Determining the track condition using soil properties - part 1

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Abstract. The track condition in conventional ballast construction is decisively determined by the condition of the sub-ballast and the subgrade. An indicator for the assessment of the track condition is the track modulus, which can be calculated by measuring the rail deflection under load or according to the soil properties. The track modulus describes the ratio between vertical loading and elastic deflection of the sleeper support. This is the first of two papers concerning a theoretical approach to estimate the track modulus of the soil by soil properties. The approach’s advantage is to be completely independent from the superstructure condition. Therefore the components of the superstructure along the track axe must not know for calculating the track modulus. Borings with direct ground exploration results and ground penetrating radar measurements are used to determine the soil condition at the track level. After the allocation of the soil properties to each layer, the cone model is used to calculate this average value of the soil properties. Subsequently, the track modulus of the soil is determined by using the average value of the soil properties. Based on the value of the track modulus of the soil, the track condition can then be determined.

1. Introduction
Faults in the track geometry should be detected as soon as possible to determine the track condition. Particularly in the case of problems in the sub-ballast and subgrade, it is important to explore the condition of the ground before a maintenance measure is planned. For this purpose, a number of available methods can be used. These methods include determining the track geometry by using track recording vehicles; calculating the vertical track stiffness (ratio point load to deflection) or the track modulus (German railway literature [1], ratio contact pressure to deflection) by measuring the rail deflection under load [2-8]; and utilizing ground penetrating radar measurements in combination with direct ground explorations such as heavy dynamic penetration tests and drilling [9-13]. The combination of these results should subsequently be used to determine the ground conditions at track level in conventional ballast construction.

2. Estimation of the track modulus of the soil using soil properties
In [14] and [15] a theoretical approach was developed to estimate the track modulus of the soil by using the average values of the soil properties (Poisson’s ratio and density) of the native, inhomogeneous soil (see figure 2). It is assumed that the soil has a homogeneous, liner elastic behaviour even in inhomogeneous layer storage. For inhomogeneous layer storage, the soil layers are summarized to a single homogeneous soil according to the cone model [16, 17] (see figure 2). Poisson’s ratio and the soil density influence the stiffness and damping behaviour of the track [13]. To determine the average value
of the soil properties, borehole profiles and drilling diagrams can be used. The layer heights, the results of the drilling diagrams, the deposit density or the consistency of each soil layer, and the condition of the ballast bed must be known in advance. Subsequently, the soil properties of the Poisson’s ratio and the density can be assigned to the ballast bed and to each soil layer. The soil properties, corresponding with the determined deposit densities and consistencies of individual soil layers, were taken from [14] (see figure 1). Furthermore, the average Poisson’s ratio and soil density were calculated using the cone model [16, 17]. The intended purpose of the cone model is to consider only those ground surfaces that are significantly affected by the traffic load [16]. According to [13], it is assumed that the load distribution takes place at an angle of 60° for a track in good condition (see figure 2, load spreading angle). The stressed surfaces are in a circular area from a bird’s-eye perspective. In the cone model method, the surface of the beam on elastic foundation from the method according to Zimmermann and Winkler [18, 19] with a width of assumed equivalent longitudinal supports and a length double the value of the characteristic length is converted into the same area as a circular area with the radius of the circular replacement surface area (see equation (1.7) and figure 2) [13]. The characteristic length of the rectangular replacement of the beam on elastic foundation in the cone model is determined according to methods from Zimmermann and Winkler [18, 19], whereby the track modulus value, which is calculated from the given deflection of the rail under load, is used (see equation (1.3)). The track modulus denotes the ratio between vertical loading and elastic deflection of the sleeper support for tracks in conventional construction methods. The unit of the track modulus according to German railway literature is MN/m³, it denotes the ratio of the contact pressure under the sleeper and the elastic deflection of the rail [1]. Methods for calculating the track modulus according to German railway literature are described in, [1], [18-21]. Typical track modulus values describing the track condition are listed in table 1.

| Track condition | Track modulus of the soil (MN/m³) |
|-----------------|----------------------------------|
| Very poor       | < 50                             |
| Poor            | ≥ 50                             |
| Good            | ≥ 100                            |
| Very good       | ≥ 150                            |
| Concrete slab   | ≥ 300                            |

The equations (1.1) – (1.15) for calculating the track modulus of the soil by rail deflection or soil properties are shown below. Two numerical examples were given in table 2.

The half sleeper support surface area can be calculated by the length of the half sleeper support surface area (see figure 2) and the sleeper width.

\[ A_s = l_a \cdot b_s \]  

\[ A_s \] half sleeper support surface area \[ [m^2] \]

\[ b_s \] sleeper width \[ [m] \]

\[ l_a \] length of the half sleeper support surface area \[ [m] \]

The width of the assumed equivalent longitudinal supports depends on the half sleeper support surface area and the spacing between the sleepers.

\[ b_l = \frac{A_s}{a} = \frac{l_a \cdot b_s}{a} \]  

\[ b_l \] width of assumed equivalent longitudinal supports \[ [m] \]

\[ a \] sleeper spacing \[ [m] \]
The track modulus depends on the type of rail and sleeper used; the static, vertical force acting on the rail; and the deflection of the rail under load.

\[
C = \frac{1}{4 \cdot b_l \cdot (E_r \cdot I_r)^{\frac{1}{3}}} \left( \frac{Q}{10^3} \right)^{\frac{4}{3}} \frac{y^2}{y^3}
\]  

(1.3)

- \( C \): track modulus [MN/m³]
- \( Q \): static, vertical force acting on the rail [kN]
- \( y \): deflection of the rail under load [m]
- \( E_r \): rail modulus of elasticity [MN/m²]
- \( I_r \): rail modulus of inertia [m⁴]

The track modulus of the rail pad describes the relationship between the vertical stiffness of the rail pad and the sleeper support surface area.

\[
k_{zw} = \frac{C_{zw}}{(A_s / 2) \cdot 10^6}
\]  

(1.4)

- \( k_{zw} \): vertical stiffness of the rail pad [N/m]

The track modulus of the soil depends on the elastic elements installed at the track. After evaluating the ground penetrating radar data, it could be determined that the track moduli of the soil calculated by rail deflection were only influenced by the stiffness of the rail pad.

\[
C_b = \frac{1}{2} \frac{1}{C - C_{zw}}
\]  

(1.5)

- \( C_b \): track modulus of the soil [MN/m³]
- \( C_{zw} \): track modulus of the rail pad [MN/m³]

The elastic length can be calculated by using the rail modulus of elasticity, the rail moment of inertia, the width of the assumed equivalent longitudinal supports, and the track modulus (equation 1.3)

\[
L = \sqrt[3]{\frac{4 \cdot E_r \cdot I_r}{C \cdot b_l}}
\]  

(1.6)

- \( L \): characteristic length [m]

The radius of the circular replacement surface area depends on the characteristic length and the width of the assumed equivalent longitudinal supports.

\[
\pi r_0^2 = b_l \cdot 2 \cdot L
\]  

(1.7)

- \( r_0 \): radius of the circular replacement surface [m]

The replacement surface area of any soil layer can be calculated by using the distance from the first replacement surface area (surface of the soil) to the conical tip and the thickness of the respective soil layer using the Pythagorean Theorem (see equations (1.3), (1.4), (1.5) and (1.6)) [16]. Moving downwards from the conical tip, the truncated cone of the cone model is limited by the so-called “semi-infinite soil layer”. In the following equations, the semi-infinite soil layer is indicated with the index 0.
\[ z_{sp} = \sqrt{\frac{6 \cdot b \cdot L}{\pi}} \] (1.8)

\[ z_{sp} \] distance from the first replacement surface area to the conical tip [m]

\[ z_i = z_{sp} + \frac{d_i}{2} + d_{i+1} + \ldots + d_n \] for \( i=1, \ldots, n \) (1.9)

\[ z_i \] distance of the soil layer \( i \) to the conical tip [m]

\[ d_i \] layer height [m]

\[ z_0 = z_{sp} + \sum_i d_i \] for \( i=1, \ldots, n \) (1.10)

\[ z_0 \] distance of the lowest soil layer to the conical tip [m]

\[ A_i = \frac{z_i^2 \cdot b \cdot 2 \cdot L}{z_{sp}^2} \] for \( i=0, \ldots, n \) (1.11)

\[ A_i \] replacement surface area of any soil layer in the truncated cone [m²]

From the knowledge gained from the replacement surfaces, the distances to the conical tip and the soil properties of the individual soil layers are calculated according to the average values of the Poisson’s ratio and the soil density including the ballast bed. The equations for this process described below:

\[ \nu_m = \frac{\nu_0 z_0}{A_0} + \sum_i \frac{\nu_i d_i}{A_i} \] for \( i=1, \ldots, n \) (1.12)

\[ \nu_m \] average value of the Poisson’s ratio of the soil [-]

\[ \rho_m = \frac{\rho_0 z_0}{A_0} + \sum_i \frac{\rho_i d_i}{A_i} \] for \( i=1, \ldots, n \) (1.13)

\[ \rho_m \] average soil density [m³]

Afterwards, the average soil properties, calculated by means of the cone model, are inserted into the functions 1.14 and 1.15. The lower and upper limits of the track modulus of the soil are determined to be 20 MN/m³ for a very poor quality of the track, and 435 MN/m³ for a very good quality of the track [3, 7, 14, 15, 22, 23].

\[ v_m = 1.362 \cdot C_b^{-0.142 - 0.4} \quad C_b \in (20, 435) \] (1.14)

\[ C_b \] track modulus of the soil [MN/m³]

\[ v_m \] average Poisson’s ratio [-]

\[ \rho_m = 2.05 \cdot C_b^{0.1 - 1.91} \quad C_b \in (20, 435) \] (1.15)

\[ \rho_m \] average soil density [g/cm³]

The function of the Poisson’s ratio and the density was determined by conducting a literature review to establish the relationship between the track modulus of the soil and the associated soil properties [23]. This empirical correlation is shown in figure 1 and describes the potential regression by means of an
approximation function \( y = a \cdot x^b + c \). With the knowledge of the average value of the soil properties, the track modulus of the soil can be calculated by using equation (1.14) and equation (1.15).

**Figure 1.** Approximation functions of the Poisson's ratio and the density for different soil types [14, 23].
2.1. Numerical example
The above-described method for estimating the track modulus of the soil by using the soil properties is shown for two practical examples. In the first example, the track modulus of the soil is derived for a soft soil with near homogeneous soil conditions. In the second example, the track modulus of the soil is estimated for stiff soil layers on soft soil layers.

Table 2. Overview of the track modulus of the soil at given borehole positions.

| Input parameters                      | Borehole 1 | Borehole 2 |
|---------------------------------------|------------|------------|
| Static vertical force acting on the rail | 100 kN     |            |
| Deflection of the rail under load     | 0.00359 m  | 0.00169 m  |
The rail deflection measurement data and borehole profiles with the corresponding drilling diagrams were provided. The resonance effects from measuring the rail deflections under load do not play a major role due to the maximum measuring speed of 15 km/h. The basic value of the longitudinal sleeper structure of the beam on elastic foundation was calculated for the two examples from the track modulus by using the measured rail deflection under load according to the approach of Zimmermann and Winkler [18, 19]. If the deflection is unknown, the characteristic length can be estimated from the results of the drilling diagrams. In order to calculate the track modulus using the measured rail deflections, the following input parameters were determined: static, vertical force acting on the rail; rail type; sleeper type; and sleeper spacing. The type of sleeper was determined by means of photo recordings along the considered track sections. Under the assumption that the two borehole profiles and their corresponding drilling diagrams were made on a heavily-loaded railway track, the sleeper spacing was set to 0.6 m. Furthermore, the tracks of the examined railway were assumed as being continuous, main tracks. The rail profile UIC 60 was ascertained and the maximum axle load applied to the track amounted to 200

The table below summarizes the input parameters for the two boreholes:

| Input parameters                        | Borehole 1                  | Borehole 2                  |
|-----------------------------------------|-----------------------------|-----------------------------|
| Type of rail                            | UIC60                       | concrete sleeper            |
| Rail modulus of elasticity              | 2.1·10^5 MN/m²              |                            |
| Rail moment of inertia                  | 3.055·10^-5 m⁴              |                            |
| Vertical stiffness of the rail pad       | 6.0·10⁸ N/m                 |                            |
| Type of sleeper                         | wooden sleeper              |                            |
| Sleeper length                          | 2.60 m                      | 2.60 m                      |
| Sleeper width                           | 0.26 m                      | 0.30 m                      |
| Sleeper spacing                         | 0.60 m                      | 0.60 m                      |
| Load spreading angle                     |                             | 60°                         |
| Layer depth, Poisson’s ratio, and density|                            |                            |
| Gravel: new                             | 0.15 m                      | 0.330                       |
| Gravel: round and contaminated          | 0.20 m                      | 0.450                       |
| Silt: soft                             | 0.15 m                      | 0.475                       |
| Silt: soft                             | 1.50 m                      | 0.475                       |
| Semi-infinite soil layer                | ∞                           | 0.475                       |

The rail deflection measurement data and borehole profiles with the corresponding drilling diagrams were provided. The resonance effects from measuring the rail deflections under load do not play a major role due to the maximum measuring speed of 15 km/h. The basic value of the longitudinal sleeper structure of the beam on elastic foundation was calculated for the two examples from the track modulus by using the measured rail deflection under load according to the approach of Zimmermann and Winkler [18, 19]. If the deflection is unknown, the characteristic length can be estimated from the results of the drilling diagrams. In order to calculate the track modulus using the measured rail deflections, the following input parameters were determined: static, vertical force acting on the rail; rail type; sleeper type; and sleeper spacing. The type of sleeper was determined by means of photo recordings along the considered track sections. Under the assumption that the two borehole profiles and their corresponding drilling diagrams were made on a heavily-loaded railway track, the sleeper spacing was set to 0.6 m. Furthermore, the tracks of the examined railway were assumed as being continuous, main tracks. The rail profile UIC 60 was ascertained and the maximum axle load applied to the track amounted to 200
kN. At the positions of the two given borehole profiles, the measured rail deflection is 3.59 mm for borehole 1 and 1.69 mm for borehole 2. The deposit density (or consistency) for each individual soil layer was determined by using the guide values listed in [24]. In table 2, the calculated track moduli of the soil values are presented. The calculated track moduli are derived from the average values of the density and the Poisson’s ratio of the soil layers. In order to verify the results, the track modulus of the soil was calculated by using the measured rail deflection according to Zimmermann and Winkler [18, 19]. By comparing the track moduli of the soil values for borehole 1 listed in table 2, it becomes clear that the track modulus values of the soil are almost identical due to the near homogeneous soil layers. Because the soil in borehole 1 is not highly load-bearing, the values of the track modulus of the soil are comparatively small, which suggests a very poor track condition according to table 1 [4]. The values for the track modulus of the soil for borehole 2 are different. Although the soft soil layers and the semi-infinite soil layer have an influence on the track modulus of the soil estimated by using the soil properties, the soft layers at the ground level do not have a great influence on the load transfer of the track. The stiff soil layer compensates the most of the vertical load. This phenomenon can be seen by the comparatively large value of the track modulus of the soil calculated by using the measured rail deflection. A track modulus of soil of 67 MN/m³ indicates a poor track condition according to table 1 [4].

2.2. Recommendations for practical implementation

The explicit framework conditions for using the theoretical approach developed by [14, 15] and [23] for determining the track modulus values on the basis of soil properties are:

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**Figure 3.** Example of the borehole profiles drilling diagram.
The correlation between the track modulus of the soil and the relevant soil properties of the Poisson’s ratio and the density is valid for a cross-sleeper track in conventional construction methods.

The borehole depth of 2 meters is defined for calculating the track modulus of the soil. Therefore, the soil properties for a depth of 2 meters and the installed components of the track must be known.

Track segments with significantly-variable sleeper support surface areas (e.g. turnouts) cannot be considered using the developed approach.

Climate factors such as temperature and precipitation can have an influence on the soil properties and the vertical stiffness of the track.

The layer arrangement of soft to stiff soil cannot yet be considered in this method; the approach must be expanded in order to adapt to this purpose (see paper Part 2).

2.3. Factors influencing the calculated track modulus values

For the calculation and/or estimation of the track modulus values using the existing approaches [1, 18, 19, 20, 21] and the developed theoretical approach [14, 15, 23], there are certain factors that must be taken into account that could affect the results of the track modulus values. A partially-faulty position of a borehole may negatively influence the track modulus values. In this case, incorrect deflection data would be used for the calculation of the track modulus of the soil. Sections of the track including damaged sleepers or rail fasteners could result in an increased rail deflection under load and a smaller calculated track modulus of the soil. These track sections must be excluded from the calculation as well as when determining the track condition. Varied climate factors during the measurements, such as temperature and precipitation, could have an effect on the track stiffness. For this reason, it is recommended that all measurements be performed as close as possible in time to each other. Other factors that must be taken into account when calculating the track modulus using the measured rail deflection are the type of sleeper, sleeper spacing, type of rail pad, and the rail profile. The properties of the installed components can be determined on-site. In equation (1.12) and equation (1.13), it can be seen that the average soil properties of the complete layer package are dependent on the soil layer heights, soil properties of the individual soil layers, and the so-called replacement surface areas. It is assumed that the semi-infinite soil layer possesses the same soil properties as the final soil layer of the borehole profile. If there is a significant deviation of the soil properties between each layer, or if the soil properties and the layer arrangement are not homogeneous (for example stiff on soft soil layers or soft on stiff soil layers) the semi-infinite soil can have a strong influence on the size of the track modulus (see table 2).

3. Conclusions

In this study, a theoretical approach for determining track conditions using soil parameters was tested for practical application. The average values of the soil properties were calculated using the results of direct ground exploration, ground penetrating radar measurements, and usage of the cone model [16, 17]. Subsequently, the track modulus of the soil was determined by the equations (1.14) and (1.15), through which the track condition could be derived. It has been shown that the developed approach can be an alternative to other methods for determining and evaluating the condition of a cross-sleeper track in conventional ballast construction. The application of this approach shows how the values of the soil parameters (Poisson’s ratio and density), as well as the layer heights of the native soil, influence the track modulus of the soil. On the one hand, the benefit of using the developed method is the determination of the track condition with a completely independent process from the superstructure condition. On the other hand, the approach allows the detection of suspected voids between the sleepers and the ballast at the borehole position, depending on the track modulus of the soil or the soil properties. By knowing the soil properties along the railway track by means of ground penetrating radar data, the track condition can be determined with the developed approach. In the end, the method allows for a continuous representation of the track condition (track modulus band) along the railway track.
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