ON THE ADIABATIC WALKING OF PLASMA WAVES IN A PULSAR MAGNETOSPHERE

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ABSTRACT

The pulsar radio emission is generated in the near magnetosphere of the neutron star, and it must propagate through the rest of it to emerge into the interstellar medium. An important issue is whether this propagation affects the planes of polarization of the generated radiation. Observationally, there is sufficient evidence that the emerging radiation is polarized parallel or perpendicular to the magnetic field line planes that should be associated with the ordinary (O) and extraordinary (X) plasma modes, respectively, excited by some radiative process. This strongly suggests that the excited X and O modes are not affected by the so-called adiabatic walking that causes a slow rotation of polarization vectors. In this paper, we demonstrate that the conditions for adiabatic walking are not fulfilled within the soliton model of pulsar radio emission, in which the coherent curvature radiation occurs at frequencies much lower than the characteristic plasma frequency. The X mode propagates freely and observationally represents the primary polarization mode. The O mode has difficulty escaping from the pulsar plasma; however, it is sporadically observed as a weaker secondary polarization mode. We discuss a possible scenario under which the O mode can also escape from the plasma and reach an observer.

Key words: pulsars: general – radiation mechanisms: non-thermal

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1. INTRODUCTION

The brightness temperature (exceeding $10^{12}$ K by many orders of magnitude) deduced from the observed radio flux densities strongly implies that the pulsar radiation must be emitted coherently. Generally, the coherent pulsar radio emission can be generated by means of either a maser or a coherent curvature mechanism (e.g., Ginzburg & Zhelezniakov 1975; Ruderman & Sutherland 1975; Melikidze & Pataraya 1980, 1984; Kazbegi et al. 1991; Melikidze et al. 2000). There is general agreement that this radiation is emitted in strongly magnetized electron–positron plasma well inside the light cylinder. Many observational constraints on emission altitudes imply that the emitted radiation detaches from the ambient plasma at an altitude $r_d$ less than 10% of the light cylinder radius $R_{LC} = P_c/2\pi$ (e.g., Cordes 1978; Blaskiewics et al. 1991; Rankin 1993; Kijak & Gil 1997; Mitra & Deshpande 1999; Gangadhara & Gupta 2001; Mitra & Li 2004; Krzeszowski et al. 2009). The emission is clearly originated at a lower altitude $r_{em}$ and it must propagate through the magnetosphere to reach an observer, that is $r_{em} < r_d$. The plasma properties in the region between $r_{em}$ and $r_d$ can, in principle, influence the nature of waves, and in Section 3.4 we will discuss the effects of propagation, especially those that concern the polarization state.

Once the waves are generated in the emission region ($r \sim r_{em}$), then in the propagation region ($r_{em} < r < r_d$) they naturally split into the ordinary (O) mode and the extraordinary (X) mode, which correspond to the normal modes of wave propagation in the strongly magnetized plasma (see, e.g., Arons & Barnard 1986; Lominadze et al. 1986). The ordinary waves are polarized in the plane of the wave vector $\mathbf{k}$ and the local magnetic field $\mathbf{B}$ and their electric field has a component along both $\mathbf{k}$ and $\mathbf{B}$. Therefore, they strongly interact with plasma particles and thus encounter difficulty escaping from the magnetosphere. On the other hand, the extraordinary waves are linearly polarized perpendicular to $\mathbf{k}$ and $\mathbf{B}$. As a result, the X mode can propagate through the magnetospheric plasma almost as in vacuum.

It is important to realize that there is also observational evidence confirming the X mode to be dominant in the pulsar radiation. Lai et al. (2001), based on the X-ray image of the Vela pulsar wind nebula, were able to model the pulsar’s rotation axis as it is projected in the sky. The X-ray image showed two X-ray arcs that were interpreted as a result of outflowing relativistic particles from two diametrically opposing pulsar beams interacting with the environment around the pulsar. The bisection of the two arcs gave the direction of the rotation axis projected on the sky plane. Furthermore, they used the radio polarization property of the Vela pulsar where the linear polarization position angle (PPA) traverse is well represented by the so-called rotating vector model (RVM). In this model, the PPA of the linear polarization executes an S-shaped track across the pulses. This was interpreted by Radhakrishnan & Cooke (1969) as an electric field vector associated with the range of open dipolar magnetic field planes intersected by the observer’s line of sight. The steepest gradient (SG) point in the PPA track is usually considered as the fiducial point located in the plane containing the magnetic dipole and the rotation axis (at least in slow pulsars). Lai et al. (2001) used the value of the absolute PPA at the SG point ($P_{PA}$), and found its direction to be perpendicular to the rotation axis, which means that the electric vector emanating out of the Vela pulsar is orthogonal to the dipolar magnetic field line planes. Hence, it was the X mode (Lai et al. 2001). This is a very significant observational result, as it was possible for the first time to verify that the electric field vectors emerging from the Vela pulsar magnetosphere are perpendicular to the dipolar magnetic field line planes. The Vela

Note that this may not be true for the millisecond pulsars.
X-ray observations also showed that the proper motion direction (PM) of the pulsar is aligned with the rotation axis.

An X-ray pulsar wind nebula is not observed for a majority of the pulsars, and hence the direction of the rotation axis on the plane of the sky cannot be directly determined. However, based on several recent careful studies by Johnston et al. (2005), Rankin (2007), and Noutsos et al. (2012, 2013), a bimodal distribution of \( \Psi = (\text{PM}_v - \text{PA}_v) \) centered around 0° and 90° has been observed. Mitra et al. (2007) used this distribution, along with the assumption that the pulsar spin axis is aligned with the proper motion direction, to establish the direction of the emerging wave at \( \text{PA}_v \) with respect to the magnetic field line planes.

It is instructive to follow and generalize their arguments. The proper motion measurement of many pulsars can give the projected orientation of the rotation axis with respect to the celestial north. We can also independently measure the absolute position angle \( \text{PA}_v \) (corrected for all instrumental and propagation effects) at the fiducial point (SG point and/or profile midpoint). Then we can find the difference \( \Psi \) between these angles, and amazingly \( \Psi \) shows a bimodal distribution centered around 0° and 90°. How can this happen? It is possible only if the spin axis is either parallel or perpendicular to the direction of the proper motion and at the same time the \( \text{PA}_v \) coincides either with the fiducial plane or the plane perpendicular to it. Let us assume for the moment that \( \text{PA}_v \) is at some arbitrary angle with respect to the fiducial plane. Then we must find such a direction for the velocity vector which is either perpendicular or parallel to the polarization vector at the fiducial point. This then needs to be extended to all pulsars in the sample, in which every pulsar could have different PMs. Such fine tuning is very difficult, given that PMs and \( \text{PA}_v \) are determined by two completely independent phenomena. Hence, the most likely explanation is that the velocity vector should either be parallel or perpendicular to the rotation axis and/or \( \text{PA}_v \), should be parallel or perpendicular to the fiducial plane. The latter is a natural consequence of pulsar radio emission being excited by the soliton coherent curvature radiation, as argued by Melikidze et al. (2000), Gil et al. (2004), and Mitra et al. (2009), hereafter Papers I, II, and III, respectively. As for the former possibility, it has yet to be established by the supernova explosion theory (e.g., Tademaru & Harrison 1975; Cowsik 1998; Spruit & Phinney 1998). However, in three cases for which X-ray emission is available (Vela, B0656+14, and J0538+2817; see Rankin 2007), the rotation axis is parallel to the proper motion direction. If this can be generalized in the future, then the pulsars with \( \Psi \sim 90^\circ \) in the bimodal distribution of \( \Psi \) (see Figure 3 in Rankin 2007) represent the X mode observed as the primary polarization mode (PPM). Similarly, those with \( \Psi \sim 0^\circ \) represent the O mode observed as a secondary polarization mode (SPM).

Recently, in Paper III, the authors showcased highly polarized subpulses from single pulses for a number of pulsars, in which the position angle of the linear polarization closely followed the mean position angle traverse. They further conjectured that the observed polarization state of some subpulses represents the X mode excited by the soliton coherent curvature radiation. This conclusion was conditional on non-fulfillment of the so-called adiabatic walking condition (AWC) first introduced by Cheng & Ruderman (1979, hereafter CR79). The conclusions of Paper III can remain unnoticed, since it lacked strong and convincing arguments that the polarization direction of the generated waves cannot be changed by adiabatic walking. In this paper, we will give further arguments, both from observational and theoretical points of view, that the AWC is indeed not satisfied in pulsar magnetospheres, under conditions required for the soliton coherent curvature radiation emitted at frequencies much lower than the local characteristic plasma frequency. Thus, the X mode excited by the coherent curvature radiation can freely leave the pulsar and reach the observer. This mode represents the observationally dominant PPM. We will also present observational evidence for the O mode, excited by the curvature radiation. This mode dominates at the emission region (about six times stronger than the X mode), but it cannot freely propagate through magnetospheric plasma. We will discuss a possible scenario under which part of the O mode can escape from the magnetosphere. This mode would then correspond to the weaker PPM.

2. IMPORTANCE OF THE SINGLE-PULSE POLARIZATION PROPERTIES

In Figure 1, we show an example of the observed composite and single-pulse polarization in pulsar PSR B2045–16 observed with the Giant Meterwave Radio Telescope (GMRT; see Paper III for observation details) near Pune, India. Two highly polarized subpulses, appearing in two different single pulses in the longitude range corresponding to the leading profile component are shown in the left panels of the figure. Their PPA traverses correspond to the two orthogonal polarization modes. These subpulses represent essentially single-polarization modes close to 100% polarization each. They obviously do not suffer from any significant depolarization, and the PPA associated with the leading profile component follows the mean position angle traverse.

Without any further modeling, we are unsure about the orientation of the polarization vectors of these modes with respect to the pulsar magnetic field line planes. We can, however, adopt the strategy taken by Mitra et al. (2007), where they determine the direction of the modal PPAs with respect to the projected magnetic field lines in planes by comparing the fiducial PPA and proper motion directions, as explained in the Introduction. The proper motion of this pulsar is \( \text{PM}_v = 92^\circ \pm 2^\circ \) and the absolute value of PPA at the fiducial longitude \( \text{PA}_v = -3^\circ \pm 5^\circ \), which gives the quantity \( \Psi = (\text{PM}_v - \text{PA}_v) = 95^\circ \pm 5^\circ \) (see Rankin 2007; Morris et al. 1979, 1981). Thus, based on the assumption that the pulsar proper motion is directed along the rotation axis, the emerging electric field at the fiducial point of the dominant PPA mode is perpendicular to the dipolar magnetic field line plane. As seen in the right panel of Figure 1, the fiducial point is determined by the stronger PPM and hence this point should be associated with the perpendicular or X mode. The weaker PPM should then be associated with the parallel or O mode. As it was pointed out in Paper II, this argument can be extended to the rest of the PPA traverse which follow the RVM model, and hence the polarization state for this pulsar at each pulse phase along the PPM represents the X mode polarized perpendicularly to the magnetic field line planes.

5 This effect is well known for the vacuum case (e.g., Jackson 1975) and for the plasma environment it was generalized by Gil et al. (2004); see the last paragraph in Section 5 of their paper.
6 PSR B2045–16 is a slowly rotating pulsar with period of 1.9 s, and its position angle curve suffers almost no distortions due to the effects of aberration and retardation (see Mitra & Li 2004 for a detailed discussion), and hence follow the RVM rather accurately.
7 Recently, Noutsos et al. (2012) performed a statistical analysis for 54 pulsars to test the pulsar rotation axis and proper motion alignment, and found strong evidence for this effect, excluding the fact that these vectors are completely uncorrelated with >99% confidence.
Figure 1. Right panel shows the average pulse profile of PSR B2045–16. The top panel of this plot shows the total intensity in black, the linear polarization in dashed red lines, and the circular polarization in dotted blue lines. In the bottom panel, the red curve is the average PPA track computed using the average Stokes $U$ and $Q$, and the dotted points are the PPA histograms which are obtained by overplotting the PPA of every single pulse. The average PPA follows the more frequently occurring PPA track known as the primary polarization mode (PPM) and the other one is called the secondary polarization mode (SPM). The left panel shows two highly polarized subpulses occurring close to the leading profile component. The top subpulse is dominated by the PPM and the bottom one is dominated by the SPM. In these subpulses, the corresponding PPA are displayed in green, overlaid on the same dotted histograms as displayed in the right panel for reference. These histograms are exactly the same in all the panels of Figures 1 and 2, as they represent all position angle data taken for this pulsar. It is important to note that these two subpulses are by no means exceptional. An exemplary collection of similar cases can be viewed in Figure 2.

(A color version of this figure is available in the online journal.)

To summarize this section, let us consider a fictitious radiation mechanism that generates two orthogonal polarization modes without being parallel or perpendicular to the magnetic field line planes. To explain the observed polarization in Figure 1, one must invoke a propagation effect that rotates the modes in such a way that they emerge almost purely polarized parallel or perpendicular to the dipolar magnetic field line planes. Such a mechanism was proposed by CR79, who introduced the so-called adiabatic walking scenario. They argued that even chaotically oriented polarization directions will slowly rotate while the waves propagate away from the generation region. As a result, within the polarization limiting region (beyond which the waves detach from the plasma; see, e.g., Arons & Barnard 1986), all the chaotic polarizations will be arranged and thus the emerging polarization will consist of the two orthogonal modes. However, this does not mean that they will be polarized parallel or perpendicular to the planes of the dipolar magnetic field lines, as demonstrated in Figure 1 (a collection of such pulses is presented in Figure 2). This would require unrealistic fine tuning in every pulse longitude. Therefore, based on our observations, we conclude that this radiation is not affected by the adiabatic walking and we receive the generated polarization pattern. The only radiation mechanism that can distinguish the magnetic field line planes is the curvature radiation excited in pulsar plasma (Papers II and III). In the next section, we will demonstrate that the AWC is not satisfied in the pulsar plasma with characteristics allowing the generation of the coherent soliton curvature radiation. Therefore, the adiabatic walking does not affect the polarization of the plasma X and O waves excited by this radiation mechanism.

Before concluding this section, it is important to comment on the fact that a majority of subpulses have intermediate, low, or undetectable polarization, where the PPA does not necessarily follow the mean PPA traverse. This property is in general expected in the currently discussed coherent curvature radiation model (Papers I–III). The subpulse in this model results from the incoherent addition of a large number of independently emitting solitons. Each of these solitons excite the X and O modes in the plasma, however, the modes becomes separated (however, in this paper, we will show that the modes do not change their state of polarization), and further that the amplitude of the O mode can be significantly damped. Before the modes emerge from the plasma, the incoherent addition of a large number of such waves (with the random contribution of X and O modes both in amplitude and phase) leads to a range of observed depolarization values (detailed considerations of that scenario have been deferred to a separate paper). Only in certain occasions the plasma conditions are such that either the X or O mode emerges as the dominant radiation mode, which is observed as close to 100% polarized subpulse emission. In fact, the highly polarized subpulses presented in this paper and

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8 The polarization of one mode is parallel to the local $k$ and $B$ plane and the polarization of the other mode is perpendicular to it. This local plane in general does not coincide with the plane of magnetic field lines.

9 It is well known that the variations of the PPA across the pulse window trace the range of dipolar magnetic field line planes as predicted by the RVM model.
Figure 2. Five more examples of highly polarized subpulses shown in the same way as in Figure 1. The left panels show stronger subpulses dominated by the PPM. The right panels show weaker subpulses dominated by the SPM. However, in these panels there are also subpulses dominated by PPM, although individual pulses the single subpulses are always polarized in one of the modes in all panels presented. For further details, see Section 2.

(A color version of this figure is available in the online journal.)
Paper III are indeed rare (typically <1%) occurrences. However, they are very important to study as they can reveal the true nature of pure X and O modes.

3. PLASMA WAVES IN THE NEAR MAGNETOSPHERE AND PULSAR EMISSION MECHANISM

The coherent curvature radiation has been considered as a natural emission mechanism for the observed pulsar radiation (e.g., Ruderman & Sutherland 1975, hereafter RS75; Benford & Buschauer 1983; Melikidze & Pataraya 1984), although several key issues in this theory have remained unresolved for a long time. For example, the theory of the formation of charged bunches leading to curvature radiation in a plasma was not developed. Also, it was not clear how the radiation emerges from the magnetospheric plasma. The recent attempts to build and develop a self-consistent theory of curvature radiation for pulsar radio emission has been published in Papers I–III. Under the framework of this theory, here we recapitulate the physical processes generating the magnetospheric plasma that leads to the observed pulsar coherent radio emission.

The basic requirement of this theory is the presence of a non-stationary inner acceleration region above the polar cap. The prototype of this region was the inner vacuum gap suggested by RS75. The formation of their gap was based on the overestimated value, by at least an order of magnitude, of the binding energy of iron ions in the neutron star crust (Gil et al. 2003, and references therein). As a result, the accelerating potential drop along the magnetic field in the gap was very high, exceeding $10^{13}$ V. In such a high potential drop, the backflowing particles heat the polar cap to enormous temperatures close to $10^7$ K, which has been never observed in the form of intense thermal X-rays form polar caps. Also, the so-called subpulse drift would be too fast compared with the observed drift rates (Gil et al. 2003, and references therein). Therefore, the original RS75 model called for a revision from the very beginning.

In the mean time, new calculations of the binding energy of iron ions was published by Jones (1986) and a similar study was performed using a different and more precise method by Medin & Lai (2006, 2007). These calculations demonstrated that the vacuum gap could be formed only if the actual surface magnetic field at the polar cap was close to $10^{14}$ G, much higher than the dipolar magnetic field at the surface of most pulsars. To preserve the idea of the inner acceleration region, one must invoke a strong non-dipolar crust anchored surface field much stronger than the dipolar component at the surface. It appears that such surface magnetic fields are likely to be produced in the neutron star crust by Hall-drift instabilities (Geppert et al. 2013, and references therein).

Based on these ideas, Gil et al. (2003) proposed the partially screened gap (PSG) model for the inner acceleration region. In this model, the backflowing particles heating the polar cap cause a thermal outflow of iron ions, which partially screen the gap potential drop. The more they screen, the less the lower potential drop causes heating and therefore less screening at the same time. Thus, the thermostatic regulation will occur establishing the surface temperature at a value close to, but slightly below, the so-called critical temperature (at which iron ions would be extracted at the co-rotation-limited Goldreich–Julian charge density). As a consequence the maximum value of the potential drop will be about one order of magnitude lower than the vacuum case of RS75. This potential drop is still high enough to drive the non-stationary sparking discharge in the PSG and produce the electron positron primary plasma similar to that envisioned by RS75. The presence of an additional iron ion component does not significantly influence the production of the secondary plasma. The so-called multiplicity factor $k = n_{k}^i/n_{GJ}$ (where $n_{k}^i$ is the charge density of the secondary plasma and $n_{GJ}$ is the Goldreich–Julian charge density) lies in the range $100 < k \leq 10^5$ (for details, see Asseo & Melikidze 1998, also Szary 2013 for the application to the PSG model). The structure of the secondary plasma is formed by the sparking discharge of the PSG.

3.1. Plasma Dispersion Relation at the Radio-emission Region

Let us now consider the magnetospheric plasma waves in the radio-emission region, i.e., the tube of the open field lines consisting of electron and positron of secondary plasma at altitudes $r$ of about 50–100 stellar radii, where the coherent curvature radio emission is expected to originate. We start with the dispersion equation in the pulsar frame of reference assuming that $k$ is along the $z$-axis and $\theta$ is the angle between the local magnetic field $B$ and the wave vector $k$ (see Kazbegi et al. 1991, hereafter KMM91). The distribution function of the secondary plasma is close to a Gaussian distribution, centered at $\gamma = \gamma_s \gg 1$, having a spread of $\Delta \gamma / \gamma_s \ll 1$ (Paper II; Szary 2013). At the altitudes $r = (50–100) \times 10^6$ cm, the fractional altitude $\delta t \equiv (r/R_{c}) = 2\pi r/(cP)$ is small for typical pulsars ($\delta t = (0.01–0.1)P^{-1}$) and hence the ratio

$$\frac{\omega_p}{\omega_B} = 2.8 \times 10^{-4} \kappa^{0.5} \tilde{P}^{0.75}_{-15} \delta t^{1.5} \ll 1, \ (1)$$

where $\omega_p^2 = 4\pi e^2 \kappa n_{GJ}/m_e$ is the plasma frequency, $\omega_B = eB/m_ec$ is the cyclotron frequency, and $m_e, e$, and $c$ correspond to the mass of the electron, the charge of the electron, and the speed of light, respectively (in cgs units).

Using the conditions stated above, we can neglect all terms containing $\omega_p^2$ and hence the dispersion relation can be written in the form

$$\left(\frac{k^2 c^2}{\omega^2} - \varepsilon_{11}\right)E_1 - \varepsilon_{13}E_3 = 0, \quad (2)$$

$$\left(\frac{k^2 c^2}{\omega^2} - 1\right)E_2 = 0, \quad (3)$$

$$\varepsilon_{31}E_1 + \varepsilon_{33}E_3 = 0. \quad (4)$$

Here $E_1, E_2$, and $E_3$ are the components of electric fields along the $x$, $y$, and $z$-axes (the $y$-axis is perpendicular to the plane of $k$ and $B$), $\varepsilon_{11} = 1 - I \sin^2 \vartheta$, $\varepsilon_{13} = \varepsilon_{31} = I \sin \vartheta \cos \vartheta$, $\varepsilon_{12} = -\varepsilon_{21} = \varepsilon_{23} = -\varepsilon_{32} = 0$, $\varepsilon_{33} = 1 - I \cos^2 \vartheta$, where $I$ can be expressed as

$$I \equiv \frac{1}{2} \sum_p \omega_p^2 \int \frac{dp}{\gamma^3} \frac{f_p}{(\omega - k v \cos \vartheta)^2}, \quad (5)$$

$\alpha$ denotes the sum over electrons and positrons, $\omega_p^2 = 4\pi e^2 n_k^i/m_e$, and $p$ and $v$ are the dimensionless momentum and velocity of the particles along the magnetic field (see KMM91). Solving the above equations, we obtain the dispersion relation,

$$1 - \frac{\omega^2}{k^2 c^2} = 0, \quad (6)$$

for the X mode, which is linearly polarized along the $y$-axis, while for the O mode (polarized in the $xz$-plane), the dispersion
curve is a solution of the following equation:

\[
\frac{1}{2} \left( 1 - \frac{k^2 c^2}{\omega^2} \cos^2 \vartheta \right) \sum \frac{\omega_p^2}{\gamma^2} \int \frac{dp}{\gamma^2 \sqrt{\left( \omega - k v \cos \vartheta \right)^2}} = 1 - \frac{k^2 c^2}{\omega^2},
\]

(7)

Let us note that under condition (1), we can assume that the distribution functions for both electrons and positrons are identical. The difference between these functions is important for the soliton coherent curvature radiation as the slowly varying charge density is proportional to \( \Sigma e_a \omega_p^2 \) and vanishes if the distribution functions are identical (see Equation (A19) in Paper I). On the other hand, those components of the linear permittivity tensor, i.e., \( \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{23}, \) and \( \varepsilon_{32} \), which have a nonzero value in the case of non-identical distribution functions (see, e.g., KMM91), are proportional to \( \omega_p^{-1} \) and are negligibly small (see Equation (1)). Thus, taking into account \( \Delta \gamma / \gamma_s \ll 1 \), the distribution functions can be described by the delta-function at \( \gamma_s \), i.e., \( f_\pm \sim \delta \left( p - p_c \right) \). For the purpose of our discussion, this is a very reasonable assumption, as we are interested in the average features of the plasma, and the peculiarities of plasma distribution are irrelevant. Then, using \( \nu \approx 1 - 0.5 \gamma^{-2} \), Equation (7) can be reduced to the following form:

\[
1 - \frac{k^2 c^2}{\omega^2} = \frac{\omega_p^2}{\gamma^2} \left( 1 - \frac{c^2}{\omega^2} \cos^2 \vartheta \right) \left( \omega - k \left( 1 - \frac{1}{2 \gamma^2} \right) \cos \vartheta \right),
\]

(8)

where \( \gamma_s = (1 + \rho_s^2)^{1/2} \). Equation (8) describes the O modes (i.e., modes polarized in the plane of \( \mathbf{k} \) and \( \mathbf{B} \)) of the magnetized electron–positron plasma and has two possible solutions, which correspond to two branches: the O mode and the L mode. In the conditions under consideration (see Section 3.2 for details), i.e., \( \vartheta \ll 1 \) and \( \omega \) being in the range from 0 up to values that do not significantly exceed \( \omega_0 \) (see Equation (9) below), the waves have following features: the L mode is almost longitudinal, while the O mode is almost transverse. Equation (8) cannot be solved analytically, and to demonstrate its basic properties, the schematic representation of its solutions are shown in Figure 3. In the frequency range \( \omega \gg \omega_s \), the polarization characteristics of the L and O modes change; the L mode becomes almost electromagnetic while the O mode turns to be almost longitudinal and as a subluminal wave, as it undergoes strong Landau damping (see Section 3.5).

If \( \vartheta = 0 \) (propagation along the local magnetic field direction), Equation (7) has two independent solutions. The purely transverse solution coincides with the X mode, while the L mode coincides with the purely longitudinal Langmuir wave, the dispersion equation of which is obtained by dividing Equation (7) by Equation (6). Two values of wave frequency can be distinguished. The first point is defined by \( k = 0 \), for which the corresponding frequency is \( \omega = \omega_1 = \gamma_s^{-1} \omega_p \). The second point is defined by the condition \( \omega = k c \), i.e., the phase velocity equals \( c \) for which the corresponding frequency is

\[
\omega = \omega_c = 2 \gamma_s \omega_p.
\]

(9)

In this paper, we use the term characteristic frequency for \( \omega_c \). It is important to note that Langmuir waves cannot be exited if \( \omega_1 < \omega < \omega_0 \), since the phase velocity exceeds the speed of light (subluminal waves). On the other hand, in the frequency region \( \omega > \omega_0 \) the phase velocity is less than the speed of light (subluminal waves), thus the two-stream instability can develop, provided that the plasma distribution function has a proper shape (see Asseo & Melikidze 1998; Lominadze et al. 1986 for details).

### 3.2. Domain of Plasma Parameters

Let us overview the basic parameters of the secondary plasma flowing along the tube of open field lines in the observer’s frame of reference. We will represent them in terms of the basic pulsar parameters. The shadowed region represents the frequency range \( \omega \ll \omega_s \), characteristic for the soliton curvature radiation, while the level \( \omega > \omega_s \) corresponds to the linear Langmuir oscillations used by CR79, which, however, cannot emit any coherent curvature radiation. In the case of parallel propagation (\( \vartheta = 0 \)), the O mode coincides with the X mode, while the L mode becomes a strictly longitudinal Langmuir wave. The dispersion curve of the Langmuir waves crosses the \( \omega = k c \) line at the point \( \omega = \omega_c \).

(A color version of this figure is available in the online journal.)

![Figure 3. Schematic representation of the electron positron plasma eigenmodes in an infinitely strong magnetic field in the case of oblique (\( \vartheta \neq 0 \)) propagation. The shadowed region represents the frequency range \( \omega \ll \omega_s \), characteristic for the soliton curvature radiation, while the level \( \omega > \omega_s \) corresponds to the linear Langmuir oscillations used by CR79, which, however, cannot emit any coherent curvature radiation. In the case of parallel propagation (\( \vartheta = 0 \)), the O mode coincides with the X mode, while the L mode becomes a strictly longitudinal Langmuir wave. The dispersion curve of the Langmuir waves crosses the \( \omega = k c \) line at the point \( \omega = \omega_c \).](image)

The characteristic plasma frequency \( \nu_p \) in GHz can be written as

\[
\nu_p = \frac{\nu_0}{2 \pi} = 5.2 \times 10^{-2} \frac{1}{\nu_s} \left( \frac{P_{-15}}{p^5} \right)^{0.5} \Gamma^{-3}.
\]

(10)

The characteristic plasma frequency \( \nu_p \) in GHz can be written as

\[
\nu_p = \frac{\nu_0}{2 \pi} = 2 \times 10^{-5} \sqrt{\frac{\nu_s}{\nu_s}} \left( \frac{P_{-15}}{p^5} \right)^{0.25} \Gamma^{-1.5},
\]

(11)

with \( \nu_0 \) given by Equation (9). As the third parameter, we present the characteristic frequency of the soliton curvature coherent radiation, which corresponds to the maximum of the power spectrum \( v_{cr} \) (see Figure 2 in Paper II),

\[
v_{cr} = 1.2 \frac{c \Gamma^3}{2 \pi \rho} = 0.8 \times 10^{-9} \Gamma^3 \rho^{-0.5},
\]

(12)

where \( \rho \) is the radius of the curvature of the dipolar magnetic field lines and \( \Gamma \) is the Lorentz factor of the emitting soliton, for which the opening angle of the radiation cone \( \vartheta \sim \Gamma^{-1} \).
If, in Equation (63) by Asseo & Melikidze (1998), we assume reasonably low values for the average Lorentz factor of plasma particles $\gamma_p/10^2 \sim (0.5-0.7)$ and for the half-width of a plasma cloud $\Delta P/\sqrt{5 \times 10^3} \sim (0.2-0.4)$, we find that a corresponding distance from the center of the star $r$ ranges from a few to several stellar radii. Thus, the altitude of about 5 stellar radii seems to be quite a good approximation for the minimal altitude at which the two-stream instability can develop.

We can now briefly discuss how Figure 4 will alter for different pulsar periods. The frequencies $\nu_c$ and $\nu_B$ are much more sensitive to changing $P$ than the characteristic frequency of the curvature emission $\nu_{cr}$, which depends on period in the same way as the fractional distance. Therefore, the soliton curvature radiation can always develop in the proper region of altitudes, i.e., about 50–100 stellar radii, irrespective of the value of $P$.\footnote{However, for very long periods, the available potential drop could not be high enough to power the primary beam, which leads to “death” of radio pulsars. It is also worth mentioning that, for very old pulsars, the crustal strong surface field may not be supported by the Hall-drift instability. The absence of the crustal field leads to switching off the creation of dense electron–positron plasma, which is necessary for a mechanism of the coherent pulsar radio emission (Geppert et al. 2013).}

Figure 4 corresponds to the total power and contains no information about the polarization properties of the observed radiation. These properties will be considered in Section 3.4 in the context of possible propagation effects.

3.3. The Mechanism of Coherent Radio Emission

Let us now briefly review the spark-associated soliton model of the coherent curvature radiation from pulsars (for details, see Paper I and references therein). Like any plasma model, for pulsar radiation this model is also based on the development of some plasma instabilities. The only plasma instability that can arise at altitudes lower than 10% of the light cylinder is the two-stream instability (Lominadze et al. 1986; KMM91; Asseo & Melikidze 1998), as all other instabilities are suspended by the strong magnetic field. The two-stream instability is a result of the effective energy exchange between particles and waves, which can occur if the phase velocity of the waves $\omega/k$ is near to the velocity of the resonant particles $\nu_c$, i.e., the resonant condition $\omega/k = \nu_c \sim 0$ is satisfied. As already mentioned, the resonance is possible in the frequency region $\nu_c \sim 0\nu_d$. However, one should realize that the amplification of waves occurs only if there is an excess of particles with velocities larger than the phase velocity of the waves near the resonant point. In the opposite situation, the waves will be damped (the Landau damping). Thus, the plasma distribution function should have a proper shape (e.g., Asseo & Melikidze 1998) for a two-stream instability development. Such conditions can naturally be realized if a plasma is produced via non-stationary gap discharge. The spark discharge timescale in the PSG is a few tens of microseconds and this results in the overlapping of successive clouds of the outflowing secondary plasma. Each elementary spark-associated plasma cloud has a spread in momentum and the overlapping of particles with different momenta leads to a two-stream instability in the secondary plasma cloud (RS75; Usov 1987; Asseo & Melikidze 1998). This triggers strong Langmuir turbulence in the plasma, and, if this turbulence is strong enough, the waves become modulationally unstable. The unstable wave packet described by the nonlinear Schrödinger equation leads to the formation of a quasi-stable nonlinear solitary wave, i.e., a soliton (Pataraya & Melikidze 1980). The longitudinal (along k) size of the soliton should be much larger than the wavelength of the linear Langmuir wave. Also, it must be charged to be able to radiate coherent curvature emission. Thus, the soliton bunch must be charge separated, which can be caused either by a difference in the distribution function of electrons and positrons, or by an admixture of iron ions in the secondary plasma, or by both these effects. A sufficient number of charged solitons is formed, which can account for the observed radio luminosity in pulsars (Paper I).
of the emitted waves should be longer than the longitudinal size of the soliton $\Delta$. This is the necessary condition for the coherency of a curvature radiation process. Thus, the frequencies plotted in Figure 4 should obey the following constraints:

$$v_{cr} \ll \frac{c}{\Delta} \ll v_0 \ll v_B.$$  

(13)

It is clearly seen from Figure 4 that the observed pulsar radiation cannot be generated at altitudes exceeding 10% of the light cylinder radius (practically, the radio-emission region should be contained between one to several percent of $R_{lc}$; see the dashed area in Figure 4). This conclusion is based purely on the properties of plasma and the emission mechanism. It also corresponds perfectly to the other limits on emission heights obtained from observations and geometrical considerations (e.g., Cordes 1978; Blaskiewics et al. 1991; Rankin 1993; Kijak & Gil 1997; Mitra & Deshpande 1999; Gangadhara & Gupta 2001; Mitra & Li 2004; Krzeszowski et al. 2009).

### 3.4. The Adiabatic Walking Condition

While studying the rotation of the polarization planes of the X and O modes propagating in the pulsar magnetospheric plasma, CR79 introduced the AWC in the form of

$$\left| \frac{1}{k} \frac{\partial \phi}{\partial x} \right| \ll |\Delta N|,$$

(14)

where $\Delta N$ is a change of $N$ occurring during the propagation of waves, $\phi$ is the linear polarization angle (dimensionless) of the mode or some similar dimensionless parameter, which defines the mode polarization, and $1/k \equiv 2\pi\lambda$, where $\lambda$ is the wavelength.12 Here $N = N^{(o)} - N^{(s)}$ is the difference between the refractive indices of the O and X modes:

$$N^{(o)} = \frac{k_c}{\omega} |\text{O-mode}| \quad \text{and} \quad N^{(s)} = \frac{k_c}{\omega} |\text{X-mode}|.$$  

Remembering that the polarization vector of waves is either tangent or normal to the $\mathbf{k}$ and $\mathbf{B}$ planes, for a change of the dimensionless parameter $\phi$, we can choose $\Delta \phi = \Delta \theta / \rho$, where $\theta \sim 1/\Gamma$. It expresses the fact that the change of the angle $\theta$ between $\mathbf{k}$ and $\mathbf{B}$ during propagation causes a corresponding change in the polarization vector. Thus, the polarization plane of a wave would rotate by the angle about unity ($\Delta \phi \sim 1$), while it propagates the distance $\Delta l = \Delta \theta / \rho$, if the AWC is fulfilled (see also Paper II).

In our formalism, as seen in Equation (6), the refractive index $N^{(s)} = 1$ for all values of $k$. To obtain $N^{(o)}$, we must solve Equation (8), which, cannot be achieved analytically. However, we can make following very reasonable assumption:

$$\frac{k^2c^2}{\omega^2} \sin^2 \theta \ll 1.$$  

(15)

This condition is consistent with Equation (13), which assumes that, in the radio-emission region, the frequency of emitted waves ($v_{cr} = kc/2\pi$) should be less than $v_0 = \omega_0/2\pi$. As sin $\theta$ cannot exceed unity, the above condition is always valid for emitted waves. The solution of Equation (8) for the O mode under the condition expressed above is

$$\omega = kc \cos \theta \left( 1 - \frac{1}{2} \frac{k^2c^2}{\omega^2} \sin^2 \theta \right).$$  

(16)

Thus, it is straightforward to show that

$$N^{(o)} = \frac{k_c}{\omega} |\text{O-mode}| = \cos^{-1} \theta \left( 1 + \frac{k^2c^2}{2\omega^2} \sin^2 \theta \right).$$  

(17)

Then, assuming $\theta \ll 1$, which is a very reasonable assumption for the relativistic plasma, we obtain

$$N = N^{(o)} - 1 = \frac{1}{2} \theta^2 \left( 1 + \frac{k^2c^2}{\omega^2} \right).$$  

(18)

In order to examine the validity of the AWC, we can use $|\partial \phi / \partial x| \approx |\Delta \phi / \Delta l| = 1/\rho \theta$ and $|\Delta N| = |\theta \Delta \theta|$, therefore the AWC can be rewritten in the following form:

$$\theta^2 \left| \frac{\Delta \theta}{\theta} \right| \gg \frac{\rho}{\rho \Gamma^3}.$$  

(19)

Let us note that $\Delta \theta / \theta \sim 1$ (meaning that the AWC is fulfilled) only if $\theta^3 \gg \Gamma^{-3}$. However, $\theta \approx \Gamma^{-1}$, thus we can safely conclude that the AWC is not satisfied under our assumptions. Therefore, adiabatic walking cannot affect the polarization of waves excited by the soliton curvature radiation.

It may be interesting to determine why we have found just a conclusion opposite to that obtained by CR79. The main difference comes from the condition $k^2c^2 = \omega^2 \ll \omega_0^2$, which must be satisfied in the case of soliton curvature radiation (Paper I). In the case of the classical curvature radiation model of RS75 that was used by CR79, the characteristic frequencies should obey the following condition: $k^2c^2 = \omega^2 \approx \omega_0^2$ (since their bunches are formed by linear Langmuir waves).

In the frequency range $\omega \sim \omega_o$ (which is not allowed in the soliton mechanism; Paper I), the dispersion curve of the O mode deviates significantly from the X-mode dispersion line $\omega = kc$ compared with the frequency range of soliton curvature emission $\omega \ll \omega_o$ (marked by the shadowed area in Figure 4). It is difficult to demonstrate these differences in Figure 3 and therefore we present the numerical solution for $|\Delta N|$ in Figure 5, obtained for three values of $\theta$. These figures are equivalent to each other since Figure 5 simply represents the difference between the O mode and the X mode in Figure 3. As seen from Figure 5, below $kc/\omega_o \approx 0.1$, $|\Delta N|$ remains unchanged, so $\Delta N \sim 0$ and the AWC cannot be satisfied. This is the range of soliton coherent curvature radiation considered in this paper as the viable pulsar radio-emission mechanism. Beyond this range, $|\Delta N|$ varies so $\Delta N$ can attain relatively large values and therefore the AWC can be satisfied. This range was considered by CR79, who used RS75 radiation model, in which charged bunches formed by linear Langmuir waves emitted coherent curvature radiation. However, this radiation mechanism should not be realized physically, as argued for the first time by Lominadze et al. (1986) and summarized in Paper I. It is worthwhile to recapitulate these arguments here. The emission of waves (by means of curvature radiation) with frequency close to the local plasma frequency $\omega_p = \omega_o$ is impossible, because one cannot simultaneously fulfill the following two conditions: (1) the timescale of the radiative process must be significantly shorter

12 This is the sufficient condition of CR79 (their Equation (2)). The necessary (adiabatic) condition of CR79 (their Equation (1)) is always satisfied if plasma properties vary slowly enough.
than the plasma oscillation periods, which means $\omega_{\text{cr}} \gg \omega_o$, and (2) the linear characteristic dimension of the bunches must be shorter than the wavelength of the radiated wave, which means $k_{\text{cr}} \ll k_o = \omega_o/c$. On the other hand $\omega_o = k_c c$ and $\omega_{\text{cr}} = k_{\text{cr}} c$. Thus, it is impossible to simultaneously satisfy the above two conditions and therefore bunching associated with high-frequency Langmuir plasma waves cannot be responsible for the coherent pulsar radio emission (see also Melrose & Gedalin 1999).

In the soliton model of coherent curvature radiation considered in this paper, the soliton-like bunches are formed due to the nonlinear evolution of Langmuir wave packets and thus the sizes of the bunches are naturally much larger than the Langmuir wavelength. Hence, the condition $k^{(\text{sol})}_{\text{cr}} \ll k_o$ is always fulfilled (here the superscript “sol” corresponds to soliton). At the same time, the soliton lifetime must be much longer than the period of Langmuir waves ($\Delta \tau \gg 2\pi/\omega_o$). However, $\omega_{\text{cr}}^{(\text{sol})} \lesssim \Delta \tau/2\pi$ or $\omega_{\text{cr}}^{(\text{sol})} \ll \omega_o$. As we have shown, this condition implies that the AWC cannot be satisfied for the plasma modes excited by the soliton curvature radiation mechanism. Thus, the excited plasma waves can retain their initial polarization, while they propagate through the magnetospheric plasma.

### 3.5. O-mode Properties

The group velocity of plasma waves describes the velocity and the direction of energy transfer and is defined as

$$\mathbf{v_g} = \mathbf{i} \frac{\partial \omega}{\partial k_\parallel} + \mathbf{j} \frac{\partial \omega}{\partial k_\perp}. \quad (20)$$

Here $\mathbf{i}$ and $\mathbf{j}$ are unit vectors directed along and across the external magnetic field vector. As the dispersion law of the X mode is $\omega = k_c$, the group and phase velocities of the X mode are equal to each other, thus $v_g = c$ and $\mathbf{v}_g \parallel \mathbf{k}$.

The dispersion law of the O mode can be expressed as $\omega = k_c c (1 - k_c^2 c^2/2\omega_0^2)$, thus the group velocity of the O mode can be calculated as

$$v_g = i c \left(1 - \frac{k_c^2 c^2}{2\omega_0^2}\right) - j k_c k_c c^3/2\omega_0^2. \quad (21)$$

Taking into account that both $k_\parallel$ and $k_\perp$ are much less than $\omega_o/c$, it is straightforward to obtain $v_g \approx i c$. Thus, the group velocity of the O mode is directed along the external magnetic field, i.e., the O mode is ducted along $\mathbf{B}$ preserving the direction of the wave vector and eventually decays as a result of the Landau damping (Arons & Barnard 1986). Thus, under normal conditions, the O mode cannot escape from the magnetosphere.

Let us consider under what special conditions a fraction of the generated O mode could escape and reach the observer. Such a possibility can be illustrated by Figure 3, which presents wave modes in a strongly magnetized plasma. As stated in Section 2, the solution of Equation (8) corresponds to two types of waves: superluminal (L mode) and subluminal (O mode). In the case of oblique (nonzero $\vartheta$) propagation, both are mixed transverse–longitudinal waves (see, e.g., Arons & Barnard 1986; KMM91). Which of those two features actually dominates depends on the value of the ratio $E_1/E_3$ (recall that $E_3$ is along $\mathbf{k}$ and $E_1$ is in the plane of $\mathbf{k}$ and $\mathbf{B}$). If $E_1/E_3 \gtrsim 1$, then the O mode (polarized in the yz-plane) is mostly transverse and thus almost electromagnetic in nature. In the opposite case, if $E_1/E_3 \ll 1$, the waves are longitudinal plasma oscillations, thus they are non-electromagnetic in nature. Therefore, if one can demonstrate that $E_1/E_3 \gg 1$ in the radio-emission region, then the L mode (provided it can be excited by some physical mechanism; some possibility will be discussed later) can escape as the observed orthogonal mode with the electric field lying in the plane of curved magnetic field lines.

Let us now estimate the value of $E_1/E_3$ within our scenario. It follows from Equation (2):

$$\frac{E_1}{E_3} = \frac{\varepsilon_3}{k_3 c^2/\omega_0^2 - \varepsilon_1} = \frac{I \sin \vartheta \cos \vartheta}{I \sin^2 \vartheta - (1 - k_c^2 c^2/\omega_0^2)}. \quad (22)$$

Substituting $I$ from Equation (8), Equation (22) becomes

$$\frac{E_1}{E_3} = -\left(\frac{\omega^2}{k_c c^2}\right) \frac{\sin \vartheta}{\left(\frac{\omega^2}{k_c c^2} - 1\right) \cos \vartheta}. \quad (23)$$

For the superluminal L mode $\omega > k_c$ and under small $\vartheta$ approximation (remembering that $\vartheta \gamma_s > 1$), the solution of Equation (8) is

$$\frac{\omega}{k_c} = 1 + \frac{1}{2} \frac{1}{\vartheta^2 \gamma_s^2} \left(\frac{\omega_s^2}{k_c c^2}\right), \quad (24)$$

which is valid provided that

$$\frac{1}{\vartheta^2 \gamma_s^2} \left(\frac{\omega_s^2}{k_c c^2}\right) \ll 1. \quad (25)$$

Incorporating Equation (24) into Equation (23), we obtain

$$\frac{E_1}{E_3} = \frac{k_c c^2}{\omega_s^2} 2\vartheta^3 \gamma_s^4 \gg 1. \quad (26)$$

As mentioned earlier, this condition means that the L mode is almost electromagnetic in nature.

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13 It worth mentioning that the L mode cannot be directly excited by curvature radiation as it is suppressed by Razin’s effect (see Paper II).
Below we describe a scenario of how this electromagnetic L mode can possibly be excited. The shaded region in Figure 3 represents the frequency range of waves generated by means of soliton coherent curvature radiation. Let us recall that our pulsar model is conditional on a non-stationary sparking discharge of the inner accelerating region (PSG). Therefore, the resultant secondary plasma must be inhomogeneous in space. In such plasma, there should be many regions with steep density gradients along and across the magnetic field lines. If the O mode reaches the steep gradient of the plasma density, where the number density (and hence the characteristic frequency $\omega_n$) changes rapidly, then the L-mode curve moves toward the shaded region (shown schematically in Figure 3). Under such a condition, the O mode can be linearly coupled with the L mode and escape as electromagnetic waves (e.g., Arons & Barnard 1986; KMM91). The L mode also preserves the polarization properties of the O modes since both the L and O modes should be linearly polarized in the same plane, which is orthogonal to the polarization plane of the X mode. Thus, the observed SPM (i.e., the O mode) will be polarized in the plane of the curved magnetic field lines. It is worth realizing that most of the generated power is contained in the O mode (about six times stronger than the X mode; see footnote 5). Therefore, only a small fraction of this mode must escape to explain the observed level of the weaker SPM.

Finally, it is important to emphasize that the escape of the L mode should occur before the O mode is damped. This problem, however, is beyond the scope of this work and will be addressed in a forthcoming paper.

4. CONCLUSIONS

In this paper, we have shown the observational evidences that the linearly polarized waves emerge from the magnetosphere either parallel or perpendicular to the magnetic field line planes. We associate these waves with the X and O modes excited by soliton coherent curvature radiation in the secondary plasma. The above conclusion is true if the waves emerge from the magnetosphere as they are generated, preserving their polarization properties. In other words, the AWC should not hold in the pulsar magnetosphere within the soliton coherent curvature radiation model.

We have demonstrated that properties of the pulsar plasma as well as features of the soliton coherent curvature radiation emission mechanism provide the proper conditions for the generation of pulsar radio emission at altitudes well below 10% of the light cylinder radius (see Figure 4 and Sections 3.2 and 3.3), which is in good agreement with observations. Then, in Section 3.4, we proceeded to show that the difference between the refractive indexes of O and X modes $N = N^o - N^x$ is negligibly small in this emission region (see Figure 5) for the frequency range $\omega \ll \omega_n$. For which the soliton coherent curvature radiation can operate. This implies that $N \sim 0$ and hence the AWC given by Equation (14) cannot be satisfied. It is important to note that our conclusion differs from that of CR79, since they calculated the AWC condition at $\omega \sim \omega_n$, while we calculated the AWC condition at $\omega \ll \omega_n$. The excited waves can hence retain their initial polarization as they propagate in the magnetospheric plasma. The X mode, which is electromagnetic in nature, escapes from the pulsar magnetosphere and represents the PPM highly polarized subpulses showcased by Paper III. Although observationally there exist highly polarized pulses in the SPM, they cannot be related directly to the O mode, as under normal circumstances this mode gets damped and cannot escape. As a possible interpretation of the SPM, we suggest in Section 3.5 that if there are steep density gradients in the radiation excitation region, then the O mode couples to the L mode and can emerge as the SPM polarized in the planes of the curved magnetic field lines.

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