Comparative study on current recommendations on technologies for the Teaching of Mathematics in Brazil and Portugal

Marcelo de Oliveira Dias\textsuperscript{a}  
Leonor Santos\textsuperscript{b}

\textsuperscript{a} Universidade Federal Fluminense, Instituto do Noroeste Fluminense de Educação Superior, Rio de Janeiro, Brazil.  
\textsuperscript{b} Universidade de Lisboa, UIDEF do Instituto de Educação, Lisboa, Portugal.

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ABSTRACT

Background: The general guidelines of the prescribed Mathematics curricula in force in Brazil and Portugal emphasise the use of digital technologies for Mathematics Education to meet the demands of the globalised world. Objectives: To analyse perspectives expressed in the documents and speeches that circulated in the reform processes of the curricular guidelines. Design: Comparative research for the analysis of recent reforms concerning the recommendations on the use of technologies. Setting and participants: Literature review and documentary analysis of the curriculum guidelines in Brazil for the final years of elementary school in Brazil and for the last year of the 2nd cycle and the entire 3rd cycle in Portugal. Data collection and analysis: Qualitative documentary analysis, the categories of analysis emerged from the literature review and trends that are under discussion and will compose the reference framework of the Mathematics Project 2030. Results: In the Digital Literacy category, we observed an emphasis on digital applications aiming at changing the framework, representation and unclear communication of the real objectives; and in the Computational Thinking category, we observed a focus on building algorithms in a limited and very specific way and using language, aiming at the development of mathematical logical thinking in structuring digital applications, in formulating and solving problems. Conclusions: The study raised the need to discuss issues related to the clear definition of objectives, the emphasis on the adoption of software primarily for illustration, limits on the connection between computational thinking and algebraic language, and the focus on rigour in the development of skills for control and management.

Keywords: Comparative research, Prescribed mathematics curricula in force, Brazil and Portugal, Digital Technologies, Mathematics Education.

Corresponding author: Marcelo de Oliveira Dias. Email: marcelo_dias@id.uff.br
Estudo comparado sobre recomendações vigentes sobre tecnologias para o Ensino de Matemática no Brasil e em Portugal

RESUMO
Contexto: As orientações gerais dos currículos prescritos de Matemática vigentes no Brasil e Portugal enfatizam à utilização de tecnologias digitais para a Educação Matemática, em uma tentativa de atender às demandas do mundo globalizado. Objetivo: Analisar perspectivas expressas nos documentos e nos discursos que circularam nos processos de reforma das orientações curriculares. Design: Investigação comparativa para a análise das recentes reformas no que tange a recomendações acerca da utilização de tecnologias. Coleta e análise de dados: Análise qualitativa documental, considerando orientações curriculares vigentes que se referem aos anos finais do ensino fundamental no Brasil, correspondendo em Portugal ao último ano do 2.º ciclo e todo o 3.º ciclo. As categorias de análise emergiram da revisão de literatura e de tendências que se encontram em discussão e irão compor o quadro de referência do Projeto Matemática 2030. Resultados: Na categoria Literacia Digital, evidenciou-se a ênfase em aplicações digitais visando mudanças de quadro, representação e comunicação sem clareza dos reais objetivos; e na categoria Pensamento Computacional, foco na construção de algoritmos de forma limitada e muito específica e uso de linguagem, visando o desenvolvimento do pensamento lógico matemático na estruturação de aplicações digitais, na formulação e solução problemas. Conclusões: O estudo suscitou a necessidade de discussão de questões relativas à definição clara de objetivos, a ênfase na adoção de software prioritariamente para ilustração, limites na ligação entre pensamento computacional e a linguagem algébrica, bem como o foco ao rigor no desenvolvimento de destrezas para controle e gestão.

Palavras-chave: Investigação comparativa, Currículos prescritos de Matemática vigentes, Brasil e Portugal, Tecnologias Digitais, Educação Matemática.

INTRODUCTION

This article aims to problematise the recent educational reforms in Brazil and Portugal, relating to the use of digital technologies (DT) in mathematical learning. To this end, the following questions were raised: What are the general guidelines of the prescribed documents in force in both countries for mathematics regarding the use of DTs? What are the similarities and specificities between the mathematics programs of the two countries? These issues were problematised from the understanding that such documents emphasise this perspective in an attempt to meet the demands of the globalised world: students who communicate and solve situations and problems mathematically through DTs, which curricular programs intend to account for in their educational pathways.

We agree with Macedo’s (2000, p. 171) view of the prescribed curriculum, i.e., as “[...] a document that legitimises the school existence itself, even knowing that the real curriculum far transcends the official document [...].” This choice is justified by the complexity of factors involved in the elaboration and development of curricula in different contexts, which the programmes intend to account for in their curriculum and educational pathways.

This article appropriates this definition to analyse how the DTs are configured in the programmes and the approaches of the curricular guidelines given by both countries.
Thus, it focuses on the general recommendations contained in the curriculum guidelines in force in each one.

In Brazil, the *Base Nacional Comum Curricular – BNCC* (National Common Curricular Base), by prescribing one of the general competencies that should permeate its components, refers to the digital culture:

> Using digital communication and information technologies in a critical, meaningful, reflective, and ethical way in the various everyday practices (including school practices) when communicating, accessing and disseminating information, producing knowledge and solving problems. (Brazil, 2017, p. 63)

In Portugal, Art. 6 of Decree-Law No. 55/2018, on the purpose of the curriculum and its promotion, establishes principles, values and areas of competence that must obey the development of the curriculum due to globalisation and technological development, aiming to prepare students who will be young and adults in 2030:

> To achieve this purpose, and without prejudice to the autonomy and flexibility exercised by the school, the conception of the curriculum underlies the following principles. Promotion of learning within the discipline of Information and Communication Technologies. (Presidência do Conselho de Ministros, 2018, p. 2931)

The country’s guidelines foresee that the competencies defined for compulsory education are guaranteed, prescribing them as one of the principles of learning in the context of the discipline Information and Communication Technologies (ICT). This presupposes literacy in ICT for the proper use of tools, reinforced Decree-Law No. 55/2018 of 2018, in Art. 12, Curricular autonomy and flexibility, paragraph 4: “In the 2nd and 3rd cycles, the basic curricular matrices integrate the Citizenship and Development component and, as a rule, the ICT component” (Presidência do Conselho de Ministros, 2018, p. 2933).

In Brazil, the document prescribes the DTs with the need to use the ICTs critically and reflectively for the production of new knowledge and problem-solving. In Portugal, capacity building involving ICTs in the context of Autonomy and Curricular Flexibility is contributing to promoting learning.

Thus, the curriculum guidelines of both countries, when considering the incorporation of DTs in the curriculum components, recommending work perspectives involving Digital Literacy (DL) and Computational Thinking (CT) for the new generations, raise implications for the development of mathematics programs, which justifies the relevance of this study and the choice of Brazil and Portugal.
COUNTRY EDUCATIONAL CONTEXTS

The organisation of the Brazilian and Portuguese educational systems

The current structure of Brazilian education stems from the **Lei de Diretrizes e Bases** - LDB (Law of Guidelines and Bases of Education) (Law No. 9.394/96), which is linked to the general guidelines of the Federal Constitution of 1988, as well as to the respective Constitutional Amendments in force. The Portuguese version derives from the **Lei de Bases do Sistema Educativo** - LBSE (Basic Law of the Educational System) (Law No. 46/86), and its 4th version is the most recent (Law No. 85/2009).

According to Art. 21 of the LDB, Basic Education in Brazil is formed by Early Childhood Education and Elementary and High School (Figure 1). Education systems can unfold elementary school into teaching-learning cycles, observing its rules. Education became mandatory for children aged 4 through 17 due to the change made in the LDB through Law No. 12.796, of April 4, 2013. This regulation formalised the change made to the Federal Constitution through Constitutional Amendment No. 59 in 2009.

The Portuguese Educational System, according to the LBSE (Art. 6), includes Early Childhood Education and Basic, Secondary and Higher Education. Compulsory schooling consists of Primary and Secondary Education (children aged 6 through 17). Basic Education lasts nine years. It is structured, according to Art. 8, in three sequential cycles of 4, 2 and 3 years (Figure 1).

Figure 1
Organisation of the educational system of Brazil and Portugal

| COUNTRY | AGE CORRESPONDING TO STUDENTS PER PERIOD |
|---------|-----------------------------------------|
|         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| Day care|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Preschool|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Early Childhood Education|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Elementary School: Early Years|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Elementary School: Final Years|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| High School|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Integrated* - have completed Elementary School.|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Concomitant* - attending the second year of preparatory high school.|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

| ADMISSION OF NON-PERIODISED STUDENTS |         |
|--------------------------------------|---------|
| Elementary School Early and Final Years|         |
| Youth and Adult Education (EJA): Minimum 15 years old |         |
| Special Education                     |         |
| Initial and continuing education courses (FIC) or Vocational Qualification * - short courses or courses at elementary school level for young people and adults who have not completed elementary school at the expected age. |         |
| High School                           |         |
| Propaedeutics- Youth and Adult Education (EJA): Minimum of 18 years old. |         |
| Subsequent professional * (requirement to have already completed preparatory High School (PROEJA) - minimum of 18 years old. |         |

| Early Childhood Education | Basic Education | Secondary Education |
|---------------------------|----------------|---------------------|
| Day care                  | 1st Cycle      | 2nd Cycle          |
| Preschool                 | Adult Education: | Adult Education:   |
|                           | . Literacy Education | . Literacy Education |
|                           | . Short-Term Actions | . Short-Term Actions |
|                           | Recurring Education (3rd cycle) | Recurring Education (3rd cycle) |
|                           | CEF – Education and Training Courses (Level 2) | CEF – Education and Training Courses (Level 2) |
|                           | PCA – Alternative Curriculum Pathways | PCA – Alternative Curriculum Pathways |
|                           | PIEF – Integrated Education and Training Plan | PIEF – Integrated Education and Training Plan |
|                           | Humanistic Scientific | Humanistic Scientific |
|                           | Courses Professional | Courses Professional |
|                           | Courses (Level II) | Courses (Level II) |
|                           | Specialised Artistic Teaching Course | Specialised Artistic Teaching Course |
|                           | EFA | EFA |
|                           | Courses (NS) | Courses (NS) |
|                           | Technological Courses (Level III) | Technological Courses (Level III) |

Caption: *Level of professional qualification.

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Elementary School in Brazil, according to Art. 32 of the LDB, aims at the basic training of the citizen, through, among other aspects: (i) the development of the ability to learn, having as primary means the full mastery of reading, writing, and calculation; (ii) the understanding of the natural and social environment, the political system, technology, arts and values on which society is based; (iii) the development of learning capacity, with a view to the acquisition of knowledge, skills and the formation of attitudes and values.

In Portugal, Art. 7 of LBSE highlights the objectives of Basic Education. Among others, they are: ensuring a standard general formation that guarantees the discovery and development of interests and skills, reasoning ability, memory and critical spirit, creativity, moral sense, and aesthetic sensitivity, promoting individual achievement in harmony with the values of social solidarity and creating conditions to encourage all students to be successful in school and education.

In the organisation of the educational systems, we identified similarities regarding the emphasis on the purposes of general education, the promotion of attitudes and ethical values, and the compulsory high school. As a specificity, the organisation in cycles in elementary school is optional in Brazil. In Portugal, it is mandatory. Preschool in Brazil is compulsory, unlike Portugal, where education is mandatory only from Basic Education.

In this study, the Final Years of Elementary Education in Brazil and Basic Education in Portugal were considered regular, of a non-professional and propedeutic qualification nature, since they are configured in a mandatory stage where knowledge considered essential for the area of Mathematics should be developed.

**Contexts of reform of curriculum guidelines in force in Brazil and Portugal**

Dias (2016, p. 39) warns that “the researcher cannot dispense with knowing satisfactorily the political situation that led to the production of a particular document,” so we will bring a brief description of the legal aspects, and the bases that support the current documents prescribed in force in the countries.

The BNCC was provided for in the Federal Constitution for Elementary Education and expanded in the *Plano Nacional de Educação* – PNE (National Education Plan) for High School, to rework and make meaning of basic education in Brazil. With its approval:

> […] the teaching networks and private schools will have the task of constructing curricula based on the essential learning established, thus moving from the propositional normative plan to the action plan and curriculum management, which involves the whole set of decisions and actions defining the curriculum and its dynamics. (Brazil, 2017, p. 20)
Based on these constitutional frameworks, Item IV of Art. 9 of the LDB states that:

[...] it is incumbent upon the Union to establish, in collaboration with the States, the Federal District and the Municipalities, competencies and guidelines for Early Childhood Education, Elementary School and High School, which will guide the curricula and their minimum contents, to ensure common basic training. (Lei de Diretrizes e Bases da Educação, 1996 *apud* Brazil, 2017, p. 10)

From this item, the BNCC highlights as clear two decisive concepts for the entire development of the curricular issue in Brazil, based on two notions considered as foundational: what is or is not common-basic, and the essential learning as a focus.

The first, already anticipated by the Constitution, establishes the relationship between what is common-basic and what is diverse in curriculum matters: competencies and guidelines are common, curricula are diverse. The second refers to focus. By saying that the curriculum contents are at the service of the development of competencies, it guides the definition of essential learning, and not only of the minimum content to be taught. (Brazil, 2017, p. 11)

Autonomous teams and a complex process of sending suggestions for analysis and promotion of state debates were created. In 2017, the 3rd (final) version of the BNCC (Brazil, 2017) for the Early and Final Years of Elementary School was approved, being implemented as of 2019.

The BNCC (Brazil, 2017) proposes five correlated thematic units that guide the formulation of skills to be developed throughout Elementary Education. This document defines competence as the “mobilisation of knowledge (concepts and procedures), skills (practices, cognitive and socio-emotional), attitudes, and values to solve complex demands of everyday life, the full exercise of citizenship and the world of work” (Brazil, 2017, p. 8).

In this sense, objects of knowledge and prescribed skills represent the crucial points of the BNCC proposal for the development of essential mathematical competencies. Teaching through competence starts from the general considerations that are comprehensive for all areas of knowledge, but there are still specific recommendations for Mathematics area:

Mathematical knowledge is necessary for all Basic Education students not only for its broad application in contemporary society, but also for its potential in the formation of critical citizens, aware of their social responsibilities. (Brazil, 2017, p. 263)
The reform focused on competencies that can serve a reductionist management model, which primarily focus on performativity. In Ball’s view (2010, p. 38), the “performances – of individual subjects or organisations - serve as measures of productivity or results, as forms of quality presentation or moments of promotion or inspection.” These competencies are pointed out as a solution to the problems of mathematics teaching in Brazil, but surrounded by turbulent processes, where the architecture of the curriculum proposal was apparently democratic, but which represents a vertically imposed global model with perspectives that generate resistance.

In Portugal, according to the Direção Geral de Educação – DGE (Directorate-General for Education), the last Review of the Curricular Structure, legitimised in Decree-Law No. 139/2012, of July 5, as well as in Order No. 5306/2012, of April 18, provides for improvements in the quality of teaching and learning through a culture of rigour and excellence since Basic Education. To achieve this end regarding Basic Education Mathematics, the Metas Curriculares de Matemática do Ensino Básico – MCMEB (Basic Education Mathematics Curricular Goals) (Bivar, Grosso, Oliveira, & Timóteo, 2012) were elaborated, listing the general objectives, specified by descriptors, “drafted concisely and pointing to accurate and evaluable performances” (Brazil, 2017, p. 1).

The Curricular Goals were built based on the thematic contents expressed in the Programa de Matemática do Ensino Básico – PMEB (Basic Education Mathematics Programme) of 2007 (Ponte et al., 2007). In this document, several general objectives and their descriptors were designed, establishing links between content without evident mutual relationship. The item “A Matemática como um todo coerente” (Mathematics as a coherent whole) emphasises that:

[…] In addition to the situations that are explicitly illustrated in the Curricular Goals, others can be treated within the scope of exercises and problems. These activities are conducive to the understanding that Mathematics consists of a complex network of relationships that gives it a very particular unity. (Bivar et al., 2012, p. 5)

In the Curricular Goals, “the contents are organised, in each cycle, by domains. The desirable articulation between the content domains and the objectives stated above is materialised” (Bivar et al., 2012, p. 5). A year later, the Programa e Metas Curriculares de Matemática do Ensino Básico - PMCMEB (Basic Education Mathematics Programme and Curricular Goals) (Bivar, Grosso, Oliveira, & Timóteo, 2013) emerged, renamed as Programa de Matemática do Ensino Básico - PMEB (Basic Education Mathematics Programme).

Thus, the Directorate-General for Education points out that the PMEB (Bivar et al., 2013) was built based on the thematic contents expressed in the 2007 PMEB (Ponte et al., 2007). It also points out that the organisation of these contents in a hierarchy that is announced as coherent and consistent caused gaps between this Programme and the Curricular Goals, legal and mandatory regulations.
Subsequently, the Orientações de Gestão Curricular para o Ensino Básico – OGCEB (Curricular Management Guidelines for Basic Education) (DGE, 2016) were configured as the guidelines for the discipline of Mathematics, governed by PMEB (Bivar et al., 2013). These documents introduce general methodological guidelines, as well as proposals for flexibility and content management, with indications that should be considered according to the school context. On July 6, 2018, the document Aprendizagens Essenciais – AE (Essential Learnings) was released through Decree-Law No. 55/2018. Its Art. 17 states that:

> Essential Learnings constitute basic curriculum guidance, for planning, conducting, and evaluating teaching and learning, in each year of schooling or training, curriculum component, disciplinary area, and discipline. (Ministério de Educação, 2018, p. 2934)

According to the DGE, the AEs (Ministério de Educação, 2018) are curriculum guidance based on the planning, execution, and assessment of teaching and learning, leading to the development of skills enrolled in the students’ profile (Perfil do Aluno – PA) at the end of the compulsory schooling. For each year of schooling, the AEs (Ministério de Educação, 2018), built from the PMEB (Bivar et al., 2013), which remains in force, establish the knowledge, skills, and attitudes students should develop.

The DGE, understanding that there is a problem of diversity of curriculum documents unanimously recognised in Portugal, tried to identify, discipline by discipline and year by year, the essential set of contents, skills and attitudes, resulting in the AE (Ministério de Educação, 2018). It also states that there was no revocation of documents in force, nor the consequent adoption of new manuals and that the AEs (Ministério de Educação, 2018) are the “common curricular denominator” for all students, constituting a common basis of reference, mainly for external review.

From the above, we can conclude that in both countries, the re-elaborations of the curricular guidelines for mathematics were inserted in complex reform processes that signal control management through performances. In Brazil, the current guidelines elaborated in controversial complexes focus on mathematical skills and competencies to be developed throughout Basic Education. In Portugal, the contents organised hierarchically generated inconsistencies between the different curriculum documents, which according to the DGE, sought to provide an answer through a new curriculum document, the AE (Ministério de Educação, 2018), not accompanied by the revocation of previous documents.

**LITERATURE REVIEW**

Several studies show that the strategic use of the DTs can support the learning of mathematical procedures, as well as the development of advanced competencies...
(Gadanidis & Geiger, 2010; Roschelle et al., 2010; Suh & Moyer, 2007), constituting one of the key points of contemporary curriculum reforms.

In this sense, one of the topics for discussion in the 24th International Commission on Mathematical Instruction (ICMI) Study, entitled “Implementação de reformas dos currículos de Matemática dentro e em diferentes contextos e tradições / Implementing reforms of mathematics curricula within and in different contexts and traditions,” was “Reformas Curriculares na Matemática escolar: desafios, mudanças e oportunidades / Curriculum Reforms in School Mathematics: Challenges, Changes, and Opportunities.” The work developed raised some questions such as “What are the types of resources and their roles (e.g., technologies) in the curricula reform and implementation?” (ICMI, 2017, p. 11), which place particular emphasis on the role of resources such as the DTs in curriculum reforms in different contexts. The theme “Globalisation, internationalisation and its impacts on the curricular reforms of Mathematics” reinforces that “these influences seem to lead increasingly towards a ‘convergence’ in the reforms of Mathematics curricula. Similarities and specificities can be observed through Comparative Studies” (ICMI, 2017, p. 12).

During this event, Azrou (2018), discussing the globalisation of systems and three reforms in Algeria, found that, according to an ancient tradition, in post-colonial countries and also in developing countries, educational systems and reforms are imported from Western countries, motivated mainly by political reasons, where reforms that emphasise technology are not always well seen. The researcher “suggests that it would be important to think about how to achieve the same goals with a reform, even if we use different configurations and specificities of educational systems in two different countries” (Azrou, 2018, p. 430).

But those international influences are still broader, as is the case with those involving organisations such as the Programme for International Student Assessment (PISA) and the Organisation for Economic Co-operation and Development (OECD). For example, a study by Kirwan and Hall (2015) showed that the reform in Ireland emphasised the need for technology development at the suggestion of the OECD.

Similarly, the Mathematics Curriculum Document Analysis (MCDA) project is underway, which is part of the OECD project “Future of Education and Skills, Education 2030” (OECD, 2018), which aims to investigate the extent to which countries incorporate broad perspectives on Mathematical Literacy and 21st century skills in their current curriculum. This project will use a framework that is under discussion in conjunction with PISA 2021, which created the Center for Curriculum Redesign. For the analysis of curricular reforms in Mathematics, with regard to the use of DT, it considers two categories: Digital Literacy (DL) and Computational Thinking (CT).

In the literature, DL perspectives are configured as awareness, attitude, and ability to use digital solutions and facilities to identify, access, manage, integrate, assess, analyse, and synthesise the DT, build new knowledge, create media expressions, and communicate with others and, beyond their usability, use them critically in everyday life. (Jenkins, Purushotma, Weigel, Clinton, & Robison, 2009; Martin 2006). Sápiras and Vecchia
(2016) highlight the close relationship between the DL and Mathematics Education by proposing a focus on the ability of multitasking, which, according to Jenkins et al. (2009), is configured as the ability to analyse the environment, to perceive essential details using different resources simultaneously.

The CT concept, according to Wing’s definition, is closely associated with the ideas of Problem Solving, systems design and understanding of behaviours guided by fundamental concepts of Computer Science (Wing, 2006). Regarding the development of the CT in Mathematics Education, Wing (2006) suggests that it should be approached from the perspective of: Conceptualising instead of programming; Fundamental and non-utilitarian ability; Complementing and combining Mathematics and Engineering, that is, “Mathematics as a basis for innovation for economic growth via Science, Technology and Engineering” (PISA, 2016, p. 4); Generating ideas, not artefacts; For everyone, anywhere. However, the relationship between the contents of Mathematics and Computing is still far from identical (Barcelos & Silveira, 2012).

Although there are indications of the transfer of competencies between the two domains, it is necessary to map the body of knowledge of both areas. The articulation between CT and Mathematics requires clear identification of the moments in which this relationship can occur throughout the school curriculum (Barcelos & Silveira, 2012). The approach of the CT in Basic Education is important because it is a stage in which several priorities, ideologies and philosophies fight for attention (Barcelos & Silveira, 2012; Computer Science Teachers Association (CSTA) & International Society for Technology in Education (ISTE), 2011; Muñoz, Villarroel, & Silveira, 2015).

The literature review highlights the need for this investigation and the adoption of the DL and CT categories to analyse the curricular prescriptions for the Mathematics teaching for the current and future generations of young Brazilian and Portuguese people. The study can reinforce issues such as the need for the development of critical reflection, appropriation of choices for exploration and dynamisation beyond the contents, decision-making, and passage between two domains. In this sense, based on recent reforms in the programs prescribed, we proposed to conduct a comparative study of the perspectives on DT contained in those documents.

METHODOLOGY

Reflecting on the methodological bases of Comparative Education, Pilz (2012) suggests that, to carry out the studies, the researcher must establish significant criteria or determine the differences so that different realities can be compared. Pilz, Krisanthan, Michalik, Zenner e Li (2016, p. 128) reinforce that “the interpretation of these comparative results requires caution, since the link between results and explanation is mainly hypothetical at this level.”

We adopted Pilz’s (2012) perspective, which summarises the following methodological phases of the comparative study: (1st) Descriptive phase – observations
and descriptions; (2nd) Explanatory phase - introduces interpretation aiming to explain and understand; (3rd) Juxtaposition phase – first attempt of comparison, offering the national finding defined in the context of the comparison criteria selected for evaluation and analysis from side to side; (4th) Comparative phase - hypotheses are tested using systematic comparison, relations between countries are evaluated by reference to the comparison criterion and conclusions can be drawn.

Regarding the delimitation of the essential research method, we elaborated a documentary analysis of the prescribed curricula of the two countries, which, according to Sharma, is defined as:

A way to collect qualitative information from a primary or original source of written, printed, and recorded materials to answer research questions in interpretative case studies. The documents provide evidence of authentic or real activities carried out in social and human thought organisations. (Sharma, 2013, p. 3)

We planned, then, a documentary research, looking for an organisation on the assumptions that support the official documents in force, and what recommendations about the use of DTs they bring to Mathematics Teaching in Brazil and Portugal. To this end, we considered the prescriptions contained in these curricular documents of the two countries listed in Figure 2 for Basic Education as a criterion for comparison.

Figure 2
Documents analysed for Basic Education in Brazil and Portugal

| Brazil | Portugal |
|--------|----------|
| • National Common Curricular Base, Mathematics area - Final Years of Elementary School, 2017. | • Mathematics Programmes and Curricular Goals - Basic Education: 2nd cycle (6th year- final) and 3rd cycle, 2013. |
| | • Curricular Management Guidelines for the Mathematics Programme and Curricular Goals for Basic Education (OGCPMCMEB, in Portuguese acronym), 2016. |
| | • Essential Learnings/Articulation with Student Profile, 2018. |

The present study will analyse the curriculum guidelines of the two countries and their intentions, discourses of the Sociedade Brasileira de Computação (Brazilian Computing Society) (SBC) (2018), which issued notes on the BNCC (Brazil, 2017) in Brazil, and standpoints of the Grupo de Trabalho de Matemática (Mathematics Working Group) (GTM) (2019), which was assigned the mission of developing a set of recommendations on teaching, learning, and assessment in the discipline of Mathematics in Portugal, aiming to characterise the trends expressed in curriculum reforms that have been carried out in these countries to guide the teaching of new generations.
We adopted categories that emerged from trends expressed in the literature review and in key points for curricular analyses that are being discussed under the Mathematics Curriculum Document Analysis (MCDA) subproject, part of the OECD project “Future of Education and Skills, Education 2030” (OECD), which aims to carry out a broad comparative study of global prescriptions to help countries find answers about what knowledge, skills, attitudes, and values are necessary for students to prosper and shape their world, as well as ways in which educational systems can effectively develop them, focusing on:

[...] technologies that have not yet been invented and solving social problems that have not yet been anticipated. Education can equip students with agency, skills, and a sense of purpose to shape their own lives and contribute to those of others. Therefore, change is imminent. (OECD, 2018, p. 1)

The project also intends to support countries in addressing common challenges to curricula implementation and in identifying critical success factors. Its strand 1 refers to the development of a learning framework for Mathematics 2030, and strand 2 refers to the Analysis of International Curriculum Programmes, aiming to build a knowledge base that will allow countries to make curriculum design processes more systematic. This means supporting international peer learning and stakeholder discussions.

Thus, the categories of analysis defined in this study were based on the Mathematics 2030 project and the literature review, and were adopted to the contexts under analysis (Table 1):

| Table 1 | Analytical categories adopted |
|-----------------------------------------------|
| **CATEGORY C1:** Digital Literacy (DL) | DL refers to the ability to use knowledge, understandings, skills, and dispositions to use digital equipment effectively, consciously and appropriately inside and outside school. Students with this ability can provide, create, and communicate information and concepts (Jenkins et al., 2009; Martin 2006). They can adapt to technological changes and use technologies to achieve a purpose and communicate with others using those devices. |
| **CATEGORY C2:** Computational Thinking (CT) | The CT involves the formulation and resolution of problems carried out through technologies. Programming is referred to as a fundamental skill (Wing, 2006) and coding to build knowledge through the understanding and skills related to the language, standards, processes, and systems necessary to instruct/direct devices such as computers and robots. It should complement and combine Mathematics and Engineering, generating ideas and being accessible to all in any context (Wing, 2006). The relationship between Mathematics and Computing curricular contents are still far from identical (Barcelos & Silveira, 2012). |
The analytical categories Digital Literacy (DL) and Computational Thinking (CT) aim to highlight similarities and specificities in the prescriptions of the two countries that will follow.

DATA PRESENTATION AND ANALYSIS

**Descriptive and Explanatory Phases**

In this article, we prioritised the analysis of general perspectives contained in the prescribed curricula for the Final Years of Elementary School in Brazil and Portugal, the final year of the 2nd cycle and the entire 3rd cycle. The non-comparison year by year of the competencies prescribed in the curriculum of the two countries is justified by the limited space available in the article.

In Brazil, work with the DTs is referred to in the BNCC as a resource to support mathematical learning:

> In addition to the different didactic and material resources, such as checkered meshes, abacuses, games, calculators, spreadsheets, and dynamic geometry software, it is important to include the history of Mathematics as a resource that can arouse interest and represent a meaningful context for Mathematics learning and teaching. However, those resources and materials need to be integrated into situations that favour reflection, contributing to the systematisation and formalisation of the mathematical concepts. (Brazil, 2017, p. 292)

The BNCC (Brazil, 2017) suggests resources such as calculators, spreadsheets, and dynamic geometry software, highlighting the need to insert the History of Mathematics and reflective processes in the approach of concepts, configuring a general and vague recommendation of its objectives.

The PMEB (Bivar et al., 2013) for the 2nd cycle of Portugal refers to the use of DTs in the construction of figures in Geometry:

> As this is an indispensable step for the serious and rigorous study of geometry in later teaching cycles, students should be able to relate the different properties studied to those they already know and that are relevant in each situation. Students are also asked to perform various tasks involving the use of drawing and measuring instruments (ruler, square, compass and protractor, and dynamic geometry programmes), and it is desirable that they master the rigorous constructions and recognise some of the mathematical results behind the different procedures. (p. 13)
The PMEB (Bivar et al., 2013) emphasise the importance of mastering and carrying out rigorous constructions in Geometry using DTs, which refer to a plane ontology of performance and regulation that serve as measures of productivity or results, to present quality, promotion, or inspection (Ball, 2010).

On the resources for the work and the problems inherent to the prescribed perspective, the recent document entitled “Recomendações para a melhoria das aprendizagens dos alunos em Matemática /Recommendations for the improvement of students’ learning in Mathematics” (Canavarro et al., 2019), the Mathematics Working Group indicated that:

Regarding the resources to be adopted, the programme refers exclusively and very carefully to technology, considering that it “can seriously condition and compromise the learning and assessment” of Mathematics. Thus, it grants that technology be used with criteria, in some very specific situations, without compromising the manual mastery of algebraic procedures or the making of graphic representations, which the program considers to be basic, justifying that “only memorisation and cumulative understanding of concepts, techniques, and mathematical relationships allow progressively more complex knowledge and solve progressively more demanding problems” (Bivar et al., 2013, p. 29 apud Canavarro et al., 2019, p. 100)

The OGCEB (DGE, 2016) highlight that students, inserted in the digital age, must use applications for mathematical learning, presenting some examples and their potential.

Scratch, which, beyond being an initiation to a programming language, involves mathematical logical thinking, estimation, coordinates in reference and variables, among other aspects; numerical applets (for example, numerical straight lines) and algebraic applets (sequence generators, multiple representations, algebraic modelling, etc.); Excel, as one of the possible digital applications, as it allows the transition between the numerical and algebraic approach, especially with reproduction in table providing multiple representations. (DGE, 2016, p. 4)

This document recommends initiation to the programming language aiming to develop logical thinking, coordinates in referential, numerical, and algebraic applets, and Excel to treat the change from the numerical to the algebraic frame.

On this document, GTM studies (Canavarro et al., 2019) infer that:

[...] the OGC propose some methodological suggestions and adopt a more favourable attitude to the use of technology. Also, they change the approach to some content and, in some way, introduce others that do not fit in descriptors inserted in the Curriculum Goals. (Canavarro et al., 2019, p. 72)
Regarding the use of Dynamic Geometry software, GTM (Canavarro et al., 2019) concluded that OGCEB (DEB, 2016):

[...they also return to the reference to technology, expanding the references, albeit with limitations, to dynamic geometry software, but continuing in the preferential logic of illustration: “Dynamic geometry programmes are precious resources for classes, especially to identify numerous situations that illustrate relationships to be analysed later in a more judicious manner.” (GT, 2016, p. 5 apud Canavarro et al., 2019, p. 102)

The analysis emphasises both the resumption of the perspectives of work with geometric software related to the Mathematics Programme of Basic Education of 2007, and the predominance of the illustrative emphasis of this perspective that should also focus on the analysis of geometric relations critically (Canavarro et al., 2019).

The AEs (ME, 2018) emphasise the learning objectives corresponding to essential learning practices in the Algebra theme, where students should “Use digital technology, namely interactive applications, specific computer programs and calculator” (Ministério de Educação, 2018, p. 11) to solve equations. However, they do not present specific indications for its use to resolve, prove, or compare representations in the passage between the numerical and algebraic frames.

The GTM report (Canavarro et al., 2019) highlights that the AEs (ME, 2018) reduce the number of concepts proposed by the PMEB (Bivar et al., 2013) and adhere to some options similar to the OGCEB’s (DEB, 2016), not only with regard to vertical flexibility but also regarding the introduction of new concepts and processes, with indications of essential learning practices, where they deduced on this methodological point referring to the Algebra theme that:

Although the essential practices refer to the use of digital technology, it is not clear in any passage whether or not solving the equations can be performed using technology and what role it plays. It will be very different to use technology to prove algebraic resolutions and compare with handmade graphical representations or to use technology as a generator of solutions to be selected with criteria and as an experience base that supports conjectures. Thus, the AEs also reveal difficulties in clearly defining their approach to Mathematics. (Ministry of Education, 2018, p. 63)

The BNCC for Mathematics emphasises mathematical processes of problem-solving, research, and development, considered potentially rich for the addition of fundamental skills for mathematical literacy (reasoning, representation, communication,
Another aspect to be considered is that algebra learning, as well as those related to other fields of mathematics (numbers, geometry, and probability and statistics), can contribute to the development of students’ computational thinking, considering that they need to be able to translate a given situation into other languages, such as transforming problem situations, presented in their mother tongue, in formulas, tables, and graphs, and vice versa. Associated with computational thinking, the importance of the algorithms and flowcharts, which can be the object of study in the Math classes, is noteworthy. An algorithm is a finite sequence of procedures that allows you to solve a given problem. Thus, the algorithm is the decomposition of a complex procedure into its simplest parts, relating them and ordering them, and can be represented graphically by a flowchart. The algorithmic language has points in common with algebraic language, especially with the concept of variable. Another skill related to algebra that maintains a close relationship with computational thinking is the identification of patterns to establish generalisations, properties, and algorithms. (Brazil, 2017, p. 269)

The document provides perspectives on CT development, where students should translate a problem situation into specific computer language, create algorithms and flowcharts, highlighting the intersections between algorithmic and algebraic language (variable concept). This should be done by identifying patterns, generalisations, and properties.

In a technical note issued, the Brazilian Computing Society (SBC, 2018) questioned such perspectives of work on CT in the Algebra unit:

Computational thinking does not aim to translate a given situation into another language, or turn problem situations into tables and graphs. Computational thinking is a skill related to the construction of solutions to problems involving the description and the generalisation of solution processes, their automation and analysis. It uses languages to describe the solutions, but the emphasis is on the process of building the solution. Algorithms can be represented by flowcharts, but, as discussed earlier, this is not the most appropriate representation. There is a range of other visual languages with much more desirable characteristics, from the didactic pedagogical point of view, that can be used for this purpose. The analogy between Algebra and Algorithm is quite questionable. Algebra is an area of mathematics that studies symbolic manipulations, allowing relationships between quantities to be described generically, through the use of variables, terms, and equations. The concept of variable in Algebra is used to enable the syntactic expression of relationships without the need for listing concrete instances, i.e., a variable is a name that we use to reference any value. In Computing, the concept of variable is diverse, it may be similar to the algebraic concept (which is the case, for example, in functional paradigms), and may represent a place or position of
memory where a value is stored (in imperative paradigms). The fact that we use variables both in the construction of Algorithms and in Algebra does not make these two areas similar. The objectives are completely different. (SBC, 2018, p. 3)

SBC pointed out limits in the connection between CT development and algebraic language, problem situations, and graphical representations, in which the former emphasises the solution processes. The flowchart was pointed out as an outdated language, suggesting other visual languages. The link between Algebra and Algorithm is questioned because the notion of a variable in Algebra is linked to the expression of relationships and, in Computation, it is diverse and broader.

In Portugal, AE also highlight the importance of developing in students a positive attitude towards mathematics, as a science that integrates and has its socio-cultural values for humanity concerning its role in the development of technology and other areas.

For this purpose, students are expected, throughout basic education, to develop interest in Mathematics and confidence in their knowledge and mathematical abilities, besides persistence, autonomy, and willingness to deal with situations that involve Mathematics in their academic career and that they may face in their life in society; the ability to appreciate aesthetic aspects of Mathematics and to recognise and value the role of Mathematics in the development of other sciences, technology, and other domains of human activity; and the ability to recognise and value Mathematics as an element of humanity’s cultural heritage. (Ministério de Educação, 2018, p. 3)

In summary, the BNCC (Brazil, 2017) points to the use of DTs and other resources for representation, systematisation, and formalisation of mathematical concepts. PMEB refers to the use of dynamic geometry software combined with other instruments for students to develop mastery in rigorous geometric constructions and recognise related results.

Similarly, the BNCC (Brazil, 2017), the AE (Ministério de Educação, 2018) and the OGCEB (DGE, 2016) point to DTs as tools for changing frames (primarily numerical frames for algebraic frames), representation, systematisation, and formalisation of concepts that will focus on digital literacy of students in various contexts (Gadanidis & Geiger, 2010; Roschelle et al., 2010; Suh & Moyer, 2007). Those curricular documents prescribe the use of devices and applications that presuppose knowledge, understanding, skills, and willingness to use the DTs effectively, aiming to provide, create, and communicate information and concepts (Jenkins et al., 2009; Martin, 2006).

About the development of CT, the similarities in the programmes of the two countries focus on the transition between numerical and algebraic languages through multiple representations in problem situations. Specifically, the BNCC (Brazil, 2017) brings skills inherent to the development of the CT from the perspective of the development
of algorithms in the Algebra unit, which is considered very specific and sometimes dissociated from the object of knowledge pointed out and surpassed by SBC (2018). It corroborates with SBC (2018) that the construction of algorithms with the inclusion of concepts such as “flowchart” in Elementary School not only impairs the development of CT, but can undoubtedly bring significant problems to algorithm learning.

The AEs (Ministério de Educação, 2018) recommend the use of devices and applications aiming at understanding and skills for digital equipment capable of promoting the creation, communication, forecasting, and description of solutions (Jenkins et al., 2009; Martin 2006). However, they do not clarify their real objectives (Canavarro et al., 2019). The OCPMCMEBs (DGE, 2016) bring perspectives on initiation to a programming language, development of mathematical thinking, and digital applications, which presuppose the development of competencies related to the CT.

The descriptive and explanatory analysis allowed us to infer that the annual comparison would only be possible considering the BNCC (Brazil, 2017) in Brazil and the AE (Ministério de Educação, 2018) in Portugal, since the PMEB (Bivar et al., 2013) do not present specific prescriptions for DTs in the years investigated. Next, we show the juxtaposition and comparison of the general curricular guidelines carried out for Basic Education.

**Juxtaposition and Comparative Phases**

This session synthesises the juxtaposition and comparison phases, looking for both the hypotheses from the data presented in the previous stages and the research questions. The similarities and specificities are highlighted, seeking to prospect to point out trends in Mathematics Education printed in the curriculum documents in force in Brazil and Portugal regarding the prescriptions on the use of the DTs. The following are analyses related to the documents, where general perspectives of what is expected of the students will be stated, as shown in Figure 3 below.
Figure 3

Similarities and specificities regarding the use of DTs in the documents in force in Brazil and Portugal

| CURRICULUM GUIDELINES | SIMILARITIES | SPECIFICS |
|-----------------------|-------------|-----------|
| BNCC/PMEB             | They prescribe DT as a fundamental element for the development of mathematical activities in social and cultural contexts, which involves the consideration of the very history of the area, and presuppose reflective processes in which students present DL, without stopping definitions of their real intentions. | They recommend DTs such as interactive applications, specific computer programs, and calculator as a tool for changing frames (numerical for algebraic), representations, systematisation, and formalisation of concepts, which will focus on students having DL digital literacy, however without a clear definition of their objectives. |
| BNCC/AE               | It prescribes the DTs and other combined resources for representation, systematisation, and formalisation of concepts. Limits in the connection between CT development and algebraic language, problem situations, and graphical representations, in which the former emphasises the solution processes. The flowchart is an outdated language, and the use of other visual languages is suggested. The link between Algebra and Algorithm is questioned because the notion of a variable in Algebra is linked to the expression of relationships and, in Computation, is diverse and broader. | For the 2nd cycle, the document refers to dynamic geometry software combined with other instruments, which should be used for students to develop mastery in rigorous geometric constructions and recognise the mathematical results behind the procedures performed in the process. This perspective prioritises students' performativity and control. |
| BNCC                  | Guided initiation to programming language, development of mathematical thinking, and digital applications (applets). They adopt a more favourable attitude to the use of technology concerning PMMEB, but focus on illustration rather than inciting critical analyses of geometric relationships. |
| PMEB                  | It suggests the use of devices and applications that presuppose understanding and skills to use digital equipment appropriately, allowing students to be able to provide, create, and communicate information and concepts in varied contexts, and predict and describe solutions. |
| OGCPMEB               | | |
| AE                    | | |

The analysis of the documents helped us to infer perspectives that meet category C1, DL, thus approaching the spirit of the OECD (2018). In terms of similarities, the BNCC (Brazil, 2017) and the AE (Ministério de Educação, 2018) highlight the use of DTs in multiple sociocultural contexts (Gadanidis & Geiger, 2010; Roschelle et al., 2010; Suh & Moyer, 2007) and that students should resort to DTs for changes in frame, representation, formalisation, and communication, but without clear definitions of their real intentions.

Specifically, the BNCC (Brazil, 2017) presents the adoption of DTs combined with other resources; the OGCPMEBs (DGE, 2016) rescue a work perspective that is more favourable to the use of technology concerning the PMMEBs (Bivar et al., 2013), and the AEs (Ministério de Educação, 2018) resume the approach of devices and applications, without clearly explaining their intentions (Canavarro et al., 2019). The PMEBs (Bivar et al., 2013) bring to the 2nd cycle perspectives of working with dynamic geometry software combined with other instruments, with emphasis on mastery and mathematical procedures in rigorous geometric constructions, which presuppose performances – of subjects aiming at results, as forms of quality presentation that serve as instruments for rankings in external assessment (Ball, 2010).
Regarding the CT category, similarly to the BNCC (Brazil, 2017), the AE (Ministério de Educação, 2018) and the OGCPMCMEB (DGE, 2016) emphasise the perspective of formulating problem situations aiming at the transition between the numerical and algebraic approach. Specifically, the BNCC (Brazil, 2017) reinforces the development of the CT through skills that require the construction of algorithms with very specific language and sometimes dissociated from the prescribed object of knowledge. AEs (Ministério de Educação, 2018) refer to the use of devices, applications, and other digital equipment in an appropriate manner, where students can provide, create, and communicate information and concepts to predict and describe solutions without a clear definition of their objectives (Canavarro et al., 2019).

The OGCPMCMEBs (DGE, 2016) highlight Scratch for development, besides initiation to a programming language, coding, and skills related to mathematical logical thinking, estimates, etc.; and the use of numerical applets, bringing Excel as a possibility of digital applications.

**CONCLUSIONS**

The comparative study showed more similarities than specificities regarding the current curricular guidelines of Brazil and Portugal on the approach to the use of DTs in Mathematics Education, indicating convergences in recent curricula reforms (ICMI, 2017).

In the prospective phase, we analysed the trends for Mathematics Education concerning the analytical categories for the DTs inspired by the framework of the MCDA Project and literature review. The analysis of the skills related to category C1, regarding similarities, allowed us to infer that the BNCC (Brazil, 2017) and the AE (Ministério de Educação, 2018) prescribe the DTs as an essential tool for the development of mathematical activities in social and cultural contexts (Gadanidis & Geiger, 2010; Roschelle et al., 2010; Suh & Moyer, 2007), where students should be involved in reflective processes involving DL capabilities and considering the History of Mathematics.

Specifically, the OGCPMEEBs (DGE, 2016) bring perspectives for students to use interactive applications with specific software for exploration and digital communication with an emphasis on illustration over critical processes.

In Portugal, AEs (Ministério de Educação, 2018) guide the use of devices and applications without a clear explanation of their intentions (Canavarro et al., 2019) and the OGCPMCMEBs (DGE, 2016) prescribe the use of dynamic geometry software, combined with other instruments for rigorous constructions that refer to performance (Ball, 2010), use of applets, and initiation to a programming language (OECD, 2018).

In category C2, we identified that the similarity in the analysis of the documents in force in the two countries was related to the emphasis on the transition of numerical and algebraic language through multiple representations. The specificities in the prescriptions
of countries allow us to infer isolated perspectives, but they should be considered in the 
reforms of future programmes.

In the BNCC (Brazil, 2017), the emphasis on the development of algorithms in 
the unit “Geometry” was prescribed in a very specific way (6th grade) and sometimes 
without connection to objects of knowledge. In the OGCPMCMsEBs (DGE, 2016), 
we inferred the initiation of programming language through Scratch language, with 
perspectives of development of mathematical logical thinking, and the use of applets for 
digital applications for problem formulation and resolution.

The investigation made it possible to reinforce the focus of curriculum guidelines 
in force in both countries on performance models (Ball, 2010), which is configured as a 
global trend of management and control regarding the intentionalities of contemporary 
curriculum reforms.

As a contribution, the research allowed us to elucidate issues related to rigour and 
mastery in the use of DTs, predominance in the use of software in a more illustrative 
perspective, and construction of algorithms in specific and limited language, limits in 
the connection between CT and algebraic language, elements that should be the focus 
of reflections by researchers in Mathematics Education and Computing areas (Barcelos 
& Silveira, 2012; Barcelos, Munôz, Villarroel & Silveira, 2015; CSTA & ISTE, 2011; 
Wing, 2006) and elaborators involved in the reform processes of curriculum documents 
in Brazil and Portugal.

Certainly, the debate does not end here, and the analysis of specific competencies in 
the body of curricular prescriptions for Elementary Education in both countries (Azrou, 
2018) may raise other important issues.

AUTHORSHIP CONTRIBUTION STATEMENT

M.O.D. and L.S. conceived the idea presented. M.O.D. and L.S. developed the 
theory. M.O.D. adapted the methodology to this context, created the models and collected 
the data. M.O.D. and L.S. analysed the documentary data. All authors actively participated 
in the discussion of the results, reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the 
corresponding authors, M.O.D. and L.S., upon reasonable request.
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