Application of the Bayesian Source Location Using Seismic and Acoustic Observations in Inversion of Surface Explosion Source Locations

Liangyong Zhang\textsuperscript{1,2}, Xubin Liang\textsuperscript{2}, Tongdong Wang\textsuperscript{2}, Houlin Fang\textsuperscript{2}, Shiying Tang\textsuperscript{2}, Yunze Liu\textsuperscript{2} and Dezhi Zhang\textsuperscript{*}\textsuperscript{1}

\textsuperscript{1} College of Meteorology and Oceanography, National University of Defense Technology, Changsha, Hunan, 410073, China
\textsuperscript{2} Northwest Institute of Nuclear Technology, Xi’an, Shaanxi, 710024, China
* Corresponding author’s e-mail: zhangdezhi@nint.ac.cn

Abstract. The dropping point of a projectile is an important parameter for evaluating the range weapon performance. A Bayesian jointed source location method by using seismic and acoustic observations is developed. This jointed method is to take advantage of signals in the atmosphere and seismic wave signals in the ground and reduce the influence of atmospheric factors, geological factors, topographical factors, and seismic phase identification factors on the location accuracy. Therefore, it is a high-precision location method. Firstly, this paper combined the four-element cross acoustic array and the single seismic measurement points to measure the acoustic signals/seismic waves of the surface explosion. Then, the paper obtained the azimuth information through sound arrays and signal arrival information through the seismic measurement points and performed an inversion of the explosive source location based on the Bayesian theory. The results show that the Bayesian location method has a high precision due to the full consideration of the error model and a-priori information. In addition, the Bayesian jointed source location method can provide more constraints on the location of the explosive source by integrating seismic wave data and acoustic data, which makes the probability density distribution of the explosive source more concentrated and the location result more robust.

1. Introduction

The dropping position of a projectile is an important parameter for evaluating the range weapon performance. Generally, the seismic waves and acoustic data are used for locating the dropping point\textsuperscript{[1-5]}. The method of locating the target by using seismic waves is to detect the seismic waves transmitted in the ground, which is less affected by environmental factors such as wind speed, wind direction, and temperature. However, the surface is complex, with different types of geological composition and surface shapes, and the larger the detection range the more inhomogeneous the surface is. It has obvious effects on the transmission speed, path and energy dissipation of seismic waves. At the same time, seismic waves are rich in variety, and the different composition and topography of the surface can produce various forms of seismic waves such as longitudinal, transverse, and Rayleigh waves, which raises high requirements for seismic wave phase identification\textsuperscript{[5-9]}. Acoustic source location is characterized by the simple structure, easy deployment and low cost by using the differences of received signals of each unit of microphone arrays to determine the orientation of the sound sources\textsuperscript{[10-12]}. Compared with location method using seismic waves, the waveforms of acoustic signals are relatively
simple. Only longitudinal waves exist, and no other types of fluctuations are generated due to reflection and scattering. However, the accuracy of acoustic location is susceptible to environmental factors such as temperature, wind speed, and wind direction \[10, 13\]. By making full use of acoustic signals in the atmosphere and seismic wave signals in the surface, the influence of atmospheric factors, geological factors, topographical factors and seismic phase identification factors on the location accuracy can be reduced, and a high location accuracy can be maintained under a variety of interference conditions \[5, 14-16\].

At present, few studies have been carried out on the Bayesian joint location method and the technical method is still immature. Hutchenson and VanDeMark et al \[2-4\] developed the Seismic Acoustic Impact Monitoring Assessment System by using main event-based seismic localization method and the acoustic multi-array cross-localization method. However, the system is still in the stage of improvement and refinement, and the joint localization method has not been reported. Bhattacharyya et al. \[14\] estimated the distance between the explosive source and the measurement points by using the arrival time difference of P-wave and acoustic wave based on the shock data of the Watusi small explosion test. Xiao Weiguo et al. \[15\] also used a similar joint method to achieve rapid localization of the target impact point. Che et al. \[16\] conducted a preliminary study on joint localization based on small surface explosions, used the area of the explosive source location obtained by the seismic localization method as a grid search area for sub-acoustic localization, used the residual error of the predicted and observed values as the target function and combined the azimuthal cross-localization method to locate the explosive source in a precise way. Xiong Chao et al. \[5\] used a weighted fusion algorithm for the joint localization of the dropping position of a projectile, simulated the projectile dropping point by performing surface explosions, and obtained the optimal weight of the sensor measurements based on the minimum mean square error so as to get the target location.

However, the above research is only in a preliminary exploration stage and cannot solve the joint localization problem under complex conditions. A data fusion method based on the probability theory \[17\], Bayesian theory can better constrain the target location by fully considering the error model and a-priori information, and localization based on Bayesian theory is a very promising method for high-precision localization \[18-22\]. In this paper, a four-element cross array and single seismic wave measurement points are combined to measure the acoustic signals/seismic waves of surface explosions, and the azimuthal information is obtained by acoustic arrays and the arrival information is obtained by seismic wave measurement points. The position of explosions is inverted based on Bayesian theory and compared with the real data for analysis.

2. Theories

By substituting the target azimuth and delay obtained by the acoustic signal/seismic wave measurement points into the error model, a joint likelihood function of the azimuth and delay can be obtained. The probability distribution of the target position can be obtained by using Bayesian theory. Assume the azimuth and delay of n arrays and l single measurement points are

\[
\theta = [\theta_1, \cdots, \theta_n], \quad t = [t_1, \cdots, t_l] \tag{1}
\]

Use d and m to represent the measured value and the source parameter to be estimated, respectively.

\[
d = [t, \theta], \quad m = \{x_0, y_0, v\} \tag{2}
\]

where \(x_0\) and \(y_0\) are the target positions and \(v\) is the wave velocity.

The posterior probability density function (PDF) based on the Bayesian theory is

\[
P(m|d) = c(d) P(m) P(d|m) \tag{3}
\]

where \(c(m)\) is the marginal function, \(P(m)\) is the a-priori information, and \(P(d|m)\) is the likelihood function. The a-priori information includes the wave velocity and explosive source location, and if the wave velocity and explosive source location are related, a hierarchical prior is used \[23\]

\[
P(m) = p(v|x_0, y_0) p(x_0, y_0) \tag{4}
\]
The a-priori information about the wave velocity is determined from atmospheric and geological models, wave propagation models, and signal classes. In a simple case, assume that the wave velocity is uniformly distributed.

Assume that the time delay and azimuth are uncorrelated, the joint likelihood function for multiple arrays and single measurement points is \[ P(d \mid m) = \prod_{i=1}^{n} \Theta(\theta_i \mid m) \prod_{j=1}^{l} \Phi(t_{ij} \mid m) \] (5)

where, \( i \) and \( j \) are the sequence numbers of the array and single measurement points, respectively.

The errors of azimuth and time delay obey Gaussian distribution with standard deviations of \( \sigma_\theta \) and \( \sigma_t \), respectively, and the likelihood function can be obtained

\[ \Theta(\theta_i \mid m) = \frac{1}{\sqrt{2\pi\sigma_\theta^2}} \exp \left[-\frac{1}{2} \left( \frac{\gamma_i - \theta_i}{\sigma_\theta} \right)^2 \right] \] (6)

\[ \gamma_i = \theta_i - \arctan \left( \frac{y_i - y_b}{x_i - x_b} \right) \] (7)

\[ \Phi(t_{ij} \mid m) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp \left[-\frac{1}{2} \left( \frac{\varepsilon_i - t_{ij}}{\sigma_t} \right)^2 \right] \] (8)

\[ \varepsilon_i = t_{ij} - \left( \frac{d_i}{v_i} - \frac{d_j}{v_j} \right) \] (9)

The error in azimuth originates from measurement error and model error, and assuming that measurement error and model error are uncorrelated, the azimuthal variance is \[ \sigma_\theta^2 = \sigma_{\theta,\text{meas}}^2 + \sigma_{\theta,\text{mod}}^2 \] (10)

In practice, the azimuthal variance can be obtained empirically by analyzing GT events (known events).

The time delay consists of wave propagation time, measurement error and model error, and the model error is closely related to the wave velocity model, which yields the time delay relationship as \[ t_{ij} = \frac{d_i}{v_i + \varepsilon_i(v_i,\sigma)} - \frac{d_j}{v_j + \varepsilon_i(v_j,\sigma)} + \varepsilon_{\text{meas}} \] (11)

Get the first-order quantity:

\[ t_{ij} = \frac{d_i}{v_j} \left( 1 - \frac{\varepsilon_i(v_j,\sigma)}{v_j} \right) - \frac{d_j}{v_i} \left( 1 - \frac{\varepsilon_i(v_i,\sigma)}{v_i} \right) + \varepsilon_{\text{meas}} \] (12)

Summarized into

\[ \varepsilon_i = \varepsilon_{\text{mod}} + \varepsilon_{\text{meas}} \] (13)

\[ \varepsilon_{\text{mod}} = \frac{d_j}{v_j} \varepsilon_i(v_j,\sigma) - \frac{d_i}{v_i} \varepsilon_i(v_i,\sigma) \] (14)

The wave velocity error and the measurement error obey Gaussian distribution. Therefore, we can get

\[ \varepsilon_i \sim N \left( 0, \left( \frac{d_i}{v_i} \right)^2 + \left( \frac{d_j}{v_j} \right)^2 \sigma_i^2 + \sigma_{\text{meas}}^2 \right) \] (15)
\[
\sigma_j = \sqrt{\left(\frac{d_j}{v_j}\right)^2 \sigma_v^2 + \left(\frac{d_i}{v_i}\right)^2 \sigma_{\text{meas}}^2}
\]  

where \(d_i\) is the distance from the source to measurement point \(i\), and \(\sigma_v\) and \(\sigma_{\text{meas}}\) are the standard deviations of the wave velocity and measurement error, respectively.

3. Experiment

The ground surface is covered by snow and semi-shrubs (see figure 1). The terrain is relatively flat. There is basically no wind, and there is little variation in ambient temperature and atmospheric pressure. The explosive equivalent energy is about 30 kgTNT, and the explosion is to take place on the ground, and the acoustic signal/seismic wave measurement points were arranged around the circumference of the explosive center (see figure 2). A four-element cross array, which is used to obtain the target azimuth, is used in acoustic signal measurement points (see figure 3). The seismic wave measurement points are single measurement points used for obtaining signal arrival information.

4. Analysis

4.1. A Bayesian approach to acoustic source localization

The common methods for time-delay estimation include the direct cross correlation (DCC) \[^{24}\] method, the standard cross correlation (SCC) \[^{25}\] method, the small wavelet transform-based cross-correlation (SWT-SCC) \[^{26}\] method, and a modified version of average magnitude difference function (MAMDF)
All these methods are used to estimate the arrival delay of signals received by sound arrays from A01 to A04. Figure 4 shows the estimation results.

In this paper, we applied the SWT-SCC method to decompose the measured signal in six layers by using “sym6” wavelet, and then performed a correlation analysis on the decomposed signals. The figure shows that the time-delay estimate obtained by the SWT-SCC method is basically the same as the actual measurement data. And the error obtained by other methods is quite large, which indicates that the SWT-SCC method is suitable for estimating the delay of sound signals of surface explosions. Due to a double-peaked structure of the waveform at the measurement point (see figure 5), the first peak has a relatively smaller amplitude compared to the second peak but the rising edge of the waveform is steeper. The SWT-SCC technique can decompose the high-frequency components of the first peak, thus improving the accuracy of time-delay estimation.

(a) Delay of array 2 relative to array 1          (b) Delay of array 3 relative to array 1
(c) Delay of array 4 relative to array 1

Figure 4. Delay estimate

Figure 5. Typical waveform of a sound signal measurement point (A01)
By substituting the delay estimated by the SMT-SCC method into the azimuth calculation formula of the four-element cross array \(^{28}\), the target azimuth is obtained (see table 1). As can be seen from the table, the above method has a good estimation performance with a maximum absolute error of less than 2\(^\circ\).

Table 1. Azimuth estimation result (northerly angle/°)

| Acoustic signal measurement points | Measured values/° | Estimated values based on the SMT-SCC method/° | Absolute error/° | Standard error/° |
|-----------------------------------|-------------------|-----------------------------------------------|------------------|-----------------|
| A01                               | 322.25            | 324.15                                        | 1.90             |                 |
| A02                               | -0.08             | 0.72                                          | 0.80             |                 |
| A03                               | 252.13            | 252.78                                        | 0.65             |                 |
| A04                               | 30.85             | 31.20                                         | 0.35             |                 |

Figure 6 and table 2 show the predicted and measured results of the explosive source position based on azimuthal data for Bayesian acoustic localization. As can be seen from the figure and table, the estimated position deviates from the actual explosive source position, which is about 11.5 m away. The estimation error of the point on a scale of 300 m (closest distance of the acoustic signal/seismic wave measurement points to the center of the explosive source position) is 3.8%.

Figure 6. Acoustic source localization results

The values in the color bar are normalized by the maximum probability density; the red pentagram indicates the estimated location; the yellow square indicates the real location; the lower triangle indicates acoustic signal measurement points.

Table 2. Estimated results obtained by using Bayesian acoustic localization

| Localization results North (m) | East (m) |
|--------------------------------|----------|
| Actual positions               | 8.4      |
|                                 | 7.8      |

4.2. A Bayesian approach to source localization by using seismic waves

After pre-exploration, the relationship between the average apparent velocity of seismic waves within 2.5 km near the center of the explosion position and the distance to measurement points is shown in figure 7. The Bayesian source localization was performed based on a-priori relationship of the average apparent velocity of seismic waves, and wave arrival information. The localization results of the
explosive source are shown in figure 8 and table 3. As can be seen from the figure and table, the estimated position deviates from the actual center of the explosion, which is 9.4 m away. The estimation error of the point on a scale of 300 m is 3.1%, which is less than that of acoustic source localization, but the probability density distribution is more discrete compared with that of acoustic source localization.

Figure 7. Relationship between the average apparent velocity and the distance to the measurement points

Figure 8. Localization results obtained by using seismic waves
The values in the color bar are normalized by the maximum probability density; the red pentagram indicates the estimated location; the yellow square indicates the real location; the upper triangle indicates seismic wave measurement points

Table 3. Estimated results obtained by seismic localization

| Localization results | North (m) | East (m) |
|----------------------|-----------|----------|
| Actual positions     | 0         | 0        |
|                      | 7.2       | -6.0     |
4.3. A jointed approach to localization by using both seismic waves and acoustic signals
Based on the a-priori relationship of the velocity of seismic waves, the wave arrival information, and azimuth information, a jointed Bayesian inversion of the explosive source position was performed. The results are shown in figure 9 and table 4. As can be seen from the figure and table, the estimated position is 10.1 m away from the actual center of the explosion, and the estimation error of the point on a scale of 300 m is about 3.4%, which is between the estimation errors of acoustic source localization and seismic localization. However, the probability density distribution is more concentrated than that of acoustic and seismic localization. This is because the joint approach to source localization integrates seismic wave information and acoustic data, which imposes more constraints on the explosive source position, and makes the probability density distribution more concentrated and the localization results more robust.

![Figure 9. Joint localization results](image)

The values in the color bar are normalized by the maximum probability density; the red pentagram indicates the estimated location; the yellow square indicates the real location; the lower triangle indicates acoustic signal measurement points; the upper triangle indicates seismic wave measurement points

| Table 4. Estimated results of joint localization |
|-----------------------------------------------|
| North (m) | East (m) |
| Estimated results | 7.6 | 6.7 |
| Actual positions | 0 | 0 |

5. Conclusion
In this paper, we performed an inversion of the surface explosion location based on the Bayesian theory. The results show that the Bayesian localization method has a high precision because the error model and a-priori information are taken into the full consideration. In addition, the Bayesian joint localization method can provide more constraints on the explosion source location by fusing seismic wave data and acoustic data, which makes the probability density distribution of the explosion source location more concentrated and the localization results more robust. In addition, the SWT-SCC method is applicable to the time-delay estimation of acoustic signals of surface explosion and it can be combined with the orientation method of the four-element cross array to have a high accuracy of azimuth estimation.
Acknowledgements
The authors express their sincere gratitude to Lu Qiang, Xiao Weigu, and Li Xin who gave selfless help during the experiments. Also, we thank the National Natural Science Foundation of China for funding the project (12072290).

References
[1] Vraca, M.S., Pokrajac, I. (2017) Application of the algorithm for time of arrival estimation of N-waves produced by projectiles of different calibers. In: 173rd Meeting of Acoustical Society of America and 8th Forum Acusticum. Boston. 30(055008), pp. 1-12.
[2] Vandemark, T.F., Johnson, L.B., Pitark, A., et al. (2013) Evaluation of seismic-acoustic analysis methods for a real-time UXO monitoring system. Journal of Environmental and Engineering Geophysics, 18(1): 71-85.
[3] Hutchenson, K.D., Conner, R.B., Johnson, L.B., et al. (2015) Evaluation and current results of the Seismic Acoustic Impact Monitoring Assessment (SAIMA) system. Journal of Environmental Engineering Geophysics, 20(1): 89-100.
[4] Vandemark, T.F., Conner, R.B., Johnson, L.B., et al. (2010) Technical overview of the Seismic Acoustic Impact Monitoring Assessment (SAIMA) system. In: 23rd Symposium on the Application of Geophysics to Engineering and Environmental Problems (SAGEEP). Keystone. pp. 1065-1076.
[5] Xiong, C., Zheng, J., Zhang, B.W., et al. (2014) Landing point location based on fusion of acoustic and seismic signal. Journal of Detection & Control, 36 (1): 6-10.
[6] Bondár, I., Myers, S.C., Engdahl, E.R. (2015) Earthquake location. In: Beer, M., Kougoumtzoglou, I.A., Patelli, E., Au, S.K. (Eds.), Encyclopedia of earthquake engineering. Springer Berlin Heidelberg, Berlin. pp. 661-676.
[7] Gesret, A., Desassis, N., Noble, M., et al. (2015) Propagation of the velocity model uncertainties to the seismic event location. Geophysical Journal International, 200: 52-66.
[8] Gibbons, S.J., Pabian, F., Nasholm, S.P., et al. (2017) Accurate relative location estimates for the North Korean nuclear tests using empirical slowness corrections. Geophysical Journal International, 208: 101-117.
[9] Myers, S.C., Johannesson, G., Hanley, W. (2009) Incorporation of probabilistic seismic phase labels into a bayesian multiple-event seismic locator. Geophys. J. Int., 177: 193-204.
[10] Damarla, T. (2015) Battelfield acoustics. Springer International Publishing AG Switzerland, Cham.
[11] Du, Y.W., Cheng, J.C., Ouyang, R.B, et al. (2017) Dictionary of physics. China Science Publishing & Media LTD, Beijing.
[12] Sallai, J., Hedgecock, W., Volgyesi P., et al. (2011) Weapon classification and shooter localization using distributed multichannel acoustic sensors. Journal of Systems Architecture, 57(10): 869-885.
[13] Cheinet, S., Broglin, T. (2015) Sensitivity of shot detection and localization to environmental propagation. Applied Acoustics, 93: 97-105.
[14] Bhattacharyya, J., Bass, H.E., Drob, D.P., et al. (2002) Description and analysis of infrasound and seismic signals recorded from the Watusi explosive experiment. In: 25th Seismic Research Review - Nuclear Explosion Monitoring: Building the Knowledge Base, Tucson. pp. 587-596. september 2002[C]. 2002.
[15] Xiao W.G., Tang, Y.K., Jin P., et al. (2011), Research on location methods of microseismic monitoring. https://wap.cnki.net/touch/web/Conference/Article/ZGDW201111002025.
[16] Che, I., Kang, I.B., Shin, J.S. (2009) Seismo-acoustic location method for small-magnitude surface explosions. Earth Planets Space, 61: e1-e4.
[17] Han, C.Z., Zhu, H.Y., Duan Z.Z. Multi-source information fusion. Tsinghua University Press, Beijing.
[18] Fagan, D.K., Taylor, S.R., Schult F.R., et al. (2009) Using ancillary information to improve
hypocenter estimation: bayesian single event location (BSEL). Pure and Applied Geophysics, 166(4): 521-545.

[19] Pasyanos, M.E., Myers, S.C. (2018) The coupled location depth yield problem for North Korea's declared nuclear tests. Seismological Research Letters, 89(6): 2059-2067.

[20] Pyle, M.L., Myers, S.C., Walter, W.R., et al. (2015) Accurate local event locations in Rock Valley, Nevada, using a bayesian multiple-event method. Bulletin of the Seismological Society of America, 105(2A): 706-718.

[21] Blom, P.S., Marcillo, O., Arrowsmith, S.J. (2015) Improved bayesian infrasonic source localization for regional infrasound. Geophys. J. Int., 203: 1682-1693.

[22] Modrak, R.T., Stephen, A. (2010) A bayesian framework for infrasound location. Geophys. J. Int., 181: 399-405.

[23] Marcillo, O., Arrowsmith, S., Whitaker, R., et al. (2014) Using physics-based priors in a Bayesian algorithm to enhance infrasound source location. Geophysical Journal International, 196: 375-385.

[24] Gao, G.W., Zhu, W.W., Li Q.Z. (2016) Study on coordinates measurement technology of impact points based on acoustic detection. Science Technology and Engineering, 16(13): 230-234.

[25] Liu, M., Zeng Y.M., Zhang M., et al. (2016) A proposed time delay estimation in speech signal based on second correlation. Journal of Applied Acoustics, 35(3): 255-264.

[26] Sun, S.X., Lv, Y.X., Gu, X.H., et al. (2008) Multiscale time-delay estimation of multi-microphones with correlative noises. Journal of Vibration and shock, 27(12): 160-166.

[27] Pertilä, P. (2009) Acoustic source localization in a room environment and at moderate distances. Tampere University of Technology, Tampere.

[28] Ge, T.L. (2020) Research and implementation of robot sound source localization system based on TDOA. Chongqing Normal University, Chongqing.