

How Far Can the SO(10) Two Higgs Model Describe the Observed Neutrino Masses and Mixings?

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Can the SO(10) model with one 10 and one 126 Higgs scalars give the observed masses and mixings of quarks and charged leptons without any other additional Higgs scalars? Recently, at least, for quarks and charged leptons, it has been demonstrated that it is possible. However, for the neutrinos, it is usually said that parameters which are determined from the quark and charged lepton masses cannot give the observed large neutrino mixings. This problem is systematically investigated, and it is concluded that the present data cannot exclude SO(10) model with two Higgs scalars although it cannot give the best fit values of the data.

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I. INTRODUCTION

SO(10) GUT model seems to us the most attractive model when we take the unification of the quarks and leptons into consideration. However, in order to reproduce the observed quark and lepton masses and mixings, usually, a lot of Higgs scalars are brought into the model. So it is the very crucial problem to know the minimum number of the Higgs scalars which can give the observed fermion mass spectra and mixings. A model with one Higgs scalar is obviously ruled out for the description of the realistic quark and lepton mass spectra. Two Higgs models were initially discussed by Mohapatra et al. [1].

In the previous paper [1], we discussed 2 Higgs scalars, {10 and 126} case and {10 and 120} case, and showed that they reproduce quark-lepton mass matrices unlike the conventional results [2]. One of new points of our approach is that we adopt general forms of Yukawa couplings allowable in the SO(10) framework. However, we did not argue there about the neutrino mass matrix since it may incorporate additional assumptions like the seesaw mechanism etc.

One of the merits of the SO(10) model is that it includes a right-handed Majorana neutrinos in the fundamental representation and naturally leads to the seesaw mechanism. Also some papers claim that two Higgs model ({10 and 126+120}) does not reproduce the large mixing angle of the atmospheric neutrino deficit [3]. So in this paper we apply our method developed in the previous paper to the neutrino mass matrix, fitting the other parameters of the quark-lepton mass matrices. Our model has the two Higgs scalars {10 and 126} both of which are symmetric with respect to the family index. Therefore mass matrices are symmetric whose entries are complex valued. We do not adopt another choice {10 and 120}. For it does not involve the mass term of the right-handed Majorana neutrinos which are the ingredients of the seesaw mechanism.

We begin with the short review of our previous work [1]. In the case where two Higgs scalars, φ_{10} and φ_{126}, are incorporated in the SO(10) model, the mass matrices of quarks and charged leptons have the following forms

\[ M_u = c_0 M_0 + c_1 M_1, \quad M_d = M_0 + M_1, \quad M_e = M_0 - 3M_1. \]  (1.1)

Here \( M_0 \) and \( M_1 \) are the mass matrices generated by the Higgs scalars \( \phi_{10} \) and \( \phi_{126} \), respectively. Also \( c_0 \) and \( c_1 \) are the ratios of VEV’s,

\[ c_0 = v_u^0/v_d^0 = \langle \phi_{10}^0 \rangle / \langle \phi_{126}^0 \rangle, \]
\[ c_1 = v_u^1/v_d^1 = \langle \phi_{10}^1 \rangle / \langle \phi_{126}^1 \rangle. \]  (1.2)

and \( \phi^0 \) and \( \phi^1 \) denote Higgs scalar components which couple with up- and down-quarks, respectively. Eliminating \( M_0 \) and \( M_1 \) from Eq. (1.1), we obtain

\[ M_\tau = c_d M_d + c_u M_u, \]  (1.3)

where

\[ c_d = \frac{3c_0 + c_1}{c_0 - c_1}, \quad c_u = \frac{4}{c_0 - c_1}. \]  (1.4)

Since \( M_u, M_d, \) and \( M_\tau \) are complex symmetric matrices, they are diagonalized by unitary matrices \( U_u, U_d, \) and \( U_\tau \), respectively, as

\[ U_u^T M_u U_u = D_u, \quad U_d^T M_d U_d = D_d, \quad U_\tau^T M_\tau U_\tau = D_\tau, \]  (1.5)

where \( D_u, D_d, \) and \( D_\tau \) are diagonal matrices given by

\[ D_u \equiv \text{diag}(m_u, m_c, m_t), \quad D_d \equiv \text{diag}(m_d, m_s, m_b), \]
\[ D_\tau \equiv \text{diag}(m_e, m_\mu, m_\tau). \]  (1.6)
Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_q$ is given by

$$V_q = U_u^T U_d^* ,$$  \hspace{1cm} (1.7)

where $\kappa = c_u/c_d$. By eliminating the parameter $c_d$, we have two equations for the parameter $\kappa$:

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^3}{m_\mu^2 m_\tau^2 m_e^2} = (1.9)^3, \hspace{1cm} (1.12)$$

and

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^2}{2(m_\mu^2 m_e^2 + m_\tau^2 m_e^2)} = \left(\frac{(1.9)^2}{(1.1)^2} - \frac{(1.9)^3}{(1.1)^3}\right), \hspace{1cm} (1.13)$$

where $\frac{(1.9)^3}{(1.1)^3}$, for instance, means the right-hand side of Eq.(1.9) to the third power. Let us denote the parameter values of $\kappa$ evaluated from Eqs.(1.12) and (1.13) as $\kappa_A$ and $\kappa_B$, respectively. If $\kappa_A$ and $\kappa_B$ coincide with each other, then we have a possibility that the SO(10) GUT model can reproduce the observed quark and charged lepton mass spectra. If $\kappa_A$ and $\kappa_B$ do not so, the SO(10) model with one 10 and one 126 Higgs scalars is ruled out, and we must bring more Higgs scalars into the model.

Note that Eqs. (1.9), (1.10), and (1.11) can constrain only the absolute value of $c_d = |c_d e^{i\sigma}|$. The argument of the parameter $c_d$ can be determined by taking neutrino sector into consideration. In the previous paper [1], we have found that only for the signs of the masses

$$(m_t, m_c, m_u; m_b, m_s, m_d; m_b, m_s, m_d, m_\mu, m_\tau)$$

$$= (+, +, +; +, -, -; +, \pm, \pm) \hspace{1cm} (a), \hspace{1cm} (1.14)$$

and

$$= (+, -, -, +; +, -, -; +, \pm, \pm) \hspace{1cm} (b), \hspace{1cm} (1.15)$$

there are solutions which gives $\kappa_A = \kappa_B$, and the corresponding parameter values $|c_d|, \kappa$ are

$$(|c_d|, \kappa) = (3.156986, -0.019296 e^{2.64172+i}), \hspace{1cm} (1.16)$$

$$= (3.03577, -0.019398 e^{-2.99570+i}) \hspace{1cm} \text{for (a)}, \hspace{1cm} (1.17)$$

and

$$(|c_d|, \kappa) = (3.13307, -0.019314 e^{2.71464+i}), \hspace{1cm} (1.18)$$

$$= (3.00558, -0.01942 e^{3.10014+i}) \hspace{1cm} \text{for (b)}, \hspace{1cm} (1.19)$$

and $m_e = 76.3$ [MeV] for input $\theta_3 = 0.0420$ [rad] and $\delta = 60^\circ$ at $\mu = m_Z$ ($m_Z$ is the neutral weak boson mass). For the relation between the values at $\mu = m_Z$ and those the relation (1.3) is re-written as follows:

$$(U^T e U)^T D e (U^T e U) = c_d V_d V^T + c_u D_u, \hspace{1cm} (1.8)$$

Therefore, we obtain the independent three equations:

$$\text{Tr} D e D e^T = |c_d|^2 \text{Tr} \left[ (V_q D_d V_q^T + \kappa D_u) (V_q D_d V_q^T + \kappa D_u)^4 \right], \hspace{1cm} (1.9)$$

$$\text{Tr} (D e D e^T)^2 = |c_d|^4 \text{Tr} \left[ (V_q D_d V_q^T + \kappa D_u) (V_q D_d V_q^T + \kappa D_u)^4 \right], \hspace{1cm} (1.10)$$

$$\det D e D e^T = |c_d|^6 \det \left[ (V_q D_d V_q^T + \kappa D_u) (V_q D_d V_q^T + \kappa D_u)^4 \right], \hspace{1cm} (1.11)$$

at $\mu = \Lambda_X$ ($\Lambda_X$ is a unification scale), see Ref. [1]. The purpose of the present paper is to investigate whether these solutions can give reasonable values for observed neutrino masses and mixings or not.

II. THE NUMBER OF PARAMETERS IN THE SO(10) MODEL WITH TWO HIGGS SCALARS

As we have discussed in the previous section, among four freedoms of complex $\{c_o, c_1\}$ or $\{c_d, \kappa\}$, we have been able to fix the three of them, $\kappa$ and $|c_d|$. This is not accidental. Let us discuss the situation in detail in the SO(10) two Higgs model.

In the previous paper, by using the relation (1.3), we have investigated whether there is a set of parameters which can give the 13 observable quantities $D_e, D_u, D_d, V_q$ or not. We can rewrite Eq.(1.8) as

$$A^T D e A_e = c_d (V_q D_d V_q^T + \kappa D_u), \hspace{1cm} (2.1)$$

where

$$A_e = U^T e U, \hspace{1cm} (2.2)$$

$$c_d = |c_d| e^{i\sigma}. \hspace{1cm} (2.3)$$

The quantities $D_e, D_u, D_d, V_q$ are inputs, and the quantities $|c_d|, \kappa$, and $A_e$ are the parameters which should be fixed from those observed quantities. In general, an $n \times n$ unitary matrix for $n$ generations has $n^2$ parameters. Therefore, the number of the parameters is

$$N(\text{eqs}) = N(A_e) + N(c_d) + N(\kappa) = n^2 + 2 + 2. \hspace{1cm} (2.4)$$

On the other hand, the number of equations is

$$N(\text{eqs}) = n(n + 1), \hspace{1cm} (2.5)$$

because Eq.(2.1) is symmetric. Therefore, the number of the unfixed parameters is given by

$$N_{\text{free}} = N(\text{pmt}) - N(\text{eqs}) = 4 - n = 1, \hspace{1cm} (2.6)$$
for \( n = 3 \), i.e., the 13 observed quantities fix the parameters \( |c_d|, \kappa, \) and \( A_e \), but 1 parameter \( \sigma \) remains as an unknown parameter.

In the present paper, we will try to predict neutrino masses

\[ D_\nu = U_\nu^T M_\nu U_\nu, \]  
\[ (2.7) \]

and mixing matrix

\[ V_\ell = U_\ell^T U_\ell^*, \]  
\[ (2.8) \]

by using the observed quantities \( D_\nu, D_u, D_d, \) and \( V_\ell \) and the parameter values \( |c_d|, \kappa, \) and \( A_e \) fixed by Eq.\( (2.1) \).

SO(10) GUT asserts that the Dirac neutrino mass matrix \( M_D \) is given by the form

\[ M_D = c_0 M_0 - 3c_1 M_1, \]  
\[ (2.9) \]

and Majorana mass matrices of the left-handed and right-handed neutrinos, \( M_L \) and \( M_R \), are proportional to the matrix \( M_1 \):

\[ M_L = c_L M_1, \quad M_R = c_R M_1, \]  
\[ (2.10) \]

where \( M_0 \) and \( M_1 \) are related to the quark and charged lepton mass matrices \( M_u, M_d, \) and \( M_e \) as follows:

\[ M_0 = \frac{3M_d + M_e}{4}, \]  
\[ (2.11) \]

\[ M_1 = \frac{M_d - M_e}{4}. \]  
\[ (2.12) \]

Then the neutrino mass matrix derived form the seesaw mechanism becomes

\[ M_\nu = M_L - M_D M_R^{-1} M_D^T \]

\[ = c_L M_1 \]

\[ -c_R^{-1}(c_0 M_0 - 3c_1 M_1) M_1^{-1}(c_0 M_0 - 3c_1 M_1)^T. \]  
\[ (2.13) \]

In the present paper we adopt \( c_L = 0 \). Also we may ignore the phase of \( c_R \) which does not affect the observed values. Therefore, we can rewrite Eq.\( (2.13) \) as

\[ |c_R| A_\nu^T D_\nu A_\nu = \tilde{M}_D M_1^{-1} \tilde{M}_D^T, \]  
\[ (2.14) \]

similarly to Eq.\( (2.1) \), where

\[ \tilde{M}_D = c_0 \tilde{M}_0 - 3c_1 \tilde{M}_1, \]  
\[ (2.15) \]

\[ \tilde{M}_0 = \frac{1}{4}(3 \tilde{M}_d + \tilde{M}_e), \]  
\[ (2.16) \]

\[ \tilde{M}_1 = \frac{1}{4}(\tilde{M}_d - \tilde{M}_e). \]  
\[ (2.17) \]

with

\[ \tilde{M}_d = U_d^T M_d U_d = V_d D_d V_d^T, \]  
\[ (2.18) \]

\[ \tilde{M}_e = U_e^T M_e U_e = A_e^T D_e A_e \]

\[ = c_d(V_q D_q V_q^T + \kappa D_u). \]  
\[ (2.19) \]

Differently from the previous work, the quantities \( D_\nu, \) and \( V_\ell \) are unknown parameters at the present stage. Since

\[ V_\ell = A^*_\nu D^*_\nu, \]  
\[ (2.20) \]

\[ N(A_e) = N(V_\ell) = n^2. \]  
\[ (2.21) \]

Of course, the unknown parameters in \( A_\nu \) contain the \( n \) unphysical parameters which cannot be determined because of the rephasing in the fields \( \epsilon_L \). Therefore, the number of the unknown parameters is

\[ N(\text{pmt}) = N(D_\nu) + N(A_\nu) + N(|c_R|) + N(\sigma) \]

\[ = n + n^2 + 1 + 1 = n^2 + n + 2 \]  
\[ (2.22) \]

and from the number of equations \( N(\text{eqs}) = n(n + 1) \) in Eq.\( (2.14) \), we obtain the number of the unfixed parameters as

\[ N_{\text{free}} = N(\text{pmt}) - N(\text{eqs}) \]

\[ = (n^2 + n + 2) - n(n + 1) = 2. \]  
\[ (2.23) \]

This means that we can predict neutrino masses and mixing completely if we give the two values \( |c_R| \) and \( \sigma \). The numerical predictions will be investigated in the next section.

### III. NUMERICAL RESULTS

Here we discuss the numerical results of the neutrino mass spectrum and neutrino mass matrix. For example, we use the set in Eq.\( (1.18) \). Even if the other sets are used, our results are scarcely changed. The allowed values of neutrino mass square differences and lepton flavor mixing angles depict complicated tracks with moving \( \sigma \equiv \arg c_d \) (Fig. 2). This figure shows a general tendency that the lepton flavor mixing angles \( \theta_{12} \) and \( \theta_{23} \) get larger as \( \sigma \) approaches to \( 3\pi/2 \). For an illustration we take \( \sigma = 149\pi/100 \), then these values become

\[ \frac{\Delta m^2_{12}}{\Delta m^2_{13}} = 0.15, \quad \frac{\Delta m^2_{23}}{\Delta m^2_{13}} = 0.85, \]

\[ \sin^2(2\theta_{12}) = 0.76, \quad \sin^2(2\theta_{23}) = 0.75, \]

\[ \sin^2(2\theta_{13}) = 0.16. \]  
\[ (3.1) \]

There still remain a little bit discrepancies between our results and experiments. However our results are much improved in comparison with those by Babu-Mohapatra \[ \text{[1]} \] in which they obtained \( \sin \theta_{12} = 0 - 0.3, \sin \theta_{13} = 0.05 \), and \( \sin \theta_{23} = 0.12 - 0.16 \). The purpose of the present paper is to study the general tendency of the fittings and not to pursue the precise data fitting, for the data themselves are not affirmative and we have theoretical ambiguities not incorporated in the present data fitting like the renormalization group effect.

In the choice of Eq.\( (3.1) \), we have

\[ |c_d| = 3.16 \]  
\[ (3.2) \]

\[ c_0 = \frac{1 - c_d}{c_u} = 54.84 e^{-0.24^{\circ}i}, \]  
\[ (3.3) \]

\[ c_1 = \frac{3 + c_d}{c_u} = 70.54 e^{+1.90^{\circ}i}. \]  
\[ (3.4) \]

In this case, Eq.\( (2.11) \) - Eq.\( (2.13) \) are re-written in the basis of \( M_u = D_u \) (see Eq.\( (1.8) \)) as
\[ M_0 = \frac{3V_q D_d V_q^T + c_d (\kappa D_u + V_q D_d V_q^T)}{4} \]
\[ = 2.1646 \times 10^3 e^{+10.48^\circ i} \begin{pmatrix} -0.00405 e^{-57.29^\circ i} & -0.00753 e^{-56.24^\circ i} & -0.00533 e^{+56.46^\circ i} \\ -0.00753 e^{-56.24^\circ i} & -0.02986 e^{-51.59^\circ i} & +0.06358 e^{-57.64^\circ i} \\ -0.00533 e^{+56.46^\circ i} & +0.06358 e^{-57.64^\circ i} & +1.00000 \end{pmatrix} \text{[MeV]}, \quad (3.5) \]

\[ M_1 = \frac{V_q D_d V_q^T - c_d (\kappa D_u + V_q D_d V_q^T)}{4} \]
\[ = 9.5127 \times 10^2 e^{-24.44^\circ i} \begin{pmatrix} -0.00715 e^{+95.23^\circ i} & -0.01333 e^{+96.54^\circ i} & +0.00944 e^{+38.23^\circ i} \\ -0.01333 e^{+96.54^\circ i} & -0.04878 e^{+90.73^\circ i} & +0.11247 e^{+95.13^\circ i} \\ +0.00944 e^{+38.23^\circ i} & +0.11247 e^{+95.13^\circ i} & +1.00000 \end{pmatrix} \text{[MeV]}, \quad (3.6) \]

\[ |c_R|M_\nu = (c_0 M_0 - 3c_1 M_1)M_1^{-1}(c_0 M_0 - 3c_1 M_1)^T \]
\[ = -4.6628 \times 10^6 e^{-52.17^\circ i} \begin{pmatrix} +0.1163 e^{+26.89^\circ i} & +0.2165 e^{+28.06^\circ i} & -0.1536 e^{-30.53^\circ i} \\ +0.2165 e^{+28.06^\circ i} & +0.8193 e^{+28.00^\circ i} & -1.9276 e^{+29.52^\circ i} \\ -0.1536 e^{-30.53^\circ i} & -1.9276 e^{+29.52^\circ i} & +1.00000 \end{pmatrix} \text{[MeV]}, \quad (3.7) \]

Let us choose the free parameter \(|c_R|\) so as to result in small neutrino masses, for example when \(|c_R| = 3.2 \times 10^{14}\), we have \(\Delta m_{23}^2 = 1.5 \times 10^{-3}[\text{eV}^2]\).

Here there arises a question what makes the two flavor mixing angles large. We need to investigate the mixing matrices \(U_\nu\) and \(U_\nu\) which diagonalize \(M_\nu\) and \(M_\nu\), respectively. Those are obtained as

\[ U_\nu = \begin{pmatrix} +0.863 & +0.504 e^{+9.46^\circ i} & -0.022 e^{+56.66^\circ i} \\ -0.493 e^{-9.82^\circ i} & +0.834 & -0.248 e^{-16.63^\circ i} \\ -0.110 e^{-21.40^\circ i} & +0.223 e^{-18.10^\circ i} & +0.960 \end{pmatrix}, \quad (3.8) \]

\[ U_\nu = \begin{pmatrix} +0.992 & -0.092 e^{-15.94^\circ i} & -0.088 e^{+12.86^\circ i} \\ +0.049 e^{+76.86^\circ i} & +0.724 & -0.688 e^{-16.08^\circ i} \\ +0.117 e^{+9.80^\circ i} & +0.683 e^{+16.74^\circ i} & +0.721 \end{pmatrix}. \quad (3.9) \]

Here, \(|U_{\nu 11}|, |U_{\nu 12}|, |U_{\nu 21}|, |U_{\nu 22}| \gtrsim 0.5\) for the charged lepton mass matrix and \(|U_{\nu 23}|, |U_{\nu 32}|, |U_{\nu 33}| \gtrsim 0.7\) for the neutrino mass matrix. Therefore the components of the lepton flavor mixing matrix become \(|V_{i11}|, |V_{i12}|, |V_{i21}|, |V_{i22}|, |V_{i23}|, |V_{i23}| \gtrsim 0.5\):

\[ V_i = \begin{pmatrix} +0.844 e^{+2.10^\circ i} & -0.494 e^{-9.95^\circ i} & +0.206 e^{+23.61^\circ i} \\ +0.527 e^{+3.26^\circ i} & +0.696 e^{-8.84^\circ i} & -0.488 e^{+24.97^\circ i} \\ +0.098 e^{-15.78^\circ i} & +0.521 e^{-27.43^\circ i} & +0.848 e^{+6.32^\circ i} \end{pmatrix}. \quad (3.10) \]

The mixing angle \(\theta_{23}\) becomes larger, while the mixing angle \(\theta_{12}\) smaller, if we take the smaller value of \(|m_\ell|\) or \(|m_d|\), or larger \(|m_e|\), or \(|m_b|\), \(|m_s|\) than their center values.

As a simple example, the shift of \(|m_d|\) and \(|m_s|\) causes the change of mixing angles and neutrino mass square differences as depicted in Fig.2. Fig.2 shows that the \(\theta_{23}\) and \(\theta_{13}\) can approach the 99\% C.L. of SK \(\text{[SK]}^+\text{CHOZ}\) but \(\theta_{12}\) and \(\Delta m_{12}^2\) are out of the range of 99\% – 99.9\% C.L. of SOLAR \(\text{[SOLAR]}^+\text{CHOZ}\).
FIG. 1. The relation between our results and the two-flavor oscillation analysis [6] when $\sigma$ is moved. (a) The circles and triangles indicate the values of $\Delta m_{23}^2/\Delta m_{13}^2$ and $\Delta m_{12}^2/\Delta m_{13}^2$ at every $\pi/2$ of $\sigma$. (b) The circles, triangles, and stars indicate the values of $\sin^2 2\theta_{12}, \sin^2 2\theta_{13},$ and $\sin^2 2\theta_{23}$ at every $\pi/2$ of $\sigma$, respectively. (c) The circles, triangles, and stars indicate the values of $(\Delta m_{23}^2, \sin^2 2\theta_{23}), (\Delta m_{12}^2, \sin^2 2\theta_{12}),$ and $(\Delta m_{12}^2, \sin^2 2\theta_{13})$ at every $\pi/2$ of $\sigma$. Here we have set $\Delta m_{23}^2 = 1.5 \times 10^{-3} [eV^2]$ in every case.
FIG. 2. The relation between $3\nu$ Oscillation analyses by G.L.Fogli et al. [10] and by us for $\Delta m_{23}^{2} = 1.5 \times 10^{-3}$[eV$^{2}$]. (a) For SK+CHOOZ. (b) For SOLAR+CHOOZ. The circles indicate our solutions for Eq.(3.1). The solid line through them is the track as $m_{d}$ is varied. It goes from the experimental limit that $|m_{d}|$ moves over the range, 4.03 - 5.29 MeV [5]. At that time, $|m_{s}|$ simultaneously changes over the range, 76.3 - 76.2 MeV so as to satisfy the relations (1.12) and (1.13). If we take the smaller $|m_{d}|$ with the fixed $\sigma$, the solution in (a) moves rightward and the solution in (b) does left-upward (Table I (i)). Since the minimum $|m_{d}|$ for (b) gives bad fitting, we have changed $\sigma$ from $149\pi/100$ to $146\pi/100$, which is denoted by star (Table I (ii)). Thus our result approaches the 99%C.L. of SK+CHOOZ and 99.9%C.L. of SOLAR+CHOOZ.

IV. DISCUSSION

Since there are only two basic matrix $M_{0}$ and $M_{1}$ in this model, the number of parameters in Eq.(2.1) and (2.14) is

\[\begin{align*}
D_{u}, D_{d}, D_{c}, D_{\nu} & = 3 \times 4 = 12 \\
\epsilon_{d}, |\epsilon_{R}|, \kappa & = 2 + 1 + 2 = 5 \\
V_{\alpha}, A_{c}, A_{\nu} & = 4 + 9 + 9 = 22 \\
\sum & = 39
\end{align*}\]

and the number of equations is $N(\text{eqs}) = 12 \times 2 = 24$. Therefore the number of free parameters is $N(\text{pmt}) - N(\text{eqs}) = 39 - 24 = 15$. On the other hand, the number of the physical parameters which can be determined by experiments is

\[\begin{align*}
m_{u}, m_{c}, m_{t} & = 3 \\
m_{d}, m_{s}, m_{b} & = 3 \\
\text{CKM: } & \theta_{12}, \theta_{23}, \theta_{13}, \delta, \delta_{c} & = 4 \\
m_{e}, m_{\mu}, m_{\tau} & = 3 \\
m_{\nu}, m_{\nu}, m_{\nu}, m_{\nu} & = 3 \\
\text{MNS: } & \theta_{12}, \theta_{23}, \theta_{13}, \delta, \beta, \rho & = 6 \\
\sum & = 22
\end{align*}\]

where $\beta$ and $\rho$ are Majorana phases in the Maki-Nakagawa-Sakata (MNS) matrix because of no rephasing in the neutrino fields $\nu_{L}$. To sum up the matter, we discuss the consistency test about 22 physical parameters by using only 15 free parameters. The consistency test in the quark sector is good, as shown in our previous paper. In the lepton sector, the test is not so bad when we
adopt the MSW large mixing angle solution of solar neutrino deficit, and this model favors the normal hierarchy of neutrino mass spectrum.

Also we can predict the yet unobserved values such as the averaged neutrino masses \( \langle m \rangle_{\alpha \beta} \) and Jarlskog parameter in the lepton part. The averaged neutrino masses appear in the reactions where the Majorana neutrinos propagate in the intermediate states. They are

\[
\langle m \rangle_{\alpha \beta} = \sum_{j=1}^{3} U_{\alpha j} U_{\beta j} m_j,
\]

where \( \alpha \) and \( \beta \) are \( e, \mu, \tau \). They correspond to neutrinoless double beta decay \( 11 \) for \( \alpha = \beta = e, \mu - e \) conversion \( (\mu^{-} + (A, Z) \rightarrow e^{+} + (A, Z - 2) \) for \( \alpha = \mu, \beta = e \), and \( K \) decay \( (K^{-} \rightarrow \pi^{+} + \mu^{-}) \) for \( \alpha = \beta = \mu \) etc. In Fig.3 we have depicted \( \sigma \) dependence of \( \langle m \rangle_{\alpha \beta} / \sqrt{\Delta m_{23}^2} \).

In the case of Eq.(3.1), these values become as follows.

\[
\frac{\langle m \rangle_{\alpha \beta}}{\sqrt{\Delta m_{23}^2}} \approx \begin{pmatrix}
0.032 & 0.065 & 0.30 \\
0.096 & 0.59 & 0.67
\end{pmatrix}.
\]

For instance, if we input \( \Delta m_{23}^2 = 1.5 \times 10^{-3} \) [eV\(^2\)], \( \langle m \rangle_{ee} \) becomes 0.0012 [eV]. This is smaller than the experimental value of the next generation experiments such as GENIUS \( 13 \), CUORE \( 14 \), and MOON \( 15 \). Jarlskog parameter \( 16 \) appears in three generations

\[
P(\nu_e \rightarrow \nu_\mu) - P(\nu_\mu \rightarrow \nu_e) = J \frac{\Delta E_{21} \Delta E_{32} \Delta E_{31}}{\Delta E_{21}^M \Delta E_{32}^M \Delta E_{31}^M} \\
	imes \sin \left( \frac{\Delta E_{21}^M L}{2} \right) \sin \left( \frac{\Delta E_{32}^M L}{2} \right) \sin \left( \frac{\Delta E_{31}^M L}{2} \right)
\]

with

\[
J = \text{Im}(V_{122} V_{122}^* V_{113} V_{123}).
\]

Here we have adopted the notation

\[
\Delta E_{jk} = E_j - E_k = \frac{\Delta m_{jk}^2}{2E} \\
\Delta E_{jk}^M = E_j^M - E_k^M
\]

with

\[
U \text{diag}(E_1, E_2, E_3) U^{-1} + \text{diag}(0, 0, 0) = U^M \text{diag}(E_1^M, E_2^M, E_3^M)(U^M)^{-1}
\]

The \( \sigma \) dependence of \( J \) is depicted in Fig.4. For Eq.(4.1), it takes

\[
J \simeq 0.00015.
\]

However, it needs careful consideration that \( J \) drastically changes at \( \sigma \simeq 3\pi/2 \). \( \langle m \rangle_{\alpha \beta} \) and \( J \) in the cases of Table I (i) and (ii) discussed in Fig.2 are also listed in Table II (i) and (ii). In this paper we have discussed how far the SO(10) two Higgs scalar model describes the quark-lepton masses and mixing parameters. We can conclude that this model cannot be rejected within the existing data. It should be remarked that the whole parameters can be decided from the existing data in principle.
The circles indicate the values of $\sigma$ for the Super-Kamiokande in Neutrino 2000 et al. [6] Particle Data Group, D. E. Groom [4] B. Brahmachari and R.N. Mohapatra, Phys. Rev. D58 [3] K. Oda, E. Takasugi, M. Tanaka and M. Yoshimura, Phys. Rev. [2] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 70 [1] K. Matsuda, Y. Koide, and T. Fukuyama, Phys. Rev. D59 055001 (1999).

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\( |m_d| = 4.03[\text{MeV}] 
\)

\( |m_d| = 76.3[\text{MeV}] \), \( \sigma = 149\pi/100 
\)

\( \langle \Delta m^2_{12} \rangle / \langle \Delta m^2_{13} \rangle = 0.43 \), \( \langle \Delta m^2_{23} \rangle / \langle \Delta m^2_{13} \rangle = 0.57 \), \( \sin^2(2\theta_{12}) = 0.52 \), \( \sin^2(2\theta_{13}) = 0.91 \), \( \sin^2(2\theta_{13}) = 0.17 \)

\( |m_d| = 4.03[\text{MeV}] 
\)

\( |m_d| = 76.3[\text{MeV}] \), \( \sigma = 146\pi/100 
\)

\( \langle \Delta m^2_{12} \rangle / \langle \Delta m^2_{13} \rangle = 0.20 \), \( \langle \Delta m^2_{23} \rangle / \langle \Delta m^2_{13} \rangle = 0.80 \), \( \sin^2(2\theta_{12}) = 0.54 \), \( \sin^2(2\theta_{13}) = 0.88 \), \( \sin^2(2\theta_{13}) = 0.20 \)

\( \langle \Delta m^2_{12} \rangle / \langle \Delta m^2_{13} \rangle = 0.43 
\)

\( \langle \Delta m^2_{23} \rangle / \langle \Delta m^2_{13} \rangle = 0.96 
\)

\( \langle \Delta m^2_{12} \rangle / \langle \Delta m^2_{13} \rangle = 0.064 
\)

\( \langle \Delta m^2_{23} \rangle / \langle \Delta m^2_{13} \rangle = 0.10 
\)

\( \langle \Delta m^2_{12} \rangle / \langle \Delta m^2_{13} \rangle = 0.71 
\)

\( J = 0.0091 
\)

\( J = -0.0114 
\)

**Table I.** Our solution (the second and third lines) from the input parameters (the first line). The result of (i) is obtained when we move \( |m_d| \) from 4.69[MeV] to 4.03[MeV], (ii) is the result when we move \( |m_d| \) as (i) and, furthermore, \( \sigma \) from 149\pi/100 to 146\pi/100. These data fitting corresponds to Fig.2.

\( \langle \Delta m^2_{12} \rangle / \langle \Delta m^2_{13} \rangle = 0.039 
\)

\( \langle \Delta m^2_{23} \rangle / \langle \Delta m^2_{13} \rangle = 0.086 
\)

\( |m^e|/\sqrt{\Delta m_{23}^2} = 0.19 
\)

\( |m^e|/\sqrt{\Delta m_{13}^2} = 0.52 
\)

\( |m^\tau|/\sqrt{\Delta m_{13}^2} = 0.064 
\)

\( |m^\mu|/\sqrt{\Delta m_{13}^2} = 0.10 
\)

\( |m^\tau|/\sqrt{\Delta m_{13}^2} = 0.54 
\)

\( J = -0.0114 
\)

**Table II.** The values of averaged neutrino masses and the Jarlskog parameter for the case (i) and (ii) in Table I.