Fragmentation-fraction ratio $f_{\Xi_b}/f_{\Lambda_b}$ in $b$- and $c$-baryon decays

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Abstract

We study the ratio of fragmentation fractions, $f_{\Xi_b}/f_{\Lambda_b}$, from the measurement of $\Xi^{0}_b \rightarrow \Xi^{+}_c \pi^{-}$ and $\Lambda^{0}_b \rightarrow \Lambda^{+}_c \pi^{-}$ with $\Xi^{+}_c/\Lambda^{+}_c \rightarrow p K^- \pi^+$. With the branching fraction $\mathcal{B}(\Xi^{+}_c \rightarrow p K^- \pi^+) = (2.2 \pm 0.8)\%$ obtained under the U-spin symmetry, the fragmentation ratio is determined as $f_{\Xi_b}/f_{\Lambda_b} = 0.054 \pm 0.020$. To reduce the above uncertainties, we suggest to measure the branching fractions of $\Xi^{+}_c \rightarrow p K^*0$ and $\Lambda^{+}_c \rightarrow \Sigma^{+} K^*0$ at BESIII, Belle(II) and LHCb.

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I. INTRODUCTION

Bottom quarks can be produced at the high energy colliders, such as LHC and Tevatron, and then hadronized into $B$ mesons and $b$-baryons. The probability of a bottom quark fragments into a certain weakly decaying $b$-hadron is called the fragmentation fractions, i.e. $f_{u,d,s} \equiv B(b \rightarrow B^-, B^0, B_s^0)$, $f_{\Lambda_b} \equiv B(b \rightarrow \Lambda^0_b)$, $f_{\Xi_b} \equiv B(b \rightarrow \Xi^0_b, \Xi^+_b)$ and $f_{\Omega_b} \equiv B(b \rightarrow \Omega^-_b)$. As non-perturbative effects, the fragmentation fractions can only be determined by experiments in some phenomenological approaches.

The $B$-meson fragmentation fractions have been measured by LEP, Tevatron and LHC with a relatively high precision \[1, 2\]. However, the current understanding of $b$-baryon productions is still a challenge. The total fragmentation fraction of $b$-baryon is the sum of all the weakly-decaying $b$-baryons,

$$f_{\text{baryon}} = f_{\Lambda_b} \left( 1 + 2 \frac{f_{\Xi^-}}{f_{\Xi^0}} + \frac{f_{\Omega^-}}{f_{\Lambda_b}} \right) = f_{\Lambda_b} (1 + \delta),$$

where the isospin symmetry is assumed as $f_{\Xi^-} = f_{\Xi^0} = f_{\Xi_b}$, and $\delta = 2 \frac{f_{\Xi^-}}{f_{\Xi^0}} + \frac{f_{\Omega^-}}{f_{\Lambda_b}}$ is the correction from $f_{\Lambda_b}$ to $f_{\text{baryon}}$. The averages of the total baryon production fractions are \[2\]

$$f_{\text{baryon}} = \begin{cases} 0.084 \pm 0.011, & \text{at LEP}, \\ 0.196 \pm 0.046, & \text{at Tevatron}, \end{cases}$$

which are inconsistent with each other, and of large uncertainties.

The total fraction of $b$-baryons has not been determined by LHCb because of its lack of measurements on $\Xi^0_b$ and $\Omega^-_b$. It has been found that the ratio $f_{\Lambda_b}/f_d$ depends on the $p_T$ of the final states \[3-6\]. At LHCb, the kinematic averaging ratio is \[5\]

$$\left| \frac{f_{\Lambda_b}}{f_u + f_d} \right|_{\text{LHCb}} = 0.240 \pm 0.022.$$  

It is required for the information of $f_{\Xi_b}$ and $f_{\Omega_b}$ to determine the other fragmentation fractions at LHCb due to the constraint of

$$f_u + f_d + f_s + f_{\text{baryon}} = 1.$$  

Since the production of $\Omega^-_b$ is suppressed compared to those of $\Xi^0_b$ by the production of an additional strange quark, the determination of $f_{\Xi_b}/f_{\Lambda_b}$ is essential to understand the productions of $b$-baryons and $B$ mesons.

So far, only Refs. \[7, 8\] have predicted the ratio $f_{\Xi_b}/f_{\Lambda_b}$, both based on the processes of $\Xi^0_b \rightarrow J/\psi \Xi^-$ and $\Lambda^0_b \rightarrow J/\psi \Lambda$ with the data given by CDF and D0. At LHCb, the productions
TABLE I: List of measurements related to the fragmentation fraction ratio $f_{\Xi_b}/f_{\Lambda_b}$.

| Measurements                                                                 | $f_{\Xi_b}/f_{\Lambda_b}$ |
|------------------------------------------------------------------------------|-----------------------------|
| $f_{\Lambda_b} \cdot B(\Lambda_b^0 \rightarrow J/\psi \Lambda) = (5.8 \pm 0.8) \times 10^{-5}$ [1] (CDF,D0) | 0.11 $\pm$ 0.03 [7]        |
| $f_{\Xi_b} \cdot B(\Xi_b^- \rightarrow J/\psi \Xi^-) = (1.02^{+0.26}_{-0.21}) \times 10^{-5}$ [1] (CDF,D0) | 0.108 $\pm$ 0.034 [8]      |
| $\frac{f_{\Xi_b}}{f_{\Lambda_b}} \cdot B(\Xi_b^- \rightarrow \Lambda_b^0 \pi^-) = (5.7 \pm 1.8^{+0.8}_{-0.5}) \times 10^{-4}$ [9] (LHCb) | 0.29 $\pm$ 0.10 (MIT bag model) [11] |
| $\frac{B(\Xi_b^+ \rightarrow \pi^-)}{B(\Lambda_b^0 \rightarrow \pi^-)} \cdot \frac{B(\Xi_b^+ \rightarrow pK^- \pi^+)}{B(\Lambda_b^0 \rightarrow pK^- \pi^+)} = (1.88 \pm 0.04 \pm 0.03) \times 10^{-2}$ [10] (LHCb) | 0.08 $\pm$ 0.03 (diquark model) [11] |

The production with the charm-baryon involving method is of the most high precision. The ratio $f_{\Xi_b}/f_{\Lambda_b}$ can be obtained as long as the related branching fractions are determined. Among them, the absolute branching fraction of $\Xi_b^+ \rightarrow pK^- \pi^+$ has never been measured [1], thus is of the largest ambiguity. In this work, we determine $f_{\Xi_b}/f_{\Lambda_b}$ with $B(\Xi_b^+ \rightarrow pK^- \pi^+)$ obtained under the $U$-spin symmetry.

This article is organized as follows. In Sec. II, we introduce the status of $f_{\Xi_b}/f_{\Lambda_b}$. The branching fraction of $\Xi_b^+ \rightarrow pK^- \pi^+$ and $f_{\Xi_b}/f_{\Lambda_b}$ are obtained in Sec. III and IV, respectively. Sec. V is the summary.

II. STATUS OF $f_{\Xi_b}/f_{\Lambda_b}$

In some literatures, it is usually assumed that the difference between the productions of $\Xi_b$ and $\Lambda_b$ is from the strange quark and up or down quarks [10, 12],

$$\frac{f_{\Xi_b}}{f_{\Lambda_b}} \sim \frac{f_s}{f_u}, \quad \text{or} \quad \frac{f_{\Xi_b}}{f_{\Lambda_b}} \sim 0.2. \quad (5)$$

However, since the fragmentation fractions are non-perturbative effects, they can only be extracted from experimental data. In this section, we introduce the status of $f_{\Xi_b}/f_{\Lambda_b}$ by means of the relevant measurements.
\textbf{A. } \Xi_b^- \rightarrow J/\psi \Xi^- \text{ v.s. } \Lambda_b^0 \rightarrow J/\psi \Lambda

So far, the only theoretical analysis on \( \frac{f_{\Xi_b}}{f_{\Lambda_b}} \) are performed in Refs. \cite{7,8} based on the experimental data of \( \Xi_b^- \rightarrow J/\psi \Xi^- \) and \( \Lambda_b^0 \rightarrow J/\psi \Lambda \). In Ref. \cite{1}, the relevant results averaging the measurements by CDF and D0 \cite{13,16}, are given as

\[
\begin{align*}
    f_{\Lambda_b} \cdot B(\Lambda_b^0 \rightarrow J/\psi \Lambda) &= (5.8 \pm 0.8) \times 10^{-5}, \\
    f_{\Xi_b} \cdot B(\Xi_b^- \rightarrow J/\psi \Xi^-) &= (1.02^{+0.26}_{-0.21}) \times 10^{-5}. 
\end{align*}
\]

The fragmentation fraction ratio of \( \frac{f_{\Xi_b}}{f_{\Lambda_b}} \) can be obtained unless the ratio of branching fractions of \( \Xi_b^- \rightarrow J/\psi \Xi^- \) and \( \Lambda_b^0 \rightarrow J/\psi \Lambda \) is known.

Both \( \Xi_b^- \rightarrow J/\psi \Xi^- \) and \( \Lambda_b^0 \rightarrow J/\psi \Lambda \) are the \( b \rightarrow c \bar{c}s \) transitions with the spectators of \( (ds - sd)/\sqrt{2} \) and \( (ud - du)/\sqrt{2} \), respectively. Therefore, the two processes are related to each other under the flavor \( SU(3) \) symmetry. The width relation of

\[
\Gamma(\Lambda_b^0 \rightarrow J/\psi \Lambda) = \frac{2}{3} \Gamma(\Xi_b^- \rightarrow J/\psi \Xi^-),
\]

is given by Voloshin \cite{7}. Using the experimental data in (6), the ratio of the fragmentation fractions can then be obtained as \cite{7}

\[
\frac{f_{\Xi_b}}{f_{\Lambda_b}} = 0.11 \pm 0.03. \tag{8}
\]

Hsiao \textit{et al} express the decay amplitudes of \( \Xi_b^- \rightarrow J/\psi \Xi^- \) and \( \Lambda_b^0 \rightarrow J/\psi \Lambda \) in the factorization approach \cite{8}. They relate the form factors of \( b \)-baryon to light baryon octet transitions based on the \( SU(3) \) symmetry, and obtain the ratio of branching fractions \( \frac{B(\Xi_b^- \rightarrow J/\psi \Xi^-)}{B(\Lambda_b^0 \rightarrow J/\psi \Lambda)} = 1.63 \pm 0.04 \). Utilizing the data in Eq. \cite{6}, the authors give a result similar to Eq. \cite{8},

\[
\frac{f_{\Xi_b}}{f_{\Lambda_b}} = 0.108 \pm 0.034. \tag{9}
\]

\textbf{B. } Heavy-flavor-conserving decay \( \Xi_b^- \rightarrow \Lambda_b^0 \pi^- \)

The LHCb collaboration has observed the first heavy-flavor-conserving \( \Delta S = 1 \) hadronic weak decay \( \Xi_b^- \rightarrow \Lambda_b^0 \pi^- \) \cite{9}, with

\[
\frac{f_{\Xi_b}}{f_{\Lambda_b}} \cdot B(\Xi_b^- \rightarrow \Lambda_b^0 \pi^-) = (5.7 \pm 1.8^{+0.8}_{-0.9}) \times 10^{-4}. \tag{10}
\]

In Ref. \cite{9}, \( f_{\Xi_b}/f_{\Lambda_b} \) is assumed to be bounded between 0.1 and 0.3, and then obtain the branching fraction of \( \Xi_b^- \rightarrow \Lambda_b^0 \pi^- \) lie in the range from \((0.57 \pm 0.21)\%\) to \((0.19 \pm 0.07)\%\). On the contrary, the fragmentation ratio \( f_{\Xi_b}/f_{\Lambda_b} \) can be obtained if \( B(\Xi_b^- \rightarrow \Lambda_b^0 \pi^-) \) is determined.
In Ref. [11], the branching fraction of $\Xi_b^- \to \Lambda_0^b \pi^-$ is calculated in the MIT bag model and the diquark model,

$$B(\Xi_b^- \to \Lambda_0^b \pi^-) = \begin{cases} 2.0 \times 10^{-3}, & \text{MIT bag model,} \\ 6.9 \times 10^{-3}, & \text{diquark model.} \end{cases}$$

(11)

Subsequently, we can obtain the ratio of fragmentation fractions according to Eq.(10) as,

$$\frac{f_{\Xi_b^-}}{f_{\Lambda_b^0}} = 0.29 \pm 0.10, \quad \text{MIT bag model,}$$

(12)

$$\frac{f_{\Xi_b^-}}{f_{\Lambda_b^0}} = 0.08 \pm 0.03, \quad \text{diquark model.}$$

(13)

**C. $\Xi_b^0 \to \Xi_c^+ \pi^-$ v.s. $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ via $\Xi_c^+ / \Lambda_c^+ \to pK^- \pi^+$**

In the above two methods, the experimental measurements are of large uncertainties, as seen in Eqs. (6) and (10). In the decay of $\Xi_b^- \to J/\Psi \Xi^-$, the efficiency of reconstruction of $\Xi^-$ with $\Xi^- \to \Lambda \pi^-$ and $\Lambda \to p \pi^-$, is very small in the hadron colliders [13, 14]. On the other hand, the branching fraction of $\Xi_b^- \to \Lambda_0^b \pi^-$ is expected to be very small.

Compared to the above processes, the relative production ratio between $\Xi_b^0 \to \Xi_c^+ \pi^-$ and $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ has been measured by LHCb with much higher precision [10],

$$\frac{f_{\Xi_b^0}}{f_{\Lambda_b^0}} \cdot \frac{B(\Xi_b^0 \to \Xi_c^+ \pi^-)}{B(\Lambda_b^0 \to \Lambda_c^+ \pi^-)} \cdot \frac{B(\Xi_c^+ \to pK^- \pi^+)}{B(\Lambda_c^+ \to pK^- \pi^+)} = (1.88 \pm 0.04 \pm 0.03) \times 10^{-2}. \quad (14)$$

As long as the branching fractions of the relevant $b$- and $c$-baryon decays are known, Eq. (14) could provide a good determination of $f_{\Xi_b^0} / f_{\Lambda_b^0}$. In Ref. [10], with naively expected values of $B(\Xi_b^0 \to \Xi_c^+ \pi^-) / B(\Lambda_b^0 \to \Lambda_c^+ \pi^-) \approx 1$ and $B(\Xi_c^+ \to pK^- \pi^+) / B(\Lambda_c^+ \to pK^- \pi^+) \approx 0.1$, it can be obtained that $f_{\Xi_b^0} / f_{\Lambda_b^0} \approx 0.2$.

The branching fraction of $\Xi_b^0 \to \Xi_c^+ \pi^-$ has not been directly measured in experiment. $B(\Xi_b^0 \to \Xi_c^+ \pi^-)$ and $B(\Lambda_b^0 \to \Lambda_c^+ \pi^-)$ are equal to each other in the heavy quark limit and the flavor $SU(3)$ symmetry. In literatures, only Refs. [17] and [18] calculate both the branching fractions of $\Xi_b^0 \to \Xi_c^+ \pi^-$ and $\Lambda_b^0 \to \Lambda_c^+ \pi^-$. With the transition form factors in the non-relativistic quark model, the ratio of branching fractions involving the factorizable contribution can be obtained in [17]:

$$\frac{B(\Xi_b^0 \to \Xi_c^+ \pi^-)}{B(\Lambda_b^0 \to \Lambda_c^+ \pi^-)} = \frac{\Gamma(\Xi_b^0 \to \Xi_c^+ \pi^-)}{\Gamma(\Lambda_b^0 \to \Lambda_c^+ \pi^-)} = \frac{0.33 a_1^2 \times 10^{10} s^{-1}}{0.31 a_1^2 \times 10^{10} s^{-1}} = 1.07, \quad (15)$$
where the difference in the lifetimes is neglected since \( \tau(\Xi^0_c)/\tau(\Lambda^0_c) = 1.006 \pm 0.021 \), and \( a_1 = C_1 + C_2/3 \) is the effective Wilson coefficient. The deviation from unity results from the mass difference between \( m_{\Xi_c} + m_{\Xi_c} \) and \( m_{\Lambda_c} + m_{\Lambda_c} \), i.e. the \( SU(3) \) breaking effect. In the soft-collinear effective theory, the non-factorizable contributions from the color-commensurate and the \( W \)-exchange diagrams are suppressed by \( \mathcal{O}(\Lambda_{QCD}/m_b) \) \(^{19}\). In Ref. \(^{18}\) in a relativistic three-quark model, it is found that the non-factorizable contributions amount up to 30% of the factorizable ones, with the ratio of \( B(\Xi^0_c \to \Xi^{+}_c \pi^-)/B(\Lambda^0_c \to \Lambda^{+}_c \pi^-) = 1.25 \). Therefore, even without a reliable study in a QCD-based approach, it can still be expected that the deviation of the ratio from unity is under control.

The absolute branching fraction of \( \Lambda^+_c \to pK^-\pi^+ \) has been well measured by Belle and BESIII \(^{20,21}\), with \( B(\Lambda^+_c \to pK^-\pi^+) = (6.35 \pm 0.33)\% \) \(^{1}\). However, branching fraction of \( \Xi^+_c \to pK^-\pi^+ \) is of large ambiguity. The ratio of \( B(\Xi^+_c \to pK^-\pi^+)/B(\Lambda^+_c \to pK^-\pi^+) \approx 0.1 \) used in \(^{10}\), is only naively assumed by the Cabibbo factor. In the next section, we will obtain the branching fraction of \( \Xi^+_c \to pK^-\pi^+ \) via \( U \)-spin analysis, and then determine \( f_{\Xi_c}/f_{\Lambda_c} \).

### III. BRANCHING FRACTION OF \( \Xi^+_c \to pK^-\pi^+ \)

The understanding of charmed baryon decays are still of high deficiency both in theory and in experiment. So far, there has not been any measurement on the absolute branching fraction of \( \Xi^0_{c/} \) decays \(^{1}\). The ratio of \( B(\Xi^+_c \to pK^-\pi^+)/B(\Xi^+_c \to \Xi^-\pi^+\pi^+) \) has been measured to be \( 0.21 \pm 0.04 \) \(^{11,22,23}\). But it is still unknown for the absolute branching fraction of \( \Xi^+_c \to pK^-\pi^+ \).

It is first found in Ref. \(^{24}\) that \( B(\Xi^+_c \to pK^-\pi^+) = (2.2 \pm 0.8)\% \) can be obtained from the measured ratio of \( B(\Xi^+_c \to pK^{*0})/B(\Xi^+_c \to pK^-\pi^+) = 0.54 \pm 0.10 \) \(^{22}\), and the \( U \)-spin relation between \( \Xi^+_c \to pK^{*0} \) and \( \Lambda^+_c \to \Sigma^+K^{*0} \). We show the \( U \)-spin analysis in detail in the present work.

The decays of \( \Xi^+_c \to pK^{*0} \) and \( \Lambda^+_c \to \Sigma^+K^{*0} \) are both singly Cabibbo-suppressed modes, with the transition of \( c \to (s\bar{s} - d\bar{d})u \) where the minus sign between \( s\bar{s} \) and \( d\bar{d} \) comes from the Cabibbo-Kobayashi-Maskawa matrix elements, \( V_{cd}V_{ud} = -V_{cs}V_{us} \). Note that the \( U \)-spin doublets are \((|d\rangle, |s\rangle)\) and \((|\bar{s}\rangle, -|\bar{d}\rangle)\). The effective Hamiltonian of \( c \to (s\bar{s} - d\bar{d})u \) changes the \( U \)-spin by \( \Delta U = 1, \Delta U_3 = 0 \), i.e. \( |H_{\text{eff}}\rangle = \sqrt{2}|1,0\rangle \). \( \Xi^+_c \) and \( \Lambda^+_c \) form a \( U \)-spin doublet of \( (\Lambda^+_c, \Xi^+_c) \). We have

\[
H_{\text{eff}}|\Xi^+_c\rangle = \sqrt{2}\left| 1, 0; \frac{1}{2}, -\frac{1}{2}\right\rangle = \frac{2}{\sqrt{3}}\left| 3, 2; -\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left| 1, 2; -\frac{1}{2}\right\rangle,
\]

\[
H_{\text{eff}}|\Lambda^+_c\rangle = \sqrt{2}\left| 1, 0; \frac{1}{2}, \frac{3}{2}\right\rangle = \frac{2}{\sqrt{3}}\left| 3, 2; 2\right\rangle - \sqrt{\frac{2}{3}}\left| 1, 2; 2\right\rangle.
\]
FIG. 1: The topological diagrams of $\Xi^+_c \to pK^0$ and $\Lambda^+_c \to \Sigma^+ K^*$. The top and bottom diagrams are color-commensurate and $W$-exchange ones, respectively.

The $U$-spin representations of the $|pK^0\rangle$ and $|\Sigma^+ K^*\rangle$ states are

$$|pK^0\rangle = \left|\frac{1}{\sqrt{2}}, \frac{1}{2}; 1, -1\right\rangle = \frac{1}{\sqrt{3}} \left|\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}} \left|\frac{1}{2}, 1, \frac{1}{2}\right\rangle,$$  

$$|\Sigma^+ K^*\rangle = \left|\frac{1}{2}, -\frac{1}{2}; 1, 1\right\rangle = \frac{1}{\sqrt{3}} \left|\frac{3}{2}, 1, \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}} \left|\frac{1}{2}, 1, \frac{1}{2}\right\rangle.$$  

The decay amplitudes are then

$$A(\Xi^+_c \to pK^0) = \langle pK^0 | H_{\text{eff}} | \Xi^+_c \rangle = \frac{2}{3} A_{3/2} - \frac{2}{3} A_{1/2},$$

$$A(\Lambda^+_c \to \Sigma^+ K^*) = \langle \Sigma^+ K^* | H_{\text{eff}} | \Lambda^+_c \rangle = \frac{2}{3} A_{3/2} - \frac{2}{3} A_{1/2},$$

where $A_{3/2}$ and $A_{1/2}$ are the amplitudes of $U$-spin of 3/2 and 1/2, respectively. It is clear that the amplitudes satisfy

$$A(\Xi^+_c \to pK^0) = A(\Lambda^+_c \to \Sigma^+ K^*).$$

This relation can also be seen from the topological diagrams in FIG. 1.

According to the relation in Eq. (22), the branching ratio of $\Xi^+_c \to pK^0$ can be obtained by

$$B(\Xi^+_c \to pK^0) = \frac{m^2_{\Lambda^+_c}}{m^2_{\Xi^+_c}} \frac{\tau_{\Xi^+_c}}{\tau_{\Lambda^+_c}} \frac{|p_c(m_{\Xi^+_c}, m_p, m_{K^*})|}{|p_c(m_{\Lambda^+_c}, m_\Sigma, m_{K^*})|} \cdot B(\Lambda^+_c \to \Sigma^+ K^*),$$

where $|p_c(M, m_1, m_2)| = \sqrt{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]/2M}$. The data of masses and lifetimes are taken from PDG: $m_{\Xi^+_c} = 2468 \text{ MeV}$, $m_{\Lambda^+_c} = 2286 \text{ MeV}$, $m_p = 938 \text{ MeV}$, $m_\Sigma = 1189 \text{ MeV}$, $m_{K^*} = 892 \text{ MeV}$, $\tau_{\Xi^+_c} = (4.42 \pm 0.26) \times 10^{-13} \text{ s}$, $\tau_{\Lambda^+_c} = (2.00 \pm 0.06) \times 10^{-13} \text{ s}$. Besides,
\( B(\Lambda_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-) = (4.57 \pm 0.29)\% \) \[1\] \[21\], and the branching ratios are \[22\] \[25\]

\[
\frac{B(\Lambda_c^+ \rightarrow \Sigma^+ K^{*0})}{B(\Lambda_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-)} = 0.078 \pm 0.022, \tag{24}
\]

\[
\frac{B(\Xi_c^+ \rightarrow pK^{*0})}{B(\Xi_c^+ \rightarrow pK^- \pi^+)} = 0.54 \pm 0.10. \tag{25}
\]

Then we can obtain

\[
B(\Xi_c^+ \rightarrow pK^- \pi^+) = (2.2 \pm 0.8)\%. \tag{26}
\]

The uncertainty is dominated by the ratios of branching fractions of \( \Lambda_c^+ \) and \( \Xi_c^+ \) decays in Eqs. \[24\] and \[25\].

The central value of \( B(\Xi_c^+ \rightarrow pK^- \pi^+) \) at the order of percent, is larger than the typical order of \( 10^{-3} \) of the ordinary singly Cabibbo-suppressed processes, such as \( B(\Lambda_c^+ \rightarrow \Sigma^+ K^{*0}) = (3.6 \pm 1.0) \times 10^{-3} \). This can be clarified by Eq. \[23\]. Firstly, the lifetime of \( \Xi_c^+ \) is two times larger than that of \( \Lambda_c^+ \). Secondly, the phase space of \( \Xi_c^+ \rightarrow pK^{*0} \) is larger than that of \( \Lambda_c^+ \rightarrow \Sigma^+ K^{*0} \) by another factor of two, i.e. \( |p_c(m_{\Xi_c}, m_p, m_\pi^0)| = 828 \text{ MeV} \) and \( |p_c(m_{\Lambda_c}, m_{\Sigma}, m_{K^*})| = 470 \text{ MeV} \). Due to the larger lifetime and phase space, the branching fraction of \( \Xi_c^+ \rightarrow pK^{*0} \) is then at the order of percent, \( (1.2 \pm 0.4)\% \).

The understanding of the dynamics of charmed baryon decays is still a challenge at the current stage. Recent theoretical studies are mostly based on the flavor \( SU(3) \) analysis \[26\]–\[30\] and the current algebra \[31\]. They have not yet been applied to the singly Cabibbo-suppressed charmed baryon decays into a light baryon and a vector meson. Therefore, it is not available to estimate the \( U \)-spin breaking effects in the above analysis of Eq. \[26\]. In the \( D \)-meson decays, the \( U \)-spin breaking effects, or say the \( SU(3) \) breaking effects, are mainly from the transition form factors and decay constants in the factorizable amplitudes, the difference between \( u\bar{u}, d\bar{d} \) and \( s\bar{s} \) produced from vacuum in the \( W \)-exchange and \( W \)-annihilation amplitudes, and the Glauber strong phase with pion in the non-factorizable contributions \[32\]–\[33\]. In Fig. \[1\] both amplitudes in the \( \Xi_c^+ \rightarrow pK^{*0} \) and \( \Lambda_c^+ \rightarrow \Sigma^+ K^{*0} \) decay are non-factorizable. The vacuum production of \( d\bar{d} \) and \( s\bar{s} \) in the \( W \)-exchange diagrams would be a main source of \( U \)-spin breaking. In the modes involving a vector meson and a pseudoscalar meson in the final states of \( D \)-meson decays, the difference between \( d\bar{d} \) and \( s\bar{s} \) production in the \( W \)-exchange diagrams can be seen from \( \chi_d^F e^{i\phi_d^F} = (0.49 \pm 0.03) e^{i(92 \pm 4)^\circ} \) and \( \chi_s^F e^{i\phi_s^F} = (0.54 \pm 0.03) e^{i(128 \pm 5)^\circ} \) \[34\] where \( \chi \) and \( \phi \) are the magnitude and strong phase of the non-perturbative parameters in the \( W \)-exchange diagrams, and the subscripts \( d \) and \( s \) denotes the quark flavor of \( q\bar{q} \) produced from the vacuum. It can be found that the \( U \)-spin breaking effects are
not large in $D \to VP$ modes. The $W$-exchange diagrams in charmed baryon decays are similar to the ones in charmed meson decays, with an additional spectator quark. It can be expected that the $U$-spin breaking effects between $\Xi_c^+ \to p K^*$ and $\Lambda_c^+ \to \Sigma^+ K^*$ would not be large, and thus the prediction of $\mathcal{B}(\Xi_c^+ \to p K^- \pi^+ ) = (2.2 \pm 0.8)\%$ would be under control.

The process of $\Xi_c^+ \to p K^- \pi^+$ with all the charged final particles is widely used to study the properties of, or to search for some heavier baryons. The mass and lifetime of $\Xi_c^0$ are measured with the most high precision via $\Xi_c^0 \to \Xi^+_c \pi^-$, $\Xi^+_c \to p K^- \pi^+$ [10]. New states of $\Xi_c^0(\frac{3}{2}^+)$ and $\Xi_c^0(\frac{3}{2}^+)$ are observed in the $\Xi_c^0 \pi^-$ spectrum with $\Xi_c^0 \to \Xi^+_c \pi^-$, $\Xi^+_c \to p K^- \pi^+$ [35]. Five new $\Omega_c^0$ resonances are observed in the final states of $\Xi^+_c K^-$ with $\Xi^+_c \to p K^- \pi^+$ [36]. It is suggested to search for the doubly charmed baryons in the decay of $\Xi_c^{++} \to \Xi_c^+ \pi^+$ with $\Xi_c^+ \to p K^- \pi^+$ [24, 37].

In this work, the ratio of fragmentation fractions $f_{\Xi_b}/f_{\Lambda_b}$ can be obtained as long as the branching fraction of $\Xi_c^+ \to p K^- \pi^+$ is determined by Eq. (14).

IV. $f_{\Xi_b}/f_{\Lambda_b}$ AND ITS IMPLICATIONS

Utilizing the prediction of $\mathcal{B}(\Xi_c^+ \to p K^- \pi^+ )$ in Eq. (26), the measured value of $\mathcal{B}(\Lambda_c^+ \to p K^- \pi^+ ) = (6.35\pm0.33)\%$ [1] and the reasonable theoretical ratio $\mathcal{B}(\Xi_c^0 \to \Xi^+_c \pi^-)/\mathcal{B}(\Lambda_c^0 \to \Lambda^+_c \pi^-) \approx 1$, the ratio of the fragmentation fraction for $b$ quark into $\Xi_c^0$ and $\Lambda_c^0$ can be obtained from Eq. (14) as

$$\frac{f_{\Xi_c^0}}{f_{\Lambda_c^0}} = 0.054 \pm 0.020.$$ (27)

The uncertainty is mainly from the branching fraction of $\Xi_c^+ \to p K^- \pi^+$ in Eq. (26). This result of $f_{\Xi_b}/f_{\Lambda_b}$ is much smaller than the naive estimation of $f_s/f_u$ or 0.2 in Eq. (5), and the MIT bag model for the branching fraction of $\Xi_b^- \to \Lambda_b^0 \pi^-$ in Eq. (12). The central value of our result in Eq. (27) is one half of those obtained via $\Xi_b^- (\Lambda_b^0) \to J/\Psi \Xi^-(\Lambda)$ in Eqs. (8) and (9). Only the prediction in the diquark model for $\Xi_b^- \to \Lambda_b^0 \pi^-$ in Eq. (13) is consistent with our result within the uncertainties, while the central value is larger as well.

The total $b$-baryon fraction can be obtained by the ratio $f_{\Xi_b}/f_{\Lambda_b}$ in Eq. (27). The production of $\Omega_b^-$ is doubly suppressed by two strange quarks, estimated as 15% of the $\Xi_b$. [38]. It is smaller than the error of $f_{\Xi_b}/f_{\Lambda_b}$ in Eq. (27), and thus can be neglected in the total fraction of $b$-baryons. We then have

$$f_{\text{baryon}} = f_{\Lambda_b} + 2f_{\Xi_b} + f_{\Omega_b} \approx f_{\Lambda_b} + 2f_{\Xi_b} = (1.11 \pm 0.04)f_{\Lambda_b}.$$ (28)
It is equivalent that the correction $\delta$ in Eq. (1) is $\delta = 0.11 \pm 0.04$, which is smaller than the estimation of $\delta = 0.25 \pm 0.10$ in Ref. [38].

With the result of $f_{\Xi_b}/f_{\Lambda_b}$ in Eq. (27), the branching fraction of $\Xi_b^0 \rightarrow \Lambda_b^0 \pi^-$ can be determined from Eq. (10),

$$B(\Xi_b^0 \rightarrow \Lambda_b^0 \pi^-) = (1.06 \pm 0.54)\%.$$  \hspace{1cm} (29)

This is consistent with the diquark model, but larger than the MIT bag model, seen in Eq. (11).

The precision of our result of $f_{\Xi_b}/f_{\Lambda_b}$ in Eq. (27) can be significantly improved by the measurements of $\Xi_c^+ \rightarrow pK^{*0}$ and $\Lambda_c^+ \rightarrow \Sigma^+ K^{*0}$ at LHCb, BESIII and Belle II. The large uncertainty of $B(\Xi_c^+ \rightarrow pK^- \pi^+)$ in Eq. (26), inducing the major uncertainty of $f_{\Xi_b}/f_{\Lambda_b}$, is dominated by two ratios of branching fractions: $B(\Xi_c^+ \rightarrow pK^{*0})/B(\Xi_c^+ \rightarrow pK^- \pi^+) = 0.54 \pm 0.10$ by FOCUS in 2001 [22] and $B(\Lambda_c^+ \rightarrow \Sigma^+ K^{*0})/B(\Lambda_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-) = 0.078 \pm 0.022$ measured by FOCUS in 2002 [25]. For the former, a more precise measurement can be performed by LHCb with partial wave analysis. At LHCb with the data of 3.3 fb$^{-1}$, there are already $1 \times 10^6$ events of $\Xi_c^+ \rightarrow pK^- \pi^+$ [36], which is four orders of magnitude larger than 200 events in Ref. [22]. The latter can be improved by the BESIII or Belle(II) experiments, which have recently performed a dozen measurements of $\Lambda_c^+$ decays [20, 21, 39-46], especially the observation of some singly or doubly Cabibbo-suppressed processes [44-46]. With the updated measurements of $\Xi_c^+ \rightarrow pK^{*0}$ and $\Lambda_c^+ \rightarrow \Sigma^+ K^{*0}$ in the near future, $f_{\Xi_b}/f_{\Lambda_b}$ could be of higher precision.

V. SUMMARY

In this work, we study the ratio of fragmentation fractions $f_{\Xi_b}/f_{\Lambda_b}$ with the data of $\Xi_c^0 \rightarrow \Xi_c^+ \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$, $\Xi_c^+/\Lambda_c^+ \rightarrow pK^- \pi^+$ at LHCb, which is the most precise measurement related to the fragmentations of $\Xi_b$ and $\Lambda_b$, seen in Table. [1] The least known branching fraction of $\Xi_c^+ \rightarrow pK^- \pi^+$ is obtained under the $U$-spin symmetry, $B(\Xi_c^+ \rightarrow pK^- \pi^+) = (2.2 \pm 0.8)\%$. The ratio $f_{\Xi_b}/f_{\Lambda_b}$ is then determined to be $f_{\Xi_b}/f_{\Lambda_b} = 0.054 \pm 0.020$. This is the first analysis of $f_{\Xi_b}/f_{\Lambda_b}$ using the LHCb data. It helps to understand the production of $b$-baryons. To improve the precision, we suggest to measure the ratios of branching fractions $B(\Lambda_c^+ \rightarrow \Sigma^+ K^{*0})/B(\Lambda_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-)$ and $B(\Xi_c^+ \rightarrow pK^{*0})/B(\Xi_c^+ \rightarrow pK^- \pi^+)$ at BESIII, Belle(II) and LHCb using the current data set or in the near future.
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