is 1, and \( k=2 \) is only for one power plant. However, regardless of the value of \( k \), \( k \) should be a unique and definite integer for any runner, otherwise it does not conform to the physical meaning illustrated in Figure 1.

Table 2 shows numbers of diametrical nodes \( k \) and frequencies of rotor-stator interference \( f_s \) under different number combinations of runner blades \( Z_r \) and guide vanes \( Z_g \) in literature [4]. In the table, \( N \) is the rotational speed, \( k \) is the number of diametrical nodes, and \( m \) is the amplification factor of
interference frequency generated by the rotating runner to the stationary guide vanes. Obviously, $k$ in the table is not unique.

### Table 1. Common number combinations of runner blades and guide vanes in pumped storage power plant in China.

| Item | Numbers of runner blades, guide vanes and diametrical nodes |
|------|------------------------------------------------------------|
| $Z_r$ | 7 7 7 9 9 9 10 5                                      |
| $Z_g$ | 16 20 24 20 22 26 16 22                                  |
| $k$   | 1 1 1 1 1 1 2 1                                        |

### Table 2. Frequencies and numbers of diametrical nodes of hydraulic interference under different combinations of $Z_g$ and $Z_r$ in literature [4].

| $f_r=Z_g N$ | $f_r=m Z_r N$ | $k$ | $f_r=Z_g N$ | $f_r=m Z_r N$ | $k$ |
|-------------|---------------|-----|-------------|---------------|-----|
| 6N          | -14           |     | 7N          | -13           |     |
| 12N         | -8            |     | 14N         | -6            |     |
| 20N         | -2            | 20N | 21N         | +1            |     |
| 24N         | +4            |     | 28N         | +8            |     |
| 30N         | +10           |     | 35N         | +15           |     |

When $Z_g=20$, $Z_r=6$, $k$ is -14, -8, -2, +4, and +10, respectively and $m$ is correspondingly equal to 1, 2, 3, 4, and 5. When $Z_g=20$, $Z_r=7$, $k$ is -13, -6, +1, +8, and +15 respectively and $m$ is correspondingly equal to 1, 2, 3, 4, and 5.

In short, since $k$ can be randomly selected without considering the physical meaning, equation (13) is always balanced, there are infinite options and rotor-stator interference is inevitable. That is because the value selection of $k$ is inconsistent with the definition of diametrical nodes and number of diametrical nodes $k$.

In the model test of the pump turbine, measurements of pressure fluctuations are mostly carried out in vaneless area. Most dominant frequency is blade-passage frequency and individual frequency is rotational frequency. And also the dominant frequency of $2x$ or $3x$ blade-passage frequency is not found. That is to say, actually $m$ is equal to 1.

4.5. Analysis of parameters $n$ and $m$

The physical meaning and the reason for setting integers $n$ and $m$ in equation (13), which are any integer [5], are not explained in literature [4] ~ [7]. But it is impossible to generate pressure pulsations and vibrations of some frequencies for no reason. Therefore, it is lack of scientific basis that, $n$ and $m$ are cited in formula derivation in literature [4], and both $n$ and $m$ are ordinal numbers from 1 to infinity in literature [6] and [7].

Combined with the actual analysis, it is proved that $n$ and $m$ are only equal to 1. In measurements of pressure fluctuations and stresses in the runner of the Francis turbine, the dominant frequency is mostly rotational frequency, but the frequency of $2Z_g N$, $3Z_g N$ doesn’t appear. Even Mr. Hiroshi Tanaka, who presents the equation (13), believes $n$ is 1 generally.
4.6. Extensive Analysis

Obviously, there are some errors in the concept, theoretical analysis, formula derivation or final formula for the rotor-stator interference and it brings confusions in practice. The correct understanding of the mutual interference between runner blades and guide vanes of pump turbine should be as below.

1) Interference from guide vanes on rotating runner

The blade channels rotate with the runner, hence the influence from runner blades on the flow in channels changes with locations, and doesn’t change with time. The influence from runner blades can be expressed as a cosine function \( A_r \cos (Z_r \theta_r + \theta_0) \) approximatively and it is the function of the phase angle \( \theta_r \) of blade channel’s position.

The influence from guide vanes on the blade channels is obviously different. Although guide vanes are in stationary area, the uneven pressure distribution at the outlet of guide vanes, due to its relative motion to the runner, will be transformed into pressure fluctuations, which can be expressed as a cosine function \( A_g \cos (2\pi Z_g N t + \phi_0) \) approximatively.

Under the combined disturbance of runner blades and guide vanes, the pressure fluctuation \( P_r (\theta_r, t) \) in blade channels can be expressed as follows.

\[
P_r (\theta_r, t) = A_r \cos(Z_r \cdot \theta_r + \theta_0) + A_g \cos(2\pi Z_g \cdot N \cdot t + \phi_0)
\]

Therefore, under normal circumstances, in blade channels there may be pressure fluctuations, the dominant frequency of which is rotational frequency or multiplying rotational frequency by the number of guide vanes, but no pressure fluctuation with blade-passage frequency.

2) Interference from runner blades on static vaneless area

The vaneless area at the high-pressure side of runner is stationary, and the influence from guide vanes on the area only changes with position, and doesn’t change with time. The influence can be expressed as a cosine function \( B_g \cos (Z_g \phi_g + \phi_0) \) approximatively and it is the function of the phase angle \( \phi_g \) of guide vane’s position.

The influence from runner blades on the flow in vaneless area rotates with the runner and it generates pressure fluctuations which changes with time and can be expressed as a cosine function \( B_r \cos (2\pi Z_r N t + \theta_0) \) approximatively.

Under the combined disturbance of runner blades and guide vanes, the pressure fluctuation \( P_s (\phi_g, t) \) in vaneless area can be expressed as:

\[
P_s (\phi_g, t) = B_r \cos(Z_r \phi_g + \phi_0) + B_g \cos(2\pi Z_g \cdot N \cdot t + \phi_0)
\]

Therefore, under normal circumstances, in vaneless area there may be pressure fluctuation with rotational frequency or blade-passage frequency, but neither pressure fluctuation the main frequency of which is multiplying rotational frequency by the number of guide vanes, nor pressure fluctuation with the dominant frequency of \( m \) times the blade-passage frequency.

3) Brief introduction of vibration frequency of powerhouse in pumped storage power plant

As mentioned above, the vibration frequency of most pumped storage power plants with severe vibrations is not blade passage frequency, but 2 or 3 times the blade passage frequency. However, through the previous analysis, it is known that the dominant frequency of the pressure fluctuations in vaneless area is the blade passage frequency, rather than 2 or 3 times the blade passage frequency. What is the reason?

It is found that pressure fluctuations with multiple blade-passage frequency appear in the flow area of guide vanes and stay vanes. Pressure fluctuations with 2 or 3 times the blade passage frequency are measured in the areas between guide vanes and stay vanes and also between two guide vanes in some pump turbines. The waveform and frequency of the pressure fluctuations are consistent with that of the powerhouse vibrations and severe powerhouse vibrations are caused by the pressure fluctuations. The pressure fluctuations are generated by the cavitation due to separation flow around the inlet of guide vanes, and in some power plants, even due to that around stay vanes. Affected by the uneven pressure on the high pressure side of runner blades, the cavitation cavity generates an expansion-contraction cycle and a pressure fluctuation with blade-passage frequency is formed beside each guide vane. The pressure
fluctuations between adjacent guide vanes have a phase difference, which is \( \Delta \phi = 2\pi/n_z \). The combined result of \( Z_q \) pressure fluctuations with blade-passage frequency is that, pressure fluctuations with \( n_z \) times the blade-passage frequency are formed in the areas of guide vanes and head covers, and vibrations of unit and powerhouse with the same frequency are formed. If there is no cavitation cavity at the guide vanes and stay vanes, there is no such pressure pulsation, so it is an additional pressure fluctuation. For some power plants, such as Qingyuan Pumped Storage Power Plant in China, the vibrations of unit and powerhouse is very weak and the dominant frequency of vibration is the blade-passage frequency, because no cavitation cavity is formed at the guide vanes.

\[ n_z = \text{int}(Z_q/Z_r) \]  

(20)

If cavitation appears at both stay vanes and guide vanes simultaneously and the two rows of cascades are staggered, pressure fluctuations and vibrations with \( n_z \) times the blade-passage frequency may be formed. \( n_c \) is the integer coverage factor of the runner blades to the two rows of cascades. The calculation formula is as below.

\[ n_c = \text{int}(2Z_q/Z_r) \]  

(21)

5. Conclusion

Based on above discussion, conclusions are as below.

(1) The pressure distribution before and after guide vanes is not uniform due to the turbulent flow around guide vanes, but it doesn’t change with time in the stationary coordinate and there is no pressure fluctuation with a dominant frequency, which is the product of guide vane number and rotational frequency.

(2) Whether it is the disturbance from rotating coordinate to stationary coordinate or vice versa, there is only one sequence of pressure fluctuation waves at one side and the other side is pressure field which is not uniform. The necessary requirements is not met for pressure waves to form interference, hence rotor-stator interference doesn’t exist.

(3) In some literatures about rotor-stator interference, multiplication is adopted to combine the two pressures (or pressure fluctuations) respectively from rotating and stationary coordinates. This combined method is not reasonable, and the addition should be used to combine the two disturbance excitations.

(4) Normally there is only pressure fluctuation with a dominant frequency which is the product of guide vane number and rotational frequency in runner channel, and only pressure fluctuation with blade-passage frequency in vaneless area. The integer \( n \) and \( m \) in the equations of rotor-stator interference is 1. In some pumped storage plant, vibrations with a dominant frequency which is multi-fold blade-passage frequency appear, because cavitation occurs around the guide vanes and stay vanes due to separated flow and vortex, and the additional pressure fluctuations are produced by the cavitation cavity stirred by runner blades.

(5) The researched objects are not consistent with application objects during formula derivation of rotor-stator interference in literature [4] and there are mistakes, such as without consideration of pump conditions, taking the steady factor as the unsteady one, confused definition of dimetrical nodes, without fully consideration of effects from guide vanes. It doesn’t match actual situations during practical applications of the equations. It proves that the equations are not the necessary requirements for rotor-stator interference and it is not effective to avoid or mitigate the vibrations of unit and powerhouse based on the equations of rotor-stator interference.

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