The strangeness of this equation is that the appearance of a non-covariant condition is not caused by any physical considerations, but only with the sole purpose of "simplifying" the gravitational field equation.

Let us note that the Newtonian limit $\sqrt{-g} \to 1$, a necessary principle of the general theory of relativity, is also satisfied for the equation without the condition $\sqrt{-g} = 1$. If we exclude this condition from equation (1):

$$\frac{\partial T_{\mu \nu}}{\partial x^\alpha} - \Gamma^{\beta}_{\mu \alpha} \Gamma^\beta_{\nu \alpha} = \kappa \left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right)$$

This shortened field equation will satisfy all the necessary principles of general relativity if we prove the covariance of the right-hand side of the equation and remember that Einstein had already proven the statement that in the Newtonian limit the equation turns into the Poisson equation. Thus, equation (2) is a complete equation of the gravitational field, satisfying all the principles introduced by Einstein into the general theory of relativity. We will solve equation (2) using numerical methods.

**THE SCHWARZSCHILD PROBLEM FOR A POINT SOURCE**

The Schwarzschild problem for a point source field for a shortened field equation is solved in the same way as the Einstein equation.

We are looking for solutions in the form

$$ds^2 = s(r) dt^2 - p(r) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2$$

Solving the shortened equation for empty space

$$\frac{\partial \Gamma^\mu_{\nu \beta}}{\partial x^\alpha} - \Gamma^{\gamma}_{\mu \alpha} \Gamma^\gamma_{\nu \beta} = 0$$

Nonzero Christoffel symbols of equation (3) 2
\[
\begin{align*}
\Gamma_{11}^1 &= \frac{p'}{2p}, & \Gamma_{22}^1 &= -\frac{r}{p}, & \Gamma_{33}^1 &= -\frac{r\sin(\theta)^2}{p} \\
\Gamma_{14}^4 &= \frac{s'}{2s}, & \Gamma_{44}^4 &= \frac{s'}{2p}, & \Gamma_{33}^2 &= -\frac{\sin(\theta)\cos(\theta)}{p}
\end{align*}
\]

From these data and equation (4), which allows us to find a nonlinear system of ordinary differential equations:

\[
\begin{cases}
\left(\frac{p'}{sp}ight)' - \frac{s^p - s^p}{p} - \frac{s^p}{p} = 0 \\
\left(\frac{s'}{sp}ight)' - \frac{s^p - s^p}{p} - \frac{s^p}{p} = 0
\end{cases}
\]

where the prime denotes the derivative with respect to \(r\). Eliminating the value \(\frac{p'}{p}\) from which we obtain a third-order equation:

\[
\left(\frac{s''}{s'} - \frac{s'}{s}\right)' - \frac{1}{2} \left(\frac{s''}{s'} - \frac{s'}{s}\right)^2 - \frac{4}{r^2} - \frac{s's'}{2s^2} = 0
\]

We were unable to find an analytical solution to this equation. A graphical numerical solution is presented in Fig. 1. The resulting solution is a metric in which the parameter \(g_{\theta\theta}\) can be considered as a measure of gravitational potential. In a region remote 1 from the field source, the solution differs slightly from the Schwarzschild one. When approaching the source, the potential \(\phi\) quickly decreases to extreme positive values of the order of \(10^{-20}\). Moreover, a flat plateau with such values penetrates into the region of the Schwarzschild solution, which has no solutions \(r < r_s\) at all. However, the solution by numerical methods is unstable, and the size of the potential well, depending on the position of the boundary conditions, the area of calculations, the algorithm and the number of points, determines the size of the plateau or the location of premature termination of the program. We assume that this is caused by the inadequacy of the Schwarzschild solution as an asymptotic solution for the initial conditions.

When solving the equation numerically, asymptotic proximity to the Schwarzschild solution was used. This made it possible to use the Schwarzschild solution values \(1 - \frac{1}{r}\) as boundary conditions close to the expected solution. As expected, in the region of relatively small values the solution follows the Schwarzschild solution. Then, similarly to the Schwarzschild solution, it quickly decreases (Fig. 1). But the difference from the Schwarzschild solution, the solution to equation (4) remains positive, although extremely small. A potential well is observed with an almost limiting potential value, which is equal to \(\phi = -c^2\) (Fig. 2).

In Fig. 2 it can be seen that the calculated curve smoothly passes into a plateau. This confirms the rule that proper time must be strictly greater than zero, and

\[
\frac{s''}{s'} - \frac{s'}{s} = \frac{p'}{p}
\]

This can be integrated with respect to \(r\):

\[
\ln \frac{s'}{s} = \ln p + C_1
\]

where \(C_1\) is the integration constant. Let’s transform this equation

\[
\frac{s'}{s} = pe^{C_1}
\]

Let’s substitute this result into the second equation of system (5):

\[
\left(\frac{p'}{2p}\right)' - \frac{p'p'}{4p^2} - \frac{2}{r^2} - C_2 \frac{p^2}{4} = 0
\]

and \(C_2\) is the new constant. It is clear that it is necessary to know the numerical value of the constant \(C_2\) in order to apply numerical methods to solve the equation. This possibility is realized simultaneously with the initial conditions at point \(r_0\). Since we assume the asymptotic equality of the solution to equation (7) to the Schwarzschild solution \(p_s = r/\left(r - 1\right)\) the boundary (initial) conditions are determined:

\[
\begin{align*}
r_0; & \\
p_0 = p_s(\ r_0) ; & \\
p'_0 = p'_s(\ r_0) ; & \\
p''_0 = p''_s(\ r_0)
\end{align*}
\]

here \(r_0\) is the position point of the boundary conditions. If we substitute boundary conditions (8) into equation (7), we obtain an equation from which we can calculate the value of \(C_2\). A simplified value of this quantity can be estimated by eliminating derivatives

\[
C_2 = -\frac{8}{p_0^2 r_0^2}
\]

Since in the Newtonian limit \(r \to \infty\), then \(C_2 = 0\). Now we can proceed to the numerical solution of equation (7). We remember that when solving the Einstein equation, the relation \(g_{11} = (g_{00})^{-1}\) is satisfied. We see something similar here when solving the truncated equation.
FIG. 1. Dependence of $g_{00}$ on the ratio $\frac{r}{r_g}$. Schwarzschild solutions of the Einstein equation (dotted line) and a numerical solution of the exact gravitational field equation (6) (solid line).

FIG. 2. Enlarged section of Fig. 1, the horizontal section of the solution to equation (8) has a positive value, but so small that the numerical solution program cannot cope and stops.

CONCLUSION

Einstein’s empty space has finally received the status of an independent physical object with parameters specified by the energy-momentum-stress tensor. The Einstein equation is a good approximation only for not too large gravitational fields, so when calculating extremely large fields it must be replaced by the exact gravitational field equation:

$$R_{\mu
u} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

Formally, the inclusion of the tensor $f_{ab}$ in the Einstein equation leads to a truncated equation, the solution of which must be a smooth metric tensor. Moreover, the law of conservation of energy-momentum follows directly from the equation of the gravitational field. For some applications, solutions with large fields are important. There is confidence that the fundamental tensor entirely belongs to the original Riemannian space and does not contain unacceptable values. In the solutions of the new shortened gravitational field equation, apparently there are no solutions that go beyond the Riemannian space and that give rise to hypotheses about the structure of singular objects, like black holes. The structure of the observed compact heavy objects “black holes” is quite explainable by the presence of potential wells in the solutions in smooth Riemann space (Fig. 2).

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

* aritrasanyal1@gmail.com
† valerymorozov@hotmail.com

[1] Morozov V. B. Exact equation of the gravitational field based on the Einstein separation of the Ricci tensor. Parana J. Sci. Educ., v.8, n.1, (16-20), January 7, 2022.
Theorem 3

As the determinant of the metric tensor $g$ approaches the value of the Minkowski metric, equation (1) approaches Einstein’s equation (2).

Proof. As shown, $B_{\mu\nu} \to 0$ as $g_{\mu\nu} \to \eta_{\mu\nu}$, so:

$$\frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\alpha} - \Gamma^\alpha_{\mu\beta} \Gamma_{\nu}^\beta \approx \frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\alpha} - \Gamma^\alpha_{\mu\beta} \Gamma_{\nu}^\beta + B_{\mu\nu} = R_{\mu\nu}.$$ 

This establishes the asymptotic equivalence of equation (1) and Einstein’s equation.

Theorem 3

From the gravitational field equation (1), the complete law of conservation of matter and the gravitational field follows.
Proof. The covariant derivative of the Einstein tensor is:

\[ G_{\nu;\mu} = 0. \]

Hence, from equation (2), the general conservation law is:

\[ T_{\nu;\mu} + f_{\nu;\mu} = 0. \]