Z⁰-Boson Decays in a Strong Electromagnetic Field *

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Abstract

The probability of Z⁰-boson decay to a pair of charged fermions in a strong electromagnetic field, Z⁰ → ̅ff, is calculated. On the basis of a method that employs exact solutions to relativistic wave equations for charged particles, an analytic expression for the partial decay width Γ(κ) = Γ(Z⁰ → ̅ff) is obtained at an arbitrary value of the parameter κ = eM³Z³√−(F_{μν}q^ν)^2, which characterizes the external-field strength. The total Z⁰-boson decay width in an intense electromagnetic field, Γ_Z(κ), is calculated by summing these results over all known generations of charged leptons and quarks. It is found that, in the region of relatively weak fields (κ < 0.06), the field-induced corrections to the standard Z⁰-boson decay width in a vacuum do not exceed 2%. As κ increases, the total decay width Γ_Z(κ) develops oscillations against the background of its gradual decrease to the absolute-minimum point. At κ_{min} = 0.445, the total Z⁰-boson decay width reaches the minimum value of Γ_Z(κ_{min}) = 2.164 GeV, which is smaller than the Z⁰-boson decay width in a vacuum by more than 10%. In the region of superstrong fields (κ > 1), Γ_Z(κ) grows monotonically with increasing external-field strength. In the region κ > 5, the t-quark-production process Z⁰ → ̅tt, which is forbidden in the absence of an external field, begins contributing significantly to the total decay width of the Z⁰-boson.

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1. INTRODUCTION

It is not an exaggeration to say that $Z^0$-bosons along with their charged partners, $W^\pm$-bosons, have been at the focus of attention in high-energy physics over more than the past two decades. The properties of these particles, which are mediators of weak interaction between leptons and quarks, were studied in detail in experiments at the LEP and SLC electron-positron colliders. In a series of experiments performed between 1989 and 1995 at CERN [1], enormous statistics of observations of $Z^0$-boson production and decays were collected. In all, approximately $2 \cdot 10^6$ events associated with leptonic modes of $Z^0$-boson decay and more than $1.5 \cdot 10^7$ events of transformation of these unstable particles into hadrons were detected. The results of these investigations made it possible to determine, among other things, the mass of the $Z^0$-boson, $M_Z = 91,1876 \pm 0,0021$ GeV, and its total decay width, $\Gamma_Z = 2,4952 \pm 0,0023$ GeV, to an extremely high degree of precision [2]. Many other parameters of the process $e^+e^- \rightarrow Z^0 \rightarrow \bar{f}f$ were also analyzed both in the vicinity of the $Z$-resonance and far from it [3]. The precision of experimental data reached in recent years has given a motivation to theoretical physicists for performing formidable work on calculating radiative corrections to the processes involving $Z^0$-boson production and decay (see, for example, [4–7] and references therein).

Not only does $Z^0$-boson physics provide a precision test for Standard Model predictions concerning particle interactions, but it is also a unique tool for seeking manifestations of new physics (beyond the Standard Model), which will replace sooner or later the generally accepted $SU(3) \times SU(2) \times U(1)$ scheme. Despite the impressive successes of the Standard Model in adequately describing a formidable set of experimental data, there is no doubt among the physics community that the modern model of fundamental interactions cannot be conclusive because of a number of fundamental
theoretical drawbacks. These include, first, an enormous number (more than 19!) of independent input parameters — coupling constants, the masses of quark and leptons, the parameters of the Cabibbo-Kobayashi-Maskawa mixing matrix, etc; second, the instability of the mass of the as-yet-undiscovered Higgs boson with respect to quadratically divergent radiative corrections and the allied hierarchy problem; and, third, the isolated character of gravitational interactions, which do not fit in the existing quantum model. This is not the whole story, however. Paradoxically as it is, the main drawback of the Standard Model at the present time is that it is in perfect agreement with all experimental data accumulated thus far, providing no hint as to the nature of new physics that will have to replace it. This kind of ”recess” in the experimental development of physics theory caused a flash of creative activity in theoretical physics in attempts at guessing vague outlines of a new theory of everything. By no means do Grand Unification theories, supersymmetry, supergravity, superstrings, supermembranes, and $M$-theory exhaust the list of ideas that require an experimental verification. The questions that a future theory should answer include that of the origin of particle masses, that of why our space-time is four-dimensional, that of why several generations of leptons exist, that of how many flavors quarks have, and that of how many neutrino flavors exist. Even today, physicists try to find answers to many of these questions by analyzing available experimental data.

For example, the total invisible width of $Z^0$-bosons, $\Gamma_{\text{inv}} = 499,0 \pm 1,5$ MeV, with respect to the decays of these particles through channels that are rather difficult to detect plays an important role in $Z^0$-boson physics. In the experiments performed at the LEP accelerator, the invisible width $\Gamma_{\text{inv}}$ was determined indirectly as the difference of the total decay width of the $Z^0$-boson and its total width with respect to all observed decays. In the Standard Model, the invisible decay width of the $Z^0$-boson
is interpreted as the partial width with respect to decays to neutrinos of various flavors:

\[ \Gamma_{\text{inv}} = \sum_{i} \Gamma(Z^0 \rightarrow \nu_i \bar{\nu}_i) = N_{\nu} \Gamma_{\nu \bar{\nu}} \]

This quantity can be considered as some kind of a counter of neutrino generations, and its precise measurement is of particular interest for new physics (see, for example, [8]). It is intriguing that the present-day experimental value of the number of neutrinos from the invisible decay width of the \( Z^0 \)-boson, \( N_{\nu} = 2.92 \pm 0.06 \) [2], differs somewhat from the result expected in the Standard Model, \( N_{\nu} = 3 \).

Exotic channels of \( Z^0 \)-boson decay frequently provide unique tests for some fundamental principles of modern physics. For example, analysis of the forbidden (in a vacuum) \( Z^0 \)-boson decays to photons (\( Z^0 \rightarrow \gamma \gamma \)) and gluons (\( Z^0 \rightarrow gg \)) makes it possible to establish a quantitative measure of the principle of quantum indistinguishability of integer-spin particles — Bose symmetry for photons and gluons [9]. According to the Landau-Yang theorem [10], a massive vector particle cannot decay to massless vector states. In an external electromagnetic field, however, the reactions \( Z^0 \rightarrow \gamma \gamma \) and \( Z^0 \rightarrow gg \) become possible [11].

The problem of searches for new possible manifestations of supersymmetry is also tightly related to \( Z^0 \)-boson physics. There is still a hope for finding relatively light superparticles produced in rare channels of \( Z^0 \)-boson decay. As an example, one can consider the reaction \( Z^0 \rightarrow \tilde{g} \tilde{g} \), in which a pair of light gluinos, \( \tilde{g} \), having a mass of about \( m_{\tilde{g}} = 12 \div 16 \text{ GeV} \) appear. This process, which is extensively discussed in the literature (see [12] and references therein), is of special interest in view of prospects for observing new-physics manifestations at next-generation \( p\bar{p} \)-colliders. Projects of experiments with \( Z^0 \)-bosons at the Large Hadron Collider(LHC) also caused interest in studying the properties of these particles in the presence of a
quark-gluon plasma, which is quite an unusual environment of [13].

In view of the aforesaid, it is of particular interest to discuss some other new phenomena in $Z^0$-boson physics that manifest themselves under unusual conditions. In this study, we investigate changes in partial decay widths of the $Z^0$-boson in the presence of strong electromagnetic fields. Interest in such investigations is explained both by astrophysical applications and by applications in the physics of relativistic particles channeling through single crystals (so-called channeling of particles [14]). It is well known that the strength of electric fields generated by the axes and planes of single crystals may reach formidable values of $(E \geq 10^{10} \text{ V/m})$ [15] over macroscopic distances. At the same time, it is well known that astrophysics studies exotic objects (neutron stars, white dwarfs) that, at the latest stages of their evolution, can undergo a strong compression, and this leads to a significant increase in the magnetic field strength inside them $(H \geq 10^8 \div 10^{13} \text{ G})$ [16]). At such enormous values of the external field strength, the physics of quantum processes changes significantly. External fields frequently remove the forbiddance of specific reactions whose occurrence is impossible in a vacuum.

The first investigations into the physics of relativistic particles in the presence of a strong external field were performed within quantum electrodynamics (see, for example, [17–19]). Later, the technique of calculations that was developed there was also used in non-Abelian gauge theories [20]. Respective calculations rely on the method of exact solutions to the wave equations for charged particles, which makes it possible to take into account the interaction with the electromagnetic field beyond standard perturbation theory. In this case, the wave functions for all charged particles and their propagators are modified in quite a unwieldy manner, depending on the configuration of an external field [21]. Although the resulting expressions are cumbersome and although the calculations are quite in-
volved, the ultimate results, which are valid at arbitrarily high values of
the external-electromagnetic-field strength, carry information that cannot
be obtained by perturbative methods, and this is an obvious advantage of
the method. From this point of view, investigation of quantum processes
in superstrong fields provides a unique possibility of analyzing the self-
consistency of the physical theory globally in the case where an expansion
in a small parameter is impossible.

The method of calculations that is used below is based on the crossed-
field model, which was successfully applied in the previous studies of the
present author to studying photino-pair production [22] and to analyzing
the decays of $W$-bosons in an external field [23].

2. PROBABILITY FOR $Z^0$-BOSON DECAY
   IN EXTERNAL FIELDS

In the leading order of perturbation theory in the coupling constants $g_V$
and $g_A$, the matrix element of $Z^0$-boson decay to a pair of charged leptons,
$\ell^{\pm}$, is given by

$$S_{fi} = i \int d^4 x \overline{\Psi}_{\ell^-}(x, p) \gamma^\mu (g_V + \gamma^5 g_A) \Psi_{\ell^+}(x, p') Z_\mu(x, q).$$

(1)

The presence of an external electromagnetic field is taken into account via
choosing specific wave functions $\Psi_{\ell^-}(x, p)$ and $\Psi_{\ell^+}(x, p')$ for the charged
leptons $\ell^{\pm}$. This approach, which, in particle physics, is referred to as
the Furry representation, yields, for the probability of $Z^0$-boson decay,
an expression in which electromagnetic interactions are taken into ac-
count exactly, beyond standard perturbation theory in the electromagnetic
coupling constant $\alpha = e^2/4\pi \simeq 1/137$. The explicit form of the
wave functions for charged particles in an external electromagnetic field
depends on the field configuration. In the present study, we restrict our-
selves to the crossed-field configuration, in which case the strength tensor
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]
has the form
\[ F_{\mu\nu} = C(k_\mu a_\nu - k_\nu a_\mu). \tag{2} \]
It is assumed in this case that the external-electromagnetic-field potential
\[ A_\mu(x) \]
is chosen in the gauge
\[ A_\mu(x) = a_\mu C(k_\nu x^\nu), \tag{3} \]
Here, the unit constant 4-vector \( a_\mu \) determines the spatial field configuration and satisfies the conditions
\[ a_\mu a^\mu = -1, \quad F_{\mu\nu} = (a_\mu F_{\nu\lambda} - a_\nu F_{\mu\lambda}) a^\lambda. \tag{4} \]
A crossed field is a particular case of the field of a plane electromagnetic wave having an isotropic wave 4-vector \( k_\mu \) (\( k^2 = 0 \)). The invariant parameter \( C \) characterizes the crossed-field strength and has the dimensions of energy. Recall that, in an arbitrary reference frame, a crossed field is a superposition of constant electric and magnetic fields whose strength vectors are orthogonal and are equal in absolute value: \( E = H = k_0 C \). Moreover, the two relativistic invariants of the electromagnetic field are zero:
\[ F_{\mu\nu} F^{\mu\nu} = F_{\mu\nu} \tilde{F}^{\mu\nu} = 0. \tag{5} \]
In the crossed-field model, the choice of wave 4-vector \( k_\mu \) is quite arbitrarily. It is only necessary that this vector be isotropic, have dimensions of energy, and satisfy the conditions
\[ k_\mu a^\mu = 0, \quad F^{\mu\lambda} F_{\lambda\nu} = C^2 k^\mu k_\nu. \tag{6} \]
In particular, the wave 4-vector \( k^\mu \) can be expressed in terms of the electromagnetic-field-strength tensor and the constant electromagnetic-potential 4-vector \( a_\lambda \) as
\[ k^\mu = -\frac{1}{C} \cdot F^{\mu\lambda} a_\lambda. \tag{7} \]
In dealing with the problem of $Z^0$-boson decay, it is advisable to express the wave vector $k^\mu$ in terms of the $Z^0$-boson 4-momentum $q^\nu$ and the electromagnetic-field-strength tensor $F^{\mu\lambda}$ (see next section) as

$$k^\mu = \frac{e^2 \Delta}{2M_Z^6 \chi^2} \cdot F^{\mu\lambda} F_{\lambda\nu} q^\nu.$$  \hspace{1cm} (8)

In an arbitrary constant uniform electromagnetic field, the probability of the decay $Z^0 \rightarrow \ell^+ \ell^-$ is a function of three invariant dimensionless parameters, $P(Z^0 \rightarrow \ell^+ \ell^-) = P(\chi, a, b)$. These parameters are determined by the strength tensor of an external macroscopic field as

$$\chi = \frac{e}{M_Z^3} \sqrt{- (F_{\mu\nu} q^\nu)^2},$$ \hspace{1cm} (9)

$$a = -\frac{e^2}{4M_Z^4} F^{\mu\nu} \tilde{F}_{\mu\nu},$$ \hspace{1cm} (10)

$$b = \frac{e^2}{4M_Z^4} F^{\mu\nu} F_{\mu\nu}.$$ \hspace{1cm} (11)

In the crossed-field model, the parameter $\chi$ has the simplest form

$$\chi = \frac{eC}{M_Z^3} (q_{\mu} k^\mu),$$ \hspace{1cm} (12)

the square of the wave vector (8) having the form

$$k^2 = \frac{\Delta^2}{4M_Z^2} \left( \frac{a^2}{\chi^4} - \frac{2b}{\chi^2} \right).$$ \hspace{1cm} (13)

In a crossed field, the two parameters (10) and (11) of the external-field strength vanish by virtue of the equality in (5), $a = b = 0$; therefore, the right-hand side of Eq. (13) also vanishes. For a constant uniform electromagnetic field of general form, the condition that the wave vector in (8) is isotropic is not satisfied: $k^2 \neq 0$. Even in this case, however, we can use the semiclassical crossed-field model, which ensures a satisfactory description of the decay probability in an electromagnetic field of general
form, provided that \( a \ll \kappa^2 \) and \( b \ll \kappa^2 \). The closer to zero the right-hand side of Eq. (13) and the higher the precision to which the condition of isotropicity is satisfied, the more precise the results that are obtained in this approximation. The above constraints are obviously satisfied in the region of relatively weak electromagnetic fields for \( a \ll 1 \) and \( b \ll 1 \). Thus, the decay probability \( P(Z^0 \to \ell^+\ell^-) \) in a crossed field will correspond to the first leading term \( P_0 \) in the semiclassical expansion of the total decay probability \( P(\kappa, a, b) \) in a power series in the vectors of the external-field strengths at small values of the parameters \( a \) and \( b \); that is,

\[
P(\kappa, a, b) = P_0(\kappa, 0, 0) + a\frac{\partial P}{\partial a}(\kappa, 0, 0) + b\frac{\partial P}{\partial b}(\kappa, 0, 0) + \ldots \quad (14)
\]

This relation between the probabilities of quantum processes in a crossed field and a constant electromagnetic field of general form was obtained for the first time within quantum electrodynamics [18].

The wave functions for charged fermions in the field of a plane electromagnetic wave were obtained as far back as 1937 by D.M.Volkov (see, for example, [24]). In the case of a crossed field, these exact solutions to the Dirac equations for leptons \( \ell^\pm \) in an external field have the form

\[
\Psi_{\ell^{-}}(x, p) = \exp\left[-ipx + \frac{ieC(pa)}{2(pk)}(kx)^2 - \frac{ie^2C^2}{6(pk)}(kx)^3\right] \times
\]

\[
\times\left\{1 - \frac{e(kx)}{4(pk)}(F_{\mu\lambda}\gamma^\mu\gamma^\lambda)\right\} \frac{u(p)}{\sqrt{2p_0}V}; \quad (15)
\]

\[
\Psi_{\ell^{+}}(x, p') = \exp\left[ip'x + \frac{ieC(p'a)}{2(p'k)}(kx)^2 - \frac{ie^2C^2}{6(p'k)}(kx)^3\right] \times
\]

\[
\times\left\{1 + \frac{e(kx)}{4(p'k)}(F_{\mu\lambda}\gamma^\mu\gamma^\lambda)\right\} \frac{u'(p')}{\sqrt{2p'_0}V}; \quad (16)
\]

The spin part of these wave functions normalized to a three-dimensional spatial volume \( V \) is determined by the constant Dirac bispinors \( u(p) \) and
The antilepton wave function \( \Psi_{\ell^+}(x, p') \) differs from expression (15) only by the reversal of the sign of the electric charge \( e \): \( e \rightarrow -e \). In the expression for the matrix element of the decay in (1), the antilepton wave function appears in the charge-conjugate form (16), which can be derived on the basis of the Dirac-conjugate bispinor \( \bar{\Psi}_{\ell^+}(x, p') \) with the aid of the charge-conjugation matrix \( U_C = -i\gamma^0\gamma^2 \); that is,

\[
\Psi_{\ell^+}^c(x, p') = U_C \bar{\Psi}_{\ell^+}^T(x, p') \tag{17}
\]

As for the \( Z^0 \)-boson, its wave function does not change in an electromagnetic field because it is electrically neutral:

\[
Z_\mu(x, q) = \exp\left(-iqx\right) \frac{v_\mu(q)}{\sqrt{2q_0V}}. \tag{18}
\]

The spin states of the \( Z^0 \)-boson are characterized by the complex polarization 4-vector \( v_\mu(q) \) satisfying standard conditions for massive vector fields; that is,

\[
v_\mu(q)q^\mu = 0, \quad v_\nu^*(q)v^\nu(q) = -1, \tag{19}
\]

\[
\sum_{\sigma=1}^3 v_\mu(q, \sigma)v^\nu_\sigma(q, \sigma) = -g_{\mu\nu} + q_\mu q_\nu/M_Z^2.
\]

We substitute the particle wave functions (15), (16), and (18) into expression (1) for the \( S \)-matrix element and perform integration of \( |S_{fi}|^2 \) over the phase space of final leptons. Upon summation over the spin states of the lepton-antilepton pair and averaging over polarizations of the \( Z^0 \)-boson, we arrive at

\[
P(Z^0 \rightarrow \ell^+\ell^- | \mathcal{N}) = \frac{(g_V^2 + g_A^2)M_Z^2}{12\pi^2q_0} \int_0^1 du \left\{ \left[ 1 - (1 + 3\lambda)\frac{m_\ell^2}{M_Z^2} \right] \Phi_1(z) - \frac{2\mathcal{N}^2}{[u(1-u)]^{1/3}} \left[ 1 - 2u + 2u^2 + (1 + \lambda)\frac{m_\ell^2}{M_Z^2} \right] \Phi'(z) \right\}. \tag{20}
\]
The probability of $Z^0$-boson decay is expressed in terms of the Airy functions $\Phi'(z)$ and $\Phi_1(z)$ (for necessary details concerning these special mathematical functions, see the Appendix) depending on the argument

$$z = \frac{m^2_\ell - M^2_Z u(1-u)}{M^2_Z [xu(1-u)]^{2/3}}. \tag{21}$$

The couplings of the $Z^0$-boson to the charged leptons of all three generations, $e^\pm, \mu^\pm,$ and $\tau^\pm,$ can be expressed in terms of the Weinberg angle $\theta_W$ and the electroweak coupling constant $g = e/\sin\theta_W$ as

$$g_V = -\frac{g(1-4\sin^2\theta_W)}{4\cos\theta_W}, \quad g_A = -\frac{g}{4\cos\theta_W}. \tag{22}$$

The dimensionless parameter $\lambda$ in expression (20) is given by

$$\lambda = \frac{g_A^2 - g_V^2}{g_A^2 + g_V^2} = \frac{1 - (1 - 4\sin^2\theta_W)^2}{1 + (1 - 4\sin^2\theta_W)^2}, \tag{23}$$

at $\sin^2\theta_W = 0, 23$, it is very close to unity ($\lambda \simeq 0.987$).

### 3. KINEMATICS OF $Z^0$-BOSON DECAY IN A CROSSED FIELD

The fact that the law of energy-momentum conservation for particles involved in the decay $Z^0 \rightarrow \ell^+\ell^-$ does not have a conventional form,

$$q_\mu + k_\mu = p_\mu + p'_\mu. \tag{24}$$

is a distinctive feature of processes occurring in an external electromagnetic field. Along with the $Z^0$-boson and final-lepton energy-momentum 4-vectors $q_\mu, p_\mu(\ell^-),$ and $p'_\mu(\ell^+),$ expression (24) also involves the wave vector $\mathbf{q}$ determining the energy contribution of the external field. It should be noted that, in this case, we are dealing with asymptotic momenta that charged particles possess at rather large distances, where the effect of the
external field is negligible, rather than with their dynamical 4-momenta in a crossed field. This is the meaning that the parameters \( p_\mu \) and \( p'_\mu \) appearing in the Volkov solutions (15) and (16) to the Dirac equation in a crossed field have. It is indeed extremely difficult to observe the kinematics of the decay \( Z^0 \to \ell^+\ell^- \) in the field itself because the charged-lepton 4-momenta are not integrals of the motion. As soon as the leptons escape from the region of the external field, their energies and momenta do not change any longer, so that one can measure them experimentally. In this case, the 4-momenta of all particles satisfy the ordinary kinematical conditions

\[
p^2 = p'^2 = m_\ell^2, \quad q^2 = M_Z^2, \quad k^2 = 0,
\]

and this makes it possible to represent the isotropic vector \( k^\mu \) in the form

\[
k^\mu = \frac{\Delta}{2(q_\alpha F_\alpha^\beta F_\beta^\sigma q^\sigma)} \cdot F_\mu^\lambda F_\lambda^\nu q^n,
\]

where the parameter \( \Delta \) characterizes the energy imbalance of the reaction \( Z^0 \to \ell^+\ell^- \) in the external field,

\[
\Delta = 2(qk) = (p + p')^2 - q^2 = 2m_\ell^2 + 2(p p') - M_Z^2
\]

Expression (24) arises in a natural way as the argument of the four-dimensional Dirac delta function in calculating the \( S \)-matrix element (1). In performing integration over the phase space of final leptons, there also occurs summation of the external-field contributions characterized by various values of the parameter \( \Delta \). This is the circumstance that distinguishes the kinematics of \( Z^0 \)-boson decay in an external field from a similar process involving a real photon, \( \gamma + Z^0 \to \ell^+\ell^- \), where the 4-momentum \( k_\mu \) is fixed and is independent of the final-lepton momenta. The physical meaning of the above formulas can most easily be understood in the \( Z^0 \)-boson rest frame, where the law of energy-momentum conservation in (24) can be expressed in terms of the following 4-vectors:

\[
q^\mu = (M_Z; 0), \quad p^\mu = (\varepsilon_1; p_1), \quad p'^\mu = (\varepsilon_2; p_2), \quad k^\mu = (k^0; k).
\]
We chose the axes of the three-dimensional Cartesian coordinate system in such a way that the vectors of the electric- and magnetic-field strength, $\mathbf{E}$ and $\mathbf{H}$, are directed along the $x$ and $y$ axes, respectively. The direction of the 3-momentum $\mathbf{k}$ formed by the spatial components of the wave 4-vector $k^\mu$ (26) then coincides with the positive direction of the $z$ axis, which is parallel to the vector product $[\mathbf{E} \times \mathbf{H}]$. In the Cartesian coordinate system chosen above, the orientation of the lepton ($\ell^-$) and antilepton ($\ell^+$) 3-momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ with respect to the $z$ axis can be described by two azimuthal angles $\vartheta_1$ and $\vartheta_2$ ($0 \leq \vartheta_{1,2} \leq \pi$). Figure 1 shows schematically the three-dimensional configuration formed by all of the aforementioned vectors.

If the decay $Z^0 \rightarrow \ell^+\ell^-$ occurs in the vacuum and if the external electromagnetic field is inoperative, the lepton and antilepton have momenta that are equal in absolute value but are oppositely directed and which
can be expressed in terms of their masses and the \( Z^0 \)-boson mass. The total energy taken away by the lepton-antilepton pair is shared between the particles in equal parts and its total amount is equal to the rest energy of the \( Z^0 \)-boson,

\[
p_1 = p_2 = \frac{M_Z}{2} \left( 1 - \frac{4m_e^2}{M_Z^2} \right)^{1/2}, \quad \varepsilon_1 = \varepsilon_2 = \frac{M_Z}{2}, \quad \vartheta_1 + \vartheta_2 = \pi. \tag{29}
\]

In a crossed field, each of the equalities in (29) is in general violated, and the energy \( \varepsilon_1 \) that is taken away in the decay by the lepton is not equal to the antilepton energy \( \varepsilon_2 \). Only fulfillment of the following conservation laws is guaranteed:

\[
\varepsilon_1 + \varepsilon_2 - M_Z = p_1 \cos \vartheta_1 + p_2 \cos \vartheta_2, \tag{30}
\]

\[
p_1 \sin \vartheta_1 - p_2 \sin \vartheta_2 = 0. \tag{31}
\]

The vectors \( p_1, p_2, \) and \( k \) lie in the same plane, but, in this case, it is not mandatory that the lepton and the antilepton fly apart along the same straight line, \( \vartheta_1 + \vartheta_2 \neq \pi \). The deficit in the ordinary law of energy-momentum conservation is compensated in an external field by the isotropic wave vector \( k^\mu \) (8).

Formula (30) represents the three-dimensional form of the conservation law of the covariant projections of the 4-momenta of all particles on to the direction of the wave vector \((qk) = (pk) + (p'k)\), while relation (31) is the conservation law for the ordinary three-dimensional projections of the particle momenta on to the axis that is orthogonal to the vector \( k \) and which lies in the plane spanned by the 3-vectors \( p_1, p_2, \) and \( k \).

We can now interpret physically the variable \( u \) appearing in the integrand on the right-hand side of (20) and in the argument of the Airy functions in (21). This variable is associated with the angles \( \vartheta_1 \) and \( \vartheta_2 \) of divergence of the lepton-antilepton pair with respect to the \( z \) axis in the
rest frame of the initial $Z^0$-boson:

$$u = \frac{(p_k)}{(q_k)} = \frac{\varepsilon_1 - p_1 \cos \vartheta_1}{M_Z} = 1 - \frac{\varepsilon_2 - p_2 \cos \vartheta_2}{M_Z}$$  \hspace{1cm} (32)$$

If the decay $Z^0 \to \ell^+ \ell^-$ is unaffected by an external field, then, by virtue of relations (29), the variable $u$ can be represented in the form

$$u = \frac{1}{2} \left[ 1 - \left( 1 - \frac{4m_\ell^2}{M_Z^2} \right)^{1/2} \cos \vartheta_1 \right] = \frac{1}{2} \left[ 1 + \left( 1 - \frac{4m_\ell^2}{M_Z^2} \right)^{1/2} \cos \vartheta_2 \right].$$  \hspace{1cm} (33)$$

The interval of its values admissible from the point of view of decay kinematics in a vacuum is determined by the inequality

$$u_1 \leq u \leq u_2, \quad \text{where} \quad u_{1,2} = \frac{1}{2} \left[ 1 \mp \left( 1 - \frac{4m_\ell^2}{M_Z^2} \right)^{1/2} \right].$$  \hspace{1cm} (34)$$

The limiting transition $\kappa \to 0$ in (20) reproduces exactly well-known results for the ordinary probability of the decay $Z^0 \to \ell^+ \ell^-$ without any field. The kinematically allowed region of values of the variable $u$ (34) corresponds to negative values of the argument of the Airy functions in (21); therefore, the limiting transition to zero values of the external-electromagnetic-field strength is implemented by means of the formal substitution of the Heaviside step $\theta$-function for the Airy function $\Phi_1(z)$:

$$\Phi_1(z) \to \pi \theta(-z)$$  \hspace{1cm} (35)$$

In the limit $\kappa \to 0$, the differential probability of $Z^0$-boson decay with respect to the variable $u$ does not depend on the direction of divergence of leptons and has the form

$$\frac{dP}{du}(Z^0 \to \ell^- \ell^+) = \frac{M_Z^2}{12\pi q_0} \left[ g_V^2 + g_A^2 + \frac{2m_\ell^2}{M_Z^2}(g_V^2 - 2g_A^2) \right]$$  \hspace{1cm} (36)$$

As a matter of fact, integration with respect to the invariant variable $u$ now reduces to multiplying expression (36) by the phase space of this
variable, \((u_2 - u_1)\). As a result, we obtain the well-known formula for the leptonic-decay width of the \(Z^0\)-boson in a vacuum,

\[
\Gamma(Z^0 \to \ell^+ \ell^-) = \frac{G_F M_Z^3}{12\pi \sqrt{2}} \sqrt{1 - 4\delta^2_\ell} \left(1 - c_\ell - \delta^2_\ell(1 + 2c_\ell)\right),
\]

where we have introduced the notation

\[
\delta_\ell = \frac{m_\ell}{M_Z}, \quad c_\ell = 4\sin^2\theta_W - 8\sin^4\theta_W \approx 0, 497
\]  

(37)

In a crossed electromagnetic field, the kinematics of \(Z^0\)-boson decay changes, first of all, owing to the increase in the phase space of possible states of final leptons \(\ell^\pm\). In the three-dimensional space of the momenta \((p_x, p_y, p_z)\) of one of the leptons — for example, \(\ell^-\) — the kinematically allowed region of the decay \(Z^0 \to \ell^+ \ell^-\) in a vacuum [see (29)] can be represented as a three-dimensional sphere, each point of this sphere corresponding to one of the final states of this lepton. In an external electromagnetic field, all possible final states of the lepton \(\ell^-\) already fill some volume in momentum space. At a fixed value of the azimuthal angle \(\vartheta_1\), the momentum of the lepton \(\ell^-\) can take any values from the interval

\[
p_{\text{max}}(\vartheta) = \frac{M_Z}{\sin^2 \vartheta} \left(\cos \vartheta + \sqrt{1 - \delta^2_\ell \sin^2 \vartheta}\right),
\]

(39)

The energy \(\varepsilon_1\) of this particle will be bounded by the inequality

\[
0 \leq p_1 < p_{\text{max}}(\vartheta_1), \quad \text{where} \quad p_{\text{max}}(\vartheta) = \frac{M_Z}{\sin^2 \vartheta} \left(\cos \vartheta + \sqrt{1 - \delta^2_\ell \sin^2 \vartheta}\right).
\]

The energy \(\varepsilon_1\) and \(\vartheta_1\) determine unambiguously decay kinematics in a crossed field. The energy \(\varepsilon_2\) and the azimuthal angle \(\vartheta_2\) of the antilepton \(\ell^+\) can be expressed in terms of these variables with the aid of the kinematical relations (30) and (31) and the standard relativistic relations

\[
\varepsilon_1 = \sqrt{p^2_1 + m^2_\ell}, \quad \varepsilon_2 = \sqrt{p^2_2 + m^2_\ell}.
\]

(41)
After some simple algebra, we obtain

\[ \varepsilon_2 = M_Z - \varepsilon_1 + \frac{M_Z (\varepsilon_1 - M_Z / 2)}{M_Z - \varepsilon_1 + p_1 \cos \vartheta_1} \quad (42) \]

\[ \vartheta_2 = \arccos \left( \frac{\varepsilon_1 + \varepsilon_2 - M_Z - p_1 \cos \vartheta_1}{\sqrt{\varepsilon_2^2 - m_\ell^2}} \right) \quad (43) \]

An analysis of the resulting formulas shows that, as a rule, the leptons \( \ell^\pm \) diverge at arbitrary angles \( \vartheta_1 \) and \( \vartheta_2 \) with respect to the z axis. This is true for almost all of the azimuthal angles, with the exception of the case where the lepton and antilepton move in the direction parallel to the z axis. In the case of \( \sin \vartheta_1 \neq 0 \) and \( \sin \vartheta_2 \neq 0 \), the lepton and antilepton energies \( \varepsilon_1 \) and \( \varepsilon_2 \), respectively, can be expressed in terms of the emission angles \( \vartheta_1 \) and \( \vartheta_2 \) as

\[ \varepsilon_1 \approx p_1 \approx \frac{M_Z \sin \vartheta_2}{\sin \vartheta_1 + \sin \vartheta_2 - \sin(\vartheta_1 + \vartheta_2)} \quad (44) \]

\[ \varepsilon_2 \approx p_2 \approx \frac{M_Z \sin \vartheta_1}{\sin \vartheta_1 + \sin \vartheta_2 - \sin(\vartheta_1 + \vartheta_2)} \quad (45) \]

These relations were obtained in the relativistic approximation, where \( \varepsilon_1 \gg m_\ell, \varepsilon_2 \gg m_\ell \) and where the parameter \( \delta_\ell \) (38) can be disregarded.

We can now draw qualitative conclusions on a typical behavior of the energies of the lepton-antilepton pair almost over the entire azimuthal-angle range \( 0 < \vartheta_{1,2} < \pi \). It was indicated above that, at a fixed value of the emission angle \( (\vartheta_1 = \text{const}) \) of the lepton \( \ell^- \), its energy can vary from the rest energy to the maximum value \( \varepsilon_{\text{max}}(\vartheta_1) \) (40). Concurrently, the emission angle \( \vartheta_2 \) of the antilepton \( \ell^+ \) can take an arbitrary value between zero and \( \pi \). As for its energy \( \varepsilon_2 \), one can see that, as the energy \( \varepsilon_1 \) increases, \( \varepsilon_2 \) first decreases from the value

\[ \varepsilon_0 = \frac{M_Z}{2} \left( 1 - 2\delta_\ell + 2\delta_\ell^2 \right) \approx \frac{M_Z}{2} (1 - \delta_\ell) \quad (46) \]

to its local minimum

\[ \varepsilon_2 = \frac{M_Z \cos (\vartheta_1/2)}{1 + \cos (\vartheta_1/2)}, \quad (47) \]
Figure 2: Lepton energy $\varepsilon_1$ as a function of the antilepton emission angle $\vartheta_2$ with respect to the $z$ axis at $\vartheta_1 = 2\pi/3$.

whereupon it increases monotonically to infinity as the angle $\vartheta_2$ tends to zero, while the lepton energy $\varepsilon_1$ tends to its maximum value $\varepsilon_{\text{max}}(\vartheta_1)$. The above relationship between the energies of the lepton-antilepton pair and the directions of lepton and antilepton emission with respect to the $z$ axis is illustrated by the graphs in Figs. 2 and 3. It should be noted that the minimum value of the antilepton energy $\varepsilon_2$ (47) is reached at the emission angle of $\vartheta_2 = \pi - \vartheta_1/2$, in which case the energy of the other particle, $\ell^-$, is

$$\varepsilon_1 = \frac{M_Z/2}{1 + \cos (\vartheta_1/2)}. \quad (48)$$

If the lepton escapes in the direction parallel to the $z$ axis, expressions (44) and (45) are inapplicable, so that one must use the exact kinematical relations (30) and (31) or their consequences (42) and (43). In this case, two situations are possible — that in which the leptons fly apart in opposite
directions and that in which the leptons move in the same direction.

In the first case (at $\vartheta_1 = 0$ and $\vartheta_2 = \pi$ or at $\vartheta_1 = \pi$ and $\vartheta_2 = 0$), the energy of the lepton moving along the positive direction of the $z$ axis can take any value from the rest energy $\varepsilon_{\text{min}} = m_\ell$ to infinity. The energy of the second lepton, which flies in the opposite direction, is virtually constant — it ranges between the initial value $\varepsilon_0$ (46) and its maximum value

$$\varepsilon_\pi = \lim_{\vartheta \to \pi} \varepsilon_{\text{max}}(\vartheta) = \frac{M_Z}{2} \left(1 + \delta_\ell^2\right). \tag{49}$$

which is very close to $\varepsilon_0$. The case where both particles $\ell^+$ and $\ell^-$ produced in $Z^0$-boson decay move in the same direction opposite to the positive direction of the $z$ axis ($\vartheta_1 = \vartheta_2 = \pi$) is also possible. In this case, their energies $\varepsilon_1$ and $\varepsilon_2$ can vary from the rest energy to the limiting value $\varepsilon_0$ (46). In addition, the following relation can be obtained for this case from

Figure 3: Antilepton energy $\varepsilon_2$ versus the antilepton emission angle $\vartheta_2$ with respect to the $z$ axis at a fixed value of the lepton emission angle ($\vartheta_1 = 2\pi/3$).
in the relativistic approximation (for $\delta_\ell \to 0$):

$$\varepsilon_1 + \varepsilon_2 = \frac{M_Z}{2}. \quad (50)$$

Thus, one can see that, in relation to the kinematics of the process $Z^0 \to \ell^+\ell^-$ in a vacuum [see (29)], the physics of $Z^0$-boson decay in an external electromagnetic field is richer in the number of possible final lepton states. From the mathematical point of view, this manifests itself in the fact that all values of the invariant variable $u$ (32) from the domain $0 < u < 1$ are now admissible. Moreover, it is noteworthy that, in a crossed field, the physical meaning of this variable changes because it now characterizes two independent kinematical quantities simultaneously - for example, $\varepsilon_1$ and $\vartheta_1$. Disregarding the cases in which leptons fly apart in the direction parallel to the $z$ axis, we can express the variable $u$ in terms of the azimuthal angles $\vartheta_1$ and $\vartheta_2$ alone by using formulas (44) and (45). The result is

$$u \simeq \frac{\tan(\vartheta_1/2)}{\tan(\vartheta_1/2) + \tan(\vartheta_2/2)}. \quad (51)$$

An analysis of the integrand on the right-hand side of (20) shows that, in an external field, not all of the directions of divergence of the lepton and antilepton are equiprobable. The graphs in Fig. 4 that represent the dependence of the differential decay width of the $Z^0$-boson on the invariant variable $u$ corroborate that, even in a relatively weak external field, there are noticeable deviations from a uniformly distributed probability of the leptonic decay mode in a vacuum [see (36)]. In the case where leptons produced in the process $Z^0 \to \ell^+\ell^-$ fly apart at specific angles ($\vartheta_1, 2 \neq 0, \pi$) in diametrically opposite directions ($\vartheta_1 + \vartheta_2 \approx \pi$), the invariant variable $u$ takes values of $u \approx 0, 5$. In this region, which is kinematically allowed for $Z^0$-boson decay in a vacuum, the deviations of the differential decay width $\Gamma'_u(u, z)$ from the corresponding static value of $\Gamma'_u(u, 0) \approx 83$ MeV are in fact unobservable. If one of the leptons flies away at a specific angle $\vartheta$ with
Figure 4: Differential width with respect to the decay $Z^0 \rightarrow \ell^- \ell^+$ as a function of the invariant angular variable $u$ (32) in a weak crossed field (at $\kappa = 0.01$)

respect to the $z$ axis and at an energy close to the maximum value $\varepsilon_{\text{max}}(\vartheta)$ in (40) and if the second lepton moves along the direction nearly parallel to this axis (in the positive direction), the variable $u$ takes values in the region around $u \approx 0$ or $u \approx 1$. In this case, one can observe oscillations of the differential decay width of the $Z^0$-boson that are characterized by a growing amplitude, which increases and decreases the static vacuum value in (36) by more than 15%. Similar oscillations of probabilities of quantum processes in an external electromagnetic field are characteristic of all particle transformations allowed in a vacuum and were described in detail in the literature [18,19].
4. LEPTONIC DECAY WIDTH OF THE $Z^0$-BOSON

The integral representation of the partial width of the $Z^0$-boson with respect to its decay to a pair of charged leptons in an external electromagnetic field can be derived from expression (20) for the probability by going over to the rest frame ($q_0 = M_Z$) and by making the change of the integration variable $x = u(1 - u)$. The result is

$$
\Gamma(Z^0 \to \ell^+ \ell^- | \kappa) = \frac{(g_\ell^2 + g_\lambda^2) M_Z}{6\pi^2} \int_0^{1/4} \frac{dx}{\sqrt{1 - 4x}} \left\{ \left[ 1 - \delta_\ell^2 (1 + 3\lambda) \right] \Phi_1(z) - 2 \left( \frac{\kappa^2}{x} \right)^{1/3} \left[ 1 - 2x + \delta_\ell^2 (1 + \lambda) \right] \Phi'(z) \right\},
$$

(52)

where the argument of the Airy functions is now determined by the expression

$$
z = \frac{\delta_\ell^2 - x}{(\kappa x)^{2/3}}.
$$

(53)

Let us now consider the asymptotic estimates obtained from this formula at various values of the external-field-strength parameter $\kappa$ (12).

In weak electromagnetic fields such that $\kappa \ll \delta_\ell^2 \ll 1$, the partial decay width of the $Z^0$-boson can be written as the sum of two terms,

$$
\Gamma(Z^0 \to \ell^+ \ell^- | \kappa) = \Gamma(Z^0 \to \ell^+ \ell^-) + \Delta \Gamma(\kappa).
$$

(54)

The first term in this expression is the $Z^0$-boson width with respect to the leptonic decay $Z^0 \to \ell^+ \ell^-$ in a vacuum [see expression (37) above]. The experimental value of this quantity is known to a precision of 0.1%: $\Gamma(Z^0 \to \ell^+ \ell^-) = 83,984 \pm 0,086$ MeV [2]. The second term characterizes the effect of the external electromagnetic field. In the leading order of the expansion in the small parameters $\kappa$ and $\delta_\ell$, this term can be represented.
in the form

\[
\Delta \Gamma(\varkappa) = -\frac{G_F M_Z^3}{4\pi \sqrt{6}} \left[ \delta_\ell^2 (1 - 6c_\ell) \varkappa \cos \left( \frac{1}{3\varkappa} \right) - \right.

\left. (1 - c_\ell) \varkappa^2 \sin \left( \frac{1}{3\varkappa} \right) + \frac{8\varkappa^2}{\sqrt{3}(1 - c_\ell)} \right] \quad (55)
\]

Similar calculations of the probability of \(Z^0\)-boson decay in a magnetic field were performed in \([25]\), where the contribution of the virtual electron-positron loop to the amplitude for elastic \(Z^0\)-boson scattering was considered, its imaginary part being directly related to the \(Z^0 \to \ell^+\ell^-\) decay width being studied. It is worth noting, however, that the results of the present calculations [expressions \((52)\) and \((55)\)] differ somewhat from their counterparts in \([25]\).

For the parameter of the external-electromagnetic-field strength, we will now consider the region specified by the condition \(\varkappa \gg \delta_\ell^2\). We use the fact that the masses of the leptons of all three generations are much smaller than the \(Z^0\)-boson mass and go over to the limit \(\delta_\ell \to 0\) in formulas \((52)\) and \((53)\). We can then calculate analytically the integral of the Airy function on the right-hand side of \((52)\) at arbitrary values of the external-field-strength parameter \(\varkappa\). The final expression for the leptonic-decay width of the \(Z^0\)-boson is written in terms of the Bessel functions \(J_\nu(w)\) carrying the noninteger indices \(\nu = \pm 1/6\) and \(\nu = \pm 5/6\) and depending on the argument \(w = (6\varkappa)^{-1}\). We now consider a universal function \(R(\varkappa)\) that describes the degree of influence of the external field on the leptonic mode of \(Z^0\)-boson decay:

\[
R(\varkappa) = \frac{\Gamma(Z^0 \to \ell^+\ell^- | \varkappa)}{\Gamma(Z^0 \to \ell^+\ell^-)}. \quad (56)
\]

In the massless-lepton approximation \((\varkappa \gg \delta_\ell^2)\), this function, which is a relative width of the \(Z^0\)-boson with respect to the decay \(Z^0 \to \ell^+\ell^-\) in an
Figure 5: Oscillations of the partial width of the $Z$-boson with respect to its decay to a pair of charged leptons in a weak electromagnetic field.

electromagnetic field, can be represented in the form

$$R(\kappa) = \frac{1}{3} \left[ 1 + R_1(\kappa) + R_2(\kappa) + R_3(\kappa) \right],$$

where the following notation has been introduced:

$$R_1(\kappa) = \frac{\pi (1 + 3\kappa^2)}{12\kappa} \left[ J_{1/6}^2 \left( \frac{1}{6\kappa} \right) + J_{-1/6}^2 \left( \frac{1}{6\kappa} \right) \right],$$

$$R_2(\kappa) = \frac{\pi (1 - 15\kappa^2)}{12\kappa} \left[ J_{5/6}^2 \left( \frac{1}{6\kappa} \right) + J_{-5/6}^2 \left( \frac{1}{6\kappa} \right) \right],$$

$$R_3(\kappa) = \frac{\pi (2 + 15\kappa^2)}{6} \left[ J_{1/6} \left( \frac{1}{6\kappa} \right) J_{-1/6} \left( \frac{1}{6\kappa} \right) - J_{-5/6} \left( \frac{1}{6\kappa} \right) J_{5/6} \left( \frac{1}{6\kappa} \right) \right].$$
The graphs representing the dependence of the relative leptonic-decay width (56) normalized to unity at zero external field on the invariant field-strength parameter \( \kappa \) (9) are on display in Figs. 5, 6, 7. In relatively weak fields (such that \( \kappa < 0.06 \)), there arise oscillations of the probability of the decay \( Z^0 \to \ell^+\ell^- \), their amplitude increasing quadratically with the parameter \( \kappa \). In this region, one can use the asymptotic expansion for the Bessel functions and approximate formula (57) by the expression

\[
R(\kappa) = 1 - \frac{16}{3} \kappa^2 + \frac{3\kappa^2}{\sqrt{3}} \sin\left(\frac{1}{3\kappa}\right) + \frac{65\kappa^3}{\sqrt{3}} \cos\left(\frac{1}{3\kappa}\right) + \\
\quad + \frac{880\kappa^4}{3\sqrt{3}} \sin\left(\frac{1}{3\kappa}\right) - \frac{160}{3} \kappa^4 + \ldots \quad (61)
\]
Figure 7: Leptonic-decay width of the $Z^0$-boson as a function of the external-field-strength parameter $\kappa$ in the vicinity of the absolute-minimum point.

It is obvious from Figs. 5 and 6 that the maximum deviation of the decay width oscillating in an external field from its vacuum value in (37) does not exceed 2%. A further increase in the parameter of the external-electromagnetic-field strength to values in the region $\kappa > 0.06$ leads to a complete disappearance of oscillations and a monotonic decrease in the relative width $R(\kappa)$ down to the absolute minimum $R_{\text{min}} = 0.843$ in the vicinity of the point $\kappa_{\text{min}} = 0.213$ (see Fig. 7). In the region of strong fields (such that $\kappa > 0.213$), the relative width of the $Z^0$-boson with respect to its leptonic decay mode grows monotonically with the field strength. In the region $\kappa \gg 1$, this growth can be approximated by the asymptotic...
expression.

\[ R(\kappa) = \frac{15 \Gamma^4(2/3)}{14\pi^2} (3\kappa)^{2/3} + \frac{1}{3} + \frac{3 \Gamma^4(1/3)}{110\pi^2 (3\kappa)^{2/3}} + \ldots \]  

(62)

5. HADRONIC MODES OF Z\(^0\)-BOSON DECAY

The results obtained in the preceding section admit a straightforward generalization to the case of the decay \(Z^0 \to \bar{f}f\), where, by \(f\), we denote an arbitrarily charged fermion, \(q_f\) standing for the absolute value of its electric charge. As is well known, the charge is \(q_f = 2/3\) for the up quarks \((u, c, t)\) and \(q_f = 1/3\) for the down quarks \((d, s, b)\). It is necessary to consider that, for the \(Z \bar{f}f\) vertex, the coupling constants \(g_V\) and \(g_A\) also depend on the absolute value of the electric charge \((q_f)\). In the Glashow-Weinberg-Salam model of electroweak interactions, they are determined by the relations

\[ |g_V(f)| = \frac{g(1 - 4q_f \sin^2 \theta_W)}{4 \cos \theta_W}, \quad |g_A(f)| = \frac{g}{4 \cos \theta_W}. \]  

(63)

Upon making the formal substitution \(e \to q_f e\) or \(\kappa \to q_f \kappa\) in (20), the differential \(Z^0 \to \bar{f}f\) decay width as a function of the invariant variables \(\kappa\) (9) and \(u\) (32) assumes the form

\[ \frac{d\Gamma}{d\kappa}(Z^0 \to \bar{f}f) = \frac{G_F M_Z^3}{12\pi^2 \sqrt{2}} \left\{ [1 - c_f - \delta_f^2 (1 + 2c_f)] \Phi_1(z_f) - 2 \left[ \frac{q_f^2 \kappa^2}{u(1 - u)} \right]^{1/3} \left[ (1 - c_f)(1 - 2u + 2u^2) + \delta_f^2 \right] \Phi'(z_f) \right\}. \]  

(64)

In this expression, use is made of the same notation as in (21) and (38):

\[ \delta_f = \frac{m_f}{M_Z}, \quad c_f = 4q_f \sin^2 \theta_W (1 - 2q_f \sin^2 \theta_W), \]  

(65)

\[ z_f = \frac{\delta_f^2 - u(1 - u)}{[q_f \kappa u(1 - u)]^{2/3}}. \]  

(66)
The limiting transition to an infinitely weak external field ($\kappa \to 0$) leads to reproducing the well-known results for the partial widths of the $Z^0$-boson with respect to its decay to light quarks; that is,

$$\Gamma(Z^0 \to \bar{u}u) = \frac{G_F M_Z^3}{4\pi \sqrt{2}} \left[ 1 - \frac{8}{3} \sin^2 \theta_W + \frac{32}{9} \sin^4 \theta_W \right].$$  \hspace{1cm} (67)

$$\Gamma(Z^0 \to \bar{d}d) = \frac{G_F M_Z^3}{4\pi \sqrt{2}} \left[ 1 - \frac{4}{3} \sin^2 \theta_W + \frac{8}{9} \sin^4 \theta_W \right].$$  \hspace{1cm} (68)

These relations are obtained from (37) upon the formal substitution $c_\ell \to c_f$ and $\delta_\ell \to \delta_f$ and the multiplication by the common factor $N_c = 3$ (number of color quark states). The inclusion of radiative corrections leads to various modifications of expressions (67) and (68), whose explicit form depends on the choice of input parameters in electroweak theory and renormalization scheme for divergent loop diagrams [4, 7]. For example, Sirlin, who was the author of one of the pioneering studies in these realms [26], proposed the parametrization

$$G_F = \frac{\pi \alpha}{\sqrt{2} M_W^2 \sin^2 \theta_W (1 - \Delta r)} \frac{1}{\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}},$$  \hspace{1cm} (69)

which has been widely used since then. Here, $\Delta r$ includes all radiative corrections. For the sake of simplicity, we restrict ourselves to the case of massless quarks ($\delta_f \ll \kappa$) and, without going in to details of the calculation of radiative corrections, try to analyze the effect of external electromagnetic fields on the hadronic decay modes of the $Z^0$-boson. Without allowance for the external-field effect, the width

$$\Gamma_{had} = \sum_{i=1}^{5} \Gamma(Z^0 \to \bar{q}_i q_i) \simeq 2\Gamma(Z^0 \to \bar{u}u) + 3\Gamma(Z^0 \to \bar{d}d)$$

agrees with the experimentally measured value of $\Gamma(Z^0 \to \text{hadrons}) = 1744, 4 \pm 2, 0$ MeV [2] to a high degree of precision. We use this approximation in the region of relatively strong electromagnetic fields ($\kappa \sim 1$)
and estimate the width of the $Z^0$-boson with respect to the hadronic decay mode as

$$\Gamma_{\text{had}}(\kappa) = 2\Gamma(Z^0 \to \bar{u}u) R\left(\frac{2}{3} \kappa\right) + 3\Gamma(Z^0 \to \bar{d}d) R\left(\frac{1}{3} \kappa\right),$$

(70)

where the function $R(\kappa)$ is determined by expressions (56) – (60). Figure 8 shows the results of numerical calculations performed for the total width of the $Z^0$-boson with respect to hadronic decay modes in an external field on the basis of expression (70) with allowance for the production of only five quarks ($u, d, s, c,$ and $b$). In this region, the contribution of the heavy $t$ quark is insignificant. This contribution will be considered individually in the next section.

One can see from Fig. 8 that, in just the same way as the width with
respect to leptonic decay modes (see Fig. 7), the width with respect to the hadronic modes of $Z^0$-boson decay in an intense external field behaves nonmonotonically. As the parameter $\kappa$ increases, the width $\Gamma_{\text{had}}(\kappa)$ first decreases, reaching the absolute minimum of $\Gamma_{\text{had}}(\kappa_{\text{min}}) = 1497$ MeV at the point $\kappa_{\text{min}} = 0.501$, where upon it begins growing. In a weak electromagnetic field ($\kappa \ll 1$), the gradual decrease in the hadronic-decay width $\Gamma_{\text{had}}(\kappa)$ is accompanied by quite complicated oscillations.

6. TOTAL DECAY WIDTH OF THE $Z^0$-BOSON AND $t$-QUARK CONTRIBUTION

In a vacuum, $Z^0$-boson decay to $t$ quarks via the process $Z^0 \rightarrow \bar{t}t$ is forbidden by the energy-conservation law because the mass of the heavy $t$ quark is rather high, $m_t = 174.2 \pm 3.3$ GeV [2]. The presence of an external electromagnetic field removes this forbiddance, opening new channels of $Z^0$-boson decay. In principle, the production of arbitrary charged fermions whose masses satisfy the relation $\delta_f = m_f/M_Z > 1/2$ becomes possible. The missing energy for these reactions comes from the external field, and this resembles tunneling through a potential barrier in nonrelativistic quantum mechanics. At small values of the electromagnetic-field strength (such that $\kappa \ll 1$), the $Z^0 \rightarrow \bar{f}f$ decay width is exponentially small:

$$\Gamma(Z^0 \rightarrow \bar{f}f) = \frac{G_F M_Z^3}{8\pi \sqrt{6}} \left(8\delta_f^2 + 1 - 6c_f\right) \delta_f^2 q_f \exp \left[-\frac{(4\delta_f^2 - 1)^{3/2}}{3q_f \kappa}\right]. \quad (71)$$

This formula was derived from expression (64) upon integration with respect to the invariant variable $u$ (32) in the approximation $z_f \gg 1$ (66). Substituting the $t$-quark charge $q_f = 2/3$ and performing summation over three color states, we obtain an asymptotic estimate for the partial width
of the $Z^0$-boson with respect to its decay to $t$-quarks in a relatively weak electromagnetic field; that is,

$$\Gamma(Z^0 \rightarrow \bar{t}t) = \frac{G_F m_t^2 M_Z^2}{4\pi \sqrt{6}} \left[8m_t^2 + (1 - 16\sin^2\theta_W + \frac{64}{3}\sin^4\theta_W) M_Z^2\right] \times$$

$$\Gamma \left[\frac{4m_t^2 - M_Z^2}{2\kappa M_Z^3} \right]^3. (72)$$

The results of numerical calculations show that the above approximate formula for $\Gamma(Z^0 \rightarrow \bar{t}t)$ leads to an error not greater than 4% for all values of the external-field-strength parameter from the region $\kappa \leq 5$. As the parameter $\kappa$ grows, the deviation of the results obtained according to the asymptotic formula (72) from the precise value of the partial width of the $Z^0$-boson with respect to its decay to $t$-quarks in an external electromagnetic field (for the description of the curves, see the main body of the text).
$Z^0$-boson with respect to the decay $Z^0 \rightarrow \bar{t}t$ in an external field increases [see Fig.9, where the dotted curve corresponds to the calculation according to formula (72), while the solid curve represents the results of a numerical integration of expression (64)].

The $t$-quark contribution to the total decay width of the $Z^0$-boson remains negligible (less than 1%) up to strength-parameter values of about $\varkappa = 6, 4$. Figure 10 displays the total decay width of the $Z^0$-boson as a function of the parameter $\varkappa$ in the region of strong fields ($\varkappa \leq 10$). The solid curve corresponds to the total decay width of the $Z^0$-boson with allowance for the $t$-quark contribution, while the dotted curve represents the same quantity calculated without taking into account the $t$-quark contribution. One can clearly see a trend toward the growth of both the absolute
value of the total decay width of the $Z^0$-boson and the fraction of the $t$-quark contribution in this decay width. As the external-field strength increases, the process $Z^0 \to \bar{t}t$, which is forbidden in a vacuum, becomes dominant in superstrong electromagnetic fields. By way of example, it can be indicated that, in the $Z^0$-boson decay width at $\kappa = 100$, the fraction associated with $t$-quarks is as large as 50%. This circumstance clearly demonstrates how an external electromagnetic field can change drastically the standard physics of quantum processes in a vacuum and serve as some kind of a catalyst for a number of new and nontrivial phenomena.
7. CONCLUSIONS

The effect of a strong electromagnetic field on the probability of $Z^0$-boson decay has been investigated. The present calculations have been performed within the crossed-field model, which makes it possible to trace characteristic variations in the modes of $Z^0$-boson decay to known leptons and quarks. We have found that, in the region of relatively strong fields ($\varkappa \sim 1$), all partial decay widths $\Gamma(Z^0 \to \bar{f}f)$ decrease in the same manner by about 12 to 15%. This decrease can be described by a universal function $R(\varkappa)$ [see (56)–(60)] depending only on the energy-momentum of the $Z^0$-boson and on the external-electromagnetic-field strength. This circumstance becomes quite obvious if we consider that all known fermions observed in $Z^0$-boson decays have masses at least an order of magnitude smaller than the $Z^0$-boson mass. Therefore, it comes as no surprise that, in the region of strong electromagnetic fields, the massless-fermion approximation works well, also making it possible to calculate the dependence of the total decay width $\Gamma_Z(\varkappa)$ of the $Z^0$-boson on the external-field-strength parameter $\varkappa$ [9].

The results of the present calculations have revealed that, in just the same way as in the case of the $W$-boson [23], the total decay width of the $Z^0$-boson in a strong electromagnetic field depends nonmonotonically on the strength parameter $\varkappa$, this dependence featuring the point of a local minimum, $\Gamma_Z(\varkappa_{\text{min}}) = 2,164$ GeV, at $\varkappa_{\text{min}} = 0,445$. The graph in Fig. 11, which represents the behavior of $\Gamma_Z(\varkappa)$ in the region $\varkappa \leq 1$, illustrates this observation.

For the sake of comparison, it can be indicated that the analogous $W$-boson-decay process $W^- \to \ell \bar{\nu}_\ell$ in an external field is also characterized by a nonmonotonic dependence of the decay width on the parameter $\varkappa$ with the point of a local minimum at $\varkappa_W = 0,6112$ [23].
Figure 11: Minimal value of the total decay width of the $Z^0$-boson in an external field.

Upon rescaling this value to the $Z^0$-boson energy scale, it corresponds to $\kappa = \kappa_W (M_W/M_Z)^3 = 0.419$. Thus, the probabilities for the decays of gauge bosons in an external field, $Z^0 \rightarrow \bar{f} f$ and $W^- \rightarrow \ell \bar{\nu}_\ell$, take their minimum values at nearly the same field strength. This agreement is not accidental since the two reactions in question proceed under similar kinematical conditions and since the masses of the $W^\pm$ and $Z^0$ bosons are almost indistinguishable in order of magnitude. It is also intriguing that, in the region around $\kappa \sim 1$, the presence or the absence of the initial-gauge-boson electric charge is immaterial because the change in the decay width is due primarily to the increase in the phase space of final particles.

In the region of superstrong fields ($\kappa \geq 10$), a sizable contribution to the total decay width of the $Z^0$-boson comes from the $t$-quark-production
process $Z^0 \to \bar{t}t$, which is forbidden in a vacuum by conservation laws. As the external-field strength increases, this reaction becomes dominant in relation to other modes of $Z^0$-boson decay, this being due to a very large mass of the $t$-quark. The external field serves as some kind of a catalyst for super heavy-particle-production processes, which cannot occur under ordinary conditions. In view of the aforesaid, it would be of interest to discuss searches for new (as-yet-undiscovered) hypothetical particles (supersymmetry and so on) among new $Z^0$-boson decay modes that arise in an external electromagnetic field. Although external-field strengths necessary for directly observing such reactions are not yet available in experiments (see, for example, [21]), these investigations are of course very interesting from the point of view of fundamental principles of physical theory.
In the present study, we have employed the special mathematical functions $\Phi(z)$, $\Phi'(z)$ and $\Phi_1(z)$ generically referred to as Airy functions. The function $\Phi(z)$ is a particular solution to a second-order linear differential equation for specific initial conditions:

$$\Phi''(z) - z\Phi(z) = 0, \quad \Phi(0) = \frac{\pi}{3^{2/3} \Gamma(2/3)}, \quad \Phi'(0) = -\frac{\pi}{3^{1/3} \Gamma(1/3)}.$$  \hspace{1cm} (A.1) 

It has the well-known integral representation

$$\Phi(z) = \int_0^\infty \cos \left(zt + \frac{t^3}{3}\right) dt.$$ \hspace{1cm} (A.2) 

Two other functions $\Phi'(z)$ and $\Phi_1(z)$ can be expressed in terms of this function as

$$\Phi_1(z) = \int_z^\infty \Phi(t) dt, \quad \Phi'(z) = \frac{d\Phi(z)}{dz}.$$  \hspace{1cm} (A.3) 

At small values of the argument $z$ ($z \ll 1$), the Airy function $\Phi(z)$ can be calculated on the basis of the following expansion in a numerical series:

$$\Phi(z) = \frac{1}{3^{2/3}} \sum_{n=0}^\infty \Gamma\left(n + \frac{1}{3}\right) \sin\left(\frac{2\pi}{3} + \frac{2\pi n}{3}\right) \frac{3^{n/3}}{n!} \cdot z^n.$$  \hspace{1cm} (A.4) 

At large values of the argument ($z \gg 1$), the Airy function can be evaluated by using the asymptotic expressions

$$\Phi(z) = \frac{1}{2z^{1/4}} \exp \left( -\frac{2}{3} z^{3/2} \right) \sum_{n=0}^\infty (-1)^n \frac{\Gamma(3n + 1/2)}{(2n)!} \frac{1}{9^n} \cdot \frac{1}{z^{3n/2}},$$ \hspace{1cm} (A.5) 

$$\Phi(-z) = \frac{1}{z^{1/4}} \sum_{n=0}^\infty \sin \left( \frac{2}{3} z^{3/2} + \frac{\pi}{4} - \frac{\pi n}{2} \right) \frac{\Gamma(3n + 1/2)}{(2n)!} \frac{1}{9^n} \cdot \frac{1}{z^{3n/2}}.$$  \hspace{1cm} (A.6) 

The properties of the Airy functions are well known and can be found in mathematical handbooks (see, for example, [27]).
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