Magnetic vortex as a ground state for sub-micron antiferromagnetic particles

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Abstract

For submicron particles shaped as any axisymmetric body and made with standard canted antiferromagnet like hematite or iron borate, the ground state may comprise of a magnetic vortex with topologically non-trivial distribution of the sublattice magnetization \(\vec{l}\) and planar coreless vortex-like structure for the net magnetization \(\vec{M}\). For antiferromagnetic particles in the vortex state, in addition to low-frequency modes, there are high frequency modes with frequencies over the range of hundreds gigahertz, including a mode localized in the region of the radius 30-40 nm near a vortex core.

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The magnetic properties of submicron ferromagnetic (FM) particles of the shape of circular cylinder (magnetic dots) are today under great consideration mainly due to their potential applications [1]. Circular dots possess the equilibrium magnetic configuration which corresponds to a vortex structure just above the single domain length scale, with the radius $R > R_{\text{crit}} \sim 100-200$ nm. The FM vortex state consists of an in-plane flux-closure magnetization distribution and a central core with radius of order of 20-30 nm magnetized perpendicularly to the dot plane. Reducing of the magnetostatic energy comes at the cost of a large exchange energy near the vortex core, as well as the magnetostatic energy caused by the core magnetization. For data storage purposes, vortex state is of considerable interest, because the low magnetic stray field reduce the particles interaction and leads to a high magnetic stability of the data written. The magnon modes for dots in virtue of effects of spatial quantization possess a discrete spectrum. The possibility to manage localization and interference of magnons spawned an idea of so-called magnonics based on usage of these modes for developing a new generation of microwave devices with submicron active elements [2]. These particles also provide an ideal experimental system for studying static and especially dynamic properties of relatively simple topologically non-trivial magnetic structures which are fundamentally interesting objects in the research area of magnetism.

All previous studies of magnetic vortices caused by magnetic dipole interaction were carried out on the magnetic particles made with soft FM with high magnetization $M_s$ like permalloy with $4\pi M_s \sim 1$ T. In this Letter we have shown that the vortices can be a ground state for sub-micron particles made with another important class of magnetic materials, antiferromagnets (AFM), with easy-plane (EP) anisotropy and Dzyaloshinskii-Morya interaction (DMI).

For AFM, exchange interaction between neighboring spins facilitates antiparallel spin orientation, which leads to the structure with two antiparallel magnetic sub-lattices, $\vec{M}_1$ and $\vec{M}_2$, $|\vec{M}_1| = |\vec{M}_2| = M_0$. As typical AFM we can mention hematite $\alpha$-Fe$_2$O$_3$, iron borate FeBO$_3$, and orthoferrites, see [3]. These materials are characterized by high temperatures of magnetic ordering and have unique physical properties: orthoferrites and iron borate are transparent in optical range and have a strong Faraday effect, the magnetoelastic coupling is quite high in hematite and iron borate [3]. They possess small but non–zero net magnetization caused by a weak non-collinearity of sublattices (sublattice canting) originated from DMI.
To describe the structure of AFM, it is convenient to introduce irreducible combinations of the vectors $\vec{M}_1$ and $\vec{M}_2$, the net magnetization $\vec{M} = \vec{M}_1 + \vec{M}_2 = 2M_0\vec{m}$ and the vector of sublattice magnetization, $\vec{l} = (\vec{M}_1 - \vec{M}_2)/2M_0$. The vectors $\vec{m}$ and $\vec{l}$ are subject to constraint $(\vec{m} \cdot \vec{l}) = 0$, $\vec{m}^2 + \vec{l}^2 = 1$. As $|\vec{m}| \ll 1$, the vector $\vec{l}$ could be considered as a unit vector. The mutual orientation of sublattices is determined by a sum of the energy of uniform exchange $W_{ex} = H_{ex}M_0\vec{m}^2$, and the DMI energy, $W_{DM} = 2M_0H_D(\vec{d} \cdot (\vec{m} \times \vec{l}))$, the unit vector $\vec{d}$ is directed along the symmetry axis of a magnet. The parameters $H_{ex} \sim 3 \cdot 10^2 - 10^3$ T and $H_D \sim 10$ T are exchange field and DMI field, respectively. Using this energy and dynamical equations for $\vec{M}_1$ and $\vec{M}_2$, one can find,

$$\vec{M} = M_{DM} (\vec{d} \times \vec{l}) + \frac{2M_0}{\gamma H_{ex}} \left( \vec{l} \times \frac{\partial \vec{l}}{\partial t} \right), \quad M_{DM} = \frac{2H_D M_0}{H_{ex}},$$

(1)

where the first term gives the static value of AFM net magnetization $M_{DM}$, comprising a small parameter, $H_D/H_{ex} \sim 10^{-2}$, second term describes the dynamic canting of sublattices, see for details [3]. $M_{DM}$ is much smaller than $M_0$ or the value of $M_s$ for typical FM, but the role of the magnetostatic energy caused by $M_{DM}$ could be essential, and could lead to the appearance of a domain structure for AFMs [2, 4]. We will show that for formation of equilibrium vortices in AFM even have some advantage compared with soft FM.

The dynamical properties of AFM are essentially different comparing with FM. A spin dynamics of an AFM can be easily described in the framework of so-called sigma-model equation ($\sigma$-ME), a dynamical equation for the vector $\vec{l}$ only, with the magnetization $\vec{M}$ being a slave variable [3]. In contrast to the Landau – Lifshitz equation for a FM magnetization, the $\sigma$-ME contains a dynamical term with a second time derivative of $\vec{l}$, combined with gradients of $\vec{l}$ in the Lorentz-invariant form $d^2\vec{l}/dt^2 - c^2\nabla^2\vec{l}$. For this reason, for AFM two magnon branches (instead of one, for FM) exist. The chosen speed $c = \gamma \sqrt{AH_{ex}/M_0}$ plays roles of both magnon speed and the limit speed of domain walls, it is determined by exchange interaction only and attains tens km/s, $c \simeq 1.4 \cdot 10^4$ m/s for iron borate and $c \simeq 2 \cdot 10^4$ m/s for orthoferrites [3]. For both modes the elliptic polarization of the oscillations of $\vec{M}_1$ and $\vec{M}_2$ is such that the oscillations of the vector $\vec{l}$ have linear polarization [3]. For EP AFM these two branches are low-frequency quasi-ferromagnetic (QFM) branch and a high-frequency quasi-antiferromagnetic (QAF) branch. QFM magnons involves the oscillations of the vectors $\vec{l}$ and $\vec{M}$ in the EP, with weak deviation of $\vec{M}$ from the EP caused by last summand in (1). The second QAF branch corresponds to the out-of-plane oscillations of $\vec{l}$.
with the dispersion law \( \omega_{\text{QAF}}(\vec{k}) = \sqrt{\omega_g^2 + c^2 \vec{k}^2} \), where \( \vec{k} \) is the magnon wave vector. The gap of QAF branch, \( \omega_g = \gamma \sqrt{2H_{\text{ex}}H_\text{a}} \) contains large value \( H_{\text{ex}} \) and attains hundreds GHz. Thus both magnon frequency and domain wall speed for AFM dynamics, comparing with FM, contain a large parameter \( \sqrt{H_{\text{ex}}/H_\text{a}} \sim 30 - 100 \), \( H_{\text{ex}} \) and \( H_\text{a} \) are exchange field and anisotropy field, respectively, which can be referred as exchange amplification of dynamical parameters of AFM. The frequencies of AFM magnon modes \( \omega_g \) reaches hundreds GHz, with values 170 GHz for hematite, 100-500 GHz for different orthoferrites and 310 GHz for iron borate [5]. Recent studies showed a possibility to excite spin oscillations of non–small amplitude for orthoferrites [6] and iron borate [7] with the use of ultra-short laser pulses.

Spin distribution for AFM can be described by the energy functional of the form \( W[\vec{l}] + W_m \). Here \( W[\vec{l}] \) describes the energy of non-uniform exchange and the anisotropy energy through only the vector \( \vec{l} \); \( W_m \) is magnetic dipole energy,

\[
W_m = -\frac{1}{2} \int \vec{M} \cdot \vec{H}_m d^3 x,
\]

where \( \vec{H}_m \) is demagnetization field caused by AFM magnetization [1]. The sources of \( \vec{H}_m \) can be considered as formal “magnetic charges”, both volume charges equal to \( \text{div}\vec{M} \) and surface charges equal \( -\vec{M} \cdot \vec{n} \), see monographs [4, 8] for general consideration and [9] for application to vortices.

For a pure uniaxial model of an AFM which is applicable for hematite and iron borate, and to some extend for orthoferrites, \( W[\vec{l}] \) can be presented as,

\[
W[\vec{l}] = \frac{1}{2} \int \left[ A(\nabla \vec{l})^2 + K \cdot \vec{l}_z^2 \right] d^3 x ,
\]

where \( A \) is the non-uniform exchange constant and \( K \) is anisotropy constant, the \( xy \)-plane is the EP for spins. Variation of the energy \( W[\vec{l}] \) gives a general two-dimensional (2D) vortex solution for the vector \( \vec{l} \) of the form

\[
\vec{l} = \vec{e}_z \cos \theta + \sin \theta [\vec{e}_x \cos(\chi + \varphi_0) + \vec{e}_y \sin(\chi + \varphi_0)]
\]

where \( \theta = \theta(r) \), \( r \) and \( \chi \) are polar coordinates in an EP of a magnet, the vector \( \vec{e}_z \) is the hard axis, the value of \( \varphi_0 \) is arbitrary, see [10, 11]. The function \( \theta(r) \) exponentially tends to \( \pi/2 \) at \( r \gg l_0 \), with the characteristic size \( l_0 = \sqrt{A/K} \), and in the center of the vortex (at \( r = 0 \)) \( \sin \theta(0) = 0 \). Near the vortex core \( \vec{l} \) deviates from the EP which leads to the loss of the anisotropy energy. The state [11] is non-uniform, what corresponds to the
loss in the exchange energy. Therefore, for the EP model without taken into account the magnetic dipole interaction the appearance of a vortex costs some energy, i.e. the vortex corresponds to excited states of AFM. Vortex excitations are important for description of thermodynamics of 2D AFM, see [10].

For small particles made with canted AFM the energy loss caused by a vortex can be compensated by the energy of magnetic dipole interaction. To explain this, note that for a uniform distribution state the contribution $W_m$ unavoidably results in a loss of the system energy, which is proportional to the particle volume $V$ [4, 8]. The energy of the uniform state could be estimated as

$$E^{(\text{homog})} = 2\pi NM_D^2 V = 2\pi NM_0^2 (2H_D/H_e)^2 V,$$

where $N$ is the effective demagnetizing factor in the direction perpendicular to the particle axis [4, 8]. In contrast, for the vortex state with a chosen value of $\sin \varphi_0 = 0$, with $\vec{M} \propto (\vec{d} \times \vec{l})$, one can find

$$\vec{M} = \sigma M_D \cdot \sin \theta (-\vec{e}_x \sin \chi + \vec{e}_y \cos \chi),$$

where $\sigma = \cos \varphi_0 = \pm 1$. A unique property of the state is that it can also exactly minimize the energy of the magnetic dipole interaction $W_m$, giving $\vec{H}_m = 0$ in the overall space. Indeed, the projection of $\vec{M}$ on the lateral surface of any axisymmetric body with the symmetry axis parallel to $z$-axis, as well as $\text{div} \vec{M}$, equal to zero. Moreover, in virtue of symmetry of DMI ($\vec{M} \cdot \vec{d} = 0$), the distribution of the magnetization $\vec{M}$ is purely planar (in contrast to $\vec{l}$) and the out of plane component of $\vec{M}$ is absent. In the vicinity of the vortex core the length of vector $\vec{M}$ decreases, turning to zero in the vortex center, see. Fig. 1. Such feature is well known for domain walls in some orthoferrites [12]. Thus the AFM vortex is the unique spin configurations which do not create demagnetization field in a singly connected body (for FM with $|\vec{M}| = \text{const}$ a configuration with $\vec{H}_m \equiv 0$ is possible only for a magnetic rings having the topology of torus).

Let us compare the energies of the vortex state and the uniform state for the AFM particle shaped as a cylinder with the height $L$ and the radius $R$. For the vortex state $\vec{H}_m = 0$ overall the volume of the particle, and the vortex energy is determined by the simple formula

$$E_v = \pi AL \ln \left( \frac{R}{\rho_0} \right),$$

where $\rho_0 = L/2$.
FIG. 1: (Colour online) The structure of the AFM vortex (schematically). The vectors \( \vec{l} \) (thin blue arrows) and \( \vec{M} \) (thick red arrows; present not in scale) are depicted for the area of the core (dashed circle) and far from it (dotted circle); the full red dot in the origin indicates the value \( \vec{M} = 0 \) for the state with \( \vec{l} \) perpendicular to the plane.

where \( \rho \approx 4.1 \) is the numerical parameter. For long cylinder with \( L \gg R \) the value of \( N \simeq 1/2 \) and the vortex state becomes favorable if the radius \( R \) exceeds some critical value \( R_{\text{crit}} \),

\[
R \geq R_{\text{crit}} = 2l_{\text{dip}} \sqrt{\ln \left( \frac{l_{\text{dip}}}{l_0} \right)}, \quad l_{\text{dip}} = \sqrt{\frac{A}{4\pi M_{\text{DM}}^2}}, \quad (8)
\]

where \( l_{\text{dip}} \) determines the spatial scale corresponding to the magnetic dipole interaction.

In the case of a thin disk, \( L \ll R \) the demagnetization field energy could be revealed as \( E^{(\text{homog})} = 2\pi RL^2 M_{\text{DM}}^2 \ln(4R/L) \), and the vortex state is energetically favorable for \( RL \geq (RL)_{\text{crit}} = 2l_{\text{dip}}^2 \).

For concrete estimations we take the parameters of iron borate, \( A = 0.7 \cdot 10^{-6} \text{ erg/cm}, \ K = 4.9 \cdot 10^6 \text{ erg/cm}^3 \) and \( 4\pi M_{\text{DM}} = 120 \text{ Oe} \). Then we obtain that \( l_0 = 3.8 \text{ nm} \), i.e. the core size is of the same order of magnitude as for typical FM (for permalloy \( l_0 = 4.8 \text{ nm} \)). The value \( l_{\text{dip}} \) is essentially higher, for iron borate \( l_{\text{dip}} = 220 \text{ nm} \). Combining these data one finds for the long cylinder \( R_{\text{crit}} = 0.9 \mu\text{m} \). For a thin disk sample the characteristic scale has submicron value, \( \sqrt{(RL)_{\text{crit}}} = 0.4 \mu\text{m} \). The similar estimations are obtained for orthoferrites, and somewhat higher values for hematite. Thus, despite the fact that characteristic values for the dipole length \( l_{\text{dip}} \) for FM and AFM differ hundredfold, the characteristic critical sizes differ not so drastically (for permalloy \( R_{\text{crit}} \sim 100-200 \text{ nm} \)). It is caused by the aforementioned fact that the magnetic field created by the vortex core
is completely absent for the AFM vortex. The situation here is common to that for FM nanorings, where the vortex core is absent and the vortex state is more stable that for FM dots.

Despite that the vortex core size in FM dots is rather small, the core contribution to $W_m$ for ferromagnetic particles of rather big radius $R \geq 0.5 \, \mu m$ is negligible, but it becomes essential for small particles with $R$ close to the critical size. Note as well that the vortex core magnetic field in the FM destroys a purely 2D distribution of $\vec{M}$ like (4), and the core size changes over the thickness of the particle. For the AFM vortex the value of $\vec{H}_m$ equals exactly zero, and truly 2D distribution of $\vec{l}$ and $\vec{M}$, independent on a coordinate $z$ along the body axis, see (6, 4), is possible.

Since magnon spectra of bulk FM and AFM differ significantly, one can expect an essential difference for magnon modes for vortex state AFM and FM particles. Remind briefly the properties of normal modes for disk shaped vortex state FM particles. For such samples, the presence of discrete spectrum of magnon modes, characterized by the principal number (the number of nodes) $n$ and the azimuthal number $m$, is well established [13, 14]. This spectrum includes a single low-frequency mode of precessional motion of a vortex core ($n = 0$, $m = 1$) which has the frequency in subGHz region [15], a set of radially symmetrical modes with $m = 0$ [16], and also a system of slightly splinted doublets with the azimuthal numbers $m = \pm |m|$, with frequencies $\omega_{|m|,n} \neq \omega_{-|m|,n}$, but $\omega_{|m|,n} - \omega_{-|m|,n} \ll \omega_{|m|,n}$ [9]. The same classification is valid for vortices for local EP FM [11]. Wysin had demonstrated the direct correspondence of gyroscopic character of vortex dynamics and doublet splinting [17].

For an AFM vortex state particle each of two magnon branches, QFM and QAF, produce a set of discrete modes with given $n$ and $m$, however their properties are different compared to that for a FM dot. The aforementioned formal Lorentz-invariance of spin dynamics of AFM manifests itself for motion of a AFM vortex core: the dynamical equation for the core coordinate $\vec{X}$ possess an inertial term, $M_v d^2 \vec{X} / dt^2$, where the effective vortex mass $M_v = E_v/c^2$ [17, 18]. For this reason, the vortex core dynamics is not a precession, as for the gyroscopic Thiele equation for FM vortices [19, 20, 21], but rectilinear oscillations, $\vec{X}(t) = \vec{a} \cos(\omega_v t + \phi_0)$ degenerated with respect to the direction $\vec{a}$ and $\phi_0$, with the frequency $\omega_v = \sqrt{\kappa/M_v}$, $\kappa$ determine the restoring force $\vec{F} = -\kappa \vec{X}$ for the vortex. For EP AFM model with $W_m = 0$ such dynamics has been observed by direct numerical simulations [10, 22]. For the vortex state particle with $R > R_{\text{crit}}$ the value of $\kappa$ is determined by the demagnetizing
field, \( \kappa = 10 \cdot 4\pi M_{DM}^2 L^2 / 9R \) \cite{15}, and

\[
\omega_v = \frac{2cM_{DM}\sqrt{10L}}{3\sqrt{AR\ln(pR/l_0)}}.
\] (9)

A simple estimate gives that \( \omega_v \), as for FM vortex, is in subGHz region, but with different (approximately square root, instead of linear for FM vortex) dependence on the aspect ratio \( L/R \). The other modes from this set far from the vortex core are characterized approximately by in-plane oscillations of \( \vec{l} \) and \( \vec{M} \). As their frequencies are small, \( \omega \ll \gamma H_{DM} \), for these modes the magnetization \( \vec{M} \) is determined mainly by the in-plane static contribution \( \Pi \), and the formulae for the demagnetization field energy for FM vortices can be used. For these modes the frequencies are of the order of a few GHz, with approximately square root dependence on the aspect ratio \( L/R \), the details will be present elsewhere. The absence of gyroscopical properties for the \( \sigma \)-ME is also manifested in the fact that for a AFM vortex the modes with the azimuthal numbers \( m = |m| \) \( m = -|m| \) are degenerated, i.e. splitting of doublets with \( m = \pm |m| \), typical for the FM vortex, is absent \cite{22,23}.

For a vortex state AFM particle the high-frequency QAF branch of magnons begets a set of discrete modes with frequencies of the order of \( \omega_g \), i.e. hundreds GHz. For these modes far from the vortex core oscillations of the vector \( \vec{l} \) are out of plane. For their description the dipole interaction is not essential and the results obtained earlier for the vortex in EP AFM \cite{22,23} can be used. The mode frequencies \( \omega_{n,m} \) are close to \( \omega_g \), and the difference \( \omega_{n,m} - \omega_g \) decreasing as the dot radius increase as \( c^2/\omega_g R^2 \), with one exception: within the set of radially-symmetrical modes with \( m = 0 \) a truly local mode is present, with the localization area of the order of \( 5l_0 \) and with the frequency \( \omega_1 \sim 0.95\omega_g \) independent on \( R \) \cite{23}.

The usage of QAF modes for vortex state AFM particles, particularly the truly local mode, would allow application of the idea of magnonincs for higher frequencies till 0.3 THz. A developed theory would be applied for other AFM systems like a FM bilayer dot containing two thin FM films with AFM interaction between them, described by the field \( H_{ex} \). If \( H_{ex} \) is large enough, \( H_{ex} > 4\pi M_s \), anti-phase oscillations of magnetic moments of the layers produce high frequency modes with frequencies of order of \( \sqrt{\gamma H_{ex} \omega_{m,n}^{FM}} \), where \( \omega_{m,n}^{FM} \) are the frequencies of modes for a single layer dot.

To conclude, for submicron particles of typical canted AFM the ground state comprises topologically non-trivial spin distribution. The magnetization of each sublattices \( \vec{M}_1 \) and
\( \vec{M}_2 \) are characterized by a vortex state with a standard out of plane structure, but the net magnetization \( \vec{M} = \vec{M}_1 + \vec{M}_2 \) form the planar vortex, where the magnetization in the vortex center turns to zero. Vortex state AFM particles possess a rich variety of normal magnon modes, from rectilinear oscillations of the vortex core position with sub-GHz frequency till out of plane modes with frequencies of order of hundreds GHz, including truly local mode.

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