Relaxation and dephasing in a flux qubit

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We report detailed measurements of the relaxation and dephasing time in a flux-qubit measured by a switching DC SQUID. We studied their dependence on the two important circuit bias parameters: the externally applied magnetic flux and the bias current through the SQUID in two samples. We demonstrate two complementary strategies to protect the qubit from these decoherence sources. One consists in biasing the qubit so that its resonance frequency is stationary with respect to the control parameters (optimal point); the second consists in decoupling the qubit from current noise by choosing a proper bias current through the SQUID. At the decoupled optimal point, we measured long spin-echo decay times of up to 4\,\mu s.

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A long-standing problem for the use of superconducting circuits as quantum bits (qubits) in a quantum computer is their relatively short dephasing time compared to the requirements of many-qubit quantum computation. Dephasing is due to the coupling of the qubit’s degrees of freedom with the many fluctuating uncontrollable ones commonly denoted as the environment. From the perspective of quantum information, it is crucial to quantitatively identify the various dephasing sources and to find strategies to overcome these, either by reducing the amount of fluctuations or by protecting the qubit against it. An important step in this direction has been accomplished in\textsuperscript{1}. The authors showed that dephasing can be significantly reduced by biasing the qubit at an optimal point where its resonance frequency is stationary with respect to its control parameters - in that case, gate voltage and magnetic flux.

In this letter we report detailed measurements of the relaxation and dephasing times in a flux-qubit as a function of its bias parameters for two different samples. Our measurements allow us to identify certain dephasing mechanisms and quantify their effect on the qubit. We find that energy relaxation is dominated by spontaneous emission towards the measuring circuit impedance. Dephasing is mainly caused by noise in the external magnetic flux biasing the qubit, thermal fluctuations of our measuring circuit, and low-frequency noise originating from microscopic degrees of freedom, probably causing critical current noise in the qubit junctions. We moreover demonstrate strategies to efficiently fight each of these noise sources.

Our flux-qubit consists of a micron-size superconducting loop intersected with three Josephson junctions. When the total phase across the three junctions is close to \(\pi\), the loop has two low-energy eigenstates (ground state \(|0\rangle\) and excited state \(|1\rangle\)) well separated from the higher-energy ones, which can thus be used as a qubit\textsuperscript{2,3}. The flux-qubit is characterized by two parameters: the minimum energy separation \(\Delta\) between \(|0\rangle\) and \(|1\rangle\), and the persistent current \(I_p\). Around \(\gamma Q = \pi\), the energy separation between these two levels depends on \(\gamma Q\) and can be written as \(E_1 - E_0 \approx hf_Q = h/\Delta^2 + e^2/\lambda,\) where \(e \equiv (I_p/e)(\gamma Q - \pi)/(2\pi).\) The qubit is inductively coupled to a SQUID detector (with a coupling constant \(M\)), which is biased at a current \(I_b\). The phase drop \(\gamma Q\) has two origins: the magnetic flux threading the qubit loop \(\Phi_x\), and the currents in the SQUID loop which depend on \(I_b\). Thus, we can write \(e = \pi(\Phi_x) + \lambda(I_b)\).

The coupling of \(e\) to fluctuating sources leads to decoherence. Noise in the magnetic flux \(\Phi_x\) or in the bias current \(I_b\) induces fluctuations of the qubit frequency \(f_Q\) and thus dephasing. A first strategy to protect the qubit from decoherence consists in biasing it at \(e = 0\) so that \(df_Q/de = 0\). This is the optimal point strategy, which was first invented and demonstrated in\textsuperscript{1}. An additional possibility is to decouple the external noise from the variable \(e\), by canceling the sensitivity coefficients \(dn/de\) and \(d\lambda/dI_b\). The flux noise can not be decoupled since \(dn/de = 1\) for constant \(M\). As we will show below, the bias current noise can be decoupled by biasing the SQUID at a current \(I_b^*\) such that \(d\lambda/dI_b(I_b^*) = 0\), which is the decoupling condition. At the decoupled optimal point, \((e = 0\) and \(I_b = I_b^*)\) we expect that the qubit coherence is best preserved, since the qubit is sensitive to noise to second order, and to bias current noise to fourth order. We also note that a strong dependence of the dephasing time on the bias current would be clear experimental evidence that current noise, and not flux noise, is the factor limiting the quantum coherence.

In the two samples, shown in figures\textsuperscript{\(\text{H}\)} and \textsuperscript{\(\text{I}\)}, the qubit loop is merged with its measuring SQUID. The dependence of \(e\) on the bias current \(I_b\) arises from the way this bias current redistributes in the SQUID and eventually generates a phase shift across the qubit junctions via the superconducting line shared by the qubit and the SQUID. The detailed configuration of the shared line is related to the specific fabrication process. We use 2-angle shadow evaporation so that the lines consist effectively of 2 layers. This induces a large asymmetry in the coupling if the qubit loop contains an odd number of junc-
The qubit is strongly coupled \[ \Phi \] on both the external flux $\Phi$. Studying the dependence of the qubit Larmor frequency $\omega_L$ through the SQUID during the application of the microwave pulse. The complete pulse sequence is depicted in figure 2a. In figure 2b, the measured qubit resonance frequency for sample A is shown as a function of the external flux $\Phi_x$, for three different values of $I_{bpl}$. We observe that for each value of the bias current, a specific value of external flux $\Phi_x^{(0)}(I_{bpl})$ realizes the optimal point condition.

We fitted all the curves with the formula $f_Q = \sqrt{\Delta^2 + |\lambda(I_b)| + 2I_p(\Phi_x - \Phi_0)/h^2}$ for different values of $I_b$. The obtained curves $\lambda(I_b)$ are shown in figure 2c and 2d for both samples. The decoupling occurs at $I_{bpl}^* = 2.9 \pm 0.1 \mu A$ for sample A and $I_{bpl}^* = 180 \pm 20 \mu A$ for sample B. Note that although the SQUID critical current is similar in both samples, the decoupling current is much closer to 0 in sample B due to the presence of the fourth junction which restores the symmetry of the coupling $\Phi_x$. We biased our qubit at the decoupled optimal point by setting $I_b = I_{bpl}^*$ and $\Phi_x = \Phi_x^{(0)}(I_{bpl}^*)$. The qubit line shape under these conditions is shown in figure 3a for sample A and 3c for sample B. For sample A, we could fit it with a Lorentzian of width $\Delta = 3.1 \pm 0.5 MHz$ (FWHM). This width yields a dephasing time $T_2 = 1/\pi \Delta \approx 100 ns$ consistent with the Ramsey fringe measurements as discussed below. For sample B, the line was split, due to the action of a strongly coupled two-level fluctuator. We fitted it by the sum of two Lorentzians of widths 7 and 6MHz. We note that in addition to the fluctuator responsible for the splitting of the line, the value of the qubit frequency at the optimal point $\Delta$ exhibited occasional jumps of around 100MHz. Also the width of the line changed significantly in time. This indicates that dephasing was probably dominated by some low-frequency noise due to one or more strongly coupled microscopic fluctuators, likely generating critical current noise. We
stress that we had no evidence for such instabilities with sample A.

We first studied the dependence of the dephasing time as a function of $\epsilon$ while keeping $I_b = I_b^*$. For sample A, we measured Ramsey fringes \[\text{FIG. 3: (a) Ramsey fringe signal measured with sample A. From top to bottom:} - \text{temporal sequence of microwave pulses corresponding to a Ramsey experiment. - Ramsey signal at the optimal decoupled point. - Qubit resonant frequency as a function of } \epsilon. - \text{Dephasing time } T_2 \text{ as a function of } \epsilon \text{ around the optimal point (full squares, the lines are a guide to the eye), and fit to the data (dotted curve) assuming that dephasing was caused by } 1/f \text{ flux noise } S_{\Phi_x} = 3 \cdot 10^{-12}/f [\Phi_0^2/Hz]. (b) Spin-echo signal measured with sample B. From top to bottom: Temporal sequence of microwave pulses corresponding to a spin-echo experiment. - Spin-echo signal at the optimal decoupled point. - Qubit resonant frequency as a function of } \epsilon. - \text{(full squares): Spin-echo time } T_{echo} \text{ as a function of } \epsilon \text{ (the lines are a guide to the eye). (dashed curve): calculated dephasing from thermal fluctuations of the photon number in the SQUID plasma mode.}
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the qubit resonance frequency and thus dephasing. It has been shown \[\text{that treating the thermal fluctuations of the plasma mode as a weak classical perturbation leads to a strong underestimate because of the neglect of quantum correlations between the qubit and the oscillator. Instead, we numerically integrated the master equation for the joint density matrix of the “qubit-plasma mode” system \[\text{FIG. 3}: \text{FIG. 3: (a) Ramsey fringe signal measured with sample A. From top to bottom:} - \text{temporal sequence of microwave pulses corresponding to a Ramsey experiment. - Ramsey signal at the optimal decoupled point. - Qubit resonant frequency as a function of } \epsilon. - \text{Dephasing time } T_2 \text{ as a function of } \epsilon \text{ around the optimal point (full squares, the lines are a guide to the eye), and fit to the data (dotted curve) assuming that dephasing was caused by } 1/f \text{ flux noise } S_{\Phi_x} = 3 \cdot 10^{-12}/f [\Phi_0^2/Hz]. (b) Spin-echo signal measured with sample B. From top to bottom: Temporal sequence of microwave pulses corresponding to a spin-echo experiment. - Spin-echo signal at the optimal decoupled point. - Qubit resonant frequency as a function of } \epsilon. - \text{(full squares): Spin-echo time } T_{echo} \text{ as a function of } \epsilon \text{ (the lines are a guide to the eye). (dashed curve): calculated dephasing from thermal fluctuations of the photon number in the SQUID plasma mode.}
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relaxation occurs by spontaneous emission towards the circuit impedance seen by the qubit. A weaker dependance is observed for sample A, which could indicate that additional environmental modes at the qubit frequency are involved. The dephasing time $T_2$ measured in sample A is strongly dependent on $I_b$. This indicates that dephasing at the optimal point is limited by noise in the bias current. For both samples, the dephasing time measured at the optimal decoupled point is similar to or larger (sample A) than the relaxation time, so that dephasing was partly limited by relaxation. To quantify the pure dephasing contribution, we calculated $T_{\phi}$, defined as $T_{\phi}^{-1} = (T_2^{-1} - (2T_1)^{-1}$ for sample A (full square curve in figure 4b), and calculated similarly $T_{\phi}^{\text{echo}}$ for sample B (full square curve in figure 4b). Our calculations taking into account the thermal fluctuations in the plasma mode are shown as the dashed curve in figures 4b and d. They are in qualitative agreement with the data, although systematically overestimating the dephasing time by a factor typically 5 compared to the measurements.

In conclusion, we presented detailed measurements of the relaxation and dephasing times as a function of bias parameters for two flux-qubit samples. We showed that the optimal point concept already demonstrated for the quantunm circuit [8] is also valid for the flux-qubit design. Making use of the SQUID geometry of our detector, we could moreover decouple the qubit from current fluctuations by biasing the SQUID at a specific current $I_b^*$. We showed that adding a fourth junction to the qubit loop enhances the symmetry of the coupling, thus lowering the value of $I_b^*$. We showed that low-frequency noise limits the dephasing time, but that spin-echo techniques provide a powerful tool to fight it. We observed remarkably long decay times of the echo signal of 4.1 µs, limited by relaxation. We provided quantitative evidence that at the optimal point dephasing is induced by the thermal fluctuations of the photon number in the plasma mode of our SQUID detector. These results indicate that long coherence times can be achieved with flux qubits.

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