A short, and definitely not a complete representation of \(\eta\) production processes on hadrons is given. First of all, the different ways of obtaining the \(\pi N \rightarrow \eta N\) and \(\pi N \rightarrow \eta N\) amplitudes are presented. After that, an overview of results obtained using these amplitudes as input for calculating processes like: \(NN \rightarrow \eta NN\), \(pd \rightarrow \eta^3He\), \(\pi d \rightarrow \eta NN\) and \(\eta d\), \(\eta^3He\) and \(\eta^4He\) as well as \(\eta\)-light nuclei bond states, will be given. The experimental and theoretical results will be reviewed. The opened problems and the way how to solve them will be presented.

1 Introduction

The problem of determining whether there exists a resonance in the \(N^*\) system is a very nontrivial one. In addition to the problem of the definition what an \(N^*\) resonance is, there is a problem of coupling different resonances to the different channels. For example, \(S_{11}(1650)\) MeV resonance does not at all couple to the photoproduction channel, so it is practically invisible in all processes involving \(\eta\) photoproduction. The second example is the fourth \(P_{11}\) resonance, which is not visible in any process which does not involve \(\eta\) production in hadronic reactions. Therefore, looking at photoproduction processes only it is not sufficient to see all possible \(N^*\) resonances. Consequently, the only method which can be applied selfconsistently to obtain all resonances in all channels is a multichannel, multiresonance, unitary coupled channel model developed by \(\ddagger\) and maximizing and updating the input to the model. In order to value the strength of the used method, the amount of the worldwide work involved in the analysis, the mutual agreement of the results and the competence of the authors, we have decided to rank the publications by a number of stars, very similar to the method used by Particle Data Group (PDG). One star indicates the pioneering attempts, while four star denotes the general world interest and a significant level of agreement reached. The estimate is just a personal judgement of the authors of the article and anyone is welcomed to modify it.

2 \(\pi N \rightarrow \eta N\) and \(\eta N \rightarrow \eta N\) models

\(\ddagger\)See the general discussion on the B(arion)R(esonance)A(nalysys)G(roup)-BRAG workshop preceding this workshop
2.1 Coupled channel models

Multiresonance, coupled channel and unitary models offer the best possibility to treat all the channels simultaneously, without the problem of not seeing the resonances which couple poorly to one of the channels. The framework has been elaborated by Cutkosky, and has been used by most of the modern approaches. It is essential to remind the reader to use the original article by Cutkosky where the full formalism is explained. The results of the original model insignificantly differ from the predictions of KH80 group, and represent the state of the art of knowledge of 80es in $\pi$-N physics. Later on, in 90es the chain of articles started to use the same formalism and exploit the new data to make more precise conclusions.

The idea of all three approaches was basically the same: to use the well known and tested formalism, and to introduce the new knowledge about $\eta$ production processes in order to obtain more reliable information about $N^*$ resonances. The essence of the formalism was to obtain the "three dimensional" T-matrix shown in Fig.1 for three channels and has been used in. In that case the chosen channels ($\pi - N$, $\eta - N$ and the third effective two body channel $\pi^2 - N$ are given on the x-axes, while partial waves are given on the y-axes). The whole formalism allows the separation of the T-matrix in partial wave amplitudes which are indicated by vertical planes in the 3-D T-matrix. However, the main problem in this formalism was a numerical minimization procedure which tended to explode in number of fitting parameters for bigger number of channels if experimental observables were fitted, because all partial waves are automatically mixed. Each of three references have tried to overcome that problem in a different way. In refs. authors have used three coupled channels only, but have chosen to fit the $\pi - N$ elastic T-matrices, and...
experimental observables. In ref\textsuperscript{3} the $\pi$-N elastic partial wave T-matrices from different sources\textsuperscript{1,2} have been used for the first channel, the second channel-$\eta$N channel have only been represented by the $S_{11}(1535)$ resonance, and the whole known data set for continuum pion production $\pi N \rightarrow \pi \pi N$ have been used to represent the third channel. The number of parameters was acceptable, and the minimization has revealed results very similar to PDG group, but more constrained in other but $\pi N$ elastic channel. The second PWA\textsuperscript{4} has as well used the $\pi-N$ elastic T-matrices from the same sources as\textsuperscript{3}, but have chosen to use the whole set of measured total and differential cross sections for the $\pi N \rightarrow \eta N$ process.

In both cases the number of parameters to be fitted was quite big (of the order of 100), but the minimization procedure was still under full control. However, the drawback of both of these approaches was that it was not forseeable to increase the number of coupled channels because the number of fitting parameters would explode beyond control if one uses MINUIT program. The third approach\textsuperscript{5} has avoided the problems in that program by not fitting the experimental data but T-matrices obtained from different sources. In that way they have been able to fit partial wave by partial wave (or plane by plane in Fig.1), and that has significantly reduced the number of input parameters and allowed them to use much more then three channels. However, the choice of input T-matrices remains an opened question to be discussed and tested. The all three analysis show a fair level of agreement and self consistence, and we dare to say that T-matrices for $\pi - N$ and $\eta - N$ channel are quite confidently determined and can be used as the input for the calculation of more complicated processes.

2.2 Quark model and coupled channel model ****

In ref\textsuperscript{6} the importance of multichannel approach has been illustrated. Namely, the possibility of existence of the fourth $P_{11}$ resonance has been reported in spite of the fact that it has not been seen in any previous single channel analysis, or even in one three coupled channel analysis which included the channel into which that resonance does not couple\textsuperscript{3}. First the existence of that resonance has been predicted in the quark model\textsuperscript{7}, but at the same time it was seen in the three coupled channel analysis\textsuperscript{4}. The agreement of the findings of both, theoretical and phenomenological analysis are striking, so it is quite likely that the number of $P_{11} N^*$ resonances should be increased to four. The result is a good example of theoretical prediction confirmed by the "experimental" partial wave analysis.
Table 1: Resonance parameters of the phenomenological and the quark models. The states are defined by the latest values given by the PDG group and other parameters are defined in the text. Errors can be find in original publications.

2.3 $\eta N$ S-wave scattering length

Another example of importance of the $\eta$ production in hadronic channels for understanding the structure of $N^{*}$ resonances is the need of the existence of the second $S_{11}(1650)$ resonance (the resonance which is extremely poorly coupled to the photoproduction channels) for the complete understanding of the $\eta N$ S-wave scattering length. Namely, the problem of extremely poorly determined value of the real part of the $\eta N$ S-wave scattering length has been known for years, and the limits have been $0.2 \text{ fm} \leq \text{Real}(a_{\eta N}) \leq 0.98 \text{ fm}$. That ambiguity was directly prohibiting the estimate of the likelihood of formation of the $\eta$-light nuclei bound states, because the existence of these bound states was directly correlated to the value of the real part of the $\eta N$ scattering length as it can be seen in Fig.2.

The spread in possible values of the real part of the $\eta N$ S-wave scattering length is given in Fig.2, and can be easily understood. The problem has been extensively addressed in ref.10. As it has been explained, any single resonance model (containing only one resonance in the S-wave) with the addition of the quite reliably measured and remeasured slope of the $\pi^- p \rightarrow \eta n$ total cross...
section near threshold can only give the values of the real part of the \(\eta N\) scattering length fixed below \(\approx 0.4\) fm. For any value bigger then that, the existence of the second \(S_{11}\) resonance (1650 in addition to 1535) has to be assumed. The ambiguity has finally been resolved by getting the overlapping results from two calculations based on entirely different formalisms. It is indicative that both approaches have to include the existence of the second \(S_{11}\) resonance. As the both publications used completely different formalisms (Cutkosky formalism and K-matrix formalism), and results coincide within the error bars, we conclude that the real part of the \(\eta N\) S-wave scattering length is quite well determined now. The existence of the second \(S_{11}\) resonance for the overall understanding of the results given in Fig.2. is, henceforth, established. Let us just remind the reader that the second \(S_{11}\) resonance is not seen in photoproduction processes. On the basis of these results for the real part of the \(\eta N\) S-wave scattering length the predictions for the existence of the bound states in different \(\eta\) -light nuclei are given in Fig.2.

![Figure 2: \(\eta N\) S-wave scattering length. The symbols for all extracted values of the \(\eta N\) scattering length are taken over from reference\(^1\). The only addition are the values extracted in ref.\(^4\) - crossed empty circles and\(^11\) - crossed full circles. Lines given on the figure indicate for which values there is a probability for the \(\eta\)-light nuclei bound states\(^{12}\).](image)

---

5
2.4 Other $\pi N \rightarrow \eta N$ and $\eta N \rightarrow \eta N$ models

Numerous other models have been produced with the aim to extract PW $T$-matrices for $\eta$ meson production in hadronic reactions \cite{13,14,15,16,17}. All of them suffer from some of the drawbacks: they are either single resonance, or simplified in some way. However, they are valuable to be looked at in order to see other approaches and possible simplification valuable for some specific cases.

3 $NN \rightarrow NN\eta$ processes

The knowledge of elementary PW $\pi N$ and $\eta N$ amplitudes have been tested in calculations involving more then two bodies in order to test the reliability and self consistence of obtained partial waves. One of the simplest examples is the $\eta$ production in nucleon-nucleon scattering. The process has been investigated experimentally \cite{18,19,20,21} and theoretically \cite{8,22,23,24,25,26}. However, even the initial agreement in theoretical calculations which mechanism is dominating has not been reached. It is generally agreed that in addition to the Born term the final state interaction should be added, but the combination of exchanged mesons which are described in different models varies. Therefore, more theoretical and experimental effort should be done in order to bring the problem to the general agreement.

4 $\pi d \rightarrow NN\eta$ processes

The testing of elementary PW $\pi N$ and $\eta N$ amplitudes is as well attempted for $\eta$ production processes in $\pi d$ reaction. The experiments are scarce \cite{27} and theoretical calculations are just being developed \cite{28,29,30}. Among reproducing the various experimental quantities like total and differential cross sections, the idea of extracting the $\pi^0$-$\eta$ mixing angle using ratios of $\pi^- d \rightarrow \eta nn$ and $\pi^+ d \rightarrow \eta pp$ has been suggested \cite{28}. However, the experimental analysis \cite{27} has not yet been finished, therefore the comparison with the theoretical predictions is in a way "hanging in the air".

5 Bound states of $\eta$ mesons

The attempts of finding indications of bound states in $\eta^3\text{He}$ system \cite{31,32,33} have been done. Results are, according to my belief, still opened to reader’s interpretation.

The same statement stands for finding $\eta^4\text{He}$ bound states what has been attempted in refs. \cite{24,33,34,35}, as well for finding $\eta$-light nuclei bound states in refs. \cite{37,38,39}. 


As a final conclusion we tend to offer the statement:

The existence of any $N^*$ resonance have to be confirmed in all channels, therefore, coupled channel models offer the best possibility to establish them unambiguously.

References

1. R.E.Cutkosky, C.P.Forsyth, R.E.Hendrick, and R.L.Kelly, *Phys. Rev.* D **20**, 2839 (1979).
2. G. Höhler, in *Elastic and Charge Exchange Scattering of Elementary Particles*, edited by H. Schopper, Landolt-Börnstein, New Series, Group X, Vol.9,Part 2b (Springer-Verlag, Berlin 1983).
3. D.M.Manley and E.M.Salesky, *Phys. Rev.* D **45**, 4002 (1992) and references therein.
4. M.Batinić, I. Dadić, I.Šlaus, A.Švarc B.M.K. Nefkens and T.-S.H. Lee, *Physica Scripta* **58**, 15 (1998) and references therein.
5. T.P.Vrana and S.A.Dytman, and T.-S.H.Lee, nucl-th/9910012 and nucl-th/9702033 available at http://xxx.lanl.gov and references therein.
6. S.Capstick, T.-S.H.Lee, W.Roberts, and A.Švarc, *Phys. Rev.* C **59**, R3002 (1999) and references therein.
7. S.Capstick, $N^*$ *Physics and Nonperturbative Quantum Chromodynamics*, Proceedings of the Joint ECT$^*$ /JLAB Workshop, Trento, Italy, May 18-29, 1998, Few Body Systems, Supplement 11 (1998) 86, SpringerWi-enNewYork.
8. M.Batinić and A.Švarc, *Physica Scripta* **56**, 321 (1997) and references therein.
9. C.Casso et al *The European Physical Journal* C**3**, 1 (1998), and former issues in *Phys. Rev. D*.
10. M.Batinić and A.Švarc, *Few Body Syst.* **20**, 69 (1996) and references therein.
11. A.M.Green and S.Wycech, *Phys. Rev.* C **55**, R2167 (1997) and references therein.
12. S. Wycech, *Workshop on Physics with the WASA Detector*, Sätra Brunn, June 17-19, 1996, Sweden
13. R.S.Bhalerao and L.C.Liu, *Phys. Rev. Lett.* **54**, 856 (1985)
14. C.Benhold and H.Tanabe, *Nucl. Phys.* A **530**, 625 (1991) and references therein.
15. M.Arima, K.Shinizu, and K.Yazaki, *Nucl. Phys.* A **543**, 613 (1992) and references therein.
16. V.V.Abaev and B.M.K.Nefkens, *Phys. Rev.* C 53, 385 (1996) and references therein.
17. J.Denschlag, L.Tiator, and D.Drechsel, nucl-th/9802063
18. A.M.Bergdolt, G.Bergdolt, O.Bing, A.Bouchakov, F.Brochard, F.Hibou, A.Moalem, A.Taleb, M.P.Combes-Comets, P.Courtat, R.Gacougnolle, Y.Le Bornec, E.Loireux, F.Reide, B.Tatischeff, N.Willis, M.Boivin, B.M.K.Nefkens, and F.Ploiun, *Phys. Rev.* D 48, R2969 (1993) and references therein.
19. E.Chivassas, G.Dellacasa, N.De Marco, C.De Oliviera Martinis, M.Gallio, P.Guaita, A.Musso, A.Picotti, E.Scomparin, and E.Vercellin, *Phys. Lett.* B 232, 270 (1994) and references therein.
20. H.Calén, J.Dyring, K.Fransson, L.Gustafsson, S.Häggström, B.Höistad, A.Johansson, T.Johansson, S.Kullander, A.Mörtsell, R.Ruber, U.Shuberth, J.Zlomačuk, C.Ekström, K.Kilian, W.Oelert, T.Sefzick, R.Bilger, W.Brodowski, H.Clement, G.J.Wagner, A.Bondar, A.Kuzmin, B.Shwartz, V.Sidorov, A.Sukhanov, A.Kupšć, P.Marciniowski, J.Stepaniak, V.Dunin, B.Morosov, A.Povtorejko, A.Zernov, J.Zabi ćerowski, A.Turowiecki, and Z.Wilhelmi, *Phys. Rev.* C 58, 2667 (1998) and references therein.
21. H.Calén, J.Dyring, G.Fäl dt, K.Fransson, L.Gustafsson, S.Häggström, B.Höistad, A.Johansson, J.Ohansson, T.Johansson, A.Kuzmin, B.Shwartz, V.Sidorov, A.Sukhanov, A.Zernov, A.Kupšć, P.Marciniowski, J.Stepaniak, J.Zabi ćerowski, A.Turowiecki, Z.Wilhelmi, and C.Wilkin, *Phys. Lett.* B 458, 190 (1999) and references therein.
22. T.Vetter, A.Engel, T.Biró, and U.Mosel, *Phys. Lett.* B 263, 153 (1991) and references therein.
23. J.M.Laget and F.Wellers, *Phys. Lett.* B 257, 254 (1991) and references therein.
24. C.Wilkin, *Phys. Rev.* C 47, R938 (1993) and references therein.
25. G.Fäl dt and C.Wilkin, *Nucl. Phys.* A 604, 441 (1996), nucl-th/9612019 and references therein.
26. V.Bernard, N.Kaiser, and Ulf-G.Maißner, nucl-th/9806013 (accepted for publ. in *Eur.Phys.J.* A)
27. B.Nefkens et al., proposal for the E890 experiment, *to be published*
28. M.Batinić et al. Few Body XV, Groenigen 1997
29. H.Garcialazo and M.T.Peña, *Phys. Rev.* C 59, 2389 (1999) and references therein.
30. H. Garcialazo and M. T. Peña, *to be published* and references therein.
31. G. Fälldt and C. Wilkin, *Nucl. Phys. A* 587, 769 (1995) and references therein.
32. G. Fälldt, TSL/ISV-96-0143 Uppsala University preprint and references therein.
33. N. Willis, Y. Le Bornec, A. Zghiche, C. Wilkin, R. Wurzinger, O. Bing, M. Boivin, P. Courtat, R. Gacougnolle, F. Hibou, J. M. Martin, F. Plouin, B. Tatischef, and J. Yonnet, *Phys. Lett. B* 406, 14 (1997) and references therein.
34. R. Frascaria, F. Roudot, R. Wurzinger, M.-A. Duval, J. Ernst, L. Godzahl, F. Hinterberger, R. Jahn, R. Joosten, T. von Oepen, and W. Spang, *Phys. Rev. C* 50, R537 (1994) and references therein.
35. C. Wilkin, *Phys. Lett. B* 331, 276 (1994)
36. S. Ceci, D. Hrupec, and A. Švarc, *J. Phys. G: Nucl. Part. Phys.* 25, L35 (1999)
37. C. Benhold and H. Tanabe, *Phys. Lett. B* 243, 13 (1990)
38. A. Fix and H. Arenhövel, [nucl-th/9703002](https://arxiv.org/abs/nucl-th/9703002)
39. J. Kulpa, S. Wycech, and A. M. Green, [nucl-th/9807020](https://arxiv.org/abs/nucl-th/9807020)