No Ghost State in the Brane World

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Abstract

We discuss the role of the trace part of metric fluctuations $h_{MN}$ in the Randall-Sundrum scenario of gravity. Without the matter, this field ($h = \eta^{MN} h_{MN}$) is a gauge-dependent term, and thus it can be gauged away. But, including the uniform source $\tilde{T}_{MN}$, this field satisfies the linearized equation $\Box_4 h = 16\pi G_5 T_\mu^\mu$. This may correspond to the scalar $\xi^5$ in the bending of the brane due to the localized source. Considering the case of longitudinal perturbations ($h_{5\mu} = h_{55} = 0$), one finds the source relation $\tilde{T}_\mu^\mu = 2\tilde{T}_{35}$, which leads to the ghost states in the massive modes. In addition, if one requires $T_{14} = 2(T_{22} + T_{33})$, it is found that in the limit of $m_h^2 \to 0$ we have the massless spin-2 propagation without the ghost state. This exactly corresponds to the same situation as in the intermediate scales of Gregory-Rubakov-Sibiryakov (GRS) model.

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I. INTRODUCTION

Recently, there have been lots of interest in the phenomenon of localization of gravity proposed by Randall and Sundrum (RS) \[1\] (for previous relevant work see references therein). RS assume a single positive tension 3-brane and a negative bulk cosmological constant in the five dimensional spacetime. By considering metric fluctuations from a background which is isomorphic to sections of \(AdS_5\), they have shown that it reproduces the effect of four dimensional gravity localized on the brane without the need to compactify the extra dimension due to the “warping” in the fifth dimensional space. In more detail, the solution to linearized equations in the five dimensions results in a zero mode, which can be identified with the four dimensional massless graviton, and the massive continuum Kaluza-Klein (KK) modes. Surprisingly, the wavefunctions of the massive continuum KK modes are suppressed at the brane for small energies, and thus ordinary gravity localized on the brane is reproduced at large distances.

On the other hand, Gregory, Rubakov and Sibiryakov (GRS) \[2\] have recently considered a brane model which is not asymptotically \(AdS_5\), but Minkowski flat. In the GRS model, however, the ordinary 4D Newton potential is reproduced at intermediate scales only not because of the massless zero mode, but for the resonance of zero mass in the continuum KK spectrum \[2-5\]. In Ref. \[4\], however, it is pointed out that the \(m_h \to 0\) limit of a massive graviton propagator does not reproduce the massless graviton propagator due to the mismatch of the number of polarizations. In this sense the GRS model of “quasi-localization” of gravity would differ from the RS model. Contrary to it, Csáki, Erlich and Hollowood \[6\] recently have argued that in the presence of localized source at \(z = 0\) the bending of the brane exactly compensates for the effects of the extra polarization in the massive graviton propagator. Thus the graviton propagator at intermediate scales is equivalent to the massless propagator of the Einstein theory just as in the RS scenario. However, at ultra large scales this effective theory includes scalar anti-gravity \[7\]. This problem may be cured by the RG analysis \[8\]. Also the authors in Ref. \[9\] point out that the mechanism to cancel the unwanted extra polarization leads to the presence of ghost. In order to have a well-defined theory, the ghost should disappear.

In this paper, we investigate the non-traceless metric fluctuations in the presence of uniform source along \(z\)-axis in the RS model. We introduce the trace field \((h)\) here instead of \(\xi^5\) in Ref. \[10\]. It shows that massive graviton modes contain ghost states which can be removed by assuming a further condition on the matter source. Our work corresponds to an alternative realization of the results in Ref. \[6\].

II. LINEARIZED PERTURBATIONS

The Randall-Sundrum model with a single domain wall (or brane) \[1\] perpendicular to the infinite fifth direction can be described by the following action:

\[
I = \int d^4 x \int_{-\infty}^{\infty} dz \left[ \frac{1}{16\pi G_5} \sqrt{-\hat{g}} (\hat{R} - 2\Lambda) - \sqrt{-\hat{g}_B} \sigma(z) + \mathcal{L}_M \right].
\]  

(1)

Here \(G_5\) is the five dimensional Newton’s constant, \(\Lambda\) the bulk cosmological constant of five dimensional spacetime, \(\hat{g}_B\) the determinant of the metric describing the brane, and \(\sigma(z) =...\)
σδ(z), σ the tension of the brane. \( I_M = \int d^4x dz \mathcal{L}_M \) denotes the matter action, and it contributes only in the linearized level. In this paper, we use the signature \((-,+,+,+,+),\) which means.

If we introduce a conformal factor as follows
\[
d s^2 = \hat{g}_{MN} dx^M dx^N = H^{-2} g_{MN} dx^M dx^N,
\]
the field equation becomes
\[
G_{MN} + 3 \frac{\nabla_M \nabla_N H}{H} - 3g_{MN} \left[ \frac{\nabla_P \nabla_P H}{H} - 2 \frac{\nabla_P H \nabla_P H}{H^2} \right] = 8\pi G_5 \left[ - \frac{\Lambda}{8\pi G_5 H^2} g_{MN} - \frac{\sqrt{-g_B}}{\sqrt{-g}} H^2 \sigma(z) g_{\mu\nu} \delta^\mu_M \delta^\nu_N - \frac{2}{\sqrt{-g}} \delta I_M \right]
\]
with the Einstein tensor \( G_{MN} \) constructed from the metric \( g_{MN} \). Now it is straightforward to see that, in the absence of matter source except for the domain wall itself (i.e., \( \delta I_M / \delta \hat{g}_{MN} = 0 \)), the most general solution having a four dimensional Poincaré symmetry is
\[
d s^2 = H^{-2}(z)(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2),
\]
where \( H = k|z| + 1, \Lambda = -6k^2(<0), \) and \( \sigma = 3k/4\pi G_5 \).

Let us consider metric fluctuations around this background spacetime as follows: \( g_{MN} = \eta_{MN} + h_{MN} \).

Defining \( \bar{h}_{MN} = h_{MN} - \frac{1}{2} \eta_{MN} h \) where \( h = \eta^{MN} h_{MN} \), the linearized perturbation equation of Eq. (3) is
\[
- \frac{1}{2} \Box \bar{h}_{MN} + \partial_M \partial^P \bar{h}_{NP} - \frac{1}{2} \eta_{MN} \partial^P \partial^Q \bar{h}_{PQ} - \frac{3}{2} \partial^P H \left( \partial_M h_{NP} + \partial_N h_{MP} - \partial_P h_{MN} \right) - 3 \eta_{MN} \left[ \left( - \frac{\partial^P \partial^Q H}{H} + 2 \frac{\partial^P H \partial^Q H}{H^2} \right) h_{PQ} - \frac{\partial^Q H}{H} \partial^P h_{PQ} \right] - 3 \left( \frac{\partial^P h}{H} + 2 \frac{\partial_P H \partial^P H}{H^2} \right) h_{MN}
\]
\[
+ 8\pi G_5 H^{-2} \left\{ \frac{\Lambda}{8\pi G_5} h_{MN} + |H| \sigma(z) \left[ \frac{1}{2} (\eta^{\alpha\beta} h_{\alpha\beta} - \eta^{PQ} h_{PQ}) \eta_{\mu\nu} \delta^\mu_M \delta^\nu_N + \delta^\mu_M \delta^\nu_N h_{\mu\nu} \right] \right\} = 8\pi G_5 \bar{T}_{MN},
\]
where the linearized five dimensional matter source \( \bar{T}_{MN} = -\delta (2\delta I_M / \sqrt{-g} \delta \hat{g}^{MN}) \) is included, and \( \Box = \eta^{MN} \partial_M \partial_N \).

Taking the harmonic gauge condition,
\[
\partial^M \bar{h}_{MN} = 0 \quad \text{or} \quad \partial^M h_{MN} = \frac{1}{2} \partial_N h,
\]
the linearized field equation becomes
\[
\Box \bar{h}_{MN} + \frac{3}{2} \frac{\partial^P H}{H} \left( \partial_M h_{NP} + \partial_N h_{MP} - \partial_P h_{MN} \right) + \eta_{MN} \frac{12k^2}{H^2} h_{55} + 12k \delta(z) \left[ (h_{MN} - \eta_{MN} h_{55}) - (h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h_{55}) \delta^\mu_M \delta^\nu_N \right] = -16\pi G_5 \bar{T}_{MN}.
\]
In the case of $h = 0$ and $\tilde{T}_{MN} = 0$, this leads to the equation (5) in Ref. [12]. In components, it becomes

$$\Box + 3 \frac{\partial H}{H} \partial_5 \bar{h}_{55} + \frac{12k^2}{H^2} \bar{h}_{55} - \left( \frac{\partial H}{H} \partial_5 + \frac{4k^2}{H^2} \right) \bar{h} = -16\pi G_5 \tilde{T}_{55}, \quad (9)$$

$$\Box + \frac{12k}{H} \delta(z) \bar{h}_{55} + 3 \frac{\partial H}{H} \partial_5 \bar{h}_{55} - \frac{\partial_5 H}{H} \partial_\mu \bar{h} = -16\pi G_5 \tilde{T}_{5\mu}, \quad (10)$$

$$\Box - 3 \frac{\partial H}{H} \partial_5 \bar{h}_{\mu\nu} + 3 \frac{\partial H}{H} \left( \partial_\mu \bar{h}_{5\nu} + \partial_\nu \bar{h}_{5\mu} \right) + \eta_{\mu\nu} \left( \frac{12k^2}{H^2} - \frac{6k}{H} \delta(z) \right) \bar{h}_{55} + \eta_{\mu\nu} \left( \frac{4k^2}{H^2} + \frac{2k}{H} \delta(z) \right) \bar{h} = -16\pi G_5 \tilde{T}_{\mu\nu}, \quad (11)$$

where $\bar{h} = \eta^{MN} \bar{h}_{MN} = -\frac{3}{2} h$.

For longitudinal metric fluctuations (i.e., $h_{55} = h_{5\mu} = 0$), one has

$$h = \eta^{\mu\nu} h_{\mu\nu} + h_{55} = h_\mu = -\bar{h}_\mu, \quad \bar{h}_{55} = -\frac{1}{2} h_\mu, \quad \bar{h}_{5\mu} = 0, \quad \bar{h} = 3 \bar{h}_{55}. \quad (12)$$

The harmonic gauge condition in Eq. (4) gives

$$\partial^\mu \bar{h}_{\mu\nu} = 0, \quad \partial_5 \bar{h}_{55} = 0. \quad (13)$$

Consequently, we have $\partial_5 \bar{h} = \partial_5 h = \partial_5 h_\mu = \partial_5 \bar{h}_\mu = 0$. Using these results, we find from Eq. (10) that $\tilde{T}_{5\mu} = 0$. And other two linearized equations become

$$\Box \bar{h}_{55} = -16\pi G_5 \tilde{T}_{55}, \quad (14)$$

$$\Box - 3 \frac{\partial H}{H} \partial_5 \bar{h}_{\mu\nu} = -16\pi G_5 \tilde{T}_{\mu\nu}. \quad (15)$$

Acting $\partial^\mu$ on Eq. (13) and using the gauge condition Eq. (13) lead to the source conservation law

$$\partial^\mu \tilde{T}_{\mu\nu} = 0. \quad (16)$$

This is a relic of the 4D general covariance on the brane. By taking the trace of Eq. (13), we also find

$$\Box_4 h_\mu = 16\pi G_5 \tilde{T}_{\mu} \quad \text{with} \quad \Box_4 = \eta^{\mu\nu} \partial_\mu \partial_\nu. \quad (17)$$

This means that the trace can propagate on the brane if one includes the matter source. Note, however, this corresponds to a massless scalar propagation. Considering $h_\mu = -2 \bar{h}_{55}$, the consistency between Eq. (14) and Eq. (17) requires the following relation

$$\tilde{T}_{55} = \frac{1}{2} \tilde{T}_{\mu}. \quad (18)$$

This is exactly the stabilization condition implemented in Refs. [13, 14]. From Eqs. (13) and (17) one obtains additional constraints as

$$\partial_5 \tilde{T}_{\mu} = \partial_5 \tilde{T}_{55} = 0. \quad (19)$$
For our purpose, let us choose here the uniform source along $z$-axis

$$\tilde{T}_{MN} = \begin{pmatrix} \frac{T_{\mu\nu}(x)}{L} & 0 \\ 0 & \frac{T_{55}(x)}{L} \end{pmatrix}, \quad (20)$$

which satisfies Eq. (19). Here the size $L$ of the extra space is still finite as is shown by

$$L = 2 \int_{0}^{\infty} \sqrt{g_{55}} dz = 2 \int_{0}^{\infty} \frac{dz}{kz + 1} = \frac{2}{k} \ln[kz + 1]_{0}^{\infty} \propto \frac{1}{k}. \quad (21)$$

We note that $\ln[k \cdot \infty + 1]$ is still finite but it is very small compared with $1/k$. This is because $k$ is allowed for up to very small quantity as the Plank scale ($10^{-34}$ cm). Then we find

$$8\pi G_{5} \tilde{T}_{MN} = 8\pi G \left( \begin{array}{cc} T_{\mu\nu}(x) & 0 \\ 0 & T_{55}(x) \end{array} \right), \quad (22)$$

where $G = G_{5}/L$ is the four dimensional Newton’s constant.

Using Eq. (17), Eq. (15) takes the form

$$(\Box - m_{h}^{2}) h_{\mu\nu} = -16\pi G_{5} (\tilde{T}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \tilde{T}), \quad (23)$$

where $\tilde{T} = \eta^{\rho\sigma} \tilde{T}_{\rho\sigma} = \tilde{T}_{\rho}$. Here the mass squared $m_{h}^{2}$ is defined by the Schrödinger-like equation

$$\left[ -\frac{1}{2} \partial_{5}^{2} + \frac{15k^{2}}{8H^{2}} - \frac{3k}{2H} \delta(z) \right] \psi(z) = \frac{1}{2} m_{h}^{2} \psi(z) \quad (24)$$

with $h_{\mu\nu}(x,z) = H^{3/2} \psi(z) \hat{h}_{\mu\nu}(x)$.

Now we examine the graviton propagator on the brane at $z = 0$ by considering only $h_{\mu\nu}(x,0) \sim \hat{h}_{\mu\nu}(x)$, which satisfies

$$(\Box - m_{h}^{2}) \hat{h}_{\mu\nu} = -16\pi G (T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T). \quad (25)$$

It requires the bilinear forms of the source with the inverse propagator to isolate the physical modes. As the present analysis is on the classical level, we express $\hat{h}_{\mu\nu}$ in terms of source. Taking Fourier transformation to momentum space results in

$$\hat{h}_{\mu\nu}(p) = \frac{16\pi G}{p^{2} + m_{h}^{2}} \left[ T_{\mu\nu}(p) - \frac{1}{2} \eta_{\mu\nu} T(p) \right]. \quad (26)$$

Then the one graviton exchange amplitude for the source $T_{\mu\nu}$ is given by

$$A^{class} = \frac{1}{4} \hat{h}_{\mu\nu}(p) T^{\mu\nu}(p) = \frac{4\pi G}{p^{2} + m_{h}^{2}} (T^{\mu\nu} T_{\mu\nu} - \frac{1}{2} T^{2}). \quad (27)$$

In order to study the massive states, it is best to use the rest frame in which

$$p_{1} \neq 0, \quad p_{2} = p_{3} = p_{4} = 0. \quad (28)$$
Considering Eqs. (16) and (28) leads to the following source relations
\[ T_{11} = T_{12} = T_{13} = T_{14} = 0. \] (29)
Thus, one obtains
\[ T^{\mu \nu}T_{\mu \nu} - \frac{1}{2} T^2 = |T_{+2}|^2 + |T_{-2}|^2 + |T_{+1}|^2 + |T_{-1}|^2 + T_{44}[\frac{1}{2}T_{44} - (T_{22} + T_{33})], \] (30)
where the first two terms correspond to the exchange of graviton with \( T_{\pm 2} = \frac{1}{2}(T_{22} - T_{33}) \pm iT_{23}, \) and the third and fourth terms are the exchange of the graviphoton with \( T_{\pm 1} = T_{23} \pm iT_{24}. \) We note here that the last term in the above equation is not positive definite. This means that there exist ghost states (negative norm states). However, if one requires
\[ T_{44} = 2(T_{22} + T_{33}), \] (31)
one immediately finds that
\[ T^{\mu \nu}T_{\mu \nu} - \frac{1}{2} T^2 = |T_{+2}|^2 + |T_{-2}|^2 + |T_{+1}|^2 + |T_{-1}|^2 \] (32)
with all positive norm states.

In the limit of \( m^2_h \to 0, \) the graviphoton propagation can be decoupled from the brane \([16].\) Hence we can neglect \( |T_{\pm 1}|^2 \)-terms. Finally the amplitude takes the form
\[ A^{\text{class}}_{m^2_h \to 0} = \lim_{m^2_h \to 0} \frac{4\pi G}{p^2 + m^2_h} [T_{+2}|^2 + |T_{-2}|^2], \] (33)
which corresponds to the massless spin-2 amplitude. This is our key result. Although this is based on the second RS model, the results obtained above are directly applicable to the situation in the intermediate scales of the GRS model \([2,17].\)

### III. DISCUSSION

We resolve the problem raised in the mechanism to cancel the unwanted extra polarization in the quasi-localization of gravity. This is done with introducing both the trace \((h)\) and the uniform source \((\tilde{T}_{MN})\) at the linearized level. In the conventional RS approach, the trace \((h)\) is just a gauge-dependent scalar and hence it can be gauged away. However, including the uniform matter source, this plays the role of \(\xi^5\) in the brane bending model \([10].\) This is because \(h (\xi^5)\) satisfy the nearly same massless equations of \(\Box_4 h = 16\pi G S^\mu_5 (\Box_4 \xi^5 = \frac{8\pi G}{6} S^\mu_4\) in Ref. \([10]).\) And the comparison of the equation \(\tilde{h}_{\mu \nu} = h_{\mu \nu} - \frac{1}{2} \eta_{\mu \nu} h\) with \(h_{\mu \nu} = h^{(m)}_{\mu \nu} + 2k \eta_{\mu \nu} \xi^5\) in Ref. \([10]\) confirms the close relationship between \(h\) and \(\xi^5.\)

If \(T_{\mu}^\mu = 0,\) one finds from Eq. (27) that the massive spin-2 states have 5 polarizations with all positive norm states \([12].\) In the case of \(h \neq 0, T_{\mu}^\mu \neq 0,\) requiring the additional condition \(T_{44} = 2(T_{22} + T_{33})\) in Eq. (31), we find the massless spin-2 state with 2 polarizations in the limit of \(m^2_h \to 0.\) In this case the ghost states disappear.

Now we wish to comment a recent paper by Kogan and Ross \([14].\) They require only \(\tilde{T}_{55} = \frac{1}{2} T_{55}^\mu\) in Eq. (18). This condition can be interpreted as follows: \(\tilde{T}_{\mu}^\mu - 2T_{55}\) is the source
for the scalar radion. In the case of a mechanism that stabilizes the extra dimension, the source for the constant mode of this scalar is identically zero. In our case this comes from the consistency between Eqs. (14) and (17) with $h_\mu^\mu = -2h_{55}$. In Ref. [14], Kogan and Ross neither choose the massive frame of Eq. (28) nor use the source conservation law in Eq. (16). These two are essential steps for obtaining the massive spin-2 amplitude of Eq. (33). In the massless frame of $p_1 = p_4, p_2 = p_3 = 0$ with $p^\mu T_{\mu\nu} = 0$ [15,18], $T_{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2$ exactly reduces to $|T_{+2}|^2 + |T_{-2}|^2$ without the ghost states. But in the massive frame it leads to Eq. (30). Hence one finds the ghost states. To eliminate these, we require a further condition as Eq. (31).

We are still under the harmonic gauge in Eq. (7). We note that the expression for $A^{\text{class}}$ is simplified by referring it to an appropriate Lorentz frame. Usually one chooses the rest frame $p_\mu = (p,0,0,0)$ to see the massive states [13]. On the other hand, we use the light-cone frame of $p_\mu = (p,0,0,p)$ for the massless states. The ghost-free condition requires a further relation among diagonal elements of $T_{MN}$. We are free from the ghost provided that $T_\mu^\mu = 3(T_{22} + T_{33}) = 2T_{55}$.

Finally, we comment on our source $T_{MN}$ in Eq. (20). In the brane-bending approach [10], the authors choose a localized source on the brane as $T_{\mu\nu}(x,z) = T_{\mu\nu}(x)\delta(z)$. Here we choose $T_{\mu\nu}(x,z) = T_{\mu\nu}(x)/L$. But, up to the integration over $z$, these two expressions lead to the same one. In the case of “brane-bending,” the source is located on the brane at $z = 0$ whereas our source is uniformly distributed along the extra dimension $z$.

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