B-L-violation in Softly Broken Supersymmetry and Neutrinoless Double Beta Decay

M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko

*Max-Planck-Institut für Kernphysik, P.O. 10 39 80, D-69029, Heidelberg, Germany

*Joint Institute for Nuclear Research, Dubna, Russia

Abstract

We prove a low-energy theorem valid for any model of weak scale softly broken supersymmetry. It claims that the neutrino Majorana mass, the B-L violating mass of the sneutrino and the neutrinoless double beta decay amplitude are intimately related to each other such that if one of them is non-zero the other two are also non-zero and, vice versa, if one of them vanishes the other two vanish as well. The theorem is a consequence of the underlying supersymmetry and independent of the mechanisms of neutrinoless double beta decay and (s-)neutrino mass generation.

Neutrinos are special among the known fermions in the sense that - being electrically neutral - they could be either Dirac or Majorana particles. Experimentally at present only upper limits on neutrino masses have been firmly established, but there are also an accumulating number of hints for non-zero neutrino masses from, for example, the solar and atmospheric neutrinos (for a recent review see [1]) as well as from recent LSND results [2]. While the discovery of any non-zero neutrino mass would present a major breakthrough, unfortunately none of the above experiments could tell whether the neutrino is a Dirac or a Majorana particle.

From a theoretical point of view Majorana neutrinos are clearly preferred. In particular, Grand Unified Theories (GUTs) most naturally lead to Majorana neutrinos. A Majorana mass for the neutrinos could quite elegantly explain the observed smallness of neutrino masses via the see-saw mechanism [4]. Also various 1-loop contributions to the neutrino self-energy, allowed in extensions of the SM [5]-[10], induce a small Majorana mass for neutrinos. Nevertheless only experiments can finally settle the question about the nature of the neutrino. The importance of neutrinoless double beta (0νββ) decay derives from the fact that it is sensitive to the Majorana nature of neutrinos.
That there is a generic relation between the amplitude of neutrinoless double beta ($0\nu\beta\beta$) decay and the (B-L)-violating Majorana mass of the neutrino has been recognized about 15 years ago [3]. A general theorem relating these two observables has been proven in [3]. It states that if any of the two quantities - the Majorana neutrino mass or the neutrinoless double decay amplitude - vanishes the other one vanishes necessarily too and, vice versa, if one of them is non-zero the other one must also differ from zero. Recall, that $0\nu\beta\beta$-decay is strictly forbidden if the neutrino is a Dirac particle having only a (B-L)-conserving Dirac mass. This theorem is valid for any gauge model with spontaneously broken symmetry at the weak-scale, independent of the mechanism of $0\nu\beta\beta$-decay. The simple neutrino exchange mechanism given in Fig.1(a) illustrates the above theorem explicitly. In this case the $0\nu\beta\beta$-decay amplitude is directly proportional to the small Majorana mass $m^\nu_M$ of the neutrino.

Weak-scale softly broken supersymmetry implies new particles with masses of the order of $\sim M_W$ and new low-energy interactions. In view of this fact the $0\nu\beta\beta$-decay amplitude may non-trivially depend not only on the Majorana neutrino mass, as claimed by the above mentioned theorem [3], but also on certain SUSY parameters. In SUSY models the neutrino ($\nu$) has a scalar superpartner the sneutrino (\tilde{\nu}). Given that they are components of the same superfield there could be a certain interplay between the neutrino and sneutrino mass terms in a low-energy theory as a relic of the underlying supersymmetry. Such a relation indeed exists [11] and provides the basis for the present paper.

In the present note we prove a low-energy theorem establishing an intimate relation between the neutrino Majorana mass, the (B-L)-violating sneutrino mass and the $0\nu\beta\beta$-decay amplitude. This theorem can be regarded as a SUSY generalization of the above cited theorem [3] proven for non-supersymmetric gauge theories. Our considerations use only the general structure of the low-energy effective Lagrangian assuming weak scale softly broken supersymmetry. The proof is based on symmetry arguments and thus generally valid, independent of mechanisms of neutrinoless double beta decay and (s-)neutrino mass generation.

As shown in [11], and as will also be demonstrated below, a self-consistent form of the neutrino and sneutrino mass terms is

$$\mathcal{L}^{\nu\tilde{\nu}}_{\text{mass}} = -\frac{1}{2}(m^\nu_M \overline{\nu^c} \nu + h.c.) - \frac{1}{2}(\tilde{m}_M^2 \tilde{\nu}_L \tilde{\nu}_L + h.c.) - \tilde{m}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L.$$

(1)

where $\nu = \nu^c$ is a Majorana field. The first two terms violate the global (B-L) symmetry while the last one respects it. The first term is a Majorana mass term of the neutrino. We call the second term a "Majorana"-like mass,
while the third one is referred to as a "Dirac"-like sneutrino mass term. This reflects an analogy with Majorana and Dirac mass terms for neutrinos. The Dirac neutrino mass term \( m_D^\nu (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \) could also be included in Eq. (1) but it is not required by the self-consistency arguments. Note that \( \tilde{m}_M^2 \) is not a positively defined parameter. The further proof does not depend on the mechanism of mass generation in the low-energy theory. For the sake of simplicity and without any loss of generality we ignore possible neutrino mixing.

The low-energy theorem we are going to prove consists of three statements. Two statements touch upon the set of three (B-L)-violating quantities: the neutrino Majorana mass \( m^\nu_M \), the "Majorana"-like sneutrino mass \( \tilde{m}_M \) and the amplitude of 0\( \nu \beta \beta \)-decay \( R_{0\nu\beta\beta} \). The third statement relates this set of (B-L)-violating quantities to the (B-L)-conserving "Dirac"-like sneutrino mass \( \tilde{m}_D \).

Statement 1: If one of the three quantities \( m^\nu_M, \tilde{m}_M, R_{0\nu\beta\beta} \) vanishes, then the two others vanish, too.

Statement 2 is an inverse to statement 1: If at least one of the three quantities \( m^\nu_M, \tilde{m}_M, R_{0\nu\beta\beta} \) is non-zero, then the two others are non-zero, too.

Statement 3: In the presence of \( \tilde{m}_M^2 \neq 0 \) in Eq. (1) there must exist a "Dirac"-like (B-L)-conserving sneutrino mass term with \( \tilde{m}_D^2 \geq |\tilde{m}_M^2| \).

Let us turn to the proof of the first two statements. It is relatively easy to see that if at least one of the quantities is non-zero the two others are generated in higher orders of perturbation theory as demonstrated in Fig.1, where only dominant diagrams are shown. Internal lines in these diagrams are neutralinos \( \chi_i \), gluinos \( \tilde{g} \), charginos \( \chi^\pm \), selectron \( \tilde{e} \), u-squark \( \tilde{u} \) and sneutrino \( \tilde{\nu} \). The latter is to be identified with the B-L-violating “Majorana” propagator proportional to \( \tilde{m}_M^2 \). The sneutrino “Majorana” propagator was explicitly derived in ref. [11] and, for the sake of self-consistency of the current paper, is repeated below.

The various diagrams lead to relations among the three (B-L)-violating observables, which we write down schematically

\[
    z_i = \sum_{i \neq j} a_{ij} \cdot z_j + A_i. \tag{2}
\]

Here, \( z_i \) can stand for \( z_i = m^\nu_M, \tilde{m}_M^2, R_{0\nu\beta\beta} \). The coefficients \( a_{ij} \) correspond to contributions of the diagrams in Fig.1(a)-(f) so that \( i, j = a, b, c, d, e, f \). Terms \( A_i \) represent any other possible contributions. The explicit form of \( a_{ij} \) and \( A_i \) is not essential in the following. Important is only the presence of a correlation between \( m^\nu_M, \tilde{m}_M, R_{0\nu\beta\beta} \), expressed by eq. (2).
Now we are going to prove that if $z_i = 0$, then $z_{i_2} = z_{i_3} = 0$ (the same will be true for any permutation). On the basis of Fig. (1) and Eqs. (2) one can expect such properties of the set of observables $z_i$. Indeed $z_i = 0$ in the left-hand side of Eq. (2) strongly disfavors $z_{j_2} \neq 0$ and $z_{j_3} \neq 0$, because it requires either all the three terms in the right-hand sides to vanish or their net cancelation. The latter is "unnatural". Even if such a cancelation would be done by hand, using (unnatural) fine-tuning of certain parameters, in some specific order of perturbation theory, it would be spoiled again in higher orders of perturbation theory. The cancelation of all terms in the right-hand side of Eqs. (2) in all orders of perturbation theory could only be guaranteed by a special unbroken symmetry. Let us envisage this possibility in details.

The effective Lagrangian of a generic model of weak scale softly broken supersymmetry contains after electro-weak symmetry breaking the following terms [12]

\[
\mathcal{L} = -\sqrt{2}g\epsilon_i \cdot \bar{\nu}_L \chi_i - g\epsilon_i^\pm \cdot \bar{\nu}_L \chi_i^\pm \bar{\nu}_L - g\epsilon_i^+ \cdot \bar{\nu}_L \chi_i^+ \bar{\nu}_L + \frac{g}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L + \bar{\nu}_L \gamma^\mu d_L)W_{\mu}^+ + g \cdot \chi_i^\gamma^\mu (O_{ij}^L P_L + O_{ij}^R P_R) \chi_j^+ W_{\mu}^- + \ldots + h.c.
\]

Dots denote other terms which are not essential for further consideration. Here, $\bar{\nu}_L$ and $\bar{e}_L$ represent scalar superpartners of the left-handed neutrino $\nu_L$ and electron $e_L$ fields. The chargino $\chi_i^\pm$ and neutralino $\chi_i$ are superpositions of the gaugino and the higgsino fields. The contents of these superpositions depends on the model. Note that the neutralino is a Majorana field $\chi_i^c = \chi_i$. The explicit form of the coefficients $\epsilon_i$, $\epsilon_i^\pm$ and $O_{ij}^{LR}$ is also unessential. For the case of the MSSM one can find them, for instance in [12]. Eq. (3) is a general consequence of the underlying weak scale softly broken supersymmetry and the spontaneously broken electro-weak gauge symmetry.

The Lagrangian (3) does not posses any continuous symmetry having non-trivial B-L transformation properties. Recall, that $U(1)_{B-L}$ is assumed to be broken since we admit B-L-violating mass terms in Eq. (1). However, there might be an appropriate unbroken discrete symmetry. Let us specify this discrete symmetry group by the following field transformations

\[
\begin{align*}
\nu &\rightarrow \eta_\nu \nu, & \bar{\nu} &\rightarrow \eta_{\bar{\nu}} \bar{\nu}, & e_L &\rightarrow \eta_e e_L, & \bar{e}_L &\rightarrow \eta_{\bar{e}} \bar{e}_L, \\
W^+ &\rightarrow \eta_W W^+, & \chi_i &\rightarrow \eta_{\chi_i} \chi_i, & \chi^+ &\rightarrow \eta_{\chi^+} \chi^+.
\end{align*}
\]

Here $\eta_i$ are phase factors. Since the Lagrangian (3) is assumed to be invariant under these transformations one obtains the following relations

\[
\eta_{\nu}^* \eta_{\bar{\nu}} \eta_{\chi_i} = 1, \quad \eta_e \eta_{\chi^+} \eta_{\bar{\nu}}^* = 1, \quad \ldots
\]
\[ \eta_\nu \eta_w \eta_{\nu}^* = 1, \quad \eta_\nu^* \eta_{\chi}^+ \eta_{\chi_i}^* = 1, \quad \text{....} \]

Dots denote other relations which are not essential here. The complete set of these equations defines the admissible discrete symmetry group of the Lagrangian in Eq. (3).

Let us find the transformation property of the operator structure responsible for $0\nu\beta\beta$-decay under this group. At the quark level $0\nu\beta\beta$-decay implies the transition $dd \rightarrow uuee$, described by the effective operator

\[ O_{0\nu\beta\beta} = \alpha_i \cdot \bar{u} \Gamma_i^{(1)} d \cdot \bar{u} \Gamma_i^{(1)} d \cdot \bar{e} \Gamma_i^{(2)} e, \]

where $\alpha_i$ are numerical constants, $\Gamma_i^{(k)}$ are certain combinations of Dirac gamma matrices. The $0\nu\beta\beta$-decay amplitude $R_{0\nu\beta\beta}$ is related to the matrix element of this operator

\[ R_{0\nu\beta\beta} \sim <2e^-(A,Z+2)|O_{0\nu\beta\beta}|(A,Z)>, \]

where $(A,Z)$ is a nucleus with the atomic weight $A$ and the total charge $Z$. The operator in Eq. (6) transforms under the group (4) as follows

\[ O_{0\nu\beta\beta} \rightarrow \eta_{0\nu\beta\beta} O_{0\nu\beta\beta} \]

with

\[ \eta_{0\nu\beta\beta} = \eta_d^* \eta_u \eta_e \]

Solving Eqs. (5), (9), one finds

\[ \eta_{\nu}^2 = \eta_\nu^2 = \eta_{0\nu\beta\beta}^2. \]

This relation proves the statements 1,2. To see this we note that the observable quantity $z_i = (m_{\nu}^2, \tilde{m}_M, R_{0\nu\beta\beta})$ is forbidden by this symmetry if the corresponding discrete group factor is non-trivial, i.e. $\eta_i^2 \neq 1$. Contrary, if $\eta_i^2 = 1$, this quantity is not protected by the symmetry and appears in higher orders of perturbation theory, even if it is not included at the tree-level. Relation (10) claims that if one of the $z_i$ is forbidden then the two others are also forbidden and, vice versa, if one of them is not forbidden they are all not forbidden. Thus, statements 1,2 are proven.

One can derive the following interesting corollary from statements 1,2.

**Corollary 1:** If the (B-L)-violating sneutrino ”Majorana” mass term is for certain reason absent in the low-energy theory, i.e. $\tilde{m}_M^2 = 0$, then the neutrino has no Majorana mass and neutrinoless double beta decay is forbidden.
Now let us turn to the statement 3. Consider the last two terms of Eq. (11) which we denote as \( \mathcal{L}_{\text{mass}}^0 \) and use the real field representation for the complex scalar sneutrino field \( \tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2} \), where \( \tilde{\nu}_{1,2} \) are real fields. Then

\[
\mathcal{L}_{\text{mass}}^0 = -\frac{1}{2}(\tilde{\mu}_M^2 \tilde{\nu}_L \tilde{\nu}_L + \text{h.c.}) - \tilde{\mu}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L = -\frac{1}{2}\tilde{\nu}_1^2 - \frac{1}{2}\tilde{\nu}_2^2
\]

(11)

where \( \tilde{\mu}_{1,2}^2 = \tilde{\mu}_D^2 \pm |\tilde{\mu}_M^2| \). Assume the vacuum state is stable. Then \( \tilde{\mu}_{1,2}^2 \geq 0 \), i.e. \( \tilde{\mu}_D^2 \geq |\tilde{\mu}_M^2| \), otherwise the vacuum is unstable and subsequent spontaneous symmetry breaking occurs via non-zero vacuum expectation values of the sneutrino fields \( \langle \tilde{\nu}_i \rangle \neq 0 \). The broken symmetry in this case is the R-parity. It is a discrete symmetry defined as \( R_p = (-1)^{3B+L+2S} \), where \( S, B \) and \( L \) are the spin, the baryon and the lepton quantum number.

This completes the proof of the theorem consisting of the above-given three statements, and moreover shows that a self-consistent structure of mass terms of the neutrino-sneutrino sector is given by Eq. (11).

Let us finally present the explicit form of the above mentioned (B-L)-violating “Majorana” propagator \( \Delta_{\nu}^M(x, y) \) for the sneutrino \( \tilde{\nu} \). It can be derived by the use of the real field representation as in eq. (11). For comparison we also give the (B-L)-conserving “Dirac” \( \Delta_{\nu}^D(x, y) \) sneutrino propagator,

\[
\Delta_{\nu}^D(x, y) = -\frac{i}{2}(\Delta_{\tilde{\mu}_1}(x, y) + \Delta_{\tilde{\mu}_2}(x, y)),
\]

(12)

\[
\Delta_{\nu}^M(x, y) = -\frac{i}{2}(\Delta_{\tilde{\mu}_1}(x, y) - \Delta_{\tilde{\mu}_2}(x, y)),
\]

(13)

where

\[
\Delta_{\tilde{\mu}_i}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{\tilde{\mu}_i^2 - k^2 - i\epsilon}
\]

(14)

is the ordinary propagator for a scalar particle with mass \( \tilde{\mu}_i \). Using the definition of \( \tilde{\mu}_{1,2} \) as in Eq. (11) one finds

\[
\Delta_{\nu}^D(x) = \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{\mu}_D^2 - k^2}{(\tilde{\mu}_1^2 - k^2 - i\epsilon)(\tilde{\mu}_2^2 - k^2 - i\epsilon)} e^{-ikx},
\]

(15)

\[
\Delta_{\nu}^M(x) = -\tilde{\mu}_M^2 \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{(\tilde{\mu}_1^2 - k^2 - i\epsilon)(\tilde{\mu}_2^2 - k^2 - i\epsilon)}.
\]

(16)

It is seen that in absence of the (B-L)-violating sneutrino “Majorana”-like mass term \( \tilde{\mu}_M^2 = 0 \) the (B-L)-violating propagator \( \Delta_{\nu}^M \) vanishes while the (B-L)-conserving one \( \Delta_{\nu}^D \) becomes the ordinary propagator of a scalar particle with
mass $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_D$. According to Eq. (11) the parameter $\tilde{m}_M^2$ describes a splitting in the sneutrino mass spectrum. This mass splitting parameter can be probed by searching for (B-L)-violating exotic processes such as neutrinoless double beta decay as discussed in the present note. It is obvious from Eq. (2) corresponding to the diagram in Fig.1(f) that certain constraints on $\tilde{m}_M^2$ can also be obtained from the experimental upper bound on the neutrino mass. Probably, $\tilde{m}_M^2$ can also be constrained by accelerator searches for supersymmetry. However this possibility might require unrealistic energy resolution for detectors if the above mentioned $0\nu\beta\beta$-decay and/or $m_\nu$ constraints on $\tilde{m}_M^2$ turned out to be too stringent. We are going to analyze these questions in a separate paper.

In summary, we have proven a low-energy theorem for weak scales softly broken supersymmetry relating the (B-L)-violating mass terms of the neutrino and the sneutrino as well as the amplitude of neutrinoless double beta decay. This theorem can be considered as a supersymmetric generalization of the well know theorem [3] relating only neutrino Majorana mass and the neutrinoless double beta decay amplitude.

ACKNOWLEDGMENTS

We thank V.A. Bednyakov, for helpful discussions. The research described in this publication was made possible in part (S.G.K.) by Grant GNTP 315NU-CLON from the Russian ministry of science. M.H. would like to thank the Deutsche Forschungsgemeinschaft for financial support by grants kl 253/8-2 and 446 JAP-113/101/0.

References

[1] A.Yu. Smirnov, Plenary talk given at 28th International Conference on High energy physics, 25 - 31 July 1996, Warsaw, Poland; hep-ph/9611465

Y. Suzuki, Plenary talk at the same Conference.

[2] The LSND Collaboration, C. Athanassopoulos et al., Phys.Rev.Lett. 77, 3082 (1996); ibid. 75, 2650 (1995).

[3] J. Schechter and J.W.F. Valle, Phys.Rev. D 25, 2951 (1982); J.F. Nieves, Phys.Lett. B 147, 375 (1984); E. Takasugi, Phys.Lett. B 149, 372 (1984); B. Kayser, in Proc. of the XXIII Int. Conf on High Energy Physics, ed. S. Loken (World Scientific Singapore, 1987), p. 945; S. Petcov, in Proc.
of 86’ Massive Neutrinos in Astrophysics and in Particle Physics, ed. O. Fackler and J. Trần Than Ván (Editions Frontieres, Gif-sur-Yvette, France, 1986), p. 187; S.P. Rosen, UTAPHY-HEP-4 and hep-ph/9210202.

[4] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, ed. F. van Nieuwenhuizen and D.Freedman, (North Holland, Amsterdam, 1979), p.315; T. Yanagida, Proc. of the Workshop on Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979; S. Weinberg, Phys.Rev.Lett. 43, 1566 (1979).

[5] I. H. Lee, Nucl. Phys. B 246, 120 (1984); Phys.Lett. B 138, 121 (1984).

[6] R. Mohapatra, S. Nussinov and J.W.F. Valle, Phys.Lett. B 165, 417 (1985).

[7] R. Barbieri et al. Phys. Lett. B 238, 86 (1990).

[8] L. Hall and M. Suzuki, Nucl.Phys. B 231, 419 (1984).

[9] E. Ma and D. Ng, Phys. Rev. D 41, 1005 (1990).

[10] A. Zee, Phys.Lett. B 93, 389 (1980).

[11] M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, hep-ph/9701253.

[12] H.E. Haber and G.L.Kane, Phys.Rep. 117, 75 (1985); H.P. Nilles, Phys.Report. 110, 1 (1984).

**Figure Captions**

Fig.1.: Lowest order perturbation theory diagrams representing the relation between the neutrino Majorana mass \( m'_{\nu} \), the "Majorana"-like (B-L)-violating sneutrino mass \( \tilde{m}_M \), and the amplitude of neutrinoless double beta decay \( R_{0\nu\beta\beta} \). (a) the neutrino contribution and (b) an example of sneutrino contribution to the \( 0\nu\beta\beta \)-decay amplitude \( R_{0\nu\beta\beta} \). \( 0\nu\beta\beta \)-vertex contribution to (c) the neutrino Majorana mass and (d) to the "Majorana"-like sneutrino mass; (e) neutrino contribution to the sneutrino "Majorana"-like mass and (f) sneutrino contribution to the neutrino Majorana mass. Crossed (s)neutrino lines correspond to the B-L-violating propagators.
Figure 1.a

\[ \text{d} \rightarrow \text{u} \]

\[ \text{W} \]

\[ \nu \times \]

\[ \text{W} \]

\[ \text{d} \rightarrow \text{u} \]
