Research Article
Wavelet-Based MPNLMS Adaptive Algorithm for Network Echo Cancellation

Hongyang Deng¹ and Miloš Doroslovački²

1 Freescale Semiconductor, 7700 W. Parmer Lane, Austin, TX 78729, USA
2 Department of Electrical and Computer Engineering, The George Washington University, 801 22nd Street, N.W. Washington, DC 20052, USA

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The $\mu$-law proportionate normalized least mean square (MPNLMS) algorithm has been proposed recently to solve the slow convergence problem of the proportionate normalized least mean square (PNLMS) algorithm after its initial fast converging period. But for the color input, it may become slow in the case of the big eigenvalue spread of the input signal’s autocorrelation matrix. In this paper, we use the wavelet transform to whiten the input signal. Due to the good time-frequency localization property of the wavelet transform, a sparse impulse response in the time domain is also sparse in the wavelet domain. By applying the MPNLMS technique in the wavelet domain, fast convergence for the color input is observed. Furthermore, we show that some nonsparse impulse responses may become sparse in the wavelet domain. This motivates the usage of the wavelet-based MPNLMS algorithm. Advantages of this approach are documented.

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1. INTRODUCTION

With the development of packet-switching networks and wireless networks, the introduced delay of the echo path increases dramatically, thus entailing a longer adaptive filter. It is well known that long adaptive filter will cause two problems: slow convergence and high computational complexity. Therefore, we need to design new algorithms to speed up the convergence with reasonable computational burden.

Network echo path is sparse in nature. Although the number of coefficients of its impulse response is big, only a small portion has significant values (active coefficients). Others are just zero or unnoticeably small (inactive coefficients). Several algorithms have been proposed to take advantage of the sparseness of the impulse response to achieve faster convergence, lower computational complexity, or both. One of the most popular algorithms is the proportionate normalized least mean square (PNLMS) algorithm [1, 2]. The main idea is assigning different step-size parameters to different coefficients based on their previously estimated magnitudes. The bigger the magnitude, the bigger step-size parameter will be assigned. For a sparse impulse response, most of the coefficients are zero, so most of the update emphasis concentrates on the big coefficients, thus increasing the convergence speed.

The PNLMS algorithm, as demonstrated by several simulations, has very fast initial convergence for sparse impulse response. But after the initial period, it begins to slow down dramatically, even becoming slower than normalized least mean square (NLMS) algorithm. The PNLMS++ [2] algorithm cannot solve this problem although it improves the performance of the PNLMS algorithm.

The $\mu$-law PNLMS (MPNLMS) algorithm proposed in [3–5] uses specially chosen step-size control factors to achieve faster overall convergence. The specially chosen step-size control factors are really an online and causal approximation of the optimal step-size control factors that provide the fastest overall convergence of a proportionate-type steepest descent algorithm. The relationship between this deterministic proportionate-type steepest descent algorithm and proportionate-type NLMS stochastic algorithms is discussed in [6].

In general, the advantage of using the proportionate-type algorithms (PNLMS, MPLMS) is limited to the cases when the input signal is white and the impulse response to be identified is sparse. Now, we will show that we can extend the
advantageous usage of the MPLMS algorithm by using the wavelet transform to cases when the input signal is colored or when the impulse response to be identified is nonsparse.

2. WAVELET DOMAIN MPNLMS

2.1. Color input case

The optimal step-size control factors are derived under the assumption that the input is white. If the input is a color signal, which is often the case for network echo cancellation, the convergence time of each coefficient also depends on the eigenvalues of the input signal’s autocorrelation matrix. Since, in general, we do not know the statistical characteristics of the input signal, it is impossible to derive the optimal step-size control factors without introducing more computational complexity in adaptive algorithm. Furthermore, the big eigenvalue spread of the input signal’s autocorrelation matrix slows down the overall convergence based on the standard LMS performance analysis [7].

One solution of the slow convergence problem of LMS for the color input is the so-called transform domain LMS [7]. By using a unitary transform such as discrete Fourier transform (DFT) and discrete cosine transform (DCT), we can make the input signal’s autocorrelation matrix nearly diagonal. We can further normalize the transformed input vector by the estimated power of each input tap to make the autocorrelation matrix close to the identity matrix, thus decreasing the eigenvalue spread and improving the overall convergence.

But, there is another effect of working in the transform domain: the adaptive filter is now estimating the transform coefficients of the original impulse response [8]. The number of active coefficients to be identified can differ from the number of active coefficients in the original impulse response. In some cases, it can be much smaller and in some cases, it can be much larger.

The MPNLMS algorithm works well only for sparse impulse response. If the impulse response is not sparse, that is, most coefficients are active, the MPNLMS algorithm’s performance degrades greatly. It is well known that if the system is sparse in time domain, it is nonsparse in frequency domain. For example, if a system has only one active coefficient in the time domain (very sparse), all of its coefficients are active in the frequency domain. Therefore, DFT and DCT will transform a sparse impulse response into nonsparse, so we cannot apply the MPNLMS algorithm.

Discrete wavelet transform (DWT) has gained a lot of attention for signal processing in recent years. Due to its good time-frequency localization property, it can transform a time domain sparse system into a sparse wavelet domain system [8]. Let us consider the network echo path illustrated in Figure 1. This is a sparse impulse response. From Figure 2, we see that it is sparse in the wavelet domain, as well. Here, we have used the 9-level Haar wavelet transform on 512 data points. Also, the DWT has the similar band-partitioning property as DFT or DCT to whiten the input signal. Therefore, we can apply the MPNLMS algorithm directly on the transformed input to achieve fast convergence for color input.

The proposed wavelet MPNLMS (WMPNLMS) algorithm is listed in Algorithm 1, where \( x(k) \) is the input signal vector in the time domain, \( L \) is the number of adaptive filter coefficients, \( T \) represents DWT, \( x_{T,l}(k) \) is the input signal vector in the wavelet domain, \( x_{T,T,l}(k) \) is the \( i \)th component of \( x_{T,l}(k) \), \( \tilde{w}_T(k) \) is the adaptive filter coefficient vector in the wavelet domain, \( \tilde{w}_{T,l}(k) \) is the \( l \)th component of \( \tilde{w}_T(k) \), \( \hat{y}(k) \) is the output of the adaptive filter, \( d(k) \) is the reference signal, \( e(k) \) is the error signal driving the adaptation, \( \tilde{\sigma}_e^2(k) \) is the estimated average power of the \( i \)th input tap in the wavelet domain, \( \alpha \) is the forgetting factor with typical value 0.95, \( \beta \) is the step-size parameter, and \( \delta_p \) and \( \rho \) are small positive numbers used to prevent the zero or extremely small adaptive
filter coefficients from stalling. The parameter $\varepsilon$ defines the neighborhood boundary of the optimal adaptive filter coefficients. The instant when all adaptive filter coefficients have crossed the boundary defines the convergence time of the adaptive filter. Definition of the matrix $T$ can be found in [9, 10]. Computationally efficient algorithms exist for calculation of $x_T(k)$ due to the convolution-downsampling structure of DWT. The extreme case of computational simplicity corresponds to the usage of the Haar wavelets [11].

$$x(k) = [x(k)x(k-1) \cdots x(k-L+1)]^T$$

$$x_T(k) = T x(k)$$

$$\hat{y}(k) = x_T^2(k) \hat{w}_T(k)$$

$$e(k) = d(k) - \hat{y}(k)$$

For $i = 1$ to $L$

$$\hat{a}_{i,j}^2(k) = a \hat{a}_{i,j}^2(k-1) + (1-a)x_{i,j}^2(k)$$

End

$$D(k+1) = \text{diag}[\hat{a}_{i,j}^2(k), \ldots, \hat{a}_{i,j}^2(k)]$$

$$\hat{w}_T(k+1) = \hat{w}_T(k) + \beta D^{-1}(k+1)G(k+1)x_Te(k)$$

$$G(k+1) = \text{diag}[g_1(k+1), \ldots, g_L(k+1)]$$

$$F(\hat{w}(k)) = \ln(1+\mu |\hat{w}(k)|), \quad 1 \leq l \leq L, \mu = 1/\epsilon$$

$$y_{\text{min}}(k+1) = \rho \max \{\delta_p, F(\hat{w}_1(k)), \ldots, F(\hat{w}_L(k))\}$$

$$y(k+1) = \max \{y_{\text{min}}(k+1), F(|\hat{w}(k)|)\}$$

$$g_l(k+1) = \frac{y_l(k+1)}{(1/L) \sum_{i=1}^{L} y_l(k+1)}, \quad 1 \leq l \leq L.$$

**Algorithm 1: WMPNLMS algorithm.**

To evaluate the performance of the WMPNLMS algorithm, we use a 512-tap wavelet-based adaptive filter (covering 64 ms for sampling frequency of 8 KHz) to identify the network echo path illustrated in Figure 1. The input signal is generated by passing the white Gaussian noise with zero-mean and unit-variance through a lowpass filter with one pole at 0.9. We also add white Gaussian noise to the output of the echo path to control the steady-state output error of the adaptive filter. The WMPNLMS algorithm uses $\delta_p = 0.01$ and $\rho = 0.01$. $\beta$ is chosen to provide the same steady-state error as the MPNLMS and SPNLMS algorithms. From Figure 3, we can see that the proposed WMPNLMS algorithm has noticeable improvement over the time domain MPNLMS algorithm. Note that SPNLMS stands for the segmented PNLMS [5]. This is the MPNLMS algorithm in which the logarithm function is approximated by linear segments.

### 2.2. Nonsparse impulse response case

In some networks, nonsparse impulse responses can appear. Figure 4 shows an echo path impulse response of a digital subscriber line (DSL) system. We can see that it is not sparse in the time domain. It has a very short fast changing segment and a very long slow decreasing tail [11]. If we apply the MPNLMS algorithm on this type of impulse response, we cannot expect that we will improve the convergence speed. But if we transform the impulse response into wavelet domain by using the 9-level Haar wavelet transform, it turns into a sparse impulse response as shown in Figure 5. Now, the WMPNLMS can speed up the convergence.

![Learning curves](image-url)

**Figure 3:** Learning curves for wavelet- and nonwavelet-based proportionate algorithms.
convergence due to the sparseness of the impulse response in the wavelet domain and the algorithm’s proportionate adaptation mechanism. The wavelet-based NLMS algorithm also identifies a sparse impulse response, but does not speed up the convergence by using the proportionate adaptation mechanism. Compared to the computational and memory requirements listed in [5, Table IV] for the MPNLMS algorithm, the WMPNLMS algorithm, in the case of Haar wavelets with $M$ levels of decomposition, requires $M + 2L$ more multiplications, $L − 1$ more divisions, $2M + L − 1$ more additions/subtractions, and $2L − 1$ more memory elements.

3. CONCLUSION

We have shown that by applying the MPNLMS algorithm in the wavelet domain, we can improve the convergence of the adaptive filter identifying an echo path for the color input. Essential for the good performance of the WMPNLMS is that the wavelet transform preserve the sparseness of the echo path impulse response after the transformation. Furthermore, we have shown that by using the WMPNLMS, we can improve convergence for certain nonsparse impulse responses, as well. This happens since the wavelet transform converts them into sparse ones.

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