Improving ant colony optimization algorithm with epsilon greedy and Levy flight

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Abstract
Ant colony optimization (ACO) algorithm is a meta-heuristic and reinforcement learning algorithm, which has been widely applied to solve various optimization problems. The key to improving the performance of ACO is to effectively resolve the exploration/exploitation dilemma. Epsilon greedy is an important and widely applied policy-based exploration method in reinforcement learning and has also been employed to improve ACO algorithms as the pseudo-stochastic mechanism. Levy flight is based on Levy distribution and helps to balance searching space and speed for global optimization. Taking advantage of both epsilon greedy and Levy flight, a greedy–Levy ACO incorporating these two approaches is proposed to solve complicated combinatorial optimization problems. Specifically, it is implemented on the top of max–min ACO to solve the traveling salesman problem (TSP) problems. According to the computational experiments using standard TSPLIB instances, greedy–Levy ACO outperforms max–min ACO and other latest TSP solvers, which demonstrates the effectiveness of the proposed methodology.

Keywords Ant colony optimization · Epsilon greedy · Levy flight · Levy distribution

Introduction

Ant colony optimization (ACO) algorithm is a meta-heuristic algorithm based on the ants’ foraging behaviour. ACO algorithm was first proposed in Dorigo’s doctoral dissertation [11] and more details of this algorithm were provided in paper [14]. The ACO implementation to solve the traveling salesman problem (TSP) and a survey of other ACO applications were presented in paper [12].

Since the birth of the ACO algorithm, there were many researchers conducted in-depth studies and proposed various improved versions. For instance, some variants of ACO such as elite ant colony algorithm [14], rank-based ant colony algorithm [6], max–min ant colony optimization algorithms [42] and ant colony system algorithm [12] were developed. These improved ACOs focused on either selecting which best solutions for pheromone updates or improving the candidate selection mechanism. Furthermore, ACO algorithm was applied not only for solving TSP [1,12,17,28,48] but also for other optimization problems such as vehicle routing problem (VRP) [5,18,33,38,39,53], quadratic assignment problem (QAP) [9,19], and job-shop scheduling problem (JSP) [24,25,54].

In the latest ACO survey papers [13,29], the authors stated that the most current research activities in this area were focusing on (1) incorporating ACO with other meta-heuristics such as simulate annealing [3,31,32], genetic algorithm [8], and tabu search [15], particle swarm optimization [21,30], and (2) various applications of ACO. Although there are many theoretical studies on the convergence [4,22,41], runtime and parameter tuning [10,34,43,55] for ACO, few improved ACO was proposed after the year 2006. Max–min ACO and ant colony system are still state-of-the-art ACOs according to Dorigo and Sttzle’s survey paper.
ACO 2019 [13], so max–min ACO is selected in the benchmark of this paper.

ACO is a kind of reinforcement learning algorithm [12, 16]. In Fig. 1, the ants, pheromone, road network in ACO are equivalent to the agents, reward, environment in Reinforcement Learning. “One of the challenges that arise in reinforcement learning, and not in other kinds of learning, is the trade-off between exploration and exploitation” [44]. The proposed greedy–Levy ACO is designed to resolve the exploration/exploitation dilemma.

In reinforcement learning, important exploration strategies include epsilon greedy ($\epsilon$-greedy) and Boltzmann exploration (Softmax) policies [44]. The $\epsilon$-greedy policy adopts the current best selection from candidates with the probability of $\epsilon$, and a random selection with the probability of $1 - \epsilon$. Ant colony system (ACS) algorithm employs the $\epsilon$-greedy policy and achieves better performance [12]. Greedy–Levy ACO integrates $\epsilon$-greedy policy and employs the Levy flight mechanism attempting to improve the $\epsilon$-greedy policy further.

In swarm intelligence and evolutionary computation, local search focuses on the feasible neighborhood space of the current solution and seeks better solutions. For a local search, there is no guarantee to obtain a global optimum. Global search tries to explore all feasible solutions in the solution space to achieve the global optimum; however, it is quite time-consuming if not impossible. Combing global and local search is an important research method for swarm intelligence and evolutionary computation [20,35,36,47]. In this paper, the proposed algorithm attempts to balance global and local search by applying epsilon greedy and Levy flight in ACO so that better solutions can be found more efficiently.

The random mechanism is embedded in ACO algorithms, and the probability distribution for choosing the next node to be visited plays a key role in solving a TSP. Among the continuous distributions, one attracting our attention has the property called fat tailed or heavy tailed, which means the tail of this distribution is thicker than others such as normal or exponential distribution. In a fat-tailed distribution case, the tail portion will have a higher probability to be chosen by ACO algorithms for the diversity purpose. Furthermore, the increase in diversity of solutions would be more likely to find an optimal solution. Specifically, Levy distribution possesses the fat-tailed feature and it may help to deliver diversified solutions efficiently.

Levy flight [40] is a type of random walking pattern conforming to Levy distribution named after the French mathematician Paul Levy. The step length of Levy flight follows the fat-tailed distribution. Steps have isotropic random directions when walking in a multi-dimensional space. Many animals’ foraging movements have Levy flight features, e.g., they spend most feeding time around a current food source, and occasionally need long-distance travel to find the next food source efficiently [45,46]. Levy flight has also been applied to improve other meta-heuristic algorithms such as particle swarm optimization (PSO) [23,26,50], artificial bee colony algorithm [2], cuckoo search algorithm [51,52], etc. Levy flight mechanism has been employed in spatial search approaches, i.e. PSO and cuckoo search algorithm, but cannot directly be applied in ACO without any special design.

In this paper, both $\epsilon$-greedy policy and Levy flight approaches are employed in the proposed greedy–Levy ACO aiming to improve searching speed and efficiency and resolve the exploration/exploitation dilemma.
Methods

Candidate selection mechanism in ACO

ACO algorithm consists of three major steps including constructing a solution, optional daemon actions, and pheromone updates. Constructing a solution means to find feasible solutions regarding a specific problem. Daemon actions are some optional operations, i.e., certain local search to improve current solution quality. Updating pheromone according to the solution quality is important for fast convergence and guiding the upcoming ants to seek better solutions in ACO.

Most ACOs employ a uniform-distribution-based random number between 0 and 1 as the probability to select one of the candidates when constructing a feasible solution. In the following discussions, a candidate means a node to be visited by an ant at a certain stage when solving a TSP, and the probability formula to select the next node from candidates is listed in formula (1); the parameter $\tau$ represents pheromone value and $\eta$ represents the attractiveness which in most occasions is the reciprocal of edge length between node $i$ and $j$, $\alpha$ and $\beta$ are exponents for $\tau$ and $\eta$, respectively.

$$P_{ij} = \begin{cases} \frac{(\tau_{ij})^\alpha(\eta_{ij})^\beta}{\sum(\tau_{is})^\alpha(\eta_{is})^\beta}, & j, s \in \text{allowed} \\ 0, & \text{otherwise} \end{cases}.$$ (1)

The selection probabilities for candidates are exponentially distributed due to the power function of attractiveness $\eta$ in formula (1). The list of candidates sorted in descending order of their selection probabilities is called candidate list in the following discussions. The selection probability for a candidate declines quickly with the decreasing of its attractiveness and close to zero if it is not located in the front part of the candidate list. Figure 2 depicts the selection probabilities computed for 5 nodes in the candidate list of a TSP instance with 100 nodes, where the x-axis represents indices of nodes in the candidate list. Since nodes are sorted in descending order of their selection probabilities, a node with a smaller index means its selection probability is relatively higher than those nodes with greater indices. Selection probabilities of most nodes are less than 1% in Fig. 2 which means they almost have no chance to be chosen. This limits the selection for candidates who have a lower probability to be selected and compromise the exploration of solution spaces. The purpose of this paper is to improve the exploration strategy and, therefore, improves the diversity of solutions for achieving a better solution faster.

$\epsilon$-Greedy policy

ACO is one type of reinforcement learning algorithm [12,16]. The $\epsilon$-greedy and Softmax policies are important exploration strategies in reinforcement learning [44]. The $\epsilon$-greedy policy defines the policy of the selection probability $p$ in formula (2), which focuses on the exploitation with a probability of $\epsilon$ by using the best candidate while conducting the exploration with a probability of $1 - \epsilon$ by applying $P_{ij}$ defined in formula (1). The $\epsilon$-greedy policy balances the exploration and exploitation well via parameter $\epsilon$ and is widely employed in many artificial intelligent algorithms [35–37,49]. The $\epsilon$-greedy policy is also adopted in an improved ACO named ant colony system (ACS) as the pseudo-stochastic mechanism [12]. The original $\epsilon$-greedy policy uses uniform distribution to select a candidate in the case of $1 - \epsilon$. Levy flight mechanism will be employed to improve the case of $1 - \epsilon$ in this paper.

$$p = \begin{cases} \text{arg} \ max\{(\tau_{ij})^\alpha(\eta_{ij})^\beta\}, & \text{if } p \leq \epsilon \\ P_{ij}, & \text{otherwise} \end{cases}. $$ (2)
Levy flight and Levy distribution

The left part of Fig. 3 shows the difference between Levy, normal and Cauchy distributions. Levy distribution is a fat-tailed one in which possibility values at the tail of its curve is larger than other distributions. The right part of Fig. 3 from paper [45] depicts the Brownian motion upon uniform distribution and Levy flight following Levy distribution. The search area covered by Levy flight is much larger than the one by Brownian motion within the same 1000 steps. Part b in the right part of Fig. 3 illustrates the detailed trajectory of corresponding Brownian motion bouncing mainly around the current spot with a step length of 1. In Levy flight, flying distance is defined as the step length in this paper and is sometimes greater than 1.

The standard Levy distribution is given in formula (3):

\[
S = \begin{cases} 
\frac{\mu}{\nu^{\frac{\beta}{\nu}}}, & \text{if } S > 1 \\
1, & \text{else}
\end{cases}, \quad \mu \sim N(0, \sigma^2_{\mu}), \nu \sim N(0, \sigma^2_{\nu}).
\]

(3)

Formula (3) illustrates how the step length S is computed, which is the most important part of Levy flight. Parameters \(\mu\) and \(\nu\) follow a normal distribution while \(\beta\) is a fixed parameter. Step length is a non-negative random number following Levy distribution and is associated with the direction that is uniformly distributed in two- or three-dimensional depending on particular applications. There is no direction needs to be considered when Levy flight is one-dimensional. The step length will be applied as an altering ratio of selection probability to choose a candidate in greedy–Levy ACO.

Integration of epsilon greedy and Levy flight mechanism

Computation of original Levy flight using formula (3) is complicated and cannot directly be employed in ACO. In the proposed greedy–Levy ACO, a Levy flight conversion formula is designed for candidate selection mechanism using formula (6).

Formula (4) is an improved version of formula (3), two normal distribution-based random numbers \(\mu\) and \(\nu\) are required in the latter while only one uniform distribution-based random variable \(P_{\text{Levy}}\) is required in the former to reduce computational cost. Altering ratio \(A\) and Levy flight threshold \(P_{\text{threshold}}\) are fixed parameters in formula (4). Formula (5) is specially designed to ensure that the altered selection probability still ranges between 0 and 1. Formula (6) is derived by combining formulas (4) and (5).

\[
S_{\text{new}} = \begin{cases} 
\frac{1 \times \frac{1 - P_{\text{threshold}}}{1 - P_{\text{Levy}}}}{1}, & \text{if } S_{\text{new}} \geq 1 \\
1, & \text{else}
\end{cases},
\]

(4)

\[
1 - P_{\text{new}} = \frac{1}{S_{\text{new}}} \times (1 - P_{\text{now}}),
\]

(5)

\[
P_{\text{new}} = \begin{cases} 
1 - A \times \frac{1 - P_{\text{Levy}}}{P_{\text{threshold}}}, & \text{if } P_{\text{Levy}} \geq P_{\text{threshold}} \\
1 - P_{\text{now}}, & \text{else}
\end{cases},
\]

(6)

- \(S_{\text{new}}\): new step length for Levy flight, \(S_{\text{new}} \geq 1\);
- \(A\): altering ratio for Levy flight, \(A \geq 0\);
- \(P_{\text{threshold}}\): parameter for Levy flight threshold, \(0 < P_{\text{threshold}} < 1\);
- \(P_{\text{Levy}}\): probability for turning on/off Levy flight altering, a uniform distribution based random number, \(0 < P_{\text{Levy}} < 1\);
- \(P_{\text{now}}\): original selection probability before Levy flight altering, a uniform distribution-based random number, \(0 < P_{\text{now}} < 1\);
– $P_{\text{new}}$: final selection probability after Levy flight altering, $0 < P_{\text{new}} < 1$.

Formula (2) of $\epsilon$-greedy presents two scenarios in selecting a candidate, (a) choosing the candidate with the maximum probability when $p \leq \epsilon$, and (b) selecting a candidate randomly when $p > \epsilon$. Formula (7) is the core design in this paper by replacing the random selection in formula (2) with the Levy flight mechanism in formula (6).

$$P_{ij} = \begin{cases} \arg \max \{(\tau_{ij})^\alpha (\eta_{ij})^\beta), & \text{if } p \leq \epsilon \\ 1 - A \times \frac{1-P_{\text{levy}}}{1-P_{\text{threshold}}} \times \left(1 - \frac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum (\tau_{ij})^\alpha (\eta_{ij})^\beta}\right), & \text{if } p > \epsilon, P_{\text{levy}} \geq P_{\text{threshold}} \\ 0, & \text{else.} \end{cases}$$

(7)

Since the candidate selection mechanism is the only difference between greedy–Levy ACO and max–min ACO, we present the pseudocode of candidate selection mechanism for them in Algorithms 1 and 2, respectively. We refer the interesting reader to the max–min ACO paper [42] for details.

**Algorithm 1** The candidate selection mechanism in greedy-Levy ACO

1: Sort the nodes in the candidate list by their probabilities of being selected
2: Remove the nodes cannot be selected in the candidate list, i.e. visited node
3: Generate a uniform random number $P_{\text{now}}$ between 0 and 1
4: if $P_{\text{now}} \leq \epsilon$ then
5: return The next node with the max probability of being selected from the candidate list.
6: else
7: Generate another uniform random number $P_{\text{ levy}}$ between 0 and 1
8: if $P_{\text{ levy}} \geq P_{\text{threshold}}$ then
9: $P_{\text{new}} = 1 - A \times \frac{1-P_{\text{levy}}}{1-P_{\text{threshold}}} + P_{\text{now}}$
10: else
11: $P_{\text{new}} = P_{\text{now}}$
12: end if
13: return The next node be selected using $P_{\text{new}}$ from the candidate list
14: end if

**Algorithm 2** The candidate selection mechanism in max–min ACO

1: Generate a uniform random number $P_{\text{now}}$ between 0 and 1
2: return The next node be selected using $P_{\text{now}}$ from the candidate list

### Computational experiments

#### Data and environment settings

Computational experiments were carried out to benchmark the proposed greedy-Levy ACO against max–min ACO and the last TSP solvers.

The code of greedy–Levy ACO was implemented on the top of publicly available source code [http://www.aco-metaheuristic.org/aco-code/](http://www.aco-metaheuristic.org/aco-code/) developed by Thomas Stützle (the author of max–min ACO). The source code covers max–min ACO and other improved ACO algorithms including basic ant system, elitist ACO, rank-based ACO, best–worst ACO and ant colony system. Though TSP is a classic and well-studied combinatorial problem, it is an NP-hard problem used quite often for ACO performance benchmark. Furthermore, max–min ACO source code used in this paper is merely designed for TSP and supports the standard TSPLIB instances with best-known solutions. Every benchmark case ran 100 trials for each TSPLIB instance considering the stochastic property of ACO algorithm, with same parameter setting for all trials on the same computer. Twelve instances including ch150, kroA200, kroB200, gr202, ts225, tsp225, pr226, gr229, gil262, a280, pr299 and lin318 from TSPLIB [http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/tsp](http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/tsp) were selected for the benchmark. Parameters used both in max–min and greedy–Levy ACOs were the same, specifically, the parameter vector (pheromone evaporation rate, alpha, beta, population size) was set to be (0.1, 1, 2, 50). The maximum running time for each trial was 86,400 s attempting to obtain the best-known solutions. Source code and running script of max–min ACO and greedy–Levy ACO are available at [https://github.com/akeylui/greedylevyacotosp/](https://github.com/akeylui/greedylevyacotosp/).

Both max–min ACO and greedy–Levy ACO in this paper employ a pheromone re-initialization mechanism. It monitors the solution procedure continuously and resets all pheromones to their max value when it finds the pheromones are concentrated on few edges and believes the solution procedure is trapped at a local optimum. This mechanism attempts to escape local optima and find the best-known solution within predefined iterations and/or computational time. It helps max–min ACO and greedy–Levy ACO to find the best-known solutions for many small- or medium-scale
TSPLIB instances including all ones used in this paper when enough time and iterations are given. During the benchmark, both max–min ACO and greedy–Levy ACO employ other improvement procedures including nearest neighbor and the 3-opt local search. The iterations when the best-known solutions are obtained were recorded for comparison in the experiment as an algorithm performance metric, which includes the iterations of 3-Opt local search which are the major iterations in max–min ACO and greedy–Levy ACO.

The computing environment for the benchmark was Windows 10 × 64, CPU 8 cores at 2.7 GHz, Memory 32 GB. The programming language for the implementation is C to keep consistent with max–min ACO algorithm.

**Parameters tuning for greedy–Levy ACO**

Larger epsilon greedy threshold, smaller Levy flight threshold and Levy flight altering ratio will speed up the convergence, however, easy to fall into local optimal solutions. On the other hand, more exploration will help to seek more solution space and yet require prolonged searching time or more iterations. It is critical to find the most reasonable parameter setting to balance the exploration/exploitation and converge to the optima faster.

Additional computational experiments were conducted for tuning the parameters specific for greedy–Levy ACO, namely epsilon greedy threshold \( \epsilon \), Levy flight threshold \( P_{\text{threshold}} \) and Levy flight altering ratio \( A \) in formula (7). Twelve instances (ch150, kroA200, kroB200, gr202, ts225, tsp225, pr226, gr229, giil262, a280, pr299 and lin318) were chosen from the TSPLIB instances to perform parameters tuning. For each parameter setting to be evaluated, 100 trials were carried out, and a few metrics are applied to determine the best parameter setting.

**Parameter tuning**

Parameter \( \epsilon \) in formula (2) varying with value 0, 0.5, 0.6, 0.7, 0.8, 0.9 were evaluated. The larger the \( \epsilon \) value is, the less exploration is. Epsilon greedy will be switched off if \( \epsilon \) value is set to 0.

Parameter Levy flight threshold in formula (2) varying from 0 to 1 with a step length of 0.05 was evaluated and analyzed. The smaller Levy flight threshold is, the more exploration is. Parameter Levy flight altering ratio in formula (2) varying from 0 to 2 with a step length of 0.2 was evaluated as well. The larger Levy flight altering ratio is, the more exploration is. Levy flight will be switched off if Levy flight threshold value is set to 0 or altering ratio is set to 0.

Greedy–Levy ACO degenerates to max–min ACO when epsilon greedy and Levy flight are both switched off.

There were 726 parameter combinations for epsilon, Levy flight threshold and altering ratio in the experiment. Each parameter combination ran 100 trials for all instances and all trials found the best-known solutions.

To measure the effectiveness of certain parameter setting, the metrics called iteration improvement percentage was defined as followed:

For the given set of parameters, let \( N_{\text{mm}(i)} \) be the average of iterations to find the best-known solution of the \( i \)th instance in 100 trials for max–min ACO while \( N_{g(i)} \) be the one required for greedy–Levy ACO, then the average of iteration improvement percentage over \( K \) instances is defined as \( \frac{\sum_{i=1}^{K} (1 - N_{g(i)}/N_{\text{mm}(i)})}{K} \), and \( K \) is 12 in our experiment. It is conceivable that the higher the average iterations improvement percentage is, the better the performance of greedy–Levy ACO for the given parameter setting.

The average of iteration improvement percentage for 726 parameter combinations is illustrated in Fig. 4. No. 609 parameter combination is the best one as shown in Fig. 4 where epsilon is 0.9, Levy flight threshold is 0 and altering ratio is 0.4, and this parameter setting is employed for later computational experiments in the following sections.

**Benchmarks between max–min ACO and greedy–Levy ACO**

The computational experiments were conducted by employing instances ch150, kroA200, kroB200, gr202, ts225, tsp225, pr226, gr229, giil262, a280, pr299 and lin318 from the TSPLIB, and the results using the suggested parameter setting for greedy–Levy ACO and max–min ACO during the experiments are plotted in Fig. 5.

The histograms in Fig. 5 depict the iterations where the best-known solutions of the tested TSP instances are obtained. The x-axis represents the experiment trials while the y-axis indicates the iterations when the best-known solution is reached. The best-known solutions for all instances were obtained in the experiment. By glancing at these histograms, it appears that the required average of iterations to reach the best-known solutions using greedy–Levy ACO is...
Fig. 5 Benchmark for 12 instances
Table 1: Statistical testing ($P$ value) for the benchmark analyses

| Instance | Wilcoxon   | Rank sums | Mann–Whitney $U$ |
|----------|------------|-----------|------------------|
| ch150    | 6.03E–04   | 5.29E–03  | 2.68E–03         |
| a280     | 0.3173     | 0.9972    | 0.5              |
| kroA200  | 0.2818     | 0.8523    | 0.4275           |
| kroB200  | 0.1201     | 0.2096    | 0.1054           |
| gr202    | 0.0112     | 0.0274    | 0.0138           |
| pr226    | 3.04E–07   | 3.92E–08  | 2.00E–08         |
| ts225    | 3.09E–03   | 1.35E–03  | 6.81E–04         |
| tsp225   | 3.29E–03   | 2.36E–03  | 1.20E–03         |
| gr229    | 2.86E–05   | 4.35E–07  | 2.21E–07         |
| pr299    | 1.02E–04   | 6.12E–04  | 3.10E–04         |
| gil262   | 3.12E–04   | 1.13E–04  | 5.74E–05         |
| lin318   | 9.53E–09   | 1.05E–10  | 5.37E–11         |

$P$ value: $< 0.05^*$; $< 0.01^{**}$; $< 0.001^{***}$

Table 2: Benchmark for max–min ACO and greedy–Levy ACO

| Instance | Max–min ACO | Greedy–Levy ACO | Improvement percentage |
|----------|-------------|-----------------|-----------------------|
|          | Average iterations | Iteration variance | Average iterations | Iteration variance | Average iterations (%) | Iteration variance (%) |
| ch150    | 1087.2           | 1160.61         | 731.33                | 522.8509          | 32.73                | 54.95                |
| a280     | 4788.89          | 5565.808        | 4042.42               | 3094.291          | 15.59                | 44.41                |
| kroA200  | 3546.18          | 2201.672        | 2921.12               | 833.6642          | 17.63                | 62.13                |
| kroB200  | 5311.75          | 3320.334        | 3956.49               | 1210.959          | 25.51                | 63.53                |
| gr202    | 37,367.48        | 39,292.75       | 21,829.34             | 13,8805.4         | 66.14                | 50.17                |
| pr226    | 35,757.25        | 38,332.23       | 6260.73               | 8624.176          | 82.49                | 77.50                |
| ts225    | 11,380.65        | 7249.736        | 8343.69               | 5525.991          | 26.69                | 23.78                |
| tsp225   | 5292.14          | 1514.43         | 4386.75               | 936.5603          | 17.11                | 38.16                |
| gr229    | 38,1295.5        | 27,8537         | 12,9097.82            | 13,8805.4         | 66.14                | 50.17                |
| pr299    | 31,672.3         | 20,896.26       | 20,279.83             | 13,403.4          | 35.97                | 35.86                |
| gil262   | 31,992.94        | 27,604          | 18,980.36             | 15,854.17         | 40.67                | 42.57                |
| lin318   | 99,586.9         | 80,655.82       | 21,902.39             | 17,059.07         | 78.01                | 78.85                |
| Average  | 40.01            | 50.24           |                      |                     |                     |                     |

lower than max–min ACO. It may imply that greedy–Levy ACO can achieve the best-known solutions faster than max–min ACO does.

To validate the above statement statistically, three non-parametric tests called Wilcoxon, rank sums and Mann–Whitney $U$ tests were applied to check if greedy–Levy ACO and max–min ACO perform similarly. For each TSP instance, the iterations of these two algorithms to reach the best-known solutions for all 100 trials were input to the function scipy.stats.wilcoxon(), scipy.stats.ranksums() and scipy.stats.mannwhitneyu() in Python packages. The outcomes are presented in Table 1. $P$ value for a280 and kroA200 is litter higher that means the performances of the compared algorithms are not so differently statistical in solving this instance because the iterations finding the best-known solution is small (about 2000–6000 iterations). For other instances, the performances of underlying algorithms are significantly different since the associated $P$ values are less than 0.05 or even 0.001. Together with the histograms, we conclude that greedy–Levy ACO can achieve best-known solutions faster than max–min ACO.

To demonstrate that the proposed algorithm can indeed reach the best-known solutions faster, further statistical analyses are conducted upon the achieved results. In Table 2, the computational results are listed for greedy–Levy ACO and max–min ACO. The following metrics are applied to both approaches:

- Average iterations: the average iterations to reach the best-known solution which used to evaluate the performance of the algorithm.
Table 3: Benchmark for ant colony system and greedy–Levy ACO

| Instance | Ant colony system | Greedy–Levy ACO | Improvement percentage |
|----------|-------------------|-----------------|------------------------|
|          | Average iterations | Iteration variance | Average iterations | Iteration variance | Average iterations (%) | Iteration variance (%) |
| ch150    | 1456.26           | 1250.36         | 664.38                | 507.6224           | 54.38                  | 59.40                   |
| a280     | 4796.58           | 5291.796        | 2574.6               | 3403.09           | 37.29                  | 20.64                   |
| kroA200  | 3401.82           | 1758.022        | 2101.12              | 1298.4029         | 12.48                  | 47.19                   |
| kroB200  | 4968.78           | 2680.551        | 2814.883             | 1412.987          | 16.17                  | 47.29                   |
| gr202    | 41,052.22         | 42,848.12       | 25,744.6             | 34,003.09         | 37.29                  | 20.64                   |
| pr226    | 43,402.3          | 45,222.89       | 23,979.12            | 19,811.32         | 37.29                  | 39.07                   |
| tsp225   | 11,328.6          | 7534.245        | 6576.96              | 4590.283          | 41.94                  | 39.07                   |
| gr229    | 38,359.66         | 26,804.65       | 16,907.28            | 12,542.05         | 55.92                  | 53.21                   |
| Gil262   | 98,783.58         | 102,143.3       | 23,979.12            | 19,811.32         | 75.73                  | 80.60                   |
| Average  | 42.76             | 51.01           |                       |                    |                        |                        |

Table 4: Iterations for greedy–Levy ACO to achieve the best-known solution

| Trials | berlin52 | ch150 | eil51 | eil76 | eil101 | korA100 | korB200 | lin105 | rat99 | st70 |
|--------|----------|-------|-------|-------|--------|----------|----------|--------|-------|------|
| 1      | 135      | 711   | 142   | 505   | 1075   | 188      | 3976     | 188    | 307   | 139  |
| 2      | 155      | 512   | 136   | 515   | 599    | 176      | 4069     | 182    | 770   | 146  |
| 3      | 131      | 2079  | 133   | 383   | 1377   | 184      | 2391     | 203    | 758   | 160  |
| 4      | 141      | 1322  | 394   | 640   | 294    | 173      | 3549     | 190    | 445   | 285  |
| 5      | 139      | 508   | 883   | 654   | 303    | 164      | 2260     | 183    | 635   | 145  |
| 6      | 139      | 820   | 268   | 255   | 146    | 167      | 3187     | 197    | 805   | 144  |
| 7      | 137      | 1162  | 541   | 508   | 1062   | 353      | 4501     | 192    | 787   | 147  |
| 8      | 156      | 501   | 139   | 131   | 1078   | 354      | 3474     | 177    | 475   | 140  |
| 9      | 145      | 1044  | 133   | 400   | 932    | 174      | 2487     | 193    | 326   | 138  |
| 10     | 135      | 1047  | 134   | 270   | 1209   | 356      | 3609     | 198    | 314   | 144  |
| 11     | 136      | 1497  | 134   | 374   | 447    | 157      | 1097     | 181    | 786   | 145  |
| 12     | 148      | 1166  | 139   | 130   | 1047   | 332      | 2900     | 203    | 315   | 144  |
| 13     | 135      | 351   | 265   | 519   | 1509   | 182      | 2201     | 199    | 315   | 143  |
| 14     | 144      | 692   | 138   | 385   | 443    | 177      | 3915     | 213    | 787   | 138  |
| 15     | 141      | 670   | 141   | 466   | 297    | 185      | 225      | 204    | 160   | 135  |
| 16     | 145      | 168   | 140   | 251   | 636    | 334      | 2970     | 187    | 308   | 138  |
| 17     | 140      | 507   | 121   | 269   | 459    | 181      | 3018     | 198    | 475   | 413  |
| 18     | 145      | 1171  | 268   | 630   | 939    | 176      | 3279     | 201    | 1646  | 149  |
| 19     | 151      | 1138  | 136   | 521   | 896    | 351      | 2981     | 194    | 1214  | 152  |
| 20     | 139      | 827   | 135   | 381   | 1531   | 171      | 3174     | 185    | 646   | 141  |
| Max iterations | 156 | 2079 | 883 | 654 | 1531 | 356 | 4501 | 213 | 1646 | 413 |
| Average iterations | 141.85 | 894.65 | 226 | 409.35 | 813.95 | 226.75 | 2963.15 | 193.4 | 613.7 | 164.3 |

∗The bold number means achieve the best-known solution within 1000 iterations

- Iteration variance: the variance of iterations to reach the best-known solution which applied to judge the stabilization of the algorithm.

Table 2 also presents the improvement magnitude of greedy–Levy ACO over max–min ACO for all tested TSP instances concerning the above metrics. Taking the results shown in Tables 1 and 2 into account, we may conclude that...
### Table 5: Best solutions for greedy–Levy ACO in 1000 iterations

| Trials | berlin52 | ch150 | eil51 | eil76 | eil101 | korA100 | korB200 | lin105 | rat99 | st70 |
|--------|----------|-------|-------|-------|--------|----------|----------|--------|-------|------|
| 1      | 7542     | 6528  | 426   | 538   | 631    | 21,282   | 29,409   | 14,379 | 1211  | 675  |
| 2      | 7542     | 6528  | 426   | 538   | 629    | 21,282   | 29,451   | 14,379 | 1211  | 675  |
| 3      | 7542     | 6549  | 426   | 538   | 630    | 21,282   | 29,394   | 14,379 | 1211  | 675  |
| 4      | 7542     | 6549  | 426   | 538   | 629    | 21,282   | 29,492   | 14,379 | 1211  | 675  |
| 5      | 7542     | 6528  | 426   | 538   | 629    | 21,282   | 29,463   | 14,379 | 1211  | 675  |
| 6      | 7542     | 6528  | 426   | 538   | 629    | 21,282   | 29,486   | 14,379 | 1211  | 675  |
| 7      | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,492   | 14,379 | 1211  | 675  |
| 8      | 7542     | 6528  | 426   | 538   | 629    | 21,282   | 29,486   | 14,379 | 1211  | 675  |
| 9      | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,446   | 14,379 | 1211  | 675  |
| 10     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,417   | 14,379 | 1211  | 675  |
| 11     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 12     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 13     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 14     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 15     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 16     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 17     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 18     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 19     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |
| 20     | 7542     | 6544  | 426   | 538   | 630    | 21,282   | 29,420   | 14,379 | 1211  | 675  |

**Best** 7542 6528 426 538 629 21,282 29,409 14,379 1211 675

**Average** 7542 6532.6 426 538 629.6 21,282 29,458.35 14,379 1211.1 675

**Worst** 7542 6549 426 538 632 21,282 29,550 14,379 1212 675

The greedy–Levy logic does help to achieve best-known solutions faster.

Ant colony system (ACS) is another state-of-the-art ACO algorithm applied with epsilon greedy strategy [13]. The result of the benchmark between ACS and greedy–Levy ACO is presented in Table 3. The epsilon parameter for ACS is 0.9 which is same in greedy–Levy ACO. The performance of greedy–Levy ACO was better than ACS from the benchmark.

### Benchmarks against the latest TSP solver

To benchmark the proposed algorithm against state-of-the-art TSP solvers developed based on ACO, PSO–ACO–3Opt [30] and PACO–3Opt [21] that had been cited the most lately in Google scholar are selected. The algorithms hybridize particle swarm optimization, ant colony optimization and 3-Opt. The corresponding paper presented computational experiments for a set of TSP instances, in which each instance was solved with 20 trials and within max 1000 iterations per trial to evaluate the performance. Specifically, the TSPLIB instances berlin52, ch150, eil101, eil51, eil76, kroA100, kroB200, lin105, rat99 and st70 were employed to perform the corresponding computational experiments.

To conduct a fair comparison, the proposed greedy–Levy ACO was applied to solve the same 10 instances mentioned above, where the best-known solutions were obtained. For each instance, the iterations to achieve the best-known solution for the corresponding 20 trials are listed in Table 4. The best solutions found in 1000 iterations are presented in Table 5. Furthermore, 3-Opt in both greedy–Levy ACO and max–min ACO were enabled.

PSO–ACO–3Opt and PACO–3Opt algorithm could not reach all the best-known solutions within the given 1000 iterations according to the outcomes in papers [21,30]. The details of the performance comparison between PSO–ACO–3Opt/PACO–3Opt and greedy–Levy ACO within 1000 iterations are presented in Table 6. Given 1000 iterations, the proposed greedy–Levy ACO can achieve better solutions comparing to PSO–ACO–3Opt/PACO–3Opt.

### Conclusions

In this paper, the proposed greedy–Levy ACO algorithm was developed on the top of max–min ACO by applying $\epsilon$-greedy policy and Levy flight mechanism. The parameters of greedy–Levy ACO were tuned carefully using associated instances. The computational experiments reveal the superiority of the proposed greedy–Levy ACO. It is observed that greedy–Levy ACO can reach the best-known solutions with...
Table 6

| Best solutions obtained in 1000 iterations by compared algorithms |
|---------------------------------------------------------------|
| Instance | Best-known | PSO–ACO–3Opt | PACO–3Opt | Greedy–Levy |
|          | Solution   | Best | Average | Worst | Error% | Best | Average | Worst | Error% | Best | Average | Worst | Error% | Best | Average | Worst | Error% |
| Berlin52 | 7542       | 7542 | 7542    | 7543.2 | 0.02   | 7542  | 7542    | 7542  | 0.00   | 7542  | 7542    | 7542  | 0.00   |
| Ch150   | 6528       | 6528 | 6528    | 6538   | 0.08   | 6528  | 6528    | 6528  | 0.00   | 6528  | 6528    | 6528  | 0.00   |
| Eil51   | 426        | 426  | 426     | 426.45 | 0.11   | 426   | 426     | 426   | 0.00   | 426   | 426     | 426   | 0.00   |
| Eil76   | 538        | 538  | 538     | 538.3  | 0.16   | 538   | 538     | 538   | 0.00   | 538   | 538     | 538   | 0.00   |
| kroA100 | 21,282     | 21,282 | 21,282 | 21,301 | 0.01   | 21,282 | 21,282 | 21,282 | 0.00   | 21,282 | 21,282 | 21,282 | 0.00   |
| kroA200 | 29,368     | 29,368 | 29,368 | 29,468 | 0.02   | 29,368 | 29,368 | 29,368 | 0.00   | 29,368 | 29,368 | 29,368 | 0.00   |
| Lin105  | 14,393     | 14,393 | 14,393 | 14,393 | 0.00   | 14,393 | 14,393 | 14,393 | 0.00   | 14,393 | 14,393 | 14,393 | 0.00   |
| Rat99   | 1224       | 1224 | 1224    | 1224   | 0.00   | 1224  | 1224    | 1224  | 0.00   | 1224  | 1224    | 1224  | 0.00   |
| St70    | 676        | 676  | 676     | 676    | 0.00   | 676   | 676     | 676   | 0.00   | 676   | 676     | 676   | 0.00   |
| Average | 0.49       | 0.40 | 0.05    |        |        | 0.00   | 0.40    | 0.05  |        | 0.00   | 0.05    |        |        |

fewer iterations comparing to max–min ACO algorithm (an average 40.01% deduction in iterations and 50.24% deduction in iteration variance for all tested TSPLIB instances). Further, other ACO researchers may be able to replicate the experiments or improve their ACO algorithm by integrating greedy–Levy mechanism based on the uploaded open-source code.

For future research, we are planning to apply greedy–Levy ACO to other optimization problems, i.e. rich VRP [7,27].

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