A Note on the Superstring BRST Operator

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Abstract

We write the BRST operator of the N=1 superstring as

\[ Q = e^{-R} \left( \frac{1}{2\pi i} \oint dz \gamma^2 b \right) e^R \]

where \( \gamma \) and \( b \) are super-reparameterization ghosts. This provides a trivial proof that \( Q \) is nilpotent.

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1 Introduction

Superstring theory in ten dimensions is a critical $N = 1$ superconformal theory. It can be quantized using a nilpotent BRST operator

$$Q = \frac{1}{2\pi i} \oint dz \left[ c(T_m + \frac{1}{2}T_g) + \gamma(G_m + \frac{1}{2}G_g) \right]$$ (1)

where $[T_m, G_m]$ are the $c = 15$ $N=1$ superconformal generators and $[T_g, G_g]$ are the $c = -15$ $N=1$ superconformal generators constructed from a pair of fermionic ghosts $[b, c]$ and a pair of bosonic ghosts $[\beta, \gamma]$. Physical states are described by vertex operators in the cohomology of $Q$ and, in order to construct vertex operators for the spacetime fermions, it is convenient to fermionize the bosonic ghosts as

$$\beta = \partial \xi e^{-\phi}$$ (2)
$$\gamma = \eta e^\phi,$$ (3)

where $\eta$ and $\xi$ are free fermions and $\phi$ is a chiral boson.$[1]\)$

Because the above fermionization involves $\partial \xi$ rather than $\xi$, it is not possible to write the zero mode of $\xi$ in terms of the $[\beta, \gamma]$ ghosts. The Hilbert space without the $\xi$ zero mode is called the “small” Hilbert space, while the Hilbert space including the $\xi$ zero mode is called the “large” Hilbert space. The small Hilbert space can be defined as operators annihilated by $\oint dz \eta$, and one can show that any such operator can be constructed out of the original $[\beta, \gamma]$ ghosts. Since physical vertex operators should be in the small Hilbert space, they must be annihilated by both $Q$ and $\oint dz \eta$. For consistency, this requires that $Q$ should not only be nilpotent, but should also anti-commute with $\oint dz \eta$.

In this letter, we will construct a similarity transformation $R$ such that $Q = e^{-R}(\frac{1}{2\pi i} \oint dz \gamma^2 b)e^R$. (Note that the first term of $R$ was constructed in $[2]\).$ Since $\gamma^2 b$ is nilpotent, this trivially proves that $Q$ is nilpotent. Furthermore, since $\oint dz \gamma^2 b$ has trivial cohomology, it proves that $Q$ has trivial cohomology in the large Hilbert space.

However, $R$ does not commute with $\oint dz \eta$, so $Q$ has non-trivial cohomology in the small Hilbert space as expected. Also, $e^{-R}(\frac{1}{2\pi i} \oint dz \gamma^2 b)e^R$ only anti-commutes with $\oint dz \eta$ in the critical dimension, so one cannot use the nilpotent $e^{-R}(\frac{1}{2\pi i} \oint dz \gamma^2 b)e^R$ to quantize the superstring when $D \neq 10$. 

1
2 Similarity transformation

After fermionizing the $[\beta, \gamma]$ ghosts as in (3) and bosonizing $\xi = e^\chi$ and $\eta = e^{-\chi}$, the BRST charge of (1) can be written as $Q = \frac{1}{2\pi i} \oint dz \, j_{BRST}$ where

$$j_{BRST} = e^{T_m} - b \partial c - \partial^2 \phi - \frac{1}{2}(\partial \phi)^2 + \frac{1}{2} \partial^2 \chi + \frac{1}{2} (\partial \chi)^2 \right] + e^{\phi-\chi} G_m - b e^{2(\phi-\chi)} + \partial^2 c + \partial (c \partial \chi).$$

(4)

We will now show that

$$j_{BRST} = e^{-R} j_0 e^R$$

(5)

where

$$j_0 = -be^{2(\phi-\chi)},$$

(6)

$$R = \frac{1}{2\pi i} \oint dz \, [c G_m e^{-\phi} e^\chi - \frac{1}{4} \partial (e^{-2\phi}) e^{2\chi} c \partial c].$$

(7)

Note that $j_{BRST}$ was used in [3] as the fermionic generator $G^+$ of a twisted N=2 superconformal algebra. Using (5), $j_{BRST}$ is trivially nilpotent since $j_0$ has no poles with itself.

To prove (5), we use the expansion

$$e^{-R} j_0 e^R = \sum_{n=0}^{\infty} \frac{1}{n!} j_n,$$

$$j_n = [j_{n-1}, R],$$

(8)

where, for $R = \frac{1}{2\pi i} \oint dz \, r(z)$, the commutator is computed following the rule

$$[j_{n-1}(y), R] = \frac{1}{2\pi i} \oint dz \, j_{n-1}(y) r(z).$$

(9)

If $D$ is the spacetime dimension of the superstring (i.e. $G_m(y) G_m(z) \rightarrow D(y-z)^{-3} + ...$), the $n = 1$ term of (8) is given by

$$j_1 = e^\phi e^{-\chi} G_m + b c \partial c - \frac{3}{2} \partial^2 c + \partial c (5 \partial \phi - 4 \partial \chi)$$

$$+ c [\frac{3}{2} \partial^2 \phi - 3 (\partial \phi)^2 - 2 (\partial \chi)^2 - \partial^2 \chi + 5 \partial \phi \partial \chi].$$

(10)
the $n = 2$ term is given by

$$j_2 = 2 c T_m + \frac{D}{2} [\partial^2 c + 2 \partial c (\partial \chi - \partial \phi)$$

$$+ c (\partial^2 \chi - \partial^2 \phi + (\partial \phi)^2 + (\partial \chi)^2 - 2 \partial \phi \partial \chi)]$$

$$- e^{-\phi} e^{\chi} G_m c \partial c + \frac{5}{4} e^{-2\phi} e^{2\chi} c \partial c \partial^2 c, \quad (11)$$

the $n = 3$ term is given by

$$j_3 = 3 e^{-\phi} e^{\chi} G_m c \partial c - \frac{3 D}{4} e^{-2\phi} e^{2\chi} c \partial c \partial^2 c, \quad (12)$$

and the $n = 4$ term is given by

$$j_4 = \frac{3 D}{2} e^{-2\phi} e^{2\chi} c \partial c \partial^2 c. \quad (13)$$

The terms for $n > 4$ in the expansion vanish identically since the OPE between $j_4$ and $R$ has no single poles.

It is straightforward to check that $j_{BRST}$ of (4) is equal to

$$j_0 + j_1 + \frac{1}{2!} j_2 + \frac{1}{3!} j_3 + \frac{1}{4!} j_4$$

when $D = 10$, so we have proven (5). Note that when $D \neq 10$, the integral of (14) contains the term $(10 - D) \oint dz (\frac{1}{2} c \partial \phi \partial \chi + \frac{1}{16} e^{2(\chi - \phi)} c \partial c \partial^2 c)$. Since this term does not anti-commute with $\oint dz \eta = \oint dz e^{-\chi}$, the integral of (14) can only be used as a BRST charge when $D = 10$ for the reasons stated in the introduction.

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