On Bounds of Spectral Efficiency of Optimally Beamformed NLOS Millimeter Wave Links

Rakesh R T, Student Member, IEEE, Debarati Sen, Member, IEEE, Goutam Das

Abstract—Antenna beamforming is an indispensable feature for millimeter wave (mmWave) wireless communications in order to compensate for the severe path loss incurred due to high frequency operation. In this paper, we introduce a novel framework to evaluate the spectral efficiency (SE) of non-line-of-sight (NLOS) mmWave links with optimal analog beamforming. Optimality here implies the joint selection of antenna beams at the transmitter and receiver which simultaneously maximize the received power. We develop a generalized mathematical framework based on the extended Saleh-Valenzuela channel model to embody the impact of optimal analog beamforming into the performance metrics for NLOS mmWave links. We further utilize the sparsity in practical mmWave channels to develop a simplified framework in order to evaluate SE of beamformed directional links. Simulation results reveal that the proposed model is sufficiently accurate to characterize practical outdoor beamformed mmWave channels. In addition, by neglecting small scale fading we arrive at closed form expressions for SE, and prove that the approximation holds good for mmWave channels which exhibit a high degree of multi-path sparsity.

Index Terms—MmWave Communication, Directional Antenna, Optimal Analog Beamforming, Spectral Efficiency.

I. INTRODUCTION

RECENT advances in technology have paved the way for emergence of wideband millimeter wave (mmWave) communications providing a viable option to meet the future demand for multi-Gbps data rates [1]. However, high frequency mmWave transmission incurs significantly large path loss during signal propagation, and thereby limits the transmission range. To overcome this bottleneck, directional antennas with beamforming capability are employed for signal transmission and/or reception [2]. The objective of beamforming protocol is to steer the antenna beams at the transmitter and receiver nodes of a link such that the transmission rate is maximized [2]. This is achieved by optimizing the signal-to-noise ratio (SNR) or signal-to-interference plus noise ratio (SINR) [3] at the receiver.

Beamforming protocols essentially enable spatial filtering of multi-path signal components based on the defined optimality criteria [3]. The quality and reliability of the link therefore depends on the beamformed directional channel and in this context, statistical modeling of beamformed directional channels is essential to accurately obtain mmWave network performance metrics such as coverage probability, spectral efficiency (SE) etc. The schemes proposed in [4], [5] which evaluate the performance of mmWave networks with analog beamforming [5] simply model the beamformed directional channel by a random gain component assuming that the channel is frequency flat. This is similar to the model used for conventional sub-6 GHz systems where channel gain is obtained as the product of a Rayleigh or Nakagami-m random variable which accounts for small scale fading effect, and a path loss term that models the large scale fading effect. Similarly, a recent work on coverage analysis for mmWave line-of-sight (LOS) links with antenna beamforming [6] approximates the beamformed directional channel by a random gain component based on the uniformly random single path (UR-SP) assumption. However, in non-LOS (NLOS) mmWave channels the power content of multi-path components [7] are comparable, and thus the modeling approaches considered for beamformed directional channels in existing literature are not applicable. Therefore, a new mathematical framework is required which embodies the impact of optimal beamforming for performance study of NLOS mmWave links.

In this paper, we develop a mathematical framework to statistically model NLOS mmWave links with optimal analog beamforming in order to evaluate the SE of noise limited NLOS mmWave links. We assume that the optimal transmitter-receiver antenna beam pair is chosen from a set of non-overlapping antenna beams spanning the 360° azimuth space such that the received signal power is maximized. The omnidirectional propagation characteristics of the channel is represented by the extended Saleh-Valenzuela (S-V) spatial channel model [7]–[9]. We further derive the bounds on SE by adopting a tractable analytical approach based on two simplified versions of the channel model: (i) by assuming that there exists at most one propagation path per receiver antenna beam with its presence modeled by a probability value, and (ii) by neglecting small scale fading for each multi-path. The paper has two main contributions: (i) we introduce a novel modeling approach to study the statistical behavior of optimal analog antenna beamforming in NLOS mmWave links, and (ii) we obtain the cumulative distribution function (CDF) of the received signal power, which is later utilized to evaluate SE of a NLOS mmWave link for the aforementioned channel model variants.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a system model consisting of an outdoor mmWave link with the transmitter and receiver nodes separated by a distance $d$. The nodes are assumed to be equipped with directional antennas with beamforming capability. We further assume that direct LOS connectivity between the transmitter and receiver is blocked and hence the beamformed link is established through NLOS multi-path components (Fig. 1). We approximate the antenna radiation
pattern by a sectored model \[4\] with zero side lobe gain. Let \(\theta_{3dB,t}\) and \(\theta_{3dB,r}\) denote antenna half power beamwidth (HPBW) of the transmitter and receiver, respectively. The transmitter and receiver main lobe antenna gain values can approximately be calculated as \(G_{m,t} = \frac{2\pi}{\theta_{3dB,t}}\) and \(G_{m,r} = \frac{2\pi}{\theta_{3dB,r}}\) \[4\]. We further assume that the nodes select the optimal antenna beam pair that maximizes the SNR at the receiver node (out of \(M_t\) and \(M_r\) number of non-overlapping beams at the transmitter and the receiver nodes, respectively).

\[
P_R = P_tCd^{-\alpha} \sum_{i=1}^{L} |h_i|^2 G_T(\Theta_{rcf} - \Theta_t)G_R(\Phi_{rcf} - \Phi_t),
\]

where \(P_t\) represents the transmit power, \(c\) denotes the intercept point from the path loss formula, and \(\alpha\) denotes the path loss exponent. \(|h_i|^2\) is the small scale fading amplitude which is generally modeled as Rayleigh or Nakagami-\(m\) random variable \[7\]–\[8\]. \(G_T(\cdot)\) and \(G_R(\cdot)\) represent the antenna gain of the transmitter and receiver antennas respectively with corresponding antenna pointing angles \(\Theta_{rcf}\) and \(\Phi_{rcf}\). \(\Theta_t\) and \(\Phi_t\) are the angle of departure (AOD) and angle of arrival (AOA) of the \(i\)-th multi-path component.

The number of multi-path components denoted by \(L\) is a random variable with its average value denoted by \(\lambda_0\) \[7\]. Under the assumption of the sectored radiation pattern model and unit transmit power, the signal power received by the \(i\)-th antenna beam pair can be obtained as \[1\] as \(P_{R,i} = cd^{-\alpha} \sum_{i=1}^{n_i} |h_i|^2 G_{m,t}G_{m,r}\), where \(n_i\) denotes the number of multi-path components located inside the antenna main lobe of the receiver corresponding to the \(i\)-th antenna beam pair. We assume that \(n_i\) is a Poisson random variable with average number of multi-path components \(\lambda_d = \lambda_0/B\), where \(B\) denotes the total number of available transmitter-receiver beam pairs \((B = M_tM_r)\). It should be noted that in practice the average received signal power varies with antenna beam orientation angle \[7\]–\[9\], and therefore \(\lambda_d\) as well as \(\alpha\) are functions of antenna beam orientation angle. As of now due to lack of availability of empirical data to capture this variation, we assume \(\lambda_d\) and \(\alpha\) to be constant \[7\]–\[9\] which incidentally also lends mathematical tractability for analysis. It may also be noted that \(\lambda_d\) and \(\alpha\) could be obtained by making use of the analytical model reported in our prior work \[10\]. However, this modeling approach is presently out of scope of this paper. The small scale fading gain \(|h_i|^2\) is assumed to be Nakagami-\(m\) distributed with mean power equal to \(1/\lambda_0\), which ensures that \(\mathbb{E}[|h_i|^2]\) \(\approx 1\), where \(\mathbb{E}[\cdot]\) denotes the expectation operator. The received power corresponding to the \(i\)-th antenna beam pair can thus be expressed as, \(P_{R,i} = cd^{-\alpha} \lambda_0^{-1} \sum_{i=1}^{n_i} |h_i|^2 G_{m,t}G_{m,r}\) with \(\mathbb{E}[|h_i|^2] = 1\). The optimal transmitter and receiver antenna beams (thick lined sectors in Fig. 1) are jointly selected based on the maximum received signal power criteria, and therefore the optimal received signal power is calculated as,

\[
P_{R,\text{opt}} = \max \left( P_{R,1}, P_{R,2}, ..., P_{R,B} \right).
\]

In practice mmWave multi-path components are sparse in time as well as the angular dimension \[8\]. Consequently, for receiver nodes which operate with narrow antenna HPBW, the probability of receiving multiple propagation components inside the antenna main lobe is negligible. For such scenarios, we simplify the proposed model by assuming that at most one multi-path component can be received within the receiver antenna main lobe. The presence of the multi-path component inside an antenna beam can be modeled by a Bernoulli random variable with success probability \(p\). The value of \(p\) can be computed as, \(p = 1 - \exp(-\lambda_0)\). Based on this model, the received power corresponding to \(i\)-th antenna beam pair can be expressed as \(P_{R,i} = \Pi'(p) |h_i|^2 G_{m,t}G_{m,r}\lambda_0^{-1}cd^{-\alpha}\), where \(\Pi'(\cdot)\) denotes the Bernoulli random variable corresponding to the \(i\)-th antenna beam pair. In addition, small scale fading gain value \(|h_i|^2\) is assumed to be a random variable with unit mean Rayleigh distribution, an assumption which not only provides tractability in analysis, but also guarantees performance bounds for SE of NLOS mmWave links.

**III. CALCULATION OF SPECTRAL EFFICIENCY**

Based on the expression for ergodic capacity for wireless networks using Hamdi’s lemma \[11\], the spectral efficiency for an mmWave link can be expressed as,

\[
SE = \mathbb{E} \left[ \ln \left( 1 + \frac{P_{R,\text{opt}}}{\sigma^2} \right) \right] = \int_0^{\infty} \left[ 1 - \mathcal{M}_{\sigma^2}(s) \right] ds,
\]

where \(\mathcal{M}_{\sigma^2}(s) = \mathbb{E}[\exp(-sP_{R,\text{opt}}/\sigma^2)\] denotes the moment generating function (MGF) of the optimal received signal power normalized by noise power \(\sigma^2\). Utilizing \[4\], we derive the expression for SE for the following cases of interest:

**A. Case 1: SE under extended S-V channel model with the assumption of Nakagami-\(m\) distributed \(|h_i|^2\).**

The variability of the power received in each antenna beam pair is only due to the parameters \(|h_i|^2\) and \(n_i\). Therefore, the power maximization in \[3\] is equivalent to the calculation of normalized received signal power corresponding to the optimal antenna beam pair, i.e.,

\[
P'_{R,\text{opt}} = \max \left( P'_{R,1}, P'_{R,2}, ..., P'_{R,B} \right),
\]

![Fig. 1: A typical node deployment scenario](image)
where $P_{R,i} = \sum_{i=1}^{n_i} |h_i|^2$, $i \in \{1, ..., B\}$. In this section, we first evaluate the CDF of $P_{R,\text{opt}}$. We note that $P_{R,i}$ for the $i$-th antenna beam pair follows a compound Poisson distribution due to the fact that $|h_i|^2$ and $n_i$ are Gamma and Poisson random variables, respectively, and it shows a mixed behavior of continuous and discrete random variables. Accordingly, $P_{R,i}$ is a continuous random variable if $n_i > 0$; and a discrete random variable if $n_i = 0$. This condition also implies that $P_{R,\text{opt}}$ is a continuous random variable if $\exists i$, where $n_i > 0, \forall i \in \{1, ..., B\}$. Therefore, we proceed with the derivation for the CDF of $P_{R,\text{opt}}$ in two exclusive parts; the first of which deals with the continuous case ($\exists i$, where $n_i > 0, \forall i \in \{1, ..., B\}$) and the second part deals with the discrete case ($n_i = 0, \forall i \in \{1, ..., B\}$) only. Hence, the CDF of $P_{R,\text{opt}}$ with $\exists i$, where $n_i > 0, \forall i \in \{1, ..., B\}$, is calculated as,

$$P_{\text{opt}}(P^*) = P\left((P_{R,1}^* \leq P^*) \cap (P_{R,2}^* \leq P^*) \cap \ldots \cap (P_{R,B}^* \leq P^*) \right)$$

$$= \sum_{i=1}^{\infty} \left(1 - e^{-\lambda d}\right)^{B-i} \left(1 - \lambda d\right)^{B-i}$$

$$= \frac{\sum_{i=1}^{\infty} \exp(-\lambda d)^{B-\lambda d} P_{R}^*}{1 - \exp(-\lambda d)}$$

(6)

Without loss of generality, we now calculate $P(P_{R,i}^* \leq P^*)$ using the property of sum of independent Gamma random variables. Conditioned on the number of received multi-path components, $n_i$, the probability density function (PDF) of $P_{R,i}^*$ is given by,

$$P(P_{R,i}^* = x|n_i) = \frac{(n_i)^{m}x^{n_i}e^{-\lambda d}x}{\Gamma(n_i)}$$

(7)

Therefore, the CDF of $P_{R,i}^*$, $\exists i$, where $n_i > 0, \forall i \in \{1, ..., B\}$ is calculated as,

$$P(P_{R,i}^* \leq P^*) = \int_0^{P^*} P(n_i)^{m}x^{n_i}e^{-\lambda d}x$$

$$= e^{-\lambda d} \sum_{k=1}^{\infty} \frac{\lambda d}{k!} \gamma(k, n_i, P^*)$$

$$= e^{-\lambda d} \sum_{k=1}^{\infty} \frac{\lambda d}{k!} \gamma(k, n_i, P^*)$$

(8)

where $\gamma(x, y)$ denotes the lower incomplete Gamma function with parameters $x$ and $y$. Hence, $P_{\text{opt}}(P^*)$ is obtained by replacing $P(P_{R}^* \leq P^*)$ with (8) in (6). The discrete probability component of the CDF of $P_{R,\text{opt}}$ is determined by the condition $n_i = 0, \forall i \in \{1, ..., B\}$. Therefore, $P(P_{R}^* = 0, \forall i \in \{1, ..., B\}) = e^{-\lambda d}$. To calculate SE, we derive the MGF of received signal power $P_{R,\text{opt}}$ based on the discrete and continuous components of the CDF as,

$$M_{\text{sig}}(s) = e^{-B\lambda d} + (1 - e^{-B\lambda d}) \int_0^{\infty} e^{-spP^*} dP_{\text{opt}}(P^*),$$

where $\rho = \lambda_0^{-1}G_{\text{m,x}}G_{\text{m,y}}\frac{e^{-\lambda d}}{\rho}$. The integral form in (9) is known as the Riemann–Stieltjes integral which satisfies the property of integration by parts,

$$\int_0^{\infty} e^{-spP^*} dP_{\text{opt}}(P^*) = e^{-spP^*} P_{\text{opt}}(P^*) |_{p^*=\infty}$$

$$- e^{-spP^*} P_{\text{opt}}(P^*) |_{p^*=0}$$

$$- \int_0^{\infty} P_{\text{opt}}(P^*) (-sp)e^{-spP^*} dP^*$$

(10)

To overcome numerical stability issues in the calculation of SE, we replace the limiting condition $P^* = \infty$ with $P^* = M_{\infty}$, where $M_{\infty}$ denotes a large positive number. This approximation has a practical relevance in the sense that $P^*$ cannot achieve an extremely large value (tending to $\infty$). Therefore, we obtain $P_{\text{opt}}(P^*) = \exp(-spM_{\infty})$ with $P_{\text{opt}}(P^*) \approx 1$ and $P_{\text{opt}}(P^*) |_{P^*=0} = 0$. Thus, (10) is modified into,

$$\int_0^{\infty} e^{-spP^*} dP_{\text{opt}}(P^*) = e^{-spM_{\infty}} + \int_0^{\infty} P_{\text{opt}}(P^*) spe^{-spP^*} dP^*$$

(11)

Consequently, (9) can be updated as,

$$M_{\text{sig}}(s) = e^{-B\lambda d} + (1 - e^{-B\lambda d}) \left[ \exp(-spM_{\infty}) \right.$$  

$$\left. + \int_0^{\infty} P_{\text{opt}}(P^*) spe^{-spP^*} dP^* \right]$$

(12)

where $P_{\text{opt}}(P^*)$ is obtained in (6). The corresponding SE is computed by substituting (12) in (3).

**B. Case 2: SE under extended S-V channel model with the assumption of at most one multi-path component per receiver antenna beam and Rayleigh distributed $|h|^2$.**

In this section, we evaluate SE of a mmWave link based on the assumption that no more than one signal propagation component of the transmitted signal arrives within each of its antenna beams. Since, small scale fading amplitude of the signal component is modeled as Rayleigh random variable, small scale fading power, $|h|^2 \sim \exp(1)$. $P_{R,\text{opt}}$ is again a mixed random variable, and therefore, the CDF of $P_{R,\text{opt}}$ conditioned on the case that at least one antenna beam pair receives the multi-path component of the transmitted signal is calculated as,

$$P_{\text{opt}}(P^*) = \sum_{i=1}^{B} \frac{(B-i)!}{(1-p)^{B-i}} P_{i}^* \left(1 - e^{-P^*} \right)^i$$

$$\frac{1 - (1-p)^B}{1 - (1-p)^B},$$

(13)

where $1 - e^{-P^*}$ is the CDF of an exponential random variable with unit mean. Consequently, the PDF corresponds to $P_{\text{opt}}(P^*)$ is obtained as,
Finally, the MGF of the received signal \( P_{R_{\text{opt}}} \) is determined by combining the PDF of its discrete and continuous components.

\[
M_{\text{sig}}(s) = (1-p)^B + (1-(1-p)^B) \int_0^\infty \frac{(1-pe^{-Ps})^{B-1}}{1-(1-p)^B} \times Bpe^{-Ps}e^{-sp}dP^s
\]

where (b) is due to Taylor series expansion of \( e^{-Ps} \).

The integration in (15) with variable transformation \( e^{-Ps} = u \), results in,

\[
M_{\text{sig}}(s) = (1-p)^B + B \sum_{i=0}^{B-1} \left( \frac{B-1}{i} \right) \frac{(-1)^i p^{i+1}}{s \rho + i + 1} \int_0^\infty \frac{e^{-s}ds}{s}.
\]

(16)

Therefore, the SE of the link under noise is calculated by substituting (16) in (3) as,

\[
SE = \int_0^\infty \left[1 - (1-p)^B - B \sum_{i=0}^{B-1} \left( \frac{B-1}{i} \right) \frac{(-1)^i p^{i+1}}{s \rho + i + 1}\right] e^{-s}ds.
\]

Now, we simplify (17) by transforming the term \( 1 - (1-p)^B \) into,

\[
1 - (1-p)^B = 1 - \sum_{i=1}^{B} \left( \frac{B}{i} \right) p^i (-1)^i
\]

\[
= B \sum_{i=1}^{B} \left( \frac{B-1}{i-1} \right) \frac{p^{i+1}}{i} (-1)^{i+1}
\]

\[
= B \sum_{i=0}^{B-1} \left( \frac{B-1}{i} \right) \frac{p^{i+1}}{i+1} (-1)^{i+1},
\]

(18)

and thus, (17) can be modified as,

\[
SE = B \sum_{i=0}^{B-1} \left( \frac{B-1}{i} \right) (-p)^{i+1} \int_0^\infty \frac{1}{s \rho + i + 1} \frac{e^{-s}ds}{s}.
\]

\[
= \sum_{i=0}^{B-1} \left( \frac{B-1}{i} \right) (-p)^{i+1} \frac{e^{-s}}{s \rho + i + 1} \int_0^\infty ds.
\]

\[
= \sum_{i=0}^{B-1} \left( \frac{B-1}{i} \right) (-p)^{i+1} \frac{e^{(i+1)\rho/s}}{i+1} E_1 \left( \frac{i+1}{\rho} \right),
\]

(19)

\[
E_1(.) \text{ denotes the exponential integral.}
\]

C. Case 3: SE under extended S-V channel model with the assumption of at most one multi-path component per receiver antenna beam and neglected small scale fading for \( \alpha \)

In this section, we obtain closed form expression for SE by assuming at most one multi-path component per receiver antenna beam and ignoring small scale fading for multi-path component. Based on the aforementioned assumptions, the MGF of the received signal is approximated as,

\[
M_{\text{sig}}(s) \approx \sum_{k=0}^{n_{\text{opt}}} \exp(-sp) \rho_{\text{opt}}(k),
\]

where \( n_{\text{opt}} \) denotes the maximum number of multi-path components received out of the possible antenna beam pairs, \( B, \rho_{\text{opt}}(k) \) represents the probability mass function of \( n_{\text{opt}} \). Further, \( p_{\text{opt}}(0) \approx 1 - p_{\text{opt}}(1) = (1-p)^B \). Hence, SE can be calculated by substituting \( M_{\text{sig}}(s) \) in (3) as,

\[
SE \approx \int_0^\infty \left[1 - (1-p)^B \right] \frac{1 - \exp(-sp)}{s} e^{-s}ds.
\]

(20)

IV. PERFORMANCE EVALUATION

Extensive Monte Carlo numerical simulations were performed to validate the analysis presented in the preceding section. We set the system parameters as \( c \delta^2 / \sigma^2 = 0.01, m = 1, 2, M_\infty = 100, \) and a variable number of antenna beam pair \( B \) is selected. Firstly, we present the analytical and the simulated plots for \( P_{\text{opt}}(P^*) \) (analytical expression from (6) in Fig. 2(a) for \( \lambda_o = 1.9, \) a value experimentally reported in (7). We choose three different values for \( B \) to capture the variation in antenna beam. The analytical CDFs are evaluated by replacing the infinite sum in (6) by its first 10 terms. The error between analytical and simulation plots is negligibly small for all the values of \( B \), which shows that the approximated analytical CDF \( P_{\text{opt}}(P^*) \) matches with the simulation.

In Fig. 2(b), we compare the analytical SE plots for Case 1 \( (m = 2) \) (articulated in Section III-A) and the corresponding simulation plots. An omni-directional channel is generated in each iteration of the simulation with Rician distributed multi-path amplitude for shape parameter \( K_s = 5 \) dB as reported in (8). Antenna radiation pattern is then applied (based on discrete set of antenna pointing directions) to obtain optimally beamformed directed channel. Two different patch antenna radiation patterns with antenna HPBW, \( \theta_{\text{dB}} = 14.5^0 \) and \( 33^0 \) are used for simulation. Correspondingly, we choose \( B = 625 \) and \( B = 121 \) for the analytical plots assuming that each node employs the same antenna HPBW \( (14.4^0 \text{ and } 32.73^0, \text{ respectively}) \) for communication. As shown in Fig. 2(b), a maximum error of only 0.15 bps/Hz between the analytical and simulation plots is observed due to the ideal sector antenna radiation pattern and the channel fading assumptions. Hence, the proposed analytical framework can be utilized to readily model optimal analog beamforming in practical mmWave channels.

Finally, we compare the plots of analytically evaluated SE for different case studies of antenna beamforming (articulated in Section III) for varying \( B \) and \( \lambda_o = 1.9 \) in Fig. 2(c). For comparison, we also illustrate the result generated for the case where the directional channel is simply modeled as a Nakagami-\( m \) \( (m = 2) \) distributed random variable, an
assumption made in [4], [5]. This is depicted by the plot with the legend ‘Case 4’ in Fig. 2(c). We adopt two experimentally reported values of $\lambda_0$ (i.e. 1.9 and 3.3) from [7]. As can be seen from Fig. 2(c), the SE plot for the Case 4 shows a deviation of 0.3 bps/Hz and 0.7 bps/Hz from the SE curves corresponding to Case 1 with $m = 1$ for $\lambda_0 = 1.9$ and 3.3, respectively which indicates that Case 1 is a well suited model for performance evaluation of NLOS mmWave links with antenna beamforming. Also, as evident from Fig. 2(c), the error between the plots corresponding to Case 1 and Case 2 is negligibly small for both the values of $\lambda_0$. We note that the assumption of at most one multi-path component per receiver antenna beam is strongly valid for practical mmWave channels. Finally, the idealistic assumption of zero fading leads to significant error in the calculation of SE in Case 3 as $\lambda_0$ increases (Fig. 2(c)). This error which is observed both, for Case 1 as well as Case 3, is due to the increase in average received signal power with respect to the average power per multi-path component $\rho$ as a result of antenna beamforming (subject to same transmit power in all cases). However, the change in average received signal power is relatively less in the low $\lambda_0$ regime in which Case 3 essentially approximates Case 1 to a fair degree of precision.

V. CONCLUSION

In this paper, we present a novel methodology to incorporate optimal analog beamforming into the framework for evaluation of SE of NLOS mmWave links. We establish that the simplistic assumption of Rayleigh or Nakagami-$m$ probability distribution for the beamformed directional channel gain is inadequate to characterize NLOS mmWave beamformed channels. As evident from the analysis and ensuing numerical results, the modeling methodology presented in this paper provides a more holistic approach by considering the impact of antenna beamforming on the SE bounds for NLOS mmWave channels. We also demonstrate that the assumption of at most one multi-path component per receive antenna beam holds reasonably good to aid modeling of practical outdoor mmWave channels. In addition, we also observe that the derived closed form expression for SE under the assumption of negligible small scale fading closely approximates the SE achievable in sparse mmWave channels.

As future work, it would be interesting to explore the scenario where average number multi-path components per antenna beam and the path loss exponent is considered to be a function of the antenna beam orientation angle.

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