Hadronic loop contributions to $J/\psi$ and $\psi'$ radiative decays into $\gamma\eta_c$ or $\gamma\eta'_c$

Gang Li$^{1,3}$ and Qiang Zhao$^{2,1,3}$

1) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P. R. China
2) Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom
3) Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, P. R. China

(Dated: October 23, 2008)

Intermediate hadronic meson loop contributions to $J/\psi$, $\psi' \to \gamma\eta_c$ ($\gamma\eta'_c$) are studied apart from the dominant M1 transitions in an effective Lagrangian approach. Due to the property of the unique antisymmetric tensor coupling in $V \to VP$, the hadronic loop transitions provide explicit corrections to the M1 transition amplitudes derived from the naive “quenched” $c\bar{c}$ transitions via the coupling form factors. This mechanism interfering with the M1 transition amplitudes naturally accounts for the deviations from the Godfrey-Isgur model predictions in $J/\psi$ and $\psi' \to \gamma\eta_c$. It also predicts a small branching ratio of $\psi' \to \gamma\eta'_c$, which can be examined by experimental measurements at BES and CLEO-c.

I. INTRODUCTION

Charmonium spectrum and decays of charmonium states are an ideal place for studying the strong interaction dynamics in the interplay of perturbative and non-perturbative QCD regime. In the past decades there have been significant progresses on the measurement of charmonium spectrum and their decays, which provide important constraints on phenomenological approaches.

As the first charmonium state discovered in the history, $J/\psi$ has been one of the most widely studied states in both experiment and theory. As a relatively heavier system compared with light $q\bar{q}$ mesons, the application of a nonrelativistic potential model (NR model) including color Coulomb plus linear scalar potential and spin-spin, spin-orbit interactions, has provided a reasonably good prescription for the charmonium spectrum. This success is a direct indication of the validity of the naive “quenched” $c\bar{c}$ quark model scenario as a leading approximation in many circumstances. A relativised version was developed by Godfrey and Isgur [2] (GI model), where a flavor-dependent potential and QCD-motivated running coupling are employed. In comparison with the nonrelativistic model, the GI model offers a reasonably good description of the spectrum and matrix elements of most of the $u, d, s, c$ and $b$ quarkonia [2, 4].

On the other hand, there also arise apparent deviations in the spectrum observables which give warnings to a simple $q\bar{q}$ treatment and more complicated mechanisms may play a role. As pointed out in Ref. [4], the importance of mixing between quark model $q\bar{q}$ states and two meson continua may produce significant effects in the spectrum observables. By including the meson loops, the quark model is practically “unquenched”. This immediately raises questions about the range of validity of the naive “quenched” $q\bar{q}$ quark model scenario, and the manifestations of the intermediate meson loops in charmonium spectrum and their decays. These issues become an interesting topic in the study of charmonium spectrum with high-statistic charmonium events from experiment. An example is the newly identified state $X(3872)$ and a possible assignment for it as a mixture of $c\bar{c}$ and $DD^*$ [3, 4], or open charm effects [5].

In the recent years, the intermediate meson loop is investigated in a lot of meson decay channels [8, 9, 10, 11, 12, 13, 14, 15, 16] as one of the important non-perturbative transition mechanisms, or known as final state interactions (FSI). In particular in the energy region of charmonium masses, with more and more data from Belle, BaBar, CLEO-c and BES, it is widely studied that intermediate meson loop may account for apparent OZI-rule violations [11, 12, 13, 14, 15, 16] via quark-hadron duality argument [17, 18, 19].

In this work, we shall study the radiative decays of $J/\psi$ and $\psi'$ into $\gamma\eta_c$ and $\gamma\eta'_c$. In the naive $q\bar{q}$ scenario, this type of decays is dominantly via magnetic dipole (M1) transitions which flip the quark spin. For $J/\psi \to \gamma\eta_c$ and $\psi' \to \gamma\eta'_c$, where the initial and final state $c\bar{c}$ are in the same multiplet, the spatial wavefunction overlap is unity at leading order, while $\psi' \to \gamma\eta_c$ will vanish due to the orthogonality between states of different multiplets. In this sense, the former decays are “allowed” while the latter is “hindered”. However, the inclusion of relativistic corrections from the quark spin-dependent potential will induce a nonvanishing overlap between states of different multiplets such that the decay of $\psi' \to \gamma\eta_c$...
is possible. Theoretical studies of the heavy quarkonium M1 transitions with relativistic corrections are various in the literature. Relativistic quark model calculations show that a proper choice of the Lorentz structure of the quark-antiquark interaction in a meson is crucial for explaining the $J/\psi \rightarrow \gamma \eta_c$ data. Systematic investigation of the M1 transitions in the framework of nonrelativistic effective field of QCD has been reported in Ref. [30], where relativistic corrections of relative order $v^2$ are included. For $J/\psi \rightarrow \gamma \eta_c$ the authors found $\Gamma_{J/\psi \rightarrow \gamma \eta_c} = (1.5 \pm 1.0)$ keV, which is in a good agreement with the data, but with quite large estimated uncertainties from higher-order relativistic corrections. Taking into account the transitions of $\psi' \rightarrow \gamma \eta_c$, the overall results for the M1 transitions still turn out to be puzzling. As studied by Barnes, Goffrey and Swanson [4] in the NR model and the relativistic GI model, although the models provide an overall consistent description of most of the existing charmonium states, theoretical results for the M1 transition have significant discrepancies compared with the experimental data [3]. For example, in both NR and GI model, the predicted partial decay widths for $J/\psi \rightarrow \gamma \eta_c$ are as large as about two times of the experimental value, while for $\psi' \rightarrow \gamma \eta_c$, the theoretical predictions are about one order of magnitude larger than the data [3]. For $\psi' \rightarrow \gamma \eta_c$, although the predicted partial decay widths 0.17-0.21 keV are smaller than the experimental upper limit ($< 0.67$ keV), it is possible that the M1 transition is very different from the experimental measurement.

Therefore, it is likely that there exist additional mechanisms beyond the $c\bar{c}$ transitions. This consideration thus prompts us to explore possible sources which can contribute to the charmonium radiative decay and cause deviations from the NR and GI model predictions, among which the intermediate meson loop transitions could be a natural mechanism.

As follows, we first brief the calculations from the NR and GI models for the M1 transitions, and then introduce the formalisms for the intermediate meson loop contributions in Section II. The results and discussions will be presented in Section III.

II. M1 TRANSITION IN NR AND GI MODEL

The detailed study of the M1 transition was given by Barnes et al. in Ref. [4], and here we quote their standard formula to incorporate the intermediate meson loop contributions which is to be introduced later.

In Ref. [4], the partial decay width via M1 transition is evaluated by

$$\Gamma_{M1}(n^{2S+1}L_J \rightarrow n^{2S'+1}L_J' + \gamma) = \frac{4}{3} \frac{2J' + 1}{2L + 1} \delta_{LL'} \delta_{S,S'} \frac{\epsilon_i^2}{m_c^2} |\langle \psi_f | \psi_i \rangle|^2 E_i \gamma E_f M_i,$$

(1)

where $n$ and $n'$ are the main quantum number of the initial and final state charmonium meson; $S$ ($S'$), $L$ ($L'$) and $J$ ($J'$) are the initial (final) state spin, orbital angular momentum and total angular momentum. $E_i$ and $E_f$ denote the final state photon and meson energy, respectively, while $M_i$ is the initial $c\bar{c}$ meson mass. $|\psi_i\rangle$ and $|\psi_f\rangle$ are the spatial wavefunctions of the initial and final state $c\bar{c}$ mesons, respectively.

In the GI model, phase space factor $E_f/M_i$ is not included though it is close to unity in many considered cases. In both GI and NR model, a recoil factor $j_0(\nu r/2)$ is included. We quote the results from Ref. [4] for future comparison.

In order to incorporate the intermediate meson loop contributions, we derive the effective $V\gamma P$ couplings due to the M1 transition from Eq. (1) by defining

$$\mathcal{M}_{f1}(M1) \equiv \frac{g_{V\gamma P}}{M_i} \varepsilon_{\alpha \beta \mu \nu} P^\alpha P^\beta P^\mu \varepsilon_{\gamma},$$

(2)

where $P_i$ and $P_{\rho}$ are four-vector momentum of the initial meson and final state photon, respectively, and $\varepsilon_i$ and $\varepsilon_{\gamma}$ are the corresponding polarization vectors. From Ref. [4], we know that these extracted effective $g_{V\gamma P}$ couplings for $J/\psi, \psi' \rightarrow \gamma \eta_c$ apparently overestimate the experimental data. Thus, the introduction of the intermediate meson loop contributions, which unquench the naive $c\bar{c}$ configurations, is supposed to cancel the M1 transition amplitudes via destructive interferences.
III. INTERMEDIATE MESON LOOP CONTRIBUTIONS

The inclusion of the intermediate meson loops in meson decays somehow “unquenches” the naive quark model. A full consideration of such an effect requires systematic coupled channel calculations for e.g. the charmonium mass spectrum [34]. An interesting feature arising from the low-lying charmonia, such as \( \eta_c, \eta_c', J/\psi, \) and \( \psi', \) is that their masses are lower than the open charmed meson decay channels. As a consequence, the lowest open charmed meson decay channels are expected to be dominant if they are the leading contributions to the sum over all intermediate virtual states are from those having less virtualities.

It should be pointed that intermediate states involving flavor changes turn out to be strongly suppressed. One reason is because of the large virtualities involved. The other is because of the OZI rule suppressions. Therefore, intermediate state contributions such as \( \rho \pi \) etc., are negligibly small.

Following the above consideration, we thus investigate \( D\bar{D}(D^*) \), \( D\bar{D}^*(D^*) \) and \( D\bar{D}^*(D) \) loops as the major contributions to \( J/\psi \to \gamma \eta_c \), and \( \psi' \to \gamma \eta_c, \gamma \eta_c' \) as illustrated in Fig. 1. We stress that although some of the vertices in the loop may violate gauge invariance, such as \( J/\psi DD \), the overall antisymmetric property is retained for the loops. The loop contributions hence only provide corrections to the \( VVP \) coupling strength for the external fields, but not change their antisymmetric tensor structure, no matter \( V \) is a massive vector meson or photon. Apart from the transitions in Fig. 1, the contact transitions in Fig. 2 will also contribute to the decay amplitude. We show that the processes of Fig. 2 are gauge invariant by themselves. In brief, due to the property of the antisymmetric tensor coupling of \( VVP \), where both \( V \) and \( P \) are external fields here, hadronic loop corrections are guaranteed to be gauge invariant in this effective Lagrangian approach.

The detailed formulation is given in the following subsections.

A. Intermediate \( D\bar{D}(D^*) + c.c. \) loop

The transition amplitude for an initial vector charmonium (\( J/\psi \) or \( \psi' \)) decay into \( \gamma \eta_c \) or \( \gamma \eta_c' \) via \( D\bar{D}(D^*) \) can be expressed as follows:

\[
\mathcal{M}_{f1} = \int \frac{d^4 p_2}{(2\pi)^4} \sum_{D_{pol}} \frac{T_1 T_2 T_3}{a_1 a_2 a_3} \mathcal{F}(p_2^2),
\]

where the vertex functions are

\[
\begin{align*}
T_1 &= ig_1 (p_1 - p_3) \cdot \varepsilon_i \\
T_2 &= \frac{ig_2}{m_2} \varepsilon_{\alpha \beta \mu \nu} P_\gamma^\alpha \varepsilon_{\beta \gamma} p_2^\mu \varepsilon_2^\nu \\
T_3 &= ig_3 (P_f + p_3) \cdot \varepsilon_2
\end{align*}
\]

where \( g_1, g_2, \) and \( g_3 \) are the coupling constants at the meson interaction vertices (see Fig. 1). The four vectors, \( P_1, P_\gamma, \) and \( P_f \) are the momenta for the initial vector, final state \( \gamma \) and pseudoscalar meson, respectively, while four-vector momenta, \( p_1, p_2, \) and \( p_3 \) are for the intermediate mesons, respectively, and \( a_1 = p_1^2 - m_{\pi}^2, a_2 = p_2^2 - m_{\rho}^2, \) and \( a_3 = p_3^2 - m_{\eta_c}^2 \) are the denominators of the propagators of these intermediate mesons.

As being studied in Ref. [16], this loop diverges logarithmically. Thus, a form factor to suppress the divergence and take into account the momentum-dependence of the vertex couplings is included:

\[
\mathcal{F}(p^2) = \left( \frac{\Lambda^2 - m_{\pi}^2}{\Lambda^2 - p^2} \right)^n,
\]

where \( n = 1, 2 \) correspond to monopole and dipole form factors, respectively. An empirical argument applied here is that in the \( P \)-wave \( V \to VP \) decay the form factor favors a dipole form. We hence deduce the loop transition amplitudes with a dipole form factor.
Substitute the vertex couplings of Eq. (11) into Eq. (3), the integral has an expression:

\[
\mathcal{M}_{fi} = \int \frac{d^4p_2}{(2\pi)^4} \sum_{D^*_{pol}} \left[ ig_1(p_1 - p_3) \cdot \varepsilon_1 \left( \frac{\Lambda_{m_2}^L \varepsilon_{\alpha \beta \mu \nu} \delta_{\gamma \delta}}{p_1^2 - m_1^2} \right) [ig_3(P_f + p_3) \cdot \varepsilon_2] \right] \mathcal{F}(p_2^2) .
\]

(6)

With a dipole form factor, we have

\[
\mathcal{M}_{fi} = \frac{\tilde{g}_a}{M_f} \varepsilon_{\alpha \beta \mu \nu} P^\alpha_\gamma \varepsilon_\gamma^\beta P^\mu_\nu \varepsilon_\nu^\nu,
\]

(7)

where

\[
\tilde{g}_a \equiv -\frac{g_1 g_2 g_3 M_i}{m_2} \int_0^1 dx \int_0^{1-x} dy \frac{2}{(4\pi)^2} \left[ \log \frac{\Delta(m_1, m_3, \Lambda)}{\Delta(m_1, m_3, m_2)} \right]
\]

\[\Delta(a, b, c) \equiv -(M_i^2 - M_f^2)(1 - x - y)x + M_i^2 x^2 + a^2(1 - x - y) - (M_f^2 - b^2)x + yc^2 .\]

(8)

where the function \( \Delta \) is defined as

In the intermediate meson exchange loop, coupling \( g_2 \) can be determined via the experimental information for \( D^0 \to D^0 \gamma (D^0 \to D^0 \gamma) \), i.e.

\[
g_2^2 = \frac{12\pi M_i^2}{\Gamma_\text{tot}} \Delta_{D^0 \to D^0 \gamma},\]

(9)

where \( \Gamma_{D^0 \to D^0 \gamma} = (38.1 \pm 2.9)\% \times \Gamma_{\text{tot}} \) is given by experiment[3]. We neglect the contributions from the charged meson exchange loop since \( \Gamma_{D^{\pm} \to D^0 \gamma} = (1.6 \pm 0.4)\% \times 96 \text{ keV} \) is about two orders of magnitude smaller than \( \Gamma_{D^{0} \to D^0 \gamma} \).

For coupling constant \( g_1 \), especially \( g_{J/\psi DD^*} \), there are several methods suggested in the literature including quark model using heavy quark effective theory approach[35], QCD sum rule[36, 37], SU(4) symmetry and vector meson dominance (VMD) model[38]. They typically give a value of order one for \( g_{J/\psi DD^*} \). In this work, we adopt \( g_{J/\psi DD^*} = 7.20 \) which is consistent with the value from Ref. [35].

For the \( g_{D^*D^0} \) coupling, we assume

\[
g_{D^*D^0} = g_{J/\psi DD^*} .
\]

(10)

B. Intermediate \( D^0 \bar{D}^*(D^*) \) + c.c. loop

As shown by Fig.[1(b)], the transition amplitude from the intermediate \( D^0 \bar{D}^*(D^*) \) + c.c. loop can be expressed the same form as Eq. (3) except that the vertex functions change to

\[
\begin{align*}
T_1 &\equiv \frac{f_{D^0}}{M_f} \varepsilon_{\alpha \beta \mu \nu} P^\alpha_\gamma \varepsilon_1^\beta p_3^\mu \varepsilon_3^\nu , \\
T_2 &\equiv \frac{f_{D^*}}{M_f} \varepsilon_{\alpha \beta \mu \nu} p_2^\alpha \varepsilon_2^\beta p_3^\mu \varepsilon_3^\nu , \\
T_3 &\equiv \frac{f_{D^*}}{M_f} \varepsilon_{\alpha \beta \mu \nu} p_2^\alpha \varepsilon_2^\beta p_3^\mu \varepsilon_3^\nu
\end{align*}
\]

(11)

where \( f_{1,2,3} \) are the coupling constants. With a dipole form factor the integration gives

\[
\mathcal{M}_{fi} = \frac{\tilde{g}_b}{M_f} \varepsilon_{\alpha \beta \mu \nu} P^\alpha_\gamma \varepsilon_\gamma^\beta P^\mu_\nu \varepsilon_\nu^\nu,
\]

(12)

where

\[
\tilde{g}_b \equiv \frac{f_{1} f_{2} f_{3}}{m_2 M_f} \int_0^1 dx \int_0^1 dy \int_0^{1-x-y} dz (1-x-y-z) \frac{2}{(4\pi)^2} \left( \frac{A}{\Delta_1^2} - \frac{B}{\Delta_1^2} \right) ,
\]

(13)
with
\[ A = \frac{1}{4}[1 - x - \frac{z}{2}(M_i^2 - M_f^2) + xM_f^2], \]
\[ B = -\frac{x}{4}(M_i^2 - M_f^2)[z(1 - x)(M_i^2 - 3M_f^2) - (M_i^2 - M_f^2)], \]
\[ \Delta_1 = -xz(M_i^2 - M_f^2) + zm_i^2 + ym_f^2 + xm_f^2 + (1 - x - y - z)\Lambda^2. \] (15)

In the above equation the intermediate meson masses \( m_{1,2,3} \) are from the \( DD^*(D^*) \) loop, which are different from those in Eq. (7). \( f_{1,2,3} \) denotes the corresponding vertex coupling constants.

In the \( DD^*(D^*) + \text{c.c.} \) loop, the coupling constant \( g_{J/\psi D^*D} \) is related to \( g_{J/\psi D\bar{D}} \) by the relation of the heavy quark mass limit [15]:
\[ g_{J/\psi D^*D} = g_{J/\psi D\bar{D}}/\bar{M}_D, \] (16)
where \( \bar{M}_D \) corresponds to the mass ratio of \( M_D/M_{D^*} \). Similarly, we have \( g_{\psi' D^*D} = g_{\psi' D\bar{D}}/\bar{M}_D \).

For \( \psi' \rightarrow J/\psi \eta_c \) and \( \gamma \eta_c' \), we assume that \( g_{\psi' D\bar{D}} = g_{J/\psi D\bar{D}} \cdot g_{D^0 D^0 \eta_c} = g_{J/\psi D^*D}, \) and \( g_{D^0 D^0 \eta_c'} = g_{\psi' D^*D} \), which are consistent with the \( ^3P_1 \) model [4]. In Table I the values of the coupling constants are listed.

C. Intermediate \( DD^*(D^*) + \text{c.c.} \) loop

The transition amplitude from the intermediate \( DD^*(D^*) + \text{c.c.} \) loop can contribute via charged intermediate meson exchange. Treating the intermediate mesons as fundamental degrees of freedom, we eventually neglect the contributions from the non-zero magnetic moments of the \( D \) mesons. The charge-neutral loop is thus suppressed due to the vanishing \( D^0 D^0 \gamma \) electric coupling. Therefore, we only consider the charged meson loop contributions as shown by Fig. 1(c). The transition amplitude can also be expressed in a form as Eq. (3) with the vertex functions
\[
\begin{align*}
T_1 &\equiv \frac{if_1' f_2' f_3'}{M_i} \varepsilon_{i\alpha} \varepsilon_{i\alpha} P^\alpha \varepsilon_i^\beta \varepsilon_i^\gamma f_3^\mu \varepsilon_3^\nu, \\
T_2 &\equiv if_2' (p_1 - p_2) \cdot \varepsilon_\gamma, \\
T_3 &\equiv if_3' (P_f - p_2) \cdot \varepsilon_3,
\end{align*}
\] (17)
where \( f_{1,2,3}' \) are the coupling constants and \( \mathcal{F}(p_2^2) \) is the form factor. With a dipole form factor the integration gives
\[ \mathcal{M}_{f_i} = \tilde{g}_c \frac{f_1' f_2' f_3'}{M_i} \varepsilon_{i\alpha} \varepsilon_{i\alpha} P^\alpha \varepsilon_i^\beta \varepsilon_i^\gamma f_3^\mu \varepsilon_3^\nu, \] (18)
where
\[ \tilde{g}_c = f_1' f_2' f_3' \int_0^1 dx \int_0^{1-x} dy \frac{2}{(4\pi)^2} \left[ \log \frac{\Delta(m_1, m_3, \Lambda)}{\Delta(m_1, m_3, m_2)} \right. \\
\left. - \frac{y(\Lambda^2 - m_2^2)}{\Delta(m_1, m_3, \Lambda)} \right]. \] (19)

It is interesting to note that the integral of the \( DD^*(D^*) + \text{c.c.} \) loop has a similar form as that of \( DD(D^*) \). However, we expect that contributions from this loop integral will be relatively suppressed since coupling \( f_2' \) is taken as the unit charge \( e = (4\pi\alpha_e)^{1/2} \).

D. Contact diagrams

The contact diagrams of Fig. 2(a) and (b) (as an example in \( J/\psi \rightarrow \gamma \eta_c \)) arise from gauging the strong \( J/\psi(\psi') D^* D \) and \( \eta_c(\eta_c') D^* D \) interaction Lagrangians containing derivatives. The general form of the
transition amplitude of Fig. 2(a) and (b) can be expressed as follows:

$$\mathcal{M}_{fi} = \int \frac{d^4 p_2}{(2\pi)^4} \sum_{D^+_\text{pol}} \frac{T_i T_2}{a_1 a_2} \mathcal{F}(p_2^2),$$

(20)

where $\mathcal{F}(p_2^2)$ is the form factor as before, and $T_i(i = 1, 2)$ are the vertex functions. For Fig. 2(a), the expressions of $T_i(i = 1, 2)$ are:

$$
\begin{align*}
T_1 &= \frac{h_{1e}}{M_i} \varepsilon_{\alpha\beta\mu\nu} (\varepsilon^\alpha \varepsilon^\beta \varepsilon^\nu \varepsilon^\gamma + p_i^\alpha \varepsilon_i^\beta \varepsilon_i^\mu \varepsilon_i^\nu), \\
T_2 &= 2ih_2 P^i \varepsilon_i,
\end{align*}

(21)

where $h_{1,2}$ represent the $J/\psi(\psi')D^* D$ and $\eta_c(\eta'_c)D^* D$ coupling constants, respectively, and their values have been given in Table I.

Using the Feynman parameter scheme, the amplitude for Fig. 2(a) can be reduced to

$$M_{fi} = \frac{2ih_1 h_2 e e_{\alpha\beta\mu\nu}}{M_i} \varepsilon_{\alpha\beta\mu\nu} \int \frac{d^4 p_2}{(2\pi)^4} \left[ \varepsilon^\alpha \varepsilon^\beta \varepsilon_i^\mu p_i^\nu + p_i^\alpha \varepsilon_i^\beta \varepsilon_i^\mu \varepsilon_i^\nu \right] \left( -P_f^\nu + \frac{p_f^\rho p_f^\sigma}{m_f^2} \right) (m_2^2 - \Lambda^2)^2

(22)

$$

with

$$\triangle_2 = x^2 M_f^2 - x M_f^2 + x m_1^2 + y m_2^2 + (1 - x - y) \Lambda^2,$$

(24)

and

$$\tilde{g}_4 = h_1 h_2 e \int_0^1 dx \int_0^{1-x} dy \frac{1}{(4\pi)^2} \left( \frac{1}{3 \triangle_2^2} - \frac{1}{6 \triangle_2} \right).$$

(25)

For Fig. 2(b), the vertex functions are:

$$
\begin{align*}
T_1 &= \frac{h_{1e}}{M_i} \varepsilon_{\alpha\beta\mu\nu} P^\alpha_i \varepsilon_i^\beta \varepsilon_i^\mu \varepsilon_i^\nu, \\
T_2 &= 2eh_2 \varepsilon_i \varepsilon_i,
\end{align*}

(26)

The amplitude can then be reduced to

$$M_{fi} = \frac{-2ih_1 h_2 e e_{\alpha\beta\mu\nu}}{M_i} \varepsilon_{\alpha\beta\mu\nu} P^\alpha_i \varepsilon_i^\beta \varepsilon_i^\mu \varepsilon_i^\nu \int \frac{d^4 p_2}{(2\pi)^4} \left( p_i^\rho p_i^\sigma - m_i^2 \right) \left( p_i^2 - m_i^2 \right) (p_f^2 - \Lambda^2)^2

(27)

$$

with

$$\triangle_3 = x^2 M_f^2 - x M_f^2 + x m_1^2 + y m_2^2 + (1 - x - y) \Lambda^2.$$

(29)

Note that the integrand has an odd power of the internal momentum, the amplitude will vanish and has no contribution to the VVP coupling.

The above deduction shows that only Fig. 2(a) has nonvanishing contributions to the transition amplitude. Meanwhile, gauge invariance is also guaranteed for the contact diagrams. The divergence of the loop integral is eliminated by adding the dipole form factor as in Fig. 1.
IV. RESULTS AND DISCUSSIONS

Proceed to numerical results from the intermediate meson exchange loops, the undetermined quantities include the cut-off energy $\Lambda$ in the dipole form factor and the relative phases among those amplitudes. The transition amplitude accommodating the M1 and intermediate meson exchange loops, i.e. $DD(D^*)$, $DD^*(D^*)$, and the contact term, can then be expressed as

$$\mathcal{M}_f = \frac{1}{M_f} [g_{\gamma\gamma p} + \tilde{g}_0 e^{i\delta_b} + \tilde{g}_3 e^{i\delta_c} + \tilde{g}_4 e^{i\delta_d}] \epsilon_{\alpha\beta\mu\nu} F^{\alpha}_{\gamma} F^{\beta}_{\gamma} F^{\mu}_{\eta} F^{\nu}_{\eta},$$

where $g_{\gamma\gamma p}$ is a real number and fixed to be positive. Couplings $\tilde{g}_0$, $\tilde{g}_3$, and $\tilde{g}_4$, calculated by the loop integrals can be complex numbers in principle. In this interested case, since the decay threshold of the intermediate mesons are above the initial meson ($J/\psi$ and $\psi'$) masses, the absorptive part of the loop integrals vanishes as a consequence. However, there might exist relative phases among those transition amplitudes. We hence include possible relative phases $\delta_{a,b,c,d}$ in the above expression.

Note that the loop contributions are supposed to provide cancellations to the M1 amplitude which is real. We thus simply take $\delta_{a,b,c,d} = 0$ or $\pi$. In this way, we have several phase combinations which are to be examined in the numerical calculation. Yet there is still a free parameter $\Lambda$ to be constrained.

We find that a reasonable constraint on the model can be achieved by requiring a satisfactory of the following conditions: i) For either constructive ($\delta = 0$) or destructive phases ($\delta = \pi$), the same value of $\Lambda$ is needed to account for $J/\psi \to \gamma \eta_c$, $\psi' \to \gamma \eta_c$ simultaneously. ii) The value of $\Lambda$ is within the commonly accepted region, 1.5 $\sim$ 2.5 GeV. iii) The prediction for $\psi' \to \gamma \eta'_c$, with the same $\Lambda$ is well below the experimental upper limit, $BR(\psi' \to \gamma \eta'_c) < 2.0 \times 10^{-3}$.

Imposing the above conditions on fitting the $\Lambda$ parameter, we obtain $\Lambda = 2.39$ GeV as the best fit with $\delta_{a,b,c,d} = \pi$, i.e. contributions from the loop integrals provide cancellations to the M1 transition amplitudes and there is no need for abnormal relative phases among the intermediate meson exchanges.

The numerical results for the intermediate meson exchanges have some predominant features. We find that the $DD(D^*)$ and $DD^*(D^*)$ loops have relatively large contributions while the $DD^*(D)$ loop is quite small. The contributions from the contact term are negligibly small in $J/\psi \to \gamma \eta_c$ and $\psi' \to \gamma \eta'_c$, while relatively large in $\psi' \to \gamma \eta_c$.

In Table I the fitted branching ratios are listed and compared with the GI model M1 transitions. We also list the exclusive contributions from the triangle diagrams of Fig. 1 and contact diagrams of Fig. 2 as a comparison. For $J/\psi \to \gamma \eta_c$, we find that the magnitude of the meson loop amplitude is smaller than the M1 amplitude, while for $\psi' \to \gamma \eta_c$, the absolute loop amplitude turns to be larger than the M1. With $\Lambda = 2.39$ GeV, we obtain $\Gamma(J/\psi \to \gamma \eta_c) = 1.59$ keV which is located at the upper limit of the experimental data, $\Gamma_{\text{exp}}(J/\psi \to \gamma \eta_c) = (1.21 \pm 0.37)$ keV. For $\psi' \to \gamma \eta_c$, we have $\Gamma(\psi' \to \gamma \eta_c) = 0.86$ keV, which is agree well with the data, $\Gamma_{\text{exp}}(\psi' \to \gamma \eta_c) = (0.88 \pm 0.13)$ keV. Taking into account the still-large uncertainties with the data for $J/\psi \to \gamma \eta_c$, the inclusion of the intermediate meson loop contributions significantly improves the theoretical results.

With the fixed $\Lambda$, the partial decay width for $\psi' \to \gamma \eta'_c$ is calculated as a prediction. The pure M1 transition predicts $\Gamma_{M1}(\psi' \to \gamma \eta'_c) \simeq 0.17$ keV, while the hadronic loops contribute $\Gamma_{HL}(\psi' \to \gamma \eta'_c) \simeq 0.054$ keV. The cancellation from the hadron loops thus leads to $\Gamma_{\text{all}}(\psi' \to \gamma \eta'_c) \simeq 0.032$ keV which is well below the experimental upper limit, 0.67 keV. Note that in all these three channels, the hadronic loop cancellations from the real part of the amplitudes possess the same relative sign to the M1 amplitudes. This makes the decay of $\psi' \to \gamma \eta'_c$ extremely interesting. As the pure M1 transition still predicts a sizeable partial width about 0.17 keV while our hadronic loop cancellation predicts a much smaller value, improved measurement of this quantity will help us gain further insights into the decay mechanisms.

For other relative phases, we find that there does not exist a common value for $\Lambda$ to fit the data for $J/\psi \to \gamma \eta_c$ and $\psi' \to \gamma \eta_c$ simultaneously. To summarize, in this work we have studied the hadronic meson loop contributions to the $J/\psi$ and $\psi'$ radiative decays into $\gamma \eta_c$ or $\gamma \eta'_c$. In the framework of effective Lagrangian phenomenology, the intermediate meson exchange loops provide corrections to the leading couplings extracted from potential quark models. In comparison with the NR and GI model, the meson loop contributions turn to cancel the NR and GI amplitudes. It is interesting to see that the meson loop contributions in $J/\psi \to \gamma \eta_c$ is smaller than the M1 transition in magnitude, while in $\psi' \to \gamma \eta_c$, the situation is opposite. Note that the pure M1
contribution in $\psi' \rightarrow \gamma \eta_c$ is about one order of magnitude larger than the experimental data, the meson loop contributions turn out to be even larger. This mechanism suggests significant cancellations between the M1 and meson loop amplitudes. It raises questions on the naive $q \bar{q}$ solution for the meson spectrum, and could be a manifestation of the limit of the quenched quark model scenario.

As a prediction from this model, we calculate the partial decay width of $\psi' \rightarrow \gamma \eta_c'$. It gives a value about one order of magnitude smaller than the experimental upper limit. Improved measurement of this decay channel is strongly recommended.

It is interesting to note that our model results are similar to those from a relativistic quark model calculation by Ebert, Faustov and Galkin [27], who find that a proper choice of the Lorentz structure of the quark-antiquark interaction in a meson is crucial for accounting for the M1 transition data. In our approach we extract the effective couplings from the NR and relativised GI model and then combine it with the gauge invariant meson loop corrections. The validity of this approach is guaranteed by the property of the unique antisymmetric tensor coupling for $VVP$ fields. In the framework of effective Lagrangian phenomenology the corrections to the leading contributions are introduced as coupling form factors.

Although we also observe strong sensitivities of the hadronic loop contributions to the cut-off energy $\Lambda$, the advantage of this approach is that the number of parameters is limited. In fact, there is little freedom for the effective couplings at vertices. By a coherent study of $J/\psi$ and $\psi' \rightarrow \gamma \eta_c$, we find that the constraint on the $\Lambda$ value is very tight. Certainly, it should be noted that our treatment of the relative phases is empirical though the favor of a destructive phase between the M1 transition amplitudes and hadron loops turns to be consistent with what one naturally expects. Note that it has been shown in Ref. [23] that a proper modification of the color Coulomb potential strength will simultaneously account for the branching ratios for $J/\psi \rightarrow \gamma \eta_c$ and $\psi' \rightarrow \gamma \eta_c$. It seems to support that the intermediate hadronic meson loops are responsible, at least partly, for such a modification, and hence break down the naive $q \bar{q}$ scenario.

The study of non-perturbative effects arising from intermediate meson loops in heavy quarkonium decays has attracted a lot of attention recently. Although such approaches still experience large uncertainties from the divergent behavior of the loop integrals, we expect that improved experimental measurements with high statistics, such as at BES and CLEO-c, provide more and more stringent constraints on the hadronic loops. Thus, insights into the effective degrees of freedom within hadrons and their decay mechanisms can be gained. This requires systematic analysis of both spectroscopy and coupled channels for which more and more theoretical efforts are undergoing.

Acknowledgement

Q.Z. would like to thank T. Barnes, K.T. Chao, Y. Jia and B.S. Zou for very useful discussions. This work is supported, in part, by the U.K. EPSRC (Grant No. GR/S99433/01), National Natural Science Foundation of China (Grant No.10675131 and 10491306), and Chinese Academy of Sciences (KJCX3-SYW-N2).

[1] T. Appelquist, A. De Rujula, H. D. Politzer and S. L. Glashow, Phys. Rev. Lett. 34, 365 (1975).
[2] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[3] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 1 (2006).
[4] T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D 72, 054026 (2005) [arXiv:hep-ph/0505002].
[5] N. A. Tornqvist, Phys. Lett. B 590, 209 (2004) [arXiv:hep-ph/0402237].
[6] E. S. Swanson, Phys. Lett. B 588, 189 (2004) [arXiv:hep-ph/0311229].
[7] E.J. Eichten, K. Lane and C. Quigg, Phys. Rev. D 69, 094019 (2004).
[8] X.Q. Li, D.V. Bugg and B.S. Zou, Phys. Rev. D 55, 1421 (1997).
[9] H.Y. Cheng, C.K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005).
[10] V.V. Anisovich, D.V. Bugg, A.V. Sarantsev, and B.S. Zou, Phys. Rev. D 51, R4619 (1995).
[11] Q. Zhao and B. S. Zou, Phys. Rev. D 74, 114025 (2006) [arXiv:hep-ph/0606196].
Q. Zhao, Phys. Lett. B 636, 197 (2006) [arXiv:hep-ph/0602216].
[12] J. J. Wu, Q. Zhao and B. S. Zou, Phys. Rev. D 75, 114012 (2007) [arXiv:0704.3652 [hep-ph]].
[13] X. Liu, X.Q. Zeng, and X.Q. Li, Phys. Rev. D 74, 074003 (2006).
[14] Q. Zhao, B. S. Zou and Z. B. Ma, Phys. Lett. B 631, 22 (2005) [arXiv:hep-ph/0508088].
[15] G. Li, Q. Zhao and B.S. Zou, [arXiv:0706.0384 [hep-ph]].
[16] H.J. Lipkin, Phys. Rev. Lett. 13, 590 (1964); 14, 513 (1965); Phys. Rep. 8C, 173 (1973); Nucl. Phys. B244, 147(1984); B291, 720 (1987); Phys. Lett. B 179, 278 (1986).
[17] P. Geiger and N. Isgur, Phys. Rev. D 47, 5050 (1993).
[18] H.J. Lipkin and B.S. Zou, Phys. Rev. D 53, 6693 (1996).
[19] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. D. Lane and T. M. Yan, Phys. Rev. Lett. 34, 369 (1975) [Erratum-ibid. 36, 1276 (1976)].
[20] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D 17, 3090 (1978) [Erratum-ibid. D 21, 313 (1980)].
[21] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D 21, 203 (1980).
[22] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D 28, 2908 (1983).
[23] V. Zambetakis and N. Byers, Phys. Rev. D 30, 1924 (1984).
[24] X. Zhang, K.J. Sebastian and H. Grotch, Phys. Rev. D 47, 054005 (2003).
[25] H. Grotch, D.A. Owen and K.J. Sebastian, Phys. Rev. D 30, 1924 (1984).
[26] X. Zhang, K.J. Sebastian and H. Grotch, Phys. Rev. D 44, 1606 (1991).
[27] D. Ebert, R.N. Faustov and V.O. Galkin, Phys. Rev. D 67, 014027 (2003).
[28] X. Zhang, K.J. Sebastian and H. Grotch, Phys. Rev. D 44, 1606 (1991).
[29] H. Grotch, D.A. Owen and K.J. Sebastian, Phys. Rev. D 30, 1924 (1984).
[30] X. Zhang, K.J. Sebastian and H. Grotch, Phys. Rev. D 44, 1606 (1991).
[31] D. Ebert, R.N. Faustov and V.O. Galkin, Phys. Rev. D 67, 014027 (2003).
[32] T.A. Lahde, Nucl. Phys. A 714, 183 (2003).
[33] S. Godfrey and J. L. Rosner, Phys. Rev. D 64, 097501 (2001) [Erratum-ibid. D 66, 059902 (2002)] [arXiv:hep-ph/0105273].
[34] N. Brambilla, Y. Jia and A. Vairo, Phys. Rev. D 73, 054005 (2006).
[35] J. J. Dudek, R.G. Edwards, D.G. Richards, Phys. Rev. D 73, 074507 (2006).
[36] E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, [arXiv:hep-ph/0701208].
[37] N. Brambilla et al. [Quarkonium Working Group], [arXiv:hep-ph/0412158].
[38] T. Barnes, seminar at Peking University, June 5, 2007; Private communications.
[39] A. Deandrea, G. Nardulli and A. D. Polosa, Phys. Rev. D 68, 034002 (2003) [arXiv:hep-ph/0302273].
[40] R. D. Matheus, F. S. Navarra, M. Nielsen and R. Rodrigues da Silva, Phys. Lett. B 541, 265 (2002) [arXiv:hep-ph/0206198].
[41] M. E. Bracco, M. Chiapparini, F. S. Navarra and M. Nielsen, Phys. Lett. B 605, 326 (2005) [arXiv:hep-ph/0410071].
[42] Z. W. Lin and C. M. Ko, Phys. Rev. C 62, 034903 (2000) [arXiv:nucl-th/9912046].
[43] Y. Oh, W. Liu and C. M. Ko, Phys. Rev. C 75, 064903 (2007) [arXiv:nucl-th/0702077].

| Coupling constants | $|g_{J/\psi DD\gamma}|$ | $|g_{\psi' D\gamma}|$ | $|g_{D^* D\gamma}|$ | $|g_{D^* D\eta_c}|$ | $|g_{D^* D\eta_c'}| |g_{D^* D\gamma}|$ | $|g_{D^* D\gamma}|$
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| numerical value   | 7.20            | 7.20            | 4.34            | 4.34            | 3.64            | 3.64            | 6.86            |

TABLE I: The absolute values of coupling constants for the effective vertex interactions. Their relative phases are determined by the SU(4) flavor symmetry.

![Diagram](attachment:fig1.png)

FIG. 1: Schematic diagrams for $J/\psi \rightarrow \gamma \eta_c$ via (a) $D \bar{D}(D^*)$, (b) $D \bar{D}^*(D^*)$ and (c) $D \bar{D}^*(D^*)$ intermediate meson loops. Similar processes occur in $\psi' \rightarrow \gamma \eta_c$ and $\gamma \eta_c'$. 
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Initial meson & $J/\psi(1^3S_1)$ & $\psi'(2^3S_1)$ \\
\hline
Final meson & $\eta_c(1^1S_0)$ & $\eta'_c(2^1S_0)$ & $\eta_c(1^1S_0)$ \\
\hline
$\Gamma_{NR}^{M1}$ (keV) & 2.9 & 0.21 & 9.7 \\
$\Gamma_{GI}^{M1}$ (keV) & 2.4 & 0.17 & 9.6 \\
$\Gamma_C$ (keV) & $\sim 0$ & $\sim 0$ & 0.04 \\
$\Gamma_{Tri}$ (keV) & 0.096 & 0.063 & 17.91 \\
$\Gamma_{HL}$ (keV) & 0.083 & 0.054 & 16.20 \\
$\Gamma_{all}$ (keV) & 1.59 & 0.032 & 0.86 \\
$\Gamma_{exp}$ (keV) & $1.21 \pm 0.37$ & $< 0.67$ & $0.88 \pm 0.13$ \\
\hline
\end{tabular}
\caption{Radiative partial decay widths given by different processes are listed: $\Gamma_{NR}^{M1}$ and $\Gamma_{GI}^{M1}$ are the M1 transitions in the NR and GI model, respectively \cite{4}; $\Gamma_{Tri}$ are inclusive contributions from the triangle diagrams (Fig. 1); $\Gamma_C$ are from contact diagrams (Fig. 2); $\Gamma_{HL}$ denote the inclusive contributions from all the intermediate hadronic loops; while $\Gamma_{all}$ are coherent results including the M1 in the GI model and intermediate hadronic loops. The experimental data are from PDG2006 \cite{3}. The results are obtained at the cut-off energy $\Lambda = 2.39$ GeV.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{diagram.png}
\caption{The contact diagrams considered in $J/\psi \rightarrow \gamma \eta_c$. Similar diagrams are also considered in $\psi' \rightarrow \gamma \eta_c(\eta'_c)$.}
\end{figure}