COSMOLOGICAL MAGNETIC FIELDS AND CMBR POLARIZATION

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Abstract

A simple introduction to physics of CMBR polarization and the Faraday rotation of the latter in cosmic magnetic field is presented. The content of the lecture is the following: 1. Description of polarization of photons. 2. Polarization field of CMBR. 3. Faraday effect. 4. Cosmic magnetic fields. 5. Faraday rotation of CMBR polarization.
1 Description of polarization of photons

The polarization state of an electromagnetic wave is determined by the vector of its electric field which is orthogonal to the direction of the propagation. The polarization density matrix is defined as the average value of bilinear combinations of the components of the electric field of the propagating wave [1]:

$$\rho_{ij} = \langle E_i E_j^* \rangle$$  \hspace{1cm} (1)

Evidently, $\rho_{ij}$ is the 2nd rank tensor in 2-dimensional $(x, y)$-space if photon propagates along $z$. This tensor has the following well known invariants:

1. Trace, which is equal to intensity of radiation:

$$T = \delta_{ij} \rho_{ij} = |E_x|^2 + |E_y|^2$$  \hspace{1cm} (2)

2. Helicity:

$$V = \epsilon_{ij} \rho_{ij}$$  \hspace{1cm} (3)

Non-zero helicity $V$ corresponds to circularly polarized photons when the average value of the photon spin projection on the direction of the photon propagation is non-vanishing.

As any $2 \times 2$-matrix $\rho_{ij}$ can be decomposed in terms of the unit matrix, $I$ and three Pauli matrices, $\sigma_k$, $k = 1, 2, 3$. The coefficients $\xi_k$ in front of the latter are called the Stokes parameters:

$$\rho_{ij} = T(I/2 + \xi_k \sigma_k)$$  \hspace{1cm} (4)

For a particular and practically interesting case of photons with a fixed frequency the components of electric fields can be written as:

$$E_x = E_0 e_x \exp[-i\omega t + i\beta_x]$$
$$E_y = E_0 e_y \exp[-i\omega t + i\beta_y],$$  \hspace{1cm} (5)

where the components $\epsilon_k$ of the photon polarization vector satisfy the normalization condition $\epsilon_x^2 + \epsilon_y^2 = 1$.

The Stokes parameters for this case can be expressed as:

$$\xi_2 = e_x e_y \sin(\beta_x - \beta_y)$$  \hspace{1cm} (6)

where $\xi_2$ is invariant with respect to rotations and pseudoscalar with respect to mirror reflection. It describes circular polarization, i.e. the photon helicity:

$$\lambda = sk/\omega$$

The other two Stokes parameters $\xi_1$ and $\xi_3$ describe linear polarization:

$$\xi_3 = (\epsilon_x^2 - \epsilon_y^2)/2$$
$$\xi_1 = e_x e_y \cos(\beta_x - \beta_y)$$  \hspace{1cm} (7)
They transform under coordinate rotation in \((x, y)\)-plane by angle \(\phi\) as:

\[
\begin{align*}
\xi'_1 &= \xi_1 \cos 2\phi - \xi_3 \sin 2\phi \\
\xi'_3 &= \xi_1 \sin 2\phi + \xi_3 \cos 2\phi
\end{align*}
\] (8)

Making proper rotation one can always arrange vanishing of one of these Stokes parameters, e.g.

\[
\xi_1 = 0
\] (9)

Using the transformation law one can check that the following combinations of the Stokes parameters are the eigen-vectors of rotation:

\[
\xi_3 \pm i\xi_1 \to \exp \left[ \pm 2i\phi \right] (\xi_3 \pm i\xi_1)
\] (10)

### 2 Polarization by Thomson scattering

As is well known, see e.g. the book \[1\], elastic scattering of unpolarized photons on unpolarized electrons

\[
\gamma + e \to \gamma' + e'
\] (11)

produces polarized photons. The reaction amplitude can be written as

\[
A = e'_i A_i,
\] (12)

where \(e'_i\) is the polarization vector of photons in the final state. The polarization matrix of the scattered photons, up to normalization factor, is expressed as:

\[
\rho_{ij} \sim A_i A^*_j
\] (13)

If we choose the coordinate \(z\) in the direction of \(\gamma'\) and \(x\) in the reaction plane and denote by \(\theta\) the scattering angle then the only non-zero Stokes parameter would be equal to

\[
\xi_3 = \frac{\sin^2 \theta}{\omega/\omega' + \omega'/\omega - \sin^2 \theta} \approx \frac{\sin^2 \theta}{1 + \cos^2 \theta}
\] (14)

where \(\omega\) and \(\omega'\) are the energies of the initial and final photons respectively and the approximation of non-relativistic electrons is made.

The result can be easily understood. The only non-vanishing combination involving \(e'\) in the amplitude is:

\[
e' k \sim \sin \theta
\] (15)

Hence, from eq. follows that

\[
\xi_3 \sim (\sin \theta)^2
\] (16)

The other Stokes parameter, which may be non-zero, vanishes by the choice of coordinate direction, \(\xi_1 = 0\), while \(\xi_2 = 0\) due to parity conservation.

The degree of polarization of the scattered radiation is proportional to the differential Thomson cross-section, which is equal to \[1\]:

\[
\frac{d\sigma}{d\Omega} = \frac{3}{16\pi} \frac{8\pi\alpha^2}{3m_e^2} (1 + \cos^2 \theta)
\] (17)
3 Polarization of CMBR

According to the previous section the photons of CMBR could become polarized due to Thomson scattering on the electrons in cosmic plasma. However, the polarization must vanish in *homogeneous and isotropic* world. It is evidently true, simply because there is no preferred direction in such a world. In realistic cosmology small but non-zero density perturbations existed in the primeval plasma prior to hydrogen recombination. As we see below, the polarization of CMBR would be induced by the the inhomogeneities in the distribution of photons, i.e. by the angular fluctuations of their temperature. This effect was noticed many years ago, the pioneering papers include refs. [2]. For review and history see e.g. [3].

We assume that circular polarization vanishes. As we mentioned above, see also ref. [4], such polarization cannot be induced due to parity conservation. Bounds on possible new interactions violating parity from the absence of circular polarization of CMB are discussed in ref. [5]. With the present day accuracy they are very far from the usual weak interactions.

As we have seen above, in the usual case of the linearly polarized CMBR radiation field the intensity of polarization is described by two functions, $Q = T\xi_3$ and $U = T\xi_1$, where $T$ is the total intensity of radiation with frequency $\omega$:

$$\hat{\rho} = \begin{pmatrix} Q & U \\ -U & Q \end{pmatrix}$$

The total polarization originated as a result of the Thomson scattering should be obtained by the integration of expression (14) with account of (17) over all angles $d\Omega = d\cos\theta d\phi$ with rotation around $z$ to the common coordinate system:

$$Q - iU = \frac{\sigma_T}{\sigma_N} \int d\omega \sin^2\theta \exp[2i\phi] T(\theta, \phi)$$

where $\sigma_N$ is some normalization area over which the data are averaged. Thus one can see that the polarization of the scattered radiation is proportional to quadrupole moment of the incoming radiation.

4 Properties of CMBR polarization field

For description of the CMBR polarization is convenient to know that in addition to the invariants (2) and (3) there exist two more invariants which can be constructed by differentiating of the polarization matrix. One of them is a scalar:

$$S = \partial_i \partial_j \rho_{ij},$$

while the other is a pseudoscalar:

$$P = \epsilon_{ijk} \partial_i \partial_j \rho_{jk}$$

If density perturbations are created by a purely scalar field, then the only way to construct a vector out of this scalar is to apply the operator of differentiation, i.e. gradient,
A second rank tensor can be either obtained by the second order derivatives, $\partial_j \partial_i$ or by multiplication by the invariant Kronecker tensor $\delta_{ij}$. In particular, the traceless part of the polarization tensor should have the form:

$$\rho_{ij} = (2\partial_i \partial_j - \delta_{ij} \partial^2) \Psi$$

(21)

where $\Psi$ is a scalar function of coordinates.

One can easily check that for the case of scalar perturbations the pseudoscalar invariant \[20\] vanishes:

$$P = 0$$

(22)

Thus we arrive to an important conclusion that a non-zero $P$ is an indication for tensor perturbations or, more generally, for something beyond scalar perturbations. A short list of those includes:

1. **Vector perturbations**, e.g. magnetic fields. The relevant part of the polarization matrix can be written in the form:

$$\rho_{ij} = \partial_i V_j + \partial_j V_i$$

(23)

where $\mathbf{V}$ describes a vector perturbation field. Correspondingly, $P$ may be non-vanishing and equal to:

$$P_V = \epsilon_{ij} \partial^2 \partial_i V_j$$

(24)

2. **Tensor perturbations**, which might be produced by gravitational waves. In this case the CMBR polarization matrix may contain the contribution:

$$\rho_{ij} = \partial_i h_{3j} + \partial_j h_{3i}$$

(25)

and the pseudoscalar invariant takes the form:

$$P_T = \epsilon_{ij} \partial^2 \partial_i h_{3j}$$

(26)

3. The pseudoscalar invariant may be non-zero even in the case of purely scalar perturbations but in the second or higher orders \[6, 7\]. The polarization matrix would be proportional to the products of gradients of two different scalar functions $\Psi_1$ and $\Psi_2$:

$$\rho_{ij} = \partial_i \Psi_1 \partial_j \Psi_2 + \partial_i \Psi_2 \partial_j \Psi_1,$$

(27)

where e.g. $\Psi_2 = \partial_t \Psi_1$. The $P$-invariant is equal to:

$$P_2 = \epsilon_{ij} \left( \partial_i \partial^2 \Psi_1 \partial_j \Psi_2 + \partial_i \partial^2 \Psi_2 \partial_j \Psi_1 \right)$$

(28)

The decomposition of the polarization matrix according to the magnitude of the scalars $S$ and $P$ is considered in ref. \[8\]. It is analogous to magnetic/electric or B/E - decomposition \[9\] but the former looks more natural because no vector field of electric or magnetic type is associated with $2 \times 2$ polarization matrix.

However, there may be other, non-primordial, sources for non-zero $P$ due to propagation of CMB photons through the large scale structure. They are analyzed in ref. \[10\].
5 Eigenvectors of polarization matrix

In some works polarization fields is characterized by the “vector”

\[ \mathbf{W} = (Q, U) \]

However, \( \mathbf{W} \) is not a vector but some mixture of components of a second rank tensor. Though such a description is possible, it is not convenient [11]. Field lines of such “vector” change under rotation in a non-trivial way. We see portraits of famous astronomers and the portrait of Louis XIV in this beautiful room. If someone takes one or other portrait and rotates it, or ever simpler, just turns one’s own head, the picture would surely remain the same. This would not be true for the pictures of CMBR polarization field presented in terms of “vector” \( \mathbf{W} \).

Much more convenient is to present polarization maps in terms of field lines of the eigenvectors \( \mathbf{N} \) of the polarization matrix \( \rho_{ij} \) [12, 4]. An interesting feature of such maps is unusual singularity points of the field lines of the eigenvectors \( \mathbf{N} \). As is known from the classical analytical mechanics, dynamical systems may normally have the following three singularity types: saddle, focus, and knot. However, the eigenvectors \( \mathbf{N} \) are not analytic near singularity, where \( |\mathbf{N}| = 0 \). Because of that the character of singularities changes. There still remain three types of them but now they look differently, see fig. 1. We suggested the following names for them: saddle (the same as above but now it has three, instead of four, straight line asymptotic), beak, and comet [12, 4]. An analysis of statistical properties of these singularities is performed in the papers [4, 11, 12, 13].

In fig. 2 the simulated map of CMBR polarization is presented, according to ref. [4]. One can easily see the singular points corresponding to the three types indicated in fig. 1. In contrast to description of the direction of the polarization field in terms of \( \mathbf{W} \), the map of field lines of \( \mathbf{N} \) remains the same independently of the angle at which one observes it. On the other hand, as we mentioned above, different singular points of the map of the field lines of \( \mathbf{W} \) transform into each other under rotation and the general picture becomes completely different.

6 Faraday effect

The Faraday effect is a rotation of the polarization plane of linearly polarized photons in magnetic field \( \mathbf{B} \), due to interaction with electrons in the medium. Such rotation is generated by the breaking of the reflection symmetry in presence of magnetic field. In a medium without reflection symmetry refraction indices for left- and right-handed photons are different, \( n_+ \neq n_- \). Linearly polarized electromagnetic wave can be decomposed into two rotationally polarized ones rotating in opposite directions. We assume that the electric field of the initial wave was directed along axis \( x \). Then using the simple identity:

\[ 1 = (1 + i)/2 + (1 - i)/2 \quad (29) \]

we expand \( E_x \) in terms of two helicity eigenstates of the incoming photon:

\[ E^{(in)} = E_x = (E_+ + E_-)/2 \quad (30) \]
Figure 1: Flux lines for three different types of singular points: (a) saddle, (b) beak, and (c) comet. Dashed lines show peculiar solutions, separatrix.

Figure 2: Simulated map of CMB polarization eigen-vector field $\mathbf{N}$ Solid lines show the flux line behavior near singular points where polarization vanishes.
Each helicity state propagates independently and acquires phase proportional to the distance of propagation:

$$E^{(\text{fin})}_{\pm} = \exp(i k_{\pm} l) \ E^{(\text{in})}_{\pm}$$  \hspace{1cm} (31)

Since the refraction index for different helicity states is different by assumption, then $k_{\pm} = k_0 \pm \Delta k$ and

$$E^{(\text{fin})}_x = E^{(\text{in})}_x \exp[i k_0 l] \cos(\Delta kl)$$

$$E^{(\text{fin})}_y = E^{(\text{in})}_x \exp[i k_0 l] \sin(\Delta kl)$$

The relative phase remains zero. It means that the initially linearly polarized wave remains linearly polarized but the polarization plane is rotated by the rotation angle:

$$\Phi = \arctan \left[ \frac{E^{(\text{fin})}_y}{E^{(\text{fin})}_x} \right] = \Delta kl$$

Thus the calculation of rotation angle is reduced to the calculation of the refraction index of ionized gas - this is the cosmic medium where photons of CMBR propagate. The equation of motion of electrons, with charge $(-e)$, in superimposed external (large) magnetic field $B_0$ and (weak) electromagnetic wave $E \exp(i \omega t)$ reads:

$$\ddot{\mathbf{r}} = eB_0 \times \dot{\mathbf{r}} - eE \exp[i \omega t]$$  \hspace{1cm} (32)

To solve this equation we need to decompose the propagating wave in terms of helicity states:

$$E = C_+ (n_x + i n_y) + C_- (n_x - i n_y)$$  \hspace{1cm} (33)

for which the equation diagonalizes and can be solved as

$$x_{\pm} = \frac{eE_{\pm}}{m \omega (\omega \mp \omega_B)}$$  \hspace{1cm} (34)

where $\omega_B = eB_0/m$.

The electric polarization moment of the plasma is easily found:

$$\mathcal{P}_{\pm} = -N_e e x_{\pm}$$  \hspace{1cm} (35)

and correspondingly the dielectric constant is

$$\epsilon_{\pm} = 1 + 4 \pi \mathcal{P}/E = 1 + \frac{4 \pi e^2 N_e}{m \omega (\omega \mp \omega_B)}$$

The plasma refraction index for different helicity states of photons is given by the standard expression: $n_{\pm} = \sqrt{\epsilon_{\pm}}$ and thus the differential Faraday rotation angle is equal to:

$$\frac{d\phi}{dl} = \frac{2 \pi N_e e^3 B_0}{m^2 \omega^2}$$  \hspace{1cm} (36)

where $m$ is the electron mass, $e^2 = \alpha = 1/137$, and $N_e$ is the number density of electrons. Usually the results is presented in terms of frequency $\nu = \omega/(2\pi)$ or wave length $\lambda = 1/\nu$. 

8
7 Cosmic magnetic fields

The origin of large scale cosmic magnetic fields remains one of cosmological mysteries. They surely exist in galaxies with the coherence scale of a few kpc and rather large field strength:

\[ B_{\text{gal}} = \text{a few } \mu \text{G}, \] (37)

for a review see [14], more recent data can be found e.g. in ref. [15]. Less certain are the observational data in favor of existence of intergalactic magnetic fields at the scales \( \sim 1 \) Mpc. Still there are rather convincing indications in favor of the latter with the amplitude \( B_{\text{ig}} \sim 10^{-3} B_{\text{gal}} \). It is rather interesting that the amplitude of these fields is related to the galactic field by the inverse ratio of the corresponding scales squared. This would be so if the adiabatic compression took place and the field was amplified as \( B \sim 1/l^2 \). Since the ratios of scales are

\[ \frac{l_{\text{gal}}^{(\text{in})}}{l_{\text{gal}}} \sim 10^2, \]
\[ \frac{l_{\text{ig}}^{(\text{in})}}{l_{\text{ig}}} \sim 3, \] (38)

one would expect \( B_{\text{gal}} \sim 10^3 B_{\text{ig}} \), if galactic and intergalactic magnetic fields have a common origin and galactic dynamo amplification was not effective. The latter might be quite efficient. According to different estimates [16], the amplification factor could be as large as \( 10^{15 \pm 5} \). If galactic dynamo indeed amplified primordial seed fields by this large factor, it means that the original fields were too weak to be observable by the Faraday rotation of the CMBR polarization. Otherwise, if \( B_{\text{ig}} \sim 10^{-9} \) Gauss, the effect may be noticeable.

There is rather strong evidence accumulated recently [17] in favor of quite strong magnetic fields in galactic clusters with the magnitude close to those in galaxies. If this is indeed true, then the chances of observation of the Faraday rotation of CMBR polarization would be even higher.

A vast literature exists on possible mechanisms of generation of primordial magnetic fields (for a review see e.g. [18]). Different mechanisms can be roughly speaking separated into the following classes:

1. Galactic processes, stellar phenomena and reconnection of field lines.
2. Processes during structure formation.
3. Processes during the recombination epoch; vorticity, \( \nabla \times V \), may be generated in the second order in density perturbations.
4. Processes in the early universe:
   a) inflation, might produce small fields but at large scales;
   b) phase transitions could create large fields but at small scales.

All these mechanisms, except for the first one, might create noticeable fields at the CMBR decoupling which can be potentially observable by the Faraday rotation of the CMBR polarization. This effect was first discussed in ref. [19] and attracted considerable attention [20] [21] [22] during the last years.
8 An estimate of the rotation angle

The dominant contribution to the rotation angle comes from the period near the hydrogen recombination epoch. Indeed, the dependence of the rotation angle on the cosmic scale factor is:

\[ d\Phi \sim \lambda^2 N_e B a d\eta \sim a^2 \frac{1}{a^3} \frac{1}{a^2} a \sim \frac{1}{a^2}, \]

(39)

where \( \eta \) is the conformal time, the magnetic field is assumed to evolve as \( B \sim 1/a^2 \) (adiabatic compression and no dynamo amplification). Thus the rotation is dominated by an early epoch. Before the recombination the photon mean free path along which the rotation angle could be “accumulated”, \( l_{\text{free}} \), is very small and practically \( \langle \Phi \rangle = 0 \). After, the recombination the number density of free electrons, \( N_e \), drops down by the factor \( 10^{-5} \). Thus the most favorable period for the rotation of the polarization plane in large scale magnetic fields is the recombination epoch.

The differential rotation angle is equal to:

\[ \frac{d\Phi}{d\eta} = x_e N_e e^2 a \]

\[ \frac{1}{2\pi m^2 \nu^2} B n \]  

(40)

where \( x_e \) is ionization fraction and \( n \) is the unit vector in the direction of propagation of radiation. By assumption of adiabatic compression \( B a^2 = \text{const} = B_0 a_0^2 \), i.e. it is equal to the present day value.

The optical depth is expressed through the Thomson cross-section \( \sigma_T \) and the number density of free electrons and is equal to

\[ \frac{d\tau}{d\eta} = N_e \sigma_T a \]

(41)

The total rotation angle (for homogeneous field along photon propagation) is easily estimated:

\[ \Phi = \frac{3\lambda^2 B_0 \cdot n}{16\pi^2 \epsilon} \int d\tau \exp (-\tau) = \frac{3\lambda^2 B_0 \cdot n}{16\pi^2 \epsilon} \]

(42)

(here the usual in this field, but unusual for particle physics, convention \( e^2 = \alpha \) is used).

Numerically we find:

\[ \Phi \approx 2^\circ \left( \frac{B_0}{10^{-9} \text{Gauss}} \right) \left( \frac{30 \text{ GHz}}{\nu_0} \right)^2 \]

(43)

(it helps to know that 1 Gauss = 6.9 \cdot 10^{-14} \text{ MeV}^2).

9 Statistical properties of magnetic fields and rotation angle

We express all the quantities below in terms of the present day values, in particular:

\[ B_0(x) = a^2(\eta) B(x, \eta). \]  

(44)
As usually the field amplitude is expanded in terms of its Fourier modes:

$$B_0(x) = \frac{1}{(2\pi)^3} \int d^3k e^{-ikx} b_0(k)$$

Below we omit sub-zero for simplicity of notations.

The correlator of the field amplitudes is:

$$\langle B_i(x_1)B_j(x_2) \rangle = C_{ij}(|x_1 - x_2|)$$

(45)

The correlator depends on the difference of the coordinates because of the average homogeneity and isotropy. Correspondingly the Fourier modes should be delta-correlated. Their correlator can be expressed through two scalar functions $A(k)$ and $S(k)$ [23]:

$$\Pi_{ij} = \langle b_i(k_1)b_j^*(k_2) \rangle = 2 (2\pi)^3 \delta(k_1 - k_2) \left[ (\delta_{ij} - \kappa_i\kappa_j) S(k) + i\epsilon_{ijl}\kappa_l A(k) \right]$$

(46)

where $\kappa_i = k_i/|k|$.

The energy of magnetic field depends only on $S(k)$:

$$\int d^3x B_j^2 = \frac{2}{\pi^2} \int dk k^2 S(k)$$

(47)

while $A(k)$ determines the so called helicity of magnetic field [24].

The correlator of the rotation angle can be expressed through the correlator of magnetic field as:

$$\langle \Phi(n)\Phi(m) \rangle = \left( \frac{3}{16\pi^2} \right)^2 \int d\eta g(\eta) \int d\eta' g(\eta') \langle [B_0(\Delta \eta \cdot n) \cdot n](B_0(\Delta \eta' \cdot m) \cdot m) \rangle$$

where $g(\eta) = (d\tau/d\eta) \exp[-\tau(\eta)]$ and $\delta \eta = \eta - \eta_0$.

Using eq. (46) one finds

$$\langle (B \cdot n)(B \cdot m) \rangle = \frac{1}{2(2\pi)^3} \int d^3k \left\{ [(nm) - (n \cdot \kappa)(m \cdot \kappa)] S(k) + i [\{n \times m \cdot \kappa\} A(k)] \exp[-ik(n\Delta\eta - m\Delta\eta')] \right\}$$

(48)

It can be checked that the term containing $A$ vanishes [21, 22]. It prevents from measurements of helicity of magnetic field through Faraday rotation, in contrast to the results of earlier papers [25].

Performing the necessary integration we obtain:

$$\langle (Bn)(Bm) \rangle = \left[ (nm)C_\perp(r) + (nr/r)(mr/r)(C_\parallel(r) - C_\perp(r)) \right],$$

(49)

where $r = n\Delta\eta - m\Delta\eta'$, and

$$C_\perp(r) = \frac{2}{3(2\pi)^3} \int_0^\infty dk E_B(k) \left[ j_0(kr) - \frac{j_2(kr)}{2} \right]$$

$$C_\parallel(r) = \frac{2}{3(2\pi)^3} \int_0^\infty dk E_B(k) \left[ j_0(kr) + j_2(kr) \right],$$

(50)
Here \( j_i(x) \) are the spherical Bessel functions of the \( i^{th} \) order and \( \mathcal{E}_B(k) \) is the magnetic power spectrum:

\[
E_B = \int^\infty_0 n f t y_0 dk \mathcal{E}_B(k),  \\
\mathcal{E}_B(k) = 2\pi k^2 S(k)
\]

The function \( S(k) \) is usually parametrized as

\[
S(k) = S_0 k^{n_S} \exp\left[-(k/K)^2\right]
\]

For this simple form of \( S(k) \) the integral can be taken analytically. The correlation function of the Faraday rotation measure, according to ref. \[21\] is presented in fig. 3. The results are not particularly sensitive to the power \( n_S \).

![Figure 3: Faraday rotation measure correlation \( RR'(\theta) \) as a function of the separation angle \( \theta \). The three lines correspond to the magnetic field spectral index \( n_S = 2 \) (solid line), \( n_S = 4 \) (dashed line) and \( n_S = 6 \) (dotted line). The correlation length of the magnetic field is taken as \( \xi = 20 \) Mpc.](image)

### 10 Conclusion

1. We do not understand how large scale cosmic magnetic fields have been formed. If \( B_{gal} \) and \( B_{ig} \) have the same origin and galactic dynamo did not operate, impact of primordial fields could be observable in CMBR polarization. Another way around,
an observation of the Faraday rotation of the CMBR polarization can bring an important information about primordial magnetic fields and help to solve the problem of their origin.

2. $P$-type (or $B$-type) polarization created by magnetic fields may mimic gravitational waves but they could be possibly distinguished due to different frequency dependence.

3. Eigenvector description of the polarization field may be useful. Their statistical properties may depend upon the strength of the primordial magnetic fields. This problem deserves further consideration.

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