Evidence for Negative Stiffness of QCD Strings

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QCD strings are color-electric flux tubes between quarks with a finite thickness determined by the dimensionally transmuted coupling constant, and thus with a finite curvature stiffness. Contrary to an earlier rigid-string model by Polyakov and Kleinert, and motivated by the properties of a magnetic flux tubes in type-II superconductors, we put forward the hypothesis that QCD strings have a negative stiffness. We set up a new string model with this property and show that it is free of the three principal problems of rigid strings — particle states with negative norm, nonexistence of a lowest-energy state, and wrong high-temperature behavior of string tension — thus making it a better candidate for a string description of quark forces than previous models.

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Color-electric flux tubes between quarks have a thickness of the order of the dimensionally transmuted coupling constant. They should therefore display a finite resistance to a change in extrinsic curvature. To account for this property, a higher-derivative stiffness term was added to the Nambu-Goto (NG) action of an infinitely thin string. The resulting Polyakov-Kleinert (PK) string seemed to render a more relevant description of gluon forces in QCD than the former NG string. Particularly attractive was the fact that a PK model containing only a stiffness term is asymptotically free at short distances and can generate the string tension spontaneously. For other properties see.

Unfortunately, however, the PK model has also serious consistency problems. First of all, there exists an unphysical ghost pole in the propagator. The pole is generated by the second derivatives with respect to the string coordinates in the action. Second, the PK model shares with all higher-derivative theories the lack of a lowest-energy state. Third, the squared string tension of the PK model has an unphysical imaginary part at high temperatures or low β = 1/T, even though this model unlike the NG string displays the correct power behavior β−2 for β → 0 consistent with asymptotic freedom (except, however, for an imaginary factor i) .

It is the purpose of this letter to propose a new string model in which the problems of the PK string are absent while the attractive features are preserved. This new model has a negative extrinsic curvature stiffness. It was inspired by properties of magnetic flux tubes in type-II superconductors and analogous properties of Nielsen-Olesen vortices in relativistic gauge models. In the London limit, these have a nontrivial energy spectrum, from which several calculations have deduced a negative stiffness . By analogy, we hypothesize that the stiffness QCD strings is also negative. The action we propose for a description of such a string is

\[ \mathcal{A} = \frac{c - 1}{2} M^2 \int d^2 \xi \sqrt{g} g^{ij} D_i x^\lambda \frac{1}{c - e^{k^2/\mu^2}} D_j x^\lambda, \]

where \( x^\lambda(\xi) \) with \( \lambda = 0, 1, \ldots, d - 1 \) are the string coordinates in a \( d \)-dimensional euclidean spacetime parametrized by \( \xi^i \), \( i = 0, 1 \), and \( g_{ij} = \partial_i x^\lambda \partial_j x^\lambda \) is the induced metric on the world surface of the string with \( g^{ij} \) being its inverse and \( g = \det(g_{ij}) \). Covariant differentiation with respect to \( \xi^i \) is denoted by \( D_i \), and \( D^2 = D_i D^i \) is Laplace-Beltrami operator. The physics of this model is governed by two mass scales \( M \) and \( \mu \), and a dimensionless constant \( c \).

Due to nonzero radius of thickness the interaction between surface elements of the string has a nonlocal character, just like in a magnetic flux tube. In momentum space, the quadratic part of the action gives rise to the free propagator

\[ G(k^2) = \frac{1}{c - 1} k^2 + \frac{c - e^{-k^2/\mu^2}}{k^2}. \]

For small \( k \), this has an expansion

\[ G(k^2) = \frac{1}{k^2} + \frac{k^2/\Lambda^2}{k^2} + \ldots, \]
with the mass parameter $\Lambda^2 \equiv (c - 1)\mu^2$. The stiffness $\alpha$ is the inverse coefficient of $k^4/2$ in the action and has the negative value $\alpha = -\Lambda^2/M^2$.

The propagator contains only a single pole at $k^2 = 0$. This is to be contrasted with the PK model where

$$G(k^2) = \frac{1}{k^2(1 + k^2/\Lambda^2)}$$

(4)
corresponds to the positive stiffness $\alpha = \Lambda^2/M^2$ and contains an unphysical pole with negative residue at $k^2 = -\Lambda^2$.

The inverse propagator (3) is plotted in Fig. 1 for sample parameters $\mu$ and $c$. The fluctuations with large momenta are more violent than in the PK string with the opposite curvature stiffness, but less violent than in a NG string with reduced tension $(c - 1)M^2/c$, which is eventually approached.

The above string is inspired by the properties of a vortex line in a type-II superconductor [13], which has a diameter of the order of the penetration depth governing the parameter $1/\Lambda$ of the model. The inverse mass $1/\mu$ corresponds to the coherence length which goes to zero in the London limit.

In QCD, the parameters of the model have the following physical origin. As long as there are no quarks, QCD gives rise to a string with only one length scale which enters into both the string tension and the thickness of the flux tube. Thus it determines the two parameters $M$ and $\Lambda$ with some ratio between them, which is a consequence of the detailed dynamics of the gluon system. As quarks are included, this ratio is modified and an additional length scale arises at which the vacuum polarization of the gluon propagator becomes important and changes the stiffness properties of the flux tube. This length scale is proportional to some average over the inverse quark masses, and accounted for by the model parameter $1/\mu$.

For practical calculations, it is convenient to approximate (3) by (5), and cut all momentum integrals off at $k^2 = \mu^2$.

To have a physical penetration depth, we must take $\Lambda$ to be smaller than the cutoff $\mu$, or $1 < c < 2$.

The action associated with the approximate Green function (5) is

$$A_1 = \frac{1}{2} M^2 \int d^2\xi \sqrt{g} g^{ij} \frac{1}{1 - D^2/\Lambda^2} D_i x^\lambda D_j x^\lambda .$$

(5)

FIG. 1. The inverse propagators of the model (solid curve) and of the approximation (dashed lines) for $\mu = 10$ and $c = 1.2$. The dotted curve shows the behavior in a PK string with the opposite value of the curvature stiffness, the dash-dotted curve corresponds to a NG string with reduced tension $(c - 1)M^2/c$. We see that the large-$k$ fluctuations are much more violent than in the PK string, but less violent than in a NG string, which is eventually approached.

We shall now derive the high-temperature limit of the total string tension, which can be done exactly in the limit $d \to \infty$. As usual in this limit, we make the metric field $g_{ij}$ an independent fluctuating field, and force it to be equal to the induced metric $\partial_i x^\lambda \partial_j x^\lambda$ by means of Lagrange multipliers $\lambda^{ij}$, adding to (3) a term

$$A_2 = \frac{1}{2} M^2 \int d^2\xi \sqrt{g} \lambda^{ij} \left( \partial_i x^\lambda \partial_j x^\lambda - g_{ij} \right) .$$

(6)

It is straightforward to calculate the partition function of the string at a finite temperature $T$ anchored to two static quarks separated by a large distance $R$. The world sheet of the string is periodic in the imaginary-time direction with
a period $\beta = T^{-1}$. Choosing coordinates with $\xi^1$ running from 0 to $R$, and $\xi^0$ from 0 to $\beta$, we parametrize the world sheet as

$$x^\lambda (\xi) = (\xi^0, \xi^1, x^a (\xi)),$$

with $x^a (\xi)$, $a = 2, 3, \ldots, d-1$, describing the transverse fluctuations which occur quadratically in the extended action $A_1 + A_2$. These can be integrated out producing a trace log with a factor $d-2$ which in the limit $d \to \infty$ suppresses the fluctuations of $g_{ij}$ and $\lambda^{ij}$, so that the saddle point approximation provides us with an exact partition function. At the saddle point, we may assume diagonal forms for the metric and the Lagrange multipliers:

$$g_{ij} = \rho_i \delta_{ij}, \quad \lambda^{ij} = \lambda_i g^{ij},$$

with the constants $\rho_i$ and $\lambda_i$. The resulting effective action $A_{\text{eff}} = A_{\text{eff}}^{\text{mf}} + A_{\text{eff}}^{\text{loop}}$ consists of a mean-field term

$$A_{\text{eff}}^{\text{mf}} = \frac{R \beta M^2}{2} \sqrt{\rho_0 \rho_1} \left[ \left( \frac{\lambda_0}{\rho_0} + \frac{\lambda_1}{\rho_1} \right) + (\lambda_0 + \lambda_1) \right],$$

which also appears in NG and PK string models, and a one-loop contribution

$$A_{\text{eff}}^{\text{loop}} = \frac{d-2}{2} R \sqrt{\rho_1} \sum_{n=0}^{\infty} \int \frac{dq_1}{2\pi} \ln \left[ \left( \frac{4\pi^2 n^2 + q_1^2}{\rho_0 \beta^2} + q_1^2 \right) \right]^{-1} + \lambda_0 \left( \frac{4\pi^2 n^2}{\rho_0 \beta^2} + \frac{q_1^2}{\rho_1} \right) \right].$$

Here the temporal components $q_0$ are summed over all thermal Matsubara frequencies $q_0 = (q_0)_n \equiv 2\pi n/\beta \sqrt{\rho_0}$, $n = 0, \pm 1, \ldots$.

The effective string tension is defined by $M_{\text{eff}}^2 = F/\beta$, where $F = A_{\text{eff}}/R$ is the free energy per unit length evaluated at the extremal values of $\rho_i$ and $\lambda_i$. At low temperatures, $\beta \gg 1/\Lambda$ and the one-loop term \([\Box]\) is very small compared to the mean-field term \([\Box]\), so that the extremum of $A_{\text{eff}}$ lies at $\rho_0 = \rho_1 = 1$, $\lambda_0 = \lambda_1 = 0$, where $F = \beta M^2$. At high temperatures, on the other hand, $\beta \ll 1/\Lambda$ and the one-loop term \([\Box]\) becomes essential. In this limit, it can easily be evaluated analytically.

First, however, let us make an instructive observation: If we assume for a moment that the cutoff $\mu^2$ is artificially small, so that $q_0^2 + q_1^2 \ll \Lambda^2$, we can neglect the denominator of the first term in Eq. \([\Box]\) and see that the reduced one-loop action \([\Box]\) takes the NG form. By the same calculation as in Ref. \([14]\) we then find at high temperatures the squared free energy per unit length

$$F^2 \approx F_{\text{NG}}^2 (\beta) = \beta^2 M^4 \left[ 1 - \frac{(d-2)\pi}{3M^2 \beta^2} \right] \approx \frac{(d-2)\pi M^2}{3}. \quad (10)$$

This is in bad qualitative disagreement with the behavior derived from QCD in the limit of large-$N$ by Polchinski \([13]\):

$$F_{\text{QCD}}^2 (\beta) \approx -\frac{2\pi^2 (\beta) N}{\pi^2 \beta^2}. \quad \quad (11)$$

In contrast to this, the present model has the correct high-temperature behavior. Since the cutoff $\mu^2$ is, of course, much larger than $\Lambda^2$, the negative curvature stiffness $\alpha = -\Lambda^2/M^2$ can take effect. In the high-temperature limit $\beta \ll 1/\Lambda$, the first term in the logarithm \([\Box]\) is small compared to the second-derivative term, and one-loop action \([\Box]\) reduces to

$$A_{\text{eff}}^{\text{loop}} \approx \frac{d-2}{2} R \sqrt{\rho_1} \lambda_1 \left( \Lambda - \frac{\pi}{3\beta} \sqrt{\rho_0} \right). \quad (12)$$

The first term comes only from zero Matsubara frequency, in the large $\mu^2$-limit, the second contains the full spectral sum in \([\Box]\), but approximated by the leading second-derivative term. Adding to \([12]\) the mean-field term \([\Box]\), the resulting high-temperature effective action $A_{\text{eff}}$ has a saddle point at
\[ \rho_0^{-1} = 2 - \lambda_1 \frac{\lambda_0 - \lambda_1}{\lambda_0 + \lambda_1}, \quad \rho_1^{-1} = 2 + \lambda_0 \frac{\lambda_0 - \lambda_1}{\lambda_0 + \lambda_1}, \]  

where \( \lambda_0 \) and \( \lambda_1 \) are determined from

\[ \frac{2 \beta M^2}{(d-2)\Lambda} \sqrt{\rho_0 \lambda_1} (\lambda_0 + \lambda_1) = 1, \quad \sqrt{\rho_0 \lambda_1} (\rho_0^{-1} - 1) = \frac{(d-2)\pi}{6\beta^2 M^2} \rho_0^{-1}. \]

In terms of \( \lambda_1 \), the associated squared free energy per unit length reads simply

\[ F^2 = M^4 \beta^2 \frac{\rho_0}{\rho_1} (\lambda_1 + 1)^2. \]

The deconfinement temperature \( T_d \) is determined by the vanishing of \( 1/\rho_1 \), which yields

\[ \frac{T_d}{M_{\text{tot}}(0)} = \frac{2}{(d-2)} \left[ b(|\alpha|)|\alpha| \left( 1 + \sqrt{\frac{(d-2)|\alpha|}{8\pi}} \right) \right]^{-1/2}. \]

Here \( M_{\text{tot}}(0) = M \sqrt{\nu + 1} \) is the total string tension of at infinite \( \beta \), and the parameter \( \nu \) is related to the stiffness \( \alpha \) as follows

\[ \nu = \sqrt{\frac{(d-2)|\alpha|}{8\pi}}. \]

The parameter \( b(|\alpha|) \) is an unique real positive root of the polynomial equation of 9th order which exists for a reasonable range of the mass parameter \( \Lambda \geq 8.25M \). Numerical studies show that the deconfinement temperature depends only weakly on the stiffness parameter \( \alpha \) and lies in our model always around \( T_d/M_{\text{tot}}(0) \approx 0.932 \). This is somewhat larger than the estimates derived from rigid strings.

Equations (13) and (14) determine \( \lambda_1 \) via a polynomial of 14th order. In the above range of the mass parameter, \( \Lambda \geq 8.25M \), there are real roots for all \( \beta \). At high-temperatures, these have the asymptotic behavior

\[ \lambda_1 = \text{const.} \times \beta^0, \quad |\text{const.}| > 2; \quad \lambda_0 = \frac{(d-2)^2 \pi^2}{36 \beta^2} \frac{(\lambda_1 - 2)^2}{\lambda_1 (\lambda_1 - 1)^2}. \]

Inserted into (13), they yield the diagonal elements of the metric. The resulting high-temperature behavior of (15) is, therefore,

\[ F^2 \approx -\frac{\pi^2 (d-2)^2}{36 \beta^2} \frac{(\lambda_1 - 2)(\lambda_1 + 1)^2}{\lambda_1 (\lambda_1 - 1)^2}. \]

This this agrees in sign and \( \beta^{-2} \)-behavior with the QCD result (11), if we neglect the weak \( \beta \)-dependence of the running coupling constant \( g(\beta) \) in (11).

Note that the two strings are not immediately comparable since the result (11) was obtained in the \( N \to \infty \)-limit where the QCD string becomes infinitely thin and has therefore no curvature stiffness. At large but finite \( N \), however, it does acquires a finite thickness, as we know from Monte Carlo simulations, and the behavior (11) can be modified at most by small terms of order \( 1/N \). In this regime the agreement between our result (18) and the QCD result (11) is definitely significant.

We consider this agreement as an important evidence for a negative stiffness of hadronic strings, which thus behave very similar to vortex lines in type-II superconductors. Our conclusion should be verified in lattice gauge simulations.

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