Integrated Possibilistic Linear Programming with Beta-Skewness Degree for a Fuzzy Multi-Objective Aggregate Production Planning Problem Under Uncertain Environments

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ABSTRACT
This study proposes an improved Fuzzy Programming (FP) approach to optimise multi-objective Aggregate Production Planning (APP) problem under uncertain environments. The proposed approach integrates the concept of Possibilistic Linear Programming (PLP) with Beta-Skewness Degree that decision-makers can manipulate the best level of data fuzziness as well as maintain such fuzziness in the optimisation process (by not turning it to deterministic data too early). The effectiveness of the proposed approach is demonstrated through a case study by minimising the highest overall deviation from the ideal solution of total costs under imprecise operating costs, customer demand, labour level, and machine capacity. Our comparative result clearly shows that the obtained solution outperforms the solutions from traditional defuzzification methods. The proposed approach also helps decision-makers not only to know and optimise the most likely situation, but also realise the outcomes in the optimistic and the pessimistic business situations so that decision makers can prepare and take necessary actions for future uncertainty.

1. Introduction
APP is a process by which decision-maker decides the suitable levels of capacity, production, subcontracting, inventory, stock-outs, and pricing over 3–18 months that can deal with uncertain customer requirements. Most organisations generate APP by considering minimise total costs: (1) Production costs that are related to purchasing materials and overhead, (2) Costs that are associated with changes in production rate such as hiring and firing costs and overtime compensation, and (3) Costs that are related to inventory such as holding and ordering costs. Fuzziness of data for an APP has been an important, challenging, and incentive working area for researchers and also one of the difficult issues that decision-makers have to face in planning. Generally, the uncertain data that can be occurred in APP could stem from two pathways: (1) Environmental uncertainty derived from a supplier’s performance and a customer’s behaviour in term of supply and demand and (2) System uncertainty derived from the unreliability of operations and processes [1]. Without

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effective techniques to handle these uncertainties, the obtained outcome from the plan cannot present their true fuzzy values since such uncertainties are normally assumed to be relatively constant and hence defuzzified too early in the optimisation process.

In this study, an improved FP; integrating the concept of PLP and fuzzy multi-objective Linear Programming (LP) with Beta-Skewness Degree, is proposed to optimise a fuzzy multi-objective APP problem under imprecise operating costs, customer demand, labour level, and machine capacity. As the main contribution of this study, the proposed approach can help decision-makers to manipulate the best levels of fuzziness of these data by not defuzzifying imprecise data earlier in the optimisation algorithm. The problem can then be solved for optimality to determine the best levels of fuzziness to be defuzzified to all fuzzy data otherwise the problem can quickly become the deterministic one after the early process of defuzzification as normally presented in the traditional defuzzification methods. The effectiveness of the proposed approach that outperforms existing approaches is demonstrated through the result comparison and managerial implications of a case study, which minimises the highest overall deviation from the ideal solution of minimum total costs in all three scenarios (most likely case, optimistic case and pessimistic case). In addition, from the obtained results under these three scenarios, decision-makers can be aware of all possible situations and can well-prepare themselves in advance for their business situations.

2. Literature Review

A review of past experiences and previous studies has shown that there is a great deal of scope and scope both in terms of APP and optimisation algorithms under uncertainty. To obtain the optimal results of the studies, classification concerning the subject of previous studies has been presented as follows:

2.1. Aggregate Production Planning

APP is a plan that determine appropriate levels of capacity, production, subcontracting, inventory, stock outs, and backordering over 3–18 months. Most organisations attempt to generate an effective APP that meets customer demand and has minimum total costs. APP was studied and researched in terms of demand and supply planning to enhance its cost and productivity by examining seven common forecasting tools for demand planning side and exercising classical chase and level aggregate planning strategy for the supply planning side [2].

Optimisation with APP can be classified into two types in terms of the number of objective functions. Firstly, with a single objective, a problem can provide the best solution, which corresponds to the minimum or maximum value of a single objective function that lumps all different objectives into one. It is useful as a model that provides decision makers with insights into the nature of the problem, but usually cannot provide a set of alternative solutions that trade different objectives against each other. For instance, APP for process industries under oligopolistic competition is studied in term of maximise the profit which a competitive version of the traditional APP model with capacity constraint is formulated by Karmarka and Rajaram [3]. APP of multiple products under demand uncertainty also studied in term of minimise the total costs which consider wastage cost and incentives for workforce [4].
Secondly, with multiple objectives, multiple objectives optimisation is used to interact among different objectives to give a set of compromised solutions, largely known as the trade-off non-dominated, non-inferior, or Pareto-optimal solutions. It is more likely to identify a wider range of these alternatives, and its model is more realistic. For instance, ranking methods of fuzzy numbers and tabu search are proposed for solving fuzzy multi-objective APP problem by Baykasoglu and Gocken [5]. The multi-objective model for multi-product multi-site APP in a green supply chain is designed to incorporate profit and green principle by using Analytical Hierarchy Process (AHP) [6].

2.2. Optimisation Under Uncertainty

Uncertainty of the data is one of the main troubles that decision-makers have to face in APP. Customer demand, unreliability of suppliers and their own operations, etc. are subject to uncertainty in real life. In addition, their performances could be obscure as they are not known in advance and cannot be estimate accurately. LP or a deterministic mathematical model cannot easily take such fuzziness into account. To handle uncertain data, a theory of fuzzy sets was first introduced by Bellman and Zadeh in 1970 [7]. Later it was further developed into a single or multiple objective LP problem. The following methods have attempted to capture the fuzziness of data for the optimisation algorithm.

2.2.1. Possibilistic Linear Programming

PLP is an approach that can be used to incorporate fuzzy data based on triangular or trapezoidal distributions. It is normally used to convert fuzzy data into crisp data that are included in the objective function. Wang and Liang [8] introduced PLP for APP problem. It yields an efficient APP compromise solution and overall degree of decision-maker satisfaction with determined goal values. Özgen et al. [9] proposed a two-phase PLP methodology for multi-objective supplier selection and order allocation problems. They applied the AHP to a multi-objective PLP model to evaluate and choose suppliers and to determine the optimum order quantities for each supplier. Kabak and Ülengin [10] applied the PLP approach for supply chain networking decisions. To maximise the total profit of an organisation, they proposed a PLP model with fuzzy demand, yield rate, costs, and capacities to make strategic resource-planning decisions.

2.2.2. Fuzzy Programming

FP is an extension of conventional programming where the aspiration level of each objective is unity. The achievement of the highest degree (unity) of the fuzzy goals of a problem is to solve multi-objective problems with imprecisely defined model parameters in a decision-making environment. This is sometimes called fuzzy mathematical programming with vagueness, where there is flexibility in the given target values of objective functions and/or the elasticity of constraints.

There are several methods of FP in the literature, such as weighted max–min, weighted additive, and Zimmermann. Amid et al. [11] applied a weighted additive fuzzy multi-objective model to a supplier selection problem under price breaks in a supply chain. The weighted additive FP model with equivalent crisp single-objective linear programming is generated to reduce dimensions of the system and complexity of computational.
Lachhwani and Poonia [12] studied a mathematical solution of the multilevel fractional programming problem with an FP in a large hierarchical decentralised organisation. The FP approach is used for achieving the highest degree of each membership goal by minimising the negative deviational variables.

Turgay and Taskin [13] applied FP to a health-care organisation to minimise the system costs while patients are satisfied. The FP model is identified and prioritised for strategic planning and resource allocation. Das et al. [14] proposed a mathematical model for solving a fully FP problem with trapezoidal fuzzy numbers. Their approach integrated the multi-objective LP and lexicographic ordering method. Then, Barik and Biswas [15] employed an FP approach for solid waste management under multiple uncertainties to minimise the net system cost of sorting and transporting the wastes and to maximise the revenue generated from different treatment facilities. Su and Ciou [16] developed a fuzzy multi-objective LP with an interactive two-phase PLP approach for solving remanufacturing production system problems with multiple goals under fuzzy environments. Taghizadeh et al. [17] also developed an interactive FP approach for solving multi-period multi-product production planning problem under imprecise environments to simultaneously minimise total production costs, rates of changes in labour levels, and maximising machine utilisation, while considering individual production routes of parts, inventory levels, labour levels, machine capacity, warehouse space, and the time value of money.

2.3. Defuzzification Methods

A defuzzification method can be used to convert fuzzy values to crisp values. Traditionally, at the beginning of the fuzzy linear programming process, fuzzy data need to be defuzzified to crisp values. However, if this process has been done too early, the characteristics of data fuzziness disappear and turn into deterministic problems. Some typical defuzzification methods and their applications are reviewed next.

2.3.1. Weighted Average Method

The weighted average method can be used to convert fuzzy constraints where one side of an equation has imprecise value by assigning weights. A possibility distribution can be used as the degree of occurrence of an event with imprecise data.

\[
 w^0E^0 + w^mE^m + w^pE^p
\]  

The triangular possibility distribution of fuzzy numbers \( \tilde{E} = (E^0, E^m, \text{and } E^p) \), where \( E^0, E^m, \) and \( E^p \) are the optimistic value, the most likely value, and the pessimistic value of \( \tilde{E} \), respectively, estimated by decision-maker. The weighted average method is used to defuzzify a fuzzy number by assigning weights \( (w^0, w^m, \text{and } w^p) \) to the optimistic value, the most likely value, and the pessimistic value of \( \tilde{E} \), as shown in Equation (1). The weights \( w^0, w^m, \) and \( w^p \) can be determined by decision-maker, based on their experience and \( w_1 + w_2 + w_3 = 1 \).

Liang [18] applied the weighted average method to defuzzify the fuzzy data in constraints of APP problems with multiple imprecise objectives by using the weights of optimistic \( (w^0) \), most likely \( (w^m) \), and pessimistic \( (w^p) \) values equal to 0.25, 0.5, and 0.25, respectively. Vluymans et al. [19] developed a clear strategy for the weighting scheme selection based upon the underlying characteristics of the data. Their present a strategy...
to select a suitable weighting scheme, based on fuzzy rough sets. Their weighting scheme selection process allows users to take full advantage of the versatility and performance improvements over the traditional fuzzy rough set approaches.

2.3.2. Fuzzy Ranking Method
Fuzzy ranking is another defuzzification method that can be used to convert fuzzy constraints on both sides of an equation that have imprecise values by separating them into three equations.

\[
E^o \leq F^o \tag{2}
\]
\[
E^m \leq F^m \tag{3}
\]
\[
E^p \leq F^p \tag{4}
\]

The triangular possibility distribution of fuzzy numbers \( \tilde{E} = (E^o, E^m, \text{ and } E^p) \) and \( \tilde{F} = (F^o, F^m, \text{ and } F^p) \) can be used, where \( E^o, E^m, \text{ and } E^p \) and \( F^o, F^m, \text{ and } F^p \) are the optimistic values, the most likely values, and the pessimistic values of \( \tilde{E} \) and \( \tilde{F} \) estimated by a decision maker, respectively. The fuzzy ranking method is used to defuzzify the fuzzy numbers by separating them into three cases, i.e. optimistic value, the most likely value, and the pessimistic value of \( \tilde{E} \) and \( \tilde{F} \), as shown in Equations (2)–(4).

Wang and Liang [7] applied the fuzzy ranking method to defuzzify the fuzzy data; labour level and machine capacity, in the constraints of APP problem under uncertain environments by dividing those fuzzy constraints into three equations. Roldán and Herrera [20] introduced this technique for ranking fuzzy numbers, which is applicable to the whole set of fuzzy numbers. Their developed technique is easy to compute and interpret in practice. It also overcomes certain shortcomings that appear when it is applied to other complex algorithms.

2.3.3. Beta-Skewness Degree
The Beta-Skewness Degree is another defuzzification method that can be used to convert fuzzy constraints to crisp constraints where \( \beta \) is the minimum degree of skewness of constraints that the decision makers are willing to admit.

According to Figure 1, the triangular possibility distribution of fuzzy numbers \( \tilde{D} = (D^o, D^m, \text{ and } D^p) \) where \( D^o, D^m, \text{ and } D^p \) are the optimistic values, the most likely values, and the pessimistic values of \( \tilde{D} \), respectively, estimated by decision-maker.

- If \( \beta = 0 \), this means that the value of \( D \) is on the right side (the triangle with the grey area) of the triangular distribution.
- If \( \beta = 1 \), this means that the value of \( D \) is on the left side (the triangle with dash lines) of the triangular distribution.
- If \( \beta = 0.5 \), this means that the value of \( D \) is in the middle of the left and right sides (the triangle with red line) of the triangular distribution.

\[
\left[ \left( 1 - \beta \right) \frac{E^p + E^m}{2} \right] + \left( \beta \frac{E^m + E^o}{2} \right) \tag{5}
\]
Equations (5) and (6) can be used to control the fuzziness of data on one side and both sides in the constraints, respectively. They are also used to find the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of each objective functions.

Here, the value of $\beta$ can be specified by decision-makers. For instance, the triangular possibility distribution of fuzzy numbers are $\tilde{E} = (E_o, E_m, E_p)$ and $\tilde{F} = (F_o, F_m, F_p)$, where $E_o$, $E_m$, and $E_p$ and $F_o$, $F_m$, and $F_p$ are the optimistic values, the most likely values, and the pessimistic values of $\tilde{E}$ and $\tilde{F}$ estimated by a decision-maker, respectively.

Parra et al. [21] proposed a method for solving several-objective problem that has parameters, which are often given by the decision makers in imprecise way through a fuzzy compromise programming approach. The Beta-Skewness Degree is used to handle the fuzzy parameters of constraints by suggesting a solution that is close to the ideal solution. Cheng, Huang, and Cai [22] proposed a method for solving Fully Fuzzy Linear Programming (FFLP) problems where all parameters and variables were fuzzy numbers. The Beta-Skewness Degree is used to convert fuzzy constraints into crisp constraints by considering an expected value. In our study, the Beta-Skewness Degree is further applied so that $\beta$ is added as a decision variable. The model is solved for optimality, to determine the best level of fuzziness to be defuzzified to each fuzzy constraint.

3. Optimisation Methodologies

In this part, the proposed approach; fuzzy multi-objective PLP with Beta-Skewness Degree, is explained according to flow chart as shown in Figure 2. Its functions consist of 3 steps: (1) PLP, (2) Fuzzy multi-objective PLP with Beta-Skewness Degree, (3) Defuzzification method, for optimisation algorithm. Firstly, PLP is used to set the objective functions and capture the imprecise and fuzzy operating costs. Then, fuzzy multi-objective PLP with Beta-Skewness Degree that consists of two phases; FP-Phase 1: The multi-objective PLP model is used to calculate the boundaries of each objective function, and FP-Phase 2: FP is used to calculate the optimal solution in which the optimisation with multiple objectives needs to compromise among possible conflicting objectives, is applied. This step can normally be done by
maximising the overall satisfaction level or minimising the overall deviation from the ideal solution. At the last step, several FP methods, such as Zimmermann’s method (when there is no requirement of assigning weights to each objective: weightless) or the weighted additive method (when different weights can be assigned to each objective: unequal weights), can be applied. In this study, minimising the highest overall deviation from the ideal solution with an equal weight for each objective is demonstrated. However, decision makers can select any suitable method based on their preferences and business situations.

3.1. Step 1: Possibilistic Linear Programming (PLP)

The PLP model is introduced to find the optimal result in each scenario, subject to imprecise costs based on the triangular distribution which is a usual possibility distribution (found in applications of fuzzy sets). The triangular distribution is an optimal transformation of the uniform probability distribution. It is the upper envelope of all the possibility distributions, a transformation from symmetric probability densities with the same support (Dubois et al. [23]).

A triangular fuzzy number can be used to express the vagueness and uncertainty of information and to represent fuzzy terms in information processing within an interval. It has been applied in many fields such as risk evaluation, performance evaluation, forecasting, matrix games, decision-making, and spatial representation. In principle, membership functions can be different shapes, but in practice, trapezoidal and triangular membership functions are the most frequently used (Zhang, Ma, and Chen [24]).

\[
f(x) = \begin{cases} 
\frac{2\text{a^o} - \text{a}^m}{(\text{a}^m - \text{a}^o)(\text{a}^o - \text{a}^p)} & \text{a}^o < x < \text{a}^p \\
\frac{2\text{a}^m - x}{(\text{a}^m - \text{a}^o)(\text{a}^m - \text{a}^p)} & \text{a}^p < x < \text{a}^m \\
\end{cases}
\]  

Equation (7) expresses the function of the triangular distribution that defines the optimistic (a^o), the most likely (a^m), and the pessimistic (a^p) values as a real number, a^o ≤ a^m ≤ a^p.
According to Figure 3, three prominent points; optimistic value point \((a^o)\), most likely value point \((a^m)\), and pessimistic value point \((a^p)\), are used to describe the triangular distribution as follows:

- The optimistic value \((a^o)\) is the value that yields the best case. There is a low likelihood that the possibility degree \(= 0\), if normalised.
- The most likely value \((a^m)\) is the value that yields the normal or general case. There is a low likelihood that the possibility degree \(= 1\), if normalised.
- The pessimistic value \((a^p)\) is the value that yields the worst case. There is a low likelihood that the possibility degree \(= 0\), if normalised.

Here, minimise the total costs of APP are concerned which the fuzzy total costs in all three scenarios are selected to be the objective function. Because of uncertain operating costs, the objective function can be divided into 3 sub-objective functions based on triangular distribution including with three prominent points: the optimistic cost \((z^o)\), the most likely cost \((z^m)\), and the pessimistic cost \((z^p)\) that are used for minimising the total costs as shown in Figure 4. PLP approach is used to defuzzified uncertain operating costs by pushing these three values; (1) Minimizing the most likely total costs (minimising \(z^m\)), (2) Maximising the lower total costs (maximising \(z^m - z^o\)), and (3) Minimising the higher total costs (minimising \(z^p - z^m\)), toward the left.
3.2. Step2: Fuzzy Multi-Objective PLP with Beta-Skewness Degree

To optimise APP problem under uncertain environments, a two-phase approach is applied. The first phase deals with the multi-objective LP model that can be used to calculate the boundaries of each objective functions. The second phase deals with FP that can be used to converts the fuzzy multiple-objective PLP values to single-objective LP values by considering minimising the highest overall deviation from the ideal solution in this study.

3.2.1. Phase 1: Multi-Objective LP Model

The crisp multi-objective LP model is stated as follows:

Minimize \( Z = [Z_1, -Z_2, Z_3] \)

\[
Z_1 = Z^m, \quad Z_2 = Z^m - Z^o, \quad Z_3 = Z^p - Z^m
\]  

(8)

A Multiple Objective Linear Programming (MOLP) problem can be converted into a single-goal LP problem by setting the criteria of solutions: Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of all objective functions. These can be used to be the boundaries of each objective function by the LP model, to obtain the maximum and minimum solutions of each objective. The objective functions used to calculate the PIS and NIS values are expressed as follows:

\[
Z_{PIS}^1 = \text{minimize } Z^m, \quad Z_{NIS}^1 = \text{maximize } Z^m
\]

\[
Z_{PIS}^2 = \text{maximize } Z^m - Z^o, \quad Z_{NIS}^2 = \text{minimize } Z^m - Z^o
\]

\[
Z_{PIS}^3 = \text{minimize } Z^p - Z^m, \quad Z_{NIS}^3 = \text{maximize } Z^p - Z^m
\]

Subject to:

\[ v \in F(v) \]  

(9)

Linear membership functions for minimising the most possible value of the imprecise total costs \( Z_1 \), maximising the possibility of obtaining lower total costs \( Z_2 \), and minimising the risk of obtaining higher total costs \( Z_3 \) are expressed in Equations (10)–(12), respectively.

\[
d_1(x) = \begin{cases} 
  \frac{1}{Z_{NIS}^1 - Z_{PIS}^1} & \text{if } Z_1 < Z_{PIS}^1 \\
  \frac{Z_{NIS}^1 - Z_{PIS}^1}{Z_{NIS}^1 - Z_{PIS}^1} & \text{if } Z_{PIS}^1 \leq Z_1 \leq Z_{NIS}^1 \\
  0 & \text{if } Z_1 > Z_{NIS}^1
\end{cases}
\]

(10)

\[
d_2(x) = \begin{cases} 
  \frac{1}{Z_{NIS}^2 - Z_{PIS}^2} & \text{if } Z_2 > Z_{PIS}^2 \\
  \frac{Z_{NIS}^2 - Z_{PIS}^2}{Z_{NIS}^2 - Z_{PIS}^2} & \text{if } Z_{NIS}^2 \leq Z_2 \leq Z_{PIS}^2 \\
  0 & \text{if } Z_2 > Z_{NIS}^2
\end{cases}
\]

(11)

\[
d_3(x) = \begin{cases} 
  \frac{1}{Z_{NIS}^3 - Z_{PIS}^3} & \text{if } Z_3 < Z_{PIS}^3 \\
  \frac{Z_{NIS}^3 - Z_{PIS}^3}{Z_{NIS}^3 - Z_{PIS}^3} & \text{if } Z_{PIS}^3 \leq Z_3 \leq Z_{NIS}^3 \\
  0 & \text{if } Z_3 > Z_{NIS}^3
\end{cases}
\]

(12)
3.2.2. Phase 2: Fuzzy Programming

In this study, the APP problem under uncertain environments is optimised by considering minimise the highest overall deviation from the ideal solution; PIS, that can be calculated from symmetric and asymmetric fuzzy programming approaches. Zimmermann’s method can be used for symmetric fuzzy decision-making where there is no difference in the importance of the weights of objectives and constraints (weightless). In contrast, the weighted additive method is an asymmetric fuzzy decision-making method where the objectives and constraints may not be equally important and can be assigned with different weights.

**Zimmermann’s Method:** it was first developed by Zimmermann [25] in 1978 for solving multi-objective LP problems. It minimises the highest deviation of objective values which can guarantee that all deviation levels of objectives are lower than the deviation level of the highest objective. The mathematical model of the Zimmermann’s method can be expressed as follows:

Minimize $L_\infty = d$

Subject to:

\[
\begin{align*}
    x & \in F(x) \\
    d_1 & \leq d \\
    d_2 & \leq d \\
    d_3 & \leq d
\end{align*}
\]  

where $d_1$, $d_2$, and $d_3$ are the deviations of objective values from their ideal solutions for objectives 1, 2, and 3, respectively. $F(x)$ denotes the feasible region for the constraints of the equivalent crisp model.

3.3. Step3: Defuzzification Method with Beta-Skewness Degree

The Beta-Skewness Degree is a decision variable that can be used to find the best level of fuzziness of data to be defuzzified as express in Equations (14) and (15). Equation (14) is used to defuzzify fuzzy constraints where one side of an equation has imprecise value and Equation (15) is used to defuzzify fuzzy constraints where both sides of an equation has imprecise value.

\[
\left[ \left( 1 - \beta_k \right) \cdot \frac{E^o + E^m}{2} \right] + \left( \beta_k \cdot \frac{E^m + E^o}{2} \right) 
\leq \left[ \left( 1 - \beta_k \right) \cdot \frac{F^p + F^m}{2} \right] + \left( \beta_k \cdot \frac{F^m + F^o}{2} \right) 
\]

For instance, the triangular possibility distribution of fuzzy numbers are $\tilde{E} = (E^o, E^m, E^p)$ and $\tilde{F} = (F^o, F^m, F^p)$, where $E^o$, $E^m$, and $E^p$ and $F^o$, $F^m$, and $F^p$ are the optimistic values, the most likely values, and the pessimistic values of $\tilde{E}$ and $\tilde{F}$ estimated by a decision maker, respectively. Each fuzzy constraint can have different levels of the Beta-Skewness Degree depending on its pattern and level of fuzziness.
4. Case Study

An APP problem with two types of products that are planned to be manufactured in the next four months is used to demonstrate all of optimisation methodologies: (1) LP, (2) Fuzzy multi-objective PLP with the defuzzification methods (weighted average and fuzzy ranking), and (3) Fuzzy multi-objective PLP with Beta-Skewness Degree. The customer demand, related operating costs, labour level, and machine capacity are uncertain, which are summarised in Tables 1–3.

- The initial inventory level of Products 1 and 2 in the first period are 400 and 200 units, respectively. The ending inventory level of Products 1 and 2 in the fourth period are 300 and 200 units, respectively.
- The costs of hiring and firing are imprecise with ($8, $10, $11) and ($2.0, $2.5, $3.2) per worker per hour, respectively.
- The initial labour level is 300 person-hours.
- The labour hours which are used to produce Products 1 and 2 are (0.03, 0.05, 0.07) and (0.05, 0.07, 0.09) person-hours per unit in any period, respectively.
- The hours of machine usage per unit are also fuzzy with (0.09, 0.10, 0.11) and (0.07, 0.08, 0.09) machine-hours for Products 1 and 2 in any period, respectively.
- The required warehouse spaces for Products 1 and 2 are 2 ft² and 3 ft² per unit, respectively.

| Table 1. Forecasted demand of Products 1 and 2 in Months 1–4. |
|---------------------------------------------------------------|
| Product          | Month | 1                           | 2                           | 3                           | 4                           |
|                  |       | (900,1000,1080)              | (2750,3000,3200)             | (4600,5000,5300)             | (1850,2000,2100)             |
|                  |       | (900,1000,1080)              | (450,500,540)                | (2750,3000,3200)             | (2300,2500,2650)             |
| $\hat{D}_{1t}$ (units) |       |                             |                             |                             |                             |
| $\hat{D}_{2t}$ (units) |       |                             |                             |                             |                             |

| Table 2. Related operating costs of Products 1 and 2. |
|-----------------------------------------------------|
| Product | 1 | 2 |
| $r_{nt}$ ($/unit) | (17,20,22) | (8,10,11) |
| $o_{nt}$ ($/unit) | (26,30,33) | (12,15,17) |
| $s_{nt}$ ($/unit) | (22,25,27) | (10,12,13) |
| $i_{nt}$ ($/unit) | (0.27,0.30,0.32) | (0.13,0.15,0.16) |
| $b_{nt}$ ($/unit) | (35,40,44) | (16,20,23) |

| Table 3. Maximum labour level, machine capacity, and warehouse space in Months 1–4. |
|-------------------------------------------------------------|
| Month | $\hat{L}_{\text{max}}$ (person-hours) | $\hat{M}_{\text{max}}$ (machine-hours) | $W_{S_{\text{max}}}$ ($\text{ft}^2$) |
|-------|------------------------------------------|------------------------------------------|---------------------------------|
| 1     | (175,300,320)                           | (360,400,430)                           | 10,000                          |
| 2     | (175,300,320)                           | (450,500,540)                           | 10,000                          |
| 3     | (175,300,320)                           | (540,600,650)                           | 10,000                          |
| 4     | (175,300,320)                           | (450,500,540)                           | 10,000                          |
5. Problem Formulation

5.1. Problem Description

An APP problem with \( N \) types of products that are planned to be manufactured in the next \( T \) months is used to demonstrate all of optimisation methodologies: (1) LP, (2) Fuzzy multi-objective PLP with the defuzzification methods (weighted average and fuzzy ranking), and (3) Fuzzy multi-objective PLP with Beta-Skewness Degree. For this demonstration, the customer demand, related operating costs, labour level, and machine capacity are uncertain.

5.2. Problem Notations

The notations that are used to formulate the mathematical model of the APP problem are expressed which the symbol (\( \tilde{\cdot} \)) refers to ambiguous data that are used in this numerical case study.

Indexes

- \( N \)  Types of products (\( n = 1, \ldots, N \))
- \( T \)  Periods (\( t = 1, \ldots, T \))
- \( J \)  Number of fuzzy objectives (\( j = 1, \ldots, J \))
- \( K \)  Number of fuzzy constraints (\( k = \) customer demand, labour level, and machine capacity)

Parameters

- \( \tilde{D}_{nt} \)  Forecast demand of product \( n \) in period \( t \) (units)
- \( \tilde{r}_{nt} \)  Cost of regular time production per unit of product \( n \) in period \( t \) ($/unit)
- \( \tilde{s}_{nt} \)  Cost of subcontracting per unit of product \( n \) in period \( t \) ($/unit)
- \( \tilde{S}_{nt} \)  Cost of subcontracts per unit of product \( n \) in period \( t \) ($/unit)
- \( \tilde{S}_{nt} \)  Cost of inventory per unit of product \( n \) in period \( t \) ($/unit)
- \( \tilde{b}_{nt} \)  Cost of backordering per unit of product \( n \) in period \( t \) ($/unit)
- \( \tilde{h}_t \)  Cost of hiring per worker in period \( t \) ($/person-hour)
- \( \tilde{f}_t \)  Cost of firing per worker in period \( t \) ($/person-hour)
- \( \tilde{L}_{tmax} \)  Maximum labour level available in period \( t \) (person-hours)
- \( \tilde{M}_{tmax} \)  Maximum machine capacity available in period \( t \) (machine-hours)
- \( \tilde{M}_{Hnt} \)  Machine’s hour usage per unit of product \( n \) in period \( t \) (machine-hours/unit)
- \( \tilde{L}_{Hnt} \)  Labour’s hour usage per unit of product \( n \) in period \( t \) (person-hours/unit)
- \( \tilde{W}_{S_{tmax}} \)  Maximum warehouse space available in period \( t \) (\( ft^2 \)/unit)
- \( \tilde{W}_{s_{nt}} \)  Warehouse space for product \( n \) in period \( t \) (\( ft^2 \)/unit)

Decision Variables

- \( RQ_{nt} \)  Regular time production quantity of product \( n \) in period \( t \) (units)
- \( OQ_{nt} \)  Overtime production quantity of product \( n \) in period \( t \) (units)
- \( SQ_{nt} \)  Subcontracting quantity of product \( n \) in period \( t \) (units)
IQ_{nt} \quad \text{Inventory quantity of product } n \text{ in period } t \text{ (units)}

BQ_{nt} \quad \text{Backorder quantity of product } n \text{ in period } t \text{ (units)}

H_t \quad \text{Number of workers hired in period } t \text{ (person-hour)}

F_t \quad \text{Number of workers fired in period } t \text{ (person-hour)}

\beta_k \quad \text{Best level of fuzziness of the fuzzy constraint } k \text{ to be defuzzified } (k = \text{customer demand, labour level, and machine capacity})

Related Notations

\tilde{Z} \quad \text{Total costs (\$)}

d_j(v) \quad \text{Linear membership function of the fuzzy objectives } (j = 1, \ldots, J)

5.3. Mathematical Model

The mathematical models of the APP problem are formulated based on three optimisation methodologies: (1) LP, (2) Fuzzy multi-objective PLP with the defuzzification methods (weighted average and fuzzy ranking method), and (3) Fuzzy multi-objective PLP with Beta-Skewness Degree, as follows:

5.3.1. LP Model

Generally, LP can be used as a mathematical tool for APP to create optimal APP. It can be used to find the optimal solution of the constant model which can be generated as below.

Objective Function: Minimising the total costs is a common objective function of the APP problem. However, the coefficients of costs can be imprecise due to incomplete or misleading information. The total costs are the sum of the production cost and the costs of changes in labour levels over the planning horizon T. The objective function is proposed as:

\[
\text{Minimize total costs} = \sum_{n=1}^{N} \sum_{t=1}^{T} (\text{Regular Time Production Costs} + \text{Overtime Production Cost} + \text{Subcontracting Cost} + \text{Inventory cost} + \text{Backordering cost}) + \sum_{t=1}^{T} (\text{Hiring cost} + \text{Layoff cost})
\]

\[
\text{Min} \tilde{Z} = \sum_{n=1}^{N} \sum_{t=1}^{T} (\tilde{r}_{nt} RQ_{nt} + \tilde{b}_{nt} SQ_{nt} + \tilde{s}_{nt} SQ_{nt} + \tilde{i}_{nt} IQ_{nt} + \tilde{b}_{nt} BQ_{nt}) + \sum_{t=1}^{T} (\tilde{h}_{t} H_{t} + \tilde{f}_{t} F_{t})
\]  

(16)

Regular time production cost (\tilde{r}_{nt}), Overtime production cost (\tilde{b}_{nt}), Subcontracting cost (\tilde{s}_{nt}), Inventory cost (\tilde{i}_{nt}), Backordering cost (\tilde{b}_{nt}), Hiring cost (\tilde{h}_{t}), and Lay off cost (\tilde{f}_{t}) are imprecise coefficients with the triangular possibility distribution.

Constraint: There are four categories of constraints: (1) Carrying inventory, (2) Labour level, (3) Machine capacity, and (4) Warehouse capacity, that are related to APP.
• Carrying Inventory Constraint

\[
\text{Demand} = \text{Previous Ending Inventory} - \text{Previous Backordering Units} + \text{Regular Time Production Units} + \text{Overtime Production Units} + \text{Subcontracting Units} - \text{Current Ending Inventory} + \text{Current Backordering units}
\]

\[
\tilde{D}_{nt} = IQ_{nt-1} - BQ_{nt-1} + RQ_{nt} + OQ_{nt} + SQ_{nt} - IQ_{nt} + BQ_{nt} \quad \forall N, \forall T \quad (17)
\]

In reality, the demand of a customer cannot be forecasted precisely, so \(\tilde{D}_{nt}\) is used for the imprecise forecasted demand of the \(n\)th product in period \(t\). Equation (17) represents the sum of regular and overtime production units, inventory levels, and subcontracting and backorder levels, which must be equal to the amount of the forecasted demand.

• Labor Level Constraints

Previous Labour Level + Hiring – Firing – Current Labour Level = 0

\[
\sum_{n=1}^{N} LH_{nt-1} (RQ_{nt-1} + OQ_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} LH_{nt} (RQ_{nt} + OQ_{nt}) = 0 \quad \forall T \quad (18)
\]

Current Labour Level ≤ Maximum Available Labor Level

\[
\sum_{n=1}^{N} LH_{nt} (RQ_{nt} + OQ_{nt}) \leq \tilde{L}_{tmax} \quad \forall T \quad (19)
\]

where Equation (18) is the labour level constraint that represents the labour level in period \(t\), equal to the labour level in period \(t-1\) plus new hired workers, minus the laid off workers in period \(t\). Generally, the level of labour cannot exceed the maximum available level of labour in each period, as shown in Equation (19). The maximum available level of labour can be imprecise due to uncertain possible demand, supply, and skill of labour in the market at each moment.

• Machine Capacity Constraint

Hours of Machine Usage ≤ Maximum Available Machine Capacity

\[
\sum_{n=1}^{N} \tilde{MH}_{nt} (RQ_{nt} + OQ_{nt}) \leq \tilde{M}_{tmax} \quad \forall T \quad (20)
\]

where \(\tilde{MH}_{nt}\) and \(\tilde{M}_{tmax}\) are fuzzy data of hours of machine usage per unit of the \(n\)th product, and maximum available machine capacity in period \(t\), respectively. In Equation (20), the hours of machine usage per unit of the \(n\)th product are less than or equal to the maximum available machine capacity in period \(t\). Similarly, the maximum available machine capacity can be imprecise (in reality) as the available machine hours could be affected by the availability and the working conditions of machines in each moment.
• Warehouse Capacity Constraint

Warehouse Space Usage ≤ Maximum Available Warehouse Space

\[ \sum_{n=1}^{N} w_{sn t} Q_{nt} \leq W_{s_{max}} \quad \forall T \] (21)

Equation (21) represents the warehouse capacity (limitation) in each period \( t \). The warehouse space per unit of \( n \)th product in period \( t \) is less than or equal to the maximum available warehouse space in period \( t \).

• Non-negativity Constraint

\[ R_{Q_{nt}}, O_{Q_{nt}}, S_{Q_{nt}}, I_{Q_{nt}}, B_{Q_{nt}}, H_{t}, F_{t} \geq 0 \quad \forall N, \forall T \] (22)

5.3.2. Fuzzy Multi-Objective PLP with the Defuzzification Methods (Weighted Average and Fuzzy Ranking Method)

This method can help decision makers to handle fuzzy data by using traditional weighted average and fuzzy ranking methods to defuzzify fuzzy data to be constant data. In this study, PLP is used to classify the imprecise operating costs into three scenarios with three objective functions, as seen in Equations (23)–(25). The defuzzification methods, weighted average and fuzzy ranking, are used to defuzzify the fuzzy customer demand, labour level, and machine capacity in the constraints. The weighted average method can be used to convert fuzzy constraints where one side of an equation has imprecise value by assigning weights and fuzzy ranking can be used to convert fuzzy constraints on both sides of an equation that have imprecise values by separating them into three equations.

Objective Function:

Minimising the most likely total costs (Min \( Z_1 = Z^m \))

\[ = \sum_{n=1}^{N} \sum_{t=1}^{T} [r_{nt}^m R_{Q_{nt}} + o_{nt}^m O_{Q_{nt}} + s_{nt}^m S_{Q_{nt}} + i_{nt}^m I_{Q_{nt}} + b_{nt}^m B_{Q_{nt}}] \]

\[ + \sum_{t=1}^{T} (h_{t}^m H_{t} + f_{t}^m F_{t}) \] (23)

Maximising the possibility of obtaining the lower total costs (Max \( Z_2 = (Z^m - Z^o) \))

\[ = \sum_{n=1}^{N} \sum_{t=1}^{T} [(r_{nt}^m - r_{nt}^o) R_{Q_{nt}} + (o_{nt}^m - o_{nt}^o) O_{Q_{nt}} + (s_{nt}^m - s_{nt}^o) S_{Q_{nt}} \]

\[ + (i_{nt}^m - i_{nt}^o) I_{Q_{nt}} + (b_{nt}^m - b_{nt}^o) B_{Q_{nt}}] \]

\[ + \sum_{t=1}^{T} [(h_{t}^m - h_{t}^o) H_{t} + (f_{t}^m - f_{t}^o) F_{t}] \] (24)

Minimising the risk of obtaining the higher total costs (Min \( Z_3 = (Z^p - Z^m) \))

\[ = \sum_{n=1}^{N} \sum_{t=1}^{T} [(r_{nt}^p - r_{nt}^m) R_{Q_{nt}} + (o_{nt}^p - o_{nt}^m) O_{Q_{nt}} + (s_{nt}^p - s_{nt}^m) S_{Q_{nt}} \]

\[ + (i_{nt}^p - i_{nt}^m) I_{Q_{nt}} + (b_{nt}^p - b_{nt}^m) B_{Q_{nt}}] \]

\[ + \sum_{t=1}^{T} [(h_{t}^p - h_{t}^m) H_{t} + (f_{t}^p - f_{t}^m) F_{t}] \]
\[
\begin{align*}
+ (p_{nt} - i_{nt}^{m})IQ_{nt} + (b_{nt}^{p} - b_{nt}^{m})BQ_{nt} \\
+ \sum_{t=1}^{T} [(h_{t}^{p} - h_{t}^{m})H_{t} + (f_{t}^{p} - f_{t}^{m})F_{t}] 
\end{align*}
\] (25)

**Constraints:**

- **Carrying Inventory Constraint**

\[
\begin{align*}
w_{1}^{p}D_{nt}^{p} + w_{2}^{m}D_{nt}^{m} + w_{3}^{o}D_{nt}^{o} = IQ_{nt-1} - BQ_{nt-1} + RQ_{nt} + OQ_{nt} + SQ_{nt} - IQ_{nt} \\
+ BQ_{nt} \forall N, \forall T 
\end{align*}
\] (26)

The fuzzy customer demand is defuzzified by the weighted average method where \(w_{1}^{p}\), \(w_{2}^{m}\), and \(w_{3}^{o}\) are the assigned weights for customer demand for pessimistic, most likely, and optimistic cases, respectively (in this study: \(w_{1}^{p} = 0.33\), \(w_{2}^{m} = 0.33\), and \(w_{3}^{o} = 0.33\)). As a result, Equation (17) of the LP model is replaced by Equation (26).

- **Labor Level Constraints**

\[
\begin{align*}
\sum_{n=1}^{N} LH_{nt-1}(RQ_{nt-1} + OQ_{nt-1}) + H_{t} - F_{t} - \sum_{n=1}^{N} LH_{nt}(RQ_{nt} + OQ_{nt}) = 0 \forall T 
\end{align*}
\] (27)

\[
\begin{align*}
\sum_{n=1}^{N} LH_{nt}^{p}(RQ_{nt} + OQ_{nt}) \leq L_{nt}^{p} \forall T 
\end{align*}
\] (28)

\[
\begin{align*}
\sum_{n=1}^{N} LH_{nt}^{m}(RQ_{nt} + OQ_{nt}) \leq L_{nt}^{m} \forall T 
\end{align*}
\] (29)

\[
\begin{align*}
\sum_{n=1}^{N} LH_{nt}^{o}(RQ_{nt} + OQ_{nt}) \leq L_{nt}^{o} \forall T 
\end{align*}
\] (30)

- **Machine Capacity Constraint**

\[
\begin{align*}
\sum_{n=1}^{N} MH_{nt}^{p}(RQ_{nt} + OQ_{nt}) \leq M_{nt}^{p} \forall T 
\end{align*}
\] (31)

\[
\begin{align*}
\sum_{n=1}^{N} MH_{nt}^{m}(RQ_{nt} + OQ_{nt}) \leq M_{nt}^{m} \forall T 
\end{align*}
\] (32)

\[
\begin{align*}
\sum_{n=1}^{N} MH_{nt}^{o}(RQ_{nt} + OQ_{nt}) \leq M_{nt}^{o} \forall T 
\end{align*}
\] (33)

The fuzzy labour level and fuzzy machine capacity constraints are defuzzified by the fuzzy ranking method where the labour level and machine capacity constraints (Equations (19) and (20)) are divided into three constraints for the optimistic case, most likely case, and pessimistic case, as shown in Equations (28)–(30) and Equations (31)–(33), respectively.
values of $L_{t_{\text{max}}}^0$, $L_{t_{\text{max}}}^m$, and $L_{t_{\text{max}}}^p$ and $M_{t_{\text{max}}}^0$, $M_{t_{\text{max}}}^m$, and $M_{t_{\text{max}}}^p$ in this study are shown in Table 3.

- Warehouse Capacity Constraint

$$\sum_{n=1}^{N} w_{snt}Q_{nt} \leq W_{S_{t_{\text{max}}}} \quad \forall T \quad (34)$$

$$R_{Qnt}, O_{Qnt}, S_{Qnt}, I_{Qnt}, B_{Qnt}, H_t, F_t \geq 0 \quad \forall N, \forall T \quad (35)$$

**Phase1-MOLP Model**: The MOLP model can be used to find the boundaries of each objective function with the PIS and the NIS that can be formulated as follows:

$$Z_{PIS}^1 = \text{minimize } Z^m, \quad Z_{NIS}^1 = \text{maximize } Z^m$$

$$Z_{PIS}^2 = \text{maximize } Z^m - Z^o, \quad Z_{NIS}^2 = \text{minimize } Z^m - Z^o$$

$$Z_{PIS}^3 = \text{minimize } Z^p - Z^m, \quad Z_{NIS}^3 = \text{maximize } Z^p - Z^m$$

Subject to:

$$v \in F(v) \quad (36)$$

Equations (37)–(39) are the linear membership functions of each objective function that are used to calculate the deviation value of each objective function. Linear membership functions for minimisation of goals and maximisation of goals are given as follows:

$$d_1(x) = \begin{cases} 
1 & \text{if } Z_1 = Z_{NIS}^1 \\
\frac{Z_{PIS}^1 - Z_1}{Z_{NIS}^1 - Z_{PIS}^1} & \text{if } Z_{PIS}^1 \leq Z_1 \leq Z_{NIS}^1 \\
0 & \text{if } Z_1 = Z_{PIS}^1 
\end{cases}$$

$$d_2(x) = \begin{cases} 
1 & \text{if } Z_2 = Z_{NIS}^2 \\
\frac{Z_{PIS}^2 - Z_2}{Z_{NIS}^2 - Z_{PIS}^2} & \text{if } Z_{PIS}^2 \leq Z_2 \leq Z_{NIS}^2 \\
0 & \text{if } Z_2 = Z_{PIS}^2 
\end{cases}$$

$$d_3(x) = \begin{cases} 
1 & \text{if } Z_3 = Z_{NIS}^3 \\
\frac{Z_{PIS}^3 - Z_3}{Z_{NIS}^3 - Z_{PIS}^3} & \text{if } Z_{PIS}^3 \leq Z_3 \leq Z_{NIS}^3 \\
0 & \text{if } Z_3 = Z_{PIS}^3 
\end{cases}$$

**Phase2-FP Model**: Here, minimisation of the maximum deviation is demonstrated to find the minimum value of the highest deviation of objective values, which can guarantee that all deviation levels of the objectives are lower than the deviation level of the highest objective. Being weightless, the minimax algorithm can be expressed as follows:

Minimise the maximum deviation (Minimise $L_{\infty} = d$)

Subject to:

$$x \in F(x)$$
\[ d_1 \leq d \]
\[ d_2 \leq d \]
\[ d_3 \leq d \]  

(40)

5.3.3. Fuzzy Multi-Objective PLP with Beta-Skewness Degree

This proposed model is similar to the fuzzy multi-objective PLP with the defuzzification methods. However, there is no aforementioned defuzzification method in the optimisation algorithm. \( \beta \) is the minimum degree of skewness of the constraints that a decision maker is willing to admit. In fact, \( \beta \) (decision variable) is added to the model to find the best level of data fuzziness to defuzzify each fuzzy constraint. It can help decision makers to handle fuzzy data by using the Beta-Skewness Degree to control the fuzziness of data that is used in the constraints. In this model, Equations (23)–(25) are also used as objective functions and the Beta-Skewness Degree is used to defuzzify the fuzzy constraints of customer demand, labour level, and machine capacity.

Constraints:

- **Carrying Inventory Constraint**

\[
\left[ \left( (1 - \beta_{\text{customer demand}}) \star \frac{D^P_{nt} + D^m_{nt}}{2} \right) + \left( \beta_{\text{customer demand}} \star \frac{D^m_{nt} + D^P_{nt}}{2} \right) \right] = IQ_{nt-1} - BQ_{nt-1} + RQ_{nt} + OQ_{nt} + SQ_{nt} - IQ_{nt} + BQ_{nt} \quad \forall N, \forall T
\]  

(41)

Fuzzy customer demand is defuzzified by using the beta-skewness degree where \( \beta_{\text{customer demand}} \) is a decision variable that is used to control the fuzziness of the customer demand. As a result, Equation (17) of the LP model is replaced by Equation (41).

- **Labor Level Constraints**

\[
\sum_{n=1}^{N} L_{nt-1} (RQ_{nt-1} + OQ_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} L_{nt} (RQ_{nt} + OQ_{nt}) = 0 \forall T
\]  

(42)

\[
\sum_{n=1}^{N} \left[ \left( (1 - \beta_{\text{labor level}}) \star \frac{L^P_{nt} + L^m_{nt}}{2} \right) + \left( \beta_{\text{labor level}} \star \frac{L^m_{nt} + L^P_{nt}}{2} \right) \right] (RQ_{nt} + OQ_{nt}) \leq \left[ \left( (1 - \beta_{\text{labor level}}) \star \frac{L_{tmax}^P + L_{tmax}^m}{2} \right) + \left( \beta_{\text{labor level}} \star \frac{L_{tmax}^m + L_{tmax}^P}{2} \right) \right] \quad \forall T
\]  

(43)

- **Machine Capacity Constraint**

\[
\sum_{n=1}^{N} \left[ \left( (1 - \beta_{\text{machine capacity}}) \star \frac{MH^P_{nt} + MH^m_{nt}}{2} \right) \right. \\
+ \left. \left( \beta_{\text{machine capacity}} \star \frac{MH^m_{nt} + MH^P_{nt}}{2} \right) \right] (RQ_{nt} + OQ_{nt}) \leq \left[ \left( (1 - \beta_{\text{machine capacity}}) \star \frac{M_{tmax}^P + M_{tmax}^m}{2} \right) \right.
\]  

(44)
\[
+ \left( \beta_{\text{machine capacity}} \cdot \frac{M_{\text{max}}^m + M_{\text{max}}^c}{2} \right) \quad \forall T
\] (44)

The fuzzy labour level and fuzzy machine capacity constraints are defuzzified by using the beta-skewness degree where the fuzziness of the labour level and machine capacity (Equations (21) and (22)) is controlled by \( \beta_{\text{labor level}} \) and \( \beta_{\text{machine capacity}} \), respectively. The labour level and machine capacity constraints are replaced by Equations (43) and (44), respectively.

- Warehouse Capacity Constraint

\[
\sum_{n=1}^{N} w_{nt} I_{nt} \leq W_{\text{Stmax}} \quad \forall T
\] (45)

\[
R_{nt}, Q_{nt}, S_{nt}, I_{nt}, B_{nt}, H_{t}, F_{t} \geq 0 \quad \forall N, \forall T
\] (46)

**Phase1-MOLP model:** Equations (47)–(49) are used to find the deviation from the ideal solution (which is the PIS) of each objective, similar to Phase 1 of fuzzy multi-objective PLP with the defuzzification method. However, Equations (50)–(52) are added to this model to control the deviation of each \( \beta \)-skewness degree of each fuzzy constraint.

\[
d_1(x) = \begin{cases} 
1 & \text{if } Z_1 = Z_1^{\text{PIS}} \\
\frac{Z_1^{\text{PIS}} - Z_1}{Z_1^{\text{NIS}} - Z_1^{\text{PIS}}} & \text{if } Z_1^{\text{PIS}} \leq Z_1 \leq Z_1^{\text{NIS}} \\
0 & \text{if } Z_1 = Z_1^{\text{NIS}} 
\end{cases}
\] (47)

\[
d_2(x) = \begin{cases} 
1 & \text{if } Z_2 = Z_2^{\text{PIS}} \\
\frac{Z_2^{\text{PIS}} - Z_2}{Z_2^{\text{NIS}} - Z_2^{\text{PIS}}} & \text{if } Z_2^{\text{PIS}} \leq Z_2 \leq Z_2^{\text{NIS}} \\
0 & \text{if } Z_2 = Z_2^{\text{NIS}} 
\end{cases}
\] (48)

\[
d_3(x) = \begin{cases} 
1 & \text{if } Z_3 = Z_3^{\text{PIS}} \\
\frac{Z_3^{\text{PIS}} - Z_3}{Z_3^{\text{NIS}} - Z_3^{\text{PIS}}} & \text{if } Z_3^{\text{NIS}} \leq Z_3 \leq Z_3^{\text{PIS}} \\
0 & \text{if } Z_3 = Z_3^{\text{NIS}} 
\end{cases}
\] (49)

\[
d_4(x) = \frac{1 - \beta_{\text{customer demand}}}{1 - \text{specified } \beta_{\text{customer demand}}}
\] (50)

\[
d_5(x) = \frac{1 - \beta_{\text{labor level}}}{1 - \text{specified } \beta_{\text{labor level}}}
\] (51)

\[
d_6(x) = \frac{1 - \beta_{\text{machine capacity}}}{1 - \text{specified } \beta_{\text{machine capacity}}}
\] (52)

where the specified \( \beta_{\text{customer demand}} \), \( \beta_{\text{labor level}} \), and \( \beta_{\text{machine capacity}} \) can be determined by decision makers based on their preferences, to limit the range of the degree of skewness of each constraint that the decision makers are willing to admit. In this case, specified \( \beta_{\text{customer demand}} \), specified \( \beta_{\text{labor level}} \), and specified \( \beta_{\text{machine capacity}} \) are equally set at 0.25.
Phase2-FP (FP) model: It is generated similarly to the previous model of fuzzy multi-objective PLP with the defuzzification methods, as seen in Equation (40). However, additional constraints of the linear membership function of the Beta-Skewness Degree (Equation (53)) are added to the model as follows:

Minimise the maximum deviation (Minimise $L_\infty = d$)

Subject to:

\[
\begin{align*}
    x &\in F(x) \\
    d_1 &\leq d \\
    d_2 &\leq d \\
    d_3 &\leq d \\
    d_4 &\leq d \\
    d_5 &\leq d \\
    d_6 &\leq d
\end{align*}
\]  
\tag{53}

6. Results

In this part, the results of each optimisation methodologies: (1) LP, (2) Fuzzy multi-objective PLP with the defuzzification methods (weighted average and fuzzy ranking method), and (3) Fuzzy multi-objective PLP with Beta-Skewness Degree, are presented and compared to demonstrated the effectiveness and advantage of the proposed approach that outperform the exist approaches.

6.1. Linear Programming

LP is an optimisation approach that can be used to find an ideal optimal solution of the APP model with no uncertainty and the obtained LP solution is used as a benchmark for comparison in this study. The ideal optimal solution of LP is expressed in Table 4.

Based on Table 4, the overall deviation ($d$) of LP is the minimum value of the maximum deviation of the objective functions. This is equal to 0% since there is no uncertainty in the model and the minimum total costs ($Z_1$) is $291,401 while there are no lower and higher total costs in this deterministic model.

| Table 4. The ideal optimal solution of LP. |
|--------------------------------------------|
| Overall deviation ($d$)                     | 0%                   |
| Minimum possible value of the lower total costs ($Z_2 = Z^m - Z^o$) | –                   |
| Minimum possible value of the most likely total costs ($Z_1 = Z^m$) | $291,401$ |
| Minimum possible value of the higher total costs ($Z_3 = Z^p - Z^m$) | –                   |
| Deviation of the minimum most likely total costs from its PIS ($d_1$) | 0%                   |
| Deviation of the minimum lower total costs from its PIS ($d_2$) | 0%                   |
| Deviation of the minimum higher total costs from its PIS ($d_3$) | 0%                   |
6.2. Fuzzy Multi-Objective PLP with Defuzzification Methods

In this model, PLP is used to identify the possible range of the fuzzy operating costs in each scenario. The customer demand is defuzzified by the weighted average method (see Equation (26)) and labour level and machine capacity are defuzzified by the fuzzy ranking method (see Equations (28)–(30)). The optimal solution of fuzzy multi-objective PLP with the defuzzification methods is shown in Table 5.

Based on the results obtained from fuzzy multi-objective PLP with the defuzzification methods (shown in Table 5), the overall deviation \( d \) is 4.85%. This is higher than the overall deviation from LP (0%) as it handles the fuzziness of data. Referring to the aforementioned percentages of the deviation of each objective function and constraint, the minimum value of the most likely total costs \( Z_1 \) is $299,340, the maximum value of the lower total costs \( Z_2 \) is $45,990, and the minimum value of the higher total costs \( Z_3 \) is $28,350. As a result, decision makers can know their possible range of total costs from each scenario. However, it is found that this method requires the defuzzification of all fuzzy data prior to the optimisation process, as it requires constant data in the calculation. As a result, the fuzziness of data would disappear earlier in the process.

6.3. Fuzzy Multi-Objective PLP with Beta-Skewness Degree

To maintain and incorporate the fuzziness of data in the process, the fuzzy multi-objective PLP with the beta-skewness degree is proposed to replace the defuzzification method. The Beta-Skewness Degree is used as a decision variable to judge the best level of data fuzziness to be defuzzified for each fuzzy constraint while maintaining the same objective functions. Its results are shown in Table 6.

All fuzzy data; customer demand, labour level, and machine capacity, are not defuzzified earlier with the defuzzification methods by the decision makers. The Beta-Skewness Degree is used as a decision variable to handle and find the best level of fuzziness to be defuzzified for each fuzzy constraint. It is found that the results outperform the methods with defuzzification by the decision makers. While the overall deviation is reduced to 1.2%, all costs from all three scenarios can be reduced by about 6.13%–7.3%, as compared to the model with the defuzzification methods. In this instance, all three values of \( \beta \) are set to about 0.72–0.78. They tend to push the fuzzy customer demand, labour level, and machine capacity to yield the best solution, toward the lower values (dash line areas as seen in Figure 1).

Table 5. The optimal solution of fuzzy multi-objective PLP with the defuzzification methods.

|                        | Fuzzy multi-objective PLP with the defuzzification methods |
|------------------------|------------------------------------------------------------|
| Overall deviation \( d_j \) | 4.85%                                                      |
| Minimum possible value of the lower total costs \( Z_2 = Z^m - Z^o = $45,990 \) | $253,350                                                   |
| Minimum possible value of the most likely total costs \( Z_1 = Z^m \) | $299,340                                                   |
| Minimum possible value of the higher total costs \( Z_3 = Z^p - Z^m = $28,350 \) | $327,690                                                   |
| Deviation of the minimum most likely total costs from its PIS \( d_1 \) | 4.85%                                                      |
| Deviation of the minimum lower total costs from its PIS \( d_2 \) | 4.85%                                                      |
| Deviation of the minimum higher total costs from its PIS \( d_3 \) | 2.75%                                                      |
Table 6. The optimal solution of fuzzy multi-objective PLP with Beta-Skewness Degree.

| Fuzzy multi-objective PLP with Beta-Skewness Degree | Overall deviation \((d)\) | Overall deviation \((d)\) | Overall deviation \((d)\) | Overall deviation \((d)\) | Overall deviation \((d)\) |
|---------------------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| Overall deviation \((d)\)                        | 1.2%                     | 7.3% lower than the previous model with the defuzzification methods | 6.5% lower than the previous model with the defuzzification methods | 6.13% lower than the previous model with the defuzzification methods |
| Minimum possible value of the lower total operating costs | $234,840 \((Z_2 = Z^m - Z^o = $45,130)\) | Minimum possible value of the lower total operating costs | $279,830 \((Z_1 = Z^m)\) | Minimum possible value of the lower total operating costs | $307,600 \((Z_3 = Z^p - Z^m = $27,770)\) |
| Deviation of the minimum most likely total operating costs from its PIS \((d_1)\) | 1.1%                     | Deviation of the minimum lower total operating costs from its PIS \((d_2)\) | 1.2%                     | Deviation of the minimum higher total operating costs from its PIS \((d_3)\) | 0.93%                     |
| Deviation of beta-skewness degree of customer demand from its PIS \((d_4)\) | 0.33%                    | Deviation of beta-skewness degree of labour level from its PIS \((d_5)\) | 0.35%                    | Deviation of beta-skewness degree of machine capacity from its PIS \((d_6)\) | 0.38%                    |
| Selected decision variables                       | 0.78                     | 0.75                     | 0.72                     | 0.72                     |

6.4. Result Comparison

The outcomes in Table 7 can be used to highlight the benefits of the defuzzification process with the Beta-Skewness Degree. It is found that when all fuzzy constraints are defuzzified earlier in the process by the decision makers’ preference, the best obtained result deviates from the ideal solution by up to 4.9%. It is likely that the total costs of the plan are higher than the optimal total costs solved by LP under the deterministic data, which depends on the weights assigned by the decision makers as these weights could favour optimistic or pessimistic conditions. However, when one fuzzy dataset is not defuzzified and searched for the best level of fuzziness by the Beta-Skewness Degree, the overall deviation of the plan gradually improves from 3.4% with one beta-skewness degree to 1.2% with all three Beta-Skewness Degree. The total costs of the plan from all 3 scenarios also decreases gradually as more constraints are not defuzzified early, but searched for their best levels of fuzziness to be defuzzified. This highlights the advantages of maintaining the fuzziness of data in the optimisation algorithm. Otherwise, the defuzzification process would turn them into constant and deterministic values by the decision makers’ preference. The benefits of having flexibility in manipulating the vagueness of data and finding their optimal levels would disappear in the process.

Based on Table 8, to satisfy the customer demand, it is found that all methods prefer to use subcontracting rather than overtime hiring due to the current cost structure. Under the deterministic model with LP, while all data are constant (most likely values are used in the calculation), more workers are hired to produce the products in-house as the number of
Table 7. Comparing results.

| Method                                                                 | Overall deviation \((d_j)\) | Minimum possible value of the lower total operating costs \((Z_2 = Z^m - Z^o)\) | Minimum possible value of the most likely total operating costs \((Z_1 = Z^m)\) | Minimum possible value of the higher total operating costs \((Z_3 = Z^p - Z^m)\) |
|------------------------------------------------------------------------|-----------------------------|---------------------------------------------------------------------------------|---------------------------------------------------------------------------------|---------------------------------------------------------------------------------|
| Linear Programming (LP)                                                 | 0%                         | –                                                                               | $291,401                                                                        | –                                                                               |
| Fuzzy multi-objective PLP with the defuzzification methods              | 4.9%                       | $253,350 ($45,990)                                                              | $299,340                                                                        | $327,690 ($28,350)                                                            |
| Fuzzy multi-objective PLP with the beta-skewness degree (not defuzzified fuzzy customer demand) | 3.4%                       | $235,930 ($45,180)                                                              | $281,110                                                                        | $308,980 ($27,870)                                                            |
| Fuzzy multi-objective PLP with the beta-skewness degree (not defuzzified fuzzy customer demand and labour level) | 2.3%                       | $235,280 ($45,070)                                                              | $280,350                                                                        | $308,140 ($27,790)                                                            |
| Fuzzy multi-objective PLP with the beta-skewness degree (not defuzzified fuzzy customer demand, labour level, and machine capacity) | 1.2%                       | $234,840 ($45,130)                                                              | $279,830                                                                        | $307,600 ($27,770)                                                            |

Table 8. Analysis of total costs from each optimisation and defuzzification method.

|                          | Linear Programming (LP) | Fuzzy multi-objective PLP with the defuzzification methods | Fuzzy multi-objective PLP with the \(\beta\)-skewness degree (not defuzzified fuzzy customer demand, labour level, and machine capacity) |
|--------------------------|-------------------------|------------------------------------------------------------|-------------------------------------------------------------------------------|
|                          | M                       | O               | M               | P               | O               | M               | P               |
| Costs related to product |                         |                 |                 |                 |                 |                 |                 |
| Regular time production cost \((\tilde{r}_{nt})\)                    | $281,650                | $179,562        | $212,689        | $234,322        | $223,729        | $266,590        | $293,249        |
| Overtime production cost \((\tilde{b}_{nt})\)                       | $0                      | $0              | $0              | $0              | $0              | $0              | $0              |
| Subcontracting cost \((\tilde{c}_{nt})\)                           | $7,625                  | $73,133         | $85,873         | $92,467         | $9,686          | $11,619         | $12,587         |
| Backordering cost \((\tilde{b}_{nt})\)                             | $0                      | $0              | $0              | $0              | $0              | $0              | $0              |
| Inventory cost \((\tilde{h}_{in})\)                               | $1,761                  | $389            | $445            | $475            | $1,286          | $1,446          | $1,544          |
| Costs related to worker                                           |                         |                 |                 |                 |                 |                 |                 |
| Hiring cost \((\tilde{h}_{i})\)                                    | $220                    | $0              | $0              | $0              | $0              | $0              | $0              |
| Firing cost \((\tilde{f}_{i})\)                                    | $145                    | $266            | $333            | $426            | $139            | $175            | $220            |
| Total costs                                                         | $291,401                | $253,350        | $299,340        | $327,690        | $234,840        | $279,830        | $307,600        |

Workers still do not reach the maximum labour level. When the customer demand, labour level, and machine capacity are uncertain and defuzzified by the defuzzification methods; weighted average and fuzzy ranking, with the multi-objective PLP, more sub-contracting is recommended as the maximum labour levels and machine capacity have already reached the maximum levels. The result also shows that the most likely total costs are slightly higher than those under the deterministic conditions, as the algorithm tries to balance the achievements from all three objectives. With the proposed fuzzy multi-objective PLP with the beta-skewness degree, the model suggests the best levels of fuzziness to be defuzzified.
The best levels are toward the left side (with lower customer demand, labour level, and machine capacity). More in-house production can be done with less sub-contracting and firing than those recommended by the defuzzification methods; weighted average and fuzzy ranking. This results in the lowest total costs of all 3 scenarios.

7. Managerial Implications

In general, the approach introduced in this study provides the following advantages in respect to other two exist approaches as below.

7.1. As Compared to the Deterministic LP Model

The decision-makers can optimise and realise the outcomes (total costs in this study) in all three possible situations (optimistic, most likely, and pessimistic). Under the obtained decision variables, the company can also be well aware of their required workforce level, inventory on hand and maximum machine capacity required under all three scenarios so that the company can prepare and take necessary budgeting and financial actions for future uncertainty.

7.2. As Compared to the FP with Defuzzification Methods; Weighted Average and Fuzzy Ranking, Model

The proposed approach avoids defuzzifying the fuzzy data earlier in the process but searches for the best level of fuzziness by the Beta-Skewness Degree of each fuzzy constraint. This is proven to improve the outcomes of all 3 scenarios as well as bring the result under the uncertainties closer to an ideal optimal result than those of LP under no uncertainty and the FP method with the defuzzification methods; weighted average and fuzzy ranking. Even though, one may argue that the level of fuzziness of some constraint such as customer demand cannot be controlled by the company or those of maximum levels of labour and machine capacity are difficult to control, having known their optimal levels definitely helps the company to put its attempts to run its operations more towards the obtained $\beta$ values. For instance, if the optimal level of fuzziness of the customer demand is relatively on the high side of the demand level. The company can perhaps put its effort to its sales team to work harder or spends more promotion or advertising budgets to increase its amount of sales and vice versa. Then, the justification between higher spending and benefits gained can be evaluated for its worthiness.

8. Conclusion

APP is intermediate-term planning that decision makers have to decide the appropriate levels of capacity, production, subcontracting, inventory, stock outs, and pricing to meet fluctuating forecasted demand. In many practical applications, the parameters in APP problem may not be known precisely. This study was then carried out to help decision makers to prepare optimal production strategies for possible uncertain business situations.
To deal with the fuzziness embodied in the APP problem, an improved FP approach; integrating fuzzy multi-objective PLP with the Beta-Skewness Degree, was proposed. The proposed method softened the rigidity, making fuzzier the perception based on defuzzifying the uncertainties using the Beta-Skewness Degree. Its computational performances and application were demonstrated through an APP case study under imprecise customer demand, labour level, and machine capacity while attempting to minimise the highest overall deviation from the ideal solution with all three scenarios (most likely case, optimistic case and pessimistic case).

The numerical example indicated superior outcomes, compared with LP and FP with traditional defuzzification methods; weighted average and fuzzy ranking. The proposed approach avoids defuzzifying the fuzzy data earlier in the process but searches for the best level of fuzziness by the Beta-Skewness Degree of each fuzzy constraint. Even though, not all level of fuzziness of data can be controlled but having known their optimal levels definitely helps decision-makers to put their attempts to run and prepare their businesses better for unforeseeable situations.

The proposed approach still has some limitations that can be researched and developed further. Firstly, our numerical model only focus on minimising the overall deviation from the ideal solution of the minimum total costs. However, it can be further developed to the multi-objective model with the capability of compromising the deviation or satisfaction of other kinds of the objectives based on the business situations and requirements. Secondly, the proposed approach can be applied to other APP models, which allow more imprecise parameters, as they may affect the results of planning. Thirdly, the fuzzy numbers that were used in our numerical model were only represented by the triangular distribution. Other forms of distribution could also be applied. Lastly, the proposed approach can be developed further by applying with other fuzzy programming methods such as the weighted additive method or the weighted max–min method in which the overall preference of decision makers can be input in the model as the relative importance of criteria. As a result, the objectives and constraints may not be equally important and can be assigned with different weights.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

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