Tau lepton distributions in semileptonic B decays

Work supported in part by KBN grants 2P30207607 and PB659/P03/95/08 and by EEC grant ERB-CIPD-CT94-0016.

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Abstract

Analytic formulae are given for order $\alpha_s$ perturbative QCD corrections to the double differential distribution of the $\tau$ lepton energy and the invariant mass of the $\tau +$ antineutrino system in semileptonic decays of the bottom quark. The corresponding distribution for B meson decays are obtained by combining these results and the nonperturbative QCD corrections calculated by Falk et al. The moments of the energy distribution of the $\tau$ lepton are calculated.
1 Introduction

The theoretical description of the semileptonic decays of beautiful hadrons based on the Heavy Quark Effective Theory (HQET) \cite{1, 2, 3, 4, 5, 6} or the heavy quark mass expansion\cite{7, 8, 9} and on perturbative QCD seems to be quite satisfactory. The recent ALEPH measurement of the semileptonic branching ratio for the baryon $\Lambda_b$\cite{10} indicates that the problems of the present theory to explain the short lifetime of $\Lambda_b$ are associated with the sector of nonleptonic decays. In contrast to the nonleptonic widths, the semileptonic widths of beautiful hadrons seem to follow the theoretical predictions. This conclusion is strengthen by an old observation that, despite the large differences in the lifetimes, the semileptonic widths of charmed hadrons are fairly similar. It is clear that the theory of the hadronic processes involving heavy quarks is still to be improved. Another failure is the prediction of a large $\Lambda_b$ polarization in $Z^0$ decays. In fact the measurement\cite{11} of $\Lambda_b$ polarization via its semileptonic decay channels\cite{12, 13, 14} gives a much smaller value than predicted in the heavy quark mass limit\cite{15}. All this indicates that further developments in the theory of nonleptonic processes are expected whereas the theory of semileptonic decays is firm and stable. Therefore the latter processes can be used in determination of parameters of the standard model like the masses $m_b$ and $m_c$ of the charm and bottom quarks and the strong coupling constant $\alpha_s$ at relatively low scales. Recent determination\cite{16} of the Cabibbo-Kobayashi-Maskawa matrix element $V_{cb}$ is a spectacular example. It has been argued\cite{13, 17, 18} that moments of the lepton spectra can be used in determination of $\alpha_s$, $m_b$, and $m_c$. Semileptonic decays of the bottom quark are particularly promising in this respect because in addition to the channels with practically massless charged leptons (electron and muon) in the final states the channel is kinematically allowed with the $\tau$ lepton, whose mass is appreciable in comparison to $m_b$ and $m_c$. This process can be also a window on a new physics\cite{19, 20, 21}. The theoretical description of inclusive semileptonic decays of the $b$ quark into the $\tau$ lepton in the framework of the standard model is therefore necessary. The nonperturbative QCD corrections to the total rate and the $\tau$ lepton energy spectrum have been calculated by a number of groups\cite{22, 23, 24}. Perturbative QCD corrections to the total semileptonic decay rate are known. In \cite{22} a numerical approach was used based on an important work by Hokim and Pham\cite{25}; see also Bagan et al.\cite{26}. In addition in\cite{17} an analytical formula has been obtained for the distribution of the
invariant mass of the virtual W (i.e. the τν̄τ system) which after performing
one-dimensional numerical integration gives the correction to the semileptonic
rate in perfect agreement with the results of \cite{22}. The case of perturbative
QCD corrections to the τ lepton energy spectrum is less satisfactory. In ref.\cite{27}
formulae are given for these corrections as a one-dimensional integral. In the
present article we give another representation of these corrections. We have
calculated analytically the double differential distribution of the τ energy and
the mass of the τν̄τ system. In the limit $m_\tau \to 0$ our formulae reduce to the
well-established results for the massless case\cite{28,29}. After numerical integra-
tion over the energy of τ they are in perfect agreement with the analytical
formula given in\cite{17}. We were unable, however, to find agreement between our
results and those of ref.\cite{27} for the corrections to the τ energy distribution.
Our numerical results for the τ energy distribution have been given in \cite{30}. In
the present article we calculate the moments of this distribution.

The article is organized as follows: in Sec.2 kinematical variables are de-
defined which are used throughout. In Sec.3 the calculation of QCD corrections
is briefly described. In Sec.4 we give the analytic result for the double differen-
tial distribution of the τ energy and the invariant mass of the τν̄τ system. This
is the main result of the paper. In Sec.5 the moments of the τ lepton energy
distribution are calculated numerically and tabulated. In Sec.6 the results of
this article are summarized.

# 2 Kinematics

## 2.1 Kinematical variables

The purpose of this section is to define the kinematical variables which are used
in this paper. We describe also the constraints imposed on these variables for
three and four-body decays of the heavy quark.

The calculation is performed in the rest frame of the decaying b quark. Since
the first order perturbative QCD corrections to the inclusive process are
taken into account, the final state can consist either of a produced quark c,
a lepton τ and an anti-neutrino ν̄τ or of the three particles and a real gluon.
The four-momenta of the particles are denoted in the following way: $Q$ for the
b quark, $q$ for the c quark, $\tau$ for the charged lepton, $\nu$ for the corresponding
anti-neutrino and $G$ for the real gluon. By the assumption that all the particles
are on-shell, the squares of their four-momenta are equal to the squares of masses:

\[ Q^2 = m_b^2, \quad q^2 = m_c^2, \quad \tau^2 = m_\tau^2, \quad \nu^2 = G^2 = 0. \]  

(1)

The four-vectors \( P = q + G \) and \( W = \tau + \nu \) characterize the quark–gluon system and a virtual intermediating \( W \) boson respectively. We define a set of variables scaled in the units of mass of the heavy quark \( m_b \):

\[ \rho = \frac{m_c^2}{m_b^2}, \quad \eta = \frac{m_\tau^2}{m_b^2}, \quad x = \frac{2E_\tau}{m_b^2}, \quad t = \frac{W^2}{m_b^2}, \quad z = \frac{P^2}{m_b^2}. \]  

(2)

We choose \( m_b \) as the unit of mass (i.e. \( Q^2 = 1 \)) and introduce light-cone variables describing the charged lepton:

\[ \tau_\pm = \frac{1}{2} \left( x \pm \sqrt{x^2 - 4\eta} \right). \]  

(3)

The system of the \( c \) quark and real gluon is characterized by the following quantities:

\[ P_0(z) = \frac{1}{2}(1 - t + z), \]
\[ P_3(z) = \sqrt{P_0^2 - z} = \frac{1}{2}[1 + t^2 + z^2 - 2(t + z + tz)]^{1/2}, \]
\[ P_\pm(z) = P_0(z) \pm P_3(z), \]
\[ Y_p(z) = \frac{1}{2} \ln \frac{P_+(z)}{P_-(z)} = \ln \frac{P_+(z)}{\sqrt{z}}, \]  

(4)

where \( P_0(z) \) and \( P_3(z) \) are the energy and the length of the momentum vector of the system in the \( b \) quark rest frame, \( Y_p(z) \) is the corresponding rapidity.

Similarly for the virtual boson \( W \):

\[ W_0(z) = \frac{1}{2}(1 + t - z), \]
\[ W_3(z) = \sqrt{W_0^2 - t} = \frac{1}{2}[1 + t^2 + z^2 - 2(t + z + tz)]^{1/2}, \]
\[ W_\pm(z) = W_0(z) \pm W_3(z), \]
\[ Y_w(z) = \frac{1}{2} \ln \frac{W_+(z)}{W_-(z)} = \ln \frac{W_+(z)}{\sqrt{t}}, \]  

(5)
¿From kinematical point of view the three body decay is a special case of the four body one with the vanishing gluon four-momentum, what is equivalent to a substitution $z = \rho$ in the previous formulae. It is convenient to use in this case the following variables:

\[ p_0 = P_0(\rho) = \frac{1}{2}(1 - t + \rho), \quad p_3 = P_3(\rho) = \sqrt{p_0^2 - \rho}, \]
\[ p_\pm = P_\pm(\rho) = p_0 \pm p_3, \quad w_\pm = W_\pm(\rho) = 1 - p_\pm, \]
\[ Y_p = \mathcal{Y}_p(\rho) = \frac{1}{2} \ln \frac{p_+}{p_-}, \quad Y_w = \mathcal{Y}_w(\rho) = \frac{1}{2} \ln \frac{w_+}{w_-}. \]  

(6)

We express also the scalar products which appear in the calculation by the variables $x$, $t$ and $z$:

\[ Q \cdot P = \frac{1}{2}(1 + z - t), \quad \tau \cdot \nu = \frac{1}{2}(t - \eta), \]
\[ Q \cdot \nu = \frac{1}{2}(1 - z - x + t), \quad \tau \cdot P = \frac{1}{2}(x - t - \eta), \]
\[ Q \cdot \tau = \frac{1}{2} x, \quad \nu \cdot \tau = \frac{1}{2}(1 - x - z + \eta). \]  

(7)

All of the written above products are scaled in the units of the mass of the $b$ quark.

### 2.2 Kinematical boundaries

The allowed ranges of $x$ and $t$ for the three-body decay are given by inequalities:

\[ 2\sqrt{\eta} \leq x \leq 1 + \eta - \rho = x_{\text{max}}, \]  

(8)

\[ t_1 = \tau_- \left( 1 - \frac{\rho}{1 - \tau_-} \right) \leq t \leq \tau_+ \left( 1 - \frac{\rho}{1 - \tau_+} \right) = t_2 \]

(9)

(a region A). In the case of the four-body process the available region of the phase space is larger than the region A. The additional, specific for the four body decay area of the phase space is denoted as a region B. Its boundaries are the following:

\[ 2\sqrt{\eta} \leq x \leq x_{\text{max}}, \quad \eta \leq t \leq t_1. \]  

(10)
We remark, that if the charged lepton mass tends to zero than the region B vanishes.

One can also parameterize the kinematical boundaries of \( x \) as functions of \( t \). In this case we obtain for the region A:

\[
\eta \leq t \leq (1 - \sqrt{\rho})^2, \quad w_- + \frac{\eta}{w_-} \leq x \leq w_+ + \frac{\eta}{w_+},
\]

and for the region B:

\[
\eta \leq t \leq \sqrt{\eta} \left( 1 - \frac{\rho}{1 - \sqrt{\eta}} \right), \quad 2\sqrt{\eta} \leq x \leq w_- + \frac{\eta}{w_-}.
\]

The upper limit of the mass squared of the \( c \)-quark — gluon system is in the both regions given by

\[
z_{\text{max}} = (1 - \tau_+)(1 - t/\tau_+),
\]

whereas the lower limit depends on a region:

\[
z_{\text{min}} = \begin{cases} \rho & \text{in the region A,} \\ (1 - \tau_-)(1 - t/\tau_-) & \text{in the region B.} \end{cases}
\]

3 Calculation of QCD corrections

The QCD corrected differential rate for \( b \to c + \tau^- + \bar{\nu} \) reads:

\[
d\Gamma = d\Gamma_0 + d\Gamma_{1,3} + d\Gamma_{1,4},
\]

where

\[
d\Gamma_0 = G_F^2 m_b^5 |V_{\text{CKM}}|^2 \mathcal{M}_{0,3}^\tau d\mathcal{R}_3(Q; q, \tau, \nu)/\pi^5
\]

in Born approximation,

\[
d\Gamma_{1,3} = \frac{2}{3} \alpha_s G_F^2 m_b^5 |V_{\text{CKM}}|^2 \mathcal{M}_{1,3}^- d\mathcal{R}_3(Q; q, \tau, \nu)/\pi^6
\]

comes from the interference between the virtual gluon contribution and Born amplitudes, and

\[
d\Gamma_{1,4} = \frac{2}{3} \alpha_s G_F^2 m_b^5 |V_{\text{CKM}}|^2 \mathcal{M}_{1,4}^- d\mathcal{R}_4(Q; q, \tau, \nu)/\pi^7
\]
describes a real gluon emission. $V_{\text{CKM}}$ is the Cabibbo–Kobayashi–Maskawa matrix element associated the $b$ to $c$ or $u$ quark weak transition. Lorentz invariant $n$-body phase space is defined as

$$d\mathcal{R}_n(P; p_1, \ldots, p_n) = \delta^{(4)}(P - \sum p_i) \prod_i \frac{d^3p_i}{2E_i}. \quad (19)$$

In Born approximation the rate for the decay into three body final state is proportional to the expression

$$\mathcal{M}_{0,3}^- = F_0(x,t) = 4q \cdot \tau Q \cdot \nu = (1 - \rho - x + t)(x - t - \eta), \quad (20)$$

where the quantities describing the $W$ boson propagator are neglected. The three-body phase space is parameterized by the use of Dalitz variables:

$$d\mathcal{R}_3(Q; q, \tau, \nu) = \frac{\pi^2}{4} dx \, dt. \quad (21)$$

The matrix element $\mathcal{M}_{1,3}^-$ was evaluated in the fifties and has the following form:

$$\mathcal{M}_{1,3}^- = -[H_0 q \cdot \tau Q \cdot \nu + H_+ \rho Q \cdot \nu Q \cdot \tau + H_- q \cdot \nu q \cdot \tau$$

$$+ \frac{1}{2} \rho(H_+ + H_-) \nu \cdot \tau + \frac{1}{2} \eta \rho(H_+ - H_- + H_L) Q \cdot \nu$$

$$- \frac{1}{2} H_L \eta q \cdot \nu], \quad (22)$$

where

$$H_0 = 4(1 - Y_p p_0/p_3) \ln \lambda + (2p_0/p_3) \left[ \text{Li}_2 \left( 1 - \frac{w_+}{p_+ w} \right) \right.$$ 

$$- \text{Li}_2 \left( 1 - \frac{w_-}{w_+} \right) - Y_p (Y_p + 1) + 2(\ln \sqrt{\rho} + Y_p) (Y_w + Y_p)$$

$$+ [2p_3 Y_p + (1 - \rho - 2t) \ln \sqrt{\rho}] / t + 4,$$

$$H_{\pm} = \frac{1}{2} \left[ 1 \pm (1 - \rho) / t \right] Y_p / p_3 \pm \frac{1}{t} \ln \sqrt{\rho},$$

$$H_L = \frac{1}{t} (1 - \ln \sqrt{\rho}) + \frac{1 - \rho}{t^2} \ln \sqrt{\rho} + \frac{2}{t^2} Y_p p_3 + \frac{\rho Y_p}{t p_3}. \quad (23)$$

The virtual correction in the interference term is renormalized and hence $\mathcal{M}_{1,3}$ is ultraviolet convergent. However, the infrared divergences are left. They are
regularized by a small mass of gluon denoted by $\lambda_G$. According to Kinoshita–Lee–Naunberg theorem, the infrared divergent part should cancel with the infrared contribution of the four-body decay amplitude integrated over suitable part of the phase space.

The rate from real gluon emission is proportional to

$$\mathcal{M}_{1,4}^- = \frac{B_1^-}{(Q \cdot G)^2} - \frac{B_2^-}{Q \cdot G \cdot P \cdot G} + \frac{B_3^-}{(P \cdot G)^2},$$

where

$$B_1^- = q \cdot \tau [Q \cdot \nu (Q \cdot G - 1) + G \cdot \nu - Q \cdot \nu Q \cdot G + G \cdot \nu Q \cdot G],$$

$$B_2^- = q \cdot \tau [G \cdot \nu Q \cdot q - q \cdot \nu Q \cdot G + Q \cdot \nu (q \cdot G - Q \cdot G - 2 q \cdot Q)] + Q \cdot \nu (Q \cdot \tau q \cdot G - G \cdot \tau q \cdot G),$$

$$B_3^- = Q \cdot \nu (G \cdot \tau q \cdot G - \rho \cdot \tau \cdot P).$$

The four-body phase space is decomposed as follows:

$$dR_4(Q; q, G, \tau, \nu) = dz \, dR_3(Q; P, \tau, \nu) \, dR_2(P; q, G).$$

The four-momentum of the $c$ quark is substituted by $P - G$ and the integration of $\mathcal{M}_{1,4}^-$ over $dR_2(P; q, G)$ is performed. Lorentz invariance allows to reduce all of appearing integrals to scalar ones:

$$I_n = \int dR_2(P; q, G)(Q \cdot G)^n.$$

The formulae for $I_n$ have been presented in [29].

In the next step Dalitz parametrization (21) of the three-body phase space $dR_3(Q; P, \tau, \nu)$ is employed. The expression obtained after integrations has one part

$$\text{const } F_0(x, t) I_{\text{div}},$$

where

$$I_{\text{div}} = I_{-2} - (1 - t + \rho)I_{-1}/(P \cdot G) + \rho I_0/(P \cdot G)^2,$$

which is infrared divergent when $(x, t)$ belongs to the region A and the rest which is infrared finite. Since the method used in this calculation is exactly
the same as this used in the previous ones\cite{28, 29}, we do not go into the details. The important point is, that the infrared divergent part is regularized by a small gluon mass $\lambda_G$ and then contains terms proportional to $\ln \lambda_G$. The infrared divergent terms from three- and four-body contributions to the decay rate cancel out and the limit of vanishing gluon mass $\lambda_G \to 0$ is performed. This procedure yields well defined double differential distributions of lepton spectra which are described below.

4 Analytic results

The double differential unpolarized distribution of the $\tau$ energy and $\tau\bar{\nu}$ invariant mass squared from the $b$ quark decay with first order perturbative QCD corrections reads\footnote{A Fortran version of the formulae given in this section is available upon request from jezabek@hpjmiady.ifj.edu.pl or leszekm@thp1.if.uj.edu.pl.}

$$
\frac{d\Gamma}{dx\,dt} = \left\{ \begin{array}{ll}
12\Gamma_0 \left[ F_0(x, t) - \frac{2\alpha_s}{3\pi} F_1^A(x, t) \right] & \text{for } (x, t) \text{ in A,} \\
12\Gamma_0 \frac{2\alpha_s}{3\pi} F_1^B(x, t) & \text{for } (x, t) \text{ in B,}
\end{array} \right.
$$

(30)

where

$$
\Gamma_0 = \frac{G_F m_b^5}{192\pi^3} |V_{CKM}|^2,
$$

(31)

$$
F_0(x, t) = (1 - \rho - x + t)(x - t - \eta)
$$

(32)

and

$$
F_1^A(x, t) = F_0 \Phi_0 + \sum_{n=1}^{5} D_n^A \Phi_n + D_6^A,
$$

(33)

$$
F_1^B(x, t) = F_0 \Psi_0 + \sum_{n=1}^{5} D_n^B \Psi_n + D_6^B.
$$

(34)
as follows

\[ \Phi_0 = \frac{2p_0}{p_3} \left[ \operatorname{Li}_2 \left( 1 - \frac{1 - \tau^+}{p_+} \right) + \operatorname{Li}_2 \left( 1 - \frac{1 - t/\tau^+}{p_+} \right) \right. \\
- \operatorname{Li}_2 \left( 1 - \frac{1 - \tau^+}{p_-} \right) - \operatorname{Li}_2 \left( 1 - \frac{1 - t/\tau^+}{p_-} \right) \\
+ \operatorname{Li}_2 (w_-) - \operatorname{Li}_2 (w_+) + 4Y_p \ln \sqrt{\rho} \\
\left. + 4 \left( 1 - \frac{p_0 Y_p}{p_3} \right) \ln (z_{\text{max}} - \rho) - 4 \ln z_{\text{max}} \right], \\
\Phi_1 = \operatorname{Li}_2 (w_-) + \operatorname{Li}_2 (w_+) - \operatorname{Li}_2 (\tau_+) - \operatorname{Li}_2 (t/\tau_+), \\
\Phi_2 = \frac{Y_p}{p_3}, \\
\Phi_3 = \frac{1}{2} \ln \sqrt{\rho}, \\
\Phi_4 = \frac{1}{2} \ln (1 - \tau_+), \\
\Phi_5 = \frac{1}{2} \ln \left( 1 - \frac{t}{\tau_+} \right), \tag{35} \\
\Psi_0 = 4 \left( \frac{p_0 Y_p}{p_3} - 1 \right) \ln \left( \frac{z_{\text{max}} - \rho}{z_{\text{min}} - \rho} \right) + 4 \ln \left( \frac{z_{\text{max}}}{z_{\text{min}}} \right) \\
+ 2p_0 \left[ \operatorname{Li}_2 \left( 1 - \frac{1 - \tau^+}{p_-} \right) + \operatorname{Li}_2 \left( 1 - \frac{1 - t/\tau^+}{p_-} \right) \right. \\
- \operatorname{Li}_2 \left( 1 - \frac{1 - \tau^+}{p_+} \right) - \operatorname{Li}_2 \left( 1 - \frac{1 - t/\tau^+}{p_+} \right) \\
+ \operatorname{Li}_2 \left( 1 - \frac{1 - \tau^-}{p_+} \right) + \operatorname{Li}_2 \left( 1 - \frac{1 - t/\tau^-}{p_+} \right) \\
\left. - \operatorname{Li}_2 \left( 1 - \frac{1 - \tau^-}{p_-} \right) - \operatorname{Li}_2 \left( 1 - \frac{1 - t/\tau^-}{p_-} \right) \right], \\
\Psi_1 = \operatorname{Li}_2 (\tau_+) + \operatorname{Li}_2 (t/\tau_+) - \operatorname{Li}_2 (\tau_-) - \operatorname{Li}_2 (t/\tau_-), \\
\Psi_2 = \frac{1}{2} \ln (1 - \tau_-), \\
\Psi_3 = \frac{1}{2} \ln (1 - t/\tau_-), \\
9
\[ \Psi_4 = \frac{1}{2} \ln(1 - \tau_+), \]
\[ \Psi_5 = \frac{1}{2} \ln(1 - t/\tau_+). \]  

We introduce \( C_1 \ldots C_5 \) to simplify the formulae for \( D_n^A \) and \( D_n^B \):

\[
\begin{align*}
C_1 &= -2xt + x + t + t^2 - \eta x + \eta t + 5\eta + \rho(x - t - \eta), \\
C_2 &= \sqrt{x^2 - 4\eta(6 - 2x + 2t - t^2/\eta + \eta - 2\rho)}, \\
C_3 &= -5 - x + 2x^2 - 3t + 5t^2 - 6xt + xt^2/\eta - 2\eta x + \eta t \\
&\quad + 11\eta - \eta^2 + \rho(4 + 7x - 11t - 7\eta) + \rho^2, \\
C_4 &= -4xt - 3x + 2x^2 + 11t + 2t^2 - xt^2/\eta + 2\eta x/t - 2\eta x \\
&\quad - 10\eta/\eta + 3\eta + 3\eta^2 + 6\eta^2/t + \rho(5x - 7t) \\
&\quad + 2\eta x/t + 8\eta/\eta - 11\eta - 4\eta^2/t^2 + \rho^2(2 - \eta/\eta)\eta/t, \\
C_5 &= \frac{1}{2} \sqrt{x^2 - 4\eta} \left[ 5 - 2t^2/\eta - 5\eta/t - \eta + 3t - 3\rho(1 - \eta/t) \right. \\
&\quad + \eta \rho (1 + \rho)(1 - \eta/t) + \rho(t - \eta) \left. \right] / 1 - x + \eta, \\
D_1^A &= C_1, \\
D_2^A &= (-5 + 2x - 3t + \rho)p_3^2 - 2\rho t + 2\eta[-4p_3^2/t^2 + [x - 7t] \\
&\quad + t^2 - \rho(x + t][p_3^2/t^2]) + \eta^2[(3 + \rho)p_3^2/t^2 + 2\rho/t], \\
D_3^A &= -4p_3^2 - 4 - 2xt + 2x + 4t^2 + 2\rho(2 + 3x - 7t) \\
&\quad + 2\eta[4(x - 1 + \rho)p_3^2/t^2 + 6 + x/t - x - 6/t] \\
&\quad + \rho(-6 + x/t + 6/t)] + 2\eta^2[-2p_3^2 + 2 - 2t - \rho(2 + t)]/t^2, \\
D_4^A &= -C_2 - C_3, \\
D_5^A &= C_2 - C_4, \\
D_6^A &= \frac{1}{4}(5x + 9xt + 4t - 6t^2 - 2xt^2/\eta - \eta x/t + \eta x - 4\eta/t - 2\eta t \\
&\quad - 22\eta + 6\eta^2/t) + \frac{1}{4}\rho(-3x + 5t - \eta x/t + 9\eta/t + 5\eta \\
&\quad - \eta^2/t^2 + 2\eta^2/t) - \frac{1}{4} \rho(t - \eta)(1 - \eta/t) - \frac{1}{4}\rho^2\eta/t(3 + \eta/t) \\
&\quad - \frac{1}{4}\rho\eta (1 + \rho)(t - \eta/t)(1 - \eta/t) / x - t - \eta/t - \frac{1}{2}C_5, \\
\end{align*}
\]
\[ D_B^1 = C_1, \]
\[ D_B^2 = C_2 - C_3, \]
\[ D_B^3 = -C_2 - C_4, \]
\[ D_B^4 = C_2 + C_3, \]
\[ D_B^5 = -C_2 + C_4, \]
\[ D_B^6 = C_5. \]  

One can perform the limit \( \rho \to 0 \), what corresponds to the decay of the bottom quark to an up quark and leptons. The formulae, which are much simpler in this case, are presented in the same manner as previously the full results:

\[
\frac{d\tilde{\Gamma}}{dx \; dt} = \begin{cases} 
12\Gamma_0 \left[ \bar{F}_0(x,t) - \frac{2\alpha_s}{3\pi} \bar{F}_1^A(x,t) \right] & \text{for } (x,t) \text{ in A,} \\
12\Gamma_0 \frac{2\alpha_s}{3\pi} \bar{F}_1^B(x,t) & \text{for } (x,t) \text{ in B,} 
\end{cases}
\]  

where

\[ \bar{F}_0(x,t) = (1 - x + t)(x - t - \eta) \]  

and

\[ \bar{F}_1^A(x,t) = \bar{F}_0 \bar{\Phi}_0 + D_1^A \bar{\Phi}_1 + D_2^A \bar{\Phi}_{2s3} + D_4^A \bar{\Phi}_4 + D_5^A \bar{\Phi}_5 + D_6^A, \]

\[ \bar{F}_1^B(x,t) = \bar{F}_0 \bar{\Psi}_0 + \sum_{n=1}^5 D_n^B \bar{\Psi}_n + D_6^B, \]

where

\[
\bar{\Phi}_0 = 2 \left[ \text{Li}_2 \left( \frac{\tau_+ - t}{1 - t} \right) + \text{Li}_2 \left( \frac{1}{1/t - 1} \right) + \text{Li}_2(t) \right] + \frac{\pi^2}{2}
\]

\[
+ \ln^2(1 - \tau_+) + 2 \ln^2(1 - t) + \ln^2(1 - t/\tau_+)
\]

\[-2 \ln(1 - t) \ln z_{\text{max}},
\]

\[
\bar{\Phi}_1 = \frac{\pi^2}{12} + \text{Li}_2(t) - \text{Li}_2(\tau_+) - \text{Li}_2(t/\tau_+),
\]

\[
\bar{\Phi}_{2s3} = \frac{2 \ln(1 - t)}{1 - t},
\]

\[
\bar{\Phi}_4 = \Phi_4,
\]

\[
\bar{\Phi}_5 = \Phi_5.
\]
and

\[ \tilde{\Psi}_0 = 2 \left[ \text{Li}_2(\frac{\tau_- - t}{1 - t}) + \text{Li}_2(\frac{1/\tau_- - 1}{1/t - 1}) - \text{Li}_2(\frac{\tau_+ - t}{1 - t}) 
- \text{Li}_2(\frac{1/\tau_+ - 1}{1/t - 1}) + \ln(1 - t) \ln\left(\frac{z_{\text{max}}}{z_{\text{min}}}\right) \right] 
- \ln\left(1 - \frac{t}{\tau_+}/(1 - \frac{t}{\tau_-})\right) \ln[(1 - \frac{t}{\tau_+})(1 - \frac{t}{\tau_-})], \]

\[ \tilde{\Psi}_n = \Psi_n \quad n = 1 \ldots 5. \] (45)

The formulae for \( C_n \) and \( D_n^A \) are more compact than (37) and (38):

\[ C_1 = -\eta x + \eta t + 5\eta - 2xt + x + t^2, \]
\[ C_2 = \sqrt{x^2 - 4\eta (6 - t^2/\eta + \eta - 2x + 2t)}, \]
\[ C_3 = -5 + xt^2/\eta - 2\eta x + \eta t + 11\eta - \eta^2 - 6xt - x + 2x^2 - 3t + 5t^2, \]
\[ C_4 = -xt^2/\eta - 2\eta x - 10\eta/t + 3\eta t + 3\eta + 5\eta^2/t^2 - 6\eta^2/t 
- 4xt - 3x + 2x^2 + 11t + 2t^2, \]
\[ C_5 = \frac{1}{2} \sqrt{x^2 - 4\eta \left(5 - 2t^2/\eta - 5\eta/t - \eta + 3t\right)}, \] (46)
\[ D_1^A = C_1, \]
\[ D_{23}^A = -\frac{5}{4} + \frac{1}{2} \eta x/t^2 - \eta x/t + \frac{1}{2} \eta x - \frac{1}{2} \eta/t^2 - \frac{3}{2} \eta/t - \frac{5}{2} \eta t + \frac{9}{2} \eta 
+ \frac{3}{4} \eta^2/t^2 - \frac{5}{4} \eta^2/t + \frac{1}{4} \eta^2 - xt + \frac{1}{2} xt^2 + \frac{1}{2} x + \frac{7}{4} t 
+ \frac{1}{4} t^2 - \frac{3}{4} t^3, \]
\[ D_4^A = \sqrt{x^2 - 4\eta \left(-6 + t^2/\eta - \eta + 2x - 2t\right) + 5 - xt^2/\eta + 2\eta x 
- \eta t - 11\eta + \eta^2 + 6xt + x - 2x^2 + 3t - 5t^2}, \]
\[ D_5^A = \sqrt{x^2 - 4\eta \left(6 - t^2/\eta + \eta - 2x + 2t\right) + xt^2/\eta - 2\eta x/t + 2\eta x 
+ 10\eta/t - 3\eta t - 3\eta - 5\eta^2/t^2 + 6\eta^2/t + 4xt + 3x - 2x^2 
- 11t - 2t^2}, \]
\[ D_6^A = \sqrt{x^2 - 4\eta \left(-\frac{5}{4} + \frac{1}{2} t^2/\eta + \frac{5}{4} \eta/t + \frac{1}{4} \eta - \frac{3}{4} t\right) - \frac{1}{2} xt^2/\eta} \]
\[-\frac{1}{4} \eta x/t + \frac{1}{4} \eta x - \eta/t - \frac{1}{2} \eta t - \frac{11}{2} \eta + \frac{3}{2} \eta^2/t + \frac{9}{4} xt + \frac{5}{4} x \]

+ t - \frac{3}{2} t^2. \tag{47}

The coefficients $D^B_n$ are expressed by $C_n$ in the same way as $D^B_n$ are expressed by $C_n$ in the set of equations (39).

The listed above results have been tested by comparison with the calculation made earlier in simpler cases. One of the cross checks was arranged by fixing the mass of the produced lepton to zero. Our results are in this limit algebraically identical with those for the massless charged lepton[28, 29]. On the other hand one can numerically integrate the calculated double differential distribution over $x$, with the limits given by the kinematical boundaries:

$$
\int_{\sqrt{w}}^{w+\eta/w+} \frac{d\Gamma}{dxdt}(x,t;\rho,\eta) = \frac{d\Gamma}{dt}(t;\rho,\eta). \tag{48}
$$

Obtained in such a way differential distribution of $t$ agrees with recently published[17] analytical formula describing this distribution. This test is particularly stringent because one requires two functions of three variables ($t, \rho$ and $\eta$) to be numerically equal for any values of the arguments. We remark, that for higher values of $t$ only the region A contributes to the integral (48) and for lower values of $t$ both the regions A and B contribute. This feature of the test is very helpful — the formulae for $F_1(x,t)$, which are different for the regions A and B can be checked separately. For completeness we quote the formulae derived in [17]:

$$
\frac{d\Gamma}{dt} = \Gamma_0 \left(1 - \frac{\eta}{t}\right)^2 \left\{ (1 + \frac{\eta}{2t}) \left[ F_0(t) - \frac{2\alpha_s}{3\pi} F_1(t) \right] \\
+ \frac{3\eta}{2t} \left[ F_0^s(t) - \frac{2\alpha_s}{3\pi} F_1^s(t) \right] \right\}, \tag{49}
$$

where

$$
F_0(t) = 4p_3 \left[ (1 - \rho)^2 + t(1 + \rho) - 2t^2 \right], \tag{50}
$$

$$
F_0^s(t) = 4p_3 \left[ (1 - \rho)^2 - t(1 + \rho) \right], \tag{51}
$$

$$
F_1(t) = A_1 \Psi + A_2 Y_w + A_3 Y_\rho + A_4 p_3 \ln \rho + A_5 p_3, \tag{52}
$$

$$
F_1^s(t) = B_1 \Psi + B_2 Y_w + B_3 Y_\rho + B_4 p_3 \ln \rho + B_5 p_3 \tag{53}
$$
and

\[ \Psi = 8 \ln(2p_3) - 2 \ln t + [2 \ln_2(w_2) - 2 \ln_2(w_1) + 4 \ln_2(2p_3/p_+)] \\
- 4 \ln_2 \ln(2p_3/p_+) - \ln p_+ \ln w_2 + \ln p_+ \ln w_1 ] 2p_0/p_3, \quad (54) \]

\[ A_1 = F_0(t), \]

\[ A_2 = -8(1 - \rho) \left[ 1 + t - 4t^2 - \rho(2 - t) + \rho^2 \right], \]

\[ A_3 = -2 \left[ 3 + 6t - 21t^2 + 12t^3 - \rho(1 + 12t + 5t^2) \right. \]
\[ + \left. \rho^2(11 + 2t) - \rho^3 \right], \]

\[ A_4 = -6 \left[ 1 + 3t - 4t^2 - \rho(4 - t) + 3\rho^2 \right], \]

\[ A_5 = -2 \left[ 5 + 9t - 6t^2 - \rho(22 - 9t) + 5\rho^2 \right], \quad (55) \]

\[ B_1 = F_0^s(t), \]

\[ B_2 = -8(1 - \rho) \left[ (1 - \rho)^2 - t(1 + \rho) \right], \]

\[ B_3 = -4(1 - \rho)^4/t - 2(-1 + 3\rho + 15\rho^2 - 5\rho^3) \]
\[ + 8(1 + \rho)t - 6(1 + \rho)t^2, \]

\[ B_4 = -4(1 - \rho)^3/t - 2(1 - \rho)(1 - 11\rho) + 6(1 + 3\rho)t, \]

\[ B_5 = -6(1 - 3\rho)(3 - \rho) + 18t(1 + \rho). \quad (56) \]

The formula (49) consists of two parts with transparent physical meaning\[17\]. The first term in the curly bracket containing the function \( F_0(t) \) corresponds to the real \( t \to bW \) decay in the limit of vanishing \( b \) quark mass. The other one is related to a fictitious decay \( t \to bH_+ \) of a heavy quark \( t \) to the quark \( b \) and a charged Higgs particle \( H_+ \). The formula \[32]\ describing the first order QCD correction \( F_1(t) \) has been confirmed by many groups, cf. note added in ref. \[29\]. On the other hand the formula for \( F_1^s(t) \) has been obtained only in \[33\] and it still requires an independent check. Our calculation confirms the result (51) of ref. \[33\].

5 Moments of \( \tau \) energy distribution

It has been argued in \[13, 17, 18\] that moments of charged lepton energy distribution from \( b \)-quark decay are valuable source of knowledge about the
physical parameters involved in the process and are only weakly affected by soft processes. Similar to the moments are the observables which are used in \[34\] to find the values of some nonperturbative parameters of HQET.

Thus, in the notation of \[18\]

\[
M_n = \int_{E_{\text{min}}}^{E_{\text{max}}} E^n \frac{d\Gamma}{dE} \, dE, \tag{57}
\]

\[
r_n = \frac{M_n}{M_0}, \tag{58}
\]

where \(E_{\text{min}}\) and \(E_{\text{max}}\) are the lower and upper limits for \(\tau\) energy and \(M_n\) involve both perturbative and nonperturbative QCD corrections to \(\tau\) energy spectrum. The nonperturbative \(1/m_b^2\) corrections to the charged lepton spectrum from semileptonic \(B\) decays have been derived in the framework of HQET\[22, 23, 24\] and even \(1/m_b^3\) corrections are known\[35\]. Up to order of \(1/m_b^2\) the corrected heavy lepton energy spectrum can be written in the following way:

\[
\frac{1}{\Gamma_0} \frac{d\Gamma}{dx} = f_0(x) - \frac{2\alpha_s}{3\pi} f_1(x) + \frac{\lambda_1}{m_b^2} f^{(1)}_{np}(x) + \frac{\lambda_2}{m_b^2} f^{(2)}_{np}(x), \tag{59}
\]

where \(\lambda_1\) and \(\lambda_2\) are the HQET parameters corresponding to the \(b\) quark kinetic energy and the energy of interaction of the \(b\) quark magnetic moment with chromomagnetic field produced by a light quark in the meson \(B\). The functions \(f^{(1,2)}_{np}\) can be easily extracted from the formula (2.11) in \[22\].

Following ref.\[18\] we expand the ratio \(r_n\):

\[
r_n = r_n^{(0)} \left( 1 - \frac{2\alpha_s}{3\pi} \delta^{(\rho)}_n + \frac{\lambda_1}{m_b} \delta^{(1)}_n + \frac{\lambda_2}{m_b^2} \delta^{(2)}_n \right), \tag{60}
\]

where \(r_n^{(0)}\) is the lowest approximation of \(r_n\),

\[
r_n^{(0)} = \left( \frac{m_b}{2} \right)^n \frac{f_{2\sqrt{\eta}}^{1+\eta-\rho} f_0(x) x^n \, dx}{f_{2\sqrt{\eta}}^{1+\eta-\rho} f_0(x) \, dx}. \tag{61}
\]

Each of the \(\delta^{(i)}_n\) is expressed by integrals of the corresponding correction function \(f^{(i)}(x)\) and the tree level term \(f_0(x)\)

\[
\delta^{(i)}_n = \frac{f_{2\sqrt{\eta}}^{1+\eta-\rho} f^{(i)}(x) x^n \, dx}{f_{2\sqrt{\eta}}^{1+\eta-\rho} f_0(x) \, dx} - \frac{f_{2\sqrt{\eta}}^{1+\eta-\rho} f^{(i)}(x) \, dx}{f_{2\sqrt{\eta}}^{1+\eta-\rho} f_0(x) \, dx}. \tag{62}
\]
where the index \( i \) may denote any type of the described above corrections. The coefficients \( \delta_n^{(i)} \) depend only on two ratios of masses of the charged lepton and the \( c \) quark to the mass \( m_b \). It is convenient to introduce the functional dependence of the \( \delta_n^{(i)} \) on the masses in the following way:

\[
\delta_n^{(i)}(m_b, m_c, m_\tau) = \delta_n^{(i)} \left( \frac{m_b}{m_\tau}, \frac{m_c}{m_b} \right).
\]  

(63)

Since the values of \( b \) and \( c \) quark masses are not known precisely, we calculated the \( \delta \)'s in a reasonable range of them. We have chosen \( 4.4 \text{ GeV} \leq m_b \leq 5.2 \text{ GeV} \) and \( 0.25 \leq m_c/m_b \leq 0.35 \). In this range the dependence of the function \( \delta_n^{(i)} \left( \frac{m_b}{m_\tau}, \frac{m_c}{m_b} \right) \) on the variables can be approximated by a second order polynomial with the maximal relative error smaller than 1%. We propose:

\[
\delta(p, q) = a + b(p - p_0) + c(q - q_0) + d(p - p_0)^2 \\
+ e(p - p_0)(q - q_0) + f(q - q_0)^2,
\]  

(64)

where \( p = m_b/m_\tau, p_0 = 4.75\text{GeV}/1.777\text{GeV} = 2.6730, q = m_c/m_b, q_0 = 0.28 \) and the polynomial coefficients are fitted for each of the \( \delta_n^{(i)} \) separately. Such a parameterization marks out the realistic masses of quarks: \( m_b = 4.75 \text{ GeV} \) and \( m_c = 1.35 \text{ GeV} \), for which the coefficient \( \delta_n^{(i)} = a_n^{(i)} \).

The results of the fits of formula (64) to the calculated numerically coefficients \( \delta_n^{(i)}(m_b/m_\tau, m_c/m_b) \) are listed in Table 1 for all the three types of \( \delta_n^{(i)} \) and \( n = 1 \ldots 5 \). With use of these results it is possible to estimate the order of relative correction to the ratio \( r_n \) (63), including reasonable values of \( \alpha_s, \lambda_1 \) and \( \lambda_2 \). The value of \( \lambda_3 \) may be easily determined from a measured \( B - B^* \) splitting, and \( \alpha_s \) should be of the order of \( \alpha_s(m_b) \). We assume \( \alpha_s \approx 0.3, \lambda_2 = 0.12 \text{ GeV}^2 \), keep the mass of the \( b \) quark fixed to 4.75 GeV and the mass of the \( c \) quark equal to 3.35 GeV. It was claimed in [34] that \( \lambda_1 \) may be constrained \( \lambda_1 = -0.35 \pm 0.05 \text{ GeV}^2 \) but we choose more conservative estimate \( -0.60 \text{ GeV}^2 \leq \lambda_1 \leq -0.15 \text{ GeV}^2 \). Thus the perturbative correction to \( r_n/r_n^{(0)} \) is about \( -0.0015 \) \((-0.0075) \) for \( n = 1 \) \((n = 5) \) whereas the nonperturbative corrections are larger: for the ”kinetic energy” part \( 0.008 \pm 0.004 \,(0.06 \pm 0.03), n = 1 \,(n = 5) \) and for the ”chromomagnetic” one \(-0.01 \,(-0.05), n = 1 \,(n = 5) \). Although the two nonperturbative terms partly cancel each other, the perturbative correction to the moments are smaller than the nonperturbative contributions. The ratios \( r_n \) may be used to fix the value of \( \lambda_1 \) if suitable precise measurements are performed.
Table 1. Polynomial coefficients defined by the formula (64) characterizing how the three different types of corrections $\delta_n^{(i)}$ depend on quark masses.

A. The perturbative correction.

|   | $a$     | $b$     | $c$       | $d$      | $e$       | $f$     |
|---|---------|---------|-----------|----------|-----------|---------|
| $\delta_1^{(p)}$ | 0.0213  | 0.0097  | -0.1059   | -0.0061  | -0.0202  | 0.178   |
| $\delta_2^{(p)}$ | 0.0443  | 0.0216  | -0.2213   | -0.0108  | -0.0454  | 0.363   |
| $\delta_3^{(p)}$ | 0.0687  | 0.0355  | -0.3457   | -0.0157  | -0.0808  | 0.554   |
| $\delta_4^{(p)}$ | 0.0946  | 0.0512  | -0.4781   | -0.0193  | -0.1163  | 0.746   |
| $\delta_5^{(p)}$ | 0.1219  | 0.0684  | -0.6168   | -0.0259  | -0.1538  | 0.934   |

B. The nonperturbative correction corresponding to kinetic energy of the $b$ quark.

|   | $a$     | $b$     | $c$       | $d$      | $e$       | $f$     |
|---|---------|---------|-----------|----------|-----------|---------|
| $\delta_1^{(1)}$ | -0.500  | 0.0     | 0.0       | 0.0      | 0.0       | 0.0     |
| $\delta_2^{(1)}$ | -1.105  | -0.081  | 0.220     | 0.011    | 0.016     | 0.26    |
| $\delta_3^{(1)}$ | -1.823  | -0.254  | 0.680     | 0.035    | 0.039     | 0.84    |
| $\delta_4^{(1)}$ | -2.664  | -0.526  | 1.396     | 0.074    | 0.064     | 1.77    |
| $\delta_5^{(1)}$ | -3.633  | -0.904  | 2.377     | 0.131    | 0.068     | 3.11    |

C. The nonperturbative correction corresponding to the interaction of the $b$ quark spin with chromomagnetic field.

|   | $a$     | $b$     | $c$       | $d$      | $e$       | $f$     |
|---|---------|---------|-----------|----------|-----------|---------|
| $\delta_1^{(2)}$ | -1.731  | -0.732  | 2.784     | 0.212    | -0.498    | -1.71   |
| $\delta_2^{(2)}$ | -3.565  | -1.561  | 5.824     | 0.437    | -0.975    | -3.45   |
| $\delta_3^{(2)}$ | -5.496  | -2.473  | 9.069     | 0.681    | -1.506    | -5.13   |
| $\delta_4^{(2)}$ | -7.517  | -3.453  | 12.472    | 0.961    | -2.141    | -6.72   |
| $\delta_5^{(2)}$ | -9.621  | -4.485  | 15.987    | 1.279    | -2.955    | -8.21   |
6 Summary

The results of this article can be summarized in the following way:
(i) an analytical expression has been presented for the double differential distribution \( d\Gamma/dx dt \) of the \( \tau \) lepton energy and the invariant mass of the leptons in \( b \to q\tau\bar{\nu}_\tau \) inclusive weak decay including first order perturbative QCD corrections. For \( m_\tau = 0 \) the results of [28, 29] are rederived
(ii) after numerical integration the distribution \( d\Gamma/dt \) is obtained in perfect agreement with the analytic result of [17] which has been derived in a completely different way
(iii) when combined with the non-perturbative QCD corrections calculated in [22, 23, 24] the results of this article describe inclusive decays of beautiful hadrons into \( \tau \) lepton and anything
(iv) the energy spectrum and a few its moments are calculated for \( B \) into \( \tau \) transitions.

Acknowledgments
MJ would like to thank Wolfgang Hollik, Frans Klinkhamer and Hans Kühn for their warm hospitality during his stay in Karlsruhe. A part of the calculations presented in this article was done in Asper Center for Theoretical Physics in summer 1995. LM is very grateful to Professor Kacper Zalewski for numerous discussions clarifying many points of the Heavy Quark Effective Theory.

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