UH model considering cohesion effects

Yangping Yao i) and Ningbo Wang ii)

i) Professor, School of Transportation Science and Engineering, Beihang University, 37, Xueyuan Road, Beijing, 100191, China.
ii) Ph.D Student, ditto.

ABSTRACT

Based on the analysis of isotropic compression behavior of clay affected by cohesion, the parameter considering cohesion is introduced into logarithmic function to describe the isotropic compressibility of normally consolidated cohesive clay. Combining with the coordinate translation method, the UH model considering cohesion effects is established in translational stress space. The proposed model can describe mechanical behavior of overconsolidated cohesive clay, including isotropic compression, shear yielding and critical state. Meanwhile, it has the same form in translational stress space as the UH model in real stress space and can be easily extended. Comparison with existing test results shows that the model can reasonably describe the basic mechanical behavior of cohesive clays.

Keywords: cohesion, isotropic compression, stress space translation, UH model

1 INTRODUCTION

Cohesion has significant influence on the stress-strain relationship and strength of clay, especially under low confining pressure. It is used to determine the construction parameters and be an important factor to evaluate the safety of geotechnical engineering, such as the calculation of Rankine soil pressure, judging the stability of tunnel working face, the anti-crack design in earth-rock dam core wall, and the slope stability control. The consideration of cohesion in the above several aspects, which mainly focus on the strength characteristics and destruction behavior of rock and soil materials, can well serve for geotechnical engineering. Nevertheless, geotechnical engineering involves a dynamic stress-strain development process, such as the tamping foundation, the slope slipping, the settlement of high embankment of airport, etc. So more attention should be given on the effect of cohesion on the development of stress-strain relationship, especially for some geotechnical engineering in which the prediction and deformation control are considered more important than strengthen. Therefore, how to reasonably consider the influence of cohesion in constitutive model is of great importance.

The constitutive model without consideration of cohesion effects have been well developed, such as the subloading surface model (Hashiguchi K. 1989), the bounding surface model (Dafalias, Y.F. 1986), the Pender’s model (Pender M. J. 1978) and Sun’s model (Sun et al. 2007a, 2007b). By introducing a path-independent unified hardening parameter into the modified Cam clay model, Yao et al. (2008a, 2009, 2012a, 2015.) proposed a unified hardening model (UH), for overconsolidated clays. And the UH model has been extended to consider many factors including particle crushing (Yao et al. 2008b), temperature (Yao et al. 2011a,2013) and material anisotropy (Yao et al. 2008, 2012b, 2017). In view of the significant difference on compressibility and shear yielding behavior between cohesive clay and normally consolidated saturated clay, some scholars have established constitutive models considering cohesion effects. Wei (2015) abstracted the general form of yield surface based on the test results of normally consolidated clay. Matsuoka et al. (1992) introduced stress parameter relevant to cementation into SMP criterion to reflect the influence of cohesion on soil deformation and strength. Yao et al. (2011b) established the constitutive model that can describe the influence of cohesion on shear yielding based on UH model by adopting the coordinate translation method. Yuan et al. (2011) proposed a modified Cam-clay model considering cohesion and S-D effect.

The above models was established on the basis of the yield surface affected by cohesion, without concerning the effect of cohesion on isotropic compressibility. The normally compression line (NCL) was still described by straight line in \( e - \ln p \) plane and its expression is

\[ e = N - \lambda \ln p \]  

where \( e \) is the slope of the NCL in \( e - \ln p \) plane. The NCL provides the stress-strain relation of normally
compressed clay under isotropic compression condition. According to Eq. (1), when the mean stress increases from the initial $p_{x0}$ to $p_x$, the total volumetric strain is

$$\varepsilon_v = \frac{\lambda \ln \frac{p_x}{p_{x0}}}{1 + \varepsilon_0}$$

(2)

The elastic volumetric strain is calculated by

$$\varepsilon_v^e = \frac{\kappa}{1 + \varepsilon_0} \frac{p_x}{p_{x0}}$$

(3)

Combining Eq. (3) with Hooke's law, the elastic modulus can be obtained

$$E = \frac{3(1-\nu)(1+\varepsilon_0)}{\kappa} p$$

(4)

It can be seen that Eq. (1) also implies the elastic modulus of soil, which is proportional to the confining pressure. When confining pressure approaches zero, the elastic modulus towards to zero too. It is clear that the formula is inapplicable to cohesive clay which has a relatively high deformation modulus and bearing capacity even under extremely low confining pressure.

In this paper, the isotropic compressibility of normally consolidated cohesive clay is analyzed, then the expression of NCL is proposed by introducing the parameter relevant to cohesion in the $e-\ln p$ plane with reference to the description of sand NCL in the CSUH model (Yao et al. 2019). Combining with the coordinate translation method, The UH model considering cohesion effects is established in the translational stress space.

### 2 ISOTROPIC COMPRESSION OF COHESIVE CLAY AND ITS DESCRIPTION

Compared with normally consolidated saturated clay, natural soil in general has cohesion due to the effect of a variety of physical and chemical factors, including granular cementation, matric suction(Sun et al. 2007c), thus making soil particles tend to be connected and changing the mechanical behavior of clay such as compressibility and strength(Liu et al. 2013).

Fig. 1 shows the isotropic compression of normally consolidated natural Bangkok clay (Balasubramaniam et al. 1981) It can be seen that the normal compression line in $e-\ln p$ space has two significant features, suggest the following:

1. The NCL is a curve other than a straight line.
2. The slope of the tangency point on NCL increases with the increase of confining pressure and gradually approaches to a constant.

In order to describe curved NCL of cohesive clay in the $e-\ln p$ plane, a parameter relevant to cohesion is introduced in the $e-\ln p$ plane and the expression of the NCL is obtained as

$$e = Z - \lambda \ln \left( \frac{p + \sigma_0}{1 + \sigma_0} \right)$$

(5)

where $Z$ is the void ratio on the NCL when $p=1$ kPa; $\lambda$ is the slope of the NCL in the $e-\ln (p+\sigma_0)$ plane, $\sigma_0$ is a parameter corresponding to cohesion and can be describe as

$$\sigma_0 = \frac{c}{\tan \phi}$$

(6)

where $c$ is cohesion, $\phi$ is internal friction angle and can be given as following formula under triaxial compression test

$$\phi = \arcsin \left( \frac{3M}{6+M} \right)$$

(7)

where $M$ is the critical state critical state stress ratio of normally consolidated saturated clay.

Fig. 2 shows the compression test data of intact loess (Chen et al. 2018) and prediction results of Eq. (5). Among them, the dotted line is the prediction result of Eq. (1) and the solid is the prediction result of Eq. (5). Basic physical parameters of material are shown in Table 1

| samples     | Z/N  | $\lambda$ | $\sigma_0$ |
|-------------|------|-----------|------------|
| intact loess| 1.03 | 0.15      | 70         |

Eq. (3) has the form just the same as the expression of ICL of sands in CSUH model. However, its meaning is totally different for $\sigma_0$ is used to describe the magnitude of compressibility is changed compared with normally consolidated saturated clay. Meanwhile, it implies that the NCL of cohesive clay is a straight line in $e-\ln (p+\sigma_0)$ plane and the effect of cohesion on initial void ratio is considered also.
It can be seen that the NCL of cohesive clay is curved in $e$-$\ln p$ plane and Eq. (5) can describe this trend reasonably, while the prediction result of Eq. (1) is quite different from the test data.

According to Eq. (5), when the mean stress increases from the initial $p_{x0}$ to $p_x$ on the NCL, the total volumetric strain is

$$
\varepsilon_v = \frac{\lambda}{1+\varepsilon_0} \ln \left( \frac{p_x + \sigma_0}{p_{x0} + \sigma_0} \right)
$$

The elastic volumetric strain is

$$
\varepsilon'_e = \frac{\kappa}{1+\varepsilon_0} \left( \frac{p_x + \sigma_0}{p_{x0} + \sigma_0} \right)
$$

where $\kappa$ is the slope of the unloading line in the $e$-$\ln (p+\sigma_0)$ plane. Combining Eq. (8) with Eq. (9), the plastic volumetric strain $\varepsilon_p$ can be derived as

$$
\varepsilon_p = \frac{\lambda - \kappa}{1+\varepsilon_0} \left( \frac{p_x + \sigma_0}{p_{x0} + \sigma_0} \right)
$$

Combining Eq. (9) with Hooke’s law, the elastic modulus can be calculated by

$$
E = \frac{3(1-2\nu)(1+\varepsilon_0)}{\kappa} \left( p + \sigma_0 \right)
$$

Eq. (11) implicitly express that the elastic modulus of cohesive clay is determine not only confining pressure $p$ but also cohesion $c$. Under extremely low confining pressure, $\sigma_0$ is the decisive factor of $E$ and determines the bearing capability of soil mass with cohesion. Compared with Eq. (4), it can be seen that Eq. (11) is more reasonable and applicable in practice.

3 STRESS-STRAIN RELATIONSHIP UNDER TRIAXIAL COMPRESSION

3.1 Yield surface

By combining the orthogonal rule and the function of plastic work done, the elliptic yield surface of normally consolidated saturated clay is obtained in MCC model. The endpoints of yield surface is character by isotropic yield stress and the intersection of the critical state line with $p$ coordinate axis. Cohesive clay have a higher peak strength due to cohesion, and can suffers tension stress. Therefore, the critical state line shifts on the $p$-$q$ plane, thus enlarging the yield surface, as shown by the dotted line in Fig. 3. Then, the yield function can be expressed as

$$
f = 1 + \frac{\hat{\eta}^2}{M^2} \frac{\hat{\sigma}}{\hat{p}} = 0
$$

where

$$
\begin{align*}
\hat{p} &= p + \sigma_0 \\
\hat{q} &= q \\
\hat{\eta} &= \frac{\hat{q}}{p} \\
\hat{\sigma} &= p_x + \sigma_0
\end{align*}
$$

$M$ is the critical state stress ratio in the translational $\hat{p}$-$\hat{q}$ plane. From Eq. (10), $\hat{p}_s$ can be solved as

$$
\hat{p}_s = \hat{p}_{x0} \exp \left( \frac{\varepsilon_p}{c_p} \right)
$$

$c_p = (\lambda - \kappa) / (1+\varepsilon_0)$; $\varepsilon_0$ is the initial void ratio. Substituting Eq. (14) into Eq. (12), the following yield function of the normally compressed cohesive clay can be obtained

$$
f = \ln \hat{p} + \ln \left( 1 + \frac{\hat{\eta}^2}{M^2} \right) - \ln \hat{p}_{x0} - \frac{\varepsilon_p}{c_p} = 0
$$

Eq. (11) has same form in the translational $\hat{p}$-$\hat{q}$ plane as the yield function of MCC in the $p$-$q$ plane. The yield function of the normally compressed cohesive clay adopts the plastic volumetric strain $\varepsilon_p$ as the hardening parameter.

3.2 Hardening parameter

The unified hardening parameter $H$ is introduced to replace the plastic volumetric strain $\varepsilon_p$ directly for overconsolidated cohesive clay and is written as follows
\[ H = \int dH = \int \frac{M_f - \eta^4}{M^4 - \eta^2} d\varepsilon_{\varepsilon}^{p} \] (16)

where \( M_f \) is potential failure stress ratio in the translational \( \hat{p} - \hat{q} \) plane and can be calculated by

\[ M_f = 6 \left[ \frac{12(3-M)}{M^2} \exp\left( -\frac{\xi}{\xi - \kappa} \right) + 1 \right]^{-1} \] (17)

where \( \xi \) is the vertical distance between of point A and point C, as shown in Fig. 4, and can be written as

\[ \xi = e_{\eta} - e \] (18)

where \( e \) is the current void ratio; \( e_{\eta} \) is the void ratio on anisotropic compression line under the current \( p \) and can be expressed as

\[ e_{\eta} = N - \lambda \ln \left( \frac{\hat{p}}{1 + \sigma_0} \right) - \left( \lambda - \kappa \right) \ln \left( 1 + \frac{\hat{\eta}^2}{M^2} \right) \] (19)

Therefore, the yield function for overconsolidated cohesive clay can be written as

\[ f = \ln \hat{p} + \ln \left( 1 + \frac{\hat{\eta}^2}{M^2} \right) - \ln \frac{\hat{p}_{0}}{c_p} - \frac{H}{c_p} = 0 \] (20)

### 3.3 Stress-strain relationship

1. Elastic strain increment

   The elastic strain increment can be calculated by

   \[
   \begin{align*}
   \varepsilon_{\varepsilon}^{\varepsilon} &= \frac{1 + \nu}{E} \delta_{\varepsilon} - \frac{\nu}{E} \delta_{\sigma_{0}} \\
   \delta_{\varepsilon} &= \delta_{\sigma_{0}} + \sigma_{0}
   \end{align*}
   \] (21)

   where \( \nu \) is the Poisson’s ratio, \( \delta_{\sigma_{0}} \) is the increment of stress, \( \delta_{\varepsilon} \) is Kronecker’s delta, and \( E \) is elastic modulus and can be calculated by Eq. (11).

2. Plastic strain increment

   By combining the associated flow rules, the plastic strain increment can be expressed as

   \[
   d\varepsilon_{\eta}^{p} = \Lambda \frac{\partial g}{\partial \sigma_{\eta}}
   \] (22)

   where the plastic factor can be written as

   \[
   \Lambda = c_p \frac{\partial f}{\partial \hat{p}} + \frac{\partial f}{\partial \hat{q}}
   \] (23)

   where

   \[
   \begin{align*}
   \frac{\partial f}{\partial \hat{p}} &= \frac{M^2 - \hat{\eta}^2}{\hat{p}(M^2 + \hat{\eta}^2)} \\
   \frac{\partial f}{\partial \hat{q}} &= \frac{\hat{\eta}}{\hat{p}(M^2 + \hat{\eta}^2)}
   \end{align*}
   \] (24)

3. Elastoplastic stress-strain relationship in \( \hat{p} - \hat{q} \) plane

   The stress-strain relations in \( \hat{p} - \hat{q} \) plane are expressed as

   \[
   \begin{bmatrix}
   d\hat{p} \\
   d\hat{q}
   \end{bmatrix} = \begin{bmatrix}
   K \cdot A_1 & 3KG \cdot A_1 \\
   3KG \cdot A_1 & 3G \cdot A_1
   \end{bmatrix} \begin{bmatrix}
   d\varepsilon_{\varepsilon}^{\varepsilon} \\
   d\varepsilon_{\eta}^{p}
   \end{bmatrix}
   \] (25)

   where \( K \) is the elastic bulk modulus, \( K = E/(3(1-2\nu)) \); \( G \) is the elastic shear modulus, \( G = E/(2(1+2\nu)) \); \( A_1 \), \( A_2 \), \( A_3 \) and \( A_4 \) can be derived as follows

   \[
   \begin{align*}
   A_1 &= \frac{12G_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)}{K_{\varepsilon, \Omega}(M^2 - \hat{\eta}^2) + 12G_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)} \\
   A_2 &= \frac{-2K_{\varepsilon, \Omega} \eta \hat{\eta}(M^2 - \hat{\eta}^2)}{K_{\varepsilon, \Omega}(M^2 - \hat{\eta}^2) + 12G_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)} \\
   A_3 &= \frac{K_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)}{K_{\varepsilon, \Omega}(M^2 - \hat{\eta}^2)^2 + 12G_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)} \\
   A_4 &= \frac{K_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)}{K_{\varepsilon, \Omega}(M^2 - \hat{\eta}^2) \eta \hat{\eta} + 12G_{\varepsilon, \Omega} \eta^2 + \hat{p}(M^4 - \hat{\eta}^4)}
   \end{align*}
   \] (26)

4. Elastoplastic stress-strain relationship in \( p-q \) plane

   By adopting the assumption that \( c \) is invariant, the parameter \( \sigma_0 \) is constant. Therefore, the increment of \( p \) and \( q \) can be calculated by

   \[
   \begin{bmatrix}
   dp \\
   dq
   \end{bmatrix} = \begin{bmatrix}
   K \cdot A_1 & 3KG \cdot A_1 \\
   3KG \cdot A_1 & 3G \cdot A_1
   \end{bmatrix} \begin{bmatrix}
   d\varepsilon_{\varepsilon}^{\varepsilon} \\
   d\varepsilon_{\eta}^{p}
   \end{bmatrix}
   \] (27)

   It can be seen that stress-strain relationship has same form in the translational \( \hat{p} - \hat{q} \) plane as the UH model in \( p-q \) plane, and it can be easily extended in three-dimensional stress space by combining the transformed stress method and failure criteria.

### 4 Model Verification

#### 4.1 Parameter determination

There are six parameters in the UH model considering cohesion effects: \( M, \lambda, \kappa, \nu, Z, \) and \( \sigma_0 \).
Among them, the first four parameters are inherited from the original UH model. \( Z \) is the void ratio on the NCL at \( p = 1\) kPa. \( \sigma_0 \) can be determined by Eq. (6).

### 4.2 Model validation

1. One-dimensional compression

A series of one-dimensional compression tests were carried out on normally consolidated and overconsolidated Shanghai clay (He et al. 2018). Fig. 5 shows the comparison between the test data and the predictions of UH model considering cohesion effects. The model parameters are listed in Table 2. It can be seen that the predictions of normally consolidated and overconsolidated cohesive clay agree with the test data very well.

Table 2. Summary of model parameters in this paper.

| Material       | \( M \) | \( \lambda \) | \( \kappa \) | \( \nu \) | \( Z \) | \( \sigma_0 \) |
|----------------|--------|--------------|-------------|--------|--------|--------------|
| Shanghai clay  | 1.4    | 0.21         | 0.03        | 0.2    | 1.4    | 30           |
| Silty clay     | 1.14   | 0.078        | 0.01        | 0.1    | 0.92   | 43           |

Fig. 5. Comparisons between one-dimensional compression test results for Shanghai clay (He et al. 2018) and model simulations.

2. Drained triaxial tests

Fig. 6, shows the drained compression test data (Estabragh et al. 2008) and the predictions of Silty clay with different OCR at different initial confining pressure. From this figure, good agreement between the model predictions and test data can be observed, especially under low confining pressure.

### 5 CONCLUSIONS

1. The isotropic compressibility of normally consolidated cohesive clay is summarized and a curved line in \( e\)–ln \( p \) plane of NCL is obtained. Then, the Eq. of NCL is proposed by introducing the parameter considering cohesion into logarithmic function.

2. By adopting the coordinate translation method, the UH model considering cohesion effects is established in translational stress space, and can describe mechanic behavior of cohesive soil, including isotropic compression, one-dimensional compression, shear yielding and critical state. Good agreement between the model predictions and test data is observed, especially under low confining pressure. Meanwhile, it has same form in the translational stress space as the UH model in the real stress space so that it can be easily extension.

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