Naturalised Supersymmetric Grand Unification

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ABSTRACT: We construct a simple model of an $SU(5)$ GUT with gauge mediated supersymmetry breaking from a metastable vacuum of a hidden sector. All mass parameters and hierarchies of our model are generated dynamically from retrofitting. This includes the $\mu$-parameter and the GUT scale. However, as typical for simple $SU(5)$ GUT models, proton longevity remains a problem.
1. Introduction

The mechanism whereby supersymmetry is broken in Nature is once again the subject of intense scrutiny. Of particular importance has been the realization by Intriligator, Seiberg and Shih (ISS) that (for appropriate choices of flavours and colours) the simplest SQCD models have SUSY breaking metastable minima [1]. Such models are phenomenologically acceptable provided the decay time from the metastable to the supersymmetric vacuum is sufficiently long. Furthermore, it was argued that the early Universe is naturally driven to such metastable minima and remains trapped there [2–6]. Metastability allows for the presence of supersymmetric ‘true’ vacua in the theory and thereby allows one to evade several stringent constraints on supersymmetry breaking. These include the Nelson–Seiberg theorem [7] which requires an $R$-symmetry leading to unwelcome phenomenological consequences, such as vanishing gaugino masses or the presence of $R$–axions.

Accepting metastable SUSY breaking minima [1] (for earlier work see [8–10]) leads to a far broader class of SUSY breaking models which is very appealing. This fact is exploited in the retrofitting approach of [11, 12] which – in the light of the ISS model – generalizes and greatly improves upon earlier models of metastable SUSY breaking. The approach begins with a model that has an exact $R$-symmetry, and breaks it with terms that are generated dynamically and are thus small. The models are metastable, but the fact that $R$-symmetry is still approximately conserved is enough to ensure that the global SUSY preserving minima are far away in field space and hence the SUSY breaking minima are long lived. There have since been a number of discussions of how such metastable SUSY breaking might be mediated to the Standard Model, including direct mediation [1, 13, 14], breaking within the visible sector [15] and gauge mediation [11, 12, 16–18].

Our purpose in this paper is to examine the consequences of these developments for Grand Unification. In particular, the retrofitting programme seeks to explain all mass scales dynamically by the confinement of hidden gauge sectors. As well as the SUSY breaking scale itself, one would naturally like to obtain explanations for other dimensionful parameters such as the $\mu$-term of the MSSM (as in e.g. [12]). Grand Unified Theories (GUTs) are of course full of dimensionful parameters: the GUT scale; the SUSY breaking scale; the $\mu$-term of the effective low energy theory. In the simplest $SU(5)$ GUTs, the latter is especially bothersome, requiring a fine-tuning between mass parameters to one part in $10^{14}$, the so-called doublet-triplet mass-splitting problem (for a review see [19]). One is led to ask whether GUTs can be made more natural in the light of metastability: is it possible to retrofit a GUT model with broken supersymmetry entirely, so that no dimensionful parameters have to be chosen by hand?

Basing our analysis on the simplest examples of gauge mediation developed in refs. [16, 17] we will argue that it is. The former paper outlined a simple model of gauge mediation, whereas the latter showed how it can be retrofitted, with all mass terms being generated dynamically. However, neither considered the coupling to, or parameters of the MSSM, such as for example the $\mu$-parameter. Our objective in the present work will be to completely retrofit this parameter as well as the other parameters required for GUT and SUSY breaking itself: in other words to construct a theory whose
GUT breaking, SUSY breaking, messenger scale and \( \mu \)-term are all generated by the dynamics. We will be able to generate and explain within this approach the three key scales of the visible sector: the GUT scale \( \sim 10^{16} \div 10^{17} \) GeV, the electro-weak and the supersymmetry breaking scales, both \( \sim 10^2 \) GeV. In particular, our model predicts a relation between the GUT and the electroweak scale,

\[
M_{\text{GUT}}^2 \sim 4\pi (\mu_{\text{MSSM}} M_p^3)^{1/2}.
\]

For this preliminary study we will be considering the simplest case which is a minimal \( SU(5) \) GUT (for a review see [20]). These models are known to conflict with bounds on the decay rate of the proton because of large dimension-5 operators mediated by Higgs triplets. Indeed in the model we present here, the Higgs triplets are lighter than usual (although still relatively close to the GUT scale) so that the proton decay rate is significantly worse. Nevertheless the model is an encouraging first step on the road to a fully consistent retrofitted GUT. We discuss in a later subsection how the model or similar GUT models may be developed in order to make it more realistic.

2. The model

We want to construct a simple and predictive model which combines and inter-relates the ideas of supersymmetric Grand Unification [21], supersymmetry breaking by a metastable vacuum [1], and naturalness achieved through retrofitting [12].

Following the general set-up of [17] we consider a model made up of three sectors.

1. The first is the \( R \)-sector whose main rôle is to dynamically generate all mass-parameters in the effective Lagrangian of the full model. This is achieved via a version of the retrofitting approach of [11, 12] which will be reviewed shortly. In our model this sector is described by a strongly coupled confining SQCD theory with the dynamical scale \( \Lambda_R \). In the full theory \( \Lambda_R \) triggers the dynamical generation of masses as in [17]. In addition, in our model the \( N_f \times N_f \) meson superfield \( \tilde{Q}_R Q_R \) of the \( R \)-sector will play the rôle of the adjoint Higgs of the GUT sector.

2. The second sector is responsible for supersymmetry breaking. It is described by the SQCD in a free magnetic phase, known as the ISS model [1]. This model contains a long-lived metastable vacuum which breaks supersymmetry, and will be referred to as the metastable susy-breaking, or MSB-sector.

3. The visible sector is the \( SU(5) \) susy GUT-sector. The \( SU(5) \) gauge group arises from gauging the flavour \( SU(N_f = 5) \) symmetry of the \( R \)-sector, and the adjoint Higgs field \( \Phi_{\text{GUT}} \) is identified with the traceless part of the \( R \)-sector mesons \( \tilde{Q}_R Q_R \). The GUT-sector is coupled to the MSB-sector via messenger fields \( f \) and \( \tilde{f} \) which are in the fundamental and the anti-fundamental of the \( SU(5) \) gauge group. Hence supersymmetry breaking is mediated to the GUT theory via gauge mediation.
In what follows we will see that this model delivers a supersymmetric Grand Unified Theory with calculable soft susy-breaking terms (arising from interactions with the MSB-sector). The model is fully natural and all the mass-scales of the theory are generated in terms of appropriate combinations of the two dynamical scales $\Lambda_R, \Lambda_{MSB}$ and the Planck scale $M_p$. In particular, by choosing $\Lambda_R$ and $\Lambda_{MSB}$ our model can naturally generate the desired values of the electro-weak, supersymmetry breaking, and the GUT scale.

2.1 Interactions between the sectors

Now we proceed to specify the interactions between the three sectors of the model. These are introduced through the superpotentials $W_1, W_2$ and $W_3$ with one property in common: they couple bilinears from one sector to a bilinear from another and as such are represented by lowest-dimensional non-renormalizable operators suppressed by $M_p$. For simplicity of presentation, in equations (2.1), (2.6), (2.10) we will include only the interactions which are necessary for our model. Other interactions will be discussed in the Appendix. The superpotential $W_1$ is responsible for the retrofitting \cite{12,17} and couples the singlet bilinear made of the gauge-strength superfield $W_R$ of the confining R-sector to the singlet bilinears of the MSB- and the GUT-sectors:

$$W_1 = \text{tr}(W_R^2) \left( \frac{1}{\hat{g}_R^2} + \frac{a_1}{16\pi^2M_p^2} \text{tr}(\tilde{Q}_{MSB}Q_{MSB}) + \frac{a_2}{16\pi^2M_p^2} \text{tr}(\tilde{f}f) + \frac{a_3}{16\pi^2M_p^2} \text{tr}(\tilde{H}H) \right), \quad (2.1)$$

where $\tilde{Q}_{MSB}, Q_{MSB}$ are the (anti)-fundamental quark superfields of the MSB sector, $\tilde{f}, f$ and $\tilde{H}, H$ are the messengers and the Higgs fields transforming in the (anti)-fundamental of the $SU(5)$ GUT. The factors of $1/16\pi^2$ on the right hand side of (2.1) indicate that these contributions come from loop effects in the underlying theory at the scale $M_p$. The constants $a_i$ are undetermined in the low-energy effective theory; they are generically of order one (which we will interpret as being in the range $10^{-3} \div 10^1$). These are the leading-order higher-dimensional operators which involve interactions between $WW$ and the matter-field bilinear gauge singlets. Operators of even higher dimension will be suppressed by extra powers of the Planck mass $M_p$ and will not be relevant for our analysis.

The R-sector is described by a non-Abelian gauge theory. We will take it to be an SQCD theory with the gauge group $SU(N_c)$ and $N_f$ flavours of quarks $\tilde{Q}_R, Q_R$ with $N_f < N_c - 1$. The quark fields $\tilde{Q}_R, Q_R$ develop (large) VEVs which break the gauge group to $SU(N_c - N_f)$. The resulting ‘low-energy’ theory of the R-sector is the pure $SU(N_c - N_f)$ SYM with the dynamical scale $\Lambda_R$ (plus colour-singlet meson fields $\tilde{Q}_R Q_R$). The SYM theory is strongly-coupled at the scale $\Lambda_R$ and develops a gaugino condensate,

$$\langle W_R^2 \rangle = \langle \lambda_R^2 \rangle = \Lambda_R^3. \quad (2.2)$$

This effect in the superpotential (2.1) generates masses $m_{Q_{MSB}}, m_f$ and $m_H$ of the order $\sim \Lambda_R^2/M_p^2$ for the appropriate chiral matter fields. This mass generation is the retrofitting mechanism of \cite{12} as explored recently in \cite{17} in the ISS model building context. A novel feature of our model compared to \cite{17} is the fact that in our context not only the MSB-quarks and the messengers, but also the GUT Higgs fields $H$ and $\tilde{H}$ get a retrofitted mass $m_H$ which gives rise to the $\mu_{MSSM}$ parameter of the
Standard Model, 
\[ \mu_{\text{MSSM}} \equiv m_H = \frac{a_3}{16\pi^2} \frac{\Lambda_R^3}{M_p^2}. \]

(2.3)

The generation of the quark masses \( m_{Q_{\text{MSB}}} \sim \Lambda_R^3 / M_p^2 \) is a key ingredient for the metastable suSy breaking [1] in the MSB sector. The relevant scale is [17]:
\[ \mu_{\text{MSB}}^2 \equiv \Lambda_{\text{MSB}} m_{Q_{\text{MSB}}} = \frac{a_1}{16\pi^2} \frac{\Lambda_{\text{MSB}}^3 \Lambda_R}{M_p^2}. \]

(2.4)

In the context of our model, the generation of \( \mu_{\text{MSSM}} \) in (2.3) and \( \mu_{\text{MSB}} \) in (2.4) are the only relevant effects of the retrofitted superpotential (2.1). The value of \( \Lambda_R \gtrsim 10^{14} \) GeV is then chosen, so as to give
\[ \mu_{\text{MSSM}} = \frac{a_3}{16\pi^2} \frac{\Lambda_R^3}{M_p^2} \gtrsim 10^2 \div 10^3 \text{GeV}, \]

as required for electro-weak symmetry breaking.

Although the messenger fields \( f, \tilde{f} \) also get a contribution to their masses from \( W_1 \), the dominant contribution to \( m_f \) comes from a second class of interactions between gauge singlets from different sectors. These couple the messenger fields of the GUT sector and the quark bilinears from the hidden sectors;
\[ W_2 = \frac{b_1}{M_p} \text{tr}(\tilde{f}f) \text{tr}(\tilde{Q}_{\text{MSB}}Q_{\text{MSB}}) + \frac{b_2}{M_p} (\tilde{f}f)(\tilde{Q}_{R}Q_{R}), \]

(2.6)

with constants \( b_1, b_2 \). These terms are ultimately responsible for the mediation of suSy-breaking from the MSB-sector to the GUT-sector, and specifically for the generation of Majorana gaugino masses. The traces in (2.6) are over gauge and flavour indices of each sector. Furthermore, as mentioned earlier, the flavour symmetry \( SU(N_f = 5) \) of the R-sector is gauged, and this makes the R-meson field \( \tilde{Q}_{R}Q_{R} \) an adjoint plus a singlet under the GUT \( SU(5) \) gauge group,
\[ \Phi_{\text{GUT}}^{ij} = \frac{1}{(\tilde{Q}_{R}Q_{R})^2} \tilde{Q}_{R}^i Q_{R}^j, \quad i, j = 1 \ldots N_f = 5. \]

(2.7)

We will show in the next subsection that the VEV for \( \tilde{Q}_{R}^i Q_{R}^j \) is generated dynamically in the R-sector of our theory and is of the form
\[ \tilde{Q}_{R}^i Q_{R}^j = M_{\text{GUT}}^2 \text{diag}(+1, +1, +1, -1, -1), \]

(2.8)

The mass term for the messengers arises from the last term\(^1\) in (2.6). Using (2.8) we find
\[ m_f = b_2 \frac{M_{\text{GUT}}^2}{M_p}. \]

(2.9)

\(^1\)The structure of the last term in (2.6) is a short-hand for a generic interaction, consistent with an unbroken \( SU(5) \), \( f \cdot (c_1 \text{tr}(\tilde{Q}_{R}Q_{R}) + c_2 \tilde{Q}_{R}Q_{R}) \cdot f \), where \( c_{1,2} \) are constants of order 1.
The third class of interactions couples the Higgs (anti)-fundamental fields of the GUT sector to the adjoint (plus a singlet) Higgs which arises from mesons of the R-sector. It has the form,

\[ W_3 = \frac{\kappa}{M_p} H \cdot \left( \text{tr}(\tilde{Q}_R Q_R) + \tilde{Q}_R Q_R \right) \cdot \bar{H}. \]  

(2.10)

These two terms are included to raise the mass of the Higgs triplet fields and do not give any additional mass to the doublets. In order for this to be the case we require the couplings to be precisely equal as shown. The doublet-triplet splitting will be discussed in more detail below.

### 2.2 R-sector and the generation of the GUT scale

In our approach all mass-parameters should be generated dynamically. An important point then is to explain how the GUT scale \( M_{\text{GUT}} \sim 10^{16} \div 10^{17} \text{ GeV} \) is generated alongside the much lower \( \mu_{\text{MSSM}} \) scale in (2.5). In this sub-section we will show that this hierarchy of scales is naturally explained by the dynamics of the R-sector of our model.

As already mentioned, the R-sector is given by an SQCD with \( N_c > N_f + 1 \), with the number of flavours being set to \( N_f = 5 \). The quarks are exactly massless since in the general set-up which we follow no tree-level masses can be put in by hand. As is well-known, there is a nonperturbative Affleck-Dine-Seiberg superpotential [22] in this theory which leads to run-away vacua and renders the theory inconsistent, unless there is a mechanism to prevent the run-away and stabilize the vacua. Without loss of generality and naturalness, this is easily achieved by adding a leading-order higher-dimensional operator to the Lagrangian,

\[ \frac{d}{2M_p} \text{tr}(\tilde{Q}_R Q_R)^2, \]  

(2.11)

where \( d \) is a constant, so that the total superpotential for the meson fields of the R-sector is,

\[ W_R = (N_c - N_f) \left( \frac{\Lambda_{\text{SQCD}}^{3N_c - N_f}}{\det_{N_f} (\tilde{Q}_R Q_R)} \right)^{\frac{1}{N_c - N_f}} + \frac{d}{2M_p} \text{tr}(\tilde{Q}_R Q_R)^2. \]  

(2.12)

The dynamical scale \( \Lambda_{\text{SQCD}} \) appearing in the Affleck-Dine-Seiberg superpotential above, is the scale of the full SQCD theory of the R-sector, and should be distinguished from the dynamical scale \( \Lambda_R \) of the ‘low-energy’ \( SU(N_c - N_f) \) pure SYM. The relation between \( \Lambda_{\text{SQCD}} \) and \( \Lambda_R \) will be determined below.

In terms of the meson field \( M_{ij} = \tilde{Q}_R^i Q_R^j \) the F-flatness condition on (2.12) gives an equation for diagonal components (without loss of generality we work in the basis where \( \langle M_{ij} \rangle \) is diagonal),

\[ \langle M_{ii} \rangle^2 = \frac{M_p}{d} \left( \frac{\Lambda_{\text{SQCD}}^{3N_c - N_f}}{\det_{N_f} M} \right)^{\frac{1}{N_c - N_f}}, \]  

(2.13)

which holds for each value of \( i = 1, \ldots, N_f = 5 \). Since the right hand side of (2.13) does not depend on \( i \) it follows that all the values of \( \langle M_{ii} \rangle^2 \) must be equal to each other. However this does not necessarily
imply that the VEVs of the meson field itself are all the same. For $N_f = 5$ there are three inequivalent discrete solutions of (2.13), the first one is

$$\langle M_{ij} \rangle = \langle M \rangle \, \text{diag}(+1,+1,+1,+1,+1) \implies SU(5),$$

(2.14)

the second solution breaks $SU(5)$ down to $SU(4)$,

$$\langle M_{ij} \rangle = \langle M \rangle \, \text{diag}(+1,-1,-1,-1,-1) \implies SU(4),$$

(2.15)

while the third solution is precisely what we require, it corresponds to a spontaneous breakdown of $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$,

$$\langle M_{ij} \rangle = \langle M \rangle \, \text{diag}(+1,+1,+1,-1,-1) \implies SU(3) \times SU(2) \times U(1).$$

(2.16)

The vacuum expectation value of the meson field in (2.13) should now be expressed in terms of the dynamical scale $\Lambda_R$ for the effective pure SYM $SU(N_c - N_f)$ theory. This is easily achieved by matching the gauge couplings of the SQCD and the SYM theories at the scale $\sqrt{M}$,

$$\Lambda_{SQCD}^{3N_c - N_f} = \Lambda_R^{3(N_c - N_f)} \, M^{N_f}. \quad (2.17)$$

Inserting into Eq. (2.13) gives

$$\langle M \rangle = \frac{1}{\sqrt{d}} \sqrt{\Lambda_R^3 M_p}, \quad (2.18)$$
in terms of $\Lambda_R$, which is precisely what we are after.

Finally, we need to define a canonically normalised meson field $\Phi_{GUT}$ in terms of the dimension-two meson field we were using so far. There are essentially two dimensionful parameters, $\sqrt{\langle M \rangle}$ and $\Lambda_R$ in the QCD theory of the R-sector, which obey $\sqrt{\langle M \rangle} \gg \Lambda_R$. The first parameter sets the scale where the full $SU(N_c)$ is broken down to $SU(N_c - N_f)$, and the second parameter, is the confinement scale of the $SU(N_c - N_f)$ SYM. The mesons describe the Higgsing of $N_f$ of the $N_c$ colors. Under the remaining $SU(N_c - N_f)$ they are colour neutral and they do not take part in the confinement of the $SU(N_c - N_f)$. Hence, the appropriate scale is the scale $\sqrt{\langle M \rangle}$ at which the gauge group is Higgsed \(^2\) is

$$\Phi_{GUT}^{ij} = \frac{1}{\sqrt{\langle M \rangle}} \tilde{Q}_R^i Q_R^j. \quad (2.19)$$

In total we have

$$\langle \Phi_{GUT}^{ij} \rangle = M_{GUT} \, \text{diag}(+1,+1,+1,-1,-1), \quad (2.20)$$

\(^2\)In the first version of this paper we argued that we can expand the Kahler potential for the mesons as $K \sim \text{const} \langle M \rangle^{1/2} M^T M + \text{const} \langle M \rangle^{1/2} M^T M$. If the constant in the first term is non-zero the first term dominates and we would have to normalise with $\Lambda_R$. However, since the mesons do not couple to the remaining $SU(N_c - N_f)$ the first term vanishes. Another way to see that the normalisation (2.19) is the right one is to note that the masses of $SU(5)$ vector bosons $m_v \sim g(Q_R) \sim g(\langle M \rangle)^2$ should be the same whether we think of the $SU(5)$ being higgsed by quarks $Q_R$ or by mesons $\Phi_{GUT}$. 

\[ - 6 - \]
where
\[ M_{GUT}^2 \sim \langle M \rangle = \frac{1}{\sqrt{d}} \sqrt{\Lambda_R^3 M_p}. \tag{2.21} \]
Eliminating \( A_R \) with (2.5) we arrive at a relation between \( \mu_{\text{MSSM}} \) and \( M_{GUT} \) as anticipated in the Introduction;
\[ M_{GUT}^2 = \frac{4\pi}{\sqrt{a_3 d}} (\mu_{\text{MSSM}} M_p^3)^{1/2}. \tag{2.22} \]
Taking \( M_p \sim 10^{19} \text{ GeV} \) and \( \mu_{\text{MSSM}} \sim 10^2 \div 10^3 \text{ GeV} \) we find
\[ M_{GUT} \sim 10^{15} \div 10^{17} \text{ GeV}, \tag{2.23} \]
if we choose the constants \( a_3, d \) in the range \( 10^{-3} \div 10^1 \).

### 2.3 Metastable supersymmetry breaking

The MSB sector is described by the ISS [1] model which is an SQCD with \( N_f \) flavours of classically massless quarks and \( N_c + 1 \leq N_f < 3N_c/2 \). The quarks \( \tilde{Q}_{MSB}, Q_{MSB} \) generate masses dynamically via the interactions (2.1) with the R-sector as explained above.

Following ISS [1] we introduce canonically normalised fields
\[ \Phi_{MSB} = \frac{\tilde{Q}_{MSB} Q_{MSB}}{\Lambda_{MSB}}. \tag{2.24} \]
The magnetic description of the gauge theory, then has a classical
\[ W_{cl} = h \text{tr}_{N_f} \Phi_{MSB} \tilde{\varphi} - h \mu_{MSB}^2 \text{tr}_{N_f} \Phi_{MSB}, \tag{2.25} \]
and dynamical superpotential
\[ W_{dyn} = N \left( h N_f \frac{\det_{N_f} \Phi_{MSB}}{\Lambda_{MSB}^{N_f - 3N}} \right)^{1/4}, \tag{2.26} \]
where \( N = N_f - N_c \) and \( h \) is a constant. Moreover, \( \tilde{\varphi} \) and \( \varphi \) are the magnetic quarks made up from suitable combinations of \( \tilde{Q}_{MSB} \) and \( Q_{MSB} \). Using the normalisation (2.24) one easily translates the retrofitted mass term for \( \tilde{Q}_{MSB} \) and \( Q_{MSB} \), \( m_{Q_{MSB}} \sim \Lambda_R^3/(16\pi^2 M_p^2) \) into \( \mu_{MSB}^2 \) as given in Eq. (2.4).

In the metastable vacuum near \( \Phi_{MSB} = 0 \) supersymmetry is broken by the rank condition at the scale \( \mu_{MSB} \). In particular, we have
\[ \text{tr}(F_{\Phi_{MSB}^ij}) \sim \mu_{MSB}^2. \tag{2.27} \]

This supersymmetry breaking is then gauge mediated to the GUT sector by the messengers \( \tilde{f}, f \) and the interaction to \( \Phi_{MSB} \) arising from the first part of Eq. (2.6). The usual one-loop diagram
with messengers propagating in the loop, generates Majorana mass terms for the gauginos of the GUT-sector,

\[ m_\lambda \sim b_1 \frac{g^2}{16\pi^2} \frac{\Lambda_{MSB}}{M_p} \frac{\text{tr}(F_{\Phi_{MSB}})}{m_f} \sim \frac{g^2}{16\pi^2} \frac{a_1 b_1}{a_3 b_2} \left( \frac{\Lambda_{MSB}}{M_{GUT}} \right)^2 \mu_{MSSM} . \]  

(2.28)

In the above equation \( \Lambda_{MSB} \) is a free parameter, and it can always be set such that the values of the gaugino masses are in the desired range,

\[ m_\lambda \sim 1 \text{ TeV} . \]  

(2.29)

Stability of the MSB sector requires that the messengers are non-tachyonic [16],

\[ b_1 \frac{\Lambda_{MSB}}{M_p} \mu_{MSB}^2 < m_f^2 = \left( b_2 \frac{M_{GUT}^2}{M_p} \right)^2 , \]  

(2.30)

and that tunneling to a possible supersymmetric vacuum with \( \langle \tilde{f} \rangle, \langle f \rangle \neq 0 \) is slow,

\[ \frac{b_2 M_{GUT}^2}{b_1 \Lambda_{MSB}} \gg \mu_{MSB} . \]  

(2.31)

Both conditions can be fulfilled in our model. Similarly, possible flavor changing effects caused by gravity mediation can be made small for a suitable choice of constants,

\[ m_{3/2} = \frac{\mu_{MSB}^2}{M_p} \lesssim 10^{-2} m_\lambda . \]  

(2.32)

### 2.4 Doublet-triplet splitting and proton decay

Let us return to the Higgs sector and in particular the Higgs triplets. First we should mention that the main issue with minimal \( SU(5) \) GUTs is that they can predict too rapid proton decay because of dimension-5 operators generated by terms of the form \( QQQL \) or \( U^c U^c D^c E^c \) in the effective tree-level superpotential ( [30,31], see [20,32] for a review). This question is also important for our model as we shall now see.

The Higgs triplets are made heavy by the effective operator

\[ W_3 = \kappa \frac{M_{GUT}}{M_p} H \cdot (\text{tr}(\Phi_{GUT}) + \Phi_{GUT}) \cdot \tilde{H} , \]  

(2.33)

where \( \kappa \) represents an unknown constant, and their masses are therefore of order

\[ m_{H_3,\tilde{H}_3} \approx \kappa M_{GUT}^2 / M_p . \]  

(2.34)

Note that the effective mass is proportional to \( \text{tr}(\Phi_{GUT}) + \Phi_{GUT} = 2 \text{diag}(1, 1, 1, 0, 0) \), so that the combined coupling shares some features with the Dimopoulos-Wilczek form as discussed widely in the context of \( SO(10) \) [23–27]. Indeed our model is rather more natural than standard minimal \( SU(5) \) for
precisely the same reason as $SO(10)$, namely because the meson field $M$ is not traceless. We would also argue that the requirement that the two couplings in $W_3$ be identical could conceivably be met by the underlying physics and is a less distasteful fine-tuning than that which occurs in minimal $SU(5)$.

(Nota that the renormalization of both couplings is identical in the fully supersymmetric theory.)

In order to see if one can avoid dimension-5 operators that are too large, we must consider how the spectrum influences the possible values of $M_{GUT}$. For reference we now collect the relevant mass-scales. First as we have seen $M_{GUT}$ itself is set by $\Lambda_R$ and subsequently $\mu_{MSSM}$ to lie in the range $10^{15.5} \lesssim M_{GUT} \lesssim 10^{17}$ GeV. The mass spectrum of the Higgs sector of in model is as follows. The fundamental Higgses $H$ and $\tilde{H}$ split into a doublet and a triplet parts

\[ m_{H_2, H_2} \sim \mu_{MSSM}, \]
\[ m_{H_3, \tilde{H}_3} \approx \frac{\kappa M_{GUT}^2}{M_p}. \]

At and below the GUT scale the light degrees of freedom contained in the elementary quarks $Q_R$ and $\tilde{Q}_R$ of the R-sector are naturally packaged into the composite R-meson Higgs $\Phi_{GUT}$. It contains the unrealised Goldstone bosons (eaten by the massive GUT vector bosons) as well as the weak-triplet fields $\sigma_3$, colour-octet fields $\sigma_8$ and the singlet. In our model the masses $m_{\sigma_3}$, $m_{\sigma_8}$ and $m_1$ are the same and given by

\[ m_{\sigma_3, \sigma_8, 1} \approx \frac{\Lambda_R^3}{M_{GUT}^2} \approx 2d \frac{M_{GUT}^2}{M_p}. \]

Here $H_2$ and $H_3$ denote the doublet and the triplet parts of the fundamental Higgs $H$ of the $SU(5)$. One requires $m_{H_3, \tilde{H}_3} \gtrsim 7 \times 10^{16}$ GeV in order to avoid proton decay [29]. This can be achieved with a moderately large value of $\kappa \sim 10$ and a GUT scale at the high end of the range, but the weak-triplets and colour-octets get only $F$-term masses which are significantly less than $M_{GUT}$. Indeed, recall that for the range of $M_{GUT}$ that we are considering, we have $10^{-6} \lesssim a_3d \lesssim 1$ as determined by $\mu_{MSSM}$, but $d$ itself can be kept as an essentially free parameter.

As discussed in [33–37] unification at values of $M_{GUT}$ that are greater than the canonical value of $10^{16}$ GeV are possible if $m_{\sigma_3, \sigma_8, 1} \ll M_{GUT}$ which is generically true for this model. A general analysis of the gauge-coupling RGE’s in $SU(5)$ yields two relations that we should satisfy in order to preserve unification [35–37];

\[ M_{GUT} = M_{GUT}^0 \left( \frac{M_{GUT}^0}{m_{\sigma_3, \sigma_8}} \right)^{\frac{1}{2}}, \]
\[ m_{H_3, \tilde{H}_3} = m_{H_3, \tilde{H}_3}^0 \left( \frac{m_{\sigma_3}}{m_{\sigma_8}} \right)^{\frac{5}{2}}, \]

where $M_{GUT}^0 = m_{H_3, \tilde{H}_3} = 10^{16}$ GeV are the values at the usual unification scale when one assumes a desert between $M_{SUSY}$ and $M_{GUT}$. In terms of the coupling $d$ (which appears in (2.37)) the first requirement (2.38) is rather interesting: it becomes

\[ M_{GUT} \approx \frac{(M_{GUT}^0)^{\frac{3}{2}} M_p^{\frac{1}{2}}}{d^{\frac{1}{4}}}. \]
It is a remarkable fact that in our model the determination of \( \mu_{\text{MSSM}} \) fixes \( M_{\text{GUT}} \propto (a_3 d)^{-\frac{1}{4}} \) as we have already noted in (2.22). Thus as long as we set \( a_3 \) so that we get \( m_{\sigma_3, \sigma_8} = M_{0_{\text{GUT}}} \) at the usual unification scale of \( 10^{16} \) GeV, we may treat \( d \) as an independent parameter which splits \( M_{\text{GUT}} \) and \( m_{\sigma_3, \sigma_8} \) in the right way. The required value of \( a_3 \) can be taken to be \( a_3 d \sim 1 \) for "usual" unification at \( M_{0_{\text{GUT}}} \) (note that the values are extremely sensitive to adjustments in \( M_{0_{\text{GUT}}} \) so the discussion at this point is very qualitative), and since \( m_{\sigma_3, \sigma_8} = M_{0_{\text{GUT}}} \) for this unification, we have \( d = M_p/M_{0_{\text{GUT}}} \approx 10^3 \) and hence \( a_3 \approx 10^{-3} \). We can then scale \( d \) independently and the first relation (2.38) is always satisfied. In particular \( d = 10^{-3} \) then gives \( M_{\text{GUT}} \sim 10^{17} \) GeV and \( m_{\sigma_3, \sigma_8} \sim 10^{12} \) GeV.

Unfortunately in the model presented here we have \( m_{\sigma_3} = m_{\sigma_8} \) so the second relation (2.39) requires \( m_{H_3, \bar{H}_3} = 10^{16} \) GeV. In other words making the triplets heavy enough to avoid proton decay by adjusting \( \kappa \) is incompatible with exact gauge unification for this model. It is unclear whether more precise study of this issue including for example two-loop effects would change this conclusion.

In addition, of course, if one is willing to go to product "GUT" groups, such as Pati-Salam models, or models based on flipped \( SU(5) \), then the doublet-triplet mass splitting problem can be easily avoided. In the latter case for example, the GUT symmetry is broken by VEVs of a \( 10 \) and \( \bar{10} \) rather than an adjoint Higgs, and the doublet and triplet masses are automatically split. Unfortunately in this case one would have to abandon the adjoint of \( SU(N_f) \) which arose rather nicely from the confinement of \( SU(N_c) \). Also it is unclear how \( 10 \)'s and \( \bar{10} \)'s would appear as composite fields in the superpotential of the \( R \)-sector.

Given the similarity of the coupling to the Dimopoulos-Wilczek solution to the doublet-triplet problem, a natural avenue to explore [38] in this class of models is embedding the \( SU(5) \) structure within \( SO(10) \). In fact all the main results of this paper can be straightforwardly generalised to an \( SO(10) \) Grand Unified Theory, and are not specific to the minimal \( SU(5) \). The reader is referred to [20] for further references to the doublet-triplet mass-splitting problem.

3. Discussion

We have presented an extremely compact formulation of a supersymmetric Grand Unified \( SU(5) \) theory. Our model has the following features:

Supersymmetry is broken spontaneously by a long-lived metastable vacuum state of a hidden MSB sector. This supersymmetry breaking is communicated to the GUT theory via gauge mediation and generates gaugino masses which can be made \( \sim 10^2 \div 10^3 \) GeV. Squark, slepton and higgsino mass splittings follow from this in the standard gauge mediation way.

The model is fully natural with all mass-parameters generated dynamically via the retrofitted couplings to the gluino condensate of the \( R \)-sector. In particular, by choosing the dynamical scale of the
R-sector to be $\Lambda_R \sim 10^{14}$ GeV, we generate the $\mu$-parameter of the Standard Model, $\mu_{MSSM} \sim 10^2 \div 10^3$ GeV, which in turn generates the required electro-weak symmetry breaking scale $\sim 10^2$ GeV.

Remarkably, the GUT scale $M_{GUT} \sim 10^{15} \div 10^{17}$ GeV $\gg \mu_{MSSM}$ is also dynamically generated in our model. This follows from the fact that the adjoint Higgs required in the GUT sector is identified with the traceless part of the meson matrix of the R-sector. The GUT $SU(5)$ group arises from gauging the $SU(5)$ flavour group of the R-sector, and we show that the required spontaneous breaking of $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ does occur at the scale $M_{GUT} \sim (\Lambda_R^3 M_p/d)^{1/4} \sim 10^{15} \div 10^{17}$ GeV.

Hence we have presented a simple and natural (modulo proton decay) model of susy GUT which can explain the values of the symmetry-breaking scales and their hierarchies. The model is weakly coupled and fully calculable including the soft-susy breaking terms.

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A. Bounds on additional Planck suppressed operators

In Sect. 2 we have made a certain selection among the Planck suppressed operators. In this appendix we will look at the interactions up to $1/M_p^2$ that we have neglected so far.

Let us start with the operators that involve gauge fields as well as matter fields as in (2.1). In Eq. (2.1) we have neglected three types of operators,

$$\Delta W_1 = \frac{\lambda_1}{16\pi^2 M_p^2} \left[ \text{tr} \left( W_{MSB}^2 \right) \text{tr}(Q_RQ_R) \right],$$

$$\Delta W_2 = \frac{\lambda_2}{16\pi^2 M_p^2} \left[ \text{tr} \left( W_{MSB}^2 \right) \text{tr} (X X) \right],$$

$$\Delta W_3 = \frac{\lambda_3}{16\pi^2 M_p^2} \left[ \text{tr} \left( W_{GUT}^2 \right) \text{tr} (X X) \right],$$

where the $X$ is symbolic for all possible matter fields $Q_R, Q_{MSB}, f, H$. $\Delta W_2$ and $\Delta W_3$ are harmless because $\text{tr}(W_{MSB}^2)$ and $\text{tr}(W_{GUT}^2)$ do not acquire significant vacuum expectation values. $\Delta W_1$ gives a mass of the order of $\Lambda_R^3/(16\pi^2 M_p^2) \sim \mu_{MSSM}$ to the $Q_R$ fields. However, this term has to be compared to the second term of (2.12) which also appears in the F-term for the GUT-field. Inserting the vacuum expectation value for $Q_RQ_R \sim M_{GUT}^2$ we find that $\Delta W_1$ is suppressed by a factor of $(M_{GUT}/(4\pi M_p))^2 \lesssim 10^{-5}$ compared to $W_R$. Therefore $\Delta W_1$ is harmless as well. Overall,

$$\lambda_1, \lambda_2, \lambda_3 \text{ can be } \mathcal{O}(1).$$
The second class of possible additional operators involves four matter fields as in (2.6) or (2.10) and is suppressed by one power of $1/M_p$. We have the following possibilities,

$$\Delta W_4 = \frac{\lambda_4}{M_p} \left[ \text{tr}(\tilde{Q}_R Q_R) \text{tr}(\tilde{Q}_{MSB} Q_{MSB}) \right],$$  \hspace{1cm} (A.5)

$$\Delta W_5 = \frac{\lambda_5}{M_p} \left[ \text{tr} \left( [\tilde{Q}_{MSB} Q_{MSB}]^2 \right) + c_5 \left[ \text{tr}(\tilde{Q}_{MSB} Q_{MSB}) \right]^2 \right],$$  \hspace{1cm} (A.6)

$$\Delta W_6 = \frac{\lambda_6}{M_p} \left[ \text{tr}(\tilde{Q}_{MSB} Q_{MSB}) \text{tr}(\tilde{HH}) \right],$$  \hspace{1cm} (A.7)

$$\Delta W_7 = \frac{\lambda_7}{M_p} \left[ \text{tr} \left( |\tilde{HH}|^2 \right) + c_7 \left[ \text{tr}(\tilde{HH}) \right]^2 \right],$$  \hspace{1cm} (A.8)

$$\Delta W_8 = \frac{\lambda_8}{M_p} \left[ \text{tr}(\tilde{HH} \tilde{ff}) + c_8 \text{tr}(\tilde{HH}) \text{tr}(\tilde{ff}) \right],$$  \hspace{1cm} (A.9)

$$\Delta W_9 = \frac{\lambda_9}{M_p} \left[ \text{tr} \left( |\tilde{ff}|^2 \right) + c_9 \left[ \text{tr}(\tilde{ff}) \right]^2 \right].$$  \hspace{1cm} (A.10)

Inserting the VEV $\langle \tilde{Q}_R Q_R \rangle \sim \Lambda_R M_{GUT}$ we find that $\Delta W_4$ gives an additional contribution,

$$\Delta \mu^2_{MSB} = \lambda_4 \frac{M_{GUT}^2}{M_p} \lambda_{MSB} \gtrsim 10^{11} \lambda_4 \mu^2_{MSB}.$$  \hspace{1cm} (A.11)

To keep our MSB scale at the desired\textsuperscript{3} value we therefore have to require,

$$\lambda_4 \lesssim 10^{-11}.$$  \hspace{1cm} (A.12)

Interactions of the type $\Delta W_5$ have two undesirable effects since they lead to linear terms in the potential through $F_{\Phi_{MSB}} = \mu_{MSB}^2 + \lambda_5 (\Lambda_{MSB}^2 / M_p) \Phi_{MSB} + \ldots$. This can either directly destabilise the metastable minimum or cause a shift in the messenger mass $M_f$ that, in turn again destabilizes the SUSY breaking vacuum. This leads to the constraint [16],

$$\frac{\lambda_5 \Lambda_{MSB}^2}{M_p} \lesssim \min \left[ 0.1 \mu_{MSB}, 10^{-2} \frac{b_1 M_{GUT}^2 M_p}{b_2 \Lambda_{MSB}^2} \right],$$  \hspace{1cm} (A.13)

where $\lambda_f$ and $\tilde{\lambda}_f$ are the constants of order one in front of the first and second term in Eq. (2.6). The first part of Eq. (A.13) is the more constraining and leads to

$$\lambda_5 \lesssim 10^{-2}.$$  \hspace{1cm} (A.14)

An interaction of type $\Delta W_6$ would turn the Higgs fields into messengers. At first this looks like a very nice feature. Unfortunately, it also leads to a very large mass term for the Higgs field. This mass

\textsuperscript{3}One might consider the possibility that $\Delta W_4$ gives indeed the dominant contribution to $\mu_{MSB}$. However, it turns out that the Landau pole of the MSB-sector, $\Lambda_{MSB}$, is then too close to $\mu_{MSB}$.\hfill

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comes again from the contribution to the $F_{\phi_{MSB}} = \mu_{MSB}^2 + \lambda_6 (M_{GUT}/M_p) \tilde{H} H + \ldots$. The cross terms lead to a contribution of

$$\Delta m_H^2 = 2\lambda_6 \mu_{MSB}^2 \frac{M_{GUT}}{M_p}.$$  \hfill (A.15)

For the Higgs doublet that is part of $H$ and $\tilde{H}$ the mass must be of the order of the electroweak scale and we need

$$\lambda_6 \lesssim 10^{-15}. \hfill (A.16)$$

Neither $\tilde{H}, H$ nor $f, f$ acquire any significant (bigger than the electroweak scale) expectation values. Therefore the remaining interactions $\Delta W_7, \Delta W_8$ and $\Delta W_9$ provide only additional Planck mass suppressed higher order interactions between the Higgses and the messengers. These interactions are not very constrained and

$$\lambda_7, \lambda_8, \lambda_9 \text{ can be } O(1). \hfill (A.17)$$

Overall the discussion of this appendix shows that the interactions $\Delta W_4, \Delta W_5$ and $\Delta W_6$ should be highly suppressed or, preferably, prevented by some mechanism of the underlying theory. All other terms can appear with their natural coefficients of order one.

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