\( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) Violation and New Thermalization Scenario in Heavy Ion Collisions

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Abstract: The violation of local \( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) invariance in QCD has been a subject of intense discussions for the last couple of years as a result of very interesting ongoing results coming from RHIC. Separately, a new thermalization scenario for heavy ion collisions through the event horizon as a manifestation of the Unruh effect, has been also suggested. In this paper we argue that these two, naively unrelated phenomena, are actually two sides of the same coin as they are deeply rooted into the same fundamental physics related to some very nontrivial topological features of QCD. We formulate the universality conjecture for \( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) odd effects in heavy ion collisions analogous to the universal thermal behaviour observed in all other high energy interactions.
1. Introduction. Motivation

The main goal of this paper is to argue that two, naively unrelated, phenomena:
1. local $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD as studied at RHIC[1, 2, 3, 4, 5]; and
2. universal behaviour of multihadron production described by a universal hadronization temperature $T_H \sim (150 - 200)\text{ MeV}$

are in fact tightly related, as they describe different sides of the same fundamental physics. Before we present our arguments suggesting the common nature of these two phenomena, we review each effect separately as it is conventionally treated today. Our next step is to take a fresh look at these effects and present some arguments suggesting that both these phenomena are in fact originated from the same fundamental physics and both are related to the very deep and nontrivial topological features of QCD.
1.1 Local $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD. Charge separation effect

The charge separation effect [6, 7] can be explained in the following simple way. Let us assume that an effective $\theta(x,t)_{\text{ind}} \neq 0$ is induced as a result of some non-equilibrium dynamics as suggested in refs. [8, 9, 10, 11, 12, 13]. The $\theta(x,t)_{\text{ind}}$ parameter enters the effective lagrangian as follows, $L_\theta = -\theta q$ where $q \equiv \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma}$ such that local $\mathcal{P}$ and $\mathcal{CP}$ invariance of QCD is broken on the scales where correlated $\theta(x,t)_{\text{ind}} \neq 0$ is induced. As a result of this violation, one should expect a number of $\mathcal{P}$ and $\mathcal{CP}$ violating effects taking place in the region where $\theta(x,t)_{\text{ind}} \neq 0$. In particular, one should expect the separation of electric charge along the axis of magnetic field $\mathbf{B}$ or along the angular momentum $\mathbf{l}$ if they are present in the region with $\theta(x,t)_{\text{ind}} \neq 0$.

This area of research became a very active field in recent years mainly due to very interesting ongoing experiments [1, 2, 3, 4, 5]. There is a number of different manifestations of this local $\mathcal{P}$ and $\mathcal{CP}$ violation, see [14, 15, 16, 17, 18, 19, 20, 21, 22] and many additional references therein. In particular, in the presence of an external magnetic field $\mathbf{B}$ or in case of the rotating system with angular velocity $\Omega$ there will be induced electric current directed along $\mathbf{B}$ or $\Omega$ correspondingly, resulting in separation of charges along those directions as mentioned above. One can interpret the same effects as a generation of induced electric field $\mathbf{E}$ directed along $\mathbf{B}$ or $\Omega$ resulting in corresponding electric current flowing along $\mathbf{J} \sim \mathbf{B}$ or $\mathbf{J} \sim \Omega$ directions. All these phenomena are obviously $\mathcal{P}$ and $\mathcal{CP}$ odd effects. Non-dissipating, induced vector current density has the form:

$$\mathbf{J} = (\mu_L - \mu_R) \frac{e\mathbf{B}}{2\pi^2},$$

where $\mathcal{P}$ odd effect is explicitly present in this expression as the difference of chemical potentials of the right $\mu_R$ and left $\mu_L$ handed fermions is assumed to be nonzero, $(\mu_L - \mu_R) \neq 0$. The combination $(\mu_L - \mu_R)$ can be thought as $\dot{\theta}(t)$ after a corresponding $U(1)_A$ chiral time-dependent rotation is performed, see also [17] for a physical interpretation of the relation $(\mu_L - \mu_R) = \dot{\theta}$.

Originally, formula (1.1) has been derived in [23], though in condensed matter context. In QCD context formula (1.1) has been used in applications to neutron star physics where magnetic field is known to be large, and the corresponding $(\mu_L - \mu_R) \neq 0$ can be generated in neutron star environment as a result of continuous $\mathcal{P}$ violating processes happening in nuclear matter[24, 25]. It has been also applied to heavy ion collisions where an effective $(\mu_L - \mu_R) \neq 0$ is locally induced. The effect was estimated using the sphaleron transitions generating the topological charge density in the QCD plasma [14, 15]. The effect was coined as “chiral magnetic effect” (CME) [14, 15]. Formula (1.1) has been also derived a numerous number of times using numerous variety of techniques such as: effective lagrangian approach developed in [26]; explicit computation approach developed in [27]; direct lattice computations [18, 19]. In addition, the effect has been studied in holographic models of QCD [28, 29, 30]. While there is a number of subtitles in holographic description of the effect [29, 30], it is fair to say: on the theoretical side the effect is a well established phenomenon. It remains to be seen if this phenomenon is related in anyway to what has been actually experimentally observed [1, 2, 3, 4, 5].

1.1 Local $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD. Charge separation effect

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This area of research became a very active field in recent years mainly due to very interesting ongoing experiments [1, 2, 3, 4, 5]. There is a number of different manifestations of this local $\mathcal{P}$ and $\mathcal{CP}$ violation, see [14, 15, 16, 17, 18, 19, 20, 21, 22] and many additional references therein. In particular, in the presence of an external magnetic field $\mathbf{B}$ or in case of the rotating system with angular velocity $\Omega$ there will be induced electric current directed along $\mathbf{B}$ or $\Omega$ correspondingly, resulting in separation of charges along those directions as mentioned above. One can interpret the same effects as a generation of induced electric field $\mathbf{E}$ directed along $\mathbf{B}$ or $\Omega$ resulting in corresponding electric current flowing along $\mathbf{J} \sim \mathbf{B}$ or $\mathbf{J} \sim \Omega$ directions. All these phenomena are obviously $\mathcal{P}$ and $\mathcal{CP}$ odd effects. Non-dissipating, induced vector current density has the form:

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1.2 Universal hadronization temperature $T_H \sim (150 - 200) \text{ MeV}$

Naively unrelated story goes as follows. We start from the following general observation: over the years, hadron production studies in a variety of high energy collision experiments have shown a remarkably universal feature, indicating a universal hadronization temperature $T_H \sim (150 - 200) \text{ MeV}$. From $e^+e^-$ annihilation to $pp$ and $p\bar{p}$ interactions and further to collisions of heavy nuclei, with energies from a few GeV up to the TeV range, the production pattern always shows striking thermal aspects, connected to an apparently quite universal temperature around $T_H \sim (150 - 200) \text{ MeV}$ [31]. While experimentally it is well established phenomenon, it is very difficult to understand its nature as number of incident particles in $e^+e^-$ annihilation as well as in $pp$ and $p\bar{p}$ interactions is not sufficient even to talk about statistical averages. This observation motivated a number of early attempts [32] to interpret the resulting spectrum of particles as the Unruh radiation [33, 34, 35] when the event horizon emerges as a result of strong interactions. The modern, QCD based formulation of this idea has been developed quite recently in refs.[36, 37, 38, 39, 40, 41, 42, 43]. See also alternative approaches [46, 47, 48] leading to the same “apparently thermal” aspects of the produced particles.

The key ingredient of the approach suggested in refs. [36, 37, 38, 39, 40, 41, 42, 43] is as follows: an observer moving with an acceleration $a$ experiences the influence of a thermal bath with an effective temperature

$T = \frac{a}{2\pi}$ \hspace{1cm} (1.2)

which is conventional Unruh effect [33]. The corresponding acceleration parameter $a$ in QCD is determined by the so-called “saturation scale” $Q_s$ [44, 45] as suggested in refs. [36, 37, 38, 39], or by the string tension $\sigma$ as advocated in refs. [40, 41, 42, 43],

$a \simeq Q_s \text{ or } a \simeq \sqrt{2\pi\sigma}$. \hspace{1cm} (1.3)

The problem of calculating of the effective acceleration “$a$” is obviously very hard problem of strongly interacting QCD. This problem of computation “$a$” is not addressed in the present paper. We simply assume that such a description exists, in which case the relation (1.3) explains the puzzle on why the temperature $T$ given by eq. (1.2) is so universal, as it it almost independent on type of processes and the energy of colliding particles, as it is entirely determined by the fundamental $\Lambda_{QCD}$ scale. This “apparent thermalization” originates from the event horizon in an accelerating frame: the incident hadron decelerates in an external colour field, which causes the emergence of the causal horizon. Quantum tunnelling through this event horizon then produces a thermal final state of partons, in complete analogy with the thermal character of quantum Unruh radiation [33, 34, 35]. One should emphasize that the Planck spectrum in this approach is not resulted from the kinetics when the thermal equilibrium with temperature $T$ given by eq. (1.2) is reached due to the large number of collisions. Rather, the Planck spectrum in high energy collisions is resulted from the stochastic tunnelling processes when no information transfer occurs. In such circumstances the spectrum must be thermal. Such interpretation would naturally explain another puzzle with the thermal spectrum in $e^+e^-$, $pp$ and $p\bar{p}$ high energy collisions when the statistical thermalization could never been reached in those systems.
We adopt this viewpoint, and we have nothing new to add to the computations presented in refs. [36, 37, 38, 39, 40, 41, 42, 43] in strongly coupled QCD. However, we interpret that the Planck spectrum observed in high energy collisions somewhat differently in comparison with papers mentioned above. Namely, we interpret the observed spectrum as the result of complete reconstruction of the QCD vacuum state in accelerating frame “a”, rather than due to any specific properties of its excitations – the partons. Our interpretation does not change any previous results within this framework. However, the new interpretation will play a crucial role when we apply the same logic and the same technique for discussions of local $\mathcal{P}$ and $\mathcal{C}\mathcal{P}$ violation in QCD, which is the main subject of the present paper.

• Our original contribution is the study of some specific topological fluctuations, which we believe are responsible for local $\mathcal{P}$ and $\mathcal{C}\mathcal{P}$ violating processes observed at RHIC. Those vacuum topological fluctuations will be changed along with many other vacuum fluctuations in accelerating frame “a”. These changes of the ground state due to the acceleration as we shall see are describable in terms of the Veneziano ghost which solves the $U(1)_A$ problem in QCD [49, 50, 51, 52]. This key degree of freedom (the Veneziano ghost) has $\eta’−$ quantum numbers, and plays the role similar to $\theta$ parameter from section 1.1. Its $0^{-+}$ quantum numbers, as we shall see, play the crucial role in linking two naively unrelated problems outlined in two sections above: local $\mathcal{P}$ and $\mathcal{C}\mathcal{P}$ violation in QCD, section 1.1, and universal hadronization temperature, section 1.2.

The paper is organized as follows. In section 2 we discuss the nature of universality of hadronization temperature observed in numerous high energy experiments. We adopt the basic logic and philosophy of refs.[36, 37, 38, 39, 40, 41, 42, 43]. We shall explicitly demonstrate the observed thermal spectrum with temperature (1.2) can be interpreted as the direct consequence of the basic features of known Bogolubov’s coefficients in the accelerating frame for a system moving with effective acceleration “a”. The temperature will be universal for all types of produced particles: massive or massless, charged or neutral, scalars, spinors, or vectors. Such a universality is similar to universal features of the Unruh radiation [33, 34, 35]. This result will be our basic explanation for the thermal spectrum observed in $e^+e^−$, $pp$ and $p\bar{p}$ high energy collisions when the statistical thermalization (due to the conventional collisions) can not be ever reached in those systems.

In section 3 we review the resolution of the $U(1)_A$ problem and structure of the $\theta$ vacua [49, 50, 51, 52] by constructing the corresponding effective lagrangian. We pay special attention to the structure of the Veneziano ghost, its contribution to the topological susceptibility with a “wrong sign” (which is a key element in resolution of the $U(1)_A$ problem), the unitarity, anomalous Ward Identities and other important properties of QCD. We formulate the physics of topological fluctuations (described by the Veneziano ghost in our framework) in such a way that the relevant technique can be easily generalized for the case of accelerating frame.

Section 4 is devoted to analysis of the Veneziano ghost in the accelerating frame. We shall explicitly compute the Bogolubov’s coefficients to demonstrate that the Veneziano ghost contribution to energy (being identically zero in Minkowski space) does not vanish

\footnote{not to be confused with conventional Fadeev Popov ghosts which appear in covariant quantization of non-abelian gauge theories}
anymore in accelerating frame, in the Rindler space. Therefore we identify the local $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD as a result of fluctuations of the vacuum $0^{-+}$ ghost field in the accelerating frame. We conclude with section 5 where we list the possible tests discriminating this framework from the previously suggested mechanisms. The readers interested in applications only may skip sections 2, 3, 4 and immediately jump to section 5.

2. Universal hadronization temperature as the Unruh effect

As we mentioned above, we adopt the basic logic and philosophy of refs.\[36, 37, 38, 39, 40, 41, 42, 43]. Essentially, the previously developed picture can be explained in a simplified way as follows: the incident parton decelerates in an external colour field. The causal horizon emerges as a result of this strong interaction. Quantum tunnelling through the emergent event horizon then produces a thermal final state of partons. The hard part of the problem, the computation of the acceleration “$a$” is not addressed in the present work. The acceleration must be an universal number (which we assume to be the case), not sensitive to a colour representation of the incident particles. Once the universal acceleration is reached, the produced particles will automatically have a thermal spectrum.

Our interpretation of this physics is somewhat different: instead of tunnelling of real particles we rather speak about changes of the ground state. Initially, the ground state was a pure quantum state in Minkowski space. In accelerating frame the Rindler observers (who do not ever have access to the entire space-time as a result of emerging horizon) would see the same ground state as a mixed state (rather than pure state) filled by particles with Planck spectrum. This new interpretation will be quite important when we apply the same technique in section 4 for discussions of the topological vacuum fluctuations in accelerating frame because there will be no real asymptotic states which would correspond to those topological vacuum fluctuations. Indeed, as we argue below, the Veneziano ghost does not contribute to absorptive parts of any correlation functions, but only to the real parts. According to our logic these topological vacuum fluctuations in accelerating frame will serve as a $\mathcal{P}$ and $\mathcal{CP}$ odd background for physical fluctuations of quarks and gluons. As we argue below, these topological vacuum fluctuations will be eventually responsible for the $\mathcal{P}$ and $\mathcal{CP}$ violating correlations observed at RHIC.

As we mentioned above, the computation of the acceleration parameter “$a$” is a hard problem of strongly interacting gauge theory which is not addressed in the present work. Instead, we adopt the entire framework and treat the acceleration “$a$” as a free parameter of the theory. This parameter, in principle, could have had any value. In nature “$a$” is not really a free parameter, but expressed in terms of $\Lambda_{QCD}$ as eq. (1.3) states. However, to simplify things, we will be working in the limit $1 \text{ GeV} \gg a \gg m_q$ where our arguments can be made precise, though numerically $a \simeq 1 \text{ GeV}$ as estimate (1.3) shows. Also: we have nothing new to add to the previous arguments [36, 37, 38, 39, 40, 41, 42, 43] suggesting that the acceleration (1.3) and the corresponding temperature (1.2) are universal for different processes. Rather we take a given accelerating parameter assuming $a \ll 1 \text{ GeV}$ and study the changes in the ground state (in comparison with Minkowski vacuum state) which occur due to the acceleration. In our analysis in what follows we consider the radiation of a single
massless scalar field to demonstrate the most important features of the accelerating frame; generalization to vector and spinor fields in the Rindler space is also known, but shall not be discussed in the present work.

2.1 Rindler space

We follow notations [35] in our analysis and separate the space time into four quadrants $F$ (future), $P$ (past), $L$ (left wedge) and $R$ (right wedge). We will choose the origin such that these regions are defined by $t > |x|$, $t < -|x|$, $x < -|t|$ and $x > |t|$ respectively. While no single region contains a Cauchy surface, the union of the left and right regions $L$ and $R$ plus the origin does contain many Cauchy surfaces, for example $t = 0$. We will write the Minkowski metric with the sign convention

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2,$$

In the quadrant $R$, called the right Rindler wedge, one may define the coordinates $(\xi^R, \eta^R, y, z)$ via the transformations

$$t = \frac{e^{a\xi^R}}{a} \sinh a \eta^R, \quad x = \frac{e^{a\xi^R}}{a} \cosh a \eta^R$$

where $a$ is a dimensional constant. We may define coordinates $(\xi^L, \eta^L, y, z)$ in the left Rindler wedge $L$ in a similar way with the signs of both $t$ and $x$ reversed [35]. In these new coordinates the metric is presented as

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2) - dy^2 - dz^2.$$  

Without losing any generalities we ignore in what follows a trivial dependence on $y, z$ coordinates. It is important to emphasize that the coordinates $(\eta^R, \xi^R)$ cover only a quadrant of Minkowski space, namely the wedge $x > |t|$. Lines of constant $\xi$ are hyperbolae

$$x^2 - t^2 = a^{-2} e^{2a\xi} = \text{world lines.}$$

They represent the world lines of uniformly accelerated observers with proper acceleration given by

$$ae^{-a\xi} = \text{proper acceleration.}$$

Thus, lines of large positive $\xi$ (far from $x = t = 0$) represent weakly accelerated observers, while large negative $\xi$ correspond a high proper acceleration. The observer’s proper time $\tau$ is

$$\tau = \eta e^{a\xi} = \text{observer’s proper time.}$$

Uniformly accelerated observers will be referred to as Rindler observers. It is important to emphasize that $L$ and $R$ regions are separated by the event horizons such that no events in $L$ can be witnessed in $R$ and vice versa. In different words, regions $L$ and $R$ represent two causally disjoint universes. Left wedge $L$ with $x < |t|$ is obtained by changing the signs in eq. (2.2). The sign reversals in $L$ mean that increasing $t$ corresponds to decreasing $\eta$ which implies that time-like Killing vector being $+\partial_\eta$ in $R$ becomes $-\partial_\eta$ in $L$ in contrast with
Minkowski space where $\partial_t$ is time-like Killing vector in entire space. This feature plays an important role in selection of positive frequency modes in $L$ and $R$ wedges as discussed below.

In terms of these variables the picture of high energy collision (in very simplified way) can be represented as follows (detail derivations and explanations are presented below in next subsection). Two energetic particles approach the interaction region ($x = t = 0$) along the light cone from $x = t = -\infty$ and $x = -t = \infty$. When the colliding particles have non-vanishing space-like transverse momenta $k^2 = -k^2_\perp$ their world lines are located off the light cone as described in [36]. Due to the interaction the initial particles experience acceleration (2.5) which we assume to be a constant “a” to simplify our computations. In this case one can switch to the Rindler frame where the question is formulated as follows: how is the initial ground state expressed as a Fock state in the Rindler frame? As we shall see below it will not be a ground state any more in the accelerating frame. Rather it will be a superposition of excited states which include both: the $L$ and $R$ components separated by the horizon. We shall see that the corresponding excitations have the thermal spectrum as the direct consequence of the Bogolubov’s coefficients’ properties. It is a different way to explain the same physics which was described in refs.[36, 37, 38, 39, 40, 41, 42, 43] as a tunnelling through the event horizon.

### 2.2 Quantum Fields in Rindler space

As we mentioned above, the dynamics along $y, z$ directions is trivial, and we ignore it in what follows to simplify the notations. The wave equation of a free massless field $\phi(t, \vec{x})$ possesses standard orthonormal mode solutions

$$u_k = \frac{1}{\sqrt{4\pi \omega}} e^{-i\omega t + i k x}.$$  \hspace{1cm} (2.7)

such that we can expand it in terms of complete orthonormal basis $u_k(t, \vec{x})$

$$\phi(t, \vec{x}) = \sum_k \left[ b_k u_k(t, \vec{x}) + b_k^\dagger u_k^*(t, \vec{x}) \right].$$  \hspace{1cm} (2.8)

The commutation relations take the form,

$$[b_k, b_{k'}] = 0, \ [b^\dagger_k, b^\dagger_{k'}] = 0, \ [b_k, b^\dagger_{k'}] = \delta_{kk'},$$  \hspace{1cm} (2.9)

while the ground state in Minkowski space $|0\rangle_M$ is defined as usual

$$b_k |0_M\rangle = 0, \ \forall k.$$  \hspace{1cm} (2.10)

The number operator $N$ and the Hamiltonian $H$ for $\phi$ field in these notations have the standard form

$$N = \sum_k b_k^\dagger b_k, \ \ H = \sum_k \omega_k b_k^\dagger b_k,$$  \hspace{1cm} (2.11)
where normal ordering is implied (but not explicitly shown) in all formulae presented below, such that the ground state satisfies the standard conditions

\[ \langle 0_M|H|0_M \rangle = 0, \quad \langle 0_M|N|0_M \rangle = 0. \quad (2.12) \]

We want to describe the ground state defined by eq. (2.10) in terms of the Rindler coordinates. For the metric (2.3) the corresponding modes are:

\[ R_u^k = \frac{1}{\sqrt{4\pi\omega}} e^{i k \xi^R - i \omega \eta^R} \quad \text{in } R, \quad R_u^k = 0 \quad \text{in } L \quad (2.13) \]

\[ L_u^k = \frac{1}{\sqrt{4\pi\omega}} e^{i k \xi^L + i \omega \eta^L} \quad \text{in } L, \quad L_u^k = 0 \quad \text{in } R \quad (2.14) \]

The set (2.13) is complete in region R, while (2.14) is complete in L, but neither is complete in on all of Minkowski space. However, both sets together are complete. The sign difference corresponds to the fact that a right moving wave in R moves towards increasing value of \( \xi \), while in L it moves toward decreasing value of \( \xi \). In any case, these modes are positive frequency modes with respect to the time-like Killing vector \( +\partial_\eta \) in R and \( -\partial_\eta \) in L. The fact that (2.13) and (2.14) have the same functional form as (2.7) is a consequence of the conformal triviality of the system. Thus the Rindler modes (2.13) and (2.14) represent a good basis for quantizing the \( \phi \) field, as good as the Minkowski basis (2.7). Therefore, one can use modes (2.13) and (2.14) to expand the field \( \phi \)

\[ \phi = \sum_k \frac{1}{\sqrt{4\pi\omega}} (b_k^L e^{i k \xi^L + i \omega \eta^L} + b_k^L e^{-i k \xi^L - i \omega \eta^L} + b_k^R e^{i k \xi^R - i \omega \eta^R} + b_k^R e^{-i k \xi^R + i \omega \eta^R}). \quad (2.15) \]

where \( b_k^L, b_k^R \) satisfy the following commutation relations,

\[ [b_k^R, b_k^R^{\dagger}] = 0, \quad [b_k^R, b_k^{\dagger}] = 0, \quad [b_k^R, b_k^R] = \delta_{kk'}, \quad (2.16) \]

and similar for \( L-\) Rindler wedge operators \( b_k^L \). The Rindler vacuum state is defined in terms of these operators as follows,

\[ b_k^R|0_R \rangle = 0, \quad \forall k. \quad (2.17) \]

We need to compute the corresponding Bogolubov’s coefficients which relate two alternative expansions (2.8) and (2.15) in order to answer the question: how is the initial ground state expressed as a Fock state in the Rindler frame? The simplest way to compute these coefficients is to note that although \( R_u^k \) and \( L_u^k \) are not analytic, the two combinations

\[ \exp \left( \frac{\pi \omega}{2a} \right) R_u^k + \exp \left( -\frac{\pi \omega}{2a} \right) L_u^{*-k} \]

\[ \exp \left( -\frac{\pi \omega}{2a} \right) R_u^{*-k} + \exp \left( \frac{\pi \omega}{2a} \right) L_u^k \]

are analytic and bounded[33]. These modes share the positivity frequency analyticity properties of the Minkowski modes (2.7), therefore, they must also share a common vacuum
The Minkowski vacuum state is determined in terms of these operators as 
\[
\eta | \rangle_{\text{ob}} \quad \text{(2.8)}
\]
Obviously that the ground state in $R$-wedge, see below precise definition. Therefore, instead of expansion (2.8) with modes (2.7) we can expand $\phi$ in terms of (2.18) as
\[
\phi = \sum_{k} \frac{1}{\sqrt{4\pi \omega}} \frac{1}{\sqrt{(e^{\pi \omega/a} - e^{-\pi \omega/a})}} \left[ b_k^1 (e^{\frac{2\pi}{2\pi} + ik\xi R - i\omega \eta R} + e^{-\frac{2\pi}{2\pi} + ik\xi L - i\omega \eta L}) 
\right.
\]
\[
+ b_k^2 (e^{\frac{2\pi}{2\pi} + ik\xi R - i\omega \eta R} + e^{-\frac{2\pi}{2\pi} + ik\xi L + i\omega \eta L})
\]
\[
\left. + b_k^{11} (e^{\frac{2\pi}{2\pi} - ik\xi R + i\omega \eta R} + e^{-\frac{2\pi}{2\pi} - ik\xi L + i\omega \eta L})
\right]
\]
\[
\left. + b_k^{21} (e^{\frac{2\pi}{2\pi} - ik\xi L - i\omega \eta L} + e^{-\frac{2\pi}{2\pi} - ik\xi R - i\omega \eta R})\right],
\tag{2.19}
\]
where $b_k^1, b_k^2$ satisfy the following commutation relations,
\[
\left[ b_k^{(1,2)}, b_{k'}^{(1,2)} \right] = 0, \quad \left[ b_k^{(1,2)}, b_{k'}^{(1,2)} \right] = 0, \quad \left[ b_k^{(1,2)}, b_{k'}^{(1,2)} \right] = \delta_{kk'}.
\tag{2.20}
\]
The Minkowski vacuum state is determined in terms of these operators as
\[
b_k^1 |0_M\rangle = 0, \quad b_k^2 |0_M\rangle = 0, \quad \forall k.
\tag{2.21}
\]
This equation replaces eq. (2.10) as it defines the same ground state $|0_M\rangle$ because both sets share a common vacuum state as explained above. Matching coefficients in (2.15) with (2.19) one finds the Bogoliubov’s coefficients [33, 35],
\[
b_k^1 = \frac{e^{-\pi \omega/2a} b_k^{11} + e^{\pi \omega/2a} b_k^{21}}{\sqrt{e^{\pi \omega/a} - e^{-\pi \omega/a}}} \quad b_k^R = \frac{e^{-\pi \omega/2a} b_k^{21} + e^{\pi \omega/2a} b_k^{11}}{\sqrt{e^{\pi \omega/a} - e^{-\pi \omega/a}}},
\tag{2.22}
\]
Now consider an accelerating Rindler observer at $\xi = \text{const}$. As we mentioned above, such an observer’s proper time is proportional to $\eta$, see eq. (2.6). The vacuum for this observer is determined by (2.17) as this is the state associated with the positive frequency modes with respect to $\eta$. A Rindler observer in $R$ will measure the energy using the Hamiltonian $H^R$ and the number operator $N^R$ which are given by
\[
N^R = \sum_k b_k R^\dagger b_k^R, \quad H^R = \sum_k \omega_k b_k R^\dagger b_k^R.
\tag{2.23}
\]
Similar expressions are also valid for a $L$-Rindler observer. The Hamiltonian $H^R$ and the number operator $N^R$ in the $R$-Rindler accelerating frame have the same form as for conventional Minkowski expressions (2.11). However, they are expressed in terms of the different operators which select a different ground state $|0_R\rangle$ as defined by eq. (2.17). It is obvious that the ground state in $R$-wedge, $|0_R\rangle$ satisfies the standard conditions,
\[
\langle 0_R | H^R | 0_R \rangle = 0, \quad \langle 0_R | N^R | 0_R \rangle = 0,
\tag{2.24}
\]
as it should.

However, if the initial system is prepared as the Minkowski vacuum state $|0_M\rangle$ defined by (2.21) (or what is the same (2.10)) a Rindler observer using the same expression for the number operator (2.23) will observe the following number of particles in mode $k$,
\[
\langle 0_M | N^R | 0_M \rangle = \langle 0_M | b_k^R b_k^R | 0_M \rangle = \frac{e^{-\pi \omega/a}}{(e^{\pi \omega/a} - e^{-\pi \omega/a})} = \frac{1}{(e^{2\pi \omega/a} - 1)},
\tag{2.25}
\]

where we used the Bogolubov’s coefficients (2.22) to express $b_k^R$ in terms of $b_k^{(1,2)}$.

This is the central formula of this section and represents nothing but the conventional Unruh effect [33, 35]. In context of high energy collisions the Planck spectrum given by eq. (2.25) was described in refs. [36, 37, 38, 39, 40, 41, 42, 43] as a tunnelling through the event horizon. In our description the same physical effect is resulted from restriction of Minkowski vacuum $|0_M\rangle$ to a single Rindler wedge region where it becomes a thermal state (rather than a pure quantum state) with temperature $T = \frac{a}{4\pi}$. This structure is precisely resulted from expression of the Minkowski ground state $|0_M\rangle$ in terms of the excited states in $L$ and $R$ regions when the combination $b_k^{R}b_{-k}^{L}$ which includes the operators from causally disconnected regions $L$ and $R$, enters formula for $|0_M\rangle$ as can be seen from explicit construction:

$$
|0_M\rangle = \prod_k \frac{1}{\sqrt{1 - e^{-2\pi\omega/a}}} \exp \left[ e^{-\pi\omega/a} b_k^{R}b_{-k}^{L} \right] |0^R\rangle \otimes |0^L\rangle ,
$$

(2.26)

where we take into account that the operators in the $L, R$ basis correspond to the decompositions with support in only one wedge such that the right hand side is represented by the tensor product $|0^R\rangle \otimes |0^L\rangle$. The crucial point in this relation is the fact that the operators from different causally disconnected regions $L$ and $R$ enter the same expression (2.26), and therefore, there is a correlation between causally disconnected regions $L$ and $R$. However, as discussed in ref. [33], one can not use these correlations to send signals.

In context of high energy collisions the picture advocated in refs. [36, 37, 38, 39, 40, 41, 42, 43] when two causally disconnected regions are connected as a result of the tunnelling through the event horizon, manifests itself in eq. (2.26) by emergence of the combination $b_k^{R}b_{-k}^{L}$ when $L$ and $R$ components are simultaneously present in eq. (2.26). The expression (2.26) also shows that the Planck spectrum observed in high energy collisions can be interpreted as a result of preparation of the ground state $|0_M\rangle$ even before collision develops. This interpretation naturally explains the puzzle with rapid “thermalization” observed in all high energy collisions. Many other observed consequences (such as dependence on transverse momenta $k_\perp^2$, saturation scale at low energies and/or peripheral AA collisions, dependence on strange mass quark, and many others) also find their simple explanations in this framework. They have been discussed in those references in great details, and we have nothing new to add to those discussions as the basic logic of our approach and the one advocated in refs. [36, 37, 38, 39, 40, 41, 42, 43] is the same as both pictures naturally lead to the Planck spectrum (2.25).

Still, we want to add one more comment which my shed some light on mysterious “apparent thermalization” effect resulted from the acceleration. The “apparent thermalization” can be understood as entanglement type behaviour when the Planck spectrum emerges as a result of the description in terms of the density matrix in $R$ region by “tracing out” over the degrees of freedom associated with inaccessible states in $L$-region. In different words, the Bogolubov transformations (2.22) describe a construction when a total system is divided into two subsystems with the horizon separating them. This is the deep physics reason why the Planck spectrum (2.25) emerges for a subsystem. It is known that a number of nontrivial physics effects (including the entanglement) are described
by the common surface separating such two sub-systems, i.e. by the horizon separating $L$ and $R$ Rindler wedges. The particle production in the framework advocated in refs. [36, 37, 38, 39, 40, 41, 42, 43] occur exactly from this horizon region, while in our framework it can be interpreted as a result of entanglement. One should also note that typical quark and gluon vacuum fluctuations develop in this accelerating environment characterized by temperature (1.2), not in laboratory frame, and not in center of mass frame. Once acceleration ceases to exist, a detector in laboratory frame will measure a particle distribution according to the temperature (1.2) as the acceleration ends almost instantaneously, and produced particles do not have time to make any adjustments to a new environment. In this respect, it is very similar to measurements of CMB (cosmic micro wave background) radiation as the CMB photons being produced at the last scattering at temperature $T \simeq 2.7$ K nevertheless do not change their properties during the next $\sim 14$ billion years.

To conclude this section: we have not derived anything new which was not previously known. However, we presented the “new thermalization” scenario advocated in refs. [36, 37, 38, 39, 40, 41, 42, 43] as a result of reconstruction of the vacuum state in accelerating system. This new interpretation will be a crucial element in section 4 when we argue that some very specific topological fluctuations are responsible for violation of local $\mathcal{P}$ and $\mathcal{C}\mathcal{P}$ invariance in QCD observed at RHIC. We would not be able to use the quasi-classical technique developed in refs. [36, 37, 38, 39, 40, 41, 42, 43] to discuss corresponding fluctuations as no classical trajectories of real particles propagating in external classical colour fields exist for these type of fluctuations. This is because the relevant topological vacuum fluctuations, as we discuss below, are not related to any absorptive parts of any physical correlation functions. Rather, the topological fluctuations which will be main subject of this paper may contribute only to the real parts of the correlation functions (such as topological susceptibility) see next section. In Minkowski space similar contributions normally treated as the subtraction constants. In the present case of the accelerating frame, the corresponding “subtraction constant” becomes a “subtraction function” which depends on acceleration and which is sensitive to the global properties of the space-time. As we shall see below, the approach developed in the present section is well suited to attack this problem as everything in section 4 will be formulated precisely in appropriate terms of vacuum fluctuations.

3. The $\theta$ vacua, $U(1)_A$ effective lagrangian and the Veneziano ghost in Minkowski spacetime

The main goal of this section is to single out (identify) the fields which describe the topological fluctuations in QCD. It turns out that these relevant fields (in a specific gauge) can be represented as (pseudo)scalar colour-singlet fields such that one can immediately apply the technique developed in previous section 2 to describe the corresponding vacuum fluctuations in accelerating frame. As we shall argue below in section 4 those vacuum topological fluctuations in accelerating frame may be responsible for $\mathcal{P}$ and $\mathcal{C}\mathcal{P}$ violation observed at RHIC. It is quite obvious that those configurations must be related to $\theta$ dependence, topological charge density, and other related problems.
Therefore, we start this section by reviewing the $\theta$ dependence in QCD and the standard resolution of $U(1)_A$ problem \cite{49,50,51,52}. We follow \cite{53} to identify the relevant for the present work degrees of freedom (Veneziano ghost) which saturates topological correlation functions. After integrating the ghost out (as was done in the original paper \cite{50}) one reproduces the $\eta'$ mass, which was the main result of \cite{50}. We keep the Veneziano ghost explicitly as it can not be integrated out in accelerating frame. Moreover, it will play a central role in our following discussions when we consider the accelerating frame relevant for description of high energy collisions. As we shall argue in section 4 the corresponding topological fluctuations in accelerating frame might be the pivotal source of $\mathcal{P}$ and $\mathcal{CP}$ violating fluctuations observed at RHIC.

### 3.1 The Lagrangian and the ghost

The starting point for our analysis will be the Lagrangian as proposed in \cite{3.1} The Lagrangian and the ghost

The constant $\eta$ reproduces the standard resolution of $U(1)_A$ problem \cite{49,50,51,52}. We follow \cite{53} to identify the relevant for the present work degrees of freedom (Veneziano ghost) which saturates topological correlation functions. After integrating the ghost out (as was done in the original paper \cite{50}) one reproduces the $\eta'$ mass, which was the main result of \cite{50}. We keep the Veneziano ghost explicitly as it can not be integrated out in accelerating frame. Moreover, it will play a central role in our following discussions when we consider the accelerating frame relevant for description of high energy collisions. As we shall argue in section 4 the corresponding topological fluctuations in accelerating frame might be the pivotal source of $\mathcal{P}$ and $\mathcal{CP}$ violating fluctuations observed at RHIC.

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“Veneziano ghost”. Indeed, a physical state of mass \( m_G \), momentum \( k \to 0 \) and coupling \( \langle 0|q(G) = c_G \) contributes to the topological susceptibility with the sign which is opposite to (3.3),

\[
\lim_{k \to 0} \int d^4xe^{ikx} \left( T \{ q(x), q(0) \} \right) \sim \lim_{k \to 0} \langle 0|q(G) \rangle \sim \frac{c_G^2}{m_G^2} \geq 0. \tag{3.4}
\]

However, the positive sign for \( b \) (and negative sign for the topological susceptibility (3.3)) is what is required to extract the physical mass for the \( \eta' \) meson, \( m_{\eta'}^2 \simeq b/2N_c \neq 0 \), see the original reference [50] for a thorough discussion.

One can interpret the field \( A_{\mu \nu \rho} \) as a collective mode of the original gluon fields, which in the infrared leads to a pole in the unphysical subspace and provides a finite contribution with a wrong sign to the topological susceptibility (3.3). We know about the existence of this very special degree of freedom and its properties from the resolution of the famous \( U(1)_A \) problem: integrating out the \( q \) field (as shown below) provides the mass for the \( \eta' \) meson. Of course \( b = 0 \) to any order in perturbation theory because \( q(x) \) is a total divergence \( q = \partial_{\mu} K^{\mu} \). However, as we learnt from [49, 51], \( b \neq 0 \) due to the non-perturbative infrared physics; in fact, in the chiral limit \( m_{\eta'}^2 \sim b \).

One can integrate out the scalar field \( q \) since there is no kinetic term associated to it. This is indeed the procedure followed by Di Vecchia and Veneziano in their original paper [50], the outcome of which, as follows from (3.1), is

\[
\mathcal{L} = \mathcal{L}_0 + \frac{1}{2} \partial_{\mu} q^{\prime} \partial^{\mu} q^{\prime} - \frac{b f q^{\prime 2}}{4N_c} \left( \theta - \frac{q^{\prime}}{f q^{\prime}} \right)^2 + N_f m_q |\langle \bar{q} q \rangle| \cos \left[ \frac{q^{\prime}}{f q^{\prime}} \right], \tag{3.5}
\]

where all the dependence on the three-form \( A_{\nu \rho \sigma} \) has disappeared. This formula explicitly shows that \( \eta' \) receives a non-vanishing mass in the chiral limit, \( m_{\eta'}^2 \simeq b/2N_c \neq 0 \) due to the non-zero magnitude of the coefficient \( b \), which enters (3.1) and (3.3). This formula also reproduces the notorious Witten-Veneziano relation for the topological susceptibility in pure gluodynamics if one substitutes \( b = 2N_c m_{\eta'}^2 \) into the expression (3.3) for the topological susceptibility.

As we mentioned above, we want to keep the ghost hidden in \( A_{\mu \nu \rho} \) explicitly. We shall now choose the Lorenz-like gauge

\[
(\partial^\rho A_{\mu \nu \rho}) \simeq (\partial_\mu K_\nu - \partial_\nu K_\mu) = 0, \quad K_\mu \equiv \epsilon_{\mu \nu \rho \sigma} A^{\nu \rho \sigma}, \quad q = \partial_\mu K_\mu, \tag{3.6}
\]

in which we will carry out our manipulations. It is the same gauge which was discussed in the original paper [50]. We choose to work only with the longitudinal part of the \( K_\mu \) field because only this longitudinal part determines the topological density \( q = \partial_\mu K_\mu \), and eventually leads to a non-vanishing contribution to the topological susceptibility (3.3). Therefore, we write the longitudinal part of \( K_\mu \) as

\[
K_\mu \equiv \partial_\mu \Phi, \tag{3.7}
\]

such that the expression for the topological density takes the form

\[
q = \partial_\mu K_\mu = \Box \Phi, \tag{3.8}
\]
where $\Phi$ is a new scalar field of mass dimension 2. We notice that the gauge condition (3.6) is automatically satisfied with our definition (3.7). Now our Lagrangian (3.1) can be expressed in terms of the $\Phi$ field as follows

$$
\mathcal{L} = \mathcal{L}_0 + \frac{1}{2} \partial_\mu \eta' \partial^\mu \eta' + N_f m_q |\langle \bar{q} q \rangle| \cos \left[ \frac{\eta'}{f_{\eta'}} \right]
+ \frac{1}{2 m_{\eta'}^2 f_{\eta'}^2} \Phi \Box \Phi + \left( \frac{\eta'}{f_{\eta'}} \right) \Box \Phi - \theta \Box \Phi,
$$

(3.9)

where we plugged in the coefficient $b \rightarrow 2N_c m_{\eta'}^2$ as the Witten-Veneziano relation requires.

If we integrate out the $2\Phi$ field we return to the expression (3.5) which describes the physical massive $\eta'$ field alone.

As usual, the presence of 4-th order operator $\Phi \Box \Phi$ is a signal that the ghost is present in the system and may be quite dangerous. However, we know from the original form (3.1) that the system is unitary, well defined etc, in different words, it does not present any problem associated with the ghost. One can redefine all fields in such a way that the final Lagrangian can be expressed as follows [53]:

$$
\mathcal{L} = \frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} + \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1
- \frac{1}{2} m_{\eta'}^2 \hat{\phi}^2 + N_f m_q |\langle \bar{q} q \rangle| \cos \left[ \frac{\phi + \phi_2 - \phi_1}{f_{\eta'}} \right],
$$

(3.10)

where all fields have now canonical dimension one in four dimensions. We claim that the Lagrangian (3.10) is that part of QCD which describes long distance physics in our context. Notice that (3.10) is exactly identical to that proposed by Kogut and Susskind (KS) in [55] for the 2d Schwinger model, see also [57] in given context. The unitarity and other important properties of QFT are satisfied in our 4d system (3.10) in the same way as they are satisfied in [55] for the 2d Schwinger model as will be reviewed in the next subsection 3.2. The Veneziano ghost in QCD is represented in our notations by the $\phi_1$ field in (3.10) and it is always accompanied by its companion, the massless field $\phi_2$. These two fields cancel each other in every gauge invariant matrix element once the auxiliary (similar to Gupta-Bleuler [58, 59]) conditions on the physical Hilbert space are imposed, see below.

Using the explicit expression for the Green’s function and the expression for the topological density $q = \partial_\mu K^\mu = \Box \Phi$ after simple algebraic manipulations one can represent the topological susceptibility for QCD in model (3.1) in the chiral limit $m_q \rightarrow 0$ in the following way,

$$
\chi_{QCD} \equiv i \int d^4 x \langle T\{ q(x), q(0) \} \rangle_{QCD} = -\frac{f_{\eta'}^2 m_{\eta'}^2}{4} \cdot \int d^4 x \left[ \delta^4(x) - m_{\eta'}^2 D^c(m_{\eta'} x) \right]
$$

(3.11)

where $D^c(m_{\eta'} x)$ is the Green’s function of a free massive particle with standard normalization $\int d^4 x m_{\eta'}^2 D^c(m_{\eta'} x) = 1$. In this expression the $\delta^4(x)$ represents the ghost contribution with the required “wrong” sign while the term proportional to $D^c(m_{\eta'} x)$ represents the $\eta'$ contribution. The ghost’s contribution can be also thought as the Witten’s contact
The density of the topological susceptibility $\chi(r) \sim \langle q(r), q(0) \rangle$ as function of separation $r$ such that $\chi_{QCD} \equiv \int dr \chi(r)$, adapted from [60]. Plot explicitly shows the presence of the contact term with the “wrong sign” (narrow peak around $r \simeq 0$) represented by the Veneziano ghost in our framework.

term not related to any propagating degrees of freedom. The topological susceptibility $\chi_{QCD}(m_q = 0) = 0$ vanishes in the chiral limit as a result of exact cancellation of two terms entering (3.11) in complete accordance with WI. When $m_q \neq 0$ the cancellation is not complete and $\chi_{QCD} \simeq m_q \langle \bar{q}q \rangle$.

One should emphasize that the presence of the contact term (described by the Veneziano ghost in our framework) $\sim \delta^4(x)$ in eq. (3.11) is well established and well understood phenomenon in QCD. In particular, it has been studied on the lattice, see e.g. [60] and references therein\(^2\) where a narrow peak around $r \simeq 0$ and a smooth behaviour in extended region of $r \sim fm$ with the opposite signs have been seen as a result of numerical computations. We reproduce Fig.1 for illustration purposes adapted from ref.[60] where these crucial elements are explicitly present on the plot. As we mentioned previously, the main goal of the present paper is to study precisely this (unphysical and non-propagating) effective degree of freedom leading to $\sim \delta^4(x)$ in eq. (3.11) (and represented by a narrow peak at $r \sim 0$ on Fig. 1) when we accelerate our system. As we discuss in great details in [54] the ghost does not contribute to absorptive parts of any correlation functions (after all, it is not an asymptotic degree of freedom). However, it does contribute to the real part as plot on Fig.1 explicitly shows.

As we shall argue below, the topological nature of the ghost and its $0^{-+}$ quantum numbers may play a crucial role in understanding of local $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD observed at RHIC [1, 2, 3, 4, 5]. Before we proceed with our description of the Veneziano ghost in accelerating frame we would like to demonstrate that the Veneziano ghost is harmless (e.g. it does not violate unitarity) in spite of its negative sign in the Lagrangian

\(^2\)A warning signal with the signs: the physical degrees of freedom in Euclidean space (where the lattice computations are performed) contribute to topological susceptibility $\chi_{QCD}$ with the negative sign, while the contact term (the Veneziano ghost) contributes with the positive sign, in contrast with our discussions in Minkowski space.
3.2 Unitarity and the ghost

We follow Kogut and Susskind construction \cite{55} in order to demonstrate the unitarity of our system when the Veneziano ghost $\phi_1$ and its partner $\phi_2$ explicitly present in the Lagrangian (3.10). The cosine interaction term (3.10) includes vertices between the ghost and the other two scalar fields, but it can in fact be shown in complete analogy with \cite{55} that, once appropriate auxiliary (similar to Gupta-Bleuler \cite{58,59}) conditions on the physical Hilbert space are imposed, the unphysical degrees of freedom $\phi_1$ and $\phi_2$ drop out of every gauge-invariant matrix element, leaving the theory well defined, i.e., unitary and without negative normed physical states, just as in the Lorentz invariant quantization of electromagnetism. Specifically, this is achieved by demanding that the positive frequency part of the free massless combination $(\phi_2 - \phi_1)$ annihilates the physical Hilbert space:

$$\left(\phi_2 - \phi_1\right)^{(+)} |H_{\text{phys}}\rangle = 0.$$  \hspace{1cm} (3.12)

With this additional requirement the quantum theory built on the Lagrangian (3.10) is well defined in any respect, and the physical sector of the theory exactly coincides with (3.5) which was obtained by a trivial integrating out procedure \cite{50}.

In particular, one can explicitly check that the expectation value for any physical state in fact vanishes as a result of the subsidiary condition (3.12):

$$\langle H_{\text{phys}} | H | H_{\text{phys}} \rangle = 0.$$  \hspace{1cm} (3.13)

In different words, all these “dangerous” states which can produce arbitrary negative energy do not belong to the physical subspace defined by eq. (3.12). Therefore, the main conclusion is that the description in terms of the ghost is equivalent to well-known standard procedure of integrating out the ghost field leading to well-known expression (3.5) for the effective low energy lagrangian.

In the next section we want to study the dynamics of the ghost fields in the accelerating frame. The corresponding technique for a non-interacting (pseudo)scalar field has been previously developed and presented in section 2. In what follows we consider the chiral limit $m_q \to 0$ such that the lagrangian describing the ghost field and its partner (3.10) is precisely represented by the combination of two massless non-interacting fields\footnote{The fluctuations of the physical massive $\eta'$ field can be obviously neglected. It will be ignored in what follows.} in which case the corresponding Bogolubov’s coefficients have been previously computed (2.22). Numerically, the chiral limit in fact implies that the acceleration parameter $a \gg m_q$. In addition, as we mentioned previously, we want to keep “$a$” as a free parameter of the theory in spite of the fact that in nature it is fixed by the string tension (1.3). Therefore, in all discussions below we consider the following hierarchy of scales

$$1 \gev \gg a \gg m_q$$  \hspace{1cm} (3.14)

in order to separate the effects topological fluctuations due to the acceleration “$a$” from conventional QCD fluctuations with typical scales $\sim \gev$. 

}\hspace{1cm} (∗∗)
4. $\mathcal{P}$ and $\mathcal{CP}$ violating fluctuations in accelerating frame

Our goal here is to understand the behaviour of the system (3.10) in the chiral limit $m_q = 0$ in accelerating frame. These fields have quantum numbers $0^+ -$ and their long wave fluctuations may produce observable $\mathcal{P}$ and $\mathcal{CP}$ odd effects as discussed below. There are many other fluctuations, of course, in the system due to conventional quarks and gluons. However, the ghost field $\phi_1$ and its partner $\phi_2$ are unique degrees of freedom in many respects, and their fluctuations, hopefully, can be separated from all other vacuum fluctuations.

The main point is as follows. As we discussed above, the Bogolubov’s coefficients (2.22) have the property that they are exponentially suppressed for $\omega \gg a$. Therefore, the typical wave lengths of fluctuations related to acceleration are $\lambda \geq a^{-1}$. If $a \approx 1$ GeV as estimation (1.3) suggests, it would be very difficult to disentangle these fluctuations from conventional fluctuations of quarks and gluons with the same typical scale $\lambda \approx \text{GeV}^{-1}$. However, we work in the limit $a \ll 1$ GeV where such separation (at least theoretically) is a possibility\textsuperscript{4}. In case $a \ll 1$ GeV all colour fields will still fluctuate with typical $\lambda \approx \text{GeV}^{-1}$ as a result of confinement which implies that all fluctuations are effectively gapped. It is in a drastic contrast with fluctuations of colourless ghost field $\phi_1$ and its partner $\phi_2$. The ghost remains massless even when interactions are present, as its pole (in unphysical Hilbert space) is topologically protected as discussed above in section 3. Indeed, non vanishing contribution to the topological susceptibility (3.3) constructed from the operators which are total derivatives $q = \partial_{\mu}K^\mu$ may only come from (unphysical) massless pole. In different words, the typical wave lengths of $\phi_1$ and $\phi_2$ fields related to acceleration are $\lambda \geq a^{-1}$ for arbitrary small $a$. Based on this comment, we ignore in this section the conventional fluctuations of quarks and gluons with $\lambda \approx \text{GeV}^{-1}$; we return to them in section 5 when we produce some numerical estimates by comparing the energetics of the ghost fields $\phi_1$, $\phi_2$ and conventional quarks and gluons.

4.1 Veneziano ghost in the accelerating frame at 1 GeV $\gg a \gg m_q$

As we mentioned above, in the region 1 GeV $\gg a \gg m_q$ the problem is reduced to free massless fields $\phi_1$ and $\phi_2$ with GB like constraint. Therefore, one can explicitly use the formalism developed earlier in section 2. In particular, one can compute the Bogolubov’s coefficients (2.22), construct the Hamiltonian and the number operator for the ghost field $\phi_1$ and its partner $\phi_2$ in accelerating frame as it was done previously, see eq.(2.23). The next step is to compute the corresponding expectation values when the system is being prepared as Minkowski vacuum $|0_M\rangle$ evolves in the accelerating background. Technically it is exactly the same problem of our previous computations of the Planck spectrum (2.25) detected by a Rindler observer in a model of a single massless particle.

We start with expansion of the ghost field $\phi_1$ and its partner $\phi_2$ using the modes (2.13)\textsuperscript{4}.

\textsuperscript{4}In fact, we argue in the next section 5 that in heavy ion collisions such conditions indeed could be achieved experimentally.
(2.14) as we have done previously (2.15),

\[
\begin{align*}
\phi_1 &= \sum_k \frac{1}{\sqrt{4\pi \omega}} \left( a_k^L e^{i k \xi^L + i \omega \eta^L} + a_k^L e^{-i k \xi^L - i \omega \eta^L} + a_k^R e^{i k \xi^R - i \omega \eta^R} + a_k^R e^{-i k \xi^R + i \omega \eta^R} \right), \\
\phi_2 &= \sum_k \frac{1}{\sqrt{4\pi \omega}} \left( b_k^L e^{i k \xi^L + i \omega \eta^L} + b_k^L e^{-i k \xi^L - i \omega \eta^L} + b_k^R e^{i k \xi^R - i \omega \eta^R} + b_k^R e^{-i k \xi^R + i \omega \eta^R} \right)
\end{align*}
\]

(4.1)

The Rindler vacuum state is defined as usual,

\[
a_k^R |0_R\rangle = 0, \quad b_k^R |0_R\rangle = 0, \quad \forall k.
\]

(4.2)

In Minkowski space we can proceed exactly along the same line as we have done in section 2. Namely, instead of expansion with modes (2.7) we can expand \( \phi_1 \) and \( \phi_2 \) in terms of (2.18) as follows:

\[
\begin{align*}
\phi_1 &= \sum_k \frac{1}{\sqrt{4\pi \omega}} \cdot \frac{1}{\sqrt{(e^{\pi \omega/\alpha} - e^{-\pi \omega/\alpha})}} \left[ a_k^1 \left( e^{\frac{\pi \omega}{2 \alpha} + i k \xi^R - i \omega \eta^R} + e^{-\frac{\pi \omega}{2 \alpha} + i k \xi^L - i \omega \eta^L} \right) \\
&\quad + b_k^2 \left( e^{\frac{\pi \omega}{2 \alpha} + i k \xi^L - i \omega \eta^L} + e^{-\frac{\pi \omega}{2 \alpha} + i k \xi^R + i \omega \eta^R} \right) \\
&\quad + a_k^1 \left( e^{-\frac{\pi \omega}{2 \alpha} - i k \xi^R + i \omega \eta^R} + e^{\frac{\pi \omega}{2 \alpha} - i k \xi^L + i \omega \eta^L} \right) \\
&\quad + a_k^2 \left( e^{-\frac{\pi \omega}{2 \alpha} - i k \xi^L - i \omega \eta^L} + e^{\frac{\pi \omega}{2 \alpha} - i k \xi^R - i \omega \eta^R} \right) \right], \\
\phi_2 &= \sum_k \frac{1}{\sqrt{4\pi \omega}} \cdot \frac{1}{\sqrt{(e^{\pi \omega/\alpha} - e^{-\pi \omega/\alpha})}} \left[ b_k^1 \left( e^{\frac{\pi \omega}{2 \alpha} + i k \xi^R - i \omega \eta^R} + e^{-\frac{\pi \omega}{2 \alpha} + i k \xi^L - i \omega \eta^L} \right) \\
&\quad + b_k^2 \left( e^{\frac{\pi \omega}{2 \alpha} + i k \xi^L - i \omega \eta^L} + e^{-\frac{\pi \omega}{2 \alpha} + i k \xi^R + i \omega \eta^R} \right) \\
&\quad + b_k^1 \left( e^{-\frac{\pi \omega}{2 \alpha} - i k \xi^R + i \omega \eta^R} + e^{\frac{\pi \omega}{2 \alpha} - i k \xi^L + i \omega \eta^L} \right) \\
&\quad + b_k^2 \left( e^{-\frac{\pi \omega}{2 \alpha} - i k \xi^L - i \omega \eta^L} + e^{\frac{\pi \omega}{2 \alpha} - i k \xi^R - i \omega \eta^R} \right) \right]
\end{align*}
\]

(4.3)

where \( b_k^1, b_k^2 \) satisfy the following commutation relations,

\[
\begin{align*}
[b_k^{(1,2)}, b_{k'}^{(1,2)}] &= 0, \quad [t_k^{(1,2)}, b_{k'}^{(1,2)}] = 0, \quad [b_k^{(1,2)}, b_{k'}^{(1,2)}] = \delta_{kk'}, \quad (4.4)
\end{align*}
\]

whereas \( a_k^1, a_k^2 \) for the ghost field \( \phi_1 \) satisfy

\[
\begin{align*}
[a_k^{(1,2)}, a_{k'}^{(1,2)}] &= 0, \quad [a_k^{(1,2)}, a_{k'}^{(1,2)}] = 0, \quad [a_k^{(1,2)}, a_{k'}^{(1,2)}] = -\delta_{kk'}, \quad (4.5)
\end{align*}
\]

where again the sign minus appears in these commutation relations. The Minkowski vacuum state is determined as usual

\[
a_k^1 |0\rangle = 0, \quad a_k^2 |0\rangle = 0, \quad b_k^1 |0\rangle = 0, \quad b_k^2 |0\rangle = 0, \quad \forall k.
\]

(4.6)

The Bogolubov’s coefficients for \( \phi_1 \) and \( \phi_2 \) fields relating the description in Minkowski and Rindler spaces can be computed exactly in the same way as it was done before, see eq. (2.22),

\[
\begin{align*}
a_k^L &= \frac{e^{-\pi \omega/2\alpha} a_k^1 + e^{\pi \omega/2\alpha} a_k^2}{\sqrt{e^{\pi \omega/\alpha} - e^{-\pi \omega/\alpha}}} \quad a_k^R = \frac{e^{\pi \omega/2\alpha} a_k^1 + e^{-\pi \omega/2\alpha} a_k^2}{\sqrt{e^{\pi \omega/\alpha} - e^{-\pi \omega/\alpha}}} \quad (4.7) \\
b_k^L &= \frac{e^{-\pi \omega/2\alpha} b_k^1 + e^{\pi \omega/2\alpha} b_k^2}{\sqrt{e^{\pi \omega/\alpha} - e^{-\pi \omega/\alpha}}} \quad b_k^R = \frac{e^{\pi \omega/2\alpha} b_k^1 + e^{-\pi \omega/2\alpha} b_k^2}{\sqrt{e^{\pi \omega/\alpha} - e^{-\pi \omega/\alpha}}}.
\end{align*}
\]
Now consider an accelerating Rindler observer at $\xi = \text{const.}$ As we discussed previously, such an observer’s proper time is proportional to $\eta$. The vacuum for this observer is determined by (4.2) as this is the state associated with the positive frequency modes with respect to $\eta$. A Rindler observer in R wedge will measure the energy and particle density using the Hamiltonian $H^R$ and density operator $N^R$ which are given by (a similar formula applies for L wedge as well),

$$H^R = \sum_k \omega_k \left( b^R_k b^R_k - a^R_k a^R_k \right), \quad N^R = \sum_k \left( b^R_k b^R_k - a^R_k a^R_k \right).$$

The subsidiary condition (3.12) defines the physical subspace for accelerating Rindler observer

$$\left( a^R_k - b^R_k \right) |\mathcal{H}^R_{\text{phys}}\rangle = 0,$$

such that the exact cancellation between $\phi_1$ and $\phi_2$ fields holds for any physical state defined by eq. (4.9), i.e.

$$\langle \mathcal{H}^R_{\text{phys}} | H^R | \mathcal{H}^R_{\text{phys}} \rangle = 0 \quad (4.10)$$
as it should.

However, if the system is prepared as the Minkowski vacuum state $|0_M\rangle$ defined by eq. (4.6) a Rindler observer using the same expressions for the number operator and Hamiltonian (4.8) will observe the following amount of energy in mode $k$,

$$\langle 0 | \omega_k \left( b^R_k b^R_k - a^R_k a^R_k \right) |0\rangle = \frac{2\omega e^{-\pi\omega/a}}{(e^{\pi\omega/a} - e^{-\pi\omega/a})} = \frac{2\omega}{(e^{2\pi\omega/a} - 1)},$$

where we used the Bogolubov’s coefficients (4.7) to express $a^R_k, b^R_k$ in terms of $a^{(1,2)}_k, b^{(1,2)}_k$. This formula (up to a numerical coefficient) has been reproduced in ref.[56] by using a different technique. This is the central result of this section and is a direct analog of Planck spectrum given by eq. (2.25) discussed previously for the conventional massless particle with the only difference of factor 2 in front which is result of extra degeneracy: we have two degrees of freedom $\phi_1$ and $\phi_2$ instead of one scalar field $\phi$ from section 2. The crucial point here is as follows. No cancellation between the ghost $\phi_1$ and its partner $\phi_2$ could occur in the expectation value (4.11), in net contrast with eq. (4.10). Technically, a “non-cancellation” of unphysical degrees of freedom (4.11) in accelerating frame is a result of opposite sign in commutator (4.5) along with negative sign in Hamiltonian (4.8).

We will discuss this important point in great details in the next subsection in non-technical, intuitive way. However, we want to emphasize that this result (4.11) should not be interpreted as actual emission of ghost modes, as they are not the asymptotic states in Minkowski spacetime in the remote past and future, and therefore they can not propagate to infinity in contrast with conventional Unruh effect, see appendix A for details. Rather, one should interpret (4.11) as an additional time dependent contribution to the vacuum energy in accelerating background in comparison with Minkowski space-time. This extra energy is entirely ascribable to the presence of the unphysical (in Minkowski space) degrees of freedom. We interpret the extra contribution to the energy observed by the Rindler
observer as a result of formation of a specific configuration, the “ghost condensate” [54], rather than a presence of “free particles” prepared in a specific mixed state which can be detected. This extra term should be treated as a result of very unique vacuum fluctuations, not related to any absorptive contributions. The observational effects of this extra vacuum contribution will be discussed in the next section 5.

4.2 Interpretation

- As explained above the nature of the effect (4.11) is the same as the conventional Unruh effect [33] discussed in section 2 when the Minkowski vacuum $|0_M\rangle$ is restricted to the Rindler wedge with no access to the entire space time. A pure quantum state $|0_M\rangle$ becomes a thermo mixed state as a result of this quantum restriction. The result (4.11), by definition, implies that the states which were unphysical (in Minkowski space) lead to physically observable phenomena, though it can not be interpreted in terms of pure states of individual particles, see Appendix A for details. The effect is obviously IR in nature, and it is basically due to the presence of the horizon which itself dynamically emerges as a result of strong interactions as advocated in refs. [36, 37, 38, 39, 40, 41, 42, 43],

- One can explicitly see why the cancellation of unphysical degrees of freedom $\phi_1$ and $\phi_2$ in Minkowski space fail to hold for the accelerating Rindler observer (4.11). The selection of the physical Hilbert subspace (3.12) is based on the properties of the operator which selects positive-frequency modes with respect to Minkowski time $t$. At the same time the Rindler observer selects the physical Hilbert space (4.9) by using positive-frequency modes with respect to observer’s proper time $\eta$. These two sets are obviously not equivalent, as e.g. they represent a mixture of positive and negative frequencies modes defined in R- and L- Rindler wedges. At the same time, the Rindler observers do not ever have access to the entire space time. Therefore, from the Rindler’s view point the cancellation in Minkowski space can be only achieved if one uses both sets (L and R). Of course, using the both sets would contradict to the basic principles as the R-Rindler observer does not have access to the L wedge even for arbitrary small acceleration parameter $a$.

- This is not the first time when unphysical (in Minkowski space) ghost contributes to a physically observable quantity. The first example is the famous resolution of the $U(1)_A$ problem in QCD, see section 3. As long as we work in Minkowski spacetime the two constructions (based on the Veneziano ghost [49, 50] and on the Witten’s subtraction constant [51]) are perfectly equivalent as the subsidiary condition (3.12) ensures that the ghost degrees of freedom are decoupled from the physical Hilbert subspace, leaving both schemes with the identical physical spectrum. In different words, these unphysical degrees of freedom do not contribute to absorptive parts, but only to the real parts of the correlation functions. In Minkowski space such a contribution is normally represented by a “subtraction constant”, while in a time dependent background this subtraction constant becomes a “subtraction function” which depends on acceleration. We advocate the ghost-based technique to account for this physics because the corresponding description can be easily generalized for accelerating background, while a similar generalization (without the ghost, but with explicit accounting for the infrared behaviour at the boundaries/horizon) is unknown and likely to be much more technically complicated. In different words, the
description in terms of the ghost is a matter of convenience in order to effectively account for the boundary/horizon effects in topologically nontrivial sectors of the theory.

• One should emphasize that the Veneziano ghost we are dealing with in this paper is very different from all other ghosts, including the conventional Fadeev Popov ghosts. The Veneziano ghost is not an asymptotic state, it does not propagate, it does not contribute to the absorptive parts of the correlation functions, as explained in Appendix A, though, it does fluctuate and does contribute to the energy through the boundary/horizon conditions (similar to the Casimir effect). The spectrum of these fluctuations is very different from conventional Fadeev Popov ghosts (when momenta could be arbitrary large in order to cancel the corresponding unphysical polarizations of the gauge fields). A typical frequency of the Veneziano ghost is determined by the horizon scale $\omega \sim a$, while the higher frequency modes $\omega \gg a$ are exponentially suppressed.

• The unique feature of the Veneziano ghosts is due to its close connection to the topological properties of the theory. Indeed, the topological density operator $q$ is explicitly expressed in terms of the Veneziano ghost $\phi_1$ as follows, $q \sim \tilde{\Box} \Phi \sim \left(\tilde{\Box} \phi - \tilde{\Box} \phi_1\right)$ such that the contact term (representing the real, not absorptive part of the topological susceptibility) $\sim \delta^4(x)$ in eq. (3.11) is saturated by the ghost. One should also note that the appearance of the ghost degree of freedom in the formalism can be traced from the induced $\tilde{\phi}^2$ term (3.1) which contains $\Phi \tilde{\Box}^2 \Phi$ operator (3.9). As is known the $\Box^2$ operator can be always re-written in terms of a degree of freedom with a negative kinetic term. This explains the origin and uniqueness of the Veneziano ghost and its relation to topological features of the theory. A number of very nontrivial properties of this ghost which are discussed in this paper are intimately related to its topological nature.

5. Observations of the $\mathcal{P}$ and $\mathcal{CP}$ odd fluctuations at RHIC

The goal of this section is to apply our previous formal analysis to the very concrete subject: we want to interpret the recent RHIC experimental results [1, 2, 3, 4, 5] as violation of local $\mathcal{P}$ and $\mathcal{CP}$ symmetries in QCD. The key point of all our previous discussions can be formulated in one line: QCD supports the topologically nontrivial unique vacuum fluctuations (Veneziano ghost) in the accelerating system. The fluctuations are IR in nature, sensitive to the horizon scale $\lambda \geq a^{-1}$ for arbitrary small $a$, they do not propagate, do not contribute to the absorptive parts of the correlation functions, but they do contribute to the real portion of the correlation functions. Their IR nature is protected by topology: they remain gapless even in the presence of the strong confined forces. Such a property is in huge contrast with conventional fast quark and gluon fluctuations which have a sharp cutoff at wavelengths $\lambda \sim \Lambda_{QCD}^{-1}$. These topological fluctuations have $0^{-+}$ quantum numbers, and in all respects very similar to the induced, slowly fluctuating $\theta_{\text{ind}}$ discussed in section 1.1 with $\dot{\theta}_{\text{ind}} \sim a$. We know about the existence of the Veneziano ghost from the resolution of the $U(1)_A$ problem when it saturates the contact term in the topological susceptibility. In the accelerating frame this contact term becomes a “subtraction function” and the
corresponding topological fluctuations lead yet to another observable phenomena as we shall argue below.

5.1 The basic picture

In what follows we assume that $1 \text{ GeV} \gg a \gg m_q$ such that we can separate the topological fluctuations with very large wave lengths $\lambda \geq a^{-1}$ which carry $0^+\mp$ quantum numbers from conventional fluctuations of quarks and gluons with typical $\lambda \sim 1 \text{ GeV}^{-1}$. We also neglect the interacting term $\sim m_q$ in eq. (3.10) such that our consideration of free fields in accelerating frame leading to (4.11) is justified.

Our basic picture in this regime can be formulated as follows. The conventional quark and gluon fluctuations with typical $\lambda \sim 1 \text{ GeV}^{-1}$ are propagating in the environment of very slow topological fluctuations with wave lengths $\lambda \geq a^{-1}$. These slow topological fluctuations can be thought as $\mathcal{P}$ and $\mathcal{CP}$ odd environment for the fast conventional fluctuations with typical $\lambda \sim 1 \text{ GeV}^{-1}$. The fast conventional fluctuations are distributed according to the Planck formula as discussed in section 2 and described by eq. (2.25). While this formula was derived for a massless scalar particle for illustration purposes, it is known that a similar thermal distribution is expected to hold for vector and spinor fields as well.

This picture for the fast fluctuations is equivalent to the “new thermalization” scenario advocated in refs. [36, 37, 38, 39, 40, 41, 42, 43] as it produces hadrons distributed according to the thermal law determined by temperature (1.2). The only new element of this work is the observation that these conventional fast fluctuations with typical $\lambda \sim 1 \text{ GeV}^{-1}$ are propagating in the $\mathcal{P}$ and $\mathcal{CP}$ odd environment described by new type of topological fluctuations with very large wave lengths $\lambda \geq a^{-1}$. We emphasize that the soft topological fluctuations can not propagate to infinity by themselves as they are not asymptotic states; rather they produce the $\mathcal{P}$ and $\mathcal{CP}$ odd environment for conventional fast fluctuations which eventually will be observed as the hadrons produced in this odd environment. Our main conjecture is that the $\mathcal{P}$ and $\mathcal{CP}$ odd fluctuations observed at RHIC [1, 2, 3, 4, 5] is a direct consequence of this odd environment. In next subsection 5.2 we present a number of qualitative consequences of the entire framework supporting this basic picture. Before we do so, we want to compare the energetics of slow topological fluctuations with conventional fast fluctuations of quarks and gluons.

Our starting formula is the Planck spectrum for the Veneziano ghost and its partner (4.11) valid for $a \gg m_q$. Number density of the $\mathcal{P}$ odd domains with size $\lambda \simeq \frac{2\pi}{\omega}$ is given by

$$dN_\omega = \frac{d^3k}{(2\pi)^3} \frac{2}{(e^{2\pi\omega/a} - 1)}, \quad (5.1)$$

while the total contribution to the energy associated with these soft fluctuations is

$$E_{\text{ghost}} \simeq \int d^3k \frac{2\omega}{(2\pi)^3} \frac{e^{2\pi\omega/a}}{(e^{2\pi\omega/a} - 1)} = \frac{\pi^2}{15} \left(\frac{a}{2\pi}\right)^4. \quad (5.2)$$

This number should be compared with standard contribution to the thermal energy associated with the fast fluctuations of $N_f$ light quark flavours and $N_c^2 - 1$ gluons at temperature
\( T = (a/2\pi), \)

\[ E_{q+g} \simeq \frac{\pi^2}{15} \left( \frac{a}{2\pi} \right)^4 \left[ (N_c^2 - 1) + \frac{7N_cN_f}{4} \right]. \]  

(5.3)

Therefore, the relative energy associated with slow ghost fluctuations with 0\(^{+-}\) quantum numbers in comparison with conventional fast fluctuations of quarks and gluons is estimated to be

\[ \kappa \equiv \frac{E_{\text{ghost}}}{E_{q+g}} \sim \frac{1}{(N_c^2 - 1) + \frac{7N_cN_f}{4}}, \]  

(5.4)

which is numerically \( \sim 0.05 \). The effect is parametrically small at large \( N_c \) and proportional \( \sim 1/N_c^2 \) which is a typical suppression for any phenomena related to topological fluctuations. The effects related to the ghost obviously vanish at \( a = 0 \) as eq. (5.2) states.

This limit corresponds to the transition to Minkowski space when the Veneziano ghost is decoupled from physical Hilbert space (3.12). The factor \( \kappa \) essentially counts number of fluctuating degrees of freedom which lead to the \( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) odd environment. However, these degrees of freedom are not the asymptotic states, and they do not propagate to infinity as explained above, and they do not contribute to the absorptive parts of any correlation functions.

The information about the \( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) odd environment must be transferred to the conventional propagating degrees of freedom which can be observed and analysed. In the ideal world with \( a \ll 1 \text{ GeV} \) all strong interactions can be treated as fast fluctuations in slow varying background of the Veneziano ghost \( \phi_1 \) and its parter \( \phi_2 \) at nonzero acceleration \( a \ll 1 \text{ GeV} \). As we mentioned above, such background can be thought as slowly varying effective \( \theta_{\text{ind}} \neq 0 \). The spectral properties of these fluctuations are determined by eq. (5.1) while its energetics is determined by eq. (5.2). Therefore, in the limit \( a \ll 1 \text{ GeV} \) we can apply our previous knowledge about physical properties of hadrons in \( \theta_{\text{ind}} \neq 0 \) background. In particular, all previous estimates on \( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) odd effects reviewed in section 1.1 (including the charge separation effect as a result of anomalous interaction of slow varying \( \theta_{\text{ind}} \neq 0 \) with electromagnetic field) remain valid in this limit when \( \mathcal{P} \) odd domain is much larger in size than conventional QCD fluctuations. Therefore, we shall not elaborate on this point in the present work. Instead, we concentrate below on immediate qualitative consequences of the picture developed in this work when acceleration “\( a \)” being the key parameter of the system is parametrically small.

5.2 Observational consequences. The universality.

- 1. First immediate consequence of the developed picture is the presence of \( \mathcal{P} \) and \( \mathcal{C}\mathcal{P} \) odd fluctuations in any accelerating system with \( a \neq 0 \) including all energetic \( e^+e^- \), \( pp \) and \( p\bar{p} \) interactions when the thermal aspects corresponding to the universal temperature around \( T_H \sim (150 - 200) \text{ MeV} \) have been already observed. According to the entire logic of our framework the presence the thermal aspects in observations is resulted from the acceleration “\( a \)” when the QCD vacuum structure is completely reconstructed. The
corresponding reconstruction, among many other things, leads to topological fluctuations which play the role of the $P$ and $CP$ odd environment where hadrons are being produced. These topological fluctuations will be developed in all energetic $e^+e^-$, $pp$, $p\bar{p}$ and heavy ion collisions. However, the heavy ion collisions are unique in comparison with other types of energetic interactions as they allow to study the dependence of these odd effects as function of centrality which, as we argue below, effectively corresponds to variation of the acceleration parameter “$a$” with centrality.

2. As we reviewed in section 1.2 the acceleration realized in nature for $e^+e^-$, $pp$, $p\bar{p}$ (not heavy ions) collisions is a universal number given by (1.3) and close to $a \simeq 1$ GeV, rather than a free parameter “$a$”. A typical domain size where $P$ and $CP$ odd fluctuations will be developed is determined by eq. (5.1). The fluctuations will be order of $\lambda \simeq 2\pi/a \sim \text{fm}$ for $a \simeq 1$ GeV which is about the size of conventional fluctuations of quarks and gluons. In such circumstances the correlations similar the ones studied at RHIC [1, 2, 3, 4, 5] are expected to be suppressed for $e^+e^-$, $pp$, $p\bar{p}$ collisions as the formation of different hadrons most likely will occur in different $P$ odd domains rather than in one large domain. Nevertheless, the effect being suppressed for $e^+e^-$, $pp$, $p\bar{p}$ collisions, still does not vanish. The intensity of the corresponding correlations for $e^+e^-$, $pp$, $p\bar{p}$ collisions is predicted to have the same intensity as in heavy ion collisions for the most central events, see below.

3. This conclusion (on suppression of the correlations for $e^+e^-$, $pp$, $p\bar{p}$ collisions) changes drastically when we consider the heavy ion collisions. In this case it has been argued [40] that the temperature (1.2), and therefore, the acceleration “$a$” will be reduced for non-central collisions, which for small angular momentum $J$ can be approximated as follows [40],

$$a(J) \simeq a_{J=0} \left(1 - cJ^2\right), \quad c > 0,$$

(5.5)

where $c$ is some positive constant. Reduction of the acceleration “$a$” will increase a typical domain size where $P$ and $CP$ odd fluctuations develop as eq. (5.1) suggests. At the same time, the conventional fluctuations responsible for the formation of hadrons are not much affected by the reduction of “$a$” as they keep a typical (for Minkowski space) scale $\sim \text{fm}$ determined by confinement forces rather than by acceleration “$a$”. When the size of the $P$ odd domain becomes sufficiently larger than $\sim \text{fm}$ scale the strength of correlations should drastically increase as quite a few particles could be formed in the same large $P$ odd domain. Even a slight reduction of “$a$” (which corresponds to moving toward the least central collisions), may produce some drastic changes in strength of correlations as dependence on “$a$” is exponential, see eq. (5.1). Strong dependence on centrality is indeed supported by observations [5], though the acceleration parameter “$a$” is obviously not identically the same variable as centrality defined in [1, 2, 3, 4, 5].

4. Another immediate consequence of the developed picture is that the correlations should demonstrate the universal behaviour similar to the universality discussed in section 1.2 as the source for the both effects (“new thermalization” scenario and $P$ and $CP$ odd effects in QCD) is the same as argued in this paper. In particular, the effect should not depend on energy of colliding ions. Indeed, the size of the $P$ odd domains as well as the spectrum of the formed particles is determined exclusively by the acceleration “$a$” and should not depend on energy of colliding ions. Such independence on energy is indeed supported by observations
where correlations measured in Au+Au and Cu+Cu collisions at $\sqrt{s_{NN}} = 62$ GeV and $\sqrt{s_{NN}} = 200$ GeV are almost identical and independent on energy [5]. These similarities in behaviour of the correlations is definitely a strong argument supporting entire framework based on universality and common origin of both effects as formulated in introductory section 1. Based on this universality we predict that the corresponding correlations at the LHC energies would demonstrate a similar strength and a similar features found at RHIC.

5. One should emphasize that the universal features as formulated above are related exclusively to the portion of the “apparent thermalization” of the system as a result of acceleration “a”. Those aspects are expected to be universal for all high energy collisions: from $e^+e^-$ to $AA$. In case of heavy ion collisions however, in addition to those “apparent thermalization” aspects there are very real thermodynamical features of the system resulting from the conventional collisions which are normally described using the hydrodynamical equations. This conventional “hydro” portion of the dynamics, of course is not universal. This portion, for example, is not present in $pp$ collisions, and must be subtracted from analysis when comparison of $AA$ with $pp$ collisions is made in order to test the universality conjecture.

6. The arguments presented above on universal behaviour do not explicitly depend on the strength of the magnetic field which is a key player in CME, see eq. (1.1). This is a consequence of the same universal behaviour discussed above. The direction of $\vec{B}$ (or angular momentum $\vec{L}$) does play a role of selecting the reaction plane, while the absolute value of $|\vec{B}|$ is less important parameter in our arguments. The corresponding $|\vec{B}|$-dependence is implicitly hidden in the magnitude of acceleration parameter “a” which is a function of many other things, including centrality, $|\vec{B}|$, etc. Therefore, one should not expect a strong dependence of the effect on charges $Z_i$ of colliding ions which would lead to very different magnetic fields $|\vec{B}|$ for Au+Au and Cu+Cu collisions. Observed similarity in behaviour for Au+Au and Cu+Cu is another manifestation of universality discussed in item 4 above. A relatively mild $Z_i$ dependence of the effect is indeed consistent with observations [5].

7. The arguments presented above on universality of the correlation strength do not depend on transverse momenta $k_{\perp}^2$, even for relatively large $k_{\perp} > 1$ GeV. This is a consequence of the same universal behaviour discussed in items 4 and 5. Indeed, the entire picture described above, assumes that all hadrons are formed with $k_{\perp}$ determined by the thermal distribution in the $P$ odd background (5.1). The spectrum of both: slow and fast fluctuations is the result of preparation of the vacuum state $|0_M\rangle$ in the accelerating frame even before the collision develops as explained above and expressed by eq. (2.26). Therefore, one should not expect strong dependence on $|k_{\perp,\alpha} - k_{\perp,\beta}|$ in the correlations for particles $\alpha$ and $\beta$ even for large $|k_{\perp,\alpha} - k_{\perp,\beta}| \geq 1$ GeV as the corresponding distributions are not much affected by $P$ odd fluctuations (5.1). This consequence of the universality is also consistent with observations [5] where it is found that the correlation depends very weakly on $|k_{\perp,\alpha} - k_{\perp,\beta}|$.

8. A picture outlined above assumes an ideal world with $a \ll 1$ GeV when a very large $P$ odd domain is formed with size $\sim (2\pi)/a$ which is much larger than conventional QCD fluctuations with typical sizes $\sim \text{fm}$. In reality, one should expect some deviations from this
universal behaviour due to a number of complications in the real (rather than ideal) world, e.g. finite size of the system. In particular, the strength of the correlations is expected to increase when $|k_{\perp,\alpha} + k_{\perp,\beta}|$ increases for finite (rather than very large) $P$ odd domain. This is due to the fact that the probability to form two fm$^{-3}$ size particles within one finite size domain is larger if the particles have smaller sizes, and correspondingly larger $|k_{\perp,\alpha} + k_{\perp,\beta}|$. This deviation from the universality apparently consistent with observations [5] where it is found that the correlation in fact increases with $|k_{\perp,\alpha} + k_{\perp,\beta}|$ even for relatively large $k_{\perp} > 1$ GeV. Such a behaviour naively contradicts to a conventional intuition that all non-perturbative effects must be suppressed for large $k_{\perp} > 1$ GeV but in fact it has a simple and natural explanation within our framework as suggested above. Another manifestation of the same finite size effect would be a sharp cutoff in the correlations when the centrality continues to increase. This pure geometrical effect occurs when the available overlapping portion of the colliding nucleus and the size of a $P$ odd domain become approximately equal in sizes. The observation of the corresponding peak in strength of the correlations gives a precise experimental tool to measure the maximal effective size of $P$ odd domains for a given system. This feature, of course, can not have an universal description as it is related to the finite size effects, and therefore, is sensitive to specific properties of colliding nuclei.

To conclude: The qualitative consequences which follow from the picture outlined above apparently consistent with all presently available data. In reality "a" is not a small number, and the size of $P$ odd domain is not very large even for non-central collisions. The finite size effects and other non-universal features may lead to some corrections from the universal picture as we mentioned in item 8 above. Moreover, those corrections themselves are not expected to follow the universal behaviour. Much work needs to be done before a qualitative picture sketched above becomes a quantitative description of the correlations observed at RHIC [1, 2, 3, 4, 5].

6. Conclusion. Future Directions.

In this paper we adopt the approach suggested in refs. [36, 37, 38, 39, 40, 41, 42, 43] and treat the universality observed in all high energy collisions as a result of the Unruh radiation characterized by a single parameter "a". The problem of computation the acceleration "a" is not addressed in the present paper. It is is obviously very hard problem of strongly interacting QCD. Instead, we study the topological fluctuations (represented in our framework by the Veneziano ghost) in the given accelerating background "a". Such a treatment is a consistent description in large $N_c$ limit when the influence of these degrees of freedom on acceleration "a" itself is negligible. There is a number of immediate consequences from this picture relevant for analysis of the correlations observed at RHIC [1, 2, 3, 4, 5] and which were outlined in section 5.2. We formulate the universal properties for the $P$ odd effects, similar to well-known universal thermal behaviour of the spectrum studied in all high energy collisions. We observe that our predictions are consistent with all presently available data.
We formulate below some possible directions for the future study which may confirm or rule out this entire framework.

- It would be very desirable (if not crucial) to understand the connection between the acceleration parameter “$a$” which is the key element of the present paper and familiar parameters such as centrality, initial energy and the charges of the colliding ions in realistic (rather than ideal) world. Such knowledge would allow us to (quantitatively) test the basic conjecture on universality of the $P$ odd effects formulated in section 5.2.
- It would be very interesting to study the $P$ odd correlations for other high energy collisions, beyond the heavy ion systems. As we mentioned in the text the $P$ odd domains are also produced in $e^+e^-$, $pp$ and $p\bar{p}$ systems, though the size of the produced $P$ odd domain would be quite small as it is determined by $2\pi/a$ with $a$ given by eq. (1.3). If one uses the same correlation [1] which has been used for analysis of heavy ion system, the universality arguments suggest that the intensity of the correlations in $e^+e^-$, $pp$ and $p\bar{p}$ systems would be exactly the same as measured in heavy ions in most central collisions when the effective acceleration “$a$” is determined by the same expression for all systems (1.3).
- The Veneziano ghost which is the subject of this paper may in fact lead to another IR effect demonstrating the sensitivity to the boundaries (in addition to sensitivity to the horizon scale studied in this work). Specifically, the Veneziano ghost which is protected by topology and which saturating the subtraction term in the topological susceptibility, may lead to the Casimir type effects as argued in [61] though no massless physical degrees of freedom are present in the system. This effect can be exactly computed in 2d QED which is known to be the model with a single massive degree of freedom [55]. Still, the Casimir like effect is present in this system [57]. The Casimir type effects in 4d QCD appear to be present on the lattice where the power like behaviour $(1/L)^{\alpha}$ as a function of the total lattice size $L$ has been observed in measurements of the topological susceptibility [62]. Such a behaviour is in huge contrast with exponential $\exp(-L)$ decay law which one normally expects for any theories with massive degrees of freedom.
- The obtained results may have some profound consequences for our understanding of physics at the largest possible scales in our universe as a result of dynamics of the same (protected by topology) Veneziano ghost. Namely, the Casimir type effects in 4d QCD may be observable using the CMB analysis as suggested in ref. [63].
- Another manifestation of the same physics is as follows. The dark energy observed in our universe might be a result of mismatch between the vacuum energy computed in slowly expanding universe with the expansion rate $H$ and the one which is computed in flat Minkowski space. If true, the difference between two metrics would lead to an estimate $\Delta E_{vac} \sim H\Lambda_{QCD}^3 \sim (10^{-3}\text{eV})^4$ which is amazingly close to the observed value today[53]. As explained in the text the typical wavelengths $\lambda$ associated with this energy density are of the order of the inverse Hubble parameter, $\lambda \sim (2\pi/H) \sim 10 \text{ Gyr}$, and therefore, these modes do not clump on distances smaller than $H^{-1}$, in contrast with all other types of matter. This makes these fluctuations to be the perfect dark energy candidates [53].
- Also, a nature of the magnetic field in the universe with characteristic intensity of around

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5I am thankful to Misha Polikarpov who brought the paper [62] to my attention.
a few $\mu G$ correlated on very large scales and observed today is still unknown. One can argue that the very same Veneziano ghost which is the subject of the present work may in fact induce the large scale magnetic field with correlation length $\sim (2\pi/H)$ as a result of anomalous interaction similar to the one which leads to the charge separation and chiral magnetic effects (1.1). In the case of heavy ions the correlation length for charge separation effect and CME is determined by the size of the $\mathcal{P}$ odd domain $\sim (2\pi/a)$, see section 1.1, while in case of expanding universe the correlation length $\sim (2\pi/H)$. More than that, the corresponding induced magnetic field in the universe is expected to be helical (i.e. $\int d^3x \vec{A} \cdot \vec{B} \neq 0$) and would naturally have the intensity $B \simeq \frac{\alpha}{2\pi} \sqrt{H \Lambda_{QCD}^3} \sim nG$, which by simple adiabatic compression during the structure formation epoch, could explain the field observed today at all scales, from galaxies to superclusters [64].

• Finally, the cosmological observations on the largest scales exhibit a solid record of unexpected anomalies and alignments, apparently pointing towards a large scale violation of statistical isotropy. These include a variety of CMB measurements, as well as alignments and correlations of quasar polarization vectors over huge distances of order of 1 Gpc. The only comment I would like to make here that such anomalies may in fact be originated from the same fundamental topological $\mathcal{P}$ odd fluctuations studied in this work, see [65] for the details.

To conclude: the development of the early Universe is a remarkable laboratory for the study of most nontrivial properties of particle physics such as $\mathcal{P}$ odd effects on the scale of the entire universe. What is more remarkable is the fact that the very same phenomena can be, in principle, experimentally tested in heavy ion collisions, where a similar environment can be achieved.

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A. The Veneziano ghost is not an asymptotic state.

The main goal of this Appendix is to argue that while the Veneziano ghost leads to a number of profound effects in Minkowski space (the resolution of the $U(1)_A$ problem) as well as in the accelerating frame (present work), it nevertheless is not a conventional propagating asymptotic state. In different words, it does not contribute to absorptive parts of any correlation functions. However, it does contribute to the real parts of the correlation functions. In Minkowski space such kind of contributions normally treated as the subtraction constants. In the present case of the accelerating frame, the corresponding
“subtraction constant” becomes a “subtraction function” which depends on acceleration
and which is sensitive to the global properties of the space-time.

The crucial observation for future analysis is as follows: the fields $\phi_1, \phi_2$ which
are originated from unphysical (in Minkowski space) degrees of freedom can couple
to other fields only through a combination $(\phi_1 - \phi_2)$ as a consequence of the original
gauge invariance (3.10). Precisely this property along with Gubta-Bleuler auxiliary
condition (3.12) provides the decoupling of physical degrees of freedom from unphysical
combination $(\phi_2 - \phi_1)$ as discussed in great details in [55].

We follow analysis of ref. [34] to study the propagating features of fields in
accelerating frame. To achieve this goal we replace a single physical field $\Phi$ from
ref. [34] by specific combination $(\phi_2 - \phi_1)$ fields for our system (3.10). It leads to
some drastic consequences as instead of conventional expectation values such as
$<0|a_k...a_{k'}|\Phi|0>$ from ref. [34]
we would get $<0|(a_k - b_k)...(a_{k'} - b_{k'})|0>$ = 0. The relevant matrix elements
vanish as a result of the corresponding commutation relation $[(a_{k'} - b_{k'}, a_k - b_k)] = 0$.
Furthermore, as $[H, (a_k - b_k)] = (a_k - b_k)$ the structure $(a_k - b_k)$ is preserved such that
$a_k$ and $b_k$ never appear separately. Based on this observation, one can argue that
the same property holds for any other operators which constructed from the
combination $(\phi_2 - \phi_1)$. In different words, no actual radiation of real particle occurs
in our case in contrast with real Unruh radiation [34], i.e the absorptive parts
of relevant correlation functions always vanish. Therefore, there are
some fluctuating degrees of freedom in the system observed by a Rindler observer without
radiation of any real particles. In many respects, this feature is similar to the Casimir
effect though spectral density distribution describing the fluctuations of the vacuum energy
has a nontrivial $\omega$ dependence in contrast with what happens in the Casimir effect.

Another way to arrive to the same conclusion is to consider the particle detector moving
along the world line described by some function $x^\mu(\tau)$ where $\tau$ is the detector’s proper
time. In the case for the Rindler space the corresponding $\tau$ is identified with $\eta$ defined by
formula (2.2). As is known, the corresponding analysis in the lowest order approximation
is reduced to study of the positive frequency Wightman Green function defined as
\begin{equation}
D^+(x,x') = \langle 0|\Phi(x), \Phi(x')|0\rangle, \tag{A.1}\end{equation}
while the transition probability per unit proper time is proportional to its Fourier transform,
\begin{equation}
\sim \int_{-\infty}^{+\infty} d(\Delta\tau) e^{-i\omega\Delta\tau} D^+(\Delta\tau) \tag{A.2}\end{equation}

where we use notations from [35]. In case of inertial trajectory for massless scalar field
the positive frequency Wightman Green function is given by
\begin{equation}
D^+(\Delta\tau') = -\frac{1}{4\pi^2} \frac{1}{(\Delta\tau' - i\epsilon)^2} \tag{A.3}\end{equation}
and the corresponding Fourier transform (A.2) obviously vanishes. No particles are
detected as expected. In case if the detector accelerates uniformly with acceleration $a$ the
corresponding Green’s function is given by \[ \text{(A.4)} \]

\[
D^+(\Delta \tau') = -\frac{1}{4\pi^2} \sum_k \frac{1}{(\Delta \tau - i2\epsilon + 2i\pi k a)^2}.
\]

As there are infinite number of poles in the lower -half plane at \( \Delta \tau = -2i\pi \frac{k}{a} \) for positive \( k \) the corresponding Fourier transform (A.2) leads to the known result \( \sim \omega[\exp(2\pi \omega/a) - 1]^{-1} \).

In our case the detector- field interaction is described by the combination \( \phi_1 - \phi_2 \) rather by a single field \( \Phi \) discussed above. Therefore, the relevant response function in our case is described by the positive frequency Green’s function defined as

\[
\sim \langle 0 | \left( \phi_1(x) - \phi_2(x) \right) \left( \phi_1(x') - \phi_2(x') \right) | 0 \rangle, \tag{A.5}
\]

which replaces eq. (A.1). One can easily see that this Green’s function given by eq. (A.5) identically vanishes as the consequence of the opposite signs in commutation relations describing \( \phi_1 \) and \( \phi_2 \) fields, in complete agreement with the arguments presented above. Therefore, the Rindler observer will see an extra energy (4.11) without detecting any physical particles. One can rephrase the same statement by saying that the ghost and its partner do not contribute to the absorptive part of the Green’s function, but do contribute to its real part. In Minkowski space the contribution to the real part of the topological susceptibility is nothing but the well known subtraction constant which has been precisely studied and measured on the lattice, see Fig.1. The ghost- based technique we advocate in this paper is well suited to study the corresponding physics in accelerating or time dependent background.

References

[1] S. A. Voloshin, Phys. Rev. C 70, 057901 (2004) [arXiv:hep-ph/0406311].
[2] I. V. Selyuzhenkov [STAR Collaboration], Rom. Rep. Phys. 58, 049 (2006) [arXiv:nucl-ex/0510069].
[3] S. A. Voloshin [STAR Collaboration], arXiv:0806.0029 [nucl-ex], arXiv:1006.1020 [nucl-th].
[4] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 103, 251601 (2009) [arXiv:0909.1739 [nucl-ex]].
[5] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. C 81, 054908 (2010) [arXiv:0909.1717 [nucl-ex]].
[6] D. Kharzeev, Phys. Lett. B 633, 260 (2006) [arXiv:hep-ph/0406125].
[7] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A 797, 67 (2007) [arXiv:0706.1026 [hep-ph]].
[8] D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998) [arXiv:hep-ph/9804221].
[9] D. Kharzeev and R. D. Pisarski, Phys. Rev. D 61, 111901 (2000) [arXiv:hep-ph/9906401].
[10] I. E. Halperin and A. Zhitnitsky, Phys. Lett. B 440, 77 (1998) [arXiv:hep-ph/9807335].
[11] T. Fugleberg, I. E. Halperin and A. Zhitnitsky, Phys. Rev. D 59, 074023 (1999) [arXiv:hep-ph/9808469].
[12] K. Buckley, T. Fugleberg and A. Zhitnitsky, Phys. Rev. Lett. 84, 4814 (2000) [arXiv:hep-ph/9910229].
[13] K. Buckley, T. Fugleberg and A. Zhitnitsky, Phys. Rev. C 63, 034602 (2001) [arXiv:hep-ph/0006057].
[14] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008) [arXiv:0711.0950 [hep-ph]].
[15] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 78, 074033 (2008) [arXiv:0808.3382 [hep-ph]].
[16] D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 80, 034028 (2009) [arXiv:0907.5007 [hep-ph]].
[17] D. E. Kharzeev, Annals Phys. 325, 205 (2010) [arXiv:0911.3715 [hep-ph]].
[18] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, arXiv:0907.0494 [hep-lat]; arXiv:0909.1808 [hep-ph]; arXiv:0909.2350 [hep-ph].
[19] M. Abramczyk, T. Blum, G. Petropoulos and R. Zhou, arXiv:0911.1348 [hep-lat].
[20] V. Skokov, A. Illarionov and V. Toneev, arXiv:0907.1396 [nucl-th].
[21] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Nucl. Phys. A 836, 311 (2010) [arXiv:0912.2961 [hep-ph]].
[22] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. Lett. 104, 212001 (2010) [arXiv:1002.2495 [hep-ph]].
[23] A. Yu. Alekseev, V. V. Cheianov and J. Fröhlich, Phys. Rev. Lett. 81, 3503 (1998).
[24] J. Charbonneau and A. Zhitnitsky, Phys. Rev. C 76, 015801 (2007) [arXiv:astro-ph/0701308].
[25] J. Charbonneau and A. Zhitnitsky, JCAP 1008, 010 (2010) [arXiv:0903.4450 [astro-ph.HE]].
[26] D. T. Son and A. R. Zhitnitsky, Phys. Rev. D 70, 074018 (2004) [arXiv:hep-ph/0405216].
[27] M. A. Metlitski and A. R. Zhitnitsky, Phys. Rev. D 72, 045011 (2005) [arXiv:hep-ph/0505072].
[28] G. Lifschytz and M. Lippert, Phys. Rev. D 80 (2009) 066005, [0904.4772]; H. U. Yee, JHEP 0911, 085 (2009) [arXiv:0908.4189 [hep-th]]; A. Rebhan, A. Schmitt and S. A. Stricker, JHEP 1001, 026 (2010) [arXiv:0909.4782 [hep-th]]; A. Gorsky, P. N. Kopnin and A. V. Zayakin, arXiv:1003.2293 [hep-ph]; A. Gyunther, K. Landsteiner, F. Pena-Benitez and A. Rebhan, arXiv:1005.2587 [hep-th].
[29] V. A. Rubakov, arXiv:1005.1888 [hep-ph].
[30] L. Brits, J. Charbonneau, [arXiv:1009.4230 [hep-th]].
[31] R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147; Nuovo Cim. A 56 (1968) 1027.
[32] A. Salam and J. Strathdee, Phys. Lett. B 66 (1977) 143; S. Barshay and W. Troost, Phys. Lett. B 73 (1978) 437; A. Hosoya, Progr. Theoret. Phys. 61 (1979) 280; M. Horibe, Progr. Theoret. Phys. 61 (1979) 661;
[33] W. G. Unruh, Phys. Rev. D 14, 870 (1976);
[34] W. G. Unruh and R. M. Wald, Phys. Rev. D 29, 1047 (1984).
[35] N. D. Birrell and P. C. W. Davies, *Quantum Fields In Curved Space*, Cambridge Univ. Pr., 1982.

[36] D. Kharzeev and K. Tuchin, Nucl. Phys. A 753, 316 (2005) [arXiv:hep-ph/0501234].

[37] D. Kharzeev, Nucl. Phys. A 774, 315 (2006) [arXiv:hep-ph/0511354].

[38] D. Kharzeev, E. Levin and K. Tuchin, Phys. Rev. C 75, 044903 (2007) [arXiv:hep-ph/0602063].

[39] D. Kharzeev, Eur. Phys. J. A 29, 83 (2006).

[40] P. Castorina, D. Kharzeev and H. Satz, Eur. Phys. J. C 52, 187 (2007) [arXiv:0704.1426 [hep-ph]].

[41] H. Satz, Eur. Phys. J. ST 155, 167 (2008).

[42] F. Becattini, P. Castorina, J. Manninen and H. Satz, Eur. Phys. J. C 56, 493 (2008) [arXiv:0805.0964 [hep-ph]].

[43] P. Castorina, D. Grumiller and A. Iorio, Phys. Rev. D 77, 124034 (2008) [arXiv:0802.2286 [hep-th]].

[44] L. D. McLerran and R. Venugopalan, Phys. Rev. D 49, 2233 (1994) [arXiv:hep-ph/9309289].

[45] L. D. McLerran and R. Venugopalan, Phys. Rev. D 49, 3352 (1994) [arXiv:hep-ph/9311205].

[46] P. Steinberg, Acta Phys. Hung. A 24, 51 (2005) [arXiv:nucl-ex/0405022].

[47] N. Evans and A. Tedder, Phys. Rev. Lett. 100, 162003 (2008) [arXiv:0711.0300 [hep-ph]].

[48] Y. Hatta and T. Matsuo, Phys. Rev. Lett. 102, 062001 (2009) [arXiv:0807.0098 [hep-ph]].

[49] G. Veneziano, Nucl. Phys. B 159, 213 (1979).

[50] P. Di Vecchia and G. Veneziano, Nucl. Phys. B 171, 253 (1980).

[51] E. Witten, Nucl. Phys. B 156, 269 (1979).

[52] C. Rosenzweig, J. Schechter and C. G. Trahern, Phys. Rev. D 21, 3388 (1980).

[53] F. R. Urban and A. R. Zhitnitsky, Nucl. Phys. B 835, 135 (2010) [arXiv:0909.2684 [astro-ph.CO]].

[54] A. R. Zhitnitsky, Phys. Rev. D 82, 103520 (2010), arXiv:1004.2040 [gr-qc].

[55] J. B. Kogut and L. Susskind, Phys. Rev. D 11, 3594 (1975).

[56] Nobuyoshi Ohta, “Dark Energy and QCD Ghost”, arXiv:1010.1339[astro-ph.CO]

[57] F. R. Urban and A. R. Zhitnitsky, Phys. Rev. D 80, 063001 (2009) [arXiv:0906.2165 [hep-th]].

[58] S. Gupta, Proc. Phys. Soc. A 63, 681, (1950).

[59] K. Bleuler, Helv. Phys. Acta 23, 567 (1950).

[60] C. Bernard et al, PoS (LATTICE 2007) 310 [arXiv:0710.3124 hep-lat].

[61] F. R. Urban and A. R. Zhitnitsky, Phys. Lett. B 688, 9 (2010) [arXiv:0906.2162 [gr-qc]].

[62] F. V. Gubarev, S. M. Morozov, M. I. Polikarpov and V. I. Zakharov, JETP Lett. 82, 343 (2005) [Pisma Zh. Eksp. Teor. Fiz. 82, 381 (2005)] [arXiv:hep-lat/0505016]

[63] F. R. Urban and A. R. Zhitnitsky, JCAP 0909, 018 (2009) [arXiv:0906.3546 [astro-ph.CO]].
[64] F. R. Urban and A. R. Zhitnitsky, Phys. Rev. D 82, 043524 (2010) [arXiv:0912.3248 [astro-ph.CO]].

[65] F. R. Urban, A. R. Zhitnitsky, “The $\mathcal{P}$-Parity Odd Universe, Dark Energy and QCD,” [arXiv:1011.2425 [astro-ph.CO]].