Method for determining parameters of the high energy materials burning rate law

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Abstract. Method for determining parameters of the burning rate law including combustion of high energy material samples in a manometric bomb is proposed. This method is based on solving the inverse problem of internal ballistics in an integral formulation. Unlike the prior versions embodiments of the experimental setup, the principle of ignition of a HE sample by a heated spiral is used herein. In the prior versions, the ignition of the sample was carried out with black powder, which introduced errors due to gas formation during the burning of the powder. In addition, heat-insulated inserts are installed on inner walls of manometric bomb to prevent heat losses. The results of the method verification for model high energy material are discussed. Verification of the proposed method for identifying the parameters of the burning rate law was carried out for the model HEM composition with known values of thermophysical and ballistic characteristics. The integral method allows determining parameters of the burning rate law $u, v$ with the error of not more than 3-4% by experimental dependence $p(t)$ measured with a random error of 10%. At the same time errors of determination $u, v$ by the differential method is 14-32%.

1. Introduction

Burning rate is one of the fundamental characteristics of high energy materials (HEMs) – gunpowder, explosives, solid propellants for rocket engines and gas generators. To calculate work processes in various technical systems, data on the parameters of the burning rate law of specific compositions HEM, are needed. Theoretical determination of these parameters is not possible. Some experimental methods are used to identify them [1-7]. One of the widespread method is the use of a manometric variable-pressure bomb (Viel’s bomb) [8-10]. When a test sample is burned, the time dependence of the pressure in the manometric bomb is measured using a low-inertia pressure sensor during combustion. Parameters of the burning rate law are determined by the measured diagram by solving the inverse problem of internal ballistics.

Currently, there are two main approaches to the processing of ballistic experiment data – differential and integral [11-13]. In the first approach, burning rate law is determined by numerically differentiating of dependence $p(t)$ in the second – the equations of the mathematical model of the process are integrated based on the experimental data table $\{t_i, p_i\}$ using simplifying assumptions. With numerical differentiation of experimental data, the possibility of obtaining acceptable results is
limited due to their high sensitivity to experimental errors. Satisfactory results for the differential approach can be obtained only after performing a data smoothing operation – experimental results \{t_i, p_i\}. Besides, most of the data preprocessing techniques introduce certain distortions into the experimental results.

The present work aims to develop a new integral method for solving the inverse problem of internal ballistics for determining the parameters of the burning rate law based on the results of pressure measurement in the Viel’s bomb during the combustion of high-energy material sample.

The paper is structured as follows: integral method based on the use of a direct functional relationship between pressure and time is described in section 2; Verification of the proposed method for identifying the parameters of the burning rate law for gunpowder H is presented in section 3; Section 4 presents an experiment; Section 5 discussed the factors affecting the errors of determination the parameters of the burning rate law; Section 6 presented the conclusions derived from the results of the study.

2. Integral method

In the pressure range \( p = (0.1\div20) \) MPa HEMs burning rate law (the dependence of the stationary burning rate on pressure) is approximated by the formula [14]:

\[
ue(p) = u_1 \left( \frac{p}{p_{atm}} \right)^v ,
\]

where \( u_1 \) – is the linear burning rate at atmospheric pressure \( p_{atm} \), \( v \) – is the exponent in the burning rate law.

The integral method proposed in the article is based on the use of a direct functional relationship (based on conservation laws) between pressure \( p \) and time \( t \), the initial data are directly the data obtained from the experiment. Integral characteristics are less sensitive to experimental errors, since deviations of opposite signs are compensated. In this case, the experimental dependences \( p = p(t) \) can be used without preliminary transformation (smoothing), therefore, the values of the desired parameters of the burning rate law (1) are determined with greater accuracy.

Note that the definition of ”integral" in the name is because the experimental dependencies used are integrals of the differential equations of internal ballistics – in this case, equations of the energy conservation law.

The method is based on solving the inverse problem of internal ballistics for the combustion process of a HEM sample in a Viel’s bomb. Samples with a known constant burning surface area (cylindrical samples armored along the side surface and one end face) are considered. Ignition of the sample is carried out from a heated wire spiral along the entire combustion surface at the same time, which excludes the effect of the products of combustion of the igniter on the combustion process. The walls of the manometric bomb are covered with heat-insulating material to reduce heat loss during combustion.

Without taking into account heat losses and changes in the volume of the manometric bomb \( W \) due to the burning of the HEM sample, the change in the internal energy of the gaseous products of combustion is determined by the equation [14]:

\[
\frac{d}{dt} \left( \rho WC_v T \right) = C_v T_v S_T \rho_T u(p) ,
\]

where \( \rho, T, C_v \) – is density, temperature and isochoric specific heat of combustion products, \( T_v \) – is combustion temperature HEM sample at a constant volume, \( S_T \) – is combustion surface area, \( \rho_T \) – is HEM sample density.

Given the equation of state of Mendeleev-Clapeyron \( p = \rho RT \) and the burning rate law (1), equation (2) can be represented as:
\[
\frac{dp}{dt} = B u_1 \left( \frac{p}{p_{\text{atm}}} \right)^\nu,
\]

(3)

where \( B = S \rho_p k f_p / W = \text{const} \), \( f_p = R T_p \) – is power of gunpowder for HEM, \( T_v = k T_p \) – combustion temperature HEM sample at constant pressure, \( R, k \)– gas constant and an isentropic exponent of combustion products.

Integrating equation (3) by the method of separation of variables with the initial condition \( p = p_0 \) at \( t = t_0 \), we obtain the expression for the dependence \( p(t) \) in the combustion process:

\[
p(t) = \left[ p_0^{1-\nu} + (1-\nu) \frac{B}{p_{\text{atm}}^\nu} u_1 (t-t_0) \right]^{\frac{1}{1-\nu}}.
\]

(4)

From equation (4), we can obtain the expression for parameter \( u_1 \):

\[
u \quad u_1 = \frac{[p_{\text{atm}}^\nu (t) - p_0^\nu]}{(1-\nu) B (t-t_0)}.
\]

(5)

The result of an experiment on burning a HEM sample in a manometric bomb is a table of pressure values over time \( \{t_i, p_i\}, (i = 1, \ldots, n) \). To identify the parameters of the burning rate law (1), the minimum is determined from \( \nu \) the functional \( F(\nu) \):

\[
\min_\nu F(\nu) = \min_\nu \left\{ \frac{1}{\bar{u}_1} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (u_{i1} - \bar{u}_1)^2} \right\}, \quad \bar{u}_1 = \frac{1}{n} \sum_{i=1}^{n} u_{i1}.
\]

(6)

The essence of the proposed method based on the use of functional (6) is that the scatter of the number of parameter values \( u_{i1}(p, t, \nu) \) calculated by equation (5) for the experimental data \( \{t_i, p_i\} \) is minimized when the parameter \( \nu \) is varied.

The presented methodology is implemented in the MS Excel software package. To minimize the scatter of the calculated values for \( u_1 \), objective function \( F(\nu) \) is carried out by the tool "Solver" in the non-linear method, the generalized reduction gradient [12] for varying values of the parameter \( \nu \). The identification parameters of the burning rate law (1) by the functional \( F(\nu) \) uses all the information of the experimental dependence \( p(t) \).

3. Verification of method

Verification of the proposed method for identifying the parameters of the burning rate law was carried out for the model HEM composition with known values of thermophysical and ballistic characteristics: \( \rho_p = 1600 \text{ kg/m}^3, \ f_p = 0.703 \text{ MJ/kg}, \ k = 1.245, \ C_V = 1.206 \text{ kJ/(kg K)}, \ T_p = 2368 \text{ K}, \ R = 297 \text{ J/(kg K)}, u_1 = 1.71 \times 10^{-3} \text{ m/s}, \nu = 0.41 \).

At the first stage, the direct problem of internal ballistics was solved – calculation of \( p(t) \) in a manometric bomb for a model HEM sample with a diameter of 20 mm. Results of calculation by equation (4) are shown in Figure 1 (line). In the second stage, the inverse problem of internal ballistics was solved for the "experimental" dependence \( p(t) \), which was obtained by superimposing random perturbations (± 10%) on the calculated (Figure 1, points).
Figure 1. Dependences \( p(t) \): 1 – computed (line); 2 – «experimental» (points).

The inverse problem was solved by three methods: integral method, differentiation of approximation \( p(t) \) by power function and direct numerical differentiation. The results of the inverse problem solution are given in Table 1.

**Table 1.** The results of the inverse problem solution.

| Parameters | Integral method | Differentiation of approximation \( p(t) \) by power function | Direct numerical differentiation |
|------------|----------------|---------------------------------------------------------------|---------------------------------|
| \( v \)    | 0.42±0.01      | 0.34±0.06                                                     | 0.35±0.05                       |
| \( u_1, \text{ mm/s} \) | 1.64±0.06      | 2.26±0.73                                                     | 2.08±0.44                       |
| Definition error \( v \), % | 2.5            | 17                                                           | 14                              |
| Definition error \( u_1 \), % | 3.5            | 32                                                           | 21                              |

4. **Experimental setup**

The scheme of experimental setup for the implementation of the proposed method is shown in Figure 2 [15].

Figure 2. Schematic of the manometric bomb.
The manometric bomb with a volume $W = 40 \text{ cm}^3$ consists of a cylindrical body 1 with two removable covers 2, 3. The inner surface of the cylindrical body 1 and covers 2, 3 are thermally insulated with textolite inserts 4. A ball valve 5 is installed in cover 3 for pressure relief after the experiment. The test HEM sample 6 (20 mm in diameter and 5 mm in height) is armored with a non-combustible composition 7 on the side surface and one end. The sample was ignited by a flat nichrome spiral 8 arranged over the entire end surface of the sample and connected to the power supply through electrodes 9. The pressure in the manometric bomb was measured by a type LKH-412 strain gauge 10 with the signal amplified by a type AT-1-8 digital amplifier 11 and recorded on a personal computer 12. The sampling rate of the sensor was 1 kHz.

The photograph and designation of the main elements of the experimental setup are shown in Figure 3.

![Figure 3. Photo of the experimental setup.](image)

Unlike the prior versions embodiments of the experimental setup, the principle of ignition of a HEM sample by a heated spiral is used herein. In the prior versions, the ignition of the sample was carried out with black powder, which introduced errors due to gas formation during the burning of the powder. In addition, heat-insulated inserts are installed on inner walls of manometric bomb to prevent heat losses.

5. Discussion of results

Thus, the proposed method of identifying the parameters of the burning rate law makes it possible to determine their values when burning a HEM sample in a manometric bomb for a given range of pressures. The integral method allows determining parameters of the burning rate law $u_1, v$ with the error of not more than 3-4% by experimental dependence $p(t)$ measured with a random error of 10%. At the same time errors of determination $u_1, v$ by the differential method is 14-32%.

It should be noted that these errors are realized at strict fulfillment of assumptions taken at the solution of direct problem, in particular at the absence of heat losses in manometric bomb.
6. Conclusion
Method for determining parameters of the burning rate law including combustion of high energy material samples in a manometric bomb is proposed. This method is based on solving the inverse problem of internal ballistics in an integral formulation. The invention proposes a new scheme of an experimental setup for implementation of the proposed method, which provides for the improvement of accuracy of determining $u_t$, $v$. The results of the method verification for model high energy material are discussed.

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Appendices
HEM – high energy material

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