A Simple Expression for Heavy to Light Meson Semileptonic Decays Form Factors

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Like the two-photon and two-gluon decays of the $P$-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ charmion state for which the Born term produces a very simple decays amplitude in terms of an effective Lagrangian with two-quark local operator, the Born term for the processes $cd \to (\pi, K)\ell\nu$ and $bd \to (\pi, K)\ell\nu$, could also produce the $D$ and $B$ meson semileptonic decays with the light meson $\pi, K$ in the final state. In this approach to heavy-light meson form factors with the $\pi, K$ meson treated as Goldstone boson, a simple expression is found for the decays form factors, given as: $f_+(0)/(1-q^2/(m^2_{D,B}+m^2_\pi))$, with $H=D, B$ for $D, B \to \pi$ form factors, and $f_+(0)/(1-q^2/(m^2_{D,B}+m^2_K))$ for $B, D \to K$ form factor. The purpose of this paper is to show that this expression for the form factors could describe rather well the $q^2$- behaviour observed in the BaBar, Belle and BESIII measurements and lattice simulation. In particular, the $D \to K$ form factors are in good agreement with the measured values in the whole range of $q^2$ showing evidence for $SU(3)$ breaking with the presence of $m^2_K$ term in the quark propagator, but some corrections to the Born term are needed at large $q^2$ for $D, B \to \pi$ form factors.

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I. INTRODUCTION

The semileptonic heavy to light meson decays form factor as in the $D, B \to \pi\ell\nu$ decays, is given by the $V-A$ current matrix elements between heavy and light meson state. The precise knowledge of these form factors is required for an exclusive determination of the charmonium state. The precise knowledge of these form factors could be fitted with a BK parametrization[7] with form factor from lattice simulation and BaBar measurements. It is shown that the $B \to \pi$ form factor from lattice simulation and BaBar measurements could be fitted with a BK parametrization[7] with $\alpha = 0.52 \pm 0.05 \pm 0.03$ given in [8].

$$f^+(q^2, \alpha) = \frac{f_0}{(1-q^2/m^2_{D,B})/(1-\alpha q^2/m^2_D)} (1)$$

For $D$ meson semileptonic decays, the BaBar and BESIII measurements[6,9] show that the $D \to \pi, K$ form factor, could be fitted with an effective two-pole :

$$f^+(q^2, \alpha) = \frac{f_0}{(1-q^2/m^2_{D,B})/(1-\alpha q^2/m^2_D)} (2)$$

or a single-pole parametrization :

$$f^+(q^2, \alpha) = \frac{f_0}{(1-q^2/m^2_{pole})} (3)$$

with $m^2_{pole} = (1.906 \pm 0.029 \pm 0.023)$GeV in the BaBar measurements[8] for $D \to \pi$ form factor and in the BESIII measurements, $m^2_{pole} = (1.911 \pm 0.012 \pm 0.004)$GeV for $D \to \pi$, and $m_{pole} = (1.921 \pm 0.010 \pm 0.007)$GeV/ for $D \to K$ form factor which seem to disagree with the higher pole mass value given by $D^*$ dominance as noted by BESIII[6]. The lower value for the pole mass is also found by Belle[5] which give a value of $1.82 \pm 0.04 \pm 0.03$GeV, but a higher value for pole mass of $(1.93 \pm 0.05 \pm 0.03)$GeV close to the BESIII value is obtained earlier[11]. Other data for $D \to K$ form factor are given in TABLE VI of Ref. [11], with the pole mass values for CLEO and FOCUS around 1.9GeV close to the BESIII values.

The problem is to obtain a theoretical expression with this pole-dominance $q^2$-behaviour for these form factors. As will be shown in the following, it is quite straightforward to obtain the pole dominance term for the form factors by noting that the form factor for semileptonic decay with pion or light hadron in the final state is, at the quark level, a $cd \to \pi\ell\nu$ and $bd \to \pi\ell\nu$ process, with the light pseudo scalar meson $\pi, K$ treated as the Goldstone boson of the chiral $SU(3) \times SU(3)$ symmetry, rather than a two-body bound state which gives a rather small $B^0 \to \pi^+\ell^-\nu$ branching ratio$[12]$. Like the two-photon and two-gluon decays of the $P$-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ charmion state$[13,15]$, the Born term from this process then gives the semileptonic decay amplitude and the $D \to \pi$ and $B \to \pi$ form factors in terms of the light pseudo scalar meson-quark coupling $g_{\pi qq}$ given by the Goldberger-Treiman relation$[16]$, the $cd \to \pi\ell\nu$ and $bd \to \pi\ell\nu$ process then produce the form factors. The use of the pion-quark coupling to obtain the semileptonic decay amplitudes is similar to the study of strong and radiative decays of vector mesons with an essentially the same Born term for the radiative decay $\rho^- \to \pi^-\gamma$ from which the extracted pion-quark coupling consistent with the theoretical value given by the bag model$[13]$ to within 50%. Thus it is possible to obtain the semileptonic de-
cays amplitude with pion in the final state treated as a Goldstone boson of chiral symmetry with the pion-quark coupling given by the Goldberger-Treiman relation. The purpose of this paper is to show that, from a similar Born term, we could obtain the semileptonic decay form factors of $B,D$ meson with a light pseudo-scalar meson in the final states from the Born term for the $cd \rightarrow \pi \ell \nu$ and $bd \rightarrow \pi \ell \nu$ process. We find that the $q^2$-dependence given by the Born term describes quite well the form factor measurements by BaBar, Belle and BESIII experiments. In particular, the value of the BESIII fitted pole mass $\bar{m}_K$ in $D \rightarrow K \ell \nu$ is very close to the effective pole mass of the Born term, showing the presence of the $m'_K$ term in the quark propagator.

II. EFFECTIVE LAGRANGIAN FOR $D \rightarrow \pi \ell \nu$ AND $B \rightarrow \pi \ell \nu$

Like the $c\bar{c} \rightarrow \gamma \gamma$ annihilation in the two-photon decay of $P-$wave charmonium state which, at the tree-level approximation, proceeds through the Born term, the reactions $c+d \rightarrow \pi \ell \nu$ and $b+d \rightarrow \pi \ell \nu$, can occur through a similar Born term, with the exchange of an $u$ quark which combines with $d$ quark to produce a pion in the final state. We have:

$$O_\mu = \bar{v}(p_d)V_\mu u(p_b)$$

with

$$V_\mu = \frac{1}{i} \left[ (-ig_{\pi qq}\gamma_5)i\frac{q^\mu - q^\mu + m_u}{(p-p_d)^2 - m_u^2}(-ig\gamma_\mu) \right]$$

with $p_b$ and $p_d$ the $b$ and $d$ momenta of the $B$ meson with mass $m_B$ and $q$ and $p$ are respectively the final state electron-antineutrino system and the pion momentum as shown in FIG.1

![Diagram showing Born term for the semileptonic B decay form factor](image)

For the quark mass in the quark propagator, we use the constituent quark mass given in [11], with

$$m_b = 4.88 \text{ GeV}, m_c = 1.5 \text{ GeV}$$
$$m_s = 0.5 \text{ GeV}, m_u = m_d = 0.3 \text{ GeV}$$

To obtain the form factor, as with $B_s \rightarrow \gamma \gamma$ decay [20], we work in the weak binding approximation with $b,d$ quark taken at rest in the $B^0$ meson. This immediately gives us the effective Lagrangian for $B,D \rightarrow \pi \ell \nu$. Using the Dirac equation and letting the $b$ and $\bar{d}$ quark mass of the Born term, with $m_d = m_u$, we have:

$$V_\mu = \frac{1}{i} \left[ (-ig_{\pi qq}\gamma_5)i\frac{\not{q} - \not{p} + m_u}{(p^2 - 2m_q m_\pi)}(-ig\gamma_\mu) \right]$$

Putting $V_\mu$ into $(O_1)$ and replacing $\bar{v}(p_d)$ and $u(p_b)$ with the quark field $\bar{u}$ and $b$ in $(O_1)$, we obtain the following local operator for the vector current matrix element in $B \rightarrow \pi \ell \nu$ decays:

$$O_P = \frac{2m_B(d\gamma_5)p_\mu}{(m_B^2 + m_s^2 - q^2)}$$

showing the appearance of the pole at $q^2 = (m_B^2 + m_s^2)$ generated by the Born term. For $D \rightarrow K$ form factor, the pole is at $q^2 = (m_B^2 + m_K^2)$. This result explains the success of the single-pole or two-pole fits of the BaBar and BESIII data as shown below. With

$$< \pi(p)|V_\mu(0)|B(p_B) > = f_+(q^2)(p_B + p)_\mu$$

and with $m_b + m_d = m_B$, $<0|\bar{d}\gamma_5|B) = m_B f_B$, the matrix element $<0|O_P|B >$ then gives us immediately the form factor for $B \rightarrow \pi \ell \nu$ decay.

Thus by putting the heavy quark at rest, we are able to obtain the effective Lagrangian for the semileptonic decays $B,D \rightarrow \pi \ell \nu$ which then gives us the form factors in a very simple expression. This is not the case for the radiative decays of light vector meson, e.g the $\rho$ meson, for which the exact calculation given in [21] is also obtained with expressions far more involved than the simple result in the static approximation mentioned earlier [17].

Using the pion-quark coupling for a constituent quark in the $B$ meson obtained from the Goldberger-Treiman relation for the pion-nucleon coupling constant with $g_A = 3/4$ [11] and dropping the weak coupling constant $g$ not relevant for our purpose, we have:

$$f_+(q^2) = \left( \frac{f_B}{f_\pi} \right) g_A \frac{1}{(1 + m_s^2/m_B^2)} \left( 1 - \frac{1}{(m_B^2 + m_s^2)} \right)$$

for $q^2 = 0$, we have:

$$f_+(0) = \left( \frac{f_B}{f_\pi} \right) g_A \frac{1}{(1 + m_s^2/m_B^2)}$$

and similar expression for $D \rightarrow K$ form factor.

There are also possible suppression of the quark-pion coupling due to the off-shell effects of the quark propagator, as the momentum of the $u-$quark in the Born term gets large for small $q^2$, the value of $f_+(q^2)$ would be suppressed for small $q^2$. This explains the small value of $f_+(0)$ for semileptonic $B$ decay. For $f_+(0)$ the QCD sum rule calculations give $0.23 \pm 0.02$ close to the values $0.24 \pm 0.02$ and $0.26 \pm 0.025$ [23,24]. The QCDSF Collaboration lattice calculation [25] also obtained $f_+(0) =$.
difference is negligible for the term \( q^2/m_B^2 \) and \( q^2/m_{B^*}^2 \), one could then replace \( m_B^2 \) by \( m_{B^*}^2 \) without affecting the BaBar BK fit, thus making the BK parametrization consistent with the Born term. What is new in this paper is that the Born term could generate this \( q^2 \)-dependence which seems impossible to obtain otherwise. As shown in FIG. 2 and FIG. 3 for \( D^0 \to \pi^- \) and \( D^+ \to \pi^0 \) form factor, the Born term plotted in the upper curve is slightly above the lower curve obtained with the BESIII fit[6] obtained with \( f_+^2(0) = 0.6372 \pm 0.0080 \pm 0.0044 \), \( M_{pole} = 1.911 \pm 0.012 \pm 0.004 \) GeV for \( D^0 \to \pi^- \) and \( f_+^2(0) = 0.6631 \pm 0.092 \pm 0.0041 \) GeV for \( D^+ \to \pi^0 \) form factors respectively, while the middle curve, obtained by adding a small polynomial term to the Born term, now almost coincides with the lower curve. A similar small polynomial term is also needed to fit the \( D^+ \to \pi^0 \) form factor BESIII data shown in FIG. 4 of Ref. [18].

The Bess data[3], as mentioned earlier, obtained a single pole mass \( m_{pole}(K^-\ell^+\nu) = 1.82 \pm 0.04 \pm 0.03 \) GeV for \( D^- \to K^-\ell^+\nu \) decay, lower than the value of 1.935 GeV of BESIII data, while for \( D^+ \to \pi^0 \) decay, \( m_{pole}(\pi^-\ell^+\nu) = 1.97 \pm 0.08 \) stat \( \pm 0.04 \) syst GeV is slightly above the BESIII value of 1.898 \pm 0.020 \pm 0.003 \) GeV BK fit with \( \alpha = 0.52 \pm 0.08 \) stat \( \pm 0.06 \) syst, while for the \( D^+ \to \pi^0 \), a similar BK fit with \( \alpha = 0.10 \pm 0.21 \) stat \( \pm 0.10 \) syst, making the fit close to the single pole model. This is consistent with a small correction we have for the \( D^+ \to \pi^0 \) form factor shown in FIG. 2. In the BaBar measurements, the simple pole mass for the \( D^0 \to \pi^- \) form factor is 1.906 \pm 0.029 \pm 0.023) very close to the BESIII value of 1.911 \pm 0.012 \pm 0.004), for \( D^0 \to K^- \), the pole mass is 1.884 \pm 0.012 \pm 0.016) rather below the BESIII value of 1.921 \pm 0.010 \pm 0.007). The Fermilab Lattice Collaboration, MILC Collaboration and HPQCD Collaboration, three-flavor QCD Calculations[20] also have lattice results for \( D \to \pi \) and \( D \to K \) fitted with BK parametrization with \( B^+ \) pole and obtain results for the CKM matrix elements in agreement with experiments.

These calculations show that the \( B \to \pi \) form factor is strongly suppressed at \( q^2 = 0 \). In term of \( f_+(0) \), we have:

\[
\begin{align*}
    f_+(q^2)_{D\pi} &= \frac{f_+(0)_{D\pi}}{1 - q^2/(m_D^2 + m_\pi^2)} \\
    f_+(q^2)_{DK} &= \frac{f_+(0)_{DK}}{1 - q^2/(m_D^2 + m_K^2)} \\
    f_+(q^2)_{B\pi} &= \frac{f_+(0)_{B\pi}}{1 - q^2/(m_B^2 + m_\pi^2)}
\end{align*}
\]

for the Born term contribution to the form factor. These are essentially the same to those used in the parametrization of the form factors measured at BaBar, Belle and BESIII. As the Born term is of pure kinematic origin, there is no \( D^*, B^* \) pole term in our expression. This explains the fact that the single-pole fits for \( D \to K, \pi \) form factors do not have a \( D^* \) pole, consistent with the quark propagator pole term. For the \( B \to \pi \) form factor, the mass difference between \( B^* \) and \( B \) is 45.78 \pm 0.35 \) MeV, the

FIG. 2: The Born term (upper curve), the Born term with a small polynomial term to fit the \( D^0 \to \pi^- \) form factor BESIII data (lower curve) which are given in FIG. 9 of Ref. [6].

FIG. 3: The Born term (upper curve), the Born term with a small polynomial term to fit the \( D^+ \to \pi^0 \) form factor BESIII data (lower curve) which are given in FIG. 6 of Ref. [18].

FIG. 4: The Born term (lower curve) and the fit to the \( D^0 \to K^- \) BES measured form factor (upper curve) given in FIG. 8 of Ref. [18].

For the \( D^0 \to K^- \) form factor, as shown in FIG. 4, the lower curve (Born term) is in excellent agreement
with the fit to the BESIII data (upper curve) $^\text{8}$ with $f^+_{K}(0) = 0.7768 \pm 0.0026 \pm 0.0036$, $\mathcal{M}_{\text{pole}} = 1.921 \pm 0.010 \pm 0.007\text{GeV}$ $V_{cs} = 0.97343 \pm 0.00015$ and $V_{cs} = 0.97343 \pm 0.00015$. This good agreement could be explained by the $m_K^2$ term in the $u$–quark propagator. If we replace the factor $q^2/(m_D^2 + m_K^2)$ by $q^2/m_{\text{eff}}$, with $m_{\text{eff}} = \sqrt{m_D^2 + m_K^2}$ as the effective mass in the pole term, then $m_{\text{eff}} = 1.931\text{GeV}$, very close to the pole mass of the BESIII fit $^\text{6}$, $\mathcal{M}_{\text{pole}} = 1.921 \pm 0.010 \pm 0.007\text{GeV}$. This explains the agreement between the two curves in FIG. 4. Agreement is also found between the Born term and the BESIII fit $^\text{18}$ for the $D^+ \rightarrow K^0$ form factor obtained with $f^+_{K}(0) = 0.7094 \pm 0.0035 \pm 0.0111$, $\mathcal{M}_{\text{pole}} = 1.935 \pm 0.017 \pm 0.006\text{GeV}$ shown in FIG. 5, very close to the effective mass $m_{\text{eff}} = 1.931\text{GeV}$ in the Born term. This dependence on $m_K^2$ in both $D^0 \rightarrow K^-$ and $D^+ \rightarrow K^0$ form factor shows evidence for the dominance of the Born term for the $D \rightarrow K$ semileptonic decay form factors.

For the $B \rightarrow \pi$ form factor, there is also a correction to the Born term to compensate for a suppression at large $q^2$ induced by $f^+(0)_{B\pi}$ mentioned above. The reason is that, for small $q^2$, the quark in the Born term quark propagator is highly virtual and is far off the mass shell, the pion-quark coupling has a suppressed form factor, making the $B \rightarrow \pi$ or $D \rightarrow \pi$ form factor suppressed by the suppression of $f^+(0)_{B\pi}$. To compensate for this suppression, a correction term $(1 + a q^2/(m_B^2 + m^2_x))$ for $B \rightarrow \pi$ is needed to bring the form factor in agreement with data at large $q^2$.

With these corrections included, the middle curve of FIG. 6 is now in agreement with data and almost coincides with the lower curve obtained with a BK fitted to the BaBar data $^\text{8}$. Thus the $D \rightarrow K, \pi$ and $B \rightarrow \pi$ form factors with the Born term as the main contribution, and assuming the same correction term for $D^0 \rightarrow \pi^-$ and $D^+ \rightarrow \pi^0$ form factor, are now in agreement with data and are given by:

$$f^+(q^2)_{D\pi} = f^+(0)_{D\pi} \frac{(1 - 0.15q^2/(m_D^2 + m^2_x))}{(1 - q^2/(m_D^2 + m^2_x))}$$

$$f^+(q^2)_{DK} = f^+(0)_{DK} \frac{(1 - q^2/(m_D^2 + m^2_K))}{(1 - q^2/(m_D^2 + m^2_K))}$$

$$f^+(q^2)_{B\pi} = f^+(0)_{B\pi} \frac{(1 - 0.70q^2/(m_B^2 + m^2_x))}{(1 - q^2/(m_B^2 + m^2_x))}$$

(13)

III. CONCLUSION

In conclusion, we have shown that, the tree-level Born term for the process $c + d \rightarrow \pi \nu$ and $b + d \rightarrow \pi \nu$ in semileptonic decays of a heavy meson to a light meson in the final state is found to describe rather well the $q^2$-dependence of the $D \rightarrow \pi, D \rightarrow K$ and $B \rightarrow \pi$ form factors. In particular, the $D^0 \rightarrow K^-$ and $D^+ \rightarrow K^0$ form factors show possible evidence for the $K$ meson mass term in the quark propagator and the simple $q^2$-dependence generated by this Born term.

- Note added: After completion of this paper, I found a previous paper $^\text{27}$ in which a B-meson pole $q^2$-dependence term $1/(1 - q^2/m_B^2)$ for the $B \rightarrow \pi$ form factor is also obtained.
