Effect of spatial dispersion on the spectrum of inter-edge magnetoplasmons in the two-dimensional heterogeneous system.

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1 Abstract

The present paper is devoted to the theoretical study of the spectrum of low-frequency electronic density oscillations (plasmon waves) running along the boundary of two contacting two-dimensional electronic systems in the perpendicular magnetic field. For the first time, such waves were predicted in the Institute of Radio-engineering and Electronics of RAS (V.A. Volkov, S.A. Mikhailov, 1992), studied experimentally in the papers of foreign investigations and called inter-edge magnetoplasmons (IEMP). In a specific case when the two-dimensional system is finite and has the half-plane shape, the internal boundary becomes external and these plasmons go over into the well-known edge magnetoplasmons.

The existing theory of edge and inter-edge magnetoplasmons is built with the neglect of spatial dispersion of the conductivity. The dispersion becomes essential when the electric potential changes significantly over the radius of the electron Larmor orbit or cyclotron radius $R_c$. Here we attempt to derive and analyze the IEMP dispersion equation involving spatial dispersion of conductivity.

The approximations are made as follows. The inner boundary is assumed to be a straight line at which the electron concentration changes weakly but stepwise. The same refers to all components of the tensor magnetoconductivity. The jump of the diagonal conductivity is considered small compared with that of the Hall magnetoconductivity. The latter is neglected everywhere with the exception of cutting nonphysical divergencies. We neglect the spatial dispersion of the Hall conductivity. The IEMP frequency is small compared with the cyclotron frequency. Finally, we solve a linearized system of quasi hydrodynamic equations of electron motion in the self-consistent field describing small fluctuations of concentration, potential, and current with the neglect of retardation.

The main result is the following. The existence of the spatial dispersion results in analogue of the so-called geometric resonance in the shortwave limit. The IEMP frequency as a function of wave vector oscillates with the period determined by a ratio of electron cyclotron radius to the plasmon wavelength. For long wavelengths, the IEMP dispersion curve coincides with the known result.
2 Introduction.

Edge magnetoplasmons represent collective excitations in the two-dimensional electron system, which propagate along the edge boundary of the system and are localized beside it. The edge magnetoplasmons are experimentally discovered on the surface of liquid helium in the magnetic field normal to the helium surface. As is stated in [5], edge magnetoplasmons have some important properties, provoking recent interest.

(1) They have a gapless spectrum \( \omega_{\text{emp}} \sim q \ln 1/|q|, \) \( \vec{q} \) being directed along the boundary of the sample. Their frequency is much smaller than the cyclotron one. Plasmons propagate along the boundary in the direction depending on the angle between the outer normal to the edge and the direction of magnetic field. The EMP frequency is proportional to the electron concentration and inversely proportional to the magnetic field \( B \) and the size of the sample. In the finite sample the wavevector is discrete \( q = 2\pi/P \) where \( P \) is the perimeter of the sample. Depending on these parameters, the EMP frequency can vary within large range from infrared frequencies in submillimetric samples of semiconductor heterostructures, e.g., quantum dots, rings, threads, antidots, down to microwave and even audio-frequencies (kHz) in the rarefied two-dimensional electronic system of macroscopic scale (cm) on the liquid helium surface.

(2) In strong magnetic fields \( \omega_c \tau >> 1 \) the EMP damping is very small both for \( \omega_{\text{emp}} \tau >> 1 \) and for \( \omega_{\text{emp}} \tau << 1 \). This is the property that gives an opportunity to observe the EMP experimentally at the frequencies lower than 1 GHz.

(3) The EMP frequency is determined by the Hall motion of electrons and is proportional to the Hall conductivity of the two-dimensional electron system. Since the quantum Hall effect is observed not only for direct current but also for microwave frequencies, the EMP can be used as a powerful instrument of studying both integer and fractional Hall effect. So, the EMP spectroscopy can be used as a method of studying edge electronic states that play a significant role in creating the Hall effect.

(4) The EMP charge is strongly localized beside the boundary of a sample over the length comparable with the width of edge electronic states.

The first experimental observation of the main IEMP properties is made in [2] in which the properties of electrons on the liquid helium surface are studied. It is found that the direction of the IEMP motion depends on the sign of the difference between the electron concentrations on the right-
hand and left-hand sides of the boundary. In the limit of strong fields the
frequency is proportional to the difference in the electron concentrations and
inversely proportional to the magnitude of magnetic field. The results are in
a qualitative agreement with the theoretical predictions.

In [4] it is found that the width of the IEMP curves increases for the
temperatures below the melting temperature of electron Coulomb crystal.

The IEMP damping and width are determined by the mean diagonal
conductivity in the boundary region. Under crystallization the conductivity
$\sigma_{xx}$ increases drastically for moderate intensive fields. The IEMP plasmons
are localized smaller than the EMP ones. Thus the IEMP plasmons are more
sensitive to crystallization in the high electron density region than the EMP
ones. Such vanishing cannot be achieved for the EMP plasmons.

In work [1] it is shown that, unlike edge magnetoplasmons, inter-edge
magnetoplasmons even in the collisionless limit have the damping connected
with emitting bulk plasmons to the region of lower concentration. The damp-
ing is weak for the significant difference in the concentrations on the left and
right sides. There are two modes, upper and lower, in the IEMP spectrum. The
upper mode has the frequency larger than the cyclotron one.

The upper branch has a strong nondissipative damping connected with
the emission of bulk 2D magnetoplasmons to the region of lower concentra-
tion. This effect takes place only in the two-dimensional system.

The damping is weak only for a significant difference in the electron con-
centrations on the right and left sides from the boundary $N^r >> N^l$ and
large frequency

$$\omega_r(q_y) \gg \omega_c, \quad \omega_r(q_y) = \sqrt{\frac{2\pi N^r e^2 q_y}{m^*}}$$

as compared with the cyclotron one.

The lower mode does not exist for such fields at which $\alpha \omega_c \tau \leq 1$. It is
well-defined for

$$\omega_c \tau \gg \frac{1}{\alpha} > 1,$$

where $\tau$ is the decay time of momentum and $\alpha = (N^r - N^l)/(N^r + N^l)$.

The IEMP frequency in any geometry with the unhomogeneous conduc-
tivity is proportional to the inhomogeneity of the Hall conductivity. The Hall
conductivity is inversely proportional to the magnitude of magnetic field.
Thus the IEMP frequency proves to be small compared with the cyclotron
one.
For calculating spectrum in the geometry with the nonuniform conductivity which changes abruptly at the boundary of a disc, we use the method in which the diagonal conductivity is replaced by some effective quantity $\sigma_{\text{eff}}$ and the inhomogeneity in $\delta\sigma_{xx}$ is considered as a small correction. The quantity $\delta\sigma_{xx}$ should not be small in comparison with $\sigma_{xx}$ but only with $\delta\sigma_{xy}$.

The approximate method $\delta\sigma_{xx} = 0$ is based on the fact that inhomogeneity in the diagonal conductivity has a relatively weak effect on the spectrum. Taking inhomogeneity into account is necessary to avoid a logarithmic divergence in the dispersion equation. The averaged quantity $\sigma_{\text{eff}}$ determines the scale of spatial dispersion in the permittivity and is determined from the condition of the current continuity at the boundary of half-planes.

With the help of analyzing an exact solution for the geometry of two half-planes we show an applicability $\delta\sigma_{xx} = 0$ not only for weak inhomogeneity but also it is stated that the error of finding the spectrum at $\alpha \leq 1$ does not exceed 8.5%.

The spectrum in the geometry of two half-planes with the different conductivities reads:

$$\omega(q_y) = \frac{2\delta\sigma_{xy}(\omega)q_yF[q_y\tilde{t}(\omega)]}{\kappa},$$

where $F(z)$ is determined in the following way:

$$F(z) = \int_0^{\pi/2} \frac{dt}{\sin t + z} = \begin{cases} \ln \frac{2}{z}, & 0 < z \ll 1, \\ \frac{\pi}{2z}, & z \gg 1. \end{cases}$$

Here

$$\tilde{t}(\omega) = \frac{l'^r(\omega) + l'^l(\omega)}{2}, \quad l'^r(\omega) = \frac{2\pi i\sigma_{xx}^r(\omega)}{\omega}, \quad l'^l(\omega) = \frac{2\pi i\sigma_{xx}^l(\omega)}{\omega}$$

and $\kappa$ is the permittivity of the ambient medium. It is shown that the spectrum in the system of more complicated geometry such as a square or rectangle can be obtained with the help of quantization rule, replacing $q_y$ with $2n/P$.

### 3 A set of the problem

Let the 2D system of two half-planes with the different tensors of two-dimensional conductivity $\sigma_{\alpha\beta}(q, \omega)$ and $\sigma_{\alpha\beta}^r(q, \omega)$ for the right and left half-planes, respectively, is placed in a strong magnetic field. We seek for the
dispersion equation for spectrum $\omega(q_y)$, $q_y$ being the wavevector of oscillations along the boundary of half-planes. In addition, $\delta\sigma_{xx}(q, \omega) = |\sigma^l_{xx}(q, \omega) - \sigma^r_{xx}(q, \omega)|$, $\delta\sigma_{xy} = |\sigma^l_{xy} - \sigma^r_{xy}|$. We make the following approximations:
(1) conductivity and concentration change abruptly at the boundary of half-planes;
(2) $\sigma_{xy}(q, \omega) = \sigma_{xy}(0, \omega)$;
(3) $\delta\sigma_{xy} = |\sigma^l_{xy} - \sigma^r_{xy}| > > \delta\sigma_{xx}$;
(4) lack of retardation;
(5) quasi hydrodynamic approximation.

For simplicity, the effective electron mass is the same everywhere. The permittivity of the environment equals unity. Let us introduce the coordinate axes in which the $y$-axis is directed along the boundary of half-planes and the region $x > 0$ refers to the right half-plane.

We have the following equation in such coordinates:

$$1 = \frac{q_y \delta \sigma_{yx}(0, \omega)}{\omega} \int_{-\infty}^{+\infty} dq_x \frac{d\phi(\vec{q}, \omega)}{\epsilon(\vec{q}, \omega)}, \quad (3)$$

where $\epsilon(q, \omega) = 1 + q_l (\epsilon_eff(q, \omega)) = \frac{2\pi i \sigma_{eff}(q, \omega)}{\omega}$ ($\sigma_{eff}(q, \omega)$ will be defined later), $\sigma_{xy} = -\sigma_{yx}$, $q = \sqrt{q_x^2 + q_y^2}$.

Let us introduce the following notations, such as: $\rho$ is the 2D density of electrons, $\vec{q} = (q_x, q_y)$ is the 2D wavevector, $\vec{r} = (x, y)$, $\varphi(\vec{r}, t)$ is a fluctuation of the potential in point $\vec{r}$ at moment $t$, $\vec{j}(\vec{r}, t)$ is the 2D vector of the current density, $\omega$ is the frequency of oscillations, $\sigma(\vec{r}, R, t)$ is the kernel of the 2D operator of conductivity, $l^l(\omega, \omega) = 2\pi i \sigma^l(q, \omega)/\omega$, $l^r(q, \omega) = 2\pi i \sigma^r(q, \omega)/\omega$, $\delta l(q, \omega) = |l^r(q, \omega) - l^l(q, \omega)|$, $\delta N = |N^r - N^l|$, $\delta J_0^2(qR) = |J_0^2(qR^r) - J_0^2(qR^l)|$, $m^*$ is the effective electron mass, $a_B^*$ is the Bohr radius of an electron with effective mass $m^*$, and $R^l, R^r$ is the cyclotron radius of the left and right half-planes, respectively.

4 Derivation of the dispersion equation.

We use the following equations:
(1) continuity equation of current in the Fourier representation:

$$\vec{q}\vec{j}(\vec{q}, \omega) = \omega \rho(\vec{q}, \omega), \quad (4)$$
(2) the 2D Poisson formula in the Fourier representation:

\[ \varphi(\vec{q}, \omega) = \frac{2\pi \rho(\vec{q}, \omega)}{q} \]  

(5)

(3) the 2D Ohm law:

\[ j(\vec{r}, t) = -\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{\alpha\beta}(\vec{r}, \vec{R}, t) \frac{\partial \varphi(\vec{R}, t)}{\partial x_{\beta}} d^2 \vec{R}, \]  

(6)

Neglecting both the spatial dispersion of the Hall conductivity and the inhomogeneity of diagonal conductivity, we obtain:

\[ \begin{cases} 
  j_x(\vec{r}, \omega) = -\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{\text{eff}}(\vec{r}, \vec{R}, \omega) \frac{\partial \varphi(\vec{R}, \omega)}{\partial x} d^2 \vec{R} - \sigma_{xy}(\vec{r}, \omega) \frac{\partial \varphi(\vec{r}, \omega)}{\partial y}, \\
  j_y(\vec{r}, \omega) = -\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{\text{eff}}(\vec{r}, \vec{R}, \omega) \frac{\partial \varphi(\vec{R}, \omega)}{\partial y} d^2 \vec{R} + \sigma_{xy}(\vec{r}, \omega) \frac{\partial \varphi(\vec{r}, \omega)}{\partial x}.
\end{cases} \]  

(7)

Assuming that \( \sigma_{xy}(\vec{r}, \omega) = -\sigma_{yx}(\vec{r}, \omega) = \sigma_{xy}^1(\omega) \theta(x) + \sigma_{xy}^2(\omega) \theta(-x) \), one may find the current in the Fourier components:

\[ \begin{cases} 
  j_x(\vec{q}, \omega) = -i \sigma_{\text{eff}}(\vec{q}, \omega) q_x \varphi(\vec{q}, \omega) - \frac{iq_y}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp(iq_x x) \sigma_{xy}(\omega) \varphi(x, q_y) dx, \\
  j_y(\vec{q}, \omega) = -i \sigma_{\text{eff}}(\vec{q}, \omega) q_y \varphi(\vec{q}, \omega) + \frac{iq_x}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp(iq_x x) \sigma_{xy}(\omega) \varphi(x, q_y) dx - \frac{\delta \sigma_{xy}(0, \omega) q_y \varphi(\vec{q}, \omega)}{\sqrt{2\pi}}.
\end{cases} \]  

(9)

Substituting this into the continuity equation, one arrives at:

\[ \delta \sigma_{xy}(0, \omega) q_y \varphi(x = 0, q_y, \omega) = \frac{q \omega \varphi(\vec{q}, \omega)}{\sqrt{2\pi}} \left[ 1 + \frac{2\pi i \sigma_{xx}(q, \omega) q}{\omega} \right]. \]  

(10)

Performing the inverse Fourier transform in \( q_x \) at \( x = 0 \), we have:

\[ \omega(q_y) = q_y \delta \sigma_{yx}(0, \omega) \int_{-\infty}^{+\infty} \frac{dq_x}{q_{\text{eff}}(q, \omega)}. \]  

(11)
5 Analysis of the dispersion equation in the limiting cases and the diagram of the IEMP spectrum.

We take the following expression for Hall conductivity $\sigma_{yx}$ from [3]. For $qR \ll 1$,

$$\sigma_{yx}(q, \omega) = \frac{N e^2}{m^* \omega_c} \left[ \frac{1}{1 - \frac{\omega^2}{\omega_c^2}} + \sum_{n=2}^{\infty} \frac{n(\omega^2)}{(n!)^2(n^2 - \frac{\omega^2}{\omega_c^2})} \right]. \tag{12}$$

It is evident that $\sigma_{yx}(q, 0) = \frac{N e^2}{m^* \omega_c}$. Then we evaluate the magnitude of the region for wave vectors $q$ in which we can neglect the spatial dispersion. Let $\omega/\omega_c = 0.5$ and in the case $qR \sim 13$ the second term of a Fourier series for the Hall conductivity depends on $q$. We use the known expression:

$$\epsilon(q, \omega) = 1 + \frac{4m^* e^2}{q R^2} \sum_{m=1}^{\infty} \frac{m^2 J_m^2(qR)}{m^2 - \frac{\omega^2}{\omega_c^2}}, \tag{13}$$

where $R$ is some effective cyclotron radius of the system. For $\omega/\omega_c \ll 1$, this expression significantly simplifies:

$$\epsilon(q, 0) = 1 + \frac{2}{qa^*_B} [1 - J_0^2(qR)], \tag{14}$$

with the help of

$$1 = J_0^2(qR) + 2 \sum_{m=1}^{\infty} J_m^2(qR). \tag{15}$$

For $qR \gg 1$, $\epsilon(q, \omega) \sim 1$ and thus the integral in the dispersion equation is logarithmic. In order to avoid divergence for small $q_B \ll 1/R$, we neglect the spatial dispersion of diagonal conductivity:

$$\epsilon_{eff}(q, \omega) = 1 + q l_{eff}(0, \omega), \tag{16}$$

$$l_{eff}(0, 0) = \frac{l_r + l^t}{2} = \frac{R^2}{a^*_B}, \quad R^2 = \frac{R^2_r + R^2_t}{2}. \tag{17}$$
For large $q_y \gg 1/R$, we cut the upper bound at $\frac{1}{\delta_l(q_y,0)}$. Then we have the following spectrum $\omega(q_y)$:

$$
\frac{\omega(q_y)}{\omega_c} = \begin{cases} 
\frac{4\delta N q_y e^2}{m^* \omega_c^2} \arctan \sqrt{\frac{(a_B^2 - q_y R^2)}{(a_B^2 + q_y R^2)}}, & q_y R^2 < 1, \\
\frac{4\delta N q_y e^2}{m^* \omega_c^2} \arctan \sqrt{\frac{(a_B^2 - q_y R^2)}{(a_B^2 + q_y R^2)}}, & q_y R^2 > 1, \\
\frac{2q_y \delta N e^2}{m^* \omega_c^2} \int_0^{25 \omega_c^2 (q_y R)} dq_y, & q_y R \gg 1.
\end{cases}
$$

For $\frac{1}{\delta_l(0,0)} < q_y < \frac{1}{R}$, $\frac{\omega(q_y)}{\omega_c}$ asymptotically tends to $\frac{\delta N}{2N_{eff}}$. When $q_y \gg \frac{1}{R}$, peaks are observed due to vanishing $\delta_l(q_y) = 0$ at some $q_y$.

To plot the dispersion curve, we take the following values in the CGS system for the constants used: $m^* = 0.2 \times 9.1 \times 10^{-28}$g, $a_B^* = 25 \times 10^{-9}$cm, $N^l = 2 \times 10^{12}$cm$^2$, $N^r = 2.4 \times 10^{12}$cm$^2$, $\omega_c = 2 \times 10^{11}$s$^{-1}$.

## 6 Conclusion

We have studied the effect of spatial dispersion on the spectrum of inter-edge magnetoplasmons. From the Fig.2 one can see that the periodic oscillations are observed for $q_y \sim n/R$. For $q_y \ll 1/R$, the spectrum coincides with that in the lack of the spatial dispersion of diagonal conductivity. The physical reason for the resonances in the IEMP spectrum is the cyclotron resonance when the frequency of electron rotation in the magnetic field equals the IEMP frequency or differs by a multiple factor from it.
Figure 1: The IEMP dispersion curve $\omega(q_y R)/\omega_c$ with involving spatial dispersion in the system of two half-planes with inhomogeneous conductivity $q_y R \ll 1$. The $y$-axis represents dimensionless quantity $\omega(q_y R)/\omega_c$. 
Figure 2: The IEMP dispersion curve $\frac{\omega(q_y R)}{\omega_c}$ with involving spatial dispersion in the system of two half-planes with the unhomogeneous conductivity. The $y$-axis represents dimensionless quantity $\frac{\omega(q_y R)}{\omega_c}$.
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