Alpha decay and proton-neutron correlations

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We study the influence of proton-neutron ($p$-$n$) correlations on $\alpha$-decay width. It is shown from the analysis of alpha $Q$ values that the $p$-$n$ correlations increase the penetration of the $\alpha$ particle through the Coulomb barrier in the treatment following Gamow's formalism, and enlarges the total $\alpha$-decay width significantly. In particular, the isoscalar $p$-$n$ interactions play an essential role in enlarging the $\alpha$-decay width. The so-called "alpha-condensate" in $Z \geq 84$ isotopes are related to the strong $p$-$n$ correlations.

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The $\alpha$ decay has long been known as a typical decay phenomenon in nuclear physics $^1$. Various microscopic approaches to estimating the formation amplitude of the $\alpha$ cluster have been proposed $^2$ $^3$ $^4$ $^5$. The calculations $^6$ $^7$ $^8$ showed that $J=0$ proton-proton ($p$-$p$) and neutron-neutron ($n$-$n$) pairing correlations cause substantial $\alpha$-cluster formation on nuclear surface. This suggests that the BCS approach with a pairing force offers a promising tool to describe the $\alpha$ decay. Proton-neutron ($p$-$n$) correlations are also significantly important for the $\alpha$-decay process in a nucleus $^9$ $^{10}$. The effect of the $p$-$n$ correlations on the $\alpha$-formation amplitude was studied by a generalization of the BCS approach including the $p$-$n$ interactions $^{11}$, though it was shown that the enhancement of the formation amplitude due to the $p$-$n$ interactions is small. The authors of Ref. $^{12}$ pointed out that continuum part of nuclear spectra plays an important role in formation of $\alpha$ cluster. On the other hand, a shell model approach including $\alpha$-cluster-model terms $^{13}$ gave a good agreement with the experimental decay width of the $\alpha$ particle from the nucleus $^{212}$Po. It is also interesting to investigate the effect of deformation on the $\alpha$-decay width. According to Ref. $^{14}$, the contribution of deformation improves theoretical values for deformed nuclei such as $^{244}$Pu.

The $p$-$n$ interactions are expected to become strong in $N \approx Z$ nuclei because valence protons and neutrons in the same orbits have large overlaps of wavefunctions $^{14}$. In fact, this can be seen in peculiar behavior of the binding energy at $N=Z$. The double differences of binding energies are good indicators to evaluate the $p$-$n$ interactions $^{15}$ $^{16}$ $^{17}$. We have recently studied $^{17}$ various aspects of the $p$-$n$ interactions in terms of the double differences of binding energies, using the extended $P+QQ$ force model $^{18}$. The concrete evaluation confirmed that the $p$-$n$ correlations become very strong in the $N \approx Z$ nuclei. It was shown in Ref. $^{14}$ that the isoscalar ($T=0$) $p$-$n$ pairing force persists over a wide range of $N \geq Z$ nuclei. One of the double differences of binding energies was also discussed as a measure of $\alpha$-particle superfluidity in nuclei $^{21}$ $^{22}$. (We abbreviate the $\alpha$-like correlated four nucleons in a nucleus to “$\alpha$-particle” in italic letters. The “$\alpha$-particle” is not a free $\alpha$ particle but a correlated unit in a nucleus.) We expect that the $p$-$n$ correlations must play an important role in the barrier penetration of the $\alpha$ decay.

Experimental evidence of the $\alpha$ clustering appears in the systematics of alpha $Q$ values ($Q_{\alpha}$) $^{23}$, i.e., a large $Q_{\alpha}$ value coincides with a large $\alpha$-decay width in the vicinity of the shell closures $Z=50$, $Z=82$ and $Z=126$ $^{23}$. The $Q_{\alpha}$ value is essentially important for penetration $^{24}$ $^{25}$ $^{26}$ $^{27}$. It is known that if experimental $Q_{\alpha}$ values are used for the $\alpha$ decay between ground states, Gamow’s treatment $^1$ describes qualitatively well the penetration of the $\alpha$ particle through the Coulomb barrier, even though the $\alpha$-particle is assumed to be “a particle” in the nucleus. The penetration probability is expected to be sensitive to the $p$-$n$ component of the $Q_{\alpha}$ value. How much is the $p$-$n$ correlation energy included in the $Q_{\alpha}$ value? What is the role of the $p$-$n$ correlations in the barrier penetration? In this paper, we study these things and the effect of the $p$-$n$ correlations on the $\alpha$ decay.

The total $\alpha$-decay width is given by the well-known formula $^{29}$,

$$\Gamma = 2P_L \frac{\hbar^2}{2M_\alpha r_c^2} g_\alpha^2(r_c),$$

(1)

where $L$ and $M_\alpha$ denote, respectively, the angular momentum and the reduced mass of $\alpha$ particle, and $r_c$ channel radius. The $\alpha$ decay width depends on the two factors, the penetration factor $P_L$ and the $\alpha$ formation amplitude $g_\alpha(r_c)$. The $\alpha$ penetration is known as a typical phenomenon of “quantum tunneling” in quantum mechanics.

Since the $\alpha$-decay width depends sensitively upon the $Q_{\alpha}$ value, we first discuss the $Q_{\alpha}$ value, which is written in terms of the binding energy $B(Z,N)$ as follows:

$$Q_{\alpha}(Z,N) = B(Z-2,N-2) - B(Z,N) + B_\alpha,$$

(2)

where $B_\alpha$ is the binding energy of $^4$He. Experimental mass data show that the $Q_{\alpha}$ values are positive for $\beta$-stable nuclides with mass number greater than about 150. The $Q_{\alpha}$ value remarkably increases at nuclei above the closed shells, $N =50$, 82 and 126. This is attributed
to dramatic increase of separation energy at the closed shells with large shell gap. The $Q_\alpha$ value becomes largest (about 10 MeV) above $N = 128$, and the $\alpha$ particle can penetrate a high Coulomb potential barrier (which is 25 MeV for $^{212}$Po, and though it prohibits the emission of the $\alpha$ particle classically).

It has been shown in Refs. \cite{16, 17, 19} that the double difference of binding energies defined by

$$
\delta V^{(2)}(Z, N) = -\frac{1}{4}[B(Z, N) - B(Z, N - 2) - B(Z - 2, N) + B(Z - 2, N - 2)],
$$

is a good measure for probing the $p$-$n$ correlations. We have recently studied global features of the $p$-$n$ correlations in $A = 40 \sim 165$ nuclei by calculating the values of $\delta V^{(2)}$ with the extended $P + QQ$ force model accompanied by an isoscalar ($T = 0$) $p$-$n$ force \cite{15}. The analysis has revealed that the $T = 0$ $p$-$n$ pairing interaction makes an essential contribution to the double difference of binding energies $\delta V^{(2)}$. The graph of $\delta V^{(2)}$ as a function of $A$ exhibits a smooth curve of $40/A$ on average and deviations from the average curve $40/A$ are small, in the region of the mass number $80 < A < 160$ \cite{10, 17, 19}. Observed $\delta V^{(2)}$ can be reproduced with the use of semiempirical mass formula based on the liquid-drop model and the symmetry energy term is the main origin of $\delta V^{(2)}$ \cite{29}. However, the parameters of the liquid-drop model are adjusted to experimental binding energies and the liquid-drop model does not give sufficient information about correlations in many-nucleon systems \cite{30}. Our analysis \cite{17, 19} indicates that the symmetry energy in the liquid-drop model is attributed dominantly to the $J$-independent $T = 0$ $p$-$n$ pairing force when considering in the context of correlations. (In SO(5) symmetry model, the contributions of the $J$-independent $T = 0$ $p$-$n$ pairing and the $J = 0$ isovector ($T = 1$) pairing forces to $\delta V^{(2)}$ are estimated to be 73% and 27%, respectively.)

In Fig. 1 we show the values of $\delta V^{(2)}$ observed in isotopes with proton number $Z = 84 \sim 100$. This figure displays dramatic deviations from the average curve $40/A$ in contrast with that for $80 < A < 160$ shown in Ref. \cite{10}. The large deviations, however, seem to be different from those in $N = Z$ nuclei with $N < 30$ discussed in Refs. \cite{10, 17}, because nuclei with $N > 128$ which has a large number of excess neutrons are in a very different situation from the $N = Z$ nuclei. The large deviations from the average curve $40/A$ for $N > 128$ cannot be explained by only the symmetry energy or the $J$-independent $T = 0$ $p$-$n$ pairing force which smoothly varies with nucleon number and is almost insensitive to the shell effects. Nuclei with large $\delta V^{(2)}$ in Fig. 1 are simply those with short half-lives (i.e., large $\alpha$-decay widths), above the double-closed-shell nucleus $^{208}$Pb. The plots of $\delta V^{(2)}$ for the Po and Rn nuclei extracted from Fig. 1 are shown in Figs. 2 (a) and 2 (b). We can see dramatic changes of $\delta V^{(2)}$ at $N = 128$ both in the Po and Rn nuclei.

It is important to note that $\delta V^{(2)}$ is largest for $^{212}$Po with one $\alpha$-particle in Fig. 2 (a) and $^{216}$Rn with two $\alpha$-particles in Fig. 2 (b) (and so on), outside the double-closed shell core $^{208}$Pb. The peaks are intimately related to the even-odd staggering of proton or neutron pairs from the "alpha condensate" point of view discussed by Gambhir et al. \cite{20}. They defined the following quantities for the correlations between pairs:

$$
V_{\text{even}}^{\text{pair}}(A) = \frac{1}{2}(B(Z - 2, N) + B(Z, N - 2))
$$

FIG. 1: The experimental double difference of binding energies, $\delta V^{(2)}$ for nuclei with proton number $Z = 84 \sim 100$ and neutron number $N = 110 \sim 157$ as a function of neutron number $N$ along neutron chain.

FIG. 2: The experimental double difference of binding energies, $\delta V^{(2)}$ for (a) the Po nuclei and (b) the Rn nuclei as a function of neutron number $N$. 
\[ \Delta_n(Z, N) = \frac{(-1)^N}{2} (B(Z, N + 1) - 2B(Z, N) + B(Z, N - 1)), \]

defines the \( Q_\alpha \) value as
\[ Q_\alpha(Z, N) = Q_{pn} + Q_{pair} + Q_S + B_\alpha, \]
\[ Q_{pn} = 4\delta V^{(2)}(Z, N), \]
\[ Q_{pair} = 2((-1)^Z\Delta_n(Z, N - 1) + (-1)^Z\Delta_p(Z - 1, N)), \]
\[ Q_S = 2(S_n(Z, N) + S_p(Z, N)). \]

Since \( \delta V^{(2)} \) represents the \( p-n \) correlations \[17, 19\], the \( p-n \) component \( Q_{pn} \) corresponds to the \( p-n \) correlation energy of each \( \alpha \)-particle. Here, note that the number of \( p-n \) bonds in an \( \alpha \)-particle is four as illustrated in Fig. 4.

The \( p-p \) and \( n-n \) pairing component \( Q_{pair} \) is given by the proton and neutron odd-even mass differences.

The upward discontinuity of \( Q_\alpha \) value at \( N = 128 \) is highest for \( ^{210}\text{Pd} \), \( ^{211}\text{Bi} \) and \( ^{212}\text{Po} \), and decreases monotonously both in lighter and heavier elements when \( |Z - 82| \) increases. Similar behavior is observed in systematics of separation energy, namely this increase comes mainly from the neutron separation energy \( S_n \). The magic character of \( Q_\alpha \) seems to be strongest at \( Z = 82, N = 128 \), though it occurs near other doubly magic or submagic nuclei. The special increase of the \( Q_\alpha \) value at \( N = 128 \) is attributed, in the first place, to the single-particle energy gaps in the magic nuclei. However, a similar systematics is also observed in the double difference of binding energies \( \delta V^{(2)}(Z, N) \) in Figs. 11 and 2. Since \( Q_{pn} \) is proportional to \( \delta V^{(2)}(Z, N) \), the \( p-n \) component \( Q_{pn} \) must contribute to the \( \alpha \) decay. In fact, if we remove \( Q_{pn} \) from the experimental \( Q_\alpha \) value and assume \( g_L(r_c) = 1.0 \), the common logarithm of the decay constant \( \log_{10}\lambda \) in the Wentzel-Kramers-Brillouin (WKB) approximation is largely reduced as shown in Fig. 4.

Figure 4 shows the significant influence of \( Q_{pn} \) on the \( \alpha \) decay. Thus the \( p-n \) correlation energy \( Q_{pn} \) increases the \( \alpha \)-decay width, though it is smaller than the separation energy and the odd-even mass difference. The previous analysis using the extended \( P + QQ \) force model tells us that \( \delta V^{(2)} \) mainly corresponds to the \( J \)-independent

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**FIG. 3:** Even-odd staggering along the alpha-line as a function of mass number. The solid circles denote \( V^\text{even}_{\text{pair}}(A) \) for even pair number and the open circles \( V^\text{odd}_{\text{pair}}(A - 2) \) for odd pair number.

\[ -B(Z, N), \]
\[ V^\text{odd}_{\text{pair}}(A - 2) = B(Z - 2, N - 2) - \frac{1}{2}(B(Z - 2, N) + B(Z, N - 2)). \]
$T = 0$ $p$-$n$ interactions. Therefore, Eq. 5 testifies that the $\alpha$-decay transition is enhanced by the $p$-$n$ correlations through the $Q_\alpha$ value. The $T = 0$ $p$-$n$ interaction is crucial to the $\alpha$-decay phenomenon.

Although we have approximated as $g_L(r_c) = 1.0$ in the above consideration, the $\alpha$ decay width depends on the $\alpha$ formation amplitude $g_L(r_c)$ as well as the $Q_\alpha$ value. The $\alpha$ formation amplitude is very important for the $\alpha$ decay from the viewpoint of nuclear structure. The almost studies of the $\alpha$ decay have been concentrated on this problem. We can get a rough estimation of the $\alpha$ decay width using $g_L(r_c) = 1.0$ in the largest limit. This assumption means a situation that an $\alpha$ particle is moving in potential between the daughter nucleus and the $\alpha$ particle. The values of $\log_{10} \lambda$ calculated using the experimental $Q_\alpha$ values and $g_L(r_c) = 1.0$ are plotted also in Fig. 5, where the channel radius $r_c$ is taken to be beyond the touching point of the daughter nucleus and $\alpha$ particle, that is, $r_c = 1.2A^{1/3} + 3.0$ fm. The values agree quite well with the experimental ones in nuclei with $Z = 84 \sim 100$, $N = 110 \sim 154$. In particular, the agreement is good for nuclei with large value of $ZQ^{-1/2}$. It is notable that the effect of the $\alpha$-formation amplitude on the $\alpha$-decay width is smaller than that of the $p$-$n$ correlations mentioned above. The $\alpha$ decay is fairly well understood in terms of tunneling in quantum mechanics when we use the experimental $Q_\alpha$ values. There are, however, still differences between the calculation and experiment. These discrepancies should be improved by appropriate evaluation of the $\alpha$-formation amplitude. The correlated unit, $\alpha$-particle, in the nucleus can be regarded as the $\alpha$ particle only with some probability and the realistic formation amplitude is not 1.0.

There are several approaches to calculation of the formation amplitude, the shell model, the BCS method, the hybrid (shell model+$\alpha$-cluster) model, etc. The effect of continuum states on the $\alpha$ decay is known to be very large. One needs therefore very large shell model basis to obtain the experimental values of $\alpha$-formation amplitude. The hybrid model by Varga et al., which treats a large shell model basis up to the continuum states through the wavefunction of the spatially localized $\alpha$ cluster, explains well the experimental decay width. We can estimate the experimental $\alpha$-formation amplitude from the ratio

$$g^{2}_{\exp}(r_c) = \frac{\lambda_{\exp}}{\lambda_{\text{cal}}(g^2(r_c) = 1)},$$

where $\lambda_{\exp}$ is the experimental $\alpha$-decay constant and $\lambda_{\text{cal}}(g^2(r_c) = 1)$ denotes the $\alpha$-decay constant calculated using $g^2(r_c) = 1$ in the WKB approximation. Figures 6(a) and 6(b) show the experimental $\alpha$-formation amplitude $g^{2}_{\exp}$ as functions of $N$ and $Z$, respectively. A remarkable feature is that $g^{2}_{\exp}$ is quite small when $N$ or $Z$ is a magic number, and becomes larger in the middle of major shell. A typical example is $g^{2}_{\exp} = 0.020$ in $^{212}$Po which is known as a spherical nucleus. This value is very close to that obtained with the hybrid model and the BCS approach. On the other hand, nuclei in mid-shell exhibit typical rotational spectra, and are considered to be deformed nuclei. The enhancement of $g^{2}_{\exp}$ may be closely related to deformation. In fact, $g^{2}_{\text{cal}}$ is con-
considerably improved by introducing the deformation \[12\], while a spherical BCS method cannot explain the experiment. The effect of the deformation on the $\alpha$-formation amplitude seems to be remarkably large. We end our discussion by commenting that the dynamical correlations due to the $p$-$n$ interactions in addition to the static contribution to the $Q_\alpha$ value are probably driving correlations of the $\alpha$-particle \[21\].

In conclusion, we have shown that the nuclear correlations reveal themselves in the $\alpha$ decay through the $Q_\alpha$ value which affects the $\alpha$ penetration factor. We estimated the effects of the $p$-$n$ correlations on the $\alpha$-decay transition from the experimental double difference of binding energies $\delta V^{(2)}$. The $p$-$n$ correlations related to the $Q_\alpha$ value increase the rate of the $\alpha$-decay transition, and plays an important role particularly on the penetration process. However, nuclei with $N > 128$ have large deviations from the average curve $40/A$ of $\delta V^{(2)}$ which cannot be explained by the symmetry energy or the $J$-independent $T = 0$ $p$-$n$ pairing force. This suggests that there would be another interactions or correlations to describe the specific feature of $\delta V^{(2)}$ in this region. This problem is should be studied further. The “$\alpha$-condensate” point of view suggests that the strong $p$-$n$ correlations in $A > 208$ nuclei cause the $\alpha$-like $2p - 2n$ correlations. The $\alpha$-like correlations are important for the penetration as well as the formation of $\alpha$-particle.

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