Abstract. The origin of anomalous electron thermal turbulence from spatial gradients in magnetized plasmas is described. Laboratory experiments demonstrating key features of drift waves are reviewed. The turbulent electromagnetic fields produce an anomalous transport that scales with both the gradient parameters and microscopic plasma scale length parameters. The change from the micro-scale dominated gyro-Bohm to the macro-scale dominated Bohm scaling laws is discussed. The close correlations between the electron turbulent transport theory and the confinement properties measured in the steady state hot electron plasmas produced in tokamak devices are presented.
1. Introduction

Plasma occurs in states of turbulence under a wide range of conditions including space and astrophysical plasmas as well as those produced in magnetohydrodynamic MHD stable laboratory confinement devices. The strength of the turbulence increases as the plasma is driven farther away from thermodynamic equilibrium. While there are many ways to drive the plasma away from equilibrium with particle beams, laser beams and radio frequency (RF) waves, a universally occurring departure from equilibrium is the existence of spatial gradients across an ambient magnetic field. The problem posed is then a classic one of determining the fluxes of particles, energy and momentum across an ambient magnetic field due to gradients in thermodynamic variables of density $n_a$, temperature $T_a$ and flow velocity $u_a$.

Plasma distributions that are driven away from the thermodynamic equilibrium of a spatially uniform Maxwell–Boltzmann velocity distribution with densities $n_a$ and temperatures $T_a$ for the charge particles species $(e_a, m_a)$ are said to have a free energy density $W_f$ available to drive plasma turbulence. For the first free energy example, we estimate the energy density associated with the relative cross-field drift velocity $u$ between the ions and the electrons driven by the pressure gradient. The cross-field drift is required to maintain force balance in the non-uniform magnetized plasma with $j \times B = \nabla p$ when $j_\perp = \sum a e_a n_a u_\perp a$ and $n_e = \sum Z_i n_i$ and $p = \sum_i p_i + p_e$. The free energy density is $W_f = \frac{1}{2} \sum a m_a n_a u_\perp a^2$. The condition for the onset of instability is determined by linear stability analysis which specifies the relations between the system parameters $\rho_a/L_{Ta}$, $T_e/T_i$, $m_i/m_e$ for various forms (different modes or branches of $\tilde{E}_j \tilde{E}_j(k) = 0$) of $q$ unstable plasma waves. Here $\rho_a = c(m_a T_a)^{1/2}/e_a B$ is the thermal gyroradius. The nature of the nonlinear saturated state depends on how far into the unstable domain the system parameters reside which typically varies with space and time as the plasma turbulence reacts on the plasma distributions to push the system back toward one of the marginally stable states. The turbulence provides a mechanism for self-organization toward a relaxed dynamical state often containing a mixture of waves, vortices and zonal flows. The best high temperature, steady state plasma confinement experiments are produced in tokamaks with large temperature...
gradients from the core plasma that is often more than 20 million kelvin surrounded by a room temperature wall no more than one metre away. These high temperature gradients drive two well known types of temperature gradient instability called the ITG and ETG modes for ion or electron temperature gradient instabilities. We will concentrate in this work on recent theoretical and experimental research on the electron temperature gradient driven turbulent transport. While much is known about these turbulence mechanisms there is still a need for more detailed formulae to make reliable predictions for operational regimes of future large tokamaks. Thus, we review the recent results on electron transport for the purpose of showing more clearly the areas that need further investigations to make predictions for the next generation of large tokamaks. Figure 1 shows a schematic diagram of the temperature gradient that operates a Carnot engine through a plasma convection to liberate energy $W$ over a correlation length $l_c$. The plasma simulations of turbulence show that there are many vortices and wave structures that carry out this transport of plasma energy across the magnetic field much faster than one would calculate from collisional transport mechanisms in the plasma. This rapid transport of thermal energy is what has prevented the last generation of tokamaks from reaching the burning plasma state when they are operating with deuterium and tritium fuels. We now define the correlation functions, and correlation times and lengths needed to discuss plasma turbulence and show a simple example of their measurement in a controlled steady state laboratory plasma.

For complex structured signals $\varphi(x, t)$, typical of what is meant by the term turbulence, the standard measure of the coherence of the signals is the two-point, two-time correlation function

$$C_{12}(x_1, t_1, x_2, t_2) = \langle \varphi(x_1, t_1)\varphi(x_2, t_2) \rangle$$

where the average $\langle \rangle$ is the time average taken over a time period $T$ that contains many oscillations of the associated field. The correlation functions for the plasma electric potential $\varphi$ and the fluctuations of the electron density $\delta n_e = n_e(x, t) - \langle n_e \rangle$ are the principal structure functions measured in laboratory plasma turbulence research. The dependence of $C_{12}$ on $r = x_1$ and $\tau = t_2 - t_1$ is strong, decaying to small values for large $r, \tau$ and weak on $R = (r_1 + r_2)/2$ and $t = (t_1 + t_2)/2$ for typical turbulent systems. In the physics idealization of homogeneous,
stationary turbulence $C_{12} \rightarrow C_{12}(r, \tau)$ independent of $R$ and $t$. It is evident that $C_{12}(r, \tau)$ is a maximum at $r = 0$, $\tau = 0$ where $C_{12}(0, 0) = \langle \varphi^2 \rangle$. The space and time separation where $C_{12}$ falls to low values for large $|r| \geq \ell_c$ or $|\tau| \geq \tau_c$ defines the correlation length $\ell_c$. These properties provide the definition of the correlation distances $\Delta x_c$, $\Delta y_c$, $\Delta z_c$ and correlation time $\tau_c$ by choosing a critical point for the fall-off of $C_{12}/C_{12}(0, 0)$ often taken as the 1/e-point. Often a theoretically more useful definition of the correlation length $\Delta x_L$ or time $\tau_c$ is the integral scale length such that $\Delta x_L C_{12}(0, 0) = \int_{-\infty}^{\infty} dx C_{12}(x, \tau)$ and $\tau_c C_{12}(\tau = 0) = \int_0^\infty C_{12}(\tau) d\tau$. Typically the subscripts on $C_{12}$ are dropped and the particular fields used in the correlation functions are implied.

A clear example of the plasma turbulence correlation function for the electrostatic potential $\varphi(x, t)$ turbulence is shown in figure 2 from the experiment of [97]. In figure 2 frame (a) the decay of the two-point correlation function of separation $\Delta x$ of the two-space points along the direction of the plasma current $j = -eux$ driving the turbulence is shown for all other separations ($\Delta x = \Delta y = \tau = 0$) equal to zero. In figure 2 frame (b) the constant level contours of $C_{12}$ of the correlation function for separations in $\Delta z$ along the current and in an orthogonal direction $\Delta y$ are shown. Note that the central maximum (approaching 40 units) occurs at the $\Delta y = \Delta z = 0$ point and the correlation length ($\Delta y_c \approx 6$ mm) perpendicular to the current is longer than the correlation parallel to the current $\Delta z = 4$ mm. This anisotropy of the turbulence is a characteristic of plasmas due to the preferred directions of unstable wave propagation associated with the geometry and the magnetic fields in the plasma systems.

For broad-band turbulence there is a simple rule for estimating the correlation time $\tau_c$ and correlation lengths $\Delta r_c$ from the spectral width of the power spectrum $I(k\omega)$ defined by the Fourier transform of the correlation function $C_{12}(r, \tau)$. The correlation time is given by $\tau_c = 1/\Delta\omega$ and the correlation length $\Delta z_c = 1/\Delta k_z$ where $\Delta\omega = \omega_{\text{max}} - \omega_{\text{min}}$ and $\Delta k_z = k_{\text{max}} - k_{\text{min}}$ are the widths of the highest levels of the fluctuation power spectrum $I(k, \omega)$. As a example, the ion acoustic turbulence in the Stenzel experiment is shown in figure 3 where a spectrum consistent with turbulence theory has been reported. The spectral width from the 1/e point is approximately 400 kHz, consistent with the directly measured $\tau_c = 2\mu s$ correlation time. Physically, the correlation time $\tau_c$ is the maximum time interval over which the field, in this case the electrostatic potential, maintains a given structure. In the next correlation time the complexity, or structure, of the field is qualitatively different.

In section 2 the case of plasma turbulence produced by the spatial gradients from its confinement is described. The confinement or trapping of a plasma is produced both in the laboratory and space/astrophysics by magnetic fields that cause the charged particles to gyrate with radius $\rho_a = m_a v_a/q_a B$ around the local magnetic field $B(x)$. The confinement along $B$ occurs either due to the large increase of $|B|$ giving rise to the magnetic mirror effect as in Earth’s magnetosphere or due to the field lines forming closed, nested toroidal surfaces as in solar current loops and the laboratory tokamak device. From over 30 years of laboratory research in the tokamak confinement studies of plasma there is a detailed understanding of the intrinsic, irreducible plasma turbulence that develops from spatial gradients. This turbulence is generically called drift wave turbulence and is driven by the cross-field gradients of the plasma density $\vec{\nabla} n = -(n/L_n) \hat{e}_z$ and temperature $\nabla T = -(T/L_T) \hat{e}_z$. While there are many detailed forms known for the turbulence depending on the plasma parameters, there are three generically distinct types of plasma cross-field diffusivity $D$ (m$^2$ s$^{-1}$) of particles and $\chi$ (m$^2$ s$^{-1}$) for thermal diffusivity. The three functionally distinct forms are

1. the Bohm diffusivity $D_B = \alpha B (T_c/e B)$,
Figure 2. Characterization of the turbulent electrostatic potential by the two-point correlation function $C_{12}$ in equation (1). (a) Decay of the correlation function with separation of the two points along an axial line parallel to the current; (b) oscillatory decay of the correlations for two points in the $y$–$z$ plane where the $\hat{y}$ direction is perpendicular to the current and $\hat{z}$ is the parallel direction (courtesy of Stenzel).

(2) the gyro-Bohm diffusivity from drift waves $D_{dw} = \alpha_{dw}(\rho_i/L_T)(T_e/eB)$ where $\rho_i = (m_iT_i)^{1/2}/eB$ is the thermal ion gyroradius and

(3) the collisional turbulence diffusivities $D_r = \alpha_{rg}v_e\rho_e^2(L_s^2/L_T R_e)$ where $v_e$ is the electron–ion collision frequency associated with resistivity $E_\parallel = \eta j_\parallel$ along the magnetic field.

The Bohm diffusivity varies as $T/B$ and is documented for the Joint European Torus (JET) in [105, 24]. A modified form, called Taroni–Bohm, of the Bohm scaling of transport has been widely accepted as a standard empirical transport formula for the two large tokamaks JET and JT-60U for many years. The modification is required to get the dependence on the plasma current produced magnetic field to dominate over the externally applied magnetic field. The numerical coefficient $\alpha_B$ is a low coefficient $\alpha_B \sim 1/200$. In contrast, the drift wave transport formulae vary as $T^{3/2}/B^2L$ and are documented in theory, e.g., [45] and gyro-kinetic simulations with a coefficient $\alpha_{dw} \sim 0.3$ [15]. The third functional form of the diffusivity is that arising from resistive interchange instabilities where $D_r$ varies as $\chi_{rg} \sim n/T_e^{1/2}B^2$ and has a coefficient $\alpha_{rg}$ of order unity [30, 92, 87, 107]. The coefficients $\alpha_B$, $\alpha_{dw}$ and $\alpha_{rg}$ are weak functions of many detailed plasma parameters such as $T_e/T_i$, $L_T/R$, $\beta = 2\mu_0p/B^2$ and more. Just as in neutral fluid turbulence, there are many degrees of freedom excited in plasma turbulence and there is difficulty in determining the details of these formulae either through theory or numerical simulations. Nonetheless, the years of experience with the tokamak confinement programme have led to rather firm general conclusions about the turbulent diffusivities. The most successful
approach to modelling the transport has been called the multi-mode approach, where all relevant instabilities are assessed at each set of plasma system parameters. Now we give some details of the spatial gradient driven turbulence in magnetized plasma.

2. Spatial gradient driven turbulence in magnetized plasma

In nonuniform, magnetized plasmas the ion acoustic waves are modified into two branches with different parallel phase velocities due to the presence of the diamagnetic currents \( j_a = e_a n_a u_{da} \) required from the \( j_a \times B = \nabla p_a \) force balance. Here, each charged particle species is designated by the subscript \( a \). The relevant drift velocities \( u_{da} \) for driving the plasma turbulence are the small diamagnetic drift velocities \( u_{da} = T_a / e_a B L_{pa} = (\rho_a / L_{pa}) v_{Ta} \) where \( L_{pa}^{-1} = -\partial_x \ln p_a(x) \).

Even for small values of \( \rho_a / L_{pa} \) these diamagnetic currents drive low-frequency \((\omega \ll e_a B / m_a)\) waves with \( k \) almost parallel to \( B \times \nabla p_a \) unstable. These waves are called drift waves and their effect is to produce a cross-field transport of particle, energy and momentum through the turbulent \( E \times B \) drifts.

2.1. Drift waves in the laboratory

The collisional drift waves with growth rates determined by the electrical resistivity \( \eta \) and thermal diffusivity \( \chi_e \) were the first drift waves to be discovered in [9] and thoroughly investigated in [35]. The identification was made in low temperature steady state plasmas produced by thermal (contact) ionization of alkali elements (principally caesium and potassium) in long cylindrical devices with closely spaced Helmholtz coils. Correlations between the observed potential-density waves with the properties predicted by the linear dispersion relation and the single-wave finite amplitude formulae [36] were used to establish that the radially localized, 10 kHz rotating wave structures were the drift waves. The dimensionless density \( \tilde{n} / n \) and potential \( \tilde{\phi} / T_e \) waves are approximately equal amplitude sinusoidal oscillations with \( \tilde{n} \) leading \( \tilde{\phi} \) by \( 30^\circ \)–45° in phase. Figure 4 shows the drift wave potential and density iso-lines. Vortex dynamics has also been
observed in the plasmas produced in these devices, called $Q$-machines in the [35] experiment. Here $Q$ stands for quiet, meaning that the plasma fluctuations are not as intense and broad band as in the toroidal devices of that period. In the experiments of Pécsei et al [77, 78], externally excited vortices of like signs were shown to coalesce into one vortex. Vortices of opposite signs were reported to interact with each other forming a dipole vortex pair.

A variety of drift-type instabilities relevant to toroidal magnetic fusion devices, including the trapped electron (TE) modes by Prager et al [82], the trapped ion instability by Slough et al [95] and the collisionless curvature driven trapped particle mode by Scarmozzino et al [91] have been produced and identified in the Columbia Linear Machine (CLM).

The drift wave driven by the radial ion temperature gradient in a collisionless cylindrical plasma was demonstrated in the modified CLM by Sen et al [93] by using biased wire screens to create a $T_{\parallel}(r)$ gradient sufficient to excite an $m = 2, 10$ kHz (in the plasma frame) drift wave oscillation. The toroidal ITG mode driven by the magnetic curvature was also produced and identified by Chen and Sen [10] in the same machine.

Drift waves were found in the transient plasmas produced in the multipole confinement machines that were both linear and toroidal devices with strongly varying $B$-fields from parallel conductors carrying large currents from external power supplies. The theory for the drift waves in the multipole takes into account the localization of the unstable oscillations to regions of unfavourable gradient-$B$ and curvature particle drifts and the shear in the helical $B(x)$-field [74]. These experiments provided further evidence for the universal appearance of drift waves in confinement geometries. The correlations of drift wave theory with the multipole and spherator experiments are described in the review article [43], section 3.3. The main result to be noted here is that the experiments show that increasing the magnetic shear reduces the fluctuation amplitudes [75]. The multipole devices are unique in being able to continuously vary the magnetic shear parameter strength from zero to the order of unity. Even with the strongest magnetic shear, however, the fluctuations were not eliminated.

The magnetic shear plays a central role in the linear and nonlinear theory of the cross-field transport consistent with the role of shear on the fluctuations measured in these experiments. In recent theory and experiments for tokamak confinement devices the combined roles of
$E_r \times B$ sheared flows and magnetic shear are known to produce enhanced confinement regimes [7, 29, 103]. The improved confinement occurs over narrow radial regions giving rise to new confinement regimes with internal transport barriers [63, 67, 98]. The principal tools available for producing these changes in transport are the control of the drift wave turbulence in the system parameters through the programming of the plasma current to control the magnetic shear in $B(x)$ and the programming of the neutral beam injectors to control the mass flow shear in the plasma flow velocity $u(x)$. The timing of the auxiliary power and the momentum injection with respect to the ohmic transformer waveform is used to determine the plasma regime created.

In tokamaks the identification of drift waves in the core plasma came from the microwave scattering experiments [71] and infrared CO$_2$ laser scattering experiments [101, 102]. These measured fluctuations were explained in the context of drift waves existing at the mixing-length level of saturation [52] taking into account the response of the TEs in the drift-wave dissipation. Subsequently, many experiments around the world have observed the universal appearance of a broad band of drift wave fluctuations with $\omega/2\pi \simeq 50$–500 kHz at $k_\perp = 1$–15 cm$^{-1}$ in toroidal confinement devices for both the tokamak and helical-stellarator systems. Many fluctuation and transport studies in toroidal confinement facilities around the world, including TFTR, Alcator, Tore Supra (TS), TEXT, ATF, Heliotron, JFT2M and ASDEX were undertaken in the 1980s and 1990s that have referred to these initial findings of drift wave turbulence and the associated radial transport. ASDEX Upgrade experiments have drawn attention to the role of a critical electron temperature gradient in the electron power balance analysis [89, 90]. In the present work we concentrate on the recent electron transport experiments on TS where long time steady state conditions are achieved with a well known power deposition profile and electron temperature profile. About 90\% of the power goes through the electrons in the transport zone between the core and the edge plasma. This makes the system particularly simple compared to those regimes where the ions and electrons share the power transport in some complex manner. In addition there have been detailed microwave scattering experiments with polarized beams in these experiments which leads to knowledge of the electron density fluctuations and a less accurate but useful measurement of the magnetic fluctuations due to a change in the polarization vector of the scattered microwave beams.

### 2.2. Conditions for transport and propagation of disturbances

Now we analyse the motion of charged particles in the $E \times B$ convection. For the small, localized excess of ion charge shown in figure 4 the convection

$$v_E = \frac{cE \times B}{B^2}$$

(2)

rotates plasma clockwise around the potential maximum $\varphi > 0$ which is also the density and electron pressure maximum in the adiabatic response. The motion is clockwise when viewed down the magnetic field line $B$. Now, if the ambient plasma is uniform ($\partial_x n_a = \partial_x T_i = 0$) across the convection zone then the cell rotates without plasma transport. When the plasma has an $x$-gradient of density (pressure), however, there is a rapid transport of the structure along the symmetry direction $\hat{y}$ with a small diffusive transport across an $x = \text{constant}$ surface. The speed of the localized structure in figure 4 along the symmetry direction is approximately the electron diamagnetic drift speed $v_{de} \equiv cT_e/eBL_n$ where $L_n^{-1} = -\partial_r \ln N$. The analytical description of
the net convective flux particle and thermal fluxes across a given surface $S$ is given by

$$
\Gamma_e = \frac{1}{S} \int_S n_a v_E \cdot d\alpha = -D_{11} \frac{dn_e}{dx} - D_{12} \frac{dT_e}{dx},
$$

(3)

$$
q_e = \frac{3}{2S} \int_S n_a T_a v_E \cdot d\alpha = -D_{21} \frac{dn_e}{dx} - n_e D_{22} \frac{dT_e}{dx}.
$$

(4)

In the absence of the phase shift $\psi_{\delta n, \varphi} = 0$ in figure 4, the particle transport vanishes. For different phase shifts the off-diagonal terms in equations (3) and (4) can add to or subtract from the diagonal terms. The situation is shown in figure 4 where the heat flux is shown versus the electron temperature gradient. In section 3 we show that the electron power balance studies show that the off-diagonal term produces an inward heat flux contribution that gives a critical gradient above which the heat flux rises to a high level. It should be understood that there are also collisional transport processes that would contribute to $\Gamma_e$ and $q_e$ but that they are in generally smaller by one to two orders of magnitude than the turbulent fluxes [94]. The collisional fluxes also exhibit off-diagonal structures [4].

In the proper set of flux-driving gradient variables the transport matrix has Onsager symmetry [94] (Sugama et al [99]). The matrix is positive definite with $D_{11}D_{22} - D_{12}D_{21} > 0$. The positive definiteness of the matrices relating conjugate pairs of driving forces and transport fluxes is developed in detail by Sugama in a series of works on turbulent transport. In typical particle and power balance experimental studies of steady state discharges the particle and thermal fluxes $\Gamma$ and $q$ are determined from the input source of particles and energy and the resulting measured profiles of $n$, $T$ to infer the required diffusivities $D_{ij}$. For different signs of $D_{12}$ there are three types of thermal flux versus temperature gradient relation that occur in plasmas. Type I shows a process of transport that starts above a critical gradient, in type II processes the flux vanishes when the gradient vanishes and in type III the flux is finite when the gradient vanishes. The type III flux occurs when the turbulence is driven by other gradients such as the ion temperature or electron density gradient for ITG or TEM driven fluctuations. Figure 5 shows these relationships schematically. The type I flux–gradient relation applies to ETG turbulent transport where one of the linear theoretical formulae gives the critical gradient.

Consider the simple case shown in figure 4. For the positive electric potential structure in figure 4 the clockwise $E \times B$ rotation brings higher density $N_\perp$ (and higher pressure $N_\perp T_e$)
plasma to the right and lower density $N_\text{e}$ (pressure) to the left resulting in a shift of the maximum density and potential, linked through the electron response by $\delta n_\text{e} \approx n_\text{e}(e\varphi/T_\text{e})$, to the right. The speed of the translation is proportional to the gradient of the density $L_n^{-1} = -\partial_i \ln N$ and inversely proportional to the strength of the magnetic field $B$. The speed also increases with electron temperature $T_\text{e}$ since the potential fluctuation $e\varphi$ scales up with $T_\text{e}$. For a negative potential structure the $E \times B$ rotation is anticlockwise, but the structure moves to the right with same speed (in the limit of small $e\varphi/T_\text{e}$) since now lower density plasma is brought to the right shifting the minimum in that direction. Now, the ion density at this location builds up in the time $\delta t$ equal to that of the original electron maximum $\delta n_\text{e} = N(e\varphi/T_\text{e})$ when the condition

$$\delta n_i = -\frac{\delta t c\varphi}{B\delta y} \frac{\partial N}{\partial x} = N \frac{e\varphi}{T_\text{e}}$$

is satisfied. In the last step we use quasi-neutrality taking $\delta n_i = \delta n_\text{e} = N(e\varphi/T_\text{e})$ which is valid for fluctuations that are large compared to the Debye length. During the time $\delta t$ the convection moves the maximum of the structure to the right by $\delta y = v_{de}\delta t$ where

$$v_{de} = \frac{\delta y}{\delta t} = -\frac{cT_\text{e}}{eBN} \frac{\partial N}{\partial x}.$$  

The $x$-displacement of the plasma during this motion is $\xi_x = v_x\delta t = -\delta t \varphi/B\delta y$. When this displacement becomes comparable to $\delta x$ the motion is nonlinear leading to the formation of nonlinear vortex structures. Locally, the plasma is mixed over the length $\delta x$ in one rotation period when the amplitude reaches the mixing length level $\xi_x = \int dt' v_{Ex} = \delta x = l_e$. The nonlinear problem is treated in *Chaos and Structures in Nonlinear Plasmas* by Horton and Ichikawa [48].

2.3. Drift wave diffusivities and the ion inertial scale length

It is conventional in the study of drift waves and transport to introduce gradient scale lengths and reference diffusivities. Thus, the length $L_n$ is defined as the density gradient scale length through the relation $1/L_n = -\partial_i \ln N$. The temperature gradient scale length $L_T$ is defined similarly. The spacetime scales of the waves lead to two different dimensional scalings for the plasma diffusivities. The reference diffusivities are the Bohm diffusivity

$$D_B = \frac{T_\text{e}}{eB},$$

and the drift wave diffusivity

$$D_{dw} = \left(\frac{\rho_s}{L_n}\right) \left(\frac{T_\text{e}}{eB}\right).$$

also commonly called the gyro-Bohm diffusivity in reference to the factor $\rho_s/L_n \ll 1$. Here $\rho_s = (m_iT_\text{e})^{1/2}/eB$ is the effective gyro-radius parameter for hot electrons of $T_\text{e} \gtrsim T_i$. Clearly, the scalings of the Bohm and gyro-Bohm diffusivities are markedly different with $D_B \propto T_e/B$ independent of the system size while $D_{dw} = T_{e3/2}/B^2L$ decreasing with the system size. In short, the Bohm (7) scaling arises from mesoscale drift wave structures $\Delta x = (\rho_s, L_T)^{1/2}$ and thus is expected near marginal stability [28, 62, 69, 104]. With a minimal model of ITG, Ottaviani and Manfredi [76] investigate the $\rho_s$ scaling for a constant thermal flux through a turbulent annulus. They observe an inverse cascade to larger elliptical vortices but find that the flux
Table 1. Plasma drift wave parameters.

| Machine         | TFTR       | TS        |
|-----------------|------------|-----------|
| Magnetic field (T) | 4.8        | 2.7       |
| Major/minor radii (m) | 2.45/0.80 | 2.30/0.75 |
| Electron temperature (keV) | 6         | 5         |
| Density $n_e$ (cm$^{-3}$) | $4.0 \times 10^{13}$ | $3.6 \times 10^{13}$ |
| Gradient length $L_n$ (cm) | 20        | 20        |
| Drift velocity $v_d$ (cm$^{-1}$) | $3 \times 10^5$ | $1 \times 10^5$ |
| $k$ scattering experiment (cm$^{-1}$) | 1–20       | 1.5–15    |
| $\omega$ scattering experiment (kHz) | 10–500     | 10–1000   |
| Fluctuations    | 0.01 to 0.1|           |

scaling remains gyro-Bohm. When the convective cell size reduces to scale as $\Delta x = r_s$ the drift wave diffusivity (8), commonly called gyro-Bohm, applies.

In defining the dimensionless gyro-radius parameter $\rho_s$, it is usual to replace the spacetime varying length $L_n$ with the relatively constant value $a$ of the plasma minor radius. Thus, a key issue is the scaling of plasma confinement with [79, 108]

$$\rho^* \equiv \frac{\rho_s}{a}.$$  (9)

Drift wave theory is able to account for confinement scaling either as $D_B$ or $\rho_s D_B$. Transport dependent on $\rho_s$ depends on the average mass $m_i$ of the working gas ions since $\rho_s = (m_i T_e)^{1/2}/eB$. The transition between the Bohm and the gyro-Bohm scaling is a difficult problem, both theoretically and experimentally, that has received recent attention. Simulations by Furnish et al [26] give one picture of the transition and those of Lin et al [69] give the results of even larger and higher resolution simulations. Both authors report that there is a transition at certain values of $\rho_s$ which measures the ratio of the micro-scale size to the global size of the system. The role of large scale computing in settling such issues is made clear by Lin et al [69]. On the experimental side of the scaling problem, the scaling studies of Perkins et al [79] and Petty et al [80] (Erba et al [22]) present evidence for the Bohm-like scaling of the turbulent transport. More recent power balance studies in the JET discharge up to 7 MA of plasma current use a model that adds the Bohm and gyro-Bohm contributions. This is called the JETTO model and is now widely used in transport predictions. Roughly, the JETTO model is obtained with

$$\chi_e = \alpha_q a^2 (a/L_p)D_B$$

and

$$\chi_i = \alpha_i \chi_e + \chi_{neo}$$

with $\alpha_q = 2.1 \times 10^{-4}$ and $\alpha_i = 3.0$ (Erba et al [24]). Here $1/q(r) = RB_0/rB_T$ gives the local pitch of the helical magnetic sideline.

The relevant system parameters for TS and the large tokamak fusion test reactor (TFTR) are given in table 1. The fluctuation measurement at wavenumbers $k_\perp \lesssim 1$ cm$^{-1}$ requires the techniques of reflectometry [20, 72] and the indirect method of beam emission spectroscopy as in the experiment [21]. Finally, it is important to point out parallels with other areas of physics. The closest and most important parallel to plasma drift waves is the analogy with the Rossby waves and vortices in geophysical atmospheric and oceanographic disturbances with periods long compared to the rotational period of the planet. Hasegawa and Mima [32] and Hasegawa et al [31] develop the limit in which the two models become isomorphic. The correspondence is due to the Coriolis force having the same mathematical form as the Lorentz force. The analogy was also recognized by [81], which led to the first rotating parabolic water tank experiments by Antipov et
al [1, 2] in Kurchatov, and Antinova [3] in Tbilisi. This aspect of the drift wave–Rossby problem is found in the article [46] in the special issue of Chaos devoted to such geophysical vortex structures. A recent high resolution simulation of geophysical vortex turbulence is in [109].

2.4. Resistive drift wave and interchange turbulence

The collisional drift wave is a paradigm for anomalous transport that has been extensively investigated with many different modellings. A particularly simple 2D model, called the [34] model, with an adiabaticity parameter α has been investigated by Wakatani and Hasegawa [107], Krommes and Hu [65], Sugama et al [100], Gang et al [27], Koniges et al [64], Biskamp et al [6] and Wakatani [115]. To understand the origin of the simple α-model and to appreciate its limits we briefly present the 3D resistive drift model. For finite resistivity η = meνe/ne2, the parallel current carried by the electrons in equation (10) yields

\[ j_\parallel = - (n_e e^2 / m_e v_e) \nabla (\varphi - \frac{T_e}{n_e} \ln n) \]

using the isothermal approximation δp_e = T_e δn_e. The collisional drift wave equation follows from the divergence of the current \( \nabla \cdot j = 0 \) with the divergence of the polarization current \( j_p \), balancing the divergence of \( j_\parallel \) through

\[ \nabla \cdot j_p = - \nabla \cdot j_\parallel = \eta^{-1} \nabla^2 (\varphi - \frac{T_e}{n_e} \ln n) \]

and the electron continuity equation. In other words the dynamical equation for the field aligned vorticity \( \zeta = b \cdot \nabla \times v_E = c \nabla_\perp^2 \varphi / B \) is given by the conservation of charge in the quasi-neutrality limit. The rotational part of the plasma momentum for the vorticity \( \nabla^2 \varphi \) is equivalent to the current closure equation. The vorticity equation and the electron continuity equation give, in dimensional form,

\[
\frac{m_i n_c}{B_0} \frac{d}{dt} \nabla^2 \varphi = \frac{B_0}{c} \nabla j_\parallel + \hat{z} \cdot \nabla p_e \times \nabla \Omega, \tag{10}
\]

\[
\frac{d}{dt} (n_0 + n_1) = \frac{e}{c} T_e n_0 \nabla \cdot \left( \frac{n_i}{n_0} - \frac{e \varphi}{T_e} \right) \times \nabla \Omega, \tag{11}
\]

where \( \nabla \Omega \) is the effective g-force used to relate the curvature and gradient-B effects to the classical Rayleigh–Taylor instability. The computation of \( \Omega(r) \) for the average curvature of the magnetic field line is extensively used in stellarator/heliotron research [8]. The derivatives on the left-hand sides of equations (10) and (11) are the \( E \times B \) convective derivatives defined by \( df/dt = \partial_t f + v_E \cdot \nabla f \).

The coupled vorticity and density equations (10) and (11) have a conserved potential vorticity \( \zeta \) given by

\[
\zeta = \frac{m_i c^2}{e B^2} \nabla_\perp^2 \varphi - \ln n_0 - \frac{n_1}{n_0} - \Omega, \tag{12}
\]

which generalizes the conserved vorticity \( \nabla_\perp^2 \varphi \) in a 2D Euler fluid. It is useful to first consider the dimensionless form of the model equations (10) and (11) in global coordinates before using the local drift wave units \( \rho_s \) and \( L_n / c_s \). Using the minor radius \( a \) for the cross-field \( B_0 \hat{z} \) dimensions, the major radius \( R \) for the dimensionless \( z / R \rightarrow z \) and time in units \( \omega_{ci} t (\rho_s / a)^2 \rightarrow \tilde{t} \) (equivalent to \( (c T_e / e B a^2) \tilde{t} \rightarrow \tilde{t} \)) one finds that the natural amplitude variables are \( e \varphi / T_e = \varphi \) and \( n_1 / n_0 = n \), and the dimensionless parameters of the model are \( \epsilon = a / R \) and \( \rho = \rho_s / a \), \( v = v_e / \omega_{ce} \).

The dimensionless model is then

\[
\rho^2 \frac{d}{dt} \nabla_\perp^2 \varphi = \frac{e^2}{v} \nabla_\parallel^2 (n - \varphi) - g \frac{\partial n}{\partial y} + \mu \nabla^2 \varphi, \tag{13}
\]

\[
\frac{dn}{dt} = \frac{e^2}{v} \nabla_\parallel^2 (n - \varphi) + \partial_x \ln n_0 \frac{\partial \varphi}{\partial y} - g \frac{\partial}{\partial y} (n - \varphi) + D \nabla^2 n, \tag{14}
\]
where \( g = \frac{d\Omega}{dr} \). This 3D model has resistive drift waves driven by the density gradient \((\partial_s n_0)^2\) through the charge separation from finite \( k_2^2 \rho_s^2 \) and the resistive interchange driven modes from \( \omega_D \) through the charge separation from finite \( k_2^2 \rho_s^2 \) and the resistive interchange driven modes from \( \omega_D = \frac{(c_b T/eB)(d\Omega/dr)}{d_t} \) is the averaged grad-B/curvature drift frequency. The linear eigenmodes are of two types: localized to the rational surfaces where \( k_\parallel = 0 \) and global modes \([41, 100]\).

The electric potential has the important property of developing an \( m = 0/n = 0 \) component with a well defined circular null surface. This \( \varphi_{0,0}(r, t) = 0 \) surface partially blocks the turbulent losses from the core of the cylindrical model. For stellarators the \( m = 1, n = 1 \) rational surface is near the edge of the plasma and the dominant modes in this simulation are \( m = 3/n = 2 \) and \( m = 2/n = 1 \) fluctuations and the \( m = 0/n = 0 \) background profile for \( v_\theta = -c E_r/B \).

These simulations with \( v_e/\omega_{ce} = 1.4 \times 10^{-4} \) are too collisional to apply to the edge of tokamaks with \( I > 1 \text{ MA} \) confinement devices (where \( v_e/\omega_{ce} \lesssim 10^{-6} \)).

Wakatani et al \([106]\) extend the investigation of the model \((13), (14)\) to include an externally imposed electric field \( E_r(r) \) exceeding the strength of self-consistently generated field from the \( m = 0/n = 0 \) modes. The \( E_r < 0 \) field suppresses the turbulence during the growth phases, but produces only a weak reduction of the flux in the saturated state. The collisionality dependence of the particle flux is shown to increase with \( v_e/\omega_{ce} < 10^{-3} \) and then to increase as \( v_e^{1/3} < 10^{-3} \). The numerical treatment of the stabilizing role of sheared flows is subtle in that the problem of resolving the low \( k \) modes giving fluctuating sheared flow requires a high density of small \( k_\parallel \) modes. Hallatschek and Biskamp \([30]\) have carried out convergence studies and conclude that often the role of shear flow damping of the turbulence is over-estimated since the \( L_x \) box size is not taken large enough to have adequate resolution of the low \( k_\parallel \) condensation of turbulent energy. With resistive interchange turbulence Hallatschek carries out high resolution simulations and finds a condition for sufficient density of the low \( k \) modes. The nature of this turbulence interaction with the shear flow is investigated for toroidal ITG modes by Li and Kishimoto \([68]\). These authors confirm earlier theoretical studies that show there is a bursting or intermittent nature to the shear generation through the turbulent Reynolds stress. Thus, the \( \chi_0 \) and the level of the turbulence generated component of the sheared flows undergo relaxation oscillations controlled by the strength of the instability and the magnitude of the shear flow damping. This generation of zonal flow behaviour persists but is much weaker for the short scale electron temperature gradient driven turbulence.

In the widely investigated 2D model of the Hasegawa–Wakatani equations \((13), (14)\) the operator \( \nabla^2_\parallel \to -k_\parallel^2 \) or \( -1/L_x^2 \), where \( k_\parallel \) is the relevant mean parallel wavenumber and \( L_x \) is the connection length to the divertor end plates in the scrape-off layer (open field lines) modelling. For the interior tokamak field lines this reduction seriously limits the applicability due to losing the information on the closeness of the helical pitch of the magnetic field to the twists of the fluctuations following the toroidal direction. This resonance of the field pitch to that of the fluctuations is a key player in numerous effects including

1. the condensation of the turbulence to large scale zonal flows described above and
2. the response of the density as adiabatic or MHD-fluid-like.

For 2D turbulence models, the parameter \( \alpha = k_\parallel^2 T_e/m_e v_e \omega_0 \) measures the parallel electron diffusion in a characteristic wave period \((1/\omega_0)\).

The spacetime units are changed to the local scales of \( \rho_s \) and \( L_n/c_s \) in these 2D studies.
The standard form of the Hasegawa–Wakatani 2D model is then

\[
\frac{d}{dt} (\nabla^2 \phi) = \alpha (\phi - n) + \mu \nabla^4 \phi, \tag{15}
\]

\[
\frac{dn}{dt} = -\kappa \frac{\partial \phi}{\partial y} + \alpha (\phi - n) + D \nabla^2 n, \tag{16}
\]

where the viscosity \(\mu\) and \(D\) are taken small, but finite to absorb all fluctuation energy reaching the smallest resolved space scales in the simulation system. The system’s strong turbulence features at small \(\alpha\) with \(\alpha/\omega \sim 1\) where \(k, \omega, \gamma\) are taken at the peak of the energy spectrum. Here the over-bars on \(k, \omega, \gamma\) denote a mean value near the peak of the energy spectrum \(E_k\). One can show that \(\bar{k} \approx \alpha^{1/3}, \bar{\gamma} \approx \alpha^{1/3}\) and that \(E_k \approx \bar{\gamma}^2/\bar{k}^3 \approx 1/\alpha^{1/3}\) (Hu et al [117]). In the large \(\alpha\) limit the density \(n \to \phi(1 + O(1/\alpha))\) approaches the adiabatic limit and a weaker turbulence appears with \(E_k \approx \bar{\gamma} \bar{\omega}/\bar{k}^3 \approx 1/\alpha\) since \(\bar{\gamma} \sim 1/\alpha\), and \(\bar{k} = \alpha^0\) and \(\bar{\omega} = \alpha^0\) independent of \(\alpha\). These \(\alpha\)-scalings in the small \(\alpha\) and large \(\alpha\) regimes have been verified by direct numerical simulation and the statistical closure method.

In the quasi-2D equations (15) and (16), the new parameter \(\alpha = k^2 T_e/m_e v_e \omega_0\) determines the properties of the waves. For \(\alpha \gtrsim 1\) the electrons tend to the Boltzmann distribution \(\tilde{n} = e\phi/T_e\) and the Hasegawa–Mima equation is recovered. The Hasegawa–Mima equation is isomorphic with the Rossby wave equation for high Rossby number geostrophic flow in the mid-latitudes [109]. There is a strong dual cascade in this equation with anisotropy in the north–south or radial direction. The anisotropy leads to zonal flows.

Plasma turbulence appears as ubiquitous as plasma itself. In space, solar and astrophysical plasmas show many varied forms of plasma turbulence ranging from large scale MHD turbulence [5] to the smallest Debye length scale Langmuir turbulence. For example, the magnetic energy released during solar flares heats and accelerates the plasma in the solar corona. Electrons on the open coronal magnetic field lines caused by large scale MHD reconnection events stream at relativistic speeds into the interplanetary plasma. The electrons beams drive Langmuir turbulence creating intermittent bursts of radio noise known as type III radio sources. The peculiar intermittency of Langmuir turbulence called nonlinear wave collapse was first described in [112]. The phenomenon of nonlinear wave collapse is reviewed by [86] for a wide range of laboratory and space physics settings. The general theoretical analysis of wave turbulence for plasmas and neutral fluids is given in [113].

### 2.5. Short wavelength drift wave turbulence

Short wavelength fluctuations \(k \rho_i \gg 1\) with finite electron inertia are driven unstable by the electron temperature gradient. The modes are electron analogues of the better studied ion temperature gradient modes often called ITG modes. Their properties are developed in [66, 47]. The short wavelength turbulence produces the electron thermal diffusivity given by

\[
\chi_e = 0.3 \left( \frac{r}{R} \right)^{1/2} \frac{v_e}{R} \frac{c^2}{\omega_{pe}^2},
\]

which explains the widely observed \(\tau_E \propto n_E a^2 R\) scaling of the energy confinement time. Recently, there have been many simulation studies of the short wavelength turbulence to understand the coupling to the electromagnetic fluctuations that produce coherent structures on the scale of the collisionless skin depth \(c/\omega_{pe}\). In typical tokamaks this length is of order
a few millimetres and is smaller than the standard drift-wave turbulence that is on the scale of several centimetres. The reason the smaller scale turbulence can compete in its ability to transport plasma is that the correlation times are smaller since the modes involve electron dynamics. Studies of the electron transport in a spectrum of ETG electromagnetic waves shows the stochastization of the guiding centre orbits and the rapid transport of the electron thermal energy [61]. Recent self-consistent field simulations include Idomura et al [55], Jenko et al [57, 58], Dorland et al [19] and Li and Kishimoto [68]. It is found that, unlike in the analogous case of ITG turbulence, the turbulent electron heat flux significantly exceeds the simple mixing length estimate, using the scale length that maximizes the growth rate. The mechanism is identified as the formation of highly elongated radial vortices (‘streamers’), instead of zonal flows as in the case of ITG, when the perturbations develop nonlinearly. The streamers lead to very effective cross-field transport while the zonal flows reduce it. This results in the discussion on the differences between ITG and ETG turbulence. The electromagnetic secondary instabilities in ETG turbulence are investigated in recent theoretical work of Holland and Diamond [40]. The possibilities of magnetic secondary instabilities (zonal magnetic fields and magnetic streamers) are studied as novel potential mechanisms for electron transport regulation and enhancement, respectively. A crucial issue raised in these works is that of pattern selection for both ITG and ETG turbulence, that is, whether zonal modes or streamers are preferentially generated. At this time, these issues are unsolved and remain open challenges to the magnetic fusion community.

In the efforts to understand the differences and possible correlation between the short wavelength \( (k \rho_e \sim 1) \) and the intermediate wavelength \( (k \rho_i \sim 1) \) instabilities and turbulence, the instabilities of the continuous wavelength spectrum from the short to the intermediate are studied by Smolyakov et al [96] (Hirose and Elia [37]). The unstable modes of \( k_z \rho_{i,e} > 1 \) are identified as short wavelength ITG and ETG modes, respectively. In contrast to the conventional ITG and ETG modes, the new modes require both ion and electron temperature gradient higher than certain thresholds. In addition, the short wavelength modes in toroidal geometry require a minimum magnetic shear as a driving force. It is claimed that the short wavelength ITG mode driven turbulences may induce higher electron thermal transport than the ETG turbulence does and, therefore, are responsible for the anomalous electron thermal transport experimentally observed.

The dependence of the critical temperature gradient on other plasma parameters such as temperature ratio \( (T_e/T_i) \), magnetic shear and safety factor for the toroidal ETG instability is studied and formulae are given by Jenko et al [57] and Dong et al [17, 18]. An interesting point from Dong et al [16] is that the critical electron temperature gradient increases from \( R/L_{Te} \sim 3 \) to \( \sim 10 \) dramatically when the temperature ratio \( T_e/T_i \) increases from 1/3 to 3. This is in great favour of \( \alpha \) particle heated burning plasmas if it is verified by future high \( T_e/T_i \) experiments. In addition, a brief estimate for ASDEX Upgrade and TS experiments [38, 89, 90] on the critical gradient is given and compared with the results from solving the integral eigenvalue problem for the ETG modes. The theoretical results are in the range of the experimental observations.

The electron thermal transport experiments on eight tokamak devices (ASDEX Upgrade, COMPASS-D, FT-U, JET, TCV, TS, RTP and AUG) are summarized by [89]. The critical gradients, above which the measured electron thermal diffusivity and the calculated growth rates of drift instabilities increase dramatically, are identified as \( R/L_{Te} \sim 8–12 \) that falls right into the range of calculated critical electron temperature gradient for toroidal ETG instabilities by Dong et al [17].
Tokamak fusion test reactor (TFTR) discharges with high core temperatures $T_e \lesssim 8$ keV, $T_i \lesssim 25$ keV) from the improved confinement regime (enhanced reversed shear) and high neutral beam heating power (28 MW) have small scale fluctuations at $k_\perp \approx 0.85 \omega_{pe}/c \sim 5 \rho_i^{-1} \approx 9$ cm$^{-1}$ [110]. These electron density fluctuations $\langle \delta n_e^2 \rangle_k$ are measured by scattering a microwave beam with $|\Delta k| = k_\perp = 8.9$ cm$^{-1}$ from the core plasma continuously in time. Power balance studies are then performed to determine the electron thermal diffusivity $\chi_e(r, t)$ required to give the measured $n_e, T_e(r, t)$ profiles from the fraction of the beam power deposited into the electrons. The resulting $\chi_e(r, t)$ is shown to track the fluctuation level over a period of one second while $\chi_e$ varies from 0.5 to 4 m$^2$ s$^{-1}$.

A related instability based on the electron inertia in the nonlinear Ohm’s law and a single pressure field driving interchange instability in the unfavourable magnetic curvature is called the current diffusive ballooning mode. Yagi and Horton [111] develop the properties of this turbulence, estimating the thermal diffusivity as

$$\chi = f(s) \frac{c^2 q^2 v_A}{\omega_{pe}^2 R} \left(-R \frac{d\beta}{dr}\right)^{3/2},$$

where $f(s)$ is a complicated function of magnetic shear $s$ obtained from the ballooning mode calculation of $\langle k_x^2 \rangle$. Evaluation of the current diffusive $\chi$, in the form given by Fukuyama et al [25], is compared with a standard ITG transport model for a high beta poloidal JT-60U discharge in [50]. The importance of the electron transport at the $c/\omega_{pe}$ scale has been pointed out and investigated by many authors: Ohkawa [73], Kadomtsev and Pogutse [59], Lee et al [66], Horton et al [45, 47, 51], Connor [13], Itoh et al [56], Fukuyama et al [25], Idomura et al [55], Hirose and Elia [37], Holland and Diamond [40], Dong et al [17, 18] and Horton [42]. It is the authors’ view that these fluctuations are the standard mechanism, albeit independently understood, for electron thermal transport. If this is indeed the case, then the electron fluctuations may be responsible for holding the electron temperature down in discharges where the ion confinement is improved dramatically. Low electron temperatures in the large D–T fusion experiments are one of the main reasons that those experiments fell short of expectations. Due to the importance of the electron turbulent transport, we discuss the TS experiments in the next section. TS has optimal plasma conditions for the study of turbulent electron transport.

3. Tore Supra hot electron plasma transport data analysis

Tokamak discharges with core electron heating dominating ion heating provide valuable models for the transport regimes in a burning fusion reactor where alpha particle slowing down through electron collisions is the dominant heating power $P_{ae}$ (MW m$^{-3}$) for thermal plasmas. The long time steady state discharges produced in TS with dominant, core localized electron heating provide a unique opportunity for the study of electron transport under conditions similar to burning plasmas in the fusion reactor of a tokamak confinement system. In general, RF heating systems in TS provide flexibility of driving up the plasma temperature and of controlling plasma current profiles. The fast wave electron heating (FWEH) provides high performance discharges with the largest increase of the core electron temperature over the ohmic temperature $T_{e,OH}$, compared with the alternative ion cyclotron resonant heating (ICRH) and the lower hybrid (LH) wave heating mechanisms. FWEH shows the longest plasma energy confinement times $\tau_E$ [39]. The universal feature of toroidal confinement arising from temperature gradient driven turbulence controlling the diffusivities is that the total electron stored energy $W_e$ and the global energy...
confinement time $\tau_E$ show a strong degradation as the total input heating power $P$, taken as the sum of ohmic and injected RF powers, increases. This behaviour is shown in the standard empirical L-mode scaling law for the global energy confinement time as a function of the system parameters. A large international database supports the standard L-mode laws of tokamak operation. Improved confinement is then measured by defining the $H$-factor through the ratio $H = \tau_E / \tau_L$. Here $H$ stands for ‘high confinement’ and the $H$-factor is the ratio of the improved $\tau_E$ to the standard $\tau_L$. By varying current profile and peaking density profile tokamaks can achieve $H$-factor from 1.6 to 2.0 and even higher for special advanced operational scenarios. For TS with FWEH, the discharge parameters range over $2 \text{ MW} \leq P \leq 10 \text{ MW}$ with $0.4 \text{ MA} \leq I_p \leq 0.9 \text{ MA}$ at two values of the toroidal magnetic field $B_\phi = 2.0$ and 2.8 T. The scaling results show that an improved confinement factor $H \simeq 1.6$ is obtained when the global energy confinement time is compared with the ITER97-L-mode [60] thermal energy confinement scaling law. The improved confinement arises from controlling magnetic shear through RF heating. The current profiles that produce low central magnetic shear and high outer confinement zone ($\rho = r/a > 0.6$) shear produced the largest $H$-factors. The ETG thermal diffusivity $\chi_e$ formula, based on the existence of critical electron temperature gradient, is shown to be consistent with the power balance $\chi_{PB}$ when $\nabla T_e$ exceeds the critical value $(\nabla T_e)_{\text{crit}}$, by a factor of two or more [38]. There is clear evidence in both the power balance $\chi_e$ and the measured density and magnetic fluctuations for a critical electron temperature gradient of about 3 keV m$^{-1}$ in the FWEH database. The critical gradient is observed to increase with magnetic shear and be independent of the magnetic field, that is consistent with what is known from electromagnetic drift wave theory [47, 51]. The electromagnetic drift wave turbulence theory successfully interprets the high power FWEH TS database of more than 40 well documented discharges. The working gas is typically helium and the plasma pressure satisfies $m_i \beta_i / m_e \approx 40$ at the mid-radius $r = a/2$. Thus, $v_A \ll v_e$ and the drift wave is electromagnetic, with an associated $\delta B_x / B \sim 10^{-5}$ fluctuation.

### 3.1. Electron transport theory

The existence of a broad band of drift-type fluctuations in TS is documented by the laser scattering experiments [14]. The long wavelength end of the scattering measures the region of the ITG-TE spectrum while the short wavelength end measures the ETG type of turbulence. If one assumes that the modes are electrostatic in nature, then the stability analysis of Ross et al [88] applies. One finds that the modes are unstable for both zero electron temperature gradient and for a finite electron temperature gradient. There is a strong increase of the short wavelength electrostatic growth rate with $\eta_e$ that leads those authors to state that for $\eta_e > 2/3$ the short wavelength modes are strongly unstable. They search for stabilization by including the ion–ion collisions.

In contrast, the ion temperature gradient is the dominant controlling parameter for the long wavelengths as is easily seen in the work of Rewoldt and Tang [85]. In that work the growth rate is shown to have a substantial value down to zero temperature gradient ($\eta_i = \eta_e = 0$) for the long wavelength modes. The Rewoldt and Tang stability analysis is historically interesting in that it shows clearly the improved confinement properties of the proposed high field burning plasma experiment called the Compact Ignition Tokamak or CIT for short. This same theme has emerged again and a new more advanced high field compact ignition tokamak is shown to lead to ignition in Hu et al [53]. Rewoldt and his collaborators show how the growth rates and ratios of the particle and thermal fluxes vary for realistic tokamak models. Due to the renewed interest in the compact high field ignition experiments a timely work to revisit is [85] in which the growth
rates were worked out for the CIT tokamak. In this work they used MHD equilibria coupled with BALDUR transport values of $\eta_i$ and $\eta_e$ to study the collision and the $\eta_i$ dependences of the complete matrix eigenvalue problem for coupled ITG and TE modes. They clearly show that the growth rates are greatly suppressed for the high density regime of a compact ignition tokamak in their figures 1 and 2. For high densities, where the bounce frequency of TEs is lower than the collision frequency, the growth rate threshold appears in $\eta_i$ as in the classical adiabatic electron ITG theory. When the density is lowered to that typical of the standard 5 T field tokamaks like TFTR and JET, the drift waves remain unstable with a substantial growth rate even at zero values of the $\eta_i$ parameter due the density gradient. This is due to the wave changing to rotate in the electron diamagnetic direction and being destabilized with the TE resonant wave interactions. The ballooning mode eigen-functions are reported along with the ratios of the particle flux and the electron heat flux divided by the larger ion thermal flux. By taking these ratios of the turbulent fluxes the uncertainty in the amplitude of the fluctuations is reduced although not entirely eliminated due to the nonlinear shift of the spectrum to wavelengths longer than those that maximize the growth rate. The subject was studied again by Dong et al [16]. In toroidal collisionless high temperature plasmas, ITG and TE modes are shown to be weakly (strongly) coupled when both the temperature gradients and the driving mechanism of the TE are moderate to strong (weak but finite). In the regime of strong coupling, there is an single hybrid mode unstable for all ITG in plasmas with positive magnetic shear. In the weak coupling case, two independent unstable modes, one in the ion and the other in the electron diamagnetic direction, are found to coexist. In either situation, a negative magnetic shear exerts a strong stabilizing influence; the stabilizing effect is considerably enhanced by the presence of trapped particles. It is predicted that for plasmas of given parameters, it will be much hard to simultaneously excite the two modes in a toroidal magnetic field with negative shear.

In view of recent short wavelength theory and simulations, it is clear that these small scale modes are a key mechanism for producing the universally observed anomalous electron thermal losses in tokamaks that were found from the beginning of tokamak history (Kadomtsev [116]). We analyse the turbulent electron heat loss in TS under the hypothesis that the short wavelength electromagnetic fluctuations arise from the well known mechanisms of the temperature gradient driven toroidal drift wave instabilities [47, 51]. The overview of the drift wave fluctuation spectrum given in figure 6 of Horton et al [47] is evaluated here in detail for TS to show the multiple spacetime scales. Four important wavelength scales shown along the $x$-axis in that figure are computed for a typical Tore Supra plasma (TS shot no 19542). Now we investigate the properties of these fluctuations in Tore Supra. Two of the characteristic cross-field wavelengths are given by the wavenumbers $k^w_\parallel$ that maximize the linear theory growth rates $\gamma_{k^w_\parallel, ITG}^{ITG}$ and $\gamma_{k^w_\parallel, ETG}^{ETG}$ for the ion and electron temperature gradient driven instabilities respectively. The third major scale length corresponds to the wavelengths that mark the transition from the short scale $\nabla T_i$ and $\nabla T_e$ directly driven turbulence that is well described within the framework of quasi-two-dimensional plasma turbulence to the longer wavelength regime where the three dimensionality of the turbulence dominates. The 2D to 3D transitional cross-field scale length is given by the conditions $\omega_{\parallel, ITG} = k^w_\parallel v_i$ for ITG and by $\omega_{\parallel, ETG} = k^w_\parallel v_e$ for ETG where $k^w_\parallel = 1/q R$ is fixed by the toroidal geometry and $q = r B_T / R B_\theta$. The inverse cascade of the quasi-two-dimensional system is arrested at this $k^w_\parallel$-scale since the fluctuations become intrinsically three dimensional at this and larger cross-field scale lengths. The FLR-fluid simulations [45] show that the $k^w_\parallel$ wavenumber spectrum changes shape, developing a flat local maximum at this transitional scale where $k^w_\parallel \sim \ell_c^{-1}$. Physically, it is clear that the nonlinear dynamics of fluctuations changes character when the time to propagate

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the ion or electron acoustic wave around the torus is shorter than the corresponding drift wave period. The nonlinear matrix elements for the mode coupling become dominated by \( k_\parallel^2 \) rather than \( k_\perp^2 \) when the turbulence scale satisfies \( k_\perp < \ell_c^{-1} \). Evidently, high \( q \) extends the range of the quasi-two-dimensional turbulence producing a higher turbulent thermal diffusivity through the extended range of the inverse cascade \( \ell_c = q R / \Delta k_T L_T = q \ell_c R / L_T \). Further discussion of this inverse cascade effect is given in Ottaviani and Manfredi [76]. With the inverse cascade ITG turbulence spectrum, two successful applications with interpretative simulations for discharges in the ITER profile database are reported by Redd et al [84]. Subsequently, Erba et al [23] confirm and extend these investigations. These inverse cascaded transport formulae are called the OHE (Ottaviani–Horton–Erba) model [70] for the ion transport channel. Simulations of ETG show the same type of inverse cascade effect which appears generic to the \( E \times B \) convective nonlinearities. Thus the same type of formula larger than that for mixing length estimate must be included in turbulent electron transport. For ETG there are two larger scales where the turbulent energy can accumulate. The electron transport driven by the ETG turbulence is less well documented than the ITG turbulence. An experiment supporting the role of the ETG turbulence is found in [110]. The key formulae are that the maximum growth rate is given by

\[
\gamma_{\text{ETG}}^{\max} = \frac{v_e}{(L_T R)^{1/2}}.
\]

Figure 6. The (a) temperature and (b) density profiles from \( t = 5 \) to 6.35 s in TS shot no 19542. (Reprinted with permission from Horton et al 2002 Phys. Plasmas 7 1494–510, copyright 2002, American Institute of Physics.)
where \( v_e = (T_e/m_e)^{1/2} \), and occurs at \( k_y \gg k_x \) with
\[
k_y^m = \frac{1}{\rho_e} \left( \frac{1 - 2L_{ne}/R}{1 + \eta_e} \right)^{1/2},
\]
where \( \rho_e = v_e/\omega_{ce} \). The electrostatic inverse cascade limit defines the electron mixing length \( \ell_{c,e} \) as
\[
\ell_{c,e}^{es} = q\rho_e \frac{R}{L_{te}}.
\]
The derivation of this electrostatic scale \( \ell_{c,e} \) applies for low plasma \( \beta_{pe} \). For higher \( \beta_{pe} \) the collisionless skin depth \( \delta_e = c/\omega_{pe} \) is the relevant breakpoint in the spectrum. If we compare the formula for \( \ell_{c,e} \) in equation (19) with
\[
\ell_{c,e}^{em} = \delta_e = \frac{c}{\omega_{pe}},
\]
then we find the critical \( \beta_e^{\text{crit}} = L_{Ti}^2/q^2 R^2 \) or \( \beta_p^{\text{crit}} = 2L_{Ti}^2/a^2 \) for the transition defined by \( \ell_{c,e}^{es} = \ell_{c,e}^{em} \). The experiments have \( \ell_{c,e}^{em} > \ell_{c,e}^{es} \) for \( r/a < 0.3 \). The scale length \( \delta_e \) in equation (20) arises from the solution of the fluctuating component of Ampere’s law
\[
\nabla_\perp^2 \tilde{A}_\parallel = -4\pi \tilde{j}_e^\parallel /c = 4\pi n_0 e \tilde{u}_\parallel /c,
\]
as explained in [47]. Here \( \tilde{u}_\parallel \) is the fluctuating parallel electron flow velocity. Three 3D partial differential equations are given and solved for simulations of the finite \( \beta_e \) ETG turbulence in Horton (1990b). The results show that at the scale \( \ell_{c,e} \) the cross-field turbulence has become approximately isotropic in the \( x-y \) plane. The vortex or coherent structures are much larger than the electron gyroradius scale. The toroidal gyrokinetic simulations of Idomura et al [55] reconfirm these findings of large scale structures supporting large thermal fluxes. The region between \( \ell_{c,e} \) and \( 1/k_y^m \) is sufficiently large that we may look for a spectral index to roughly characterize this region. It is to be noted, however, that in general there is some degree of linear growth and damping throughout \( k \)-space so that Kolmogorov exponents are not strictly defined, since there are no source/sink-free transport regions in \( k \)-space. For completeness we note that the corresponding ITG time-space scales are
\[
\gamma_{\text{max}}^{\text{ITG}} = \frac{c_s}{(L_{Ti}R)^{1/2}},
\]
\[
k_y^m = \frac{1}{\rho_i} \left( \frac{1 - 2L_n/R}{1 + \eta_i} \right)^{1/2},
\]
and
\[
\ell_{c,i} = q\rho_i \frac{R}{L_{ti}}.
\]
Note that for flat density profiles where \( 2L_n/R \rightarrow 1 \) another formula applies for \( k_y^m \). The derivation and interpretation of these time-spaces is given in [70, 84]. The ion temperature does not become high enough for finite ion \( \beta \) effect to be important in those experiments. For plasma states well above the critical gradient for the instability the threshold is relatively unimportant and the turbulent diffusivities are given by
\[
\chi_e = C_e \ell_{c,e}^{2\gamma_{\text{max}}^{\text{ETG}}}
\]
and

\[ \chi_i = C_i \ell^2 \gamma_{\text{ITG}}. \]  

(26)

The coefficients \( C_i = 0.014 \) from [70] for a JET discharge and \( C_e = 0.1 \) is determined [51] for a Tore Supra discharge. The theoretical meaning of \( C_i \) and \( C_e \) is the fraction to which the ITG and ETG turbulence reach the levels given by the mixing length level of \( e \phi / T_e \sim \ell_c / L_T \). The [39] power balance studies indicate that magnetic shear \( s = r q' / q \) is an important parameter. Strong magnetic shear leads to electron Landau damping and is known to provide a threshold gradient above the adiabatic value. Hoang et al. [39, 38] investigate the question of the experimental evidence for the existence of a critical electron gradient \( (\nabla T_e)_c \). Hoang et al. [39] found evidence for a critical electron temperature gradient near \( (\nabla T_e)_c \sim 3 \text{ keV m}^{-1} \) and attempted to correlate the \( (\nabla T_e)_c \) inferred from heating experiments with the Rebut et al. [83] model. Serious difficulties with the toroidal magnetic field dependence and magnetic shear independence of the RLW formula are reported. Thus the emphasis has changed to the thresholds for ITG and ETG turbulence. There are well developed formulae for critical threshold gradients for the ITG and ETG turbulence since these values are from the linear dispersion relation. Here we examine the effects of magnetic shear dependence of the \( (\nabla T_e)_c \) formula. The resulting ETG electron diffusivity is

\[ \chi_e = C_e q^2 \left( \frac{R}{L_{T_e}} \right)^{3/2} \left( \frac{e B}{c \rho_e} \right) \left[ \frac{T_e}{L_{T_e}} - C_L \left( \frac{|s| T_e}{q R} \right) \left( 1 + \frac{T_e}{T_i} \right) \right], \]  

(27)

with two parameters \( C_e \) and \( C_L \). Formula (27) is the straightforward generalization of the ion turbulent thermal diffusivity formula in the OHE model to the electron turbulence. The dependence of equation (27) on \( q^2 \) and \( R / L_{T_e} \) reproduces features shown in [39] for \( \chi_e \) from power balance. The critical gradient also agrees in magnitude (a few kilo-electronvolts per metre) and in the observed increase of \( (\nabla T_e)_c \) with magnetic shear reported from both the power \( \chi_e \) and the fluctuation levels in Tore Supra. Recent documentation of this agreement between theory and electron power balance inferred values of the critical gradient are presented in [38]. For the core region of the discharge the electrostatic \( \chi_e \) in equation (27) transforms to an electromagnetic turbulent transport given by

\[ \chi_{e}^{em} = C_{e}^{em} \frac{e^2}{\omega_{pe} (L_{T_e} R)^{1/2}} \frac{v_e}{\beta_{pe} > \beta_{crit}}, \]  

(28)

Because of the fewer electromagnetic transport simulations, less is known about the details of equation (28). The independence of \( \chi_{e}^{em} \) from \( B_T \) appears strange but is explained by the requirements that \( \beta_{e}^{\text{crit}} = L_{T_e}^2 / q^2 R^2 \) \( (\beta_{pe} > \beta_{crit} \sim (L_{T_e} / a)^2) \) and \( q \geq 1 \). Analysing the difficult region between the high and low beta regimes we find that a good working formula to span the two regimes is given by

\[ \chi_{e}^{em} = \left[ 1 + \left( \frac{C_e^{em} \beta_e}{\beta_{e}^{crit}} \right) \right] \chi_e^{ex}. \]
Table 2. TS high \( \beta_p \) experiment no 19542 at \( t = 6 \) s.

| \( R/a \) (m) | 2.3/0.75 |
|-------------|----------|
| \( B_\phi \) (T) | 2.7 |
| \( I_p \) (MA) | 0.69 |
| \( P_{RF} \) (MW) | 3.4 |
| \( n_e(0) \) (m\(^{-3}\)) | \( 3.6 \times 10^{19} \) |
| \( T_e(0)/T_i(0) \) (keV) | 4.3/0.55 |
| \( n_D\tau_T T_e \) (m\(^{-3}\) s keV) | \( 3.7 \times 10^{18} \) |
| \( v_\phi(0) \) (km s\(^{-1}\)) | -20 |

3.2. Theory comparison with power balance analysis

3.2.1. A reference TS shot. Here we analyse TS shot no 19542 for the time interval \( t = 5.0–6.35 \) due to the previous detailed study in [39] and its availability on the ITER profile database. The characteristics of this 0.685MA/2.71 T discharge are given in table 2. The profiles of the electron temperature \( T_e \), ion temperature \( T_i \) and electron density are shown in figure 6. In the fast wave RF heating the power is absorbed by electron Landau damping (\( \omega_{RF} \approx k_{RF} v_e \)) and is exponentially localized (\( P_{RF} e^{-(r/L_p)} \)) to the core of this discharge with the 1/e width \( L_p = 4.3 \) cm. In this discharge the net integrated power deposited is 3.4 MW. The strong localization of the deposited power is important for producing the high electron temperature gradient shown in figure 7 that reaches a maximum value of 12 keV m\(^{-1}\) at \( r/a \approx 0.3 \). Also shown in figure 7 is the critical electron temperature gradient computed from equation (27). We see that in the core out to \( r/a \approx 0.8 \) the electron temperature gradient exceeds this critical gradient by a factor from five to two in going from \( r/a = 0.3 \) to 0.8. Thus, in our first analysis we choose to compute the thresholdless \( \chi_e \) formula with one parameter \( C_e \) to the power balance \( \chi_e \).

The steady state power balance equations in the core plasma are

\[
\frac{1}{r} \frac{\partial}{\partial r} (r q_e) = P_{RF}^e + j_\parallel E_\parallel - Q_{ei},
\]

(29)

\[
\frac{1}{r} \frac{\partial}{\partial r} (r q_i) = P_{RF}^i + Q_{ei}.
\]

(30)

Beyond \( r > 0.8 \) the radiation losses in the electron channel and the atomic physics processes in the ion channel may become dominant. Thus, we limit the transport analysis to \( \rho_1 < \rho < \rho_2 \) typically choosing \( \rho_1 = 0.1 \) and \( \rho_2 = 0.8 \). The electron–ion energy transfer term \( Q_{ei} \) is subdominant to \( P = P_{RF}^e + P_{\Omega e}^e \), but is the dominant input term in the ion power balance equation. More than 90% of the RF input power is coupled to the electron, and since \( T_e \gg T_i, Q_{ei} \) depends mainly on \( T_e \). The integration of equations (29) and (30) from \( \rho = 0 \) to \( \rho \) gives the power balance fluxes shown in figure 8. The peak flux \( q_e \) occurs at the edge of the FEW profile and then falls approximately as \( 1/r \) until \( r/a \approx 0.74 \) where the lower temperature \( T_e \) provides a strong sink into the ion channel. The power balance thermal diffusivities are defined as

\[
\chi_{e,\text{PB}}^{\text{PB}} = -\frac{q_e}{n_e(dT_e/dr)}
\]

(31)

\[
\chi_{i,\text{PB}}^{\text{PB}} = -\frac{q_i}{n_i(dT_i/dr)}
\]

(32)
Figure 7. (a) The gradient of the electron temperature compared with critical gradient \((\nabla T_e)_c = -1.88(|s|T_e/qR)(1 + T_e/T_i)\) as in equation (27). (b) The gradient of the ion temperature compared with critical gradient \((\nabla T_i)_c = -1.7(|s|T_i/qR)(1 + T_i/T_e)\) for ITG modes. All the profiles are for \(t = 5\ s\) to \(6.35\ s\) in Tore Supra shot no 19542. (Reprinted with permission from Horton et al 2002 Phys. Plasmas 7 1494–510, copyright 2002, American Institute of Physics.)
Figure 8. (a) The deposition profiles for the fast wave heating $P_{ICRHE}(r)$ are highly peaked in the core as given in the $U$-files. The heating of the ions is from the electron–ion coupling $Q_{ei}(r)$ with a small fraction from the fast wave heating. (b) The steady state power balance heat fluxes. All the profiles are for $t = 5$ s to 6.35 s in Tore Supra shot no 19542. (Reprinted with permission from Horton et al 2002 Phys. Plasmas 7 1494–510, copyright 2002, American Institute of Physics.)
following equation (4) of Hoang et al \[39\].

The values obtained for the power balance diffusivities \(31\) and \(32\) were compared with the computations from equations \(26\) and \(27\). In constructing this comparison we choose \(C_e\) and \(C_i\) by minimizing the mean-square deviation of the model \(\chi\) values with the power balance \(\chi\) values. The resulting values are \(C_e = 0.1\) and \(C_i = 0.015\) with the mean square deviations given by \(\sigma_{C_e} = 0.3\) and \(\sigma_{C_i} = 0.5\). The value of the \(C_e\) is consistent with earlier theoretical estimates. The value of \(C_i\) is consistent with that value obtained for the JET shot no 19649, which is used in calibration of the OHE model \[70\] to within the limits of the large errors in \(\chi_i\).

The probability distribution functions (PDFs) of \(\chi_{\text{expt}} / \chi_{\text{model}} = \chi_{PB} / \chi_{em}\) are not Gaussian, but have a skewness to large values of \(\chi_{PB} / \chi_{i}\) and \(\chi_{PB} / \chi_{em}\). These heavy tails on the PDFs lead to the large values of \(\sigma_{C_e}\) and \(\sigma_{C_i}\).

3.2.2. TS-98 database. A database of approximately 50 discharges has been assembled and used for power balance analysis. The toroidal magnetic field \(B_\phi\) and plasma current \(I_p\) range from a minimum of 2.1 T/0.4 MA to a maximum of 2.8 T/0.9 MA and the electron density from \(1.2 \times 10^{19}\) m\(^{-3}\) with a range of mixtures of deuterium and helium as working gas. Figure 9 shows the range of the key instability quantities \(L_{Te} / R\) and \(L_{ne} / R\) over the database. Also shown are the stability boundaries in the two limiting cases of peaked density profile, given by the oblique line of slope \(\eta_e = 2/3\), and flat density profile, which is given by the vertical line at abscissa \(L_{Te} / R = \left[\pi / 2(1 + Z_{\text{eff}} T_{\text{e}} / T_i)\right]^{-1}\). Clearly almost all the database points fall in the unstable region. We will show below that the wide range of variations is reduced to a narrow spread of values by the electromagnetic \(\chi_{EM}\) formula.

For each of the independent time slices distributed over different TS shots, we obtain a coefficient \(C_{es}\) in equation \(27\) and a coefficient \(C_{em}\) in equation \(28\) by minimizing the square deviation between the model diffusivities and the ones obtained from power balance equations, as we did in the analysis of the reference shot in the previous section. In this way we get two distributions from the two sets of coefficients, with 48 samples in each for various values of \(\nu = 1\) and 2 in the exponent of \(q\) in \(\chi_{es}\) and \(\chi_{em}\) formulae. The mean square deviation of the data from the ETG model is minimized for \(\nu = 0–1\) for the \(\chi_{em}\) and \(\nu = 0\) for the \(\chi_{es}\) models, and \(C_{em}\) has much smaller relative deviation than \(C_{es}\) for all variations of \(\nu\). The best fit values are \(C_{em} = 0.1\) and 0.15 with error measure for the \(\chi_{es}\) more than twice that of the \(\chi_{em}\) formula. The results suggest that the electromagnetic turbulence formula (equation \(28\)) gives a better model for the turbulent electron thermal diffusivity when only one correlation length formula is used. Predictive simulations have confirmed this suggestion from figure 10.

Finally, we find that in terms of global energy confinement time scaling, the electromagnetic \(\chi_{ETG}\) formula reproduces pretty well the improved confinement which is almost of H-mode quality. This is shown in figure 10, where the \(H\)-factor \(H = W_{\text{tot}} / W_{\text{L}}\) of these discharges ranges between \(H = 1.4\) and 1.7. Here \(W_L\) is the ITER L-mode scaling \[60\]

\[
W_{\text{L}} = 0.023 \kappa^{-0.64} R^{1.83} (R/a)^{0.06} f_p^{0.06} B_\phi^{0.03} n_e^{-0.40} M_{\text{eff}}^{0.20} P^{0.27}.
\]  \(33\)

In contrast, we find that the low \(\beta_{\text{pe}}\), electrostatic \(\chi_{ETG}\) formula predicts L-mode quality performance, which is too pessimistic for these discharges.
3.3. Interpretive transport simulations

Now, we test the theory for the electron transport making simulations in the LOCO transport code in the interpretive configuration. By interpretive simulations one means transport runs on a database of shots with theoretical formulae used for $\chi_e$ and $\chi_i$ that yield the evolution of $T_e(r,t)$ for the model to be compared with the measured profiles. The comparisons are more demanding than the steady state power balance since these are a ramp-up and ramp-down of the driving RF heating power. For this model of electron thermal transport the measured electron density is used in the simulation to eliminate uncertainties associated with the particle transport processes. Such runs allow one to interpret and assess the spacetime evolution of the temperature profiles given by the theoretical electron thermal diffusivity formula. Here we show one comparison from the database that demonstrates well the accuracy to which the spacetime evolution of the electron temperature profile is reproduced by the $\chi_e^{ETG}$ formula. In [51,38] many more discharges are analysed and statistical distributions of the deviations between theory and the power balance inferred $\chi_e$ are presented.

3.3.1. Shot 18368, $I_p = 0.65$ MA, $P_{\text{IRC}} = 3$ MW → 6 MW. This discharge has a step-up over a period $\Delta t = 0.5$ s ($\sim 5 \tau_E$) from the RF driving level of 3 to 6 MW shown in figure 11(a).
The net stored electron energy increases from 0.2 to 0.35 MJ with the model following closely the measured increase of electron thermal energy. The tracking of the modelled $T_e$ profile at $r/a = 0, 0.3$ and 0.7 with the measured profile is shown in figure 11(b). The model over-predicts the core temperature rather significantly. There are some large 100 ms oscillations on the core temperature which are not in the model $T_e$ and may account for the discrepancy. The model tracks the measured profile well at $r/a = 0.3$ and 0.7 through the step-up in the RF power.

The top panel of figure 12 shows that the model prediction for the stored energy tracks well the measured stored energy over the step-up in the RF power. The core oscillations show up as $\Delta W_e \leq 50$ kJ excursions of the model $W_e$ above that from the measured profiles for which $\Delta W_e \leq 300$ kJ.

This and other similar interpretive simulations establish that the theoretical $\chi_e$ model tracks well both the radial and time variations of the electron temperature profile in the RF heated plasma. The histograms for ratios of $C_e = \chi_e^{PB}/\chi_e$ (model) are shown for the FWEH database of 48 shots in [51] and for a larger set of competing theoretical models in [38]. The conclusion is that the ETG transport formulae work well to provide a predictive tool for determining the electron temperature in tokamaks.
Figure 11. Interpretive simulation for shot 18368. (a) Applied FWEH heating ramp from 3 to 6 MW for $\Delta t = 0.5$ s at $t = 5.8$ s. (b) The rise and saturation of $T_e$ at $r/a = 0, 0.3$ and 0.7 from the experiment (solid), and from the model without (dashed) and with (dotted) a critical gradient term. (c) Evolution of $\tau_E$ from the experiment (solid) and from the model without (dashed) and with (dotted) a critical gradient term. (Reprinted with permission from Horton et al 2002 Phys. Plasmas 7 1494–510, copyright 2002, American Institute of Physics.)

3.4. Summary

The theoretical and experimental status of turbulent plasma transport in tokamaks is reviewed with an emphasis on the electron turbulent transport. The degree of the electron anomaly is the largest and, historically, the longest standing transport anomaly in the controlled fusion research programme. We show that recent developments in theory and simulations allow the formula for $\chi_e$ to be used and tested for confidence with steady state electron data. While the best approach for describing turbulent transport is explicitly that of the multi-mode approach where all possible instabilities are assessed and applied as dictated by theory, we report here tests of the electron transport in plasma where the power flow is dominantly through the electron channel and thus most sensitive to the $\chi_e$ formula. The power balance analysis of the hot electron TS plasmas is unusually clear due to the direct, local deposition of RF power into core electrons through the FWEH mode of operation. As a consequence, there is a multi-megawatt thermal flux conducting through the electron channel in the radial region from $\rho = \rho_1 \simeq 0.2$ to $\rho_2 \simeq 0.7$. Since the electron temperature profile is accurately measured for these discharges there is a well defined electron thermal diffusivity $\chi_e$ called the power balance thermal diffusivity.

The data analysis presented here shows clear evidence for the short scale length electron temperature gradient drift wave turbulence, providing a basic physics explanation for the
mechanism producing the observed thermal diffusivity. This hypothesis is tested with transport codes with the local heating deposition profiles and edge boundary conditions given. The results show that the ETG formula is able to reproduce well the accurately known electron temperature profiles. The theory gives that the turbulent correlation scale length changes character in going from the high electron pressure core to the lower pressure outer zone plasma. The degree to which the data support the $\chi_e$ formula based on the correlation lengths $\ell_{c,e}^m = c/\omega_{pe}$ and $\ell_{c,e}^{cs} \approx q\rho_e R/L_{Te}$ is presented. The interpretation and predictions resulting from theoretical turbulence $\chi_e$ formulae are then compared with the well documented empirical formula based on the database—the Taroni–Bohm formula. The conclusion is that it is the electromagnetic turbulent electron thermal diffusivity that explains well the power balance $\chi_{PB}^e$. This explanation is consistent with the report of Colas et al [12] that reports the presence and role of internal magnetic fluctuations in electron heat transport. The Colas et al [12] transport analysis uses the internal magnetic fluctuations $\delta B_r$ measured with the cross-polarization scattering technique [114]. The magnetic turbulence at short wavelength $k_\perp = 12.6$ cm$^{-1}$ is measured at $r/a = 0.55$. Power scans show that the intensity at this wavenumber and position increases as $(\delta B_r/B)^2 \approx 10^{-8}(8\text{ m}^{-1}/L_{Te})$ in experiments where $\nabla T_e \leq 7$ keV m$^{-1}$. The corresponding maximum electron thermal flux is 0.1 MW m$^{-2}$ which exceeds considerably the values shown in figure 8 for shot TS19542 ($t = 6$ s). The $\chi_e$ formulae used by Colas et al [12] are not self-consistent collective electromagnetic field transport formulae: rather, they are the test particle

Figure 12. Interpretive simulation for shot 18368. (a) The stored electron energy $W_e$, from the experiment (solid) and from the simulations (dashed). (b) The temperature profile at $t = 6.5$ s, from the experiment (circles) and from the simulations (solid). (Reprinted with permission from Horton et al 2002 Phys. Plasmas 7 1494–510, copyright 2002, American Institute of Physics.)
diffusivities of electrons in a given spectrum of purely magnetic fluctuations. More realistic test particle diffusivities for electrons are in [61], keeping both the electric fields and the magnetic fluctuations. The motion of the electron guiding centre is strongly influenced by both the electric and magnetic components of the fluctuations. For very low fluctuation levels the Coulomb collisional pitch-angle scattering and the electromagnetic drift wave fluctuation spectrum act together to determine the electron diffusion in tokamak geometry. In very low amplitude fluctuation limit, one recovers [61] the neoclassical banana-plateau diffusion coefficient. At the fluctuation levels given by the mixing length amplitudes, the anomalous transport formulae for the electron turbulent transport in equations (26) and (27) are recovered.

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