Solution for strain-softening surrounding rock reinforced by grouted bolts

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Abstract
To evaluate the reinforcement effectiveness of the grouted rock bolts, a dimensionless parameter’s approach (i.e. bolt density $\beta$) is adopted. Following the Mohr–Coulomb and generalized Hoek–Brown failure criteria, the concept of the neutral point for the grouted rock bolts is introduced, and the determination of the equivalent plastic zone is divided into three categories (i.e. minimal yielding, major yielding, and excessive yielding). The potential plastic zone is sufficiently subdivided into a large number of concentric annuli through the finite difference approximation of equilibrium and compatibility equations. A simple approach for circular tunnel in strain-softening surrounding rock incorporating the neutral point of the grout rock bolts is proposed to derive the solutions of stress and displacement. The proposed approach is validated with the published results. With the increase in bolt density $\beta$, the plastic radius would reduce 10%–30%, and the radial displacements would decrease by 10%–40%.

Keywords
Strain-softening surrounding rock, grouted rock bolts, bolt density parameter, neutral point, stepwise approach

Date received: 25 March 2016; accepted: 27 January 2017

Academic Editor: Jose Ramon Serrano

Introduction
Grouted rock bolts have been widely used in all civil engineering fields because of its simple structure, convenient construction, remarkable reinforcement effectiveness, low cost, and engineering adaptability, especially in cracked and softening rock masses. The grouted rock bolts can effectively improve the strength and stabilization of the surrounding rock through convergence control. Although most researchers have contributed to the stress and displacement of tunnels and have solved many technical problems in practice, the effects of strain-softening characteristic on the stress and displacement of a tunnel, reinforced with grouted rock bolts, have yet to be defined clearly. And the method of ground reaction curve of circular tunnel excavated in strain-softening rock mass was initially proposed by Brown et al. based on finite difference method.

The effects of the grouted rock bolts on rock mass have been studied by many researchers. Stille and Stille et al. presented a closed-form elastic–plastic solution of the grouted rock bolts. Five different approaches of bolt performance are considered in this model, and the results agree well with the measured data. Indraratna and Kaiser developed an analytical model in which the elastic–plastic concept was adopted and a proper interaction mechanism between the ground and the grouted rock.
bolts was considered. Peila and Oreste\textsuperscript{6} presented a new convergence–confinement approach able to model a reinforced rock mass zone surrounding an opening zone with new improved properties. Oreste and Peila\textsuperscript{7} presented a new procedure for the computation of the convergence–confinement curves of a bolted tunnel. Fahimifar and Sorouch\textsuperscript{8} developed solutions for the stress and displacement of the tunnels that were excavated in softening rock mass and reinforced by the grouted rock bolts. Ranjbarnia et al.\textsuperscript{9} proposed an analytical solution to determine the stress and displacement of a tunnel that was excavated in a brittle and strain-softening rock mass and reinforced with active grouted rock bolts. This solution is based on the nonlinear peak strength criterion. Osgoui and Oreste\textsuperscript{10} developed an elastic–plastic analytical solution of an axisymmetrical problem for a circular tunnel reinforced by the grouted rock bolts. This improved model was based on the approach of Indraratna and Kaiser,\textsuperscript{5} and the nonlinear Hoek–Brown (H–B) failure criterion was adopted. B Liu et al.\textsuperscript{11} presented an analytical design method for the truss-bolt system in reinforcing underground fractured rock roofs in coal mines. Carranza-Torres\textsuperscript{12} proposed the method of quantifying the mechanical contribution of rock bolts installed systematically around tunnels. Reinforcement of the effects of using rock bolt on rock mass with cross-flaws was explored by B Zhang et al.\textsuperscript{13} X Li et al.\textsuperscript{14} proposed an analytical model of shear behavior of a fully grouted cable bolt subjected to shearing to effectively evaluate the cable shear behavior and the influence of fully grouted cable bolts on joint shear resistance. A comprehensive study including laboratory tests and numerical modeling was performed to investigate factors of rock bolt fracture under varying loading conditions by H Kang et al.\textsuperscript{15}

In the analyses of engineering problems, the Mohr–Coulomb (M–C) and H–B failure criteria have been used widely by most scholars such as Carranza-Torres,\textsuperscript{16} Park and Kim,\textsuperscript{17} and Yang and Yin.\textsuperscript{18} Lee and Pietruszczak\textsuperscript{19} developed a numerical procedure for calculating the stress and radial displacements of a circular tunnel excavated in strain-softening rock mass, which obeyed the M–C and generalized H–B failure criteria. Park et al.\textsuperscript{20} proposed the pressure-induced inflatable pipe method. The concept of cavity expansion for tunnel reinforcement was used. The interaction between tunnel supports and ground convergence was also investigated analytically and numerically in this approach. Zou and Li\textsuperscript{21} proposed an improved numerical approach to analyze the stability of the strain-softening surrounding rock with the consideration of the hydraulic–mechanical coupling and the variation in elastic strain in the plastic region. In those literatures, the analytical and numerical solutions based on the plane strain assumption are discussed with different constitutive models (e.g. elastic–plastic model, elastic–brittle–plastic model, and strain-softening model) combined with the linear M–C or generalized H–B failure criteria. Although the solutions of the strain-softening surrounding rock based on M–C or H–B failure criteria are obtained by most researchers, the effect of the grouted rock bolts and the position of the neutral point was seldom considered.

The main objective of this article is to explore solutions for the stress and displacement of circular openings excavated in strain-softening surrounding rock. In the presented solutions, a numerical stepwise procedure that considers the reinforcement effectiveness and the neutral point of the grouted rock bolts is adopted and modified, where the deterioration of strength in the plastic region is considered. The improved model which considers the strain-softening behavior and is compatible with M–C and generalized H–B failure criteria was proposed based on model of Indraratna and Kaiser.\textsuperscript{5}

### Theory and methodologies

#### Statement of the problem

As shown in Figure 1, in a continuous, homogeneous, isotropic, initially elastic rock mass, a circular opening with a radius ($a$) is excavated. The cross section of the tunnel is subjected to uniform hydrostatic pressure ($s_0$). After excavating, an internal support pressure ($p_{in}$) acts on the tunnel wall surface in the radial direction. Grouted rock bolts are installed systematically over the periphery of the opening. In the system of the cylindrical polar coordinates, the stress and displacement of the surrounding rock are only related to radius, so the gravity field can be ignored.

![Figure 1. Axisymmetric model and division of the zones around tunnel.](image-url)
During excavation of tunnel, $p_{in}$ is gradually reduced. When the internal support pressure ($p_{in}$) is less than a critical support pressure, plastic region will appear around the surrounding rock. After this initial yielding, the rock strength drops gradually following the post-yield softening behavior.

The strain-softening model is used, and the numerical stepwise procedure is improved for the solution for stress and strain in the plastic region.

The stress–strain relationship of the strain-softening rock mass is shown in Figure 2. Figure 2 displays the idealized relationships of $\sigma_1 / C_0$ versus $\epsilon_1$. Both the decrease in strength from the peak and the continued deformation at the residual region are accompanied by plastic dilation.

Several assumptions below have been made to determine the solution of stress and displacement of the strain-softening surrounding rock incorporating the reinforcement effectiveness and the neutral point of the grouted rock bolts. The rock mass around the circular opening is regarded as an isotropic, continuous, and permeable medium which obeys the M–C or generalized H–B failure criteria under the plane strain condition. The strain-softening constitutive model that follows a non-associated flow rule is employed for formula derivation. The elastic strain in the plastic and softening regions of the surrounding rock obeys Hooke’s law.

Considering the reinforcement effectiveness and the neutral point of the grouted rock bolts, some assumptions are also needed.

1. Axisymmetric rock bolt patterns consisting of grouted rock bolts are installed with $S_T$ spacing around the circumference and with $S_L$ spacing along the longitudinal axis of the tunnel as shown in Figure 3.
2. The deformations between the rock mass and the grouted rock bolts are compatible.
3. The strength and deformation parameters deteriorate with the development of plastic deformation.

**Bolt density parameter and neutral point**

As shown in Figure 3, a circular tunnel is reinforced by grouted rock bolts. The bolt density parameter proposed by Indraratna and Kaiser is adopted to describe the global intensity parameter of the reinforced surrounding rock as follows

$$\beta = \frac{\pi d \lambda'}{S_T \theta} = \frac{\pi d \lambda' a}{S_T S_T}$$

where $\beta$ is dimensionless. It reflects the relative density of the rock bolts and takes into consideration the shear stress on the bolt surface, which opposes the rock mass displacements near the tunnel wall. The diameter of grouted rock bolts is $d$ and the tunnel radius is $a$. $\lambda'$ is the friction factor which is analogous to the coefficient of friction.

The number of the grouted rock bolts is presumably distributed uniformly on the tunnel surface in both the circumferential and longitudinal directions to maintain model symmetry. The horizontal distance between the two adjacent bolts, in the inner surface of the tunnel, is denoted by $S_T$. The longitudinal space along the axis of the tunnel is represented by $S_L$.

The shear stress is related to the first derivative of the axial stress. Hence, one point where the value of $\tau_Z$ is 0 is defined as the neutral point that exists where the axial stress attains the maximum, as shown in Figure 3. The location of the neutral point is given by Indraratna and Kaiser

$$r = L \ln \left(1 + \frac{L}{a}\right)$$

The shear stress is characterized by the division of the rock bolts, a pick-up length and an anchor length, on either side of the neutral point. By the division of the rock bolts, pick-up side and anchor side, the shear stress is characterized as presented in Figure 4. The location of the neutral point is given by Indraratna and Kaiser

$$\rho = \frac{L}{\ln(1 + (L/a))}$$

$\rho$ is also the anchor length.
Three categories of the equivalent plastic zone

The plastic zone around the surrounding rock reinforced by the grouted rock bolts is redefined as the equivalent plastic zone. Because the location of the plastic zone is related to the neutral point and the bolt length, three cases (as shown in Figure 5) are analyzed and calculated for the determination of the equivalent plastic zone radius ($R$) and corresponding classifications are shown below.

Case I: minimal yielding: $R^* < \rho < (a + L)$;
Case II: major yielding: $\rho < R^* < (a + L)$;
Case III: excessive yielding: $R^* > (a + L)$.

Failure criterion

The M–C failure criterion is given by

$$\sigma_1 = \sigma_3 N + Y$$  \hspace{1cm} (4)

where $\sigma_1$ is the major principal stress; $\sigma_3$ is the minor principal stress; $Y$ and $N$ are the strength parameters of the rock mass ($Y = 2c \cos \phi / (1 - \sin \phi)$, $N = (1 + \sin \phi) / (1 - \sin \phi)$), where $c$ and $\phi$ are the cohesion and internal friction angle, respectively.

The following generalized H–B failure criterion is adopted by

$$\sigma_1 = \sigma_3 + \sigma_c \left( \frac{m \sigma_3}{\sigma_c} + s \right)^a$$  \hspace{1cm} (5)

where $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses, respectively. $\sigma_c$ is the uniaxial compressive strength of the rock mass. $a$, $m$, and $s$ are the strength parameters of the generalized H–B failure criterion.

The strength and deformation parameters of the strain-softening rock mass are evaluated based on plastic deformation and are controlled by the plastic deviatoric strain

$$\phi^p = e_1^p - e_3^p$$  \hspace{1cm} (6)

where $e_1^p$ and $e_3^p$ are the major and minor plastic strains, respectively.

The bilinear function that adopts the plastic shear strain to describe the physical parameters of the surrounding rock mass is presented by

$$\omega(\phi^p) = \begin{cases} \omega_0 - (\omega_p - \omega_0) \frac{\phi^p}{\gamma_r}, & 0 < \phi^p < \gamma_r^p \\ \omega_0, & \gamma_r^p \leq \phi^p \end{cases}$$  \hspace{1cm} (7)

where $\omega$ represents a strength parameter, such as $c$, $\phi$, $m$, $s$, and $a$; $\gamma_r^p$ is the critical deviatoric plastic strain from which the residual behavior is observed first, and which should be identified through experimentation. The subscripts $p$ and $r$ represent the peak and residual values, respectively.

Stress and strain solutions based on the M–C failure criterion

Stress and strain solutions in elastic region

The solutions of stress, strain, and displacement in the elastic zone considering the reinforced rock are given by Indraratna and Kaiser as follows

$$\begin{align*}
\sigma_r &= \sigma_0 \left( 1 - \left( \frac{\rho}{R^*} \right)^2 \right) + \sigma_{rr} \left( \frac{\rho}{R^*} \right)^2 \\
\sigma_\theta &= \sigma_0 \left( 1 + \left( \frac{\rho}{R^*} \right)^2 \right) - \sigma_{rr} \left( \frac{\rho}{R^*} \right)^2 \\
e_r^p &= \frac{1}{r} \left( (\sigma_r - \sigma_0) - \sigma_{rr} (\rho / R^*)^2 \right) \\
e_\theta^p &= \frac{1}{r} \left( (\sigma_\theta - \sigma_0) - \sigma_{rr} (\rho / R^*)^2 \right)
\end{align*}$$  \hspace{1cm} (8)

$$u = \frac{1}{E} \left[ \left( \sigma_r - \sigma_0 \right) - \nu (\sigma_\theta - \sigma_0) \right]$$  \hspace{1cm} (9)

$$w = \frac{1}{E} \left[ \left( \sigma_\theta - \sigma_0 \right) - \nu (\sigma_r - \sigma_0) \right]$$  \hspace{1cm} (10)
The relationship of the radial stress and the tangential stress at the interface between the elastic rock and the equivalent plastic zone is presented by

\[
\sigma_\theta - \sigma_r = 2(\sigma_0 - \sigma_r) \quad (11)
\]

Combining equations (4) and (11) leads to

\[
\sigma_{rr} = \frac{2\sigma_0 - Y}{N + 1} \quad (12)
\]

where \(\sigma_{rr}\) is the radial stress at the interface between the elastic zone and the equivalent plastic zone.

**Stress and strain solutions of excessive yielding**

\(R^* > (a + L)\)

In zone I, the plastic zone is beyond the reinforced zone. The stress equilibrium equation of an element near an unsupported opening can be represented by

\[
\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (13)
\]

In a bolted element, as shown in Figure 6, the stress equilibrium equation is given by

\[
\frac{d\sigma_r}{dr} + \frac{1 - N(1 + \beta)\sigma_r}{r} = \frac{Y(1 + \beta)}{r} \quad (14)
\]

Equations (13) and (14) describe the stress equilibrium condition of the unreinforced and reinforced segments, respectively. If the terms \(N(1 + \beta)\) and \(Y(1 + \beta)\) are replaced by the equivalent strength parameters \(N^*\) and \(Y^*\), respectively, equation (14) for the bolted parts can be simplified to

\[
\frac{d\sigma_r}{dr} + \frac{(1 - N^*)\sigma_r}{r} = \frac{Y^*}{r} \quad (15)
\]

where \(N^* = N(1 + \beta)\) and \(Y^* = Y(1 + \beta)\).

If the bolt density parameter is introduced into the stress equilibrium equation, the radial stress at the interface between the elastic rock and the equivalent plastic zone can be modified by

\[
\sigma_{rr} = \frac{2\sigma_0 - \frac{Y^*}{N^* + 1}}{N^* + 1} \quad (16)
\]

Considering the reinforcement effectiveness of the grouted rock bolts, the residual strength parameters of rock mass can be represented by \(N^*_r = N_r(1 + \beta)\) and \(Y^*_r = Y_r(1 + \beta)\).

The total plastic region is presumably divided into \(n\) connect annuli which are bounded by two annuli of the radius \(r_{(i-1)}\) and \(r_{(i)}\). The radius of the first ring is \(r_{(0)} = R\) which is at the interface between the elastic region and the equivalent plastic zone. To satisfy the equilibrium condition, the thickness of the annulus is determined automatically by the numerical process. In Figure 7, the \(i\)th annulus which is determined by the outer radius \(r_{(i-1)}\) and the inner radius \(r_{(i)}\) adjacent to the elastic region is given. The slight increment of the radial stress is regarded as the known increment. So the \((n-1)\) iterations of brittle–plastic analysis are conducted on the inner plastic annulus until the residual strength is reached.

\[
\sigma_r \text{ on both the inner and outer boundaries of the plastic zone are assigned, and } \sigma_r \text{ decreases monotonically from } \sigma_{rr} \text{ at } r = R \text{ to } p_{in} \text{ at } r = r_0. \text{ The increment of radial stress is as follows}
\]

\[
\Delta\sigma_r = \frac{p_{in} - \sigma_{rr}}{n} \quad (17)
\]

So the radial stress components for \(i\)th radius can be

\[
\Delta\sigma_{r(i)} = \frac{p_{in} - \sigma_{rr}}{n} \quad (18)
\]

For a large magnitude \(n\), the corresponding tangential stress is given by

---

**Figure 6.** Equilibrium considerations for bolt–ground interaction.

**Figure 7.** Normalized plastic region with the finite number annuli.
\[ \sigma_{\theta(i)} = N^*_r \sigma_{r(i)} + Y^*_r \]  
(19)

where \( N^*_r \) and \( Y^*_r \) are the equivalent strength parameters.

So the tangential stress components can be represented by

\[ \Delta \sigma_{\theta(i)} = \sigma_{\theta(i)} - \sigma_{\theta(i-1)} \]  
(20)

Under the plane strain condition, the elastic strain increments can be obtained using Hooke’s law as follows

\[
\begin{align*}
\Delta \varepsilon_{\theta(i)} &= \frac{1 + \nu}{E} [(1 - \nu)\sigma_{r(i)} - \nu \sigma_{\theta(i)}] \\
\Delta \sigma_{\theta(i)} &= \frac{1 + \nu}{E} [(1 - \nu)\sigma_{\theta(i)} - \nu \sigma_{r(i)}]
\end{align*}
\]  
(21)

The stress equilibrium differential equation that considers the reinforcement effectiveness of the grout rock bolts for the \( i \)th annulus is derived using equation (15) and expressed as equation (22)

\[ \sigma_{r(i)} - \sigma_{r(i-1)} - \frac{(1 - N^*_r)\sigma_{\theta(i)}}{\rho(i)} = \frac{Y^*_r}{\rho(i)} \]  
(22)

where \( \sigma_{r(i)} = (1/2)(\sigma_{r(i)} + \sigma_{r(i-1)}) \), \( \rho(i) = r(i)/R^* \), and \( \rho(i) = (1/2)(\rho(i) + \rho(i-1)) \). The normalized inner radius \( \rho(i) = r(i)/R \) can be expressed as

\[ \rho(i) = \frac{X + 1}{X - 1} \]  
(23)

where \( X = ((1 - N^*_r)\sigma_{\theta(i)} + Y^*_r)/(\sigma_{\theta(i)}) \).

The compatibility equation can be written in the general form as follows

\[ \frac{d\varepsilon_\rho}{d\rho} + \frac{\varepsilon_\rho - \varepsilon_\theta}{\rho} = 0 \]  
(24)

where \( \varepsilon_\rho = du/dr \) and \( \varepsilon_\theta = u/r \).

The total strain in the plastic zone that is the sum of the elastic strain and the plastic strain is given by

\[
\begin{align*}
\varepsilon_{\theta(i)} &= \varepsilon_{\theta(i)}^e + \varepsilon_{\theta(i)}^p \\
\varepsilon_{\rho(i)} &= \varepsilon_{\rho(i)}^e + \varepsilon_{\rho(i)}^p
\end{align*}
\]  
(25)

Combining equations (22) and (23) leads to

\[ \Delta \varepsilon_{\rho(i)} = A_2 - A_3 \]  
(26)

where \( A_1 = (1/\Delta \rho(i)) + (1 + k_{(i-1)}(1/\rho(i))) \), \( A_2 = (\sigma_{r(i)}^e/\Delta \rho(i)) - (\Delta \sigma_{\theta(i-1)} - \Delta \rho_{\theta(i-1)})/(\rho(i)) \), \( A_3 = (\sigma_{\theta(i-1)}^e - \sigma_{\theta(i)}^e)/(\rho(i)) \), and \( k_{(i-1)} = ((1 + \sin \phi_{(i-1)})/(1 - \sin \phi_{(i-1)})) \).

Using equation (6), the plastic deviatoric plastic strain can be expressed by

\[ \gamma_{\rho(i)}^p = \gamma_{\rho(i-1)}^p + (\Delta \varepsilon_{\rho(i)}^p - \Delta \varepsilon_{\rho(i-1)}^p) \]  
(27)

So the total strain at \( i \)th ring is given by

\[
\begin{align*}
\varepsilon_{\theta(i)} &= \varepsilon_{\theta(i-1)} + \Delta \varepsilon_{\theta(i)}^p + \Delta \varepsilon_{\theta(i)}^e \\
\varepsilon_{\rho(i)} &= \varepsilon_{\rho(i-1)} + \Delta \varepsilon_{\rho(i)}^p + \Delta \varepsilon_{\rho(i)}^e
\end{align*}
\]  
(28)

Through the relationship between strain and displacement, the displacement at \( i \)th annulus can be obtained as

\[ U_i = \varepsilon_i r_i \]  
(29)

Thus, the stress and displacement solutions of the reinforced rock in this plastic region are obtained.

Zone II: \( r < a < L \)

This is the middle plastic zone which is confined between the neutral point and the bolt end. Consequently, the rock mass is subjected to the negative shear stress of the rock bolts. So the rock mass has weak values of the strength parameters as follows

\[ N' = N(1 - \beta), Y' = Y(1 - \beta) \]  
(30)

If \( N' \) is greater than the residual strength of rock mass \( N^*_r \), the rock mass strength first drops to \( N' \) and then continues to decrease until \( N^*_r = N^*_r \). If \( N' \) is less than the residual strength of rock mass, as this plastic zone becomes a residual plastic zone, the rock mass strength no longer changes. So the rock mass strength parameters maintain the residual strength \( N_r \).

The first annulus in zone II is the interface between the plastic zones I and II, and the boundary conditions of stress and strain in this region can be obtained. Then as with zone I the stress and strain can be calculated by stepwise method.

Zone III: \( a < r < \rho \)

This yielded zone is confined by a range between the tunnel boundary and the neutral point. It falls within the pick-up length of the rock bolts and is stabilized by the positive shear stress. The residual plastic zone is reinforced and its strength parameters are effectively increased, and the strength parameters maintain the residual strength as follows

\[ N_r^* = N_r(1 + \beta), Y_r^* = Y_r(1 + \beta) \]  
(31)

The boundary condition of the stress and strain in region II is used as the initial condition of region III. Then the calculation steps are the same as region II.

Since \( \varepsilon_\rho = u/r \), the displacement at each ring can be obtained by
\[ U_{(i)} = \varepsilon_0 \rho_{(i)} \]  

The procedure explained above can be repeated for \( n \) times until \( \sigma_{r(i)} \) reaches the internal support pressure. Therefore, we can obtain the equivalent plastic radius of zone III as follows

\[ R = \frac{r_0}{\rho_{(n)}} \]  

**Stress and strain solutions of major yielding**

\( (\rho < R^* < (a + L)) \)

**Zone I: \( r > a + L \)**

Zone I is in elastic state and beyond the rock bolts. The stresses in elastic zone are given by

\[
\begin{align*}
\sigma_r &= \sigma_0 \left[ 1 - \left( \frac{a + L}{r} \right)^2 \right] + \sigma_L \left( \frac{a + L}{r} \right)^2 \\
\sigma_\theta &= \sigma_0 \left[ 1 + \left( \frac{a + L}{r} \right)^2 \right] - \sigma_L \left( \frac{a + L}{r} \right)^2
\end{align*}
\]  

where \( \sigma_L = \sigma_0 [1 - ((R)/(a + L))] + \sigma_{\text{R}}((R)/(a + L))^2 \).

**Zone II: \( R^* < r < a + L \)**

This is the inner elastic zone within the reinforced zone. The stress in this zone is given by

\[
\begin{align*}
\sigma_r &= \sigma_0 \left[ 1 - \left( \frac{R^*}{r} \right)^2 \right] + \sigma_{\text{R}} \left( \frac{R^*}{r} \right)^2 \\
\sigma_\theta &= \sigma_0 \left[ 1 + \left( \frac{R^*}{r} \right)^2 \right] - \sigma_{\text{R}} \left( \frac{R^*}{r} \right)^2
\end{align*}
\]  

**Zone III: \( \rho < r < R^* \)**

This is the outer plastic zone beyond the neutral point. The strength parameters of rock mass begin to decrease from maximum values \( N^* \) and \( Y^* \). If \( N^* \) is greater than the residual strength of rock mass \( N^*_r \), the rock mass strength first drops to \( N^*_r \) and then continues to decrease until \( N^*_0 = N^*_r \). If \( N^* \) is less than the residual strength of rock mass, as this plastic zone becomes a residual plastic zone, the rock mass strength no longer changes. Then the rock mass strength parameters maintain the residual value \( N^*_r \).

The stress and strain of rock mass are solved by the same method explained in section “Stress and strain solutions of excessive yielding \( (R^* > (a + L)) \).”

**Zone IV: \( a < r < \rho \)**

This yielded zone is confined between the neutral point and tunnel boundary, and the same approach in section “Stress and strain solutions of excessive yielding \( (R^* > (a + L)) \)” can be used.

**Stress and strain solutions of minimal yielding**

\( (R^* < \rho < (a + L)) \)

**Zone I: \( r > (a + L) \)**

This is the outermost elastic zone, and the stress is given by

\[
\begin{align*}
\sigma_r &= \sigma_0 \left[ 1 - \left( \frac{a + L}{r} \right)^2 \right] + \sigma_L \left( \frac{a + L}{r} \right)^2 \\
\sigma_\theta &= \sigma_0 \left[ 1 + \left( \frac{a + L}{r} \right)^2 \right] - \sigma_L \left( \frac{a + L}{r} \right)^2
\end{align*}
\]  

where \( \sigma_L = \sigma_0 [1 - ((\rho)/(a + L))] + \sigma_{\text{R}}((\rho)/(a + L))^2 \).

**Zone II: \( \rho < (a + L) \)**

The anchor length of the rock bolts is contained in this elastic zone. The stress can be represented by

\[
\begin{align*}
\sigma_r &= \sigma_0 \left[ 1 - \left( \frac{\rho}{r} \right)^2 \right] + \sigma_{\text{R}} \left( \frac{\rho}{r} \right)^2 \\
\sigma_\theta &= \sigma_0 \left[ 1 + \left( \frac{\rho}{r} \right)^2 \right] - \sigma_{\text{R}} \left( \frac{\rho}{r} \right)^2 \\
\sigma_\rho &= \sigma_0 \left[ 1 - \left( \frac{R^*}{\rho} \right)^2 \right] + \sigma_{\text{R}} \left( \frac{R^*}{\rho} \right)
\end{align*}
\]  

**Zone III: \( R^* < r < \rho \)**

This elastic zone is confined to the end of the pick-up length of the rock bolts. The stress is given by

\[
\begin{align*}
\sigma_r &= \sigma_0 \left[ 1 - \left( \frac{R^*}{r} \right)^2 \right] + \sigma_{\text{R}} \left( \frac{R^*}{r} \right)^2 \\
\sigma_\theta &= \sigma_0 \left[ 1 + \left( \frac{R^*}{r} \right)^2 \right] - \sigma_{\text{R}} \left( \frac{R^*}{r} \right)^2
\end{align*}
\]  

**Zone IV: \( a < r < \rho \)**

This region is the plastic zone. The stress and strain of rock mass are solved by the same method explained in section “Stress and strain solutions of excessive yielding \( (R^* > (a + L)) \).”
Stress and strain solutions based on the H–B failure criterion

Stress and strain solutions in elastic region

Taking excessive yielding \(R^* > a + L\) as an example, its equivalent plastic radius exceeds the length of the rock bolts. In the plastic zone, the strength parameters of the reinforced rock mass are increased to equivalent parameters and are as follows

\[
\begin{align*}
    m_b^* &= (1 + \beta)m_b \\
    s &= (1 + \beta)s \\
    \sigma_{ci}^* &= (1 + \beta)\sigma_{ci}
\end{align*}
\]

The relationships of the radial and tangential stresses at the interface between the elastic rock and the equivalent plastic zone are presented by

\[
2(\sigma_0 - \sigma_r) = \sigma_\theta - \sigma_r = \sigma_c \left( \frac{\sigma_r}{\sigma_c} + s \right)^{a_1}
\]

Considering the reinforcement effectiveness of the grouted rock bolts, the radial stress at the interface between the elastic rock and the equivalent plastic zone can be modified as follows

\[
2(\sigma_0 - \sigma_r) = \sigma_{ci}^* \left( \frac{m_b^* \sigma_r}{\sigma_{ci}^*} + s^* \right)^{a_1}
\]

The stress equilibrium differential equation that considers rock bolts’ effectiveness for the i-th annulus is expressed as equation (45)

\[
\frac{\sigma_{r(i)} - \sigma_{r(i-1)}}{p(i) - p(i-1)} - \frac{2H^{*H-B}}{p(i)} = 0 \tag{45}
\]

where \(p(i) = r(i)/R^*\), \(\rho(i) = (1/2)(\rho(i) + \rho(i-1))\), and \(H^{*H-B} = \sigma_{ci}^* (m_b^* \sigma_r/\sigma_{ci}^*) + s^*\).

The normalized inner radius \(p(i) = r(i)/R^*\) can be given by

\[
\rho(i) = \frac{2H^{*H-B} + \Delta\sigma_r}{2H^{*H-B} - \Delta\sigma_r}
\]

The increment of the tangential strain is represented by

\[
\Delta\varepsilon^p_{\theta(i)} = \frac{A_2 - A_3}{A_1}
\]

where \(A_1 = (1/\Delta\rho(i)) + (1 + k(i-1)/\rho(i))\), \(A_2 = (\Delta\varepsilon^p_{\theta(i)}/\Delta\rho(i)) - (H(i)/2G\rho(i))\), \(A_3 = ((\varepsilon^p_{\theta(i)} - \varepsilon^p_{\theta(i)/\rho(i)})/k(i-1) = (1 + \sin \phi(i-1)/1 - \sin \phi(i-1)), \) and \(H(i) = (1/2)(H(i)^{*H-B} + H(i)^{*H-B})\).

The displacement at each ring can be obtained using the relationship between the strain and displacement. We can use the same method in section “Stress and strain solutions in plastic region” to calculate the stress and displacements of another two categories: the major yielding \(\rho < R^* < a + L\) and the minimal yielding \((R^* < \rho < a + L)\).

Validations

To validate the accuracy of the proposed approach based on the M–C and the generalized H–B failure criteria, the results of the proposed approach for \(\gamma_p = 0\) are compared with those by Indraratna and Kaiser\(^5\) and Osgoui and Oreste\(^10\) respectively. The results of the proposed solution based on the M–C failure criterion incorporating the neutral point of the grouted rock bolts for the cases of \(\beta = 0, \beta = 0.15, \) and \(\beta = 0.291\) are compared, respectively. The detailed parameters are given by Indraratna and Kaiser\(^5\) as follows: \(a = 0.13 \text{ m}, \ L = 0.1 \text{ m}, \ E = 1500 \text{ MPa}, \ \nu = 0.25, \ \sigma_0 = 14 \text{ MPa}, \ \sigma_c = 3.5 \text{ MPa}, \ \phi_p = 32^\circ, \ \phi_r = 27^\circ, \ Y_p = 3.5, \ Y_r = 3.125, \ F_m = 0.8 \text{ MPa}, \) and \(\gamma_p = 0.008. \) The results are shown in Figure 8(a)–(c).

The results of the proposed solution based on the generalized H–B failure criterion incorporating the neutral point of the grouted rock bolts for the cases of \(\beta = 0 \) and \(\beta = 0.132\) based on the generalized H–B failure criterion are also compared, respectively, corresponding to \(a = 2.0 \text{ m}, \ E = 5.7 \text{ GPa}, \ \nu = 0.3, \ \sigma_0 = 10 \text{ MPa}, \ \sigma_c = 30 \text{ MPa}, \ \sigma_r = 27 \text{ MPa}, \ m_p = 1.7,\)
Figure 8. Stress curve comparison for different $\beta$: (a) $\beta = 0$ and $\gamma_p = 0$, (b) $\beta = 0.145$ and $\gamma_p = 0$, (c) $\beta = 0.291$ and $\gamma_p = 0$, (d) $\beta = 0$ and $\gamma_p = 0$, and (e) $\beta = 0.132$ and $\gamma_p = 0$. 
The parameters above are referred to Osgoui and Oreste,\textsuperscript{10} and the results are shown in Figure 8(d) and (e). As is demonstrated in Figure 8, for the cases of $\gamma_p = 0$, the results of the proposed approach are consistent with those presented by Indraratna and Kaiser\textsuperscript{5} and Osgoui and Oreste,\textsuperscript{10} because the elastic–brittle–plastic model ($\gamma_p = 0$) is just a special case of strain-softening model. Hence, the solutions of Indraratna and Kaiser\textsuperscript{5} and Osgoui and Oreste\textsuperscript{10} are the special cases of the proposed approach based on the M–C and generalized H–B failure criteria, respectively.

**Numerical analysis and discussion**

**Comparison with elastic–brittle–plastic model**

To study the effect of strain-softening in combination with the grouted rock bolts on the stress and displacement, the stress and strain curves are plotted to compare the strain-softening model with the elastic–brittle–plastic model proposed by Indraratna and Kaiser\textsuperscript{5} and Osgoui and Oreste,\textsuperscript{10} respectively.

Figures 9–11 depict the differences between the predicted results of the strain-softening model and elastic–brittle–plastic model with different bolt densities. It is shown from Figures 9 and 11 that stresses presented by the proposed model are larger than those based on the elastic–brittle–plastic model at the same point in the tunnel. Moreover, as is shown in Figures 14 and 15, the plastic radius of the proposed model would be reduced by 10% and 12%, respectively, for $\beta = 0.145$ and 0.291. The probable reason is that the strength parameters of strain-softening model are larger than those of the elastic–brittle–plastic model in some annuli of the plastic zone. As can be seen from Figure 9(c) that with the increase in the bolt densities, the stress curves of the proposed model are more agreeable with the curves based on the ideal elastic–plastic model in comparison with the elastic–brittle–plastic model.

It can be found from Figure 10(a) and (b) that the tangential strains presented by the proposed model are...
smaller than those presented by Indraratna and Kaiser\textsuperscript{5} at the same point. Moreover, as shown in Figure 10(c), when the bolt densities increase, the results are closer to those based on the ideal elastic model in contrast to the elastic–brittle–plastic model.

It can be noted that the displacement in Figure 12 presented by the proposed model is smaller than those presented by Osgoui and Oreste.\textsuperscript{10} For example, the radial displacement decreases from 18.2 to 15.3 mm if $b = 0$. The reduction is due to the fact that the strength parameters of strain-softening model are larger than those of the elastic–brittle–plastic model in some annuli of the plastic zone. Therefore, this model is more accurate than the elastic–brittle–plastic model.

It can be noted that the displacement in Figure 12 presented by the proposed model is smaller than those presented by Osgoui and Oreste.\textsuperscript{10} For example, the radial displacement decreases from 18.2 to 15.3 mm if $b = 0$. The reduction is due to the fact that the strength parameters of strain-softening model are larger than those of the elastic–brittle–plastic model in some annuli of the plastic zone. Therefore, this model is more accurate than the elastic–brittle–plastic model.

**Effects of the bolt density parameters**

Figure 10(b) illustrates that the stress grows with the increasing bolt densities at the same point. The plastic radius would drop correspondingly for a larger $b$. For example, the plastic radius of the proposed model would be reduced by 10% if $b$ increases from 0.145 to 0.291. Moreover, we can see from Figure 18 that tangential strain reduced sharply when $b$ increased from 0 to 0.291 at the same point, and the strength of the surrounding rock reinforced by the grouted rock bolts is improved. Therefore, the effect of the bolt density parameters is significant for the surrounding rock reinforcement.

Several examples are conducted to highlight the effects of bolt density parameters that incorporate the neutral point of the grouted rock bolts on stress and strain.

To examine the effects of bolt density parameters under M–C failure criterion, three cases are performed: (1) $\beta = 0$, $\gamma_p = 0.008$; (2) $\beta = 0.145$, $\gamma_p = 0.008$; and (3) $\beta = 0.291$, $\gamma_p = 0.008$. The input data are the same as those listed in section “Validations.”
The results are displayed in Figures 13.

For several given bolt densities, the radial displacements with different hydrostatic pressures and internal support pressures are shown in Figure 14(a) and (b). The tendency that the displacement of the reinforced surrounding rock decreases with the β increasing is provided in Figure 14. For example, the displacement reduction is 25% as β increases from 0 to 0.145 for $\sigma_0 = 12$ MPa. The displacement of the reinforced surrounding rock corresponding to a larger β is smaller. One change that the displacement decreases about 55% as β increases from 0 to 0.291 for $p_{wp} = 0.4$ MPa can be seen. As shown in Figure 14, the radial displacement decreases with the increasing internal support pressure. So the bolt density β has a significant effect on the surrounding rock reinforcement.

Based on the H–B failure criterion, two cases are carried out corresponding to (1) $\beta = 0$ and $\gamma_p = 0.047$
and (2) $\beta = 0.132$ and $\gamma_p = 0.0047$ to examine the effects of the bolt density parameters. The input data are listed in section “Validations.”

The results are displayed in Figure 15(a)–(c).

It can be seen from Figure 15(a) that stress increases with the increasing bolt densities at the same point. The plastic radius would be reduced correspondingly for a larger $\beta$. For example, the plastic radius of the proposed model would be reduced from 4.35 to 3.75 m if $\beta$ increases from 0 to 0.132. Therefore, the effects of the bolt density parameters are significant in the surrounding rock reinforcement. With different hydrostatic pressures, displacements in the tunnel wall and the equivalent plastic radius vary with different bolt density parameters as shown in Figures 15(b) and 15(c). It can be seen that the greater the bolt density parameters, the smaller the radial displacements in the tunnel wall and equivalent plastic radius. For example, the displacement decreases from 25 to 15.5 mm as $\beta$ increases from 0 to 0.325 for $\sigma_0 = 14$ MPa.

As shown in Figure 15(d), the radial displacement in the tunnel wall decreases when the increasing internal support pressure increases. The displacement of the reinforced surrounding rock corresponding to a larger $\beta$ is smaller. For instance, the displacement decreases about 30% as $\beta$ increases from 0 to 0.25 for $p_{in} = 0$ MPa. Therefore, the effect of the bolt density on the surrounding rock reinforcement is great.

**Effects of the strain-softening parameters**

To identify the effect of $\gamma_p$ on the stress, strain, and displacement field under M–C and H–B failure criteria, three different $\gamma_p$ are analyzed, respectively. Meanwhile, several examples are taken to explain the effect.

It can be seen from Figures 16(a) and 17(a) that the stress increases with the increase in $\gamma_p$ at the same point. The plastic radius would be reduced correspondingly for a larger $\gamma_p$. Moreover, the plastic radius would be reduced by 10% for $\beta = 0.291$. Figures 16(a) and 17(a) and (b) illustrate the reduction in tangential strain and radial displacement for different $\gamma_p$, respectively. Figure 17(b) reflects that the radial displacement decreases from 14 to 11.5 mm if $\gamma_p$ increased from 0 to 0.008.

Hence, it would be easy to note that $\gamma_p$ has a significant effect on the surrounding rock reinforcement. The effects of $\gamma_p$ on the stress are significant, and the equivalent plastic radius decreases with the increase in $\gamma_p$. For example, the equivalent plastic radius for $\gamma_p = 0$ decreases from 4.05 to 3.5 m if $\gamma_p$ is equal to 0.008.

**Comparison with in situ monitoring data**

**General situation of engineering.** Sanchahe Tunnel is located in the slope transition zone between Yunnan Guizhou Plateau and Guangxi hilly. The pile number of the left tunnel of Sanchahe Tunnel is ZK74 + 277–ZK75 + 245, the surrounding rock of the tunnel is mainly the plastic gravel containing silty clay, the length is 968 m, and the maximum buried depth is about 91.3. The zone ZK74 + 277–ZK75 + 615 is the key monitoring object; because the surrounding rock is soft and the stability is pretty poor, the surrounding rock properties are poor, such as joint and fissure, rock mass fragmentation, and the shape of the loose soil. The vertical section profile of ZK74 + 277–ZK75 + 615 is shown in Figure 18.

**Measurement of radial displacement.** The ground deformation of this tunnel is monitored and measured by multi-point borehole extensometers. Multi-point borehole extensometers are installed within the tunnel ground.
and they measure the displacement of the tunnel wall relative to the fixed end-point of each borehole extensometer along the direction of radius (Figure 19(a)).

The scheme of radial displacement measurement is presented as follows:

1. First, determination of the location and depth of drilling; next, drilling and instrument installation; finally, grouting and sealing.
2. The measurement section locates in a representative geological section; we chose ZK74 + 290, where the surrounding rock is soft and the stability is pretty poor.
3. To facilitate the calculation and analysis, each measurement section should be laid on eight multi-point borehole extensometers around the tunnel wall; each borehole extensometer could measure six points along the radius. And the arrangements of multi-point borehole extensometers should be close to the anchor.
4. The measurement frequency is once a day.
5. Calculate the radial displacements of six points along the radius by averaging the measured data of eight multi-point borehole extensometers.

Result analyses. Based on geological investigation report, the recommended parameters of rock mass are shown as follows: $a = 5.6 \text{ m}$, $L = 3.8 \text{ m}$, $E = 1500 \text{ MPa}$, $\nu = 0.35$, $\sigma_0 = 15 \text{ MPa}$, $\sigma_C = 3.5 \text{ MPa}$, $P_{\text{in}} = 1.5 \text{ MPa}$, $\gamma_p = 0.008$, $m_p = 2.0$, $a_p = 0.50$, $s_p = 0.0045$, $m_r = 1.0$, $a_r = 0.55$, and $s_r = 0.0025$; these are estimated approximately. In order to verify the accuracy of the model, we collected the monitoring data about the relationship between the radial displacement and the radius.

Figure 15. Results for different $\beta$ based on the H–B failure criterion: (a) stress field $\beta$, (b) equivalent plastic radius, and (c) ground reaction curves.
Based on the M–C failure criterion, the results of the improved model that considering the bolt density $b$ and the neutral point of the grout rock bolts and the strain-softening model are compared with the monitoring data of Sanchahe Tunnel. As shown in the figure, for the relationships between the radial displacements and the radius for the case of $b = 0.18$, when $r/a = 2$, the calculation result is slightly larger than the monitoring result, which represents that the model is reliable. However, when $r/a < 2$, the monitoring result is much smaller than the calculation result, most probably because of the following: $s_0 = 15$ MPa and $P_{in} = 1.5$ MPa are given by substantial engineering experience; not the real value, but there are some temporary supports for tunnel wall to reduce the radial displacement; the deformation of multi-point borehole extensometers is not coordinated with the surrounding rock deformation.

Based on the generalized H–B failure criterion, for the relationships between the radial displacements and the radius for the case of $\gamma_p = 0.08$, when $r/a < 2$, the tendency of the calculation curve is nearly the same as the monitoring curve, which represents that the model is also reliable. Comparing Figure 20 with Figure 21, the curve of the calculation result based on the M–C failure criterion is more consistent with the monitoring data curve. This is because the strength parameters of the generalized (H–B) failure criteria are estimated approximately, which can cause slight differences with the actual value.

**Conclusion**

Incorporating the strain-softening characteristic of the surrounding rock and the neutral point of the grouted rock bolts, the equivalent plastic zone is divided into three categories (i.e. minimal yielding, major yielding, and excessive yielding). The potential plastic zone is sufficiently subdivided into a large number of
concentric annuli; the bolt density parameters are introduced, the solutions of stress and displacement of circular tunnel are presented, and the corresponding calculation approach is proposed as well. Based on the elastic–brittle–plastic model, the proposed model is validated by the special case as $\gamma_p = 0$. Moreover, the stress presented by the proposed model is larger than those presented by elastic–brittle–plastic model at the same point. The comparison results show that the radial displacements in the tunnel wall and the plastic radius will be effectively reduced with the increase in the bolt density parameter $\beta$ and strain-softening parameter $\gamma_p$.

In practical tunnel engineering, the proposed approach can be widely used to evaluate the reinforcement effectiveness due to its simple practicality. In our

**Figure 18.** The vertical section profile of Sanchahe Tunnel: (a) the vertical section profile of Sanchahe Tunnel at ZK74+277-75+275 and (b) the vertical section profile of Sanchahe Tunnel at ZK74+895-ZK75+275.

**Figure 19.** The in situ monitor site: (a) installation of multi-point borehole extensometers and (b) arrangement of measuring points.

**Figure 20.** Comparison based on M–C failure criterion.
future work, we will compare our results to in situ monitoring data. Furthermore, the model can be extended to a similar problem in other projects, such as slope engineering, foundation pit engineering, and mining engineering.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors are grateful to the 973 Program (2013CB036004) and National Natural Science Foundation of China (no. 51208523).

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Appendix 1

Notation

\[ a \] radius of the tunnel opening [L]
\[ a_p \] parameter of H–B failure criterion for peak strength [-]
\[ a_r \] parameter of H–B failure criterion for residual strength [-]
\[ c \] cohesive strength
\[ d \] diameter of bolt [-]
\[ e \] elastic parts of strain [-]
\[ E \] Young’s modulus of the rock mass [FL^{-2}]
\[ m \] parameter of H–B failure criterion for peak strength [-]
\[ m_r \] parameter of H–B failure criterion for residual strength [-]
\[ m^* \] parameter of H–B failure criterion for peak strength after reinforced [-]
\[ m_r^* \] parameter of H–B failure criterion for residual strength after reinforced [-]
\[ N' \] parameter of M–C failure criterion for weak values [-]
\[ N'^* \] parameter of M–C failure criterion for peak strength after reinforced [-]
\[ N_r^* \] parameter of M–C failure criterion for residual strength after reinforced [-]
\[ m \] parameter of M–C failure criterion for weak values [-]
\[ m^* \] parameter of M–C failure criterion for peak strength after reinforced [-]
\[ m_r^* \] parameter of M–C failure criterion for residual strength after reinforced [-]
\[ p \] plastic parts of strain [-]
\[ p_{in} \] internal support pressure [FL^{-2}]
\[ r \] radial distance from the center of the tunnel opening [L]
\[ R^* \] equivalent plastic zone radius [L]
\[ s \] parameter of H–B failure criterion for peak strength [-]
\[ s_r \] parameter of H–B failure criterion for residual strength [-]
\[ s^* \] parameter of H–B failure criterion for peak strength after reinforced [-]
\[ s_r^* \] parameter of H–B failure criterion for residual strength after reinforced [-]
\[ S_L \] longitudinal spacing along the axial of the tunnel [L]
\[ S_T \] axial spacing between the bolts [L]
\[ U \] radial displacement [L]
\[ Y' \] parameter of M–C failure criterion for weak values [-]
\[ Y_r^* \] parameter of M–C failure criterion for residual strength after reinforced [-]
\[ \beta \] bolt density parameter [-]
\[ \gamma^p \] deviatoric strain [-]
\[ \gamma_r^p \] critical deviatoric plastic strain [-]
\[ \epsilon_r \] radial strain [-]
\[ \epsilon_0 \] circumferential stain [-]
\[ \epsilon_0^p \] major plastic strains [-]
\[ \epsilon_3 \] minor plastic strains [-]
\[ \eta \] constant of proportionality (coupling constant).
\[ \lambda' \] friction factor [-]
\[ v \] Poisson’s ratio of the rock mass [-]
\[ \rho_0 \] location of the neutral point [L]
\[ \sigma_c \] uniaxial compressive strength of the rock [FL^{-2}]
\[ \sigma_c^* \] parameter of H–B failure criterion for peak strength after reinforced [-]
\[ \sigma_{cr}^* \] parameter of H–B failure criterion for residual strength after reinforced [-]
\[ \sigma_r \] radial stress [FL^{-2}]
\[ \sigma_{R} \] radial stress at the interface between the elastic rock and the equivalent plastic zone [FL^{-2}]
\[ \sigma_0 \] circumferential stress [FL^{-2}]
\[ \sigma_0 \] hydrostatic stress [FL^{-2}]
\[ \sigma_1 \] major principal stresses [FL^{-2}]
\[ \sigma_3 \] minor principal stresses [FL^{-2}]
\[ \tau_2 \] shear stress along the bolt [FL^{-1}]
\[ \phi \] internal friction angle
\[ \varphi \] dilation angle [-]