UNIVERSALITY-BREAKING EFFECTS IN DIS AND DRELL-YAN

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Several properties of high energy hadronic collisions are illustrated by comparing DIS and the Drell-Yan process within a scalar QED model. Diffraction and transverse momentum broadening within the target system are found to be non-universal. Here these effects are simply due to the Coulomb phase shift in the Drell-Yan production amplitude.

1 Introduction and summary

In high-energy hadron-hadron collisions, QCD factorization theorems valid for leading-twist inclusive cross sections are known to fail in hard diffraction. For instance, in diffractive jet production, \( A + B \rightarrow A + \text{jets} + X \), there exist non-factorizable leading-twist contributions to the cross section [1]. Those contributions arise when the second hard gluon exchange, necessary to leave hadron \( A \) intact, occurs either with the active parton of the hard jet production subprocess, or with a spectator parton of the projectile hadron \( B \). This type of process, called ‘coherent hard diffraction’ [2], cannot be expressed in terms of universal diffractive parton distributions and thus breaks factorization.

In general, observables which are differential in target fragmentation can violate universality. One way to break universality is to constrain the target to be diffractively scattered. Another is to measure the transverse momentum...
between some of the target fragments. In this talk I present a model comparing DIS and Drell-Yan (DY), where both universality-breaking effects are due to the infrared sensitive Coulomb phase factor in the DY production amplitude. This factor leads to a suppression of diffractive events and to less transverse momentum broadening in DY compared to DIS.

### 2 Model for DIS and DY parton distributions

The parton distributions are modelled within scalar QED, the simplest gauge theory where factorization and universality can be tested. For the DIS forward Compton amplitude we use the model of Ref. [3] which is pictured in Fig. 1a. The target ‘quark’ is a scalar of mass \( M \) and the ‘quark’ and ‘antiquark’ of momenta \( p_1 \) and \( p_2 \) scalars of mass \( m \). The model for DY production [4] is directly obtained (Fig. 1b) by crossing the lines of the virtual photon \( \gamma^*(q) \) and of the struck scalar ‘quark’ \( q(p_1) \), and by the replacement \( q^2 = -Q^2 < 0 \rightarrow q^2 = Q^2 > 0 \). We work in a target rest frame where\(^1\) \( q = (\mp M x_B, q^-, 0\perp) \) for DIS and DY respectively, with \( q^- \equiv 2\nu = Q^2/M x_B \).

![Figure 1: SQED model for the quark distribution in DIS (a) and DY (b).](image)

We briefly recall the main features of the model. Since one concentrates on Coulomb rescatterings, extracting in the Bjorken limit the leading-twist cross

\(^1\)We use the light-cone variables \( k^\pm = k^0 \pm k^z \) to define a momentum \( k = (k^+, k^-, \vec{k}_\perp) \).
section requires focusing on the aligned jet kinematical region. Thus most of the incoming energy $\nu$ is transferred to the struck scalar quark in DIS (Fig. 1a), i.e. $p_1^- \simeq q^-$. Note that in the chosen Lorentz frame the momentum $K$ (see Fig. 1) satisfies $K^+ > 0$, giving for the hard subprocess $\gamma^* q \to q$ in DIS and $\bar{q} q \to \gamma^*$ in DY. The hard vertex is given by the bare virtual photon coupling $e$, i.e. the hard scale $\nu$ does not flow in internal propagators. Thus the contribution to $\sigma_{DIS}$ or $\sigma_{DY}$ we calculate is directly interpreted as a contribution to the scalar quark distribution in the target $f_{q/T}(x)$ probed in DIS or DY, and evaluated at $x = x_B^2$. Diagrams contributing to the $Q^2$ evolution of $f_{q/T}$ are not included. Our model thus describes $f_{q/T}(x,Q_0)$ at an initial soft scale $Q_0$. This is sufficient to study questions related to universality.

The features I will discuss in the next sections are most easily obtained after resumming Coulomb exchanges. This resummation can be done explicitly in the limit $x_B \ll 1$. In transverse coordinate space the production amplitudes $\mathcal{M}_{DIS}(\gamma^* T \to q(p_1)\bar{q}(p_2)T(p'))$ and $\mathcal{M}_{DY}(\bar{q}(p_1)T \to \gamma^* q(p_2)T(p'))$ are simply related by a phase factor (see Eq. (39) of Ref. [3] and Eq. (B6) of Ref. [4]):

$$
\mathcal{M}_{DY}(\vec{r}_\perp,\vec{R}_\perp) = -e^{i\phi_\lambda(R_\perp)}\mathcal{M}_{DIS}(\vec{r}_\perp,\vec{R}_\perp)
$$

$$
\phi_\lambda(R_\perp) = g^2 \int \frac{d^2 \vec{l}_\perp}{(2\pi)^2} \frac{e^{i\vec{R}_\perp \cdot \vec{l}_\perp}}{l_\perp^2 + \lambda^2} = \frac{g^2}{2\pi} K_0(\lambda R_\perp)
$$

For further convenience we give [3, 4]:

$$
\mathcal{M}_{DIS}(\vec{r}_\perp,\vec{R}_\perp) = M p_2^- \psi_{\gamma^*}(r_\perp) T_{qq}(\vec{r}_\perp,\vec{R}_\perp)
$$

$$
\psi_{\gamma^*}(r_\perp) = \frac{eQ}{\pi} K_0(m_{||}\cdot r_\perp)
$$

$$
m_{||}^2 = p_2^2 M x_B + m^2
$$

$$
T_{qq}(\vec{r}_\perp,\vec{R}_\perp) = 2\sin(W_\lambda/2) e^{-iW_\lambda/2}
$$

$$
W_\lambda(\vec{r}_\perp,\vec{R}_\perp) = \phi_\lambda(R_\perp) - \phi_\lambda(|\vec{R}_\perp + \vec{r}_\perp|)
$$

In the above equations $\vec{r}_\perp$ and $\vec{R}_\perp$ are the variables conjugate to $\vec{p}_{2\perp}$ and $\vec{l}_\perp$, where $l = p - p'$ is the total Coulomb momentum exchange, $g$ denotes the coupling of the exchanged Coulomb photons to the scalar quarks, and we introduced a finite photon mass $\lambda$ as an infrared regulator.

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\footnote{Indeed $x = K^+/p^+ \simeq q^+/p^+ = x_B$ in DIS and DY respectively.}
From Eq. (1) the DIS and DY cross sections integrated over \( \vec{r}_\perp \) and \( \vec{R}_\perp \) are identical. Our model is thus consistent with the universality of the inclusive scalar quark distribution, namely \( f_{q/T}^{DIS}(x) = f_{q/T}^{DY}(x) \). In particular leading-twist shadowing effects are the same in DIS and DY. One can easily show that the \( K_\perp \)-dependent distribution is also universal, \( f_{q/T}^{DIS}(x, K_\perp) = f_{q/T}^{DY}(x, K_\perp) \).

3 Violation of universality in diffractive events

The transverse coordinate space DIS and DY amplitudes are identical up to the phase shift \( \phi_\lambda(R_\perp) \), which arises because the DY process involves the scattering of a charge instead of a dipole in DIS. In Eq. (3) \( \psi_{\gamma^*}(r_\perp) \) is the \( \gamma^* \to q\bar{q} \) wavefunction and \( T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp) \) the \( q\bar{q} \) dipole scattering amplitude. We recover the expression of \( \sigma_{DIS} \) in terms of the \( q\bar{q} \) dipole cross section \( \sigma_{q\bar{q}} \) [5]:

\[
\sigma_{DIS} = \sigma_{DY} \propto \int d^2 \vec{r}_\perp |\psi_{\gamma^*}(r_\perp)|^2 \sigma_{q\bar{q}}(r_\perp) \tag{8}
\]

\[
\sigma_{q\bar{q}}(r_\perp) = \int d^2 \vec{R}_\perp |T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp)|^2 \tag{9}
\]

In our model, the diffractive cross section is identified with \( C \)-even exchanges, which correspond (for \( x_B \ll 1 \)) to the imaginary part of the production amplitude. In DIS the \( |t| = l_\perp^2 = 0 \) diffractive cross section reads:

\[
\frac{d\sigma_{DIS}^{diff}}{d l_\perp^2} \bigg|_{l_\perp^2 = 0} \propto \int d^2 \vec{r}_\perp |\psi_{\gamma^*}(r_\perp)|^2 \left| \int d^2 \vec{R}_\perp \text{Im} T_{q\bar{q}} \right|^2 \tag{10}
\]

\[
\propto \int d^2 \vec{r}_\perp |\psi_{\gamma^*}(r_\perp)|^2 \sigma_{q\bar{q}}(r_\perp)^2 \tag{11}
\]

where the second line is obtained from the unitarity relation following from (6):

\[
2\text{Im} T_{q\bar{q}} = -4\sin^2(W_\lambda/2) = -|T_{q\bar{q}}|^2 \tag{12}
\]

Comparing (8) to (11) one observes the close relationship between the total and \( t = 0 \) diffractive DIS cross sections [5].

From (1) this relation does not hold in the DY case, as can be directly seen by replacing in (10) \( \text{Im} T_{q\bar{q}} \) by

\[
\text{Im} T_{DY} = \text{Im} \left[ -e^{i\phi_\lambda(R_\perp)} T_{q\bar{q}} \right] \neq \text{Im} T_{q\bar{q}} \tag{13}
\]
We easily obtain from (6), (7) and (13):

\[ \int d^2 \tilde{R}_\perp \text{Im} T_{DY} = \int d^2 \tilde{R}_\perp \left\{ \cos[\phi_\lambda(\tilde{R}_\perp)] - \cos[\phi_\lambda(\tilde{R}_\perp + \tilde{r}_\perp)] \right\} = 0 \] (14)

One thus finds that at leading-twist the \( t = 0 \) diffractive DY cross section vanishes. As a consequence the diffractive scalar quark distribution is non-universal. This is due to the presence of the Coulomb phase in (13), which spoils the DIS unitarity relation (12).

4 Violation of universality in momentum-broadening

Now we fix the total transverse momentum exchange \( l_\perp \). Since the soft quark of momentum \( p_2 \) belongs to the outgoing target system (see Fig. 1), \( l_\perp \) is a variable internal to the target structure, different from the probed transverse momentum \( K_\perp \). We give below a heuristic argument why fixing \( l_\perp \) breaks universality and refer to Ref. [4] for a proof.

Remembering that \( \tilde{I}_\perp \) is conjugate to \( \tilde{R}_\perp \) we have, using (1):

\[ \tilde{M}_{DY}(-\tilde{r}_\perp, \tilde{I}_\perp) = -\int d^2 \tilde{R}_\perp e^{-i\tilde{R}_\perp \tilde{I}_\perp + i\phi_\lambda(\tilde{R}_\perp)} M_{DIS}(\tilde{r}_\perp, \tilde{R}_\perp) \] (15)

At fixed \( \tilde{I}_\perp \), \( \tilde{M}_{DY} \) is suppressed because the infrared sensitivity of \( \phi_\lambda(\tilde{R}_\perp) \) makes the phase factor rapidly oscillating. Indeed, when \( \lambda \to 0 \), \( \tilde{R}_\perp \sim 1/\lambda \to \infty \). Strictly speaking, since \( \phi_\lambda(\tilde{R}_\perp) \propto g^2 \), this heuristic argument holds only beyond leading order. In Ref. [4] the \( O(g^4) \) shadowing correction to the Born DY cross section is calculated, and the typical values of \( l_\perp \) contributing to \( \sigma_{DY} - \sigma_{Born}^{DY} \) are indeed shown to scale with \( \lambda \):

\[ \langle l_\perp \rangle_{DY} \sim \lambda \ll \langle l_\perp \rangle_{DIS} \sim m \] (16)

Thus leading-twist transverse momentum broadening within the target is non-universal, and is suppressed in DY compared to DIS in the present model.

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