Darwinian Selection and Non-existence of Nash Equilibria.

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We study selection acting on phenotype in a collection of agents playing local games lacking Nash equilibria. After each cycle one of the agents losing most games is replaced by a new agent with new random strategy and game partner. The network generated can be considered critical in the sense that the lifetimes of the agents are power law distributed. The longest surviving agents are those with the lowest absolute score per time step. The emergent ecology is characterized by a broad range of behaviors. Nevertheless, the agents tend to be similar to their opponents in terms of performance.

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It has been argued that biological evolution is driven by a combination of natural selection and self-organization \[1\,2\]. Mutations act at random at the level of genotype while selection acts at the phenotype. Darwinian selection is expected to occur because some phenotypes are more viable than others \[3\]. The viability or fitness of a given phenotype is, of course, not an absolute quantity but depends on context and the environment the phenotype is exposed to \[4\]. The environment (physical as well as biotic) is to a large extent produced by co-existing phenotypes, and hence the environment and the corresponding fitness are self-organized emergent properties of the ecology. Moreover, fitness or vigor is necessarily a relative quantity. An organism can become so vigorous that it removes its own foundation of existence; a balance very relevant e.g. to host-parasite systems. For organisms to be able to coexist for extended periods of time they need to develop phenotypes which are in a kind of restrained poise.

Here we analyze Darwinian selection acting on phenotype in a model consisting of an adaptive network of agents of different strategies mutually competing in local zero-sum games lacking Nash equilibria. We let selection act by removing the agents with the smallest score (most negative) after a round of games. Thus the viability of an agent is a function of his phenotypical behavior in a specific environment. When the local games are of opposing interest with only a single winner we find that the agents organize into a system where agents are homogeneous in terms of success but heterogeneous in terms of activity. The agents achieve longevity by organizing themselves in a way that minimize the absolute value of their score. By this strategy they avoid to be amongst the worst performers themselves; moreover low absolute score allows the agents to retain well tuned partners by not forcing many lost games on to their opponents, which would lead to the elimination of the partner.

Our model is closely related to previous network models. Kauffman introduced Boolean networks to study the relationship between evolution and self-organization \[1\].

Collective adaptive agents striving to be among the minority were studied in mean-field models by Arthur \[2\] and Challet and Zhang \[6\]. Aspects of Darwinian evolution were added to the minority game by Zhang \[7\]. This line of approach were further developed by Paczuski, Bassler and Corral (PBC) who considered agents playing the minority game \[8\]. The goal of agents is to be amongst the global minority though they are linked only to a subset of all the agents. The strategy of the worst performing agent is randomly changed after each cycle while the links of the network remain unchanged \[8\].

Here we modify the model introduced by PBC to a Local Darwinian Network model (LDN) in which we allow agents to play competitive local games and to adapt their connections as well as their strategies. Viewed from the perspective of evolutionary ecology it appears to be more natural to consider agents performing local games (organisms or species entangled in a web of mutual competitions). The renewal of the properties of the worst performing agent should be thought of as representing either the “mutation” of an individual or, if agents are thought of as representing species, as a species being superseded by an invading species with different affinities. In either case it is natural to update the set of interaction links. Further, from a statistical mechanics or complex systems perspective we expect systems coupled globally to more readily enter a critical state, hence it is of interest to explore the criticality of purely local systems.

The LDN net is critical for \(K = 2\) as is the original Kauffman net \[1\]. The PBC net is critical for \(K = 3\) \[8\]. Here \(K\) is the number of independent arguments of the agents’ Boolean strategy functions.

The homogeneity parameter \(P\) of an agent measures the fraction of 0’s and 1’s – whichever is in majority – in the Boolean output assigned to the \(K^2\) distinct input states of the agent. For \(K = 2\), the homogeneity parameter is constrained to one of the values 1/2, 3/4 or 1 where in the last case the agent will not switch output signal at all. In contrast to PBC, we fix the homogeneity parameter of each agent to 3/4 ensuring that the network is logically entirely homogeneous with respect to the output of the agents. It follows that any discrepancy in the switching behavior of two competing agents is of dynamical origin.
The model – Consider N agents, each assigned a Boolean signal $S_i = 0$ or 1, for $i = 1,\ldots,N$. The dynamics consists of two types of updates: a) the agents playing rounds of games, b) and the act of Darwinian selection/mutation.

a) The games are performed in the following way. Each agent is defined as aggressor (A) of one other randomly assigned agent (thus agents participate on average in two games) who acts as opponent (O) in a zero sum game. In each time step A and O compare their Boolean signals $S_A$ and $S_O$. If $S_A = S_O$ the aggressor A scores +1 and the opponent scores –1. If $S_A \neq S_O$ the aggressor A scores –1 and the opponent scores +1. The game is identical to the simple “coin guessing” game where one of two persons is to pick the hand – left or right - that is holding the coin. The game has the property that, when considering only pure (non-randomized) strategies, there exists no Nash equilibrium. No Nash equilibrium exists in the sense that it will always be possible for either player to benefit by changing his strategy if the opponent sticks to his strategy. In our simulations, two agents are never allowed to act as mutual aggressors in which case both agents could receive a zero score. Neither do we allow agents to act as aggressor for themselves. The fact that agents cannot collaborate in order to achieve neutral (zero) scores will be referred to as an opposing interests property of the network.

The signals used for the zero-sum games are generated in the following way. The output signal at a given time step $S_i(t)$ of agent $i$ is determined deterministically from the output at the previous time step from $K$ other source agents $S_i(t-1), S_{i2}(t-1), \ldots, S_{iK}(t-1)$

$$S_i(t) = f_i(S_{i1}(t-1), S_{i2}(t-1), \ldots, S_{iK}(t-1)),$$

where $f_i$ is a randomly chosen Boolean function associated with agent $i$. For $K = 2$ the function is such that for exactly three of the functions four input configurations $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$ the same Boolean output is assigned, whereas the signal is different for the fourth configuration. This ensures $P = 3/4$.

One can think of a specific choice for the functional form of $f_i$ as the genotype of agent number $i$. The K source-agents associated with $i$ can include the opponent of $i$ but will not include agent $i$. The source-agents and the opponents are to be thought of as the environment of an agent.

For fixed assignments of source-agents as well as aggressor and opponents Eq. 1 is now parallel updated $\forall i$. Since the state space of the net is finite and the dynamics is deterministic, periodic orbits will always be reached (though the maximum length of a period is huge $2^N$). We measure the average score per time step of the agents over a round of games consisting of either simulating the transient plus one periodic cycle or by performing $10^4$ parallel updates which ever is the shortest. We denote this sequence of deterministic updates a test cycle. The score gained by an agent characterizes the success of his phenotypical behavior.

b) The Darwinian update is done in the following way. After testing the performance of the agents, the worst performing agent is replaced by a new agent with a new Boolean function (i.e., genotype) chosen completely at random. (In case the worst performance is shared by more then one agent, then one of them is chosen at random for replacement.) The new agent is randomly assigned an opponent and a set of K source-agents. Intuitively, this means that agents belonging to ecological niches that no longer exist are removed from the system, whereas new candidate niches are sampled fortuitously by agents who themselves have random properties. However, the wiring of those agents that attack an agent being replaced is not changed. These agents now attack the new agent as we proceed with a new round of games, or test cycle, as described under a).

In this letter, we use a Non-Darwinian version of the model as benchmark allowing us to isolate the effect of the Darwinian updating. In the Non-Darwinian model, at the end of each test cycle, we randomly pick an agent for replacement. Agents are in this situation not punished for performing poorly and are all subject to the same probability of being replaced.

Results - The initial configuration consists of randomly assigned input links, Boolean functions and game partners. As the sequence of test cycles and Darwinian replacements are repeated the model organizes gradually into a stationary state. The critical properties of the network are indicated in Fig. 1 where we exhibit the probability of the length of transients $p(t)$ and of the length of periodic attractors $p(a)$ respectively. The distributions do exhibit scale free power law like behavior, though an accurate determination of the precise functional form (and precise values of the exponents) is unfortunately not possible. The behavior of $p(t)$ for $K = 2$ for large $N$ appears to be consistent with $p(t) \sim t^{-\alpha}$ for $\alpha \simeq 1.4$. The behavior of $p(a)$ for $K = 2$ for large $N$ appears to consistent with $p(a) \sim a^{-\beta}$ for $\beta \simeq 1.2$. The behavior of the Non-Darwinian version of the network is similar as shown in the figures. Hence the Darwinian move is not crucial for the existence of the power law like behavior of $p(t)$ and $p(a)$. Nevertheless we shall see below that the Darwinian selective move has a significant effect on the characteristics of the agents. A transient and corresponding first occurrence of periodic attractor make up a test cycle. We find that the distribution of test cycles also appears to be power law like. Notice that in order to find the largest observations in Fig. 1 we iterated the network more than the maximum number of updates of a test cycle and then reentered the state at $10^4$ updates to perform the Darwinian update.

We define the lifetime $l$ of an agent as the number of updates survived at the moment the agent is removed from the system. As exhibited in Fig. 2 we find that under Darwinian updating the network develops what appears to be a power law like distribution of lifetimes $p(l)$ with increasing system size, consistent with $p(l) \sim l^{-\gamma}$ for $\gamma \simeq 0.9$. Notably the magnitude of the power law
exponent is smaller than one. Since test cycles are numerically truncated at a finite length of $10^4$ updates, the distribution of lifetimes is somewhat distorted for lengths longer than the numerical cutoff. Because of logarithmic binning the effect is not visible in Fig. 2. We find that for all considered values of $N = 64, 128$ and 1024 the lifetime distribution for the Non-Darwinian model (the circles in Fig. 2) are approximately linear as function of the logarithm of the argument, i.e. not at all described by a power law.

Next we address the behavior of the agents in the stationary critical state. We denote by the lifetime yield of an agent the score averaged over the lifetime of the agent. The yield of an agent characterizes the efficiency of the agent’s phenotypical behavior. Fig. 3 shows that longevity is directly related to near zero values of the agent’s lifetime yield. Thus the selective pressure on the agents in the model leads to a finely tuned ecology, where agents tend to avoid winning too much, which might induce annihilation of their partners. Instead agents manage to keep their partners and live long by entering into near neutral patterns.

Fig. 3 is a histogram of yield accumulated over entire test cycles of the Darwinian and benchmark ecologies respectively. The accumulated yield of an agent is the average score of the agent over the length of his life so far. (Notice that agents were not born at the same time.) The standard deviation $\delta Y$ of the distribution of accumulated yield is a measure of the equality in success in the network. We find a smaller $\delta Y$ in the Darwinian ecology for all considered system sizes $N = 128, 256$ and 1024.

Indeed, one would expect that removal of the worst performing agents should reduce the abundance of poorly performing agents with large negative average scores. But we observe that the abundance of well performing agents with large positive scores is also somewhat reduced. In the Darwinian model agents are thus clustered around the zero score. Notice that given opposing interests of agents, the collected environment of an agent receives the same score as the agent but with opposite sign. In the Darwinian model, agents are therefore similar to their respective environments in terms of success. An interesting property of e.g. the Minority Game (MG) model is that any favorable configuration for a particular agent is inherently unstable as other agents act to reverse the situation. A similar situation holds for LDN as well but unlike MG the environments of the agents are heterogeneous.

Let us now describe in more detail how the agents manage to achieve low absolute scores and thereby survive many test cycles. For the benchmark and Darwinian model respectively, we have investigated the switching
activity of the agents. By a switch we mean the change from sending signal 0 to 1 or vice versa. By a distinct switching activity we mean a certain observed number of switches of one or more agents over a test cycle. The number \( W(N, t + a) \) of observed distinct activities cannot be larger than the number of agents in the ecology \( N \) nor the length of the test cycle \((t + a)\). We have investigated the dependence of \( W \) on \((t + a)\) for system sizes \( N = 128, 256 \) and 1024. We find that for all cycle lengths \( 2 \leq (t + a) < 10^4 \), the Darwinian ecology exhibits a larger ratio \( W/(t + a) \). This may suggest that the Darwinian updating of the network produces a greater variety of phenotypes.

We now discuss the robustness of the model. Increasing the number of opponents per agent does not appear to change the test cycle distribution. We do not observe criticality in case the input wiring of the agents is unchanged as the agents are replaced. We point out that, in general, testing the stationarity of the model is numerically challenging. One can consider an alternative version of the network where the homogeneity parameter is not fixed. In this situation, PBC find that when agents compete globally, the homogeneity parameter self-organizes to a distinct value as the network enters a critical state. In contrast, for the LDN model we find that the homogeneity parameter decreases from the initial unbiased value making the transients and periodic orbits very long, tough still, perhaps, power law distributed. This version of the model is difficult to handle numerically.

As pointed out by PBC, an interesting question is – what games lead to complex, scale free states under Darwinian selection? We speculate that the game should inherently prevent agents from achieving mutual gain while allowing the losing agents to improve their unfavorable situation by a change of behavior.

We suggest that the "coin-guessing" game of the LDN model is a suitable representation of such situations where agents tend to become equal in strength to their opponents implying their exact "task" becomes irrelevant. For example, the game is similar to many situations in the business world, e.g. the trade of a stock involving one seller and one buyer. From a dynamical point of view, it is significant that either the buyer or seller in a trade will necessarily become a winner and the other agent a loser as the price of the stock diverges from its value when the deal was made. (If the price of the stock goes up the seller would have been better off on the other side of the trade). As in the LDN model, the determinant of who is the winner is of complex dynamical origin.

The score distribution of the Darwinian model might be compared to e.g. the distribution of performances of fund managers. Interestingly, extensive investigations have given little evidence that fund managers can consistently beat relevant market indexes (see p. 368 in [9]). Similarly, the LDN model has the property that successful agents tend to be short-lived.

We have presented a Local Darwinian Network model which exhibits critical behavior without global interactions. The model demonstrates that – given that agents act under opposing interest with their respective environments – Darwinian phenotypical selection of the worst performer together with genotype mutations can lead to an ecology where the distribution of lifetimes lacks a characteristic scale. The agents of the emergent ecology are heterogeneous with respect to their activity but in terms of success they are nearly equal to their respective environments. Longevity is achieved by adapting to a near neutral yield which ensures the stability of the fellow partners.

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