First-Class Functions for First-Order Database Engines

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Abstract
We describe query defunctionalization which enables off-the-shelf first-order database engines to process queries over first-class functions. Support for first-class functions is characterized by the ability to treat functions like regular data items that can be constructed at query runtime, passed to or returned from other (higher-order) functions, assigned to variables, and stored in persistent data structures. Query defunctionalization is a non-invasive approach that transforms such function-centric queries into the data-centric operations implemented by common query processors. Experiments with XQuery and PL/SQL database systems demonstrate that first-order database engines can faithfully and efficiently support the expressive "functions as data" paradigm.

1. Functions Should be First-Class
Since the early working drafts of 2001, XQuery’s syntax and semantics have followed a functional style: functions are applied to form complex expressions in a compositional fashion. The resulting XQuery script’s top-level expression is evaluated to return a sequence of items, i.e., atomic values or XML nodes [8].

Ten years later, with the upcoming World Wide Web Consortium (W3C) XQuery 3.0 Recommendation [28], functions themselves now turn into first-class items. Functions, built-in or user-defined, may be assigned to variables, wrapped in sequences, or supplied as arguments to and returned from higher-order functions. In effect, XQuery finally becomes a full-fledged functional language. Many useful idioms are concisely expressed in this "functions as data" paradigm. We provide examples below and argue that support for first-class functions benefits other database languages, PL/SQL in particular, as well.

This marks a significant change for query language implementations, specifically for those built on top of (or right into) database kernels. While atomic values, sequences, or XML nodes are readily represented in terms of the widespread first-order database data models [9], this is less obvious for function items. Database kernels typically lack a runtime representation of functional values at all. We address this challenge in the present work.

In query languages, the “functions as data” principle can surface in various forms.

Functions as Values. XQuery 3.0 introduces name#n as notation to refer to the n-ary function named name: math:pow#2 refers to exponentiation while fn:concat#2 denotes string concatenation, for example. The values of these expressions are functions—their types are of the form function(t1) as t2 or, more succinctly, t1 -> t2—which may be bound to variables and applied to arguments. The evaluation of the expression

let $exp := math:pow#2 return $exp(2, 3) yields 8, for example.

1[... XQuery is a functional language in which a query is represented as an expression.” [11, §2]

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2Our design follows Michael Kay’s proposal for maps in XSLT 3.0. Of two entries under the same key, we return the entry inserted first (this is implementation-dependent: http://www.w3.org/TR/xslt-30/#map).
declare function map:empty() {
  function($x) { () }
}
}

declare function map:entry($k, $v) {
  function($map) {
    function($x) { if ($x = $k) then $v else $map($x) }
  }
}

declare function map:new($es) {
  fold-right(function($f, $x) { $f($x) }, map:empty(), $es)
}

function

declare function map:remove($map, $k) {
  function($x) { if ($x = $k) then () else $map($x) }
}

unwrap

map:get

map

Figure 2: Direct implementation of maps as functions from keys to values (XQuery 3.0).

(: wrap(), unwrap(): see Section 4.1 :)

declare function map:empty() {
  element map()
}

declare function map:entry($k, $v) {
  element entry {
    element key { wrap($k) }, element val { wrap($v) }
  }
}

declare function map:new($es) {
  element map ($es)
}

declare function map:get($map, $k) {
  unwrap($map/child::entry[child::key = $k][1]/
  child::val/child::node())
}

declare function map:remove($map, $k) {
  element map ($map/child::entry[child::key != $k] }
}

Figure 3: A first-order variant of XQuery maps.

An alternative, regular first-order implementation of maps is shown in Figure 3. In this variant, map entries are wrapped in pairs of key val XML elements. A sequence of such pairs under a common parent forms a complex map. Map lookup now requires an additional function map:get—to e.g., with $m as above: map:get($m, 2)—that uses XPath path expressions to traverse the resulting XML element hierarchy. (We come back to wrap and unwrap in section 4.1.)

We claim that the functional variant in Figure 2 is not only shorter but also clearer and arguably more declarative, as it represents a direct realization of the “a map is a function” premise. Further, once we study their implementation, we will see that the functional and first-order variants ultimately lead the query processor to construct and traverse similar data structures (Section 4.1). We gain clarity and elegance and retain efficiency.

Functions in Data Structures. Widely adopted database programming languages, notably PL/SQL [4], treat functions as second-class citizens: in particular, regular values may be stored in table cells while functions may not. This precludes a programming style in which queries combine tables of functions and values in a concise and natural fashion.

The code of Figure 4 is written in a hypothetical dialect of PL/SQL in which this restriction has been lifted. In this dialect, the function type $t_1 \rightarrow t_2$ reads $\text{FUNCTION}(t_1)$ returns $t_2$ and $\text{FUNCTION}(\text{e}) \text{ BEGIN END}$ denotes a literal function with argument $e$ and body $\text{e}$.3

The example code augments a TPC-H database [34] with a configurable method to determine order completion dates. In lines 18 to 25, table COMPLETION is created and populated with one possible configuration that maps an order status (column $\text{c\_orderstatus}$) to its particular method of completion date computation. These methods are specified as functions of type $\text{FUNCTION} (\text{ORDERS} \rightarrow \text{DATE})$ held in $\text{c\_completion}$, a functional column: while we directly return its $\text{o\_orderdate}$ value for a finalized order (status 'F') and respond with an undefined NULL date for orders in processing ('P'), the completion date of an open order ('O') is determined by function $\text{item\_dates(GREATEST)}$: this function consults the commitment and shipment dates of the order’s items and then returns the most recent of the two (since argument $\text{comp}$ is $\text{GREATEST}$).

Function $\text{item\_dates}$ itself has been designed to be configurable. Its higher-order type $\text{DATE} \times \text{DATE} \rightarrow \text{DATE}$ indicates that $\text{item\_dates}$ returns a function to calculate order completion dates once it has been supplied with a suitable date comparator (e.g., $\text{GREATEST}$ in line 25). This makes $\text{item\_dates}$ a $\text{curried}$ function which consumes its arguments successively (date comparator first, order second)—a prevalent idiom in function-centric programming [6].

Note that the built-in and user-defined functions $\text{GREATEST}$ and $\text{item\_dates}$ are considered values as are the two literal func-

3We are not keen to propose syntax here. Any notation that promotes first-class functions would be fine.

4Type $\text{ORDERS}$ denotes the type of the records in table $\text{ORDERS}$.

5Built-in SQL function $\text{GREATEST}$ (LEAST) returns the larger (smaller) of its two arguments.
This work deliberately pursues a non-invasive approach that enables off-the-shelf database systems to support the function-centric style of queries we have advocated in Section 1. If these existing first-order query engines are to be used for evaluation, it follows that we require a first-order representation of functional values. Closures [5, 23] provide such a representation. We very briefly recall the concept here.

The XQuery 3.0 snippet of Figure 5 defines the higher-order grouping function \( \text{group-by} \) by which receives the grouping criterion in terms of the functional argument \( \$\text{key} \): a group is the sequence of those items \( \$x \) in \( \$\text{seq} \) that map to the same key value \( \$\text{key} (\$x) \). Since XQuery implicitly flattens nested sequences, group-by cannot directly yield the sequence of all groups. Instead, group-by returns a sequence of functions each of which, when applied to zero arguments, produces "its" group. The sample code in lines 11 to 14 uses group-by to partition the first few elements of the Fibonacci series into odd/even numbers and then wraps the two resulting groups in XML group elements.

**Closures.** Note that the inline function definition in line 8 captures the values of the free variables \( \$f_{\text{ib}} \), \( \$\text{key} \), and \( \$\text{seq} \) which is just the information required to produce the group for key \( \$x \). More generally, the language implementation will represent a functional value \( f \) as a bundle that comprises

1. the code of \( f \)'s body
2. its environment, i.e., the bindings of the body's free variables at the time \( f \) was defined.

Together, code and environment define the closure for function \( f \).

In the sequel, we will use

\[ \ell = x_1, \ldots, x_n \]

to denote a closure whose environment contains \( n \geq 0 \) free variables \( x_1, \ldots, x_n \), bound to the values \( x_1, \ldots, x_n \).

Label \( f \) identifies the code of the function's body (in the original work on closures, code points were used instead [5]). In the example of Figure 5, two closures are constructed at line 8 (there are two distinct grouping keys \( \$k \equiv 0.1 \) that represent instances of the literal function. If we order the free variables as \( \$k \), \( \$\text{key} \), \( \$\text{seq} \), these closures read

\[ \delta_1 (0\ 1\ 1\ 1\ 2\ \ldots) \quad \text{and} \quad \delta_2 (1\ 1\ 1\ 1\ 2\ \ldots) \].

(the two closures share label \( \delta_1 \) since both refer to the same body code \( \$\text{seq} [\$\text{key}(.) = \$x k] \). Observe that

* closures may be nested: \( \$\text{key} \) is bound to closure \( \delta_2 \) with empty environment, representing the literal function \( (\$x) \ (\$x \mod 2) \) (defined in line 12) whose body has no free variables, and
* closures may contain and share data of significant size; both closures contain a copy of the \( \$f_{\text{ib}} \) sequence (since free variable \( \$\text{seq} \) was bound to \( \$f_{\text{ib}} \)).

We will address issues of closure nesting, sharing, and size in Sections 4 and 5.

The key idea of defunctionalization, described next, is to trade functional values for their closure representation—ultimately, this leaves us with an equivalent first-order query.

### 3. Query Defunctionalization

**Query defunctionalization** is a source-level transformation that translates queries over first-class functions into equivalent first-order queries. Here, our discussion revolves around XQuery but defunctionalization is readily adapted to other query languages, e.g., PL/SQL (see Section 3.1).

The source language is XQuery 3.0, restricted to the constructs that are admitted by the grammar of Figure 6 (these restrictions aid brevity—defunctionalization is straightforwardly extended to cover the full XQuery 3.0 specification). Notably, the language subset includes

- two kinds of expressions that yield functional values (literal functions of the form \( \text{function}(\$x_1, \ldots, \$x_n) \{ e \} \) as well as named function references \( \text{name} \# n \)), and

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1. If we agree on a variable order, there is no need to save the variable names \( v_i \) in the environment.
dynamic function calls of the form $e_1(e_2, \ldots, e_n)$, in which expression $e$ evaluates to an $n$-ary function that is subsequently applied to the appropriate number of arguments.

The transformation target is a first-order dialect of XQuery 1.0 to which we add closure construction and elimination. A closure constructor $\llbracket \llbracket x_1, \ldots, x_n \rrbracket \rrbracket$ builds a closure with label $\ell$ and an environment of values $v_1, \ldots, v_n$. Closure elimination, expressed using case $\cdots$ of, discriminates on a closure’s label and then extracts the environment contents: from the $b$ branches in the expression

$$\begin{align*}
\text{case } e \text{ of } & \\
\begin{cases} 
\text{if } \llbracket \llbracket x_1, \ldots, x_n \rrbracket \rrbracket \Rightarrow e_1 \\
\ldots \\
\text{if } \llbracket \llbracket v_1, \ldots, v_n \rrbracket \rrbracket \Rightarrow e_b
\end{cases} 
\end{align*}$$

if $e$ evaluates to the closure $\llbracket \llbracket x_1, \ldots, x_n \rrbracket \rrbracket$, case $\cdots$ of will pick the $i$th branch and evaluate $e_i$ with the variables $\langle v_{i,j} \rangle$ bound to the values $x_j$. We discuss ways to express the construction and elimination of closures in terms of regular query language constructs in Section 4.

Figure 7 shows the relevant excerpt of the resulting target language. In a sense, this modified grammar captures the essence of defunctionalization: functional values and dynamic function calls are traded for the explicit construction and elimination of first-order closures. The translation can be sketched as follows:

(1) A literal function is replaced by a closure constructor whose environment is populated with the bindings of the free variables referenced in the function’s body. The body’s code is wrapped inside a new top-level surrogate function $\ell$ whose name also serves as the closure label.

(2) A reference to a function named $\ell$ is replaced by a closure constructor with empty environment and label $\ell$.

(3) A dynamic function call (now equivalent to an application of a closure with label $\ell$ to zero or more arguments) is translated into a static function call to a generated dispatcher function. The dispatcher receives the closure as well as the arguments and then uses closure elimination to forward the call to function $\ell$, passing the environment contents (if any) along with the arguments.

Appendix A elaborates the details of this transformation, including the generation of dispatchers, for the XQuery case. A syntax-directed top-down traversal identifies the relevant spots in a given program at which closure introduction or elimination has to be performed according to the cases (1) to (3) above. All other program constructs remain unchanged. The application of defunctionalization to the XQuery program of Figure 5 yields the code of Figure 8. We find the expected surrogate functions $\ell_1, \ell_2, \ldots$, dispatchers (dispatch_n), and static dispatcher invocations. Overall, the resulting defunctionalized query adheres to the target language of Figure 7, i.e., the query is first-order. Once we choose a specific implementation for closure construction and elimination, we obtain a query that may be executed by any XQuery 1.0 processor.

### 3.1 Query Defunctionalization for PL/SQL

Query defunctionalization does not need to be reinvented if we carry it over to PL/SQL. Much like for XQuery, the defunctionalization transformation for a PL/SQL dialect with first-class functions builds on three core cases (see above and Figure 21 in Appendix A):

1. The creation of function literals (applies in lines 9, 23, and 24 of the PL/SQL example in Figure 4).
2. References to named function values (GREATEST in line 25).
3. Dynamic function application (applies in lines 10 and 30).

Applied to the example of Figure 4 (order completion dates), defunctionalization generates the output of Figure 9. The resulting code executes on vanilla PL/SQL hosts; we show a PostgreSQL 9 dialect here, minor adaptations yield syntactic compatibility with Oracle.

PL/SQL operates over typed tables and values and thus requires the generation of typed closures. In the present example, we use $\tau_{t_1 \rightarrow t_2}$ to denote the type of closures that represent functions of type $t_1 \rightarrow t_2$. (For now, $\tau$ is just a placeholder—Section 4 discusses suitable relational implementations of this type.) As expected, we find higher-order function item_dates to accept and return values of such types $\tau$ (line 35).
CREATE FUNCTION \( f_1 \) (o ORDERS, comp \( \text{TDATE} \times \text{TDATE} \rightarrow \text{TDATE} \)) RETURNS DATE AS
BEGIN
RETURN (SELECT dispatch_2(comp, MAX(li.l_commitdate), MAX(li.l_1))
          FROM LINEITEM li
          WHERE li.l_orderkey = o.o_orderkey);
END;

CREATE FUNCTION \( f_2 \) (o ORDERS) RETURNS DATE AS
BEGIN
RETURN o.o_orderdate;
END;

CREATE FUNCTION \( f_3 \) (o ORDERS) RETURNS DATE AS
BEGIN
RETURN NULL;
END;

CREATE FUNCTION dispatch_1(c.c_completion, o) AS completion
BEGIN
CASE o.o_orderstatus
WHEN 'O'
RETURN (SELECT dispatch_2(comp, MAX(li.l_commitdate), MAX(li.l_shipdate))
          FROM ORDERS li
          WHERE li.l_orderkey = o.o_orderkey);
END;

CREATE FUNCTION dispatch_2(c.c_completion, o) AS completion
BEGIN
CASE c.c_completion
WHEN 'F'
RETURN GREATEST(c.c_completion, o.o_orderstatus);
END;

CREATE TABLE COMPLETION (
  c_orderstatus CHAR(1),
  c_completion \( \text{TORDERS} \rightarrow \text{TDATE} \) AS completion
);

SELECT o.o_orderkey,
  c_orderstatus,
  dispatch_if(c.c_completion, o) AS completion
FROM ORDERS o, COMPLETION c
WHERE o.o_orderkey = c.c_orderstatus;

Figure 9: PL/SQL code of Figure 4 after defunctionalization.

Likewise, PL/SQL defunctionalization emits typed dispatchers dispatch_i each of which implement dynamic function invocation for closures of a particular type: the dispatcher associated with functions of type \( t_1 \rightarrow t_2 \) has the PL/SQL signature
\[ \text{FUNCTION}(t_1) \rightarrow t_2 \]. With this typed representation come opportunities to improve efficiency. We turn to these in the next section.

Tables of Functions. After defunctionalization, functional values equate first-order closure values. This becomes apparent with a look at table COMPLETION after it has been populated with three functions (in lines 45 to 48 of Figure 9). Column \( c\_completion \) holds the associated closures (Figure 10). The closures with labels \( f_2 \) and \( f_3 \) represent the function literals in lines 23 and 24 of Figure 4: both are closed and have an empty environment. Closure \( f_1 \), representing the function literal defined at line 9 of Figure 4, carries the value of free variable comp which itself is a (date comparator) function. We thus end up with a nested closure.

Tables of functions may persist in the database like regular first-order tables. To guarantee that closure labels and environment contents are interpreted consistently when such tables are queried, update and query statements need to be defunctionalized together, typically as part of the same PL/SQL package [4, §10] (whole-query transformation, see Appendix A). Still, query defunctionalization is restricted to operate in a closed world: the addition of new literal functions or named function references requires the package to be defunctionalized anew.

4. Representing (Nested) Closures

While the defunctionalization transformation nicely carries over to query languages, we face the challenge to find closure representations that fit query runtime environments. Since we operate non-invasively, we need to devise representations that can be expressed within the query language’s data model itself. (We might benefit from database engine adaptations but such invasive designs are not in the scope of the present paper.)

Defunctionalization is indifferent to the exact method of closure construction and elimination provided that the implementation can (a) discriminate on the code labels \( \ell \) and (b) hold any value of the language’s data model in the environment.

If the implementation is typed, we need to (c) ensure that all constructed closures for a given function type \( t_1 \rightarrow t_2 \) share a common representation type \( \tau \) (cf. our discussion in Section 3.1).

Since functions can assume the role of values, (b) implies that closures may be nested. We encountered nested closures of depth 2 in Figure 10 where the environment of closure \( f_1 \) holds a closure labeled \( \ell_4 \). For particular programs, the nesting depth may be unbounded, however. The associative map example of Section 1 creates closures of the form
\[
\ell_1 \leftrightarrow k_1 \leftarrow v_1 \ell_2 \leftrightarrow k_2 \leftrightarrow v_2 \ldots \ell_n \leftrightarrow k_n \leftrightarrow v_n
\]
where the depth is determined by the number \( n \) of key/value pairs \((k_i, v_i)\) stored in the map.

Here, we discuss closure implementation variants in terms of representation functions \( \text{C}[-] \) that map closures to regular language constructs. We also point out several refinements.

4.1 XQuery: Tree-Shaped Closures

For XQuery, one representation that equates closure construction with XML element construction is given in Figure 11. A closure with label \( \ell \) maps to an outer element with tag \( \ell \) that holds the environment contents in a sequence of \( \text{env} \) elements. In the environment, atomic items are tagged with their dynamic type such that closure elimination can restore value and type (note the calls to function \( \text{wrap}() \) and its definition in Figure 12): item 1 of type \text{xs:integer} is held as \text{<atom>\text{<integer>1</integer>}}</atom>.

Item sequences map into sequences of their wrapped items, XML nodes are not wrapped at all.

Figure 10: Table of functions: \( \text{COMPLETION} \) holds closures of type \( \text{TORDERS} \rightarrow \text{TDATE} \) in column \( c\_completion \).

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Since PL/SQL lacks parametric polymorphism, we may assume that the \( t_i \) denote concrete types. Type specialization [33] could pave the way for a polymorphic variant of PL/SQL, one possible thread of future work.
Closure elimination turns into an XQuery \texttt{typeswitch}() on the outer tag name while values in the environment are accessed via XPath child axis steps (Figure 13). Auxiliary function \texttt{unwrap()} (obvious, thus not shown) uses the type tags to restore the original atomic items held in the environment.

In this representation, closures nest naturally. If we apply $C[]$ to the closure (+) that resulted from key-value map construction, we obtain the XML fragment of Figure 14 whose nested shape directly reflects that of the input closure.

**Refinements.** The above closure representation builds on inherent strengths of the underlying XQuery processor—element construction and tree navigation—but has its shortcomings: XML nodes held in the environment lose their original tree context due to XQuery’s copy semantics of node construction. If this affects the defunctionalized queries, an environment representation based on \textit{by-fragment} semantics [36], preserving document order and ancestor context, is a viable alternative.

Further options base on XQuery’s other aggregate data type: the item sequence: closures then turn into non-empty sequences of type \texttt{item}(). While the head holds label $\ell$, the tail can hold the environment’s contents: $(\ell, x_1, \ldots, x_n)$. In this representation, neither atomic items nor nodes require wrapping as value, type, and tree context are faithfully preserved. Closure elimination accesses the $x_i$ through simple positional lookup into the tail. Indeed, we have found this implementation option to perform particularly well (Section 5). Due to XQuery’s implicit sequence flattening, this variant requires additional runtime effort in the presence of sequence-typed $x_i$ or closure nesting, though (techniques for the flat representation of nested sequences apply [25]).

Lastly, invasive approaches may build on engine-internal support for aggregate data structures. Saxon [25], for example, implements an appropriate tuple structure that can serve to represent closures.\footnote{http://dev.saxonica.com/blog/mike/2011/07/#000186}

### 4.2 PL/SQL: Typed Closures

Recall that we require a fully typed closure representation to meet the PL/SQL semantics (Section 3.1). A direct representation of closures of, in general, unbounded depths would call for a \textit{recursive} representation type. Since the PL/SQL type system reflects the flat relational data model, recursive types are not permitted, however.

**Figure 13:** XQuery closure elimination: \texttt{typeswitch()} discriminates on the label, axis steps access the environment.

**Figure 14:** XML representation of the nested closure (+). $t_{key}$ and $t_{val}$ denote the types of keys and values, respectively.

**Figure 15:** Relational representation for closures, general approach ($\gamma$ denotes an arbitrary but unique key value).

Instead, we represent closures as \textit{row values}, built by constructor \texttt{ROW()}, \textit{i.e.}, native aggregate record structures provided by PL/SQL. Row values are first-class citizens in PL/SQL and, in particular, may be assigned to variables, can contain nested row values, and may be stored in table cells (these properties are covered by feature S024 “support for enhanced structured types” of the SQL:1999 standard [31]).

Figure 15 defines function $C[]$ that implements a row value-based representation. A closure $\langle x_1, \ldots, x_n \rangle$ of type $\tau_1 \rightarrow \tau_2$ maps to the expression \texttt{ROW}(\ell, \gamma). If the environment is non-empty, $C[]$ constructs an additional row to hold the environment contents. This row, along with key $\gamma$ is then appended to binary table $\mathbf{ENV}_{t_1 \rightarrow t_2}$ which collects the environments of all functions of type $t_1 \rightarrow t_2$. Notably, we represent non-closure values $x$ as is ($C[x] = x$), saving the program to perform \texttt{wrap()}/\texttt{unwrap()} calls at runtime.
Adding native support for first-class functions to a first-order query (see Figure 10).

Environments matches well-known relational encodings of tree-shaped structures. Does it Function? (Experiments)

5. Does it Function? (Experiments)

Adding native support for first-class functions to a first-order query processor calls for disruptive changes to its data model and the associated set of supported operations. With defunctionalization and its non-invasive source transformation, these changes are limited to the processor’s front-end (parser, type checker, query simplification). Here, we explore this positive aspect but also quantify the performance penalty that the non-native defunctionalization approach incurs.

XQuery 3.0 Test Suite. Given the upcoming XQuery 3.0 standard, defunctionalization can help to carry forward the significant development effort that has been put into XQuery 1.0 processors. To make this point, we subjected three such processors—Oracle 11g (release 11.1) [24], Berkeley DB XML 2.5.16 [1] and Sedna 3.5.161 [15]—to relevant excerpts of the W3C XQuery 3.0 Test Suite (XQTS).\footnote{A pre-release is available at http://dev.w3.org/cvsweb/2011/QT3-test-suite/misc/HigherOrderFunctions.xml} All three engines are database-supported XQuery processors; native support for first-class functions would require substantial changes to their database kernels.

Instead, we fed the XQTS queries into a stand-alone preprocessor that implements the defunctionalization transformation as described in Section 3. The test suite featured, e.g.,
• named references to user-defined and built-in functions, literal functions, sequences of functions, and
• higher-order functions accepting and returning functions.
All three systems were able to successfully pass these tests.

Closure Size. We promote a function-centric query style in this work, but ultimately all queries have to be executed by data-centric database query engines. Defunctionalization implements this transition from functions to data, i.e., closures, under the hood. This warrants a look at closure size.

Turning to the XQuery grouping example of Figure 5 again, we see that the individual groups in the sequence returned by group-by are computed on-demand: a group’s members will be determined only once its function is applied ($g()$ in line 14). Delaying the evaluation of expressions by wrapping them into (argument-less) functions is another useful idiom available in languages with first-class functions [7], but there are implications for closure size: each group’s closure captures the environment required to determine its group members. Besides $\$key$ and $\$k$, each environment includes the contents of free variable $\$seq$ (the input sequence) such that the overall closure space requirements are in $O(\gamma \cdot \$seq)$ where $\gamma$ denotes the number of distinct groups. A closure representation that allows the sharing of environments (Section 4.2) would bring the space requirements down to $O(\gamma \cdot \$seq)$ which marks the minimum size needed to partition the sequence $\$seq$.

Alternatively, in the absence of sharing, evaluating the expression $\$seq[\$key(.) = \$k]$ outside the wrapping function computes groups eagerly. Figure 18 shows this alternative approach in which...
The bracketed part has been changed from Figure 5. A group’s closure now only includes the group’s members (free variable $\text{group}$, line 9 in Figure 18) and the overall closure sizes add up to $O(|\text{seq}|)$ as desired. Closure size thus should be looked at with care during query formulation—such “space leaks” are not specific to the present approach, however [30].

With defunctionalization, queries lose functions but gain data. This does not imply that defunctionalized queries use inappropriate amounts of space, though. In our experiments we have found function-centric queries to implicitly generate closures whose size matches those of the data structures that are explicitly built by equivalent first-order formulations.

To illustrate, recall the two XQuery map variants of Section 1. Given $n$ key/value pairs $(k_i, v_i)$, the function-centric variant of Figure 2 implicitly constructs the nested closure shown in Figure 14: a non-empty map of $n$ entries will yield a closure size of $10 \cdot n$ XML nodes. In comparison, the first-order map variant of Figure 3 explicitly builds a key/value list of similar size, namely $1 + 9 \cdot n$ nodes (Figure 19). Further, key lookups in the map incur almost identical XPath navigation efforts in both variants, either through closure label discrimination by dispatchers or, in the first-order case, the required calls to $\text{map:get}$.

### Native vs. Dispatched Function Calls

As expected, the invocation of functions through closure label discrimination by dispatchers introduces measurable overhead if compared to native function calls. To quantify these costs, we performed experiments in which 10$^8$ native and dispatched calls were timed. We report the averaged wall-clock times of 10 runs measured on a Linux host, kernel version 3.5, with Intel Core i5 CPU (2.6 GHz) and 8 GB of primary memory.

Both, function invocation itself and closure manipulation contribute to the overhead. To assess their impact separately, a first round of experiments invoked closed functions (empty environment). Table 1a documents the cost of a dispatched PL/SQL function call—i.e., construction of an empty closure, static call to the 

### Table 1: Performing 10$^8$ invocations of closed functions (native vs. dispatched calls). Wall-clock time measured in ms.

|          | Oracle native | Oracle dispatch | PostgreSQL native | PostgreSQL dispatch | BaseX native | BaseX dispatch | Saxon native | Saxon dispatch |
|----------|---------------|-----------------|--------------------|---------------------|--------------|---------------|--------------|---------------|
| 10 500   | 10 500        | 11 860          | 11 860             | 10 500              |
| 2 132    | 2 132         | 2 132           | 2 132              | 2 132              |
| 1 500    | 1 500         | 1 500           | 1 500              | 1 500              |
| 729 413  | 729 413       | 729 413         | 729 413            | 729 413            |
| 38 543   | 38 543        | 38 543          | 38 543             | 38 543             |

(a) Unary PL/SQL function.  
(b) Literal XQuery function.

### Table 2: 10$^8$ invocations and elimination of closures of varying size (1/5/10 free variables). Wall-clock time measured in ms.

| # Calls | Line | Query/Function | ENV | Inline |
|---------|------|---------------|-----|--------|
| 1       | 50   | SELECT o_orderkey, ... | 47 874 | 40 093 |
| 15 000  | 19   | dispatch_1()    | 44 249 | 35 676 |
| 73 2044 | 1    | $f_1()$         | 22 748 | 22 120 |
| 73 2044 | 3    | $L\text{select}_1 dispatched_2()$ | 9 270  | 9 363 |
| 73 2044 | 28   | $L\text{dispatch}_2()$ | 3 554  | 3 450 |
| 729 413 | 9    | $f_2()$         | 2 942  | 2 856 |
| 38 543  | 14   | $f_3()$         | 1 155  | 149    |

### Table 3: Profiles for the PL/SQL program of Figure 9: environment tables vs. closure inlining. Averaged cumulative time measured in ms. Line numbers refer to Figure 9.

### Environment Tables vs. Closure Inlining

Zooming out from the level of individual function calls, we assessed the runtime contribution of dynamic function calls and closure elimination in the context of a complete PL/SQL program (Figure 9). To this end, we recorded time profiles while the program was evaluated against a TPC-H instance of scale factor 1.0 (the profiles are based on PostgreSQL’s `pg_stat_statements` and `pg_stat_user_functions` views [2]). Table 3 shows the cumulative times (in ms) over all query and function invocations: one evaluation of `dispatch_1()`, includ-
ing the queries and functions it invokes, takes 44 429 ms/1 500 000 ≈ 0.03 ms on average (column ENV). The execution time of the top-level SELECT statement defines the overall execution time of the program. Note that the cumulative times do not add up perfectly since the inevitable PL/SQL interpreter overhead and the evaluation of built-in functions are not reflected in these profiles.

Clearly, dispatch_1() dominates the profile as it embodies the core of the configurable completion database. For more than 50% of the overall 1 500 000 orders, the dispatcher needs to eliminate a closure of type \texttt{ORDERSDATE} and extract the binding for free variables from its environment before it can invoke surrogate function \( f_1() \). According to Section 4.2, closure inlining is applicable here and column Inline indeed shows a significant reduction of execution time by 18% (dispatch_2()) does not benefit since it exclusively processes closures with empty environments.)

Simplifications. A series of simplifications help to further reduce the cost of queries with closures:
* Identify \( f_1 \) and \( f \) (do not build closures with empty environment).

* If Dispatch\((n)\) is a singleton set, dispatch\(_n\) becomes superfluous as it is statically known which case branch will be taken.

* When constructing \( f_1, \ldots, f_n \), consult the types of the \( e_i \) to select the most efficient closure representation (recall our discussion in Section 4).

For the PL/SQL program of Figure 9, these simplifications lead to the removal of dispatch_2() since the functional argument \( \text{comp} \) is statically known to be \( \text{GREATEST} \) in the present example. Execution time is reduced by an additional 11% (see column Simplified above). We mention that the execution time now is within 19% of a first-order formulation of the program—this first-order variant is less flexible as it replaces the join with (re-)configurable function table \texttt{COMPLETION} by an explicit hard-wired \texttt{CASE} statement, however.

Avoiding Closure Construction. A closer look at the “native” row of Table 2 shows that a growing number of free variables only has moderate impact on BaseX* and Saxon’s native implementations of dynamic function calls: in the second-round experiments, both processors expand the definitions of free variables inside the called function’s body, effectively avoiding the need for an environment. Unfolding optimizations of this kind can also benefit defunctionalization.

The core of such an inlining optimizer is a source-level query rewrite in which closure construction and elimination cancel each other out:

\[
\text{let } v_1 := e_1, \ldots, v_n := e_n \Rightarrow e \quad \Rightarrow \quad v_n := e_n \rightarrow e
\]

As this simplification depends on the closure label \( f \) and the environment contents \( e_1, \ldots, e_n \) to be statically known at the case \( \ldots \) site, the rewrite works in tandem with unfolding transformations:

* Replace \( \\text{let} \)-bound variables by their definitions if the latter are considered simple (e.g., literals or closures with simple environment contents).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Query/Function} & \textbf{Simplified} & \textbf{Simplified} & \textbf{Simplified} \\
\hline
\texttt{SELECT o_orderkey,} & 36010 & 31851 & 18023 \\
\text{dispatch_1()} & 4770 & 2923 & 154 \\
\hline
\end{tabular}
\caption{Example of Table 2: execution times for the simplified \texttt{SELECT} of Figure 18 (defunctionalization and unfolding rewrite applied).}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{} & \textbf{Oracle} & \textbf{Berkeley DB} & \textbf{Sedna} \\
\hline
\textbf{defunctionalization} & 5.03 & 20.60 & 2.56 \\
\textbf{+ unfolding} & 4.99 & 9.29 & 1.31 \\
\textbf{+ simplifications} & 1.28 & 7.45 & 0.98 \\
\hline
\end{tabular}
\caption{Table 4: Impact of unfolding and simplifications on the evaluation of group-by \( (\text{\$seq, function(\$x) \{ \$x mod 100 \}}) \) for \( |\text{\$seq}| = 10^4 \). Averaged wall-clock time measured in seconds.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure20.jpg}
\caption{First-order XQuery code for the example of Figure 18 (defunctionalization and unfolding rewrite applied).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure21.jpg}
\caption{Target query (first-order XQuery) compared to the original query (second-order XQuery).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure22.jpg}
\caption{Execution time comparison of the original and optimized query (second-order XQuery).}
\end{figure}

...
From the start, the design of FQL [10] relied on functions as the primary query building blocks: following Backus’ FP language, FQL offers functional forms to construct new queries out of existing functions. Buneman et al. describe a general implementation technique that evaluates FQL queries lazily. The central notion is that of suspensions, pairs \((f, x)\) that represent the yet unevaluated application of function \(f\) to argument \(x\). Note how group-by in Figure 8 mimics suspension semantics by returning closures (with label \(f_i\)) that only get evaluated (via \(\text{dispatch}_0\)) once a group’s members are required.

A tabular data model that permits function-valued columns has been explored by Stonebraker et al. [32]. Such columns hold QUEL expressions, represented either as query text or compiled plans. Variables may range over QUEL values and an \(\text{exec}(e)\) primitive is available that spawns a separate query processor instance to evaluate the QUEL-valued argument \(e\) at runtime.

Finally, the Map-Reduce model [13] for massively distributed query execution successfully adopts a function-centric style of query formulation. Functions are not first-class, though: first-order user-defined code is supplied as arguments to two built-in functions \text{map} and \text{reduce}—Map-Reduce builds on higher-order function constants but lacks function variables.

Defunctionalized XQuery queries that rely on an element-based representation of closures create XML fragments (closure construction) whose contents are later extracted via child axis steps (closure elimination). When node construction and traversal meet like this, the creation of intermediate fragments can be avoided altogether. Such fusion techniques have been specifically described for XQuery [22]. Fusion, jointly with function inlining as proposed in [16], thus can implement the case \(\cdots\) of cancellation optimization discussed in Section 5. If cancellation is not possible, XQuery processors can still benefit from the fact that node identity and document order are immaterial in the remaining intermediate fragments [19].

7. Closure

We argue that a repertoire of literal function values, higher-order functions, and functions in data structures can lead to particularly concise and elegant formulations of queries. Query defunctionalization enables off-the-shelf first-order database engines to support such a function-centric style of querying. Cast in the form of a syntax-directed transformation of queries, defunctionalization is such a function-centric style of querying. Cast in the form of a syntax-directed transformation of queries, defunctionalization is sufficiently self-contained such that it can be carried over to other languages and systems.

Acknowledgment. We dedicate this work to the memory of John C. Reynolds († April 2013).

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Appendix

A. Defunctionalization for XQuery

This appendix elaborates the details of defunctionalization for XQuery 3.0. The particular formulation we follow here is a deliberate adaptation of the transformation as it has been described by Tolmach and Oliva [33].

We specify defunctionalization in terms of a syntax-directed traversal, \( Q[e] \), over a given XQuery 3.0 source query \( e \) (conforming to Figure 6). In general, \( e \) will contain a series of function declarations which precede one main expression to evaluate. \( Q \) calls on the auxiliary \( D[\cdot] \) and \( E[\cdot] \) traversals to jointly transform declarations and expressions—this makes \( Q \) a *whole-query transformation* [26] that needs to see the input query in its entirety. All three traversal schemes are defined in Figure 21.

\( E \) features distinct cases for each of the syntactic constructs in the considered XQuery 3.0 subset. However, all cases but those labeled (1)–(3) merely invoke the recursive traversal of subexpressions, leaving their input expression intact otherwise. The three cases implement the transformation of literal functions, named function references, and dynamic function calls. We will now discuss each of them in turn.

Case (1): Literal Functions. Any occurrence of a literal function, say \( f = \text{function}(x_1, \ldots, x_n) \cdot \ell \), is replaced by a closure constructor. Meta-level function \( label() \) generates a unique label \( \ell \) which closure elimination will later use to identify \( f \) and evaluate its body expression; see case (3) below. The evaluation of \( e \) depends on its free variables, i.e., those variables that have been declared in the lexical scope enclosing \( f \). We use meta-level function \( fvl() \) to identify these variables \( x_1, \ldots, x_n \) and save their values in the closure’s environment. At runtime, when the closure constructor is encountered in place of \( f \), the closure thus captures the state required to properly evaluate subsequent applications of \( f \) (recall Section 2).

Note that defunctionalization does not rely on functions to be pure: \( e \) to properly evaluate subsequent applications of \( f \) on its free variables, \( e \) body expression constructor. Meta-level function \( label() \) with label \( e \) replaces the dispatcher for arity \( n \) contained literal functions of arity \( n \).

To illustrate, consider the following XQuery 3.0 snippet, taken from the group-by example in Figure 5:

```xquery
for $x$ in distinct-values($keys$)
    return
        function() { $seq = [key(.):=$x$] }.
```

We have \( fvl() \{ [key(.):=$x$] \} = \$x$ \cdot $key$, $seq$. According to \( E \) and case (1) in particular, the snippet thus defunctionalizes to

```xquery
for $x$ in distinct-values($keys$)
    return $x$:[key:$seq$]
```

where \( \ell_1 \) denotes an arbitrary yet unique label.

If we assume that the free variables are defined as in the example of Figure 5, the defunctionalized variant of the snippet will evaluate to a sequence of two closures:

\[
(\ell_1 \cdot (0, \ldots, 1, 2, \ldots)), \ (\ell_2 \cdot (0, \ldots, 1, 2, \ldots)).
\]

These closures capture the varying values \( 0, 1 \) of the free iteration variable \( \$x \) as well as the invariant values of \( \$key \) (bound to a function and thus represented in terms of a closure with label \( \ell_2 \)) and \( \$seq = (0, \ldots, 1, 2, \ldots) \).

Since we will use label \( \ell_1 \) to identify the body of the function literal \( \text{function}() \{ [key(.):=$x$] \} \), case (1) saves this label/body association in terms of a case \( \cdot \) of branch (see the assignment to \( \text{branch} \) in Figure 21). We will shed more light on \( \text{branch} \) and \( \text{lifited} \) when we discuss case (3) below.

Case (2): Named Function References. Any occurrence of an expression \( name\#n \), referencing function \( name \) of arity \( n \), is replaced by a closure constructor with a unique label \( \ell \). In XQuery, named functions are *closed* as they are exclusively declared in a query’s top-level scope—either in the query prolog or in an imported module [28]—and do not contain free variables. In case (2), the constructed closures thus have empty environments. As before, a case \( \cdot \) of branch is saved that associates label \( \ell \) with function \( name \).

Case (3): Dynamic Function Calls. In case of a dynamic function call \( (e_1, \ldots, e_n) \cdot \ell \), we know that expression \( e \) evaluates to some functional value (otherwise \( e \) may not occur in the role of a function and be applied to arguments). Given our discussion of cases (1) and (2), in a defunctionalized query, \( e \) will thus evaluate to a closure, say \( [x_1; \ldots; x_m] \cdot \ell \cdot (m > 0) \), that represents some function \( f \).

In the absence of code pointers, we delegate the invocation of the function associated with label \( \ell \) to a *dispatcher*, an auxiliary routine that defunctionalization adds to the prolog of the transformed query. The dispatcher (i) receives the closure as well as \( e_1, \ldots, e_n \) (the arguments of the dynamic call) as arguments, and then (ii) uses case \( \cdot \) of to select the branch associated with label \( \ell \). (iii) The branch unpacks the closure environment to extract the bindings of the \( m \) free variables (if any) that were in place when \( f \) was defined, and finally (iv) invokes a surrogate function that contains the body of the original function \( f \), passing the \( e_1, \ldots, e_n \) along with the extracted bindings (the surrogate function thus has arity \( n + m \)).

Re (i) and (ii). In our formulation of defunctionalization for XQuery, a dedicated dispatcher is declared for all literal functions and named function references that are of the same arity. The case \( \cdot \) of branches for the dispatcher for arity \( n \) are collected in set \( Dispatch(n) \) when \( E \) traverses the input query (cases (1) and (2) in Figure 21 add a branch to \( Dispatch(n) \) when an \( n \)-ary functional value is transformed). Once the traversal is complete, \( Q \) adds the dispatcher routine to the prolog of the defunctionalized query through \( declare\_dispatch(n, Dispatch(n)) \). This meta-level function, defined in Figure 22, emits the routine \( dispatch_n \) which receives closure \( $cl\oslash \) and \( n \) arguments of the original dynamic call. Discrimination on the label \( \ell \) in set \( dispatch_n \) selects the associated branch. Because \( dispatch_n \) dispatches calls to any \( n \)-ary function in the original query, we declare it with a polymorphic signature featuring XQuery’s most polymorphic type \( \text{item}() \). The PL/SQL variant of defunctionalization, discussed in Section 3.1, relies on an alternative approach that uses typed dispatchers.

Any occurrence of a dynamic function call \( (e_1, \ldots, e_n) \) is replaced by a static call to the appropriate dispatcher \( dispatch_n \).

Figure 8 (in the main text) shows the defunctionalized query for the XQuery group-by example of Figure 5. The original query contained literal functions of arity 0 (in line 8) as well as arity 1 (in line 12). Following case (1), both have been replaced by closure constructors (with labels \( \ell_1 \) and \( \ell_2 \), respectively, see lines 16 and 20 in Figure 8). \( f(vx) \{ vx \mod 2 \} \) is closed: its closure (label \( \ell_2 \)) thus contains an empty environment. Dynamic calls to both functions have been replaced by static calls to the dispatchers \( dispatch_0 \) or \( dispatch_1 \). For the present example, \( Dispatch(0) \) and \( Dispatch(1) \) were singleton sets such that both dispatchers contain case \( \cdot \) of expressions with one branch only. (For an example of a dispatcher with three branches, refer to the PL/SQL function \( dispatch_1 \) in Figure 9, line 19.)

---

11Note that \( E[\cdot] \) defines a separate case for static function calls of the form \( name\#n \).
\(D[\text{declare function } name(x_1, \ldots, x_n) \{ e \}] = \text{declare function } name(x_1, \ldots, x_n) \{ E[e] \}
\)

\[
\begin{align*}
E[\text{for } v \in e_1 \text{ return } e_2] &= \text{for } v \in E[e_1] \text{ return } E[e_2] \\
E[\text{let } v := e_1 \text{ return } e_2] &= \text{let } v := E[e_1] \text{ return } E[e_2] \\
E[\text{if } (e_1) \text{ then } e_2 \text{ else } e_3] &= \text{if } (E[e_1]) \text{ then } E[e_2] \text{ else } E[e_3] \\
E[\text{element } n \{ e \}] &= \text{element } n \{ E[e] \} \\
E[\text{case } e_1] &= E[e_1] = E[e_2] \\
E[\text{name } (e_1, \ldots, e_n)] &= \text{name } (E[e_1], \ldots, E[e_n]) \\
E[\text{function } (x_1 \text{ as } t_1, \ldots, x_n \text{ as } t_n) \text{ as } \{ e \}] &= \{ E[v_1, \ldots, E[v_m]] \text{ Dispatch } = \text{Dispatch } \cup \{ \text{branch} \} \\
&\quad \text{Lifted } = \text{Lifted } \cup \{ \text{lifted} \} \\
&\quad \text{where } \\
&\quad \ell = \text{label}(n) \\
&\quad \{E[v_1, \ldots, E[v_m]] = \text{for } \text{function } (x_1 \text{ as } t_1, \ldots, x_n \text{ as } t_n) \text{ as } \{ e \} \}
\end{align*}
\]

\[
\begin{align*}
E[\text{name } \# n] &= E[\text{label}(n)] \\
&\quad \text{Dispatch } = \text{Dispatch } \cup \{ \text{branch} \} \\
&\quad \text{where } \\
&\quad \ell = \text{label}(n) \\
&\quad \text{branch } = E[\text{name } (n)] \Rightarrow \text{name } (E[b_1, \ldots, E[b_n]) \\
E[e (e_1, \ldots, e_n)] &= \text{dispatch } _n (E[e], E[e_1], \ldots, E[e_n]) \\
Q[d_1; \ldots; d_n; e] &= \forall i \in \text{dom} (\text{Dispatch}) \Rightarrow \text{dispatch } _i (i, \text{Dispatch } (i)) \Rightarrow \text{Lifted } \Rightarrow D[d_1]; \ldots; D[d_n]; E[e]
\end{align*}
\]

Figure 21: Defunctionalization of XQuery 3.0 function declarations (D), expressions (E) and queries (Q).

\[
declare \text{dispatch } (n, \{ \text{case } 1, \ldots, \text{case } k \}) \equiv \\
\begin{align*}
&1 \text{ declare function } \text{dispatch } _n ( \text{\} \\
&2 \text{ \$ clos as closure,} \\
&3 \text{ \$ b } i \text{ as item }()*, \text{ \$ b } n \text{ as item }() \text{ as item }()* \\
&4 \text{ \{ } \\
&5 \text{ \text{ case } \$ clos of} \\
&6 \text{ \text{ case } 1} \\
&7 \text{ \text{ : } } \\
&8 \text{ \text{ case } k} \\
&9 \text{ \) } \\
\end{align*}
\]

Figure 22: Declaring a dispatcher for n-ary functional values.

Re (iii) and (iv). Inside its dispatcher, the case branch for the closure \(x_1, \ldots, x_m\) for function \(f\) invokes the associated surrogate function, also named \(\ell\). The original arguments \(e_1, \ldots, e_n\) are passed along with the \(x_1, \ldots, x_m\). Surrogate function \(\ell\) incorporates \(f\)'s body expression and can thus act as a “stand-in” for \(f\). We declare the surrogate function with the same argument and return types as \(f\)—see the types \(t\) and \(t_1, \ldots, t_n\) in case (3) of Figure 21. The specific signature for \(\ell\) ensures that the original semantics of \(f\) are preserved (this relates to XQuery’s function conversion rules [28, §3.1.5.2]).

While \(f\) contained \(m\) free variables, \(\ell\) is a closed function as it returns the \(m\) bindings as explicit additional function parameters (surrogate function \(\ell\) is also known as the lambda-lifted variant of \(f\) [21]). When case (1) transforms a literal function, we add its surrogate to the set \(\text{Lifted}\) of function declarations. When case (2) transforms the named reference \(\text{name}\# n\), \(\text{Lifted}\) remains unchanged: the closed function \(\text{name}\) acts as its own surrogate because there are no additional bindings to pass. Again, once the traversal is complete, \(Q\) adds the surrogate functions in set \(\text{Lifted}\) to the prolog of the defunctionalized query. Returning to Figure 8, we find the two surrogate functions \(\ell_1\) and \(\ell_2\) at the top of the query prolog (lines 1 to 4).