Research on inversion method with multi-transceiver distance in underground space based on apparent conductivity

Zhong Mingyou1, Zhang Bo2*, Bian Leixiang1*, Rong Xiaoli1, Cui Chenli1, Li Jiayang1, Han Songtong1
1 Nanjing University of Science and Technology, Nanjing 210094, China
2 Army Engineering University of PLA, Nanjing 210001, China
*Corresponding author’s e-mail: lxbian@163.com, emczhangbo@163.com

Abstract. The layered ground under the excitation of a harmonic magnetic field will generate a secondary magnetic field. On the condition of low induction number, the apparent conductivity can be obtained by using the normalized secondary field measured by the instrument directly. By discretizing the integral of relative response function density, the expression of contribution value of conductivity of each layer to the apparent conductivity can be obtained. By using multiple sets of transceiver distance, we can obtain different apparent conductivity observation equations, then the conductivity and thickness of each layer can be solved by Gauss-Newton method. A three-layer medium model containing a V-shaped tunnel is designed for inversion simulation calculations. The results show that the higher the SNR of observed signal, the more stable the inversion result. The maximum relative error of thickness is about 10%.

1. Introduction
The measurement of apparent conductivity of the earth is widely used in geophysical exploration techniques, such as soil mapping[1], groundwater exploration[2], and dam maintenance[3]. According to the measurement results of apparent conductivity, the spatial structure and electrical parameters of the underground medium layer can be described more accurately. The method of measuring apparent conductivity to explain the electrical parameters of underground medium has been applied in many areas. Xu[4] used the EM34 conductivity meter to measure conductivity of the earth in the field, and the experimental results showed that the abnormal value of apparent conductivity was consistent with the position of river trench; Martini et al.[1] measured the apparent conductivity of the soil and draw a moisture map of the soil in the survey area; Hulin et al.[5] demarcated the scope of archaeological remains by this method.

Using apparent conductivity for data interpretation, the cumulative response function and relative response function density are very effective tools. McNeil[6] proposed the basic principles of apparent conductivity measurement by using fixed transceiver distance in the frequency domain electromagnetic method, and the concepts of cumulative response function and relative response function density. This concept is used in the forward calculation and measurement data interpretation[7][8]. However, in most cases, current methods based on apparent conductivity measurement and data interpretation only care about the conductivity of the medium layer while ignoring the thickness of the medium layer, which is also important for underground space structures. In addition, McNeil[6] thought that apparent conductivity is the integration of the conductivity of infinite thin-layer medium. For a multi-layer medium model, there are many parameters to be solved
and the nonlinear equation has no analytical solution. In this paper, based on previous studies, the integration of relative response function density is discretized to obtain the relationship between apparent conductivity of multi-layer model and the actual conductivity of each layer, and we used Gauss-Newton method to solve the nonlinear equation, the actual conductivity and thickness of each layer are obtained in the same calculation.

2. EM theory

2.1. Apparent conductivity

We consider a transmitting coil on the horizontal earth surface and a receiving coil at the position \( \mathbf{r} \) away from it, as shown in Figure 1. When the harmonic current \( \mathbf{i} = \mathbf{I}_e e^{jwt} \) is introduced into the transmitting coil, the generated harmonic primary field \( \mathbf{H}_p \) will excite the underground medium layer, and the medium will generate the induced current, which generates the secondary field \( \mathbf{H}_s \) to modulate the primary field. The receiving coil at point P receives both primary field \( \mathbf{H}_p \) and secondary field \( \mathbf{H}_s \).

![Figure 1. Schematic diagram of the relationship between primary and secondary fields](image)

According to Xu's derivation[4], the secondary field response from the observation point at the same horizontal height as the vertical magnetic dipole and spaced apart by \( \mathbf{r} \) is:

\[
H_z = \frac{-m}{4\pi r^3} \left\{ 9 - [9 - 9ikr - 4(kr)^2 + i(kr)^3]e^{ikr} \right\} 
\]

(1)

Where \( m \) is the magnetic moment of the dipole, \( k \) is the wave number of the electromagnetic wave passing through the medium, and \( k = \sqrt{\frac{\omega}{\mu_0 \sigma}} \), \( i = \sqrt{-1} \). Equation (1) is a complex function of wave number \( k \) and transceiver distance \( r \), and it is hard to solve it without any simplification.

As we all known, the skin depth is an important concept in the field of electromagnetic exploration. It is defined as the propagation distance when the amplitude of electromagnetic waves attenuates to 1/e of the ground amplitude when they propagate in underground medium. The calculation formula of skin depth \( \delta \) is:

\[
\delta = \sqrt{\frac{2}{\sigma \mu_0 \omega}} 
\]

(2)
Hence

\[ kr = \sqrt{2i\left(\frac{r}{\delta}\right)} = \sqrt{2iB} \]  

(3)

The ratio \( \frac{r}{\delta} \), intercoil spacing divided by the skin depth, is defined as the induction number \( B \)[6]. Equation (1) can be regarded as a function of the induction number \( B \). When \( B \ll 1 \), that is, the distance between the transmitter and the receiver is far less than the skin depth, the exponential function \( e^{ikr} \) in formula (1) is expanded into Taylor’s series, and the high-order term is ignored:

\[ H_z \approx H_p[1 + \left(\frac{ikr}{4}\right)^2] \]  

(4)

Subtract \( H_p \) from equation (4), the result is the value of the secondary field in the receiving coil, and the normalized secondary field[9] is:

\[ \frac{H_z - H_p}{H_p} \approx -\frac{i\mu_0\omega\sigma r^2}{4} \]  

(5)

\[ \sigma_a = \left| \frac{4}{\mu_0\omega r^2} \right| \frac{H_z - H_p}{H_p} \]  

(6)

It can be considered that when the transmitting coil is located on the ground and the transceiver distance \( r \) is much smaller than the skin depth \( \delta \) of the electromagnetic field, there is a linear relationship between the ratio of the primary field to the secondary field with the apparent conductivity[9]. By measuring the normalized secondary field, we can obtain the value of the apparent conductivity of the earth directly.

2.2. Multi-transceiver distance inversion method

The underground medium is considered to be formed by an infinite number of thin layers with a thickness of \( dz \), so we can obtain the secondary field in the receiving coil by calculating the induced magnetic field caused by the current loop in all the thin layers. By using the low induction number condition, McNeill[10][11] has given the cumulative response function of the vertical dipole:

\[ R_v(z) = \frac{1}{\sqrt{4\pi z^2 + 1}} \]  

(7)

In equation (7), the relative depth \( z \) is the ratio of the depth \( h \) to the transceiver distance \( r \). The derivative of cumulative response function to relative depth \( z \) is the relative response function density:

\[ \Phi_v(z) = \frac{dR_v(z)}{dz} = \frac{4z}{(4z^2 + 1)^{1/2}} \]  

(8)

The relative response function density is relative contribution of the unit thickness of layer to the secondary field. Then the apparent conductivity obtained at the observation point is the sum of the relative contribution of each layer of medium to the secondary field:

\[ \sigma_a = \int_{-\infty}^{0} \sigma \Phi_v(z)dz \]  

(9)

As for the three-layers model, equation (8) can be discretized into:
\[
\sigma_a = \sigma_1 R_v(z_1) + \sigma_2 [R_v(z_2) - R_v(z_1)] + \sigma_3 [1 - R_v(z_1)]
\]

Equation (9) is the basic theory of inversion. There are five parameters to be solved for the three-layer model, which are the conductivity of each layer (i.e. \(\sigma_1, \sigma_2, \sigma_3\)) and the thickness of the first two layers (i.e. \(h_1, h_2\)). We can obtain different apparent conductivity equations by changing the transceiver distance \(r\):

\[
\begin{align*}
\sigma_a(r_1) &= \sigma_1 R_v(h_1 / r_1) + \sigma_2 [R_v(h_2 / r_1) - R_v(h_1 / r_1)] + \sigma_3 [1 - R_v(h_1 / r_1)] \\
\sigma_a(r_2) &= \sigma_1 R_v(h_2 / r_2) + \sigma_2 [R_v(h_3 / r_2) - R_v(h_2 / r_2)] + \sigma_3 [1 - R_v(h_2 / r_2)] \\
\sigma_a(r_3) &= \sigma_1 R_v(h_3 / r_3) + \sigma_2 [R_v(h_4 / r_3) - R_v(h_3 / r_3)] + \sigma_3 [1 - R_v(h_3 / r_3)] \\
\sigma_a(r_4) &= \sigma_1 R_v(h_4 / r_4) + \sigma_2 [R_v(h_5 / r_4) - R_v(h_4 / r_4)] + \sigma_3 [1 - R_v(h_4 / r_4)] \\
\sigma_a(r_5) &= \sigma_1 R_v(h_5 / r_5) + \sigma_2 [R_v(h_6 / r_5) - R_v(h_5 / r_5)] + \sigma_3 [1 - R_v(h_5 / r_5)]
\end{align*}
\]

Gauss Newton method is a good choice for solving this kind of nonlinear equations. The Gauss Newton method expands the nonlinear equation into Taylor’s series and ignores the high-order term. The nonlinear equation is approximately linearized to obtain the solution of the equation. The main steps of this method are as follows:

1. Establish a nonlinear equation \(F(x) = 0\);
2. Get the Jacobian matrix \(JF(x)\) of nonlinear equations;
3. Get the iteration relationship \(x' = x - [JF(x)]^{-1}F(x)\);

3. Simulation and discussion

3.1. Simulation model

In order to verify the effectiveness of the inversion algorithm, we designed a model shown in Figure 2 for simulation analysis. The model is divided into three layers, the first layer is soil, the conductivity is \(\sigma_1 = 0.001\) S/m; the second layer is a V-shaped tunnel with 5 continuous steps at both ends of the tunnel, each step is 20m in width and 3m in height. The tunnel is filled with air, so the conductivity of the second layer medium is \(\sigma_2 = 0\) S/m; the third medium is rock layer with conductivity \(\sigma_3 = 0.0005\) S/m. The height of V-shaped tunnel (i.e. the thickness of the second medium) is 20m.

![Figure 2. Schematic diagram of underground space structure with V-shaped tunnel](image)

10000 measuring points are set in XOY horizontal plane evenly, and the interval between adjacent measuring points is 4m. Considering that there is noise signal in the observed signal of the instrument in the actual measurement process, the theoretical electromagnetic response of the layered model is superimposed with the noise of a certain signal-to-noise ratio (SNR) as the actual observation data of the instrument, and then the conductivity and thickness of each measuring point in the model are calculated by using the multi-transceiver distance inversion algorithm.
3.2. Results and discussion

The three-layer medium model shown in Figure 2 requires five sets of observation equation to solve. Set the magnetic moment $m=400\text{Am}^2$, the excitation frequency $f=1\text{kHz}$, and the transceiver distance of the five groups are $r=10\text{m}$, $15\text{m}$, $20\text{m}$, $25\text{m}$ and $30\text{m}$ respectively. When the SNR of the observation signal is $30\text{dB}$ and $40\text{dB}$ respectively, the depth distribution results of the first two layers of medium are calculated as shown in Figure 3. The minus indicates that the medium layer is located below the surface of the earth.

![Result at 30dB](image)

(a) result at 30dB

![Result at 40dB](image)

(b) result at 40 dB

Figure 3. Results of the first two layers of medium depth at different SNR

From the results in Figure 3, it can be seen that the inversion results under the two SNR conditions are basically consistent with the thickness of the model, and can clearly reflect the structural characteristics of the underground V-shaped tunnel. The higher the SNR, the more "smooth" the inversion results, inversion results of some observation points under $30\text{dB}$ have a large deviation from the true value, and the inversion results of each observation point under $40\text{dB}$ are more stable.

By analyzing the survey line with $Y=200\text{m}$, the relative error of the depth inversion results is shown in Figure 4. According to the results in the figure, the relative errors at different measuring points of the model are also different. The relative error is large in the range of $X<100\text{m}$ and $X>300\text{m}$, which can reach $10\%$, in the range of $100\text{m}<X<300\text{m}$, the relative error of the first layer of medium is about $5\%$, and the relative error of the second layer of medium is about $8\%$. From the overall results, the error of the depth inversion result of the second layer is larger than that of the first layer, and the error when the SNR is $40\text{dB}$ is smaller than that when the SNR is $30\text{dB}$.

![Relative error at 30dB](image)

(a) result at 30dB

![Relative error at 40dB](image)

(b) result at 40 dB

Figure 4. Relative error of depth results on $Y=200\text{m}$ survey line
In addition, we draw the conductivity distribution results on the XOZ profile according to the inversion results, as shown in Figure 5. It can be seen that there are three kinds of medium alternating on the profile, the structure and thickness information of each layer can be roughly judged according to the distribution of conductivity. Comparing with the case of SNR is 30dB, the result of conductivity is more accurate when SNR is 40dB, under the condition of the same depth and the same layer, fluctuation of the inversion result is smaller.

![Conductivity Distribution](image)

Figure 5. Distribution of conductivity inversion results on XOZ profile

4. Conclusion
In this paper, based on the theoretical formula of apparent conductivity under the condition of low induction number, the expression of the conductivity contribution of each layer to the apparent conductivity is obtained by discretizing the integral of response function density, and we used Gauss Newton method to solve the conductivity and thickness of underground medium layer. In order to verify the effectiveness of the inversion algorithm, we designed a three-layer medium inversion model with V-shaped tunnel, and calculated the results when instrument observation signal is 30dB and 40dB respectively. From the results obtained, the error is smaller when the SNR is higher. The inversion results of thickness and conductivity clearly show the stratification of different medium in the model.

The multi-transceiver distance inversion method based on apparent conductivity does not require grounded electrodes. The procedure of this method is simple and easy to operate, which is appropriate for practise field measurement. In view of these advantages, this method can be widely used in engineering and environmental issues such as dam quality inspection, riverbed or groundwater exploration.

Acknowledgments
This work was supported by National Natural Science Foundation (61973165), National Natural Science Foundation Youth Fund (61801511) and Jiangsu Provincial Natural Science Foundation Youth Fund (BK20180580). Thanks to all colleagues, experts and editorial department for your support.

References
[1] Martini E, Werban U, Wollschläger U, et al. (2015) On the use of repeated EMI measurements for mapping soil moisture and implications for soil mapping. In: First Conference on Proximal Sensing Supporting Precision Agriculture. Italy.1-5.
[2] Oladunjoye, M., Adabanija, M., & Adeboye, O. (2013). Groundwater prospecting and exploration in a low potential hard rock aquifer: case study from Ogbomoso North, southwestern Nigeria. Journal of Environment and Earth Science, 3(14), 84-102.

[3] Pan D Z, Xu D M. (2006) Application of EM34-3 type earth conductivity detector for detecting dam leakage. Journal of Heliongjiang Hydraulic Engineering College.33(3):43-45.

[4] Xu C, Bai D H, Ding Y F, et al. (2010) Research of the electromagnetic terrain conductivity measurement at low induction numbers. Progress in Geophys.25(4):1372-1379.

[5] Hulin G, Maneuvrier C, Vincent J B, et al. (2015) What exists beneath the place where Conrad Schlumberger achieved the first (1912) electrical prospection experiment. In:Near Surface Geoscience 2015-21st European Meeting of Environmental and Engineering Geophysics. Italy.1-5.

[6] McNeil, J D. (1980) Electromagnetic Terrain Conductivity Measurement at Low Induction Numbers. Geonics Limited, Ontario.

[7] De Andrade, F. C. M. & Fischer, T. . (2017). Generalised relative and cumulative response functions for electromagnetic induction conductivity meters operating at low induction numbers. Geophysical Prospecting, 66(3), 595-602.

[8] Callegary, J. B., Ty P. A. Ferré, et al. (2007). Vertical spatial sensitivity and exploration depth of low-induction-number electromagnetic-induction instruments. Vadose Zone Journal Vzj, 6(1), 158-167.

[9] Piao H R, Li X B. (1988) Modified out of phase electromagnetic method. Chinese Journal of Geophysics. 31(5):594-600.

[10] McNeil, J D. (1983) EM34-3 survey interpretation techniques. Geonics Limited, Ontario.

[11] McNeil, J D. (1985) EM34-3 Measurements at Two Inter-coil Spacings to Reduce Sensitivity to Near-surface Material. Geonics Limited, Ontario.