Low-Redshift Cosmic Baryon Fluid on Large Scales and She-Leveque Universal Scaling

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We investigate the statistical properties of cosmic baryon fluid in the nonlinear regime, which is crucial for understanding the large-scale structure of the universe. With the hydrodynamic simulation sample of the Universe in the cold dark matter model with a cosmological constant, we show that the intermittency of the velocity field of cosmic baryon fluid at redshift $z = 0$ in the scale range from the Jeans length to about 16 h$^{-1}$ Mpc can be extremely well described by She-Leveque's universal scaling formula. The baryon fluid also possesses the following features: (1) for volume weight statistics, the dissipative structures are dominated by sheets, and (2) the relation between the intensities of fluctuations is hierarchical. These results imply that the evolution of highly evolved cosmic baryon fluid is similar to a fully developed turbulence.

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Introduction.—The formation and evolution of large-scale structure of the universe is governed by the gravitational clustering of cosmic matter, in which about 72\% is dark energy, 24\% cold dark matter, and 4\% baryon. Most baryonic matter is in the form of gas. Therefore, the evolution of the density and velocity fluctuations of cosmic baryon fluid is dominated by the underlying gravitational potential of dark matter. In the linear regime, the baryon fluid follows the mass and velocity fields of collisionless dark matter point by point. In the nonlinear regime, however, as first pointed out by Shandarin and Zeldovich in their early study of structure formation, the dynamical behavior of cosmic matter clustering on large scales is similar to turbulence [1]. They emphasized that the motion of self-gravitation matter in the expanding universe is like that of noninteracting matter moving by inertia. In other words, cosmic matter underwent a scale-free evolution somewhat like fully developed turbulence in inertial range.

Nevertheless, the dynamical difference between the structure formation of cosmic matter and turbulent flow in incompressible fluid is obvious. The latter is rotational in general [2], while the former is irrotational, because vorticities do not grow during the clustering [3]. In turbulence, energy passes from large to the smallest eddies, while cosmic baryonic gas falls into massive halos to form structures, including light-emitting objects.

Yet, the turbulencelike behavior of cosmic baryon fluid has been gradually noticed in the last decade. Although the evolution of cosmic baryon fluid is governed by the Naiver-Stokes equation, the dynamics of growth modes of the fluid can be sketched by a stochastic force driven Burgers’ turbulence [4]

\[
\frac{\partial \varphi}{\partial t} + \frac{1}{2a^2}(\nabla \varphi)^2 - \frac{\nu}{a^2} \nabla^2 \varphi = \phi, \tag{1}
\]

where $\varphi$ is velocity potential, $\nu = (1/a)\nabla \varphi$, $a$ is the scale function of the cosmological expansion, and $\nu$ is from the Jeans diffusion. The stochastic term $\phi$ is actually the gravitational potential of the dark matter mass density perturbation, i.e., $\nabla^2 \phi = 4\pi G a^2 \bar{\rho}_{dm} \delta_{dm}$, and $\delta_{dm} = (\bar{\rho}_{dm} - \bar{\rho}_{dm})/\bar{\rho}_{dm}$. The stochastic property of $\delta_{dm}$ is given by the correlator $\langle \delta_{dm}(k) \delta_{dm}(k') \rangle = P(k)\delta_{kk'}$ and the power spectrum $P(k) \propto k^{-\alpha}$. For the flat cold dark matter model with a cosmological constant ($\Lambda$CDM), the index is initially $\alpha = 2.3$ in the scale range considered. A basic feature of the fluid described by Eq.(1) is that the Burgers’ turbulence is developed when the Reynolds number is large [5]. For instance, the probability distribution functions of the velocity differences across a distance $r$, $\delta v_r \equiv \{v(x + r) - v(x) \cdot r/r\}$, are scaling, when $r$ is larger than the Jeans length [6]. These results indicate that some dynamical features of the cosmic baryon fluid at low redshifts in the nonlinear regime are turbulencelike.

Therefore, an important question is how well and why the dynamical state of cosmic baryon fluid at low redshifts can be described as turbulence. Although the cosmic baryon fluid has been a central topic in cosmology for a long time [7], no turbulencelike behavior has been studied. In this Letter, we investigate this problem in the context of the universal scaling law of fully developed turbulence. Using numerical samples, we first show that the universal scaling does exist in the cosmic baryon fluid, and then analyze the underlying physics of this scaling.

Intermittent exponent.—The universal property of fully developed turbulence is measured by the structure functions $S_p(r)$ and intermittent exponent $\zeta_p$, defined as

\[
S_p(r) \equiv \langle \delta v_r^p \rangle \sim r^{\zeta_p}. \tag{2}
\]

Based on dimensional argument of hierarchical evolution, Kolmogorov in 1941 predicted that for fully developed turbulence on scales of inertial range, the intermittent
exponent is $\zeta_p = p/3$. Experimental and numerical results do not, however, support the $p/3$ law. It should be attributed to intermittency; i.e., turbulence field in the inertial range is characterized by stronger non-Gaussianity on smaller scales. A remarkable development was made by She and Leveque (SL hereafter). They proposed that the non-Gaussian behavior of fully developed turbulence is determined by the hierarchical structure originated from the Navier-Stokes equation, and the $p/3$ law should be replaced by

$$\zeta_p/\zeta_3 = [1 - C(1 - \beta^3)]p/3 + C(1 - \beta^p),$$

where $C$ is the Hausdorff codimension of the most dissipative structures, and parameter $\beta$ is given by the hierarchical evolution (see below). The SL formula Eq.(3) is in excellent agreement with various experiments of turbulence, including also turbulence in compressible fluid. The SL scaling law of structure function is considered to be universal for characterizing the fully developed turbulence.

To investigate the universal scaling of cosmic baryon fluid, we use the cosmological hydrodynamic simulation samples produced by the code WIGEON (Weno for Intergalactic medium and Galaxy Evolution and formatiON) This is a hybrid hydrodynamic/$N$-body simulation, consisting of the WENO algorithm for baryonic fluid, and $N$-body simulation for particles of dark matter. The baryon fluid obeys the Navier-Stokes equation, and is gravitationally coupled with collisionless dark matter. We use the $\Lambda$CDM cosmological model with parameters given by the recent observations of the cosmic microwave background radiation. The linear power spectrum of mass density perturbations is taken from the fitting formulas of Eisenstein & Hu. The atomic processes in the plasma of hydrogen and helium of primordial composition, including ionization, radiative cooling and heating, are modeled in the same way as in

The simulations were performed in a periodic, cubic box of size $64 \, h^{-1}$Mpc with a $512^3$ grid and an equal number of dark matter particles. The simulations start at a redshift $z = 99$. A uniform UV-background of ionizing photons is switched on at $z = 6$ to heat the gas and reionize the universe. The temperature of the baryonic gas generally lies in the range $10^4 - 10^6$ K, and the speed of sound $v_s$ in the baryonic gas is only a few km s$^{-1}$ to a few tens km s$^{-1}$. The Jeans length $\lambda_J$ yields a term like the viscosity $\mu \approx v_s \lambda_J$. On the other hand, the bulk velocity of the baryonic gas is of the order of hundreds of km s$^{-1}$. Therefore, the Reynolds number would be larger than $\sim 100$ if the scales under consideration are larger than the Jeans length $\lambda_J$, which is in the range $0.1 - 0.3 \, h^{-1}$Mpc for redshifts $z < 4$

These samples are successful to model the power spectrum and non-Gaussian features of the observed transmitted flux of Ly$\alpha$. Hence it would be suitable to study whether the universal scaling law Eq.(3) is available for cosmic baryon fluid. For this purpose, we randomly sampled 10,000 one-dimensional sub-samples, with each one containing 512 data points. At each point of a line, the peculiar velocity and mass density of the baryonic gas is recorded.

We calculated the moments of velocity difference $\delta v_r$ for $r$ from 1 to $16 \, h^{-1}$ Mpc, of which the lower limit is larger than the Jeans length, and the upper limit is due to the size of the simulation box. The intermittent exponent $\zeta_p$ for the sample at redshift $z = 0$ is shown in Fig.1. The error bars are the variance of the samples. The intermittency of cosmic baryon fluid is excellently fitted by the scaling Eq.(3) on all orders considered, if the parameters of SL formula [Eq.(3)] are taken to be $C = 1$ and $\beta^3 = 1/3$.

Our sample is very different from experimental and numerical samples used to test the SL formula. The success shown in Figure 1 further supports that the hypotheses used to derive the SL formula Eq.(3) are universal so that it would also be available for cosmic baryon fluid. In this sense, the non-Gaussian behavior of the baryon fluid in an expanding universe at redshift $z = 0$ can essentially be described as a fluid satisfying the universal scaling for fully developed turbulence.

**Singular dissipative structures.**—Simply speaking, the underlying physical picture of the SL formula is as follows. In the inertial range, the fluid evolution is governed by scale-covariant interactions from the Navier-Stokes equations. The kinetic energy of fluid is dissipated in singular structures, and the energy dissipation on different scales satisfies hierarchical relation.

Although the clustering of baryon fluid is governed by the gravity of the background mass field, the expansion of the Universe eliminates the gravity of the uniformly distributed matter. The peculiar motion of baryon fluid feels only the gravity given by the random fluctuations of the distributed dark matter. In a nonlinear regime, baryon fluid decouples from dark matter. It is similar to the decoupling of a passive substance from the underlying field during nonlinear evolution. The decoupling
leads to the velocity of baryon fluid generally being less or not larger than the velocity of dark matter. Therefore, baryon fluid is actually driven by a random force of the dark matter gravity. The potential of the random gravity is of power law, i.e., scale free. In the nonlinear regime, the power of fluctuations transfers from larger to smaller scales. Therefore, in the scale range of 1 to about 10 $h^{-1}$ Mpc the evolution of cosmic fluid is similar to turbulence in inertial range.

Since $C = 1$ gives a best fitting (Fig.1), the singular dissipative structures should be two-dimensional sheets. It is well known that in structure formation sheets are dominant in terms of volume weight statistics. For incompressible fluid, the singular structures are traced by the amplitude of vorticity $|\nabla \times \mathbf{v}|$. Since the Burgers turbulence is caused by shock wave, the singular dissipative structures are traced by the divergence of baryon fluid $\nabla \cdot \mathbf{v}$. Figure 2 presents a typical $\nabla \cdot \mathbf{v}$ contour of baryonic fluid in a slice with area $16 \times 16$ $h^{-2}$ Mpc$^2$ and thickness $2$ $h^{-1}$ Mpc at $z = 0$. One can clearly see the filamentary structures in two dimensions, and therefore, they are most likely to be sheets in three dimensions. In Fig.2, we also plot the difference of velocity fields between dark matter and baryon matter. It shows that the decoupling between the velocity fields of baryon fluid and dark matter occurred on the entire field.

**Hierarchical relation.**—The parameter $\beta$ of Eq. (3) describes the hierarchical evolution of the cosmic baryon matter. The hierarchical clustering is well known in the theory of large-scale structure formation of the Universe. It has been proposed that the hierarchical clustering is given by the relation $\langle \delta \rho_i^N \rangle \propto \langle \delta \rho_l^2 \rangle^{N-1}$, where $\delta \rho_l$ is the density fluctuation field of the cosmic matter on scale $l$. In hydrodynamics, this relation is the so-called pure hierarchical relation. It has been shown, however, that the observed intermittency is not subject to the pure hierarchical relation if the proportional coefficient between $\langle \delta \rho_i^N \rangle$ and $\langle \delta \rho_l^2 \rangle^{N-1}$ is scale independent.

In the universal scaling scheme, the hierarchy is not described by mass density fluctuation $\delta \rho$, but by a hierarchical relation of structure functions as

\[ F_{p+1}(r) = A_p F_p(r)^{\beta'} F_{\infty}(r)^{1-\beta'}, \quad (4) \]

where $F_p(r) \equiv S_{p+1}(r)/S_p(r)$, and number $A_p$ are scale $r$ independent. $F_{p+1}(r)$ describes the intensity of fluctuations. The larger the $p$ of $F_p(r)$, the higher the intensity of fluctuations. The most intermittent structures are described by $F_{\infty}(r)$. Equation (4) describes the hierarchical relation between fluctuations with different intensities. Equation (4) is invariant with respect to a translation in $p$. The parameter $\beta'$, which should be in the range $0 < \beta' < 1$, is to measure the degree of intermittency of turbulent flow. The smaller the $\beta'$, the stronger the intermittency. For $\beta' = 1$, the field is not intermittent.

From Eq. (4), we have

\[ \frac{F_{p+1}(r)}{F_2(r)} = \frac{A_p}{A_1} \left( \frac{F_p(r)}{F_1(r)} \right)^{\beta'}. \quad (5) \]

If $A_p$ is $p$ independent, we should have

\[ \ln F_{p+1}(r)/F_2(r) = \beta' \ln F_p(r)/F_1(r) + \text{const}. \quad (6) \]

Substituting Eq. (3) into Eq. (6), we have $\beta' = \beta$. Equation (6) does not contain term $F_{\infty}(r)$, and therefore, it can be directly used to test the assumption of hierarchy. Figure 3 presents the relation of $\log[F_{p+1}(r)/F_2(r)]$ vs $\log[F_p(r)/F_1(r)]$, which is a perfectly straight line. The slope is $\beta' = 0.69 \pm 0.02$, which is also perfectly in agreement with the parameter $\beta = (1/3)^{1/3} = 0.69$. Therefore, the assumption of a constant $A_p$ ($p$ independence) is tenable. The hierarchical relation Eq. (4) with constant $A_p$ and $\beta = 0.69$ provides a good description of the hierarchical structures of cosmic baryon fluid on the scales of the inertial range.

**Discussion and conclusion.**—We showed that all the moments of the fluctuations of velocity field of cosmic baryon fluid at $z = 0$ on scales larger than the Jeans length have a power-law dependence on scales, and the intermittent exponents obey the universal law, which depends only on (1) the dimension of the most singular
dissipative structures and (2) the hierarchical relation between structures with various intensities of fluctuations. This result strongly indicates that the highly nonlinear evolution would lead to the cosmic baryon fluid reaching a statistically quasiequilibrium state satisfying the universal scaling as that of fully developed turbulence. It should be emphasized that the turbulence described by Eq. (1) is stirred at all scales, not only at the largest scale as in the conventional model of turbulence. The SL scaling is reasonable not only for the conventional energy cascade but also the case of Eq. (1).

In view of this picture, we can say that in the highly nonlinear regime, the statistical properties, especially the intermittent behavior, of the velocity fluctuations are actually independent of the details of the dissipative processes. The state depends only on the dimension of dissipative structures and the hierarchical relation index.

We believe that some statistical features, which have already been recognized in large-scale structures, would be directly in consequence of the universal properties of cosmic baryon fluid.

The velocity and mass density field on scales larger than the Jeans length are the basic environment of the formation of the luminous objects. Therefore, the universal properties of the nonlinear regime provide common frame of studying the dynamical and thermodynamical evolution of structure formation. For instance, the redshift dependence of the scaling behavior would be useful for understanding the evolution of cosmic clustering. The details will be reported in the near future.

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