SOME 4-TOTAL MEAN CORDIAL GRAPHS DERIVED FROM WHEEL

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Abstract. In this paper we investigate the 4-total mean cordial labeling behaviour of helm, closed helm, flower graph, sunflower graph, gear graph, subdivision of wheel, web graph.

Keywords: wheel; helm; flower graph; gear graph; web graph.

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1. INTRODUCTION

Graphs in this paper are finite, simple and undirected. k-total mean cordial labeling of graphs have been introduced in [3] and they investigate the 4-total mean cordial labeling behaviour of path, cycle, star, bistar, wheel, subdivision of star, subdivision of bistar, subdivision of comb, subdivision of crown, subdivision of doublecomb, subdivision of jellyfish, subdivision of ladder, subdivision of triangular snake in [3, 4]. In this paper, we investigate the 4-total mean cordial labeling behaviour of helm, closed helm, flower graph, sunflower graph, gear graph, subdivision of wheel, web graph. Terms are not defined here follow from Harary [2] and Gallian [1].

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2. 4-TOTAL MEAN CORDIAL GRAPH

Definition 2.1. Let $G$ be a graph. Let $f : V(G) \rightarrow \{0, 1, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $f(uv) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$. $f$ is called $k$-total mean cordial labeling of $G$ if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with $x, x \in \{0, 1, \ldots, k-1\}$. A graph with admit a $k$-total mean cordial labeling is called $k$-total mean cordial graph.

3. PRELIMINARIES

Definition 3.1. The graph $W_n = C_n + K_1$ is called a wheel, where $C_n$ is the cycle $u_1u_2\ldots u_nu_1$, $V(K_1) = \{u\}$.

Definition 3.2. The Helm $H_n$ is the graph obtained from the wheel $W_n$ with $V(H_n) = V(W_n) \cup \{v_1 : 1 \leq i \leq n\}$ and $E(H_n) = E(W_n) \cup \{u_iv_i : 1 \leq i \leq n\}$.

Definition 3.3. Closed Helm $CH_n$ is a graph obtained from the helm $H_n$ with vertex set $V(CH_n) = V(H_n)$ and $E(CH_n) = E(H_n) \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_nv_1\}$.

Definition 3.4. A flower graph is the graph obtained from a helm $H_n$ by joining each pendent vertices $v_i$ to the central vertex $u$ of the helm.

Definition 3.5. The sunflower graph $SF_n$ is obtained by taking a wheel $W_n = C_n + K_1$ where $C_n$ is the cycle $u_1u_2\ldots u_nu_1$, $V(K_1) = \{u\}$ and new vertices $v_1, v_2, \ldots, v_n$ where $v_i$ is join by the vertices $u_iu_{i+1}$ (mod $n$).

Definition 3.6. The web graph $Wb_n$ is the graph obtained from a closed helm $CH_n$ with $V(Wb_n) = V(CH_n) \cup \{x_i : 1 \leq i \leq n\}$ and $E(Wb_n) = E(CH_n) \cup \{v_ix_i : 1 \leq i \leq n\}$.

Notation 1. $[x]$ denote the greatest integer $\leq x$.

4. MAIN RESULTS

Theorem 4.1. The Helm graph $H_n$ is 4-total mean cordial for all values of $n$.

Proof. Take the vertex set and edge set of $H_n$ as in definition 3.2. Clearly, $|V(H_n)| + |E(H_n)| = 5n + 1$.

Assign the label 2 to the central vertex $u$. 

Case 1. \( n \equiv 0 \pmod{4} \).

Let \( n = 4r, r \in \mathbb{N} \). Now we consider the vertices \( u_1, u_2, \ldots, u_n \). Assign the labels 0, 0, 2, 3 respectively to the vertices \( u_1, u_2, u_3, u_4 \). Then we assign the label 0, 0, 2, 3 respectively to the vertices \( u_5, u_6, u_7, u_8 \). We now assign the label 0, 0, 2, 3 respectively to the vertices \( u_9, u_{10}, u_{11}, u_{12} \). Continuing in this process until we reach the vertex \( u_n \). In this case, the vertices \( u_{4r-3}, u_{4r-2}, u_{4r-1}, u_{4r} \) receive the labels 0, 0, 2, 3. Now we move to the vertices \( v_1, v_2, \ldots, v_n \). Assign the labels 0, 1, 2, 3 respectively to the vertices \( v_1, v_2, v_3, v_4 \). Now we assign the label 0, 0, 2, 3 respectively to the vertices \( v_5, v_6, v_7, v_8 \). We now assign the label 0, 1, 2, 3 respectively to the vertices \( v_9, v_{10}, v_{11}, v_{12} \). Proceeding like this process until we reach the vertex \( v_n \). Clearly, the vertices \( v_{4r-3}, v_{4r-2}, v_{4r-1}, v_{4r} \) receive the labels 0, 1, 2, 3 respectively.

Case 2. \( n \equiv 1 \pmod{4} \).

Let \( n = 4r + 1, r \in \mathbb{N} \). Assign the labels to the vertices \( u_i, v_i \) \((1 \leq i \leq 4r)\) as in case 1. Next we assign the labels 2, 0 respectively to the vertices \( u_{4r+1}, v_{4r+1} \).

Case 3. \( n \equiv 2 \pmod{4} \).

Let \( n = 4r + 2, r \in \mathbb{N} \). As in case 1, assign the label to the vertices \( u_i, v_i \) \((1 \leq i \leq 4r)\) as in case 1. Now we assign the labels 3, 0, 2 respectively to the vertices \( u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2} \).

Case 4. \( n \equiv 3 \pmod{4} \).

Let \( n = 4r + 3, r \in \mathbb{N} \). Label the vertices \( u_i, v_i \) \((1 \leq i \leq 4r)\) as in case 1. Finally we assign the labels 3, 2, 0, 1, 0 respectively to the vertices \( u_{4r+1}, u_{4r+2}, u_{4r+3}, v_{4r+1}, v_{4r+2}, v_{4r+3} \).

This vertex labeling \( f \) is a 4-total mean cordial labeling follows from the Table 1

| Nature of \( n \) | \( t_{mf} (0) \) | \( t_{mf} (1) \) | \( t_{mf} (2) \) | \( t_{mf} (3) \) |
|------------------|----------------|----------------|----------------|----------------|
| \( n = 4r \)     | 5r             | 5r             | 5r+1           | 5r             |
| \( n = 4r + 1 \) | 5r+1           | 5r+2           | 5r+2           | 5r+1           |
| \( n = 4r + 2 \) | 5r+3           | 5r+2           | 5r+3           | 5r+3           |
| \( n = 4r + 3 \) | 5r+4           | 5r+4           | 5r+4           | 5r+4           |

Table 1

Case 5. \( n = 3 \).

A 4-total mean cordial labeling is given in Table 2
Theorem 4.2. The closed helm $CH_n$ is 4-total mean cordial for all values of $n$.

Proof. Take the vertex set and edge set of $CH_n$ as in definition 3.3. Clearly, $|V(CH_n)| + |E(CH_n)| = 6n + 1$.

Assign the label 0 to the central vertex $u$.

Case 1. $n \equiv 0 \pmod{4}$.

Let $n = 4r, r \geq 1$. Now we consider the vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Then we assign the label 2 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. We now move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 1 to the $r$ vertices $v_1, v_2, \ldots, v_r$. Next we assign the label 2 to the $r$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. Finally we assign the label 3 to the $2r$ vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{4r}$.

Case 2. $n \equiv 1 \pmod{4}$.

Let $n = 4r + 1, r \geq 1$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Next we assign the label 2 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Now we assign the label 3 to the vertex $u_{4r+1}$. Next we assign the label 2 to the $r$ vertices $v_1, v_2, \ldots, v_r$. Then we assign the label 1 to the $r$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$ and assign the label 0 to the vertex $v_{4r+1}$. Finally we assign the label 3 to the $2r$ vertices $v_{2r+1}, v_{2r+3}, \ldots, v_{4r+1}$.

Case 3. $n \equiv 2 \pmod{4}$. Let $n = 4r + 2, r \geq 1$. Assign the label 0 to the $2r + 1$ vertices $u_1, u_2, \ldots, u_{2r+1}$. Next we assign the label 2 to the $2r + 1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{4r+2}$. Now we assign the label 1 to the $r$ vertices $v_1, v_2, \ldots, v_r$. Then we assign the label 2 to the $r + 1$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r+1}$. Finally we assign the label 3 to the $2r + 1$ vertices $v_{2r+2}, v_{2r+3}, \ldots, v_{4r+2}$.

Case 4. $n \equiv 3 \pmod{4}$.

Let $n = 4r + 3, r \geq 1$. Assign the label 0 to the $2r + 1$ vertices $u_1, u_2, \ldots, u_{2r+1}$. Then we assign the label 2 to the $2r + 1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{4r+2}$. Now we assign the label 1 to the vertex $u_{4r+3}$. Next we assign the label 1 to the $r$ vertices $v_1, v_2, \ldots, v_r$. Then we assign the label 2 to the $r$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$.
vertices \(v_{r+1}, v_{r+2}, \ldots, v_{2r}\). Now we assign the label 3 to the \(2r + 2\) vertices \(v_{2r+1}, v_{2r+2}, \ldots, v_{4r+2}\). Finally we assign the label 0 to the vertex \(v_{4r+3}\).

This vertex labeling \(f\) is 4-total mean cordial labeling follows from the Table 3.

| Nature of \(n\) | \(t_{mf}(0)\) | \(t_{mf}(1)\) | \(t_{mf}(2)\) | \(t_{mf}(3)\) |
|-----------------|----------------|----------------|----------------|----------------|
| \(n = 4r\)     | 6\(r\)         | 6\(r + 1\)     | 6\(r\)         | 6\(r\)         |
| \(n = 4r + 1\) | 6\(r + 1\)     | 6\(r + 2\)     | 6\(r + 2\)     | 6\(r + 2\)     |
| \(n = 4r + 2\) | 6\(r + 3\)     | 6\(r + 3\)     | 6\(r + 4\)     | 6\(r + 3\)     |
| \(n = 4r + 3\) | 6\(r + 4\)     | 6\(r + 4\)     | 6\(r + 5\)     | 6\(r + 5\)     |

**TABLE 3**

### Case 5. \(n = 3\).

A 4-total mean cordial labeling is given in Table 4.

| Value of \(n\) | \(u\) | \(u_1\) | \(u_2\) | \(u_3\) | \(v_1\) | \(v_2\) | \(v_3\) |
|----------------|------|--------|--------|--------|--------|--------|--------|
| \(n = 3\)     | 0    | 0      | 2      | 1      | 0      | 3      | 3      |

**TABLE 4**

\[\square\]

**Theorem 4.3.** Flower graph \(FL_n\) is 4-total mean cordial for all \(n\).

**Proof.** Take the vertex set and edge set of \(FL_n\) as in definition 3.4. Note that \(|V(FL_n)| + |E(FL_n)| = 6n + 1\). Assign the label 1 to the central vertex \(u\).

#### Case 1. \(n \equiv 0 \pmod{4}\).

Let \(n = 4r, r \in \mathbb{N}\). Consider vertices \(u_1, u_2, \ldots, u_n\). Assign the label 0 to the \(2r\) vertices \(u_1, u_2, \ldots, u_{2r}\).

Then we assign the label 2 to the \(r\) vertices \(u_{2r+1}, u_{2r+2}, \ldots, u_{3r}\). Next we assign the label 3 to the \(r\) vertices \(u_{3r+1}, u_{3r+2}, \ldots, u_{4r}\). Now we move to the vertices \(v_1, v_2, \ldots, v_n\). Assign the label 0 to the \(r + 1\) vertices \(v_1, v_2, \ldots, v_{r+1}\). Next we assign the label 1 to the \(r - 1\) vertices \(v_{r+2}, v_{r+3}, \ldots, v_{2r}\).

Finally we assign the label 3 to the \(2r\) vertices \(v_{2r+1}, v_{2r+2}, \ldots, v_{4r}\).

#### Case 2. \(n \equiv 1 \pmod{4}\).

Let \(n = 4r + 1, r \in \mathbb{N}\). Assign the labels to the vertices \(u_i, v_i\) (\(1 \leq i \leq 4r\)) as in case 1. Next we assign the labels 3,0 respectively to the vertices \(u_{4r+1}, v_{4r+1}\).
**Case 3.** \( n \equiv 2 \pmod{4} \).

Let \( n = 4r + 2, \ r \in \mathbb{N} \). Label the vertices \( u_i, v_i \ (1 \leq i \leq 4r) \) as in case 1. Now we assign the labels 3,0,0,0 respectively to the vertices \( u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2} \).

**Case 4.** \( n \equiv 3 \pmod{4} \).

Let \( n = 4r + 3, \ r \in \mathbb{N} \). As in case 1, assign the label to the vertices \( u_i, v_i \ (1 \leq i \leq 4r) \). Finally we assign the labels 3,3,2,0,0,0 respectively to the vertices \( u_{4r+1}, u_{4r+2}, u_{4r+3}, v_{4r+1}, v_{4r+2}, v_{4r+3} \).

Table 5 shows that the vertex labeling \( f \) is a 4-total mean cordial labeling.

| Order of \( n \) | \( t_{mf} (0) \) | \( t_{mf} (1) \) | \( t_{mf} (2) \) | \( t_{mf} (3) \) |
|------------------|----------------|----------------|----------------|----------------|
| \( n = 4r \)     | 6r + 1         | 6r             | 6r             | 6r             |
| \( n = 4r + 1 \) | 6r + 2         | 6r + 1         | 6r + 2         | 6r + 2         |
| \( n = 4r + 2 \) | 6r + 3         | 6r + 3         | 6r + 4         | 6r + 3         |
| \( n = 4r + 3 \) | 6r + 4         | 6r + 5         | 6r + 5         | 6r + 5         |

**Case 5.** \( n = 3 \).

A 4-total mean cordial labeling is given in Table 6.

| \( n \) | \( u \) | \( u_1 \) | \( u_2 \) | \( u_3 \) | \( v_1 \) | \( v_2 \) | \( v_3 \) |
|---------|-------|-------|-------|-------|-------|-------|-------|
| \( n = 3 \) | 2     | 0     | 0     | 3     | 0     | 1     | 3     |

**Table 6**

**Theorem 4.4.** The sunflower graph \( SF_n \) is 4-total mean cordial, for all \( n \).

**Proof.** Take the vertex set and edge set of \( SF_n \) as in definition 3.5. Clearly that \(|V(SF_n)| + |E(SF_n)| = 6n + 1\). Assign the label 0 to the central vertex \( u \).

**Case 1.** \( n \) is even.

We consider the vertices \( u_1, u_2, \ldots, u_n \). Assign the label 0 to the \( \frac{n}{2} \) vertices \( u_1, u_2, \ldots, u_{\frac{n}{2}} \). Then we assign the label 1 to the \( \frac{n}{2} \) vertices \( u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_n \). We now move to the vertices \( v_1, v_2, \ldots, v_n \).

Next assign the label 2 to the \( \frac{n-2}{2} \) vertices \( v_1, v_2, \ldots, v_{\frac{n-2}{2}} \). Now we assign the label 3 to the \( \frac{n+2}{2} \) vertices \( v_{\frac{n}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n \).
Case 2. $n$ is odd.

Assign the label 0 to the $\frac{n-1}{2}$ vertices $u_1, u_2, \ldots, u_{n-1}$. Next we assign the label 2 to the $\frac{n+1}{2}$ vertices $u_{n+1}, u_{n+2}, \ldots, u_n$. Assign the label 2 to the $\frac{n-3}{2}$ vertices $v_1, v_2, \ldots, v_{n-3}$. We now assign the label 3 to the $\frac{n+1}{2}$ vertices $v_{n-1}, v_{n+1}, \ldots, v_{n-1}$. Finally we assign the label 0 to the vertex $v_n$.

This vertex labeling $f$ is 4-total mean cordial labeling follows from the Table 7:

| Nature of $n$ | $t_{mf}(0)$ | $t_{mf}(1)$ | $t_{mf}(2)$ | $t_{mf}(3)$ |
|--------------|-------------|-------------|-------------|-------------|
| $n$ is even   | $\frac{3n}{2}$ | $\frac{3n}{2}$ | $\frac{3n}{2}$ | $\frac{3n+2}{2}$ |
| $n$ is odd    | $\frac{3n+1}{2}$ | $\frac{3n+1}{2}$ | $\frac{3n-1}{2}$ | $\frac{3n+1}{2}$ |

Table 7

Theorem 4.5. The Gear graph $G_n$ is 4-total mean cordial for every $n$.

Proof. Take the vertex set and edge set of the wheel $W_n$ as in definition 3.1. Let $v_i$ be the vertex which subdivide the edge $u_i u_{i+1}$ $(1 \leq i \leq n-1)$ and $v_n$ be the vertex which subdivide the edge $u_n u_1$. Clearly $|V(G_n)| = 5n + 1$.

Assign the label 2 to the central vertex $u$.

Case 1. $n \equiv 0 \pmod{4}$.

Let $n = 4r$, $r \in \mathbb{N}$. Consider the rim vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Then we assign the label 1 to the vertex $u_{2r+1}$. Now we assign the label 2 to the $r - 1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{3r}$. We now assign the label 3 to the $r$ vertices $u_{3r+1}, u_{3r+2}, \ldots, u_{4r}$. Now we move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 0 to the $r$ vertices $v_1, v_2, \ldots, v_r$. Next we assign the label 1 to the $r$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. We now assign the label 2 to the $r$ vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$. Finally we assign the label 3 to the $r$ vertices $v_{3r+1}, v_{3r+2}, \ldots, v_{4r}$.

Case 2. $n \equiv 1 \pmod{4}$.

Let $n = 4r + 1$, $r \in \mathbb{N}$. As in the Case 1, assign the label to the vertices $u_i, v_i$ $(1 \leq i \leq 4r)$. Finally assign the labels 0, 3 to the vertices $u_{4r+1}, v_{4r+1}$.

Case 3. $n \equiv 2 \pmod{4}$.

Let $n = 4r + 2$, $r \in \mathbb{N}$. Label the vertices $u_i, v_i$ $(1 \leq i \leq 4r + 1)$ as in Case 2. Next assign the labels 2, 0 to the vertices $u_{4r+2}, v_{4r+2}$.
**Case 4.** \( n \equiv 3 \pmod{4} \).

Let \( n = 4r + 3, r \in \mathbb{N} \). Assign the label to the vertices \( u_i, v_i \) \((1 \leq i \leq 4r + 1)\) as in Case 2. Finally we assign the labels 3,2,0,0 to the vertices \( u_{4r+2}, u_{4r+3}, v_{4r+2}, v_{4r+3} \).

The Table 8, establish that this vertex labeling \( f \) is a 4-total mean cordial labeling of gear \( G_n \).

| Order of \( n \) | \( t_{mf}(0) \) | \( t_{mf}(1) \) | \( t_{mf}(2) \) | \( t_{mf}(3) \) |
|------------------|----------------|----------------|----------------|----------------|
| \( n = 4r \)     | 5r             | 5r+1           | 5r             | 5r             |
| \( n = 4r+1 \)   | 5r+1           | 5r+2           | 5r+2           | 5r+1           |
| \( n = 4r+2 \)   | 5r+3           | 5r+3           | 5r+3           | 5r+2           |
| \( n = 4r+3 \)   | 5r+4           | 5r+4           | 5r+4           | 5r+4           |

**Table 8**

**Case 5.** \( n = 3 \).

A 4-total mean cordial labeling for this case is given in Table 9

| Value \( n \) | \( u \) | \( u_1 \) | \( u_2 \) | \( u_3 \) | \( v_1 \) | \( v_2 \) | \( v_3 \) |
|---------------|-------|--------|--------|--------|-------|-------|-------|
| \( n = 3 \)   | 2     | 0      | 1      | 3      | 0     | 0     | 3     |

**Table 9**

\( \square \)

**Theorem 4.6.** The subdivision of the wheel \( W_n, S(W_n) \) is 4-total mean cordial for all values of \( n \).

**Proof.** Take the vertex set and edge set of the wheel \( W_n \) as in definition 3.1. Let \( x_i \) be the vertex which subdivide the edge \( uu_i \) \((1 \leq i \leq n)\) and \( y_i \) be the vertex which subdivide the edge \( u_iu_{i+1} \) \((1 \leq i \leq n-1)\) and \( y_n \) be the vertex which subdivide the edge \( u_nu_1 \). Clearly, \( |V(W_n)| + |E(W_n)| = 7n + 1 \).

Assign the label 1 to the central vertex \( u \). Now we consider the vertices \( x_1, x_2, \ldots, x_n \). Assign the label 0 to the \( n \) vertices \( x_1, x_2, \ldots, x_n \).

**Case 1.** \( n \equiv 0 \pmod{4} \).

Let \( n = 4r, r \in \mathbb{N} \). Consider the vertices \( u_1, u_2, \ldots, u_n \). Assign the labels 0 to the \( r \) vertices
Next we assign the label 2 to the $r$ vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Now we consider the vertices $y_{1}, y_{2}, \ldots, y_{n}$. Assign the label 3 to the $r$ vertices $y_{1}, y_{2}, \ldots, y_{r}$. Now we assign the label 0 to the $r$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r}$. Then we now assign the label 2 to the $2r - 1$ vertices $y_{2r+1}, y_{2r+2}, \ldots, y_{4r-1}$. Finally we assign the label 1 to the vertex $y_{4r}$.

**Case 2.** $n \equiv 1 \pmod{4}$.

Let $n = 4r + 1$, $r \in \mathbb{N}$. Assign the labels 0 to the $r$ vertices $u_{1}, u_{2}, \ldots, u_{r}$. Next we assign the label 2 to the $r$ vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r + 1$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r+1}$. Next we assign the label 3 to the $r$ vertices $y_{1}, y_{2}, \ldots, y_{r}$. Now we assign the label 0 to the $r + 1$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r+1}$. Then we now assign the label 2 to the $2r - 2$ vertices $y_{2r+2}, y_{2r+3}, \ldots, y_{4r-1}$. Now we assign the label 3 to the vertex $y_{4r}$. Finally we assign the label 2 to the vertex $y_{4r+1}$.

**Case 3.** $n \equiv 2 \pmod{4}$.

Let $n = 4r + 2$, $r \in \mathbb{N}$. We now assign the label 0 to the $r$ vertices $u_{1}, u_{2}, \ldots, u_{r}$. Next we assign the label 2 to the $r$ vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r + 2$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r+2}$. Assign the label 3 to the $r$ vertices $y_{1}, y_{2}, \ldots, y_{r}$. Now we assign the label 0 to the $r + 1$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r+1}$. Then we now assign the label 2 to the $2r - 1$ vertices $y_{2r+2}, y_{2r+3}, \ldots, y_{4r}$. Finally we assign the labels 3,1 to the vertices $y_{4r+1}, y_{4r+2}$.

**Case 4.** $n \equiv 3 \pmod{4}$.

Let $n = 4r + 3$, $r \in \mathbb{N}$. Assign the labels 0 to the $r$ vertices $u_{1}, u_{2}, \ldots, u_{r}$. Next we assign the label 2 to the $r$ vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 3 to the $2r + 3$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r+3}$. Assign the labels 3 to the $r$ vertices $y_{1}, y_{2}, \ldots, y_{r}$. Now we assign the label 0 to the $r + 2$ vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r+2}$. Then we now assign the label 2 to the $2r - 2$ vertices $y_{2r+3}, y_{2r+4}, \ldots, y_{4r}$. Finally we assign the labels 3,3,1 to the vertices $y_{4r+1}, y_{4r+2}, y_{4r+3}$.

This vertex labeling $f$ is a 4-total mean cordial labeling follows from the Tabel 10

**Case 5.** $n = 3$.

A 4-total mean cordial labeling for this case is given in Table 11

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**Theorem 4.7.** The web graph $Wb_{n}$ is 4-total mean cordial, for all $n$. 

\[\square\]
\[ n = 4r \quad \begin{array}{cccc} t_{mf}(0) & t_{mf}(1) & t_{mf}(2) & t_{mf}(3) \\ 7r & 7r & 7r & 7r + 1 \\ 7r + 2 & 7r + 2 & 7r + 2 & 7r + 2 \\ 7r + 3 & 7r + 4 & 7r + 4 & 7r + 4 \\ 7r + 5 & 7r + 6 & 7r + 6 & 7r + 6 \end{array} \]

**TABLE 10**

\[
\begin{array}{cccccccc}
\text{Value of } n & u & u_1 & u_2 & u_3 & x_1 & x_2 & x_3 & y_1 & y_2 & y_3 \\
n = 3 & 1 & 0 & 3 & 3 & 0 & 0 & 0 & 2 & 2 & 2 \\
\end{array}
\]

**TABLE 11**

Proof. Take the vertex set and edge set of the web graph \( Wb_n \) as in definition 3.6. Clearly that
\[ |V (Wb_n)| + |E (Wb_n)| = 8n + 1. \]

Assign the label 2 to the central vertex \( u \).

We now consider the vertices \( u_1, u_2, \ldots, u_n \). Assign the label 0 to the \( n \) vertices \( u_1, u_2, \ldots, u_n \). Then we consider the vertices \( v_1, v_2, \ldots, v_n \). We now assign the label 2 to the \( n \) vertices \( v_1, v_2, \ldots, v_n \). We now move to the vertices \( x_1, x_2, \ldots, x_n \). Finally we assign the label 3 to the \( n \) vertices \( x_1, x_2, \ldots, x_n \). Clearly \( t_{mf}(0) = t_{mf}(1) = 2n, t_{mf}(2) = 2n + 1 \) and \( t_{mf}(3) = 2n \). □

**CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

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