Probing dynamics of dark energy with latest observations

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Abstract We examine the validity of the \(\Lambda\)CDM model and probe the dynamics of dark energy with the latest astronomical observations. Using the \(\Omega_m(z)\) diagnosis, we find that various kinds of observational data are in tension within the \(\Lambda\)CDM framework. We then allow for dynamics of dark energy and investigate the constraint on dark energy parameters. We find that for two different kinds of parameterisations of the equation of state parameter \(w\), a combination of current data mildly favours an evolving \(w\), although the significance is not sufficient for it to be supported by Bayesian evidence. A forecast of the DESI survey shows that the dynamics of dark energy could be detected at the 7\(\sigma\) confidence level and would be decisively supported by Bayesian evidence, if the best-fit model of \(w\) derived from current data is the true model.

Key words: cosmology — dark energy — large scale structure

1 INTRODUCTION

The accelerating expansion of the Universe revealed by type Ia supernovae (SNIa) is one of the most significant discoveries in modern cosmology (Riess et al. 1998; Perlmutter et al. 1999). In the framework of general relativity, cosmic acceleration in the late Universe is due to dark energy (DE), a yet unknown energy component contributing to about two thirds of the total energy budget of the Universe. From astronomical observations, measurements of the equation of state (EoS) parameter \(w\), which is the ratio of pressure to energy density of DE, can shed light on the nature of DE, as different DE models can be characterised by \(w\). For example, the cosmological constant \(\Lambda\), which is one of the most popular DE models, predicts that \(w = -1\), while in dynamical dark energy (DDE) models including quintessence (Peebles & Ratra 1988), phantom (Caldwell 2002), quintom (Feng et al. 2005) and so on, \(w\) evolves with redshift \(z\). Hence, reconstructing the \(w(z)\) function from observations, including cosmic microwave background (CMB), SNIa and large scale structure (LSS) measurements, is an efficient way to test DE models.

Performing a consistency check for the \(\Lambda\)CDM model, which has the least number of model parameters compared with DDE models in general, using observations is a common starting point for phenomenological studies of DE. Interestingly, recent studies show that different kinds of observational data are in tension within the framework of the \(\Lambda\)CDM model (Zhao et al. 2017a; Font-Ribera et al. 2014; Sahni et al. 2014; Battye et al. 2015; Aubourg et al. 2015; Planck Collaboration et al. 2016a; Addison et al. 2016; Bernal et al. 2016; Di Valentino et al. 2016, 2017; Solà et al. 2017a; Solà et al. 2017b). In particular, Zhao et al. (2017a) quantify the tension using the Kullback-Leibler divergence (Kullback & Leibler 1951) and use a nonparametric DDE model to successfully relieve the tension. Their analysis basically shows that the tension within \(\Lambda\)CDM can be interpreted as a signal of dynamics of DE at a 3.5\(\sigma\) confidence level (CL).

In this paper, we perform a complementary study to Zhao et al. (2017a). We first reinvestigate the ten-
sion between different datasets using the $O_m$ (Zunckel & Clarkson 2008; Sahni et al. 2008) diagnosis, and then reconstruct $w(z)$ following a parametric approach. We quantify the significance of $w \neq -1$ and perform model selection using Bayesian evidence on current and simulated future observational data.

This paper is organised as follows. In the next section we present the methodology used for quantifying tension among datasets, for performing DE model parameter inference and for model selection. We also describe datasets used in this work.

## 2 METHOD AND DATA

In this section, we present the methodology used for quantifying tension among datasets, for performing DE model parameter inference and for model selection. We also describe datasets used in this work.

### 2.1 The $O_m$ Diagnosis

The quantity $O_m$ is defined as follows (Zunckel & Clarkson 2008; Sahni et al. 2008),

$$O_m(z) = \frac{[H(z)/H_0]^2 - 1}{(1 + z)^3 - 1},$$

(1)

where $H(z)$ and $H_0$ are the Hubble parameter measured at redshift $z$ and 0 respectively. It is a useful diagnosis of any deviation from the $\Lambda$CDM model simply because $O_m(z) = \Omega_m$ in $\Lambda$CDM. Thus any non-constancy of $O_m(z)$ signals that $w \neq -1$, if flatness of the Universe is assumed.

Observationally, $H_0$ can be directly measured in the local Universe, and $H(z)$ can be estimated from CMB, baryonic acoustic oscillations (BAO) redshift surveys using either galaxy (gBAO) or Lyman-$\alpha$ forest (LyQFB), and from the relative age of old and passively evolving galaxies following a cosmic chronometer approach (observational Hubble parameter data (OHD)).

### 2.2 Parametrisations of the Universe

In this work, we consider two kinds of parametrisations of $w(a)$, where $a$ is the scale factor of the Universe\(^1\).

#### Parametrisation I – Polynomial expansion

(Planck Collaboration et al. 2016b)

$$w(a) = \sum_{i=0}^{N_p} w_i (1 - a)^i,$$

(2)

where $N_p$ defines the order of the polynomial expansion. Note that $N_p = 0$ and $N_p = 1$ are the $w$CDM model, in which $w$ is a constant, and the Chevallier-Polarski-Linder (CPL) model (Chevallier & Polarski 2001; Linder 2003) respectively, and including higher order terms allows more general behaviour of $w(a)$. In this work, we consider cases with $N_p < 5$.

#### Parametrisation II – Oscillatory function

Although Parametrisation I allows for oscillatory behaviours of $w(a)$ in general, it requires a large number of terms in order to properly approximate a periodic oscillatory function, e.g., a cosine function. Therefore we consider another kind of parametrisation as

$$w(a) = w_0 + w_1 (1 - a)^{w_2} \cos(w_3 a + w_4).$$

(3)

This is a general cosine function that allows its mean, amplitude, period and phase to be free parameters. It is similar to the functional form used in Feng et al. (2006) but is more general in that the $(1 - a)^{w_2}$ term allows the amplitude to vary with the scale factor.

Our parametrisation of the Universe is thus,

$$P \equiv \{\omega_b, \omega_c, \Theta_s, \tau, n_s, A_s, w_0, \ldots, w_4, \mathcal{N}\},$$

(4)

where $\omega_b$ and $\omega_c$ are the baryon and cold dark matter physical densities respectively, $\Theta_s$ is the angular size of the sound horizon at decoupling, $\tau$ is the optical depth, $n_s$ and $A_s$ are the spectral index and the amplitude of the primordial power spectrum respectively, and $w_0, \ldots, w_4$ denote the above-mentioned DE EoS parameters. We marginalise over nuisance parameters $\mathcal{N}$ such as the intrinsic supernova luminosity, galaxy bias, etc.

### 2.3 Observational Datasets Used

The datasets we consider in this work include the gBAO measurements that utilise the BOSS DR12 sample at nine effective redshifts (Zhao et al. 2017b; Wang et al. 2016), the LyQFB measurements (Delubac et al. 2015), the 6dFRS (Beutler et al. 2011) and SDSS main galaxy sample (Ross et al. 2015) BAO measurements, the WiggleZ galaxy power spectra (Parkinson et al. 2012), the recent estimate of the Hubble constant $H_0$ obtained from local measurements of Cepheids (Riess et al. 2016) ($H_0$), the recent OHD measurements of $H(z)$ (Moresco et al. 2016), the JLA sample of SNIa (Betoule et al. 2014), the weak lensing shear angular power spectra from CFHTLenS (Heymans et al. 2013) and the Planck 2015 CMB temperature and polarisation angular power spectra (Planck Collaboration et al. 2016a).

\(^1\) For more parametrisations of $w(a)$, see Pantazis et al. (2016).
Specifically, we obtain, and green horizontal bands respectively in Figure 1.

We present our results in Table 1 and in Figures 1–3.

### 2.4 Parameter Estimation and Model Selection

We use a modified version of CAMB (Lewis et al. 2000) to calculate observables, and include DE perturbations following the approach developed in Zhao et al. (2005). We perform a Markov Chain Monte Carlo (MCMC) global fitting of parameters listed in Equation (4) to a combination of datasets described in Section 2.3 using a modified version of CosmoMC (Lewis & Bridle 2002), and use the PolyChord (Handley et al. 2015) plug-in of CosmoMC to compute Bayesian evidence for the model selection.

### 3 RESULT

We present our results in Table 1 and in Figures 1–3.

The quantity $O_{m}(z)$ is estimated using $H(z)$ measurements from Planck 2015, gBAO, OHD and LyαFB, with the recent $H_0$ measurement presented in Riess et al. (2016). To check the constancy of $O_{m}(z)$ using each individual kind of dataset, and the consistency between different kinds of data, we fit constants to the $O_{m}(z)$ measurements from Planck 2015, gBAO and OHD separately, and show the 68% CL constraints in cyan, blue and green horizontal bands respectively in Figure 1. Specifically, we obtain,

$$O_{m}(\text{Planck 2015}) = 0.266 \pm 0.013,$$

$$O_{m}(\text{gBAO}) = 0.165 \pm 0.032,$$

$$O_{m}(\text{OHD}) = 0.229 \pm 0.026,$$

For the purpose of forecast, we simulate future gBAO data assuming a DESI$^2$ sensitivity following DESI Collaboration et al. (2016), and also consider a future space-based supernova mission described in Astier et al. (2011).

### Table 1

| Parameterisation I | $w_0$ | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $\sqrt{\Delta \chi^2}$ | $\Delta \ln E$ |
|--------------------|-------|-------|-------|-------|-------|------------------------|----------------|
| $-1.02 \pm 0.04$(0.01) | 0     | 0     | 0     | 0     | 0     | 0.4(0.8)              | $-2.3(-3.4) \pm 0.3$ |
| $-1.08 \pm 0.10$(0.05) | 0.26 \pm 0.40(0.21) | 0     | 0     | 0     | 0     | 0.7(5.2)              | $-3.9(6.1) \pm 0.3$ |
| $-1.18 \pm 0.17$(0.08) | 1.50 \pm 1.75(0.67) | $-2.34 \pm 0.31(1.14)$ | 0     | 0     | 0     | 1.1(5.4)              | $-7.1(3.1) \pm 0.3$ |
| $-1.07 \pm 0.17$(0.10) | $-1.42 \pm 2.04(1.22)$ | 12.1 \pm 1.02(3.75) | $-17.7 \pm 1.26(3.32)$ | 0     | 1     | 1.8(5.6)              | $-8.4(0.5) \pm 0.3$ |
| $-1.00 \pm 0.18$(0.09) | 0.38 \pm 2.72(1.59) | $-15.8 \pm 21.24(9.29)$ | 72.0 \pm 62.3(20.0) | $-79.6 \pm 55.0(13.4)$ | 2.2(6.0) | $-8.8(0.0) \pm 0.3$ |

| Parameterisation II | $w_0$ | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $\sqrt{\Delta \chi^2}$ | $\Delta \ln E$ |
|--------------------|-------|-------|-------|-------|-------|------------------------|----------------|
| $-1.03 \pm 0.04$(0.03) | 4.98 \pm 2.87(0.61) | 5.38 \pm 2.43(0.39) | 13.3 \pm 6.42(0.40) | 0     | 2.6(7.4) | $-2.2(14.0) \pm 0.3$ |
| $-1.03 \pm 0.05$(0.03) | 4.77 \pm 2.86(0.64) | 5.61 \pm 2.46(0.41) | 13.8 \pm 7.57(0.84) | 4.90 \pm 2.84(1.82) | 2.6(7.5) | $-2.0(14.2) \pm 0.3$ |

$Om(\text{LyαFB}) = 0.226 \pm 0.020$. (8)

It is true that neither the Planck 2015, gBAO nor OHD dataset shows a significant deviation from a constant $O_{m}$ given the level of uncertainty, however, the derived $O_{m}$ from Planck 2015, gBAO and OHD are different at larger than 2$\sigma$ CL. Furthermore, the $O_{m}$ values derived here are all smaller than $\Omega_{m}$ derived from Planck 2015 alone in the $\Lambda$CDM model (Planck Collaboration et al. 2016a), which is $\Omega_{m} = 0.315 \pm 0.013$. This to some extent is due to the fact that the $H_0$ value used here, which is $73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$, is significantly larger than that derived from Planck 2015, which is $67.31 \pm 0.96$ km s$^{-1}$ Mpc$^{-1}$. All these discrepancies among datasets suggest that the $\Lambda$CDM model may need to be extended.

For more general DE models parametrised by Equations (2) and (3), we derive constraints on model parameters, which are shown in Table 1. For the polynomial expansion case, we increasingly add higher order terms to the $w$CDM model in the global fitting. We find that the $\chi^2$ can be reduced by 4.8 at most for the $N_p = 4$ model.

For the purpose of model selection, we also evaluate the logarithmic Bayesian factor,

$$\Delta \ln E \equiv \ln E_{\text{DDE}} - \ln E_{\text{LCDM}},$$

where

$$E \equiv \int d^n \theta P(\theta)$$

denotes the Bayesian evidence, which is an integral of the probability distribution function of $n$-dimensional parameters $\theta$. We find that $\Delta \ln E$ is negative for all cases, meaning that neither of these DDE models is favoured over the $\Lambda$CDM model. The $N_p = 4$ case, in which $w(z)$ is parametrised with five free parameters, is found to be not equal to $-1$ at 2.2$\sigma$ CL, and the Bayesian factor is
**Fig. 1** The measured $O_m$ from various kinds of data: gBAO (blue squares), OHD (green circles) and LyαFB (red triangle). The horizontal cyan, green and blue bands show the 68% CL allowed values for a constant $O_m$ fitted to Planck 2015, OHD and gBAO, respectively. The black solid curve shows $O_m$ derived from the best-fit $w(z)$ model. See text for details.

**Fig. 2** Blue bands: the mean with 68% CL error of the reconstructed $w(z)$ using Parametrisation I for different orders of the polynomial. The grey band in the $N_p = 4$ panel shows the nonparametric $w(z)$ reconstruction result in Zhao et al. (2017a).

**Fig. 3** Same as Fig. 2 but for Parametrisation II. The upper and lower panels show the reconstruction result with and without the $w_4$ parameter fixed respectively.
as low as $\Delta \ln E = -8.8 \pm 0.3$, which strongly indicates current data do not support extending $\Lambda$ in this parametrisation.

For Parametrisation II, we show results with and without the phase $w_4$ fixed, and find that whether $w_4$ varies or not does not change the result: $\chi^2$ is reduced by 6.8 (a 2.6$\sigma$ signal of $w \neq -1$) by four additional parameters with a Bayesian factor $\Delta \ln E = -2.2 \pm 0.3$. Although this model is not supported by the Bayesian evidence, it is much less disfavoured than the $N_p = 4$ model in Parametrisation I, and it fits the data better.

In Figures 2 and 3, we reconstruct $w(z)$ using constraints on DE parameters we obtained. As shown, the best-fit $w(z)$ models with all five DE parameters varied, which are shown in the far right panel of Figure 2, and in the lower panel in Figure 3, crosses $-1$ during evolution, and exhibits a certain level of oscillations with respect to redshift $z$, which is consistent with the prediction of the model of oscillating quintom (Feng et al. 2006). We compare this result to the nonparametric reconstruction presented in Zhao et al. (2017a). As shown, our result is consistent with that in Zhao et al. (2017a) within 1$\sigma$ CL.

To reinvestigate the tension among various datasets in DDE models, we over-plot $O_m$ for the best-fit DDE model as parametrised by Equation (3) (black solid). As shown, it is consistent with all datasets, signalling a release of tension among datasets.

To assess whether the best-fit $w$ model found in this work will be supported by future observations, we take the best-fit $w$ model as a fiducial model, create mock BAO and supernovae data assuming a DESI survey (DESI Collaboration et al. 2016) and a future space-based supernova mission (Astier et al. 2011) combined with Planck 2015 data, and repeat our analysis. We find that for Parametrisation I, models having $N_p = 1, 2$ will be supported by Bayesian evidence, with a signal of $w \neq -1$ at 5$\sigma$ CL. Although the $N_p = 3, 4$ models fit data better, they are not especially preferred over the $\Lambda$CDM model even for the future data. On the other hand, future data support the oscillation model much more significantly. Namely, those models will be detected at more than 7$\sigma$ CL with a large Bayesian factor of $\Delta \ln E = 14 \pm 0.3$.

4 CONCLUSIONS AND DISCUSSION

We revisit the consistency among various kinds of recent observations using the $O_m$ diagnosis, and confirm that tension exists among Planck 2015, gBAO, OHD, Ly$\alpha$FB and the new $H_0$ measurement in the $\Lambda$CDM model.

We therefore investigate the dynamics of DE and perform parametric reconstruction of $w(z)$ with two kinds of parametrisations using a combination of current datasets and simulated future data. We find that an oscillatory $w(z)$ across $-1$ during the evolution is mildly favoured by a combination of current observations at a CL of 2.6$\sigma$ based on the improvement in $\chi^2$. This model can relieve the tension well among datasets. It is true that this is not sufficient for it to be supported by Bayesian evidence, however, for future galaxy surveys with a sensitivity similar to DESI and space-based supernova surveys, the best-fit model derived in this work will be detected at a CL of 7$\sigma$, and will be decisively supported by Bayesian evidence.

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Y. C. Zhang et al.: Probing Dynamics of Dark Energy

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