What is more important for radiated power from cells - size or geometry?

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Abstract. The cytoskeleton, especially microtubules, is potential source of electrodynamic field of living eukaryote cells. Microtubule network is a very dynamic structure which is composed of highly polar molecules - tubulin heterodimers. Microtubules have their eigenmode vibrations in frequency range from kHz to GHz. We approximated electrical properties of tubulin heterodimer as an elementary electric dipole and described the longitudinal mechanical oscillation of one microtubule by spatial modulation function. Mechanical oscillations of tubulin heterodimers were approximated as a chain of rigid particles and described by the spatial modulation function. An optical branch was used as it is more efficient for generation of electromagnetic field. We also created asymmetrical model of microtubule network of dividing cell and symmetrical model of microtubule network of non-dividing cell. The field around oscillating microtubule was calculated as a vector sum of contributions from all elementary electric dipoles. Finally, we performed calculations with several cell sizes and numbers of microtubules in models and performed comparative analysis.

1. Introduction
Microtubules are cellular structures assumed to be the generators of electrodynamic field [1, 2, 3]. Several experimental works have reported the electrical oscillations of living cells in the frequency region kHz-GHz [4, 5, 6]. Although a theoretical analysis of radiated power from oscillators located in cellular membrane [1], and simple microtubule network [7] has been performed, no works reported parametric analysis of the the dependence of radiated electromagnetic power on the cell size. Yet it is clear that these kind of theoretical predictions are necessary to estimate required dimensions and sensitivity of the sensors for the measurement of cellular electrodynamic activity. In this paper, we provide first results from calculations of radiated electromagnetic power from symmetric radial distribution of microtubule network and the distribution of the microtubules during the cell division for several sizes of a cell.

2. Approximation of microtubule
Microtubules are mostly composed of 13 protofilaments. The basic subunit of one protofilament is tubulin heterodimer. Tubulin heterodimer contains two parts α- and β- tubulin dimer and has strong static dipole moment of 337 D (1.12 x 10^-27 Cm) in direction of microtubule axis (Figure 1b). Microtubule network changes structure during the cell cycle. Mitotic spindle is
created during mitosis. This structure of microtubule network ensures accurate distribution of genetic information (Figure 2). Increased intensity of electric field has been experimentally observed around dividing cells [8, 5].

Figure 1. a) Elementary electric dipole in Cartesian coordinate system, b) Approximation of electrical properties of tubulin heterodimer

Electric properties of tubulin heterodimer were approximated by an elementary electric dipole. Field of such dipole is described by these three equations 1 - 3.

$$H_\varphi = -\frac{Idl}{4\pi} k^2 \sin(\varphi_p)(\frac{1}{jkr} + \frac{1}{(jkr)^2})e^{-jkr}$$

(1)

$$E_r = -\frac{Idl}{4\pi} 2Zk^2 \cos(\varphi_p)(\frac{1}{(jkr)^2} + \frac{1}{(jkr)^3})e^{-jkr}$$

(2)

$$E_\varphi = -\frac{Idl}{4\pi} Zk^2 \sin(\varphi_p)(\frac{1}{jkr} + \frac{1}{(jkr)^2} + \frac{1}{(jkr)^3})e^{-jkr}$$

(3)

where $E$ is electric field intensity, $H$ is magnetic field intensity, $I$ is equivalent current, $dl$ is length of the dipole, $Z$ is wave impedance, $k$ is propagation constant, $\omega$ is angular frequency, $p$ is dipole moment, $j$ is imaginary unit ($j^2 = -1$) and other symbols are according to Figure 1a.

All equations include:

$$Idl = j\omega \hat{p}$$

(4)

3. Models of microtubule network
We created two types of models of microtubule network in program Matlab - symmetric and asymmetric model. The symmetric model is model of non-dividing cell (Figure 3). This model has symmetric and uniform distribution of microtubules on sphere 1. In symmetric model,

1 Sphere is model of centrosome with diameter 200 nm
There is the same length and number of elementary electric dipoles in each microtubule. Each size of symmetric model of microtubule network has different number of microtubules. The second model of microtubule network is a model of mitotic spindle. The polar and kinetochore microtubules have different length and different numbers of elementary electric dipoles. Astral microtubules have the same length and number of elementary electric dipoles. Each size of asymmetric model of microtubule network has the same number of microtubules - 100 astral microtubules (50 from each centrosome) and 200 polar and kinetochore microtubules (100 from each centrosome). The symmetric and asymmetric models are composed of two types of points - green and red. The red point represents the centre of gravity of one tubulin heterodimer and the green point describes the direction of dipole moment \(^2\) (Figure 1 and 5).

**Figure 3.** Model of symmetric microtubule network. Each coloured line represents one microtubule.

**Figure 4.** Model of mitotic spindle, asymmetric model.

The dimensions, numbers of microtubules and numbers of elementary electric dipoles in models of four sizes of microtubule networks are in the tables 2 and 1. Comparison of sizes of models are in Figure 6.

**Figure 5.** Detail of models of microtubule network. Red points represent centres of gravity of tubulin heterodimers and green points describe the direction of dipole moment of tubulin heterodimers.

\(^2\) the end point of dipole moment of one tubulin heterodimer
Table 1. Parameters of models of symmetric microtubule network. HET - heterodimer, PT - protofilament

| model NO. | diameter [µm] | MTs in one model | HETs in one PT | HETs in one model |
|-----------|---------------|-----------------|----------------|------------------|
| 1)        | 3             | 121             | 175            | 275 275          |
| 2)        | 5             | 116             | 300            | 452 400          |
| 3)        | 7             | 114             | 425            | 629 850          |
| 4)        | 10            | 158             | 613            | 1 259 102        |

Table 2. Parameters of models of asymmetric microtubule network. HET - heterodimer, PT - protofilament, K - kinetochore, P - polar

| model NO. | A [µm] | B [µm] | K+P MTs in model | HETs in K+P MTs | astral MTs | HETs in 1 astral PT | HETs in 1 astral MT | HETs in model |
|-----------|--------|--------|------------------|-----------------|------------|---------------------|---------------------|--------------|
| 5)        | 4.28   | 1.00   | 200              | 448 058         | 50+50      | 80                  | 104 000             | 552 058      |
| 6)        | 6.88   | 1.67   | 200              | 730 808         | 50+50      | 134                 | 174 200             | 905 008      |
| 7)        | 9.48   | 2.33   | 200              | 1 014 078       | 50+50      | 188                 | 244 400             | 1 258 478    |
| 8)        | 17.85  | 6.69   | 200              | 2 162 862       | 50+50      | 269                 | 349 700             | 2 512 562    |

Figure 6. Comparison of sizes of models of microtubule networks sorted by number of elementary electric dipoles in model from a) to h). The models are in scale.

4. Parameters of calculations
We used two different surrounding media: lossy and lossless (physiological solution and lossless medium with permittivity of water). The parameters of surrounding media are in Table 4 (εr
Table 3. Parameters of surrounding medium around models of microtubule networks for 8 frequencies. $\varepsilon_r$ and $\sigma$ have subscript 1 for lossy medium and subscript 2 for lossless medium.

| $f$ [Hz] | 1 k, 1 M | 1 G | 10 G | 42 G | 100 G |
|----------|----------|-----|------|------|-------|
| $\varepsilon_{r1}$ [-] (relative permittivity) | 81 | 81 | 71 | 20 | 10 |
| $\sigma_{1}$ [S/m] (conductivity) | 1 | 1.33 | 17.22 | 39.67 | 68.50 |
| $\varepsilon_{r2}$ [-] (relative permittivity) | 81 | 81 | 71 | 20 | 10 |
| $\sigma_{2}$ [S/m] (conductivity) | 0 | 0 | 0 | 0 | 0 |

and $\sigma$ have subscript 1 for lossy medium and subscript 2 for lossless medium). We introduced a longitudinal shift between neighbouring protofilaments 4.92 nm (corresponds to lattice A). Mechanical longitudinal oscillation of tubulin heterodimers in protofilament is represented by oscillations of dipole moments in direction of microtubule axis ($p_{\text{static,axis}} = 337D$ static part of one dipole moment). The amplitude of kHz oscillations is estimated: $p_{\text{oscil,1}} = \frac{1}{2}p_{\text{static,axis}}$. Pelling [9, 10] measured oscillations of yeast cell wall in range 1 - 10 nm in kHz region, which was attributed to interaction of motor proteins with cytoskeleton. Therefore we used 1 nm amplitude for the microtubule oscillations in kHz region. In frequency region from MHz to GHz we calculated with amplitude of oscillation: $p_{\text{oscil,2}} = \frac{0.1}{2}p_{\text{static,axis}}$. This value is assumed to be of the level of thermal motion amplitude of proteins [11].

Oscillations of tubulin heterodimers in one protofilament were approximated by longitudinal oscillations of the chain of rigid two type particles, optical branch ([12]). Optical branch is able to generate and absorb electromagnetic field more effectively than acoustic branch. Movement of particles (tubulin heterodimers) and corresponding oscillations of the dipole moment are described by spatial modulation function:

$$p_{\text{mod}} = p_{\text{oscil}} \cdot \sin(dis)$$  \hspace{1cm} (5)

where $p_{\text{mod}}$ is the modulated dipole moment, $p_{\text{oscil}}$ is the oscillatory component of the dipole moment and $dis$ is basically distance along the distance in direction of microtubule axis. Modulation function is specific for each mode, minimal mode number is zero, maximal mode number (max) depends on the number of tubulin heterodimers, i.e. length of microtubule. The mode zero represents oscillations of all dipoles in phase and the max mode represents oscillations of neighbouring dipoles in antiphase. Our latest works showed that mode zero is more effective for generation electromagnetic field than higher modes. We assumed that all microtubules oscillate at mode zero in calculations in this paper.

The calculation of power around models was realised as a vector sum of contribution from all subunits (elementary electric dipoles) in one point of field of evaluation (PoFE). We repeated this calculations for all PoFEs. These PoFEs are distributed according to distribution of charges on the spherical surface with the condition of the smallest potential energy. The sphere has centre in the middle of model (Figure 7). In every PoFEs we calculate a radial part of Poynting vector - S and then power is

$$P = \sum_{j}^{np} S_j \cdot A_{\text{PoFE}}$$  \hspace{1cm} (6)

$$A_{\text{PoFE}} = \frac{4\pi r^2}{np}$$  \hspace{1cm} (7)
where \( P \) is power from one PoFE, \( A_{\text{PoFE}} \) is surface corresponding to one PoFE, \( r \) is radius sphere PoFEs, \( np \) number of PoFEs (in our calculations we used 81). We calculate also the electric intensity in planes parallel to axis of Cartesian coordinate system.

\[ \text{Figure 7. PoFEs - points of field evaluation (red dots) around model of mitotic spindle.} \]

5. Results

We present selected results of calculations of radiated power and electric intensity around eight models of microtubule networks. In calculations we used approximation of electrical properties of tubulin heterodimer by elementary electric dipole (equations: 1 - 3). The protofilaments oscillate in mode 0 which is the most effective mode for generation of electromagnetic field (all dipoles oscillate in phase). The frequencies of oscillations were considered to be from kHz to GHz region. Lossy and lossless medium was considered.

Figures 8 and 9 depict comparison of radiated power from eight models of microtubule networks.

\[ \text{Figure 8. P [log}_{10}(W)] - power generated by models of microtubule networks. Comparison of models. Oscillation frequency is 1 GHz, lossy medium and mode 0.} \]

A legend of pictures is sorted by numbers of elementary electric dipole of models (Figure 6). Variation of the starting distance different beginning of curves is caused by different sizes of models. The calculations of power start close to the border of microtubule network (symmetric: diameter/2, see Table 1 and asymmetric: \( A/2 \), see Table 2). The calculated power is higher in lossy medium than lossless medium at the cell wall but decreases faster with distance in radial direction from the model. This effect is caused by phase asymmetry between electric and magnetic field component in lossy medium. The phase shift is \( \pi/2 + \phi_k \) in lossy medium and \( \pi/2 \) in lossless medium (\( \phi_k \) is a phase of propagation constant \( k \) of electromagnetic field). The phase shift in lossy medium causes that the product of the near field components (\( E \sim \frac{1}{(jkr)^{3/2}} \)) and
$H \sim \frac{1}{(jk\tau)^2}$ is not zero and therefore contributes to the radiation energy in the radial direction. We use only far field components ($E \sim \frac{1}{jk\tau}$ and $H \sim \frac{1}{(jk\tau)^2}$) for the calculation of radiated power in lossless medium since the near field components cancel out perfectly. The Figure 9 shows radiated power in dependence on radial distance from the models. The difference between the value of power of models 1 and 8 is one order of magnitude. Model 8 is composed of 10 times more elementary electric dipole than model 1. This dependence is the same for other excitation frequencies. The dependence of excitation frequency on radiated power is depicted in Figure 10. The higher is the frequency, the higher is power in radial direction. Steeper decrease of power is observed in the distance where the field loses its near field character.

$\text{Figure 9. } P \left[\log_{10}(W)\right]$ - power generated from models of microtubule networks. Comparison of models. Oscillation frequency is frequency 1 GHz, lossless medium and mode 0.

$\text{Figure 10. } P \left[\log_{10}(W)\right]$ - power generated from model of microtubule network (model 8). Comparison of excitation frequencies. Lossy medium and mode 0.

Figures 12 - 15 depict the electric intensity in logarithmic scale in three orthogonal sections through origin of the Cartesian coordinate system (centre of model) of model 5.

6. Discussion and conclusion
Comparison of calculated radiated power from the eight models gives answer to the question "What is more important for radiated power from cells - size or geometry?" For calculations based on our models, the correct answer is size of the cell. More specifically, the number of elementary electric dipoles (tubulin heterodimers) but the size corresponds to the mass of tubulin heterodimers in the cell. It should be noted here that geometrical and phase symmetry of radiating sources is expected to play a role in the total radiated power. Here we presented model with apparent geometrical symmetry (non dividing cell) and phase symmetry (all dipoles oscillate in phase). However, symmetry is not perfect due to the helicity of microtubules,
Figure 11. $E_{\log_{10}(V/m)}$ - electric intensity around model 5. Every tenth elementary electric dipole in protofilament is depicted in this model figure. Black points represent centre of gravity of tubulin heterodimer and green points show direction of dipole moment.

Figure 12. $E_{\log_{10}(V/m)}$ - electric intensity around model 5.

Figure 13. $E_{\log_{10}(V/m)}$ - electric intensity around model 5 (ZX - plane).

Figure 14. $E_{\log_{10}(V/m)}$ - electric intensity around model 5 (YX - plane).

therefore the total number of radiators (tubulin dipoles) has greater influence on the radiated power than the geometry of the microtubule network.
Figure 15. $E[\log_{10}(V/m)]$ - electric intensity around model 5 (equatorial plane)

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