Electromagnetic field angular momentum in condensed matter systems

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(November 23, 2018)

Abstract

Various electromagnetic systems can carry an angular momentum in their E and B fields. The electromagnetic field angular momentum (EMAM) of these systems can combine with the spin angular momentum to give composite fermions or composite bosons. In this paper we examine the possibility that an EMAM could provide an explanation of the fractional quantum Hall effect (FQHE) which is complimentary to the Chern-Simons explanation. We also examine a toy model of a non-BCS superconductor (e.g. high T_c superconductors) in terms of an EMAM. The models presented give a common, simple picture of these two systems in terms of an EMAM. The presence of an EMAM in these systems might be tested through the observation of the decay modes of a charged, spin zero unstable particle inside one of these systems.

PACS numbers : 71.10.Pm , 73.40.Hm , 74.20.-z

Typeset using REVTEX

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I. INTRODUCTION

The \textbf{E} and \textbf{B} fields of certain electromagnetic configurations carry a field angular momentum. This usually leads to interesting, but not particularly physically relevant, results. For example, it was noted long ago \cite{1} that an electric charge plus magnetic monopole carried an EMAM in its fields which is independent of the distance between the charges \cite{2}. However, monopoles are currently still only theoretical constructs, and therefore the physical usefulness of this result is minimal. The more realistic point charge/point magnetic dipole system also has an EMAM, which is inversely proportional to the distance between the charge and dipole \cite{3}. On applying this charge/dipole result to various atomic scaled systems it is found that classically this EMAM tended to be small. For example, the EMAM arising from the charge of the proton and the intrinsic magnetic dipole moment of the electron in a hydrogen atom, is on the order of $10^{-5}\hbar$ \cite{4}. The small size of this EMAM is due to the fact that the charge/point dipole system field angular momentum depends inversely on the distance between the charge and dipole, and the typical atomic size (0.1 nm) is “large” in this context. One can also consider the EMAM of atomic scaled systems arising from the interaction between the charge of a nucleus and magnetic dipole moments generated by the orbital motion of the electrons, such as in hydrogen in an excited $L \neq 0$ state. The EMAM of this last example is exactly zero \cite{4}. One place where a field angular momentum has found an application is in the semi-classical approach to diamagnetism \cite{5}.

In nuclear scaled systems where distances are five orders of magnitude smaller than for atomic systems, there is a better chance that such an EMAM could play a role \cite{5}, and perhaps may have some connection to the proton spin problem.

In this paper we wish to examine the possibility that even though the EMAM apparently does not play a role in individual atomic sized systems, it may nevertheless be important in certain condensed matter systems. The idea is that despite condensed matter systems having distance scales which are of the atomic scale or larger, this can be countered by having either an externally applied magnetic field, or by having many contributions to the
EMAM coming from the large number of particles in a typical condensed matter system. In effect the “large” distances are compensated for by a large number of contributors to the EMAM. In the following two sections we will apply these ideas to the fractional quantum Hall system [4], and in a toy model for non-BCS superconductors.

II. FQHE IN TERMS OF EMAM

In the Chern-Simons models of the FQHE composite objects are formed which consist of electrons bound to fictitious Chern-Simons flux tubes [7] [8]. When these composite fermions are placed in an external magnetic field there is a partial cancellation or screening of the Chern-Simons flux by the external magnetic flux. For certain values of the external magnetic field this leads to these composites of a charge + Chern-Simons flux tube undergoing the integer quantum Hall effect with respect to the uncanceled or unscreened part of the external magnetic field. These explanations have their theoretical origins in work by Wilczek [9] where the electric charge sits outside the region of magnetic flux. In this paper we take the electron as sitting inside the magnetic flux tube, so that the system develops an EMAM which combines with the spin of the electron to give rise to a composite fermion which then undergoes the IQHE. This EMAM explanation of the FQHE is meant to be complimentary to the Chern-Simons explanation of the FQHE (Ref. [10] gives an overview of the aspects Chern-Simons theory of the FQHE which will be relevant in this paper). The possible advantage of the EMAM explanation is that we do not need to postulate a fictitious Chern-Simons flux – the composite fermion arises directly from the combination of the EMAM plus the spin of the electron.

In this paper we will work mainly in the mean field approximation where the interaction between electrons is ignored. In this approximation the Chern-Simons approach to the FQHE starts with a Hamiltonian of the form

$$H_{mf} = \sum_i \left[ \frac{\mathbf{p}_i + \frac{\mathbf{e}}{c} \mathbf{A}(\mathbf{r}_i) - \frac{\mathbf{e}}{c} \mathbf{a}(\mathbf{r}_i)}{2m_b} \right]^2$$

(1)
where $r_i$ is the position of the $i^{th}$ electron; $A(r_i)$ is the external vector potential associated with the external magnetic field, and $a(r_i)$ is the Chern-Simons vector potential; $m_b$ is the effective mass of the electron in the material. Usually there is an electron-electron interaction term of the form $\sum_{i<j} V(r_i - r_j)$, with $V$ being a Coulomb potential, for example. In the mean field approximation this term is dropped. The original Hamiltonian for the system is of the form given in Eq. (1) except with no $a(r_i)$ term. This term comes into the Hamiltonian in the following way. If $\Psi(r_1, r_2, \ldots, r_N)$ is a solution to the original Hamiltonian ($H_0\Psi = E\Psi$) with no $a(r_i)$, then the phase transformed wavefunction

$$\Phi(r_1, r_2, \ldots, r_N) = \left[ \prod_{i<j} e^{-i2m\theta(r_i - r_j)} \right] \Psi(r_1, r_2, \ldots, r_N) \quad (2)$$

will be a solution to the Hamiltonian in Eq. (1). In Eq. (2) $m$ is an integer and $2m$ is the (even) number of Chern-Simons flux tubes attached to the electron. $\theta(r_i - r_j)$ gives the angle between the vector $r_i - r_j$ and the $\hat{x}$ axis and is defined modulo $2\pi$ ($\hat{z}$ is taken as perpendicular to the 2D system and $\hat{x}$ is in the plane). This transformation can be thought of as a singular gauge transformation since $\theta$ is nonsinglevalued. The Chern-Simons vector potential is then given by

$$a(r_i) = i\nabla_i \left[ \prod_{j} e^{-i2m\theta(r_i)} \right] \quad (3)$$

Since this vector potential is a gradient one would think that the “magnetic” field associated with it is zero, $b_{CS}(r) = \nabla \times a(r) = 0$. However, due to the singular character of the transformation in Eq. (2) one finds that $b_{CS}(r) = 2\pi(2m)n(r)$, where $n(r)$ is the local electron density which is a sum of delta function that spike at the location of each electron. In the mean field approximation $n(r)$ is replaced by $n_e \approx const.$, the average density.

In this Chern-Simons mean field approach the FQHE is described as an integer quantum Hall effect (IQHE) for the composite fermions of the electrons + Chern-Simons flux tubes. When the external magnetic field is imposed it partially cancels or screens the Chern-Simons field so that the composite fermions see a reduced, effective magnetic field of $B_{eff} = B_{ext} - b_{CS}$ with respect to which these composite fermions then undergo the IQHE. Although
this mean field description is very crude (since it ignores the inter-electron interactions) it
nevertheless gives the Jain series\(^\text{[11]}\) for the fractional quantum Hall states, and it provides
a starting point for more accurate studies such as using the random phase approximation
(RPS)\(^\text{[10]}\).

We now want to present a similar picture of the FQHE, but without the need of intro-
ducing the fictitious Chern-Simons potential. In our approach the composite fermions arise
through the combining of the intrinsic spin of the electron with the EMAM produced by
the external magnetic flux and the charge of the electron which sits \emph{inside} a magnetic flux
tube. That such a complimentary view is possible is suggested by Ref.\(^\text{[9]}\) where the charge
+ flux tube system is equivalently examined using both singular gauge transformation arg-
uments (Chern-Simons type description) \emph{and} angular momentum arguments (EMAM type
description).

The angular momentum carried in the electric and magnetic fields can be written as\(^\text{[2]}\)
\[
\mathbf{L}_{em} = \frac{1}{4\pi c} \int r \times (\mathbf{E} \times \mathbf{B}) d^3 r
\]

We consider an electron located in the \(z = 0\) plane whose electric field in cylindrical coordi-
nates takes the usual Coulomb form
\[
\mathbf{E} = \frac{-e}{r^3} (\rho \hat{\rho} + z \hat{z})
\]

The external magnetic field, \(\mathbf{B}\), is taken to be of uniform strength, \(B_0\), inside an infinitely long
cylinder of radius \(R\), and zero outside this cylinder. The \(\mathbf{B}\) field points along the positive \(z\)
axis, and the axis of the cylinder goes through the electron. In cylindrical coordinates
\[
\mathbf{B} = \begin{cases} 
B_0 \hat{z} & r \leq R \\
0 & r > R 
\end{cases}
\]

From Eqs.\(^\text{(3)}\) and \(\text{(3)}\) \(\mathbf{E} \times \mathbf{B} \propto \hat{\phi}\). When this is combined with \(r = \rho \hat{\rho} + z \hat{z}\) in \(r \times (\mathbf{E} \times \mathbf{B})\)
one gets components in the \(\hat{z}\) and \(\hat{\rho}\) directions. The \(\hat{\rho}\) component will vanish upon doing
the \(\phi\) integration in Eq.\(^\text{(4)}\). The remaining \(\hat{z}\) term gives on combining Eqs.\(^\text{(3)}\) - \(\text{(3)}\)
where $\Phi$, is the flux of the magnetic tube. From Eq. (6) the direction of this field angular momentum is determined by the direction of $\mathbf{B}$ and the sign of the charge. If $\Phi$ equals some integer multiple, $n$, of the quanta of magnetic flux ($\Phi_0 = 2\pi \hbar c/e$) then $L_{zm}^m = n\hbar$. The spin of the electron is $S_z = \pm \hbar/2$ so the combination of the EMAM plus the electron’s spin yields a composite fermion with a half integer angular momentum of $(n \pm \frac{1}{2})\hbar$. Thus for certain values the external magnetic flux the charge of the electron will “absorb” an integer multiple of the magnetic flux quanta, $\Phi_0$, and generate an EMAM. This EMAM, when combined with the intrinsic spin of the electron, gives a composite fermion, which then undergoes the IQHE with respect to the remaining, “unabsorbed” flux, $\Phi_{ext} - n\Phi_0$. The phase factor of the wave function of this composite fermion equals $(-1)^{1+2n}$, which explains the Jain series \[11\] in a simple way. Also according to Laughlin \[12\] the excited-state of a 2D electron gas in a strong magnetic field has fractional charge. Thus if we replace $e$ with $\frac{1}{3}e$ in (7) we obtain $L_{zm}^m = \frac{1}{3}\hbar$ when $\Phi = \Phi_0$. This yields fractional statistics for the excited-state of the 2D electron gas in a strong magnetic field, which has been observed experimentally \[13\].

Eq. (7) is for a free electron, but we can extend our considerations slightly beyond the mean field approximation by including screening effects. For an electron inside some material the electric field will be screened past a certain distance as follows:

$$
E = -\frac{e}{\epsilon_s r^2}(1 + \lambda r)e^{-\lambda r}\hat{r},
$$

where $\lambda^{-1}$ is the screening length and $\epsilon_s$ is the dielectric constant. There are two cases connected with this effective screening distance. First if the screening length is much larger than the radius of the magnetic flux tube ($\lambda^{-1} \gg R$) then Eq. (7) and all the developments leading up to it should still hold approximately. The second case is when the screening length is much shorter than the radius of the magnetic flux tube ($\lambda^{-1} \ll R$). In this case one can repeat all the developments up to Eq. (7), but now one finds that

$$
L_{zm}^m = \frac{eB_0}{2c} \left( \frac{2}{\lambda \sqrt{\epsilon_s}} \right)^2 = \frac{eB_0 R_{eff}^2}{2c} = \frac{e\Phi_{eff}}{2\pi c},
$$

(9)
This result is similar to that of Eq. (7) except with the replacement $R \rightarrow R_{\text{eff}} = 2/\lambda \sqrt{\epsilon_s}$, and $\Phi \rightarrow \Phi_{\text{eff}}$. The difference is that in Eq. (7) the radius is set by the arbitrary, external choice of $R$, while in Eq. (9) the effective radius, $R_{\text{eff}}$, is set by the screening length of the material.

As in the previous case we want $L_{z}^{\text{em}} = n\hbar$, so one can ask if Eq. (9) can be satisfied for reasonable values of the magnitudes of $B_0$ and $R_{\text{eff}} = 2/\lambda \sqrt{\epsilon_s}$. As an example take $n = 1$ so that $L_{z}^{\text{em}} = \hbar$. Eq. (9) then implies that we want $eB_0 R_{\text{eff}}^2/2c = \hbar$. For the FQHE one has magnetic fields on the order of $B_0 \approx 100$ kilogauss or larger. The FQHE is associated with semiconductor materials such as silicon or gallium-arsenic. For silicon the screening length is on the order of $\lambda^{-1} = 24$ nm and $\epsilon_s = 12$ which implies $R_{\text{eff}} = 2/\lambda \sqrt{\epsilon_s} = 1.39 \times 10^{-6}$ cm. Using these numbers in Eq. (9) gives an EMAM of $1.5 \times 10^{-27}$ erg-sec which is of order $\hbar$ as desired.

The EMAM explanation of the FQHE is a simple, alternative way of looking at this effect, which is complimentary to the Chern-Simons approach in the mean field approximation. A strong point of the EMAM explanation is that it does not require one to postulate a Chern-Simons flux; one only needs the real external magnetic flux and the internal spin of the electron.

**III. SUPERCONDUCTORS IN TERMS OF EMAM**

Ordinary superconductors such as Mercury or Tin are well explained by BCS theory. However, with the discovery of high temperature superconductors, and heavy-fermion superconductors one finds systems for which it is no longer certain that BCS theory is the full explanation. In particular it is not clear how the BCS model can lead to the binding of Cooper pairs at the large temperatures found in high $T_c$ materials. One possibility is that the phonons in high $T_c$ materials have some strong self coupling similar to that of gluons in QCD, leading to a much stronger binding. In this section we examine a toy model of a 1D and 2D atomic lattice. The atom at each lattice site is assumed to possess a magnetic
dipole moment with a magnitude of the order of the Bohr magneton, \( \mu_B = e \hbar / 2m_e c \). We then consider an electron placed in this lattice of atomic magnetic dipoles. The charge, \(-e\), of the electron in conjunction with the magnetic dipoles at each lattice site will generate an EMAM. If the sum total EMAM is of the right size then the combination of electron spin plus total EMAM will result in an integer angular momentum making the electron effectively a boson, which can then condense into the superconducting state. Heuristically one can think of this as the electron “binding” with (i.e. sitting in) some local region of the lattice, leading to a bosonic composite with an integer angular momentum.

We first examine the EMAM carried in the field of an electron with charge, \(-e\), and a single point magnetic dipole, \( \mathbf{m} \). Without loss of generality we place the magnetic dipole at the origin and align its magnetic moment along the z-axis. Its magnetic field takes the standard form \( \mathbf{B} = \frac{1}{r^3} [3(\mathbf{m} \cdot \hat{r})\hat{r} - \mathbf{m}] \). Placing the electron at some arbitrary point \( \mathbf{R} \) gives an electric field of the form \( \mathbf{E} = -e \frac{\mathbf{E'}}{r'^2} \) where \( r' = r - R \). Using these two fields in Eq. (4) yields an EMAM whose z-component is

\[
(L_{em})_z = \pm \frac{e|m|}{cR} (1 - \cos^2 \theta)
\]  

\( \theta \) is the angle between \( \mathbf{m} \) and \( \mathbf{R} \). The \( \pm \) sign indicates that the direction of \((L_{em})_z\) depends on the direction of \( \mathbf{m} \) and on the sign of the charge. We now want to apply Eq. (10) to an electron sitting in a 1D antiferromagnetic lattice of atomic magnetic moments. We will take the magnetic moments to alternate between \( \mathbf{m}_a = a \mu_B \hat{z} \) and \( \mathbf{m}_b = -b \mu_B \hat{z} \) on neighboring lattice sites; \( \mu_B \) is the Bohr magneton; \( a, b \) are positive constants of order unity; \( \hat{z} \) is perpendicular to the line of the 1D lattice. The lattice spacing between neighboring \( \mathbf{m}_a \)'s and \( \mathbf{m}_b \)'s is \( d \). Since the magnetic moments are taken to be aligned along the z-axis, \( \theta = \pi/2 \) in Eq. (10). The EMAM due to the \( n^{th} \) plus \((n + 1)^{th}\) lattice site is}

\[
(L_{em})_{zn} = \frac{e|m_a|}{cR_n} - \frac{e|m_b|}{cR_{n+1}} = \frac{r_o}{d} \left( \frac{a}{n} - \frac{b}{(n + 1)} \right) \frac{\hbar}{2}
\]  

where \( r_o = e^2 / m_e c^2 \) is the classical electron radius; \( \mathbf{m}_a \) is located at the \( n^{th} \) lattice site while \( \mathbf{m}_b \) is located at the \((n + 1)^{th}\) lattice site. There will also be an EMAM from the interaction
between the electron and the magnetic moment of the lattice site on which it is located. From Eq. (10) it may appear as if this would lead to a divergent EMAM. If the electron and the magnetic dipole are placed on the same lattice site then apparently \( R \to 0 \) in Eq. (10) implying that \((L_{em})_z \to \infty\). This assumes that the point dipole expression for the magnetic field (i.e. \( B = \frac{1}{4\pi r^3}[3(m \cdot \hat{r})\hat{r} - m] \)) is valid for all \( r \). However, once \( r \) takes values which are “inside” the electronic orbitals of the atomic magnetic dipole in question this expression for \( B \) is no longer valid. If one models these electronic orbitals as effective currents then as the external electron moves closer to the origin of the atom (\( r = 0 \)) then the EMAM coming from the “inside” magnetic field tends to cancel the EMAM coming from the “outside” magnetic field. For a multi-electron atom this is difficult to see in detail, however in ref. [4] it was shown that for the hydrogen atom the EMAM arising from the magnetic field of an electron in a non-zero orbital angular momentum state and the electric field arising from the charge of the proton was exactly zero. This was due to a cancellation between the different EMAMs that resulted from the different magnetic fields in the regions “inside” and “outside” of the electron’s orbit. Generally as the external electron is moved closer to the atomic magnetic dipole the EMAM will tend to increase until the electron is at a distance which is just outside of the outer electronic orbitals of the atom. As the electron moves inside the electronic orbitals the net EMAM will decrease due to the cancellation between the EMAMs from the different magnetic fields inside and outside the electronic orbitals. In dealing with the EMAM from the electron and the atomic magnetic moment which are at the same lattice site we will assume that the dipole moment is \( m_b \), and that the electron is at a distance, \( d_0 \), which is roughly the atomic radius of the magnetic moment. This will make the contribution from this lattice site as large as possible. However, for this 1D case we will find that whether or not one maximizes the EMAM from this closest lattice site, that the net field angular momentum is still too small to play a significant role. Putting this all together the net EMAM for this 1D lattice is

\[
L_{em}^{(1D)} = -\frac{r_0 b}{d_0} \left( \frac{\hbar}{2} \right) + \frac{r_0 \hbar}{d} \sum_{n=1}^{\infty} \left( \frac{a}{2n - 1} - \frac{b}{2n} \right)
\]
The first term is the maximum possible contribution to the EMAM arising from the inter-
action between the electron and the lattice site on which it sits. A factor of two in the
second term comes from summing the lattice sites to the right and left. The sum in Eq.
(12) is divergent unless \( a = b \). We will assume that \( a \neq b \). In physical situations this sum
is cut off from above for several reasons: the screening of the electric field of the electron,
the finite size of the sample, or the appearance of defects of the crystalline lattice which
would alter the ideal antiferromagnetic ordering which we have assumed. Depending on this
cut-off length, \( L_{em} \) could take on any value up to \( \infty \). As a specific example assume \( a = 4 \),
and \( b = 2 \). Take the lattice spacing as \( d = 2 \times 10^{-10} \) m, take \( d_0 \simeq 0.53 \times 10^{-10} \) m (the Bohr
radius), and \( r_o \simeq 2.82 \times 10^{-15} \) m. Finally we assume that the sum in Eq. (12) is cut off after
10\(^{20}\) magnetic dipole lattice sites. With this the sums in Eq. (12) can be approximated by
\( (a - b) \int_{10^{19}}^{10^{20}} \frac{dx}{x} \simeq 46.1 \). With these numbers \( L_{em} \simeq 0.001 \frac{A}{\mu} \) which is small compared to the
fundamental unit of angular momentum (\( \frac{\hbar}{2} \)). The cut-off of 10\(^{20}\) lattice sites is arbitrary,
however, because of the logarithmic behavior of the integral approximation for the sum in
Eq. (12) even taking the cut-off at 10\(^{10}\) lattice sites would not make \( L_{em} \) of the order \( \hbar \). Thus given this small magnitude for \( L_{em} \) it is unlikely that the EMAM plays any significant
role in this 1D case.

Next we consider a 2D lattice in which all the magnetic moments are aligned in the same
direction. This is somewhat similar to the 2D Ising model of magnetic moments on a lattice.
For this 2D case one would expect \( L_{em} \) to become larger since there are more neighboring
dipoles for the electron to interact with. We take the dipoles with \( \mathbf{m}_a = a \mu_B \hat{z} \) and \( a = 4 \)
as before. Under these conditions the EMAM along the z-axis from the dipole located at
\( x = nd \) and \( y = md \) is

\[
(L_{em}^z)_{n,m} = \frac{e|\mathbf{m}|}{\sqrt{(nd)^2 + (md)^2}} = \frac{r_0a}{d\sqrt{n^2 + m^2}} \frac{\hbar}{2} \tag{13}
\]

Summing the first \( N \) sites along the \( x \) and \( y \) directions gives a total EMAM of

\[
L_{em} = \sum_{n,m=-N}^{N} (L_{em}^z)_{n,m} = \frac{2r_0a}{d} \left( \frac{\hbar}{2} \right) \sum_{n,m=1}^{N} \frac{1}{\sqrt{n^2 + m^2}} \tag{14}
\]
There should also be a contribution coming from the interaction between the electron and the dipole located on the lattice site at which the electron is located, exactly as in the 1D lattice case (i.e. one should add a term to Eq. (14) like the first term in Eq. (12)). This term, by itself, gives only a small contribution, and we will therefore ignore it. The double sum in Eq. (14) can be approximated by the double integral $\int_1^N \int_1^N [(dx)(dy)/\sqrt{x^2 + y^2}] \simeq \int_0^{2\pi} (d\phi) \int_1^N (rdr) \frac{1}{r} = 2\pi (N - 1)$ where $r = \sqrt{x^2 + y^2}$. In replacing the Cartesian integral over $dxdy$ by the polar integral over $rdrd\phi$ we are ignoring the contributions from the dipoles at the corners of the square region indicated by the sum in Eq. (14). This is actually physically more realistic, since the dipoles which influence the charge should have a circular, not a square, cut-off. For an infinite 2D lattice (i.e. $N \to \infty$) the field angular momentum will diverge (this time linearly rather than logarithmically) unless the integral is cut-off from above. Physically this cut-off will arise due to the screening of the electric field of the electron. Here we will simply put in this screening distance cut-off by hand to see roughly the size needed to generate an EMAM of the order $\hbar/2$. Setting the cut-off in the above integral approximation of the sum as $N = 1500$, and taking the other parameters as in the 1D example, gives an EMAM of $L_{em} \simeq 1.06(\hbar/2)$. Multiplying this $N$ with the lattice spacing of $d = 2 \times 10^{-10}$ m implies a screening distance of $3 \times 10^{-7}$ m. The combination of this EMAM plus the internal spin of the electron yields a composite with integer spin. This bosonic quasi-particle of electron plus the local region of the magnetic dipole lattice in which it sits, could condense into a superconducting state. Although the screening distance cut-off in the above example (which was set by hand) was large, it was not so large as to be out of the question for some materials. For example, it is only somewhat larger than the screening length of silicon discussed at the end of section II (this is only for comparison since silicon is not a high temperature superconductor). This does imply that metallic materials or materials with small screening lengths would not be viable under the above mechanism.

One may wonder if a ferromagnetic 3D lattice might not work even better at giving a substantial EMAM. There are two arguments as to why this proposed mechanism would not work better for a 3D ferromagnetic lattice. First, the EMAM of the magnetic dipole
and electric charge is maximized when $\theta = \pi/2$ as occurs when the electron is in the same plane as the dipoles \textit{and} if the dipoles have their moments oriented perpendicular to this plane. For a 3D lattice of magnetic dipoles, the dipoles which are not coplanar with the electron will give a smaller EMAM. In addition the EMAM from the magnetic dipoles above the plane of the electron will tend to cancel against the EMAM from the magnetic dipoles below the plane of the electron. Second, the size of the EMAM depends closely on $N$ from Eq. (14) which in turn is connected with the screening distance of the electric field in the material. For ferromagnetic materials, such as iron, these screening distances are small and so $N$ would also in general be too small to generate a large EMAM.

¿From the above estimates we can see that for a 1D lattice it is unlikely that the EMAM could play any significant role since the number of dipole lattice sites ($N$) which must contribute is just too large. For a 2D lattice the number of lattice sites needed to give an EMAM of order $\hbar/2$ is much smaller (since the EMAM dependence goes from $ln(N) \rightarrow N$ on going from $1D \rightarrow 2D$). However, $N$ (or the screening length) must be “fine tuned” in order to get an EMAM of exactly some integer number times $\hbar/2$. Also the high $T_c$ materials to which we want to apply this mechanism are generally antiferromagnetic rather than ferromagnetic as in the simple 2D case we considered. To address this one could group pairs of anti-aligned lattice sites. The closer lattice site would contribute more than the further lattice site due to the $1/r$ dependence of the EMAM in Eq. (11) so that each pair would give some net EMAM, just as in the 1D antiferromagnetic case. Each successive pair of sites would also contribute an EMAM in the same direction as the closest pair. For such an anti-ferromagnetic 2D lattice one would have to increase the number of lattice sites so that the total EMAM would have the right magnitude to give a net angular momentum which had integer (bosonic) values. Thus the case for this alternative mechanism of generating a superconducting state may appear only marginally viable. Nevertheless, this idea does have two interesting features:

1. There is no need to postulate any kind of exotic binding mechanism for Cooper pairs
which persists up to the temperatures observed in high $T_c$ materials. As long as the electron is “bound” to (i.e. sits in) an \textit{ordered} lattice, as discussed above, then it will effectively act as a boson which can condense into a superconducting state. In this picture the reason for the disruption of the superconducting state is the disordering of the lattice and/or a changing of the screening length as the temperature increases. The increase of the temperature can also influence $L_{em}$ through changes of $R$ and $\theta$ in Eq. (10).

2. It provides a common picture for high $T_c$ materials and the fractional quantum Hall systems. In the arguments given above both systems are explained in terms of the combination of EMAM with intrinsic spin angular momentum to transform fermions into composite fermions or composite bosons.

One final problem in using this EMAM model for high $T_c$ materials is that experiments indicate that the charge of the order parameter in high $T_c$ materials is still $-2e$, which is an indication of Cooper pairing. In the EMAM picture, as in the holon/spinon picture \[16\], the order parameter charge equals the charge of the electron, $-e$. One possible resolution to this is that in certain situations the electric charge of an object may be shifted through the effects of the background or vacuum. An example of this is the $\Theta$ vacuum effect \[17\] where a magnetic charge placed in a vacuum with a non-zero, CP violating parameter ($\Theta$) will have its electric charge shifted by $-e\Theta/2\pi$. The present situation is different from the $\Theta$ vacuum example, since in that case it is a hypothetical monopole which develops a shifted electric charge. Nevertheless, in the present context one could postulate that if the high $T_c$ materials behaved as an “exotic” background that the effective charges of the electrons could be shifted via their interaction with this background. In this regard it should be mentioned that recently it has been observed that high $T_c$ materials do violate $P$ and $T$ invariance \[18\].
IV. POSSIBLE EXPERIMENTAL TEST FOR THE EMAM MECHANISM

An experimental test, which in principle would probe the existence of an EMAM is to observe the decay of a low speed, unstable, charged and zero spin particle, such as a pion ($\pi^\pm$), inside the magnetic field environment of one of these systems. Considering the example of the positively charged pion, in vacuum it decays into an antimuon and a muon neutrino. These decay products move in opposite directions, with anti-aligned spins in order to conserve momentum and angular momentum. If the pion is placed inside a quantum Hall system at a quantum Hall step, or in a strong magnetic field, then it will behave as an effective boson, but now with a non-zero spin coming from the EMAM. If the magnetic field is such that the EMAM associated with the pion is $\hbar$ then the effective spin of the pion will be spin $1 = \text{spin } 0 + \text{EMAM}$. The decay of this new “spin 1” pion will be altered from the decay of the pion in free space. In free space when the pion decays the positively charged antimuon, $\mu^+$, and the muon neutrino, $\nu_\mu$, move off back to back in any direction. and have their spins anti-aligned. In the case when the pion is implanted into a quantum Hall system, an EMAM develops, and the direction in which the decay products move off becomes restricted. In the above example the initial angular momentum of the system was $\hbar$ and pointed along the direction of the magnetic field. Thus the decay products also had their angular momentum (EMAM plus the spins of both the antimuon and muon neutrino) aligned along or against the direction of the magnetic field. Neutrinos in the Standard Model are purely left handed, which means that their spin and their momentum point in opposite directions (If neutrinos turn out to have a small mass, as recent experiments indicate, then this is only approximately true. However, in the case of the pion decay the neutrino is very well approximated as being purely left handed). Since the spin of the neutrino is required to be along or against the direction of the magnetic field this means that its momentum must also lie along the line of the magnetic field. Thus in the presence of an EMAM the antimuon and muon neutrino should decay predominately with their momenta along the direction of the magnetic field. If there is no EMAM then the two decay products should be able to
move off in any direction since there is no angular momentum quantization axis picked out before hand by virtue of the fact that the pion is spin zero.

V. DISCUSSION AND CONCLUSIONS

In this article we have given an alternative, simple picture of the fractional quantum Hall effect. We looked at the EMAM which arises when an electron sits inside a magnetic flux tube. The size of the EMAM depends on the amount of flux, \( \Phi \), in which the electron is embedded. For certain values of the flux the EMAM takes on integer multiples of \( \hbar \). This EMAM then combines with the intrinsic spin of the electron to yield a composite fermion. The FQHE occurs when some number of electrons sits inside a flux tube of the appropriate strength to give rise to an EMAM, which when added to the internal spin of the electrons gives the combined system a net half integer spin making it effectively a composite fermion.

Next a toy model for non-BCS superconductivity was given in terms of an EMAM. By considering an electron sitting in a 1D or 2D lattice with anti-aligned/aligned atomic, magnetic dipoles at each site it was found that an EMAM occurred which, when combined with the internal spin of the electron, could yield an integer angular momentum. The combination of the electron plus the “local” region of the lattice in which it was located, acted effectively as an integer spin, bosonic quasi-particle, which could condense into a superconducting state. The main weakness of this argument, even for the 2D lattice, was the large number of lattice sites (\( \approx N^2 \approx 10^6 \)) which needed to be included in the “local” neighborhood of the electron in order to get an EMAM of the correct order to transform the electron into a boson. The chief advantage of the EMAM mechanism for high \( T_c \) superconductivity is that it does not require some exotic binding mechanism to form Cooper pairs, but rather it is connected with the magnetic ordering of the lattice.

The presence of an EMAM might be tested by the observation of the decay of unstable, charged, zero spin particles, such as pions, within a quantum Hall system or a strong magnetic field. In such an environment the direction of the decay products of the unstable
particles would tend to be restricted by the direction of the magnetic field, which determines the direction of the EMAM.

VI. ACKNOWLEDGMENT

It is pleasure to thank K. Ruebenbauer for useful discussions in connection with certain aspects of this work.
REFERENCES

[1] J.J. Thomson, *Elements of the Mathematical Theory of Electricity and Magnetism* (Cambridge U.P., Cambridge, 1904), 3rd Ed., Sec. 284

[2] J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd Ed., p. 251

[3] D. Singleton, Phys. Lett. B427, 155, (1998); Am. J. Phys. 66, (1998)

[4] J. Dryzek and D. Singleton, Am. J. Phys., 67, 930 (1999)

[5] S.L. O’Dell and R.K.P. Zia, Am. J. Phys., 54, 32 (1986)

[6] D.C. Tsui, H.L. Stormer and A.C. Gossard, Phys. Rev. Lett. 48 1559 (1982).

[7] S.M. Girvin and A.H. MacDonald, Phys. Rev. Lett., 58, 1252 (1987)

[8] D.H. Lee and C.L. Lane, Phys. Rev. Lett., 64, 1313 (1990)

[9] F. Wilczek, Phys. Rev. Lett., 48, 1144 (1982); Phys. Rev. Lett., 49, 957 (1982)

[10] Steven H. Simon, cond-mat/9812186

[11] J.K. Jain, Phys. Rev. Lett. 63, 199 (1989)

[12] R.B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983)

[13] A.M. Chang, M.A. Paalanen, H.L. Strörmer, J.C.M. Hwang, D.C. Tsui, Surface Science, 142, 173 (1984)

[14] G.R. Stewart, Rev. Mod. Phys., 56, 755 (1984)

[15] V.D. Dzhunushaliev, Phys. Rev. B 54, 10121 (1996)

[16] R.B. Laughlin, Science, 242, 525 (1988)

[17] E. Witten, Phys. Lett. B86, 283 (1979)

[18] Y. Maeno, *et. al.*, Nature 372, 532 (1994); G.M. Luke, *et. al.*, Nature 394, 558 (1998); K. Krishana, *et. al.*, Science, 277, 83 (1997)