Optimization of Approach Trajectory Considering the Constraints Imposed on Flight Procedure Design

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Abstract

Due to recent growth of air traffic demand, inefficiency in flight is becoming a more and more important issue. However, present flight trajectory is not completely optimized because of the existence of difficulties in flight procedure design. This paper describes a method to optimize the approach procedure in terms of fuel efficiency. In proposed method, route is parameterized by position of each waypoint and speed above them. Then assuming constant acceleration and point mass model, total fuel consumption is calculated. Optimization is conducted with constraints imposed upon flight procedure design by using genetic algorithm. We applied the method to approach procedure to a real airport and more efficient trajectory is obtained.

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Peer-review under responsibility of Chinese Society of Aeronautics and Astronautics (CSAA)

Keywords: Optimization; Air Traffic Management; Genetic Algorithm; Flight Procedure; Area Navigation;

1. Introduction

The steady growth of air traffic demand would cause lack of airspace capacity and bring a lot of problems such as delays, reduction in airline economic efficiency, increase in the emission of greenhouse gases and so on. Therefore, efficiency in the usage of airspace has to be enhanced. Thanks to the spread of area navigation (RNAV), flexibility in flight procedure design has been improved and made it possible to design more efficient flight procedure. Despite

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doi:10.1016/j.proeng.2014.12.534
this increase of design flexibility, many rules and constraints are still imposed upon flight procedure and make it difficult. Current design process deals with this by using specific software which detects the violation against the rules and the constraints. However it is not designed according to appropriate indices and there may be rooms for improvement. In order to solve this problem, this paper aims to establish a method for optimal flight procedure design.

Required Navigation Performance Authorization Required approach (RNP AR APCH) is one of the approach procedures based on RNAV and has been introduced to more and more airports in Japan. Although this procedure requires aircrafts to have more precise navigational performance, it enables more flexible and efficient route design. Hence we focus on this approach procedure.

Similar research was conducted by some researchers. Richter expressed RNAV route in a sequence of straight line and arc, then positions of waypoints and radii of arcs were set as optimization parameters which were optimized by solving optimal control problem [1]. Hartjes developed an extension of a tool called NOISHHH which had been developed for noise abatement RNAV trajectory design [2,3]. Both of them showed results that were more realistic and practical. However there are other rules which we have to consider when designing flight procedure. So we add more constraints than these previous studies and aim to create more realistic route.

The rest of the paper is structured as follows: In Section II we discuss how we formulate and solve optimization problem, route parameterization, constraints and objective function are described in this Section. Section III presents an example of optimization by applying proposed method to a real airport. Finally, concluding remarks are made in Section IV.

**Nomenclature**

| Symbol | Description |
|--------|-------------|
| $C_D$  | Drag coefficient |
| $C_{D0}$ | Parasitic drag coefficient |
| $C_{D2}$ | Induced drag coefficient |
| $C_{f1}, C_{f2}$ | Fuel flow parameters given in BADA |
| $C_L$ | Lift coefficient |
| $D$ | Drag force |
| $g$ | Gravity acceleration |
| $h_{path}$ | Altitude of path |
| $h_{terrain}$ | Terrain elevation |
| $L$ | Lift force |
| $l_i$ | Length of the $i$-th segment |
| $m$ | Aircraft mass |
| $m_{fin}$ | Final aircraft mass |
| $m_{ini}$ | Initial aircraft mass |
| $n$ | Number of waypoints |
| $R$ | Rate of turn |
| $r$ | Radius of arc |
| $r_{RF}$ | Radius of RF leg |
| $T$ | Thrust |
| $T_{max}$ | Maximum thrust |
| $T_{min}$ | Minimum thrust |
| $v_i$ | Reference speed at the $i$-th waypoint |
| $V_{min}$ | Minimum airspeed restriction |
| $V_{RAS}$ | True airspeed |
| $V_{TW}$ | Tail wind component for calculation of turn radius |
| $x$ | $x$ coordinate of aircraft |
| $x$ | Optimization variables |
| $x_i$ | $x$ coordinate of the $i$-th waypoint |
| $y$ | $y$ coordinate of aircraft |
2. Methods Used

2.1. Parameterization of air route

In RNP AR APCH procedure, route is expressed by a sequence of waypoints and legs. Leg is a component that connects two waypoints and we can use two types of leg, TF (Track to Fix) and RF (Radius to Fix) as shown in Fig.1. Approach procedure can be divided into three segments, each is named as initial, intermediate and final approach segment.

We are considering two elements for optimization parameters. As stated above, current route is represented by a sequence of waypoints that are predetermined by the authorities. So the lateral location of waypoints (i.e. $x$ and $y$ coordinate of waypoints) should be included into parameters. Although aircraft speed along the trajectory is not defined strictly, aircraft motion like bank angle and rate of turn which depends on speed is limited. Hence, we set reference speed on each waypoint and this is also added to optimization parameters. Here, reference speed stands for the speed which aircraft passes above the waypoint at. Those reference speeds can change within a certain speed range during optimization process and are used to determine the thrust power and aircraft motion. In calculation, the position and speed of the first and the last waypoint are not parameterized because it has to be fixed. So the optimization variables are like (1).

$$x = [x_2, y_2, v_2, \ldots, x_{n-1}, y_{n-1}, v_{n-1}]$$

In addition to that, leg type and number of waypoints would affect the shape of route. Of course the more the number of waypoints becomes, the more efficient the procedure would become. However it would also become winding and complex. So it is hard to determine an appropriate evaluation index. Besides, they are represented by integer values and sometimes hard to handle in a single optimization process. Therefore we use them as input parameters prepared before computation.

In terms of vertical profile, we are assuming constant descent gradient and therefore the altitude of a certain position in the path is determined by along-track distance from runway threshold.

Fig.1. Structure of route and definition of parameters
2.2. Calculation of fuel consumption

For the calculation of fuel consumption and aircraft motion, Base of Aircraft Data (BADA) [4] provided by EUROCONTROL is used. BADA includes data of many commercial aircrafts and each data is composed of mass, engine thrust, aerodynamics, fuel flow and so on. In this paper, point mass model equation shown in equation (2) is used to calculate motion and fuel consumption. Thrust $T$, lift coefficient $C_L$ and bank angle $\phi$ are treated as control variables in this equation. Here, drag coefficient can be written like equation (3) by using $C_{D0}$, $C_{D2}$ which are provided in BADA. As mentioned above, lateral flight path and speed when aircraft passing above each waypoint are uniquely determined if optimization variables are given. Assuming acceleration and descent gradient in each segment are constant, equations (4-6) are obtained. Substituting equations (4-6) into the last three equations of (2), three transcendental equations are obtained. Then, control variables can be determined by solving these equations.

$$\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{V}_{TAS} \\
\dot{\psi} \\
\dot{\gamma}
\end{bmatrix} =
\begin{bmatrix}
V_{TAS} \cos \gamma \sin \psi \\
V_{TAS} \cos \gamma \cos \psi \\
V_{TAS} \sin \gamma \\
(T - D)/m - g \sin \gamma \\
L \sin \phi / m V_{TAS} \cos \gamma \\
(L \cos \phi / m - g \cos \gamma)/V_{TAS}
\end{bmatrix}$$

$$C_D = C_{D0} + C_{D2} \times C_L^2$$

$$\dot{V}_{TAS} = 0.5 \times (v_{t+1}^2 - v_t^2)/(l_i \cos \gamma_i)$$

$$\dot{\psi} = \begin{cases} 
0 & (\text{straight segment}) \\
V_{TAS} \cos \gamma / r & (\text{arc segment})
\end{cases}$$

$$\dot{\gamma} = 0$$

Using the thrust calculated above, fuel flow is achieved because it is represented in a function of thrust and true airspeed in equation (7). Finally, we can get total fuel consumption by numerically integrating (7) and subtracting final mass from initial mass. So the objective function can be written like equation (8).

$$\dot{m} = -C_{f1}T(1 + V_{TAS}/C_{f2})$$

$$f(x) = m_{ini} - m_{fin}$$

| Segment   | RNP value [NM] | MOC [m] |
|-----------|----------------|---------|
| Initial   | 1              | 300     |
| Intermediate | 1            | 150     |
| Final     | 0.3            | See [5] |

Fig. 2. Segment Width and minimum obstacle clearance (MOC)
2.3. Constraints

Here we discuss constraints taken into account in this paper. First, we have to consider the clearance from obstacles. The width of protection area and minimum obstacle clearance (MOC) are defined in Fig. 2. The procedure has to be designed so as not to let obstacles inside of dotted area in Fig. 2. RNP value and MOC applied to each segment are listed in Table 1. MOC in final approach segment is not constant value and special calculation is needed. Although we don’t go into a detail of that calculation, it is described in [5]. We evaluate the clearance from obstacles like the following. We set points on the path at equal intervals in the forward direction and also set points in the cross direction. Here, the number of points we set is 100 in forward direction and 7 in cross direction, so the total number of evaluation point is 700. At every point we calculate an elevation from terrain model, path altitude and MOC, then evaluate clearance like (9). Terrain model is constructed from the data provided by Geospatial Information Authority of Japan. The data has approximately 10m resolution (0.4" grid spacing for latitude and longitude). After converting the data from geographic coordinate system to Cartesian coordinate system, the grid shape isn’t rectangle and becomes scattered data. Therefore scattered data interpolation method in [6] is adopted in this paper.

Other constraints on segment length $l_i$, rate of turn $R$, bank angle $\phi$, turn angle over fly-by waypoint $\theta_{FB}$ and turn radius of RF leg $r_{RF}$ shown in equations (10-14) are also imposed. $DTA$ in (10) stands for Distance of Turn Anticipation and is defined in Fig. 1. $V_{TW}$ in (14) is tailwind component that has to be considered in flight procedure design and its value is dependent on altitude and listed in [5]. $V_{min}$ in (14) is minimum airspeed restriction listed in Table 2. For example, if RF leg is in final approach segment and the procedure allows flight of category D aircraft, $V_{min}$ is 165 KIAS. In this paper, minimum airspeed restriction of category D is used as $V_{min}$ in order to create more widely used route. Besides, listed values in Table 2 are used as upper speed limit. In order not to create infeasible path, maximum thrust and minimum thrust are considered and inequality constraints are imposed at both ends of each leg. In addition to that, RF leg has to be smoothly linked with previous and following leg. Constraints on turning angle at RF waypoint is imposed so that it becomes zero.

\[
\begin{align*}
  h_{path} - MOC - h_{terrain} &\geq 0 \\
  l_i &\geq 2 \times RNP + DTA \\
  R &\leq 3^\circ/sec \\
  \phi &\leq 20^\circ \\
  \theta_{FB} &\leq 90^\circ \\
  r_{RF} &\geq \max(2 \times RNP, (V_{min} + V_{TW})^2/g \tan 20^\circ) \\
  T_{min} &\leq T \leq T_{max}
\end{align*}
\]  

Table 2. Airspeed restriction (in KIAS)

| Approach Segment | Category C | Category C (Minimum) | Category D | Category D (Minimum) |
|------------------|------------|----------------------|------------|----------------------|
| Initial          | 240        | 210                  | 250        | 210                  |
| Intermediate     | 240        | 180                  | 250        | 180                  |
| Final            | 160        | 140                  | 185        | 165                  |
2.4. Optimization method

There are a lot of approaches in optimization but they can be mainly classified into two groups. One is gradient based method and the other is heuristic algorithm. Gradient based method can search a local optimal solution and requires that objective function and constraints are differentiable although its computational cost is relatively low and its solution is mathematically certified. On the other hand, heuristic algorithm doesn’t require gradients of functions so that it is easier to handle and also it can search global optimal solution. Of course there are some drawbacks in these algorithms. Because of the broadness of their search, the most serious drawback is considered to be their high computational cost. However we are focusing on flight procedure design which is predetermined and fixed to each airport and it doesn’t require so fast speed like real time simulation. In addition to that, functions may become multimodal in the above formulation because terrain model is taken into account. Therefore we adopt heuristic algorithm, especially genetic algorithm. In order to handle constraints in genetic algorithm, lagrangian barrier method described in [7] is used.

3. Numerical Example

We tested usefulness of proposed method by applying it to approach procedure design of a real airport. In this paper Kumamoto airport is selected for test. There are two reasons why we choose it. The first reason is that it is surrounded by mountains and is suitable for testing the obstacle clearance evaluation. The second reason is that it has already have RNP AR route and we can make comparison between optimal solution and current procedure. In the calculation, we used B737-800 as aircraft model.

The example scenario is like the following. We use eight waypoints in order to construct the procedure and the first waypoint is assigned to IAF, the second waypoint to IF and the third waypoint to FAF. This is the same as current procedure shown in Fig. 3. The speed and altitude at the last waypoint are 140 KIAS and 50 feet above runway threshold respectively. In terms of initial speed we tested two cases, 240 KIAS and 210 KIAS, because those are used as airspeed restriction of category C aircraft in initial approach segment in Table 2. Besides, the change of flap setting is considered. We assume that the flap setting is changed from approach setting into landing setting at FAF. Altitude restriction that initial altitude is over 8000ft should be added to the constraints like the real procedure. However in order not to change the gravitational potential energy at initial state, initial altitude should be the same even if the parameters are changed. Therefore we assume the profile that begins from level flight at 8000 ft and changes into continuous descent with constant descent angle. If the route is too short and cannot reach 8000 ft at the initial waypoint, the route is treated as infeasible solution. In this example, we have $3 \times 6 = 18$ optimization variables because initial and final state are fixed and only 6 waypoints are considered. Used leg types are the same as current procedure.
Fig. 3. Current RNP AR Procedure to Kumamoto Airport [8]

Fig. 4. Current and optimal trajectories
Optimization results are shown in Fig. 4 and Fig. 5. The first straight segment of optimal route becomes shorter than the current one and therefore the total path length becomes shortened. In case initial speed is 240 KIAS, the straight segment also becomes shorter than current route but a little bit longer than the case that initial speed is 210 KIAS. This is because longer distance is required in order to obtain enough deceleration for airspeed limitation if the initial speed is high. Besides, the latter part of routes is almost the same because of the constraints to keep clearance from obstacles and to construct a straight-in approach route. The fuel consumption, path length and flight time are listed in Table 3. From this result, the optimal route in which initial speed is 210 KIAS seems to be the best because of its shorter flight time and less fuel consumption. Furthermore, considering the upper level flight, it would require less fuel than the case which initial speed is 240KIAS because it allows to lose more kinetic energy.

4. Conclusion

This paper presented the optimization method for the approach procedure design, especially RNP AR approach procedure design within the flight procedure rules. The method is based on genetic algorithm and is tested in the approach procedure to Kumamoto airport, a real airport which is surrounded by mountains. In the example case, flight procedure is optimized with respect to fuel efficiency. We can conclude that shorter the distance becomes, fuel efficiency tends to be improved. However the feasibility of the route is dependent on deceleration performance of aircrafts, so we have to carefully consider the accuracy of aircraft model for practical use. Also, appropriate initial speed should be given by taking into account the en-route traffic flow. It should be noted that the route is generally used by several types of aircraft in real situation and we should optimize not fuel consumption of a single aircraft but the expected value of fuel consumption, although only a single type of aircraft is used in this paper.

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