COMPACTNESS OF $\Box_b$ ON A CR MANIFOLD

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ABSTRACT. This note is aimed at simplifying current literature about compactness estimates for the Kohn-Laplacian on CR manifolds. The approach consists in a tangential basic estimate in the formulation given by the first author in [Kh10] which refines former work by Nicoara [N06]. It has been proved by Raich [R10] that on a CR manifold of dimension $2n-1$ which is compact pseudoconvex of hypersurface type embedded in $\mathbb{C}^n$ and orientable, the property named “$(CR-P_q)$” for $1 \leq q \leq \frac{n-1}{2}$, a generalization of the one introduced by Catlin in [CS], implies compactness estimates for the Kohn-Laplacian $\Box_b$ in degree $k$ for any $k$ satisfying $q \leq k \leq n-1-q$. The same result is stated by Straube in [S10] without the assumption of orientability. We regain these results by a simplified method and extend the conclusions in two directions. First, the CR manifold is no longer required to be embedded. Second, when $(CR-P_q)$ holds for $q=1$ and, in case $n=1$, under the additional hypothesis that $\bar{\partial}_b$ has closed range on functions) we prove compactness also in the critical degrees $k=0$ and $k=n-1$.

MSC: 32F10, 32F20, 32N15, 32T25

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1. INTRODUCTION AND STATEMENTS

Let $M$ be a compact pseudoconvex CR manifold of hypersurface type of real dimension $2n-1$ endowed with the Cauchy-Riemann structure $T^{1,0}M$. We choose a basis $L_1, ..., L_{n-1}$ of $T^{1,0}M$, the conjugated basis $\bar{L}_1, ..., \bar{L}_{n-1}$ of $T^{0,1}M$, and a transversal, purely imaginary, vector field $T$. We also take a hermitian metric on the complexified tangent bundle in which we get an orthogonal decomposition $\mathbb{C}TM = T^{1,0}M \oplus T^{0,1}M \oplus \mathbb{C}T$. We denote by $\omega_1, ..., \omega_{n-1}, \bar{\omega}_1, ..., \bar{\omega}_{n-1}, \gamma$ the dual basis of 1-forms. We denote by $\mathcal{L}_M$ the Levi form defined by $\mathcal{L}_M(L, \bar{L}') := d\gamma(L, \bar{L}')$ for $L, L' \in T^{1,0}M$. The coefficients of the matrix $(c_{ij})$ of $\mathcal{L}_M$ in the above basis are described through Cartan formula as

$$c_{ij} = \langle \gamma, [L_i, \bar{L}_j] \rangle.$$
We denote by $\mathcal{B}^k$ the space of $(0, k)$-forms $u$ with $C^\infty$ coefficients; they are expressed, in the local basis, as $u = \sum_{|J|=k} u_J \omega_J$ for $\omega_J = \omega_{j_1} \wedge \cdots \wedge \omega_{j_k}$. Associated to the Riemannian metric $(\cdot, \cdot)_z$, $z \in M$ and to the element of volume $dV$, there is a $L^2$-inner product $(u, v) = \int_M (u, v)_z dV$. We denote by $(L^2)^k$ the completion of $\mathcal{B}^k$ under this norm; we also use the notation $(H^s)^k$ for the completion under the Sobolev norm $H^s$. Over the spaces $\mathcal{B}^k$ there is induced by the de-Rham exterior derivative a complex $\bar{\partial}_b : \mathcal{B}^k \to \mathcal{B}^{k+1}$. We denote by $\bar{\partial}_b^* : \mathcal{B}^k \to \mathcal{B}^{k-1}$ the adjoint and set $\square_b = \bar{\partial}_b \partial_b^* + \partial_b^* \bar{\partial}_b$. Let $\varphi$ be a smooth function and denote by $(\varphi_{ij})$ the matrix of the Levi form $\mathcal{L}_\varphi = \frac{1}{2}(\partial_b \partial_b^* - \partial_b^* \partial_b)(\varphi)$ in the basis above. Let $L^2_\varphi$ be the $L^2$ space weighted by $e^{-\varphi}$ and denote by $L^2_{\varphi,J} = L_j - \varphi_j$, for $\varphi_j := L_j(\varphi)$, the $L^2_{\varphi,J}$-adjoint of $-\bar{L}_j$. The following is the tangential version of the celebrated Hörmander-Kohn-Morrey basic estimate. We present here the refinement by Khanh [Kh10] of a former statement by Nicoara [N06]. Le $z_0 \in M$; for a suitable neighborhood $U$ of $z_0$ and a constant $c > 0$, we have
\[
\|\bar{\partial}_b u\|_{\varphi}^2 + \|\partial_b^* u\|_{\varphi}^2 + c\|u\|_{\varphi}^2 \\
\geq \sum_{|K| = k-1}^t \sum_{ij} (\varphi_{ij} u_{iK}, u_{jK})_\varphi - \sum_{|J|=k}^q \sum_{j=1} (\varphi_{jj} u_J, u_J)_\varphi \\
+ \sum_{|K|=k-1}^t \sum_{ij} (c_{ij} Tu_{iK}, u_{jK})_\varphi - \sum_{|J|=k}^q \sum_{j=1} (c_{jj} Tu_J, u_J)_\varphi \\
+ \frac{1}{2} \left( \sum_{j=1}^q \|L^2_{\varphi,J} u\|_{\varphi}^2 + \sum_{j=q_{b-1}+1}^{n-1} \|\bar{L}_j u\|_{\varphi}^2 \right),
\] for any $u \in \mathcal{B}^k_c(U)$ where $q_o$ is any integer with $0 \leq q_o \leq n - 1$. We introduce now a potential-theoretical condition which is a variant of the “P-property” by Catlin [C84]. In the present version it has been introduced by Raich [R10] and Straube [S10].

**Definition 1.1.** Let $z_o$ be a point of $M$ and $q$ an index in the range $1 \leq q \leq n - 1$. We say that $M$ satisfies property $(CR - P_q)$ at $z_o$ if there is a family of weights $\{\varphi^q\}$ in a neighborhood $U$ of $z_o$ such that, if $\lambda_{1}^q \leq \cdots \leq \lambda_{n-1}^q$ are the ordered eigenvalues of the Levi form $\mathcal{L}_\varphi^q$, we have
\[
(1.2) \begin{cases}
|\varphi^q(z)| \leq 1, & z \in U \\
\sum_{j=1}^q \lambda_j^q(z) \geq \epsilon^{-1}, & z \in U \text{ and } \ker \mathcal{L}_M(z) \neq \{0\}.
\end{cases}
\]

It is obvious that $(CR - P_q)$ implies $(CR - P_k)$ for any $k \geq q$.

**Remark 1.2.** We restrict our considerations to the unit sphere. Outside a neighborhood $V_c$ of $\ker d\gamma$, the sum $\sum_{j=1}^q \lambda_j^q$ can get negative; let $-b_c$ be a bound from below. Now, if $c_\epsilon$
is a bound from below for $d\gamma$ outside $V_\epsilon$, by setting $a_\epsilon := \frac{c^{-1} + b_\epsilon}{qc_\epsilon}$, we have,

$$\sum_{j \leq q} \lambda_j^\epsilon + qa_\epsilon c_\epsilon \geq \epsilon^{-1}$$
onumber

on the whole $U$.

Again, (1.3) for $q$ implies (1.3) for any $k \geq q$.

Related to this notion there is the main result of the paper

**Theorem 1.3.** Let $M$ be a compact pseudoconvex CR manifold of hypersurface type of dimension $2n-1$. Assume that property $(CR - P_q)$ holds for $1 \leq q \leq \frac{n-1}{2}$ over a covering $\{U\}$ of $M$. Then we have compactness estimates

$$\|u\|^2 \leq \epsilon(\|\bar{\partial}_b u\|^2 + \|\bar{\partial}_b^* u\|^2) + C_\epsilon \|u\|_{-1}^2$$

for any $u \in D_k^\epsilon \cap \bar{\partial}_b^k$ with $q \leq k \leq n-1 - q$,

where $D_k^\epsilon$ and $\bar{\partial}_b^k$ are the domains of $\bar{\partial}_b^\epsilon$ and $\bar{\partial}_b$ respectively. The proof of this, as well as of the theorem which follows, is given in Section 2. Let $H_k = \ker \bar{\partial}_b \cap \ker \bar{\partial}_b^*$ be the space of harmonic forms of degree $k$. As a consequence of (1.4), we have that for $q \leq k \leq n-1 - q$, the space $H_k$ is finite-dimensional, $\bar{\partial}_b$ is invertible over $H_k$ (cf. [N06] Lemma 5.3) and its inverse $G_k$ is a compact operator. When $k = 0$ and $k = n-1$ it is no longer true that it is finite-dimensional. However, if $q = 1$, we have a result analogous to (1.4) also in the critical degrees $k = 0$ and $k = n-1$.

**Theorem 1.4.** Let $M$ be a compact, pseudoconvex CR manifold of hypersurface type of dimension $2n-1$. Assume that property $(CR - P_q)$ holds for $q = 1$ over a covering $\{U\}$ of $M$ and, in case $n = 2$, make the additional hypothesis that $\bar{\partial}_b$ has closed range. Then

$$\|u\|^2 \leq \epsilon(\|\bar{\partial}_b u\|^2 + \|\bar{\partial}_b^* u\|^2) + C_\epsilon \|u\|_{-1}^2$$

for any $u \in H_k^\perp$, $k = 0$ and $k = n-1$.

In particular, $G_k$ is compact for $k = 0$ and $k = n-1$.

2. Proofs

**Proof of Theorem 1.3.** We choose a local patch where a local frame of vector fields is found in which (1.1) is fulfilled. The key point is to specify the convenient choices of $q_o$ and $\varphi$ in (1.1). Let $1 = \psi^+ + \psi^- + \psi^0$ be a conic, smooth partition of the unity in $\mathbb{R}^{2n-1}$ dual to the space to which $U$ is identified in local coordinates. Let id = $\Psi^+ + \Psi^- + \Psi^0$ be the microlocal decomposition of the identity by the pseudodifferential operators with symbols $\psi^\pm$ and let $\zeta$ be a cut-off function. We decompose a form $u$ as

$$u^\pm = \zeta \Psi^\pm u \quad u \in B^k_c(U), \quad \zeta_{\text{supp } u} \equiv 1.$$  

For $u^+$ we choose $q_o = 0$ and $\varphi = \varphi^\epsilon$. We also need to go back to Remark 1.2. Now, if $a_\epsilon$ has been chosen so that (1.3) is fulfilled, we remove $T$ from our scalar products observing
that, for large $\xi$, we have $\xi_{2n+1} > a_\epsilon$ over $\text{supp } \psi^+$. In the same way as in Lemma 4.12 of [N06], we conclude that for $k \geq q$

$$
\sum_{|K|=k-1}^\prime \sum_{i,j=1}^{n-1} ((c_{ij} T + \phi_{ij}^e) u_{iK}^+, u_{jK}^+)_{\varphi^e} \geq \sum_{|K|=k-1}^\prime \sum_{i,j=1}^{n-1} ((a_{ij} c_{ij} + \phi_{ij}^e) u_{iK}^+, u_{jK}^+)_{\varphi^e} - C\|u^+\|_{\varphi^e}^2 - C_\epsilon \|\tilde{\Psi}^0 u^+\|_{\varphi^e}^2 - C_\epsilon \|\tilde{\Psi}^0 u^+\|_{\varphi^e}^2,
$$

where $\tilde{\Psi}^0 > \Psi^0$ in the sense that $\tilde{\Psi}^0|_{\text{supp } \psi^0} \equiv 1$. Note that there is here an additional term $-C_\epsilon \|u^+\|_{\varphi^e}^2$ with respect to [N06]. Reason is that $(c_{ij} \xi_{2n-1} + \phi_{ij}^e)$ can get negative values, even on $\text{supp } \psi^+$, when $\xi_{2n-1} < a_\epsilon$. Integration in this compact region, produces the above error term. It follows

$$
(2.2) \quad \|u^+\|_{\varphi^e}^2 \leq \epsilon (\|\tilde{\partial}_b u^+\|_{\varphi^e}^2 + \|\tilde{\partial}_b^* u^+\|_{\varphi^e}^2) + C_\epsilon \|u^+\|_{\varphi^e}^2 + C_\epsilon \|\tilde{\Psi}^0 u^+\|_{\varphi^e}^2,
$$

where $\kappa_{ij}$ is the Kronecker symbol. We also notice that

$$
|\tilde{\partial}_{b,\chi}(\varphi^e)|^2 \leq 2|\tilde{\partial}_b^e|^2 + 2\chi^2 \sum_{|K|=k-1}^\prime \sum_{j=1}^{n-1} \varphi_j^e u_{jK}^2.
$$

Remember that $\{\varphi^e\}$ are uniformly bounded by 1. Thus, if we choose $\chi = \epsilon^{(t-1)}$, we have that $\tilde{\chi} \geq 2\chi^2$ for $t = \frac{1}{2} \varphi^e$. For this reason, with this modified weight, we can replace the weighted adjoint $\tilde{\partial}_{b,\varphi^e}$ by the unweighted $\tilde{\partial}_b^e$ in (2.2). By the uniform boundedness of the weights, we can also remove them from the norms and end up with the estimate

$$
(2.3) \quad \epsilon^{-1} \|u^+\|_{\varphi^e}^2 \leq \|\tilde{\partial}_b^e u^+\|_{\varphi^e}^2 + \|\tilde{\partial}_b u^+\|_{\varphi^e}^2 + C_\epsilon \|\tilde{\Psi}^0 u^+\|_{\varphi^e}^2,
$$

for $u^-$, we choose $q_0 = n-1$ and $\varphi = -\varphi^e$. Observe that for $|\xi|$ large we have $-\xi_{2n-1} \geq a_\epsilon$ over $\text{supp } \psi^-$ (cf. [N06] Lemma 4.13); thus, we have in the current case, for $k \leq n-1 - q$

$$
\sum_{|K|=k-1}^\prime \sum_{i,j=1}^{n-1} ((c_{ij} T - \varphi_{ij}^e) u_{iK}^-, u_{jK}^-)_{\varphi^e} - \sum_{|J|=k}^\prime \sum_{j=1}^{n-1} ((c_{jj} T - \varphi_{jj}^e) u_{jK}^-, u_{jK}^-)_{\varphi^e}
$$

$$
\geq - \sum_{|K|=k-1}^\prime \sum_{i,j=1}^{n-1} ((a_{ij} c_{ij} + \varphi_{ij}^e) u_{iK}^-, u_{jK}^-)_{\varphi^e} - C\|u^-\|_{\varphi^e}^2 - C_\epsilon \|u^-\|_{\varphi^e}^2 - C_\epsilon \|\tilde{\Psi}^0 u^-\|_{\varphi^e}^2,
$$

$$
\geq \epsilon^{-1} \|u^-\|_{\varphi^e}^2 - C\|u^-\|_{\varphi^e}^2 - C_\epsilon \|u^-\|_{\varphi^e}^2 - C_\epsilon \|\tilde{\Psi}^0 u^-\|_{\varphi^e}^2.
$$
Thus, we get the analogous of (2.2) for $u^+$ replaced by $u^-$ and, removing again the weight from the adjoint $\bar{\partial}_b^{\ast}$ and from the norms, we conclude
\begin{equation}
(2.4) \quad \|u^\pm\|^2 \leq \epsilon(\|\bar{\partial}_b u^\pm\|^2 + \|\bar{\partial}_b^\ast u^\pm\|^2) + C_\epsilon\|u^\pm\|^2_{-1,\nu} + C_\epsilon\|\tilde{\Psi}^0 u\|^2, \quad k = 0, \ldots, n - 1 - q.
\end{equation}
In addition to (2.3) and (2.4), we have elliptic estimates for $u^0$
\begin{equation}
(2.5) \quad \|u^0\|^2_1 < \|\bar{\partial} u^0\|^2 + \|\bar{\partial}_b^\ast u^0\|^2 + \|u\|^2_{-1}.
\end{equation}
We put together (2.3), (2.4) and (2.5) and notice that
\begin{equation}
(2.6) \quad \|\bar{\partial}_b(\zeta \Psi^0 u)\|^2 \leq \|\zeta \Psi^0 \bar{\partial}_b u\|^2 + \|\bar{\partial}_b, \zeta \Psi^0 u\|^2 \\
\quad \leq \|\Psi^0 \bar{\partial}_b u\|^2 + \|\zeta \Psi^0 u\|^2 + \|\tilde{\Psi}^0 u\|^2,
\end{equation}
for $\zeta \succ \zeta$ and $\tilde{\Psi}^0 \succ \Psi^0$. The similar estimate holds for $\bar{\partial}_b$ replaced by $\bar{\partial}_b^\ast$. Since $\zeta_{\text{supp} u} = 1$, then
\begin{equation}
\|u\|^2 \leq \sum_{+, -} \|\zeta \Psi^0 u\|^2 + Op^{-\infty}(u) \\
\quad \leq \epsilon \sum_{+, -} (\|\bar{\partial}_b u\|^2 + \|\bar{\partial}_b^\ast u\|^2) + C_\epsilon\|u\|^2_{-1},
\end{equation}
and therefore
\begin{equation}
(2.7) \quad \|u\|^2 \leq \epsilon(\|\bar{\partial}_b u\|^2 + \|\bar{\partial}_b^\ast u\|^2) + C_\epsilon\|u\|^2_{-1}, \quad q \leq k \leq n - 1 - q.
\end{equation}

We pass now to consider $u$ globally defined on the whole $M$ instead of a local patch $U$. We cover $M$ by $\{U_\nu\}$ so that in each patch there is a basis of forms in which the basic estimate holds. In the identification of $U_\nu$ to $\mathbb{R}^{2n-1}$, we suppose that the microlocal decomposition which yields (2.7) is well defined. We then apply (2.7) to a decomposition $u = \sum_{\nu} \zeta_\nu u$ for a partition of the unity $\sum_{\nu} \zeta_\nu = 1$ on $M$, observe that $[\bar{\partial}_b, \zeta_\nu]$ and $[\bar{\partial}_b^\ast, \zeta_\nu]$ are 0-order operators and thus give rise to an error term which is controlled by $\epsilon^{-1}\|u\|^2$ and get (2.7) for any $u \in \mathcal{B}^k$. Finally, by the density of smooth forms $\mathcal{B}^k$ into Sobolev forms $(H^1)^k$, (2.7) holds in fact for any $u \in D_{\bar{\partial}_b}^k \cap D_{\bar{\partial}_b^\ast}^k$. The proof is complete.

\[\square\]

\textit{Proof of Theorem 1.4.} When $q = 1$, we observe that we have the estimate for $u^-$ in degree $k = 0$ and for $u^+$ in degree $k = n - 1$ (cf. [KN06] Lemma 3.3). We prove how to get the estimate for $u^+$ in degree $k = 0$ (the one for $u^-$ when $k = n - 1$ being similar). Now, if $n > 2$, we have, as a consequence of (2.7), that $\bar{\partial}_b^\ast$ has closed range on 1-forms. In particular
\[\mathcal{H}^{0, \perp} = (\ker \bar{\partial}_b)^\perp = \text{range} \bar{\partial}_b^\ast.\]
Thus, if \( u \in \mathcal{H}^{0, \perp} \), then \( u = \bar{\partial}^*_b v \) for some \( v \in (L^2)^1 \). Moreover, we can choose the solution \( v \) belonging to \((\ker \bar{\partial}^*_b)^\perp \subset \text{range} \bar{\partial}_b \subset \ker \bar{\partial}_b \). Also, since \( v \in (\ker \bar{\partial}^*_b)^\perp \subset \mathcal{H}^{0, \perp} \), then (cf. [N06] Lemma 5.3)
\[
\|v\|_0^2 < \|\bar{\partial}^*_b v\|^2 + \|\bar{\partial}_b v\|^2 + \|\bar{\psi} v\|_{-1}^2 = \|u\|_0^2.
\]

We use the microlocal decomposition of \( u \) and \( v \) and notice that \( u^+ = \bar{\partial}^*_b(v^+) + [\bar{\partial}^*_b, \bar{\psi}^+]v \) and that \([\bar{\partial}^*_b, \bar{\psi}^+]v = \bar{\psi}^0 v \). Hence
\[
\|u^+\|^2 \leq (u^+, \bar{\partial}^*_b(v^+)) + (u^+, \bar{\psi}^0 v)
\]
\[
\leq (\bar{\partial}_b(u^+), v^+) + (\bar{\psi}^0 u, v)
\]
\[
\leq (\|\bar{\partial}_b(u^+)\| \epsilon(\|\bar{\partial}^*_b(v^+)\| + \|\bar{\partial}^*_b v\|) + \|\bar{\psi}^0 v\|) + (\|\bar{\psi}^0 u\| \|u\|)
\]
\[
\leq (\|\bar{\partial}_b(u^+)\| \epsilon\|u^+\| + \epsilon(\|\bar{\partial}_b(u^+)\| \|\bar{\psi}^0 v\| + \|\bar{\partial}_b u\| \|u\|)
\]
\[
\leq \epsilon(\|\bar{\partial}_b(u^+)\|)^2 + \epsilon\|u^+\|^2 + s.c.\|u\|^2 + l.c.\|\bar{\psi}^0 u\|^2,
\]
where s.c. and l.c. denote a small and large constant respectively. If we take summation over \(+, -, 0\), we end up with
\[
\|u\| \leq \epsilon\|\bar{\partial}_b u\| + l.c.\|\bar{\psi}^0 u\|.
\]

When \( n = 2 \) and only estimates for \( v^+ \) and not for the full \( v \) in degree 1 are provided by Theorem 1.3, we use the extra assumption that \( \bar{\partial}_b \) has closed range. Thus \( \bar{\partial}^*_b \) has also closed range, we can write \( u \in \mathcal{H}^{0, \perp} \) as \( u = \bar{\partial}^*_b v \) for \( v \) satisfying \( \bar{\partial}_b v = 0 \) and \( \|v\| < \|u\| \) and the proof goes through in the same way as above by using (2.2) for \( u^+ \).

\[ \square \]

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