Testing the constancy of Spearman’s rho in multivariate time series

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Abstract A class of tests for change-point detection designed to be particularly sensitive to changes in the cross-sectional rank correlation of multivariate time series is proposed. The derived procedures are based on several multivariate extensions of Spearman’s rho. Two approaches to carry out the tests are studied: the first one is based on resampling and the second one consists of estimating the asymptotic null distribution. The asymptotic validity of both techniques is proved under the null for strongly mixing observations. A procedure for estimating a key bandwidth parameter involved in both approaches is proposed, making the derived tests parameter-free. Their finite-sample behavior is investigated through Monte Carlo experiments. Practical recommendations are made and an illustration on trivariate financial data is finally presented.

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1 Introduction

Let $X_1, \ldots, X_n$ be a multivariate times series of $d$-dimensional observations and, for any $i \in \{1, \ldots, n\}$, let $F_i$ denote the cumulative distribution function (c.d.f.) of $X_i$.

We are interested in procedures for testing $H_0 : F_1 = \cdots = F_n$ against $\neg H_0$. Notice that the aforementioned null hypothesis can be simply rewritten as

$$H_0 : \exists F \text{ such that } X_1, \ldots, X_n \text{ have } c.d.f. F. \quad (1)$$

Such statistical procedures are commonly referred to as tests for change-point detection (see, e.g., Csörgő and Horváth 1997, for an overview of possible approaches).

The majority of tests for $H_0$ developed in the literature deal with the case $d = 1$. We aim at developing nonparametric tests for multivariate time series that are particularly sensitive to changes in the dependence among the components of the $d$-dimensional observations. The availability of such tests seems to be of great practical importance for the analysis of economic data, among others. In particular, assessing whether the dependence among financial assets can be considered constant or not over a given time period appears crucial for risk management, portfolio optimization and related statistical modeling (see, e.g., Wied et al. 2014; Dehling et al. 2014, and the references therein for a more detailed discussion about the motivation for such statistical procedures).

The above context, rather naturally, suggests to address the informal notion of dependence through that of copula (see, e.g., Nelsen 2006). Assume that $H_0$ in (1) holds and that, additionally, the common marginal c.d.f.s $F_1, \ldots, F_d$ of $X_1, \ldots, X_n$ are continuous. Then, from the work of Sklar (1959), the common multivariate c.d.f. $F$ of the observations can be written as

$$F(x) = C\{F_1(x_1), \ldots, F_d(x_d)\}, \quad x \in \mathbb{R}^d,$$

where the function $C : [0, 1]^d \rightarrow [0, 1]$ is the unique copula associated with $F$. It follows that $H_0$ can be rewritten as

$$H_0,_{m \cap} H_0,_{c} : \exists F_1, \ldots, F_d \text{ such that } X_1, \ldots, X_n \text{ have marginal } c.d.f. \text{s } F_1, \ldots, F_d, \quad (2)$$

$$H_0,_{c} : \exists C \text{ such that } X_1, \ldots, X_n \text{ have copula } C. \quad (3)$$

Several nonparametric tests designed to be particularly sensitive to certain alternatives under $H_0,_{m \cap} \neg H_0,_{c}$ were proposed in the literature. Tests for the constancy of Kendall’s tau (which is a functional of $C$) were investigated by Gombay and Horváth (1999) (see also, Gombay and Horváth 2002) and Quessy et al. (2013) in the case of serially independent observations. A version of the previous tests adapted to a very general class of bivariate time series was proposed by Dehling et al. (2014). Recent multivariate alternatives are the tests studied in (Bücher et al. 2014, see also the references therein) based on Cramér–von Mises functionals of the sequential empirical copula process.