A study on nanoparticle transport in a micro blood vessel

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This study focuses on the simulation of a nanoparticle loaded blood flow in a micro-channel. The Euler-Euler approach is used to describe the multi component behavior of blood. Nanoparticles are modeled by a concentration equation. The influence of the magnetic field gradient on particle transport is investigated for different Reynolds numbers.

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1 Introduction

Magnetic Drug Targeting is a new technique for delivering drugs to specific locations in patient’s body. Magnetic nanoparticles are therefore coated with drugs, injected into the bloodstream and then guided or forced to enter tissue by using a magnetic field. CFD is a helpful tool to design such processes, but for micro blood vessels standard methods are not applicable. The reason is, that the blood cell concentration at the vessel walls decreases, known as the Fährnus–Lindqvist effect. Since nanoparticle transport is highly dependent on local blood cell concentration (hematocrit), this effect has to be considered.

2 Modeling Approaches

An approach capable of modeling all relevant effect is a combination of models presented in [1] and [2]. To model the blood flow an Euler-Euler method for laminar flows as presented in [1] is used. It considers plasma and red blood cells as interacting continua and is based on volume averaged quantities denoted with an overbar in the following equations:

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \bar{U}) = 0, \quad \text{RBC}: \phi = \alpha, \quad \text{plasma}: \phi = \beta, \quad \alpha + \beta = 1, \]  

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \bar{U} \mathcal{U}_\phi) + \nabla \cdot (\phi \mathbf{T}_\phi) = -\frac{\phi}{\rho_\phi} \nabla p - \frac{\mathbf{M}_\phi}{\rho_\phi}, \quad \nabla \cdot (\alpha \mathbf{U}_\alpha + \beta \mathbf{U}_\beta) = 0, \]  

\[ \mathbf{T}_\phi = 2 \eta_\phi \gamma_\phi \left[ \bar{D}_\phi - \frac{1}{3} (\nabla \cdot \bar{U}_\phi) \mathbf{I} \right], \quad \gamma_\phi = \sqrt{2 \tau r (\bar{D}_\phi^2) \mathbf{I}}, \quad \bar{D}_\phi = \frac{1}{2} \left[ \nabla \mathbf{U}_\phi + (\mathbf{U}_\phi)\mathbf{T} \right], \]  

\[ \mathbf{M}_\alpha = -\mathbf{M}_\beta = 18 \eta_\beta \left( 1 + 36.4\alpha \right) \bar{D}_\alpha \beta \frac{1}{2\pi d_\alpha} \frac{1}{\sqrt{3}\beta} \mathbf{D}_\beta \cdot \mathbf{U}_r, \quad \mathbf{U}_r = \mathbf{U}_\alpha - \mathbf{U}_\beta. \]

Eq. (1) is the transport equation for each components volume fraction, \( \phi \) is used as a placeholder. The first part of eq. (2) is the momentum equation. It uses the stress tensor \( \mathbf{T}_\phi \) as defined in eq. (3) based on the generalised Newtonian fluid. The viscosity \( \eta_\beta \) generally depends on the shear rate \( \gamma_\phi \) and the volume fraction \( \phi \). Interphase momentum transfer \( \mathbf{M}_\phi \), dependent on the relative velocity \( \bar{U}_r \) and the RBC diameter \( d_\alpha = 8 \mu \text{m} \), is considered by the Batchelor-drag and Saffman-lift model in eq. (4). The second part of eq. (2) is the volumetric continuity equation stating that the mixture velocity field is free of divergence.

To model nanoparticle transport the scalar transport eq. (5) is used. The terms \( U_{Drift,\alpha}, D_B, D_c \) and \( f(\alpha) \) in eq. (6) and (7) originate from [1] and represent the effect of nanoparticles drifting out of regions with high hematocrit and shear rates.

\[ \frac{\partial c}{\partial t} + \nabla \cdot (c (\bar{U} - U_{Drift})) = \nabla \cdot (D_c \nabla c), \quad U_{Drift} = U_{Drift,\alpha} + U_{Drift,\text{mag}} \]  

\[ U_{Drift,\alpha} = (D_B + \xi \alpha \gamma_\alpha) \nabla f(\alpha), \quad U_{Drift,\text{mag}} = -\frac{d_c^2}{36\eta} \frac{\mu_0 \chi}{(1 + \frac{\chi}{\alpha})^3} \nabla (|\mathbf{H}|^2) \]  

\[ D_c = (D_B + \xi \alpha \gamma_\alpha) (1 + f(\alpha)), \quad \eta = \alpha \eta_\alpha (\gamma_\alpha, \alpha) + \beta \eta_\beta \]

A drift of the nanoparticles by the application of a magnetic field is modelled by \( U_{Drift,\text{mag}} \) as proposed in [2]. Its direction is given by the magnetic field \( \mathbf{H} \) due to the term \( \nabla (|\mathbf{H}|^2) \) of eq. (6). Here \( d_c \) represent nanoparticle diameter of 250 nm, the magnetic susceptibility \( \chi = 20 \). The usage of the mixture viscosity \( \eta \) increases the resistance against the movement by the magnetic field in regions of high \( \alpha \). To perform calculations the approaches were implemented in foam – extend – 4.0. Therefore the solver twoPhaseEulerFoam was modified.

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1 RBC are actually considered as moderately viscous non-Newtonian, plasma low viscous Newtonian. The models presented in [1] were applied.
3 Test Case

The test case is a 100 µm 2D micro-channel as used in [3]. Its height corresponds to a typical arterioles diameter. At the outlet a block profile for the horizontal velocity is set, a no slip boundary condition is applied at the walls. At the inlet there is the pressure reference, a constant hematocrit of $\alpha = 0.3$ and the reference value of nanoparticle concentration which is $c = 1$. The unnamed boundary conditions are of the homogeneous Neumann type.

![Fig. 1: Sketch of the micro channel. In the region denoted with arrows the magnetic field gradient is set.](image)

In the area above the patch of evaluation $|\nabla (|H|^2)|$ is set to a constant value that is varied in the studies. Multiplied with the magnetic drift direction $n_{\text{Drift, mag}}$ this is substituted in eq. (6) for $|\nabla (|H|^2)|$. All calculations start from the steady state that was calculated without applying the magnetic field. If the effect of magnetic drift velocity is strong enough, the nanoparticle concentration will increase at the patch of evaluation. As an integral value here $c$ is averaged over the length of the patch, named $\tau_{\text{wall}}$. The steady state value is used for studies over $|\nabla (|H|^2)|$.

4 Results

Fig. 2 (left) shows the temporal progress of the nanoparticle concentration in the region of the applied magnetic field. The Fahreus effect increases the blood cell concentration in the center of the channel. Therefore the walls are enriched with nanoparticles since they are displaced out of the center. Thus $c$ is increased by 60% at the walls in reference to the inlet. By applying the magnetic field gradient, $c$ will decrease at the upper wall and increase at the lower wall. The example shows the $c$-field for $Re = 5$ and $|\nabla (|H|^2)| = 2\times10^4$ A m$^{-3}$ over time. It takes 22 ms to reach the steady state.

![Fig. 2: (left): Temporal progress of the nanoparticle field. (right): Influence of $Re$ on the steady state nanoparticle concentration.](image)

Fig. 2 (right) shows the result of a study for an increasing Reynolds number. It can be seen a certain magnetic field gradient is needed to overpower the convectional transport of nanoparticles with the blood flow. As the Reynolds number increases, the magnetic field gradient needs to be increased to be able to enrich an arterioles wall with nanoparticles.

References

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