Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

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Abstract. We simulate the neutrino and $\gamma$-ray emissions of the galaxy which are originated from the hadronic scattering of cosmic rays (CRs) with the interstellar medium (ISM). Rather than assuming a uniform CR density, we estimate the spatial distribution of CR nuclei by means of numerical simulations. We consider several models of the galactic magnetic field and of the ISM distribution, finding only a weak dependence of our results on their choice. We find that by extrapolating the predicted $\gamma$-ray spectra down to a few GeV we get a good agreement with EGRET measurements. Then, we can reliably compare our predictions with available observations above the TeV both for the $\gamma$-rays and the neutrinos. We confirm that the excesses observed by MILAGRO in the Cygnus region and by HESS in the galactic centre ridge cannot be explained without invoking significant CR overdensities in these regions. Finally, we discuss the perspectives that a km$^3$ neutrino telescope based in the Northern Hemisphere has to measure the diffuse emission from the inner galaxy.

Keywords: cosmic rays, neutrino and gamma astronomy

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1. Introduction

It is well known that the interaction of cosmic ray (CR) nuclei with the interstellar medium (ISM) should give rise to diffuse $\gamma$-ray and neutrino emissions [1, 2]. Several satellites (see [3] for a review), especially EGRET [4, 5], have already observed the $\gamma$-ray diffuse emission of the galaxy up to $\sim$10 GeV. Around the GeV, the hadronic origin of a large fraction of that emission is testified by its good correlation with the distribution of the interstellar hydrogen. GLAST observatory [6] should soon provide a deeper insight into the nature of that emission, as it will probe the $\gamma$-ray sky with much better sensitivity and angular resolution than EGRET and extend the explored energy window up to 300 GeV. Above that energy, however, satellite observatories can hardly provide significant data.

Ground based experiments are divided in two main groups: extensive air shower arrays and atmospheric Cherenkov telescopes. In the former class, experiments like MILAGRO [7] and TIBET [8] can probe large regions of the sky at energies larger than a few TeV with an angular resolution of several degrees. These experiments have already provided interesting constraints on the diffuse emission from the galactic disc [9, 10] and, in the case of MILAGRO, a diffuse emission from the Cygnus region has been recently observed [11]. Unfortunately, none of these experiments covers a sky window encompassing the galactic...
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV centre (GC). Atmospheric Cherenkov telescopes, like HESS [12] and MAGIC [13], are sensitive to $\gamma$-rays with energy in the range $0.1 \text{ TeV} \lesssim E \lesssim 100 \text{ TeV}$ and have a very good angular resolution (better than $0.1^\circ$). However, although these instruments can search for a diffuse emission in some limited regions of the sky, they are mainly dedicated to observing quite localized sources.

Neutrino telescopes (NTs) may soon provide a new valuable probe of high energy CR physics which can be complementary to $\gamma$-ray observatories. The large detection volume, large field of view, available observation time, and relatively good angular resolution (which should be better than $1^\circ$ for water Cherenkov NTs looking for up-going muon neutrinos) make, at least in principle, these instruments well suited to search for the diffuse neutrino emission of the galaxy.

The AMANDA telescope, which is operating at the South Pole, has already put an upper limit on the $\nu_\mu$ flux from the GP [14] (see section 4) and ICECUBE [15] should significantly improve this limit.

However, in this context the most promising NTs are those based in the Northern Hemisphere (ANTAARES [16], NEMO [17] and NESTOR [18] and especially their planned km$^3$ upgrade, the Km3NeT project [19]) as they will be sensitive to up-going $\nu_\mu$s coming from the GC region. Furthermore, since these instruments will be deployed in water, their angular resolution should be better than the NT at the South Pole, improving their chances of disentangling the signal from the background.

From the theoretical point of view, several calculations of the $\gamma$-ray and neutrino diffuse fluxes above the TeV have been already performed. In [20]–[22] the CR distribution in the galactic disc was assumed to be homogeneous so that the secondary $\nu$ and $\gamma$-ray fluxes come out to be proportional to the gas column density (in [20,22] a uniform gas distribution was assumed). While that was a very reasonable approximation to start with, a more accurate analysis should account for the inhomogeneous CR distribution which arises due to the non-uniform scatter of sources. It should be noted that, since CR sources are likely to be related to supernova remnants (SNRs) and these objects are most abundant in the gas (target) rich regions, the secondary $\gamma$-ray and neutrino emissions may be significantly enhanced with respect to the case in which a uniform CR distribution is assumed. The CR homogeneity assumption was released in more recent works, where the high energy nucleus distribution was modelled by solving the spatial diffusion equation [23,24]. These papers, however, were mainly addressed to modelling the diffuse $\gamma$-ray emission below the TeV, and did not consider neutrinos. In [25] a different, and promising, approach was considered to solve the diffusion equation and to model the neutrino emission up to the PeV. In that paper, however, several approximations were made which did not allow an angular accuracy better than $10^\circ$ to be reached.

The aim of this work is to model both the diffuse $\gamma$-ray and neutrino emissions of the galaxy above the TeV with a better accuracy. This is necessary in order to be able to establish the detection perspectives of air shower arrays and neutrino telescopes and to interpret correctly forthcoming observations. Our approach is comprehensive as we carefully account both for the spatial distribution of CR nuclei and for that of the ISM.

In section 2 we start by discussing the main properties of the ISM. We pay special attention to the distribution of SNR (see section 2.1) and to that of the atomic and molecular hydrogen (section 2.3). In both cases we adopt models which we apply for the first time in this context. Details of gas models and their comparison with those adopted
in previous works are given in appendix A. In section 3 we model the distribution of the CR nuclei in the galaxy by solving numerically the diffusion equation. Our approach is similar to that sketched in [25], that we fully exploit here. Differently from [23, 24], we use expressions for the diffusion coefficients as determined from Monte Carlo simulations of charged particle propagation in turbulent magnetic fields [26, 27]. This allows us to get more detailed CR distributions above the TeV and to test how much the large uncertainties in the knowledge of the turbulent component of the GMF (see section 2.2) affect our predictions. In section 4 we combine the simulated CR distributions and the gas models to map the expected $\nu_\mu$ and $\gamma$-ray emissions. Then, in section 5 we compare our predictions with available experimental results. In section 6 we briefly discuss the perspectives that a km$^3$ NT to be built in the Mediterranean sea has to detect the muon neutrino emission from the inner galaxy. Finally, in section 7 we summarize our conclusions.

2. The spatial structure of the ISM

In order to assess the problem of the propagation of CRs and their interaction with the ISM we need the knowledge of three basic physical inputs, namely:

(i) the distribution of supernova remnants (SNRs), which we assume to trace that of CR sources;
(ii) the properties of the galactic magnetic field (GMF) in which the propagation occurs;
(iii) the distribution of the diffuse gas providing the target for the production of $\gamma$-rays and neutrinos through hadronic interactions.

Since all these inputs are affected by large uncertainties and systematics, we need to estimate how their poor knowledge is transferred in the resulting $\gamma$-ray and $\nu$ emission. Therefore, we compare the results obtained using several models. In the following we assume cylindrical symmetry and adopt the Sun galactocentric distance $r_\odot = 8.5$ kpc.

2.1. The SNR distribution in the galaxy

The rate of galactic SN explosions, as inferred from observations of SNs in external galaxies similar to the Milky Way [28], is roughly

$$ R_I \sim \frac{1}{250} \text{yr}^{-1} \quad R_{II} \sim \frac{1}{60} \text{yr}^{-1} $$

respectively for type I and type II SNs. The total rate is $\simeq 1/48$ yr$^{-1}$ which is in reasonably good agreement with records of historical SNs. Since we will normalize the CR injection rate by requiring the simulated CR flux to meet the observed value at the Earth position, we are only interested in the $R_I$ and $R_{II}$ ratio.

What is even more crucial to our analysis is the spatial distribution of type I and II SNs. Since type-Ia and core-collapsed SNs originate from different stellar populations their spatial distributions, as well as their rates, are different.

Several methods to determine the SNR distribution in the galaxy are discussed in the literature. One of the most commonly adopted methods is that to estimate the SNR distances on the basis of the surface brightness–distance ($\Sigma$–$D$) relation [29]. Such an analysis does not cover the GC region. Furthermore, several doubts have been raised
on the accuracy of this method as it is plagued by a number of systematics concerning
the completeness of the available SNR catalogue and the proper handling of selection
effects [30,31]. Here we adopt an SNR distribution which is inferred from observations
of related objects, such as pulsars or progenitor stars, as done e.g. in [32], which, in our
opinion, is a safer approach.

In [32] the spatial distribution of type-Ia SNRs adopted was assumed to follow that of
old disc stars which have an exponential scale length \( \approx 4.5 \text{ kpc} \) along \( r \) and an exponential
scale height \( \approx 300 \text{ pc} \). Therefore,
\[
R_I(r, z) = K_I \exp \left( -\frac{r - r_\odot}{4.5 \text{ kpc}} - \frac{|z|}{0.3 \text{ kpc}} \right),
\]
where \( K_I \) is a normalization factor. Although type-Ia SNs are globally less frequent than
core-collapse SNs, their rate is dominating in the inner few kpc of the galaxy.

To trace core-collapse originated SNRs one may use either HII regions, which are
produced by their luminous progenitors, or pulsars, which are a likely left-over of the
collapse. The pulsar distribution at birth was estimated to be [32]–[34]
\[
R_{II}(r, z) = K_{II} f(z) \begin{cases} 
3.55 \exp \left[ -\left( \frac{r - 3.7 \text{ kpc}}{2.1 \text{ kpc}} \right)^2 \right], & r < 3.7 \text{ kpc} \\
\exp \left[ -\frac{r^2 - r_\odot^2}{(6.8 \text{ kpc})^2} \right], & r > 3.7 \text{ kpc}
\end{cases}
\]
where
\[
f(z) = 0.79 \exp \left[ -\left( \frac{z}{0.2 \text{ kpc}} \right)^2 \right] + 0.21 \exp \left[ -\left( \frac{z}{0.6 \text{ kpc}} \right)^2 \right].
\]
While the absolute values of \( K_I \) and \( K_{II} \) are irrelevant here, their ratio is needed to
normalize the relative weights of type-I and type-II SNR distributions. By requiring that
\( R_{II}/R_I = 4.2 \), as follows from (1), we find \( K_{II}/K_I \approx 7.3 \).

In figure 1 we show the total SNR radial distribution as obtained in this way and
compare it with that given in [29]. Both distributions have been normalized so to take
the same value at the solar circle, where observations are most reliable. It is evident
that in the inner galaxy the SNR rate distribution which we adopt in this work is
significantly higher than that given in [29]. Rather, we have a relatively good agreement
with the distribution adopted in [35] (see also [36]), which is based on pulsars, and with
independent observations of the 1809 keV line of \( ^{26}\text{Al} \), which is thought to be a reliable
tracer of SNs [37,38].

We assume the spatial distribution of the CR sources to coincide with that of SNRs
and the CR energy spectrum at source to be everywhere a power-law with exponent \( \beta \)
(\( \beta \), which refers to the injection spectrum, should not be confused with the slope of the
observed CR spectrum). Therefore, the source term in our diffusion equation will be
\[
Q(E, r, z) = q(r, z) E^{-\beta} = K (R_I(r, z) + R_{II}(r, z)) E^{-\beta}.
\]
Its absolute normalization \( K \) and spectral slope \( \beta \) will be fixed so as to reproduce the
observed spectrum at Earth below the knee. For protons [39]
\[
I_p(E) \approx 8.7 \times 10^{-6} \left( \frac{E}{1 \text{ TeV}} \right)^{-2.7} \text{ (TeV cm}^2 \text{ s sr)}^{-1}.
\]
2.2. Regular and random magnetic fields

The Milky Way, as well as other spiral galaxies, is known to be permeated by large-scale, so called *regular*, magnetic fields. The orientation and strength of these fields is measured mainly by means of Faraday rotation measurements (RMs) of polarized radio sources. From these observations it is known that the regular field in the disc of the galaxy is prevalently oriented along the disc plane and it seems to follow the galactic arms as observed in other spiral galaxies. According to [40], its strength at the Sun position is \( B_0 = B_{\text{disc} \text{reg}}^r(r_\odot, 0) = 2.1 \pm 0.3 \, \mu\text{G} \) while at smaller radii \( B_{\text{disc} \text{reg}}^r(r) = B_0 \exp\{- (r - r_\odot)/r_B\} \) where \( r_B = 8.5 \pm 4.7 \, \text{kpc} \). A \( 1/r \) profile seems to give the worst fit of data. Unfortunately, observations are not significant for \( r < 3 \, \text{kpc} \). Most likely [40] the regular field in the disc has a bi-symmetric structure (BSS) with a counterclockwise field in the spiral arms and clockwise in the interarm regions. Concerning its vertical behaviour, it is generally assumed that \( B_{\text{disc} \text{reg}}^r \) decreases exponentially for increasing values of \( |z| \) with a scale height of a few hundred parsecs. There is increasing evidence that the field is symmetric for \( z \to -z \) (BSS-S) [41].

Superimposed on the regular field a random, or turbulent, component of the GMF is known to be present. In the disc, this component is comparable to, or even larger than, the regular one. Indeed, the locally observed rms value of the total field is about \( 6 \pm 2 \, \mu\text{G} \), which is two to four times larger than \( B_{\text{reg}}(r_\odot, 0) \). From polarimetric measurements of stellar light and RMs of close pulsars it has been inferred that the GMF is chaotic on all scales below \( L_{\text{max}} \sim 100 \, \text{pc} \). The power spectrum of the GMF fluctuations is poorly known. Observational data, obtained from RM of pairs of close pulsars, are compatible with a Kolmogorov spectrum, i.e. \( B^2(k) \propto k^{-5/3} \), though with a very large uncertainty (see e.g. [42] and references therein).
What is most relevant here is the galactic magnetic halo (MH) since most CR propagation takes place outside the disc. The regular component of the MH has been studied by means of RMs of polarized extra-galactic radio sources. These observations showed that $B_{\text{halo}}$ is mainly azimuthal, the same as $B_{\text{disc}}$, and that its vertical scale height is $z_r \simeq 1.5$ kpc [43]. The radial behaviour of $B_{\text{halo}}$ is poorly known and it is generally assumed that it traces that of $B_{\text{disc}}$. Here we share the same attitude. It is worth noticing that the vertical symmetry of the MH might be opposite with respect to that in the disc: this fact may have relevant consequences for the propagation of CRs with $E/Z \gtrsim 10^{15}$ eV [27].

In the following we assume a symmetric structure so that the regular component of the MH can be considered as an extension of $B_{\text{disc}}$ and we can combine both in a unique simple structure:

$$B_{\text{reg}}(r, z) = B_0 \exp \left\{ \frac{r_r - r_{\odot}}{r_B} \right\} \frac{1}{2 \cosh(z/z_r)}, \quad (7)$$

with $z_r = 1.5$ kpc and $r_B$ left as a free parameter. Similarly to [27], we use a cosh function to regularize the exponential vertical profile of the regular and turbulent components of the GMF at $z = 0$.

Concerning the properties of the random MF component in the halo, very little is known from observations. Both the vertical extension of the radio halo and the isotopic relative abundances of CR suggest that the vertical scale height of the random fields is significantly larger than that of the regular one, $z_t \simeq 3$–5 kpc or larger. It is also likely that the turbulence strength increases in the regions with the highest star-forming activity. We account for this possible radial dependence of $B_{\text{turb}}$ by means of the parameter $\sigma(r) \equiv \langle B_{\text{ran}} \rangle_{\text{rms}}(r, 0)/B_{\text{reg}}(r, 0)$. Therefore, we write

$$B_{\text{ran}}(r, z) = \sigma(r) B_{\text{reg}}(r, 0) \frac{1}{2 \cosh(z/z_t)} . \quad (8)$$

Due to the relatively small number of extra-galactic radio sources, the maximal scale of magnetic fluctuations $L_{\text{halo}}^\text{max}$ and their power spectrum in the MH are practically unknown. In the following we assume that $L_{\text{halo}}^\text{max}$ is the same as in the disc and consider only Kolmogorov and Kraichnan ($B^2(k) \propto k^{-3/2}$) power spectra.

2.3. The gas distribution

The diffuse gas accounts for about 10–15% of the total mass of the galactic disc (not including dark matter) and its chemical composition is dominated by hydrogen (about 90.8% by number and 70.4% by mass) and helium (9.1% by number and 28.1% by mass). Hydrogen is shared in three main components [32]: ionized (HII, total mass $M_{\text{HII}} \simeq 1 \times 10^9 M_\odot$), atomic (HI, $M_{\text{HI}} \simeq 6 \times 10^9 M_\odot$) and molecular ($H_2$, $M_{\text{H}_2} \simeq 1$–2 $\times 10^9 M_\odot$). While all these components have comparable masses, their spatial distributions are quite different. The much hotter HII has a scale height along the vertical axis considerably larger than the other hydrogen components ($h_{\text{HII}} \simeq 1$ kpc). Therefore, its contribution to the $\gamma$-ray and neutrino emissivity from the galactic plane is subdominant and can be neglected in the following.

The construction of galactocentric radial density profiles for HI and $H_2$ intrinsically needs model dependent assumptions (see appendix A for further details). Therefore, large
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

Figure 2. Left panel: the HI volume density along the galactic plane \((z = 0)\) is plotted versus the radius \(r\). Green crosses display the binned distribution as given in [44] while the continuous line (green) is our fit of these points (model A). Blue crosses are derived from [46]. The dashed (red) line is our fit of these points with the distribution given in [48] for \(r > 3\) kpc (model B). Right panel: the radial profile of the HWHM scale height \((z_{1/2})\) for the same models.

Uncertainties are involved in this operation. Here we consider two models, which we call gas models A and B, both for the atomic and molecular hydrogen, that will be discussed in detail in appendix A.

Model A. This has been developed by Nakanishi and Sofue (NS) in [44] for HI and by the same authors in [45] for H\(_2\).

Model B. We construct model B by suitably combining the results of different analyses which have been separately performed for the disc and the galactic bulge. For the H\(_2\) and HI distributions in the bulge we use a detailed 3D model recently developed by Ferriere et al [46] on the basis of several observations. For the molecular hydrogen in the disc we use the well known Bronfman et al model [47]. For the HI distribution in the disc, we adopt the Wolfire et al [48] two-dimensional model. Although the Ferriere et al model is not cylindrically symmetric, we verified this by fitting their 3D distribution with a cylindrically symmetric one and assuming a Gaussian vertical profile peaked on the GP; we get \(\gamma\)-ray and neutrino fluxes that, when integrated over windows larger than 1 degree squared, differ very little from those obtained using the complete 3D models. For this reason, in the following we work only with averaged 2D distributions.

In figures 2 and 3 we show the volume density radial and vertical profiles of HI and H\(_2\) obtained with models A and B together with the continuous fits we use in our analysis without reporting observational errors, as they are typically much smaller than systematic ones (the difference between models A and B will provide a glimpse of the extent of these uncertainties).

While the gas densities in models A and B differ relatively little close to the solar circle (what is most relevant here is H\(_2\)), the main discrepancies arise in the galactic bulge. Indeed, this is the region where the uncertainties on the gas velocity are the largest. This discrepancy, however, has little consequence on the gas column density (see figure 4), as it should since this quantity is almost directly related to the observed CO emission.
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

Figure 3. Left panel: the H$_2$ volume density along the galactic plane as a function of $r$. Green crosses display the binned distribution as given in [45] and the continuous (green) line is our continuous fit of these points (model A). Red and blue crosses are derived (see text) from [47] and [46] respectively. The dashed (red) line is our combined fit of all these points (model B). Right panel: the radial profile of the HWHM scale height ($z_{1/2}$) for the same models.

Figure 4. Profiles of the hydrogen nuclei (HI + 2H$_2$) column density are shown along the galactic plane (left panel) and along the $l = 0$ line (right panel). The three diagrams correspond to model A (dashed, blue line), model B (continuous, red line) and the hydrogen distribution adopted in [21]. Column densities are averaged over $1^\circ \times 1^\circ$ angular bins.

In the following we will use model B as our reference model since, in the central region of the GE, it provides a better fit of the $^{12}$CO emission survey [49]. In figure 4 we also compare the hydrogen column density distributions obtained with models A and B with that used in [21], finding that the former are more narrowly peaked along the GP. In the following we will assume that helium is distributed in the same way as all hydrogen nuclei.

3. Numerical simulation of CR diffusion in the galaxy

The ISM is a quite turbulent magneto-hydro-dynamic (MHD) environment. Since the Larmor radius of high energy nuclei,

$$r_L(E) = \frac{E}{ZeB_{\text{reg}}} \simeq 0.1 \left( \frac{E}{10^2 \text{ TeV}} \right) \left( \frac{B_{\text{reg}}}{1 \mu\text{G}} \right) \text{ pc},$$

is sufficiently small, the CRs are likely to diffuse significantly as a result of magnetic turbulent fluctuations. In order to calculate their diffusion in the MHD turbulence, we need to know the three-dimensional distribution of the magnetic fluctuations (or the magnetic turbulent strength).

$$B_{\text{turb}}(\mathbf{r}) = B_{\text{reg}} \left( \frac{\mathbf{r}}{r_{\text{reg}}} \right) \left( \frac{r_{\text{reg}}}{\mathbf{r}} \right) \left( \frac{\mathbf{r}}{r_L(E)} \right),$$

where $B_{\text{reg}}$ is the magnetic field in the region of scale $r_{\text{reg}}$.

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is much smaller than the maximal scale of the magnetic field fluctuations \( L_{\text{max}} \sim 100 \text{ pc} \), the propagation of these particles takes place in the spatial diffusion regime. The diffusion equation which describes such a propagation is (see e.g. [50])

\[ \nabla_i J_i(E, r, z) \equiv - \nabla_i \left( D_{ij}(r, z) \nabla_{j} N(E, r, z) \right) = Q(E, r, z) \quad (10) \]

where \( N(E, r, z) \) is the differential CR density averaged over a scale larger than \( L_{\text{max}} \), \( Q(E, r, z) \) is the CR source term (5) and \( D_{ij}(r, z) \) are the spatial components of the diffusion tensor. We assume cylindrical symmetry and use Einstein notation for repeated indices. Equation (10) does not contain energy loss/gain terms. This is justified by the fact that in the high energy range relevant for our problem (\( E_{\nu(\gamma)} > 1 \text{ TeV} \) i.e. \( \langle E \rangle \gtrsim 10 \text{ TeV} \)) nucleus energy losses and re-acceleration are expected to be negligible (see e.g. [51, 24]).

Under the assumption of cylindrical symmetry the diffusion equation takes the simpler form [50]

\[
\left\{- \frac{1}{r} \partial_r [r D_\perp \partial_r] - \partial_z [D_\perp \partial_z] + u_r \partial_r + u_z \partial_z \right\} N(E, r, z) = Q(E, r, z),
\quad (11)
\]

where \( D_\perp \) and \( D_A \) are respectively the diffusion coefficient in the direction perpendicular to \( B_{\text{reg}} \) and the antisymmetric (Hall) coefficient, while \( u_r \) and \( u_z \) are the drift velocities as defined in appendix B. Equation (11) will be derived in appendix B.

Several simulations of the diffusion coefficients in high turbulence MHD media have been performed [52, 26]. We adopt here the expressions for these coefficients which have been derived in [53] and used in [25]. These expressions provide \( D_\perp \) and \( D_A \) as functions of the regular magnetic field and of the turbulence level \( \sigma \) at that point. With respect to other works, where the CR density has been simulated by using a mean value of the diffusion coefficients as derived from the observed secondary/primary ratio of CR nuclear species, our approach offers the advantage of providing the diffusion coefficients point by point. This allows us to test how the expected neutrino and \( \gamma \)-ray emissions depend on the properties of the turbulent components of the GMF. We verified, by means of dedicated runs, that the simulated escape time of nuclei found with our numerical code is compatible with that estimated from observations of the \( B/C \) ratio measured at energies around the GeV (see e.g. [24]).

Not all the advantages of getting a realistic spatial distribution of CR by solving the diffusion equation, however, were fully exploited in [25]. In that work, in fact, the author solved equation (11) analytically, which was possible only considering an oversimplified spatial dependence of the diffusion coefficients. For example, \( D_\perp \) was assumed to be spatially homogeneous.

In this work we solve numerically\(^4\) (11) under more general conditions. We impose the boundary conditions \( N = 0 \) on the cylindrical surface \( (r = 30 \text{ kpc}, z = z_t) \) where the turbulent halo is supposed to end and CRs escape to infinity without further diffusing. For any given value of the particle rigidity the code provides a bi-dimensional histogram \( \psi(r, z) \) mapping the particle spatial distribution in the stationary limit.

\(^4\) The code is based on a previous Fortran version written by J. Candia, which we translated into C++ and improved in several places.
3.1. Models

In order to verify how much the large uncertainties in the knowledge of the GMF properties may affect our predictions for the CR distribution, we consider and test several models.

Our first result of these tests is that the CR spatial distribution and spectrum are almost independent of the radial and vertical length-scales of the regular magnetic field. Therefore the large observational uncertainties in $r_\text{B}$ and $z_\text{r}$ do not affect our final results. In the following we will always assume $r_\text{B} = 8.5 \text{ kpc}$ and $z_\text{r} = 1.5 \text{ kpc}$.

More significant is the effect of changing the turbulent halo scale height $z_t$, as this parameter determines the length over which CRs have to diffuse before escaping to infinity (here we always assume that $z_\text{r} < z_t$, as suggested from observations; see section 2.2). Indeed, we found that in the inner galaxy, i.e. where sources are most abundant, the CR density increases with $z_t$. We considered several values of this parameter, finding that $z_t = 3 \text{ kpc}$ provides the best matching of EGRET data (see section 5).

In table 1 we summarize the main features of all models that will be discussed in this paper.

In figures 5 and 6 we show the high energy proton fluxes along two significant sections of the galactic halo: the galactic plane and a plane perpendicular to it at $r = r_\odot$. The effect of changing the SNR distribution is evident when comparing models 0 and 1 as both have the same GMF turbulent spectrum (Kolmogorov) and strength ($\sigma = 1$ everywhere on the GP). It is evident that the adoption of the SNR distribution given in [32] gives rise to a $\sim 30\%$ increase in the CR density in the GC region with respect to that derived following [29].

We also investigate the effect of changing the turbulence level uniformly in the magnetic halo, finding a marginal effect at energies below the PeV. More interesting is the effect of assuming a radially dependent turbulence strength $\sigma(r)$. In model 2 we assume that quantity to follow the same radial profile of SNR as shown in the upper curve of figure 1. This choice is justified by the well known argument according to which MHD turbulence in the ISM is powered by SN ejecta. By comparing proton flux profiles in figures 5, 6 for models 1 and 2 (as both are obtained by using the same value of $\sigma = 1$ at the solar circle), we find that a radially dependent $\sigma$ gives rise to a smoothing of the CR density distribution with respect to that of sources. That is to be expected as regions which are poor in sources are more easily filled by CRs coming from more active regions if the turbulence strength is locally smaller. The effect is less evident at higher energies since in this case CRs escape more rapidly along the $z$ axis.

In the above we assumed a Kolmogorov power spectrum (i.e. $\gamma = 5/3$) for the turbulent component of the GMF. It is interesting to investigate how the CR spatial distribution and spectrum change when adopting a Kraichnan spectrum ($\gamma = 3/2$). In model 3 we assumed that $\sigma = 1$ like SNR for $r < r_\odot$, and a Kraichnan spectrum for $r > r_\odot$. The effect of changing the scale height $z_t$ in model 3 is larger than in previous cases, and it is evident that in the inner galaxy the CR density is significantly decreased.

We also investigated the effect of assuming that the radial profile of the GMF is the same as that of the regular field, noting that in this case the CR density is increased by a factor of $\sim 50\%$. This is consistent with the idea that the GMF is generated by the interaction of SNR and the regular field. Finally, we found that the effect of changing the turbulence level uniformly in the magnetic halo is much smaller than in the previous cases.
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

**Figure 5.** The proton flux profiles at $E = 10$ TeV are plotted along the galactic mid-plane (left panel) and along the $z$-axis at $r = r_\odot$ (right panel). The dash–dotted line (orange), continuous (red), dotted (green), and dashed (blue) correspond to the models 0, 2, 1, 3 respectively (see text). All fluxes have been normalized to the observed value at the Sun position.

**Figure 6.** The same as figure 5 for $E = 1$ PeV.

distribution is affected by adopting a different spectrum. A reasonable possibility is to assume a Kraichnan spectrum (model 3) which is characterized by $\gamma = 3/2$. Noticeably, in this case $D_\perp \propto E^{1/2}$ so that the value of the spectral index at the sources, which is required to fit the observed proton spectrum at the Earth, is $\beta = 2.7 - 0.5 = 2.2$. This is in good agreement with the value measured for the $\gamma$-ray spectrum of several SNRs (see e.g. [54] and references therein). Since the Kraichnan spectrum is harder than Kolmogorov’s, it gives rise to a tighter confinement of CRs in the nearby sources, explaining the higher density in the inner galaxy which we observe for this model in figures 5 and 6.

The deformation of the vertical profiles observed for all models at high energy is due to the Hall diffusion, which gives rise to a drift in a direction perpendicular to both $B_{\text{reg}}$ (along $\hat{\phi}$) and the radial component of $\nabla \Phi_p$.

Finally, we also considered the effect of changing the symmetry of the regular magnetic field with respect to the galactic plane (model 4). In all previous cases a symmetric (S) configuration (i.e. $B_{\text{reg}}$ does not reverse its sign for $z \to -z$) was considered. Since observations have not settled what the actual symmetry is yet, it is interesting to consider the possible implications of its change. In [50, 27] it has already been shown that if the magnetic halo is A-symmetric Hall diffusion gives rise to a significant increase of the CR
density in the GC region at energies above the PeV. We confirm the existence of this effect. The consequences for the neutrino spectrum were investigated in [25]. However, as they are negligible in the energy range considered in this work we will disregard the A-symmetric models in the following.

As we mentioned, the diagrams in this section refer to protons only. The corresponding flux profiles for composite nuclei can be obtained by a simple energy rescaling \( (E \rightarrow E/Z) \) and by using the proper normalization given by the observed flux of each nuclide. Therefore,

\[
\frac{dn_Z(E; r, z)}{dE} = \sum_z I_p(E_0) f_Z \left( \frac{E}{E_0} \right)^{-\alpha_Z} \psi(E/Z; r, z)
\]

where \( f_Z \) and \( \alpha_Z \) are respectively the locally observed fraction at 1 TeV of the species with charge \( Z \) with respect to protons and their spectral slope. Recent compilations of measurements of both quantities can be found in [39]. We conclude this section by noticing that, although the CR energy spectrum is not exactly the same over the whole galaxy, this has negligible implications for the following results.

4. Mapping the \( \gamma \)-ray and neutrino emission

We start by considering \( \gamma \)-ray production by the decay of neutral pions which are generated by the interaction of the nucleonic component of CRs with the ISM (mainly hydrogen and helium). Here we do not consider a possible contribution to the diffuse \( \gamma \)-ray flux which may be originated by inverse-Compton (IC) scattering of relativistic electrons with the background radiation (see e.g. [51,23]). Bremsstrahlung is negligible at the energies considered in this work. Due to the low density of the ISM practically all mesons decay before interacting with matter. Since at these high energies constituent nucleons interact independently from one another, here we need only to consider elementary inelastic nucleon–nucleon scattering. Furthermore, we can safely assume proton–neutron invariance. In the energy range considered in this work \( (E_{\gamma} < 100 \text{ TeV}) \), the attenuation of the \( \gamma \)-ray flux due to pair-production scattering onto the CMB radiation can be neglected (see e.g. [21,22]).

High energy neutrinos \( (E > 1 \text{ TeV}) \) are prevalently originated by the decay of charged pions and kaons which are generated by the same hadronic scattering process as considered for the production of \( \gamma \)-rays. At the source only electron and muon neutrinos are produced. During the propagation over galactic distances neutrino oscillations redistribute the neutrino budget equally among all lepton families. In the following we will be interested only in the \( \nu_\mu + \bar{\nu}_\mu \) flux reaching the Earth.

Under the assumption that the primary proton spectrum is a power law and that the differential cross-section follows a scaling behaviour (which is well justified at the energies considered in this paper), the \( \gamma \)-ray (muon neutrino) emissivity can be written as (see e.g. [1,2])

\[
Q_{\gamma}(\nu_\mu + \bar{\nu}_\mu)(E_{\nu}; r, z) = I_N \frac{dn_p(E_{\nu}, r, z)}{dE} \sigma_{ppcn_H}(r, z) Y_{\gamma}(\nu_\mu + \bar{\nu}_\mu)(\alpha).
\]

Here \( \alpha \) is the primary CR spectral index and \( \sigma_{pp} \simeq 3.3 \times 10^{-26} \text{ cm}^{-2} \) is the pp inelastic cross-section at 1 TeV [55]. The \( \gamma \)-ray (muon neutrino) yield \( Y_{\gamma}(\nu_\mu + \bar{\nu}_\mu)(\alpha) \), as determined
Figure 7. The $\gamma$-ray and $\nu_\mu + \bar{\nu}_\mu$ yields are represented as a function of the primary nuclei spectral index $\alpha$. Neutrino oscillations are accounted for.

in [56], is shown in figure 7.\footnote{An almost 10% contribution to the photon emissivity coming from $\eta$ decay, which was not considered in [56], has been included here.} Those values are in agreement with previous results (see e.g. [57]–[59]) within 20%. Above a few GeV, yields are practically independent of energy. For $\alpha = 2.7$ the $\gamma/\nu_\mu + \bar{\nu}_\mu$ flux ratio is 3.1.

The factor $f_N$ in (13) represents the contribution from all other nuclear species both in the CR and the ISM. We find

$$f_N \approx \sum_{Z,A} f_Z \frac{\alpha_1}{\alpha_Z} \left( \frac{E_\nu}{E_\nu^0} \right)^{\alpha_1 - \alpha_Z} \left( 1 + 4 \frac{n_{\text{He}}}{n_{\text{H}}} \right) A^{1-\alpha_Z} \approx 1.4,$$

which is in good agreement with the value found in other works (see e.g. [60,61]). Here we used experimental values of $f_Z$ and $\alpha_Z$ as given in [39] and the helium/hydrogen ratio in the ISM $n_{\text{He}}/n_{\text{H}} \approx 0.09$ [32]. All other nuclear components in the ISM give a negligible contribution.

The differential $\gamma$-ray (neutrino) flux reaching the Earth is given by the line integral

$$I_{\gamma(\nu_\mu + \bar{\nu}_\mu)}(E_\nu; \ b, l) = \frac{1}{4\pi} \int Q_{\gamma(\nu_\mu + \bar{\nu}_\mu)}(E_\nu; \ b, l, s) \ ds,$$

where $s$ is the distance from the Earth and $(l, b, s)$ are related to $(r, z, \phi)$ through

$$z = s \sin b \quad r = \sqrt{(s \cos b \cos l - r_\odot)^2 + (s \cos b \sin l)^2},$$

in cylindrical symmetry. Finally, the integrated fluxes are determined by integrating the power law spectrum over the energy up to 1 PeV.

For the sake of clarity, in the following we will show flux diagrams as obtained only with our model 3B (model 3 for the CR distribution and B for the gas), which is our preferred model. As we mentioned in section 2.3, gas model B gives the best matching of CO surveys, while CR model 3 has to be preferred because, by adopting gas model B, it best reproduces EGRET observations above a few GeV (see section 5.1). At the end of
In this section we will briefly discuss how our predictions would change on adopting different models.

In figure 8 we show two representative sections of the neutrino flux profile above 1 TeV. In order to show how the expected signal may depend on the experimental angular resolution, in the same figure we draw the flux averaged over angular bins of different sizes. It is evident that due to the narrowly peaked behaviour of the gas density along the GP ($b = 0$) the averaged flux which may be measured from these regions should change significantly on varying the angular resolution.

In figure 9 we compare our results with those obtained in [21] and [22], which have been derived assuming a uniform CR density. We also show the flux as obtained by using...
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

**Figure 10.** The all-sky map of $\Phi_{\nu_{\mu} + \bar{\nu}_{\mu}}(E_{\nu} > 1 \text{ TeV})$ is shown in galactic coordinates (the GC is in the centre). This map has been obtained with HEALPix [62]. The flux distribution is projected in such a way that all pixels in this map correspond to the same solid angle. The corresponding $\gamma$-ray flux can be obtained by multiplying this diagram by 3.1.

the same gas model B but a uniform CR density. By comparing these profiles the reader can see that the flux we expect from the central region of the GP is significantly larger than in [21]. This discrepancy is due, for a large fraction, to the SNR overdensity in the molecular ring region.

Finally, in figure 10 we represent a full sky map of $\Phi_{\nu_{\mu} + \bar{\nu}_{\mu}}(E_{\nu} > 1 \text{ TeV}; b, l)$ obtained using model 3B. This figure has been produced with the HEALPix package [62]. We conclude this section by discussing how our predictions would change if different models for the gas and CR distributions were adopted. We find that by using gas model A rather than B, the $\gamma$-ray and neutrino fluxes along the GP grow at most by a factor of two. We find comparable displacements by adopting any of the other CR models considered in section 3. Therefore, we conclude that, under the hypothesis on which this analysis is based, our results are only very slightly model dependent.

5. Comparison with available observations

5.1. Gamma-rays

First of all, we would like to compare our results with EGRET observations. This is required to put our results on more solid ground and to verify that our choice of the preferred model is consistent with those observations. Here we consider only EGRET

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6 The reader should also note that in [21] the $N_H \rightarrow \Phi_{\gamma}(E > 1 \text{ TeV}; b, l)$ conversion factor is $6.02 \times 10^{-33}$, while we have $8.87 \times 10^{-33} \ (\text{TeV s sr}^{-1})$.

7 See http://healpix.jpl.nasa.gov
measurements of the diffuse emission along the GP between 4 and 10 GeV \[5, 63\]. Such a comparison requires a considerable extrapolation of our previous findings. We are allowed to do this since, already for \(<10\) TeV, nucleus propagation takes place deep into the spatial diffusion regime so that the energy dependence of the diffusion coefficients does not change going to lower energies (see e.g. \[26, 27\]). As a consequence, in the stationary regime the CR distribution above 10 GeV is given by a uniform \(\propto E^{-2.7}\) rescaling of that shown in figure 3. We neglect small corrections due to the variations of the pp scattering cross section with the energy.

In table 2 and in figure 11 we compare respectively theoretical mean fluxes and flux profiles along the GP with EGRET measurements \[63\]. Considering the large uncertainties involved in the modelling of the CR and gas distribution, the agreement is rather good. It should be noticed that in the energy range 4–10 GeV the IC contribution to the photon flux from the GP is expected to be subdominant (see e.g. \[51, 24\]). In figure 12 we also show our prediction for the \(\gamma\)-ray differential spectrum between 1 GeV and 100 TeV in the region \(73.5^\circ \leq l \leq 76.5^\circ, \ |b| \leq 1.5^\circ\) and we compare it with EGRET \[63\] and MILAGRO measurements \[11\] in the same region\(^8\). All the theoretical fluxes shown in this section have been derived using CR model 3 and gas model B. As we mentioned in the previous section, while we prefer gas model B because it provides the best description of CO surveys, once we have made this choice, model 3 is that which gives the best fit of EGRET data. Anyhow, passing from one model to another, our predictions change by a factor of two at most.

Reassured by these findings, we are now ready to compare our results with measurements made at higher energies. Several air shower array experiments looked for

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\(^8\) The best fit of the spectral slope observed by EGRET above 1 GeV is \(-2.6\) \[4\]. This is compatible with that adopted in our paper within errors.
the γ-ray diffuse emission of the galaxy above the TeV. The most interesting results are those of TIBET [10] and MILAGRO [9,11]. Both experiments probed different regions of the GP. In all these regions only upper limits were found except in Cygnus, where MILAGRO found a significant excess on the background [9,11]. In table 2 we compare our predictions, as obtained with our preferred model 3B (see section 4), with these measurements. With the exception of Cygnus (see also the discussion about the GC ridge at the end of this subsection), in all other regions we predict fluxes which are significantly below the experimental limits. Therefore, there is still room for a large IC contribution.

Concerning the excess in the Cygnus region observed by MILAGRO, we confirm the conclusion [11,64] that it cannot be explained by the interaction of the diffuse component of galactic CR with the gas in that region (see figure 12). Indeed, we find that a CR local overdensity of about 20 is required to explain this signal in terms of hadronic emission.

Table 2. In this table our predictions for the mean γ-ray flux in some selected regions of the sky are compared with some available measurements. Since measurements’ errors are much smaller than theoretical uncertainties they are not reported here.

| Sky window       | $E_\gamma$ (GeV) | Our model $\Phi_{\gamma}(>E_\gamma)$ (cm$^2$ s sr)$^{-1}$ | Measurements $\Phi_{\gamma}(>E_\gamma)$ (cm$^2$ s sr)$^{-1}$ |
|------------------|------------------|-------------------------------------------------|-------------------------------------------------
| $|l| < 10^\circ$, $|b| < 2^\circ$ | 4 GeV            | $\approx 4.7 \times 10^{-6}$                       | $\approx 6.5 \times 10^{-6}$ [63]              |
| $20^\circ \leq l \leq 55^\circ$, $|b| < 2^\circ$ | 3 TeV            | $\approx 5.7 \times 10^{-11}$                     | $\lesssim 3 \times 10^{-10}$ [10]              |
|                 | 4 GeV            | $\approx 4.4 \times 10^{-6}$                     | $\approx 5.3 \times 10^{-6}$ [63]              |
| $73.5^\circ \leq l \leq 76.5^\circ$, $|b| \leq 1.5^\circ$ | 12 TeV           | $\approx 2.9 \times 10^{-12}$                     | $\approx 6.0 \times 10^{-11}$ [11]             |
|                 | 4 GeV            | $\approx 2.4 \times 10^{-6}$                     | $\approx 3.96 \times 10^{-6}$ [63]             |
| $140^\circ < l < 200^\circ$, $|b| < 5^\circ$ | 3.5 TeV          | $\approx 5.9 \times 10^{-12}$                     | $\lesssim 4 \times 10^{-11}$ [9]               |
|                 | 4 GeV            | $\approx 5.9 \times 10^{-7}$                     | $\approx 1.2 \times 10^{-6}$ [63]              |
A similar effect has to be invoked to explain HESS measurement of the diffuse \( \gamma \)-ray TeV emission from the GC ridge region \[65\]. In fact, even accounting for the detailed three-dimensional gas model recently developed in \[46\] for the GC region, we find that in the region \( |b| < 0.8^\circ, |l| < 0.3^\circ \) the expected \( \gamma \)-ray flux for \( E > 1 \) TeV is \( \sim 2.4 \times 10^{-9} \) cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\), which is almost 10 times smaller than the HESS measurement. It is worth noticing that the slope of the spectrum measured by HESS is close to \(-2.3\), which is quite different from that of galactic CR. This probably means that primary particles in the GCR have a local origin. It is evident that the analysis performed in this work, as well as in related papers \[23,51,24\], cannot account for this kind of local emissions.

5.2. Neutrinos

The only available upper limit on the neutrino flux from the galaxy has been obtained by the AMANDA-II experiment \[14\]. Being located at the South Pole, AMANDA cannot probe the emission from the GC. In the region \( 33^\circ < l < 213^\circ, |b| < 2^\circ \), and assuming a spectral index \( \alpha = 2.7 \), their present constraint is

\[
I_{\nu^+ + \bar{\nu}^+} < 6.6 \times 10^{-4} \left( \frac{E}{1 \text{ GeV}} \right)^{-2.7} \text{ (GeV cm}^2 \text{ s sr)}^{-1}, \tag{17}
\]

which implies \( \Phi_{\nu^+ + \bar{\nu}^+} (>1 \text{ TeV}) < 3.1 \times 10^{-9} \) (cm\(^2\) s sr\(^{-1}\)). According to our model the expected flux in the same region is \( \Phi_{\nu^+ + \bar{\nu}^+} (>1 \text{ TeV}) \simeq 4.2 \times 10^{-11} \) (cm\(^2\) s sr\(^{-1}\)).

6. The expected neutrino signal in the Northern Hemisphere

In the energy range 1–100 TeV water (or ice) Cherenkov neutrino telescopes are best suited to look for up-going muons produced by charged-current interactions of muon neutrinos in the Earth. While the Earth offers almost a complete shielding from up-going atmospheric muons below the horizon, an unavoidable background is given by atmospheric neutrinos. Several estimates have been made of the atmospheric neutrino flux, based on different assumptions modelling hadronic interactions (see e.g. \[66\]–\[68\]). Above the TeV, all calculations almost agree, predicting an averaged flux

\[
F_{\nu^+ + \bar{\nu}^+} (E_{\nu}) \simeq 4.6 \times 10^{-8} \left( \frac{E_{\nu}}{1 \text{ TeV}} \right)^{-3.7} \text{ TeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}, \tag{18}
\]

though a \( \sim 40\% \) uncertainty remains due to the experimental error on the primary CR spectrum and the theoretical error modelling strange particle production.

Since, as is evident from our previous results, the expected neutrino flux from the galaxy is significantly smaller than such background, a suitable procedure has to be adopted to disentangle the signal. One possible approach is to search only for neutrinos with \( E \gtrsim 100 \) TeV \[25\]. This however may be hard to do due to the very low flux at energies this high and to the Earth’s opacity (only Earth skimming neutrinos or shower-like events produced by down-going neutrinos can be detected in this case). In our opinion a more promising strategy is to search for up-going muon neutrinos above 1–10 TeV. Their arrival direction can be reliably reconstructed with an angular resolution as good as 0.5°.
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

in water, and this information may be used to identify the galactic emission as a localized excess of events.

While it is quite evident that the most natural search window (on-source region) is a narrow band along the galactic equator (see figure 10), its optimal sizes have to be chosen by taking into account the angular extension of the source. In the previous section we showed that most of the neutrino flux from the galaxy should be concentrated in the region |l| < 50°, |b| < 1°. Since the Gaussian width of the signal \( \sigma_{\text{sig}} \approx 2° \) is comparable to the expected angular resolution of neutrino telescopes, some attention has to be paid when choosing the latitude width of the search window. By assuming that both the line spread function of the experiment and the signal profile along \( b \) are Gaussian, having widths \( \sigma_{\text{lsf}} \) and \( \sigma_{\text{sig}} \), respectively, the optimal search window width is approximatively given by (see e.g. [14]) \( \Delta b \approx (1 - 2) \left( \sigma_{\text{lsf}}^2 + \sigma_{\text{sig}}^2 \right)^{1/2} \). Since for a km\(^3\) water NT the expected line spread function is \( \sigma_{\text{lsf}} \approx 0.5° \), we think that \( \Delta b = 3° \) is a reasonable value to adopt.

Although to perform a detailed calculation of the expected signal in a given NT is beyond the purposes of this work, here we perform a simplified estimate, which is only intended to give the reader a feeling of the chances that forthcoming experiments have to achieve a positive detection. The expected muon detection rate of neutrinos coming from this window is

\[
\hat{N}_\nu(> E_\nu) = \int_{\Delta l} \int_{\Delta b} \int_{E_\nu} dE \ A^{\text{eff}}(E)v(b, l) \frac{dN_\nu(E; l, b)}{dE},
\]

where \( A^{\text{eff}}(E) \) is the effective area of the neutrino telescope and \( v(l, b) \) is the visibility function (i.e. the fraction of time that a point in a sky with galactic coordinates \( (l, b) \) spends above the visibility horizon). For example, \( v(0, 0) = 0.67 \) at the ANTARES geographical position.

Since the effective area is relatively weakly dependent on the arrival direction of the neutrinos, we adopt here a mean value obtained by averaging over all possible nadir angles. As a reference we use \( A^{\text{eff}}(E_\nu) \) as provided by the ANTARES collaboration\(^9\) (see e.g. [69]) and assume that for a km\(^3\) experiment it will be 40 times larger. This is a rapidly growing function of the energy for \( E \lesssim 100 \text{ TeV} \). It is interesting to observe that the product \( A^{\text{eff}}(E)(dN_\nu(E; l, b))/dE \), which is related to the detection efficiency, is peaked at about 1 TeV for a \( \alpha = 2.7 \) power law spectrum.

In figure 13 we show the integrated muon neutrino detection rate as a function of the minimum energy to be expected in a km\(^3\) NT placed at the same geographical position as ANTARES. The upper curve delimiting the signal band corresponds to model 3A, while the lower to model 2B. We compare this rate with the atmospheric neutrino detection rate expected in ANTARES [70] multiplied by a factor of 40 to account for the larger effective area. Our calculation accounts for the angular dependence of the expected atmospheric neutrino detection rate.

According to figure 13, the background is about 50 times larger than the signal above 1 TeV in the \( |l| < 30° \) region, and about five times above 10 TeV. Therefore, an excess may be detectable for \( E \gtrsim 10 \text{ TeV} \) only after a considerable number of years.

\(^9\) Here we use an effective area as provided by the ANTARES collaboration for point-like sources. According to a recent analysis [70], which adopts a suitable rejection strategy for the residual atmospheric muon background, the effective area for an all-sky diffuse flux differs very little from that for point-like sources. A dedicated analysis for the kind of emission considered in this work has not been performed yet.
Diffuse neutrino and gamma-ray emissions of the galaxy above the TeV

Figure 13. The expected $\nu_\mu + \bar{\nu}_\mu$ detection rate (red band, between continuous lines) from the regions $|l| < 50^\circ$, $|b| < 1.5^\circ$ (left), $|l| < 30^\circ$, $|b| < 1.5^\circ$ (centre), $|l| < 10^\circ$, $|b| < 1.5^\circ$ (right), is compared with the background due to atmospheric neutrinos from the same region (light blue band, between dashed lines). An effective area 40 times larger than ANTARES has been adopted. $E$ is the neutrino energy.

7. Conclusions

We have performed a comprehensive calculation of the neutrino and $\gamma$-ray flux distributions which are expected to be originated by the interaction of CR nuclei with the interstellar gas.

Our computation improves and updates previous analyses under several aspects.

Modelling the spatial distribution of CR in the galaxy, we accounted for a spatial dependence of the diffusion coefficients. This approach allowed us to test how different models of the turbulent component of the GMF affect the CR distribution and the secondary $\gamma$-ray flux and to choose our preferred model as that which best reproduces EGRET $\gamma$-ray observations above a few GeV. Interestingly, we found that such a model also gives the best agreement with the hard $\gamma$-ray spectra observed for several SNRs.

Having assumed that CR sources are distributed like SNRs, we adopted a detailed model of the spatial distribution of these objects.

Concerning the gas distribution, we considered two models, both for HI and H$_2$, which we applied for the first time in this framework. Although these models have significant differences in some regions, we showed that they give rise to neutrino and $\gamma$-ray fluxes which, when averaged over windows larger than few degrees squared, coincide within a factor of two. We also found that the column density profile across the GP is almost the same for these two models, while it differs significantly from that adopted in previous works [21, 25]. As a consequence, we predict the neutrino and $\gamma$-ray emissions to be more narrowly peaked along the GP. This improves significantly the perspectives to disentangle the signal from the almost isotropic background if the experimental angular resolution is better than 1°–2°.

We compared our predictions with the available observations.

Concerning the $\gamma$-rays, we found that above the TeV our predicted fluxes are below the experimental limits by less than an order of magnitude. Since our analysis does not account for a possible IC contribution, which may be not negligible above the TeV, it is possible that those experiments will get a positive signal in the next few years.
The comparison of the experimental results with our predictions, as well as with those of previous works (see e.g. [23, 51, 24]), will help to understand which fraction of this emission is of leptonic origin.

Concerning the signal observed by MILAGRO in the Cygnus region [9, 11, 64], we confirm that it cannot be explained without invoking a significant CR overdensity, or a large IC contribution, which may be due to a local concentration of sources. A similar effect takes place in the GC region. We found that in order to explain the excess observed by HESS from the GC ridge [65] the CR flux must exceed our prediction by a factor of about 10. This is not unexpected since the distribution of star forming regions, hence of CR sources, is known to be quite clumpy. It is understood that the kind of simulation performed in this work, as well as in previous ones [23, 51, 24], can only model the mean CR distribution smoothed on scale of several hundred parsecs.

We also extrapolated our results down to a few GeV, where the IC scattering emission is expected to be subdominant. We found that in the most dense regions the flux distribution agrees reasonably well with EGRET observations. This result encourages us to improve the accuracy of our analysis by using, for example, more detailed models of the gas distribution, so as to be able to simulate what GLAST may observe above the GeV at least in some limited regions of the sky.

Going back to neutrinos, we compared our predictions for the muon neutrino flux from the GP with the experimental limit recently established by AMANDA-II [14]. We found that our predicted flux is almost two orders of magnitude below that limit. Unfortunately, ICECUBE [15] will also hardly be able to get a positive signal.

Since a neutrino telescope placed in the Northern Hemisphere may have better chances, we investigated this possibility in some detail. Assuming that such an instrument will be placed at the same position as ANTARES [16] and have a 40 times larger effective area, we estimated the expected signal and the background along the galactic Equator. We found that the detection of the smooth component of the diffuse emission may require more than 10 years of data taking. It should be noted, however, that our analysis provides only a lower limit to the expected emission from the galactic plane. As we mentioned above, several observations suggest that the CR and gas distributions may be more clumpy than considered in this work. Furthermore, fluctuations of these quantities are likely to be spatially correlated. This may lead to a significant enhancement of the neutrino flux from some regions as may be the case for the GC ridge [56, 54] and the Cygnus region [71, 72]. In a forthcoming paper we will try to develop our analysis in order to model at least some of these effects.

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Appendix A. Models of the gas distributions

The HI and H$_2$ density distributions are generally inferred from radio observations of the 21 cm line and $^{12}$CO rotational line emissions respectively. The galactocentric radial density profiles are estimated by converting the line-of-sight velocity (which is determined from the Doppler shift) into heliocentric distance by means of the galactic rotation curve (see e.g. [32]). This operation is impossible for the whole galaxy and it always involves large uncertainties so that some phenomenological models have to be invoked.

We are going here to describe in detail the models sketched in section 2.3.

Model A. This has been developed by Nakanishi and Sofue (NS) in [44] for HI and by the same authors in [45] for H$_2$. For both gas components NS developed 3D models $(r, z, \phi)$, which, however, do not cover the entire galaxy. By averaging over the azimuth they get 2D distributions which are approximatively symmetric with respect to the galactic plane. NS assumed that the vertical profiles of the HI and H$_2$ densities have the form $n(r, z) = n(r, 0) \text{sech}^2\left\{\log(1 + \sqrt{2}) \frac{z}{z_{1/2}(r)}\right\}$, which is almost coincident with a Gaussian peaked at the galactic plane half-width $h(r) = z_{1/2}(r)/\sqrt{\ln(2)}$. The quantity $2 \cdot z_{1/2}(r)$ is usually called the full width at half maximum (FWHM) scale height. Then, they derived binned radial distributions for $z_{1/2}(r)$ and the gas density $n(r, 0)$ along the galactic plane.

Model B. This is the product of the combination of results of different analyses which have been separately performed for the disc and the galactic bulge. For the H$_2$ and HI distributions in the bulge we use a detailed 3D model recently developed by Ferriere et al [46] on the basis of several observations. In that model both H$_2$ and HI are concentrated in two main structures. The central molecular zone (CMZ) appears as a quite dense $500 \times 30$ pc wide (sizes at half maximum density) ellipse on the plane of the sky (for HI the vertical extension is 90 pc) containing almost $2 \times 10^7 M_\odot$ in H$_2$ and $1 \times 10^6 M_\odot$ in HI. It gives rise to a pronounced peak in the gas column density in the direction of the GC. The holed galactic bulge disc (GBD) is a 3 kpc long and 1 kpc wide toroidal structure. Its mass is comparable to that in the CMZ ($\sim 3 \times 10^7 M_\odot$ in H$_2$ and $3 \times 10^6 M_\odot$ in HI) but it is spread over a much larger volume so that the gas density in that region is much smaller than in the CMZ. Furthermore, the GBD is significantly inclined with respect to the plane of the sky so that its contribution to the gas column density along the line of sight is quite small. For the molecular hydrogen in the disc we use the Bronfman et al model [47]. Although such a model is almost two decades old, it was shown [32] that it still provides a good description of recent CO surveys [49]. Once averaged over the azimuth, the distribution of Bronfman et al is well approximated by a disc with a Gaussian vertical profile symmetric with respect to the galactic plane. Since in [47] $r_\odot = 10$ kpc was adopted, we correct the gas densities and the scale heights given in that paper to make them compatible with the value $r_\odot = 8.5$ kpc we use in this work.
Furthermore, we accounted for the different values of the \( \text{H}_2 - \text{CO} \) conversion factor, which is \( X = 0.5 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1} \text{ s} \) in [46] and \( X = 2.8 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1} \text{ s} \) in [47]. Even by taking into account a possible increase of \( X \) with \( r \) (see e.g. equation (7) in [46]), these values are too different to be compatible. We assume that for \( 2 < r < 10 \), \( X = 1.2 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1} \text{ s} \), as this is the mean value which one gets in that region by taking \( X = 0.5 \) at \( r = 0 \) and assuming that it grows like \( \exp(r/7.1) \) as argued in [73]. Hence, we correct the density of Bronfman et al by multiplying it by the factor \( 1.2/2.8 \).

Appendix B. Numerical solution of the diffusion equation

We will derive here equation (11).

The diffusion equation which describes the propagation of CRs in a turbulent medium is equation (10),

\[
\nabla_i J_i(E,r,z) \equiv - \nabla_i (D_{ij}(r,z) \nabla_j N(E,r,z)) = Q(E,r,z).
\] (B.1)

Cylindrical symmetry will be assumed.

The diffusion tensor can be conveniently decomposed into

\[
D_{ij} = (D_{\|} - D_{\perp}) b_i b_j + D_{\|} \delta_{ij} + D_A \epsilon_{ijk} b_k
\] (B.2)

where \( b_i \) are the components of the regular magnetic field versor. The symmetric components \( D_\| \) and \( D_\perp \) are the diffusion coefficients along and perpendicular to \( B_{\text{reg}} \), while \( D_A \) is the antisymmetric (Hall) diffusion coefficient. All these coefficients are functions of the energy though with different behaviours. For a fixed value of \( L_{\text{max}} \) and for \( r_L \ll L_{\text{max}} \), \( D_\| \) and \( D_\perp \) are proportional to \( E^{2-\gamma} \) while \( D_A \propto E \) (see e.g. [50, 53]), where \( \gamma \) is the power-law index of the GMF turbulent fluctuations as defined in section 2.2. The Hall diffusion becomes dominant only for very high values of the rigidity, i.e. for \( r_L(E) \gtrsim 0.1 L_{\text{max}} \).

As motivated in section 2.2 and similarly to what was done in other works [50, 23, 27], we can approximate the regular component of the magnetic field to be azimuthally oriented, i.e. \( b_\phi = \pm 1 \) and \( b_r = b_z = 0 \). Under such an assumption \( D_\| \) becomes not physically relevant and the diffusion equation takes the simpler form [50]

\[
\left\{ -\frac{1}{r} \partial_r [r D_\perp \partial_r] - \partial_z [D_\perp \partial_z] + u_r \partial_r + u_z \partial_z \right\} N(E,r,z) = Q(E,r,z),
\] (B.3)

where we defined

\[
u_r \equiv -\partial_z D_A
\] \[
u_z \equiv \frac{1}{r} \partial_r (D_A r).
\] (B.4)

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