Uncertainties in nuclear transition matrix elements for neutrinoless $\beta\beta$ decay II: the heavy Majorana neutrino mass mechanism

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Employing four different parametrizations of the pairing plus multipolar type of effective two-body interaction and three different parametrizations of Jastrow-type of short range correlations, the uncertainties in the nuclear transition matrix elements $M^0_{\nu \nu}$ due to the exchange of heavy Majorana neutrino for the $0^+ \rightarrow 0^+$ transition of neutrinoless double beta decay of $^{94}$Zr, $^{96}$Zr, $^{98}$Mo, $^{100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128}$Te and $^{150}$Nd isotopes in the PHFB model are estimated to be around 35%. Excluding the nuclear transition matrix elements calculated with Miller-Spenger parametrization of Jastrow short range correlations, the uncertainties are found to be smaller than 20%.

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I. INTRODUCTION

In addition to establishing the Dirac or Majorana nature of neutrinos, the observation of $(\beta\beta)_{0\nu}$ decay is a convenient tool to test the lepton number conservation, possible hierarchies in the neutrino mass spectrum, the origin of neutrino mass and CP violation in the leptonic sector. Further, it can also ascertain the role of various gauge models associated with all possible mechanisms, namely the exchange of light neutrinos, heavy neutrinos, the right handed currents in the left-right symmetric model (LRSM), the exchange of sleptons, neutralinos, squarks and gluinos in the R$\nu$-violating minimal super symmetric standard model, the exchange of leptoquarks, existence of heavy sterile neutrinos, compositeness, extradimensional scenarios and Majoron models, allowing the occurrence of $(\beta\beta)_{0\nu}$ decay. Stringent limits on the associated parameters have already been extracted from the observed experimental limits on the half-life of $(\beta^-\beta^-)_{0\nu}$ decay [1] and presently, all the experimental attempts are directed for its observation. The experimental and theoretical studies devoted to $(\beta\beta)_{0\nu}$ decay over the past decades have been recently reviewed by Avignone et al. [2] and references there in.

Presently, there is an increased interest to calculate reliable NTMEs for $(\beta^-\beta^-)_{0\nu}$ decay due to the exchange of heavy Majorana neutrinos, in order to ascertain the dominant mechanism contributing to it $\eta_3$ $\eta_4$. The lepton number violating $(\beta^-\beta^-)_{0\nu}$ decay has been studied by Vergados by taking a Lagrangian consisting of left-handed as well as right-handed leptonic currents $\eta_5$. In the QRPA, the $(\beta^-\beta^-)_{0\nu}$ decay due to the exchange of heavy Majorana neutrinos has been studied by Tomoda $\eta_6$. The decay rate of $(\beta^-\beta^-)_{0\nu}$ mode in the LRSAM has been derived by Doi and Kotani $\eta_7$. Hirsch et al. $\eta_8$ have calculated all the required nuclear transition matrix elements (NTMEs) in the QRPA and limits on the effective light neutrino mass $\langle m_{\nu}\rangle$, heavy neutrino mass $\langle M_N\rangle$, right handed heavy neutrino $\langle M_R\rangle$, $\langle \lambda \rangle$, $\langle \eta \rangle$ and mixing angle $\tan \xi$ have been obtained. The heavy neutrino mechanism has also been studied in the QRPA without $\eta_9$ and with pn-pairing $\eta_{10}$. In the heavy Majorana neutrino mass mechanism, Šimkovic et al. $\eta_{11}$ have studied the role of induced weak magnetism and pseudoscalar terms and it was found that they are quite important in $^{48}$Ca nucleus. The importance of the same induced currents in both light and heavy Majorana neutrino exchange mechanism has also been studied using the pn-QRPA $\eta_{12}$ as well as SRQRPA $\eta_{3}$.

In spite of the remarkable success of the large scale shell model (LSSM) calculations of Strassbourg-Madrid group $\eta_{13}$, there is a necessity of large configuration mixing to reproduce the structural complexity of medium and heavy mass nuclei. On the other hand, the QRPA and its extensions have emerged as successful models by including a large number of basis states and in correlating the single-β GT strengths and half-lives of $(\beta^-\beta^-)_{2\nu}$ decay in addition to explaining the observed suppression of $M_{2\nu}$ $\eta_{14,15}$. In the mass region $90 \leq A \leq 150$, there is a subtle interplay of pairing and quadrupolar correlations and their effects on the NTMEs of $(\beta^-\beta^-)_{0\nu}$ decay have been studied in the interacting shell model (ISM) $\eta_{16,17}$, deformed QRPA model $\eta_{18,21}$, and projected-Hartree-Fock-Bogoliubov (PHFB) model $\eta_{22,24}$.

The possibility to constrain the values of the gauge parameters using the measured lower limits on the $(\beta^-\beta^-)_{0\nu}$ decay half-lives relies heavily on the model dependent NTMEs. Different predictions are obtained by employing different nuclear models, and within a given model, varying the model space, single particle energies (SPEs) and effective two-body interaction. In addition, a number of issues regarding the structure of NTMEs,
namely the effect of pseudoscalar and weak magnetism terms on the Fermi, Gamow-Teller and tensorial NTMEs \cite{24,25}, the role of finite size of nucleons (FNS) as well as short range correlations (SRC) vis-a-vis the radial evolution of NTMEs \cite{16,26,28} and the value of the axial-vector coupling constant $g_A$ are also the sources of uncertainties and remain to be investigated.

It was observed by Vogel \cite{29} that in case of well studied $^{76}$Ge, the calculated decay rates $T^D_{1/2}$ differ by a factor of 6-7 and consequently, the uncertainty in the effective neutrino mass $\langle m_\nu \rangle$ is about 2 to 3. Thus, the spread between the calculated NTMEs can be used as the measure of the theoretical uncertainty. In case the $(\beta\beta)_{0\nu}$ decay of different nuclei will be observed, Bilenky and Grifols \cite{30} have suggested that the results of calculations of NTMEs of the $(\beta\beta)_{0\nu}$ decay can be checked by comparing the calculated ratios of the corresponding NTMEs-squared with the experimentally observed values.

Bahcall et al. \cite{31} and Avignone et al. \cite{32} have calculated averages of all the available NTMEs, and their standard deviation is taken as the measure of theoretical uncertainty. On the other hand, Rodin et al. \cite{33} have calculated nine NTMEs with three sets of basis states and three realistic two-body effective interactions of charge dependent Bonn, Argonne and Nijmegen potentials in the QRPA as well as RQRPA and estimated the theoretical uncertainties by making a statistical analysis. It was noticed that the variances are substantially smaller than the average values and the results of QRPA, albeit slightly larger, are quite close to the RQRPA values. Faessler and coworkers have further studied uncertainties in NTMEs due to short range correlations using unitary correlation operator method (UCOM) \cite{26} and self-consistent coupled cluster method (CCM) \cite{27}.

The PHFB model has the advantage of treating the pairing and deformation degrees of freedom on equal footing and projecting out states with good angular momentum. However, the single $\beta$ decay rates and the distribution of GT strength, which require the structure of the intermediate odd $Z$-odd $N$ nuclei, can not be studied in the present version of the PHFB model. In spite of this limitation, the PHFB model in conjunction with pairing plus quadrupole-quadrupole (PQQ) \cite{34} has been successfully applied to reproduce the lowest yrast states, electromagnetic properties of the parent and daughter nuclei, and the measured $(\beta\beta)^{(-)}_{2\nu}$ decay rates \cite{35,36}.

In the PHFB formalism, the existence of an inverse correlation between the quadrupole deformation and the size of NTMEs $M_{2\nu}$, $M_{(0\nu)}$ and $M_{(0\nu)}^{(0\nu)}$ has been observed \cite{22,23}. Further, it has been noticed that the NTMEs are usually large for a pair of spherical nuclei, almost constant for small deformation, suppressed depending on the difference in the deformation $\Delta \beta_2$ of parent and daughter nuclei and having a well defined maximum when $\Delta \beta_2 = 0$ \cite{22,23}.

In Ref. \cite{37}, a statistical analysis was performed for extracting uncertainties in eight (twelve) NTMEs for $(\beta^+\beta^-)_{0\nu}$ decay due to the exchange of light Majorana neutrino, calculated in the PHFB model with four different parameterizations of pairing plus multipolar type of effective two-body interaction \cite{23} and two (three) different parametrization of Jastrow type of SRC \cite{27}. In confirmation with the observation made by Simkovic et al. \cite{27}, it was noticed that the Miller-Spencer type of parametrization is a major source of uncertainty and its exclusion reduces the uncertainties from 10%-15% to 4%-14%. Presently, the same procedure has been adopted to estimate the theoretical uncertainties associated with the NTMEs $M_{N}^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay due to the exchange of heavy Majorana neutrino. In Sec. II, a brief discussion of the theoretical formalism is presented. The results for different parameterizations of the two-body interaction and SRC vis-a-vis radial evolution of NTMEs are discussed in Sec III. In the same section, the averages as well as standard deviations are calculated for estimating the theoretical uncertainties. Finally, the conclusions are given in Sec. IV.

II. THEORETICAL FORMALISM

In the charged current weak processes, the current-current interaction under the assumption of zero mass neutrinos leads to terms which, except for vector and axial vector parts, are proportional to the lepton mass squared, and hence negligible. However, it has been reported by Simkovic et al. \cite{24,25} that the contribution of the pseudoscalar term is equivalent to a modification of the axial vector current due to PCAC and greater than the vector current. The contributions of pseudoscalar and weak magnetism terms in the mass mechanism can change $M_{(0\nu)}$ up to 30% and the change in $M_{(0\nu)}^{(0\nu)}$ is considerably larger. In the shell-model \cite{16,38}, IBM \cite{39} and GCM+PNAMP \cite{40}, the contributions of these pseudoscalar and weak magnetism terms to $M_{(0\nu)}$ have been also investigated. However, it has been reported by Suhrmann and Civitarese \cite{41} that these contributions are relatively small and can be safely neglected. Therefore, the investigation of this issue is of definite interest and is reported in the present work.

In the two nucleon mechanism, the half-life $T^\nu_{1/2}$ for the $0^+ \rightarrow 0^+$ transition of $(\beta^-\beta^-)_{0\nu}$ decay due to the exchange of heavy Majorana neutrino between nucleons having finite size is given by \cite{6,7}

$$T^{\nu}_{1/2} (0^+ \rightarrow 0^+) = \left( \frac{m_p}{\langle M_N \rangle} \right) \quad G_{01} |M_{(0\nu)}^{(0\nu)}|^2, \quad (1)$$

where $m_p$ is the proton mass and

$$\langle M_N \rangle^{-1} = \sum_{i} U_{ei}^2 m_i^{-1}, \quad m_i > 1 \text{ GeV}, \quad (2)$$

and in the closure approximation, the NTMEs $M_{N}^{(0\nu)}$ is of the form \cite{12,20,27}.
where

\[
M_N^{(0\nu)} = -M_{Fh} + M_{GT h} + M_{T h},
\]

(3)

and

\[
M_a = \sum_{n,m} \left\langle \hat O_{\alpha nm} \hat\tau_n^+ \tau_m^+ \right| \hat O_{\alpha nm} \right\rangle
\]

(4)

with

\[
O_{Fh} = H_{Fh}(r_{nm}),
\]

(5)

\[
O_{GT h} = \sigma_n \sigma_m H_{GT h}(r_{nm}),
\]

(6)

\[
O_{T h} = [3(\sigma_n \cdot \hat r_{nm})(\sigma_m \cdot \hat r_{nm}) - \sigma_n \cdot \sigma_m] H_{GT h}(r_{nm})
\]

(7)

The exchange of heavy Majorana neutrinos gives rise to short ranged neutrino potentials, which with the consideration of FNS are given by

\[
H_{\alpha h}(r_{nm}) = \frac{2R}{(m_p m_e)\pi} \int f_{\alpha h}(q r_{nm}) h_\alpha(q) q^2 dq
\]

(8)

where \( f_{\alpha h}(q r_{nm}) = j_0(q r_{nm}) \) for \( \alpha = F \) as well as \( GT \).

Further, the \( h_F(q), h_{GT}(q) \) and \( h_T(q) \) are written as

\[
h_F(q) = \left( \frac{g_v}{g_A} \right)^2 \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^4 \]

(9)

\[
h_{GT}(q) = \frac{g_A^2(q^2)}{g_A^2} \left[ 1 - \frac{2}{3} \frac{g_p(q^2)q^2}{g_A(q^2)^2 m_p} + \frac{1}{3} \frac{g_p^2(q^2)q^4}{g_A^2(q^2)^2 4 m_p^2} + \frac{2}{3} \frac{g_M^2(q^2)q^2}{g_A^2(q^2)^2 4 m_p^2} \right]
\]

\[
\approx \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^4 \left[ 1 - \frac{2}{3} \frac{q^2}{(q^2 + m_{\pi}^2)^2} + \frac{1}{3} \frac{q^4}{(q^2 + m_{\pi}^2)^2} \right] + \left( \frac{g_v}{g_A} \right)^2 \frac{\kappa^2 q^2}{6 m_p^2} \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^4
\]

(10)

\[
h_T(q) = \frac{g_A^2(q^2)}{g_A^2} \left[ \frac{2}{3} \frac{g_p(q^2)q^2}{g_A(q^2)^2 2 m_p} - \frac{1}{3} \frac{g_p^2(q^2)q^4}{g_A^2(q^2)^2 4 m_p^2} + \frac{1}{3} \frac{g_M^2(q^2)q^2}{g_A^2(q^2)^2 4 m_p^2} \right]
\]

\[
\approx \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^4 \left[ \frac{2}{3} \frac{q^2}{(q^2 + m_{\pi}^2)} - \frac{1}{3} \frac{q^4}{(q^2 + m_{\pi}^2)^2} \right] + \left( \frac{g_v}{g_A} \right)^2 \frac{\kappa^2 q^2}{12 m_p^2} \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^4
\]

(11)

where the form factors are given by

\[
g_A(q^2) = g_A \left( \frac{\Lambda_A^2}{q^2 + \Lambda_A^2} \right)^2
\]

\[
g_M(q^2) = \kappa g_v \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^2
\]

\[
g_p(q^2) = \frac{2 m_p g_A(q^2)}{(q^2 + m_{\pi}^2)} \left( \frac{\Lambda_A^2 - m_{\pi}^2}{\Lambda_A^2} \right)
\]

(12)

with \( g_v = 1.0, g_A = 1.254, \kappa = \mu_p - \mu_n = 3.70, \Lambda_V = 0.850 \text{ GeV}, \Lambda_A = 1.086 \text{ GeV} \) and \( m_{\pi} \) is the pion mass.

Substituting Eq. (10), Eq. (11) in Eq. (3), there is one term, associated with \( h_F, \) Eq. (9), contributing to \( M_{Fh}, \) while \( M_{GT h} \) has four terms, denoted by \( M_{GT - AA}, M_{GT - AP}, M_{GT - PP} \) and \( M_{GT - MM} \), which correspond to the four terms in \( h_{GT}, \) Eq. (10). The tensor contribution, \( M_{T h}, \) has three terms, denoted by \( M_{T - AP}, M_{T - PP} \) and \( M_{T - MM} \), which correspond to the three terms in \( h_T, \) Eq. (11). Their contributions to the total nuclear matrix element are discussed in Sec. III.

The short range correlations (SRC) arise mainly from the repulsive nucleon-nucleon potential due to the exchange of \( \rho \) and \( \omega \) mesons and have been incorporated by using effective transition operator \([42]\), the exchange of \( \omega \)-meson \([43]\), UCOM \([29, 14]\) and the self-consistent CCM \([27]\). The SRC can also be incorporated phenomenologically by Jastrow type of correlations with Miller-Spencer parametrization \([43]\). Further, it has been shown in the self-consistent CMM \([27]\) that the SRC effects of Argonne and CD-Bonn two nucleon potentials are weak and it is possible to parametrize them by Jastrow type of correlations within a few percent accuracy. Explicitly,

\[
f(r) = 1 - ce^{-ar^2}(1 - br^2)
\]

(13)

where \( a = 1.1, 1.59 \) and \( 1.52 \text{ fm}^{-2}, b = 0.68, 1.45 \) and \( 1.88 \text{ fm}^{-2} \) and \( c = 1.0, 0.92 \) and \( 0.46 \) for Miller-Spencer parametrization, CD-Bonn and Argonne V18 NN potentials, respectively. In this work the NTMEs \( M_N^{(0\nu)} \) are calculated in the PHFB model for the above mentioned three sets of parameters for the SRC, denoted as SRC1, SRC2 and SRC3, respectively.

In Fig.1, we plot the neutrino potential \( H_N(r, \Lambda) = H_{Fh}(r, \Lambda) f(r) \) with the three different parametrizations of SRC. It is noticed, that the potentials due to FNS and FNS+SRC3 are peaked at the origin where as the peaks
The calculation of $M_N^{(0\nu)}$ in the PHFB model has been discussed in our earlier work \[22, 37\] and one obtains the following expression for NTMEs $M_N^{(0\nu)}$ of $(\beta^-\beta^-)_{0\nu}$ decay \[37\].

$$M_N^{(0\nu)} = [n_{J\nu=0}n_{J\nu=0}]^{-1/2} \int_0^\pi d\theta n_{Z,N}(Z+2,N-2)(\theta) \sum_{\alpha\beta\gamma\delta} (\alpha|O_{\alpha}|\beta) \gamma\delta$$

and the expressions for calculating $n_f^l$, $n_{J\nu=0}(Z+2,N-2)(\theta)$, $f_{Z,N}$ and $F_{Z,N}(\theta)$ are given in Refs. \[22, 37\].

The calculation of matrices $f_{Z,N}$ and $F_{Z,N}(\theta)$ requires the amplitudes $(u_{im},v_{im})$ and expansion coefficients $C_{ij,m}$, which specify the axially symmetric HFB intrinsic state $|\Phi_0\rangle$ with $K = 0$. Presently, they are obtained by carrying out the HFB calculations through the minimization of the expectation value of the effective Hamiltonian given by \[22\]:

$$H = H_{sp} + V(P) + V(QQ) + V(HH)$$

where $H_{sp}$, $V(P)$, $V(QQ)$ and $V(HH)$ denote the single particle Hamiltonian, the pairing, quadrupole-quadrupole and hexadecapole-hexadecapole part of the effective two-body interaction, respectively. The $HH$ part of the effective interaction $V(HH)$ is written as \[22\]:

$$V(HH) = -\left(\frac{\chi_4}{2}\right) \sum_{\alpha\beta\gamma\delta} \sum_\nu (-1)^\nu (\alpha|\nu Y_{4,\nu}(\theta,\phi)|\gamma) (\beta|\nu Y_{4,-\nu}(\theta,\phi)|\delta) \ a_{\alpha\beta}^\dagger a_{\delta\gamma}$$

with $\chi_4 = 0.2442 \chi_2 A^{-2/3} b^{-4}$ for $T = 1$, and twice of this value for $T = 0$ case, following Bohr and Mottelson \[40\].

In Refs. \[22, 35, 36\], the strengths of the like particle components $\chi_{pp}$ and $\chi_{nn}$ of the $QQ$ interaction were kept fixed. The strength of proton-neutron ($pn$) component $\chi_{pn}$ was varied so as to reproduce the excitation energy of the $2^+$ state $E_{2^+}$ for the considered nuclei, namely $^{94,96,98,100,102}$Zr, $^{94,96,98,100,104}$Mo, $^{96,100,104}$Ru, $^{104,110}$Pd, $^{110}$Cd, $^{128}$Te, $^{128,130}$Xe, $^{150}$Nd and $^{150}$Sm as closely possible to the experimental values. This is denoted as $PQQ1$ parametrization. Alternatively, one can employ a different parametrization of the $\chi_{2pn}$, namely $PQQ2$ by taking $\chi_{2pp} = \chi_{2nn} = \chi_{2pn}/2$ and the ex-
citation energy $E_{2^+}$ can be reproduced by varying the $\chi_{2qq}$. Adding the $HH$ part of the two-body interaction to $PQQ$ and $PQQQ$ and by repeating the calculations, two more parameterizations of the effective two-body interactions, namely $PQQHH1$ and $PQQHH2$ were obtained [37].

The four different parameterizations of the effective pairing plus multipolar correlations provide us four different sets of wave functions. With three different parameterizations of Jastrow type of SRC and four sets of wave functions, sets of twelve NTMEs $M_N^{(0\nu)}$ are calculated for estimating the associated uncertainties in the present work. The uncertainties associated with the NTMEs $M_N^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay are estimated statistically by calculating the mean and the standard deviation defined by

$$M_N^{(0\nu)} = \frac{\sum_{i=1}^{k} M_N^{(0\nu)}(i)}{N}$$

and

$$\Delta M_N^{(0\nu)} = \frac{1}{\sqrt{N-1}} \left[ \sum_{i=1}^{N} \left( M_N^{(0\nu)} - M_N^{(0\nu)}(i) \right)^2 \right]^{1/2}$$

III. RESULTS AND DISCUSSIONS

The model space, SPE’s, parameters of $PQQ$ type of effective two-body interactions and the method to fix them have already been given in Refs. [22, 32, 33]. It turns out that with $PQQ1$ and $PQQ2$ parameterizations, the experimental excitation energies of the $2^+$ state $E_{2^+}$ [45] can be reproduced within about $2\%$ accuracy. The electromagnetic properties, namely reduced $B(E2; 0^+ \rightarrow 2^+)$ transition probabilities, deformation parameters $\beta_2$, static quadrupole moments $Q(2^+)$ and gyromagnetic factors $g(2^+)$ are in overall agreement with the experimental data [48, 49].

A. Short range correlations and radial evolutions of NTMEs

In the approximation of finite size of nucleons with dipole form factor (F) and finite size plus SRC (F+S), the theoretically calculated twelve NTMEs $M_N^{(0\nu)}$ using the four sets of HFB wave functions generated with $PQQ1$, $PQQHH1$, $PQQQ$ and $PQQHH2$ parameterizations of the effective two-body interaction and three different parameterizations of Jastrow type of SRC for $^{94,96}Zr$, $^{98,100}Mo$, $^{104}Ru$, $^{110}Pd$, $^{128,130}Te$ and $^{150}Nd$ isotopes due to the exchange of heavy Majorana neutrino exchange. (a), (b), (c) and (d) denote $PQQ1$, $PQQHH1$, $PQQ2$ and $PQQHH2$ parameterizations, respectively. See the footnote in page 3 of Ref. [37] for further details.

| Nuclei | F | F+S |
|--------|---|-----|
| $^{94}Zr$ |  |  |
| (a) | 236.9498 | 77.5817 | 138.2606 | 191.3897 |
| (b) | 220.3794 | 72.4285 | 128.7496 | 178.0783 |
| (c) | 205.8370 | 72.9303 | 124.3248 | 168.5705 |
| (d) | 211.0437 | 68.9323 | 122.9710 | 170.3572 |
| $^{96}Zr$ |  |  |
| (a) | 177.7479 | 56.4900 | 109.4434 | 152.8831 |
| (b) | 185.5251 | 59.5338 | 107.2877 | 149.3117 |
| (c) | 170.8199 | 54.2382 | 98.4051 | 137.2870 |
| (d) | 175.4730 | 56.0746 | 101.2963 | 141.1240 |
| $^{98}Mo$ |  |  |
| (a) | 355.1915 | 117.0804 | 208.2494 | 287.5615 |
| (b) | 346.1118 | 116.4967 | 204.5667 | 281.0515 |
| (c) | 358.5109 | 118.0563 | 210.1150 | 290.2080 |
| (d) | 343.4160 | 115.2077 | 202.6977 | 278.7158 |
| $^{100}Mo$ |  |  |
| (a) | 365.8004 | 122.2000 | 215.8888 | 296.9869 |
| (b) | 361.8877 | 122.6611 | 214.7455 | 294.4297 |
| (c) | 368.4056 | 123.2364 | 217.5391 | 299.1598 |
| (d) | 328.9795 | 111.4446 | 195.1601 | 267.5689 |
| $^{104}Ru$ |  |  |
| (a) | 274.0700 | 89.7666 | 160.7925 | 222.1151 |
| (b) | 264.9015 | 88.1515 | 156.2893 | 215.1076 |
| (c) | 258.2796 | 84.6746 | 151.6002 | 209.3600 |
| (d) | 247.0603 | 83.2308 | 145.8435 | 200.6645 |
| $^{110}Pd$ |  |  |
| (a) | 424.6601 | 140.3359 | 249.6835 | 344.1817 |
| (b) | 379.9404 | 127.4915 | 224.6563 | 308.6907 |
| (c) | 407.2163 | 134.6824 | 239.4733 | 330.1888 |
| (d) | 390.3539 | 130.6314 | 230.5392 | 316.9996 |
| $^{128}Te$ |  |  |
| (a) | 190.5325 | 63.2437 | 111.5413 | 154.1796 |
| (b) | 231.8024 | 77.4559 | 136.7936 | 188.1983 |
| (c) | 220.7156 | 73.5158 | 130.0810 | 179.0960 |
| (d) | 235.4814 | 78.6367 | 138.9052 | 191.1366 |
| $^{130}Te$ |  |  |
| (a) | 236.0701 | 81.5493 | 141.3497 | 192.7610 |
| (b) | 231.5921 | 79.3844 | 138.1901 | 188.8492 |
| (c) | 233.0024 | 80.4020 | 139.4400 | 190.2194 |
| (d) | 230.5282 | 78.9288 | 137.9057 | 187.9675 |
| $^{150}Nd$ |  |  |
| (a) | 163.8037 | 55.8968 | 97.8169 | 133.6912 |
| (b) | 130.1364 | 43.8840 | 77.3178 | 105.9993 |
| (c) | 160.2720 | 54.6713 | 95.6942 | 130.8005 |
| (d) | 131.9781 | 44.6741 | 78.5433 | 107.5175 |

The contribution of conventional Fermi matrix elements $M_{Fh} = M_{F-VV}$ is about $20\%$ to the total matrix element.

(ii) The Gamow-Teller matrix element is noticeably modified by the inclusion of the pseudoscalar and weak magnetism terms in the hadronic currents. While $M_{GT-PP}$ increases the absolute value of

100$Mo$ are presented in Table II for $PQQ1$ parametrization. From the inspection of Table II the following observations emerge.
TABLE II: Decomposition of NTMEs for the ($\beta^-
\beta^-$)$_{0\nu}$ decay of $^{100}$Mo including finite size effect (F) and SRC (F+S) for the PQQ1 parametrization.

| NTMEs       | F            | F+S          |
|-------------|--------------|--------------|
| $M_F$       | 68.6223      | 35.8191      |
| $M_{GT-\alpha\alpha}$ | -370.5960    | -44.5650     |
| $M_{GT-\alpha\beta}$ | 174.4640     | 43.3631      |
| $M_{GT-\alpha\beta}$ | -66.3082     | -8.3767      |
| $M_{GT-\alpha\beta}$ | -41.7693     | 16.3949      |
| $M_{F}$     | -300.2095    | -93.1837     |
| $M_{F}$     | 9.4369       | 9.2393       |
| $M_{F}$     | 1.2567       | 1.1163       |
| $M_{F}$     | 7.0314       | 6.8028       |
| $M_N^{(0\nu)}$ | 365.8004     | 122.2000     |

$M_{GT-\alpha\alpha}$, $M_{GT-\alpha\beta}$ has a significant contribution with opposite sign in all cases. The term $M_{GT-\alpha\beta}$ is smaller than others, and the introduction of short range correlations changes its sign.

(iii) The tensor matrix elements have a very small contribution, smaller than 2%, to the total transition matrix elements.

(iv) The inclusion of short range correlations changes the nuclear matrix elements significantly, whose effects are large for the Gamow-Teller and Fermi matrix elements but small in the case of tensor ones.

(v) The Miller-Spencer parameterization of the short range correlations, SRC1, cancels out a large part of the radial function $H_N$, as shown in Fig. 1. The same cancellation reduces the calculated matrix elements to about one third of its original value. The other two parameterizations of the short range correlations, namely SRC2 and SRC3, have a sizable effect, which is in all cases much smaller than SRC1.

With respect to the point nucleon case, the change in $M_N^{(0\nu)}$ is about 30%–34% due to the FNS. With the inclusion of effects due to FNS and SRC, the NTMEs change by about 75%–79%, 58%–62% and 43%–47% for F+SRC1, F+SRC2 and F+SRC3, respectively. It is noteworthy that the SRC3 has practically negligible effect on the finite size case. Further, the maximum variation in $M_N^{(0\nu)}$ due to PQQHH1, PQQ2 and PQQHH2 parametrization with respect to PQQ1 interaction are about 24%, 18% and 26% respectively.

In the QRPA [26,27], ISM [16] and PHFB [28,37], the radial evolution of $M_N^{(0\nu)}$ due to the exchange of light Majorana neutrino has already been studied. In both QRPA and ISM calculations, it has been established that the contributions of decaying pairs coupled to $J = 0$ and $J > 0$ almost cancel beyond $r \approx 3$ fm and the magnitude of $C_N^{(0\nu)}$ for all nuclei undergoing ($\beta^-\beta^-$)$_{0\nu}$ decay have their maximum at about the internucleon distance $r \approx 1$ fm. These observations were also made in the PHFB model [28,37]. Similarly, the radial evolution of $M_N^{(0\nu)}$ can be studied by defining

$$M_N^{(0\nu)} = \int C_N^{(0\nu)}(r) \, dr$$

The radial evolution of $M_N^{(0\nu)}$ has been studied for four cases, namely F, F+SRC1, F+SRC2 and F+SRC3. To make the effects of finite size and SRC more transparent, we plot them for $^{100}$Mo in Fig. 2. In case of finite sized nucleons, the $C_N^{(0\nu)}$ are peaked at $r \approx 0.5$ fm and with the addition of SRC1 and SRC2, the peak shifts to about 0.8 fm. However, the position of peak is shifted to 0.7 fm for SRC3. In Fig. 3, we plot the radial dependence of $C_N^{(0\nu)}$ for six nuclei, namely $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd and the same observations remain valid. Also, the same features in the radial distribution of $C_N^{(0\nu)}$ are noticed in the cases of PQQ2, PQQHH1 and PQQHH2 parametrizations.

B. Uncertainties in NTMEs

The uncertainties associated with the NTMEs $M_N^{(0\nu)}$ for ($\beta^-\beta^-$)$_{0\nu}$ decay are estimated by preforming a statistical analysis by using Eqs. (17) and (18). In Table IV, sets of twelve NTMEs $M_N^{(0\nu)}$ of $^{94,96}$Zr, $^{98,100}$Mo, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes are displayed, which are employed to calculate the average values $\overline{M_N^{(0\nu)}}$ as well as uncertainties $\Delta \overline{M_N^{(0\nu)}}$ tabulated in Table III for the bare axial vector coupling constant $g_A = 1.254$ and quenched value of $g_A = 1.0$. 

![Fig. 2: Radial dependence of $C_N^{(0\nu)}(r)$ for the ($\beta^-\beta^-$)$_{0\nu}$ decay of $^{100}$Mo isotope.](image_url)
It turns out that in all cases, the uncertainties $\Delta M^{(0\nu)}$ are about 35% for $g_A = 1.254$ and $g_A = 1.0$. Further, we estimate the uncertainties for eight NTMEs $M_N^{(0\nu)}$ calculated using the SRC2, and SRC3 parameterizations and the uncertainties in NTMEs reduce to about 16% to 20% with the exclusion of Miller-Spenser type of parametrization. In Table IV, average NTMEs for case II along with NTMEs calculated in other models have been presented. It is noteworthy that in the models employed in Refs. [6, 8, 9], effects due to higher order currents have not been included. We also extract lower limits on the effective mass of heavy Majorana neutrino $\langle M_N \rangle$ from the largest observed limits on half-lives $T_{1/2}^{(0\nu)}$ of $(\beta^- \beta^-)_{0\nu}$ decay. The extracted limits are $\langle M_N \rangle > 5.67^{+0.94}_{-0.94} \times 10^7$ GeV and $> 4.06^{+0.64}_{-0.64} \times 10^7$ GeV, from the limit on half-life $T_{1/2}^{(0\nu)} > 3.0 \times 10^{24}$ yr of $^{130}$Te [56] for $g_A = 1.254$ and $g_A = 1.0$, respectively.

**FIG. 3:** Radial dependence of $C_N^{(0\nu)}(r)$ for the $(\beta^- \beta^-)_{0\nu}$ decay of $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes. In this Fig., (a), (b), (c) and (d) correspond to F, F+SRC1, F+SRC2 and F+SRC3, respectively.

**IV. CONCLUSIONS**

We have employed the PHFB model, with four different parameterizations of pairing plus multipole effective two body interaction, to generate sets of four HFB intrinsic wave functions, which reasonably reproduced the observed spectroscopic properties, namely the yrast spectra, reduced $B(E2:0^+ \rightarrow 2^+)$ transition probabilities, static quadrupole moments $Q(2^+)$ and $g$-factors $g(2^+)$ of participating nuclei in $(\beta^- \beta^-)_{2\nu}$ decay, as well as their $M_{2\nu}$ [35, 36]. Considering three different parameterizations of Jastrow type of SRC, sets of twelve NTMEs $M_N^{(0\nu)}$ for the study of $(\beta^- \beta^-)_{0\nu}$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes in the heavy Majorana neutrino mass mechanism have been calculated.

The study of effects due to finite size of nucleons and SRC reveal that in the case of heavy Majorana neutrino exchange, the NTMEs change by about 30%–34% due to finite size of nucleons and the SRC1, SRC2 and SRC3...
TABLE III: Average NTMEs $\langle M_N^{(0)} \rangle$ and uncertainties $\Delta M_N^{(0)}$ for the $(\beta^- \beta^-)_\nu$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes. Both bare and quenched values of $g_A$ are considered. Case I and Case II denote calculations with and without SRC, respectively.

| Emitters | Case I | Case II |
|----------|--------|---------|
| $^{94}$Zr | 1.254 | 126.2146 | 44.9489 | 152.8378 | 27.1912 |
| 1.0 | 142.9381 | 49.1752 | 172.1620 | 29.3965 |
| $^{96}$Zr | 1.254 | 100.5313 | 36.8858 | 122.5048 | 21.9029 |
| 1.0 | 114.4851 | 40.3246 | 138.6328 | 23.5263 |
| $^{98}$Mo | 1.254 | 202.5006 | 71.6345 | 245.3957 | 41.8882 |
| 1.0 | 230.1520 | 78.3244 | 280.5688 | 49.1588 |
| $^{100}$Mo | 1.254 | 205.7333 | 73.0792 | 250.1870 | 43.1199 |
| 1.0 | 235.0696 | 79.9883 | 282.7964 | 47.1334 |
| $^{104}$Ru | 1.254 | 150.5572 | 53.9389 | 182.7216 | 34.1045 |
| 1.0 | 170.8075 | 59.0467 | 207.1750 | 34.3939 |
| $^{110}$Pd | 1.254 | 231.4743 | 82.4924 | 280.5868 | 49.1588 |
| 1.0 | 263.4339 | 90.3033 | 317.3947 | 53.0150 |
| $^{128}$Te | 1.254 | 126.8285 | 45.3819 | 153.7370 | 29.4676 |
| 1.0 | 143.9772 | 50.6942 | 173.5263 | 31.8554 |
| $^{130}$Te | 1.254 | 136.8566 | 46.9164 | 164.5378 | 27.2226 |
| 1.0 | 154.3797 | 51.2511 | 185.2849 | 29.1907 |
| $^{150}$Nd | 1.254 | 85.5467 | 31.4473 | 103.4294 | 20.9802 |
| 1.0 | 97.3640 | 34.5024 | 117.0160 | 22.8729 |

change them by 75%–79%, 58%–62% and 43%–47%, respectively. Further, it has been noticed through the study of radial evolution of NTMEs that the FNS and SRC play a more crucial role in the heavy than in the light Majorana neutrino exchange mechanism.

Finally, a statistical analysis has been performed by employing the sets of twelve NTMEs $M_N^{(0)}$ to estimate the uncertainties for $g_A = 1.254$ and $g_A = 1.0$. It turns out that the uncertainties are about 35% for all the considered nuclei. Exclusion of Miller-Spenser parametrization of Jastrow type of SRC, reduces the maximum uncertainties to a value smaller than 20%. The best extracted limit on the effective heavy Majorana neutrino mass $\langle M_N \rangle$ from the available limits on experimental half-lives $T^{(0)}_{1/2}$ using average NTMEs $\langle M_N^{(0)} \rangle$ calculated in the PHFB model is $> 5.67^{+0.94}_{-0.94} \times 10^7$ GeV and $> 4.06^{+0.64}_{-0.64} \times 10^7$ GeV for $^{130}$Te isotope.

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TABLE IV: Average NTMEs $\overline{M}_N^{(0\nu)} \left( = (g_A/1.254)^2M_N^{(0\nu)} \right)$ for the $(\beta^{-}\beta^{-})_{0\nu}$ decay of $^{94,96}\text{Zr}$, $^{98,100}\text{Mo}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ isotopes. Both bare and quenched values of $g_A$ are considered. The superscripts $a$ and $b$ denote the Argonne and CD-Bonn potentials.

| $\beta^{-}\beta^{-}$ emitters | $g_A$ | $\overline{M}_N^{(0\nu)}$ | QRPA | QRPA | QRPA | QRPA | SRQRPA$^a$ | SRQRPA$^b$ | $T_{1/2}^{0\nu}$ (yr) | Ref. | $(m_N)$ (GeV) |
|-----------------------------|------|-----------------|-----|-----|-----|-----|-----------|-----------|----------------|-----|----------------|
| $^{94}\text{Zr}$           | 2.54 | 152.84±27.19    | 1.0 | 109.48±18.69 | 99.062 | 9.2×10$^{21}$ | [51] | 2.57×10$^{10}$ |     | 1.34×10$^{4}$      |
| $^{96}\text{Zr}$           | 2.54 | 122.50±21.92    | 1.0 | 88.16±14.96 | 99.062 | 9.2×10$^{21}$ | [51] | 2.68×10$^{6}$ |     | 1.34×10$^{4}$      |
| $^{98}\text{Mo}$           | 2.54 | 245.40±41.89    | 1.0 | 176.33±28.61 | 1.0×10$^{14}$ | [52] | 9.70×10$^{6}$ |     | 1.34×10$^{4}$      |
| $^{100}\text{Mo}$          | 2.54 | 250.19±43.71    | 155.960 | 333.0 | 56.914 | 76.752 | 259.8 | 404.3 | 4.6×10$^{23}$ | [53] | 3.43×10$^{-7}$ |
| $^{110}\text{Pd}$          | 2.54 | 280.57±49.16    | 1.0 | 201.84±29.97 | 191.8 | 310.5 | 2.47×10$^{7}$ |     | 1.75×10$^{4}$      |
| $^{128}\text{Te}$          | 2.54 | 153.74±29.47    | 122.669 | 303.0 | 101.233 | 1.1×10$^{23}$ | [55] | 2.06×10$^{6}$ |     | 1.75×10$^{4}$      |
| $^{130}\text{Te}$          | 2.54 | 164.54±27.22    | 108.158 | 267.0 | 92.661 | 239.7 | 384.5 | 3.0×10$^{24}$ | [56] | 5.67×10$^{7}$ |
| $^{150}\text{Nd}$          | 2.54 | 103.43±20.98    | 153.085 | 422.0 | 1.8×10$^{22}$ | [57] | 5.99×10$^{-8}$ |     | 4.31×10$^{4}$      |

[27] P. Šimkovic, A. Faessler, H. Mütter, V. Rodin, and M. Stauf, Phys. Rev. C 79, 055501 (2009).
[28] P. K. Rath, R. Chandra, K. Chaturvedi, P. K. Raina, and J. G. Hirsch, Phys. Rev. C 80, 044303 (2009).
[29] P. Vogel, in Current Aspects of Neutrino Physics, edited by D. O. Caldwell (Springer, 2001) Chap. 8, p. 177; arXiv: nucl-th/0005020.
[30] S. M. Blenken and J. A. Grifols, Phys. Lett. B550, 154 (2002).
[31] John N. Bahcall, Hitoshi Murayama, and C. Peña-Garay, Phys. Rev. D 70, 033012 (2004).
[32] F. T. Avignone III, G. S. King III, and Yu. G. Zdesenko, New Journal of Physics 7, 6 (2005).
[33] V. A. Rodin, A. Faessler, P. Šimkovic, and P. Vogel, Phys. Rev. C 68, 044302 (2003).
[34] M. Baranger and K. Kumar, Nucl. Phys. A110, 490 (1968).
[35] R. Chandra, J. Singh, P. K. Rath, P. K. Raina, and J. G. Hirsch, Eur. Phys. J. A 23, 223 (2005).
[36] S. Singh, R. Chandra, P. K. Rath, P. K. Raina, and J. G. Hirsch, Eur. Phys. J. A 33, 375 (2007).
[37] P. K. Rath, R. Chandra, K. Chaturvedi, P. K. Raina, and J. G. Hirsch, Phys. Rev. C 82, 064310 (2010).
[38] M. Horoi and S. Stoica, Phys. Rev. C 81, 024321 (2010).
[39] J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009).
[40] T. R. Rodríguez and G. Martínez-Pinedo, Phys. Rev. Lett. 105, 252503 (2010).
[41] J. Suhonen and O. Civitarese, Phys. Lett. B668, 277 (2008).
[42] H. F. Wu, H. Q. Song, T. T. S. Kuo, W. K. Cheng, and D. Strottman, Phys. Lett. B162, 227 (1985).
[43] J. G. Hirsch, O. Castaños, and P. O. Hess, Nucl. Phys. A582, 124 (1995).
[44] M. Kortelainen and J. Suhonen, Phys. Rev. C 76, 024315 (2007); M. Kortelainen, O. Civitarese, J. Suhonen, and J. Toivainen, Phys. Lett. B647, 128 (2007).
[45] A. Miller and J. E. Spencer, Ann. Phys. (NY) 100, 562 (1976).
[46] A. Bohr and B. R. Mottelson, Nuclear Structure Vol. I (World Scientific, Singapore, 1998).
[47] M. Sakai, At. Data Nucl. Data Tables 31, 399 (1984).
[48] P. Raghavan, At. Data Nucl. Data Tables 42, 189 (1989).
[49] S. Raman, C. W. Nestor Jr., and P. Tikkanen, At. Data Nucl. Data Tables 78, 1 (2001).
[50] R. Arnold et al., Nucl. Phys. A658, 299 (1999).
[51] J. Argyriades et al., Nucl. Phys. A847, 168 (2010).
[52] J. H. Frenlin and M. C. Walters, Proc. Phys. Soc. Lond. A 65, 911 (1952).
[53] R. Arnold et al., Phys. Rev. Lett. 95, 182302 (2005).
[54] R. G. Winter, Phys. Rev. 85, 687 (1952).
[55] C. Arnaudoli et al., Phys. Lett. B557, 167 (2003).
[56] C. Arnaudoli et al., Phys. Rev. C 78, 035502 (2008).
[57] J. Argyriades et al., Phys. Rev. C 80, 032501(R) (2009).