Numerical Simulations of Turbulent Non-premixed Combustion in a Regenerative Furnace

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1. Introduction

Economic competition in the steel industry has increased the demand for fuel-efficient uses in furnaces. In addition, due to stringent environmental laws the reduction of combustion-generated pollutants has become equally important. Unfortunately, fuel efficiency and pollution control can be difficult to attain simultaneously. For example, preheating the incoming combustion air can increase fuel efficiency but the resulting elevated temperatures would promote NOx emissions.

Although experimental techniques have been relied on almost exclusively for furnace research and development, they have been proven time-consuming and expensive. Fortunately, advances in numerical techniques and computing power have given birth to computational fluid dynamics (CFD), which have reduced the need for full-scale, prototype testing. The majority of CFD research that has been done to date in the area of combustion has focused primarily on simple flames. The most relevant research that has been done on simple flames is on turbulent, non-premixed types, such as the works of Jones and Whitelaw1, Mechitoua and Viollet2, Chen et al3, Cook4, Kent and Bilger5, Magnusson and Hjertager6, Chen and Kollmann7, Smith et al8, Spalding9, Janicka and Kollmann10. Significantly less research has been done on gas-fired furnaces. This is not surprising considering the complexity associated with such systems and the amount of computational effort required to model them. The previous work by Zhang et al11 compared the numerical simulations with measurements from an experiment furnace. The results were very promising. However, only specific numerical models were validated. Further advances in research will require the validation of additional models. The objective of this work therefore, is to validate the different models against experimental results in order to obtain the most viable design tool for simulating turbulent, non-premixed combustion in a regenerative gas-fired furnace with a moving slab. Through validation, the range of applicability of different models can be limited and an indispensable design tool for studying new furnace configurations with different operating conditions can be realized.

The moment closure method is used in this work to simulate the turbulent non-premixed combustion process in the regenerative furnace. The assumed-shape probability density function (PDF) method is applied to account for the effects of turbulent fluctuations on combustion. The combustion model based on the chemical equilibrium concept is employed to specify the instantaneous thermochemical state of the combustion mixture and to take into account the intermediate species formation and dissociation. Based on the composition of the fuel and air, the combustion process involves 15 chemical species, namely, CH, CH2, CH3, CH4, C2H4, CO, CO2, H, H2, H2O, N, N2, O, O2, and OH.

2. Experimental Furnace

The experimental regenerative continuous slab reheat furnace developed by the NKK Corporation is selected in this study for the validation of the numerical simulations. The configuration of the furnace is shown in Fig. 1. The inner dimensions are 8×4×3 m3 which are on the same order of magnitude as in the full-size industrial furnace. The steel slabs are 0.11 m thick and 3.5 m wide and are low-carbon steel (0.08% C). They are fed continuously into the bottom centre of the furnace from left to right in the x-direction.

Four pairs of burners are used in the experimental furnace as shown in Figs. 1 and 2. They are located 1.5 m from the furnace floor. The burners are paired up so that one burner from a pair is in a firing mode while the other, located on the opposite wall, is in an exhaust mode. The ex-
haust/firing cycles are reversed every 30 sec. Adjacent burners on the same wall are alternated when fired. In this way, heat transfer to the slab occurs more uniformly. In the numerical simulation, the first and third burners on the front wall and the second and fourth burners on the back wall are in the firing mode as shown in Fig. 2. The furnace also contains an auxiliary exhaust outlet on the left-hand side of the furnace wall as shown in Fig. 1. The combustion air, fuel and exhaust gases flow directions, and the slab moving direction are shown in Fig. 2. Although not shown in Fig. 1, the bottom of the furnace is equipped with water-cooled skids, and four cooling pipes equally located along the x-direction and three cooling pipes equally located along the y-direction in order to cool the skids. These cooling pipes extract heat at a rate of 271 kW. The operating conditions are listed in Table 1.

The regenerative low-NOx burners are used to increase the furnace thermal efficiency and to reduce the NO production rate. By using regenerative burners, the combustion air is preheated using the heat in the exhaust gases. Therefore, the fuel consumed to heat the combustion air can be reduced and the furnace thermal efficiency can be increased. Each burner contains a ceramic honeycomb regenerator that absorbs heat from the flue gases during the exhaust cycle and releases it to the combustion air during the firing cycle. The burners are also low NOx burners. The configuration of the burner is depicted in Fig. 3. Each burner consists of one air injection nozzle and two fuel injection nozzles. The combustion air and fuel are injected separately into the furnace at high velocities in order to achieve good mixing of the combustion air with the flue gases before combustion occurs. Consequently, the local oxygen concentration in the primary reaction zone is reduced. Moreover, the mixing of the fuel and combustion air is suppressed at the initial stage of combustion, resulting in a lower peak flame temperature. The drop in the maximum flame temperature and local oxygen concentration effectively suppresses the formation of NO.

3. Conservation Equations

Since the flows in furnaces are turbulent and reactive, the Favre-averaged form of conservation equations is used in this study to account for the effects of density change. The steady three-dimensional conservation equations of mass, momentum and energy can be expressed as,

\[ \frac{\partial}{\partial x_i} (\overline{\rho u_i}) = 0 \]  

\[ \frac{\partial}{\partial x_i} (\overline{\rho u_i} u_j) = \frac{\partial }{\partial x_j} \left[ \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} - \frac{2}{3} \frac{\partial \overline{u_i} \overline{u_j}}{\partial x_i} \right] - \frac{\partial p}{\partial x_j} + \frac{\partial }{\partial x_j} \left( -\overline{p u_i u_j} \right) \]  

\[ \frac{\partial}{\partial x_i} (\overline{\rho h}) = \frac{\partial }{\partial x_i} \left( \frac{\mu_s}{\alpha_h} \frac{\partial \overline{h}}{\partial x_i} \right) + S_h \]

where \( x_i \) is the position vector, \( u_i \) velocity vector, \( \rho \) density, \( p \) pressure, \( \mu \) molecular viscosity, \( h \) static enthalpy, \( \alpha_s \) turbulent Prandtl number, and \( -\overline{p u_i u_j} \) Reynolds stresses. \( S_h \)
consists of sources of enthalpy due to chemical reaction and radiation. The quantities with a tilde over them are the Favre-averaged values. A quantity with an overbar represents the Reynolds-averaged value. The Reynolds stresses in the momentum equation must be modeled in order to close the equation. Models relating to theses stresses are categorized as turbulence models because they contain physical information about the statistical behavior of turbulent flows. Models relating in particular to the determination of the radiation contribution to the source term in the energy equation are categorized as radiation models.

Besides above conservation equations, the moving steel slabs in the furnace are modeled by solving the following conduction equation.

$$\frac{\partial}{\partial x_i}\left(\rho u_i C_p T_f\right) = \frac{\partial}{\partial x_i}\left(\lambda \frac{\partial T_f}{\partial x_i}\right)$$

where the subscript $s$ denotes the properties of the moving slabs, $C_p$ is the specific heat and $\lambda$ is the thermal conductivity.

4. Combustion Model

A reaction mechanism is used to obtain reaction rates that are related to the source terms in the species conservation equations. It must therefore be specified whenever species conservation equations are solved. When a large number of species are involved, description of the combustion reaction mechanism becomes exceedingly complex. For the combustion system in this study, the fuel that injected into the furnace is the by-product of the steel production process and is composed of a mixture of a large number of gases as shown in Table 1. Therefore, a reaction mechanism for this case would be difficult to determine. To avoid specifying the reaction mechanism, it is possible to use a combustion model involving the solution of a conserved scalar from which individual species concentrations are derived. This combustion model is called the mixture fraction/PDF model. Both of them do not require knowledge of the reaction mechanism. The first option is the flame sheet approximation where it is assumed that the chemistry is infinitely fast. Hence, fuel and air never coexist. As soon as they mix, the reaction proceeds to the right and they become burned. Such a description of the system chemistry yields a straight-line relationship between the mixture fraction and species mass fractions. However, the flame sheet approximation requires the reaction stoichiometry. This limits its application to the prediction of single step reactions where intermediate species formation and dissociation effects are not considered. Often use of this approximation in modeling combustion in furnaces with preheated air results in a serious over prediction of peak flame temperature. The second option for describing the system chemistry does not require the reaction stoichiometry and can therefore predict the formation of intermediate species and take into account dissociation effects. This option is referred to as the equilibrium assumption and it assumes that the chemistry is fast enough for chemical equilibrium to always exist at the molecular level. With this option, species concentrations or mass fractions are obtained from the mixture fraction by minimizing Gibbs free energy.

With the system chemistry defined, the instantaneous thermochemical state of the mixture in a turbulent non-premixed combustion system is a function of the instantaneous mixture fraction and enthalpy only, i.e.,

$$\phi = \phi(f, h)$$

where $\phi$ may denote an instantaneous species concentration, temperature or fluid density.

Temporal fluctuations can be accounted for by using a joint PDF, $P(f, h)$. However, the computation of $P(f, h)$ is not practical for engineering applications. If we assume that the heat losses do not significantly affect the turbulent enthalpy fluctuations, we have $P = P(f)$. Then, the Favre-averaged values of $\phi$ can be calculated by

$$\tilde{\phi} = \int_0^1 P(f)\phi(f, \hat{h})df$$

The shape of the probability density function depends on the nature of the fluctuations that are encountered as a result of turbulence. Normally, the shape is different for every situation but several shapes have been observed experimentally more often than others. These common shapes have been expressed in terms of mathematical functions and are the only ones used in practice to obtain mean values from instantaneous values. There are two mathematical functions for describing the shape of the PDF in FLUENT. The first function is the double delta function. It is the simplest of the two available functions and is therefore computationally less intensive. The second function is the $\beta$-function that might be more accurate than the double delta function.

5. Auxiliary Models

All auxiliary models tested in this study are from FLUENT. To save the space, the equations describing the mod-
els, which are available from the FLUENT User’s Manual, will not be presented in the paper.

5.1. Turbulence Models

Three turbulence models are available in FLUENT. The first two, the \( k-e \) and the renormalization group (RNG) \( k-e \) models, are very similar. Both of them are two-equation models that base the approximation of the Reynolds stresses on two turbulence quantities, turbulent kinetic energy \((k)\) and its dissipation rate \((\varepsilon)\), that can be obtained from the solution of their transport equations. The third turbulence model is the Reynolds stress model (RSM). It is much more rigorous in its evaluation than the other two, involving the solution of transport equations for each of the Reynolds stresses.

5.1.1. The \( k-e \) Model

The \( k-e \) model is based on the eddy-viscosity hypothesis, Boussinesq hypothesis, which relates the Reynolds stresses to the mean velocity gradients.

\[
-\bar{p}u'u_j = -\frac{2}{3}k\delta_{ij} + \mu_i \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} \mu_i \frac{\partial \bar{u}_i}{\partial x_j} \delta_{ij} \quad ... (9)
\]

where

\[
\mu_i = \rho C_{\mu} \frac{k^2}{\varepsilon} \quad \text{and} \quad C_{\mu} = 0.09
\]

The solutions of two equations, \( k \) and \( \varepsilon \) transport equations, are required and the turbulent viscosity is determined from \( k \) and \( \varepsilon \). The assumption of isotropy is made when deriving the \( k \) and \( \varepsilon \) transport equations. The turbulent kinetic energy \((k)\) and its dissipation rate \((\varepsilon)\) transport equations are given as

\[
\frac{\partial}{\partial x_i}(\bar{p}u'_i k) = \frac{\partial}{\partial x_j}\left( \frac{h_i}{\sigma_k} + \frac{\partial \bar{u}_i}{\partial x_j} \right) + G_k - \rho \bar{e} \quad ... ... (10)
\]

\[
\frac{\partial}{\partial x_i}(\bar{p}u'_i \varepsilon) = \frac{\partial}{\partial x_j}\left( \frac{h_i}{\sigma_\varepsilon} \frac{\partial \bar{u}_i}{\partial x_j} + \frac{G_k}{C_{\varepsilon} k} - C_{\varepsilon} \frac{\varepsilon^2}{k} \right) \quad ... (11)
\]

where

\[
G_k = \mu_i \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j}, \quad C_1 = 1.44, \quad C_2 = 1.92,
\]

\[
\sigma_i = 1.0 \quad \text{and} \quad \sigma_\varepsilon = 1.3
\]

The \( k-e \) model has been successful in predicting the flow field for a wide variety of turbulent cases, however, the model is not without drawbacks. For one thing, it is not suitable for flows that are significantly affected by turbulence because of the assumption of isotropy. Also, the model is semi-empirical in nature. This means that the constants appearing in its equations are limited to the range of parameters for which the experiments were performed.

5.1.2. The RNG \( k-e \) Model

The RNG \( k-e \) model is based on the same principle as the \( k-e \) model and the relationship between the Reynolds stresses and the mean velocity gradients is still the framework for this model. The main difference lies in the way the viscosity is determined. Instead of using a semi-empirical approach as in the \( k-e \) model, the viscosity is obtained using a RNG technique. The transport equations for \( k \) and \( \varepsilon \) that the RNG \( k-e \) model utilizes are given as

\[
\frac{\partial}{\partial x_i}(-\bar{p}\bar{u}_i k) = \frac{\partial}{\partial x_j}\left( \mu_i \frac{\partial k}{\partial x_j} + \mu_i \frac{S^2 - \varepsilon}{\rho} \frac{\partial \bar{p}}{\partial x_j} \bar{e} \right) \quad ... (12)
\]

\[
\frac{\partial}{\partial x_i}(-\bar{p}\bar{u}_i \varepsilon) = \frac{\partial}{\partial x_j}\left( \mu_i \frac{\partial \varepsilon}{\partial x_j} + \frac{C_1}{k} \mu_i S^2 - C_{\varepsilon} \frac{\varepsilon^2}{k} - R \right) \quad ... (13)
\]

where

\[
\mu_i = \rho C_{\mu} \frac{k^2}{\varepsilon} \quad \text{and} \quad \mu_{eff} = \mu + \mu_i
\]

\( C_1, C_2 \) and \( C_{\mu} \) are now analytically derived constants having values of 1.42, 1.68, and 0.085, respectively. For high-Reynolds-number flows \((\mu / \mu_{eff} < 1)\), \( \alpha_i = \alpha_\varepsilon = 1.393 \). The R term in the transport equation for \( \varepsilon \) (Eq. (13)) represents the effect of the mean rate of strain on the dissipation rate and is defined as

\[
R = \frac{C_{\varepsilon} \eta^3 (1 - \eta / \eta_0)}{1 + \beta \eta^3} \rho \bar{e} \quad ... (14)
\]

where \( \eta = S / \bar{e} \), \( \eta_0 = 4.38 \) and \( \beta = 0.012 \). The S term in Eqs. (12) and (13) is the modulus of the mean rate-of-strain tensor, defined as

\[
S = \sqrt{2S_{ij} S_{ij}} \quad ... ... (15)
\]

where the mean strain rate \( S_{ij} \) given by

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]

The RNG model is clearly an improvement over the standard \( k-e \) model. The RNG model also accounts for low Reynolds number effects and the effect of swirl on turbulence and provides an analytical formula for turbulent Prandtl numbers. However, even with all these improvements, the model still cannot be applied to situations where the flow is anisotropic.

5.1.3. The Reynolds Stress Model (RSM)

Unlike the \( k-e \) and RNG \( k-e \) models where the isotropic eddy-viscosity hypothesis is used to model the Reynolds stresses, in RSM, the Reynolds stresses are obtained from the solution of their transport equations.

\[
\bar{u}_k \frac{\partial}{\partial x_k}(u'_i u'_j) = - \frac{\partial}{\partial x_i} \left[ \bar{u}'_i u'_j + \frac{p}{\rho} (\delta_{ij} u'_i + \delta_{ij} u'_j) \right]
\]

\[
- \frac{\mu}{\rho} \frac{\partial}{\partial x_i} \left( \bar{u}'_i u'_j + \bar{u}'_j u'_i \right) \frac{\partial \bar{u}_i}{\partial x_j}
\]

\[
+ \frac{p}{\rho} \left[ \frac{\partial u'_i}{\partial x_k} + \frac{\partial u'_j}{\partial x_k} \right] - 2 \mu \frac{\partial}{\partial x_i} \left( \bar{u}'_i u'_j \right) \frac{\partial \bar{u}_i}{\partial x_j} \quad ... ... (16)
\]

Several terms in Eq. (16) constitute additional unknowns. In order to obtain closure of the equation set, these unknowns must be modeled within the RSM itself.

\[
- \frac{\partial}{\partial x_k} \left[ (u'_i u'_j) + \frac{p}{\rho} (\delta_{ij} u'_i + \delta_{ij} u'_j) \right] - \frac{\mu}{\rho} \frac{\partial}{\partial x_i} \left( u'_i u'_j \right) \frac{\partial \bar{u}_i}{\partial x_j}
\]

\[
= \frac{\partial}{\partial x_k} \left( \frac{\mu_i}{\rho \sigma_i} \frac{\partial (u'_i u'_j)}{\partial x_j} \right) \quad ... ... (17)
\]
The constant $E$ in this case is 9.793 while the Van Driest constant, $A$, is 26. The dimensionless thermal conduction sublayer thickness, $y_f^*$, can be obtained by equating Eqs. (22) and (23). There is also an additional standard wall function for determining the value of $\varepsilon$ in the near-wall region based on the assumption that the production of $k$ and $\varepsilon$ are in equilibrium.

$$
\varepsilon_p = \frac{c_{\mu}^{3/4} k_p^{1/2}}{\kappa y_p} \hspace{1cm} \text{(24)}
$$

In the near-wall region, the turbulent kinetic energy is obtained from its transport equation and the following imposed boundary condition

$$
\frac{\partial k}{\partial n} = 0 \hspace{1cm} \text{(25)}
$$

5.2.2. Nonequilibrium Wall Functions Model

The nonequilibrium wall function for mean velocity in the fully turbulent region is sensitized to pressure gradients.

The nonequilibrium wall function for $y^* > 11.225$ is given by

$$
\frac{\dot{U} C_{\mu}^{1/4} k_p^{1/2}}{\tau_w / \rho} = \frac{1}{\kappa} \ln \left( E_{y^*} \right) \hspace{1cm} \text{(26)}
$$

where

$$
\dot{U} = \frac{1}{2} \frac{dp}{dx} \left[ \frac{y_v}{y_v} \ln \left( \frac{y}{y_v} \right) - \frac{y - y_v}{y_v} + \frac{y_v^2}{y} \right], \quad y_v = \frac{\mu y_v^*}{\rho C_{\mu}^{1/4} k_p^{1/2}} \text{ and } y_v^* = 11.225
$$

To account for the nonequilibrium effects, a two-layer concept is employed in the nonequilibrium wall functions model with two distinct functions for each of the viscous and fully turbulent layers. The resulting nonequilibrium wall functions for the turbulent quantities are given below

$$
\tau = \left\{ \begin{array}{ll}
0, & y < y_v \\
\tau_w, & y > y_v
\end{array} \right. \hspace{1cm} \text{and } k = \left\{ \begin{array}{ll}
\frac{y}{y_v} \left( \frac{y}{k} \right)^2, & y < y_v \\
\frac{2 \mu k}{\rho y_v} \left( \frac{y}{k} \right)^{1/2}, & y > y_v
\end{array} \right. \hspace{1cm} \text{(27)}
$$

where $C_1 = \kappa C_{\mu}^{-3/4}$.

The non-equilibrium wall functions model also achieves increased improvement over the standard wall functions model in cases of highly nonequilibrium flows. These types of flows cause the proportions of the viscous sublayer and fully turbulent layer to vary along the wall boundary. Such variations render the local equilibrium hypothesis invalid.

5.3. Radiation Models

Radiation models approximate the radiation contribution to the source term in the energy conservation equation. All radiation models employ the radiative transfer equation (RTE) in one form or another. The two radiation models that are available within FLUENT, are the discrete transfer...
radiation model (DTRM) and the P-1 radiation model. In the discrete transfer model, the radiation contribution to the source term is determined by solving the RTE for a series of rays. In the P-1 radiation model, the problem of finding the radiation contribution is reduced to solving an equation for the radiation temperature through various substitutions involving the RTE.

5.3.1. The DTRM

This model is derived based on the assumptions that scattering is relatively small in comparison to absorption and emission, and the radiating gases are gray. The radiation transfer equation used by the DTRM is

\[
\frac{dl}{ds} = \frac{\alpha \sigma T^4}{\pi} - \alpha l
\]

where \(l\) is the radiation intensity, \(\alpha\) is the absorption coefficient, and \(\sigma\) is the Stefan–Boltzmann constant. Equation (28) is integrated along a series of rays emanating from the faces of each discrete control volume. Integration leads to the change in radiation intensity along each ray and the sum of these changes constitutes the source term in the energy conservation equation. The absorption coefficient can be taken as a constant. For increased accuracy, the DTRM may be utilized with an effective emissivity submodel, which takes into account the non-gray property of the radiating gas. This submodel dispenses with the use of an absorption coefficient in favor of averaged values of absorptivity and emissivity.

5.3.2. The P-1 Radiation Model

The P-1 radiation model is actually the lowest order approximation of the more general P-N model. In the P-1 model, \(N\) is set equal to 1 as suggested by the name of the model. The mean radiation flux, \(\bar{q}\), is obtained by the following equation.

\[
\nabla \sigma = a \phi - 4 \alpha \sigma T^4
\]

where \(\phi\) is determined by the following transport equation,

\[
\nabla (1 - \nabla \phi) - a \phi + 4 \alpha \sigma T^4 = 0
\]

where \(\Gamma = 1/(3(a + \sigma_s))\) and \(\Phi_s\) is the scattering coefficient.

The P-1 model gives good results in many circumstances without making the problem computationally intensive. Unlike the DTRM, the P-1 radiation model takes into account the effects of scattering. For this reason, the P-1 model may be preferred in cases where scattering cannot be neglected. However, the P-1 model also has some limitations. For one, it may overpredict the radiation fluxes in problems with localized sources of heat. In addition, there is no option to take into account the non-gray property of gases as in the DTRM. Therefore, the effective emissivity model cannot be employed with the P-1 radiation model.

5.4. Variable Radiation Parameters Models

The absorption coefficient in the DTRM and the P-1 radiation model can be constant or it can be made to vary with composition. Two variable absorption coefficient models are available in FLUENT. Modak’s model is based on Hottel charts that were developed from experiments. The expression for the absorption coefficient is obtained from a direct curve fit of these charts. The weighted-sum-of-gray-gases mode (WSGGM) relates the absorption coefficient to the emissivity. In the limit of small path length, the gray gas approximation becomes valid and the general expression for the absorption coefficient reduces to a different form. Hence, in the WSGGM, two functions for the absorption coefficient are employed. In addition to the above models, there is a third variable radiation parameter model. This is the effective emissivity submodel mentioned earlier. This model relates the absorptivity to the emissivity and the expression for the emissivity is borrowed from the WSGGM.

5.4.1. Modak’s Model

In this model, the emissivity for a gas is determined from correlated data. The absorptivity is related to the emissivity. The emissivity, \(\varepsilon\), for a gas is given by

\[
\varepsilon = \varepsilon_e + \varepsilon_w - \Delta \varepsilon_{cw}
\]

where \(\varepsilon_e\) and \(\varepsilon_w\) are the emissivities for carbon dioxide and water vapor, respectively. They are functions of the gas temperature and partial pressure, and mean beam length. The correction factor, \(\Delta \varepsilon_{cw}\), in Eq. (30) accounts for the reduction in emission that is associated with the mutual absorption of radiation between carbon dioxide and water vapor when both of them are present in a mixture with other non-radiating gases. Good results have been obtained with this model regardless of its empirical nature. However, Modak’s model is applicable and can be used successfully in cases where temperature is higher than 2 000 K, and is only available for use with the DTRM.

5.4.2. The WSGGM

In this model, the absorption coefficient is related to the emissivity as

\[
a = \frac{-\ln(1 - \varepsilon)}{s}
\]

Here, the emissivity, \(\varepsilon\), is approximated using weighting factors.

\[
\varepsilon = \sum_{i=0}^{j} a_{ij}(T)[1 - \exp(-\kappa_i ps)]
\]

where \(a_{ij}\) is the emissivity weighting factors for the \(i\)th fictitious gray gas; \(\kappa_i\) is the absorption coefficient for the \(i\)th fictitious gray gas; \(p\) is the sum of the partial pressures of all absorbing gases; and \(s\) is the path length. The path length can be specified either as a characteristic cell size or as a mean beam length. Using the characteristic cell size is more appropriate in cases where the primary interest is in the distribution of radiation around a source. The mean beam length would be a better choice if the objective were to determine radiation exchange between walls. The WSGGM has a wider range of applicability than Modak’s model. In particular, it is not restricted to the range of temperatures.
applications where radiation is the primary mode of heat transfer to the load. This is because in-flame soot exhibits nearly blackbody behavior that is responsible for increases in radiation heat losses from the flame. In industrial furnace applications therefore, the challenge is to promote soot production in such a way that would enhance radiative heat transfer to the load without increasing soot emissions that could pose potential health risks.

Different models describing the mechanism have been proposed. Application of these models results in the prediction of soot concentrations as well as the effect of soot on radiative heat transfer. Two models within FLUENT for predicting soot concentration are empirically based. Both of them require information about the combustion system but only one is applicable for a wide range of hydrocarbons and does not require input that has to be experimentally obtained specifically for the system being modeled. This model is the two-step Tesner model in which the formation of soot is assumed to occur in two stages. In the first stage, radical nuclei are generated from the gaseous fuel. In the second stage, soot is generated or formed from the nuclei. The model therefore predicts the concentration of the nuclei, n, and through the dependency on these nuclei, the concentration of the soot, c. Both concentrations are obtained from the solution of their transport equations

\[
\frac{\partial}{\partial x_i} (\bar{\rho}u_i n) = \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{\sigma_{\mu}} n \right) + p R_n \quad \ldots \ldots \ldots \ldots \ldots \ldots (32)
\]

\[
\frac{\partial}{\partial x_i} (\bar{\rho}u_i c) = \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{\sigma_{\mu}} c \right) + p R_s \quad \ldots \ldots \ldots \ldots \ldots \ldots (33)
\]

The terms R_n and R_s appearing in the above equations are the net rates of nuclei generation and soot generation.

6. Numerical Procedure

A control volume approach\(^{14}\) is used in which a nonstaggered grid arrangement and a sequential procedure of the SIMPLE (semi-implicit method for pressure-linked equations) algorithms\(^{15}\) are used to transform the governing differential equations into computer solvable, algebraic equations. Interpolation is accomplished via a first-order power-law scheme. Solutions of the resulting algebraic equations are obtained with the line Gauss-Seidel (LGS) solver and multigrid acceleration.

7. Results and Discussion

Total 13 different cases listed in Table 2 were considered in this study. The first case consists of the k-ε turbulence model, the standard wall function model, the P-1 radiation model with the WSGGM and a path length that is based on the characteristic cell size, and the mixture fraction/PDF combustion model that employs a β PDF. This is the case used in the previous study by Zhang et al.\(^{11}\) and is used as the control case in this study from which the all other cases are derived. As can be seen, the models in each case are almost identical to Case 1 except for one model that separates it from the control case. Converged solutions were obtained for the first 11 cases. Therefore, only the results for the first 11 cases will be presented.

7.1. Velocity and Temperature Distributions

The velocity and temperature distributions on the horizontal plane that cuts through the air injection ports and the temperature distribution for the slab were obtained for the 11 cases. The distributions are similar from case to case and the corresponding contour maps that are shown for Case 1 in Figs. 4 and 5 are representative of the type of conditions that are obtained with the different cases.

Figure 4 shows the velocity distribution. From this figure, it can be seen that the jet streams from the burners maintain a direction that is almost straight toward the opposite end of the wall. Very little deviation occurs from a straight path since the air is injected into the burners at a high velocity and the distance between inlet and exhaust ports is relatively short. Distinct recirculation zones between the jet streams and between the jet stream and the wall can also be seen from this figure. These recirculation zones are induced by the interaction between two opposing, high velocity jet streams and flow impingement on the walls.

The temperature distribution on the horizontal plane that cuts through the air injection ports is depicted in Fig. 5. In this figure, the main reaction zones are represented by high levels of temperature. These temperatures general tend to be higher downstream from the slab inlet and are a consequence of the heat that is transferred to the slab as it moves along the furnace floor. Maximum temperatures for the various cases range anywhere between 1758 K to 1832 K. Figure 5 also shows the temperature distribution for the slab with a clearance of 0.25 m on either side (represented by additional horizontal lines on the top and bottom). The figure shows that the temperature of the slab increases from one end of the furnace to the other as it absorbs heat from the flue gases. The figure also shows that the change in the

| Case | Description |
|------|-------------|
| Case 1 | k-ε model, standard wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 2 | RING model, standard wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 3 | RSM model, standard wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 4 | RSM with directional diffusion, standard wall function, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 5 | k-ε model, nonequilibrium wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 6 | k-ε model, standard wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 7 | k-ε model, standard wall function model, P-1 model with a constant absorption coefficient of 0.5, mixture fraction/PDF model with a β PDF |
| Case 8 | k-ε model, standard wall function model, P-1 model with WSGGM and a path length that is based on the mean beam length, mixture fraction/PDF model with a β PDF |
| Case 9 | k-ε model, standard wall function model, DTRM with Modak’s model, mixture fraction/PDF model with a β PDF |
| Case 10 | k-ε model, standard wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF, Tesner model with C_P, defined as the fuel |
| Case 11 | k-ε model, standard wall function model, P-1 model with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF, Tesner model with C_H, defined as the fuel |
| Case 12 | k-ε model, standard wall function model, DTRM with WSGGM and a path length that is based on the characteristic cell size, mixture fraction/PDF model with a β PDF |
| Case 13 | k-ε model, standard wall function model, DTRM with the effective emissivity submodel, mixture fraction/PDF model with a β PDF |
temperature along the longitudinal direction gradually diminishes as the slab advances. This is indicated by the increase in the distance between the temperature contours. Depending on the case, the final temperature at which the slab exits the furnace ranges from 1 585 K to 1 621 K.

### 7.2. Energy Flow Rates

The energy flow rates were compared with the experimental data. The results are summarized in Table 3 for Cases 1–11. All of the values are in kilowatts except for the quantities in parentheses and square brackets. The quantities in parentheses represent the percentage error between the predictions and the measurements. The quantities in the square brackets represent percentage difference between the predicted energy output rate and the energy input plus energy generation rates, i.e.,

\[
\left( \frac{\text{Predicted Energy Output Rate}}{\text{Energy Input Rate} + \text{Energy Generation Rate}} \right) \times 100
\]

The quantities in the last column without the square brackets are the difference between the predicted energy output rate and the energy input plus energy generation rates. All the energy output values are determined based on the measured temperatures and the thermal properties of the flue gases and slab.

The predicted energy output rates for the flue gases at the burner outlets and for the slab are in very good agreement with the experimental data. However, high percentage errors that fall in the range of 73.5% and 89.1% are noticed for the auxiliary exhaust. The predicted flue gas temperature at the auxiliary outlet is too high, which causes the pre-
dicted energy output at the auxiliary outlet about 300 kW higher the experimental value. As shown in Table 3, the overall energy output is overpredicted by 337 to 382 kW for different cases. This can be attributed to the factor that the output energy is overpredicted by about 300 kW at the auxiliary outlet.

Table 3 also shows that the predicted heat transfer rate to the slab is particularly sensitive to the radiation models used in the simulation since 98% of the heat is transferred to the slab by radiation. In the first six cases (Cases 1–6), the radiation model used is P-1 model with WSGGM and a path length that is based on the characteristic cell size and the errors for the heat transferred to the slab are less than 1% (The experimental uncertainty associated with the slab temperature measurement is less than 0.5%). Case 7 uses P-1 model with a constant absorption coefficient of 0.5 and the error is similar to the errors in the first six cases. Cases 8 and 9 employ P-1 model with WAGGM and a path length that is based on the mean beam length and DTRM with Modak’s model, respectively. They underpredict the heat transfer rates by 3.5% and 4%, respectively. The comparison shows that the cell based approach for defining the path length is more accurate than the mean beam length method. The high percentage error using DTRM with Modak’s model may be attributed to the factor that Modak’s model is applicable when temperature is higher than 2 000 K and the maximum temperature in the experimental furnace is less than 2 000 K.

Relatively high percentage errors for the heat transfer to the slab are seen in Cases 10 and 11. These cases were expected to predict the highest heat transfer rates because of the additional soot models. Instead, the soot models underpredicted the heat transfer to the slab by 1.44% for Case 10 and 1.38% for Case 11. These discrepancies can be attributed to a number of weaknesses in the soot models. Probably the greatest weakness in the Tesner soot model is its inability to base the calculations on a mixture of gases. It only allows one type of gas to be specified as the fuel, which lends itself to error when a mixture of gases is part of the problem definition. The Tesner soot model is also semi-empirical in a nature and unless experimental data are defined explicitly, the default soot modeling parameters are set for the combustion of acetylene. Although, these parameters are generally applicable for a wide range of hydrocarbons, they still introduce a certain level of uncertainty to the results. The generalized soot model that estimates the effect of soot on radiation is also a weak model. The model’s expression for the absorption coefficient of soot is based on the results of two other approximations. This also reduces the accuracy of the model.

7.3. Slab Temperature Distribution

The predicted centerline slab temperature distribution was also compared with the experimental data. The results for Case 1 are presented in Fig. 6. This figure shows good agreement between predicted and experimental results. In fact, the agreement is almost the same from case to case. At the slab inlet, the temperature is slightly overpredicted, and at the slab outlet, it is slightly underpredicted. The discrepancy at the slab inlet lies in the problem definition and in the fact that the slab speed is an average that is based on a 10-minutes interval. In reality, one slab (0.5 m long) is pushed into the furnace in 10 sec and 9 min and 50 sec elapse before the next slab is introduced.

7.4. Computational Effort

On average, a converged solution was obtained after four and a half days on a Silicon Graphics Indy (SGI) workstation using version 4.44 of FLUENT. The SGI is a 133 MHz

| Case | Fuel | Air | Fuel Heating Value | Fuel at Burner | Fuel at Auxiliary | Slab | Cooling Water | Waste | Overall Energy Balance |
|------|------|-----|-------------------|----------------|------------------|------|--------------|------|-----------------------|
| 1    | 8.75 | 2137.58 | 3279.66 | 2230.00 | 2387.00 | 2191.00 | 590.00 | - |                     |
| 2    | 8.75 | 2137.58 | 3279.66 | 2279.29 | 671.96 | 2171.66 | 590.00 | 61.20 | 373.23 (9.88%) |
| 3    | 8.75 | 2137.58 | 3279.66 | 2287.19 | 721.90 | 2166.93 | 590.00 | 19.13 | 382.16 (7.04%) |
| 4    | 8.75 | 2137.58 | 3279.66 | 2299.34 | 731.64 | 2191.72 | 590.00 | 21.12 | 386.59 (9.76%) |
| 5    | 8.75 | 2137.58 | 3279.66 | 2299.96 | 690.75 | 2177.19 | 590.00 | 59.07 | 368.96 (8.60%) |
| 6    | 8.75 | 2137.58 | 3279.66 | 2247.20 | 683.02 | 2178.41 | 590.00 | 61.56 | 337.20 (9.21%) |
| 7    | 8.75 | 2137.58 | 3279.66 | 2266.49 | 767.17 | 2162.86 | 590.00 | 73.99 | 387.83 (7.15%) |
| 8    | 8.75 | 2137.58 | 3279.66 | 2344.09 | 223.52 | 2114.34 | 590.00 | 38.57 | 366.52 (8.75%) |
| 9    | 8.75 | 2137.58 | 3279.66 | 2347.64 | 694.41 | 2103.36 | 590.00 | 49.35 | 361.87 (8.67%) |
| 10   | 8.75 | 2137.58 | 3279.66 | 2310.05 | 671.35 | 2159.35 | 590.00 | 58.57 | 366.33 (8.75%) |
| 11   | 8.75 | 2137.58 | 3279.66 | 2306.66 | 673.69 | 2160.85 | 590.00 | 58.61 | 366.82 (8.76%) |

Fig. 6. Slab temperature distribution at y=2 m.
The accuracy between the P-1 model and DTRM cannot be compared. (4) The use of β-PDF and Delta PDF does not affect the results very much. Therefore, the Delta PDF is the better choice because it is computational less intensive. (5) The relatively large errors that were obtained when the soot models were introduced. In summary, the predicted results are sensitive to the radiation models used in the simulations. However, they are not sensitive to the turbulence models and wall functions used in the simulations. This is due to the fact that based on the predicted results, 98% of the heat is transferred by radiation in the reheat furnace. Thus, the radiation models play a very important role in the simulations. However, the flow field does not have significant effect on the heat transfer in the reheat furnace. This is why the models related to the flow field, such as turbulence models, have little effect on the prediction of the heat transfer.

The three best cases in terms of both accuracy and computational effort are Cases 1, 6 and 7. These three cases, which employ the $k-\varepsilon$ turbulence model, the Delta PDF, and a constant absorption coefficient, showed very good agreement between the predictions and the measurements. They are associated with models that require the least amount of computational effort from their class. Overall, the work was successful in showing the potential of CFD in reproducing the velocity and temperature fields.

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### Table 4

List of cases in order of increasing computational effort for each class of models.

| Model Class  | Order | Case | Model Description                      |
|--------------|-------|------|----------------------------------------|
| Turbulent Models | 1     | 1    | $k-\varepsilon$ model                  |
|              | 2     | 2    | RNG $k-\varepsilon$ model              |
|              | 3     | 3    | RSM                                    |
|              | 4     | 4    | RSM with directional diffusivity       |
| Near-wall Treatment | 1     | 1    | Standard wall function                 |
|              | 2     | 5    | Nonequilibrium wall function           |
| Radiation Models | 1     | 7    | P-1 model with a constant absorption coefficient |
|              | 2     | 8    | P-1 model with WSGGM and a path length based on the mean beam length |
|              | 3     | 9    | DTRM with Modak’s model                |
|              | 4     | 10   | Tenser model with $C_{MN}$ defined as the fuel |
| Soot Models  | 5     | 11   | Tenser model with $C_{MN}$ defined as the fuel |

IP22 processor with 64 MB of main memory, a data cache size of 16 KB, and a secondary cache size of 512 KB. It is configured with IRIX 6.2 as a dataless client so that the operating system and applications are all loaded over the network.

Table 4 lists all of the cases, in order of increasing computational effort for each class of models. Since the number of iterations required to reach convergence is dependent on underrelaxation factors, the order was based on the complexity of the models. The type and number of equations in each model gives a good indication of the efficiency with which the model can predict the results in relation to the other models. As can be seen from the table, the complexity of the models in some instances appears to be the same. This is true in the case of the soot models and the P-1 radiation models that use the WSGGM based on the mean beam length and the characteristic cell size.

### 8. Conclusions

This work provided an evaluation of the numerical models within FLUENT in terms of their predictive capability in simulating combustion in a regenerative, slab reheat furnace. Total 13 cases employing different numerical models were considered. Based on the evaluation, a number of conclusions can be made. (1) The use of different turbulence models, $k-\varepsilon$ model, RNG $k-\varepsilon$ model, RSM and RSM with directional diffusivity, does not affect the simulation results significantly. Therefore, the $k-\varepsilon$ model is the best choice since it requires the least amount of computational effort amount the models. (2) The standard wall function and nonequilibrium wall function give similar results. So, the standard wall function is the better choice due to its simplicity. (3) The radiation models that define the path length in terms of the characteristic cell size should take precedence over models that define it in terms of the mean beam length. Modak’s model is not suitable for the combustion process considered in this study since the maximum temperature is less than 2,000 K. Unfortunately, the converged results using the DTRM with WSGGM and a path length that is based on the characteristic cell size and DTRM with the effective emissivity submodel could not be obtained.