Fluctuations and Correlations of Conserved Charges near the QCD Critical Point

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(Dated: October 6, 2010)

Abstract

We study the fluctuations and correlations of conserved charges, such as the baryon number, the electric charge and the strangeness, at the finite temperature and nonzero baryon chemical potential in an effective model. The fluctuations are calculated up to the fourth-order and the correlations to the third-order. We find that the second-order fluctuations and correlations have a peak or valley structure when the chiral phase transition takes place with the increase of the baryon chemical potential; the third-order fluctuations and correlations change their signs during the chiral phase transition and the fourth-order fluctuations have two maximum and one minimum. We also depict contour plots of various fluctuations and correlations of conserved charges in the plane of temperature and baryon chemical potential. It is found that higher order fluctuations and correlations of conserved charges are superior to the second-order ones to be used to search for the critical point in heavy ion collision experiments.

PACS numbers: 12.38.Mh, 24.60.Ky, 11.30.Rd, 25.75.Nq

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I. INTRODUCTION

Studies of QCD thermodynamics and QCD phase diagram have attracted lots of attentions in recent years. People believe that deconfined quark gluon plasma (QGP) is formed in ultrarelativistic heavy ion collisions [1–8]. Various field theory models studies [9–17] indicate that there is a critical point in the QCD phase diagram in the plane of temperature and baryon chemical potential [18], which separates the first-order phase transition at high baryon chemical potential from the continuous crossover at high temperature. Although there is no definite evidence that the QCD critical point also exists in the lattice QCD calculations, due to the sign problem at finite chemical potential, some lattice groups find that the QCD critical point maybe exist in the phase diagram, based on extrapolating results at small $\mu/T$ (ratio of the chemical potential and the temperature) to those at large $\mu/T$ [19–21]. In the meantime, experiments with the goal to search for the QCD critical point are planned and underway at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL) and at the Super Proton Synchrotron (SPS) at CERN in Geneva [22–24].

In order to map the QCD phase diagram, locating the QCD critical point is a crucial and vital task. It has been known that the event-by-event fluctuations of various particle multiplicities are enhanced in heavy ion collisions that freeze out near the critical point. Therefore the QCD critical point can be found through the non-monotonic behavior of various fluctuation observables as a function of varying control parameters [25–34]. Particularly, fluctuations of conserved charges, such as the baryon number, electric charge, and strangeness, deserve more attentions. On the one hand, the fluctuations of conserved charges are sensitive to the structure of the thermal strongly interacting matter and behave differently between the hadronic and QGP phases [15, 25, 26, 30, 33]. On the other hand, since the conserved charges are conserved through the evolution of the fire ball, the fluctuations of conserved charges can be measured in heavy ion collision experiments.

In our previous work [36], we have studied the fluctuations and correlations of conserved charges in the 2+1 flavor Polyakov–Nambu–Jona-Lasinio (PNJL) model at finite temperature and zero chemical potential. We made an interesting comparison with the recent lattice calculations which were performed with an improved staggered fermion action at two values of the lattice cutoff with almost physical up and down quark masses and a physical value for
the strange quark mass \[37\]. It has been seen that our calculated results are well consistent with those obtained in lattice calculations, which indicates that the 2+1 flavor PNJL model is well applicable to study the fluctuations and correlations of conserved charges. The validity of this effective model is expected, since the critical behavior of the QCD phase transition is governed by the universality class of the chiral symmetry, which is kept in this model. Furthermore, compared with the conventional Nambu–Jona-Lasinio model, the PNJL model not only has the chiral symmetry and its dynamical breaking mechanism, but also includes the effect of color confinement through the Polyakov loop \[17, 38–40\].

In this work, we will extend our previous work \[36\] to the cases with nonzero baryon chemical potential. Such an extension is quite nontrivial, which allows us to use the fluctuations and correlations of conserved charges to explore the critical behavior of the QCD critical point and to find out the location of the critical point. Furthermore, when the baryon chemical potential does not vanish, the fluctuations and correlations of odd order develop finite values, which are identical to zero at vanishing baryon chemical potential. In this paper we shall pay our most attentions on studies of the non-monotonic behavior of the fluctuations and correlations of conserved charges near the QCD critical point and depict the contour plots of various fluctuations and correlations in the plane of temperature and baryon chemical potential.

This paper is organized as follows. In Sec. II we introduce the fluctuations and correlations of conserved charges and simply review the formalism of the 2+1 flavor PNJL model. In Sec. III we give our calculated numerical results of the fluctuations of conserved charges. In Sec. IV we present the numerical results of the correlations among conserved charges. Our summary and conclusions are given in Sec. V.

II. FLUCTUATIONS AND CORRELATIONS

In this section we focus on the cumulants of the conserved charge multiplicity distributions, which can be expressed as the derivative of the pressure \((P)\) of a thermodynamical system with respective to its chemical potentials corresponding to conserved charges, i.e.

\[
\lambda_{ijk}^{BQS} = \frac{\partial^{i+j+k}(P/T^4)}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k}, \tag{1}
\]
where $T$ is the temperature, and $\mu_{B,Q,S}$ are the chemical potentials for baryon number, electric charge, and strangeness, respectively. They are related with the quark chemical potentials through the following relations,

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \quad \text{and} \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S. \quad (2)$$

where $\mu_{u,d,s}$ are the chemical potentials for $u$, $d$, and $s$ quarks, respectively. Denoting the ensemble average of conserved charge number $N_X$ ($X = B, Q, S$) with $\langle N_X \rangle$, we can obtain the second and higher order fluctuations of the conserved charges as follow

$$\chi_X^2 = \frac{1}{VT^3}\langle \delta N_X^2 \rangle, \quad (3)$$

$$\chi_X^3 = \frac{1}{VT^3}\langle \delta N_X^3 \rangle, \quad (4)$$

$$\chi_X^4 = \frac{1}{VT^3}\left(\langle \delta N_X^4 \rangle - 3\langle \delta N_X^2 \rangle^2\right), \quad (5)$$

where $\delta N_X \equiv N_X - \langle N_X \rangle$ and $V$ is the volume of the system. In the same way, we can also obtain mixed cumulants of the conserved charge distributions, which are known as the correlations among conserved charges. For example,

$$\chi_{XY}^{11} = \frac{1}{VT^3}\langle \delta N_X \delta N_Y \rangle, \quad (6)$$

$$\chi_{XY}^{12} = \frac{1}{VT^3}\langle \delta N_X \delta N_Y^2 \rangle, \quad (7)$$

$$\chi_{XYZ}^{111} = \frac{1}{VT^3}\langle \delta N_X \delta N_Y \delta N_Z \rangle, \quad (8)$$

It should be noted that when the chemical potentials are vanishing, i.e. $\mu_{B,Q,S} = 0$, the fluctuations and correlations of conserved charges in Eq.(1) (also known as generalized susceptibilities) are nonvanishing only when $i + j + k$ is even, but when $\mu_B$ has finite values, which is the case discussed in this paper, the generalized susceptibilities also develop finite values when $i + j + k$ is odd.

We now adopt the 2+1 flavor Polyakov-loop improved NJL model to study the fluctuations and correlations of conserved charges near the QCD critical point, which is a nontrivial extension to our previous work \[36\], where the fluctuations and correlations of conserved charges were calculated in the 2+1 flavor PNJL model at finite temperature but with vanishing chemical potentials. It is interesting to notice that the calculated results in \[36\] are well consistent with those obtained in lattice calculations \[37\], which shows that the 2+1 flavor PNJL model is well applicable to study the cumulants of conserved charge multiplicity.
distributions. Before a detailed study, let us give a brief review on the 2+1 flavor PNJL model for completeness. Details about the model can be found in Ref. [17].

The Lagrangian density for the 2+1 flavor PNJL model is given as

\[
\mathcal{L}_{\text{PNJL}} = \bar{\psi} (i \gamma_\mu D^\mu + \gamma_0 \hat{\mu} - \hat{m}_0) \psi + G \sum_{a=0}^{8} \left[ (\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} i \gamma_5 \tau_a \psi)^2 \right] \\
- K \left[ \text{det}_f(\bar{\psi}(1 + \gamma_5)\psi) + \text{det}_f(\bar{\psi}(1 - \gamma_5)\psi) \right] - \mathcal{U}(\Phi, \Phi^*, T),
\]

where \( \psi = (\psi_u, \psi_d, \psi_s)^T \) is the three-flavor quark field, and

\[
D^\mu = \partial^\mu - iA^\mu \quad \text{with} \quad A^\mu = \delta_0^\mu A^0, \quad A^0 = g A_4 \frac{\lambda_a}{2} = -iA_4,
\]

where \( \lambda_a \)'s are the Gell-Mann matrices in color space and \( g \) is the gauge coupling strength. \( \hat{m}_0 = \text{diag}(m_0^u, m_0^d, m_0^s) \) is the three-flavor current quark mass matrix. Throughout this work, we take \( m_0^u = m_0^d \equiv m_0^l \), while keep \( m_0^s \) being larger than \( m_0^l \), which breaks the \( SU(3)_f \) symmetry. The matrix \( \hat{\mu} = \text{diag}(\mu_u, \mu_d, \mu_s) \) denotes the quark chemical potentials which are related with the conserved charge chemical potentials through relations in Eq. (2).

In the above PNJL Lagrangian, \( \mathcal{U}(\Phi, \Phi^*, T) \) is the Polyakov-loop effective potential, which is expressed in terms of the traced Polyakov-loop \( \Phi = (\text{Tr}_c L)/N_c \) and its conjugate \( \Phi^* = (\text{Tr}_c L^\dagger)/N_c \) with the Polyakov-loop \( L \) being a matrix in color space given explicitly by

\[
L(\vec{x}) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] = \exp[i\beta A_4],
\]

with \( \beta = 1/T \) being the inverse of temperature and \( A_4 = iA^0 \).

In our work, we use the Polyakov-loop effective potential which is a polynomial in \( \Phi \) and \( \Phi^* \) [41], given by

\[
\frac{\mathcal{U}(\Phi, \Phi^*, T)}{T^4} = - \frac{b_2(T)}{2} \Phi \Phi^* - \frac{b_3}{6} (\Phi^3 + \Phi^*^3) + \frac{b_4}{4} (\Phi \Phi^*)^2,
\]

with

\[
b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 + a_3 \left( \frac{T_0}{T} \right)^3.
\]

The parameters in the effective potential are fitted to reproduce the thermodynamical behavior of the pure-gauge QCD obtained from the lattice simulations, and their values are given in Table II. The parameter \( T_0 \) is the critical temperature for the deconfinement phase transition to take place in pure-gauge QCD and \( T_0 \) is chosen to be 270 MeV according to the lattice calculations.
TABLE I: Parameters for the Polyakov-loop effective potential $U$

|   | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $b_3$ | $b_4$ |
|---|------|------|------|------|------|------|
|   | 6.75 | −1.95 | 2.625 | −7.44 | 0.75 | 7.5 |

In the mean field approximation, the thermodynamical potential density for the 2+1 flavor quark system is given by

$$\Omega = -2N_c \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left\{ E_f^0 \theta(L^2 - p^2) + \frac{T}{3} \ln \left[ 1 + 3\Phi e^{-2(E_f^0 - \mu_f)/T} + 3\Phi e^{-3(E_f^0 - \mu_f)/T} \right] + \frac{T}{3} \ln \left[ 1 + 3\Phi e^{-(E_f^0 + \mu_f)/T} + 3\Phi^* e^{-2(E_f^0 + \mu_f)/T} + e^{-3(E_f^0 + \mu_f)/T} \right] \right\} + 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) - 4K\phi_u \phi_d \phi_s + U(\Phi, \Phi^*, T), \quad (14)$$

where $\phi_i$'s ($i = u, d, s$) are the quark chiral condensates, and the energy-momentum dispersion relation is $E_f^0 = \sqrt{p^2 + M_i^2}$, with the constituent mass being

$$M_i = m_i^0 - 4G\phi_i + 2K\phi_j \phi_k. \quad (15)$$

Minimizing the thermodynamical potential in Eq. (14) with respective to $\phi_u, \phi_d, \phi_s, \Phi,$ and $\Phi^*$, we obtain a set of equations for the minimal conditions, which can be solved as functions of temperature $T$ and three conserved charge chemical potentials $\mu_B, \mu_Q,$ and $\mu_S$.

We will use the method of Taylor expansion to compute the fluctuations and correlations of conserved charges in the PNJL model. Before numerical calculations, we should fix the five parameters in the quark sector of the model, whose values used usually in the literatures are those obtained in Ref. [47], $m_0^i = 5.5$ MeV, $m_0^s = 140.7$ MeV, $GA^2 = 1.835$, $KA^5 = 12.36$, and $\Lambda = 602.3$ MeV, which are fixed by fitting the observables $m_\pi = 135.0$ MeV, $m_K = 497.7$ MeV, $m_{\eta'} = 957.8$ MeV, and $f_\pi = 92.4$ MeV.

### III. NUMERICAL RESULTS OF FLUCTUATIONS OF CONSERVED CHARGES

In this section, we are going to present the numerical results for the conserved charge fluctuations in the 2+1 flavor PNJL model. It is found that the QCD critical point is located at about $T_c = 160$ MeV and $\mu_{Bc} = 819$ MeV ($\mu_Q = \mu_S = 0$) with input parameters
FIG. 1: (color online). Quadratic (top), cubic (middle), and quartic (bottom) fluctuations of baryon number as functions of the baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) with several values of temperature in the PNJL model.

given above. This QCD critical point separates the first-order chiral phase transition at high baryon chemical potential from the continuous crossover at high temperature. Our attentions are paid to investigate the behaviors of the conserved charge fluctuations near the critical point, especial their singular behaviors, which shed light on the universal sym-
metry property of the QCD critical point. Furthermore, the singular behaviors of conserved charge fluctuations are also very helpful for searching for the QCD critical point in experiments [34]. In Fig. 1 we show the quadratic, cubic, and quartic fluctuations of baryon number as functions of $\mu_B$ ($\mu_Q = \mu_S = 0$) at several values of temperature calculated in the PNJL model. From the top panel of Fig. 1 one can see that $\chi^B_2$ has a peak structure when the chiral phase transition takes place, and this peak becomes sharper and narrower while moving toward the QCD critical point. We can also clearly notice that the $\chi^B_2$ diverges at the critical point, which is explicitly seen from the curve with the temperature of 160 MeV. According to the definition of cumulants of the conserved charge multiplicity distributions in Eq. (1), we have 

$$\chi^B_3 = \frac{\partial \chi^B_2}{\partial (\mu_B/T)}.$$  

Since $\chi^B_2$ develops a cusp during the chiral phase transition, $\chi^B_3$ changes its sign there, which is clearly shown in the middle panel of Fig. 1. This structure of $\chi^B_3$ was also found by Asakawa et al. [32], who argued that the two sides of the QCD phase boundary can be distinguished by the sign of $\chi^B_3$, therefore the third cumulants carry more information than the second ones. One can also find that $\chi^B_3$ diverges at the QCD critical point. Furthermore, we also calculate the fourth-order fluctuations of the baryon number, and the results are shown in the bottom panel of Fig. 1. Based on the above analysis, it is expected that $\chi^B_4$ carries even more information than $\chi^B_3$, since $\chi^B_4$ has two positive maxima and one negative minimum as shown in Fig. 1. Furthermore, we find that all amplitudes of $\chi^B_2$, $\chi^B_3$, and $\chi^B_4$ grows rapidly when moving toward the QCD critical point and finally diverge there.

In Fig. 2 we plot contours of the quadratic, cubic, and quartic fluctuations of the baryon number as functions of $T$ and $\mu_B$ calculated in the PNJL model. It is clearly seen that the chiral phase transition line in each of the three contour plots is obvious, and more important is that the region near around the QCD critical point also becomes manifest in the three plots, where the contour lines are dense. Therefore, our calculations demonstrate that by employing the second and higher order cumulants of baryon multiplicity distributions, it is possible to search for the QCD critical point. We should emphasize that in heavy ion collision experiments finite size and time effects should also be included [28, 34]. Comparing higher order cumulants $\chi^B_3$ and $\chi^B_4$ with the quadratic one $\chi^B_2$, we observe that the former are superior to the latter in the search for the QCD critical point. This is because only when the location is very near the chiral phase transition line, $\chi^B_3$ and $\chi^B_4$ are nonvanishing, while $\chi^B_2$ still has finite value when the location is far from the chiral phase transition line and in the
FIG. 2: (color online). Contour plots of quadratic (top), cubic (middle), and quartic (bottom) fluctuations of the baryon number as functions of temperature $T$ and baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) in the PNJL model.

Chiral symmetry restored phase as shown in the top panel of Fig. 2. We should emphasize that in the top panel of Fig. 2, there is a gray region in the lower right corner, which is because the quadratic baryon number fluctuation in this region have relative large value. However, this region does not correspond to any singular behavior, since $\chi_B^2$ in this region is not divergent and it has a weak dependence on the temperature and baryon chemical...
potential. This characteristic is quite different from that in the region near the QCD critical point, where $\chi_B^3$ changes rapidly with $T$ and $\mu_B$, and diverges at the QCD critical point, which is reflected by dense contour lines near the critical point in our plots.

![Graphs showing ratios of cubic to quadratic (left panel) and those of quartic to quadratic (right panel) baryon number fluctuations as functions of baryon chemical potential $\mu_B$ at several values of temperature in the PNJL model.]

**FIG. 3:** (color online). Ratios of cubic to quadratic (left panel) and those of quartic to quadratic (right panel) baryon number fluctuations as functions of baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) at several values of temperature in the PNJL model.

In Fig. 3 we show $\chi_B^3/\chi_B^2$ and $\chi_B^4/\chi_B^2$ versus baryon chemical potential at several values of temperature calculated in the PNJL model. From Eq. (3) to Eq. (5) we find

\begin{align}
\frac{\chi_B^3}{\chi_B^2} &= \frac{\langle \delta N_B^3 \rangle}{\langle \delta N_B^2 \rangle}, \\
\frac{\chi_B^4}{\chi_B^2} &= \frac{\langle \delta N_B^4 \rangle - 3\langle \delta N_B^2 \rangle^2}{\langle \delta N_B^2 \rangle^2}.
\end{align}

In fact, $\chi_B^3/\chi_B^2$ and $\chi_B^4/\chi_B^2$ are the skewness and kurtosis of the baryon multiplicity distributions, respectively, which can be extracted from event-by-event fluctuations in heavy ion collision experiments. Furthermore, these ratios are intensive quantities, i.e., being independent of the volume of the system. In Fig. 3 we show that $\chi_B^3/\chi_B^2$ and $\chi_B^4/\chi_B^2$ have the same structure as $\chi_Q^3$ and $\chi_Q^4$, respectively, i.e., $\chi_B^3/\chi_B^2$ change its sign during the chiral phase transition and there are two positive maxima and one negative minimum on $\chi_B^4/\chi_B^2$. Furthermore, it is observed that the amplitudes of $\chi_B^3/\chi_B^2$ and $\chi_B^4/\chi_B^2$ grow rapidly when moving toward the QCD critical point and become divergent at the critical point.

Fig. 4 shows the quadratic, cubic, and quartic cumulants of the electric charge multiplicity distributions as functions of $\mu_B$ calculated in the PNJL model. It is seen that $\chi_Q^0$
FIG. 4: (color online). Quadratic (top), cubic (middle), and quartic (bottom) fluctuations of electric charge as functions of baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) with several values of temperature in the PNJL model.

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are similar to $\chi_3^B$ and $\chi_4^B$, respectively. We should emphasize that once the location deviates from the chiral phase transition line, the higher order fluctuations of electric charge approach zero rapidly, while the second-order fluctuation $\chi_2^Q$ still has finite value even the location is far away from the chiral phase transition line.

FIG. 5: (color online). Contour plots of quadratic (top), cubic (middle), and quartic (bottom) fluctuations of electric charge as functions of temperature $T$ and baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) in the PNJL model.

In Fig. 5 we show the contour plots of the quadratic, cubic, and quartic fluctuations of electric charge as functions of temperature and baryon chemical potential calculated in the PNJL model. From the three plots one can easily recognize the chiral phase transition line, which shows the same features as the contour plots of baryon number fluctuations given
in Fig. 2. However, we find that employing the quadratic fluctuations of electric charge to search for the QCD critical point is not easy, since the critical point in the top panel of Fig. 5 is not obvious. While for the higher order fluctuations of electric charge, it is seen that the QCD critical point in the contour plots of $\chi_3^Q$ and $\chi_4^Q$ is quite distinct. Therefore, higher order fluctuations of electric charge are more appropriate for being used to search for the QCD critical point in heavy ion collision experiments.

Fig. 6 shows the quadratic, cubic, and quartic fluctuations of strangeness versus the baryon chemical potential at several values of temperature calculated in the PNJL model. It is noticed that the second-order and higher order fluctuations of strangeness are also enhanced when moving toward the QCD critical point, which are the same as the fluctuations of baryon number and electric charge. However, contributions to the singularity of the strangeness fluctuations from the QCD critical point are much less than those to the singularity of the baryon number or electric charge fluctuations. In the same way, it is found that higher order fluctuations of strangeness are superior to the second-order one in search for the QCD critical point.

IV. NUMERICAL RESULTS OF CORRELATIONS OF CONSERVED CHARGES

In this section, we are considering the correlations between or among conserved charges in the 2+1 flavor PNJL model. We will show that higher order correlations of conserved charges are sensitive to the critical behaviors related to the QCD critical point and therefore are very appropriate for being employed to search for the critical point. In Fig. 7 we show the dependence of the second-order correlations, i.e., $\chi_{11}^{BQ}$, $\chi_{11}^{BS}$, and $\chi_{11}^{QS}$ on the baryon chemical potential at several values of temperature in the PNJL model. It is found that all the second-order correlations have a non-monotonic behavior as functions of the baryon chemical potential. There is a peak structure on $\chi_{11}^{BQ}$ during the chiral phase transition; $\chi_{11}^{BS}$ is negative in the whole chemical potential region and has a minimum at the phase transition; and $\chi_{11}^{QS}$ also has a negative minimum but its value is positive in the chiral symmetry restored phase. Comparing $\chi_{11}^{BQ}$ with $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$ one can see that $\chi_{11}^{BQ}$ does not vanish only when the thermodynamical system is very near the chiral phase transition, while the magnitude of $\chi_{11}^{BS}$ and $\chi_{11}^{QS}$ increases with the baryon chemical potential in the chiral symmetric phase as shown in the middle and bottom panels of Fig. 7. This is because in the chiral symmetric
FIG. 6: (color online). Quadratic (top), cubic (middle), and quartic (bottom) fluctuations of strangeness as functions of baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) with several values of temperature in the PNJL model.

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FIG. 7: (color online). Second-order correlations between baryon number and electric charge (top), baryon number and strangeness (middle), electric charge and strangeness (bottom) as functions of the baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) with several values of temperature in the PNJL model.

functions of temperature and baryon chemical potential calculated in the PNJL model. It is found that the chiral phase transition line in the three plots is distinct, but the QCD critical point in the plot of $\chi_{11}^{BQ}$ is more apparent than that in $\chi_{11}^{BS}$ or $\chi_{11}^{QS}$. 

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FIG. 8: (color online). Contour plots of the second-order correlations $\chi^{BQ}_{11}$ (top), $\chi^{BS}_{11}$ (middle), and $\chi^{QS}_{11}$ (bottom) as functions of temperature $T$ and baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) in the PNJL model.

In Fig. 9 we plot the third-order correlations $\chi^{BQ}_{21}$, $\chi^{BQ}_{12}$, $\chi^{BS}_{21}$, $\chi^{BS}_{12}$, $\chi^{QS}_{21}$, and $\chi^{QS}_{12}$ as functions of the baryon chemical potential at several values of temperature calculated in the PNJL model. It is seen that all these third-order correlations change their signs at the chiral phase transition, which are the same as the third-order fluctuations of conserved charges. More concretely, $\chi^{BS}_{21}$ and $\chi^{QS}_{21}$ change their signs from negative to positive with the increase of the baryon chemical potential during the chiral phase transition, while other correlations in Fig. 9 changes their signs in the opposite direction. Furthermore, one can observe that the oscillating amplitudes of these third-order correlations all increase rapidly when moving
FIG. 9: (color online). Third-order correlations $\chi^{BQ}_{21}$ (top-left), $\chi^{BQ}_{12}$ (top-right), $\chi^{BS}_{21}$ (middle-left), $\chi^{BS}_{12}$ (middle-right), $\chi^{QS}_{21}$ (bottom-left), and $\chi^{QS}_{12}$ (bottom-right) as functions of the baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) with several values of temperature in the PNJL model.

toward the QCD critical point and diverge at the critical point.

Fig. 10 shows the contour plots of the third-order correlations $\chi^{BQ}_{21}$, $\chi^{BQ}_{12}$, $\chi^{BS}_{21}$, $\chi^{BS}_{12}$, $\chi^{QS}_{21}$, and $\chi^{QS}_{12}$ as functions of temperature and baryon chemical potential calculated in the PNJL model. Comparing Fig. 10 and Fig. 8, one can easily notice that the QCD critical point in Fig. 10 is much more obvious than that in Fig. 8, which means that higher order correlations are more sensitive to the critical behavior of the QCD critical point than the quadratic correlations, and are better to be used for exploring the critical behavior of the QCD critical point in heavy ion collision experiments. As for the six contour plots in Fig. 10 it is easily seen that the QCD critical point in $\chi^{BQ}_{21}$, $\chi^{BS}_{21}$, and $\chi^{QS}_{21}$ is more distinct than that in $\chi^{BQ}_{12}$, $\chi^{BS}_{12}$, and $\chi^{QS}_{12}$. 

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FIG. 10: (color online). Contour plots of the third-order correlations $\chi^{BQ}_{21}$ (top-left), $\chi^{BQ}_{12}$ (top-right), $\chi^{BS}_{21}$ (middle-left), $\chi^{BS}_{12}$ (middle-right), $\chi^{QS}_{21}$ (bottom-left), and $\chi^{QS}_{12}$ (bottom-right) as functions of temperature $T$ and baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) in the PNJL model.

In Fig. 11 we present the last third-order correlation $\chi^{BQS}_{111}$, i.e., the correlation among the baryon number, electric charge, and the strangeness, as a function of the baryon chemical potential with several values of temperature calculated in the PNJL model. We see that $\chi^{BQS}_{111}$ changes its sign from negative to positive during the chiral phase transition and diverges at the QCD critical point. Furthermore, it is noticed that only when the thermodynamical system is near the chiral phase transition, $\chi^{BQS}_{111}$ has nonvanishing value. We also show the corresponding contour plot of the $\chi^{BQS}_{111}$ as the function of $T$ and $\mu_B$ in Fig. 12 and it is seen that the QCD critical point in the contour plot of $\chi^{BQS}_{111}$ is very obvious. Therefore, $\chi^{BQS}_{111}$ is an ideal probe to search for the QCD critical point in heavy ion collision experiments.
FIG. 11: (color online). Third-order correlation among baryon number, electric charge, and strangeness, i.e., $\chi_{111}^{BQS}$ as a function of the baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) with several values of temperature in the PNJL model.

FIG. 12: (color online). Contour plot of the third-order correlation $\chi_{111}^{BQS}$ as the function of temperature $T$ and baryon chemical potential $\mu_B$ ($\mu_Q = \mu_S = 0$) in the PNJL model.
V. SUMMARY AND DISCUSSIONS

We have studied the fluctuations and correlations of conserved charges, i.e., the baryon number, the electric charge and the strangeness, in the 2+1 flavor Polyakov–Nambu-Jona-Lasinio model at finite temperature with nonzero baryon chemical potential. More attentions have been paid on the studies of the non-monotonic behavior of the fluctuations and correlations of conserved charges near the QCD critical point. The fluctuations are calculated up to the fourth-order and the correlations to the third-order.

It has been shown that the second-order fluctuations and correlations have a peak or valley structure when the chiral phase transition takes place with the increase of the baryon chemical potential. As for the higher order fluctuations and correlations of conserved charges, it has been seen that the third-order fluctuations and correlations change their signs during the chiral phase transition and the fourth-order fluctuations have two maximum and one minimum. Furthermore, it has been noticed that the absolute values of the extrema of the fluctuations and correlations at the chiral phase transition increase rapidly when the thermodynamical system moves toward the QCD critical point (in heavy ion collision experiments, which means that the freeze-out point moves toward the QCD critical point), and finally the fluctuations and correlations of conserved charges diverge at the critical point.

We have also explicitly demonstrated the critical behavior by depicting contour plots of the fluctuations and correlations of conserved charges. In these contour plots, one can clearly figure out the chiral phase transition line. Comparing with the second-order fluctuations and correlations, we have found that higher order cumulants, such as the third- and fourth-order fluctuations and the third-order correlations discussed in this paper, are more sensitive to the critical behavior of the QCD critical point. Therefore, we arrive at the conclusion that the higher order fluctuations and correlations of conserved charges are superior to the second-order ones to be used to search for the critical point in heavy ion collision experiments. Particularly, we would like to address that among all the fluctuations and correlations discussed in this paper, the numerical calculations within the 2+1 flavor PNJL model indicate that $\chi^{BQ}_{21}$, $\chi^{BS}_{21}$, $\chi^{QS}_{21}$, and $\chi^{BQS}_{111}$ are the most valuable probes for exploring the QCD critical point.
Acknowledgements

W. J. F. acknowledges financial support from China Postdoctoral Science Foundation No. 20090460534. Y. L. W. is supported in part by the National Science Foundation of China (NSFC) under the grant No. 10821504.

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