We present a comprehensive study of white dwarf collisions as an avenue for creating type Ia supernovae. Using a smooth particle hydrodynamics code with a 13-isotope, α-chain nuclear network, we examine the resulting 56Ni yield as a function of total mass, mass ratio, and impact parameter. We show that several combinations of white dwarf masses and impact parameters are able to produce sufficient quantities of 56Ni to be observable at cosmological distances. We find that the 56Ni production in double-degenerate white dwarf collisions ranges from sub-luminous to the super-luminous, depending on the parameters of the collision. For all mass pairs, collisions with small impact parameters have the highest likelihood of detonating, but 56Ni production is insensitive to this parameter in high-mass combinations, which significantly increases their likelihood of detection. We also find that the 56Ni dependence on total mass and mass ratio is not linear, with larger-mass primaries producing disproportionately more 56Ni than their lower-mass secondary counterparts, and symmetric pairs of masses producing more 56Ni than asymmetric pairs.

Key words: hydrodynamics – nuclear reactions, nucleosynthesis, abundances – supernovae: general – white dwarfs

Online-only material: color figures

1. INTRODUCTION

While the preferred mechanism for type Ia supernovae (SNe Ia) involves a single white dwarf star accreting material from a non-degenerate companion (Whelan & Iben 1973; Nomoto 1982; Hillebrandt & Niemeyer 2000), recent observational evidence suggests that a non-negligible fraction of observed SNe Ia may derive from double-degenerate progenitor scenarios. Scalzo et al. (2010) observed the supernova SN 2007if photometrically, and assuming no host galaxy extinction, they found 1.6 ± 0.1 $M_\odot$ of 56Ni with 0.3–0.5 $M_\odot$ of unburned carbon and oxygen forming an envelope. This 56Ni yield implies a progenitor mass of 2.4 ± 0.2 $M_\odot$, which is well above the Chandrasekhar limit—the maximum mass for a non-rotating white dwarf (Chandrasekhar 1931; Pfannes et al. 2010; Yoon & Langer 2004, 2005). It follows that two white dwarfs must have been involved in the event that produced SN 2007 if, since a single white dwarf cannot accrete enough material to reach this mass without either exploding as an SN Ia or collapsing to form a neutron star (Yoon et al. 2007). Furthermore, spectroscopic observations by Tanaka et al. (2010) suggest SN 2009dc produced $\gtrsim 1.2 M_\odot$ of 56Ni, depending on the assumed dust absorption. This also implies a progenitor mass $\sim 1.4 M_\odot$ as 0.92 $M_\odot$ of 56Ni is the greatest yield a Chandrasekhar mass can produce (Khokhlov et al. 1993).

Howell et al. (2006) inferred from their observations of SN 2003fg that $\sim 1.3 M_\odot$ of 56Ni was produced, as did Hicken et al. (2007) in their observations of SN 2006gz. There is a growing body of evidence supporting double-degenerate SNe Ia progenitor scenarios. Since any supernova arising from a double-degenerate progenitor scenario may not fit the standard templates for SNe Ia, these transients must be filtered out if SNe Ia are to remain as premier cosmological tools. To that end, we must develop models that give clear and detectable signatures of double-degenerate SNe Ia to distinguish them from standard SNe Ia.

Currently, models of double-degenerate progenitors are split into two, dynamically different scenarios. In the first, white dwarfs in close binaries lose angular momenta through gravitational radiation, ultimately merging into a thermally supported super-Chandrasekhar object or detonating outright (Iben & Tutukov 1984; Webbink 1984; Benz et al. 1989a; Yoon et al. 2007; Pakmor et al. 2010). In the second, two white dwarfs collide in dense stellar systems such as globular cluster cores (Raskin et al. 2009; Rosswog et al. 2009; Lorén-Aguilar et al. 2009, 2010), where white dwarf number densities can be as high as $\approx 10^4$ pc$^{-3}$. This follows from conservative estimates for the average globular cluster mass, $10^6 M_\odot$ (Brodie & Strader 2006), and for the average globular cluster core radius, 1.5 pc (Peterson & King 1975), taken together with the Salpeter initial mass function (Salpeter 1955). Assuming cluster velocity dispersions on the order of 10 km s$^{-1}$, this allows for 10–100, $z \lesssim$ 1 collisions per year. Observations by Chomiuk et al. (2008) of globular clusters in the nearby S0 galaxy NGC 7457 have detected what is likely to be an SN Ia remnant. Given the difficulty in distinguishing SNe Ia as residing in galaxy field stars or in globular clusters in front of or behind their host galaxies (Pfahl et al. 2009), the frequency with which these can occur warrants investigation.

Numerical simulations of white dwarf collisions were pioneered in Benz et al. (1989b) using a smooth particle hydrodynamics (SPH) code. They concluded from their results that white dwarf collisions were of little interest as the 56Ni yields were small. However, their simulations employed an approximate equation of state (EOS) for white dwarfs and resolutions were low, relative to what is possible with current computing resources. Moreover, as will be discussed, the infall velocities and velocity gradients play a crucial role in the final 56Ni yields.

More recently, Raskin et al. (2009), Rosswog et al. (2009), and Lorén-Aguilar et al. (2010) revisited collisions using up-to-date SPH codes and vastly more particles ($8 \times 10^5$, $2 \times 10^6$, and $4 \times 10^6$, respectively). In Raskin et al. (2009), a single mass...
pair \((0.6 \, M_\odot \times 2)\) was explored with three impact parameters, whereas in Rosswog et al. (2009), several mass pairs were examined in direct, head-on collisions. Both of these papers aimed at establishing double-degenerate collisions as SNe Ia progenitors, finding that \(^{56}\text{Ni}\) is indeed produced prodigiously in such collisions, lending credence to their candidacy. Loret-Aguilar et al. (2010) examined one mass pair \((0.6 \, M_\odot + 0.8 \, M_\odot)\), but at a number of different impact parameters, ranging from those that resulted in direct collisions to those that resulted in eccentric binaries, aimed at establishing the parameters of white dwarf coalescence arising from collisional dynamics.

In this paper, we revisit the three impact parameters studied in Raskin et al. (2009) using a variety of mass pairings. Using 22 combinations of masses and impact parameters, we aim to answer five key questions: how does \(^{56}\text{Ni}\) production depend on

1. the total mass of the system?
2. the mass ratio of the two stars?
3. the impact parameter?
4. the infall velocities of the constituent stars?
5. tidal effects?

While the last two of these questions can be eliminated with robust initial conditions, they are nevertheless important details that are sometimes overlooked. Armed with this information, we will be able to make some conclusions about the observability of different combinations of collision parameters, and to determine whether the resulting transient of any particular collision is as luminous as an SN Ia.

The structure of this paper is as follows. In Section 2, we discuss the details of our initial conditions and our new hybrid burning nuclear network. In Section 3, we give the details of the results of each simulation that resulted in a detonation along with a study of the effect of numerical parameters on the \(^{56}\text{Ni}\) yield in Section 3.1.2, and in Section 4, we discuss those that resulted in remnants. Finally, in Section 5, we summarize our results and conclusions.

2. METHOD

2.1. Particle Setsups and Initial Conditions

As in Raskin et al. (2009), we employ a version of a three-dimensional SPH code called SNSPH (Fryer et al. 2006). SPH codes are particularly well suited to these kinds of simulations as the white dwarf stars involved are very dense and moving very rapidly. Advecting rapidly moving, isothermal, cold white dwarfs in Eulerian, grid-based codes introduces perturbations that can be challenging to overcome. Moreover, because many of our simulations are grazing impacts, conservation of angular momentum is crucial to the final outcomes, for which SNSPH excels (Fryer et al. 2006).

In our previous work, we used a Weighted Voronoi Tessellations method (WVT; Diehl & Statler 2006) for our particle setups. This method arranges particles in a pseudo-random spatial distribution with thermodynamic quantities that are consistent with the chosen EOS. The default operation for this method is to allow the masses of particles to vary in order to keep their sizes, or smoothing lengths \(h\), constant throughout the initial setup. This approach has its advantages when it comes to spatial arrangement, but one disadvantage is that it produces a uniform level of refinement in the initial conditions regardless of where most of the mass resides. The result is that much higher particle counts are required to reach convergence.

To remedy this, we modified the WVT method to keep mass fixed, varying \(h\) consistent with the density profiles of white dwarf stars. This has the effect of concentrating resolution where most of the mass resides, vastly reducing the required particle counts for convergence. In fact, whereas in Raskin et al. (2009), we showed that convergence of the \(^{56}\text{Ni}\) yield was reached at approximately \(10^6\) particles, using constant mass particles, we reach convergence with only 200,000. A convergence test on particle count of our fiducial case, \(0.64 \, M_\odot \times 2\) with zero impact parameter, is discussed in Section 3.1.2.

A further modification we have added to our previous approach is an isothermalization step in our initial conditions. When mapping one-dimensional profiles for cold white dwarfs onto a resolution limited, three-dimensional particle setup, there is often a relaxation time, during which the stars oscillate before finding equilibrium. For white dwarf masses of \(\approx 0.6 \, M_\odot\), this settling time is short, but for larger masses, the oscillations can continue for several minutes or hours. These repeated gravitational contractions heat the interiors of the stars until they can no longer be considered “cold” white dwarfs. Therefore, we relaxed each individual star in a modified version of SNSPH that artificially cools the stars by keeping each particle at a constant temperature during the stars’ oscillations until they reach a cold equilibrium. Figure 1 shows a temperature profile for a \(0.64 \, M_\odot\) white dwarf that has been passed through this isothermalization routine, indicating an isothermal temperature of \(\approx 10^7\) K throughout.

As in our previous work, the initial conditions for the positions and velocities of the white dwarf stars in our simulations were generated using a fourth-order Runge–Kutta solver with an adaptive time step that integrates simple kinematic equations. The impactor star was initially given a small velocity comparable to the velocity dispersion of globular cluster cores, \(\sigma = 10 \, \text{km} \, \text{s}^{-1}\). The solver places the stars at \(0.1 \, R_\odot\) apart with the proper velocity vectors expected for free fall from large initial separations with a given velocity dispersion.

Figure 2 compares the initial conditions of the \(0.64 \, M_\odot \times 2\), head-on collision to the velocity gradient that is introduced by tidal forces. The relative velocity of the centers of mass can be predicted analytically for a zero impact parameter collision with \(v_c = (2GM_1 + M_2)/\Delta r^{1/2}\), where \(M_1\) are the masses of the constituent white dwarfs and \(\Delta r\) is the separation of their centers of mass.

As Figure 2 demonstrates, when the stars are allowed to free fall in SNSPH from larger separations, such as the separation of \(0.1 \, R_\odot\) used throughout this paper, the velocity gradients that arise from tidal distortions are non-negligible. As will be shown

![Figure 1. Temperatures and densities of particles lying on the x-axis in a 0.64 M_\odot white dwarf. This star was created using WVT and isothermalized to 10^7 K after ~5 minutes. (A color version of this figure is available in the online journal.)](image-url)
Figure 2. Velocity evolution from our initial conditions to the moment of first contact, indicating strong velocity gradients induced by tidal forces. The shaded area denotes the spread in relative x-velocities and $v_c$ is the relative velocity of the centers of mass.
(A color version of this figure is available in the online journal.)

Figure 3. NSE distributions for $\rho = 1.67 \times 10^9$ g cm$^{-3}$ and $Y_e = 0.5$ in an $\alpha$-centric nuclear network. Proton and neutron mass fractions are plotted for reference. At $T_D \approx 6$, $^4$He begins to dominate the isotope distribution.
(A color version of this figure is available in the online journal.)

Table 1
Initial Velocities of Each Component Star in the Head-on Cases of Each Mass Pair for Initial Separations of 0.1 $R_{\odot}$

| No. | $m_1 (M_{\odot})$ | $m_2 (M_{\odot})$ | $v_1 \times 10^3$ km s$^{-1}$ | $v_2 \times 10^3$ km s$^{-1}$ |
|-----|-----------------|-----------------|------------------|------------------|
| 1   | 0.64            | 0.64            | 1.10             | 1.10             |
| 2   | 0.64            | 0.81            | 1.31             | 1.03             |
| 3   | 0.64            | 1.06            | 1.58             | 0.95             |
| 4   | 0.81            | 0.81            | 1.24             | 1.24             |
| 5   | 0.81            | 1.06            | 1.51             | 1.15             |
| 6   | 0.96            | 0.96            | 1.35             | 1.35             |
| 7   | 1.06            | 1.06            | 1.41             | 1.41             |
| 8   | 0.50            | 0.50            | 0.97             | 0.97             |

Note. All velocities are relative to the center of mass.

in Section 3.1.2, the magnitudes of these velocities and their spreads play an important role in the final outcomes and the $^{56}$Ni yields of each simulation as they determine how much kinetic energy is converted to thermal energy, and thusly, when carbon ignition occurs. A shooting method was used to determine the necessary, initial vertical separation that resulted in the final impact parameter that we desired at the moment of impact. Initial velocities for all of our collision scenarios with zero impact parameter are given in Table 1. Likewise, all of our stars are initialized with 50% $^{12}$C and 50% $^{16}$O throughout. This approximates typical carbon–oxygen white dwarf compositions, and we use the Helmholtz free-energy EOS (Timmes & Arnett 1999; Timmes & Swesty 2000).

2.2. Hybrid Burner

Most large, hydrodynamic codes use some form of a hydrostatic nuclear network (e.g., Eggleton 1971; Weaver et al. 1978; Arnett 1994; Fryxell et al. 2000; Starrfield et al. 2000; Herwig 2004; Young & Arnett 2005; Nonaka et al. 2008). That is, the thermodynamic conditions present at the start of a burn calculation are not altered until the next hydrodynamic time step, which oftentimes is controlled by abundance or energy changes from the burn calculation rather than a pure Courant condition. The effect of this is to fix the temperature-dependent reaction rates throughout the hydrodynamic time step to what they were at its start.

There are several reasons for running a simulation in this way, the most important of which is to avoid a decoupling of the nuclear network from the hydrodynamic calculation. However, for limited spatial, mass and time resolutions, this approximation—that the thermodynamic conditions do not change rapidly enough during a burn to warrant a sub-cycle recalculation of the nuclear reaction rates—fails in regimes where the nuclear reactions are strongly temperature dependent, such as at temperatures where photo-disintegration is the dominant nuclear process. As Figure 3 shows, the nuclear statistical equilibrium (NSE) state for material with $\rho = 1 \times 10^9$ g cm$^{-3}$ at $T_D \approx 7$ has most of the heavy isotopes photo-disintegrating back to $^4$He. In such a regime, the material undergoing photo-disintegration experiences what amounts to an abrupt phase change through a strongly endothermic reaction. In nature, this reaction should rapidly cool the material before complete photo-disintegration, allowing these liberated $\alpha$-particles to react with other isotopes. However, a hydrostatic burn will overestimate the timescale for this cooling as it assumes that a full hydrodynamic time step is necessary for relevant pressure or temperature changes. With the temperature remaining fixed over an artificial long time, this approach results in the nuclear network removing far too much internal energy, $u$, to be physical.

Typically, one attempts to limit the impact of such a phase change by relying on a global time-step minimization scheme of the form

$$\Delta_t_{n+1} = \min \left[ \Delta_t, \Delta_t \times f_u \times \left( \frac{u^n}{u^n - u^{n-1}} \right) \right],$$

where the subscript $n$ refers to the iteration number, $\Delta_t$ is the Courant time, $u^i$ is the specific internal energy of the $i$th particle, and $f_u$ is a dimensionless parameter which constrains the maximum allowable change in energy. Our global time step is also controlled in this manner. In practice, the conditions immediately prior to photo-disintegration will fix the next time step, $\Delta_t_{n+1}$, of order $10^{-5}$ s for $f_u = 0.30$. However, even this time step is too large to capture the relevant temperature changes affecting the reaction rates on timescales of order $10^{-12}$ s.

The alternative approach to a hydrostatic burn is to use a “self-heating/cooling” nuclear network that simultaneously integrates an energy equation and the abundance equation self-consistently (see, e.g., Müller 1986). When applied to a particle code like SPH, this type of calculation keeps $\rho$ fixed, but updates temperature in a fashion that is consistent with the EOS and the new internal energy at each sub-cycle. The ordinary differential equation (ODE) that governs a hydrostatic burn calculation of the abundance of an isotope $Y_i$ assuming the mass diffusion gradients are negligible, is of the
form

\[
\dot{Y}_i = \sum_j C_i R_{ij} Y_j + \sum_{j,k} \frac{C_i}{C_j C_k} \rho N_A R_{ijk} Y_j Y_k + \sum_{j,k,l} \frac{C_i}{C_j C_k C_l} \rho^2 N_A^2 R_{ijkl} Y_j Y_k Y_l, \tag{2}
\]

where the coefficients \( C_{i,...,l} \) specify how many particles of the \( i \)th species are created or destroyed, and \( R_{i,...,l} \) are the temperature-dependent reaction rates for each of the different reaction types. The first term describes weak reactions (\( \beta \)-decays and electron captures) and photo-disintegrations, the second describes two-body reactions of the type \( ^{12}\text{C}(\alpha,\gamma)^{16}\text{O} \), and the third term describes three-body reactions, such as \( ^4\text{He}(2\alpha,\gamma)^{12}\text{C} \). The energy generation ODE takes the form

\[
\dot{\epsilon} = -N_A \sum_i \ddot{Y}_i m_i c^2, \tag{3}
\]

where \( m_i \) is the rest mass of the \( i \)th isotope (see, e.g., Benz et al. 1989b). The density and temperature equations are simply

\[
\frac{d\rho}{dt} = \rho \frac{\partial}{\partial \rho} \left( \frac{P}{\rho^2} \right) = \frac{\partial s}{\partial T}, \tag{4}
\]

where \( \frac{du}{dt} \) is the change in specific energy and \( ds/dt \) is the change in specific entropy. In accordance with \( \rho = 0 \) and employing the identity \( T s = \dot{\epsilon} \), this reduces to

\[
\frac{\partial u}{\partial T} \frac{dT}{dt} = \dot{\epsilon}
\]

\[
\frac{\partial}{\partial T} \frac{dT}{dt} = -\frac{\dot{\epsilon}}{c_v}
\]

where \( c_v \) is the specific heat capacity at a constant volume. Equations (2), (3), and (5) are evolved simultaneously and self-consistently (Müller 1986).

At very high spatial resolutions and small time steps, the self-heating approach would be identical to the hydrostatic approach. As Figure 4 shows for the energy, temperature, and composition of a particle after a representative time step as determined by Equation (1) with \( f_u = 0.30 \). The solid lines show the implicit integrations from a hydrostatic calculation, while the dotted lines indicate those for a self-heating calculation. In both cases, \( \rho \) is kept constant, while in the self-heat calculation, the temperature, and thus the nuclear reaction rates are recalculated at each implicit integration step, consistent with the first law of thermodynamics.

(A color version of this figure is available in the online journal.)

![Figure 4](image-url)
grazing collisional impacts, and fully grazing/glancing impacts. Table 2 summarises the $^{56}\text{Ni}$ yields of each of our simulations. In this table, the impact parameter, $b$ (the vertical separation between the cores of both white dwarfs at the moment of impact) is given as the fraction of the radius of the primary white dwarf. Thus, the $b = 0$ column shows the yields for head-on impacts, the $b = 1$ column indicates a full white dwarf radius and $b = 2$ indicates two white dwarf radii, or a fully grazing impact.

Dursi & Timmes (2006) examined the shock-ignited detonation criteria for carbon in a white dwarf using numerical models. They derived a relationship between the density of the carbon fuel and the minimum radius of a burning region that will launch a detonation. For a carbon abundance of $X_{\text{C}} \sim 10^{-7} \text{g cm}^{-3}$, their results suggest a minimum burning region is unshocked, carbon–oxygen material and is indicated number density in the $\rho$–$T$ plane, which is in fact at lower densities than the surrounding shocked medium. While the whole of the shocked region continues to heat up, causing more material to ignite near the center, the steep pressure gradient prevents the energy liberated by burning to initiate a detonation. For these burning particles, with silicon ash behind them and higher pressure carbon in front, the energy they deposit in their neighbors is insufficient to greatly alter their energy-generation rate. Instead, the burning region grows only as fast as material is heated to the critical temperatures needed for carbon ignition, $T_0 \approx 3$. The separation of material into three distinct phases is clearest in the left two panels of Figure 5, which plots particle number density in the $\rho$–$T$ plane. The lower, more populated region is unshocked, carbon–oxygen material and is indicated in green. The less populated middle region, also represented with green at $T_0 \approx 1$, is a shocked material that has yet to reach the critical conditions for carbon ignition, and the upper, sparse region is material that has begun burning carbon to silicon-group elements, represented in red. The pressure gradient slopes positively in all directions away from the geometric center where this early burning begins, which is in fact at lower densities than the surrounding shocked medium. While the whole of the shocked region continues to heat up, causing more material to ignite near the center, the steep pressure gradient prevents the energy liberated by burning to initiate a detonation. For these burning particles, with silicon ash behind them and higher pressure carbon in front, the energy they deposit in their neighbors is insufficient to greatly alter their energy-generation rate. Instead, the burning region grows only as fast as material is heated to the critical temperatures needed for carbon ignition, $T_0 \approx 1$, by the conversion of kinetic energy to thermal energy.

Approximately 2 s after the stars first collide, sufficiently high temperatures and densities are reached at the edges of the shocked region to initiate carbon-burning. In these locations,
the pressure gradient slopes negatively in all directions. The liberated energy is free to break out, initiating detonations at the ignition points. The sound speeds in these zones are raised higher than the infalling speeds due to the rapid rise in temperature, and $^{56}$Ni begins to appear in large quantities, indicated in blue in the left panels of Figure 6. Sustained detonation fronts then propagate through the unburned material, as well as the silicon “ash” that lies in the shocked region. As Figure 7 shows, significantly more $^{56}$Ni is produced during this phase. In Figure 7, it is also evident that the shocks overtake one another inside the contact zone, shocking the material a second time and producing yet more $^{56}$Ni. Less than 1 s after the detonations began, the entire system has become unbound, freezing out the nuclear reactions, as can be seen in Figure 8. The final $^{56}$Ni yield for this simulation was $0.51\, M_\odot$.

In the $b = 1$ simulation, the added angular momentum distorted the shocked region between the two stars, resulting in detonations lighting off-center and off-axis as compared to the $b = 0$ case. As Figure 9 shows, the detonation waves traveling through the densest portions of the shocked regions where the sound speed is highest twist the material into a uniquely anisotropic configuration. Moreover, because much of the material is traveling nearly perpendicular to the shock, the density in the pre-detonation, shocked region is lower than in the $b = 0$ case. This reduces $^{56}$Ni production by about 7% to $0.47\, M_\odot$.

Since the post-explosion expansion phase is homologous, the pattern of isotopes present at the moment the system becomes unbound is not altered by the expansion. Therefore, the velocities plotted in Figure 10 for several isotopes in the $b = 0$ and $b = 1$ cases are directly related to the radial distribution of the isotopes.

The velocity structure preserves the isotopic segregation expected behind the burning front, with a progression from complete burning of carbon and oxygen to iron-peak elements, though silicon-group elements, and finally ending with an unburned or only partially burned carbon and oxygen envelope. This layered structure is in agreement with the observations of Scalzo et al. (2010) and others of SNe Ia suspected of having been produced from double-degenerate progenitor scenarios.

The $b = 2$ scenario for this mass pair did not feature a detonation, and instead, resulted in a hot remnant embedded in a disk. Details of this simulation and its outcome will be discussed in Section 4.

### 3.1.2. Variations of Parameters

In order to assess the impact of the time step on the nuclear yields, we compared three simulations of the $0.64\, M_\odot \times 2$ mass pairing described in Section 3.7. This is due, in large part, to the kinetic energy at impact, which is related directly to the infall speed. With greater infall speeds, the shocked region heats...
sufficiently to initiate a detonation earlier, and the detonations begin nearer to the central region (the $y$–$z$ plane).

We tested this mechanism with a $0.64 M_\odot \times 2$ collision scenario by giving the constituent stars an artificially high infalling
velocity to reproduce the kinematic energies associated with collisions of larger masses. In that test, the critical temperatures for carbon ignition were reached in locations nearer to the y–z plane, but still displaced enough that the pressure gradient was favorable to a detonation. In this case, the $^{56}$Ni production was actually depressed, resulting in only $0.39 \, M_{\odot}$, due to an early detonation coupled with altered shock conditions.

We also tested the effect of velocity gradients (tidal distortions) on the final $^{56}$Ni yield by running a $0.64 \, M_{\odot} \times 2$, $b = 0$ collision with an initial separation of only $0.048 \, R_{\odot}$ with the commensurate relative velocities. In that simulation, the $^{56}$Ni yield was also depressed, slightly, to $0.48 \, M_{\odot}$. The combination

**Figure 8.** Same format as Figure 5, at a later time in the simulation. (A color version of this figure is available in the online journal.)

**Figure 9.** Two-dimensional slice of interpolated densities through the $x$–$y$ plane of the $b = 1$ case of two $0.64 \, M_{\odot}$ white dwarfs colliding. Four snapshots at different times are shown. Arrows in the top left panel indicate the directions of motion of each star. (A color version of this figure is available in the online journal.)

**Figure 10.** Masses of several isotopes at logarithmically spaced velocity bins for the $b = 0$ and $b = 1$ cases of mass pairing $1, 0.64 \, M_{\odot} \times 2$. (A color version of this figure is available in the online journal.)
counts from $10^4$ particles total to $2 \times 10^6$. As Figure 11 demonstrates, convergence was reached at $2 \times 10^5$ particles. This compares favorably to previous convergence estimates in Raskin et al. (2009) that concluded $\sim 10^6$ particles were needed for convergence using equal-$h$ particle setups.

Early work on numerical simulations of white dwarf collisions carried out by Benz et al. (1989b) did not have the benefit of modern computational resources to reach these kinds of resolutions. Consequently, the $^{56}$Ni yields in those simulations were comparatively quite low.

### 3.2. Mass Pair 2—0.64 $M_\odot$ + 0.81 $M_\odot$

For asymmetrical collisions involving 0.64 $M_\odot$ and 0.81 $M_\odot$ white dwarfs, the higher kinetic energy with which they collide results in several, almost immediate detonations near the center in the $b = 0$ scenario. As Figure 12 shows, these detonation shocks superimpose to form a single, nearly spherical shock front that raises the sound speed above the infall speed for material in the 0.64 $M_\odot$ star, but the pressure gradient leftward of the detonation (region 1) stalls the detonation shock, which can be seen as a higher density, laminar shock at the rightmost edge of region 1 in Figure 12.

However, region 1 does not maintain its lenticular shape as the two stars are moving at different speeds relative to this shocked region. This allows the detonation shock to travel through this region at $\approx$ Mach 1, eventually reaching fresh carbon outside of region 1. This fresh carbon ignites explosively, creating a second detonation (region 3 in Figure 13), which sends leading shocks back through region 1 and into region 2, shocking it a second time and eventually catching up with the first detonation shock.

Most of the $^{56}$Ni in this scenario is produced in the more massive star, as Table 4 demonstrates. Since only low-density

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**Figure 11.** Convergence of the $^{56}$Ni yield with particle count for simulations employing equal mass particles (blue, 0.64 $M_\odot \times 2$) and equal $h$ particles (red, 0.6 $M_\odot \times 2$ from Raskin et al. 2009). The dashed, vertical line indicates the number of particles used in simulations throughout this paper. (A color version of this figure is available in the online journal.)

**Figure 12.** Same format as Figure 5 for mass pair 2, 0.64 $M_\odot + 0.81 M_\odot$, and $b = 0$. Material shocked by the collision is labeled as region 1. Material behind the first detonation shock is labeled as region 2. (A color version of this figure is available in the online journal.)
Figure 13. Same format as Figure 5 for mass pair 2, $0.64 M_\odot + 0.81 M_\odot$, and $b = 0$, at a later time in the simulation. Material shocked by the collision is labeled as region 1. Material behind the first detonation shock is labeled as region 2, and material behind the second detonation shock is labeled region 3.

(A color version of this figure is available in the online journal.)

Table 4

| $b$ | Isotope | 0.81 ($M_\odot$) | 0.64 ($M_\odot$) | Total ($M_\odot$) |
|-----|---------|-----------------|-----------------|-----------------|
| 0   | $^{12}$C | 0.21            | 0.03            | 0.24            |
|     | $^{16}$O | 0.25            | 0.14            | 0.39            |
|     | $^{28}$Si| 0.12            | 0.27            | 0.39            |
|     | $^{56}$Ni| 0.13            | 0.02            | 0.14            |
| 1   | $^{12}$C | 0.02            | 0.03            | 0.05            |
|     | $^{16}$O | 0.07            | 0.14            | 0.21            |
|     | $^{28}$Si| 0.12            | 0.25            | 0.37            |
|     | $^{56}$Ni| 0.49            | 0.04            | 0.53            |

portions of the $0.64 M_\odot$ star had entered the shocked region before the detonation, most of its contribution to the total output is in Si-group elements.

In the $b = 1$ case, the pre-detonation, shocked region reaches much higher densities, and the oblique angle at which the white dwarf stars enter the shocked region allows more material to become strongly shocked by the detonation. The detonation shock also twists around the peculiar density contours inside the shocked region, shocking much of the material several times, as is seen in the bottom right panel of Figure 14. The $0.81 M_\odot$ star experiences a near complete burn of all of its carbon and oxygen. However, as in the $b = 0$ case, most of the $0.64 M_\odot$ star remains unshocked at the time of the detonation breakout. As before, the $0.64 M_\odot$ star contributes mostly Si-group elements to the total output, as shown in Table 4.

The velocity profiles for the $b = 0$ and $b = 1$ cases of mass pair 2, shown in Figure 15, reinforce the observation that $^{56}$Ni is created in a confined region in the $b = 0$ case, mainly in the densest portions of the shocked material from the $0.81 M_\odot$ star. Carbon and oxygen, together, dominate the total output by mass, while in the $b = 1$ case, $^{56}$Ni is the dominant isotope, followed by $^{28}$Si.
3.3. Mass Pair 3—0.64 $M_\odot + 1.06 M_\odot$

As with mass pair 2, the $b = 0$ case of mass pair 3 experiences a detonation of material in the 0.64 $M_\odot$ star very quickly after the stars first collide. However, owing to the greater potential well into which the 0.64 $M_\odot$ star is falling, the sound speed of the material shocked by the detonation is less than the infall velocity as shown in the top left panel of Figure 16.

After $\approx 0.7$ s, as the core of the 1.06 $M_\odot$ star enters the shocked region, a second detonation lights on the left edge of the shocked region. This powers a shock that travels through both stars, catching up with the shock from the first detonation in the 0.64 $M_\odot$ star. The material in the 0.64 $M_\odot$ star burns mostly to $^{28}$Si, while what burns in the 1.06 $M_\odot$ star burns almost entirely to $^{56}$Ni, due to its higher density. The contributions from each star to the total elemental abundances are given in Table 5.

In simulations of mass pair 3 that introduced a non-zero impact parameter, the 1.06 $M_\odot$ star was simply too compact to be significantly disrupted by a collision with a 0.64 $M_\odot$ star. In both the $b = 1$ and $b = 2$ cases, most of the 1.06 $M_\odot$ star survived the collision, while completely disrupting the 0.64 $M_\odot$ star. Details of those simulations are given in Section 4.

3.4. Mass Pair 4—0.81 $M_\odot \times 2$

The symmetrical mass pair, 0.81 $M_\odot \times 2$, is quite unlike the 0.64 $M_\odot \times 2$ mass pairing discussed above. For the 0.81 $M_\odot \times 2$ mass pair with $b = 0$, several detonations occur in the $y$–$z$ plane simultaneously and almost immediately after impact, owing to the higher temperatures reached in the shocked region from the higher infall speeds. As Figure 17 shows, these detonations superimpose and produce copious amounts of $^{56}$Ni as they travel through the much denser material present inside the 0.81 $M_\odot$ stars. This denser material allows for a significantly greater

**Table 5**

| Isotope | 1.06 ($M_\odot$) | 0.64 ($M_\odot$) | Total ($M_\odot$) |
|---------|-----------------|-----------------|-----------------|
| $^{12}$C | 0.39            | 0.02            | 0.41            |
| $^{16}$O | 0.40            | 0.15            | 0.55            |
| $^{28}$Si | 0.05           | 0.24            | 0.29            |
| $^{56}$Ni | 0.19           | 0.07            | 0.26            |

Figure 16. Same format as Figure 5 for mass pair 3, 0.64 $M_\odot + 1.06 M_\odot$, and $b = 0$. (A color version of this figure is available in the online journal.)
conversion of carbon and oxygen to $^{56}$Ni. Therefore, with only a 26% increase in total mass of the system over the $0.64 \, M_\odot \times 2$ scenario, there is a 64% increase in $^{56}$Ni production to $0.84 \, M_\odot$.

Having denser and more compact stars also reduces the sensitivity of the $^{56}$Ni yield to impact parameter. Indeed, with two $0.81 \, M_\odot$ stars, both the $b = 1$ and $b = 2$ simulations resulted in detonations and significant $^{56}$Ni production, $0.84 \, M_\odot$ and $0.65 \, M_\odot$, respectively. Differences in the $^{56}$Ni yield for the two non-zero impact parameter simulations stem mainly from the amount of material that burns to $^{28}$Si before the detonations occur, with the $b = 2$ scenario featuring much more material burning at lower temperatures to silicon before the detonation. The high activation energy of $^{28}$Si prevents much of that material from being converted to $^{56}$Ni.

3.5. Mass Pair 5—$0.81 \, M_\odot + 1.06 \, M_\odot$

The $0.81 \, M_\odot + 1.06 \, M_\odot$ mass pair follows a very similar pattern to that of mass pair 2 ($0.64 \, M_\odot + 0.81 \, M_\odot$). Intermediate impact parameters allow more material to enter the shocked region before detonation, and so there is a rise in $^{56}$Ni production in the $b = 1$ case over $b = 0$. However, because both stars involved in the collision are denser than their counterparts in mass pair 2, much more $^{56}$Ni is produced overall. Contributions to the total yield in the $b = 0$ and $b = 1$ simulations are given in Table 6.

3.6. Mass Pairs 6 and 7—$0.96 \, M_\odot \times 2$ and $1.06 \, M_\odot \times 2$

The $0.96 \, M_\odot \times 2$ and $1.06 \, M_\odot \times 2$ simulations were essentially similar to the $0.81 \, M_\odot \times 2$ simulations with the exception that the greater the mass of the constituent stars, the less sensitive the $^{56}$Ni yield was to impact parameter. Indeed, both mass pairs 6 and 7 produced almost the same yield in all three tested collision scenarios.

What distinguishes the $1.06 \, M_\odot \times 2$ mass pair from all the others attempted is that the $^{56}$Ni yield is super-Chandrasekhar in all cases. Were such explosions observed, there would be no doubt that a double-degenerate progenitor scenario of some kind was responsible. The resulting $1.71 \, M_\odot$ of $^{56}$Ni from the $1.06 \, M_\odot \times 2$ simulations appear strikingly similar to the $1.7 \, M_\odot$ of $^{56}$Ni derived from the observations of Scalzo et al. (2010).

3.7. Mass Pair 8—$0.50 \, M_\odot \times 2$

Finally, we studied symmetric collisions of low-mass, $0.50 \, M_\odot$ white dwarfs. Table 2 demonstrates that the $b = 0$ collision scenario for this mass pairing produced less than $0.01 \, M_\odot$ of $^{56}$Ni despite having resulted in a detonation. In this case, the energy generated from even mild carbon-burning was sufficient to unbind the stars. As in the $0.64 \, M_\odot \times 2$, $b = 0$ scenario, the lower velocities with which the stars collide result in a late
dwarfs is warranted. However, further investigation involving helium white dwarfs as the component stars of this mass pair were not attempted with carbon–oxygen transients. Other simulations introducing impact parameters higher than the 0.64 \( M_\odot \) white dwarfs that entered into the collision (\( r_{\text{rem}} \approx 2.5 \times 10^8 \) cm), larger than the 0.64 \( M_\odot \) white dwarfs.

The compact remnant was slightly less massive at \( m_\text{rem} \approx 1.06 \times 10^9 \) \( M_\odot \) than the 0.64 \( M_\odot \) white dwarfs that entered into the collision (\( r_{\text{rem}} \approx 6.98 \times 10^6 \) cm). Carbon ignition nominally takes place at approximately \( (7–8) \times 10^8 \) K (e.g., Gasques et al. 2007), but recent phenomenological models (e.g., Jiang et al. 2007) have suggested a strongly reduced, low-energy astrophysical S-factors for carbon fusion reactions that potentially reduce carbon ignition temperatures to \( \sim 3 \times 10^8 \) K, especially at densities of \( 10^9 \) g cm\(^{-3}\). A lower carbon-burning threshold would be of interest to future studies of collision remnants.

Since the system started in a bound state (\( T \lesssim V \), where \( T \) in this case is the total kinetic energy and \( V \) is the total gravitational potential energy) and since any energy gained from nuclear processes is negligible, most of the material cannot escape the system and the disk remains bound to the compact core. It will eventually cool and collapse onto the surface of the compact object. However, the hot core may accelerate parts of the disk to escape velocity via radiative processes, and so the calculation of the final mass of the resultant white dwarf is beyond the scope of this paper. Suffice it to say, the final mass will not exceed the Chandrasekhar limit as only \( 1.28 \times 10^9 \) \( M_\odot \) of material is available.

The compact remnant core after 100 s featured a nearly constant density of \( \rho \approx 10^6 \) g cm\(^{-3}\). It was surrounded by a thick, Keplerian disk \( \approx 2.0 \times 10^{10} \) cm in radius with a scale radius of \( r_0 \approx 2.3 \times 10^9 \) cm. The compact object at the center of the disk is not strictly a white dwarf since much of its pressure support is thermal (\( T \approx 5 \times 10^{9} \) K). Indeed, since degeneracy pressure support necessitates that more massive white dwarfs are smaller than less massive ones, this object at \( \approx 0.8 \, M_\odot \) is far too large to be wholly degenerate (\( r_{\text{rem}} \approx 2.5 \times 10^8 \) cm), larger than the 0.64 \( M_\odot \) white dwarfs that entered into the collision (\( r_{0,64} \approx 6.98 \times 10^6 \) cm). It is clear from the velocity profile of the most abundant isotopes from this collision, given in Figure 18, that carbon and oxygen remain mostly unburned in this scenario. This seems to suggest that collisions of low-mass white dwarfs \(( M \lesssim 0.6 \, M_\odot )\) of the CO variety would not produce observable transients. Other simulations introducing impact parameters higher than the 0.64 \( M_\odot \) white dwarfs that entered into the collision (\( r_{0,64} \approx 6.98 \times 10^6 \) cm). Carbon ignition nominally takes place at approximately \((7–8) \times 10^8 \) K (e.g., Gasques et al. 2007), but recent phenomenological models (e.g., Jiang et al. 2007) have suggested a strongly reduced, low-energy astrophysical S-factors for carbon fusion reactions that potentially reduce carbon ignition temperatures to \( \sim 3 \times 10^8 \) K, especially at densities of \( 10^9 \) g cm\(^{-3}\). A lower carbon-burning threshold would be of interest to future studies of collision remnants.

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For the \( b = 2 \) case of mass pair 2, 0.64 \( M_\odot \) + 0.81 \( M_\odot \), the compact remnant was slightly less massive at \( \approx 0.75 \, M_\odot \). However, since the total mass of the system is super-Chandrasekhar, the final remnant mass will result in a super-Chandrasekhar white dwarf. Again, this final mass will depend greatly on radiative processes, and the likelihood of producing an SN Ia will hinge on the accretion rate of the disk onto the core.

The simulations of mass pair 3, 0.64 \( M_\odot \) + 1.06 \( M_\odot \), resulted in remnants in both the \( b = 1 \) and \( b = 2 \) cases as the 1.06 \( M_\odot \) white dwarf was too compact for the star to be much affected by a grazing collision with a 0.64 \( M_\odot \) white dwarf. In the \( b = 1 \) case,
some of the atmosphere of the 1.06 \( M_\odot \) star was stripped away to join the material from the disrupted 0.64 \( M_\odot \) star in the disk, while in the \( b = 2 \) case, the 1.06 \( M_\odot \) star was nearly unaffected by the collision. Figure 20 illustrates that the remnant core of the \( b = 2 \) simulation masses \( \approx 1.0 \ M_\odot \), and the central densities are essentially unchanged from the 1.06 \( M_\odot \) progenitor.

Some features worth noting in the \( b = 2 \) profiles in Figure 20 are the indications of a cold core (\( T_{\text{core}} \approx 2 \times 10^8 \) K) surrounded by a hot envelope (\( T_{\text{env}} \approx 9 \times 10^8 \) K), and the presence of a strong overdensity in a part of the disk, which causes large spreads in density and temperature for the disk material. This suggests that while the 0.64 \( M_\odot \) star is no longer a gravitationally bound object, it has nevertheless not been completely disrupted.

The radial profiles of the \( b = 2 \) simulation of the 0.81 \( M_\odot \) + 1.06 \( M_\odot \) mass pair, shown in Figure 21, indicate a very similar structure, with a high-temperature envelope surrounding a cold core and an overdensity in a part of the disk. These inhomogeneities in the disk would most likely vanish after many crossing-times; however, it is highly unlikely that radiative processes or collisional excitations within the disk could remove as much as 0.4 \( M_\odot \). This suggests that after a Kelvin–Helmholtz timescale, the remnant from the \( b = 2 \) collision of mass pair 5 will almost certainly be super-Chandrasekhar.

Some of the properties of these remnants are very similar to the simulated remnant explored in Yoon et al. (2007), which was produced by the merger of 0.9 \( M_\odot \) and 0.6 \( M_\odot \) white dwarfs. In that simulation, the remnant also featured a cold core (\( T_{\text{core}} \approx 1 \times 10^8 \) K) surrounded by a hot envelope (\( T_{\text{env}} \approx 6 \times 10^8 \) K), embedded inside a thick, Keplerian disk. Using a one-dimensional stellar evolution code, they found that such systems can indeed evolve on timescales \( \sim 10^5 \) yr toward becoming SNe Ia.

5. DISCUSSION

White dwarf collisions are not typically regarded as SNe Ia progenitors, and therefore, they have been relatively unexplored theoretically. Here we have conducted a comprehensive suite of simulations of such collisions examining the dependence of their \( ^{56}\text{Ni} \) yield on total mass, mass ratio, and impact parameter. Our results suggest that white dwarf collisions are a viable avenue for producing SNe Ia with brightnesses that range from sub-luminous to super-luminous.

In fact, in more than 75% of our simulations, collisions resulted in detonations, and in all but the least massive combination of stars, significant amounts of \( ^{56}\text{Ni} \) were produced. We found that even mass pairs that are below the Chandrasekhar limit featured explosive nuclear burning, with the 0.64 \( M_\odot \) \times 2 mass pair producing \( ^{56}\text{Ni} \) in quantities comparable to standard SNe Ia. Moreover, the most massive combinations of stars produced super-luminous quantities of \( ^{56}\text{Ni} \), regardless of the impact parameter, greatly increasing their likelihood of detection. The \( ^{56}\text{Ni} \) yields from these collisions are consistent with those of observed SNe Ia with super-Chandrasekhar mass progenitors.

Asymmetric mass pairs generally produced less \( ^{56}\text{Ni} \) in head-on collisions than symmetric pairs. At middling impact parameters, much more \( ^{56}\text{Ni} \) was produced; however, at high impact parameters, there was little or none. This is due primarily to the delicate balance which must be struck between the dynamics of the impact and the binding energy of the less massive star in order to establish a stalled shock region that can lead to a detonation. At high impact parameters, the less massive star is typically unbound by the collision before much or any \( ^{56}\text{Ni} \) is produced.

For combinations of masses and impact parameters that did not detonate, the end result always featured a compact, semidegenerate object surrounded by a bound, thick disk of carbon and oxygen. Many of these systems were super-Chandrasekhar, and over Kelvin–Helmholtz timescales, these, too, are candidate progenitors for producing SNe Ia.

Our results have shown that \( ^{56}\text{Ni} \) production in white dwarf collisions is a nonlinear process that depends on several factors, including infall velocities and tidal distortion effects. Foremost among parameters to be explored in future studies is the composition of the constituent white dwarfs. Helium has a much lower activation energy than carbon or oxygen, and combinations of stars that include helium white dwarfs would almost certainly produce interesting and different results. Other avenues to be explored include the impact of more detailed modeling of the isotopic profiles in the progenitor stars, and the possibility of sparse hydrogen atmospheres. The results of these studies will shed further light on the contribution of double-degenerate collisions to the observed population of SNe Ia.

This work was supported by the National Science Foundation under grant AST 08-06720, by the National Aeronautics and Space Administration under NESSF grant PVY0401, and by a grant from the Arizona State University chapter of the GPSA. All simulations were conducted at the Ira A. Fulton High Performance Computing Center at Arizona State University. We thank James Rhoads and Sumner Starrfield for insightful discussions, and our anonymous referee for useful suggestions and feedback.
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