Dispersion-compensated waveguide for difference-frequency generation in quantum-cascade laser

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Abstract. The waveguide structure of a semiconductor quantum-cascade laser is presented, which provides compensation of material dispersion of the refractive index. This structure can be used to generate radiation with a frequency of 1.4 THz due to nonlinear light conversion.

1. Introduction
There has been a steady interest to sources of terahertz radiation over the past decades. This is due to the wide prospects for the use of terahertz radiation for the needs of communications, medicine, spectroscopy, visualization and sensing, etc. [1-2]. One of the most attractive methods for generating terahertz radiation is difference frequency generation in quantum cascade lasers [3-4]. However, this approach appears to be inefficient due to the phase mismatch resulting from the uncompensated dispersion of the refractive index. This is why the maximum THz output power of such lasers is limited to 1s mW in pulsed and few 10s of µW in continuous wave mode [6].

In this work, we study the possibility of difference frequency generation in a dual-frequency quantum-cascade laser, emitting in the 8 μm spectral region. As a prototype, we use the laser, described in [6-7], that provides stable generation of radiation with wavelengths λ₁ = 7800 nm and λ₂ = 8100 nm in a wide temperature range and discuss a waveguide design enabling compensation of dispersion of the refractive index. The difference frequency in this case turns to be equal 1.4 THz, and it can be calculated by formula

\[ \omega_3 = \frac{2\pi c}{\lambda_1} - \frac{2\pi c}{\lambda_2}, \]  

(1)

where c is the speed of light in vacuum.

The difference frequency generation occurs while radiation propagates through the waveguide structure. The structure is an active region placed between the claddings which consist of different semiconductor layers, differing both in the materials used, their doping levels, and thicknesses.

The main factor limiting the efficiency of nonlinear light conversion is the coherence length, that is, the distance along which the field amplitudes have the same signs. When propagating at distances exceeding the coherence length, the radiation intensity reduces due to wave interference. In the case of ideal coherence (also called phase matching), the coherence length becomes infinite. In real situations, the material dispersion of the refractive index leads to the fact that the coherence length has finite value, usually not exceeding 10s of μm. However, the use of semiconductor metamaterials in many cases makes it possible to compensate the dispersion of the refractive index and thus increase the coherence length. Two approaches for difference frequency generation in semiconductor metamaterial are
described in [8]. In one case, the difference frequency wave propagates perpendicularly to the pump radiation fluxes. In this case phase matching condition must be applied only to the pump radiation waves:
\[ \mathbf{k}_1 = \mathbf{k}_2. \] (2)

In another case, all three light fluxes propagate through the structure in one direction, and the wavenumbers satisfy the relation
\[ \mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3. \] (3)

From our point of view, the first approach is the most suitable for our purposes.

2. Waveguide structure description

In experiment [7], the active region of the quantum cascade laser consisted of fifty cascades based on alternating In\textsubscript{0.53}Ga\textsubscript{0.47}As quantum wells and Al\textsubscript{0.48}In\textsubscript{0.52}As barriers (for a detailed description, see [7]). The lower waveguide cladding is an In\textsubscript{0.53}Ga\textsubscript{0.47}As layer with a free electron concentration of 5\cdot10^{16} cm\textsuperscript{-3} and a thickness of 500 nm. The upper cladding consists of a metamaterial 3.9 μm thick, which is described below, as well as two In\textsubscript{0.53}Ga\textsubscript{0.47}As contact layers: with a concentration of 1\cdot10^{17} cm\textsuperscript{-3} and a thickness of 100 nm and with a concentration of 1\cdot10^{19} cm\textsuperscript{-3} and a thickness of 20 nm. The metamaterial consists of alternating layers of In\textsubscript{0.53}Ga\textsubscript{0.47}As doped with 1\cdot10^{17} cm\textsuperscript{-3} and In\textsubscript{0.53}Ga\textsubscript{0.47}As with a concentration of about 2\cdot10^{18} cm\textsuperscript{-3}. The number of pairs of such layers and their thicknesses will be determined below by calculating the waveguide dispersion.

3. Results and discussion

Metamaterial is a set of alternating semiconductor layers differing in thickness and doping levels. The dispersion of waves in such metamaterial is calculated in [8,9]. It is shown that such a material can be represented as a material with refractive index depending on the direction of the light propagation. The corresponding dependence is shown in the figure 1 [8].

![Figure 1](image_url)

**Figure 1.** The dependence of the light propagation angle \( \varphi \) in metamaterial on the difference frequency \( f_3 \) values while the coherence length is 1 cm.
In this work, we use a similar approach to compensate material and waveguide dispersion in the structure of a quantum cascade laser. We show that for efficient difference frequency generation, the metamaterial must have refractive indices \( n_1 = 3.384 \) at \( \lambda_1 = 7800 \) nm and \( n_2 = 3.392 \) at \( \lambda_2 = 8100 \) nm. This follows from the calculation of waveguide dispersion. The dependence of effective index \( N_{\text{eff}} \) of refraction on the metamaterial refractive index is shown on figure 1.

![Figure 1.](image1)

**Figure 2.** The dependence of the effective index \( N_{\text{eff}} \) of refraction on the index of refraction of metamaterial \( n \) at the wavelength 8100 nm.

The effective refraction index of waveguide under study is 3.33 for both wavelengths \( \lambda_1 \) and \( \lambda_2 \). Thus, the problem is reduced to calculating the parameters of metamaterial which provides refractive index values \( n_1 \) and \( n_2 \). The solution of wave equation for metamaterial in Bloch’s approximation is described in [9]. The dispersion equation has the form

\[
P(\omega, q) = R(\omega, k_z),
\]

where \( k_0 = \omega/c \),

\[
P(\omega, q) = \frac{\left(\varepsilon_1(\omega) + \varepsilon_2(\omega)\right)k_0^2 - 2q^2}{2\kappa_1(\omega)\kappa_2(\omega)} \left[sh(a\kappa_1(\omega))ch(b\kappa_2(\omega)) + ch(a\kappa_1(\omega))sh(b\kappa_2(\omega))\right].
\]

\[
R(\omega, k_z) = \cos k_z(a + b).
\]

The solution of this equation allows one to determine the thicknesses \( a \) and \( b \) of doped and undoped layers of metamaterial. Our calculations show that \( a \) must be 130 nm and \( b \) must be 20 nm. The number of periods is 26, so the metamaterial thickness is 3.9 \( \mu \)m. The tolerance of the waveguide structure to variation of its parameters is the subject of further study.
4. Conclusions
It is shown that the use of metamaterials as waveguide claddings of a quantum-cascade laser allows to significantly increase the efficiency of difference frequency generation due to compensation of material dispersion. The structure proposed generates radiation of 1.4 THz frequency when pumped at wavelengths of 7800 and 8100 nm. The difference frequency is generated in the direction perpendicular to the alternating waveguide layers. The calculated values of metamaterial parameters are presented.

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