In this Comment are corrected some results that are obtained in [1]. Instead of using the phase invariance and the time dilation in the derivation of the expressions for the Doppler shift and the aberration as in [1] we use the Lorentz transformations (LT) of the four-dimensional (4D) wave vector. Otherwise, all notations are kept the same as in [1].

Let us assume that the spectrograph is at rest in the laboratory inertial frame of reference (IFR) $S$. The light source is at rest in the IFR $S'$ which is moving with velocity $v$ relative to $S$ along the common $x,x'$-axis. The components of the wave 4-vector in $S$ are $k^\mu = (\omega/c)(1, \cos \phi, \sin \phi, 0)$ and in $S'$ they are $k'^\mu = (\omega'/c)(1, \cos \phi', \sin \phi', 0)$ for which it holds $k'^\mu k'_\mu = k'^\nu k'_\nu = 0$. Here, $\phi$ ($\phi'$) is the angle between the wave direction of propagation and $x$- ($x'$-)axis, i.e. relative to $v$. The components of $k^\mu$ can be obtained by the LT of the components $k'^\mu$ which yields

$$k^\mu = \left[ \frac{\gamma \omega'}{c} (1 + \beta \cos \phi'), \frac{\gamma \omega'}{c} (\cos \phi' + \beta), \frac{\omega'}{c} \sin \phi', 0 \right]. \quad (1)$$

Hence, the Doppler shift is given as

$$\omega = \gamma \omega' (1 + \beta \cos \phi') \quad (2)$$

whereas the equations that describe the change in the direction of wave propagation are

$$\cos \phi = \frac{\cos \phi' + \beta}{1 + \beta \cos \phi'} \quad (3)$$

and

$$\sin \phi = \frac{\sin \phi'}{\gamma (1 + \beta \cos \phi')} \quad (4)$$

In fact, if the observation of the unshifted line (i.e. of the frequency $\omega' = \omega_0$ from the atom at rest) is performed at an observation angle $\phi'$ in $S'$, the rest frame of the emitter, then the same light wave (from the same but now moving atom) will have the shifted frequency $\omega$ and will be seen at an observation angle $\phi$ (generally different

From $\phi'$ in $S$, the rest frame of the spectrometer. In astronomy the angular shift

$$\Delta = \phi' - \phi \quad (5)$$

is dubbed aberration.

The inverted relations between IFRs $S$ and $S'$ are obtained by mere interchange of $\phi$ and $\phi'$ and $\beta$ by $-\beta$. Thus, the inverted Doppler effect and cosine, Eqs. (2) and (6), read

$$\omega' = \gamma \omega (1 - \beta \cos \phi) \quad (6)$$

and

$$\cos \phi' = \frac{\cos \phi - \beta}{1 - \beta \cos \phi} \quad (7)$$

respectively. We emphasize that Eqs. (6) and (7) have been derived by Einstein in his fundamental work on special relativity theory (SRT) [2] and may also be found in many textbooks on SRT like e.g. [3].

Keywords: Doppler effect, special relativity theory, zero-frequency shift, aberration of light.

FIG. 1. (Color online.) Angles $\phi$ (blue), $\phi'$ (red), and $\phi + \phi'$ (black) as well as the absolute value of the aberration $|\phi' - \phi|$ as a function of $\omega'/\omega$. Results due to Eq. (8) are shown by the full-line curves while those of Eq. (9) by the dash-dotted-line curves. The aberration, Eq. (10) is shown by the dashed-line curves, in magenta for IFR $S$ and in cyan for IFR $S'$.
the emitter IFR obviously unphysical behavior of such a wave because in receding from the observer at \( \phi = \pi \).

Equally unphysical is the case of a light wave which is head-on approaching the observer, \( \phi = \pi \).

The author states \( \phi + \phi' = \pi \) does come? In fact, it will be shown below (see Fig. 2) that for \( \phi' + \phi = \pi \) the aberration \( \Delta \) is at maximum. Also, he claims the zero shift taking place at \( \phi = \phi_{zfs} \), where the aberration reaches a maximum, which is an entirely contradictory statement because for \( \phi = \phi' \) the aberration, Eq. (4), vanishes. For \( \phi \) and \( \phi' \) of Eq. (8) the resulting \( \Delta \) is shown by the dashed curve. Because \( \Delta \) changes the sign for \( \omega' > \omega \), in order to fit into the figure frame, in Fig. 1 is displayed the absolute value \( |\Delta| \).

According to the principle of relativity the physical reality should not depend on the concrete IFR and coordinate basis used in describing it. The most simple way to verify the correctness of an expression is to interchange the IFRs, i.e. \( S \) and \( S' \). In that case Eq. (8) becomes

\[
\cos \phi = \frac{-\beta \cos \phi'}{1 + \beta \cos \phi'}
\]

and its predictions for \( \phi, \phi' \), and \( \phi + \phi' \) are shown by the dash-dotted line curves in Fig. 1. All three considered physical quantities \( \phi, \phi' \), and \( \phi + \phi' \) display an entirely different feature: these \( \phi \) and \( \phi' \) are mirror symmetric about \( \pi/2 \) relative to their previous graphs while their sum is symmetric about the angle \( \pi \). One again has \( \phi_{zfs} = \phi'_{zfs} \) but its value is \( \pi \) minus the previous one that was obtained from Eq. (8). Although the aberration curve seems to be unchanged that is due to its absolute value. Namely, with Eq. (9) \( \Delta > 0 \) for \( \omega' > \omega \) and \( \Delta < 0 \) for \( \omega' < \omega \).

Figure 2 displays the correct observables \( \phi, \phi', \Delta \), and \( \phi + \phi' \) obtained by using Eqs. (3), (7), and (5), respectively. Inverting the role of the IFRs \( S \) and \( S' \) gives the identical results as it should be (the full, Eq. (3), and dashed curves, Fig. 2, are laying over each other). Because \( \phi \) and \( \phi' \) are monotonically increasing functions of \( \omega' / \omega \) such is also \( \phi + \phi' \). The aberration \( \Delta \) is indeed maximal at \( \omega' = \omega \) where \( \phi + \phi' = \pi \) and \( \phi'_{zfs} = \pi - \phi_{zfs} \).

Another way to verify the correctness of Eqs. (11) to (14) is to use a geometric approach to SRT from [1]. As seen from Sec. 7.2 in [4] an abstract coordinate-free wave vector \( k^\mu \) is represented in \( S(S') \) by the coordinate-based geometric quantity (CBGQ) \( k^\mu e_\mu (k'^\mu e'_\mu) \) comprising both the components \( k^\mu \) and the 4D basis vectors \( e_\mu \). Any CBGQ is an invariant 4D quantity under the LT since the components transform by the LT and the basis vectors by the inverse LT leaving the whole CBGQ unchanged; it is the same physical quantity for relatively moving inertial observers. It can be easily seen that with (14) it holds that \( k^\mu e_\mu = k'^\mu e'_\mu \), which proves the validity of (14), i.e., of Eqs. (2), (3) and (4) and at the same time it disproves Eq. (8).

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