Natural \( \alpha \)-Attractors from \( \mathcal{N} = 1 \) Supergravity via flat Kähler Manifolds

Tony Pinhero\(^1\,*\)

\(^1\)Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B. T. Road, Kolkata 700108, India.

We present \( \alpha \)-attractor models for inflation based on \( \mathcal{N} = 1 \) supergravity with flat Kähler manifolds. The function form of the associated Kähler potential in these models are logarithmic square in nature and has a visible shift symmetry in its composite canonical variables. The scalar potential \( V \) with respect to these field variables has an infinitely long dS valley of constant depth and width at large values of inflaton field \( \psi \) and attains a Minkowski minimum at small \( \psi \). We illustrate this new framework with a couple of examples.

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I. INTRODUCTION

One can give the best definition to cosmological \( \alpha \)-attractors \([1–15]\) in terms of the non-canonical real field variable \( \phi \), through a toy Lagrangian as,

\[
L = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} \phi \partial_{\mu} \partial^{\mu} \phi - V(\phi) \right]
\]  

(1)

One of the key properties of this theory is that, inflationary observational predictions are to a large extent determined by the field space metric or simply, by the geometry of the moduli space, but not by the potential term. This means that in the leading order approximation in \( 1/N_\epsilon \), where \( N_\epsilon \) is the number of e-folds, the observational predictions are stable under significant modifications of the inflationary potential. The reason for such observational predictions of these theories is that their kinetic term has a second order pole. To understand this in detail, we provide a brief technical discussion of these models \([4]\). Models of the form defined in (1) can be generally written in terms of the field variable \( \rho \) as,

\[
L = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} K_E(\rho) (\partial \rho)^2 - V_E(\rho) \right]
\]  

(2)

Now, if we assume that the pole of the kinetic term is located at \( \rho = 0 \), the Laurent expansion of the \( K_E \) term is as follows

\[
K_E = \frac{a_p}{\rho^p} + \ldots, \quad V_E = V_0 (1 + c \rho + \ldots)
\]  

(3)

Where \( a_p \) is the residue at the leading order pole of order \( p \). With the use of these information, one can derive the expression for the spectral index \( n_s \) and tensor-to-scalar ratio in the leading order approximation of \( 1/N_\epsilon \) \([4]\):

\[
n_s = 1 - \frac{p}{p - 1} \frac{1}{N_\epsilon}, \quad r = \frac{8 c_{\rho}^{-1} a_p^{1 - \frac{1}{p - 1}}}{(p - 1)^{\frac{1}{p - 1}}} \frac{1}{N_\epsilon^2}
\]  

(4)

In this expression, note that the spectral index completely depends upon the order of the pole whereas the tensor-to-scalar ratio depends not only on the leading pole and its corresponding residue. One can see that the contribution from the potential term \( c \) vanishes for the pole of order two in the expression of \( r \). The Lagrangian defined in (1) has a leading pole of order two with residue \( \frac{2}{3} a \), and hence the observational predictions are insensitive to the structure of the potential. As a result, (4) boils down to

\[
n_s = 1 - \frac{2}{N_\epsilon}, \quad r = \frac{12a}{N_\epsilon^2}
\]  

(5)

for the theory (1). These predictions are exactly matching with the latest observations made by Planck \([16, 17]\).

It is well-known that these models (1) can be successfully embedded in different Supergravity theories. In particular, the following equivalent choices of the Kähler potential \([5, 6]\)

\[
K = -3a \log (1 - ZZ^*) \text{ or } K = -3a \log (T + T^*), \quad (6)
\]

along with different choices of superpotentials embeds these models in \( \mathcal{N} = 1 \) Supergravity. Here \( Z \) and \( T \) are the inflaton superfields in terms of the disk variables and half plane variables respectively. The particular choice of the superpotential depends on whether the stabilizer field \( S \) is inside or outside the logarithmic term of the Kähler potential. Such an embedding is also possible with slight modifications in the above defined Kähler potentials (see \([7–9]\)). All these models are based on the logarithmic Kähler potentials and this logarithmic Kähler potential is the signature of the hyperbolic geometry in supergravity. For instance, the Kähler metric based on the Kähler potential in terms of the disk variables \( Z \) is given \([6]\) as

\[
ds^2 = \frac{3a}{(1 - ZZ^*)^2} dZ dZ^*
\]  

(7)

and the curvature of corresponding Kähler manifold is given by \( R_{\text{Kähler}} = -2/3a \). If we decompose the field \( Z \)
in terms of its real field variables \( Z = (x + iy)/\sqrt{3\alpha} = r e^{i\theta}/\sqrt{3\alpha} \), this metric takes the form

\[
d s^2 = \frac{dx^2 + dy^2}{(1 - \frac{x^2 + y^2}{3\alpha})^2} = \frac{dr^2 + r^2d\theta^2}{(1 - \frac{r^2}{3\alpha})^2}.
\]

(8)

This resembles the 2d metric of a Poincaré disc of the hyperbolic geometry with constant negative curvature \( \mathcal{R}_{\text{Poincaré}} = -2/3\alpha \). Thus, one can claim that \( \alpha \)-attractor models in \( \mathcal{N} = 1 \) Supergravity are based on the Poincaré disc or the half-plane model of hyperbolic geometry \([6]\). Based on these geometrical aspects, the observational predictions (5) can be read as,

\[
n_s = 1 - \frac{2}{N_e}, \quad r = \mathcal{R}_E^2 \frac{4}{N_e^2}
\]

(9)

where \( \mathcal{R}_E = \sqrt{3\alpha} \) is the radius of the Poincaré disc. The relation of this radius to the curvature of the Poincaré disc is given by \( \mathcal{R}_E^2 = -2 / \mathcal{R}_{\text{Kähler}} \). Thus, the primary focus of \( \alpha \)-attractor models is to understand the geometry of the scalar manifold from the observations rather than achieving the traditional goal of reconstruction of the inflationary potential.

After all these success of \( \alpha \)-attractor models, recently some models \([18, 19]\) come up with the almost identical predictions of \( \alpha \)-attractors and these models have a nearly flat kähler geometry origin in \( \mathcal{N} = 1 \) supergravity. At certain limits of these theories, these models boil down to the conformal attractor models \([1, 2]\) which are the part of the \( \alpha \)-attractor family at \( \alpha = 1 \). From this perspective, we ask whether the \( \alpha \)-attractor models can have a flat kähler geometry origin in \( \mathcal{N} = 1 \) supergravity and for any value of \( \alpha > 0 \), these models are stable.

In the following section, we present two models of \( \alpha \)-attractors in \( \mathcal{N} = 1 \) supergravity, each based on a logarithmic square Kähler potential, and we show that associated Kähler manifold of each of these potentials is geometrically flat.

II. \( \alpha \)-ATTRACTION FROM FLAT KÄHLER GEOMETRY

A. Model-I

We start by defining a Kähler potential of the form

\[
K = -\frac{3\alpha}{8} \log^2 \left[ \frac{(1 + \Phi)(1 - \Phi^*)}{(1 - \Phi)(1 + \Phi^*)} \right] + SS^* - \zeta(SS^*)^2
\]

(10)

where \( \Phi \) is the inflaton superfield (non-canonical) and \( S \) is the chiral multiplet which serves the job of the stabilizer field or the nilpotent superfield. During inflation, the inflaton partner \( \Phi - \Phi^* \) and the stabilizer field attains zero vev and hence the Kähler potential vanishes. This is obviously due to the inflaton shift symmetry in the Kähler potential and this shift symmetry is explicitly visible when one writes the above Kähler potential in its series form as

\[
K = -\sum_{n=\text{odd}}^{N} K^{(n)} (\Phi^n - \Phi^{*n}) \right]^2 + SS^* - \zeta(SS^*)^2
\]

(11)

with

\[
K^{(n)} = \frac{1}{n} \sqrt{\frac{3\alpha}{2}}, \quad N \to \infty.
\]

(12)

The index \( n \) takes only odd positive integer values and \( K^{(n)} \) are the dimensionless coupling constants for the self interactions of chiral superfields. This Kähler potential is invariant under the following shift transformation:

\[
\sum_{n=\text{odd}}^{N} K^{(n)} \Phi^n \rightarrow \sum_{n=\text{odd}}^{N} K^{(n)} \Phi^n + C_N
\]

(13)

Because of the invariance (13), one can consider \( \sum_{n=\text{odd}}^{N} K^{(n)} \Phi^n \) as a composite field \( \Phi \), which transforms under a Nambu-Goldstone like shift symmetry. This shift symmetry generalizes the shift symmetries proposed in \([20-22]\) in the context of supergravity realization of the chaotic inflation and running kinetic inflation. Due to this shift symmetry, the real component of the composite field \( \Phi \) will be absent in the Kähler potential (11), and this real component \( \text{Re} \left[ \sum_{n=\text{odd}}^{N} K^{(n)} \Phi^n \right] = \text{Re} \left[ \Phi \right] \) can be identified as the inflaton scalar field \([21, 22]\). Since physics is invariant under field transformation, one can also proceed the same analysis in terms of the real non-canonical variables. In such a scenario, real part of \( \Phi \) can be identified as inflaton in terms of the non-canonical chiral field \( \Phi \). In order to explicitly show that the non-canonical kinetic term arising from this model also has a pole of order two with respect to the real field variable, we continue our investigation in terms of the non-canonical variables from the Kähler potential defined in (10). This investigation and the subsequent analysis based on the series Kähler potential (11) is relegated to Appendix A. The Kähler potential defined in (10) can give the following kinetic term for inflation

\[
\frac{1}{\sqrt{g}} L_{\text{kin}} = -\frac{3\alpha}{(1 - \Phi^2)(1 - \Phi^{*2})^2} \partial_{\mu} \Phi \partial_{\mu} \Phi^* - \left(1 - 4\zeta S^* S\right) \partial_{\mu} S \partial_{\mu} S^*
\]

(14)

As in the traditional way, we also consider a small breaking term of the shift symmetry in the superpotential,
\[ W = mS \Phi \] (15)

for the successful inflation. This small breaking term ensures a tree-level mass for the field \( \Phi \) through F-term of the stabilizer field \( S \). When \( m \to 0 \), we have an enhanced shift symmetry so that our model is natural according to 't Hooft's sense [23]. The superpotential considered in (15) is completely reserved for the construction of T-model of \( \alpha \)-attractors. However, to construct a general \( \alpha \)-attractor model, one can also define a superpotential of the form,

\[ W = mSf(\Phi) \] (16)

where \( f(\Phi) \) is the general function of the chiral superfield \( \Phi \). Now, let us decompose these complex chiral superfields \( \Phi \) and \( S \) into real scalar fields

\[ \Phi = \frac{1}{\sqrt{6\alpha}} (\phi + i\chi), \quad S = \frac{1}{\sqrt{2}} (s + i\beta) \] (17)

Based on the F-term potential of \( N = 1 \) SUGRA,

\[ V = e^K \left( D_\phi W K^{ij} D_\chi W^* - 3 |W|^2 \right). \] (18)

and with the kinetic term (14), one will end up with the total Lagrangian for the inflation real field \( \phi \) along the inflationary trajectory \( \chi = s = \beta = 0 \) as

\[ L = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{(1 - \phi^2/6\alpha)^2} \frac{1}{2} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right] \] (19)

with

\[ V(\phi) = \begin{cases} 
    m^2 \phi^2/6\alpha, & \text{for } W = mS \Phi \\
    m^2 \phi^2/(\sqrt{6\alpha}), & \text{for } W = mSf(\Phi).
\end{cases} \] (20)

The Lagrangian (19) with (20) defines the complete \( \alpha \)-attractor model for inflation in terms of non-canonical real field variable \( \phi \).

Next, we study the stability of the inflationary trajectory with respect to the small fluctuations of the fields \( \chi \) and \( S \). In order to check this stability, one can calculate the canonical masses of all the fields orthogonal to the inflaton direction \( \phi \) based on the F-term potential of SUGRA (18) for the theory (10) with (15) or (16) at \( \chi = S = 0 \) as

\[ m^2_{\chi} = 6H^2 (1 + \epsilon - \eta/2) \] (21)

\[ m^2_\phi = 12H^2 (\zeta + \epsilon/4) \] (22)

and the canonical mass of the inflaton is

\[ m^2_\phi = 3H^2 \eta \] (23)

Here we have used \( V(\phi) = 3H^2 \) where \( H \) is the Hubble’s constant, and \( \epsilon \) and \( \eta \) are the slow-roll parameters of inflation in terms of non-canonical variable \( \phi \). In order to get the canonical masses for both inflaton \( \phi \) and its partner \( \chi \), we have multiplied by the factor \((1 - \phi^2/6\alpha)^2\) to the second derivatives of their potentials. From (21) it is evident that for any value of \( \alpha \), \( m^2_{\chi} > \mathcal{O}(H) \), i.e., the field \( \chi \) is heavy and reaches its minimum quickly. Moreover from (22), for \( \zeta \geq 1/12 \), \( m_\phi \to \mathcal{O}(H) \), is also heavy and vanish. Therefore, during inflation all these fields are stabilized at the inflationary trajectory \( \chi = S = 0 \).

Now, we focus on the properties of supergravity potential (18) based on our model. For the simplest case, we consider T-model potential based on (10) and (15) in terms of the non-canonical real variables \( \Phi = (\phi + i\chi)/\sqrt{6\alpha} \). From Fig. (1), it is evident that existence of flat direction is not visible in terms of these non-canonical variables \( \phi \) and \( \chi \) for the theory (10) and (15) due to the existence of the non-canonical kinetic term in it. In order to see the flat direction explicitly in the potential, we have to switch to more suitable variables, say, canonical variables for both \( \phi \) and \( \chi \) fields. So we will use the variables \( \Phi = \tanh \Phi \), which is related to \( \Phi \) as \( \Phi = \tanh \Phi \). In terms of this new variables kinetic terms for both the fields are canonical, i.e.,

\[ L_{kin} = \frac{1}{2} \left( \partial_\phi^2 + \partial_{\bar{\phi}}^2 \right) \] (24)

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1 It is well known that, due to the shift symmetry (13) in terms of its canonical composite variables \( \Phi \), \( m_{\phi\phi}(\Phi) > \mathcal{O}(H) \) for any choice of the superpotential [24]. Since physics is invariant under the field transformation, it is well expected that the canonical mass of non-canonical field \( m_{\phi\phi} \) is \( \mathcal{O}(H) \). However, we provide the explicit calculations for the same here.
for any values of both \( \psi \) and \( \theta \). Further, the potential (18) at \( S = 0 \) takes the form

\[
V = m^2 e^\theta \left| \tanh \left( \frac{\psi + i\theta}{\sqrt{6\alpha}} \right) \right|^2
\]

\[
= m^2 \cosh \sqrt{\frac{2}{3\alpha} \psi - \cos \sqrt{\frac{2}{3\alpha} \theta} e^\theta}
\]

This potential is depicted in the Fig.(2), which is the three dimensional plot of T-model potential with symmetric shoulders and visible flat directions. One can see that this scalar potential has a Minkowski minimum at small values of the inflaton field \( \psi \) and has infinitely long dS valley at large values of \( \psi \) with constant depth and width. Thus, this potential closely resembles to the one obtained from the theory based on the hyperbolic geometry of \( \alpha \)-attractors [25].

Finally, we focus on the most important part of our model, which is the geometry of the \( \text{Kähler} \) manifold. It is quiet evident that because of the shift symmetry (13) of the \( \text{Kähler} \) potential (11), \( \text{Kähler} \) manifold is flat in terms of its canonical variables. More clearly, due to the shift symmetry, the series \( \text{Kähler} \) potential attains a canonical form in terms of its composite canonical fields \( \hat{\Phi} = \sum_{n=odd}^N K^{(n)} \) given by

\[
K = k (\hat{\Phi} - \Phi^*)^2 + SS^* - \zeta(SS^*)^2
\]

(26)

where \( k \) is some arbitrary constant. As an another example, one can also write the \( \text{Kähler} \) potential (10) in a more convenient form as

\[
K = -\frac{3\alpha}{2} \left( \tanh^{-1} \Phi - \tanh^{-1} \Phi^* \right)^2 + SS^* - \zeta(SS^*)^2.
\]

(27)

This \( \text{Kähler} \) potential is also invariant under the following shift symmetric transformation:

\[
\tanh^{-1} \Phi \to \tanh^{-1} \Phi + C
\]

(28)

In this case, we choose composite canonical field \( \hat{\Phi} = \tanh^{-1} \Phi \). This is why we obtained canonical kinetic terms for both \( \psi \) and \( \theta \) fields with an infinite dS valley potential earlier with these composite fields (see 24 and 25). In terms of this canonical field \( \hat{\Phi} \), the above \( \text{Kähler} \) potential takes the same form as (26) which is the well known \( \text{Kähler} \) potential proposed in [20] with flat \( \text{Kähler} \) geometry. Thus, our \( \text{Kähler} \) manifold is also flat in terms of its canonical variables. As physics is invariant under field transformation, we expect same for non-canonical variables as well. However, we explicitly calculate the curvature for \( \text{Kähler} \) manifold in terms of its non-canonical variable \( \Phi \) from the \( \text{Kähler} \) potential (10). Based on this \( \text{Kähler} \) potential, metric of the moduli space is defined as

\[
d\bar{s}^2 = g_{\Phi\Phi^*} d\Phi d\Phi^*
\]

(29)

where

\[
g_{\Phi\Phi^*} = K_{\Phi\Phi^*} = \frac{3\alpha}{(1 - \Phi^2)(1 - \Phi^*2)}.
\]

(30)

Now, we proceed to compute the non-vanishing Levi-Civita connection coefficients, Riemannian tensors, and the curvature of the moduli space associated with this \( \text{Kähler} \) metric. They are given as

\[
\Gamma^{\Phi}_{\Phi\Phi} = \frac{2\Phi}{(1 - \Phi^2)}, \quad \Gamma^{\Phi^*}_{\Phi\Phi^*} = \frac{2\Phi^*}{(1 - \Phi^*2)},
\]

(31)

\[
\mathcal{R}^{\Phi\Phi}_{\Phi\Phi^*} = \partial\Phi \partial\Phi^* \Gamma^{\Phi}_{\Phi\Phi} = 0.
\]

(32)

Since all components of these Riemannian tensors vanish, curvature of the \( \text{Kähler} \) manifold is \( \mathcal{R}_{\text{Kähler}} = 0 \). Alternatively, from the definition of curvature of \( \text{Kähler} \) manifold via the metric:

\[
\mathcal{R}_{\text{Kähler}} = -\delta^{-1}_{\Phi\Phi^*} \partial\Phi \partial\Phi^* \log g_{\Phi\Phi^*} = 0.
\]

(33)

From the above, we conclude that geometry associated with our \( \text{Kähler} \) manifold is flat.

So far, we have \( \alpha \)-attractor models in supergravity based on the hyperbolic \( \text{Kähler} \) geometry of the Poincaré disk or half plane with the logarithmic \( \text{Kähler} \) potentials. This logarithmic \( \text{Kähler} \) potential is the signature of the hyperbolic \( \text{Kähler} \) geometry of those models. In such models, parameter \( \alpha \) is interpreted as the reciprocal of the curvature of the \( \text{Kähler} \) manifold. In contrast, we present an \( \alpha \)-attractor model based on the square of the logarithmic \( \text{Kähler} \) potential with a vanishing curvature. However, this leaves us with the following question -
We also consider a shift symmetry breaking superpotential
\[ W = \sqrt{\lambda} S \Phi^2 \]  
(38)

or generally,
\[ W = S f(\Phi^2). \]  
(39)

One may see that this model is related to the previous model (11) and (15) (or (16)) by a scaled transformation in the field \( \Phi \to \Phi^2 \). Since this scaling is non-linear and these models are not related by any Kähler transformations, one can consider these as two different models. The Kähler potential defined in (35) produces the following kinetic term as
\[ \frac{1}{\sqrt{-g}} L_{kin} = - \frac{12\lambda}{(1 - \Phi^4) (1 - \Phi^4)} \partial_\mu \Phi \partial^\mu \Phi - (1 - 4\zeta S S) \partial_\mu S \partial^\mu S \]  
(40)

decomposing these complex variables into real variables as follows
\[ \Phi = \frac{1}{(6\alpha)^{\frac{1}{4}}} (\phi + i\chi), \quad S = \frac{1}{\sqrt{2}} (s + ib). \]  
(41)

With the help of (18), for the theory (40) and (38) (or 39), one can write the total Lagrangian for inflation along \( S = \chi = 0 \) as
\[ L = -\sqrt{-g} \left[ \frac{1}{2} R - \frac{4\phi^2}{(1 - \frac{\phi^4}{6\alpha})} \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \]  
(42)

where
\[ V = \left\{ \frac{\lambda}{6\alpha} \phi^4, \quad \text{for } W = \sqrt{\lambda} S \Phi^2 \right\} \]  
(43)

\[ + \frac{\lambda}{6\alpha} \phi^2 \left( \frac{\phi^2}{\sqrt{6\alpha}} \right), \quad \text{for } W = S f(\Phi^2). \]

Note that the kinetic term has four poles of order two at \( ((6\alpha)^{\frac{1}{4}}, -(6\alpha)^{\frac{1}{4}}, i(6\alpha)^{\frac{1}{4}}, -i(6\alpha)^{\frac{1}{4}}) \) and has a leading pole of order two with residue \( \frac{\lambda}{6\alpha} \). This is the required condition in kinetic term for the \( \alpha \)-attractors in its non-canonical variables so that the observational predictions are to a large extent determined by this term, rather than by the potential. In order to switch to the canonical variable, we solve the equation
\[ \frac{2\phi}{(1 - \frac{\phi^4}{6\alpha})} \partial_\mu \phi = \partial^\mu \phi \]  
(44)

which gives
\[ \phi = (6\alpha)^{\frac{1}{4}} \tanh^{\frac{1}{2}} \left( \frac{\psi}{\sqrt{6\alpha}} \right) \]  
(45)
Thus, the total Lagrangian for the $\alpha$-attractor model in terms of canonical variable $\psi$ reads

$$L = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} \partial_{\mu} \psi \partial^{\mu} \psi - \frac{f^2}{\alpha} \left( \tanh \left( \frac{\theta}{\sqrt{\alpha}} \right) \right) \right].$$

(M46)

Masses of all stabilized heavy fields along inflationary trajectory $S = \chi = 0$ are given by

$$m^2_R = 6H^2 \left( 1 - \frac{3}{4} \epsilon + \eta / 2 \right), \quad m^2_\rho = m^2_\pi = 12H^2 \left( \zeta + \epsilon / 8 \right).$$

(M47)

(M48)

These masses are slightly different compared to the masses of Model-I. The scalar potential for the model (36) and (38) in terms of the variables (41) is shown in Fig.(4).

As in the earlier case, here also the flat directions are not visible in terms of these variables. Migrating to more adequate variables $\Phi = \tanh^{1/2} \Phi$ with $\hat{\Phi} = (\psi + i \theta) \sqrt{\alpha}$, we get exactly the same potential (25) that we obtained in the previous model, which has visible flat directions. It is shown in Fig.(2). Moreover, the kinetic term is canonical for inflaton and its partner (see 24).

Finally, the metric of the moduli space obtained from the Kähler potential (35),

$$ds^2 = \frac{12\alpha}{(1 - \Phi^4)^2} d\Phi d\Phi^*$$

(M49)

gives non-vanishing connection coefficients as follows:

$$\Gamma^{\Phi}_{\Phi\Phi} = \frac{1 + 3\Phi^4}{\Phi (1 - \Phi^4)}, \quad \Gamma^{\Phi^*}_{\Phi^*\Phi^*} = \frac{1 + 3\Phi^{*4}}{\Phi^* (1 - \Phi^{*4})}.$$\n
(M50)

However, all components of Riemannian tensors vanish and hence, the geometry of the Kähler manifold is flat. As before, the parameter $\alpha$ here can be related to the coupling constants as

$$K^{(2n)} = \frac{n}{6a} \frac{1}{n^{2n-1}}$$

(M51)

and its behaviour is shown in Fig.(5).

III. CONCLUDING REMARKS

We have presented a couple of $\alpha$-attractors models based on $\mathcal{N} = 1$ supergravity with the use of logarithmic square Kähler potentials. The associated geometry of such Kähler manifolds have been found to be flat where all components of Riemannian tensors vanish. The masses of all fields which are orthogonal to inflaton direction have been stabilized during inflation.

The scalar potential $V$ in its canonical form seems to have an approximately similar kind of behavior compared to the potential which is obtained from the theory based on the hyperbolic geometry of the $\alpha$-attractors.

In contrast to the supergravity $\alpha$-attractor models based on the hyperbolic geometry in the literature, we have explicitly shown that one can obtain the same $\alpha$-attractors from the flat Kähler manifolds. Therefore, it is evident that cosmological $\alpha$-attractor models can emerge not only from the hyperbolic Kähler manifolds but also from the geometry of flat Kähler manifolds.

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Appendix A: Derivation of Model-I in series form

The Kähler potential defined in (11) can give the following kinetic term for inflation as

$$\frac{1}{\sqrt{-g}} L_{\text{kin}} = -2 \sum_{n=odd}^{N} K^{n}(n) \Phi^{n-1} \partial \Phi = \partial \Phi \quad \text{(A3)}$$

which yields,

$$\sum_{n=odd}^{N} K^{n}(n) \Phi^{n-1} \partial \Phi = \hat{\Phi} \quad \text{(A4)}$$

Here the values of $K^{n}(n)$ are adopted from (A2). Note that, Left hand side of the above (A4) represents the series expansion of $\sqrt{\frac{3}{\alpha}} \tanh^{-1}\left(\frac{\phi}{\sqrt{3\alpha}}\right)$ for large values of $N$. Hence the chiral field $\Phi$ can be represented in terms of canonically normalized field $\hat{\Phi}$ as,

$$\Phi = \sqrt{\frac{3}{\alpha}} \tanh\left(\frac{\phi}{\sqrt{3\alpha}}\right) \quad \text{(A5)}$$

Thus in terms of canonical fields total Lagrangian for inflation takes the form,

$$L = \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \hat{\Phi} \partial^{\mu} \hat{\Phi} \right] \left[ \tanh\left(\frac{\phi}{\sqrt{3\alpha}}\right) \right]^{2} \quad \text{(A6)}$$

Decomposing $\hat{\Phi}$ into real and imaginary parts as $\hat{\Phi} = \frac{1}{\sqrt{2}} (\hat{\psi} + i \hat{\chi})$ and at $\chi = 0$ Lagrangian for inflation in terms of the real field $\hat{\psi}$ as,

$$L = \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \hat{\psi} \partial^{\mu} \hat{\psi} \right] \left[ \tanh\left(\frac{\phi}{\sqrt{6\alpha}}\right) \right]^{2} \quad \text{(A7)}$$

Which defines the T-model of $\alpha$-attractors. For the general case, or for the superpotential of (16), Lagrangian reads the form by using (A5) as,

$$L = \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \hat{\psi} \partial^{\mu} \hat{\psi} - f^{2} \left(\sqrt{3\alpha} \tanh\left(\frac{\phi}{\sqrt{6\alpha}}\right)\right) \right] \quad \text{(A8)}$$

This (A8) defines the $\alpha$-attractor Lagrangian in terms of the real canonical variable $\hat{\psi}$. However physics is invariant under field transformation, one can also derive the above same Lagrangian in terms of the real non-canonical variable. So, in such a scenario, real part of $\Phi$ can be identified as inflaton in terms of the non-canonical chiral field $\Phi$. By decomposing $\Phi$ in terms of real and imaginary components $\Phi = \frac{1}{\sqrt{2}} (\phi + i\chi)$, and by assuming along flat direction $S = \chi = 0$, the (A1) can be written as

$$\frac{1}{\sqrt{-g}} L_{\text{kin}} = - \left[ \sum_{n=odd}^{N} \frac{1}{(6\alpha) \frac{n}{2}} \phi^{n-1} \right] \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \quad \text{(A9)}$$

For large values of $N$, and for the field $\phi$ satisfies the condition $\phi^{2} < 6\alpha, \alpha$, one can identifies the series defined in the (A9) is a the Taylor’s series expansion of the term $1/\left(1 - \frac{\phi^{2}}{6\alpha}\right)^{2}$. Thus the final kinetic term is

$$\frac{1}{\sqrt{-g}} L_{\text{kin}} = - \left[ \sum_{n=odd}^{N} \frac{1}{(6\alpha) \frac{n}{2}} \phi^{n-1} \right] \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \quad \text{(A10)}$$

Now for the superpotential defined in (15) or for (16), one can write the potential in terms of real field $\phi$ for $S = \chi = 0$ as,

$$V = \frac{m^{2}}{2} \phi^{2} \quad \text{(A11)}$$

or

$$V = f^{2} \left(\frac{\phi}{\sqrt{2}}\right) \quad \text{(A12)}$$

respectively. Clubbing together (A10) and (A11) we get the T-model Lagrangian in real canonical variable as

$$L = \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \right] \left[ \tanh\left(\frac{\phi}{\sqrt{3\alpha}}\right) \right]^{2} \quad \text{(A13)}$$

or, generally

$$L = \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - f^{2} \left(\sqrt{3\alpha} \tanh\left(\frac{\phi}{\sqrt{6\alpha}}\right)\right) \right] \quad \text{(A14)}$$

Which defines the $\alpha$-attractor Lagrangian before switching to canonical variable.
Appendix B: Derivation of Model-II in series form

Kinetic term for the Kähler potential (36) is

\[
\frac{1}{\sqrt{-g}} L_{\text{kin}} = -2 \sum_{n=odd} K^{(2n)} 2n \Phi^{2n-1} \sum_{n=odd} K^{(2n)} 2n \Phi^{2n-1} \partial_\mu \Phi_n \partial^\mu \Phi_n
\]

with (51). Decomposing the field \( \Phi \) into real and imaginary parts as \( \Phi = \frac{1}{\sqrt{2}} (\psi + i\chi) \) and considering along the flat direction \( X = \chi = 0 \), we get

\[
\frac{1}{\sqrt{-g}} L_{\text{kin}} = - \left( \sum_{n=odd} \left( \frac{2}{6\alpha} \Phi^{2n-1} \right) \right)^2 \frac{1}{2} \partial_\mu \phi \partial^\mu \phi
\]

This can be explicitly write in the form,

\[
\frac{1}{\sqrt{-g}} L_{\text{kin}} = \left( 1 + \frac{1}{6\alpha} \phi^4 + \frac{1}{(6\alpha)^2} \phi^8 + \frac{1}{(6\alpha)^3} \phi^{12} + \ldots \right)^2 \times \frac{4\phi^2}{2} \partial_\mu \phi \partial^\mu \phi
\]

Now for very large values of \( N \) and if the field \( \phi \) satisfies the condition \( \phi^2 < 6\alpha \), one can recognize the series defined in (B3) as the coefficient of \( \frac{4\phi^2}{2} \partial_\mu \phi \partial^\mu \phi \) is as the Taylor’s series expansion of the term \( \frac{1}{1 - \phi^2} \). Thus the final kinetic term in terms of real non-canonical variable takes the form

\[
\frac{1}{\sqrt{-g}} L_{\text{kin}} = - \frac{4\phi^2}{2} \partial_\mu \phi \partial^\mu \phi
\]

and the scalar potential reads

\[
V = \left\{ \frac{A^4}{4} \psi^4, \right. \text{ for } W = \sqrt{A} S \Phi
\]

\[
\left. \left( f^2(\phi^2), \text{ for } W = S f(\phi^2) \right) \right\}
\]

Clubbing (B4) and (B5) together, we get the total Lagrangian for inflation in non-canonical real variable as,

\[
L = \sqrt{-g} \left[ \frac{1}{2} R - \frac{4\phi^2}{2} \partial_\mu \phi \partial^\mu \phi - f^2(\phi^2) \right]
\]

Solve the equation for canonical normalization

\[
\frac{2\phi}{1 - \frac{\phi^2}{6\alpha}} \partial_\mu \phi = \partial \psi
\]

one yields,

\[
\phi = (6\alpha)^{\frac{1}{2}} \tanh \left( \frac{\psi}{\sqrt{6\alpha}} \right)
\]

So, finally in terms if this canonical field \( \psi \), the Lagrangian reads

\[
L = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - f^2 \left( \frac{3\alpha}{2} \tanh \left( \frac{\psi}{\sqrt{6\alpha}} \right) \right) \right]
\]

Which defines total Lagrangian for \( \alpha \)-attractors.
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