Simplified Infinite Fin Method to Model the Heat Transfer Associated with Slab Edges and Balconies

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Abstract. As building codes become more stringent in terms of thermal performance of building envelopes, and higher insulated wall assemblies are becoming more common, the heat flow due to major thermal bridges can contribute to a significant portion of the total heat transfer through a building façade. Characterizing different thermal bridging elements is essential not only to capture the thermal resistance of wall assemblies and understand the thermal efficiency of buildings, but also in terms of understanding the impact of each thermal bridging element and mitigation strategies that can be used. Numerical simulations are used widely to characterize different thermal bridging elements. However, not all designers have access, technical skills or time to complete numerical simulations to calculate the heat transfer loss through thermal bridges. In this study we propose an analytical method to integrate the effect of adding a slab edge/balcony/eyebrow into a clear-field wall assembly. The additional heat transfer due to the slab edge is calculated by considering the slab edge to be an infinite fin. The additional heat transfer is integrated into the clear-field as a quasi-convective heat transfer coefficient. The overall thermal resistance of the wall assembly is calculated by employing the parallel path method. Comparing the results obtained from this method with the numerical simulations which were benchmarked against guarded hot box results, an overall deviation of 1 to 8 percent was observed.

1. Introduction
Canada is working towards reducing its greenhouse gas emissions by at least 40 percent below 2005 level by 2030. In North America, residential and commercial buildings account for more than one third of all energy use and greenhouse gas emissions [1]. Improved energy conservation in buildings has been recognized as an important and cost-effective approach to reduce energy consumption and to mitigate greenhouse gas emissions. Most, if not all, governments and utilities have a mandate to encourage energy conservation in buildings and local jurisdictions have been adopting increasingly more stringent building energy efficiency standards. Space conditioning is one of the largest components of energy use in commercial, institutional, and residential buildings in North America. Building enclosure thermal performance is a critical consideration for reducing space heating loads and will be an increasingly important factor as authorities strive for lower energy consumption in buildings.
It is well understood that the thermal performance of building envelopes can be significantly affected by thermal bridging. Thermal bridges are localized areas of high heat flow through walls, roofs and other insulated building envelope components. Thermal bridging is caused by highly conductive elements that penetrate the thermal insulation or may also be caused by misaligned planes of thermal insulation. These paths allow heat flow to bypass the insulating layer thereby reducing the effectiveness of the insulation in providing resistance to heat loss to the building exterior. This heat flow can often be significant and seemingly well-insulated buildings may fail to meet energy code requirements, designer intent, or occupant expectations. Windows are normally considered as the largest thermal bridge in buildings because the thermal performance is often very low compared to the surrounding (as an example an R-2 aluminium frame window within an R27 insulated wall). After windows and doors, exposed concrete slab edges and balconies account for the second greatest source of building enclosure heat loss in multi-storey buildings [2].

Balconies, intermediate floor intersections, and exposed concrete slab edges are common components of condominium and other multi-storey buildings due to the market demand. These exposed components increase the heat transfer between conditioned spaces and the exterior since they not only increase the exposed surface area to the unconditioned space, but also they often interrupt the continuity of the exterior insulation. They can also create cold interior surface temperatures which may lead to reduction in occupant and increased risk of condensation/fungal growth. The available market solutions are merely being used because of the availability, cost and the lack of data regarding the their effects on the energy savings versus the added expense [2].

Slab edges, balconies and eyebrows can be categorized as linear thermal bridge elements. The linear thermal bridges are affected by a number of factors such as geometry, the material properties, the ambient temperature variation [3] as well as the convective and radiative heat transfer coefficients at the surfaces [4,5]. The in situ measurements of thermal behaviour of building elements is conducted by employing heat flux and temperature sensors [6-9]. Whereas, laboratory tests can be performed in more controlled environments and tests can be repeated [10]. Several numerical software tools are also widely being used to calculate the linear transmittance in different scenarios. Deque et al. [11] employed a two-stage modelling approach, the first involved accurately modelling the heat transfer in thermal bridges and then the reduced models were integrated in the other components of the envelope to be simulated. Ge and Baba [12] investigated the dynamic effect of thermal bridges on the energy performance of a residential building through three different methods: equivalent U-value, equivalent wall and direct 2D/3D modelling methods.

Different methods such as hand calculation methods (ASHRAE Fundamental [13] and National Energy Building Code of Canada [14]), Guarded Hot Box testing (ASTM 1363 [15]), in situ testing (ASTM 1155 [16]) and computer simulations (ISO 10211[17]) are being used to calculate the thermal bridging effects. These methods are known to be within 10 percent accuracy of the actual thermal behavior [18]. There are also thermal bridging catalogues such as the BC Hydro thermal bridging guide [19] that can provide the required parameters to account for different types of thermal bridges. However, the catalogues cannot account for any deviation in the geometry and material change from those mentioned in the catalogue so the number of cases that they can be used for is limited.

2D and 3D simulations are widely being used to quantify the linear and point transmittance of slab edges and balconies. However, not all practitioners have access to, nor the skills or time to
complete the computer simulations. In this study, we investigate the effect of adding a slab edge, intermediate floor intersection or a balcony on the overall thermal resistance of a Clearfield wall. Using the superposition method, the slab edge was considered as an infinite fin and the additional heat transfer due to the fin was calculated analytically and then integrated as a quasi-convective heat transfer coefficient to the Clearfield wall. The parallel path method was then employed to include the additional heat transfer into the overall thermal resistance of the wall assemblies. Different wall assemblies were studied and less than eight percent error was found between the proposed method and simulation results.

2. Methodology: Modelling slab edges as fins

In this section we discuss the correlations that are used to model a slab edge as an infinite fin. We investigate a wall assembly with a slab edge as can be seen in Figure 1 right. Using the parallel path method one can separate the heat flow paths into two paths as depicted in Figure 1 left. The first path is through the base wall assembly without the linear thermal bridge, and the second path is through the exterior building components, the fin and into the ambient climate. Our goal is to model the slab edge as an infinite fin and replace the fin by translating the additional heat transfer due to the extended surface into a heat transfer coefficient at the intersection of the fin base and the wall. The heat transfer coefficient can then be used in the parallel path method with the Clearfield wall U-value to obtain the overall thermal resistance of the assembly.

Starting with the equation for the parallel path method where the total thermal transmittance of the assembly, $U_{Assembly}$ (W m$^{-2}$ K$^{-1}$), can be written as,

$$U_{Assembly} = A_{O,f}U_O + A_{Bridge,f}U_{Bridge}$$

Where: $U_O = 1/R_O$ and $U_{Bridge} = 1/R_{Bridge}$ (subscript ‘O’ denotes the Clearfield path and ‘Bridge’ denotes the thermal bridge path) are the thermal transmittance values of the two paths through the wall (W m$^{-2}$ K$^{-1}$); $A_{O,f}$ and $A_{Bridge,f}$ are the fractional surface areas of the two parallel paths based on dividing each path surface area by the total area ($A_{total}$).

$$R_O = R_{Out,Film} + R_A + R_B + R_{Inner,Film}$$
$$R_{Bridge} = R_{Out,Film} + R_A + R_B + R_{Fin}$$

$$A_{O,f} = \frac{A_O}{A_{total}} = \frac{(2 x Z_1)}{(2 x Z_1) + Z_2}$$
$$A_{Bridge,f} = \frac{A_{Bridge}}{A_{total}} = \frac{(Z_2)}{((2 x Z_1) + Z_2)}$$

The external and internal convection coefficients can also be written as nominal resistances,

$$R_{Out,Film} = \frac{1}{h_{out}}$$ and $$R_{Inner,Film} = \frac{1}{h_{in}}$$

where $h_{out}$ and $h_{in}$ are 34 and 8.4 W m$^{-2}$ K$^{-1}$ respectively from (ASHRAE Handbook of Fundamentals [13]).

The heat transfer rate using the convective heat transfer at the tip of the fin boundary condition can be written as [19]:

$$q_f = \sqrt{hPk_Ac} \theta_b \frac{\sinh mL + \frac{h}{mk} \cosh mL}{\cosh mL + \frac{h}{mk} \sinh mL}$$
Where $h$ is the convection coefficient of internal air ($\text{W}\text{ m}^{-2}\text{K}^{-1}$), $P$ is the perimeter of the tip of the fin (m), $k$ is the thermal conductivity of the fin material ($\text{W}\text{ m}^{-1}\text{K}^{-1}$) and $A_C$ is the cross sectional area of the fin tip ($\text{m}^2$). $\theta_b$ is the difference in temperature between the base of the fin and the surrounding ambient temperature (K).

$$P = 2w + 2t$$

Where $w$ and $t$ are the width and thickness at the tip of the fin respectively. Assuming a nominal distance into the page where $w$ equals 1 m, the perimeter term can be rewritten as,

$$P = 2 + 2t$$

The same can be done for the cross sectional area of the fin: $A_C = wt$ ($\text{m}^2$), which becomes: $A_C = t$ ($\text{m}^2$).

In order to simplify the heat transfer rate for the fin, we investigated the possibility of using an infinite fin approach. The infinite fin is considered a fin of sufficient length such that as the length $L \to \infty$ the temperature difference between the tip and the surrounding atmosphere is zero $\theta_L \to 0$. Using this assumption, the heat transfer rate from the infinite fin $q_f$ (W) can be expressed as [19]:

$$q_f = \sqrt{hPkA_C\theta_b}$$

As it can be seen, the difference between the boundary condition of the actual fin and the infinite fin approach is the second part of the equation, related to the fin with a convective heat transfer boundary condition:

$$\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}$$

To determine the effect of the infinite fin assumption (i.e. neglecting the equation above) we plotted the “convective fin coefficient”, a term to describe the ratio of $q_f/q_{f,\infty}$, against the concrete slab edge length. For a 10 cm slab edge this coefficient is 1.017 and for a 20 cm slab edge it reduces to 1.00058 – so within 0.06% of the value if the infinite fin approach is not assumed. Hence, the assumption to model slab edges and balconies in buildings as an infinite fins is considered valid since these features are expected to be longer than 20 cm. It also means that the temperature of the section of a concrete slab edge longer than 20 cm is essentially the same as the ambient temperature and therefore does not affect the heat transfer through the wall.
To calculate the fin heat transfer one needs to calculate $\theta_b$, which requires the base temperature at the intersection of the fin and wall assembly and is unknown. In order to solve this problem, we integrated the fin heat transfer into a quasi-convection heat transfer coefficient at the base area. The heat transfer from the fin can be represented as an equal amount of heat transfer due to convection from the spot at the base of the fin. The heat transfer due to convection can be written as:

$$q_{\text{convection}} = hA\Delta T$$

Combining with the fin heat transfer, one can calculate the quasi-convective heat transfer coefficient as:

$$h_{\text{no_fin}} = \frac{\sqrt{hPA_c}}{A_c}$$

And the resistance of the fin $R_{\text{Fin}}$ can be rewritten as:

$$R_{\text{No,Fin}} = \frac{1}{h_{\text{no_fin}}}$$

This is represented in the schematic (left) and resistance diagram (right) shown in Figure 3. The total thermal resistance for each of the paths through the wall can now be determined. With the two thermal resistance values, the thermal transmittance values can be calculated, followed by the thermal transmittance and subsequent thermal resistance for the assembly.

**Figure 2.** Convective fin coefficient versus concrete slab edge length

**Figure 3:** Left – Schematic diagram of wall without fin Right – Updated resistance diagram for wall without fin
3. Results and discussion
To investigate the validity of the infinite fin approach, results from simulation using COMSOL Multiphysics are compared with the results employing the infinite fin and parallel path approach for two cases. Case 1 has a continuous interior insulation layer; whereas, in Case 2 the slab edge has penetrated the exterior insulation. The geometries of the test cases can be seen in Figure 1 with the material properties and component dimensions listed in Table 1. The results of the numerical simulations of the full 2D geometry and calculations using the infinite fin assumption for both Case 1 and Case 2 are shown in Table 2. Comparing the results between the calculations and simulations shows that the infinite fin method better estimates the thermal resistance for the assembly in Case 1 when compared to the simulation results. This is due to the continuous insulation layer in Case 1 which results in less lateral heat transfer from the slab edge to the surrounding structure as compared to Case 2. The reduction in lateral heat transfer is closer to the one-dimensional heat transfer assumption of parallel path calculation method. In other words, the slab edge caused two sources of additional heat transfer in Case 2: a) extended exposed area of the wall to the fin; and, b) an interruption in the continuous insulation layer. For Case 2 the increase in lateral heat transfer resulted in the infinite fin assumption resulted in overestimating the thermal resistance by approximately 5%. To test the parallel path – infinite fin approximation, four assemblies from The BC Hydro Thermal Bridging Guide [18] were analysed:

- 5.2.16: Exterior and Interior Insulated 3 5/8” x 1 5/8” Steel Stud (16” o.c.)-Drained EIFS Wall Assembly – Intermediate Floor Intersection
- 5.2.18: Exterior Insulated 3 5/8” x 1 5/8” Steel Stud (16” o.c.) Wall Assembly with Thermally Broken ISO Clip System Supporting Horizontal Sub-girts – Intermediate Concrete Floor Intersection
- 6.2.1: Exterior Insulated Concrete Drained EIFS Wall Assembly – Intermediate Floor Intersection
- 6.2.6: Interior Insulated Concrete Mass Wall with 1 5/8” x 1 5/8” Steel Studs (16” o.c.) Supporting Interior Finish – Continuous Concrete Intermediate Floor Intersection

The schematics of these walls can be found in BC Hydro Thermal Bridging Guide [18]. The calculated results are compared with the simulation results and can be seen in Table 3. The assemblies that had thermal transmittance values closest to the simulated results were externally insulated assemblies without highly conductive internal components (i.e. steel studs), as seen in the assemblies of Case 1 and 6.2.1. For assemblies with highly conductive internal wall components the method becomes less accurate due increased lateral heat transfer. Overall however, using this approach allows for a reasonable approximation of the thermal transmittance of complex assemblies including intersections or balconies without the need to perform computer simulations. The thermal transmittance of assemblies of 5.2.16 and 5.2.18 are reasonably predicted within 7% of the simulated thermal transmittance despite having highly conductive steel studs causing greater amounts of lateral heat transfer. The highest discrepancy is observed for the case where the interior insulation is interrupted and the concrete slab is in contact with the concrete wall (6.2.5). One should note that errors reported in Table 3 are associated with both using the parallel path and infinite fin method assumptions.
Table 1: Material and geometry data for testing parallel path and infinite fin assumption

|                | Case 1 | Case 2 |
|----------------|--------|--------|
| a (W/m² K⁻¹)  | 0.04   | 1.8    |
| b (m)          | 1.8    | 0.04   |
| c (m)          | 1.8    | 1.8    |
| Thickness (m)  | 0.1    | 0.1    |
| Z1 (m)         | 1.22   | -      |
| Z2 (m)         | 0.203  | 0.203  |

Table 2: Results for simulated and calculated Case 1 and Case 2 assemblies

|                | Case 1 | Case 2 |
|----------------|--------|--------|
| U₀ [W/(m²K)]  | 0.37   | 0.37   |
| R₀ [W/(m²K)]  | 2.705  | 2.705  |
| U [W/(m²K)]   | 0.37   | 0.732  |
| R [W/(m²K)]   | 2.702  | 1.366  |
| Difference U  | 0.02%  | 4.97%  |
| Difference R  | 0.0%   | 0.0%   |

Table 3: Calculated results for BC Hydro assemblies

| BC Hydro assembly | U₀ [W/(m²K)] | R₀ [m²K/W] | U [W/(m²K)] | R [m²K/W] | U assembly [W/m²K] | R assembly [m²K/W] | % difference R value |
|------------------|--------------|------------|-------------|-----------|--------------------|--------------------|----------------------|
| 5.2.16           | 0.34         | 2.93       | 0.39        | 2.55      | 0.37               | 2.72               | 6.62%                |
| 5.2.18           | 0.35         | 2.87       | 0.37        | 2.71      | 0.35               | 2.89               | 6.68%                |
| 6.2.1            | 0.32         | 3.10       | 0.33        | 3.02      | 0.33               | 3.08               | 1.86%                |
| 6.2.5            | 0.44         | 2.26       | 0.74        | 1.36      | 0.68               | 1.47               | 8.13%                |

4. Conclusions

An analytical approach in combination with the parallel path calculation method is proposed to calculate the effective thermal resistance of wall assemblies containing linear thermal bridges such as slab edges/balconies/eyebrows. This method can be used when designers do not have access to a numerical simulation tool or do not have the expertise or time to conduct simulations. The additional heat transfer due to the linear thermal bridge element was calculated as the heat transfer from an infinite fin. It has been shown for concrete slab edges longer than 10 cm, the error associated with assuming the fin to be infinite is less than 1.1 percent. So one can simplify the fin heat transfer rate correlation and consider all the balconies and slab edges to be infinite fins. Since the fin base temperature is required to calculate the fin heat transfer rate and it is unknown we converted the additional heat transfer to a quasi-convective heat transfer coefficient at the fin base (intersection). The overall thermal resistance of the wall...
assembly was then calculated employing the parallel path calculation method. Applying this method on different wall assemblies and comparing the results with the numerical simulation led to one to eight percent deviation, demonstrating the potential effectiveness of this assumption in simplifying the calculation of heat transfer due to a slab edge or balcony.

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