Multiscaling Differential Contraction Integral Method for Inverse Scattering Problems With Inhomogeneous Media

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Abstract—Practical applications of microwave imaging often require the solution of inverse scattering problems with inhomogeneous backgrounds. Toward this end, a novel inversion strategy, which combines the multiscaling (MS) regularization scheme and the difference contraction integral equation (DCIE) formulation, is proposed. Such an integrated approach mitigates the nonlinearity and the ill-posedness of the problem to obtain reliable high-resolution reconstructions of the unknown scattering profiles. The arising algorithmic implementation, denoted as MS-DCIE, does not require the computation of Green’s function of the inhomogeneous background, and thus, it provides an efficient and effective way to deal with complex scenarios. The performance of the MS-DCIE is assessed by means of numerical and experimental tests in comparison with competitive state-of-the-art inversion strategies as well.

Index Terms—Difference contraction integral equation (DCIE), inhomogeneous media, inverse scattering (IS), microwave imaging, multiscaling (MS).

I. INTRODUCTION

IN ENGINEERING, microwave imaging applications are concerned with both homogeneous and inhomogeneous backgrounds [1]. As for the latter, through-wall imaging [2], [3], [4], nondestructive evaluation (NDE) [5], [6], [7], [8], scanning microscopy [9], cross-well induction logging [10], marine magnetotelluric surveys [11], and biomedical imaging [12], [13], [14] are popular real-world applications. Regardless of the type of background, the behavior of time-harmonic electromagnetic (EM) fields and their interactions with the environment (i.e., scatterers, host medium, and receivers) can be faithfully described in terms of either the wave equation, which is a partial differential equation (PDE) or the corresponding integral equation (IE).

Well-known examples of inverse scattering (IS) approaches formulated within the Lippmann–Schwinger IE (LSIE) are the methods based on the Born [15], [16], the Rytov [17], the extended Born [18], and the distorted Born [19], [20], [21], [22] approximations or Newton-type techniques [23], [24], [25]. PDE-based methods (e.g., [26], [27]) have been extensively studied as well. In principle, all these methods can handle IS problems (ISPs) with inhomogeneous media, but their efficiency and accuracy might be underwhelming since a repeated solution of the corresponding forward (FW) scattering problem is required to iteratively update the unknown contrast [28]. In addition, some approaches rely on linear approximations, which do not hold true for highly contrasted objects when multiple-scattering phenomena are nonnegligible [29].

To avoid multiple calls of an FW solver and to enable the application of different regularization techniques, such as the total variation [30] and multiplicative regularizations [31], [32], [33], by also reducing the nonlinearity of the ISP at hand due to new theoretical formulations (e.g., the contraction IE (CIE) [34], [35]) for dealing with strong contrasts or electrically large scatterers, an alternative class of inversion strategies has been considered. Namely, the modified gradient method [36], the contrast source inversion (CSI) method [37], [38], and the subspace-based optimization method (SOM) [32], [39], [40], [41] have been proposed where the equivalent source is now the “secondary” unknown instead of the EM field, while the contrast is still the “primary” quantity to be determined.

To address the ISPs with inhomogeneous scenarios, an effective and widely adopted strategy in many practical scenarios (e.g., NDE [8]) is to exploit some a priori
information on the EM characteristics of the background. In this framework, the CSI [42] and the SOM [43] methods adopt a PDE-based modeling for computing Green’s function of the inhomogeneous background so that the inversion is carried out within the LSIE framework. Despite their successful application, these approaches suffer from the heavy computational burden of numerically determining the inhomogeneous-media Green’s function without using acceleration techniques as for homogeneous backgrounds (e.g., the conjugate gradient fast Fourier transform (CG-FFT) [44] or the fast multipole method (FMM) [45]). Conversely, learning approaches proved to retrieve both qualitative [46], [47] and quantitative [48], [49], [50] features of the imaged domain with almost real-time performance. More specifically, learning-by-examples [47], [49] and deep learning [48], [50] strategies have been recently introduced for efficiently solving ISPs starting from the gathered information in a properly built training set of input–output pairs.

Otherwise, the difference CIE (DCIE) method [51] exploits an alternative ISP formulation that only needs the closed-form expression of Green’s function for the homogeneous background. However, the arising inversion process has to be stabilized [34], [35] by means of an effective regularization technique such as the multiscaling (MS) strategy, which has proved able to handle both 2-D [29] and 3-D [52] scenarios as well as aspect-limited configurations [53], [54], [55], [56]. Moreover, it has been successfully combined with different optimization methods [29] and formulations [57], [58].

Generally speaking, the MS strategy is a multizoom metalevel scheme aimed at identifying, at each step, the region of the investigation domain where the unknown scatterers are supposed to be most likely located, referred to as a region of interest (RoI). In this latter, the reconstruction of the object descriptors, whose number is kept close to the amount of available information from the scattering data/measurements, is then performed with a suitable inversion method. In this way, while mitigating the ill-posedness and the nonlinearity of the successive inversions, multiresolution imaging of the scenario is yielded, the higher spatial resolution being only in the RoI identified at each step.

In this work, the DCIE formulation is extended to the MS scheme for defining a new inversion strategy able to effectively and efficiently address imaging problems involving unknown objects in inhomogeneous backgrounds. Such integration is not straightforward since the differential formulation considers the EM interactions generated by the unknown objects also outside the RoI. Indeed, external equivalent currents are generated because of the coupling between unknown scatterers and the inhomogeneous background and they cannot be neglected without compromising the validity of the IS model. Therefore, an ad hoc RoI estimation technique is proposed to derive a novel and customized ISP solution approach, namely, the MS-DCIE. This method features the advantages of both the DCIE inversion (i.e., computationally fast and weakly nonlinear) and the MS scheme (i.e., reduced ill-posedness and multiresolution reconstruction).

The outline of this article is given as follows. The ISP is mathematically formulated within the DCIE framework in Section II, while Section III details the MS-DCIE inversion strategy. In Section IV, a representative set of numerical and experimental test cases are presented and discussed to assess the reliability and effectiveness of the proposed method. Finally, some concluding remarks are drawn in Section V.

II. ISP FORMULATION (DCIE FRAMEWORK)

Let \( \mathcal{H} \) be a square investigation domain of side \( L_\mathcal{H} \) and characterized by a known and inhomogeneous permittivity distribution \( \varepsilon_H(\mathbf{r}) \), which is univocally described by the contrast function \( \tau_H(\mathbf{r}) \triangleq \varepsilon_H(\mathbf{r})/\varepsilon_B - 1 \), \( \varepsilon_B \) and
the object given by (2) require the evaluation of medium with permittivity \(\varepsilon\) 

\[ J^{(v)}_{\Delta}(\mathbf{r}) \triangleq J^{(v)}(\mathbf{r}) - J^{(v)}_{H}(\mathbf{r}) \]  

(6)

\( J^{(v)}_{H}(\mathbf{r}) \) being the equivalent current induced on the investigation domain \( \mathcal{H} \) with contrast \( \tau \) and radiating in the free-space background with \( \tau_{B}(\mathbf{r}) = 0 \) i.e., \( J^{(v)}(\mathbf{r}) \triangleq \tau(\mathbf{r})\xi^{(v)}(\mathbf{r}) \) and \( J^{(v)}_{H}(\mathbf{r}) \triangleq \tau_{H}(\mathbf{r})\xi^{(v)}(\mathbf{r}) \), so that, after some simple manipulations, it turns out that

\[ \xi^{(v)}_{\Delta}(\mathbf{r}_{m}) = \int_{\mathcal{H}} G_{B}(\mathbf{r}_{m}, \mathbf{r}')J^{(v)}_{\Delta}(\mathbf{r}')d\mathbf{r}' \]  

(7)

\[ J^{(v)}_{\Delta}(\mathbf{r}) = \tau_{\Delta}(\mathbf{r})\left[\xi^{(v)}_{\Delta}(\mathbf{r}) + \int_{\mathcal{H}} G_{B}(\mathbf{r}, \mathbf{r}')J^{(v)}_{\Delta}(\mathbf{r}')d\mathbf{r}'\right] \]

\[ \tau_{H}(\mathbf{r})\int_{\mathcal{H}} G_{B}(\mathbf{r}, \mathbf{r}')J^{(v)}_{\Delta}(\mathbf{r}')d\mathbf{r}'. \]  

(8)

By introducing the auxiliary parameter \( \beta(\mathbf{r}) \) [34] and defining the modified contrast function \( \chi \) \[ \chi(\mathbf{r}) \triangleq (\beta(\mathbf{r})\tau(\mathbf{r}))/\beta(\mathbf{r})\tau(\mathbf{r} + 1) \], (8) is then rewritten in the so-called DCIE form [51]

\[ \beta(\mathbf{r})J^{(v)}_{\Delta}(\mathbf{r}) \]

\[ = \chi(\mathbf{r})\left[\int_{\mathcal{H}} G_{B}(\mathbf{r}, \mathbf{r}')J^{(v)}_{\Delta}(\mathbf{r}')d\mathbf{r}' + \beta(\mathbf{r})J^{(v)}_{\Delta}(\mathbf{r}') + \beta(\mathbf{r})J^{(v)}_{H}(\mathbf{r}) + \xi^{(v)}(\mathbf{r})\right] \]

\[ \chi(\mathbf{r})\left[\int_{\mathcal{H}} G_{B}(\mathbf{r}, \mathbf{r}')J^{(v)}_{\Delta}(\mathbf{r}')d\mathbf{r}' + \beta(\mathbf{r})J^{(v)}_{\Delta}(\mathbf{r}') + \beta(\mathbf{r})J^{(v)}_{H}(\mathbf{r}) + \xi^{(v)}(\mathbf{r})\right] \]

(9)

where \( \chi(\mathbf{r}) \) \[ \chi(\mathbf{r}) \triangleq (\beta(\mathbf{r})\tau(\mathbf{r}))/\beta(\mathbf{r})\tau(\mathbf{r} + 1) \] and \( \chi(\mathbf{r}) \) \[ \chi(\mathbf{r}) \triangleq (\beta(\mathbf{r})\tau(\mathbf{r}))/\beta(\mathbf{r})\tau(\mathbf{r} + 1) \] are the host medium and the differential modified contrast, respectively. Although (8) and (9) model the same phenomena, the \( \beta \) term offers a way to weight the impact of the multiple scattering effects, thus reducing the nonlinearity of the ISP by letting the local wave effect dominate the global one [34], [59].

Within the above DCIE formulation, the solution of the original ISP is then recast to that of determining the distribution of the differential modified contrast \( \chi_{\Delta} \) (i.e., the primary unknown) and the \( V \) differential equivalent currents, \[ \{J^{(v)}_{\Delta} \}; v = 1, \ldots, V \} \] (i.e., the secondary unknowns), which fulfill (7) and (9).

Toward this end, (7) and (9) are first discretized by partitioning the investigation domain \( \mathcal{H} \) into \( N \) square subunits, \( \{\mathcal{H}_{n}; n = 1, \ldots, N\} \) \[ \mathcal{H} = \sum_{n=1}^{N} \mathcal{H}_{n} \], centered at \( \{\mathbf{r}_{n}; n = 1, \ldots, N\} \) and using \( M(N) \) Dirac’s test functions to sample (7) and (9) at the \( M(N) \) locations of the probes in the observation (investigation) domain \( \{\mathbf{r}_{m}; m = 1, \ldots, M\} \). The following numerical forms of (7) and
(9) are then derived
\[
\bar{\mathcal{E}}^{(v)}_{\text{sca, }\Delta} = \bar{\mathcal{E}}_{\text{ext}}^{(v)} \bar{\mathcal{J}}_{\Delta} \bar{T}^{(v)} + \bar{\mathcal{E}}_{\text{ext}}^{(v)} \bar{\mathcal{J}}_{\Delta} \hat{\bar{T}}^{(v)} + \bar{\mathcal{E}}_{\text{ext}}^{(v)} \bar{\mathcal{J}}_{\Delta} \bar{T}^{(v)} \tag{10}
\]

where $\bar{\mathcal{E}}^{(v)}_{\text{sca, }\Delta} = \{\mathcal{E}^{(v)}_{\text{sca, }\Delta}(\mathbf{r}_m); \, m = 1, \ldots, M\}$, $\bar{\mathcal{E}} = \{\mathcal{E}(\mathbf{r}_n); \, n = 1, \ldots, N\}$, $\bar{\mathcal{J}}_{\Delta} = \{\mathcal{J}(\mathbf{r}_n); \, n = 1, \ldots, N\}$, $\bar{\mathcal{H}} = \{\mathcal{H}(\mathbf{r}_n); \, n = 1, \ldots, N\}$, and $\bar{T}_{\Delta}^{(v)} = \{T_{\Delta}^{(v)}(\mathbf{r}_n); \, n = 1, \ldots, N\}$, while $\bar{T}$ stands for the elementwise multiplication. Moreover, $\mathcal{G}_{\text{ext}}(\bar{\mathcal{E}}_{\text{ext}})$ is the $N \times N(M \times N)$ internal (external) Green’s matrix whose $(\rho, q)_\text{th}(m, n)_\text{th}$ entry is given by $\mathcal{G}_{\text{ext}}(\bar{\mathcal{E}}_{\text{ext}}) = j(k_B^2/4) \int \mathcal{H}_{\Delta}(k_B^2 \mathbf{r}_n - \mathbf{r}')d\mathbf{r}'(\bar{\mathcal{E}}_{\text{ext}} = j(k_B^2/4) \int \mathcal{H}_{\Delta}(k_B^2 \mathbf{r}_n - \mathbf{r}')d\mathbf{r}'$, $k_B^2$ being the wavenumber ($k_B \equiv (2\pi/\lambda_B)$) and $\mathcal{H}_{\Delta}(.)$ being the 0th-order Hankel function of the second kind.

### III. ISP Solution (MS-DCIE Inversion Method)

The solution of the inverse problem formulated in Section II (i.e., the estimation of the spatial distribution within $\mathcal{H}$ of both the differential modified contrast, $\bar{\mathcal{H}}$, and the differential equivalent currents, $\{\bar{T}_{\Delta}(v); \, v = 1, \ldots, V\}$) is addressed with an approach based on the application of the MS scheme to the DCIE formulation of the ISP. More specifically, the scattering-data inversion is carried out by means of an iterative strategy that performs $S$ successive “zooming” steps. At each $s$th ($s = 1, \ldots, S$, $s$ being the step index) step, the spatial distribution of the generic unknown $\mathcal{E}$ ($\mathcal{E} \in \{\mathcal{H}, \bar{T}_{\Delta}(v); \, v = 1, \ldots, V\}$) in the corresponding RoI, $\mathcal{S}$, which is the portion of the investigation domain $\mathcal{H}$ where $\mathcal{E}(\mathbf{r}_n) \neq 0$, is retrieved by means of an inversion algorithm as proposed in (10) and (11). Such a reconstruction is then exploited to improve the RoI estimate by also enhancing the spatial resolution of the retrieval. The process is repeated until a data-matching convergence criterion holds true.

The implementation of such a multilevel process needs: 1) to define the RoI for both the primary, $\mathcal{S}_{\Delta, \mathcal{H}}$, and the secondary, $\mathcal{S}_{\Delta}$, unknowns (Section III-A); 2) to choose an inversion method to process, at each $s$-step ($s = 1, \ldots, S$), the scattering data for determining the spatial distribution of the generic unknown $\mathcal{E}$ in the corresponding RoI, $\mathcal{S}_{\Delta}$ (Section III-b); 3) to define a suitable cost function that faithfully links the ISP at hand with its mathematical formulation within the DCIE framework so that the actual ISP solution coincides with the global minimum of the cost function itself (Section III-C); 4) to customize the multilevel MS strategy to both such a formulation (i.e., problem unknowns and cost function) and the integration with the optimization level (Section III-D). These items will be detailed or briefly recalled in the following.

#### A. RoI Definition

To properly address this issue, let us first recall the case of the MS as applied to the CIE formulation for the ISP with homogeneous media, where the CIE unknowns are the modified contrast function, $\mathcal{C}$, and the $V$ equivalent currents, $\{\mathcal{T}_\mathcal{V}(v); \, v = 1, \ldots, V\}$, whose supports, namely $\mathcal{S}_\mathcal{V}$ and $\mathcal{S}_\mathcal{A}$, coincide with the extension $\mathcal{S}_{\mathcal{A}, \mathcal{V}}$ of the unknown object $\mathcal{O}$, which is modeled by $\mathcal{T}$ ($\mathcal{S}_\mathcal{V} \equiv \mathcal{S}_{\mathcal{A}, \mathcal{V}}$). Therefore, the RoIs of both the primary unknown and the secondary one, which are identified at each $s$th ($s = 1, \ldots, S$) MS step (i.e., $\mathcal{S}_\mathcal{V}$ and $\mathcal{S}_\mathcal{A}$), are the same region where the unknown object is most likely to be present [58] (i.e., $\mathcal{S}_\mathcal{V} = \mathcal{S}_{\mathcal{A}, \mathcal{V}}$).

Otherwise, the problem unknowns for the DCIE formulation are the differential modified contrast, $\mathcal{C}$, and the $V$ differential equivalent currents, $\{\mathcal{T}_\mathcal{V}(v); \, v = 1, \ldots, V\}$, $\mathcal{S}_{\mathcal{A}}$ and $\mathcal{S}_{\mathcal{V}}$ being the corresponding supports. While $\mathcal{S}_{\mathcal{A}}$ is equal to the area occupied by the unknown scatterer and the standard RoI definition applies ($\mathcal{S}_{\mathcal{A}} = \mathcal{S}_{\mathcal{A}, \mathcal{V}}$), $\mathcal{S}_{\mathcal{V}}$ might also span outside the object region $\mathcal{O}$ (i.e., $\mathcal{S}_{\mathcal{V}} \supset \mathcal{S}_{\mathcal{A}, \mathcal{V}}$) since the equivalent currents, $\{\mathcal{T}_\mathcal{V}(v); \, v = 1, \ldots, V\}$, as well as the differential ones, $\{\mathcal{T}_\mathcal{V}(v); \, v = 1, \ldots, V\}$, are here induced also in the external inhomogeneous host medium. As detailed in Appendix B, the RoI of $\mathcal{T}_\mathcal{V}(v)$ at the $s$th ($s = 1, \ldots, S$) MS step turns out to be

$$\mathcal{S}_{\mathcal{A}}^{(s)} = \mathcal{S}_{\mathcal{A}}^{(s)} \cup \mathcal{S}_{\mathcal{V}}^{(s)}.$$  

#### B. SOM Inversion

Concerning the inversion method, the SOM [60] is adopted for the following reasons. First, the scattering operator $\mathcal{G}_{\text{ext}}$ in (10) is compact, and thus, the ISP at hand is ill-posed [61]. In particular, the ISP is not unique because of the (possible) presence of nonradiating components of the induced equivalent currents, which do not contribute to the scattered field $\xi_{\text{sca, }\Delta}$ collected in the observation domain external to $\mathcal{H}$. To recover uniqueness, it is then necessary to consider these components during the inversion process as the SOM does. Second, due to the properties of the DCIE formulation, the ISP nonlinearity is mitigated, and thus, the use of a deterministic fast inversion method, instead of computationally demanding “bare” global optimization techniques [62], could be profitable.

According to the guidelines in [39], the SOM is customized to the DCIE formulation, toward the integration within the MS processing scheme, as follows. The $s$th ($v = 1, \ldots, V$) differential equivalent current $\mathcal{T}_{\mathcal{V}}^{(s)}$ is decomposed into two parts, namely, the deterministic component, $\mathcal{T}_{\mathcal{DP}}^{(s)}$, and the ambiguous one, $\mathcal{T}_{\mathcal{AP}}^{(s)}$, which includes the nonradiating terms

$$\mathcal{T}_{\Delta}^{(s)} = \mathcal{T}_{\mathcal{DP}}^{(s)} + \mathcal{T}_{\mathcal{AP}}^{(s)}.$$  

The former, $\mathcal{T}_{\mathcal{DP}}^{(s)}$ ($v = 1, \ldots, V$), is computed from (10) by applying the singular value decomposition (SVD) to the external Green’s matrix $\mathcal{G}_{\text{ext}}$ [63]. It turns out that

$$\mathcal{T}_{\mathcal{DP}}^{(s)} = \sum_{n=1}^{N_s} \mathcal{U}_{n}^* \xi_{\text{sca, }\Delta}^{(s)} \mathcal{W}_{n},$$  

where $*$ stands for conjugate transposition, while $\cdot$ denotes the scalar product. Moreover, $\{\mathcal{U}_{n}; \, n = 1, \ldots, N\}$ are the $N$ singular values of $\mathcal{G}_{\text{ext}}$, while $\{\mathcal{U}_{n}; \, n = 1, \ldots, N\}$ and

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{\bar{W}_n; n = 1, \ldots, N} are the \(M\)-size left-singular vectors and the \(N\)-size right-singular vectors, respectively. In (14), \(N_{\text{th}}\) is the SVD truncation threshold, which is adaptively set as follows [57]:

\[
N_{\text{th}} = \arg \min_N \left\{ \sum_{n=1}^{N} \sigma_n - \alpha \right\}
\]

(15)

\(\alpha (0 < \alpha \leq 1)\) being a real user-defined calibration parameter, as detailed in Section IV.

The ambiguous current component, \(\hat{\mathbf{J}}^{(v)}_{\Delta, \text{AP}} (v = 1, \ldots, V)\), is related to the smaller singular values of \(\hat{G}_{\text{ext}}\) and it is yielded by the linear combination of the remaining \((N - N_{\text{th}})\) right-singular vectors

\[
\hat{\mathbf{J}}^{(v)}_{\Delta, \text{AP}} = \sum_{n=1}^{N - N_{\text{th}}} \mathbf{e}^{(v)}_{n-N_{\text{th}}} \bar{W}_n
\]

(16)

where \(\mathbf{e}^{(v)} = \{\mathbf{e}^{(v)}_n; n = 1, \ldots, (N - N_{\text{th}})\}\) is the unknown complex algebraic vector of the weights of the \(v\)th \((v = 1, \ldots, V)\) ambiguous current, while \(\bar{W}\) is the corresponding \((N - N_{\text{th}}) \times V\) size matrix (\(\bar{W} = \{\mathbf{e}^{(v)}_n; v = 1, \ldots, V\}\)).

C. Cost Function Definition

The cost function \(\Psi\) quantifies the error in fulfilling (10) and (11) and it is defined as

\[
\Psi(\mathbf{c}, \mathcal{X}_\Delta) = \sum_{i=1}^{V} \left[ \frac{\Psi_{\text{data}}^{(v)}(\mathbf{c}^{(v)})}{\| \mathbf{c}^{(v)} \|_2^2} + \frac{\Psi_{\text{state}}^{(v)}(\mathbf{c}^{(v)}, \mathcal{X}_\Delta)}{\| \mathbf{c}^{(v)} \|_2^2} \right]
\]

(17)

where \(\| \cdot \|\) is the \(\ell_2\)-norm, while the data equation mismatch, \(\Psi_{\text{data}}^{(v)}\), is derived from (10)

\[
\Psi_{\text{data}}^{(v)}(\mathbf{c}^{(v)}) = \left\| \bar{g}_{\text{ext}} \left[ \hat{\mathbf{J}}^{(v)}_{\Delta, \text{DP}} + \sum_{n=N_{\text{th}}+1}^{N} \mathbf{e}^{(v)}_{n-N_{\text{th}}} \bar{W}_n \right] - \mathbf{c}^{(v)} \right\|_2^2
\]

(18)

and the state equation mismatch \(\Psi_{\text{state}}^{(v)}\) stems from (11)

\[
\Psi_{\text{state}}^{(v)}(\mathbf{c}^{(v)}, \mathcal{X}_\Delta) = \left\| \Psi_{\text{AP}}^{(v)}(\mathbf{c}^{(v)}, \mathcal{X}_\Delta) - \Psi_{\text{DP}}^{(v)}(\mathcal{X}_\Delta) \right\|_2^2
\]

(19)

D. MS Implementation

The proposed MS approach first detects the presence/position of a scatterer inside the investigation domain. Then, it smartly reallocates the ISP unknowns by adaptively improving the resolution only where needed. As a consequence, the ratio between the unknowns and the nonredundant data is kept as low as possible, thus mitigating the nonlinearity and reducing the occurrence of local minima, enabling more robust exploitation of deterministic local search algorithms [29], [55], [61]. Moreover, the progressively acquired information on the scattering scenario is successfully exploited to counteract the ill-posedness, providing suitable initializations at the higher resolution steps. Within the DCIE framework, the algorithmic implementation of the MS scheme as integrated into the SOM-based inversion can be described through the following multistep iterative \((i\) being the iteration index) process (summarized in the flowchart of Fig. 2, where the novelties with respect to the single-resolution strategy [51] have been highlighted).
1) Initialization: Initialize the MS step index \( s = 1 \) and the RoI for both the unknowns to the whole investigation domain \( S_\Delta^{(1)} = \delta_\Delta = \mathcal{H} \) - Fig. 3(a).

2) MS Loop:

   a) Unknowns Setup \((i=0)\): If \( s = 1 \), then reset the unknowns (i.e., \( \chi_\Delta^{(i-1)} = \emptyset \) and \( \chi_\Delta^{(i)} = \emptyset \)). Otherwise (i.e., \( s > 1 \)), map the current solution from the previous zooming step into the current zooming discretization grids of the RoIs \( \{ S_\delta^{(i)}; \varphi \in \{ \Delta_x, \Delta_y \} \} \) (i.e., \( \chi_\Delta^{(i)} = \Phi_\Delta([\chi_\Delta^{(i-1)}]; S_\delta^{(i)}) \) and \( [\chi_\Delta^{(i)}]_0 = \Phi_\Delta([\chi_\Delta^{(i-1)}]; S_\delta^{(i)}) \) being equal to \( [\chi_\Delta^{(i-1)}]_0 = \chi_\Delta^{(i)} + \sum_{n=N_{n-1}+1}^{N_{n}} [c_\omega^{(i)}(r_{n-n_{n-1}})]_0 \) with \( \Phi_\delta \) being the mapping operator from the grid of \( S_\delta^{(i-1)} \) to the finer one of \( S_\delta^{(i)} \).

   b) Scattering-Data Inversion: Compute the DCIE regularization parameter vector \( \beta^{(i)} = \{ \beta(r_n) ; n = 1, \ldots, N \} \) by setting its \( N \) entries to the value

   \[
   \beta^{(i)} = \gamma \times \max_{r_\delta \in \delta_\Delta} \left| \int_{S_\Delta} G_{\text{im}}(r_n, r') dr' \right| \tag{22}
   \]

   where \( \gamma \) is a control parameter [34]. Retrieve the \( S \)th set of unknowns \( (\chi_\Delta^{(i)}, \xi_\Delta^{(i)}) \) within the corresponding RoI \( S_\delta^{(i)} \) by solving the following optimization problem:

   \[
   (\chi_\Delta^{(i)}, \xi_\Delta^{(i)}) = \arg \min_{\chi_\Delta, \xi_\Delta} [\Psi(\chi_\Delta, \xi_\Delta)] \tag{23}
   \]

   with \( I \) iterations of the deterministic (local search) Polak–Ribiere version of the conjugate gradient (CG) algorithm [64] (i.e., \( \chi_\Delta^{(i)} = \chi_\Delta^{(i-1)} + \beta^{(i)} \xi_\Delta^{(i)} \)) starting from \( \chi_\Delta^{(i)} = \emptyset \) and \( \xi_\Delta^{(i)} = \emptyset \). Update the trial differential current \( [\chi_\Delta^{(i)}]_0 \) through (13) by using (14) and (16) with \( c_{n-n_{n-1}}^{(i)} \) and go to the “Termination.”

   c) Step Check: Halt the MS loop if the maximum number of zooming steps is reached [i.e., \( s = S \) - Fig. 3(d)], define the estimated solution by setting \( \chi_\Delta^{(i)} = \chi_\Delta^{(i)} \) and \( [\chi_\Delta^{(i)}]_0 = \beta^{(i)} \xi_\Delta^{(i)} \) as well as \( \beta^{(i)} = \beta^{(i)} \), and go to the “Termination.”

   d) Roll Update: Apply the “filtering and clustering” operations [29] on \( \chi_\Delta^{(i)} \) to determine the corresponding new RoI \( S_\delta^{(i+1)} \) [Fig. 3(b) and (c)], by defining its center \( r_\Delta^{(i+1)} = (x_\Delta^{(i+1)}, y_\Delta^{(i+1)}) \), and side \( L_\Delta^{(i+1)} \), as follows:

   \[
   S_\Delta^{(i+1)} = \frac{\sum_{n=1}^{N_{\Delta_{n}}} \chi_\Delta^{(i)}(r_n)}{\sum_{n=1}^{N_{\Delta_{n}}} \chi_\Delta^{(i)}(r_n)}, \tag{24}
   \]

   \((\zeta \in \{ x; y \})\) and

   \[
   L_\Delta^{(i+1)} = 2 \times \frac{\sum_{n=1}^{N_{\Delta_{n}}} \| r_\Delta^{(i+1)} - r_\Delta^{(i)} \| \chi_\Delta^{(i)}(r_n)}{\sum_{n=1}^{N_{\Delta_{n}}} \chi_\Delta^{(i)}(r_n)} \tag{25}
   \]

   Identify the new RoI for \( \chi_\Delta^{(i)} \) (i.e., \( S_\Delta^{(i+1)} \)) through (12).

3) Roll Check: Terminate the MS loop if the zooming factor \( \eta^{(i)} \), which is defined as

   \[
   \eta^{(i)} = \frac{L_\Delta^{(i+1)} - L_\Delta^{(i)}}{L_\Delta^{(i+1)}} \tag{26}
   \]

   is below a user-defined threshold \( \eta^{(i)} \) and set the problem solution to the current trial one (i.e., \( \chi_\Delta^{(i)} = \chi_\Delta^{(i)} \) and \( \beta^{(i)} = \beta^{(i)} \)). Otherwise, update the MS loop index (i.e., \( s \leftarrow (s + 1) \) and restart the “MS Loop.”

4) Termination: Given \( \chi_\Delta^{(i)} \), compute the modified contrast \( \chi^{(i)} \) (i.e., \( \chi^{(i)} = \chi^{(i)} + \chi^{(i)} \)) to output the estimated contrast vector \( \chi^{(i)}(r_n) ; n = 1, \ldots, N \) whose \( n \)th entry is equal to

   \[
   \chi^{(i)}(r_n) = \frac{\chi^{(i)}(r_n)}{\chi^{(i)}(r_n)} \tag{27}
   \]

IV. NUMERICAL AND EXPERIMENTAL RESULTS

An extensive set of tests has been carried out to assess the reliability and performance of the proposed IS method. The most representative ones have been selected and reported in the following to: 1) provide some general rules for calibrating the control parameters of the MS-DCIE; 2) show the effectiveness of such an implementation of the MS scheme to yield high-accuracy reconstructions when dealing with inhomogeneous scenarios; and 3) provide a comparative analysis with other state-of-the-art approaches.

To quantitatively estimate the accuracy of the reconstructions, let us consider two metrics. The one is a local metric and it is the local error function, \( \mathcal{E} \), defined as

\[
\mathcal{E}(\tau) = \frac{\tau(\tau - \tau^{(i)}(\tau))}{\tau(\tau + 1)} \tag{28}
\]

\( \tau \) and \( \tau^{(i)} \) being the actual and the reconstructed contrast, respectively. The other metric gives global/integral information on the imaging process and it is given by

\[
\Xi = \frac{1}{A} \sum_{r_n \in A} |\mathcal{E}(r_n)| \tag{29}
\]

where \( A \) is the area of the region where the error figure is evaluated. More specifically, \( \Xi = \Xi_{\text{int}} \), \( \Xi = \Xi_{\text{opt}} \), and \( \Xi = \Xi_{\text{ext}} \) if \( A = \mathcal{H}, A = S_\Delta, \) and \( |\mathcal{H} \setminus S_\Delta| \), respectively.

Unless otherwise stated, the side of the square investigation domain \( \mathcal{H} \) has been set to \( L_\mathcal{H} = 3 \times \lambda_\mathcal{H} \), where \( \lambda_\mathcal{H} \) is the wavelength in the external background. Such a benchmark scenario has been probed by \( \nu = 27 \) directions, while the scattering data have been collected in \( M = 27 \) uniformly spaced locations on a circle of radius \( \rho = 2.2 \times \lambda_\mathcal{H} \) and centered in the host medium region \( \mathcal{H} \). In all numerical tests, the scattered field samples in (10) have been synthetically generated by solving the FW scattering problem with a method of moments (MoM) solver and discretizing the investigation domain with \( NFW = 80 \times 80 \) square cells. Moreover, the data samples have been blurred with an additive Gaussian noise, characterized by a signal-to-noise ratio (SNR), to model realistic measurement conditions. As for the data
inversion, a coarser discretization grid has been assumed (i.e., \( S \) and a remaining area of contrast \( \tau_3 \) thickness \( 0 \) a known host domain composed by a centered square ring of \( \varepsilon_r \) of the unknown scatterer) [Fig. 1(c)], located at \((\tau_d,0)\) differential contrast \( 7 \) an off-centered square scatterer \( 0 \). B \( \lambda \varepsilon \) reported hereinafter for illustration purposes. More in detail, analysis has been performed and the representative results which controls the DCIE regularization at each MS step.

Which determines the SVD truncation threshold \( N \) depends on the values of two control parameters: \( \alpha \) and \( \gamma \).

The behavior of the MS-DCIE inversion method mainly with \( B \) \( \lambda \varepsilon \) the MS zooming threshold according to \([56]\).

The optimal setup for \( \alpha \) and \( \gamma \) has then been chosen according to the following rule:

\[
\zeta^* = \frac{\int_{\text{SNR}} \arg \min_{\zeta} \left\{ \sum_{i=1}^{M} \left| \frac{\varepsilon_{\text{tot}}(\zeta)}{\text{SNR}} \right| d\text{SNR} \right.}{\int_{\text{SNR}} d\text{SNR}}
\]

\((\zeta = \{\alpha; \gamma\})\) and the result has been \((\alpha^*, \gamma^*) = (0.4, 1.4)\).

**B. Numerical Assessment**

Once calibrated, the performance of the MS-DCIE has been assessed in comparison with competitive state-of-the-art inversion approaches. First, the comparison with the single-resolution DCIE\(^1\) has been carried out. The test case refers to the “Square” scatterer in Section IV-A and the scattering data have been blurred with \( \text{SNR} = 20 \) [dB]. Fig. 5 shows the evolution of the cost function, \( \Psi \), the total reconstruction error, \( \varepsilon_{\text{tot}} \), and the MS zooming factor, \( \eta^{(s)} \), during the MS process. Moreover, the colormaps of the retrieved differential contrast, \( \tau_\Delta \), are reported in Fig. 6 as well. As it can be observed, the estimated \( \tilde{S}_{\Delta} \)-RoI shrinks around the actual object position [Fig. 6(a)–(c)] until there is an almost perfect match with the scatterer support [Fig. 6(d)]. This occurs at \( s = 4 \) when the value of \( \eta^{(s)} \) falls below the threshold \( \eta_{\text{min}} \) (Fig. 5) and the inversion process is stopped. It is worth noticing that during the MS procedure, both the cost function and

\(^1\)According to the guidelines in \([34]\), the regularization parameter has been set to \( \beta = 2 \), while the investigation domain \( \mathcal{H} \) has been partitioned in \( N_{\text{DCIE}} = 46 \times 46 \) square cells.

Fig. 5. Numerical assessment (“Square” object \( \tau_\Delta = 2 \), \( M = V = 27 \), \( \text{SNR} = 20 \) [dB], and MS-DCIE). Plots of the cost function, \( \Psi \), the total reconstruction error, \( \varepsilon_{\text{tot}} \), and the MS zooming factor, \( \eta^{(s)} \), versus the MS step index \((s = 1, \ldots, S)\). has been retrieved by applying the MS-DCIE strategy with different values of the control parameters and for different SNRs. The analysis outcomes are summarized in Fig. 4 where the behavior of the total reconstruction error \( \varepsilon_{\text{tot}} \) versus \( \alpha \) [Fig. 4(a)] and \( \gamma \) [Fig. 4(b)] is shown in correspondence with different noise levels on the scattering data. It turns out that the \( \alpha \) value has a low impact on the MS-DCIE performance, \( \varepsilon_{\text{tot}} \) being almost flat or with a limited range of variations for a given value of \( \text{SNR} \). On the contrary, the \( \gamma \) parameter has a more notable effect on the reconstruction accuracy since it “controls” the nonlinearity of the inverse problem at hand \([34]\).
the reconstruction error decrease (Fig. 5) by pointing out the enhancement of the inversion accuracy in correspondence with a better fitting with the scattering data. The real and imaginary parts of the contrast profile \( \tau_{\text{opt}} \) derived by the MS-DCIE [Figs. 6(e) and 7(a)] are then compared with those retrieved with the DCIE method [Figs. 6(f) and 7(b)]. Both pictorially and quantitatively, it is evident that the MS-based inversion significantly improves the reconstruction inside (outside) the object support. Indeed, the internal/external error reduces of about \( \Theta \Xi_{\text{int}} = 77\% (\Theta \Xi_{\text{ext}} = 52\%) \) \( (\Theta \Xi \triangleq (\Xi_{\text{DCIE}} - \Xi_{\text{MS-DCIE}}) / \Xi_{\text{DCIE}}) \) with respect to the single-resolution approach. Moreover, it is worth observing that the MS-DCIE yields a more precise estimation of the imaginary part of the contrast \( \text{Im}[\tau_{\text{opt}}] = 0 \), whereas nonnegligible artifacts arise in the DCIE result [Fig. 7(a) versus Fig. 7(b)]. Furthermore, despite the iterative nature of the inversion process and the additional performed operations (e.g., the RoI definition/updating), the overall inversion time of the MS-DCIE \( (T_{\text{MS-DCIE}} \approx 42 \text{ [s]}) \) is noticeably lower than that of the DCIE \( (T_{\text{DCIE}} \approx 70 \text{ [s]}) \). Such a higher computational efficiency with respect to single-resolution methods is in accordance with the reference literature [25], [54], [58], as it comes from the significantly lower amount of retrieved unknowns at each MS step \( (i.e., N_{\text{MS-DCIE}} = 900 \text{ versus } N_{\text{DCIE}} = 2116) \).

The comparative assessment has been extended next to the strategies based on the DLSIE formulation [51], [58], which are referred in the following as MS-DLSIE and DLSIE, respectively, and the results from the analysis on scenario “Circular-Ring” object, shown in the inset of Fig. 8, are

Fig. 8. Numerical assessment (“Circular Ring” object \( [\tau_\lambda = 2], M = V = 27 \), and SNR = 20 [dB]). Plots of the total reconstruction error, \( \Xi_{\text{tot}} \), versus SNR.

Fig. 9. Numerical assessment (“Circular Ring” object \( [\tau_\lambda = 2], M = V = 27 \), and SNR = 20 [dB]). Maps of the local error, \( \Xi \), yielded by (a) MS-DCIE, (b) DCIE, (c) MS-DLSIE, and (d) DLSIE.
reported hereinafter. Fig. 8 plots the values of $\tau$ the actual differential contrast, $\text{SNR} = 20 \text{ [dB]}$, and $\text{SNR} = 20 \text{ [dB]}$. Plots of the global reconstruction errors versus the number of measurement points and views.

Fig. 11. Numerical assessment (“Circular Ring” object, $M = V = 27$, and SNR = 20 [dB]). Plots of the global reconstruction errors as a function of the actual differential contrast, $\tau$.

Fig. 12. Numerical assessment (“Circular Ring” object, $M = V = 27$, and SNR = 20 [dB]). Maps of the local error, $E$, yielded by (a) and (b) MS-DCIE and (c) and (d) DLSIE when (a) and (c) $\tau = 3$ and (b) and (d) $\tau = 4$.

Fig. 13. Numerical assessment (“Circular Ring” object, $M = V = 27$, and SNR = 20 [dB]). Maps of the local error, $E$, yielded by (a) MS-DLSIE and (b) DLSIE.

The proposed inversion method has been validated when dealing with imaging setups consisting in a lower number of antennas as well. Toward this end, the number of views and measurement points has been reduced from $M = V = 27$ down to $M = V = 16$, and the corresponding error metrics have been reported in Fig. 10. The results indicate good robustness of the MS-DCIE despite the reduction of the available data for the inversion. Quantitatively, the total error increases by 56% ($\approx \frac{\Xi_{\text{tot}}|_{M=V=16} - \Xi_{\text{tot}}|_{M=V=27}}{\Xi_{\text{tot}}|_{M=V=27}}$) for the MS-DCIE, while a larger degradation (i.e., 79%) is observed for the DCIE in correspondence with a decrease by 41% in the number of sensors (Fig. 10).

The performance of the MS-DCIE has been assessed also against the scatterer permittivity, thus verifying its accuracy against a higher nonlinearity of the ISP. Toward this end, the actual contrast of the circular ring object has been varied in the range from $\tau = 2$ (i.e., $\varepsilon = 3\varepsilon_0$) up to $\tau = 4$ (i.e., $\varepsilon = 5\varepsilon_0$) while keeping the noise level to SNR = 20 [dB]. The plots of the reconstruction indexes in Fig. 11 point out that the inversion becomes more and more difficult when the differential contrast becomes stronger and stronger since all errors get larger, but they also point out the effectiveness of the MS in dealing with stronger scatterers with an average improvement of the total (external) error, $\Xi_{\text{tot}}|_{\text{ext}}$, of about 45% (48%) with respect to the DCIE single-resolution strategy. Moreover, the higher the contrast is, the greater is the advantage of using the MS for imaging the unknown object support since the internal error gap $\Theta\Xi_{\text{int}}$ grows as $\tau$ tends to $\tau = 4$ (i.e., $\Theta\Xi_{\text{int}}|_{\tau=4} = 12\%$, $\Theta\Xi_{\text{int}}|_{\tau=3} = 37\%$, and $\Theta\Xi_{\text{int}}|_{\tau=2} = 51\%$). These outcomes are highlighted by the error maps related to $\tau = 2$ [Fig. 9(a) and (b)], $\tau = 3$ [Fig. 12(a) and (c)], and $\tau = 4$ [Fig. 12(b) and (d)]. For the sake of completeness, Fig. 13 reports the error maps.
Fig. 14. Numerical assessment (“Circular Ring” object \(\text{Re}\{\tau_A\} = 2\), \(M = V = 27\), and \(\text{SNR} = 20\) [dB]). Plots of the global reconstruction errors as a function of the actual differential conductivity, \(\sigma_A\).

"Circular Ring" Object, \(\text{Re}\{\tau_A\} = 2\), \(M = V = 27\), and \(\text{SNR} = 20\) [dB]

![Graph showing reconstruction errors](image)

Fig. 15. Numerical assessment (“Circular Ring” object \(\text{Re}\{\tau_A\} = 2\), \(\sigma_A = 10^{-1}\) [S/m], \(M = V = 27\), and \(\text{SNR} = 20\) [dB]). Maps of (a) and (c) real and (b) and (d) imaginary part of the local error, \(E\), yielded by (a) and (b) MS-DCIE and (c) and (d) DCIE.

obtained for \(\tau_A = 4\) by the MS-DLSIE [Fig. 13(a)] and the DLSIE [Fig. 13(b)]. These results confirm the superior performance of the proposed strategy in dealing with such a strong scatterer (having a nonnegligible size as well), due to its capability of mitigating the negative effects of multiple-scattering phenomena yielded by combining the CIE and the MS strategy [34], [58]. Quantitatively, the MS-DCIE turns out to provide the lowest reconstruction error, being \(\Sigma_{\text{tot}}^{\text{MS-DCIE}} = 6.49 \times 10^{-2}\) [Fig. 12(b)] versus \(\Sigma_{\text{tot}}^{\text{MS-DLSIE}} = 12.41 \times 10^{-2}\) [Fig. 13(a)].

Another numerical experiment has been devoted to assess the MS-DCIE when dealing with lossy scatterers \((\sigma_A \neq 0\) [S/m]). By keeping the ring target of Fig. 8, its differential contrast has been set to \(\text{Re}\{\tau_A\} = 2\) \((\Rightarrow \text{Re}\{\tau_A\} = 3\epsilon_0)\), while the actual scatterer conductivity has been varied within the range \(10^{-3}\) [S/m] \(\leq \sigma_A \leq 1\) [S/m] \((f = 300\) [MHz]). Unlike the dependence on the scatter permittivity in Fig. 11, the accuracy improvement of the MS strategy reduces as the object conductivity increases (e.g., \(\Theta \Sigma_{\text{tot}}^{\text{MS-DCIE}}|_{\sigma_A=10^{-1}[\text{S/m}]} = 56\%\), \(\Theta \Sigma_{\text{tot}}^{\text{DCIE}}|_{\sigma_A=10^{-2}[\text{S/m}]} = 51\%\), \(\Theta \Sigma_{\text{tot}}^{\text{MS-DCIE}}|_{\sigma_A=10^{-3}[\text{S/m}]} = 44\%\), and \(\Theta \Sigma_{\text{tot}}^{\text{DCIE}}|_{\sigma_A=10^{-4}[\text{S/m}]} = 24\%\) —Fig. 14). However, it is worth noticing that locally, the MS-DCIE still significantly outperforms the DCIE in retrieving the imaginary part of the contrast distribution by better detailing the contours and the support of the unknown circular ring as highlighted by the error maps in Fig. 15 (\(\sigma_A = 10^{-1}\) [S/m]).

The next numerical experiment is aimed at evaluating how an inaccurate knowledge of the host medium contrast, \(\tau_H(r)\), affects the MS-DCIE reconstruction. Toward this end, the a priori information on the host medium has been supposed to be affected by an uncertainty proportional to the constant \(\delta\)

\[
\hat{\tau}_H(r) = \tau_H(r) \times (1 + \delta)
\]

(31)

Fig. 16. Numerical assessment (“Square” object \(\tau_A = 2\), \(M = V = 27\), and \(\text{SNR} = 20\) [dB]). Plots of the global reconstruction errors versus the a priori knowledge uncertainty, \(\delta\).

"Square" Object, \(\text{SNR}=20\) [dB]

![Graph showing reconstruction errors](image)

Fig. 17. Numerical assessment (“Square” object \(\tau_A = 2\), \(M = V = 27\), and \(\text{SNR} = 20\) [dB]). Maps of the local error, \(E\), yielded by (a), (c), and (e) MS-DCIE and (b), (d), and (f) DCIE when (a) and (b) \(\delta = 5\%\), (c) and (d) \(\delta = 20\%\), and (e) and (f) \(\delta = 80\%\).

"Square" Object, \(\text{SNR}=20\) [dB]
and the analysis has been carried out on the “Square object” scenario with $r_\Delta = 2$ ($\Rightarrow \varepsilon_u = 3\varepsilon_0$) and SNR = 20 [dB]. By varying the value of $\delta$ from 0% up to 100%, it is not surprising that all error indexes get worse when the a priori information on the host medium is more and more imprecise (Fig. 16). However, the MS-DCIE always yields better reconstructions as quantitatively confirmed by the total error gap $\Theta \Xi_{\text{tot}}$ values (i.e., $\Theta \Xi_{\text{tot}} \geq 32\%$) even though the improvements with respect to the DCIE diminish as the host medium knowledge is getting more inaccurate (i.e., $\Theta \Xi_{\text{tot}} |_{\delta=0.05} = 71\%$—Fig. 17(a) versus Fig. 17(b), $\Theta \Xi_{\text{tot}} |_{\delta=0.2} = 60\%$—Fig. 17(c) versus Fig. 17(d), and $\Theta \Xi_{\text{tot}} |_{\delta=0.8} = 45\%$—Fig. 17(e) versus Fig. 17(f)).

The following analysis is aimed at investigating the MS-DCIE performance when dealing with more complicated/arbitrary environments as well as to assess the robustness of the optimal parameters setting (i.e., $\alpha^*$ and $\gamma^*$) deduced from the preliminary calibration. Toward this end, a highly detailed host medium consisting of the MTT-website “QR code” has been considered [Fig. 18(a)]. The MS-DCIE [Fig. 18(c)] and DCIE [Fig. 18(d)] outcomes are shown in Fig. 18 in terms of the corresponding local error maps. It turns out that, differently from the single-resolution strategy, the proposed MS approach successfully retrieves the location, shape, and composition of the object, as quantitatively confirmed by the internal error (i.e., $\Theta \Xi_{\text{int}} = 77\%$—Fig. 18(c) versus Fig. 18(d)). Similar conclusions hold for a host medium resembling the “Taiji” symbol with a spatially continuous variation of the contrast [Fig. 18(b)]. By comparing the local error maps obtained with the MS-DCIE [Fig. 18(d)] and the DCIE [Fig. 18(f)] methods, the advantage of the MS approach is remarkable, yielding significantly lower distortions both inside ($\Theta \Xi_{\text{int}} = 85\%$) and outside ($\Theta \Xi_{\text{ext}} = 93\%$) the support of the unknown scatterer.

Finally, the robustness of the proposed approach to inaccuracies in the knowledge of the inhomogeneous host medium is further investigated in the following. Toward this aim, a set of numerical experiments has been performed considering the “Taiji” scenario [Fig. 18(b)] and blurring the a priori known host medium with an additive Gaussian noise, resulting in an imprecise $\tau_H (r)$ knowledge with SNR equal to $\Lambda$. The global error metrics are shown in Fig. 19 as a function of $\Lambda$, which has been varied in the range $\Lambda \in [10, 100]$ [dB]. The MS-DCIE is always capable of correctly estimating the unknown object location and composition regardless of the imprecise a priori knowledge of the background, always yielding lower errors with respect...
to the DCIE. For completeness, the reconstructions for the most challenging case of \( \Delta = 10 \) [dB] are shown in Fig. 20. Despite the nonnegligible inaccuracies in modeling the actual host medium [Fig. 20(a) versus Fig. 18(b)], the local error map of the MS-DCIE clearly indicates a higher reconstruction accuracy compared to the single-resolution strategy [\( \Theta \oint = 81\% \)]—Fig. 20(b) versus Fig. 20(c)].

### C. Experimental Assessment

To complete the validation of the MS-DCIE with real data, the results of this section refer to the experimental measurements from the Institut Fresnel for the “FoamDielIntTM” object [65]. In this test case, the square investigation domain of side \( L_H = 0.4 \) [m] has been illuminated by a ridged-horn antenna from \( V = 8 \) different angular directions and the electric-field data samples have been collected by another horn antenna moved in \( M = 241 \) uniformly distributed locations on a circle of radius \( \rho = 1.67 \) [m]. Within the investigation domain, there is an inner cylinder with a diameter of \( D_1 = 31 \) [mm] and relative permittivity \( \varepsilon_1 = 3 \), which is centered at \( (x_0, y_0) = (-5 \) [mm], 0 [mm]), and it is surrounded by a larger centered cylinder of diameter \( D_2 = 80 \) [mm] having a relative permittivity equal to \( \varepsilon_2 = 1.45 \) [Fig. 21(a)].

In order to test the differential formulation of the proposed MS method for inhomogeneous media, the outer cylinder has been assumed to be part of the known host medium with contrast \( \tau_Y(r) \) [Fig. 21(b)] so that the differential contrast \( \tau_A(r) \) of the inner cylinder is the unknown to be retrieved [Fig. 21(c)].

As for the a priori knowledge on the host medium, it is worthwhile to note that the permittivity of the second/larger cylinder (\( \varepsilon_2 = 1.45 \)) is known with a precision of about \( \pm 0.15 \) [65], which corresponds to an uncertainty in (31) of approximately \( \delta = \pm 10\% \).

For comparison purposes, such real data have been processed with the IP methods based on both DCIE and DLSIE formulations as well as single-resolution and multiresolution strategies. The data inversion outcomes in terms of the total reconstruction error, \( \zeta_{\text{tot}} \), versus the dataset frequency, \( f \), are summarized in Table I. Generally, MS-based implementations yield lower reconstruction errors when compared to their single-step counterparts. Moreover, the obtained results further verify the robustness of the optimal parameters setting drawn from the preliminary numerical calibration in Section IV-A when dealing with an experimental imaging scenario. The same holds true for the DCIE formulation versus the DLSIE-based one due to the mitigation of the nonlinearity enabled by the introduction of the modified contrast in (9) [51]. Always, the MS-DCIE outperforms the other alternative techniques by confirming the conclusions drawn from the numerical analyses in Section IV-B.

As for the local accuracy of the reconstructions, let us consider the error maps for some representative cases. For instance, Figs. 22 and 23 show the local error distribution, \( \| \varepsilon(r) \| \), yielded when processing the datasets at \( f = 7 \) [GHz] (Fig. 22) and \( f = 8 \) [GHz] (Fig. 23). As one can observe, the DLSIE-based methods fail at correctly estimating the contrast of the unknown scatterer as highlighted by the higher values of the error index in Figs. 22(c) and (d) and 23(c) and (d). On the contrary, the single-resolution DCIE reconstructions provide a more reliable estimation of the unknown cylinder permittivity [Figs. 22(b) and 23(b)], even though the retrieved distributions still present several artifacts outside the scatterer support along with nonnegligible errors on the scatterer edges. As expected, the MS-DCIE provides the smoothest error distribution in the whole investigation domain [Figs. 22(a) and 23(a)] and it obtains a careful representation of the target edges.

### Table I

| \( f \) [GHz] | \( \zeta_{\text{tot}} \) [\( \times 10^{-4} \)] |
|-------------|-----------------|
| 6.0         | 4.57            | 6.39 | 8.28 | 12.47 |
| 7.0         | 5.01            | 6.01 | 8.31 | 12.93 |
| 8.0         | 4.69            | 8.31 | 10.91| 12.32 |
| 9.0         | 4.39            | 6.70 | 8.84 | 11.34 |

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TABLE II

EXPERIMENTAL ASSESSMENT (“FoamDielIntTM” OBJECT
$[\tau_1 = 2, \tau_2 = 0.45], L_{\text{tot}} = 0.2 \text{ m}, V = 8, \text{ and } M = 241$)—TOTAL
RECONSTRUCTION ERROR, $\xi_{\text{tot}}$

| $f \text{ [GHz]}$ | MS-DCIE | DCIE | MS-DLSIE | DLSIE |
|------------------|---------|------|---------|-------|
| 6.0              | 1.81    | 5.55 | 3.21    | 10.18 |
| 7.0              | 1.88    | 6.54 | 9.12    | 11.58 |
| 8.0              | 1.71    | 6.38 | 10.71   | 11.24 |
| 9.0              | 1.96    | 5.24 | 5.65    | 6.24  |

V. CONCLUSION

A novel inversion method, named MS-DCIE, has been developed to address the ISPs with inhomogeneous media. The proposed approach combines the DCIE formulation with the MS inversion strategy and it has proved to be reliable and effective in a wide range of scenarios and under different conditions. As a matter of fact, the effectiveness of the developed IS technique has been tested against both numerical and experimental scattering data by considering lossless and lossy profiles as well as varying the object contrast. The effects of some uncertainty on the a priori knowledge of the host medium have been evaluated as well.

Future works, beyond the scope of this article, will be aimed at extending the formulation of both the FW and inverse problems to deal with arbitrary (non-TM) illuminations and fully 3-D geometries [25]. The exploitation of nonsquare domains/pixels and adaptive meshes as well as the customization of the proposed implementation to biomedical scenarios of great applicative interest will be the object of future studies.

APPENDIX A

With reference to Fig. 1(a) (scenario “with object”) and according to the LSIE theory, the EM phenomena are described by the “data equation”

$$\xi_{\text{sca}}(\mathbf{r}_m) = \int_{\mathcal{H}} G_B(\mathbf{r}_m, \mathbf{r}) \tau(\mathbf{r}) \xi^{(v)}(\mathbf{r}) d\mathbf{r}$$

and the “state equation”

$$\xi^{(v)}(\mathbf{r}) = \xi_{\text{inc}}^{(v)}(\mathbf{r}) + \int_{\mathcal{H}} G_B(\mathbf{r}, \mathbf{r}') \tau(\mathbf{r}') \xi^{(v)}(\mathbf{r}') d\mathbf{r}.$$  (32)

Analogously, the LSIE equations for the scenario “without the object” [Fig. 1(a)] turn out to be

$$\xi_{\text{sca},H}(\mathbf{r}_m) = \int_{\mathcal{H}} G_B(\mathbf{r}_m, \mathbf{r}) \tau_H(\mathbf{r}) \xi_{H}^{(v)}(\mathbf{r}) d\mathbf{r},$$  (34)

$$\xi_{H}^{(v)}(\mathbf{r}) = \xi_{\text{inc}}^{(v)}(\mathbf{r}) + \int_{\mathcal{H}} G_B(\mathbf{r}, \mathbf{r}') \tau_H(\mathbf{r}) \xi_{H}^{(v)}(\mathbf{r}') d\mathbf{r}.\quad (35)$$

By subtracting (34) from (32) and (35) from (33), it turns out that

$$\xi_{\text{sca},\Lambda}(\mathbf{r}_m) = \int_{\mathcal{H}} G_B(\mathbf{r}_m, \mathbf{r}) \left[ \tau(\mathbf{r}) \xi^{(v)}(\mathbf{r}) - \tau_H(\mathbf{r}) \xi_{H}^{(v)}(\mathbf{r}) \right] d\mathbf{r},$$

$$\xi^{(v)}(\mathbf{r}) = \xi_{H}^{(v)}(\mathbf{r}) + \int_{\mathcal{H}} G_B(\mathbf{r}, \mathbf{r}') \left[ \tau(\mathbf{r}') \xi^{(v)}(\mathbf{r}') - \tau_H(\mathbf{r}') \xi_{H}^{(v)}(\mathbf{r}') \right] d\mathbf{r}.$$  (37)

where $\xi_{\text{sca},\Lambda}$ is the differential scattered field given by the difference between the scattered field with, $\xi_{\text{sca}}^{(v)}$, and without, $\xi_{\text{sca},H}$, the unknown object $[\xi_{\text{sca},\Lambda}(\mathbf{r}_m) = \xi_{\text{sca}}^{(v)}(\mathbf{r}_m) - \xi_{\text{sca},H}(\mathbf{r}_m); m = 1, \ldots, M; v = 1, \ldots, V]$.

Finally, since $\tau(\mathbf{r}) = \tau_S(\mathbf{r}) - \tau_T(\mathbf{r})$ and after simple manipulations, (4) and (5) are yielded.

APPENDIX B

The support $S_\varphi$ of a function $\varphi$, referred here as RoI of $\varphi$, is defined as the region of the investigation domain $\mathcal{H}$ where the value of the function is nonnull

$$S_\varphi \triangleq \{ \mathbf{r} : \varphi(\mathbf{r}) \neq 0 \}.\quad (38)$$
Since the difference equivalent current, $J^{(v)}_J$, is given by (6), then its value is nonzero only if either $J^{(v)}_J(r)$ or $J^{(v)}_\Lambda(r)$ are nonzero, vice versa $J^{(v)}_\Lambda(r) = 0$ if $J^{(v)}_J(r) = 0$ and $J^{(v)}_J(r) = 0$. Therefore, the following relation on the support $S_{\Delta J}$ of $J^{(v)}_\Lambda$ holds true:

$$S_{\Delta J} \subseteq (S_J \cup S_{J_H}). \quad (39)$$

Moreover, the $v$th ($v = 1, \ldots, V$) equivalent current, $J^{(v)}(r)$, is zero if the contrast $\tau(r)$ is zero being $J^{(v)}(r) \triangleq \tau(r) J^{(v)}(r)$, and thus, $S_J \subseteq S_{\tau}$. Analogously, $S_{J_H} \subseteq S_{\tau}$ since $J^{(v)}_H(\tau) \triangleq \tau_H(\tau) J^{(v)}(r)$. Accordingly, (39) can be rewritten as follows:

$$S_{\Delta J} \subseteq (S_J \cup S_{J_H}). \quad (40)$$

Moving to the modified contrasts, given by $\chi(\tau) \triangleq (\beta(\tau) \tau(r))/(\beta(\tau) \tau(r) + 1)$ and $\chi(\tau) \triangleq (\beta(\tau) \tau_H(\tau))/(\beta(\tau) \tau_H(\tau) + 1)$ subject to $\beta(\tau) \neq 0$, it turns out that $S_{\tau} \equiv S_J$ and $S_{\tau_H} \equiv S_J$. By combining these latter conclusions with (40), one yields that

$$S_{\Delta J} \subseteq (S_J \cup S_{J_H}) \quad \text{(41)}$$

which assumes the form (12) by observing that $S_J \equiv (S_{\Delta J} \cup S_{J_H})$ from the definition of $\chi(\Delta J)(r)$ [i.e., $\chi(\Delta J)(r) \triangleq \chi(\tau) - \chi_H(\tau)$].

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