Strength Model of Unsaturated Clays with Adsorptive Effects

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Abstract. The interactions between liquid phase and solid matrix in unsaturated clays can be divided into capillary effects and physicochemical effects according to their formation mechanism. The physicochemical effects (that is, the adsorption effects) become significant at low saturation degrees. However, current strength models for unsaturated soils almost are established on the basis of capillary mechanism, and the contribution of adsorption effects to soil strength is neglected. A binary-medium shear strength model is firstly proposed, involving the capillary and adsorption effects. The unsaturated clay is assumed to consist of two ideal parts, i.e., the ideal capillarity part and the ideal adsorption part, and then the corresponding ideal strength formulas are established. Furthermore, a participation function is used to reflect the degrees of capillary effects and adsorption effects. Finally, predictions are performed on the shear strength of unsaturated clays. After comparing predicted results and exiting test results, it is shown that the proposed model can well describe the strength of unsaturated clays.

1. Introduction
The strength theory of unsaturated soil is of great significance in many engineering problems such as slope stability, bearing capacity of foundation and soil pressure. Different from the saturated soil, the presence of suction will increase the strength of unsaturated soil, however, the increase of saturation may significantly reduce the bearing capacity of soil and slope safety factor[1]. In the past few decades, many scholars have conducted relevant research and experiments on the strength of unsaturated soils, and these studies mainly focus on the shear strength. So far, most studies on the shear strength of unsaturated soils can be divided into two categories: one is Bishop strength formula of single stress state variable[2] and another is Fredlund strength formula of double stress state variable[3]. Using soil water characteristic curve and shear strength parameters of saturated soils, these strength formulas predict the shear strength of unsaturated soils directly or indirectly. All these strength formulas contain suction term, reflecting the contribution of suction to shear strength. Wheeler[4] pointed out that: two unsaturated soil samples with the same net stress, matrix suction and void ratio but different saturations, can be significantly different on the mechanical behaviors and the interaction force between soil particles (so-called effective stress). That is to, all kinds of external or internal stresses are the same, but the strength and deformation of unsaturated soils are different due to the different saturation (which leads to different internal structure of unsaturated soil). Therefore, when discussing the influence of matric suction on the properties of unsaturated soil, the influence of saturation must be considered in addition to matric suction[5]. The influence of the degree of saturation to stress-strain relationship in unsaturated soils has been obtained more and more attention in recent years. However, it should be noted that most current strength formula of unsaturated soils are developed based on the
capillary model. In fact, the capillary model is invalid at low degree of saturation. Thus, it is not suitable to establish the strength formula when the degree of saturation of unsaturated sols is low, and the adsorptive matrix suction should be involved. In addition, the adsorptive part is significant in the clay (such as expensive soils) even at high saturation. Therefore, a strength theory, which considers the different roles of the capillary effect and adsorptive effect to soil strength, should be established.

In view of the problems existing in the strength theory of unsaturated soil, the different impacts of capillary effects and adsorptive effects on the strength of unsaturated clay are comprehensively considered. In this paper, a shear strength model which can comprehensively consider the capillary effect and the adsorptive effects is established.

2. Shear strength model of unsaturated clays

When discussing the strength of unsaturated soil, the concept of suction should be clearly understood. The suction usually includes matrix suction and solute suction. When the chemical concentration of pore water is not considered, the effect of solute suction can be ignored, and the matrix suction is mainly concerned. Matrix suction is generally composed of capillary part and adsorptive part. The suction used in engineering usually refers to the matrix suction, which is closely related to water content or saturation, and is usually characterized by soil water characteristic curve. It should be noted that the matrix suction in the soil water characteristic curve includes not only the capillary part, but also the adsorptive part. The capillary part can be expressed as $s = u_w - u_g$ (where $u_g$ is pore gas pressure, $u_w$ is pore water pressure), while the adsorptive part is caused by the physical-chemical interaction between solid and liquid phases. Gens\cite{6} pointed out that when the matrix suction is large, the degree of liquid phase adsorption to solid phase is larger, but it should not be considered that the traditional negative pore water pressure is large. The two parts of matrix suction are obviously different in concept, and the mechanism and influence on the strength of unsaturated soil are also different.

So far, most of the strength formulas of unsaturated soil are based on capillary effects, and the matrix suction varies with the saturation in the range of $0 \sim 1$, which is equivalent to assuming that capillary effects plays a major role in the whole range of saturation. But in fact, the capillary effects is very small at low saturation, so the adsorptive effects should be considered in order to establish a more reasonable strength formula. The influence of capillary effects and adsorptive effects on unsaturated soil behaviors mainly depends on soil type and water content. For non-cohesive soil or soil with higher saturation, the capillary part plays a controlling role; while for clay with higher plasticity index or low saturation, the adsorptive part plays a controlling role\cite{7}. Therefore, when establishing the strength model, we should consider the effect of adsorptive action and its difference from capillary action.

In order to consider the difference between the two effects effectively, the unsaturated soil is abstracted into two ideal parts, namely ideal capillary part and ideal adsorptive part, and then the corresponding ideal strength formulas are established respectively. Following the modeling method of binary medium model\cite{8-10}, the strength theory of binary medium for unsaturated soil is established, that is, combining two ideal strength formulas by introducing participation variables.

2.1. Ideal capillary strength formula

The capillary matrix suction can be expressed as $s = u_w - u_g$. However, due to the influence of gasification, the tension in pore water cannot be greater than a certain limit value, which represents the critical phase change value of water from liquid state to gas state. The limit value can be expressed as $s_{u_g}$. When the matrix suction is greater than $s_{u_g}$, the pore water vaporizes and the capillary mechanism becomes invalid. Baker and Frydman\cite{11} analyzed the change of suction measured by different types of soil with water content and dry density, and gave the range of $s_{u_g}$ was $100 \sim 400kPa$, by summarizing the research results for $s_{u_g}$. It can be seen that the scope of the capillary mechanism is relatively limited. The capillary mechanism in the soil fails at higher suction levels, and the strength of the soil is
controlled by the adsorptive effect. However, most of the existing strength formulas of unsaturated soil ignore this and cannot accurately predict the strength under higher suction levels.

In the ideal capillary situation, we assume that the mechanical and hydraulic behaviors of soil are only related to the capillary effects between the solid-liquid phase, that is, the influence of physicochemical interaction between the solid-liquid phases on soil strength is ignored, and only the influence of capillary effects is considered. In the actual soil, when the suction is less than air intake value, the soil is nearly saturated, which can be approximated as an ideal capillary condition. Under this assumption, referring to the existing strength theories based on macro capillarity, the ideal capillary strength formula can be expressed as follows:

\[ \tau_f = c' + (\sigma - u_a)\tan\phi' + \tau_c \]  

Where, \( c' \) is the effective cohesion, \( \phi' \) is the effective internal friction angle, \( \sigma \) is the normal stress, \( u_a \) is the pore gas pressure, and \( \tau_c \) is the additional strength caused by capillary effects. It is noteworthy that the effect of suction on the strength of unsaturated soil is different from the effective stress caused by load. It cannot be simply regarded as an increase in effective stress or pressure, but should be considered separately. In this paper, the added value of strength caused by capillary effects is regarded as friction strength.

When describing capillarity, in addition to suction, the number and range of suction action, that is saturation, should also be considered\[12\]. In this paper, \( S_c \) is used to represent the capillary effects, and the additional strength can be expressed as:

\[ \tau_c = (u_a - u_w)S_c \tan \alpha \]  

Where, \( \alpha \) is the friction angle corresponding to capillarity, describing the contribution of capillarity to the additional strength; Oberg\[13\] also gave a similar form of additional strength, and considered that \( \tan \alpha = \tan \phi' \).

Substituting formula (2) into formula (1), the ideal capillary strength formula is:

\[ \tau_f = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)S_c \tan \alpha \]  

2.2. Ideal adsorptive strength formula

For clay with higher plasticity index or low saturation, the traditional capillary model is no longer applicable. It is necessary to consider the complex interaction between solid and liquid phases, that is, the so-called physicochemical effects. The contribution of \( (u_a - u_w) \) type matrix suction to the strength has been invalid, thus how to determine the additional strength caused by adsorptive effects is the key to establish the ideal adsorptive strength formula. Zhou\[14\] proposed a soil water characteristic curve model that consider capillary effects and adsorptive effects separately, but consider the strength of unsaturated soil is only controlled by capillary effects. Karube\[15\] pointed out that with the increase of suction, the additional strength caused by suction increases nonlinearly; and when the suction in the soil is large enough, the additional strength caused by adsorptive suction tends to a certain value.

In the case of ideal adsorption, it is assumed that the interaction between solid and liquid phases in soil is only caused by physicochemical mechanism, that is, the influence of capillary effects between solid and liquid phases on soil strength is ignored, and only physicochemical effects is considered. Under this assumption, the effect of adsorption on the strength of unsaturated soil is regarded as an increase in additional strength, rather than an increase in effective stress or pressure. The ideal adsorptive strength formula can be expressed as:

\[ \tau_{ad} = c' + (\sigma - u_a)\tan\phi' + \tau_{ad} \]  

Where, \( \tau_{ad} \) is the additional strength caused by adsorption.
According to the description of capillary gasification value in the previous paper, when the matrix suction is greater than $s_w$, the capillary mechanism becomes invalid. At this time, the strength of soil will be controlled by adsorption, and the influence of adsorption will be reflected by the change of saturation. Therefore, the additional strength of adsorption which is established with matric suction is quantitative and saturation is independent variable, can be expressed as:

$$\tau_{ad} = s_w \zeta(S_r)$$  \hspace{1cm} (5)

Where, $s_w$ is capillary gasification value, $\zeta(S_r)$ is a function of saturation.

In the actual soil, the capillary effects is already very small at low saturation, and the main effect is absorption. According to this situation, the strength test results of unsaturated soil under different lower saturation can be selected to determine the function form of $\zeta(S_r)$.

In view of the influence of absorption on the mechanical properties of unsaturated soil under low saturation, more and more scholars have started the relevant theoretical and experimental research. By summarizing the existing shear test results of unsaturated soil under low saturation[16-18], a fitting analysis of the variation of additional strength is carried out, as shown in Fig. 1-4.

![Figure 1. Fitting analysis of test results on Madrid grayclay[16].](image1)

![Figure 2. Fitting analysis of test results on Madrid clay sand[16].](image2)

![Figure 3. Fitting analysis of test results on Speswhite kaolin clay[17].](image3)

![Figure 4. Fitting analysis of test results on silty clay[18].](image4)
According to the soil water characteristic curve of the test soil samples, $s_m$ is determined respectively, and the corresponding mechanical parameters and fitting results are given in the figure. Based on the fitting analysis of the test results, it is found that the additional strength increases nonlinearly with the decrease of saturation, and the growth rate decreases and tends to be gentle; when the saturation is small enough, the additional strength tends to a certain value. Finally, it is determined that: $\xi(S_s) = a(1-S_s^b)$, $a,b$ is fitting parameter. So, the ideal capillary strength formula can be obtained as:

$$\tau_{sc} = c' + (\sigma - u_w)\tan \phi' + s_m(1 - S_s^b)$$ (6)

2.3. Binary medium strength model

The unsaturated soil is abstracted into two ideal parts in the previous sections, and the ideal capillary and adsorptive strength formulas are then established respectively. However, both capillary and adsorption effects coexist between liquid and solid matrix in unsaturated soil. Thus, it is necessary to consider the real soil as a mixture of two ideal parts.

A participation function $\xi$ is introduced to reflect the proportions of capillary effect and adsorptive effect between solid and liquid phases in unsaturated soils. $\xi$ can be defined as the share rate of ideal capillary part to the overall shear strength of soil, then $(1 - \xi)$ is the share rate of ideal adsorptive part to the overall shear strength of soil. The overall shear strength of unsaturated clay $\tau_f$, therefore, can be expressed as follows:

$$\tau_f = \xi \tau_{sc} + (1 - \xi) \tau_{ad}$$ (7)

Where, $\tau_{sc}$ is the shear strength of idea capillary part, $\tau_{ad}$ is the shear strength of idea adsorptive part. The experimental study shows that with the decrease of water content, the deviation between real soil behavior and the ideal behavior under ideal capillary condition is greater$^{[10]}$. The participation function is related to saturation in this paper and can be expressed as follows:

$$\xi = \left\{ \left[ 1 - \frac{1 - S_s}{1 - S_s^m} \right] \right\}^\gamma$$ (8)

where $S_s^m$ is the saturation corresponding to capillary gasification value, $\gamma$ is the fitting parameter, $\{\}$ is McCauley bracket. $S_s = 1$, that is, $\xi = 1$ means that the soil is completely in the ideal capillary condition, and the shear strength is completely controlled by capillary action. $S_s \leq S_s^m$, that is, $\xi = 0$ means that the soil is completely in the ideal adsorptive condition, and the shear strength is completely controlled by the adsorptive action. $S_s^m < S_s < 1$, that is, $0 \leq \xi \leq 1$ means that the soil is in the intermediate condition between the two ideal situations, and the shear strength is jointly undertaken by capillary and adsorptive actions.

Substituting the ideal capillary strength formula (3), the ideal adsorptive strength formula (Eq. (6)) and the participation function formula (Eq. (8)) into Eq. (7), the shear strength of the whole soil can be obtained:

$$\tau_f = c' + (\sigma - u_w)\tan \phi' + \tau_s$$ (9)

where $\tau_s$ is the additional strength of unsaturated soil caused by matric suction and can be expressed as:

$$\tau_s = \xi \tau_c + (1 - \xi) \tau_{ad} = \xi(u_w - u_{w,0})S_s \tan \alpha + (1 - \xi)s_m(1 - S_s^b)$$ (10)
When VG model is adopted for soil water characteristic curve, the relationship between $S_r$ and $(u_a - u_n)$ can be represented as follows:

$$S_r = \frac{1 - S_{r}^{res}}{\left[1 + \left(\alpha (u_a - u_n)^n\right)^{1/n}\right]^{1/n} + S_{r}^{res}}$$  \hspace{1cm} (11)$$

Where $\alpha$ and $n$ are the fitting parameters, $S_{r}^{res}$ is the residual saturation.

A compressive shear strength model of unsaturated clay, completely considering the influences of capillary effect and adsorptive effect on soil strength, is then established.

### 3. Model prediction and verification

In order to verify the rationality of the model, the prediction results of the strength model are compared with the corresponding test data. Firstly, the compression shear strength model of unsaturated clay is verified, and the strength test results of escario\textsuperscript{[16]} and Tarantino\textsuperscript{[17]} are selected. Among them, Madrid grayclay\textsuperscript{[16]} is composed of 72% clay, 27% silt and 1% sand, with liquid limit and plastic limit of 57% and 29% respectively; speswhite kaolin clay\textsuperscript{[17]} is composed of 80% clay and 20% silt, with liquid limit and plastic limit of 64% and 32% respectively. The net stress of the two clays is 300kPa and 600kpa respectively in the shear test.

The mechanical indexes and model parameters of test clay are shown in Table 1. The parameters in the ideal capillary strength formula can be obtained from the shear test results of saturated soil; the parameters in the ideal viscosity strength formula can be obtained from the soil water characteristic curve and the shear test results at low saturation; the parameters in the participation function are used to control the prediction results of the strength model. At the same time, according to the soil water characteristic curve, the saturation in the strength model can be transformed into matrix suction to reflect the variation of soil strength with matrix suction.

| soil sample            | $c'$ (kPa) | $\phi'$ (°) | $s_n$ (kPa) | $S_{r}^{\alpha}$ | $a$  | $b$  |
|------------------------|------------|-------------|-------------|------------------|-----|-----|
| Madrid gray clay       | 30         | 25.3        | 300         | 0.814            | 1.628 | 1.76 |
| Speswhite kaolin clay  | 14.8       | 16.89       | 300         | 0.766            | 0.4471 | 3.307 |

The comparison of strength test data and model prediction results of unsaturated clay are illustrated in Figures 5 and 6. It can be seen that the growth rate of soil strength gradually decreases with the increase of matrix, and the shear strength of soil tends to a certain value when the matric suction in soil is large enough (Figure 5 is drawn in the logarithmic coordinate of matrix suction in order to reflect the prediction effect). This feature are described by using the variation of the participation function $\xi$ in this model. In addition, the influence of the parameters in the participation function $\gamma$ on the bending degree of the strength curve is also discussed. The values of parameters $\gamma$ are taken as 2.0, 1.0 and 0.5 respectively. The comparison with the three verification results show that the bending degree of soil strength curve with matrix suction increases gradually with the decrease of parameters $\gamma$. This is because the parameter $\gamma$ is directly proportional to the participation function. Therefore, under a given water content, the smaller the parameter $\gamma$, the greater the contribution of adsorptive effect to the soil strength.
4. Conclusions
A binary medium shear strength model is established for unsaturated clay in this paper. The different influences of capillary effect and adsorptive effect between liquid and solid matrix on soil strength are considered in the proposed model. This model consists of ideal capillary strength formula and ideal adhesion strength formula, and the proportion of these two effects can be reflected by a participation function. Compared with the test results of unsaturated clay with the prediction results given by the proposed model, the unsaturated clay shear model can better describe the shear strength characteristics of soil in a wide range of suction. When the matric suction is large enough, the compressive shear strength of soil tends to a certain value.

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References
[1] Liu Yan, Zhao Chenggang, Cai Guoqing, et al. Research progress of unsaturated soil mechanics[J]. Mechanics in Engineering, 2015, 37(4): 457–465.
[2] Bishop A W. The principle of effective stress[J]. TekniskUkeblad, 1959, 106(39): 113–143.
[3] Fredlund D G, Morgenstern N R, Widger R A. The shear strength of unsaturated soils[J]. Canadian Geotechnical Journal, 1978, 15: 313–321.
[4] Wheeler S J, Sharma R S, Buisson M S R. Coupling of hydraulic hysteresis and stress-strain behaviour in unsaturated soils[J]. Géotechnique, 2003, 53(1): 41–54.
[5] Zhao Chenggang, Wei Changfu, Cai Guoqing. Development and challenge for soil mechanics[J]. Rock and Soil Mechanics, 2011, 32(12): 3521–3540.
[6] Gens A. Soil-environment interactions in geotechnical engineering[J]. Géotechnique, 2010, 60(1): 3–74.
[7] Zhao Chenggang, Li Jian, Liu Yan, et al. Discussion on some fundamental problems in unsaturated soil mechanics[J]. Rock and Soil Mechanics, 2013, 34(7): 1825–1831.
[8] Shen Zhuijiang. Breakage mechanics for geological materials: An ideal brittle-elasto-plastic model[J]. Chinese Journal of Geotechnical Engineering, 2003, 25(3): 253–257.
[9] Liu Enlong, Shen Zhuijiang. Binary medium model for structured soils[J]. Journal of Hydraulic Engineering, 2005, 36(4): 391–395.
[10] Li J, Zhao C G, Cai G Q, et al. A model considering solid-fluid interactions stemming from capillarity and adsorption mechanisms in unsaturated expansive clays[J]. Science Bulletin, 2014, 59(26):3314–3324.

[11] Baker R, Frydman S. Unsaturated soil mechanics: Critical review of physical foundations[J]. Engineering Geology, 2009, 106(1-2): 26–39.

[12] Gallipoli D, Gens A, Sharma R, et al. An elasto-plastic model for unsaturated soil incorporating the effect of saturated degree on mechanical behaviour. Géotechnique, 2003, 53(1): 123–135.

[13] Oberg A, Sallfors G. Determination of shear strength parameters of unsaturated silts and sands based on the water retention curve. Geotechnical Testing Journal, 1997, 20(1):40–48.

[14] Zhou A, Huang R, Sheng D. Capillary water retention curve and shear strength of unsaturated soils[J]. Canadian Geotechnical Journal, 2016, 53.

[15] Karube D, Kawai K. The role of pore water in the mechanical behavior of unsaturated soils[J]. Geotechnical and Geological Engineering, 2001, 19(3):211–241.

[16] Escario V, Juca J. Strength and deformation of partly saturated soils. In Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering (ICSMFE), Rio de Janeiro, 1990: 43–46.

[17] Tarantino A, Tombolato S. Coupling of hydraulic and mechanical behaviour in unsaturated compacted clay[J]. Géotechnique, 2005, 55(4):307–317.

[18] Shen Xizhong, Guan Xinjian, Lan Yuan. Calculation of effective strength indexes of unsaturated low liquid limit clay[J]. Rock and Soil Mechanics, 2007, 29(Supp.1): 207–210.