Fast Optimal Dispatch of Renewable Energy Sources in Distribution Networks

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Abstract. With the increasing penetration of distributed renewable energy sources such as photovoltaic and wind power generators in distribution networks, overvoltage problems occurred as a result of the reversed power flow. The randomness and intermittence of power outputs of the renewable energy sources make the operations of distribution systems complex at the same time. This paper proposes a simplified optimal dispatch algorithm for fast renewable energy sources power control to overcome voltage problems and permit better performance for distribution systems. Case study demonstrates the accuracy and efficiency of the optimal dispatch method proposed.

1. Introduction
Distributed renewable energy sources (RESs) have brought many problems to distribution systems while producing clean energy. Overvoltage problems may occur when power generations of RESs far exceed load demands [1]. Meanwhile, the strong fluctuation and uncertainty of active power outputs of RESs restrict their further application.

Inverter-interfaced RESs have a certain ability to generate reactive power in addition to active power. Reactive power can be used for voltage regulation and network loss reduction [2]. Proper active power curtailment is also benefit to grid operation when necessary [3]. Dealing with operational problems of distribution systems with appropriate dispatch and control of RESs instead of resorting to energy storage, reactive power compensation or other devices has great economical advantages.

Different from the dispatch of generators in transmission systems, which is run every five or ten minutes, distributed RESs in distribution systems require second-level dispatch for the strong variation of their active power outputs and the rapid change of load power[4]. The development of measurement, communication and inverter control technologies makes the second-level dispatch possible, while difficulties of solving large scale, nonlinear program problems still remain, which is the main concern of this paper.

A simplified method is proposed in this paper for the fast calculation of optimal RESs dispatch problem. Taylor expansion is used here to turn the non-convex dispatch problem into linear program problem. Fast speed and high accuracy is guaranteed for the simplified method proposed by this paper. Case study is given at last, which demonstrates that applying the aforementioned dispatch method to RESs enormously improves the performance of distributed systems.
2. Optimal RESs Dispatch Problem Formulation

2.1. Model of distribution systems

Consider a distribution system comprising \( n + 1 \) nodes, where node 0 is the root node with constant voltage. Usually the secondary side of transformers in distribution systems is considered as the root node. If node \( i \) is a node of the system, then let node \( i-1 \) be the node next to node \( i \), closer to node 0. Let line \( i \) be the line between node \( i \) and node \( i-1 \). There are loads and/or RESs connected to all nodes. Schematic diagram of the distribution circuit is shown as Figure 1.

\[
\begin{align*}
    P_i &= P_{i-1} - r_i \frac{P_{i-1}^2 + Q_{i-1}^2}{V_{i-1}^2} + P_i^g - P_i^l \\
    Q_i &= Q_{i-1} - x_i \frac{P_{i-1}^2 + Q_{i-1}^2}{V_{i-1}^2} + Q_i^g - Q_i^l
\end{align*}
\]

(1) (2)

Where \( P_{i-1} \) and \( Q_{i-1} \) are the active and reactive power transmitted from node \( i-1 \) to node \( i \), and \( P_i \) and \( Q_i \) are those transmitted from node \( i \) to further node. \( P_i^g \) and \( Q_i^g \) are the active and reactive power of RES generation, and \( P_i^l \) and \( Q_i^l \) are those of loads connected to node \( i \) respectively. \( V_{i-1} \) is the voltage magnitude of node \( i-1 \). \( r_i \) and \( x_i \) are the resistance and reactance of line \( i \).

2.2. The DistFlow equations

The DistFlow equations are efficient power flow equations for radial distribution networks and are widely used in optimal power flow problem formulation of distribution systems. The Equations are:

\[
\begin{align*}
    P_i &= P_{i-1} - r_i \frac{P_{i-1}^2 + Q_{i-1}^2}{V_{i-1}^2} + P_i^g - P_i^l \\
    Q_i &= Q_{i-1} - x_i \frac{P_{i-1}^2 + Q_{i-1}^2}{V_{i-1}^2} + Q_i^g - Q_i^l
\end{align*}
\]

(1) (2)

The network power loss is

\[
    F_2 = \sum_{i=1}^{n} \frac{P_i^2 + Q_i^2}{V_{i-1}^2}
\]

(4)

Applying equation (1) and equation (2) as equal constraints, while constraining the voltage within nominal operational limits and active and reactive power output of RESs within generating limits, we have the following optimization problem

\[
\text{problem 1: } \min k_1 F_1 + k_2 F_2
\]

(5)
s.t. equation (1), (2),
\[ V \leq V_i \leq \bar{V} \]
\[ 0 \leq p_i^g \leq p_i^{g_{\text{max}}} \]
\[ q_i^g \leq q_i^g \leq \bar{q}_i^g \]
\[ i = 1, 2, \ldots, n \]

Where \( k_1 \) and \( k_2 \) are weighting coefficients of \( F_1 \) and \( F_2 \). \( V \) and \( \bar{V} \) are the minimum and maximum voltage limit. \( q^g \) and \( \bar{q}^g \) are the lower and upper reactive power generation limits of RES \( i \).

Problem 1 is a nonlinear problem basically as a result of equation (1) and (2). Solving a nonlinear problem is usually time-consuming and requires a high performance of computing facilities. It is particularly serious for large scale distribution systems with a huge number of RESs, which makes second level dispatch hard to achieve.

3. Linear Problem Formulation

When dispatch periods are significant short, changes of variables such as voltage, load power are extremely small. Under such a premise, taylor expansion is used to simplify problem 1 and makes it linear.

Assuming that with measurement, all variable including voltage and power is knowable in real time. Variables of last dispatch period is described as \( P_i^{t-1}, Q_i^{t-1}, V_i^{t-1} \). For objective function equation (4), with taylor expansion and neglecting the higher order, we have

\[ F_2 = F_2^0 + \sum_{i=1}^{n} \frac{\partial F_2}{\partial p_i^g} \Delta P_i + \frac{\partial F_2}{\partial Q_i^g} \Delta Q_i + \frac{\partial F_2}{\partial V_i^{t-1}} \Delta V_i \]

\[ \text{Where} \quad F_2^0 \text{ is the value of } F_2 \text{ with} \]
\[ P_i = P_i^{t-1}, Q_i = Q_i^{t-1}, V_i = V_i^{t-1}, \quad i = 1, 2, \ldots, n \]

And \( \Delta P, \Delta Q, \Delta V \) are increments compared with \( P_i^{t-1}, Q_i^{t-1}, V_i^{t-1} \), described as

\[ \Delta P_i = P_i - P_i^{t-1}, \Delta Q_i = Q_i - Q_i^{t-1}, \Delta V_i = V_i - V_i^{t-1}, \quad i = 1, 2, \ldots, n \]

For equation (1) and (2), define

\[ f_{p,i} = P_i - P_{i-1} + r_i \frac{P_i^{t-1} + Q_i^{t-1}}{V_i^{t-1}} - p_i^g + p_i' \]

\[ f_{q,i} = Q_i - Q_{i-1} + x_i \frac{P_i^{t-1} + Q_i^{t-1}}{V_i^{t-1}} - q_i^g + q_i' \]

\[ i = 1, 2, \ldots, n \]

Then, with the same method we have

\[ f_{p,i} = f_{p,i}^0 + \frac{\partial f_{p,i}}{\partial P_i^{t-1}} \Delta P_i + \frac{\partial f_{p,i}}{\partial Q_i^{t-1}} \Delta Q_i + \frac{\partial f_{p,i}}{\partial V_i^{t-1}} \Delta V_i + \frac{\partial f_{p,i}}{\partial p_i^g} \Delta p_i^g \]

\[ i = 1, 2, \ldots, n \]

Where \( f_{p,i}^0 \) and \( f_{q,i}^0 \) are the value of \( f_{p,i} \) and \( f_{q,i} \) with variable values measured in last dispatch period. \( \Delta p_i^g, \Delta q_i^g \) are the increment of \( p_i^{g_{t-1}}, q_i^{g_{t-1}} \).
Equation (9), (14) and (15) are all linear equations. Consequently, a simplified linear optimal dispatch problem is formulated, as

\[ \text{problem 2: } \min k_1F_1 + k_2F_2 \]

s.t. equation (14), (15), (6), (7), (8)

After the problem being solved, updates control variables of all RESs, as

\[ P_i' = P_i^{\text{pre}} + \Delta P_i \]

\[ Q_i' = Q_i^{\text{pre}} + \Delta Q_i \]

\[ i = 1, 2, \ldots, n \]

It is worth noting that taking increments \( \Delta P, \Delta Q, \Delta V, \Delta p^g, \Delta q^g \) as variables, problem 2 is a linear program problem. There are many mature methods for solving linear program problems, like the simplex method and the interior point method. Compared with problem 1 which is nonlinear, problem 2 is much more efficient in calculation and is potential for second level dispatch application.

4. Case Study

IEEE 33-node distribution system \[^5\] and IEEE 118-node distribution system \[^6\] are used here. Suppose that there are RESs connected to all nodes of distribution system. All RESs' power outputs are based on real photovoltaic and wind power generations data. A 24 hour simulation is made of system dispatch with RESs applying both optimization model of problem 1 and problem 2 to make a comparison on accuracy and speed of the two methods.

4.1. Comparison on calculation accuracy

The mathematical model of problem 1 is the accurate expression of optimal RESs dispatch problem, so its solution is theoretically optimal. Compared with the value of objective function of problem 1, the average error of problem 2 is 2.69% in the 33-node system simulation and 2.91% in the 118-node system simulation. Comparison on accuracy of 33-node system simulation is shown in figure 2.

4.2. Comparison on calculation speed

| Average calculation time/s | Problem 1 | Problem 2 |
|----------------------------|-----------|-----------|
| 0.05                       |           |           |
| 0.10                       |           |           |
| 0.20                       |           |           |
| 0.25                       |           |           |
| 0.30                       |           |           |

Figure 2. Comparison on accuracy.
Comparison on calculation speed is shown in table 1. It is obvious that calculation of a nonlinear program problem is time-consuming especially with a large system. Computation speed of problem 2, which is a linear program problem, is much faster than the nonlinear one, and less affected by the system scale.

5. Conclusion
This paper proposed a simplified linear program method for fast optimal RESs dispatch. Compared with the origin nonlinear problem, the calculation speed of the method conducted is significantly fast while high computation accuracy is guaranteed. As a result, the simplified optimal dispatch algorithm has a better performance for second level distributed RESs control in distribution systems.

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