We report on a first study of 4-jet production in a complete high-energy factorization (HEF) framework \cite{1, 2}. We include and discuss contributions from both single-parton scattering (SPS) and double-parton scattering (DPS) and compare to the measured data. The DPS HEF result is considerably smaller than the one obtained with collinear factorization. The mechanism leading to this difference is of kinematical nature. In contrast to the collinear approach, the HEF approach nicely describes the distribution of the $\Delta S$ variable, which involves all four jets and their angular correlations.

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1. Introduction

So far, complete \((n \geq 4)\)-jet production via single-parton scattering (SPS) was discussed only within collinear factorization. Results up to next-to-leading (NLO) precision can be found in\(^3\),\(^4\). Here we report on recent study of production of four jets including SPS processes and Double-parton scattering (DPS) within high-energy \((k_T)\)-factorization (HEF)\(^5\),\(^6\),\(^7\),\(^8\)\(^1\). Double-parton scattering (DPS) was claimed to have been observed for the first time at the Tevatron\(^9\). In the LHC era, with much higher collision energies available, the field has received a new impulse and several experimental and theoretical studies address the problem of pinning down DPS effects. Even just from purely theoretical point of view, the problem is quite subtle\(^11\). As for the non perturbative side, it is in principle necessary, when considering a double-parton scattering, to take into account the correlations between the two partons coming from the same protons and involved in the scattering processes. Such an information should be encoded in a set of double parton distribution functions (DPDFs), generalising usual parton distribution functions (PDFs). Some successful attempts to generalize the usual evolution and to have relevance for phenomenology are becoming to appear only recently\(^13\),\(^14\),\(^15\),\(^16\).

In the meanwhile, phenomenological and experimental studies of double-parton scattering rely on factorized Ansatz for the DPDFs, which amount to neglecting momentum correlations between partons and introducing an effective cross section, \(\sigma_{\text{eff}}\). The latter quantity is usually extracted from experimental data. In the present approach we will use the factorized Ansatz and concentrate on the difference between leading-order collinear and high-energy-factorization results. The latter includes effectively higher-order corrections. For most of high-energy reactions the single-parton scattering dominates over the double-parton scattering. The extraordinary example is double production of \(c\bar{c}\) pairs\(^12\). For four-jet production, disentangling the ordinary SPS contributions from the DPS corrections can be quite challenging for several reasons: first of all, it is necessary to define sufficiently sensitive, process-dependent observables, w.r.t. which the DPS differential cross section manifestly dominates at least in some corners of phase space.

2. Single-parton scattering production of four jets

The HEF factorization formula for the calculation of the inclusive partonic 4-jet cross section at the Born level reads

\[
\sigma_{4-\text{jets}}^B = \sum_{i,j} \int \frac{d\gamma_1}{\gamma_1} \int \frac{d\gamma_2}{\gamma_2} \int d^2k_1 d^2k_2 \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F) \times \frac{1}{2 \pi} \prod_{l=1}^4 \frac{d^3k_l}{(2\pi)^32E_l} \Theta_{4-\text{jet}}(2\pi)^4 \delta \left( x_1 P_1 + x_2 P_2 + \bar{k}_{T1} + \bar{k}_{T2} - \sum_{l=1}^4 k_l \right) |\mathcal{M}(i^*, j^* \to 4\text{ part})|^2.
\]

(2.1)

Here \(\mathcal{F}_i(x_k, k_{T_k}, \mu_F)\) is a TMD (Transversal Momentum Dependent)\(^19\) distribution function for a given type of parton carrying \(x_{1,2}\) momentum fractions of the proton and evaluated at the

\(^1\)for overview of the framework see \(^9\)
factorization scale $\mu_F$. The index $l$ runs over the four partons in the final state, the partonic center of mass energy squared is $\hat{s} = 2x_1x_2P_i \cdot P_j$; the function $\Theta_{4\text{-}jet}$ takes into account the kinematic cuts applied and $\mathcal{M} \to 4 \text{ part}$ is the gauge invariant matrix element for $2 \to 4$ particle scattering with two initial off-shell legs calculated numerically with a numerical package [18]. It includes symmetrization effects due to identity of particles in the final state and new degrees of freedom introduced via $\vec{k}_T$, which are the parton’s transverse momenta, i.e. the momenta perpendicular to the collision axis. The formula is valid when the $x$’s are not too large and not too small when complications from nonlinearities may eventually arise [17].

3. Double-parton scattering production of four jets

Single-parton scattering contributions are expected to be dominant for high momentum transfer, as it is highly unlikely that two partons from one proton and two from the other one are energetic enough for two hard scatterings to take place, as the behaviour of the PDFs for large $x$ suggests. However, as the cuts on the transverse momenta of the final state are softened, a window opens to possibly observe significant double parton scattering effects, as often stated in the literature on the subject. First of all, let us recall the formula usually employed for the computation of DPS cross sections, adjusting it to the 4-parton final state,

$$\frac{d\sigma^0_{4\text{-}jet,DPS}}{d\xi_1d\xi_2} = \frac{m}{\sigma_{eff}} \sum_{i_1,j_1,k_1,l_1} \frac{d\sigma^B(i_1j_1 \rightarrow k_1l_1)}{d\xi_1} \frac{d\sigma^B(i_2j_2 \rightarrow k_2l_2)}{d\xi_2}, \quad (3.1)$$

where the $\sigma(ab \rightarrow cd)$ cross sections are obtained by restricting formula (2.1) to a single channel and the symmetry factor $m$ is 1 unless the two hard scatterings are identical, in which case it is $1/2$, so as to avoid double counting them. Above $\xi_1$ and $\xi_2$ stand for generic kinematical variables for the first and second scattering, respectively. The effective cross section $\sigma_{eff}$ can be interpreted as a measure of the transverse correlation of the two partons inside the hadrons, whereas the possible longitudinal correlations are usually neglected. In this paper we use $\sigma_{eff}$ provided by the CDF, D0 collaborations and recently confirmed by the LHCb collaboration $\sigma_{eff} = 15 \text{ mb}$, when all SPS mechanisms of double charm production are included.

3.1 Comparison to CMS data

As discussed in Ref. [21], so far the only experimental analysis of four-jet production relevant for the DPS studies was realized by the CMS collaboration [20]. The cuts used in this analysis are $p_T > 50 \text{ GeV}$ for the first and second jets, $p_T > 20 \text{ GeV}$ for the third and fourth jets, $|\eta| < 4.7$ and the jet cone radius parameter $\Delta R > 0.5$.

It goes without saying that the LO result with soft cuts applied needs refinements from NLO contributions. For this reason, in the following we will always perform comparisons only to data (re)normalised to the total (SPS+DPS) cross sections. What is interesting in the HEF result, compared to collinear factorization, is the dramatic damping of the DPS contribution. In Figs. 1 and 2 we compare the predictions in HEF to the CMS data. Here both the SPS and DPS contributions are normalized to the total cross section, i.e. the sum of the SPS and DPS contributions. In all cases the renormalized transverse momentum distributions agree with the CMS data. However, the absolute cross sections obtained in this case within the HEF approach are too large.
we present our HEF prediction for the normalized to unity distribution in the 3 jets. In addition we show the ratio of the SPS HEF result to the CMS data.

Figure 1: Comparison of the LO collinear and HEF predictions to the CMS data for the 1st and 2nd leading jets. In addition we show the ratio of the SPS HEF result to the CMS data.

Not only transverse momentum dependence is interesting. The CMS collaboration extracted also a more complicated observables [20]. One of them, which involves all four jets in the final state, is the $\Delta S$ variable, defined in Ref. [20] as the angle between pairs of the harder and the softer jets, where $\vec{p}_T (j_i, j_k)$ stands for the sum of the transverse momenta of the two jets in arguments.

In Fig. 3 we present our HEF prediction for the normalized to unity distribution in the $\Delta S$ variable. Our HEF result approximately agrees with the experimental $\Delta S$ distribution. In contrast the LO collinear approach leads to $\Delta S = 0$, i.e. a Dirac-delta peak at $\Delta S = 0$ for the distribution in $\Delta S$. For the DPS case this is rather trivial. The two hard and two soft jets come in this case from the same scatterings and are back-to-back (LO), so each term in the argument of $arccos$ is zero (jets are balanced in transverse momenta). For the SPS case the transverse momenta of the two jet pairs (with hard jets and soft jets) are identical and have opposite direction (the total transverse momentum of all four jets must be zero from the momentum conservation). Then it is easy to see that the argument of $arccos$ is just -1. This means $\Delta S = 0$. The above relations are not fulfilled in the HEF approach. The SPS contribution clearly dominates and approximately gives the shape of
the $\Delta S$ distribution. It is anyway interesting that we describe the data via pQCD effects within our HEF approach which are in Ref. [20] described by parton-showers and soft MPIs.

![Graph showing the comparison of HEF predictions to CMS data for $\Delta S$ spectrum.](graph.png)

**Figure 3:** Comparison of the HEF predictions to the CMS data for $\Delta S$ spectrum. In addition we show the ratio of the (SPS+DPS) HEF result to the CMS data.

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