The NLO corrections to the space-like scalar pion form factors in $k_T$ factorization

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(Dated: November 12, 2014)

In this paper, by employing the $k_T$ factorization theorem, we calculated the space-like scalar pion form factor $Q^2 F(Q^2)$ for $\pi \rightarrow \pi$ transition at the leading order(LO) and the next-to-leading order (NLO). We find that (a) the term proportional to $\phi^A(x_1)\phi^P(x_2)$ in hard kernel $H_a^{(0)}$ provides the absolutely dominant contribution to $Q^2 F(Q^2)$ at the leading order; (b) all the infrared singularities in the NLO quark level diagrams either canceled by themselves or can be absorbed into the NLO pion meson wave functions, an infrared finite factor which describes the NLO correction to the LO result can then be extracted out; (c) the NLO correction has an opposite sign with those LO contribution and can lead to about 10% decrease to the LO prediction for $Q^2 F(Q^2)$ in the considered region $Q^2$; and (d) in the large region of $Q^2$, the pQCD predictions for $Q^2 F(Q^2)$ can be enhanced by about 50% if one uses the full pion distribution amplitudes instead of the asymptotic ones.

PACS numbers: 11.80.Fv, 12.38.Bx, 12.39.St, 13.20.He

I. INTRODUCTION

The $k_T$ factorization theorem\cite{1–3}, as the foundation of the perturbative QCD (pQCD) factorization approach to deal with the B meson hadronic and semileptonic decays \cite{4–12}, has been widely studied and explored at next-to-leading-order (NLO) recently \cite{13–18}. In the pQCD factorization approach, one can calculate perturbatively the contributions from annihilation diagrams in the two-body hadronic B meson decays, this is the well-known advantage of the pQCD factorization approach. The large branching ratios of the pure annihilation decays $B^0_s \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow K^+K^-$ have been measured by both CDF and LHCb collaboration \cite{19–21} and agree well with the pQCD predictions \cite{22–24}.

The power counting of the pQCD factorization approach is different from other popular factorization approaches. In the pQCD approach, most NLO contributions to the two-body hadronic B meson decays become available now. The NLO vertex corrections, the NLO contributions from quark loops and the chromo-magnetic penguin operator $O_{8g}$, for example, have been evaluated several years ago in Refs. \cite{25–27}. The NLO twist-2 and twist-3 contributions to the $B \rightarrow \pi$ transition form factors have been calculated very recently in Refs. \cite{14–17}. These NLO contributions can provide a great help to interpret the so-called “$K\eta'$” puzzle and “$K\pi$” puzzle in the framework of the standard model(SM) \cite{25, 28–30}.

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The still missing NLO parts in the pQCD approach are the NLO contributions to the non-factorizable spectator diagrams and the annihilation diagrams, which are most likely small in size according to general arguments [25, 28–30]. For those “Tree” dominated decays, such as $B^0 \rightarrow \pi^+\pi^-$, this general expectation works well, as proved by explicit calculations in Refs. [29–31], since those still missing NLO pieces are the next order corrections to the small LO quantities for such decay modes. For other decay modes without the dominant “Tree” contributions, however, the things may be rather different. For the $B^0 \rightarrow \pi^0\pi^0$ decay, for example, the contribution from the “emission-diagrams” and “annihilation-diagrams” at LO level are similar in size as listed explicitly in Table III of Ref. [31], while the NLO corrections from all currently known sources can provide about $\sim 90\%$ enhancement to the central value of the LO result for the CP-averaged branching ratio, but the resultant pQCD prediction $Br(B^0 \rightarrow \pi^0\pi^0) = 0.23^{+0.19}_{-0.15} \times 10^{-6}$ [31] is still much smaller even than the new measured value from Belle [32]: $Br(B^0 \rightarrow \pi^0\pi^0) = (0.90 \pm 0.12 \pm 0.10) \times 10^{-6}$ at 6.7$\sigma$. One possible way to resolve this “$\pi\pi$” puzzle is to evaluate the still missing NLO corrections from the spectator diagrams and the annihilation diagrams, and take them into account in the calculation for the considered decay modes.

In this paper, we concentrate on the calculation for the NLO space-like scalar pion form factors, which correspond to the transition process $\pi \rightarrow \pi$ with the insertion of scalar weak interaction vertex. This calculation is necessary for us to evaluate the still missing NLO correction from the annihilation diagrams in the pQCD factorization approach, and may also provide some help to interpret the “$\pi\pi$” puzzle.

In this paper, we will present the first calculation for the space-like scalar pion form factors up to NLO in the $k_T$ factorization theorem. All the infrared (IR) singularities, such as the soft divergences generated from exchanging a massless gluon between two on-shell external quark lines and/or the collinear divergences generated from emitting a massless gluon from a light paralleled external line, are regulated by the off-shell transverse momentum $k_T$. We will verify that all the IR divergences obtained from the NLO calculations can be absorbed into the NLO wave functions completely as in Refs. [14, 16]. The NLO pion meson wave functions have been written down from the factorization processes and have been proved to have the universal format as in Refs. [33–36]. We will calculate the convolutions of the LO hard kernel and the NLO wave functions, as well as the quark level diagrams for the NLO corrections to the LO scalar transition process $\pi \rightarrow \pi$, and finally get an IR finite NLO hard kernel by making the difference of these two sets. With the appropriate choice of the renormalization scale $\mu$ and the factorization scale $\mu_f$, say setting them as the internal hard scale $t$ as postulated in Refs. [14, 16], the NLO corrections are proved under control.

This paper is organized as follows. In Sec. II, we give a brief review about the evaluations of the LO diagrams for the scalar transition process $\pi \rightarrow \pi$, and show the LO pQCD predictions for the $Q^2$-dependence of the form factor $Q^2F(Q^2)$. In Sec. III, $O(\alpha_s^2)$ QCD quark diagrams for the process will be calculated. The convolutions of $O(\alpha_s)$(NLO) effective diagrams for the pion wave functions and $O(\alpha_s)$(LO) hard kernel would also be presented in this section, then the $k_T$-dependent NLO hard kernel will be obtained. Sec. IV contains the numerical analysis of the NLO correction to the space-like scalar pion form factors. Section V contains the conclusions.

II. LEADING ORDER ANALYSIS

Because the vertex in Fig. 1 are scalar in nature, the scalar pion form factors at LO level can be written directly from the hard kernels of the sub-diagrams in Fig. 1. The momentum of the initial and final state pion meson are defined as $p_1 = (p_1^+, 0, 0_T)$ and $p_2 = (0, p_2^-, 0_T)$ respectively, and
FIG. 1. The LO quark diagrams for the space-like scalar pion form factors for transition $\pi \rightarrow \pi$, with the symbol $\bullet$ representing the insertion of the scalar interaction vertex.

$q = p_1 - p_2$ is the momentum transfer in the scalar vertex. In this paper, we just concentrate on the space-like region where $Q^2 > 0$, with $Q^2 \equiv -q^2 = 2p_1 \cdot p_2$. We also define that the anti-quarks carry the momentum $k_1 = (x_1 p^+_1, 0, k_{1T})$ and $k_2 = (0, x_2 p^-_2, k_{2T})$ for the initial and the final pion mesons respectively, with $x_1$ and $x_2$ being the momentum fractions. The following hierarchy is postulated in the small-x region as in Res. [14, 16]:

$$Q^2 \gg x_1 Q^2 \sim x_2 Q^2 \gg x_1 x_2 Q^2 \gg k_{1T}^2 \sim k_{2T}^2.$$ \hfill (1)

We use the Fierz identity in Eq. (2) and the $SU(3)_c$ group identity in Eq. (3) to factorize the fermion flow and the color flow. The identity matrix $I$ in the Fierz identity is a 4-dimensional matrix and $(i, j, l, k)$ are the Lorentz index, while the identity matrix $I$ in the $SU(3)_C$ group is a 3-dimensional matrix and $(i, j, l, k)$ are color index.

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma^5)_{ik}(\gamma^5)_{lj} + \frac{1}{4}(\gamma^\alpha)_{ik}(\gamma^\alpha)_{lj} + \frac{1}{4}(\gamma^5\gamma^\alpha)_{ik}(\gamma^5\gamma^\alpha)_{lj} + \frac{1}{4}(\gamma^\alpha\gamma^5)_{ik}(\gamma^\alpha\gamma^5)_{lj} + \frac{1}{8}(\sigma^{\alpha\beta}\gamma^5)_{ik}(\sigma^{\alpha\beta}\gamma^5)_{lj},$$ \hfill (2)

$$I_{ij}I_{lk} = \frac{1}{N_c}I_{ij}I_{lk} + 2(T^c)_{ij}(T^c)_{lk}.$$ \hfill (3)

Because the weak vertex in Fig. 1(a) is proportional to identity $I$ in the Lorentz space, then we can obtain the hard kernel $H^{(0)}_a$ by sandwiching Fig. 1(a) with the following two sets of structures of pion wave functions:

$$\left( \frac{\gamma_5 p_1}{4N_c}, \frac{\gamma_5 p_2}{4N_c}, \frac{\gamma_5 (\not{p}_- + \not{p}_+)}{4N_c} \right) ; \left( \frac{\gamma_5 n_+}{4N_c}, \frac{\gamma_5 n_-}{4N_c} \right),$$ \hfill (4)

where $n_+ = (1, 0, 0_T)$ and $n_- = (0, 1, 0_T)$ denote the unit vector along with the positive and negative $z$-axis direction. Of course, one can write down $H^{(0)}_a$ directly with the initial and the final
pion meson wave functions\cite{7,37-40} in Eqs. (5,6) with the chiral mass of pion $m_{0\pi} = 1.74$ GeV.

$$\Phi_{\pi}(p_1, x_1) = \frac{1}{\sqrt{6}} \{ \phi_1 \gamma_5 \phi_\pi^A(x_1) + m_{0\pi} \gamma_5 \left[ \phi_\pi^P(x_1) - (\not{\gamma}_\mu \not{\gamma}_+ - 1) \phi_\pi^T(x_1) \right] \}, \quad (5)$$

$$\Phi_{\pi}(p_2, x_2) = \frac{1}{\sqrt{6}} \{ \gamma_5 \phi_2 \phi_\pi^A(x_2) + \gamma_5 m_{0\pi} \left[ \phi_\pi^P(x_2) - (\not{\gamma}_\mu \not{\gamma}_+ - 1) \phi_\pi^T(x_2) \right] \}. \quad (6)$$

Then the LO contributions to the hard kernel from Fig. 1(a) can be written as

$$H_\alpha^{(0)}(x_1, x_2, Q^2) = \frac{8\pi \alpha_s C_F m_{0\pi} Q^2}{(p_2 - k_1)^2(k_1 - k_2)^2} \cdot \left\{ 2\phi_\pi^A(x_1)\phi_\pi^P(x_2) + x_1 \left[ \phi_\pi^P(x_1) - \phi_\pi^T(x_1) \right] \phi_\pi^A(x_2) \right\}. \quad (7)$$

where $\alpha_s$ is the strong coupling constant, $C_F = 4/3$ is the color factor. It is not difficult to find the end-point behavior of the LO hard kernel for Fig. 1(a):

$$H_\alpha^{(0)}(x_1, x_2, Q^2)|_{\text{end-point}} \to (16\pi \alpha_s C_F m_{0\pi} Q^2) \cdot \left\{ \frac{(1 - x_1)}{x_1 x_2} + (1 - x_2) \right\}, \quad (8)$$

where the first and second term describes the end-point behavior of the corresponding term in Eq. (7). It is easy to verify that the second term in Eq. (8) is strongly suppressed by a factor of $\frac{x_1 x_2(1 - x_2)}{(1 - x_1)}$ relative to the first term. The first term proportioned to $\phi_\pi^A(x_1)\phi_\pi^P(x_2)$ in Eq. (7), consequently, will provide the dominate contribution when compared with the second term in Eq. (7). The numerical results as illustrated by the curves in Fig. 2 confirmed this point directly.

With the LO hard kernel $H_\alpha^{(0)}$ in Eq. (7), one can derive the corresponding space-like scalar pion form factor at the LO level in the form of

$$Q^2 F(Q^2)|_{\text{LO}} = 8\pi m_{0\pi} C_F Q^4 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \alpha_s(t) \cdot e^{-2S_\pi(t)} \cdot h(x_1, x_2, b_1, b_2) \cdot \left\{ 2\phi_\pi^A(x_1)\phi_\pi^P(x_2)S_\pi(x_2) + x_1 \left[ \phi_\pi^P(x_1) - \phi_\pi^T(x_1) \right] \phi_\pi^A(x_2) \right\}, \quad (9)$$

where the function $S_\pi(t)$ and the threshold resummation function $S_\pi(x)$ are the same ones as being used in Refs. [7, 16]. In numerical calculation we choose $c = 0.4$ in the function $S_\pi(x)$. The hard function $h(x_1, x_2, b_1, b_2)$ in Eq. (9) can be written as the following form

$$h(x_1, x_2, b_1, b_2) = K_0 \left( \sqrt{x_1 x_2} Q b_1 \right) \left[ \theta(b_1 - b_2) I_0 \left( \sqrt{x_2} Q b_2 \right) K_0 \left( \sqrt{x_2} Q b_1 \right) + (b_1 \leftrightarrow b_2) \right], \quad (10)$$

where the function $K_0$ and $I_0$ are the modified Bessel function. Following Refs. [14, 16], we here also choose $\mu = \mu_f = t$ in the numerical calculations:

$$\mu = \mu_f = t = \max \left( \sqrt{x_1} Q, \sqrt{x_2} Q, 1/b_1, 1/b_2 \right). \quad (11)$$

In this paper we just consider the sub-diagram Fig. 1(a) in detail, because the other sub-diagrams can be obtained by simple kinetic transitions as we have argued in Ref. [16]. The LO hard kernel of Fig. 1(b/c/d) can be obtained from $H_\alpha^{(0)}(x_1, x_2, Q^2)$ by simple replacements of $x_i \leftrightarrow x_j$, $x_i \to (1 - x_i)$ and $x_j \to (1 - x_j)$ with $i \neq j = (1, 2)$. Taking Fig. 1(b) as an example, we make
the direct analytical evaluations for Fig. 1(b) and find the LO hard kernel $H_b^{(0)}(x_1, x_2, Q^2)$ and its
end-point behavior:

$$
H_b^{(0)}(x_1, x_2, Q^2) = \frac{8\pi\alpha_s C_F m_0 Q^2}{(p_1 - k_2)^2(k_1 - k_2)^2} \cdot \left\{ 2\phi^{P}_{\pi}(x_1)\phi^{A}_{\pi}(x_2) + x_2\phi^{A}_{\pi}(x_1) \left[ \phi^{P}_{\pi}(x_2) - \phi^{T}_{\pi}(x_2) \right] - \frac{(1-x_2)}{x_1 x_2} + (1-x_1) \right\},
$$

(12)

$$
H_b^{(0)}(x_1, x_2, Q^2)_{\text{end-point}} = 16\pi\alpha_s C_F m_0 Q^2 \cdot \left\{ \frac{(1-x_2)}{x_1 x_2} + (1-x_1) \right\}.
$$

(13)

They are identical with the ones in Eqs. (7,8) after an exchange of $x_1 \leftrightarrow x_2$.

The LO hard kernel $H_c^{(0)}(x_1, x_2, Q^2)$ of Fig. 1(c) and $H_d^{(0)}(x_1, x_2, Q^2)$ of Fig. 1(d), furthermore, can also be obtained from $H_a^{(0)}$ by simple replacements: $x_1 \rightarrow (1-x_1)$ and $x_2 \rightarrow (1-x_2)$ for $H_c^{(0)}$, and $x_1 \rightarrow (1-x_2)$ and $x_2 \rightarrow (1-x_1)$ for $H_d^{(0)}$. By making the direct analytical calculations we find the following expressions for $H_c^{(0)}$ and $H_d^{(0)}$:

$$
H_c^{(0)}(x_1, x_2, Q^2) = \frac{8\pi\alpha_s C_F m_0 Q^2}{(p_2 - p_1 + k_1)^2(p_2 - k_2 - p_1 + k_1)^2} \cdot \left\{ 2\phi^{A}_{\pi}(x_1)\phi^{P}_{\pi}(x_2) + (1-x_1) \left[ \phi^{P}_{\pi}(x_1) - \phi^{T}_{\pi}(x_1)\phi^{A}_{\pi}(x_2) \right] \right\},
$$

(14)

$$
H_d^{(0)}(x_1, x_2, Q^2) = \frac{8\pi\alpha_s C_F m_0 Q^2}{(p_1 - p_2 + k_2)^2(p_2 - k_2 - p_1 + k_1)^2} \cdot \left\{ 2\phi^{P}_{\pi}(x_1)\phi^{A}_{\pi}(x_2) + (1-x_2) \phi^{A}_{\pi}(x_1) \left[ \phi^{P}_{\pi}(x_2) + \phi^{T}_{\pi}(x_2) \right] \right\}.
$$

(15)

To check the validity of the exchanging symmetry, we used the following symmetric relations of the two-parton distribution amplitudes (DA’s):

$$
\phi^{A}(x_i) = \phi^{A}(1-x_i), \quad \phi^{P}(x_i) = \phi^{P}(1-x_i), \quad \phi^{T}(x_i) = -\phi^{T}(1-x_i).
$$

(16)

Taking the above exchanging symmetry into account, we get to know that only the sub-diagram Fig. 1(a) in Fig. 1 is independent, which will simplify our calculation for the NLO corrections greatly.

In Fig. 2, we show the $Q^2$-dependence of the LO space-like scalar pion form factor $Q^2 F(Q^2)$ for Fig. 1(a), to support our previous theoretical arguments. In this figure, the contributions from the three different terms as given in Eq. (7) are also plotted explicitly: the upper dot-dashed curve with the label ”LO1”, shows the contribution from the first term proportional to $2\phi^{A}_{\pi}(x_1)\phi^{P}_{\pi}(x_2)$ in the LO hard kernel $H_a^{(0)}$, the short-dashed and dots curve with the label ”LO2” and ”LO3” shows the contribution from the second term and the third term in $H_a^{(0)}$ respectively, and finally the solid line denotes the total LO contribution. One can see from Fig. 2 that the first term in Eq. (7) does provide the absolutely dominant contribution (larger than 90%), while the contribution from the third term is tiny in size and can be neglected safely.

In the numerical calculations, we integrate for the partons’ momentum fractions $(x_1, x_2)$ over the range of $x_\pi = [0, 1]$, and find that the main contribution comes from the small $x_1, x_2 \sim 0.1$ region [41, 42]. We obtained the LO theoretical predictions as shown in Fig. 2(a) by using the asymptotic pion DA’s in the integration:

$$
\phi^{A}_{\pi}(x) = \frac{3 f_\pi}{\sqrt{6}} x(1-x), \quad \phi^{P}_{\pi}(x) = \frac{f_\pi}{2\sqrt{6}}, \quad \phi^{T}_{\pi}(x) = \frac{f_\pi}{2\sqrt{6}}(1-2x),
$$

(17)
one can see easily that the first term in the LO hard kernel with the pion decay constant $f_\pi = 0.13$ GeV. For the LO theoretical predictions as shown in Fig. 2(b), on the other hand, the ordinary full pion DA’s with high order terms as given in Refs. [39, 43] are used:

$$\phi_\pi^A(x) = \frac{3 f_\pi}{\sqrt{6}} x(1 - x) \left[ 1 + a_2^\pi C_2^A(u) + a_4^\pi C_4^A(u) \right],$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2 \sqrt{6}} \left[ 1 + \left( 30 \eta_3 - \frac{5}{2} \rho_\pi^2 \right) C_2^P(u) - 3 \left( \eta_3 \omega_3 + \frac{9}{20} \rho_\pi^2 (1 + 6 a_2^\pi) \right) C_4^P(u) \right],$$

$$\phi_\pi^T(x) = \frac{f_\pi}{2 \sqrt{6}} (1 - 2x) \left[ 1 + 6 \left( 5 \eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho_\pi^2 - \frac{3}{5} \rho_\pi^2 a_2^\pi \right) (1 - 10x + 10x^2) \right],$$

where the Gegenbauer moments $a_i^\pi$, the parameters $\eta_3$, $\omega_3$ and $\rho_\pi$ are adapted from Refs. [39, 43]:

$$a_2^\pi = 0.25, \quad a_4^\pi = -0.015, \quad \rho_\pi = m_\pi/m_\pi^0, \quad \eta_3 = 0.015, \quad \omega_3 = -3.0.$$  \tag{19}

The relevant Gegenbauer polynomials $C_{2,4}^{3/2}(2x - 1)$ and $C_{2,4}^{3/2}(2x - 1)$ can be found easily in Refs. [39, 43].

From the expressions of the LO hard kernels $H_a^{(0)}$, $H_b^{(0)}$, $H_c^{(0)}$ and $H_d^{(0)}$ as given in Eqs. (7,12,14,15) and the LO pQCD predictions for $Q^2 F(Q^2)$ as illustrated in Fig. (2), one can see the following points:

1. The LO hard kernel $H^{(0)}$ only receive the contributions from the three cross productions of the DA’s with different twists for the initial and final pion wave function. Take $H_a^{(0)}$ as an example, the contributions come from the terms proportional to $\phi^A(x_1)\phi^P(x_2)$, $\phi^P(x_1)\phi^A(x_2)$ and $\phi^T(x_1)\phi^A(x_2)$ respectively, as listed in Eq. (7).

2. From Fig. 2 one can see easily that the first term in the LO hard kernel $H_a^{(0)}$ provides the absolutely dominant contribution (larger than 90%) to the form factor $Q^2 F(Q^2)$: 0.25 ≤ "LO" ≤ 0.75 numerically. In the whole region of $1 < Q^2 < 30$ GeV$^2$, the contribution from the second (third) term is smaller than 0.05 (0.02). The contribution from the third term is indeed tiny in size and can be neglected safely. This fact does support our previous argument from the analysis for the end-point behavior of the three terms in Eqs. (8,13).
(3) Since the LO contributions from the second and third term of $H^{(0)}_a(x_1,x_2,Q^2)$ are already very small, it is reasonable for us to consider the NLO contributions to the space-like scalar pion form factor from the dominant first term proportioned to $\phi^A(x_1)\phi^P(x_2)$ in $H^{(0)}_a$ only in the next section, which would simplify our calculations significantly.

(4) By comparing the curves as shown in Fig. 2(a) and Fig. 2(b), one can see that the pQCD predictions for the form factors have a clear dependence on the choice of the pion DA’s being used in the integration. For a given $Q^2 = 20$ GeV, for example, we find the following numerical results:

$$Q^2 F(Q^2) = \begin{cases} 
0.48, & \text{asymptotic pion DA’s used} \\
0.68, & \text{full pion DA’s used}.
\end{cases}$$

(20)

III. NLO CORRECTIONS

In this section we will calculate the $O(\alpha_s^2)$ quark level diagrams as well as the convolutions of the effective diagrams for the $O(\alpha_s)$ wave functions and the LO ($O(\alpha_s)$) hard kernel in the 't Hooft-Feynman gauge, and try to find the IR finite NLO corrections to the space-like scalar pion form factor in the $k_T$ factorization theorem. From the discussions at the end of last section, we get to know that it is reasonable to only consider the NLO corrections to the LO hard kernel $H^{(0)}_a(x_1,x_2,Q^2)$ with its first term only, i.e.

$$H^{(0)}_{a,1}(x_1,k_{1T};x_2,k_{2T};Q^2) = \frac{16\pi\alpha_s C_F m_0 Q^2}{(p_2 - k_1)^2(k_1 - k_2)^2} \phi^A_\pi(x_1)\phi^P_\pi(x_2).$$

(21)

Under the hierarchy as shown in Eq. (1), only those terms which don’t vanish in the limits of $x_i \to 0$ and $k_{iT} \to 0$ should be kept.

A. NLO Contributions of the QCD Quark Diagrams

We first calculate the NLO ($O(\alpha_s^2)$) corrections to Fig. 1(a) in the $k_T$ factorization theorem in this subsection. These NLO corrections include the self-energy diagrams, the vertex diagrams, the box and pentagon diagrams, as illustrated in Figs. (3,4,5) respectively. We will use the dimensional reduction scheme[44] to extract the ultraviolet (UV) divergences, and use the transverse momentum for the external light quarks in Eq. (22) to regulate the IR divergences in loops. Following the method being used in Refs. [14, 16] we make the same definitions:

$$\delta_1 = \frac{k_{1T}^2}{Q^2}, \quad \delta_2 = \frac{k_{2T}^2}{Q^2}, \quad \delta_{12} = \frac{-(k_1 - k_2)^2}{Q^2}.$$  

(22)

Following the standard procedure we calculate the one loop self-energy Feynman diagrams as
shown in Fig. 3 and find the following NLO self-energy corrections:

\[
G_{3a}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{\delta_1 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
G_{3b}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{\delta_1 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
G_{3c}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{\delta_2 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
G_{3d}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{\delta_2 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
G_{3e}^{(1)} = -\frac{\alpha_s C_F}{4\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{x_1 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
G_{3f+3g+3h+3i}^{(1)} = \frac{\alpha_s}{4\pi} \left[ \left( 5 - \frac{2}{3} N_f \right) \left( \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{\delta_{12} Q^2 e^{\gamma_E}} \right) \right] H^{(0)},
\]

where \(1/\epsilon\) represents the UV pole term, \(\mu\) is the renormalization scale, \(\gamma_E\) is the Euler constant, \(N_f\) is the number of the active quarks flavors, and \(H^{(0)} = H_{a,1}^{(0)}(x_1, k_{1T}; x_2, k_{2T}; Q^2)\) has been defined in Eq. (21). For the sake of simplicity, we will use the abbreviation \(H^{(0)}\) instead of the term \(H_{a,1}^{(0)}(x_1, k_{1T}; x_2, k_{2T}; Q^2)\) to denote the LO hard kernel throughout the text unless otherwise stated explicitly. The Figs. 3(f,g,h,i) denote the self-energy corrections to the exchanged gluon itself.

It’s easy to find that all these self energy corrections are equal to the self energy corrections for the pion electromagnetic form factors\([14, 16]\), because these self energy diagrams just correct the light quark fields, while don’t involve the inner structure of the initial and final mesons. The additional factor \(1/2\) is considered for self energy diagrams Fig. 3(a,b,c,d) because of the freedom to choose the most outside vertex of the radiative gluon.

The vertex correction diagrams with three-point loop integrals are plotted in Fig. 4, the NLO
corrections from these five vertex diagrams are summarized in the following form:

\[
\begin{align*}
G^{(1)}_{4a} &= \frac{\alpha_s C_F}{4\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{Q^2 e^{\gamma_E}} - 2 \ln x_1 \ln \delta_1 - 2 \ln \delta_1 - 2 \ln x_1 - \frac{5\pi^2}{2} + 2 \right] H^{(0)} , \\
G^{(1)}_{4b} &= -\frac{\alpha_s}{8\pi N_c} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{x_1 Q^2 e^{\gamma_E}} + 1 \right] H^{(0)} , \\
G^{(1)}_{4c} &= -\frac{\alpha_s}{8\pi N_c} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{\delta_1 Q^2 e^{\gamma_E}} - \ln \frac{\delta_1}{\delta_1} \ln \frac{\delta_2}{\delta_1} - \ln \frac{\delta_1}{\delta_1} - \ln \frac{\delta_2}{\delta_1} - \frac{\pi^2}{3} + 2 \right] H^{(0)} , \\
G^{(1)}_{4d} &= \frac{\alpha_s N_c}{8\pi} \left[ \frac{3}{\epsilon} + 3 \ln \frac{4\pi \mu^2}{\delta_1 Q^2 e^{\gamma_E}} - \ln \frac{\delta_1}{\delta_1} - \ln \frac{\delta_2}{\delta_1} + 6 \right] H^{(0)} , \\
G^{(1)}_{4e} &= \frac{\alpha_s N_c}{8\pi} \left[ \frac{3}{\epsilon} + 3 \ln \frac{4\pi \mu^2}{x_1 Q^2 e^{\gamma_E}} - \ln x_2 \ln \delta_2 - \ln \delta_2 \\
&\quad + \ln x_1 \ln x_2 + \ln x_2 - \frac{\pi^2}{3} + 6 \right] H^{(0)} .
\end{align*}
\] (24)

All these five vertex diagrams would have IR divergences at the first sight. The radiated gluon in Fig. 4(a) would generate the collinear divergence when it’s parallel to the initial momentum \( p_1 \). Fig. 4(b) would include the collinear divergence at \( l \parallel p_2 \) region. Fig. 4(c) would generate both the soft and collinear divergence because the radiated gluon is attached to the external light quark lines, then the double logarithm would appear. The radiated gluon in Fig. 4(d) would generate the collinear divergences from \( l \parallel p_1 \) and \( l \parallel p_2 \) regions, while the gluon in Fig. 4(e) could generate the collinear divergence in the \( l \parallel p_1 \) region. But the detailed calculations show that the collinear singularity in Fig. 4(b) is forbidden by the kinetics, so \( G^{(1)}_{4b} \) is IR finite.

The box and pentagon diagram in Fig. 5 are more complicated because they would involve four-point and five-point integrals. But the sub-diagrams Figs. 5(a,e) are reducible diagrams and their contributions will be canceled completely by the relevant effective diagrams to be evaluated in the next subsection, so we can set them to be zero here safely. Then we just need to calculate three four-point diagrams Figs. 5(b,d,f) and one five-point diagram Fig. 5(c). From the evaluations
of the Feynman diagrams in Fig. 5 we find the following NLO corrections:

\begin{align}
G_{5a,5e}^{(1)} &= 0,
G_{5b}^{(1)} &= -\frac{\alpha_s N_c}{8\pi} \left[ \ln \delta_1 - \ln \delta_{12} - 1 \right] H^{(0)}, \\
G_{5c}^{(1)} &= -\frac{\alpha_s}{8\pi N_c} \left[ \ln \delta_1 \ln \delta_2 - 2 \ln x_1 \ln \delta_1 - \ln \delta_1 + \frac{1}{2} \ln^2 \delta_{12} - \ln^2 x_2 - \frac{5}{12} \pi^2 \right] H^{(0)}, \\
G_{5d}^{(1)} &= \frac{\alpha_s}{8\pi N_c} \left[ \ln \delta_1 \ln \delta_2 - 2 \ln x_1 \ln \delta_1 + \ln \delta_2 - \ln x_1 - \frac{\pi^2}{3} - 1 \right] H^{(0)}, \\
G_{5f}^{(1)} &= -\frac{\alpha_s}{8\pi N_c} \left[ \ln \frac{\delta_1}{\delta_{12}} \ln \frac{\delta_2}{\delta_{12}} - \ln x_2 \ln \delta_2 + \frac{1}{2} \ln^2 \delta_{12} \\
&+ \ln x_1 \ln x_2 - \frac{3}{2} \ln^2 x_2 - \frac{\pi^2}{3} - 1 \right] H^{(0)}. \quad (25)
\end{align}

The three sub-diagrams Figs. 5(c,d,f) all generate the double logarithms, because the two endpoints of the radiated gluon is attached to the external lines, which could result in the soft and collinear singularities. The Fig. 5(b) contains only the collinear divergence in the \( l \parallel p_1 \) region because one end-point of the radiated gluon is attached to the internal gluon. For the remaining IR singularities generated in Figs. (3,4,5), we can sort them into two groups as shown in Eqs. (26,27) by using the phase space splicing method \[45\]: one is from the region \( l \parallel p_1 \) and the other is from the region \( l \parallel p_2 \).

\begin{align}
G_{IR1}^{(1)} &= \frac{\alpha_s C_F}{4\pi} \left[ -2 \ln x_1 \ln \delta_1 - 4 \ln \delta_1 \right] H^{(0)}, \\
G_{IR2}^{(1)} &= \frac{\alpha_s C_F}{8\pi} \left[ -2 \ln x_2 \ln \delta_2 - 4 \ln \delta_2 \right] H^{(0)}. \quad (27)
\end{align}

As for the UV divergences, they are forbidden for the Feynman diagrams in Fig. 5 from the surface divergence analysis. While the UV divergences in the NLO quark level diagrams in Figs. (3,4) can be summed up and written in the form of

\[ \frac{\alpha_s}{4\pi} \left( 11 - \frac{2}{3} N_f \right) \frac{1}{\epsilon}. \quad (28) \]

Such UV divergence is the same one as that appeared in the pion electromagnetic form factors \[14, 16\].
B. Convolutions of the NLO Wave Functions With the LO Hard Kernel

As argued in Refs. [16, 33–35], the IR divergences of the NLO corrections from the quark level Feynman diagrams in Figs. (3,4,5) can be absorbed into the non-perturbative wave functions which are universal. Based on this argument, we will make a convolution of the NLO wave functions with the LO hard kernel $H^{(0)}$, and find that the resultant IR part should cancel the IR divergences appeared in the NLO amplitude $G^{(1)}_{\text{IR1}}$ and $G^{(1)}_{\text{IR2}}$ as given in Eqs. (26,27). The twist-2 part of the initial pion wave function $\Phi_{\pi,A}(x_1, k_{1T}; x'_1, k'_{1T})$ and the twist-3 part of the final state pion wave function $\Phi_{\pi,P}(x'_2, k'_{2T}; x_2, k_{2T})$ can be defined by the non-local matrix elements [16, 33–35],

\[
\Phi_{\pi,A}(x_1, k_{1T}; x'_1, k'_{1T}) = \int \frac{dy^- d^2y_T}{(2\pi)^2} e^{-iy^+_1 P_1^+ y^- + ik'_1 T \cdot y_T} \\
\cdot \langle 0 | \bar{q}(y)\gamma_5 \gamma_\mu W_y(n_1) \Gamma_{n_1; 0} W_0(n_1) q(0) | \bar{\pi}(P_1 - k_1) d(k_1) \rangle,
\]

\[
\Phi_{\pi,P}(x'_2, k'_{2T}; x_2, k_{2T}) = \int \frac{dz^+ d^2z_T}{(2\pi)^2} e^{-iz^+_2 P_2^+ z^- + ik'_{2T} T \cdot z_T} \\
\cdot \langle 0 | \bar{q}(z) W_z(n_2) \Gamma_{n_2; z, 0} W_0(n_2) | u(P_2 - k_2) d(k_2) \rangle,
\]

where $y = (0, y^- , y_T)$ and $z = (z^+ , 0, z_T)$ are the light-cone coordinates of the anti-quark field $\bar{q}$, $W_y(n_1)$ and $W_z(n_2)$ with the choice of $n_i^2 \neq 0$ to avoid the light-cone singularity [15, 36, 46] are the Wilson line integrals:

\[
W_y(n_1) = \mathcal{P} \exp[-ig_s \int_0^\infty d\lambda n_1 \cdot A(y + \lambda n_1)],
\]

\[
W_z(n_2) = \mathcal{P} \exp[-ig_s \int_0^\infty d\lambda n_2 \cdot A(z + \lambda n_2)],
\]

where the symbol $\mathcal{P}$ denotes the path ordering operator.

We firstly consider the convolutions of the $\mathcal{O}(\alpha_s)$ twist-2 initial pion wave functions $\Phi^{(1)}_{\pi,A,i}$, as shown in Fig. 6, with the $\mathcal{O}(\alpha_s)$ hard kernel $H^{(0)}$ in Eq. (21),

\[
\Phi^{(1)}_{\pi,A} \otimes H^{(0)} \equiv \sum_{i=a}^h \int dx'_1 d^2k'_{1T} \Phi^{(1)}_{\pi,A,i}(x_1, k_{1T}; x'_1, k'_{1T}) H^{(0)}(x'_1, k'_{1T}; x_2, k_{2T}).
\]

The reducible effective diagram Fig. 6(c) carry all the NLO contributions from the reducible diagrams Fig. 5(a), so we can also set it’s contribution to be zero safely. The convolutions of the
NLO initial wave functions $\Phi_{\pi,s}^{(1)}$, and the LO hard kernel $H^{(0)}$ are summarized as

$$\Phi_{\pi,A,b}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu_r^2}{\delta_1 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},$$

$$\Phi_{\pi,A,c}^{(1)} \otimes H^{(0)} = 0,$$

$$\Phi_{\pi,A,d}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu_r^2}{\delta_1 Q^2 e^{\gamma_E}} - \ln^2 \left( \frac{\delta_1 r_{Q1}}{x_1} \right) - 2 \ln \left( \frac{\delta_1 r_{Q1}}{x_1} \right) - \frac{\pi^2}{3} + 2 \right] H^{(0)},$$

$$\Phi_{\pi,A,e}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[ \ln^2 \left( \frac{\delta_1 r_{Q1}}{x_1} \right) + \frac{\pi^2}{3} \right] H^{(0)},$$

$$\Phi_{\pi,A,f}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu_r^2}{\delta_1 Q^2 e^{\gamma_E}} - \ln^2 \left( \frac{\delta_1 r_{Q1}}{x_1} \right) - 2 \ln \left( \frac{\delta_1 r_{Q1}}{x_1} \right) - \frac{\pi^2}{3} + 2 \right] H^{(0)},$$

$$\Phi_{\pi,A,g}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[ \ln^2 \left( \frac{\delta_1 r_{Q1}}{x_1} \right) - \frac{\pi^2}{3} \right] H^{(0)},$$

$$(\Phi_{\pi,A,h}^{(1)} + \Phi_{\pi,A,i}^{(1)} + \Phi_{\pi,A,j}^{(1)}) \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\epsilon} + \ln \frac{4\pi \mu_r^2}{Q^2 e^{\gamma_E}} - \ln \delta_1 \right] H^{(0)},$$

where $r_{Q1} = Q^2 / \xi_1^2$ and the scale $\xi_1^2 \equiv 4(n_1 \cdot p_1)^2 / |n_1|^2 = Q^2 |n_1^- / n_1^+|$ are introduced to regularize the light cone singularity. We can find that the double logarithms only generated from the effective diagrams without the loop momentum $l$ flowing into the LO hard kernel, such as the case in Figs. 6(d) and 6(f), because the effective diagrams with the soft loop momentum flowing into the LO hard kernel are highly suppressed by the dynamics. These double logarithms are canceled each other completely, resulting in single logarithms only. These single logarithms will be canceled by the IR singularity as given in Eq. (26) from the NLO quark level diagrams.

The remaining convolutions to be treated are those between the $O(\alpha_s)$ hard kernel $H^{(0)}$ and the $O(\alpha_s)$ twist-3 final state pion wave functions $\Phi_{\pi,P,i}^{(1)}$ as shown in Fig. 7.

$$H^{(0)} \otimes \Phi_{\pi,P,i}^{(1)} \equiv \sum_{i=a}^h \int d\pi^2 d\pi^2 k_{2T} H^{(0)}(x_1, k_{1T}; x_2, k_{2T}) \Phi_{\pi,P,i}^{(1)}(x_2', k_{2T}', x_2, k_{2T}).$$

FIG. 6. The effective $O(\alpha_s)$ diagrams for the twist-2 initial $\pi$ meson wave functions.
We can also set the convolution of the $H^{(0)}$ and Fig. 7(c) zero with the same reason as for the Fig. 6(c). Then all the convolutions of the effective diagrams in Fig. 7 read as

\[
H^{(0)} \otimes \Phi_{\pi,P,a}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\varepsilon} + \ln \frac{4\pi \mu^2}{\delta_2 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
H^{(0)} \otimes \Phi_{\pi,P,b}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\varepsilon} + \ln \frac{4\pi \mu^2}{\delta_2 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)},
\]

\[
H^{(0)} \otimes \Phi_{\pi,P,c}^{(1)} \equiv 0,
\]

\[
H^{(0)} \otimes \Phi_{\pi,P,d}^{(1)} = \frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\varepsilon} + \ln \frac{4\pi \mu^2}{\xi_2^2 e^{\gamma_E}} - \ln^2 \left( \delta_2 r Q^2 \right) - 2 \ln \left( \delta_2 r Q^2 \right) - \frac{\pi^2}{3} + 2 \right] H^{(0)},
\]

\[
H^{(0)} \otimes \Phi_{\pi,P,e}^{(1)} = \frac{\alpha_s C_F}{8\pi} \left[ \ln^2 \left( \frac{\delta_2 r Q^2}{x_2} \right) + \frac{\pi^2}{3} \right] H^{(0)},
\]

\[
H^{(0)} \otimes \Phi_{\pi,P,f}^{(1)} = \frac{\alpha_s C_F}{8\pi} \left[ \frac{1}{\varepsilon} + \ln \frac{4\pi \mu^2}{\xi_2^2 e^{\gamma_E}} - \ln^2 \left( \frac{\delta_2 r Q^2}{x_2^2} \right) - 2 \ln \left( \frac{\delta_2 r Q^2}{x_2^2} \right) - \frac{\pi^2}{3} + 2 \right] H^{(0)},
\]

\[
H^{(0)} \otimes \Phi_{\pi,P,g}^{(1)} = \frac{\alpha_s C_F}{8\pi} \left[ \ln^2 \left( \frac{\delta_2 r Q^2}{x_2^2} \right) - \frac{\pi^2}{3} \right] H^{(0)},
\]

\[
H^{(0)} \otimes (\Phi_{\pi,P,h}^{(1)} + \Phi_{\pi,P,i}^{(1)} + \Phi_{\pi,P,j}^{(1)}) = \frac{\alpha_s C_F}{4\pi} \left[ \frac{1}{\varepsilon} + \ln \frac{4\pi \mu^2}{Q^2 e^{\gamma_E}} - \ln \delta_1 \right] H^{(0)},
\]

where $r Q^2 = Q^2 / \xi_2^2$ with the scale $\xi_2^2 \equiv 4(n_2 \cdot p_2)^2 / |n_2^2| = Q^2 |n_2^+ / n_2^-|$. The double logarithms in Eq. (36) are also canceled each other as the case in Eq. (34), and the remaining single logarithms can also been canceled by the IR singularity in Eq. (27). When compared with the convolutions of the irreducible diagrams in Figs. 6(d,e,f,g), there is an additional factor 1/2 for those of the irreducible diagrams Fig. 7(d,e,f,g), since the twist-3 final state wave functions $\Phi_{\pi,P,i}^{(1)}$ have different spin structure from the twist-2 initial state wave function $\Phi_{\pi,A,i}^{(1)}$.

C. The NLO Hard Kernel

The $k_T$ factorization theorem states that the NLO hard kernel can be obtained by taking the difference of the NLO quark level diagrams and the convolutions of LO hard kernel with NLO.
wave functions [16, 33–35], i.e.,
\[ H^{(1)}(x_1, k_{1T}; x_2, k_{2T}) = G^{(1)}(x_1, k_{1T}; x_2, k_{2T}) \]
\[ - \sum_{i=0}^{h} \int d^2 k_{1T}^\prime \Phi^{(1)}_{\pi,A_i}(x_1, k_{1T}; x_1', k_{1T}') H^{(0)}(x_1', k_{1T}'; x_2, k_{2T}) \]
\[ - \sum_{i=0}^{h} \int d^2 k_{2T}^\prime H^{(0)}(x_1, k_{1T}; x_2', k_{2T}') \Phi^{(1)}_{\pi,P_i}(x_2', k_{2T}'; x_2, k_{2T}). \] (37)

Besides the contributions from the reducible diagrams, we here sum up all \( G_i^{(1)} \) as given in Eqs. (23,24,25) to obtain the NLO corrections \( G^{(1)} \) from the quark level diagrams in Figs. (3,4,5) for \( N_f = 6 \) and find the result,
\[ G^{(1)} = \frac{\alpha_s C_F}{8\pi} \left[ \frac{29}{2} \left( \frac{1}{\epsilon} + \ln \frac{4\pi \mu^2}{Q^2e^{\gamma_E}} \right) - 4 \ln \delta_1 (\ln x_1 + 1) - 2 \ln \delta_2 (\ln x_2 + 1) - \frac{1}{4} \ln^2 \delta_{12} \right. \]
\[ + \left. \frac{9}{4} \ln \delta_{12} + \frac{1}{2} \ln x_1 \ln x_2 + \frac{5}{8} \ln^2 x_2 - \frac{43}{4} \ln x_1 + \frac{9}{4} \ln x_2 - \frac{267\pi^2}{48} + \frac{65}{2} \right] H^{(0)}. \] (38)

By summing up all convolutions as listed in Eqs. (34,36) for Figs. (6,7) without the reducible diagrams, we find the total result:
\[ \Phi^{(1)}_{\pi,A} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[ \frac{4}{\epsilon} + 4 \ln \frac{4\pi}{e^{\gamma_E}} + 4 \ln \frac{\mu^2}{Q^2} - 2 \ln (\delta_1 r_{Q1})(\ln x_1 + 2) \right. \]
\[ + \ln^2 x_1 - 2 \ln \delta_{12} + 4 \ln (x_1 r_{Q1}) - \pi^2 + 4 \right] H^{(0)}, \] (39)
\[ H^{(0)} \otimes \Phi^{(1)}_{\pi,P} = \frac{\alpha_s C_F}{8\pi} \left[ \frac{4}{\epsilon} + 4 \ln \frac{4\pi}{e^{\gamma_E}} + 4 \ln \frac{\mu^2}{Q^2} - 2 \ln (\delta_2 r_{Q2})(\ln x_2 + 2) \right. \]
\[ + \ln^2 x_2 - 2 \ln \delta_{12} + 4 \ln (x_2 r_{Q2}) - \pi^2 + 4 \right] H^{(0)}. \] (40)

The UV divergence in Eq. (38), which would determine the renormalization-group (RG) evolution of the strong coupling constant \( \alpha_s \), is the same one as that in the pion electromagnetic form factor as given in Refs. [14, 16]. The bare coupling constant \( \alpha_s \) in Eqs. (38,39,40) can be rewritten as
\[ \alpha_s = \alpha_s(\mu_f) + \delta Z(\mu_f)\alpha_s(\mu_f), \] (41)
with the counter-term \( \delta Z(\mu_f) \) defined in the modified minimal subtraction scheme (\( \overline{MS} \)). We can insert the \( \alpha_s \) in Eq. (41) into Eqs. (21,38,39,40) to regularize the UV poles in Eq. (37) through the term \( \delta Z(\mu_f) H^{(0)} \), and then the UV poles in Eqs. (39,40) are regulated by the counter-term of the quark field and by an additional counter-term in Eq. (41).

One should be careful that the internal quark with the tiny momentum fraction \( x_1 \) would be on-shell, which would then generate an additional double logarithm \( \ln^2 x_1 \), so we must subtract this jet function as described in Eq. (42) to obtain the real NLO hard kernel.
\[ J^{(1)} H^{(0)} = -\frac{1}{2} \alpha_s(\mu_f) C_F \left[ \ln^2 x_1 + \ln x_1 + \frac{\pi^2}{3} \right] H^{(0)}. \] (42)
After renormalizing the UV divergences and subtracting the jet function, one can obtain the NLO hard kernel for Fig. 1(a) by combing the results as given previously in Eqs. (37,38,39,40) together:

\[ H^{(1)}(x_i, \mu, \mu_f, Q^2) \equiv F^{(1)}(x_i, \mu, \mu_f, Q^2)H^{(0)}, \]  

(43)

with

\[
F^{(1)}(x_i, \mu, \mu_f, Q^2) = \frac{\alpha_s(\mu_f)C_F}{8\pi} \left[ \frac{21}{2} \ln \frac{\mu^2}{Q^2} - 8 \ln \frac{\mu_f^2}{Q^2} - \frac{1}{4} \ln^2 \delta_{12} + \frac{33}{4} \ln \delta_{12} + \frac{1}{2} \ln x_1 \ln x_2 
- \frac{3}{8} \ln^2 x_2 - \frac{71}{4} \ln x_1 - \frac{7}{4} \ln x_2 - \frac{107}{48} \pi^2 + \frac{41}{2} \right] H^{(0)},
\]

(44)

here one has made the choice for \( r_{Q_1} = r_{Q_2} \equiv 1 \) as in Refs. [14, 16].

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section we will calculate the NLO corrections to the space-like scalar pion form factor in the \( k_T \) factorization theorem numerically. From the expression of the NLO hard kernel \( H^{(1)}(x_i, \mu, \mu_f, Q^2) \) as given in Eq. (43), one can define the space-like scalar pion form factor for Fig. 1(a) up to NLO as the form of

\[
Q^2 F(Q^2)|_{\text{NLO}} = 8\pi m_\pi C_F Q^4 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \alpha_s(t) \cdot e^{-2S_n(t)} \cdot h(x_1, x_2, b_1, b_2) 
\cdot \left\{ 2\phi_\pi^A(x_1)\phi_\pi^P(x_2)S_t(x_2) \left[ 1 + F^{(1)}(x_i, \mu, \mu_f, Q^2) \right] + x_1 \left[ \phi_\pi^P(x_1) - \phi_\pi^T(x_1) \right] \phi_\pi^A(x_2) \right\}.
\]

(45)

where the function \( F^{(1)}(x_i, \mu, \mu_f, Q^2) \) describes the NLO contribution to the space-like scalar pion form factor and has been defined in Eq. (44).

In this paper, we evaluate the NLO scalar pion form factor for Fig. 1(a) only and can find the others from Figs. 1(b,c,d) by simple replacements of \( x_i \) or \( 1 - x_i \) as described in previous section, due to the following two reasons:

(i) Since the initial and final state meson are the same pion meson, which is a \( q\bar{q} \) bound state and also a Nanbu-Goldstone boson, then there is an exchange symmetry of the momentum fractions for those four LO sub-diagrams in Fig. 1, as we have demonstrated in Section. II. This symmetry imply that the NLO hard kernels from the sub-diagrams Figs. 1(b,c,d) all can be obtained from that of Fig. 1(a) by simple kinematic replacements too. For example, the NLO correction \( F_b^{(1)}(x_i, \mu, \mu_f, Q^2) \) to the dominant first term proportional to \( \phi_\pi^A(x_2)\phi_\pi^P(x_1) \) in \( H_b^{(0)}(x_1, x_2, Q^2) \) in Eq. (12) can be obtained from \( F^{(1)}(x_i, \mu, \mu_f, Q^2) \) in Eq. (44) by simple replacements of \( x_1 \leftrightarrow x_2 \), and is of the form

\[
F_b^{(1)}(x_i, \mu, \mu_f, Q^2) = \frac{\alpha_s(\mu_f)C_F}{8\pi} \left[ \frac{21}{2} \ln \frac{\mu^2}{Q^2} - 8 \ln \frac{\mu_f^2}{Q^2} - \frac{1}{4} \ln^2 \delta_{12} + \frac{33}{4} \ln \delta_{12} 
+ \frac{1}{2} \ln x_2 \ln x_1 - \frac{3}{8} \ln^2 x_2 - \frac{71}{4} \ln x_2 - \frac{7}{4} \ln x_1 - \frac{107}{48} \pi^2 + \frac{41}{2} \right] H_b^{(0)},
\]

(46)
while the space-like scalar pion form factor from Fig. 1(b) up to NLO level can be written as the form of

\[ Q^2F(Q^2)_{\text{h,NLO}} = 8\pi m_0 C_F Q^4 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \alpha_s(t) \cdot e^{-2S_{\pi}(t)} \cdot h(x_2, x_1, b_2, b_1) \]

\[ \times \left\{ 2\phi^P_\pi(x_1)\phi^A_\pi(x_2)S_t(x_1) \left[ 1 + F_b^{(1)}(x_i, \mu_2, Q^2) \right] \right. \]

\[ + x_2\phi^A_\pi(x_1) \left[ \phi^P_\pi(x_2) - \phi^T_\pi(x_2) \right] \left\} . \]

Explicit calculations confirmed this exchanging symmetry clearly.

(ii) In heavy to light decay processes, only the time-like scalar pion form factor, which would appear in the factorizable annihilation diagrams of B meson decays, is the physical quantity, while the space-like scalar pion form factor is not. So it’s enough for us to calculate analytically the NLO correction to its LO part for Fig. 1(a) only. One can obtain the results for other three sub-diagrams by simple replacements of \( x_i \) when it is necessary.

In Fig. 8, we plot the \( Q^2 \)-dependence of the pQCD predictions for the form factor \( Q^2F(Q^2) \) for Fig. 1(a). The pQCD predictions as shown in Fig. 8(a) and 1(b) are obtained when the asymptotic or the full pion DA’s are used in the integration. The dot-dashed and dotted curve shows the LO contribution and the NLO correction respectively, while the solid curve refers to the total pQCD predictions after the inclusion of the NLO corrections. In Fig. 9, we also show the \( Q^2 \)-dependence of the pQCD predictions for the form factor \( Q^2F(Q^2) \) for Fig. 1(b). The curves in Fig. 9 have the same meaning as those in Fig. 8.

From the numerical results as illustrated in Fig. 8 and Fig. 9, we find the following points:

(i) As shown by the dots line in Fig. 8, the NLO correction to the LO pQCD prediction for \( Q^2F(Q^2) \) is negative in sign and very small in magnitude in the whole considered region of \( Q^2 \). The inclusion of the NLO corrections can produce about \( \sim 10\% \) decrease to the LO result in the \( Q^2 \) region considered.

(ii) By comparing the curves in Fig. 8(a) and Fig. 8(b), one can see clearly the effects of the high order terms, such as the terms with the non-zero Gegenbauer moments \( a_2^\pi \) and \( a_4^\pi \), in
FIG. 9. The same as Fig. 8, but the pQCD predictions come from the evaluation of Fig. 1(b).

the pion DA's. At the lower limit of $Q^2 \sim 1 \text{ GeV}^2$, the numerical values of LO and NLO pQCD predictions for form factors in Fig. 8(a) and Fig. 8(b) become identical, but the curves in Fig. 8(b) become larger more rapidly than those in Fig. 8(a) along with the increase of $Q^2$. In the region of $Q^2 > 20 \text{ GeV}^2$, the inclusion of the high order terms in pion DA's can produce about 50% enhancement to its LO pQCD part.

(iii) As illustrated by the curves in Figs. (8,9), the LO and NLO contributions to the form factor $Q^2 F(Q^2)$ from Fig. 1(1a) and 1(b) are indeed identical, which is what we expect based on the exchanging symmetry as discussed in Section II. For Fig. 1(c) and 1(d), we also have the same result.

V. CONCLUSION

In this paper, we made the first calculation for the NLO contribution to the space-like scalar pion form factor in the $k_T$ factorization theorem. The external light quarks are all set off-shell by $k_T^2$ to regulate the IR divergences which would appear in the NLO calculations. We calculated both the NLO quark-level diagrams and the convolutions of the LO hard kernel $H^{(0)}$ with the NLO wave functions to obtain the NLO hard kernel $H^{(1)}$. Because all quarks in this process are massless, then all the IR divergences in these two type diagrams can be described by the logarithms $\ln^2(k_T)$. The QCD dynamics ensures that the contribution from the radiated soft gluon is highly suppressed by $1/Q^2$ in the perturbative theory, our LO and NLO numerical calculations confirmed this point by showing that the double logarithms $\ln^2(k_T)$ generated from the soft kinetic region are canceled completely between the quark-level diagrams and the effective diagrams. We then prove that all the remaining collinear divergences from the quark-level diagrams are also canceled by those from the effective diagrams at NLO level, which is also the basic requirement of the $k_T$ factorization theorem.

We made the numerical evaluations for the space-like scalar pion form factor $Q^2 F(Q^2)$ up to NLO by using the asymptotic and the full pion DA's in the integration, respectively. Based on our analytical calculations and numerical results, we found the following points:

(i) At the leading order, the first term proportional to $\phi_1^A(x_1)\phi_F^P(x_2)$ in $H^{(0)}_A(x_1, x_2, Q^2)$ provides the dominant contribution, larger than 90% of the total LO result, to the considered
form factor $Q^2 F(Q^2)$. So it is reasonable for us to study the NLO corrections to the dominant first term in Eq. (7) only.

(ii) The NLO correction has an opposite sign with the LO contribution but are small in magnitude, and can result in about 10% decrease to the LO pQCD prediction for the form factor $Q^2 F(Q^2)$ in the considered $Q^2$ region.

(iii) At the lower limit of $Q^2 \to 1$ GeV, the pQCD predictions for $Q^2 F(Q^2)$ approaches the same value whether we use the asymptotic or the full pion DA's in the integrations. In the larger $Q^2$ region, say $Q^2 > 20$ GeV$^2$, the high order terms of pion DA’s can provide about 50% enhancement to its counterpart obtained by employing the asymptotic pion DA’s in the integration.

(iv) There is an exchanging symmetry between the LO hard kernels $H^{(0)}_{a,b,c,d}$. The contribution to the form factor $Q^2 F(Q^2)$ from the four sub-diagrams of Fig. 1 are identical.

Although the NLO correction to the space-like scalar pion form factor is only about 10% of its LO counterpart, the NLO correction to the time-like scalar pion form factor might be large in size and may play an important role in our effort to understand the $B \to \pi^0 \pi^0$ puzzle. So it is reasonable for us to extend current studies in this paper to the case for the evaluation of the NLO corrections to the time-like scalar pion form factor and study the relevant phenomenological consequences in our next work.

VI. ACKNOWLEDGMENT

The authors would like to thank Hsiang-nan Li and Cai-Dian Lü for long term collaborations and valuable discussions. This work is supported by the National Natural Science Foundation of China under Grant No.11235005 and by the Project on Graduate Students Education and Innovation of Jiangsu Province under Grant No. CXZZ13-0391.

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