LATTICE STUDY OF NUCLEON PROPERTIES WITH
DOMAIN WALL FERMIONS

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Domain wall fermions (DWF) are a new fermion discretization scheme with greatly
improved chiral symmetry. Our final goal is to study the nucleon spin struc-
ture through lattice simulation using DWF. In this paper, we present our current
progress on two topics toward this goal: 1) the mass spectrum of the nucleon
excited states and 2) the iso-vector vector and axial charges, $g_V$ and $g_A$, of
the nucleon.

1. Introduction

The RIKEN-BNL-Columbia-KEK QCD Collaboration has been pursuing
the domain-wall fermion (DWF) method in lattice quantum chromodynamics
(QCD). In DWF an extra fifth dimension is added to the lattice. By manip-
ulating the domain-wall structure of the fermion mass in this fifth dimension,
we control the number of light fermion species in the other four space-time
dimensions. These light fermions possess exact chiral symmetry in the limit
of an infinite fifth dimension. In particular: 1) fermion near-zero mode effects
are well understood, 2) explicit chiral symmetry breaking induced by a finite
extra dimension is described by a single residual mass parameter, which is
very small in the present calculation, in the low-energy effective lagrangian,
and 3) non-perturbative renormalization works well.

DWF is a promising new approach for treating fermions on the lattice. However,
we need several tests of DWF in the baryon sector to reach our final
goal of establishing the spin structure of the nucleon from first principles.
Here we report our recent studies of the mass spectrum of the nucleon and
its excited states and the nucleon matrix elements of the iso-vector vector
and axial charges in quenched lattice QCD with DWF. Although most of the
latter results are preliminary, the conclusive results in the former subject have
been reported in Ref.[4].

2. Nucleon excited states

First, we discuss the mass spectrum of the nucleon $N$ and its excited states
(the negative-parity nucleon $N^*$ and the positive-parity first excited nucleon
$N'$) by means of a systematic investigation utilizing two distinct interpolating
operators $B_1^\pm$ and $B_2^\pm$. For an explanation of those operators, see Ref.[4]. Our quenched DWF calculation was employed on lattice with size $16^3 \times 32 \times 16$, gauge coupling $\beta = 6/g^2 = 6.0$, and domain wall height $M_5 = 1.8$. Additional details of our simulation can be found in Ref.[4].

In Fig.1 we show the low-lying nucleon spectrum as a function of the quark mass, $m_f$ in lattice units ($a^{-1} \approx 1.9$ GeV set from $aM_{\rho} = 0.400(8)$ in the chiral limit). $B_1^+$ gives the ground-state nucleon mass $N$ (cross). The $N^*$ mass estimates (square and diamond) are extracted from both $B_1^-$ and $B_2^-$. The corresponding experimental values for $N$ and $N^*$ are marked with lower and upper stars. Both $N^*$ mass estimates extracted from two distinct operators agree with each other. The large $N$-$N^*$ mass splitting is clearly evident.

In contrast to the negative parity operators, we find that the mass estimates from a second operator, $B_2^+$, are considerably larger than the ground state obtained from $B_1^+$. This suggests that $B_2^+$ has negligible overlap with the nucleon ground state and provides a signal for the positive-parity excited nucleon $N'$. To justify this possibility, we employ a sophisticated approach which utilizes the transfer matrix of a $2 \times 2$ correlation function constructed from both $B_1^+$ and $B_2^+$. The diagonalization of the transfer matrix yields the excited state. Fig.2 shows a comparison of the fitted mass from $\langle \langle B_2^+(t)\bar{B}_2^+(0) \rangle \rangle$ (circle) and the estimated mass from the average effective mass given by the smaller eigenvalue of the transfer matrix (bullet). The cross symbol corresponds to the nucleon ground state mass evaluated from the larger eigenvalue of the transfer matrix, which is quite consistent with the fitted mass from $\langle \langle B_1^+(t)\bar{B}_1^+(0) \rangle \rangle$.

Another important conclusion can be drawn from Fig.1 and Fig.2. In the heavy quark mass region, the ordering of the negative-parity nucleon ($N^*$) and the positive-parity excited nucleon ($N'$) is inverted relative to experiment. This remarkable result was originally reported in our early paper and subsequently confirmed in Ref.[6]. Further systematic calculation is required to determine whether this ordering switches to the observed ordering as one approaches the chiral limit.

3. Nucleon matrix elements

The nucleon (iso-vector) axial charge $g_A$ is a particularly interesting quantity. We know precisely the experimental value $g_A = 1.2670(35)$ from neutron beta decay. Why does $g_A$ deviate from unity in contrast to vector charge, $g_V = 1$? The simple explanation is given by the fact that the axial current is only partially conserved in the strong interaction while the vector current is exactly conserved. Thus, the calculation of $g_A$ is an especially relevant test of
the ratio between two- and three-point functions are regularized in different schemes. The operators are related by a renormalization factor $Z$ usually estimated in perturbation theory $(P)$. For the axial current, the three-point function is averaged over $i = 1, 2, 3$. The lattice estimates of vector and axial charges can be derived from the ratio between two- and three-point functions

$$g_{\text{v}}^\text{lattice} = \frac{G_{V}^{\mu}(t, t') - G_{V}^{\mu}(t, t')}{G_{A}^\gamma(t)} ,$$

where $G_{V}^\gamma(t) = \text{Tr}[P_{V} \sum_{\vec{x}, \vec{x}'} (TB_{1}(\vec{x}, t)J_{\mu}^{\nu}(\vec{x}', t)B_{1}(0, 0))]$.

Recall that in general lattice operators $O_{\text{lat}}$ and continuum operator $O_{\text{con}}$ are regularized in different schemes. The operators are related by a renormalization factor $Z_{C}$: $O_{\text{con}}(\mu) = Z_{C}(\mu)O_{\text{lat}}(a)$. This implies that the continuum value of vector and axial charges are given by $g_{c} = Z_{c}g_{\text{v}}^\text{lattice}$. In the case of conventional Wilson fermions, the renormalization factor $Z_{A}$ is usually estimated in perturbation theory ($Z_{A}$ differs from unity because of explicit symmetry breaking). For DWF, the conserved axial current receives no renormalization. This is not true for the lattice local current. An important advantage with DWF, however, is that the lattice renormalizations, $Z_{V}$ and $Z_{A}$, of the local currents are the same so that the ratio $(g_{\text{v}}/g_{\text{v}}^\text{lattice})$ directly yields the continuum value $g_{A}$.

Our preliminary results are analyzed on 200 quenched gauge configurations at $\beta = 6.0$ on a $16^{3} \times 32 \times 16$ lattice with $M_{5} = 1.8$. We choose a fixed separation in time of the nucleon interpolating operators, $t = t_{\text{source}} - t_{\text{sink}}$ and $t' < t$, with currents inserted in between. We take $t_{\text{source}} = 5$ and $t_{\text{sink}} = 21$.

In Fig.3 we show the dependence of the vector renormalization, $Z_{V} = 1/g_{V}^\text{lattice}$ on the location of current insertions. A good plateau is observed so that $Z_{V}$ is certainly well behaved. The value $0.763(5)$ at $m_{f} = 0.02$ (obtained by averaging over time slices denoted by the solid line in Fig.3) agrees well with $Z_{A} = 0.7555(3)$, which was obtained from a completely different calculation of meson two-point correlation functions based on the relation $\langle A_{\mu}^{\text{conserved}}(t)q\gamma_{5}q(0)\rangle = Z_{A} \langle A_{\mu}^{\text{local}}(t)q\gamma_{5}q(0)\rangle$. 

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For the axial charge, $g_A^{\text{lattice}}$, plateaus are seen for $10 \leq t \leq 16$ in Fig.4, so the charge ratios $(g_A/g_V)^{\text{lattice}}$ at each $m_f$ are averaged in this time slice range. We find that there is a fairly strong dependence of $(g_A/g_V)^{\text{lattice}}$ on $m_f$ as be shown in Fig.5. A simple linear extrapolation to $m_f = 0$ yields $g_A = 0.62(13)$, which is roughly 1/2 the experimental value\[. However, a simple linear ansatz may not describe the data in which case the result in the chiral limit may be even smaller.

To compare to results using Wilson fermions, we plot $g_A$ versus the square of the $\pi$-$\rho$ mass ratio in Fig.6. Our result is given by the bullets. We also include two heavier mass points (with large errors) from an earlier simulation using a larger separation between the source and the sink. The triangles and diamonds are from Wilson simulations at relatively strong\[ and weak\[ coupling. At first glance, the DWF and Wilson fermions seem to be in rough agreement except for the lightest point. However, our DWF results have a strong mass dependence. This may be a finite volume effect; the Wilson results at strong coupling were simulated on a lattice with a volume which is twice ours.

A couple of comments on the mass dependence of our data are in order. First, it is interesting to note that our results appear consistent with the value $5/3$ (marked as star) in the heavy quark limit, while the others seem inconsistent with this limit. Second, our results may also be consistent with vanishing in the chiral limit. This can be explained through the axial Ward-Takahashi identity which governs $g_A$. If the PCAC relation $m_\pi^2 \propto m_f$ is modified, for example by chiral logarithms, the nucleon matrix element of the pseudoscalar density does not develop a pole as $m_f \to 0$, and the r.h.s of the identity vanishes in the chiral limit. Thus, $g_A$ must also vanish as $m_f \to 0$. Indeed, we already know that the PCAC relation for the pion mass is modified in the quenched approximation by two effects: zero-modes of the Dirac operator and the quenched chiral logarithm\[. Further investigation of the Ward-Takahashi identity is under way.

4. Conclusions

We have explored several nucleon properties in quenched lattice QCD using domain wall fermions toward our final goal of studying the spin structure of the nucleon from first principles.

Our quenched DWF calculation reproduces very well the large mass splitting between the nucleon $N(939)$ and its parity partner $N^*(1535)$\[. We have also calculated the mass of the first positive-parity excited state $N^*(1440)$ by the diagonalization of a $2 \times 2$ matrix correlator and confirmed that it is heavier than the negative-parity excited state $N^*(1535)$\[. A remaining puzzle is
whether or not a switching of $N^*$ and $N'$ occurs close to the chiral limit.

A preliminary calculation of iso-vector vector and axial charges shows that all the relevant three-point functions are well behaved. $Z_V$ determined from the nucleon matrix element of the vector current agrees closely with that from an NPR study of quark bilinears and a direct calculation using meson correlation functions. This indicates that $g_V = 1$ and $Z_V = Z_A$ are mutually satisfied in our quenched DWF calculation. However, a linear extrapolation of $g_A$ to the chiral limit yields a value which is a factor of two smaller than the experimental value. We are currently investigating the Ward-Takahashi identity that governs $g_A$ to shed light on this behavior. We also plan to check related systematic effects arising from finite lattice volume and quenching (for example quenched chiral logarithms, zero modes, and the absence of the full pion cloud), especially in the lighter quark mass region.

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References

1. An early review of domain wall fermions is given in T. Blum, Nucl. Phys. B (Proc. Suppl.) 73, 167 (1999); For a recent review see P. Vranas, Nucl. Phys. B (Proc. Suppl.) 94 177 (2001).
2. T. Blum, et al., hep-lat/0007038.
3. T. Blum, et al., hep-lat/0102005.
4. S. Sasaki, T. Blum and S. Ohta, hep-lat/0102010; S. Sasaki, Nucl. Phys. B (Proc. Suppl.) 83, 206 (2000) and hep-ph/0004252.
5. M. Lüchsher and U. Wolff, Nucl. Phys. B339, 222 (1990).
6. F.X. Lee, hep-lat/0011000; D. Richards, hep-lat/0011025.
7. T. Blum, S. Ohta and S. Sasaki, Nucl. Phys. B (Proc. Suppl.) 94, 295 (2001).
8. M. Fukugita et al., Phys. Rev. Lett. 75, 2092 (1995).
9. M. Göckeler et al., Phys. Rev. D53, 2317 (1996).
Figure 1. \( N \) and \( N^* \) (square and diamond) masses versus the quark mass \( m_f \) in lattice units. Note the large \( N-N^* \) mass splitting which is within 10\% (in the chiral limit) of the experimental value (bursts).

Figure 2. The mass of the positive-parity excited state (circles) is too high compared to the nucleon ground state (cross) to account for the observed splitting.
Figure 3. Dependence of vector renormalization, $Z_V = 1/g_V^{\text{lattice}}$, on $t'$, at $m_f = 0.02$. A good plateau is observed.

Figure 4. The lattice axial charge, $g_A^{\text{lattice}}$, at $m_f = 0.02$. A good plateau is seen in the range $10 \leq t \leq 16$. 
Figure 5. Dependence of \((g_A/g_V)_{\text{lattice}}\) on \(m_f\).

Figure 6. \(g_A\) versus \((m_\pi/m_\rho)^2\).