Flat central density profile and constant dark matter surface density in galaxies from scalar field dark matter

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Accepted 2012 January 20. Received 2012 January 13; in original form 2011 November 10

ABSTRACT

Using the scalar field dark matter (SFDM) model, it is proposed that galaxies form by condensation of a scalar field (SF) very early in the Universe, forming Bose–Einstein condensate (BEC) drops (i.e. in this model, the haloes of galaxies are gigantic drops of SF). Here, as in the Λ cold dark matter (ΛCDM) model, large structures form by hierarchy, and thus all the predictions of the LCDM model at large scales are reproduced by the SFDM model. This model predicts that all galaxies must be very similar and must exist for larger redshifts than in the LCDM model. In this paper, we show that BEC dark matter haloes fit the high-resolution rotation curves of a sample of 13 low-surface-brightness galaxies. We compare our fits to those obtained using Navarro–Frenk–White and pseudo-isothermal (PI) profiles. We have found better agreement with the SFDM and PI profiles. The mean value of the logarithmic inner density slopes is \( \alpha = -0.27 \pm 0.18 \). As a second result, we find a natural way to define the core radius with the advantage of being model-independent. Using this new definition in the BEC density profile, we find that the recent observation of the constant dark matter central surface density can be reproduced. We conclude that, in light of the difficulties that the standard model is currently facing, the SFDM model could be a worthy alternative to enable us to continue exploring further.

Key words: galaxies: fundamental parameters – cosmology: observations – dark matter.

1 INTRODUCTION

In the Λ cold dark matter (ΛCDM or LCDM) model, which is known as the standard model of cosmology, the formation of structures in the Universe is through a hierarchical process of growth of structures. This means that small structures, such as small haloes of galaxies, merge to form larger structures, such as galaxy clusters and superclusters haloes. In this scenario, the Universe contains around 96 per cent of an unknown form of energy, which is usually called dark matter (DM) and dark energy. The LCDM model can successfully describe cosmological observations such as the large-scale distribution of galaxies, the temperature variations in the cosmic microwave background radiation and the recent acceleration of the Universe (Coles 2005; Peebles, Lyman & Bruce 2009; Lahav & Liddle 2010; Guo et al. 2011).

However, recent observations in far and nearby galaxies have shown that the model faces serious conflicts when trying to explain the galaxy formation at small scales (Friedmann 2011; for a review, see Robles & Matos, in preparation). For instance, in the LCDM simulations, the haloes present rising densities towards the central region, behaving as \( \rho \sim r^{-1} \) well within 1 kpc (Graham et al. 2006; Navarro et al. 2010). However, various observations suggest that the rotation curves (RCs) are more consistent with a constant central density (Kuzio de Naray, McGaugh & de Blok 2008; de Blok 2010). This is most commonly known as the cusp/core problem (de Blok 2010).

Studying a wide range of galaxies of different morphologies and with magnitudes in the interval \( -22 \leq M_b \leq -8 \), Donato et al. (2009) fitted their RCs using a Burkert profile for the DM (Burkert 1995). They found that

\[
\log(\mu_0 / M_\odot \, \text{pc}^{-2}) = 2.15 \pm 0.2
\]

remains approximately constant, where

\[
\mu_0 = \rho_0 r_0.
\]

Here, \( \rho_0 \) is the central DM density and \( r_0 \) is the core radius. Similar results were found by Kormendy & Freeman (2004) and Spano et al. (2008). In Boyarsky et al. (2009, 2010) they found a similar result for \( \mu_0 \) in LSB galaxies and extended the relation to galaxy clusters. Exploring further the constant value of \( \mu_0 \) for the DM, Gentile et al. (2009) found that within \( r_0 \) the DM central surface density in terms of the mass inside it \( M(<r_0) \) is \( (\Sigma)_{0,\text{DM}} = M(<r_0) / 4\pi r_0^2 \approx 72^{+12}_{-17} \, \text{M}_\odot \text{pc}^{-2} \). The gravitational acceleration due to DM felt by a test particle at the radius \( r_0 \) was found to be

\[
g_{\text{DM}}(r_0) = G\pi(\Sigma)_{0,\text{DM}} = 3.2^{+1.8}_{-1.2} \times 10^{-9} \, \text{cm s}^{-2}.
\]

Additionally, they reported that the acceleration due to the luminous matter at \( r_0 \) is \( g_{\text{star}}(r_0) = 5.7^{+0.8}_{-0.6} \times 10^{-10} \, \text{cm s}^{-2} \).

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In the LCDM model, the galaxies have evolved through numerous mergers and they have grown in different environments. The star formation and basic properties of the galaxies are not expected to be common factors between them. Therefore, it seems very unlikely that both the constancy of $\mu_0$ and the core in the central regions of galaxies can be explained in this model.

These problems can be used to test alternative DM models. There are other models that do not include DM but instead modify the Newtonian force law $F = ma$. One of these models, proposed by Milgrom (Sanders 2009; Milgrom 2010), is called MOND. In this modification, Newton’s law is $F = ma\mu(x/a)$, where the fixed acceleration scale $a_0$ divides the Newtonian and MONDian regimes. For $x \ll 1$, we have the MONDian regime, and for $x \gg 1$, we recover the usual Newtonian acceleration, $a_0 \sim 1.2 \times 10^{-10}$ m s$^{-2}$.

Recently, the scalar field dark matter (SFDM) model has received much attention. When the SF contains a self-interaction, this model is also called the Bose–Einstein condensate (BEC) DM model; both names are used in the literature and are interchangeable. The main idea is simple (Guzmán & Matos 2000). In the SFDM model, it is proposed that galaxies form by condensation of a SF with an ultra-light mass of the order of $m_0 \sim 10^{-22}$ eV. Therefore, when we mention the SFDM or BEC model in this paper, we are describing a SF that condenses somehow and becomes BEC DM. From this mass, it follows that the critical temperature of condensation $T_c \sim 1/m_0^{5/3}$ TeV is very high. Thus, BEC drops form very early in the Universe. It has been proposed that these drops are the haloes of galaxies (see Matos & Uréña 2001), that is, haloes are gigantic drops of SF. However, as in the LCDM model, large structures form by hierarchy (Matos & Uréña 2001; Suárez & Matos 2011), and thus all predictions of the LCDM model at large scales are reproduced by SFDM. In other words, in the SFDM model, the haloes of galaxies do not form hierarchically, but they form at the same time and in the same way when the Universe reaches the critical temperature of condensation of the SF – in a similar way as water drops form in clouds. From this, it follows that all galaxies must be very similar, because they formed in the same manner and at the same moment.

Therefore, from this model, we should expect well-formed galaxy haloes to exist at larger redshifts than in the LCDM model. In this model, scalar particles with this small mass are such that their wave properties may lead to the cusp and the high number of small satellites (Hu, Barkana & Gruzinov 2000); this is another problem that is still present in the LCDM model (Klypin et al. 1999; Robles & Matos, in preparation). Summarizing, it is remarkable that with only one free parameter, the ultra-light scalar field mass ($m_0 \sim 10^{-22}$ eV), we find the following:

(i) The SFDM model fits the evolution of the cosmological densities (Matos, Vazquez & Magaña 2009).
(ii) The SFDM model fits the RCs of large galaxies (Bernal, Matos & Núñez 2008; Harko 2011a) and low-surface-brightness (LSB) galaxies (Guzmán & Lora-Clavijo 2011).
(iii) With this mass, the critical mass of collapse for a real SF is $10^{12}$ $M_\odot$ (i.e. that observed in galaxy haloes; Alcubierre et al. 2002).
(iv) The SF has a natural cut-off, and thus the substructure in clusters of galaxies is avoided naturally. With a SF mass of $m_0 \sim 10^{-22}$ eV, the amount of substructure is compatible with that observed (Matos & Uréña 2001; Suárez & Matos 2011).
(v) We expect that in the SFDM model galaxies are formed earlier than in the CDM model, because they form BECs at a critical temperature $T_c \gg$ TeV. So, if the SFDM model is right, we should see large galaxies at large redshifts with similar features.
(vi) Recently, it has been demonstrated that SFDM haloes maintain satellite galaxies going around large galaxies for enough time to explain the existence of old stars in the satellites, provided that the mass of the SF is $m_0 \sim 10^{-22}$ eV (Lora et al. 2011).

This idea was first considered by Sin (1994) and Ji & Sin (1994), and it was independently introduced by Guzmán & Matos (2000). In the BEC model, DM haloes can be described in the non-relativistic regime, where DM haloes can be seen as a Newtonian gas. If we consider a SF self-interaction, we need to add a quartic term to the SF potential. In this case, the equation of state of the SF is that of a polytope of index $n = 1$ (Harko 2011a; Suárez & Matos 2011). Different issues concerning BEC DM haloes and the cosmological behaviour of the BEC model have been studied by Colpi, Shapiro & Wasserman (1986), Gleiser (1988), Bohmer & Harko (2007), Harko (2011a), Matos & Guzmán (2001), Chavanis (2011) and Chavanis & Luca (2011).

It is a fact that any model trying to become a serious alternative to the LCDM model not only has to succeed in reproducing observations in which the standard model fails, but it also has to keep the solid description at a large scale. For this reason, our aim here is to test the BEC model with the two observations mentioned above: the cusp/core problem (Bohmer & Harko 2007) and the constant DM central surface density. In order to do this, we have used the Thomas–Fermi approximation and a static BEC DM halo to fit the RCs of a set of galaxies. However, so far there has been no comparison between the density profile and the data. In this paper, we fill this blank by fitting the RCs of 13 high-resolution LSB galaxies. Additionally, we compare the fits to two characteristic density profiles: the cuspy Navarro–Frenk–White (NFW) profile, which results from $N$-body simulations using the LCDM model, and the pseudo-isothermal (PI) core profile. The comparison allows us to show that the model is in general agreement with the data and with a core in the central region. For our second result, we have found that the meaning of a core is ambiguous. In order to clarify the meaning and to unify the description, we propose a new definition for the core and core radius, which allows us to decide when a density profile is cusp or core. Using this definition in the BEC model discussed above, we find that the BEC model can reproduce the constant value of $\mu_0$. As a cross-check, we used the PI profile and we have found our results to be in very good agreement with observations. This argues in favour of the model and our definition.

In Section 2, we describe the density profiles that we use to fit the RCs, and we give our definition of the core and core radius. In Section 3, we fit the galaxy data and we obtain the fitting parameters and the core radius for the PI and BEC density profiles. In Section 4, we discuss our results, and in Section 5 we give our conclusions.

2 DARK MATTER HALO DENSITY PROFILES

In this section, we provide the DM profiles that are used in the analysis. In Section 2.4, we briefly describe the usual meaning of the core and we establish a new definition for it.

2.1 BEC profile

Bohmer & Harko (2007) have considered the case in which the DM is in the form of a static BEC and the number of DM particles in the ground state is very large. Following Bohmer & Harko (2007), and assuming the Thomas–Fermi approximation (Dalforno et al. 1999),
which neglects the anisotropic pressure terms that are relevant only in the boundary of the condensate, we give the system of equations describing the static BEC in a gravitational potential $V$ as

$$\nabla p \left( \frac{\rho_p}{m} \right) = -\rho \nabla V$$

(4)

and

$$\nabla^2 V = 4\pi G \rho,$$

(5)

with the following equation of state

$$p(\rho) = U_0 \rho^2.$$

(6)

Here, $U_0 = (2\pi^2 \hbar^2 a)/m^3$, $\rho$ is the mass density of the static BEC configuration and $p$ is the pressure. Because we are considering zero temperature, $p$ is not a thermal pressure but instead it is produced by the strong repulsive interactions between the ground-state bosons. Assuming spherical symmetry and denoting $R$ as the radius at which the pressure and density are zero, the density profile takes the form (Bohmer & Harko 2007)

$$\rho_b(r) = \frac{\rho_0}{k^2} \frac{\sin(kr)}{kr},$$

(7)

where $k = \sqrt{Gm^3\hbar^2 a}/R$, $\rho_0 = \rho_b(0)$ is the BEC central density, $m$ is the mass of the DM particle and $a$ is the scattering length. The mass at radius $r$ is given by

$$m(r) = 4\pi\rho_0^2 \frac{\sin(kr)}{k^2} \frac{\sin(kr)}{kr} - \cos(kr).$$

(8)

Thus, the tangential velocity $V_B$ of a test particle at a distance $r$ is

$$V_B^2(r) = \frac{4\pi G \rho_0}{k^2} \left( \frac{\sin(kr)}{kr} - \cos(kr) \right).$$

(9)

The logarithmic slope of a density profile is defined as

$$\alpha = \frac{d(\log \rho)}{d(\log r)}.$$

(10)

Using equation (7) in equation (10), we obtain (Harko 2011b)

$$\alpha(r) = -\left[ 1 - \frac{\pi \sigma}{R} \cot \left( \frac{\pi \sigma}{R} \right) \right].$$

(11)

Additionally, the logarithmic slope of the RC is defined (Harko 2011b) by

$$\beta = \frac{d(\log V)}{d(\log r)}.$$

(12)

From equation (9), we obtain

$$\beta = \frac{1}{2} \left[ 1 - \frac{(\pi \sigma/R)^2}{1 - (\pi \sigma/R) \cot(\pi \sigma/R)} \right].$$

(13)

### 2.2 Pseudo-isothermal profile

All the empirical core profiles that exist in the literature fit two parameters: a scale radius and a scale density. A characteristic profile of this type is

$$\rho_{\text{PI}} = \frac{\rho_{0\text{PI}}}{1 + (r/R_c)^2},$$

(14)

which is the PI profile (Begeman, Broeils & Sanders 1991). Here, $R_c$ is the scale radius and $\rho_{0\text{PI}}$ is the central density. The RC is

$$V(r)_{\text{PI}} = \sqrt{4\pi G \rho_{0\text{PI}}^3 R_c^2 \left[ 1 - \frac{R_c}{r} \arctan \left( \frac{r}{R_c} \right) \right]}.$$

(15)

### 2.3 Navarro–Frenk–White profile

The NFW profile emerges from numerical simulations that use only CDM and are based on the LCDM model (Dubinski & Carlberg 1991; Navarro, Frenk & White 1996, 1997). In addition to this, we have chosen this profile because it is representative of what is called the cuspy behaviour ($\alpha \approx -1$) in the centre of galaxies as a result of DM. The NFW density profile (Navarro et al. 1997) and the RC are given respectively by

$$\rho_{\text{NFW}}(r) = \frac{\rho_1}{(r/R_c)(1 + r/R_c)^2},$$

(16)

and

$$V_{\text{NFW}}(r) = \sqrt{4\pi G \rho_1 R_c^3 \left[ \ln \left( 1 + \frac{r}{R_c} \right) - \frac{r/R_c}{1 + r/R_c} \right]}.$$

(17)

Here, $\rho_1$ is related to the density of the Universe at the moment the halo collapsed and $R_c$ is a characteristic radius.

### 2.4 Meaning of the core radius and cusp/core discrepancy

In the large-scale simulations that use collisionless CDM, the inner regions of DM haloes show a density distribution described by a power law $\rho \sim r^\alpha$ with $\alpha \approx -1$. Such behaviour is what is now called a cusp. However, observations in mainly dwarf and LSB galaxies seem to prefer a central density as $\rho \sim r^\beta$. This discrepancy between observations and the CDM model is called the cusp/core problem. Among the empirical profiles most frequently used to describe the constant density behaviour in these galaxies are the PI profile (Begeman et al. 1991), the isothermal (I) profile (Athanassoula, Bosma & Papaioannou 1987) and the Burkert profile (Burkert 1995). Even though their behaviour is similar in the central region and is specified by the central density fitting parameter, their second parameter (called the core radius) does not represent the same idea. For instance, in the PI profile (equation 14), we see that the core radius is the distance in which the density is half the central density. For the Burkert profile, the core radius $R_c$ is when $\rho_b(\text{back}) = \rho_b(\text{back})/4$. For an isothermal profile (Spano et al. 2008), $\rho_b(\text{back}) = \rho_b(\text{back})/2^{1/2}$. Hence, we see an ambiguity in the meaning of the core radius – all core radii have the same name but the interpretation depends on the profile. If we want to compare the central density of LSB galaxies with that of NFW, it usually suffices to have a qualitative comparison. So far, this is what we have been doing by fitting empirical profiles. However, high-resolution RCs demand a more quantitative comparison. Indeed, if we want to test models by fitting RCs, we have to know the specific meaning and size of the core in order to be able to tell if a model is consistent with a cusp or not, by making a direct comparison with the data.

For this reason, we ask what is the core and where is it? To solve the previous ambiguity and to unify the concept for future comparisons, we have found that a good definition for the core is a region where the density profile presents logarithmic slopes $\alpha \geq -1$. The core radius can be defined as the radius at which the core begins (i.e. for a radius smaller than the core radius, we have $\alpha \geq -1$). This means that its value $r'$ is determined by the equation

$$\alpha(r') = -1.$$

(18)

The advantages of this definition are that the interpretation is independent of the profile chosen (also note that it applies to the total density profile and it is not restricted to that of DM) and that,
by virtue of the same definition, we can directly tell if a DM model profile is cored or cuspy. With our new definition, the specific distance at which the core radius occurs still depends on the profile chosen, but now there is only one physical interpretation. In the following, when we refer to both the core and core radius, we adopt the previous interpretation.

Applying the definition to equation (7), we obtain the core radius for the BEC profile $R_b$. For comparison, we use equation (14) because it turns out that $R_b$ corresponds to the core radius, as defined above. Finally, fitting the NFW profile provides a direct comparison between a cusp and core, and hence to the cusp/core problem.

3 FITS AND DATA

We see from equation (7) that the BEC model satisfies $\rho \sim r^0$ near the origin, but a priori this does not imply consistency with the observed RCs. Therefore, we fit the profiles in Section 2 to 13 high-resolution observed RCs of a sample of LSB galaxies. The RCs were taken from a subsample of de Blok et al. (2001). We have chosen galaxies that have at least three values within ~1 kpc, that do not present bulbs and where the quality in the RC in Hα is as good as defined in McGaugh, Rubin & de Blok (2001). The RCs in this paper omit galaxies presenting high asymmetries and, in the error bars, we include experimental errors in the velocity measurement, inclination and small asymmetries. Because the DM is the dominant mass component for these galaxies, we adopt the minimum disc hypothesis, which neglects the baryon contribution to the observed RC. In order to show that neglecting the effect of baryons is a good hypothesis for LSB and dwarf galaxies, we include in Fig. 1 two representative examples (F568-3 and F583-1). For these galaxies, we plot the contribution of the gas, the disc and the DM separately. We carried out the fitting first, considering the total contribution and then using only DM (the last two rows in Tables 1 and 2). We found no substantial differences in our values, as can be seen from our results in Tables 1 and 2. Because the other galaxies belong to the category of DM-dominated galaxies, as other authors have shown (de Blok et al. 2001; Kuzio de Naray et al. 2008), neglecting baryons in our analyses will not substantially modify our results.

As the difference between a core and a cusp is most notable only for data values within 1 kpc, and given that in the interval 1–10 kpc, the slopes of the core and cusp profiles are very similar (which can lead to the wrong conclusion that cuspy halos are consistent with observations), we determined the logarithmic slope and the uncertainty following de Blok et al. (2001), with the difference that we fit only the data within 1 kpc and that there is no need for an uncertain 'break radius'.

Table 1 lists the fitting parameters of the profiles of Section 2. We also include the values of the logarithmic slope and its uncertainty. The value $R_1$ denotes the nearest radius to 1 kpc where a data point is given. We obtain $\alpha$ by fitting values inside $R_1$, and we also report the core radius for the BEC profile $R_b$ in order to compare it with $R_c$. Table 2 reports both the value of equation (3) for the BEC profile and the logarithm of equation (2) for PI and BEC profiles.

In section 4 we show our fits to the RC data and the density profiles.

4 DISCUSSION

In Fig. 2, we plot $\beta$ using three different values of $R$. From this figure, we see a common behaviour. We find that $\beta$ is a decreasing function of $r$ and that it is zero before $R$, which tells us that equation (9) always reaches a maximum before $R$. We also notice that there is a region in which $V_B \sim r$, and we can take this region to be when $0.9 \leq \beta \leq 1$. If we use $\beta = 0.9$ as an upper bound for the region in which the linear behaviour ($V_B \sim r$) remains valid, we obtain an upper bound radius of $r \approx 0.31 R$. This means that for values of $r \leq 0.31 R$, we expect $V_B \sim r$. The latter can be used as a test for the BEC model by fitting the RCs and verifying this solid-body behaviour within the above-mentioned region. The fits of the RCs in Fig. 3 prove that the solid-body-like behaviour characterized by a linear increase of the velocity in the central region is satisfied by the BEC model – in fact, it is more consistent with the core PI and BEC profiles than the cuspy NFW profile.

If we now turn to the density profiles, our fits within $R_1$ give an average value of $\alpha = -0.27 \pm 0.18$, consistent with that obtained by de Blok et al. (2001), $\alpha = -0.2 \pm 0.2$, and with $\alpha = -0.29 \pm 0.07$ reported by Oh Se-Heon et al. (2011), who have analysed seven dwarf galaxies from the Hα Nearby Galaxy Survey (THINGS). The case of ESO 1870510 might be considered to be consistent with the NFW profile. However, it is the innermost value that considerably decreases $\alpha$. Because it is an irregular galaxy, more central data near the innermost region are required to discard the possibility of any violent event that might have caused such a slope value.

The density profiles corresponding to the RC fits are also shown in Fig. 3 for each galaxy. We also show in Fig. 3 the core radius in the BEC and PI profiles. The continuous grey arrow is the fit that determines $\alpha$ (the size of the arrow denotes the fitted region and is bounded by $R_1$). From Fig. 3 we see that the BEC fits slightly deviate for the farthest data points as a result of the finite size of the radius $R$ that is fixed by the same data. This discrepancy is because the halo might be more extended than the value $R$. As a matter of a fact, the more extended the ‘flat’ outer region is in the RCs, the more conspicuous the discrepancy. The main reason for this comes from

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**Figure 1.** Baryon contribution to the RC for F568-3 and F583-1. We denote the observed data by black dots with error bars, DM with blue asterisks, the disc with cyan solid squares and the gas with magenta open squares. We fit the figures on the left assuming the minimum disc hypothesis, while for the figures on the right we subtract in quadrature the baryons and we fit only the DM. We notice that the baryonic component is not dominant in the outer regions and that the difference in the fits is barely noticeable.
is not expected to correlate in Table 1, we did not find a tendency in units of $\times 10^{-3}$ M$_{\odot}$ pc$^{-3}$. $^a$ $\rho_1$ is in units of $\times 10^{-3}$ M$_{\odot}$ pc$^{-3}$. $^b$ Both $\mu_0^{PI}$ and $\mu_0^{B}$ are in units of M$_{\odot}$ pc$^{-2}$.

Table 2. Quantities derived from the parameters in Table 1.

| Galaxy         | $\mu_0^{PI}$ | $\mu_0^{B}$ | $\mu_0^{DM}$ |
|----------------|--------------|-------------|--------------|
|                 | (M$_{\odot}$ pc$^{-2}$) | (M$_{\odot}$ pc$^{-2}$) | ($\times 10^{-3}$ cm s$^{-2}$) |
| ESO1200211      | 1.42 ± 0.13  | 1.60 ± 0.50 | 0.908        |
| ESO1870510      | 1.72 ± 0.78  | 1.68 ± 0.58 | 2.15         |
| ESO3020120      | 2.05 ± 0.52  | 2.00 ± 0.51 | 2.17         |
| ESO3050090      | 1.76 ± 0.59  | 1.81 ± 0.60 | 4.58         |
| ESO4880049      | 2.22 ± 0.11  | 2.16 ± 1.16 | 6.63         |
| U4115           | 2.14 ± 0.40  | 2.03 ± 0.23 | 4.93         |
| U11557          | 1.92 ± 0.31  | 1.82 ± 0.54 | 3.03         |
| U11611          | 2.97 ± 0.13  | 2.40 ± 0.42 | 11.3         |
| U11748          | 3.21 ± 0.19  | 3.05 ± 0.78 | 50.4         |
| U11819          | 2.42 ± 0.30  | 2.37 ± 0.25 | 10.8         |
| U11583          | 1.88 ± 0.25  | 1.83 ± 0.53 | 3.08         |
| F568-3          | 2.03 ± 0.35  | 2.03 ± 0.53 | 4.91         |
| F583-1          | 1.91 ± 0.23  | 1.91 ± 0.65 | 3.66         |
| F583-3 (DM)     | 1.95 ± 0.51  | 1.95 ± 0.68 | 4.09         |
| F583-1 (DM)     | 1.89 ± 0.13  | 1.88 ± 0.59 | 3.45         |

Fig. 2, where we infer that the RC speed always presents a maximum value followed by a continuous decrease. This means that to avoid the above-mentioned discrepancy we need the BEC RC profile to remain approximately constant after its maximum. From Fig. 1, we can see that the total RC is dominated by the DM contribution, especially in the outer regions. Hence, unless the baryons become the dominant component in the outer regions, which does not seem to be observed, it is unlikely that adding the baryonic contribution to the RCs in our galaxies will solve the discrepancy.

One solution to keep the BEC RC constant after its maximum is to make finite temperature corrections to equation (14) (Harko & Madaras 2012). This would alleviate the latter problem in LSB and dwarf galaxies but not in larger galaxies. Other authors have proposed including vortex lattices (Zimmer 2011; Rindler-Daller & Shapiro 2012) and adding more nodes (Ji & Sin 1994; Sin 1994) in the solution of equations (4) and (5). Nevertheless, it can be shown (Guzmán & Ureña-López 2003) that a system of many nodes is unstable, and therefore no final conclusion has yet been reached.

When comparing the BEC and PI core radii, we find a general difference of $\sim 2$ kpc. The core size in the PI profile is approximately 2 kpc smaller than the BEC core size, but the PI central density is larger. In U4115, U11557 and U11583, both profiles are very similar, which results in similar core and central density values. This can also be taken as a consistency check for our core definition.

For our second test, we use $R_B$ to calculate equation (2). We have already seen that $R_c$ and $R_B$ are generally different and that $R_B$ is not a fit parameter. Hence, a priori, $R_B$ is not expected to correlate with $\rho_0^B$. However, with the values from Table 1, we obtain

$$\log \left( \frac{\mu_0^{II}}{M_{\odot} \text{pc}^{-2}} \right) = \log \rho_0^B \, R_B = 2.05 \pm 0.56$$

(19)

and

$$\log \left( \frac{\mu_0^{PI}}{M_{\odot} \text{pc}^{-2}} \right) = \log \rho_0^{PI} \, R_c = 2.08 \pm 0.46$$

(20)

for the average values in the BEC and PI profiles, respectively. We see that there is excellent agreement of equation (19) with equation (20), which is used as a cross-check, and with equation (1), where a much larger sample was used. The agreement shows that the BEC model is capable of reproducing the constancy of the value $\mu_0$, something that is not possible in the NFW profile because of the cuspy nature.
Figure 3. Observed LSB galaxy RCs and density profiles with the best halo fits. Below each RC is its density profile along with the fits. We show the PI (green dashed line), BEC (red solid line) and NFW (black double-dotted line) DM halo profiles. The observational data are shown with error bars. The grey (continuous) arrow denotes the best fit to the data within $R_1$ and the vertical arrows denote the PI (blue dash–dotted line) and BEC (magenta dotted line) core radii.
In Fig. 4, we plot the above values for each galaxy. We define the DM central surface density (mentioned in Section 1) for the BEC profile by

$$\langle \Sigma_{0,\text{DM}}^B \rangle = \frac{M_{\leq R_B}}{\pi r_B^2},$$

(21)

where $M_{\leq R_B}$ is obtained from equation (8) evaluated at $R_B$. From Table 2, we see that for U11748 the value $\log \mu_{B,0}$ is considerably higher than the rest and has the largest uncertainty. For this reason, in the following analysis we omit both this value and the smallest value, which corresponds to ESO 1200211. By doing this, we obtain an average value for equation (19) of $\langle \langle \Sigma_{0,\text{DM}}^B \rangle \rangle \approx 191.35 \, M_\odot \, \text{pc}^{-2}$. For the acceleration felt by a test particle located in $R_B$ due to DM only, we have

$$g_{\text{DM}}(r_B) \approx 5.2 \times 10^{-9} \, \text{cm} \, \text{s}^{-2},$$

which is broadly consistent with equation (3).

The fact that all galaxies present approximately the same order of magnitude in $g_{\text{DM}}(R_B)$ might suggest that $R_B$ represents more than a transition towards a constant density. It can give us information about the close relation between DM and the baryons. Moreover, in view of the lack of a unique core radius, we can interpret the transition in the DM distribution as the effect of crossing a certain acceleration scale instead of a radial length-scale. Such an interpretation reminds us of that given in the MOND model, but with the big difference that the acceleration scale found is for DM and is not a postulate of the model.

To determine which interpretation causes the transition, either an acceleration scale or a length-scale, we need to study the properties of larger samples of galaxies that have been observed with new telescopes.

5 CONCLUSIONS

In this paper, we find that the BEC model gives a constant density profile that is consistent with the RCs of DM-dominated galaxies. The profile is as good as one of the most frequently used empirical core profiles, but with the advantage of coming from a solid theoretical frame. We fit the data within 1 kpc and we find a logarithmic slope $\alpha = -0.27 \pm 0.18$ in perfect agreement with a core. It is important to note that the cusp in the central regions is not a prediction that comes from first principles in the CDM model – it is a property that is derived by fitting simulations that use only DM.

We have established the ambiguity present in the usual interpretation of the core radius. We have proposed a new definition for the core and core radius that takes away the ambiguity and that has a clear meaning, which allows for a definite distinction when a density profile is either core or cusp.

Using our definition, we find that the core radius in the BEC profile is, in most cases, over 2 kpc larger than the core radius in the PI profile. We have assumed that DM particles are bosons and that a great number of these are in the ground state in the form of a condensate. This has led to good results for our sample of galaxies, but it might be necessary to consider more than these simple hypotheses.

As a second result, and as a direct consequence of our core definition, we have been able to obtain the constant value of $\mu_0$, which is proportional to the central surface density. This result is one of several conflicts that jeopardize the current standard cosmological model.

If we continue to observe even more galaxies with core behaviour, this model can be a good alternative to the LCDM model.
Figure 4. Plot of $\log(\mu_{h}^B/M_{\odot}\text{ pc}^{-2})$ and $\log(\mu_{\rho I}^P/M_{\odot}\text{ pc}^{-2})$ for each galaxy, $N$ denotes the galaxy according to Table 1. Here, we observe that these values remain approximately constant in both profiles, and this serves as a cross-check for our definition of $R_0$ in the BEC profile. The green dashed line represents the mean values given in equations (19) and (20).

ACKNOWLEDGMENTS

This work was partially supported by CONACYT Mexico under grants CB-2009-01, no. 132400 and 10101/131/07 C-234/07 of the Instituto Avanzado de Cosmología (IAC) collaboration (http://www.iac.edu.mx/).

REFERENCES

Alcubierre M., Siddhartha Guzmán F., Matos T., Núñez D., Ureña L. A., Wiederhold P., 2002, Class. Quant. Grav., 19, 5017
Athanassoula E., Bosma A., Papaianniou S., 1987, A&A, 179, 2340
Begeman K. G., Broeils A. H., Sanders R. H., 1991, MNRAS, 249, 523
Bernal A., Matos T., Núñez D., 2008, Rev. Mex. Astron. Astrofis. Ser. Conf., 44, 149
Bohmer C. G., Harko T., 2007, J. Cosmol. Astropart. Phys., 06, 025
Boyarsky A., Ruchayskiy O., Jakubovskiy D., Macciò A. V., Malyshev D., 2009, preprint (arXiv:0911.1774v1)
Boyarsky A., Neronov A., Ruchayskiy O., Tkachev I., 2010, Phys. Rev. Lett., 104, 191301
Burkert A., 1995, ApJ, 447, L25
Chavanis P. H., 2011, Phys. Rev. D, 84, 043531
Chavanis P. H., Delfini L., 2011, Phys. Rev. D, 84, 043532
Coles P., 2005, Nat, 433, 248
Colpi M., Shapiro S. L., Wasserman I., 1986, Phys. Rev. Lett., 57, 2485
Dalfovo F., Giorgini S., Pitaevskii L. P., Stringari S., 1999, Rev. Mod. Phys., 71, 463
de Blok W. J. G., 2010, Adv. Astron., Article ID 789293
de Blok W. J. G., McGaugh S. S., Bosma A., Rubin V. C., 2001, ApJ, 552, L23
Donato F. et al., 2009, MNRAS, 397, 1169
Dubinski J., Carlberg R. G., 1991, ApJ, 378, 496
Friedmann D. E., 2011, J. Cosmol., 13 (arXiv:0912.1668v4)
Gentile G., Famey B., Zhao H., Salucci P., 2009, Nat, 461, 627
Gleiser M., 1988, Phys. Rev. D, 38, 2376
Graham A. W., Merritt D., Moore B., Diemand J., Terzić B., 2006, AJ, 132, 2701
Guo Q. et al., 2011, MNRAS, 413, 101
Guzmán F. S., Lora-Clavijo F. D., 2011, MNRAS, 416, 3083
Guzmán F. S., Matos T., 2000, Class. Quant. Grav., 17, L9
Guzmán F. S., Ureña-López L. A., 2003, Phys. Rev. D, 68, 024023
Harko T., 2011a, MNRAS, 413, 3095
Harko T., 2011b, J. Cosmol. Astropart. Phys., 05, 022
Harko T., Madarassy E. J. M., 2012, J. Cosmol. Astropart. Phys., 01, 020
Hu W., Barkana R., Gruzinov A., 2000, Phys. Rev. Lett., 85, 1158
Ji S. U., Sin S. J., 1994, Phys. Rev. D, 50, 3650
Klypin A., Kravtsov A. V., Valenzuela O., Prada F., 1999, ApJ, 522, 82
Kormendy J., Freeman K. C., 2004, in Ryder S. D., Psano D. J., Walker M. A., Freeman K. C., eds, Proc. IAU Symp. Vol. 220, Dark Matter in Galaxies. Cambridge Univ. Press, Cambridge, p. 377
Kuzio de Naray R., McGaugh S. G., de Blok W. J. G., 2008, ApJ, 676, 920
Lahav O., Liddle A. R., 2010, in Nakamura K. et al. (Particle Data Group), The Review of Particle Physics, J. Phys. G, 37, 075021 and 2011 partial update for the 2012 edition (arXiv:1002:3488v1)
Lora V., Magaña J. A., Bernal A., Sanchez-Salcedo F. J., Grebel E. K., 2011, J. Cosmol. Astropart. Phys., in press (arXiv:1110.2684v2)
Matos T., Guzmán F. S., 2001, Class. Quantum Grav., 18, 5055
Matos T., Ureña-López L. A., 2001, Phys Rev. D, 63, 063506
Matos T., Vazquez A., Magaña J. A., 2009, MNRAS, 389, 13957
McGaugh S. G., Rubin V. C., de Blok W. J. G., 2001, AJ, 122, 2381
Milgrom M., 2010, in Alimi J.-M., Fusia A., eds, AIP Conf. Proc. Vol. 1241, Invisible Universe. American Institute of Physics, New York, p. 139 (arXiv:0912.2678v2)
Navarro J. F., Frenk C. S., White S. D. M., 1996, ApJ, 462, 563
Navarro J. F., Frenk C. S., White S. D. M., 1997, ApJ, 490, 493
Navarro J. F. et al., 2010, MNRAS, 402, 21
Oh S.-H., Brook C., Governato F., Brinks E., Mayer L., de Blok W. J. G., Brooks A., Walter F., 2011, AJ, 142, 24
Peebles P. J. E., Lyman A. P., Jr, Bruce R. P., 2009, Finding the Big Bang. Cambridge Univ. Press, Cambridge
Rindler-Daller T., Shapiro P. R., 2012, MNRAS, in press (doi:10.1111/j.1365-2966.2012.20588.x) (arXiv:1106.1256v4)
Sanders R. H., 2009, Adv. Astron., Article ID 752439
Sin S. J., 1994, Phys. Rev. D, 50, 3650
Spano M., Marcelin M., Amram P., Carignan C., Epinat B., Hernandez O., 2008, MNRAS, 383, 297
Suárez A., Matos T., 2011, MNRAS, 416, 87
Zinner N. T., 2011, Phys. Res. Int., 734543 (arXiv:1108.4290v1)

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