Testability requirement uncertainty analysis in the sensor selection and optimization model for PHM

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Abstract. Prognostics and health management (PHM) has been an important part to guarantee the reliability and safety of complex systems. Design for testability (DFT) developed concurrently with system design is considered as a fundamental way to improve PHM performance, and sensor selection and optimization (SSO) is one of the important parts in DFT. To address the problem that testability requirement analysis in the existing SSO models does not take test uncertainty in actual scenario into account, fault detection uncertainty is analyzed from the view of fault attributes, sensor attributes and fault-sensor matching attributes qualitatively. And then, quantitative uncertainty analysis is given, which assigns a rational confidence level to fault size. A case is presented to demonstrate the proposed methodology for an electromechanical servo-controlled system, and application results show the proposed approach is reasonable and feasible.

1. Introduction
Catastrophes caused by aerospace system faults in recent years impel people to explore fault mechanisms and the corresponding countermeasures. Prognostics and Health Management (PHM), which generally combines sensing and interpretation of environmental, operational, and performance-related parameters to assess the health of a system and predict remaining useful life [1], is significant to improve complex system safety and reliability[2-3]. Obviously, information sensing and test are the foundation of PHM [1, 4-5], and some studies and applications also show the PHM performance mainly depends on test information rather than on the adopted models or algorithms [6]. Testability is a design characteristic which allows the status (operable, inoperable, or degraded) of an item to be determined and the isolation of faults within the item to be performed in a timely manner [7]. The design scheme which satisfies system testability requirements is design for testability (DFT). The traditional DFT is developed through an ad hoc heuristic process rather than through a system method (William, 20078), which will result in high cost, long period, etc. At present, model-based DFT is becoming popular, which supports for concurrent design very well, and sensor selection and optimization (SSO) is one of the important contents in DFT.

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With the great development of PHM methodology, many researchers and institutes have paid attention to SSO problems for PHM. NASA has studied sensor optimization configuration technology for engine health management since 2005, and proposed a famous systematical sensor selection strategy (S4), and the researchers also studied some experiment validation and verification for health monitoring and management of some aerospace systems such as turbo engine, RS-68 rocket engine [5, 9-11]. Cheng, S. et al studied sensor selection and optimization for PHM systematically. They also proposed the state-of-art sensor systems for PHM and further discussed the emerging trends in technologies of sensor systems for PHM [1, 4, 12]. Kwon, D. et al also paid much attention to sensor selection for PHM [13]. Xu, Z. et al proposed a fault tolerant sensor architecture and realized the architecture through the design of a dual mode humidity/pressure MEMS sensors with an integrated temperature function for health and usage monitoring [14]. Novis, A. et al analyzed the characteristics of sensor system used in real PHM environment in order to improve system diagnostic capability [15]. Baer, W. G. et al constructed an open standard smart sensor structure, and designed a sensor system for PHM [16].

In the existing SSO models, testability requirement model is mainly based on the assumption that: if a sensor relates to a fault, the fault will be detected by the sensor with probability one when the fault occurs. The certainty assumption is usually applicable for digital systems due to its good modularity and fault propagation certainty. However, for complex systems usually consisting of mechatronics, electronics and hydraulics, fault detection probability is closely related to test uncertainty. Generally, test uncertainty may arise from two major sources: 1) environment uncertainty is a common type of uncertainty source, which usually includes temperature, vibration, humidity and electromagnetic interference; 2) measurement uncertain is usually introduced during the measurement process, which often consists of sensor reliability, human error, calibration bias. Test uncertainty needs to be addressed since uncertainty is always related to the measurement performance and is a key factor for SSO for complex system PHM.

2. Fault detection uncertainty analysis

2.1. Qualitative analysis

2.1.1. Fault attributes analysis. Fault detection probability is obviously affected by fault characteristics. As we know, if fault symptom is distinctive and stable, the fault will be detected with high probability; besides, fault detection probability will increase with the decrease of fault detection threshold.

(1) Fault sensitivity is the relative quantity of fault statistical features. Fault statistical features are generally divided into time-domain features and frequency-domain features. Amplitude, peak, root-mean-square, margin factor, kurtosis are typical time-domain features, while frequency, wavelet information entropy and power spectrum are typical frequency-domain features. Fault sensitivity is very important for fault early state detection and fault prognostics. Take the pulse fault and wear fault of engine as examples to demonstrate the quantitative description of fault sensitivity. Suppose the statistical features of the two types of faults are amplitude, then, pulse fault sensitivity is formulated by [17]:

$$S_p = \frac{\left( \theta(k_p) - \frac{1}{M_2 - M_1} \sum_{k=M_1}^{M_2} \theta(k) \right)}{\frac{1}{M_2 - M_1} \sum_{k=M_1}^{M_2} \theta(k)} \times 100\%$$

where \(S_p\) is pulse fault sensitivity, \(M_1\) and \(M_2\) are sample points of feature \(\theta\) after starting engine three seconds and before appearing pulse respectively, \(\theta(k)\) is the amplitude of the \(k\)-th sample point of \(\theta\), \(\theta(k_p)\) is the amplitude of \(\theta\) when pulse appears.
Based on the same example, wear fault sensitivity is formulated by:

\[
S_w = \frac{1}{L_2 - L_1} \sum_{k=1}^{L_2} \left| \frac{\theta(k) - 1}{M_2 - M_1} \sum_{k=M_1}^{M_2} \theta(k) \right| \times 100\%
\]

(2)

where \(S_w\) is wear fault sensitivity, \(M_1, M_2\) and \(\theta(k)\) have the same meanings with the above equation, \(L_1\) and \(L_2\) are sample points of feature \(\theta\) when appearing wear fault and when closing engine respectively, \(\theta(k_w)\) is the amplitude of \(\theta\) during fault wearing process.

(2) Fault stability is the fluctuation degree of fault statistical features, which can be described by stable behaviour quantity [18]. Supposing the sample size of certain fault statistical feature \(\theta\) is \(N\), i.e., \(\Theta=\{\theta_i, i=1,2,\cdots,N\}\), \(\theta_i\) is the \(i\)-th measurement value of the statistical feature \(\theta\). Normalize \(\Theta\):

\[
\Gamma = \frac{\theta_i}{\max\{\theta_i\}}
\]

Then, the stable behaviour quantity of \(\Theta\) can be represented as:

\[
S_{\theta} = \frac{\sum_{i=1}^{N} [|\gamma_i - \overline{\gamma}| \ln(1 - |\gamma_i - \overline{\gamma}|)]}{\max \{\sum_{i=1}^{N} [|\gamma_i - \overline{\gamma}| \ln(1 - |\gamma_i - \overline{\gamma}|)]\}}
\]

(4)

\(S_{\theta} \in [0,1]\). \(S_{\theta}=0\) means the statistical feature \(\theta\) is most stable, while \(S_{\theta}=1\) means the statistical feature \(\theta\) is most unstable. Generally, good fault stability is propitious to fault detection and further to fault prognostics.

(3) Fault resolution is the minimum measurable variation of fault symptoms. For each fault, minimum observable symptoms can be determined based on expert knowledge or past experiences.

(4) Time to failure (TTF) is the time duration between the initiation of a fault and the time when the failure occurs. For sudden faults (intermittent faults or instantaneous faults), the TTF is approximate to zero.

(5) Fault symptom duration is time duration of fault statistical features, which usually equals to TTF.

(6) Fault detection threshold is the minimal fault symptom quantity which enables the fault detectable. Generally, for the same fault severity level, the smaller the fault detection threshold is, the higher the fault detection probability is. Of course, the probability of false alarm will increase.

2.1.2. Sensor attributes analysis. Obviously, sensor attributes will have direct impacts on fault detection probability.

(1) Sensor resolution is the capability that a sensor can measure the minimum variation of inputs.

(2) Sensor signal to noise ratio (SSNR): a high SSNR implies fault detection uncertainty is small, while a low SSNR implies that it is hard for the sensor to detect the fault. Obviously, sensors with a low SSNR are not suitable for fault early state detection, so SSO for PHM should take the sensors with higher SSNR into account first.

(3) Sensor failure rate is the probability that a sensor can not execute the stated functions during the state time span and at the stated conditions. It is a reliability index of sensors and a function of time.

(4) Sensor sensitivity is the ratio of output variation at sensor static condition to the corresponding input variation.

2.1.3. Fault-sensor matching attributes analysis. As we know, a sensor is good for a fault does not mean the sensor is suitable for another fault. So it is necessary to analyze fault-sensor matching attributes.
(1) Sensor fault detection timeliness (SFDT) is the ratio of the time span between the initiation of a fault (potential failure) and the detection of the fault by the sensor (time to detection, TTD) to the duration between the initiation of the fault and the time when the failure occurs (TTF), vide figure 1.

\[ SFDT = \frac{TTD}{TTF} \] (5)

A low SFDT means that the sensor can detect a fault occurrence at early stage, which is very useful for fault prognostics; while a high SFDT means the sensor needs a long time to declare a fault occurrence. If SFDT \( \geq 1 \), which implies that the sensor detects a fault when the fault leads to a failure, and fault prognostics becomes insignificant. So SSO for PHM should take SFDT into account.

(2) Sensor fault detection sensitivity (SFDS) is the ratio of a sensor variation of per unit sensor resolution to a fault variation of per unit fault resolution.

\[ SFDS = \frac{\Delta S}{\Delta s} \] (6)

where \( \Delta S \) is sensor measurement variation, \( \Delta s \) is sensor resolution; \( \Delta F \) is fault symptom variation, \( \Delta f \) is fault resolution.

(3) Sensor fault trackability (SFT) is the ratio of the time span of fault symptom trackable (symptom tracking time, STT) to the time to failure (TTF), vide figure 2.

\[ SFT = \frac{STT}{TTF} \] (7)

Generally, when a fault occurs, a sensor can track the fault once it detects the fault until the fault evolves to failure. Sensors of these properties can be selected for fault prognostics and health evaluation. However, for certain sensors, they will return to normal measurement state after detecting and tracking the fault symptoms for a period of time span, so they are not suitable for PHM.

2.2. Quantitative analysis

Based on the uncertainty analysis stated above, one can see that fault detection probability depends on many factors including fault attributes, sensor attributes and fault-sensor matching attributes, etc. In order to analyze fault detection uncertainty quantitatively, sensor fault detection probability (SFDP) is normally defined.

Definition 1: SFDP is the extent to which a sensor can detect the presence of a particular fault, which is also called as true positive detection probability.
SFDP represents the proportion of fault conditions that a candidate sensor identifies correctly, and is used to evaluate the sensor performance. Compared to SFDP, sensor false alarm probability (SFAP) represents the proportion of non-fault conditions that a candidate sensor incorrectly identifies as faults, which is also called as false positive detection probability. Note that the false negative detection probability is the complement of the true positive detection probability and that the true negative detection probability is the complement of the false positive detection probability. These two quantities fully describe the fault detection uncertainty as shown in figure 3.

\[
\text{Figure 3. Fault detection matrix.}
\]

It is possible that a fault can be detected by more than one sensor, so fault total detection probability (FTDP) should be defined based on SFDP.

Definition 2: FTDP is defined as the extent to which the sensor scheme can detect the presence of a particular fault.

SFDP quantitative uncertainty analysis is as follows [19]. Suppose the actual fault size (e.g. crack size) is \( a \), and the sensor measurement is \( a' \), then:

\[
a' = g(a, P) + \varepsilon
\]

where \( P \) is the measurement condition parameters such as material properties, temperature fluctuations and sensor locations, \( g \) is a nonlinear function relating actual defect size to sensor measurement, and \( \varepsilon \) is the measurement noise. A commonly used model is [20]:

\[
\log a' = \beta_0 + \beta_1 \cdot \log a + \varepsilon
\]

where \( \varepsilon \) is normally distributed with zero mean and variance \( \sigma^2 \). Based on the model, SFDP can be calculated as:

\[
SFDP(a, a_{\text{thr}}) = \Phi\left(\frac{\log a - \mu_1}{\sigma_1}\right)
\]

where \( \Phi \) is the standard normal cumulative distribution function (CDF), \( a_{\text{thr}} \) is the fault detection threshold, and

\[
\mu_1 = \frac{\log a_{\text{thr}} - \beta_0}{\beta_1}, \sigma_1 = \frac{\sigma}{\beta_1}
\]

Parameters \( \beta_0, \beta_1 \) and \( \sigma \) can be estimated by a maximum likelihood estimator (MLE). Obviously, \( SFDP(a, a_{\text{thr}}) = P(a' > a_{\text{thr}} | a) \). However, the actual defect size \( a \) is usually unknown, so confidence bounds on \( a \) should be analyzed in order to justify the reasonableness and dependability of measurement value \( a' \). According to (9), the logarithm actual defect size distribution is:

\[
\log a = \frac{\log a' - \beta_0}{\beta_1} - \frac{\varepsilon}{\beta_1}, \varepsilon \sim N(0, \sigma^2)
\]
Obviously, log \( a \) is a normal distribution with mean \((\log a' - \beta_0)/\beta_1\) and variance \(\sigma^2/\beta_1^2\) when the sensor measurement is \(a'\). \(\beta_0\), \(\beta_1\) and \(\sigma\) are estimated in the SFDP estimation process. The confidence bounds of \(a\) can be easily calculated using the lognormal CDF.

3. Testability requirement model under uncertainty test

3.1. Testability requirement description for PHM

Fault diagnostics and fault prognostics are two key technologies in PHM. Testability for PHM should enable system faults detectable, isolable and predictable. Three universal testability indexes including fault detectable rate (FDR), fault isolatable rate (FIR) and fault predictable ratio (FPR) are defined to describe the testability requirements for PHM. In Ref. [21], FDR and FIR are defined as follows.

Definition 3: FDR is the ratio of the number of faults detected correctly by sensors to the total number of system faults during the stated time span.

Definition 4: FIR is the ratio of the number of faults isolated correctly to no more than the stated replaceable units by sensors during the stated time span to the number of the detected faults during the same time span.

Generally, a fault is detectable doesn’t mean the fault is predictable. Whether a fault is predictable or not depends on two aspects: one is the fault should be progressive in nature; the other is the fault should be a key component’s fault.

Definition 5: possible predictable fault (PPF) is a progressive key fault.

A fault satisfying definition 5 may not be predictable, and fault predictability is also related to timely detection and evolution track. If a fault is detected by a sensor when or after the fault leads to a failure, fault prognostics becomes insignificant; further, if the evolution process of a fault can not be tracked by a sensor, (data driven-based) fault prognostics may not be realized.

Definition 6: predictable fault (PF) is a PPF of which the early state is detectable and the evolution process is trackable.

Definition 7: FPR is the ratio of the number of PFs determined correctly by sensors to the total number of PPFs of system during the stated time span.

3.2. Testability requirement modeling based on fault-sensor dependency matrix

Given the fault set is \(F=\{f_1, f_2, \ldots, f_m\}\), and the corresponding failure rate vector is \(\lambda=\{\lambda_1, \lambda_2, \ldots, \lambda_m\}\). The complete sensor set used for selection is \(T=\{t_1, t_2, \ldots, t_n\}\), and the corresponding sensor failure rate vector is \(FR=\{r_1, r_2, \ldots, r_n\}\). A matrix \(B=\{b_{ij}\}_{m \times n}\) is used to denote fault-sensor dependencies. The rows of \(B\) correspond to faults, and the columns correspond to sensors. Element \(b_{ij}\) is a two-tuple, \(b_{ij}=(u,v)\). And we suppose that if a sensor can detect the early state of a fault, it also means the sensor can track the fault evolution process. Then, if sensor \(t_j\) can detect fault \(f_i\) and its early state, then \(b_{ij}=(1,1)\). If sensor \(t_j\) can detect fault \(f_i\) but can not detect its early state, \(b_{ij}=(1,0)\). If sensor \(t_j\) can’t detect fault \(f_i\) nor its early state, then \(b_{ij}=(0,0)\), \((b_{ij}=0\) for short). Generally, if a sensor can detect early state of a fault, it also means that the sensor can detect the fault, so the case \(b_{ij}=(0,1)\) will not exist.

Given \(\cup\) denotes Boolean variable OR operation. And \(\oplus\) denotes set XOR operation, when the two set are different, the operation result is true. \(b_{ij}(k)\) denotes the k-th item of the two-tuple \(b_{ij}=(u,v)\), \(k=1,2\). \(T_f\) and \(T_p\) denote the sensor sets which can detect fault \(f_i\) and fault \(f_j\) respectively, i.e., \(T_f=\{t_j|b_{ij}(1)=1, t_j\}\). \(T_f\) is also called fault features of fault \(f_i\). \(F_{PP}\) denotes system PPFs. Given the ambiguity group size is \(L\), then, the determinable faults set \(F_{DP}\), isolable faults set \(F_I\) and predictable faults set \(F_P\) are formulated respectively by:
bij\((k) = 1\) has two meanings: one is that sensor \(ti\) relates to fault \(fi\); the other is that sensor \(ti\) can detect fault \(fi\) with probability one when fault \(fi\) occurs. However, as stated previously, fault detection uncertainty is existing objectively in actual scenario. In engineering applications, fault attributes, sensor attributes and fault-sensor matching attributes can be divided into sensor function attributes and performance attributes. Function attributes mainly refer to sensor reliability, which can be featured by sensor failure rate \(rj\) \((j = 1, 2, \ldots, n)\). Performance attributes mainly include SSNR, SFDS, SFDT and SFT, which can be featured by \(\rho_{ij}\) [6].

\[
\rho_{ij} = \begin{cases} 
(1 + e^{-10(SFDS_{ij} - 0.5)})^{-1} \times (1 + e^{-(SSNR_{ij} - 0.5)})^{-1} & \text{if } SFDT_{ij} < 1 \\
(1 - SFDT_{ij})^{0.5} \times (SFT_{ij})^{0.2} & \text{if } SFDT_{ij} \geq 1 
\end{cases}
\]

where \(SFDS_{ij}\) denotes the detection sensitivity of sensor \(ti\) to fault \(fi\), \(SSNR_{ij}\) denotes SNR of sensor \(ti\), \(SFDT_{ij}\) denotes the detection timeliness of sensor \(ti\) to fault \(fi\), \(SFT_{ij}\) is trackability of sensor \(ti\) to fault \(fi\).

Impact of sensor function attributes on detectability and predictability of fault \(fi\) can be formulated by:

\[
\begin{align*}
R_i^1 &= 1 - \prod_{j \in T}^{} \rho_{ij}b_j(1) \\
R_i^2 &= 1 - \prod_{j \in T}^{} \rho_{ij}b_j(2)
\end{align*}
\]

Impact of sensor performance attributes on detectability and predictability of fault \(fi\) can be formulated by:

\[
\begin{align*}
P_i^1 &= \sum_{j \in T}^{} \rho_{ij}b_j(1) / \sum_{j \in T}^{} b_j(1) \\
P_i^2 &= \sum_{j \in T}^{} \rho_{ij}b_j(2) / \sum_{j \in T}^{} b_j(2)
\end{align*}
\]

According to (15) and (16), the total detectable and predictable probability of fault \(fi\) can be formulated by:

\[
\begin{align*}
FTDP_i^1 &= R_i^1 \times P_i^1 \\
FTDP_i^2 &= R_i^2 \times P_i^2
\end{align*}
\]

According to definition 3, 4 and 7, testability requirement model for PHM (FDR, FIR and FPR) under uncertainty test can be formulated by:
The main goal of SSO is that the designed sensor scheme should satisfy the needs of system required FDR, FIR and FPR.

4. Case demonstration
Take certain electromechanical servo control system (ESCS) as an example to demonstrated the proposed methodology. The ESCS mainly consists of power, controller, driver, motor and reducer, and the corresponding failure modes are shown in table 1.

| Module     | Failure mode       | Fault mechanism                          | Failure rate | Detection method | Criticality |
|------------|--------------------|------------------------------------------|--------------|------------------|-------------|
| Power      | Abnormal output ($f_1$) | Cable failure, element aging and damage | $1.0 \times 10^{-5}$ | Voltage          | II          |
|            | Abnormal output ($f_2$) | Hard ware failure                         | $0.1 \times 10^{-5}$ | Logical output   | II          |
| Controller | Abnormal output ($f_3$) | Electrical element aging, over-current damage | $1.0 \times 10^{-5}$ | State signal     | II          |
| Driver     | Non-uniform gap between stator and rotor ($f_4$) | Manufacturing error, improper operation | $1.0 \times 10^{-5}$ | Vibration, current, flux | III         |
| Motor      | Open in stator coil ($f_5$) | Cable connector release, bad welding, mechanical stress, intensive current density | $1.0 \times 10^{-5}$ | Current, rotating speed | II          |
|            | Short in stator coil ($f_6$) | Moisture, insulation aging, over-voltage | $1.0 \times 10^{-5}$ | Temperature      | II          |
|            | Grounding in stator coil ($f_7$) | Insulation aging                          | $0.1 \times 10^{-5}$ | Temperature, current | II          |
|            | Bear wearing ($f_8$) | Fatigue, bad lubrication                  | $15.0 \times 10^{-5}$ | Vibration        | II          |
| Reducer    | Gear fatigue wear ($f_9$) | Bad lubrication, alternate stress          | $20.0 \times 10^{-5}$ | Vibration        | II          |
|            | Bear fatigue wear ($f_{10}$) | Bad lubrication, fatigue stress            | $15.0 \times 10^{-5}$ | Vibration        | II          |
|            | No output ($f_{11}$) | Jammed, hard fault                         | $1.0 \times 10^{-5}$ | Rotating speed   | II          |
According to definition 5, The ESCS PPFs is \( F_{pp} = \{ f_1, f_2, f_3, f_8, f_9, f_{10} \} \). The sensors and its attributes for selection are shown in table 2.

**Table 2.** The sensors and its attributes for selection.

| Sensor                                      | Failure rate (/1000h) | Cost | Limit number |
|---------------------------------------------|-----------------------|------|--------------|
| Power level signal detection \((t_1)\)     | 0.01                  | 1    | 10           |
| Controller level signal detection \((t_2)\)| 0.01                  | 5    | 10           |
| Driver level signal detection \((t_3)\)    | 0.01                  | 10   | 10           |
| Motor vibration sensor \((t_4)\)           | 0.001                 | 200  | 10           |
| Motor stator current sensor \((t_5)\)      | 0.001                 | 150  | 10           |
| Motor rotating speed optical-electricity encoder \((t_6)\) | 0.01                  | 1000 | 10           |
| Motor stator temperature sensor \((t_7)\)  | 0.01                  | 100  | 10           |
| Reducer vibration sensor \((t_8)\)         | 0.001                 | 200  | 10           |
| Reducer rotating speed optical-electricity encoder \((t_9)\) | 0.01                  | 1000 | 10           |

The ESCS fault-sensor dependency matrix is shown in table 3.

**Table 3.** The fault-sensor dependency matrix of ESCS.

|        | \( t_1 \) | \( t_2 \) | \( t_3 \) | \( t_4 \) | \( t_5 \) | \( t_6 \) | \( t_7 \) | \( t_8 \) | \( t_9 \) |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( f_1 \) | \( (1,1) \) | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         |
| \( f_2 \) | 0         | \( (1,1) \) | 0         | 0         | 0         | 0         | 0         | 0         | 0         |
| \( f_3 \) | 0         | 0         | \( (1,1) \) | 0         | 0         | 0         | 0         | 0         | 0         |
| \( f_4 \) | 0         | 0         | 0         | \( (1,1) \) | \( (1,1) \) | 0         | 0         | 0         | 0         |
| \( f_5 \) | 0         | 0         | 0         | 0         | \( (1,0) \) | \( (1,0) \) | 0         | 0         | 0         |
| \( f_6 \) | 0         | 0         | 0         | 0         | 0         | \( (1,0) \) | \( (1,0) \) | 0         | 0         |
| \( f_7 \) | 0         | 0         | 0         | 0         | \( (1,0) \) | 0         | \( (1,0) \) | 0         | 0         |
| \( f_8 \) | 0         | 0         | 0         | \( (1,1) \) | 0         | \( (1,0) \) | 0         | 0         | 0         |
| \( f_9 \) | 0         | 0         | 0         | 0         | 0         | 0         | \( (1,1) \) | \( (1,0) \) | \( (1,0) \) |
| \( f_{10} \) | 0         | 0         | 0         | 0         | 0         | 0         | 0         | \( (1,1) \) | 0         |
| \( f_{11} \) | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | \( (1,0) \) |

Supposing all the sensor SNR is 10dB, sensor resolution is 0.01, and all the fault resolution is 1, fault symptom duration time equals to time to failure. The detection sensitivity of all the sensors to all the faults is 0.9. According to (14), \( \rho_{ij} \) \((i=1,2,...,11, j=1,2,...,9)\) calculation results are shown in table 4.

**Table 4.** The ESCS \( \rho_{ij} \) results.

|        | \( t_1 \) | \( t_2 \) | \( t_3 \) | \( t_4 \) | \( t_5 \) | \( t_6 \) | \( t_7 \) | \( t_8 \) | \( t_9 \) |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( f_1 \) | 0.9211    | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         |
| \( f_2 \) | 0         | 0.9316    | 0         | 0         | 0         | 0         | 0         | 0         | 0         |
| \( f_3 \) | 0         | 0         | 0.9053    | 0         | 0         | 0         | 0         | 0         | 0         |
| \( f_4 \) | 0         | 0         | 0         | 0.8216    | 0.8783    | 0.8504    | 0         | 0         | 0         |
| \( f_5 \) | 0         | 0         | 0         | 0         | 0.9571    | 0.9159    | 0         | 0         | 0         |
| \( f_6 \) | 0         | 0         | 0         | 0         | 0         | 0.9159    | 0.9106    | 0         | 0         |
| \( f_7 \) | 0         | 0         | 0         | 0         | 0         | 0.9571    | 0         | 0.9106    | 0         |
According to (15)-(17), fault detection uncertainties can be calculated, as shown in table 5.

Table 5. The related calculation results under uncertainty test.

|     | $R_i^1$ | $P_i^1$ | $SFDP_i^1$ | $R_i^2$ | $P_i^2$ | $SFDP_i^2$ |
|-----|---------|---------|------------|---------|---------|------------|
| $f_1$ | 0.9999  | 0.9211  | 0.9210     | 0.9999  | 0.9211  | 0.9210     |
| $f_2$ | 0.9999  | 0.9316  | 0.9315     | 0.9999  | 0.9316  | 0.9315     |
| $f_3$ | 0.9999  | 0.9053  | 0.9052     | 0.9999  | 0.9053  | 0.9052     |
| $f_4$ | 1.0000  | 0.8501  | 0.8501     | 1.0000  | 0.8501  | 0.8501     |
| $f_5$ | 1.0000  | 0.9365  | 0.9365     | 0.0000  | 0.0000  | 0.0000     |
| $f_6$ | 1.0000  | 0.9133  | 0.9133     | 0.0000  | 0.0000  | 0.0000     |
| $f_7$ | 1.0000  | 0.9338  | 0.9338     | 0.0000  | 0.0000  | 0.0000     |
| $f_8$ | 1.0000  | 0.8755  | 0.8755     | 0.9999  | 0.8672  | 0.8671     |
| $f_9$ | 1.0000  | 0.8796  | 0.8796     | 0.9999  | 0.8700  | 0.8699     |
| $f_{10}$ | 0.9999 | 0.8810  | 0.8809    | 0.9999  | 0.8810  | 0.8809     |
| $f_{11}$ | 0.9999 | 0.9510  | 0.9509    | 0.0000  | 0.0000  | 0.0000     |

According to (18), the testability requirements for the ESCS PHM under uncertainty test are: FDR=0.9951, FIR=0.9827 and FPR=0.9992. One can see that the testability level of the ESCS is very good and hence can provide sufficient state information for the ESCS PHM.

5. Conclusions

To address the uncertainty existing in the testability requirements modeling for PHM in engineering applications, fault detection uncertainty is analyzed systematically from fault attributes, sensor attributes, fault-sensor matching attributes respectively. Further, quantitative uncertain analysis is described, which can assign a rational confidence level to the actual defect size. Based on the uncertainty analysis, testability requirement model for PHM is proposed. Due to considering the fault prognostics requirements for testability (FPR) and fault detection uncertainty, the model can be used to sensor selection and localization for PHM very well. At last, a ESCS is taken as an example to illustrate the presented approach. The application results show the proposed method is feasible and reasonable and can guide SSO for PHM in real applications.

References

[1] Cheng S, Azarian M and Pecht M 2010 J. Sensors 10 5774
[2] Kalgren P W, Byington C S, Roemer M J and Watson M J 2006 Defining PHM—a lexical evolution of maintenance and logistics IEEE Systems Readiness Technology Conf. (Anaheim, CA, USA)
[3] Orsagh R F, Brown D W, Kalgren P W and Byington C S 2006 Prognostic health management for avionic systems IEEE Aerospace Conf. (Big Sky, MT)
[4] Cheng S, Tom K, Thomas L and Pecht M 2010 J. IEEE Sensors Journal 10 856
[5] Santi L M, Sowers T S and Aguilar R B 2005 Optimal sensor selection for health monitoring systems (NASA/TM-2005-213955)
[6] Zhang G F 2005 Optimum sensor localization/selection in a diagnostic/prognostic architecture Ph. D. thesis (Georgia: Georgia Institute of Technology)
[7] MIL-STD-2165 26 JANUARY 1985
[8] William A M, George K and Louis M S 2007 Sensor selection and optimization for health assessment of aerospace systems (NASA/TM-2007-214822)
[9] Sowers S, Kopasakis G and Simon D L 2008 Application of the systematic sensor selection strategy for turbofan engine diagnostics (NASA/TM-2008-215200)
[10] Maul W A and Kopasakis G 2007 Sensor selection and optimization for health assessment of aerospace systems (NASA/TM-2007-214822)
[11] Simon D L and Garg S 2009 A systematic approach to sensor selection for aircraft engine health estimation (NASA/TM-2009-215839)
[12] Cheng S, Tom K and Pecht M 2010 J. IEEE Transactions on Devices and Materials Reliability 10 374
[13] Kwon D, Azarian M H and Pecht M 2011 J. IEEE Sensors Journal 11 1236
[14] Xu Z, Koltsov D and Richardson A 2010 Design and simulation of a multi-function MEMS sensor for health and usage monitoring Prognostics & System Health Management Conf. (PHM’10 Macau, China)
[15] Novis A and Powrie H E G 2006 PHM sensor implementation in the real world-a status report Proc. IEEE Aerospace Conf. (Montana, USA)
[16] Baer W G and Lally R W 2000 An open-standard smart sensor architecture and system for industrial automation Proc. IEEE Aerospace Conf. (Big Sky, MT, USA)
[17] Xie G J 2006 Research on the real time fault detection technology and system for liquid-propellant rocket engine turbopump Ph. D. thesis (Changsha: National Universtiy of Defense Technology)
[18] Liu G and Qu L S 2002 J. Journal of Vibration Engineering 15 106
[19] Tang L, Kacprzynski G J and Goebel K 2008 Methodologies for uncertainty management in prognostics Proc. IEEE Aerospace Conf. (Big Sky, MT, USA)
[20] Nondestructive evaluation system reliability assessment (MIL-HKBK-1823)
[21] Tian Z and Shi J Y 2003 System testability design, analysis and verification (Beijing: Beihang University Press)