Research on multi-criteria decision-making problem using an interval-valued intuitionistic fuzzy soft information

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Abstract
Graph theory is a very beneficial mathematical tool to handle complex problems in various domains. Mathematical frameworks based on Interval-valued intuitionistic fuzzy soft information achieve more accuracy, compatibility, and flexibility of real-time applications related to other traditional fuzzy-based modelling techniques. This study demonstrates the application of interval-valued intuitionistic fuzzy soft information (IVIFSI) in a real-time scenario to handle decision-making problems in the multi-criteria environment. According to the interval score function, some advanced notions related to IVIFSI such as judgement matrix, interval exponential matrix, and possibility degree matrix are defined. Based on these parameters, a ranking technique is proposed for making a decision. In the end, a real-time scenario is given where there is an investor who wants to invest a certain amount of money in the best Indian industries for safe investments. The proposed framework provides the procedure to select an appropriate industry for investment business based on specified multiple criteria.

Keywords
Decision making problem, intuitionistic fuzzy.

AMS Subject Classification
03E72.

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Article History: Received 01 February 2020; Accepted 29 March 2020

1. Introduction
Handling decision making problems through mathematical modelling and assessing uncertainties related to the real-time applications has received grand attention of the research community especially in the domain of applied mathematics, computational intelligence, and decision support system. For modelling real-world applications, there is a necessity to manage data contain high levels of uncertainty and fuzziness. Uncertainties arise from several sources which are heterogeneous in nature and cannot be solved by an individual mathematical model. Over the past forty years, Fuzzy sets, introduced by Zadeh, have paved the way towards the modelling of vagueness, uncertainty, and imprecision information in real-time applications [1]. Based on the basic definition stated by Zadeh, Kauffman developed the initial idea of fuzzy graphs [2]. Fuzzy graphs are now accepted as a mature technology for manipulating perceptions (e.g., perceptions of comfort, safety, health, and size) involving an ambiguous and uncertain environment. It is a weighted graph which provides a normalized relational strength over a fuzzy subset [3]. It is swiftly entering into the majority of engineering fields such as computer science, communication system, electrical and electronics engineering, information coding, image processing, and data mining where the degree of intrinsic data of the
system fluctuates with various levels of accuracy.

Consequently, Bhattacharya [4] presented some valuable insights on fuzzy graphs. Then, Mordeson and Nair studied some operations on fuzzy graphs [5]. Recently, fuzzy graphs having gained more attention from several researchers. Akram et al. extended the notion of fuzzy graphs to interval-valued fuzzy graph [6], intuitionistic fuzzy graphs [7], intuitionistic fuzzy hypergraphs structures [8], and bipolar fuzzy graphs [9]. Samanta et al. presented some concepts on fuzzy planar graphs, irregular bipolar fuzzy graphs, completeness and regularity of the generalized fuzzy graph, and vague graphs [10–12]. However, the main challenge in implementing a fuzzy graph is that a membership degree in [0, 1] is allocated to every individual component in the set for a specific problem. Later, it was agreed that a single membership function might not reflect the uncertainty of a real-time scenario and the intricacy of the information completely. In order to overcome this drawback, Molodtsov proposed soft sets as a novel mathematical tool for handling uncertainty problems.

The soft set delivers a parameterized perspective for modelling and assessing uncertainties [13]. The usage of the soft set is increasing hastily and several scholars are aiming at real-world problems with incomplete and vague data. Molodtsov’s tactic revealed the applicability of soft sets to many more domains and achieved some important results successively improved by other researchers like Maji et al. [14] and Aktas and Cagman [15], and others. Ali et al. discussed some interesting properties of soft sets [16]. Sezgin and Atagun investigated the theoretical characteristics of the soft sets by presenting some basic operations on soft sets [17]. Nowadays, soft sets are finding numerous applications to solve problems with uncertainties.

Off late, several interesting advancements in soft sets have proposed along with the concept of graphs. Ali et al. presented the neighborhoods and soft sets based representation of graph theory [18]. Akram and Nawaz introduced the notion of soft graphs [19, 20]. The concept of the fuzzy soft set also gives a parameterized aspect of soft computing as well as uncertainty modeling. Maji et al. developed the idea of fuzzy soft graphs by integrating the concepts of the soft graphs and fuzzy graphs [21]. Indeed, the concept of the fuzzy soft graph is more generalized than that of the fuzzy graph and the soft graph.

Atanassov conferred the intuitionistic fuzzy set theory, which is the generalization of Zadeh’s fuzzy set [22]. An intuitionistic fuzzy set is pigeonholed by the degree of membership and the degree of non-membership of attributes. The initial idea of the intuitionistic fuzzy graph (IFG) was also developed by Atanassov [23] in 1999. Later, Karunambigai and Parvathy presented IFGs as a special case of Atanassov’s IFG [24]. Since IFG can define the uncertainty of a real-world problem more sensibly and carefully than the fuzzy graphs, large numbers of studies on the IFG have been carried out in recent years.

On the other hand, with the hasty growth of society, the IFGs cannot describe the uncertainty effectively when we experience the conditions where the degree of membership and the non-membership of attributes can be described as a value ranges (i.e., interval) rather than as an exact real number. Atanassov and Gargov integrated the concept of interval number theory with the intuitionistic fuzzy set, which can successfully adapt the dynamic conditions of real-world problems [25]. It facilitates an extra competence to handle ambiguous data and handle non-statistical ambiguity by describing the fluctuations of membership and non-membership degrees and has acted as a key element to model the imprecise system and gained more attention from academician as well as investigators.

The IVIFSG has been found applications in several fields like decision making [26], pattern recognition [27], medical diagnosis [28], graph theory [29], etc. Interval-valued intuitionistic fuzzy soft graph (IVIFSG) is a fusion of IFG and interval theory to represent the uncertainty in the description of the items (vertices) or their relationships (edges) effectively. The theory of IVIFSG generalizes Zadeh’s fuzzy sets with the membership degree $U_x(a)$, the non-membership degree $S_X(a)$ and hesitancy degree (i.e., unknown degree $H_X(a)$) [22, 23, 25]. Based on the definition of IVIFSG, $U_x(a)$, $S_X(a)$ and $H_X(a)$ are intervals, where $U_x(a)$ represents the range of supporting agent (membership function), $S_X(a)$ represents the range of conflicting agent (non-membership function), and $H_X(a)$ represents the range of missing agent. Furthermore, the inferior of $U_x(a)$ denoted as $I(U_x(a))$, is the strong supporting agent of event $X$, the inferior of $S_X(a)$ denoted as $I(S_X(a))$, is the strong conflicting agent of event $X$, the inferior of $H_X(a)$ is the strong missing agent of event $X$, the superior of $H_X(a)$ is the maximum missing agent of the event $X$, and $S(H_X(a)) - I(H_X(a))$ represents the convertible missing agent, where $I(U_x(a)) + I(S_X(a)) + S(H_X(a)) = 1$.

Atanassov has separated the transformable missing agent into two sections: $S(U_x(a)) - I(U_x(a))$ being the missing agent who can be transformed into supporting agent, and $S(S_X(a)) - I(S_X(a))$ being the missing agent which can be transformed into the conflicting agent, where $S(U_x(a)) - I(U_x(a)) + S(S_X(a)) - I(S_X(a)) = S(H_X(a)) - I(H_X(a))$. So, the IVIFSG proposed by Atanassov is based on the point estimate, which means that these intervals can be considered as the result of empirical analysis. But, the magnitudes of the missing agent transformed into the supporting agent or the conflicting agent may not be constants. For instance, $S(U_x(a)) - I(U_x(a))$ is an invariant for one trial, but it may be changed for other cases. Therefore, based on the interval estimate, we propose an IVIFSG model to satisfy the requirements of the real-world problem. This study explores some fundamental operations of IVIFSG and some important characteristics of IVIFSG such as an interval score function, judgement matrix, interval exponential matrix, and possibility degree matrix. Based on these new parameters, a prioritization technique is proposed for making a decision. In this paper,
the properties of IVIFSIs are used to find a required solution from a finite number of possible options evaluated on multiple attributes.

2. Preliminaries and Notations

We start this section by presenting some fundamental notions associated with the interval number theory and IVIFSIs, which will be utilized in this paper.

2.1 Interval Number Theory

In general, Interval number theory is used to represent ambiguous data and/or decision variables [30]. The concept of interval number and its corresponding operations which will be worthwhile in this study are given as follows.

Definition 2.1. Let us assume $R$ is the set of all real numbers, and $a^-, a^+ \in R$. An interval number is denoted as:

$$\tilde{a} = [a^-, a^+]$$

where $a^-$ is a lower limit, $a^+$ is an upper limit of the $\tilde{a}$ and $0 \leq a^- \leq a^+ \leq 1$. The set of all interval-numbers is designated by $I[0, 1]$. The interval $[a^-, a^+]$ is identified with the number $a \in [0, 1]$. The interval values between the lower and upper limits of a fuzzy number provide its degree of ambiguity.

Definition 2.2. If $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+] \in I[0, 1]$, then the addition and subtraction operations of these two interval numbers are defined as follows:

$$\tilde{a} \oplus \tilde{b} = [a^- + b^-, a^+ + b^+]$$  \hspace{1cm} (2.1)

$$\tilde{a} \ominus \tilde{b} = [a^- - b^+, a^+ - b^-]$$  \hspace{1cm} (2.2)

Implication: Given a set of interval numbers $\tilde{a}_1 = [a_1^-, a_1^+]$, $\tilde{a}_2 = [a_2^-, a_2^+]$, ..., $\tilde{a}_i = [a_i^-, a_i^+]$ then the summation of these interval numbers is defined as

$$\sum_{i=1}^{n} (\tilde{a}_i) = \sum_{i=1}^{n} (a_i^-), a_i^+ \right) \right]$$

$$\sum_{i=1}^{n} (\tilde{a}_i) = \sum_{i=1}^{n} (a_i^-), a_i^+ \right)\}$$

From the above implication, we can realize that the interval of $\sum_{i=1}^{n} (\tilde{a}_i)$ is the summation of the interval of every number. The fuzziness increases with the aggregation of interval numbers.

Definition 2.3. If $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+] \in I[0, 1]$, then the multiplication and division operations between two interval numbers are defined as follows:

$$\tilde{a} \oplus \tilde{b} = \left[ (a^- b^-, a^- b^+), (a^+ b^-, a^+ b^+) \right], \max (a^- b^-, a^- b^+, a^+ b^-, a^+ b^+)$$

$$\tilde{a} \ominus \tilde{b} = [a^- b^+ \left[ \frac{1}{b^-}, \frac{1}{b^+} \right], \text{ provided } [b^-, b^+] \neq 0$$

2.2 Interval-valued Intuitionistic Fuzzy Soft Information

Definition 2.4. Intuitionistic fuzzy soft information $\tilde{X}$ in $A$ can be defined as follows [23]:

$$\tilde{X} = \{(a, U_\tilde{X}(a), S_\tilde{X}(a)); x \in A\}$$  \hspace{1cm} (2.6)

where $U_\tilde{X}(a)$ and $S_\tilde{X}(a)$ denote the degree of membership and the non-membership of attributes of the element $a$ to the set $X$ respectively; $A \neq \emptyset$ be a given set; $U_\tilde{X} : A \rightarrow [0, 1]$ and $S_\tilde{X} : A \rightarrow [0, 1]$ meet the constraint $0 \leq U_\tilde{X}(a) + S_\tilde{X}(a) \leq 1$, for each $a \in A$.

Definition 2.5. If $\tilde{X}$ be an intuitionistic fuzzy number, then it can be defined as [31]:

1. It is an intuitionistic fuzzy subset.
2. For any $a_i \in R$ such that $U_\tilde{X}(a_i) = 1$. That is $S_\tilde{X}(a_i) = 0$
3. It has a convex set for the membership function $U_\tilde{X}(a)$, where

$$U_\tilde{X}(\delta a_1 + (1 - \delta) a_2) \geq \min (U_\tilde{X}(a_1), U_\tilde{X}(a_2))$$  \hspace{1cm} \forall a_1, a_2 \in R, \delta \in [0, 1]

4. It has a concave set for the non-membership function $S_\tilde{X}(a)$ where

$$S_\tilde{X}(\delta a_1 + (1 - \delta) a_2) \leq \max (S_\tilde{X}(a_1), S_\tilde{X}(a_2))$$  \hspace{1cm} \forall a_1, a_2 \in R, \delta \in [0, 1]

Definition 2.6. The interval-valued intuitionistic fuzzy information $\tilde{X}$ over a given set $A$ can be denoted as follows [23]:

$$\tilde{X} = \{(a, U_\tilde{X}(a), S_\tilde{X}(a)); a \in A\}$$

where $C \subseteq [0, 1]$ is the set of all closed subintervals of the interval, $U_\tilde{X} : C \subseteq [0, 1], S_\tilde{X} : C \subseteq [0, 1]$ with the condition $0 \leq SU_\tilde{X}(a) + SS_\tilde{X}(a) \leq 1$, $\forall a \in A$.

$U_\tilde{X}(a)$ and $S_\tilde{X}(a)$ are closed intervals for each $a \in A$. The lower and upper limit of $U_\tilde{X}(a)$ and $S_\tilde{X}(a)$ are $U_\tilde{X}(a)$, $U_\tilde{X}(a)$, $S_\tilde{X}(a)$, and $S_\tilde{X}(a)$ correspondingly. The interval-valued intuitionistic fuzzy information $\tilde{X}$ can be defined as

$$\tilde{X} = \{(a, [U_\tilde{X}(a), U_\tilde{X}(a)]), [S_\tilde{X}(a), S_\tilde{X}(a)]; a \in A\}$$

where $0 \leq U_\tilde{X}(a) + S_\tilde{X}(a) \leq 1, U_\tilde{X}(a) \geq 0, S_\tilde{X}(a) \geq 0$.

The hesitancy degree of the IVIFSIs of $a \in A$ in $\tilde{X} = ([U_\tilde{X}(a)U_\tilde{X}(a)], [S_\tilde{X}(a), S_\tilde{X}(a)])$ is

$$H_\tilde{X}(a) = 1 - U_\tilde{X}(a) - S_\tilde{X}(a)$$

$$H_\tilde{X}(a) = \left(1 - U_\tilde{X}(a) - S_\tilde{X}(a), 1 - [U_\tilde{X}(a) - S_\tilde{X}(a)]$$
Some arithmetic operations on IVIFS are given as follows:

Let \( \tilde{X} = ([U^-_X, U^+_X], ([S^-_X, S^+_X]) \) and \( \tilde{Y} = ([U^-_Y, U^+_Y], ([S^-_Y, S^+_Y]) \) be two IVIFNs then,

\[
\tilde{X} \oplus \tilde{Y} = \left( [U^-_X + U^-_Y - U^-_X U^-_Y U^+_X + U^+_X U^+_Y], [S^-_X S^-_Y + S^+_X S^+_Y] \right)
\]

(2.7)

\[
\tilde{X} \otimes \tilde{Y} = \left( [U^-_X U^-_Y, U^+_X U^+_Y], [S^-_X + S^-_Y - S^-_X S^-_Y S^+_X S^+_Y + S^+_X + S^+_Y - S^+_X S^+_Y] \right)
\]

(2.8)

**Definition 2.7.** The individual interval-valued intuitionistic judgement matrix \( \tilde{J} = ([\tilde{U}_{i1}, \tilde{U}_{i2}, \ldots, \tilde{U}_{in}]_{n \times 1}, [\tilde{S}_{i1}, \tilde{S}_{i2}, \ldots, \tilde{S}_{in}]_{n \times 1}) \) can be defined as follows:

\[
\tilde{J} = \left( ([\tilde{U}_{11}, \tilde{U}_{12}, \ldots, \tilde{U}_{1n}], [\tilde{S}_{11}, \tilde{S}_{12}, \ldots, \tilde{S}_{1n}]], \ldots, ([\tilde{U}_{n1}, \tilde{U}_{n2}, \ldots, \tilde{U}_{nn}], [\tilde{S}_{n1}, \tilde{S}_{n2}, \ldots, \tilde{S}_{nn}] \right)
\]

(2.9)

where \( i (i = 1, 2, 3, \ldots, n) \) and \( j (j = 1, 2, 3, \ldots, n) \) are the criterion number. The individual interval-valued intuitionistic judgement matrix plays an important role in decision making process. The reciprocal value of \( \left( [\tilde{U}_{ij}, \tilde{U}_{ij}^+], [\tilde{S}_{ij}, \tilde{S}_{ij}^+] \right) \) is \( \left( [\tilde{S}_{ji}, \tilde{S}_{ji}^+] \right) \).\( \tilde{U}_{ij} \) is the amount of money in one of the best Indian industries for safe investments. Assume there are five potential industries for investment business as given below.

\[
I_n = \{I_1, I_2, I_3, I_4, I_5\}
\]

(2.10)

To prove the effectiveness of the application of interval-valued intuitionistic fuzzy soft information in a real-time scenario, we will consider a multi decision-making problem where there is an investor who needs to invest a certain amount of money in one of the best Indian industries for safe investments. Assume there are five potential industries for investment business as given below.

\[
I_n = \{I_1, I_2, I_3, I_4, I_5\}
\]
industry. The statistical information about industries given in this section is adapted from [32].

In India, agriculture is the main source of revenue for approximately 58% of the population. In the Financial Year (FY) 2018, the GVA (Gross Value Added) estimate by agriculture, fishing and forestry is US$ 271.00 billion (B) (i.e., Rs. 18.53 trillion). The food industry in India is poised for the massive development, raising its involvement towards global food trade every year owing to its mammoth aptitude for value addition, mainly in food processing companies. The grocery and food market in India is ranked as 6th largest in the world, with the contribution of 70% of the trades. Indian food processing sector contributes 32% of the overall food market, one of the major industries in this country and is graded 5th for export, consumption, production, and anticipated progress. It contributes about 6% of total industrial investment, 13% of exports, 8.39% of GVA in agriculture, and 80% of GVA in manufacturing.

In the crop year of 2017-18, food grain yield is expected at 284.83 million tonnes (MT). In 2018-19, the Indian Government is aiming food grain yield of 285.2 MT. Meat production was projected at 7.4 MT in the year of 2017, whereas milk production was 165.4 MT. Since September 2018, the overall seeded area with Kharif harvests in India achieved 105.78M hectares. India is the second leading fruit producer in the world. The yield of horticulture is projected at 314.7 MT in 2018-19. Overall Indian agricultural exports increased to 16.45% between the financial years 2010 and 2018 to achieve US$ 38.21B in the year 2018. The agriculture exports were estimated at US$ 38.54B in FY2019. India is also the leading producer, exporter, and consumer of spices and spice products. Indian tea trades touched a 36 year high of 240.68M kgs in FY17 whereas coffee trades achieved record 3.95 MT in FY18. Indian spice exports touched US$ 3.1B during 2017–18. Indian food and grocery retail trade were worth US$ 380B in FY17.

The Indian automobile sector became the world’s fourth-largest with trades growing 9.5% year-on-year to 4.02M units (exclusive of two-wheelers) in 2017. It was the seventh leading producer of marketable automobiles in 2018. The Two Wheelers sector takes over the market with respect to volume due to an increase in young and middle-class people. Additionally, the increasing interest of the industries in discovering the rural markets further supported the development of this segment. To continue with the increasing demand, many automobile manufacturers have started participating profoundly in different parts of the industry during the last few years. The automobile companies have concerned Foreign Direct Investment (FDI) influxes worth US$ 21.38 B between the period of April 2000 and March 2019.

Indian healthcare domain has become one of the leading segments in terms of employment and revenue. Healthcare domain includes clinical lab tests, medical equipment, hospitals, health insurance, medical tourism, and telemedicine. The healthcare industry in India is rising at a brisk pace owing to its firming coverage, facilities and growing disbursement by private and public stakeholders. In India, the healthcare domain is divided into two most important sectors – private and public. The public (i.e. Government) healthcare domain includes limited secondary and tertiary care organizations in major cities and aiming at delivering rudimentary clinical services through Primary Healthcare Centres (PHCs) in rural regions. The private organizations deliver most of the secondary, tertiary and quaternary care services with the main focus on tier I, tier II, and metro cities. India’s competitive advantage lies in its great pool of proficient medical experts. The country is also cost-competitive as related to its peers in Asia as well as Western countries. The cost of Indian surgical treatment is approximately one-tenth of that in Western Europe or US.

The healthcare industry can upturn three-fold to Rs 8.6 trillion by 2022. The country is realizing 22 to 25% progress in medical tourism and the healthcare sector is projected to achieve US$ 9B by 2020. There is substantial space for improving clinical facilities considering that medical expenditure as a percentage of Gross Domestic Product (GDP) is increasing. The government’s spending on the healthcare domain has increased from 1.2% to 1.4%. Government of India is planning to raise public health expenditure to 2.5% of the GDP by 2025. The Indian global sourcing market remains increasing at a greater leap related to the IT-Business Process Management (IT-BPM) sector. India is the important global sourcing destination, accounting for around 55% market share of the US$ 185-190B global services sourcing business in 2017-18.

IT & ITeS (Information Technology & Information Technology Enabled Services) industries in India have set up over 1,000 international distribution centres in 80 countries. India has become the digital competences hub of the world about 75% of global digital talent present in the country. Indian IT & ITeS sector increased to US$ 181B in 2018-19. Exports from the industry grew to US$ 137B in FY19 whereas native returns (together with hardware) increased to US$ 44B. During the financial year of 2018-19, the expenditure on the Indian IT sector is estimated to increase by 9% to achieve US$ 87.1B. Income from the digital sector is estimated to include 38% of the predicted US$ 350B industry profits by 2025. Indian IT’s core skills and strengths have gained significant attention from several countries. Indian computer hardware and software organizations have attracted increasing Foreign Direct Investment (FDI) influxes worth US$ 37.23B from April 2000 to March 2019 and rank second in an influx of FDI. Top Indian IT companies such as Wipro, Infosys, Tech Mahindra, and TCS are expanding their contributions and showcasing their most important concepts of artificial intelligence, blockchain to users by innovation hubs, research and development centres.

At present, India is ranked as the world’s second-biggest telecommunications market with 1.20B user base and has recorded robust development in the last decade. The mobile
The reformist and liberal policies of the Indian Government have been a key element in consort with robust user demand in the hasty development in the telecom division. The Government has facilitated easy market availability to telecom tools and fair and proactive governing bodies that have guaranteed accessibility of services to telecom user at reasonable charges. The deregulation of Foreign Direct Investment (FDI) standards has enabled the industry one of the top 5 Indian employment opportunity maker. India is the second biggest market in the world in terms of the number of internet users. Since 2018, India has 604.21M internet users. Furthermore, in 2017, the country exceeded the USA to become the second biggest market in the world in terms of total app downloads. India continued as the fastest emergent market for Google Play downloads in the second and third quarter of 2018. Over the next five years, the massive growth of mobile-phone penetration and reduction in data expenses will add 500M new subscribers, generating openings for new businesses in the country.

After analyzing the above statistical facts, the investor must select the most appropriate enterprise based on the following five criteria:

\[ C_n = \{C_1, C_2, C_3, C_4, C_5\} \]

where \(C_1\) represents market size; \(C_2\) represents customer satisfaction; \(C_3\) represents social-economic and ecological impact; \(C_4\) represents annual returns; and \(C_5\) represents risk factors. The decision-maker evaluates these five possible industries \(I_n (n = 1, 2, 3, 4, 5)\) based on their decisions as shown in Table 2. In Table 2, EE represents exactly equal. It is represented by \([0.5, 0.5], [0.5, 0.5]\) and it has the identical reciprocal value \((0.5, 0.5), [0.5, 0.5]\).

### Table 2. The comparison matrices for the main criteria

| \(I_1\) | \(I_2\) | \(I_3\) | \(I_4\) | \(I_5\) |
|---|---|---|---|---|
| \(I_1\) EE | VH, VH, MH | MH, MH, H | MH, EE, ML | AH, VH, VH |
| \(I_2\) EE | L, ML, MH | VL, VL, L | MH, EE, VH |
| \(I_3\) EE | ML, EE, L | VH, HH, MH |
| \(I_4\) EE | VH, VH, MH |
| \(I_5\) EE |

#### Step 2:

The semantic information estimated in Table 2 is transformed into their respective IVIFSI by exploiting Table 1. In this process, the separate IVIFSI judgement matrix \(J_{ij}^n\) for each decision-maker is achieved. The IVIFSI judgement matrix is given in Table 3.

The aggregated value is calculated using Equations (2.10) and (2.11) as follows: The evaluation criteria for \(I_1\) and \(I_2\) comparison are

\[ VH = ([0.65, 0.75], [0.20, 0.25]), \text{ } MH = ([0.55, 0.65], [0.30, 0.35]) \] and \(H = ([0.60, 0.70], [0.25, 0.30])\).

\[ J_{i1} = \left[ \begin{align*}
1 - ((1 - 0.65) \times (1 - 0.65) \times (1 - 0.55))^\frac{1}{2} \\
1 - ((1 - 0.75) \times (1 - 0.75) \times (1 - 0.65))^\frac{1}{2} \\
(0.20 \times 0.20 \times 0.30)^\frac{1}{2}, (0.25 \times 0.25 \times 0.35)^\frac{1}{2}
\end{align*} \right]
= \left[ \begin{align*}
0.62, 0.72 \\
0.23, 0.28
\end{align*} \right]
\]

The evaluation value for \(I_1\) and \(I_3\) comparison are \(MH = ([0.55, 0.65], [0.30, 0.35]), \text{ } EE = ([0.50, 0.50], [0.50, 0.50])\) and \(ML = ([0.30, 0.35], [0.55, 0.65])\).

\[ J_{i1} = \left[ \begin{align*}
1 - ((1 - 0.55) \times (1 - 0.55) \times (1 - 0.60))^\frac{1}{2} \\
1 - ((1 - 0.65) \times (1 - 0.65) \times (1 - 0.70))^\frac{1}{2} \\
(0.30 \times 0.30 \times 0.25)^\frac{1}{2}, (0.35 \times 0.35 \times 0.30)^\frac{1}{2}
\end{align*} \right]
= \left[ \begin{align*}
0.57, 0.67 \\
0.28, 0.33
\end{align*} \right]
\]

The evaluation value for \(I_1\) and \(I_4\) comparison are \(MH = ([0.55, 0.65], [0.30, 0.35]), \text{ } EE = ([0.50, 0.50], [0.50, 0.50])\) and \(ML = ([0.30, 0.35], [0.55, 0.65])\).

\[ J_{i1} = \left[ \begin{align*}
1 - ((1 - 0.55) \times (1 - 0.55) \times (1 - 0.30))^\frac{1}{2} \\
1 - ((1 - 0.65) \times (1 - 0.50) \times (1 - 0.35))^\frac{1}{2} \\
(0.30 \times 0.50 \times 0.55)^\frac{1}{2}, (0.35 \times 0.50 \times 0.65)^\frac{1}{2}
\end{align*} \right]
= \left[ \begin{align*}
0.48, 0.52 \\
0.44, 0.55
\end{align*} \right]
\]

The evaluation value for \(I_1\) and \(I_5\) comparison are \(AH = ([0.70, 0.80], [0.15, 0.20]), VH = ([0.65, 0.75], [0.20, 0.25])\).
Table 3. Calculation of aggregated IVIFSI judgement matrix $\tilde{J}_{g_{ij}}$

|   | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ |
|---|---|---|---|---|---|
| $I_1$ | $[0.50,0.50],[0.50,0.50]$ | $[0.62,0.72],[0.62,0.72]$ | $[0.57,0.67],[0.57,0.67]$ | $[0.28,0.33],[0.28,0.33]$ | $[0.48,0.52],[0.48,0.52]$ |
| $I_2$ | $[0.23,0.28],[0.23,0.28]$ | $[0.50,0.50],[0.50,0.50]$ | $[0.38,0.46],[0.38,0.46]$ | $[0.46,0.54],[0.46,0.54]$ | $[0.22,0.27],[0.22,0.27]$ |
| $I_3$ | $[0.50,0.50],[0.50,0.50]$ | $[0.38,0.46],[0.38,0.46]$ | $[0.50,0.50],[0.50,0.50]$ | $[0.38,0.46],[0.38,0.46]$ | $[0.63,0.73],[0.63,0.73]$ |
| $I_4$ | $[0.28,0.33],[0.28,0.33]$ | $[0.46,0.54],[0.46,0.54]$ | $[0.25,0.30],[0.25,0.30]$ | $[0.50,0.50],[0.50,0.50]$ | $[0.22,0.27],[0.22,0.27]$ |
| $I_5$ | $[0.25,0.30],[0.25,0.30]$ | $[0.60,0.70],[0.60,0.70]$ | $[0.23,0.28],[0.23,0.28]$ | $[0.62,0.72],[0.62,0.72]$ | $[0.50,0.50],[0.50,0.50]$ |

$0.25)$ and $VH =([0.65, 0.75], [0.20, 0.25])$

$\tilde{J}_{g_{15}} = \left\langle \frac{1 - ((1 - 0.70) \times (1 - 0.65) \times (1 - 0.65))^{\frac{1}{2}}}{1 - ((1 - 0.80) \times (1 - 0.75) \times (1 - 0.75))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{13}} = \left\langle \frac{0.15 \times 0.20 \times 0.20^{\frac{1}{2}}}{0.20 \times 0.25 \times 0.25^{\frac{1}{2}}} \right\rangle$

$L =([0.25, 0.30], [0.60, 0.70])$, $ML =([0.30, 0.35], [0.55, 0.65])$ and $MH =([0.55, 0.65], [0.30, 0.35])$

$\tilde{J}_{g_{23}} = \left\langle \frac{1 - ((1 - 0.25) \times (1 - 0.30) \times (1 - 0.55))^{\frac{1}{2}}}{1 - ((1 - 0.30) \times (1 - 0.35) \times (1 - 0.65))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{24}} = \left\langle \frac{0.60 \times 0.55 \times 0.30^{\frac{1}{2}}}{0.70 \times 0.65 \times 0.35^{\frac{1}{2}}} \right\rangle$

The evaluation value for $I_2$ and $I_3$ comparison are

$VH =([0.65, 0.75], [0.20, 0.25])$, $VL =([0.20, 0.25], [0.65, 0.75])$ and $L =([0.25, 0.30], [0.60, 0.70])$.

$\tilde{J}_{g_{25}} = \left\langle \frac{1 - ((1 - 0.25) \times (1 - 0.20) \times (1 - 0.25))^{\frac{1}{2}}}{1 - ((1 - 0.25) \times (1 - 0.25) \times (1 - 0.30))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{24}} = \left\langle \frac{0.65 \times 0.65 \times 0.60^{\frac{1}{2}}}{0.75 \times 0.75 \times 0.70^{\frac{1}{2}}} \right\rangle$

The evaluation value for $I_2$ and $I_4$ comparison are

$MH =([0.55, 0.65], [0.30, 0.35])$, $EE =([0.50, 0.50], [0.50, 0.50])$ and $VH =([0.65, 0.75], [0.20, 0.25])$

$\tilde{J}_{g_{25}} = \left\langle \frac{1 - ((1 - 0.55) \times (1 - 0.50) \times (1 - 0.65))^{\frac{1}{2}}}{1 - ((1 - 0.65) \times (1 - 0.65) \times (1 - 0.65))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{24}} = \left\langle \frac{0.30 \times 0.50 \times 0.20^{\frac{1}{2}}}{0.35 \times 0.50 \times 0.25^{\frac{1}{2}}} \right\rangle$

The evaluation value for $I_2$ and $I_5$ comparison are

$ML =([0.30, 0.35], [0.65, 0.65])$, $EE =([0.50, 0.50], [0.50, 0.50])$ and $L =([0.25, 0.30], [0.60, 0.70])$.

$\tilde{J}_{g_{35}} = \left\langle \frac{1 - ((1 - 0.35) \times (1 - 0.50) \times (1 - 0.30))^{\frac{1}{2}}}{1 - ((1 - 0.35) \times (1 - 0.50) \times (1 - 0.30))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{34}} = \left\langle \frac{0.55 \times 0.50 \times 0.60^{\frac{1}{2}}}{0.65 \times 0.50 \times 0.70^{\frac{1}{2}}} \right\rangle$

The evaluation value for $I_3$ and $I_4$ comparison are

$VH =([0.65, 0.75], [0.20, 0.25])$, $H =([0.60, 0.70], [0.25, 0.30])$ and $MH =([0.55, 0.65], [0.30, 0.35])$

$\tilde{J}_{g_{35}} = \left\langle \frac{1 - ((1 - 0.65) \times (1 - 0.60) \times (1 - 0.55))^{\frac{1}{2}}}{1 - ((1 - 0.75) \times (1 - 0.70) \times (1 - 0.65))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{34}} = \left\langle \frac{0.20 \times 0.25 \times 0.30^{\frac{1}{2}}}{0.25 \times 0.30 \times 0.35^{\frac{1}{2}}} \right\rangle$

The evaluation value for $I_3$ and $I_5$ comparison are

$VH =([0.65, 0.75], [0.20, 0.25])$, $VH =([0.65, 0.75], [0.20, 0.25])$ and $MH =([0.55, 0.65], [0.30, 0.35])$

$\tilde{J}_{g_{35}} = \left\langle \frac{1 - ((1 - 0.65) \times (1 - 0.60) \times (1 - 0.55))^{\frac{1}{2}}}{1 - ((1 - 0.75) \times (1 - 0.70) \times (1 - 0.65))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{34}} = \left\langle \frac{0.20 \times 0.25 \times 0.30^{\frac{1}{2}}}{0.25 \times 0.30 \times 0.35^{\frac{1}{2}}} \right\rangle$

The evaluation value for $I_4$ and $I_5$ comparison are

$\tilde{J}_{g_{45}} = \left\langle \frac{1 - ((1 - 0.65) \times (1 - 0.65) \times (1 - 0.55))^{\frac{1}{2}}}{1 - ((1 - 0.75) \times (1 - 0.75) \times (1 - 0.65))^{\frac{1}{2}}} \right\rangle$

$\tilde{J}_{g_{44}} = \left\langle \frac{0.20 \times 0.20 \times 0.30^{\frac{1}{2}}}{0.25 \times 0.25 \times 0.35^{\frac{1}{2}}} \right\rangle$

Step 3: Using Equation (2.12), the score judgement matrix $\tilde{S}_{g_{ij}}$ is calculated and given in Table 4. The score judgement
values are calculated from their \( J_{gij} \) values as follows:

\[
\tilde{S}_{g12} = \left[ (0.62 - 0.28), (0.72 - 0.28) \right] = [0.34, 0.44]
\]
\[
\tilde{S}_{g13} = \left[ (0.57 - 0.33), (0.67 - 0.28) \right] = [0.24, 0.39]
\]
\[
\tilde{S}_{g14} = \left[ (0.48 - 0.55), (0.52 - 0.44) \right] = [-0.07, 0.08]
\]
\[
\tilde{S}_{g15} = \left[ (0.67 - 0.23), (0.77 - 0.18) \right] = [0.44, 0.59]
\]
\[
\tilde{S}_{g23} = \left[ (0.38 - 0.54), (0.46 - 0.46) \right] = [-0.16, 0.00]
\]
\[
\tilde{S}_{g24} = \left[ (0.22 - 0.73), (0.27 - 0.63) \right] = [-0.51, -0.36]
\]
\[
\tilde{S}_{g25} = \left[ (0.57 - 0.35), (0.65 - 0.31) \right] = [0.22, 0.34]
\]
\[
\tilde{S}_{g34} = \left[ (0.36 - 0.61), (0.39 - 0.55) \right] = [-0.25, -0.16]
\]
\[
\tilde{S}_{g35} = \left[ (0.60 - 0.30), (0.70 - 0.25) \right] = [0.30, 0.45]
\]
\[
\tilde{S}_{g45} = \left[ (0.62 - 0.28), (0.72 - 0.23) \right] = [0.34, 0.49]
\]

The interval exponential matrix is calculated using Equation (2.13) and presented in Table 4. For example, the interval exponential value [1.40, 1.55] for \( I_1 \) and \( I_2 \) is calculated as follows

\[
\exp_{12} = \left[ e^{0.34}, e^{0.44} \right] = [1.40, 1.55]
\]

Similarly, the pairwise interval exponential values for other criteria are calculated and tabulated as shown in Table 5.

**Step 4:** The priority vector of the interval exponential matrix is estimated by Equation (2.14) and given in Table 6.

**Step 5:** Each element of possibility degree matrix

\[
D = (d_{ij})_{n \times n}
\]

is estimated based on the priority vector of the interval exponential matrix by relating the calculated weights in Step 4 using Equation (2.15). For example, the possibility degree value for \( I_1 \) and \( I_2 \) equals to 1.000. It is estimated as

\[
D(\tilde{w}_1 > \tilde{w}_2) = d_{12} = \frac{\max(0, 0.279 - 0.157) - \max(0, 0.223 - 0.195)}{(0.279 - 0.223) + (0.195 - 0.157)} = \frac{0.122 - 0.028}{0.056 + 0.038} = 1
\]

The possibility degree values for all the criteria are calculated and tabulated in Table 7.

**Steps 7:** In Table 7, we rank the weight of attributes using Equation (2.16) and then normalize the calculated weights by Equation (2.17). For example, the ranked and normalized
weights for $I_i$ are estimated as follows:
\[
w_1 = \frac{\sum_{j=1}^{5} d_{ij} - 1}{n} + 0.5 = \frac{(0.5 + 1 + 1 + 0.591 + 1) - 1}{5} + 0.5 = 1.118
\]
\[
w_1^N = \frac{w_1}{\sum_{i=1}^{5} w_i} = \frac{1.118}{4.900} = 0.228
\]

**Table 8.** Normalized weights vector

| $I_i$ | $W_i$ | $W_i^N$ |
|-------|-------|---------|
| $I_1$ | 1.118 | 0.228   |
| $I_2$ | 0.648 | 0.132   |
| $I_3$ | 0.757 | 0.154   |
| $I_4$ | 1.974 | 0.402   |
| $I_5$ | 0.403 | 0.082   |
| Total | 4.900 |         |

**Step 8:** From the result of the evaluation, Industry $I_4$ which has maximum priority weight (i.e., 0.402) is selected as the best option to invest money.

### 4. Conclusion

Graph theory is finding several applications in different science and technology domains such as biotechnology, communication networks, information coding, operations research and many other domains of science and technology. The Intuitionistic fuzzy soft information plays an important role as a numerical tool for modeling a real-world problem with multiple attributes. This study demonstrates the application of IVIFSI in a real-time scenario to handle decision-making problems in the multi-criteria environment. According to the interval score function, some advanced notions related to IVIFSI such as judgement matrix, interval exponential matrix and possibility degree matrix of IVIFSI are defined. Based on these new parameters, a prioritization technique is proposed for making a decision. In the end, a real-time scenario is given where there is an investor who wants to invest a certain amount of money in the best Indian industries for safe investments. The proposed framework provides the procedure to select an appropriate industry for investment business based on specified multiple criteria.

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