Aging phenomena in spin glasses: theory, experiment, and simulation

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Abstract

We study numerically temperature-shift and field-shift aging protocols on the 3-dimensional (3D) Ising Edwards-Anderson (EA) spin-glass (SG) model focusing on respectively the temperature-chaos nature and the stability under a static field of the SG phase. The results of the latter strongly support the droplet theory which predicts the instability of the SG phase under the field. They are also discussed in relation with the experimental studies.

Key words: Spin Glass; Aging Phenomena; Droplet Theory
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1. Introduction

More than a decade the dispute on the nature of ordering in spin glasses between the mean-field theory based on the replica symmetry breaking in Parisi’s solution of the infinite-ranged model and the droplet theory based on a phenomenological scaling argument on the short-ranged model has remained unsettled [1]. Currently, the following two explicit predictions by the droplet theory [2,3] have been extensively studied. 1) The equilibrium SG states at two temperatures with difference $\Delta T$ are uncorrelated in length scales larger than the overlap length $L_{\Delta T}$, i.e., the so-called temperature($T$)-chaos nature of the SG phase, and 2) in the equilibrium and thermodynamic limits the SG phase is broken by a static field $h$ of infinitesimal strength, thereby introduced is the crossover length $L_h$. The latter separates characteristics of droplet excitations by their size $L$, such that they are dominated by the Zeeman energy for $L > L_h$ and by the SG free energy gap for $L < L_h$.

One may imagine that the existence of such a characteristic length $L_{\Delta T}$ ($L_h$) could be easily checked by the temperature($T$)-shift (field($h$)-shift) aging protocol, where the temperature (field) is changed once on a way to equilibrium. However, this is rather hard because of the extremely slow spin dynamics in the SG state: within a time-window of relative magnitude of $10^{4-5}$, which is the case not only in simulations but also in ordinary experiments, the SG correlation length, or mean domain size, $R(t)$, is considered to grow only by a factor two or even less as will be discussed below. Therefore, no simulation nor experiment on such an aging protocol alone has not yet succeeded to settle the problem mentioned above. A possible strategy to overcome this difficulty is to examine a set of $T$- ($h$-) shift protocols with $L_{\Delta T}$ ($L_h$) systematically changed relatively.
to $R(t)$ within the available time-window, to look for a possible scaling behavior in the length scales involved, and to check whether the behavior is compatible with the droplet theory or not [4].

In the present work we numerically study aging dynamics in the 3D Ising EA SG model with Gaussian nearest-neighbor interactions with zero mean and variance $J$. The latter is used as the unit of energy and temperature. In this unit the SG transition temperature is estimated as $T_c \simeq 0.95$ [5,6]. Along the above-mentioned strategy we have examined a number of $T$- and $h$-shift processes with various $\Delta T$ and $h$ as well as the (waiting) time, $t_w$, of the shift. For further details of the model and the Monte Carlo (MC) method of simulation the reader may refer to our previous works [7,8,9,10,11].

In the next section we investigate the $T$-chaos nature through $T$-shift processes, and in §3 the stability of the SG phase under a field through $h$-shift processes. The last section is devoted to discussions.

2. $T$-shift aging processes

Following the idea of ‘twin-experiments’ in [4], we examine here negative (positive) $T$-shift aging processes. A system is instantaneously quenched at time $t = 0$ from $T = \infty$ to $T_1$ ($T_2$) below $T_c$, the temperature is decreased (increased) to $T_2$ ($T_1$) at $t = t_w^{\text{sh}}$ ($t_p^{\text{sh}}$), and then let the system to age at $T_2$ ($T_1$). Throughout the process we measure the imaginary part of the ac susceptibility of frequency $\omega$, $\chi''(\omega; t)$ [8]. Its typical behavior in a negative $T$-shift process is shown in Fig. 1. By the $T$-shift at $t = t_w^{\text{sh}}$, $\chi''(\omega; t)$ rapidly decreases, and it merges to the $T_2$-isothermal curve from below. If, however, the branch of $\chi''(\omega; t)$ at $t > t_w^{\text{sh}}$ is shifted to the right by a proper amount, $t_{\text{sh}}$, as indicated by the open circles in the figure, it merges to the $T_2$-isothermal curve from above at $t = t_{\text{mr}}$. The inset of the figure explains how to determine $t_{\text{sh}}$: when an amount of the shift is smaller than $t_{\text{sh}}$, the shifted branch crosses with the $T_2$-isothermal curve (triangles in the inset), while a larger shift delays the merging significantly (squares). Here we call the time required for $\chi''(\omega; t)$ to merge to the isothermal curve in this sense the crossover time and denote it as $t_{\text{cr}}^{\text{sh}}$, i.e., $t_{\text{cr}}^{\text{sh}} = t_{\text{mr}} - (t_w^{\text{sh}} + t_{\text{sh}})$. [This characteristic time is called the effective age in [4] and the effective waiting time in [11].] For a positive $T$-shift process we can similarly define the crossover time $t_{\text{cr}}^{\text{sh}}$ [11]. A set of results for $t_w$ and $t_{\text{cr}}$ thus obtained are shown in Fig. 2.

After the first quench of a negative $T$-shift process, the SG ordering, or the mean domain size

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{$\chi''(\omega; t)$ in a negative $T$-shift process with $t_w^{\text{sh}} = 512$, $T_1 = 0.7, T_2 = 0.4$, and $t_w = 2\pi/\omega = 64$ (solid circles). The continuous curves represent $\chi''(\omega; t)$ in the isothermal aging at $T = T_1$ and $T_2$. The open circles are the properly shifted branch of $\chi''(\omega; t)$ at $t > t_w^{\text{sh}}$. Shown in the inset are how the shifted branches merge to the $T_2$-isothermal curve (see the text).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{$t_{\text{cr}}^{\text{sh}}$ ($t_p^{\text{sh}}$) vs. $t_{\text{cr}}^{\text{sh}}$ ($t_p^{\text{sh}}$) for the negative (positive) $T$-shift processes between three pairs of temperatures. The lines represent the expected behavior from Eq.(1) combined with Eq.(3) as explained in the text.}
\end{figure}
time range, which we call the transient regime, sub-domains of a mean size $L(t - t_w)$ in local equilibrium of $T_2$ grows within each domains of a mean size $R(t_w)$ in local equilibrium of $T_1$. If $R(t_w)$ is enough smaller than the overlap length $L_{\Delta T}$ introduced in §1, we naively expect that $L(t - t_w)$ catches up $R(t_w)$ at $t - t_w \simeq t_{cr}$, or,$$L(t_{cr}) = R_{T_1}(t_{cr}) = R_{T_1}(t_w),$$to hold, where $R_{T_1}(t)$ is the growth law of SG domains in the isothermal aging at $T$. By the first equality above we imply that the growth mechanisms of $L(t - t_w)$ at $T$ and $R_{T_1}(t)$ are common, i.e., they are due to thermal activation processes. The same equation as above but with $T_1$ and $T_2$ interchanged is expected to hold for the corresponding positive $T$-shift process. This expectation is in fact confirmed by the result in Fig. 2 that the data $t'_{cr}$ vs. $t_w$ and $t''_w$ vs. $t_{cr}$ lie on a common curve for a small $\Delta T = T_1 - T_2 (= 0.1)$. We call the interpretation so far described the cumulative memory scenario for $T$-shift processes with small $|\Delta T|$.

For later discussions let us here mention explicit expressions for the growth law of $R_{T_1}(t)$. Within the accuracy of simulations, the numerical results of $R_{T_1}(t)$ [7,12,13] are well fitted either to the logarithmic law proposed in the droplet theory [14]$$R_{T_1}(t)/l_0 \simeq \left[ T \frac{\ln(t/t_0)}{\Delta_g} \right]^{1/\psi},$$or to a power law$$R_{T_1}(t)/l_0 \simeq (t/t_0)^{1/z(T)}.$$

Above, $l_0$ and $t_0$ are the characteristic length and time scales, and $\Delta_g$ and $\psi$ the characteristic scale of energy barrier of droplet excitations and the associated exponent, respectively. In Eq.(3) we set $l_0 = 1$ lattice distance and $t_0 = 1$ MC step per spin, and the exponent $1/z(T)$ is given by [7]$$1/z(T) \simeq bT/T_c,$$with $b \simeq 0.16$ in the range $0.7 \gtrsim T/T_c \gtrsim 0.4$.

For a large $\Delta T (= 0.3)$, the sets of data of negative and positive $T$-shift processes in Fig. 2 differ significantly from each other. Also the second equality of Eq.(1) is violated for the negative $T$-shift process shown in Fig. 1. We tentatively attribute this deviation from the cumulative memory scenario to the temperature-chaos nature of the equilibrium SG phase described in §1. If $L_{\Delta T}$ is smaller than $R_{T_1}(t_w)$, then in the time range after the $T$-shift in which $L(t - t_w) > L_{\Delta T}$ is satisfied, aging dynamics looks as if the system is already in the $T_2$-isothermal aging state. This is because, by definition of $L_{\Delta T}$, the longer range order than $L_{\Delta T}$ developed at $T = T_1$ is irrelevant to the SG order at $T = T_2$. Therefore $t_{cr}$ is given by the condition $L(t_{cr}) \simeq L_{\Delta T}$, and is independent of $t_w$ in this case.

However, $t_w$ in the present simulation is not large enough to reproduce such a drastic phenomenon. We consider that the negative $T$-shift process observed here is in the weak chaos regime proposed by Jönnson et al [15]. We also note that the second equality of Eq.(1) is little violated in the positive $T$-shift with $\Delta T = 0.3$. This asymmetric appearance of the chaos effect contradicts the prediction of the droplet theory [2] as well as the recent experiment on the Heisenberg spin glass AgMn [4].

3. h-shift aging processes

The h-shift processes we mainly discuss here are as follows. After instantaneous quench to $T$, the system is aged under a zero field until $t = t_w$, when a field $h$ is switched on. We then observe the induced (zero-field-cooled) magnetization $M (= M_{ZFC})$ and $\chi''(\omega; t)$ in the whole process. We study them exclusively at $T = 0.6$ where the aging dynamics is considered not affected by the critical dynamics close to $T_c$.

It is well established that, at least for a small $h$, the logarithmic growth rate of $M$ defined by $S(t') = \partial(M/h)/\partial\ln t'$ with $t' = t - t_w$ exhibits a peak at $t' \sim t_w$. More generally, the peak position of $S(t')$ plays the same role as $t_{cr}$ discussed in the previous section. It gives us a characteristic time scale of crossover from the isothermal aging state
under $h = 0$ to that under a finite $h$, and so is denoted also by $t_{cr}$. In Fig. 3 we show $S(t')$ with $t_w = 4096$ for various values of $h$. For a small $h (= 0.1)$, as mentioned just above, $t_{cr} \simeq t_w$ is ascertained. When $h$ increases, however, $t_{cr}$ significantly decreases. In Fig. 4 the resultant $t_{cr}$ for various $h$ and $t_w$ are presented.

Now we convert the relations between $t_{cr}$ and $t_w$ into those between the characteristic length scales $R_{cr}^{eff} = R_T(t_{cr}, h)$ and $R_T(t_w)$. The effective crossover length $R_{cr}^{eff}$ is the mean size of subdomains grown under $h$ in the period of $t_{cr}$ and has to be estimated by the growth law under $h$. Since, however, it is quite time-consuming to simulate the latter due to the presence of the paramagnetic component, we approximate it by $R_T(t_{cr}, h = 0)$, i.e., by Eq.(3) in the present work.

The obtained sets of $R_{cr}^{eff}$ and $R_T(t_w)$ for different $h$ are normalized by the crossover length $L_h$ introduced in §1. It is given by [2]

$$L_h/l_0 \simeq (h/\Upsilon)^{-\delta},$$

with $\delta = (\frac{d}{2} - \theta)^{-1}$, where $\Upsilon$ is the unit of free-energy gap. In evaluating explicitly $L_h$ by Eq.(5) we simply put $\Upsilon$ unity and $\delta = 0.77$ with $d = 3$ and $\theta = 0.2$ [7,16]. The resultant scaling plot of $R_{cr}^{eff}/L_h$ vs. $R_T(t_w)/L_h$ is presented in Fig. 5. One sees in the figure that most of the data tend to lie on a single curve. If we take into account the fact that we have not adjusted the parameters involved at all, the scaling behavior obtained is quite satisfactory.

We interpret the scaling behavior shown in Fig. 5 as dynamical crossover from the SG aging state to the paramagnetic state. Actually, in the range $x \ll 1$, where $R_T(t)$ measured within the time window of our simulation is much smaller than $L_h$, various properties which are usually regarded as those of the SG state are observed. Among them are the so-called $\omega t$-scaling of the ac susceptibility in the isothermal aging [1] and the sum rule $M_{ZFC}(t) + M_{TR}(t) = M_{FC}(t)$ [17], where $M_{FC}$ is the induced magnetization observed when $h$ is applied just after the quench to $T$ and $M_{TR}$ is the thermoremanent magnetization observed after $h$ in the field-cooled process is switched off at $t = t_w$. In the range $x \gg 1$ corresponding to a large field of $h \gg 1.0$, on the contrary, the complete saturation of $M_{ZFC}$ and $M_{FC}$ to a common value is observed within the time window of our simulation. This is

\begin{center}
\includegraphics[width=0.4\textwidth]{fig3.png}
\caption{Fig. 3. $S(t')$ with $t_w = 4096$ plotted vs $\ln t'$. The data are for $h = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.75, 1.0 and 2.0 from right to left.}
\end{center}

\begin{center}
\includegraphics[width=0.4\textwidth]{fig4.png}
\caption{Fig. 4. Plot $t_{cr}$ vs. $t_w$. The data points are for $h = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.75 and 1.0 from top to bottom, and the line represents $t_{cr} = t_w$.}
\end{center}

\begin{center}
\includegraphics[width=0.4\textwidth]{fig5.png}
\caption{Fig. 5. $R_{cr}^{eff}/L_h$ vs $R_T(t_w)/L_h$. The data at small $x$ are fitted to $y = x - cx^{1+1/\delta}$ with $c = 0.15$, which is the formula proposed in the weak chaos scenario in [15].}
\end{center}
the expected property of the paramagnetic state. Combined with these observations which will be discussed elsewhere, we conclude that the result of Fig. 5 is a strong support of the droplet picture for the $h$-shift aging process and so the instability of SG state in the equilibrium and thermodynamic limits.

Lastly let us try to extend the scaling behavior obtained above by our simulation on the Ising EA model to the one in real Ising spin glasses such as Fe$_{0.5}$Mn$_{0.5}$TiO$_3$, focusing on the crossover point defined by the condition $x = R_T(t_w)/L_h = 1$. Here we use the power growth law of Eq.(3) with $t_0 = 10^{-12}$ s. Then, with the same approximations, such as $R_T(t, h) = R_T(T, h = 0)$, as those already introduced, the condition is reduced to

$$h^{-\delta} \simeq (t/t_0)^{\beta t/T}.$$  

(6)

Also using $b = 0.16$ and $\delta = 0.77$ as in the simulation, we obtain, for example, $h \simeq 0.6T$ for $T/T_c = 0.6$ and $t = 10^2$ s. A set of these figures is compared with $h \simeq 1.3$ T, $T_{\chi}/T_c \simeq 0.6$ and $t_{\chi\psi} \sim 10^2$ s, which are picked up from the paper by Katori and Ito [18], who determined the critical temperature $T_{\chi}$ of Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ as the one where the $M_{ZFC}$ starts to deviate from $M_{FC}$. In a certain range of $T_{\chi}$, they found the relation $h \propto (1 - T_{\chi}/T_c)^{\alpha}$ with $\alpha \simeq 1.5$ which is the expected behavior of the de Almaid-Thouless phase transition [20] in the infinite-ranged SG model. However, the rather good coincidence of the above two sets of vaules, combined with the dependence of $h$ on $T$ from Eq.(6) with a fixed $t$ equal to the measuring time $t_{\chi\psi}$ in their temperature sweeping experiment (not shown), suggests that $T_{\chi}$ they measured could be interpreted as the dynamical crossover point with $t_{\chi\psi} \simeq t_{\chi\psi}$.

Another experiment on Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ under large static field $h$ was done by Mattsson et al. [19], who measured the ac susceptibility. They found two regimes in its behavior which are separated by the the freezing temperature $T_{f}(\omega, h)$. Although what they measured is droplet excitations almost in equilibrium but not domains in aging, we may use Eq.(6) with $t = t_{\psi} = 2\pi/\omega$, i.e., $T_{f}$ is the temperature at which the mean size of droplets which can respond to the ac field of period $\omega$ is equal to $L_h$. Then Eq.(6) yields, for example, $h \simeq 3.0$ T with $t_{\psi} = 10^{-2}$ s and $T_{f}/T_c = 0.5$, while the corresponding experimental value is $h = 2.2$ T.

Although we do not want to insist strongly the semi-quantitative agreement of the experimental results with a simple extension of the simulational results, we may claim from the above argument that $t_{\chi\psi}$ for field-shift processes under $h$ of a few Oe is beyond the astronomical time.

4. Discussions

On the interpretation of aging phenomena in spin glasses by means of the droplet picture, the length scales of certain domains and droplets play a central role. In the above arguments, they are $\xi_{\Delta T}$, $L_h$, $R_T(t_{\psi})$, and $L(t - t_{\psi})$. In the numerical study so far carried out [7,12,13], $R_T(t)$ is replaced by the coherence length of the real replica overlap function which has been explicitly evaluated. Since the overlap function is obtained by averaging over sites in each sample and then over samples, detailed properties of each domains, such as whether they are fractal or compact, are averaged out, and so they are beyond the scope of the present simulation. Neither the nature of subdomains has been directly pursued by simulation yet. But its introduction with the subdomains-within-domain scenario [9,11] helps us very much to interpret the observed phenomena in the transient regime after the $T$- or $h$-shift as well as the memory effect observed in the $T$- or $h$-cycling processes [1]. For example, even in the cummulative regime of Fig. 5 with $R_{\chi\psi} \simeq R_T(t_{\psi})$, rejuvenation-like behavior (or a jump-up) in $\chi'(\omega; t)$ has been observed both in experiment [21] and the present simulation (not shown).

Another fundamental ingredient of the analysis based on the droplet picture is the growth law of $R_T(t)$, which is frequently used as the conversion formula from the time scale to the length scale. However it is hard to accurately determine it due to the following circumstances. Let us consider the ratio $r = R_T(t_{\max})/R_T(t_{\min})$, where $t_{\max}$ ($t_{\min}$) is the maximum (minimum) time scale of observation. The results of our simulation with $t_{\min} \simeq 10$ and $t_{\max} \simeq 10^5$ yield, say at $T/T_c = 0.6$, $r \simeq 2.3$. If Eqs.(3) and (4) with $t_0 = 10^{-12}$ s and $b = 0.16$ are
extended to the time scale of ordinary experiments, i.e., $t_{\text{min}} \sim 10$ s and $t_{\text{max}} \sim 10^5$ s, the identical $r$ is obtained. When the logarithmic law of Eq. (2) is applied to the experimental data, $r$ becomes even smaller [4]. We may say that, both in experiments and simulations, we measure a very limited portion of an equilibration process, at least the length scale of SG ordering is concerned. This is the reason of difficulty in determining the growth law of $R_T(t)$ accurately.

In order to investigate the stability of the equilibrium SG state against perturbations such as the temperature change $\Delta T$ and a static field $h$, thereby overcoming the above-mentioned difficulty, we have carried out a set of shift-aging protocols where the perturbation of various strengths, $P$ ($\Delta T$ or $h$), is applied after the waiting time $t_w$ which are also systematically changed. For each process specified by $(P, t_w)$, we have extracted the crossover time $t_{cr}$ between the state aging to an unperturbed SG state and the one to the perturbed SG, or non-SG state. The length scales $R_{eff}(P; t_{cr})$ associated with $t_{cr}$ are then found to be scaled nicely by the characteristic length scale associated with $P$. This strategy, first carried out in [4], is expected to be most efficient to judge the stability of the equilibrium SG state by actual experiments.

Irrespectively of the strategy mentioned above, experiments to extract $R_T(t)$ with referring to another length scale independently of the growth law itself are certainly very much required. To our knowledge, however, there has been only two such experiments which use a length scale associated with the static field effect. One is due to Mattsson et al [19] already mentioned above and the other due to Joh et al [22]. The latter result prefers the power law of Eq.(3) with the value of $b$ in $1/z(T)$ nearly equal to our numerical results. From this point of view, an experiment on SG fine particle systems may be of interest.

To conclude we have presented and discussed the results of our simulation on the $T$- and $h$-shift aging processes in the 3D Ising EA SG model. In the $T$-shift process only a precursor of the temperature-chaos effect has been observed, while the results on the $h$-shift process strongly support the droplet picture that the SG state under a finite field is unstable in the equilibrium and thermodynamic limits, though it takes astronomic time for the SG state to be broken if $h$ is small.

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