Extracting $\gamma$ and Penguin Topologies through
CP Violation in $B^0_s \rightarrow J/\psi K_S$

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Abstract

The $B^0_s \rightarrow J/\psi K_S$ decay has recently been observed by the CDF collaboration and will be of interest for the LHCb experiment. This channel will offer a new tool to extract the angle $\gamma$ of the unitarity triangle and to control doubly Cabibbo-suppressed penguin corrections to the determination of $\sin 2\beta$ from the well-known $B^0_d \rightarrow J/\psi K_S$ mode with the help of the $U$-spin symmetry of strong interactions. While any competitive determination of $\gamma$ is interesting, the latter aspect is particularly relevant as LHCb will enter a territory of precision which makes the control of doubly Cabibbo-suppressed Standard-Model corrections mandatory. Using the data from CDF and the $e^+e^-$ $B$ factories as a guideline, we explore the sensitivity for $\gamma$ and the penguin parameters and point out that the $B^0_s-\bar{B}^0_s$ mixing phase $\phi_s$, which is only about $-2^\circ$ in the Standard Model but may be enhanced through new physics, is a key parameter for these analyses. We find that the mixing-induced CP violation $S(B^0_s \rightarrow J/\psi K_S)$ shows an interesting correlation with $\sin \phi_s$, which serves as a target region for the first measurement of this observable at LHCb.
1 Introduction

As the LHCb experiment at CERN’s Large Hadron Collider (LHC) has just started to take first physics data, we are entering a new territory in precision flavour physics. A particularly exciting feature of the LHCb research programme is the exploration of decays of $B_s$ mesons. Among the various $B_s$ decays with a promising physics potential is the $B^0_s \to J/\psi K_S$ mode, which has recently been observed by the CDF collaboration at the Tevatron [1].

In Ref. [2], it was pointed out that the CP-violating asymmetries of this channel offer a probe for extracting the angle $\gamma$ of the unitarity triangle (UT). Moreover, it was noted that also penguin topologies can be determined, which limit the theoretical precision of the measurement of $\sin 2\beta$, where $\beta$ is another UT angle, by means of the mixing-induced CP violation in the “golden” decay $B^0_d \to J/\psi K_S$. In this strategy, the $U$-spin flavour symmetry of strong interactions is used, which is a subgroup of $SU(3)_F$ relating down and strange quarks to each other, and allows us to relate the hadronic parameters of the $B^0_s \to J/\psi K_S$ and $B^0_d \to J/\psi K_S$ modes to one another. In Fig. [1] we show the decay topologies of the $B^0_s \to J/\psi K_S$ channel; interchanging all down and strange quarks, we obtain the $B^0_d \to J/\psi K_S$ topologies.

![Decay topologies](image)

Figure 1: Decay topologies contributing to the $B^0_s \to J/\psi K_S$ channel: “tree” (left) and “penguin” (right) diagrams.

In view of the start of LHCb, we would like to have a closer look at the prospects for analysing the $B^0_s \to J/\psi K_S$ channel at this experiment. Here we will give a special emphasis to the extraction of the hadronic penguin parameters and the control of the corresponding uncertainty in the measurement of $\sin 2\beta$ from $B^0_d \to J/\psi K_S$. In Ref. [3], using data on the $B^0_d \to J/\psi \pi^0$ channel (see also [1]), it was pointed out that such effects tend to lower a tension in the fit of the UT between measurements of its side $R_b \propto |V_{ub}/V_{cb}|$ and $\sin 2\beta$. Following these results, we will explore the prospects for such an analysis by means of the $B^0_s \to J/\psi K_S$ channel as an analysis of $B^0_d \to J/\psi \pi^0$ is considered to be more challenging at LHCb.

Due to the low branching fraction of the $B^0_s \to J/\psi K_S$ mode, LHCb might be able to get only very crude first information on the corresponding CP asymmetries by the end of 2011, even though a signal should be clearly visible by then. In order to explore the
full potential of this decay, we will therefore assume two milestones for our feasibility study:

- an integrated luminosity of 6 fb\(^{-1}\), which the LHCb experiment could realistically collect by the end of the second data taking period of 2014–15 (with the LHC having reached the design centre-of-mass energy \(\sqrt{s} = 14\) TeV);

- an integrated luminosity of 100 fb\(^{-1}\), which is the current target for a subsequent LHCb upgrade.

For the measurement of \(\sin 2\beta\) through \(B^0_d \to J/\psi K_S\), \((\sin 2\beta)_{J/\psi K_S}\), LHCb expects precisions of 0.022 with 2 fb\(^{-1}\)\([5]\), and 0.003–0.010 for 100 fb\(^{-1}\), depending on to be determined systematic errors \([6]\). In view of these impressive numbers, we have to take penguin effects into account in order to match the experimental with the theoretical precision. Such an analysis may eventually also allow us to pin down new-physics (NP) effects in \(B^0_d - \bar{B}^0_d\) mixing; the current data still leave space for NP contributions as large as about 50\% of the SM contribution, with a CP-violating NP phase below 10\° (see, for instance, Ref. \([7]\)).

The outline of this paper is as follows: in Section 2, we discuss the strategy to extract \(\gamma\) and the penguin parameters from the \(B_{d,s} \to J/\psi K_S\) modes. In Section 3, we have a look at the picture for the relevant observables emerging from the current data, and, after introducing the framework for the LHCb feasibility study in Section 4, discuss then in Section 5 the prospects for the resulting determination of \(\gamma\). In Section 6, we focus on the extraction of the penguin parameters, and discuss then the control of their impact on the measurement of \((\sin 2\beta)_{J/\psi K_S}\) in Section 7. Finally, we summarise our conclusions and give a brief outlook in Section 8.

2 The Basic Strategy

The \(B^0_s \to J/\psi K_S\) transition originates from a \(\bar{b} \to \bar{c}cd\bar{d}\) quark-level process and has a decay amplitude of the following structure:

\[
A(B^0_s \to J/\psi K_S) = \lambda_{c(d)}^{(c)} \left[ A_T^{(c)} + A_P^{(c)} \right] + \lambda_{u(d)}^{(d)} A_P^{(u)} + \lambda_{t(d)}^{(d)} A_P^{(t)} ,
\]

where \(A_T^{(c)}\) and \(A_P^{(j)}\) denote strong amplitudes that are related to the “tree” and “penguin” topologies (with internal \(j \in \{u, c, t\}\) quarks), respectively, which are illustrated in Fig. 1. On the other hand, the

\[
\lambda_{q(d)}^{(d)} \equiv V_{qd} V_{qb}^* \]

factors contain the relevant combinations of elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. If we make use of the unitarity of the CKM matrix and apply the Wolfenstein parametrisation \([8]\) as generalised in Ref. \([9]\), we can write \((1)\) as

\[
A(B^0_s \to J/\psi K_S) = -\lambda A \left[ 1 - ae^{i\theta} e^{i\gamma} \right].
\]

Here

\[
A \equiv \lambda^2 A \left[ A_T^{(c)} + A_P^{(c)} - A_P^{(t)} \right]
\]

(4)
and
\[ ae^{i\theta} \equiv R_b \left[ \frac{A_P^{(u)} - A_P^{(t)}}{A_T^{(c)} + A_P^{(c)} - A_P^{(t)}} \right] \]
are CP-conserving, “hadronic” parameters, where
\[ A \equiv \frac{|V_{cb}|}{\lambda^2} = 0.809 \pm 0.026 \quad \text{and} \quad R_b \equiv \left( 1 - \frac{\lambda^2}{2} \right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|, \]
with \( \lambda \equiv |V_{us}| = 0.22521 \pm 0.00083 \) are CKM parameters. It should be noted that \( R_b \) is one of the sides of the UT.

The \( B_s^0 \rightarrow J/\psi K_S \) channel is a decay into a CP eigenstate with eigenvalue \(-1\), and offers a time-dependent CP asymmetry of the following structure:
\[ a_{CP}(t) \equiv \frac{\Gamma(B_s^0 \rightarrow J/\psi K_S) - \Gamma(\bar{B}_s^0 \rightarrow J/\psi K_S)}{\Gamma(B_s^0 \rightarrow J/\psi K_S) + \Gamma(\bar{B}_s^0 \rightarrow J/\psi K_S)} = \frac{C(B_s^0 \rightarrow J/\psi K_S) \cos(\Delta M_s t) - S(B_s^0 \rightarrow J/\psi K_S) \sin(\Delta M_s t)}{\cosh(\Delta \Gamma_s t/2) - \mathcal{A}_{\Delta \Gamma}(B_s^0 \rightarrow J/\psi K_S) \sinh(\Delta \Gamma_s t/2)}, \]
where \( C(B_s^0 \rightarrow J/\psi K_S) \) and \( S(B_s^0 \rightarrow J/\psi K_S) \) describe direct and mixing-induced CP violation, respectively, and \( \mathcal{A}_{\Delta \Gamma}(B_s^0 \rightarrow J/\psi K_S) \) is accessible thanks to the width difference \( \Delta \Gamma_s \) of the \( B_s \)-meson system. In terms of CP-violating phases and the hadronic parameters introduced above, these observables read as follows:
\[ C(B_s^0 \rightarrow J/\psi K_S) = \frac{2a \sin \theta \sin \gamma}{1 - 2a \cos \theta \cos \gamma + a^2} \]
\[ S(B_s^0 \rightarrow J/\psi K_S) = \frac{\sin \phi_s - 2a \cos \theta \sin(\phi_s + \gamma) + a^2 \sin(\phi_s + 2\gamma)}{1 - 2a \cos \theta \cos \gamma + a^2} \]
\[ \mathcal{A}_{\Delta \Gamma}(B_s^0 \rightarrow J/\psi K_S) = \frac{\cos \phi_s - 2a \cos \theta \cos(\phi_s + \gamma) + a^2 \cos(\phi_s + 2\gamma)}{1 - 2a \cos \theta \cos \gamma + a^2}, \]
where \( \phi_s \) is the CP-violating \( B_s^0-\bar{B}_s^0 \) mixing phase, which is in the Standard Model (SM) given by \( \phi^\text{SM}_s = -2\lambda^2 \eta = -(2.12 \pm 0.11)^\circ \) but may well be enhanced by NP effects \[7\].

It should be noted that these quantities are not independent from one another, satisfying the relation
\[ [C(B_s^0 \rightarrow J/\psi K_S)]^2 + [S(B_s^0 \rightarrow J/\psi K_S)]^2 + [\mathcal{A}_{\Delta \Gamma}(B_s^0 \rightarrow J/\psi K_S)]^2 = 1. \]

However, we have still another independent observable at our disposal, which is the CP-averaged rate \[2\]
\[ \langle \Gamma(B_s^0 \rightarrow J/\psi K_S) \rangle \equiv \Phi^{s}_{J/\psi K_S} \times |\mathcal{N}|^2 \times \left[ 1 - 2a \cos \theta \cos \gamma + a^2 \right], \]
where \( \Phi^{s}_{J/\psi K_S} \) denotes a phase-space factor, and \( \mathcal{N} \equiv -\lambda \mathcal{A} \).

In the case of the decay \( B_d^0 \rightarrow J/\psi K_S \), the counterpart of the penguin parameter \( ae^{i\theta} \) enters with the doubly Cabibbo-suppressed parameter \( \epsilon \equiv \lambda^2/(1 - \lambda^2) = 0.053 \) as follows:
\[ A(B_d^0 \rightarrow J/\psi K_S) = \left( 1 - \frac{\lambda^2}{2} \right) \mathcal{A} \left[ 1 + \epsilon \lambda e^{i\phi_s} e^{i\gamma} \right], \]
where the hadronic parameters are defined in analogy to the $B^0_s \to J/\psi K_S$ case \cite{2}, and the primes remind us that we are dealing with a $b \to s$ mode. The corresponding observables can be obtained from the expressions given above by simply making the following replacements:

$$a \to -a', \quad \theta \to \theta', \quad A \to A', \quad N \to N' \equiv -\frac{1}{\sqrt{\epsilon}} \frac{A'}{A} N.$$  \hfill (14)

Concerning the mixing-induced CP asymmetry $S(B^0_d \to J/\psi K_S)$, we have to replace, in addition, the $B^0_s - \bar{B}^0_s$ mixing phase by its $B_d$-meson counterpart $\phi_d$, which is in the SM given by $\phi^\text{SM}_d = 2\beta$.

Thanks to the $\epsilon$ suppression of the penguin parameter in (13), the $B^0_d \to J/\psi K_S$ decay is referred to as the “golden” channel for the measurement of $\phi_d$. In the $B$-factory era of the last decade, it was justified in terms of accuracy to simply neglect this term. On the other hand, we are entering a new territory for precision $B$ physics in the LHC era which makes it mandatory to take the penguin effects into account. This was already anticipated in 1999 in Ref. \cite{2}, where it was proposed to use the $B^0_s \to J/\psi K_S$ channel to accomplish this task and to extract the angle $\gamma$ of the UT, exploiting the feature that the decays $B^0_s \to J/\psi K_S$ and $B^0_d \to J/\psi K_S$ are related to each other through an interchange of all down and strange quarks, i.e. by the $U$-spin flavour symmetry of strong interactions. The relevant observables are the CP asymmetries $C(B^0_s \to J/\psi K_S)$ and $S(B^0_s \to J/\psi K_S)$ as well as the following ratio of the CP-averaged rates:

$$H \equiv \frac{1}{\epsilon} \frac{\langle \Gamma(B^0_s \to J/\psi K_S) \rangle}{\langle \Gamma(B^0_d \to J/\psi K_S) \rangle} \left| \frac{A'}{A} \right|^2 \frac{\Phi^d_{J/\psi K_S}}{\Phi^s_{J/\psi K_S}} = \frac{1 - 2a \cos \theta \cos \gamma + a^2}{1 + 2\epsilon a' \cos \theta' \cos \gamma + \epsilon^2 a'^2}.$$  \hfill (15)

The $U$-spin flavour symmetry implies

$$a = a', \quad \theta = \theta', \quad A' = A.$$  \hfill (16)

as well as

$$A' = A.$$  \hfill (17)

The three observables depend then on $\gamma$ as well as $a$ and $\theta$. Consequently, we can convert the measured values of $C(B^0_s \to J/\psi K_S)$, $S(B^0_s \to J/\psi K_S)$ and $H$ into $\gamma$, $a$ and $\theta$. In Sections 5 and 6, we discuss in detail how the implementation of this strategy works in practise at the LHCb experiment.

### 3 Picture from Current Data

The CDF collaboration has recently reported the observation of the $B^0_s \to J/\psi K_S$ channel, with the following result \cite{1}:

$$\frac{\text{BR}(B^0_s \to J/\psi K_S)}{\text{BR}(B^0_d \to J/\psi K_S)} = 0.0405 \pm 0.0070(\text{stat.}) \pm 0.0041(\text{syst.}) \pm 0.0050(\text{frag.}), \quad (18)$$
where the last error is associated with the ratio $f_s/f_d$ of the $B_s$ and $B_d$ fragmentation functions. Using the value $\text{BR}(B_d^0 \rightarrow J/\psi K^0) = (8.71 \pm 0.32) \times 10^{-3}$ of the Particle Data Group (PDG) [10] yields

$$\text{BR}(B_s^0 \rightarrow J/\psi K^0) = (3.53 \pm 0.61 \text{(stat.)} \pm 0.35 \text{(syst.)} \pm 0.43 \text{(frag.)} \pm 0.13 \text{(PDG)}) \times 10^{-5}.\quad (19)$$

It is interesting to compare this result with the branching ratio of the $B^0_d \rightarrow J/\psi \pi^0$ channel. If we neglect penguin annihilation and exchange topologies, which contribute to $B^0_d \rightarrow J/\psi \pi^0$ but have no counterpart in $B^0_s \rightarrow J/\psi \bar{K}^0$ and are expected to play a minor role (which can be probed through $B^0_s \rightarrow J/\psi \pi^0$), the $SU(3)$ flavour symmetry applied to the spectator $d$ and $s$ quarks implies

$$\Xi_{SU(3)} \equiv \frac{\text{BR}(B^0_s \rightarrow J/\psi \bar{K}^0)}{2 \text{BR}(B^0_d \rightarrow J/\psi \pi^0)} \frac{\tau_{B_d}}{\tau_{B_s}} \frac{\Phi_{J/\psi \pi^0}}{\Phi_{J/\psi \bar{K}^0}} \xrightarrow{SU(3)} 1,\quad (20)$$

where the factor of 2 is associated with the wave function of the $\pi^0$, whereas the $\tau_{B_d}$ and $\tau_{B_s}$ denote the $B_d$ and $B_s$ lifetimes, respectively. Using the PDG values $\tau_{B_d}/\tau_{B_s} = 1.036 \pm 0.018$ and $\text{BR}(B^0_d \rightarrow J/\psi \pi^0) = (1.76 \pm 0.16) \times 10^{-5}$, we obtain

$$\Xi_{SU(3)} = 1.01 \pm 0.25,\quad (21)$$

which agrees very well with the $SU(3)$ expectation in (20), although the error is still sizable.

In the following analysis, we will use the hadronic parameters extracted from the CP violation data for the $B^0_d \rightarrow J/\psi \pi^0$ channel as a guideline, using the numerical values obtained in Ref. [3]. Making the same assumptions as in (20), we have then

$$a \in [0.15, 0.67], \quad \theta \in [174, 213]^{\circ},\quad (22)$$

which correspond to $\gamma = (65 \pm 10)^{\circ}$. The numerical value in (21) gives us confidence in this procedure. The observables of the $B^0_s \rightarrow J/\psi K_S$ channel then read as

$$H = 1.53 \pm 0.31,\quad (23)$$

and

$$C \equiv C(B^0_s \rightarrow J/\psi K_S) = C(B^0_d \rightarrow J/\psi \pi^0) = -0.10 \pm 0.13.\quad (24)$$

Concerning the mixing-induced CP violation $S \equiv S(B^0_s \rightarrow J/\psi K_S)$, we can calculate this observable with the help of Eq. (9) as a function of the $B^0_s$–$B^0_s$ mixing phase $\phi_s$ for given values of $H$, $C$ and $\gamma$. The corresponding formulae are given as follows:

$$2a \cos \theta = \frac{1 - H + (1 - \epsilon^2 H)a^2}{(1 + \epsilon H) \cos \gamma}\quad (25)$$

$$1 - 2a \cos \theta \cos \gamma + a^2 = \left[\frac{(1 + \epsilon)(1 + \epsilon a^2)}{1 + \epsilon H}\right] H,\quad (26)$$

where

$$a^2 = P \pm \sqrt{P^2 - Q}\quad (27)$$
with

\[ P = \frac{2 \left[ (1 + \epsilon H) \sin \gamma \cos \gamma \right]^2 - (1 - H)(1 - \epsilon^2 H) \sin^2 \gamma - \epsilon \left[ (1 + \epsilon) H C \cos \gamma \right]^2}{\left[ (1 - \epsilon^2 H) \sin \gamma \right]^2 + \left[ \epsilon (1 + \epsilon) H C \cos \gamma \right]^2} \quad (28) \]

\[ Q = \frac{\left[ (1 + \epsilon) H C \cos \gamma \right]^2 + \left[ (1 - H) \sin \gamma \right]^2}{\left[ (1 - \epsilon^2 H) \sin \gamma \right]^2 + \left[ \epsilon (1 + \epsilon) H C \cos \gamma \right]^2}. \quad (29) \]

In these expressions, we have used the U-spin relation \([16]\) but did not make any other approximation. If we assume the SM, we arrive at the following prediction:

\[ S(B_s^0 \rightarrow J/\psi K_S)|_{SM} = 0.54 \pm 0.14 \cdot 0.25 |H - 0.00| |C + 0.16| |\gamma = 0.54 \pm 0.21. \quad (30) \]

As we have noted above, NP may well enter the $B_s^0 \rightarrow J/\psi K_S$ channel through contributions to $B^0_s-B^0_s$ mixing, yielding a value of $\phi_s$ different from $\phi_s^{SM}$. The most recent compilation of the CDF and DØ collaborations using measurements of CP violation in $B^0_s \rightarrow J/\psi\phi$ \([11]\) can be found in Refs. \([12]\) and \([13]\), respectively. Unfortunately, the situation is not conclusive, although the CDF and DØ data are consistent with each other: the CDF collaboration finds the ranges $\phi_s \in [-59.6^\circ, -2.29^\circ] \sim -30^\circ \vee [-177.6^\circ, -123.8^\circ] \sim -150^\circ$ (68% C.L.), while the DØ collaboration gives a best fit value around $\phi_s \sim -45^\circ$, taking also information from the dimuon charge asymmetry and the measured $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$ branching ratio into account.

Interestingly, we can also probe this kind of NP through the mixing-induced CP violation in $B_s^0 \rightarrow J/\psi K_S$. In Fig. 2 we show the correlation between $S(B_s^0 \rightarrow J/\psi K_S)$ and $\sin \phi_s$, which can be determined from the time-dependent angular analysis of the $B_s^0 \rightarrow J/\psi\phi$ channel \([11]\). In this figure, we assume that NP is only present in $B^0_s-B^0_s$ mixing and have used the formulae give above to calculate curves for different values of $H$ and $\gamma$. We observe the following features:

- Thanks to the penguin contributions, we can distinguish between the twofold ambiguity for $\phi_s$ originating from the measured value of $\sin \phi_s$, in particular also between $\phi_s \sim 0^\circ$ (as in the SM) and $\phi_s \sim 180^\circ$.
- For $\phi_s \sim -30^\circ$ and $\phi_s \sim -45^\circ$, the mixing-induced CP asymmetry is $\sim 0$ while it approaches $-1$ for $\phi_s \sim -150^\circ$. Consequently, we could then also clearly identify the scenarios corresponding to the CDF and DØ data discussed above.

In view of these observations, already first experimental information on the mixing-induced CP violation in $B_s^0 \rightarrow J/\psi K_S$ would be very interesting to complement the analyses of CP violation in $B^0_s \rightarrow J/\psi\phi$. Needless to note that this feature relies on sizable penguin contributions to the $B_s^0 \rightarrow J/\psi K_S$ channel.

4 Framework for the LHCb Feasibility Study

With an anticipated integrated luminosity of $1 \text{ fb}^{-1}$ by the end of 2011, the LHCb experiment expects to be able to measure $\sin \phi_s$ with a precision between 0.04 and
Figure 2: Correlation between $\sin\phi_s$ and $S(B_s^0 \to J/\psi K_S)$ for different values of $H$ (left panel, with $\gamma = 65^\circ$) and different values of $\gamma$ (right panel, with $H = 1.53$). The impact of a variation of $C = -0.10 \pm 0.13$ is small.

0.07 $^{[14]}$. The $B_s^0 \to J/\psi K_S$ decay should then already be clearly visible with the following uncertainties:

$$\frac{\text{BR}(B_s^0 \to J/\psi K_S)}{\text{BR}(B_d^0 \to J/\psi K_S)} = 0.0403 \pm 0.0065\text{(stat.)} \pm 0.0029\text{(frag.)},$$

(31)

based on an estimated event yield of 60 000 $B_s^0 \to J/\psi K_S$ and 700 $B_s^0 \to J/\psi K_S$ decays, and where the error on the fragmentation fractions is based on the method proposed in Ref. $^{[15]}$. Concerning the CP-violating observables, we expect a sensitivity of $\Delta C = \Delta S = 0.34$, which might allow us to get first information on these quantities.

In order to explore the feasibility of the $B_s^0 \to J/\psi K_S$ measurement at LHCb with higher integrated luminosities, we perform a toy Monte Carlo (MC) analysis. For this study we assume the following situation: Already with an integrated luminosity of 1 fb$^{-1}$ we can pin down the $B_s^0 - \bar{B}_s^0$ mixing phase. Hence, for the two milestone scenarios it will be well known. In the following discussion we will consider the SM case $\phi_s = -2.12^\circ$.

Secondly, the used signal and background yields are extrapolated from Ref. $^{[5]}$. The number of $B_d^0 \to J/\psi K_S$ events listed there is scaled, using the measured $B_s^0 \to J/\psi K_S$ branching fraction, to give an expected yield of 4000 and 70 000 $B_s^0 \to J/\psi K_S$ events for the integrated luminosities of 6 fb$^{-1}$ and 100 fb$^{-1}$, respectively. These event yields are based on an estimated inclusive $J/\psi$ cross section of $\sigma_{\text{incl.}J/\psi} = 262 \, \mu\text{b}$ at $\sqrt{s} = 14$ TeV, the LEP fragmentation functions published by HFAG $^{[16]}$, and a $b\bar{b}$ cross section at $\sqrt{s} = 14$ TeV of $\sigma_{b\bar{b}} = 568 \, \mu\text{b}$, which was extrapolated from the recently measured $b\bar{b}$ cross section of $\sigma_{b\bar{b}} = (284 \pm 20 \, \text{(stat.)} \pm 49 \, \text{(syst.)}) \, \mu\text{b}$ at $\sqrt{s} = 7$ TeV $^{[17]}$.

For the MC analysis the reconstructed mass and lifetime distribution of both signal and background are modelled. The former is used to obtain the errors on the CP observables $C$ and $S$, while the latter is used for the determination of the statistical error on $H$. The error on $H$ coming from the ratio $f_s/f_d$ of fragmentation functions,
| Parameter | Value  | Parameter                  | Value  |
|-----------|--------|----------------------------|--------|
| $\sigma_{b\bar{b}}$ | 568 $\mu$b | $B_s$ lifetime resolution | 0.041 ps |
| $\sigma_{\text{Incl.}J/\psi}$ | 262 $\mu$b | mistag fraction | 0.36 |
| $f_d/f_s$ | 3.55 $\pm$ 0.26 | $B$ mass resolution | 15.2 MeV/c$^2$ |

Table 1: Overview of the parameters used by the toy MC.

The modelled $B_s$ lifetime distribution, given in the left panels of Fig. 3, has three major components: the $B_s^0 \to J/\psi K_S$ signal (plotted for $B_s$ and $B_s$ combined), an inclusive $J/\psi$ background and a combinatoric $b\bar{b}$ background. The $B_s$ and $\bar{B}_s$ signals are described individually using the time evolution equations (11) and (12) in Ref. [2], respectively, and include a dilution factor due to mistagging of the original $B$ meson. The inclusive $J/\psi$ sample contains both prompt and non-prompt (e.g. from $B$ decays) $J/\psi$ candidates, which results in an asymmetric distribution. The combinatoric $b\bar{b}$ background describes all other background from $b\bar{b}$ events; other sources of background are considered negligible. All lifetime distributions are convoluted with a Gaussian resolution model, which causes some events to have a negative lifetime. The resolution for the $B_s^0 \to J/\psi K_S$ signal is $\sigma = 0.041$ ps [5].

The modelled mass distribution, also given in Fig. 3, describes the two $B$ mass peaks ($B_d$ on the left, and $B_s$ on the right and in the insert) and an exponential background for both the inclusive $J/\psi$ and combinatoric $b\bar{b}$ sample.

The signal yield is small compared to the number of background events, as can be seen from the inserts in Fig. 3, making it hard to distinguish the $B_s$ mass peak from the background. This is the main reason for choosing the high luminosity scenarios of 6 fb$^{-1}$ and 100 fb$^{-1}$. Nonetheless, the lifetime distribution shows that the background is predominantly prompt. The long lifetime region ($t > 2$ ps) is therefore sufficiently sensitive to $B_s^0 \to J/\psi K_S$ in order to extract $C$ and $S$. This is confirmed by the plot of the time-dependent CP asymmetry $a_{CP}(t)$ defined in equation (7) and given in Fig. 4 for our two milestone scenarios.

The toy MC study is mainly used to estimate the errors on the relevant observables. The actual values of these observables are determined by their dependency on $a$, $\theta$ and $\gamma$, given in equations (8), (9) and (15). The input values for $a$, $\theta$ and $\gamma$ were taken from the $B_d^0 \to J/\psi \pi^0$ analysis whose result was already given in (22). We take the central values of these intervals,

$$a = 0.41, \quad \theta = 194^\circ, \quad \gamma = 65^\circ, \quad (32)$$

which result in the following picture for the statistical uncertainties at LHCb:

$$6 \text{ fb}^{-1}: \Delta H = 0.069, \quad \Delta S = 0.14, \quad \Delta C = 0.14, \quad (33)$$

$$100 \text{ fb}^{-1}: \Delta H = 0.052, \quad \Delta S = 0.035, \quad \Delta C = 0.035. \quad (34)$$

In principle these statistical errors could depend on the input value for $a$, $\theta$ and $\gamma$, but we will assume them constant for the remainder of this paper.
Figure 3: The lifetime distribution for an originally tagged $B_s$ meson (left) and the total mass distribution (right) for the decay $B^0_s \rightarrow J/\psi K_S$ and its background, as used in the MC analysis. This shows good sensitivity to the signal in the long lifetime region. The number of events correspond to a data sample of 6 fb$^{-1}$ (top) and 100 fb$^{-1}$ (bottom) respectively.
Figure 4: Toy MC simulation of the time-dependent CP asymmetry $a_{CP}(t)$ defined in equation (7) for the 6 fb$^{-1}$ (top) and 100 fb$^{-1}$ (bottom) scenarios.
For a more detailed discussion of the experimental feasibility study, the reader is referred to Ref. [18].

5 Extraction of $\gamma$ at LHCb

First information about $\gamma$ could in principle be obtained from $H$ as the general structure of this observable in (15) implies the following inequality [19]:

$$H \geq [1 - 2\epsilon \cos^2 \gamma + \mathcal{O}(\epsilon^3)] \sin^2 \gamma$$

(35)

However, as the experimental data indicate a value of $H > 1$, the corresponding bound on $\gamma$ is not effective.

As discussed in Ref. [2], using the $B_s^0 - \bar{B}_s^0$ mixing phase as an input, we can convert the $B_s^0 \to J/\psi K_S$ CP asymmetries $C$ and $S$ into a contour in the $\gamma - a$ plane, which is theoretically clean. Using the $U$-spin symmetry, we can determine another contour from $H$ and $S(B_s^0 \to J/\psi K_S)$. Thanks to the $U$-spin relation in (16), the intersection of these curves allows us then to extract $\gamma$ and the penguin parameter $a$. Knowing these parameters, the CP-conserving strong phase $\theta$ can be extracted as well. In Fig. 5 we illustrate the corresponding situation in the $\gamma - a$ plane for our LHCb study.

Figure 5: Determination of $\gamma$ and $a$ from intersecting contours. Situation resulting from our LHCb feasibility study for 6 fb$^{-1}$ (left) and 100 fb$^{-1}$ (right).

The angle $\gamma$ can also be extracted, together with $a$ and $\theta$, from a simultaneous $\chi^2$ fit to $C$, $S$ and $H$ using the expressions in (8), (9) and (15). The corresponding numerical results read as follows:

$$6 \text{ fb}^{-1}: \quad \gamma = (65 \pm 10) \degree, \quad a = 0.410 \pm 0.079, \quad \theta = (194 \pm 17) \degree,$$  

(36)

$$100 \text{ fb}^{-1}: \quad \gamma = (65.0 \pm 3.2) \degree, \quad a = 0.410 \pm 0.023, \quad \theta = (194.0 \pm 4.1) \degree.$$  

(37)

Here we give only the statistical errors, which show that LHCb should have sensitivity to extract $\gamma$ from the $U$-spin-related $B_{s,d} \to J/\psi K_S$ decays. Using other avenues to extract
γ, in particular theoretically clean strategies employing $B_s^0 \to D_s^+ K^\pm$ and similar modes, LHCb expects the following accuracies for γ:

\[ 6 \text{ fb}^{-1} : \quad \gamma = (65.0 \pm 3.1)° \quad [14], \quad 100 \text{ fb}^{-1} : \quad \gamma = (65.0 \pm 1.0)° \quad [6], \quad (38) \]

which should be compared with the current situation emerging from $B \to D^{(*)} K^{(*)}$ analyses:

\[ \gamma = (71^{+21}_{-25})° \quad (\text{CKMfitter} \ [20]), \quad (78 \pm 12)° \quad (\text{UTfit} \ [21]). \quad (39) \]

Consequently, LHCb should make significant progress with the determination of γ with respect to the unsatisfactory current situation.

Concerning the $B_{s,d} \to J/\psi K_S$ strategy, we find a good sensitivity for this angle in our feasibility study. We have also to deal with theoretical uncertainties originating from $U$-spin-breaking effects. As far as the corrections to (16) are concerned, they actually play a very minor role for the determination γ as the $a'$ terms in (15) are strongly suppressed by the tiny $\epsilon$ parameter in [2]; numerically, they give a correction for γ well below 1°. However, $U$-spin-breaking effects enter also through the ratio $|A'/A|$, which affects the determination of $H$ as defined in (15). We find that an uncertainty of 10% for the relevant form-factor ratio $F_{B_d^0 K^0}(M_{J/\psi}; 1^-)/F_{B_s^0 K^0}(M_{J/\psi}; 1^-)$, as considered in Ref. [3], corresponds to an error of about $\Delta \gamma_{\text{FF}} = 12°$, which essentially matches the statistical error of 10° for our 6 fb$^{-1}$ scenario in (36). For an LHCb upgrade with 100 fb$^{-1}$, a form-factor uncertainty of 2.5% would be required in order to match (37), which serves as the ultimate benchmark for future calculations of this quantity. We hope that already by the end of the second LHCb data taking period, i.e. 2014–15, we will have a better understanding of $SU(3)$ breaking through studies of various other $U$-spin- and $SU(3)$-related $B$-meson decays, as well as through improved theoretical studies, including in particular also non-factorisable $U$-spin-breaking corrections to $|A'/A|$. The current data from the Tevatron do not indicate sizable non-factorisable $SU(3)$-breaking effects [22], which is also supported by [21].

An important aspect of the determination of γ through the $B_{s,d} \to J/\psi K_S$ strategy is that it may reveal a NP contribution to the $B_s^0 \to J/\psi K_S$ decay amplitude. This would be indicated by a value of γ in conflict with the clean determinations through the pure “tree” decays summarised in (38). In the following discussion, we assume that we have negligible NP effects of this kind, i.e. assume the SM expressions for the decay amplitudes in (3) and (13).

6 Clean Determination of the $B_s^0 \to J/\psi K_S$ Penguin Parameters at LHCb

The major application of the $B_s^0 \to J/\psi K_S$ channel at LHCb will be the extraction of the hadronic penguin parameters $(a, \theta)$ and to take them into account in the determination of the $B^0_d$–$\bar{B}^0_d$ mixing phase $\phi_d$ from $(\sin 2\beta)_{J/\psi K_S}$. Since our goal is to minimise here the $U$-spin-breaking corrections, we shall refrain from using the $H$ observable and will use γ as given in (38) as an input. Following Ref. [3], we can then convert the CP asymmetries $S$ and $C$ into theoretically clean contours in the $\theta–a$ plane; their intersection allows us
then to extract \(a\) and \(\theta\). In Fig. 6 we illustrate the corresponding situation arising from our LHCb feasibility study. In order to guide the eye, we show also the contour corresponding to \(H\), which would be interesting to obtain insights into \(U\)-spin-breaking effects. Performing again a \(\chi^2\) fit yields the following results:

\[
6 \text{ fb}^{-1}: \ a = 0.41 \pm 0.14 \,(\text{stat.}) \pm 0.02 \,(\gamma), \quad \theta = [194 \pm 16 \,(\text{stat.}) \pm 0.8 \,(\gamma)]^\circ, \ 
\]

\[
100 \text{ fb}^{-1}: \ a = 0.41 \pm 0.03 \,(\text{stat.}) \pm 0.01 \,(\gamma), \quad \theta = [194.0 \pm 4.0 \,(\text{stat.}) \pm 0.3 \,(\gamma)]^\circ. \ 
\]

We observe that LHCb is expected to be able to determine the \(B^0_s \to J/\psi K_S\) parameters with impressive precision. It should be emphasised that these parameters rely only on the SM expression for the decay amplitude and are theoretically clean.

7 Controlling Penguin Effects in the Measurement of \((\sin 2\beta)_{J/\psi K_S}\) at LHCb

The values of the penguin parameters \(a\) and \(\theta\) are interesting from the point of view of strong interaction dynamics. However, they have also an important practical application for testing the Kobayashi–Maskawa mechanism of CP violation. Using the \(U\)-spin relation \(16\), we can relate the \(B^0_s \to J/\psi K_S\) parameters to their \(B^0_d \to J/\psi K_S\) counterparts, which allows us then to control the hadronic uncertainty of the measurement of the \(B^0_d-B^0_d\) mixing phase \(\phi_d\) through the mixing-induced CP violation in \(B^0_d \to J/\psi K_S\). The corresponding, generalised expression reads as follows \(3\):

\[
S(B^0_d \to J/\psi K_S) = \sin(\phi_d + \Delta \phi_d),
\]

\[
\frac{S(B^0_d \to J/\psi K_S)}{\sqrt{1 - C(B^0_d \to J/\psi K_S)^2}} = \sin(\phi_d + \Delta \phi_d), \ 
\]
where

\[
\sin \Delta \phi_d = \frac{2\epsilon a \cos \theta \sin \gamma + \epsilon^2 a^2 \sin 2\gamma}{N \sqrt{1 - C(J/\psi K_S)^2}} \quad (43)
\]

\[
\cos \Delta \phi_d = \frac{1 + 2\epsilon a \cos \theta \cos \gamma + \epsilon^2 a^2 \cos 2\gamma}{N \sqrt{1 - C(J/\psi K_S)^2}} \quad (44)
\]

with \( N \equiv 1 + 2\epsilon a \cos \theta \cos \gamma + \epsilon^2 a^2 \), so that

\[
\tan \Delta \phi_d = \frac{2\epsilon a \cos \theta \sin \gamma + \epsilon^2 a^2 \sin 2\gamma}{1 + 2\epsilon a \cos \theta \cos \gamma + \epsilon^2 a^2 \cos 2\gamma}. \quad (45)
\]

The current value of the mixing-induced CP violation is given by

\[
S(B_d^0 \rightarrow J/\psi K^0) = 0.655 \pm 0.024, \quad (46)
\]

while the direct CP asymmetry is currently measured as

\[
C(B_d^0 \rightarrow J/\psi K^0) = -0.003 \pm 0.020. \quad (47)
\]

These numbers are averages of the BaBar and Belle data over the \( B_d^0 \rightarrow J/\psi K_S \) and \( B_d^0 \rightarrow J/\psi K_L \) final states, as obtained by the Heavy Flavour Averaging Group [16]. The value in (47) implies that the deviation of the square root in (42) from one is at most at the level of 0.0002, i.e. completely negligible.

Using these formulae, we can convert the contour plots in Fig. 6 into curves in the \( \theta - \Delta \phi_d \) plane, as shown in Fig. 7. In this analysis, we have uncertainties from \( U \)-spin-breaking corrections to (16), which we parametrise as follows:

\[
a = \xi a', \quad \theta = \theta' + \Delta \theta. \quad (48)
\]
If we vary $\xi \in [0.85, 1.15]$ and $\Delta \theta \in [-20^\circ, 20^\circ]$, and perform a $\chi^2$ fit to $C$ and $S$, we obtain

$$6 \text{ fb}^{-1} : \Delta \phi_d = \left[ -2.23 \pm 0.78 \text{ (stat.)} \pm 0.12 \left( \gamma \right) \pm 0.29(\xi) \pm 0.33(\Delta \theta) \right]^\circ, \quad (49)$$

$$100 \text{ fb}^{-1} : \Delta \phi_d = \left[ -2.23 \pm 0.19 \text{ (stat.)} \pm 0.04 \left( \gamma \right) \pm 0.29(\xi) \pm 0.33(\Delta \theta) \right]^\circ. \quad (50)$$

In order to study the dependence on the input values of the penguin parameters for our analysis, we show the correlation between the hadronic shift $\Delta \phi_d$ and its statistical error at LHCb in Fig. 8. The corresponding curves show nicely that we can actually well determine the $\Delta \phi_d$ correction for a wide range of $(a, \theta)$ that should contain the “true” values of these parameters. It should also be noted that already a small penguin contribution with $a = 0.1$ gives a correction of $\Delta \phi_d \sim -0.5^\circ$.

In order to fully appreciate the results of this study, we have to look at the precision for $(\sin 2\beta)_{J/\psi K_S}$ at LHCb: we expect 0.022 for 2 fb$^{-1}$ [5], and 0.003–0.010 (depending on to be determined systematic errors) for 100 fb$^{-1}$ [6]. We extrapolate these numbers into 0.014 for 6 fb$^{-1}$, which should be compared with [46]. For a central value of $S(B_d^0 \to J/\psi K^0) \sim 0.655$, these numbers correspond to errors of about 1$^\circ$ and (0.2–0.8)$^\circ$ for the phase itself. In order to match these very impressive precisions, we definitely have to control the doubly Cabibbo-suppressed penguin contributions in $B_d^0 \to J/\psi K_S$. Looking at the numerical results in [49] and [50], we observe that we can actually achieve this goal. Already with 6 fb$^{-1}$ we have to control the penguin effects; for an upgrade of LHCb this is absolutely necessary to fully exploit the tremendous statistics. Measurements along these lines may eventually allow us to resolve CP-violating NP contributions to $B_d^0 \to B_{d}^{0*}$ mixing.

Let us finally note that we can – in addition to the direct CP violation in the $B_d^0 \to J/\psi K_{S,L}$ channels [3] – add another observable to this analysis. It is the direct
CP asymmetry of the $B^+_u \rightarrow J/\psi\pi^+$ decay, which we get from the $B^0_s \rightarrow J/\psi K_S$ topologies in Fig. 1 by replacing the strange spectator quark by an up quark. The resulting contours in Fig. 7 are similar to those related to $C$. Already with 1 fb$^{-1}$, as expected by the end of 2011, we find from a feasibility study similar to that described above that the $B^+_u \rightarrow J/\psi\pi^+$ mode should be clearly visible at LHCb, with a statistical error of $C(B^+_u \rightarrow J/\psi\pi^+) = 0.005$. However, the production asymmetry between $B$ and $\bar{B}$ states at the LHC (which, unlike the Tevatron, is a $pp$ collider) is expected to be non-negligible and will have to be measured and corrected for. This is likely to induce irreducible systematic errors at a few per-cent level for the measurement of $C(B^\pm_u \rightarrow J/\psi\pi^\pm)$, which have to be studied in further detail.

8 Conclusions

As was pointed out about one decade ago, the decay $B^0_s \rightarrow J/\psi K_S$ is the $U$-spin partner of the “golden” mode $B^0_d \rightarrow J/\psi K_S$ and allows an extraction of the UT angle $\gamma$ and of the corresponding penguin parameters. In the summer of 2010, the CDF collaboration has announced the first observation of the $B^0_s \rightarrow J/\psi K_S$ transition, with a branching ratio in agreement with an $SU(3)$ relation to the $B^0_d \rightarrow J/\psi\pi^0$ decay.

We have calculated a correlation between the mixing-induced CP violation in the $B^0_s \rightarrow J/\psi K_S$ channel, $S$, and $\sin \phi_s$, which serves as a “target region” for the first CP violation measurement in this decay. We observe an interesting pattern for values of the $B^0_s$–$\bar{B}^0_s$ mixing phase $\phi_s$ in the region of the current measurements from the Tevatron: we can distinguish between the twofold solutions for $\phi_s$ arising from the measurement of $\sin \phi_s$, with $S \sim 0$ and $S \sim -0.8$ for $\phi_s \sim -30^\circ$ and $\phi_s \sim -150^\circ$, respectively, whereas we expect $S \sim +0.5$ in the SM. Thanks to improved measurements of input parameters, these predictions can be sharpened in the future.

By the end of 2011, i.e. the end of the first LHCb data taking period, we expect a clear signal for $B^0_s \rightarrow J/\psi K_S$, with very crude first information on the corresponding CP asymmetries. Extrapolating from published LHCb studies, we have performed a detailed feasibility study for the measurement of the $B^0_s \rightarrow J/\psi K_S$ observables, yielding about 4000 and 70000 signal events for 6 fb$^{-1}$ and 100 fb$^{-1}$, respectively. These integrated luminosities refer to the end of the second LHCb data taking period of 2014–15 and a subsequent LHCb upgrade. Using input parameters as suggested by current data, we have shown that the $B_{s,d} \rightarrow J/\psi K_S$ strategy offers indeed another determination of $\gamma$ for LHCb. However, we conclude that the most important application of this method will be the determination of the corresponding hadronic penguin parameters and their control in the measurement of $(\sin 2\beta)_{J/\psi K_S}$, which will allow us to match the corresponding experimental precision. Studies along these lines may eventually allow us to resolve NP in $B^0_d$–$\bar{B}^0_d$ mixing.

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