Magnetic skyrmions as host of neutrino mass spectroscopy

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ABSTRACT

Large magneto-electric effect in multi-ferroics and topological protection against decays of excited magnetic skyrmions are an ideal setting for detecting radiative emission of neutrino pair (RENP). We propose a new RENP scheme using multi-ferroic magnetic skyrmion condensates in order to determine the absolute neutrino mass including the mass hierarchy patterns, the type of neutrino mass (Majorana vs Dirac), one of CP violation parameters, and to detect relic 1.9 K cosmic neutrino. We critically examine the possibility of feasible experiments, and show what are required as good targets for neutrino physics.

Keywords
Neutrino mass, Majorana particle, Relic neutrino, CP violation, Skyrmion, Multi-ferroics
Introduction

The discovery of neutrino oscillation [1] has established that at least two of three neutrinos have finite masses with their mass eigenstates mixed in weak decay of nuclei and elementary particles. Combined with the cosmological microwave fluctuation and simulation of structure formation [1], the heaviest neutrino has a mass in the range $m_3 = 60 \sim 200\text{ meV}$, the next heaviest of $m_2 = \sqrt{m_3^2 - (50\text{meV})^2}$, and the lightest of $m_1 = \sqrt{m_2^2 - (10\text{meV})^2}$ for the normal hierarchy (NH) mass pattern. For the inverted hierarchy (IH) mass pattern two measured mass difference values, $(50\text{meV})^2$ and $(10\text{meV})^2$, are interchanged in the formula. Neutrino oscillation experiments are sensitive only to mass squared differences and they left important questions unanswered: (1) absolute value of neutrino masses, (2) nature of neutrino masses, either of Dirac type or of Majorana type, (3) CP violation (CPV) parameters (one phase for Dirac and three phases for Majorana). There are on-going experiments [3], [4], [1] that attempt to partially answer questions, (1) and (2), using nuclear targets. So far results of time consuming efforts have yielded only limits on neutrino parameters. One of the major goals in the next round of oscillation experiments is to measure one of CPV phases. One may add to this list (4) detection of cosmic relic neutrino of 1.9 K [2] which clarifies the state of universe at one second after the big bang.

Atomic targets are better suited than nuclear targets to answer these questions, since available energy differences $< O(10)\text{eV}$ are closer to neutrino masses. But detection rate when events occur incoherently are much less at unobservable levels. It was recently suggested [5], [6] that a prepared macro-coherence of states enhances rates to circumvent the smallness of rates in the de-excitation process $|e\rangle \rightarrow |g\rangle + \gamma + \nu_i \bar{\nu}_j$; radiative emission of neutrino pair (RENP) with $\nu_i, \bar{\nu}_j, i, j = 1, 2, 3$ mass eigenstates. The target phase coherence between an excited state $|e\rangle$ and the ground state $|g\rangle$ may be realized by good-quality excitation laser. Experiments [7] in QED second order process of para-hydrogen, two-photon emission from a vibrationally excited state, confirmed that the coherence close to necessary levels is achieved, with rate enhancement of order $10^{18}$. The project of macro-coherent RENP was termed neutrino mass spectroscopy [6], [2], since it covers all four raised questions, $(1) \sim (4)$. Note that one does not need any specific theory of neutrino mass beyond the standard theory of particle physics except introduction of the neutrino mass term, in order to advance this project. There is however a serious problem envisaged in RENP schemes using a gas due to small available target atom numbers, hence it is desirable to use solid environments for detection. But various de-coherence processes are expected to be very fast in solids in conflict with the macro-coherence.

In the present work we propose to solve the de-coherence problem in solids by a topology protection. A series of recent experiments in condensed matter physics discovered topologically stable magnetic skyrmions in a variety of magnetic systems [3]. Magnetic skyrmions are characterized by integer Z topological charges, $\int d^2\tau \bar{n} \cdot \partial_j \bar{n} \times \partial_y \bar{n}/4\tau$ with $\bar{n} = \vec{S}/S$ the unit spin vector, as defined by mapping of a sphere onto two dimensional plane. The topology protection realizes a long relaxation time, which is proposed below to use in applications to fundamental physics, which may be called topological RENP experiment. Calculated event rates are in proportion to squared magneto-electric strength, hence insulating magnetic skyrmions are candidate hosts.

We use in the present work the natural unit such that $\hbar = c = k_B = 1$ to simplify theoretical formulas. In this unit $1\text{ eV} = 1/(1.97 \times 10^{-5}\text{cm}) = 3/(1.97 \times 10^{-15}\text{sec})$ and $1\text{ K} = 0.086\text{ meV}$.

Multi-ferroic coupling in RENP

Neutrino pair emission from atomic electron occurs with a hamiltonian density, $\nu_i^\dagger \vec{\sigma} \nu_j \cdot e^j \vec{e}$, neutrino-pair spin times electron spin $\vec{S} = \vec{\sigma}/2$, written in terms of field operators $\nu_i^\dagger$. This four-Fermi interaction consists of added W- and Z-exchange contributions. Mass eigenstate fields $\nu_i$’s are related to the electron neutrino $\nu_e$ that appears in beta decay by $\nu_i = \sum_\alpha U_{\alpha i} \nu_\alpha$, where $(U_{\alpha i}), \alpha = e, \mu, \tau$, is the $3 \times 3$ unitary mixing matrix. Relevance of the electron spin in the hamiltonian suggests how the magnetic order in solids may help in RENP. Since it is practically impossible to detect neutrinos in atomic experiments, one needs detection of photons to probe neutrino properties. For this the weak interaction is connected to QED electric dipole interaction, $e \vec{E} \cdot e^j \vec{r}_e$ in the second order perturbation theory, to give a product of spin and dipole, $\vec{S}_i e^j \vec{r}_e$ for electronic transition. The RENP probability amplitude
is then given by
\[ \frac{2G_F}{\sqrt{2}} a_{ij} \nu_i \bar{\nu}_j \cdot \langle g \rangle \sum_n \left( \frac{\bar{\nu}_i | n \rangle \langle n | \bar{\nu}_j}{\epsilon_e - \epsilon_n - \omega} + \frac{\bar{\nu}_j | n \rangle \langle n | \bar{\nu}_i}{\epsilon_g - \epsilon_n + \omega} \right) | e \rangle \cdot \vec{E}_\gamma, \quad a_{ij} = U_{ei}^* U_{ej} - \frac{1}{2} \delta_{ij}, \tag{1} \]

(with \( G_F = 1.17 \times 10^{-5} \text{GeV}^{-2} \) the Fermi coupling constant) using the energy conservation \( \epsilon_g + \omega = \epsilon_e - E_i - E_j \) to eliminate the neutrino pair energy \( E_i + E_j, E_i = \sqrt{p_i^2 + m_i^2} \) in favor of the photon energy \( \omega \). We shall give an example of states, \( | e \rangle, | n \rangle, | g \rangle \) later.

The summed bracket quantity in eq.(1) may be rewritten by introducing an average intermediate state energy \( \bar{\epsilon}_n \) and by multiplying the magnetic moment \( g \mu_B \) times the squared number density \( N^2 \) of excited atoms, to give
\[ N^2 (\frac{2m_e}{g})^2 \sum_n \left( \frac{\bar{\nu}_i | n \rangle \langle n | \bar{\nu}_j}{\epsilon_e - \epsilon_n - \omega} + \frac{\bar{\nu}_j | n \rangle \langle n | \bar{\nu}_i}{\epsilon_g - \epsilon_n + \omega} \right) \equiv h(\omega) \vec{M} \vec{P} \] \tag{2}
where \( \vec{M}, \vec{P} \) are magnetization and electric polarization of target medium, respectively, assumed to be uniform in crystals.

A large product \( MP \) may emerge in multi-ferroic materials, objects in which both ferro-electric and ferro(anti-ferro)-magnetic orders co-exist \[9, 10\]. Calculated event rates of RENP in multi-ferroics depend on an overall factor given by \( G^2 \frac{2m_e}{gN} V^2 \) (a target volume) times a power of atomic level spacing between \( | e \rangle \) and \( | g \rangle \), \( \epsilon_{eg} \). Taking typical values for \( M, P, N, \epsilon_{eg} \), along with the phase space volume of neutrino pair, gives total RENP rates of order,
\[ \frac{6}{(2\pi)^4} \frac{G^2}{gN} (\frac{2m_e}{g} \frac{M \mu_B V}{(5 \times 10^{-8} \text{cm})^3})^2 \epsilon_{eg}^7 \sim 1.0 \times 10^{-5} \text{sec}^{-1} (\frac{V}{\text{cm}^3})^2 (\frac{5 \times 10^{-8} \text{cm}}{N})^3 (\frac{M}{\mu_B (5 \times 10^{-8} \text{cm})^3})^2 (\frac{P}{\mu C/m^2})^2 (\frac{\epsilon_{eg}}{eV})^7. \tag{3} \]

**RENP spectrum** There are six distinct pair thresholds denoted by \((ij)\) at \( \omega_{ij} = \epsilon_{eg} - m_i - m_j \), rate rises being determined by neutrino mass mixing parameters, including CP violating phases. Distinction of Majorana from Dirac neutrinos becomes possible by exploiting the very nature of Majorana particle being its own anti-particle; \( \nu_i = \bar{\nu}_i \). Theoretical prediction of Majorana-pair production is governed by anti-symmetric wave functions of two identical fermions, which gives spectra different from Dirac-pair of distinguishable neutrino and anti-neutrino \[5\].

For spectrum calculation recall that the neutrino helicity and the momentum \( (\vec{p}_i) \) are summed due to their detection being impossible, to give the neutrino phase space integral, including the Pauli blocking of relic neutrinos,
\[ \int \frac{d^3 p_i d^3 p_j}{(2\pi)^5} \delta(\epsilon_{eg} - \omega - E_i - E_j)(1 - \delta_M \frac{m_i m_j}{E_i E_j}) = \frac{4}{(2\pi)^3} \epsilon_{eg}^5 \left( G(\frac{\epsilon_{eg}}{\epsilon_{eg}}, \frac{m_i}{\epsilon_{eg}}, \frac{m_j}{\epsilon_{eg}}) - \delta_M H(\frac{\epsilon_{eg}}{\epsilon_{eg}}, \frac{m_i}{\epsilon_{eg}}, \frac{m_j}{\epsilon_{eg}}) \right) \tag{4} \]
\[ G(x, a, b) = \int_a^{1-x-b} dy \left((y^2 - a^2)((1-x-y)^2 - b^2)\right)^{1/2} y(1-x-y)(1-f_i)(1-f_j), \tag{5} \]
\[ H(x, a, b) = ab \int_a^{1-x-b} dy \left((y^2 - a^2)((1-x-y)^2 - b^2)\right)^{1/2} (1-f_i)(1-f_j). \tag{6} \]

where \( f_i = 1/(\epsilon_{pi}/T_0 + 1) \) is the Fermi-Dirac distribution function of \( \epsilon_{pi} = \sqrt{E_i^2 - m_i^2} \), expressed in terms of integration variables \( x, y, a, b \), with \( T_0 = 1.9 \text{K} \) the relic neutrino temperature \[2\]. The Majorana/Dirac distinction appears in the \( \delta_M \) term of eq.(4): \( \delta_M = 1 \) for Majorana neutrinos and \( = 0 \) for Dirac neutrinos.
The differential spectrum rate is given by

$$\frac{d^2\Gamma}{d\omega d\Omega} = \frac{6}{(2\pi)^4} G_F^2 \left( \frac{2m_e}{g} \right)^2 \left( \frac{MPV}{N} \right)^2 \sin^2 \theta \cos^2 \varphi \epsilon_{eg}^5 F(\omega; m_i),$$

$$F(\omega; m_i) = \omega^3 |h(\omega)|^2 \sum_{ij} \left( |a_{ij}|^2 G^{\omega}_{\epsilon_{eg}} + \epsilon_{eg} \right) - \delta_M \Re(a_{ij}) H(\omega) \sin^2 \theta.$$  

\(\theta\) is the photon emission angle measured from \(\vec{P}\)-direction, and \(\varphi\) is the photon linear polarization \((\vec{e})\) angle from the plane made of \(\vec{P}\) and photon wave vector such that \(\vec{P} \cdot \vec{e} = P \sin \theta \cos \varphi\). Note that except the average value \(\bar{\epsilon}_n\) in \(h(\omega)\) of eq.(2) and \(\epsilon_{eg}\), RENP rate formula contains measured quantities of target atoms such as \(M, P, N\). The massless limit of neutrinos without the relic effect is easily worked out, to give the integral, \(\epsilon_{eg}^5 G(\omega/\epsilon_{eg}, 0, 0) \sum_{ij} |a_{ij}|^2 = (\epsilon_{eg} - \omega)^5/40\). The global spectrum shape is of the form, \(\omega^3|h(\omega)|^2(\epsilon_{eg} - \omega)^3d\omega\), which is modified by finite neutrino mass function \(F(\omega; m_i \neq 0)\).

Before we discuss a specific target example of spectrum shapes, we demonstrate importance of relic neutrino effects in Fig(1). The ratio of two spectrum rates [12], rate including the Pauli blocking \(1 - f_i\) to rate without it, is plotted for \(\epsilon_{eg} = 30\) meV. Actual experimental data cannot disentangle relic neutrino effect of 1.9 K, hence this ratio may effectively be regarded as experimental values divided by a hypothetical result calculated without the relic neutrino. Mass determination to sensitivity >\(O(1)\) meV and the Majorana/Dirac distinction is not difficult for \(\epsilon_{eg} = 30\) meV. We find it remarkable that the topological RENP greatly improves the sensitivity of neutrino mass spectroscopy including the relic neutrino detection. In the rest of presentation we include the important effect of 1.9 K relic neutrino [13].

**An example of RENP scheme in multi-ferroic skyrmion lattice** Most magnetic skyrmions appear in metals, but recently, insulating multi-ferroic skyrmion lattice (SkL) has been discovered in Cu₂OSeO₃ as its host [14], [15], [8]. Calculations in [16], [17] indicate that both parallel and perpendicular \(\vec{M}\), \(\vec{P}\)'s of large values may arise consistently with experimental results. Measured values of this SkL are \(P_0 = 0.2 \mu C/m^2\), \(M_0 = 0.1 \mu B/Cu^{2+}\), \(N = 1/(8.9 \times 10^{-8} \text{cm})^{-3}\). The unit cell of Cu₂OSeO₃ is made of four bypyramids of altogether 16 Cu²⁺ magnetic ions. According to density functional theory (DFT) calculation [17] the ground state consists of four spin triplets \(|1\rangle\) of Cu²⁺. When one of bypyramids is excited to either of two singlets \(|0\rangle, i = 1, 2\) of energy \(\sim 24\) meV, a quintet \(|2\rangle\) of energy \(\sim 29\) meV, or two triplets \(|3\rangle\), \(i = 1, 2\) of energy \(\sim 39\) meV, they form the first groups of excited skyrmions. In RENP rate estimate we take as relevant states, \(|\epsilon\rangle = |2\rangle, |n\rangle = |1\rangle, |g\rangle = |1\rangle\) in eq.(2).

We illustrate numerical results of calculated RENP spectral shapes in Fig(2). Due to the closeness of this system \((\epsilon_{eg} = 29\) meV) to neutrino masses in the 10 meV range, there are good sensitivities both to Majorana/Dirac distinction and to one of Majorana CPV phases. For determination of neutrino properties and parameters the spectrum shape \(F(\omega; m_i)\) of eq.(3) is all one needs. But the feasibility of RENP does depend on absolute rate values. In order to obtain absolute rates, one should multiply numbers in Fig(2) by \(4 \times 10^{-9} \text{sec}^{-1}/\text{eV}\). Hence the example of Cu₂OSeO₃ gives rates too small to be detectable. Possible improvements shall be discussed later.

Rate increase with the level spacing \(\epsilon_{eg}\) is dramatic: the total rate increases with a high power of \(\epsilon_{eg} \propto \epsilon_{eg}^7\) if \(\epsilon_{ne} < \epsilon_{eg}\), as numerically confirmed. Hence it is critically important to find more insulating skyrmions of larger energy gaps to achieve larger rates. Or one should find more appropriate excited levels in Cu₂OSeO₃, for instance 150 meV or a slightly higher.

**Role of topology** We assume that both the excited initial lattice state and the final one in RENP are topologically stable with skyrmion condensates. Difference (including the vanishing one) of topological quantum numbers is transfered to those of emitted particles, since the overlap of wave functions of emitted particles and emitting atoms is very small. This becomes possible if three unit vectors \(\vec{n}_i\) or RENP particles carry a topological integer value of \(\int d^2r \vec{n}_i \cdot \partial_x \vec{n}_2 \times \partial_y \vec{n}_3 / 4\pi\). Three spin vectors \(\vec{n}_i\) can be in the same plane: the neutrino pair is emitted perpendicular to each other with their spins almost (exactly for the
Figure 1: Theoretically calculated ratio of two rates: with the relic Pauli blocking and without it: \( \epsilon_{eg} = 30 \) meV with \( \bar{\epsilon}_r = 40\)meV in \( f(\omega) \) of eq.(2), all assuming CP conservation and Majorana NH neutrinos. Assumed parameters are the smallest mass 1 meV, 1.9 K in solid black, 5 meV, 1.9 K in thin dashed black, 1 meV, 1.9 K x 1.5 in dashed red, 1 meV, 1.9 K x 0.5 in dash-dotted blue. Left panel for NH and right panel for IH.

Figure 2: Left panel. Spectrum sensitivity to Majorana and Dirac neutrino masses in NH case. Dirac mass 1 meV in solid black, 5 meV in dotted black, Majorana mass 1 meV in dashed red, and 5 meV in dash-dotted blue. All cases assume CP conservation. Right panel. Spectrum sensitivity to Majorana CPV phases. NH Majorana of the smallest neutrino mass 1 meV is assumed: \( (\alpha, \beta, \delta) = (0, 0, 0) \) in solid black, \( (\pi/8, 0, 0) \) in dashed red, \( (\pi/6, 0, 0) \) in dash-dotted blue, and \( (\pi/4, 0, 0) \) in dotted black.
massless case) along emitted directions and photon polarization out of the emission plane, corresponding to \( \theta = \pi/2, \varphi = 0 \) in the rate formula. This restriction gives a reduction of neutrino phase space integration by some amount, but within a tolerable range of reduction.

The topological conservation law, on the other hand, restricts possible modes and configurations of QED backgrounds. The major QED background is three photon emission, with polarization constraints similar to the RENP case. Although the QED background has much larger rates than RENP, specified characteristics of background events helps much to reject backgrounds from the RENP signal.

Comments and prospects In the rate estimate given above, we took modest \( P, M, N \) values of target \( \text{Cu}_2\text{OSeO}_3 \). There are a few rooms for improvements if we use better targets (yet to be identified): (1) larger gaps, for instance \( \epsilon_{eg} = 2.9 \text{ eV} = 100 \times \text{Cu}_2\text{OSeO}_3 \) value gives \( 10^{14} \) larger rate, (2) increase of \( 10^3 \) larger \( P \) to \( 200 \mu \text{C/m}^2 \) gives \( 10^6 \) larger rate, (3) decrease of spin density \( N \) by 10 gives \( 10^2 \). There exist multi-ferroics without skyrmion structure satisfying requirements, (2) and (3). Taken altogether, (1) \( \sim \) (3) increase the rate by \( 10^{22} \), presumably in the feasibility range of order \( 0.5 \text{ sec}^{-1} \).

On theoretical fronts there are a number of works to be done: (1) DFT calculation of excited skyrmions to search for a level spacing \( \epsilon_{eg} = a \times 100 \text{ meV} \) (the best sensitivity to neutrino mass determination expected), (2) detailed study of QED background rejection including effects of topology protection.

In summary, multi-ferroic skyrmions provide the unique opportunity of unraveling the conundrum of neutrinos deeply related to physics beyond the standard theory and fundamental problems of cosmology.

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In the topological RENP scheme here no momentum conservation among three emitted particles, $\gamma, \nu_1, \nu_2$, is not imposed, which gives spectrum shapes given by eq. (7), different from other RENP schemes of [6]. Contributions to RENP rates with the momentum conservation taken into account is found to be much smaller than rates given here.

The reason the Pauli blocking distortion appears in a wider energy region of Fig(1) than the case of [2] is that the phase space restriction without the momentum conservation [11] is much less severe. The Fig(1) type of plot makes it possible to use the relic neutrino effect for identification of neutrino mass thresholds.

Strictly, one should include the stimulated cosmic microwave background (CMB) emission effect by multiplying $1 + f_{\text{cmb}}(\omega)$, $f_{\text{cmb}}(\omega) = 1/(e^{\omega/T_{\text{cmb}}} - 1)$, $T_{\text{cmb}} \sim 3K$ to the rate formula, eq. (7). But this has effects only at $\omega < T_{\text{cmb}}$ where rates are very small. Hence we ignore this factor.

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