Large Magellanic Cloud Near-infrared Synoptic Survey. IV. 
Leavitt Laws for Type II Cepheid Variables

Anupam Bhardwaj1,2, Lucas M. Macri3, Marina Rejkuba2, Shashi M. Kanbur4, Chow-Choong Ngeow5, and Harinder P. Singh1
1 Department of Physics & Astrophysics, University of Delhi, Delhi 110007, India; anupam.bhardwaj@gmail.com, abhardwaj@eso.org
2 European Southern Observatory, Karl-Schwarzschild-Straße 2, D-85748, Garching, Germany
3 Mitchell Institute for Fundamental Physics & Astronomy, Department of Physics & Astronomy, Texas A&M University, College Station, TX 77843, USA
4 State University of New York, Oswego, NY 13126, USA
5 Graduate Institute of Astronomy, National Central University, Jhongli 32001, Taiwan

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Abstract

We present time-series observations of Population II Cepheids in the Large Magellanic Cloud at near-infrared (JHKs) wavelengths. Our sample consists of 81 variables with accurate periods and optical (VI) magnitudes from the OGLE survey, covering various subtypes of pulsators (BL Herculis, W Virginis, and RV Tauri). We generate light-curve templates using high-quality I-band data in the LMC from OGLE and Ks-band data in the Galactic bulge from VISTA Variables in Via Láctea survey and use them to obtain robust mean magnitudes. We derive period–luminosity (P–L) relations in the near-infrared and Period–Wesenheit (P–W) relations by combining optical and near-infrared data. Our P–L and P–W relations are consistent with published work when excluding long-period RV Tauris. We find that Pop II Cepheids and RR Lyraes follow the same P–L relations in the LMC. Therefore, we use trigonometric parallax from the Gaia DR1 for VY Pyx and the Hubble Space Telescope parallaxes for k Pav and 5 RR Lyrae variables to obtain an absolute calibration of the Galactic Ks-band P–L relation, resulting in a distance modulus to the LMC of µL,M C = 18.54 ± 0.08 mag. We update the mean magnitudes of Pop II Cepheids in Galactic globular clusters using our light-curve templates and obtain distance estimates to those systems, anchored to a precise late-type eclipsing binary distance to the LMC. We find that the distances to these globular clusters based on Pop II Cepheids are consistent (within 2σ) with estimates based on the Mv − [Fe/H] relation for horizontal branch stars.

Key words: distance scale – Magellanic Clouds – stars: variables: Cepheids

Supporting material: machine-readable tables

1. Introduction

Classical Cepheid variables are Population I stars used as standard candles for the extragalactic distance scale, thanks to their high luminosities and a well-defined period–luminosity relation (PLR) (the “Leavitt Law”; Leavitt & Pickering 1912). They are the primary distance indicator used in the most accurate and precise determination of the Hubble constant to date (Riess et al. 2016). Type II Cepheids (hereafter T2Cs) are low-mass, Pop II stars that can be found in globular clusters and disk, bulge, and halo environments (Wallertzer 2002; Sandage & Tammann 2006). Classical Cepheid variables and T2Cs follow different PLRs, with the latter more than a magnitude fainter than the former at similar periods. T2Cs are further classified based on their periods as BL Herculis (BLH, 1 days ≤ P ≤ 4 days), W Virginis (WVI6, 4 days ≤ P ≤ 20 days), and RV Tauri (RVT, P > 20 days). The classification of RVT is often ambiguous because they show irregular light curves, but they are considered a subtype of T2Cs (Sandage & Tammann 2006; Feast et al. 2008). Soszyński et al. (2008) suggested another subtype, peculiar W Virginis (PWX, 4 days ≤ P ≤ 10 days), with distinct light curves that are mostly brighter and bluer than WVI.

The PLRs of T2Cs at optical bands have been studied extensively (Nemec et al. 1994; Alcock et al. 1998; Kubiak & Udalski 2003; Majaess et al. 2009; Schmidt et al. 2009, and references therein). These relations exhibit possible nonlinearities, which, coupled with fainter absolute magnitudes (relative to Classical Cepheids), limits their use as potential distance indicators. The third phase of the Optical Gravitational Lensing Experiment (OGLE-III) presented optical light curves and PLRs for T2Cs in the Galactic bulge and the Magellanic Clouds (MCs) in Soszyński et al. (2008, 2010, 2011). They found that bulge T2Cs are dominated by short-period BLH stars, which are more luminous than their counterparts in the MCs, and that LMC RVT stars lie above the PLR followed by the shorter-period classes. Theoretical studies of T2Cs based on pulsating models and evolutionary calculations by Bono et al. (1997) found that their masses decrease with increasing period and they follow a period–luminosity–amplitude relation in the B band.

Over the past decade, the increased availability of large-format and higher-quality near-infrared (NIR) detectors has made it possible to study increasingly larger samples of Cepheids at longer wavelengths, where PLRs are less sensitive to extinction and metallicity (Madore & Freedman 1991). Matsumaga et al. (2006, 2009, 2011) derived NIR PLRs for T2Cs in Galactic globular clusters (GGCs) and the MCs. The authors found evidence for different slopes in the PLRs of each system, as well as a varying frequency of each subtype. Feast et al. (2008), Groenewegen et al. (2008), and Ciechanowska et al. (2010) discussed the application of NIR PLRs of T2Cs, and Groenewegen et al. (2008) estimated a distance to the Galactic center of R0 = 7.94 ± 0.37 kpc using these variables. Recently, Ripepi et al. (2015) presented JK observations of T2Cs in the MCs from the VISTA survey of the Magellanic Clouds system (VMC).
This paper is the fourth in a series of articles based on observations obtained by the Large Magellanic Cloud Near-infrared Synoptic Survey (LMCNISS; Macri et al. 2015, hereafter Paper I). Paper I presented survey details and the absolute calibration of NIR PLRs for Classical Cepheids. Bhardwaj et al. (2016a, 2016b, hereafter Papers II and III) derived PWRs for Classical Cepheids, studied possible nonlinearities in the Leavitt Laws, and estimated Cepheid-based distances to Local Group galaxies. In this paper we focus on the NIR observations of T2Cs and their corresponding relations.

The rest of the paper is structured as follows: Section 2 describes the observations, data reduction, and photometry of T2Cs; Section 3 discusses the variation of light-curve parameters as a function of period and wavelength and the construction of templates; Section 4 contains the derivation of NIR PLRs and PWRs, a comparison to published work, and an estimate of the distance to the LMC; Section 5 presents template fits to observations of T2Cs in GGCs and the resulting distance estimates; Section 6 summarizes our results.

2. The Data

In Paper I we carried out a time-series survey of 18 deg² in the central region of the LMC at $JHK_s$ wavelengths using the 1.5 m telescope at the Cerro Tololo Inter-American Observatory and the CPAPIR camera. Observations were carried out in queue mode by the SMARTS consortium during 32 nights from 2006 November to 2007 November. The survey products include measurements for more than $3.5 \times 10^6$ sources, including $\sim 1500$ Classical Cepheids. Interested readers should refer to Paper I for details of the data reduction, time-series photometry, magnitude calibration, and artificial star simulations used to derive crowding corrections. Given the fainter nature of T2Cs, their crowding corrections were more significant than those of Classical Cepheids and ranged from 0.001 mag to 0.10, 0.08, and 0.27 mag in $JHK_s$, respectively.

We cross-matched the LMCNISS catalog against OGLE-III (Soszyński et al. 2008) and identified 81 T2Cs with periods ranging from 1 to 68 days; 70 of these have $JHK_s$ measurements, while the remaining 11 only have data in $J$ and/or $H$ band. The sample consists of 16 BLH, 31 WVI, 12 PWV, and 22 RVT stars.

3. Light-curve Analysis

We compiled multiwavelength data available in the literature in order to study the variation in T2C light-curve structure as a function of period and bandpass. The sources used were as follows: optical photometry from OGLE-III for 203 objects in the LMC (Soszyński et al. 2008), and 357 variables in the bulge (Soszyński et al. 2011), Galactic globular clusters (GGCs; Matsunaga et al. 2006), and the LMC (Soszyński et al. 2008; Ripepi et al. 2015). “TW” represents the amplitudes for T2Cs in the LMC based on our observations.

The NIR times-series photometry for these objects is presented in Table 1. We adopt the period ($P$), time of maximum brightness in $I$ band ($T_{max}$), and optical ($V$) mean magnitudes from OGLE-III.

### Table 1

| ID       | Band | MJD     | Phase | Mag   | σ    |
|----------|------|---------|-------|-------|------|
| T2CEP-025 | J    | 42,620  | 0.879 | 13.633 | 0.021 |
| T2CEP-025 | J    | 42,753  | 0.881 | 13.609 | 0.021 |
| T2CEP-025 | H    | 42,623  | 0.879 | 13.513 | 0.018 |
| T2CEP-025 | H    | 42,756  | 0.881 | 13.451 | 0.040 |
| T2CEP-025 | $K_s$ | 42,625  | 0.879 | 13.348 | 0.015 |
| T2CEP-025 | $K_s$ | 42,758  | 0.881 | 13.370 | 0.032 |

Note. ID: T2CEP-NNN, from OGLE-III catalog of LMC Type II Cepheids (Soszyński et al. 2008); MJD = JD -2,450,000; phase is determined using the period and time of maximum brightness in $I$ band from OGLE-III. The fifth column represents magnitude in a NIR band, and the sixth column lists its associated uncertainty. The entire table is available online as supplemental material; sample time series in $JHK_s$ for T2Cs are shown here for guidance regarding its content. (This table is available in its entirety in machine-readable form.)
because the number of observations is small and most of the models presented in Bhardwaj et al. 2015 cannot have sufficient phase coverage or photometric accuracy to construct light-curve templates for these types of variables. For example, the light curves of T2Cs in the LMC currently available from the VMC survey have an average of 12 epochs in the $K_s$ band, while the $J$-band photometry is limited to only a couple of epochs (Ripepi et al. 2015). T2Cs in GGCs have 9–40 observations per light curve, but the sample is limited to 46 stars (Matsunaga et al. 2006). There are no other NIR time-series studies on T2Cs in the literature, thus limiting the sample size and the phase coverage for each period range. Therefore, we use OGLE-III LMC I-band data for the purpose of constructing templates. We also use $K_s$-band photometry from VVV to construct an alternative set of templates for comparison. Figure 2 displays the $I$-band Fourier parameters for T2Cs in the LMC and Galactic bulge. Note that the coefficients associated with the lower-order Fourier amplitudes ($R_{21}$ and $R_{31}$) and phases ($\phi_{21}$ and $\phi_{31}$) contain most of the quantitative information about light-curve structure (Simon & Lee 1981; Bhardwaj et al. 2017). The mean Fourier parameters exhibit similar variations as a function of period for both LMC and Galactic bulge samples in the $I$ band. Therefore, it is reasonable to assume that the light-curve structure in the $K_s$ band is also similar for both populations, as metallicity effects are less pronounced at longer wavelengths. The final calibrator sample consists of 170 $I$-band and 225 $K_s$-band light curves in the LMC and Galactic bulge, respectively. We divide the $I$- and $K_s$-band light curves into 10 period bins, as the mean Fourier parameters vary significantly as a function of this parameter. The period is the best observable to trace the changes in light-curve structure of Type II Cepheids in the LMC at $I$ band (left) and the Galactic bulge at $K_s$ band (right). The binning step size is 0.025 in phase, with the average value and standard deviation of the mean displayed with diamond symbols and error bars. The solid red lines represent seventh-order Fourier fits. The adopted range of periods (in days) for each bin is labeled in the top right corner of the left panel.

### Table 2

| Bin | $P$ (days) | $N_I$ | $N_{K_s}$ |
|-----|------------|------|----------|
| 1   | 1–2        | 48   | 69       |
| 2   | 2–3        | 11   | 30       |
| 3   | 3–5        | 10   | 18       |
| 4   | 5–7        | 15   | 8        |
| 5   | 7–9        | 21   | 21       |
| 6   | 9–11       | 12   | 22       |
| 7   | 11–13      | 9    | 14       |
| 8   | 13–15      | 12   | 16       |
| 9   | 15–20      | 15   | 13       |
| 10  | >20        | 17   | 14       |

Note. Period range for each bin is provided in the second column. $N_I$ and $N_{K_s}$ represent the number of stars in each bin with good-quality light curves in $I$ and $K_s$ band, respectively.

We fit each UHK$_s$ light curve with a fourth-order Fourier sine series, $m = m_0 + \sum_{i=1}^{4} A_i \sin(2\pi \phi + \phi_i)$ (Bhardwaj et al. 2015), where $m$ is the observed magnitude, $A_i$ is the amplitude of each term, and $\phi$ represents the corresponding phase. The series is kept to $i \leq 4$ because the number of observations is small and most light curves have large gaps in phase. Figure 1 shows the total amplitude of each variable in each band as a function of period. We also plot the amplitudes derived by Ripepi et al. (2015) for comparison. The amplitude increases as a function of period for WVI stars, while it exhibits the opposite behavior for RVT variables. The $I$-band amplitudes are the best determined since those light curves are of much higher quality. The short-period BLH stars are fainter, and the long-period RVT stars exhibit irregular light curves. Therefore, the amplitudes for these variables display a greater scatter as compared to WVI stars. A more detailed discussion on variation of light-curve parameters as a function of period and wavelength for (Classical) Cepheids is presented in Bhardwaj et al. (2017).

#### 3.1. Templates for Type II Cepheids

The NIR photometry of T2Cs available in the literature does not have sufficient phase coverage or photometric accuracy to construct light-curve templates for these types of variables. For example, the light curves of T2Cs in the LMC currently available from the VMC survey have an average of 12 epochs in the $K_s$ band, while the $J$-band photometry is limited to only a couple of epochs (Ripepi et al. 2015). T2Cs in GGCs have 9–40 observations per light curve, but the sample is limited to 46 stars (Matsunaga et al. 2006). There are no other NIR time-series studies on T2Cs in the literature, thus limiting the sample size and the phase coverage for each period range. Therefore, we use OGLE-III LMC I-band data for the purpose of constructing templates. We also use $K_s$-band photometry from VVV to construct an alternative set of templates for comparison. Figure 2 displays the $I$-band Fourier parameters for T2Cs in the LMC and Galactic bulge. Note that the coefficients associated with the lower-order Fourier amplitudes ($R_{21}$ and $R_{31}$) and phases ($\phi_{21}$ and $\phi_{31}$) contain most of the quantitative information about light-curve structure (Simon & Lee 1981; Bhardwaj et al. 2017). The mean Fourier parameters exhibit similar variations as a function of period for both LMC and Galactic bulge samples in the $I$ band. Therefore, it is reasonable to assume that the light-curve structure in the $K_s$ band is also similar for both populations, as metallicity effects are less pronounced at longer wavelengths. The final calibrator sample consists of 170 $I$-band and 225 $K_s$-band light curves in the LMC and Galactic bulge, respectively. We divide the $I$- and $K_s$-band light curves into 10 period bins, as the mean Fourier parameters vary significantly as a function of this parameter. The period is the best observable to trace the changes in light-
The adopted period bins and the number of stars in each bin are listed in Table 2. The number of stars per bin ranges from 8 to 69, with a significant fraction consisting of BLH stars having $P \lesssim 2$ days. The median photometric uncertainty per bin is $\sim 0.01$ and $\sim 0.1$ mag for $I$ and $K_s$ band, respectively.

Figure 4. Representative NIR light curves of six Type II Cepheids in the LMC. The left, middle, and right panels show short-, intermediate-, and long-period BLH, WVI, and RVT stars, respectively. $J$-, $H$-, and $K_s$-band light curves are plotted using blue, black, and red symbols, respectively. The solid and dashed lines represent the $I$- and $K_s$-band templates, respectively, fit to the data in each band.

### Table 3

| Bin | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $\phi_5$ | $\phi_6$ | $\phi_7$ | $\sigma$ |
|-----|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|----------|----------|---------|
| I   |       |       |       |       |       |       |       |          |          |          |          |          |          |          |         |
| 1   | 0.361 | 0.088 | 0.033 | 0.019 | 0.010 | 0.002 | 0.002 | 3.138    | 3.164    | 2.835    | 2.822    | 2.855    | 3.295    | 0.045    | 0.005   |
| 2   | 0.408 | 0.091 | 0.042 | 0.030 | 0.026 | 0.017 | 0.011 | 2.944    | 3.412    | 3.429    | 3.462    | 3.693    | 3.914    | 4.382    | 0.002   |
| 3   | 0.365 | 0.135 | 0.061 | 0.027 | 0.010 | 0.002 | 0.002 | 2.981    | 3.426    | 3.841    | 4.250    | 4.589    | 5.563    | 5.104    | 0.003   |
| 4   | 0.352 | 0.052 | 0.009 | 0.000 | 0.004 | 0.003 | 0.003 | 2.879    | 3.716    | 3.801    | 1.192    | 5.833    | 4.261    | 4.756    | 0.005   |
| 5   | 0.362 | 0.046 | 0.011 | 0.006 | 0.005 | 0.003 | 0.001 | 2.881    | 3.898    | 5.045    | 1.545    | 3.287    | 4.830    | 6.009    | 0.004   |
| 6   | 0.357 | 0.009 | 0.016 | 0.011 | 0.003 | 0.003 | 0.001 | 2.986    | 4.778    | 1.775    | 3.465    | 0.874    | 2.383    | 2.968    | 0.005   |
| 7   | 0.332 | 0.035 | 0.012 | 0.016 | 0.013 | 0.007 | 0.003 | 2.928    | 3.099    | 2.589    | 2.903    | 5.821    | 1.984    | 2.012    | 0.002   |
| 8   | 0.446 | 0.043 | 0.041 | 0.027 | 0.004 | 0.001 | 0.001 | 3.079    | 5.385    | 2.518    | 3.832    | 0.974    | 2.593    | 6.001    | 0.002   |
| 9   | 0.471 | 0.028 | 0.030 | 0.028 | 0.011 | 0.001 | 0.002 | 3.030    | 4.583    | 2.997    | 3.836    | 3.793    | 3.646    | 2.417    | 0.001   |
| 10  | 0.318 | 0.055 | 0.020 | 0.006 | 0.002 | 0.004 | 0.002 | 3.044    | 3.860    | 4.047    | 4.748    | 6.039    | 1.181    | 2.433    | 0.007   |
| K   |       |       |       |       |       |       |       |          |          |          |          |          |          |          |         |
| P   |       |       |       |       |       |       |       |          |          |          |          |          |          |          |         |
| 1   | 0.381 | 0.100 | 0.025 | 0.012 | 0.005 | 0.005 | 0.002 | 2.895    | 3.661    | 3.681    | 3.801    | 3.629    | 3.714    | 3.394    | 0.009   |
| 2   | 0.421 | 0.071 | 0.015 | 0.025 | 0.012 | 0.005 | 0.002 | 2.957    | 4.093    | 4.509    | 4.984    | 4.043    | 0.622    | 1.530    | 0.008   |
| 3   | 0.405 | 0.073 | 0.028 | 0.016 | 0.006 | 0.003 | 0.003 | 2.914    | 4.032    | 4.910    | 0.408    | 3.104    | 2.779    | 3.042    | 0.005   |
| 4   | 0.351 | 0.043 | 0.018 | 0.023 | 0.012 | 0.024 | 0.021 | 2.895    | 4.366    | 4.554    | 2.869    | 4.454    | 4.199    | 0.961    | 0.013   |
| 5   | 0.365 | 0.030 | 0.010 | 0.010 | 0.007 | 0.013 | 0.011 | 2.885    | 4.292    | 4.832    | 0.594    | 4.471    | 5.421    | 4.553    | 0.008   |
| 6   | 0.362 | 0.026 | 0.017 | 0.028 | 0.007 | 0.013 | 0.011 | 2.958    | 4.837    | 2.379    | 4.647    | 3.568    | 2.370    | 2.487    | 0.013   |
| 7   | 0.365 | 0.060 | 0.022 | 0.008 | 0.018 | 0.018 | 0.004 | 2.897    | 4.832    | 2.356    | 3.440    | 1.580    | 5.186    | 2.610    | 0.013   |
| 8   | 0.371 | 0.020 | 0.028 | 0.004 | 0.016 | 0.011 | 0.012 | 3.091    | 3.158    | 2.789    | 0.581    | 5.449    | 4.873    | 5.638    | 0.013   |
| 9   | 0.357 | 0.047 | 0.017 | 0.013 | 0.011 | 0.007 | 0.017 | 3.129    | 3.903    | 3.743    | 2.693    | 5.985    | 1.475    | 5.316    | 0.012   |
| 10  | 0.297 | 0.051 | 0.009 | 0.023 | 0.019 | 0.020 | 0.017 | 2.874    | 3.766    | 1.938    | 6.167    | 4.367    | 0.966    | 4.343    | 0.017   |

Inno et al. (2015) recently derived NIR templates for Classical Cepheids and suggested that setting the zero phase of the light curves to the epoch of mean brightness would avoid problems in estimating the precise maximum for bump Cepheid light curves with poor phase coverage. Although the calibrating sample of T2Cs has very good phase coverage in both $I$ and $K_s$ band, we...
adopt the same phasing strategy to avoid any complications. We first fit light curves with a fourth-order Fourier series and determine the phase corresponding to mean magnitude along the rising branch. We normalize the light curves to zero mean and unity amplitude and merge those within an adopted period bin. We then adopt a seventh-order Fourier series as the optimum fit, following previous work on templates by Soszyński et al. (2005) and Inno et al. (2015). The residuals from these fits follow a normal distribution, and we recursively remove 3σ outliers to increase the robustness of our results. The merged light curves and the Fourier series fits are displayed in Figure 3, while the Fourier coefficients are listed in Table 3. \( T_1, T_2, \ldots, T_{10} \) represent the merged light-curve templates in each bin. Typical standard deviations of the template fits are \(<0.01\) mag and 0.01 in \( I \) and \( K_s \) band, respectively. It is evident that the progression of the normalized and merged templates in each bin is similar for both bands, although the \( K_s \)-band templates are based on a significantly smaller number of data points. While the seventh-order series fits result in some wiggles for the \( K_s \)-band templates, it has no impact on the derived mean magnitudes. For example, the Fourier fits for the \( T_4 \) and \( T_4 \) sets could have been done at lower order, but we decided to retain the same expansion to facilitate the comparison with \( I \)-band templates.

### 4. Leavitt Laws for Type II Cepheids

We phase the NIR light curves of T2Cs in the LMC using the OGLE-III values of \( P \) and \( T_{1,\text{max}} \) (Soszyński et al. 2008). We use the same technique described above to set the zero phase of the light curves to the time of mean light in the rising branch. We fit the templates from Table 3 and solve independently for each amplitude and a possible phase shift in the time of mean light relative to \( I \) band. The amplitudes derived through this procedure show similar trends to those obtained via Fourier fit, and no significant phase shifts are seen. Figure 4 shows representative light curves for each T2C class.
of Cardelli et al. (1989) with $R_V = 3.23$ and individual reddening values from the map of Haschke et al. (2011). The total-to-selective absorption ratios per unit of $E(V-I)$ are $R_J = 0.69$, $R_H = 0.43$, and $R_K = 0.28$ (Paper II).

We fit PLRs of the following form:

$$m_\lambda = a_\lambda [\log(P) - 1] + b_\lambda,$$

where $m_\lambda$ is the extinction-corrected mean magnitude, $\lambda$ represents the $JHK_s$ bands, $a$ is the slope, and $b$ is the zero-point at $P = 10$ days. We fit PLRs to the entire sample, as well as to subsamples of faint (BLH+WVI) and bright (PWV+RVT) variables, iteratively removing $2\sigma$ outliers in all cases. As most of the outliers are likely due to blends or additional crowding effects, they appear on the bright side of the PLRs (see also discussion in Matsunaga et al. 2009; Ripepi et al. 2015). We adopt this threshold throughout the paper to have a stronger constraint on slopes and zero-points. Since the samples are small, a higher sigma-clipping threshold marginally changes the slopes (by less than half of their quoted uncertainties), and the typical increase in the number of stars and the dispersion is less than $10\%$. Figure 5 displays the results of the fits, which are also summarized in Table 5. We also note that a detailed statistical analysis on P–L relations was presented in Paper III to test Classical Cepheid data for nonlinearity under various assumptions, such as independent identically distributed observations, normality of residuals, and homoskedasticity. We also performed White’s test (White 1980) for a Type II Cepheid sample and found that our data provide evidence of homoskedasticity under a $95\%$ confidence interval.

Previous studies at optical and NIR wavelengths (see Soszyński et al. 2008; Matsunaga et al. 2009; Ripepi et al. 2015) have suggested that PWV and RVT stars lie above the PLR defined by the shorter-period BLH and WVI stars. A single PLR fit to our entire sample also gives evidence that WVI stars are mostly found below the regression line, especially in $J$ band. We use the $F$-test as described in Paper III, to quantify the statistical significance of nonlinearities in the slopes of the PLRs for various subsamples. We find a considerable difference between the PLR slopes for RVTs and BLH+WVI variables when considered separately.

**Table 6**

Comparison of Type II Cepheid P–L Relations

| Band | $a$  | $\sigma_a$ | $\sigma_{m}$ | $N$ | Loc | Src | $|T|$  | $p(t)$ |
|------|------|------------|--------------|-----|-----|-----|------|--------|
| $J$  | -2.374 | 0.058 | 0.259 | 30 | LMC | TW | ... | ... |
| $J$  | -2.163 | 0.044 | 0.210 | 137 | LMC | M09 | 3.017 | 0.003 |
| $J$  | -2.190 | 0.040 | 0.130 | 120 | LMC | R14 | 2.557 | 0.011 |
| $J$  | -2.092 | 0.116 | 0.330 | 47 | SMC | M11 | 2.344 | 0.021 |
| $J$  | -2.230 | 0.053 | 0.160 | 46 | GGC | M06 | 1.591 | 0.114 |
| $H$  | -2.261 | 0.061 | 0.208 | 72 | LMC | TW | ... | ... |
| $H$  | -2.316 | 0.043 | 0.200 | 136 | LMC | M09 | 0.746 | 0.457 |
| $H$  | -2.214 | 0.148 | 0.320 | 25 | SMC | M11 | 0.357 | 0.722 |
| $H$  | -2.344 | 0.050 | 0.150 | 46 | GGC | M06 | 0.996 | 0.321 |
| $K_s$ | -2.483 | 0.089 | 0.190 | 56 | LMC | TW | ... | ... |
| $K_s$ | -2.278 | 0.047 | 0.210 | 129 | LMC | M09 | 1.933 | 0.055 |
| $K_s$ | -2.385 | 0.030 | 0.090 | 120 | LMC | R14 | 1.312 | 0.191 |
| $K_s$ | -2.113 | 0.105 | 0.290 | 45 | SMC | M11 | 2.609 | 0.011 |
| $K_s$ | -2.408 | 0.047 | 0.140 | 46 | GGC | M06 | 0.768 | 0.444 |
| $K_s$ | -2.240 | 0.140 | 0.410 | 39 | GB | G08 | 1.399 | 0.165 |

**Note.** Loc: LMC/SMC: Large/Small Magellanic Cloud; GGC: Galactic globular clusters; GB: Galactic bulge. Src: TW: this work; M06: Matsunaga et al. (2006); G08: Groenewegen et al. (2008); M09: Matsunaga et al. (2009); M11: Matsunaga et al. (2011); R15: Ripepi et al. (2015). $|T|$ represents the observed value of the t-statistic, and $p(t)$ gives the probability of acceptance of the null hypothesis (equal slopes).
However, we find consistent slopes between PWV+RVT stars and BLH+WVI variables. Table 5 summarizes our findings. We note that the PLR dispersions are reduced by 12%, 2%, and 3% in \( JHK_s \) respectively, when using template-fit instead of Fourier-fit magnitudes.

4.1. Comparison with Published PLRs

We compare our PLRs with previous work carried out by Matsunaga et al. (2006), Groenewegen et al. (2008), Matsunaga et al. (2009, 2011), and Ripepi et al. (2015). We use the \( t \)-test as discussed in Paper II, to compare the slopes given their uncertainties and the dispersion of the underlying relations. The observed \( t \)-statistic is compared with theoretical values, calculated from the \( t \)-distribution at the 95% confidence interval (see Paper II for details). In brief, the null hypothesis that the two slopes are the same is rejected if the observed \( t \)-statistic (\( T \)) is greater than the theoretical (\( t \)) value. Table 6 lists the various slopes and the results of the test statistic. The probability (\( p(t) \)) of the acceptance of null hypothesis is also provided, and \( p(t) < 0.05 \) suggests that the two slopes under consideration are not equal.

We find that the slope of the \( J \)-band PLR for our entire sample is not consistent with those derived by Matsunaga et al. (2009) and Ripepi et al. (2015) for T2Cs in the LMC, but it is consistent in \( H \) and \( K_s \) bands. We note that those studies did not consider RVTs since they were found to lie well above the single regression line relative to shorter-period T2Cs at optical wavelengths. This deviation is not significant in our sample in \( K_s \) band. The slope of the \( J \)-band PLR for BLH+WVI stars is \( -2.100 \pm 0.107 \), consistent with published work given the uncertainties. The slopes of the LMC PLRs are not consistent with their SMC counterparts in \( J \) and \( K_s \) bands from Matsunaga et al. (2011) but are in agreement in all bands with those from globular clusters (Matsunaga et al. 2009). Furthermore, the slope of the LMC \( K_s \) band PLR is also consistent with the corresponding relation based on Galactic bulge variables (Groenewegen et al. 2008).

We also compare our T2C NIR mean magnitudes with values found in the literature. We find 76 objects in common with Matsunaga et al. (2011) and 62 in common with Ripepi...
We note that the former are single-epoch JHK$_s$ measurements in the InfraRed Survey Facility (IRSF; Kato et al. 2007) system, while the latter are mean magnitudes in the VISTA system. We apply the relevant color transformations from IRSF to the Two Micron All Sky Survey (2MASS) following Kato et al. (2007) and from VISTA to 2MASS as derived by Cambridge Astronomy Survey Unit (2016). Figure 6 shows the difference in our magnitudes with respect to VISTA and IRSF as a function of color. The K$_s$-band mean magnitudes in this work are consistent with those from VISTA given their uncertainties. The agreement with IRSF is also good except for a few stars with J - K$_s > 0.7$ mag. The J-band magnitudes from our work and IRSF are in good agreement, while a mild trend is seen in H band.

4.2. Near-infrared P–L and P–W Relations for OGLE-III Sample of Type II Cepheids

We compile JHK$_s$ magnitudes for T2Cs in the LMC that also have VI mean magnitudes from OGLE-III. We give preference to our NIR measurements, except for BLH stars in K$_s$ band. If measurements are not available in our database, we use JK$_s$ mean magnitudes from Ripepi et al. (2015) or phase-corrected single-epoch magnitudes from Matsunaga et al. (2011) as the lowest-priority source. We thus obtain NIR magnitudes in at least one band for 197 out of the 203 OGLE T2Cs in the LMC. All measurements are transformed into the 2MASS system. We derive PLRs for each class of variable following Equation (1), as well as from IRSF to the Two Micron All Sky Survey (2MASS) following Kato et al. (2007) and from VISTA to 2MASS as derived by Cambridge Astronomy Survey Unit (2016). Figure 6 shows the difference in our magnitudes with respect to VISTA and IRSF as a function of color. The K$_s$-band mean magnitudes in this work are consistent with those from VISTA given their uncertainties. The agreement with IRSF is also good except for a few stars with J - K$_s > 0.7$ mag. The J-band magnitudes from our work and IRSF are in good agreement, while a mild trend is seen in H band.
Table 8
Type II Cepheids and RR Lyrae with Parallaxes and Pulsation Distances

| ID    | Type | \( \log P \) (days) | \( K_{\alpha} \) (mag) | \( E_{BV} \) (mag) | \( \pi \) (mas) | LKH (mag) | [Fe/H] (dex) | \( \sigma_{[Fe/H]} \) |
|-------|------|---------------------|-------------------------|---------------------|----------------|-----------|-------------|-----------------|
| VY Pyx | BLH  | 0.093               | 5.72                    | 0.05                | 3.85 ± 0.28  | −0.01     | −0.01       | 0.15            |
| SW Tau | BLH  | 0.200               | 7.95                    | 0.28                | 1.57 ± 0.04  | ...       | 0.22         | B−W             |
| V553 Cen | BLH  | 0.314               | 6.86                    | 0.00                | 1.85 ± 0.05  | ...       | 0.24         | B−W             |
| \( \kappa \) Pav | WVI  | 0.958               | 2.78                    | 0.02                | 5.57 ± 0.28  | −0.02     | 0.00         | 0.13            |
| XZ Cyg | RRab | −0.331              | 8.72                    | 0.10                | 1.67 ± 0.17  | −0.09     | −1.44        | 0.20            |
| UV Oct | RRab | −0.266              | 8.30                    | 0.09                | 1.71 ± 0.10  | −0.03     | −1.74        | 0.11            |
| RR Lyr | RRab | −0.247              | 6.49                    | 0.03                | 3.77 ± 0.13  | −0.02     | −1.39        | 0.13            |
| SU Dra | RRab | −0.180              | 8.62                    | 0.01                | 1.42 ± 0.16  | −0.11     | −1.80        | 0.20            |
| RZ Cep | RRe  | −0.511              | 7.88                    | 0.08                | 2.54 ± 0.19  | −0.05     | −1.77        | 0.20            |

Note. \( E_{BV} = E(B − V) \). B−W distances from Feast et al. (2008) are converted to parallaxes for relative comparison.

Table 9
Estimates of the LMC Distance Modulus

| Source | \( \mu \) (mag) |
|--------|-----------------|
| T2C \( \pi \) | 18.54 ± 0.11 |
| RRL \( \pi \) | 18.55 ± 0.10 |
| T2C B−W | 18.41 ± 0.09 |

4.3. Distance to the LMC Using HST Parallaxes

It can be seen that RRLs nicely follow the PLR of T2Cs, which is shallower than those obeyed by Classical Cepheids.

We use trigonometric parallaxes for two T2Cs and five RRLs in the solar neighborhood, obtained with the Hubble Space Telescope (HST; Benedict et al. 2011) and Gaia (Lindegren et al., 2016), to calibrate the zero-point of our PLRs and estimate a distance to the LMC. We also use distance estimates for two T2Cs (V553 Cen and SW Tau) determined via the Baade–Wesselink (B−W) method (Feast et al. 2008). Table 8 summarizes the distance estimates for all calibrators along with magnitudes and other properties adopted from Feast et al. (2008) and Benedict et al. (2011). We include the LKH correction (Lutz & Kelker 1973) and “fundamentalize” the period of first-overtone RRL by adding \( \Delta \log(P) = 0.127 \). Lastly, RRL magnitudes are corrected for metallicity effects using the recent \( P − L_{K} − [Fe/H] \) relation of Muraveva et al. (2015).

The P2Rs formed as a combination of \( I \) band and one of \( J, H, \) or \( K_{s} \) band exhibit a smaller dispersion than other NIR relations. Similarly, \( W_{\lambda, \lambda} \) displays a significantly smaller dispersion than \( W_{\lambda, H} \) and \( W_{\lambda, K_{s}} \), presumably due to the dominant sample of template-fit mean magnitudes in \( I \) and \( K_{s} \) band instead of the phase-corrected single-epoch magnitudes in \( H \) band. We compare the P2R slopes with those of Ripepi et al. (2015) and find a consistent value for \( W_{\lambda, K_{s}} \), while our value for \( W_{\lambda, I} \) is marginally steeper. However, we note that the Ripepi et al. (2015) results are based on a single regression fit to BLH+WVI variables and those are consistent with our results for the same subsample.

4.3. Distance to the LMC Using HST Parallaxes

The short-period T2Cs reside in the same instability strip that extends a few magnitudes above the horizontal branch and includes RR Lyrae (RRLs; Sandage & Tammann 2006). It has been suggested that RRLs follow the same PLRs as short-period T2Cs (Sollima et al. 2006; Feast et al. 2008; Ripepi et al. 2015). We therefore further extend the expanded PLRs of Section 4.2 with NIR measurements of RRLs in the LMC from Borissova et al. (2009) and Muraveva et al. (2015), as shown in Figure 9.

\begin{equation}
W_{\lambda, \lambda} = m_{\lambda} - R_{\lambda}^{\lambda} (m_{\lambda} - m_{\lambda}),
\end{equation}

where \( m_{\lambda} \) is the mean magnitude in one of \( V, I, J, H, K_{s} \), and \( \lambda > \lambda_{2} \). We use the Cardelli et al. (1989) reddening law and assume a value of total-to-selective absorption of \( R_{V} = 3.23 \). The resulting absorption ratios in other bands are \( R_{I}^{J} = 1.63, R_{K}^{H} = 0.69 \), \( R_{K}^{H} = 1.92, R_{J}^{I} = 0.41, R_{V}^{I} = 0.22, R_{V}^{H} = 0.13, R_{V}^{I} = 0.92, R_{I}^{H} = 0.42, R_{K}^{H} = 0.24 \) (see Paper II). The Wesenheit magnitudes are fit with log \( (P) \) as an independent variable. The resulting PLRs and P2Rs are plotted in Figures 7 and 8, respectively, and summarized in Table 7.

The P2Rs formed as a combination of \( I \) band and one of \( J, H, \) or \( K_{s} \) band exhibit a smaller dispersion than \( W_{I, H} \) and \( W_{I, K_{s}} \), presumably due to the dominant sample of template-fit mean magnitudes in \( I \) and \( K_{s} \) band instead of the phase-corrected single-epoch magnitudes in \( H \) band. We compare the P2R slopes with those of Ripepi et al. (2015) and find a consistent value for \( W_{I, K_{s}} \), while our value for \( W_{I, I} \) is marginally steeper. However, we note that the Ripepi et al. (2015) results are based on a single regression fit to BLH+WVI variables and those are consistent with our results for the same subsample.
deviation of the mean added in quadrature. We note that the LMC distance modulus based on B−W distances is smaller than those based on trigonometric parallaxes, similar to the results of Feast et al. (2008). Therefore, we calculate a mean distance modulus to the LMC based only on the two independent calibrations that rely on parallaxes: \( \mu = 18.54 \pm 0.08 \) mag.

## 5. Distances to GGCs

Matsunaga et al. (2006) published NIR light curves for T2Cs in 26 GGCs and derived the corresponding PRs. However, the definition of mean magnitude adopted by the authors was a simple mean of maximum and minimum values, which may bias the results since the light curves are neither sinusoidal nor fairly well sampled. Therefore, we use our templates to fit their data and obtain robust mean magnitudes. The resulting light curves are displayed in Figure 12 in the Appendix, while the mean magnitudes are listed in Table 10. Figure 10 displays the difference in mean magnitudes obtained via these two approaches, showing that the results from Matsunaga et al. (2006) were significantly biased toward larger values.

In order to obtain distance estimates to these globular clusters, we perform an absolute calibration of the LMC PRs using the distance modulus derived by Pietrzyński et al. (2013) using late-type eclipsing binaries, \( \mu = 18.493 \pm 0.048 \) mag. This estimate is significantly more precise and accurate than the one we obtained in Section 4.3 using a few trigonometric parallaxes and was also adopted in Papers I and II to calibrate the Classical Cepheid PRs and PWRs.

We correct the GGC photometry for interstellar extinction using the tabulated \( E(B−V) \) values and the Cardelli et al. (1989) extinction law. We use the absolute calibration of the LMC PRs to determine distances to each T2C in Table 10 and compute weighted averages for clusters with more than one variable. The error budget includes uncertainties in (1) mean magnitudes derived from template fits, (2) absolute calibration, and (3) eclipsing binary distance to the LMC, added in
quadrature. The results are presented in Table 11, along with the estimates by Matsunaga et al. (2006), who used the $M_V - [\text{Fe/H}]$ relation. The bottom panel of Figure 10 shows the difference between the two approaches; the distances are in agreement within $2\sigma$ in almost all cases.

6. Conclusions

We summarize the results of this work as follows:

1. We present time-series observations of 81 Type II Cepheids in the LMC at $JHK_s$ wavelengths, based on the survey of Maci et al. (2015). The $JHK_s$ data complement the photometry from the VMC survey (Ripepi et al. 2015), while the $H$-band time-series observations are presented for the first time. We develop templates using high-quality and well-sampled light curves of variables in the LMC (observed in $I$ band by OGLE) and the Galactic bulge (observed in $K_s$ band by VVV).

2. We derive robust mean magnitudes based on template fits and obtain PLRs for each class of variable. Our relations are consistent with published work based on variables in the LMC, GGCs, and the Galactic bulge.

3. We compile NIR magnitudes for the entire sample of OGLE-III Type II Cepheids and derive new PWRs by combining optical and NIR data. The slopes of the $W_{\alpha Ks}$ and $W_{VJ}$ relations are consistent with the findings of Ripepi et al. (2015); in the latter case, when the comparison is restricted to BL Her and W Virginis stars.

4. We use the Gaia DR1 parallax for VV Pxy and the HST parallaxes for $k$ Pav and five RR Lyrae variables to obtain an absolute calibration of the zero-point of the $P$--$L$ relations. This yields an estimate of the LMC distance modulus of $\mu_{\text{LMC}} = 18.54 \pm 0.08$ mag, in very good agreement with the more accurate and precise estimate by Pietrzyński et al. (2013). Our estimate is also consistent with recent results based on Classical Cepheids (Monson et al. 2012; Paper II).

5. We update the mean magnitudes for Type II Cepheids in 26 GGCs using our light-curve templates and estimate distances to these systems. Our findings are in good agreement with estimates based on horizontal branch stars by Matsunaga et al. (2006).

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Software: MPCURVEFIT (Markwardt 2009).

Appendix

Template-fit Light Curves

The three panels of Figure 11 present the result of template fits to the $JHK_s$ light curves of Type II Cepheids in the LMC.
based on data from Macri et al. (2015). Figure 12 shows the same, but for variables in GGCs based on data from Matsunaga et al. (2006). Stars are arranged by decreasing period. The $J$- (blue) and $K_s$-band (red) light curves have been offset by $+0.25$ and $-0.5$ mag, respectively, for visualization purposes. $H$-band light curves are shown in violet color. The solid and
Figure 11. (Continued.)
Phase

Figure 11. (Continued.)
dashed lines represent the best-fit \( I \)- and \( K_s \)-band-based templates for each band. The star “ID” and “Period (in days)” are also provided on the top of each light curve.

The template fits are performed using least-squares minimization in the *IDL MPCURVEFIT* routine (Markwardt 2009). Templates can be used to \( JHK_s \) light curves with poor phase

**Figure 12.** Template fits to light curves of Type II Cepheids in Galactic globular clusters, based on data from Matsunaga et al. (2006).
coverage in order to obtain robust mean magnitudes. In the case of single-epoch observations, accurate amplitude ratios for Type II Cepheids will be required to best fit observations. For short-period Type II Cepheids, light curves exhibit large scatter with respect to the template fits, which provide a robust estimate of the uncertainty associated with mean magnitudes.

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