Roche lobe effects on the atmospheric loss of “Hot Jupiters”

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Abstract. Observational evidence of a hydrodynamically evaporating upper atmosphere of HD209458b (Vidal-Madjar et al. 2003; 2004) and recent theoretical studies on evaporation scenarios of “Hot Jupiters” in orbits around solar-like stars with the age of the Sun indicate that the upper atmospheres of short-periodic exoplanets experience hydrodynamic blow-off conditions resulting in loss rates of the order of about $10^{10}$–$10^{12}$ g s$^{-1}$ (Lammer et al. 2003; Yelle 2004; Baraffe et al. 2004; Lecavelier des Etangs et al. 2004; Jaritz et al. 2005, Tian et al. 2005; Penz et al. 2007). By studying the effect of the Roche lobe on the atmospheric loss from short-periodic gas giants we found, that the effect of the Roche lobe can enhance the hydrodynamic evaporation from HD209458b by about 2 and from OGLE-TR-56b by about 2.5 times. For similar exoplanets which are closer to their host star than OGLE-TR-56b, the enhancement of the mass loss can be even larger. Moreover, we show that the effect of the Roche lobe raises the possibility that “Hot Jupiters” can reach blow-off conditions at temperatures which are less than expected ($< 10000$ K) due to the stellar X-ray and EUV (XUV) heating.

Key words. exoplanets – Roche lobe – atmospheric loss

1. Introduction

Since the observation of an extended and evaporating upper atmosphere around the Jovian-like exoplanet HD209458b by Vidal-Madjar et al. (2003), different blow-off scenarios for evaporating hydrogen-rich atmospheres are discussed in the literature (Sasselov 2003; Lammer et al. 2003; Lecavelier des Etangs et al. 2004; Yelle 2004; Vidal-Madjar et al. 2004; Grießmeier et al. 2004; Baraffe et al. 2004; Erkaev et al. 2005; Tian et al. 2005; Penz et al. 2006).

Because hydrogen-rich upper atmospheres of short-periodic gas giants in orbits with semi-major axes less than $< 0.1$ AU can be heated to temperatures of up to $10000–20000$ K by X-rays and EUV (XUV) radiation of the central star (Lammer et al. 2003; Yelle 2004), they experience hydrodynamic blow-off. Yelle (2004) studied in detail the photochemistry of the thermospheres of “Hot Jupiters” and found that the lower thermosphere may be cooled primarily by infra red radiative emissions from H$_3$ molecular ions which are created by photoionization of H$_2$ molecules and related ion chemistry (Yelle 2004).

Recently, Lecavelier des Etangs et al. (2004) and Jaritz et al. (2005) discussed the Roche lobe effects on hydrodynamic mass loss from the “Hot Jupiters” atmospheres and argued that for some close-in exoplanets, due to the expected high exospheric temperatures, the exobase level $r_{\text{exo}}$ can reach the Roche lobe $r_{\text{Ro}}$ before classical hydrodynamic blow-off conditions may develop and, thus, affect their atmospheric loss rates. The Roche lobe is defined as the last equipotential surface around a planet where its gravitational potential energy is zero. Below this boundary, the surfaces of constant potential encompass the host star. Because atmospheric particles at and beyond the Roche lobe can escape unhampered, Lecavelier des Etangs et al. (2004) argued that the Roche lobe can be seen as an equivalent to the exobase level. However, there is an important difference between the Roche lobe and the exobase, since at the Roche lobe and above it a planetary atmosphere is not bound to its planet by gravitational forces.
and can freely escape, whereas at the exobase level (where $r_{\text{ex}} < r_{\text{RI}}$) gas temperature should be above some critical value for blow-off to occur.

In the typical blow-off scenarios considered, for example, by Watson et al. (1981) and Jaritz et al. (2005) it is assumed that the stellar X-ray and EUV radiation is absorbed mainly at the so-called expansion radius $r_1$ (Watson et al., 1981) or at $r_{\text{exp}}$ (Jaritz et al., 2005) which is located for most “Hot Jupiters” at a planetocentric distance close to $r_{\text{RI}}$ or above. Therefore, Jaritz et al. (2005) proposed that, depending on a planetary and stellar mass, planetary size, stellar type and orbital distance, both scenarios, the classical hydrodynamic blow-off, and the so-called “geometrical blow-off”, as expected by Lecavelier des Etangs et al. (2004), may occur.

However, hydrodynamic model simulations by Yelle (2004) which include photochemistry of a hydrogen-rich atmosphere, indicate that the optical density for the stellar XUV radiation absorption $\tau_{\text{XUV}}$ by evaporating hydrogen is $\ll 1$ at $r_{\text{RI}}$ or at distances corresponding to Watson’s $r_1$, so that the main part of the XUV radiation is absorbed at lower altitudes. This is also in agreement with a recent study by Kulikov et al. (2006) who showed that in a terrestrial type planetary atmosphere even at very high XUV fluxes the bulk of the XUV radiation is absorbed at a distance $r_{\text{XUV}}$ (where $\tau_{\text{XUV}} = 1$) which is much closer to the planetary radius $r_{\text{pl}}$ than $r_1$. At altitudes which are much higher than $r_{\text{pl}}$ or $r_{\text{XUV}}$, the absorption optical depth $\tau_{\text{XUV}}$ of an evaporating atmosphere is much less than 1. And below $r_{\text{XUV}}$, where the optical depth $\tau_{\text{XUV}} \gg 1$, in situ XUV atmospheric heating becomes negligible.

The aim of this paper is to show that for some of the observed close-in “Hot Jupiters’ the Roche lobe can be fairly close to a planet and its exobase level, which can substantially enhance the mass loss of a planetary atmosphere as compared to the classic blow-off conditions, and may thus have important evolutionary implications.

## 2. Roche lobes of close-in exoplanets

We consider two spherical masses, $M_{\text{pl}}$ (exoplanet) and $M_{\text{star}}$ (star), separated by an orbital distance $d$, which rotate around their common center of mass. In the rotating coordinate system, the energy per unit mass of a test particle in the ecliptic plane is given by Paczyński (1971)

$$\Phi = \frac{GM_{\text{pl}}}{r_a} - \frac{GM_{\text{star}}}{r_b} - \frac{G(M_{\text{pl}} + M_{\text{star}})s^2}{2d^2},$$

(1)

where $s$ is the distance from a given point to the center of mass, $r_a$ and $r_b$ are distances to the centers of the exoplanet and the star, respectively, and $G$ is Newton’s gravitational constant.

The first term in eq. (1) represents the potential of the exoplanet and the second the potential of the star. The third term is the result of the orbital motion of the whole system. By introducing dimensionless quantities $\delta = M_{\text{pl}}/M_{\text{star}}$, $\lambda = d/r_{\text{pl}}$, $\eta = r_a/r_{\text{pl}}$, we analyze the variation of the potential along the axis which connects the exoplanet with its host star

$$\Phi(\eta) = \Phi_0 \left[ \frac{1}{\eta} - \frac{1}{\delta(1 - \eta)} - \frac{1 + \delta}{\delta} \left( \frac{1}{1 + \delta} - \eta \right)^2 \frac{1}{2\lambda^3} \right],$$

(2)

where

$$\Phi_0 = GM_{\text{pl}}/r_{\text{pl}}.$$

(3)

For this potential, there are two locations, the Lagrangian point L1 and the Lagrangian point L2 which are saddle points. Both points are rather close to each other for a small ratio of mass (Gu et al., 2003)

$$r_{\text{L1L2}} = \left(\frac{\delta}{3}\right)^{1/3} \left[ 1 + \frac{1}{3} \left( \frac{\delta}{3} \right)^{1/3} \right] d,$$

(4)

therefore, we consider as the Roche lobe boundary $r_{\text{RI}}$, approximately

$$r_{\text{RI}} \approx \left(\frac{\delta}{3}\right)^{1/3} d.$$

(5)

The potential difference between the Roche lobe boundary and the planetary radius (surface) can be written as

$$\Delta\Phi = \Phi_0 \frac{(\eta - 1)}{\eta} \left[ 1 - \frac{1}{\delta} \frac{\eta}{(\lambda(1 + \eta) - \eta)} - \frac{(1 + \delta)\eta(1 + \eta)}{2\lambda^3} \right].$$

(6)
at the Roche lobe boundary 2004) were used for the calculation of OGLE-TR-132 b (Bouchy et al. 2004; Moutou et al. 2004), OGLE-TR-10 b (Bouchy et al. 2005) and TreS-1 (Alonso et al. 2004), OGLE-TR-113 b (Bouchy et al. 2004), OGLE-TR-111 b (Pont et al. 2004), OGLE-TR-56 b (Burrows et al. 2004; Baraffe et al. 2004), OGLE-TR-132 b (Moutou et al. 2004), OGLE-TR-113 b (Bouchy et al. 2004), OGLE-TR-111 b (Pont et al. 2004), OGLE-TR-10 b (Bouchy et al. 2005) and TreS-1 (Alonso et al. 2004) were used for the calculation of $r_{\text{RI}}$ and $K$.

Table 1. Factors $1/K$ and $K$ for 7 “Hot Jupiters” where the planetary mass and radius is known. The planetary and stellar parameters for HD209458 b Barman et al. 2002, Vidal Madiar et al. 2003, OGLE-TR-56 b Burrows et al. 2004, Baraffe et al. 2004, OGLE-TR-132 b Moutou et al. 2004, OGLE-TR-113 b Bouchy et al. 2004, OGLE-TR-111 b Pont et al. 2004, OGLE-TR-10 b Bouchy et al. 2005 and TreS-1 Alonso et al. 2004 were used for the calculation of $r_{\text{RI}}$ and $K$.

| Exoplanet   | star type | $M_\star [M_\odot]$ | $M_{\text{pl}} [M_{\text{Jup}}]$ | $r_{\text{pl}} [r_{\text{Jup}}]$ | $r_{\text{RI}} [r_{\text{Jup}}]$ | $1/K$ | $K$ | $d [\text{AU}]$ |
|-------------|------------|----------------------|-----------------------------------|---------------------------------|---------------------------------|------|-----|----------------|
| HD209458 b  | G0V        | 1.05                 | 0.69                              | 1.43                            | 2.10                            | 0.47 | 0.045 |                  |
| OGLE-TR-56 b| G          | 1.04                 | 1.45                              | 1.23                            | 3.0                             | 0.41 | 0.023 |                  |
| OGLE-TR-132 b| F          | 1.34                 | 1.01                              | 1.15                            | 3.5                             | 0.52 | 0.031 |                  |
| OGLE-TR-113 b| K          | 0.77                 | 1.35                              | 1.08                            | 3.6                             | 0.57 | 0.023 |                  |
| TreS-1      | K0V        | 0.87                 | 0.75                              | 1.08                            | 4.9                             | 0.67 | 0.023 |                  |
| OGLE-TR-111 b| G or K    | 0.82                 | 0.53                              | 1.00                            | 5.8                             | 0.74 | 0.047 |                  |
| OGLE-TR-10 b| G or K     | 1.22                 | 0.57                              | 1.24                            | 3.7                             | 1.9  | 0.042 |                  |

By assuming $d \gg r_{\text{RI}} > r_{\text{pl}}$ and $M_{\text{star}} \gg M_{\text{pl}}$ what means $\lambda \gg \eta > 1$ and $\delta \ll 1$, we can simplify the expression for the potential difference

$$\Delta \Phi = \Phi_0 \left( \frac{\eta - 1}{\eta} \right) \left[ 1 - \frac{3(1+\eta)}{2\delta \lambda^3} \right].$$

Introducing the dimensionless quantities $\lambda$ and $\eta$ in eq. (5), we can further simplify eq. (7) for the potential difference to

$$\Delta \Phi = \Phi_0 \frac{(\eta - 1)^2(2\eta + 1)}{2\eta^3}.$$  

3. Implications for atmospheric blow-off

By considering a hydrodynamic regime of atmospheric escape we apply the energy conservation equation

$$\Gamma \left[ m \Delta \Phi + \frac{m v^2}{2} + \frac{5}{2} k (T_{\text{RI}} - T_0) \right] = \int_{r_{\text{pl}}}^{r_{\text{RI}}} q r^2 dr,$$

where $\Gamma$ is the loss rate of particles per steradian, $q = q_{\text{XUV}} - q_{\text{IR}}$, with the XUV volume heating rate $q_{\text{XUV}}$ and the cooling rate $q_{\text{IR}}$ due to IR emitting molecules like $H_2^+$ (Yelle, 2004), $m$ is the mass of the evaporating particles, and $v$ is the outflow bulk velocity at the Roche lobe boundary $r_{\text{RI}}$, $k$ is the Boltzmann constant, $T_{\text{RI}}$ is the temperature of hydrogen at the Roche lobe $r_{\text{RI}}$ and $T_0$ is the temperature at a distance close to $r_{\text{pl}}$, which is about the effective radiative temperature $T_{\text{eff}}$ of the exoplanet. Substituting the potential difference $\Delta \Phi$ into (9), we obtain

$$\Gamma = \frac{\int_{r_{\text{pl}}}^{r_{\text{RI}}} q r^2 dr}{\frac{m M_{\text{star}} G K}{r_{\text{pl}}} + \frac{m v^2}{2} + \frac{5}{2} k (T_{\text{RI}} - T_0)},$$

where the factor $K$ is

$$K = \frac{(\eta - 1)^2(2\eta + 1)}{(2\eta^3)} < 1.$$  

Neglecting the kinetic energy term $m v^2/2$ and also the thermal energy $5k(T_{\text{RI}} - T_0)$ and introducing the planetocentric distance $r_1$

$$r_1 = \left( \frac{\int_{r_{\text{pl}}}^{r_{\text{RI}}} q r dr}{I_{\text{XUV}}} \right)^{\frac{1}{2}},$$

we can obtain the energy limited escape rate equation similar to that derived by Watson et al. (1981), which corresponds to the distance below which the incoming stellar XUV radiation $I_{\text{XUV}}$ is absorbed by an evaporating atmosphere

$$\Gamma = \frac{r_1 r_{\text{RI}}^2 I_{\text{XUV}}}{m M_{\text{pl}} G K}.$$  

Fig. 1 illustrates the difference between the XUV absorption radius $r_1$ of Watson et al. (1981), the radius $r_{\text{XUV}}$ where the bulk XUV flux is absorbed and the energy absorption function $q_{\text{XUV}}$ has its peak, and the Roche lobe $r_{\text{RI}}$. Depending on the planetary...
and stellar parameters, \( r_1 \) can be located outside the Roche lobe. Due to the low number density of the evaporating hydrogen at planetary distances around and above \( r_{\text{RI}} \), the optical density of the XUV radiation \( \tau_{\text{XUV}} \) is \( \ll 1 \) and the main part of the stellar XUV radiation is absorbed at \( r_{\text{XUV}} \) which is close to \( r_{\text{pl}} \). This altitude \( r_{\text{XUV}} \) corresponds generally to the thermosphere and ionosphere.

If the bulk of the stellar XUV radiation is absorbed in the thermosphere at altitudes \( r_{\text{XUV}} < r_{\text{RI}} \), the particle loss can be enhanced substantially by the effect of the Roche lobe which manifests itself in the loss enhancement factor \( 1/K \) in eq. (13). Table 1 and Fig. 2 show the Roche lobe induced mass loss enhancement factor \( 1/K \) (and also \( K \)) as a function of \( r_{\text{RI}} \) in planetary radii \( r_{\text{pl}} \).

One can see from Table 1 that OGLE-TR-56b at an orbital distance of about 0.023 AU experiences the strongest enhancement of the Roche lobe affected evaporation resulting in a factor of about 2.4, while the Roche lobe induced enhancement of mass loss at HD209458b at 0.045 AU is about 2. As one can see from Fig. 2, the mass loss of “Hot Jupiters” which orbit at \( \leq 0.02 \) AU around their host stars, could be dramatically larger, because \( r_{\text{RI}} \) would move closer to \( r_{\text{pl}} \) and that enhances the evaporation. One more important effect due to the Roche lobe on “Hot Jupiters”, which experience hydrodynamic conditions, manifests itself in the potential energy difference which is less than the thermal energy at the exobase for blow-off to occur

\[
m\Delta\Phi \leq kT_{\text{max}},
\]

Here \( T_{\text{max}} \) is the maximum exobase temperature produced by the XUV heating. This equation yields

\[
\frac{GmM_{\text{pl}}K}{kT_{\text{max}}} \leq 1.
\]

One can see that the effect of the Roche lobe helps to satisfy this condition for lower temperatures than expected for the classic blow-off (that is for \( K = 1 \)). In other words, if the blow-off temperature for an exoplanet without the effect of the Roche lobe \( (K \approx 1) \) is, for example, 10000 K, a similar exoplanet, but which is closer to its host star, may start to evaporate hydrodynamically due to the Roche lobe effect at about 5000 K if the factor \( K \) is \( \approx 0.5 \) like it is for OGLE-TR-132b (see Table 1).

This result is very important, because it indicates that the effect of the Roche lobe can enhance the possibility that “Hot Jupiters” may reach hydrodynamic blow-off conditions more easily, even if their atmospheres have a high amount of molecules like \( \text{H}_2 \), which act as IR-coolers in the thermosphere. Both effects, the enhanced mass loss and the higher probability that “Hot
Jupiters’ reach hydrodynamic blow-off conditions at very close orbital distances to their host stars, may enhance the evaporation rate. The results of our study have to be included in the statistical mass-radius analysis of hot exoplanets expected to be detected during the CoRoT mission in the near future.

4. Conclusions

Our study shows that hydrodynamically driven atmospheric mass loss from “Hot Jupiters” at close orbital distances that are much less than 0.05 AU, may be strongly enhanced due to the Roche lobe effect as compared to non-affected exoplanets. We found that the mass loss due to the Roche lobe can be enhanced several times if the Roche lobe is located closer to a planet at a distance of a few planetary radii. Furthermore, our study indicates that the Roche lobe effect may also help “Hot Jupiters” to attain hydrodynamic blow-off conditions even if their exospheric temperatures are lower than those required for the blow-off to occur in the case of a classic Newtonian gravitational potential of a planet. Both effects may have a strong impact on the atmospheric evolution of short periodic hydrogen-rich gas giants.

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