Is Non-minimal Inflation Eternal?

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Abstract: The possibility that the non-minimal coupling inflation could be eternal is investigated. We calculate the quantum fluctuation of the inflaton in a Hubble time and find that it has the same value as in the minimal case in the slow-roll limit. Armed with this result, we have studied some concrete non-minimal inflationary models including the chaotic inflation and the natural inflation while the inflaton is non-minimally coupled to the gravity and we find that these non-minimal inflations could be eternal in some parameter regions.

Keywords: Cosmology, Inflation, Eternal inflation, Non-minimal Coupling.
1. Introduction

Inflation has been remarkably successful in explaining the properties of the universe and the origin of the primordial perturbation [1], which is thought of as the seed of the large scale structures. In the context of new inflation, the early inflating universe is driven by a scalar field called inflaton, which is at first living at an unstable state like the false vacuum state, then it slowly rolls down to a stable state like the true vacuum state. An interesting phenomenon in the inflationary scenario is the eternal inflation, which means the inflation never ends.

The eternal new inflation was first discovered by Steinhardt [2], and Vilenkin [3] showed that new inflationary models are generically eternal. As we known, the decay of the false vacuum to a true vacuum is an exponential process, however, it also exponentially expands when it decays during the inflation time. In fact, the rate of exponential expansion is always much faster than the rate of exponential decay in any successful inflationary models. Therefore, the false vacuum never disappears and the total volume of the false vacuum grow exponentially with time when inflation starts, see ref [4].

Actually, some inflationary models like chaotic inflation does not have a false vacuum state, but it can also be eternal, which is called slow-roll eternal inflation [5]. In these models, the inflaton is classically rolling down the hill, and the change in the field during some time interval is influenced by the quantum fluctuations, which can drive the field upward or downward relative to the classical trajectory. There is always some probability that the classical evolution is smaller than the quantum fluctuations, then the inflaton will fluctuate up and not down. Therefore, this process will continues forever and inflation will never ends. The recent progress on eternal inflation, see ref. [6] and for recent reviews see ref. [4, 7].

There is a class of inflationary models called non-minimal inflations, in which the inflaton ($\varphi$) is non-minimally coupled to the gravity and terms like $f(R, \varphi)$ are introduced in the effective action, where $R$ is the Ricci scalar. The cosmological effects of the non-minimal inflationary models have been well studied. It shows that the power spectrum is generally blue, and the problem of getting a running spectral index is eased in the non-minimal inflation [8]. For other recent studies on the non-minimal inflation, see ref. [4]. However, the eternal property of them has not studied in these literatures. In this paper, we have investigated that whether the non-minimal coupling inflation could be eternal or not, and we focus on the simplest model with $f = \zeta R \varphi^2$. 
The paper is organized as follows. In Section 2, we briefly review the background dynamics for the inflaton and the metric, impose the slow-roll condition and define three slow-roll parameters including the ordinary two and a new one. In Section 3, we calculate the quantum fluctuation and the classical motion of the inflaton during a Hubble time. Then, in Section 4, we focus on some concrete non-minimal inflation models and find that they could be eternal in some parameter regions. In the last section, we give some discussion the conclusions.

2. Background dynamics

The Lagrange density of the inflaton non-minimally coupled to gravity is given by

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) - \frac{1}{2} f(R) \varphi^n, \quad (2.1)$$

where $f$ is a function of the scalar curvature. The total action reads

$$S = \frac{1}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}, \quad (2.2)$$

where we have set $8\pi G = M_p^{-2} = 1$. In a flat FRW universe with the unperturbed metric

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2), \quad (2.3)$$

and the scalar curvature reads

$$R = 6 \left( \dot{H} + 2H^2 \right), \quad (2.4)$$

where $H = \dot{a}/a$ is the Hubble parameter and the over dot denotes the derivative with respect to the co-moving time $t$. By varying the action (2.2) we obtain the field equations

$$3H^2 = \frac{1}{2} \dot{\varphi}^2 + V + \frac{1}{2} f \varphi^2 + 3H^2 \left[ \frac{d}{dt} \left( \frac{f' \varphi^2}{H} \right) - f' \varphi^2 \right], \quad (2.5)$$

$$-2\dot{H} = \varphi^2 + H^3 \frac{d}{dt} \left( \frac{f' \varphi^2}{H^2} \right) - \frac{d^2}{dt^2} \left( f' \varphi^2 \right), \quad (2.6)$$

where prime denotes the derivative with its argument, i.e. $f' = df/dR$ and $V' = dV/d\varphi$ in the following E.O.M. of the scalar field

$$\ddot{\varphi} + 3H \dot{\varphi} + V' + f \varphi = 0. \quad (2.7)$$

To get an enough long time inflation, we impose the following slow-roll conditions

$$|\dot{H}| \ll H^2, \quad |\dot{\varphi}| \ll 3H |\dot{\varphi}|, \quad (2.8)$$

which leads to

$$|f' \varphi^2 \dot{H}| \ll V, \quad |V'' + f - \frac{3H f' \dot{R}}{f} \frac{f \varphi}{V' + f \varphi}| \ll 9H^2, \quad (2.9)$$

and we also assume $\dot{\varphi}^2/2 \ll V$ to simplify the following calculation. Moreover, if $f$ is a monomial of $R$, e.g. $f \sim R^n$, then $3H (\log f)' \dot{R} \approx 6n \dot{H}$. Therefore, in the case of $|f \varphi| \ll |V'|$ or $|f \varphi| \gg |V'|$, the third term on the L.H.S. of the second equation in (2.9) can be neglected and it simplifies to

$$|f' \varphi^2 \dot{H}| \ll V, \quad |V'' + f| \ll 9H^2. \quad (2.10)$$
With these conditions in mind, the equations of motion are simplified

\[ 3H^2 = \frac{1}{2} \left( f - 6f'H^2 \right) \varphi^2 + V, \]  
\[ 3H\dot{\varphi} = -V', \quad |f\varphi| \ll |V| \quad \text{(Case I.)}, \]  
\[ 3H\dot{\varphi} = -f\varphi, \quad |f\varphi| \gg |V'| \quad \text{(Case II.)}. \]  

The traditional the dimensionless slow-roll parameters are given by

\[ \epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = M_p^2 \frac{V''}{V}, \]  

and we define another new a new dimensionless slow-roll parameter as

\[ \Delta \equiv M_p^2 \frac{f}{V}, \]  

where we have recovered the Planck mass to indicate these parameters being dimensionless. Then the slow-roll conditions (2.10) becomes

\[ \epsilon \Delta \varphi^2 \ll 1, \quad \eta + \Delta \ll 1 \]  

where we have used \( f' \sim f/R \) and \( V' \sim V/\varphi \). Therefore, if \( \epsilon, \eta, \Delta \ll 1 \), the slow-roll conditions in (2.10) is met. In the following, we will consider a simple class of model with \( f = \xi R \). Thus, the Friedmann equation (2.11) becomes

\[ 3H^2 = \frac{V}{1 - \xi \varphi^2}, \]  

and the new slow-roll parameter is

\[ \Delta = \frac{4 \xi (2 - \epsilon)}{(1 - \xi \varphi^2)}. \]  

Therefore, \( \Delta \ll 1 \) requires \( \xi \ll 1 \) in this model. In the case of \( |f\varphi| \ll |V'| \), i.e. \( \Delta \varphi^2 \ll 1 \), the slow-roll parameter is

\[ \epsilon = \frac{V'}{2V^2} \left[ 1 - \left( 1 - \frac{2V}{V' \varphi} \right) \xi \varphi^2 \right], \]  

while in the case of \( |f\varphi| \gg |V'| \), i.e. \( \Delta \varphi^2 \gg 1 \), it becomes

\[ \epsilon = \frac{f\varphi V'}{2V^2} \left[ 1 - \left( 1 - \frac{2V}{V' \varphi} \right) \xi \varphi^2 \right]. \]  

It should be noticed that in the latter case, the condition \( \Delta \varphi^2 \gg 1 \) means we must have large field inflation, i.e. \( \varphi^2 \gg M_p^2 \), and in both case, \( \xi \varphi^2 \sim \mathcal{O}(1) \) is required, otherwise it reduces to the minimal coupling case. Therefore, in the following, we will only consider large field inflation.

3. Fluctuation and motion of the inflaton

In order to calculate the quantum fluctuation of the inflaton, we should expand the action (2.2) to the second order. The action approach guarantees the correct normalization for the quantization of fluctuations and it is convenient to work in the ADM formalism. We write the metric as

\[ ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt), \]  

where \( N \) is the lapse function and \( N^i \) is the shift vector and the action (2.2) becomes

\[ S = \frac{1}{2} \int dtdx^3 \sqrt{h} \left[ NR^{(3)} + N^{-1} (E_{ij}E^{ij} - E^2) + N^{-1} (\dot{\varphi} - N^i \partial_i \varphi)^2 - Nh^{ij} \partial_i \varphi \partial_j \varphi - N \left( 2V + f \varphi^2 \right) \right], \]
where \( h = \det h_{ij} \) and the symmetric tensor \( E_{ij} \) is defined as
\[
E_{ij} = \frac{1}{2} \left( h_{ij} - \nabla_i N_j - \nabla_j N_i \right), \quad E = E^i_i.
\]
Here \( R^{(3)} \) is the three-dimensional Ricci curvature which is computed from the metric \( h_{ij} \). Note that the extrinsic curvature is \( K_{ij} = E_{ij}/N \). We will work in the spatially flat gauge and neglect the tensor perturbations:
\[
\varphi(t, x) = \bar{\varphi}(t) + \delta \varphi(t, x), \quad h_{ij} = \alpha^2 \delta_{ij},
\]
where \( \bar{\varphi}(t) \) is the background value of the scalar field and \( \delta \varphi \) is a small fluctuation from the background value. In the ADM formalism, one can think of \( N \) and \( N^i \) as Lagrange multipliers, and to get the action for \( \zeta \), one need to solve the constraint equations for \( N \) and \( N^i \) and plug the result back in the action.

The equations of motion for \( N^i \) and \( N \) are the momentum and Hamiltonian constraints
\[
\nabla_i \left[ \left( 1 - f' \varphi^2 \right) N^{-1} \left( E^i_j - \delta^i_j E \right) \right] - N^{-1} \left( \varphi - N^i \partial_i \varphi \right) \partial_j \varphi = 0 \quad \text{(3.5)}
\]
\[
R^{(3)} - \left( 1 - 2 f' \varphi^2 \right) N^{-2} \left( E^i_j E^{ij} - E^2 \right) - N^{-2} \left( \varphi - N^i \partial_i \varphi \right)^2 - 2V - f \varphi^2 - h^{ij} \partial_i \varphi \partial_j \varphi = 0 \quad \text{(3.6)}
\]
By decomposing \( N^i \) into \( N^i = \partial^i \psi + N^i_\gamma \) where \( \partial_i T^i_\gamma = 0 \) and \( N = 1 + N_1 \), where \( N_1, N_\gamma, \psi \sim \mathcal{O}(\delta \varphi) \), and plugging these expansion into the eqs. (3.5) and (3.6) for \( N \) and \( N^i \), one can obtain the solutions in the first order of \( \zeta \). In the case of \( f = \xi R \), we get the first order solutions in Appendix A, and in the following we will still use \( \varphi \) to denote the background value \( \bar{\varphi} \):
\[
N_1 = \frac{\delta \varphi}{1 - \xi \varphi^2} \left( \frac{\varphi}{2H} - 2 \xi \varphi \right), \quad N^i_\gamma = 0,
\]
and
\[
(1 - \xi \varphi^2) \partial^2 \psi = N_1 \frac{\varphi^2}{2H} - \frac{\varphi}{2H} \partial \varphi - \left( \frac{3 \varphi}{2H} - 3 \xi \varphi + \frac{V'}{2H^2} \right) H \partial \varphi
\]
with suitable boundary conditions. We also get the exact background dynamic equation
\[
3H^2(1 - \xi \varphi^2) = \frac{1}{2} \varphi^2 + V, \quad \text{(3.9)}
\]
which is consistent with eq.(2.17) in the slow-roll limit. To find the quadratic action for \( \delta \varphi \), we need plug eqs.(3.5) and (3.6) in the action and expand it to the second order. However, we can see that these expressions for \( N \) and \( N^i \) are subleading in slow-roll (\( \dot{\varphi} \ll H^2 \)) and large field \( (\varphi^2 \gg M_P^2) \) inflation compared to \( \delta \varphi \). So, it is enough to consider just the action (2.2) for \( \delta \phi \) in the de Sitter background and we get the second order action
\[
S_2 = \frac{1}{2} \int d^4 x a^3 \left[ \delta \dot{\varphi}^2 - (\nabla \delta \varphi)^2 - V'' \delta \varphi^2 - 12 \xi H^2 \delta \varphi^2 \right], \quad \text{(3.10)}
\]
and the perturbation equation is
\[
\delta \varphi_k + 3H \delta \dot{\varphi}_k + \frac{k^2}{a^2} \delta \varphi_k = 0, \quad \text{(3.11)}
\]
where we have used \( \eta \ll 1 \) and \( \Delta \ll 1 \) and \( \delta \varphi_k \) is the Fourier transform of \( \delta \varphi \). Thus, the quantum fluctuation of \( \delta \varphi \) in a Hubble time has the same value as in minimal case [8]
\[
\delta \varphi \approx \frac{H}{2\pi},
\]
while usually the classical motion of the inflaton during one Hubble time is given by
\[
|\delta \varphi| \approx \varphi H^{-1} \sim \frac{V'}{3H^2} \left( 1 + \Delta \varphi^2 \right),
\]
and the condition for eternal inflation to happen is roughly \( \delta \varphi > |\delta \varphi| \).
4. Eternal non-minimal inflation

In this section, we focus on some concrete large field inflation models. The first example is

\[ V(\varphi) = \frac{1}{2}m^2\varphi^2, \quad (4.1) \]

where \( m \) is the mass of inflaton. Then, the condition of the eternal inflation is

\[ \frac{2(1 + \Delta \varphi^2)}{\varphi} < \frac{H}{2\pi} \sim m\varphi. \quad (4.2) \]

where we have used \( \xi^2 \sim O(1) \). Then, for the case of \( \Delta \varphi^2 \ll 1 \), it requires

\[ \varphi > \varphi_c = M_p\sqrt{\frac{m}{M_p}}, \quad \varphi \ll \Delta^{-1/2}M_p, \quad (4.3) \]

where we have recovered the Planck mass and \( \varphi_c \gg M_p \) is the critical value of inflaton to be eternal. While for the case of \( \Delta \varphi^2 \gg 1 \), it requires

\[ \Delta < \frac{m}{M_p}, \quad \varphi \gg \Delta^{-1/2}M_p, \quad (4.4) \]

to become eternal inflation. Therefore, in both case, the inflation could never be eternal if \( m < \Delta M_p \).

The second example considered here is

\[ V(\varphi) = \lambda\varphi^n, \quad (4.5) \]

with \( n > 2 \). Then, the condition of the eternal inflation is

\[ \frac{n(1 + \Delta \varphi^2)}{\varphi} < \frac{H}{2\pi} \sim \lambda^{1/2}\varphi^{n/2}. \quad (4.6) \]

Then, for the case of \( \Delta \varphi^2 \ll 1 \), it requires

\[ \varphi > \varphi_{c1} = n^{2/(n+2)}\frac{M_p}{\lambda^{1/(n+2)}}, \quad \varphi \ll \Delta^{-1/2}M_p, \quad (4.7) \]

So, if \( \lambda < \Delta^{(n+2)/2} \), the inflation could never be eternal. While for the case of \( \Delta \varphi^2 \gg 1 \), it requires

\[ \varphi > \varphi_{c2} = (n)^{2/(n-2)}\left(\frac{\Delta^2}{\lambda}\right)^{1/(n-2)}M_p, \quad \varphi \gg \Delta^{-1/2}M_p, \quad (4.8) \]

to become eternal inflation and it is always an eternal inflation if \( \lambda > \Delta^{(n+2)/2} \).

The above two examples both belong to the class of the chaotic inflation, which is a typical large field inflation, while the inflaton is non-minimal coupled to the gravity. Another elegant inflationary model is called the natural inflation where the potential takes the following form

\[ V(\varphi) = V_0 \left[ \cos \left( \frac{\varphi}{y} \right) + 1 \right], \quad (4.9) \]

and this model can be of the small field or large field type depending on the parameter \( g \). If \( 2\pi g > M_p \), namely the periodicity of the inflaton larger than the Planck scale, it would be a large field inflation. Then, the condition of the inflation being eternal is

\[ F(\varphi)^{2/3} < V(\varphi), \quad \varphi \ll \Delta^{-1/2}M_p, \quad (4.10) \]
for the case of $\Delta \varphi^2 \ll 1$, and

$$F(\varphi)^{2/3}(\Delta \varphi^2)^{2/3} < V(\varphi), \quad \varphi \gg \Delta^{-1/2} M_p,$$

(4.11)

for the case of $\Delta \varphi^2 \gg 1$. Here we have defined the function

$$F(\varphi) = -\frac{V_0}{g} \sin \left( \frac{\varphi}{g} \right).$$

(4.12)

**Figure 1:** Case $\Delta \varphi^2 \ll 1$. The eternal and non-eternal regions are illustrated with $V_0 = 10^{-4}$, and $2\pi g = 10^3$ in the Planck units.

**Figure 2:** Case $\Delta \varphi^2 \gg 1$. The eternal and non-eternal regions are illustrated with $V_0 = 2 \times 10^{-4}$, $2\pi g = 10^4$, and $\Delta = 10^{-6}$ in the Planck units.

To illustrate the region in which the inflation could be eternal, one should solve eqs.(4.10) and (4.11). Since they do not have analytical solutions, we have solved them numerically and illustrated the results in Figs.(1) and (2) with particular parameters to give an example. It should be noticed that, in Fig.(1), $\varphi \ll \Delta^{-1/2}$, and if $\Delta$ is not small enough, there would be no eternal region. For example, let $\Delta = 10^{-6}$, not all the areas in Fig.(1) are valid since $\varphi \ll 10^3$ is required. In other words, in the case of $\Delta \varphi^2 \ll 1$, the inflation could be eternal only if $\Delta$ is much small. In Fig.(2), we have set $\Delta = 10^{-6}$ for numerical calculation, and then $\varphi \gg 10^3$ is required in this case.
5. Conclusions

Many inflationary models have eternal properties like the new inflation and the chaotic inflation. In this paper, we have studied the eternal properties of non-minimal inflationary models, in which there is a non-minimal coupling between the inflaton and the gravity. We have calculated the quantum fluctuation of the inflaton at the first order, and find that it has the same value as in the minimal case, namely, the quantum fluctuation of inflaton in a Hubble time is roughly proportional to the Hubble parameter during the inflation. If the quantum fluctuation overcome the classical motion of the inflaton, the inflation could never end.

We have studied some concrete non-minimal inflationary models including the chaotic inflation with power law potentials and the natural inflation, derived the conditions under which the inflation could be eternal and found that in some parameter spaces, it could be eternal inflation. Our results could be simply generalized to the case of $f \sim R\varphi^n$, where $n = 2$ is studied in this paper.

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A. First order solution

In this model, the constraint eqs. (3.5) and (3.6) becomes

$$
\partial_i \left[ (1 - \xi \varphi^2)N^{-1} (E^i_j - \delta^i_j) \right] - N^{-1} (\dot{\varphi} - N^j \partial_j \varphi) \partial_i \varphi = 0, \tag{A.1}
$$

$$
(1 - \xi \varphi^2) \left[ R^{(3)} - N^{-2} (E^i_j E^{ij} - E^2) \right] - N^{-2} (\dot{\varphi} - N^j \partial_j \varphi)^2 - 2V - h^i_j \partial_i \varphi \partial_j \varphi = 0, \tag{A.2}
$$

with the ansatz

$$
N = 1 + N_1, \quad N^i = \partial^i \psi + N^i_T, \quad \partial_i N^i_T = 0, \quad N_i = h_{ij} N^j, \tag{A.3}
$$

and the spatially flat metric

$$
h_{ij} = a^2 \delta_{ij}, \quad h^{ij} = a^{-2} \delta^{ij}, \quad \sqrt{h} = \sqrt{\det h_{ij}} = a^3, \quad \Gamma^i_{ij} = 0, \quad R^{(3)} = 0. \tag{A.4}
$$

After some length calculations, we find

$$
E^{ij} E_{ij} - E^2 = -6H^2 + 4H \partial^2 \psi, \tag{A.5}
$$

and

$$
E^i_j - \delta^i_j E = -2\delta^i_j H - \partial^i \partial_j \psi + \delta^i_j \partial^2 \psi - \delta_{ji} \partial^i N^l_T. \tag{A.6}
$$

Thus, eqs. (3.3) and (3.6) becomes

$$
2H(1 - \xi \varphi^2) \partial_j N_1 + 4H \xi \varphi \partial_j \dot{\varphi} - (1 - \xi \varphi^2) \delta_{ji} \partial^2 N^l_T = \dot{\varphi} \partial_i \delta \varphi, \tag{A.7}
$$

$$
-\left(1 - \xi \varphi^2 \right) 4H \partial^2 \psi - 12H^2 \left( \frac{\dot{\varphi}}{2H} - \xi \varphi \delta \varphi \right) = -2N_1 \dot{\varphi}^2 + 2\dot{\varphi} \delta \dot{\varphi} + 2V' \delta \varphi. \tag{A.8}
$$

From eq. (A.7), one finds

$$
N_1 = \frac{\delta \varphi}{1 - \xi \varphi^2} \left( \frac{\dot{\varphi}}{2H} - 2\xi \dot{\varphi} \right), \quad \partial^2 N^l_T = 0, \tag{A.9}
$$

so with appropriate choice of boundary conditions one can justifiably set $N^l_T = 0$. Plugging the solution of $N_1$ into eq. (A.8) we obtain the result (3.3).
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