Casting the Coronal Magnetic Field Reconstruction Tools in 3D Using the MHD Bifrost Model

Gregory D. Fleishman1, Sergey Anfinogentov2, Maria Loukitcheva1,3,4, Ivan Mysh’yakov2, and Alexey Stupishin3

1 Physics Department, Center for Solar-Terrestrial Research, New Jersey Institute of Technology Newark, NJ, 07102-1982, USA
2 Institute of Solar-Terrestrial Physics (ISZF), Lermontov st., 126a, Irkutsk, 664033 Russia
3 Saint Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034 Russia
4 Max-Planck-Institut für Sonnensystemforschung, Justus-von-Liebig-Weg 3, D-37077 Göttingen, Germany

Received 2017 January 27; revised 2017 March 17; accepted 2017 March 18; published 2017 April 11

Abstract

Quantifying the coronal magnetic field remains a central problem in solar physics. Nowadays, the coronal magnetic field is often modeled using nonlinear force-free field (NLFFF) reconstructions, whose accuracy has not yet been comprehensively assessed. Here we perform a detailed casting of the NLFFF reconstruction tools, such as \( \pi \)-disambiguation, photospheric field preprocessing, and volume reconstruction methods, using a 3D snapshot of the publicly available full-fledged radiative MHD model. Specifically, from the MHD model, we know the magnetic field vector in the entire 3D domain, which enables us to perform a “voxel-by-voxel” comparison of the restored and the true magnetic fields in the 3D model volume. Our tests show that the available \( \pi \)-disambiguation methods often fail in the quiet-Sun areas dominated by small-scale magnetic elements, while they work well in the active region (AR) photosphere and (even better) chromosphere. The preprocessing of the photospheric magnetic field, although it does produce a more force-free boundary condition, also results in some effective “elevation” of the magnetic field components. This “elevation” height is different for the longitudinal and transverse components, which results in a systematic error in absolute heights in the reconstructed magnetic data cube. The extrapolations performed starting from the actual AR photospheric magnetogram are free from this systematic error, while other metrics are comparable with those for extrapolations from the preprocessed magnetograms. This finding favors the use of extrapolations from the original photospheric magnetogram without preprocessing. Our tests further suggest that extrapolations from a force-free chromospheric boundary produce measurably better results than those from a photospheric boundary.

Key words: magnetohydrodynamics (MHD) – Sun: chromosphere – Sun: corona – Sun: general – Sun: magnetic fields – Sun: photosphere

Supporting material: animations

1. Introduction

The magnetic structure of the solar corona plays a key role in all solar activities, yet direct measurements of the coronal magnetic field are extremely difficult to make. Instead, the field strength and direction are measured at the photospheric (or possibly chromospheric) boundary; specifically, the vector fields are measured from the full Stokes polarized intensity of Zeeman-sensitive spectral lines with the circular polarization providing the line-of-sight field strength and the linear polarization providing the transverse field. Then, to assess the coronal magnetic field, these measured photospheric fields are extended into the corona through potential or force-free field extrapolations. The extrapolations were first performed by Sakurai (1981); nowadays, these have reached a high level of sophistication (see reviews by Sakurai 1989; Amari et al. 1997; Wiegelmann 2008; Wiegelmann & Sakurai 2012; Wiegelmann et al. 2014).

Although most of the extrapolation methods work well in reproducing available analytical NLFFF solutions (e.g., Low & Lou 1990), which are static and relatively morphologically simple, the assessment of the methods using realistic dynamical cases having a lot of spatial complexity is still limited (De Rosa et al. 2009). Since a straightforward “voxel-by-voxel” comparison between the NLFFF reconstructed data cube and real data cannot be performed given the lack of the coronal magnetic field diagnostics, a number of indirect tests is typically employed. These can be a comparison of a subset of selected magnetic field lines with EUV loops or by computing various metrics describing the field “force-freeness.” However, comparison with bright EUV loops allows for only a morphological comparison and only for a small fraction of the volume, where such loops are present, but does not allow quantitative assessment of the magnetic field vector components. Force-free metrics computed for various instances of the extrapolated data cubes are comparably imperfect and, thus, they do not offer an easy way of favoring one extrapolation method over another. DeRosa et al. (2015) considered the effect of the spatial resolution of the base magnetogram on the quality of the coronal magnetic field reconstruction with alternative extrapolation methods. They found that the result of an extrapolation is sensitive to the spatial resolution of the input base map and the alternative codes yield noticeably different magnetic data cubes. Thus, the casting of competing extrapolation methods using available coronal diagnostics remains inconclusive, since no true comparison of the reconstructed and real magnetic fields can be performed in the three-dimensional (3D) domain. Nevertheless, direct tests of the performance of the NLFFF coronal reconstruction tool using modern, highly realistic, spatially complex 3D MHD models of a fragment of the solar atmosphere can be performed and thus, are called for.

In this study we employ a publicly available outcome (en024048_hiion, Carlsson et al. 2016, http://sdc.uio.no/search/simulations) of the advanced 3D radiation
magnetohydrodynamic (RMHD) code Bifrost (Gudiksen et al. 2011) to perform comprehensive tests of the various steps in coronal magnetic modeling: (i) $\pi$-disambiguation of the measured transverse magnetic field, (ii) preprocessing of the photospheric magnetic field, and (iii) NLFFF extrapolation from the bottom boundary up to the coronal volume. In each step, we make a comparison between available alternative codes: either two different methods or two alternative implementations of the same method for the steps mentioned above.

2. RMHD Model Data Cube

Bifrost is a flexible and massively parallel code for general purposes developed by Gudiksen et al. (2011). The Bifrost-based RMHD Model of Carlsson et al. (2016), en024048_hion, represents realistic simulations of the outer solar atmosphere with a magnetic field topology similar to an enhanced network area on the Sun. The simulation data cubes used in this work were obtained from the Hinode Science Data Centre Europe in the publicly available time span of 1590 s.

2.1. Overview of the Data Cube

The simulation covers a physical extent of $24 \times 24 \times 16.8$ Mm, with a grid of $504 \times 504 \times 496$ cells, extending from $-2.4$ Mm below the photosphere, which corresponds to $Z = 0$ Mm, to $14.4$ Mm above the photosphere. It encompasses the upper convection zone, photosphere, chromosphere, and the lower corona. The horizontal axes have an equidistant grid spacing of 48 km; the vertical grid spacing is nonuniform, with a spacing of 19 km between $Z = -1$ Mm and $Z = 5$ Mm. The spacing increases toward the top of the computational domain to a maximum of 98 km. The magnetic topology is defined by two opposite-polarity patches separated by 8 Mm, with an average unsigned strength of $\sim 50$ G in the photosphere, representing two patches of the quiet-Sun network. The simulation includes optically thick radiative transfer in the photosphere and low chromosphere; parameterized radiative losses in the upper chromosphere, transition region, and corona; thermal conduction along magnetic field lines; and an equation of state that includes the effects of non-equilibrium ionization of hydrogen. The Bifrost code solves the equations of resistive MHD on a staggered Cartesian grid but the published data cubes for en024048_hion have the variables specified at the cell centers.

One of the simulation snapshots, snapshot 385, has already been used in a series of papers to investigate the formation of the IRIS diagnostics (Leenaarts et al. 2013a, 2013b; Pereira et al. 2013, 2015; Lin & Carlsson 2015; Rathore & Carlsson 2015; Rathore et al. 2015) and of other chromospheric lines (Leenaarts et al. 2012, 2015; Štěpán et al. 2012, 2015; de la Cruz Rodríguez et al. 2013), as well as to model continuum free–free emission from the solar chromosphere (Loukitcheva et al. 2015a) with the perspective of ALMA observations (Loukitcheva et al. 2015b).

As has been mentioned, the available en024048_hion data cubes are specified on a nonuniform grid in the vertical direction, although most of the NLFFF approaches employ a regular grid, which has a number of computational advances, including uniform precision of the spatial derivatives. In order to make the en024048_hion model compatible with the extrapolated data cubes, the original model grid was transformed to a uniform grid (with a 48 km step along all three axes) by linear interpolation in the vertical direction.

Given that the applicability of the force-free magnetic modeling relies on the dominant role of the Lorentz force in the overall balance of forces, we start with an analysis of the height distribution of the plasma parameter $\beta = p_{\text{kin}}/p_B$ in the model volume. In Figure 1, we plot the range of $\beta$ values as a function of the model height using the original nonuniform grid and demarcate a number of characteristic levels for further references. Level $H = 0$ corresponds to the nominal photosphere in the model data cube. Since the model was created to describe an enhanced network on the quiet Sun rather than a typical active region (AR), the $\beta$ values at the level of the nominal photosphere are, not surprisingly, much larger than those for an AR (Gary 2001). Such a boundary condition, having an overall weak magnetic field with small-scale inserts of a stronger field, represents a big challenge for the currently available methods of coronal force-free modeling designed for ARs with relatively large-scale areas of strong magnetic field, where the plasma parameter $\beta$, although larger than one, is not exceptionally large.

Therefore, to emulate a photospheric boundary representative of a typical AR, we select a higher level where the distribution of the plasma $\beta$ is similar to that in a typical AR (Gary 2001); we call this level a “typical AR $\beta$ photosphere,” or just the $\beta$-photosphere for short. Specifically, we define this layer as the lowest layer above the nominal photosphere where 95% of the nodes have $\beta < 10^2$. This level roughly corresponds to $H_{\beta_{\text{ph}}} \approx 1$ Mm; the exact height varies between the data cubes obtained with different binning factors (see Section 2.2). Then, we define a chromospheric interface as the lowest layer where 95% of the nodes have $\beta < 10^{-1}$, which is supposed to offer an ideal boundary condition for force-free modeling. This level roughly corresponds to the height $H_{\beta_{\text{ch}}} \approx 2.5$ Mm. We return to a more specific definition of these levels in the next section, while introducing rebinned data cubes with a lower spatial resolution.
2.2. Setup for the Testing

From the original magnetic data cube, we initially created a new uniform data cube with full resolution over the \( x \), \( y \) coordinates and a regular grid in the \( z \) direction, with the voxel height of 48 km equal to the pixel sizes in the \( x \), \( y \) directions, while entirely discarding the subphotospheric part of the data cube. This yields a new full-resolution data cube with a regular grid having cubic voxels, convenient for further manipulation and analysis, which we refer to as the “original regular” data cube hereafter.

This new original data cube was then rebinned to produce lower-resolution data cubes\(^5\) with binning factors \( n = 2, 3, 4, 6, 7, 9 \). Apparently, a lower-resolution voxel includes \( n^3 \) original voxels. Therefore, for each new voxel, we use the mean values of the magnetic field components to represent these values in the center of the new voxel such that

\[
\overline{B}_\alpha = \frac{1}{n^3} \sum_{i=1}^{n^3} B_{\alpha}[i], \quad \alpha = x, y, \text{ or } z, \tag{1}
\]

and the standard deviations \( \delta B_{\alpha} \) is

\[
\delta B_{\alpha}^2 = \frac{1}{n^3 - 1} \sum_{i=1}^{n^3} (B_{\alpha}[i] - \overline{B}_\alpha)^2, \quad \alpha = x, y, \text{ or } z. \tag{2}
\]

This set of the data cubes with different resolutions represents the input for our tests. The distributions of \( z \)-component of the magnetic field for different binning factors at the nominal and \( \beta \)-photospheres and at the chromosphere are shown in Figures 2–4.

2.3. Volume Properties of the Regular Data Cubes

There are a number of numerical characteristics, similar to those introduced by Wheatland et al. (2000) and Schrijver et al. (2006), used to evaluate to what extent the reconstructed magnetic field matches the force-free criterion:

\[
\theta = \arcsin \left( \frac{\sum_i \sigma_i}{N} \right), \quad \theta_j = \arcsin \left( \frac{\sum_i |j||\sigma_i|}{\sum_i |j||\sigma_i|} \right),
\]

\[
\sigma_i = \frac{|j \times \overline{B}|}{|j||\overline{B}|}, \tag{3}
\]

and the divergence-free criterion:

\[
f = \frac{1}{N} \sum_i \frac{|\nabla \overline{B}_i|}{6 |\overline{B}_i|} dx, \tag{4}
\]

where \( \overline{B}_i \) is the model field; \( j \) is the corresponding electric current density, with the summation performed over the voxels of the computational grid (excluding boundaries); \( dx \) is the grid spacing; \( \sigma_i \) is the sine of the angle between the magnetic field

---

\(^5\) All of these data cubes with regular spacing as well as standard deviations are available at our project Web site: http://www.ioffe.ru/LEA/SF_AR/files/Magnetic_data_cubes/Bifrost/index.html.
and the current density at the $i$th node of the computational grid; $\theta$ is the angle averaged over all nodes, where in the ideal case of a force-free field it must be zero; $\theta_j$ is a similar metric but weighted with the electric current, which means that contributions coming from subdomains with strong currents dominate this metric; and $f$ is a parameter characterizing the accuracy of the magnetic field being divergence-free.

Here we compute these metrics for the original regular data cube and for the data cubes obtained after the rebinning. Table 1 presents the metrics calculated for the data cube set. For each data cube, we present three sets of metrics. The first one is calculated for the entire data cube volume starting from the nominal photosphere. The other two are for subdomains that start at either the $\beta$-photosphere or chromosphere levels. Independently of the binning factor, each metric displays the same trend as the function of the subdomain used for the computation: from the full data cube to the chromosphere-limited subdomain, $\theta$ shows a minor decrease, while $\theta_j$ decreases significantly, which is not surprising. Indeed, $\theta_j$ is weighted with the current density, while the strongest currents are concentrated near the bottom boundary. If this bottom boundary is located below the chromosphere, then the contribution from the non-force-free field region dominates. Above the chromosphere, the field corresponds to a force-free configuration with a reasonably high accuracy, though not exactly. It is interesting that the divergence-free parameter $f$ also improves when only the subdomain located above the chromosphere is taken into account, which, perhaps, originates from regridding the original nonuniform vertical grid to the uniform one used here. In addition, the prominent small-scale structure of the magnetic field at the lower levels results in more significant errors in the differential schemes used here to compute the spatial derivatives.

All metrics get worse with increasing binning factor. The reason for this is the unavoidable increase of numerical errors of the derivative computation. Indeed, rebinning the model data causes distortions and thus, an increase of the divergence-free metrics $f$.

### 3. $\pi$-disambiguation Tests

Reconstruction of the coronal magnetic field typically starts from the magnetic vector data on the photospheric or, less commonly, chromospheric levels of the solar atmosphere. The magnetic vector measurements are performed using spectropolarimetry of the Zeeman splitting of magnetic-sensitive optical or IR lines, with the circular polarization providing the line-of-sight component and the linear polarization providing the transverse component of the magnetic field vector. Apparently, only the azimuth angle, but not the direction of the transverse component, is being measured. This uncertainty of the transverse component direction is commonly called the $\pi$-ambiguity (180° ambiguity), and thus a method resolving this $\pi$-ambiguity is called for.
3.1. \( \pi \)-disambiguation Methods

A number of disambiguation methods, which differ in their approaches, performance, and speed, have been proposed (Metcalf 1994; Georgoulis 2005; Crouch 2013; Gosain & Pevtsov 2013; Rudenko & Anfinogentov 2014). Detailed comparisons of different methods can be found in comprehensive reviews (Metcalf et al. 2006; Leka et al. 2009). For our tests, we selected the minimum energy (ME) solution (Metcalf 1994); the so-called super fast and quality disambiguation (SFQ) method (Rudenko & Anfinogentov 2014), also known as the new disambiguation method (NDA); and the acute angle (AA) method, which makes a choice by comparing the azimuth angle with that of a "reference" field (Metcalf et al. 2006). The reference field can be a potential or linear force-free field, for example, but we employ a simple version of the AA method, which utilizes a naive approach where the true transverse field direction is selected to be closer to the potential field.

The ME disambiguation (http://www.cora.nwra.com/AMBIG/) is based on simulated annealing (Metropolis et al. 1953) and is regarded as providing the highest disambiguation quality over all methods. However, this method is rather computationally expensive. Our implementation of the ME method follows that of Metcalf (1994), with an improvement suggested by Leka et al. (2009), namely with the initial "temperature" varying from pixel to pixel depending on the maximum temperature among all possible combinations of the transverse field orientation of the neighbor pixels. In addition, in this implementation, for any pixel keeping its transverse field orientation for a reasonably long time (typically, during 100 consecutive iterations), the ambiguity is treated as "resolved" and the algorithm does not change its state over the remaining iterations. Thus, the number of analyzed pixels decreases with the number of iterations, which reduces the annealing time.

The SFQ method (Rudenko & Anfinogentov 2014) includes two steps: a preliminary disambiguation and a "cleaning" procedure. The former is a local comparison of the observed ambiguous field with the reference (potential) one. The comparison procedure is similar to that in the AA method. The discontinuities in this first-order approximate solution are then cleaned out with an iterative procedure. On each iteration, the transverse magnetic field is compared with its local average and inverted in those pixels where the alternative value is closer to the smoothed field.

3.2. Performance Metrics for the \( \pi \)-disambiguation Methods

To test the quality of the \( \pi \)-disambiguation methods, we used the following metrics (Metcalf et al. 2006), pixel error \( E_{\text{pix}} \) and flux error \( E_{\text{flux}} \):

\[
E_{\text{pix}} = \frac{N_{\text{err}}}{N},
\]

\[
E_{\text{flux}} = \frac{\Phi_{\text{err}}}{\Phi_{\text{true}}},
\]

where \( N_{\text{err}} \) and \( \Phi_{\text{err}} \) are the number of erroneous pixels and the flux error, respectively, and \( N \) and \( \Phi_{\text{true}} \) are the total number of pixels and the true flux, respectively.
### Table 1
Numerical Characteristics of Different Domains Depending on Binning Factor

| Binning Factor | Volume Above | Layer # | $\theta_0$ | $\theta_f$ | $f \times 10^6$ |
|---------------|-------------|--------|----------|----------|----------------|
| 1             | Nominal     | 0      | 19.23    | 48.60    | 568            |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 17 | 17.21    | 18.65    | 158            |
|               | chromosphere | 42     | 16.23    | 5.59     | 107            |
| 2             | Nominal     | 0      | 18.59    | 46.00    | 1573           |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 8  | 16.65    | 18.57    | 401            |
|               | chromosphere | 21     | 15.55    | 5.97     | 245            |
| 3             | Nominal     | 0      | 18.48    | 43.93    | 2594           |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 6  | 16.32    | 14.70    | 609            |
|               | chromosphere | 14     | 15.37    | 6.41     | 410            |
| 4             | Nominal     | 0      | 18.62    | 42.41    | 3315           |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 4  | 16.67    | 16.53    | 934            |
|               | chromosphere | 11     | 15.49    | 6.84     | 583            |
| 6             | Nominal     | 0      | 19.12    | 39.53    | 3690           |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 3  | 16.94    | 13.53    | 1315           |
|               | chromosphere | 7      | 15.87    | 7.60     | 951            |
| 7             | Nominal     | 0      | 19.40    | 38.24    | 3671           |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 3  | 16.93    | 11.37    | 1415           |
|               | chromosphere | 6      | 16.12    | 7.94     | 1135           |
| 9             | Nominal     | 0      | 20.03    | 36.13    | 4057           |
|               | photosphere |        |          |          |                |
|               | $\beta$-photosphere | 2  | 17.82    | 13.63    | 1939           |
|               | chromosphere | 5      | 16.64    | 8.58     | 1482           |

Note. For each particular layer, the “layer” column shows its level in the corresponding rebinned grid. The BIFROST photosphere always corresponds to zero level.

$$E_{\text{flux}} = \frac{\sum_{\text{err}} B_r(x, y)}{\sum_{\text{all}} B_r(x, y)}$$

Where $N$ is the total number of pixels and $N_{\text{err}}$ is the number of pixels where the disambiguation method fails, $B_r(x, y)$ is the absolute value of the transverse magnetic field in pixel $(x, y)$, and $\sum_{\text{err}}$ and $\sum_{\text{all}}$ are the summations over those pixels where the disambiguation failed and over all pixels in the magnetogram, respectively, as well as the typical computation time.

### 3.3. Comparison of the Alternative Approaches

The $\pi$-disambiguation methods described above have been applied to all reference layers and all rebinned data cubes. Here we summarize their performance.

For the nominal photosphere (see Table 2), all tested methods fail to provide acceptable results. Both flux and pixel errors are of the order of 20%–30%, with the only exceptions being the large bin factors and SFQ method, where the errors are about 10%; see Figure 5. In most cases (but not always), both methods work better in areas of stronger magnetic field but fail in areas of weak small-scale magnetic field. This failure originates from the fact that none of the methods is capable of correctly processing the very small magnetic elements with the size of the order of one pixel (salt and pepper patterns). This might have important implications for the $\pi$-disambiguation of the magnetic field in the quiet-Sun areas, which is, in particular, a substantial part of the full-disk disambiguation in the SDO/HMI pipeline.

Table 3 shows the test results for the level of a typical AR $\beta$-photosphere. At this level, the magnetic field becomes noticeably smoother than that at the nominal photosphere, and all methods improve their recovery success rate by at least an order of magnitude. AA pixel errors exceed 5% and the flux errors exceed 2%. SFQ and ME results are much better. On the finest grid, the best quality is provided by SFQ, while the output of the ME method contains an extended area where the transverse flux is inverted. For the grids with lower resolutions (bin 6–9), this artifact disappears and ME outperforms SFQ. However, both SFQ (all bins) and ME (bins 3–9) give flux errors of 1% or less and thus, their results can be used as boundary conditions for NLFFF extrapolations. In principle, it is tempting to use the $\pi$-disambiguation mismatches between the different disambiguation methods to reprocess those pixels, where two methods give opposite results, to improve the solution. However, as clearly seen from Figure 6, there are a number of pixels where both methods fail to recover the field azimuth, even though the total number of those “bad” pixels is very small. Figure 6 demonstrates that the failures are not random but occur in the areas where the field is either weak or almost vertical or both.

At the chromospheric level (Table 4), the magnetic field is more horizontal and smooth enough, so it is disambiguated perfectly by ME (there are errors in several pixels only for bins 4 and 6). The SFQ method also works excellently and fails only in several tens of pixels giving flux errors from 0.0001% to 0.01%, depending on the binning. AA also performs better on the smooth chromospheric (rather than the photospheric) field with the flux error about 1.5% for all bins.

These tests validate both ME and SFQ disambiguation methods at the $\beta$-photosphere and the chromosphere. This permits us to assume that the $\pi$-ambiguity has been resolved perfectly while performing further tests on the field preprocessing and extrapolation. Note that in this present article we do not study the dependence of the disambiguation accuracy on the position of the data cube at the solar disk. Most of the disambiguation approaches tend to generate more errors toward the solar limb. Here, the SFQ method has an advantage. According to tests on real magnetograms observed close to the disk center and artificially rotated toward the limb, it maintains a high accuracy even in those areas (Rudenko & Anfinogentov 2014), while other methods often produce artifacts in the transverse field components.
4. Preprocessing Tests

Even photospheric magnetic vector data with a perfectly resolved $\pi$-ambiguity represent a substantial challenge for coronal magnetic field reconstruction. The problem is that coronal modeling relies on the force-freeness of the magnetic field in the corona, which is dominated by the magnetic pressure (the plasma $\beta$ is less than 1), while the photospheric magnetic field is not force-free, given that $\beta \gg 1$ there. To overcome this mismatch, it has been proposed (Wiegelmann et al. 2006; Fuhrmann et al. 2007, 2011; Jiang & Feng 2014) that the measured photospheric magnetogram be modified in such a way that a new “preprocessed” magnetogram is representative of a higher chromospheric height and thus, more force-free than the original photospheric one. An appropriately performed preprocessing can facilitate the coronal field reconstruction by providing a boundary condition that is more suitable for an NLFFF extrapolation. The tests performed using available chromospheric diagnostics (Jing et al. 2010) confirm that the preprocessing is doing reasonably well; here we perform tests of the various available approaches to preprocessing using the same modeling data cubes as above.

4.1. Preprocessing Approaches

Wiegelmann et al. (2006) proposed preprocessing the photospheric vector magnetogram data to drive the observed non-force-free photospheric data toward a suitable boundary condition in the chromosphere by minimizing a functional $L_{\text{prep}}$:

$$L_{\text{prep}} = \mu_1L_1 + \mu_2L_2 + \mu_3L_3 + \mu_4L_4,$$

where

$$L_1 = (\Sigma_p(\hat{B}_x \cdot \hat{B}_y))^2 + (\Sigma_p(\hat{B}_y \cdot \hat{B}_z))^2 + (\Sigma_p(\hat{B}_z \cdot \hat{B}_x - \hat{B}_x \cdot \hat{B}_z))^2,$$

$$L_2 = (\Sigma_p(\hat{B}_y^2 - \hat{B}_x^2 - \hat{B}_z^2))^2 + (\Sigma_p(\hat{B}_z^2 - \hat{B}_x^2 - \hat{B}_y^2))^2 + (\Sigma_p(\gamma \hat{B}_z \cdot \hat{B}_y - x \hat{B}_y \cdot \hat{B}_z))^2,$$

$$L_3 = \Sigma_p(\hat{B}_x - B_x)^2 + \Sigma_p(\hat{B}_y - B_y)^2 + \Sigma_p(\hat{B}_z - B_z)^2,$$

$$L_4 = \Sigma_p(\Delta \hat{B}_x)^2 + \Sigma_p(\Delta \hat{B}_y)^2 + \Sigma_p(\Delta \hat{B}_z)^2.$$
Here, minimizing the $L_1$ and $L_2$ terms ensures that the final magnetogram corresponds, as closely as possible, to the force-free and torque-free conditions, respectively. The $L_3$ term is responsible for the similarity of the preprocessed data to the original magnetogram, while the $L_4$ term is responsible for the smoothing. The summations $\Sigma_p$ represent the surface integrals over all available grid nodes $p$ at the bottom boundary; $\Delta$ stands for the two-dimensional (2D) Laplace operator. The idea is to minimize $L_{\text{prep}}$ at once, so that all $L_n$ terms are getting smaller simultaneously. The weights $\mu_n$ are unknown a priori; Wiegelmann et al. (2006) tested various choices for $\mu_n$ and came up with a strategy on how to choose these weights; the default set of the weights is $\mu = [1, 1, 10^{-3}, 10^{-2}]$. Minimization of the functional is performed iteratively using the Newton-Raphson scheme (Press et al. 2007). Hereafter, we refer to our implementation of the preprocessing method developed by Wiegelmann et al. (2006) as TW preprocessing.

Fuhrmann et al. (2007) proposed another form of functional (5) to be minimized. The distinctions are in the form of the $L_4$ term (the smoothing was performed using a median filter instead of the differential Laplace operator in Wiegelmann et al. 2006), in varying the field only inside a given range defined by a field threshold value instead of considering $L_3$, and in the minimization method selected—annealing simulation.

Here, minimizing the $L_1$ and $L_2$ terms ensures that the final magnetogram corresponds, as closely as possible, to the force-free and torque-free conditions, respectively. The $L_3$ term is responsible for the similarity of the preprocessed data to the original magnetogram, while the $L_4$ term is responsible for the smoothing. The summations $\Sigma_p$ represent the surface integrals over all available grid nodes $p$ at the bottom boundary; $\Delta$ stands for the two-dimensional (2D) Laplace operator. The idea is to minimize $L_{\text{prep}}$ at once, so that all $L_n$ terms are getting smaller simultaneously. The weights $\mu_n$ are unknown a priori; Wiegelmann et al. (2006) tested various choices for $\mu_n$ and came up with a strategy on how to choose these weights; the default set of the weights is $\mu = [1, 1, 10^{-3}, 10^{-2}]$. Minimization of the functional is performed iteratively using the Newton-Raphson scheme (Press et al. 2007). Hereafter, we refer to our implementation of the preprocessing method developed by Wiegelmann et al. (2006) as TW preprocessing.

Fuhrmann et al. (2007) proposed another form of functional (5) to be minimized. The distinctions are in the form of the $L_4$ term (the smoothing was performed using a median filter instead of the differential Laplace operator in Wiegelmann et al. 2006), in varying the field only inside a given range defined by a field threshold value instead of considering $L_3$, and in the minimization method selected—annealing simulation.

The Astrophysical Journal, 839:30 (27pp), 2017 April 10

Fleishman et al.

Table 4

| Pixel Error, % | Flux Error, % | Computation Time |
|----------------|--------------|------------------|
| SFQ | ME | AA | SFQ | ME | AA | SFQ | ME |
| Bin | | | | | | | |
| 2 | 0.005 | 0 | 5 | 0.0001 | 0 | 1 | 0.73 s | 6 m03 s |
| 3 | 0.007 | 0 | 4.9 | 0.0003 | 0 | 1.4 | 0.3 s | 2 m35 s |
| 4 | 0.03 | 0.013 | 4.6 | 0.006 | 0.0004 | 1.3 | 0.13 s | 1 m28 s |
| 6 | 0.06 | 0.014 | 4.7 | 0.014 | 0.0005 | 1.4 | 0.07 s | 40 s |
| 7 | 0.06 | 0 | 4.6 | 0.02 | 0 | 1.3 | 0.05 s | 30 s |
| 9 | 0.03 | 0 | 4.4 | 0.01 | 0 | 1.3 | 0.04 s | 19 s |

The iteration step $s$ is being optimized itself during the minimization given that $s$ can be considered as one of the arguments of the functional; the optimum step is selected by the standard “golden section” method (Press et al. 2007) at each iteration step.
Fuhrmann et al. (2011) have shown that the Wiegelmann et al. (2006) method gives a smoother preprocessed field. Since both methods have similar natures and show comparable final results, we only consider the Wiegelmann et al. (2006) method for further testing.

Jiang & Feng (2014) proposed a slightly different approach to the preprocessing that consists of two distinct steps. The first step produces a potential extrapolation starting from the photospheric vertical magnetic field component \( \hat{B}_z \). Then, at a certain level, typically one pixel above the photosphere, the \( \hat{B}_z \) component is taken from the potential extrapolation, while a functional similar to Equation (4) is formed for the transverse field components only and is being minimized at the second step of the method. An apparent advantage of this approach is the ability to control the height level, to which the preprocessed magnetogram must correspond. For our tests, we used the preprocessing code provided by the authors Jiang & Feng (2014), with the default set of the weights \( \mu = [1, 1, 10^{-3}, 1] \) and with a few cosmetic modifications needed to ease the running of the code. In what follows, we refer to the preprocessing method of Jiang & Feng (2014) as JF preprocessing.

### 4.2. Performance Metrics for the Preprocessing Codes

Preprocessing methods are supposed to modify the magnetic field vector measured at the photosphere to make it more compatible with the force-freeness of the magnetic field model in the corona. The corresponding modification of the photospheric field could either result in the removal of the force component from the magnetic field distribution without any noticeable change in the field strength, or, in addition to this removal, may yield a corresponding decrease of the field strength emulating an effective “elevation” of the field distribution up to a higher level where the magnetic field becomes a force-free one.

Thus, we assess the preprocessing methods in the following three respects:

1. Has the magnetic field become significantly more force-free after preprocessing?
2. How close is the preprocessed field to the magnetic field in the BIFROST model?
3. Does the preprocessing cause an effective “elevation” of the field distribution up to a higher level?

The force-freeness of the preprocessed magnetic field is assessed using the \( L_1 \) and \( L_2 \) terms from Equation (5). In this work, we quantitatively measure the preprocessing efficiency by calculating the logarithmic ratio of the \( L_1 \) and \( L_2 \) parameters before and after the preprocessing: \( \log_{10} \frac{L_1^p}{L_1^o} \) and \( \log_{10} \frac{L_2^p}{L_2^o} \). The higher values of these parameters indicate a more efficient removal of the force-carrying magnetic field component.

The consistency of the preprocessed magnetic field with the model magnetic field at a given level of the data cube can straightforwardly be assessed using the \( L_3 \) metrics in Equation (5). To assess if any effective elevation has happened, we calculate this metric using the Bifrost data from the same level and a few higher levels. We also use a few more useful metrics defined below:

1. Normalized discrepancy between the preprocessed and observed field components:

   \[ L_3 = \sqrt{\frac{L_3}{\sum_p B^2}}. \]  

2. Normalized discrepancy between the absolute value of the preprocessed and observed fields:

   \[ E_B = \sqrt{\frac{\sum_p (\bar{B} - B)^2}{\sum_p B^2}}. \]  

3. Average angle between the preprocessed and observed fields:

   \[ A = \arccos \left( \frac{\sum_p \bar{B} \cdot B}{\bar{B} \cdot B} \right). \]  

4. Slope of the regression dependence \( \bar{B} = SB + b \):

   \[ S = \frac{\sum_p \bar{B}B - \sum_p \bar{B} \sum_p B}{\sum_p B^2 - (\sum_p B)^2/N}. \]
5. Correlation coefficient:

\[
R = \frac{\sum_p \tilde{B} \cdot \tilde{\sum}_p \tilde{B} \cdot \tilde{B} / N}{\sqrt{\sum_p \tilde{B}^2 - (\tilde{\sum}_p \tilde{B})^2} / N} \frac{\sqrt{\sum_p \tilde{B}^2 - (\tilde{\sum}_p \tilde{B})^2} / N}{\sum_p \tilde{B}^2 - (\tilde{\sum}_p \tilde{B})^2} / N.
\] (10)

A better preprocessing method is expected to give lower values of \(L_3\), while the preferable values of the slope \(S\) and the correlation coefficient \(R\) are those closer to unity. If the preprocessed and the original fields are substantially different, \(L_3\) will have a larger value. For instance, if \(\tilde{B} \equiv 0\), the \(L_3\) metric will be equal to unity for any \(\tilde{B}\). Therefore, we will interpret the preprocessing results with \(L_3 \geq 0.5\) as unacceptable. Additionally, we calculate metrics (6)–(10) for the model magnetic field taken at the height of one voxel to nail down a possible effective elevation that can be a side or intended effect of the preprocessing.

4.3. Comparison of the Alternative Approaches

Tables 5–7 show the metrics describing how force-free are the boundary conditions produced by the two codes. For the nominal photosphere, all results have \(L_3 > 0.5\), meaning that the preprocessed field is strongly different from the original magnetogram. Therefore, we interpret the preprocessing results obtained for the nominal photosphere as incorrect and do not analyze them in what follows. For all levels and binning factors, our implementation of the TW preprocessing (Wiegelmann et al. 2006) outperforms the JF preprocessing code (Jiang & Feng 2014), providing significantly more force-free results. However, the \(L_3\) metrics are nearly the same for both codes, meaning that both solutions are comparably close to the input magnetic field.

Detailed quantitative comparisons of the preprocessing results and the magnetic field in the Bifrost model are given in Tables 8 and 9. We separately assess the full vector and the

**Table 8**

| Bin level | Full Vector | Longitudinal Component | Transverse Components |
|-----------|-------------|------------------------|-----------------------|
| 2         | JF          | TW                     | JF                    | TW                                      |
| +0        | 0.25        | 0.16                   | 11.55                 | 10.85                                   |
| +1        | 0.14        | 0.13                   | 22.62                 | 22.44                                   |

**Table 9**

| Bin level | Full Vector | Longitudinal Component | Transverse Components |
|-----------|-------------|------------------------|-----------------------|
| 2         | JF          | TW                     | JF                    | TW                                      |
| +0        | 0.08        | 0.03                   | 2.93                  | 1.23                                    |
| +1        | 0.05        | 0.05                   | 4.12                  | 3.48                                    |

The Astrophysical Journal, 839:30 (27pp), 2017 April 10

Fleishman et al.
longitudinal and transverse components at two levels: the same level as the initial magnetogram and one level higher. The $E_B$ metric (Equation (7)) indicates the discrepancy between the absolute values of the preprocessed and modeled magnetic fields, while the difference in the directions of the fields is shown by the $A$ metric (Equation (8)). This metric shows that there is an effective field elevation caused by either of the JF (all bins) or TW (bins 4–9) preprocessing methods. The effect is more pronounced for JF preprocessing especially at low spatial resolution (bin 9). However, the mean angle between the preprocessed and Bifrost fields shows a different behavior. A one-level elevation does not improve the mean angle metric for JF preprocessing, while the TW method demonstrates a slight improvement of this metric at bins 7 and 9. Being applied to the chromospheric level (even though this might not be needed in practice), JF preprocessing again demonstrates effective elevation in $E_B$, while the TW method shows no improvement at a higher level in either absolute value discrepancy or in angle.

The JF and TW methods preprocess longitudinal and transverse magnetic field components differently. Therefore, we assess the field components separately. For both longitudinal and transverse components, we calculate the slope (Equation (9)) and correlation (Equation (10)) metric. The former characterizes the absolute value of the field while the latter assesses the spatial structuring of the field.

Both slope and correlation metric show the presence of an effective elevation in the longitudinal component preprocessed using the JF method. Indeed, such a behavior is rather expected because the JF preprocessing fixes the longitudinal component to be equal to the potential field extrapolation results and does not optimize it. The transverse component is, however, elevated by a height exceeding one voxel size (but less than two voxel sizes). Therefore, there is a mismatch in the heights to where the longitudinal and transverse components are elevated in the JF method. This mismatch is primarily responsible for the mismatch in the angle between the preprocessed and the (one level up) model field mentioned above.

After the TW preprocessing, the value of the longitudinal field becomes underestimated for the original level ($S < 1$) but overestimated for the +1 voxel level ($S > 1$), meaning that some effective elevation is present, but the elevation height is somewhere between 0 and 1 voxel. In contrast, the transverse component displays an elevation very close to the one-voxel height; thus, the TW preprocessing also results in a mismatch in the elevation heights for the longitudinal and transverse components, the same as with the JF method.

The JF preprocessing demonstrates a better cross-correlation metric $R$ for the longitude component for all levels and binning factors, both at the photospheric and chromospheric heights. The cross-correlation coefficient for the transverse field is worse for both methods at all levels and heights. At high resolutions (bins 2–4), JF and TW show very similar results, while at low resolution (bins > 4), JF demonstrates a higher correlation with the modeled field. This behavior is also demonstrated in Figure 7 in the form of two-dimensional histograms showing the scatter of the preprocessed field components versus the modeled ones.

Our tests show that both methods have their own advantages and disadvantages. The TW preprocessing improves the force-freeness metrics by many orders of magnitude, but significantly changes the field structure, especially for lower spatial resolutions; see Figure 8. It also effectively elevates the field, but the elevation height seems to differ from one voxel and differs for the longitudinal and transverse components. The JF preprocessing saves the field structure for low-resolution grids better and more distinctly elevates the longitudinal field to the height of one voxel, but the output is not as force-free as that for the TW method; in addition, the transverse component is

---

**Figure 7.** Dependence of the magnetic field preprocessed with the JF (left panel) and TW (right panel) preprocessing codes on the initial field in the $\beta$-photosphere for bin = 9.
effectively elevated by a height larger than the size of one voxel. These mismatches in the elevation heights for the field components revealed in both methods will then have a negative impact on the fidelity of the NLFFF extrapolations.

5. NLFFF Tests

The different approaches and algorithms of NLFFF reconstruction proposed so far include the vertical integration method, the boundary integral method, the Euler potential method, the Grad-Rubin methods, and various kinds of evolutionary methods, such as magnetofrictional and optimization methods (see, e.g., the brief overview in Aschwanden 2005, Section 5.3.3). Correspondingly, a variety of computing codes employing one version of these methods or another have been implemented. Although it is certainly interesting to cast all or most of the available NLFFF reconstruction methods versus realistic MHD models, such a study would be highly excessive for a single paper. Here we focus on two different implementations of the optimization method\(^7\) (Wheatland et al. 2000) performed by our team members Alexey Stupishin (hereafter AS, following Wiegelmann 2004) and Ivan Myshyakov (hereafter IM, following Rudenko & Myshyakov 2009). The IM code was used by Kaltman et al. (2015) and Livshits et al. (2016), while the AS code was used by Kaltman et al. (2012), Bogod

---

\(^7\) We do not consider the modifications proposed by Wiegelmann & Inhester (2010) and Wiegelmann et al. (2012) to address imperfections of real data because our paper attempts to evaluate the best achievable performance of the NLFFF extrapolation codes themselves, i.e., those not affected by the possible negative influence of the measurement errors or lack of data.
Table 10
Performance of the Magnetic Field Reconstruction Methods

| Bin | Impl | Chromosphere | $\beta$-photosphere | $\beta$-photosphere, JF Preprocessed | $\beta$-photosphere, TW Preprocessed |
|-----|------|--------------|---------------------|-------------------------------------|-------------------------------------|
|     |      | $\theta^\circ$ | $\theta_j^\circ$ | $\theta_m^\circ$ | $\theta_m^\circ$ | $\theta_j^\circ$ | $\theta_m^\circ$ | $\theta_j^\circ$ | $\theta_m^\circ$ | $\theta_j^\circ$ | $\theta_m^\circ$ | $\theta_j^\circ$ | $\theta_m^\circ$ | $\theta_j^\circ$ | $\theta_m^\circ$ |
| 3   | IM   | 9.3          | 4.5                | 17.8                | 11.4                | 16.4          | 11.1                | 30.5                | 20.9                | 11.5          | 13.7                | 28.2                | 19.5                | 12.4          | 13.7                | 26.7                | 18.8                |
|     | AS   | 24.4         | 8.7                | 22.2                | 12.7                | 33.8          | 14.9                | 24.5                | 20.6                | 35.9          | 19.0                | 24.5                | 19.5                | 33.8          | 18.2                | 24.3                | 20.1                |
| 4   | IM   | 10.7         | 5.4                | 18.1                | 11.5                | 18.9          | 13.4                | 34.3                | 22.3                | 13.0          | 16.5                | 28.5                | 21.2                | 13.7          | 16.9                | 27.1                | 20.3                |
|     | AS   | 24.9         | 9.4                | 21.9                | 12.4                | 34.5          | 16.3                | 24.5                | 20.7                | 36.6          | 21.3                | 24.3                | 20.0                | 34.5          | 20.8                | 24.1                | 20.5                |
| 6   | IM   | 10.6         | 5.9                | 16.0                | 10.0                | 18.2          | 13.4                | 27.2                | 17.4                | 13.7          | 16.9                | 23.4                | 15.8                | 15.1          | 18.9                | 22.6                | 16.4                |
|     | AS   | 26.8         | 10.7               | 21.6                | 12.0                | 34.3          | 18.1                | 23.4                | 18.2                | 39.8          | 25.5                | 23.4                | 15.4                | 34.2          | 24.2                | 23.0                | 16.9                |
| 7   | IM   | 11.0         | 6.3                | 15.8                | 9.8                 | 14.3          | 10.6                | 21.3                | 13.2                | 12.5          | 14.2                | 20.2                | 13.5                | 14.0          | 17.9                | 19.4                | 14.2                |
|     | AS   | 28.3         | 11.5               | 21.6                | 11.8                | 39.1          | 21.1                | 23.0                | 17.1                | 42.0          | 25.9                | 23.0                | 14.4                | 38.8          | 27.7                | 22.4                | 15.1                |
| 9   | IM   | 11.8         | 7.0                | 15.1                | 9.4                 | 17.7          | 14.7                | 24.2                | 15.2                | 16.2          | 20.2                | 20.7                | 13.9                | 19.4          | 26.2                | 20.0                | 14.6                |
|     | AS   | 30.1         | 13.3               | 22.1                | 12.2                | 43.0          | 26.1                | 24.0                | 18.9                | 46.1          | 32.6                | 24.1                | 14.9                | 42.8          | 35.0                | 23.2                | 15.8                |

Note. “Bin” is the binning factor. “Impl” is the implementation of the optimization method. The following four wide columns contain the numerical characteristics of the reconstructed field. The column title provides information on the starting layer and whether preprocessing was applied.
et al. (2012), and Yasnov et al. (2016) to address various problems in magnetic modeling.

5.1. Description of the NLFFF Optimization Methods

In both implementations used in this paper, the NLFFF reconstructions are performed following the optimization method (Wheatland et al. 2000). The main idea of the optimization method is to transform some trial configuration of the magnetic field (usually a potential extrapolation from the bottom boundary) to a final force-free field configuration. This is achieved by minimization of the following, positively defined, functional:

\[
L = \int_V \left[ B^{-2} \left( (\nabla \times B) \times B \right)^2 + |\nabla \cdot B|^2 \right] w(x, y, z) dV,
\]

where \( w(x, y, z) \) is a "weight" function defined below.

It is obvious that if the magnetic field \( B \) has a force-free configuration everywhere in the volume of interest \( V \) then \( L \) must be zero. In practice, the solution with \( L = 0 \) is hardly achievable, so minimization of the functional leads to an approximate force-free solution for the magnetic configuration. Solving the minimization problem (see appendices in Wheatland et al. 2000 and Wiegelmann 2004 for the detailed math) yields two equations, which optimize the magnetic field in the inner volume and on the boundaries of the computational domain.

The two implementations of the optimization method used in our study are different in how the boundary zone is treated. The IM implementation takes into account both optimization equations as detailed by Rudenko & Myshyakov (2009), such as the magnetic field being allowed to reconfigure everywhere in the volume of interest including its top and side boundaries; \( w \equiv 1 \) in this implementation. The magnetic field at the bottom boundary, which represents the input magnetogram, remains fixed during the optimization. Having a variable magnetic field at the top and side boundaries is potentially helpful, because, given that the NLFFF reconstruction is initiated with a potential configuration, fixing the potential field at these boundaries may

Figure 10. 2D histograms of the \( B_x \) reconstruction obtained using two methods, IM and AS, from the chromosphere and the \( \beta \)-photosphere without preprocessing, in 3D volume. The buffer zone is discarded everywhere.
have a negative impact on the force-freeness of the reconstructed magnetic field.

The alternative, the AS implementation, follows the Wiegelmann (2004) approach in selecting the weight function: \( w(x, y, z) = w_x(x)w_y(y)w_z(z) \), where the factors \( w_x(x) \) and \( w_y(y) \) are both equal to 1 in the internal area of the simulation cube \((0.1-0.9) \cdot L_x \). In the buffer zone outside this internal area, the factors \( w_x(x) \) and \( w_y(y) \) decrease toward zero with the cosine functions. The other factor \( w_z(z) = 1 \) for the heights \((0-0.9) \cdot L_z \) and then goes to zero with the cosine function. The magnetic field at the top and side boundaries is frozen to be the potential one, the same one used to initialize the optimization. After each successful iteration, the step is increased by a factor of 1.03, while after each unsuccessful iteration the step is reduced by a factor of 0.8. The optimization ends when the step has become less than 0.01 of the initial step.

One modification of the AS implementation relative to the original method is the selection of the initial approximation for the magnetic field (which follows a multigrid extension proposed by Metcalf et al. 2008): for the sparsest grid (bin 9 in our tests), the initial approximation is the potential field, while for each subsequent denser grid, the initial field is taken as an appropriate interpolation of the final (NLFFF) state of the previous grid. The mentioned modifications optimize the simulation time but have almost no effect on the final metrics of the reconstructed NLFFF cube.

Alternative implementations have different time performances. Both codes spend comparable times on a single iteration. For example, in the case of bin 9, the IM code does \( \approx 770 \) iterations per second, while the speed of the AS code is \( \sim 200 \) iterations per second (calculations were performed on a four-core 3.4 GHz processor). However, the AS code has to make only about 2,500 iterations in total to get the final result for bin = 9 due to a dynamically changed time step; the number of required iterations becomes significantly smaller (several hundreds) for the higher-resolution grids as they use the interpolated solution of the scarcer grid as the initial condition, which is a much better approximation of reality than the potential extrapolation used for the bin = 9 case. The IM code has a fixed time step, makes \( 10^5 \) iterations in total and

![Figure 11. 2D histograms of the \( B_y \) reconstruction obtained using two methods, IM and AS, from the chromosphere and the \( \beta \)-photosphere without preprocessing, in 3D volume. The buffer zone is discarded everywhere.](image-url)
Figure 12. Model $B_z$ field distributions (left column) and performance of the IM (next two columns) and AS (two right columns) NLFFF extrapolations ($B_z$ component) from the chromospheric level (bin = 3) at three levels (their heights are shown in the panel titles); second and fourth columns: residual between the extrapolated and the model field; third and fifth columns: relative error. The residual is within 2–3 G at all levels, while the relative error increases with height, because the field strength decreases with height. The relative error is also bigger along the “neutral lines,” where the field is close to zero. The results for other components and other binning factors are similar to those shown in this figure. The animation of this figure shows the same information but for all layers of the reconstructed $B_z$ data cubes for the IM and AS extrapolations. Each frame of the animation shows two four-panel blocks pertaining to the IM and AS extrapolations, respectively, at a given layer: (a) the model field, (b) the restored field, (c) the residual, and (d) the relative error.

(An animation of this figure is available.)

Figure 13. Model $B_y$ field distributions (left column) and performance of the IM (next two columns) and AS (two right columns) NLFFF extrapolations ($B_y$ component) from the chromospheric level (bin = 3) at three levels (their height are shown in the panel titles); second and fourth columns: residual between the extrapolated and the model field; third and fifth columns: relative error. The residual is within 2–3 G at all levels, while the relative error increases with height, because the field strength decreases with height. The relative error is also bigger along the “neutral lines,” where the field is close to zero. The results for other components and other binning factors are similar to those shown in this figure. The animation of this figure shows the same information but for all layers of the reconstructed $B_y$ data cubes for the IM and AS extrapolations. Each frame of the animation shows two four-panel blocks pertaining to the IM and AS extrapolations, respectively, at a given layer: (a) the model field, (b) the restored field, (c) the residual, and (d) the relative error.

(An animation of this figure is available.)
even more for lower bin factors. Therefore, the AS implementation requires a considerably shorter computational time.

5.2. Metrics for the Evaluation of the NLFFF Method Performance

There are two questions we want to answer about the performance of our NLFFF extrapolation codes: (1) how close the final data cubes are to the targeted force-free state and (2) how well they reproduce the original field in the entire 3D domain. To address the first question, we use the same metrics as we have used to evaluate the force-freeness of the original model field itself, Equations (3)–(4).

To assess how close the NLFFF extrapolated data cube is to the corresponding model data cube, we use "angular" metrics similar to Equation (3):

\[
\theta_m = \arccos \left( \frac{\sum_i N_i \tau_i}{N} \right), \quad \theta_{mj} = \arccos \left( \frac{\sum_i N_i j_i \tau_i}{\sum_i N_i j_i} \right),
\]

\[
\tau_i = \frac{B_{NLFFF,i} \cdot B_i}{|B_{NLFFF}| |B_i|},
\]

where \( j \) is the electric current density computed for the reconstructed field, the summation is performed over the voxels of the analyzed volume subdomain, \( \tau_i \) is the cosine of the angle between the restored and model magnetic field at the \( i \)th node of the computational grid, \( \theta_m \) is the angle averaged over all nodes which in the ideal case of the force-free field must be zero, \( \theta_{mj} \) is a similar metric but weighted with the restored electric current which ensures that the contribution from nodes with a strong electric current dominates this metrics. For a voxel-to-voxel inspection, we compute the local error (residual) \( \Delta_\alpha[i] \), the local relative error \( \delta_\alpha[i] \), and the local normalized residual \( \chi^2_\alpha[i] \) as

\[
\Delta_\alpha[i] = B_{NLFFF,\alpha}[i] - \overline{B}_\alpha[i],
\]

\[
\delta_\alpha[i] = \frac{B_{NLFFF,\alpha}[i] - \overline{B}_\alpha[i]}{\langle B_\alpha[i] \rangle}, \quad \alpha = x, y, \text{ or } z,
\]

\[
\chi^2_\alpha[i] = \frac{(B_{NLFFF,\alpha}[i] - \overline{B}_\alpha[i])^2}{\delta B^2_\alpha[i]}, \quad \alpha = x, y, \text{ or } z,
\]

where \( i \) is the number of a given voxel, \( \langle B_\alpha \rangle = \sqrt{B^2_\alpha + \delta B^2_\alpha} \), \( \overline{B}_\alpha[i] \) and \( \delta B^2_\alpha \) are defined for each binning factor by Equations (1) and (2), while \( B_{NLFFF,\alpha} \) is the corresponding component of the magnetic field obtained from the extrapolation. Here, to compute the relative error, we take into account that after cube rebinning, the magnetic field in each voxel is
Table 11
Normalized rms Error at a Given Level for Bin Factor 9

| Level, Mm | Chromo δrms(B) | IM | AS | χ2-photo | IM | AS | Chromo δrms(Bx) | IM | AS | χ2-photo | IM | AS | Chromo δrms(By) | IM | AS | χ2-photo | IM | AS | Chromo δrms(Bz) | IM | AS | χ2-photo | IM | AS |
|----------|----------------|----|----|----------|----|----|----------------|----|----|----------|----|----|----------------|----|----|----------|----|----|----------------|----|----|----------|----|----|
| 0.86     | ...            | ... | ... | 0.00     | ... | ... | 0.00          | ... | ... | 0.00     | ... | ... | 0.00          | ... | ... | 0.00     | ... | ... |
| 1.29     | ...            | ... | ... | 13.22    | ... | ... | 41.50         | ... | ... | 48.41    | ... | ... | 64.89         | ... | ... | 84.15    | ... | ... |
| 1.71     | ...            | ... | ... | 11.83    | ... | ... | 54.58         | ... | ... | 76.79    | ... | ... | 150.87        | ... | ... | 34.28    | ... | ... |
| ...      | ...            | ... | ... | 12.60    | ... | ... | 45.84         | ... | ... | 98.33    | ... | ... | 177.45        | ... | ... | 32.92    | ... | ... |
| 2.34     | 2.13          | 12.26| 15.80| 19.70    | 46.90| 79.76| 48.00         | 184.75| 280.19| 32.75    | 28.91| 43.18| 56.51         | ... | ... |
| 2.86     | 4.31          | 12.03| 17.02| 24.16    | 33.32| 46.40| 79.76         | ... | ... | 71.85    | 125.21| 147.31| 178.25        | ... | ... |
| 3.39     | 5.33          | 12.84| 16.35| 25.46    | 38.36| 47.57| 91.95         | ... | ... | 67.50    | 157.94| 230.89| 42.88         | ... | ... |
| 4.29     | 6.25          | 13.75| 16.65| 33.59    | 57.48| 65.65| 113.74        | ... | ... | 75.95    | 129.38| 184.94| 72.54         | ... | ... |
| 6.00     | 7.08          | 14.79| 16.05| 33.71    | 68.16| 62.19| 125.21        | ... | ... | 85.61    | 169.44| 212.02| 84.86         | ... | ... |
| 7.57     | 7.93          | 17.46| 18.16| 35.51    | 84.52| 61.14| 138.04        | ... | ... | 114.93   | 203.77| 252.55| 94.02         | ... | ... |
| 6.43     | 9.63          | 17.86| 18.65| 37.03    | 81.69| 60.85| 139.94        | ... | ... | 138.18   | 226.76| 261.13| 98.34         | ... | ... |
| 8.79     | 10.29         | 18.81| 18.67| 40.70    | 93.39| 68.74| 153.62        | ... | ... | 160.61   | 257.38| 290.81| 95.84         | ... | ... |
| 7.29     | 11.34         | 19.22| 18.02| 44.19    | 106.77| 77.61| 165.73        | ... | ... | 138.80   | 221.37| 243.29| 80.96         | ... | ... |
| 7.71     | 12.30         | 19.05| 18.73| 46.95    | 119.11| 84.25| 177.37        | ... | ... | 148.52   | 228.13| 251.31| 101.62        | ... | ... |
| 8.14     | 13.17         | 21.74| 20.19| 49.63    | 132.08| 92.43| 189.03        | ... | ... | 170.13   | 246.65| 266.38| 95.58         | ... | ... |
| 8.57     | 14.41         | 21.74| 20.89| 53.43    | 145.35| 100.12| 198.74        | ... | ... | 189.90   | 260.63| 282.46| 92.08         | ... | ... |
| 9.00     | 15.50         | 21.85| 20.78| 57.27    | 158.31| 109.08| 210.81        | ... | ... | 200.99   | 267.05| 287.85| 85.29         | ... | ... |
| 9.43     | 16.92         | 25.86| 25.21| 61.65    | 171.14| 119.76| 220.42        | ... | ... | 217.77   | 287.73| 314.15| 84.87         | ... | ... |
| 9.86     | 18.70         | 28.49| 27.76| 66.06    | 181.68| 127.53| 228.34        | ... | ... | 243.57   | 325.13| 496.97| 84.62         | ... | ... |
| 10.29    | 20.69         | 41.73| 33.76| 68.89    | 189.89| 136.12| 231.95        | ... | ... | 258.39   | 339.71| 539.40| 195.19        | ... | ... |
| 10.71    | 23.21         | 47.38| 36.83| 72.72    | 197.35| 147.00| 236.25        | ... | ... | 258.66   | 316.44| 528.81| 215.54        | ... | ... |
| 11.14    | 26.26         | 54.07| 39.79| 79.75    | 210.62| 162.30| 245.62        | ... | ... | 282.73   | 328.77| 564.24| 238.96        | ... | ... |
only known to an accuracy of $\mathbf{B}_0 \pm \delta R_x$. Thus, in the denominators of $\delta_B[i]$ in Equation (13), we use $\langle \mathbf{B}_0 \rangle$ rather than $\mathbf{B}_0$; otherwise, in “singular” points, where $\mathbf{B}_0$ is very close to zero in the denominator, such a metric would artificially underestimate the accuracy. However, to compute a similar metric for the absolute value of the magnetic field vector, we do not add any $\delta B$ because the absolute value is never very close to zero in the analyzed volume.

To characterize the extrapolation performance in a given subdomain, which can be, for example, a given layer or the entire data cube, we use the normalized rms residual $\Delta_{\text{rms}}$, the normalized rms error $\delta_{\text{rms}}$, and the “effective $\chi^2$” metrics defined as

$$\Delta_{\text{rms},\alpha} = \frac{\sum_{i=1}^{N_{\text{vox}}} \delta^2_B[i]}{\sum_{i=1}^{N_{\text{vox}}} B_0^2[i]}^{\frac{1}{2}}, \quad \alpha = x, \ y, \ or \ z,$$

$$\delta_{\text{rms},\alpha} = \frac{1}{N_{\text{vox}}} \sum_{i=1}^{N_{\text{vox}}} \delta_B^2[i], \quad \alpha = x, \ y, \ or \ z,$$

$$\chi^2_{\text{eff},\alpha} = \frac{1}{N_{\text{vox}}} \sum_{i=1}^{N_{\text{vox}}} \chi^2_{B}[i], \quad \alpha = x, \ y, \ or \ z,$$

where the summation is performed over the subdomain of the data cube used for the analysis; $N_{\text{vox}}$ is the total number of voxels in the selected subdomain. The normalized rms residual, Equation (15), is similar to metric (7), which has been used to evaluate the preprocessing performance; this metric gives more weight to the voxels, where the magnetic field is strong. In contrast, metric (16) gives equal weight to any voxel, with either strong or weak field. The $\chi^2$ metric weights the voxel in accordance with the uncertainty to which the magnetic field is known in the given voxel.

5.2.1. NLFFF Extrapolations from the Chromospheric Boundary

We expect that any extrapolation approach will perform best if the bottom boundary condition is close to being force-free. Thus, testing the extrapolation performance from the nearly force-free chromospheric layer will characterize the true potential of the given extrapolation code itself. For this reason, we begin our tests from the NLFFF data cubes extrapolated from the chromospheric level.

---

Figure 15. Relative rms residual in a layer as a function of height for the NLFFF reconstructions obtained using two methods, IM and AS, from the chromosphere and the $\beta$-photosphere without preprocessing. The side buffer zones are discarded everywhere, while the height of the top buffer zone is shown by the dashed vertical line.

---

8 All reconstructed data cubes obtained in our study are available at our project Web site: http://www.ioffe.ru/LEA/SF_AR/files/Magnetic_data_cubes/Extrapolations-Bifrost/index.html.
Figure 16. Model $B_z$ field distributions (left column) and performance of the IM (next two columns) and AS (two right columns) NLFFF extrapolations ($B_z$ component) from the $\beta$-photospheric level ($\text{bin} = 3$) at three levels (their heights are shown at the panel titles); second and fourth columns: residual between the extrapolated and the model field; third and fifth columns: relative error. The residual is within $2$–$3$ G at all levels, while the relative error increases with height, because the field strength decreases with height. The relative error is also bigger along the “neutral lines,” where the field is close to zero. The results for other components and other binning factors are similar to those shown in this figure. The animation of this figure shows the same information but for all layers of the reconstructed $B_z$ data cubes for the IM and AS extrapolations. Each frame of the animations shows two four-panel blocks pertaining to the IM and AS extrapolations, respectively, at a given other binning factors are similar to those shown in this figure.

Table 12

| Level, Mm | Chromo $\Delta_{\text{rms}}(B)$ | $\beta$-photo $\Delta_{\text{rms}}(B)$ | Chromo $\Delta_{\text{rms}}(B_x)$ | $\beta$-photo $\Delta_{\text{rms}}(B_x)$ | Chromo $\Delta_{\text{rms}}(B_y)$ | $\beta$-photo $\Delta_{\text{rms}}(B_y)$ | Chromo $\Delta_{\text{rms}}(B_z)$ | $\beta$-photo $\Delta_{\text{rms}}(B_z)$ |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.86      | ...             | 0.00            | ...             | 0.00            | ...             | 0.00            | ...             | 0.00            |
| 1.29      | ...             | 5.25            | 0.07            | 5.52            | 0.67            | ...             | 6.82            | 7.52            |
| 1.71      | ...             | 4.40            | 6.14            | ...             | 6.27            | ...             | 9.64            | 12.00           |
| 2.14      | 0.00            | 0.00            | 0.00            | 7.35            | 10.83           | 0.00            | 11.37           | 14.58           |
| 2.57      | 0.82            | 0.81            | 4.25            | 4.67            | 1.13            | 1.06            | 7.67            | 11.16           |
| 3.00      | 1.60            | 1.56            | 4.78            | 4.73            | 2.21            | 2.05            | 8.39            | 12.57           |
| 3.43      | 2.43            | 2.10            | 5.95            | 4.72            | 3.19            | 4.09            | 9.20            | 14.17           |
| 3.86      | 3.28            | 2.70            | 6.53            | 5.08            | 3.91            | 5.22            | 10.21           | 15.88           |
| 4.29      | 4.14            | 3.30            | 7.63            | 5.32            | 4.51            | 6.86            | 11.50           | 18.44           |
| 4.71      | 5.01            | 3.86            | 8.82            | 5.90            | 5.21            | 8.71            | 13.16           | 21.11           |
| 5.14      | 5.91            | 4.50            | 10.08           | 6.30            | 6.10            | 10.66           | 15.01           | 24.28           |
| 5.57      | 6.78            | 5.08            | 11.33           | 6.96            | 6.96            | 12.62           | 16.85           | 27.32           |
| 6.00      | 7.66            | 5.67            | 12.56           | 7.36            | 7.87            | 14.53           | 18.87           | 30.52           |
| 6.43      | 8.55            | 6.16            | 13.72           | 7.94            | 8.88            | 16.59           | 21.09           | 33.55           |
| 6.86      | 9.40            | 6.66            | 14.84           | 8.34            | 9.95            | 18.61           | 23.25           | 36.47           |
| 7.29      | 10.17           | 7.11            | 15.95           | 9.06            | 10.85           | 20.63           | 24.97           | 38.90           |
| 7.71      | 10.85           | 7.58            | 17.06           | 9.71            | 11.64           | 22.56           | 26.41           | 41.22           |
| 8.14      | 11.47           | 8.11            | 18.19           | 10.71           | 12.40           | 24.69           | 28.04           | 43.80           |
| 8.57      | 12.04           | 8.78            | 19.45           | 11.71           | 13.20           | 26.71           | 29.95           | 46.53           |
| 9.00      | 12.61           | 9.61            | 20.94           | 13.09           | 13.95           | 28.73           | 31.88           | 49.22           |
| 9.43      | 13.30           | 10.71           | 22.75           | 14.52           | 14.68           | 30.73           | 33.73           | 51.53           |
| 9.86      | 14.16           | 12.06           | 25.00           | 16.28           | 15.40           | 32.88           | 35.33           | 53.28           |
| 10.29     | 15.22           | 13.68           | 27.81           | 18.13           | 16.23           | 35.15           | 37.03           | 54.73           |
| 10.71     | 16.64           | 15.52           | 31.34           | 20.20           | 17.12           | 37.58           | 38.81           | 55.97           |
| 11.14     | 18.58           | 17.64           | 35.72           | 22.40           | 18.19           | 39.55           | 40.66           | 56.64           |

(An animation of this figure is available.)
Table 10 presents the angular metrics that characterize the force-freeness of the reconstructed field (the angles $\theta$ and $\theta_j$) and its closeness to the original model field (the angles $\theta_m$ and $\theta_{mj}$) computed for the entire 3D volume above the bottom boundary and excluding the top and side buffer zones. It is interesting that the IM code provides a more force-free magnetic field than the one in the model: the corresponding values of $\theta$ and $\theta_j$ in Table 10 are systematically lower than those in Table 1. This implies that the dynamics and the finite gas pressure in the coronal volume described by the MHD model produce a measurable deviation of the coronal magnetic field from the force-free state, while the NLFFF optimization drives the magnetic data cube toward another, more force-free, solution. Nevertheless, the extrapolated field is reasonably close to the model one; see the $\theta_m$ and $\theta_{mj}$ metrics. In contrast, the field restored with the AS code is less force-free than the model one, which is likely a negative effect of the fixed top and side boundary conditions and the buffer zone employed in this method.

Likewise in the preprocessing tests, we are looking whether the restored values of the magnetic field components (or the absolute values) correlate with the model ones and what the scatter is around the cross-correlation curves. Figures 9–11, two left columns, display these cross-correlation plots in the form of 2D histograms superimposed on the $y = x$ diagonal line. These plots suggest that the performance of the extrapolations improves for higher binning factors; in particular, the “clouds” in the area of poorer restored weak field values (around zero) are bigger for the smaller binning factors. Comparison between the two methods suggests that the IM approach works better for small magnetic field values, while for a large magnetic field, both methods perform comparably, although sometimes the AS method performs marginally better. We will return to this comparison later.

Figures 12 and 13 give a clear visual idea of the optimization code performance in three layers: one close to the bottom of the cube, another one in a middle height, and the last one close to the top buffer zone in the AS code (for a fair comparison of the methods, we exclude the same buffer zone in both AS and IM cases); see animated figure for all layers. The error (second and fourth columns) of the magnetic field reconstruction slightly increases with height: although the scale of the error variation is almost the same at all of the three presented levels, $-2 \, \text{G} \lesssim \Delta_z \lesssim 2 \, \text{G}$, the areas occupied by blue or red colors...
indicating bigger errors) increase with height. As a result, the relative error (third and fifth columns) increases with height noticeably: the green area, where the relative error is within ±10%, becomes smaller with height. The relative error becomes large at neutral lines (i.e., where the magnetic field is about zero) at any layer, even at the one closest to the bottom.

At low heights, the two competing methods perform comparably well: although the AS method shows a smaller error in the middle of the plot where the magnetic field is reasonably large, it also produces some artifacts close to the boundaries, which is not surprising given that the method employs a buffer zone at the boundary regions. It is, however, interesting, that at intermediate heights the AS method provides a bigger green area in the plot, where the relative error of the field reconstruction is within 10%, than the IM method, although the IM method outperforms the AS one at higher heights. A qualitatively similar picture is observed for two other components of the magnetic field: $B_y$ (Figure 13) and $B_z$ (not shown).

The fact that we created our model data cubes by rebinning the original data cube implies that the magnetic field value $B_i$ in a given voxel is only known to an accuracy of $B_i \pm \delta B_i$, defined by Equations (1) and (2); thus, even the most precise field reconstruction would only recover the field to this accuracy. Stated another way, an ideally perfect reconstruction would have $\chi^2_{\alpha} [ i ] \sim 1$ and $\chi^2_{\text{eff,} \alpha} \sim 1$. We evaluated the $\chi^2_{\text{eff,} \alpha}$ metrics for our extrapolation data cubes and found them to be much larger than 1 in all cases. Inspection of the inputs used to compute the $\chi^2_{\text{eff,} \alpha}$ metric shows that the $\delta B_i$ from Equation (2) are very small, implying that the magnetic field in the model volume is known with a very high accuracy even after rebinning. No reconstruction can be performed with such high accuracy; so the formal $\chi^2$ test fails.

However, the reconstruction often provides an accuracy of 10%–30% (see Figures 12, 13), which is fully acceptable for most practical applications, even though it is not as perfect as the accuracy of the original model field. To further quantify that, Figures 14(a) and (b) show the height dependence for the normalized rms error of the absolute value and all components of the magnetic field for a representative set of three different binnings. A few observations can be made from this figure. First, the performance of both methods improves from small to large bin factors. Given that Figures 12 and 13 contain domains

Figure 18. 2D histograms of the $B_i$ reconstruction obtained using two methods, IM and AS, from the $\beta$-photosphere with preprocessing, in 3D volume. The buffer zone is discarded everywhere.
with local errors larger than 70%, the rms error can be rather large, especially at greater heights. Second, reconstruction of the $B_y$ component is not as good as for the $B_x$ or $B_z$ components. This is an outcome of the asymmetry of the original model, in which the $B_y$ values are systematically smaller than other components. Third, the absolute value of the magnetic field is recovered much better than any single component of the field, which looks unexpected, but as we show below, this is an outcome of the large errors coming from the weak field contributions. Finally, we find that the IM code works on average better than the AS code, while extrapolation is being made starting from the chromospheric magnetogram; see Table 11 for a level-by-level comparison of the normalized rms error in the case of bin $= 9$. We believe that this is a direct outcome of taking into account the equations for the force-free field boundaries of the modeling cube: having an almost force-free field bottom boundary condition at the chromospheric level is more consistent with force-free boundaries (IM) than with the buffer zone with the potential boundaries (AS).

The absolute values of the normalized rms errors are rather large, which is the outcome of the large errors recovering small (close to zero) values of the field components. To explicitly demonstrate this, we turn to the normalized rms residual, which is weighted with the strong field contributions. Figures 15(a) and (b) display the height dependence of the normalized rms residuals for the IM and AS codes respectively; see Table 12 for a level-by-level comparison of the normalized rms residual in the case of bin $= 9$. This plot and the table confirm that the magnetic field is recovered more accurately in the voxels with the large magnetic field. In particular, the absolute value of the field and its components are now restored with comparable accuracies (although the transverse components often display a bigger error than the longitudinal component). This metric only slightly depends on the grid resolution because the most numerous voxels with a weak field (which are more numerous for higher-resolution data cubes) only have a weak contribution to this metric. The two methods recover the absolute value and $B_z$ component comparably well, but the IM code outperforms the AS code in recovering the transverse components (especially $B_x$). In all cases, the normalized rms residuals for the absolute value $|B|$ and the vertical component $B_z$ are within 20% while for the transverse components, these are within 40%. Overall, we can conclude that these extrapolations from the force-free chromospheric level work rather well.

Figure 19. 2D histograms of the $B_y$ reconstruction obtained using two methods, IM and AS, from the photosphere with preprocessing, in 3D volume. The buffer zone is discarded everywhere.
5.2.2. NLFFF Extrapolations from the $\beta$-photospheric Boundary

For the extrapolation from the $\beta$-photosphere boundary (without preprocessing) we computed the same metrics as for the chromospheric case. The angular metrics in Table 10 generally show the same trends as for the chromosphere, but with larger values of the angles, which is expected. Again, the IM code often produces more force-free data cube than the model one. Figures 9–11(c) and (d) display the cross-correlation between the restored and model fields in the volume above the $\beta$-photosphere (the buffer zone excluded). Although the scatter around the $y = x$ diagonal is larger than in the chromospheric case, the distribution follows the $y = x$ dependence remarkably well for all components of the magnetic field. This is highly important because this tells us that NLFFF extrapolations, even when starting from a non-force-free photospheric boundary, preserve the correct height scale, which is essential for all tasks that include the model-to-data comparison.

Not surprisingly, the normalized rms error and residual are bigger, while extrapolating from the photosphere compared with the chromospheric case. In particular, Figures 15(c) and (d) show that the normalized rms residual metrics for the photospheric case are roughly a factor of two to three larger than those for the chromospheric case. A qualitative difference between the former and the latter is that now we see a relatively big jump at the curves just above the bottom boundary. This jump is an immediate result from the conflict between the forced photospheric boundary, which is not allowed to change, and the implied force-free condition above this level: in fact, a few more layers above the $\beta$-photosphere are also not force-free; thus, the restored field noticeably deviates from the model one there. Figure 16 gives a clear visual idea of the optimization code performance in three layers: one close to the bottom of the cube, another one in a middle height, and the last one close to the top buffer zone in the AS code (for a fair comparison of the methods, we exclude the same buffer zone in both AS and IM cases); see the animated figure for all layers.

Another interesting observation is that here the AS method works better than the IM one for the absolute value $|B|$ and the vertical component $B_z$, although not as well for the transverse components, especially for $B_x$, while the $B_y$ component is recovered comparatively imperfectly by both methods. This mismatch in the accuracy of the restoration of various components of the magnetic field is a likely cause of the large angle error in the AS extrapolation code. We conclude that the extrapolation from the ($\beta$-)photospheric level works acceptably well for many practical applications, even though measurably less perfect than the extrapolation from the chromospheric
| Level, Mm | IM   | AS   | IM   | AS   | IM   | AS   | IM   | AS   | IM   | AS   | IM   | AS   | IM   | AS   | IM   | AS   |
|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1.29     | 11.74| 11.74| 18.52| 18.52| 49.39| 49.39| 59.91| 59.91| 62.80| 62.80| 43.53| 43.53| 56.00| 56.00|     |      |
| 1.71     | 10.20| 10.62| 15.88| 16.19| 57.55| 60.68| 63.15| 63.47| 61.15| 64.76| 42.61| 40.57| 49.65| 47.24|     |      |
| 2.14     | 9.26 | 11.65| 14.66| 17.41| 57.71| 63.22| 66.38| 72.56| 63.11| 128.83| 40.51| 42.27| 47.20| 53.71|     |      |
| 2.57     | 8.86 | 12.26| 13.99| 18.46| 55.86| 62.28| 68.84| 71.48| 66.53| 174.11| 37.28| 42.30| 43.73| 52.01|     |      |
| 3.00     | 8.69 | 12.49| 13.30| 18.31| 59.76| 64.16| 73.87| 67.36| 86.77| 244.61| 36.86| 43.26| 45.01| 45.72|     |      |
| 3.43     | 8.79 | 12.83| 12.71| 18.49| 63.75| 67.47| 79.79| 71.74| 115.00| 295.16| 42.79| 58.30| 54.18| 57.00|     |      |
| 3.86     | 9.24 | 12.91| 12.32| 18.34| 68.76| 76.11| 87.46| 74.34| 119.52| 238.25| 44.34| 58.69| 57.04| 56.78|     |      |
| 4.29     | 9.81 | 13.12| 11.95| 18.28| 77.17| 86.33| 101.38| 84.83| 119.39| 192.68| 48.39| 78.24| 59.27| 78.25|     |      |
| 4.71     | 10.40| 13.17| 11.42| 18.37| 92.92| 98.20| 125.24| 95.05| 120.45| 193.75| 51.69| 75.26| 57.69| 71.12|     |      |
| 5.14     | 11.06| 13.13| 10.92| 18.17| 86.43| 105.57| 119.66| 101.92| 136.51| 237.17| 58.54| 82.58| 58.61| 84.14|     |      |
| 5.57     | 11.84| 13.23| 10.69| 18.36| 82.65| 117.65| 116.60| 112.03| 173.13| 268.30| 70.34| 86.36| 64.69| 89.18|     |      |
| 6.00     | 12.71| 13.46| 10.75| 18.53| 76.01| 120.13| 109.49| 115.10| 206.64| 285.48| 76.77| 88.02| 69.53| 92.58|     |      |
| 6.43     | 13.74| 13.87| 11.15| 19.07| 78.13| 133.96| 106.72| 129.77| 243.42| 312.53| 77.84| 89.20| 71.81| 95.07|     |      |
| 6.86     | 14.67| 14.01| 11.54| 19.34| 84.33| 146.48| 104.80| 141.80| 210.74| 263.12| 78.33| 97.72| 74.96| 103.99|     |      |
| 7.29     | 15.73| 14.64| 12.25| 20.48| 87.35| 158.28| 100.91| 154.88| 225.33| 266.84| 79.19| 105.43| 74.85| 113.87|     |      |
| 7.71     | 16.96| 15.73| 13.23| 21.98| 92.34| 169.14| 108.47| 166.54| 254.39| 284.09| 86.63| 122.39| 77.93| 132.37|     |      |
| 8.14     | 18.36| 16.89| 14.46| 23.76| 95.86| 179.78| 111.08| 179.02| 274.37| 302.49| 101.12| 144.44| 84.03| 157.78|     |      |
| 8.57     | 19.88| 18.29| 15.87| 25.53| 100.29| 200.13| 113.97| 189.64| 287.52| 330.73| 115.77| 162.48| 90.98| 177.24|     |      |
| 9.00     | 21.59| 19.83| 17.45| 27.68| 106.53| 200.27| 121.84| 200.94| 306.10| 355.13| 116.86| 160.11| 89.20| 175.69|     |      |
| 9.43     | 23.76| 21.92| 19.41| 30.11| 112.49| 206.55| 129.09| 208.11| 328.16| 374.39| 123.33| 163.78| 90.70| 179.46|     |      |
| 9.86     | 26.38| 24.33| 21.73| 33.08| 119.47| 211.15| 135.16| 215.00| 325.05| 380.54| 143.46| 180.35| 104.73| 198.94|     |      |
| 10.29    | 29.27| 26.70| 24.18| 35.67| 127.75| 215.23| 143.72| 229.97| 303.74| 356.55| 168.32| 198.96| 123.37| 219.53|     |      |
| 10.71    | 32.69| 29.16| 27.12| 38.61| 136.22| 224.39| 164.62| 233.03| 321.87| 380.88| 204.36| 219.73| 157.59| 243.95|     |      |
| 11.14    | 36.57| 31.54| 30.78| 41.03| 150.26| 233.82| 196.81| 246.27| 330.97| 402.63| 248.93| 243.80| 202.33| 270.82|     |      |
level; see Tables 11 and 12 for the level-by-level comparisons of the normalized rms error and residual in the case of bin = 9.

5.2.3. NLFFF Extrapolations from a Preprocessed Boundary

Finally, we tested extrapolations from the photospheric level after its preprocessing; specifically, we used both preprocessed methods for both IM and AS extrapolation codes. Now, unlike the cases without preprocessing, we include the bottom (preprocessed) layer in the metrics computation given that it was modified compared to the original model field. Table 10 shows that there is no coherent behavior of the metrics versus model grid resolution. Sometimes, but not always, the overall field becomes more force-free than without preprocessing (better θ metric), while becoming less force-free in the subvolume with a strong electric field (worse θ metric).

For the IM code, the use of any preprocessing improves agreement between the restored and model field for smaller binning factors (higher grid resolution): the TW preprocessing works better for bins 3 and 4, while the JF one works better for larger bin factors, where the extrapolation with the preprocessor shows a marginal improvement (if any) compared with the extrapolation without preprocessing.

However, the price we pay for these marginally improved angular metrics is the corrupted height scale, which is vividly demonstrated by Figures 17–19. Specifically, the magnetic field extrapolated from the JF preprocessed boundary condition shows a nice y = x pattern for the Bz component, while it deviates from that for both Bx and By components, which underestimate the magnetic field in the volume. In contrast, the use of the TW preprocessed boundary condition overestimates the magnetic field components, but not equally: the Bz component is clearly more strongly overestimated than the transverse ones, which almost follow the y = x regression in some cases.

Figure 20 reveals one more problem with the field reconstruction starting from a preprocessed boundary: the field restoration error is large starting from the very bottom layer with the normalized rms residual exceeding 10%–15% in many cases, especially for the case with TW preprocessing. We believe that this happens because the magnetic field becomes overly smooth due to the preprocessing (this smoothing is stronger for the TW preprocessing—see Section 4 and specifically, Figure 8; the amount of smoothing can be reduced by lowering the μd parameter; however, it is well beyond our study to optimize the choice of the preprocessing parameters).

At somewhat higher layers, where the field in the model is itself smooth, the metric decreases and thus, the field is more accurately reconstructed at intermediate heights compared with low heights; see Tables 13 and 14 for quantitative level-by-level comparisons of the normalized rms error and residual in the case of bin = 9 reconstruction from the preprocessed boundaries. Overall, the entire range of tests does not justify preprocessing the photospheric boundary given that a marginal or no improvement in the volumetric metrics is achieved at the expense of the height scale corruption.

6. Discussion and Conclusions

Here we have demonstrated that a realistic MHD model in the presented case—an en0240408 hion simulation (Carlsson et al. 2016) obtained with the Bifrost code (Gudiksen et al. 2011)—can be very efficiently used to cast various tools used for coronal magnetic field reconstruction. In particular, we have evaluated the performance of the π-disambiguation codes, magnetogram preprocessing codes,
and NLFFF extrapolation codes developed following the optimization method (Wheatland et al. 2000).

We have found that the currently used $\pi$-disambiguation codes work pretty well at the AR photosphere and chromosphere, but often fail at the quiet-Sun photosphere. This can become important when the question of the magnetic field at the quiet Sun is specifically addressed. Here we are primarily interested in the performance of the reconstruction tools in ARs; thus, we assumed that the $\pi$-ambiguity has been perfectly resolved.

Then, we have assessed the performance of two different preprocessing approaches aimed to improve the bottom boundary condition toward force-freeness. Although the tested preprocessing codes do produce a more force-free boundary, there is an unwelcome by-product of these preprocessings—the poorly controlled elevation of the magnetic vector components by exactly one voxel that ensure the consistent elevation of the magnetic vector.

On the other hand, comparison between the volumetric metrics of the magnetic data cubes extrapolated from the photospheric level either with or without preprocessing shows they are not much different from each other, while extrapolation without preprocessing preserves the correct height scale. From this perspective, we conclude that the use of NLFFF extrapolation from the actual photospheric magnetogram (without any preprocessing, at least when noise in the data does not represent a problem) is preferable. We have to note, however, that this conclusion is based only on the tests performed with the “standard” parameters $\mu_i$, $i = 1 \ldots 4$, specified for the two alternative preprocessing methods. It is very possible that there are combinations of the parameters $\mu_i$ that ensure the consistent elevation of the magnetic vector components by exactly one voxel (or another integer number of voxels) along with suppressing noise in the data, which might be helpful, but searching for such optimized combinations is clearly beyond the scope of this study.

Finally, we compared the results of NLFFF extrapolation using two different versions of the optimization method—one that uses the weighting function and the other that employs the full set of NLFFF equations (including the one for the top and side boundaries). Although we found the metrics of the two codes to be comparable, the codes that utilize the full set of the NLFFF equations still work systematically better than the one with the weighting function. We believe that this is because the use of the full set of equations is much more consistent with the assumption of field force-freeness than the presence of the boundary buffer zone with the potential boundary conditions at the top and side boundaries.

This work was supported in part by NSF grants AGS-1250374, AGS-1262772, and AST-1312802, and NASA grants NNX14AK66G, and NNX14AC87G to New Jersey Institute of Technology and RPBR grants 15-02-01077, 15-02-01089, 15-02-03717, 15-02-03835, 15-02-08028, 16-02-00254, 16-02-00749, and 16-32-00315. This study was supported by the Program of basic research of the RAS Presidium No. 9. The authors acknowledge the Marie Curie PIRSES-GA-2011-295272 RadioSun project.

References

Amar, T., Aly, J. J., Luciani, J. F., Boulmezaoud, T. Z., & Mikic, Z. 1997, SoPh, 174, 129
Ashchwanden, M. J. 2005, Physics of the Solar Corona. An Introduction with Problems and Solutions (2nd ed.; Chichester: Praxis)
Bogod, V. M., Stupishin, A. G., & Yasnov, L. V. 2012, SoPh, 276, 61
Carlsson, M., Hansteen, V. H., Gudiksen, B. V., Leenaarts, J., & De Pontieu, B. 2016, A&A, 585, A4
Crouch, A. D. 2013, SoPh, 282, 107
de la Cruz Rodríguez, J., De Pontieu, B., Carlsson, M., & Roupppe van der Voort, L. H. M. 2013, ApJL, 764, L11
De Rosa, M. L., Schrijver, C. J., Barnes, G., et al. 2009, ApJ, 696, 1780
DeRosa, M. L., Wheatland, M. S., Leka, K. D., et al. 2015, ApJL, 811, 107
Fuhrmann, M., Seehafer, N., & Valori, G. 2007, A&A, 476, 349
Fuhrmann, M., Seehafer, N., Valori, G., & Wiegelmann, T. 2011, A&A, 526, A70
Gary, G. A. 2001, SoPh, 203, 71
Georgoulis, M. K. 2005, ApJL, 629, L69
Gosain, S., & Petvtsov, A. A. 2013, SoPh, 283, 195
Gudiksen, B. V., Carlsson, M., Hansteen, V. H., et al. 2011, A&A, 531, A154
Jiang, C., & Feng, X. 2014, SoPh, 289, 63
Jiang, T., Yuan, Y., Wiegelmann, T., et al. 2010, ApJL, 719, L56
Kaltman, T. I., Bogod, V. M., Stupishin, A. G., & Yasnov, L. V. 2012, ApJ, 756, 790
Kaltman, T. I., Kochanov, A. A., Myshyakov, I. I., et al. 2015, Ge&Ae, 55, 1124
Leenaarts, J., Carlsson, M., & Roupppe van der Voort, L. 2012, ApJ, 749, 136
Leenaarts, J., Carlsson, M., & Roupppe van der Voort, L. 2015, ApJ, 802, 136
Leenaarts, J., Pereira, T. M. D., Carlsson, M., Uitenbroek, H., & De Pontieu, B. 2013a, ApJ, 772, 89
Leenaarts, J., Pereira, T. M. D., Carlsson, M., Uitenbroek, H., & De Pontieu, B. 2013b, ApJ, 772, 90
Leka, K. D., Barnes, G., Crouch, A. D., et al. 2009, SoPh, 260, 83
Lin, H.-H., & Carlsson, M. 2015, ApJ, 813, 34
Livshits, M. A., Grigoryeva, I. Y., Myshyakov, I. I., & Rudenko, G. V. 2016, ApR, 60, 939
Loutkitcheva, M., Solanki, S. K., Carlsson, M., & White, S. M. 2015a, A&A, 575, A15
Loutkitcheva, M., Solanki, S. K., White, S. M., & Carlsson, M. 2015b, in ASP Conf. Ser. 499, Revolution in Astronomy with ALMA: The Third Year, ed. D. Jono et al. (San Francisco, CA: ASP), 349
Low, B. C., & Lou, Y. Q. 1990, ApJ, 352, 343
Metcalfe, T. R. 1994, SoPh, 155, 235
Metcalfe, T. R., Leka, K. D., Barnes, G., et al. 2006, SoPh, 237, 267
Metcalfe, T. R., De Rosa, M. L., Schrijver, C. J., et al. 2008, SoPh, 247, 269
Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. 1953, JCP, 21, 1087
Pereira, T. M. D., Carlsson, M., De Pontieu, B., & Hansteen, V. 2015, ApJ, 806, 14
Pereira, T. M. D., Leenaarts, J., De Pontieu, B., Carlsson, M., & Uitenbroek, H. 2013, ApJ, 778, 143
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 2007, Numerical Recipes: The Art of Scientific Computing (3rd ed.; New York: Cambridge Univ Press)
Rathore, B., & Carlsson, M. 2015, ApJL, 811, 80
Rathore, B., Carlsson, M., Leenaarts, J., & De Pontieu, B. 2015, ApJ, 811, 81
Rudenko, G. V., & Anfinogentov, S. A. 2014, SoPh, 289, 1499
Rudenko, G. V., & Myshyakov, I. I. 2009, SoPh, 257, 287
Sakurai, T. 1989, SSRv, 51, 11
Schrijver, C. J., De Rosa, M. L., Metcalf, T. R., et al. 2006, SoPh, 235, 161
Séspán, J., Trujillo Bueno, J., Carlsson, M., & Leenaarts, J. 2012, ApJL, 758, L43
Séspán, J., Trujillo Bueno, J., Leenaarts, J., & Carlsson, M. 2015, ApJ, 803, 65
Wheatland, M. S., Sturrock, P. A., & Roumeliotis, G. 2000, ApJ, 540, 1150
Wiegelmann, T. 2004, SoPh, 219, 87
Wiegelmann, T. 2008, IJGRA, 113, A03S02
Wiegelmann, T., & Inhester, B. 2010, A&A, 516, A107
Wiegelmann, T., Inhester, B., & Sakurai, T. 2006, SoPh, 233, 215
Wiegelmann, T., & Sakurai, T. 2012, LRR, 9, 5
Wiegelmann, T., Thalmann, J. K., Inhester, B., et al. 2012, SoPh, 281, 37
Wiegelmann, T., Thalmann, J. K., & Solanki, S. K. 2014, A&AReVs, 22, 78
Yasnov, L. V., Karlicky, M., & Stupishin, A. G. 2016, SoPh, 291, 2037