Hydrodynamic elliptic and triangular flow in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76A$ TeV

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It is shown that a simultaneous comparison of both elliptic and triangular flow from (2 + 1)-dimensional viscous fluid dynamics with recent measurements in Pb + Pb collisions at the Large Hadron Collider (LHC) favors a small specific shear viscosity ($\eta/s_{QGP} \approx 1/(4\pi)$) for the quark–gluon plasma. Using this viscosity value, the relative magnitude of the elliptic and triangular flow is well described with Monte Carlo Glauber (MC-Glauber) initial conditions while Monte Carlo Kharzeev–Levin–Nardi (MC-KLN) initial conditions require twice as large viscosity to reproduce the elliptic flow and then underpredict triangular flow by about 30%.

1. Introduction

Much attention has been given recently to the extraction of the shear viscosity to entropy density ratio (i.e. the specific shear viscosity $\eta/s$) of the quark–gluon plasma (QGP) from elliptic flow data in relativistic heavy-ion collisions [1–15]. A major road block in this effort is insufficient knowledge of the initial shape of the thermalized fireball created in these collisions, whose initial ellipticity is uncertain by about 20% [17–21]. This induces an ized fireball created in these collisions, whose initial ellipticity is insufficient knowledge of the initial shape of the thermal-

relativistic heavy-ion collisions [1–15]. A major road block in this presents such an analysis.

Our study is based on a (2 + 1)-dimensional viscous hydro-
dynamic model with longitudinal boost-invariance, describing numerically the transverse evolution of the heavy-ion collision fireball near midrapidity. As in past work [4–6,19–21] we explore two different types of fluctuating initial conditions for the entropy and energy density profiles, generated from Monte Carlo versions of the Glauber and KLN models (see [19] and references therein for details of the implementation used here). We first generate averaged initial conditions from ensembles of lumpy initial conditions using these two models, then evolve them hydrodynamically as a substitute for event-by-event calculations. The validity of this approach will be discussed below.

The MC-KLN calculations were done using a Monte Carlo sample of initial state profiles with identical properties as those used in [16] for the calculation of transverse momentum spectra and elliptic flow in 2.76A TeV Pb–Pb collisions at the LHC. For the $x$ dependence of the gluon structure function in the MC-KLN model we used the power $\lambda = 0.28$ [19]; the normalization factor for the initial entropy density was fixed by hand to reproduce the measured charged hadron multiplicity density $dN_{ch}/dy$ for the 5% most central collisions [39]; the measured dependence of $dN_{ch}/dy$ on collision centrality [40] is then automatically reproduced reasonably well by the model [16] (see Fig. 1(a)). MC-KLN runs were done with $\eta/s = 0.2$ which, for this type of initial conditions, was shown to yield a good overall description of the measured transverse momentum spectra and elliptic flow in 200A GeV Au–Au collisions at RHIC [16] and gave an impressively accurate prediction for the unidentified and identified charged hadron spectra and elliptic flows in 2.76A TeV Pb–Pb collisions at the LHC [16,41].

For the MC-Glauber runs we generated a new set of initial configurations that differs from those used for 200A GeV Au–Au collisions in [6] by the wounded nucleon to binary collision ratio. Taking the initial entropy density $s(r_{⊥}; b) = \kappa (1/2)n_{NN}(r_{⊥}; b) + n_{BC}(r_{⊥}; b)$, we determine $\kappa$ and $x$ by a two-parameter fit to the ALICE data [40] shown in Fig. 1(a). Due to viscous heating during the hydrodynamic evolution, which itself depends on collision
centrality, the fit value for x depends on the assumed shear viscosity. For MC-Glauber initial conditions we took $\eta/s = 0.08$ which was shown in [13,25,35] to provide a reasonable description of the charged hadron $v_2(p_T)$ and $v_3(p_T)$ data measured by ALICE; this results in $x = 0.118$ for Pb–Pb collisions at the LHC (instead of 0.14 for Au–Au at RHIC [6]).

Both models have additional parameters that potentially affect the initial geometric deformation, such as the nucleon size and density profile. We here use disc-like nucleons with a radius controlled by the nucleon–nucleon cross section [19]. The effects of these parameters are being studied systematically and will be reported in a longer paper; so far, we have not encountered anything that could change the conclusions drawn here.

In [21] we showed that, due to similar fluctuation mechanisms, the MC-KLN and MC-Glauber models generate similar third-order eccentricities $\varepsilon_3$ whereas the ellipticity $\varepsilon_2$, which is mostly controlled by collision geometry, is about 20% larger in the MC-KLN model. Event-by-event ideal [21] and viscous [38,42] hydrodynamic simulations with both realistically fluctuating [21,42] and doubly-deformed Gaussian initial conditions [38] (with simultaneously non-zero $\varepsilon_2$ and $\varepsilon_3$ eccentricities) have shown that the hydrodynamic conversion efficiencies for translating initial spatial eccentricities into final flow anisotropies [30,43,44], although different for $v_2/\varepsilon_2$ and $v_3/\varepsilon_3$, are very similar in the MC-KLN and MC-Glauber models. The similar $\varepsilon_3$ and different $\varepsilon_2$ of these models should thus be straightforwardly reflected in analogous differences in $v_2$ and $v_3$ [37,38], allowing for an experimental distinction between the models.

The initial profiles are hydrodynamically evolved with equation of state (EOS) s95p-PCE [15] which matches numerical results from lattice QCD at high temperatures to a hadron resonance gas at low temperatures [45] and implements chemical freeze-out at $T_{\text{chem}} = 165$ MeV. The latter is an important improvement over the work presented in [12,13,28,35] since accounting for the chemical non-equilibrium evolution in the late hadronic stage strongly affects the elliptic flow $v_2$ [46]. The hydrodynamic output is converted to final hadron distributions along an isothermal decoupling surface with $T_{\text{dec}} = 120$ MeV, using the Cooper–Frye prescription. Event-by-event viscous hydrodynamic simulations with full inclusion of unstable resonance decays are at present numerically too costly for systematic flow studies over a range of viscosities, collision energies, centralities, and collision systems. A recent event-by-event study by Schenke et al. [35] with a restricted set of resonances showed that, compared to a full calculation, $v_2$ ($v_3$) was overpredicted by 10–15% (25–30%). This is larger than the difference in these observables seen [21,38,42] between event-by-event and single-shot calculations. For this reason we use “single-shot hydrodynamics”, where the fluctuating initial conditions are averaged before the hydrodynamic evolution instead of afterwards, but include the full cascade of resonance decays in the final state. We rotate each lumpy initial condition according to its second or third order participant plane orientation, $\psi_2$ or $\psi_3$ [21,38], to generate two sets of smooth initial conditions. The averaged $v_2$- and $\psi_2$-rotated profiles are used to calculate the elliptic and triangular flow, respectively. This differs from [28] where a suitably normalized $\cos(3\phi)$ modulation is superimposed on optical Glauber or KLN initial conditions by hand. We emphasize that our single-shot approach is used as an approximation to obtain what should be calculated event by event. Our procedure allows us to make direct statements about the connection between elliptic and triangular flow and the fluctuating initial eccentricities in the MC-Glauber and MC-KLN models even though we don’t use the fluctuating initial profiles directly for the dynamical evolution. The validity of the substitution of single-shot for event-by-event hydrodynamics, at the level of precision needed here, was initially explored in [21] and is justified in detail in a forthcoming systematic analysis [42].

2. Transverse momentum spectra

Fig. 1(b) shows the charged hadron $p_T$-spectra for 2.76A TeV Pb–Pb collisions at different centralities, for both MC-Glauber ($\eta/s = 0.08$) and MC-KLN ($\eta/s = 0.2$) initial conditions. For the

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1 For each event there are three equivalent choices for $\psi_3$; we checked that, since $\psi_2$, $\psi_3$ show almost no correlation [21,30,47], nearly identical $\varepsilon_3$ values, $p_T$-spectra and triangular flow are obtained with different $\psi_3$ selections for the averaging process.

2 Using that method, it was observed already in [28] that RHIC $v_3$ data favored a small $\eta/s$ and conflicted with the value $\eta/s = 0.16$ used in conjunction with KLN initial eccentricities, and in [25] the same conclusion was drawn from a comparison of the calculations in [28] with triangular flow measured in 2.76A TeV Pb–Pb collisions at the LHC.

3 We checked that the $p_T$-spectra from averaged $\psi_2$- and $\psi_3$-rotated initial profiles are almost identical; this ensures that the discussion below of the $p_T$-dependent elliptic and triangular flows, which are obtained from differently averaged initial conditions, is not distorted by different $p_T$-distributions of the final hadrons carrying the flow.
most central (0–5%) collisions the spectra from both models agree well with published ALICE data. In more peripheral collisions the MC-KLN spectra are harder than those from MC-Glauber initial conditions. This is a consequence of larger radial flow caused by larger transverse viscous pressure gradients in the MC-KLN case where the fluid is taken to have 2.5 times larger shear viscosity than in the MC-Glauber simulations, in order to obtain the same elliptic flow [46,6]. In peripheral collisions these viscous effects are stronger than in more central collisions where the fireball is larger [49]. As shown in [21,50], event-by-event evolution of fluctuating initial conditions generates, for small values of $\eta/s$, flatter hadron spectra than single-shot hydrodynamics, especially in peripheral collisions, due to stronger radial flow driven by hot spots in the fluctuating initial states. Proper event-by-event evolution of the latter is therefore expected to reduce the difference between the MC-Glauber and MC-KLN curves in Fig. 1(b) since this effect is relatively strong for $\eta/s = 0.08$ (MC-Glauber) [21] but almost absent for $\eta/s = 0.2$ (MC-KLN) [42].

### 3. $p_T$-integrated elliptic and triangular flow

In Fig. 2 we compare our $p_T$-integrated $v_2$ and $v_3$ as functions of centrality with ALICE $v_2(2)$, $v_2(4)$, $v_3(2)$, and $v_3(4)$ data, extracted from 2- and 4-particle correlations [25]. For both models, $v_{2,3}$ from averaged smooth initial conditions lie between the experimental $v_{2,3}(2)$ and $v_{2,3}(4)$ values. This is consistent with the theoretical expectation [51,52] that $v_4(2)$ ($v_4(4)$) is shifted up (down) relative to the average flow by event-by-event flow fluctuations and was also found in [8,13,16]. Upon closer inspection, however, and recalling that ideal single-shot hydrodynamics with smooth initial condition was shown [21] to generate $v_2$ similar to $v_2$ from the corresponding event-by-event evolution, it seems that the MC-KLN is favored since it produces $v_2$ results closer to the $v_2$ data. Unfortunately, similar reasoning using $v_3$ argues against the MC-KLN model. To eliminate the interpretation difficulties associated with a comparison of average flows from single-shot evolution of averaged initial conditions with data that are irreducibly affected by naturally existing event-by-event fluctuations, we proceed to a comparison of eccentricity-scaled flow coefficients.

Assuming linear response of $v_{2,3}$ to their respective eccentricities $\varepsilon_{2,3}$ (which was found to hold with reasonable accuracy for $v_2$ and $v_3$ but not for higher order anisotropic flows [21]), we follow [53] and scale the flow $v_{2,3}$ from single-shot hydrodynamics by the eccentricity $\varepsilon_{2,3}$ of the ensemble-averaged smooth initial energy density, while scaling the experimental $v_{2,3}(2)$ and $v_{2,3}(4)$ data by the corresponding fluctuating eccentricity measures $\varepsilon_{2,3}(2)$ and $\varepsilon_{2,3}(4)$, respectively, calculated from the corresponding models. In [42] we justify this procedure for $v_{2,3}(2)$ and $v_{2,3}(4)$ for viscous hydrodynamics (see [21] for event-by-event ideal fluid dynamics) and also show that it fails for $v_3(4)/v_3(4)$ since this ratio is found to differ strongly from $v_3(\varepsilon)$.

The eccentricity-scaled elliptic and triangular flow coefficients for the MC-KLN and MC-Glauber models are shown in Figs. 3a, b and 3c, d, respectively, and compared with the corresponding data from ALICE. The first thing to note is the impressively accurate agreement between the experimentally measured $v_2(2)/v_2(2)$ and $v_4(2)/v_4(4)$, showing that for elliptic flow the idea of scaling "each flow with its own eccentricity" [53] works very well. The same is not true for $v_3(2)/v_3(2)$ and $v_3(4)/v_3(4)$ for which the experimental values do not at all agree (not shown), nor are they expected to [42]. Secondly, both $v_2(2)/v_2(2)$ and $v_4(2)/v_4(4)$ measured by ALICE agree well with the viscous hydrodynamic calculations, for both the MC-Glauber and MC-KLN models, confirming that for each model the correct value of $\eta/s$ has been used as far as elliptic flow is concerned.

We emphasize that the conclusions up to this point, based on the analysis of elliptic flow alone, agree with what has by now been firmly established [6,16,35]: to reproduce experimental $v_2$ measurements, the MC-Glauber model must be coupled with $\eta/s = 0.08$ while MC-KLN initial profiles require $\eta/s = 0.16–0.20$. Since $\eta/s$ is fixed from $v_2$, their predictions for triangular flow...
provide the power to discriminate between these models experimentally. This is shown in the bottom panels of Fig. 3. Clearly, with the viscosities needed to reproduce $v_2$, the MC-KLN model badly disagrees with the experimental $v_3$ data. The measured triangular flow is simply too large to accommodate a specific shear viscosity as large as 0.2. Within the present approach, the only possibility to avoid this conclusion is that somehow the MC-Glauber and MC-KLN models both underpredict the initial third-order eccentricity $\varepsilon_3$ by about 50%. Although we have not gone through the numerical expense of a systematic study trying to fit $\eta/s$ to the $v_3$ data independently, the schematic study in [41] tells us that we must reduce $\eta/s$ by about a factor 2 (to a value around 0.1) if we want to fit $v_3$ with the MC-KLN model. This is close to the value of 0.08 needed with the Glauber model to reproduce $v_2$ and, as seen in Fig. 3d, also $v_3$. For $\eta/s = 0.08$, viscous hydrodynamics with MC-Glauber initial conditions is seen to reproduce the ALICE data quite well over the entire range of collision centralities, even if the measured centrality dependence of $v_2(2)/\varepsilon_3(2)$ is a bit steeper than the calculated one.

Fig. 3 can be summarized by stating that the ALICE data for the $p_T$-integrated elliptic and triangular data strongly favor MC-Glauber initial conditions and, by implication, a small value of $\eta/s \simeq 0.08$ for the specific QGP shear viscosity.

4. $p_T$-differential elliptic and triangular flow

We close this Letter by cross-checking, at one collision centrality (30–40%) where $v_3(p_T)$ data are available [25], the $p_T$-differential anisotropic flows. The corresponding comparison between data and theory is shown in Fig. 4; as in Fig. 3 we compare the eccentricity-scaled flows, plotting $v_{2,3}/\varepsilon_{2,3}$ for the models and $v_2(4)/\varepsilon_2(4)$ ($v_3(2)/\varepsilon_3(2)$) for the elliptic (triangular) flow data. As seen in the upper panels, both initial state models describe the measured elliptic flow well up to $p_T \sim 1$–1.5 GeV/$c$; at larger $p_T$, they overpredict $v_2(p_T)$ for charged particles – a problem noticed before [14,16] and possibly related to an imperfect model description of the measured final chemical composition [41]. The disagreement at larger $p_T$ is worse for MC-Glauber initial conditions; this is likely related to our earlier observation in Fig. 1(b) that our MC-Glauber $p_T$-spectra are steeper than the MC-KLN ones in peripheral collisions – an artifact of our single-shot approach and possibly remedied by a proper event-by-event hydrodynamical simulation.

Fig. 4b shows again the disagreement between theory and experiment for triangular flow when we use MC-KLN initial conditions: the model strongly underpredicts the data at all $p_T$, i.e. it gives the wrong slope for $v_3(p_T)$. With MC-Glauber initial conditions and correspondingly lower shear viscosity $\eta/s = 0.08$ (Fig. 4d), the measured $v_3(p_T)$ is well described up to $p_T \sim 1$ GeV/$c$ but overpredicted at larger $p_T$. Again, the latter can be at least partially attributed to the fact that MC-Glauber $p_T$-spectrum from our single-shot hydrodynamic approach is too steep at this collision centrality, which can in future studies be corrected by performing the hydrodynamic evolution properly event by event.

5. Summary

Using a well-calibrated single-shot viscous hydrodynamic approach without hadronic cascade afterburner but properly implementing hadronic chemical freeze-out at $T_{chem} \approx 165$ MeV and including a full set of resonance decays, we have shown that a combined analysis of the ALICE data for elliptic and triangular flow from 2.76A TeV Pb-Pb collisions leads to a strong preference for initial conditions from the Monte Carlo Glauber model, combined with a low value for the QGP shear viscosity $\eta/s \simeq 0.08$, and disfavors the considerably larger viscosities of $\eta/s \sim 0.2$ that are required to reproduce the measured elliptic flow when assuming the more eccentric Monte Carlo KLN initial profiles. Final confirmation of these conclusions will require a proper event-by-event evolution of the fluctuating initial density profiles and coupling viscous hydrodynamics to a microscopic description of the dilute late hadronic stage where viscous hydrodynamics breaks down [54], and a similar analysis of recently published PHENIX data at lower RHIC energies [23]. Given the large magnitude of the under-prediction $v_3$ in the MC-KLN model observed here we doubt, however, that such more sophisticated approaches will be able to reverse the conclusions drawn here.

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