1. Introduction

Giant radio halos (size $\sim 1 \, h_{50}^{-1} \, \text{Mpc}$, GRHs elsewhere) are the most spectacular evidence for non-thermal components (relativistic particle and magnetic field) in the intra cluster medium (ICM). They are diffuse synchrotron radio emission extending on Mpc scale, have no obvious connection with the cluster galaxies, but are rather associated with the ICM (e.g., Giovannini & Feretti 2000). Such a synchrotron radio emission requires a population of GeV relativistic electrons (and possibly positrons) and cluster magnetic fields on $\mu$G levels. An independent evidence of the existence of cluster magnetic fields in the ICM is given by the rotation measurement studies of radio galaxies located within or behind the clusters (e.g., Giovoni & Feretti 2004). These cluster magnetic fields on $\mu$G levels.

An independent evidence of the existence of cluster magnetic fields in the ICM is given by the rotation measurement studies of radio galaxies located within or behind the clusters (e.g., Giovoni & Feretti 2004). One possibility to explain the presence of GeV electrons diffused on Mpc scale is given by the so-called re-acceleration model. In this model the radio emitting electrons injected in the ICM (by AGN, starbursts, supernovae and/or galactic winds, and hadronic collisions) are re-accelerated in situ by some kind of turbulence generated in the cluster volume during cluster mergers (e.g., Brunetti et al. 2001, 2004; Petrosian 2001; Fujita et al. 2003; Brunetti & Blasi 2005). Observations indicate that the detection rate of GRHs shows an abrupt increase with increasing the X-ray luminosity of the host clusters. In particular about 30-35% of the galaxy clusters with X-ray luminosity larger than $10^{45}$ erg/s show diffuse non-thermal radio emission (Giovannini & Feretti 2002); these clusters have also high temperature ($kT > 7$ keV) and large mass ($\gtrsim 2 \times 10^{15} M_\odot$). Furthermore, GRHs are always found in merging clusters (e.g., Buote 2001). Although the re-acceleration model seems to reproduce the observational features of the diffuse radio emission, a theoretical investigation of the statistical properties of the GRHs in galaxy clusters in the framework of this model has not yet carried out extensively. Only recently, Cassano & Brunetti (2005; CB05 elsewhere), have calculated the expected occurrence of GRHs as a function of the cluster mass and dynamical status in the framework of the re-acceleration model. CB05 follow cluster formation using the Press & Schechter (1974; PS hereafter) formalism and assume that a fraction, $\eta_h$, of the $PdV$ work done by the merging subclusters in going through the main one is injected in the form of fast magnetosonic (MS) waves which accelerate relativistic electrons in the ICM. They show that GRHs are naturally expected only in the more massive clusters with an expected occurrence (at $z<0.2$) which can be reconciled with the observed one under viable assumptions ($\eta_h \simeq 0.24 - 0.34$). More recently, Cassano, Brunetti, Setti (2006, CBS06 elsewhere) extended this investigation and calculated the expected evolution with cosmic time of the fraction of galaxy cluster with GRHs, the expected luminosity function of GRHs and their number counts.
In CBS06 it was assumed a scaling of the rms magnetic field with cluster mass \( B \propto M^b \) (\( b < 2 \)) and it was shown that the available radio-X correlations can be accounted for by assuming \( b \geq 0.5 - 0.6 \). A scaling of the magnetic field with cluster mass is indeed expected by numerical MHD simulations (e.g., Dolag et al. 2004) where the seed field is amplified by the effect of shear flows driven by the cluster formation process. These simulations show that the resulting magnetic field strengths depend on the final cluster mass and/or temperature as \( B \propto T^\alpha \) with \( \alpha \sim 2 \) (Dolag et al. 2002). Here we focus our attention on the possibility to make constraints on the value of the magnetic field strength in galaxy clusters by comparing the observed and the predicted statistical properties of GRHs. In addition, we show how expectations for future observations will be powerful tools to test this re-acceleration model and also to provide additional constraints on \( B \) in the ICM. We focus our attention on GRHs only (size \( \sim 1 \ h_{50}^{-1} \) Mpc, GRH elsewhere). The adopted cosmology is: \( \Lambda \)CDM (\( H_o = 70 \) Km s\(^{-1}\) Mpc\(^{-1}\), \( \Omega_{\alpha,m} = 0.3 \), \( \Omega_\Lambda = 0.7 \), \( \sigma_8 = 0.9 \)).

2. X-ray–radio correlations and constraining \( B \)

It is well known that the radio power of GRHs scales with cluster mass, X-ray luminosity and temperature (e.g., Feretti 2003). Using a sample of 17 galaxy clusters CBS06 found a correlation between the radio power at 1.4 GHz and the cluster virial mass \( P_{1.4} \propto M_{\alpha,M}^{\alpha_M} \), with \( \alpha_M = 2.9 \pm 0.4 \).

By assuming that electrons are re-accelerated in the ICM, the aim of this Section is to constrain the magnetic field strength in galaxy clusters and its dependence on cluster mass by comparing this correlation with that expected by the electron re-acceleration model. CB05 derived an expected trend between the bolometric radio power, \( P_R \), and the virial cluster’s mass in the framework of the particle acceleration model. Assuming a scaling of \( B \) with the cluster mass of the form \( B = B_{\langle M \rangle} (M/\langle M \rangle)^b \) (with \( b > 0 \) and \( B_{\langle M \rangle} \) being the rms magnetic field associate to a given cluster mass, here \( \langle M \rangle \approx 1.6 \times 10^{15} M_\odot \)) one has:

\[
P_R \propto \frac{M_{\alpha}^{2-\Gamma} B_{\langle M \rangle}^{2} \cdot (M_{\nu}/\langle M \rangle)^{2b}}{(B_{\langle M \rangle}^{2} \cdot (M_{\nu}/\langle M \rangle)^{2b} + B_{\text{cmb}}^{2})^2}
\]  

(1)

where \( B_{\text{cmb}} = 3.2(1+z)^2 \mu G \) is the equivalent magnetic field strength of the CMB and \( \Gamma \) is defined by \( b \propto T^{\Gamma} \); we use \( \Gamma \approx 2/3 \) (virial case) or \( \Gamma \approx 0.56 \) (e.g., Nevalainen et al. 2000). The expected value of the slope of Eq. 1 depends on \( B_{\langle M \rangle} \) and \( b \). CBS06 show that Eq. 1 can be used also for the scaling between the monochromatic (observed) radio power at 1.4 GHz and the cluster mass, so that the rms magnetic field and \( b \) can be constrained by matching the slope of Eq. 1 with the observed one, \( \alpha_M = 2.9 \pm 0.4 \). In Fig. 1 we report the expected \( \alpha_M \) as a function of \( B_{\langle M \rangle} \) for \( \Gamma \approx 2/3 \) for different values of \( b \) (\( b = 0.5 \) to 1.7, see caption) together with the region spanned by the observed 1 \( \sigma \) range. It is clear that the observed values of the slope cannot be reconciled with the expected one for \( b < 0.5 - 0.6 \) and that larger values of \( B_{\langle M \rangle} \) are required by increasing \( b \).

In Fig. 2 we report the allowed region of parameters in the plane \( (B_{\langle M \rangle}, b) \) (shadowed area) which is obtained by requiring that the expected slope of Eq. 1 is consistent with the observed one at 1\( \sigma \) level. An additional lower limit on \( B_{\langle M \rangle} \) (vertical arrow in Fig. 2) can be inferred in order to not overproduce, via IC scattering of the CMB photons, the
fluxes of the hard-X ray excesses observed in a few clusters (e.g., Fusco-Femiano et al. 2003). These values of $B_{<M>}$ should be considered as lower limits because the IC emission may come from a more external region with respect to the synchrotron emission (e.g., Brunetti et al. 2001; Kuo et al. 2003; Colafrancesco et al. 2005) and also because additional mechanisms may contribute to the hard-X ray fluxes. We use the value of the magnetic field derived for the Coma cluster, $B_{IC} \simeq 0.2 \mu G$ (Fusco-Femiano et al. 2004) and obtain the lower bound of B with cluster mass from the scaling law $B = B_{<M>} (M/ <M>)^{b}$.

The resulting $(B_{<M>} , b)$ region spans a wide range of values of B and b. However, given a fixed slope of the B–M scaling the constraints on the value of B are relatively tight. We stress again that this region refers to the case in which electrons are re-accelerated in the ICM (re-acceleration model). In this case we can identifies two allowed regimes: a sub-linear scaling ($b > 1$) with relatively high values of B and a sub-linear scaling ($b < 1$) with lower values of B.

3. Magnetic field and the occurrence of GRHs

In their theoretical approach CB05 and CBS06 identified GRHs with those objects in their synthetic cluster population with a synchrotron break frequency $\nu_b > 200$ MHz in a region of 1 Mpc $h_{70}^{-1}$ size. The break frequency can be expressed as a function of the cluster mass and of the rms field in the emitting volume, $\nu_b \propto M^b (CBS06)$:

$$\nu_b \propto M^{2-\Gamma+b} \frac{B_{<M>} < M >^{-b} \eta_t^2}{(B_{<M>}^2 <M>^{2b} + B_{cmb}^2)^2} \tag{2}$$

The assumption that B depends on the cluster mass should affect the value and dependence with cluster mass of the synchrotron break frequency and thus the occurrence of GRHs. Here we are interested in understanding the effect of assuming different $(B_{<b>}, b)$ configurations in the resulting probability to form GRHs in the context of the particle acceleration model. Eq.2 has two different behaviors in the case of IC dominance ($B << B_{cmb}$) and in the case of synchrotron dominance ($B > B_{cmb}$) in galaxy clusters. Typically in the case of a sub-linear scaling ($b < 1$) it is $B << B_{cmb}$, and an increase of $\dot{B}$ does not significantly affect the synchrotron losses. In this case one has: $\nu_b \propto M^{2-\Gamma+b} (1 + z)^{-8}$ and the probability to form GRHs in these clusters increases with mass ($2 - \Gamma + b > 0$ always) and decreases with $z$. On the other hand, in the case of a super-linear scaling ($b > 1$) the value of the mass for which $B$ becomes equal to $B_{cmb}$, $M_* (equipartition mass)$, is within the cluster mass range. For $M > M_*(z)$ it is $\nu_b \propto M^{2-\Gamma-3b}$, and the particle energy losses would increase as the mass becomes larger. In this case the probability to form GRHs would decrease with cluster mass and the occurrence of GRHs with $z$ is mainly driven by the cosmological evolution of the cluster-merger history (which drives the injection of turbulence) rather than by the dependence of the IC losses with $z$. An example of this different behavior is reported in Fig.3, where we plot the probability as a function of the cluster mass at $z \leq 0.1$ for a sub-linear case (solid line, see caption) and for a super-linear case (dashed line, see caption). In the super-linear case, the peak of the probability is expected at the $equipartition$ mass $M \sim M_*$. From the analysis of the fraction of the clusters hosting GRH with cluster mass, it is thus possible to constrain the value of $M_*$ and thus of $(B_{<M>}, b)$ in galaxy clusters.

4. Number counts of GRHs and magnetic field

CBS06 derived number counts of GRH in the framework of the re-acceleration model. The number counts are obtained by combining the PS mass function of clusters with the expected radio power–mass scaling (Sect. 2) and with the probability to form GRHs in a given cluster mass bin. Since both the probability and slope of the radio power–mass scaling (Fig.1) depend on the scaling mass $M \sim M_*$, the expected number counts should depend on the strength and scaling of the magnetic field in galaxy clusters. Fig.4 shows the expected number counts in the cases of a super-linear and a sub-linear scaling of the rms magnetic field with cluster mass. The two shadowed regions are obtained by combining the expectations with $b > 1$ (upper region) and $b < 1$ (lower region) by assuming the region of $(B_{<M>}, b)$ reported in Fig.2. The black points are the number counts of GRHs for $z < 0.2$. They are obtained by making use of the radio data from the analysis of the 1.4 GHz NVSS radio survey by Giovannini et al. (1999) and by accounting for the incompleteness of their sky-coverage. For fluxes larger than 30 mJy the expected number counts are close to the counts obtained from the present observations ($z \leq 0.2$ GRHs dominate at these fluxes), while at lower fluxes present surveys fail in catching the bulk of GRHs. At these fluxes our expectations are very
of these GRHs in sublinear case should be a factor
considered with future deeper radio surveys. On the other hand, the
In this contribution we have shown that the observed correla-
5. Conclusions

Fig. 4. Total number of expected GRHs above a given radio
flux at 1.4 GHz from a full sky coverage and data points for
sensitive to the scaling of B with the cluster mass. In partic-
ular, we note that assuming a superlinear scaling of $B$ with
cluster mass, up to $\sim 100$ GRHs are expected to be dis-
covered with future deeper radio surveys. On the other hand, the
number of these GRHs in sublinear case should be a factor
of $\sim 2$ smaller. It is clear that future deep radio surveys will
allow to provide constraints on $B$ in galaxy clusters.

5. Conclusions

In this contribution we have shown that the observed correla-
tions between radio and X-ray properties of galaxy clusters,
which, however, generally sample regions which are

We made use of the statistical calculations developed in
CBS06, which are based on the re-acceleration of a seed pop-
ulation of relativistic electrons by MS waves injected in the
ICM during merger events (CB05) and which assume a de-
pendence of the rms magnetic field intensity on cluster mass,
$B = B_{<M>}(M/ < M >)^{b}$. The main results of this paper are:

i) A comparison of the expected correlation between the
radio power and the cluster mass with the observed one al-
ows the definition of a permitted region of the parameter
space ($B_{<M>,b}$) in galaxy clusters, with typically small val-
ues of $B$ for sublinear scaling ($b < 1$) and higher values of
$B$ for superlinear scaling ($b > 1$). A lower bound is found at
$b \sim 0.5 - 0.6$, while a lower bound $B_{<M>} = 0.2 \mu$G is also
obtained from IC arguments (Sect. 2). A superlinear scaling
of $B$ with mass, as expected by MHD simulations (Dolag et
al. 2004) falls within the allowed region. The values of $B$ in
the superlinear case are close to (slightly smaller than) those
obtained from rotation measurements (e.g., Govoni & Feretti
2004), which, however, generally sample regions which are
even more internally placed than those spanned by GRHs.

ii) We have shown that the occurrence of GRHs as a func-
tion of mass and redshift is sensitive to $B$ in galaxy clusters.
This probability depends on the merging history of clusters
and on the relative importance of the synchrotron and IC
losses, which indeed depends on the value of the magnetic
field. We show that typically in the case of sublinear scal-
ing of the magnetic field with cluster mass ($b \sim 0.6 - 0.9$)
the probability to have GRHs increases with cluster mass and
decreases with redshift, whereas in the case of superlinear
scalings ($b \sim 1.2 - 1.7$) more complex behaviors of the prob-
ability with mass and redshift are present.

iii) We have derived the integral number counts of GRHs
at 1.4 GHz in both the superlinear and sublinear cases. We
find that at higher fluxes ($> 30 - 40$ mJy) the predicted num-er counts are dominated by the $z \lesssim 0.2$ clusters discovered
in the NVSS. We estimate that the number of GRHs which
would be discovered below 30 mJy by future deeper radio
surveys (by LOFAR, LWA, and SKA) will be up to $\sim 100$ if
superlinear scalings of the mass with $B$ hold, while they will
be up to $\sim 50$ in the case of sublinear scaling.

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