QCD Sum Rule for $\frac{1}{2}^-$ Baryons

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Abstract

The masses of the flavor octet and singlet baryons with negative parity and spin $\frac{1}{2}$ are calculated using the QCD sum rule. We find that the chiral symmetry breaking vacuum condensates cause the mass splitting of the positive and negative baryons. The present sum rule reproduces the observed masses of these baryons within 10%, and predicts the mass of the excited $\Sigma$ baryon with $J^P = \frac{1}{2}^-$ at 1.63 GeV. We confirm that the negative-parity state, $\Lambda_S^-$, is the ground state in the flavor singlet baryon spectrum.

1 Introduction

The negative parity excited states of baryons have been understood fairly well in the (nonrelativistic) quark model as one-quark excited states belonging to the SU(6) 70 representation [1]. The observed spectrum tends to agree with the prediction, although some of the states, such as $\Lambda(1405)$, $N(1535)$, have irregular masses and non-natural decay rates. Many refinements were proposed to achieve a quantitative agreement.

Yet, our understanding of the hadron physics insists that the role of the chiral symmetry must be important even in the baryon states. Indeed, the chiral symmetry suggests that the positive and negative parity states are paired into a parity doublet and the pair would be degenerate when the chiral symmetry is restored. Because the nonrelativistic quark model does not observe the chiral symmetry, a different approach is anticipated to understand the chiral structure of baryons, both the positive and negative parity states.

The technique of the QCD sum rule relates the hadron properties to the QCD parameters and is a powerful tool to extract the hadron properties from QCD, the first principle of the strong interaction [2, 3]. The QCD sum rule for the baryon was first proposed by Ioffe [4]. In the QCD sum rule a correlation function, such as $\Pi(p) = i \int d^4xe^{ipx}\langle0|\bar{J}(x)J(0)|0\rangle$, is calculated, where $J(x)$ is an interpolating field (IF) that couples to the state of the hadron in question. The nonperturbative effects, such as the quark condensate $\langle\bar{q}q\rangle$
and $\langle \pi_GGG \rangle$, are included as the power corrections in the theoretical side. The quark condensate $\langle \bar{q}q \rangle$, which is the order parameter of the chiral symmetry breaking, gives effects of the chiral symmetry breaking to the hadron spectrum in the QCD sum rule.

In our previous paper\cite{5}, the technique to extract masses of negative-parity baryon in the QCD sum rule was proposed and the masses of the nucleons with positive and negative parity are calculated. It is important to separate contribution of the negative-parity baryon ($B_-$) from that of the positive-parity baryon ($B_+$), since the IF for baryon couples to the states of the $B_-$, although we define the IF $J_B$ has positive parity. In order to separate the $B_-$ contribution we use the “old-fashioned” correlation function defined as

$$\Pi(p) = i \int d^4xe^{ipx}\theta(x_0)\langle 0|J_B(x)\bar{J}_B(0)|0 \rangle,$$

and construct sum rules in the complex $p_0$-space in the rest frame ($\vec{p} = 0$). Our approach is suitable for investigating the mass splitting, because $B_+$ and $B_-$ can be treated simultaneously in this sum rule. In the present paper, we extend the technique to the hyperons and the flavor singlet baryons $\Lambda_S$.

In Sec.2 the formulation of our approach is explained and we see that the chiral symmetry breaking is responsible for the mass splitting of $B_+$ and $B_-$. In Sec.3 we present the method to extract the masses and to determine the parameters in the theoretical side. In Sec.4 we give the numerical results for the masses of the baryons and the parameters in the theoretical side. We also discuss the relation of the mass splitting to the chiral symmetry breaking vacuum condensate. We show that the negative parity baryon is the lowest state in the flavor singlet spectrum and that the QCD sum rule predicts that the spin-parity of $\Xi(1690)$ is $\frac{1}{2}^-$. A summary is given in Sec.5.

2 Formulation

It is important to choose an appropriate interpolating field (IF) in the correlation function in the QCD sum rule. The IF should have the same quantum numbers as the baryon in question, so that it creates or annihilates a single particle state of the baryon from the vacuum. For the spin $\frac{1}{2}$ octet baryon two independent IFs can be constructed without a derivative \cite{2}. The IF for the nucleon is, for instance, written as

$$J_{N}(x) = \varepsilon_{abc}[(u_a(x)C d_b(x))\gamma_5 u_c(x) + t(u_a(x)C \gamma_5 d_b(x))u_c(x)],$$

where $a$, $b$ and $c$ are color indices, $C = i\gamma_2\gamma_0$ (standard notation) is for the charge conjugation and $t$ is a real parameter representing the mixing of two independent IFs. If we choose $t = -1$ and use the Fierz transformation, the IF (2) is reduced to the Ioffe’s IF \cite{4}. In the previous paper \cite{5}, we found that $J_N$ with $t = 0.8$ is appropriate for the nucleon resonance. For the $\Sigma$ baryon, we replace a d-quark by an s-quark in eq.(2) and obtain

$$J_{\Sigma^+}(x) = \varepsilon_{abc}[(u_a(x)C s_b(x))\gamma_5 u_c(x) + t(u_a(x)C \gamma_5 s_b(x))u_c(x)].$$

(3)
Similarly for the $\Xi$ baryon, replacing $u$-quark by $s$-quark in eq. (2), we obtain

$$J_{\Xi^{-}}(x) = \varepsilon_{abc}[(s_a(x)C\bar{d}_b(x))\gamma_5s_c(x) + t(s_a(x)C\gamma_5\bar{d}_b(x))s_c(x)]. \quad (4)$$

The IF for $\Lambda$ is more complicated:

$$J_{\Lambda}(x) = \varepsilon_{abc}[(d_a(x)CS_b(x))\gamma_5u_c(x) + (s_a(x)Cu_b(x))\gamma_5d_c(x) - 2u_a(x)C\bar{d}_b(x)\gamma_5s_c(x) + t\{[d_a(x)C\gamma_5s_b(x)]u_c(x) + (s_a(x)C\gamma_5u_b(x))d_c(x) - 2u_a(x)C\gamma_5\bar{d}_b(x)s_c(x)]\} \quad (5)$$

The IF for the flavor singlet baryon $\Lambda_S$ is given by the flavor antisymmetric combination of the quark operators:

$$J_{\Lambda_S}(x) = \varepsilon_{abc}[(u_a(x)C\gamma_5\bar{d}_b(x))s_c(x) - (u_a(x)C\bar{d}_b(x))\gamma_5s_c(x) - (u_a(x)C\gamma_5\gamma_\mu\bar{d}_b(x))\gamma^\mu s_c(x)]. \quad (6)$$

This IF is unique and has no parameter such as $t$ in the octet IF.

We now explain the technique of treating the $B_-$ in the QCD sum rule according to the Ref. [1]. First, we observe that the IFs given in eqs. (2)–(6) annihilate not only the positive parity baryon state, but also a single particle state of the negative parity baryon [4, 6]. Since the parity of the fermion state can be reversed by multiplying $\gamma_5$, the IF with negative parity may be obtained as

$$J_-(x) \equiv i\gamma_5J(x). \quad (7)$$

Then we expect

$$\langle 0|J_-(x)|B_- \rangle = \lambda_-u_-(x), \quad (8)$$

where $|B_- \rangle$ denotes a single particle state of the negative parity baryon, $\lambda_-$ is the coupling strength and $u_-(x)$ is the (corresponding) Dirac spinor. From eqs. (2) and (8), we obtain

$$\langle 0|J(x)|B_- \rangle = -\langle 0|i\gamma_5J_-(x)|B_- \rangle = -i\lambda_-\gamma_5u_-(x). \quad (9)$$

Thus, $J(x)$ couples also to the negative-parity baryon. This short exercise tells us that the conventional sum rule for the ground state baryon already contains the negative parity baryon state as a part of the continuum spectrum.

Our task is to separate the negative parity contribution properly. According to the previous paper [4], we use the “old-fashioned” correlation function (1). In the phenomenological side, if the lowest energy states of $B_+$ and $B_-$ are picked up and the rest is regarded as a continuum, the imaginary part of the “old-fashioned” correlation function is written as

$$\text{Im} \Pi(p_0) = (\lambda_+)^2\frac{\gamma_0+1}{2}\delta(p_0-m_+) + (\lambda_-)^2\frac{\gamma_0-1}{2}\delta(p_0-m_-) + \cdots \text{(continuum)} \quad (10)$$

$$\equiv \gamma_0A(p_0) + B(p_0). \quad (11)$$

In this expression, the zero-width pole approximation is applied. Although the resonances have significant widths (eg. about 150 MeV for $N(1535)$),
we expect that the approximation is valid because the IFs for baryons are composed of three quarks and may not couple strongly with $q\bar{q}$ states.

The difference of the contributions of $B_+$ and $B_-$ in eq (11) is the sign of the chiral odd part. Therefore $A(p_0) + B(p_0)$ has only $B_+$ contribution and $A(p_0) - B(p_0)$ includes only $B_-$ contribution. In this way we can separate the $B_-$ contribution from the ground state baryon. Moreover we construct the sum rule in the $p_0$ complex plane, because the function $A(p_0)$ defined in eq.(11) is not an analytic function of $p_2$, while the functions $A(p_0)$ and $B(p_0)$ is analytic in the upper half $p_0$-space.

We take the lowest mass pole and approximate others as a continuum whose behavior above a threshold $s_0^\pm$ is same as the theoretical side. Then we obtain sum rules for the positive- and negative-parity baryons:

$$\int_0^{s_0^+} [A_{OPE}(p_0) + B_{OPE}(p_0)] \exp \left[ -\frac{p_0^2}{M^2} \right] dp_0 = (\lambda_+)^2 \exp \left[ -\frac{m^2}{M^2} \right], \quad (12)$$

$$\int_0^{s_0^-} [A_{OPE}(p_0) - B_{OPE}(p_0)] \exp \left[ -\frac{p_0^2}{M^2} \right] dp_0 = (\lambda_-)^2 \exp \left[ -\frac{m^2}{M^2} \right], \quad (13)$$

where $M$ is the Borel mass and $A_{OPE}$ and $B_{OPE}$ are calculated based on QCD using the OPE at the region where $p_0$ is large. It should be noted that the OPE is valid for large $p_0$ even if $\vec{p} = 0$ because the singularity of the correlation function resides at the light cone [8]. The explicit forms of $A_{OPE}$ and $B_{OPE}$ are given in appendix for the flavor octet and the singlet baryons up to dimension six, $O(m_s)$ and $O(\alpha_s)$. The odd higher dimensional terms in the OPE do not contribute to the sum rule after the Borel transform, since their imaginary parts are proportional to the odd rank derivatives of the delta-function with respect to $p_0$. It is easy to show that the integral vanishes after partial integration.

Note that the difference of the sum rules for $B_+$ and $B_-$ is the chiral odd term $B(p_0)$, which is proportional either to the quark condensate $\langle \bar{q}q \rangle$ or to the mixed condensate $\langle \bar{q}g\sigma \cdot Gq \rangle$. If the chiral symmetry is restored at high temperature, for instance, the $B(p_0)$ term goes to zero in the chiral limit. Then the sum rules (12) and (13) are identical, and will predict the same masses for the positive and negative parity baryons. This situation is similar to that in the linear sigma model for parity-doublet baryons proposed by DeTar and Kunihiro [4]. There the positive and negative parity nucleons are assumed to form a parity doublet and the Lagrangian has a chiral invariant mass term. Under the restoration of the chiral symmetry, the nucleons have the same mass, while in the spontaneous symmetry broken phase the mass splitting is proportional to the nonvanishing vacuum expectation value of sigma. In another article [10], we have shown that this similarity to the linear sigma model is confirmed also in the $\pi NN^*$ coupling in the QCD sum rule.

We note several other approaches of the QCD sum rule for $B_-$. In the case that $B_-$ is the lowest energy state in the considering spectrum, the mass of $B_-$ is extracted in the usual sum rule. In Ref. [4], Ioffe pointed out that the negative-parity resonance is the ground state in the spectrum of the baryon with spin $J = \frac{3}{2}$ and isospin $T = \frac{1}{2}$ and calculated its mass. Liu also applied the QCD sum rule to $\Lambda(1405)$ and concluded that for the $\Lambda(1405)$ the
IF consisting of three quarks and a flavor octet quark-antiquark pair is important [11]. In his analysis, however, the continuum term is not considered. We obtain the realistic mass of Λ(1405) in the three-quark sum rule with a continuum term. Some other approaches treat B− with an IF which does not couple to the positive-parity ground state baryon. In Ref. [6] an optimized IF for N− was proposed by requiring that the chiral odd correlation function, in which the B+ contribution and the B− contribution have different sign, becomes negative. Lee and Kim also investigated the mass of N(1535) [12] and Λ(1405) [13] in the QCD sum rule. They proposed a new IF with a covariant derivative, expecting that it has a large overlap with the nonrelativistic quark wave function of N(1535). They chose the IF so that it does not couple to the ground state nucleon [12]. We, however, employ the nonderivative IF in the present study because our main interest is to study the mechanism of B++−B− mass splitting.

3 Determination of the B− masses

We have three phenomenological parameters, the mass \(m_{B\pm}\), the threshold \(s_\pm\) and the coupling strength \(\lambda_\pm\) to be determined from the sum rules (12) and (13). Unlike the standard sum rule we have only one sum rule for each baryon. Therefore we are forced to solve the system of three equations, eq. (12), the first and the second derivatives of (12) with respect to the Borel mass. In general, when we differentiate the sum rule with respect to the Borel mass, the reliability of the QCD sum rule is lost, since the derivative picks up the a factor \(p^2\), which may enhance the contribution of the continuum. But we should judge the reliability from the stability of the results against the Borel mass. One sees that the baryon masses show enough stability in our analysis.

The theoretical side depends on the QCD parameters, such as the quark mass and the gauge coupling constant, and also on the other parameters that describe the properties of the nonperturbative vacuum of QCD, such as the quark and gluon condensates. We take the chiral limit for the up and down quarks, i.e. \(m_q = 0\), where we use the symbol \(q\) for the up and down quarks. We introduce the strange quark mass \(m_s\), \(\chi \equiv \langle \bar{s}s\rangle/\langle \bar{q}q\rangle\) and \(\chi_5 \equiv \langle \bar{s}g\sigma\cdot Gs\rangle/\langle \bar{q}g\sigma\cdot Gq\rangle\) for the flavor SU(3) symmetry breaking. The gluon condensate is fixed to \(\langle \alpha_s\pi GG\rangle = (0.36\text{ GeV})^4\) since the coefficient is small in comparison with the other terms due to a suppression factor of \(1/(2\pi)^2\) [14]. The vacuum saturation is assumed for evaluating the matrix element of the four-quark operators, i.e. \(\langle (\bar{q}q)^2\rangle = \langle \bar{q}q\rangle^2\). These parameters have some uncertainty, which depends on the truncation in the OPE. In order to remove this uncertainty, we use our sum rules in the following way. The value of \(\langle \bar{q}q\rangle\) and \(m_s^2 \equiv \langle \bar{s}g\sigma\cdot Gs\rangle/\langle \bar{q}q\rangle\) are determined so that the sum rules (12) and (13) for the nucleon reproduce the observed masses of \(N_+\) and \(N_-\). In doing so we require that the prediction of the sum rule at \(M \simeq m_B\) coincides with the observed mass within 5% and also that for the Borel stability variation of the predicted mass against \(M\) in the region \(m_B \sim m_B + 0.5\text{ GeV}\) is less than 10%. In the same way, the values of \(m_s\), \(\chi\) and \(\chi_5\) are determined so that the
sum rules \(^{(12)}\) for the hyperons give the observed masses of the \(\Lambda_+, \Sigma_+\) and \(\Xi_+\).

4 Results and Discussion

The determined parameters in the theoretical side are given in Table 1. The masses of the positive-parity hyperons, \(\Lambda_+, \Sigma_+\) and \(\Xi_+\) are sensitive to the \(SU(3)\) breaking parameters \(m_s, \chi\) and \(\chi_5\). Therefore these parameters are determined well. As we shall see later, the value of \(\langle \bar{q}q \rangle\) is determined from the mass splitting of \(B_+\) and \(B_-\). The up and down quark condensate agrees well to the “standard value” \(\langle \bar{q}q \rangle = (-0.225 \pm 0.025\text{GeV})^3\), which was estimated in the chiral perturbation \(^{(13)}\), and the value \(\langle uu \rangle = -(0.230 \pm 0.015\text{GeV})^3\) that was estimated in the QCD sum rule for the octet and the decuplet baryons \(^{(16)}\). The value \(m_0\) is also consistent with \(m_0^2 = 0.5 \sim 1.0\text{ GeV}^2\) estimated in the sum rules for baryon \(^{(17, 18)}\) and \(m_0^2 = 1.1 \pm 0.1\text{ GeV}^2\) in the lattice calculation \(^{(13)}\). The instanton contribution leads to the mixed condensate somewhat larger, \(m_0^2 = 1.4\text{ GeV}^2\) \(^{(13, 21)}\). The reason for this large value is that the higher dimensional operators induced by the instanton reduce \(m_0\) effectively \(^{(22)}\). Although the strange quark mass \(m_s\) is somewhat smaller than the update analysis \(^{(21)}\), our results of the \(B_-\) masses are insensitive to \(m_s\). The sum rules for the baryons gives \(\chi \sim 0.8\) \(^{(16, 23)}\). The value of the \(\chi_5\) is expected to be close to \(\chi\), because both \(\chi\) and \(\chi_5\) are related to the flavor \(SU(3)\) breaking in QCD. In Ref. \(^{(23)}\), however, the sum rule for \(\Omega\) baryon suggests \(\chi_5 = 1.4\).

The masses of the flavor octet and singlet baryons calculated in the QCD sum rule are shown in Table 2. The observed masses are reproduced fairly well. The masses of the \(\Lambda_-\), \(\Sigma_-\), \(\Xi_-\), \(\Lambda_{S-}\) and \(\Lambda_{S+}\) are the prediction without adjustable parameters. These masses are taken at the Borel mass \(M \sim m_B\). The \(M\) dependence of the masses are shown in Fig. 1 for the octet \(B_+\), in Fig. 3 for the octet \(B_-\), in Fig. 4 for \(\Lambda_{S-}\) and in Fig. 5 for \(\Lambda_{S+}\). All masses are stable against the Borel mass \(M\). The excited \(\Xi\) baryon with \(J^P = \frac{3}{2}^-\) has not been identified by experiment, but resonances with unknown spin and parity are found at 1690 MeV and 1950 MeV. The prediction of our sum rule prefers \(\Xi(1690)\). Our result suggests that the masses of the \(B_-\) tend to be degenerate. This is the result of two different origins of the mass difference. The strange quark mass raises the hyperon masses, while the quark condensate widens the mass splitting of \(B_+\) and \(B_-\). Because the strange quark condensate is smaller than the up and down quark condensate, the effect of the strange quark mass is partly canceled in the negative parity baryons.

The \(\langle \bar{q}q \rangle\) dependence of the masses of \(B_-\) is shown in Fig 6, where for each value of \(\langle \bar{q}q \rangle\) the other parameters in the theoretical side, \(m_0, m_s, \chi\) and \(\chi_5\) are adjusted so that the masses of \(N_-, \Lambda_-, \Sigma_-\) and \(\Xi_-\) are reproduced, while the mixed condensate \(\langle \bar{q}g \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle\) and the strange quark condensate \(\langle \bar{ss} \rangle = \chi \langle \bar{q}q \rangle\) are varied along with the quark condensate \(\langle \bar{q}q \rangle\). We see that the quark condensate pushes up the masses of the \(N_-\) and \(\Sigma_-\) as long as the masses of \(N_+\) and \(\Sigma_+\) are fixed. Therefore the magnitude of the quark condensate determines the scale of the mass splitting of \(B_+\) and \(B_-\). The
masses of $\Lambda_-$ and $\Xi_-$ behave similarly against the quark condensate as in $N$ and $\Sigma$.

It is extremely interesting to observe that the QCD sum rule predicts the flavor-singlet $\Lambda_S$ spectrum in the reversed order. Namely, the baryon $\Lambda_{S-}$ is lighter than the positive parity $\Lambda_{S+}$. This is consistent with the quark model prediction that the Pauli principle forbids all quarks occupying the ground s-wave state. In the correlation function $B$ for the $\Lambda_S$, there is no dimension five term, $\langle \bar{q}g\sigma \cdot Gq \rangle$. If we put the dimension five term in the $B$ correlation function by hand and calculate the masses of the $\Lambda_{S+}$ and $\Lambda_{S-}$, then we find that the increase of the dimension five term raises the mass of $\Lambda_{S-}$ and lowers that of $\Lambda_{S+}$. Thus we conclude that the absence of the mixed condensate term in the $\Lambda_S$ sum rule causes the reversed order of $\Lambda_{S+}$ and $\Lambda_{S-}$. We also confirm that the $\langle \bar{q}g\sigma \cdot Gq \rangle$ terms is essential in raising the $B_-$ masses as was stressed in Ref. [13].

5 Summary

We have proposed a technique to estimate the masses of negative-parity baryon resonances $B_-$ in the QCD sum rule. It is important to separate the $B_-$ contribution from the positive-parity state $B_+$. We find that when the chiral symmetry is restored, the masses of $B_+$ and $B_-$ become degenerate. This is quite natural from the chiral symmetry point of view and seems to suggest that N(1535) (or in general $B_-$) is the chiral partner of N(940) (or $B_+$). We have calculated the masses of the flavor singlet and octet $B_+$ and $B_-$, and have confirmed that the sum rule reproduces the observed masses. The mass of $\Xi$-baryon is predicted 1.63 GeV and it may be assigned to the observed $\Xi(1690)$ for which the spin-parity is not yet known. We find that the flavor singlet baryon with negative parity is the ground state and that the predicted mass is close to $\Lambda(1405)$. We confirm that the magnitude of the chiral symmetry breaking vacuum condensate, such as $\langle \bar{q}q \rangle$ and $\langle \bar{q}g\sigma \cdot Gq \rangle$, determines the scale of the mass splitting of $B_+$ and $B_-$. In the present analysis, we have not calculated the next-to-leading order of $\alpha_s$ or higher dimensional terms in the theoretical side. Although they may modify our numbers slightly, we believe that the qualitative features of our calculation will not change. They are left for future analysis.

Appendix

The “old fashioned” correlation function is defined by

$$\Pi(p) = i \int d^4xe^{ix\cdot p}\theta(x_0)\langle 0|J(x)\bar{J}(0)|0\rangle.$$  \hspace{1cm} (14)

$$\text{Im}\Pi(p_0, \vec{p} = 0) \equiv \gamma_0 A(p_0) + B(p_0).$$  \hspace{1cm} (15)

The functions $A(p_0)$ and $B(p_0)$ defined in eq. (15) up to dimension six and $\mathcal{O}(m_s)$ neglecting $\alpha_s$ are given for each baryon in the following.
N
\[ A(p_0) = \frac{5 + 2t + 5t^2}{2^{10} \pi^4} p_0^5 \theta(p_0) \] (16)
\[ + \frac{5 + 2t + 5t^2}{2^9 \pi^2} p_0 \theta(p_0) (\frac{\alpha_s}{\pi}) GG \]
\[ - \frac{5 + 2t - 7t^2}{12} \delta(p_0) \langle \bar{q}qqq \rangle, \]
\[ B(p_0) = \frac{5 + 2t - 7t^2}{32\pi^2} p_0^2 \theta(p_0) \langle \bar{q}q \rangle \] (17)
\[ - \frac{3(1 - t^2)}{32\pi^2} \theta(p_0) \langle \bar{q}g\sigma \cdot Gq \rangle. \]

A
\[ A(p_0) = \frac{5 + 2t + 5t^2}{2^{10} \pi^4} p_0^5 \theta(p_0) \] (18)
\[ + \frac{5 + 2t + 5t^2}{2^9 \pi^2} p_0 \theta(p_0) (\frac{\alpha_s}{\pi}) GG \]
\[ + \left( \frac{1 + 4t - 5t^2}{48\pi^2} m_s \langle \bar{q}q \rangle + \frac{5 + 2t + 5t^2}{64\pi^2} m_s \langle \bar{s}s \rangle \right) p_0 \theta(p_0) \]
\[ + \left( \frac{-13 - 2t - 11t^2}{36} \langle \bar{q}qqq \rangle - \frac{1 + 4t - 5t^2}{18} \langle \bar{s}s\bar{q}q \rangle \right) \delta(p_0) \]
\[ + \left( \frac{-5 + 2t - 7t^2}{96\pi^2} \delta(p_0) + \frac{1 - t^2}{32\pi^2} \theta(p_0) \langle \bar{q}g\sigma \cdot Gq \rangle \right) m_s \langle \bar{q}g\sigma \cdot Gq \rangle \]
\[ - \frac{1 + t + t^2}{48\pi^2} m_s \langle \bar{s}g\sigma \cdot Gs \rangle \delta(p_0) \]
\[ B(p_0) = -\frac{13 - 2t - 11t^2}{3 \cdot 2^8 \pi^4} m_s p_0^4 \theta(p_0) \] (19)
\[ + \left( \frac{1 + 4t - 5t^2}{48\pi^2} \langle \bar{q}q \rangle + \frac{13 - 2t - 11t^2}{96\pi^2} \langle \bar{s}s \rangle \right) p_0^2 \theta(p_0) \]
\[ - \frac{1 - t^2}{32\pi^2} (\langle \bar{q}g\sigma \cdot Gq \rangle + 2 \langle \bar{s}g\sigma \cdot Gs \rangle) \theta(p_0). \]

Σ
\[ A(p_0) = \frac{5 + 2t + 5t^2}{2^{10} \pi^4} p_0^5 \theta(p_0) \] (20)
\[ + \frac{5 + 2t + 5t^2}{2^9 \pi^2} p_0 \theta(p_0) (\frac{\alpha_s}{\pi}) GG \]
\[ + \left( \frac{3(1 - t^2)}{16\pi^2} m_s \langle \bar{q}q \rangle + \frac{5 + 2t + 5t^2}{64\pi^2} m_s \langle \bar{s}s \rangle \right) p_0 \theta(p_0) \]
\[ + \left( \frac{1 - 2t + t^2}{12} \langle \bar{q}qqq \rangle - \frac{1 - t^2}{2} m_s \langle \bar{q}qqq \rangle \delta(p_0) \right) \]
\[ + \frac{1 - t^2}{32\pi^2} \left( -8 \delta(p_0) + \frac{3}{p_0} \theta(p_0) \langle p_0^2 - m_s^2 \rangle \right) m_s \langle \bar{q}g\sigma \cdot Gq \rangle \]
\[ B(p_0) = \frac{1 - 2t + t^2}{2s\pi^4} m_s p_0^4 \theta(p_0) \]

\[ + \left( \frac{3(1 - t^2)}{16\pi^2} \langle \bar{q}q \rangle - \frac{1 - 2t + t^2}{32\pi^2} \langle \bar{s}s \rangle \right) p_0^2 \theta(p_0) \]

\[ - \frac{3(1 - t^2)}{32\pi^2} \theta(p_0) \langle \bar{q}g \sigma \cdot Gq \rangle. \]

\[ \Xi \]

\[ A(p_0) = \frac{5 + 2t + 5t^2}{210\pi^4} p_0^5 \theta(p_0) \]

\[ + \frac{5 + 2t + 5t^2}{29\pi^2} p_0 \theta(p_0) \frac{\sqrt{s}}{\pi} GG \]

\[ + \left( \frac{3(1 - t^2)}{16\pi^2} m_s \langle \bar{q}q \rangle + \frac{3(1 + 2t + t^2)}{32\pi^2} m_s \langle \bar{s}s \rangle \right) p_0 \theta(p_0) \]

\[ + \left( \frac{1 - 2t + t^2}{12} \langle \bar{s}s \bar{s}s \rangle - \frac{1 - t^2}{2} \langle \bar{s}s \bar{q}q \rangle \right) \delta(p_0) \]

\[ + \frac{1 - t^2}{32\pi^2} \left( -4\delta(p_0) + \frac{3}{p_0} \theta(p_0^2 - 4m_s^2) \right) m_s \langle \bar{q}g \sigma \cdot Gq \rangle \]

\[ - \frac{1}{192\pi^2} m_s \langle \bar{s}g \sigma \cdot Gs \rangle \delta(p_0) \]

\[ B(p_0) = \frac{-3(1 - t^2)}{27\pi^4} m_s p_0^4 \theta(p_0) \]

\[ + \left( \frac{3(1 - t^2)}{16\pi^2} \langle \bar{s}s \rangle - \frac{1 - 2t + t^2}{32\pi^2} \langle \bar{q}q \rangle \right) p_0^2 \theta(p_0) \]

\[ - \frac{3(1 - t^2)}{32\pi^2} \theta(p_0) \langle \bar{s}g \sigma \cdot Gs \rangle. \]

\[ \Lambda_S \text{ (Flavor Singlet)} \]

\[ A(p_0) = \frac{3}{28\pi^4} p_0^5 \theta(p_0) \]

\[ + \frac{3}{27\pi^2} p_0 \theta(p_0) \frac{\alpha_s}{\pi} GG \]

\[ + \frac{1}{16\pi^2} m_s \left( 4\langle \bar{q}q \rangle + 3\langle \bar{s}s \rangle \right) p_0 \theta(p_0) \]

\[ - \frac{1}{3} \left( \langle \bar{q}g \bar{q}q \rangle + 2\langle \bar{s}s \bar{q}q \rangle \right) \delta(p_0) \]

\[ - \frac{1}{16\pi^2} m_s \left( \langle \bar{q}g \bar{g} \sigma \cdot Gq \rangle + 3\langle \bar{s}g \sigma \cdot Gs \rangle \right) \delta(p_0) \]

\[ B(p_0) = \frac{-1}{64\pi^4} m_s p_0^4 \theta(p_0) \]

\[ + \frac{1}{8\pi^2} \left( 2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right) p_0^2 \theta(p_0) \]
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Table 1: The determined QCD parameters

| ⟨qq⟩ | m₀   | mₛ   | χ  | χₛ |
|------|------|------|----|----|
| (-0.244 GeV)³ | 0.9 GeV | 0.1 GeV | 0.75 | 0.8 |
Table 2:  

| Baryon | $N_{+}$ | $\Lambda_{+}$ | $\Sigma_{+}$ | $\Xi_{+}$ | $N_{-}$ | $\Lambda_{-}$ | $\Sigma_{-}$ | $\Xi_{-}$ | $\Lambda_{S-}$ | $\Lambda_{S+}$ |
|--------|--------|--------------|--------------|-----------|--------|--------------|--------------|-----------|----------------|----------------|
| Sum rule | 0.94   | 1.12         | 1.21         | 1.32      | 1.54   | 1.55         | 1.63         | 1.63      | 1.31           | 2.94           |
| Exp.    | 0.94   | 1.12         | 1.19         | 1.32      | 1.535  | 1.67         | 1.62         | ——        | 1.405          | ——             |

Figure 1: The Borel mass $M$ dependence of the octet $B_+$ masses. The solid line denotes $M = m_B$. 
Figure 2: The Borel mass $M$ dependence of the octet $B_-$ masses. The solid line denotes $M = m_B$. The lines for masses of $\Sigma_-$ and $\Xi_-$ are almost overlapped.

Figure 3: The Borel mass $M$ dependence of the singlet $\Lambda_S^-$ masses. The solid line denotes $M = m_B$. 
Figure 4: The Borel mass $M$ dependence of the singlet $\Lambda_{S+}$ masses. The solid line denotes $M = m_B$.

Figure 5: The $\langle \bar{q}q \rangle$ dependence of the masses of $N_-$ and $\Sigma_-$. The other QCD parameters are fixed so that the masses of the ground state baryons $N_+, \Lambda_+, \Sigma_+$ and $\Xi_+$ are reproduced for each value of $\langle \bar{q}q \rangle$. The value of the mixed condensate $\langle \bar{q}g \sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ and the strange quark condensate $\langle \bar{s}s \rangle = \chi \langle \bar{q}q \rangle$ are varied along with the quark condensate $\langle \bar{q}q \rangle$. 