Semi-Analytic Model for Intergalactic Gamma-Ray Cascades in Extragalactic Magnetic Fields

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Abstract: Primary gamma rays emitted by extragalactic sources, such as blazars, will generate electromagnetic cascades in intergalactic space. These cascades proceed via electron-positron pair production and inverse Compton scattering on cosmic background radiation, mainly the cosmic microwave background (CMB) and extragalactic background light (EBL) fields. The existence of an extragalactic magnetic field (EGMF) could deflect electron-positron pair trajectories and scatter the cascade photons, possibly creating a halo around the source while suppressing the cascade flux collected by a detector. We develop a semi-analytic model for the cascade process and apply it to combined GeV-TeV data on high-frequency-peaked BL Lacertae objects (HBLs) from the Fermi Large Area Telescope (LAT) and ground-based Cherenkov telescopes, comparing observation results with model predictions using a robust statistical framework. Lower limits with different confidence levels on the field strength of the EGMF derived from this procedure are discussed under various assumptions about the source livetime.

Keywords: Gamma-Ray Astronomy, Extragalactic Magnetic Field, Electromagnetic Cascade, BL Lacertae Objects, Extragalactic Background Light, Fermi Gamma-Ray Space Telescope, VERITAS

1 Introduction

Extragalactic blazars emit gamma rays in both high-energy (HE, 100 MeV ≲ E ≲ 300 GeV) and very-high-energy (VHE, E ≥ 100 GeV) bands. Due to the existence of the extragalactic background light (EBL), which spans over the optical to far-infrared wavelength range, gamma rays with energies above 10 GeV may be absorbed and produce electron-positron pairs [1]. An electromagnetic cascade then develops via inverse Compton scattering of the e± pairs on the cosmic microwave background (CMB) and subsequent secondary pair productions. In the presence of an extragalactic magnetic field (EGMF), the charged pairs in the cascade will be deflected, spreading the cascade photons in both the spatial and temporal distributions [2, 3]. The characteristic angular spread could create an apparent halo around the point source, and the time delay of cascade photons could appear in the observation of gamma-ray bursts or flaring blazars. Therefore, gamma-ray astronomy of extragalactic sources provides a useful probe into the EGMF strength and configurations.

According to previous studies [4, 5] this method will be sensitive to EGMF below 10⁻¹⁴ Gauss, much lower than what any other measurement has achieved. For instance, from Faraday rotation measurements on extragalactic radio sources [6,7] or analysis of CMB anisotropy [8,9] only an upper limit ≈ 10⁻⁹ Gauss on the field strength is obtained. The new EGMF window below 10⁻¹⁴ Gauss is particularly interesting for the understanding of astrophysical magnetic fields. A primordial field within the window could be responsible for generating the galactic and intra-cluster magnetic fields [12], and its own origin can be related to either the inflationary era or phase transitions in the early universe [13]. On the other hand, if the EGMF strength turns out to be zero, the astrophysical fields would have to be coming from seed fields produced locally via the Biermann battery mechanism or related processes [14]. Hence a lower limit on the EGMF instead of a measurement could be already useful in clarifying the origin of all the magnetic fields we have in the universe today.

One way for obtaining the lower limit, using gamma rays as a probe, would be to compare the cascade flux from VHE emission of a source with the actual HE observed spectrum [15,16]. If the HE measured flux is lower than the zero-field cascade flux prediction, the EGMF would have to be non-zero to dilute the cascade photons into a spreading angle and thus suppress the collected flux. Deriving an EGMF lower limit in this way requires a realistic model of the electromagnetic cascade correlated with EGMF. Existing simplified analytic models [17] and Monte Carlo simulations [18] set a lower bound for the field strength at 10⁻¹⁶ to 10⁻¹⁵ Gauss assuming the studied sources to be active with unlimited livetime or 10⁻¹⁹ to 10⁻¹⁷ Gauss for the sources to be active for only ~ 3 years of simultaneous HE and VHE observations [19,20,21]. In this work we model the cascade semi-analytically and use our model predictions to place a lower limit on the EGMF using a sys-
tematic framework, with the blazar RGB J0710+591 as an example for application and data analysis.

2 Model Description

The geometry of the cascade is shown in Fig. 1. Primary gamma rays emitted by a blazar at distance \( L \) from the earth are absorbed after going through distance \( L' \). The electron-positron pairs get deflected by the EGMF to angle \( \theta_d \) and upscatter CMB to secondary photons directed toward the detector at an incidence angle \( \theta_s \). The emission angle of the primary photon at the source with respect to the line of sight is \( \theta_s = \theta_d - \theta_c \). The difference in path length between the secondary photon and a direct photon that goes from the source to the observer in a straight line is

\[
\Delta L = c\Delta T = L' + \sqrt{L^2 + L'^2 - 2LL'\cos \theta_s} - L \tag{1}
\]

where \( \Delta T \) is the time delay of the secondary photon and \( c \) is the speed of light.

For a primary photon with energy \( \epsilon \), we assume the electron and positron produced in the pair-production process each carry \( \epsilon/2 \). Then the pair starts to lose energy continuously via inverse Compton scattering on the CMB while being deflected by the EGMF at the same time. An electron with Lorentz factor \( \gamma_c \) on average scatters CMB photons to energy \( 4\gamma_c^2\epsilon_0/3 \), where \( \epsilon_0 \) is the average CMB photon energy \( \sim 6.4 \times 10^{-3} \text{ eV} \) [22]. The energy loss process is then described by

\[
\frac{d\gamma_c mc^2}{dt} = -\frac{4}{3} \epsilon_0 n_{\text{CMB}} \sigma_T \gamma_c^2 \tag{2}
\]

where \( n_{\text{CMB}} \) is the CMB photon number density \( \sim 411 \text{ cm}^{-3} \) and \( \sigma_T \) is the Thomson cross section \( \sim 6.65 \times 10^{-26} \text{ cm}^2 \). The Lorentz deflection process is, on the other hand

\[
\frac{d\theta_d}{dt} = \frac{c}{r_l} = \frac{eB}{\gamma_c mc} \tag{3}
\]

where \( r_l = \gamma_c mc^2/eB \) is the Larmor radius, \( m_c \) is the electron rest mass, and we have assumed the EGMF \( B \) to be perpendicular to the electron momentum. Eqs. 2 and 3 combine to give the deflection angle for an electron/positron to go from Lorentz factor \( \gamma_c \) to \( \gamma_e \):

\[
\theta_{d0} = 3 \frac{eB}{\epsilon_0 n_{\text{CMB}} \sigma_T} (\gamma_e^{-2} - \gamma_c^{-2}) \tag{4}
\]

which is generalized to

\[
\theta_d = \text{arccos} \left( \sin^2 \theta_f \cos \theta_{d0} + \cos^2 \theta_f \right) \tag{5}
\]

when the angle \( \theta_f \) between \( B \) and the electron momentum is other than \( \pi/2 \).

Combined with the geometry in Fig. 1, Eq. 5 uniquely determines \( \theta_s \) and \( \theta_c \) for a given set of \( B, \gamma_c, \gamma_e, L' \), and \( \theta_f \), provided that \( \theta_c < \pi/2 \). The number of secondary photons between energies \( E \) and \( E + dE \) produced by the electron going from \( \gamma_c \) to \( \gamma_e \) can be calculated from the CMB spectrum, replacing the CMB photon energy with \( 3E/(4\gamma_e^2) \):

\[
dN(E, \gamma_e) = \frac{dE}{8\gamma_e^4} \int \frac{dE'}{8\gamma_e^4} \frac{1}{L^3} \gamma_e^2 \left( e^{3E/(4\gamma_e^2kT)} - 1 \right)
\]

\[
= \frac{81\pi E^2 m_e^2 d\gamma_e}{32h^3 c^3 \gamma_e^4 \epsilon_0 n_{\text{CMB}} e^{3E/(4\gamma_e^2kT)} - 1} dE
\]

\[
\frac{dN(E)}{dE} = \frac{81\pi E^2 m_e^2}{16\hbar^3 c \epsilon_0 n_{\text{CMB}}} \int \frac{d\gamma_e}{\gamma_e^8} \left( e^{3E/(4\gamma_e^2kT)} - 1 \right)
\]

\[
\times \int_0^{\pi/2} d\theta_f g(\theta_f) \int dE' L' \cos \theta_s/\lambda_e f(\epsilon, \theta_s)
\]

\[
\times \exp \left(-\sqrt{L^2 + L'^2 - 2LL'\cos \theta_s} /\lambda_e \right) \tag{6}
\]

where \( \lambda_e \) is the mean free path of a gamma-ray photon at energy \( \epsilon \), depending on the specific EBL profile. \( g(\theta_f) \) is the probability distribution of \( \theta_f \), which is \( \sin \theta_f \) for a randomly pointing field. \( f(\epsilon, \theta_s) \) is the intrinsic spectrum of the source, and we integrate over \( \epsilon \) starting from \( 2\gamma_e m_e c^2 \) to 200 TeV, as primary photons beyond that energy are mostly absorbed within 1 Mpc away from the blazar, and quickly deflected away by the relatively large magnetic field there. The upper limit on \( \gamma_e \) is 100 TeV/(\( m_e c^2 \)) and a lower limit at \( 10^3 \) is also placed as a practical matter for the numerical integration, since there is negligible CMB density beyond 3 meV and we are not interested in secondary flux below 100 MeV. The integration limits on \( L' \) are enforced through observational cuts on \( \Delta T \) and \( \theta_c \) as shown in Fig. 1.

The blazar intrinsic emission \( f(\epsilon, \theta_s) \), the photon mean free path \( \lambda(\epsilon) \), and the EGMF directional profile \( g(\theta_f) \) are the inputs to the cascade model. In practice we model the blazar emission as boosted isotropic radiation [23]

\[
f(\epsilon, \theta_s) = f_0 (1 - \beta \cos \theta_s)^{-\alpha-1} e^{-\epsilon/\epsilon_0} + f_0 (1 + \beta \cos \theta_s)^{-\alpha-1} e^{-\epsilon/\epsilon_0} \tag{8}
\]

where the second term models a counter jet. To obtain a conservative prediction on the cascade flux we choose the optical depth profile of [24] which is relatively transparent for VHE gamma rays. \( g(\theta_f) \) is taken as \( \sin \theta_f \) as we have no prior assumption on the EGMF configuration.
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3 Model Application and EGMF Constraint

As an example we consider the high-frequency-peaked BL Lacertae object (HBL) RGB J0710+591 located at red-shift \( z = 0.125 \). The predictions of the total flux as a sum of both the direct and cascade photons within the instrument point-spread function (PSF) normalized to the observed data are shown in Fig. 2 for different EGMF strengths and assumptions on source livetime, with \( \alpha = 1.5, \Gamma = 1/\sqrt{1-\beta^2} = 10, \) and \( E_0 = 25 \text{ TeV} \) in Eq. 8.

The VHE data are from VERITAS measurements \(^1\) and the HE data points are extracted from public Fermi Large Area Telescope (LAT) data between August 2008 and January 2011 using unbinned likelihood analysis in the Fermi Science Tools v9r18p6 with the instrument response functions (IRFs) P6_V3_DIFFUSE \(^2\), galactic diffuse emission model gll\_iem\_v02 \(^3\) and isotropic background model isotropic\_iem\_v02 \(^4\). The ~ 3-year period with simultaneous HE-VHE data sets a lower limit on the livetime of this source.

By requiring that the HE total flux not exceed the Fermi LAT measured spectrum, we can roughly see the EGMF strength \( B \) has a lower limit between \( 10^{-16} \) and \( 10^{-15} \) Gauss for the unlimited livetime case, or between \( 10^{-18} \) and \( 10^{-17} \) Gauss for the livetime assumption of 3 years. A more systematic lower limit could be derived by fitting the total flux to the measured data points with free parameters of normalization \( f_0 \), index \( \alpha \) and cutoff energy \( E_0 \). To take into account of complexities in blazar modeling (e.g., \(^2\)) we also fit with a broken power law at 80 GeV and add one more free parameter \( \alpha_{\text{break}} \) below the break energy, only to find that the resulting lower limit is not greatly affected. Restricting the range of \( \alpha \) and \( \alpha_{\text{break}} \) to be no harder than the physically motivated hardness limit \( 1.5 \) \(^27\) \(^28\) and the range of \( E_0 \) to be (0.1 TeV, 100 TeV), we plot the minimum \( \chi^2 \) from the fit as a function of EGMF strength in Fig. 3 at various source livetime limits. As expected all the curves converge at low EGMF strengths or large livetimes.

The EGMF lower limits at different confidence levels are derived by finding the point where \( \chi^2 \) exceeds its minimum value in each curve by \( \Delta \chi^2 \) in Fig. 3 which is just a variant of the profile likelihood method for determining confidence intervals. Two sample confidence levels (90% and 95%) are given by requiring \( \Delta \chi^2 \) to be 2.72 and 3.84 \(^29\), respectively. We show these lower limits versus the blazar livetime in Fig. 4. At livetimes below \( 10^{-4} \) years, i.e., when the \( \Delta T \) constraint is dominating over the Fermi LAT PSF constraint on \( \theta_c \), we have the EGMF lower limit scaling with \( \Delta T \) as \( B \sim \sqrt{\Delta T} \), consistent with Eqs. 1 and 4. The nominal lower limit at 95% confidence level is \( B \gtrsim 2 \times 10^{-16} \text{ Gauss} \) if the source has unlimited livetime and \( B \gtrsim 3 \times 10^{-18} \text{ Gauss} \) if the source has the minimum livetime \( \sim 3 \) years.

4 Conclusions

The 95% lower limits on EGMF strength inferred by the semi-analytic cascade model for both source livetime assumptions are consistent with the results from full Monte Carlo simulations of the cascade in \(^21\) on the same blazar, 1. Near the completion of this proceeding the Fermi team released an updated IRF P6\_V11\_DIFFUSE. We re-analyzed the LAT data with the new IRF and applied the cascade model within the same analysis framework to find that the final constraint on EGMF strength was not significantly affected.

2. http://fermi.gsfc.nasa.gov/ssc/
demonstrating that the model correctly takes into account the important geometrical and physical aspects involved in the cascade calculation. The limits are conservative as we choose a comparably transparent EBL model and also assume the electron-positron pairs to be always going in one coherent magnetic field domain, which is only valid when the EGMF coherence length \( \lambda_B \gtrsim 1 \text{ Mpc} \). If \( \lambda_B \lesssim 1 \) Mpc instead, the charged pairs would random walk through the EGMF domains [30], and the deflection angle would become smaller, causing a larger cascade flux and a more stringent lower limit on \( B \).

Because we ignore cosmological expansion and evolution, the model is only applicable to sources within a redshift of 0.2. However most of the TeV blazars already detected are located well in this range, so we have a pool of candidate sources with moderate size. We choose RGB J0710+591 as an example because it has simultaneous data in GeV and TeV and does not have any variability detected in either energy band. The caveat with this one-source study is that the EGMF along the line of sight to the source may not be representative of the overall field strength, e.g., when there is a filament of intra-cluster magnetic field along the direction. Therefore a statistically more reliable constraint should be obtained by studying an unbiased set of blazars and using the systematic framework presented here to combine the results on single sources. As the gamma-ray telescopes continue to monitor the sky, more and more blazars with simultaneous GeV-TeV baseline spectra will be observed and we will be approaching the goal of probing the all-sky EGMF.

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