Einstein-Podolsky-Rosen Correlations in Deuteron Photodisintegration

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Abstract. Einstein-Podolsky-Rosen correlations have so far been measured only between pairs of photons and between pairs of protons. It is proposed to measure these correlations between the proton and the neutron emerging from the breakup of the deuteron induced by gamma rays near threshold. The feasibility of the experiment is discussed. Polarimeters with substantially higher overall efficiency than the presently reported value of about $10^{-4}$ are needed in order to get enough events.

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1 Introduction

The interesting feature of the Einstein-Podolsky-Rosen (EPR) correlations between spatially separated particles is that they imply a violation of Bell’s inequality and hence reveal a certain fundamental nonlocality of nature [1]. This is predicted by quantum mechanics and has by now been confirmed in quite a number of experiments [2]. Nevertheless, all those experiments are only concerned with pairs of photons, except one, which is concerned with pairs of protons [3]. In view of the importance of nonlocality for our understanding of nature and in view of the fact that specific shortcomings of the individual experiments still permit loopholes to escape from the acceptance of nonlocality [4], it is desirable to investigate a larger variety of physical situations. Besides this I have a special reason for suggesting this experiment because I have proposed an alternative to the Copenhagen interpretation in quantum mechanics in which I predict that no EPR correlations will show up between elementary particles that are not identical and not particle-antiparticle pairs [5]. Be that as it may, the experiment will be interesting in itself.

A number of proposals which involve particles other than photons and protons have already been made, though not yet performed. The N and the O atom obtained by (pre)dissociation of an NO molecule have been considered in [6], and the Na atom pairs obtained by dissociation of Na$_2$ molecules in [7]. And there exists by now quite a number of more or less detailed proposals concerned with particle-antiparticle pairs such as $e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$, $\bar{K}K$, $\Lambda\bar{\Lambda}$ and $B^0\bar{B}^+$ [2,8,9].

The proposed experiment differs from the above-mentioned ones in that it is concerned (1) not with atoms but with elementary particles, and (2) with elementary particles that are not particle-antiparticle pairs.
2 Typical Experimental Situation

A typical experimental arrangement for measuring EPR correlations are two spin-\(1/2\) particles which originate from a collision or decay in a spin singlet state and fly apart in opposite directions. Particle 1 then enters the Stern-Gerlach-type apparatus A and particle 2 apparatus B. Particle 1 is deflected either upward or downward with respect to the spin-reference axis \(a\) of apparatus A and shows up with either spin up \((r_A = +1)\) or spin down \((r_A = -1)\). The same applies to particle 2 in apparatus B with axis \(b\). \(a\) and \(b\) are unit vectors. If \(E(a, b)\) is the average of the product \(r_A r_B\) the Bell inequality (in one of its many equivalent forms) reads [1]

\[
|E(a, b) + E(a', b') + E(a', b) - E(a, b')| \leq 2. \tag{1}
\]

Let \(P(r_A, r_B|\vartheta)\) be the probability of particle 1 being pushed into a spin-\(r_A\) state and particle 2 into a spin-\(r_B\) state, where \(\vartheta\) \((0 \leq \vartheta \leq \pi)\) is the angle between the axes \(a\) and \(b\). Standard quantum mechanics predicts the expressions

\[
P(r_A, r_B|\vartheta) = \frac{1}{4}(1 - r_A r_B \cos \vartheta), \tag{2}
\]

\[
E(a, b) = -ab = -\cos \vartheta \tag{3}
\]

for the singlet state [1]. This can lead to a violation of Bell’s inequality for certain choices of \(\vartheta\) and is therefore locally inexplicable [1].

3 The Deuteron Case. Singlet State

The above situation can easily be arranged in the photodisintegration of the deuteron induced by photons with LS (laboratory system) energies \(E_\gamma\) above the threshold of 2.226 MeV. Up to 2.3 MeV the total cross section rises as [10,11]

\[
\sigma = 675 \frac{\sqrt{E_\gamma/E_u - 2.226}}{(E_\gamma/E_u)(E_\gamma/E_u - 2.149)} \text{ mb}, \quad E_u = 1 \text{ MeV}. \tag{4}
\]

The cross section reaches its maximum of 2.5 mb at about 4.4 MeV, and at 22 MeV it has decreased to 0.5 mb. Up to \(\approx 2.4\) MeV formation of the spin singlet state (M1 transition, \(3S \rightarrow 1S\), formula (4)) prevails, then that of the triplet state (E1 transition, \(3S \rightarrow 3P\)) [11,12].

Let us first assume that only the singlet state contributes. The modification due to some contribution of the triplet state will be considered later. To measure the spin correlation one may follow the procedure of the proton-proton scattering experiment [3]. In that experiment protons of 13 MeV kinetic LS energy were scattered by a hydrogen target. Each of the two protons emerging from the scattering was slowed down to 6 MeV and then entered a spin-component analyzer (polarimeter). In this analyzer it was scattered by a carbon foil and then registered by one of two detectors (R or L). These detectors lay in a plane which contained the direction of the proton entering the analyzer. Each detector formed a fixed
angle of 50° with that direction, and the plane with the detectors could be rotated around that direction. The coincidences between the detectors of one analyzer with the detectors of the other were counted \((N_{LL}, N_{LR}, \text{etc.})\). The expression

\[
E_{\text{exp}} = \frac{N_{LL} + N_{RR} - N_{RL} - N_{LR}}{N_{LL} + N_{RR} + N_{RL} + N_{LR}},
\]

after some corrections, represents the experimental value for \(E(a, b)\). It was measured for various angles \(\vartheta\) between the detector planes of the two analyzers (in the center-of-momentum system (CMS) of the protons) and was compared with the maximum value \(E_{\text{max}}(\vartheta)\) compatible with Bell’s inequality. \(E_{\text{max}}(\vartheta)\) was calculated for a selected number of angles, observing invariance of \(E\) under reflection and rotation. A smooth interpolation formula is [8]

\[
E_{\text{max}}(\vartheta) = 2\vartheta/\pi - 1.
\]

This formula, by the way, coincides with the spin correlation formula calculated for the two fragments of an exploding classical bomb [13]. The maximal difference between \(|E(\vartheta)|\) of the formulas (3) and (6) is 0.21 and occurs at \(\vartheta = 40°\). The experimental values confirmed the quantum mechanical formula (3) and for some values of \(\vartheta\) were definitely outside the range (6) allowed by Bell’s inequality. In the deuteron experiment the role of the two protons after scattering is taken over by the proton and neutron after disintegration and the same procedure of comparing \(|E(\vartheta)|\) with \(|E_{\text{max}}(\vartheta)|\) may be adopted here.
4 The Triplet Contribution

If the nucleons emerging from deuteron disintegration are in a spin triplet state, the spin correlation formulas are more complicated. Quantum mechanics in this case predicts

\[ E(a, b) = a_z b_z \quad \text{in } |1, +1\rangle \quad \text{and} \quad |1, -1\rangle \] (7)

\[ E(a, b) = ab - 2a_z b_z \quad \text{in } |1, 0\rangle \] (8)

where \(a_z\) and \(b_z\) are the components of \(a\) and \(b\) in some preferred direction. This direction, that is, the exact wave function or density matrix of the triplet state must also be known, and this requires additional experimental efforts. One might therefore restrict oneself to situations where the nucleons emerge in a pure singlet state. As mentioned in the introduction this occurs when the \(\gamma\)-ray energy is only little above threshold. On the other hand, the cross section for disintegration goes down rapidly when \(E_\gamma\) approaches the threshold, formula (4). One may therefore retain somewhat higher \(\gamma\)-ray energies, where the cross section is larger, but the contribution from the triplet state is still small. Observing that the angular distribution of the nucleons in the singlet state is isotropic and in the triplet state follows a \(\sin^2 \Theta\) law (\(\Theta = \text{angle between the proton and the direction of the } \gamma\text{ rays}\)) one may restrict oneself to protons and neutrons.

We want to consider this point more quantitatively. An estimate of the relative triplet contribution as a function of the photon energy \(E_\gamma\) can be extracted from the empirical and theoretical data presented in [11,12,15,16]: in the range of \(E_\gamma = 2.3 - 5\) MeV the CMS differential cross section can be written as

\[ \frac{d\sigma}{d\Omega} = a_M + a_E + (b_M + b_E) \sin^2 \Theta \] (9)

where the index M denotes the singlet and E the triplet contribution, and

\[ a_M/a_E = 48 \times (E_\gamma/E_u - 2.226)^{-1.75} \] (10)

\[ b_E/a_E = 6910 \times (E_\gamma/E_u)^{-1.89} \] (11)

\[ b_M = 0 \] (12)

\[ E_u = 1\text{ MeV} \]

represent simple numerical fits. From this one obtains, for example, that the relative triplet contribution in forward direction \(\Theta = 0^\circ\)

\[ \frac{(d\sigma/d\Omega)_E}{(d\sigma/d\Omega)_E + (d\sigma/d\Omega)_M} = (1 + a_M/a_E)^{-1} \] (13)

at \(E_\gamma = 5\) MeV reaches 11%.

When a finite opening (half) angle \(\alpha\) around \(\Theta = 0^\circ\) is taken into account, the relative triplet contribution becomes larger. One has to replace \(d\sigma/d\Omega\) by \(\sigma/\Omega = \int_0^\alpha \int_0^{2\pi} (d\sigma/d\Omega) \sin \Theta d\Theta d\varphi/ \int_0^\alpha \int_0^{2\pi} \sin \Theta d\Theta d\varphi\). Using (9) one obtains

\[ \frac{\sigma/\Omega}{\sigma/\Omega} = a_M + a_E + (b_M + b_E) \times \left(\frac{2}{3} + \frac{1}{3} \cos^3 \alpha - \cos \alpha\right) / (1 - \cos \alpha) \] (14)
where the error of the approximation (15) is less than 5% up to $\alpha = 25^\circ$. From (15) and (12) one obtains now the relative triplet contribution at an opening angle $\alpha$ around the forward direction

$$
\frac{(\sigma/\Omega)_E}{[(\sigma/\Omega)_E + (\sigma/\Omega)_M]} = \left(1 + \frac{a_M}{a_E} \frac{1}{1 + (b_E/a_E)(\alpha^2/2)}\right)^{-1}.
$$

(16)

Using (10) and (11) and choosing an opening angle of $\alpha = 2^\circ$ one obtains a contribution of 13% at $E_\gamma = 5$ MeV, which is not much larger than the contribution of 11% obtained for $\alpha = 0^\circ$ from (13).

Now, a contribution of less than 18% of the triplet state is still acceptable because one usually expects the correlations to be in accordance with quantum theory, which in this case predicts values for $|E(a, b)|$ that are still larger than $|E_{\text{max}}(a, b)|$ for some angles $\vartheta$. This is seen in the following way: the inclusion of the triplet state may lead to a smaller value of $|E(a, b)|$ that that calculated for the singlet state only. Let us write (in obvious notation)

$$
E_{\text{st}}(a, b) = (1 - r)E_s(a, b) + rE_t(a, b)
$$

and let us assume the extreme case that $E_t(a, b)$ is just the negative of $E_s(a, b)$ (cf. formula (8) with $a_zb_z = 0$). $r$ is the relative overall contribution of the triplet state. With formula (3) this gives us $E_{\text{st}}(a, b) = E_{\text{st}}(\vartheta) = (2r - 1)\cos \vartheta$, and with $E_{\text{max}}(\vartheta)$ from formula (6) we obtain the difference

$$
\Delta = |E_{\text{st}}(\vartheta)| - |E_{\text{max}}(\vartheta)| = (1 - 2r)\cos \vartheta + 2\vartheta/\pi - 1
$$

(17)

where the last expression holds for $r \leq 0.5$, $\vartheta \leq 90^\circ$. For $r = 0$ (singlet only) formula (17) results in $\Delta = 0.21$ ($\vartheta = 40^\circ$). For larger $r$ $\Delta$ decreases. It reaches zero at $r = 0.18$ ($\vartheta = 90^\circ$), but it is still $\Delta = 0.07$ for $r = 0.1$ ($\vartheta = 50^\circ$). Thus, the better the experimental accuracy the smaller the value of $\Delta$ that can be resolved, and the larger the value of $r$ that can still lead to a demonstration of a violation of Bell’s limit. Of course, with this procedure, considering only an upper limit of the triplet contribution, we cannot demonstrate that the violation is just the one predicted by quantum mechanics. Nevertheless a violation of Bell’s inequality can in principle be demonstrated.
5 The Rate of Events

If the experiment is done in the way discussed above, comparing $|E(\theta)|$ with $|E_{\text{max}}(\theta)|$, the accuracy is determined by that of $E_{\text{exp}}$ from formula (5). The $N_{\text{LL}}$ etc. are numbers of coincidences, and the higher these numbers the better the statistical accuracy. In the following we shall only consider this statistical accuracy, that is, the rate of coincidence events. This is always an important point in experiments of this nature.

The rate is proportional to the product of the incoming $\gamma$-ray flux, the number of target deuterons, the photodisintegration cross section, and the square of the overall efficiency of the polarization analyzers. In order to obtain a rough estimate let us assume that the deuteron experiment is done in exactly the same way as the pp-scattering experiment [3], so that we only have to replace the flux of incoming protons ($5 \times 10^{18}/\text{m}^2\text{s}$) by that of incoming photons ($10^{18}/\text{MeVs}$, [17], assuming a 1 MeV $\gamma$-ray energy range and assuming that all of the beam’s cross section can be utilized) and the pp cross section ($5 \times 10^{-29}\text{m}^2$) by the photodisintegration cross section ($2.5 \times 10^{-31}\text{m}^2$, maximum value). The number of target particles is assumed to be the same in both experiments ($1.34 \times 10^{19}$). These numbers result in coincidence rates in the deuteron experiment that are smaller by a factor $10^{10}$ than those in the pp experiment. The main reason for the low coincidence rates, both in the deuteron and in the pp experiment, is the low “transmission” or “overall polarimeter (analyzer) efficiency”. This efficiency is defined as the number of particles that are deflected and registered in one of the detectors of the analyzer divided by the total number of particles that entered the analyzer. It is of the order of $10^{-4}$ for the considered protons and neutrons of a few MeV [18]. This value contributes quadratically because the number of coincidences between two equal analyzers is proportional to the square of their efficiency.

Since the polarimeters utilize the scattering of nucleons in carbon foils their low efficiencies are mainly due to the small cross section for this process, which is of the order of a barn ($10^{-28}\text{m}^2$) for nucleons of a few MeV. As far as I can see there might still be hope to obtain larger efficiencies when one considers nucleons of lower energy, down to 100 keV or even less. At these energies the nucleon-carbon cross sections increase considerably, especially for protons, and they exhibit strong fluctuations and large bumps at nuclear resonances. So one might perhaps find special energy values where these cross sections are large and lead to high efficiencies. Or one may be able to invent novel types of polarimeters for these slow nucleons. (Stern-Gerlach magnets for neutrons?). This, together with an effort to get a high flux of incoming $\gamma$ rays and a large number of target deuterons, might lead to sufficient events in a reasonable time.

Another aspect of a low efficiency is that it permits a loophole to escape from accepting nonlocality [3]. Even if the efficiency is high enough to lead to a sufficient number of events it may not be high enough to close that loophole. I think, however, that even so the experiment would be interesting enough.

Slow nucleons from deuteron photodisintegration means $\gamma$ rays near thresh-
old, with the additional advantage of a pure singlet state. Here, in very good approximation it is \( E_\gamma = 2.226 \text{MeV} + 2E_{\text{kin}} \), where \( E_{\text{kin}} \) is the kinetic energy of one nucleon, hence \( E_\gamma > 2.4 \text{MeV} \) for \( E_{\text{kin}} > 100 \text{keV} \). Relativistic kinematics [19] shows that for \( 2.4 \text{MeV} < E_\gamma < 35 \text{MeV} \) the velocity of the nucleons in the CMS is always more than 10 times the velocity of the CMS in the LS. So, in the CMS↔LS transformation the angles never change by more than 10\% and the kinetic energies never by more than 20\%.

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