Consistent Prediction for Direct CP Violation and $\Delta I = 1/2$ Rule

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Abstract

The theoretical status of direct CP violation $\varepsilon'/\varepsilon$ is briefly reviewed. Special attention is paid to the recent new consistent predictions for both the ratio $\varepsilon'/\varepsilon$ and the $\Delta I = 1/2$ rule within the standard model. In particular, two matching conditions resulting from the matching between the QCD and chiral perturbation theory (ChPT), and also some algebraic relations of chiral operators are found to be very useful. It is of interest that the new predictions are no longer sensitive to the strange quark mass, and are also renormalization scale and scheme independent in the leading QCD and chiral loop approximation with large $N_c$ approach. The new prediction for the direct CP violation with the value $\varepsilon'/\varepsilon = (20 \pm 4) \times 10^{-4}[\text{Im} \lambda_t/1.2 \times 10^{-4}]$ is consistent with the most recent experimental results reported by the NA48 and KTeV groups.

I. INTRODUCTION

One of puzzles in particle physics is the origin and mechanism of CP violation [1]. In the standard model (SM), CP violation is described by the Kobayashi-Maskawa (KM) [2] phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix. Such a phase can arise from the explicit CP violation imposed in the Yukawa coupling constants of SM or originate from spontaneous CP violation [3] in some extended models of SM. One of the simplest models is the extension of SM with two Higgs doublets (S2HDM) [4] motivated from spontaneous CP violation. In such an S2HDM, rich CP-violating sources can be induced from a single relative CP-violating phase of vacuum. These sources have been classified into four types [5]: (i) induced single KM phase $\delta_{KM}$ or Wolfenstein parameter $\eta$ [6]; (ii) induced new type of flavor-dependent CP-violating phases $\delta_f$ via charged and neutral scalar interactions; (iii) induced super-weak type CP violation [7] via flavor-changing scalar interactions; (iv) induced neutral scalar-pseudoscalar mixing CP violation. When the source of CP violation is dominated by the induced KM phase, the phenomena of CP violation in S2HDM would be similar to the SM.

To make a consistent prediction for the direct CP-violating parameter $\varepsilon'/\varepsilon$ caused by the KM CP-violating phase, it is necessary to understand simultaneously the longstanding puzzle of the $\Delta I = 1/2$ rule in the kaon decays as they involve the long-distance evolution
of common hadronic matrix elements, where the low energy dynamics of QCD shall play a crucial role for a consistent analysis. During the past few years, both theoretical and experimental efforts on direct CP violation in the kaon decays have been made important progresses. On the experimental side, two improved new experiments [7,8] with higher precision have reported results which are consistent each other at the $1 - \sigma$ level. On the theoretical side, several groups [9–21] have made detailed calculations for the ratio $\varepsilon'/\varepsilon$. Recently, there have been some interesting developments [9] which are mainly based on QCD of quarks and chiral perturbation theory (ChPT) [22,23] at low energies for mesons. Consequently, it has reached an agreement between the experimental results and the theoretical predictions.

To reach a consistent theoretical prediction for the direct CP-violating parameter $\varepsilon'/\varepsilon$, it must satisfy, at least, the following simple criteria: (i) the prediction should be renormalization scale independent; (ii) the $\Delta I = 1/2$ rule can be reproduced. To satisfy the first criteria (i), one should be able to solve the matching problem between perturbative QCD used to calculate the Wilson coefficient functions $c_i(\mu)$ and effective theories applied to evaluate the hadronic matrix elements of operators $<|Q_i|>(\mu)$. To reach the second one (ii), the effective theory should well describe the low energy dynamics of QCD in weak kaon decays. One of such attractive effective theories is the chiral perturbation theory (ChPT) inspired from $1/N_c$ expansion [24,25] which has actually provided a successful description on many processes [23]. In ref. [9], we have further shown that the ChPT with a functional cutoff momentum scheme can lead to a consistent prediction for the direct CP-violating parameter $\varepsilon'/\varepsilon$ and the $\Delta I = 1/2$ rule.

II. BRIEF REVIEW ON THEORETICAL STATUS

The present status of theory versus experiment is summarized as follows:

| References       | $\varepsilon'/\varepsilon [10^{-3}]$ |
|------------------|--------------------------------------|
| World Average    | 17.2 ± 1.8                           |
| NA48 [7]         | 15.3 ± 2.6                           |
| KTeV [8]         | 20.7 ± 2.8                           |
| Beijing [4]      | 20 ± 4 ± 5                           |
| Dortmund [10]    | 6.8 → 63.9 (S)                        |
| Dubna [11]       | −3.2 → 3.3 (S)                       |
| Granada-Lund [12]| 34 ± 18                              |
| Montpellier [13] | 4 ± 5                                |
| Munich [14]      | $9.2^{+7.5}_{-14.0}$ (G)             |
|                  | 1.4 → 32.7 (S)                       |
| Roma [15]        | $8.1^{+10.4}_{-9.5}$ (G)             |
|                  | −13.0 → 37.0 (S)                     |
| Taipei [16]      | 7 → 16                               |
| Trieste [17]     | 22 ± 8 (G)                           |
|                  | 9 → 48 (S)                           |
| Valencia [18]    | 17 ± 9                               |
It is seen that great theoretical efforts have been made by many groups as a consistent prediction for $\varepsilon'/\varepsilon$ and $\Delta I = 1/2$ rule play an important role for understanding the origin and mechanism of CP violation as well as the low energy dynamics of QCD. Where different approaches have been adopted to obtain the above theoretical results:

Beijing’s [9]: The ChPT inspired from $1/N_c$ expansion has been adopted to calculate the chiral loop contributions. The calculating scheme is based on the one with cutoff momentum $M$ suggested by Bardeen, Buras and Gerard [26], but it is generalized by taking the cutoff momentum as the function of the renormalization scale $\mu$, i.e., $M = M(\mu)$, instead of naively identifying the cut-off momentum to the QCD running scale $\mu$.

Dortmund’s [10]: Chiral loops are calculated by separating the factorized and non-factorized diagrams [28]. The $\Delta I = 1/2$ rule may be reproduced [29]. While the matching procedure needs to be improved as the presence of the quadratic cutoff may induce a matching scale instability.

Dubna’s [11]: Chiral loops are regularized via the heat-kernel method in the ENJL framework up to $O(p^6)$. The renormalization scheme dependence needs to be improved. A phenomenological fit of the $\Delta I = 1/2$ rule also results in a big uncertainty on the matrix element $\langle Q_6 \rangle$.

Granada-Lund’s [12]: The X-boson method within the ENJL model is used to improve the matching between long- and short-distance components. The $\Delta I = 1/2$ rule can be reproduced. Both renormalization scale and scheme dependences have been improved. While Non-FSI chiral corrections need to be further considered.

Montiepeller’s [13]: Effects to the matrix element $\langle Q_6 \rangle$ from the $\bar{q}q$ component of the scalar meson was considered by using QCD sum rule. The question is how to explicitly separate the $\bar{q}q$ from gluon component of the scalar meson.

Munich’s [14]: A phenomenological $1/N_c$ approach is adopted, where some of the matrix elements were obtained by fitting the $\Delta I = 1/2$ rule at $\mu = m_c = 1.3$ GeV. The matrix elements $\langle Q_6 \rangle$ and $\langle Q_8 \rangle_2$ relevant to the direct CP-violating parameter $\varepsilon'/\varepsilon$ remain undetermined and are taken around their leading $1/N_c$ values.

Roma’s [15]: Present lattice results remain unreliable as large renormalization uncertainties at the matching scale between the lattice and continuum results. The present lattice calculations can only use the lowest order chiral perturbation theory to evaluate the $K \to \pi\pi$ amplitude. The recent result for $\varepsilon'/\varepsilon$ was estimated by taking $B_6$ to be the VSA result varied by a 100% error.

Taipei’s [16]: Effective Hamiltonian approach was adopted in conjunction with generalized factorization for hadronic matrix elements. The non-factorizable effects were considered by introducing phenomenological parameters. Some assumptions are needed to fix the phenomenological parameters.

Trieste’s [17]: The chiral quark model has been used to evaluate the hadronic matrix elements which were assumed to be matched to the NLO short-distance Wilson coefficients at $\mu = 0.8$ GeV. The $\Delta I = 1/2$ rule can be reproduced at that point. The problem of renormalization scale and scheme dependence needs to be improved.
Valencia’s [18]: A dispersive analysis à la Omnès is used to obtain the FSI effects relative to the leading $1/N_c$ amplitudes. The FSI effects alone cannot reproduce the $\Delta I = 1/2$ rule. More comments on this approach may be found in ref. [30].

In most of the approaches, the matching procedure remains the main problem. This is because the energy scale $M$ of long-distance operator evolution from meson loops must in general be smaller than the chiral symmetry breaking scale $\Lambda_f$, i.e., $M < \Lambda_f \sim 1$ GeV, while the energy scale $\mu$ of the short-distance operator evolution from perturbative QCD should be above the confining scale, i.e., $\mu > 1$ GeV. Naively identifying the scale $M$ in ChPT to the scale $\mu$ in perturbative QCD may become inappropriate. In the following, we will focus our discussions on the recent new consistent predictions for the direct CP-violating parameter $\varepsilon'/\varepsilon$ and the $\Delta I = 1/2$ rule presented in ref. [9].

III. LOW ENERGY DYNAMICS OF QCD

To treat the low energy dynamics of QCD, our starting points are mainly based on the following basic considerations:

• In the large $N_c$ limit but with the combination $\alpha_s N_c \equiv \alpha_0$ being held fixed. The QCD loop corrections which are proportional to $\alpha_s$ are then corresponding to a large $N_c$ expansion, $\alpha_s \sim 1/N_c$ [24].

• Chiral flavor symmetry $SU(3)_L \times SU(3)_R$ is supposed to be broken dynamically due to attractive gauge interactions, namely the chiral condensates $\langle \bar{q}q \rangle$ exist and lead to the Goldstone-like pseudoscalar mesons $\pi$, $K$, $\eta$. The chiral symmetry breaking scale $\Lambda_f$ is characterized by the condensate, $\Lambda_f \sim 4\pi \sqrt{-2\langle \bar{q}q \rangle / r} \sim 1$ GeV with $r = m_{\pi_0}^2 / \hat{m}$ ($\hat{m} = (m_u + m_d)/2$).

• The low energy dynamics of QCD in large $N_c$ limit is considered to be described by the chiral Lagrangian. The ChPT is going to be treated as a cut-off effective field theory which may be regarded as a consistent theory in a more general sense [31]. The cut-off momentum $M$ is expected to be below the chiral symmetry breaking scale $\Lambda_f$.

• The chiral meson loop contributions are characterized by the powers of $p^2/\Lambda_f^2$ with $\Lambda_f = 4\pi f$. Here $f^2 \sim -2\langle \bar{q}q \rangle / r \sim N_c$ is at the leading $N_c$ order and fixed by the $\pi$ decay coupling constant $f \sim F_\pi$. Thus the chiral meson loop contributions are also corresponding to a large $N_c$ expansion of QCD, $p^2/\Lambda_f^2 \sim 1/N_c \sim \alpha_s$. Therefore both chiral loop and QCD loop contributions must be matched each other, at least in the sense of large $N_c$ limit. Thus the final physical results should be independent of the renormalization scale and the calculating schemes.

• The cut-off momentum $M$ of loop integrals is in general taken to be a function of $\mu$, i.e., $M \equiv M(\mu)$ [3], instead of naively identifying it to the renormalization scale $\mu$ appearing in the perturbative QCD in large $N_c$ limit. $M \equiv M(\mu)$ may be regarded as a functional cut-off momentum, its form is determined by the matching between the Wilson coefficients of QCD and hadronic matrix elements evaluated via ChPT.
In a word, we are going to treat the ChPT with functional cut-off momentum $M(\mu)$ scheme, as a low energy effective field theory of QCD in the large $N_c$ approach.

**IV. CHIRAL REPRESENTATIONS AND RELATIONS OF OPERATORS**

The general form of the chiral Lagrangian is constructed based on the chiral flavor symmetry $SU(3)_L \times SU(3)_R$ and can be expressed in terms of the expansion of the momentum $p$ and the quark mass $m_q$ to the energy scale $\Lambda_\chi = O(1)$ GeV at which nonperturbative effects start to play the rule. For our purpose, here we only use the chiral Lagrangian which is relevant to the $K \to \pi\pi$ decays (for the most general one, see ref. [23])

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{4} \left\{ tr(D_\mu D^\mu U) + \frac{m_q^2}{4N_c} tr(ln U^\dagger - ln U)^2 + r tr(M U^\dagger + U M^\dagger) \right. \\
\left. + r \frac{\chi_5}{\Lambda_\chi^2} tr \left( D_\mu U^\dagger D^\mu U (M U^\dagger + U^\dagger M) \right) \right. \\
\left. + r^2 \frac{\chi_8}{\Lambda_\chi^2} tr \left( M U^\dagger M U + MU^\dagger MU \right) \right\}$$

with $D_\mu U = \partial_\mu U - ir_\mu U + iU l_\mu$, and $M = \text{diag}(m_u, m_d, m_s)$. $l_\mu$ and $r_\mu$ are left- and right-handed gauge fields, respectively. The unitary matrix $U$ is a non-linear representation of the pseudoscalar meson nonet given as $U = e^{i\Pi/f}$ with $\Pi = \pi a \lambda_a$ and $tr(\lambda_a \lambda_b) = 2\delta_{ab}$. Here we keep the leading terms at large $N_c$ limit except the anomaly term which arises from the order of $1/N_c$. Note that in order to make clear for two independent expansions, namely $1/N_c$ expansion characterized by $p^2/\Lambda_\chi^2$ in the large $N_c$ limit, and the momentum expansion described by $p^2/\Lambda_\chi^2$, it is useful to introduce the scaling factor $\Lambda_\chi \approx 1$ GeV and to redefine the low energy coupling constants $L_i$ introduced in ref. [23] via $L_i = \chi_i f^2/4\Lambda_\chi^2$ and $H_j = \kappa_j f^2/4\Lambda_\chi^2$, so that the coupling constants $\chi_i$ ($i = 3, 5, 8$) and $\Lambda_\chi$ are constants in the large $N_c$ limit and the whole Lagrangian is multiplied by $f^2$ and is of order $N_c$ except the U(1) anomalous term. Numerically one sees that $\chi_i = O(1)$ for $\Lambda_\chi = 1$ GeV.

with the above chiral Lagrangian, the quark currents and densities can be represented by the chiral fields

$$\bar{q}_L \gamma^\mu q_L \equiv \frac{\delta \mathcal{L}}{\delta (l_\mu(x))_{ij}} = -f^2 2 \{ U^\dagger \partial^\mu U \\
\left. -r \frac{\chi_5}{2\Lambda_\chi^2} \left( \partial^\mu U^\dagger M - M^\dagger \partial^\mu U + \partial^\mu U^\dagger M U - U^\dagger MU \partial^\mu U \right) \right\}_{ij}$$

$$\bar{q}_R q_L \equiv -\frac{\delta \mathcal{L}}{\delta M_{ji}} = -r \frac{f^2}{4} \left( U^\dagger + \frac{\chi_5}{\Lambda_\chi^2} \partial_\mu U^\dagger \partial^\mu U U^\dagger + 2r \frac{\chi_8}{\Lambda_\chi^2} U^\dagger M U^\dagger + r \frac{\kappa_2}{\Lambda_\chi^2} M^\dagger \right)_{ij} \right.$$
with $Q_i$ the four quark operators. Note that only seven operators are independent as the linear relation $Q_4 = Q_2 - Q_1 + Q_3$. $Q_7$ and $Q_8$ originate from electroweak penguin diagrams. $c_i(\mu)$ are Wilson coefficient functions $c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$. Where $\tau = -\lambda_t/\lambda_u$ with $\lambda_q = V_{qs}^* V_{qd}$. The hard task is for calculating the hadronic matrix elements $\langle Q_i(\mu) \rangle_I$ for $\mu < \Lambda_\chi < 1$ GeV which is at the order of chiral symmetry breaking scale. This is because perturbative QCD becomes unreliable in such a low energy scale. It has been shown that the ChPT with functional cut-off momentum could provide a powerful way to evaluate $\langle Q_i(\mu) \rangle_I$ when $\mu < \Lambda_\chi$. The procedure is as follows: Firstly one represents the current or density four quark operators $Q_i$ by bosonized chiral fields from the chiral Lagrangian, i.e., $Q_i^\chi$, then calculate loop contributions by using the functional cut-off momentum scheme. Finally, one matches the two results obtained from QCD and ChPT with functional cut-off momentum by requiring scale independence of the physical results. $Q_i^\chi$ can be written as the following form

\[ Q_1^\chi + H.c. = -f^4 \, \text{tr} \left( \lambda_6 U^\dagger \partial_\mu U \right) \text{tr} \left( \lambda^{(1)} U^\dagger \partial^\mu U \right) + O(1/\Lambda_\chi^2), \]
\[ Q_2^\chi + H.c. = -f^4 \, \text{tr} \left( \lambda_6 U^\dagger \partial_\mu U \lambda^{(1)} U^\dagger \partial^\mu U \right) + O(1/\Lambda_\chi^2), \]
\[ Q_3^\chi + H.c. = -f^4 \, \text{tr} \left( \lambda_6 U^\dagger \partial_\mu U \right) \text{tr} \left( U^\dagger \partial^\mu U \right) + O(1/\Lambda_\chi^2), \]
\[ Q_4^\chi + H.c. = -f^4 \, \text{tr} \left( \lambda_6 \partial_\mu U^\dagger \partial^\mu U \right) + O(1/\Lambda_\chi^2), \]
\[ Q_5^\chi + H.c. = -f^4 \, \text{tr} \left( \lambda_6 U^\dagger \partial_\mu U \right) \text{tr} \left( U \partial^\mu U^\dagger \right) + O(1/\Lambda_\chi^2), \]
\[ Q_6^\chi + H.c. = +f^4 \left( \frac{r^2 \chi_5}{\Lambda_\chi^2} \right) \text{tr} \left( \lambda_6 \partial_\mu U^\dagger \partial^\mu U \right) + O(1/\Lambda_\chi^2), \]
\[ Q_7^\chi + H.c. = -\frac{1}{2} Q_5^\chi - \frac{3}{2} f^4 \, \text{tr} \left( \lambda_6 U^\dagger \partial_\mu U \right) \text{tr} \left( \lambda^{(1)} U \partial^\mu U^\dagger \right) + O(1/\Lambda_\chi^2), \]
\[ Q_8^\chi + H.c. = -\frac{1}{2} Q_6^\chi + f^4 r^2 \frac{3}{4} \text{tr} \left( \lambda_6 U^\dagger \lambda^{(1)} U \right) \]
\[ + f^4 r^2 \frac{3}{4} \frac{\chi_5}{\Lambda_\chi^2} \text{tr} \lambda_6 \left( U^\dagger \lambda^{(1)} U \partial_\mu U^\dagger \partial^\mu U + \partial_\mu U^\dagger \partial^\mu U U^\dagger \lambda^{(1)} U \right) \]
\[ + f^4 r^2 \frac{3}{4} \frac{\chi_8}{\Lambda_\chi^2} 2 r \text{tr} \lambda_6 \left( U^\dagger \lambda^{(1)} U U \cdot \lambda \right) + O(1/\Lambda_\chi^2). \]

with the matrix $\lambda^{(1)} = \text{diag}(1,0,0)$. Thus loop contributions of the chiral operators $Q_i^\chi$ can be systematically calculated by using ChPT with functional cut-off momentum.

For $K \to \pi \pi$ decay amplitudes and direct CP-violating parameter $\varepsilon' / \varepsilon$, the most important chiral operators are $Q_1^\chi$, $Q_2^\chi$, $Q_6^\chi$ and $Q_8^\chi$. In fact, the chiral operators $Q_3^\chi$ and $Q_5^\chi$ decouples from the loop evaluations at the $p^2$ order, i.e.

\[ Q_5^\chi = Q_3^\chi = 0 \quad (6) \]

which can explicitly be seen from the above chiral representations due to the traceless factor $\text{tr} \left( U \partial^\mu U^\dagger \right) = 0$ when ignoring the singlet U(1) nonet term which is irrelevant to the Kaon.
decays. Here $U \partial^\mu U^\dagger = A_\mu \chi^a$ may be regarded as a pure gauge. As a consequence, it implies that at the lowest order of $p^2$, we arrive at two additional algebraic chiral relations

$$Q_4^\chi = Q_2^\chi - Q_1^\chi = -f^4 \tr \left( \lambda_6 \partial_\mu U^\dagger \partial^\mu U \right) + O(1/\Lambda_\chi^2).$$

(7)

and

$$Q_6^\chi = -\left( \frac{r^2 \chi_5}{\Lambda_\chi^2} \right) (Q_2^\chi - Q_1^\chi) = \left( \frac{r^2 \chi_5}{\Lambda_\chi^2} \right) f^4 \tr \left( \lambda_6 \partial_\mu U^\dagger \partial^\mu U \right)$$

(8)

Notice that the mass parameter $r$ is at the same order of the energy scale $\Lambda_\chi$, and $\chi_5$ is at order of unit, thus the leading non-zero contribution of $Q_6^\chi$ is at the same order of $Q_2^\chi$ and $Q_1^\chi$.

The above algebraic chiral relations mentioned also as Wu-relations in the literature [32] were first derived in ref. [27], they have been checked from an explicit calculation up to the chiral one-loop level by using the usual cut-off regularization [26]. It was based on this observation that the ratio $\varepsilon'/\varepsilon$ was predicted [27,33] to be large enough ($\varepsilon'/\varepsilon = (1-3) \times 10^{-3}$) for observation. In ref. [33], it has further been shown that the algebraic Wu-relations of the chiral operators survive from loop corrections and can provide a consistent prediction for both the direct CP-violating parameter $\varepsilon'/\varepsilon$ and the $\Delta I = 1/2$ rule when applying for an appropriate matching procedure between QCD and ChPT.

V. MATCHING APPROACH

The short-distance evolution of quark operators is performed from perturbative QCD. When the energy scale $\mu$ is high, $m_W > \mu > m_b$, there are eleven independent operators $Q_i$ ($i = 1, \cdots, 11$). When the energy scale $\mu$ runs down to below the bottom quark mass $m_b$ and above the charm quark mass $m_c$, i.e., $m_b > \mu > m_c$, the operator $Q_{11}$ decouples and operator $Q_{10}$ is given by the linear combination $Q_{10} = -2Q_1 + 2Q_2 + Q_3 - Q_4$. Once the energy scale $\mu$ goes down to below $m_c$ but above the confining scale or the energy scale $\Lambda_\chi$, i.e., $m_c > \mu > \Lambda_\chi$, two operators $Q_9$ and $Q_4$ become no longer independent and are given by the linear combination $Q_9 = Q_2 + Q_1$ and $Q_4 = Q_3 + Q_2 - Q_1$. Thus there are only seven independent operators below $m_c$ and above $\Lambda_\chi$. The one-loop QCD corrections of the quark operators at the energy scale just above the energy scale $\Lambda_\chi$ is given by

$$Q_1(\mu_Q) = Q_1(\mu) - 3\frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_2(\mu) + O(1/N_c),$$

(9)

$$Q_2(\mu_Q) = Q_2(\mu) - 3\frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_1(\mu)$$

$$-\frac{1}{3} \frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_4(\mu) - \frac{1}{3} \frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_6(\mu) + O(1/N_c),$$

(10)

$$Q_3(\mu_Q) = Q_3(\mu) - 3\frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_2(\mu)$$

$$-\frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_4(\mu) - \frac{\alpha_s}{4\pi} \ln\left( \frac{\mu^2_Q}{\mu^2} \right) Q_6(\mu) + O(1/N_c),$$

(11)
$$Q_6(\mu_Q) = Q_6(\mu) - \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_4(\mu)$$

$$+ [3(N_c - 1/N_c) - 1] \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_6(\mu) + O(1/N_c) ,$$

(12)

$$Q_8(\mu_Q) = Q_8(\mu) + [3(N_c - 1/N_c) - 1] \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_8(\mu) ,$$

(13)

and

$$Q_3(\mu_Q) = Q_3(\mu) - \frac{11}{3} \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_4(\mu) - \frac{2}{3} \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_6(\mu) + O(1/N_c) ,$$

(14)

$$Q_5(\mu_Q) = Q_5(\mu) + 3 \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_6(\mu) + O(1/N_c) ,$$

(15)

$$Q_7(\mu_Q) = Q_7(\mu) + 3 \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu_Q^2}{\mu^2}\right) Q_8(\mu) + O(1/N_c) .$$

(16)

It is then clear that

i) In the large $N_c$ limit, $Q_1, Q_2, Q_4,$ and $Q_6$ form a complete set of operators under QCD corrections.

ii) The evolution of $Q_8$ is independent of other operators and only caused by loop corrections of the density.

iii) The operator $Q_3$ is given by the linear combination $Q_3 = Q_4 - (Q_2 - Q_1)$. The operator $Q_5$ is driven by the operator $Q_6$, and the operator $Q_7$ is driven by the operator $Q_8$.

When the energy scale $\mu$ approaches to the confining scale, or $\mu < \Lambda_\chi \sim \Lambda_F \sim 1$ GeV, as we have discussed in the above sections, long-distance effects have to be considered. The evolution of the operators $Q_i(\mu)$ when $\mu < \Lambda_\chi$ is supposed to be carried out by the one of the chiral operators $Q_i^\chi(M(\mu))$ in the framework of the functional cut-off ChPT truncated to the pseudoscalars. To be treated at the same approximations made in the short-distance operator evolution of QCD, we should only keep the leading terms (i.e., quadratic terms of functional cut-off momentum) and take the chiral limit, i.e., $m_K^2, m_\pi^2 << \Lambda_F^2$. The evolution of the operators $Q_1^\chi$ and $Q_2^\chi$ is simply given by

$$Q_1(\mu) \to Q_1^\chi(M(\mu)) = Q_1^\chi(M(\mu')) - \frac{2(M^2(\mu) - M^2(\mu'))}{\Lambda_F^2} Q_2^\chi(M(\mu')) ,$$

(17)

$$Q_2(\mu) \to Q_2^\chi(M(\mu)) = Q_2^\chi(M(\mu')) - \frac{2(M^2(\mu) - M^2(\mu'))}{\Lambda_F^2} Q_1^\chi(M(\mu'))$$

+ \frac{M^2(\mu) - M^2(\mu')}{\Lambda_F^2} (Q_2^\chi - Q_1^\chi)(M(\mu')) ,$$

(18)

where $\Lambda_F = 4\pi F = 1.16$ GeV with $F$ the renormalized one of $f$. The operators $Q_i^\chi (i = 4, 6, 8)$ can be written as follows

$$Q_4(\mu) \to Q_4^\chi(M(\mu)) = (Q_4^\chi - Q_4^\chi)(M(\mu)) ,$$

(19)

$$Q_6(\mu) \to Q_6^\chi(\mu, M(\mu)) = \left[1 + 3(N_c - 1/N_c) \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{\mu^2_{\chi}}\right)\right] Q_6^\chi(\mu, M(\mu)) ,$$

(20)

$$Q_8(\mu) \to Q_8^\chi(\mu, M(\mu)) = \left[1 + 3(N_c - 1/N_c) \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{\mu^2_{\chi}}\right)\right] Q_8^\chi(\mu, M(\mu)) .$$

(21)
where the explicit $\mu$-dependence of the operators $Q_6^\chi(\mu, M(\mu))$ and $Q_8^\chi(\mu, M(\mu))$ arise from the running quark mass and behaves like $1/(m_q(\mu) + \hat{m}(\mu))^2$. Notice that the operators $Q_6^\chi$ and $Q_8^\chi$ decouple from the evolution, namely $Q_6^\chi = 0$ and $Q_8^\chi = 0$. Thus the independent operators are reduced once more in the long-distance operator evolution when $\mu < \Lambda_\chi$ due to the algebraic chiral operator relations.

Matching the loop results evaluated from QCD with the ones from the ChPT with functional cut-off momentum at the energy scale $\Lambda_\chi$ with large $N_c$ expansion and requiring $\mu$-independence in the large $N_c$ limit, i.e., $\frac{\partial}{\partial \mu} Q_1(\mu) = 0$, we arrive at the two matching conditions [8],

$$\mu \frac{\partial}{\partial \mu} \left( \frac{2M^2(\mu)}{\Lambda_F^2} \right) = \frac{3\alpha_s}{2\pi}, \quad (22)$$

$$Q_6^\chi(\mu_\chi, M(\mu)) = -\frac{11}{2}(Q_2^\chi - Q_1^\chi)(M(\mu)), \quad \mu < \Lambda_\chi \quad (23)$$

When combining the second matching condition with the chiral Wu-relation

$$Q_6^\chi(\mu_\chi, M(\mu)) \simeq \left( -\frac{R_x^2 \chi_5^5}{\Lambda_\chi^2} \right) (Q_2^\chi - Q_1^\chi)(M(\mu)), \quad \mu < \Lambda_\chi \quad (24)$$

$$R_\chi \equiv R(\mu \simeq \mu_\chi) \simeq m_s^2/\hat{m}(\mu_\chi) \simeq 2m_K^2/(m_s + \hat{m})(\mu_\chi), \quad (25)$$

we are able to fix the strange quark mass [9]

$$\frac{R_x^2 \chi_5^5}{\Lambda_\chi^2} = \frac{11}{2} \rightarrow m_s(\mu_\chi) \simeq 196\text{MeV}, \quad (26)$$

Note that the coupling constants $\chi_5$ and $r$ must be replaced by the corresponding renormalized ones $\chi_5^R$ and $R(\mu)$ as their loop corrections are at the subleading order. Here we have also used the result $\Lambda_\chi = 1.03\sqrt{\chi_5} \text{GeV}$ which is fixed from the ratio of the kaon and pion decay constants.

After integrating the first matching condition, we find that the $\mu$-dependence of the functional cut-off momentum $M(\mu)$ can be written as [8]

$$\frac{2M^2(\mu)}{\Lambda_F^2} \simeq \frac{3\alpha_s}{4\pi} + \frac{3\alpha_s}{4\pi} \ln(\frac{\mu^2}{\mu_0^2}), \quad (27)$$

which shows that after imposing the matching condition for the anomalous dimensions between quark operators $Q_i(\mu)$ in QCD and the corresponding chiral operators $Q_i^\chi(M(\mu))$ in the ChPT with functional cut-off momentum, the dimensionless ratio $M^2/\Lambda_F^2$ is only related to the strong coupling constant $\alpha_s$ and becomes scheme-independent, which implies that the long-distance operator evolution in ChPT with functional cut-off momentum can be carried out by using any approach. Where $\mu_0$ is the low energy scale arised as the integrating constant. In general, we have $\mu_0 > \Lambda_{QCD}$. To fix the value of $\mu_0$, we use $M_0 \simeq \mu_0$. Thus $\mu_0$ (or $\alpha_s(\mu_0)$) is determined via $\mu_0 \simeq \Lambda_F \sqrt{3\alpha_s(\mu_0)/8\pi}$. Using the definition $\alpha_s(\mu) = 6\pi/[33 - 2n_f] \ln(\mu^2/\Lambda_{QCD}^2)]$ with $n_f = 3$, $\mu_0$ is found to be (for $\Lambda_{QCD} = 325 \pm 80$ MeV)
\[ \mu_0 \simeq 435 \pm 70\text{MeV} \quad \text{or} \quad \alpha_s(\mu_0)/2\pi \simeq 0.19^{+0.06}_{-0.05} . \]  

Thus the functional cut-off momentum \( M(\mu) \) at \( \mu = \Lambda_\chi \) yields the following corresponding value

\[ M_\chi \equiv M(\mu = \Lambda_\chi) \simeq 1\text{GeV} \simeq 0.71^{+0.11}_{-0.12}\text{GeV} . \]  

which provides the possible allowed range of the energy scale where the ChPT with functional cut-off momentum can be used to describe the low energy behavior of QCD at large \( N_c \) limit.

**VI. LONG-DISTANCE EVOLUTION OF CHIRAL OPERATORS**

We may now adopt the matching conditions and algebraic Wu-relations of the chiral operators to investigate the long-distance operator evolution. The \( \Delta S = 1 \) low energy \( (\mu < \Lambda_\chi) \) effective Hamiltonian for calculating \( K \to \pi \pi \) decay amplitudes may be expressed as follows

\[ \mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \lambda_u \sum_{i=1,2,4,6,8} c_i(\Lambda_\chi) Q_i^\chi(M(\Lambda_\chi)) \]  

It is seen that the first matching condition enables us to sum over all the leading terms via renormalization group equation down to the energy scale \( \mu_0 \), and the second matching condition together with the algebraic chiral operator relations allows us to evaluate the penguin operators \( Q_i^\chi(M) \) and \( Q_i^\chi(M) \) from the operators \( Q_i^\chi(M) \) and \( Q_i^\chi(M) \). So that the operators \( Q_i^\chi(M) \) and \( Q_i^\chi(M) \) form a complete set for the operator evolution below the energy scale \( \mu \simeq \Lambda_\chi \simeq 1 \text{GeV} \), or correspondingly, below the functional cut-off momentum \( M(\mu \simeq \Lambda_\chi) \simeq 0.71^{+0.11}_{-0.12} \text{GeV} \) for \( \Lambda_{QCD} = 325 \pm 80 \text{MeV} \). By choosing a new operator basis \( Q_i^\chi(M(\mu)) = Q_i^\chi(M(\mu)) + Q_i^\chi(M(\mu)) \) with the anomalous dimension matrix for the basis \( (Q_i^\chi, Q_i^\chi) \)

\[ \gamma = \frac{\alpha_s}{2\pi} \begin{pmatrix} -9/2 & 0 \\ -3/2 & 3 \end{pmatrix} \]  

and following the standard procedure of the renormalization group evolution with the initial conditions for the Wilson coefficient functions: \( c_-(\Lambda_\chi) = c_2(\Lambda_\chi) - c_1(\Lambda_\chi) \) and \( c_+(\Lambda_\chi) = c_2(\Lambda_\chi) + c_1(\Lambda_\chi) \), we find in the leading logarithmic approximation that

\[ Q_\chi^\chi(M(\Lambda_\chi)) = \eta_\chi^{-1/2} \eta_\chi(M(\Lambda_\chi)) Q_\chi^\chi(\mu_0), \]  

\[ Q_\chi^\chi(M(\Lambda_\chi)) = \eta_\chi^{1/3} \eta_\chi(M(\Lambda_\chi)) Q_\chi^\chi(\mu_0) + \frac{1}{3} \left( \eta_\chi^{-1/2} - \eta_\chi^{1/3} \right) \eta_\chi(M(\Lambda_\chi)) Q_\chi^\chi(\mu_0), \]

with \( \eta_\chi = \alpha_s(\Lambda_\chi)/\alpha_s(\mu_0) \), and

\[ Q_\chi^\chi(\mu_0) = Q_\chi^\chi(0) + \frac{9\alpha_s(\mu_0)}{8\pi} Q_\chi^\chi(0), \]  

\[ Q_\chi^\chi(\mu_0) = Q_\chi^\chi(0) - \frac{3\alpha_s(\mu_0)}{4\pi} Q_\chi^\chi(0) + \frac{3\alpha_s(\mu_0)}{8\pi} Q_\chi^\chi(0), \]
where $\eta_-(M_\chi)$, $\eta_1(M_\chi)$ and $\eta_2(M_\chi)$ represent the finite meson mass contributions

\[
\eta_-(M_\chi) \simeq 1 + \frac{3}{4} m_K^2 - \frac{9}{2} m_\pi^2 \ln \left( 1 + \frac{M^2(\mu)}{\tilde{m}^2} \right),
\]

\[
\eta_1(M_\chi) \simeq 1 + \frac{1}{2} m_K^2 + 3 m_\pi^2 \ln \left( 1 + \frac{M^2(\mu)}{\tilde{m}^2} \right),
\]

\[
\eta_2(M_\chi) \simeq 1 + \frac{m_K^2 - \frac{3}{2} m_\pi^2}{M^2_\chi} \ln \left( 1 + \frac{M^2(\mu)}{\tilde{m}^2} \right).
\] (36)

Numerically, we use $\tilde{m} \simeq 300$ MeV, $m_K = 0.495$ GeV and $m_\pi = 0.137$ GeV. When the QCD scale takes the value $\Lambda_{QCD} = 325 \pm 80$ MeV with the corresponding low energy cut-off momentum $\mu_0 \simeq 435 \pm 70$ MeV, we have

\[
Q_\chi^-(M(\Lambda_\chi)) = (3.17^{+0.66}_{-0.43}) Q_\chi^-(0) = Q_\chi^-(M(\Lambda_\chi)),
\]

\[
Q_\chi^+(M(\Lambda_\chi)) = (0.55^{+0.09}_{-0.06}) Q_\chi^+(0) + (0.8^{+0.11}_{-0.05}) Q_\chi^-(0),
\]

\[
Q_\chi^-(\mu_\chi, M(\Lambda_\chi)) = -\frac{11}{2} Q_\chi^-(M(\Lambda_\chi)) = -(17.44^{+3.62}_{-2.37}) Q_\chi^-(0),
\]

\[
Q_\chi^+(\mu_\chi, M(\Lambda_\chi)) = \frac{33}{8} \frac{\Lambda_\chi^2}{\chi_5^2(\mu_\chi)} (Q_\chi^+ + Q_\chi^-)(0) = 19.18 (Q_\chi^+ + Q_\chi^-)(0).
\] (40)

with these analyzes, we are able to present our numerical predictions for the isospin amplitudes and the direct CP-violating parameter $\varepsilon'/\varepsilon$.

**VII. NEW PREDICTIONS FOR $\varepsilon'/\varepsilon$ AND $\Delta I = 1/2$ RULE**

The direct CP violation $\varepsilon'/\varepsilon$ in kaon decays arises from the nonzero relative phase of isospin amplitudes $A_0$ and $A_2$

\[
\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|} \left( \frac{Im A_2}{Re A_2} - \frac{Im A_0}{Re A_0} \right)
\] (41)

with $\omega = Re A_2/Re A_0 = 1/22.2$, we arrive at the following general expression

\[
\frac{\varepsilon'}{\varepsilon} = \frac{G_F}{2} \frac{\omega}{|\varepsilon| Re A_0} Im \lambda_I (h_0 - h_2/\omega)
\] (42)

where $A_I$ are the $K \to \pi \pi$ decay amplitudes with isospin $I$ with

\[
A_I \cos \delta_I = \langle \pi \pi | H_{\pi \pi}^{\Delta S=1} | K \rangle \equiv \frac{G_F}{\sqrt{2}} \lambda_u \sum_{i=1,2,4,6,8} c_i(\Lambda_\chi) Re \langle Q_i^+(M(\Lambda_\chi)) \rangle_I
\] (43)

The CP-conserving amplitudes are given by

\[
Re A_0 \cos \delta_0 = \frac{G_F}{\sqrt{2}} Re \lambda_u \sum_{i=1,2,4,6,8} z_i(\Lambda_\chi) Re \langle Q_i^+(M(\Lambda_\chi)) \rangle_0
\]

\[
\simeq \frac{G_F}{\sqrt{2}} Re \lambda_u \frac{1}{2} z_-(\Lambda_\chi) Re \langle Q^-_i(M(\Lambda_\chi)) \rangle_0 + \frac{1}{2} z_+(\Lambda_\chi) Re \langle Q^+_i(M(\Lambda_\chi)) \rangle_0
\]
\[ \cos \delta_2 = \frac{G_F}{\sqrt{2}} Re\lambda_u \sum_{i=1,2,8} z_i(\Lambda_\chi) Re\langle Q_i^\chi (M(\Lambda_\chi)) \rangle_2 \]

\[ \simeq \frac{G_F}{\sqrt{2}} Re\lambda_u \left[ \frac{1}{2} z_-(\Lambda_\chi) Re\langle Q_-^\chi (M(\Lambda_\chi)) \rangle_2 + \frac{1}{2} z_+^\chi(\Lambda_\chi) Re\langle Q_+^\chi (M(\Lambda_\chi)) \rangle_2 \right] , \]

and the hadronic matrix elements concerned for calculating the ratio \( \varepsilon' / \varepsilon \) are

\[ h_0 = (\cos \delta_0)^{-1} \sum_{i=1,2,4,6,8} y_i(\Lambda_\chi) Re\langle Q_i^\chi (M(\Lambda_\chi)) \rangle_0 (1 - \Omega_{IB}) \]

\[ \simeq (\cos \delta_0)^{-1} y_0(\Lambda_\chi) Re\langle Q_0^\chi (M(\Lambda_\chi)) \rangle_0 (1 - \Omega_{IB}) \]  

\[ h_2 = (\cos \delta_2)^{-1} \sum_{i=1,2,8} y_i(\Lambda_\chi) Re\langle Q_i^\chi (M(\Lambda_\chi)) \rangle_2 \]

\[ \simeq (\cos \delta_2)^{-1} y_8(\Lambda_\chi) Re\langle Q_8^\chi (M(\Lambda_\chi)) \rangle_2 \]

where we have taken into account the possible isospin breaking effect \( \Omega_{IB} \) with the most recent refined calculation in [33] \( \Omega_{IB} \simeq 0.16 \pm 0.03 \), which is smaller than the previously estimated value \( \Omega_{IB} \simeq 0.25 \pm 0.1 \) [35], but with a large error [36]. The CKM factors \( Re\lambda_u \) and \( Im\lambda_t \) are given in the Wolfenstein parameterization [3] as follows

\[ Re\lambda_u = Re(V^*_{us} V_{ud}) = \lambda , \quad Im\lambda_t = Im(V^*_{ts} V_{ta}) = A^2 \lambda^5 \eta \]

To obtain the numerical results, we use the following reliable values for all relevant parameters: \( \Lambda_{QCD} = 325 \pm 80 \text{ MeV} \), \( \mu_0 = 435 \pm 70 \text{ MeV} \), \( \Lambda_\chi = 1.0 \text{GeV} \), and \( \Lambda_F = 1.16 \text{GeV} \); \( z_-(\Lambda_\chi) = (z_2 - z_1)(\Lambda_\chi) = 2.181^{+0.197}_{-0.177} \), \( z_+(\Lambda_\chi) = (z_2 + z_1)(\Lambda_\chi) = 0.685 \mp 0.029 \), \( z_4(\Lambda_\chi) = -(0.012 \pm 0.003) \) and \( z_6(\Lambda_\chi) = -(0.013 \pm 0.003) \), as well as \( y_6(\Lambda_\chi) = -(0.113^{+0.024}_{-0.021}) \) and \( y_8(\Lambda_\chi)/\alpha = 0.158^{+0.040}_{-0.033} \). \( \langle Q_0^\chi(0) \rangle_0 = 36.9 \times 10^6 \text{ MeV}^3 \), \( \langle Q_0^\chi(0) \rangle_0 = 12.3 \times 10^6 \text{ MeV}^3 \),  

\[ \langle Q_8^\chi(0) \rangle_0 = 34.8 \times 10^6 \text{ MeV}^3 \text{ and } \langle Q_8^\chi(0,0) \rangle_2 = 328.8 \times 10^6 \text{ MeV}^3 \]. Note that for the Wilson coefficient functions, we only use the leading order results at one-loop level for a consistent analysis since the chiral operators have only been evaluated up to the leading order at the coefficient functions, we only use the leading order results at one-loop level for a consistent analysis. With these input values, we obtain the isospin amplitudes

\[ ReA_0 = (2.56^{+0.78}_{-0.37}) \times 10^{-4} (\cos \delta_0)^{-1} \text{ MeV} = (3.10^{+0.94}_{-0.61}) \times 10^{-4} \text{ MeV} \],

\[ ReA_2 = (0.12 \mp 0.02) \times 10^{-4} (\cos \delta_2)^{-1} \text{ MeV} = (0.12 \mp 0.02) \times 10^{-4} \text{ MeV} \]

which is consistent with the experimental data: \( ReA_0 = 3.33 \times 10^{-4} \text{ MeV} \) and \( ReA_2 = 0.15 \times 10^{-4} \text{ MeV} \). Here the final state interaction phases, \( \delta_0 = (34.2 \pm 2.2)^\circ \) and \( \delta_2 = (-6.9 \pm 0.2)^\circ \) [39] have been used. Simultaneously, it leads to a consistent prediction for the direct CP-violating parameter \( \varepsilon' / \varepsilon \).
\[ \frac{\varepsilon'}{\varepsilon} = (23.6^{+12.4}_{-7.8}) \times 10^{-4} \left( \frac{I m \lambda_t}{1.2 \times 10^{-4}} \right) \] (51)

From the above analyzes, it is noticed that

1. The main uncertainties for the predictions arise from the QCD scale \( \Lambda_{QCD} \) (or the low energy scale \( \mu_0 \)) and the combined CKM factor \( I m \lambda_t \). Nevertheless, the uncertainties from the energy scale may be reduced from comparing the predicted isospin amplitudes \( A_0 \) and \( A_2 \) with the well measured ones. It is seen that the results corresponding to the large values of \( \Lambda_{QCD} > 325 \) MeV appear not favorable.

2. Considering from the isospin amplitude \( A_2 \), the ratio \( \varepsilon'/\varepsilon \) favors the low values

\[ \frac{\varepsilon'}{\varepsilon} \simeq 16 \times 10^{-4} \left( I m \lambda_t / 1.2 \times 10^{-4} \right) \] (52)

while from the isospin amplitude \( A_0 \), it favors the high values

\[ \frac{\varepsilon'}{\varepsilon} \simeq 24 \times 10^{-4} \left( I m \lambda_t / 1.2 \times 10^{-4} \right) \] (53)

From the ratio of two amplitudes \( ReA_0 / ReA_2 \), i.e., the \( \Delta I = 1/2 \) rule, the ratio \( \varepsilon'/\varepsilon \) favors the middle values

\[ \frac{\varepsilon'}{\varepsilon} \simeq 20 \times 10^{-4} \left( I m \lambda_t / 1.2 \times 10^{-4} \right) \] (54)

3. The above results are renormalization scheme independent as the consistent matching between QCD and ChPT occurs at the leading one-loop order of \( 1/N_c \sim \alpha_s \sim 1/\Lambda_F^2 \) around the scale \( \Lambda_\chi \). The renormalization scheme dependence arises from the next-to-leading order of perturbative QCD [40], which could become substantial for some of the Wilson coefficient functions when the renormalization scale \( \mu \) runs down to around the scale \( \Lambda_\chi = 1 \) GeV. For the long-distance evolution, the scheme is fixed by the ChPT with functional cut-off momentum. For matching to this scheme, it is useful to introduce a scheme independent basis for the perturbative QCD calculation of short-distance physics. Then applying our above procedure to find out the matching conditions at the next-to-leading order \( 1/N_c^2 \sim \alpha_s^2 \sim 1/\Lambda_F^4 \). To work out the scheme independent basis in QCD, it may be helpful to adopt the method discussed in ref. [41] and use the cut-off momentum basis.

In summary, our new consistent prediction for the direct CP-violating parameter \( \varepsilon'/\varepsilon \) is

\[ \frac{\varepsilon'}{\varepsilon} = (20 \pm 4) \times 10^{-4} \left( \frac{I m \lambda_t}{1.2 \times 10^{-4}} \right) = (20 \pm 4 \pm 5) \times 10^{-4} \] (55)

where the first error from the low energy scale and the second one from the CKM factor \( I m \lambda_t \). Our prediction is consistent with the most recent results reported by the NA48 collaboration at CERN [7] and the KTeV collaboration at Fermilab [8]

\[ Re(\varepsilon'/\varepsilon) = (20.7 \pm 2.8) \times 10^{-4} \quad (2001 \text{ KTeV}) \] (56)

\[ Re(\varepsilon'/\varepsilon) = (15.3 \pm 2.6) \times 10^{-4}, \quad (2001 \text{ NA48}) \] (57)

as well as with the world average

\[ Re(\varepsilon'/\varepsilon) = (17.2 \pm 1.8) \times 10^{-4} \quad (\text{World Average 2001}) \] (58)
VIII. CONCLUSIONS

Let me briefly summarize the new predictions by Beijing group:

- We have clearly made a bi-expansion: $1/N_c \sim \alpha_s \sim M^2/\Lambda_f^2$ and $p^2/\Lambda^2 (m_q^2/\Lambda^2)$ by introducing the $N_c$-independent energy scale. The leading non-zero contributions for relevant chiral operators are all at the same order of $\sqrt{N_c}$.

- Chiral representation of operators allows us to get some useful algebraic chiral relations which reduces the independent operators and relates the ratio $\varepsilon'/\varepsilon$ to the $\Delta I = 1/2$ rule.

- Chiral loops have been calculated by using the functional cutoff momentum (FCOM) ($M(\mu)$) scheme. The form of $M(\mu)$ is determined by matching between short- and long-distance contributions.

- The leading order of $1/N_c$ between chiral one loops and QCD one loops has been found to be well matched at large-$N_c$ limit. In this sense, ChPT does well describe the low energy dynamics of QCD.

- Two useful matching conditions have been resulted, which makes the predictions for the ratio $\varepsilon'/\varepsilon$ and the $\Delta I = 1/2$ rule are insensitive to the strange quark mass.

- The results at the leading order of loops are both renormalization scale and renormalization scheme independent.

The present applications of ChPT also indicate that the ChPT with chiral Lagrangian obtained based on the chiral flavor symmetry $SU(3)_L \times SU(3)_R$ and inspired from $1/N_c$ approach is a consistent and useful effective theory. With the functional cutoff momentum scheme, the ChPT may well describe the low energy dynamics of QCD. We expect that the theoretical uncertainties can be further improved within the framework of ChPT. Nonetheless to say, the flavor symmetry has played an important role on flavor physics.\[14\]
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