Comparing parton energy loss models

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Abstract
The similarities and differences between three commonly used formalisms for radiative parton energy loss in hot strongly interacting matter are discussed. The single gluon emission spectra are evaluated for a model system consisting of a homogeneous medium with a fixed length, the ‘TECHQM brick’. Sizable quantitative differences are found and the origins of these differences are discussed.

Keywords: parton energy loss, Quark-Gluon Plasma, QCD

One of the most striking experimental results from high-energy nuclear collisions at the Relativistic Heavy Ion Collider (RHIC) is that hadron production at high transverse momentum \( p_T \) is suppressed by a factor 4–5 compared to expectations from an independent superposition of nucleon-nucleon collisions \([1–4]\). High-\( p_T \) hadrons are dominantly produced by fragmentation of high-\( p_T \) partons from hard scatterings in the early stage of the collision. The suppression of hadron production is generally attributed to energy loss of the high-momentum partons as they propagate through the hot and dense medium.

The dominant parton energy loss mechanism is expected to be radiative energy loss, although collisional energy loss also plays a role \([5, 6]\). These processes have been extensively modeled with the intent to use the measured suppression to determine medium properties like the transport coefficient or (energy) density \([5–10]\). It has been found, however, that different energy loss calculations lead to medium densities which differ by a large factor \([10]\).

There are four widely used formalisms to calculate radiative parton energy loss in hot and dense QCD matter. Each formalism uses a different set of approximations. In this contribution, three of the four formalisms will be compared using a model problem: the ‘brick problem’, as formulated by TECHQM (Theory-Experiment Collaboration in Hot Quark Matter). It turns out that the main quantitative differences between the models can be attributed to specific differences in the model assumptions and the approximations made in the calculation. The most important differences are related to the treatment of large angle radiation and assumptions about the importance of interference effects between vacuum- and medium-induced radiation and between scatterings in the medium.

1. Energy loss formalisms

The term radiative energy loss refers to gluon emission by a fast quark or gluon which is stimulated by the presence of a colored medium. It is important to realise that energetic partons also radiate in vacuum, as part of the jet fragmentation process which leads to hadronisation. Medium-induced radiation is the additional radiation that is stimulated by elastic and inelastic scatterings of the parton in the medium.

Three of the four parton energy loss formalisms can be related to a common path integral formalism, that was formulated by Baier, Dokshitzer, Mueller, Peigné and Schiff (BDMPS) \([11,12]\) and independently by Zakharov \([13]\).
The energy loss calculation is generally split into two parts. First, the single-emission spectrum is calculated using the path integral or equivalent formulas. Multiple gluon emission is calculated using an independent emission ansatz, where the number of gluons is the integral over the single-gluon spectrum. In the following discussion, we will focus on the single-gluon spectrum.

BDMPS evaluated the resulting energy loss in the approximation where the medium consists of static scattering centers and the projectile and the outgoing gluon undergo many soft scatterings with the medium (multiple-soft scattering approximation). Technically, this corresponds to using the saddle point approximation of the corresponding path integral. The original BDMPS calculation takes a limit in which the medium length goes to infinity. Later, Salgado and Wiedemann (SW) \[14, 15\] improved the calculation to include finite-length effects. Multiple gluon emission is calculated using a poisson ansatz.

The energy loss path integral can also be evaluated using an opacity expansion, i.e. by organising the calculation in terms of the number of dominant scatterings. This calculation was proposed by Gyulassy, Levai, and Vitev (GLV) \[16\] and independently by Wiedemann \[17\]. In most applications, the further approximation is made that only one scattering dominates the dynamics (single hard scattering approximation). Multiple gluon emission is calculated using a poisson ansatz.

Arnold, Moore, and Yaffe used a similar path integral, but evaluate the entire problem using Hard-Thermal Loop improved finite temperature field theory \[13\]. In this calculation, the full multiple scattering is taken into account and the medium is a thermalised Quark-Gluon plasma with dynamical scattering centers. For this calculation, only results in the infinite-length limit exist. Multiple gluon emission is calculated using coupled rate equations.

The fourth commonly used parton energy loss formalism is the Higher-Twist formalism, as formulated by Wang and Guo \[19, 20\]. In this formalism the medium properties enter as gluon field strength correlators into a higher twist calculation of gluon radiation. Multiple gluon emission is calculated using an extension of the DGLAP evolution, which is a well-established theoretical tool for parton fragmentation.

2. Large angle radiation

All of the currently used radiative energy loss formalisms have been derived in the soft collinear limit, in which the transverse momentum of the radiated gluon $k_\perp$ is much smaller than the total gluon momentum $k$, which in turn
is much smaller than the energy \( E \) of the incoming parton \( k_\perp \ll k \ll E \). The calculated gluon radiation spectra, however, have significant radiation probabilities for large angles, leading to important uncertainties in the calculated gluon rates.

To illustrate this, consider the gluon radiation spectrum from the first order opacity expansion

\[
x \frac{dN}{dk_\perp^2 dx} = \frac{2C_R \alpha_s}{\pi} \frac{L}{\lambda} \frac{1}{k_\perp^2} \int_0^{q_{\text{max}}} d^2q \frac{\mu^2}{2 \pi (q^2 + \mu^2)^2} \times \frac{2 k \cdot q (k - q)^2 L^2}{16 x^2 E^2 + (k - q)^4 L^2},
\]

where \( L \) is the length of the medium, \( \mu \) the Debye screening mass, \( E \) the parton energy, \( k \) the transverse momentum vector of the emitted gluon, and \( q \) the transverse momentum vector exchanged with the medium. The momentum fraction \( x \) is taken to be the light-cone fraction \( x = k^+/E^+ \) in the work by Gyulassy, Levai, and Vitev [16], while it is taken to be a fraction in Minkowski space \( x \approx k/E \) by Salgado and Wiedemann [15]. Figure 1 shows the radiation spectrum Eq. (1) for a medium with length \( L = 5 \) fm and a temperature \( T = 300 \) MeV (\( \mu^2 = g^2 T^2 = 0.34 \) GeV\(^2\), \( \lambda = 1.0 \) fm, \( q_{\text{max}} = \sqrt{3\mu T} \)). The emitted gluon spectrum peaks at small gluon momentum \( k \) and at finite transverse momentum \( k_\perp \sim \mu \). Also shown in the figure are lines indicating the kinematic limits (perpendicular radiation) \( k_\perp = k \) and \( k_\perp = k^+ \) in Minkowski and light-cone coordinates. These limits are different because \( x^+ \approx x_E \) and \( k \approx 1/2 k^+ \) only in the small-angle limit, and the difference between \( k \) and \( 1/2 k^+ \) becomes significant for large-angle radiation.

The right panel of Fig. 2 shows the gluon radiation spectrum as a function of gluon momentum \( k \) or half the light-cone momentum \( k^+ \) which is obtained by integrating the double-differential spectrum over \( k_\perp \) within the kinematic limits. A large difference in the radiation probability at small momenta \( k \) results from the choice of Minkowski or light-cone coordinates. This large difference is a measure of the uncertainty associated with applying the calculation beyond the soft-collinear limit for which it was derived. The associated uncertainty is large because the calculated radiation spectrum has significant intensity at large angles. This was qualitatively pointed out already in [14]. For a more extensive exploration of the differences between the opacity expansion formalism used by GLV and SW, see [21].

### 3. AMY and the multiple soft scattering approximation

Both the AMY formalism [18] and the multiple-soft scattering formalism formulated by BDMPS [11] and later extended by SW (ASW-MS formalism) [14, 15] start from a path-integral formalism in which all scatterings in the
medium are taken into account. The main conceptual difference between the two calculations is that the AMY formalism uses Hard Thermal Loop field theory, which implies an equilibrated Quark-Gluon Plasma at high temperature as the medium model, while BDMPS use static scattering centers [22].

However, in addition to this conceptual difference, there are important implementation choices which affect the gluon emission rates. This is illustrated in Fig. 2 where the gluon emission spectra for AMY, BDMPS and ASW-MS are compared for two path lengths (L = 2 and 5 fm). For this comparison, the transport coefficient $\hat{q}$ which is used to set the medium density in the BDMPS and ASW-MS calculations was calculated using the $q \ll T$ limit of the HTL scattering rate $d\Gamma/d^2q = C_R/(2\pi)^2 g\mu^2 T/q^2(q^2 + \mu^2)$, which is also used in the AMY calculation.

Comparing first the AMY (green dash-dotted line in Fig. 2) curves with the BDMPS (dashed) calculation, one sees that the results are similar for $L = 5$ fm, while for short path lengths ($L = 2$ fm), AMY generates more radiation. This is due to the fact that AMY uses the limit $L \to \infty$ which ignores the interference between radiation in the vacuum and the medium, and the increased effect of finite formation times at small length [23]. The ASW-MS calculation (blue curve in Fig. 2) is based on the BDMPS formalism, but takes into account finite-length effects. These effects include the interference between medium-induced and vacuum radiation as well as the large-angle cut-off described in the previous section, although these aspects are not easily separated in the calculation [14]. The effect of the finite-length corrections is a pronounced reduction of the radiation at small gluon energies $k$, which is similar to the effect seen in Fig. 1.

4. Putting everything together

Figure 3 shows the gluon radiation for two variants of the opacity expansion (DGLV and ASW-SH), the HTL-based calculation (AMY), and the multiple-soft scattering approximation (ASW-SH). The band for the multiple-soft scattering approximation indicates the uncertainty associated with the calculation of the transport coefficient $\hat{q}$ from the temperature $T$. The figure clearly shows that the AMY calculation produces the most radiation, while the multiple-soft scattering approximation (ASW-MS) gives the least. The results form the and the opacity expansions (ASW-SH and DGLV) are between those extremes. The differences are also path-length dependent.

The largest differences occur at small $k$, where the effect of large-angle radiation is most pronounced. The AMY calculation does not take into account the kinematic bound in this regime, which leads to the diverging behavior as
The other calculations implement the kinematic bound, but the associated uncertainties are large, because the emission rate near the boundary can be large (see Fig. 1).

A more detailed discussion of the path-length dependence of the emission can be found in [24], where the different calculations are compared to a numerical solution of the full path integral. In that paper one can clearly see that for short path lengths the $N = 1$ opacity expansion provides the most accurate result, while the AMY calculation overestimates the radiation and the BDMPS/ASW-MS calculation underestimates the radiation (see also [24]). For longer path lengths, on the other hand, the $N = 1$ opacity expansion produces too much radiation, because it does not take into account the interference between scattering centers, while the AMY and BDMPS/ASW-MS results are in good agreement with the path-integral formalism.

5. Conclusion and outlook

The side-by-side comparisons of single gluon emission spectra from medium-induced parton energy loss using three different energy loss formalisms as presented here clearly show sizeable differences in the emission rates.

Some of the differences are due to incomplete treatment of interference effects, either between medium-induced and vacuum radiation (AMY) or between scattering centers ($N = 1$ opacity expansion). Such approximations may be appropriate in limits of short or long path lengths. A quantitative evaluation of the radiation process in heavy ion collisions, however, requires calculations for short as well as long path length. Future calculations should therefore aim to include all interference terms that are known. It also important to include a realistic scattering potential, that includes reasonable probabilities for large momentum transfer, which reduces the formation time significantly [24]. This is currently done in the opacity expansions and AMY, but not in the multiple soft scattering approximation ASW-MS.

Another important source of uncertainty in the current calculations is the collinear approximation that is used the derivation of the radiation spectra. All calculations, except AMY, implement a cut-off to limit radiation to the kinematic bound at large angles ($k_\perp < k$), but the underlying formalisms generate large probability for large-angle radiation at small momenta, thus violating the soft collinear approximation. This could be systematically addressed using calculations that go beyond the collinear limit. Some work in this direction already exists, mostly in the form of Monte-Carlo approaches which can in principle use the full 2-to-3 matrix elements instead of the collinear limit for in-medium scattering and radiation [25–27].

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