ON THE THEORY OF GAMMA RAY BURSTS AND HYPERNOVAE: The Black Hole Soft X-ray Transient Sources

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Abstract

We show that a common evolutionary history can produce the black hole binaries in the Galaxy in which the black holes have masses of $\sim 5 - 10 M_\odot$. In with low-mass, $\lesssim 2.5 M_\odot$, ZAMS (zero age main sequence) companions, the latter remain in main sequence during the active stage of soft X-ray transients (SXTs), most of them being of K or M classification. In two intermediate cases, IL Lupi and Nova Scorpii with ZAMS $\sim 2.5 M_\odot$ companions the orbits are greatly widened because of large mass loss in the explosion forming the black hole, and whereas these companions are in late main sequence evolution, they are close to evolving. Binaries with companion ZAMS masses $\gtrsim 3 M_\odot$ are initially “silent” until the companion begins evolving across the Herzsprung gap.

We provide evidence that the narrower, shorter period binaries, with companions now in main sequence, are fossil remnants of gamma ray bursters (GRBs). We also show that the GRB is generally accompanied by a hypernova explosion (a very energetic supernova explosion). We further show that the binaries with evolved companions are good models for some of the ultraluminous X-ray sources (ULXs) recently seen by Chandra in other galaxies.

The great regularity in our evolutionary history, especially the fact that most of the companions of ZAMS mass $\lesssim 2.5 M_\odot$ remain in main sequences as K or M stars can be explained by the mass loss in common envelope evolution to be Case C; i.e., to occur only after core He burning has finished. Since our argument for Case C mass transfer is not generally understood in the community, we add an appendix, showing that with certain assumptions which we outline we can reproduce the regularities in the evolution of black hole binaries by Case C mass transfer.

Key words: black hole physics — stars: binaries: close — accretion — gamma-ray bursts
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1 INTRODUCTION

The discovery of afterglows to gamma-ray bursts (GRBs) has greatly increased the possibility of studying their physics. Recent observations strongly suggest a connection between GRBs and supernovae, with indication that the supernovae in question are especially energetic and of type Ib/c, i.e., core collapses of massive stars which have lost their hydrogen envelope (see Van Paradijs et al.[1], and references therein). This supports suggestions by Woosley[2] and Paczyński[3] for the origin of GRBs in stellar core collapses. The hydrodynamics of a jet escaping from a star and causing its explosion was explored in detail by MacFadyen & Woosley[4], who showed that contrary to accepted wisdom, a fairly baryon-free, ultra-relativistic jet could plow through the collapsing star and emerge with large Lorentz factors. The powering of the outflow by coupling of high magnetic fields to the rotation of the black hole [5], first suggested by Paczyński[3] in the context of GRBs, was worked out in detail by Van Putten[6,7]. Li has also discussed the deposition of energy from a black hole into the accretion disk in a recent series of papers[8].

Building on these thoughts, we have modeled both the powering of a GRB by black-hole rotation and the stellar evolution pathways that set up favorable conditions for that mechanism [9]. An essential ingredient in this model is a rapidly rotating black hole, and it is this aspect that we focus on in the present paper.

A massive star in a close binary will spin faster than a massive single star for a number of reasons: first, when the hydrogen envelope is lifted off by spiral-in, it will cease to serve as a sink of angular momentum for the core. Second, the tidal friction concomitant with the spiral-in process will spin up the inner region, giving it a larger angular momentum than the same region in a single star [10]. Third, tidal coupling in the close binary will tend to bring the primary into corotation with the orbital period. This latter process is not very efficient in the short post spiral-in life of the binaries we consider, but its effect does probably matter to the outer layers of the helium star, which can be important for our work. With its more rapid rotation, the helium star then
forms a black hole with a large Kerr parameter, which immediately after its formation (in a few seconds) begins to input power into its surroundings at a very high rate. This, then, powers both a GRB [9] and the expulsion of the material that was centrifugally prevented from falling into the black hole. In fact, Van Putten[6,7] estimates that the power input into that material exceeds that into the GRB and Li[11] also finds that more energy can be extracted by the disk than by the GRB. It should be noted that an initially less rapidly rotating black hole could be spun up by disk accretion quite rapidly, and start a similar process after some accretion has taken place [4,9]. Some implications of such more complicated sequences of events are discussed by Lee et al.[12].

More than dozen soft X-ray transient (SXT) black hole binaries were observed in our Galaxy. By far the most famous one is Nova Scorpii 94. Israelian et al.[13] found the overabundances of heavy elements in the subgiant companion star in Nova Scorpii, and suggested that it is a relic of hypernova explosion. Observed high system velocity and the quasi periodic oscillations support the hypernova explosion associated with rapidly rotating black hole. In a more recent analysis [14], we found the correlation between the black hole masses and the reconstructed pre-explosion orbital periods of SXTs. In binaries with preexplosion orbital period less than 12 hours, rapidly rotating black holes are formed in the core of helium stars by the prompt collapse, and the outer part of the helium star is held from immediate collapse by the angular momentum support. Based on these, we suggest that the SXTs with preexplosion orbital period less than or similar to 12 hours are the relics of GRBs and the hypernovae.

Since the afterglows have thus far only been seen for long GRBs (duration $\gtrsim 2s$), we shall concentrate on the mechanism for this subclass. The shorter bursts (duration $\lesssim 2s$) may have a different origin; specifically, it has been suggested that they are the result of compact-object mergers and therefore offer the intriguing possibility of associated outbursts of gravity waves. (Traditionally, binary neutron stars have been considered in this category [15,16]. More recently, Bethe & Brown [17] have shown that low-mass black-hole, neutron-star binaries, which have a ten times greater formation rate and are stronger gravity-wave emitters, may be the more promising source of this kind.)

The plan of this paper is as follows. In Sec. 2, the observational indications of the GRB and supernova/hypernova association are discussed.

In Sec. 3, we give the simple argument for the energetics of GRBs and Hypernovae. Our simple argument is based on the Blandford-Znajek mechanism, and we discuss various channels of the power output from the black hole-accretion disk system. Woosley’s Collapsar model is discussed as a progenitor of GRBs and Hypernovae.
In Sec. 4, we summarize the observational indications of SXTs, especially Nova Scorpii which is by far the most interesting SXT. We also discuss our recent discoveries on the empirical correlation between the orbital periods and black hole masses of X-ray transient black hole systems.

The evolution of the SXTs is discussed in Sec. 5. Based on the Case C mass transfer, which is essential part of the evolution, we could pin down the common envelope efficiency.

Using the evolution scenario in the previous section, in Sec. 6, we reconstructed the pre-explosion orbital period and the black hole masses at the time of formation. We found a regularity in the reconstructed mass-period relations.

In Sec. 7, we discuss a schematic model which explains the correlation between the black hole masses and the preexplosion periods. In SXTs with short pre-explosion orbital periods, less than or similar to 12 hours, rapidly rotating black holes are formed. Based on these results, we suggest that the progenitors of black holes in SXTs are the sources of GRBs and Hypernovae if their preexplosion spin periods are less than or similar to 12 hours. In this model we assumed that the tidal interaction synchronized the orbital period and spin period of the black hole progenitor.

In Sec. 8, we discuss other observational issues of SXTs. We give estimates on the chemical composition of a few X-ray transients. We discuss the untralusuminous X-ray sources in our Galaxy. We also discuss the population synthesis of SXTs and GRBs.

Our final discussion and conclusion follows in Sec. 9.

In the Appendix, we discuss the Case C mass transfer in detail, which is essential part of the evolution of the black hole progenitors. We also show when there will be deviation from Case C, indicating that these will not affect our main results.

2 GRB AND SUPERNOVA/HYPERNOVA ASSOCIATION: OBSERVATIONS

An important recent clue to the origin of GRBs is the probable association of some of them with ultra-bright type Ibc supernovae [18–21]. Recently, there appeared a beautiful example of the GRB-hypernova association [22]. In this paper the light curve showed a bump, reddened by the many metal lines, rising on that from the GRB afterglow after several days. This bump was fit well by red shifting the known hypernova light curve from SN1998bw, previously
interpreted as a hypernova explosion, to the relevant redshift (0.36).

The very large explosion energy implied by fitting the light curve of SN 1998bw, which was associated with GRB 980425[18], indicates that a black hole was formed in this event[23]. This provides two good pieces of astrophysical information: it implicates black holes in the origin of GRBs, and it demonstrates that a massive star can explode as a supernova even if its core collapses into a black hole.

The GRB and accompanying hypernova have a common explanation in the model of Brown et al.[9] and Lee et al.[14]. Both are powered by the rotational energy of the black hole, the hypernova through the closed field lines coupling the black hole to the accretion disk.

In a review by Tsvi Piran[24] it is pointed out that the actual GRB energy is narrowly distributed around a “mere” $\sim 10^{51}$ ergs. Hypernova calculations which reproduce the spectra of 1998bw seem to need $\sim 10^{52}$ ergs, ten times more (see the results in Fig. 4 of Brown et al.[9].) We have shown that the total energy from the rotational energy of the black hole is $\sim 10^{53}$ ergs [9,14].

3 ENERGETICS OF GRB AND HYPERNOVA

3.1 Simple circuitry

We start from the viewpoint that the GRB is powered by electromagnetic energy extraction from a spinning black hole, the so-called Blandford-Znajek[5] mechanism. This was worked out in detail by Lee et al.[25,26], who built on work by Thorne et al.[27] and Li[28]. They have shown that with the circuitry in a 3+1 dimensional description using the Boyer-Lindquist metric, one can have a simple pictorial model for the BZ mechanism. Although our numbers are based on the detailed review of Lee et al.[25], which confirms the original Blandford-Znajek paper[5], we illustrate our arguments with the pictorial treatment of Thorne et al.[27] in “The Membrane Paradigm”. Considering the time as universal in the Boyer-Lindquist metric, essential electromagnetic and statistical mechanics relations apply in their 3+1 dimensional manifold. We summarize their picture in our Fig. 1.

The simple circuitry which involves steady state current flow is, however, inadequate for describing dissipation of the black hole rotational energy into the accretion disk formed from the original helium envelope. In this case the more rapidly rotating black hole tries to spin up the inner accretion disk through
Fig. 1. The black hole in rotation about the accretion disk. A circuit, in rigid rotation with the black hole is shown. This circuit cuts the field lines from the disk as the black hole rotates, and by Faraday’s law, produces an electromotive force. This force drives a current. More detailed discussion is given in the text.

the closed field lines coupling the black hole and disk. Electric and magnetic fields vary wildly with time. Using the work of Blandford & Spruit[29] we show that this dissipation occurs in an oscillatory fashion, giving a fine structure to the GRB, and that the total dissipation should furnish an energy comparable to that of the GRB to the accretion disk. We use this energy to drive the hypernova explosion.

Not any black-hole system will be suitable for making GRB: the black hole must spin rapidly enough and be embedded in a strong magnetic field. Moreover, the formation rate must be high enough to get the right rate of GRB even after accounting for substantial collimation of GRB outflows. We argue that the systems known as black-hole transients are the relics of GRBs, and discuss the recent evidence from high space velocities and chemical abundance anomalies that these objects are relics of hypernovae and GRBs; we especially highlight the case of Nova Scorpii 1994 (GRO J1655−40).

The surface of the black hole can be considered as a conductor with surface resistance $R = 4\pi/c = 377$ ohms. A circuit that rotates rigidly with the black hole can be drawn from the loading region, the low-field region up the axis of rotation of the black hole in which the power to run the GRB is delivered, down a magnetic field line, then from the North pole of the black hole along the (stretched) horizon to its equator. From the equator we continue the circuit through part of the disk and then connect it upwards with the loading region. We can also draw circuits starting from the loading region which pass along only the black hole or go through only the disk, but adding these would not change the results of our schematic model.

Using Faraday’s law,
\[ V = \int [\vec{v} \times \vec{B}] \cdot d\vec{l}, \]  

we can integrate downwards along our current loop. Because this law involves \( \vec{v} \times \vec{B} \) the integrals along the field lines make no contribution. We do get a contribution \( V \) from the integral from North pole to equator along the black hole surface. Further contributions to \( V \) will come from cutting the field lines from the disk. We assume the field \( B \) to be weak enough in the loading region to be neglected.

The GRB will be powered by

\[ \dot{E}_{\text{GRB}} = I_{\text{BH}+D}^2 R_L \]  

where \( R_L \) is the resistance of the loading region, and

\[ I_{\text{BH}+D}^2 = \left( \frac{V_D + V_{\text{BH}}}{R_D + R_{\text{BH}} + R_L} \right)^2. \]  

The load resistance has been estimated in various ways and for various assumptions [30–32]. All estimates agree that to within a factor of order unity \( R_L \) is equal to \( R_{\text{BH}} \). The disk resistance \( R_D \) is usually omitted.

In a similar fashion, some power will be deposited into the disk

\[ \dot{E}_{\text{disk}} = I_{\text{BH}+D}^2 R_D \]  

but this equilibrium contribution will be small because of the low disk resistance \( R_D \).

Blandford & Spruit[29] and Van Putten[7] have shown that important dissipation into the disk comes through magnetic field lines coupling the disk to the black hole rotation. As shown in Fig. 2 these lines, anchored in the inner disk, thread the black hole.

The more rapidly rotating black hole will provide torques, along its rotation axis, which spin up the inner accretion disk, in which the closed magnetic field lines are anchored. With increasing centrifugal force the material in the inner disk will move outwards, cutting down the accretion. Angular momentum is then advected outwards, so that the matter can drift back inwards. It then delivers more matter to the black hole and is flung outwards again. The situation

\footnote{Whether this is really so, or just wishful thinking is not clear to us. It is well known that such impedance matching maximizes the power output.}
Fig. 2. Magnetic field lines, anchored in the disk, which thread the black hole, coupling the disk rotation to that of the black hole. The more rapidly rotating black hole torques up the accretion disk delivering rotational energy into it. This energy is changed to heat by the viscosity, powering the hypernova explosion.

is like that of a ball in a roulette wheel (R.D. Blandford, private communication). First of all it is flung outwards and then drifts slowly inwards. When it hits the hub it is again thrown outwards. The viscous inflow time for the fluctuations is easily estimated to be

$$\tau_{\text{visc}} \sim \Omega_{\text{disk}}^{-1} \left( \frac{r}{H} \right)^2 \alpha_{\text{vis}}^{-1}$$

(5)

where $H$ is the height of the disk at radius $r$, and $\alpha_{\text{vis}}$ is the usual $\alpha$-parameterization of the viscosity. We choose $\alpha_{\text{vis}} \sim 0.1$, $r/H \sim 10$ for a thin disk and then arrive at $\tau_{\text{visc}} \sim 0.1$ s. We therefore expect variability on all time scales between the Kepler time (sub-millisecond) and the viscous time, which may explain the very erratic light curves of many GRBs.

We suggest that the GRB can be powered by $\dot{E}_{\text{GRB}}$ and a Type Ibc supernova explosion by $\dot{E}_{\text{SN}}$ where $\dot{E}_{\text{SN}}$ is the power delivered through dissipation into the disk. To the extent that the number of closed field lines coupling disk and black hole is equal to the number of open field lines threading the latter, the two energies will be equal. In the spectacular case of GRB 980326 [19], the GRB lasts about 5 s, which we take to be the time that the central engine operates. We shall show that up to $\sim 10^{53}$ erg is available to be delivered into the GRB and into the accretion disk, the latter helping to power the supernova (SN) explosion. This is more energy than needed and we suggest that injection of energy into the disk shuts off the central engine by blowing up the disk and thus removing the magnetic field needed for the energy extraction from the black hole. If the magnetic field is high enough the energy will be delivered in a short time, and the quick removal of the disk will leave the black hole still spinning quite rapidly.

3.2 Energetics of GRBs

The maximum energy that can be extracted from the BZ mechanism [25] is

$$(E_{\text{BZ}})_{\text{max}} \approx 0.09 \ M_{\text{BH}} c^2.$$

(6)
This is 31% of the black hole rotational energy, the remainder going toward increasing the entropy of the black hole. We have chosen here (see preceding footnote)

\[ \epsilon_\Omega = \frac{\Omega_{\text{disk}}}{\Omega_H} = 0.5 \]  

(7)

which maximizes the BZ power.

For a 7\(M_\odot\) black hole, such as that found in Nova Sco 1994 (GRO J1655−40),

\[ E_{\text{max}} \simeq 1.1 \times 10^{54} \text{ erg} \]  

(8)

We estimate below that the energy available in a typical case will be an order of magnitude less than this. Without collimation, the estimated gamma-ray energy in GRB 990123 is about 4.5 \(\times\) \(10^{54}\) erg [33]. The BZ scenario entails substantial beaming, so this energy should be multiplied by \(d\Omega/4\pi\), which may be a small factor \(\sim 10^{-2}\).

The BZ power can be delivered at a maximum rate [25] of

\[ P_{\text{BZ}} = 6.7 \times 10^{50} \left( \frac{B}{10^{15}\text{G}} \right)^2 \left( \frac{M_{\text{BH}}}{M_\odot} \right)^2 \text{ erg s}^{-1} \]  

(9)

so that high magnetic fields are necessary for rapid delivery.

The above concerns the maximum energy output into the jet and the disk. The real energy available in black-hole spin in any given case, and the efficiency with which it can be extracted, depend on the rotation frequency of the newly formed black hole and the disk or torus around it. The state of the accretion disk around the newly formed black hole, and the angular momentum of the black hole, are somewhat uncertain. However, the conditions should be bracketed between a purely Keplerian, thin disk (if neutrino cooling is efficient) and a thick, non-cooling hypercritical advection-dominated accretion disk (HADAF) [34]. Let us examine the result for the Keplerian case. In terms of

\[ \tilde{a} = \frac{J_c}{M^2 G} \]  

(10)

we find the rotational energy of a black hole to be

\[ E_{\text{rot}} = f(\tilde{a}) M c^2 \]  

(11)
where

$$f(\tilde{a}) = 1 - \sqrt{\frac{1}{2}(1 + \sqrt{1 - \tilde{a}^2})}.$$  \hfill (12)

For a maximally rotating black hole one has $\tilde{a} = 1$.\footnote{As an aside, we note a nice mnemonic: if we define a velocity $v$ from the black-hole angular momentum by $J = MR_{\text{Sch}}v$, so that $v$ carries the quasi-interpretation of a rotation velocity at the horizon, then $\tilde{a} = 2v/c$. A maximal Kerr hole, which has $R_{\text{event}} = R_{\text{Sch}}/2$, thus has $v = c$. For $\tilde{a} \lesssim 0.5$, the rotation energy is well approximated by the easy-to-remember expression $E_{\text{rot}} = \frac{1}{2}Mv^2$.}

We begin with a neutron star in the middle of a Keplerian accretion disk, and let it accrete enough matter to send it into a black hole.

In matter free regions the last stable orbit of a particle around a black hole in Schwarzschild geometry is

$$r_{\text{iso}} = 3R_{\text{Sch}} = 6\frac{GM}{c^2}. \hfill (13)$$

This is the marginally stable orbit $r_{\text{ms}}$. However, under conditions of hyper-critical accretion, the pressure and energy profiles are changed and it is better to use $r_{\text{iso}} \gtrsim 2R_{\text{Sch}}$.\hfill (14)

With the equal sign we have the marginally bound orbit $r_{\text{mb}}$. With high rates of accretion we expect this to be a good approximation to $r_{\text{iso}}$. The accretion disk can be taken to extend down to the last stable orbit [9].

We take the angular velocity to be Keplerian, so that at the radius of $2R_{\text{Sch}}$

$$v^2 = \frac{GM}{2R_{\text{Sch}}} = \frac{c^2}{4}, \hfill (15)$$

or $v = c/2$. The specific angular momentum is then

$$l \geq 2R_{\text{Sch}}v = 2\frac{GM}{c} \hfill (16)$$

which in Kerr geometry indicates $\tilde{a} \sim 1$. Had we taken one of the slowest-rotating disk flows that are possible, the advection-dominated or HADAF
case [36,34], which has $\Omega^2 = 2\Omega_{K}^2/7$, we would have arrived at $\tilde{a} \sim 0.54$, so the Kerr parameter will always be high. Later in Sec. 7, we discuss that the Kerr parameter for the SXTs with orbital period shorter than or similar to 0.5 days, at the time of black hole formation, is very large, close to the maximum rotation.

Further accretion will add angular momentum to the black hole at a rate determined by the angular velocity of the inner disk. The material accreting into the black hole is released by the disk at $r_{\text{iso}}$, where the angular momentum delivered to the black hole is determined. This angular momentum is, however, delivered into the black hole at the event horizon $R_{\text{Sch}}$, with velocity at least double that at which it is released by the disk, since the lever arm at the event horizon is only half of that at $R_{\text{Sch}}$, and angular momentum is conserved. With more rapid rotation involving movement towards a Kerr geometry where the event horizon and last stable orbit coincide at

$$r_{\text{iso}} = R_{\text{event}} = \frac{GM}{c^2}.$$  \hfill (17)

Although we must switch over to a Kerr geometry for quantitative results, we see that $\tilde{a}$ will not be far from its maximum value of unity. Again, for the lower angular-momentum case of a HADAF, the expected black-hole spin is not much less.

Van Putten [37,7] has listed many channels through which the rotational energy of the black hole can be dissipated.

We note that in the scenario we developed here, the black hole rotational energy will depend only on the binary period just before black hole formation, which, in turn, is determined chiefly by companion mass [14]. However, the power, the rate at which the energy is delivered, is proportional to $B^2$, which will vary from star to star. If $B^2$ is low, so that the central engine must work for a long time, then neutrino losses will be somewhat increased.

Finally we note that an extensive numerical calculation by Koide et al.[38] supports in a general way that the rotational energy can be extracted from black holes as in Blandford-Znajek, although the energy comes out in somewhat different forms.

3.3 *Hypernova explosion*

In Brown et al.[9] the Blandford-Znajek mechanism was described in a schematic way; first of all a rapidly rotating black hole evolves in the core of the He star in a binary. The He star had been set into rotation as the H envelope of the
massive black hole progenitor was taken off by the companion star in common envelope evolution and further spun up by tidal interactions as described by Lee et al.[14]. The rotational energy of the black hole was used both to power the GRB and the hypernova explosion, the latter in the accretion disk resulting from the inner part of the remnant helium star.

The way in which the rotating black hole powers the hypernova explosion was developed by Van Putten[6,7]. The black hole rotates much faster than the accretion disk, so the closed magnetic field lines threading the black hole at one end and frozen in the matter of the accretion disk at the other end torque the latter up, delivering angular momentum and energy into the latter. Li[11] has shown that the efficiency for the black hole to deliver energy into the accretion disk is greater than that into the GRB. As much as \( \sim 10^{52} \) ergs can be provided for the hypernova explosion, if the black hole rotates sufficiently fast.

Brown et al.[9] showed that the outer part of the spinning He star can be supported by centrifugal force for a viscous time scale, Eq. (5) which will be much longer than the \( \tau_{\text{visc}} \) for the inner disk because of the lower \( \Omega \) and smaller \( \alpha \) far out in the He star. (The \( \alpha \) will be smaller because of the lower magnetic field \( B \)[39].) For thin disks this can be two orders of magnitude longer than the dynamical time which we discuss later. By this time the hypernova explosion is assumed to reach the matter and expel it. Once the energy from the black hole is converted into heat by viscosity, the hypernova explosion is triggered in a relatively short time. Given the \( 10^{52} \) ergs energy estimated in Brown et al.[9], and mass \( 2 \times 10^{34} \) g, we find a velocity

\[
v = \sqrt{\frac{2E}{M}} = 10^9 \text{ cm s}^{-1}.
\]  

(18)

With a He-star radius \( \lesssim 1R_\odot \) it takes the shock \( \sim 50 \) s, roughly the viscous time scale, to go through the outer He star, although the subsequent expansion develops over a much longer time.

The empirical relationship between preexplosion period, which was reconstructed through evolutionary arguments, and black hole mass was adduced by Lee et al.[14] to support this argument. The close relationship with Woosley’s failed supernova model for GRBs [2] was noted.

Because with black hole formation the He star fell into an accretion disk this model is clearly aspherical. Podsiadlowski et al.[40] investigated both spherical and aspherical models for matter deposition onto the F-star companion. They found that with the \( 16M_\odot \) He star non-spherical models they calculate, all produce either an unacceptable overabundance of O and Mg or an unacceptable underabundance of S and Si, depending on where the cut-off below which
matter can be mixed into the ejecta occurs. However, they find for spherical models that 10 to 16 $M_\odot$ He cores are the most probable, consistent with the 11 $M_\odot$ He star found by Lee et al.[14]. O and Mg are formed in quiescent burning in stars and their abundance in the presupernova model is roughly linear with ZAMS mass, and decreasing it from 40$M_\odot$ to 30$M_\odot$ will reduce these elements. The amounts of Si and S depend sensitively on the energy of the explosion. We believe that aspherical calculations should be made in the 30$M_\odot$ region of ZAMS masses.

We believe Nova Scorpii should be evolved like the other SXTs, it being exceptional because of its low preexplosion period, and its loss of nearly half of its mass in the explosion forming the black hole. Its system velocity is large in a Blaauw-Boersma type kick, because the companion is relatively massive, $\sim 2M_\odot$ at the time of explosion as we detail later, so the center of gravity is somewhat away from the black hole where the mass is lost.

The SXTs with low-mass main sequence companion do not have large system velocities [41], at least not radial ones, which would be seen. Their black hole evolution should not be influenced appreciably by the companion star. On the other hand, the Blaauw-Boersma kick is roughly proportional to the companion mass and the mass loss being at the position of the black hole, so that the system velocity depends on the difference in position of the black hole and center of gravity of the system.

We suggest that the neutron star kick velocity may be suppressed by the rapid rotation of the system in which it is born. A possible model for the neutron star kick velocity in supernovae is that it arises from the stochastic sowing of convection cells necessary for the supernova explosion. Because of the randomness, the number of cells in one hemisphere can be different from the number in the other, and this can power the kick velocity in a jet-like fashion. In the accretion disk preceding neutron star formation, the Bernoulli number is positive and convection carries matter out at angles $\theta \sim 30^\circ - 45^\circ$ [4]. Whereas the MacFadyen-Woosley[4] treatment is numerical, the underlying accretion disk equations are symmetrical in polar angle and do not involve the angle of rotation about the axis, so one would not expect any substantial inbalance in the two hemispheres as in the supernova problem.

3.4 Previous models

3.4.1 Collapsar

Woosley[2], and MacFadyen & Woosley[4] suggested the Collapsar model as a source of GRBs. In this model the center of a rotating Wolf-Rayet star evolves into a black hole, the outer part being held out by centrifugal force. The latter
Fig. 3. Time evolution of BH mass and angular momentum taken from Fig. 19 of MacFadyen & Woosley[4]. The upper panel shows the increase in the Kerr parameter for various models for the disk interior to the inner boundary at 50 km. “Thin” (dash-dot), neutrino-dominated (thick solid) and advection dominated (short dash) models are shown for initial Kerr parameter $\tilde{a}_{\text{init}} = 0.5$. The lower panel shows the growth of the gravitational mass of the black hole. The short-dashed line shows the growth in baryonic mass of the black hole since for a pure advective model no energy escapes the inner disk.

...evolves into an accretion disk and then by hypercritical accretion spins the black hole up.

Our schematic model has the advantage over numerical calculations that one can see analytically how the scenario changes with change in parameters or assumptions. However, our model is useful only if it reproduces faithfully the results of more complete calculations which involve other effects and much more detail than we include. We here make comparison with Fig.19 of MacFadyen & Woosley[4]. Accretion rates, etc., can be read off from their figure which we reproduce as our Fig.3. MacFadyen & Woosley prefer $\tilde{a}_{\text{initial}} = 0.5$ (We have removed their curve for $\tilde{a}_{\text{initial}} = 0$). This is a reasonable value if the black hole forms from a contracting proto-neutron star near breakup. MacFadyen & Woosley find that $\tilde{a}_{\text{initial}} = 0.5$ is more consistent with the angular momentum assumed for the mantle than $\tilde{a}_{\text{initial}} = 0$. (They take the initial black hole to have mass $2M_\odot$; we choose the Brown & Bethe[42] mass of $1.5M_\odot$.) We confirm this in the next section.

After 5 seconds (the duration of GRB980326) the MacFadyen & Woosley black hole mass is $\sim 3.2M_\odot$ and their Kerr parameter $\tilde{a} \sim 0.8$, which gives $f(\tilde{a})$ of our Eq.(12) of 0.11. With these parameters we find $E = 2 \times 10^{53}$ erg, available for the GRB and SN explosion.
One can imagine that continuation of the MacFadyen & Woosley curve for $M_{BH}(M_\odot)$ would ultimately give something like our $\sim 7M_\odot$, but the final black hole mass may not be relevant for our considerations. This is because more than enough energy is available to power the supernova in the first 5 seconds; as the disk is disrupted, the magnetic fields supported by it will also disappear, which turns off the Blandford-Znajek mechanism.

Power is delivered at the rate given by Eq.(9). Taking a black hole mass relevant here, $\sim 3.2M_\odot$, we require a field strength of $\sim 5.8 \times 10^{15}$ G in order for our estimated energy ($4 \times 10^{52}$ erg) to be delivered in 5 s (the duration of GRB 980326). For such a relatively short burst, we see that the required field is quite large, but it is still not excessive if we bear in mind that magnetic fields of $\sim 10^{15}$ G have already been observed in magnetars[43,44]. Since in our scenario we have many more progenitors than there are GRBs, we suggest that the necessary fields are obtained only in a fraction of all potential progenitors.

Thus we have an extremely simple scenario for powering a GRB and the concomitant SN explosion in the black hole transients, which we will discuss in Sec.7. After the first second the newly evolved black hole has $\sim 10^{53}$ erg of rotational energy available to power these. The time scale for delivery of this energy depends (inversely quadratically) on the magnitude of the magnetic field in the neighborhood of the black hole, essentially that on the inner accretion disk. The developing supernova explosion disrupts the accretion disk; this removes the magnetic fields anchored in the disk, and self-limits the energy the B-Z mechanism can deliver.

MacFadyen & Woosley point out that “If the helium core is braked by a magnetic field prior to the supernova explosion to the extent described by Spruit & Phinney[45] then our model will not work for single stars.” Spruit & Phinney argue that magnetic fields maintained by differential rotation between the core and envelope of the star will keep the whole star in a state of approximately uniform rotation until 10 years before its collapse. As noted in the last section, with the extremely high magnetic fields we need the viscosity would be expected to be exceptionally high, making the Spruit & Phinney scenario probable. Livio & Pringle[46] have commented that one finds evidence in novae that the coupling between layers of the star by magnetic fields may be greatly suppressed relative to what Spruit & Phinney assumed. However, we note that even with this suppressed coupling, they find pulsar periods from core collapse supernovae no shorter than 0.1 s. Independent evidence for the fact that stellar cores mostly rotate no faster than this comes from the study of supernova remnants: Bhattacharya[47,48] concludes that the absence of bright, pulsar-powered plerions in most SNRs indicates that typically pulsar spin periods at birth are no shorter than 0.03–0.05 s. Translated to our black holes, such spin periods would imply $\dot{a} \lesssim 0.01$, quite insufficient to power a GRB.
3.4.2 Coalescing low-mass black holes and helium stars

Fryer & Woosley\cite{49} suggested the scenario of a black hole spiraling into a helium star. This is an efficient way to spin up the black hole.

Bethe & Brown\cite{17} evolved low-mass black holes with helium star companion, as well as binaries of compact objects. In a total available range of binary separation $0.04 < a_{13} < 4$, low-mass black-hole, neutron-star binaries were formed when $0.5 < a_{13} < 1.4$ where $a_{13}$ is the initial binary separation in units of $10^{13}$ cm. The low-mass black hole coalesces with the helium star in the range $0.04 < a_{13} < 0.5$. Binaries were distributed logarithmically in $a$. Thus, coalescences are more common than low-mass black-hole, neutron-star binaries by a factor of \( \ln(0.5/0.04)/\ln(1.9/0.5) = 1.9 \).

In Bethe & Brown\cite{17}, the He-star, compact-object binary was disrupted $\sim 50\%$ of the time by the He-star explosion. This does not apply to the coalescence. Thus, the rate of low-mass black-hole, He-star mergers is 3.8 times the formation rate of low-mass black-hole, neutron-star binaries, or

\[
R = 3.8 \times 10^{-4} \text{ yr}^{-1}
\]  

(19)

in the Galaxy. The estimated empirical rate of GRBs, with a factor of 100 for beaming, is $10^{-5} \text{ yr}^{-1}$ in the Galaxy\cite{50}. Thus, the number of progenitors is more than adequate.

In Bethe & Brown\cite{17} the typical black hole mass was $\sim 2.4M_\odot$, somewhat more massive than their maximum assumed neutron star mass of $1.5M_\odot$. As it enters the helium star companion an accretion disk is soon set up and the accretion scenario will follow that described above, with rotating black holes of various masses formed.

The problem with this scenario is, however, that usually a substantial hydrogen envelope will be left around the black hole, helium star system. The reason the black hole coalesces with the He star is that its drop in gravitational binding is insufficient to expel the hydrogen envelope. If an appreciable envelope is left, it would be difficult for a jet to return a highly relativistic $\Gamma$ while plowing through the overlying matter \cite{4}. A layer of surface hydrogen is allowed as long as its radius is not too large, but even a modest total amount of hydrogen will interfere with a jet. The supernova, however, might still look like an exceptionally powerful type Ibc event. Our estimated number of coalescing low-mass black-hole, He star systems is so large that some interesting SN explosions and perhaps even GRBs should nonetheless survive.
Table 1
Abundances in the secondary of Nova Scorpii[13]. [Xi/H] are the logarithmic abundances relative to solar abundances.

|     | N   | O   | Mg  | Si  | S   | Ti  | Fe  |
|-----|-----|-----|-----|-----|-----|-----|-----|
| [Xi/H] | 0.45 | 1.00 | 0.90 | 0.90 | 0.75 | 0.90 | 0.10 |
| error | 0.50 | 0.30 | 0.40 | 0.30 | 0.20 | 0.40 | 0.20 |

4 SOFT X-RAY TRANSIENTS: OBSERVATIONS

4.1 Hypernova explosion in Nova Scorpii

Far and away the most interesting binary to date is Nova Scorpii although GRS 1915+105 is now growing in importance since the companion star has been found (see later). Nova Sco 1994 is a black hole transient X-ray source. It consists of a black hole of $\sim 5.4 M_\odot$ and a subgiant of current mass of about $1.45 M_\odot$. Their separation is $15.2 R_\odot$. Israelian et al. (1999) have analyzed the spectrum of the subgiant and have found that the $\alpha$-particle nuclei O, Mg, Si and S have abundances 6 to 10 times the solar value (see Table 1). This indicates that the subgiant has been enriched by the ejecta from a supernova explosion; specifically, that some of the ejecta of the supernova which preceded the present Nova Sco (a long time ago) were intercepted by star B, the present subgiant. Israelian et al.[13] estimate an age since accretion started from the assumption that enrichment has only affected the outer layers of the star. We here reconsider this: the time that passed since the explosion of the progenitor of the black hole is roughly the main-sequence lifetime of the present subgiant companion, which given its mass of $\sim 2 M_\odot$ will be about 1 Gyr. This is so much longer than any plausible mixing time in the companion that the captured supernova ejecta must by now be uniformly mixed into the bulk of the companion. This rather increases the amount of ejecta that we require the companion to have captured. (Note that the accretion rate in this binary is rather less than expected from a subgiant donor, though the orbital period leaves no doubt that the donor is more extended than a main-sequence star[51]. It is conceivable that the high metal abundance has resulted in a highly non-standard evolution of this star, in which case one might have to reconsider its age.)

The presence of large amounts of S is particularly significant. Nomoto et al.[52] have calculated the composition of a hypernova from an $11 M_\odot$ CO core, see Fig. 4. This shows substantial abundance of S in the ejecta. Ordinary supernovae produce little of this element, as shown by the results of Nomoto et al.[52] in Fig. 4. The large amount of S, as well as O, Mg and Si we consider the strongest argument for considering Nova Sco 1994 as a relic of a hypernova,
Fig. 4. The isotopic composition of ejecta of the hypernova \((E_K = 3 \times 10^{52} \text{erg})\) (left) and the normal supernova \((E_K = 1 \times 10^{51} \text{erg})\) (right) for a 16\(M_\odot\) He star, from Nomoto et al.\[52\]. Note the much higher sulphur abundance in the hypernova.

and for our model, generally.

Fig. 4 also shows that \(^{56}\text{Ni}\) and \(^{52}\text{Fe}\) are confined to the inner part of the core, below the mass cut. However, as known from SN 1987A, \(^{56}\text{Ni}\) (which decays into \(^{52}\text{Fe}\)) gets out through Rayleigh-Taylor instabilities. Maeda et al.\[53\] as described by Nomoto et al.\[54\] find that in the aspherical (pancake-type) explosion appropriate for the hypernova, the \(^{56}\text{Ni}\) goes out preferentially perpendicular to the pancake. Thus, the low amount of Fe deposited on the F-star companion in Nova Scorpii has a natural explanation because we see the binary edge on. On the other hand, not much of a GRB was seen in 1998bw, which is interpreted in terms of us seeing this binary sideways. We would, however, be expected to see a lot of \(^{56}\text{Ni}\). We are grateful to Garik Israelian for pointing this out to us.

The solar abundance of oxygen is about 0.01 by mass, so with the abundance in the F star being 10 times solar, and oxygen uniformly mixed, we expect \(0.1 \times 2.5 = 0.25M_\odot\) of oxygen to have been deposited on the companion. \([\text{Si/O}]\) is 0.09 by mass in the Sun, and \([\text{S/O}]\) is 0.05, so since the over-abundances of all three elements are similar we expect those ratios to hold here, giving about \(0.02M_\odot\) of captured Si and \(0.01M_\odot\) of captured S. We therefore need a layer of stellar ejecta to have been captured which has twice as much Si as S, at the same time as having about 10 times more O. From Fig. 4, we see that this occurs nowhere in a normal supernova, but does happen in the hypernova model\[52\] at mass cuts of 6\(M_\odot\) or more. This agrees very nicely with the notion that a hypernova took place in this system, and that the inner 7\(M_\odot\) or so went into a black hole. In Sec. 8 the chemical deposition will be discussed in more detail.
A further piece of evidence that may link Nova Sco 1994 to our GRB/hypernova scenario are the indications that the black hole in this binary is spinning rapidly. Zhang et al.[55] argue from the strength of the ultra-soft X-ray component that the black hole is spinning near the maximum rate for a Kerr black hole. However, studies by Sobczak et al.[56] show that it must be spinning with less than 70% maximum. Gruzinov[57] finds the inferred black hole spin to be about 60% of maximal from the 300 Hz QPO. Strohmayer[58] also found the 450 Hz QPO, which also support the rapidly rotating black hole in Nova Sco. Our estimates of the last section indicate that enough rotational energy will be left in the black hole so that it will still be rapidly spinning[14].

We have already mentioned the unusually high space velocity of $-106 \pm 19$ km s$^{-1}$. Its origin was first discussed by Brandt et al.[59], who concluded that significant mass must have been lost in the formation of the black hole in order to explain this high space velocity: it is not likely to acquire a substantial velocity in its own original frame of reference, partly because of the large mass of the black hole. But the mass lost in supernova is ejected from a moving object and thus carries net momentum. Therefore, momentum conservation demands that the center of mass of the binary acquire a velocity; this is the Blaauw–Boersma kick[60,61]. Note that the F-star companion mass is relatively large among the black-hole transient sources, so the center of mass is somewhat away from the black hole and one would expect a large kick. Nelemans et al.[41] estimate the mass loss in this kick to be $5-10M_\odot$.

In view of the above, we consider it well established that the black hole in Nova Sco 1994 is the relic of a hypernova. We believe it highly likely that some of the other black-holes in the transient X-ray sources are also hypernova remnants. Later in Sec. 7 we will show that the SXTs with short orbital period at the time of black hole formation are good candidates for fossil remnants of GRBs.

### 4.2 An empirical correlation between orbital period and black-hole mass

We have collected data from the literature on black-hole binaries in our Galaxy. In Table 2 we collect data of those in which the mass function is known, and some manner of mass estimate for both the black hole and companion can be given. In Table 3 we list the properties of two key systems in more detail. In Fig. 5 we show the masses of the black holes as a function of orbital period. While the ranges of black-hole masses for main-sequence and evolved systems overlap, the latter tend to have higher masses; the exception is Nova Scorpii 1994, which we shall see later is a natural but rare case of the general evolution scenario that we describe in this paper. In Fig. 6, we show the donor masses as a function of orbital period. They show a more obvious trend of more massive donors in evolved systems. As we shall see, this is a natural consequence of the
Fig. 5. Black hole mass as a function of present orbital period of 14 SXTs. Note that the orbital period is on a logarithmic scale. SXTs with subgiant or giant companions are indicated by big open circles (denoted as “Nu” for nuclear evolution). Filled squares indicate SXTs with main-sequence companions (denoted as “AML” for angular momentum loss). The vertical dotted line is drawn to indicate the possible existence of different classes according to evolutionary path of the binary, as discussed in section 5. 4U 1543−47 is marked with both symbols, since we believe it to be right on the borderline between main-sequence and evolved; for the purpose of modeling, it can be treated as evolved.

fact that only evolved systems can come into Roche contact in wide binaries, and more massive donors are more likely to come into contact via nuclear evolution. (The various curves are explained in section 5.)

In the following sections, we shall argue that the correlation between black-hole mass and period also has physical meaning: the shorter the orbital period, the more rapidly rotating the helium star progenitor to the black hole. Rapid rotation centrifugally prevents some fraction of the helium star to collapse into a black hole, resulting in a smaller black hole mass. The reason why the correlation in Fig. 5 is weak is that evolution of the binary since formation of the black hole has washed out the relation. Properly, we should consider the correlation between pre-explosion orbital period and post-explosion black hole mass. Much of our work presented here is concerned with understanding the evolution of these binaries, and using this knowledge to find the systems for which we can reconstruct those parameters. Using that subset, we show much
Table 2
Parameters of black hole binaries in our Galaxy with measured mass functions. Binaries are listed in order of increasing orbital period. All systems except Cyg X-1 (steady X-ray source) are SXTs. XN indicates X-ray Nova. Earlier observations (Greene et al. 2001) gave the black hole mass in Nova Scorpii as 6.3 ± 0.5\(M_\odot\). New analyses of the light curve by Beer & Podsiadlowski (2001) give a somewhat smaller mass 5.4 ± 0.3\(M_\odot\) and 1.45 ± 0.35\(M_\odot\) for the companion. References: a) McClintock et al. 2001[62], b) Wagner et al. 2001[63], c) Bailyn et al. 1998[64], d) Filippenko et al. 1999[65], e) Gelino et al. 2001a[66], f) Harlaftis et al. 1996[67], g) Filippenko & Chornock 2001[68], h) Gelino et al. 2001b[69], i) Orosz et al. 1998[70], j) Orosz 2002[71], k) Orosz et al. 2002[72], l) Beer & Podsiadlowski 2001[73], m) Orosz et al. 2001[74], n) Herrero et al. 1995[75], o) Shahbaz et al. 1994[76], p) Shahbaz et al. 1996[77], q) Greiner et al. 2001[78].

| X-ray names     | type       | \(P_{\text{orb}}\) (day) | \(d\) (kpc) | \(i\) (degree) | \(M_{\text{opt}}\) (\(M_\odot\)) | Ref. |
|-----------------|------------|------------------------|-------------|----------------|-------------------------------|-----|
| XTE J1118+480   | K7V-M0V    | 0.1699\(\times\)10(4)  | 6.1(3)      | 0.09–0.5       | a,b                           |     |
| KV Ursae Majoris|            | 701(10)                | 1.9(4)      | 81(2)          | 6.0–7.7                       |     |
| XN Persei 92    | M0 V       | 0.2127(7)              | 1.15–1.27   | 0.10–0.97      | c                            |     |
| GRO J0422+32    | 390.6(65)  | 305–3.29               |            | 3.4–14.0       |                              |     |
| XN Vel 93       | K6-M0      | 0.2852                 | 3.4–14.0    |              | d                            |     |
| MM Velorum      | 475.4(59)  | 78                     |             | 3.64–4.74     |                              |     |
| XN Mon 75       | K4 V       | 0.3230                 | 2.83–2.99   |                |                              |     |
| A 0620–003      | 433(3)     | 1.164(114)             | 40.75(300)  | 11.0(19)       |                              |     |
| XN Vel 88       | K5 V       | 0.3441                 | 5.01(12)    | 0.26–0.59      | c,f                          |     |
| GS 2000+251     | 520(16)    | 2                      | 47–75       | 6.04–13.9      |                              |     |
| XTE 1859+226    | 570(27)    | 0.380(3)               | 7.4(11)     |                | g                            |     |
| V406 Vulpeculae |            |                        |             |                |                              |     |
| XN Muscae 91    | K4 V       | 0.4326                 | 2.86–3.16   | 0.56–0.90      | c,h                          |     |
| GS 1124–683     | 406(7)     | 5.04(15)               | 6.95(6)     |                |                              |     |
| XN Ophiuchi 77  | K3 V       | 0.5213                 | 4.44–4.86   | 0.3–0.6        | c                            |     |
| H 1705–250      | 420(30)    | 5.5                    | 60–80       | 5.2–8.6        |                              |     |
| IL Lupi         | A2 V       | 1.1164                 | 0.252(11)   | 1.3–2.6        | i,j                          |     |
| 4U 1543–47      | 129.6(18)  | 9.1(11)                | ~ 22        | 2.0–9.7        |                              |     |
| XTE J1550–564   | G8IV-K4III | 1.552(10)              | 6.86(71)    | 1.31^{+0.33}_{-0.37} | k                            |     |
| V381 Normae     | 349(12)    | 4.7–5.9 (?)            | 70.8–75.4   | 10.56^{+1.92}_{-0.88} |                              |     |
| XN Scorpii 94   | F6III      | 2.6127(8)              | 2.64–2.82   | 1.1–1.8        | c,l                          |     |
| GRO J1655–40    | 227(2)     | 3.2                    | 67–71       | 5.1–5.7        |                              |     |
| V4641 Sagittarii| B9III      | 2.817                  | 2.74(12)    | 6.53^{+1.03}_{-1.08} | m                            |     |
| XTE J1819–254   | 211.0(31)  | 9.59^{+2.72}_{-2.38}   | 9.61^{+2.08}_{-0.88} | c,o,p                      |     |
| Cyg X-1         | O9.7Iab    | 5.5996                 | 0.25(1)     | ~ 17.8         | n                            |     |
| 1956+350         | 74.7(10)   | 2.5                    | ~ 10.1      |               |                              |     |
| V404 Cygni      | K0 IV      | 6.4714                 | 6.02–6.12   | 0.57–0.92      | c,o,p                        |     |
| GS 2023+338     | 208.5(7)   | 2.2–3.7                | 52–60       | 10.3–14.2      |                              |     |
| GRS 1915+105    | K-MIII     | 33.5(15)               | 9.5(30)     | 1.2(2)         | q                            |     |
| V1487 Aquilae   | 140(15)    | 12.1(8)                | 70(2)       | 14(4)          |                              |     |
Table 3
Parameters for Nova Scorpii [73] and V4641 Sgr [74].

| Parameter                      | Nova Scorpii | V4641 Sgr  |
|-------------------------------|--------------|------------|
| Orbital period (days)         | 2.623        | 2.817      |
| Black hole mass ($M_\odot$)   | 5.4 ± 0.3    | 9.61$^{+2.08}_{-0.88}$ |
| Companion mass ($M_\odot$)    | 1.45 ± 0.35  | 6.53$^{+1.6}_{-1.03}$ |
| Total mass ($M_\odot$)        | 6.85         | 16.19$^{+3.58}_{-1.94}$ |
| Mass ratio                    | 0.27         | 1.50 ± 0.13 |
| Orbital separation ($R_\odot$) | 15.2         | 21.33$^{+1.25}_{-1.02}$ |
| Companion radius ($R_\odot$)  | 4.15         | 7.47$^{+0.53}_{-0.47}$ |
| Distance (kpc)                | 3.2          | 9.59$^{+2.72}_{-2.19}$ |

better agreement between our model predictions and the observed relation between reconstructed period and mass; this supports our evolutionary model and has ramifications for the origin of GRBs.

5 THE EVOLUTION OF SOFT X-RAY TRANSIENTS

5.1 Prior to the formation of the black hole

Following on the work of Brown et al.[79], who showed the importance of mass loss of helium stars in binaries determines the final outcome of binary evolution, Brown et al.[80], Wellstein & Langer[81], and Brown et al.[82] showed that massive helium stars could evolve into high-mass black holes only if they were covered with hydrogen during most of their helium core burning era (Case C mass transfer in binaries). In case A or B mass transfer in binaries (Roche Lobe overflow in main-sequence or red-giant stage) the Fe core that was left was too low in mass to go into a high-mass black hole. Brown et al.[82] showed that high-mass black holes could be formed only if the mass was taken off of the black hole progenitor after helium core burning was finished; i.e. Case C mass transfer.

Brown et al.[83] showed that with the Schaller et al. evolution, this could happen only in the neighborhood of ZAMS mass $20M_\odot$, definitely not at $25M_\odot$ and higher, and then only in a narrow range of initial supergiant radii $\sim 1000 - 1175M_\odot$. Because of the wind losses, the mass transfer would begin as Roche lobe overflow only in Case B mass transfer, with these higher main sequence masses, because the giant radii of these stars exceed their radii in
Fig. 6. Companion mass as a function of present orbital period of 13 SXTs. XTE 1859+226 is not included because the companion mass is not well determined[68]. Symbols of SXTs are the same as in Fig. 5. Line III indicates the orbital period for which a companion of that mass fills its Roche lobe on the ZAMS. No system can exist above and to the left of this line for a significant duration. Lines I and II are the upper period limit for systems that can come into contact while the donor is on the main sequence. For high masses (line II) this limit is set by the period where the evolution time of the companion is too short to allow the orbit to shrink significantly before it leaves the main sequence. For low masses (line I), where the donor never evolves off the main sequence within a Hubble time, the limit is set by period for which the shrinking time scale of the orbit equals the Hubble time. The dot-dashed line indicates the point where a system that starts its life on lines I/II comes into Roche contact. For very low masses, this equals line III, because the donor never moves significantly away from its ZAMS radius, whereas for very high masses it equals line II, because the orbit cannot shrink before the companion evolves off the main sequence. At intermediate masses, the companion expands somewhat while the orbit shrinks, and fills its Roche lobe at a larger period than line III. Systems that become SXTs with main-sequence donors within a Hubble time must start between line III and line I/II. At the start of mass transfer, they must lie in the narrow strip between line III and the dot-dashed line.

the supergiant phase. However, the helium star progenitors in at least three of the evolved binaries seem to be too massive for the $20 - 23M_\odot$ ZAMS progenitors used by Brown et al.[83]: the black hole in V404 Cyg is probably at least $10M_\odot$ [64,76,77], and the black hole in Nova Scorpii is of mass $\sim 5.4\pm0.3M_\odot$ [73] and the mass loss in black hole formation is $\gtrsim 5M_\odot$ [41] so that
the progenitor of the helium star must have been $\sim 11M_\odot$. From Table 3, the black hole in V4641 Sgr is of mass $9.61^{+2.08}_{-0.88}M_\odot$ [74]. The tentative conclusion from the above is that at least these binaries with evolved companions seem to have come from helium cores of $\sim 11M_\odot$, or ZAMS mass $\sim 30M_\odot$. With high wind mass loss rates as proposed by Schaller et al.[84], such massive stars have larger radii as giants than as supergiants, thus making Case C mass transfer impossible. However, since radii and mass loss rates of evolved stars are very uncertain, Lee et al.[14] took the view that the need for $\sim 11M_\odot$ helium cores implies that their progenitors, $30M_\odot$ main-sequence stars, do expand enough to allow Case C mass transfer. This was done by reduction of mass loss during He burning, the extension being made by hand.

Podsiadlowski et al.[40] studied mass deposition on the Nova Scorpii companion very thoroughly. However, they used as black hole progenitor a $16M_\odot$ He star, corresponding to a ZAMS $40M_\odot$ star. After having modified wind losses by hand in order to extend progenitors up to ZAMS $\sim 30M_\odot$, we are hardly in a position to criticize this procedure, but we feel uncomfortable with high-mass black holes following from such high ZAMS masses. Brown et al.[79] calculated that 4U 1700-37, a low-mass compact object, had as progenitor a ZAMS $40 \pm 10M_\odot$ star. Ankay et al.[85] argue that the progenitor of the compact object is $\gtrsim 35M_\odot$. Reynolds et al.[86] modeled the X-ray spectrum of 4U 1700-37, obtained with Beppo SAX and report the presence of a possible cyclotron feature. If real, this observation yields a magnetic field of about $5 \times 10^{12}G$, so that 4U 1700-37 must be a neutron star. Brown et al.[79] interpreted the low-mass compact object from the region of ZAMS mass $\sim 40M_\odot$ as meaning that the envelopes of stars in this mass range had such large wind loss rates that only low-mass compact objects could be formed. In any case, Lee et al.[14] showed that Nova Scorpii could be evolved from a ZAMS $\sim 30M_\odot$ star, as long as wind losses were cut down sufficiently to enable Case C mass transfer, so we suggest that stars as massive as ZAMS $40M_\odot$ may end up as low-mass compact objects.

We note that a low-mass compact object would fit into the Brandt et al.[59] scheme of a two step process in which a neutron star with a kick velocity is first formed, and then accretes enough matter to go into a black hole. Podsiadlowski et al.[40] evolve Nova Scorpii from a different mechanism than the other black hole SXTs, which have small system velocities, as compared with the $106 \text{ km s}^{-1}$ of Nova Scorpii. We prefer to evolve Nova Scorpii similarly to the other SXTs, although it is special because of its large system velocity, which we interpret as indicating that it lost a lot of mass in the explosion, substantially more than the other black hole transient source binaries.

Some uncertainty in the evolution of all compact X-ray binaries is the phase of spiral-in that occurred in their evolution: these binaries are initially very wide, and when the primary fills its Roche lobe and transfers mass to the secondary,
the mass transfer leads to instability, resulting in the secondary plunging into the primary’s envelope. Next, dissipation of orbital energy of the secondary causes the primary’s envelope to be ejected, and the orbit to shrink. Following the original work by Webbink[87], Brown et al.[83] write the standard formula for common envelope evolution as

\[
\frac{G M_p M_e}{\lambda R} = \frac{G M_p M_e}{\lambda r_L a_i} = \alpha_{ce} \left( \frac{G M_{He} M_d}{2a_f} - \frac{G M_p M_d}{2a_i} \right)
\]

(20)

where \(M_p\) is the total mass of the BH progenitor star just before the common envelope forms, \(M_e\) is the mass of its hydrogen envelope, \(M_{He}\) is the mass of its core, \(a_i\) and \(a_f\) are the initial and final separation, before and after the common envelope, respectively, and \(r_L \equiv R_L / a\) is the dimensionless Roche-lobe radius. This equation essentially relates the loss of orbital energy of the secondary to the binding energy of the ejected envelope. The parameter \(\lambda\) is a shape parameter for the density profile of the envelope. It can vary greatly between stars [88], but for the extended, deeply convective giants we deal with in Case C mass transfer it is always close to 7/6. (See also Appendix C of Brown et al.[89].) The parameter \(\alpha_{ce}\) accounts for the efficiency with which orbital energy is used to expel the envelope, and may also account for some other effects such as extra energy sources and the possibility that each mass element of the envelope receives more than the minimum energy needed to escape (see, e.g., Bhattacharya & Van den Heuvel[90] and references therein).

Given the parameters of the system at first Roche contact, when spiral-in starts, the final separation is determined by the product of \(\lambda\) and \(\alpha_{ce}\), the efficiency of the energy conversion. In general, these parameters are only the simplest recipe prescription for the complex hydrodynamical interaction during spiral-in. While we therefore cannot predict the value of \(\lambda\alpha_{ce}\) from first principles, we can try to find its value from constraints in some systems, and then assume it is the same for all similar systems.

To calibrate the spiral-in efficiency, we need to find systems in which we can also estimate the orbital separation just after spiral-in well. This is complicated by the fact that mass transfer has taken place since the spiral-in. Most SXTs have small mass ratios, and for such small mass ratios the orbital separation is fairly sensitive to the amount of mass transferred, making it hard to derive the post-spiral-in separation from the present one. The exception is V4641 Sgr, in which the present mass ratio is close to 1 (Lee et al.[14]). Since the initial mass ratio could not have been significantly greater than 1 (since that would result in unstable mass transfer), and furthermore the orbital period changes very little with mass transfer for nearly equal masses, we can fairly approximate the post-spiral-in separation by the present one. Lee et al.[14] showed the predicted ranges of post-spiral-in orbital periods for different values of \(\lambda\alpha_{ce}\), and a value quite close to 0.2 was indicated. For 4U 1543–47 (IL Lupi), they found that it
is near the boundary between evolved and main-sequence evolution. To place it there, they found from the reconstructed orbital period in their Fig 6 that $\lambda_{ce} \sim 0.2$ is also consistent with the properties of this system.

Now if we look at Eq. (20), we see that $a_f$ scales almost linearly with the donor (companion) mass $M_d$. The envelope mass $M_e$ is roughly $0.7M_{\text{giant}}$ (Bethe & Brown[17]) and we use

$$M_{He} = 0.08(M_{\text{giant}}/M_\odot)^{1.45}M_\odot$$

so that

$$a_f \propto \frac{M_d}{M_\odot} \left(\frac{M_{\text{giant}}}{M_\odot}\right)^{-0.55} a_i$$

assuming $\lambda_{ce}$ to be constant and with neglect of the small term in $a_i^{-1}$ in the r.h.s of Eq. (20). From the possible ranges for the Case C mass transfer, we note that the 20% possible variation in $a_i$ results in the same percentage variation in $a_f$. Because the actual ZAMS mass can be anywhere in the range $20 - 30M_\odot$ there can be an additional $\sim 25\%$ variation in $a_f$ with giant mass, as compared with the linear dependence on $M_d$. In view of the modest size of these variations at a given donor mass, we make the approximation in the rest of the paper that the pre-explosion orbital separation depends only on $M_d$, and scales linearly with $M_d$. This simple scaling and the modest amount of scatter around it are partly the result of the weak dependences on initial parameters in Eq. (22), but chiefly the result of the fact that our model uses Case C mass transfer. This constrains the Roche contact to first occur when the orbital separation of the binary is in a very narrow range, e.g., $\sim 1700R_\odot$ for ZAMS $20M_\odot$ star and $1M_\odot$ companion, as we make clear in the Appendix.

In short, the general properties of SXTs and the specific cases of V4641 Sgr and IL Lupi favor $\lambda_{ce} \sim 0.2$, which we therefore adopt as a general efficiency for the evolution of other transient sources. This then makes it possible to make quite specific predictions for the prior evolution of many of the other SXTs.

### 5.2 Expected regularities

From the above theory, certain regularities follow for the system behavior as a function of companion mass. First, the binding energy relation for spiral-in (Eq. (20)) shows that very nearly $a_f \propto M_d$, with not much variation due to other aspects of the systems (see previous section). Furthermore, the relation
between Roche lobe radius and donor mass when $M_d \ll M_{BH}$ implies that $R_L/a_f \propto M_d^{1/3}$ (e.g., Eggleton[91]). As a result, the Roche lobe radius of the donor just after spiral-in will scale with donor mass as $R_L \propto M_d^{4/3}$. On the other hand, the donor radius itself depends on its mass only as $R_d \propto M_d^{8/3}$. Therefore, a low-mass donor overfills its Roche lobe immediately after spiral-in. In the donor mass range we consider ($M_d \gtrsim 0.7 M_\odot$) it does not overfill its Roche lobe by much, so we assume that the system adjusts itself quickly by transfer of a small amount of mass to the He star, which widens the orbit until the donor fills its Roche lobe exactly. Above this minimum mass, there will be a range of donor masses that are close enough to filling their Roche lobes after spiral-in that they will be tidally locked and will come into contact via angular momentum loss (AML). Above this, there will be a range of mixed evolution, where both AML and nuclear evolution (Nu) play a role. Finally, for the most massive donors, $M_d > 2.5 M_\odot$, the post-spiral-in orbits will be too wide for AML to shrink them much, so mass transfer will be initiated only via nuclear expansion of the donor. Of course, the ranges of Case C radii of stars and variations of primary masses will ensure that the boundaries between these regions are not sharp: near the boundaries the fate of the system depends on its precise initial parameters.

5.3 Comments on Case C mass transfer

Brown et al.[80] and Wellstein & Langer [81] motivated the need for Case C mass transfer, mass transfer following core He burning, in order to “trick” the companion in a binary into evolving through the important part of its lifetime as a single star, and then only removing its H envelope when that evolution was finished. The point of the authors mentioned was that “naked” He cores, evolved in binaries in either Case A or Case B mass transfer, blew away, ending up in Fe cores that were too low in mass to evolve into high-mass black holes. We go through this argument in detail in the Appendix.

An extensive comparison of “naked” and “clothed” He star evolution was carried out by Brown, et al.[82]. The important question of mass loss during the Wolf Rayet stage is addressed in this section. Earlier calculations used mass losses deduced from Wolf-Rayet winds, driven chiefly by free-free scattering. These depended on the square of the matter density, and because of clumpiness were too large. As discussed in Brown et al. Moffat & Robert[92] arrive at a wind loss rate 2.1 times smaller than that used by Woosley et al.[93], obtained from the polarization of the Thomson scattering which is linear in the density. This value of wind loss is roughly consistent with that measured from the period change in V404 Cyg.

The conclusion of Brown et al.[82] was that even with the reduced wind losses,
enough of the naked He star would blow away that the Fe cores following Case A or Case B mass transfer were insufficient in mass to evolve into high-mass black holes.

Nelemans & van den Heuvel[94] use mass loss rates by Nugis & Lamers[95] which are substantially lower than those used by Wellstein & Langer[81] in the evolution of the CO cores evolved by Heger and described in Brown et al.[82]. They obtain much more massive final He star masses.

However, the final Fe core mass is not a linear function of the final He star masses, as discussed by Fryer, et al.[97]. In Table 4 we give results from Fryer et al. for a ZAMS 60$M_\odot$ star. The different mass loss rates are scaled to those of Braun[96]. The Braun rate is essentially that of Woosley et al.[93]. Fryer et al. discuss the nonmonotonic nature of the Fe core masses as function of He core in terms of the way in which C shell burning takes place. Remnant masses include fallback following supernova explosions of energies $\sim 10^{51}$ ergs. In our case of hypernova explosions of much higher energies, provided by magnetohydrodynamical effects, we expect little fallback. However, the results of Fryer et al. do support the suggestion of Nelemans & van den Heuvel[94] that if mass loss rates are decreased sufficiently, high mass black holes can be formed in supernova explosions following Case A or B mass transfer, because of fallback. We believe that Case C mass transfer is necessary in the evolution of high mass black holes in the transient sources, however. Indeed, from the dynamics of the situation resulting from the low mass companion, De Kool et al.[98] evolved A0620 in Case C mass transfer. Regularities that result from Case C mass transfer seem to be realized in the transient sources with main sequence companions[83]. In hypernova explosions the magnetohydrodynamics effects should be sufficient to expel matter that does not immediately fall into the black hole, so that fallback will be rather different from that in supernova explosions as outlined by Lee et al.[14].

It is clear that with low metallicity winds can be near the lower end of those in Table 4. We believe that it is no coincidence that there are two continuously

| Mass Loss Rate* | Final $M_{He}$ | Final Fe core | Remnant mass |
|-----------------|---------------|---------------|--------------|
| 1/1             | 3.132         | 1.352         | 1.35         |
| 1/2             | 4.389         | 1.305         | 1.17         |
| 1/3             | 6.108         | 1.606         | 2.11         |
| 1/4             | 7.550         | 1.749         | 5.20         |
| 1/6             | 10.750        | 1.497         | 10.7         |
shining black hole binaries LMC X-1 and LMC X-3 in the LMC which has low metallicity, whereas only Cygnus X-1 is known in the Galaxy.

6 RECONSTRUCTING THE PRE-EXPLOSION ORBITS

6.1 Evolution of SXTs with evolved companions

Nova Sco 94 (GRO J1655−40):

The most extensive evolutionary studies have been made for Nova Scorpii. Starting from the work of Regős et al.[51] who make the case that the companion is in late main sequence evolution, Beer & Podsiałowski[73] carry out extensive numerical calculations of the evolution, starting with a pre-explosion mass of \(2.5M_\odot\) and separation of \(\sim 6R_\odot\). More schematically we arrived at a pre-explosion mass of \(1.91M_\odot\) and separation of \(5.33R_\odot\) [14]. We consequently have an 0.4 day pre-explosion period. With \(\sim 6M_\odot\) mass loss in the explosion[41], nearly half the system mass, the binary period increases to 1.5 day, well beyond the period gap. This is also the period required if the common-envelope efficiency in this binary was again 0.2 [14]. This explains why Nova Sco is the only system with a black-hole mass in the lower end of the range: its evolution really places it among the narrow-orbit systems. Generally, the mass loss during explosion is mild, and does not change which category a system belongs to. But in those exceptional cases where the mass loss comes close to half the total mass, the orbit widens very much and converts an AML system to a nuclear-evolution system. After explosion the binary evolves to its present period by nearly conservative mass transfer. Our estimate is that \(0.41M_\odot\) is transferred from the donor to the black hole. Brown et al.[99] first made the case that Nova Scorpii was the relic of a GRB.

IL Lupi:

IL Lupi may be one of the most interesting binary after Nova Scorpii; it is quite similar to the latter. Its present period is just at middle of the period “gap”, i.e., binaries with the same companion mass but shorter periods will lose angular momentum by gravitational waves and magnetic braking, shortening their periods as they lose angular momentum, whereas those with longer periods will move outwards, widening their orbit, as they evolve transferring mass to the black hole. Orosz et al.[70] find the companion to be a late (estimated \(3.93 \times 10^8\) yrs post ZAMS) main sequence A-star. Recently over-abundances of Mg in the companion star of IL Lupi have been observed [71]. In analogy with the case of the overabundances in Nova Scorpii[13,14,34], this indicates that there was an explosion at the time of black hole formation in this sys-
tem, in which some of the material ejected from the core of the helium-star progenitor to the black hole ended up on the companion. Based on these observations and our given efficiency \(\lambda_{cc} = 0.2\), one can start with \(11M_\odot\) He star and \(1.7M_\odot\) companion as a possible progenitor of IL Lupi. From the lower boundary of the curve with \(\lambda_{cc} = 0.2\) in Fig. 6 of Lee et al.[14] the period would be 0.5 days. By losing \(4.2M_\odot\) during the explosion, the binary orbit would be widened to 1.12 day. The period had to be shortened to 0.8 day by magnetic braking and gravitation wave radiation before the mass transfer started. Conservative transfer of \(0.23M_\odot\) from the companion to the black hole would bring the period from 0.8 day to the present 1.1164 day.

**V4641 Sgr:**

As we discussed in section 5, this system is our calibrator for the spiral-in efficiency \(\lambda_{cc} = 0.2\) (Lee et al.[14]). The present mass ratio of V4641 Sgr is close to 1. Since the initial mass ratio could not have been significantly greater than 1 (since that would result in unstable mass transfer), and furthermore the orbital period changes very little with mass transfer for nearly equal masses, we assume that its present state is very close to the one immediately following spiral-in [14].

**GRS 1915+105:**

Recently Greiner et al.[78] have determined the period and black hole mass of GRS 1915+105 to be 33.5 day and \(14\pm4M_\odot\). Interestingly, we can evolve a system with properties very close to this by simply starting from V4641 Sgr and following its future evolution with conservative mass transfer \((P_{\text{orb}} \propto \mu^3\), where \(\mu\) is the reduced mass); allowing for \(4.6M_\odot\) to be transferred from the donor to the black hole, we have

\[
P_{1915} = \left( \frac{9.61 \times 6.53}{14.21 \times 1.93} \right)^3 P_{4641} = 33.7 \text{ day.}
\]

This would give a companion mass of 1.93\(M_\odot\), as compared with the Greiner et al.[78] mass of \(M_d = 1.2 \pm 0.2M_\odot\). However, the mass transfer cannot be completely conservative because of loss by jets, etc., as evidenced by the microquasar character of this object. Furthermore the above \(M_d\) is viewed as a lower limit by Greiner et al. because the donor is being cooled by rapid mass loss, but its mass is estimated by comparison with non-interacting stars. We thus believe our evolution to be reasonable. We position the pre-explosion period and black hole mass of GRS 1915+105 at the same point as V4641 Sgr. Since mass transfer and widening of the orbit always occur together, the effect of this post-explosion evolution is to introduce a weak secondary correlation between orbital period and companion mass in the long-period regime, where such a correlation is not expected to arise from the pre-explosion evolution.
**V404 Cyg:**

The black hole in V404 Cyg appears to be somewhat more massive than in IL Lupi, so we begin with a similar mass companion, but a $10M_\odot$ black hole, which would have a period of 0.63 day. Again, we neglect mass loss in the explosion, although a small correction for this might be made later. Conservative transfer of $1M_\odot$ from the donor to the black hole then brings the period to

$$0.63 \text{ day} \left( \frac{1.7M_\odot \times 10M_\odot}{0.7M_\odot \times 11M_\odot} \right)^3 = 6.7 \text{ day} \quad (24)$$

close to the present 6.47 day period. Here we take $11M_\odot$ and $0.7M_\odot$ as current masses in V404 Cyg [71]. The black hole in V404 Cyg seems to be somewhat more massive than the others in the transient sources, with the exception of that in GRS 1915+105. In both cases we achieve the relatively high black hole masses and periods by substantial accretion onto the black hole.

**GRO J1550–564:**

The high mass black hole in J1550–564, $10.56M_\odot$ [72], is slightly less massive than the assumed black hole mass of V404 Cyg, and the companion is more massive than V404 Cyg with short period, 1.552 days. So, we start from the same initial conditions just derived for V404 Cyg (Fig. 7), and end up with the present system via simple conservative mass transfer.

**Cygnus X-1:**

Cyg X-1 is usually not considered to have come from the same evolutionary path as the SXTs, since it is a persistent X-ray source with a much more massive donor. But with the discovery of objects with relatively massive donors in the SXT category, such as V4641 Sgr, it is worth considering the implications of our model for it. Cyg X-1 has been shown to have an appreciable system velocity[100] although it may be only $1/3$ the 50 km s$^{-1}$ given there, depending on the O-star association (L. Kaper, private communication). The evolution of Cyg X-1 may have been similar to that of the transient sources, the difference being in the copious mass loss from the companion O9I star, causing the black hole to accrete and emit X rays continuously. If we scale to Nova Scorpii to obtain the initial binary separation, we find

$$a_f = \frac{17.8M_\odot}{1.91M_\odot} \times 5.33 \ R_\odot = 50 \ R_\odot \quad (25)$$

somewhat larger than the present binary separation of $40R_\odot$. (We would obtain $38R_\odot$ if we scaled from the Beer & Podsiałowski [73] companion mass
Fig. 7. Reconstructed pre-explosion orbital period vs. black hole masses of SXTs with evolved companions. The reconstructed pre-explosion orbital periods and black hole masses are marked by filled circles, and the current locations of binaries with evolved companions are marked by open circles.

of 2.5\( M_\odot \) for Nova Scorpii.) Given uncertainties in the mass measurements, we believe it possible for Cyg X-1 to be accommodated in this scheme. Some sort of common envelope envelope evolution seems to be necessary to narrow the orbit in the evolution involving the necessarily very massive progenitor stars[82].

6.2 Discussion of the hypercritical accretion onto black holes

If we assume the time scale for the transfer, \( \sim 4 \times 10^5 \) yr for a 6.5\( M_\odot \) giant[84,101], we would need a transfer rate of \( 1.2 \times 10^{-5} \) yr\(^{-1} \), roughly 240 times Eddington limit for an 10\( M_\odot \) black hole, where the Eddington limit is \( \dot{M}_{\text{Edd}} = 8\pi G M / \kappa_{\text{es}} c \sim 5 \times 10^{-8} M_\odot \) yr\(^{-1} \) and \( \kappa_{\text{es}} \) is opacity.

It seems that the rate is extremely high. We would make the point, however, that hypercritical inflow, in which the photons are carried in with the adiabatic inflow, is much easier in black holes where the matter can easily go over the event horizon, than onto neutron stars, where accretion is onto the surface. The trapping radius for hypercritical accretion was derived in Eq. (2.10) of
Brown[102]

\[ r_{tr} \simeq 0.6R_s\dot{m} \]  

(26)

where \( R_s \) is the Schwarzschild radius and \( \dot{m} = \dot{M}/\dot{M}_{Edd} \). Although this looks quite small, for accretion onto the neutron star \( r_{tr} \) must be taken to be much larger. That is, as described in Brown et al.[34] in some detail, the accreting matter is initially of too low temperature to cool by neutrinos, and it simply accumulates on the neutron star, forming an accretion shock. The accretion shock has to pile up a lot of \( \gamma = 4/3 \) matter so that the central region gets hot enough to emit neutrino pairs. Now the incoming matter can only be trapped outside of the accretion shock so that \( r_{tr} \gtrsim r_{sh} \), where[102]

\[ r_{sh} \simeq 2.6 \times 10^8 \text{ cm } (\dot{M}/M_{\odot} \text{ yr}^{-1})^{-0.37} \]  

(27)

It is this shock radius, which is relatively large compared with the dimensions of the neutron star that make the minimum rate for hypercritical accretion as large as

\[ \dot{m} \simeq 10^4 \quad \text{for neutron star.} \]  

(28)

In the case of the black hole accretor the matter can flow adiabatically across the event horizon, once its angular momentum has been brought down as described above by the accretion disk.

Hypercritical accretion in an accretion disk is somewhat different from the spherical situation and we may have to be more careful than in Brown et al.[34] where the accretion rate of \( \sim 1 M_{\odot} \text{ yr}^{-1} \) meant \( \dot{M} \sim 10^8 \dot{M}_{Edd} \) and the photons were clearly trapped and carried in with the adiabatic inflow. Here we consider 100—1000 \( \dot{M}_{Edd} \). With an accretion disk present, the photons will be trapped when the time to carry them into the center of the accretion disk is less than the time for the photons to diffuse out nearly spherically, since the disk will be both geometrically and optically thick, with nearly equal \( H \) and \( R \).

In Brown et al.[34] we found that the inward radial velocity in the disk was

\[ |v_r| = (3/7)\alpha v_K \]  

(29)

where \( v_K \) was the Keplerian velocity \( \sqrt{GM/r} \), a factor of \( \sqrt{2} \) less than the free fall velocity assumed for the photons. The viscous disk time is, thus,

\[ \tau_{\text{visc}} = \frac{r}{|v_r|} \].  

(30)
For the electron diffusion the optical depth is

\[ \tau_{es} = \int_{r}^{\infty} \rho \kappa_{es} dr = \frac{2R \dot{m}}{\sqrt{r} r_s} \tag{31} \]

where \( r_s = \frac{2GM}{c^2} \) and \( \dot{M}_{\text{edd}} = 4\pi cR/\kappa_{es} \). This gives, in random walk, a dynamical time for diffusion of

\[ \tau_{\text{diff}} \simeq \frac{h}{c} \frac{h}{\lambda_{es}} = \frac{h^2}{c} \kappa_{es} \rho. \tag{32} \]

We obtain the trapping radius by setting

\[ \tau_{\text{visc}} = \tau_{\text{diff}} \tag{33} \]

where we have used the random walk diffusion time, we find the trapping radius

\[ r_{tr} = 0.5 \dot{m} R, \tag{34} \]

in agreement with Begelman & Meier[103]. This should be compared with \( r_{tr} = 2R \dot{m} \) for the spherical situation[104], which was lowered to \( r_{tr} = 0.6R_s \dot{m} \) in Brown[102]. The decrease came from replacing random walk by a solution of the diffusion equation.

We[17,102] pointed out that in common envelope evolution a neutron star can accrete \( \sim 1M_\odot \text{yr}^{-1} \), because in hypercritical accretion the photons are swept in by the adiabatic inflow. However, for hypercritical accretion a lower limit of \( \dot{M} = 10^4 \dot{M}_{\text{edd}} \) is needed. This results from the pressure of the radiation, as does the usual limit of \( \dot{M}_{\text{edd}} \). However, if there is no surface onto which the material accretes and radiates away its binding energy but just the event horizon as on a black hole, this lower limit need not be applicable. The matter can simply be swept over the event horizon in the adiabatic inflow.

The matter must first be “processed”, i.e., have most of its angular momentum removed, before it can go into the black hole. This “processing” should be carried out by the accretion disk. One might think that the luminosity from the disk should not exceed Eddington; otherwise the pressure might remove the outer parts of the disk. In Grimm et al.[105] the average luminosity for 1915+105 is that of a hydrogen accretion rate of \( \sim 3 \times 10^{-8} M_\odot \text{yr}^{-1} \) onto a neutron star, whereas that for V4641 Sgr is \( \sim 100 \) times less. In both cases there are jets, indicating super Eddington accretion at times.
We interpret the jets to result from the accretion onto the accretion disk being so great at times that the accretion disk cannot accept it all. That 1915+105 is much more active than V4641 Sgr is quantified by the $\sim 100$ times greater average luminosity. However, at high accretion rates we would expect the disk to be optically thick and that the photons would be carried by the matter over the event horizon so there may be some limit to the disk luminosity, with increasing accretion.

As Greiner et al. wrote [78], the black hole in 1915 may be spinning rapidly. We show that generally $\sim 10^{53}$ ergs is available in the spin energy of the black hole, but that only $\sim 10^{52}$ ergs is used in the GRB and hypernova explosion, mostly in the latter.

We believe that in the short time $\sim 5-10$ sec central engines enough energy will be transferred from the rapidly rotating black hole to the accretion disk to power the hypernova explosion. The latter dismantles the accretion disk so that no further rotational energy can be transferred, leaning the black hole in rapid rotation.

### 6.3 Problems with the close (AML) systems

Reconstruction of the AML binaries is more complicated, because they have lost angular momentum through magnetic braking and gravitational waves, so that their present positions as plotted in Fig. 5 are not those at pre-explosion time. As with the evolved companions, matter will have been accreted onto the black hole, so the black hole masses will be somewhat greater than just following the explosion. As noted earlier, the binaries with less massive companions with separation $a_f$ at the end of common envelope evolution overfill their Roche Lobes. The outer part of the companion, down to the Roche Lobe $R_L$ is transferred onto the He star. This mass transfer widens the orbit to $R_L$, possibly overshooting. Unless much mass is lost in the explosion when the black hole is formed, the Roche lobe radius is unchanged by the formation of the black hole, and corresponds to line III in Fig. 6.

Brown et al. [83] explored the evolution of ZAMS $1.25M_\odot$ stars under magnetic braking, gravitational waves and mass transfer to the black hole. We adapt the same methods to make a more detailed study of the AML. First of all we construct (Fig. 6) the lower limit on the companion mass for evolution in a Hubble time, giving the dot-dashed line there. All binaries with companions in main sequence at the beginning of mass transfer must lie between the dot-dashed line and line III in that figure. The fact that the AMLs tend to lie below the dot-dashed line implies both mass loss from the companion and accretion onto the black hole. Therefore, all these systems have shrunk their orbits
Fig. 8. Present orbital period vs. black hole masses of SXTs. The deviations from the theoretical curves are substantial due to the post-explosion evolution of the binaries. The arrows on the AML systems point to an approximate post-explosion location if the donor mass was initially $1.5 M_\odot$, and $0.7 M_\odot$ has now been transferred to the black hole. The solid lines indicate the possible ranges of black hole masses with polytropic index $n = 3$ (radiative), for given pre-explosion spin periods which are assumed the same as the pre-explosion orbital period. Here we used $R_{He} = 0.22 (M_{He}/M_\odot)^{0.6} R_\odot$. For comparison, the results with a “scaled” He core ($9.15 M_\odot$) of Woosley’s $25 M_\odot$ star at the beginning of $^{12}$C burning with $T_c = 5 \times 10^8$ K, appropriate for Case C mass transfer, are plotted as a dashed line[106]. In this plot, we scaled the radius of Woosley’s core, $R_{\text{Woosley}} \sim 3 \times 10^{10}$ cm, by a factor 2. As can be seen, the AML systems can plausible originate from systems within the curves, and thus are consistent with our theory. However, since they could have originated anywhere between the open square and their current location, they do not strongly test the theory.

and increased their black-hole mass since the formation of the black hole, by amounts that cannot be determined well. In Fig. 8, however, we show where the four shortest period AMLs would have come from, had they lost $0.7 M_\odot$ from an initial $1.5 M_\odot$. From our earlier discussion about the $a_f$ following common envelope evolution we saw that binaries with companions which stayed in main sequence were favored to come from companion masses less than $2 M_\odot$, and from Fig. 6, we see that they would chiefly have companion ZAMS mass greater than $\sim 1 M_\odot$, so that most of them would initially have periods of 0.4–0.7 days (which follows from the separations obtained from our Eq. (22)). In trying to
Fig. 9. Evolution of a binary with $7M_\odot$ black hole and $2M_\odot$ companion for the initial period of 0.54 day. The solid line marks the evolution in case the companion star adjust itself as it loses mass; the dashed line traces the evolution in case the mass loss does not affect the internal time scale of the companion star, so that it follows the same time evolution as an undisturbed $2M_\odot$ star.

To understand the detailed evolution of the AML we begin from a binary with a $2M_\odot$ companion which just fills its Roche Lobe following common envelope evolution. We then follow its evolution under the two different assumptions made in Brown et al.[83]: (1) That its time of evolution is always given by its initial $2M_\odot$ mass, i.e., ignoring effects of mass loss on the internal evolution time (dashed lines in Fig. 9 and right dashed line in Fig. 10) (2) That the evolution of the star proceeds according to its adjusted mass (solid lines in Fig. 9 and left dashed line in Fig. 10). Since mass loss drives the companion out of thermal equilibrium, these two extremes bracket the outcome of a full stellar model calculation.

In summary, the AML systems have had the information on their post-explosion parameters partly erased by subsequent evolution, in a manner that we cannot undo. Therefore, they can only provide a crude consistency check on the mass-period relation for black holes in SXTs, rather than provide precise constraints.
Fig. 10. Evolutionary tracks of a binary with $9M_\odot$ black hole and $2M_\odot$ companion for the initial period of 0.54 day. The two evolution possibilities are as in Fig. 9. Left (right) dashed line corresponds to the solid (dashed) lines in Fig. 9.

7 ANGULAR MOMENTUM AND ITS CONSEQUENCES FOR THE MASS AND SPIN OF THE BLACK HOLE

It is, in general, a difficult and unsolved problem to calculate the angular momentum of a stellar core at any given time. Even if we make the usual assumption that the rotation is initially solid-body, and not very far away from the maximal stable rotation frequency, the viscous coupling between the various layers of the star as it evolves is poorly known, and thus it is hard to be very quantitative. The general trend, however, is that the core will shrink and the envelope expand. In absence of viscous coupling, every mass element retains its angular momentum, and hence the core spins up as the envelope spins down, setting up a strong gradient in rotation frequency between the core and the envelope. Viscosity will then act to reduce this gradient, transporting angular momentum from the core to the envelope, but the efficiency of this process is very uncertain[45,46].

As we noted above (section 1), in our scenario, a number of effects will increase the angular momentum of the core relative to a similar core of a single star: (1) during spiral-in, the matter somewhat inside the orbit of the secondary is spun up by tidal torques[10]; (2) the removal of the envelope halts the viscous
slowdown of the core by friction with the envelope; (3) during the post-spiral-
in evolution, tidal coupling will tend to spin the helium star up even closer to
the orbital period than was achieved by the first effect. This will not be a very
strong effect because the duration of this phase is short, but it will affect
the outer parts of the helium star somewhat, and this is the most important part
(see below).

The net result of all these effects will be that the helium star will spin fairly
rapidly, especially its envelope. The core is not so crucial to our argument
about the fraction of the star that can fall into the black hole, since the few
solar masses in it will not be centrifugally supported even in quite short orbits.
For the purpose of a definite calculation, we therefore make the following
assumptions: (1) the helium star co-rotates with the orbit before explosion
and is in solid-body rotation; (2) the mass distribution of the helium star with
radius is given by a fully radiative zero-age helium main sequence star. This
latter approximation is, of course, not extremely good. However, what counts
is the angular momentum as a function of mass, so the fact that the mass
distribution has changed from helium ZAMS to explosion would be entirely
inconsequential if no redistribution of angular momentum had taken place in
the interim. As we saw above, any redistribution of angular momentum would
take the form of angular momentum transport toward the outer layers. This
means that relative to our ideal calculations below, a better calculation would
find more angular momentum in the outer layers, and therefore somewhat
smaller black hole masses than the ones we calculate.

We now investigate how much mass will be prevented from falling into the
black hole by the angular momentum of the He star, under the above assump-
tions of solid-body rotation with a period equal to that of the binary. If we
assume that angular momentum is conserved during the collapse, we can get
the cylindrical radius $R_c$ within which matter is not centrifugally prevented
from falling into the black hole:

$$R_c^2 \Omega = \tilde{l}(\hat{a}) \frac{G M_c}{c}$$

where $\tilde{l}(\hat{a})$ is the dimensionless specific angular momentum of the marginally
bound orbit for a given Kerr parameter $\hat{a}$, and $M_c$ is the total mass inside the
cylinder of radius $R_c$. The Kerr parameter becomes

$$\hat{a} = \frac{I_c \Omega}{G M_c^2 / c} = k^2 \tilde{l}(\hat{a})$$

where $I_c$ is the total moment of inertia inside the cylinder of radius $R_c$, $I_c = k^2 M_c R_c^2$. $M_c$ gives an estimate of the final black hole mass. Combining these
relations with a profile of angular momentum and mass versus radius using
Fig. 11. The Kerr parameter of the black hole resulting from the collapse of a helium star synchronous with the orbit, as a function of orbital period. The conditions are the same as in Fig. 8, as is the meaning of the three curves. Woosley’s helium core is of mass $9.15 M_{\odot}$ from a ZAMS $25 M_{\odot}$ star [106]. Note that the result depends very little on the mass of the helium star, or on whether we use a simple polytrope or a more sophisticated model. The plot illustrates that rapidly rotating black holes needed for powering GRBs originate only from originally short-period SXTs.

the assumptions listed above, we can calculate the expected black hole mass and Kerr parameter as a function of SXT period before explosion.

In Fig. 8 we show the predicted relation between orbital period and black-hole mass for different helium star masses in our model. We compare these with the present properties of all SXTs for which the required parameters are known. The properties are consistent with the theoretical relations, but do not confirm it very strongly due to the evolutionary changes discussed in section 6. Specifically, the AML systems lie above and to the left of the curves, because their orbits shrunk and their black holes accreted mass since the formation of the black hole. However, as we saw in section 6.3, plausible amounts of conservative mass transfer since the explosion would place the systems among the theoretical post-explosion curves (indicated by the open squares and arrows).

To test the theory more strongly, we show in Fig. 7 only those systems for which the pre-explosion properties could be reconstructed (section 6). We com-
pare the observed points with ideal polytropic helium stars of $7\, M_\odot$ and $11\, M_\odot$, and with a full model calculation obtained from Woosley[106]. By coincidence, the curves converge near the region of the shortest-period observed systems, so that the uncertainty in helium star mass is not of great importance to the outcome. A helium star mass in the lower end of the range $(7–9\, M_\odot)$ may be somewhat preferred for these systems. For periods above 1 day, angular-momentum support is not important, and the mass of the final black hole will be very close to that of the helium star, and thus varies somewhat from system to system. As we can see, the reconstructed pre-explosion properties lie much closer to the theoretical predictions.

As a corollary, we find that systems with very large velocities, like Nova Sco, will be rare: at the shortest pre-explosion orbits, where much mass is ejected, the companion mass tends to be small. Then the center of mass of the binary is close to that of the helium star, which strongly limits the systemic velocity induced by the mass loss. On the other hand, for the widest systems, where the companion tends to be massive enough to allow a significant systemic velocity induced by mass loss, the mass loss itself becomes too small to induce much of a systemic velocity.

An important result for our proposed relation between SXTs and hypernovae and GRBs is shown in Fig. 11. This figure shows the expected Kerr parameter of the black hole formed in our model. We see that for the short-period systems, this Kerr parameter is very large, 0.7–0.9. This means that we are justified in adding only the mass that immediately falls in to the black hole, because as soon as the rapidly rotating black hole is formed, it will drive a very large energy flux in the manner described by Brown et al.[9]. This both causes a GRB and expels the leftover stellar envelope. The systems with longer orbital periods do not give rise to black holes with large Kerr parameters, and thus are presumably not the sites of GRBs.

8 OTHER OBSERVATIONAL ISSUES

8.1 Deposition of metals in the companions of SXTs

We now model the deposition of the expelled matter onto the companion star in a highly schematic way.

With a preexplosion separation in Nova Scorpii of $5.33\, R_\odot$, and ejection of $6\, M_\odot$ of matter, with an F-star radius of $1.68\, R_\odot$, in a spherically symmetrical explosion the fraction of solid angle subtended by the companion at the time of explosion was
\[
\Omega = \frac{\pi(1.68R_\odot)^2}{4\pi(5.33R_\odot)^2} = 0.025
\]

which with \(6M_\odot\) matter deposited would give

\[
\Delta M_{dep} \sim 0.15M_\odot
\]

Because of the smaller separation of \(5M_\odot\) and the greater companion mass at the time of explosion, this is \(\sim 5\) times greater than that estimated by Brown et al.\[9\]. Less than a factor of 2 would now be needed, following their estimates, for sufficient metal deposition off the companion. As motivated there, such a factor should come from the asphericity of the explosion.

We shall estimate the ratios of matter deposited on IL Lupi and V4641 Sgr as compared with Nova Scorpii, assuming the factor for asymmetry enhancement to be the same in these cases. Since the thermohaline time scale is short compared with the lifetime of the companion star, we shall divide the amount of deposited matter by the volume of the star in order to estimate the anomaly in metal deposition.

The ratio of anomaly in IL Lupi to Nova Scorpii is thus estimated to be

\[
R_{\text{IL Lupi}/\text{Nova Sco}} = \left(\frac{4.2M_\odot}{6M_\odot}\right)\left(\frac{1.91M_\odot}{1.7M_\odot}\right)\left(\frac{1.53R_\odot}{1.68R_\odot}\right)^2\left(\frac{5.33R_\odot}{6.19R_\odot}\right)^2
\]

\[
\simeq 0.48
\]

where the anomaly is assumed to be proportional to

\[
\text{anomaly} \propto \left(\frac{\Delta M_{\text{explosion}}}{M_d}\right)\left(\frac{R_d}{a}\right)^2.
\]

The ratio of anomaly in V4641 Sgr to Nova Scorpii is estimated as

\[
R_{\text{V4641 Sgr}/\text{Nova Sco}} = \left(\frac{1.6M_\odot}{6M_\odot}\right)\left(\frac{1.91M_\odot}{6.53M_\odot}\right)\left(\frac{4.49R_\odot}{1.68R_\odot}\right)^2\left(\frac{5.33R_\odot}{21.3R_\odot}\right)^2
\]

\[
\simeq 0.04
\]

Our estimates thus show that the anomaly should be detectable in IL Lupi, the oxygen abundance being \(\sim 5\) times normal, but only marginally so in V4641 Sgr, \(\sim 40\%\).
8.2 Ultraluminous X-ray source

An interesting byproduct of our black hole evolution in binaries is their possible and promising use to explain ultraluminous X-ray sources (ULXs) in external galaxies. These have been extensively observed by the Chandra X-ray observatory\[107\]. We follow here the discussion of King et al.\[108\]. “A key to understand their nature may be that they appear to occur preferentially, although not exclusively, in regions of star formation.” The initial explanation of these ULXs was that they came from $\sim 100M_\odot$ black holes; the disadvantage of this explanation was that such objects are not seen in our Galaxy and no one knows how to evolve them.

King et al.\[108\] suggest that the ULX’s originate from the same sort of binaries we have evolved above, the black hole SXTs. The high luminosity arises when they cross the Herzsprung gap, as GRS 1915+105 does at this time. The time scale for this is the thermal time scale

$$\tau_{\text{th}} = \frac{3 \times 10^7}{(M/M_\odot)^2} \text{yr}, \quad (42)$$

i.e. $7 \times 10^5$ yrs for the $6.5M_\odot$ companion in V4641 Sgr which becomes something like GRS 1915+105 after crossing much of the Herzsprung gap according to our scenario\[14\].

The initial problem with the SXTs as explanation was that luminosities of $\sim 10$ times Eddington (Here we use Eddington to mean that limit for a neutron star with $\sim 20\%$ efficiency, midrange for the efficiencies of 6 to 42% for nonrotating and rapidly rotating black hole, respectively.) were required. The initially proposed $\sim 100M_\odot$ black holes were brought down to $\sim 10M_\odot$ ones by the realization that accretion could proceed at $\dot{M} ~ 10\dot{M}_{\text{Edd}}$ because of instabilities in the accretion disk which make the disk “porous” \[109–111\]. Since the Eddington limit goes linearly with the Schwarzschild radius $R_s$ and $R_s$ linearly with the mass, $M_{\text{BH}}$, a $10M_\odot$ star accreting at $10\dot{E}_{\text{Edd}}$ would look like a $100M_\odot$ black hole accreting at Eddington.

Taking the Schaller et al.\[84\] models we have estimated the mass loss in going from V4641 Sgr to GRS 1915+105 in assumed conservative mass transfer as in Lee et al.\[14\]. The assumed $5M_\odot$ companion goes out of equilibrium in the mass transfer, which should be calculated in an evolutionary code. We hope to get an estimate by simply assuming the mass going over the Roche Lobe to be transferred to the black hole, and readjusting the Roche Lobe according to the changed mass ratio. We find the mass transfer to be at a rate of $\dot{M} \sim 10^{-5}M_\odot$ per year, about 200 times the $\sim 5 \times 10^{-8}M_\odot$ yr$^{-1}$ assumed for Eddington.
Belloni et al.[112] find the emission from 1915+105 to be rather complicated in detail. They interpret the \( \sim 1 \) sec appearance and disappearance of emission from an optically thick inner accretion disk as coming from fluctuations in the inner radius of the disk from \( \sim 20 - 80 \) km, the lower radius during bursts and the upper during quiescence.

We cannot make a detailed model, but support that the accretion is hyper-critical most of the time, as would be necessary to build up the black hole mass of 1915+105 from that V4641 Sgr over \( 4 - 5 \times 10^5 \) years.

King et al.[108] emphasize the importance of beaming in building up the high apparent luminosities which are observed. Our evolution of the SXTs as fossil remnants of GRBs involve initially large effects of beaming in order to produce the GRBs, and, as noted earlier, the binaries should be left in considerable rotation. In fact, jets are seen in 1915+105, as well as evidence of rotation.

Our evolution[14] where the separation \( a_f \) between donor (companion) and black hole following common envelope evolution is proportional to \( m_d \) (Eq. (5) of Lee et al.[14]), is helpful in understanding which binaries are good progenitors of ULXs. The “silent partners” of Brown et al.[80] become SXTs only after they evolve, because only then can their outer matter cross the Roche Lobe. Their companions are the more “massive” ones and consequently pour more matter into the black hole.

We can obtain a limit on the companion mass necessary for a ULX from the evolution of Nova Scorpii by Beer & Podsiadlowski[73]. Their F-star companion was initially of \( 2.5M_\odot \), and began evolving in main sequence, with resulting loss of mass to the black hole. The calculation of Beer & Podsiadlowski is a fully evolutionary one. None the less we find that we can reproduce their results by conservative mass transfer, in which the period of the binary goes as the cube of the reduced mass. This is not surprising, in that the quiescent luminosity in Nova Scorpii is \( \sim 2.5 \times 10^{32} \) ergs, a factor of \( \sim 10^6 \) less than Eddington[113]. During bursts the luminosity is higher, \( \sim 10^{48} \) ergs. What we learn from this is that rapid mass transfer – although less than Eddington – is going on silently between companion star and black hole while the observable effects are small, except during outbursts. We extend this scenario now to the Herzsprung gap, where both mass transfer and outbursts are much greater. The calculated rate of Beer & Podsiadlowski[73] for the (future) crossing of the Herzsprung Gap is \( \sim 4 \times 10^{-8} M_\odot \text{yr}^{-1} \), above \( \dot{M}_{\text{Edd}} \) for their \( 5.3M_\odot \) black hole. According to Lee et al.[14], the reason that Nova Scorpii (and IL Lupi) can begin transferring mass in main sequence is that they are kicked into relatively large orbits in the explosion which forms the black hole due to large mass losses. (Companions in both binaries show substantial metal abundances that would have come from the explosion.)
In general we found most of the binaries with initial companion mass $< 2.5 M_\odot$ stay in main sequence[14] during the active stage of SXTs. We interpret Nova Scorpii and IL Lupi as having companions in between the main sequence ones and those of the silent partners, as the companion mass increases. Thus, we estimate $m_d \sim 3 M_\odot$ as minimum mass for the “silent partner” companion.

A $3 M_\odot$ companion could be stripped down to its $0.4 M_\odot$ He core in transferring matter after expansion to the Herzsprung gap. Using the thermal time scale of $3.3 \times 10^6$ yrs this means maximum average rate of mass transfer of $0.9 \times 10^{-6} M_\odot$ yr$^{-1}$ or $\sim 20 M_{\text{Edd}}$ for a $10 M_\odot$ black hole. Thus an actual rate of $\dot{M} \sim 10 M_{\text{Edd}}$ is reasonable for such a star, and progenitor binaries with $3 M_\odot$ (and more massive) companion should give ULXs.

The highly super Eddington (hypercritical) accretion with more massive companions, as in V4641 Sgr and 1915+105 is unlikely to produce higher luminosity, because the high density infalling matter can very efficiently trap the photons and carry them adiabatically inwards with the adiabatic inflow. We propose to divide the accretion disk into the outer thin part with $\dot{m} \ll 10$ and the inner thick part with $\dot{m} \gg 10$. (We basically extend $\dot{M}_{\text{Edd}}$ to $10 \dot{M}_{\text{Edd}}$ to take into account the effects of inhomogeneities, assuming a ten fold increase.) Photons in the outer thin disk are emitted; these in the inner thin disk are trapped and carried inward. The two regions are effectively separated by (approximately) stationary matter.

Given the complexities of accretion disks, this may seem to be an oversimplified approximation. Indeed, the complication induced by inhomogeneities in the thin disk has forced an increase in $\dot{M}_{\text{Edd}}$ to an effective $10 \dot{M}_{\text{Edd}}$, where the factor 10 is an estimate from Begelman[111]. The concept of photon trapping is, however, an exact translation of neutrino trapping in supernova explosions, which is discussed in Bethe et al.[114]. There are, however, instabilities which lead to an oscillatory in the accretion, which we mention below. A major assumption is that the superposition of these on our above simple model does not change the average accretion rate.

Whereas SXTs such as 1915+105 may well produce many of the observed ULXs, this cannot be the whole story, since they are also observed in elliptical galaxies. King et al.[108] also suggest unstable accretion during a thermal time scale of a radiative star onto a neutron star such as occurs in Her X-1, the companion being more massive than the neutron star, as a source. Whereas the companions may be sufficiently massive in spiral galaxies to produce the ULXs, it is not clear that they can do so in elliptical galaxies. We have not dealt with the relevant binaries in this paper.
8.3 Population synthesis of SXTs and GRBs

The 14 SXTs in Table 2 separate nearly equally into those with unevolved main sequence and those with evolved companions. This immediately tells us that there are many unobserved black-hole binaries, since the lifetime for the evolved companions is nearly two orders of magnitude shorter than that for the main sequence ones. The former are the “silent partners” referred to in Brown et al.[80].

Nearly all observed black hole transient X-ray sources are within 10 kpc of the sun. Extrapolating to the entire Galaxy, a total of $\sim 10^4$ black-hole transients with main-sequence K companions has been suggested[80].

The lifetime of a K-star in a black hole transient X-ray source may be the $\sim 10^{10}$ yr lifetime of the K-star[115] but it is in Roche contact only $\sim 10^9$ years[83] In this case the birth rate of the observed transient sources would be

$$\lambda_K = \frac{10^4}{10^9} = 10^{-5} \text{ per galaxy yr}^{-1}. \quad (43)$$

We see no reason why low-mass companions should be preferred, so we assume that the formation rate of binaries should be independent of the ratio

$$q = \frac{M_{B,i}}{M_{A,i}}. \quad (44)$$

In other discussions of binaries, e.g., in Portegies Zwart & Yungelson[116], it has often been assumed that the distribution is uniform in $q$. This is plausible but there is no proof. Since all primary masses $M_A$ are in a narrow interval, 20 to 30 $M_\odot$, this means that $M_B$ is uniformly distributed between zero and some average $M_A$, let us say $25 M_\odot$. Then the total rate of creation of binaries of our type is

$$\lambda = \frac{25}{1.25} \lambda_K = 2 \times 10^{-4} \text{ galaxy}^{-1} \text{ yr}^{-1} \quad (45)$$

where we have used $1.25 M_\odot$ for the initial mass of the companion which is stripped down to a K-star. This is the rate of mergers of low mass black holes with neutron stars which Bethe & Brown[17] have estimated to be

$$\lambda_m \simeq 2 \times 10^{-4} \text{ galaxy}^{-1} \text{ yr}^{-1}. \quad (46)$$

These mergers have been associated speculatively with short GRBs, while formation of our binaries is supposed to lead to “long” GRBs[117]. We conclude
that the two types of GRB should be equally frequent, which is not inconsistent with observations. In absolute number both of our estimates eqs. (45) and (46) are substantially larger, possibly by a factor $\sim 100$, than the observed rate of $10^{-7}$ galaxy$^{-1}$ yr$^{-1}$; this is natural, since substantial beaming is expected in GRBs produced by the Blandford-Znajek mechanism\cite{118}. Although we feel our mechanism to be fairly general, it may be that the magnetic fields required to deliver the BZ energy within a suitable time occur in only a fraction of the He cores. Also, from our above estimates, only the binaries with preexplosion periods $P \lesssim 1$ day would be expected to deliver enough rotational energy to power a strong GRB. Thus the realistic $\lambda$ could easily come down to $\sim 10^{-5}$ galaxy$^{-1}$ yr$^{-1}$.

With our assumed distribution uniform in $q$, the silent partners with $M_d \gtrsim 3M_\odot$ fill up nearly the entire interval; i.e., $\Delta q \sim 1$. For the binaries with main sequence companions $M_d \lesssim 2.5M_\odot$, so our interval up to $\sim 25M_\odot$ would give an order of magnitude more, roughly $10^5$. However, they will be observed only for the thermal time Eq. (42) which is only a fraction

$$F = \frac{3 \times 10^7/(M/M_\odot)^2}{10^{10}/(M/M_\odot)^{2.5}} = 3 \times 10^{-3}(M/M_\odot)^{1/2},$$

(47)

of their total lifetime, which we take to be

$$F \sim 10^{-2}.$$  

(48)

This brings the birth rate of ULXs down from that of GRBs, Eq. (45), to

$$\lambda_{ULX} \sim 2 \times 10^{-6} \text{galaxy}^{-1} \text{yr}^{-1}$$  

(49)

in rough agreement with King et al.\cite{108}.

In our Galaxy we see only the incipient ULX V4641 Sgr which is beginning to cross the Herzsprung gap and GRS 1915+105 which is near the end of its crossing. The latter is not beamed towards us. Both are within 12 kpc, about 1/4 of the way to the edge of the Galaxy which we take as two dimensional. Thus, we would estimate $\sim 16 - 32$ times more in the entire Galaxy. With a beaming factor of 10, this would imply $\sim 1 - 2$ ULXs from the SXTs in our Galaxy.

We should not forget about the two black hole binaries LMC X-3 and LMC X-1 in the Large Magellanic Cloud, which, like Cyg X-1, shine continuously, with luminosities of $\sim 2 - 3 \times 10^{38}$ ergs sec$^{-1}$\cite{107} These might well be ULXs were they beamed in our direction. Because of its low metallicity, winds in the LMC may be substantially less than in the Galaxy, for which we argue
that the strong winds during He core burning reduce the final Fe core size in
giants below the mass necessary for later formation of high mass black holes.
Including the LMC black holes doubles our Galactic estimate and suggests
that there might be a correlation in the number of ULXs with metallicity.

9 DISCUSSION AND CONCLUSION

Our work here has been based on the Blandford-Znajek mechanism of ex-
tracting rotational energies of black holes spun up by accreting matter from
a helium star. We present it using the simple circuitry of “The Membrane
Paradigm”[27]. Energy delivered into the loading region up the rotational axis
of the black hole is used to power a GRB. The energy delivered into the
accretion disk powers a SN Ib explosion.

We also discussed black-hole transient sources, high-mass black holes with low-
mass companions, as possible relics for both GRBs and Type Ib supernova
explosions, since there are indications that they underwent mass loss in a
supernova explosion. In Nova Sco 1994 there is evidence from the atmosphere
of the companion star that a very powerful supernova explosion (‘hypernova’) occurred.

We have shown that there is an observed correlation between orbital period
and black-hole mass in SXTs. We have modeled this correlation as resulting
from the spin of the helium star progenitor of the black hole: if the pre-
explosion orbit has a short period, the helium star spins rapidly. This means
that some part of its outer envelope is centrifugally prevented from falling into
the black hole that forms at the core. This material is then expelled swiftly,
leading to a black hole mass less than the helium star mass. As the orbital
period is lengthened, the centrifugal support wanes, leading to a more massive
black hole. The reason for swift expulsion of material held up by a centrifugal
barrier is the fact that black holes formed in our scenario naturally have high
Kerr parameters (Fig. 11). This implies that they input very high energy
fluxes into their surrounding medium via the Blandford-Znajek mechanism,
and thus power both a GRB and the expulsion of the material that does not
immediately fall in.

However, because the correlation is induced between the orbital period before
explosion and the black-hole mass, its manifestation in the observed correlation
between BH mass and present orbital period is weakened due to post-explosion
evolution of the binaries. We therefore considered the evolution in some detail,
and for a subset of the systems were able to reconstruct the pre-explosion
orbital periods. The correlation between pre-explosion period and black hole
mass (Fig. 7) is in much better agreement with our model than the original
one between present period and black hole mass (Fig. 5). We developed a
quantitative model for the relation between period and mass, and showed
that it fits the subset of reconstructible SXT orbits.

Nova Scorpii stands out as the most extreme case of mass loss, nearly half of
the total system mass, and, therefore, a great widening in the orbit which gets
its period well beyond the gap between shrinking and expanding orbits. From
Fig. 7 we see that its black hole mass is far below the polytropic line for its
$M_{\text{He}} = 11M_{\odot}$ progenitor. We believe that in the case of this binary a short
central engine time of several seconds was able to furnish angular momentum
and energy to the disk quickly enough to stop the infall of some of the interior
matter not initially supported by centrifugal force; i.e., the angular momentum
was provided in less than a dynamical time. In other words, the Blandford-
Znajek mechanism that drives the GRB not only expelled the matter initially
supported for a viscous time by angular momentum, but actually stopped the
infall within a dynamical time.

Since we can also compute the Kerr parameters of the black holes formed via
our model, we find that the short-period systems should have formed black
holes with Kerr parameters in the range 0.7–0.9. This makes them prime
candidates for energetic hypernovae and GRBs, and thus provides further
support for our earlier study in which we posited that SXTs with black-hole
primaries are the descendants of GRBs. We can now also refine this statement:
SXTs with short orbital periods before the formation of the black hole have
given rise to a GRB in the past.

We estimate the progenitors of transient sources to be formed at a rate of 200
GEM (Galactic Events per Megayear). Since this is much greater than the
observed rate of GRBs, there must be strong beaming and possible selection
of high magnetic fields in order to explain the discrepancy.

We believe that there are strong reasons that a GRB must be associated with
a black hole, at least those of duration several seconds or more discussed here.
Firstly, neutrinos can deliver energy from a stellar collapse for at most a few
seconds, and sufficient power for at most a second or two. Our quantitative
estimates show that the rotating black hole can easily supply the energy as
it is braked, provided the ambient magnetic field is sufficiently strong. The
black hole also solves the baryon pollution problem: we need the ejecta that
give rise to the GRB to be accelerated to a Lorentz factor of 100 or more,
whereas the natural scale for any particle near a black hole is less than its
mass. Consequently, we have a distillation problem of taking all the energy
released and putting it into a small fraction of the total mass. The use of a
Poynting flux from a black hole in a magnetic field[5] does not require the
presence of much mass, and uses the rotation energy of the black hole, so it
provides naturally clean power.
Of course, nature is extremely inventive, and we do not claim that all GRBs will fit into the framework outlined here. We would not expect to see all of the highly beamed jets following from the BZ mechanism head on, the jets may encounter some remaining hydrogen envelope in some cases, jets from lower magnetic fields than we have considered here may be much weaker and delivered over longer times, etc., so we speculate that a continuum of phenomena may exist between normal supernovae and extreme hypernovae/GRBs.

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A The Case for Case C Mass Transfer

The proportionality of the separation \( a_f \) following common envelope evolution to \( a_i \), scaled with donor mass \( M_d \) makes it possible for us to understand why the 6 binaries with shortest periods in Table 2 all have K or M companions in main sequence. (Not much is known about V406 Vulpeculae and we assume MM Velorum to be in main sequence.) This would be a striking coincidence, the progenitor binaries all having black hole progenitor masses of ZAMS 20 – 30\( M_\odot \) and the companion ZAMS < 2.5\( M_\odot \), unless there were some underlying reason for this regularity. We have emphasized in the text that in Case C mass transfer the orbital separations for the above progenitor binaries in Roche Lobe contact are at \( \sim 1700R_\odot \) (\( \pm \sim 10\% \)), \( \sim 8 \) AU, for ZAMS 20\( M_\odot \) black hole progenitor and 1\( M_\odot \) companion star.

The energy to remove the envelopes of the supergiants is furnished in common envelope evolution by the drop in gravitational binding energy of the companion, as it drops from \( a_i \) to \( a_f \). This energy is thus linearly proportional to the companion mass which we label \( M_d \). The reason that the companion masses are so low is that the gravitational binding of the supergiant envelope goes inversely with its radius, as \( R^{-1} \), so that for \( R \sim 5 \) AU (about 2/3 of the orbital separation) the envelope binding is very low. We believe that to be the principle, although complications enter through the variation in ZAMS masses of the supergiants.

We were led to Case C mass transfer because in Case B mass transfer (Roche Lobe over flow) so much of the naked He core blows away that the resulting
Fe core is not massive enough to form high-mass compact objects[82].

The question is, if Case C mass transfer keeps the He star clothed during He burning so that the resulting Fe core is massive enough to collapse into a high-mass black hole, then why wouldn’t late Case B mass transfer work? In fact, we find that very late Case B with transfer in the last $\sim 6 \times 10^4$ yrs of the He core burning, probably would work.

We should confess that our Case C mass transfer (or very late Case B) does not have support in the results of present stellar evolution except for the ZAMS $20M_\odot$ star[84]. We have based it on the regularities in the SXTs. We now turn the argument around and say how present stellar evolution must be modified in order to lead to these regularities.

We show in Fig. A.1 the results of the Schaller et al.[84] stellar evolution for a ZAMS $20M_\odot$ star. It is seen that the main increase in radius comes after the start of He core burning (which begins while H shell burning is still going on). With further He core burning there is a flattening off of the radius versus burning stage and then a further increase in radius towards the end of and following He core burning. Our model requires that mass transfer take place during this last period of increase in radius, so that the orbital separation ($\sim 3/2$ of the giant radius) is well localized $a_{i,RLOF} \sim 1700R_\odot$ at the time of Roche Lobe contact, or $a_{i,t=0} \sim 1500R_\odot$ initially.

Portegies Zwart et al.[119] have pointed out that wind loss from the giant preceding common envelope evolution is important and we follow their development in identifying the “No RLOF” part of the curve in Fig. A.1. Because of the wind loss the binary widens. The Roche Lobe overflow will take place during the very rapid increase in radius of the giant in the beginning of He core burning, or in very late Case B or in Case C mass transfer. The binary has widened too much by the time the giant has reached the flat part of the $R$ vs stage curve. In fact, it cannot transfer mass during this stage, because it will have already come to Roche contact during early Case B. This is made clear by the shaded area in the solid line in Fig. A.1. Brown et al.[83] have shown that the SXTs with main sequence companions can be evolved with a $1 - 1.25M_\odot$ companion mass, so the above results may be directly applicable. For higher mass companions, this shaded area becomes smaller because the effect of the winds is smaller. This makes the intermediate Case B mass transfer possible. However, in this case, high-mass black holes may not form because the Fe core is not massive enough to form high-mass compact objects as we discussed above[82].

Now, in fact, the curve of radius vs burning stage for the next massive star, of ZAMS $25M_\odot$, by Schaller et al.[84] does not permit Roche Lobe contact during Case C at all, the winds having widened the binary too much by the
Fig. A.1. Radius of black hole progenitors ($R$) and the initial orbital separations ($a_i$) of the progenitors of X-ray transient binaries with a $1M_\odot$ companion. A) The lower dotted curves ($R$) corresponds to the radius of the black hole progenitors taken from Schaller et al.[84]. That for the $25M_\odot$ star is similar but for the $30M_\odot$ the radius does not increase following the end of He core burning. B) From the mass of the primary at the tabulated point one can calculate the semimajor axis of a binary with a $1M_\odot$ secondary in which the primary fills its Roche Lobe, and this semimajor axis is shown in the upper dot-dashed curve ($a_{i, RLOF}$). C) The solid curves ($a_{i, t=0}$) correspond to the required initial separations after corrections of the orbit widening due to the wind mass loss, $a_{i, t=0} = a_{i, RLOF} \times (M_p + M_d)/(M_{p,0} + M_d)$ where $M_p$ is the mass of the black hole progenitor at a given stage and $M_{p,0} = 20M_\odot$ is the ZAMS mass of the black hole progenitor. Primaries at the evolutionary stages marked by the shaded area cannot fill their Roche Lobe for the first time at that stage, but have reached their Roche Lobe at an earlier point in their evolution.

The lack of increase in $R$ for the more massive stars is due to the cooling effect by strong wind losses. As shown by Lee et al.[14] giant progenitors as massive as $30M_\odot$ are necessary as progenitors of some of the black holes in the SXTs, especially for the binaries with evolved companions, in order to furnish the high mass black hole masses. These authors reduce wind losses by hand, forcing the resulting curve of $R$ vs burning stage to look like that for a ZAMS $20M_\odot$ shown in Fig. A.1 during the He core burning where the effect of wind loss is important. In other words, in order to get the observed regularities in...
the evolution of SXTs, especially Eq. (22) which gives the linear dependence on $M_d$ of the preexplosion separation of the binary, we must manufacture $R$ vs burning stage curves for which mass transfer can be possible both early in Case B and in Case C. With early Case B mass transfer, or intermediate Case B mass transfer if it occurs, the winds during He core burning are so strong that not enough of an Fe core is left to result in a high-mass black hole, rather, a low-mass compact object results[82].

This story is somewhat complicated, but there have been many years of failures in trying to evolve black holes in binaries without taking into account the effects of binarity (mass transfer in our model) on the evolution. On the other hand, de Kool et al.[98] had no difficulty in evolving A0620 in Case C mass transfer. The necessity in a similar evolution for the other black hole binaries was, however, not realized at that time.

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