A SEMICLASSICAL APPROACH TO FUSION REACTIONS

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The semiclassical method of Alder and Winther is generalized to study fusion reactions. As an illustration, we evaluate the fusion cross section in a schematic two-channel calculation. The results are shown to be in good agreement with those obtained with a quantal Coupled-Channels calculation. We suggest that in the case of coupling to continuum states this approach may provide a simpler alternative to the Continuum Discretized Coupled-Channels method.

1. Introduction

The importance of Coupled-Channels effects on the fusion cross section has been investigated by several authors. These studies have established that the main effect of the coupling of the entrance channel with other bound channels is to produce a pronounced enhancement of the fusion cross section at sub-barrier energies. A more complicated situation arises when the reaction involves weakly bound nuclei. In such cases, the elastic channel is strongly coupled with the breakup channel, which corresponds to states of three or more bodies in the continuum. The total fusion cross section, $\sigma_F$, is then the sum of different processes: the complete fusion cross section, $\sigma_{CF}$, where all projectile's and target's nucleons merge into a compound system, and incomplete fusion cross sections, $\sigma_{ICF_i}$, where only

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the $i^{th}$ fragment of the projectile fuses with the target while the remaining ones come out of the interaction region.

A recent review of the experimental and theoretical work on the fusion of unstable or weakly bound nuclei can be found in ref.\textsuperscript{2}. The first theoretical studies\textsuperscript{3,4,5} used schematic models, which stress particular aspects of the fusion process. More recently, sophisticated quantum Coupled-Channels calculations have been performed\textsuperscript{6,7}. These calculations approximate the continuum by a discrete set of states, according to the Continuum Discretized Coupled-Channels method (CDCC). Those calculations led to the conclusion that in collisions with very heavy targets the coupling to the continuum has a strong influence on the complete fusion cross section. The progress in the experimental study of these collision is more recent, since only recently unstable beams at barrier energies became available\textsuperscript{8,9,10,11}. Besides, measurements at sub-barrier energies are very hard to perform, owing to the low intensity of the unstable beams. Although some recent measurements of the fusion cross section in collisions of unstable beams from heavy targets show an enhancement at sub-barrier energies\textsuperscript{8,11}, more data are needed for a final conclusion. On the other hand, data on the fusion cross section in reactions induced by light weakly bound stable projectiles have been available for a longer time\textsuperscript{12}.

The importance of the details of the CDCC basis in calculations of the fusion cross section, pointed out in ref.\textsuperscript{7}, indicates that a simple approximation for the breakup channel can only be used for very qualitative estimates, like that of ref.\textsuperscript{13}. A reasonable alternative is the use of the semiclassical method of Alder-Winther (AW)\textsuperscript{14}. This method was originally proposed to study Coulomb excitation of collective states and it was later generalized to other nuclear reactions, including the excitation of the breakup channel\textsuperscript{15}. More recently, it has been used to study the breakup of $^8$B in the $^8$B + $^{58}$Ni collision\textsuperscript{16} for a comparison with the CDCC calculations of Nunes and Thompson\textsuperscript{17}. The discretization of the continuum space was carried out in the same way as in ref.\textsuperscript{17} and the results obtained with the AW approximation were shown to be in good agreement with those of the CDCC method. In the present work, we show how the AW method can be used to evaluate the complete fusion cross section in collisions of weakly bound projectiles and discuss its validity in a schematic two-channel example.

This paper is organized as follows: in section 2 we introduce the Alder-Winther method and show how it can be used to evaluate the complete fusion cross section. An application to a schematic model that mimics the $^6$He + $^{238}$U is made. In section 3 we present the conclusions of this work.
2. The Alder- Winther method.

Let us consider a collision described by the projectile-target separation vector, \( \mathbf{r} \), and the relevant intrinsic degrees of freedom of the projectile, represented by \( \xi \). For simplicity, we neglect the internal structure of the target. The projectile’s Hamiltonian is

\[
h = h_0(\xi) + V(\mathbf{r}, \xi),
\]

where \( h_0(\xi) \) is the intrinsic Hamiltonian and \( V(\mathbf{r}, \xi) \) represents the projectile-target interaction. The eigenvectors of \( h_0(\xi) \) are given by

\[
h_0 | \phi_\alpha \rangle = \varepsilon_\alpha | \phi_\alpha \rangle.
\]

The Alder- Winther method is implemented as follows. First, one uses classical mechanics for the variable \( \mathbf{r} \). In its original version, a Rutherford trajectory \( \mathbf{r}_l(t) \) was used. The trajectory depends on the collision energy, \( E \), and the angular momentum, \( l \). In our case, we use the solution of the classical equation of motion with the potential \( V(\mathbf{r}) = \langle \phi_0 | V(\mathbf{r}, \xi) | \phi_0 \rangle \), where \( | \phi_0 \rangle \) is the ground state of the projectile. Using the trajectory, the coupling interaction becomes a time-dependent interaction in the \( \xi \)-space. That is, \( V(\xi, t) \equiv V(\mathbf{r}_l(t), \xi) \). Then the dynamics in the intrinsic space is treated as a time-dependent quantum mechanics problem, according to the Schrödinger equation

\[
h \psi(\xi, t) = [h_0(\xi) + V(\xi, t)] \psi(\xi, t) = i\hbar \frac{\partial \psi(\xi, t)}{\partial t}.
\]

Expanding the wave function in the basis of intrinsic eigenstates,

\[
\psi(\xi, t) = \sum_\alpha a_\alpha(l, t) \phi_\alpha(\xi) e^{-i\varepsilon_\alpha t/\hbar},
\]

and inserting the expansion in eq.(3), one obtains the Alder- Winther equations

\[
i\hbar \dot{a}_\alpha(l, t) = \sum_\beta \langle \phi_\alpha | V(\xi, t) | \phi_\beta \rangle e^{i(\varepsilon_\alpha - \varepsilon_\beta)t/\hbar} a_\beta(l, t).
\]

These equations should be solved with initial conditions \( a_\alpha(l, t \to -\infty) = \delta_{\alpha 0} \), which means that before the collision \( t \to -\infty \) the projectile was in its ground state. The final population of channel \( \alpha \) in a collision with angular momentum \( l \) is \( P_l(\alpha) = |a_\alpha(l, t \to +\infty)|^2 \) and the cross section is

\[
\sigma_\alpha = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(\alpha).
\]

A similar procedure can be used to derive angular distributions.
The AW method can be extended to evaluate the fusion cross section as follows. The starting point is the general expression for the fusion cross section in multi-channel scattering

$$\sigma_F = \sum_{\alpha} \sigma_F^{(\alpha)}; \quad \sigma_F^{(\alpha)} = \frac{\pi}{k^2} \sum_l (2l + 1) P_l^{F}(\alpha),$$

(7)

with the fusion probability for the $l^{th}$-partial-wave in channel $\alpha$ given by

$$P_l^{F}(\alpha) = \frac{4k}{E} \int dr \ |u_l(k,r)|^2 W_F^F(r).$$

(8)

Above, $u_l(k,r)$ represents the radial wave function for the $l^{th}$-partial-wave in channel $\alpha$ and $W_F^F$ is the absolute value of the imaginary part of the optical potential in this channel arising from fusion. To use the AW method to evaluate the fusion cross section, we make the approximation

$$P_l^{F}(\alpha) \simeq T_l \ |a_{\alpha}(l,t_{ca})|^2.$$  

(9)

In the above equation, $T_l$ is the tunneling probability and $|a_{\alpha}(l,t_{ca})|^2$ is the probability that the projectile is in the state $|\phi_{\alpha}\rangle$ when the system reaches closest approach.

We have performed a preliminary calculation for a two-channel case, studying the scattering of $^6$He projectiles on a $^{238}$U target, at near barrier energies. The weakly bound $^6$He nucleus dissociates into $^4$He and two neutrons, with threshold energy $B = 0.975$ MeV. The elastic channel is strongly coupled to the breakup channel and this coupling has a strong influence on the fusion cross sections. Here we represent the breakup channel by a single effective channel and evaluate the complete fusion cross section using the semiclassical method mentioned above. In this approximation, the complete fusion cross section corresponds to the contribution from the elastic channel to eq.(7). For simplicity, we neglect the excitation energy and the spin of the effective channel. We adopt a form factor with the radial dependence of the electric dipole coupling and the strength is chosen arbitrarily, in such a way that the coupling modifies the cross section predicted by the one dimension penetration barrier appreciably. In figure 1, we compare results obtained with the AW method with those of a coupled channel calculation and also with those obtained with the neglect of channel coupling. We adopt Wood-Saxon shapes for the real and imaginary potentials, with the parameters: $V_0 = -50 \text{ MeV}, r_{0r} = 1.25 \text{ fm}, a_r = 0.65 \text{ fm}, W_0 = -50 \text{ MeV}, r_{0i} = 1.0 \text{ fm}, a_i = 0.65 \text{ fm}$. We conclude that the semiclassical results are very close to those of a full coupled-channel calculation.
It should be remarked, however, that this good agreement does not occur at sub-barrier energies. In this energy range the classical trajectory does not reach the barrier radius and therefore the effective barrier lowering that enhances the cross section is not accounted for.

3. Conclusions

The semiclassical Coupled-Channels theory of fusion reactions presented here is a natural extension of what has been done for other reaction channels. As it has been shown in a previous study of the breakup cross section\(^{16}\), it allows a realistic description of the breakup channel, including continuum-continuum coupling. Although the calculation presented was restricted to a schematic model two-channel model, an extension to a large set of continuum states along the lines of ref.\(^{16}\) should present no major difficulties.

It should be pointed out that the present model can be extended to calculate the fusion of the fragment that contains all or most of the charge of...
the projectile. In a way, it could be considered an improved semi-quantal version of the classical three-body model of Hinde et al.\textsuperscript{12}. Work is in progress to accommodate both complete and incomplete fusion in the theory and thus supplying a simplified albeit accurate version of the CDCC.

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