Some Electromagnetic Transition Properties of Odd-A Palladium Isotopes

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Abstract: In this study, the Palladium nuclei have been studied by interacting boson fermion model for even-odd nuclei. The isotopes under study have proton number 46 and neutron numbers 63-67. The calculations involved the eigenstates and reduce electric probability. The calculations show that the energy levels are reasonable with experimental data. The B(E2) values compared with those of experimental and are in good agreements. All calculations compared with experimental and give more data for reader.

Keywords: IBFM, Pd isotopes, eigenstates, B(E2).

1. Introduction

The nuclear model (interacting boson fermion model) its structure hinders a lot of numbers of boson which is distributed in the angular or bitals L=0 and 2 [1,2]. Moreover, the odd nucleon proton or neutron, and M-fermions involving single-molecule circles with rakish second ji = j1, j2, j3,….. [3]. The segments of the fermion precise second are the m-dimensional space of the gathering U(m) with mi = Ji(2ji + 1) [4]. The fermions creation a†i and destruction ai are administrators for the single-molecule notwithstanding. The boson creation bi† and obliteration bi are administrators for the aggregate degrees of opportunity. The fermion administrators fulfill hostile to compensation relations [5]:

\{ai, a†j\} = δij, \{a†i, ai\} = {ai, a†i} = 0

The linear finding of fermion creation a†i and destruction ai

(1)
The Hamiltonian of Interacting Boson Fermion Model had been depend on the algebraic structure, that aim of concurrent probability of dynamical limit for even-odd nuclei. In case of the single-j, the m values are \( m = 2j + 1 \), in general, a chain of algebras is:

\[ U(2j + 1) \supset SU(2j + 1) \supset SP(2j + 1) \supset SU(2) \supset O(2) \]

The aim of present study is to calculate the eigenstates and reduce electric transition probability for Palladium isotopes with mass numbers 109-113. Furthermore, the comparison are achieving between the calculations and experimental values. This study will gives the more results for reader and who is interested in the nuclear structure field.

2. Method

In the IBFM odd-A centers are depicted similar to a mixed game plan of conveying bosons and fermions, the possibility of dynamical adjusts must be summarized. Under the impediment, that both the boson and fermion states have incredible saucy power, the different social occasion binds should contain the upheaval bundle \( O(3) \) for boson and \( SU(2) \) for fermion as subgroup \([9-6]\).

\[
\begin{align*}
U^B(6) & \supset \ldots \ldots \ldots O^B(3) \\
U^F(m) & \supset \ldots \ldots SU^F(2)
\end{align*}
\]

(3)

If one of subgroups of \( U^B(6) \) is isomorphic to one of the subgroups of \( U^F(m) \), the boson and fermion bundle chains can be joined into a regular boson-fermion pack chain. Right when the Hamiltonian is composed similarly as Casimir invariants of the joined boson-fermion pack chain, dynamical boson-fermion equity rises. The odd-A centers are delineated by the coupling of the odd fermionic semi atom to a total boson place. The total Hamiltonian involves three segments and is given by the going with condition \([10-13]\):

\[ H = H_B + H_F + V_{BF} \]

which contains one-body terms only and given by

\[ H_F = \sum_{j\mu} \epsilon_j \hat{a}_j^{\dagger} \hat{a}_\mu \]

(5)

where \( \epsilon_j \) are the quasi-particle energies and \( \hat{a}_j^{\dagger} \hat{a}_\mu \) is the creation (annihilation) operator for the quasi-particle in the eigenstate \(|jm\rangle\).

\[ V_{BF} \]

is the boson-fermion interaction that describes the interaction between the odd quasi-nucleon and the even-even core nucleus \([14-17]\):

\[
V_{BF} = \sum_j A_j \left( \left( d^\dagger \times \bar{a}_j \right)^{(0)} \times \left( a_j^\dagger \times \bar{a}_j \right)^{(0)} \right)_{\ell_0} + \sum_{jj'} I_{jj'}^{(2)} \left[ Q^{(2)} \times \left( a_j^\dagger \times \bar{a}_j \right)^{(2)} \right]_{\ell_0} + \sum_{jj'} A_{jj'}^{(2)} \left[ \left( d^\dagger \times \bar{a}_j \right)^{(jj)} \times \left( d \times a_j^\dagger \right)^{(jj)} \right]_{\ell_0}.
\]

(6)

where \( Q^{(2)} \) is the core boson quadrupole operator.

The parameters \( A_j, I_{jj'} \) and \( A_{jj'}^{(2)} \) are defined by the following equations:

\[
A_j = A_0 \sqrt{2j + 1}
\]

\[
I_{jj'} = \sqrt{5} \left[ \Gamma_0 (u_j u_{j'} - \nu_j \nu_{j'}) Q_{jj'} \right]
\]

\[
A_{jj'}^{(2)} = -\sqrt{5} A_0 \left[ \left( u_j \nu_{j'} + u_{j'} \nu_j + u_j u_{j'} \right) Q_{jj'} + \left( u_j \nu_{j'} + u_{j'} \nu_j + u_j u_{j'} \right) Q_{jj'} \right] \sqrt{2j + 1}
\]

(7)

The \( A_0 \) is the monopole communication; \( \Gamma_0 \) is the quadrupole communication; \( \Lambda_0 \) is the trading of a semi molecule with one of the two fermions framing a boson. The dynamical boson-fermion evenness related with as far as ability and the single fermion (odd nucleon) involving single-molecule circles with turn \( j = 1/2, 3/2, 5/2 \). For this situation, the fermion space is disintegrated into a pseudo-orbital part with \( K = 0, 2 \) and a pseudo-turn part with \( s = 1/2 \) \([5]\).
3. Calculations Results

The calculations results can be divided to eigenstates sections and electric transition probability.

3-1 Eigenstates

According to shell model in the nuclear models the magic numbers are 50 and 82. So, Pd isotopes have 46 protons that are mean have 4 holes to fill the magic numbers. Furthermore, these nuclei have 62-66 neutron numbers that’s mean particles outside the magic number from 13 to 16 particles. These isotopes can be assumed as core in the framework of interacting boson fermion model. The $^{109-113}$Pd isotopes can be study of some properties of nuclear structure. In this section, the eigenstates of these nuclei have been calculated. To calculate it, the ODDA code [4] has been used. Hamiltonian to study these nuclei can be written as [18, 19]: $H=H_B+H_F+V_{BF}$. All parameters which is used in this calculations are presented in the tables 1 and 2. These free parameters estimated and keep as minimum adjusted and in MeV unit.

The calculated eigenstates of 1, 2, 3 band for $^{109-113}$Pd nuclei are shown in the Figure 1. In this figure, the calculated eigenstates compared with those of experimental [6-9]. It is noticed that a reasonable between them. The state with ''( )'' for 1, 2 and 3 bands according to the cases which the energy, spin, parity of corresponding levels are not well confirm experimentally.

Table 1. The parameters which are used in the ODDA codes to calculate the energy levels. All parameters in MeV unit.

|       | $^{109}$Pd | $^{111}$Pd | $^{113}$Pd |
|-------|------------|------------|------------|
| BFE   | 0.20       | 0.17       | 0.21       |
| BFQ   | 0.01       | 0.03       | 0.03       |
| BFM   | 0.00       | -0.10      | -0.07      |

Table 2. The addition parameters which used in the ODDA code. $\varepsilon_i$ parameter in MeV unit and $\nu_i^2$ without unit.

| Parameters | $^{109}$Pd | $^{111}$Pd | $^{113}$Pd |
|------------|------------|------------|------------|
| $\varepsilon_i$ | 3$s_{1/2}$ | 2$d_{5/2}$ | 2$d_{3/2}$ | 3$s_{1/2}$ | 2$d_{5/2}$ | 2$d_{3/2}$ | 3$s_{1/2}$ | 2$d_{5/2}$ | 2$d_{3/2}$ |
| 0.408      | 2.087      | 1.371      | 2.375      | 2.050      | 1.372      | 2.366      | 2.036      | 1.360      |
| $\nu_i^2$  | 0.060      | 0.082      | 0.772      | 0.061      | 0.083      | 0.779      | 0.060      | 0.083      | 0.779      |
Figure 1: (Color online) The eigenstates calculations and experimental [20] for $^{109-113}$Pd nuclei.

3-2 Electric Transition Probability

The second property of nuclear structure is electric reduced probability transition. This property will be explained in details with empirical values. Furthermore, it is give a good test of nuclear structure and the wave function of the nuclear model. The electric transition operators can be express as sum of two parts, the first part is for the bosen of the eigenstate and second part only on the fermion eigenstate [29-32], the fermion can be moved in $j=1/2$, $3/2$, and $5/2$ sub-orbital. The $(eB)$ values represented on the effective charge and calculated form experimental data and it tabulated in the table (3). The selections rules which used in the our estimated for the effective charge $(eB)$ values for fermion are $\Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 = 0$ and $\Delta (\tau_1 + \tau_2) = \pm 1$ transitions. It is for $\Delta \sigma 1 = \Delta \sigma 2 = \Delta \sigma 3 \neq 0$, $\Delta (\tau_1 + \tau_2) = \pm 1$ transitions, $eB$ ($\alpha_2 = eF$ ($f_2$) allowed, and thus expected to be
weaker, which are for $e_B \neq e_F$. At $e_B \neq e_F$ allowed only, the effective charge ($e_F$) can be reproduced from the experimental $B(E2; J_i \rightarrow J_f)$ and it express as [24].

$$B(E2; (N,1,0)\rightarrow (N+1,0), (\tau_1, \tau_2), L_f, J_f)$$

$$= (\alpha_2 - f_2^2) \frac{2}{2(N+1)(N+2)}$$

(8)

($e_B$) represented on the effective charge for fermion and it is tabulated in table (3). The $B(E2)$ results are calculated and compared with experimental value and tabulated in table (4).

Table 3. The parameters of reduce electric quadruple transition probability (in $eB$) for $^{109-113}$Pd nuclei.

| Nucleus  | $e_B (eB)$ | $e_F (eB)$ |
|----------|------------|------------|
| $^{109}$Pd | 0.0895     | -0.0128    |
| $^{111}$Pd | 0.0862     | -0.0862    |
| $^{113}$Pd | 0.0685     | -0.0685    |

Table 4. Reduce electric quadruple transition probability by IBFM and experimental values [20]. All values in $e^2b^2$ unit.

| $I_i \rightarrow I_f$ | $^{109}$Pd EXP | $^{109}$Pd IBFM-1 | $^{111}$Pd EXP | $^{111}$Pd IBFM-1 | $^{113}$Pd EXP | $^{113}$Pd IBFM-1 |
|-----------------------|----------------|-----------------|----------------|-----------------|----------------|-----------------|
| $1/2^+_1 \rightarrow 5/2^+_1$ | 0.0041 | 0.0034 | -- | 0.0006 | -- | 0.0000 |
| $3/2^+_1 \rightarrow 5/2^+_1$ | -- | 0.0000 | -- | 0.0806 | -- | 0.0015 |
| $5/2^+_1 \rightarrow 5/2^+_1$ | -- | 0.0840 | -- | 0.1196 | -- | 0.1035 |
| $7/2^+_1 \rightarrow 3/2^+_1$ | -- | 0.0407 | -- | 0.0079 | -- | 0.0023 |
| $1/2^+_1 \rightarrow 3/2^+_1$ | -- | 0.0013 | -- | 0.0138 | -- | 0.0032 |
| $7/2^+_1 \rightarrow 5/2^+_1$ | -- | 0.1112 | -- | 0.1818 | -- | 0.1371 |
| $9/2^+_1 \rightarrow 7/2^+_1$ | -- | 0.0289 | -- | 0.0134 | -- | 0.0125 |
| $11/2^+_1 \rightarrow 9/2^+_1$ | -- | 0.0407 | -- | 0.0689 | -- | 0.0521 |

4. Conclusion

The Palladium nuclei are lie in the middle nuclei region. These nuclei with neutron numbers range 63-67. The nuclear structure of these nuclei has been study by interacting boson fermion model. Some properties of nuclear structure, energy levels and reduced electric transition probability have been calculated and compared with experimental data. From this comparison, we conclude good agreements between them. This study provides the reader by more data about the nuclear structure for this nuclei understudy.
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