On the Orientifolding of Type II NS–Fivebranes

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Abstract

Dualities between certain supersymmetric gauge field theories in three and four dimensions have been studied in considerable detail recently, by realizing them as geometric manipulations of configurations of extended objects in type II string theory. These extended objects include ‘D–branes’ and ‘NS–(five)branes’. In constructing the brane configurations which realize dualities for orthogonal and symplectic gauge groups, an ‘orientifold’ was introduced, which results in non–orientable string sectors. Certain features of orientifolded NS–branes —such as their existence— were assumed in the original construction, which have not been verified directly. However, those features fit very well together with the properties of the relevant field theories, and subsequently yielded the known dualities. This letter describes how orientifolded NS–branes can exist in type II string theory by displaying explicitly that the assumed combinations of world–sheet and space–time symmetries do indeed leave the NS–brane invariant and therefore can be gauged. The resulting orientifolded NS–brane can be described in terms of background fields, and furthermore as an exact conformal field theory, to exactly the same extent as the standard NS–brane.

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1. The NS–Brane: Background Fields

The low energy limit of type II string theory describes the following potentials as space–
time background fields:

\[ \text{R–R : } A^{(1)}, A^{(3)}, A^{(5)}, A^{(7)}, A^{(9)} \]  \hspace{1cm} (1)

for type IIA, and

\[ \text{R–R : } A^{(0)}, A^{(2)}, A^{(4)}, A^{(6)}, A^{(8)} \]  \hspace{1cm} (2)

for type IIB. Common to both are the potentials

\[ \text{NS–NS : } B^{(2)}, B^{(6)} \]  \hspace{1cm} (3)

where ‘R–R’ refers to the sector arising from the world–sheet left and right moving
fermionic excitations which are integer moded (Ramond) and ‘NS–NS’ denotes those aris-
ing from the half–integer modes (Neveu–Schwarz). The superscript denotes the rank of
the potential as an antisymmetric tensor field. The fields denoted above are not all inde-
pendent, as ten dimensional Hodge duality relates them by acting on their field strengths
\[ F^{(p+2)} \leftrightarrow F^{(8–p)} \].

There are natural dynamical sources for these fields which are \( p \)-dimensional extended
objects (‘branes’) which couple electrically to a field of rank \( p+1 \) via the \( p+1 \) dimensional
world–volume they sweep out in space–time as they propagate. In the R–R sector the
most basic\(^1\) electric sources are the D0–, D2–, D4–, D6– and D8–branes, respectively
for type IIA and the D(-1)–, D1–, D3–, D5– and D7–branes respectively for type IIB. By
the Hodge duality just mentioned, the D\( p \)–brane couples magnetically to \( A^{(7–p)} \) and is
thus the ‘dual’ of the D\((6–p)\)–brane in that sense.

Meanwhile, in the NS–NS sector the electric source for the \( B^{(2)} \) field (the standard Kalb–
Ramond field) is of course the fundamental type IIA (or IIB) string itself. The electric
source for \( B^{(6)} \), (and hence a magnetic source for its Hodge partner, \( B^{(2)} \)) is a five dimen-
sional extended object\(^2\) known under various names such as ‘fivebrane’, ‘NS–fivebrane’,
‘NS–brane’, ‘type IIA fivebrane’, \textit{etc.}, depending upon context\(^3\).

Of all of the objects described thus far, the fivebrane is the least well understood in
terms of a perturbative description. The D–branes enjoy a very good description of their
perturbative dynamics because their collective coordinates arise as standard open string
excitations. Meanwhile, the NS–branes are described in the closed string sector where we
can describe the collective coordinates of extended solitonic objects much less easily. The

\(^1\) We could conceivably also use the term ‘F1–brane’ and ‘F5–brane’ as shorthand for the funda-
mental string and its magnetic dual, the NS–fivebrane. Such notation has the distinction of being
of similar form to that used for D–branes, but unfortunately is not as completely unambiguous
without clumsily appending further an ‘A’ or a ‘B’ somewhere.
NS–brane was first introduced as a Yang–Mills Euclidean instanton, dressed up with stringy fields. A direct product with six flat space–time coordinates to make it a ten dimensional solitonic solution yielded the following form for the background fields in type II string theory:

$$ds^2 = -dt^2 + \sum_{\alpha=1}^{5} dx^\alpha dx_\alpha + e^{2\Phi} \sum_{\mu=6}^{9} dx^\mu dx_\mu;$$

$$e^{2\Phi} = e^{2\Phi_0} + \frac{Q}{(x-x_0)^2};$$

$$H_{\mu\nu\kappa} = -\epsilon_{\mu\nu\kappa} \partial_\lambda \Phi,$$

where early Greek letters label \((t,x^1,\ldots,x^5)\), the coordinates along the world–volume, and later ones label \((x^6,x^7,x^8,x^9)\), the coordinates transverse to the world–volume. Also, \(x^2 = \sum_{\mu} x^\mu x_\mu\). The field \(H^{(3)}\) is the rank three field strength of \(B^{(2)}\). The scalar \(\Phi\) is the dilaton field, which determines the string coupling: \(g_{II} = e^{\Phi}\). The constant \(\Phi_0\) sets the value of the coupling at infinity. The constant \(x_0\) is the location of the centre of the fivebrane.

The number \(Q\) is the \(H^{(3)}\) magnetic charge of the brane, measured by a flux integral over an asymptotic three sphere \(S^3\) which surrounds the fivebrane:

$$Q = -\frac{1}{2\pi^2} \int_{S^3} H^{(3)}.$$  \(\text{(5)}\)

The charge is quantized in integer multiples of \(\alpha'\), the inverse string tension, the minimal integer allowed being 1.

One of the problems with describing this object in these terms is the fact that at the centre of the solution, \(x=x_0\), the dilaton field blows up, signaling the presence of a region of strong string coupling \(g_{II}\) there. We expect that perturbation theory in \(g_{II}\) departs markedly from accuracy somewhere in the core.

2. The NS–Brane: Conformal Field Theory at the Core

Although the core represents a region we must treat with care, we can approach it with some caution and learn more about it. As \(x\sim x_0\) we can write the space–time geometry as:

$$ds^2 = -dt^2 + \sum_{\alpha=1}^{5} dx^\alpha dx_\alpha + \frac{Q}{r^2} \left( dr^2 + r^2 d\Omega_3^2 \right).$$  \(\text{(6)}\)

Here, we are using a radial coordinate \(r\), and \(d\Omega_3^2\) is the line element on the unit three–sphere \(S^3\). We can introduce a new radial coordinate \(\sigma=\log_e(r/\sqrt{Q})\) to give:

$$ds^2 = -dt^2 + \sum_{\alpha=1}^{5} dx^\alpha dx_\alpha + Q \left( d\sigma^2 + d\Omega_3^2 \right).$$  \(\text{(7)}\)
with
\[ H^{(3)} = -Q\epsilon_3, \quad \text{and} \quad \Phi = -\sigma, \tag{8} \]
where \( \epsilon_3 \) is the volume element on \( S^3 \).

So we see that after a change of variables, we can examine the theory of the core of the NS–brane somewhat more closely. Infinitely far down the throat of this geometry \( (\sigma \to -\infty) \) the string coupling, \( g_{11} \), diverges, and the nature of the physics is not clear. However, as we will recall below, the physics in the approach to that limit is described in terms of familiar conformal field theories, and we might hope to learn something about the theory from this description. We will therefore (as is traditional when discussing this limit of NS–branes) tentatively ignore our concerns about the strong coupling region until such time as we learn more about precisely which aspects of the physics we lose control of describing.

The theory of the non–trivial part of the core has the geometry associated to a product of two conformal field theories\[4,5\]. The \( \sigma \) coordinate, with an associated linear dilaton, is simply a Feigin–Fuchs\[6\] conformal field theory of a free field with a background charge. The \( S^3 \) (angular) part, with the specified \( H \)–field with quantized charge \( Q=ka' \), (for \( k \) integer), is precisely a conformal field theory which can be written as an \( SU(2) \) Wess–Zumino–Novikov–Witten\[7\] (WZNW) theory at level \( k \).

Alternatively, if we include the time coordinate \( t \) from the world–volume, rescale it to \( \tau=t/\sqrt{Q} \), the \( (\tau,x^6,x^7,x^8,x^9) \) geometry of the core is the \( \sigma \to -\infty \) limit of the following metric:
\[
ds^2 = \sum_{\alpha=1}^{5} dx^\alpha dx_\alpha + Q \left( d\sigma^2 - \tanh^2 \sigma \, d\tau^2 + d\Omega_3^2 \right), \tag{9}\]
with
\[ H = -Q\epsilon_3, \quad \Phi = \log e \cosh \sigma. \tag{10} \]

This is simply the throat which appears at the extremal limit of a five dimensional magnetically charged black hole. The horizon is at \( \sigma =0 \), which is not relevant to us, as this description of the core is only good for larger negative \( \sigma \):

This is the geometry of a product of exact conformal field theory descriptions\[8\]. The \( (t,\sigma) \) theory is an \( SL(2,\mathbb{R})/U(1) \) coset at level \( k \) (describing a two–dimensional black hole\[9\]) and the angular parts are again the \( SU(2) \) WZNW model at level \( k \). Of course, in the region where the core description is valid, \( \sigma \) large and negative, the two descriptions coincide: The NS–fivebrane and the five dimensional black hole are indistinguishable far down the throat.

In checking that the conformal field theories indeed give valid superstring theory backgrounds, we should recall\[10\] that the world–sheet fermions introduced for supersymmetry (which are essentially free fermions for WZNW models and their cosets\[11,12\]), have the effect of shifting the effective level of the WZNW models from \( k \) to \( k–2 \) for the \( SU(2) \)
theory and to $k+2$ for the $SL(2, \mathbb{R})$ theory. The conformal anomaly of the curved five dimensions of the solution is therefore shifted from the naive

$$c = \frac{3k}{k+2} + \frac{3k}{k-2} - 1 + \frac{5}{2}$$

(11)

to

$$c = 5 + \frac{5}{2},$$

(12)

which is what is should be for a total of $c=15$ for a complete solution, after tensoring with the trivial conformal field theory for the $(x^1, \ldots, x^5)$ free bosons and their superpartners.

Concentrating on the angular sector for now, let us prepare for a later discussion by writing the standard action for the level $k$ SU(2) WZNW model:

$$S = -\frac{k}{4\pi} \int_{\Sigma} d^2z \text{tr}[g^{-1} \partial_z g \cdot g^{-1} \partial_{\bar{z}} g] - \frac{ik}{12\pi} \int_B d^3y \epsilon^{ijk} \text{tr}[g^{-1} \partial_i g \cdot g^{-1} \partial_j g \cdot g^{-1} \partial_k g],$$

(13)

where $\Sigma$ is the two dimensional world–sheet of the string (with topology of the sphere) and $B$ is a three dimensional ball whose boundary is $\Sigma$. We are using complex coordinates $(z, \bar{z})$ on the world–sheet, $g \in SU(2)$, and the trace is in the Lie algebra of $SU(2)$, canonically normalized. The second term is the most natural way to write the possible $B^{(2)}$ couplings in the theory, essentially working in terms of the field strength $H^{(3)}$. One can always work in terms of the potential gaining a completely two dimensional action, but at the cost of possibly introducing ‘Dirac strings’, and losing some of the manifest symmetries and topological properties of the model.

3. Orientifolding

In ref.[13], a configuration of branes in type IIA string theory was presented which yielded a geometrical realization of the duality[14,13] of four–dimensional $\mathcal{N}=1$ supersymmetric $SO(N_c)$ and $USp(N_c)$ gauge theories with $N_f$ flavours of quarks. That configuration was an orientifold generalization of that of ref.[16], the latter yielding the duality for $U(N_c)$. After a rotation[17] and a T–duality transformation, these configurations should imply an orientifold generalization of the type IIB brane configurations in ref.[18]. Those were designed to study dualities[19] of $\mathcal{N}=4$ gauge theories in three dimensions.

The gauge sector of the four dimensional field theory is supplied by the world–volume fluctuations of D4–branes. These branes are suspended a finite distance between a pair of NS–branes, which are not parallel in two of their dimensions. As as result, the transverse fluctuations of the D4–branes are frozen out of the problem, and the usual hypermultiplets representing those fluctuations are not present. The quarks are supplied by the relative fluctuations of D6– and the D4–branes. The precise configurations of the branes are listed in the table in the next section.
It is a natural step to go from realizing gauge group $U(N)$ on oriented open string sectors to building gauge groups $SO(N)$ and $USp(N)$ instead, using non-orientable string sectors. The latter may be obtained by a projection which is known as an ‘orientifold’ procedure\[20\].

Orientifold technology has been considerably refined in recent times, because using it in combination with D–brane technology, many new types of consistent string theory backgrounds may be constructed, many of which have been indispensible in the study of string duality\[21,22,23,24\].

In spirit, an orientifold is much like an orbifold. In general, it is the gauging of a combination of a discrete spacetime symmetry with world–sheet parity, $\Omega$. Together, these discrete transformations should of course form a group which we denote as, $G_\Omega$. $G_\Omega$ should obviously be a global symmetry of the starting model, otherwise it would not make sense to gauge it. Gauging $G_\Omega$ simply means to consistently project out all states which are not invariant under the symmetry.

Orientifolding has been largely carried out in the context of either toroidal compactifications, or orbifolds thereof, and so not much is known about orientifolding closed superstring backgrounds with a non–trivial distribution of curvature. In trying to extend the work of ref.\[16\] to incorporate $SO(N)$ or $USp(N)$ gauge groups in the open string sector, the authors of ref.\[13\] were forced to consider the action of the orientifold group on the NS–brane.

An immediate concern about orientifolding type II theory is the question of the very existence of a fivebrane in the resulting theory. In the most simple orientifold model, type I string theory, there is no NS–brane. This is because there is no $B^{(6)}$ potential for it to couple to. This is easy to see if one considers the basic $\sigma$–model coupling of fundamental type II strings to $B^{(2)}$:

$$\int d^2z \, B_{\mu\nu} \left( \partial_\mu x^\alpha \partial_\nu x^\alpha - \partial_\mu \bar{x}^\alpha \partial_\nu \bar{x}^\alpha \right).$$

(14)

The action of world–sheet parity is $\Omega : z \leftrightarrow \bar{z}$, and therefore $B^{(2)}$ must go to $-B^{(2)}$ under $\Omega$ in order for $\Omega$ to be a symmetry. In type IIB string theory, where the left and right supersymmetries are of the same chirality, the group $G_\Omega = \{1, \Omega\}$ is a symmetry of the theory and may be gauged. In the process, $B^{(2)}$ and its Hodge partner $B^{(6)}$ are projected out of the theory, as they are odd. There are no NS–branes in the resulting type I theory.

The analogous minimal case for type IIA string theory is to consider the combination of $\Omega$ with a reflection in one of the space–time coordinates (say $x^9$). The reflection, $R_9 : x^9 \rightarrow -x^9$, mirrors the two space–time supersymmetries into each other, and $G_\Omega = \{1, \Omega R_9\}$ is a global symmetry which can therefore be gauged, resulting in the type I' theory\[5\], which is of course $T_9$–dual to the type I theory.

The type I' theory has 16 D8–branes, giving $SO(32)$ gauge symmetry, and an ‘O8–plane’,

\[2\] Which should perhaps have been called the ‘type IA’ theory, and its $T_9$–dual cousin the ‘type IB’ theory.
located at a point in the \( x^9 \) direction, about which the type IIA theory is reflected. The D8–branes are \( T_9 \)–dual to the D9–branes defining type I theory, and the O8–plane is the space–time manifestation of combining T–duality with orientifolding: \( \Omega \leftrightarrow \Omega R_9 \); It is the fixed point set of the \( R_9 \) reflection.

By using T–duality on such flat backgrounds as above, we can deduce therefore that the world–sheet fermions (and the corresponding space–time supersymmetry they generate after the GSO projection) constrain us to consider consistent gauging of only \( \Omega \) times an even number of space–time reflections for type IIB; and times only an odd number of space–time reflections for type IIA. This gives \( \Omega n \)–planes in each theory with \( n \) odd for type IIB and even for type IIA. By extension, we can expect that this restriction will be applicable to any non–trivial backgrounds which are asymptotically flat, which includes the NS–branes under consideration.

4. Orientifolding NS–Branes

To proceed, we must learn from the example of the type IIA orientifold and find an orientifold group that is complicated enough to produce symmetries of the action which allow the couplings to the NS–NS potential to survive at least in some sectors of the theory, allowing the possibility for an NS–brane to be defined.

In ref.\[13\], it was observed that the only type of orientifolds of the configuration of ref.\[16\] which would preserve the existing \( D=4, \mathcal{N}=1 \) supersymmetry were of the type \( \{ 1, \Omega R_{45789} \} \) and \( \{ 1, \Omega R_{456} \} \). (Here \( R_{\mu \nu} \equiv R_\mu R_\nu, \text{ etc.} \) This introduces O4–planes and O6–planes respectively.

A table showing the configuration of branes and possible orientifolds is given below:

| type | \( t \) | \( x^1 \) | \( x^2 \) | \( x^3 \) | \( x^4 \) | \( x^5 \) | \( x^6 \) | \( x^7 \) | \( x^8 \) | \( x^9 \) |
|------|------|------|------|------|------|------|------|------|------|------|
| NS   | —    | —    | —    | —    | —    | —    | —    | —    | —    | —    |
| NS'  | —    | —    | —    | —    | —    | —    | —    | —    | —    | —    |
| O4   | —    | —    | —    | —    | —    | —    | —    | —    | —    | —    |
| O6   | —    | —    | —    | —    | —    | —    | —    | —    | —    | —    |
| D4   | —    | —    | —    | —    | —    | —    | —    | —    | —    | —    |
| D6   | —    | —    | —    | —    | —    | —    | —    | —    | —    | —    |

In the table, a dash ‘—’ represents a direction along an extended object’s world–volume
while a dot ‘•’ is transverse. For the special case of the D4–branes’ $x^6$ direction, where a world–volume is a finite interval, we use the symbol ‘[—]’. (A ‘•’ and a ‘—’ in the same column indicates that one object is living inside the world–volume of the other in that direction, and so they can’t avoid one another. Two ‘•’s in the same column tell us that the objects are point–like, and need not coincide in that direction, except for the specific case where they share identical values of that coordinate.)

Let us consider the first NS–brane in the table, point–like in the $(x^6, x^7, x^8, x^9)$ directions, like our NS–brane in the previous sections. All of our comments will apply equally well to the second, differently oriented one, which is denoted NS$^\prime$ in the table.

In the case where we have an O4–plane present, the spacetime reflection is $R_{45789}$. The action of reflections in the $(x^4, x^5)$–plane, where the NS–brane has no structure, will produce no physics of interest for us, so it will suffice to study $R_{789}$. Notice that the reflections are only in three of the four directions in which the brane has structure.

In the case where we have an O6–plane present, the non–trivial reflection acting on the NS–brane is $R_6$. Now, this is a reflection on just one of the four directions in which the brane has structure.

In order to proceed, it is prudent to change coordinates from the Cartesian ones we have been using for transverse directions to radial and angular coordinates $(r, \phi, \psi, \theta)$. For this, we can use Euler coordinates:

$$
\begin{align*}
    x^6 &= r \cos \left( \frac{\phi + \psi}{2} \right) \cos \frac{\theta}{2} \\
    x^7 &= r \sin \left( \frac{\phi + \psi}{2} \right) \cos \frac{\theta}{2} \\
    x^8 &= r \cos \left( \frac{\phi - \psi}{2} \right) \sin \frac{\theta}{2} \\
    x^9 &= r \sin \left( \frac{\phi - \psi}{2} \right) \sin \frac{\theta}{2},
\end{align*}
$$

where $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$ and $0 \leq \psi < 4\pi$.

In these coordinates, the metric on the unit three–sphere is:

$$
    d\Omega^2_3 = d\theta^2 + d\psi^2 + d\phi^2 + 2 \cos \theta d\phi d\psi,
$$

while the volume element is:

$$
    \epsilon_3 = \frac{1}{4\pi} \sin \theta \, d\theta d\phi d\psi.
$$

Therefore, in the core limit, the $\sigma$–model coupling of $B^{(2)}$ must be:

$$
    B_{\phi\psi} \left( \partial_z \phi \partial_{\bar{z}} \psi - \partial_z \phi \partial_{\bar{z}} \psi \right),
$$

7
where
\[
B_{\phi\psi} = \frac{Q}{4\pi}(\pm 1 - \cos \theta),
\]  
(19)

where the ± choice refers to the North and South poles, \( \theta = 0 \) and \( \pi \), respectively.

Now let us study the action of \( R_{789} \) and \( R_6 \) in these coordinates. Upon examination of the coordinate transformations (13) above, we see that:

\[
R_{789} : \begin{align*}
\psi &\rightarrow -\pi - \phi \\
\phi &\rightarrow \pi - \psi
\end{align*}
\]

\[
R_6 : \begin{align*}
\psi &\rightarrow \pi - \phi \\
\phi &\rightarrow \pi - \psi
\end{align*}
\]

leaving all other coordinates changed in each case.

Looking at the coupling of the NS–brane’s \( B^{(2)} \)–field, we see that in each case, combining the reflection with parity \( \Omega : z \leftrightarrow \bar{z} \) leaves the combination (\( \partial_z \phi \partial_{\bar{z}} \psi - \partial_{\bar{z}} \phi \partial_z \psi \)) invariant. Therefore, we need to have \( B^{(2)} \) be even under \( \Omega R_{789} \) or \( \Omega R_6 \) which it is, because of its antisymmetry under exchange of indices and by virtue of it being odd under \( \Omega \).

Notice that we could have also considered \( \Omega R_{6789} \) and \( \Omega R_{89} \), thus acting only on an even number of directions transverse to the fivebrane. In these cases we have:

\[
R_{89} : \begin{align*}
\psi &\rightarrow \psi - \pi \\
\phi &\rightarrow \phi + \pi
\end{align*}
\]

\[
R_{6789} : \begin{align*}
\psi &\rightarrow \psi - 2\pi \\
\phi &\rightarrow \phi
\end{align*}
\]

again leaving all other coordinates changed in each case. So we see that \( R_{89} \) and \( R_{6789} \) are manifestly symmetries before acting with \( \Omega \) and therefore will result in \( B^{(2)} \) being odd under \( \Omega R_{89} \) and \( \Omega R_{6789} \), and hence it will be projected out in constructing the gauged theory.

Notice that at this level it seems to be consistent to have an orientifold with \( \text{both O}6 \)– and \( \text{O}4 \)–planes present, in the way they are aligned in the table. (Such configurations were considered recently in ref.\[25\], following ref.\[13\].) The candidate orientifold group \( G_\Omega \) would have elements \( \Omega R_{45789} \) and \( \Omega R_{456} \). By closure of the group algebra (and necessarily the operator product expansion) this leads to the presence of the pure orbifold symmetry \( R_{6789} \), which is just the overall reflection symmetry \( \psi \rightarrow \psi - 2\pi \) of the model.

Although we have discussed the detailed form of \( B^{(2)} \) only in the core limit, our conclusions about the global symmetries extend to the complete NS–brane, as the asymptotic region is smoothly connected to the throat region of the core, developing a less trivial dependence on the radial coordinate in the fields (see (4)) as this happens. At any radius, the \( SO(4) \) rotational symmetry guarantees that we can always do the decomposition and analysis of
this section, and so we conclude that the couplings of the complete NS–brane admit the operations $\Omega R_6$ and $\Omega R_{789}$ as global symmetries.

5. The Orientifolded NS–Brane: The Theory at the Core

In the WZNW model, the interesting reflection symmetries have a very natural interpretation in terms of the group element $g$. In Euler coordinates, it is written:

$$g = e^{i\phi \sigma_3/2} e^{i\theta \sigma_2/2} e^{i\psi \sigma_3/2} = \begin{pmatrix}
e^{i\frac{\phi+\psi}{2}} \cos \frac{\theta}{2} & e^{i\frac{\phi-\psi}{2}} \sin \frac{\theta}{2} \\
e^{-i\frac{\phi+\psi}{2}} \sin \frac{\theta}{2} & e^{-i\frac{\phi-\psi}{2}} \cos \frac{\theta}{2}
\end{pmatrix}$$

(22)

(where the $\sigma_i$ are the Pauli matrices), from which we see:

$$R_6: g \rightarrow -g^{-1}$$

$$R_{789}: g \rightarrow g^{-1}.$$  

(23)

Upon combining these with $\Omega: z \leftrightarrow \bar{z}$, these are manifestly symmetries of the WZNW action ([13]).

It would be very interesting to carry out the complete study of this gauged model, defining a new theory with non–orientable sectors. Notice, for example that the conserved left– and right–moving Lie algebra valued currents,

$$J(z) = kg^{-1} \partial_z g \quad \text{and} \quad \bar{J}(\bar{z}) = k \partial_{\bar{z}} \bar{g}g^{-1},$$

(24)

in terms of which much of the physics of the conformal field theory is determined, are mapped into each other under the orientifold groups.\(^3\)

6. Closing Remarks

In closing, we have demonstrated that the assumption made in ref.[13] was correct. It is indeed possible to define a consistent orientifold projection on the theory of an NS–brane by combining world–sheet parity, $\Omega$, with certain spacetime reflections: We can introduce orientifold planes where one or three of the four transverse NS–brane directions are also transverse to the orientifold plane. As there is no non–trivial structure in the directions along the NS–brane’s world–volume, it is clear by T–duality that we can have

\(^3\) It should be noted here that an early example of a study of a non–trivial orientifold was carried out in ref.[26], in the context of gauging spacetime symmetries of the two dimensional geometry of the (bosonic) stringy black hole background defined by the $SL(2, \mathbb{R})/U(1)$ coset.
orientifold planes with $n$ extended directions from $n = 1, \ldots, 8$ intersecting the NS–brane consistently, providing that only one or three transverse coordinates are acted upon. Taking into account our earlier restriction (due to spacetime supersymmetry) on the number of reflections allowable, $n$ has to be even in type IIA and odd in type IIB, just like the brane dimensions.

This demonstration serves to identify the global orientifold symmetries, denoted $G_{\Omega}$, which should be gauged to find the projected theory. It would be an interesting task to study the full projected conformal field theory. This is a very tractable problem, for the WZNW model organizes the conformal field theory very well. It will be particularly interesting to study such orientifolded NS–branes in a compact situation where associated fluxes cannot leak off to infinity. There, additional consistency checks will arise from computing the Klein bottle diagram which now appears in the theory at one loop. Any tadpole divergences associated with this diagram might conceivably be canceled with cylinder and Möbius–strip diagrams arising from introducing D–branes to obtain consistent backgrounds. In this way, a whole new class of interesting non–trivial orientifold + D–brane + NS–brane backgrounds can be constructed which will certainly be interesting for studies of the properties of both string and field theory.

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