CP Trajectory Diagram;  
— A tool for pictorial representation of CP and matter effects in neutrino oscillations —

Hisakazu Minakata \(^a\) and Hiroshi Nunokawa \(^{b,c}\)

\(^a\)Department of Physics, Tokyo Metropolitan University  
1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan  
\(^b\)Instituto de Física Teórica, Universidade Estadual Paulista  
Rua Pamplona 145, 01405-900 São Paulo, SP Brazil  
\(^c\)Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas  
P.O. Box 6165, 13083-970 Campinas SP Brazil

Abstract

We introduce “CP trajectory diagram in bi-probability space” as a powerful tool for pictorial representation of the genuine CP and the matter effects in neutrino oscillations. Existence of the correlated ambiguity in a determination of CP violating phase $\delta$ and the sign of $\Delta m^2_{31}$ is uncovered. Principles of tuning beam energy for a given baseline distance are proposed to resolve the ambiguity and to maximize the CP-odd effect. We finally point out, quite contrary to what is usually believed, that the ambiguity may be resolved with 50% chance in the super-JHF experiment despite its relatively short baseline of 300 km.

1 Introduction

Probing into CP violation in the lepton sector is one of the most challenging goals in particle physics. Long baseline neutrino oscillation experiments are the prime candidates for observational means for detecting such effect. However, it has been known since sometime ago that the earth matter effect acts as a contamination to the measurement of genuine CP violating effects due to the leptonic Kobayashi-Maskawa phase in such experiments [1]. Therefore, it is of crucial importance to achieve a complete understanding of the features of

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interplay between the genuine CP and the matter effects to have a realistic design of experiments for measuring the CP violating phase.

We describe in this article a new powerful tool which we call “CP trajectory diagram in bi-probability space” [2]. It enables us to represent pictorially the three effects, the effects of (a) genuine CP violation due to the sin\(\delta\) term, (b) CP conserving cos\(\delta\) term, and (c) fake CP violation due to earth matter, separately in a single diagram. By using the CP trajectory diagram we observe that there is a two-fold ambiguity in the determination of \(\delta\) which is related with the sign of \(\Delta m^2_{13}\). As described in detail in ref. [2], this is a remnant of the approximate degeneracy in the vacuum oscillation probability under the transformations \((\delta \rightarrow \pi - \delta)\) and \((\Delta m^2_{13} \rightarrow -\Delta m^2_{13})\).

We then discuss principles of tuning beam energy for a given baseline distance to resolve the ambiguity, and to maximize the CP-violating effect. Finally, we point out that the ambiguity may be resolved with 50 \% chance in the super-JHF experiment [3] with a megaton class water Cherenkov detector. It is quite contrary to the conventional belief that the sign of \(\Delta m^2_{13}\) can not be determined with such a short baseline as \(L = 300\) km. In a companion article, which is a report for the Proceedings of TAUP2001 [4], we discuss further physical implications of our results. In particular, we explore the possibility of an \textit{in situ} simultaneous measurement of \(\delta\) and the sign of \(\Delta m^2_{13}\) in a single experiment.

2 CP trajectory diagram in bi-probability space

We now introduce the CP trajectory diagram in bi-probability space spanned by \(P(\nu) \equiv P(\nu_\mu \rightarrow \nu_e)\) and \(P(\bar{\nu}) \equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)\). Suppose that we compute the oscillation probability \(P(\nu)\) and \(P(\bar{\nu})\) with a given set of oscillation and experimental parameters. Then, we draw a dot on the two-dimensional plane spanned by \(P(\nu)\) and \(P(\bar{\nu})\). When \(\delta\) is varied we have a set of dots which forms a closed trajectory, closed because the probability must be a periodic function of \(\delta\), a phase variable.

In fig. 1 plotted is the contours of oscillation probabilities \(P(\nu)\) and \(P(\bar{\nu})\) which is drawn by varying the CP violating phase \(\delta\) from 0 to \(2\pi\). They are averaged over the Gaussian neutrino energy distribution with peak energy of 500 MeV and width 100 MeV. As you might have guessed, these diagrams are elliptic. It is so exactly in vacuum and in a good approximation even in matter at relatively short baseline and as far as the mass hierarchy \(|\Delta m^2_{12}/\Delta m^2_{13}| \ll 1\) is the case [2].

What does CP trajectory diagram actually represent? There is a very simple
answer to this question. The lengths of major and minor axes represent the coefficients of CP violating \( \sin \delta \) and CP conserving \( \cos \delta \) terms, respectively, in the neutrino oscillation probability \( P(\nu_\mu \to \nu_e) \). Whereas the distance between two (positive and negative \( \Delta m^2_{13} \)) ellipses gives the size of the matter effect. The last point can be explicitly verified by running the same calculation with twice larger matter effects, as done in ref. [2]. Therefore, you can see the size of these three terms just by eye when it was projected onto the CP trajectory diagram.

\[
L = 295 \text{ km}, \quad <E_\nu> = 0.5 \text{ GeV}
\]

![Diagram](image)

Fig. 1: CP trajectory in the bi-probability (given in %) plane for the baseline \( L = 295 \text{ km} \) and energy \( <E> = 500 \text{ MeV} \). As indicated in the figures, the solid and the dashed lines are for \( \Delta m^2_{13} > 0 \) and \( \Delta m^2_{13} < 0 \) cases, respectively, and the dotted and the dash-dotted lines correspond to the same signs of \( \Delta m^2_{13} \) as above but with matter effect switched off. The mixing parameters are fixed as \( \Delta m^2_{13} = \pm 3 \times 10^{-3} \text{ eV}^2 \), \( \sin^22\theta_{23} = 1.0 \), \( \Delta m^2_{12} = 5 \times 10^{-5} \text{ eV}^2 \), \( \sin^22\theta_{12} = 0.8 \), \( \sin^22\theta_{13} = 0.05 \). We take \( \rho Y_e = 1.4 \text{ g/cm}^3 \) where \( \rho \) is the matter density and \( Y_e \) is the electron fraction.

It is obvious from fig. 1 that the approximate degeneracy \( (\delta \to \pi - \delta) \) and \( (\Delta m^2_{13} \to -\Delta m^2_{13}) \) which exists in the vacuum case is lifted by the matter effect; matter helps! However, it is also clearly seen in fig. 1 that there are remaining degeneracies if the sign of \( \Delta m^2_{13} \) is not known \( a \ priori \).
The above discussion assumes that the value of $\theta_{13}$ is known prior to the experiment. If the value is not known in advance there is another ambiguity, the $(\delta - \theta_{13})$ ambiguity, as discussed in detail in ref. [5]. Therefore, there exits a combined ambiguity, $(\delta - \text{sign of } \Delta m_{13}^2)$ times $(\delta - \theta_{13})$, which in the worst case can be $4$-fold. The ambiguity was referred to as the “clover-leaf ambiguity” in TAUP2001 [4].

3 Principle of choosing beam energies for long-baseline neutrino oscillation experiments

If one wants to resolve the two-fold ambiguity $(\delta - \text{sign of } \Delta m_{13}^2)$ in a single experiment one cannot tune the experimental parameters so that the $\sin \delta$ term is maximal because then the trajectory shrinks to a straight line, closing the possibility of resolution of the ambiguity. Then, there should be a compromise. First, one can obtain the conditions to maximize lengths of major and minor axes, which lead to [2]

\[
\left( \frac{E}{1 \text{ GeV}} \right)_{\cos \delta} = 1.13, 0.47, 0.29 \left( \frac{L}{300 \text{ km}} \right) \left( \frac{\Delta m_{13}^2}{3 \times 10^{-3} \text{ eV}^2} \right), \tag{1}
\]

\[
\left( \frac{E}{1 \text{ GeV}} \right)_{\sin \delta} = 0.62, 0.24, 0.14 \left( \frac{L}{300 \text{ km}} \right) \left( \frac{\Delta m_{13}^2}{3 \times 10^{-3} \text{ eV}^2} \right). \tag{2}
\]

It is shown in ref. [2] that one can compromise these two requirements in the energy region of $0.5$ to $2$ GeV.

4 Possibility of simultaneous determination of $\delta$ and $\Delta m_{13}^2$ in the super-JHF experiment

We now address our final subject, probably the most important one, the possibility of simultaneous determination of $\delta$ and $\Delta m_{13}^2$ in the super-JHF experiment. We assume that $\theta_{13}$ is known in a reasonable accuracy prior to this experiment. In fig. 2 plotted is the CP trajectory diagram in the number of appearance events plane for $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ channels. We assume a water Cherenkov detector of fiducial volume $0.9$ Mton, and $4$ MW of proton beam power which is planned in the JHF experiment in its phase II [3]. We use the off-axis (OA) $3$ degree beam whose neutrino energy peaks at $E \sim 0.5$ GeV. We refer ref. [2] for a detailed explanation of how the computation of number of events is done and the results for alternative choice of beams.

One can clearly see from fig. 2 that at least a half of the parameter space
fulfilling the condition \( \sin \delta \cdot \Delta m^2_{13} < 0 \) does not suffer from the ambiguity problem. Then, with the 50% chance (thanks to nature’s kind setting!) they will be able to determine the CP violating angle \( \delta \) and the sign of \( \Delta m^2_{13} \) simultaneously in the super-JHF experiment.

Fig. 2: CP trajectory in event number plane \( N(e^-) - N(e^+) \) for OA beam 3 degree and the baseline \( L = 295 \) km. The dotted circles correspond to 3 \( \sigma \) statistical uncertainty.

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