1. Introduction

At present large amount of technical equipment which create engineering networks are hidden under the ground surface, for example telecommunication and power cables, gas and water pipes, sewerage pipes etc. Construction of new underground networks, their repairs, construction of other objects, earthworks etc. require precise marking out of the engineering networks in a given place. Combinations of geodetic and electromagnetic methods are used to determine the position.

One of frequently used electromagnetic methods of the position determining is the marking by markers. Essentially, the marker is a resonant circuit, which is tuned on the concrete frequency different for different marked equipment (83 kHz for gas pipes, 101.4 kHz for telecommunication cables, 121.6 kHz for sewerage pipes, 145.7 kHz for water pipes and 169.8 kHz for power cables). The marker is buried under the ground together with engineering network equipment and marks important points (connection, turning, crossing, point of reparation) or route. The marker can be whenever localized by special device – locator.

2. Principle of marker – locator system operation

The marker is resonant circuit connected according to Fig. 1. The locator periodically transmits an excitation signal which excites damped harmonic oscillations in the marker. Frequency of damped oscillations is given by a marker tuning and this signal is received and evaluated by a locator (Fig. 2).

Signals recorded by a digital oscilloscope directly in the locator are depicted in Fig. 3, where trace #1 shows the excitation current in the locator antenna and trace #2 shows received response from the marker, i.e. damped harmonic oscillations. The marker is localized by a method which is shown in Fig. 4.

3. Dependence of received signal from marker on distance from locator

The transmitting antenna of locator is essentially a circular loop coil with radius $r_{LT}$ (with area $S_{LT}$) and with $N_{LT}$ turns. The excitation current $I_{e}$ flows by this coil and excites magnetic field. Intensity of the magnetic field in the distance $x$ from the antenna and in its axis direction is given (according to [1]) by formula

$$H_{L}(x) = \frac{N_{LT} H S_{LT}}{2 \pi (r_{LT}^2 + x^2)^{3/2}} \quad (1).$$
Assuming that the excitation current is harmonic and has frequency \( f \) in the marker coil which has \( N_M \) turns, area \( S_M \), inductivity \( L_M \) and its axis is identical with the locator antenna axis the voltage

\[
U_M = N_M \frac{d\Phi(x)}{dt} = \mu_0 N_M S_M H_L(x)
\]

is induced, where \( \Phi(x) \) is magnetic flux and \( \mu \) is environment permeability. This voltage causes the current

\[
I_M = \frac{U_M}{\omega L_M} = \frac{Q_M}{L_M} \frac{\mu_0 N_M S_M H_L(x)}{L_M}
\]

in the marker coil, where \( Q_M \) is the quality factor of marker resonant circuit. The current \( I_L \) creates magnetic field with intensity

\[
H_M(x) = \frac{N_M I_M S_M}{2\pi r_M^2 + x^2} \frac{1}{3}
\]

or, after combining (1), (3), (4) and (5)

\[
U_L = \mu_0 N_{LR} S_{LR} Q_{LR} H_M(x)
\]

where separate symbols denote:

- \( N_{LR} \) receiving locator coil number of turns
- \( S_{LR} \) area of receiving locator coil
- \( Q_{LR} \) quality factor of locator input tuned circuit.

The marker voltage \( U_M \) and receiving coil voltage \( U_L \) calculated from (2) and (6) are graphically depicted in figures 5 and 6 in dependence on distance \( x \). For calculation these parameters were used:

- marker parameters
  - number of turns \( N_M = 18 \)
  - inductivity \( L_M = 169 \mu H \)
  - coil radius \( r_M = 0.1 m \)
  - coil area \( S_M = 0.0628 m^2 \)
  - quality factor \( Q_M = 70 \)

- transmitting locator coil parameters (shall be chosen subject to [2])
  - number of turns \( N_{LT} = 30 \)
  - coil radius \( r_{LT} = 0.1 m \)
  - coil area \( S_{LT} = 0.0628 m^2 \)
  - excitation current \( I_L = 0.2 A \)

- receiving locator coil parameters
  - number of turns \( N_{LR} = 50 \)
  - coil radius \( r_{LR} = 0.075 m \)
  - coil area \( S_{LR} = 0.03534 m^2 \)
  - quality factor \( Q_{LR} = 15 \)

- other parameters
  - permeability \( \mu = 4\pi \times 10^{-7} H/m \)
  - frequency \( \omega = 637 \text{ krad/s} (f = 101.4 \text{ kHz}) \).

As (2) depends on (1), it is obvious that the marker induced voltage depends reverse proportionally on the third power of distance \( x \). Similarly, it is obvious that voltage on the receiving coil, i.e. induced back from excited marker depends reverse proportionally on the sixth power of distance \( x \). Moreover, these simplified calculations do not consider that the amplitude of marker voltage falls exponentially with time (damped harmonic oscillations of marker).

4. Calculation of voltage induced into marker by current shock wave

Markers can mark not only non-metallic, but also metallic pipes and cables in an open field. For this reason we must consider the influence of the current waves which spread in a metal conductor on the current and voltage loading of marker components. This shock wave can be created by lightning current or short circuit in power cable. Amplitude of lightning current can achieve up to 50 kA [3]. Considering the ground and metallic conductor
damping factor, we assume about current wave with amplitude 2 kA.

Time dependence of such shock wave corresponds approximately with Fig. 7 and can be mathematically modelled by subtraction of two exponentials [3]

\[ i(t) = I(e^{-t/t_1} - e^{-t/t_2}) \]  

(7),

where \( T_h \) is front time, \( T_m \) is half-rear time and \( I_{MAX} \) is maximum value of current in the time \( T_h \). For measuring and calculation the current wave with \( T_h = 8 \, \mu s \) and \( T_m = 20 \, \mu s \) are being used. Corresponding parameters \( t_1, t_2 \) and \( I \) must be calculated from a non-linear equation system

\[ \frac{t_2 - t_1}{t_2 - t_1} \ln \left( \frac{t_2}{t_1} \right) = T_m \]  

(8a)

\[ I \left( e^{-t_2/t_1} - e^{-t_2/t_1} \right) = I_{MAX} \]  

(8b)

\[ I \left( e^{-t_2/t_1} - e^{-t_2/t_1} \right) = I_{MAX} \]  

(8c).

The numerical solution of the system (8) for given \( T_h = 8 \, \mu s, T_m = 21.5 \, \mu s \) (if \( T_m = 20 \, \mu s \) the system diverges) and \( I_{MAX} = 2000 \, A \) gives the parameters of shock wave model: \( t_1 = 7.18 \, \mu s, t_2 = 8.95 \, \mu s \) and \( I = 24.7 \, kA \).

We assume that the markers of circular and square shape lay in the distance \( r \) from a straight infinitely long conductor by which the current wave \( i(t) \) with parameters described above flows (Fig. 8). The current \( i(t) \) creates in marker area magnetic flux

\[ \Phi(t) = \int_{S_M} \frac{\mu N_M}{2\pi r} i(t) dS_M = \int r^2 - a \frac{\mu N_M}{2\pi} \frac{i(t)}{2} a \cdot dx = \]  

\[ = \frac{\mu a N_M}{2\pi} \ln \frac{2r + a}{2r - a} i(t) = M(t) \]  

(9),

where the circular marker was replaced by a square marker with equivalent area for simplifying the surface integral calculation,

\[ a = \sqrt{\pi \cdot r_M} \]  

(10)

is length of equivalent square marker side and

\[ M = \frac{\mu a N_M}{2\pi} \ln \frac{2r + a}{2r - a} \]  

(11)

is mutual inductivity of a straight conductor and marker coil.

As the marker is essentially resonant circuit, the time dependence of current \( i_M(t) \) induced into the marker circuit by influence of current wave \( i(t) \) (formula (7)) can be calculated from a differential equation.
The current induced in the marker by current wave was experimentally measured with utilization of a pulse generator generating the current pulse with parameters $T_1 = 8 \mu s$, $T_2 = 20 \mu s$, $I_{MAX} = 2000 A$. The current shape was recorded by a digital oscilloscope as a voltage fall on 0.47 Ω resistor and is in Fig. 10. The maximum value of current recalculated from voltage (360 mV) and resistance (0.47 Ω) is 0.76 A.

5. Conclusion

The mathematical calculations described in this paper were used in marker locator development, which was realized by the Department of Control and Information systems for the Komplex company as the contract no. EF/28/97.

The calculation of voltage induced from an excited marker back to the receiving antenna showed that this voltage is reverse proportionally dependent on the sixth power of distance. This fact limits practically achievable distance from which the marker can be localized to the value of 2.5, maximum 3 m.

The calculation and measuring of voltage and current in the marker induced from current wave flowing by a metal conductor of marked equipment showed that these influences cannot be neglected. For increasing the marker reliability and lifetime (marker lifetime must be greater than 40 years) the marker capacitor must be high voltage and current type. Therefore the capacitors with working voltage of 1500 V are used. Similarly, insulation of the marker coil conductor must be carefully chosen.

At present the development of a new marker type based on RFID (Radio Frequency Identification) technology is in progress. The RFID technology enables to supplement new function of marker – identification of underground device by a numeric code.

References

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