INTERACTIVE GAMES AND REPRESENTATION
THEORY. II. A SECOND QUANTIZATION

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This short note is devoted to a second quantization of a classical picture described in [1].

1.1. Interactive games and intention fields.

Definition 1 [1]. An interactive system (with $n$ interactive controls) is a control system with $n$ independent controls coupled with unknown or incompletely known feedbacks (the feedbacks, which are called the behavioral reactions, as well as their couplings with controls are of a so complicated nature that their can not be described completely). An interactive game is a game with interactive controls of each player.

Below we shall consider only deterministic and differential interactive systems. For simplicity we suppose that $n = 2$. In this case the general interactive system may be written in the form:

\[ \dot{\varphi} = \Phi(\varphi, u_1, u_2), \]

where $\varphi$ characterizes the state of the system and $u_i$ are the interactive controls:

\[ u_i(t) = u_i(u_i^c(t), [\varphi(\tau)]_{\tau \leq t}), \]

i.e. the independent controls $u_i^c(t)$ coupled with the feedbacks on $[\varphi(\tau)]_{\tau \leq t}$. One may suppose that the feedbacks are integrodifferential on $t$.

Proposition [1]. Each interactive system (1) may be transformed to the form (2) below (which is not, however, unique):

\[ \dot{\varphi} = \tilde{\Phi}(\varphi, \xi), \]

where the magnitude $\xi$ (with infinite degrees of freedom as a rule) obeys the equation

\[ \dot{\xi} = \Xi(\xi, \varphi, \tilde{u}_1, \tilde{u}_2), \]

where $\tilde{u}_i$ are the interactive controls of the form $\tilde{u}_i(t) = \tilde{u}_i(u_i^c(t); \varphi(t), \xi(t))$ (i.e. the feedbacks are on $\xi(t)$ as well as on $\varphi(t)$ and are differential on $t$).

Remark 1. One may exclude $\varphi(t)$ from the feedbacks in the interactive controls $\tilde{u}_i(t)$. 
Definition 2 [1]. The magnitude $\xi$ with its dynamical equations (3) and its contribution into the interactive controls $\tilde{u}_i$ will be called the intention field.

Remark 2. The theorem holds true for the interactive games.

Remark 3. In practice, the intention fields may be often considered as a field-theoretic description of subconscious individual and collective behavioral reactions. The intention fields realize a “virtual dynamical memory” for the transmission of cognitive data in the scheme of the accelerated nonverbal cognitive computer and telecommunications [2].

1.2. Second quantization of intention fields and inverse problem of representation theory.

Starting the classical picture sketched above one is able to perform its second quantization. It means that the intention fields are quantized. The quantum picture is more realistic for a description of processes of the information transmission by intention fields in interactive games (cf.[3]).

The second quantization is deeply related to some inverse problems of the representation theory.

Definition 3 [1]. The main inverse problem of representation theory for the interactive system (1) (or for the interactive game) is

1. to write the system (1) in the form (2);
2. to determine the geometrical and algebraical structure of the intention field;
3. to find the algebraic structure, which “governs” the dynamics (3).

Remark 4. The solution of the main inverse problem of representation theory for the interactive system may use a posteriori data on the system.

So to perform a second quantization of the intention field it is sufficient to solve the dynamical inverse problem of representation theory [4] for the interactive system (game) (1).

Remark 5. This article as well as [1] may be regarded as a realization of some ideas of [5] on the mathematics beyond the conventional one based on the well-known forms of perception such as vision. Thus, the theory of interactive games plays a role for the subconscious individual and collective behavioral reactions analogous to geometry for a visual perception.

Remark 6. I suppose that it is reasonable to describe ‘Chi’ (‘Tsi’) of Chinese tradition and its manifestations as (perhaps, quantum) intention fields (cf.[3]), at least partially. Apparently, in concrete situations we should have deal with complex couplings of intention fields with physical magnitudes (i.e. the complexes of $\xi$ and $\phi$).

References

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