A REANALYSIS OF SMALL-SCALE VELOCITY DISPERSION IN THE CIA1 SURVEY

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ABSTRACT

The velocity dispersion of galaxies on scales of \( r \sim 1 \, h^{-1} \) Mpc, \( \sigma_{12}(r) \), may be estimated from the anisotropy of the galaxy-galaxy correlation function in redshift space. We present a reanalysis of the CIA1 survey in which we correct an error in the original analysis of Davis & Peebles. We give a detailed breakdown of how the value of \( \sigma_{12}(r) \) depends on various aspects of the way in which corrections for infall into the Virgo Cluster are applied. We find that \( \sigma_{12}(r) \) is extremely sensitive to the details of these corrections. We conclude that a robust value of \( \sigma_{12} \) cannot be obtained from this survey.

Subject headings: cosmology: observations — galaxies: clusters: general — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION

Davis & Peebles (1983, hereafter DP83) calculated the velocity dispersion of galaxies, \( \sigma_{12}(r) \), on scales of \( r \sim 1 \, h^{-1} \) Mpc for the CIA1 redshift survey, a survey containing 1840 redshifts covering 1.83 sr in the north Galactic hemisphere (Huchra et al. 1983). Their result, \( \sigma_{12}(1) = 340 \pm 40 \) km s\(^{-1}\) on scales of \( 1 \, h^{-1} \) Mpc, became the standard by which \( N \)-body simulations were judged for perhaps 10 years, and a primary argument against the cold dark matter scenario for structure formation with the assumption that galaxies trace the mass fluctuations in an unbiased way, which yields much higher velocities on this scale (e.g., Davis et al. 1985; Gelb & Bertschinger 1994). The same calculation was done on the Southern Sky Redshift Survey (SSRS1; da Costa et al. 1991), with results of \( \sigma_{12}(1) \sim 300 \) km s\(^{-1}\) (Davis 1988), in apparent agreement with the CIA1 result. It is only relatively recently that there have been attempts to reproduce the results of DP83 (Mo, Jing, & Borner 1993; Zurek et al. 1993, 1994) and to perform this analysis on new, larger redshift surveys (Fisher et al. 1994a, 1994b; Marzke et al. 1995; Guzzo et al. 1995). It is now apparent that there is a large variation in \( \sigma_{12} \) between different surveys (see Table 1), as anticipated by the results of Mo et al. (1993).

Mo et al. (1993) calculated \( \sigma_{12} \) on various samples of galaxies drawn from the CIA1, SSRS1, IRAS, and CIA slice redshift surveys. Zurek et al. (1993, 1994) also reanalyzed the CIA1 redshift survey in an attempt to reproduce the results of DP83 and, in addition, calculated the same statistic on mock redshift surveys drawn from \( N \)-body simulations of a cold dark matter model. The present paper is a confirmation and an extension of these studies. The numerical values of \( \sigma_{12} \) obtained by the two groups for CIA1 differed significantly from each other and from DP83. This is in keeping with the conclusions reached by both groups and by this paper: the \( \sigma_{12} \) statistic is extremely sensitive to the presence of rich clusters in the sample and to corrections for infall toward the Virgo Cluster. In addition, both groups stated that they obtain values much higher than DP83 for CIA1 unless they remove the Virgo Cluster from the sample or remove the peculiar velocity component due to infall into Virgo.

Despite this, the \( \sigma_{12}(1) = 340 \) km s\(^{-1}\) value continues to be quoted without any discussion of these issues. With this paper, we hope to clarify the source(s) of the discrepancy and remove the element of speculation that was, of necessity, present in the previous papers. Because this work was done with access to the computer code used in the original calculation, we are able to clarify some details of the calculation of DP83 that are crucial to understanding how the quoted value was obtained. There are several different corrections that were applied to the data in DP83, and we show exactly how the different corrections (even the order in which they are applied) affect the results. In addition, we discuss the effects of an error that we discovered in the computer program used to obtain the results in DP83, which partially explains the discrepant results. We hope that this will help to lay any confusion or controversy to rest.

We also reproduce the earlier results for SSRS1 (Davis 1988; Mo et al. 1993) and investigate how removing clusters affects \( \sigma_{12} \). The effects of removing clusters are investigated further in a separate paper using mock catalogs extracted from \( N \)-body simulations (Somerville, Primack, & Nolthenius 1997).

2. METHOD

In this section, we describe briefly the method used to extract the pairwise velocity dispersion \( \sigma_{12} \) from the redshift-space correlation function \( \xi(r_p, \pi) \). Readers should refer to DP83 and Fisher et al. (1994a, 1994b) for more details.

The correlation function in redshift space, \( \xi(r_p, \pi) \), is estimated by counting the number of pairs in a bin in \( r_p \) (separation perpendicular to the line of sight) and \( \pi \) (separation parallel to the line of sight). It is normalized by constructing a catalog of Poisson-distributed points with the same selection function and angular limits as the data, and by counting pairs between the data and the Poisson catalog:

\[
1 + \xi(r_p, \pi) = \frac{n_D \, DD(r_p, \pi)}{n_R \, DR(r_p, \pi)},
\]

where \( DD \) is the number of pairs between data and data, and \( DR \) is the number of pairs between the data catalog and a Poisson catalog. The quantities \( n_R \) and \( n_D \) are the minimum variance-weighted densities (see Davis & Huchra 1982) of the Poisson and data catalogs, respectively.
Let $F(w|v)$ be the distribution function of velocity differences $w$ for pairs of galaxies with vector separation $r$, and let $f(w_3|r)$ be the velocity distribution function averaged over the directions perpendicular to the line of sight. The first moment of $F(w|v)$, $\bar{v}_{13}(r)$, is the mean streaming velocity relative to the Hubble flow, and from isotropy it must be a function of the magnitude of $r$ only. Correspondingly, the first moment of $f(w_3|r)$ is $\langle w_3 \rangle = y \bar{v}_{13}(r)/r$, where $y$ is the component of $r$ along the line of sight.

A reasonable form for the distribution function $f(w_3|r)$, parameterized by its moments, is

$$f(w_3|r) \propto \exp \left\{ -\frac{\|w_3(r) - \langle w_3(r) \rangle\|^2}{\sigma_{13}(r)} \right\}.$$  

(2)

It has been found empirically from studying observations and N-body simulations (Peebles 1976; Fisher et al. 1994b; Zurek et al. 1994; Marzke et al. 1995) that on small scales, an exponential form ($n = 1$) fits the data better than a Gaussian ($n = 2$) or any higher power of the argument. Adopting this form for $f(w_3|r)$, and using $r^2 = r_p^2 + y^2$ and $w_3 = r - y$, we have

$$1 + \xi(r_p, \pi) = \frac{H_o}{\sqrt{2}} \int \frac{dy}{\sigma_{13}(r)} \left[ 1 + \xi(r) \right]$$

$$\times \exp \left\{ -\frac{\|w_3 - \langle w_3 \rangle\|^2}{\sigma_{13}(r)} \right\}.$$  

(3)

An approximation based on self-similar solutions of the BBGKY hierarchy suggests a form for $\bar{v}_{13}(r)$ (Davis & Peebles 1977):

$$\bar{v}_{13}(r) = \frac{F H_o r}{[1 + (r/r_0)^2]},$$  

(4)

where $F$ is an adjustable parameter of the model. The assumption of stable clustering (that the collapse of the cluster is exactly balanced by the Hubble flow) leads to $F = 1$. DP83 investigated different values of $F$, but the usually quoted result is for $F = 1$. The velocity dispersion $\sigma_{13}(r)$ is obtained by fitting the model of equation (3) to $\xi(r_p, \pi)$ as estimated from equation (1).

3. REANALYSIS OF CfA1

In the CfA1 survey, the Virgo Cluster dominates the foreground of the sample. At the time when the analysis of DP83 was done, an accepted approach to correcting for the infall of our Galaxy and other galaxies into Virgo was to assume a spherical infall model. In this model, the infall velocity of any galaxy was taken to be inversely proportional to the galaxy’s distance from Virgo, and the model was scaled so that the infall of the Local Group had the then favored value of 440 km s$^{-1}$ (currently favored values are 200–300 km s$^{-1}$). Therefore, in DP83 the redshift of each galaxy was corrected both for the infall of the Local Group and for its own infall into Virgo. Each redshift was also corrected for Galactic rotation using a rotation velocity of 220 km s$^{-1}$. We will subsequently refer to the correction to each galaxy’s distance due to the infall of the Local Group toward Virgo (plus the Galactic rotation correction) as the “dipole” correction, and the correction due to each galaxy’s own infall into Virgo (plus Galactic rotation and the dipole correction) as the “inverse Virgo distance” or $1/r_V$ correction.

There are two stages of the procedure in which these corrections may be applied. First, there is the distance used in calculating an absolute magnitude, used to create a semivolume-limited catalog by eliminating all galaxies with absolute magnitude $M_B > -18.5 + 5 \log h$. Second, there is the velocity used in calculating the correlation function $\xi(r_p, \pi)$. The choice of which corrections are to be used in each of these quantities is important, as we shall see.

It could be argued that infall corrections should only be applied to the velocities used as distances for the volume limiting. The method we use here to extract $\sigma_{13}$ relies on measuring the distortion of $\bar{v}_{13}(r)$ due to peculiar velocities, so in general one would not want to use corrected velocities in the pair counting to compute $\xi(r_p, \pi)$. However, in the CfA1 sample, the shear due to the presence of the Virgo Cluster in the foreground is so large that it could be argued that some correction is needed, even in the velocities used in pair counting.

Because the inverse Virgo distance model is singular near Virgo, DP83 applied the infall corrections differently, depending on which region of the survey a galaxy was in. We define the “inner Virgo core” as all galaxies with $\theta < 6^\circ$ and $v < 2500$ km s$^{-1}$, where $\theta_p$ is the angle with respect to the center of the Virgo Cluster ($\alpha_p = 12^h 24^m 48^s$, $\delta_p = 12^\circ 67^\prime$) and $v$ is the redshift. We define the “outer core” as galaxies with $6^\circ < \theta_p < 14^\circ$ and $v < 2500$. We define the “field” as all the remaining galaxies. Note that the determination of whether a galaxy is assigned to the Virgo core depends on which of the velocities (uncorrected, dipole corrected, or
inverse Virgo distance corrected) is used for the velocity condition. DP83 used the inverse Virgo-corrected velocity.

Table 2 defines various permutations of "cuts," or ways of applying the different corrections in the different regions of the survey. The entry cut 1 shows the choices necessary to reproduce the original results of DP83. This reflects our discovery, in the course of this reanalysis, that the inner core of Virgo was inadvertently deleted in the original analysis because of a typographical error in the computer program used to obtain the results presented in DP83. The entry cut 2 is what DP83 intended to do: the mean velocity of the Virgo Cluster (\(\langle v_{\text{virgo}} \rangle = 1460 \text{ km s}^{-1}\)) is assigned to all galaxies in the inner core for the volume limiting; the dipole-corrected velocities are used in pair counting in the inner core, and in both the volume limiting and pair counting in the outer core; and the inverse Virgo distance-corrected velocities are used in the field. Note that this value of \(\langle v_{\text{virgo}} \rangle\) is the measured mean redshift of Virgo, 1020 km s\(^{-1}\), plus the dipole correction for the infall of the Local Group, 440 km s\(^{-1}\). Cut 3 is the same as cut 2, except that the inverse Virgo distance correction is not used in the pair counting. Cut 4 uses the mean Virgo redshift for the volume limiting in the inner core, but it uses only the dipole correction in all the other regions. This is to enable us to disentangle the effect of using the mean Virgo velocity in the inner core from the effect of the inverse Virgo distance correction.

We now know that a simple spherical infall model is not a very good model for the flow field around the Virgo Cluster. Even in the relatively low mass Virgo Cluster favored by the Faber-Burstein Great Attractor flow model, the redshift-distance relation is triple valued within 20° of Virgo (Nolthenius 1993). In order to study the effects of using the spherical infall model, cut 5 and cut 6 include only the dipole correction in all the regions of the survey. Cut 5 includes the correction in the pair counting, whereas cut 6 does not. Finally, we show the result obtained if no corrections at all are applied to the redshifts at any stage in the procedure. We also show the result obtained when no corrections are applied and all clusters with internal velocity dispersion greater than 500 km s\(^{-1}\) have been removed using an automated cluster-removing program (see Somerville et al. 1997).

Figure 1 shows the results for \(\sigma_{1,2}(r)\) for each of the cuts. As can be seen, the results are very sensitive to the way in which the cuts are applied. Note that the scale dependence of \(\sigma_{1,2}\) with \(r\) also changes significantly depending on the way the corrections are applied. In all cases, the slope is steeper than the original result of DP83. One can under-

### Table 2

**Different Ways of Applying Infall Corrections**

| Cut             | Volume Limit | Pair Count | \(\sigma_{1,2}(1)\) (km s\(^{-1}\)) | \(N_{\text{gal}}\) | \(N_{\text{virgo}}\) |
|-----------------|--------------|------------|-----------------------------------|---------------------|---------------------|
| Cut 1:          |              |            |                                   |                     |                     |
| Inner core ......| Deleted       | Deleted    | 346 ± 98                          | 1235                | 0                   |
| Outer core ......| Dipole       | Dipole     |                                   |                     |                     |
| Field           | \(1/r_{V}\)  | \(1/r_{V}\) |                                   |                     |                     |
| Cut 2:          |              |            |                                   |                     |                     |
| Inner core ......| \(\langle v_{\text{virgo}} \rangle\) | Dipole     | 453 ± 118                         | 1268                | 33                  |
| Outer core ......| Dipole       | Dipole     |                                   |                     |                     |
| Field           | \(1/r_{V}\)  | \(1/r_{V}\) |                                   |                     |                     |
| Cut 3:          |              |            |                                   |                     |                     |
| Inner core ......| \(\langle v_{\text{virgo}} \rangle\) | Dipole     | 666 ± 189                         | 1268                | 33                  |
| Outer core ......| Dipole       | Dipole     |                                   |                     |                     |
| Field           | \(1/r_{V}\)  | Dipole     |                                   |                     |                     |
| Cut 4:          |              |            |                                   |                     |                     |
| Inner core ......| \(\langle v_{\text{virgo}} \rangle\) | Dipole     | 737 ± 229                         | 1195                | 46                  |
| Outer core ......| Dipole       | Dipole     |                                   |                     |                     |
| Field           | Dipole       | Dipole     |                                   |                     |                     |
| Cut 5           | Dipole       | Dipole     | 646 ± 184                         | 1191                | 42                  |
| Cut 6           | Dipole       | None       | 867 ± 233                         | 1191                | 42                  |
| No corrections  | None         | None       | 618 ± 113                         | 1021                | 29                  |
| Clusters removed| None         | None       | 406 ± 85                          | 972                 | 0                   |

Note.—\(N_{\text{gal}}\) is the number of galaxies included in the volume-limited catalog. \(N_{\text{virgo}}\) is the number of galaxies in the Virgo core included in the volume-limited catalog. See the text for definition of the labels.
stand these results qualitatively as follows. In cut 6, the dipole correction assigns larger distances to all the galaxies, especially those in the Virgo core. This makes the galaxies seem more luminous, and more of them make it into the volume-limited catalog. In cut 6, 42 galaxies from the Virgo core are included in the volume-limited catalog, as compared with 29 when no corrections are used. The galaxies in the Virgo core have large pairwise velocities, and \( \sigma_{12} \) is pair-weighted, so this increases \( \sigma_{12} \) considerably. Cut 5 consists of the same galaxies as cut 6, but using the dipole-corrected velocities to calculate \( \xi(r_p, \pi) \) reduces the velocity dispersion of the Virgo Cluster and hence the measured \( \sigma_{12} \) for the sample. The differences in \( \sigma_{12} \) for cut 3, cut 4, and cut 5 are not significant—the correlation functions appear very similar, and it is only because the correlation functions are very noisy at large \( \pi \) that different values of \( \sigma_{12} \) are obtained. The values are within the (large) formal errors on the fit. This implies that using the mean Virgo redshift for the galaxies in the inner core of Virgo did not have a large effect on the results. In cut 2, galaxies with \( v > 2500 \) km s\(^{-1} \), the cutoff for the Virgo core, are assigned larger velocities because of the \( 1/r_p \) correction. This whole band of galaxies is therefore moved farther away from the Virgo core, reducing the number of pairs containing Virgo galaxies that fall in the \( 1 \) \( h^{-1} \) bin, and reducing \( \sigma_{12}(1) \). Finally, not surprisingly, removing the Virgo core altogether reduces \( \sigma_{12} \) significantly, as can be seen in cut 1.

Our analysis of the SSRS1 survey yields \( \sigma_{12}(1) \approx 323 \pm 91 \) km s\(^{-1} \) (SSRS1), which is in agreement with previous results (Davis 1988; Mo et al. 1993). The SSRS1 survey does not contain any rich foreground clusters, and therefore the original analysis was not complicated by any infall corrections. Unlike CfA1, when we remove the clusters from SSRS1 using the cluster-removal routine, the value of \( \sigma_{12}(1) \) does not change significantly (see Fig. 2). This suggests that \( \sigma_{12} \) for CfA1 is dominated by the Virgo Cluster and that the SSRS1 value is more typical of the field. However, the analysis of existing redshift surveys does not yet allow a definite conclusion. Removing clusters appears to reduce sample-to-sample variation in \( \sigma_{12} \) (see Table 3), but work on simulations (Somerville et al. 1997) suggests that this will also reduce the ability of the statistic to discriminate between different cosmological models. In addition, this work suggests that the results are likely to be sensitive to the details of how the clusters are identified and removed.

4. CONCLUSIONS

We hope that this paper has removed any confusion regarding the value of the velocity dispersion in the CfA1 survey. By performing an extensive reanalysis of the CfA1 data following DP83, with reference to the original code used for that calculation, we have identified the reasons why previous authors obtained substantially different results for this statistic. In agreement with previous studies (Mo et al. 1993; Zurek et al. 1994), we have shown that the value of the velocity dispersion is extremely sensitive to the way in which corrections for infall into the Virgo Cluster are applied. This is because the Virgo Cluster is in the foreground of the CfA1 survey and contains many intrinsically faint galaxies in a thermally hot region. Increasing the distance to these galaxies by a small amount results in the inclusion of fewer of these galaxies in the volume-limited sample. Because \( \sigma_{12} \) is pair-weighted, including or leaving out even \( \sim 10 \) galaxies from the Virgo Cluster can change \( \sigma_{12} \) by \( \sim 100–200 \) km s\(^{-1} \). In addition, including corrections for cluster infall in the calculation of the correlation function in redshift space, \( \xi(r_p, \pi) \), effectively removes part of the “finger of god” and reduces the velocity dispersion of the cluster. Once again, the pair-weighted nature of the statistic means that this will result in a significant reduction in the overall value of \( \sigma_{12} \) for the sample.

We have shown how the sensitivity of the results to these details can explain the different values for \( \sigma_{12} \) reported by other authors. The inadvertent deletion of the Virgo core in the analysis of DP83, due to an error in the computer program, also partially explains the discrepancy.

However, no infall corrections were used in recent calculations of \( \sigma_{12} \) for other redshift surveys, and yet a wide range of values for \( \sigma_{12} \) has been obtained for different surveys. We have argued, in agreement with other authors, that this is because \( \sigma_{12} \) is extremely sensitive to clusters, and...
existing redshift surveys do not sample a large enough volume of space to represent a fair sample of these relatively rare objects. One approach to solving this problem is to remove the clusters from the sample before calculating $\sigma_{12}$; however, this reduces the ability of the statistic to discriminate between cosmological models (Somerville et al. 1997). Analysis of larger volume redshift surveys will be necessary in order to obtain a robust value of $\sigma_{12}$ that is useful in discriminating between cosmological models or for estimating $\Omega_0$. In the meantime, modified velocity statistics such as the galaxy-weighted velocity dispersion (Miller, Davis, & White 1996), a density-dependent version of the pairwise velocity dispersion (Kepner, Summers, & Strauss 1996), the median velocity of groups (Nolthenius, Klypin, & Primack 1997), or other statistics designed to be less sensitive to clusters may be promising alternatives.

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