Gauge-invariant decomposition of nucleon spin

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Abstract. Based on gauge-invariant decomposition of covariant angular momentum tensor of QCD in an arbitrary Lorentz frame, we investigate the relation between the known decompositions of the nucleon spin into its constituents, thereby clarifying in what respect they are common and in what respect they are different critically. We argue that the decomposition of Bashinsky and Jaffe, that of Chen et al., and that of Jaffe and Manohar are contained in our more general decomposition, after an appropriate gauge-fixing in a suitable Lorentz frame, which means that they are all gauge-equivalent. We however point out that there is another gauge-invariant decomposition of the nucleon spin, which is closer to the Ji decomposition, while allowing the decomposition of the gluon total angular momentum into its spin and orbital parts. An advantage of the latter decomposition is that each of the four terms corresponds to a definite observable, which can be extracted from high-energy deep-inelastic-scattering measurements.

1. Introduction
After 20 years of theoretical and experimental efforts, we now believe that only about 1/3 of the nucleon spin comes from the intrinsic quark spin [1] - [4]. Unfortunately, what carry the remaining 2/3 of the nucleon spin is still a totally unanswerable question. Especially difficult question here is whether the gluon total angular momentum can be decomposed into its spin and orbital parts in a gauge-invariant way. Most people believe that the polarized gluon distribution $\Delta g(x)$ is an observable quantity from polarized DIS measurements. On the other hand, it is often claimed that there is no gauge-invariant decomposition of gluon total angular momentum into its spin and orbital parts. Because the gauge principle is one of the most important principle of physics, which demands that only gauge-invariants can be observed, how to reconcile these conflicting observations is a fundamentally important problem in nucleon spin physics.

First, we recall that there are two popular decompositions of the nucleon spin. One is the Jaffe-Manohar decomposition [5] given in the form :

$$ J_{QCD} = \int \psi^\dagger \frac{1}{2} \Sigma \psi \, d^3x + \int \psi^\dagger \mathbf{x} \times \frac{1}{i} \nabla \psi \, d^3x + \int \mathbf{E}^a \times \mathbf{A}^a \, d^3x + \int \mathbf{E}^{ai} \mathbf{x} \times \nabla \mathbf{A}^{ai} \, d^3x, $$

(1)

while the other is the Ji decomposition [6] given as follows :

$$ J_{QCD} = \int \psi^\dagger \frac{1}{2} \Sigma \psi \, d^3x + \int \psi^\dagger \mathbf{x} \times \frac{1}{i} \mathbf{D} \psi \, d^3x + \int \mathbf{x} \times (\mathbf{E}^a \times \mathbf{B}^a) \, d^3x. $$

(2)
Figure 1. Two well-known nucleon spin decompositions.

In these widely-known decompositions, only the intrinsic quark spin part is common, and the other parts are totally different. An apparent disadvantage of the Jaffe-Manohar decomposition is that each term is not separately gauge-invariant, except for the quark spin part. On the other hand, each term of the Ji decomposition is separately gauge-invariant. Note, however, that further gauge-invariant decomposition of $J^g$ into its spin and orbital parts is given up in this widely-known decomposition. An especially important observation here is that, since the quark orbital angular momenta (OAMs) in the two decompositions are apparently different, i.e.

$$L^Q \neq L^Q,$$

one must necessarily conclude that the sum of the gluon spin and OAM in the Jaffe-Manohar decomposition does not coincide with the total gluon spin in the Ji decomposition, i.e.

$$\Delta g + L^g \neq J^g.$$  

Recently, a new gauge-invariant decomposition of nucleon spin has been proposed by Chen et al. [7],[8]. The basic idea is to decompose the gluon field $A$ into two parts, the physical part $A_{phys}$ and the pure-gauge part $A_{pure}$. Imposing some additional condition, i.e. what-they-call the generalized Coulomb gauge condition, Chen et al. arrive at the decomposition of the nucleon spin in the following form :

$$J_{QCD} = \int \psi^\dagger \left( \frac{1}{2} \Sigma \psi \right) d^3x + \int \psi^\dagger \left( (\mathbf{p} - g A_{pure}) \psi \right) d^3x$$

$$+ \int E^a \times A_{phys}^a d^3x + \int E^{a3} (\mathbf{x} \times \nabla) A_{phys}^{a3} d^3x$$

$$= S^q + L^q + S^g + L^g.$$  

An interesting feature of this decomposition is that each term is separately gauge-invariant, while allowing the decomposition of the gluon total angular momentum into its spin and orbital parts. Another noteworthy feature of this decomposition is that it reduces to the gauge-variant decomposition of Jaffe and Manohar in a particular gauge, $A_{pure} = 0, A = A_{phys}$.

In a recent paper [9], however, we have shown that the way of gauge-invariant decomposition of nucleon spin is not necessarily unique, and proposed yet another gauge-invariant decomposition given as follows :

$$J_{QCD} = S^q + L^q + S^g + L^g.$$  

2
where

\[ S^q = \int \psi^\dagger \frac{1}{2} \Sigma \psi \, d^3x, \quad (7) \]

\[ L^q = \int \psi \mathbf{x} \times (\mathbf{p} - g \mathbf{A}) \psi \, d^3x, \quad (8) \]

\[ S^g = \int F^a \times A^a_{\text{phys}} \, d^3x, \quad (9) \]

\[ L^g = \int E^a \times (\mathbf{x} \times \nabla) A^a_{\text{phys}} \, d^3x + \int \rho^a (\mathbf{x} \times A^a_{\text{phys}}) \, d^3x. \quad (10) \]

The characteristic features of our decomposition are as follows. First, the quark parts of this decomposition is common with the Ji decomposition. Second, the quark and gluon spin parts are common with the Chen decomposition. A crucial difference with the Chen decomposition appears in the orbital parts. That is, although the sums of the quark and gluon OAMs in the two decompositions are the same, i.e.

\[ L^q + L^g = L'^q + L'^g, \quad (11) \]

each term is different in such a way that

\[ L^g - L'^g = -(L^q - L'^q) = \int \rho^a (\mathbf{x} \times A^a_{\text{phys}}) \, d^3x. \quad (12) \]

The difference arises from the treatment of the 2nd term of Eq.(10). We call this term the potential angular momentum term, since the QED correspondent of this term is the orbital angular momentum carried by the electromagnetic field or potential, which appears in the Feynman paradox raised in his famous textbook of classical electrodynamics [10]. We have included this term into the gluon OAM part, while Chen et al. included it into the quark OAM part.

To explain it in more detail, we first recall that that the potential angular momentum term can also be expressed as

\[ \int \rho^a (\mathbf{x} \times A^a_{\text{phys}}) \, d^3x = g \int \psi^\dagger \mathbf{x} \times A_{\text{phys}} \psi \, d^3x. \quad (13) \]

Note that this term is solely gauge-invariant, as can easily be convinced from the covariant (or homogeneous) gauge transformation property of the physical part of the gluon field \( A_{\text{phys}} \). This means that the gauge principle alone cannot say in which part of the decomposition one should include the potential angular momentum term. One has a freedom to include it into the quark OAM part, which would lead to the Chen decomposition. In fact, one sees that, in the following sum, the physical part of gluon field is exactly canceled out and the pure gauge part remains, which just corresponds to the quark OAM part of the Chen decomposition:

\[ L^q (\text{Ours}) + \text{potential angular momentum} \]

\[ = \int \psi^\dagger \mathbf{x} \times (\mathbf{p} - g \mathbf{A}) \psi \, d^3x + g \int \psi^\dagger \mathbf{x} \times A_{\text{phys}} \psi \, d^3x \]

\[ = \int \psi^\dagger \mathbf{x} \times (\mathbf{p} - g \mathbf{A}_{\text{pure}}) \psi \, d^3x = L'^q (\text{Chen}). \quad (14) \]

However, we do not recommend the Chen decomposition, because the common knowledge of standard electrodynamics tells us that the momentum appearing in the Lorentz force equation of motion, i.e.

\[ m \frac{d^2 \mathbf{x}}{dt^2} = \frac{d\mathbf{\Pi}}{dt} = e \left[ \mathbf{E} + \frac{1}{2} \left( \frac{d\mathbf{x}}{dt} \times \mathbf{B} - \mathbf{B} \times \frac{d\mathbf{x}}{dt} \right) \right]. \quad (15) \]
is the so-called dynamical momentum $\mathbf{I} = \mathbf{p} - e \mathbf{A}$ with the full gauge field $\mathbf{A}$, not the canonical momentum $\mathbf{p}$ or its nontrivial gauge-invariant extension $\mathbf{p} - e \mathbf{A}_{\text{pure}}$ [11]. By the same token, the orbital angular momentum, accompanying the mass flow of a charged particle, is the dynamical OAM, not the canonical OAM.

Since the quark part of our decomposition just coincides with the Ji decomposition, we may follow Ji's well-known program toward the full decomposition of the nucleon spin. First, extract $J^q$ and $J^g$ through GPD analyses. Next, extract the quark and gluon polarization through polarized DIS measurements and identify them with the quark and gluon spin parts in our decomposition. Then, the quark and gluon OAM can be known by subtracting $S^q$ and $S^g$ from $J^q$ and $J^g$, respectively.

What was lacking in this wishful argument is a rigorous proof of the identification of our gluon spin part with the 1st moment of the polarized gluon distribution $\Delta g(x)$. This problem is of fundamental importance, especially because we are aware of the wide-spread statement that there is no gauge-invariant decomposition of $J^g$, and that there is no gauge-invariant local operator corresponding to the 1st moment of $\Delta g(x)$. Another important problem is as follows. Since our gauge-invariant decomposition was given in a particular or fixed Lorentz frame, we could not give a definite answer to the question whether our decomposition has a frame-independent meaning or not? The confirmation of frame-independence is very important. Otherwise, the decomposition cannot be thought of as reflecting an intrinsic property of the nucleon. We can show that these two questions can be answered simultaneously, by making full use of a gauge-invariant decomposition of covariant angular-momentum tensor of QCD in an arbitrary Lorentz frame [12].

2. Gauge-invariant decomposition of covariant angular-momentum tensor

Covariant generalization of the gauge-invariant decomposition of the nucleon spin has twofold advantages. Firstly, it is vital to prove the Lorentz frame-independence of the decomposition. Secondly, it can generalize and unify the various nucleon spin decompositions in the market. To achieve this goal, we can follow a similar idea as proposed by Chen et al. [7],[8].

The starting point is the decomposition of the full gauge field $A^\mu$ into its physical and pure-gauge parts, i.e. $A^\mu = A^\mu_{\text{phys}} + A_{\text{pure}}^\mu$. Here, we impose the following conditions on those components. The first is the pure-gauge condition for $A_{\text{pure}}^\mu$:

$$F_{\mu \nu}^{\prime \prime} \equiv \partial^\mu A_{\text{pure}}^\nu - \partial^\nu A_{\text{pure}}^\mu - i g [A^\mu_{\text{pure}}, A^\nu_{\text{pure}}] = 0,$$

while the second is the gauge transformation properties for these two components:

$$A_{\text{phys}}^\mu (x) \rightarrow U(x) A_{\text{phys}}^\mu (x) U^{-1}(x),$$

$$A_{\text{pure}}^\mu (x) \rightarrow U(x) \left( A_{\text{pure}}^\mu (x) - \frac{i}{g} \partial^\mu \right) U^{-1}(x).$$

As a matter of course, these conditions are not enough to fix gauge uniquely. However, the point of our argument is that we can postpone a concrete gauge fixing until later stage, while accomplishing a gauge-invariant decomposition of $M^{\mu \nu \lambda}$ based on the above conditions only. As expected, we again find that the way of gauge-invariant decomposition is not unique. Basically, we are left with two possibilities, i.e. the decomposition (I) and the decomposition (II) [12].

The decomposition (II) is given as follows:

$$M^{\mu \nu \lambda}_{\text{QCD}} = M^{\mu \nu \lambda}_{q-\text{spin}} + M^{\mu \nu \lambda}_{q-\text{OAM}} + M^{\mu \nu \lambda}_{g-\text{spin}} + M^{\mu \nu \lambda}_{g-\text{OAM}} + \text{boost} + \text{total divergence},$$

\[19\]
with
\[ M_{q\text{-spin}}^{\mu\nu\lambda} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \bar{\psi} \gamma_\sigma \gamma_5 \psi, \]
\[ M_{q\text{-OAM}}^{\mu\nu\lambda} = \bar{\psi} \gamma^\mu (x^\nu \partial^\lambda_{\text{pure}} - x^\lambda \partial^\nu_{\text{pure}}) \psi, \]
\[ M_{g\text{-spin}}^{\mu\nu\lambda} = 2 \text{Tr} \{ F_{\mu\nu} A_{\text{phys}}^\lambda - F_{\mu\nu} A_{\text{phys}}^\lambda \}, \]
\[ M_{g\text{-OAM}}^{\mu\nu\lambda} = 2 \text{Tr} \{ F_{\mu\nu}^\alpha (x^\nu \partial^\lambda_{\text{pure}} - x^\lambda \partial^\nu_{\text{pure}}) A_{\text{phys}}^\alpha \}. \]

(20)
(21)
(22)
(23)

Here, the 1st and the 2nd terms are the quark spin and OAM terms, while the 3rd and the 4th terms are the gluon spin and OAM terms. The remaining boost and the total divergence terms do not contribute to the nucleon spin sum rule so that they can be neglected in the following discussion. Remarkably, we can show that this decomposition reduces to any ones of Bashinsky-Jaffe [13], of Chen et al. [7,8], and of Jaffe-Manohar [5], after an appropriate gauge-fixing in a suitable Lorentz frame, which means that they are all gauge-equivalent! However, they are not our recommendable decompositions, because the quark and gluon OAMs in those do not correspond to known experimental observables.

Our recommendable decomposition is the gauge-invariant decomposition (I), which is given in the following form:
\[ M^{\mu\nu\lambda} = M_{q\text{-spin}}^{\mu\nu\lambda} + M_{q\text{-OAM}}^{\mu\nu\lambda} + M_{g\text{-spin}}^{\mu\nu\lambda} + M_{g\text{-OAM}}^{\mu\nu\lambda} \]
\[ + \text{ boost} + \text{ total divergence}, \]

(24)

with
\[ M_{q\text{-spin}}^{\mu\nu\lambda} = M_{q\text{-spin}}^{\mu\nu\lambda}, \]
\[ M_{q\text{-OAM}}^{\mu\nu\lambda} = \bar{\psi} \gamma^\mu (x^\nu \partial^\lambda_{\text{pure}} - x^\lambda \partial^\nu_{\text{pure}}) \psi \neq M_{q\text{-OAM}}^{\mu\nu\lambda}, \]
\[ M_{g\text{-spin}}^{\mu\nu\lambda} = M_{g\text{-spin}}^{\mu\nu\lambda}, \]
\[ M_{g\text{-OAM}}^{\mu\nu\lambda} = M_{g\text{-OAM}}^{\mu\nu\lambda} + 2 \text{Tr} \left[ (D_\alpha F_{\mu\nu}^\alpha) (x^\nu A_{\text{phys}}^\lambda - x^\lambda A_{\text{phys}}^\nu) \right]. \]

(25)
(26)
(27)
(28)

The difference with the decomposition (II) appears in the orbital angular momentum parts. A great advantage of the decomposition (I) over the decomposition (II) is the concrete connection with high-energy deep-inelastic-scattering observables, as we shall argue in the next section.
3. Observability of our nucleon spin decomposition

Inserting our decomposition (I) into the helicity normalization condition,

\[ \frac{\langle P, s \mid W^s \mid P, s \rangle}{\langle P, s \mid P, s \rangle} = \frac{1}{2}, \] (29)

where

\[ W^\mu = -\frac{1}{2\sqrt{P^2}} \epsilon_{\mu\alpha\beta\gamma} M^{\alpha\beta} P^\gamma, \quad W^\mu s_\mu = J \cdot P / |P|, \] (30)

with \( W^\mu \) being the standard Pauli-Lubansky vector constructed from the angular-momentum tensor and the nucleon momentum, we can derive the following nucleon spin sum rule [12] :

\[ \frac{1}{2} = S_q + L_q + S_g + L_g = J_q + J_g, \] (31)

with

\[ S_q = \frac{1}{2} \Delta q, \] (32)
\[ L_q = \frac{1}{2} [ A_{q_{20}}^q(0) + B_{q_{20}}^q(0) ] - \frac{1}{2} \Delta q, \] (33)
\[ S_g = \Delta g, \] (34)
\[ L_g = \frac{1}{2} [ A_{g_{20}}^g(0) + B_{g_{20}}^g(0) ] - \Delta g. \] (35)

Here, \( A_{q_{20}}^q(0) \) and \( B_{q_{20}}^q(0) \) respectively stand for the 2nd moments of the unpolarized GPDs \( H^{q/s}(x, \xi, t) \) and \( E^{q/s}(x, \xi, t) \) with \( \xi = t = 0 \), i.e.

\[ A_{q_{20}}^q(0) = \int_{-1}^{1} x H^{q/s}(x, 0, 0) \, dx, \] (36)
\[ B_{q_{20}}^q(0) = \int_{-1}^{1} x E^{q/s}(x, 0, 0) \, dx. \] (37)

As desired, the total nucleon spin consists of four terms, corresponding to the intrinsic quark spin, the quark OAM, the intrinsic gluon spin, and the gluon OAM. Our derivation insures that this decomposition is not only gauge-invariant but also basically Lorentz frame-independent.

Crucially important to establish is the relation with actual high-energy observables. We can prove that the quark and gluon intrinsic spin parts of our recommendable decomposition precisely coincides with the 1st moments of the polarized distribution functions appearing in the polarized DIS cross sections [12].

\[ \Delta q = \int_{-1}^{1} \Delta q(x) \, dx, \quad \Delta g = \int_{-1}^{1} \Delta g(x) \, dx. \] (38)

What is more, we can verify that the following important relation holds :

\[ L_q = J_q - \frac{1}{2} \Delta q \]
\[ = \frac{1}{2} \int_{-1}^{1} x [ H^q(x, 0, 0) + E^q(x, 0, 0) ] \, dx - \frac{1}{2} \int_{-1}^{1} \Delta q(x) \, dx \]
\[ = \langle p \uparrow \mid M_{q-OAM}^{12} \mid p \uparrow \rangle, \] (39)
This identity means that the quark OAM defined as the difference between the 2nd moments of unpolarized GPDs $H + E$ and the 1st moment of polarized quark distribution just coincides with the proton matrix element of the our quark OAM operator containing full gauge covariant derivative [6]. This confirms that the quark OAM extracted from the combined analysis of GPDs and polarized PDFs is the dynamical OAM not the canonical-like OAM!

Similarly, we can prove the following identity for the gluon part:

$$L_g = J_g - \Delta g$$

$$= 1 \int_{-1}^{1} x [H^g(x,0,0) + E^g(x,0,0)] dx - \int_{-1}^{1} \Delta g(x) dx$$

$$= \langle p \uparrow | M_{g-OAM}^{012} | p \uparrow \rangle,$$  (41)

with

$$M_{g-OAM}^{012} = 2 \text{Tr} \left[ E^J (x \times D_{\text{pure}})^3 A_{\text{phys}}^J \right] : \text{canonical OAM}$$

$$+ 2 \text{Tr} \left[ \rho (x \times A_{\text{phys}})^3 \right] : \text{potential OAM term.}$$  (42)

It means that the gluon OAM extracted from the combined analysis of GPD and polarized PDF contains the potential OAM, in addition to the canonical-like OAM. It would be legitimate to call this whole part the gluon dynamical OAM.

4. Some phenomenological implications

We think it instructive to call attention to some other recent investigations related to the nucleon spin decomposition. As emphasized above, the quark orbital angular momentum extracted from the combined analysis of the unpolarized GPDs and the longitudinally polarized quark distribution functions is the dynamical orbital angular momentum not the canonical one or its nontrivial gauge-invariant extension. At least so far, we have had no means to extract the canonical orbital angular momentum purely experimentally, which also means that the difference between the dynamical and canonical orbital angular momenta is not a direct experimental observable. Nevertheless, it is not impossible to estimate the size of this difference within the framework of a certain model. In fact, Burkardt and BC estimated the difference between the orbital angular momentum obtained from the Jaffe-Manohar decomposition and that obtained from the Ji decomposition within two simple toy models, and emphasize the possible importance of the vector potential in the definition of orbital angular momentum [14]. The difference between the above two orbital angular momenta is nothing but the potential angular momentum in our terminology.

Also noteworthy is recent phenomenological investigations on the role of orbital angular momenta in the nucleon spin. In a recent paper, we have pointed out possible existence of significant discrepancy between the lattice QCD predictions [15],[16] for $L^u - L^d$ (the difference of the orbital angular momenta carried by up- and down-quarks in the proton) and the prediction of a typical low energy model of the nucleon, for example, the refined cloudy-bag model [17]. It is an open question whether this discrepancy can be resolved by strongly scale-dependent nature of the quantity $L^u - L^d$ especially in the low $Q^2$ domain as claimed in [18], or whether the discrepancy has a root (at least partially) in the existence of two kinds of quark orbital angular momenta as indicated in [19],[20]. (See also [21],[22].)
5. Summary and conclusion
To sum up, inspired by the recent proposal by Chen et al., we find it possible to make a gauge-invariant decomposition of covariant angular-momentum tensor of QCD in an arbitrary Lorentz frame. Based on this fact, we could show that our decompositions of nucleon spin are not only gauge-invariant but also practically frame-independent. We have also succeeded to convince that each piece of our nucleon spin decomposition (I) precisely corresponds to the observables that can be extracted from combined analysis of the GPD measurements and the polarized DIS measurement, thereby supporting the standardly-accepted program aiming at complete decomposition of the nucleon [23]-[25].

A practically very important lesson learned from our theoretical consideration is that the quark OAM extracted from the combined analysis of GPDs and polarized PDFs is the dynamical quark OAM, not the canonical OAM or its non-trivial “gauge-invariant extension” as advocated by Chen et al. Similarly, the gluon OAM extracted from the combined analysis of the gluon GPD and polarized gluon distribution is the dynamical gluon OAM, which contains the potential angular momentum term in addition to the canonical one. Note that, at the moment, we do not know any practical means, with which we can extract the canonical OAMs purely experimentally, i.e. model independently. Still, one should keep in mind the existence of 2 kinds of quark and gluon OAMs!

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