CARL: Conditional-value-at-risk Adversarial Reinforcement Learning

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Abstract

In this paper we present a risk-averse reinforcement learning (RL) method called Conditional value-at-risk Adversarial Reinforcement Learning (CARL). To the best of our knowledge, CARL is the first game formulation for Conditional Value-at-Risk (CVaR) RL. The game takes place between a policy player and an adversary that perturbs the policy player’s state transitions given a finite budget. We prove that, at the maximin equilibrium point, the learned policy is CVaR optimal with a risk tolerance explicitly related to the adversary’s budget. We provide a gradient-based training procedure to solve CARL by formulating it as a zero-sum Stackelberg Game, enabling the use of deep reinforcement learning architectures and training algorithms. Finally, we show that solving the CARL game does lead to risk-averse behaviour in a toy grid environment, also confirming that an increased adversary produces increasingly cautious policies.

Reinforcement Learning (RL) problems \cite{14} are usually approached by training agents to find policies that maximise the expected return of the available trajectories. This expectation maximisation approach achieved great performance in recent years on artificial domains like board games \cite{13} or video games \cite{15}. However, naively maximizing the expected return can result in unsafe behaviour in critical domains such as healthcare \cite{17} or autonomous driving \cite{8}, where it is especially important that the agent avoids risky behaviour.

This has motivated the community to design risk-sensitive algorithms, where the agent is trained to account for the possibility of catastrophic events. One popular way to proceed is to include a risk measure \cite{1} in the algorithm’s objective. A commonly used risk measure is the Conditional Value-at-Risk (CVaR), defined as the expectation over the worst $\alpha$-quantile of a distribution. The search for a CVaR-optimal policy is called CVaR-RL and can be described as

$$\pi^* := \max_{\pi} \text{CVaR}_\alpha \left[ \sum_{t=1}^{\infty} \gamma^t r_t \big| \mathcal{P} \right],$$

where actions $a_t$ are sampled from $\pi(s_t)$, next states $s_{t+1}$ are sampled from the transition function $\mathcal{P}(s_t, a_t)$, rewards $r_t$ are obtained from the reward function $R(s_{t+1})$ and $\gamma \in [0, 1]$ is a discount factor.

Existing methods for CVaR-RL \cite{12, 7} usually adopt a distributional RL perspective \cite{2}, where a critic is trained to predict the whole distribution of returns rather than only its mean. We adopt a simpler approach inspired from previous empirical \cite{10} and theoretical \cite{4} work on adversarial RL. Casting the adversarial RL framework as a two-player zero-sum game, we provide a game-theoretic analysis showing that the equilibrium of the game leads to a CVaR optimal behavior.

The method we propose, which we refer to as Cvar Adversarial Reinforcement Learning (CARL), is a game between an agent and an adversary that has a fixed budget. The adversary $\Lambda$ aims to minimize the rewards collected by the agent by perturbing the next state transition distributions. Specifically, given normal transitions $\mathcal{P}_t = \mathcal{P}(s_t, a_t)$, the adversary produces perturbed transitions $\hat{\mathcal{P}}_t = \mathcal{P}_t \circ \delta_t$, for $\circ$ the Hadamard product and adversary perturbations $\delta_t$. For each trajectory, the adversary is given a fixed budget $\eta$ which is defined as the product of all the perturbations it produced. Namely, at each time step $t$, a factor $\delta_t = \delta(s_{t+1})$ corresponding to the adversary impact on the next state sampled from $\hat{\mathcal{P}}_t$, is multiplied with previous perturbations to ensure that the budget constraint $\eta \geq \prod_{t=1}^{\infty} \delta_t$ is respected. The agent, on its hand, is tasked with the usual maximization of its return expectation, under the only difference that the expectation is now taken with respect to the perturbed state transitions $\hat{\mathcal{P}}$. In the end, CARL can be seen as maximin problem of the form

$$\max_{\pi} \min_{\Lambda} \mathbb{E} \left[ \sum_{t=1}^{\infty} \gamma^t r_t \big| \hat{\mathcal{P}} \right],$$

where actions $a_t$ are sampled from $\pi(s_t)$, next states $s_{t+1}$ are sampled from the transition function $\mathcal{P}(s_t, a_t)$, rewards $r_t$ are obtained from the reward function $R(s_{t+1})$ and $\gamma \in [0, 1]$ is a discount factor.
which corresponds to the classical expectation maximization objective in RL, with the modification that transitions are sampled from the (budgeted) perturbed transitions \( \hat{P} \).

The solution \((\pi^*, \Lambda^*)\) to the maximin problem (2) is very closely related to the CVaR-optimality objective \((1)\). Indeed, Chow et al. [4] proved that the policy \(\pi^*\) in the solution is the CVaR optimal policy and that the quantile it is optimizing for is precisely \(\alpha = \frac{1}{\eta}\). Therefore, solving the CARL game not only produces a CVaR optimal policy, but it also provides a straightforward way to select the risk sensitivity of the model by setting the adversary’s budget.

Computing the solution to (2) for players represented by neural networks is not a simple task however. Indeed, the dependence of each player’s reward on the other makes the optimization objective non-stationary [5], hampering naive gradient-based algorithms’ convergence. To this end, we propose to view CARL as a Stackelberg game [16], which allows us to derive well-defined gradient-based algorithms. Under the Stackelberg formulation, we fix a leader and a follower, where the follower is defined as being optimal with respect to the leader. This way, the follower’s parameters are implicitly defined by the leader’s parameters and we can solve the game as a bi-level optimization problem. The key to solving this problem is to make the follower quickly learn almost optimal parameters while moving the leader’s parameters in slower fashion.

In practice, we adopt a few relaxations of the exact Stackelberg game setting that have proven effective in domains like GANs [6; 9] or model-based RL [11], to name a few. Precisely, we (1) implement the almost optimal updates of the follower by doing multiple gradient steps and (2) we use the first-order approximation of the joint gradient to update the leader. A pseudocode of the overall algorithm can be seen in Algorithm 1.

**Algorithm 1 CARL Game: Practical Stackelberg Meta-Algorithm**

| Require: \(\pi_\theta\) (parametrized policy), \(\Lambda_\omega\) (parametrized adversary), \(\eta\) (perturbation budget) |
|-------------------------------------------------|
| 1: while not done do |
| 2: Get initial state \(s_t = s_0\) |
| 3: \(\eta_r = \eta\) \hspace{1cm} \(\triangleright\) Remaining trajectory budget |
| 4: while not terminal do |
| 5: \(a_t \sim \pi_\theta(s_t)\) |
| 6: \(\mathcal{P}_t = \mathcal{P}(s_t, a_t)\) |
| 7: \(\delta = \Lambda_\omega(\mathcal{P}_t, \eta_r)\) \hspace{1cm} \(\triangleright\) Subject to \(||\delta||_\infty \leq \eta\) |
| 8: \(\hat{\mathcal{P}}_t = \mathcal{P}_t \circ \delta\) |
| 9: \(s_{t+1} \sim \hat{\mathcal{P}}_t, r_{t+1} \sim \mathcal{R}(s_{t+1})\) |
| 10: \(\eta_r = \frac{\eta_r}{\eta_{s_{t+1}}}\) \hspace{1cm} \(\triangleright\) Update remaining budget |
| 11: if time to update leader then |
| 12: Update \(\theta\) according to any classical RL algorithm. |
| 13: else |
| 14: Update \(\omega\) according to any classical RL algorithm. |

We conduct experiments on a variant of the Lava environment from Gym Minigrid [3], presented in Figure 1. Our preliminary results are displayed in Figure 2. These results confirm that increasing the budget results in a safer policy, as the agent progressively learns to distance itself more from the lava. We also see that training the agent without the adversary (budget \(\eta = 1\) case) results in a policy where the agent comes close to the lava, whereas the two cases with non-empty budgets result in the agent completely avoiding those risky positions.
Figure 1: **Overview of our Lava gridworld environment.** The agent (red circle) can navigate through the environment using any of the 4 usual directions (up, down, left and right). Reaching the goal (green square) gives a large reward (+1) to the agent while stepping in the lava (wavy orange squares) gives an equally large negative reward (-1). Each step also incurs a small negative reward (-0.035). Trajectories end when the agent reaches the goal or a lava square or after 100 steps. At each time step, state transitions bring the agent to a random accessible square (in the 4 directions) with probability $p = 0.15$, otherwise moving it according to the agent’s selection.

Figure 2: **Repartition of visited states from policies learned with varying adversary budget.** Each figure represents the density distribution of states visited by an agent trained with an adversary given budget of $\eta = 1, 10, 100$, from left to right. A budget of 1 signifies that there was no adversary during training. For evaluation, we sample 100,000 trajectories using the learned policy and no adversary.
References

[1] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath. Coherent measures of risk. *Mathematical finance*, 9(3):203–228, 1999.

[2] M. G. Bellemare, W. Dabney, and R. Munos. A distributional perspective on reinforcement learning. In *International Conference on Machine Learning*, pages 449–458. PMLR, 2017.

[3] M. Chevalier-Boisvert, L. Willems, and S. Pal. Minimalistic gridworld environment for openai gym. 
https://github.com/maximecb/gym-minigrid, 2018.

[4] Y. Chow, A. Tamar, S. Mannor, and M. Pavone. Risk-sensitive and robust decision-making: a cva
r optimization approach. In *NIPS*, pages 1522–1530, 2015. URL https://proceedings.neurips.cc/paper/2015/hash/64223ccf70bbb65a3a4aceac37e21016-Abstract.html

[5] T. Fiez, B. Chasnov, and L. Ratliff. Implicit learning dynamics in stackelberg games: Equilibria characteriza-
tion, convergence analysis, and empirical study. In H. D. III and A. Singh, editors, *Proceedings of the 37th Inter-
national Conference on Machine Learning*, volume 119 of *Proceedings of Machine Learning Research*, pages 3133–3144. PMLR, 13–18 Jul 2020. URL https://proceedings.mlr.press/v119/fiez20a.html

[6] M. Heusel, H. Ramsauer, T. Unterthiner, B. Nessler, and S. Hochreiter. Gans trained by a two time-scale
update rule converge to a local nash equilibrium. *Advances in neural information processing systems*, 30, 2017.

[7] R. Keramati, C. Dann, A. Tamkin, and E. Brunskill. Being optimistic to be conservative: Quickly learning a
cvar policy. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 4436–4443, 2020.

[8] J. Levinson, J. Askeland, J. Becker, J. Dolson, D. Held, S. Kammel, J. Z. Kolter, D. Langer, O. Pink, V. Pratt, et al. Towards fully autonomous driving: Systems and algorithms. In 2011 IEEE intelligent
vehicles symposium (IV), pages 163–168. IEEE, 2011.

[9] L. Metz, B. Poole, D. Pfau, and J. Sohl-Dickstein. Unrolled generative adversarial networks. In 5th
International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Conference Track Proceedings. OpenReview.net, 2017. URL https://openreview.net/forum?id=Bydr0fCle

[10] L. Pinto, J. Davidson, R. Sukthankar, and A. Gupta. Robust adversarial reinforcement learning. In *International Conference on Machine Learning*, pages 2817–2826. PMLR, 2017.

[11] A. Rajeswaran, I. Mordatch, and V. Kumar. A game theoretic framework for model based reinforcement
learning. In H. D. III and A. Singh, editors, *Proceedings of the 37th International Conference on Machine
Learning*, volume 119 of *Proceedings of Machine Learning Research*, pages 7953–7963. PMLR, 13–18 Jul 2020. URL https://proceedings.mlr.press/v119/rajeswaran20a.html

[12] F. Schubert, T. Eimer, B. Rosenhahn, and M. Lindauer. Automatic risk adaptation in distributional
reinforcement learning, 2021.

[13] D. Silver, T. Hubert, J. Schrittwieser, I. Antonoglou, M. Lai, A. Guez, M. Lanctot, L. Sifre, D. Kumaran,
T. Graepel, et al. A general reinforcement learning algorithm that masters chess, shogi, and go through
self-play. *Science*, 362(6419):1140–1144, IEEE, 2018.

[14] R. S. Sutton, A. G. Barto, et al. *Introduction to reinforcement learning*, volume 135. MIT press Cambridge, 1998.

[15] O. Vinyals, I. Babuschkin, W. M. Czarnecki, M. Mathieu, A. Dudzik, J. Chung, D. H. Choi, R. Powell,
T. Ewalds, P. Georgiev, et al. Grandmaster level in starcraft ii using multi-agent reinforcement learning.
*Nature*, 575(7782):350–354, 2019.

[16] H. Von Stackelberg. *Market structure and equilibrium*. Springer Science & Business Media, 2010.

[17] J. Wiens, S. Saria, M. Sendak, M. Ghassemi, V. X. Liu, F. Doshi-Velez, K. Jung, K. Heller, D. Kale,
M. Saeed, et al. Do no harm: a roadmap for responsible machine learning for health care. *Nature medicine*, 25(9):1337–1340, 2019.