On the Achievable Max-Min Rates of Cooperative Power-Domain NOMA Systems

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ABSTRACT The non-orthogonal multiple access (NOMA) schemes, where the users are allowed to share the same time-frequency resource, have recently received a major interest to meet the increasing demands for higher data rates, improved user fairness, and/or lower latency applications in future dense-user wireless networks. Moreover, the achievable user rates of NOMA schemes can be further increased through the integration of these multiple access schemes with other enabling schemes such as full-duplex user cooperative communications. In this paper, the optimization of the achievable max-min rates for full-duplex cooperative NOMA (CNOMA) schemes for the two-user scenario are revisited and is extended to the three-user scenario where novel expressions for the optimal user cooperation power are derived for both scenarios. The obtained results shed new lights on the optimal transmit power allocation and the optimal user cooperation power allocation in CNOMA systems and the different design trade-offs in such systems.

INDEX TERMS Non-orthogonal multiple access schemes, cooperative communications, user cooperation, max-min rate, power allocation, cooperative water-filling, mm-wave and THz, indoor VLC.

I. INTRODUCTION
The increasing mobile data use and emerging new applications in fifth generation (5G) and beyond networks require peak data rates in the order of Gigabits per second with low latency. This motivated the move from the current congested radio bands to higher frequency bands such as millimeter wave (mm-wave) band [1], THz, and even visible light communications (VLC) bands [2]. For example, the recently licensed 28 GHz and 38 GHz bands offer more than one GHz of bandwidth which allows service providers to expand the transmission bandwidth more than the previous 4G networks [1]. The effect of the higher path loss due to higher frequency bands propagation conditions, compared to the lower frequency bands, is accommodated through the deployment of dense small cells. Moreover, the spectral efficiency in 5G and beyond-5G networks is/will be also enhanced through the further adoption of combinations of high-data rate enabling technologies and schemes such as advanced multiple-input multiple-output (MIMO) architectures, advanced interference reduction beamforming schemes, full-duplex cooperative communications, mm-wave and THz, VLC, and non-orthogonal multiple access (NOMA) schemes [3]–[6].

As compared to orthogonal multiple access (OMA) schemes such as time-division multiple access (TDMA), code division multiple access (CDMA), and orthogonal frequency division multiple access (OFDMA), NOMA schemes allow multiple users to share same resources in the same frequency, time, and space dimensions. There are several variants of NOMA in literature; however, the three major variants of NOMA are, namely, the power-domain NOMA, the code-domain NOMA, and NOMA multiplexing in multiple domains (i.e., the power domain, the code domain, and the spatial domain) [7], [8]. However, the most widely adopted one is the power-domain NOMA (PD-NOMA) where the channel gain differences among the served users are utilized to rank the users, perform fair power allocation, and allow the receivers to decode the super-imposed users’ signals using successive interference cancellation (SIC).

On the other hand, cooperative communications schemes including both user-assisted [9] and rely-assisted [10] schemes have received interest over the last two decades in both academia and industry. Most of the conducted research has considered the relay-based schemes where one or more of dedicated relays are utilized to improve the coverage,
reliability, and/or achievable rates. However, the emergence of peer-to-peer or device-to-device communications and the SIC-decoding-based NOMA schemes has revitalized interest in user-assisted cooperation schemes especially with the fact that the move from half-duplex (HD) to full-duplex (FD) cooperative communications [11], [12] would theoretically double the achievable rates and reduce the network latency. The full-duplex mode has been shown to be implementable even for the single-antenna case and a variety of self-interference management or reduction schemes have been introduced and implemented [13], [14] and references therein.

The integration of cooperative communications with NOMA was recently investigated for both relay-assisted PD-NOMA, where dedicated relays are deployed to enhance the links of the base station (BS) or the access point (AP) to the served users, and user-cooperative PD-NOMA where the strong users, being able to decode the weaker users’ signals, act as relays to improve the weak users data rates [15]–[17]. An early proposal of C-NOMA in [18] was over two transmission phases and the expressions for the outage probability were derived and analyzed to reveal the performance gains of C-NOMA over non-cooperative NOMA and OMA in terms of the diversity order and outage capacity. Further performance analysis of cooperative NOMA was carried out as in [19] and references therein.

While the optimization of achievable sum rate of cooperative NOMA systems has been well-investigated as in [16] and references therein, only few works have considered the optimization of another common fairness measure which is the max-min achievable rates. The max-min rate optimization of conventional (non-cooperative) NOMA systems was first considered in [20] where the bi-section method was utilized to obtain the max-min rates. A similar work in [21] has considered the maximization of the minimum normalized rate for conventional NOMA systems where an approximation of the objective function was utilized to convexify the optimization problem. The maximization of the minimum rate of the cell-edge users in VLC networks, in [22], using hybrid NOMA and linear zero-forcing (ZF) pre-coding schemes, was tackled by numerical convex optimization of the standard determinant maximization.

For cooperative NOMA systems, the optimal power allocation for the max-min rate in a cooperative NOMA system without a direct link between the source and the weak user was carried out in [23] and the power allocation in a cooperative NOMA system with the direct link was considered in [24] where a two-step optimization method was utilized to maximize the minimum rate of the two NOMA users. In this paper, the max-min rate optimization for two-user cooperative NOMA system is re-visited and the three-user system is investigated where the main contributions are:

- The formulations are extended to the three-user cooperative NOMA system where closed-form expressions for the optimal cooperation power are derived.
- New insights into the optimal cooperation power, such as cooperative water-filling, and the performance of cooperative NOMA systems are presented.

The obtained expressions can be utilized for radio frequency (RF) channels with dominant line-of-sight (LoS) conditions as well as indoor VLC channels. Also, the developed formulations are extensible to fading channels to derive the corresponding outage and ergodic capacity metrics.

**II. BACKGROUND AND SYSTEM MODEL**

**A. COOPERATIVE BROADCAST CHANNELS**

A basic degraded cooperative or relay broadcast channel (RBC) for single-antenna users is shown in Fig. 1. The first user (the strong user or User 1) has a stronger channel link to the transmitter than the second user (the weak user or User 2). At the transmitter, both intended messages or signals for the weak and the strong users are encoded with different power allocation proportions for each user. At the strong user receiver, the signal of the weak user has relatively a high signal-to-noise ratio (SNR) which implies that the strong user can successfully decode and subtract the weak user signal before decoding its own signal (i.e., performing SIC). Also, it performs user cooperation to improve the weak user achievable rate [25].

![FIGURE 1. A two-user cooperative relay broadcast channel.](image-url)

The received signals at the two users can be expressed as,

\[ y_1 = h_1 x + z_1, \]  
\[ y_2 = h_2 x + h_{12} x_1 + z_2, \]

where \( h_1, h_2 \), and \( h_{12} \) denote the channel gains of the links from the source to the users and the cooperative link, respectively. The signal \( x \) contains the information message intended for the two users while \( x_1 \) contains the information message sent by the first (strong) user to the second weaker user. The noise components \( z_1 \) and \( z_2 \) are independent.
Gaussian random variables with zero-mean and same unit variance \( \sigma^2 \).

The achievable rates can be expressed using the partially cooperative RBC model [25], as the following rate for the strong user

\[
R_1(\alpha) = \log_2 \left( 1 + \frac{|h_1|^2 \alpha P}{\sigma^2} \right),
\]

and the following for the weak user (this is the expression that should have been used in [24])

\[
R_2(\alpha, P_1) = \min \{R_{1-2}, R_{2-2}\} = \begin{cases} \log_2 \left( 1 + \frac{|h_1|^2 (1 - \alpha) P}{|h_1|^2 \alpha P + \sigma^2} \right), & \log_2 \left( 1 + \frac{|h_2|^2 (1 - \alpha) P + |h_1|^2 |P_1|}{|h_2|^2 \alpha P + \sigma^2} \right), \end{cases}
\]

where \( \alpha \) denotes the portion of source (BS or AP) transmit power, \( P \), that is allocated for the signal (message) intended for the strong user and \( P_1 \) denotes the cooperation power at User 1. The rate \( R_{1-2} \) denotes the achievable rate for the signal of User 2 as decoded at User 1.

III. THE COOPERATIVE PD-NOMA SYSTEM: TWO-USER CASE REVISITED

A two-user cooperative NOMA scheme has a strong user (User 1) that is close to the BS or the AP with stronger channel gain and a second user (User 2) that is farther away from the BS/AP with weaker channel gain. So, it can be modeled using the RBC model shown in Fig. 1 with an additional constraint to guarantee fairness among the served users. Both the BS/AP and the users are assumed to have single antennas and the BS/AP is assumed to know the instantaneous channel gains of the links to the users and the strong user is assumed to know the instantaneous cooperation link channel gain.\(^1\) One way to guarantee fairness is to consider the max-min rate where the smaller rates are maximized so that the users end up with equal rates. However, as the optimization here involves both the optimal transmit power allocation at the source (the BS or the AP) as well as the optimal cooperation power at the strong user, we may consider the following cases:

A. NO POWER CONTROL ON THE USER COOPERATION POWER

In this case, the cooperation power at the strong user, towards the weak user, has no pre-determined maximum value and is left to be optimized. Although this case might be theoretical, it will lead to decoupling the source transmit power allocation from the user cooperation power allocation and to closed-form expressions, in terms of the channel gains and the source transmit power, for the latter one.

\(^1\)Such assumption should be valid for a small number of users; however, it becomes more demanding for a large number of users [15] and the reader may refer to [26], [27] for more details about the effect of partial or limited channel state information (CSI) on the performance of NOMA systems.

The corresponding optimization problem may be expressed as

\[
\max_{\alpha, P_1} \min \{R_1, R_2\},
\]

s.t. \( P \leq P_T \),

\[
P_1 \geq 0,
\]

where \( P_T \) is the constraint of the maximum BS/AP transmit power. This optimization problem, as shown in [24], is neither concave nor quasi-concave and was tackled there using a two-stage optimization procedure. In here, a different approach from the one in [24] is utilized leading to more compact expressions for the optimal cooperation power.

Proposition 1: The optimal value of the cooperation power at User 1, \( P_{1,\text{opt}} \), can be obtained as

\[
P_{1,\text{opt}} = \left( \frac{1}{|h_1|^2} - \frac{1}{|h_2|^2} \right) \frac{|h_2|^2}{|h_1|^2} (1 - \alpha_{\text{opt}}) P,
\]

for \( |h_1|^2 > |h_2|^2 \),

and the optimal value of \( \alpha \) can be obtained as

\[
\alpha_{\text{opt}} = \frac{\sqrt{1 + \frac{|h_1|^2 P}{\sigma^2}} - 1}{\frac{|h_1|^2 P}{\sigma^2}}. \tag{7}
\]

Proof: The main argument in the proof is that since we know that there will be enough cooperation power to bridge the gap between \( R_{1-2} \) and \( R_{2-2} \) and hence \( \gamma_{1-2} \), which represents the signal-to-noise-plus-interference ratio (SNIR) corresponding the weak user signal when decoded at the strong user and \( \gamma_{2-2} \) (since the channel here is degraded) and yet not exceed \( \gamma_{1-2} \) due to the minimum operator there. So, the optimal value of \( \alpha \) can be determined by the solving following the quasi-concave optimization sub-problem

\[
\max_{\alpha} \min \{R_1, R_{1-2}\},
\]

s.t. \( P \leq P_T \),

which can be easily solved using the bisection method. However, targeting analytical expressions and similar to the proof in [28], Proposition 2, as \( \gamma_{1-2} = \frac{|h_1|^2 (1 - \alpha) P}{|h_1|^2 \alpha P + \sigma^2} \) is a monotonically decreasing function of \( \alpha \) while \( \gamma_{1-1} = \frac{|h_1|^2 \alpha P}{\sigma^2} \) in a monotonically increasing function of \( \alpha \), then the maximization of the minimum of the optimal rates will take place for \( \gamma_{1-2} = \gamma_{1-1} \) which will result in the closed-form expression in (7).

The next step is to determine the required optimal user cooperation power where considering the inner minimum operator, in 4, it follows that \( \gamma_{1-2} = \frac{|h_1|^2 (1 - \alpha) P}{|h_1|^2 \alpha P + \sigma^2} \geq \gamma_{2-2} = \frac{|h_2|^2 (1 - \alpha) P}{|h_2|^2 \alpha P + \sigma^2} \) and the additional term due to cooperation, \( |h_1|^2 P_1 \), as it is monotonically increasing w.r.t. \( P_1 \), should contribute to bridging the gap between \( \gamma_{1-2} \) and \( \gamma_{2-2} \) and yet not exceed \( \gamma_{1-2} \). So, we may write \( \gamma_{1-2} = \frac{|h_1|^2 (1 - \alpha) P}{|h_1|^2 \alpha P + \sigma^2} + \frac{|h_1|^2 P_1}{|h_1|^2 \alpha P + \sigma^2} \) which results in, upon
solving for $P_1$, the optimal value of the user cooperation power as in (6).

**Remark 1:** The approach used here is also similar to the primal decomposition approach in [29] where $\max_{x,y}f(x,y) = \max_{x}\{\max_{y}f(x,y)\}$.

**Remark 2:** The optimal value of $\alpha$ can be verified numerically via solving the max-min problem $\max_{\alpha} \min\{\gamma_1, \gamma_2\}$ using the bi-section method.

**Remark 3:** The user cooperation power decreases as $|h_1|^2$ and $|h_2|^2$ get comparable as expected and is interestingly proportional to the difference in "water-level" of these gains. This may be seen as a form of "cooperative" water-filling as compared to the classical "egoistic" water-filling in opportunistic scheduling. Also, the required optimal cooperation power is directly proportional to $|h_2|^2$ as expected. This is further investigated in the results section.

**Remark 4:** The user cooperation power should not exceed $P_{1, opt}$ as an upper limit to avoid the scenario where the cooperation power starts to contribute, unnecessarily, to more self-interference at the strong user [25]. This is further investigated in section V.

### B. WITH POWER CONTROL ON THE USER COOPERATION POWER

In this case, the cooperation power at the strong user is controlled or limited such that it may not exceed a certain maximum value denoted as $P_{1, max}$. Hence, the optimization problem modifies to

$$\max_{\alpha, P_1} \min\{R_1, R_2\},$$

s.t. $P \leq P_T$,

$$P_1 \leq P_{1, max},$$

$$P_1 \geq 0,$$

and $P_{1, max}$ is again the constraint of the maximum cooperation power at the strong user (User 1). This optimization problem can be tackled in a similar way to the one in (5). So, for the scenario when $P_{1, max} \geq P_{1, opt}$, the results in Proposition 1 will follow. On the other hand, if $P_{1, max} < P_{1, opt}$ (i.e., the available cooperation power is small) then the maximum of the inner minimum operator in $R_2$ can not be attained (since $R_{2-2} < R_{1-2}$ and the optimal value of $P_1, P_{1, opt}$, can be expressed as

$$P_{1, opt}' = \min(P_{1, opt}, P_{1, max})$$

where $P_{1, opt}$ is the one obtained in (6) for $\alpha_{opt}$ as in (7), and the optimal power allocation parameter for the case here, $\alpha_{opt}$, is

$$\alpha_{opt} = \begin{cases} \frac{\alpha_{opt}}{\alpha_{opt}} & \text{if } P_{1, opt} \leq P_{1, max} \\ \frac{\alpha_{opt}}{\alpha_{opt}} & \text{if } P_{1, max} < P_{1, opt} \end{cases}$$

where $a = 1, b = \frac{\sigma^2}{|h_1|^2 P} + \frac{\sigma^2}{|h_2|^2 P}$, and $c = \frac{\sigma^2}{|h_1|^2 P} + \frac{\sigma^2}{|h_2|^2 P}$. The value of $\alpha_{opt}'$ for $P_{1, max} < P_{1, opt}$

**Algorithm 1:** Overview of the Max-Min Optimization Algorithm

1. **Input** $h_1, h_2, h_{12}, P, P_{1, max}$
2. **Output:** $R_1$ and $R_2$
3. Obtain $\alpha_{opt}$ using 7
4. Compute $P_{1, opt}$ using 6
5. if $P_{1, opt} \leq P_{1, max}$
6. Compute $R_1$ and $R_2$.
7. else ($P_{1, opt} > P_{1, max}$)
8. Set $P_1 = P_{1, opt}$ (as $R_2$ can not be maximized w.r.t. $P_1$)
9. Obtain $\alpha_{opt}'$ using 11
10. Compute $R_1$ and $R_2$.

is obtained simply by solving the quasi-concave problem $\max_{\alpha} \min\{R_1, R_{2-2}\}$, as the value of $P_1$ is fixed now, where

$$\frac{|h_2|^2 (1-\alpha P) + |h_{12}|^2}{|h_1|^2 + \sigma^2}$$

is monotonically decreasing function of $\alpha$ while $\gamma_{1-1} = \frac{|h_1|^2 \alpha P}{|h_2|^2 + \sigma^2}$ is monotonically increasing function of $\alpha$, so that equating $\gamma_{1-1}$ to $\frac{|h_1|^2 (1-\alpha P) + |h_{12}|^2}{|h_1|^2 \sigma^2 + \sigma^2}$ will lead to the expression in (11).

Hence, the max-min optimization for this case can be outlined as in the following algorithm.

### IV. THE COOPERATIVE PD-NOMA SYSTEM: THE THREE-USER CASE

The three-user cooperative NOMA system, similar to the two-user cooperative CNOMA system and using the degraded RBC model, is shown in Fig. 2 where each user decodes the signals of the weaker users, in addition to its own signal, and forwards them to the weaker users in a full-duplex cooperation mode as in the two-user case. Hence, the received signals at the three users may be expressed as

$$y_1 = h_1 x + z_1,$$

$$y_2 = h_2 x + h_{12} x_1 + z_2,$$

and

$$y_3 = h_3 x + h_{13} x_1 + h_{23} x_2 + z_3,$$

where $h_1, h_2,$ and $h_{12}$ are as defined before, $h_3$ denotes the link from the source or BS to the third user (User 3), $h_{13}$ and $h_{23}$ denote the channel gains of the cooperative links from User 1 and User 2 to User 3, respectively. The signal $x$ contains the information messages, from the source or the BS, intended to the three users, the signal $x_1$ contains the cooperative information messages intended to User 2 and User 3 (upon successful decoding of their signals at User 1 which is guaranteed as $|h_1|^2 > |h_2|^2 > |h_{12}|^2$), and similarly the signal $x_2$ contains the cooperative information message intended to User 3 (upon successful decoding of its signal at User 2 which is guaranteed as $|h_2|^2 > |h_{12}|^2$), and the noise component $z_3$ is independent Gaussian random variable with zero-mean and variance $\sigma^2$. 

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where the problem may be expressed as following cases: intended for User 2. User 1 that is allocated for the "cooperative" signal (message) A three-user cooperative relay broadcast channel.

The achievable rates of the three users can be expressed as [30]

\[ R_1 = \log_2 \left(1 + \frac{|h_1|^2 \alpha_1 P}{\sigma^2}\right), \]

\[ R_2 = \min \{ \log_2 \left(1 + \frac{|h_1|^2 \alpha_2 P}{|h_1|^2 \alpha_1 P + \sigma^2}\right), \log_2 \left(1 + \frac{|h_2|^2 \alpha_2 P + |h_{12}|^2 \beta P_1}{|h_2|^2 \alpha_1 P + \sigma^2}\right) \}, \]

and

\[ R_3 = \min \{ R_{1-3}, R_{2-3}, R_{3-3} \}, \]

where

\[ R_{1-3} = \log_2 \left(1 + \frac{|h_1|^2 \alpha_3 P}{|h_1|^2 \alpha_1 P + |h_1|^2 \alpha_2 P + \sigma^2}\right), \]

\[ R_{2-3} = \log_2 \left(1 + \frac{|h_2|^2 \alpha_3 P + |h_{12}|^2 (1 - \beta) P_1}{|h_2|^2 \alpha_1 P + |h_2|^2 \alpha_2 P + |h_{12}|^2 \beta P_1 + \sigma^2}\right), \]

\[ R_{3-3} = \log_2 \left(1 + \frac{|h_3|^2 \alpha_3 P + |h_{13}|^2 (1 - \beta) P_1 + |h_{23}|^2 P_2}{|h_3|^2 \alpha_1 P + |h_3|^2 \alpha_2 P + |h_{13}|^2 \beta P_1 + \sigma^2}\right), \]

where \( \alpha_i \) denotes the portion of the BS transmit power, \( P \), that is allocated for the signal (message) intended for the \( i \)th user (as \( \alpha_i P \)) and \( \beta \) denotes the portion of the cooperation power at User 1 that is allocated for the "cooperative" signal (message) intended for User 2.

Similar to the two-user scenario, we may consider the following cases:

**A. NO POWER CONTROL ON THE USER COOPERATION POWER**

In this case, the transmit power at the stronger users, towards the weaker users, has no pre-determined maximum value and is left to be optimized. The corresponding optimization problem may be expressed as

\[ \max_{\alpha_1, \alpha_2, \alpha_3, P_1, P_2, P_3} \min \{ R_1, R_2, R_3 \}, \]

s.t. \( P \leq P_T \),

\[ P_{12} \geq 0, P_{13} \geq 0, P_2 \geq 0 \]

\[ \alpha_1 + \alpha_2 + \alpha_3 \leq 1, \]

where \( P_{12} = \beta P_1 \) and \( P_{13} = (1 - \beta) P_1 \).

This optimization problem (as (5) is a special case of it) is again neither concave nor quasi-concave and it is more involved than the one in (5) since more parameters are involved with varying dependence of some of these parameters (while \( R_{2-2} \) monotonically increases with \( P_{12} \), \( R_{2-3} \) and \( R_{3-3} \) decrease). The numerical solution of this problem would require either joint search or alternating one through determining the optimal values of \( \alpha_i \) assuming fixed values of \( P_{12} \), \( P_{13} \), and \( P_2 \) (being a quasi-concave one) and then searching for the optimal values of the user cooperation powers. However, in here, it is tackled using a more intuitive approach in a similar way to the two-user scenario in the previous section.

**Proposition 2:** The optimal values of the cooperation power at User 1, \( P_1 \), and at User 2, \( P_2 \), for given \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) can be obtained as

\[ P_{1,\text{opt}} = P_{12,\text{opt}} + P_{13,\text{opt}}, \]

where

\[ P_{12,\text{opt}} = \left(1 + \frac{\sigma^2}{|h_{12}|^2 \alpha_1 P} \right) \frac{|h_2|^2}{1 + \frac{\sigma^2}{|h_1|^2 \alpha_2 P}} \]

\[ = \left(1 + \frac{\sigma^2}{|h_{12}|^2 \alpha_1 P} \right) \frac{|h_2|^2}{|h_{12}|^2 \alpha_2 P}, \]

\[ P_{13,\text{opt}} = \left(1 + \frac{|h_{13}|^2 P_{12,\text{opt}} + \sigma^2}{|h_{13}|^2 \alpha_1 P + \sigma^2} \right) \frac{|h_1|^2}{1 + \frac{|h_{13}|^2 \alpha_2 P}{|h_{13}|^2 \alpha_2 P}} \]

\[ = \left(1 + \frac{|h_{13}|^2 P_{12,\text{opt}} + \sigma^2}{|h_{13}|^2 \alpha_1 P + \sigma^2} \right) \frac{|h_1|^2}{|h_{13}|^2 \alpha_3 P}, \]

and

\[ P_{2,\text{opt}} = \left(1 + \frac{|h_{13}|^2 P_{12,\text{opt}} + \sigma^2}{|h_{13}|^2 \alpha_1 P + \sigma^2} \right) \frac{|h_{13}|^2}{1 + \frac{|h_{13}|^2 \alpha_2 P}{|h_{13}|^2 \alpha_2 P}} \]

\[ = \left(1 + \frac{|h_{13}|^2 P_{12,\text{opt}} + \sigma^2}{|h_{13}|^2 \alpha_1 P + \sigma^2} \right) \frac{|h_{13}|^2}{|h_{23}|^2 \alpha_3 P} \]

\[ = \frac{|h_{13}|^2}{|h_{23}|^2 \alpha_3 P_1}, \]

where the values of \( \alpha_{1,\text{opt}} \), \( \alpha_{2,\text{opt}} \), and \( \alpha_{3,\text{opt}} \) are obtained using the bisection method as in Algorithm 2 in the Appendix.

**Proof:** The proof can be constructed in a similar to the proof of Proposition 1 where the optimal source power allocation parameters, \( \alpha_i \) can be analytically decoupled from
the optimization of user cooperation powers and determined through solving the quasi-concave optimization problem as explained in the Appendix. Next, one may start with maximizing $R_2$, for given values of $\alpha_1$ and $\alpha_2$, as explained before in the proof of Proposition 1 and determine the optimal value of $P_{12}$, and then utilize the fact that $R_{2,3}$, for a fixed or given value of $P_{12}$, is a monotonically increasing function of $P_{13}$ and hence the optimal value of $P_{13}$ can be obtained by setting $\gamma_{2,3} = \gamma_{1,3}$ (since $\gamma_{1,3} > \gamma_{2,3} > \gamma_{3,3}$ by degradedness). Finally, the optimal value of $P_2$ can be obtained, for fixed or given values of $P_{12}$ and $P_{13}$, through setting $\gamma_{2,3} = \gamma_{3,3}$ and solving for $P_2$ to get the expressions in (23)-(25).

Remark 5: Again in similar way the two-user scenario, the cooperation powers are proportional to the differences in the channel gains and their ratios. For example, $P_{13,\text{opt}}$ is proportional to the water-level difference of $|h_1|^2$ and $|h_2|^2$ as $P_{13,\text{opt}}$ should fill in the gap to attain $R_{2,3} = R_{1,3}$ and it increases as $P_{12,\text{opt}}$ increases since the latter tends to decrease $R_{2,3}$. A similar observation applies to $P_{2,\text{opt}}$ where it is proportional to the water-level differences of $|h_1|^2$, $|h_2|^2$, and $|h_3|^2$ and it tends to reduce as $P_{13,\text{opt}}$ increases since $P_{13,\text{opt}}$ tends to increase $R_{3,3}$ as the channel gain $|h_{13}|^2$ gets larger.

Remark 6: A possible scenario is that each user would forward only the information message of the next weaker user so that User 1 would re-encode and transmit the intended message of User 2 only ($\beta = 1$ and $P_{12} = P_1$). This will result in smaller max-min rate as it is no more possible to attain $R_{2,3} = R_{1,3}$ and the max-min rate will be upper limited by $R_{2,3}$. However, such scheme may result in larger sum rate of the three users if it were the target metric.

V. THE EFFECT OF SELF-INTERFERENCE IN FULL-DUPLEX COOPERATION: REVISITED

The derivations in the previous section have assumed no self-interference at the cooperating users as they re-encode and transmit the intended messages of the weaker users. However, in practice some self-interference is there due to full-duplex mode where the differences in the power of transmitted and received signals can be large. Again, a lot of efficient interference mitigation or reduction techniques were proposed and implemented [14]. However, the residual interference is usually modeled as an additional independent signal component so that the SNR at the User 1 of its decoded signal becomes $\gamma_{1,1,s} = \frac{|h_1|^2 aP}{\kappa |h_1|^2 P_1 + \sigma^2}$, where $|h_1|$ denotes the self-interference channel gain and $0 \leq \kappa \leq 1$ is a parameter that denote the self-interference cancellation or reduction factor so that $\kappa = 0$ refers to perfect cancellation and $\kappa = 1$ indicates that no self-interference cancellation scheme is adopted at the cooperating user.

So, the achievable rates, denoted as $R_{1,s}$ and $R_{2,s}$, can be expressed as the following for the strong user

$$R_{1,s} = \log_2 \left(1 + \frac{|h_1|^2 aP}{\kappa |h_1|^2 P_1 + \sigma^2}\right),$$

and for the weak user as

$$R_{2,s} = \min \{R_{1,2,s}, R_{2,2,s}\}
= \{ \log_2 \left(1 + \frac{|h_1|^2 (1 - a)P}{|h_1|^2 aP + \kappa |h_2|^2 P_1 + \sigma^2}\right),
\log_2 \left(1 + \frac{|h_2|^2 (1 - a)P + |h_3|^2 P_1}{|h_2|^2 aP + \sigma^2}\right) \}.$$ (27)

A. NO POWER CONTROL ON THE USER COOPERATION POWER

The corresponding optimization problem may be expressed as

$$\max \min_{\alpha, P_1} \{R_{1,s}, R_{2,s}\},$$

s.t. $P \leq P_T$,

$$P_1 \geq 0.$$ (28)

This optimization problem is again neither convex nor quasi-concave and would not allow analytical decoupling of optimizing the source power parameters, $\alpha$, from optimizing the cooperative power, as done before for self-interference-free case, since now both $R_{1,1,s}$ and $R_{1,2,s}$ and equivalently $\gamma_{1,1,s}$ and $\gamma_{1,2,s}$ are now dependent on $P_1$. To tackle this problem, a two-step procedure was proposed in [24] where it is reduced to a quasi-concave problem, for fixed $P_1$, to allow converting it to into a series of convex feasibility problems and hence the use of the bi-section method, which leads to a semi-analytical solution as in Theorem 2 in [24].

Then, the optimal value of $P_1$ is derived (the second term in eqns. 2 and 4 in [24] is incorrect and hence the expressions derived for the optimal transmit power at the strong user...
(cooperation power) need to be corrected accordingly). However,
using a more intuitive approach here, the parameters $P_1$
and $\alpha$ can be jointly optimized as in the following proposition.

**Proposition 3:** The optimal value of the cooperation power
at User 1, $P_{1, opt}$, and $\alpha_{opt}$ can be obtained by jointly solving
\begin{equation}
P_1 = \frac{-b + \sqrt{b^2 + 4ac}}{2a},
\end{equation}
where in here $a = \kappa |h_1|^2 P, b = |h_1|^2 |h_2|^2 \alpha P + \kappa |h_1|^2 |h_2|^2 (1 - \alpha) P + |h_2|^2 \sigma^2$, and $c = (1 - \alpha) P \sigma^2$.

and
\begin{equation}
\alpha = \sqrt{1 + \frac{|h_1|^2 P}{\kappa |h_1|^2 P + \sigma^2}} - 1.
\end{equation}

**Proof:** Since $\gamma_{1-2,s}$ is a monotonically decreasing function of $P_1$ while $\gamma_{2-2,s}$ is a monotonically increasing function of $P_1$, then the maximization of $R_{2,s}$ will take place at $\gamma_{1-2,s} = \gamma_{2-2,s}$ which will result in the closed-form expression in (29). Similarly, the maximization of $\min\{R_{1,s}, R_{1-2,s}\}$ will take place at $\gamma_{1-1,s} = \gamma_{1-2,s}$ which will result in the closed-form expression in (30) using the same monotonicity argument as before.

**Remark 7:** The expression in (30) reduces to the one in (7) for $\kappa = 0$ as expected. Also, the optimal value of $\alpha$ here, $\alpha_{opt}$, increases as User 1 needs more power to compensate for SINR reduction due to self-interference. Also, as the self-interference gets large, it may result in lower performance of FD CNOMA as compared to HD CNOMA and the condition for that can be obtained by solving $\frac{1}{2} R_1 \geq R_{1,s}$ to result in $\kappa |h_1|^2 P_{1, opt,s} + 1 \geq \frac{|h_1|^2 \alpha_{opt,P} \sigma^2}{1 + \frac{|h_1|^2 \alpha_{opt,P} \sigma^2}{\alpha P}}$. This condition may reduce for low source-User 1 link SNR $\frac{|h_1|^2 P}{\sigma^2}$ and to $\kappa |h_1|^2 P_{1, opt,s} + 1 \geq 2 \alpha_{opt,P} \approx 2$ (since both $\alpha_{opt}$ and $\alpha_{opt,P}$ approach 0.5 for small SNR as explained before). For large source-User 1 link SNR, the condition may reduce to $\kappa |h_1|^2 P_{1, opt,s} + 1 \geq \frac{|h_1|^2 \alpha_{opt,P} \sigma^2}{\alpha P}$.

**B. WITH POWER CONTROL ON THE USER COOPERATION POWER**

For this case, the corresponding optimization problem may be expressed as
\begin{equation}
\max \min \{R_{1,s}, R_{2,s}\}, \\
\text{s.t.} \ P \leq P_T, \\
P_1 \leq P_{1, max}.
\end{equation}

Again, in here the scenario where $P_{1, max} < P_{1, s, opt}$, where that optimal value is obtained as in Proposition 3, is the relevant one. Now, $\gamma_{2-2,s}$ will be always smaller than $\gamma_{1-2,s}$ and the optimal value of $\alpha$ can be easily obtained by setting $\gamma_{2-2,s} = \gamma_{1-1,s}$ as the monotonicity argument also applies here as before to get the following
\begin{equation}
\alpha_{s, opt} = \frac{-b + \sqrt{b^2 + 4ac}}{2a},
\end{equation}
where in here $a = |h_1|^2 |h_2|^2 P, b = P \sigma^2 (|h_1|^2 + |h_2|^2) + \kappa |h_1|^2 |h_2|^2 P P_{1, max} + |h_2|^2 \sigma^2$, and $c = |h_1|^2 |h_2|^2 P P_{1, max} (|h_2|^2 + |h_2|^2)^2 + |h_2|^2 \alpha P \sigma^2 + |h_2|^2 P_{1, max} \sigma^2$.

For the three-user scenario, the expressions for the rates, with self-interference, can be written as
\begin{equation}
R_{1,s} = \log_2 (1 + \frac{|h_1|^2 \alpha_1 P}{\kappa_1 |h_2|^2 P_1 + \sigma^2}),
\end{equation}
where \$P_1 = \frac{-b + \sqrt{b^2 + 4ac}}{2a}$, and
\begin{equation}
R_{2,s} = \min \{ \log_2 (1 + \frac{|h_1|^2 \alpha_2 P}{|h_2|^2 \alpha_1 P + \kappa_1 |h_2|^2 P_1 + \sigma^2}), \log_2 (1 + \frac{|h_2|^2 \alpha_2 P + |h_2|^2 \alpha_1 P + \kappa_2 |h_2|^2 P_1 + \sigma^2}) \},
\end{equation}
\begin{equation}
R_{1-3,s} = \log_2 (1 + \frac{|h_1|^2 \alpha_3 P + |h_2|^2 \alpha_3 P + \kappa_1 |h_2|^2 P_1 + \sigma^2}),
\end{equation}
\begin{equation}
R_{2-3,s} = \log_2 (1 + \frac{|h_2|^2 \alpha_3 P + |h_2|^2 P_1 + \kappa_2 |h_1|^2 P_1 + \sigma^2}),
\end{equation}
where $|h_{s,1}|$ and $|h_{s,2}|$ denote the self-interference channel gains at User 1 and User 2, respectively and similarly $\kappa_1$ and $\kappa_2$ denote the corresponding self-interference cancellation or reduction factors. $R_{3-3,s} = R_{3-3}$ as before since User 3 is not transmitting to others. For the optimization of the transmit power and the cooperation power parameters, a similar sequential approach to the one utilized in the proof of Proposition 2 applies using similar monotonicity arguments where, in the first step, we may set $R_{3-3} = R_{2-3,s}$, solve for $P_2$, then set $R_{2-3,s} = R_{1-3,s}$ and solve for $P_{13}$, and finally set $R_{1-2,s} = R_{2-2,s}$ and solve for $P_{12}$ using the expressions obtained before for $P_{13}$ and $P_2$. Then, in the second step, we solve for the $\alpha_1$’s; however, this will lead to very tedious analytical expressions and one has to resort to numerical solutions which is beyond the scope of this paper. Again, the self-interference will degrade the performance of the FD-CNOMA system.

**VI. RESULTS AND DISCUSSIONS**

**A. THE TWO-USER CASE**

In here, the obtained results for deterministic (non-fading) channels such as line-of-sight (LoS) dominant high frequency mm-wave and THz RF channels and LoS indoor VLC channels are presented. The results are extensible to fading channels; however, that will require the derivations of the outage capacity, for a given QoS target rate, and ergodic capacity expressions which is beyond the scope of this paper.

The optimal power allocation is shown in Fig. 3, for the two-user case $|h_1|^2 = 10, |h_2|^2 = 5$ and $\sigma^2 = 1$, where
it can be seen that the optimal value of $\alpha$, i.e., the portion of power allocated to the strong user, approaches 1/2 as the SNR of the strong user link gets small which is due to the fact that $\gamma_{1-1}$ and $\gamma_{1-2}$ becomes comparable and hence equal power allocation becomes optimal to achieve the same rate. On the other hand, the optimal value of $\alpha$ becomes small as that SNR becomes large. The explanation of this is that both the numerator $\gamma_{1-1}$ and the denominator in $\gamma_{1-2}$ need to be controlled through reducing $\alpha$ to end up with the target equal rates. In general, the ratios of $|h_{12}|^2$ and $|h_{11}|^2$ are dependent on the channel parameters such as the path loss parameters (path loss exponent, operating frequency,...) for RF channels and the Lambertian model parameters for indoor VLC systems [31] such as the distance between the light-emitting diode (LED) and the served users, the angle of irradiance, the angle of incidence, the order of Lambertian emission, and the optical filter gain.

The plots of the optimized max-min rates for cooperative NOMA as compared to non-cooperative NOMA, where no cooperation between the users takes place, for the two-user case, assuming $|h_{11}|^2 = 10$, $|h_{12}|^2 = 5$ and $\sigma^2 = 1$, are shown in Fig. 4 for both the cases of no power control on the cooperation power and with power control on the cooperation power (set as $P_{1,\text{max}} = 0.5 P_{1,\text{opt}}$). The first observation is that the user cooperation is beneficial only for the medium range of the transmit power. The explanation is that $\gamma_{1-2}$ and $\gamma_{2-2}$ become comparable for small and large $P$ and hence the role of cooperation gets negligible. The second observation is that the cooperation gain increases as $|h_{12}|^2/|h_{11}|^2$ increases and becomes negligible, for all the SNR range, when $|h_{11}|^2$ and $|h_{12}|^2$ are comparable, as expected since the cooperation power tends to “water-fill” the difference between $1/|h_{11}|^2$ and $1/|h_{12}|^2$ as explained before. The ratios of the channel gains here are chosen assuming a desirable scenario for CNOMA where a good disparity between the strong user and the weak user is there and the cooperation link (User 1-User 2 link) is relatively strong. However, for dominantly LoS RF channels in general, the ratio of the channel gains can be approximated as $|h_{11}|^2/|h_{12}|^2 \approx d_{12}^{-\nu_1}/d_{12}^{-\nu_2}$ where $d_1$ and $d_2$ denote the distances of the BS-User 1 and BS-User 2 links with the corresponding path loss exponents $\nu_1$ and $\nu_2$, respectively. So, for a given value $|h_{12}|^2/|h_{11}|^2 = \zeta_{12}$, we may write relate the distances as $d_2 = (\zeta_{12} d_1^{\nu_2})^{1/\nu_2}$. The path loss exponents values will depend on the propagation conditions of the different links; however, since User 1 is assumed to be the strong user, then $d_1 \ll d_2$ and/or the probability of LoS for the BS-User 1 link is higher such that $\nu_2 > \nu_1$, due to the effect of the multiple reflections [32] and hence the required disparity between the strong and weak user is guaranteed.

Also, the plots show interestingly that HD CNOMA, where the BS sends to User 1 and User 2 in the first time slot and then User 1 sends to the cooperative messages User 2 in the second time slot, outperforms non-cooperative or conventional NOMA for small SNR values. However, that SNR range increases (may go up to 10 dB) for smaller values of $|h_{22}|^2$, for the same ratio of $|h_{12}|^2/|h_{11}|^2$. This is due to the increased relative gain of user cooperation as the BS-to-User 2 link, $|h_{22}|^2$, gets weaker. The explanation is that the role of the term due to user cooperation $|h_{12}|^2 P_1$ becomes more evident as the term associated with $|h_{22}|^2$ gets smaller. Finally, it can be observed that self-interference-free FD CNOMA always outperforms HD CNOMA as expected.

To investigate the effect of the path loss exponents on the CNOMA system, the optimal cooperation power, as given in (6), is shown in Fig. 5 for different values of $\nu_{12}$ and hence the ratio $|h_{12}|^2/|h_{11}|^2 \approx d_{12}^{-\nu_{12}}/d_{12}^{-\nu_1}$, where $d_{12}$ and $\nu_{12}$ denote the distance of User 1-User 2 link and the corresponding path loss exponent, respectively, as it plays an important role in determining the required cooperation power. Assuming that the angle between the BS-to-User 1 link and the BS-to-User 2 link is small, it follows that $d_{12}$ will be slightly less than $d_2$ as $d_{12} \approx d_2 - d_1$ which implies that
User 3 in the third time slot, may achieve higher max-min rate than User 3 in the second time slot, and finally User 2 sends to and then User 1 sends the cooperative messages to User 2 and the super-imposed messages to three users in the first time slot slotted CNOMA, with three time slots where the BS transmits users increases. Again, it is interesting that even orthogonal or smaller, as expected, for the max-min rate as the number of users increases. The max-min achievable rates for the three user case are (as far as the condition path loss exponents, and other path loss parameters of the values in general are dependent on the respective distances, are chosen for a desirable scenario for CNOMA and their channel parameters of the corresponding links. However, as shown in Fig. 7, the cooperation gain increases as \(|h_3|^2\) decreases which is due to the fact that the weaker the channel gain of User 3, the smaller its achievable rate is and hence the common max-min rate of the three users leading to a more role of cooperation to improve that common rate. Again, the channel gains and their ratios here are chosen for a desirable scenario for CNOMA and their values in general are dependent on the respective distances, path loss exponents, and other path loss parameters of the corresponding RF links or other parameters for VLC channels (as far as the condition \(|h_1|^2 > |h_2|^2 > |h_3|^2\) is satisfied).

The max-min achievable rates for the three user case are smaller, as expected, for the max-min rate as the number of users increases. Again, it is interesting that even orthogonal or slotted CNOMA, with three time slots where the BS transmits the super-imposed messages to three users in the first time slot and then User 1 sends the cooperative messages to User 2 and User 3 in the second time slot, and finally User 2 sends to User 3 in the third time slot, may achieve higher max-min rate for small SNR as compared to conventional NOMA. Similar to the two-user scenario, the optimal cooperation power is proportional to the “water-level” differences \(\frac{1}{|h_2|^2} - \frac{1}{|h_1|^2}\) and \(\frac{1}{|h_3|^2} - \frac{1}{|h_1|^2}\) as well as the channel gains ratios \(\frac{|h_2|^2}{|h_1|^2} \cdot \frac{|h_3|^2}{|h_1|^2}\) where again these ratios are dependent on the propagation channel parameters of the corresponding links.

The tolerance of the bi-section method in Algorithm 2 is set to \(\epsilon = 0.5 \times 10^{-3}\) where such a value is numerically sufficient for the achievable rate being a less-sensitive measure as compared to outage probability or bit error rate. Finally, the computational complexity of orthogonal or slotted CNOMA and conventional NOMA are similar to that of FD-CNOMA as the bi-section method is utilized for power allocation for all of these schemes as in Algorithm 2. The CNOMA schemes will require the additional computation of the required optimal cooperation powers as given in Propositions 1 and 2 for the two-user and the three-user scenarios, respectively.
VII. CONCLUSION
As the merge of the enabling communications schemes and technologies for future wireless networks is gaining more interest in both academia and industry to meet the high-data rate applications and other desired QoS requirements, the integration of power-domain NOMA with cooperative communications is considered in this paper where the two-user cooperative NOMA system achievable max-min rate optimization is revisited and novel results are derived and extended to the three-user cooperative system. The results shed new light on the performance and design of both full-duplex and half-duplex cooperative NOMA schemes and can be utilized to investigate the performance of CNOMA in both line-of-sight high-frequency RF channels as well as VLC channels. Also, the developed formulations can be extended to multipath fading scenarios for the ergodic or outage capacity. The formulations may be extended to multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) relay broadcast channels in a similar way to [33]; however, it should be observed that some other schemes such as rate splitting might be more attractive than NOMA for these channels [34], [35].

APPENDIX
In here, the bi-section algorithm to compute the optimal power allocation coefficients is presented.

Algorithm 2 Overview of the Bi-Section Algorithm

```
Input: \|h_1\|^2, P, r_{\min} = 0, r_{\max} = R_1, \epsilon
Output: \alpha_1^*, \alpha_2^*, and \alpha_3^*
1 while \( r_{\max} - r_{\min} \geq \epsilon \) do
2 \hspace{1em} r = 0.5(\( r_{\min} + r_{\max} \))
3 end
4 Solve to find \( \alpha_1^*, \alpha_2^*, \) and \( \alpha_3^* \) if feasible then
5 \hspace{1em} \( \alpha^* = \alpha, r = r_{\min} \)
6 else
7 \hspace{1em} \( r = r_{\max} \)
8 end
```

This is known to be a quasi-concave optimization which can be efficiently solved through transforming it into a set of convex feasibility problems and using the bi-section method as in the following algorithm.

The complexity of the bi-section method is \( O(N \log \epsilon) \) where \( N \) is the number of the users and \( \epsilon \) denotes the tolerance of the bi-section method [20].

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