Cosmological consequences of short distance physics

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Inflation can act as a space-time microscope for Planck or string scale effects, leaving potentially observable traces in the primordial perturbation spectrum. I discuss two frameworks that were used recently to study this phenomenon: nonlinear dispersion and short distance uncertainty.

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1 Introduction

To the best of our current understanding, inflation – an epoch of accelerated cosmological expansion – may have taken place when the expansion rate of the universe was $H \sim 10^{14}$ GeV (but there exist models at lower values of $H$) [1, 2]. Quantum vacuum fluctuations at this energy scale are generally held responsible for producing adiabatic perturbations that are reflected in the CMBR anisotropies. It is possible that nature has provided us with a rare glimpse at physics far beyond any other means of experimental access.

The increasing precision of observations linked, directly or indirectly, to the physics of inflation presents us with two important challenges. First, we must probe the sensitivity of cosmological predictions derived in the framework of semiclassical QFT on curved or non-stationary space-times with respect to possible quantum gravitational modifications. This approach has a long and fruitful tradition in the context of the so-called trans-Planckian problem of Hawking radiation (reviewed in [3]) and was indeed inspired by it. Much of the work discussed in Sec. 2 is based on fluid analogies of black hole horizons introduced by Unruh [4] which study the effects of a nonlinear dispersion relation on outgoing Hawking quanta. Since these are often described as “sonic black holes” I will refer to this line of research as “sonic inflation” in Sec. 2.

Secondly, and perhaps even more importantly, it is possible that such modifications leave an unambiguous and detectable footprint in the cosmological data. Its concrete quantification could eventually be based on string/M theory models (or directly on the holographic principle [5]), but one can begin this ambitious task by exploring field theoretic models that mimic some general feature of short distance physics (as an analogy, think of starting with the Euler equations and add the transport terms without knowing the microscopic theory). For example, it is possible to implement a cutoff that effectively limits the resolution of short distances into the standard formalism, and re-analyze the predictions of inflation. I discuss the current status of this program in Sec. 3.

All of these studies break local Lorentz invariance in order to define the notion of a locally measurable short distance scale. So far, there is no experimental indication that Lorentz invariance might be violated in nature [6]. On the other hand, neither is there any experimental indication that it is unbroken at $\sim 10^{14}$ GeV, so one should remain open minded. As for physical motivation, there are speculations about broken Lorentz invariance in the contexts of string/D-brane interactions [7], brane cosmology [8], and quantum gravity phenomenology [9], to name a few. Violations or modifications of local Lorentz symmetry also play a vital role in varying speed of light (VSL) cosmologies, making it obvious to look at VSL from the viewpoint of short distance uncertainty; see Sec. 4 for a brief discussion.
2 Sonic inflation

The first thing an experimentalist notices as the wavelength of a fluid sound wave approaches the molecular scale is a change of the dispersion relation, typically dipping off from the linear behavior toward a slower growth of frequency $\omega$ with wavenumber $\kappa$. This fact, together with the observation that sound waves in a fluid with sonic horizon are mathematically equivalent to scalar fields in a black hole space-time, led Unruh [4] to introduce sonic black holes (or “dumb holes”) by replacing the linear dispersion relation in the mode equation of Hawking quanta with an artificial nonlinear one, chosen to mimic the fluid behavior. Interpreting the scale of the nonlinearity as the Planck scale, this framework has been used extensively to study the robustness of Hawking radiation with respect to trans-Planckian physics [10].

Similar to Hawking photons barely escaping to infinity, quantum fluctuations in an inflationary background undergo enormous redshifting before they reach super-horizon scales. Trans-Planckian effects in inflation can therefore be addressed in a nearly identical manner [11, 12]. Both scalar and tensor perturbation amplitudes are proportional to the solution of a harmonic oscillator equation,

$$\chi''_k + \omega^2 \chi_k = 0,$$

(1)

for the mode $\chi_k$ of the comoving wavenumber $k$ as a function of conformal time $\eta$, evaluated at horizon crossing, $k = aH$ ($a$ is the scale factor of a flat FRW line element and $H = a'/a^2$). Normally, $\omega^2$ is given by

$$\omega^2_0(\eta) = k^2 - \frac{a''(\eta)}{a(\eta)},$$

(2)

whereas it is written as

$$\omega^2_F(\eta) = [a(\eta) F(k/a(\eta))]^2 - \frac{a''(\eta)}{a(\eta)},$$

(3)

in the modified form. $F(k/a)$ is a smooth function with the following properties:

1. $F(k/a)$ converges to the usual physical wavenumber $\kappa = k/a$ for $\kappa \ll \kappa_c$. The critical wavenumber $\kappa_c$ marks the scale of deviations from standard physics and is presumably of order the Planck scale. It is usually assumed that $F(k/a) = \kappa$ at horizon crossing, but this need not be the case even if $H < \kappa_c$, depending on the shape of $F$ [12].

2. At around $\kappa_c$, $F$ veers off the linear path and increases, becomes constant, or decreases in what are usually called superluminous, Unruh, or subluminous dispersion relations [13].
With the exception of [14] where it is derived from $\kappa$-Poincaré algebra, $F$ is simply chosen by hand to fit these criteria. On a side note, it is even possible to create forms of $F$ that allow for an inflationary period without a scalar inflaton field [15].

As there are two dimensionful variables in this theory, $H$ and $\kappa_c$, the solutions are controlled by the dimensionless quantity

$$\sigma = \frac{H}{\kappa_c},$$

which is usually a small number. Another important role is played by the adiabaticity parameter,

$$C(\eta) = \left| \frac{\omega^\prime}{\omega^2} \right|,$$

describing the “smoothness” of $F$ with respect to the oscillations of $\chi_k$ and hence the accuracy of the WKB approximation.

As shown in [16] and confirmed in [17], $C \leq \sigma$ for all regular, monotonically growing functions $F$, while changes in the power spectrum due to nonadiabatic particle production are at most of order $C$ (but may be much smaller). Consequently, if the critical wavenumber is identified with the Planck scale, the CMBR fluctuation amplitude indicates that $\sigma \lesssim 10^{-5}$ and such changes are undetectable. However, in some string or M theory models $\sigma$ may be several orders of magnitude larger [13].

Singular dispersion relations of the form $F(\kappa) \sim \kappa(1-\kappa/\kappa_c)^{-n}$ with $0 < n < 1$ and those with $F(\kappa \gg \kappa_c) < H$ exhibit nonadiabatic behavior [10]. In the latter case, each mode undergoes another “horizon crossing” on very small length scales below which it is frozen by cosmic expansion. Ref. [19] argues that this effect produces an effective dark energy contribution for a certain class of dispersion relations (but see [20] who reach a different conclusion based on the analysis of the stress-energy tensor). A string theoretic derivation of this type of dispersion relation is offered in Ref. [21].

In order to find the initial conditions of the mode equation (1) one must choose a vacuum for the field. The adiabatic vacuum, constructed from the WKB solution of Eq. (1), can be considered a natural ground state [22]. Backreaction of Planck scale particles forces the vacuum to be close to the adiabatic one in order to be consistent with slow-roll inflation [23]. In this case, deviations of the power spectrum can be caused by nonadiabatic processes on subhorizon scales (see above) or if $\omega_F \neq \omega_0$ at horizon crossing. Alternative vacua were investigated in [11]. The authors of Ref. [24] gave the initial value problem a new twist by constructing a bouncing universe cosmology. They followed the mode evolution through the bounce where it picks up non-adiabatic corrections, showing that the spectrum may be affected even with adiabatic initial conditions.

A crucial question is whether the effects of nonlinear dispersion can be disentangled from features of the inflaton potential. As noted in [25] and further analyzed in [13], Planck scale effects might act differently on scalar and tensor perturbations and thereby violate the scalar-tensor consistency relation of slow-roll inflation. This violation could be the smoking gun for physics beyond the standard paradigm of inflation.

3
3 Short distance uncertainty

Instead of modifying the dispersion relation by hand one can use an observation made in various contexts in string theory and quantum gravity: on very general grounds, our experimental ability to probe short distances seems to be limited by the Planck or string scale [26]. The cutoff presumably arises from complicated dynamics of the underlying fundamental theory but it can also be modeled by a nonlinear correction to the canonical commutation relation:

\[ [x, p] = i\hbar(1 + \beta p^2) \]  

(6)

as discussed in [27]. The correction term results in a lower bound \( \Delta x_{\text{min}} \sim \beta^{1/2} \) for distance measurements. Furthermore, Eq. (6) belongs to a class of only very few types of short-distance structures of space-time that are admitted under very general assumptions [28]. Hilbert space representations of Eq. (6) were employed, for instance, for regularizing field theory [29] and, more recently, for analyzing the impact of short distance uncertainty on the predictions of inflation.

In Ref. [30], Eq. (6) was implemented in a scalar field theory (representing, e.g., the inflaton) on an FRW background. The action decomposes into modes of conserved quasi-wavenumber \( \tilde{k} \) which converges to the usual comoving wavenumber \( k \) for \( \tilde{k}/a \ll \beta^{-1/2} \). Each \( \tilde{k} \)-mode is generated at the time \( \eta_c \) corresponding to its “Planck scale crossing”, defined by \( a(\eta_c) = \tilde{k}(e\beta)^{1/2} \), and obeys an oscillator equation with mass and damping terms that are singular at \( \eta_c \).

Refs. [31, 32] analyzed the implications for the cosmic perturbation spectrum. As shown in Ref. [31] by transforming the \( \tilde{k} \)-mode equation to the form of Eq. (1), new effects assuming full adiabaticity shortly before horizon crossing are at most of order \( \sigma^2 \). However, the numerical evaluation of \( C \) indicates that \( C \sim \sigma \) during most of the subhorizon evolution so that, following the arguments in Sec. 2, \( O(\sigma) \) effects on the spectrum cannot be strictly ruled out even if the mode is adiabatic immediately after its creation.

A numerical analysis of the \( \tilde{k} \)-mode evolution was performed in Refs. [32]. The authors used initial conditions obtained from an analytical solution of the linearized mode equation near \( \eta_c \) (also proposed in [11]) and found new features in the power spectrum whose amplitudes are linear in \( \sigma \). These are apparently related to deviations from adiabaticity. Ref. [18], on the other hand, argues on the basis of low energy locality that the leading order effect must be \( O(\sigma^2) \).

Obviously, finding the leading order correction in \( \sigma \) is of utmost importance for the observability of cutoff effects in the CMBR, given that \( \sigma \) is probably \( \lesssim 10^{-3} \). A linear signal would make the detection of Planck scale physics in cosmological data a realistic goal for the intermediate future. But even \( O(\sigma^2) \) effects are potentially observable in certain classes of M theoretic models [18]. Much of the remaining uncertainty stems from the ambiguity in the initial conditions of the \( \tilde{k} \)-mode equation. However, in contrast with the sonic inflation approach, we now have a specific prescription for the generation of individual \( \tilde{k} \)-modes, so there is hope that the model itself picks out a preferred vacuum.
For completeness, note also that the generation of modes from a quantum gravitational “soup” plays a central role in Brout’s two-fluid model of inflation [33].

4 Varying speed of light cosmology

Varying speed of light (VSL) cosmology is one of the few serious competitors of inflation [34]. It solves the horizon problem by postulating an early cosmological epoch where, roughly speaking, information propagates faster than the current speed of light $c$. This involves some modification or violation of Lorentz symmetry whose form and implementation has been addressed in different ways (e.g., [35]). A number of possible explanations for a varying speed of light have been offered, motivated, e.g., by noncritical string theory [36], brane cosmology [37], and non-commutative geometry [38].

The framework of short distance uncertainty described in Sec. 3 provides a well-defined platform for searching for VSL effects. As shown in Ref. [39] (see also [40]), the reduction of the available phase space volume per quantum mode at short wavelengths causes the equation of state of ultrarelativistic particles to stiffen at very high densities. This leads to a stronger than usual deceleration of the scale factor which competes with a higher than usual propagation speed of the particles. The definition of the latter, however, is ambiguous: possibilities include the group and phase velocity in the high energy tail, the thermal average of the group and phase velocity, and the speed of sound. Of these three groups, only the first provides a possible solution to the cosmological horizon problem. Perhaps more worrisome is the fact that this solution involves applying Eq. (6) at super-Planckian densities where it is unlikely to fully represent the relevant physics.

To conclude, the field of short distance physics in the very early universe has opened a number of new ways to think about old problems, including the tantalizing (but still highly speculative) prospect of detecting Planck scale effects in cosmological observations. We can hope for interesting new developments by the time of Cosmo-02.

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