Proportional integral derivative controller assisted reinforcement learning for path following by autonomous underwater vehicles

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1. Introduction

The work presented in this article, is the result of an attempt to use Deep Reinforcement Learning (DRL) controllers as a substitute for conventional Proportional Integral Derivative (PID) controllers in motion control systems for Autonomous Underwater Vehicles (AUV). The main control problem tackled here is 3D path following \cite{6, 13, 22}. The theory behind AUV modeling, the control problem and DRL are introduced in section 2. Solutions to the path following problem are well documented in traditional control literature \cite{9, 12, 20}, but using a DRL approach is an active area of research. AUVs are employed in various sub-sea applications, such as seafloor mapping, pipeline inspection and research operations. The diversity of operational settings for AUVs implies that truly autonomous vehicles must be able to follow spatial trajectories, maintain a desired velocity and avoid collisions - all at the same time! Combined, the mentioned objectives are hard to solve, and the level of complexity suggests using learning controllers, such as Reinforcement Learning (RL) agents. The work presented in this article aspires to be a step towards achieving this endgoal.

The implementation of the simulation model and the utilized DRL algorithm are briefly described in section 3. The main innovative contribution is detailed in section 4, where PID assisted training is used to split the DRL controller into three separate parts (one for each actuator of the AUV). We also show that achieving a well performing 3D path-following controller through DRL is feasible, at least in theory, in section 5.

2. Theory

2.1. Governing equations of motion for an AUV

In this section we briefly present the system of equations governing the motion of an AUV. In marine systems modeling, this representation involves a transformation between different Cartesian frames. The notation used in this article to detail the equations of motion for the marine vessel, is the notation given by SNAME (1950) elaborated in Table 1 \cite{8}.

Modeling motion dynamics for an AUV involves transformation between coordinate systems. The two coordinate systems of interest are the body-frame, \{b\}, which is the body-fixed reference frame with origin at the vessel’s center of control (CO), and the North-East-Down (NED) coordinate system, \{n\}. In NED coordinates, the \(x\) axis points to true North, the \(y\) axis points to the East and the \(z\) axis points downwards, normal to Earth’s surface. The NED-frame is considered to be inertial for local navigation, so that Newton’s laws of motion still apply. In the body-frame, the \(x\) axis points along the longitudinal axis of the vessel, the \(y\) axis points along the transverse axis and the \(z\) axis is normal to the surface of the vessel. Figure 1 illustrates the relationship between the two reference frames.

The rotation of \{b\} with respect to \{n\} is described by the Euler angle rotation matrix:

\[ R^b_n(\Theta_{eb}) = \begin{bmatrix} \cos \theta \cos \phi & -\cos \theta \sin \phi & \sin \theta \\ \sin \phi \sin \theta + \cos \theta \sin \psi & \cos \phi \cos \theta - \sin \phi \sin \psi \sin \theta & \sin \phi \cos \theta - \cos \phi \sin \psi \\ -\cos \phi \sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix} \] (1)

where \(\Theta_{eb} = [\phi, \theta, \psi]\) are the Euler angles describing the vehicle’s attitude, and \(s\phi = \sin \phi, \ c\phi = \cos \phi\). Relating
The vessel’s motion is governed by the nonlinear kinetic equations given in \(\{b\}\):

\[
M\ddot{v}_b + C(v_b)v_b + D(v_b)v_b + \mathbf{g}(\eta) = \tau_{\text{control}}
\]

Table 1
Notation for marine vessels as given by SNAME (1950)

| Degree of freedom | Forces and moments | Velocities | Positions |
|------------------|--------------------|------------|-----------|
| 1 translation in the x direction (surge) | 1 | \(u\) | \(x\) |
| 2 translation in the y direction (sway) | 2 | \(v\) | \(y\) |
| 3 translation in the z direction (heave) | 3 | \(w\) | \(z\) |
| 4 rotation about x axis (roll) | 4 | \(q\) | \(\phi\) |
| 5 rotation about y axis (pitch) | 5 | \(p\) | \(\theta\) |
| 6 rotation about z axis (yaw) | 6 | \(r\) | \(\psi\) |

Figure 1: Simple illustration of BODY and NED coordinate systems. The BODY frame is obtained by rotating the NED frame by its principal axes.
where \( v_r \) is the velocity relative to the velocity of ocean currents. Initially it is assumed that there are no currents. However, as part of the motivation behind this thesis is to uncover how the machine learning controller handles the inclusion of environmental disturbances, the model is implemented such that ocean currents can easily be added.

**Mass Forces** The system’s inertia matrix, \( \mathbf{M} \), is the sum of the inertia matrix for the rigid body and the added mass. Added mass is the inertia added from the weight of fluid displaced by the vessel when moving through it. Because of the symmetry assumptions, both matrices are diagonal. However, the rigid body matrix is defined in the center of gravity, such that it must be shifted to the center of control, yielding some coupling terms:

\[
\mathbf{M} = \begin{bmatrix}
  m - X_u & 0 & 0 & 0 \\
  0 & m - Y_v & 0 & -m_z G \\
  0 & 0 & m - Z_w & 0 \\
  m_z G & 0 & 0 & 0 \\
 0 & 0 & 0 & I_x - K_p \\
  m_z G & 0 & 0 & 0 \\
 0 & 0 & 0 & I_y - M_q \\
 0 & 0 & 0 & I_z - N_r \\
\end{bmatrix}
\]

(6)

**Coriolis Forces** Naturally, the added mass will also affect the Coriolis-centripetal matrix, \( \mathbf{C}(v_r) \), which defines the forces occurring due to \( \{b\} \) rotating about \( \{n\} \). Moreover, the Coriolis-centripetal matrix could be expressed independently of the linear velocities, easing the implementation of irrotational ocean currents [8, p. 222]. In 6 degrees of freedom (DOF), these matrices are given by:

\[
\mathbf{C}(v_r) = \begin{bmatrix}
  \mathbf{0} & \mathbf{C}_{12}(v_r) & \mathbf{C}_{21}(v_r) & \mathbf{C}_{22}(v_r)
\end{bmatrix}
\]

(7)

where

\[
\mathbf{C}_{12}(v_r) = \begin{bmatrix}
  m_z G r & (m - Z_u) w_r \\
  -m_z G p + (m - Y_v) v_r & -m_z G q - (m - X_u) u_r \\
  -(m - Y_v) v_r & (m - X_u) u_r \\
  0 & 0
\end{bmatrix}
\]

\[
\mathbf{C}_{21}(v_r) = \begin{bmatrix}
  -m_z G r & (m - Z_u) w_r \\
  -(m - Z_u) w_r & -m_z G r \\
  (m - Y_v) v_r & -(m - X_u) u_r \\
  m_z G p - (m - Y_v) v_r & m_z G q - (m - X_u) u_r \\
  0 & 0
\end{bmatrix}
\]

\[
\mathbf{C}_{22}(v_r) = \begin{bmatrix}
  0 & (I_z - N_r) r \\
  -(I_y - M_q) q & -(I_x - K_p) p \\
  (I_y - M_q) q & (I_x - K_p) p \\
  0 & 0
\end{bmatrix}
\]

(8)

Combining and inserting numerical values yields the full Coriolis-centripetal matrix:

\[
\mathbf{C}(v_r) = \begin{bmatrix}
  0 & 0 & 0 & 0.18 r \\
  0 & 0 & 0 & -34 w_r \\
  -0.18 r & 34 u_r & 0.18 p - 34 v_r & 0 \\
  -34 w_r & -0.18 r & 0.18 q + 19 u_r & -1.8 r \\
  34 u_r & -19 u_r & 0 & 1.8 q \\
  34 w_r & -34 v_r & 0 & 1.8 q \\
  0.18 r & 19 u_r & 0 & -1.8 q \\
  1.8 r & -1.8 q & 0 & 0.04 p \\
  0 & 0.04 p & 0 & 0
\end{bmatrix}
\]

(9)

**Damping Forces** The components of hydrodynamic damping modelled is linear viscous damping, nonlinear (quadratic) damping due to vortex shedding and lift forces from the body and control fins. Thus, the damping matrix, \( \mathbf{D}(v_r) \), can be expressed as:

\[
\mathbf{D}(v_r) = \mathbf{D} + \mathbf{D}_n(v_r) + \mathbf{L}(v_r)
\]

(10)

The linear damping is given by

\[
\mathbf{D} = \begin{bmatrix}
  X_u & 0 & 0 & 0 & 0 & 0 \\
  0 & Y_v & 0 & 0 & Y_r & 0 \\
  0 & 0 & Z_w & 0 & Z_q & 0 \\
  0 & 0 & 0 & K_p & 0 & 0 \\
  0 & 0 & M_w & 0 & M_q & 0 \\
  0 & N_v & 0 & 0 & 0 & N_r
\end{bmatrix}
\]
The nonlinear damping is given by

\[
\mathbf{D}_n(\mathbf{v}_r) = -\begin{bmatrix}
X_{u|u}|u| & 0 & 0 & 0 \\
0 & X_{v|v}|v| & 0 & 0 \\
0 & 0 & Z_{u|u}|u| & 0 \\
0 & 0 & 0 & M_{u|u}|u| \\
0 & 0 & N_{v|v}|v| & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & Y_{r|r}|r| & 0 \\
Z_{q|q}|q| & 0 & 0 & 0 \\
M_{q|q}|q| & 0 & 0 & 0 \\
0 & N_{r|r}|r| & 0 & 0
\end{bmatrix} K_{p|r}|p|
\]  

Finally, the lift is given by

\[
\mathbf{L}(\mathbf{v}_r) = -\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & Y_{u|u} + Y_{u|b} & 0 & 0 \\
0 & 0 & Z_{u|u} + Z_{u|b} & 0 \\
0 & 0 & 0 & M_{u|u} + M_{u|b} \\
0 & N_{u|u} + N_{u|b} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & Y_{r|r} & 0 & 0 \\
Z_{q|q} & 0 & 0 & 0 \\
M_{q|q} & 0 & 0 & 0 \\
0 & N_{r|r} & 0 & 0
\end{bmatrix}
\]

**Restoring Forces** Buoyancy acting on the body initiates restoring forces and moments for the AUV. This can be considered as a virtual spring acting on the system. Based on all previous assumptions, the restoring force vector can be written as:

\[
\mathbf{G}(\eta) = \begin{bmatrix}
(W - B) \sin \theta \\
-(W - B) \cos \theta \sin \phi \\
-(W - B) \cos \theta \cos \phi \\
z_0 W \cos \theta \sin \phi \\
z_0 W \sin \theta \\
0
\end{bmatrix}
\]

**Control Inputs** There are 3 control inputs: propeller shaft speed, rudder and elevator fins denoted \(n\), \(\delta_r\), and \(\delta_z\), respectively. The control surfaces can maximally be rotated 30° in each direction, and the propeller thrust is limited such that the AUV does not violate the low-speed assumption. The control inputs are related to the control force vector according to Equation 14:

\[
\mathbf{r}_{control} = \begin{bmatrix}
1 & 0 & 0 \\
0 & Y_{u|b} u_r^2 & 0 \\
0 & 0 & Z_{u|b} u_r^2 \\
0 & 0 & 0 \\
0 & M_{u|b} u_r^2 & 0 \\
0 & N_{u|b} u_r^2 & 0
\end{bmatrix} \begin{bmatrix}
n \\
\delta_r \\
\delta_z
\end{bmatrix}
\]

For a more thorough derivation of the model and how the numerical values are calculated, the readers are referred to [1] and [8].

### 2.2. Path Following

The main control problem addressed in this work is that of path following, where the objective is to follow a pre-planned path without time-constraints. Consequently, the goal is to drive tracking-errors to zero. [8, ch. 9]. A set of \(n\) waypoints is used to represent the path, starting at the origin of the NED coordinate frame for simplicity. The path is generated by linear interpolation between the waypoints, resulting in a piecewise straight-line path. For a path defined in three-dimensional space, the parametric equations for the interpolation scheme are [3]:

\[
\begin{align*}
x_{p,i}(s) &= x_{p,i-1} + s \cos \chi_{p,i-1} \cos \psi_{p,i-1} \\
y_{p,i}(s) &= y_{p,i-1} + s \sin \chi_{p,i-1} \cos \psi_{p,i-1} \\
z_{p,i}(s) &= z_{p,i-1} - s \sin \psi_{p,i-1}
\end{align*}
\]

where subscript \(p\) signifies that the coordinate is representing the path and \(i\) denotes the waypoint index. These coordinates define the path relative to the inertial frame. The angles \(\chi_{p,i-1}\) and \(\psi_{p,i-1}\) denote the azimuth and elevation angle of the straight line between waypoints \(i-1\) and \(i\). The parametric equations are continuously differentiable with respect to \(s\), which is the along-track distance travelled on the path from waypoint \(i-1\) to \(i\).

To define the tracking-errors, the Serret-Frenet (SF) reference frame associated with each point of the path is introduced. The \(x_{SF}\) axis is tangent to the path, the \(y_{SF}\) axis normal to the path and the \(z\) axis is given by \(z_{SF} = x_{SF} \times y_{SF}\) and is thus orthogonal to the other two axes [7]. The vector \(e = [s, e, h]^T\) is defined by the along-track distance, cross-track error and vertical-track error illustrated in Figure 2. This vector points towards the closest point on the path from the vessel.

![Figure 2: The Serret-Frenet reference frame defines the components for the tracking-error vector. The control objective in path-following is to drive \(e\) and \(h\) to zero.](image)

Traditional path following controllers strive to align the velocity vector in the inertial frame with the path tangent. Instead of aiming directly at the closest point of the path, the vessel aims at a point further ahead, decided by a look-ahead
distance $\Delta$, which is set by the control designer. The vector $\epsilon$ is obtained in the SF-frame by:

$$
\epsilon = R_n^{SF}(v_p X_p)^T(p^n - p^n_p)
$$

(16)

where $p$ is the position of the vessel and $p_p$ is the closest point on the path. Now the desired azimuth and elevation angle can be calculated according to:

$$
\chi_d(e) = \chi_p + \chi_r(e), \quad v_d(h) = v_p + v_r(h)
$$

(17)



where

$$
\chi_r(e) = \arctan(-\frac{\epsilon}{\Delta}), \quad v_r(h) = \arctan(\frac{h}{\sqrt{e^2 + \Delta^2}}).
$$

(18)

The angles, $\chi_r(e)$ and $v_r(h)$, can be interpreted as corrective steering, and when driven to zero the velocity vector aligns perfectly with the tangent of the path [3]. This provides a reasonable control objective, and the challenge now lies in finding a control law that maps these errors to good actuator outputs.

A few approaches can be used to design these control laws. One is to decompose the system into a longitudinal and a lateral model and design autopilots for each of them. The coupling terms are considered disturbances. If the motion is reserved for one plane at a time, these disturbances will remain rather small and neglecting them is justified. Additionally, the system can be linearized around an equilibrium point and pole-placement or optimal control strategies can be used to find the feedback gains. [8, ch. 10]

Encarnasi and Pascoal proposed a more ambitious approach in the paper “3D Path Following for Autonomous Underwater Vehicle”. Here, they developed a nonlinear kinematic controller using Lyapunov theory, feedback linearization and backstepping. Their simulations showed impressive results for 3D path-following, both for straight-line paths and a helix. A caveat to their approach was that disturbances and saturation limits for the actuators were not considered in the analysis. [7]

2.3. Deep Reinforcement Learning

Training machines to execute tasks via RL is not a new field of research. In fact, reinforcement techniques was developed for learning control systems as early as 1965 [19]. However, the field has made some major progress in the last decades, merging together with deep neural networks to form what is now known as DRL. The advances in deep neural networks, RL itself, and the computing power of modern hardware, have made it tractable to train and implement DRL controllers to solve complex control problems such as playing Atari games or controlling robotic locomotion. [2][15]

Reinforcement Learning In RL an algorithm, known as an agent, makes an observation $s_t$ of an environment and performs an action $a_t$. The observation is referred to as the state of the system, and is drawn from the state space $S$. The action is restricted to the well-defined action space $A$. When an RL task is not infinitely long, but ends at some time $T$, we say that the problem is episodic, and that each iteration through the task is an episode.

After performing an action the agent receives a scalar reward signal $r = r(s_t, a_t)$. The reward quantifies how good it was to choose action $a_t$ when in state $s_t$. The objective of the agent is typically to maximize expected cumulative reward.

The action choices of the agent are guided by a policy $\pi(s)$, which can be either deterministic or stochastic. In the case that the learning algorithm involves a neural network, the policy is parametrized by the learnable parameters of the network, denoted by $\theta$. When the policy is stochastic and dependent on a neural network, we write $\pi(s) = \pi_\theta(a|s)$.

The value function $V_\pi(s)$ describes how valuable it is to be in state $s$ under the policy $\pi$. The Q-function $Q_\pi(s, a)$ expresses the value of performing action $a$ when in state $s$.

Mathematically, the value function and Q-function can be expressed through the return, which is shown Equation 19. The return is a measure of accumulated reward between times $t$ and $T$, where $T$ might be infinity. The discount factor $\gamma$ weights the importance of rewards that are close in time versus those distant in time.

$$
R_t^g = \sum_{k=t}^{T} \gamma^{t-k}r(s_k, a_k), \quad 0 < \gamma < 1
$$

(19)

Using the return, the value function is $V_{\pi}(s_t) = \mathbb{E}\{R_t^g | s_t; \pi\}$ and the Q-function is $Q_{\pi}(s_t, a_t) = \mathbb{E}\{R_t^g | s_t, a_t; \pi\}$.

Learning by reinforcing good choices is synonymous with how humans (and other animals) learn. RL is therefore a formal version of trial-and-error learning. The goal of many RL algorithms can be formally stated as the optimization problem in Equation 20 [16].

$$
\theta^* = \arg \max_{\theta} \mathbb{E}_{s \sim \rho, a \sim \pi_\theta} [R_t^g]
$$

(20)

Solving Equation 20 yields the optimal parameters $\theta = \theta^*$ that maximize the expected return at all times $t$, when the actions are drawn from the policy $\pi_\theta$, and the state distribution is given by $\rho_\theta$. Algorithms that aim to solve Equation 20 can be roughly divided into four categories:

- **Policy gradients method**: Maximize the objective directly through gradient ascent. [18]

- **Value-based methods**: Estimate the value function and/or the Q-function, and make a policy that increases the probability of taking actions that maximize their values. [16]

- **Actor-Critic methods**: A hybrid of policy gradient and value-based methods. The value function or Q-function is approximated by a neural network which...
acts as a critic. The actor, which is the policy, is updated as suggested by the critic through some policy gradient method [21]. This idea is illustrated in Figure 3.

- **Model-based RL**: A model of the environment is created through exploration, and the estimate is utilized to make decisions. For instance, the model can be used in optimal control or Value-based methods [5].

![Diagram](image)

**Figure 3**: Actor-critic methods are a hybrid of policy gradient and value-based methods.

**Proximal Policy Optimization** In this work, we use the actor-critic algorithm known as Proximal Policy Optimization (PPO) as suggested by Schulman et al. [2]. In this section, the general theory behind the method is presented. Define the advantage function as:

$$ A(s, a) = Q(s, a) - V(s). $$

The advantage function represents the difference in expected return by taking action \( a \) in state \( s \), as opposed to following the policy. Because both \( Q(s, a) \) and \( V(s) \) are unknown, an estimate of the advantage function, \( \hat{A}_t \), is calculated based on an estimate of the value function \( \hat{V}(s) \), which is made by the critic neural network.

An alternative for estimating the advantage function is the generalized advantage estimate (GAE), given in Equation 22 [17].

$$ \hat{A}_t = \delta_t + (\gamma \lambda) \delta_{t+1} + \cdots + (\gamma \lambda)^{T-t} \delta_{T-1} $$  \hspace{1cm} (22)

where \( \delta_t = r_t + \gamma \hat{V}(s_{t+1}) - \hat{V}(s_t) \).

Here, \( T \) is a truncation point which is typically much smaller than the duration of an entire episode. As before, \( \gamma \) is the discount factor. As the GAE is a sum of uncertain terms, the tuneable parameter \( \lambda \) is introduced to reduce variance. However, \( \lambda < 1 \) makes the GAE biased towards the earlier estimates of the advantage function. Hence, choosing \( \lambda \) is a bias-variance trade-off.

The second key component in PPO is introducing a surrogate objective. It is hard to apply gradient ascent directly to the RL objective in Equation 20. Therefore, Schulman et al. suggest a surrogate objective which is such that an increase in the surrogate provably leads to an increase in the original objective [2]. The proposed surrogate objective function is given by Equation 23.

$$ L_{CLIP} = \Delta A_t = \min \left( \frac{Q(s_t, a_{old})}{Q(s_t, a_{old})}, 1 \right) + \lambda \left( \frac{Q(s_t, a_{old})}{Q(s_t, a_{old})} - 1 \right) A_t $$ \hspace{1cm} (23)

The tuning parameter \( \epsilon \) reduces the incentive to make very large changes to the policy at every step of the gradient ascent. This is necessary as the surrogate objective only estimates the original objective locally in a so-called trust-region.

### 3. Implementation

The implementation of our solution makes use of the RL framework **OpenAI Gym**. OpenAI Gym is a Python library that was created for the purpose of standardizing the benchmarks used in RL research [4]. It provides an easy-to-use framework for creating RL environments in which custom RL agents can be deployed and trained with minimal overhead.

Stable Baselines is a Python library that provides a large set of state-of-the-art parallelizable RL algorithms compatible with the OpenAI gym framework, including PPO [11]. PPO is used in this work because of its reputable performance on continuous control problems. In fact, its performance on the OpenAI benchmark - a set of standardized test environments, created to assess and compare different RL algorithms - was so impressive that it has become the go-to RL algorithm in the OpenAI library. The algorithm in its most general form can be seen in Algorithm 1.

**Algorithm 1**: Proximal Policy Optimization, Actor-Critic style

```python
for iteration: 1,2... do
    for actor: 1,2...N do
        Run policy \( \pi_{\theta_{old}} \) for \( T \) time-steps
        Compute advantage estimate \( \hat{A}_1...\hat{A}_T \)
        (Optimize surrogate L wrt \( \theta \), with K epochs and mini-batch size \( M < NT \))
        \( \theta_{old} \leftarrow \theta \)
    end
end
```

More details of the implementation can be found in the code on Github [10].

### 4. Simulation set-up

This section provides a detailed description of the set up used to do path following simulations presented in the subsequent sections. We utilized two distinct approaches to train the RL agent to achieve the objective of path following: one
is called end-to-end training (subsection 4.1) where the agent learns through freely exploring the environment and gradually trading off exploration for exploitation. In the second approach, the agent’s learning is assisted by PID controllers (subsection 4.2). Since one of the desired features of the path-following problem is to maintain a desired cruise speed while minimizing tracking-errors, the first part of the controller design was a simple velocity controller. Because of the reduced complexity of this problem, it could act as a sanity check for the simulator code and the implementation of the RL scheme. Furthermore, because a constant reference cruise speed is part of both the approaches, the reward functions should contain similar penalizing terms for deviating from this reference. The velocity controller worked well but to save space we do not present the result from those simulations. In the two approaches presented here, the AUV is randomly initialized within a proximity of 5 meters of the first waypoint. It is desired that the AUV maintains a cruise speed of $u_d = 1.5 \text{ ms}^{-1}$. The AUV is underactuated, as it operates in 6 DOF with only 3 actuators. The simulated learning processes are described in the following sections.

### 4.1. End-to-end learning

In this scenario, the observation that the agent makes of the environment, $s$, consists of normalized measurements of the Euler angles, $\Theta_o = [\phi_o, \theta_o, \psi]^T$, the angular rates $\omega_o = [\dot{\phi}_o, \dot{\theta}_o, \dot{\psi}]^T$ and the control errors $e = [\ddot{u}_o, \ddot{\theta}_o, \ddot{\psi}_o, \dot{e}_o, \dot{h}_o]^T$. These are listed in Table 3 together with their empirical or true maximums, and reward function coefficients. Subscript o indicates that these values have been normalized by their empirical or true maximum, to ensure that values fed to the neural networks are between $-1$ and $1$. Neural networks work better with normalized data, which often improves the numerical stability of the model and reduces training time. There is no obvious empirical maximum for the vertical and cross track error, which make the normalization factors a design choice. Choosing $e_{max} = h_{max} = 25m$ is reasonable, since tracking-errors $> 25m$ indicate significantly poor performance. Errors above this threshold can therefore be uniform. As is the state-of-the art in RL and ML in general, the reward function was crafted through reasoning and modified iteratively. The reward function governing the AUV behaviour is

$$r = \alpha_1^T |\Theta_o| + \alpha_2^T |\omega_o| + \alpha_3^T |e|.$$  \hspace{0.5cm} (24)

The penalization factors, $\alpha_i$, were chosen based on the control objective and empirical trials. For instance, it is not obvious that roll and roll rate should be penalized, but this is done to avoid a behaviour where the rudder acts as the elevator fin and vise versa. Angular rates are penalized to indirectly penalize aggressive and large control inputs. In Equation 24, $| \cdot |$ denotes the element-wise absolute value. As all elements of $\alpha_i$ are negative, the reward function is always negative. The chosen penalization factors $\alpha_i$ are given in Table 3.

### 4.2. PID assisted learning

In PID assisted training, the idea is to decouple the neural network into three parts that will be trained separately, each controlling its own actuator. A cross-track controller operates the rudder fins, a vertical-track controller the elevator fins, and a velocity controller controls the propeller shaft speed. This is analogous to how traditional autopilots are designed, but the key difference is in the set-up: To not lose information about the system it should see every state and other actuator while training and in operation. In order to avoid training the neural networks together, PI/PID controllers are enabled to stabilize the two sub-processes that are not considered at the time. An example of this scheme when training the network controlling the rudder is illustrated in Figure 4. The design of the PID controllers is not central, but the PID controllers should not lead to unstable behaviour. One could argue that a controller that does not make the AUV behave perfectly is preferable to a suboptimal one during PID assisted learning. This way, the agent will be exposed to larger parts of the state and action space, increasing exploration.

With PID assistance, the agent should be able to learn a control law that is based on all available information in the system. The reward functions defined in Equation 25 and Equation 26 are shaped as Gaussian and quadratic functions, and are used for the cross- and vertical-track controllers, respectively. They are shaped differently simply to observe the difference in behavioural outcome, such as the training process and the final tracking error. As explained in [14], the quadratic function is a good candidate for continuous reward functions because of its well defined reward gradient. The quadratic function is interesting as it is both negative definite and continuously differentiable. Another attribute of

| Observation       | Max      | $\alpha$    |
|-------------------|----------|-------------|
| Roll              | $\phi_o$ | $\theta_o$ |
| Pitch             | $\theta_o$ | $\psi$ |
| Yaw               | $\phi_o$ | $\theta_o$ |
| Roll rate         | $p_o$    | $q_o$ |
| Pitch rate        | $q_o$    | $r_o$ |
| Yaw rate          | $r_o$    | $e_o$ |
| Surge error       | $\ddot{u}_o$ | $\ddot{\theta}_o$ |
| Course error      | $\dot{e}_o$ | $\dot{h}_o$ |
| Elevation error   | $\dot{\psi}_o$ | $\dot{\theta}_o$ |
| Cross track error | $\ddot{e}_o$ | $\ddot{h}_o$ |
| Vertical track error | $\dddot{e}_o$ | $\dddot{h}_o$ |

The penalization factors, $\alpha_i$, were chosen based on the control objective and empirical trials. For instance, it is not obvious that roll and roll rate should be penalized, but this is done to avoid a behaviour where the rudder acts as the elevator fin and vise versa. Angular rates are penalized to indirectly penalize aggressive and large control inputs. In Equation 24, $| \cdot |$ denotes the element-wise absolute value. As all elements of $\alpha_i$ are negative, the reward function is always negative. The chosen penalization factors $\alpha_i$ are given in Table 3.

With PID assistance, the agent should be able to learn a control law that is based on all available information in the system. The reward functions defined in Equation 25 and Equation 26 are shaped as Gaussian and quadratic functions, and are used for the cross- and vertical-track controllers, respectively. They are shaped differently simply to observe the difference in behavioural outcome, such as the training process and the final tracking error. As explained in [14], the Gaussian function is a good candidate for continuous reward functions because of its well defined reward gradient. The quadratic function is interesting as it is both negative definite and continuously differentiable. Another attribute of

| Observation       | Max      | $\alpha$    |
|-------------------|----------|-------------|
| Roll              | $\phi_o$ | $\theta_o$ |
| Pitch             | $\theta_o$ | $\psi$ |
| Yaw               | $\phi_o$ | $\theta_o$ |
| Roll rate         | $p_o$    | $q_o$ |
| Pitch rate        | $q_o$    | $r_o$ |
| Yaw rate          | $r_o$    | $e_o$ |
| Surge error       | $\ddot{u}_o$ | $\ddot{\theta}_o$ |
| Course error      | $\dot{e}_o$ | $\dot{h}_o$ |
| Elevation error   | $\dot{\psi}_o$ | $\dot{\theta}_o$ |
| Cross track error | $\ddot{e}_o$ | $\dddot{h}_o$ |
| Vertical track error | $\dddot{e}_o$ | $\dddot{h}_o$ |
the quadratic function is its relative decrease in punishment as each error approaches zero. Hence, large errors should have a greater influence when performing gradient ascent, and be prioritized during learning.

\[ r_1(\ddot{r}, e, \delta_r) = \alpha_1(1-e^{-5\ddot{r}^2}) + \alpha_2(1-e^{-5e^2}) + \alpha_3(1-e^{-5\delta_r^2}) \]  
\[ r_2(h, \delta_e) = \alpha_1 \ddot{h}^2 + \alpha_2 h^2 + \alpha_3 \delta_e^2 \]  

Table 4 details the elements of the state vector that the agent receives for cross-track control. Note that when learning vertical-track control, elevation and vertical-track error are penalized instead of course and cross-track error. Also, the position of the rudder fin replaces the observation of the elevator fin. The penalization factor for fin actuation is \( \alpha_3 = -1 \times 10^{-2} \) in both cases.

Table 4

| Observation                  | Max      | \( \alpha \) |
|-----------------------------|----------|--------------|
| Relative surge speed        | \( u_\rho = \frac{\rho}{\rho_{\text{max}}} \) ∈ [−1, 1] | 2 0           |
| Relative sway speed         | \( v_\rho = \frac{\rho}{\rho_{\text{max}}} \) ∈ [−1, 1] | 0.3 0         |
| Relative heave speed        | \( w_\rho = \frac{\rho}{\rho_{\text{max}}} \) ∈ [−1, 1] | 0.3 0         |
| Roll                        | \( \phi = \frac{\phi}{\phi_{\text{max}}} \) ∈ [−1, 1] | \( \pi \) 0 |
| Pitch                       | \( \theta = \frac{\theta}{\theta_{\text{max}}} \) ∈ [−1, 1] | \( \pi \) 0   |
| Yaw                         | \( \psi = \frac{\psi}{\psi_{\text{max}}} \) ∈ [−1, 1] | \( \pi \) 0   |
| Roll rate                   | \( p_w = \frac{p}{p_{\text{max}}} \) ∈ [−1, 1] | 1.2 0         |
| Pitch rate                  | \( q_u = \frac{q}{q_{\text{max}}} \) ∈ [−1, 1] | 0.4 0         |
| Yaw rate                    | \( r_w = \frac{r}{r_{\text{max}}} \) ∈ [−1, 1] | 0.4 0         |
| Surge error                 | \( s_w = \frac{s}{s_{\text{max}}} \) ∈ [−1, 1] | 2 0           |
| Course error                | \( \delta_c = \frac{\delta_c}{\delta_{\text{max}}} \) ∈ [−1, 1] | \( \pi \) −2e-2 |
| Elevation error             | \( \alpha = \frac{\alpha}{\alpha_{\text{max}}} \) ∈ [−1, 1] | \( \pi \) 0   |
| Cross track error           | \( e_v = \frac{e}{e_{\text{max}}} \) ∈ [−1, 1] | 25 −5e-2      |
| Vertical track error        | \( h_v = \frac{h}{h_{\text{max}}} \) ∈ [−1, 1] | 25 0          |
| Propeller shaft speed       | \( n \) | 1 0           |
| Elevator fin position       | \( \delta_e \) | 1 0           |

5. Results and discussion

In this section we present the major findings of this work. First we present the results for the end-to-end learning, and then for the PID assisted learning.

5.1. End-to-end learning

The hyperparameters used during end-to-end learning are given in Table 5. The number of steps \( T \) in algorithm 1 and large batch size \( M \) in algorithm 1 lead to long learning times on a desktop computer.

Figure 5 shows the reward value as a function of simulated time steps the AUV has spent exploring the environment. The AUV was left to explore the environment for 30 million time steps, and as seen in Figure 5, the reward peaks at around 8 million. After 10 million time steps an unlearning process is observed. It is not obvious why the agent seemingly unlearns behaviour after 10 million time steps. It is speculated that the agent discovers different possibilities for minimizing the reward function when it receives new information about the environment. The agent might observe a set of unlikely states after 10 million time steps that motivates this new approach. Note that some noise in the learning process is expected, caused by the path being randomly generated for each episode. However, the test results are simu-
Table 5

Values of hyperparameters used for training the DRL controllers.

| Hyperparameter      | Value |
|---------------------|-------|
| Learning rate       | 5e-5  |
| Discount rate       | 0.999 |
| GAE parameter       | 0.95  |
| # Actors            | 10    |
| # Steps             | 6144  |
| Epochs              | 4     |
| Batch size          | 1024  |
| Min. reward         | -500  |

Figure 6 shows the surge, desired surge, sway and heave speeds when applying the best controller for the end-to-end learning. The RL agent keeps the surge speed close to the setpoint, though an offset is observed. The four abrupt changes in the surge speed happen when the guidance system switches between waypoints.

Next, the normalized control action is seen in Figure 7. The propeller thrust is initially high, in order to accelerate the AUV to the desired surge speed. The fin movements are limited, demonstrating the effect of penalizing the angular rates, as discussed in subsection 4.1. When the guidance system switches between waypoints, the use of control inputs increases.

Figure 6: Velocity plot from end-to-end control simulation.

Figure 7: Normalized control inputs for end-to-end control simulation. ($n =$ propeller shaft speed, $\delta_r =$ rudder, $\delta_a =$ elevator)

As with the surge speed, the control errors, seen in Figure 8, experience rapid changes when the guidance system switches waypoints. None of the errors are completely eliminated, and an interpretation of the results is that the effect of one actuator disturbs the others, which yields oscillating control errors.

Figure 5: Episode reward when training the end-to-end controller. Performance peaks around 8M time steps.
The effect of changing waypoints is again displayed in Figure 9, which shows a 3D plot of the simulation. When changing waypoints, the AUV overshoots and is unable to fully reduce the errors before the guidance system targets a new waypoint. The performance is not considered as adequate to state that the end-to-end controller solves the path following problem. However, the results are promising and with more research on reward functions and penalization schemes, end-to-end control might be feasible.

The results were obtained by simulating without ocean current disturbances. Better results under ideal conditions are needed before progressing further with this method.

5.2. PID assisted learning

Hyperparameters remained as in Table 5, as the results for the end-to-end and PID assisted learning should be comparable. The achieved reward during PID assisted learning is shown in Figure 10 and Figure 11 when using the reward function from Equation 25 and Equation 26, respectively.

The episode rewards during training of the cross-track controller is displayed in Figure 10. Compared to Figure 5, the agents learning seems faster and more stable, as intended. In this learning scenario, the reward function is simpler and as the agent is only learning one control input at a time, the state space being explored is consequently smaller. There is a small dip in accumulated reward at 6M time-steps, but this is quickly solved and the learning saturates at 10M time-steps. Hence, there are no signs of the unlearning behaviour that was observed in the end-to-end case.

In Figure 11, the episodic reward during training of the vertical-track controller is seen. Here, the agent experiences no drop in reward and the performance saturates at about 5M time-steps, indicating faster learning. This leads us to believe that the quadratic reward function from Equation 26 might be better suited for this control problem.
with confidence on the basis that after 30M timesteps the latter controller’s performance is not adequate, as seen in Figure 9. PID assisted learning yields a very capable controller after 10M time-steps. This is also highlighted in the simulation results, which indicate significantly better performance on path following after combining the cross and vertical-track machine learning controllers obtained with PID assisted learning.

Note that the PID controllers are only necessary when training, but the PI controller for surge speed is used in the simulation results in subsection 5.2. There is no reason to doubt whether a velocity controller could learn equivalently to the tracking controllers, i.e. by PID assistance. However, this is not pursued as earlier experiments (not included in this article) showed that perfect velocity tracking is achievable through end-to-end learning for velocity control.

To best be able to compare the reward functions and observe their effect on learning, the disturbing current was removed during training. Additionally, the path generated in all test simulations are identical. Regardless of the networks training without disturbances, the simulation results show great performance on the path-following problem, both with and without an ocean current perturbing the system. The performance of the combined PID assisted controllers after training is presented in the next section.

### 5.2.1. Simulation Results without Current Disturbance

In addition to a greatly reduced training time, Figure 12 to 15 reveal better tracking performance than the end-to-end trained controller.

In Figure 12 we observe a typical PI setpoint regulation of the surge speed. The effects from switching waypoints on surge speed are negligible, which indicates that the disturbances from the control fins are small.

The normalized control action is pictured in Figure 13. The fins are used conservatively, as intended by the penalization term in the reward functions. The control is smooth and well behaved and the oscillatory, aggressive behaviour seen in the end-to-end simulation is not displayed here. Furthermore, the effect of waypoint switching is minuscule, in accordance with the negligible effects observed in the velocity plot.

Control errors stay very close to zero, and only see an increase when the guidance system targets new waypoints, as seen in Figure 14. Perfect surge speed regulation is observed, and the DRL controllers make the AUV follow the path accurately.

The increase in performance is truly reflected in Figure 15, where the trajectory of the AUV and the path are displayed. The increase in performance is self-evident when comparing this plot with Figure 9. There is no longer an overshoot, and the AUV stays close to desired path, even when changing waypoints.

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Figure 12: Velocity plot from PID assisted control simulation.

Figure 13: Normalized control inputs from PID assisted control simulation.

Figure 14: Normalized control errors from the PID assisted control simulation. All control errors are eliminated and the deviations are only the result of switching waypoints.
it is an encouraging finding that the AUV is able to perform so well in the presence of an ocean current with varying intensity when it has been trained under perfect conditions.

5.2.2. Simulation Results with Current Disturbance

This section presents the simulation results when employing the same controllers from Section 5.2.1 in the presence of ocean currents. The direction of the current is randomly initialized, while the intensity is simulated as a random walk within the interval $0.5$ to $1.0 \, \text{ms}^{-1}$.

When ocean currents are present, the DRL controllers make greater use of the fins, as seen in Figure 16. Especially the elevator fin is utilized more than before, due to the current having a large vertical component. This does not seem to introduce unwanted behaviour.

Figure 17 presents the control errors, which reveals a slight offset in vertical tracking. The agent has inferred that it must increase the control input in order to compensate for the current, but since it has not experienced this scenario during training, it is not able to compensate completely. However, it appears that the AUV is able to perform well in the presence of an ocean current with varying intensity when it has been trained under perfect conditions.

As previously stated, the neural networks were trained without any currents or disturbances, so it is not expected to achieve perfect tracking in these simulations. The main idea is to demonstrate that the controllers still, to a great extent, achieve path following, even when exposing them to previously unseen scenarios. Overall these results can be seen as a form of robustness testing, and the results are quite exciting.
By adding disturbances during training, robustness could be increased further. In that way, it would be possible learn to compensate for the currents by exploration and exploitation.

6. Conclusion

The 3D path following problem for AUVs was solved by utilizing DRL controllers, more specifically with the algorithm PPO. Two methods were considered: end-to-end learning, where the agent is left entirely alone to explore the solution space in its search for an optimal policy, and PID assisted learning, where the DRL controller is essentially split into three separate parts, each controlling its own actuator (rudder, elevator and thrust). When training one actuator at a time, the two others are being controlled by stable PID controllers such that the control objectives can be achieved.

End-to-end learning showed promising results, but due to a highly complex reward scheme and sensitivity to training set-up, more research is needed to obtain better performance with this method. The study of DRL in practical applications are mostly based on heuristics and experimental data, which prompts major challenges in the fusion of cybernetics and AI.

PID assisted learning gave excellent simulation results, and together with the quadratic cost function it was found to significantly reduce the number of simulated time-steps needed to train the controller. Furthermore, an advantage with the DRL approach, compared to traditional control methods, is the lack of need for an accurate underlying world model to achieve excellent results. No tuning is needed, it does that itself by exploration and exploitation. No a priori information is needed, other than that traditional control methods (PID) are able to stabilize subprocesses during learning. The results make us hopeful that extending the aforementioned methods to incorporate collision avoidance, can take AUV motion control systems one more step towards true autonomy.

Acknowledgment

The authors acknowledge the financial support from the Norwegian Research Council and the industrial partners: DNV GL, Kongsberg and Maritime Robotics of the Autosit project. (Grant No.: 295033).

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