Dark energy of the photon space, inflation of the charged black holes and universe evolution
Jae-Kwang Hwang
JJJ Physics Laboratory, Brentwood, TN 37027 USA
e-mail: jkhwang.koh@gmail.com

Abstract;
Space-time evolution of our universe is explained by using the 3-dimensional quantized space model (TQSM) based on the 4-dimensional (4-D) Euclidean space. The energy (E = cΔtΔV), charges and energy density (|q| = ρ = cΔt) and absolute time (ct) are newly defined based on the 4-D Euclidean space. The photon flat space with the constant energy density of ρ = cΔtq is proposed as the dark energy (DE). The dark energy is separated into the v DE and photon DE which create the new photon spaces with the constant energy density of ρ = cΔtq. The v DE is from the v pair production by the CPT symmetry and the photon DE is from the photon space pair production by the T symmetry. The vacuum energy crisis and Hubble’s constant puzzle are explained by the photon space with the v DE and photon DE. The big bang and inflation of the primary black hole is connected to the accelerated space expansion and big collapse of the photon space through the universe evolution. The big bang from the nothing is the pair production of the matter universe with the positive energy and the partner anti-matter universe with the negative energy from the CPT symmetry. Our universe is the matter universe with the negative charges of electric charge (EC), lepton charge (LC) and color charge (CC). This first universe is made of dark matter -, lepton -, and quark - primary black holes with the huge negative charges which cause the Coulomb repulsive forces much bigger than the gravitational forces. The huge Coulomb forces induce the inflation of the primary black holes, that decay to the super-massive black holes and particles.

Key words; Dark energy; Photon flat space; Charged black holes; Big bang and inflation; Universe evolution; Vacuum energy crisis; Hubble’s constant puzzle; 4-D Euclidean space.

1. Introduction

The standard model (SM) has been successfully applied to the particle physics and astrophysics. But still many unsolved physics problems including the inflation, dark matters and dark energy in Fig. 1 need the extension of the standard model with the revolutionary ideas. For example, the origins of the inflation and big bang have been studied by many people [1-16] in the SM model. However, what caused the inflation and big bang, and why the inflation stopped without the gravitational effect remain as the important research topics in the SM model. The space expansion is faster than the light during the inflation epoch. The graviton is not faster than the light. Therefore, once the inflation starts, the matters cannot decelerate the inflation because the matters cannot communicate by the gravitational interactions. The inflation cannot be stopped by the gravitation force between the matters. The SM model assumes that the inflation slowed down and stopped. I think that the origins of the big bang and inflation need the extended standard model. And the recent accelerated space expansion has been explained by the dark energy [17-33]. But the existence and origin of the dark energy have not been discovered in terms of the SM model. Therefore, the present 3-D quantized space model (TQSM) [34] in Fig. 2 is considered as the potential candidate of the extended standard model. The TQSM model in Figs. 2 - 4 makes the
possible answers to several problems of Fig. 1 unsolved by the standard model as shown in the next paragraph.

The huge energy of our universe comes from the creation of the CPT symmetric matter and anti-matter universes at big bang. The singularity does not exist because of the strong Coulomb force due to the huge charges. The inflation and big bang come from the huge Coulomb repulsive force with the huge charges of the primary black holes. The inflation due to the Coulomb force slowed down and stopped when the Coulomb forces due to charges of the black holes was equal to the gravitation force due to the mass of the black holes. This explains that the inflation slowed down and finally stopped. The radiation $H_0(F_\alpha)$ created during the inflation due to the Coulomb force makes the space expansion in the early universe. The space expansion velocity caused from the radiation has been decreased after the inflation stopped. The Hubble’s constant $H_0(F_\alpha)$ of the inflation is given as $H_0(F_\alpha)$ in Figs. 2-4 in the present work. After the inflation stopped, the space expansion velocity $v$ caused by the radiation has been decreasing from $v \gg c$ to $v < c$ because $H_0 = 0$. The gravitation interactions between the matters act when $v < c$. The space expansion due to the radiation was resisted by the gravitation effects and warped spaces of the matters when $v < c$. If the cosmic microwave background (CMB) radiation comes from the early universe with $v > c$, the gravitational wave effects cannot be observed on the CMB data. But If the cosmic microwave background (CMB) radiation comes from the early universe with $v < c$, the gravitational wave effects should be observed on the CMB data.
H_p in Figs. 2-4. The photon spaces between the secondary and super massive black holes were developed to the voids with the very small density of the matters.

The present work focuses only on the dark energy, inflation and universe evolution. The dark energy is the force to drive the recent accelerated space expansion. The recent accelerated space expansion was discovered and confirmed by the data obtained from the weak gravitational lensing [35], the Type Ia supernovae [36], the galax clusters [37], the cosmic microwave background [21] and baryonic acoustic oscillations [38]. The space inflation (early space expansion) of the universe after big bang is like the recent accelerated space expansion. But it is thought that the origin of the inflation is different from the origin of the recent accelerated space expansion in Figs. 2-4. The inflation was caused by the huge Coulomb repulsive force of the primary and secondary black holes with the huge charges after the big bang in terms of the 3-D quantized space model [34] in Figs. 2 and 3. The recent accelerated space expansion is caused not by the big bang energy of the huge Coulomb force, but by the photon flat space (called as the dark energy) with the neutron dark energy and photon dark energy in the present work in Fig. 4. Also, the dark energy is considered as the cosmological constant (Λ) in the general relativity and ΛCDM model based on the Minkowski space-time. In the universe evolution, the densities of the matters, radiation and dark energy related to the Hubble’s constant are analyzed. The dark energy density remains constant, but the matter density and radiation density are decreased with increasing of the universe size. The radiation is created as the result of the inflation. The matters are created from the decays of the black holes during the inflation. The space during the decelerated space expansion epoch was affected by the radiation effect (H_\text{r}(F_\text{r})) and the resistance effect (H_\text{f}) including the gravitation force (F_\text{g}) between the matters and the resisting force (F_\text{w}) of the warped matter space in Figs. 2-4.
In section 2, the dark energy and photon flat space are discussed. In section 3, the space-time geometry in 3-D quantized space model is explained in terms of the TQSM model. And in section 4 the inflation and charged black holes and in section 5 university evolution and big bang are discussed in terms of TQSM.

2. Dark energy and photon flat space

The 3-D photon flat space with the constant time width of cΔt_q is the 3-D Euclidean space in terms of the 3-D quantized space model (TQSM). Our universe is the photon flat space with the matters and particles in Fig. 2 and 3. The matters including the black holes, dark matters and elementary fermions are the warped photon spaces toward the positive time direction (positive charges) and toward the negative time direction (negative charges). The local radiation is the local photon flat space with the enhanced time width of cΔt > cΔt_q in Figs. 3 and 4. The matters and radiation are moving over the 3-D photon flat space. The electromagnetic force, gravitation force, dark matter force, weak force, and strong force act among the matters through the force carrying bosons. The huge Coulomb forces of the primary black holes and secondary black holes with the huge charges induce the inflation after the big bang of the matter universe with the positive energy in Figs. 2 and 3 (see section 4 for the inflation). The inflation makes a large effect (H_i in Hubble’s constant) on the early space expansion. And the radiation created during the inflation epoch induces the space expansion after inflation. The early space expansion (H_{ra} in Hubble’s constant) due to the radiation is decelerated by the resistance effect (H_i in Hubble’s constant) of the gravitational forces and warped spaces of the matters against the space expansion. The observed accelerated photon space expansion at the present time requires the new forces called as the dark energy (H_{DE} in Hubble’s constant). This dark energy has the constant energy density unlike the matters and the radiation. This means that the dark energy has the fundamental property the same as the photon space itself. This condition needs the creation of the new photon space. The new photon spaces can be created by the quantum space fluctuations with the energy conservation as shown in Fig. 4.
Three kinds of space quantum fluctuations are the matter CP pair production, neutrino CPT pair production and photon T pair production in Fig. 4. The new photon spaces are created by the neutrino CPT pair production ($H_\nu$ in Hubble’s constant) and photon T pair production ($H_\rho$ in Hubble’s constant) in Fig. 4. Here the time conversion symmetry (T) in the TQSM model is different from the time conversion symmetry (T) in the SM model. The time conversion symmetry (T) is based on the absolute time ($t$ in the present work) for the TQSM model and on the relative time ($t_l$ in the present work) for the SM model in Figs. 5 - 7. This is the big difference of the T definition between the TQSM and SM models. The C, P and T symmetries are shown in Fig. 7.

Three quantum space fluctuations are the matter CP pair production, neutrino CPT pair production and photon T pair production in Fig. 4. The new photon spaces are created by the neutrino CPT pair production ($H_\nu$ in Hubble’s constant) and photon T pair production ($H_\rho$ in Hubble’s constant) in Fig. 4. Here the time conversion symmetry (T) in the TQSM model is different from the time conversion symmetry (T) in the SM model. The time conversion symmetry (T) is based on the absolute time ($t$ in the present work) for the TQSM model and on the relative time ($t_l$ in the present work) for the SM model in Figs. 5 - 7. This is the big difference of the T definition between the TQSM and SM models. The C, P and T symmetries are shown in Fig. 7.
The space expansion is described as the well-known Hubble’s equation of \( v = Hx \) since the inflation. The space expansion \( (H) \) is influenced by the matters \( (H_r) \) (gravitation force \( (F_g) \)), warped-space resistance force \( (F_w) \), inflation \( (H_i) \) (Coulomb force \( (F_c) \)), radiation force \( H_{ra}(F_{ra}) \) and photon space fluctuations (dark energy) \( H_{ps}(v \ DE(H_v)) \) and photon DE \( (H_p) \) in Figs. 2-4. In other words, \( H = H_i + H_{ra} - H_r + H_{ps} \).

Fig. 6. The big bang process is compared for the space-time evolution in TQSM and space-time creation in SM. The x1x2x3 space coordinates and x0y0z0 space coordinates can be always overlapped with the corresponding time gap of \( c \Delta t \).

Fig. 7. The P, C, and T symmetries are compared.
In Fig. 8, the dark energy is the photon flat space which includes the neutrino and photon dark energies in Figs. 2-4. In other words, the new photon spaces are created by the photon space pair production by the T symmetry and by the neutron pair production by the CPT symmetry. The T symmetry makes the photon space pair of the positive energy (time momentum) and negative energy (time momentum). The CPT symmetry makes the neutrino pair of the neutrino with the positive energy and negative LC charge and anti-neutrino with the negative energy and positive LC charge. For the accelerated space expansion, the neutrino dark energy (DE) and photon dark energy play the major role. The Hubble’s constant (H) is \( H \approx H_\text{r} + H_\text{s} > H_\text{r} \) at the present time in Fig. 8.

\[
\rho_\text{DE} = c \Delta t q = \text{constant}
\]

For the accelerated space expansion (H)

\[
v = H(t)x, \quad dv/dt > 0, \quad dH/dt < 0
\]

\[-1 < \frac{dH}{H^2 dt} < 0, \quad dH_{ps}/dt < 0, \quad H < H_{ps}
\]

\[\rho_{DE} = c \Delta t q = \text{constant}
\]

At \( H_{ps} \gg H_r \), \( H \rightarrow H_{ps} \) and \( v \rightarrow H_{ps} x \)

\[H = H_{ps} \text{ can be used for the present large cosmic scale because } F_g = 0 \text{ and } F_r 0.
\]

The space expansion velocity: \( v < \infty \)

Fig. 8. The Hubble’s constant and the accelerated space expansion.

In Fig. 8, the dark energy is the photon flat space which includes the neutrino and photon dark energies in Figs. 2-4. In other words, the new photon spaces are created by the photon space pair production by the T symmetry and by the neutron pair production by the CPT symmetry. The T symmetry makes the photon space pair of the positive energy (time momentum) and negative energy (time momentum). The CPT symmetry makes the neutrino pair of the neutrino with the positive energy and negative LC charge and anti-neutrino with the negative energy and positive LC charge. For the accelerated space expansion, the neutrino dark energy (DE) and photon dark energy play the major role. The Hubble’s constant (H) is \( H \approx H_\text{r} + H_\text{s} > H_\text{r} \) at the present time in Fig. 8.
The vacuum energy density has been calculated in quantum field theory in Fig. 9. And the vacuum energy density calculated with the maximum quantum vacuum oscillation energy of the Planck energy \( (E_P = M_P c^2) \) is much bigger than the observed vacuum energy density. This indicates that the Planck energy cannot be the part of the vacuum oscillation energy at the present time. The maximum vacuum oscillation energy at the present time can be calculated from the observed vacuum density in Figs. 9 and 10. This vacuum oscillation energy is \( 3.5 \times 10^{-3} \text{ eV} \) with which only the neutrino vacuum oscillations are possible. The vacuum state is the photon flat space in the present work. The vacuum state makes the neutrino and photon fluctuations in Fig. 4. Therefore, the observed vacuum energy density supports the neutrino dark energy and photon dark energy as the quantum vacuum fluctuations in Fig. 4.

The dark energy consists of the neutrino and photon dark energies. The Hubble’s constant \( (H) \) of the photon flat space with the accelerated space expansion is, in general, \( H = H_v + H_p - H_r + H_m \). In Fig. 11, the experimental Hubble’s constant values [21, 36, 39-41] are compared. The Hubble’s constant data of \( H_{\text{CMB}} \) from the CMB radiation [21] includes all the matters with the neutrinos that came from the neutrino CTP pair production. The neutrinos that came from the neutrino CTP pair production play the role as the neutrino dark energy, too. Therefore, the neutrino dark energy density should be subtracted from the matter density in Fig. 11. This indicates that the Hubble’s constant \( (H_v) \) due to the neutrino dark energy should be added to the \( H_{\text{CMB}} \). In other word, \( H_{\text{CMB}} = H_p - H_r + H_m \) for the CMB data. And \( H = H_{\text{CMB}} + H_v = H_v + H_p - H_r + H_m \) for the Hubble’s constant data \( (H) \) [36, 39-41] as explained in Fig. 11. The present neutrino dark energy...
gives the Hubble’s constant of \( H_v = 2.1 - 5.5 \text{ km/sec/Mpc} \). The Hubble’s constant puzzle is explained by using the neutrino dark energy and photon dark energy in Figs. 4 and 11.

| From the Lambda CDM model, 
\[ H(a) = H_0 \left( \Omega_m a^{-3} + \Omega_{\text{rad}} a^{-2} + \Omega_{\text{DE}} \right)^{0.5} \]  
\[ a(t) = \left( \frac{\Omega_m}{\Omega_{\text{DE}}} \right)^{1/2} \sinh^{1/2} \left( \frac{t}{t_\Lambda} \right) \]  
\[ \Omega_{\text{DE}} = \Omega_\Lambda, \quad t_\Lambda = 2 / (3H_0\Omega_\Lambda^{0.5}) \]  
Without the ν DE effect,  
\[ H(\text{CMB}) = H_\nu - H_\nu + H_\nu = 67.7(4) \text{ km/sec/Mpc} \]  
from final Planck CMBR 2018 results.

With the ν DE effect,  
\[ \Omega_m a^{-3} \rightarrow (\Omega_m - \Omega_\nu) a^{-3} \]
Then, \( H(\text{CMB}) \) is increased by \( H_\nu \),  
\[ H(\text{CMB}) = H_0 - H_\nu + H_\nu \rightarrow H = H_0 + H_\nu - H_\nu + H_\nu \]  
\[ H_\nu = 2.1 - 5.5 \text{ km/sec/Mpc} \]  
from the observed Hubble’s constant data.

The present Hubble constant \( H(\text{DE}) \) is predicted from the cosmic microwave background radiation (CMBR) which was left over after the big bang more than \( 13 \times 10^9 \) years ago:  
\[ v = (H_\nu - H_\nu + H_\nu)x. \]
\[ H_\nu - H_\nu = 67.7(4) \text{ km/sec/Mpc} \]  
from Final Planck CMBR 2018 results.

The present Hubble constant \( H = H_\nu + H_\nu + H_\nu \) :  
\[ 72.1(20) \text{ km/sec/Mpc} \]  
from the red giant stars.  
\[ 69.8(19) \text{ km/sec/Mpc} \]  
from the red giant stars.  
\[ 73.2(13) \text{ km/sec/Mpc} \]  
from the Milky Way Cepheids.  
\[ 73.2(17) \text{ km/sec/Mpc} \]  
from the Type 1a supernova.  
\[ H_\nu > H_\nu \quad v = Hx = (H_\nu - H_\nu + H_\nu)x = (H_\nu + H_\nu - H_\nu + H_\nu)x. \]
The neutrino Hubble constant \( H_\nu \) caused by the neutrino CPT pair production is \( H_\nu = 2.1 - 5.5 \text{ km/sec/Mpc} \)  
from the above data. The Hubble constant puzzle is explained by the additional neutrino Hubble constant \( H_\nu \).

The photon flat space energy (ν and photon DEs) is the dark Energy (DE).

Fig. 11. The Hubble’s constant due to the ν dark energy is obtained as the answer to the Hubble’s constant puzzle.

3. Space-time geometry in 3-D quantized space model

The space-time geometry research is closely related to the special and general relativities, quantum mechanics, manifold mathematical physics, cosmology, and standard model. The present model is based on the Euclidean space-time, but not on the Minkowski space-time in Figs. 2, 5, 6, 7 and 10.

The time axis that is defined on the Minkowski space-time is relatively changing with the particle velocity in the Lorentz transformations of the relativity theory. But the time axis (called as the absolute time of \( ct \)) that is defined on the Euclidean space-time does not change with the particle velocity in Fig. 5. Only the space-time distance (called as the relative time of \( ct \)) is changing with the particle as shown in Fig. 5. This relative time (\( ct \)) corresponds to the time (t) of the Minkowski space-time even though the concept is very different. The modified Lorentz transformations are introduced in terms of the Euclidean space-time in Ref. 38. Therefore, there are two real times of the absolute time and relative time based on the Euclidean space-time. In the special and general relativity, there is only one real time of the relative time (t in the present work). All physical concepts in the present work are discussed based on the Euclidean space-time. In Fig. 5, the massive particle and photon are defined as the warped space and flat space on the 4-D Euclidean space-time. When the particle does not move, only the absolute time is ticking on. Then the observed relative time is the same as the absolute time. And if the particle moves with the velocity of \( v = \Delta x/\Delta t \), the observed relative time of \( c\Delta t \) is different from the absolute time of \( c\Delta t \). The relative time axis is varying following the particle velocity. Therefore, the relative time can be related with the velocity and moving space distance of the particle in the modified Lorentz-transformations [42]. The absolute time axis is fixed always as the fourth dimensional axis in the
4-D Euclidean space-time. The photon is the flat space in Fig. 5. The photon moves with the light velocity of c. The whole photon space (or whole 3-D quantized space) in Fig. 5 is moving with the light velocity of c = Δx/Δt along the x axis. The internal photons inside the flat space move with the light velocity of c = Δx/Δt within the flat space. In Fig. 5, the rest particle, moving particle and photon are compared. The photon has the zero rest mass which means the flat space. Therefore, even though the internal photon is moving along the x space direction with the constant speed of c, the photon space is not changed. It is expressed as A=B in Fig. 5. In other words, the electromagnetic wave does not change the space. But the moving massive particle changes the space as shown in Fig. 5. It indicates that the electromagnetic wave is the space fluctuations which does not change the photon space itself. Also, the photon moves along the time axis of ct like the rest particle does. Therefore, ct is equal to ct of the photon in Fig. 5. This means that the photon has both properties of the particle (photon) and wave (electromagnetic wave). This explains the particle-wave duality of the photon.

The present work is entirely based on the 4 dimension Euclidean space but not on the 4 dimension Minkowski space. Fourth dimension axis is the absolute time axis of ct. The ideas proposed by the present model are graphically explained for the readers who want to understand the basic physical concepts. In the present model, the photon is the flat space with the zero rest mass. The particles and matters with non-zero rest masses (m) including the gravitons are considered as the warped spaces. Here, the graviton has the non-zero rest mass. This tells that all the particles including the gravitons with the non-zero rest masses are the warped spaces which are created from the flat photon space. Only the photon has the zero rest mass which indicates the flat space. The rest mass energy of E = mc² is defined as the four-dimension space volume of cΔtΔx1Δx2Δx3 = cΔtΔV.

Because the 4-D volume is the only factor to define the different particles, it is reasonable to say that the 4-D volume is the rest mass energy of the corresponding particle. This four-dimension space volume of cΔtΔx1Δx2Δx3 is connected to the wave function of the quantum mechanics. This could be the origin of the quantum mechanics. The wave function is formed along the absolute time axis of ct but not along the relative time axis of ct. It will be interesting to research the relation
between the four-dimension space volume of $\Delta(ct)\Delta x_1\Delta x_2\Delta x_3$ and the particle energy in quantum mechanics. The relation between the four-dimension space volume and wave function in the quantum mechanics is discussed in another paper.

The definition of $E = c\Delta x_1\Delta x_2\Delta x_3 = c\Delta x_1\Delta x_2\Delta x_3$ is very important in the physical point of view. The particle is the warped space, and the particle energy is the warped space volume on the 4-D Euclidean space. The time momentum is $p_t = mc = E/c = \Delta tx_1\Delta x_2\Delta x_3$. From the definition of the energy, we just can observe the space volume because we live on the 3-D space. The energy and mass cannot be directly observed because it is the 4-D Euclidean space volume. We observe the energy and mass as the effect projected on the 3-D space when the particle is moving. The inertia mass of $m = F/a$ is one example. The black holes have the huge energy (or mass) and very small space volume in Figs. 10, and 12 - 14. This indicates that the $c\Delta t$ is huge for the black holes. The singularity is expected for the black holes because of the huge gravitational force in terms of the general relativity. Therefore, the black holes are very stable and could be evaporated through the Hawking radiation from the quantum vacuum fluctuations [43 – 49].

Now I propose the radical charge assumption of $|q| = c\Delta t$. In this definition, the negative charge is the space-time shape warped toward the negative time direction and the positive charge is the space-time shape warped toward the positive time axis. Under this charge definition, the black holes have the huge charge and very small space volume in Figs. 12 - 14. This means that the black holes (BH) have the stable space-time warped shape under the condition of $F_c = F_g + F_B$. Here, $F_c$, $F_g$ and $F_B$ are the Coulomb force, gravitational force, and boson force, respectively. If the extremely large black holes are created at the big bang, these primary black holes have the huge Coulomb force to meet the condition of $F_c >> F_g + F_B$. Because of the huge Coulomb force with
the huge BH surface fluctuations, these primary black holes experience the rapid space expansion and rapid time (charge) contraction that are called as the inflation. Finally, these primary black holes are changed to the stable black holes with the condition of $F_c = F_g + F_B$. The black holes are not the singularities.

4. Inflation and charged black holes

The energy, and charges (EC, LC and CC) are defined in the TQSM. The energy (E) of the particle is $E=c\Delta t\Delta V$. And the charge is defined as the $c\Delta t$ value which is positive along the positive ct time axis and negative along the negative ct time axis. The elementary fermions have the fixed spin of $s=1/2$, constant charges, constant space volume and the constant energy (or constant rest mass). However, the black holes without the internal structures have the varying physical quantities of the spins, charges, and space volumes except the total energy. The three kinds of black holes exist as the dark matter black hole with the EC charge, lepton black hole with the EC and LC charges and quark black hole with the EC, LC and CC charges. These charged black holes are the totally merged space-time states without the internal structures. These black holes have the charges and masses. The dark matter black hole, lepton black hole and quark black hole are expressed as the charge configurations of (EC), (EC,LC) and (EC,LC,CC), respectively. The black holes with the very small space volume are hugely warped along the time axis of ct. The huge time size ($c\Delta t$) means the huge charge of $q = c\Delta t$ in Fig. 10 and 12 - 14. These charged black holes have the surface fluctuations from the competition of the attractive gravitational force ($F_g$), attractive boson force ($F_B$) and repulsive Coulomb force ($F_c$). The short range (gauge) boson force ($F_B$) should be generally added as the attractive force for the black holes because many black holes have the very small space sizes. The first black hole created at the big bang is developed from the primary black hole to our present matter universe. This primary black hole should experience the space expansion (inflation) and time contraction because of the energy conservation equation of $E = c\Delta t\Delta V$. The black hole and particle with the non-zero rest masses are defined as the warped space-time. The 4-
dimensional space-time volume of \( E = c \Delta t \Delta V \) is defined as the energy of the black hole and particle. From this definition, the inflation means the huge space expansion (inflation) and huge time contraction from the energy conservation equation of \( E = c \Delta t \Delta V \).

The black hole is the particle with the very small space volume and very large mass. The typical definition of this black hole leads to the very large time of \( c \Delta t \) because of the very large energy (E) and very small space volume (\( \Delta V \)) in the equation of \( E = c \Delta t \Delta V \). In the general relativity, it is thought that the black hole does not experience the surface fluctuations and is very stable attracting everything including the photons. Eventually, the black hole leads to the singularity at its center. The black hole does not decay and emits only Hawking radiation from the quantum vacuum fluctuation. Therefore, we need the very radical and new physical concept in order to explain the inflation of the primary black hole after the big bang. The answer to this question can come out from the possibility that the black hole can be hugely charged as proposed in the present 3-dimensional quantized space model (TQSM) as shown in Figs. 12 - 14. The present TQSM model is the extended standard model. If the black hole has the huge charge that can give the huge repulsive Coulomb force large enough to overcome the attractive gravitational force and boson force, the inflation of the primary black hole created at the big bang can be explained. Now the charge is defined as \( |q| = c \Delta t \). This charge definition is used for the black hole evolution including the inflation in the present work. The inflation and decay of the primary black hole continue until the Coulomb force is similar to the sum of the gravitational force and bosons force for the black hole as shown in Figs. 12 - 14. The Coulomb forces for the leptons and quarks are much stronger than the gravitational forces. This means that the lepton black holes and quark black holes can be destroyed because of the strong Coulomb forces greater than the sum of the long-range gravitational force and short-range boson force. In the inflation process of the primary black hole, after the moment at which the space sizes of the secondary black holes become bigger than the boson force range of about \( < 10^{-18} \) m, the black holes are rapidly destroyed before the crossing point of \( F_c \approx F_g + F_B \), because the Coulomb forces are much stronger than the gravitational forces for the leptons and quarks and the Coulomb forces are still much stronger than the gravitational forces for the charged black holes. Therefore, the charged particles like electrons and protons are emitted as the aftermath of the inflation of the lepton black holes and quark black holes in Fig. 14. These electrons and protons are distributed as the normal matters in the galaxies. These normal matters are locally distributed by the gravitational force between the normal matters and dark matters (including the dark matter black holes). Now dark matter primary black hole experiences the inflation after the big bang like the lepton primary black hole and quark primary black hole. The Coulomb force between the dark matters is very weak to be neglected in the most cases. But the very weak Coulomb force of the charged dark matters play the major role for the inflation of the dark matter primary black hole and secondary black holes with the huge EC charges. The Coulomb force of the dark matter black holes with the huge EC charges is bigger than the sum of the gravitational force and short-range boson force of the dark matter black holes. In other words, \( F_c > F_g + F_B \). In the inflation process, after the moment at which the space sizes of the secondary black holes become bigger than the boson force range of about \( < 10^{-18} \) m, the black holes are slowly destroyed because the Coulomb forces are much weaker than the gravitational forces for the charged elementary dark matters and the Coulomb forces are still stronger than the gravitational forces for the charged black holes. The dark matters are emitted from the host black holes. The emitted dark matters are locally distributed around the left-over dark matter black holes. At the crossing point of \( F_c \approx F_g + F_B \), the dark matter black hole stops to decay. The dark matter black hole
with the large EC charge enough to meet the condition of $F_c \approx F_g + F_B$ remains as the super-massive black holes at the center of the galaxies in Fig. 13. Therefore, the emitted normal matters, emitted dark matters and left-over dark matter black hole form the galaxies.

In the inflation process, the black hole is decayed to the smaller black holes. These small black holes decay to the galaxy black holes (galaxy particles) which are the seed black holes of the galaxies. In order for the black hole to decay, the Coulomb force ($F_c$) should be larger than the sum of the gravitational force ($F_g$) and the short-range boson force ($F_B$). The first black hole followed by the big bang should be electrically charged with the huge charges of EC, LC and CC which have the very small space volume ($\Delta V$) and very big absolute time ($c\Delta t$) (huge charges). In order for the big bang and inflation to take place, the first condition is that the Coulomb force, $F_c$ is much bigger than the sum ($F_A$) of the gravitational force ($F_g$) and bosons force ($F_B$). Because $F_c \gg F_A$, the big bang and inflation mean the sudden increasing of the space volume and the sudden decreasing of the absolute time ($c\Delta t$) (charges). After these processes, a lot of smaller super-massive black holes are created. These super-massive black holes decays slowly to form the galaxy clusters and each galaxy. The black holes are not the singularities because of the huge repulsive Coulomb forces due to the huge charges of the black holes in Figs. 12 - 14.

![4-dimensional Euclidean space](image)

E = $mc^2$, $E_0 = m_0c^2$, $t_1 = \gamma t_2$, $t_1$: absolute time, $t_2$: relative time

$p_t = m_0c \frac{x_4}{ct} = \frac{E_0}{c} = \frac{E}{ct} = \frac{m_0c^2}{ct} = \frac{x_4}{ct}$

$p_x = m_0 \frac{c}{ct} \frac{x}{c^2} = \frac{E}{ct} = \frac{m_0c^2}{ct} = \frac{E}{c}

p_d = m_0 \frac{c}{ct} \frac{d}{c^2} = \frac{E}{ct} = \frac{m_0c^2}{ct} = \frac{E}{c}$

Note that the negative time, negative mass (energy) and negative momenta are possible. $E_0 < 0$ and $m_0 < 0$ for $p_t < 0$.

Fig. 15. 4-dimensional momenta are defined for the absolute time ($t$) along the $x_4$ axis. Here, the big bang is explained as the universe pair production (see Fig. 6). The $x_1x_2x_3$ space coordinates can be always replaced with the $x_0y0z0$ space coordinates if needed.

The big bang process is the sudden space-time creation from the nothing in the standard model as shown in Figs. 2, 6 and 15. There have been many efforts to explain this sudden 4-D space-time creation. The sudden 4-D space and time creation does not look like the natural process in terms of the CPT symmetry. It is because the partner universe is not proposed in the well-known big bang theory. In other words, the TCP symmetry is broken in the well-known big bang standard model. The corresponding space-time is based on the 4-D Minkowski space where the observed time of $t_1$ is changing with changing of the particle velocity. In this well-known big bang theory,
the observed time and energy (mass) are positive in Figs. 2, 6 and 15. This means that the time momentum of \( p_t = E/c \) is positive along the time axis of \( ct \), and our universe is moving toward the positive (forward) time direction since the birth of the universe. This explains the fact that our universe cannot be moving to the backward time direction and the observed mass (energy) are positive. At least this is true in our universe where we live. The standard model including the big bang process has been successfully established to explain many properties of the elementary particles, for a long time.

In the present work, our space-time matter universe is explained by the extension from the 4-D \( x_0y_0z_0x_4 \) Euclidean space to the 3-D \( x_1x_2x_3 \) Euclidean space on the 4-D \( x_0y_0z_0x_4 \) Euclidean space in Figs. 6 and 14. This \( x_1x_2x_3 \) space is called as the 3-D quantized space with the quantized time width of \( c\Delta t_q \) (3-D QS) or the 4-D quantized space and time (4-D QST) in Figs. 6 and 14. Then the time axis of \( ct \) is the fourth dimension \( x_4 \) axis. The quantized time width of \( c\Delta t_q \) could
be the Planck size scale or smaller quantum size. Our matter universe corresponds to the x1x2x3 quantized space in Figs. 6 and 14 because only the three space dimensions and one time dimension are observed in the real macroscopic world. Also, two E and M space fluctuations in the EM wave indicates that we live in the 3-D quantized x1x2x3 space. This is called as the space-time evolution which looks like the natural process in terms of the CPT symmetry because our matter x1x2x3 universe and the partner anti-matter x1x2x3 universe are created at the big bang by the CPT symmetry. In the president 3-D quantized space model, the universe can move toward the positive time direction or negative time direction. It is because the time momentum of \( p_t = E_0/c \) in Figs. 6 and 14, the energy of the universe with the positive time momentum is positive and the energy of the partner universe with the negative time momentum is negative. Of course, the zero time momentum of \( p_t = E_0/c = 0 \) means the nothing with the zero energy. Therefore, the universe with the positive time momentum and the partner universe with the negative time momentum are created from the nothing with the zero time momentum of \( p_t = E_0/c = 0 \) as shown in Figs. 6 and 14. The created universe is called as the x1x2x3 space with the very small time width of \( c\Delta t_q \) which could be the Planck time scale or smaller quantum size. The present big bang theory in terms of the 3-D quantized space model (TQSM) is called as the space-time evolution theory while the well-known big bang theory in terms of the standard model (SM) is called as the space-time creation theory in Fig. 2, 6, and 15 - 20.

The 3-dimensional quantized flat space is the photon space. The particle is the warped space with the velocity (v) slower than the photon velocity of c. Under this definition of the particle mass, it is proposed that the electric charge is positive for the positive warped space and negative charge for the negative warped space in the x1x2x3 space. Based on these concepts, the 3-dimensional quantized space model is developed for the elementary fermions and bosons [34]. It is surprising that several new particles including three fermionic dark matters exist in terms of the 3-dimensional quantized space model [34]. The fermion with the intrinsic spin of 1/2 is the open warped photon space along the positive or negative direction of the time axis. The boson with the intrinsic spin of 1 is the closed warped photon space along both positive and negative directions of the time axis. And the graviton with the intrinsic spin of 2 is the closed warped photon space along both positive and negative directions of the time axis and the photon with the intrinsic spin of 1 is just the flat space.

5. University evolution and big bang in terms of TQSM

The big bang theory has been developed to show the beginning moment of our matter universe. The 4-D space and time were created at one moment (big bang) in the past as shown in Figs. 2, 6 and 15 - 20. This space and time creation in the standard model means there was nothing before the big bang in Fig. 6. Even the time did not exist before the big bang moment in terms of the standard model. It seems to me that the god created everything of the space and time at the big bang moment. The CPT symmetry does not exist because there is nothing before the big bang. Because our universe is the matter dominated universe, the CP symmetry problem is unsolved in SM. Also, the time (T) symmetry before and after the big bang cannot be applied to our universe. I think that these CP and T symmetry problems are the serious problems. For the explanation of the geometrical structure of our universe, the 4-D Minkowski space was introduced in the special and general relative theories. Then the introduced time should be varied depending on the relative velocity of two space-time frames. This is called as the relative time of t which corresponds to the
relative time of \( t_1 \) in the present 3-D quantized space model (TQSM). We know in the real life that the massive particles and matters are made of the 3-D space volume at the instant time of \( t \). It means that the instant time of \( t \) has the very small-time gap of \( c\Delta t_0 \) when we observe the 3-D space volume at the time of \( t \). This is the 3-D quantized \( x1x2x3 \) space in Fig. 6. The well-known big bang theory assumes that this 3-D quantized space is flowing along the relative time axis of \( c t_1 \). In this case the time and space are warped together. There is nothing else except our universe of 3-D quantized space. The well-known big bang theory, special and general relativity theories and standard model are developed based on this single 3-D quantized space on the 4-D Minkowski space.

Therefore, I have thought of the origin of the big bang based on the 4-D Euclidean space rather than the 4-D Minkowski space. Let us think of the 3-D Euclidean quantized space which is overlapped on the 4-D Euclidean quantized space in Fig. 6. Then the 3-D quantized \( x1x2x3 \) space is moving along the 4\(^{th}\) dimension axis in the 4-D Euclidean space. The 4\(^{th}\) dimension axis is the absolute time axis of \( c t \). This 3-D quantized space is assigned as the \( x1x2x3 \) space and the 4-D Euclidean space is assigned as the \( x0x00x0ct \) space and time. It is assumed that the 3-D quantized \( x1x2x3 \) space is moving along the \( x4 \) axis with the constant speed of \( c \) because the 3-D quantized \( x1x2x3 \) space is the photon space. Then, from \( c = \Delta x4/\Delta t \), \( x4 = ct \), the \( x4 \) axis is the absolute time axis of \( c t \). Under this assumption, the mathematical 4-D Euclidean space is evolved to the physical 3-D quantized space based on the 4-D Euclidean space as shown in Fig. 6. From this physical concept, the 4-D momenta in Fig. 6 is defined for the \( x1x2x3 \) space. Then, note that, in Fig. 15, the 4-D momenta can be expressed as \( p_x = m_0c^{x4/ct} \), \( p_\chi = m_0c^{x/ct} \) and \( p_\text{tl} = m_0c^{d/ct} \). The 4\(^{th}\) dimension axis is the absolute time axis of \( c t \). Note that the negative time, negative mass (energy) and negative momenta are possible. \( E_0 < 0 \) and \( m_0 < 0 \) for \( p_t < 0 \) in Fig. 15. And the C, P and T symmetry are defined for our universe based on the 4-D Euclidean space. The T symmetry is defined from the absolute time axis of \( c t \) not from the relative time of \( c t_1 \). The negative time momentum means negative mass and negative energy. Our matter universe with the positive time momentum and anti-matter partner universe with the negative time momentum should be created following the CPT symmetry as shown in Figs. 6 and 15. Therefore, there is only the 4-D Euclidean space before the big bang and there is the 3-D quantized space based on the 4-D Euclidean space after the big bang. Because our universe has the positive time momentum and positive energy (mass) and the partner universe has the negative time momentum and negative energy (mass), these two universes are created from the nothing with the zero time momentum and zero energy (mass). Therefore, in Figs. 6 and 15, the CPT symmetry explains why our universe is the matter universe and where the energy of our universe comes from. Now we have the complete birth history of our universe in terms of the present 3-D quantized space model (TQSM).

Note that the time, \( t \) in the Lorentz transformations corresponds to \( t_1 \) in the present work. In other words, the time axis of \( c t \) in the present work is the fourth dimension axis in the 4-dimensional Euclidean space in Figs. 6, 15 and 16. And the time axis of \( c t \) in the Lorentz transformations of the Minkowski space is the distance axis of \( c t_1 \) in the present 4-dimensional Euclidean space. Therefore, the direction of the \( c t_1 \) time axis depends on the corresponding particle velocity of \( v = x/t_1 \). This means that the observed time of \( t_1 \) depends on the corresponding velocity of \( v \) as shown in the Lorentz transformations of the special relativity. Therefore, the time of \( t_1 \) is called as the relative time in the present work. The relation between the relative time of \( t_1 \) and the observed space distance of \( x \) is described as a function of \( v \) in the Lorentz transformations of the special relativity.
But the time axis direction of ct does not depend on the particle velocity. It is just the fourth dimension axis in the 4-dimensional Euclidean space. Therefore, the time of t is called as the absolute time in the present work. It is clear from the definition of the time momentum \((p_t = E_0/c = m_0c)\). Then from \(c = \Delta x/\Delta t\), \(x_4 = ct\), the x4 axis is the absolute time axis of ct. The photon with the constant speed \((c)\) is the flat space with the zero charge. It is defined that our x1x2x3 universe is the photon space which moves along the positive x4 axis with the constant speed of c in the 4-dimensional Euclidean space of x0y0z0x4. In this definition, the x4 axis becomes the absolute time axis of ct in the 4-dimensional Euclidean space of x0y0z0x4 in Figs. 6, 15 and 16. Then, this photon space has the microscopic time width of \(c\Delta t_q\) which could be the Planck length size. But the time width of the local photon space can be changed depending on the photon energy. This photon space is called as the x1x2x3 space in Figs. 6, 15 and 16.

The photon consists of many internal photons. These internal photons form the photon wave of the electromagnetic wave by the interferences. And the x4 axis is the ct time axis. The photon has the constant velocity of \(c = \Delta x/\Delta t\) along the x4 axis. And the internal photons have the constant velocity of \(c = \Delta x/\Delta t\) along the x axis on the x1x2x3 space. So, the moving distance \((\Delta x = c\Delta t)\) of the internal photon on the x1x2x3 space is the same to the moving distance \((\Delta x_4 = c\Delta t)\) of the whole photon space along the ct axis. This means that the whole photon space can be treated as the rest particle with the velocity of c along the ct axis. This axis is the absolute time axis. Also, each internal photon can be considered as the moving particle on the x1x2x3 space with the velocity of c. This internal photon is the electromagnetic wave which is closely related to the electric and magnetic fields. It is thought that each photon and electromagnetic wave can consist of several internal photons with the less energies. Therefore, the x1x2x3 space corresponding to each photon can be always defined based on the 4-dimensional Euclidean space as shown for our universe. Here our universe is the warped space of the x1x2x3 photon (flat) space.

![Three-dimensional quantized space model (TQSM) (4-D Euclidean space)](image)

**Fig. 17.** The universe evolution and black holes are explained.

Therefore, our universe is the x1x2x3 photon space positioned on the mother x0y0z0x4 space. This is the 3-dimensional quantized x1x2x3 space. Everything in the present work is based on the 4-dimension Euclidean space. Then, our universe is moving with the photon velocity of c along the x4 axis which is called as the time axis of ct. Please note that the absolute time lapse is defined as the \(\Delta t = \Delta x_4/c\). This is the absolute time which is not dependent on the particle velocity. The relative time of \(t_i\) is dependent on the particle velocity as shown in the special relativity theory.
The time of $t$ in the special relativity theory corresponds to the relative time of $t_l$ in the present work. The relative time of $t_l$ is defined as $\Delta t_l = \Delta t/c = \Delta x/v$. Our universe can be locally warped to create the particles in Fig. 16. Then the rest mass energy and electric charge (EC) are introduced as the warped space of our $x1x2x3$ universe. The 3-dimensional quantized space is the photon space. The particle is the warped space with the velocity ($v$) slower than the photon velocity of $c$. Then the rest mass of the particle is defined as the 4-dimensional volume of the warped space.

Under this definition of the particle mass, it is proposed that the electric charge is positive for the positively warped space and negative for the negatively warped space in the $x1x2x3$ space. Because other lepton charges (LC) and color charges (CC) exist in the real world, we need two more 3-D quantized spaces of the $x4x5x6$ and $x7x8x9$ spaces [34]. The $x4x5x6$ and $x7x8x9$ spaces are for the LC and CC charges, respectively. The present work including only the EC $x1x2x3$ space for simplicity can be easily extended to the research including all three spaces of the $x1x2x3$ space.
LC x4x5x6 space and CC x7x8x9 space. In the extended case, the charge \( q \) of \( (EC) \) can be, easily, replaced with the charges \( (q) \) of \( (EC,LC) \) and \( (EC, LC, CC) \).

In Fig. 17-20, the universe evolution is shown in terms of the TQSM model. The big bang is the pair creation of the CPT symmetric matter and antimatter universes from nothing. The photon spaces between the secondary and super massive black holes were developed to the voids with the very small density of the matters. It is concluded that the singularity does not exist. The inflation epoch, decelerated period and accelerated space expansion period are described. The dark energy is the photon space. And our universe was created from the nothing and will be annihilated to the nothing as shown in Fig. 2. It is indicated that the universe was decelerated by the boson force and gravitation force and has been accelerated by the dark energy in Fig. 20. The more detail can be seen in Ref. [50], too.

6. Summary and conclusions

In the present paper, the universe evolution is discussed by the decay of the charged black holes. The massive elementary fermions are defined as the warped spaces toward the positive time axis for the positive charges and toward the negative time axis for the negative charges. The photons are the flat space on the 4-D Euclidean space-time. First the energy \( (E=c\Delta t\Delta V) \), charges and energy density \( \left| q \right| = \rho = c\Delta t \) and absolute time \( (ct) \) are newly defined based on the 4-D Euclidean space. The big bang is understood by the space-time evolution of the 4-D Euclidean space but not by the sudden 4-D Minkowski space-time creation. The charged black holes are treated like the elementary fermions. The first matter universe created at the big bang is made of the negatively charged black holes with the huge energy (mass) and very small space volume (maybe smaller than the Planck scale). The origins of the inflation and big bang for our matter universe has not been discovered. In the present work, the origins of the big bang and inflation are briefly explained by using the evolution of the charged black holes in terms of the 3-dimensional quantized space.
model (TQSM) based on the 4-dimensional (4-D) Euclidean space. Note that the black holes are not the singularities because of the huge Coulomb repulsive forces with the huge BH surface fluctuations. The first primary black holes created at the big bang moment are closest to the singularities in the physical concept. This first primary black holes with the huge charges are inflated to the huge space volume with the much smaller charges.

The dark energy has been proposed to explain the accelerated space expansion. However, the problem of what the dark energy is should be solved. The dark energy is the fundamental property of the space-time because the dark energy density remains constant through the universe evolution. This indicates that the new space needs to be created by the dark energy. In the present TQMS model, the photon flat space is the background space-time. This photon space has the constant density which means that the constant time width \( c\Delta t_q = \text{constant}\). to give the density definition of \( \rho = c\Delta t_q \). This is consistent with the expected properties of the dark energy. And the quantum fluctuations of the photon space include the neutrino pair production by the CPT symmetry and photon pair production by the T symmetry. Because of the T symmetry which creates the new space on our matter universe, these two quantum fluctuations create the new spaces without changing the energy density. These new spaces drive the accelerated space expansion. This is what the dark energy means. In the present work, it is concluded that the dark energy is the photon space. Then the dark energy consists of the neutrino dark energy and photon dark energy. In the viewpoint of the Hubble’s constant, \( H_{DE} = H_{ps} = H_v + H_p \). Then the Hubble’s constants (H) including \( H_{DE} = H_{ps} = H_v + H_p \) are directly obtained from the weak gravitational lensing, the Type Ia supernovae, the galax clusters, and baryonic acoustic oscillations. And the Hubble’s constant (H(CMB)) including only the \( H_p \) term is indirectly obtained from the cosmic microwave background radiation (CMB). The Hubble’s constant puzzle is that the H(CMB) values are smaller than the H values. This can be explained by the neutrino dark energy effect of \( H_v \). The \( H_v \) value is \( H_v = H - H(CMB) = 2.1 - 5.5 \, \text{km/sec/Mpc} \).

Another problem is the vacuum density crisis. Quantum field theory gives the huge vacuum density of \( \rho(QFT) = 10^{123} \rho_{exp} \). In the present work, the vacuum state is the photon space. In other words, the vacuum energy density is the energy density of the photon space. The experimental vacuum energy density of \( \rho_{exp} \) means the energy density of the photon space. The experimental vacuum energy density of \( \rho_{exp} \) allows only the neutrino oscillations. This gives the answer to the big difference between two values of \( \rho(QFT) \) and \( \rho_{exp} \). The QFT calculation use the Planck energy as the maximum vacuum oscillation. But the experimental values give only the neutrino vacuum oscillation. The vacuum constant crisis is solved when the photon space is taken as the vacuum state. Therefore, in general, the vacuum state has only the neutrino pair productions by the CPT symmetry. In the localized photon space with the higher energy density of the radiation, the more massive particle pair productions by the CP symmetry like the pair production of the electron and positron can be allowed.

The big bang process created the matter universe with the positive energy and the partner anti-matter universe with the negative energy from the CPT symmetry. Our universe is the matter
universe with the negative charges of electric charge (EC), lepton charge (LC) and color charge (CC). This first universe is made of three dark matter -, lepton -, and quark - primary black holes with the huge negative charges which cause the Coulomb repulsive forces much bigger than the gravitational forces. The huge Coulomb forces induce the inflation of the primary black holes, that decay to the super-massive black holes.

We do not observe the rest mass of the photon. We observe the massive particles with the 3-D volumes. The massive particles and massless photons occupy the 3-D volume on the 3-D Euclidean space. Then, what is the difference between the massless photon and massive particles? The answer to this question comes from the 4-D Euclidean space. It is thought that the massive particles take the 4-D warped space that is the warped version of the flat photon space along the time axis of ct. The 4-D volume of the 4-D warped space is the rest mass energy of the massive particle. Under this new idea, the flat photon space has the zero-rest mass energy. The flat photon space has the 3-D quantized space of the physical x1x2x3 space with the very small-time width of cΔtq. Therefore, the mathematical 4-D Euclidean space is the unquantized space without the photons (space fluctuations). And the physical 4-D Euclidean space is the 4-D quantized space-time that is the 3-D quantized space of the physical x1x2x3 space with the very small-time width of cΔtq. This is called as the space-time evolution in the present work from the mathematical space to the physical space.

In the 4-D Euclidean space, all axes have the positive and negative directions. However, the space momenta along the 3-D space axes have been studied in the physical world. For the time axis, only the positive axis in the 4-D Minkowski space has been taken into consideration because we observe only the positive time direction. From the viewpoint of the 4-D momenta on the 4-D Euclidean space, the time axis should have the positive and negative time directions. If the negative time direction is allowed in the physical point of view, the well-known big bang theory should be changed to include the negative time direction. In this case, the partner universe with the negative time momentum is allowed. Note that the negative energy and negative mass are allowed from the negative time momentum. This means that the big bang is the pair creation of our matter universe and partner anti-matter universe which are the 3-D quantized spaces. This new interpretation completes the big bang theory in terms of the conserved CPT symmetry. It explains why our universe is the matter universe. It is concluded that three unsolved questions of dark energy, big bang, inflation, charged black hole decay, vacuum density crisis and Hubble’s constant puzzle are solved by using the photon space and the charged black hole in terms of the 3-D quantized space model (TQSM). I wish the present results can inspire people to study on the present topics with their own new ideas. The more details can be seen in Ref. [50], too.

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