Real-space condensation of reciprocal active particles driven by spontaneous symmetry breaking induced nonreciprocity

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Abstract

We investigate the steady-state and dynamical properties of a reciprocal many-body system consisting of self-propelled active particles with local alignment interactions that exists within a fan-shaped neighborhood of each particle. We find that the nonreciprocity can emerge in this reciprocal system once the spontaneous symmetry breaking is present, and the effective description of the system assumes a non-Hermitian structure that directly originates from the emergent nonreciprocity. This emergent nonreciprocity can impose strong influences on the properties of the system. Moreover, it can even drive a real-space condensation of active particles. Our findings pave the way for identifying a new class of physics in reciprocal systems that is driven by the emergent nonreciprocity.

Introduction.

The exceptional role played by the nonreciprocity in various many-body systems has recently attracted much attention in a wide range of fields in physics, ranging from condensed matter physics [1–11], over statistical physics [12–14], to biological physics [17–21], etc. Well-known examples include novel critical phenomena in nonreciprocal phase transitions [12], rich collective dynamical behaviors in the predator-prey systems [19–23], Tuning patterns in the activator-inhibitor systems [24–26], and non-Hermitian skin effects in the lattice models with nonreciprocal hopping [27, 28].

In fact, one can notice that the nonreciprocity giving rise to the rich nonreciprocal physics in these scenarios is directly built-in by, for instance, the nonreciprocal interactions between different types of agents. While fundamental laws of physics, such as Newton’s third law, are usually reciprocal, hence also the physical systems governed by them. In these regards, it seems that the rich nonreciprocal physics are irrelevant for ubiquitous reciprocal systems. However, in the spirit of “more is different” pioneered by P. W. Anderson [29], one could actually still expect that the nonreciprocity can emerge in reciprocal many-body systems under certain circumstances. This thus gives rise to a novel scenario for physics associated with the nonreciprocity, and raises the fundamental question of the existence and the physical consequences of the emergent nonreciprocity in reciprocal many-body systems.

In this work, we address this question for a two-dimensional (2D) reciprocal system consisting of self-propelled active particles with local alignment interactions that exists within a fan-shaped neighborhood of each particle [see Eq. (1) and the top of Fig. 1(a)]. We find that the nonreciprocity can emerge in this reciprocal many-body system and impose strong influences on the properties of the system. More specifically, we find the following. (i) Emergence of nonreciprocity that is induced by the spontaneous symmetry breaking (SSB). At the low noise level below the flocking transition, the system spontaneously breaks the rotational symmetry and induces the nonreciprocity between the active particles located relatively in the front and the ones located relatively in the back along the direction of the collective velocity [see the top of Fig. 1(b)]. In this case, the effective description of the system assumes a non-Hermitian structure that directly originates from the emergent nonreciprocity [see Eq. (2)]. (ii) Emergent nonreciprocity driven traveling band formation and real-space condensation of active particles. The traveling band is very elusive in similar related active matter systems [30–31], here we find that the emergent nonreciprocity can enhance the formation of the traveling band in a considerably much larger parameter regime (see Fig. 2). Even more remarkably, at low and intermediate noise levels, we find that a strong enough emergent nonreciprocity could even drive a real-space condensation of active particles along the direction of the collective velocity [see Fig. 3 and the leftmost panel of Fig. 1(b)].

System and model.

The system under study consists of $N$ self-propelled active particles moving in the 2D space of linear size $L$ under influences from environmental fluctuations. Each particle interacts with other particles via a local alignment interaction within its fan-shaped neighborhood [see the top of Fig. 1(a)]. The dynamics of the system is modeled by a modified Vicsek model with extrinsic noise (also known as “vectorial” noise) [35–41], where the active particles move synchronously at discrete time steps $\Delta t$ with a constant speed $v$ along the directions specified by

$$\theta_j(t + \Delta t) = \arg \sum_{k \in U_j(r, \phi)} (e^{i\theta_k(t)} + \eta e^{i\xi_k(t)}).$$

Here, for the $j$th active particle at time $t$, $\theta_j(t)$ denotes its direction of motion in the 2D space, $U_j(r, \phi)$ denotes its
fan-shaped neighborhood with $r$ being the radius of the active particle’s scope and $\phi$ being the view angle. The environmental fluctuations are modeled by the random noise term $\eta \xi_j(t)$, where $\xi_j(t)$ a random variable whose value is uniformly distributed within the interval $[-\pi, \pi]$ and $\eta \in [0, 1]$ is the level of the extrinsic noise. The numerical results presented in this work are obtained by directly simulating the dynamical Eq. (1) with periodic boundary condition imposed. We set $r = 1$ and $\Delta t = 1$, hence the distance and time are measured in the unit of $r$ and $\Delta t$, respectively. If not specified in text, the total number of particles $N = 2500$, the linear size $L = 25$, the speed $v = 1/2$, and $10^2$ stochastic trajectories are used to perform ensemble average.

**Emergent nonreciprocity and non-Hermitian effective description in the presence of the SSB.**—From the form of Eq. (1) we notice that irrespective of the value of the view angle $\phi$, each particle satisfies the same form of the dynamical equation, reflecting that it imposes influences on the other particles in the same manner, indicating that the system is reciprocal. This is in sharp contrast to the dynamical equations for systems with the built-in nonreciprocity, for instance, the predator-prey systems [19–22], the activator-inhibitor systems [24–26], the leader-follower systems [42–44], etc., where different types of agents satisfy different forms of dynamical equations, reflecting the fact that they impose influences on each other in intrinsically different ways. However, we also notice that once the view angle $\phi < 2\pi$, a transient front-back nonreciprocity between any two different particles can actually exist if one particle lies in the view of the other one but not vice versa [see for instance the top of Fig. (1a)].

As one can see from Fig. (1a), which shows the typical steady-state configurations of this reciprocal system at a relatively high noise level $\eta$ above the flocking transition, there is essentially no difference among configurations with different view angles. This indicates that the transient front-back nonreciprocity does not affect the properties of this reciprocal system in the symmetric disordered phase. Indeed, since the direction of motion of each particle changes randomly all the time in the symmetric disordered phase, there is no overall nonreciprocity presented in the system.

But the SSB of the rotational symmetry can cause the system to develop the flocking phase at a relatively low noise level [53–59], where the macroscopic number of particles moving along almost the same direction of the collective velocity $\mathbf{v}_c \equiv \sum_{j=1}^N \mathbf{v}_j$ (with $\mathbf{v}_j$ being the velocity of the $j$th particle). In this case, with the limited view angle $\phi < 2\pi$, the influences of the active particles located relatively in the front ("front-particles") along the direction of $\mathbf{v}_c$ on the ones located relatively at the back ("back-particles") are stronger than the influences of "back-particles" on "front-particles" [see for instance the top of Fig. (1b)]. This "influence-imbalance" thus indicates that in the presence of the SSB, the overall nonreciprocity emerges in the system between these two "types" of active particles. Since the smaller view angle $\phi$ results in the larger "influence-imbalance" hence the stronger nonreciprocity, one naturally expects that with a small enough view angle $\phi$, the emergent nonreciprocity could strongly affect the physical behavior of the system. Indeed, as one can see from Fig. (1b), which shows the typical steady-state configurations of this reciprocal system at a low noise level $\eta$ below the flocking transition, decreasing the view angle $\phi$ (hence increasing the strength of the emergent nonreciprocity) clearly changes the steady-state configurations of the system, i.e., from the typical flocking configuration with no additional structure, over the traveling band that is very elusive in the conventional Vicsek model [30–34], eventually to a remarkable real-space condensation line [see the leftmost panel of Fig. (1b)].

To investigate how this emergent nonreciprocity affects the system, let us first use a simple zero-dimensional effective model to describe the exchange process between the “front-particles” and “back-particles” in the system. This effective model consists of two groups of particles labeled by $F$ and $B$, whose dynamics is determined by two dynamical rules $F \lambda_{FB} B$ and $F \lambda_{BF} B$, where $F$ and $B$ denote any “front-particle” and “back-particle”, respectively. $\lambda_{FB}$ ($\lambda_{BF}$) is the rate, i.e., the probability within a unit time, for a single “front-particle” (“back-particle”) to change into a “back-particle” (“front-particle”). The
state of this model system at time $t$ is determined by the probability distribution $P(n_B, n_F; t)$ that denotes the probability for the model system in a configuration with $n_F$ “front-particles” and $n_B$ “back-particles”. With these two dynamical rules, the Fokker–Planck equation that determines the time evolution of the probability distribution $P(n_B, n_F; t)$ can be straightforwardly obtained, which assumes the explicit form

$$
\partial_t P(n_B, n_F; t) = \left( e^{-\partial_{n_B}} - e^{-\partial_{n_F}} \right) \Lambda \left( e^{\partial_{n_B} N_B} e^{\partial_{n_F} N_F} \right) P(n_B, n_F; t),
$$

with $\Lambda \equiv \begin{pmatrix} -\lambda_{BF} & \lambda_{FB} \\ \lambda_{BF} & -\lambda_{FB} \end{pmatrix}$. One immediately notices that the matrix $\Lambda$ that appears in this Fokker–Planck equation is a non-Hermitian matrix in general as long as $\lambda_{BF} \neq \lambda_{FB}$. Indeed, in the presence of the SSB, the corresponding $\lambda_{BF}$ is expected to be larger than $\lambda_{FB}$ with the limited view angle $\phi < 2\pi$ due to the emergent nonreciprocity. One thus expects that major effects of the emergent nonreciprocity on the system can be captured by the effective non-Hermitian description Eq. (2). The steady-state solution of Eq. (2) can be obtained analytically. The average number of “front-particles” $\langle n_F \rangle$ and its particle number fluctuations $\Delta n_F \equiv \langle n_F^2 \rangle - \langle n_F \rangle^2$ in the steady-state assume the form

$$
\langle n_F \rangle = \frac{N \lambda_{BF}}{\lambda_{FB} + \lambda_{BF}}, \quad \Delta n_F = \frac{N \lambda_{BF} \lambda_{FB}}{(\lambda_{FB} + \lambda_{BF})^2}.
$$

From the above steady-state properties of the “front-particles”, one can see that a large imbalance between $\lambda_{BF}$ and $\lambda_{FB}$ causes more particles to concentrate in the front, and at the same time reduces the corresponding particle number fluctuations. This thus suggests that the physical effects of the emergent nonreciprocity are to compress the particle number distribution along the direction of the collective velocity which thus facilitates the formation of the traveling band, and to make the band more stable by reducing the particle number fluctuations within it. As we shall see in the following, the system can form stable traveling band in a large parameter regime. Even more remarkably, it can manifest a novel real-space condensation of active particles at the large enough emergent nonreciprocity.

**Emergent nonreciprocity driven traveling band formation and real-space condensation.**—In the conventional Vicsek model (corresponding to Eq. (1) with the full view angle $\phi = 2\pi$), the traveling band is quite elusive and can exist only when the noise level of the system is within a very narrow interval slightly below the flocking transition [9, 34]. Its dynamics features alternate disintegration and reconstruction of the band structure [34], giving rise to large particle number fluctuations in the band region that clearly manifests its fragileness. From the above study on the non-Hermitian effective description Eq. (2) of the system in the presence of the SSB, we have seen that the emergent nonreciprocity with the limited view angle $\phi < 2\pi$ can assume the effect of reducing the particle number fluctuations within the band. This thus indicates that the stability of the traveling band can be substantially enhanced by the emergent nonreciprocity.

Fig. (a) shows the single trajectory time evolution of the traveling band width $w_b$ for the system ($N = 10^4$, $L = 50$) at the noise level $\eta = 3/5$ slightly below the flocking transition. The black and red curves correspond to the view angle $\phi = 2\pi$, $5\pi/3$, respectively. The temporal fluctuation of the traveling band width $w_b$ is clearly suppressed by the emergent nonreciprocity with $\phi = 5\pi/3$. (b) Typical steady-state configurations of the system ($N = 2500$, $L = 25$) with a fixed limited view angle $\phi = 5\pi/3$ at different noise levels ($\eta = 3/5, 2/5, 1/4, 1/10$). The emergent nonreciprocity with the limited view angle drives the formation of the traveling band in a considerably large interval of the noise level. See text for more details.

Figure 2. (a) Typical single trajectory time evolution of the traveling band width $w_b$ for the system ($N = 10^4$, $L = 50$) at the noise level $\eta = 3/5$ slightly below the flocking transition. The black and red curves correspond to the view angle $\phi = 2\pi$, $5\pi/3$, respectively. The temporal fluctuation of the traveling band width $w_b$ is clearly suppressed by the emergent nonreciprocity with $\phi = 5\pi/3$. (b) Typical steady-state configurations of the system ($N = 2500$, $L = 25$) with a fixed limited view angle $\phi = 5\pi/3$ at different noise levels ($\eta = 3/5, 2/5, 1/4, 1/10$). The emergent nonreciprocity with the limited view angle drives the formation of the traveling band in a considerably large interval of the noise level. See text for more details.
of the traveling band in a considerably large interval of emergent nonreciprocity can indeed drive the formation of the system in the presence of the SSB, which corresponds to the strongest emergent nonreciprocity. Particularly, in the extreme case where the front if the imbalance between况且 mean interaction with $\phi < \pi$, therefore $\lambda_{FB}$ is negligibly small compared to $\lambda_{FB}$ at low noise levels.

To further investigate the influence of the extrinsic noise on the existence of the real-space condensation, we calculate the view angle $\phi$ dependence of $\max[\rho(x)]$ at different noise levels. As shown in Fig. 3(b), one can see that real-space condensation generally exists at low ($\eta = 1/10$) and intermediate ($\eta = 1/5$) noise levels when the view angle $\phi$ is decreased below $\pi$. At the relatively high noise level ($\eta = 3/5$), no real-space condensation exists even if the view angle $\phi$ is decreased below $\pi$. This is due to the fact that although the “front-particles” can hardly impose any influences on the “back-particles” by the alignment interaction with $\phi < \pi$, the strong environmental fluctuations can still give a finite contribution to $\lambda_{FB}$.

Influences from environmental fluctuations are also reflected directly in the view angle $\phi$ dependence of $\max[\rho(x)]$ in the traveling band width $w_b$ as shown in Fig. 3(c). At low ($\eta = 1/10$) and intermediate ($\eta = 1/5$) noise levels, the emergent nonreciprocity induced by the view angle $\phi$ below $\pi$ is strong enough to drive $w_b$ to decrease to zero, hence the formation of the real-space condensation. At the relatively high noise level ($\eta = 3/5$) that is still below the flocking transition, the density fluctuation caused by the large extrinsic noise can prevent the traveling band width $w_b$ from decreasing to zero even in the presence of the emergent nonreciprocity. While at the high noise level ($\eta = 3/4$) above the flocking transition, the change of the view angle $\phi$ does not impose any noticeable effect on $w_b$, which is a clear manifestation of the fact that the nonreciprocity is induced by the SSB.

Finally, we remark that the emergent nonreciprocity driven traveling band formation and real-space condensation is quite robust against choices of particle densities, finite size effects, and even the type of the noise in the system (see Supplemental Material [45] for details). This naturally facilitates further direct experimental observations of these emergent nonreciprocity driven physics.

Conclusions.—Nonreciprocity can emerge in reciprocal systems and crucially influences their physical properties, as the emergent nonreciprocity in the reciprocal system of active particles with the limited view angle reveals. The SSB of the rotational symmetry of this system can induce the nonreciprocity between the active particles located relatively in the front and the ones located relatively in the back along the direction of the collective velocity with different view angles at a fixed low noise level $\eta = 1/10$. One can see that as the view angle is decreased from $2\pi$, the normalized density distribution $\rho(x)$ become narrower. In particular, with a small enough view angle $\phi$ that is slightly below $\pi$, the peak value of the normalized density distribution $\rho(x)$, denoted as $\max[\rho(x)]$, reaches 1. This indicates that all the active particles in the system condense at the same position along the direction of the collective velocity [see also the leftmost panel of Fig. 3(b)]. Indeed, the “front-particles” can hardly impose any influences on the “back-particles” by the alignment interaction with $\phi < \pi$, therefore $\lambda_{FB}$ is negligibly small compared to $\lambda_{FB}$ at low noise levels.

Further simulations of the system with a fixed limited view angle $\phi = 5\pi/3$ at different noise levels ($\eta = 3/5, 2/5, 1/4, 1/10$) show in Fig. 2(b) that the emergent nonreciprocity can indeed drive the formation of the traveling band in a considerably large interval of the noise level (with fixed $\phi = 5\pi/3$, the traveling band is generally observed at $1/10 \leq \eta \leq 3/5$).

From the study on the non-Hermitian effective description Eq. 2 of the system in the presence of the SSB, we also notice that more particles shall concentrate in the front if the imbalance between $\lambda_{BF}$ and $\lambda_{FB}$ becomes larger. Particularly, in the extreme case where $\lambda_{BF}$ is negligibly small compared to $\lambda_{FB}$, all the particles shall concentrate in the front. This indicates that they shall organize themselves in such a way that share the same position along the direction of the collective velocity, i.e., form a real-space condensation along this direction. Fig. 3(a) shows the normalized density distribution $\rho(x)$ along the direction of the collective velocity with different view angles at a fixed low noise level $\eta = 1/10$. One can see that as the view angle is decreased from $2\pi$, the normalized density distribution $\rho(x)$ become narrower. In particular, with a small enough view angle $\phi$ that is slightly below $\pi$, the peak value of the normalized density distribution $\rho(x)$, denoted as $\max[\rho(x)]$, reaches 1. This indicates that all the active particles in the system condense at the same position along the direction of the collective velocity [see also the leftmost panel of Fig. 3(b)]. Indeed, the “front-particles” can hardly impose any influences on the “back-particles” by the alignment interaction with $\phi < \pi$, therefore $\lambda_{FB}$ is negligibly small compared to $\lambda_{FB}$ at low noise levels.

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velocity. In particular, this gives rise to a remarkable real-space condensation of active particles that is absent in the corresponding reciprocal system without the emergent nonreciprocity. Since the mechanism for the nonreciprocity to emerge in reciprocal systems is quite general, we expect that there exists a large class of emergent nonreciprocity driven new physics in reciprocal systems to be identified. For instance, the flock of active spins \[46-48\] are expected to manifest similar real-space condensation driven by the emergent nonreciprocity. Moreover, the SSB in many other reciprocal active matter systems with their dynamics determined by intelligent decision-making rules, such as the intelligent materials \[49\], systems with herding dynamics \[50, 51\], and systems with quorum sensing \[52–54\], etc., might also induce the nonreciprocity and give rise to new physics. We believe our findings and predictions will stimulate further theoretical and experimental efforts in revealing the emergent nonreciprocity driven new physics in various reciprocal many-body systems.

Note added.—Upon completion of this manuscript, we became aware of the work by Loos et al. \[55\], reporting long-range order and directional defect propagation in a 2D XY model with vision cone interactions that is associated with the zero speed limit of the model investigated in this manuscript.

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Supplemental Material for “Real-space condensation of reciprocal active particles driven by spontaneous symmetry breaking induced nonreciprocity”

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Figure S1. The view angle $\phi$ dependence of the traveling band width $w_b$ in different cases. These results show that the particle density, system size, and the type of noise do not assume strong influences on the physical effects of the emergent nonreciprocity. See text for more details.

INFLUENCES OF PARTICLE DENSITY, SYSTEM SIZE, AND THE TYPE OF NOISE

For the sake of completeness, it is worth noting that several other properties in this nonequilibrium complex system might also affect its collective behaviors [1–7]. The particle density $\bar{n} = N/L^2$ is one that cannot be ignored. A high particle density $\bar{n}$ makes the system relatively robust against influences from environmental fluctuations, but as one can see from the red and black lines in Fig. S1 reducing the density $\bar{n}$ within a certain range do not prevent the real-space condensation from existing. The purple line in Fig. S1 indicates that finite size effects do not impose strong influences on the physical effects of the emergent nonreciprocity, neither. Besides, when the emergent nonreciprocity is strong enough to driven the real-space condensation, we find that larger systems (e.g., $N = 10^4$) can accommodate several traveling “lines” at the same time without forming regular wave trains, which is similar to the cases associated with the well-established traveling band [1]. Furthermore, concerning the fluctuations, the extrinsic noise characterizes influences from environmental fluctuations and assumes that active particles are totally willing to be aligned with their neighbors. There is another type of noise in the Vicsek model, namely, the intrinsic noise (also known as “angular” noise) [3], where the dynamics of active particles follows $\theta_{j}^{t+\Delta t} = (\arg \sum_{k \in U_{j}(r,\phi)} e^{i\theta_{k}^{t}}) + \tilde{\eta}_{j}^{t}$, with $\tilde{\eta}$ being the intrinsic noise strength characterizing influences from the “free will” of the active particles [3]. As one can see from the blue line in Fig. S1, the emergent nonreciprocity driven traveling band formation and real-space condensation can also be found in this case. These results show that the particle density, system size, and the type of noise do not assume strong influences on the physical effects of the emergent nonreciprocity.

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