The weak anthropic principle and the landscape of string theory

George Ellis∗
Mathematics Department, University of Cape Town, Rondebosch, Cape Town 7701, South Africa
and
Lee Smolin†
Perimeter Institute for Theoretical Physics, Waterloo, ON N2J 2W9, Canada

January 16, 2009

Abstract

We note that there is an exception to the general arguments that no falsifiable predictions can be made, on the basis of presently available data, by applying the weak anthropic principle (WAP) to the landscape of string theory. If there are infinitely more vacua in the landscape for one sign of a parameter than the other, within an anthropically allowed range, then under very weak assumptions about the probability measure one gets a firm prediction favoring that sign of that parameter. It is interesting to note that while the understanding of the string landscape is evolving, present evidence on the nature of the landscape allows such an argument to be made, leading to the conclusion that the WAP favors a negative value for the cosmological constant, \( \Lambda \), in contradiction to the result of astronomical observations. The viability of applying the WAP to string theory then requires that either there are found an infinite discretum of anthropically allowed vacua for \( \Lambda > 0 \), or the recently found infinite discretum of solutions for \( \Lambda < 0 \) be reduced to a finite value.

∗Email address: george.ellis@uct.ac.za
†Email address: lsmolin@perimeterinstitute.ca
1 Introduction

Recently it has been argued by ourselves [1, 2, 3, 4] and others that the present dominant interpretation of the Weak Anthropic Principle (WAP), whereby the existence of life is explained by random selection from an ensemble of universes with differing properties [5, 6], is by itself not falsifiable, because any predictions made from it depend on assumptions made about the ensemble of universes and the probability measure taken on it. There is a great deal of freedom in the choices that can be made here, and those assumptions determine the expectations for experiments; but they are untestable, in particular because all the other universes in the supposed ensemble are unobservable [3, 4].

These considerations are the focus of present interest because of the claims that the WAP can be applied to the “landscape” [1] of string theory [7, 8]. According to recent results, the landscape consists of an infinite set of discrete vacua, which are argued to be different possible quasistable ground states for string theory [9, 10, 11, 12]. These include on the order of $N^+ = 10^{500}$ flux compactifications which appear compatible with 3+1 non-compact dimensions and a positive cosmological constant $\Lambda$ [9]. This landscape of possibilities leads to a vast number of different predictions for post standard model physics [9], rather than a single prediction that can be verified or disproved. This has led to worries that string theory fails to be predictive.

Combining these considerations, it appears at first that the WAP places no constraints on string theory because it is expected that the Landscape will include many versions of local physics that will allow life to exist; consequently the WAP can always be fulfilled in any ensemble of universes where most of the possibilities of the Landscape are realised. The project of combining the WAP with the landscape of string theory, as proposed in [7, 8], then has been argued to be untestable.

In reply, it has been proposed that the theory makes exactly one prediction, which is that the sign of the spatial curvature be negative [8, 13]. This in principle could allow disproof of most versions of the theory, were the scalar curvature measured to be positive. However there will for the foreseeable future be considerable uncertainties that will prevent many from accepting this kind of data as knock-out disproof of either the landscape or the idea of an ensemble of universes, even if the observations come out on the side of positively curved spatial sections.¹

In this comment we point out there is another important exception to the conclusion that the combination of the WAP with the string landscape is untestable. In spite of the general difficulties, we show that there are possible distributions of vacua that allow an argument based on the WAP to be made which leads to a prediction which is stable under a large number of choices for the probability measure

¹ The best present value of the spatial curvature parameter from the combined astronomical data has shifted from $\Omega_k = 1.02 \pm 0.02$ [14], marginally indicating positively curved spatial sections, to $\Omega_k = 1.003 \pm 0.010$ [15], consistent with positive or negative values.
One case in which this can occur is if there are a finite number of anthropically allowed vacua for one sign of \( \Lambda \), but an infinite discrete set of anthropically allowed vacua for the other sign. For example, suppose that there is an infinite discretum of anthropically allowed vacua for \( \Lambda > 0 \) and only a finite discretum for \( \Lambda < 0 \). We can then argue that under a large set of possible choices for probability weights the theory will predict \( \Lambda > 0 \). This is because there would then be two discrete sets of universes where life may be possible: one, \( S_- \) (with \( \Lambda < 0 \)), is finite, and the other, \( S_+ \) (with \( \Lambda > 0 \)) is infinite. If we follow the logic of the WAP argument by choosing randomly from the infinite set \( S \equiv S_+ \cup S_- \) which is the union of them (with \( \Lambda \) taking all values), there is vanishing probability that we will end up in the finite subset \( S_- \) of negative cosmological constant universes. Thus, in this case, we would predict that \( \Lambda > 0 \). This would be most gratifying as that is what is observed [16, 17].

At present the study of the landscape is evolving, and it is too soon to draw definitive conclusions of this kind. But, for what it may be worth, it can be pointed out that at present the situation is the reverse of that just sketched. There are at present claimed to be a large, if finite discretum of vacua with values of the cosmological constant that are positive but less than anthropic bounds [9]. There is even a conjecture that the set of anthropically allowed vacua consistent with \( \Lambda > 0 \) is bounded by a finite number [12]. Thus, at present, the evidence is that the cardinality of the \( \Lambda > 0 \), anthropically allowed discretum, which we may call, \( N_+ \), is large, but finite.

At the same time, the present evidence is that the set of string vacua with negative cosmological constant is actually countably infinite [10, 11]. As is shown in [10], the number of string vacua appears to diverge as \( \Lambda \rightarrow 0 \) from below. In the vacua studied in [10] there is a condition that the compactification volume also be large, so the number anthropically allowed will be a proper subset and, perhaps even finite. However in [11] evidence is found for an infinite set of additional negative \( \Lambda \) compactifications, including some whose numbers diverge within finite ranges of macroscopic parameters. At present, our understanding is that not enough is known about this set to know whether all but a finite set can be excluded on anthropic grounds. Thus the present situation (which may certainly change) is that the best estimate for the number of negative cosmological constant anthropically allowed vacua, \( N_- \), is

\[ N_- = \infty. \]  \hspace{1cm} (1)

If further studies should confirm these estimates, then there would be important consequences for the viability of the landscape approach to string phenomenology. This is because the combination of the results and conjectures in the papers [10, 11, 12] lead, if correct, to the conclusion that the weak anthropic principle implies that the cosmological constant will be negative, which is in contradiction to the astronomically determined value [16, 17]. Consequently, if the present situ-
ation turns out to be correct[22], we will have to conclude that either the string theory landscape does not describe nature, or the Anthropic style of argument whereby the existence of life is explained by random selection from an ensemble of universes cannot be valid (perhaps because the hypothesized ensemble of universes does not in fact exist). But this is then problematic for both understandings, because on the one hand the landscape of string theory is claimed to be the natural setting for applying the anthropic style of argument [7], and on the other, anthropic arguments seem to be the only way to get reasonably unique physical predictions out of the landscape of string theory. If the combination does not work, as we argue here is implied by the current best understandings of the landscape, then both components are on weak ground.

The purpose of this note is to make this argument more carefully.

2 The weak anthropic principle

Here is a standard version of the weak anthropic principle [18]. We posit that there is an actual ensemble \( \mathcal{E} \) of universes \( u_i \), or of expanding universe domains in a larger universe, which is a discrete set; we live in one of the expanding universe domains in the ensemble. They may, for example, have been produced by eternal inflation [19], or other mechanisms. We may take the number of actual expanding universe domains \( N_U \) to be arbitrarily large, or countably infinite. On \( \mathcal{E} \) there is a measure \( f_i \) which is proportional to the probability for the universe \( u_i \) to contain intelligent life. If it is impossible for life to exist in a universe\(^2 u_i \), then \( f_i = 0 \). There is then a subensemble \( \mathcal{L} \subset \mathcal{E} \) of universes where intelligent life is possible.

In some versions, including those associated with the principle of mediocrity [20], \( f_i \) is proportional to something like “the number of civilizations” likely to exist in the universe \( u_i \). In other versions, \( f_i \) has a constant non-zero value if intelligent life is possible in \( u_i \).

We may be interested in the value of an observable, \( \mathcal{O} \) whose value in the universe \( u_i \) is \( \mathcal{O}_i \). According to the weak anthropic principle its expectation value is given by

\[
< \mathcal{O} > = \frac{\sum_i \mathcal{O}_i f_i}{\sum_i f_i}
\]

where the sum indicated by \( \sum_i' \) means sum over all the universes in the ensemble \( \mathcal{L} \) where intelligent life is possible. Alternatively, let \( Q \) be some property that a universe may or may not have. Those universe which have the property live in an ensemble \( \mathcal{E}_Q \). Let \( u_i' \) denote all \( u_i \) such that \( u_i \in \mathcal{E}_Q \cap \mathcal{L} \). Then the probability that

\(^2\)From here on we use ‘universe’ as shorthand for ‘universe or expanding universe domain’.
Q is true in a typical randomly chosen universe with intelligent life is

\[ P_Q = \frac{\sum u'_i f_i}{\sum_i f_i} \]  

The reason why it is sometimes argued that the weak anthropic principle implies directly no predictions is that these observables depend crucially on the choices made of the weights \( f_i \). However, given information about the \( f_i \), some predictions are possible, as we shall now see.

3 Anthropic constraints on the cosmological constant

A famous argument of Weinberg tells us that there is a maximum value of the cosmological constant \( \Lambda \) compatible with the existence of galaxies [6]. Since it is believed that stars form in galaxies, and planets providing viable habitats for life circle around stars, we take this value, which we call \( \Lambda_+ \) as the limiting value necessary for intelligent life.

There is, so far as we know, no cosmological or astronomical reason that life is not compatible with zero cosmological constant. Therefore, there can be no astronomical reason life is not compatible with a small negative cosmological constant, but there will be a lower limit \( \Lambda_- \) to the negative values of \( \Lambda \) compatible with intelligent life, because all universes with a negative cosmological constant recollapse, and too negative a value for \( \Lambda \) will imply extremely short life times for the universe before they recollapse, not allowing the extended times needed for the evolution of intelligent life.

There seems no reason why life is not roughly as probable in a member of the string theory landscape with slightly negative value of \( \Lambda \) than slightly positive. It is true that for the positive \( \Lambda \) members of the discretum, supersymmetry is always broken at the level of the effective supergravity description, whereas for the negative \( \Lambda \) members sometimes it is broken at that level and sometimes it is preserved. It is also possible that supersymmetry breaking is necessary for life. But even if that were the case, there is a general expectation that even in a world governed by a string theory compactification with \( N = 1 \) supersymmetry, supersymmetry can break at a much lower scale. Indeed, a common expectation has been that a phenomenologically realistic string theory would be compactified with \( N = 1 \) supersymmetry, leading to a low energy phenomenology given by the MSSM (minimal supersymmetric standard model), and that supersymmetry would be broken spontaneously at the weak scale. There seems no reason to believe that this is more or less likely for members of the discretum with slightly positive \( \Lambda \) than slightly negative \( \Lambda \).

The main relevant differences between the two cases, from the point of view of the weak anthropic principle, is that according to current understandings of the
landscape of string theory, there is a finite set of the slightly positive $\Lambda$ elements of the discretum, while the slightly negative $\Lambda$ discretum is an infinite set.

We may then reason in the following way. Consider an actually existing ensemble of universes which includes our own universe, which is selected from the ensemble because it allows life to exist. Let us define the function $f(\Lambda)$ to be the anthropic weight given to a universe in the ensemble with cosmological constant $\Lambda$, averaging over all other parameters. That is, it is the probability that intelligent life will occur in that universe. We require that $f(\Lambda)$ be a smooth, normalizable function such that if $\Lambda_i$ is the cosmological constant of the universe $u_i$,

$$f_i = f(\Lambda_i)w_i$$

(4)

Here $w_i$ is a weight that accounts for everything in the properties of the universe that affect the existence of intelligent life, except the cosmological constant. Assuming the existence of this function dependent only on $\Lambda$ is equivalent to assuming that the probability for life to occur is a separable function, so that we can consider the dependence on $\Lambda$ separately from the dependence on other parameters$^3$. This may not be true for all circumstances, but is probably true for universes close to ours in the ensemble of possibilities, that is, those with properties similar to that in which we live. In any case this is the assumption made by Weinberg in his analysis [6], and by most papers that followed that one. We will see what the result is on this basis, taken as a first approximation that allows an analysis of the probabilities. Further analyses could look into weakening this restriction; we doubt that will change the result.

For universes within the ensemble $\mathcal{L}$ of universes with life, we can assume there are minimum and maximum values $w_{\text{min}}$ and $w_{\text{max}}$ such that

$$\forall u_i \in \mathcal{L}, \ 0 < w_{\text{min}} \leq w_i \leq w_{\text{max}}.$$  

(5)

That is, given all other factors, the expectation for life does indeed depend crucially on the cosmological constant; but that dependence is not infinitely sensitive.

There are many details that go into the $w_i$ whose effects on the chance for intelligent life are difficult to estimate with present knowledge, such as whether supersymmetry breaking happens at the Planck scale or lower, or whether indeed supersymmetry breaking is necessary for life. But these details are not crucial; all we need is the condition (5).

We may also impose very weak conditions on $f(\Lambda)$ coming from our knowledge of these anthropic conditions. We define the ratio

$$r = \lim_{\epsilon \to 0^+} \frac{\int_{\Lambda_\epsilon}^\Lambda f(\Lambda) \, d\Lambda}{\int_{\Lambda_\epsilon}^{\Lambda_+} f(\Lambda) \, d\Lambda},$$

(6)

$^3$For a discussion of parametrisation of universes in relation to the anthropic principle, see [3].
As discussed above, there seems no reason that a slightly positive value of the cosmological constant should be infinitely more friendly to life than a slightly negative value. We then propose it is reasonable to assume

- **A**: \( f(\Lambda) \) is smooth and \( \int_{\Lambda_-}^{\Lambda_+} d\Lambda f(\Lambda) = 1 \).

- **B**: \( r \) is a non-zero, but finite number.

The first condition gives a normalization for \( f(\Lambda) \) and implies that \( f(\Lambda) \) is bounded. The second says that the conditions for intelligent life do not favor one sign of the cosmological constant over another sign more than can be expressed by a finite, non-zero ratio. Together with the first condition it implies that \( f(\Lambda) \) is bounded separately for positive and negative \( \Lambda \).

Given that there is very unlikely to be an anthropic reason to favor very small positive values of \( \Lambda \) over very small negative values, these seem reasonable assumptions that will in fact necessarily be fulfilled without implying any further restrictions on the physics considered. But they are the specific assumptions we will need in what follows, and we label them as such. In effect they explicate the anthropic requirements that have to be fulfilled in relation to the values of \( \Lambda \). Further analyses could look at weakening these conditions.

### 4 A prediction from combining string theory with the weak anthropic principle

Given the definitions made above, we can compute the probability that the cosmological constant is strictly greater than some positive value \( \epsilon < \Lambda_+ \) on the basis of the properties of the string theory landscape. Suppose that selection of the universe in which we live takes place in an ensemble associated with such a landscape. Let \( i(\epsilon, \Lambda_+) \) be the set of values \( i \) such that \( \epsilon \leq \Lambda_i \leq \Lambda_+ \) and \( i(\Lambda_-, \Lambda_+) \) be the set of values \( i \) such that \( \Lambda_- \leq \Lambda_i \leq \Lambda_+ \). Then on the basis of the present understandings of the landscape of string theory, summarized above, we argue as follows:

\[
P_{\Lambda > \epsilon} = \frac{\sum'_{i(\epsilon, \Lambda_+)} f_i}{\sum'_{i(\Lambda_-, \Lambda_+)} f_i}
\]  

(7)

For any finite value of \( \epsilon \) much smaller than the anthropic limit \( \Lambda_+ \) the numerator is a finite sum, albeit over a large number of elements. But the denominator is an infinite sum. Given the conditions **A** and **B** we have assumed for \( f \) it follows that each term in the sums in the numerator and denominator is finite, positive and
bounded. Then, for any positive $\epsilon$ much smaller than $\Lambda_+$ we have\textsuperscript{4}.

$$\Lambda_+ \gg \epsilon > 0 \Rightarrow P_{\Lambda > \epsilon} = 0$$  \hspace{1cm} (9)

Alternatively we can compute the expectation value of $\Lambda$. From (2) we have

$$\langle \Lambda \rangle = \sum_i f_i \Lambda_i \sum_i f_i$$  \hspace{1cm} (10)

It follows from our assumptions plus the results on the string landscape [10, 11] that

$$\langle \Lambda \rangle \leq 0.$$  \hspace{1cm} (11)

This is because there are infinitely more universes in (10) with $\Lambda \leq 0$ than for strictly positive values.

These predictions disagree with current astronomical observations, which show that $\Lambda > 0$ at present [16, 17], with the observations being compatible with the ‘dark energy’ causing the observed acceleration being a cosmological constant [21].\textsuperscript{5} Thus, one can say that given the assumptions A and B together with the current results on the string theory landscape, by using the WAP we reach a prediction that is falsified by current observations. This is the result claimed in the introduction.

We can also consider the probability $P_{\text{compatible}}$ that string theory is in the finite set, conjectured by Acharya and Douglas [12] to be compatible with all current observations. Given that this is a finite subset of string vacua, and that there is an infinite set of string vacua not compatible with current observations, one reaches the conclusion by similar reasoning that

$$P_{\text{compatible}} = 0.$$  \hspace{1cm} (12)

This reinforces the conclusion reached above. Similarly, we can consider the possibility that there is a continuous infinite set of zero cosmological constant, supersymmetric vacua.\textsuperscript{6} This would also strengthen the conclusion.

\textsuperscript{4}Here is the demonstration

$$P_{\Lambda > \epsilon} < \frac{w_{\text{max}}}{w_{\text{min}}} \sum_i f_i \Lambda_i \sum_i f_i = 0.$$  \hspace{1cm} (8)

This vanishes because the $f(\Lambda)$ are bounded, so the expression is a ratio of a finite sum of positive bounded terms to an infinite sum of positive bounded terms.

\textsuperscript{5}Even if the observations eventually show ‘dark energy’ varies with time and so is not a cosmological constant, the Landscape argument predicts there will indeed be a negative cosmological constant! This is not what is observed.

\textsuperscript{6}David Gross, Private communication.
5 Conclusions

We have found that, while it is difficult generally to extract falsifiable predictions from a combination of the WAP and the string landscape, there are exceptions in which the statistics of the string vacua are weighed so heavily towards one range of parameters, that there are predictions which are stable under a wide range of choices of probability weights. As we have seen here, the present data on the landscape, as of this date, allows such an argument to be made with regard to the sign of the cosmological constant, and that prediction happens to be in disagreement with observation.

It may very well be that this situation changes when the landscape is better explored. There are four basic possibilities, depending on whether $N_+$ or $N_-$ are finite or infinite. If both are finite or both are infinite, it may be difficult to get a firm prediction which does not depend on otherwise untestable assumptions about the probability weights. But if one is finite and the other infinite, as is the case with our present understandings, we can expect a firm prediction, one of which would be right, in that it agrees with the data, and the other wrong - it would indeed be falsified. The latter is what is in fact implied by the current status of understanding of the landscape.

ACKNOWLEDGEMENTS

We are grateful to Stephon Alexander, Michael Dine, Shamit Kachru, Washington Taylor and Jeff Murugan for correspondence and discussions on this issue. We are also grateful to Paul Davies for invitations to a workshop at Arizona State University where these issues were discussed. Research at Perimeter Institute for Theoretical Physics is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MEDT. The Cosmology Group at the University of Cape Town is supported by the NRF (South Africa) and the University of Cape Town Research Committee.

References

[1] L Smolin, Life of the Cosmos. (Oxford Univ. Press, NYC, and Weidenfeld and Nicolson, London, 1997).

[2] L Smolin, “Scientific alternatives to the anthropic principle”. arXiv: hep-th/0407213.

[3] G F R Ellis, U Kirchner and W Stoeger, “Multiverses and physical cosmology”. Mon Not Roy Ast Soc 347, 921 (2004). [arXiv: astro-ph/0305292].
[4] G F R Ellis, “Philosophy of cosmology”. In Handbook in Philosophy of Physics, Ed J Butterfield and J Earman (Elsevier, 2006), section 9.2. [arXiv: astro-ph/0602280].

[5] M J Rees, Just six numbers: The Deep Forces that shape the universe (Weidenfeld and Nicholson, London, 1999); M J Rees, Our Cosmic Habitat (Princeton and Oxford, 2003).

[6] S W Weinberg, “A priori distribution of the cosmological constant”. Phys.Rev. D61 103505 (2000) [arXiv: astro-ph/0002387]; “The cosmological constant problem”, arXiv: astro-ph/0005265 (2005).

[7] L Susskind, “The anthropic landscape of string theory”. arXiv: hep-th/0302219 (2003).

[8] L Susskind, The Cosmic Landscape: String Theory and the Illusion of Intelligent Design (New York: Little Brown, 2006).

[9] S Kachru, R Kallosh, A Linde and S Trivedi, “de Sitter vacua in string theory”. Phys. Rev. D68 (2003) 046005 [arXiv:hep-th/0301240].

[10] O DeWolfe, A Giryaets, S Kachru and W Taylor. “Type IIA Moduli Stabilization”. arXiv: hep-th/0505160 (2005).

[11] J Shelton, W Taylor, and B Wecht, “Generalized Flux Vacua”. arXiv: hep-th/0607015 (2006).

[12] B S Acharya and M R Douglas, “A Finite Landscape?”, arXiv: hep-th/0606212 (2006).

[13] B Freivogel, M Kleban, M R Martinez, and L Susskind, “Observational consequences of a landscape”, arXiv: hep-th/0505232 (2005).

[14] D N Spergel et al., “First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters”. Astrophysical Journal Supplement Series 148 175-194 (2003). [arXiv: astro-ph/0302209].

[15] M Tegmark et al, “Cosmological constraints from SDSS Luminous Red Galaxies”. arXiv: astro-ph/0608632 (2006).

[16] S Perlmutter et al., “Discovery of a supernova explosion at half the age of the universe”. Nature 391: 51 (1998).

[17] T R Choudhury and T Padmanabhan, “Cosmological parameters from supernova observations: A critical comparison of three data sets”. Astron.Astrophys. 429: 807 (2005).
[18] J Barrow and F Tipler, *The Cosmological Anthropic Principle*. (Oxford University Press, 1984).

[19] A. Linde, *Particle Physics and Inflationary Cosmology*. (Harwood Academic, 1990).

[20] J. Garriga and A. Vilenkin, *Testable anthropic predictions for dark energy*, *Phys.Rev.* **D67** (2003) 043503 [arXiv: astro-ph/0210358].

[21] H K Jassal, J S Bagla, and T Padmanabhan, “WMAP constraints on low redshift evolution of dark energy”. *Mon. Not. Roy. Astron. Soc.* **356**: L11 (2005).

[22] This paper was first drafted in late 2006, but we delayed posting in case the situation with regards to an infinity of negative cosmological constant vacua evolved so as to change the conclusions.