Threshold Top Quark Production

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Abstract. Predictions for the total cross section for $e^+e^- \rightarrow t\bar{t}$ near threshold are reviewed. The renormalization group improved results at NNLL order have improved convergence and reduced scale dependence relative to fixed order results at NNLO. Prospects for measurements of the top-quark mass, width, and Yukawa coupling are discussed.

INTRODUCTION

The production of top quark pairs is among the important projects of a future linear collider. The large top quark width $\Gamma_t \sim 1.5$ GeV makes the threshold cross section look quite different from that of charm or bottom pairs. The large value of the width prohibits the production of toponium states, and at the same time serves as an infrared cutoff from sensitivity to non-perturbative effects. Thus, perturbative methods can be used to describe the top-antitop dynamics to a very high degree of precision. This makes the threshold region an ideal place for extracting fundamental top quark parameters such as the top mass, width, and Yukawa coupling (for a light higgs).

In the threshold region $\sqrt{s} \simeq 2m_t \pm 10$ GeV, the $t$ and $\bar{t}$ move with non-relativistic velocities. Defining the energy $m_t v^2 = \sqrt{s} - 2m_t$, we see that this region of $s$ corresponds to velocities $|v| \lesssim \alpha_s$. In this region an exact treatment of QCD Coulomb singularities $(\alpha_s/v)^k$ is required, ruling out a pure $\alpha_s$ expansion. However, a combined expansion in powers of $v$ and $\alpha_s$ can be performed. Schematically, for the $e^+e^- \rightarrow t\bar{t}$ cross section, $\sigma_{t\bar{t}}(s)$, we want an expansion of the form

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left( \frac{\alpha_s}{v} \right)^k \left[ 1 + \left\{ \alpha_s, v \right\} + \left\{ \alpha_s^2, \alpha_s v, v^2 \right\} + \ldots \right].$$

The power counting for these corrections can be implemented in a simple and systematic way using the effective theory framework of Non-Relativistic QCD (NRQCD).

The leading order (LO) prediction for $R$ is shown in Fig. 1. The large top width gives the threshold cross section a smooth line-shape, with a single bump from the remnant of the 1S toponium state. The characteristics of the cross section are sensitive to the top parameters. In particular, the top mass determines the location of the rise/peak.

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FIGURE 1. Leading order predictions for $R$. Results are shown for three renormalization scales of order the momentum transfer in the Coulomb potential, $\mu = 15\,\text{GeV}$ (upper), $30\,\text{GeV}$ (middle), and $60\,\text{GeV}$ (lower).

the top width determines the slope of the rise and shape of the peak, and the overall normalization provides information on $\alpha_s(m_t)$ and the top-Yukawa coupling for a light higgs. However, at LO the prediction for $R$ suffers from considerable scale uncertainty as shown by the three curves in Fig. 1. This illustrates the importance of including higher order terms. Several groups have computed the next-to-next-to-leading order (NNLO) QCD corrections to the total cross section (see Ref. [1] for a summary and comparison of these results). Using “threshold” top-quark mass parameters, an accurate prediction for the location of the peak in the cross section was obtained. It was concluded that infrared safe top masses could be determined with a precision of $< 200\,\text{MeV}$. However, surprisingly the NNLO corrections to the cross section normalization were big, with large scale uncertainty. The residual scale dependence was estimated to make the normalization of the cross section uncertain to $\approx 20\%$ [1]. These large corrections seemed to jeopardize measurements of the top width, strong coupling, and Yukawa coupling.

To understand the source of this large scale dependence it is useful to recall that the dynamics of the top-antitop system are governed by vastly different energy scales. These quarks have mass $m_t \sim 175\,\text{GeV}$, typical momenta $p \sim m_t v \sim 25\,\text{GeV}$, and energies $E \sim m_t v^2 \sim 4\,\text{GeV}$. The scattering amplitudes therefore involve logarithms

$$\ln\left(\frac{\mu^2}{m_t^2}\right), \quad \ln\left(\frac{\mu^2}{p^2}\right), \quad \ln\left(\frac{\mu^2}{E^2}\right),$$

which are not all small for a single choice of $\mu$. In the expansion for $R$ these large logarithms appear as logs of the velocity. This suggests that a better expansion might involve a summation of these logarithms

$$R = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_j (\alpha_s \ln v)^j \times \left[ 1 + \{\alpha_s, v\} + \{\alpha_s^2, \alpha_s v, v^2\} + \ldots \right].$$

The additional summation of logarithms can be performed using renormalization group equations (RGE). This is similar to how logarithms of $m_W/m_b$ are summed for the elec-
troweak Hamiltonian. In our case the summation is complicated by the presence of two low energy scales that are coupled by the equations of motion, $E = \frac{p^2}{(2m)}$. This complication can be dealt with by using a renormalization group with a subtraction velocity $v$ [2, 3, 4, 5, 6]. In this framework there are two renormalization group parameters in the effective Lagrangian: $\mu_S$ for soft gluons and $\mu_U$ for ultrasoft gluons, where $\mu_S = m_t v$ and $\mu_U = m_t v^2$. Running from $v = 1$ to $v \sim v$ sums logarithms of $v$ and minimizes both $\ln(\mu^2 / p^2)$ and $\ln(\mu^2 / E^2)$ terms in the amplitudes. (The $\ln(\mu^2 / m^2)$ terms are minimized by matching QCD onto the effective theory near $\mu = m_t$.) In Refs. [7, 8] it was shown that the expansion in Eq. (3) is better behaved than that in Eq. (1). Furthermore, the normalization of the cross section at NNLL has significantly smaller scale uncertainty ($\approx 3\%$) than the NNLO result. This review focuses on these results; for more details and further references see Ref. [8].

Below, the steps in the renormalization group improved calculation are briefly described. This is followed by predictions for the cross section as well as a description of the prospects for measurements of the top parameters in light of the reduced theoretical uncertainties at NNLL order.

### RENORMALIZATION GROUP IMPROVED CALCULATION

The computation of the renormalization group improved cross section can be divided into three parts:

1. Matching of QCD onto an effective theory for non-relativistic top quarks. Determine the Wilson coefficients $C(v)$ at $v = 1$ ($\mu = m_t$) to the desired order in $\alpha_s(m_t)$.
2. Scaling $C(v)$ from $v = 1$ to $v = v_0 \approx C_F \alpha_s$ by calculating anomalous dimensions and using the renormalization group. This scaling sums the terms of the form $[\alpha_s \ln(v)]^j$ in Eq. (3).
3. Computing the cross section using the Lagrangian and currents renormalized at the low scale $v = v_0$.

Each of these parts will be discussed in turn.

### 1. Matching onto the Effective Theory

For non-relativistic scattering the relevant momentum regions can be classified by their typical energy and momenta ($k^0, k$). They include hard modes with momenta $\sim (m, m)$, potential modes with momenta $\sim (mv^2, mv)$, soft modes with momenta $\sim (mv, mv)$, and ultrasoft modes with momenta $\sim (mv^2, mv^2)$. Fluctuations involving hard or offshell momenta are integrated out, while effective theory fields are introduced for modes with nearly on-shell momenta. The degrees of freedom therefore include potential top and anti-top quarks ($\psi_\mu$ and $\chi_\mu$), soft gluons and light quarks ($A_\mu$ and $\phi_q$), and ultrasoft gluons and light quarks ($A_\mu$ and $\phi_{us}$). Soft energies and momenta appear as labels on the fields while ultrasoft momenta are represented by explicit coordinate...
dependence [2]. This enables us to distinguish the size of momenta, for instance a derivative \( \partial^\mu \psi_p(x) \sim m v^2 \psi_p(x) \).

In this framework the action for non-relativistic top quarks has terms

\[
\mathcal{L} = \sum_p \psi_p^\dagger(x) \left\{ iD^0 - \frac{(p-iD)^2}{2m} + \ldots \right\} \psi_p(x) + (\psi \rightarrow \chi)
\]

\[
- \sum_{p,p'} F(p,p') \left[ \psi_{p'}^\dagger(x) \psi_p(x) \chi_{p'}^\dagger(x) \chi_p(x) \right]
\]

\[
-2\pi \alpha_s(m v) \sum_{p,p',q,q'} \psi_{p'}^\dagger \left[ A_q^\alpha, A_q^\beta U_{\alpha\beta} \right] \psi_p + (\psi \rightarrow \chi),
\]

where color and spin indices are suppressed. The covariant derivatives in the first line involve only ultrasoft gluons. The function \( F(p,p') \) in the second line contributes to the potential between quarks and anti-quarks. For our purposes

\[
F(p,p') = \frac{1}{k^2} V_c(v) + \frac{\pi^2}{m |k|} V_k(v) + \frac{1}{m^2} V_2(v) + \frac{S^2}{m^2} V_3(v) + \frac{(p^2 + p'^2)}{2m^2 k^2} V_4(v),
\]

where \( k = p' - p \), \( S \) is the total spin operator, and all coefficients are in the color singlet channel. The matching for these Wilson coefficients is needed at two loops for \( V_c \), one loop for \( V_k \), and tree level for \( V_2, V_3 \) [5, 6]. Finally, the third line in Eq. (4) is an example of the type of interaction that occurs between potential quarks and soft gluons, with \( U_{\alpha\beta}(p,p',q,q') \) a matching function of the label momenta. At NNLO (and NNLL order) time ordered products of two of these soft interactions also contribute terms to the potential giving

\[
V_{\text{soft}}(p,p') = - \frac{C_F \alpha_s(m v)}{k^2} \left[ - \beta_0 \ln \left( \frac{k^2}{m^2 v^2} \right) + a_1 \right]
\]

\[
- \frac{C_F \alpha_s(m v)}{4\pi k^2} \left[ \beta_0^2 \ln^2 \left( \frac{k^2}{m^2 v^2} \right) - \left( 2 \beta_0 a_1 + \beta_1 \right) \ln \left( \frac{k^2}{m^2 v^2} \right) + a_2 \right],
\]

where \( \beta_i \) are coefficients of the QCD \( \beta \)-function and the constants \( a_i \) can be found in Ref. [9]. The complete potential is then \( V(p,p') = F(p,p') + V_{\text{soft}}(p,p') \).

We also need to take into account that the top quarks decay. We will assume that the decay products are hard and can be integrated out. At lowest order this induces the operators

\[
\mathcal{L} = \sum_p \psi_p^\dagger \frac{i}{2} \Gamma_t \psi_p + \sum_p \chi_p^\dagger \frac{i}{2} \Gamma_t \chi_p.
\]

In the Standard Model the dominant decay channel is \( t \rightarrow bW^+ \) and gives a width of \( \Gamma_t = 1.43 \text{ GeV} \) which we will use as our central value. Counting \( \Gamma_t \sim m_t v^2 \), Eq. (7) gives a consistent next-to-leading order treatment of electroweak effects for the total cross section [10, 11]. Thus, we will not include electroweak decay related effects to the same order as the QCD corrections. From partial knowledge of these corrections [12], the missing terms are expected to be at the few percent level.
Besides the effective Lagrangian we also need external currents to produce the top-antitop pair. Since we wish to describe $e^+e^- \to \{\gamma', Z'\} \to t\bar{t}$ these currents are induced by both electromagnetic and weak interactions. The relevant vector current is $J^\nu_\mu = c_1(v) \ O_{p,1} + c_2(v) \ O_{p,2}$, where

$$O_{p,1} = \psi_p^\dagger \sigma(i\sigma_2) \chi^\nu_{-p} , \quad O_{p,2} = \frac{1}{m^2} \psi_p^\dagger p^2 \sigma(i\sigma_2) \chi^\nu_{-p} ,$$

and the relevant axial-vector current is $J^\nu_\mu = c_3(v) \ O_{p,3}$, where

$$O_{p,3} = -i \frac{2m}{\sigma} \psi_p^\dagger \sigma \cdot (i\sigma_2) \chi^\nu_{-p} . \quad (8)$$

The matching for these Wilson coefficients is needed at two loops for $c_1$ and tree level for $c_{2,3}$. The two-loop matching for $c_1$ is scheme dependent [13, 14, 15], and in the $\overline{\text{MS}}$ scheme with our definition of the operators, can be found in Ref. [8].

### 2. Renormalization group scaling

To sum the $(\alpha_s \ln v)^j$ terms in $R$ we must determine the anomalous dimensions for the Wilson coefficients $\mathcal{V}_{c,k,2,s,r}$ in Eq. (5) and the current coefficients $c_{1,2,3}$. The anomalous dimensions for $\mathcal{V}_{c,2}$, $\mathcal{V}_{k,2}$, and $\mathcal{V}_{2,2,s,r}$ are required at three, two, and one loop respectively. These have been computed in Refs. [3, 5, 6, 16, 17], and due to mixing depend on the one-loop running of the HQET terms up to $1/m^2$ [18]. These anomalous dimensions can contain terms like (with $b_i$ coefficients that depend on color factors)

$$\nu \frac{\partial}{\partial \nu} \mathcal{V} = b_1 \alpha_s^2(m_t \nu) + b_2 \alpha_s(m_t \nu) \alpha_s(m_t \nu^2) + b_3 \alpha_s^2(m_t \nu) \ln \left[ \frac{\alpha_s(m \nu^2)}{\alpha_s(m \nu)} \right]$$

$$+b_4 \frac{\alpha_s^2(m_t \nu)}{2} + \ldots . \quad (9)$$

For $\mathcal{V}_{c,k,2,r}$ both soft and usoft loops contribute, so their anomalous dimensions depend on both $\alpha_s(m_t \nu)$ and $\alpha_s(m_t \nu^2)$. The anomalous dimension for $\mathcal{V}_s$ comes only from soft loops but also involves the mixing of the $\psi \sigma \cdot B \psi$ operator whose Wilson coefficient is $c_F(v)$. Of the current coefficients, $c_3$ has no anomalous dimension, while $c_2(v)$ has contributions only from ultrasoft loops [7]. More interesting is the anomalous dimension for $c_1(v)$ which starts at two-loop order from purely potential loops. At this order [2]

$$\nu \frac{\partial}{\partial \nu} \ln[c_1(v)] = -\frac{\mathcal{V}_c(v)}{16\pi^2} \left( \frac{\mathcal{V}_c(v)}{4} + \mathcal{V}_2(v) + \mathcal{V}_s(v) + 2 \mathcal{V}_c(v) + \mathcal{V}_k(v) \right) + \frac{\mathcal{V}_k(v)}{2} , \quad (10)$$

so the solution for $c_1$ depends on the solutions for the potential coefficients. At three loops there are new contributions to the anomalous dimension for $c_1(v)$ coming from mixed potential-ultrasoft and potential-soft loops. These contribute at NNLL order but are currently unknown. In Ref. [8] these unknown terms were estimated to affect the cross section at the 2% level (this rough estimate was based on the size of known terms
and using dimensional analysis and parameter dependence to estimate the size of the contributions which are unknown). Of all the Wilson coefficients the one which is the most responsible for the difference in the NNLO and NNLL cross sections is \( V_k(\nu) \) which changes by an order of magnitude between \( \nu = 1 \) to \( \nu = 0.15 \). The two-loop running of this coefficient is shown in Fig. 2.

### 3. Cross section computation

The total cross section for \( e^+ e^- \to t \bar{t} \) is given by

\[
\sigma^{\gamma Z}_{\text{tot}}(s) = \frac{4\pi\alpha^2}{3s} \left[ F^v(s) R^v(s) + F^a(s) R^a(s) \right],
\]

where \( F^v \) and \( F^a \) are trivial functions depending on the charge and weak isospin of the fermions, and \( \sin \theta_W \). \( R^v \) and \( R^a \) are determined by

\[
R^v(s) = \frac{4\pi}{s} \text{Im} \left[ c_1^2(v) \mathcal{A}_1(v,m,v) + 2 c_1(v) c_2(v) \mathcal{A}_2(v,m,v) \right],
\]

\[
R^a(s) = \frac{4\pi}{s} \text{Im} \left[ c_3^2(v) \mathcal{A}_3(v,m,v) \right],
\]

where \( \mathcal{A}_i \) are time-ordered products of effective theory currents [\( \hat{q} = (\sqrt{s} - 2m_t, 0) \)]

\[
\mathcal{A}_i(v,m,v) = i \sum_{p,p'} \int d^4x e^{i\hat{q} \cdot x} \left\langle 0 \left| T \ O_{p,1}(x) O_{p',1}^+(0) \right| 0 \right\rangle,
\]
\( A_2(v,m,v) = \frac{i}{2} \sum_{p,p'} \int d^4x e^{i\vec{q}\cdot \vec{x}} \left\langle 0 \left| T \left[ O_{p,1}(x) O_{p',2}^\dagger(0) + O_{p,2}(x) O_{p',1}^\dagger(0) \right] \right| 0 \right\rangle, \)

\( A_3(v,m,v) = i \sum_{p,p'} \int d^4x e^{i\vec{q}\cdot \vec{x}} \left\langle 0 \left| T O_{p,3}(x) O_{p',3}^\dagger(0) \right| 0 \right\rangle. \) (13)

These time-ordered products can be evaluated in terms of non-relativistic Greens functions to give

\[
A_1(v,m,v) = 18 \left[ G^c(v,m,v) + (V_2(v) + 2V'_1(v)) \delta G^\delta(v,m,v) \right.
\]

\[
+ V'_1(v) \delta G'(v,m,v) + V'_k(v) \delta G'^k(v,m,v) + \delta G^{\text{kin}}(v,m,v)] ,
\]

\[
A_2(v,m,v) = v^2 A_1(v,m,v), \quad A_3(v,m,v) = 12 G^1(v,m,v)/m^2, \] (14)

Here \( G^c \) are evaluated numerically with the \( 1/k^2 \) term in \( F(p,p') \) and \( V_{\text{soft}}(p,p') \) [19]. In Ref. [8] we analytically evaluated \( \delta G^{\delta,r,k,kin} \) with a single insertion of the corresponding potentials or \( p^4 \) kinetic energy correction. The P-wave Greens function \( G^1 \) was also evaluated in closed form. The analytic calculations enabled all ultraviolet subdivergences to be subtracted in \( \overline{\text{MS}} \) which is necessary to be consistent with the scheme dependence of the Wilson coefficients. In Eq. (14) the velocity \( v = (\sqrt{s} - 2m_t + i\Gamma_t)^{1/2}/m_t^{1/2} \) and \( m = m_t \) is the pole mass. The Greens functions depend on the subtraction velocity through \( \ln(v^2/v^2) \) and these logarithms are not large when the Greens functions are evaluated at the low scale \( v \approx v_0 \). At this scale all large logarithms have been resummed in the potential and current Wilson coefficients. Typically, \( v_0 \approx 0.15 - 0.2 \), but to numerically test the remaining scale dependence we use the larger range \( v_0 = 0.1 - 0.4 \).

**RESULTS**

Soon after the NNLO results were derived it was realized that the inherent uncertainty in the top-quark pole mass due to infrared renormalons causes problems for predictions for the peak in the cross section. Therefore, for precision predictions the pole mass is not a suitable mass parameter. The \( \overline{\text{MS}} \) top-mass provides an infrared safe alternative, however it complicates the non-relativistic power counting. Essentially, it shifts \( E \) by an amount \( \sim m_\alpha s \), which is much larger than the original size of the energy \( E \sim m_\alpha s^2 \). Both of these problems can be addressed by switching to a “threshold mass”, defined as an infrared safe mass parameter which differs from \( m_t^{\text{pole}} \) by \( \sim m_\alpha s^2 \). Several possible threshold masses were suggested, including the PS mass [20], kinetic mass [21], and 1S mass [22, 12]. In Ref. [1] it was concluded that threshold masses could be determined with a precision of \( < 200 \) MeV from the total cross section. Converting the result to an \( \overline{\text{MS}} \) top-mass would then lead to a similar precision for this parameter. In this section predictions will be given using the 1S mass parameter. For a detailed description of how the NNLL pole mass expressions are converted to the 1S mass in a manner consistent with the power counting see Ref. [8].

We begin by comparing results in the fixed order and renormalization group improved expansions. We concentrate on \( R^v \) since \( R^a \) gives only a small contribution to the cross
FIGURE 3. Comparison of $Q^2 R^v$ with fixed $M_{1S}$ mass for the fixed order and resummed expansions. The dotted, dashed, and solid curves in a) are LO, NLO, and NNLO, and in b) are LL, NLL, and NNLL order. For each order four curves are plotted for $v = 0.1, 0.125, 0.2, \text{ and } 0.4$.

section, and is essentially identical in the two approaches. The $R^v$ results are shown in Fig. 3 and use $M_{1S} = 175$ GeV, $\alpha_s(m_Z) = 0.118$ and $\Gamma_t = 1.43$ GeV. At each order in the expansions four curves are shown which correspond to $v = 0.1, 0.125, 0.2, \text{ and } 0.4$. It is clearly visible that the NNLL results in Fig. 3(b) have much smaller scale dependence than the NNLO results in Fig. 3(a). It should be noted that our NNLO results shown in Fig. 3(a) agree quantitatively with those presented in Ref. [1]. The uncertainty in these results stems to a large extent from the uncertainty in the choice of the renormalization scales in the NNLO contributions. Essentially what the anomalous dimensions and renormalization group do is remove this uncertainty. Also, more than half of the improved convergence of the NNLL result is due to the reduced size of $V_k(v=v_0)$ compared to $V_k(1)$.

From the remaining scale uncertainty and the size of some higher order QCD corrections, the uncertainty in the NNLL cross section was conservatively estimated to be $\pm 3\%$ [8]. This level of precision should enable extractions of various top parameters from the cross section with fairly good precision. In Fig. 4 we show the scale dependence of the peak position for the NNLO (dashed) and NNLL (solid) predictions. The
FIGURE 5. Variation of the NNLL cross section for (a) the inclusion of a Standard Model (SM) Higgs boson and (b) the value of the strong coupling. The relative changes are shown by red dashed lines. For (a) the lower black solid line is the decoupling limit for the Higgs boson, and the upper blue solid line is for a SM Higgs with mass $m_H = 115$ GeV.

NNLL prediction is slightly less scale dependent than the NNLO prediction until we get to small $v$. For $v < 0.15$ the larger scale dependence at NNLL is explained by the fact that these predictions depend on the coupling $\alpha_s(m_t \nu^2)$, while the NNLO predictions do not. Also shown in Fig. 4 are the NNLL predictions for the total cross section varying the width by ±10%. The size of the variations indicate that a measurement with better than 10% precision is definitely feasible. In Fig. 5 the dependence of the cross section on the top Yukawa $y_t$ (for a Higgs mass $m_H = 115$ GeV) and on $\alpha_s(m_Z)$ are shown. It looks quite promising that a ±20% variation in $y_t$ gives a larger change in the cross section than our estimate for the remaining theoretical uncertainty. It should be kept in mind that since both $y_t$ and $\alpha_s(m_Z)$ mainly effect the normalization, at some level these parameters cannot be fixed independently using only the total cross section.

CONCLUSION

In this talk I have discussed predictions for the threshold $e^+e^- \rightarrow t\bar{t}$ cross section at NNLO and NNLL order as defined by the expansions in Eq. (1) and Eq. (3). The NNLL predictions made in Refs. [7, 8] sum large logarithms of the velocity using results for the renormalization group improved Wilson coefficients from Refs. [2, 3, 5, 6]. One missing ingredient is the three loop anomalous dimension for $c_1$, for which only partial results are known. However, rough estimates indicate that this missing anomalous dimension is unlikely to affect the cross section at more than the 2% level [8]. The stability of predictions for the peak in the cross section are very similar at NNLO and NNLL, so that measurements with $\delta m_t < 200$ MeV for short distance masses are feasible with either expansion. The size of the NNLL normalization corrections and variation of the NNLL cross section for various choices of the renormalization parameter are an order of magnitude smaller than the results of earlier NNLO calculations. A conservative estimate of the remaining theoretical uncertainty in the total cross section is ±3% [8]. With such small uncertainty, measurements of top parameters with uncertainties $\delta \alpha_s(m_Z) \sim 0.002$, $\delta \Gamma_t/\Gamma_t \sim 5\%$, and $\delta y_t/y_t \sim 20\%$ appear feasible. However, realistic simulation stud-
cies should be done to see how these numbers hold up once effects such as initial state radiation, beamstrahlung, and the beam energy spread are taken into account.

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