Pion electromagnetic form factor from full Lattice QCD

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Motivation

• The electromagnetic form factor of the charged $\pi$ meson parameterises the deviations from the behaviour of a point-like particle when struck by a photon

• These deviations arise from the internal structure of the $\pi$: constituent quarks and their strong interaction

• Can be calculated in QCD, but need fully nonperturbative treatment $\rightarrow$ use Lattice QCD

• Experimental determination from $\pi - e$ scattering

• Important to work at physical pion mass
Dependence on pion mass

$\langle r^2 \rangle$ (fm$^2$)

$M_{\pi}^2$ (GeV$^2$)

ETMC, Phys. Rev. D79 (2009) 074506
Lattice configurations

- MILC $n_f=2+1+1$ HISQ lattice configurations
- HISQ action for valence quarks
- Quark masses tuned to physical masses
- $L m_\pi \approx 4$ for the coarse ($a=0.12$ fm) and fine ($a=0.088$ fm) lattices

| Set | $a$/fm | $a m_l$ | $a m_s$ | $a m_c$ | $m_\pi$/MeV | $L/a \times L_t$ | $N_{\text{conf}}$ |
|-----|--------|---------|---------|---------|-------------|-----------------|----------------|
| 1   | 0.15   | 0.00235 | 0.0647  | 0.831   | 133         | 32 $\times$ 48  | 1000           |
| 2   | 0.12   | 0.00184 | 0.0507  | 0.628   | 133         | 48 $\times$ 64  | 1000           |
| 3   | 0.088  | 0.00120 | 0.0363  | 0.432   | 128         | 64 $\times$ 96  | 223            |
Form factors = 3pt amplitudes

- Consider two currents, a 1-link spatial vector current and a scalar current.
- Use a phase at the boundary to give a quark a momentum: \( \Phi(x + \hat{e}_j L) = e^{i2\pi\theta_j} \Phi(x) \rightarrow p_j = \frac{2\pi\theta_j}{L} \).
- Tune \( \theta \) to get the desired \( q^2 \) and extract \( f_+(q^2) \) in the space-like (negative) region of \( q^2 \) near zero.

\[
q^2 = (E(\vec{p}_2) - E(\vec{p}_1))^2 - (\vec{p}_2 - \vec{p}_1) \cdot (\vec{p}_2 - \vec{p}_1)
\]
Connected and disconnected diagrams

- Writing down 3-point matrix elements gives two types of terms, connected and disconnected
- Vector current: Disconnected diagrams cancel due to charge conjugation and isospin symmetries
- Scalar current: For a full calculation of the scalar form factor need both connected and disconnected diagrams, but here we only consider connected diagrams

\[ J^{\pi_1}(\vec{p}_1) \pi(\vec{p}_2) \pi(\vec{p}_1) \pi(\vec{p}_2) \]
Fitting the correlators

- Fit 2-point and 3-point correlators simultaneously
- Multi-exponential fits to reduce systematical errors from the excited states
- Use Bayesian priors to constrain fit parameters
- Fit all $q^2$ values simultaneously to take into account the correlations
Scalar and vector form factors

\[
\langle \pi(\vec{p}_1) | J | \pi(\vec{p}_2) \rangle = Z \sqrt{4E_0(\vec{p}_1)E_0(\vec{p}_2)} J_{0,0}(\vec{p}_1, \vec{p}_2)
\]

\[
\langle \pi(\vec{p}_1) | V_i | \pi(\vec{p}_2) \rangle = f_+(q^2)(\vec{p}_1 + \vec{p}_2)_i
\]

\[
\langle \pi(\vec{p}_1) | S | \pi(\vec{p}_2) \rangle = f_0(q^2) \frac{\partial M_\pi^2}{\partial m_l}
\]

- Need renormalisation constant \( Z \) for the vector current: demand that \( f_+(0) = 1 \)
- Scalar current is absolutely normalised, but we do not have complete calculation of the matrix element (only the connected 3pt correlator) - treat the scalar current as requiring a \( Z \) factor and set \( f_0(0) = 1 \)
Results: vector form factor

\[ f_+(q^2) \]

\[ q^2 \text{ in GeV}^2 \]

Graph showing the relationship between \( f_+(q^2) \) and \( q^2 \) in GeV^2, with different fit lines for experimental data and lattice simulations.
Results: scalar form factor

Connected diagram only
Continuum extrapolation

- Fit the form factors to the pole form
  \[ f(q^2) = \frac{1}{(1 + ba^2 + ca^4 + q^2\langle r^2 \rangle / 6)} \]
  or as power series in \( q^2 \) allowing for \( a^2 \) and \( m_\pi \) dependence
  \[ f(q^2) = A_0 + \frac{1}{6}\langle r^2 \rangle q^2 + A_4 q^4 + A_6 q^6; \quad A_i = d_i (1 + b_i a^2 + c_i a^4) \]
  \[ \langle r^2 \rangle = A_2 + c_J \ln(m_\pi^2/\mu^2) \]

- The slope at \( q^2=0 \) gives the mean square of the charge radius:
  \[ \langle r_{v}^2 \rangle = -6 \frac{df_+(q^2)}{dq^2} \bigg|_{q^2=0} \]
Dependence on pion mass

\[ \langle r^2 \rangle / \text{fm}^2 \]

\[ - \ln \left( \frac{M^2_\pi}{\text{GeV}^2} \right) \]

ETMC
this work
expt NA7
Vector mean square radius

\[ \langle r^2 \rangle / \text{fm}^2 \]

- **n_f = 2 + 1 + 1**
  - \( m_{\pi}^{\text{min}} = 128 \text{ MeV} \)

- **n_f = 2 + 1**
  - \( m_{\pi}^{\text{min}} = 260 \text{ MeV} \)

- **n_f = 2**
  - \( m_{\pi}^{\text{min}} = 280 \text{ MeV} \)
  - \( m_{\pi}^{\text{min}} = 330 \text{ MeV} \)
  - \( m_{\pi}^{\text{min}} = 290 \text{ MeV} \)
  - \( m_{\pi}^{\text{min}} = 400 \text{ MeV} \)

- **expt, NA7**
- **HPQCD**
- **UKQCD/RBC**
- **ETMC**
- **Mainz**
- **QCDSF/UKQCD**
- **JLQCD/TWQCD**
Scalar mean square radius

\[ <r^2>/\text{fm}^2 \]

\( \pi\pi \) scattering & \( \chi pt \)

Connected + disconnected

Connected

JLQCD/TWQCD

HPQCD

Mainz
Charge density

• In the non-relativistic limit, \( q^2 \approx -\langle \vec{q} \rangle^2 \), the form factor \( f_+(q^2) \) can be viewed as the Fourier transform of the electric charge distribution

• The form factor is usually taken to be of pole form

\[
    f_+(q^2) = \frac{1}{(1 + q^2\langle r_V^2 \rangle/6)}
\]

or a power series in \( q^2 \)
Non-relativistic charge density

\[ \langle r_v^2 \rangle = 0.409 \text{ fm}^2 \]
Summary

- Full Lattice QCD calculation of the pion vector electromagnetic form factor
  - physical pion mass
  - can choose the $q^2$ range
  - determine the charge radius:
    our preliminary result is $\langle r_v^2 \rangle = 0.409(23) \text{ fm}^2$

- Compare with experiment - get good agreement

- The scalar form factor needs much more work
Thank you!
References

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