Extended state floating up in a lattice model: Bona fide levitation fingerprints, irrespective of the correlation length

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The evolution of extended states with magnetic field and disorder intensities is investigated for 2D lattice models. The floating-up picture is revealed when the shift of the extended state, relative to the density of states, is properly taken into account, either for white-noise or correlated disorder.

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I. INTRODUCTION

The behavior of extended (current carrying) states in two dimensional (2D) disordered systems is the key feature in the transition between two widely accepted limits: absence of delocalized states without magnetic field and the existence of them in the Quantum Hall regime. Laughlin [1] and Khmelnitzkii [2] proposed that the extended states at the center of the Landau bands should float up in energy above the Fermi level. Although appealing, this levitation, as the transition mechanism between the mentioned limits, is still controversial [3]. Perturbative approaches identify a weak levitation regime in the strong magnetic field limit [4, 5], while several numerical works report contradictory results. We mention some illustrative controversial works based on tight-binding lattice models. The non-float-up picture, where the extended states are supposed to disappear at finite B or disorder strength [6, 7], was recently reaffirmed in a proposed non-float-up phase diagram [8]. Other studies show evidences of floating up for white-noise disorder [9, 10, 11] with further disappearance of current carrying states, due to merging with states carrying negative Hall conductance (states of opposite Chern numbers: a lattice effect). A few very recent works have considered models with correlated disorders [12, 13] concluding that correlations would allow the floating up process not observed in white-noise case. Therefore, we revisit the problem, within the same framework of a 2D tight-binding lattice. The difference is that a direct comparison between different disorder models is established. We pay special attention to the deviation of the extended states relative to density of states (DOS) peaks in order to characterize the floating-up (as already suggested in references [9, 11]) and it was possible to observe and quantify bona fide levitation both in energy and in filling factor.

II. MODEL CALCULATION

The model Hamiltonian describes a square lattice of s-like orbitals, with nearest-neighbor interactions only:

\[ H = \sum_i \varepsilon_i c_i^\dagger c_i + \sum_{<i,j>} V(e^{i\phi_{ij}} c_i^\dagger c_j + e^{-i\phi_{ij}} c_j^\dagger c_i) \]  

where \( c_i \) is the fermionic operator on site \( i \). The magnetic field is introduced by means of the phase \( \phi_{ij} = 2\pi(e/h) \int_{j}^{i} \mathbf{A} \cdot \mathrm{d}\mathbf{l} \) in the hopping parameter \( V \). Considering the Landau gauge, \( \phi_{ij} = 0 \) along the \( x \) direction and \( \phi_{ij} = \pm 2\pi(x/a)\Phi/\Phi_0 \) along the \( \mp y \) direction, with \( \Phi/\Phi_0 = Ba^2e/h \) (\( a \) is the lattice constant).

Disorder is introduced by assigning random fluctuations to the orbital energy \( \varepsilon_i \), taking \( \varepsilon_i \leq |W/2| \). In the white-noise case, these energies are uncorrelated. In the correlated disorder model, a gaussian correlation \( \varepsilon_i = \frac{1}{\pi\lambda^2} \sum_j \varepsilon_j e^{-|\mathbf{R}_i-\mathbf{R}_j|^2/\lambda^2} \) [12] with correlation length \( \lambda \), is assumed. The tight-binding parameters are chosen in order to emulate the effective mass of an electron in the bottom of the GaAs conduction band: \( m^* = \hbar^2/(2|V|a^2) = 0.067m_e \). We focus the analysis on the lowest Landau levels for small magnetic fluxes \( \Phi/\Phi_0 \leq 1/20 \), a range where the lattice effects on the electronic spectra (Landau levels) are negligible. We consider squares of 40x40 sites with periodic boundary conditions.

The localization of the states is evaluated by means of the Participation Ratio (PR) [14]:

\[ PR = 1/(N \sum_{i=1}^{N} |a_i|^4) \]  

where \( N \) is number of lattice sites and \( a_i \) is the amplitude of the normalized wavefunction on site \( i \). The signature of an extended state in the PR, within each Landau band, will be a sharp peak for sufficient large systems. Both, the DOS and PR shown here are averages of 100 disorder configurations.

III. RESULTS AND DISCUSSION

In Fig.1 we show the calculated PR and the DOS for the lowest two broadened Landau bands, for a white-noise disorder amplitude of \( W/V = 2.8 \) and magnetic flux \( \Phi/\Phi_0 = 0.05 \). The floating up of the extended state (PR peak position) relative to the center of the 1\(^{st} \) Landau band, indicated by \( \delta E \), is evident.

The extended state actually goes down in energy with increasing disorder, due to the overall repulsion of states
induced by the potential fluctuation. The important feature, however, is that this repulsion is more pronounced for localized states (as proposed by Haldane and Yang [4]), resulting in a relative levitation in energy of the extended states. Fig.2 shows the equivalent results to Fig.1 for a correlated disorder of amplitude $W/V = 4.8$ and a correlation length $\lambda = 2a$. Now the levitation of the extended states is not only relative, but absolute in energy.

**FIG. 1.** DOS (continuous line) and the PR (circles) for the lowest two Landau bands for the white-noise disorder model: $W/V = 2.8$ and $\Phi/\Phi_0 = 0.05$. $\delta E$ is the energy shift of the extended state.

**FIG. 2.** Same as Fig.1 for the correlated disorder model with $W/V = 4.8$ and $\lambda = 2a$.

From Figs.1 and 2 it becomes clear that changes in the potential landscape lead to modifications in both, the DOS and the general features of the localization of the states. Concerning the DOS we note that while in the white-noise disorder the successive Landau bands have all the same broadening $\Gamma$, the correlations make $\Gamma$ dependent on Landau level index: the higher the index, the narrower the band [15]. For white-noise potential fluctuations, the PR peaks for higher Landau bands become highly asymmetric and less resolved than the lowest one, already at relatively small band superpositions. On the other hand, a smoother potential fluctuation, Fig.2, leads to well resolved PR peaks for all Landau bands. Although the shift $\delta E$ observed is smaller than that of Fig.1, for approximately the same superposition situation, it can be followed until greater superpositions and also for higher Landau levels, not only the first one.

**FIG. 3.** Extended states shift vs. $\hbar\omega_c/\Gamma$. Circles are for the white-noise case. Squares(triangles) are for the first(second) extended state for correlated disorder.

Referring to the energy shift, $\delta E$, the evolution of the critical states as a function of increasing disorder or diminishing magnetic field can be followed in Fig.3, which summarizes our extensive studies. Here the normalized shift, $\delta E/\hbar\omega_c$, are plotted as a function of $\hbar\omega_c/\Gamma$, the ratio between the energy separation of the DOS peaks and the Landau band width (determined at half height). For the white-noise case we follow $\delta E$ by either varying the disorder (filled circles) or magnetic flux (open circles). Squares and triangles are for the first and second Landau band critical states for the gaussian-correlated disordered system, respectively.

The equivalence between diminishing the magnetic field or increasing disorder for observing the floating up of the extended states is established for the white-noise case. The dependence observed is thus a general result valid for any magnetic flux or disorder values in the showed $\hbar\omega_c/\Gamma$ range. For the correlated disorder it is seen that for the 1$^{st}$ Landau level the levitation is less pronounced than that of white-noise, for equivalent $\hbar\omega_c/\Gamma$. This fact qualitatively agrees with analytical predictions [4, 5] and represents the first numerical quantitative comparison of different disorder models levitation. Also from Fig.3, a weaker levitation for the 2$^{nd}$ critical state compared to the 1$^{st}$ one is verified (notice that results shown...
in Fig.3 already take in account the narrower $\Gamma$ of 2nd Landau band, so this is a intrinsic smaller floating up process). This second fact contradicts the perturbative approaches [4,5] and initial conjectures for levitation [1, 2] that expects more pronounced shifts for higher delocalized levels. Finally, it is worth noting that for all cases $\delta E/\hbar\omega_c \propto (\hbar\omega_c/\Gamma)^{-2}$.

![Graph](image)

FIG. 4. Levitation of the filling factor vs. disorder strength for the correlated disorder case. Circles(squares) are for the first(second) extended state.

In Fig.4 we see the floating up of the filling factors, $\nu$, corresponding to the two lowest extended states as a function of disorder strength for the correlated disorder case. The values for the 2nd extended state have been rigidly shifted down by $\nu = 1$, in order to better compare them with the results for the 1st one. Again we can see the less pronounced levitation of the 2nd extended levels. The striking result is that the floating up is linear for a wide range of $W/V$, occurring already before the Landau bands superposition, which only starts for $W/V \approx 3.6$. This behavior is analogous for the white-noise case (not shown here). The relevance of this observation is that it unambiguously contradicts the “apparent levitation” concept, which credits the floating up of the filling factor for the lowest extended state exclusively to Landau band superpositions [6, 16]. Superpositions nevertheless affect the filling factor and indeed deviations from linear floating up, observed in Fig.4 for strong disorder, are due to Landau band superpositions. The levitation for the lowest state is enhanced, while the second one shows a saturation, due to the broadening dependence on the Landau level index.

IV. FINAL REMARKS

In conclusion, we could quantitatively identify the levitation of extended states in energy, relative to the Landau band centers, for both, white-noise and correlated disorder, either as a function of decreasing magnetic field or increasing disorder strength. We believe that the present results settle the controversy on the levitation in a lattice model: bona fide levitation of delocalized states occurs and a correct interpretation of the floating-up picture has to focus on the evolution of extended states positions relative to the DOS. The well established behavior of the extended states for $\hbar\omega_c/\Gamma \gtrsim 1$ suggests an intermediate situation between the Global Phase Diagram [17] (based on Laughlin conjecture [1]) and a recent proposal [8] (non-float-up phase diagram). Extending the behavior of a less pronounced levitation of higher extended states to $\hbar\omega_c/\Gamma < 1$, the present results would also support the possibility of direct transitions from $\nu > 1$ states to the insulator phase. However, the size of the system has to be continuously increased for decreasing magnetic field or increasing disorder in order to follow the extended states for $\hbar\omega_c/\Gamma < 1$ [3]. Therefore, the above conclusions can not be extended to arbitrarily low magnetic fields, and the implications of these results on the Quantum Hall Phase Diagram have to be taken carefully. Perhaps more important than the size effects, are the unavoidable lattice effects (annihilation of current carrying states with opposite Chern numbers) [10]. These lattice effects, however, are overcome by the correlation in the disorder [13] which has the property of widening the “continuum limit window” of lattice models. Finally, the analysis of $\delta E/\hbar\omega_c$ vs. $\hbar\omega_c/\Gamma$ permits a direct comparison between disorder models or even with experimental results including density of states measurements [18].

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