Efficient routing on multilayered communication networks

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Abstract – We study the optimal routing on multilayered communication networks, which are composed of two layers of subnetworks. One is a wireless network, and the other is a wired network. We develop a simple recurrent algorithm to find an optimal routing on this kind of multilayered network, where the single-channel transmission mode and the multichannel transmission mode used on the wireless subnetwork are considered, respectively. Compared with the performance of the shortest path algorithm, our algorithm can significantly enhance the transport capacity. We show that our methods proposed in this letter could take advantage of the coupling of the two layers to the most extent, so that the wireless subnetwork could sufficiently utilize the wired subnetwork for transportation.

I. Introduction. – Transportation is an important problem in natural and engineering systems. The study of transportation optimization on complex networks has attracted a lot of interest in the past decade [1, 2]. These studies include the influence of network structure on the transportation [3–7], features of different transportation models [8–13], and offer strategies to improve transportation from different viewpoints [14, 15]. The shortest path route, which takes the shortest-distance paths between the origins and the destinations, is a most common strategy for routing. However, this approach easily leads to congestion on some popular nodes having the most number of routes passing by. Therefore, to overcome this shortcoming several efficient strategies have been developed in a static scheme by focusing on the rerouting of congested flow so as to lead the system from congestion to free flowing [16, 17]. It is proved that to find the optimal routing of a transportation network, which has maximum transportation capacity without congestion, is an NP-hard problem. It is remarkable that Bassler et al. [17] recently presented routing strategies for wired networks and wireless networks respectively for the purpose of increasing the transportation capacity of these networks, which outperform many other methods. Their algorithm is a suboptimal solution and takes the running time $O(N^3 \log N)$. Hence, the transportation capacity can be much improved if one can judiciously choose the routes.

With the rapid development of communication networks, wired and wireless networks have been widely applied in many aspects of society, which include the Internet, which is based on the wired infrastructure and mobile telecommunication, which is based on the wireless framework. Despite the works introduced above, relatively little attention has been paid to the integration of the wired and wireless networks. The issue of taking the wired and wireless networks as a whole subject has become increasingly fundamental due to the widespread overlap of their applications. In this paper, we introduce an intuitive model which captures the important characteristics of the wired and wireless networks, and then we study transportation optimization on such integrated networks. Specifically, we first construct a wired network and a wireless network. Then, we choose the same number of nodes in each network and combine them pairwise to form a new one. Thus, the previous wired and wireless networks become the two layers of the new network. Notice that some characteristics of the wireless network are different from the wired one. One is the absence of the effects of rich hub connectivity and small shortest path length, which by contrast are important features of wired networks. The other is a constraint in the process of packet transmission. The constraint comes from the broadcast transmission mode of the
wireless device. When a wireless device is sending packets, other devices within the one-hop transmission range of the sender or the receiver are forbidden to send or receive information packets if they are working in the same channel, which is called medium access control. However, the wired networks do not have such constraint. After that, we propose a heuristic routing method to implement the efficient routine on the multilayered networks by considering the different features of the wired and wireless networks \[17\]. Comparing with the shortest path method, we find our method can significantly enhance the transportation capacity of the two-layered network.

The remainder of this paper is structured as follows. In sec. II, we present the model of the multilayered communication network. In Sec. III, we present our algorithm for single channel transmission mode and show the results of the algorithm. In Sec. IV, we study the general situation where the wireless networks work in the multichannel mode. In Sec. V, we give our conclusion.

II. The Model. – We initially construct two sub-networks. One, called wired layer, is a wired network with \(N_W\) nodes and the other, called wireless layer, is a wireless network with \(N_I\) nodes. The wired layer is constructed with the Positive-Feedback Preference (PFP) Model \[18\] which accurately reproduces many topological properties of AS-level Internet. The wired layer constructed by the PFP model has the effects of rich hub connectivity and small shortest path length, and has an average degree \(\approx 5.4\). For the wireless layer, we use the minimum degree geometric network model \[19\], which is appropriate in describing wireless networks. In the random geometric network model, a pair of nodes can make contact with each other when their distance is shorter than a critical value. However, this critical value needs to be provided by an external control authority which does not exist in ad hoc networks. For the minimum degree geometric network model, all the nodes are randomly distributed on a square and they can decide on their own contact radius by increasing their contact power until they have at least \(k_{\text{min}}\), mutual neighbors. Therefore, with a given \(k_{\text{min}}\), all the nodal degrees in the wireless layer will be no less than \(k_{\text{min}}\). After generating the two layers, we randomly choose a node in the wired layer and a node in the wireless layer and then merge them into a single one. The newly merged node preserves all the connections that the previous two nodes have. Thus, this node could serve as an interface of the two layers. We continually perform this process until \(N_I\) pairs of nodes from the two layers are merged to form \(N_I\) nodes. For simplicity, we refer to these \(N_I\) nodes as interfacing node which belong to the both layers, and we refer to the nodes in the wired (wireless) layer other than the interfacing nodes as wired (wireless) node. Thus, \(N_W\) (\(N_I\)) is the number of nodes in the wired (wireless) layer and the size of the whole network is \(N = N_W + N_I - N_I\).

Figure 1 offers an illustration of the network structure. In the simulations, all the averaged results and their standard deviation (which is indicated by the error bars), are obtained from 500 different realizations, if not otherwise specified.

III. Single Channel Mode. – Wireless networks have a broadcasting constraint when the nodes work on the single channel mode \[19\]. The constraint is that when a node is sending an information packet, all its incoming links are forbidden to send packets in order to avoid packet collisions. This constraint causes a big obstruction for the efficient information transmission. In the transmission process, the wireless nodes work on the first-in-first-possible-out basis. That is, a wireless node attempts to send its first-in-line packet. When the intended recipient is blocked due to the broadcasting constraint, the node then tries to send its second-in-line packet and so on, until either an idle recipient is found or the end of the queue is reached. While, for the wired networks there is no such constraint, and therefore the wired nodes simply work on the first-in-first-out basis. The interfacing nodes adopt the transmission process according to whether the intended recipient or the sender is a wired node or a wireless node.

When a new packet is added to the network at a node, it is appended at the end of the queue of this node. A packet is removed when it reaches its destination. When the routes between all pair of nodes are given, the betweenness of a node \(i\) is defined as \(B_i = \Sigma_{t \neq i} p_i(st)/p(st)\), where \(p(st)\) is the number of routes going from node \(s\) to node \(t\) and \(p_i(st)\) is the number of routes going from node \(s\) to node \(t\) passing through node \(i\). We assume that a node \(i\) has the processing capacity of handling \(C_i\) information packets per time step and different nodes may have different processing capacities. Thus, the value of the betweenness-to-capacity ratio of node \(i\), denoted as \((B/C)_i\), equals to \(B_i/C_i\).

An important consequence of the broadcast constraint...
is that a node in the wireless layer needs not only to process the information packets on itself but also to process the packets on its wireless neighbors. This situation leads to a modified version of the betweenness-to-capacity ratio [19], which is the summation of the betweenness-to-capacity ratios of the objective node and its wireless neighbors. However, the wired nodes do not need such modification. Hence, the effective betweenness-to-capacity ratio, denoted as \((B/C)_{\text{eff}}\), for different nodes is different. The effective betweenness-to-capacity ratio of a wireless node is equal to the sum of its own betweenness-to-capacity ratio and the betweennesses-to-capacity ratio of all its neighbors. The effective betweenness-to-capacity ratio of an interfacing node is equal to the sum of its own betweenness-to-capacity ratio and the betweennesses-to-capacity ratio of its neighbors connected with its wireless connections. Finally, the effective betweenness-to-capacity ratio of a wired node is equal to its own betweenness-to-capacity ratio (see Fig. 1). The maximum transportation capacity depends on the maximum effective betweenness-to-capacity ratio which is denoted as \((B/C)_{\text{eff}}\). The optimal routing is achieved when \((B/C)_{\text{eff}}\) is minimized. Adopting the algorithm that is used for single layered network [17], we extend it to the multilayered communication network to reduce the \((B/C)_{\text{eff}}\) through the following steps:

1. Assign every link a unit weight and compute the shortest path between all pairs of nodes and the betweenness of every node.

2. Calculate the effective betweenness-to-capacity ratio of all the nodes and find the node which has the highest score. (a) If the node is on the wireless layer, increase the weights of all the links sourced from this node and its neighbors by half unit weight. (b) If the node is an interfacing node, increase the weights of the links sourced from this node and its neighbors in the wireless layer by half unit weight. (c) If the node is on the wired layer, increase the weight of the links sourced from this node by half unit weight.

3. Recompute the shortest paths and the betweennesses. Go back to step 2.

4. The algorithm stops when the value of the maximum effective betweenness-to-capacity ratio is stable.

We note that the weight of a link can be regarded as the measure of the length of this link. Hence, a larger weight corresponds to a longer distance, and a packet will less likely pass through the corresponding link because of the higher cost required.

A plot of the maximum effective betweenness-to-capacity ratio \((B/C)_{\text{eff}}\) obtained from the shortest path method and the optimal algorithm versus network size is shown in Fig. 2(a). Obviously, \((B/C)_{\text{eff}}\) increases with the increasing of the size of the networks. The inset shows the ratio \(\rho\) of the results obtained from the optimal algorithm to those obtained from the shortest path algorithm. We observe that when the network size is small, the effect of the optimal algorithm is very limited. This is because of the fact that those small networks do not leave much space for the algorithm to adjust the transmission routes to reduce the \((B/C)_{\text{eff}}\). However, with the growth of the network size, there could be more alternative routes for a transmission. Therefore, it could be easier for the algorithm to find a route without passing the node that has the maximum betweenness-to-capacity ratio, which results in a considerable decrease in the \((B/C)_{\text{eff}}\). Hence, this algorithm could be more effective for a larger network, which is a good point as real communication networks are generally very large.

The interfacing nodes take an important role for the whole network. Since the routes of the inter-layer communication need to pass through the interfacing nodes, when the number of interfacing nodes is small they may be the bottleneck. Figure 2(b) shows the maximum effective betweenness-to-capacity ratio \((B/C)_{\text{eff}}\) from the optimal algorithm as a function of \(N_1\) for different network sizes. We can see that when \(N_1\) is small the value of \((B/C)_{\text{eff}}\) can be very large, which means the algorithm has limited effect in this situation because of the bottleneck effect of the interfacing nodes. However, with the increase of the \(N_1\) the value reduces quickly and the situation gets much improved.
work on the same channel is well avoided. Since nodes that work on different channels do not influence each other, the effect of the broadcast constraint can be ignored in the multichannel mode and the communication efficiency could be significantly enhanced. In the following, we study the transport optimization of the multilayered network in which the wireless layer works in the multichannel mode.

Since the broadcasting constraint can be ignored under the multichannel mode, it is not necessary to use the modified version of betweenness-to-capacity ratio as did in the previous section. In this mode, the effective betweenness-to-capacity ratio is just the betweenness-to-capacity ratio of the node itself. Thus, in the multichannel mode the algorithm works as follows:

1. Assign every link a unit weight and compute the shortest path between all pairs of nodes and the betweenness of every node.

2. Calculate the betweenness-to-capacity ratio of all the nodes and find the node which has the highest value. Increase the weight of the links reached by this node by half unit weight.

3. Recompute the shortest paths and the betweennesses. Go back to step 2.

4. The algorithm stops when the value of the maximum betweenness-to-capacity ratio is stable.

An interesting point of the multilayered network is the influence of the interfacing nodes. Since all the inter-layered transmission need to go through the interfacing nodes, we may estimate the lower bound of the maximum betweenness of the interfacing nodes. From the definition of the betweenness, we have \( \sum_{i \in \mathcal{I}} B_i = \sum_{i \in \mathcal{I}} \sum_{s \neq t} p_i(st)/p(st) \), where \( \mathcal{I} \) is the set of the interfacing nodes. This equation could be further separated into four cases. Those are (i) \( s \in \mathcal{W}, t \in \mathcal{W} \); (ii) \( s \in \mathcal{L}, t \in \mathcal{L} \); (iii) \( s \in \mathcal{W}, t \in \mathcal{L} \); and (iv) \( s \in \mathcal{L}, t \in \mathcal{W} \). Since all the routes for intra-layered communication are confined in that layer, the influence of the interfacing nodes can be very large. Figure 3 (a) shows the maximum betweenness-to-capacity ratio \((B/C)_{\text{max}}\) as a function of \( N_1 \) obtained from the shortest path algorithm (square symbols), the optimal algorithm (circle symbols), and the theoretical lower bound of the interfacing nodes \((2N_W N_L)/(N_1 C)\) (black curve), for the case of \( C = 1 \), \( N_W = 200 \) and \( N_L = 1000 \). Similarly to the single channel mode, \((B/C)_{\text{max}}\) decreases with the increase of \( N_1 \), which means there also exists the bottleneck effect in the multichannel mode when the number of interfacing nodes is small. Moreover, the circle symbols almost overlap with the black curve when \( N_1 \leq 6 \) and are just a little larger when \( N_1 > 6 \), which shows that the optimal algorithm can reduce the \((B/C)_{\text{max}}\) nearly to the theoretical lower bound and thus to the optimal level. We note that when \( N_1 = 1 \) the values of \((B/C)_{\text{max}}\) of the two algorithms are very close. This is because when there is only one interfacing node, this node has to handle all the inter-layer communications whether or not the optimal algorithm is adopted, which causes the node to have the \((B/C)_{\text{max}}\) and the value is approximated to \((2N_W N_L)/(N_1 C) = 4 \times 10^5\). Furthermore, for the optimal algorithm when \( N_1 = 2 \) the value of \((B/C)_{\text{max}}\) drops by around a half compared to the case of \( N_1 = 1 \), which means this algorithm can well distribute the information packets to the two interfacing nodes. Furthermore, we observe that when \( N_1 = 1 \) the distribution of \((B/C)_{\text{max}}\) obtained from the optimal algorithm under different realizations is squeezed in a narrow range. For example, for \( N_1 = 1 \), the ratio of the standard deviation of the distribution of \((B/C)_{\text{max}}\) with performing the algorithm to that without performing the algorithm is smaller than 0.01. This phenomenon is distinct from what is observed in the single channel mode. In the single channel mode, the distributions of the \((B/C)_{\text{max}}\) with and without performing the optimal algorithm spread in the ranges with comparable sizes, and both can be well.
fitted by the Gumbel distribution \cite{21}. The reason of this phenomenon is that in the multichannel mode, the optimal algorithm could always tune the $(B/C)_{\text{max}}$ close to the theoretical lower bound resulting in the fact that the values of $(B/C)_{\text{max}}$ for different realizations are very near to each other, which further proves the effectiveness of the algorithm.

As the maximum betweenness-to-capacity ratio of the interfacing nodes should be no less than $(2N_w N_1)/(N_1 C)$, when the value of $(B/C)_{\text{max}}$ is close to this lower bound, the routes for the intra-layered communication need to be mainly restricted in the respective layer to avoid going through the interfacing nodes. Thus, two wireless nodes cannot use the wired layer as well as its effects of rich hub connectivity and small shortest path length to shorten their distance in this situation. In order to improve the transmission efficiency by reducing the transmission distance, without increasing the value of $(B/C)_{\text{max}}$, an effective method is to increase the processing capacity of the interfacing nodes. To illustrate the effectiveness of this method we show the average transmission distance among all pairs of wireless nodes, denoted as $d_{\text{LL}}$, in Fig. 3(b). In this figure, $d_{\text{LL}}$ obtained from the shortest path algorithm as shown by the solid square symbols is the smallest one among all the cases, while $d_{\text{LL}}$ obtained from the optimal algorithm where all the nodes have $C = 1$ as shown by the solid circle symbols is much greater especially for large $N_1$. However, when we increase the processing capacity of the interfacing nodes to $C = 4$ while other nodes still have $C = 1$, the $d_{\text{LL}}$ has an obvious drop as shown by the solid triangle symbols which shows that increasing the processing capacity of the interfacing nodes could be of benefit to the wireless nodes utilizing the wired layer to reduce the transmission distance.

Moreover, we consider that if the positions of the interfacing nodes in the wireless layer are well-distributed, the wireless nodes could access their nearest interfacing nodes with the least of hops on average. Thus, this placement could further shorten the transmission distance when wireless nodes utilize the wired layer for transmission. For simplicity, we call such placement of the interfacing nodes in the wireless layer as Optimal Placement (denoted as “OP”), and correspondingly the placement in which the interfacing nodes are randomly distributed in the wireless layer as we did in the previous part of the work is called Random Placement (denoted as “RP”). To obtain the OP, we first get $N_1$ optimal positions with the algorithm introduced in the appendix. Then, in the stage of constructing multilayered network we choose $N_1$ wireless nodes who are nearest to the $N_1$ optimal positions as the wireless candidates to compose the $N_1$ interfacing nodes. In this way, the composed interfacing nodes have OP in the wireless layer. The open triangle symbols in Fig. 3(b) show the results of the OP where the interfacing nodes have $C = 4$. We observe that the $d_{\text{LL}}$ is further reduced in this case. The inset shows the corresponding $(B/C)_{\text{max}}$ for the cases that the interfacing nodes have $C = 1$ and $C = 4$. Obviously, the $(B/C)_{\text{max}}$ for the case of $C = 4$ is always smaller than that of $C = 1$. We note that $(B/C)_{\text{max}}$ of OP and RP are very close when the capacity of the interfacing nodes in both cases is the same. An example is shown in Fig. 3(a) where the open circle symbols correspond to OP and solid circle symbols correspond to RP. Therefore, by combining the methods of using the optimal algorithm, increasing the capacity of the interfacing nodes which are a small fraction of the whole network, and the OP, both the transportation capacity and average transmission distance can be much improved. Since these two factors are generally conflicting with each other, it is significant that both of them can be enhanced at the same time.

V. Conclusion. – In summary, we propose a model to describe multilayered communication networks. In the model, there are two layers. One, called wired layer, is composed of nodes connected with wired links, and the other, called wireless layer, is composed of nodes connected with wireless links. The layers are connected by merging some pairs of nodes in different layers into single ones, called interfacing nodes. Then, we present a recurrent algorithm to find optimal routing for this multilayered network for two different cases in which the wireless nodes can work in single channel mode and multichannel mode, respectively. In this algorithm, by gradually increasing the weight of the connections pointing to the node which has the maximum effective betweenness-to-capacity ratio, the burden on this node could be gradually reduced. By repeating this process, the final maximum effective betweenness-to-capacity ratio could be much suppressed and the transportation capacity could be enhanced significantly. Our results show that this algorithm is an effective navigation technique for multilayered communication networks. Since for the network on one hand the wireless layer may utilize the wired layer for transportation, on the other hand the interfacing nodes can easily become bottleneck, how to utilize the advantage of the multilayered networks while avoiding the restriction of the interfacing nodes is crucial for enhancing the transportation capacity. Our routing method proposed for the multilayered networks can squeeze the interfacing nodes to the extent that their effective betweenness-to-capacity ratio is almost the same, so that the bottleneck effect is suppressed to the most extent. By further cooperating with the methods of increasing the processing capacity and the OP of the interfacing nodes, the wireless layer may have a sufficient utilization of the wired layer for transportation. Hence, our method could serve as a good candidate of the standard technique for optimal transportation on multilayered communication networks.

Appendix. A method of placing an arbitrary number of nodes on a square in a well-distributed pattern. – Suppose there are $N$ nodes to be dealt with, we first regard the $N$ nodes as $N$ solid circles with the same radius which are required to be confined in the square. It can be proved that placing the nodes on the square so
Fig. 4: (a) An example of $N = 2$, where labels 1 and 2 indicate the centers of circle 1 and 2. We denote the candidate radius of circle 1 and 2 as $r_1$ and $r_2$, respectively. For circle 1, $x_L$, $y_L$, $y_U$ and $y_D$ are the distances between its center to the four boundaries of the square, and $d_{12}$ is the distance between the two centers. Hence, $r_1 = \min\{x_L, x_R, y_U, y_D, d_{12}/2\}$. Similarly, we can get the value of $r_2$. Thus, the value of $r_{\text{min}}$ is $r_{\text{min}} = \min\{r_1, r_2\}$. Dashed circles are those plotted by the candidate radii. (b), (c) and (d) show the examples of the cases $N = 4$, 7 and 15, respectively.

that they are well-distributed is equivalent to place these solid circles so that they can have the largest identical radius, and the positions of the centers of these circles are the positions in which to place the nodes. Following we present the algorithm of placing these solid circles so that they can have the largest identical radius:

1. Initially, randomly distribute these nodes on the square. Then, assign each node a candidate radius which is the shortest one among the distances that are from the node to the four boundaries of the square or as half as those from the node to the other nodes. The identical radius of these solid circles which satisfies the above conditions, denoted as $r_{\text{min}}$, is the smallest one among all the candidate radii (see Fig. 4(a)).

2. Disturb the position of a randomly chosen node and calculate the $r_{\text{min}}$ again. If $r_{\text{min}}$ decreases, the disturbance is discarded and the position of the disturbed node is recovered. If $r_{\text{min}}$ is unchanged, the disturbance is accepted and the position of the disturbed node is updated. If $r_{\text{min}}$ increases, the disturbance is accepted and the position of the node as well as the value of the $r_{\text{min}}$ are updated accordingly.

3. Go back to step 2 and perform a new disturbance until $r_{\text{min}}$ is stable.

After performing the algorithm, the final positions of the centers are the positions in which to place the nodes.

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