Abstract

We discuss a possible explanation of the 25 year old mystery of the large transverse spin asymmetries found in many semi-inclusive hadron-hadron reactions. We obtain the first reliable information about the transverse polarized quark densities $\Delta_T q(x)$ and we find surprising implications for the usual, longitudinal, polarized DIS. The plan of the presentation is as follows: 1) A brief reminder about the internal structure of the nucleon. 2) The transverse asymmetries. 3) Why it is so difficult to explain the asymmetries. 4) Failure and then success using a new soft mechanism. 5) Implications for polarized DIS.
1 Internal structure of the nucleon at the parton level

For each quark there are three kinds of number densities:

a) The usual $q(x)$

$$ p = xP $$

$q(x)$ is the number density of quarks with momentum fraction in the range $x \leq p/P \leq x + \Delta x$. This is mostly measured in DIS.

b) The longitudinal polarized density $\Delta q(x)$

$$ q_{\pm}(x) \text{ is the number density at } x \text{ with spin } s \text{ along (+) or opposite (-) to the spin } S (\rightarrow) \text{ of the nucleon. The new density is}$$

$$ \Delta q(x) = q_+(x) - q_-(x) . \quad (1) $$

It is measured in DIS using a longitudinally polarized nucleon target.

c) The transverse polarized density $\Delta_T q(x)$

$$ q_{\uparrow\downarrow}(x) \text{ are the number densities at } x \text{ with transverse spin } s \text{ along (\uparrow) or opposite (\downarrow) to the transverse nucleon spin } S (\uparrow\downarrow). \text{ The new density is}$$

$$ \Delta_T q(x) = q_{\uparrow}(x) - q_{\downarrow}(x) . \quad (2) $$

Note that $\Delta_T q(x)$ cannot be measured in DIS with a transversely polarized target; $g_2(x)$ does not tell us anything about $\Delta_T q(x)$.

In summary there are 3 independent functions, all equally fundamental, describing the internal structure of the nucleon: $q(x)$, $\Delta q(x)$ and $\Delta_T q(x)$. 
How can we measure $\Delta_T q(x)$? The ideal reaction would be Drell-Yan using transversely polarized beam and target, but this has never been done. It is one of the prime aims at RHIC. Can one use semi-inclusive hadron-hadron reactions with a transversely polarized target? At first sight, yes. At second sight, no. And finally, yes, but one has to introduce a new theoretical idea and thereby it seems possible to resolve the ancient puzzle of the large transverse spin asymmetries.

2 The transverse spin asymmetries

There is a mass of data on reactions of the type $A^\uparrow + B \rightarrow C + X$ for which the asymmetry $A_N$ under the reversal of the transverse spin is measured:

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}.$$  

Some examples are shown in Fig. 1 for $p^\uparrow p \rightarrow \pi X$ and $\overline{p}^\uparrow p \rightarrow \pi X$ respectively. From looking at many reactions one concludes that:
- the asymmetries are large!
- they increase with $p_T$
- they increase with $x_F$
- they seem independent of energy
- they occur in a variety of reactions.

For decades there has been no serious theoretical explanation and, as we shall see, the standard approach via perturbative QCD gives $A_N = 0$.

Figure 1: Single spin asymmetry for $p^\uparrow p \rightarrow \pi X$ versus $x_F$ and $\overline{p}^\uparrow p \rightarrow \pi X$ versus $p_T$, both at 200 GeV$^2$. Diamonds correspond to $\pi^+$, squares to $\pi^0$ and triangles to $\pi^-$. 
3 Why it is difficult to explain the asymmetries

The standard parton model picture for $A^\uparrow + B \rightarrow C + X$ at large momentum transfer is

$$A \uparrow + B \rightarrow C + X.$$ (3)

The hadronic $A_N$ depends upon the asymmetry $\hat{a}_N$ at the parton level, i.e. the asymmetry in

$$q_a^\uparrow + q_b \rightarrow q_c + q_d.$$ (4)

But

$$\hat{a}_N \propto \text{Im}\{(\text{Helicity Non-flip})^* (\text{Single flip})\}.$$ (5)

In lowest order this is doubly zero: there is no helicity flip and the amplitudes are real. Going to higher order doesn’t help. One finds, if one takes $m_q \neq 0$,

$$\hat{a}_N = \alpha_s \frac{m_q}{\sqrt{s}} f(\theta^*)$$ (6)

which gives asymmetries of less than 1%.

4 New soft mechanism

Consider, for concreteness, the reaction $p^\uparrow p \rightarrow \pi^\pm X$. Let us concentrate only on the partons in the polarized proton and follow them through the partonic diagram. We assume that the $\pi$’s come mainly from the fragmentation of quarks. The notation is the following: $f_{q/p}$ is the number density of $q$ in $p$ and $D_{\pi/q}$ the number density of $\pi$ in the fragmentation of $q$. 
Proceeding blindly to sum over all possible spins of the quarks leads to

\[
d\sigma^{\uparrow} - d\sigma^{\downarrow} = \left[ f_{q_\uparrow/p_\uparrow} - f_{q_\downarrow/p_\downarrow} \right] \cdot \hat{\sigma} \cdot D^{\pi/q} + \\
+ \left[ f_{q_\uparrow/p_\downarrow} - f_{q_\downarrow/p_\uparrow} \right] \cdot \Delta \hat{\sigma} \cdot \left[ D^{\pi/q}(s_c) - D^{\pi/q}(-s_c) \right]
\]

where \( D^{\pi/q} \) is the usual, unpolarized fragmentation function, and \( \Delta \hat{\sigma} \) will be defined presently; \( s_c \) is the polarization vector of quark \( c \). The key question is: which, if any, of these terms are non-zero?

a) With usual collinear kinematics

\[
f_{q/p_\uparrow} - f_{q/p_\downarrow} = 0
\]

Reason?

There are only two independent vectors, \( P \) and the pseudovector \( S \). We cannot construct a scalar which depends on \( S \). Similarly

\[
D^{\pi/q}(s) - D^{\pi/q}(-s) = 0
\]

Again, we cannot construct a scalar from the vector \( P \) and the pseudovector \( s \).

Thus both terms in Eq. (7) vanish in the collinear kinematics and \( A_N = 0 \).

b) With intrinsic transverse momentum

Now, apparently, we could have

\[
f_{q/p}(x, k_T) = f(x, k_T) + \tilde{f}(x, k_T) S \cdot (P \times k_T)
\]
implying

$$f_{q/p} - f_{q/p} \neq 0$$  \hspace{1cm} (11)

This mechanism was proposed by Sivers \cite{2} and further studied in \cite{3}. However, it violates time-reversal invariance, so we shall take the first term in Eq. (7) to be zero. Strangely, the analogous mechanism for the fragmentation

$$D^{\pi/q}(s) - D^{\pi/q}(-s) \neq 0$$  \hspace{1cm} (12)

does not violate time-reversal invariance. This is the Collins mechanism \cite{4}. Hence Eq. (7) becomes

$$d\sigma^\uparrow - d\sigma^\downarrow = \left[ f_{q_1/p} - f_{q_2/p} \right] \cdot \Delta \sigma \cdot \left[ D^{\pi/q_c}(s_c) - D^{\pi/q_c}(-s_c) \right]$$

$$= \left[ \Delta_T q_a \right] \cdot \left[ \frac{d\sigma}{dt}(a^\uparrow b \to c^\downarrow d) - \frac{d\sigma}{dt}(a^\uparrow b \to c^\downarrow d) \right] \cdot \Delta N D^{\pi/q_c} \cdot (z, k_T^\pi) \cdot (13)$$

In full detail \cite{5}

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \int dx_a dx_b d^2 k_T^\pi q(x_b) \Delta_T q(x_a) \times$$

$$\times \left[ \frac{d\sigma}{dt}(a^\uparrow b \to c^\downarrow d) - \frac{d\sigma}{dt}(a^\uparrow b \to c^\downarrow d) \right] \cdot \Delta N D^{\pi/q_c}(z, k_T^\pi) \cdot (14)$$

where the term $\left[ \frac{d\sigma}{dt}(a^\uparrow b \to c^\downarrow d) - \frac{d\sigma}{dt}(a^\uparrow b \to c^\downarrow d) \right]$ is calculated in PQCD. The result depends on two unknown functions: $\Delta_T q(x)$ and $\Delta N D^{\pi/q_c}$, which we can measure by trying to fit the data.

Now, as stressed earlier, the asymmetries are large, so will demand large values of $\Delta_T q(x)$ and $\Delta N D^{\pi/q_c}$. However, positivity requires that

$$|\Delta N D^{\pi/q_c}| \leq 2D^{\pi/q_c}$$  \hspace{1cm} (15)

and the Soffer bound \cite{6} restricts the magnitude of $\Delta_T q(x)$:

$$|\Delta_T q(x)| \leq \frac{1}{2} [q(x) + \Delta q(x)]$$  \hspace{1cm} (16)

where $\Delta q(x)$ is the usual longitudinal polarized quark density.

How important is the Soffer bound? In Fig. 2 we show a typical picture of $\Delta u(x)$ and $\Delta d(x)$. We see that:
Figure 2: A typical picture of $\Delta u(x)$ and $\Delta d(x)$, from LSS fit to polarized DIS experimental data.

a) $\Delta u(x)$ is positive everywhere, so that
   - $u(x) + \Delta u(x)$ is big
   - RHS of Soffer bound is large
   - not very restrictive on $\Delta_T u(x)$

b) $\Delta u(x)$ is (usually) negative everywhere, so that
   - $d(x) + \Delta d(x)$ is small
   - RHS of Soffer bound is small
   - highly restrictive on $\Delta_T d(x)$

But the measured asymmetries are such that $A_{\pi^+} \simeq -A_{\pi^-}$, so that if the $\pi^+$ come mainly from the $u$-quarks and the $\pi^-$ from $d$-quarks we expect trouble in getting a large enough asymmetry for $\pi^-$. Indeed, if we use the Gehrmann-Stirling (GS) $\Delta u(x)$ and $\Delta d(x)$ to bound $\Delta_T u(x)$ and $\Delta_T d(x)$ we obtain a catastrophic fit to the data (Fig. 3) with $\chi^2_{D.O.F} \sim 25$!

Can we escape this dilemma? There is a surprising escape route!

There is an old PQCD argument that requires for quarks, antiquarks and gluons

$$\frac{\Delta q(x)}{q(x)} \to 1 \text{ as } x \to 1$$

(17)

which implies that all $\Delta q(x)$ must become positive as $x \to 1$. But almost all fits to polarized DIS ignore this condition on the grounds that (i) Eq. (17) is incompatible with DGLAP evolution and that (ii) the data demand a negative $\Delta d(x)$. In fact, these arguments are spurious because (i) DGLAP is not valid as $x \to 1$ where one
So let us try to impose $\Delta q(x)/q(x) \to 1$ as $x \to 1$ in the fits to polarized DIS. In fact, this was done by Brodsky, Burkhardt and Schmidt (BBS) [12], but the treatment was rough and evolution was not included. This was improved upon by Leader, Sidorov and Stamenov (LSS) [11] so as to include evolution and a reasonably good fit to the polarized DIS data was achieved. In Fig. 4 we compare the GS and BBS $\Delta d(x)$. It is clear that the Soffer bound on $\Delta_T d(x)$ will be much less restrictive at large $x$ for the BBS case. Indeed, using the BBS $\Delta q(x)$ to bound the $\Delta_T q(x)$ has a dramatic effect upon our attempts to fit the $\pi^\pm$ asymmetries as can be seen in Fig. 5 where $\chi^2_{D.O.F} = 1.45$. In carrying out the fit [13] we made the following simplifications:

a) The asymmetry is largest at large $x_F \Rightarrow$ large $x$ is important. Therefore we used only $u$ and $d$ quarks.

b) Large $x_F \Rightarrow$ large $z$ in the fragmentation. Hence we assumed $u \to \pi^+$, $d \to \pi^-$ only.

c) The unknown functions were parameterized so that the bounds in Eqs. [13] and [16] are automatically satisfied. Thus we took

$$\Delta_T q(x) = N_q \left[ \frac{x^a (1 - x)^b}{a^a b^b (a + b)^{a+b}} \right] \left\{ \frac{1}{2} [q(x) + \Delta q(x)] \right\}$$

(18)
\[
\Delta_N D(z) = N_F \left[ \frac{z^\alpha (1-z)^\beta}{(\alpha+\beta)^{\alpha+\beta}} \right] \{2 D(z)\}, \tag{19}
\]

where \(N_{q,F}\) are real constants with \(|N_{q,F}| \leq 1\), and the functions in square brackets have modulus \(\leq 1\). The fit to the asymmetry data then determines a range of possible \(\Delta_T u(x)\) and \(\Delta_T d(x)\) as shown in Fig. 6.

Figure 5: Single spin asymmetry for \(p^8p \rightarrow \pi X\) as obtained by using the BBS set of distribution functions. The solid line refers to \(\pi^+\), the dashed line to \(\pi^0\) and the dash dotted line to \(\pi^-\).
5 Implications and conclusions

a) It seems that the soft Collins mechanism can explain the semi-inclusive transverse spin asymmetries if $\Delta_T u(x)$ and $\Delta_T d(x)$ are large enough in magnitude.

b) This, via the Soffer bound, seems to require $\Delta q(x)/q(x) \to 1$ as $x \to 1$.

c) For the $d$-quark this implies that $\Delta d(x)$ must change sign and become positive at large $x$.

d) This, in turn, has a significant effect upon the shape of $g_1^u(x)$ at large $x$. Fig. 7 compares the behavior of $g_1^u(x)$ for the “best” usual fit to the polarized parton densities with that from fits satisfying $\Delta d(x)/d(x) \to 1$. The exciting link between transverse asymmetries and polarized DIS emphasizes the importance of extending that polarized DIS measurements to larger $x$.

e) Some notes of caution:

(i) The Collins mechanism does not seem able to produce large enough $A_N$ at the largest $x_F$ measured. However, there does exist another kind of mechanism, outside the framework of the usual parton model, based on correlated quark-gluon densities in a hadron, which can also produce a transverse spin asymmetry. It may be that a superposition of the two mechanisms is needed.

(ii) For either of these mechanisms $A_N$ must decrease when $p_T^\pi$ becomes much

Figure 6: The allowed range of distribution functions $\Delta_T u(x)$ and $\Delta_T d(x)$ versus $x$, as determined by the fit using the BBS $d^3$ distribution functions. The dotted lines are the boundaries imposed by the Soffer inequality.
Figure 7: The neutron longitudinal asymmetry $A_n^0(x)$, as obtained by using the BBS $\square$ and (LSS)$_{\text{BBS}}$ $\square$ parametrizations (solid and dashed lines respectively), and the LSS parametrizations (dash-dotted line).

greater than the intrinsic $k_T^\pi$. So far there is no sign of such a decrease in the data.

f) Finally, we wish to re-emphasize the beautiful interplay between, at first sight, quite unrelated aspects of particle physics.

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