Particle multiplicities and fluctuations in 200 GeV Au-Au collisions

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Abstract. We use the statistical hadronization model (SHM) to describe hadron multiplicity yields and fluctuations. We consider 200 GeV Au-Au collisions, and show that both event averaged yields of stable particles and resonances, and event-by-event fluctuation of the $K/\pi$ ratio can be described within the SHM using the same set of thermal parameters, provided that the phase space occupancy parameter value is significantly above chemical equilibrium, and the freeze-out temperature is \( \sim 140 \) MeV. We present predictions that allow to test the consistency of our results.

The statistical hadronization model (SHM) [1, 2, 3] has been extensively applied to the study of soft particle production in hadronic systems. When it includes the full spectrum of hadronic resonances [4], the SHM, with judiciously chosen (fitted) parameters, quantitatively describes the abundances of all hadrons produced in heavy ion collisions at all considered reaction energies [5].

The ability of the SHM to describe not just averages, but event-by-event multiplicity fluctuations has not been widely investigated. Event-by-event fluctuations have attracted theoretical [6, 7, 8, 9] and experimental interest, both as a consistency check for existing models [6, 7] and as a way to search for new physics [8].

The objective of this work is to determine weather the SHM can describe both yields and fluctuations with the same parameters. We obtain a good fit to 200 GeV RHIC experimental data including both yields and fluctuations measurements, discuss the results in the context of the bulk properties of the matter created at RHIC, and present predictions allowing further tests of the model.

The statistical hadronization model assumes that final states are produced in proportion to their phase space size. The first and second cumulants of this probability distribution give, respectively, the average value over all events its event-by-event fluctuation.

In this work we use the Grand Canonical (GC) ensemble, as implemented in open-source software [10], to calculate fluctuations and yields. We motivate this choice by the fact that the considered RHIC experiments observe the mid-rapidity slice of the system, comprising roughly 1/8 of the total hadron multiplicity. The further assumption of (approximate) boost invariance at mid-rapidity allows to image this rapdity slice into a domain in configuration space. The matter content in this space domain is expected to be in contact and exchanging energy and conserved quantum numbers with the unobserved regions. This than creates the GC system we consider on an event-by-event basis.

If the freeze-out temperature throughout the system is the same, a rather simple model
can be used to obtain both yields and fluctuations \cite{11, 12}. However, one should note that even if practically all produced particles originate in statistical model processes, their fluctuations could comprise novel creation mechanisms, related to their formation dynamics. However, such novel mechanisms are not present in all observables, and the observables we consider here seem to follow non-equilibrium SHM calculations.

The final state particle yield can then be computed as a function of the particle properties, resonance decay tree, freeze-out temperature and fugacities (technical details are found in our recent report \cite{13}). While the temperature controls the particle yield dependence on the mass, the fugacity $\lambda$ describes both the yield of conserved quantities (such as baryon number, charge and strangeness) across all particles, and the absolute yields which depend on the degree of chemical equilibration. It is common to introduce chemical potentials associated with each conservation law, $\mu = T \ln \lambda$, while the fugacities associated with the chemical nonequilibrium condition are called $\gamma$.

Detailed balance requires that the particle fugacity be given by the conserved charge fugacity to the power of the particle’s ‘charge’, generalized to contain all conserved quantities (electrical charge, baryon number etc.). Thus, for a particle with $q, (\bar{q})$ light (anti)quarks, $s, (\bar{s})$ strange (anti)quarks and isospin $I_3$ we have

$$\lambda_{\text{eq}}^i = \lambda_q^{q-\bar{q}} \lambda_s^{s-\bar{s}} \lambda_{I_3}^{I_3}$$

However, the condition of chemical equilibrium might no longer hold when the fireball is rapidly expanding. Thus, chemical parameters acquire a kinetic (time-dependent) component parametrized in terms of phase space occupancies, $\lambda_i = \lambda_{\text{eq}}^i \gamma_q^{q-\bar{q}} \gamma_s^{s-\bar{s}}$, where $\gamma_q = 1, \gamma_s = 1$ at equilibrium. Even in chemical nonequilibrium the particle fugacity $\lambda_i$ is the parameter controlling the particle yield, and the first and second cumulants can be calculated from the partition function in the usual way \cite{14, 15}.

If the expanding system undergoes a fast phase transition from a Quark Gluon Plasma (QGP) to a hadron gas (HG), chemical non-equilibrium \cite{16} and super-cooling \cite{17} are expected to arise given the requirement that entropy has to increase while the transition occurs from a high to low entropy density phase. The virtue of a hadronization temperature near 140 MeV, and an over-saturated phase space ($\gamma_q \sim 1.5, \gamma_s \sim 2$), is a match of both the energy and entropy density between the QGP and HG phases.

Fits to experimental data at both SPS and RHIC energies support these values of $\gamma_{q,s}$. Moreover, best fit $\gamma_{q,s} > 1$ arises for a critical reaction energy \cite{5} (corresponding to the energy of the $K/\pi$ “horn” \cite{18}) and system size \cite{19}, as expected from the interpretation of $\gamma_q$ as a manifestation of a phase transition. However, even though the fits performed in \cite{5} strongly favor the chemical non-equilibrium, they do not rule out equilibrium models. The equilibrium model remains marginally compatible with data giving a less convincing but statistically still non-absurd validity. This happens since in a fit considering only particle yields, the chemical non-equilibrium phase space occupancies $\gamma_i$ and $\gamma_q$ correlate with freeze-out temperature \cite{5}, making a distinction between a higher temperature equilibrated freeze-out ($\gamma_q = 1, \gamma_s \leq 1$) scenario and a supercooled scenario where $\gamma_{q,s} > 1$ difficult. When full chemical nonequilibrium is allowed for, $\gamma_q \simeq 1.6$ and $T = 140$ MeV is found. The best fit freeze-out temperature
when full chemical equilibrium is assumed varies between studies, ranging from $T = 155$ MeV in latest SHARE based studies [5], $T = 165$ MeV for those carried out 2 years ago by STAR experimental group [20], to $T = 177$ MeV offered in the initial RHIC data exploration in which strange particles were not yet fully incorporated [21].

The study presented in [13] makes it clear that the dependence of the fluctuation

$$\sigma^2_X = \frac{\langle X^2 \rangle - \langle X \rangle^2}{\langle X \rangle}$$

on $T$ and $\gamma_q$ is different, allowing us to independently determine these two variables. A higher temperature decreases the fluctuations with respect to the Poisson value $\sigma = 1$, expected for a Boltzmann distribution, since it introduces greater particle correlations arising from increased resonance decay contribution. Increasing $\gamma_q$ rapidly increases fluctuations of quantities related to pions, due to the fact that at $\gamma_q > 1 \lambda_\pi$ rapidity approaches $e^{m_\pi/T}$, giving fluctuations an extra increase compared to yields [12, 13].

By virtue of the implied physical picture, equilibrium models generally assume a long time span between chemical (particle production) and thermal (particle scattering) freeze-out, which would alter considerably the multiplicity of directly detectable resonances. In the chemical non-equilibrium supercooled freeze-out picture, however, it is natural to assume that particle scattering after emission is negligible [16] and thus one can in most cases assume that the thermal and chemical freeze-out temperatures are the same. Hence, a reliable way to probe the re-interaction period would be instrumental for our understanding of how the fireball produced in heavy ion collisions breaks up.

We have recently shown [12] that a comparison of fluctuations to directly detected resonances probes the interval between chemical and thermal freeze-out. Consider, for example, $\sigma_{K^+/\pi^-}$ The numerator and the denominator terms in this ratio are linked by a large correlation term due to the $K^{*0}(892) \rightarrow K^+\pi^-$ decay. This correlation probes the $K^{*0}(892)$ abundance at the initial chemical freeze-out, since subsequent rescattering of $K^+ \rightarrow K\pi$ decay products or on-shell $K^*$ regeneration from in-medium $K^+\pi^-$ pairs does not alter the final abundance of $\pi^+$ and $K^-$. On the other hand, a direct measurement of the $K^{*0}(892)/K^-$ relative yield through invariant mass reconstruction measures the $K^{*0}(892)$ abundance at thermal freeze-out, after all rescattering/regeneration ceased. Hence, comparing the $K^+/\pi^-$ fluctuation to the $K^{*0}(892)/K^-$ ratio provides a gauge for the effect of the hadronic reinteraction period on particle abundances. A strong constraint arises in a model where chemical and thermal freeze-out coincide, as both observables must be described with the same set of statistical parameters determined by global yields of other particles. In this way one can argue that the $K^+/\pi^-$ fluctuation and the $K^{*0}(892)/K^-$ relative yield measured by invariant mass method offer a decisive test of the chemical non-equilibrium sudden hadronization reaction picture.

While they are phenomenologically powerful, fluctuation measurements are also vulnerable to systematic effects which need to be carefully considered. Volume and centrality fluctuations, difficult to describe in a model-independent way, can be taken care of by considering event by event fluctuations of particle ratios, where the fluctuation in volume cancels out, event by event, to first order. This leaves, however, possibly large effects due to limited experimental acceptance. These can usually be subtracted by considering fluctuations measured within fake events, created using mixed event techniques.
Such “static” fluctuation, with, by definition, no correlations between particles, can be described by a purely Poisson term, where fluctuation is governed by particle yields:

$$\sigma_{\text{stat}}^2 = \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle}$$  (4)

It can also be seen that mixed-events, made from tracks measured in a given detector, also contain the effect due to that detector’s acceptance.

Subtracting $\sigma_{\text{stat}}$ from the total fluctuation leaves the “dynamical” fluctuation term:

$$\sigma_{\text{dyn}}^2 = \sqrt{\sigma^2 - \sigma_{\text{stat}}^2}$$  (5)

which comprises the physically interesting effects such as resonance decay correlations, Bose-Einstein correlations, and eventual new dynamics. Provided certain assumptions for the detector response function hold (see appendix A of [23]), this dynamical fluctuation is a “robust”, detector-independent observable.

When using measured $\sigma_{\text{dyn}}$s in fits to experimental data, the data-sample should include $\sigma_{\text{dyn}}$, particle yields (which are needed to determine the Poisson contribution to $\sigma_{\text{stat}}$ as per Eq. (4)) and particle ratios. We have performed a fit incorporating all STAR ratios given in Ref. [20], with the exception of the $\Delta^{++}/p$, which the STAR collaboration has since begun to reevaluate, and we also for the present ignore the $\Omega/\Omega > 1$, which cannot be fitted with the SHM [13, 24]. On the other hand, we have included in the fit procedure the preliminary value of $\sigma_{\text{dyn}}^{K/\pi}$ measured by STAR [22], as well as the published yield for $\phi$ [25] and $\pi^- [26]$. We assume [26, 27] full feed-down correction for $K_S \rightarrow \pi^\pm$ and $\Lambda \rightarrow \pi$ weak decays, and no correction for $\Lambda \rightarrow p$.

The fit parameters include the overall normalization, the freeze-out temperature $T$, $\lambda_{q,s,t}$, and $\gamma_{q,s}$. We also require, by implementing them as “data-points”, strangeness, electrical charge and baryon number conservation: $\langle s - \bar{s} \rangle = 0$, $\langle Q \rangle / \langle B \rangle = (\langle Q \rangle / \langle B \rangle)_{\text{Au}} = 0.4$. The fit’s statistical significance ($P_{\text{true}}$) profile is shown in the left panel of Fig. 1. To obtain a profile for $P_{\text{true}}$ a fit was performed for each fixed parameter point on the abscissa, all fit parameters except the one shown on the abscissa were varied. It can be seen that the fit tightly constrains $\gamma_q$ well above the equilibrium value, accompanied by $T \sim 140$ MeV, in good agreement of the prediction of the supercooled hadronization scenario.

The right panel of Fig. 1 shows the sensitivity of $\sigma_{K/\pi}^{\text{dyn}}$ to $\gamma_q$ and temperature, and explains why the correlation between $T$ and $\gamma_q$ disappears when fluctuations are taken into account. As can be seen, a fit assuming the chemical equilibrium ($\gamma_q = 1, T = 155$ MeV [5]) misses $\sigma_{K/\pi}^{\text{dyn}}$ by many standard deviations. On the contrary the chemical nonequilibrium fit seems to be right where this fluctuation is measured. Introducing exact conservation for strangeness within the observed window (canonical ensemble) would decrease the theoretical $\sigma_K$ [28], thereby increasing the chemical equilibrium theory to experiment discrepancy. It is only through $\gamma_q > 1$ that $\sigma_{K/\pi}^{\text{dyn}}$ increases to the point where it becomes compatible with the experimental value.

An eye assessment of the fit’s goodness is provided by the left panel of Fig. 2. As can be seen, the fit gives an adequate description of all particle yields, including the
FIGURE 1. Left: statistical significance ($P_{\text{true}}$) profiles for freeze-out Temperature $T$ and light quark phase space occupancy $\gamma_q$ obtained in the fit shown in Fig. 2. Right: Sensitivity of $\sigma_{K/\pi}^{\text{dyn}}$ to freeze-out temperature and $\gamma_q$ for the fit in Fig. 2. At each point in the abscissa, a fit is performed, with only the particle yield data-points used, varying all fit parameters except the one on the abscissa. $\sigma_{K/\pi}^{\text{dyn}}$ is evaluated using the best fit parameters. The dot-dashed lines refer to the experimental limits [22] for the $\sigma_{K^+/\pi^+}$ (blue, lower values) and $\sigma_{K^-/\pi^-}$ (red, higher values). See [13] for further details.

resonance $(K^+(892) + K^{-}(892))/K^-$ and $\Lambda(1520)/\Lambda$. It can also adequately describe the event-by-event fluctuations of $K^+/\pi^+$ or $K^-/\pi^-$. $K^+/\pi^+$ fluctuations do not directly test the simultaneous freeze-out hypothesis, since $K^-\pi^-$ and $K^+\pi^+$ are not correlated by resonances. To test sudden freeze-out, we have used the best fit parameters to predict the yields of several resonances subject to current experimental investigation ($\rho^0, f_0(950), \Delta^{++}, \Sigma^{*+}(1385), \Xi^*(1530)$) as well as the dynamical fluctuation of the ratio of their decay products ($\pi^+/\pi^-, p/\pi^\pm, \Lambda/\pi^\pm, \Xi/\pi^\pm$). The result is shown in Fig. 2 right panel.

Note the significant difference between ratios such as $p/\pi^+$ and $p/\pi^-$, while the fluctuations of $p/\pi^+$ and $p/\pi^-$ are substantially identical. This systematics, which repeats itself when the $\Lambda/\pi$ ratio is considered, is due to the correlations provided by the leading resonance decay ($\Delta \to p\pi, \Sigma^+(1385) \to \Lambda\pi$). Thus, the combined measurement of the resonance and the ratio of decay products yields a very powerful constraint on the simultaneous freeze-out model considered here, and it will be interesting to see to what extent will the model agree with data. In particular, the difference between $\sigma_{K^+/\pi^+}^{\text{dyn}}$ and $\sigma_{K^-/\pi^-}^{\text{dyn}}$ is intriguing, since the isospin chemical potential required to reproduce it ($\lambda_{I3} \sim 0.96$) is excluded through ratios such as the $\pi^+/\pi^-$. It remains to be seen whether this result is due to experimental systematics not analyzed within the preliminary measurement, or whether additional theoretical insights are needed to describe it.

In conclusion, we have shown that the SHM can describe both the yields and event-by-event fluctuations measured so far in RHIC 200 GeV Au-Au collisions, provided that phase space is saturated above equilibrium and the system is super-cooled with respect to the phase transition temperature. We have justified this scenario in the context of a fast
phase transition from a high-entropy phase, and argued that the simultaneous description of yields and fluctuations is consistent with an explosive freeze-out, where interactions after hadronization are negligible. We have predicted experimental observables suitable for testing this model further, and eagerly await more published data to determine to what extent can the SHM account for both yields and fluctuations in light and strange hadrons produced in heavy ion collisions.

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