Optomechanical correlations and signal self-amplification in interferometric measurements

P.-F. Cohadon, P. Verlot, A. Tavernarakis, T. Briant, A. Heidmann
Laboratoire Kastler Brossel, ENS, UPMC, CNRS,
Case 74, 4 place Jussieu, F75252 Paris Cedex 05, France

Abstract. Radiation pressure exerted by light in interferometric measurements is responsible for displacements of mirrors which appear as an additional back-action noise and limit the sensitivity of the measurement. We experimentally study these effects by monitoring in a very high-finesse optical cavity the displacements of a mirror with a sensitivity at the $10^{-20} \text{m/Hz}$ level. This very high sensitivity is a step towards the observation of fundamental quantum effects of radiation pressure such as the standard quantum limit in interferometric measurements. We report the observation of optomechanical correlations between two optical beams sent into the same moving mirror cavity. We also observed a self-amplification of a signal, which is a consequence of dynamical back-action of radiation pressure in a detuned cavity, and may improve the interferometric measurement sensitivity beyond the standard quantum limit.

1. Introduction
In quantum mechanics, measurements are always related to a back-action on the measured system, which may limit the sensitivity. Interferometers are ruled by such limits: quantum effects of optomechanical coupling, the radiation-pressure coupling between a moving mirror and an incident light field, cause displacements of mirrors and lead to a standard quantum limit (SQL) for the measurement [1, 2, 3]. Such radiation-pressure effects are so weak that they haven’t been experimentally demonstrated yet, although the future generations of gravitational-wave interferometers will most probably be confronted to these limits [4, 5].

Overcoming these limits was a major motivation for the quantum optics community, and number of quantum noise reduction schemes have been proposed which rely on the use of squeezed light sent into the interferometer [6, 7, 8], or on the use of the quantum correlations induced by radiation pressure between phase and intensity fluctuations [9]. Dynamical effects of radiation pressure play also an important role in detuned signal-recycled gravitational-wave interferometers, giving the possibility for resonant amplification of the gravitational-wave signal and for reshaping the noise curves beyond the standard quantum limit [10, 11].

We present in this paper an experiment devoted to the observation of quantum effects of radiation pressure and to the test of quantum-noise reduction schemes. Our experimental setup is based on a high-finesse Fabry-Perot cavity used to monitor the small displacements of a mirror. The experiment already achieved a sufficient sensitivity to analyze the internal thermal noise of the mirror at room temperature and to fully characterize the optomechanical properties of its internal vibration modes [12, 13, 14]. Recent progress in the experimental setup and the development of a dual optical injection system allowed us to observe various radiation-pressure effects in our experiment[15, 16]. We report the observation of radiation-pressure induced
Principle of an optomechanical sensor based on a high-finesse cavity. Any displacement of the moving mirror or any optical length variation δx_{sig} of the cavity are detected through the phase shift δϕ_{out} of the field reflected by the cavity. Quantum back-action noise corresponds to the spurious displacements of the moving mirror induced by the quantum fluctuations of the intracavity radiation pressure.

correlations between the intensity of an optical beam and the phase of a second beam, both sent into the same moving mirror cavity. We also observed a self-amplification of a signal, which may allow to overcome the standard quantum limit [17] as predicted in detuned signal-recycled gravitational-wave interferometers.

2. Radiation pressure and optomechanical coupling

The optomechanical coupling is the cross-coupling between a movable mirror and the light reflected upon it. Any displacement of the mirror induces a linear phase-shift of the reflected beam, providing a possibility to read out the displacement. On the other hand, radiation pressure appears as an additional force which perturbs the mirror motion. This corresponds to the back-action of the displacement measurement. When the mirror is part of a high-finesse optical cavity (Fig. 1), the optomechanical coupling is enhanced and the phase of the reflected field becomes very sensitive to mirror displacements. At resonance, the corresponding phase-shift δϕ_{out} is given by

\[ \delta \varphi_{\text{out}} = \delta \varphi_{\text{in}} + \frac{8 F}{\lambda} (\delta x_{\text{sig}} + \delta x_{\text{m}}), \]

where \( \lambda \) is the optical wavelength, \( F \) the cavity finesse, and \( \delta \varphi_{\text{in}} \) the phase fluctuations of the incident beam. The signal \( \delta x_{\text{sig}} \) can either be a physical displacement of the mirror, such as the response to an external force, or an apparent variation of the cavity length due for example to a gravitational wave. Fluctuations \( \delta x_{\text{m}} \) represent spurious displacements of the mirror, including thermal and radiation-pressure noises. The latter corresponds to the mechanical response of the mirror to the radiation pressure exerted by the intracavity field. In the framework of linear response theory [18], the mirror response \( \delta x_{\text{rad}} [\Omega] \) to the radiation-pressure fluctuations \( \delta F_{\text{rad}} [\Omega] \) at a frequency \( \Omega \) is proportional to the mechanical susceptibility \( \chi [\Omega] \) of the mirror:

\[ \delta x_{\text{rad}} [\Omega] = \chi [\Omega] \delta F_{\text{rad}} [\Omega] = 2 \hbar k \chi [\Omega] \delta I [\Omega], \]

where \( k \) is the field wavevector, and \( \delta I [\Omega] \) the intracavity intensity fluctuations.

Neglecting thermal and classical noises, the sensitivity of the measurement is ultimately limited by the sum of two quantum noises: the phase noise \( \delta \varphi_{\text{in}} \) of the incident light beam and the radiation-pressure noise \( \delta x_{\text{rad}} \). A compromise between the two leads to the so-called standard quantum limit [1, 2, 3, 4], which corresponds to a minimum measurable length variation \( \delta x_{\text{sig}} \) equal to \( \sqrt{\hbar |\chi[\Omega]|} \). This limit has never been experimentally observed and a step towards its experimental demonstration would be the observation of the quantum radiation-pressure noise \( \delta x_{\text{rad}} \), which is responsible for correlations between the intensity and the phase of the light.
Figure 2. Experimental setup. A laser source provides two orthogonally polarized beams which are both sent into the moving mirror cavity. The signal beam is used to set the laser frequency at resonance via a Pound-Drever-Hall technique, while the meter beam can be detuned by an acousto-optic modulator (AO₂). The phase of the reflected meter beam is monitored by a homodyne detection, and the intensity of the signal beam by a photodiode. A signal generator provides either an intensity noise of the signal beam (via the electro-optic modulator EO) to test optomechanical correlations, or a modulation of the laser frequency to demonstrate the self-amplification of a signal.

To observe these optomechanical correlations, we send two beams in the cavity: the intensity fluctuations of the first, intense, signal beam drive the mirror into motion by radiation pressure, whereas the resulting position fluctuations $\delta x_{\text{rad}}$ are monitored through the phase $\delta \phi_{\text{out}}$ of the second, weaker, meter beam. As the intensity fluctuations of the signal beam are unaltered by reflection upon the cavity and as far as the radiation pressure of the meter beam is negligible, the intensity-phase correlations observable between the two reflected beams provide a direct measurement of the optomechanical correlations. Also note that this measurement actually corresponds to a quantum non-demolition (QND) measurement of the intensity of the signal beam by the phase of the meter beam, as it has already been performed with nonlinear optical media [19], but here based on the optomechanical coupling [20, 21].

3. Experimental setup

Our experimental setup (Fig. 2) is based on a high-finesse cavity built with a cylindrical input mirror and a fused-silica plano-convex end mirror which exhibits gaussian internal vibration modes [14]. We work at frequencies close to one mechanical resonance with the following optomechanical characteristics, deduced from the observation of the thermal noise at room temperature: resonance frequency $\Omega_m/2\pi = 1128$ kHz, mass $M = 72$ mg, and mechanical quality factor $Q = 760\,000$ when the cavity is operated in vacuum. The cavity is very short and compact (0.25 mm long) and we have obtained a cavity finesse up to $\mathcal{F} = 330\,000$.

A Titane-Sapphire laser working at 810 nm provides two cross-polarized signal and meter beams, both sent into the high finesse cavity. The laser is frequency stabilized on the resonance of the cavity via a Pound-Drever-Hall technique: the incident signal beam is phase-modulated at 20 MHz by a resonant electro-optical modulator (REO), and the resulting intensity modulation of the reflected beam provides the error signal. The meter beam can be arbitrarily detuned from the cavity resonance by an acousto-optic modulator (AO₂). Both beams are intensity stabilized...
and spatially filtered by a mode cleaner.

The phase fluctuations $\delta \varphi_{m}^{\text{out}}$ of the reflected meter beam are monitored by a homodyne detection, with a local oscillator derived from the incident meter beam and phase-locked in order to detect the phase quadrature. With this setup, we obtain a shot-noise-limited sensitivity to mirror displacements of $2.7 \times 10^{-20} \text{m}/\sqrt{\text{Hz}}$ at frequencies above 200 kHz. Intensity fluctuations $\delta I_{s}^{\text{out}}$ of the reflected signal beam are monitored by a high-efficiency photodiode. We have carefully eliminated unwanted optical reflections so that the optical rejection of the double-beam injection system is higher than 35 dB: the phase fluctuations of the meter beam are insulated from the intensity fluctuations of the signal beam in such a way that any observable effects of the signal beam are necessarily induced by intracavity radiation pressure.

4. Observation of optomechanical correlations

We have performed an experiment to demonstrate the optomechanical correlations induced by radiation pressure between the intensity of the signal beam and the phase of the meter beam. Since quantum radiation-pressure effects are dominated by thermal noise in our system at room temperature, we use a tiny classical intensity noise of the signal beam that mimics its quantum fluctuations but at a level at least 30 dB higher \[15, 22\]. This noise is produced by a wideband electro-optical modulator (EO in Fig. 2) driven by a digitized noise source. We generate a gaussian intensity noise centered at a frequency $\Omega_{c}$ close to the mechanical resonance $\Omega_{m}$,

$$
\delta I_{s}^{\text{in}}(t) = X_{s}^{\text{in}}(t) \cos(\Omega_{c}t) + Y_{s}^{\text{in}}(t) \sin(\Omega_{c}t),
$$

where the two slowly-varying quadratures $X_{s}^{\text{in}}$ and $Y_{s}^{\text{in}}$ are random gaussian functions.

The intensity of the reflected signal beam and the phase of the meter beam are independently acquired by two spectrum analysers, which directly extract the quadratures $X_{s}^{\text{out}}(t)$, $Y_{s}^{\text{out}}(t)$, $X_{m}^{\text{out}}(t)$, and $Y_{m}^{\text{out}}(t)$ of the reflected signal intensity and meter phase, respectively. Figure 3 presents the observed phase-space trajectories: clear correlations are obtained between the intensity noise of the signal beam (left) and the meter phase noise (right). Small discrepancies are visible between the two trajectories, mainly due to the thermal noise of the mirror which is 5 times smaller than the displacements induced by the radiation pressure. The optomechanical correlations can be made more quantitative by computing the correlation coefficient $C_{I_{s}, \varphi_{m}}$ defined as

$$
C_{I_{s}, \varphi_{m}} = \frac{\left| \langle \delta I_{s}^{\text{out}} \delta \varphi_{m}^{\text{out}*} \rangle \right|^2}{\langle |\delta I_{s}^{\text{out}}|^2 \rangle \langle |\delta \varphi_{m}^{\text{out}}|^2 \rangle},
$$

Figure 3. Phase-space trajectories of the intensity noise of the signal beam (left) and the phase noise of the meter beam (right).
Figure 4. Modulation of the meter phase in response to a signal modulation swept around the mechanical resonance frequency $\Omega_m$. Curves a to c correspond to increasing negative cavity detunings. The optomechanical sensor exhibits a self-amplification of the signal at a frequency close to the effective resonance of the mirror, down-shifted from $\Omega_m$ by optical spring. The dashed curve is a fit of curve b deduced from eq. (5).

where the brackets $\langle \ldots \rangle$ stand for a temporal average. We obtain a coefficient $C_{I_s,\phi_m} \simeq 0.96$, in perfect agreement with the theoretical value expected when radiation-pressure and thermal noises are assumed to be preponderant in the measurement [16].

5. Observation of signal self-amplification

When the cavity is detuned with respect to the laser frequency, dynamical effects of radiation pressure are responsible for a change of the mechanical behavior of the moving mirror. As the intracavity intensity follows an Airy peak when the cavity length is changed, the displacements of the mirror are coupled to the intensity fluctuations in such a way that the radiation-pressure force becomes proportional to the displacement itself. This restoring force leads to an optical spring effect which changes the mechanical susceptibility of the mirror. The resulting effective susceptibility $\chi_{\text{eff}}$ exhibits both a shift of the resonance frequency and a modification of its damping. Both effects have been observed in many experiments [23, 24, 25, 26, 27, 28], the optical damping being responsible for an efficient self-cooling of the mirror.

We demonstrate in this section that the radiation pressure is also responsible for a self-amplification of a signal, enabling the measurement sensitivity to beat the standard quantum limit. The signal is a modulation $\delta x_{\text{sig}}$ of the optical length of the cavity, experimentally obtained through a modulation of the laser frequency (Fig. 2). In a detuned cavity, the radiation-pressure force becomes proportional to this length variation, leading to a mirror displacement $\delta x_m$ proportional to $\delta x_{\text{sig}}$. According to eq. (1), one can show [17, 29] that the phase $\delta \phi_m^{\text{out}}$ reproduces the total length variation equal to

$$\delta x_m[\Omega] + \delta x_{\text{sig}}[\Omega] = \frac{\chi_{\text{eff}}[\Omega]}{\chi[\Omega]} \delta x_{\text{sig}}[\Omega]. \quad (5)$$

Depending on the ratio between the initial and effective susceptibilities, one then gets either an amplification or a compensation of the signal by the mirror motion.

The signal beam is now only used to lock the meter beam frequency at a specific detuning with respect to the optical resonance of the cavity. We use a network analyzer to sweep the signal $\delta x_{\text{sig}}$ around the mechanical resonance and to monitor the resulting phase modulation of the reflected meter beam. Curves in fig. 4 show the experimental results obtained for different detunings, together with a theoretical curve deduced from eq. (5). They clearly show a self-amplification of the signal near the effective mechanical resonance of $\chi_{\text{eff}}$, which is down-shifted
from $\Omega_m$ by the optical spring effect. We reach a self-amplification factor larger than 6 for curve $b$: dynamical back-action effects induce a motion $\delta x_m$ of the mirror in phase with the signal $\delta x_{\text{sig}}$, and with an amplitude larger than the signal itself.

6. Conclusion
We have presented an experiment based on the ultrasensitive measurement of the motion of a mirror in a high-finesse cavity. The mirror exhibits a mechanical resonance at a frequency around 1 MHz with a quality factor close to $10^6$. Our setup is then very sensitive to the intracavity radiation pressure and may be used to demonstrate quantum effects of optomechanical coupling. We have observed the optomechanical correlations induced by radiation pressure between two light beams sent into the cavity. Such correlations are still at the classical level but similar correlations are expected in the quantum regime at low temperature. Inserting the cavity in a cryogenic environment should enable to detect these quantum correlations and hence demonstrate quantum radiation-pressure noise.

We have also observed a dynamical radiation-pressure effect corresponding to a self-amplification of a signal in a detuned cavity. Although our setup is currently limited by thermal noise, such a scheme has been shown to allow to overcome the standard quantum limit [17], as it is expected in detuned signal-recycled gravitational-wave interferometers. Using the parameters of our experiment and the self-amplification factor obtained so far, sensitivity may be improved beyond the standard quantum limit by more than 9 dB for frequencies close to the effective mechanical resonance of the mirror. Further experimental work is underway to reduce the thermal noise and to reach the corresponding sensitivity.

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