Local transformation of two EPR photon pairs into a three-photon W state

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We propose and experimentally demonstrate a transformation of two EPR photon pairs distributed among three parties into a three-photon W state using local operations and classical communication. We then characterize the final state using quantum state tomography on the three-photon state and on its marginal bipartite states. The fidelity of the final state to the ideal W state is 0.778 ± 0.043 and the expectation value for its witness operator is −0.111 ± 0.043 implying the success of the proposed local transformation.

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Recent development in quantum information science has revealed most of the mysteries concerning entanglement between two qubits: We know how to prepare, characterize and quantify it. In particular, using local operations and classical communication (LOCC), any state of two qubits can be generated from a single resource of qubits in a maximally entangled state, which is called an Einstein-Podolsky-Rosen (EPR) pair. For photonic qubits, the state \(|\text{EPR}\rangle \equiv (|\text{HH}\rangle + |\text{VV}\rangle)/\sqrt{2}\) serves as an EPR pair, where \(|\text{H}\rangle\) and \(|\text{V}\rangle\) represents the horizontal and the vertical polarization, respectively. By contrast, entanglement among three or more qubits still remains as a challenge because such systems have a richer and more complex structure which originates from the existence of different ways the qubits can be entangled with each other. One of such interesting features is the presence of inequivalent classes of entangled states: The states in distinct classes cannot be interconverted using LOCC [1], even probabilistically. The simplest example is the three-qubit entanglement, where there are two distinct classes of states: the Greenberger-Horne-Zeilinger (GHZ) type of states including a standard one \(|\text{GHZ}_3\rangle = (|\text{HHH}\rangle + |\text{VVV}\rangle)/\sqrt{2}\), and the W-type states including \(|\text{W}_3\rangle = (|\text{HHV}\rangle + |\text{HVH}\rangle + |\text{VHH}\rangle)/\sqrt{3}\). A key distinction between these states is that while the loss of any one of the qubits completely destroys the entanglement in the GHZ state \(|\text{GHZ}_3\rangle\), the entanglement between the remaining two qubits survives in the W state \(|\text{W}_3\rangle\) [1]. Recently, there have been a number of theoretical proposals and experimental demonstrations for the preparation of three-qubit GHZ and W states in optical experiments [2,3,4,5,6,7,8,9,10,11].

Existence of the distinct classes implies that there is no three-qubit state that can be used as a universal resource for generating arbitrary three-qubit pure states under LOCC. For this purpose, one must look for a resource in larger systems. One of the simplest way is to distribute the resource for bipartite entanglement between one party (Charlie) and each of the other parties (Alice and Bob), resulting in state \(|\text{EPR}_\text{AC}|\text{EPR}_\text{BC}\rangle\). Starting with this resource, it is at least theoretically easy to show that Charlie can prepare three local auxiliary qubits in the desired three-qubit state, which may be an entangled state, and then faithfully send one qubit to Alice and another to Bob by quantum teleportation [12]. Since this scenario involves seven qubits in total, it is hard to carry out experiments and a more direct way of converting EPR pairs to three-qubit states is desired. For the GHZ-type states, it is easy to do this since we can convert the two EPR pairs to \(|\text{GHZ}_3\rangle\) by quantum parity check [2,12], which can be done by a polarization beam splitter and post-selection [5]. Any GHZ-type state is then produced with nonzero probability by applying a unitary operation and local filtering on each photon, which can be done with high precision. This line of strategy was further extended for the W-type states by Walther et al. [3], who experimentally demonstrated that \(|\text{W}_3\rangle\) can be approximately generated from \(|\text{GHZ}_3\rangle\) by LOCC. In this method, there is a trade-off between the success probability and the fidelity of the final state such that the fidelity approaches unity only in the limit of zero success probability, which reflects the fact that \(|\text{GHZ}_3\rangle\) and \(|\text{W}_3\rangle\) belong to distinct classes of states.

The aim of this paper is to propose and experimentally demonstrate the missing path of resource conversion, namely, direct transformation of bipartite resource \(|\text{EPR}_\text{AC}|\text{EPR}_\text{BC}\rangle\) into the W state \(|\text{W}_3\rangle\). Our scheme simply uses a polarization-dependent beam splitter (PDBS) and a photon detection to realize a desired transformation of Charlie’s two photons into one photon. We will first discuss the working principle and then describe our experimental results.

Let us assume that four photons in state \(|\text{EPR}_{12}|\text{EPR}_{34}\rangle = (|\text{HHHH}\rangle_{1234} + |\text{HHVV}\rangle_{1234} + |\text{VHHV}\rangle_{1234} + |\text{VVVV}\rangle_{1234})/2\) are distributed such that Alice has the photon in mode 1, Bob has mode 4, and Charlie has modes 2 and 3. Charlie sends his two photons to a PDBS, whose output modes are labelled as 5 and 6. Let \(\mu > 0\) be the transmission coefficient of the PDBS for the H polarization, and \(\nu > 0\) for the V polarization. The PDBS transforms \(|\text{HHVV}\rangle_{1234}\) into \(|\text{VHHV}\rangle_{1234}\) and \(|\text{HHVV}\rangle_{1234}\) into \(|\text{HHVV}\rangle_{1234}\) with probability \(\mu^2\) and \(\nu^2\), respectively. When Charlie detects a photon from mode 5, it implies that the remaining photon is in mode 6. Then we multiply the two success probability, which reflects the fact that \(|\text{GHZ}_3\rangle\) and \(|\text{W}_3\rangle\) belong to distinct classes of states.
p_H \equiv 3 \min\{|a|^2, |b|^2, |c|^2\}. Similarly, they can also generate \(|W_3\rangle\) for the case where Charlie has detected a V-polarized photon in mode 5, with an overall success probability \(p_V \equiv 3 \min\{|d|^2, |b|^2, |c|^2\}. The success probability \(p_H\) becomes largest when \(|a| = |b| = |c|\) holds, which happens when the parameters \((\mu, \nu)\) for the PDBS are chosen to be \(\mu = (5 + \sqrt{5})/10\) and \(\nu = (5 - \sqrt{5})/10\) or vice versa. For this choice, \(|a| = |b| = |c| = |d| = 1/(2\sqrt{5})\) holds, and hence both probabilities take their optimal values \(p_H = p_V = 3/20 = 15\%\) without introducing local attenuations \([14]\).

In our experiment, we recorded only the case when Charlie has detected an H-polarized photon in mode 5. We also made a sub-optimal choice of the PDBS parameters, \(\mu = (7 + \sqrt{17})/16\) and \(\nu = 1/2\). One of the reasons for this choice is that the two-photon interference for the V polarization is observed directly, which makes the alignment easier and gives us a clue about how well the two photons from different pairs are overlapped at the PDBS. Under this choice, we have \(|a| > |b| = |c|\) and we need to introduce a polarization-dependent loss for Alice’s photon in mode 1. The success probability for the ideal case is calculated to be \(p_H = 3(9 - \sqrt{17})/128 \sim 11.4\%\).

The details of our experimental setup are shown in Fig. 1. Charlie’s local operations are performed as follows. Modes 2 and 3 are overlapped at the PDBS, and polarizing beamsplitter (PBS) placed at the output mode 5 selects only the H-polarized photons. A half-wave plate (HWPc) at mode 6 interchanges H and V polarizations. On Alice’s side, a set of glass plates (GP) are placed in mode 1, which can be tilted to adjust the amount of the polarization dependent loss. The two plates are tilted in opposite directions such that the beam passing through experiences a minimal transverse shift. Successful events are signalled by four-photon coincidences using photon detectors in modes 1, 4, 5 and 6. The quarter-

\[
\begin{align*}
|c| \langle HHV \rangle_5 + b|HV V \rangle_5 + a|V H V \rangle_5 |H\rangle_5 \\
+ |d| V V V \rangle_5 + a|V H V \rangle_5 + b|V H V \rangle_5 |V\rangle_5,
\end{align*}
\]

with

\[
\begin{align*}
a &= \sqrt{\mu \nu}/2, \\
b &= -\sqrt{(1-\mu)(1-\nu)}/2, \\
c &= (2\mu - 1)/2, \\
d &= (2\nu - 1)/2.
\end{align*}
\]
wave plates (QWP), HWP and PBSs in front of the detectors in modes 1, 4 and 6 are used for verification experiments. In order to demonstrate the effectiveness of the process, we performed experiments to determine that (i) the photons from the two EPR pairs overlapped well at the PDBS, (ii) two highly entangled photon pairs $\rho_{12}$ and $\rho_{34}$ were generated by SPDC, and (iii) the final three-photon state $\rho_{146}$ was close to the W state $|W_3\rangle$.

In order to demonstrate (i), we set the UV pulses to H-polarization so that in each pass of the UV pulses through the BBOs, V-polarized photon pairs were generated. The HWPs inserted in front of the detectors were adjusted so that only V-photons arrive at the detectors. One photon from each pair was then sent to the PDBS and four-fold coincidences were recorded while the optical delay experienced by the photons in modes 2 and 3 were changed using the motorized stage M. When the temporal overlap of these two V-photons at the PDBS was achieved, Hong-Ou-Mandel dip was observed as in Fig. 2. The observed visibility is 0.885 at zero delay time.

For (ii), after setting the zero-delay time, we adjusted the UV pulses to diagonal polarization so that EPR pairs $\rho_{12}$ and $\rho_{34}$ were generated. Each pair was characterized by quantum state tomography (QST) using 16 different tomographic settings chosen from the combinations of the single photon projections, $|H\rangle$, $|V\rangle$, $|D\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$, $|R\rangle = (|H\rangle - i|V\rangle)/\sqrt{2}$ and $|L\rangle = (|H\rangle + i|V\rangle)/\sqrt{2}$, on each photon [17]. Since the PDBS has different transmission coefficients for H and V polarizations, the measurement bases for the photons in modes 2 and 3 were selected by the HWPs and QWPs inserted before the PDBS. The QWP and HWPs in mode 6 and the set of glass plates in mode 1 were adjusted so that they did not affect the polarization of the incoming photons. Coincidences were recorded in modes 1 and 6 for $\rho_{12}$, and in modes 4 and 6 for $\rho_{34}$. From these measured polarization correlations, we estimated the fidelity $F_{ij} \equiv \langle \text{EPR}|\rho_{ij}|\text{EPR}\rangle$ of each pair to the ideal EPR pair as $F_{12} = 0.967 \pm 0.002$ and $F_{34} = 0.976 \pm 0.002$. Here and henceforth, uncertainties in the fidelities and the other quantities were calculated using a Monte Carlo routine assuming Poissonian statistics of errors. We further reconstructed their density matrices $\rho_{12}$ and $\rho_{34}$, and calculated the amounts of entanglement using entanglement of formation (EOF) [18] as 0.922 ± 0.006 and 0.947 ± 0.004. The density matrices estimated using the maximum likelihood method are shown in Fig. 3.

In the last phase of the experiment (iii), we adjusted the glass plates (GP) to induce the required loss on V-photons in mode 1, and set the QWPs and HWPs in modes 2 and 3 such that they only add a constant phase shift between H and V on the incoming photons. HWP in mode 5 was also adjusted so that only H-photons arrive at the detector. HWPC in mode 6 was set to swap H and V polarizations. We post-selected the successful events with four-fold coincidences. The final three-photon state $\rho_{146}$ was characterized using 64 different tomographic settings [17] implemented by the sets of QWP, HWP and PBS in front of the detectors in modes 1, 4 and 6. We recorded coincidences for an acquisition time of 5800s at each tomographic setting. From the recorded correlations, we reconstructed the density matrix of $\rho_{146}$ using iterative maximum likelihood (IML) method [19,20]. This is shown in Fig. 4 together with the density matrix for the ideal $|W_3\rangle$. The density matrix for the ideal W state consists of only nine real nonzero terms, namely, the diagonal terms corresponding to $|HHV\rangle$, $|HVV\rangle$ and $|VVV\rangle$ and six off-diagonal elements corresponding to coherences among these terms. It is seen that the density matrix of the state prepared in our experiment has a similar structure with nine dominant elements.

Furthermore, from the reconstructed density matrix, we calculated the fidelity as $F \equiv \langle W_3 | \rho_{146} | W_3 \rangle =$

![FIG. 3: Real part of the reconstructed density matrices for the initial EPR pairs: (a) $\rho_{12}$ and (b) $\rho_{34}$.](image-url)
0.778 ± 0.043. We also calculated the entanglement-witness of this state using the operator $W_W = \frac{1}{2} I - |W_3\rangle\langle W_3|$ to distinguish it from separable and bi-separable states [21]. For an ideal W state, the expectation value of this operator is $-1/3$. We find $\text{Tr}(W_W \rho_{146}) = -0.111 ± 0.043$ for the final state in our experiment, which confirms that $\rho_{146}$ has a genuine tripartite entanglement.

One of the distinct properties of the W state is the entanglement left in the marginal state of any pair of qubits after one qubit is removed. We confirmed this by reconstructing the density matrices $\rho_{14}$, $\rho_{16}$ and $\rho_{36}$, corresponding respectively to Alice-Bob, Alice-Charlie, and Bob-Charlie marginal bipartite states. These density matrices are given in Fig. 5 together with the density matrix of the marginal bipartite state of the ideal W state. $W$ e also calculated the EOFs as $0.89$, the imperfections in the EPR pairs (polarization, this effect is expected to decrease the final fidelity only limited by the imperfection of the apparatus.

In summary, we have demonstrated a method for converting two EPR photon pairs to a three-photon W state via LOCC, using a polarization dependent beamsplitter and post-selection. The achieved final state was shown to have various characteristics of the W state. The achieved fidelity of $0.778 ± 0.043$ is higher than the value of $0.684 ± 0.024$ previously obtained via local transfromation from a GHZ state [3], signing the advantage of direct transformation that does not suffer from the fidelity-efficiency trade-off. This work extends our ability to manipulate multipartite entanglement, since our results imply that it is now possible to generate arbitrary three qubit states from a single resource of two EPR pairs via LOCC with a moderate success probability and with fidelity only limited by the imperfection of the apparatus.

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![FIG. 5: Real part of the reconstructed reduced density matrices of the experimentally obtained W state, (a) $\rho_{14}$, (b) $\rho_{16}$ and (c) $\rho_{36}$. (d) Real part of the reduced density matrix of the ideal state $|W_3\rangle$ for which $\rho_{14} = \rho_{16} = \rho_{36}$.](image)

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