Linear programming Monte Carlo method based on remote sensing for ecological restoration of degraded ecosystem

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Abstract. A linear programming Monte Carlo method (LPMC) is proposed to achieve the optimal restoration of the whole degraded coastal wetland ecosystem, i.e. the minimum restoration cost, in the case of uncertain ecological restoration costs of different degraded coastal wetlands. LPMC can comprehensively and systematically complete the dynamic analysis of coastal wetland ecosystem restoration cost, including the uncertainty analysis of ecological restoration cost and the screening of equivalent robust solutions. The applicability of this method is demonstrated by solving a practical problem of ecological restoration of coastal wetlands. The results show that this method can generate the robust optimal solution block for the globally optimal objective function and decision variables under the condition of restoring cost uncertainty, including a variety of optimal solutions adapted to the different needs.

1. Introduction
Coastal wetland is a highly productive ecosystem, providing a series of valuable services. However, despite its high benefits, coastal wetlands are one of the most threatened ecosystems in China. Therefore, the ecological restoration of coastal wetlands in China is becoming increasingly prominent. [1-3]. However, there are many uncertainties in the restoration of coastal wetlands. These uncertainties may aggravate the coastal wetland restoration complexity. Therefore, traditional optimization methods, including linear programming, will become ineffective at compensating for the lack of support from Monte Carlo simulation when there are various uncertainties in the components of the ecological restoration system. One optimal solution to strengthen the degraded coastal wetland ecosystem restoration is to introduce the optimal decision-making technology to increase the return on investment in ecological restoration. For example, Suding showed that the optimal decision-making technology can be established by using the decision-making model [4] and lately, Laughlin built such trait-based models [5]. Although the benefits of the optimal decision-making technology in the restoration of degraded ecosystems have been recognized for many years, they usually provide only the isolated numerical solutions with the poor robustness, and pay little attention to using Monte Carlo simulation to provide a set of robust solutions for the different needs. This article provides a linear programming Monte Carlo (LPMC) method to solve these problems, which can minimize the robust cost of ecological restoration investment.
2. Material and methods

The following work was conducted in 1997 in Dafeng of China. The coastal wetland area of Dafeng City is about 73,300 hectares, including 11 types of wetland utilization. Nearly cloudless TM 5 image was used to capture the area of these wetlands.

To formalize the optimization of ecological restoration, let $V_1$ and $V_2$ represent the first optimization objective to maximize the value of wetland ecosystem services and the second optimization objective to minimize the cost of wetland ecosystem restoration, respectively. The following formula is used to solve the optimization problem of ecological restoration:

$$ROI = \max \left( \frac{V_1}{V_2} \right) = \frac{\max V_1}{\min V_2}$$ (1)

We let a linear programming variable $X_{ij}$ be equal to area that is changed from the wetland area of type $i$ to the wetland area of type $j$. Let $\hat{C}_{ij}$ be a random variable representing the cost of this area change. The equation (1) can be defined as two objective functions for linear programming: $\min V_2$ and $\max V_1$ as follows.

$$\min V_2 = \sum_{i=1}^{11} \sum_{j=1}^{11} \hat{C}_{ij} X_{ij}$$ (2)

Subject to:

$$\sum_{i=1}^{11} X_{ij} = \left| X_{ij}^\text{vi} - \hat{X}_{ij} \right|, \quad \sum_{j=1}^{11} X_{ij} = \left| X_{ij}^\text{vi} - \hat{X}_{ij} \right|, \quad X_{ij} \geq 0$$ (3-5)

$$\max V_1 = \sum_{i=1}^{11} P X_{ij}$$ (6)

Subject to:

$$X_{2,2} + X_{5,5} \geq 0.2 \hat{X}, \quad X_{1,1} + X_{4,4} + X_{9,9} + X_{10,10} \geq 0.57 \hat{X}, \quad X_{3,3} + X_{11,11} \geq 0.17 \hat{X}$$ (7-9)

$$X_{1,1} + X_{2,2} + X_{3,3} + X_{4,4} + X_{5,5} + X_{8,8} + X_{10,10} + X_{11,11} \geq 0.9 \hat{X}, \quad \sum_{i=1}^{11} X_{ij} P_i - \sum_{i=8}^{11} X_{ij} P_i \leq 0$$ (10-11)

$$\sum_{i=1}^{11} X_{ij} - \sum_{j=1}^{11} X_{ij} \leq 0, \quad \sum_{i=1}^{11} X_{ij} = \hat{X}, \quad X_{ij} \geq 0, \quad \hat{C}_{ij} = 0, \quad i = 1, \ldots, 11; \quad j = 1, \ldots, 11$$ (12-16)

where $X_{ij}^\text{vi}$ = optimal planning area for $i$th wetland use type, $\hat{X}_{ij}$ = actual area for $i$th wetland use type, $X_{ij}$’s ecosystem service value, $i = 1, \ldots, 11$, $X_{1,1}$ = cultivated land area, $X_{2,2}$ = forest land area, $X_{3,3}$ = aquaculture pond area, $X_{4,4}$ = seawall bare area, $X_{5,5}$ = seawall forest area, $X_{6,6}$ = residential land area, $X_{7,7}$ = salt field area, $X_{8,8}$ = grassland area, $X_{9,9}$ = bare land area, $X_{10,10}$ = area for beach reclamation for land, $X_{11,11}$ = area of water reclamation for land, $\hat{X}$ = total area of coastal wetlands, $0.2 \hat{X} \wedge 0.57 \hat{X} \wedge 0.17 \hat{X} \wedge 0.9 \hat{X}$ = Dafeng municipal government’s plan, $P_1 = 0.92 \times 10^4$km$^2$, $P_2 = 2.61 \times 10^4$km$^2$, $P_3 = 0.04 \times 10^4$km$^2$, $P_4 = 0.20 \times 10^4$km$^2$, $P_5 = 1.31 \times 10^4$km$^2$, $P_6 = 0.00 \times 10^4$km$^2$, $P_7 = 0.04 \times 10^4$km$^2$, $P_8 = 2.32 \times 10^4$km$^2$, $P_9 = 0.01 \times 10^4$km$^2$, $P_{10} = 0.23 \times 10^4$km$^2$, $P_{11} = 0.04 \times 10^4$km$^2$, $E(\hat{C}_{1,2}) = C_{1,2} = 1.00 \times 10^4$S, $E(\hat{C}_{1,8}) = C_{1,8} = 0.40 \times 10^4$S, $E(\hat{C}_{3,2}) = C_{3,2} = 1.30 \times 10^4$S, $E(\hat{C}_{3,8}) = C_{3,8} = 0.60 \times 10^4$S, $E(\hat{C}_{9,2}) = C_{9,2} = 1.20 \times 10^4$S, $E(\hat{C}_{9,8}) = C_{9,8} = 0.50 \times 10^4$S.

Linear programming was used to find the optimal settings of decision variable $X_{ij}$. Monte Carlo method is usually used to reduce the impact of uncertainty on complex decision-making and provides an implementation framework for LPMC [6, 7]. Let $\min V_{2ij}$ be a random sample $i$ of LPMC solution $j$. This sample $i$ obeys a normal distribution with mean $\min V_{2\hat{C}_{ij}}$ and standard deviation $\sigma'_{ij}$ as follows.
\[
\min V_2 = \frac{1}{n} \sum_{i=1}^{n} \min V_{2,i}, \quad \sigma'_j = \left( \frac{1}{n-1} \sum_{i=1}^{n} (\min V_{2,i} - \min V_{2,i})^2 \right)^{1/2}
\]  

(17-18)

The confidence interval with coefficient \( \gamma_j \) for \( \min V_2 \) is \((L, U)\)

\[
L = \min V_2 - T_{n-1,j}^{-1} \left( \frac{1+\gamma_j}{2} \right) \frac{\sigma'_j}{n^{1/2}}, \quad U = \min V_2 + T_{n-1,j}^{-1} \left( \frac{1+\gamma_j}{2} \right) \frac{\sigma'_j}{n^{1/2}},
\]

(19-20)

where \( c = T_{n-1,j}^{-1} (\cdot) \) is the quantile function for the \( t \) distribution with \( n - 1 \) freedom degrees, \( L \) is for lower limit of confidence interval, and \( U \) is for upper limit of confidence interval. Here, what we are interested in is how to test the equivalence between the LP and LPMC solutions. The hypotheses about this equivalence are

\[
H_0 : \min V_2 = \min V_{2,0} \quad H_j : \min V_2 \neq \min V_{2,0}.
\]

(21-22)

For each \( \min V_{2,0} \) of LP, we can use \((L, U)\) to compute a level \( \alpha_h=1-\gamma_j \) test of the hypotheses. The test will accept \( H_j \) if \( \min V_{2,0} \) is not in \((L, U)\).

3. Results

At LPMC, the uniform distribution of ecological restoration costs of coastal wetlands is mean \( E(\hat{C}_{ij}) \) plus or minus \( E(\hat{C}_{ij}) \times 10\% \), i.e. between \( E(\hat{C}_{ij})+E(\hat{C}_{ij}) \times 10\% \) and \( E(\hat{C}_{ij})-E(\hat{C}_{ij}) \times 10\% \). LP and LPMC give the optimal solution of Dafeng coastal wetland as shown in tables 1-3. Here, the block of LPMC solution is a set of LPMC solutions with the same confidence level, that is, the solutions of these LPMCs are equivalent and can be freely selected according to different needs. Therefore, there is little difference in the cost of ecological restoration between different solutions in the same solution block, while the ecological restoration cost of solutions in the different blocks may vary greatly.

| Table 1. Ecological restoration area for LPMC block |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Type | Object | \( X_{12} \) | \( X_{13} \) | \( X_{14} \) | \( X_{15} \) | \( X_{16} \) |
| LP solution | 1193.37 | 68.70 | 0.00 | 0.00 | 42.60 | 50.77 | 6.27 |
| LPMC solution 01 | 1176.41 | 68.70 | 0.00 | 0.00 | 42.60 | 50.77 | 6.27 |
| LPMC solution 02 | 1201.52 | 17.93 | 50.77 | 0.00 | 93.37 | 0.00 | 6.27 |
| LPMC solution 03 | 1210.61 | 62.43 | 0.00 | 6.27 | 48.87 | 50.77 | 0.00 |
| LPMC solution 04 | 1209.52 | 11.66 | 50.77 | 6.27 | 99.64 | 0.00 | 6.27 |
| LPMC solution 05 | 1173.39 | 17.91 | 50.79 | 0.00 | 93.39 | 0.00 | 6.25 |
| LPMC solution 06 | 1118.01 | 68.70 | 0.00 | 0.00 | 42.60 | 50.79 | 6.25 |

| Table 2. Ecological restoration cost for LPMC block |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Type | Object | \( X_{12} \) | \( X_{13} \) | \( X_{14} \) | \( X_{15} \) | \( X_{16} \) |
| LP solution | 1193.37 | 10.00 | 13.00 | 12.00 | 4.00 | 6.00 | 6.00 |
| LPMC solution 01 | 1176.41 | 9.80 | 13.12 | 12.11 | 4.00 | 5.96 | 5.96 |
| LPMC solution 02 | 1201.52 | 10.27 | 12.17 | 12.24 | 3.95 | 6.20 | 6.20 |
| LPMC solution 03 | 1210.61 | 10.42 | 13.27 | 11.19 | 3.93 | 5.86 | 5.86 |
| LPMC solution 04 | 1209.52 | 10.72 | 12.21 | 11.31 | 3.95 | 6.10 | 6.10 |
| LPMC solution 05 | 1173.39 | 10.80 | 11.96 | 12.82 | 3.65 | 5.98 | 5.98 |
| LPMC solution 06 | 1118.01 | 9.32 | 12.67 | 12.43 | 3.95 | 5.42 | 5.42 |
While our method uses the actual distribution of uncertainties to expand the application of degraded ecosystem development and ecological restoration, that is, to maximize the ecosystem service values and minimize the ecological restoration cost while ensuring the robustness and flexibility. Therefore, LPMC solutions have better robustness and flexibility. Our research results provide a practical tool for the joint optimization of degraded ecosystem development and ecological restoration, that is, to maximize the ecosystem service values and minimize the ecological restoration cost while ensuring the robustness and flexibility of solutions. Therefore, we propose to implement Monte Carlo simulation for the wider distribution of uncertainties to expand the applications of LPMC.

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