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Bimodal behavior of post-measured entropy and one-way quantum deficit for two-qubit X states

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Abstract A method for calculating the one-way quantum deficit is developed. It involves a careful study of post-measured entropy shapes. We discovered that in some regions of X-state space the post-measured entropy $\tilde{S}$ as a function of measurement angle $\theta \in [0, \pi/2]$ exhibits a bimodal behavior inside the open interval $(0, \pi/2)$, i.e., it has two interior extrema: one minimum and one maximum. Furthermore, cases are found when the interior minimum of such a bimodal function $\tilde{S}(\theta)$ is less than that one at the endpoint $\theta = 0$ or $\pi/2$. This leads to the formation of a boundary between the phases of one-way quantum deficit via finite jumps of optimal measured angle from the endpoint to the interior minimum. Phase diagram is built up for a two-parameter family of X states. The subregions with variable optimal measured angle are around 1% of the total region, with their relative linear sizes achieving 17.5%, and the fidelity between the states of those subregions can be reduced to $F = 0.968$. In addition, a correction to the one-way deficit due to the interior minimum can achieve 2.3%. Such conditions are favorable to detect the subregions with variable optimal measured angle of one-way quantum deficit in an experiment.

Keywords X density matrix · Post-measured entropy · Unimodal and bimodal functions · One-way quantum deficit

1 Introduction

Quantum correlation is a key feature of quantum mechanics and it lies at the heart of quantum information science. Besides the quantum entanglement and discord, the one-way quantum deficit is one of the most important measures of quantum correlation [1][2][3][4]. The entanglement is identical to the discord
Definitions of quantum discord $Q$ and one-way quantum deficit $\Delta$ involve the minimization procedure to obtain the optimal measurement performed on one part of bipartite system. This procedure for the two-qubit systems with X density matrix is reduced to the minimization problem on one variable – the polar angle $\theta \in [0, \pi/2]$ (see Refs. [6,7,8,9]). Moreover, a formula for the quantum discord is presented in a partially analytic (piecewise-analytical-numerical) form \cite{10,11,12},

$$Q = \min \{ Q_0, Q_\vartheta, Q_{\pi/2} \}.$$  \hspace{1cm} (1)

Here, the subfunctions (branches) $Q_0$ and $Q_{\pi/2}$ are the analytical expressions (corresponding to the discord with optimal measurement angles equaling zero and $\pi/2$, respectively) and only the third branch $Q_\vartheta$ requires to perform numerical minimization to obtain state-dependent minimizing angle $\vartheta \in (0, \pi/2)$ if, of course, the interior minimum exists. Equations for $0$- and $\pi/2$-boundaries separating respectively the $Q_0$ and $Q_{\pi/2}$ regions with the $Q_\vartheta$ one can be written as \cite{10,11,12}.

$$Q''(0) = 0, \quad Q''(\pi/2) = 0.$$  \hspace{1cm} (2)

Here $Q''(0)$ and $Q''(\pi/2)$ are the second derivatives of the measurement-dependent discord function $Q(\theta)$ with respect to $\theta$ at the endpoints $\theta = 0$ and $\pi/2$, correspondingly. The equations (2) are based on the unimodality hypothesis for the function $Q(\theta)$ which is confirmed for different classes of X states \cite{12,13}. Notice that Eqs. (2) reflect the bifurcation mechanism of appearance of the minimum inside the interval $(0, \pi/2)$.

On the other hand, as mentioned above, there is a close connection between the one-way quantum deficit and quantum discord. Therefore it would be tempting to propose that similar properties are valid for the measurement-dependent one-way quantum deficit function $\Delta(\theta) = \tilde{S}(\theta) - S$, where $S$ is the pre-measurement entropy.

Recently, the authors \cite{14} have claimed the result which is reduced to the statement that the one-way quantum deficit $\Delta = \min_{\theta} \Delta(\theta)$ for the general X states is given by

$$\Delta = \begin{cases} 
\Delta(\theta), & \Delta''(0) < 0 \text{ and } \Delta''(\pi/2) < 0, \ \vartheta \in (0, \pi/2); \\
\min \{ \Delta(0), \Delta(\pi/2) \}, & \text{others.}
\end{cases}$$  \hspace{1cm} (3)

If the function $\Delta(\theta)$ is monotonic or has single extremum inside the interval $(0, \pi/2)$ this conclusion takes place.

In the present paper we show that the post-measured entropy and consequently the measurement-dependent one-way quantum deficit can display more general behavior which refutes the relation (3). We discuss the difficulties arisen from a new type of behavior and propose, instead of Eq. (3), the method giving the correct calculation of one-way deficit for two-qubit X states.
2 Results and discussion

Let us consider a two-parameter family of X states

$$\rho_{AB} = q_1 |\Psi^+\rangle \langle \Psi^+| + q_2 |\Psi^-\rangle \langle \Psi^-| + (1-q_1-q_2)|00\rangle \langle 00|,$$

(4)

where $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$. This family generalizes the class of special X states from Ref. [14] which corresponds to the case $q_1 = 0$.

The density matrix (4) in open form is given as

$$\rho_{AB} = \begin{pmatrix}
1 - q_1 - q_2 & 0 & 0 & 0 \\
0 & (q_1 + q_2)/2 & (q_1 - q_2)/2 & 0 \\
0 & (q_1 - q_2)/2 & (q_1 + q_2)/2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.$$  

(5)

Eigenvalues of this matrix equal

$$\lambda_1 = 1 - q_1 - q_2, \quad \lambda_2 = q_1, \quad \lambda_3 = q_2, \quad \lambda_4 = 0.$$

(6)

Owing to the non-negativity requirement for any density matrix, one obtains that the domain of definition for the parameters (arguments) $q_1$ and $q_2$ is restricted by conditions

$$q_1 \geq 0, \quad q_2 \geq 0, \quad q_1 + q_2 \leq 1.$$

(7)

Thus, the domain in plane $(q_1, q_2)$ is the triangle $T$ which is shown in Fig. 1.

One-way quantum deficit (quantum work deficit) for a bipartite state $\rho_{AB}$ is defined as the minimal increase of entropy after a von Neumann measurement on one party (without loss of generality, say, $B$) [15,16,17]

$$\Delta = \min_{\{\Pi_k\}} S(\bar{\rho}_{AB}) - S(\rho_{AB}),$$

(8)

where

$$\bar{\rho}_{AB} = \sum_k (I \otimes \Pi_k) \rho_{AB} (I \otimes \Pi_k)^+$$

(9)

is the weighted average of post-measured states and $S(\cdot)$ means the von Neumann entropy. In Eqs. (5) and (6), $\Pi_k (k = 0, 1)$ are the general orthogonal projectors

$$\Pi_k = V \pi_k V^+,$$

(10)

where $\pi_k = |k\rangle \langle k|$ and transformations $\{V\}$ belong to the special unitary group $SU_2$. Rotations $V$ may be parametrized by two angles $\theta$ and $\phi$ (polar and azimuthal, respectively):

$$V = \begin{pmatrix}
\cos(\theta/2) & -e^{-i\phi} \sin(\theta/2) \\
e^{i\phi} \sin(\theta/2) & \cos(\theta/2)
\end{pmatrix}$$

(11)

with $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$.

Using Eq. (6) one gets the pre-measured entropy

$$S(q_1, q_2) = S(\rho_{AB}) = -q_1 \log q_1 - q_2 \log q_2 - (1-q_1-q_2) \log (1-q_1-q_2).$$

(12)
Fig. 1 Triangle $T$ in the plane $(q_1, q_2)$ with vertices $(0, 0), (0, 1),$ and $(1, 0)$ is the permitted region for the parameters $q_1$ and $q_2$. Dotted lines 1 and 1' are the boundaries defined by the equation $\Delta_0 = \Delta_{\pi/2}$. Solid lines 2 and 2' are the $\pi/2$-boundaries. Dotted line 3 is the path $q_1 + q_2 = 0.75$. Crosses ($\times$) at the points $(0, 0.5)$ and $(0.5, 0)$ mark the 0-boundaries.

Eigenvalues of the matrix $\tilde{\rho}_{AB}$ are equal to

$$A_{1,2} = \frac{1}{4} \left[ 1 + (1 - q_1 - q_2) \cos \theta \pm \left[ (1 - q_1 - q_2) \cos \theta \right]^2 + (q_1 - q_2)^2 \sin^2 \theta \right]^{1/2}$$

$$A_{3,4} = \frac{1}{4} \left[ 1 - (1 - q_1 - q_2) \cos \theta \pm \left[ (1 - q_1 - q_2 - 2q_1 - 2q_2 - \sin \theta) \right]^{1/2} + (q_1 - q_2)^2 \sin^2 \theta \right].$$

It is seen that the azimuthal angle $\phi$ has dropped out from the given expressions. This is due to the fact that one pair of non-diagonal entries of the density matrix (5) vanishes. Using Eqs. (13) we arrive at the post-measured entropy (entropy after measurement)

$$\tilde{S}(\theta; q_1, q_2) \equiv S(\tilde{\rho}_{AB}) = h_4(A_1, A_2, A_3, A_4),$$

where $h_4(x_1, x_2, x_3, x_4) = -\sum_{i=1}^4 x_i \log x_i$ with additional condition $x_1 + x_2 + x_3 + x_4 = 1$ is the quaternary entropy function.

Notice that function $\tilde{S}$ of argument $\theta$ is invariant under the transformation $\theta \to \pi - \theta$ therefore it is enough to restrict oneself by values of $\theta \in [0, \pi/2]$. Moreover, the pre- and post-measured entropies $S$ and $\tilde{S}$, as functions of $q_1$ and $q_2$, are symmetric under the exchange $q_1 \leftrightarrow q_2$. 
Equations (12)–(14) define the measurement-dependent one-way deficit function \( \Delta(\theta) = \tilde{S}(\theta) - S \). Direct calculations show that for every choice of model parameters the function \( \tilde{S}(\theta) \) and hence \( \Delta(\theta) \) possess an important property, namely their first derivatives with respect to \( \theta \) identically equal zero at both endpoints \( \theta = 0 \) and \( \theta = \pi/2 \):

\[
\tilde{S}'(0) = \Delta'(0) \equiv 0, \quad \tilde{S}'(\pi/2) = \Delta'(\pi/2) \equiv 0.
\] (15)

From Eqs. (13) and (14) we get the expressions for the post-measurement entropy at the endpoint \( \theta = 0 \),

\[
\tilde{S}_0(q_1, q_2) = -(1 - q_1 - q_2) \log(1 - q_1 - q_2) - (q_1 + q_2) \log[(q_1 + q_2)/2],
\] (16)

and at the second endpoint \( \theta = \pi/2 \):

\[
\tilde{S}_{\pi/2}(q_1, q_2) = \log 2 + h\left(\frac{1}{1 - 2 q_1 - 2 q_2} \right),
\] (17)

where \( h(x) = -x \log x - (1 - x) \log(1 - x) \) is the Shannon binary entropy function. Together with Eq. (12) they supply us with explicit expressions for the one-way deficit at the endpoints: \( \Delta_0 = \Delta(0) \) and \( \Delta_{\pi/2} = \Delta(\pi/2) \). In particular, if \( q_1 \) or \( q_2 \) equals zero then \( \Delta_0 = q \log 2 \) (= \( q \), bit), where \( q = \{q_1, q_2\} \).

Solving the transcendental equation

\[
\Delta_0 = \Delta_{\pi/2}
\] (18)

or, the same, \( \tilde{S}_0 = \tilde{S}_{\pi/2} \) we find the subregions in the plane \( (q_1, q_2) \), where \( \Delta_{\pi/2} < \Delta_0 \) (restricted in Fig. 1 by dotted curves 1 and 1’ and corresponding Cartesian axes \( Oq_1 \) and \( Oq_2 \)) and where, v.v., \( \Delta_0 < \Delta_{\pi/2} \) (marked in Fig. 1 by symbol \( \Delta_0 \)). The curve 1 has two endpoints on the axis \( Oq_1 \): at \( q_1 = 0.61554 \) and \( q_1 = 1 \). Analogously for the curve 1’ (see Fig. 1).

The 0- and \( \pi/2 \)-boundaries, i.e., where respectively the second derivatives

\[
\Delta''(0) = 0 \quad \text{and} \quad \Delta''(\pi/2) = 0
\] (19)

or, the same, \( \tilde{S}''(0) = 0 \) and \( \tilde{S}''(\pi/2) = 0 \), will be needed below. As calculations yield,

\[
\tilde{S}''(\pi/2) = \frac{(q_1 - q_2)^2}{2r^3} \left[ r^2 - (1 - 2q_1 - 2q_2)^2 \right] \ln \frac{1 + r}{1 - r},
\]

\[
\frac{1 - q_1 - q_2}{1 - r^2} \left[ 1 - 2(1 - 2q_1 - 2q_2)(1 - \frac{1 - 2q_1 - 2q_2}{2r^2}) \right],
\] (20)

where

\[
r = \sqrt{(1 - q_1 - q_2)^2 + (q_1 - q_2)^2}.
\] (21)

On the other hand, calculations show that the second derivative \( \tilde{S}''(\theta) \) with respect to \( \theta \) is finite at \( \theta = 0 \) only when \( q_1 q_2 = 0 \):

\[
\tilde{S}''(0) = \frac{1 - 3q + 2q^2}{2 - 3q} \ln \frac{2(1 - q)}{q},
\] (22)
where again $q = \{q_1, q_2\}$. The roots of equation $\tilde{S}''(0) = 0$ are $1/2$ and $1$. Thus, the bifurcation 0-boundary exists only if $q_1 = 0$ or, inversely, $q_2 = 0$ (that is, only at two points on each of the Cartesian axes $Oq_1$ and $Oq_2$). The corresponding 0-boundaries $q_1 = 1/2$, when $q_2 = 0$, and $q_2 = 1/2$, when $q_1 = 0$ are shown in Fig. 1 by the crosses.

The results of numerical solution of the equation $\tilde{S}''(\pi/2) = 0$ are presented in Fig. 1 by solid lines 2 and 2'. The endpoints for the curve 2 on the axis $Oq_1$ are $q_1 = 0.67515$ and $q_1 = 1$. The curves 1 and 2 intersect at the point with coordinates $q_1 = 0.739409$ and $q_2 = 0.029686$ ($q_1 + q_2 = 0.769095$). Analogously for the curves 1' and 2' with, of course, permutation of $q_1$ and $q_2$ (see again Fig. 1).

Let us consider the behavior of post-measured entropy $\tilde{S}(\theta)$ and non-minimized one-way deficit $\Delta(\theta)$ by moving along different trajectories (paths) in the triangle $T$.

Start with the passing along the leg of triangle $T$. Figure 2 shows the evolution of shape of the post-measured entropy $\tilde{S}(\theta; q_1, 0)$ with changing the

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**Fig. 2** Post-measurement entropy $\tilde{S}$ vs $\theta$ by $q_2 = 0$ and $q_1 = 0.5$ (a), $0.55$ (b), $0.65$ (c), and $0.7$ (d).
Fig. 3 Measurement-dependent one-way quantum deficit $\Delta(\theta)$ along the line $q_1 + q_2 = 0.75$ by $q_1 = 0.72$ (1), 0.72015 (2), and 0.7205 (3). The bimodality appearing from an inflection point is clearly seen.

parameter $q_1$. The curve has the monotonically increasing behavior when the argument $q_1$ varies from $q_1 = 0$ to $q_1 = 1/2$; see Fig. 2(a). At the point $q_1 = 1/2$ a bifurcation of the minimum at $\theta = 0$ occurs. Then, when $q_1$ increases from 0.5 to 0.67515, the curve $\tilde{S}(\theta)$ has, as shown in Figs. 2(b) and (c), the interior minimum, with the function $\tilde{S}(\theta)$ being here unimodal. So, the region with variable optimal angle $\vartheta$ takes up a part $0.17515 \approx 17.5\%$ on the section $[0, 1]$ of $Oq_1$ axis and the fidelity of states at points $(0.5, 0)$ and $(0.67515, 0)$ is equal to $F = 96.8\%$. The position of such a local minimum smoothly increases from zero to $\pi/2$; see again the curves in Figs. 2(b) and (c). The values of $\tilde{S}_0$ and $\tilde{S}_{\pi/2}$ become equal at the point $q_1 = 0.61554$ (i.e., $\tilde{S}_0 = \tilde{S}_{\pi/2} = 5.7667$ bit), hence $\Delta_{\pi/2} = \Delta_0 = q_1 = 0.61554$ bit) and the depth of interior minimum is 0.01397 bit what gives a relative correction to the one-way deficit equaled $\delta\Delta = 2.4\%$. Then, at the value of $q_1 = 0.67515$, the system experiences a new sudden transition – from the branch, which is characterized by the continuously changing optimal angle $\vartheta$ in the full interval (from 0 to $\pi/2$), to the branch $\tilde{S}_{\pi/2}$ with constant optimal measurement angle equaled $\pi/2$. After this the curves of post-measured entropy exhibit monotonically decreasing behavior as illustrated in Fig. 2(d). One should emphasize here that the minimized one-way quantum deficit, $\Delta = \min_{\theta} \Delta(\theta)$, vs the model parameter $q_1$ is continuous and smooth. Nevertheless, the function $\Delta(q_1)$ has nonanalyticities at the points $q = 0.5$ and 0.67515 which manifest themselves in higher derivatives.

1 Note for comparison that in two-photon experiments one achieves now the values of fidelity $F = 99.8(2)\%$ [18] and $F = 99.8(1)\%$ [19].
Consider now the behavior of post-measurement entropy and measurement-dependent one-way deficit in the bulk area of $T$. We can inspect the total domain taking all possible straight-line trajectories $q_1 + q_2 = const \leq 1$. The behavior of the system is, obviously, symmetric relative to the middle of such trajectories. Take, for instance, the trajectory $q_1 + q_2 = 0.75$ which is shown in Fig. 1 by the straight line 3. The shape of the curve $\Delta(\theta)$ has the monotonically increasing type in the middle of this trajectory ($q_1 = q_2 = 0.375$). However, with the increase of the value of parameter $q_1$, the birth of a pair of extrema from an inflection point occurs inside the interval $(0, \pi/2)$; the situation is illustrated in Fig. 3. This phenomenon happens at the value of $q_1 = 0.72015$. According to the definition (see, e.g., Ref. [20]) a function having two extrema in some interval is called bimodal on this interval.

With further increase of the $q_1$ value a qualitatively new effect is observed. We demonstrate it by the curves $S(\theta)$ shown in Fig. 4. When the parameter $q_1$ achieves the value of 0.72159, the position of global minimum suddenly jumps through a finite step $\Delta\vartheta$ from zero to $\vartheta = 1.0409 \approx 60^\circ$ (see Fig. 4). As a result, the fracture is arisen on the continuous curve of minimized one-way quantum deficit $\Delta(q_1)$. The position of the fracture point is determined from the equation $\hat{S}_0 = \hat{S}_\vartheta$ or

$$\Delta_0 = \Delta_\vartheta. \quad (23)$$

After this the interior minimum lies lower than another minimum located at the endpoint $\theta = 0$. Notice that behavior of curve 3 in Fig. 4 leads to a contradiction with Eq. (23), i.e., the equation is incorrect for general X states.
Fig. 5  Measurement-dependent one-way quantum deficit $\Delta(\theta)$ along the line $q_1 + q_2 = 0.75$ by $q_1 = 0.722$ (a), 0.723 (b), 0.727 (c), and 0.75 (d). Minimum on the curve disappears at the endpoint $\theta = \pi/2$ through the bifurcation mechanism whereas the maximum annihilates at the endpoint $\theta = 0$ via the singularity mechanism.

With further increasing $q_1$ the interior minimum smoothly moves to the point $\theta = \pi/2$ and disappears at $q_1 = 0.72358$ when the trajectory crosses the curve 2, i.e., the $\pi/2$-boundary (see Fig. 4). The dynamics of corresponding deformations of $\Delta(\theta)$ is depicted in Fig. 5. After crossing the $\pi/2$-boundary, the behavior of $\Delta$ undergoes to the branch $\Delta_{\pi/2}$ up to the point of contact of trajectory with the Cartesian axis, i.e., up to $q_1 = 0.75$, where the interior maximum of $\Delta(\theta)$ disappears at the endpoint $\theta = 0$. This happens through a new non-bifurcation (and non-inflection) mechanism. Since the second derivative $\Delta''(\theta)$ at $\theta = 0$ diverges out of the Cartesian axes we will call this mechanism the singular one.

As a result, the one-way quantum deficit is obtained from the final equation

$$\Delta = \min\{\Delta_0, \Delta_\theta, \Delta_{\pi/2}\}, \quad (24)$$
Fig. 6 One-way quantum deficit $\Delta$ vs $q_1$ along the path $q_1 + q_2 = 0.75$ is shown by solid line. Dotted line corresponds to the branch $\Delta_{\pi/2}$. Fraction $\Delta_\vartheta$ with variable optimal measured angle lies between two arrows. The transition $\Delta_0 \leftrightarrow \Delta_\vartheta$ is displayed as a fracture on the curve $\Delta(q_1)$ whereas the $\Delta_{\vartheta} \leftrightarrow \Delta_{\pi/2}$ one is hidden — the curve is here continuous and smooth.

Table 1  Jumps of optimal measured angles, $\Delta\vartheta$, on the boundary between the phases $\Delta_0$ and $\Delta_{\pi/2}$

| $q_1$   | $q_2$   | $\Delta\vartheta$ |
|---------|---------|---------------------|
| 0.5     | 0       | 0 = 0°              |
| 0.544535| $0.55 - q_1$ | 0.1267 ≈ 7°          |
| 0.588104| $0.6 - q_1$ | 0.2470 ≈ 14°         |
| 0.631766| $0.65 - q_1$ | 0.4020 ≈ 23°         |
| 0.676082| $0.7 - q_1$ | 0.6252 ≈ 36°         |
| 0.721590| $0.75 - q_1$ | 1.0409 ≈ 60°         |
| 0.739409| 0.029686 | $\pi/2 = 90$°        |

where $\Delta_0$ and $\Delta_{\pi/2}$ are known in closed analytical forms and $\Delta_{\vartheta}$ is found numerically. The behavior of one-way deficit along the trajectory $q_1 + q_2 = 0.75$ is shown in Fig. 6.

Either totally or partially similar behavior takes place for other trajectories $q_1 + q_2 = \text{const}$ which go lower the intersection point of curves defined by equations $\Delta_0 = \Delta_{\pi/2}$ and $\Delta''(\pi/2) = 0$, i.e, when $\text{const} \leq 0.769095$. For example, in the case of trajectory $q_1 + q_2 = 0.65$, the bimodality appears at $q_1 \approx 0.631$ and a jump of optimal measurement angle from zero happens at $q_1 = 0.631766$. Values of jump angles $\Delta\vartheta$ in different cases are collected in Table 1.

A set of points where the optimal measurement angle discontinuously changes from zero to a finite value gives the jumping (or hopping) bound-
Fig. 7 A fragment of phase diagram. The boundary 1 is defined by equation $\Delta_0 = \Delta_\vartheta$, 2 is the $\pi/2$-boundary, and the boundary 3 is defined by equation $\Delta_0 = \Delta_{\pi/2}$. The black circle (●) is the intersection point of $\pi/2$-boundary with equilibrium curve of phases $\Delta_0$ and $\Delta_{\pi/2}$. (This figure represents a part of the domain of definition shown in Fig. 1).
Fig. 8  Dependence of $\Delta$ vs $q_1$ by $q_2 = 0.8 - q_1$. Arrow marks the position of a fracture at the point $q_1 = 0.769269$, where the one-way deficit undergoes from the branch $\Delta_0$ to the $\Delta_{\pi/2}$ one.

3 Summary and concluding remarks

In this paper we have found that besides the monotonic and unimodal behavior the post-measured entropy and hence the measurement-dependent one-way quantum deficit upon the measurement angle can have a new kind of behavior. Namely, these functions can exhibit the bimodal shape in the open interval $(0, \pi/2)$ for different regions in the space of X state parameters. This expands the variety of behavior for the one-way quantum deficit $\Delta$. In particular, a new state-dependent phase (fraction) which is characterized by a partial interval of optimal measured angles has been found. Instead of smooth conjugation of the branches $\Delta_0$ and $\Delta_{\pi/2}$ this leads to a fracture on the curve of one-way deficit.

New mechanism of a boundary arising between the phases via jumping the optimal measured angle on a finite step has been discovered. Instead of bifurcation conditions (19) the boundary is now determined by a relation like (23). The study of post-measured entropy shapes is the general way to determine the correct one-way quantum deficit.

This is in contrast with the behavior of conditional entropy and, consequently, measurement-dependent quantum discord in the same regions of parameter space: their behavior is restricted by monotonic and unimodal types. In any case, this rather simple and therefore attractive picture is valid for the different specific cases and subclasses of X states [10,12,13]. In particular, such a behavior of conditional entropy is confirmed for the symmetric XXZ states...
those may be written in an equivalent form as
\[
\rho_{AB} = q_1|\Psi^+\rangle\langle\Psi^+| + q_2|\Psi^-\rangle\langle\Psi^-| + q_3|00\rangle\langle00| + q_4|11\rangle\langle11|
\]
with \( q_1 + q_2 + q_3 + q_4 = 1 \).

An intriguing question remains: are there any more general shapes of curves for the post-measured entropy of X states? For instance, can this entropy have trimodal and, maybe, multimodal dependence? The answer to these and other questions should come from the future investigations of post-measurement entropy shapes in the full five-parameter X-state space.

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