A Radio Bursts Detection Method Based on Hough Transform

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ABSTRACT

We present a simple and fast method for incoherent dedispersion and fast radio burst (FRB) detection based on the Hough transform, which is widely used for feature extraction in image analysis. The Hough transform maps a point in the data maps to a straight line in the parameter space, and points on the same dispersed $f^{-2}$ curve to a bundle of lines all crossing at the same point, thus the curve is transformed to a single point in the parameter space, enabling a simple way for the detection of radio burst. By choosing an appropriate truncation threshold, the method has a complexity of $O(N_{f}N_{d})$, where $N_{f}$, $N_{d}$ are the dimension of the data in frequency, time, and dispersion measure, respectively, this is lower than other existing methods. Using simulation data of different noise levels, we studied how the detected peak varies with different truncation thresholds. We also tested the method with some real pulsar and FRB data.

Key words: radio continuum: transients, methods: data analysis

1 INTRODUCTION

Astronomical radio pulses are dispersed while traveling through the interstellar medium (ISM) or intergalactic medium (IGM) plasma. At a lower frequency the wave travels at a lower speed and arrives at a later time. This dispersion of the arrival time significantly decreases the pulse amplitude at a fixed observation time. In order to improve the detection sensitivity, dedispersion of the signal is required to compensate for the time delay induced by dispersion. A number of dedispersion and detection algorithms have been developed over the years, the computation is demanding as it often needs to be done in nearly real time. For the recently discovered fast radio bursts (FRBs), which are bright millisecond radio pulses with unknown origin and mostly non-repeating, this is especially so. The inferred FRB rate is fairly high (Lorimer et al. 2007; Thornton et al. 2013; Petroff et al. 2015a; CHIME Scientific Collaboration et al. 2017). Efficient dedispersion algorithm would be very useful for searching FRBs.

The received signal can be de-dispersed by applying frequency dependent time delays to the signal prior to integration, but the difficulty is that usually the amount of dispersion is not known, so a large number of trials with different dispersion measures have to be attempted in each search. This brute force dedispersion procedure requires expensive computations, of a complexity $O(N_{f}N_{d})$. To speed up the dedispersion process, many algorithms are developed, for example, the tree dedispersion algorithm, which has a complexity of $O(N_{f}N_{f} \log N_{f})$ (Taylor 1974), the Fast Dispersion Measure Transform (FDMT) algorithm of complexity $O(2N_{f}N_{f} + N_{f}N_{d} \log_{2} N_{f})$ (Zackay & Ofek 2014), etc.

Obviously, the optimal dedispersion algorithm should maximize the signal-to-noise ratio of the pulse, which can only be fulfilled by integrating the flux exactly along the dispersion curve in the time-frequency domain. Mathematically, detection of such a curve can be achieved by a family of transformations, for example, the Radon transform (Radon 1917) and Hough transform (Hough 1962). The Radon transform maps a curve or more generally a shape $c(p)$ in a $D$-dimensional space to a parameter space by integral projection,

$$R_{c}(p) = \int_{x \in c(p)} f(x) dx = \int_{S^{D}} f(x) \delta(C(x;p)) \, dx,$$  

where $p$ is a vector of parameters describing the shape, $C(x;p)$ is a set of constraint functions that together define the shape, and $\delta(\cdot)$ denotes the Dirac delta function in the above, or the Kronecker delta in the discrete case. The Hough transform is closely related to the Radon transform (van Ginkel et al. 2004), though in its original formulation it is inherently discrete. It was originally designed to detect straight lines in binary images, but it can be extended to...
detect more general shapes and in grey-valued images. For this purpose, we set up an N-dimensional accumulator array \( A(p) \), each dimension of it corresponding to one of the parameters of the shape to be searched. Each element of this array contains the number of "votes" in favor of the presence of a shape with the parameters corresponding to that element. The votes are obtained as follows: for each point \( x_i \) with value \( g_i = I(x_i) \) in the input image \( I(x) \), if the shape passes through it, the vote for this shape parameter is increased by an amount of \( g_i \), i.e., let
\[
A(p) \leftarrow A(p) + g_i \delta(C(x_i; p)).
\]
(2)

If a shape with parameter \( p \) is present in the image, all of the pixels that are part of it will vote for it, yielding a large peak in the accumulator array. The shape detection problem in the image space is then transformed to a simple peak finding problem in the parameter space. As we usually do not know the dispersion measure in advance, the whole parameter space (in practice a range of dispersion measures) needs to be explored. Using the fact that most points in the data are background noise, we could truncate the data according to an appropriate threshold, this will throw away most of the noise below the threshold, thus making the map sparse, and the required computation is then drastically reduced.

The use of Hough transform for radio transients detection and dedispersion was investigated in Fridman (2010), in which a dispersed pulse is approximated as a straight line in the time-frequency plane within a small bandwidth. The data is first converted to a binary image, by taking a threshold given by \( 1\sigma \) value above the mean. The method was demonstrated with the application of Hough transform to the pulsar B0329+54 data observed by LOFAR in 10 MHz bandwidth.

In this paper we study the detection of radio pulse with Hough transform. We do not make the straight line approximation but detect directly the \( f^{-2} \) pulse track curve on the time-frequency plane, hence not limited in the usable bandwidth. In the truncation we will not fix the threshold, but use robust statistical quantities based on the median and median absolute deviation (MAD) to determine the threshold value, which are more reliable and less affected by outliers and strong pulse signals presented in the data. We apply the Hough transform to the truncated gray-valued image instead of the binary image, to help suppress the noise and improve the signal-to-noise ratio in the transformed parameter space.

2 ALGORITHM

We consider the Hough transformation algorithm for incoherent dedispersion and pulse search. Our input data is a time stream of spectrum which is the short time integral of the intensity, either from a single receiver or from the synthesized beam of an array. The dispersion delay of the pulse arrival time at a frequency \( f_1 \) relative to \( f_2 \) is given by
\[
\Delta t = t_1 - t_2 = d(f_1^{-2} - f_2^{-2}).
\]
(3)

where \( t_i \) is the arrival time of signal at frequency \( f_i \) in units of ms, \( d \approx 4.15 \times \text{DM} \), DM is the dispersion measure in units of \( \text{pc} \text{cm}^{-3} \). \( f_i \) is frequencies measured in GHz. Each dispersed pulse signal arrival time at frequency \( f \) then falls on a curve
\[
t = d f^{-2} + t_0.
\]
(4)

where \( t_0 \) is the time offset of the curve, which can be uniquely determined by the two parameters \((d, t_0)\).

2.1 Hough Transform

For a data point \((t_i, f_i)\) on the curve defined by Eq. (4), we have the relation of the two parameters as
\[
t_0 = -f_i^{-2}d + t_i,
\]
(5)

which in the parameter space \((t_0, d)\) is a straight line with slope \(-f_i^{-2}\) and interception \(t_i\), so each point on the curve defined in Eq. (4) in the data maps to a straight line in this space, and all of the points on the curve map to a bundle of lines, which all cross at the same point \((t_0, d)\). The problem of detecting a \( f^{-2} \) curve in the observing data is transformed to a peak detection problem, which is both easier and more robust. Furthermore, the presence of discontinuity and outliers have little effect on the peak detection in the parameter space, as long as there are enough identifiable points on the curve. Outliers, even ones in the form of a line, will not generate peaks as high as the one corresponding to the curve since they do not have the \( f^{-2} \) function form.

To apply the Hough transform, we initialize an all zero accumulator matrix \( A(t_0, d) \) of dimensionality \( N_b \times N_d \), with DM range \([d_{\text{min}}, d_{\text{max}}]\). For each point \((t_i, f_i)\) in \( I \), we accumulate a straight line given by Eq. (5) with strength \( I(t_i, f_i) \) to the accumulator \( A \), i.e.,
\[
A \leftarrow A + I(t_i, f_i) \delta(f_i^{-2}d - t_i + t_0).
\]
(6)

This will take \( O(N_p) \) operations. We see lines corresponding to points that are on the pulse curve Eq. (5) will all cross at the point \((t_0, d)\), generating a high peak at this point in \( A \), with value about \( \mu N_b \) where \( N_b \) is the number of points on the curve and \( \mu \) is the mean value of these points.

If the source dispersion value \( d_s \) is known a priori, as in the case of known pulsars, the DM range can be very narrow, otherwise a wide range should be chosen to cover possible dispersion for the searched signal. Once we have chosen the appropriate range \([d_{\text{min}}, d_{\text{max}}]\), the range of \( t_0 \) is
\[
t_{0,\text{min}} = -d_{\text{max}} f_{\text{min}}^{-2} + t_{\text{min}},
\]
(7)
\[
t_{0,\text{max}} = -d_{\text{min}} f_{\text{max}}^{-2} + t_{\text{max}}.
\]
(8)

and \( N_b \approx N_d \). The attainable resolution of \( d \) is determined by the time and frequency resolution: from Eq. (3), for neighboring frequency \( d \approx \frac{1}{2} \Delta f^3/\Delta f \), so
\[
\Delta d = \frac{3}{2} \Delta f^2 \Delta f = \frac{3}{2} \Delta f^2 \Delta t \approx f_{\text{min}}^2 \Delta t.
\]
(9)

Conversely, given a maximum size of the data that could be stored, the maximum time duration then limits the range of dispersion to be searched in full efficiency.

2.2 Background Subtraction

Before applying the Hough transform, we first pre-process the data by subtracting out the mean of the background in the time-frequency data frame. We can apply the Hough transform to this background-subtracted data, but then the
computation is inefficient, as most of the data are just noise, while the pulse signal if present only takes a very small portion of the data. In this case, every point in the data is to be transformed, leading to a worst case computational complexity of $O(N_t N_f N_d)$, which is the same as that of the brute force dedispersion procedure. To reduce the amount of computation, we can apply a truncation threshold to filter out most of the data before doing the Hough transform. Specifically, we can record in an array the data points $(t_i, f_i)$ whose background-subtracted value $|h(t_i, f_i)|$ higher than the truncation threshold, and do Hough transform according to Eq. (6) for only these data points by searching within this array. For each such point $(t_i, f_i)$, in the accumulator array $A$ we add a value $h(t_i, f_i)$ to all data points $(d_j, t_{0j})$ for $j = 1, \cdots, N_d$ located on the straight line Eq. (5). For each $d_j$ the corresponding $t_{0j} = -f_i^{-1}d_j + t_i$.

If the truncation threshold is set appropriately such that most of the pulse is preserved, and the pulse has a maximum length in the image $I(t_i, f_i)$, i.e., it pass the two points $(t_{\text{min}}, f_{\text{max}})$ and $(t_{\text{max}}, f_{\text{min}})$, then the number of points on the curve of the pulse is $\sim \max(N_t, N_f)$, where $N_t$ and $N_f$ are the numbers of time bins and frequency bins, respectively. Plus the remaining noise and maybe outliers, the number of points remain in the sparse image is of order $O(\max(N_t, N_f))$, for each point, it takes $O(N_d)$ operations to accumulate a straight line to the accumulator array, thus lead to a total computation complexity of order $O(\max(N_t, N_f) N_d)$.

For a background noise with a Gaussian distribution $N(\mu, \sigma^2)$, which is a good approximation for receiver noise or astronomy background in a short period of time, the threshold can be set as $T = \tau \sigma$, which will remove $\sim 68\%$, $\sim 95\%$ and $99.7\%$ of the data for $\tau = 1.0, 2.0$ and 3.0 respectively. In the truncation process, some pulse signal may also be thrown away, especially for low signal-to-noise ratio (SNR) data. The threshold $T$ should be chosen to achieve optimal detect sensitivity.

If strong outliers such as radio frequency interferences (RFIs) are present in the data, the Gaussian model of noise may not be valid. The more robust median and the median absolute deviation (MAD) may be used instead (Hampel 1974; Rousseeuw & Croux 1993; Leys et al. 2013; Fridman 2008). We set

\[ \hat{\mu} = \text{median}(I), \]
\[ \hat{\sigma} = \text{MAD}(I) \equiv \text{median}(|I - \text{median}(I)|)/0.6745. \]

In practice, the median and MAD do not need to be computed every cycle, but can instead be updated after a number of cycles to reduce the amount of computation. There is a small chance that along one of the curves the data points happen to fluctuate in such a way that they generate a peak in the accumulator matrix. However, the background mean has already been subtracted, the remaining noise has a zero mean truncated Gaussian distribution, so the expectation value of $\mathbb{E}(A_{d_i}) = 0$, as the positive and negative values of the noise will typically cancel out in the accumulation.

2.3 Outliers and Intermittent Signal

Outliers such as RFIs are often present in the data, which may be much stronger than the astronomical radio pulse signal. In most cases, however, the outliers would appear as vertical (short pulse in time) or horizontal (narrow frequency band) lines in the data $I(t, f)$. Such outliers are automatically filtered out, as these data points will be mapped into lines which will cross at $d = 0$ (no time delay) or $d = \infty$ (infinite large dispersion), which would not contribute to the accumulator matrix in the reasonable range of $[d_{\text{min}}, d_{\text{max}}]$. It is not very likely that the shape of the outliers happens to appear as a $f^{-2}$ curve with parameter $(d, t_0)$ in the right range, though in rare coincidence such event could be produced, e.g., as in the case of the so called “peryton” which has a roughly $f^{-2}$ shape (Petroff et al. 2015b). A simple automated algorithm as discussed here may not be able to identify all such cases, but hopefully the algorithm could filter out most outliers such that only a small number of pulse events remain and can be further investigated in detail with human intervention.

In the real data the pulse signal may also be intermittent, i.e. it is not continuous along the entire curve, but consists a few segments with gaps between them in the original data, or in the truncated data due to the truncation process. This may cause problem for methods that depend on the continuity of the curve, but the Hough transform method described in this paper has no such limitations. As long as enough points remained on the curve, and their sum has a significantly higher value than the background level, they will be accumulated to a high peak and be detected.
3 APPLICATION TEST

In this section we test how our algorithm works with data. We first test it with simulation data (3.1), then with real pulsar and FRB data (3.2).

3.1 Simulation Data

We generate a mock sample of observing data as a superposition of Gaussian noise and a dispersed FRB signal. The noise is independent and identically distributed (iid) Gaussian with a distribution $N(0, \sigma_n^2)$, and the signal is also iid Gaussian with distribution $N(\mu, \sigma_s^2)$. The parameters for the simulated data are set as $\sigma_n = 1.0$, $\mu = 3.0$, $\sigma_s = 3.0$, and $DM = 1000\text{ pc cm}^{-3}$. The observing frequency range is 400 – 800 MHz, with 2048 frequency bins. The simulated data $I(t, f)$ is shown in the top panel of Figure 1. The simulated data is truncated with a threshold $\tau = 3.0$. The truncated data $I_m$ is very sparse, as shown in the bottom panel of Figure 1, but the signal are mostly preserved.

The Hough transform of the truncated data is shown in the range of $DM \in [800, 1200]\text{ pc cm}^{-3}$ in Figure 2, from which we see the peak is just at the right location $DM = 1000\text{ pc cm}^{-3}$. Here we show both a 2D color plot and a 3D plot for better illustration, as the strongest point in the figure is too narrow that it is hard to see in the 2D plot.

We further explore effects of different noise level $\sigma_n$. The Peak-to-Median Ratio (PMR) $PMR = \max(A)/\text{median}(A_{>0})$ can be used as a measure of the relative strength of the highest peak to the noise background level, where $A_{>0}$ is the set of points for which the accumulator $A$ have positive values. The PMR as a function of the noise level $\sigma_n$ (with $\tau = 1, 2, 3, 4$) are plotted in the top panel of Figure 3 (shown in dB scale, i.e. $10\log_{10}(PMR)$). We see that generally the PMR decreases monotonically as the noise level $\sigma_n$ increases, but up to $\sigma_n = 6.0$ the overall PMR is still sufficiently (PMR=14.3, or 11.5 dB) for detection. We also plot curves for different threshold value $\tau$. A higher threshold $\tau$ generally yields better SNR at low noise level $\sigma_n$, but at high noise level $\sigma_n$ this is reversed.

In the Bottom panel of Figure 3 we plot the PMR (shown in dB scale) as a function of the selected threshold $\tau$, for fixed noise level but different mean values. Initially the PMR value increases as $\tau$ increases, but then it decreases after reaching a peak. The truncation process does not only reduce computation complexity, but also throws away the relatively noisy data, while preserving the data where the signal is stronger, so the PMR is enhanced for some appropriate $\tau$, and an optimal sensitivity is achieved at $\tau \sim 3.5$. The effect of truncation threshold may also depend on the mean background level. Several different mean value $\mu$ are plotted. However, as the curves show, although the peak value of PMR depends much on the $\mu$ value, for low $\mu$ the peak PMR is much small (e.g. when $\mu = 0.5$, we can get a peak PMR of 117.9(20.7 dB), while for $\mu = 3$ the peak PMR is up to $\sim 1350(31.3 \text{ dB})$). However, the optimal value does not change much. Even in the extreme case when $\mu = 0$, as long as $\sigma_s$ is much higher than $\sigma_n$, the pulse can be detected by the Hough transform method, for example, when $\mu = 0$, $\sigma_s = 3.0$, while $\sigma_n = 1.0$, we obtain a PMR of 19.6 (12.9 dB) when using a truncation threshold $\tau = 3.0$. These results show that the Hough transform pulse detection method is quite robust.

Figure 2. The Hough transform of the data shown in 2d (top) and 3d (bottom) plot.

Figure 3. Top: the Peak-to-Median Ratio (PMR) shown in dB for different noise level $\sigma_n$, Bottom: the PMR (shown in dB) for different threshold $\tau$. 
Radio burst detection by Hough transform

Event Telescope DM [pc cm$^{-3}$] $S_{\text{peak,obs}}$ [Jy] $F_{\text{obs}}$ [Jy ms] Ref
FRB 010125 Parkes 790(3) 0.30 2.82 Burke-Spolaor & Bannister (2014)
FRB 010621 Parkes 745(10) 0.41 2.87 Keane et al. (2011)
FRB 010724 Parkes 375 $>30.00 \pm 10.00$ $>150.00$ Lorimer et al. (2007)
FRB 110220 Parkes 944.38(5) 1.30 $\pm 0.00$ 7.2 $^{+0.13}_{-0.11}$ Thornton et al. (2013)
FRB 110523 GBT 623.30(6) 0.60 1.04 Masui et al. (2015)
FRB 110626 Parkes 723.0(3) 0.40 0.56 Thornton et al. (2013)
FRB 110703 Parkes 1103.6(7) 0.50 2.15 Thornton et al. (2013)
FRB 140514 Parkes 553.3(3) 0.47 $^{+0.32}_{-0.28}$ 1.32 $^{+0.50}_{-0.34}$ Petroff et al. (2015a)

Table 1. FRBs used in this paper and some of their parameters.

3.2 Real Pulsar and FRB Data

We first apply our method to real observation data of three pulsars, i.e. B0329+54, B1929+10, and B2319+60 taken by the Green Bank Telescope (GBT)\(^1\). We chose a truncation threshold $\tau = 3.0$, and used a 4000×2000 accumulator $A(\theta, d)$ with a DM range $[0, 100]$ pc cm$^{-3}$. The Hough transform for an example of the data lasting about 2 seconds containing several pulses are shown in the top panels of Figure 4 for the three pulsars. The corresponding Hough transform are plotted in the bottom panels. and we have also marked the detected peaks by a red + in the transformed images.

In the data shown in the plot, there is one single pulse track for B2319+60 (right column) with DM = 94.591 pc cm$^{-3}$, while for B0329+54 (left column) there are three tracks with DM = 26.764 pc cm$^{-3}$, and many tracks for B1929+10 (middle row) with DM = 3.1832 pc cm$^{-3}$. Each pulse track in the observed data has been transformed to a bundle of lines crossing at the same point, which can be detected easily with the program (though when plotted in Figure 4 they are visually not so obvious due to the small size of the peak point, to aid the eye we marked these by a cross in the figure.) If there are more than one cross points corresponding to more than one pulse track in the observed data, they all have the same DM value, which are all very close to the values measured with other programs (e.g. those given by \texttt{psrcat} in Manchester et al. (2005)). Note there are two strong narrow frequency band RFIs in the middle of each data, but they do not show in the corresponding Hough transformed images, as the Hough transform automatically rejects them.

We also apply our method to several FRB event data, including eight FRBs observed by the Parkes telescope taken from the FRB Catalogue\(^2\) compiled by Petroff et al. (2016), and one FRB event data (FRB 110523)\(^3\) observed by the GBT (Masui et al. 2015). These FRBs are listed in Table 1. Observing data of these FRBs are shown in Fig.5, and their corresponding Hough transformed result are shown in Fig.6, with the detected peaks marked by red + signs in the transformed image.

We tried a few different truncation thresholds, then ap-

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\(^1\) https://dss.gb.nrao.edu/project/GBT14B-339/public

\(^2\) http://www.astronomy.swin.edu.au/pulsar/frbcat/

\(^3\) http://www.cita.utoronto.ca/~kiyo/release/FRB110523/
We apply the Hough transform to search for them. We found that in most cases a threshold $\tau = 3.0$ is sufficient, but for a few weaker ones lower thresholds are required, specifically, $\tau = 2.5$ for FRB 120127, $\tau = 2.0$ for FRB 010724, and $\tau = 1.0$ for FRB 110626. We see from Table 1 that FRB 120127 and FRB 110626 have very low peak intensity $S_{\text{peak,obs}}$ and integrated intensity $F_{\text{obs}}$ relative to other ones, lower thresholds are needed for their detection. The lower threshold would require larger amount of computation, while in higher threshold they might be missed. The case of FRB 010724 is somewhat different, its pulse signal is fairly strong, but it could also only be detected with a lower threshold, say $\tau = 2.0$.

This may be related to its non-uniform background noise, as can be seen clearly in Figure 5, the background noise has big difference between the left and right part of the pulse track, this affects the background mean subtraction, and further makes the pulse detection harder. Nevertheless, all FRBs can be successfully detected by using the Hough transform method, and the DM values obtained from the detected peaks are very close to the public values as listed in Table 1.

Note that in our processing, we have not make any special treatment for the RFIs or any other outliers before the Hough transform. For all data except for the FRB 110523,
we have used the raw data, the only processing besides the truncation and Hough transform described above is rebinning in time direction to reduce the amount of data. For FRB 110523 the pre-processed data is available to us, which has been calibrated and RFI flagged. We can see from the data images, many of the data has RFIs or outliers in them, usually single frequency or narrow band RFIs, some are much stronger than the pulse signals. As we discussed in Section 2.3, they should not have much impact on the detection based on the Hough transform, and this is confirmed by the results. Note that in the present treatment, the search of $A$ is conducted at the pixel level, i.e. we search the pixel with maximum $A$ value, not integrating over neighboring pixels. This may not achieve the largest sensitivity, but simplifies the computation.

4 CONCLUSION

We have presented a simple and fast radio bursts detection and incoherence dedispersion method based on Hough transform. The $f^{-2}$ burst curve in the observed time stream data is mapped to be a bundle of straight line in the transformed space which crossed the same point determined by the dispersion measure of the burst. By detecting the peak, we can detect the bursts and measure their dispersion measures. The advantage of the method is that by setting an appropriate truncation threshold, it has a low computational complexity of $O(\max(N_r, N_f) N_d)$, which is much lower than the existing algorithms. Wise choice of noise truncation threshold may also improve the detection sensitivity. The method automatically rejects most commonly encountered RFIs, making it good for online (real time) bursts detection. We have shown its effectiveness by simulation and application to the real pulsar and FRB observing data.

The truncation of data which saves much computation is particularly important for real-time detection. The computation time for several different truncation thresholds are shown in Figure 7 for FRB 010125 with data dimension of 96 x 500 and for FRB 110220 with data dimension of 1024 x 500). The computation time are measured on an Intel Xeon E5-2670 2.60 GHz CPU using program written in the C programming language. For comparison, the brute force dedispersion is also implemented in the same computing environment, the algorithms run with a single thread for the same range and resolution of dispersion measure, and the time reported is the average time for 10 runs. For the computation of the median (and also the MAD) in the Hough transform method, we have simply implemented it by first sorting the array, which is not a very fast method for median computation, more effective methods exists, for example, the median of medians algorithm (Blum et al. 1973), which finds an approximate median in linear time. If some of the faster median computing methods are used, the performance of the Hough transform method can be further improved. But even for this simple implementation, the computation time of the Hough transform method with a truncation threshold $\tau \geq 1.0$ is less than that of the brute force method, and the computation times decays exponentially with the increasing truncate threshold $\tau$.

The Hough transform method is readily applicable to online (nearly real time) processing. It can also be easily parallelized to speed up the computation, by either partition the points in the truncated image to $N$ parts and do the Hough transform for these points in each part independently, with the total accumulator given by the sum, or by partition the computation of different dispersion measures. With these advantages, the method can be a very useful tool for the search of fast radio burst and pulsars.

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Figure 7. Comparison of the computation time for the Hough transform method with different truncation threshold $\tau$ and the brute force method, Top panel for FRB 010125, Bottom panel for FRB 110220.
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