Behavior of open sets in bi-Alexandroff topological space

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Abstract
The goal of this paper is to establish the properties of which exhibit the characterization of a \( j \)-open set in bi-Alexandroff topological space and some properties of \( j \)-open set are analyzed. Also we have studied the notion of \( j \)-bi-continuous function in bi-Alexandroff topological space.

Keywords
Alexandroff space, bi-Alex open sets, bi-Alex closed sets, \( j \)-bi open set.

AMS Subject Classification
54A10.

1. Introduction

Nowadays the topological structures have many applications in our daily life. Single topology is extended to bitopology, tri topology and quad topology with usual definitions. The concept of bitopological space was first introduced by Kelly [5], and tritopological spaces were implemented by Martin M. Kovar [8]. The quad topological space was invented by Dhanya V. Mukundan [4]. These papers are pushup to study us Alexandroff topological space into bi-Alexandroff topological space.

Alexandroff topological space was first exposed by P. Alexandroff in 1937 [1]. The Alexandroff space (a space with the condition of Alexandroff) is a topological space such that arbitrary intersection of open sets is open (the union of any number of closed sets is closed), equivalently, every point has a minimal neighborhood \( V(x) \) and is that the intersection of all open sets containing \( x \), or equivalently it has an unique minimal base [2].

The Alexandroff topological space was a consequence of the important role of finite spaces in digital topology and therefore the undeniable fact that these spaces have all properties of finite spaces [7], [10]. D. Sasikala and I. Arockiarani initiated \( \lambda_{j} \)-\( \alpha \)-closed sets in generalized topological spaces in 2011 [11]. Then they also initiated decomposition of \( j \)-closed sets in bigeneralized topological space in 2012 [12].

Furthermore \( j \)-sets have motivated us to analyze \( j \)-sets in bi-Alexandroff topological space. Our purpose of this paper is to develop the basic concepts and properties of bi-Alexandroff topological space by using \( j \)-sets.

2. Preliminaries

Definition 2.1. [1] Let us consider \( X \) be a topological space, then \( X \) is an Alexandroff space if arbitrary intersection of open sets is open.

Theorem 2.2. [13] Let us consider \( X \) be a metric space, then \( X \) is an Alexandroff space if and only if \( X \) has discrete topology.

Definition 2.3. [5] If \( X \) is any set, a basis for the topology on the set \( X \) is a collection \( B \) of subsets of \( X \) called basis elements such that,

(1) For each \( x \in X \), there is at least one basis element \( B \) containing \( x \).
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(2) If \( x \) belongs to the intersection of two basis element \( B_1 \) and \( B_2 \), then there is a basis element \( B_3 \) containing \( x \) such that \( B_3 \subseteq B_1 \cap B_2 \).

**Theorem 2.4.** [13] If \( X \) is an Alexandroff space with topology \( \tau \) then,

\[
\beta = \{ S(x) \mid x \in X \}
\]

is a basis for \( \tau \).

**Definition 2.5.** [5] Let us consider \( X \) be a topological space with \( \tau \). If \( Y \) is a subset of \( X \), the collection

\[
\tau_Y = \{ Y \cap U \mid U \in \tau \}
\]

is a topology on \( Y \), called the subspace topology with this topology, \( Y \) is called a subspace of \( X \).

**Definition 2.6.** [5] If \( \mathcal{B} \) is basis for the topology on a set \( X \) then,

\[
\mathcal{B}_Y = \{ B \cap Y \mid B \in \mathcal{B} \}
\]

is a basis for the subspace topology on \( Y \).

**Definition 2.7.** [9] A subset \( S \) of a space \( X \) is said to be a pre-open set if \( A \subseteq \text{int}(c(S)) \).

**Definition 2.8.** [11] A subset \( S \) of a space \( X \) is said to be a \( j \)-open set if \( A \subseteq \text{int}(\text{precl}(S)) \).

3. bi-Alexandroff topological space

**Definition 3.1.** Let a non-empty set be \( X \), \( A_{\tau_1} \) and \( A_{\tau_2} \) are Alexandroff topologies on \( X \). Then a subset \( S \) of \( X \) is said to be a bi-Alex open set (briefly \( A_{\tau_1 \tau_2} \)-open set) if \( S \in A_{\tau_1} \cap A_{\tau_2} \) and complement is said to be a bi-Alex closed set (briefly \( A_{\tau_1 \tau_2} \)-closed set).

Obviously by the definition of bi-alex open sets it satisfies all the axioms of Alexandroff topological space, in which Alexandroff topology is denoted by \( A_{\tau_1} \cap A_{\tau_2} \) with two topologies called bi-Alexandroff topological space and denoted by \( (X, A_{\tau_1}, A_{\tau_2}) \).

**Example 3.2.** Let \( X = \{ a, b, c, d \} \), \( A_{\tau_1} = \{ X, \emptyset, \{ a \}, \{ b \}, \{ c \}, \{ d \}, \{ a, b \}, \{ a, c \}, \{ a, d \}, \{ b, c \}, \{ b, d \}, \{ a, b, c \}, \{ a, b, d \}, \{ a, c, d \}, \{ b, c, a \} \} \), \( A_{\tau_2} = \{ X, \emptyset, \{ a \}, \{ b \}, \{ c \}, \{ d \}, \{ a, b \}, \{ a, c \}, \{ a, d \}, \{ b, c \}, \{ b, d \}, \{ a, b, d \}, \{ a, b, c \} \} \).

Then bi-Alex open sets are \( X, \emptyset, \{ b \}, \{ d \}, \{ a, d \}, \{ a, b, d \} \) bi-Alex closed sets are \( X, \emptyset, \{ a, c \}, \{ a, b, c \}, \{ a, c, d \}, \{ b, c, d \} \).

**Definition 3.3.** A subset \( S \) of a bi-Alexandroff topological space \((X, A_{\tau_1}, A_{\tau_2})\) is called bi-Alex neighborhood of a point \( x \in X \) if and only if there exists a bi-Alex open set \( U \) such that \( x \in U \subseteq S \).

**Definition 3.4.** A bi-Alexandroff topological space be \((X, A_{\tau_1}, A_{\tau_2})\) and take \( S \subseteq X \). Then the intersection of bi-Alex closed sets containing \( S \) is called a bi-Alex closure of \( S \) and denoted by \( \text{bi-cl}(S) \) and the union of bi-Alex open sets contained in \( S \) is called a bi-Alex interior of \( S \) and denoted by \( \text{bi-int}(S) \).

**Theorem 3.5.** \( X \) is a bi-Alexandroff topological space if and only if for any \( x \in X \) has a minimal bi-Alex open neighborhood.

**Proof.** Assume that \( X \) is a bi-Alexandroff topology with \( y \in X \). Let \( O(y) = \{ U \subseteq X \mid U \) is an open neighborhood of \( y \) in \( A_{\tau_1} \) and \( A_{\tau_2} \} \). Let \( S(y) = \bigcap U \) for \( U \in O(y) \), then \( S(y) \) is an open neighborhood of \( y \) in \( A_{\tau_1} \) and \( A_{\tau_2} \), because \( X \) is bi-Alexandroff topology. Since \( S(y) = \bigcap U \) it is clear that \( S(y) \) is a minimal bi-Alex open neighborhood of \( y \).

Conversely, suppose that for each \( y \in X \) has a minimal bi-Alex open neighborhood \( S(y) \), then an arbitrary intersection of bi-Alex open sets \( V = \bigcap_{\alpha \in A} U_{\alpha} \), where \( U_{\alpha} \) is open in \( A_{\tau_1} \) and \( A_{\tau_2} \). If \( V \) is empty set then it completes the proof. If not, then pick \( y \in V \) and we have \( y \in U_{\alpha}, \forall \alpha \in A \). Hence \( S(y) \subseteq U_{\alpha} \forall \alpha \), because \( S(y) \) is the minimal bi-Alex open neighborhood. Therefore \( S(y) \subset V \). Hence \( V \) is open in \( A_{\tau_1} \) and \( A_{\tau_2} \). Therefore \( V \) is bi-Alex open set, since it contains an bi-Alex open set around each of it’s point.

**Definition 3.6.** Let \((X, A_{\tau_1}, A_{\tau_2})\) be a bi-Alexandroff topological space. If \( S \) is a subset of \( X \), then the collection

\[
\tau_S = \{ S \cap G \mid G \text{ is bi-Alex open in } X \}
\]

is a Alexandroff topology on \( S \) called the subspace bi-Alexandroff topology and with this topology \( \tau_S \), \( S \) is called a subspace of \( X \).

**Example 3.7.** Let \( X = \{1, 2, 3, 4\}, A_{\tau_1} = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}, A_{\tau_2} = \{X, \emptyset, \{3\}, \{4\}, \{1, 3\}, \{1, 3, 4\}, \{1, 2, 3\}\} \).

Then bi-Alex open sets are \( X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\} \). Let \( S = \{2, 3\} \), then \( \tau_S = \{\{X, \emptyset, \{2\}, \{3\}, \{2, 3\}\}\} \).

**Definition 3.8.** If \( S \) is a subspace of bi-Alexandroff topological space \( X \), we say that \( H \) is open in \( S \) if \( H \in \tau_S \).

**Lemma 3.9.** Let \( S \) is a subspace of bi-Alexandroff topological space \( X \). If \( H \) is open in \( S \) and \( S \) is bi-Alex open in \( X \), then \( H \) is bi-Alex open in \( X \).

**Proof.** Since \( H \) is open in \( S \), then by the definition of subspace bi-Alexandroff topology, we can write \( H = S \cap G \), for some set \( G \) is bi-Alex open in \( X \). Since \( S \) is bi-Alex open in \( X \) which implies that, \( S \) and \( G \) are both bi-Alex open sets in \( X \). Thus we have \( S \cap G \) is bi-Alex open set in \( X \).

**Lemma 3.10.** Let \((X, A_{\tau_1}, A_{\tau_2})\) be a bi-Alexandroff topological space and let \( \mathcal{B} \) be base for \( A_{\tau_1} \cap A_{\tau_2} \). If \( \tau_S \) is a subspace of \((X, A_{\tau_1}, A_{\tau_2})\), then \( \tau_S = \{ B \cap S \mid B \in \mathcal{B} \} \).

**Proof.** \( \mathcal{B} \) is a base for \( A_{\tau_1} \cap A_{\tau_2} \), and so \( B \subseteq A_{\tau_1} \cap A_{\tau_2} \), which implies that, \( B \subseteq \tau_S \).

Let \( H \in \tau_S \) which implies that \( H = S \cap G \), for some bi-Alex open set \( G \) in \( X \), and let \( y \in H \) from this we get \( y = G \cap S \), this means that \( y \in G \) and \( y \in S \). Since \( \mathcal{B} \) is a base for \( A_{\tau_1} \cap A_{\tau_2} \), \( y \in G \) and \( G \in A_{\tau_1} \cap A_{\tau_2} \), which implies that \( y \in B \subseteq G \), for some subset \( B \) in \( \mathcal{B} \). Then \( y \in B \cap S \subseteq S \cap G \). We have \( H = \)
The function \( f : X \to Y \) is bi-continuous, since \( f^{-1}(H) \) is bi-Alex open in \( X \) for every bi-Alex open set \( H \subseteq Y \).

**Lemma 3.13.** If \( S \) is a subspace of a bi-Alexandroff topological space \( X \), then the inclusion function \( i : S \to X \) is bi-continuous.

**Proof.** Suppose if \( G \) is a bi-Alex open in \( X \), then \( i^{-1}(G) = G \cap S \) which is bi-Alex open in \( S \) by the definition of subspace bi-Alexandroff topology.

**Lemma 3.14.** If \( f : X \to Y \) is a bi-continuous and if \( S \) is a subspace of a bi-Alexandroff topological space \( X \), then \( f|S : S \to Y \) (restricted function) is bi-continuous.

**Proof.** The function \( f|S \) equals the composite of the inclusion function \( i : S \to X \) and the function \( f : X \to Y \), both of which are bi-continuous function.

**Lemma 3.15.** If the function \( f : X \to Y \) is bi-continuous and if \( T \) is a subspace of bi-Alexandroff topological space \( Y \) containing the image set \( f(X) \), then \( g : X \to T \) obtained by restricting the range of \( f \) is bi-continuous. If \( T \) is a space having \( Y \) as a subspace bi-Alexandroff topology, then \( h : X \to T \) obtained by expanding the range of \( Y \) is bi-continuous.

**Proof.** Let the function \( f : X \to Y \) be bi-continuous. If \( f(X) \subseteq T \subseteq Y \), we prove that the function \( g : X \to T \) obtained from \( f \) is bi-continuous. Let \( V \) be bi-Alex open set in \( T \), and \( U \) be bi-Alex open set in \( Y \). Then \( V \cap U \) for some bi-Alex open set \( U \) of \( Y \). Since \( T \) contains the entire image set \( f(X) \).

\[ f^{-1}(U) = g^{-1}(V) \]

by the elementary set theory, since \( f^{-1}(U) \) is bi-Alex open set, so is \( g^{-1}(V) \).

Now to prove that \( h : X \to T \) is bi-continuous. If \( T \) has \( Y \) as a subspace bi-Alexandroff topology, note that \( h \) is the composite of the map \( f : X \to Y \) and the inclusion function \( i : Y \to T \), it completes the proof.

### 4. pre-bi open set and pre-bi closed set

**Definition 4.1.** Let us consider \( X \) be a bi-Alexandroff topological space \((X,A_{1},A_{2})\). A subset \( S \) of \( X \) is said to be pre-bi-open if \( S \subseteq \text{bi-int}(\text{bi-cl}(S)) \) and \( S \) is a pre-bi closed if the complement of \( S \) is a pre-bi-open.

**Example 4.2.** Let \( X = \{a,b,c,d\} \), \( A_{1} = \{X,\emptyset,\{b\},\{d\},\{a,d\},\{a,b,d\}\} \), \( A_{2} = \{X,\emptyset,\{b\},\{d\},\{a\},\{a,d\}\} \). Then \( A_{1} \) is bi-Alex closed sets and \( A_{2} \) is bi-Alex open set.

**Remark 4.3.** It is clear that, every bi-Alex open set is pre-bi-open and every bi-Alex closed set is pre-bi-closed set in general.

**Theorem 4.4.** Let \((X,A_{1},A_{2})\) be a bi-Alexandroff topological space in which each pre-bi open set \( a \) is bi-Alex closed then every singleton in \( X \) is either a bi-Alex open or a bi-Alex closed.

**Proof.** Let \( a \in X \), and suppose that the singleton set \( \{a\} \) is not bi-Alex open, then clearly \( \{a\} \) is not pre-bi-open, which implies that \( \{a\} \not\subseteq \text{bi-int}(\text{bi-cl}\{a\}) \), so that \( \text{bi-int}(\text{bi-cl}\{a\}) = \emptyset \). We have \( \text{bi-int}(\text{bi-cl}\{X - \{a\}\}) \supseteq \text{bi-int}(X - \text{bi-int}(\text{bi-cl}\{a\})) = X - \{a\} \). Thus \( X - \{a\} \) is pre-bi-open and hence it is bi-Alex open. Therefore \( \{a\} \) is bi-Alex closed set.

**Theorem 4.5.** \((X,A_{1},A_{2})\) is a bi-Alexandroff topological space in which every subset is a pre-bi-open if and only if every bi-Alex open set in \((X,A_{1},A_{2})\) is a bi-Alex closed set.

**Proof.** Let us consider \( U \) be a bi-Alex open set in \( X \), then we have \( X - U = \text{bi-cl}(X - U) \) which is pre-bi-open, which implies that \( \text{bi-cl}(X - \{a\}) \subseteq \text{bi-int}(\text{bi-cl}(X - \{a\})) \) so that \( \text{bi-int}(\text{bi-cl}(X - \{a\})) = X - \{a\} \). Thus \( X - \{a\} \) is pre-bi-open and \( U \) is a bi-Alex closed.
Conversely, let $S$ be a subset of $X$, then $S = \overline{\text{bi-cl}(S)}$ is bi-Alex open and hence bi-Alex closed. Thus we have $X = \overline{\text{bi-cl}(X)} = \overline{\text{bi-cl}(S)} = \overline{\text{bi-cl}(X - S)}$. So that $S \subseteq \overline{\text{bi-cl}(S)} = \overline{\text{bi-cl}(S)}$, and hence $S$ is pre-bi open.

**Theorem 4.6.** Let $(X, A_1, A_2)$ be a bi-Alexandroff topological space, then the following results are equivalent.
(i) Every pre-bi open set is bi-Alex open.
(ii) Every dense set is bi-Alex open.

Proof. (i) $\Rightarrow$ (ii). Let $S$ be a dense subset of $X$. Then $S = \overline{\text{bi-cl}(S)} = \overline{\text{bi-cl}(S)}$, and hence $S$ is pre-bi open and $S$ is bi-Alex open.

(ii) $\Rightarrow$ (i). Let $A$ be a pre-bi open subset of $X$, so that $A \subseteq \overline{\text{bi-int}(\text{pre-bi-cl}(S))}$ and hence $A$ is pre-bi open and $S$ is bi-Alex open.

**Theorem 5.4.** Arbitrary intersection of $j$-bi closed sets is $j$-bi closed.

Proof. Let $\{S_i\}_{i \in I}$ be a collection of $j$-bi closed sets in $X$, for each $i \in I$.

\[
\bigcap_{i \in I} S_i = \overline{\text{bi-cl}(\bigcap_{i \in I} S_i)} \subseteq \bigcap_{i \in I} \overline{\text{bi-cl}(S_i)} = \bigcap_{i \in I} \overline{\text{bi-cl}(S_i)} = \bigcap_{i \in I} \overline{\text{bi-cl}(S_i)}
\]

This implies that $\bigcap_{i \in I} S_i$ is $j$-bi open set.

**Remark 5.5.** We will denote the $j$-bi interior (resp. $j$-bi closure) of any subset $S$ of $X$ by $j$-bi-int($S$) (resp. $j$-bi-cl($S$)) where $j$-bi-int($S$) is the union of all $j$-bi open sets contained in subset $S$ and $j$-bi-cl($S$) is the intersection of all $j$-bi closed sets containing subset $S$.

**Theorem 5.6.** A subset $S$ of $X$ is a $j$-bi open if and only if $S = j$-bi-int($S$).

Proof. $S$ is $j$-bi open and we know that $S \subseteq S$. Therefore $S \in \{P \mid P \subseteq S, P$ is $j$-bi open$\}$. $S$ is in this collection and other remaining members in this collection is a subset of $S$ and clearly union of this collection is $S$. That is,

\[
\bigcup\{P \mid P \subseteq S, P$ is $j$-bi open$\} = S
\]

and hence $j$-bi-int($S$) = $S$.

Conversely, since $j$-bi-int($S$) is $j$-bi open set. That is $S = j$-bi-int($S$), which implies that $S$ is a $j$-bi open set.

**Theorem 5.7.** A subset $S$ of $X$ is a $j$-bi closed if and only if $S = j$-bi-cl($S$).

Proof. From the definition of $j$-bi closure, we can write $j$-bi-cl($S$) = $\cap \{P \mid P \supseteq S, P$ is $j$-bi closed$\}$. If $S$ is a $j$-bi closed then $S$ is a member in $\cap \{P \mid P \supseteq S, P$ is $j$-bi closed$\}$, and each member contains $S$. Hence $S = j$-bi-cl($S$).

Conversely, If $S = j$-bi-cl($S$), then $S$ is bi-Alex closed, since $j$-bi-cl($S$) is a $j$-bi closed set.

### 5. $j$-open set on bi-Alexandroff topological space

**Definition 5.1.** Let us consider $X$ be a bi-Alexandroff topological space $(X, A_1, A_2)$. A subset $S$ of $X$ is said to be $j$-bi open set if $S \subseteq \overline{\text{bi-int}(\text{pre-bi-cl}(S))}$ and $S$ is said to be $j$-bi closed if the complement of $S$ is a $j$-bi open set.

**Example 5.2.** Let $X = \{1, 2, 3, 4\}$, $A_1 = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}\}$. Then $\overline{\text{bi-cl}(X)} = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}\}$.

**Theorem 5.3.** Arbitrary union of $j$-bi open sets is a $j$-bi open set.

Proof. Let $\{S_i\}_{i \in I}$ be a collection of $j$-bi open sets in $X$, for each $i \in I$.

\[
\bigcup_{i \in I} S_i \subseteq \overline{\text{bi-cl}(\bigcup_{i \in I} \text{bi-int}(\text{pre-bi-cl}(S_i)))}
\]

This implies that $\bigcup_{i \in I} S_i$ is $j$-bi open set.

**Theorem 5.4.** Arbitrary intersection of $j$-bi closed sets is $j$-bi closed.

Proof. Let $\{F_i\}_{i \in I}$ be a collection of $j$-bi closed sets in a space $X$, for each $i \in I$, then $\overline{\text{bi-cl}(\text{pre-bi-cl}(F_i))} \subset F_i$, since $F_i$ is an arbitrary indexed collection of $j$-bi open sets, from above theorem 5.3 we get $\bigcap_{i \in I} F_i$ is a $j$-bi open set, since $\bigcap_{i \in I} F_i$ is a $j$-bi closed set, which implies that $\bigcap_{i \in I} F_i$ is $j$-bi open set. Hence $\bigcap_{i \in I} F_i$ is $j$-bi closed set.
bi-Alex open sets of $Y$ are $Y, \emptyset, \{v_1\}, \{v_2\}, \{v_1,v_2\}, \{v_1,v_2,v_3\}$
bi-Alex closed sets are $Y, \emptyset, \{v_2,v_3,v_4\}, \{v_1,v_3,v_4\}, \{v_1,v_4\}, \{v_1\}$, and $j$-bi open sets in $(X, A_{\tau_1}, A_{\tau_2})$ are, $X, \emptyset, \{w\}, \{w,x\}, \{w, z\}, \{w, x, y\}, \{w, y, z\}, \{z, w, x\}$

The function $f : X \rightarrow Y$ is $j$-bi-continuous, since $f^{-1}(H)$ is $j$-bi open in $X$ for every $j$-bi open set $H$ in $Y$.

**Theorem 6.3.** Let $X$ and $Y$ be bi-Alexandroff topological spaces and let the function from $X$ into $Y$. Then the following results are equivalent.

(i) $f$ is $j$-bi-continuous.

(ii) For every subset $S$ of $X$, one has $f(S) \subset f^\lambda (S)$.

(iii) For every $j$-bi closed set $F$ of $Y$, the set $f^{-1}(F)$ is $j$-bi closed in $X$.

(iv) For each $a \in X$ and each neighborhood $H$ of $f(a)$, there is a neighborhood $G$ of $a$ such that $f(G) \subset H$.

**Proof.** We prove that $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$ and then $(i) \Rightarrow (iv) \Rightarrow (i)$.

$(i) \Rightarrow (ii)$. Assume that $f$ is $j$-bi-continuous. Let $S$ be a subset of $X$. We prove that if $a \in S$, then $f(a) \in f(S)$. Let $H$ be a neighborhood of $f(a)$, then $f^{-1}(H)$ is a $j$-bi open set of $X$ containing $a$, it must intersect $S$ in some point $b$. Then $H$ intersects $f(S)$ in the point $f(b)$. So that $f(a) \in f(S)$ as desired.

$(ii) \Rightarrow (iii)$. Let $F$ be $j$-bi closed in $Y$ and let $S = f^{-1}(F)$. We have to prove that $S$ is $j$-bi closed in $X$, it is enough to prove that, $S = S$. By elementary set theory, we have $f(S) = f(f^{-1}(F)) \subset F$. Therefore if $a \in S$, then $f(a) \in f(S) \subset f(S) \subset F$, so that $a \in f^{-1}(F) = S$, thus $S \subset S$, so that $S = S$ as desired.

$(iii) \Rightarrow (iv)$. Let $a \in X$ and let it be a neighborhood of $f(a)$. Then the set $G = f^{-1}(H)$ is a neighborhood of $a$ such that $f(G) \subset H$.

$(iv) \Rightarrow (i)$. Let $H$ be a $j$-bi open set of $Y$ and let $a$ be a point of $f^{-1}(H)$. Then $f(a) \in H$; so that by hypothesis, there is a neighborhood $G_a$ of $a$ such that $f(G_a) \subset H$, then $G_a \subset f^{-1}(H)$. It follows that $f^{-1}(H)$ can be written as the union of all $j$-bi open sets $G_a$, so that it is $j$-bi open set.

**Remark 6.4.** If the condition $(iv)$ in above theorem 6.3 holds for the point $a$ of $X$, we say that $f$ is $j$-bi-continuous at the point $a$.

**7. Rules for constructing $j$-bi-continuous function**

**Lemma 7.1.** Let us consider $X$ and $Y$ be bi-Alexandroff topological spaces and $y \in Y$. The function $c_y : X \rightarrow Y$ maps all of $X$ in to the single point $y$ is $J$-bi-continuous, where $c_y$ is called constant function.

**Proof.** Let $H$ be a $j$-bi open set in $Y$. If $y \in H$, then $c_y^{-1}(H) = X$, which is $j$-bi open set. On the other hand, if $y \notin H$, then $c_y^{-1}(H) = \emptyset$, so again we get the preimage is a $j$-bi open.

**Lemma 7.2.** Let $X, Y$ and $Z$ be three bi-Alexandroff topological spaces. If the function $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are $j$-bi-continuous, then their composition map $gof : X \rightarrow Z$ is $j$-bi-continuous.

**Proof.** Let $H \subseteq Z$ be a $j$-bi open, then $(gof)^{-1} = \{a \in X | (gof)(a) \in H\}$.

$= \{a \in X | f(g(a)) \in H\}$.

$= \{a \in X | f(x) \in f^{-1}(g^{-1}(H))\}$.

$(gof)^{-1} = f^{-1}(g^{-1}(H))$.

Now, $g$ is a $j$-bi-continuous, so $g^{-1}(H)$ is $j$-bi open in $Y$ and $f$ is a $j$-bi-continuous, thus $f^{-1}(g^{-1}(H))$ is a $j$-bi open in $X$.

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