On the anomalies of gravity

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Abstract

The paper is based on the recently proposed 4-dimensional optical space theory and draws some of its consequences for gravitation. Starting with the discussion of central movement, the paper proceeds to establish a metric compatible with Newtonian mechanics which can be accommodated by the new theory and finds a correction term which can be neglected in most practical circumstances. Being effective in the very short range, the correction term affects substantially the results when continuous mass distributions are considered. The main consequence is the possibility of explaining the orbital speeds found around galaxies, without the need to appeal for a lot of dark matter. The speed of gravity is also discussed and the theory is found compatible with a gravitational speed equal to the speed of light. On the subject of black holes, it is suggested that they are just a possibility but not a geometric inevitability.

Key words: Dark matter; speed of gravity; alternative theories

1 Introduction

In a recent paper [1] we introduced the concept of 4-dimensional optical space as an alternative to the ordinary relativistic space. The main characteristics of the 4-dimensional optical space being a signature 4, with the consequent Euclidean tangential space, and geodesic arc length given by

\[ c^2(dt)^2 = g_{\alpha\beta} dx^\alpha dx^\beta, \]  

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1Greek indices take values 0 to 3, while latin indices take values 1 to 3.
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where \( c \) is the speed of light in vacuum, \( t \) is time, \( x^0 = \tau \) is a coordinate which was named *proper time* and the \( x^a \) correspond to the spatial cartesian coordinates \( x, y, z \).

It was also stated as a premise that all particles and photons should move along metric geodesics if a metric was chosen that accounted for all the relevant interactions. This principle was shown to hold for two of the main interactions in nature, namely gravity and electromagnetism.

Although the 4-D optical space theory is expected to be equivalent to general relativity in the prediction of test particle trajectories and whenever trajectories follow metric geodesics, this is not the case for massive bodies or for other situations where relativistic trajectories deviate from geodesics. In the present work we extract some important consequences of the new formalism for gravitation, which explain many of the reported puzzles of existing gravitational theory \[2\]. The paper is virtually self-contained, apart from concepts and formulations that can be found in most standard textbooks; among others we used Refs. \[3, 4, 5\].

2 General central movement problem

The problem we want to solve is that of bodies orbiting other bodies under the influence of their mutual gravitational field but it is useful to start discussing central movement in the context of the new theory in its most general formulation.

A general lagrangean for central movement must reflect the combined effect of gravitational and electrostatic fields because these are the basic interactions whose range is bigger than atomic dimensions; however it does not need to be the most general spherical solution. In view of the arguments presented in \[1\] we propose that the general form is

\[
2L = n_\tau^2 \left( \dot{x}^0 \right)^2 + n_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = n_\tau^2 \dot{\tau}^2 + n_\tau^2 \left[ \dot{r}^2 + r^2 \left( \dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2 \right) \right],
\]

(2)

with

\[
x^0 = \tau, \quad x^1 = r \sin \theta \cos \phi, \quad x^2 = r \sin \theta \sin \phi, \quad x^3 = r \cos \theta.
\]

The first Euler-Lagrange equation is simply,

\[
\frac{\partial L}{\partial \dot{\tau}} = n_\tau^2 \dot{\tau} = \text{constant}.
\]

\[2\] It is customary in relativity to normalize \( c = 1 \); in this work we use adimensional units obtained by dividing length, time and mass by the factors \( \sqrt{G\hbar/c^3} \), \( \sqrt{G\hbar/c^5} \) and \( \sqrt{\hbar c/G} \), respectively. \( G \) is the gravitational constant and \( \hbar \) is Planck’s constant divided by \( 2\pi \).
Without loss of generality we can set $\theta = \pi/2$ because we know beforehand that the trajectory is flat and we are at liberty to choose the $x^3$ axis perpendicular to the plane of the trajectory. A conservation law can be obtained from the $\phi$ equation

$$rn_\tau \dot{\phi} = J,$$

where $J$ is a constant related to angular momentum.

Finally we write the radial equation

$$n_\tau^2 \ddot{r} = n_\tau \partial_\tau n_\tau \dot{\tau}^2 + \left( r^2 n_\tau \partial_\tau n_\tau + rn_\tau^2 \right) \dot{\phi}^2. \quad (6)$$

For simplicity we will limit the discussion to circular orbits, with $\dot{\tau} = \dot{\phi} = 0$. This will quickly lead us to the general law of angular velocity dependence on distance, without the complications of non-circular orbits. Let us also replace the $t$ derivative by a derivative with respect to $\tau$ by the following relation

$$\dot{\phi} = \frac{d\phi}{d\tau} \dot{\tau} = \omega \dot{\tau}, \quad (7)$$

and obtain the equation for the angular velocity in circular motion as

$$\omega^2 = \frac{-n_\tau \partial_\tau n_\tau}{r \left( n_\tau^2 + rn_\tau \partial_\tau n_\tau \right)}. \quad (8)$$

Notice that angular velocity has been defined with respect to proper time which coincides with time measured on the gravitational center clock.

## 3 The gravitational field

Newtonian mechanics tells us that the gravitational pull force of a large body, considered fixed with mass $M$, over a much smaller body of mass $m$ is

$$\vec{f} = m \nabla V,$$

where the arrows were used to denote vectors in 3D space and the "nabla" operator has its usual 3-dimensional meaning; $V$ is the gravitational potential given by

$$V = \frac{GM}{r}; \quad (10)$$

$r$ is distance between the two bodies.

If the moving body is under the single influence of the gravitational field, the rate of change of its momentum will equal this force and so we write $d^2 \vec{r}/dt^2 = \nabla V$; $\vec{r}$ is the position vector. If we use mass scaling of the
coordinates introduced in [1], the spatial components of the momentum must appear as \(m^2\delta_{ab}\ddot{x}^b\); the gradient is also affected by the scaling and we expect it to appear as \(\partial_a V/m\). Using primed indices to denote unscaled coordinates it is
\[
\delta_{a'b'}\ddot{x}^{b'} = \partial_a V, \\
m^2\delta_{ab}\ddot{x}^b = \partial_a V.
\]
(11)

(12)

We are looking for a lagrangean of the type given by Eq. (2), but in the absence of an electrostatic field \(n_r = n_t\). In Cartesian coordinates it is
\[
2L = n_r^2\delta_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta.
\]
(13)

This lagrangean must be consistent with the non-relativistic form of the gravitational force and the resulting metric must be asymptotically flat.

If we derive the Euler-Lagrange’s equations for the 3 spatial dimensions we get
\[
n_r\delta_{a\beta}\ddot{x}^\beta = \partial_a n_t\delta_{a\beta}\dot{x}^\alpha\dot{x}^\beta = \partial_a n_r \left[ (\dot{x}^0)^2 + \delta_{ij}\dot{x}^i\dot{x}^j \right].
\]
(14)

From Eq. (13) we take \(\delta_{ij}\dot{x}^i\dot{x}^j = 1/n_r^2 - (\dot{x}^0)^2\), to be replaced above:
\[
n_r^2\delta_{a\beta}\ddot{x}^\beta = \frac{\partial_a n_r}{n_r}.
\]
(15)

It is now convenient to make the replacement \(n_r = m\eta_r\), so that the previous equation becomes
\[
m^2\eta_r^2\delta_{a\beta}\ddot{x}^\beta = \frac{\partial_a \eta_r}{\eta_r} = \dot{p}_a.
\]
(16)

If Eq. (16) is to produce the same results as Eq. (12) at appreciable distances from the central body, \(\eta_r\) must be a function of \(r\) that when expanded in series of \(1/r\) has the first two terms \(1 + M/r\), where adimensional units were used to remove the gravitational constant. An interesting possibility is the function
\[
\eta_r = e^V = e^{M/r}.
\]
(17)

The second members of the two equations are now equal and the first members will be equivalent in non-relativistic situations. So compatibility with Newtonian mechanics is ensured.
4 Angular velocity dependence on distance

We are now in position to discuss how the angular velocity of orbiting bodies is expected to depend on the distance to the central mass. Our reference is the $r^{-3/2}$ dependence from Newtonian gravitation but we wish to find some explanation for the apparent anomalies near galaxies \[2\], which people usually attribute to large amounts of dark matter.

We go back to Eq. (8) and replace both $n_\tau$ and $n_r$ by $m^2\eta_r$ to get

$$\omega^2 = \frac{M}{r^3 + Mr}.$$  
(18)

The angular velocity is shown to depend on the distance in a similar way to the Newtonian predictions at large distances, but closer to the central mass the dependence evolves gradually to $\sqrt{r}$. The range of the correcting term can be found by setting $M/r^2 = 1$; for the Sun this range is $5 \times 10^{-16}$ m. This effect becomes more important as masses grow and a range of 1 m would imply a central mass of $8 \times 10^{61}$ Kg, or about $4 \times 10^{31}$ times Sun’s mass.

Naturally, being a short range correction, the $1/\sqrt{r}$ dependence must be decisive in the way continuous mass distributions behave. A correct study of continuous mass distribution dynamics is a difficult subject, which we can turn into a more manageable one by assuming some law for the mass density. This posture is not unquestionable because the net effect of a mass element will be different whether it is stationary or moving relative to the center of mass. In spite of these limitations, we think it is very enlightening to make estimates based on pre-defined density laws.

As usual we will assume that the mass distributions are spherically symmetrical or symmetrical about one axis. When studying the movement of a mass element we only have to consider the mass inside of a sphere with radius equal to the distance of the mass element to the center of mass, for spherically symmetrical distributions, or the mass of a cylinder of the same radius, for the axis symmetric distributions.

If the mass density is $\rho$, the total mass inside a sphere of radius $r$ is given by

$$M = 4\pi \int_0^r \rho r'^2 \, dr',$$  
(19)

while for a cylinder with the same radius we have

$$M = 2\pi \int_0^r \rho r' \, dr'.$$  
(20)
We will now look at two special cases: The first one has a mass density decaying with $1/r^2$ and spherical distribution while in the second one mass density decays with $1/r$ and symmetry is axial. The previous equations become respectively

$$M = 4\pi \rho_0 r,$$  \hspace{1cm} (21)

$$M = 2\pi \rho_0 r,$$  \hspace{1cm} (22)

with $\rho_0$ some constant. In both cases the mass affecting the orbit of the mass element at distance $r$ can be expressed as $M_0 r$. If this is substituted in Eq. (17) the resultant $\eta r$ is a constant and the corresponding angular velocity is zero.

Let us look at what Newtonian mechanics predicts for the same situations: We take the expression for angular velocity $\omega^2 = M/r^3$ and replace $M$ by $M_0 r$, to get an angular velocity that decreases linearly with $1/r$. This difference is of the utmost importance when studying the dynamics around galaxies. If beyond some distance the mass density of a spiral or disk shaped galaxy shows a dependence on $1/r$, the angular velocity will be governed only by the mass in the core of the galaxy. Given the rough approximations made in this study it is conceivable that some density law can even counteract this dependence yielding an almost constant angular velocity, without the need to call for dark matter.

## 5 Speed of gravity and black holes

In general relativity gravity must act instantaneously [2], which is seen as a contradiction with Einstein’s philosophy. It appears as information being carried at cosmological distances in no time and this is certainly not in accord with the spirit of relativity.

Eq. (8) holds the key to the paradox. Angular velocity is here defined relative to proper time and proper time is synchronized all over the Universe by photons [1]; whatever carries gravity must then travel at the speed of light. In a forthcoming paper we will show that gravity is indeed intermediated by so called gravitons, which are just another view of photons. We will also show experimental proof of graviton existence and detection.

And what about black holes? Black holes can be accommodated in 4D optical space by the consideration of $\eta r$, given by

$$\frac{1}{\eta r} = 1 - \frac{M}{r}. \hspace{1cm} (23)$$
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Trajectories evaluated with the metric resulting from the definition above will almost duplicate Schwartzschild’s geodesics, except for different angular velocities. On the other hand, there is no geometrical reason to impose this solution, as was the case with general relativity \cite{6}, and we think it is good to keep an open mind but until there is evidence of their existence it is best not to consider black holes at all.

6 Conclusion

All over the past year the author has been addictive to the idea of a new formulation of general relativity, motivated by similarities with optical propagation. He has been tormenting colleagues and friends with his ideas, which were made public at the OSA meeting of October 2000 \cite{7}. The first concepts were developed into a theory named 4-dimensional optical space theory and written in the form of a paper \cite{1} in last April. The present work is the first to extract new results from this theory in order to seek an explanation for existing puzzles in the Universe.

It has been shown that the new theory is compatible with established results from Newtonian gravitation and is more effective than relativity in generalizing them to the cosmological scale. In particular it was possible to explain observed anomalies in the gravitation around galaxies without the need to appeal for large amounts of dark matter. The paper was also successful in reconciling a propagation speed for relativity with the stable orbits of planets, without the consequence of an inevitable fall into the Sun. As a final result a discussion of black holes showed that although these can be accommodated by the new theory they are not inevitable and can be left as an open question pending observational evidence.

The author believes that the present work definitely establishes the new theory as the best existing theory for gravitation but he has much wider expectations. Some of his still unpublished results show that gravity is quantized and intermediated by massless particles, analogous to photons; other results show that quantum mechanics follows directly from the theory. These results will be the subject of forthcoming papers. Ultimately the author hopes to include strong interaction and find mass as a consequence of geometry.

References
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[1] José B. Almeida. An alternative to Minkowski space-time. Submitted to Gen. Rel. Grav., http://www.arXiv.org/abs/gr-qc/0104029, 2001.

[2] Tom Van Flandern. The speed of gravity – what the experiments say. Phys. Lett. A, 250:1–11, 1998.

[3] Ray D’Inverno. Introducing Einstein’s Relativity. Clarendon Press, Oxford, 1996.

[4] J. L. Martin. General Relativity: A Guide to its Consequences for Gravity and Cosmology. Ellis Horwood Ltd., U. K., 1988.

[5] Bernard F. Schutz. A First Course in General Relativity. Cambridge University Press, Cambridge, 1990.

[6] K. Schwarzschild. On the gravitational field of a mass point according to Einstein’s theory. Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, Phys.-Math. Klasse, pages 189–196, 1916. Eng. transl., http://www.arXiv.org/abs/physics/9905030.

[7] José B. Almeida. Optical interpretation of special relativity and quantum mechanics. In OSA Annual Meeting, Providence, RI, 2000. http://www.arXiv.org/abs/physics/0010076.