Mathematical Model of Tip-hypha Anastomosis and Dichotomous Branching with Hyphal Death

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Abstract. In this paper, We studied the case of growth of kinds of fungi when blend two kinds of hyphal anastomosis and Dichotomous branched with a hyphal death. These species consume all energy. We use mathematical models as partial differential equations (PDEs) which illustrate phenomena biological for each kind. we need Some time, that's true for the growth of fungi. To get an approximate solution for this system, we will rely on the numerical solution. For this, we need Some of the steps in this solution are stationary, phase, and traveling states Solution And to determine the initial condition. we will use the code and Thus we recognize the behavior of the kinds.

1. Introduction
We constructed new models for the development of fungal mycelia. At this scale, partial differential equations representing the interaction of biomass with the underlying substrate are the appropriate choice. These models are of a complex mathematical structure, comprising both parabolic and hyperbolic parts. Thus, their analytic and numerical properties are non-trivial, and for this a group of any number of types can be expressed during the growth stages of a specific type of fungi. To facilitate discussion of these kinds the abbreviated symbols for each type are used, as in table (1), which describe some biological types where each type analyzed mathematically and given an explanation and a description of the parameters. In this paper, we will mix certain types of fungi [1, 2, 3, 4].

2. Mathematical Model
We will speak about a new kind of fungal branching with fungal death is Tip-hypha anastomosis with dichotomous branching (HY). The table below shows these kinds[6, 7]:

| Biological kind            | Symbol | Version | parameters | parameters description            |
|----------------------------|--------|---------|------------|-----------------------------------|
| Tip-hypha anastomosis      | H      | \(\sigma = -\beta_2 np\) | \(\beta_2\) | indicates to "the rate of the tip reconnection per unit length hypha per unit time". |
| Dichotomous branching      | Y      | \(\sigma = \alpha_4 n\)   | \(\alpha_4\) | indicates to "the number of the tips produced per unit time" |
| Hayphal death              | D      | \(d = \gamma p\)          | \(\gamma\)    | indicated to the rate constant for the hyphal autolysis. |

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We are able to describe hyphal growth by using the following system:

\[
\begin{align*}
\frac{\partial p}{\partial t} &= J_a - D \\
\frac{\partial n}{\partial t} &= -\frac{\partial J_a}{\partial t} + \sigma(p, n)
\end{align*}
\]  
(1)

Where \( J_a = n v \) indicated to the flux and \( D = \gamma p \) is hyphal death, and by compensation about (\( \sigma(p, n) = \alpha_1 n - \beta_2 np \)) in above system, we obtain :

\[
\begin{align*}
\frac{\partial p}{\partial t} &= n v - p \\
\frac{\partial n}{\partial t} &= -\frac{\partial (nv)}{\partial t} + (\alpha_1 n - \beta_2 np)
\end{align*}
\]  
(2)

3. Non-dimensionlisation and Stability of uniform solution

In this part, we illustrate how can lay these parameters to get dimensionless

\[
\begin{align*}
\frac{\partial \bar{p}}{\partial \tau} &= n - d \\
\frac{\partial \bar{n}}{\partial \tau} &= -\frac{\partial \bar{p}}{\partial \tau} + \alpha (n - p)
\end{align*}
\]  
(3)

Where \( \bar{p} = \frac{\alpha_1 \gamma}{\beta_3 v} \), \( \alpha = \frac{\alpha_1}{\gamma} \)

this parameter is represented "rate of hyphal tips per unit hyphal per unit length hypha per unit time". \( d \) is hyphal death where the value of \( d = 1 \) and \( \alpha n (1 - p) \) indicates to "the number of the tips produced per unit time".

Now, we find steady states from the system (3) when take the following:

\[
\begin{align*}
n - \bar{p} &= 0 & \rightarrow & & f(p, n) \\
\alpha n (1 - p) &= 0 & \rightarrow & & g(p, n)
\end{align*}
\]  
(4)

The solution of the system above, we obtain the values of \((p, n)\)-plane, the steady state are: \((1, 1), (0, 0)\). Now, we can take Jacobain for the system(4)

\[
J_{(p,n)} = \begin{bmatrix} -1 & 1 \\ \alpha n & \alpha (1 - p) \end{bmatrix}
\]  
(5)

We will get two eigenvalues of \( \lambda (\lambda_n; n = 1, 2) \)

We notice through solution the system in the point \((0, 0)\) is unstable node while in the point \((1, 1)\) is the saddle point when \( \alpha \) is non-negative. The Fig(1) illustrate that by Using "MATLAB pplane7".
4. Traveling Wave Solution and the steady states

In this part, we will speak the travelling wave solution, let \( z = x - ct \), and we impose:

\[
\begin{align*}
N(x, t) &= N(z) \\
p(x, t) &= P(z)
\end{align*}
\]

(6)

where \( P(z) \) indicate to density profiles, and \( c \) rate of propagation of colony. \( N(z) \) and \( P(z) \) positive function for \( z \). The function \( N(x, t) \), \( p(x, t) \) are traveling waves, and are moves at constant speed \( c \) in positive \( x \) direction, where \( c > 0 \) and \( \alpha = 1 \). We appearance the traveling wave solution of the system in \( t \) and \( t \) in the form (3)(4,5):

\[
\begin{align*}
\frac{dp}{dt} &= -c \frac{dN}{dz} \\
\frac{dn}{dt} &= -c \frac{dN}{dt} \\
\frac{dp}{dt} &= 1 - c \left[ \alpha N(1 - P) \right], \quad c \neq 1, \quad -\infty < x < \infty
\end{align*}
\]

(7)

Therefore becomes the system (3):

\[
\begin{align*}
\frac{dp}{dt} &= -1 \left[ N - D \right] \\
\frac{dp}{dt} &= \frac{1}{1 - c} \left[ \alpha N(1 - P) \right], \quad c \neq 1, \quad -\infty < x < \infty
\end{align*}
\]

(7)

We notice the steady states of the system (7) we get the point \((p, n) = (0, 0)\) is saddle point and \((1,1)\) is stable spiral for \( c < 1 \). See Figure (2) Using MATLAB pplane7

Figure 1. The \((p, n)\)-plane, we note that trajectories connects the saddle point \((1,1)\) and the unstable node \((0, 0)\) for \( \alpha = 2 \).
Figure 2. The (p, n)-plane, we note that trajectories connect when $c = -1.5, \alpha = 1$, the stable spiral (1, 1), and saddle point (0, 0).

5. The numerical solution
we will be using pdepe code in MATLAB "to solve the system(3). This showing the numerical solution of the branch ($p$) and tips ($n$), obtain through the initial condition ($1 \rightarrow 0$). The figures below explains this.

Figure 3. The initial condition of ($n$), and ($p$) from 1 to 0, with parameter $\alpha = 0.5$. solution to the system(3).
Figure 4. solution to the system(3) with parameters $\alpha$, and the wave speed $c$. In (a) $\alpha=0.5$, $c=2.0145$, in(b) $\alpha=1$, $c=3.0657$, and in (c) $\alpha=2$, $c=5.114$ respectively for time $t$, where (n) is the branches referred to the red line, and (p) is the tips referred to the blue line.
6. **The conclusions**

we concluded from above results that the traveling wave \( c \) increase whenever the values of \( \alpha \) increase for time \( t \).

Since "the value of \( \alpha = \frac{\alpha_1}{\gamma} \)" and we notes that \( \alpha \) directly proportional with \( \alpha_1 \) "the number of the tips produced per unit time" and inversely proportional \( \gamma \) "the rate constant for the hyphal autolysis ". That's mean from a biological point of view the growth increases whenever \( \alpha \) increases.

7. **References**

[1] Ali Hussein Shuaa Al-Taie (2018), *Energy Consumption of Lateral Branching, Tip death due to over crowding, with Exponential Function*.

[2] Ali Hussein Shuaa Al-Taie. Continuum. (2011), *models for fungal growth*.

[3] Zill, D. G. (2012). *A first course in differential equations with modeling applications*. Cengage Learning.

[4] Edelstein, L (1982), The propagation of fungal colonies: A model for tissue growth. *Journal of theoretical Biology*, J theor. Biol. 98:679-710.

[5] Kuto, K. and Yamada, Y (2004), Multiple coexistence states for a prey-predator system with cross-diffusion. *Journal of Differential Equations*, 197(2):315-348.

[6] Michael J. Markowski, (2008), *Modeling behavior in vehicular and pedestrian traffic flow*.

[7] Murray J.D. 1989. *Mathematical Biology II :Spatial Models and biomedical Application*. Springer – Verlag New York.