Current non-conservation effects in $\nu$DIS diffraction

R. Fiore* and V.R. Zoller†

* Dipartimento di Fisica, Università della Calabria and Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, I-87036 Rende, Cosenza, Italy
† ITEP, Moscow 117218, Russia

Abstract. In the neutrino DIS diffraction the charged current non-conservation gives rise to sizable corrections to the longitudinal structure function, $F_L$. These corrections is a higher twist effect enhanced at small-$x$ by the rapidly growing gluon density. The phenomenon manifests itself in abundant production of charm and strangeness by longitudinally polarized W bosons of moderate virtualities $Q^2 \lesssim m_c^2$.

Keywords: neutrino deep inelastic scattering, small-$x$, color dipoles
PACS: 13.15.+g 13.60.Hb

INTRODUCTION

Weak currents are not conserved. Here we focus on manifestations of the charmed-strange ($cs$) charged current non-conservation (CCNC) in small-$x$ neutrino DIS. For light flavors the hypothesis of the partial conservation of the axial-vector current (PCAC) [1] quantifies the CCNC in terms of observable quantities [2]. The $cs$ current non-conservation is not constrained by PCAC and we quantify the $cs$CCNC in terms of the light cone wave functions of the color dipole QCD approach. The observable highly sensitive to the CCNC effects is the so called longitudinal structure function $F_L(x, Q^2)$. Our finding is that the higher twist correction to $F_L$ arising from the $cs$CCNC appears to be enhanced at small $x$ by the BFKL [3] gluon density factor,

$$F_L^{cs} \sim \frac{m_c^2}{Q^2} \left( \frac{1}{x} \right)^\Delta.$$  

As a result, the component of $F_L(x, Q^2)$ induced by the charmed-strange current grows rapidly to small-$x$ and dominates $F_L$ at $Q^2 \sim m_c^2$ [4,5].

CCNC IN TERMS OF LCWF

In the color dipole (CD) approach to small-$x$ $\nu$DIS [6] the responsibility for the quark current non-conservation takes the light-cone wave function (LCWF) of the quark-antiquark Fock state of the longitudinal ($L$) electro-weak boson. If the Cabibbo-suppressed transitions are neglected, the Fock state expansion reads

$$|W_L^+\rangle = \Psi^{cs}|cs\rangle + \Psi^{ud}|ud\rangle + \ldots,$$
where only $ud$- and $c\bar{s}$-states (vector and axial-vector) are retained.

In the current conserving eDIS the Fock state expansion of the longitudinal photon contains only $S$-wave $q\bar{q}$ states and $\Psi$ vanishes as $Q^2 \to 0$,

$$\Psi(z, r) \sim 2\delta_{\lambda, -\bar{\lambda}} Q z (1 - z) \log(1/\varepsilon r).$$  (3)

In DIS the CCNC adds to Eq.(3) the $S$-wave mass term \cite{7,8}

$$\sim \delta_{\lambda, -\bar{\lambda}} Q^{-1} [(m \pm \mu) (1 - z) m \pm \bar{\mu}] \log(1/\varepsilon r)$$  (4)

and generates the $P$-wave component of $\Psi(z, r)$,

$$\sim i\delta_{\lambda, \bar{\lambda}} e^{-i2\lambda \phi} Q^{-1} (m \pm \mu) r^{-1}$$  (5)

(upper sign - for the axial current, lower - for the vector one). Clearly seen are the built-in divergences of the vector and axial-vector currents $\partial_\mu V^\mu \sim m - \mu$ and $\partial_\mu A^\mu \sim m + \mu$.

This LCWF describes the quark antiquark state with quark of mass $m$ and helicity $\lambda = \pm 1/2$ carrying fraction $z$ of the $W^+$ light-cone momentum and antiquark having mass $\mu$, helicity $\bar{\lambda} = \pm 1/2$ and momentum fraction $1 - z$. The distribution of dipole sizes, $r$, is controlled by the attenuation parameter

$$\varepsilon^2 = Q^2 z (1 - z) + (1 - z) m^2 + z \mu^2$$

that introduces the infrared cut-off, $r^2 \sim \varepsilon^{-2}$.

**HIGH $Q^2$**: z-SYMMETRIC $c\bar{s}$-STATES

In the color dipole representation \cite{9,10} the longitudinal structure function $F_L(x, Q^2)$ in the vacuum exchange dominated region of $x \leq 0.01$ can be represented in a factorized form

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_W} \int dz d^2 r |\Psi(z, r)|^2 \sigma(x, r),$$  (6)

where $\alpha_W = g^2 / 4\pi$ and $G_F / \sqrt{2} = g^2 / m_W^2$. The light cone density of color dipole states $|\Psi|^2$ is the incoherent sum of the vector ($V$) and the axial-vector ($A$) terms,

$$|\Psi|^2 = |V|^2 + |A|^2$$

The Eqs. (3,6) make it evident that for large enough virtualities of the probe, $Q^2 \gg m^2$, the $S$-wave components of

$$F_L^{(v)} = F_L^{ud} + F_L^{cs}$$  (7)

corresponding to the “non-partonic” configurations with $z \sim 1/2$ do dominate \cite{11} and two terms in the expansion (7) that mimics the expansion (2) do converge (see Fig. 1). To the Double Leading Log approximation (DGLAP \cite{12,13})

$$F_L^{ud} \approx F_L^{cs} \approx \frac{2}{3\pi} \alpha_S(Q^2) G(x, Q^2).$$  (8)
FIGURE 1. Two components of $F_L = F_{cs}^L + F_{ud}^L$ at $x_{bj} = 10^{-4}$ are shown by solid lines. The $S$-wave and $P$-wave contributions to $F_{cs}^L$ and $F_{ud}^L$ are represented by dotted and dashed lines, correspondingly.

The rhs of (5) is quite similar to $F_L^{(e)}$ of eDIS [12, 14] (see [15] for discussion of corrections to Double Leading Log-relationships between the gluon density $G$ and $F_L^{(e)}$).

MODERATE $Q^2$: ASYMMETRIC $c\bar{s}$-STATES AND $P$-WAVE DOMINANCE

The $S$-wave term dominates $F_L$ at high $Q^2 \gg m_c^2$. At $Q^2 \lesssim m_c^2$ the $P$-wave component takes over (see Fig.1). To evaluate it we turn to Eq. (6). For $m_c^2 \gg m_s^2$,

$$|V_L|^2 \sim |A_L|^2 \propto \left(\frac{m_c^2}{Q^2}\right)\epsilon^2 K_1(\epsilon r)$$

and corresponding $z$-distribution, $dF_{cs}^L/dz$, develops the parton model peaks at $z \to 0$ and $z \to 1$ [4]. Integrating over $z$ near the endpoint $z = 1$ in (6) yields [5]

$$\int dz |\Psi_{cs}(z,r)|^2 \approx \frac{\alpha_W N_c}{\pi^2} \frac{m_c^2}{m_c^2 + Q^2} \frac{1}{r^4}$$

(9)

for $r^2$ from $(m_c^2 + Q^2)^{-1} \ll r^2 \ll m_c^2$. This is the $r$-distribution for $c\bar{s}$-dipoles with $c$-quark carrying a fraction $z \sim 1$ of the $W^+$'s light-cone momentum.

The lowest order pQCD cross section [9]

$$\sigma(r) \approx \pi C_F \alpha_s^2 r^2 \log \left(1/r^2\right)$$

saturates for large dipoles and can be approximated by

$$\sigma(r) \approx \pi C_F \alpha_s^2 r^2 \log \left(1 + r_s^2/r^2\right).$$
The saturation radius is found to be \( r_S^2 = A/\mu_G^2 \), where \( A \approx 10 \) \cite{15} and \( \mu_G = 1/R_c \) is the inverse correlation radius of perturbative gluons. From the lattice QCD studies \( R_c \approx 0.2 - 0.3 \text{ fm} \) \cite{17}. Then, for the charmed-strange \( P \)-wave component of \( F_L \) with fast \( c \)-quark \((z \to 1) \) one gets

\[
F_{L}^{cs} \approx \frac{N_c C_F}{8} \frac{m_c^2}{Q^2 + m_c^2} \left( \frac{\alpha_S}{\pi} \right)^2 \log^2 \left[ (Q^2 + m_c^2) r_S^2 \right].
\]

Additional contribution to \( F_{L}^{cs} \) comes from the \( P \)-wave \( c \bar{s} \)-dipoles with “slow” \( c \)-quark, \( z \to 0 \). For low \( Q^2 \ll m_c^2 \) this contribution is rather small,

\[
F_{L}^{cs} \approx \frac{N_c C_F}{4} \frac{Q^2 + m_c^2}{m_c^2} \left( \frac{\alpha_S}{\pi} \right)^2 \log (r_S^2 m_c^2).
\]

If, however, \( Q^2 \) is large enough, \( Q^2 \gg m_c^2 \), corresponding distribution of dipole sizes

\[
\int dz |\Psi^{cs}(z,r)|^2 \approx \frac{\alpha_W N_c}{\pi^2} \frac{m_c^2}{m_c^2 + Q^2} \frac{1}{Q^2 r^4}
\]

valid for \((m_c^2 + Q^2)^{-1} \ll r^2 \ll m_c^2 \) and \( z \to 0 \) leads to

\[
F_{L}^{cs} \approx \frac{N_c C_F}{8} \frac{m_c^2}{Q^2} \left( \frac{\alpha_S}{\pi} \right)^2 \log^2 \left( \frac{Q^2 + m_c^2}{m_c^2} \right).
\]

Therefore, at high \( Q^2 \gg m_c^2 \) both kinematical domains \( z \to 0 \) and \( z \to 1 \) (Eqs. (13) and (10), respectively) contribute equally to \( F_{L}^{cs} \) and one can anticipate similar \( x \)-dependence of both contributions.

In the CD approach the BFKL-log \((1/x)\) evolution of \( \sigma(x,r) \) is described by the CD BFKL equation of Ref.\cite{16}. For qualitative estimates it suffices to use the DGLAP approximation. The DGLAP resummation results in the \( P \)-wave component of \( F_L \) that rises rapidly to small \( x \),

\[
F_{L}^{cs} \approx \frac{N_c C_F}{2} \frac{m_c^2}{Q^2} L(Q^2) \eta(x)^{-1} I_2 \left( 2 \sqrt{\xi(x,Q^2)} \right).
\]

In Eq.(14), which is the DGLAP-counterpart of Eq.(11), \( I_2(z) \simeq \exp(z)/\sqrt{2 \pi z} \) is the Bessel function,

\[
\xi(x,Q^2) = \eta(x)L(Q^2)
\]

is the DGLAP expansion parameter with

\[
L(k^2) = \frac{4}{\beta_0} \log[\alpha_s(\mu_G^2)/\alpha_s(k^2)],
\]

\[
\alpha_s(k^2) = \frac{4\pi}{\beta_0} \log(k^2/\Lambda^2)
\]
and $\eta(x) = C_A \log(x_0/x)$.

As for our numerical estimates (Fig. 1), we calculate nuclear and nucleon structure functions to the leading order in $\alpha_s \log(1/x)$ within the color dipole BFKL approach [18]. The full scale BFKL evolution of $F_L(x, Q^2)$ is shown in Fig. 2 of Ref. [19].

SUMMARY

Summarizing, it is shown that at small $x$ and moderate virtualities of the probe, $Q^2 \sim m_c^2$, the higher twist corrections brought about by the non-conservation of the charmed-strange current dramatically change the longitudinal structure function, $F_L$. The effect survives the limit $Q^2 \to 0$ and seems to be interesting from a point of view of feasible tests of Adler’s theorem [2] and the PCAC hypothesis.

ACKNOWLEDGMENTS

The work was supported in part by the Ministero Italiano dell’Istruzione, dell’Università e della Ricerca and by the RFBR grant 06-02-16905 and 07-02-00021.

REFERENCES

1. Y. Nambu, Phys. Rev. Lett. 4, 380-382 (1960); M. Gell-Mann and M. Levy, Nuovo Cimento 17, 705-726 (1960).
2. S. Adler, Phys. Rev. 135, B963-B966 (1964).
3. E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP 45, 199-204 (1977); I.I. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822-829 (1978).
4. R. Fiore and V.R. Zoller, JETP Lett. 87, 524-530 (2008).
5. R. Fiore and V.R. Zoller, “Full of charm neutrino DIS”, in ’08 QCD and High Energy Interactions, Proc. of 43rd Rencontres de Moriond on QCD and Hadronic Interactions, La Thuile, Italy, 2008, e-Print: arXiv:0805.2090
6. V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Phys.Lett. B 292, 181-188 (1992).
7. R. Fiore and V.R. Zoller, JETP Lett. 82, 385-389 (2005).
8. R. Fiore and V.R. Zoller, Phys.Lett. B 632, 87-91 (2006).
9. N.N. Nikolaev and B.G. Zakharov, Z.Phys. C 49, 607-618, (1991); ibid. C 53, 331-346 (1992); ibid. C 64, 631-652 (1994).
10. A.H. Mueller, Nucl. Phys. B 415, 373-385, (1994); A.H. Mueller and B. Patel, Nucl. Phys. B 425, 471-488 (1994).
11. V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Phys. Lett. B 328 143-148 (1994).
12. Yu.L. Dokshitzer, Sov. Phys. JETP 46, 641-653 (1977).
13. V.N. Gribov and L.N. Lipatov. Sov. J. Nucl. Phys. 15, 438-450 (1972); G. Altarelli and G. Parisi. Nucl. Phys. B 126, 298-318 (1977).
14. A.M. Cooper-Sarkar, G. Ingelman, K.R. Long, R.G. Roberts and D.H. Saxon, Z. Phys. C 39, 281-290 (1988); R.G. Roberts, “The structure of the proton”, Cambridge Univ. Press, 1990.
15. N.N. Nikolaev and B.G. Zakharov Phys. Lett. B332 184-190 (1994).
16. N.N. Nikolaev, B.G. Zakharov and V.R. Zoller, JETP Lett. 59, 6-12 (1994).
17. M. D’Elia, A. Di Giacomo and E. Meggiolaro, Phys. Rev. D 67, 114504-114512 (2003).
18. N.N. Nikolaev, W. Schäfer, B.G. Zakharov, V.R. Zoller, JETP Lett. 84, 537-541 (2007).
19. R. Fiore and V.R. Zoller, JETP Lett. 85, 309-314 (2007).