Kerr-breather-soliton time crystals

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Abstract: We describe Kerr-breather-soliton time crystals in which a breathing excitation is sub-harmonically locked to the repetition frequency. Nonlinear modeling explores the behavior of soliton time crystals, and we will report on progress towards their observation. © 2019 The Author(s)

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1. Introduction

Dissipative Kerr frequency combs in resonators [1], which are driven detuned and feature nonlinear interactions, support circulating soliton pulses of light that are sustained by the pump laser and can propagate indefinitely. These solitons are of interest both from the standpoint of fundamental nonlinear dynamics and because they could bring optical-frequency synthesis [2] and clock metrology [3,4] and other capabilities to the chip scale.

Under some conditions Kerr solitons can exhibit breathing behavior, in which case their amplitude, bandwidth, and duration oscillate in time over the course of many resonator round trips [5–7]. These breather oscillations emerge spontaneously, and their frequency is determined by the system parameters. Here we propose and theoretically investigate the possibility that these breather oscillations may become sub-harmonically locked to the periodic perturbation occurring at the resonator round-trip time \( T_R \) that arises from out-coupling and pumping [8]. The result of subharmonic locking is an integer ratio \( T_b/T_R = N \gg 1 \) between the breather period \( T_b \) and the round-trip time. We find that in a regime of intermediate resonator finesse \( F \sim 30-40 \), rigid subharmonic locking of the breather oscillation to the round-trip time occurs and is maintained over a range of system parameters. We observe locking with an integer locking ratio as high as \( N=20 \), and propose a way to realize the effect with higher locking ratios and at higher finesse. This system's subharmonic response to the governing equations that are invariant under discrete time translations is similar to recently proposed and reported discrete time crystals in quantum many-body systems [9,10]. We will discuss a route towards experimental realization of these breather-soliton time crystals, and explain how their incorporation into photonics systems could simplify proposed applications of microresonator solitons. Importantly, our work explores a new regime of microresonator Kerr-nonlinear dynamics in which the round-trip-time perturbation plays a significant role.

![Figure 1](image-url)

Figure 1: At left is a diagram that explains our system. A Kerr-microresonator emits soliton pulses at the repetition frequency corresponding to the round-trip time. We perform simulations, using the Ikeda map, that determine soliton propagation; the blue dots indicate the simulated soliton intensity as a function of roundtrip number. (a) Shows the breather soliton intensity outside the time crystal regime. The phase of the breather oscillation is unlocked from the repetition frequency. (b) Shows the time crystal regime in which the breather oscillation is phase locked to a subharmonic of the repetition frequency. The horizontal lines demonstrate the constant relative phase of the breather soliton time crystal.

2. Breather soliton time crystals

If a single soliton circulates in the cavity then a pulse is out-coupled once per round trip, and a train of pulses separated by the round-trip time \( T_R \) propagates away from the resonator—this is illustrated schematically in the diagram at left in Fig. 1. It has generally been assumed that the timescale over which the intracavity field evolves in
microcombs is determined by the cavity photon lifetime, and that the round-trip time plays little role beyond setting the repetition rate $f_{rep}$ of the out-coupled pulse train. This approximation emerges in the limit of high finesse and results in the mean-field Lugiato-Lefever equation (LLE) model of the nonlinear dynamics. On the other hand, to explore round-trip-time effects one can make use of an Ikeda map [8] in which the intracavity field is determined by simulation of the generalized nonlinear Schrödinger equation (GNLSE) over one round trip, and then coupling to the resonator is taken into account.

Kerr-soliton breathing is the spontaneous emergence of oscillations in the amplitude and temporal duration of a circulating soliton. These oscillations typically occur over many round trips with a breathing period $T_b >> T_R$ so that they manifest as the modulation of the pulse energies in the out-coupled pulse train. This is depicted in Fig. 1b and can be revealed in experiments that satisfy a defined outcome, e.g., by impinging the pulse train on a photodetector. We investigate the possibility of subharmonic locking of breather oscillations to the periodic perturbation at the round-trip time such that $T_b = N T_R$, where $N>1$ and even potentially much larger than one. We adopt the descriptive term breather-soliton time crystals to emphasize the connection between our investigations and recent research outside of the field of microcombs, and to make clear the nature of discrete-time-translation-symmetry violation that we observe.

We perform numerical simulations, utilizing the Ikeda-map formulism. These simulations reveal subharmonic locking of the breather oscillations to the round-trip time in a regime of intermediate finesse; the basic effect is shown in Fig. 1c. We focus on the low-finesse regime in this initial study for two reasons– it strengthens the perturbation at the round-trip time, and it reduces the time needed to conduct simulations. We present synchronization results for $T_b T_R = N$ up to 13. We explore the rigidity of subharmonic locking and find that it remains robust over a range of system parameters, and even as parameters are dynamically varied. Our simulations indicate that the strength of locking decays exponentially as the finesse is increased, so we discuss possible routes towards realization of the effect at higher finesse. We find that temporal sharpening of the breather oscillation peak through introduction of fourth-order resonator mode dispersion immediately allows us to extend the effect to $N=20$ without making substantial efforts at optimization. This locking ratio already indicates that the effect could be employed to electronically measure, for example, a microcomb repetition rate of 1 THz by measuring the frequency of the subharmonically locked breather oscillations at 50 GHz. This would allow significant simplification of proposals for integration of microcombs for applications. We will conclude by discussing possibilities for experimental realization of breather-soliton time crystals.

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3. References
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