Non Existence of Quantum Mechanical Self Replicating Machine

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Abstract

In this paper we establish the impossibility of existence of self replicating machine in the quantum world. We establish this result by three different but consistent ways of linearity of quantum mechanics, no signalling criterion and conservation of entanglement under Local Operation and Classical Communication.

1 Introduction:

It was Shannon who first introduced us to the amazing world of binary logic and information theory. After that it had been a long journey and still we are searching for the nature of information in quantum world. The most important query from the physicists point of view is to find out the answer to the question that how the laws of physics are governing the dynamics of living units? Is it classical physics which explains all the active forces of life or we have to search into the microscopic world so that quantum mechanics can give a valid explanation of it. One of the major thrust area of research is to look out how the information defined in classical world differs from quantum information and also to enlist many computational procedures which are feasible in the classical world.
but get restricted at microscopic level. Cloning and deletion which is feasible in classical information, cannot be executed with hundred percent fidelity for an unknown quantum state [1,2]. Recently it had been seen that self replication machine which is feasible in classical information theory [3] fails to self replicate two non orthogonal quantum states [4]. It may be remarked that the process of replication or cloning and self replication is not just the same thing. A Universal constructor is said to exist if it can implement copying the original state along with the stored programme by a linear operator acting on the joint Hilbert Space.

There are many physicists [5] and biologists [6] who strongly believe that quantum mechanics will solve the mysteries of living systems. It had been suggested that the working principles of enzymes could be understood from the quantum mechanical principles [7]. Quantum mechanical algorithms had been proposed for genetic evolution [8]. It had been seen that quantum mechanics play an important role during cell mutation and entanglement between the mutational state and environment can also enhance the probability of mutation [9].

In this work our basic objective is to prove the impossibility of the self replication process in quantum world from linearity of quantum mechanics, no signalling criterion and principle of conservation of entanglement. Moreover here we try to give a biological outlook of the self replication process. We also try to find out what meaning does it convey to us biologically when we say that self replication is not feasible because of the linear structure of quantum theory, no signalling condition and conservation of entanglement under LOCC operations.

2 Non existence of Self replicating machine: Linearity of Quantum mechanics

A Quantum mechanical self replicating machine may be completely specified by a quadruple \((\psi, \rho_U, C, \Sigma)\). Here \(\psi \in H_N\) is the state of the (artificial or real) living system that contains quantum information to be self replicated and \(\rho_U \in H^K\) is the programme state that carries the instructions to copy the original information (the unitary operator
\( U; U(|\psi\rangle|0\rangle) = |\psi\rangle|\psi\rangle \), is encoded in the programme state \(|P_U\rangle\). Here \(|C\rangle\) is the control unit and \(|\Sigma\rangle\) is the finite collection of the blank states \(|0\rangle|0\rangle|0\rangle\ldots|0\rangle\) in a \( M \) dimensional Hilbert space on which the original state along with the program state is to be copied.

We had already seen in [4] such a self replicating machine don’t exist in quantum world and this result is consistent with the unitarity of quantum theory.

In this section we try to find out whether the linear structure of Quantum theory supports the existence of self replicating machine. Here we will treat quantum states as a smallest state in the living system that represents either ‘artificial’ or ‘real’ life. In this process we will assume a self replicating machine for orthogonal quantum states with non orthogonal program states. Then we will try to see that whether we can self replicate an unknown qubit.

Let \(|\psi_1\rangle\) and \(|\psi_2\rangle\) are two orthogonal quantum states and let \(|P_{U_1}\rangle\) and \(|P_{U_2}\rangle\) be the programme states, where the unitary operators for copying those two orthogonal states are encoded. Here \(|P_{U_1}\rangle\) and \(|P_{U_2}\rangle\) are non orthogonal quantum states as we know that Quantum mechanical self replicating machine for orthogonal states can exist only when the program states are non orthogonal[15]. Therefore the self replicating process for these two orthogonal quantum states is given by,

\[
L[|\psi_1\rangle|0\rangle|P_{U_1}\rangle|0\rangle^{\otimes m}|C\rangle]|0\rangle^{\otimes n-(m+1)} = |\psi_1\rangle|P_{U_1}\rangle L[|\psi_1\rangle|0\rangle|P_{U_1}\rangle|0\rangle^{\otimes m}|C_1\rangle]|0\rangle^{\otimes n-2(m+1)} \tag{1}
\]
\[
L[|\psi_2\rangle|0\rangle|P_{U_2}\rangle|0\rangle^{\otimes m}|C\rangle]|0\rangle^{\otimes n-(m+1)} = |\psi_2\rangle|P_{U_2}\rangle L[|\psi_2\rangle|0\rangle|P_{U_2}\rangle|0\rangle^{\otimes m}|C_2\rangle]|0\rangle^{\otimes n-2(m+1)} \tag{2}
\]

It is important that (1-2) is not merely a cloning transformation, on the contrary it is a recursively defined transformation where the fixed unitary operator \( L \) acts on initial (parent) configuration and the same operator acts on the final child configuration after the copies have been produced.

Let us consider a non orthogonal quantum state \(|\xi\rangle\) which can be expressed as a linear superposition of orthogonal quantum states \(|\psi_1\rangle\) and \(|\psi_2\rangle\).

Let

\[
|\xi\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle
\]  

where \( \alpha^2 + |\beta|^2 = 1 \).

Let us consider the action of a quantum mechanical self replicating machine on an un-
known quantum state $|\xi\rangle$ along with the programmed state $|P_U\rangle$ on basis of the transformation defined in (1-2)

$$L[(\alpha|\psi_1\rangle|0\rangle(|P_{U_1}\rangle) + \beta|\psi_2\rangle|0\rangle(|P_{U_2}\rangle)]|0\rangle^{\otimes m}|C_1\rangle|0\rangle^{\otimes n-(m+1)} =$$

$$\alpha|\psi_1\rangle|P_{U_1}\rangle L(|\psi_1\rangle|0\rangle|P_{U_1}\rangle|0\rangle^{\otimes m}|C_1\rangle)|0\rangle^{\otimes n-2(m+1)} + \beta$$

$$|\psi_2\rangle|P_{U_2}\rangle L(|\psi_2\rangle|0\rangle|P_{U_2}\rangle|0\rangle^{\otimes m}|C_2\rangle)|0\rangle^{\otimes n-2(m+1)}$$

which is not equivalent to the original self replicating process defined as

$$L(|\xi\rangle|0\rangle|P_U\rangle|0\rangle^{\otimes m}|C_3\rangle)|0\rangle^{\otimes n-(m+1)} = |\xi\rangle|P_U\rangle L(|\xi\rangle|0\rangle|P_U\rangle|0\rangle^{\otimes m}|C_3\rangle)|0\rangle^{\otimes n-2(m+1)}$$

Since (4) and (5) can never be identical on basis of the linear structure of quantum theory. Thus we can say that construction of such a machine in quantum world for an unknown qubit is strictly impossible. In real biological systems self replication of a macroscopic species is a classical process that takes place in an open system in which decoherence is very strong and rapid. This can be probably one explanation of the feasibility of self replication process in biological world. Self replication process at the quantum level may resemble biological replication while in reality replication of a living organism may be something different and should not be confused with the former one.

3 Non existence of Self replicating machine: No signalling principle and Conservation of entanglement under LOCC

In this section we try to find out whether the fundamental principles like no-signalling criterion and conservation of entanglement under LOCC support the existence of quantum mechanical universal self replicating machine.

Let us consider two non orthogonal states given by the form,

$$|\psi_1\rangle = a|0\rangle + b|1\rangle,$$

$$|\psi_2\rangle = c|0\rangle + d \exp i\theta|1\rangle$$

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where $a, b, c, d$ are real numbers satisfying the relation $a^2 + b^2 = c^2 + d^2 = 1$ and $0 < \theta < \pi$, $a > 0$, $c > 0$ and the states $|0\rangle$ and $|1\rangle$ are orthogonal to each other. Let $|P_{U_1}\rangle$ and $|P_{U_2}\rangle$ are the programme states corresponding to the states $|\psi_1\rangle$ and $|\psi_2\rangle$ respectively.

Let us consider an entangled state shared by two distant parties Alice and Bob

$$|\chi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle|\Sigma\rangle|P_{U_1}\rangle|\Sigma\rangle^{\otimes m}|C\rangle)_B(|\Sigma\rangle^{\otimes n-(m+1)})_B + \frac{1}{\sqrt{2}}(|\psi_2\rangle|\Sigma\rangle|P_{U_2}\rangle|\Sigma\rangle^{\otimes m}|C\rangle)_B(|\Sigma\rangle^{\otimes n-(m+1)})_B$$

(8)

where Alice is in possession with the qubit 'A' and Bob is in possession with the qubit 'B'.

Let us assume that Bob is in possession with quantum mechanical universal constructor whose action on the non orthogonal quantum states is defined by the transformations,

$$L([|\psi_1\rangle|\Sigma\rangle|P_{U_1}\rangle|\Sigma\rangle^{\otimes m}|C\rangle)(|\Sigma\rangle^{\otimes n-(m+1)}) = |\psi_1\rangle|P_{U_1}\rangle L([|\psi_1\rangle|\Sigma\rangle|P_{U_1}\rangle|\Sigma\rangle^{\otimes m}|C^1])$$

$$L([|\psi_2\rangle|\Sigma\rangle|P_{U_2}\rangle|\Sigma\rangle^{\otimes m}|C\rangle)(|\Sigma\rangle^{\otimes n-(m+1)}) = |\psi_2\rangle|P_{U_2}\rangle L([|\psi_2\rangle|\Sigma\rangle|P_{U_2}\rangle|\Sigma\rangle^{\otimes m}|C^2])$$

(9) (10)

The reduced density matrix on Alice's side before the application of quantum mechanical universal constructor is given by,

$$\rho^A = \frac{1}{2}[I + |0\rangle\langle 0|(|\psi_2\rangle|\psi_1\rangle\langle P_{U_2}|P_{U_1}\rangle) + |1\rangle\langle 1|(|\psi_1\rangle|\psi_2\rangle\langle P_{U_1}|P_{U_2}\rangle)]$$

(11)

The reduced density matrix on Alice's side after the application of quantum mechanical universal constructor is given by

$$\rho^A_{U} = \frac{1}{2}[I + |0\rangle\langle 0|(|\psi_2\rangle|\psi_1\rangle^2\langle P_{U_2}|P_{U_1}\rangle^2(C^2|C^1\rangle) + |1\rangle\langle 1|(|\psi_1\rangle|\psi_2\rangle^2\langle P_{U_1}|P_{U_2}\rangle^2(C^1|C^2\rangle)]$$

(12)

Since this operation is totally local and there is no classical communication between two parties, the density matrix on Alice's side must remain unchanged. Now from equations (11) and (12) we get that the equations will be identical only when

$$|\psi_2|\psi_1\rangle\langle P_{U_2}|P_{U_1}\rangle[1 - |\psi_2|\psi_1\rangle\langle P_{U_2}|P_{U_1}\rangle(C^2|C^1\rangle) = 0$$

(13)

or

$$|\psi_1|\psi_2\rangle\langle P_{U_1}|P_{U_2}\rangle[1 - |\psi_1|\psi_2\rangle\langle P_{U_1}|P_{U_2}\rangle(C^1|C^2\rangle) = 0$$

(14)
Now the equation (13) tells us that the Self replicating machine exists under two criterions, either i) $\langle \psi_2 | \psi_1 \rangle = 0$, $\langle P_{U_2} | P_{U_1} \rangle \neq 0$. or ii) $\langle \psi_2 | \psi_1 \rangle \neq 0$, $\langle P_{U_2} | P_{U_1} \rangle = 0$. The first condition states that if the states are orthogonal, no restrictions are imposed on the program state. This clearly indicates that with finite dimensional program state and finite number of blank states orthogonal states can self replicate. This is nothing but a realization of classical universal constructor [3]. On the other hand we see that the non orthogonal quantum states can self replicate only when the program states are mutually orthogonal in a finite dimensional program Hilbert space. Since here the self replication process is totally a local operation and there is no classical communication from Bob to Alice, the reduced density matrix on Alice’s side will remain unchanged from the principle of no signalling. But we find that the equations (11) and (12) will not be identical unless either of the conditions (i) and (ii) holds. Hence we come to the conclusion that the transformations defined in (9,10) is not valid. This establishes the impossibility of the existence of quantum mechanical universal constructor from the principle of no signalling. In living organisms the concept of entanglement is nothing but interdependency between different species. The probable biological explanation to the principle of no signalling is that in absence of classical communication between two species, any local operation like self replication on one of the correlated species doesn’t change the state of the other organisms.

Let us consider the largest eigen values $\lambda^A$ and $\lambda^A_U$ of the two density matrices (11) and (12) respectively

$$\lambda^A = \frac{1}{2} + \frac{|p||q|}{2}$$

(15)

$$\lambda^A_U = \frac{1}{2} + \frac{|p|^2|q|^2|r|}{2}$$

(16)

where, $p = \langle \psi_1 | \psi_2 \rangle$, $q = \langle P_{U_1} | P_{U_2} \rangle$, $r = \langle C^1 | C^2 \rangle$.

Now $\lambda^A - \lambda^A_U = \frac{1}{2}|p||q||1 - |p||q||r||$. Since $|p| < 1$, $|q| < 1$ and $|r| < 1$, therefore $|p||q||r| \neq 1$. Hence $\lambda^A - \lambda^A_U \neq 0$. 

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Now, $\lambda^A \neq \lambda^B$ and hence the amount of entanglement of the entangled state before and after the application of self replicating machine doesn’t remains same. This violates the conservation of entanglement under LOCC. The violation of conservation of entanglement is equivalent of saying the violation of the principle of conservation of information in the closed system [13,14]. Hence once again we rule out the existence of universal self replicating machine from the principle of conservation of entanglement.

4 Conclusion:

In this paper we have considered an artificial living system, where quantum states are treated as living units carrying information. We have then analyzed the process of self replication as a microscopic phenomenon. We came to a conclusion that the process of self replication is not possible for quantum states, by we assuming either the linear structure of quantum theory, the principle of no signalling or conservation of entanglement to be valid. In reality, we know that the process of self replication is feasible in the biological world even if the laws of quantum mechanics prevail at the level of living organisms[11]. This may indicate there is something beyond these principles that may be the root. Thus by self replication of a living unit we never refer to a situation which is going to violate these principles. Indeed it remains a major question that how this restriction will look in light of no signalling and conservation of information. If we go for the explanation of biological processes like reproduction of a living unit from information theoretic viewpoint we see that the information obtained by the living unit to carry out the process of self replication already existed somewhere; thus by establishing the permanence of information [5,12].

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