Photoabsorption off nuclei with self consistent vertex corrections

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We study photoproduction off nuclei based on a self consistent and covariant many body approach for the pion and isobar propagation in infinite nuclear matter. For the first time the t-channel exchange of an in-medium pion is evaluated in the presence of vertex correction effects consistently. In particular the interference pattern with the s-channel in-medium nucleon and isobar exchange contribution is considered. Electromagnetic gauge invariance is kept as a consequence of various Ward identities obeyed by the computation. Adjusting the set of Migdal parameters to the data set we predict an attractive mass shift for the isobar of about 50 MeV at nuclear saturation density.

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I. INTRODUCTION

There is empirical evidence from photon nucleus absorption cross sections that the delta resonance changes its properties in nuclear matter substantially already at nuclear saturation density [1, 2, 3]. The microscopic description of the isobar self energy is a challenge taken up by various groups [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Naturally the study of the latter requires a solid understanding of the pion spectral function in nuclear matter [16, 17, 18, 19] and references in [16].

The phenomenological spreading potential [1] suggests a small repulsive mass shift of the isobar together with an increase of its width. Also recent data on electroproduction of isobars off helium three appear consistent with the latter interpretation [2]. With the exceptions [5, 11, 14] model computations of the isobar self energy claim results compatible with a small repulsive mass shift. On naive grounds one may reject the works [5, 11, 14], that predict a sizeable attractive mass shift for the isobar, as being unrealistic and incompatible with nuclear photo absorption data. However, the situation is not clear cut. First, one may observe that various detailed works [5, 12] adjust their model parameters as to reproduce the spreading potential [1] and therefore cannot be taken as a microscopic confirmation of a repulsive isobar mass shift. Second, one should recall the argument put forward in [5, 6, 11] that the apparent mass shift seen in photo absorption data is affected significantly by short range correlation effects. Thus an attractive isobar mass shift cannot be ruled out, since the phenomenological spreading potential [1] is effective in the sense that the latter effects were not explicitly accounted for.

The purpose of this work is a study of the photoabsorption cross section off nuclei based on a self consistent and covariant many body approach for the pion and isobar propagation in infinite nuclear matter that takes into account short range correlation effects consistently. We will apply the pion and isobar propagator as determined in [16] within a novel covariant approach where vertex effects parameterized by Migdal’s parameters are considered self consistently. Phenomenological soft form factors that would suppress vertex correction effects artificially are avoided in [16]. Scalar and vector mean fields for the nucleon and isobar are incorporated consistently.

In the isobar region the t-channel pion-exchange process is known to define a sizeable background term for the \( \gamma p \rightarrow \pi^+ n \) reaction [17]. Thus it is crucial to consider the t-channel exchange of an in-medium pion on equal footing as the in-medium exchange of the isobar when computing the photo absorption cross section of nuclei. In this work, for the first time, photo absorption is considered in the presence of short-range correlation effects in the \( \gamma\pi\pi \), \( \gamma N\Delta \), \( \gamma\pi N\Delta \), \( \pi N\Delta \) and \( \pi NN \) vertices. Electromagnetic gauge invariance is kept as a consequence of a series of Ward identities obeyed in the computation. In particular the interference of the in-medium s-channel isobar exchange and the t-channel in-medium pion exchange is considered.

The set of Migdal parameters is adjusted as to obtain agreement with nuclear photo absorption data [2]. As a firm prediction we obtain an attractive mass shift for the isobar of about 50 MeV at nuclear saturation densities. A
comprehensive discussion of the relevance of various many-body effects is given.

II. PHOTO ABSORPTION CROSS SECTION

We specify the isobar-hole model in its covariant form [16, 20, 21]. The interaction of pions with nucleons and isobars is modeled by the leading order vertices

\[
\mathcal{L}_{\text{int}} = \frac{f_N}{m_\pi} \bar{\psi} \gamma_5 \gamma^\mu (\partial_\mu \bar{\psi}) \vec{T} \psi + \frac{f_\Delta}{m_\pi} \left( \bar{\psi} \gamma_5 \gamma^\mu \tau^\nu \psi \right) + g_{11} \frac{f_\Delta}{m_\pi^2} \left( \bar{\psi} \gamma_5 \gamma^\mu \tau^\nu \psi \right) + g_{12} \frac{f_N f_\Delta}{m_\pi^2} \left( \bar{\psi} \gamma_5 \gamma^\mu \tau^\nu \psi \right) \left( \bar{\psi} \gamma^\mu \vec{T} \psi \right) + \text{h.c.},
\]

where we use \( T_i^j T_j^i = \delta_{ij} - \tau_i \tau_j / 3 \) together with the free-space values \( f_N = 0.988 \) and \( f_\Delta = 1.85 \) in this work. We consider Migdal’s short range correlation vertices as introduced in [20, 21], where it is understood that the local vertices are to be applied at the Hartree level. The Fock contribution can be cast into the form of a Hartree contribution by simple Fierz transformation. Therefore it only normalizes the coupling strength in (1) and can be omitted here.

We supplement (1) by leading order and relevant electromagnetic vertices

\[
\mathcal{L}_{\text{e.m.}} = -e A^\mu \bar{\psi} \left( \frac{1 + \gamma_5}{2} \gamma_\mu \psi \right) - e A^\mu \left( \vec{\pi} \times (\vec{\partial} \bar{\pi}) \right)_3 \psi + \frac{i f_\gamma}{2 m_\pi} \epsilon_{\mu \nu \alpha \beta} F^{\alpha \beta} \bar{\psi} (\partial^\mu \vec{\psi}) T_3 \psi + \text{h.c.} + \frac{f_\gamma}{m_\pi} \left( F_{\mu \nu} (\partial^\mu \vec{\psi}) \gamma_5 T_3 \psi \right) + \text{h.c.},
\]

with the electromagnetic field strength tensor \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The magnetic and electric coupling constants \( f_\gamma \) and \( f'_\gamma \) are determined from the photon induced pion production cross section off the proton. We compute the cross sections as defined by the diagrams of Figure 1. The contribution of the u-channel isobar exchange is much suppressed and therefore neglected. The isobar propagator is specified in [10]. It is modeled by a one-loop self energy describing the leading decay process of the isobar into a pion and a nucleon. As illustrated in Figure 2 in the isobar region the photon-proton cross sections are reasonably well described by the electric and magnetic coupling constants \( f_\gamma = 0.012 \) and \( f'_\gamma = 0.024 \). Our values are close to the ones of Pascalutsa and Phillips, \( f_\gamma \simeq 0.009 \) and \( f'_\gamma \simeq 0.021 \). While the neutral pion production is dominated by the s-channel isobar exchange contribution, the production of the charged pion shows a sizeable background contribution. Following the arguments put forward in [10], we consider the Lagrangian densities (1) to be effective and allow their parameters to enjoy a residual but smooth density dependence. The latter reflects the dynamics of modes that are integrated out and therefore not treated explicitly here.

We compute the photo absorption cross section for an ‘ideal’ infinite nucleus. Our studies will be based on the in-medium nucleon propagator parameterized in terms of scalar and vector means fields:

\[
S(p) = \frac{1}{p - \Sigma^S_N \bar{\psi} - m_N + i \epsilon} + \Delta S(p), \quad m_N = m_N^{\text{vac}} - \Sigma^S_N,
\]

FIG. 1: Feynman diagrams for the photon induced pion production.
isospin symmetric nuclear matter. The isobar propagator
\[ S_{\pi N}(\pi N) \] as a function of \( f \) based on the Lagrangian density (1). In this work we take the results of [16] and consider the pion and isobar in terms of the imaginary part of the forward Compton amplitude
\[ \alpha_{\beta} \]

in the rest frame of the bulk with \( u_\mu = (1, \vec{0}) \) one recovers with (11) the standard result \( \rho = 2k_F^3/(3\pi^2) \). We assume isospin symmetric nuclear matter. The isobar propagator \( S_{\mu\nu}(w) \), is the solution of Dyson’s equation
\[ S_{0\mu\nu}(w) = \frac{-1}{\vec{\psi} - m_\Delta + i\epsilon} \left( g^{\mu\nu} - \frac{\gamma^\mu \gamma^\nu}{3} - \frac{2 w^\mu w^\nu - \gamma^\mu w^\nu - \gamma^\nu w^\mu}{3 m_\Delta^2} \right), \]
\[ S_{\mu\nu}(w) = S_{0\mu\nu}(w - \Sigma^\Delta u) + S_{0\mu\nu}(w - \Sigma^\Delta u) \Sigma^{\alpha\beta}(w) S_{\beta\nu}(w), \]

where we allow for scalar and vector mean fields of the isobar with \( m_\Delta = m_\Delta^{\text{vac}} - \Sigma^\Delta \) as developed in [16, 26, 27]. In [16] the pion and isobar self energy \( \Sigma^{\alpha\beta}(w, u) \) were determined in a self consistent and covariant many-body approach based on the Lagrangian density [11]. In this work we take the results of [16] and consider the pion and isobar propagators as a function of \( f_N, f_\Delta \) and \( g_{ij} \) and the mean field parameters for the nucleon and isobar. For the details on the pion and isobar self energies we refer to [16]. It is the aim of the present work to find sets of parameters that lead to a faithful representation of the nuclear photoabsorption data.

The computation of the total absorption cross section is performed in the nuclear matter rest frame. Fermion motion effects are considered. We express the cross section
\[ \sigma_{\gamma A}(q_0) = \frac{4}{\rho} \int_0^{k_F} \frac{d^3p}{(2\pi)^3} \frac{\Im A_{\gamma N \rightarrow \gamma N}(q, p)}{2(p - u \Sigma_V) \cdot q}, \]
\[ p_0 = \sqrt{m_N^2 + \vec{p}^2 + \Sigma_V}, \quad q_0 = |\vec{q}|, \quad u_\mu = (1, \vec{0}), \]
in terms of the imaginary part of the forward Compton amplitude \( A_{\gamma N \rightarrow \gamma N}(p, q) \). We explore the role of intermediate \( \pi N, N\eta N \) and \( \Delta hN \) states with
\[ \Im A_{\gamma N \rightarrow \gamma N}(q, p) = \Im A_{\gamma N \rightarrow \gamma N}^{(\pi)}(q, p) + \Im A_{\gamma N \rightarrow \gamma N}^{(ph)}(q, p) + \Im A_{\gamma N \rightarrow \gamma N}^{(\text{interference})}(q, p), \]
\[ \Im A_{\gamma N \rightarrow \gamma N}^{(\pi)}(q, p) = \sum_{\lambda, i} \Tr \int \frac{d^3\ell}{(2\pi)^3} \frac{\Theta(|\vec{l}| - k_F)}{16 E_\ell} e^{\dagger}(q, \lambda) T_{\gamma N \rightarrow \gamma N}^{i,\ell,\mu}(q, l; w) \]
\[ \times (\ell + M) T_{\gamma N \rightarrow \gamma N}^{i,\mu}(q, l; w) e^{\dagger}(q, \lambda) \left( \rho^{(\pi)}(|w_0 - l_0|, \vec{w} - \vec{\ell}) \right)_{l_0 = E_\ell + \Sigma_V}, \]
In Figure 3 the in-medium generalization of the diagram's of Figure 1 are shown, where short range correlation of the pion self energy is determined by the s-wave pion-nucleon scattering length at $q_0 = (m_\pi, 0)$

The latter probes the product of the pion and particle-hole production amplitudes. Furthermore, $M = m_N - y \Sigma_V$ and $w = p + q$ and $E_l^2 = m_N^2 + \vec l^2$ with $E_l > 0$. We expect the most important contribution from (7) to result from the intermediate $\pi N$ states, where we consider an effective in-medium pion state characterized by its spectral distribution. The effects from the nucleon-hole-nucleon ($NhN$) and isobar-hole-nucleon ($\Delta hN$) states are described by a tensor spectral distribution. This is possible since we consider only resonant contributions through the isobar s-channel process, for which the production amplitudes as implied by (1) are degenerate. Due to phase-space considerations the contribution from the $NhN$ states is much larger as compared to the one of the $\Delta hN$ states, at least in the isobar region. This implies that this contribution will be roughly proportional to $(g'_{12})^2$ and will become the more important the larger this value becomes.

We begin with a detailed exposition of the pion and particle-hole spectral distributions $\rho_\pi(q)$ and $\rho_{ph}^\alpha(q)$ required in (7). The central building blocks, in terms of which they are expressed, are the short-range correlation bubbles

$$
\Pi_{\mu\nu}^{(Nh)}(q) = \frac{2 f_N^2}{m_N^2} \int \frac{d^4l}{(2\pi)^4} i \epsilon_{\mu\nu\lambda\sigma} \left[ \frac{\Delta S(l) \gamma_5 \gamma_\lambda}{l + \vec q - M + i \epsilon} \gamma_\lambda \right] \left( \frac{1}{l + \vec q} \right),
$$

$$
\Pi_{\mu\nu}^{(\Delta h)}(q) = \frac{4}{3} \frac{f_N^2}{m_N^2} \int \frac{d^4l}{(2\pi)^4} i \epsilon_{\mu\nu\lambda\sigma} \left[ \frac{\Delta S(l) S_{\mu\nu}(l + q)}{l + \vec q - M + i \epsilon} \right],
$$

where 'tr' denotes the trace in Dirac space. Note that the isobar-hole loop function in (8) is given in terms of the in-medium isobar propagator as specified in (4) by the isobar self energy. Details on the evaluation of the loop tensors (8) can be found in [10]. For the spectral distributions $\rho_\pi(q)$ and $\rho_{ph}^\alpha(q)$ of (7) we find

$$
\Pi_\pi(q) = -4 \pi \left( 1 + \frac{m_\pi}{m_N} \right) b_{\text{eff}} \rho - \sum_{i,j=1}^2 q_\mu \left( \Pi^{\mu\nu}_{ij}(q) + \left[ \Pi(q) \cdot \chi(q) \cdot \Pi(q) \right]_{ij}^{\mu\nu} \right) q_\nu,
$$

$$
\rho_\pi(q) = -3 \frac{1}{q^2 - m_N^2 - \Pi_\pi(q)},
$$

where we recall the value $b_{\text{eff}} \simeq -0.01$ fm from [10]. The latter value is needed to achieve consistency with the low-density limit, where the pion self energy is determined by the s-wave pion-nucleon scattering length at $q_0 = (m_\pi, 0)$

We continue with the specification of the pion and particle-hole production amplitudes $T^{i,i}_{\mu\nu} N \rightarrow \pi N$ and $T^{i,i}_{\mu\nu} N \rightarrow ph N$ in (7). In Figure 3 the in-medium generalization of the diagrams of Figure 4 are shown, where short range correlation

\begin{align*}
\Xi A_{\gamma N \rightarrow \gamma N}^{(ph)}(q,p) &= \sum_{\lambda,i} \text{Tr} \int \frac{d^3l}{(2\pi)^3} \frac{\Theta[\vec l - k_F]}{16 E_l} \epsilon_\lambda(q,\lambda) \epsilon_\mu(q,\lambda) T^{i,i}_{\mu\nu} N \rightarrow ph N (q,l,w) \\
&\times \left( I + M \right) T^{i,i}_{\mu\nu} N \rightarrow ph N (l,q,w) \epsilon_\nu(q,\lambda) \left( \vec \theta + M \right) \rho_{ph}^\alpha(q) (|w_0 - l_0|, \vec w - \vec l) \bigg|_{l_0 = E_l + \Sigma_V},
\end{align*}

where the 'Tr' denotes the trace in Dirac and flavor space. In this work we will neglect the interference term of the pion and particle-hole contributions. Furthermore, $M = m_N - y \Sigma_V$ and $w = p + q$ and $E_l^2 = m_N^2 + \vec l^2$ with $E_l > 0$. We expect the most important contribution in (7) to result from the intermediate $\pi N$ states, where we consider an effective in-medium pion state characterized by its spectral distribution. The effects from the nucleon-hole-nucleon ($NhN$) and isobar-hole-nucleon ($\Delta hN$) states are described by a tensor spectral distribution. This is possible since we consider only resonant contributions through the isobar s-channel process, for which the production amplitudes as implied by (1) are degenerate. Due to phase-space considerations the contribution from the $NhN$ states is much larger as compared to the one of the $\Delta hN$ states, at least in the isobar region. This implies that this contribution will be roughly proportional to $(g'_{12})^2$ and will become the more important the larger this value becomes.
effects are considered in terms of various vertex functions. A similar graphical representation holds for the particle-hole production amplitude, for which, however, we consider resonant contributions only. More explicitly, the in-medium $\gamma N \to \pi N$ and $\gamma N \to \phi h N$ amplitudes to be used in (7) take the form

$$T_{i,\mu}^\gamma N \to \pi N (\bar{q}, q; w) = \Lambda_{i,\alpha}^{(\pi N)}(\bar{q}) S^{\alpha\beta}(p + q) \Lambda_{\mu\beta}^{\gamma(\Delta)}(p, q) + \Lambda_{i,\alpha}^{(\pi N)} S(p + q) \Lambda_{\mu\beta}^{\gamma(\Delta)}(q) + \Lambda_{\mu\beta}^{\gamma(\Delta)} S(\bar{q} - q) \Lambda_{i,\alpha}^{\gamma(\Delta)}(\bar{q}) \Lambda_{\mu\beta}^{\gamma(\Delta)}(q),$$

$$T_{i,\alpha\mu}^\gamma N \to \phi h N (\bar{q}, q; w) = \frac{f_\Delta}{m_\pi} T_i^{\gamma} g_{\alpha\beta} S^{\nu\beta}(p + q) \Lambda_{i,\alpha}^{(\pi N)}(p, q),$$

(10)

where the various vertex functions are subject to the Ward identities

$$q^\mu \Lambda_{\mu\nu}^{(\gamma N)}(q) = e \frac{1 + \tau_3}{2} \gamma_\nu, \quad q^\mu \Lambda_{\mu\nu}^{(\gamma N)}(p, q) = 0,$$

$$q^\mu \Lambda_{i,\mu}^{(\gamma N)}(\bar{q}, q) = \frac{q^2}{2} \Lambda_{i,\mu}^{(\gamma N)}(\bar{q}, q) + \Lambda_{i,\mu}^{(\gamma N)}(\bar{q}, q),$$

$$q^\mu \Lambda_{i,\mu}^{(\gamma N)}(\bar{q}, q) = e \left[ \frac{1 + \tau_3}{2}, \Lambda_{i,\mu}^{(\gamma N)}(\bar{q}, q) \right] - e \left[ \frac{1 + \tau_3}{2}, \Lambda_{i,\mu}^{(\gamma N)}(\bar{q}, q) \right].$$

(11)

with the pion self energy $\Pi_\pi(q)$ of (8). Electro magnetic gauge invariance of the in-medium $\gamma N \to \pi N$ amplitude (10) is a consequence of the identities (11). The vertex functions of (10) take the form

$$\Lambda_{i,\alpha}^{(\pi N)}(\bar{q}) = \frac{f_N}{m_\pi} T_3^{\gamma} (q^\nu + \left[ \Pi(q) \cdot \chi(q) \right]^{\mu\nu})_{11} + \left[ \Pi(q) \cdot \chi(q) \right]^{\mu\nu}_{12} \gamma_5 \gamma_\nu,$$

$$\Lambda_{i,\alpha}^{(\pi N)}(\bar{q}, q) = \frac{f_N}{m_\pi} \frac{T_3^{\gamma}}{T_3^{\gamma}} (q^\nu + \left[ \Pi(q) \cdot \chi(q) \right]^{\mu\nu})_{12} g_{\alpha\nu},$$

$$\Lambda_{\mu}^{(\gamma N)}(q) = e \left[ \frac{1 + \tau_3}{2}, \Lambda_{\mu}^{(\gamma N)}(q) \right] - e \left[ \frac{1 + \tau_3}{2}, \Lambda_{\mu}^{(\gamma N)}(q) \right],$$

$$\Lambda_{i,\alpha}^{(\gamma N)}(q) = \frac{i f_\pi}{m_\pi} T_3^{\gamma} \left[ 1 + \Pi(q) \cdot \chi(q) \right]^{\alpha\nu}_{12} \Pi_{\alpha\mu}^{(\Delta h)}(q) \left[ 1 + \Pi(q) \cdot \chi(q) \right]^{\nu\mu}_{12},$$

$$\Lambda_{i,\alpha}^{(\gamma N)}(q) = e \left( q^\mu + \bar{q} \mu + \sum_{ij=1}^{2} \left( \Pi(q) \cdot \chi(q) \cdot \Pi(q) \right)^{\nu\beta}_{ij} q^\beta + \bar{q} \mu \left[ \Pi(q) \cdot \chi(q) \cdot \Pi(q) \right]^{\alpha\nu}_{ij} g_{\mu\nu} - \sum_{ik=1}^{2} \bar{q} \mu \left[ \Pi(q) \cdot \chi(q) \right]^{\alpha\mu}_{ik} \Pi_{\alpha\mu}^{(k)}(q, q) \left[ 1 + \Pi(q) \right]^{\nu\beta}_{ij} q^\beta \right),$$

$$\Lambda_{i,\mu}^{(\gamma N)}(q) = \frac{e f_N}{2 m_\pi} \left[ \tau_3, \tau_1 \right] - \gamma_5 \gamma_\mu \left( \sum_{n=1}^{2} \left[ \Pi(q) \cdot \chi(q) \right]^{\nu\beta}_{1n} g_{\mu\beta} - \sum_{j=1}^{2} \bar{q} \mu \left[ 1 + \Pi(q) \cdot \chi(q) \right]^{\alpha\mu}_{1k} \Pi_{\alpha\mu}^{(k)}(q, q) \chi_{ij}^{\nu} q^\beta \right),$$

(12)

with the loop tensors

$$\Pi_{\mu,\alpha}^{(\Delta h)}(q) = \frac{4 f_\pi^2}{3} \frac{m_\pi^2}{(2\pi)^4} i \text{tr} \Delta S(l) S_{\mu\nu}(l + q)(l + q)_{\alpha} + (q_\mu \rightarrow -q_\mu),$$

$$\Pi_{\mu,\alpha\beta}(q) = 2 \frac{f_\pi^2}{m_\pi^2} \frac{m_\pi^2}{(2\pi)^4} i \text{tr} \left\{ \gamma_5 \gamma_\mu \Delta S(l) \gamma_5 \gamma_\alpha \left[ \frac{1}{l + q - M + i \epsilon} + \frac{1}{2} \Delta S(l + q) \right] \right\}$$

$$+ \frac{1}{2} \Delta S(l + q) \Pi_{\mu,\alpha\beta}(l + q, l + q) \left( \frac{1}{l + q - M + i \epsilon} + \frac{1}{2} \Delta S(l + q) \right)$$

(11)
\[ \frac{3}{4} \Delta S(l + \bar{q}) \Gamma_\mu^{(N N)}(l + \bar{q}, l + q) \Delta S(l + q)] \right) - (\bar{q}_\mu, q_\mu) \to -(\bar{q}_\mu, q_\mu), \]

\[ \Pi_{\mu,\alpha\beta}^{(12)}(\bar{q}, q) = \frac{4}{3} \frac{f_\gamma^2}{m_h^2} \int \frac{d^4l}{(2\pi)^4} i \frac{\Gamma^{(N N)}}{\Gamma^{(\mu,\alpha\beta)}} \left( \Delta S(l) S^{\alpha\beta}(l + \bar{q}) \right) g_{\alpha\kappa} \times \Gamma^{(\mu,\alpha\beta)}(l + \bar{q}, l + q) S^{\kappa\tau}(l + \bar{q}) g_{\beta\rho} \left( (\bar{q}_\mu, q_\mu) \to -(\bar{q}_\mu, q_\mu) \right), \]

(13)

Given the vertices (12) the Ward identities (11) follow if the loop tensors \( \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) \) obey the reduced Ward identities

\[ (\bar{q} - q)_\mu \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) = \delta_{ij} \Pi_{ii}^{\alpha\beta}(q) - \delta_{ij} \Pi_{ii}^{\alpha\beta}(\bar{q}). \]

(14)

The identities (14) hold provided that the \( \gamma N N \), \( \gamma N \Delta \) and \( \gamma \Delta \Delta \) vertices in (13) satisfy the constraint equations

\[ (\bar{p} - p)^\mu \Gamma^{(N N)}_\mu(\bar{p}, p) = \bar{p} - p, \quad (\bar{p} - p)^\mu \Gamma^{(\gamma N \Delta)}_\mu(\bar{p}, p) = 0, \]

\[ (\bar{p} - p)^\mu \Gamma^{(\gamma \Delta \Delta)}_\mu(\bar{p}, p) = [S^{-1}]_{\alpha\beta}(\bar{p}) - [S^{-1}]_{\alpha\beta}(p). \]

(15)

We point out that the evaluation of \( \Pi_{22}(\bar{q}, q) \) required the evaluation of the diagrams of Figure 3 where the photon couples to the intermediate pion-nucleon state building up the isobar self energy. This leads to a self consistency issue, since the latter requires the knowledge of the \( \gamma \pi \pi \) vertex, which in turn depends on \( \Pi_{22}(\bar{q}, q) \).

To make progress we consider the following decomposition

\[ \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) = \frac{u_{\bar{q}}}{u \cdot (\bar{q} - q)} \delta_{ij} \left( \Pi_{ii}^{\alpha\beta}(q) - \Pi_{ii}^{\alpha\beta}(\bar{q}) \right) + \Delta \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q), \]

\[ (\bar{q} - q)_\mu \Delta \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) = 0, \]

(16)

where we argue that the terms \( \Delta \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) \) are suppressed by \( 1/m_N \) or \( 1/m_\Delta \) as compared to the first term in (16). This is easily seen for the '11' term. The \( \gamma N N \) vertex takes the form

\[ \Gamma^{(\gamma N N)}_\mu(\bar{p}, p) = \gamma_\mu + \frac{2 i f_\gamma}{e m_h^2} \gamma_5 \gamma_\mu e^{i\tau\tau\alpha\beta} \chi^{\rho\kappa}_{12}(\bar{p} - p) \Pi^{(\Delta h)}_{\kappa\tau,\beta}(\bar{p} - p) (\bar{p} - p)_\alpha = \gamma_\mu, \]

(17)

where the vertex corrections vanish due to the anti symmetry of the \( \epsilon \) tensor. A further possible contribution proportional to \( f_\gamma \) is obsolete also. As a consequence \( \Delta \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) \) enjoys a representation, which follows from the one of \( \Pi_{11}^{\mu,\alpha\beta}(\bar{q}, q) \) in (13), upon the replacement

\[ \Gamma^{(\gamma N N)}_\mu = \gamma_\mu \to \gamma_\mu - u_{\bar{q}} \frac{\bar{q} - q}{u \cdot (\bar{q} - q)}. \]

(18)

The \( \gamma N N \) vertex in (13) is sandwich between two nucleon propagators that are on-shell in the limit of a large nucleon mass. Since vector currents of massive particles are dominated by their zero component, our claim follows. By analogy to the nucleon case, we expect the term \( \Delta \Pi_{22}(\bar{q}, q) \) to be suppressed by \( 1/m_\Delta \) as compared to the first term in (16). Finally an explicit analysis of the term \( \Delta \Pi_{12}(\bar{q}, q) \) reveals also its suppression by \( 1/m_N \). The \( \gamma N \Delta \) vertex in (13) reads

\[ \Gamma^{(\gamma N \Delta)}_\mu(\bar{p}, p) = i (\bar{p} - p)^\tau \epsilon_{\mu\rho\sigma\tau} \chi^{\rho\kappa}_{22}(\bar{p} - p) \Pi^{(\Delta h)}_{\kappa\tau,\beta}(\bar{p} - p) \]

\[ - f_\gamma^2 \gamma_5 \left( g_{\mu\alpha} (p \cdot (\bar{p} - p)) - p_\mu (\bar{p} - p)_\alpha \right), \]

(19)

where vertex corrections proportional to \( f_\gamma^2 \) vanish identically. The suppression of \( \Delta \Pi_{12}(\bar{q}, q) \) follows upon an evaluation of the appropriate trace in (13). Thus, in the following we neglect the terms \( \Delta \Pi_{ij}^{\mu,\alpha\beta}(\bar{q}, q) \) for \( i, j = 1, 2 \). It is stressed that the term \( u \cdot (\bar{q} - q) \) in (16) does not cause any kinematical singularity for on-shell photons with \( (\bar{q} - q)^2 = 0 \).
We adjust the set of parameters to the photoabsorption data [2]. For the scalar and vector nucleon mean field we use the values $\Sigma_{S}^N = 0.35$ GeV and $\Sigma_{V}^N = 0.29$ GeV at nuclear saturation density with $k_F = 0.27$ GeV as assumed also in [27]. Following previous works [5, 10] an averaged density of 0.8 times saturation density is taken to compute the photoabsorption data. A compilation of the results can be found in Figure 4 and Figure 5. As it turns out we need an extensive scan in the parameter space was performed. We assure that given our values for the nucleon mean field for the isobar at $\Sigma_{\Delta} = 0.25$ GeV, $\Sigma_{\Delta} = 0.11$ GeV, $g'_{11} = 1.0$, $g'_{12} = 0.4$, $g'_{22} = 0.4$. (20)

An extensive scan in the parameter space was performed. We assure that given our values for the nucleon mean fields there is a well defined and localized region in parameter space that leads to an accurate reproduction of the photoabsorption data. A compilation of the results can be found in Figure 4 and Figure 5. As it turns out we need a reduction of $f_\Delta$ and an increase of $f_\gamma$ as compared to their free-space values. Extrapolated linearly up to nuclear saturation density we derive a 15% reduction of $f_\Delta$ and a 15% increase for $f_\gamma$. Attempts to describe the data with no in-medium modifications of those parameters fails as the isobar turns too broad and consequently the cross section too small. A reproduction of the data set is possible also assuming a moderate reduction of $f_N$. However, this would require an even stronger medium modification of the parameters $f_\Delta$ and $f_\gamma$. Changes in $f'_\gamma$ have only a tiny influence on the results so we keep this parameter at its free-space value.

In Figure 4 we study possible variations of $g'_{12}$ and $g'_{22}$ around the central values 0.4 of (20). The magnitudes of the Migdal parameters are dependent to some extent on the subtleties of the chosen approach. Thus we refrain from a detailed comparison with values obtained in different schemes. Keeping $g'_{11} = 1.0$ and a scalar mean field for the isobar at $\Sigma_{S}^\Delta = 0.2$ GeV, we readjust the magnitude for the vector isobar mean field. If we allow for variations larger that 0.1 in the Migdal parameters the cross section can no longer be reproduced accurately. From Figure 4 we see that with increasing values of $g'_{12}$ and $g'_{22}$ the shape of the cross section gets narrower. The best description is obtained with a parameter set that delivers also the largest over all magnitude for the cross section. Altogether we arrive at the values of $g'_{12}$ and $g'_{22}$ to be round about 0.4. In Figure 5 we illustrate the effect of lowering Migdal’s parameter $g'_{11}$ down to 0.6. As seen in the figure such a low value of $g'_{11}$ leads to a significant overshoot of the cross section at small photon energies. Though the resonance contribution itself is not affected much, the background contribution is enhanced strongly. This is shown by the dashed-dotted line which gives the result implied by all but the first diagram of Figure 3. We checked that variations of $g'_{12}$, $g'_{22}$ or the isobar mean field parameters do not lead to a significant suppression of this contribution. The only mechanism to arrive at a smaller $g'_{11}$ would be a significant reduction of $f_N$, however, at the price of an even larger reduction of $f_\Delta$. Thus we arrive at a rather large value for $g'_{11} \approx 1.0$.

In Figure 6 we study the importance of various contributions and approximations. In the left upper panel the contributions of the resonance, background and the one of the particle hole contributions are compared with the full result. The background contribution, defined by all but the first diagram of Figure 3 is essentially flat and delivers about 100 $\mu$b to the cross section. The resonance itself contributes about 200 $\mu$b in the peak while the two-particle hole...
final states deliver an additional 50 $\mu$b. As can be seen when adding up all contributions incoherently interference effects play a minor role only. We turn to the upper-right panel of Figure 6 which illustrates the importance of vertex corrections. The solid line of that panel gives our result implied by the parameter set (20) but a bare $\gamma N \Delta$ vertex in the production amplitudes $T_{\gamma N \rightarrow \pi N}$ and $T_{\gamma N \rightarrow phN}$ of (7). The neglect of short-range correlation effects in the $\gamma N \Delta$ vertex implies a significant shift of the isobar strength about 50 MeV towards lower energies. Thus the apparent peak position seen in the absorption cross section does not directly reflect the isobar contribution. A realistic prediction of the in-medium isobar mass requires the proper consideration of such effects. An even more dramatic influence of short-range correlation effects is documented by the lower-left panel of Figure 6. Here we assume again the parameters set (20) but also bare $\gamma N \Delta$ and $\pi N \Delta$ vertices. The pion and isobar propagators used are obtained within the self consistent and covariant approach [16], where correspondingly a bare $\pi N \Delta$ vertex was taken. This calculation corresponds to the dashed lines in Figure 4 of [16]. As anticipated by our previous study [16] a neglect of short-range correlation effects in the $\pi N \Delta$ vertex leads to a much broader isobar which then translates into an almost flat photoabsorption cross section. We finally turn to the lower-right panel of Figure 6. Here we focus on the background contributions. While the dashed-dotted line shows the full background contribution, the short dashed-dotted line gives the result for the background processes implied when using a bare pion propagator and bare vertices in Figure 3. The vertex correction in the background terms are essential to keep our approach consistent. An approximative treatment in which the in-medium spectral distribution of the pion is neglected would lead to a strong underestimation of the background processes. In this case the Pauli-blocking effect would cut away the low-energy cross section as can be seen from Figure 6. We emphasize that the consideration of such effects is crucial to arrive at a realistic estimate for Migdal’s parameter $g'_{11}$.

It is interesting to compare our results with previous studies. We find a qualitative agreement with the results of [5], which claimed an attractive mass shift for the isobar in nuclear matter based on a perturbative and non-relativistic many-body approach. This is in stark contrast to the more recent works [10, 15], which claim small and repulsive mass shifts of the isobar in cold nuclear matter. The differences are traced to the neglect of important short-range correlation effects and the use of a soft and phenomenological form factor in the $\pi N \Delta$ vertex [10, 15].

IV. SUMMARY

We presented a first computation of the nuclear photoabsorption cross section that considered the effect of short range-correlations effects in the $\gamma \pi \pi$, $\gamma N \Delta$, $\gamma \pi N \Delta$, $\pi N \Delta$ and $\pi N N$ vertices. We applied the self consistent and covariant many-body approach developed by the authors for the $\pi N \Delta$ systems in the presence of short-range correlation effects. In particular the in-medium interference of the s-channel isobar exchange and the t-channel pion exchange was evaluated consistently with an in-medium pion propagator. It was shown that the latter plays an important role in the determination of Migdal’s parameter $g'_{11} \simeq 1.0$, for which we obtained a rather large value. An accurate reproduction of the photoabsorption data was achieved. Based on our analysis we predict an attractive mass shift of about 50 MeV for the isobar in cold and saturated nuclear matter.

**FIG. 5:** Photoabsorption cross section for $g'_{11} = 1.0$ (solid line) and $g'_{11} = 0.6$ (dashed line). We use $\Sigma^S = -0.2$ GeV and $\Sigma^T = -0.09$ GeV. In addition we show the background contribution for the run with $g'_{11} = 0.6$ (dash dotted line). The data are taken from [2].
FIG. 6: Photoabsorption cross section using the parameter set (20). The upper-left panel shows the relevance of various contributions, the upper-right panel the effect of short-range correlations in the $\gamma N\Delta$ vertex. The lower-left panel follows if bare $\gamma N\Delta$ and $\pi N\Delta$ vertices are assumed. The solid lines give the complete calculations, the dashed lines the resonance contributions, the dashed-dotted lines the background contributions and the dotted lines the two-particle hole contributions. The lower-right panel illustrates the importance of in-medium effects on the background processes. The short-dashed dotted line provides the background contribution evaluated with bare vertices and a free-space pion. The data are taken from [2]. See the text for more details.

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