Effect of two-boson exchange on parity-violating $ep$ scattering

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Abstract

We compute the corrections from two-photon and $\gamma-Z$ exchange in parity-violating elastic electron–proton scattering, used to extract the strange form factors of the proton. We use a hadronic formalism that successfully reconciled the earlier discrepancy in the proton’s electron to magnetic form factor ratio, suitably extended to the weak sector. Implementing realistic electroweak form factors, we find effects of the order 2–3% at $Q^2 \lesssim 0.1 \text{ GeV}^2$, which are largest at backward angles, and have a strong $Q^2$ dependence at low $Q^2$. Two-boson contributions to the weak axial current are found to be enhanced at low $Q^2$ and for forward angles. We provide corrections at kinematics relevant for recent and upcoming parity-violating experiments.
Two-photon exchange corrections have recently been found to play an unexpectedly important role in elastic electron–proton scattering. Despite being of $O(\alpha)$ smaller than the Born amplitudes, $2\gamma$ exchange effects have been shown to have a strong angular dependence, which significantly influences the extraction of the electric form factor in Rosenbluth separations. Such corrections were found to resolve a major part of the discrepancy between the electric to magnetic proton form factor ratio, $G^p_E/G^p_M$, determined via the Rosenbluth and polarization transfer methods (see Ref. [1] and references therein).

Elastic $ep$ scattering has also been used to probe the strangeness content of the proton, through measurements of the strange electric and magnetic form factors. This is achieved by scattering polarized electrons from unpolarized protons, and observing the parity-violating (PV) asymmetry $A_{PV} = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$, where $\sigma_{R(L)}$ is the cross section for a right-(left-) handed electron. The numerator in the asymmetry is sensitive to the interference of the vector and axial-vector currents, and hence violates parity [2].

In view of the large $2\gamma$ contributions to electromagnetic form factors, it is natural to ask what effect the exchange of two bosons ($\gamma$ or $Z$) may have on the PV asymmetries. In particular, since the extracted strange form factors appear to be rather small [3], these two-boson exchange (TBE) contributions could affect the extraction significantly. In this paper we compute the relevant corrections to the PV asymmetry arising from the interference between single $Z$ boson and $2\gamma$ exchange amplitudes (which we denote by “$Z(\gamma\gamma)$”), and between the one-photon exchange and $\gamma-Z$ interference amplitudes (denoted by “$\gamma(Z\gamma)$”). We use the hadronic formalism developed in Ref. [1], which allows a more natural implementation of hadronic structure effects in radiative corrections at low four-momentum transfer squared $Q^2$, where PV electron scattering experiment are typically performed.

In their seminal work on electroweak radiative effects, Marciano & Sirlin [4] computed the $\gamma(Z\gamma)$ contribution at $Q^2 = 0$, both at the quark level and at the hadronic level using dipole form factors. The $Z(\gamma\gamma)$ contribution was calculated in Ref. [5] within a generalized parton distribution formalism, applicable at a scale of several GeV$^2$. More recently, Zhou et al. [6] computed the TBE effects on $A_{PV}$ within a hadronic basis using monopole form factors.

In the present analysis, we account for the finite size of the proton by using realistic electromagnetic form factors in the loop graphs, determined self-consistently from a global analysis of elastic cross section and polarization transfer data [7] including explicit $2\gamma$ ex-
change corrections. Furthermore, while an overall, factorized correction was applied to the PV asymmetry in Ref. [6], here we compute explicitly the individual TBE corrections to the proton and neutron (or to the $\sin^2 \theta_W$-dependent and independent) terms in $A_{PV}$.

In the Born approximation, the amplitude for the weak current mediated by $Z$ exchange is given by:

$$M_Z = \frac{e^2}{Q^2 + M_Z^2} \frac{1}{(4\sin^2 \theta_W \cos^2 \theta_W)^2} j_Z^\mu J_{Z\mu},$$

where $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ is the Weinberg angle, with $M_W$ ($M_Z$) the $W$ ($Z$) boson mass. For the corresponding electromagnetic one-photon exchange amplitude, $M_{\gamma}$, we refer to Ref. [1] for details. The weak leptonic current is given by a sum of vector and axial-vector terms,

$$j^\mu_Z = \bar{u}_e (g^v_{\mu} \gamma^\mu + g^a_{\mu} \gamma^\mu \gamma_5) u_e,$$

where $g^v_e = -(1 - 4 \sin^2 \theta_W)$ and $g^a_e = +1$ are the vector and axial-vector couplings of the electron to the weak current. The matrix element of the weak nucleonic matrix is given by

$$\Gamma^\mu_Z = \bar{u}_N \Gamma^\mu_Z u_N,$$

where

$$\Gamma^\mu_Z = \gamma^\mu F_1^Z + \frac{i\sigma^\mu\nu q_\nu}{2M} F_2^Z + \gamma^\mu \gamma_5 G_A^Z,$$

where $q$ is the four-momentum transfer and $M$ is the nucleon mass. Assuming isospin symmetry, the weak vector form factors $F_{1,2}^{Zp}$ for a proton target are related to the electromagnetic form factors of the proton and neutron $F_{1,2}^{\gamma p,n}$ (at tree level) by

$$F_{1,2}^{Zp} = (1 - 4 \sin^2 \theta_W) F_{1,2}^{\gamma p} - F_{1,2}^{\gamma n} - F_{1,2}^s,$$

where $F_{1,2}^s$ are the contributions from strange quarks. The small factor $(1 - 4 \sin^2 \theta_W)$ suppresses the overall contribution from the proton electromagnetic form factors. The weak axial-vector form factor of the proton is given by $G_A^{Zp} = -G_A^{\gamma p} + G_A^{s}$, where $G_A^s$ is the strange quark contribution.

In the standard model the parity-violating asymmetry $A_{PV}$ receives contributions from products of vector electron and axial-vector proton currents, and axial-vector electron and vector proton currents. It can be written as a sum of proton vector, strange, and axial-vector contributions:

$$A_{PV} = - \left( \frac{g_{\rho} Q^2}{4\sqrt{2\pi\alpha}} \right) (A_V + A_s + A_A),$$
where

\[ A_V = g_A^e \rho \left[ (1 - 4\kappa \sin^2 \theta_W) - \frac{\varepsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\sigma_{\text{red}}} \right], \quad (5a) \]

\[ A_s = -g_A^e \rho \frac{\varepsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\sigma_{\text{red}}}, \quad (5b) \]

\[ A_A = g_A^e \sqrt{\tau(1 + \tau)(1 - \varepsilon^2)} \frac{\tilde{G}_Z^p G_M^p}{\sigma_{\text{red}}}, \quad (5c) \]

where \( G_F = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W M_W^2} \) is the Fermi constant, and \( \sigma_{\text{red}} = \varepsilon (G_E^p)^2 + \tau (G_M^p)^2 \) is the reduced unpolarized cross section, with \( \varepsilon = (1 + 2(1 + \tau) \tan^2(\theta/2))^{-1} \) the photon polarization parameter and \( \tau = Q^2/4M^2 \).

The parameters \( \rho \) and \( \kappa \) in Eqs. (5) contain higher order radiative effects, such as vertex corrections, wave function renormalization, vacuum polarization, and inelastic bremsstrahlung, which have been calculated previously and are well known. At tree level, \( \rho = \kappa = 1 \). Beyond tree level, \( \rho \) and \( \kappa \) also contain contributions from the interference of the Born and TBE diagrams, which we denote by \( \Delta \rho \) and \( \Delta \kappa \), respectively. The form factor \( \tilde{G}_A^Z \) implicitly contains higher order radiative corrections for the proton axial current, as well as the hadronic anapole contributions \([2, 3]\). At tree level, and in the absence of the anapole term, \( \tilde{G}_A^Z \to G_A^Z \) above.

The contribution of the \( Z(\gamma\gamma) \) and \( \gamma(Z\gamma) \) TBE corrections to the PV cross section can be written:

\[ \Delta \sigma_{\text{TBE}} = 2 \Re \left[ \mathcal{M}_{\gamma\gamma} \mathcal{M}_Z^* + (\mathcal{M}_{\gamma Z} + \mathcal{M}_{Z\gamma}) \mathcal{M}_Z^* \right], \quad (6) \]

where \( \mathcal{M}_{\gamma\gamma} (\mathcal{M}_{\gamma Z}) \) is the two-photon (\( \gamma-Z \)) exchange amplitude. Since the asymmetry \( A_{PV} \) is constructed as a ratio of the PV cross section to the unpolarized (parity conserving) cross section, one also needs to include corrections to the latter from the interference of one and two-photon exchange amplitudes (denoted by “\( \gamma(\gamma\gamma) \)”). These have been computed in Ref. [1] within the current framework.

The \( \sin^2 \theta_W \) dependence of the TBE corrections can be obtained explicitly by calculating separately the proton and neutron contributions of Eq. (5) to the PV asymmetry. In so doing the \( \sin^2 \theta_W \)-dependent and independent parts can be evaluated and the \( \Delta \rho \) and \( \Delta \kappa \)
corrections determined from the vector part of $A_{PV}$ in Eq. (5a):

$$\Delta \rho = \frac{A_{pV}^p + A_{nV}^n}{A_{pV} + A_{nV}} - \frac{\Delta \sigma^{\gamma(\gamma\gamma)}}{\sigma_{\text{red}}},$$

$$\Delta \kappa = \frac{A_{p,V,\text{tree}}^p - A_{p,V}^p}{A_{p,V,\text{tree}}^p + A_{n,V}^n},$$

where $A_{p(n)}^V$ is the TBE contribution to $A_V$ from the proton (neutron), and $A_{p(n),\text{tree}}^V$ is the corresponding tree-level asymmetry, with $\Delta \sigma^{\gamma(\gamma\gamma)}$ the electromagnetic two-photon exchange contribution to $\sigma_{\text{red}}$. The contributions to $\Delta \rho$ arise therefore from the $\gamma(Z\gamma)$ and $Z(\gamma\gamma)$ corrections, as well as from the electromagnetic corrections $\gamma(\gamma\gamma)$. On the other hand, $\Delta \kappa$ receives contributions only from the $\gamma(Z\gamma)$ and $Z(\gamma\gamma)$ corrections.

The calculation of the TBE corrections proceeds along the same lines as that of the $2\gamma$ amplitudes in Ref. [1], with the replacement of the $\gamma N N$ vertex function by $\Gamma_\mu^\mu$ in Eq. (2). As in Ref. [1], we parameterize the form factors $F_{Zp}^{1,2}$ as sums of three monopoles, but take the proton form factors from the more recent global fit of Ref. [7], and the neutron form factors from Ref. [8]. Since the main purpose of the PV experiments is to extract strange contributions to form factors by comparing the measured asymmetry with the predicted zero-strangeness asymmetry, in all our numerical simulations we set the strange form factors to zero, $F_{1,2}^s = 0 = G_A^s$. For the axial-vector form factor we use the empirical dipole fit, $G_A(Q^2) = G_A(0)/(1 + Q^2/\Lambda_A^2)$, where $G_A(0) = 1.267$ is the axial-vector charge, and the mass parameter $\Lambda_A = 1$ GeV. Varying $\Lambda_A$ by 20% does not affect the results significantly.

In Fig. 1 we show the calculated $\Delta \rho$ and $\Delta \kappa$ corrections for $Q^2 = 0.01, 0.1$ and 1 GeV$^2$, relative to the soft-photon approximation (SPA) of Mo & Tsai [9]. (All of the following results will be relative to the SPA.) In the SPA the $Z$ exchange is factorized, which results in a cancellation of the model independent infrared contribution to the PV asymmetry. The total $\Delta \rho$ is $\approx 1\text{--}2\%$ over most of the range of $\varepsilon$, increasing at small $\varepsilon$ and small $Q^2$. At low $Q^2$ values the $Z(\gamma\gamma)$ piece largely cancels with the $\gamma(\gamma\gamma)$, so that the total is saturated mostly by the $\gamma(Z\gamma)$ contribution. At $Q^2 = 1$ GeV$^2$ the signs of the $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections change, and the $\gamma(Z\gamma)$ contribution decreases in magnitude.

The TBE corrections to $\kappa$ are somewhat smaller in magnitude, ranging from $\sim 0.2\text{--}0.3\%$ at $Q^2 = 0.01$ GeV$^2$, where the $Z(\gamma\gamma)$ contribution is dominant, to less than 0.1% at $Q^2 = 1$ GeV$^2$, where there is large cancellation between these. As seen in Eq. (7b), there is no contribution to $\Delta \kappa$ from $\gamma(\gamma\gamma)$.
FIG. 1: TBE contributions $\Delta \rho$ and $\Delta \kappa$, relative to the SPA [9], as a function of $\varepsilon$ for $Q^2 = 0.01, 0.1$ and 1 GeV$^2$. The various contributions are: $\gamma(\gamma\gamma)$ (dotted), $Z(\gamma\gamma)$ (dashed), $\gamma(Z\gamma)$ (dot-dashed), and the total (solid).

The $Q^2$ dependence of $\Delta \rho$ and $\Delta \kappa$ is illustrated in Fig. 2 for several scattering angles. A rapid $Q^2$ variation is evident for $Q^2 \lesssim 0.1$ GeV$^2$, especially at backward scattering angles. This is most pronounced in $\Delta \kappa$ and leads to a change in sign at $Q^2 \approx 0.05$ GeV$^2$ at $\theta = 150^\circ$. Because of the strong $Q^2$ dependence, estimates at $Q^2 = 0$, by Marciano and Sirlin [4] and more recently in Refs. [10, 11] in a somewhat different limit to the $Q^2 = 0$ point, are in general not sufficient to obtain a reliable correction for the actual experiments. Approximating the TBE corrections at non-zero $Q^2$ by their $Q^2 = 0$ values may therefore lead to errors in the
FIG. 2: TBE corrections $\Delta \rho$ and $\Delta \kappa$ as a function of $Q^2$ for fixed scattering angle $\theta = 10^\circ$ (dotted), $\theta = 60^\circ$ (dashed), $\theta = 110^\circ$ (dot-dashed), and $\theta = 150^\circ$ (solid).

TABLE I: Ratio $\delta = A_{PV}^{\text{TBE}}/A_{PV}^{\text{tree}}$ of TBE contributions to the proton asymmetry relative to the tree-level asymmetry (in percent) at the $Q^2$ and scattering angle $\theta$ of selected past and future experiments. The results are compared for different input form factors (empirical, dipole and monopole).

| $Q^2$ (GeV$^2$) | $\theta$ | Ref. | $\delta$ (%) |
|-----------------|---------|------|--------------|
|                 |         |      | empirical dipole monopole |
| 0.1             | 144.0$^\circ$ | [12] | 1.62 | 1.52 | 1.72 |
| 0.23            | 35.31$^\circ$ | [13] | 0.63 | 0.58 | 0.84 |
| 0.477           | 12.3$^\circ$ | [14] | 0.16 | 0.15 | 0.24 |
| 0.997           | 20.9$^\circ$ | [15] | 0.22 | 0.23 | 0.30 |
| 0.109           | 6.0$^\circ$  | [16] | 0.20 | 0.16 | 0.32 |
| 0.23            | 110.0$^\circ$| [17] | 1.39 | 1.33 | 1.52 |
| 0.03            | 8.0$^\circ$  | [18] | 0.58 | 0.47 | 0.86 |

extracted form factors.

The above behavior is qualitatively reproduced if one uses dipole form factors (for either the Dirac and Pauli, or electric and magnetic form factors, with a dipole mass of 1 GeV) or monopoles (for the electric and magnetic, with a monopole mass of 0.56 GeV [6]), instead of the empirical ones [7]. Quantitatively, however, there are significant differences between the empirical and monopole results at large $\varepsilon$, with the monopole results for $\Delta \rho$ ($\Delta \kappa$) being $\sim 30\%$ ($60\%$) larger in magnitude at $\varepsilon \sim 0.9$, with larger differences at larger $Q^2$. This
reflects the sensitivity of the loop integrals to the large-momentum tails of form factors (and hence nucleon structure effects), even at low $Q^2$.

The sensitivity to the input form factors is more clearly illustrated in Table I, where we list the values of the TBE contributions to the proton PV asymmetry relative to the tree-level asymmetry (in percent), $\delta = A_{\text{PV}}^{\text{TBE}}/A_{\text{PV}}^{\text{tree}}$, at the kinematics relevant to several past and future experiments [12, 13, 14, 15, 16, 17, 18]. The results for the empirical form factors [7] are compared with those using dipole and monopole form factors in the loop integration. Generally the effects using the empirical form factors are $\lesssim 0.5\%$ for most of the forward angle experiments, increasing to $\sim 1.5\%$ at backward angles. The results with the dipole form factors are similar, tending to be $\sim 5$–10\% smaller. With the monopole form factors [6], however, the values of $\delta$ are some 50\% larger than with the empirical, which suggests insufficient suppression of contributions from large loop momenta. The TBE corrections to $A_{\text{PV}}$ may therefore be underestimated using monopole form factors.

The impact of these differences on the strange form factors is difficult to gauge without performing a full reanalysis of the data, since in general different electroweak parameters and form factors are used in the various experiments [12, 13, 14, 15, 16, 17, 18]. However, as an estimate of the possible effects we have determined following Ref. [6] the quantity defined there as $\delta_G$, as a measure of the induced difference between the strange asymmetry extracted using the different form factors. With the parameters of this analysis we find, for example, differences of the order of 15\% between the empirical and monopole form factors for the HAPPEX kinematics [14], around 20\% for the G0 datum [15] in Table I, and over 30\% for the PVA4 kinematics [13]. Although these values should be treated as indicative only, they clearly point to the need for a more detailed reanalysis of the data including TBE effects and a careful treatment of the form factors in the loop integrations.

As discussed above, the effective axial-vector form factor $\tilde{G}_A^{Zp}$ in the axial asymmetry $A_A$ is defined to include the anapole form factor, and higher order radiative corrections. In extracting the strange form factors from data, Young et al. fit the effective $\tilde{G}_A^{Zp}$ without decomposing it into its various contributions. Alternatively, one may extract the anapole form factor from the $\tilde{G}_A^{Zp}$ by correcting for the radiative effects. In Fig. 3 we show the correction $\delta_A$ to the axial PV asymmetry, $A_A \rightarrow A_A(1 + \delta_A)$, at several $Q^2$ values. The correction is of order 1–2\% for $Q^2 \geq 0.1$ GeV$^2$, but increases rapidly for decreasing $Q^2$, especially at large $\varepsilon$, where it reaches 10\% at $\varepsilon \sim 0.95$ at $Q^2 = 0.01$ GeV$^2$. This behavior
FIG. 3: TBE correction $\delta A$ to the hadronic axial part of the PV asymmetry, for $Q^2 = 0.01$ (dashed), 0.1 (solid) and 1 GeV$^2$ (dotted).

may be related to the faster vanishing of the axial Born asymmetry compared with the TBE contribution at large $\varepsilon$.

The corrections calculated here are presented in a form that can be straightforwardly applied to the $A_{PV}$ data. The values for $\Delta \rho$ and $\Delta \kappa$ can simply be added to the existing radiative corrections contained in $\rho$ and $\kappa$, taking care to subtract any partial TBE contributions that have already been included [4]. Because the TBE effects are largest at backward angles, they will be most relevant for the SAMPLE experiment at Bates [12], and for the backward angle run of the G0 experiment [15] at Jefferson Lab. For the former, we find $(\Delta \rho, \Delta \kappa)(Q^2 = 0.1 \text{ GeV}^2, \theta = 144^\circ) = (1.950 \times 10^{-2}, 7.998 \times 10^{-4})$, while for the latter $(\Delta \rho, \Delta \kappa)(Q^2 = 0.23 \text{ GeV}^2, \theta = 110^\circ) = (1.473 \times 10^{-2}, -1.342 \times 10^{-4})$ and $(\Delta \rho, \Delta \kappa)(Q^2 = 0.62 \text{ GeV}^2, \theta = 110^\circ) = (1.261 \times 10^{-2}, 1.103 \times 10^{-4})$. In addition, even though it is at forward angles, the considerably smaller uncertainties expected in the Qweak experiment [18] at Jefferson Lab will require a careful treatment of the radiative effects. In particular, we find $(\Delta \rho, \Delta \kappa)(Q^2 = 0.03 \text{ GeV}^2, \theta = 8^\circ) = (3.755 \times 10^{-3}, -2.655 \times 10^{-4})$. A reanalysis of the entire data set on strange form factors incorporating these effects is currently in progress.
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[1] P. G. Blunden, W. Melnitchouk and J. A. Tjon, Phys. Rev. Lett. 91, 142304 (2003); Phys. Rev. C 72, 034612 (2005).
[2] M. J. Musolf et al., Phys. Rept. 239, 1 (1994).
[3] R. D. Young, J. Roche, R. D. Carlini and A. W. Thomas, Phys. Rev. Lett. 97, 102002 (2006).
[4] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 46, 163 (1981); Phys. Rev. D 22, 2695 (1980); ibid. D 27, 552 (1983); ibid. D 29, 75 (1984).
[5] A. V. Afanasev and C. E. Carlson, Phys. Rev. Lett. 94, 212301 (2005).
[6] H. Q. Zhou, C. W. Kao and S. N. Yang, Phys. Rev. Lett. 99, 262001 (2007).
[7] J. Arrington, W. Melnitchouk and J. A. Tjon, Phys. Rev. C 76, 035205 (2007).
[8] P. E. Bosted, Phys. Rev. C 51, 409 (1995).
[9] L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969).
[10] A. Aleksejevs, S. Barkanova and P. G. Blunden, arXiv:0710.3204 [physics.comp-ph].
[11] J. Erler, A. Kurylov and M. J. Ramsey-Musolf, Phys. Rev. D 68, 016006 (2003); M. J. Musolf and B. R. Holstein, Phys. Lett. B 242 (1990) 461.
[12] B. Mueller et al., Phys. Rev. Lett. 78, 3824 (1997).
[13] F. E. Maas et al., Phys. Rev. Lett. 93, 022002 (2004).
[14] K. A. Aniol et al., Phys. Rev. C 69, 065501 (2004).
[15] D. S. Armstrong et al., Phys. Rev. Lett. 95, 092001 (2005).
[16] A. Acha et al., Phys. Rev. Lett. 98, 032301 (2007).
[17] JLab experiments E04-115 and E06-008, D. Beck spokesperson.
[18] JLab experiment E05-020, R. D. Carlini et al. spokespersons.