$R_F$ Parity and Almost Massless Up Quark

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Abstract

We introduce a parity $R_F$ to introduce naturally small masses for the first family members and in particular almost massless $u$ quark toward the strong CP solution. We also discuss the phenomenological implications of this model on the proton decay and the neutrino mass. Furthermore, it is possible to embed this $R_F$ parity to local $U(1)_R$ gauge symmetry.
The massless u-quark scenario is one of the attractive solutions of the strong CP problem [1], which seems to be still alive [2], but there does not exist a compelling model for this [3]. Toward a solution of this kind, one must introduce a symmetry which distinguishes the family number and is probably broken spontaneously. If this symmetry guarantees the masslessness of the u-quark only, then the symmetry need not be broken. But, if the symmetry renders other particles such as electron and d quark massless, then it must be spontaneously broken.

The most abnormal masses in the standard model is the masses of the first family which are $10^{-5}$ times smaller than the electroweak symmetry breaking scale. Because of the smallness of these masses, the radiative generation of the first family masses were considered before, but has not led to massless u-quark [5].

Toward the solution of the gauge hierarchy problem, supersymmetry seems to be needed [6]. In this supersymmetric scenario, we encounter the unwanted R-parity violating terms in general. To forbid these, the R-parity defined as $R = (-1)^{3B+L+2S}$ is assumed to be conserved [7].

In this paper, we formulate a theory with naturally small electron and d-quark masses and almost vanishing u quark mass. Toward this purpose, let us generalize the R-parity to $R_F$ so that the family information is encoded,

$$R_F = (-1)^{3B+L+2S}(-1)^{2I_F},$$

where $B, L, S, I$ and $F$ are the baryon number, lepton number, spin, weak isospin, and the first family number, respectively. Namely,

$$F = \delta_{f1},$$

where $f = 1, 2, 3$. Then, the $R_F$ quantum numbers of chiral superfields are

$$
L_1 \quad L_2 \quad L_3 \quad E_1^c \quad E_2^c \quad E_3^c \\
+1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1
$$

$$
Q_1 \quad Q_2 \quad Q_3 \quad U_1^c \quad U_2^c \quad U_3^c \quad D_1^c \quad D_2^c \quad D_3^c \\
+1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1
$$

$$
H_1 \quad H_2 \\
+1 \quad +1
$$

where $H_1, H_2$ are the Higgs superfields with $Y = -1/2, +1/2$, respectively. All quark singlet superfields are given $R_F = -1$ so that there is no $\lambda''$ coupling. The lightest $R_F = -1$ particle is a neutrino or u-quark.

The most general $d = 3$ superpotential consistent with the $R_F$ parity is

1Note, however, that there is another school favoring $m_u \simeq 5$ MeV [3].

2H. Georgi introduced a $U(1)$ gauge symmetry for this purpose [4].
\[ W_0 = f^{(1)}_{ij} L^i_j E^c E^c_i H_1(i \neq 1) + f^{(a)}_{ij} Q^i_j U^c H_2(i \neq 1) + f^{(d)}_{ij} Q^i_j D^c H_1(i \neq 1) + \lambda_{ij} L^i_j E^c E^c_i H_1(i \neq 1) + \lambda_{ij} L^i_j Q^i_j D^c H_1(i \neq 1), \] which gives the following \( Q_{em} = 2/3 \) and \( Q_{em} = -1/3 \) quark mass matrices,

\[
M^{(2/3)} = \begin{pmatrix}
0, & H_0^0, & H_2^0 \\
0, & H_2^0, & H_3^0 \\
0, & H_0^0, & H_2^0
\end{pmatrix}, \quad M^{(-1/3)} = \begin{pmatrix}
\tilde{l}_0, & H_1^0 \text{ and } \tilde{l}_0, & H_1^0 \text{ and } \tilde{l}_0 \\
\tilde{l}_2, & H_1^0 \text{ and } \tilde{l}_0, & H_1^0 \text{ and } \tilde{l}_0 \\
\tilde{l}_0, & H_1^0 \text{ and } \tilde{l}_0, & H_1^0 \text{ and } \tilde{l}_0
\end{pmatrix},
\]

where we have suppressed the Yukawa couplings. The rows count the singlet anti-quarks, and the columns count the doublet quarks. The charged lepton matrices have the same form as the \( Q_{em} = -1/3 \) quark mass matrices.

It is obvious that \( \text{Det}M^{(2/3)} = 0 \), implying the massless u-quark. The special feature in supersymmetry is that the Higgs fields giving masses to u- and d-quarks are different. Only, \( H_2 \) can give a mass to u-quark. But d quark can obtain mass from \( H_1 \) and also from sneutrinos. This makes the difference between u- and d-quarks. In the limit of vanishing sneutrino VEV’s, the first family masses are all zero, rendering a partial hierarchy of masses among families.

However, if this vanishing sneutrino VEV’s of the second and third family persists, the model is futile due to the experimental facts on \( m_e \neq 0 \) and \( m_d \neq 0 \). Thus, we require that sneutrinos of the second and/or the third generation should develop a small vacuum expectation value so that d-quark and electron obtain masses.

The \( d = 2 \) superpotential consistent with the \( R_F \) parity is

\[
\mu H_1 H_2 + \mu L_1 H_2.
\]

Thus \( \tilde{\nu}_1 = \tilde{l}_0^0 \) develops a VEV since the scalar potential with soft terms contains

\[
m_0^2 |\tilde{l}_0^0|^2 + (A \mu_1 \frac{\tilde{l}_0^0}{\sqrt{2}} + \tilde{l}_0^0 + \text{h.c.}),
\]

where \( m_0 \) and \( A \) are of order supersymmetry breaking scale \( M_{\text{SUSY}} \). However, with the fields given in Eq. (2), \( \tilde{\nu}_{2,3} \) can never obtain VEV’s, and hence electron and d-quark remain massless. We must include other fields to give VEV’s to \( \tilde{\nu}_{2,3} \). As a minimal example, let us introduce a singlet superfield \( S \) with \( Y = 0 \) and \( R_F = -1 \) and introduce a small explicit \( R_F \) violating \( \epsilon^2 \) term in the singlet sector superpotential

\[
W_1 = M_S S^2 + \epsilon^2 S + f^3_S S L_i H_2(i \neq 1) + \frac{\lambda_{ijk} S D^c l_j U^c k}{M_P},
\]

where \( M_P \) is the Planck mass. To see the physical effects of breaking the \( R_F \) parity, we introduced the softly breaking \( \epsilon^2 \) term, which is hoped to mimic a general feature in other \( R_F \) breaking models. We require \( |\epsilon| \ll M_S \) so that \( R_F \) breaking is soft and weak, and \( M_S \) will be constrained later. For simplicity, let us consider the case \( f^3_S = 0 \) and \( f^2_S \equiv f_S \). Then, the scalar potential is described by

\[
V = V_F + V_D + V_{\text{soft}},
\]
Near \((S, \bar{\nu}_2) = (0,0)\), the relevant terms are

\[
V_F \simeq |M_S|^2 |S|^2 + \frac{|f_S|^2 v_2^2}{2} |\bar{\nu}_2|^2 + \frac{|f_S|^2 v_2^2}{2} |S|^2 + \left[ M_S \left( \epsilon^2 + f_S \frac{v_2}{\sqrt{2}} \bar{\nu}_2 \right) \right]^* S + \text{h.c.},
\]

\[
V_D \simeq \frac{M_Z^2 \cos 2\beta}{2} |\bar{\nu}_2|^2,
\]

\[
V_{\text{soft}} \simeq m_S^2 |S|^2 + m_\bar{\nu}_2^2 |\bar{\nu}_2|^2 + \left[ B_S M_S S^2 + B_\epsilon \epsilon^2 S + A_S f_S \frac{v_2}{\sqrt{2}} \bar{\nu}_2 \right] S + \text{h.c.},
\]

where \(m_S, m_\bar{\nu}_2, B_S, B_\epsilon, \) and \(A_S\) are of order supersymmetry breaking scale \(M_{\text{SUSY}}\). There appear linear terms \((\epsilon^2\) terms) for \(S\) in Eq. (9) and Eq. (11), and hence \(S\) develops a VEV. At this \(\langle S \rangle\) vacuum, \(\bar{\nu}_2\) contains linear terms also. Therefore, \(S\) and \(\bar{\nu}_2\) fields develop VEV’s in general.

Now let us try to estimate VEV’s for \(S\) and \(\bar{\nu}_i\). For this purpose, we impose the following four conditions to be satisfied:

(i) Electron and d-quark obtain \(O(1)\) MeV masses,
(ii) Neutrino mass is of order \(5 \times 10^{-2}\) eV [9],
(iii) The u-quark mass is sufficiently small, \(\delta m_u < 10^{-13}\) GeV [10], and
(iv) Proton does not decay too fast.

We find that the VEV’s of \(S\) and \(\bar{\nu}_i\) fields are typically given by

\[
\langle S \rangle \sim \frac{\epsilon^2}{M_S}, \quad \langle \bar{\nu}_i \rangle \sim \frac{f_S^i v_2 \epsilon^2}{M_{\text{SUSY}}^2}.
\]

In deriving the above VEV’s, we assume \(M_S \gg M_{\text{SUSY}}\) to get the neutrino mass consistent with the recent Super-Kamiokande data [4] as we will see. With these VEV’s, to give \(O(1)\) MeV masses to electron and d-quark, we obtain

\[
\left( |\lambda_{i1j}| \text{ or } |\lambda'_{i1j}| \right) \cdot f_S^i \cdot \epsilon^2 (i \neq 1) \sim O(10) \text{ GeV}^2,
\]

where we take \(v_2 \approx 10^2\) GeV and \(M_{\text{SUSY}} \approx 1\) TeV. Here, Eq. (13) can be satisfied for the largest \(\lambda_{i1j} (i \neq 1)\) for the electron mass and for the largest \(\lambda'_{i1j}\) for the d-quark mass.

Though the u-quark mass is zero at tree level, it can be generated radiatively when \(S\) field has a vacuum expectation value. The one-loop u quark mass is given by

\[
\delta m_u \sim \sum_{i=2,3} \frac{f_{31}^{(u)} \lambda_{i13} m_b f_S^i(S)}{16\pi^2 M_{\text{SUSY}}} \sim 3 \times 10^{-4} \frac{f_{31}^{(u)}}{(M_S/\text{GeV})} \text{ GeV},
\]

where we take \(m_b = 5\) GeV and \(M_{\text{SUSY}} = 1\) TeV. Therefore, to solve the strong CP problem, we obtain [10]

\[
\frac{f_{31}^{(u)}}{(M_S/\text{GeV})} < 3 \times 10^{-10}.
\]

Let us now proceed to discuss the proton decay rate and generation of neutrino mass, resulting from the \(R_F\) parity violation.

The VEV of \(S\) induces the conventional \(\lambda''\) couplings as follows
\[
\lambda_{ijk}'' \sim \frac{\bar{\lambda}_{ijk}''}{M_P M_S} \sim \bar{\lambda}_{Sijk}'' \left( \frac{\epsilon^2}{\text{GeV}^2} \right) \left( \frac{M_{\text{SUSY}}}{M_S} \right) \times 10^{-21}.
\] (16)

The bounds on the product of \( \lambda' \cdot \lambda'' \) from the proton stability are [11]

\[
\lambda_{1i1k} \cdot \lambda_{1i1k}'' < 10^{-24}, \quad \lambda'_{\text{any}} \cdot \lambda''_{\text{any}} < 10^{-9}.
\] (17)

And there are somewhat model-dependent bounds on single \( \lambda'' \) [12]. Noting that there are no \( \lambda'_{i1k} \) couplings in our model, the induced \( \lambda'' \) couplings are small enough to avoid the fast proton decay.

The mixing between \( S \), neutralinos and neutrino (\( \nu_2 \) in our example of \( f_3^3 = 0 \)) gives the following neutrino mass through the see-saw mechanism

\[
m_\nu \approx \frac{f_S^2 v_2^2}{M_S} + \frac{f_S^2 \epsilon^4}{M_{\text{SUSY}} M_S^2} - 2 \frac{f_S^4 v_2^4 \epsilon^4}{M_{\text{SUSY}}^3 M_S^2}.
\] (18)

The first term comes from the mixing between \( S \) and neutrino. The second term is the very well-known tree-level neutrino mass from the mixing between neutralino and neutrino under the presence of conventional R-parity violation [13]. We have neglected \( O(< \bar{\nu} >^2 / M_{\text{SUSY}}) \) which comes from the gaugino intermediate tree level diagram. Since we are interested in a generic bound on the coupling of \( S \), inclusion of this term would not change our conclusion very much. The last term of the above equation, coming from the overall mixing, can be neglected. For this neutrino mass to be consistent with the recent Super-Kamiokande data [9], \( \sqrt{\Delta m^2_{\text{atm}}} \sim 5 \times 10^{-2} \text{ eV} \), the following relations are to be satisfied

\[
\frac{f_S}{M_S} < 5 \times 10^{-15} \left( \frac{m_\nu}{5 \times 10^{-2} \text{ eV}} \right) \frac{1}{\text{GeV}}, \quad \text{and}
\frac{f_S \epsilon^2}{M_S} < 2 \times 10^{-4} \left( \frac{m_\nu}{5 \times 10^{-2} \text{ eV}} \right)^{1/2} \text{ GeV}.
\] (19)

For example, let’s think about the case that \( f_{13}^{(u)} \sim 3 \times 10^{-2} \) and \( \lambda \) or \( \lambda' \sim 10^{-3} \). In this case \( f_S \epsilon^2 \sim 10^4 \text{ GeV}^2 \) from the Eq. (13) and \( M_S > 10^8 \text{ GeV} \) from the Eq. (15). In this case the second condition of the Eq. (19) is fulfilled. Let’s take \( M_S = 10^8 \text{ GeV} \). Then, from the first condition of the Eq. (19), \( f_S < 7 \times 10^{-4} \). Let’s take \( f_S = 7 \times 10^{-4} \), then \( \epsilon \sim 4 \times 10^3 \) GeV. From all these, \( \langle S \rangle \sim 0.16 \text{ GeV} \) and \( \langle \bar{\nu} \rangle \sim 1 \text{ GeV} \). Finally \( \lambda''_{ijk} \sim 1.6 \times 10^{-19} \). We find that large enough \( M_S \) are sufficient for the massless u-quark scenario to be the solutions of the strong CP problem.

Finally, let us promote the \( Z_2 \) discrete symmetry group \( R_F \) to a subgroup of a local \( U(1)_R \) gauge group. This kind of discrete gauge symmetry is considered to be beautiful, because otherwise it is not guaranteed for the gravitational interaction to preserve the discrete symmetry [14]. To consider the anomaly, the additive \( U(1)_R \) charges of the fermionic fields are given by

\[
\begin{array}{cccccc}
L_1 & L_2 & L_3 & E_1^c & E_2^c & E_3^c \\
0 & 1 & 1 & 1 & 1 & 1
\end{array}
\]
If $U(1)_R$ with the above charges is broken to $Z_2$, we obtain the $R_F$ parity. The anomaly problem of $U(1)_R$ was considered by Ibanez and Ross \[15\]. Their anomaly free conditions can be summarized as

$$
\begin{align*}
\sum_i q^3_i &= 2m + n, \\
\sum_i q_i &= 2p + q, \\
\sum_i q_i &= 2r, \\
\sum_i q_i &= 2r', 
\end{align*}
$$

(21)

where $m, n, p, q, r,$ and $r'$ are integers. The first condition is for vanishing $U(1)^3_R$ anomaly, the second condition for $U(1)_R$–graviton–graviton anomaly, the third condition for $U(1)_R$–$SU(2)_L$ anomaly and the last condition for $U(1)_R$–$SU(3)_c$ anomaly. The first and the second conditions are trivial in our case. The last two conditions are also satisfied in our case. However, the fields given in Eq. (20) alone do not cancel $U(1)_R–U(1)_Y$ and $U(1)_R–U(1)_Y$ anomalies. This problem can be solved by introducing more singlets, which we will not specify. Thus our choice of $U(1)_R$ charges makes it possible to embed our $R_F$ symmetry to a local gauge symmetry $U(1)_R$.

In conclusion, we distinguished the families through a parity $R_F$ so that the first family members obtain naturally small masses. In particular, the up quark mass turns out to be even smaller, falling in the region of solving the strong CP problem.

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\[3\] To make the $u$-quark mass zero, we could have interchanged the $U(1)_R$ charges of $Q_1$ and $U^c_1$ fields. For this choice, however, anomaly free conditions for $U(1)_R–SU(2)_L$ and $U(1)_R–SU(3)_c$ cannot be satisfied.
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