Curve Fitting of Raspberry Pi Car-Route Based on Baseline Least Squares Method

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Abstract. Curve fitting has a wide range of applications in engineering design, computer graphics, etc. In some case traditional least square methods can’t use known conditions as fully as possible, so we introduce a notion named baseline. We firstly estimate a baseline and use the point to line distance as the parameter of the probability density as the weight value of the least square. The software we developed shows that our methods needs less running time and can makes better fitting consequence.

1. Introduction
In some cases, we want to fit some lines with some scatter. Total least squares (TLS), weighted total least squares (WTLS) [1,2,3], robust weighted total least squares (RWTLS), these methods were adopted to fit lines. However, in some special circumstances, we have a better choice to fit these lines. If we have a baseline, we can use some simple weight values to divide the point set according to different weights. The choice of baseline depends on the situation. In most circumstance, we may not use the baseline as our result.

There are some similar method such as piecewise least squares, multiple adaptation least square and moving least squares. [4,5,6]But by introducing the concept of a baseline, our software needs less running time and can makes better fitting consequence. The point to line distances were the parameters of the operators, which we choose different probability densities as weighted values.

This paper is organized as follows. In section 2 we will test every kinds of weight using in linear least square can get the best fitting line. In the section 3 we discuss the result of the experiment and
explore the main factor of the route of the car. The study may seem to have narrow application, which will be discuss in the section 4.

2. Method
Least square requires the minimum sum of squares of residuals, which is more available than sum of absolute values. Observation shows that the points are distributed near the sides of a straight line. So we can determine the mathematical model:

\[ y \approx P(x) = a + bx \]

According to least square, we need to make

\[ \delta(a, b) = \sum_{k=1}^{7} \rho_k[y_k - (a + bx_k)]^2 \]

get it’s minimum value. That means we should make

\[ \frac{\partial \delta}{\partial a} = 0 \]
\[ \frac{\partial \delta}{\partial b} = 0 \]  

(2)

There are three common probability distributions that might able to suit the condition of linear least square. Independent variable X is select as the distance from the measuring point to the benchmark line. The benchmark line comes from the real map of the inner building. The first one is uniform distribution. In this case, the weight could be represented as:

\[ \rho(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{other} \end{cases} \]  

(3)

In this case weight can be omitted

The second one is index distribution. The weight in this situation could be written as:

\[ \rho(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases} \]  

(4)

The third one is normal distribution. Its weight is this:

\[ \rho(x) = \frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]  

(5)

Bring (4) into (2), we get following equations:
In the above equations, \(a_1\) and \(b_1\) represent the parameters of the baseline. \(a\) and \(b\) represent the parameter of the mathematic model.

Bring (5) into (2), we get following equations:

\[
2 \sum_{k=1}^{7} \lambda e^{-\frac{\lambda}{\sqrt{\sigma^2+1}}} |y_k-a_1+b_1 x_k| (y_k - (a + bx_k)) x_k = 0
\]

\[
2 \sum_{k=1}^{7} \lambda e^{-\frac{\lambda}{\sqrt{\sigma^2+1}}} |y_k-a_1+b_1 x_k| (y_k - (a + bx_k)) = 0
\]

(6)

Here we use the standard normal distribution, i.e., \(\sigma = 1, \mu = 0\).

Then we calculate the accuracy of the formula using two norm.

2.1 Dataset details

We demonstrated how to use the formula by the dataset measuring in advance. A raspberry pi car is controlled to go straight in for ten seconds on a flat road. At the end of each ten seconds, we get the position coordinates by Wi-Fi. [7] Position is transformed into a point in a two-dimensional coordinate system. The baseline is converted by the actual map. Since we can’t avoid the measurement error in the real world, and the points closer to the baseline have higher probability weights, we use the baseline least square method to get the actual car’s route. In this experiment, the last 30% of the dataset is used to test the accuracy of the method.

2.2 Workflow

The overall workflow was demonstrated in Fig.1. Firstly, the system has a few prerequisite python packages, including numpy version 1.12.1, pandas version 0.19.2, Pillow version 4.1.0. These python packages are pre-install in many system, but in case not, the user may manually install them using “pip”, e.g., pip install numpy.
3. Result
Measure the y coordinate of the car in the x coordinate as follows

| Measuring point | /m  | /m  |
|----------------|-----|-----|
| 1              | 4.1 | 10.8|
| 2              | 10.0| 15.25|
| 3              | 15.1| 21.8|
| 4              | 21.0| 25.35|
| 5              | 25.0| 30.9|
| 6              | 30.1| 35.1|
| 7              | 35.0| 40.7|
| 8              | 40.1| 45.25|
The expression of the baseline is $y = x$, so as to say, $a_1 = b_1 = 1$.

Bring the measured value into equation 6, we get the image of the index distribution.

Bring the measured value into equation 7, we get the image of the Normal distribution.

|   |     |     |
|---|-----|-----|
| 9 | 45.0| 50.3|
| 10| 50.0| 55.6|
| 11| 55.1| 60.45|
| 12| 60.1| 65.05|
| 13| 65.0| 70.2|
| 14| 70.2| 75.3|
| 15| 75.1| 80.5|
| 16| 80.0| 85.4|
| 17| 85.1| 90.3|
| 18| 90.0| 95.25|
| 19| 95.0| 100.8|
| 20| 100.0|105.45|
After five groups of test, we getting the average value of the accuracy of the formula.

| Methods              | Accuracy (m) | Running time(s) |
|----------------------|--------------|-----------------|
| LS                   | 0.56         | 1.0             |
| PLS                  | 0.21         | 3.2             |
| Our_method_index     | 0.22         | 1.3             |
| Our_method_normal    | 0.19         | 1.4             |

The code of the system is available in https://github.com/vsfh/WeighedLeastQuare
4. Conclusion
This work proposed an easy-to-use method to get the proper line of several point. The accuracy of the method is better than other least square algorithms. It can also be used on some special situation except map drawing. By measuring the error between the baseline and the estimated line, we can derive the wear of the vehicle tire. It can also be applied to wind tunnel experiments. Of course, these require more experimental research and hypothesis verification.

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