Mathematical modeling of operation processes of refrigerant compressor self-acting valves of domestic refrigerators

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Abstract. The article presents a mathematical model of refrigerant compressor self-acting valves for domestic refrigerators providing a set of data characterizing compressor operating cycle: change in the cylinder pressure, change in cylinder volume, and change in gas flow entering or leaving the cylinder, change in moving trajectory of the valve plate. The technique and algorithm for calculating valves parameters in the compressor operating cycle is proposed. The presented mathematical model provides valves calculation and ensures calculation analysis of the valves depending on their parameters for a particular model of a refrigerant compressor.

1. Introduction
A general tendency of improving household refrigeration is: increasing technical level and creating new types of refrigerators using micro-compressor technology, improving designs of assembly units, reducing energy consumption, increasing durability and intensifying heat exchange, etc. [1, 2, 3]

The practice of design and production of refrigerator compressors shows that to create an optimal design of a compressor, as a rule, a number of theoretical and experimental studies are carried out, one of the main directions is the optimization of valves operation [4,5,6].

The development of a scientifically based method for calculating design parameters of the valve mechanism allows minimizing volume and energy losses, reducing shock loads [7].

2. Mathematical model of self-acting valves
The analysis of theoretical developments in the field of calculation and improvement of piston compressor valves showed that to receive complex data characterizing operating cycles of a compressor – return expansion, absorption, compression, forcing and trajectories of valves movement, it is necessary to get the description (model) of the following processes: a) pressure change in the cylinder; b) volume change in the cylinder; c) change of gas flow entering or leaving the cylinder and d) change in movement of the valve plates.

In order to simplify the task, the following assumptions are made: a) the valve plate is considered as cantilever beam of variable cross section of a trapezoidal shape with mass concentrated at the free end, having one degree of freedom, the valve stroke is limited in movement by the stop b) working body is regarded as ideal gas and gas movement energy in the cylinder is not taken into account c) pressure pulsation in cavities before and after the cylinder is absent, and d) heat transfer in the cylinder is taken into account only during compression and reverse expansion by selecting appropriate polytropic indicators, while in suction and ejection, the cylinder is considered to be an adiabatic system.

Taking into account accepted assumptions, a differential equation of pressure change in the cylinder is written as:

\[ \frac{dP}{dt} = P \left( \frac{dm}{m} - \frac{dV}{V} \right) \]  

(1)
\[ P_2 = P_1 \left( \frac{m_2}{m_1} \frac{V_1}{V_2} \right)^n \]

where \( P \) is pressure, \( V \) is volume and \( m \) is gas mass; and \( n \) is indicator of the process.

For compressors with crank-rocker transmission mechanism of rotation cylinder volume is:

\[ V_i = A_n e (1 - \cos \beta_i + 2 \alpha) \]

where \( V \) is cylinder volume; \( A_n \) is piston area; \( e \) is shaft and crank eccentricity; \( \beta_i \) is shaft rotation angle; \( \alpha \) is relative void volume.

The change in gas mass in the cylinder when the valves are open is similar to [1] and is determined by the equation of flow through the throttle

\[ \Delta m = (\alpha \zeta_p A_c)_{i-1} \cdot \sqrt{2 \rho (P_1 - P_2)_{i-1}} \cdot \Delta t, \]

Where \( \Delta m \) is t mass gas entering or leaving the cylinder during \( \Delta t \) (positive values are taken during suction, negative values are taken during discharge) \( \alpha \) is the coefficient of flow through the valve (determined experimentally); \( \epsilon_p \) is the coefficient of expansion:

\[ \epsilon_p = 1 - 0.3 \frac{P_1 - P_2}{P_1}. \]

(as adopted in accordance with the recommendations [8]); \( A_c \) is the valve seat area; \( \rho \) is the gas density in front of the valve; \( P_1, P_2 \) are the current pressure values before and after the valve; \( \Delta t \) is the time interval.

When moving in the gas flow to the valve plate, as can be seen from figure 1, the following forces act: \( F_n \) is the force of gas-dynamic flow pressure due to the pressure difference before and after the valve; \( F_y \) is plate elastic force; \( F_d \) is damping force; \( F_3 \) is valve sticking force.

When modeling the movement of free end of the valve plate, the component of gravity, due to a small mass of the plate, is not taken into account and but sticking force of the valve has real values only in the initial period of valve opening.

The equation of the motion of the free end of the valve plate in this case is written in the form:

\[ m_n \frac{d^2 h}{dt^2} = F_n - F_y - F_d - F_3. \]

Pressure force of the flow, reduced to the free end of the valve plate by analogy with the work [9], is determined from the expression:

\[ F_n = \rho_n (P_1 - P_2) A_k. \]

where \( \rho_n \) is the flow pressure coefficient taken into account reduction to the free end of the plate (determined experimentally) \( A_k \) is valve plate area; \( P_1, P_2 \) are current pressure values before and after the valve.

The damping force, in accordance with the recommendations of [8], is assumed to be proportional to the speed of the plate movement:

\[ F_d = b \frac{dh}{dt}. \]

where \( b = 2n_3 m_n \) is the damping coefficient; \( n_3 \) is the attenuation coefficient (determined experimentally); \( m_n \) is the plate mass reduced to the free end of the plate.

Reduced elastic force of the valve plate is obtained from differential equation of elastic line of the trapezoidal shape cantilever beam loaded by concentrated force at the free end.
For a triangular plate, differential equation of the elastic line is written as:

$$EJ_x \frac{d^2 h}{dx^2} = M_x.$$  \hspace{1cm} (8)

where $J_x$ is moment of section inertia in the point $x$; $E$ is transverse modulus of material elasticity; $M_x$ is bending moment in section $x$.

From similarity of triangles:

$$b_x = \frac{b_t}{l} \left(1 - \frac{x}{l}\right).$$  \hspace{1cm} (9)

The moment of section inertia in the point $x$ is equal to:

$$J_x = \frac{b_t \delta^3}{12} \left(1 - \frac{x}{l}\right).$$  \hspace{1cm} (10)

where $J_t = \frac{b_t \delta^3}{12}$ is the moment of section inertia in point $x = 0$.

Substituting values $J_x$ and $M_x$, into (8)

$$\frac{d^2 h}{dx^2} = \frac{F_t \left(1 - \frac{x}{l}\right)}{EJ_t} = \frac{F_t}{EJ_t}.$$  \hspace{1cm} (11)

Integrating the equation of elastic line twice, the dependence of deflection value of the triangular plate in any section on applied force is obtained:

$$h = \frac{F_t l x^2}{2 E J_t}.$$  \hspace{1cm} (12)

From (12) force that must be applied at the end of the triangular plate to obtain the deflection $h$ is equal to:

$$F_t = \frac{2 E J_t h}{lx^2}.$$  \hspace{1cm} (13)

To obtain a deflection $h$, force must be applied to the rectangular plate:

$$F_n = \frac{6 E J_n h}{x^2 \left(3 x - l\right)}.$$  \hspace{1cm} (14)

where $J_n = \frac{b_n \delta}{12}$ is inertia moment of the rectangular plate section.

Summing expressions (13) and (14), gives the force to be applied at the end of the trapezoidal plate to obtain desired deflection $h$:

$$F_x = \frac{2 E h \left(\frac{J_t}{l} + \frac{3 J_n}{3 l - x}\right)}{x^2 \left(3 x - l\right)}. $$  \hspace{1cm} (15)

or when $x=l$

$$F_y = \frac{(2 J_t + 3 J_n)E}{l^3} = c_n h.$$  \hspace{1cm} (16)
where 
\[ c_n = \frac{E\delta^3 (2b_1 + 3b_2)}{12l^3}. \]

The reduction of the plate mass to the point of forces application is based on the equality of kinetic energies of motion of the reduced \( m_n \) and true mass of the system \( m_0 \).

The value of reduced mass is proportional to true mass and is determined by the formula:
\[ m_n = k_m m_0. \tag{17} \]

where \( k_m \) is mass reduction coefficient.

The mass reduction coefficient depends on the law of motion velocities change of mass elements, and can be determined from the expression
\[ k_m = \frac{1}{m_0} \int \left( \frac{\delta x}{\delta} \right)^2 dm. \tag{18} \]

where \( \delta, \delta_x \) are generalized displacement points of mass reduction and arbitrary system points under static action of generalized force on it can be applied in a reduction point.

Based on (18), the mass reduction coefficient for the plate with constant cross section is:
\[ k_m = \frac{1}{l} \int_0^l \left( \frac{y_x}{y} \right)^2 dx = \frac{1}{l} \int_0^l \left[ \frac{Fx^2 (3l - x)}{6EJ} - \frac{3EJ}{Fl^3} \right] dx = \frac{1}{l} \int_0^l \left( \frac{3x^2}{2l^2} - \frac{x^3}{2l^3} \right)^2 dx. \tag{19} \]

For variable cross section plate of a trapezoidal form, mass reduction coefficient is calculated as:
\[ k_m = \frac{2}{l} \int_0^l \left( \frac{y_x}{y} \right)^2 f(x)dx. \tag{20} \]

where \( f(x) \) is the dependence of cross-sectional area on \( x \).

\[ k_m = \frac{2}{l} \int_0^l \left( \frac{Fx^2}{2EJ} \cdot \frac{2EJ}{Fl^3} \right)^2 \left( 1 - \frac{x}{l} \right) dx = \frac{2}{l} \int_0^l \left( \frac{x^2}{l^2} \right)^2 \left( 1 - \frac{x}{l} \right) dx = \frac{2}{l} \int_0^l \left( \frac{x^4}{l^4} - \frac{x^5}{l^5} \right) dx. \tag{21} \]

The value of mass reduction coefficient for a trapezoidal valve plate, calculated from expressions (19) and (21), is 0.3.

Substituting into the equation of the valve plate motion (5) dependences (6), (7), (16), we have:
\[ m_n \frac{d^2 h}{dt^2} = \rho_n \left( P_1 - P_2 \right) A_k + b \frac{dh}{dt} - c_n h - F_3. \tag{22} \]

After series of transformations and substitutions, the equation (22) can be represented as:
\[ \frac{d^2 h}{dt^2} + 2n_3 \frac{dh}{dt} + k_x^2 h = k_2, \quad k_1 = \sqrt{\frac{c_n}{m_n}}; \quad k_2 = \frac{\rho_n \left( P_1 - P_2 \right) A_k - F_3}{m_n}. \tag{23} \]

The function \( k_2 \) is constant value, which is valid for the subsequent step-by-step solution of equations (23) with a small step.

The equation (23) is a second-order differential linear inhomogeneous equation with constant coefficients.

The general solution of the differential equation (23) is equal to the sum of its particular solution of \( h_{ch} \) and general solution \( h_0 \) of the corresponding homogeneous equation [10]:
\[ h = h_0 + h_{ch}. \tag{24} \]
The characteristic equation is written as:

\[ \lambda^2 + 2n_3\lambda + k_1^2 = 0. \] (25)

The roots of this equation are:

\[ \lambda_1 = -n_3 + \sqrt{n_3^2 - k_1^2}, \lambda_2 = -n_3 - \sqrt{n_3^2 - k_1^2}. \]

In the case under consideration \( n_3 < k_1 \) general solution of the homogeneous equation is written as:

\[ h_0 = e^{-n_3t}\left(C_1 \cos\left(k_1^2 - n_3^2 t\right) + C_2 \sin\left(k_1^2 - n_3^2 t\right)\right). \] (26)

where \( C_1 \) and \( C_2 \) are constant integrations.

The particular solution of equation (23) is written in the form:

\[ h_u = A \cdot \frac{dh_u}{dt} = 0 \cdot \frac{d^2 h_u}{dt^2} = 0. \] (27)

Substitution of (27) in equation (23) gives:

\[ A = \frac{k_2}{k_1}. \] (28)

The general solution of equation (23) is written as

\[ h = e^{-n_3t}\left(C_1 \cos\left(k_1^2 - n_3^2 t\right) + C_2 \sin\left(k_1^2 - n_3^2 t\right)\right) + \frac{k_2}{k_1}, \] (29)

\[ \frac{dh}{dt} = w = -n_3e^{-n_3t}\left(C_1 \cos\left(k_1^2 - n_3^2 t\right) + C_2 \sin\left(k_1^2 - n_3^2 t\right)\right) + e^{-n_3t}\left(C_2 \left(k_1^2 - n_3^2 \cos\left(k_1^2 - n_3^2 t\right) - C_1 \left(k_1^2 - n_3^2 t\right)\right)\right) \] (30)

Permanent integration of \( C_1 \) and \( C_2 \) are determined from the following initial conditions: \( h(t)_{t=0} = h_0 \)

\( .w(t)_{t=0} = w_0. \)

Joint solution of equations (29) and (30) gives:

\[ C_1 = h_0 - \frac{k_2}{k_1}, C_2 = \frac{n_3 \left(h_0 - \frac{k_2}{k_1}\right)}{\sqrt{k_1^2 - n_3^2}}. \] (31)

After substitution of values and a number of transformations, solution of equation (23) is written in the form:

\[ h = e^{-n_3t} \times \left[ \left(h_0 - \frac{k_2}{k_1^2}\right) \cos\left(k_1^2 - n_3^2 t\right) + \frac{n_3 \left(h_0 - \frac{k_2}{k_1^2}\right)}{\sqrt{k_1^2 - n_3^2}} \times \sin\left(k_1^2 - n_3^2 t\right) \right] + \frac{k_2}{k_1^2}, \] (32)
Meanwhile calculation of reverse expansion and angle of the compressor shaft

The development of methods and algorithms for calculating compressor valves parameters

Calculation of the working process is carried out in stages with a step \( \Delta t \) which corresponds to rotation angle of the compressor shaft. Meanwhile \( \Delta \beta = 1 \) is used for calculation of reverse expansion and compression and \( \Delta \beta = 1 \) is calculation of suction and discharge.

To calculate the \( i \)-th integral, the initial equations are written in the form:

\[
P_i = P_{i-1} \left( \frac{m_i}{m_{i-1}} \frac{V_{i-1}}{V_i} \right),
\]

\[
V_i = A_n e (1 - \cos \beta_i + 2a).
\]

\[
\Delta m = (\alpha e A_c)_{i-1} \sqrt{2 \rho (P_1 - P_2)_{i-1}} \Delta t.
\]

\[
m_i = m_{i-1} \pm \Delta m.
\]

\[
h_i = e^{-n_3 \Delta t} \times
\]

\[
\left( h_{i-1} - \frac{k_{2i}}{k_1} \right) \cos \left( \sqrt{k_1^2 - n_3^2} \Delta t \right) + \frac{w_{i-1} + n_3 \left( h_{i-1} - \frac{k_{2i}}{k_1^2} \right)}{\sqrt{k_1^2 - n_3^2}} \cdot \sin \left( \sqrt{k_1^2 - n_3^2} \Delta t \right) + \frac{k_{2i}}{k_1^2};
\]

\[
w_i = e^{-n_3 \Delta t} \cos \left( \sqrt{k_1^2 - n_3^2} \Delta t \right) - \left( \frac{w_{i-1} + n_3 \left( h_{i-1} - \frac{k_{2i}}{k_1^2} \right)}{\sqrt{k_1^2 - n_3^2}} + \left( h_{i-1} - \frac{k_{2i}}{k_1^2} \right) \sqrt{k_1^2 - n_3^2} \right) \sin \left( \sqrt{k_1^2 - n_3^2} \Delta t \right);
\]

\[
t_i = t_{i-1} + \Delta t \beta_i = 360 n_v t_c, \quad k_1 = \frac{C_n}{m_n} \cdot k_{2i} = \frac{\rho_{ni-1}(P_1 - P_2)_{i-1} A_k - F_3}{m_n},
\]
where $n_p$ is the frequency of the compressor shaft rotation; $C_n$ is the valve plate stiffness; $m_n$ is the valve plate mass; $\rho_n$ is the pressure coefficient of the flow; $A_k$ valve plate area; $F_3$ is the plate sticking force.

When calculating the processes of reverse expansion and compression in the cylinder, the number $m_i$ of equation (34) takes the value of 1, and the polytropic exponent $n=n_p$ under reverse expansion, and $n=n_c$ in compression, determined from experimental diagrams of pressure change in the compressor cylinder. Calculation of suction and discharge is carried out by the indicator $n$ which is equal to the adiabatic indicator $k$ for a refrigerant agent [11].

Regarding the pressure difference in equations (36) and (38), (39), four cases are considered:

- suction $P_1-P_2 = P_{km1} - P_i$; discharge $P_1-P_2 = P_i - P_{km2}$; return leaks when closing is delayed:

for a suction valve $P_1-P_2 = P_i - P_{km}$; for discharge valve $P_1-P_2 = P_{km2} - P_i$.

When calculating displacement and speed of valves movement according to equations (38), (39), in case the valve reaches the lift limit, speed in the first reverse motion interval is calculated by the equation:

$$\omega_{i-1} = -\psi \cdot \omega_{i-1}.$$  

where $\psi$ is the speed recovery coefficient.

Speed values in simulation were taken within the following limits: $\psi = 0.35$ is partial recovery of speed, $\psi = 0$ is full damping [9].

Phased calculation with step $\Delta \beta$, begins when the piston is in the top dead point. It is conventionally assumed that the discharge valve is closed and $P_{km2}$ is pressure in the cylinder which is equal to the discharge pressure $P_i$.

The reverse expansion is described by equation (34) and it ends when the pressure in the cylinder is equal to the suction pressure $P_{km1}$. From this moment the opening of the suction valve, described by equations (38), (39), begins. Calculation of gas mass entering the cylinder is carried out according to the equation (37). In the final phase of suction, as a result of decrease in piston speed, differential pressure in the valve decreases and the valve begins to close. In case of delay in the valve closing under insufficient plate rigidity, reverse leakage occurs, reducing the compressor performance. When the valve is placed on the saddle, the calculation of suction is over. The calculation of compression and discharge is carried out similarly to reverse expansion and suction.

Reaching the angle of shaft rotation $\beta = 360$, the calculation is repeated. Obviously, the position of discharge valve and the cylinder pressure obtained from the previous cycle should be taken as initial data for recalculation, which takes into account the effect of depression on discharge and delay of valve closure on the compressor operation process.

4. Analysis of the valve mechanism parameters effect on the effectiveness of its work

Calculations and measurements of actual valve movements were carried out for the serial model of the compressor HKB6. The results of the calculations and the actual valve movements after their processing in the form of diagrams are shown in Figure 1. The analysis of the calculated and actual diagrams of valve movements shows fairly good coincidence, which indicates the efficiency of the proposed mathematical model. The angles of opening and closing of discharge and suction valves and the speed of plate movement in the moment of touching the lift stop and landing on the saddle on the calculated and actual diagrams almost completely coincide, and discrepancy of plate movement in shaft rotation angle, does not exceed 9%, due to adoption of simplifying assumptions.

The proposed mathematical model allows calculating analysis of valves operation depending on their parameters for this compressor model, but it can also be applied to other models. It should be borne in mind that the effectiveness of the valve operation depends on movement of its plate and pressure loss during valve operation.
The plate of serial discharge valve oscillates throughout ejection. The valve closure takes place prematurely, when the angle of rotation of the shaft is equal to 356°, with subsequent rebound and closing of the 3° after the piston passes the top dead center.

Increase in rigidity of the valve plate by 1.5 times from the nominal value leads to increase in the amplitude of plate oscillations and overpressure in the cylinder, and is also accompanied by increase in advance angle of the valve closure. With decrease in plate stiffness in 2 times, there is decrease in pressure loss and speed of seating the valve on the saddle, as well as the absence of valve oscillations during ejection from the cylinder.

Increase in the flow pressure coefficient ensures more complete opening of the flow section valve throughout ejection stroke, simultaneously reducing the overpressure for valve lift and landing on the saddle.

The calculation of the valve displacement diagram for different valve mass values, damping coefficients and speed recovery is important for assessing their influence on the valve operation and allows choosing optimal valve parameters.

5. Conclusion
1. A mathematical model has been developed for calculating the operating cycle of refrigerant compressor valves for household refrigeration devices. The efficiency of the proposed mathematical model is proved.

2. The calculated analysis of the influence of the valve mechanism parameters on the compressor operation cycle is given and recommendations on the choice of optimal values are proposed.

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