Comparative analysis of improved FCM algorithms for the segmentation of retinal blood vessels

Imane Mehidi1,2 · Djamel Eddine Chouaib Belkhiat1,2 · Dalel Jabri1,3

Accepted: 21 August 2022 / Published online: 3 October 2022
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Abstract
The main purpose of identifying and locating the retina vessels is to specify from the fundus image the various tissues of the vascular structure, which can be wide or tight. The classification of vessels in the retinal image often confronts several challenges, such as the low contrast accompanying the fundus image, the inhomogeneity of the background lighting, and the noise. Moreover, fuzzy c-means (FCM) is one of the most frequently used algorithms for medical image segmentation due to its effectiveness. Hence, many FCM method derivatives have been developed to improve their noise robustness and time-consuming. This paper aims to analyze the performance of some improved FCM algorithms to recommend the best ones for the segmentation of retinal blood vessels. Eight derivatives of FCM algorithm are detained in this study: FCM, EnFCM, SFCM, FGFCM, FRFCM, DSFCM_N, FCM_SICM and SSFCA. The performance analysis is conducted from three viewpoints: noise robustness, blood vessels segmentation performance, and time-consuming. At first, the noise robustness of improved FCM clustering algorithms is evaluated using a synthetic image degraded by various types and levels of noise. Then, the ability of the selected algorithms to segment retinal blood vessels is assessed based on images from the DRIVE and STARE databases after a pre-processing phase. Finally, the time consumption of each algorithm is measured. The experiments demonstrate that the FRFCM and DSFCM_N algorithms achieve better results in terms of noise robustness and blood vessels segmentation. Regarding the running time, the FRFCM algorithm requires less time than other algorithms in the segmentation of retinal images. The results of this study are extensively discussed, and some suggestions are proposed at the end of this paper.

Keywords Fuzzy c-means clustering · Retinal fundus images · Vessels segmentation · Comparison · Analysis

1 Introduction
The segmentation of the blood vessels is a high priority for interpreting medical images because the vessels analysis is crucial for examination, treatment planning, execution, and evaluation of clinical outcomes in various fields, including neurosurgery, laryngology, and ophthalmology. Generally, in the field of ophthalmology, the segmentation of vessel structures from the fundus images is often done manually, which is tedious, time-consuming, and error-prone, especially for large population screening. Also, the rating of the majority of pixels is often apparent to an ophthalmologist, except for those belonging to vessels of pathology area and those of small vessels (Niemeijer et al. 2004; Srinidhi et al. 2017; Almotiri et al. 2018). Therefore, automatic segmentation is needed, as it can reduce manual fundus photography while increasing speed and accuracy. Furthermore, retinal fundus images are often degraded by noise, uneven illumination, and blurring due to the illumination and contrast inhomogeneity during the acquisition and the complexity of retinal images in terms of blood vessel size (thick or thin). In addition, broken vessels at bifurcations/crossover points and the morphology of the eye are the characteristics of each patient.
All those challenges have an impact on the vessels classification. (Abdulsahib et al. 2021). Inevitably, there are various methods and techniques available for vessel segmentation of retinal images. In such a manner, clustering is a popular method for image segmentation, and diverse algorithms have been proposed and used medically. Clustering is an unsupervised classification approach that aims to divide data into numerous disjoint subsets based on their characteristics. Generally, clustering techniques classify the image pixels in c-clusters so that the members of the same cluster are more associated with one another. The number c of clusters is generally predefined or fixed by a validity criterion or according to a priori knowledge. The clustering algorithms of image segmentation involve k-means (Macqueen 1967), k-medoids (Bezdek et al. 1984), mean-shift (Comaniciu and Meer 2002), fuzzy c-means (FCM) (Bezdek et al. 1984), etc. Therefore, FCM-based algorithms have been popularly used in the segmentation of medical images. Such success mainly attributes to the introduction of fuzziness for the belongingness of all image pixels. This allows the clustering methods to preserve more information from the original image than the hard segmentation methods. In order to increase the performance of FCM for remove noise, some techniques have been proposed. Bias correction fuzzy c-means (BCFCM) are a type of FCM in which a bias field is added to the objective function (Ahmed et al. 2002). Despite the fact that the bias field can repair pixels distorted by noise, it has low efficiency for many types of noisy images due to the bias field’s sparsity. Based on this research, (Zhang et al. 2018) proposed a deviation sparse FCM (DSFCM) that employs regularization with sparsity constraint to ease the occurrence of a BCFCM defect. DSFCM produces accurate clustering centers, however it is susceptible to regularization parameters and so has a low robustness. In the same work, (Zhang et al. 2018) take the deviations between measured values and theoretical values into account. In (Szilagyi et al. 2003) suggested an Enhanced FCM (EnFCM) by incorporating a histogram into the goal function and applying local linear-weight filtering to each pixel. The EnFCM achieves good computational performance in image segmentation because the number of gray levels is usually significantly fewer than the number of pixels in a grayscale image. On the basis of this work, (Cai et al. 2007) proposed the fast generalized FCM (FGFCM) by incorporating a bilateral filter into the objective function. In the same context, the spatial information was introduced by (Chuang et al. 2006) into conventional FCM for better segmenting images. Recently, using morphological reconstruction and membership filtering, (Lei et al. 2018) suggested a fast and robust fuzzy c-means (FRFCM). FRFCM achieves satisfactory segmentation results and needs a short accomplishment time for various kinds of images. Similarly, (Wang et al. 2020) proposed a robust fuzzy clustering algorithm based on membership linking and filtering for the segmentation of noisy images. (Jia et al. 2020) suggested a self-sparse FCM (SSFCA) algorithm in which the regularisation under Gaussian metric is inserted in the objective function to acquire fuzzy membership with sparsity. This one leads to a decrease in the proportion of noisy features and enhanced clustering results.

Recently, some improved FCM clustering algorithms have appeared in the literature. For example, a novel fuzzy clustering method has been developed by (Song et al. 2022) to perform highly accurate clustering using incomplete data. It’s based on two ideas: (1) Utilizing a nonnegative latent factor (NFLF) model to prefill the missing data in the inputs, and (2) Integrating the distribution of inputs and the weights of local features into the objective function through sparse self-representation (SSR) and weighting allocation to focus on crucial features. Furthermore, (Xue et al. 2022) proposed a new equivalent minimization formulation for solving the fuzzy c-means problem. Then, a simple alternating iteration algorithm has been proposed to solve the new minimization problem based on an Iteratively Re-Weighted (IRW) method. This optimization method is called IRW-FCM. Moreover, the authors felt that using this method for image segmentation and facial image recognition can achieve highly satisfactory performance.

In summary, from the above review of the literature, it appears that the FCM clustering method has attracted widespread attention due to its effectiveness, and it has been the source of several derivatives. However, to the best of our knowledge, performance analysis of different derivatives of the FCM algorithm in terms of the blood vessel segmentation has not been well investigated enough, which substantiates the relevance of this study. Therefore, the particular strengths of this study can be stated as follows:

- The performance analysis of some improved FCM algorithms is conducted to recommend the best ones for the segmentation of retinal blood vessels. Eight derivatives of the FCM algorithm, having a publicly available implementation, are detained in this study: FCM, EnFCM, SFM, FGFCM, FRFCM, DSFMCN, FCM_SICM, and SSFCA.
- The present study is conducted from three viewpoints: noise robustness, blood vessels segmentation performance, and time-consuming. Different metrics are employed to perform this study, such as segmentation precision, sensitivity, specificity, F1 score, and accuracy.
- The noise robustness is evaluated based on a synthetic image corrupted by various types and levels of noise, such as Gaussian and salt & pepper noise.
- The assessment of the blood vessels segmentation is carried out using two publicly available databases, DRIVE and STARE. The detail preserving in the retina image and the contrast enhancement are achieved using a series
of pre-processing operations: contrast limited adaptive histogram equalization and bottom-hat filtering.

- The practicability of the improved FCM algorithms is compared by measuring the running times of various algorithms.
- The obtained results are extensively discussed, and the most efficient FCM algorithms are determined.
- Some avenues for reflection are addressed to motivate the development of new segmentation methods that consider the complexities of eye retina images.

The remainder of this work is arranged in the following manner. In Sect. 2, we present the background for our research. Section 3 illustrates the noise robustness analysis of improved FCM algorithms. We address in detail the Blood vessel segmentation analysis in Sect. 4. Finally, we present the conclusion in Sect. 5.

2 Background

FCM clustering technique is an unsupervised segmentation method where the image is divided into several groups or regions. It is applied generally in various decision-making obstacles and chiefly in medical image studies. The fuzzy set theory suggested by (Zadeh 1965) provides a mighty tool for obstacles and chiefly in medical image studies. The fuzzy set regions. It is applied generally in various decision-making method where the image is divided into several groups or regions. The fuzzy c-means clustering technique is an unsupervised segmentation technique widely used for the medical images segmentation (Nageswara Reddy et al. 2022) (Chaira 2022). Its performances depend on the type of the images and the targeted area. This section presents some extensions of the FCM algorithms have been proposed.

2.1 FCM

FCM clustering algorithm is a soft segmentation technique widely used for the medical images segmentation (Nageswara Reddy et al. 2022) (Chaira 2022). Its performance in procuring an optimal solution depends on the initial positions of the cluster center, the measure of membership degree for each data point, etc. In the standard FCM method, the centers are initialized randomly, and the measure of membership only uses the gray feature.

Let’s consider an image \( I \) with \( P \) pixels. Since each image pixel often implies a set of attributes (variables), \( I \) can be formulated as \( X = \{ x_1, x_2, \ldots, x_P \} \subset \mathbb{R}^P \). The FCM algorithm divides \( X \) into several clusters \( C \) by minimizing the following objective function:

\[
J_{FCM} = \sum_{i=1}^{C} \sum_{j=1}^{P} u_{ij}^m \| x_j - v_i \|^2
\]

Subject to

\[
\sum_{i=1}^{C} u_{ij} = 1, \text{ for } j \in \{1, 2, \ldots, P\}, \text{ with } 0 \leq u_{ij} \leq 1
\]

where

- \( u_{ij} \) is the degree of membership of \( x_j \) in the cluster \( C_i \).
- \( m \) is the fuzzification exponent \((m > 1)\).
- \( x_j \) is the \( i \)th of \( d \)-dimensional measured data.
- \( v_i \) is the \( d \)-dimension of the cluster.
- \( \| \cdot \| \) is the Euclidean distance.

The minimization of (1) can be realized by iteratively updating the partition matrix and the cluster centers:

\[
u_{ij}^m = \frac{1}{\sum_{q=1}^{c} \left( \frac{\| x_j - v_i \|}{\| x_j - v_q \|} \right)^\frac{2}{m-1}} \text{ and } v_i = \frac{\sum_{j=1}^{n} u_{ij}^m \cdot x_j}{\sum_{j=1}^{n} u_{ij}^m}.
\]

By presetting a nonnegative threshold \( \epsilon \), (FCM)\(^1\) iterates until \( \max_{ij} \left\{ \left\| u_{ij}^{(t+1)} - u_{ij}^{(t)} \right\| \right\} < (\epsilon) \) has been met. Here, \( t \) is an iteration index. This procedure converges to a local minimum or a saddle point of \( J_{FCM} \).

2.2 EnFCM

The enhanced fuzzy c-means (EnFCM)\(^2\) algorithm was proposed by (Szilagyi et al. 2003) on the basis of the research findings of (Ahmed et al. 2002), originally developed for the segmentation of MRI brain images. It executes clustering based on gray-level histograms (\( l \)) rather than pixels of a summed image, which lessens the computational complexity. A linearly weighted sum image (\( \xi \)) is formed in advance from the original image and its local neighbor average image in terms of:

\[
\xi_l = \frac{1}{\alpha + 1} \left( x_l + \frac{\alpha}{N_R} \sum_{j \in \mathcal{N}_l} x_j \right)
\]

where \( \xi_l \) is the gray value of the \( l \)th pixel of the image (\( \xi \)), \( x_l \) is the neighbors of \( x_l \), \( \alpha \) is the control of the effect of the neighbors’ terms (weight factor).

\(^1\) https://fr.mathworks.com/help/fuzzy/fcm.html.
\(^2\) https://github.com/marearth/fcm_m.
\( N_I \) represents the set of neighbors \( x_j \) falling into a window around \( x_i \). Next, the clustering process is running on the gray-level histogram of the newly formed image \((\xi_i)\). As a result, the objective function, in this case is represented as:

\[
J_{E_{nFCM}} = \sum_{i=1}^{c} \sum_{l=1}^{q} \gamma_l u_{il}^m (\xi_i - v_l)^2
\]

Subject to

\[
\sum_{l=1}^{q} \gamma_l = N \quad \text{and} \quad \sum_{i=1}^{c} u_{il} = 1
\]

where \( v_l \) is the prototype of the \( i_{th} \) cluster, \( u_{il} \) is the fuzzy membership of gray value \( l \) with respect to cluster \( i \), \( q \) is the number of gray levels of the addressed image, which is usually much less \( N \).

\( \gamma_l \) is the total number of pixels with a gray value of \( l \).

The \( J_{E_{nFCM}} \) is minimized using the following equations to calculate the membership partition matrix and the cluster centers:

\[
u_{il} = \left[ \sum_{j=1}^{c} \left( \frac{\xi_j - v_l}{\xi_i - v_l} \right)^2 \right]^{-\frac{1}{m-1}} \]

\[
v_l = \frac{\sum_{i=1}^{q} \gamma_l u_{il}^m s_i}{\sum_{i=1}^{q} \gamma_l u_{il}^m}.
\]

The iterative process of the EnFCM algorithm is similar to the FCM algorithm, but it is applied to the linearly weighted sum image \((\xi)\) by using Eqs. (7) and (8).

EnFCM does not automatically choose the weight factor to achieve accurate results for segmenting MRI images with noise content and MRI sequences. To avoid this, some recent research has been used multi-objective particle swarm optimization to control the weight parameter, leading to maximum segmentation accuracy (Singh et al. 2022).

### 2.3 SFCM

Chuang et al. (2006) suggested a spatial fuzzy c-means (SFCM)\(^3\) algorithm in which spatial information can be integrated into fuzzy membership functions directly using:

\[
h_{ij} = \sum_{k \in N_i} u_{ik}
\]

where \( N_i \) is the local window \((w)\) centered around the image pixel \( n \), \( h_{ij} \) is the probability that pixel \( x_j \) belongs to \( i_{th} \) cluster.

The memberships \( u_{ij} \) is updated using the following relationships:

\[
u_{ij} = \frac{u_{ij}^p h_{ij}^q}{\sum_{k=1}^{c} u_{kj}^p h_{kj}^q}
\]

where \( p, q \) are two parameters controlling the respective contribution. The weighted \( u_{ij} \) and the centroid \( v_i \) are updated as usual according to Eq. (3). In the literature, the SFCM algorithm has been presented for the first time to segment brain T1 and T2-weighted MRI images. Similarly, this algorithm has been used for segmenting cerebrospinal fluid (CSF), fat, gray matter (GM), white matter (WM), air, and bone. In addition, SFCM has been combined with level set method to increase its effectiveness in automated medical image segmentation (Li et al. 2011).

### 2.4 FGFCM

Cai et al. (2007) proposed the fast generalized fuzzy c-means clustering (FGFCM)\(^4\) that included a new factor \( S_{ij} \) as a local similarity measure. Its permits to ensure both detail-preservation and noise-immunity for image segmentation. Another advantage, it dispenses with the empirically adjusted parameter \( \alpha \) that EnFCM requires, and it ultimately achieves gray-level histogram clustering. Its definition is given as follows:

\[
S_{ij} = \begin{cases} 
\exp\left(-\max(|p_i - p_j|, |q_i - q_j|) / \lambda_s - |v_i - v_j| / \lambda_g \right) & i \neq j, \\
0 & i = j.
\end{cases}
\]

where \( i_{th} \) is the center of the local window, \( j_{th} \) is the set of the neighbors falling into the window around the \( i_{th} \) pixel, \( p_i, q_i \) is the coordinates of pixel \( i \), \( x_j \) is the gray level the value \( i \), \( \lambda_s, \lambda_g \) are two scale factors playing a role similar to factor \( \alpha \) in EnFCM, \( S_{ij} \) is a function of the local density surrounding the central pixel, and its value indicates the local window’s gray value homogeneity degree. \( \sigma_i \) is defined as:

\[
\sigma_i = \sqrt{\frac{\sum_{j \in N_i} \|x_i - x_j\|^2}{N_R}}.
\]

FGFCM algorithm integrates local and gray-level information (11) inside the objective function producing a new image \((\xi_i)\) defined as follows:

\[
\xi_i = \frac{\sum_{j \in N_i} S_{ij} x_j}{\sum_{j \in N_i} S_{ij}}.
\]

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\(^3\) https://fr.mathworks.com/matlabcentral/fileexchange/31068-spatial-fuzzy-clustering-and-level-set-segmentation.

\(^4\) https://github.com/marearth/fcm_m.
where $\xi_i$ is the gray-level value of the $i$th pixel of the image $\xi$, $s_i$ is the gray-level value of the neighbors of window center, $N_i$ is the set of neighbors falling in the local window, $S_{ij}$ is the local similarity measure between the $i$th and the $j$th pixel.

FGFCM has been tested on a real brain MRI image in order to study the effect of the window size $w$ on the algorithm’s effectiveness. A local window of $5 \times 5$ has proven to be effective.

### 2.5 FRFCM

Lei et al. (2018) suggested a fast and robust fuzzy c-means clustering algorithm (FRFCM),\(^5\) which is based on morphological reconstruction and membership filtering. They could be considered as pre-processing and post-processing, respectively. FRFCM is applied to gray-level histogram in a similar manner to EnFCM. For the data point $(x_j = R^C(x))$ and its membership value $(u_{ij})$, the objective function is minimized using the Lagrange function, where $(R^C(x))$ is the morphological closing reconstruction of original image $(x)$. The objective function is written as follows:

$$J_{FRFCM} = \sum_{j=1}^{N} \sum_{i=1}^{c} \gamma_j u_{ij}^m \|x_j - y_i\| - \lambda (\sum_{i=1}^{c} u_{ij} - 1)$$  \hspace{1cm} (14)

where $\gamma_j$ is the number of the pixels having the gray value equal to $l$, $u_{ij}$ is the fuzzy membership of gray value $l$, $x_j$ is the gray level of an image that has been reconstructed using MR $(R^C(x))$, $\lambda$ is a Lagrange multiplier.

Instead of using the distance vector (between the pixel values and cluster centers), a faster membership filtering is used. The following relationships are used to update the memberships filtering:

$$u_{ij} = \|x_j - y_i\|^{-2/(m-1)} / \sum_{r=1}^{c} \|x_j - y_r\|^{-2/(m-1)} \hspace{1cm} (15)$$

The cluster centers $(y_i)$ is updated using the following relationships:

$$y_i = \sum_{j=1}^{N} \gamma_j u_{ij}^m x_j / \sum_{j=1}^{N} \gamma_j u_{ij}^m \hspace{1cm} (16)$$

Motivated by the excellent performance of the FRFCM, (Shen et al. 2021) have suggested an optical selective encryption scheme for the medical image based on the FRFCM algorithm and face biometric.

### 2.6 DSFCM_N

In deviation-sparse fuzzy c-means with neighbor information (DSFCM_N)\(^6\) algorithm, Zhang et al. (2018) take into account the deviations $e_k$ between measured values $x_k$ and theoretical values $x_\Delta k$. Thus, the accuracy of the calculated deviations would have a great influence on the clustering results. To distinguish noise from outliers, the authors imposed sparsity constraint on the deviations between measured and theoretical values. Moreover, this sparsity could also ensure that the deviations would not diverge. The objective function is denoted as:

$$J_{DSFCM_N} = \sum_{i=1}^{N} \sum_{j=1}^{N} u_{ij}^m \frac{1}{1 + d_{kj}} \frac{1}{\|x_j - y_i\|_2^2}$$ \hspace{1cm} (17)

where $k$ is the neighbor of pixel $j$ in the local window $N_j$, $N_j$ is the local window centralized in $j$, $N_k$ is the neighbor pixels including $j$, $d_{kj}$ is the Euclidean distance between $k$ and $j$, $e_k$ is the deviation vector of pixel $k$, $l$ is the dimension of the data, $\lambda$ is a regulating vector of length $l$.

The deviation matrix $(e_{jt})$ is updated using the following relationships:

$$e_{jt} = \text{soft} \left( \sum_{i=1}^{r} \sum_{k \in N_j} \frac{(u_{ik})^m}{1 + d_{kj}} \frac{\gamma_i}{\sum_{k \in N_j} (u_{ik})^m} \sum_{r=1}^{l} \frac{\lambda_r e_{rk}^2}{\sum_{i=1}^{r} \sum_{k \in N_j} (u_{ik})^m} \right) \hspace{1cm} (18)$$

The memberships $(u_{ij})$ is updated using the following relationships:

$$u_{ij} = \left( \sum_{r=1}^{l} \frac{1}{1 + d_{kj}} \|x_k - y_i\|_2^2 \right)^{1/(m-1)} \hspace{1cm} (19)$$

\(^5\) https://github.com/jiaxhsust/Significantly-Fast-and-Robust-FCM-Based-on-Morphological-Reconstruction-and-Membership-Filtering.

\(^6\) https://github.com/zhangyuxuan1996/DSFCM_N.
The cluster centers \((y_j)\) is updated using the following relationships:

\[
y_j = \left(\sum_{j=1}^{N} u_{ij}^{m} \sum_{k \in N_j} \frac{1}{1 + d_{kj}} (x_k - e_k)\right) / \sum_{k \in N_j} \frac{1}{1 + d_{kj}} \sum_{j=1}^{N} u_{ij}^{m},
\]

(20)

Real brain MRI images were used by (Zhang et al. 2018) to test the effectiveness of DSFCM_N algorithm for segmenting white matter (GM), gray matter (GM), and cerebrospinal fluid (CSF).

### 2.7 FCM_SICM

Robust fuzzy c-means clustering algorithm with adaptive spatial, constraint, and membership (FCM_SICM)\(^7\) linking for noisy image segmentation has been proposed by (Wang et al. 2020). Firstly, the fast bilateral filter is applied to obtain local spatial and intensity information. Secondly, an absolute difference between the original image and the bilateral filtered one is used, and the reciprocal of the absolute difference image between the original image and the bilateral filtered one is used, and the reciprocal of the difference image whereas the different image itself constrain conventional FCM, as well as, the local spatial and intensity information, respectively. Finally, membership linking is reached by summing each membership degree calculated from the earlier iteration within every cluster in squared logarithmic form as the denominator of the objective function as follows:

\[
J_{FCM\_SICM}^{(t)} = \sum_{j=1}^{K} \frac{1}{\delta_j} (u_{ij}^{(t)})^m \| y_j - c^{(t)}_i \|^2 + \sum_{j=1}^{N} \Delta y_j (u_{ij}^{(t)})^m \| y_j - c^{(t)}_i \|^2 \\
\sum_{j=1}^{N} \ln^2 \left( \frac{1}{\delta_j} \sum_{j=1}^{N} \frac{1}{1 + d_{ij}} \sum_{j=1}^{N} u_{ij}^{m} \right) + 1
\]

(21)

Subject to

\[
\sum_{j=1}^{K} u_{ij}^{(t)} = 1.
\]

(22)

The center \((c_i)\) is updated using the following relationships:

\[
c^{(t)}_i = \sum_{j=1}^{N} \left( \frac{1}{\delta_j} y_j + \Delta y_j \bar{y}_j \right) (u_{ij}^{(t)})^m / \sum_{j=1}^{N} \left( \frac{1 + \Delta y_j^2}{\delta_j} \right) (u_{ij}^{(t)})^m.
\]

(23)

The membership \((u_{ij})\) is updated using the following relationships:

\[
u_{ij}^{(t)} = \frac{1}{\sum_{j=1}^{K} \frac{1}{u_{ij}^{m}} \left( \| y_j - c^{(t)}_i \|^2 + \Delta y_j \| y_j - c^{(t)}_i \|^2 \right) + \ln^2 \left( \frac{1}{\delta_j} \sum_{j=1}^{N} \frac{1}{1 + d_{ij}} \sum_{j=1}^{N} u_{ij}^{m} \right) + 1}
\]

(24)

### 2.8 SSFCA

Self-sparse fuzzy algorithm (SSFCA)\(^8\) was proposed by (Jia et al. 2020). This algorithm is based on integrating into the objective function a regularization under Gaussian metric. The objective is to obtain fuzzy membership with sparsity, which reduces a proportion of noisy features and improves clustering results. The final objective function is defined as:

\[
J_{SSFCA} = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij} \Phi^2(x_j, v_i, \Sigma_i) + \gamma \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^2
\]

(25)

where

\[
\gamma \text{ is a balance factor used for controlling the sparsity of memberships.}
\]

\[
\Sigma_i \text{ is covariance matrix.}
\]

\[
\Phi^2(x_j, v_i, \Sigma_i) \text{ is the distance function between } x_j \text{ and } v_i.
\]

By setting the value of \((\gamma)\), fuzzy memberships with varying degrees of sparsity can be obtained using the optimization approach suggested in (Huang et al. 2015) to solve Eq. (25). The memberships matrix can be rewritten as:

\[
J_{SSFCA} = \min \| u_{ij} - h_{ij} \|^2
\]

(26)

Subject to,

\[
h_{ij} = -\Phi^2(x_j, v_i, \Sigma_i) / 2\gamma.
\]

(27)

The cluster centers is updated using the following relationships:

\[
v_i = \frac{\sum_{j=1}^{n} u_{ij} x_j}{\sum_{j=1}^{n} u_{ij}}
\]

(28)

The covariance matrix is updated using the following relationships:

\[
\Sigma_i = \frac{\sum_{j=1}^{n} u_{ij} (x_j - v_i)^T (x_j - v_i)}{\sum_{j=1}^{n} u_{ij}}.
\]

(29)

\(^7\) https://github.com/wqshmzh/FCM_SICM-Algorithm.

\(^8\) https://github.com/jiaxhsut/Rust-Self-Sparse-Fuzzy-Clustering-for-Image-Segmentation.
Furthermore, after describing the FCM methods considered in this study, we emphasize the following remarks:

- All of the improved FCM algorithms are based on the fuzzy theory and they share some common settings, such as the fuzzy exponent, the convergence threshold, and the maximum iteration number. In contrast, the specific parameters for each algorithm should be tuned separately to reach satisfactory results.
- The improved FCM algorithms have been designed to correct the shortcomings that the conventional FCM has encountered. For example, the failure to consider the spatial information in the objective function.
- The similarity between SFCM, FGFCM, FRFCM, and FCM_SICM algorithms lies in the fact that they incorporate local spatial information into their objective function. In addition, these algorithms employ neighborhood information of a center pixel by using a fixed-sized window to improve the image segmentation effect.
- At the same time, DSFCM_N and SSFCA algorithms incorporate deviation and sparsity into their objective function, leading to improve the noise robustness.
- To address the issue of high computational complexity, the EnFCM algorithm performs the clustering on gray levels, while the FRFCM algorithm employed morphological reconstruction and membership filtering, making them significantly faster.

3 Noise robustness analysis

To get a better idea of the noise robustness of the improved FCM algorithms in terms of image segmentation, we conduct in this section some experiments on a well-known synthetic image. Eight derivatives of the FCM clustering algorithm are detained in this study: FCM, EnFCM, SFCM, FGFCM, FRFCM, DSFCM_N, FCM_SICM, and SSFCA. The size of the synthetic image is 269 × 260 and it includes three classes (tree intensity values 75, 100, and 175) (Lei et al. 2018), as shown in Figs. 1a and 2a. To carry out our study, the synthetic image is corrupted with varying levels of Gaussian, and salt & pepper noise. In addition, the common parameters to all methods are tuned as: the fuzzy exponent \( m = 2 \), the convergence threshold \( e = 10^5 \), and the maximum iteration number \( T = 100 \), whereas the specific parameters for each algorithm are shown in Table 1. All parameters have been carefully chosen after many experiments in order to achieve the best results, and this was taken with all FCM algorithms.

3.1 Validation

Among the most commonly used metrics in medical image field, we choose the following four parameters to evaluate the quality and reliability of improved FCM algorithms:

- \( TP \): Number of pixels in the region of interest (retinal vessels) detected correctly.
- \( TN \): Number of background pixels detected correctly.
- \( FP \): Number of background pixels detected as the region of interest pixels (retinal vessels).
- \( FN \): Number of pixels in the region of interest (retinal vessels) detected background pixels.

Using the above basic parameters, certain ratios are formulated to compare the performance of different methods:

Precision

\[
Pr = \frac{TP}{TP + FP}
\]
Table 1 Parameters used

| Methods     | Parameters                                                                 |
|-------------|-----------------------------------------------------------------------------|
| EnFCM       | $\alpha = 4.2$                                                              |
| SFCM        | $w = 5, p = 1, q = 1$                                                        |
| FGFCM       | $\alpha (\lambda_s, \lambda_g) = 6.2$                                      |
| FRFCM       | $w = 1, se = 1, shape = disk$                                               |
| DSFCM_N     | $w = 3, \lambda = 10$                                                       |
| FCM_SICM    | $\sigma_d = 0.5, \sigma_r = 2$                                             |
| SSFCA       | $\gamma = 3$                                                               |

Sensitivity

$$Sn = \frac{TP}{TP + FN}$$ \hspace{1cm} (31)

Specificity

$$Sp = \frac{TN}{TN + FP}$$ \hspace{1cm} (32)

F1 score

$$F_1 = \frac{2TP}{2TP + FP + FN}$$ \hspace{1cm} (33)

Accuracy

$$Acc = \frac{TP + TN}{TP + TN + FN + FP}$$ \hspace{1cm} (34)

3.2 Results on a synthetic image

The purpose of this subsection is to report the results of the validation experiments, as well as some analysis. Figures 1 and 2 illustrate segmentation results obtained by the different algorithms. Tables 2 and 3 give the metrics scores. Broadly speaking, from Table 2 and Fig. 1, we can notice that the FRFCM and DSFCM_N algorithms obtain better segmentation results on the synthetic image corrupted by Gaussian noise (2 and 4%). Likewise, it is consistently higher than other algorithms. FRFCM included images’ local spatial information by introducing morphological reconstruction operation to ensure noise-immunity and image detail-preservation. Whereas the DSFCM_N algorithm suppresses noises by considering the sparsity of deviations between measured and theoretical values. It does not impose any limit on the artifacts, which could achieve a better visual effect. Moreover, when testing the improved FCM algorithms on the synthetic image corrupted by salt & pepper noise (3 and 6%), the results in Table 3 and Fig. 2 demonstrate that the FRFCM and DSFCM_N algorithms are more effective, which confirms the results of the first experiment.

4 Blood vessel segmentation analysis

Retinal blood vessels consist of arteries. The veins appear as elongated features, and their tributaries are visible within the retinal image. Furthermore, retinal vessel segmentation is a topic of great interest in medical image analysis because vessel analysis is critical for diagnosis, treatment planning, implementation, and evaluation of clinical outcomes. This section deals with the analysis of vessel segmentation by using improved FCM algorithms. In addition, a quantitative comparison is carried out between the different algorithms in terms of their effectiveness and time-consuming.
Table 2 The metrics values achieved by different algorithms on the synthetic images corrupted by various ratios of Gaussian noise

| Ratio | Methods   | Pr   | Sp   | Sn   | F1   | Acc  |
|-------|-----------|------|------|------|------|------|
| 2%    | FCM       | 0.9380 | 0.9711 | 0.7269 | 0.8793 | 0.8191 |
|       | EnFCM     | 0.9848 | 0.9914 | 0.9225 | 0.9655 | 0.9526 |
|       | SFCM      | 0.95861 | 0.9794 | 0.7890 | 0.9079 | 0.8656 |
|       | FGFCM     | 0.9855 | 0.9917 | 0.9342 | 0.9701 | 0.9592 |
|       | FRFCM     | **0.9991** | **0.9994** | **0.9962** | **0.9982** | **0.9977** |
|       | DSFMC_N   | **0.9951** | **0.9970** | **0.9986** | **0.9976** | **0.9968** |
|       | FCM_SICM  | 0.9465 | 0.9759 | 0.7082 | 0.8753 | 0.8102 |
|       | SSFCA     | 0.7305 | 0.8193 | 0.8137 | 0.8172 | 0.7698 |
| 4%    | FCM       | 0.8512 | 0.9361 | 0.6066 | 0.8123 | 0.7084 |
|       | EnFCM     | 0.9564 | 0.9786 | 0.7792 | 0.9037 | 0.8588 |
|       | SFCM      | 0.8857 | 0.9521 | 0.6159 | 0.8258 | 0.7266 |
|       | FGFCM     | 0.9588 | 0.9795 | 0.7909 | 0.9086 | 0.8668 |
|       | FRFCM     | **0.9928** | **0.9957** | **0.9749** | **0.9879** | **0.9838** |
|       | DSFMC_N   | **0.9762** | **0.9854** | **0.9900** | **0.9872** | **0.9831** |
|       | FCM_SICM  | 0.8563 | 0.9401 | 0.5926 | 0.8095 | 0.7005 |
|       | SSFCA     | 0.7300 | 0.8175 | 0.8199 | 0.8184 | 0.7724 |

Bold values indicate better results than other improving FCM algorithms.

Table 3 The metrics values achieved by different algorithms on the synthetic images corrupted by various ratios of salt & pepper noise

| Ratio | Methods   | Pr  | Sp  | Sn  | F1  | Acc  |
|-------|-----------|-----|-----|-----|-----|------|
| 3%    | FCM       | 0.9753 | 0.9850 | 0.9851 | 0.9850 | 0.9802 |
|       | EnFCM     | 0.9673 | 0.9803 | 0.9673 | 0.9754 | 0.9673 |
|       | SFCM      | 0.9886 | 0.9931 | 0.9933 | 0.9932 | 0.9910 |
|       | FGFCM     | 0.9688 | 0.9812 | 0.9669 | 0.9759 | 0.9679 |
|       | FRFCM     | **0.9996** | **0.9997** | **0.9998** | **0.9998** | **0.9997** |
|       | DSFMC_N   | **0.9985** | **0.9991** | **0.9829** | **0.9930** | **0.9907** |
|       | FCM_SICM  | 0.9754 | 0.9850 | 0.9841 | 0.9847 | 0.9797 |
|       | SSFCA     | 0.9744 | 0.9844 | 0.9851 | 0.9847 | 0.9797 |
| 6%    | FCM       | 0.9519 | 0.9704 | 0.9704 | 0.9704 | 0.9611 |
|       | EnFCM     | 0.9409 | 0.9644 | 0.9408 | 0.9555 | 0.9409 |
|       | SFCM      | 0.9886 | 0.9931 | 0.9933 | 0.9932 | 0.9910 |
|       | FGFCM     | 0.9394 | 0.9636 | 0.9357 | 0.9531 | 0.9375 |
|       | FRFCM     | **0.9996** | **0.9998** | **0.9997** | **0.9997** | **0.9996** |
|       | DSFMC_N   | **0.9944** | **0.9967** | **0.9692** | **0.9864** | **0.9816** |
|       | FCM_SICM  | 0.9514 | 0.9702 | 0.9697 | 0.9700 | 0.9605 |
|       | SSFCA     | 0.9517 | 0.9704 | 0.9701 | 0.9703 | 0.9608 |

Bold values indicate better results than other improving FCM algorithms.

4.1 Complexity of retinal images

Retinal fundus images have many challenges related to the classification of vessels, including the low contrast accompanying the fundus image, the inhomogeneity of the background lighting, and the noise. The inhomogeneity and noise occur during the acquisition process of the image. The noise can alter the intensity of a pixel and affect its classification certainty. As a result, the difference in thickness between thicker and thinner vessels is higher. Another challenge depends on color variations in the retina of different subjects, which are rooted in biological characteristics. Therefore, segmentation of the retinal vascular image remains a challenge despite the numerous methods proposed in the literature.
4.2 Pre-processing method

The pre-processing stage aims to improve image quality. One of the most crucial tasks in image processing is the contrast enhancement of the input image. This operation is merely for enhancing and increasing image clarity to reach better segmentation results. Figure 3 describes the flowchart of the retinal image segmentation phases with the improved FCM algorithms. It consists of five parts: retina image from the database, green extraction channel, image enhancement, image segmentation, and vascular separation. Some techniques such as CLAHE and bottom-hat filter are incorporated in the image enhancement stage to improve the final segmentation result. In the segmentation stage, the improved FCM algorithms are applied to divide the image into 3 clusters. One of them is the region of interest. In vascular separation, the area of interest is extracted and validated with some indicators to evaluate the effectiveness and robustness of the different FCM algorithms subject to experience.

4.2.1 Retinal image components

It is well known that the image of the retina consists of three channels, red, green, and blue (Fig. 4). Among these channels, the red one is supersaturated, and the blue one is poorly lit to reveal the blood vessel. Thus, the green channel is more adapted for the vascular detection due to the significant contrast of the blood vessels. For this reason, the image analysis is achieved only on the green channel (Azzopardi et al. 2015).

4.2.2 Contrast limited adaptive histogram equalization

Improving image contrast is a process that aims to increase the intensity variations of the considered image. Contrast enhancement is an essential means of analyzing medical
images. CLAHE is a method widely used to process biomedical images and signals. Its main interest lies in increasing the visibility of the region of interest. CLAHE is an improved algorithm of adaptive histogram equalization (AHE) (Pizer et al. 1987) developed by (Zuiderveld 1994). It reduces the noise amplification problem by segmenting an image into small interrelated areas called tiles, followed by employing histogram equalization above each tile.

4.2.3 Bottom-hat filtering (BTH)

In digital image processing and mathematical morphology, bottom-hat (BTH) filtering is used for various tasks, such as background equalization, image enhancement, feature extraction, and others. It is used for the improvement of retina images, given its effective ability in enhancing the appearance of the retinal vessels that appear black. Bottom-hat filtering (BTH) process is based upon the subtracting the input image \( I \) from the result \( \phi_S(I) \) of performing a morphological closing operation \( \phi_S \) on the input image \( I \). The closure performs a dilation \( (\oplus) \) followed by Erosion \( (\ominus) \). The result is filling holes and connecting objects nearby. The equation of the bottom-hat (BTH) filtering is given as follows:

\[
BTH = \phi_S(I) - I
\]

In this study, the BTH filter is used immediately after the extraction of the green channel. Its role is to prepare and enhance the appearance of the image components, mainly the vessels. The filter settings are chosen after many experiments, and eventually, the value 11 is chosen for the structural element with the square shape as it gives satisfactory results (Gonzalez et al. 2002; Bhabatosh et al. 1977).

In order to highlight the interest in the retinal image pre-processing phases before the segmentation process, we conducted several experiments by employing the improved FCM algorithms. The main idea is to show retinal image segmentation results before and after each pre-processing step. Fig. 5(a–d) illustrates the results of this study. Some algorithms like FCM, EnFCM, SFCM, and FGFCM have poor segmentation result on the color images because they are designed to segment only the grayscale images. The images in Fig. 5a are the segmentation results performed on the three channels illustrated on one image. Moreover, the images in Fig. 5(a–c) demonstrate the inability of the improved FCM algorithms to segment retinal images and their need for pre-processing phases. The images in Fig. 5d demonstrate

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**Table 4** Characteristics of databases

| Database | DRIVE | STARE |
|----------|-------|-------|
| Number of image | 40 | 20 |
| Camera | Canon CR5 | TopCon TRV-50 |
| FoV | 45° | 35° |
| Spatial resolution | 565 × 584 | 605 × 700 |
| Format | JPEG | PPM |
| Ground truth | available | available |
that the proposed pre-processing operations (CLAHE and bottom-hat filter) have contributed to achieving the desired segmentation.

### 4.3 Experiments on retinal images

This section reviews the performance analysis of the improved FCM algorithms. As mentioned previously, eight derivatives of FCM clustering algorithm are tested in these experiments: FCM, EnFCM, SFCM, FGFCM, FRFCM, DSFCM_N, FCM_SICM, and SSFCA. The effectiveness and the efficiency of the different algorithms have been evaluated on the DRIVE and STARE databases. The retinal images are segmented into 3 clusters, representing the background, the retinal vessels, and a cluster of the other ocular tissues. The retinal vessels are the region of interest that we require to achieve. The effectiveness of each improved FCM algorithm can be summarized by its ability to accurately distinguish the vessels pixels from those of the background. The performance assessment is performed by comparing the achieved segmentation results with those made manually using the same metrics mentioned in Sect. 3.1.

#### 4.3.1 Database

There are many open public databases available for researchers, along with the ground truth images of blood vessel details, such as IDRiD (Porwal et al. 2018), CHASE_DB1 (Fraz et al. 2012), DRIVE (Staal et al. 2004) and STARE (Hoover et al. 2000). Therefore, we chose the DRIVE and STARE databases. Table 4 demonstrates some of its features.

#### 4.3.2 Results on retinal images

As mentioned above, the experiments have been conducted on two well-known open-access databases, DRIVE and STARE. Hence, the results obtained on the DRIVE database are illustrated in Table 5 and Fig. 6. As shown in Table 5, the DSFCM_N algorithm has the highest values in terms of the segmentation accuracy (Acc = 0.9522) and F1 score (F1=0.3323). This latter is followed by the conventional FCM algorithm with an accuracy value of (Acc= 0.9504) and the FRFCM algorithm (Acc=0.9499), while the highest specificity value is achieved by the SFCM algorithm (Sp = 0.9971), and the best sensitivity value is obtained by the SSFCA algorithm (Sn = 0.7289).

However, the results obtained on the STARE database are illustrated in Table 6. As indicated in this table, the FGFCM algorithm has the highest value in terms of the segmentation accuracy (Acc = 0.9517), which is a little higher compared with the FRFCM algorithm (Acc = 0.9511). Moreover, the highest F1 score is reached by the FRFCM algorithm. However, the SFCM algorithm achieved the highest specificity value (Sp = 0.9911), and the best precision value is obtained by the EnFCM algorithm (Pr = 0.8831). Fig. 7 depicts the comparison of the segmentation results on the STARE database.

It should also be mentioned that the results of this performance analysis are fully compatible with those obtained in the section dealing with noise robustness analysis. In fact, the algorithms DSFCM_N, FRFCM, and FGFCM provide the best segmentation accuracy since these algorithms use several mathematical tools such as morphological reconstruction and membership filters to suppress noises and achieve a better visual effect and segmentation accuracy. Broadly speaking, the performance in terms of segmentation accuracy of the eight improved FCM algorithms is close to each other, even though the objective functions are different. Consequently, we tend to say that there is a significant similarity between the improved FCM algorithms.

#### 4.3.3 Running time

To evaluate the practicability of improved FCM algorithms, their execution times are compared in this subsection. All experiments are executed on a workstation with an Intel®

| Methods   | Pr      | Sp      | Sn      | F1      | Acc    |
|-----------|---------|---------|---------|---------|--------|
| FCM       | 0.8913  | 0.9939  | 0.4944  | 0.3180  | 0.9504 |
| EnFCM     | 0.8831  | 0.9935  | 0.4902  | 0.3152  | 0.9497 |
| SFCM      | 0.9331  | 0.9971  | 0.4012  | 0.2805  | 0.9453 |
| FGFCM     | 0.7748  | 0.9819  | 0.5979  | **0.3374** | 0.9492 |
| FRFCM     | 0.8803  | 0.9929  | 0.4996  | 0.3187  | 0.9499 |
| DSFCM_N   | 0.8762  | 0.9922  | 0.5354  | **0.3323** | **0.9522** |
| FCM_SICM  | 0.9231  | 0.9965  | 0.4165  | 0.2870  | 0.9460 |
| SSFCA     | 0.6516  | 0.9581  | 0.7289  | 0.3440  | 0.9377 |

Bold values indicate better results than other improving FCM algorithms.
Comparative analysis of improved FCM algorithms for the segmentation of…

Fig. 6 Comparison of segmentation results on DRIVE Database. a Ground truth, b FCM result. c EnFCM result. d SFCM result. e FGFCM result. f FRFCM result. g DSFCM_N result. h FCM_SICM result. i SSFCA result

Table 6 Segmentation precision (Pr), sensitivity (Se), specificity (Sp), F1 score, and accuracy (Acc) achieved by different algorithms on the STARE database

| Methods       | Pr     | Sp     | Sn     | F1     | Acc    |
|---------------|--------|--------|--------|--------|--------|
| FCM           | 0.7322 | 0.9788 | 0.5727 | 0.3214 | 0.9474 |
| EnFCM         | 0.8831 | 0.9802 | 0.5654 | 0.3447 | 0.9482 |
| SFCM          | 0.8146 | 0.9911 | 0.4632 | 0.2952 | 0.9509 |
| FGFCM         | 0.7804 | 0.9854 | 0.5464 | 0.3214 | 0.9517 |
| FRFCM         | 0.7525 | 0.9815 | 0.5870 | 0.4201 | 0.9511 |
| DSFCM_N       | 0.7264 | 0.9742 | 0.6059 | 0.3303 | 0.9455 |
| FCM_SICM      | 0.7749 | 0.9847 | 0.5026 | 0.3048 | 0.9475 |
| SSFCA         | 0.4872 | 0.9194 | 0.7919 | 0.3016 | 0.9089 |

Bold values indicate better results than other improving FCM algorithms

Core (TM) i5-2520 2.5GHz CPU and 2G memory and by using MATLAB. Tables 7 and 8 show the running times of different algorithms measured on the DRIVE and STARE databases, respectively. The running time is the average value calculated by executing each algorithm five times on the databases.

As indicated in the tables, it is obvious that the SSFCA algorithm is the slower one (165.73 s on DRIVE and 243.31 s on STARE). Since the SSFCA integrates into the objective function the regularization under Gaussian metric to obtain fuzzy membership with sparsity. However, the running time of the algorithms SFCM, FGFCM, DSFCM_N, FCM_SICM is very close to each other. They are around of 10 to 22 seconds. The best running times are achieved by the FRFCM and EnFCM algorithms. In fact, working on gray level, which is significantly less than the number of pixels in an image, makes EnFCM fast. As regards the FRFCM algorithm, its timeliness is due to the incorporation of the morphological reconstruction and membership filtering in the objective function.

Furthermore, the FRFCM algorithm employs the concept of the histogram. Therefore, the objective function of FRFCM converges rapidly. Given the previous experiments, we feel that the FRFCM algorithm, and to a lesser degree, the DSFCM_N algorithm, represent the best possible compromise in terms of the noise robustness, the retinal vessel segmentation effectiveness, and the time-consuming.

5 Conclusion

In this paper, the performance of some improved FCM algorithms has been analyzed and compared in terms of the retinal vascular segmentation. Eight derivatives of the FCM algorithm have been detained in this study: FCM, EnFCM, SFCM, FGFCM, FRFCM, DSFCM_N, FCM_SICM and SSFCA. The performance analysis has been conducted from three viewpoints: noise robustness, blood vessels segmentation performance, and time-consuming. The algorithms have been compared on the basis of several metrics like segmenta-
Fig. 7 Comparison of segmentation results on STARE database. (a) Ground truth, (b) FCM result. (c) EnFCM result. (d) SFCM result. (e) FGFCM result. (f) FRFCM result. (g) DSFCM_N result. (h) FCM_SICM result. (i) SSFCA result.

Table 7 Comparison of execution times (in seconds) performed by different algorithms on DRIVE database

| Methods    | FCM | EnFCM | SFCM | FGFCM | FRFCM | DSFCM_N | FCM_SICM | SSFCA |
|------------|-----|-------|------|-------|-------|----------|----------|-------|
| Average time | 8.15 | 5.79  | 22.03| 17.29 | 0.32  | 10.96    | 10.93    | 165.73|

Bold values indicate better results than other improving FCM algorithms

Table 8 Comparison of execution times (in seconds) performed by different algorithms on STARE database

| Methods    | FCM | EnFCM | SFCM | FGFCM | FRFCM | DSFCM_N | FCM_SICM | SSFCA |
|------------|-----|-------|------|-------|-------|----------|----------|-------|
| Average time | 11.73 | 7.29  | 31.15| 21.80 | 0.40  | 13.78    | 14.05    | 243.31|

Bold values indicate better results than other improving FCM algorithms

As a result, the FRFCM algorithm, and to a lesser degree the DSFCM_N algorithm, represent the best possible compromise between the three comparison criteria.

As a logical follow-up to this study, it would be interesting to extend this study to address other derivatives of the FCM algorithm on the one hand (Zhang et al. 2022) (Qiao et al. 2022), and to use other databases like IDRiD (Porwal et al. 2018), CHASE_DB1 (Fraz et al. 2012) on the other hand. Although the obtained results are satisfactory, retinal vessels segmentation based on the FCM algorithms could be further improved to achieve more accurate results. As a future scope, it will be interesting to focus on improving the pre-processing phase through the use of filters whose work is appropriate to the structure of the retinal vessels, such as Jerman filtering and morphological reconstructions. Another avenue that could be explored is the use of the field-of-view mask (FoV) to create an image without background pixels on the one hand, and to decrease the time-consuming by reducing the mathematical operations on the other hand.

Data Availability Enquiries about data availability should be directed to the authors.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the original study.

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