Unpredictability and the transmission of numbers

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Abstract Curiously overlooked in physics is its dependence on the transmission of numbers. For example the transmission of numerical clock readings is implicit in the concept of a coordinate system. The transmission of numbers and other logical distinctions is often achieved over a computer-mediated communications network in the face of an unpredictable environment. By unpredictable we mean something stronger than the spread of probabilities over given possible outcomes, namely an opening to unforeseeable possibilities. Unpredictability, until now overlooked in theoretical physics, makes the transmission of numbers interesting. Based on recent proofs within quantum theory that provide a theoretical foundation to unpredictability, here we show how regularities in physics rest on a background of channels over which numbers are transmitted.

As is known to engineers of digital communications, numerical transmissions depend on coordination reminiscent of the cycle of throwing and catching by players tossing a ball back and forth. In digital communications, the players are computers, and the required coordination involves unpredictably adjusting "live clocks" that step these computers through phases of a cycle. We show how this phasing, which we call logical synchronization, constrains number-carrying networks, and, if a spacetime manifold in invoked, put "stripes" on spacetime. Via its logically synchronized channels, a network of live clocks serves as a reference against which to locate events. Such a network in any case underpins a coordinate frame, and in some cases the direct use of a network can be tailored to investigate an unpredictable environment. Examples include explorations of gravitational variations near Earth.
Keywords unpredictability · coordinate system · live clock · logical synchronization · numerical transmission

1 Introduction

Some of us, especially on the theory side, entered physics to evade surprises, to find the enduring. What, though, if the enduring is the prevalence of unpredictable surprises? By unpredictable we mean something stronger than uncertain. While uncertainty pertains to a spread in a probability measure over a given set of possibilities, unpredictability allows for the emergence of unforeseen possibilities. Unpredictability as discussed here stems from a distinction between evidence and explanations of that evidence. This distinction is reflected within quantum theory. On the blackboard of theory, given evidence (from the workbench, so to speak) is expressed as a probability measure over a set of outcomes. The probability measure is parametrized by “knob settings,” thought of as under experimental control [1]. Explanations or predictions of the probabilities are expressed on the blackboard in terms of wave functions or density operators representing a prepared quantum state, along with linear operators expressing a measurement procedure. We proved that for any given parametrized probability measure there are infinitely many quantum explanations that generate the given probabilities, but that disagree with each other about probabilities from evidence of future experiments, not yet on hand [2]. Choosing a quantum explanation of given evidence therefore requires a reach beyond logic to make a guess. Because of the need to guess, the explanation is not merely uncertain, but is unpredictable: the guess announces an unforeseen possibility.

Here we offer networks of numerical transmission as structures recognizing the role of unpredictability in the concept of location. To locate something one needs a background against which to locate it. Usually in physics that background is a coordinate frame, which for theoretical purposes is represented by a coordinate system. (One distinguishes a coordinate system as a mathematical construction from its realization as a coordinate frame involving measurement uncertainties [3].) In Sec. 2 we review how coordinate frames depend on transmissions of numerical clock readings through an unpredictable environment, so that a location is specified by its relation a background consisting of numerical clock readings and number-carrying signals, for example in the Global Positioning System (GPS). In the theories of special and general relativity, a coordinate system entails numerically expressed reference patterns clock readings, toward with one tries to steer a physical frame. In general relativity, the possible reference patterns of clock readings and number-bearing signals are constrained by a metric tensor field. With the high precision involved in the search for gravitational radiation, this metric tensor field can be unpredictable, thus subjecting any reference pattern of clock readings to unpredictability [2]. This unpredictability is essential to the concepts of live clocks and their logical synchronization reviewed in Sec. 3. The most precise frames require facing the unpredictability of the reference pattern by replacing the traditional use of a prescribed reference pattern with one that is provisional and continually adapted. Thus at a level of feedback above the steering toward a given reference pattern, the live clocks of a network detect and respond to
unpredictable failures to steer within an allowable tolerance of whatever reference pattern is invoked at the moment, followed by employing a revised reference pattern.

To show how logical synchronization differs from the synchronization defined by Einstein, in Sec. 4 we invoke the assumption of a spacetime manifold, indeed a flat spacetime, to show logical synchronization in a case of live clocks fixed to a rotating platform, where Einstein synchronization is precluded. It is noted, however, that live-clock networks as a concept make no assumption of a spacetime manifold; spacetime coordinates enter as an optional ingredient in the planning of some, but by no means all, reference patterns.

Sec. 5 summarizes the overall perspective and points to some open questions concerning the locating of events not by coordinates but in terms of number-carrying communications channels arising or constructed in particular situations and dependent on active maintenance. Such networks locate events in terms of “who’s in touch with whom” over channels linking live clocks that compute their own rate adjustments in response to measured deviations from a reference pattern of channels, that itself varies unpredictably. Indeed, the needs for changing the reference pattern give evidence for an unpredictable environment as a topic of experimental investigation.

2 Transmitted numbers in the theory of reference frames

Quantum theory presupposes coordinate systems as mathematical constructions that one relates to physical systems. Coordinate systems depend (at least locally) on Einstein’s imagined patterns of light signals propagating between imagined proper clocks. In terms of these clocks and signals Einstein defined the synchronization of proper clocks fixed to a non-rotating, rigid body in free fall (i.e., a Lorentz frame) and co-defined “time” as the readings of such proper clocks, with the implications that distance from proper clock A to proper clock B is defined, as in radar, in terms of the duration at A from the transmission of a light signal to the return of its echo from B. Specifically, according to Einstein’s definition of the synchronization of proper clocks \[4\], clock B is synchronous to clock A if at any A-reading \( t_A \), A could send a signal reaching B at B-reading \( t_B \), such that an echo from B would reach A at A-reading \( t'_A \), satisfying the criterion

\[ t_B = \frac{1}{2}(t_A + t'_A). \] (1)

By postulate, proper clocks are free of drift in frequency, so that the relation that defines synchronization can be thought of as what “would hold” if signals were transmitted, without requiring actual transmission. But when we turn from coordinate systems as mathematical entities to their realization by physical coordinate frames, drift of physical clocks, stemming from quantum uncertainty and other causes, has to be dealt with \[2\]. Dealing with it entails attending to the actual transmission of numerical clock readings. Inspired by computer-mediated digital communications systems, we reflect the need for the transmission of physical clock readings into theory by representing a real-time process-control computer that takes part in a network as a modified Turing machine stepped by a clock \[5\]. To deal with communications between Turing machines, it is necessary for the clock that steps a Turing machine to
tick at a rate that can be adjusted by commands issuing from that machine. We call such an adjustable clock in combination with the Turing machine that regulates its rate a live clock [6].

Seen from the standpoint of live clocks as actors in a network, the story of signaling implicit in Einstein’s definition of synchronization [1] can be retold as follows. A live clock $A$ transmits a signal conveying the very $A$-reading $t_A$ at which the transmission occurs. Live lock $B$ receives the number-bearing signal at $B$-reading $t_B$ and echoes back a signal, which conveys the number $t_B$, to live clock $A$, whereupon live clock $A$ records its $A$-reading $t'_A$ at its receipt of the number $t_B$ from $B$. Notice that numerical clock readings are transmitted from live clock to live clock, and that a live clock takes numerical readings of itself at the transmission and at the receipt of signals that convey clock readings. The reading of say clock $B$ at the receipt of a signal carrying a clock reading of $A$ is distinct from the $A$-reading received.

A network of live clocks acts in response to deviations of relations among its clock readings from an imagined reference. One might suppose that the live clocks of a network could employ a reference stated purely in terms of desired relations among the numerical clock readings at their transmissions and receptions of signals. This supposition, however, is wrong, because those relations provide no scale. A reference consisting of the Einstein synchronization relation illustrates the issue of scale. According to general relativity, these (blackboard) relations are progressively more precisely realizable as the clocks over a region increase their tick rates and correspondingly shrink their separations. Thus the synchronization relations have to be augmented by some local scale. In the International System of Units (SI), this scale is chosen to be a resonance of cesium 133 imagined for cesium atoms in free fall and at absolute zero temperature [2]. This specification is interpreted as defining a scale for a proper clock as conceived by Einstein [4]. Because no two clocks tick quite alike, the reference for a live clock cannot be any realization of a live clock, but must be a blackboard concept tied to a realization only to within some tolerance.

### 3 Logical synchronization

A live clock operates in a cycle of receiving unpredictable information from an environment, storing that information in memory, computing a response, and issuing that response to the environment. The cycle has subcycles, and at the finest level is composed of moments and moves of the clock-driven Turing machine that makes up the live clock. For a live clock to take part in communication, its moments and moves have to be regulated to avoid the logical conflict of a collision between writing into memory and reading from memory. (In human terms this is the collision between trying to speak and listen at the same time.) To avoid this conflict, the modified Turing machine is driven by the adjustable clock through a cycle with two phases of moves and two phases of moments, with reading from memory taking place in a phase separated from a phase of writing into memory.

A cycle of the live clock corresponds to a unit interval of the readings of its adjustable clock. A reading of a live clock can be expressed in the form $m.\phi_m$ where an integer $m$ indicates the count of cycles and $\phi_m$ is the phase within the cycle. We
choose the convention that $-1/2 < \phi_m \leq 1/2$. (It is not necessary to think of the
signals as points in time; it suffices to think of a point reference within the signal.)

3.1 Channels from one live clock to another

To express the transmission of numbers from one live clock to another, we fol-
low Shannon in speaking of a communications channel; however we augment his
information-theoretic concept of a channel \[7\] with the live-clock readings at the
transmission and reception of character-bearing signals \[2\]. Each character transmit-
ted from a live clock $A$ to a live clock $B$ is associated with a reading of live clock $A$
of the form $m.\phi_m$ at the transmission and with a reading of live clock $B$ of the form
$n.\phi_n$ at the reception. A channel from $A$ to $B$ includes a set of such pairs of readings
of the transmitting and the receiving live clocks. The necessity of avoiding a conflict
between reading and writing imposes a constraint on the phases of reception.

Restricting our attention to timing, we indicate a \textit{channel} from live clock $A$ to
live clock $B$, denoted $\overrightarrow{AB}$, as a set of pairs, each pair of the form $(m.\phi_m, n.\phi_n)$. The
first member $m.\phi_m$ is an $A$-reading at which live clock $A$ can transmit a signal
and the second member $n.\phi_n$ is a $B$-reading at which live clock $B$ can register the
reception of the signal. For theoretical purposes, it is convenient to define an \textit{endlessly
repeating channel} of the form

$$\overrightarrow{AB} = \{ (m + \ell j.\phi_A, n + \ell k.\phi_B) \}, \tag{2}$$

where $m$, $n$, $j$, and $k$ are fixed integers and $\ell$ ranges over all integers. Again for
theoretical purposes, we sometimes consider channels for which the phases are all
zero, in which case we may omit writing the phases.

\textbf{Proposition:} A character can propagate from one live clock to another only
if the character arrives within the writing phase of the receiving live clock.

When this phase constraint is met for a channel between a transmitting live clock
and a receiving live clock, we say the receiving live clock is \textit{logically synchronized}
to the transmitting live clock. Logical synchronization is analogous to the coordination
between neighboring people in a bucket brigade, or that between players tossing a
ball back and forth, where the arrival of the ball must be within a player’s ‘phase
of catching’. In this way the notion of a channel is expanded to include the clock
readings that indicate phases of signal arrivals that have to be controlled in order for
the logical synchronization of the channel to be maintained. (While in many cases
the integers in clock readings that count cycles can be definitely specified, the phases
are never exactly predictable.) We model the phase of writing at which a live clock
can receive a character as corresponding to

$$|\phi| < (1 - \eta)/2, \tag{3}$$

where $\eta$ (with $0 < \eta < 1$) is a phase interval that makes room for reading.

Logically synchronizing a channel means bringing about the condition \[3\] on
phases at which signals arrive. Once logical synchronization is acquired for a set
of channels, maintaining it typically requires more or less continually adjusting the rates of ticking and the acceleration of the live clocks, in order to steer the phases of arriving characters toward a suitable reference pattern. In the simplest case, this reference pattern entails zero phases of reception.

Relations among readings of live clocks that contribute to a reference pattern for a network include what we call echo counts, closely related to distances defined by radar:

**Definition of echo count:** Suppose that at its reading $m.0$ a live clock $A$ transmits a signal at $m$. A to a live clock $B$, and the first signal that $B$ can transmit back to $A$ after receiving $A$’s signal reaches $A$ at $m'.\phi'$; then the quantity $m'.\phi' - m.0$ will be called the echo count $\Delta_{ABA}$ at $m$.

Although the concept of a channel is applicable without reference to a spacetime manifold, in this paper we explore several questions of possible reference patterns of logically synchronized channels under the hypothesis of a flat spacetime, so that we can speak of the period of a live clock as if it could be determined by a proper clock of special relativity. In general that period can vary from tick to tick of the live clock [2]; here however, we limit ourselves to the assumption of constant proper periods. (Patterns of channels between live clocks in a curved spacetime are discussed elsewhere [2,6].)

4 Logical synchronization where Einstein synchronization is precluded

Although channels linking live clocks are defined without any assumption of a spacetime manifold, it is interesting to compare logical synchronization of live clocks with the quite different synchronization defined by Einstein in special relativity. Einstein synchronization is stated in terms of coordinates, and so to compare and contrast the logical synchronization of channels linking live clocks with Einstein’s synchronization, we assume for this section a coordinate system that assigns flat spacetime coordinates to ticks of live clocks (represented on the blackboard). In special relativity, the Sagnac effect precludes the Einstein synchronization of neighboring live clocks attached to a rotating platform. We review this situation and contrast it with several cases in which channels between neighboring live clocks can be logically synchronized, including theoretical cases in which logical synchronization with zero phases at reception is possible.

Consider $n$ live clocks $A_1, \ldots, A_n$ fixed to the nodes of a regular polygon of $n$ sides, with $n \geq 3$ rotating in its plane about its center at constant angular rate $\omega$, in a flat spacetime, with the center at rest in some Lorentz frame, relative to which we use time and space coordinates. Let the radius of the polygon be $r$. Let $T_+$ be the coordinate time duration from transmission to reception by a nearest neighbor in the direction of rotation. Let $T_-$ be the coordinate time duration from transmission to reception by nearest neighbor in direction counter to rotation. By symmetry, assume that all the live clocks tick with a common period $p$ relative to the Lorentz frame.
4.1 Forward and backward propagation times

Accounting for the rotation of a regular polygon of \( n \) sides, one finds relations among \( T_+ \), \( T_- \), the radius \( r \) of the polygon, angular rotation rate \( \omega \), and speed of light \( c \):

\[
cT_+/r = 2 \sin \left( \frac{\pi}{n} + \frac{\omega T_+}{2} \right), \tag{4}
\]

\[
cT_-/r = 2 \sin \left( \frac{\pi}{n} - \frac{\omega T_-}{2} \right), \tag{5}
\]

which implies a relation between \( T_+ \) and \( T_- \) obtained by solving these equations for \( \omega \):

\[
\frac{2}{T_+} \left[ \sin^{-1} \left( \frac{cT_+}{2r} \right) - \frac{\pi}{n} \right] = \frac{2}{T_-} \left[ \frac{\pi}{n} - \sin^{-1} \left( \frac{cT_-}{2r} \right) \right] = \omega. \tag{6}
\]

For the case of a rotating hexagonal arrangement of live clocks (so \( n = 6 \)), the ratio \( T_-/T_+ \) is plotted against \( cT_+/r \) in Fig. 1. For a hexagonal arrangement in the absence of rotation, we have that \( cT_+/r = 1 \), while \( cT_+/r \) reaches a maximum value of 2 for the (hypothetical, superluminal) rotation of the arrangement that makes the light signal go diagonally across the hexagonal pattern.

4.2 Einstein synchronization impossible

Unless the angular velocity \( \omega \) is zero, \( T_+ > T_- \). For that reason if rotating live clocks tick in coincidence relative to the Lorentz frame, they fail to satisfy the Einstein criterion \( \frac{1}{c} \). By shifting the ticks of each clock in time, however, one can arrange for \( A_2 \) to be Einstein synchronous to \( A_1 \), and so on through \( A_n \) to \( A_{n-1} \), but it is impossible to close the loop to make \( A_1 \) synchronous to \( A_n \). Thus the Einstein synchronization relation, which is transitive for clocks fixed to a Lorentz frame, is not a transitive relation for clocks on a rotating platform. For a rotating platform Einstein synchronization even in one rotational direction is impossible.

4.3 Logical synchronization for live clocks on a rotating platform

For logical synchronization the situation is different, in one way more restrictive, but in others less restrictive. In particular, logical synchronization allows for asymmetry in propagation times, that is, in cases that \( T_+ \neq T_- \). For the cases considered in the following, assume all \( n \) live clocks arranged at vertices of the rotating polygon have tick zero at frame time \( t = 0 \), and all continue to tick with the common period \( p \).

4.3.1 Case of low bandwidth based on tolerance of phasing

By making the live-clock period \( p \) sufficiently long, logical synchronization at low bandwidth can operate merely by making the allowed phase interval for reception longer than the duration for a signal to propagate from a live clock to its nearest
Fig. 1 Ratio $T_-/T_+$ vs. $T_+$ and $\omega r/c$ vs. $T_+$ (dashed), from Eq. (6).

neighbor. I.e. one arranges for the period $p$ of the live clocks to be longer than the coordinate-time interval $T_+$, so that $T_+/p$ is within the phase $(1 - \eta)/2$ allowed for logical synchronization, per Eq. (3). This of course limits bandwidth, which is proportional to

$$1/p < \frac{1 - \eta}{2T_+}. \quad (7)$$

4.3.2 Case of one-way ring of logical synchronization with zero phases of reception

More interesting are the cases of logical synchronization with zero phases. Suppose $T_+ = N_+ p$ for $N_+$ a positive integer. Then theoretically there can be $n$ channels
linking nearest neighbors in the direction of rotation:

\[ A_j A_{j+1} = \{(k, k + N_+)|k \in \mathbb{Z}\}, \quad (8) \]

where \( j \in \{1, 2, \ldots, n\} \) and we view \( A_{n+1} \) as another name for \( A_1 \). Given \( T_+ \) and any positive integer \( N_+ \), the necessary and sufficient condition is met by a clock period \( p = T_+/N_+ \).

4.4 Case of two-way ring of logical synchronization with zero phases of reception

For two way logical synchronization of nearest-neighbor channels, there is a restrictive relation between the radius of the polygon and the angular velocity of the platform. Logically synchronized channels with zero receptive phases from one live clock to its neighbor in the direction counter to rotation have the form analogous to that of Eq. (8):

\[ A_j A_{j-1} = \{(k, k + N_-)|k \in \mathbb{Z}\}, \quad (9) \]

where \( j \in \{1, 2, \ldots, n\} \) and we view \( A_0 \) as another name for \( A_n \). Given \( T_- \) and any positive integer \( N_- \), the necessary and sufficient condition is met by a clock period \( p = T_-/N_- \). Thus for two-way, 0-phase channels between nearest neighbors the period of the live clocks has the necessary and sufficient condition

\[ p = T_+/N_+ = T_-/N_- \quad (10) \]

which requires that \( T_-/T_+ \), plotted in Fig. 1 for the case \( n = 6 \), be a rational number.

4.5 Live clocks as tools of exploration

In these examples, we assumed that a given angular velocity with respect to a coordinate system is “given” and requires no action on their part. More interesting is the case in which the angular velocity is given to the live clocks as a reference pattern, and the mission of the live clocks is to adjust their accelerations and tick rates maneuver to obtain channels that correspond to this reference angular velocity. Another case is the exploration of possible channels in order to measure a rotation rate, as in a Sagnac interferometer.

5 Discussion

As a gentle illustration of logical synchronization, in the previous section we invoked the familiar assumption of a coordinate system on a spacetime manifold relative to which to describe an example of polygonal ring of rotating live clocks with nearest neighbors linked by logically synchronized channels. However, as already emphasized, the concept of logical synchronized channels does not in itself make any assumption of a spacetime manifold, let alone a coordinate system on that manifold.
Indeed the bringing about of a pattern of logically synchronized channels provides a background against which to locate events, tailored to one or another particular situation, and this background provides some or all of the services asked of a coordinate frame. That is, in a network of logically synchronized channels linking live clocks, events are located by their proximity to live-clock readings that, via these channels are related to readings of the other live clocks of the network. As discussed in [2], gravitation affects the reference patterns of channels toward which a network of live clocks can successfully steer, so that at high precision the reference patterns are themselves hypotheses arrived at in part by unpredictable guesswork, subject to revision in response to failures to come within tolerable deviations from them.

The events that are critical to location are acts of transmitting and receiving numbers, or, to put it a little more generally acts of transmitting and receiving logical distinctions, such as the distinction between 0 and 1 or the distinction between 'yes' and 'no'. It is the communication of logical distinctions in the face of an unpredictable environment that gives digital computers their power, and indeed gives life the capacity to propagate through the mechanisms of DNA replication. The communication of logical distinctions depends on regenerative amplification that reshapes signals to maintain logical distinctions while allowing for tolerances in system components [9, 8]. Besides regenerative amplification, the communication of logical distinctions requires phase management in the face of unpredictable environmental behavior. The live clocks of a network function primarily not to tell “time” in the sense of a space-time coordinate, but to regulate the phasing needed for logical synchronization.

We have stressed unpredictability of guesses that enter reference patterns, but as discussed in [2], unpredictable events are also physical, as in the detections by a photodetector. While in some experiments, one accumulates such detections passively to get an average rate of detections that can be related to a probability, in other experiments, notably the operation of an atomic clock, detections that cannot be individually predicted have to be responded to promptly, and so enter the operation of clocks that realize the SI units of the Hertz and the second.

There is lots left to do:

1. Can one retrieve some notion of a time coordinate that is available without the assumption of a spacetime manifold, based on tracing the implications of the relations between the transmissions of numbers and their receptions, as expressed by the channels of a network?
2. Regenerative amplification is found in biology, for example in the propagation of electrical spikes in nerve fibers [10]. Is phase management present in biology?
3. There are questions to ask and to answer concerning possibilities for patterns of channels among live clocks, whether in engineered systems or as found in living organisms. To get a glimpse of the issue, for any live clock \( A \), in principle there is a (likely variable) tick rate that will make it logically synchronized to signals from an arbitrary second live clock \( B \), but the issue is not so simple if live clock \( A \) wants to steer toward logical synchronization with a third live clock \( C \) in addition to \( B \).

And there are bigger questions. Quantum mechanics depends on coordinate systems which, as we have seen, depend on clocks; however, the concept of a coordinate
system abstracts the clocks out of sight. When we look into the clocks and their communication by the transmission of number-carrying signals, we find a situation readily described, as above, in terms of computer engineering or its abstract Turing machines, without the use of quantum language. Realized clock networks depend on regenerative amplification—thermodynamically non-reversible (even if logically reversible [11]) and outside of any graceful description in the language of quantum theory. Question: can some novel quantum-theoretic description (involving decoherence?) represent networks of live clocks, or is quantum mechanics irreducibly dependent on systems for locating events that are outside its descriptive reach?

Finally there is the question of accepting or rejecting unpredictability as a fundamental feature of life. One often thinks of a coordinate system working like rigid fences that organize a landscape, but if unpredictability is pervasive, if the earth on which we stand shifts, which at present levels of clock stability it always does, how is one to locate objects of interest? There can be no rigid body on which to stand. The application of physical laws that underpin predictions requires number-carrying channels. Channels operate in the face of unpredictability that no law can shut out. When channels of a network fail, as on occasion they do, the applications that depend on them fragment. Sometimes that fragmentation of a network calls us to search for a different background pattern toward which to steer. If needs to adjust our reference patterns are in the cards, it is perhaps better to be nimble. Recognizing unpredictability can be a first step toward that nimbleness.

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