On electron–proton energy exchange in strong magnetic field

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Abstract. Heating of protons in cold electron gas in strong magnetic field is studied. Calculations of heating process are performed using molecular dynamics method. Estimations of heating rate depending on initial proton energies and electron gas temperatures are made.

1. Introduction
In the experiments in CERN [1] on antihydrogen antiprotons and positrons are mixed together in magnetic trap and antiatoms are formed due to three body recombination. One of the problems in these experiments is relaxation of antiproton energy in positron gas. In the first experiments antiprotons had high initial energy and lose it during mixing in collisions with positrons. Kinetics of the collision processes is strongly influenced by confining magnetic field applied along the axis of experimental set with magnitudes of $H \sim 10^4$ Gauss.

To study the problem of energy relaxation we solve similar task of relaxation of a proton energy in ultracold electron gas in strong magnetic field. In our previous work [2] we studied energy relaxation of hot protons (with energies up to several eV) injected into cloud of ultracold electron gas with temperatures of about 15–50 K and density $n_e \sim 10^8$ cm$^{-3}$. Presence of magnetic field complicates theoretical study of kinetic processes and we used molecular dynamics method to simulate the behavior of proton-electron system.

Achievements in experimental techniques allowed production of cold antiprotons with energies of about positron temperature and may be lower. The present paper is devoted to the problem of heating of cold protons in electron gas.

2. Molecular dynamics simulation
To simulate electron–proton collisions kinetics we place electrons and protons in cubic simulation cell. We considered electron–proton system as non neutral two temperature plasma. Plasma parameters in the simulations corresponded the experimental conditions: electrons temperature $T_e = 5$–$50$ K, electrons density $n_e = 10^8$ cm$^{-3}$. In the present work we studied growth of protons energy $T_p$ from initial values near 0 K. In the experiments number of antiprotons was $10^4$ times lower than number of positrons and due to this positron temperature is nearly constant. Molecular dynamics simulation of systems with such big difference between numbers of heavy
Figure 1. Square of proton velocity at $T_e = 5$ K and $T_p(0) = 0$ K: solid line—axial velocity, dashed line—transverse velocity.

Figure 2. Square of proton velocity at $T_e = 50$ K and $T_p(0) = 0$ K: solid line—axial velocity, dashed line—transverse velocity.

and light particles is computationally difficult. We used in our simulations particle numbers ratio $N_e/N_p \sim 100$ and stopped simulations when electron temperature change is significant.

We considered non degenerated plasma. Thermal electron wavelength $\lambda \sim \hbar/\sqrt{m_e T_e}$, where $m_e$ is electron mass, for the lowest electron temperature is much less than mean distance $n_e^{-1/3}$. This allows us to use classical equation of motion to simulate particles movement.

Plasma considered here is weakly coupled. Coupling parameter for the lowest electron temperature $\gamma = n_e^{1/3}e^2/T_e \ll 1$. Coupling of protons is negligible due to very low protons density.

For weakly coupled plasma in the absence of magnetic field electron–proton relaxation time is given by Landau formula [3]

$$\tau_e = \frac{T_e^{3/2}m_p}{4n_e e^4 L_e (2\pi m_e)^{1/2}},$$

where $m_p$—proton mass, $L_e$—Coulomb logarithm. $L_e$ for weakly coupled plasma is determined by formula

$$L_e = \ln \frac{\sqrt{T_e/4\pi n_e e^2}}{e^2/T_e}.$$  

The relaxation time is relatively high, for example as may be seen from (1) $\tau_e$ is $m_p/m_e$ times higher than electrons thermalization time. Molecular dynamics simulation of system with high mass ratio for extended physical time is complicated. To speed up calculations we used variable timestep method. We used large timestep for particles that are distant from the others and decreased timestep for particles that are close to each other. Magnitude of the decrease was determined based on the distance between the particles to reach given calculation precision.

All simulations in this work were made for magnetic field $H = 10^4$ Gauss. To take into account magnetic field we used Boris method [4]. Electron subsystem is strongly magnetized—electron Lamor radius $r_L = m_e e v_e/eH$ is much less than minimal approach distance $\sim e^2/T$. Strong magnetization of electrons subsystem imposes a limit on maximum timestep—it must be less than inverse electron gyrofrequency $\omega^{-1} = (eH/m_e e)^{-1}$, that for $H = 10^4$ Gauss is
$5.6 \times 10^{-12}$ s. Typical simulation steps number reached at maximum several tens of million that corresponds to physical time of about $10^{-5}$ s.

In our simulations classical equations of motion were solved for about 100–300 particles in simulation cell using NVE ensemble. The number of particles in the cell was chosen so that Debye screening length was less than the cell size. To simulate continuous plasma we applied periodic boundary conditions to the cell. Variable timestep and rare opposite charges close collisions allowed us to substitute pure Coulomb force between particles in the equations of motion. All results are averaged over about 100 proton trajectories.

First we made calculations for zero initial proton energy $T_p = 0$ K. In figure 1 calculation results of proton velocity components growth for $T_e(0) = 5$ K and $T_p(0) = 0$ K are shown. As is seen from the figure magnetic field presence leads to different rates of heating of a proton along and transverse to magnetic field directions. For zero proton energies that difference is most significant because in this case protons are also strongly magnetized initially. Axial relaxation time is about $4.6 \times 10^{-5}$ s, transverse time is $10^{-6}$ s. Relaxation time from (1) $\tau_e = 1.9 \times 10^{-5}$ s is between axial and transverse values.

In figure 2 results for axial and transverse proton velocity components increase are shown for $T_e(0) = 50$ K and again zero proton initial energy. As in previous case axial and transverse rates are different, relaxation time for axial energy can be estimated as $9 \times 10^{-5}$ s and for transverse $3.6 \times 10^{-5}$ s. In this case proton energy grows faster than is predicted from (1)—for $T_e = 50$ K $\tau_e = 1.8 \times 10^{-4}$ s.

In figures 3 and 4 calculations results are shown for nonzero initial proton energies. In this case energy increasing rate is also different for directions along and transverse to magnetic field. As may be seen from the results of the simulations transverse velocity of proton in the first moments of time if significantly higher than axial velocity. This difference may influence energy of atoms that can be formed in recombination processes. If an atom is formed early after particles mixing start its velocity (that is equal to proton velocity) in axial direction may be about zero and because of that the atom would leave the trap practically at a right angle to magnetic field direction. Due to this trapping of atom will be complicated if an atomic trap is placed on the axis of the experimental setup.
3. Conclusion
We performed calculations of heating of protons in strongly magnetized non neutral two temperature plasma. Heating rate agrees only qualitatively with Landau formula and depends on direction. Difference in heating rates along and across the magnetic field direction leads to significant difference (up to several times) in axial and transverse proton velocity components early after plasma creation.

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