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Neutrinoless double-β decay in effective field theory: The light-Majorana neutrino-exchange mechanism

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We present the first chiral effective theory derivation of the neutrinoless double-β decay $nn \rightarrow pp$ potential induced by light Majorana neutrino exchange. The effective-field-theory framework has allowed us to identify and parametrize short- and long-range contributions previously missed in the literature. These contributions cannot be absorbed into parametrizations of the single-nucleon form factors. Starting from the quark and gluon level, we perform the matching onto chiral effective field theory and subsequently onto the nuclear potential. To derive the nuclear potential mediating neutrinoless double-β decay, the hard, soft, and potential neutrino modes must be integrated out. This is performed through next-to-next-to-leading order in the chiral power counting, in both the Weinberg and pionless schemes. At next-to-next-to-leading order, the amplitude receives additional contributions from the exchange of ultrasoft neutrinos, which can be expressed in terms of nuclear matrix elements of the weak current and excitation energies of the intermediate nucleus. These quantities also control the two-neutrino double-β decay amplitude. Finally, we outline strategies to determine the low-energy constants that appear in the potentials, by relating them to electromagnetic couplings and/or by matching to lattice QCD calculations.

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I. INTRODUCTION

The observation of neutrinoless double-β decay ($0\nu\beta\beta$) would be direct evidence of lepton number violation (LNV) beyond the standard model (SM), demonstrating that neutrinos are Majorana fermions [1], shedding light on the mechanism of neutrino mass generation, and probing a key ingredient (LNV) for generating the matter-antimatter asymmetry in the universe via “leptogenesis” [2]. The current experimental limits on the half-lives are quite impressive [3–10], at the level of $T_{1/2} > 5.3 \times 10^{25}$ yr for $^{76}$Ge [10] and $T_{1/2} > 1.07 \times 10^{26}$ yr for $^{136}$Xe [3], with next-generation ton-scale experiments aiming at two orders of magnitude sensitivity improvements.

By itself, the observation of $0\nu\beta\beta$ would not immediately point to the underlying mechanism of LNV. In an effective field theory approach to new physics, LNV arises from $\Delta L = 2$ operators of odd dimension, starting at dimension 5 [11–14]. As discussed in detail in Ref. [15], if the scale of lepton number violation, $\Lambda_{\text{LNV}}$, is in the range 1–100 TeV, short-distance effects encoded in local operators of dimension 7 and 9 provide contributions to $0\nu\beta\beta$ within reach of next-generation experiments. However, whenever $\Lambda_{\text{LNV}}$ is much higher than the electroweak scale, the only low-energy manifestation of this new physics is a Majorana mass for light neutrinos, encoded in a single gauge-invariant dimension-5 operator [11], which induces $0\nu\beta\beta$ through light Majorana-neutrino exchange [16–18].

To interpret positive or null $0\nu\beta\beta$ results in the context of this minimal extension of the SM (the three light Majorana neutrinos paradigm), it is critical to have good control over the relevant hadronic and nuclear matrix elements. Current knowledge of these is somewhat unsatisfactory [19], as (i) few of the current calculations are based on a modern effective field theory (EFT) analysis, and (ii) various approaches lead to estimates that differ by a factor of 2 to 3. In this paper we present the first end-to-end EFT analysis of $0\nu\beta\beta$ induced by light Majorana-neutrino exchange, describing the physics from the scale $\Lambda_{\text{LNV}}$ all the way down to the nuclear energy scale. The EFT framework has allowed us to identify long- and short-range contributions to $0\nu\beta\beta$ previously missed in the literature, that are, by power counting, as large as corrections usually included. The main results of our work are expressions for the leading and next-to-next-to-leading order (N$^2$LO) chiral potentials mediating $0\nu\beta\beta$, and the amplitude induced by the exchange of ultrasoft neutrinos, with momenta much smaller than the Fermi momentum.

II. EFFECTIVE THEORY FRAMEWORK

The starting point of our analysis is the weak scale effective Lagrangian, which we take to be the SM augmented by Weinberg’s $\Delta L = 2$ dimension-5 operator [11],

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \left\{ \frac{u_{\nu \beta}}{\Lambda_{\text{LNV}}} \epsilon_{ij} \epsilon_{mn} L_i^a C L_m^\beta H_j H_n + \text{H.c.} \right\}. \quad (1)$$

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where $u_{a\beta}$ is a $3 \times 3$ matrix, $L = (v_L \ e_L)^T$ is the left-handed SU(2) lepton doublet, $H$ is the Higgs doublet, $\alpha, \beta \in e, \mu, \tau$, and $i, j, m, n$ are SU(2) indices. This operator induces a Majorana mass matrix for neutrinos, of the form $m_{a\beta} = -u_{a\beta} (v^2 / \Lambda_{LNV})$, where $v = (\sqrt{2} G_F)^{-1/2} \approx 246$ GeV is the Higgs vacuum expectation value; for $\Lambda_{LNV} \gg v$ this is the well known “seesaw” relation.

Neglecting QED and weak neutral-current effects, the low-energy effective Lagrangian at scale $\mu \sim 1$ GeV is given by

$$L_{\text{eff}} = L_{\text{QCD}} - \left\{ 2 \sqrt{2} G_F V_{ud} \bar{u} L \gamma^\mu d_L \bar{e} L \gamma_\nu v_{eL} + \frac{1}{2} m_{\beta\beta} \bar{v}_e \gamma^\mu C v_{eL} - C L_O L + \text{H.c.} \right\}. \quad (2)$$

The second term in (2) represents the Fermi charged-current weak interaction. The last two terms encode LNV through the difference in electron momenta, a safe assumption given that $|p_1 - p_2| / k_F \ll 1$.

The amplitude (5) receives contributions from neutrino virtualities ranging from the weak scale all the way down to the IR scale of nuclear bound states. Roughly speaking, one can identify three regions, whose contributions can be conveniently described in terms of appropriate effective theories:

(i) A hard region with $k^2_{L} \equiv (k^0)^2 + k^2 \gg \Lambda^2 \sim 1$ GeV$^2$. This contribution is controlled by the quark-level short-distance behavior of the correlator (6). An operator product expansion analysis shows that integrating out hard neutrinos and gluons generates a local term in the effective action proportional to $O_L$, with Wilson coefficient

$$C_L(\Lambda_\chi) = \frac{8 G_F^2 V_{ud}^2 m_{\beta\beta} \alpha_s(\Lambda_\chi)}{4 \pi}. \quad (7)$$

This short-distance component is currently missing in all calculations of $0\nu\beta\beta$, which start from the nucleon-level realization of the weak currents in the correlator (6). Within such approaches, the new effect can be estimated by considering the hadronic realization of $O_L$, sensitive to pion-range and short-range nuclear effects, that has been studied in the context of TeV sources of LNV [20–24]. In what follows we adopt a chiral EFT approach, and the effect of hard modes will be encoded in local counterterms of the low-energy effective chiral Lagrangian, transforming as $O_L$ under the chiral group.$^2$

(ii) A soft and potential region with $k^2_{L} \sim k^2_F < \Lambda^2$. Here the appropriate hadronic degrees of freedom are pions and nucleons, described by chiral EFT. In analogy with the strong and electroweak interactions in the SM, integrating out pion degrees of freedom and neutrinos with soft $(k^0 \sim |k| \sim k_F)$ and potential $(k^0 \sim k^2_F / m_N, |k| \sim k_F)$ scaling of their 4-momenta generates nucleon level $\Delta L = \Delta I = 2$ potentials that mediate $0\nu\beta\beta$ between nuclear states.

(iii) Ultrasoft or “radiation” region, with neutrino momenta scaling as $k_0 \sim |k| \ll k_F$. Here the effective theory contains as explicit degrees of freedom nucleons interacting via appropriate potentials [see (ii) above], electrons, and essentially massless neutrinos, whose ultrasoft modes cannot be integrated out (similarly to gauge fields in NRQED and NRQCD [28–30]). These modes do not resolve the nuclear constituents, and this part of the amplitude is sensitive to nuclear excited states and transitions among them induced by the electroweak currents.

Contributions to $0\nu\beta\beta$ from regions (ii) and (iii) are included in all existing calculations albeit within certain approximations and not fully in the spirit of EFT. In particular, we have

$$1^\text{To obtain (5) we have approximated } \bar{e}_L(x) \gamma^\mu \gamma^\nu e_1(x) \simeq \bar{e}_L(x) \gamma^\mu \gamma^\nu e_1(x) = g^\mu\nu \bar{e}_L(x) e_1(x), \text{ which amounts to neglecting the difference in electron momenta, a safe assumption given that } |p_1 - p_2| / k_F \ll 1.$$
identified corrections that cannot be parametrized through the single-nucleon form factors. We next discuss the $0\nu\beta\beta$ amplitude in the context of chiral EFT, in which the contributions from region (i) are captured by local counterterms, the contributions from region (ii) can be explicitly evaluated and lead to appropriate potentials, and the contributions from region (iii) can be displayed in terms of non-perturbative nuclear matrix elements of the weak charged current and bound state energies.

III. CHIRAL EFT AND $0\nu\beta\beta$

We describe the low-energy realization of the GeV-scale effective Lagrangian in Eq. (2) in the framework of chiral perturbation theory ($\chi$PT) [31–34] and its generalization to multinucleon systems, chiral EFT [35–37]. Chiral symmetry and its spontaneous and explicit breaking strongly constrain the form of the interactions among nucleons and pions. Pion interactions are strongly constrain the form of the interactions among nucleons and pions. Pion interactions are derivative, allowing for an expansion in interactions between nucleons and pions. Pion interactions are strong and can be systematically included in the power counting by naive dimensional analysis, and the lowest order four-nucleon operators have scaling as $\sim g_{\pi NN}^2 m_N^2$.

Tree-level diagrams involving the above interactions and Majorana neutrino exchange generate $\Delta L = 2$ amplitudes such as $\nu \rightarrow e e$ and $nn \rightarrow ppeee$, scaling as $O(G_F^2 m_{\beta\beta})$. At the one-loop level UV divergences appear which require the introduction of $\Delta L = 2$ local operators with chiral index $\Delta = 2$. We find three independent structures with the correct transformation properties:

$$L_{\Delta L=2}^{(2)} = \frac{5}{12} F_0^4 \left[ F_{0}^{2 \pi} \bar{u} \gamma^\mu \bar{u} \gamma^\mu \right] L_{\nu L\mu}^2$$

Here $L_{\nu L\mu}^2 = uu^\dagger u^\mu$, the dots stand for terms involving more than two pions, and three $a priori$ unknown $O(1)$ low-energy constants (LECs) appear: $g_{\pi NN}^2$, $g_{VNN}^2$, and $g_{VNN}^N$.

In the mesonic and single-nucleon sector of the theory, all momenta and energies are typically $\sim p$, and the perturbative expansion of the $\chi$PT Lagrangian and power counting of loops [31] implies that the scattering amplitudes can also be expanded in $p/\Lambda_\chi$. For systems with two or more nucleons the energy scale $p^2/2m_N$ becomes relevant and the corresponding amplitudes do not have a homogeneous scaling in $p$. Therefore, the perturbative expansion of interactions does not guarantee a perturbative expansion of the amplitudes [35,36]. Indeed, the so-called “reducible” diagrams (in which the intermediate state consists purely of propagating nucleons) are enhanced by factors of $m_N/p$ with respect to the $\chi$PT power counting and need to be resummed. On the other hand, loop diagrams whose intermediate states contain interacting nucleons and pions—“irreducible”—follow the $\chi$PT power counting [35,36]. Reducible diagrams are then obtained by patching together irreducible diagrams with intermediate states consisting of free-nucleon propagators. This is equivalent to solving the Schrödinger equation with a potential $V$ defined by the sum of irreducible diagrams. For external perturbations, such as electroweak currents and $\Delta L = 2$ interactions, one can similarly identify irreducible contributions that admit an expansion in $p/\Lambda_\chi$ [41].

While the scaling of irreducible loop diagrams is unambiguous, the power counting for four-nucleon operators has been the object of much debate in the literature [41–43]. In the Weinberg power counting [35,36], the scaling is determined by naive dimensional analysis, and the lowest order four-nucleon operators in the strong and $\Delta L = 2$ sectors are given,
respectively, by $C_{S,T} \sim \mathcal{O}(F_0^{-2})$ in Eq. (8c) and $g_{\nu\nu}^{\text{NN}} \sim \mathcal{O}(1)$ in Eq. (9). While phenomenologically successful [44], the Weinberg power counting is not fully consistent. Inconsistencies appear in some channels, such as the $^1S_0$ channel, where the cutoff dependence of the solution of the Lippman-Schwinger equation cannot be absorbed by the counterterms that appear at lowest order [42,43]. Various solutions to this problem have been proposed, including treating pion exchange in perturbation theory (perturbative pion or “KSW” scheme [42,45,46]), expanding the nuclear forces around the chiral limit [47], or, for processes at low enough energy, integrating out pions and working in the pionless EFT [41,48]. In Sec. III A, we will discuss $0\nu\beta\beta$ in the Weinberg power counting, and we will extend the treatment to the pionless EFT in Section III B.

Finally, matching the chiral EFT framework to many-body quantum mechanics, one obtains the following nuclear Hamiltonian appropriate for calculating $0\nu\beta\beta$ amplitudes:

$$H_{\text{eff}} = H_0 + \sqrt{2}G_F V_{\text{ud}} \sum_{a=1}^{A} \left( g_\Delta \delta^{\mu 0} - g_\Delta^{\mu 0} \sigma^{(a)\nu} \right) x^{(a)+} \times \tilde{e}_L \gamma_\mu V_L (x_a) + 2G_F^{2} V_{\text{ud}} m_\beta \tilde{e}_L \tilde{e}_R^\nu V_r.$$  \hspace{0.5cm} (10)

The first term ($H_0$) encodes the strong interaction. In the Weinberg counting, the leading-order strong potential is given by one-pion exchange plus the contact terms $C_{S,T}$ [35,36], which in momentum space reads ($q$ is the momentum conjugate to $x_a \equiv x_a - x_b$):

$$V_{\text{strong,0}} = \frac{1}{2} \sum_{a \neq b} \left( \frac{-g_\Delta^2}{4F_\pi^2} \sigma^{(a)} \cdot q \sigma^{(b)} \cdot q \tau^{(a)} \cdot \tau^{(b)} + C_S + C_T \sigma^{(a)} \cdot \sigma^{(b)} \right).$$  \hspace{0.5cm} (11)

Here and in the following, we replace the LO couplings and decay constants by their physical values, $g_\Delta^2 \to g_\Delta^2, F_0 \to F_\pi$, etc., which can be consistently done to the order we are working in the chiral expansion.

The second term in (10) is the usual charged-current weak interaction. From now on, we set $g_V = 1$, neglecting small isospin-breaking corrections. Note that light Majorana neutrinos and electrons with ultra soft momenta are active degrees of freedom in the low-energy theory.

The third term in (10) directly mediates $\Delta L = 2$ amplitudes, and we discuss it next.

A. The $\Delta L = \Delta I = 2$ potential

The potential $V_\nu$ encodes physics from hard scales [the counterterms of Eq. (9)] as well as soft scales, obtained by integrating out pions and Majorana neutrinos with soft and potential scaling of their 4-momenta. In practice $V_\nu$ is given by the sum of “irreducible” diagrams mediating $nn \to ppe\bar{e}$ in chiral EFT. As discussed above, $V_\nu$ admits a chiral expansion:

$$V_\nu = \sum_{a \neq b} \left( V_{\nu,0}^{(a,b)} + V^{(a,b)}_{\nu,2} + \cdots \right).$$  \hspace{0.5cm} (12)

The LO neutrino potential is obtained by tree-level neutrino exchange, which involves the single-nucleon currents (see Fig. 1). In momentum space one finds [15]

$$V_{\nu,0}^{(a,b)} = \frac{1}{q^2} \left\{ \frac{1}{q^2} - g_A^2 \right\} \sigma^{(a)} \cdot \sigma^{(b)} \cdot q \cdot q \left( \frac{2m_\pi^2 + q^2}{(q^2 + m_\pi^2)^2} \right).$$  \hspace{0.5cm} (13)

Analogously to the strong-interaction case, the neutrino potential $V_\nu$ depends only on the momentum scale $q \sim k_F$ and not on infrared scales such as the excitation energies of the intermediate odd-odd nucleus in $0\nu\beta\beta$, often approximated by their average $\bar{E} - 1/2(E_1 + E_2)$. Note that the commonly used neutrino potential [16,19] reduces to $V_{\nu,0}$ when $\bar{E} - 1/2(E_1 + E_2)$ is set to zero.

At N$^2$LO in the Weinberg power counting, several new contributions arise. These consist of (a) corrections to single-nucleon currents, which are often included in the literature via momentum-dependent form factors, and (b) genuine N$^2$LO two-body effects, such as loop corrections to Fig. 1, which induce the short-range neutrino potential $V_{\nu,2}$ that has never been considered in the literature. Note that two-nucleon effects in the weak currents [49,50], which induce three-nucleon potentials in Eq. (12), start contributing to $0\nu\beta\beta$ at N$^3$LO, once one takes into account that $S \cdot v = 0$ and $v \cdot q \approx O(k_F^2/m_N)$. Including N$^2$LO corrections to the single-nucleon currents, the potential $V_{\nu,0}^{(a,b)}$ is modified as

$$V_{\nu,0}^{(a,b)} = \tau^{(a)+} \tau^{(b)+} \frac{1}{q^2} \left( \frac{g_A^2 h_F(q^2)/g_A^2 - \sigma^{(a)} \cdot \sigma^{(b)} h_{\text{GT}}(q^2)}{h_T(q^2)} \right),$$  \hspace{0.5cm} (14)

where we have introduced the tensor operator $S^{(ab)} = -(3 \sigma^{(a)} \cdot q \sigma^{(b)} \cdot q - q^2 \sigma^{(a)} \cdot \sigma^{(b)})/q^2$. The functions $h_F$, $h_{\text{GT}}$, and $h_T$ are expressed in terms of the isovector vector, axial, induced pseudoscalar, and magnetic nucleon form factors as [19]

$$h_F(q^2) = g_A^2(q^2),$$
$$h_{\text{GT}}(q^2) = g_A^2(q^2) + g_P(q^2) g_A(q^2) \frac{q^4}{3m_N}$$
$$+ g_P^2(q^2) \frac{q^4}{12m_N^2} + g_S^2(q^2) \frac{q^4}{6g_A^2m_N^2},$$
\[ h_T(q^2) = -g_F(q^2) g_A(q^2) \frac{q^2}{3m_N} - g_F^2(q^2) \frac{q^4}{12m_N^2} + g_M(q^2) \frac{q^2}{12g_\Lambda^2 m_N^2}. \]  

(15)

In the literature, the dipole parametrization of the vector and axial form factors is often used,

\[ g_V(q^2) = \left( 1 + \frac{q^2}{\Lambda_V^2} \right)^{-2}, \quad g_A(q^2) = \left( 1 + \frac{q^2}{\Lambda_A^2} \right)^{-2}, \]  

(16)

with vector and axial masses \( \Lambda_V = 850 \text{ MeV} \) and \( \Lambda_A = 1040 \text{ MeV} \). The magnetic and induced pseudoscalar form factors are then assumed to be given by

\[ g_M(q^2) = (1 + \kappa_1) g_V(q^2), \quad g_P(q^2) = -\frac{2m_N g_A(q^2)}{q^2 + m_\mu^2}. \]  

(17)

where \( \kappa_1 = 3.7 \) is the nucleon isovector anomalous magnetic moment. Expanding Eqs. (16) and (17) for small \( q^2 \), one recovers the LO and, for \( g_A(q^2) \), the N\(^2\)LO \( \chi \)PT expressions of the nucleon form factors. In the case of \( g_V, g_P, \) and \( g_M \), the N\(^2\)LO \( \chi \)PT results, given for example in Ref. [51], deviate from Eqs. (16) and (17). However, any parametrization that satisfactorily describes the observed nucleon form factors can be used in the neutrino potential (14).

The potential \( V_{\nu,2} \) is induced by one-loop diagrams with a virtual neutrino and pions contributing to \( nn \rightarrow ppee \), built out of the leading interactions of Eqs. (8). They can be separated into three classes, involving the \( \pi \pi \rightarrow ee \) (Fig. 2, upper panel), \( n \rightarrow p\pi^+ee \) (Fig. 2, lower panel), and \( nn \rightarrow ppee \) (Fig. 3) effective vertices. Note that for diagrams such as Fig. 3(A) or 3(D) we include only the two-nucleon irreducible component. We regulate the loops dimensionally, with scale \( \mu \), and subtract the divergences according to the MS scheme. The UV divergences are absorbed by the counterterms of Eq. (9), which cancel the \( \mu \) dependence of the loops and also provide finite contributions.

\[ V_{\nu,2} = (a^{(a)} + r^{(b)+}) \left( V_{VV}^{(a,b)} + V_{AA}^{(a,b)} + V_{AA}^{(a,b)} \ln \frac{m_\mu^2}{\mu_{\mu \nu}^2} + V_{CT}^{(a,b)} \right). \]  

(18)

Since we are interested in potentials that mediate \( 0^+ \rightarrow 0^+ \) nuclear transitions, we only need the parity-even contributions that arise from two insertions of the vector current (VV) or axial current (AA), and we write the N\(^2\)LO two-body potentials as

\[ V_{\nu,2}^{(a,b)} = \left( V_{VV}^{(a,b)} + V_{AA}^{(a,b)} + V_{AA}^{(a,b)} \ln \frac{m_\mu^2}{\mu_{\mu \nu}^2} + V_{CT}^{(a,b)} \right). \]  

(18)
For the contribution of two vector currents, we find
\[ \nu_{\nu\nu}^{(a,b)} = -\frac{g_A^2}{(4\pi F_\pi)^2} \frac{\sigma^{(a)} \cdot q \sigma^{(b)} \cdot q}{m_\pi^2} \left\{ \frac{2(1-\hat{q})^2}{\hat{q}^2(1+\hat{q})^2} \ln(1+\hat{q}) - \frac{2}{\hat{q}} + \frac{7 - 3\hat{q}L_\pi}{(1+\hat{q})^2} + \frac{L_\pi}{1+\hat{q}} \right\}, \] (19)
where \( \hat{q} = -q^2/m_\pi^2 \) and \( L_\pi = \ln \frac{m_\pi^2}{\Lambda_\pi^2}. \) This form agrees with Ref. [52], where the virtual photon corrections to the one-pion exchange potential were calculated. For the axial component we find
\[ \nu_{AA}^{(a,b)} = \frac{g_A^2}{(4\pi F_\pi)^2} \frac{\sigma^{(a)} \cdot q \sigma^{(b)} \cdot q}{m_\pi^2} \left\{ \frac{g_A^2}{1+\hat{q}}(L_\pi - 4) + \frac{1}{(1+\hat{q})^2} \right\} + \frac{1^{(a)} \times 1^{(b)}}{(4\pi F_\pi)^2} \left\{ \frac{3}{4} (1 - g_A^2) L_\pi + g_A^2 f_4(\hat{q}) + \frac{g_A^2}{2} f_2(\hat{q}) + f_0(\hat{q}) + 24 g_A^2 F_\pi^2 C_T [L_\pi + 1] \right\}, \] (20)
\[ \nu_{AA}^{(a,b)} = \frac{g_A^4}{(4\pi F_\pi)^2} \frac{\sigma^{(a)} \cdot q \sigma^{(b)} \cdot q}{m_\pi^2} \left\{ \frac{g_A^2}{1+\hat{q}}(L_\pi - 4) + \frac{1}{(1+\hat{q})^2} \right\} - \frac{g_A^2}{(4\pi F_\pi)^2} 48 C_T 1^{(a)} \times 1^{(b)}, \] (21)
where
\[ f_0(\hat{q}) = -\frac{1 + 8\hat{q}}{6\hat{q}} + (1 + \hat{q}) \left( \frac{1 + 8\hat{q} + \hat{q}^2}{6\hat{q}^2} \right) \ln(1+\hat{q}) - \frac{1}{24} \left( 4 + \hat{q} \right) \left( 5 + 2\hat{q} \right) g(\hat{q}), \] (22)
\[ f_2(\hat{q}) = \frac{1 + 8\hat{q}}{3\hat{q}} + \left( 1 + \hat{q} \right) \left( \frac{1 + 5\hat{q} + \hat{q}^2}{3\hat{q}^2} \right) \ln(1+\hat{q}) - \frac{1}{12} \left( 40 + 47 \hat{q} + 10\hat{q}^2 \right) g(\hat{q}), \] (23)
\[ f_4(\hat{q}) = \frac{1}{6} \left( 20 + \hat{q} - \frac{12}{4+\hat{q}} \right) \left( \frac{640 + 912\hat{q} + 375\hat{q}^2 + 46\hat{q}^3}{6\hat{q}^2(1+\hat{q})} \right) \ln(1+\hat{q}) + \frac{1}{24(4+\hat{q})} g(\hat{q}), \] (24)
and the loop function \( g(\hat{q}) \) is
\[ g(\hat{q}) = \frac{4}{\sqrt{\hat{q}(4+\hat{q})}} \arctanh \left( \frac{\hat{q}}{4+\hat{q}} \right). \] (25)

Finally, the counterterm potential reads
\[ \nu_{CT}^{(a,b)} = \frac{g_A^2}{(4\pi F_\pi)^2} \frac{\sigma^{(a)} \cdot q \sigma^{(b)} \cdot q}{m_\pi^2} \left\{ \frac{5}{6} g^{\pi\pi}_V \frac{\hat{q}}{1+\hat{q}} - g^{\pi N}_V \frac{1}{1+\hat{q}} \right\} - \frac{2g^{NN}_V}{(4\pi F_\pi)^2} \frac{1^{(a)} \times 1^{(b)}}{\hat{q}(4+\hat{q})}, \] (26)
The \( \mu \) dependence of \( g^{\pi\pi}_V, g^{\pi N}_V, \) and \( g^{NN}_V \) cancels the \( \mu \) dependence of \( L_\pi \) in Eqs. (19) and (20). We will discuss strategies to estimate the finite parts of the LECs in Sec. III D below.

B. The \( \Delta L = \Delta I = 2 \) potential in the pionless EFT
The previous discussion assumed the Weinberg power counting, that, while phenomenologically successful [44], is not formally consistent [42,43]. Few-body systems and processes characterized by scales \( p \ll m_\pi \) can be studied in pionless EFT, a low-energy EFT, in pion masses, in pion degrees of freedom are integrated out [41,48]. For physical pion masses, pionless EFT converges very well for the \( A = 2,3 \) systems, and works satisfactorily well for up to \( A = 6 \) [53]. While the application of this EFT to nuclei with \( A > 6 \) needs to be studied in more detail, it is interesting to extend the framework developed in the previous section to pionless EFT, especially in the light of a possible matching to lattice calculations of 0\( \nu \beta \beta \) matrix elements performed at heavy pion masses. A similar matching between lattice and pionless EFT for strong interaction and electroweak processes has been carried out in Refs. [27,54–59]. While the lattice QCD calculations relevant for 2\( \nu \beta \beta \) were performed at a single lattice spacing of \( a \sim 0.145 \) fm and at \( m_\pi \sim 806 \) MeV [27,59], they represent the first step for the field and are very promising. We are optimistic that in the near future lattice calculations of electroweak processes and 0\( \nu \beta \beta \) in the two nucleon system will reach control over all lattice systems, as recently achieved for the nucleon axial coupling \( g_A \) [60–63].

Pionless EFT describes physics at the scale \( p \) smaller than the cutoff of the theory \( \Lambda_\pi \sim m_\pi \). For power counting purposes, we introduce the scale \( \Lambda \sim p \ll \Lambda_\pi \). The leading-order Lagrangian is given by Eq. (8c), and the fine tuning of the S-wave nucleon-nucleon scattering lengths is accounted for by assigning the coefficients \( C_{S,T} \) the scaling
\[ C_{S,T} = O \left( \frac{4\pi}{m_N \Lambda} \right). \] (27)
Using dimensional regularization with power divergence subtraction (PDS) [46], at the scale \( \mu \) the couplings \( C_{S,T} \) can be expressed in terms of the spin-singlet \( ^1S_0 \) and spin-triplet \( ^3S_1 \).
The relevance of the ultrasoft region can be also understood diagrammatically. Indeed, while diagram 3(O) is suppressed according to
\[ (4 \pi F_0)^2 - \frac{1}{\mu} \] the loop (37) and the ultrasoft diagram 3(N) and 3(O). These diagrams are scaleless and thus are enhanced in the pionless theory [41]. The leading \( \Delta L = 2 \) potential in the pionless EFT has the form
\[ V^{(a,b)}_{\nu,0} = \tau^{(a)+}\tau^{(b)+}\left[ \frac{1}{q^2} (g_{\nu}^2 - g_{\Lambda}^2 \sigma^{(a)} \cdot \sigma^{(b)}) - \frac{2 g_{\nu}^{NN}}{(4 \pi F_0)^2} 1^{(a)} \times 1^{(b)} \right]. \] (29)

This first term comes from long-range neutrino exchange, as in Eq. (13), with the difference that the contributions of the induced pseudoscalar form factor are subleading. In addition, the scaling of the nucleon-nucleon coupling \( g_{\nu}^{NN} \), introduced in Eq. (9), is modified. This coupling connects two \( S \) waves and thus is enhanced in the pionless theory [41], scaling as
\[ g_{\nu}^{NN} = \mathcal{O}(1) \] and satisfies
\[ \frac{d}{d \ln \mu} g_{\nu}^{NN} = \frac{1 + 3 g_{\Lambda}^2}{2}. \] (34)

Equation (33), together with (28), confirms the power-counting expectation of Eq. (30).

Beyond leading order in the pionless EFT there appears a single four-nucleon operator contributing to \( V_{\nu,0}^{(a)} \) at \( N^2 \)LO, which is conveniently expressed in terms of the \( S_0 \) projectors \( P_{\pm}^{(S_0)} \),
\[ \mathcal{L} = g_{\nu,2}^{NN} \kappa \ell \ell \Sigma \mathcal{E}^{T}_{\nu} ((N^T \nabla^2 P_{\pm}^{(S_0)} N)(N^T \nabla^2 P_{\pm}^{(S_0)} N)^\dagger) + \text{H.c.}, \] (35)

with \( \kappa = (\mu_\nu^2 \ell \tau_{\ell} \tau_{\pm})/2\sqrt{2}, \) and \( \kappa \) given in Eq. (9). The LEC \( g_{\nu,2}^{NN} \) scales as
\[ g_{\nu,2}^{NN} = \mathcal{O}(\frac{A^2_{\Lambda}}{\Lambda^2_{\Lambda}}). \] (36)

Operators connecting two neutrons and two protons in the \( P \) waves are not enhanced by \( \Lambda^{-2} \), and appear at even higher order.

Additional corrections to the potential \( V_{\nu,0} \) arise from loop diagrams \( 3(N) \) and \( 3(O) \). These diagrams are scaleless and vanish in dimensional regularization. If the infrared divergence is regulated by a neutrino mass \( m_\nu \), the \( m_\nu \) dependence is canceled by the diagrams in Fig. 4, and one obtains
\[ V^{(a,b)}_{\nu,\text{loops}} = -\tau^{(a)+}\tau^{(b)+} (1^{(a)} \times 1^{(b)}) \frac{48 g_{\nu}^{NN} C_T}{(4 \pi)^2} \ln \frac{\mu^2}{\mu_{\text{us}}^2}. \] (37)

The \( \mu \) dependence in (37) is reabsorbed by a subleading term in \( g_{\nu,2}^{NN} \). Equation (37) shows that it is always possible to choose \( \mu_{\text{us}} \) so that the correction to the potential vanishes, and the effect of diagrams \( 3(N) \) and \( 3(O) \) is all encoded in the ultrasoft contribution. Note that the loop (37) and the ultrasoft amplitude are suppressed by \( p/(4 \pi m_\nu) \sim (p/\Lambda_{\text{so}}) \times 1/(4 \pi)^2 \) with respect to the LO, and are thus smaller than corrections from Eq. (35), scaling as \( p^2/\Lambda_{\text{so}}^2 \).

The relevance of the ultrasoft region can be also understood diagrammatically. Indeed, while diagram (3O) is suppressed
with respect to the LO, diagrams with an arbitrary number of insertions of $C_1$ and $C_2$ between the emission and absorption of the neutrino are not suppressed with respect to $3\langle O \rangle$. These diagrams need to be resummed, and correspond to building up the intermediate states. They are captured by the ultrasoft contribution discussed in Sec. III C below.

C. The $0\nu\beta\beta$ amplitude

Starting from the nuclear Hamiltonian of Eq. (10), one calculates the full $0\nu\beta\beta$ amplitude as the sum of two contributions,  

$$ T_{fi} = -T_{\text{lep}} \times ((V_{\nu,0})_{fi} + (V_{\nu,2})_{fi}) + T_{\text{usoft}}, $$

(38)

where we defined $T_{\text{lep}} = 4G_F^2 V_{\nu,0}^2 m_{\beta\beta}^2 L (p_c) C\bar{U}_F^T(p_c)$. The first term represents a single insertion of the $\Delta L = \Delta I = 2$ potential [third term in Eq. (10)]. On the other hand $T_{\text{usoft}}$ arises from double insertions of the weak interaction [second term in Eq. (10)], which involves the exchange of ultrasoft Majorana neutrinos, with four-momenta scaling as $k^0 \sim |k| \ll k_F$. The diagram contributing to $T_{\text{usoft}}$ is given in Fig. 6, and its expression is

$$ T_{\text{usoft}} = -\frac{T_{\text{lep}}}{4} \sum_n \int \frac{d^4-k}{(2\pi)^4} \frac{1}{|k|} \times \left[ \frac{\langle f | J_{\mu}[n] | n \rangle \langle n | J^{\mu}|i \rangle}{|k| + E_2 + E_n - E_i - i\eta} \right. $$

$$ + \left. \frac{\langle f | J_{\mu}[n] | n \rangle \langle n | J^{\mu}|i \rangle}{|k| + E_1 + E_n - E_i - i\eta} \right]. $$

(39)

Here $J_\mu \equiv V_\nu(x_0 = 0) - A_\nu(x_0 = 0) = \sum_i \epsilon^{(i)\mu} (\delta_{ij0} - g_A \delta_{ijk} \bar{g}_k^{(i)}) \delta^{(3)}(x_i)$ is the lowest-order nuclear weak current and $|n\rangle$ represent a complete set of nuclear states (eigenstates of $H_0$) with 3-momentum $\pm k + (1/2)(p_i + p_\nu)$ [the $\pm$ refer to the first and second terms in Eq. (39), respectively]. The quantum numbers of $J_\mu$ imply that, for given $0^+$ even-even initial and final states, $|n\rangle$ spans the set of eigenstates of the intermediate odd-odd nucleus. Since we are in the ultrasoft regime, we expand $\langle f | J_\mu[n] | n \rangle J^{\mu}|i \rangle$ in $k$ and keep only the $k = 0$ term, noting that finite momentum terms would produce upon integration additional positive powers of the IR scale $E_{1,2} + E_n - E_i$, and therefore additional suppression.

Evaluating the loop integral in dimensional regularization with MS subtraction, we find

$$ T_{\text{usoft}}(\mu_{\text{us}}) = T_{\text{lep}} \times \frac{1}{8\pi^2} \sum_n \langle f | J_{\mu}[n] | n \rangle \langle n | J^{\mu}|i \rangle $$

$$ \times \left\{ (E_2 + E_n - E_i) \left[ \frac{\mu_{\text{us}}}{2(E_2 + E_n - E_i)} + 1 \right] \right. $$

$$ \left. + (E_1 + E_n - E_i) \left[ \frac{\mu_{\text{us}}}{2(E_1 + E_n - E_i)} + 1 \right] \right\}, $$

(40)

The UV divergence and the associated logarithmic dependence on $\mu_{\text{us}}$ are reabsorbed by the term in the potential proportional to $V_{\nu,0}$. To verify this, using the completeness relation for the eigenstates of $H_0$ we write the term proportional to $\ln \mu_{\text{us}}$ in (40) as a double commutator [29] and evaluate it using the lowest order chiral potential in $H_0$, finding

$$ \frac{dT_{\text{usoft}}}{d\ln \mu_{\text{us}}} = -T_{\text{lep}} \times \frac{1}{8\pi^2} \langle f | [J_{\mu}, [H^0, H_0]] | i \rangle $$

$$ = -T_{\text{lep}} \times \frac{1}{8\pi^2} \langle f | [A, [A, H_0]] | i \rangle $$

$$ = -2 T_{\text{lep}} \times \sum_{a,b} \langle f | \bar{\tau}^{(a)\mu} + \tau^{(b)\mu} + \bar{\tau}^{(a)b}_{AA} | i \rangle, $$

(41a)

$$ -T_{\text{lep}} \frac{d(V_{\nu,2})_{fi}}{d\ln \mu_{\text{us}}} = +2 T_{\text{lep}} \times \sum_{a,b} \langle f | \pi^{(a)\mu} + \tau^{(b)\mu} + \bar{\tau}^{(a)b}_{AA} | i \rangle, $$

(41b)

with $\bar{\tau}^{(a)b}_{AA}$ given in (21). The $\mu_{\text{us}}$ independence of the total amplitude implied by Eqs. (41) is a non-trivial consistency check for our calculation and allows us to pick a convenient scale, such as $\mu_{\text{us}} = m_N$, which eliminates the contribution of $\bar{\tau}^{(a)b}_{AA}$. Moreover, the cancellation implies that $T_{\text{usoft}}$ has the same chiral scaling as $(V_{\nu,2})_{fi}$, and is thus two orders down compared to the leading contribution $(V_{\nu,0})_{fi}$. This suppression can also be seen by directly comparing the scaling of $T_{\text{usoft}}$ and $(V_{\nu,0})_{fi}$. In fact, $(V_{\nu,0})_{fi} \sim 1/(4\pi R_A) \sim k_F/(4\pi)$, which leads to $T_{\text{usoft}}/T_0 \sim \sum_n (E_{1,2} + E_n - E_i)/(4\pi k_F) \times \langle f | J_{\mu}[n] | n \rangle \langle n | J^{\mu}|i \rangle$. Note that the dimensionless transition matrix elements $\langle f | J_\mu[n] | n \rangle J^{\mu}|i \rangle$ also control the $2\nu\beta\beta$ decay amplitude, through a different $E_n$-dependent weighted sum [19]. The overlap matrix elements quickly die out for $E_n - E_i > 10$ MeV, as borne out in several explicit calculations using different many-body methods [65–67]. Therefore, recalling the scaling $E_n - E_i \sim k_F^2/m_N$ we recover $T_{\text{usoft}}/T_0 \sim \epsilon^2$.

3This follows by taking matrix elements of $V_{\nu,0}(r_{ab}) \propto 1/(4\pi r_{ab})$ between nuclear states normalized to unity.
In summary, in the chiral EFT framework one expects the following hierarchy of contributions to the $0\nu\beta\beta$ amplitude of Eq. (38):

1. The leading contribution is given by $T_0 = -T_{\text{lep}} \times (V_{\nu}, 0)$, with $V_{\nu}, 0$ given in Eqs. (12) and (13). This leading term is not sensitive to the intermediate states of the odd-odd nucleus. $V_{\nu}, 0$ corresponds to the standard neutrino potential [19] evaluated at $E - 1/2(E_1 + E_f) \rightarrow 0$.

2. A commonly included (but incomplete) N$^3$LO contribution is obtained by inserting momentum dependent form factors in (13), as shown in Eq. (14) and the subsequent discussion.

3. The new N$^3$LO contribution is given by $T_2 = -T_{\text{lep}} \times (V_{\nu}, 2)$, with $V_{\nu}, 2$ given in Eqs. (12), (18) and $T_{\text{soft}}(\mu)$ from Eq. (40). With the choice of renormalization scale $\mu_\text{ren} = m_\pi$, $\tilde{V}_{AA}$ drops out of the calculation. Note that $T_{\text{soft}}$ requires the same nuclear structure input needed in $\nu\beta\beta$ calculations, namely $(f|J_\mu|n)(n|J^\mu|i)$ and the excited energy levels of the intermediate nucleus ($E_\text{ex}$).

4. In the pionless EFT one should use $(V_{\nu}, 0)$ in the chiral EFT and pionless EFT ([V_{\nu}, 0] in pionless EFT), one encounters a priori unknown counterterms, which can be estimated in naive dimensional analysis. In the next section we discuss how to go beyond this rough estimate.

D. Estimating the low energy constants

Chiral EFT. Interestingly, $\pi\pi$ and $\pi N$ interactions similar to those in Eq. (9) are encountered when considering electromagnetic corrections to meson-meson and meson-nucleon interactions [68–72]. In the electromagnetic case, these operators arise from two insertions of the electromagnetic interaction, which involves the exchange of hard photons. In the case considered here, the operators are generated by the exchange of hard neutrinos. However, the neutrino propagator and weak vertices combine to give, up to a factor, a massless gauge boson propagator in Feynman gauge [see Eqs. (3) and (5)].

This formal analogy can be exploited to relate the LECs needed for $0\nu\beta\beta$ (two insertions of the $\tau^-$ weak current) to the LECs associated with the $\Delta I = 2$ component of the product of two electromagnetic currents, that belongs to the $5L \times 1R$ irreducible representation of the chiral SU(2) group. Based on this observation, we have identified the operators of Refs. [68–72] that correspond to $g_{\pi\pi}^{\tau_-}$ and $g_{\pi N}^{\tau_-}$. Explicitly, the relation between our couplings renormalized in the MS scheme and those of, e.g., Ref. [70] (which are in the modified MS scheme commonly employed in $\chi$PT [73]), is given by

$$g_{\nu}^{\pi\pi} = -\frac{4g_{\pi\pi}}{5}(4\pi)^2 \left( \kappa_5' + \frac{3}{8(4\pi)^2} \right),$$

$$g_{\nu}^{\pi N} = (4\pi F_\pi)^2 \left( \frac{g_A^2 + g_S^2}{g_A} - \frac{1 - g_A^2}{(4\pi F_\pi)^2} \right).$$

We suggest that many-body calculations be organized according to this hierarchy, with the aim of (i) comparing results of various methods order by order in chiral EFT and (ii) checking to what degree the chiral counting is respected in large nuclei.

Finally, note that in evaluating $(V_{\nu}, 2)$ in chiral EFT and pionless EFT ([V_{\nu}, 0] in pionless EFT), one encounters a priori unknown counterterms, which can be estimated in naive dimensional analysis. In the next section we discuss how to go beyond this rough estimate.
IV. CONCLUSIONS

We have presented the first comprehensive effective theory analysis of $0\nu\beta\beta$ induced by light Majorana-neutrino exchange, describing the physics from the scale $\Lambda_{\ell,\nu}$ all the way down to the nuclear energy scale. The full $0\nu\beta\beta$ amplitude receives contributions from hard, soft, potential, and ultrasoft neutrino virtualities. Starting from the quark-level description, we have performed the matching to chiral EFT. In this context, contributions from hard modes are captured by local counterterms, while the contributions from soft and potential modes can be explicitly evaluated and lead to appropriate nuclear potentials—insensitive to properties of intermediate nuclear states—for which we have derived LO and NLO expressions [see Eqs. (12), (13), (18)]. We have identified new contributions that cannot be captured by parametrizations of single nucleon form factors and by power-counting arguments as large as terms usually included in the $0\nu\beta\beta$ amplitude. The contributions from ultrasoft modes appear at NLO and can be displayed in terms of nuclear matrix elements of the weak current and excitation energies of the intermediate odd-odd nuclei, that also control the $2\nu\beta\beta$ amplitude [see Eq. (40)].

In Sec. III D we discuss strategies to determine the low-energy constants (LECs) that appear in the potentials. We have worked out a connection to the electromagnetic LECs encoding the effect of hard virtual photons in hadronic processes, that can be obtained from model estimates, lattice QCD, and, at least in principle, from data. We have also discussed a strategy to match directly to $\Delta I = 2$ hadronic amplitudes that could be calculated in lattice QCD.

While the bulk of our discussion is based on the Weinberg version of chiral EFT, in Sec. III B we also present the potential in pionless EFT to LO and NLO. We plan to study the consistency of the Weinberg power counting for $0\nu\beta\beta$ decay in future work.

In Sec. III C we discuss the hierarchy of chiral EFT contributions to the “master formula” for the $0\nu\beta\beta$ amplitude, Eq. (38), describing their relation (when applicable) to the standard treatment of $0\nu\beta\beta$ matrix elements in the literature. We advocate that many-body calculations with existing methods [78–89], as well as with methods under development [90], should be organized according to the EFT power counting scheme, isolating LO, NLO, and ultrasoft contributions. Ideally, the neutrino potential derived here should be used with nuclear wavefunctions also based on chiral EFT and computed at next-to-leading order, or higher. This is particularly important when evaluating short range potentials. Benchmark calculations of double-$\beta$ matrix elements of light nuclei [91] will help quantify the impact of the new NLO potential and ultimately assess the validity of the chiral framework.

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