On the Detection of Single-Mode Squeezed States in Kerr Nonlinear Coupler

Mohd Syafiq M. Hanapi¹, Abdel-Baset M.A. Ibrahim¹*, Rafael Julius²

¹Faculty of Applied Sciences, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia
²Faculty of Applied Sciences, Universiti Teknologi MARA (UiTM) Perak, Tapah Campus, 35400 Tapah Road, Perak, Malaysia

*Corresponding author’s email: abdelbaset@uitm.edu.my

Abstract. The generation of squeezed states of light in two guided waves Kerr nonlinear coupler (KNLC) was examined using both the analytical perturbative (AP) and the short-time approximation (STA) method. A comparative analysis between these two methods is provided. We have found that, at certain combinations of input parameters, the STA method may not be able to detect the generation of squeezed states of light in the current KNLC system. Consequently, some essential physics could be lost. On the other hand, for the AP method, all time-dependent terms are included in the mode solutions which improves its sensitivity to detect the generation of squeezed states.

1. Introduction

Squeezed states of light or simply squeezed light has special properties that allow it to surpass the standard quantum limit. Squeezed light is crucial in future technological applications. It can mainly be applied to enhance the sensitivity of optical measuring equipment [1], [2]. Furthermore, squeezed light has the potential to initiate the quantum teleportation phenomenon [3], [4]. Short-range teleportation is essential to deliver quantum computational data [5]. Hence, squeezed light is important to put the quantum computer into action. It is also important in many other applications such as communication [6]–[8], gravitational wave detection [9], [10], quantum imaging [11]–[13], and quantum error correction coding [14], [15]. Due to all these futuristic applications, both theoretical and experimental research on squeezed light was given great attention [16]–[20].

Directional coupler essentially consists of two waveguides placed closely along each other to allow the propagating field-modes to linearly couple via evanescent waves. A schematic diagram of this devices is shown in Fig. 1. The strength of linear coupling is inversely proportional to the separation between the channels. In Kerr nonlinear coupler, the propagating optical field in the waveguides produces an optical Kerr effect due to the nonlinear interaction with the guiding medium. Recently, Kerr nonlinear coupler (KNLC) became a versatile source of quantum phenomena [21]–[25]. This is because it is a simple-structured device and is easily integrated with/within other devices such as photonic chips, quantum circuits, and optical logic gates [26]–[29].

A standard method to investigate squeezed state generation in KNLC is the short-time approximation (STA) method where an analytical solution is assumed in the form of second-order Taylor series where the higher-order terms are ignored. This approximation leads to the loss of some essential physics in the short-time approximation method as will be evident in this work. Alternatively, to examine the generation of squeezed states in KNLC, we present an analytical solution to the coupled Heisenberg
equations of motion for the propagating modes in the form of Baker–Hausdorff (BH) formula. Based on BH formula, an intuitive solution of the modes could be obtained with fully deterministic time-dependent coefficients. In this method, the nonlinearity in the system is assumed to be small enough to be treated as a small perturbation, hence the name analytical-perturbative or simply the (AP) method. Previously, the AP method was employed to examine the quantum properties generated in mechanisms other than KNLC such as optomechanical systems [30], [31], light-semiconductor interaction [32], Raman process [33]–[36], four-wave mixing [37], and Bose-Einstein Condensation [38]–[40].

In this work, we aim at investigating the generation of squeezed states of light in KNLC using the AP method. In particular, we aim at comparing both methods and examine their efficiency in detecting single-mode squeezed states generated in Kerr nonlinear coupler. This comparison could provide useful physical insights on the accuracy, capability, and reliability of these two methods in detecting the nonclassical effects in quantum optical devices. In the following sections, we present the basic mathematical formulation of both the STA and AP method, a discussion of the main results, and finally, a conclusion is drawn.

2. Mathematical formulation

In this section, the complete mathematical description for KNLC is presented. The two-mode system under consideration can be characterized by the following Hamiltonian operator.

\[
\hat{H} = \hbar [\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 + g (\hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_2 \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_2 \hat{a}_1) + k (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2)]
\]

(1)

In Eq. 1, \( \hbar \) is the reduced Planck constant, and \( \hat{a}_j^\dagger (\hat{a}_j) \) is the creation (annihilation) operator of the modes. The first two terms on the right-side of Eq.1 describe the propagating terms of both modes at a frequency \( \omega_j \). The third term represents the nonlinear coupling between the modes and the Kerr medium. This interaction is controlled by the nonlinear coupling coefficient, \( g \). The final term describes the linear coupling interaction between both modes, with linear coupling strength \( k \). The overall energy in the mode interaction is conserved since the energy loss by one mode corresponds to an energy gain by the opposite mode. By substituting Eq. (1) into the general form of Heisenberg equation of motion \( \frac{d\hat{a}_j}{dt} = i[\hat{a}_j, \hat{H}] \) we obtained the following.

\[
\frac{d\hat{a}_1}{dt} = i [\omega_1 \hat{a}_1 + 2g \hat{a}_2^\dagger \hat{a}_2 + k \hat{a}_2]
\]

(2)

\[
\frac{d\hat{a}_2}{dt} = i [\omega_2 \hat{a}_2 + 2g \hat{a}_1^\dagger \hat{a}_1 + k \hat{a}_1]
\]

(3)

Figure 1. A schematic diagram of the two-channel Kerr coupler. Each waveguide is traversed by one fundamental mode. The light modes are represented by their respective annihilation operators, \( \hat{a}_1 \) and \( \hat{a}_2 \), while the quantity \( k \) symbolizes the strength of linear coupling.
Each one of these equations describes the motion of an individual light mode. The system (2)-(3) has the form of coupled nonlinear differential equations, hence analytical closed-form solutions are readily unavailable. Here, we attempt to obtain the closed-form analytical solution for such a system under certain approximations using both the STA and AP methods.

### 2.1. Short-Time Approximation (STA) Method

In this method, solutions for (2)-(3) can be derived using the McLaurin series. This method employs the short-time approximation that neglects terms beyond $t^2$. Under this approximation, the solutions will take the following form

\[
\begin{align*}
\dot{\hat{a}}_1(t) &= \hat{a}_1(t) + i\left[\hat{\alpha}_1\hat{a}_1(t) + 2\hat{g}\hat{a}_1(t)\hat{a}_1^*(t) + \hat{\kappa}\hat{a}_1(t)\right]t - \frac{1}{2}\left[\hat{\alpha}_1^2\hat{a}_1(t) + \hat{\alpha}_1\hat{\kappa}\hat{a}_1(t) + 2\hat{g}\hat{a}_1(t)\hat{a}_1^*(t)\right]t^2 \\
\dot{\hat{a}}_2(t) &= \hat{a}_2(t) + i\left[\hat{\alpha}_2\hat{a}_2(t) + 2\hat{g}\hat{a}_2(t)\hat{a}_2^*(t) + \hat{\kappa}\hat{a}_2(t)\right]t - \frac{1}{2}\left[\hat{\alpha}_2^2\hat{a}_2(t) + \hat{\alpha}_2\hat{\kappa}\hat{a}_2(t) + 2\hat{g}\hat{a}_2(t)\hat{a}_2^*(t)\right]t^2
\end{align*}
\]

For convenience in numerical simulation, solutions (4)-(5) is expressed in terms of these dimensionless parameters defined as $\hat{\alpha}_i = \alpha_i/\omega_i, \hat{\kappa} = \kappa/\omega_i$ and $\tau = \omega_i t$. These solutions only contain the term up to $t^2$ only. The absence of higher-order terms may affect the sensitivity of lead to the loss of some essential physics.

### 2.2. Analytical Perturbative (AP) Method

In Heisenberg picture, the Heisenberg equation of motion has a general solution in the form of Baker-Campbell-Hausdorff (BCH) formula $\hat{a}_j = e^{it\hat{a}_j(0)} e^{-it\hat{a}_j(0)}$. It is convenient to expand this solution using Maclaurin series as $\hat{a}_j = \hat{a}_j(0) + it[H, \hat{a}_j(0)] - \frac{t^2}{2!}[H, [H, \hat{a}_j(0)]]$ considering only the first three terms. The commutator terms in this expression can simply be obtained using the initial state of Eqs. (2)-(3). For this method, we employ the weak nonlinearity approximation, where the nonlinear coupling coefficients are restricted to have a linear power only, i.e., terms with $\hat{g}^2$ and above are neglected. Following this mathematical process, the modes solutions take the following form.

\[
\begin{align*}
\hat{a}_1 &= A_1\hat{a}_1(0) + A_2\hat{a}_2(0) + A_3\hat{a}_1^*(0)\hat{a}_2^*(0) + A_4\hat{a}_1(0)\hat{a}_2(0)\hat{a}_1^*(0) + A_5\hat{a}_1^*(0)\hat{a}_2^*(0)\hat{a}_1(0) + A_6\hat{a}_1(0)\hat{a}_2^*(0)\hat{a}_1^*(0) \\
\hat{a}_2 &= B_1\hat{a}_1(0) + B_2\hat{a}_2(0) + B_3\hat{a}_1^*(0)\hat{a}_2^*(0) + B_4\hat{a}_1(0)\hat{a}_2(0)\hat{a}_1^*(0) + B_5\hat{a}_1^*(0)\hat{a}_2^*(0)\hat{a}_1(0) + B_6\hat{a}_1(0)\hat{a}_2^*(0)\hat{a}_1^*(0)
\end{align*}
\]

However, these solutions are not complete yet, since the $A'$s and $B'$s coefficients remain unknown. These coefficients can be obtained directly from Maclaurin expansion. However, if we do so, the outcome of the AP method becomes similar to the STA method. Alternatively in the AP method, the A’s and B’s coefficients are assumed unknown functions of time with terms span from $t^0$ to $t^n$. For this reason, the AP is said to be superior to the STA method. Therefore, solutions obtained using this method are more general compared with the STA method. Now, the A’s and B's coefficients can be evaluated numerically by solving the following two sets of coupled differential equations:

\[
\begin{align*}
\frac{dA_1}{dt} &= i\left[\hat{\alpha}_1A_1 + \hat{\kappa}B_1\right] \\
\frac{dA_2}{dt} &= i\left[\hat{\alpha}_2A_2 + \hat{\kappa}B_2\right] \\
\frac{dA_1}{dt} &= i\left[\hat{\alpha}_1A_1 + 2\hat{g}\left[A_1^2 + \hat{\kappa}B_1\right]\right] \\
\frac{dA_2}{dt} &= i\left[\hat{\alpha}_2A_2 + 2\hat{g}\left[A_2^2 + \hat{\kappa}B_2\right]\right]
\end{align*}
\]
\[\alpha = \frac{1}{2} (\hat{a}_j + \hat{a}_j^\dagger) \text{ and } \hat{Y} = -\frac{i}{2} (\hat{a}_j - \hat{a}_j^\dagger), \] respectively. The variance for these quadrature operators can be derived as follow.

\[
\langle (\Delta \hat{X})^2 \rangle, \langle (\Delta \hat{Y})^2 \rangle = \frac{1}{4} \left[ 1 + 2 \left( \langle \hat{a}_j^\dagger \hat{a}_j \rangle - \langle \hat{a}_j \rangle \langle \hat{a}_j^\dagger \rangle \right) \pm \left( \langle \hat{a}_j^\dagger \rangle - \langle \hat{a}_j \rangle \right)^2 + \text{c.c.} \right] \tag{20}
\]

In this paper, we assume that the initial state of the field is coherent state. Hence, the initial expectation value of the mode-annihilation operators is defined as \( \langle \hat{a}_j(0) \rangle = \alpha_j \), where \( \alpha_j \) is the amplitude of the input coherent field. The mode solutions derived earlier from the STA method (4)-(5) and that of the AP method (6)-(7) will be used to evaluate the quadrature variances using Eq. (20). For the STA method, we substitute Eqs. (4)-(5) into (20) to get the following formulas,

\[
\langle (\Delta \hat{X})^2 \rangle, \langle (\Delta \hat{Y})^2 \rangle = \frac{1}{4} \left[ 1 \pm 2 \Re \left( i \alpha_j^2 - 2 \hat{k} \alpha_j \alpha_j^* \tau - \hat{g} \alpha_j \alpha_j^* \tau - 8 \Re \left[ \alpha_j \alpha_j^* \alpha_j^* \right] + \text{c.c.} \right) \right] \tag{21}
\]

\[
\langle (\Delta \hat{X})^2 \rangle, \langle (\Delta \hat{Y})^2 \rangle = \frac{1}{4} \left[ 1 \pm 2 \Re \left( i \alpha_j^2 - 2 \hat{k} \alpha_j \alpha_j^* \tau - \hat{g} \alpha_j \alpha_j^* \tau - 8 \Re \left[ \alpha_j \alpha_j^* \alpha_j^* \right] + \text{c.c.} \right) \right] \tag{22}
\]

where c.c. is complex conjugate. For the AP method, the quadrature variance is derived by substituting Eqs. (6)-(7) into (20), and we obtain

\[
\langle (\Delta \hat{X})^2 \rangle, \langle (\Delta \hat{Y})^2 \rangle = \frac{1}{4} \left[ 1 \pm \left[ (A_1 A_1 + A_2 A_2) \alpha_j^2 + A_1 A_1 \alpha_j \alpha_j^* + A_2 A_2 \alpha_j \alpha_j^* + \text{c.c.} \right] \right] \tag{23}
\]

\[
\langle (\Delta \hat{X})^2 \rangle, \langle (\Delta \hat{Y})^2 \rangle = \frac{1}{4} \left[ 1 \pm \left[ (B_1 B_1 + B_2 B_2) \alpha_j^2 + B_1 B_1 \alpha_j \alpha_j^* + B_2 B_2 \alpha_j \alpha_j^* + \text{c.c.} \right] \right]. \tag{24}
\]

Using these equations (21)-(24), the condition of squeezing is given by \( \langle (\Delta \hat{X})^2 \rangle < \frac{1}{4} \) or \( \langle (\Delta \hat{Y})^2 \rangle < \frac{1}{4} \). In deriving Eqs. (21)-(24), the following identities of quantum mechanics are essential to convert the
expression into the normal-ordered operator configuration: \( \hat{a}_j (\hat{a}_j^\dagger)^m = m (\hat{a}_j^\dagger)^{m-1} + (\hat{a}_j^\dagger)^m \hat{a}_j \) and 
\( \hat{a}_j^m \hat{a}_j^\dagger = m \hat{a}_j^{m-1} + \hat{a}_j \hat{a}_j^m \), where \( m \) is an integer number. Finally, these equations are solved numerically and the graphical results are presented in the next section.

3. Result and Discussion

Throughout the investigation, some of the parameters are set to be constant. These parameters are \( \alpha_1 = 1 \), \( \alpha_2 = 1 \), \( \bar{\alpha}_i = 1 \) and \( \tilde{g} = 0.01 \). For all the graphs presented here, squeezing is identified whenever the field oscillates below the shot noise. For a clear indication, the shot noise is illustrated by the horizontal ‘black solid line’. Figure 2a shows the quadrature fluctuation obtained using the analytical perturbative method. As seen, the squeezing occurs in a periodical manner, hence it is not continuous as indicated by the STA method. Plus, if squeezing exists in the first quadrature component, it will not exists in the other quadrature component at the same time, and vice versa. This is due to the Heisenberg uncertainty relation. Overall, squeezing is possible in both quadrature components. In Fig. 2b, we notice the quadratic-like behavior of quadrature variances resulted from the STA formulation without oscillation is detected. Because of this limitation, STA fails to detect the existence of squeezing in the second quadrature component. This confirms our observation that the STA method may ignore some important physics. Figure 2c shows the combination of the two previous graphs. Here, we can see a significant difference between the AP method and the STA method at long evolution distances. As time increases, the disagreement between these two results becomes more obvious. Nevertheless, the two methods highly agree with each other at the very beginning. Therefore, we can say that the accuracy of the STA method is limited for a very short time only. the failure of the STA method at a longer evolution time may be due to the elimination of terms beyond the second-order terms with \( \tau^2 \).

![Figure 2](image-url)

**Figure 2.** Time evolution of the quadrature variances for the first waveguide-mode using (a) the AP method and (b) the STA method. A combination of these two graphs is shown in (c). \( \Delta X_1^2 \) and \( \Delta Y_1^2 \) are...
first and the second quadrature component of the field mode respectively. The optical mode in the second waveguide shows exact similar results. The input parameters used \( \Delta \tilde{\omega} = 0 \) and \( \tilde{k} = 0.3 \).

In Fig. 3, squeezing for different values of linear coupling coefficient \( \tilde{k} \) is shown. Here, we only show the first quadrature component \( \Delta X_1^2 \) of the first waveguide mode. In the AP method, as \( \tilde{k} \) increases, the squeezing occurs more frequently (see Fig. 3a). This is true, since the quadrature variances exhibit dense oscillations at higher values \( \tilde{k} \) due to the rapid exchange of energy between the waveguides. For the considered time range, the maximal squeezing appears four times and six times at \( \tilde{k} = 0.1 \) and \( \tilde{k} = 0.8 \), respectively. Although the frequency of the quadrature oscillation increases, the amplitude of the graph remains unchanged. This indicates that the strength of the squeezed light is not affected by the value of \( \tilde{k} \). As mentioned earlier, energy is conserved in the interaction process, since the energy gain by one channel is balanced with the energy loss in the other channel. Therefore, by strengthening the linear coupling interaction, no extra energy will be added to the system. On the other side, in the STA method (Fig. 3b), the curve of \( \tilde{k} = 0.8 \) lies lower than the curve of \( \tilde{k} = 0.1 \). This indicates that a strong linear coupling yields a stronger squeezing compared with weak linear coupling. This outcome opposes the AP results. Furthermore, the second quadrature component \( \Delta Y_1^2 \) oscillates completely above the shot-noise level as observed previously in Fig. 2.

![Figure 3](image)

**Figure 3.** Temporal evolution of the first quadrature variance in the first waveguide mode using (a) the AP method and (b) the STA method for different values of linear coupling coefficient. Other parameters remain as in Fig. 2.

When we introduce a frequency mismatch between both modes, the pattern of the graph generated by the AP method changes (Fig. 4a). In this case, the modes propagate at different phases. Therefore, the interaction between the modes takes place differently compared with the case of frequency matching. However, this change in quadrature pattern is not predicted by the STA method (see Fig. 4b). As we can see here, both graphs are overlapped with each other to indicate that the interaction occurs similarly for both conditions. Therefore, the STA method seems not suitable to examine the squeezed light under the presence of frequency mismatch.
4. Conclusion
In conclusion, the AP method and the STA method have been used to study the detection of single-mode squeezed states of light generated in Kerr nonlinear coupler composed of two waveguides. Special focus was given to the comparison between these two mathematical methods. Both methods describe the system in Heisenberg picture in quantum mechanics, where the dynamics are characterized by Heisenberg equation of motion. In the STA method, the Heisenberg equation of motion is solved utilizing the short-time approximation, while the AP method assumes a weak nonlinearity. Mathematically, we have seen that the solution of the AP method remains more general compared with the STA method. The STA method does not show oscillations in both quadrature variances of the propagating modes which contradict the oscillatory nature of light. Besides, the existence of squeezing in the second quadrature component cannot be detected using the STA method. Nevertheless, the STA method is highly agreeable with the AP method at a very early evolution. The STA method predicts the amplification of the squeezed signal when the linear coupling is strengthened as opposing to the AP method. Furthermore, The STA method also fails to detect the effect of frequency mismatch on the propagating modes.

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