How good pre-service mathematics teacher in reading mathematical proof?

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Abstract. Mathematics learning at the college has demanded pre-service mathematics teacher to have high-level mathematical skills including mathematical proofs. Several studies have focused on improving these abilities, but very few focus on the ability to read mathematical proofs. Though this ability that is needed by students in learning mathematical proofs. The purpose of this study is to analyse the ability to read mathematical proofs of students. This research is qualitative research, data obtained through interview and test. There were two problems in geometry given to three students with different abilities. The results show that two of the three students have good the ability to reading the mathematical proof and one student has the ability to reading proof that is lacking. Even though the three students have high mathematical initial knowledge.

1. Introduction

Geometry is a branch of mathematics that is learned from elementary school to college. Its abstract nature often makes it difficult for students to learn geometry. Included in solving geometry problems such as transformation. Geometry problems generally have five [1], namely understanding and applying concepts, visualizing geometric objects, determining principles, understanding problems and mathematical proof. The most difficult for students is a mathematical proof, especially in construction and reading the mathematical proof. In this study, the focus is more on reading mathematical proof.

Besides that, a pre-service mathematics teacher at a college is required to have advanced mathematical abilities, in this case, the mathematical proof ability. However, the fact is in the field, the average value for proof ability is always low every year, this data is taken from the final examination of the geometry transformation. This course contains student competencies in drawing geometric objects to construct the proof. Several studies [2-6] have been made regarding the importance of mathematical proof. This is also expressed by Hanna [7,8], that some mathematicians consider mathematical proof to be important because it can foster formal logic.

Before going further on the construction of proof, it is also important to identify the ability of reading the mathematical proof of pre-service mathematics teachers. Reading proof is part of proof validation [9,10]. The indicator reads the proof: first, the ability to use definitions or theorems as a basis for giving reasons for correct steps of proof or improvement of symbols, narratives, premise at the stage/step of proof that is not right. Second, comparing two definitions or theorems, then choosing one definition to
be used in proving a statement. Third, the ability to examine a mathematical statement to determine the truth or to show the error of the statement using counter examples.

The importance of reading mathematical proof above shows that these abilities need to be known, especially for pre-service mathematics teacher. Therefore, this study will reveal how good pre-service mathematics teachers in reading mathematical proof.

2. Methods

This research is qualitative research that aims to analyze the ability of reading proof of pre-service mathematics teachers. The research subjects were three students of mathematics teacher candidates with different abilities, these differences in abilities were seen from initial mathematical knowledge. Researchers choose students as subjects because they have achieved high-level thinking, so they can reveal the ability to read mathematical proof. The three students contracted the subject of transformation geometry. Retrieving data through tests and in-depth interviews. Three questions related to the transformation geometry problem are given in Table 1.

| No. | The problem on transformation geometry |
|-----|---------------------------------------|
| 1.  | Problems to be proven: T: V → V is defined as follows: If P ∈ S then T (P) = P. If P ∉ S then T (P) = P’, so that the line S is the axis of the segment PP’. Prove that T is a transformation! Proof: Obviously every P on V, there is a pre-order P’, so T (P) = P’. If P ∈ S, then P’ = P and if P ∉ S then P’ is a mirror of P on the S axis. For P ∈ S, Q ∈ S and P ≠ Q, it is clear P ≠ Q’. For P ∉ S, Q ∉ S, P ≠ Q. It will be investigated the position of P ’and Q’. Suppose P’ = Q’. Because S is the PP’ line segment axis then S perpendicular to PP’ and because S is the axis of the QQ’ line segment then S is perpendicular to QQ’. Then because P’ = Q’ and both perpendicular lines S, PP’ and QQ’ coincide. As a result A = B. This is a contradiction, it should be P’Q’. Thus T is a transformation. |

From the steps of the proof above
a. What definitions, axioms or theorems are used in the verification steps above? Explain.
b. Is there a step in the proofs that is wrong / lacking in the proof? Explain.
c. If there is a step of proof that is wrong / lacking, write down the steps of proofs that you think are appropriate
d. Is the adequacy of information from the evidentiary steps fulfilled? Explain.
e. Can you prove in another way that is shorter? Explain

2. The following two geometry theorems are presented:
   i. If two triangles are congruent, the length ratio of the corresponding sides is equal.
   ii. Two triangles are congruent if the two sides and the angle flanked are equal.

From the two theorems above, which is the theorem that can be used to prove the following statement:

**An isometry preserves the magnitude of the angle between two lines.**

Explain your reasons.

3. Consider the following statements:
   i. Each isometry is the opposite isometry
   ii. If T, S Isometry, and g a line then g’ = (TS) (g) is also a line.
   iii. Every reflection on the line is the opposite isometry.
   iv. If g ‖ h and g’ = (TS) (g), h’ = (TS) (h) then g’ ‖ h’.
   v. Half a round is not a collineation.

Based on the statements above:
   a) Write the correct statements, then give the words (with pictures).
   b) Write the wrong statements, then give the reason with a counter example.
All the problems given contain three aspects of reading mathematical proof on the geometry of transformation. First, the ability to use definitions or theorems as a basis in giving reasons for the correct steps of proof or improvement of symbols, narratives, premises in the stage/step of proof that is not right. Second, comparing two definitions or theorems, then choosing one definition to be used in proving a statement. Third, the ability to examine a mathematical statement to determine the truth or to show the error of the statement using examples of denial. Each student is given each of the three problems in Table 1. Data processing is done qualitatively.

3. Results and Discussion

The following is an overview of the analysis of test results and in-depth interviews of three pre-service teachers. This analysis is carried out on each indicator reading mathematical proof.

Gina answered question number one correctly. Gina already knows the definition that must be used to prove the problem and can explain it correctly. He can identify the steps of proof that are still wrong and correct the wrong steps correctly and can know the adequacy of the information presented in question number one. Figure 1 is an example of an answer from Gina.

Figure 1. Gina's answer to question number one

Figure 1 explains that Gina knows the definitions used in the proof presented in question number one. The Gina's ability to read proof is good in the first aspect, but Gina considers that the problem given to question number one can be proven by other means, namely by using images.

Subkhan can answer question number one correctly. Subkhan can mention the definitions used in the steps of proof correctly. Subkhan can determine the steps of the wrong and lack of proof, and can correct the steps of the proof correctly and in detail. Pay attention to each symbol and notation in the verification steps. Subkhan has good abilities in the first aspect, even exceeding the expectations of the researchers.

Figure 2. Subkhan's answer to question number one.

Figure 2 explains that Subkhan can identify the lack of proof steps, and rewrite the proof steps correctly. Even Subkhan knew the type of transformation in question number one. Subkhan also explained that the steps of the proof could be carried out with the editorial of different proofs accompanied by pictures.

Suryana can answer some questions number one correctly. Suryana can write the definition used, but defines it. Not careful in seeing the proof step, can correct the step of the wrong proof, but it is not detailed. Moreover, can determine the steps of proof that are lacking, there is no other proof.
Figure 3. Suryana's answer to question number 1a.

Figure 3 illustrates that Suryana knows the definition used in the proof step in question number one, but in the second statement it is not completely written. So the definition written is wrong. An error also occurred when Suryana rewrote the proof steps for question number one. This error can be seen in figure 4.

Figure 4. Suryana's answer to question number 1e.

Figure 4 illustrates that Suryana rewrote the proof steps, but there is an error in the steps to prove the objective function. The proof step written is part of the proof step on injective function.

Gina and Subkhan answered question number two correctly. They can compare two definitions or theorems, then choose one definition to be used in proving a statement. Both even construct proof using the theorem presented. An example of Gina's answer can be seen in Figure 5.

Figure 5. Gina's answer to question number two.
Suryana answers question number two by choosing one of the definitions to be used in proving a statement but cannot give a reason for choosing the theorem (Figure 6).

![Figure 6. Suryana's answer to question number two.](image)

Gina answered question number three quite well. Gina can identify mathematical statements that have the truth or to show error statements. However, the correct mathematical statement is included with reasons or mathematical arguments that are still not quite right (see Figure 7a). Gina tends to show the correct mathematical statement using images (see Figure 7b). On the other hand Gina can identify false mathematical statements accompanied by using counter examples.

![Figure 7. Gina's answer to question number three.](image)

Unlike Gina, Subkhan answered question number three very well, the arguments given were very precise in identifying mathematical statements that had the truth, illustrations in each statement were always accompanied by clear images. Mathematical statements that have errors are accompanied by correct arguments but are not accompanied by counter examples (Figure 8).

![Figure 8. Subkhan's answer to question number three.](image)
Suryana answered part of question number three correctly. Suryana tends to identify statements only from images, does not provide arguments against mathematical statements that have truth or have errors (Figure 9).

Figure 9. Suryana's answer to question number three.

4. Conclusions
Here are the conclusions obtained in this study. The results show that two of the three students have good the ability to reading mathematical proof and one student has the ability to reading proof that is lacking. Even though the three students have high mathematical initial knowledge.

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