Properties of the Intracluster Medium Assuming an Einasto Dark Matter Profile

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Abstract

I investigate an analytical model of galaxy clusters based on the assumptions that the intracluster medium plasma is polytropic and is in hydrostatic equilibrium. The Einasto profile is adopted as a model for the spatial-density distribution of dark matter halos. This model has sufficient degrees of freedom to simultaneously fit X-ray surface brightness and temperature profiles, with five parameters to describe the global cluster properties and three additional parameters to describe the cluster’s cool-core feature. The model is tested with Chandra X-ray data for seven galaxy clusters, including three polytropic clusters and four cool-core clusters. It is found that the model accurately reproduces the X-ray data over most of the radial range. For all galaxy clusters, the data allows one to show that the model is essentially as good as that of Vikhlinin et al. and Bulbul et al., as inferred by the reduced $\chi^2$.

Key words: galaxies: clusters: general – galaxies: clusters: intracluster medium

1. Introduction

Galaxy clusters, the largest known objects with quasi-relaxed structures in the universe, are essential probes for tracing the growth of cosmic structures and testing cosmological models (Mantz et al. 2010; Allen et al. 2011; De Haan et al. 2016). Galaxies, ionized intracluster medium (ICM), and dark matter are considered to be the main components of galaxy clusters. The ICM, the diffuse plasma distributed between galaxies in a cluster, accounts for most of the baryonic constituent of galaxy clusters, with electron temperatures in the approximate range of 1–10 keV. Studies of the ICM provide insights into the formation and evolution of galaxy clusters (see, e.g., Lin et al. 2012; Barnes et al. 2016). Furthermore, the ICM yields valuable information on the structure and mass distribution of galaxy clusters. For example, one can estimate the cluster’s gas and total masses from the ICM gas density and temperature profiles, under the hypothesis of hydrostatic equilibrium, i.e., that the pressure exerted by the gas is equally counterbalanced by the total gravitational potential. From these mass measurements, it is possible to estimate the gas mass fraction. This gas mass fraction can then be used, with a constraint on the baryon matter density from Big Bang nucleosynthesis or cosmic microwave background (CMB) measurements, to set a tight constraint on the total matter density (see, e.g., Ettori et al. 2013; Mantz et al. 2014; Applegate et al. 2016).

Throughout the past few decades, various models have been employed to analyze the structure and morphology of the ICM plasma, most notably the standard isothermal $\beta$-model (Cavaliere & Fusco-Femiano 1976). It is found that the $\beta$-model provides a reasonable fit to the density structure of the ICM plasma within relatively small radii (see, e.g., Mohr et al. 1999; LaRoque et al. 2006). However, this model is limited in its ability to reproduce all observed features of the density distribution of the ICM plasma. This is the case, for example, in relaxed galaxy clusters with central-cuspy profiles (e.g., Pointecouteau et al. 2004). Moreover, deep X-ray observations with Chandra and XMM-Newton found that the mean temperature profile declines on a large scale (see e.g., Vikhlinin et al. 2005; Leccardi & Molendi 2008).

Accordingly, other approaches have been attempted to circumvent these issues. One such approach is to use the equation of state to relate the pressure ($P$), density ($\rho$), and temperature ($T$) of the ICM plasma in thermodynamic equilibrium, i.e., $f(P, \rho, T) = 0$ (Horedt 2004). Under certain physical conditions, one finds that the pressure and density are related by a polytropic equation of state as $P \propto \rho^\gamma$, where $\gamma$ is the polytropic index, with values that are expected to range from 1 for an isothermal gas to 5/3 for an adiabatic gas. With the aid of the polytropic equation of state, several studies introduced a revised version of the $\beta$-model to describe the observable properties of galaxy clusters (see, e.g., Markevitch et al. 1999; Ettori 2000). These studies found that the mass measurements of typical galaxy clusters under the assumption of the polytropic model can vary significantly within the virial regions, when compared to those derived under the isothermal assumption. However, De Grandi & Molendi (2002) concluded that the polytropic $\beta$-model does not provide a reasonable fit to the temperature measurements over the full cluster-radial range. Instead, the authors found that the temperature measurements can be better modeled by a phenomenological broken power law.

To describe a broader range of the ICM-related features, different modified versions of the $\beta$-model have been proposed to model the distribution of the ICM plasma, most notably the Vikhlinin et al. (2006) model. These authors suggested a model with 17 free parameters to reconstruct the observable properties of the ICM gas. According to this model, the three-dimensional profile of the gas density is

$$n_e^2(r) = n_{e0}^2 \frac{(r/r_c)^{1+a}}{(1 + r^2/r_c^2)^{3(1-a)/2}} \frac{1}{(1 + r^3/r_c^3)^{y/3}} + \frac{n_{e0}^2}{(1 + r^2/r_c^2)^{3/2}},$$

(1)

while the gas temperature is given by

$$T_e(r) = T_{e0} \frac{(r/r_c)^{1-a}}{(1 + r^2/r_c^2)^{y/3}} \tau_{cool}(r),$$

(2)
where \( \tau_{\text{cool}}(r) \) is a phenomenological function that can be expressed as (Allen et al. 2001)

\[
\tau_{\text{cool}}(r) = \frac{\xi + (r/r_{\text{cool}})^{2}}{1 + (r/r_{\text{cool}})^{2}},
\]

where \( 0 < \xi < 1 \) is a free parameter, measuring the amount of central cooling, \( a_{\text{cool}} \) is the shape parameter, and \( r_{\text{cool}} \) is the cooling radius. Hence, nine parameters \( (n_{e0}, r_{e1}, \beta_{1}, \alpha, r_{s}, \epsilon, n_{e02}, r_{e2}, \text{ and } \beta_{2}) \) are used to describe the gas density, and eight parameters \( (T_{e0}, \xi, r_{\text{cool}}, a_{\text{cool}}, r_{e}, a, b, \text{ and } c) \) for the gas temperature.

Although the Vikhlinin et al. (2006) model provides a good fit to the density and temperature profiles over the whole radial range, the model is no longer considered physically motivated, but rather is an ad hoc model tailored to the observed data. Moreover, there are many degeneracies between the model’s best-fitting parameters, implying less precise estimates for their values. This could cause a major issue when one deals with relatively few data points or when one attempts to extrapolate outside the range of observed data.

This has motivated more physically grounded models with a limited number of free parameters. Ascasibar & Diego (2008) presented a simple analytical model based on the assumptions that the ICM plasma is spherical symmetry and is in hydrostatic equilibrium. With only five parameters, the authors found that the model can reconstruct the gas density and temperature profiles of galaxy clusters yielded from the best-fitting parameters of Vikhlinin et al. (2006) with less than 20% discrepancy over most of the cluster-radial range.

More recently, Bulbul et al. (2010) proposed an analytical model to describe the observable properties of the diffuse ICM in galaxy clusters based on the assumptions that the ICM plasma is polytropic and is in hydrostatic equilibrium. In the polytropic state, the electron number density, \( n_{e}(r) \), and temperature, \( T_{e}(r) \), of the ICM gas are related using a simple power law (Ascasibar et al. 2003),

\[
n_{e}(r) = n_{e0} \left[ \frac{T_{e}(r)}{T_{e0}} \right]^{\beta}, \tag{4}
\]

where \( n_{e0} \) and \( T_{e0} \) are the central electron density and temperature, respectively, and \( n \) is the polytropic index. Under the assumption of hydrostatic equilibrium, one can relate the properties of the ICM gas to the total gravitational potential, \( \phi(r) \),

\[
\frac{1}{\rho_{e}} \frac{d\rho_{e}(r)}{dr} = - \frac{d\phi(r)}{dr}, \tag{5}
\]

where \( \rho_{e} = \mu n_{e} \) is the electron density, \( \mu \) is the mean mass per particle in units of the proton mass \( m_{p} \), \( P_{e}(r) = n_{e} k_{B} T_{e} \) is the electron pressure, and \( k_{B} \) is the Boltzmann constant.

From Equations (4) and (5), the polytropic temperature distribution can then be derived as a function of the gravitational potential,

\[
T_{e}(r) = - \frac{1}{n + 1} \frac{\mu m_{p}}{k_{B}} \phi(r). \tag{6}
\]

However, Bulbul et al. (2010) dropped the polytropic assumption to account for the gas cooling in the cluster center. Adopting a generalized form of the Navarro et al. (1996) profile (NFW), with density slope in the outer regions controlled by a free parameter \( \beta \), the three-dimensional gas density and temperature profiles derived by Bulbul et al. (2010) are

\[
n_{e}(r) = n_{e0} \left[ \frac{1 + r/r_{s}}{(\beta - 2) r/r_{s} + 1 + r/r_{s}} \right]^n \tau_{\text{cool}}^{-1}(r), \tag{7}
\]

and

\[
T_{e}(r) = T_{e0} \left[ \frac{1 + (r/r_{s})^{\beta-2} - 1}{(\beta - 2) r/r_{s} + 1 + (r/r_{s})^{\beta-2}} \right] \tau_{\text{cool}}(r), \tag{8}
\]

respectively, with five free parameters \( (n_{e0}, T_{e0}, r_{s}, \beta, \text{ and } n) \) to describe global cluster properties and three additional parameters \( (r_{\text{cool}}, a_{\text{cool}}, \text{ and } \xi) \) to describe the cluster’s cool-core feature.

In recent years, however, many observational studies (e.g., Beraldo e Silva et al. 2013; Umetsu et al. 2014) and high-resolution N-body simulations (e.g., Hayashi & White 2008; Dhar & Williams 2010; Dutton & Macció 2014; Klypin et al. 2016) have indicated that the Einasto profile (Navarro et al. 2004) provides a better fit to the spatial-density distribution of dark matter halos than does the NFW profile. In this paper, therefore, I adopt the Einasto profile as a model for dark matter halos instead of the generalized NFW model used by Bulbul et al. (2010). The model presented in this work represents a slight variation in respect to the model introduced by Bulbul et al. (2010). The Einasto profile, which has three parameters, has a logarithmic slope that decreases inward more gradually than the singular two-parameter profiles. Furthermore, this three-parameter model allows the density profile to be tailored to each individual halo, thereby yielding improved fits (see, e.g., Navarro et al. 2004). Adopting this profile, I derive analytical expressions for the thermodynamic properties of the ICM relevant to X-ray observation. The model is tested with X-ray data of a sample of seven galaxy clusters. All of the clusters have sufficient signal-to-noise to enable accurate analysis for the radial profiles of projected gas density and temperature. Moreover, the model is compared with the Vikhlinin et al. (2006) model (Equations (1) and (2)) and the Bulbul et al. (2010) model (Equations (7) and (8)).

This paper is structured as follows: Section 2 describes the model; testing the model with X-ray Chandra data and comparing it with previous analytical models are presented in Section 3. Finally, in Section 4, I discuss and conclude the results. Throughout this paper, I adopt a Λ CDM cosmology with \( \Omega_{\Lambda} = 0.7, \Omega_{m} = 0.3 \), and \( H_{0} = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

2. Modeling of the Properties of the Intracluster Medium

2.1. Einasto Profile

The model developed in this paper is essentially the same as the one by Bulbul et al. (2010) explained in Section 1, but now the generalized NFW profile is replaced by the Einasto profile. A generalized form of the Einasto profile for a spherical density distribution at radius \( r \) is

\[
\rho_{\text{tot}}(r) = \rho_{-2} \exp \left\{ \frac{2}{\alpha} \left[ \left( \frac{r}{r_{-2}} \right)^{\alpha} - 1 \right] \right\}, \tag{9}
\]

where \( r_{-2} \) and \( \rho_{-2} \) are the radius and density at which the logarithmic slope \( d \ln \rho_{\text{tot}}/d \ln r = -2 \), and \( \alpha \) is the shape parameter.

Since dark matter is the dominant component in galaxy clusters, the Einasto profile is a good approximation for the
total density profile. This density profile is further combined with the polytropic and hydrostatic equilibrium assumptions to derive the analytical expressions for the total mass, the electron density, and the electron temperature of the ICM gas.

2.2. Total Mass and Potential Profiles

The total mass of the galaxy cluster, which is mainly made up of dark matter, can be determined by integrating the Einasto profile (Retana-Montenegro et al. 2012)

\[ M_{\text{tot}}(r) = M_0 \gamma \left( \frac{3}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right), \quad (10) \]

where \( \gamma(a, x) \) is the lower incomplete gamma function, and

\[ M_0 = 4\pi \rho_0 r_0^2 \frac{\alpha^{3/\alpha - 1}}{2^{2/\alpha}} \exp \left( \frac{2}{\alpha} \right). \quad (11) \]

Figure 1 shows the total mass profiles for various values of the \( \alpha \) parameter. The total mass profiles are finite since the Einasto density profile cuts off exponentially at large radii.

For a spherically symmetric mass distribution, the cluster’s gravitational potential is related to the total gravitating mass by

\[ \phi(r) = \int \frac{G M_{\text{tot}}(r)}{r^2} dr, \quad (12) \]

where \( G \) is the Newtonian gravitational constant. Using Equation (10), the gravitational potential can be found as (Retana-Montenegro et al. 2012)

\[ \phi(r) = \phi_0 \left[ \left( \frac{2r_0}{\alpha r_{-2}} \right)^{-1/\alpha} \gamma \left( \frac{3}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) + \Gamma \left( \frac{2}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \right], \quad (13) \]

where

\[ \phi_0 = -4\pi G \rho_0 r_0^2 \frac{\alpha^{2/\alpha - 1}}{2^{2/\alpha}} \exp \left( \frac{2}{\alpha} \right), \quad (14) \]

and \( \Gamma(b, x) \) is the upper incomplete gamma function, which is related to the lower incomplete gamma function, \( \gamma(b, x) \), through the complete gamma function,

\[ \Gamma(b) = \Gamma(b, x) + \gamma(b, x). \quad (15) \]

2.3. Temperature and Density Profiles

Using Equations (6) and (13), the polytropic temperature profile becomes

\[ T_{\text{e, poly}}(r) = T_0 \left[ \left( \frac{2r_0}{\alpha r_{-2}} \right)^{-1/\alpha} \gamma \left( \frac{3}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) + \Gamma \left( \frac{2}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \right], \quad (16) \]

where

\[ T_0 = 4\pi G \rho_0 r_0^2 \frac{\mu m_p}{k_B} \frac{\alpha^{2/\alpha - 1}}{2^{2/\alpha}} \exp \left( \frac{2}{\alpha} \right). \quad (17) \]

Taking advantage of the power-law relation between the density and temperature of the ICM plasma (Equation (4)), the polytropic electron density is

\[ n_{\text{e, poly}}(r) = n_0 \left[ \left( \frac{2r_0}{\alpha r_{-2}} \right)^{-1/\alpha} \gamma \left( \frac{3}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \right] + \Gamma \left( \frac{2}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \gamma(n+1), \quad (18) \]

2.4. Pressure Profile

Using the ideal gas law expression, \( P(r) = n_k T_r \), and with the help of Equations (16) and (18), the pressure profile for polytropic clusters is given by

\[ P(r) = P_0 \left[ \left( \frac{2r_0}{\alpha r_{-2}} \right)^{-1/\alpha} \gamma \left( \frac{3}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \right] + \Gamma \left( \frac{2}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \gamma(n+1), \quad (19) \]

where \( P_0 = n_0 k_B T_0 \) is the central pressure.

2.5. Cool-core Component

Several studies have shown that the polytropic model can also be applied to cool-core clusters, after adopting a cool-core-correction temperature profile (see, e.g., Landry et al. 2013). The temperature profile of such clusters that feature a decline in temperature in the central region can be parameterized by

\[ T_{\text{e, cool}}(r) = T_0 \left[ \left( \frac{2r_0}{\alpha r_{-2}} \right)^{-1/\alpha} \gamma \left( \frac{3}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \right] + \Gamma \left( \frac{2}{\alpha}, \frac{2r_0}{\alpha r_{-2}} \right) \gamma(n+1) T_{\text{cool}}(r). \quad (20) \]

Recent X-ray and SZ effect observations (see, e.g., Arnaud et al. 2010; Sayers et al. 2013) have found that the ICM pressure profile, when scaled appropriately, follows a nearly universal shape, suggesting that it is relatively independent of morphology and dynamical state of the ICM gas. Accordingly, following Bulbul et al. (2010), in this work it is assumed that the pressure distribution is the same for polytropic and cool-core.
clusters (Equation (19)). The density profile for cool-core clusters, therefore, can be obtained using $n_{e,\text{cool}}(r) = \frac{L(r)}{k T_{e,\text{cool}}(r)}$,

$$n_{e,\text{cool}}(r) = n_0 \left[ \frac{2r^\alpha}{\alpha r_2^{\alpha-2}} \right]^{-1/\alpha} \frac{3}{\alpha} \frac{2r^\alpha}{\alpha r_2^{\alpha-2}} + \Gamma \left( \frac{2}{\alpha}, \frac{2r^\alpha}{\alpha r_2^{\alpha-2}} \right)^\alpha \tau_{\text{cool}}^{-1}(r).$$

To keep the hydrostatic equilibrium assumption (Equation (5)), these modified temperature (Equation (20)) and density (Equation (21)) profiles require the introduction of a modified version for the total mass enclosed in radius $r$,

$$M_{\text{tot}}(r) = M_0 \frac{3}{\alpha} \frac{2r^\alpha}{\alpha r_2^{\alpha-2}} \tau_{\text{cool}}^{-1}(r).$$

where the $\tau_{\text{cool}}^{-1}(r)$ term is significant only in the central region (see Figure 1).

2.6. Surface Brightness Profile

The X-ray surface brightness, $S_X(R)$ of a galaxy cluster along the line of sight, $dl$, is related to the electron density and temperature distributions of the ICM gas by Suto et al. (1998)

$$S_X(R) = \frac{1}{4\pi(1+z)^4} \int n_e^2(r) \Lambda_{ee}(T_e) dl,$$

where $z$ is the cluster redshift, and $\Lambda_{ee}(T_e)$ is the X-ray spectral emissivity, which depends on temperature and metallicity, and is calculated in units of counts cm$^{-3}$s$^{-1}$ using the average cluster temperature. Substituting Equations (20) and (21) into Equation (23), the radial profiles of the X-ray surface brightness profile can then be obtained by numerical integration.

Figure 2 depicts the radial profiles of the X-ray surface brightness with and without the phenomenological function for different values of the $\alpha$ parameter. The effect of the phenomenological function is observable only in the cluster’s central region.

3. Testing Model with X-Ray Observations

3.1. Data Sample

To test the model, archival Chandra X-ray data of seven galaxy clusters were analyzed. The source name, the Chandra observation identification number, the exposure time, the redshift, the Galactic absorption, and the redshift reference of the cluster sample are listed in Table 1. The selected clusters have X-ray luminosities in the 0.1–2.4 keV band of $L_X,\text{keV} > 4 \times 10^{44}$ erg s$^{-1}$, with redshifts $0.10 < z < 0.60$. These galaxy clusters are selected since they have a regular dynamical activity, and the images have sufficient signal-to-noise to enable accurate analysis for the radial profiles of the projected temperature and X-ray surface brightness. Many of these clusters have been studied in the literature. In Section 3.8, I compare the mass measurements for Abell 1835, estimated from the current work, with those reported in Landry et al. (2013).

3.2. Data Reduction

Data reduction was performed using the Chandra Interactive Analysis of Observations (CIAO) version 4.9, with the latest calibration database (CALDB) version 4.7.3. I reprocessed the Chandra data using the chandra_repro routine to perform the recommended data preparation, such as checking the source coordinate, filtering the event file to good time intervals, removing streak events, and identifying the bad pixels. This script generates an event file and a bad-pixel file. Since all observations were taken in VFAINT mode, events with significant positive pixels at the border of the event island were excluded by further filtering.

As part of the data reduction, background light curves were examined to detect and remove the flaring periods. For the background data set, the light curve is generated in the 0.3–12.0 keV energy band, following the recommendations given by Markevitch et al. (2003). The light curve is analyzed using the lc_sigma_clip routine provided by the PYTHON script light curves.py. This routine removes data points that lie outside a certain sigma value from the mean count rate (a 3$\sigma$ clip was used in this work).

3.3. Background Subtraction

An important aspect of analysis of extended objects is the background subtraction. For this purpose, blank-sky backgrounds were extracted for all observations, processed, and reprojected onto the sky to match the cluster observation. Although the background spectrum is remarkably stable, there are short-term and secular changes of the background intensity of as much as 30% because of charged-particle events. Following a method similar to that described in Vikhlinin et al. (2005), small adjustments to the background normalization were applied to increase the accuracy of the background, based on data in the energy range 9.5–12.0 keV, where the effective area of the ACIS detector and the source emission are almost zero.
Besides charged-particle events, soft X-ray emission contributes to the blank-sky-background data. This soft background, which is likely to arise from differences in the extragalactic and Galactic foreground emissions between the source and blank-sky observations, was fitted to a thermal-plasma model. The soft background model was then scaled to the cluster-sky area, and was included as a fixed background component in the cluster’s spectral analysis.

3.4. X-Ray Images and Spectra

The X-ray images of the selected clusters were created in the energy range of 0.5–7.0 keV in order to determine the radial surface brightness profiles. This energy range was selected to minimize the high-energy-background particle, which rises significantly at low and high energies. An exposure-corrected image of the cluster’s selected region was created, and passed to the source-detection tool to detect and remove point sources and extended substructures. Count values in the point-source regions were replaced with those interpolated from their background. The surface brightness profiles were then extracted in concentric annuli centered at the Chandra selected center.

Like the surface brightness profiles, X-ray spectra were also extracted in the energy range of 0.5–7.0 keV from concentric annuli centered at the Chandra selected center, after excluding point sources and extended substructures. The X-ray spectrum of every galaxy cluster was fitted to a MEKAL model (Mewe et al. 1985), modified by local Galactic absorption. The temperature, abundance, and normalization were left free in all annuli, whereas the cluster redshift, z, and the Galactic absorption, N_H, were fixed (see Table 1).

3.5. Sources of Systematic Uncertainty

The radial surface brightness and temperature profiles are subject to various sources of systematic error. The choice of the local background region is the major source of error in the calibration of the Chandra observations. I follow Bulbul et al. (2010) and consider a ±5% uncertainty in the imaging- and spectral-data analysis due to the variation of the count rates of different background regions. Contamination on the optical filter is another major source of uncertainty. This affects the spectral data by 5% (Bulbul et al. 2010), and is added in quadrature to each energy bin in the temperature data. Another possible source of uncertainty is the spatially dependent nonuniformity in the effective area of the ACIS detector. The spatial dependence of the detector efficiency scatters at a level of ±1% (Bulbul et al. 2010). This uncertainty, also, is added in quadrature to each annulus in the count rates.

3.6. Model Fitting

A Markov chain Monte Carlo (MCMC) approach is adopted, as illustrated in Bonamente et al. (2004), for the fitting process. The parameter space in this approach is explored by moving randomly from a set of parameters to another using the Metropolis-Hastings algorithm. This algorithm typically accepts a move to the new point with the likelihood higher than the old one. Hence, the algorithm gradually moves toward the highest likelihood regions, where the parameter values yield the best fit to the data.

The MCMC method is used to independently calculate the likelihood of the spatial and spectral data with the model. After binning the X-ray data, the log likelihood for the spatial data is given by

$$\ln(L_{\text{spatial}}) = -\frac{1}{2} \chi^2 - \frac{1}{2} \sum_i \ln(2\pi\sigma_i^2),$$

(24)

where \(\chi^2 = \sum_i [(D_i - M_i)/\sigma_i]^2\), \(D_i\) is the number of counts detected in bin \(i\), \(M_i\) is the number of counts predicted by the model in bin \(i\), and \(\sigma_i\) is the measured uncertainty on \(D_i\).

For the spectral data, the log likelihood, \(\ln(L_{\text{spectral}})\), is the same as that in the spatial case (Equation (24)), except here \(\chi^2 = \sum_i [(T_i - M_i)/\sigma_i]^2\), where \(T_i\) and \(M_i\) are the measured and predicted temperatures, respectively, and \(\sigma_i\) is the measured uncertainty on \(T_i\). Then, the joint likelihood of the spatial and spectral models (\(L = L_{\text{spatial}}L_{\text{spectral}}\)) is calculated, and the goodness of fit is tested using the \(\chi^2\) statistic.

Adopting the MCMC approach, radial profiles of temperature and background-subtracted surface brightness are fitted to the model (Equations (20) and (23)). Five parameters are used to model the global-cluster properties, whereas three describe the cluster’s central region. At the beginning, all clusters were fit to the model letting all parameters, including the cool-core parameters, free to vary. For three clusters (Abell 2218, Abell 2050, and MACS J0647.7+7015), however, it is found that the shape parameter \(a_{\text{cool}} \approx 1\), suggesting that there is no need for the cool-core parameters for these clusters. For these three clusters, therefore, the \(a_{\text{cool}}\) parameter is set to 1, and then the X-ray data fitted to the model, i.e., using only the global parameters. I classified such clusters as polytropic clusters, i.e., do not possess a cool-core component, whereas the remaining clusters are classified as cool-core clusters.

The values of the best-fitting parameters for the gas temperature (Equation (20)) and density (Equation (21))

| Cluster    | ObsID   | Exposure (ks) | z      | \(N_H\) (10^{20} \text{cm}^{-2}) | z Reference |
|------------|---------|---------------|--------|-------------------------------|-------------|
| Abell 2218 | 1666    | 40.7          | 0.176  | 2.57                          | Struble & Rood (1999) |
| Abell 1835 | 6880    | 117.1         | 0.253  | 2.04                          | Struble & Rood (1999) |
| Abell 2050 | 18251   | 14.9          | 0.120  | 4.66                          | Ebeling et al. (1996) |
| Abell 1689 | 7289    | 74.9          | 0.183  | 1.81                          | Struble & Rood (1999) |
| MACS J0647.7+7015 | 3196 | 19.1          | 0.584  | 5.24                          | LaRoque et al. (2003) |
| MACS J1423.8+2404 | 4195 | 114.2        | 0.545  | 2.45                          | LaRoque et al. (2003) |
| RXC J0214.8-2430 | 11757 | 19.7          | 0.161  | 7.70                          | Böhringer et al. (2004) |
profiles are listed in Table 2 (see Table 4 for their associated reduced \(\chi^2\)). The best-fitting surface brightness and temperature profiles of all clusters are presented in Figures 3 and 4. For most polytropic and cool-core clusters, the model accurately reproduces the X-ray surface brightness and temperature profiles over the most radial range. For the Abell 1835 and Abell 2050 clusters, however, the model does not fit well the temperature profile, particularly at intermediate to large radii.

### 3.7. Mass and Pressure Measurements

With the best-fitting parameters for the gas density and temperature profiles in hand, it is straightforward to determine the cluster masses. The gas mass can be obtained by integrating the density profile over a given volume,

\[
M_{\text{gas}}(r) = 4\pi \mu_e m_p \int n_e(r) r^2 dr,
\]

where \(\mu_e m_p\) is the mean mass per electron.

The total mass can be obtained using Equation (22), where the \(a_{\text{cool}}\) parameter is set to 1 for polytropic clusters. Moreover, the pressure profile can also be obtained using Equation (19). In Table 3, I present the gas and total cluster masses enclosed within radii of \(r_{2500}\) and \(r_{500}\), corresponding to densities 2500 and 500 times the critical density of the universe at the redshift of the cluster, respectively. Also listed in Table 3 are the pressure values obtained at these radii. Measurements reported in this table take account of the systematic uncertainties discussed in Section 3.5.

### 3.8. Comparison with Previous Measurements

The mass measurements for Abell 1835, from this study, are compared here with the results presented in Landry et al. (2013). I estimate the cluster masses within the same angular radii as reported in Landry et al. (2013). To allow a fair comparison of the cluster masses, their uncertainties on \(r_{2500}\) and \(r_{500}\) are adopted.

Using the Vikhlinin et al. (2006) model, Landry et al. (2013) estimated the gas mass of \(4.60^{+0.23}_{-0.25} \times 10^{13} M_\odot\) and the total mass of \(4.74^{+0.60}_{-0.55} \times 10^{14} M_\odot\) within \(r_{2500} = 161.7^{+6.5}_{-6.2}\) arcsec. These values are consistent, at the 1\(\sigma\) level, with the gas mass of \(4.53^{+0.55}_{-0.50} \times 10^{13} M_\odot\) and total mass of \(4.88^{+0.75}_{-0.40} \times 10^{14} M_\odot\) obtained in the current work for Abell 1835. Within \(r_{500} = 323.2^{+8.4}_{-8.3}\) arcsec, the predicted gas and total masses of Abell 1835 by Landry et al. (2013), using the Vikhlinin et al. (2006) model, are \(10.75^{+0.28}_{-0.29} \times 10^{13} M_\odot\) and \(7.56^{+0.62}_{-0.59} \times 10^{14} M_\odot\), respectively. These masses are about 26\% ± 6\% and 24\% ± 6\% larger than the corresponding masses of \(7.95^{+0.79}_{-0.79} \times 10^{13} M_\odot\) and \(5.73^{+0.99}_{-0.95} \times 10^{14} M_\odot\) given by this study within the same region.

Using the Bulbul et al. (2010) model, the estimates obtained in the Landry et al. (2013) work for the gas and total masses are \(4.00^{+0.16}_{-0.15} \times 10^{13} M_\odot\) and \(3.33^{+0.20}_{-0.25} \times 10^{14} M_\odot\), respectively, within \(r_{2500} = 143.8^{+2.9}_{-2.8}\) arcsec. The corresponding masses predicted in this work are \(4.10^{+0.34}_{-0.31} \times 10^{13} M_\odot\) and \(4.18^{+0.50}_{-0.41} \times 10^{14} M_\odot\). The former value is consistent well with that reported by Landry (2013), whereas the total mass is about 20\% ± 5\% larger than that given by these authors. Within \(r_{500} = 309.6^{+8.3}_{-8.0}\) arcsec, the estimated gas and total masses by Landry et al. (2013) are \(10.36^{+0.27}_{-0.27} \times 10^{13} M_\odot\) and \(6.65^{+0.54}_{-0.50} \times 10^{14} M_\odot\), respectively. The gas mass value is larger by about 24\% ± 4\% than the gas mass of \(7.85^{+0.67}_{-0.63} \times 10^{13} M_\odot\) derived by this study, but the total mass is statistically consistent with the total mass of \(5.64^{+0.76}_{-0.66} \times 10^{14} M_\odot\) estimated by the current study.

Overall, the mass measurements predicted from this study within the \(r_{2500}\) radii are in agreement with previous measurements. Within the \(r_{500}\) radii, however, some of the mass measurements are statistically inconsistent with previous measurements. Such discrepancies could be attributed to the choice of model for fitting the X-ray data. Landry et al. (2013) found that the choice of model may introduce uncertainties of 6\% ± 6\% and 10\% ± 8\%, respectively, to the measurements of the cluster gas and total masses at \(r_{500}\). The temperature profile could be another possible source for the discrepancy between these measurements. The temperature profile used in this analysis is slightly different from that used in Landry et al. (2013), and I estimated that the mass measurements for Abell 1835 could be affected by adopting the current temperature profile up to ±5\% within the \(r_{500}\) radius.

In addition to the mass comparison, the radial pressure distribution for Abell 1835 is estimated using Equation (19), with the best-fitting parameters (Table 2), and then compared with those predicted by Landry et al. (2013) using the Vikhlinin et al. (2006) and Bulbul et al. (2010) models, and assuming \(P(r) = n_e k_B T_e\). Figure 5 shows the radial pressure profiles for Abell 1835 predicted from this work and Landry et al.’s (2013) work, associated with their 68.3\% confidence interval. This figure suggests that, despite the differences in the temperature profiles adopted by the current work and Landry et al.’s (2013) work, the parameterized pressure profiles for Abell 1835 are more robust.
3.9. Comparison with Previous Models

The model proposed in the current work is compared with the Vikhlinin et al. (2006) model (Equations (1) and (2)) and the Bulbul et al. (2010) model (Equations (7) and (8)) in order to test which model provides a better fit to the X-ray data. For this purpose, the $\chi^2$ statistic is used as a metric to compare these models. Table 4 shows the combined $\chi^2_{\text{tot}}$ per degree of freedom (reduced $\chi^2_{\text{tot}}$) of the surface brightness and temperature associated with each fit for all studied galaxy clusters. As inferred by the reduced $\chi^2_{\text{tot}}$, all the models describe the data equally well. Similar to the Bulbul et al. (2010) model, however, the current model does not reproduce well the temperature profile at intermediate and outer radii for some clusters, such as Abell 1835 (Figure 4).

4. Discussion and Conclusion

In this work, I present an analytical model for the density and temperature profiles of galaxy clusters based on the assumption of hydrostatic equilibrium in the cluster’s gravitational potential. The model represents a variation of the model proposed by Bulbul et al. (2010). Here, the Einasto profile is adopted to model the spatial-density distribution of dark matter halos instead of the generalized NFW model used by Bulbul et al. (2010). This three-parameter profile is initially combined...
Figure 4. Same as Figure 3, except for cool-core clusters.
with a polytropic equation of state. Then, a cool-core correction is applied to the temperature profile and the gas density profile is derived under the assumption that the pressure profile is the same as in the polytropic case.

The model uses five parameters to describe the global properties of the ICM gas, with three additional parameters to describe the cluster’s core region. The main advantage of this model is the limited number of free parameters, which makes it simple and robust. The robustness feature is particularly important when one attempts to fit data that consist of few measurements. This can be helpful, for example, in galaxy clusters, where the X-ray count rate is low, particularly in the outskirts. Therefore, the proposed model represents a practical improvement compared to the model introduced by Vikhlinin et al. (2006), which has 17 free parameters. From a computational point of view, it is also more convenient to fit the observations to a model characterized by a few parameters. Another feature of this new model is the weak degeneracies between the best-fitting parameters, compared to the Vikhlinin et al. (2006) model (see Figures 3 and 4). This implies that more precise estimates are obtained for parameters.

The model is tested observationally with the X-ray data for polytropic and cool-core clusters. For most clusters, it is observed that the model is able to accurately fit the radial distributions of the ICM properties over the cluster’s full radial range. It is also shown that the model is essentially as good as that of Vikhlinin et al. (2006) and Bulbul et al. (2010), as indicated by the reduced $\chi^2$. Similar to the Bulbul et al. (2010) model, however, the model does not fit the temperature profile well enough at intermediate and large radii for some clusters.

Besides its application to model X-ray data, the model can be applied to Sunyaev–Zel’dovich effect observations, making it useful for various cosmological studies. The model can be used, for example, to measure cosmic distances (Bonamente et al. 2006), cluster pressure profiles (Bonamente et al. 2012), and gas mass fractions (Planck Collaboration et al. 2013). Furthermore, the model can be used to set up the initial conditions in cosmological numerical simulations, since it provides a simple and accurate description of the properties of the ICM plasma.

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### Table 3

| Cluster     | $r_\Delta$ (arcsec) | $M_{gas}$ ($10^{14}$ $M_\odot$) | $M_{tot}$ ($10^{14}$ $M_\odot$) | $P(r_\Delta)$ (10$^{-3}$ keV cm$^{-3}$) |
|-------------|---------------------|---------------------------------|---------------------------------|----------------------------------------|
| Abell 2218  | 195.5 ± 3.7         | 2.63 ± 0.24                     | 3.90 ± 0.27                     | 2.31 ± 0.32                            |
| Abell 1835  | 148.2 ± 6.4         | 4.17 ± 0.36                     | 4.63 ± 0.45                     | 4.17 ± 0.23                            |
| Abell 2050  | 253.5 ± 20.8        | 1.53 ± 0.31                     | 2.62 ± 0.55                     | 2.41 ± 0.44                            |
| Abell 1689  | 194.2 ± 13.6        | 2.61 ± 0.63                     | 3.61 ± 0.78                     | 3.70 ± 0.85                            |
| MACS J0647.7+7015 | 65.9 ± 3.1 | 1.17 ± 0.25                     | 2.01 ± 0.36                     | 3.59 ± 0.61                            |
| MACS J1423.8+2404 | 73.1 ± 7.3 | 2.38 ± 0.46                     | 2.59 ± 0.61                     | 5.22 ± 0.94                            |
| RXC J2014.8-2430 | 165.2 ± 10.7 | 2.04 ± 0.48                     | 1.59 ± 0.33                     | 2.24 ± 0.35                            |

### Table 4

| Cluster     | Reduced $\chi^2_{int}$ Values |
|-------------|-------------------------------|
|             | This Work | Vikhlinin et al. (2006) | Bulbul et al. (2010) |
| Abell 2218  | 0.92      | 0.45                  | 1.32                  |
| Abell 1835  | 1.05      | 0.96                  | 1.27                  |
| Abell 2050  | 0.52      | 0.22                  | 0.32                  |
| Abell 1689  | 0.72      | 0.76                  | 0.76                  |
| MACS J0647.7+7015 | 0.54 | 1.12              | 0.69                  |
| MACS J1423.8+2404 | 0.52 | 0.56              | 0.59                  |
| RXC J2014.8-2430 | 1.12 | 1.16              | 1.13                  |

### Figure 5

Parameterized pressure profiles for Abell 1835 predicted from this work (Equation (19)) and Landry et al. (2013) work. The lines are the best-fitting models, and the shadow regions are the 68.3% confidence intervals. The vertical dashed line indicates the $r_\Delta$ radius. The pressure profile seems robust in respect to different models.
