Abstract

Over 60 authors have worked on various versions of self-creation cosmology (SCC) since the original paper in 1982. These papers adapted the Brans Dicke theory to create mass out of the universe’s self contained scalar, gravitational and matter fields. The most recent 2002 version of the theory was concordant with all previous standard tests of GR but was falsified by the Gravity Probe B geodetic precession experiment. Here the different versions of the theory are reviewed and it is noted that the 2002 theory not only reduces to the second 1982 theory when cast into a the ‘true’ form of the scalar field stress-energy tensor but also then passes the GP-B test. Further experiments are able to resolve the SCC-GR degeneracy, one of which is briefly described. SCC may be able to explain some intriguing anomalies also, in the spherically symmetric One-Body problem, it transpires that the temporal Newtonian potential is three times larger than the spatial one. Cosmological solutions to the field equations, which have been published elsewhere, are here extended to show that the cosmological density parameter Omega = 1.

1 A résumé of SCC papers

All self-creation cosmology (SCC) papers (Barber, 1982, 2002) modify the Brans Dicke theory (Brans & Dicke, 1961) to include mass creation. The first paper (Barber, 2002) suggested two toy models, the first of which was rejected in that paper on the grounds of a gross violation of the equivalence principle and experiment while Brans (Brans, 1987) subsequently showed that it was also internally inconsistent. The second theory (SCC2), however, has led to over 80 citations. A more comprehensive version was published (Barber, 2002), that was unfortunately cast in a form in which its prediction of geodesic precession (Barber, 2006) was subsequently falsified by the Gravity Probe B (GPB) experiment. This falsification has stimulated further work. Here a corrected form is presented which fortuitously proves to be a version of the second 1982 theory. It not only passes all experimental tests to date (now including GPB) but also makes an interesting cosmological model.
Many authors have tried to include a scalar field in General Relativity (GR), not only to more fully include Mach’s Principle (such as in BD), but also to accommodate certain attempts at quantum gravity in which scalar tensor theories that have proven to describe the low energy limit. Unfortunately, one persistent problem has been the perturbation of the metric by the addition of a scalar field consequentially the experimental agreement with GR has been lost. As a consequence only a very weak field in such theories has been thought to be empirically viable, at least within the Solar System laboratory. This problem is overcome in SCC where the effect of a non-minimally connected scalar field exactly compensates for the perturbation of space-time, thus restoring its predictions to those of GR.

This non-minimal connection of the scalar field to matter in the SCC Jordan frame (JF) identifies it in Quiros’ classification scheme as a class II scalar-tensor theory (Quiros, 1999). A special feature of the theory is the JF locally conserves energy (measured in the ‘Machian’, i.e. Centre of Mass, frame of reference) whereas the Einstein frame (EF) conserves energy-momentum. This requirement, together with the principle of mutual interaction (see below), determines the coupling constant to be $\omega = -\frac{3}{2}$. As a consequence the EF of the theory proves to be canonical GR \textit{in vacuo}.

The question as to which frame is physical, that is that which relates to experimental measurement, depends in this theory on the physical standard chosen to measure time. On the one hand, in the EF, where atomic masses are constant, the time standard is set by an atomic clock and a second may be defined conventionally as the duration of exactly $9.19263177\times10^9$ periods of the radiation emitted by the transition between the two hyperfine levels of the ground state of the caesium 133 atom. On the other hand, in the JF, a second may be defined as the duration of exactly $1.604.10^{11}$ periods of the radiation corresponding to the peak of the isotropic CMB black body spectrum. As both systems of time measurement are physically significant both definitions are ‘physical’ in an experimental sense. Time is the fundamental measurement in both frames, determined by Bohr atomic frequencies in the EF and the frequency of a reference photon, carefully defined, in the JF. By definition the speed of light \textit{in vacuo} is invariant in both. The two frames are synchronized at some local event in the present epoch used to set laboratory standards but they will generally diverge at other times and locations. It is the contention of this theory that gravitational, and hence cosmological, problems have to be solved in the JF and this is used throughout unless specifically stated otherwise. Consequently ephemeris time is to be defined
in the JF and suffers a secular clock drift relative to time measured by atomic clocks.

The JF version of the theory has been described as 'semi-metric' because although there is a metric and photons do follow the geodesics of that frame, freely falling test particles do not. In vacuo an extra force acts on 'free falling particles', compensates for the perturbation of the GR metric. This results in SCC test particles following canonical GR geodesics in both frames and SCC and GR predictions are identical in all tests to date. Nevertheless, there are two further experiments that will be able to distinguish between them. In addition the theory also offers explanations for the Pioneer Anomaly and some other intriguing anomalies (Barber, 2002b).

2 Deriving the field equations

Following BD, SCC theories incorporate Mach’s Principle (MP) by assuming the inertial masses of fundamental particles are dependent upon their interaction with a scalar field $\phi \approx \frac{1}{G_N}$ coupled to the large scale distribution of matter in motion. This coupling is described by a field equation of the simplest general covariant form:

$$\Box \phi = 4\pi \lambda T_M ,$$  \hspace{1cm} (1)

where $T_M$ is the trace, $(T_M^\sigma \sigma)$, of the energy momentum tensor describing all non-gravitational and non-scalar field energy and $\lambda$ is some undetermined coupling constant of the order unity. In the spherically symmetric One Body problem of BD

$$\lim_{r \to \infty} \phi (r) = \frac{\psi}{G_N} ,$$  \hspace{1cm} (2)

where $\psi$ is of the order unity and determined by $\lambda$. (see equation 145 below).

The gravitational field equation included the energy-momentum tensor of the scalar field energy $T_{\phi \mu \nu}$ where $T_M{\mu \nu}$ and $T_{\phi \mu \nu}$ are the energy momentum tensors describing the matter and scalar fields respectively.

In both GR and BD the equation describing the interchange of energy between matter and gravitation is,

$$\nabla_\mu T^\mu_M {\nu} = 0 ,$$  \hspace{1cm} (3)

This equation, which conserves four-momentum, is a consequence of the equivalence principle, and in the theory of BD it guarantees that the scalar
field interacts with matter only by adapting the curvature of space-time and in no other way, i.e. ordinary matter is minimally coupled to the scalar field. In BD and SCC the scalar field is included in the gravitational field equation which becomes
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} [T_{M\mu\nu} + T_{\phi\mu\nu}] . \] (4)

In the 1982 SCC2 theory the scalar field was minimally coupled to the metric and therefore only interacted with the material universe by determining the gravitational coefficient \( G \) with a field equation
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{M\mu\nu} . \] (5)

As \( \phi \) was not constant then \( \nabla_\mu T^\mu_{M\nu} \neq 0 \) and ordinary matter was non-minimally connected to the scalar field. However this hypothesis was criticized by Brans, (Brans, 1987), on the basis of the difficulty of defining a metric if the paths of photons are not null-geodesics. Nevertheless SCC2 continues to provoke discussion. [A selection from 81 'other-author' papers is given below].

2.0.1 Two principles

The principle of mutual interaction What is to constrain the process if mass is indeed created out of gravitational and scalar fields? The SCC answer is the postulate of the Principle of Mutual Interaction (PMI) (Barber, 2002), which states that: “The scalar field is a source for the matter-energy field if and only if the matter-energy field is a source for the scalar field.” In more specific terms, if the source for the scalar field is the trace of the matter stress-energy tensor then the divergence of the matter stress-energy tensor should be coupled to this trace.

The conservation equation is consequently replaced in SCC with the PMI 'creation' equation of the form
\[ \nabla_\mu T^\mu_{M\nu} = f_\nu (\phi) \Box \phi = 4\pi \lambda f_\nu (\phi) T_M \] (6)

As a consequence relativistic energy such as light, which is trace free, still obeys the Equivalence Principle and Brans’ latter criticism is resolved because, at least in vacuo,
\[ \nabla_\mu T^\mu_{em\nu} = 4\pi \lambda f_\nu (\phi) T_{em} = 4\pi \lambda f_\nu (\phi) (3p_{em} - \rho_{em}) = 0 \] (7)
where $p_{em}$ and $\rho_{em}$ are the pressure and density of an electromagnetic radiation field with an energy momentum tensor $T_{em\mu\nu}$ and where $p_{em} = \frac{1}{3} \rho_{em}$. So although the equivalence principle is violated in general it is not so for photons, which still travel through empty space on (null) geodesic paths. Therefore, although the theory is not fully metric in the classical sense, as photons still do obey the equivalence principle it might be called a semi-metric theory. On the other hand particles with mass suffer a 'scalar field force' perturbing their trajectories from geodesics. In other words, ordinary matter and relativistic energy are non-minimally and minimally coupled to the scalar field respectively.

**The local conservation of energy**  A second principle, that of the principle of the local conservation of energy complements equation 6 and fully determines the theory. This postulates that the potential energy expended in moving an object in a gravitational field should translate into an increase in rest mass. If $\Phi_N (x^\mu)$ is the dimensionless Newtonian gravitational potential defined by a measurement of acceleration in a local experiment in a frame of reference co-moving with the Centre of Mass frame (CoM), with

$$\frac{d^2 r}{dt^2} = -\nabla \Phi_N (r)$$

and normalized so that $\Phi_N (\infty) = 0$, then

$$\frac{1}{m_p (x^\mu)} \nabla m_p (x^\mu) = \nabla \Phi_N (x^\mu),$$

where $m_p(x^\mu)$ is measured locally at $x^\mu$ and $c = 1$ throughout. This has the solution

$$m_p(x^\mu) = m_0 \exp[\Phi_N (x^\mu)],$$

where

$$m_p (r) \to m_0 \quad as \quad r \to \infty.$$  

This can be seen by considering time dilation observed as the gravitational red shift of light.

**The gravitational red-shift of light**  The gravitational red shift of light is now to considered in order to examine the measurement problem in both the EF and the JF. This analysis depends on the assumption that if no work
is done on, or by, a projectile while in free fall then its energy $E$, $P^0$, is conserved *when measured in a specific frame of reference*, that of the CoM of the system. In a gedanken, 'thought', experiment, construct a laboratory at the co-moving centroid, the CoM, of the system. Connect it to the outside world by a radial tube through which identical test masses and photons may be projected *in vacuo*. Launch such projectiles, with rest masses, $m_0$, at the CoM at various velocities to reach maximum altitudes $r_i$ where $r_i$ varies increasing from $R$, the radius of the central mass, to infinity. The 'rest' mass of the projectile $m_c(r)$, the 'coordinate' mass, is in general to be a function of altitude measured in the CoM frame of reference.

First consider such a photon emitted by one atom at altitude $x_2$ and absorbed by another at an altitude $x_1$. The emission and absorption frequencies of the photon, $\nu(x_2)$ and $\nu(x_1)$, are determined by comparing the arrival times of two adjacent wave fronts emitted from one point in a gravitational field at ($x_2$) and received at another at ($x_1$). The standard time dilation relationship is thereby derived

$$\frac{\nu(x_2)}{\nu(x_1)} = \left[\frac{-g_{00}(x_2)}{-g_{00}(x_1)}\right]^{\frac{1}{2}}. \quad (11)$$

Hence substituting $x_2 = r$ and $x_1 = \infty$ in equation $[11]$ where $g_{00}(x_1) = -1$, and writing $\nu(\infty)$ as $\nu_0$, yields the standard (GR) gravitational red shift relationship

$$\nu(r) = \nu_0 \left[-g_{00}(r)\right]^{\frac{1}{2}}, \quad (12)$$

where the observer is at infinite altitude receiving a photon emitted at altitude $r$.

Now consider the various projectiles. With the standard definition of proper time $\tau$ from the metric

$$d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu. \quad (13)$$

The 4-momentum vector of the projectile is defined

$$P^\mu = m_c \frac{dx^\mu}{d\tau}. \quad (14)$$

The time component of 4-momentum $P^\mu$ is the total 'relative' energy $E$ and the space components form the 'relative' 3-momentum $p$. 

6
Now from equation (13) we obtain

\[
\frac{d\tau^2}{dt^2} = -g_{00} - 2g_{i0}v^i - v^2 ,
\]

(15)

where \(v^i = \frac{dx^i}{dt}\) and \(v^2 = g_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}\).

(16)

Therefore in a spherically symmetric, non-rotating, metric with \(g_{i0} = 0\),

\[-g_{00}E^2 = m_c^2 + p^2 .\]

(17)

This is the spherically symmetric curved space-time equivalent to the SR identity

\[E^2 = m_c^2 + p^2 .\]

(18)

Now consider two of the projectiles as they momentarily reach their respective apocentres at maximum altitude \(r\), and \(r + \delta r\). As they are momentarily stationary in the CoM frame \(p = 0\). The difference between the two adjacent projectiles at their apocentres is that one has a total energy and rest mass of \(E(r)\), and \(m_c(r)\), and the other \(E(r + \delta r)\), and \(m_c(r + \delta r)\).

Expanding for small \(\delta r\), and where a prime (‘) means \(\frac{d}{dr}\), in the limit \(\delta r \to 0\) we obtain

\[
\frac{1}{2} \left[ -g'_{00}(r) \right] + \frac{E'(r)}{E(r)} = \frac{m'_c(r)}{m_c(r)} .
\]

(19)

Here two identical projectiles are compared which are separated by an infinitesimal increase in altitude. The only difference between them is the infinitesimal energy \(\delta E\) required to raise such a projectile from \(r\) to \(r + \delta r\).

Although the Newtonian potential \(\Phi_N (r)\) is defined by

\[
\nabla^2 \Phi_N (r) = 4\pi G_N T^{00} = 4\pi G_N \rho
\]

(20)

which is normalized,

\[\Phi_N (\infty) = 0 ,\]

it is actually measured in a Cavendish type laboratory experiment by the force vector acting on a body, which is given by

\[\mathbf{F} = -m_p \nabla \Phi_N (r) .\]

(21)
Then if the mass \( m (r) \) is raised a height \( \delta r \) against this force, the infinitesimal energy \( \delta E \) required is

\[
\delta E = - F \circ \delta r = m_p (r) \nabla \Phi_N (r) \circ \delta r .
\]  

(22)

That is in the radial case

\[
\delta E = m_p (r) \Phi'_N \delta r ,
\]  

(23)

where \( m_p (r) \) is that physical mass entering into the Newtonian gravitational equation. Define such physical mass, momentarily at rest, as

\[
m_p (r) = E (r) ,
\]  

(24)

so that the total ”relative” energy at an altitude \( r \) is its rest mass at that altitude, measured in the CoM frame of reference. In the limit \( \delta r \to 0 \) equations [23] and [24] become

\[
\frac{E' (r)}{E (r)} = \Phi'_N (r) ,
\]  

(25)

which when substituted in equation [19] yields

\[
\frac{1}{2} \left[ -g'_{00} (r) \right] + \Phi'_N (r) = \frac{m'_c (r)}{m_c (r)} .
\]  

(26)

This integrates directly,

\[
\frac{1}{2} \ln \left[ -g_{00} (r) \right] + \Phi_N (r) = \ln [m (r)] + k
\]  

(27)

where \( k \) is determined in the limit \( r \to \infty, g_{00} (r) \to -1, \Phi_N (r) \to 0 \) and \( m (r) \to m_0 \). The rest mass, \( m (r) \), of a projectile at altitude \( r \), evaluated in the co-moving CoM frame is therefore given by

\[
m_c (r) = m_0 \exp \left[ \Phi_N (r) \right] \left[ -g_{00} (r) \right]^\frac{1}{2} .
\]  

(28)

This is the value, \( m_c (r) \), given by an observer at infinite altitude, where Special Relativity and a ground state solution to the theory are recovered, with well defined particle rest mass \( m_0 \), 'looking down' to a similar particle at an altitude \( r \). From this expression it is obvious that with our assumption of the conservation of energy, \( P^0 \), in the CoM frame gravitational time dilation,
the factor \([-g_{00}(r)]^{\frac{1}{2}}\), applies to massive particles as well as to photons. As physical experiments measuring the frequency of a photon compare its energy with the mass of the atom it interacts with, it is necessary to compare the masses (defined by equation 28) of two atoms at altitude, \(r\) and \(\infty\), with the energy (given by equation 12) of a ”reference” photon transmitted between them. This yields the physical rest mass \(m_p(r)\) as a function of altitude

\[
\frac{m_p(r)}{\nu(r)} = \frac{m_0}{\nu_0} \exp [\Phi_N (r)] .
\]

Equation 29 is a result relating observable quantities, but how is it to be interpreted? In other words how are mass and frequency to be measured in any particular frame? In the GR EF (and BD JF) the physical rest mass of the atom is defined to be constant, hence prescribing \((\tilde{\eta}^\mu)\), with \(m_p(\tilde{r}) = m_0\). In this case equation 29 becomes

\[
\nu(\tilde{r}) = \nu_0 \left( 1 - \tilde{\Phi}_N(\tilde{r}) + ... \right) .
\]

Hence photons transmitted out of a gravitational potential well are said to exhibit a red shift which is equal to the dimensionless Newtonian potential \(\tilde{\Phi}_N\), and equal in GR, ”coincidentally”, to the time dilation effect, the factor \([-g_{00}(\tilde{r})]^{\frac{1}{2}}\). That is, compared to reference atoms they mysteriously appear to lose (potential) energy.

However in the SCC JF rest mass is given by the expression equation 10, consequently a comparison of equation 29 with the equation for rest mass in this frame yields

\[
\nu(r) = \nu_0 .
\]

Therefore in the SCC JF, in which energy is locally conserved, gravitational red shift is interpreted not as a loss of potential energy by the photon but rather as a gain of potential energy by the apparatus measuring it. Time dilation is subsumed into the change of mass of the observer’s apparatus. The ’absolute’ time of the JF is defined in a specific or ’preferred’ frame of reference, which is that one ’anchored’ to the co-moving centroid, or centre-of-mass of the system.

It is important to note that in this frame the frequency, and hence wavelength and energy, of a free photon is invariant, even when traversing curved space-time.
On the other hand, as experiments using physical apparatus refer measurements of energy and mass to the mass of the atoms of which they are composed, such observations interpret rest masses to be constant by definition. In SCC such experiments are analysed in the EF in which equation 30 describes gravitational red shift. Using either frame the gravitational red shift prediction in SCC is in agreement with GR and all observations to date.

Because physical rulers and clocks vary with atomic masses in the JF SCC interprets physical observations by using light to fulfill the fundamental role of measuring the universe. This may be seen to be a similar, but more general, method used in E.A.Milne’s theory of Kinematic Relativity. (Milne, 1935, 1948).

2.1 The SCC conformal frames of reference

Weyl’s hypothesis, (Weyl, 1918), led to the concept that the space-time manifold $M$ is not equipped with a unique metric as in GR but a class $[g_{\mu\nu}]$ of conformally equivalent Lorentz metrics $g_{\mu\nu}$. In a conformal transformation one metric transforms into a physically equivalent alternative according to

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}.$$  \hspace{0.5cm} (32)

The self creation, $(\nabla_\mu T^\mu_M \neq 0)$, of SCC requires the JF scalar field to be non-minimally connected to matter, hence it is an example of the work of Magnano and Sokolowski (Magnano & Sokolowski, 1994) who applied conformal duality to GR in order to include a scalar field as an additional source of gravity. In their case [in contrast to BD, (Dicke, 1962)] ordinary matter is non-minimally coupled to the scalar field in the JF and it is minimally coupled in the EF. In the JF particle masses and the Gravitational ‘constant’ vary, whereas in the EF they are both constant.

The Lagrangian density in the JF is given by

$$L^{SCC}[g, \phi] = \frac{\sqrt{-g}}{16\pi} \left( \phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) + L_{\text{matter}}[g, \phi],$$ \hspace{0.5cm} (33)

which, on varying the metric components produces the gravitational field equation,
\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi}{\phi} T_{M\mu\nu} + \frac{\omega}{\phi^2} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\sigma \phi g^{\mu\sigma} \nabla_\sigma \phi \right) \] (34)

\[ + \frac{1}{\phi} \left( \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right) , \]

and the conformal dual of equation (33) is, (Dicke, 1962)

\[ L^{SCC}[\tilde{g}, \tilde{\phi}] = \frac{\sqrt{-\tilde{g}}}{16\pi} \left[ \tilde{\phi} \tilde{R} + 6 \tilde{\phi} \Box \ln \Omega \right] + \tilde{L}^{SCC}_{\text{matter}}[\tilde{g}, \tilde{\phi}] \] (35)

\[ - \frac{\sqrt{-\tilde{g}}}{8\pi} (2\omega + 3) \frac{\tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \Omega}{\Omega^2} \] (36)

\[ - \frac{\sqrt{-\tilde{g}}}{16\pi} \omega \left[ 4 \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \tilde{\phi} + \tilde{g}^{\mu\nu} \tilde{g}_{\mu\phi} \tilde{g}^{\nu\phi} \right] . \]

As mass is conformally transformed according to

\[ m(x^\mu) = \Omega \tilde{m}_0 \] (37)

(see Dicke, 1962), where \( m(x^\mu) \) is the mass of a fundamental particle in the JF and \( \tilde{m}_0 \) its invariant mass in the EF then equations (31) and (37) require

\[ \Omega = \exp [\Phi_N(x^\mu)] . \] (38)

The metrics thus relate \textit{in vacuo} according to equation (32)

\[ g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \exp [2\Phi_N(x^\mu)] g_{\mu\nu} , \] (39)

where \( \tilde{g}_{\mu\nu} \) is the GR metric. Compare this to Nordström’s pre-GR attempt to develop a relativistic gravitational theory by a conformal mapping of the Minkowski metric \( \eta_{\mu\nu} \),

\[ \eta_{\mu\nu} \rightarrow g_{\mu\nu} = \exp [2\Phi_N(x^\mu)] \eta_{\mu\nu} . \] (40)

(Nordström, 1913, see also review by Brans, 1997). Whereas Nordström tried to include gravitational potential energy in Special Relativity (SR) by conformally mapping \( \eta_{\mu\nu} \) the SR metric, in SCC it is included by a conformal mapping onto \( \tilde{g}_{\mu\nu} \) the GR metric. However this mapping is only exact \textit{in vacuo}, therefore SCC is not a simple conformal mapping of GR. There is a
significant physical difference which will reveal itself as the theory develops, although the vacuum solutions of geodesic orbits are the same in both theories. Note also the dimensionless $Gm^2$ will not be invariant under translation within a gravitational field because of the real variation of mass caused by such inclusion of potential energy.

**The transformation of $\phi$**  The question how $\phi$ transforms has to be addressed if SCC is to concatenate potential energy within the definition of inertial mass. In scalar-tensor theories the conformal transformation of the scalar field is assumed to depend on the dimensionless and therefore invariant,

$$Gm^2 = \tilde{G}\tilde{m}^2$$

so by equations 2 and 37 we have

$$\tilde{\phi} = \phi\Omega^{-2}.$$  (42)

So, if the conformal transformation $\Omega$ is defined by

$$\Omega = (G\phi)^\alpha$$

then

$$\tilde{\phi} = G^{-2\alpha}\phi^{(1-2\alpha)},$$

which, as $\tilde{\phi}$ is a constant in BD, determines $\alpha = \frac{1}{2}$ and therefore

$$\Omega = \sqrt{(G\phi)}.$$  (45)

This relationship in standard scalar-tensor theory depends on the assumption that it is $Gm^2$ that is invariant under the transformation and that then completely determines the conformal factor $\Omega$. However in the SCC JF the local conservation of energy requires a different form for $\Omega$, is this in fact possible? In responding to this question we note that in this frame potential energy is to be convoluted with inertial mass and hence gravitation, therefore it is reasonable to assume the dimensionless invariant has to be enlarged to include such potential energy in the form of the dimensionless Newtonian potential $\Phi_N (x^\mu)$. The general conformal invariant therefore becomes

$$Gm^2 f (\Phi_N) = \tilde{G}\tilde{m}^2$$

(46)
The form of \( f(\Phi_N) \) is determined by the exact relationship between \( G\phi(x_\mu) \) and \( \Phi_N(x_\mu) \).

[For example, in the static dust filled universe where \( \nabla^2\phi = -4\pi \lambda \rho \), and \( \nabla^2\Phi_N = -4\pi G \rho \) the dimensionless \( f(\Phi_N) \) is determined to be (when \( \lambda = 1 \) as below)

\[
f(\Phi_N) = G_N \phi \exp(2G_N\phi).
\]

In the EF frame \( \tilde{m}(x_\mu) \) is constant, consequently so are \( \tilde{G} \) and hence \( \tilde{\phi} \) and

in this case

\[
\nabla_\alpha \tilde{\phi} = 0.
\]  

(47)

2.1.1 The SCC Conformal Einstein Frame

If the following conditions hold

i) \( \Box \ln \Omega = 0 \), ii) \( \omega = -\frac{3}{2} \) and iii) \( \nabla_\alpha \tilde{\phi} = 0 \), then the EF

Langrangian density equation 35 reduces to

\[
L^{SCC}[\tilde{g}] = \frac{\sqrt{-\tilde{g}}}{16\pi G_N} \tilde{R} + \tilde{L}^{SCC}_{\text{matter}}[\tilde{g}],
\]

where matter is now minimally connected and the conformal transformation

of the Lagrangian density reduces to that of canonical GR.

The energy-momentum tensor of matter is thereby defined by

\[
\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-\tilde{g}}} \frac{\partial}{\partial \tilde{g}^{\mu\nu}} \left( \sqrt{-\tilde{g}} \tilde{L}_{\text{matter}} \right).
\]

(49)

In SCC do these three conditions actually hold to achieve this vast simplification?

The first holds as a static vacuum condition because, under the SCC conformal factor \( \Omega \), the term \( \Box \ln \Omega \) becomes \( \Box \Phi_N(\tilde{x}^\mu) \) and in a harmonic coordinate system this reduces to \( \tilde{\nabla}^2 \Phi_N(\tilde{x}^\mu) \), which equals zero in vacuo.

The second condition, \( \omega = -\frac{3}{2} \), will shortly be shown to be the case.

The third holds when the EF is defined as that in which \( \tilde{\phi} \) is constant.

Therefore, for a static metric in vacuo, such as in the Schwarzschild solution, SCC reduces to GR. As experiments verifying GR have only tested this situation \( [R_{\mu\nu} = 0] \), they will also concur with SCC. Other tests in which condition i) does not hold will be able to resolve the degeneracy between the two theories.
2.2 Determining the 'creation' equation

In order to determine $T_{\phi \mu \nu}$ and $f_{\nu} (\phi)$ in equations 4 and 6, equation 4 is written in the following mixed tensor form

$$T_{\mu M \nu} = \frac{\phi}{8\pi} \left( R_{\nu}^{\mu} - \frac{1}{2} \delta_{\nu}^{\mu} R \right) - T_{\phi \nu}$$

and Weinberg’s method is followed, (Weinberg, 1972, pages 158-160, equations 7.3.4-7.3.12). The most general form of $T_{\phi \nu}$ using two derivatives of one or two $\phi$ fields and $\phi$ itself is

$$T_{\phi \nu} = A(\phi)g^{\mu \sigma} \nabla_{\sigma} \phi \nabla_{\nu} \phi + B(\phi)\delta_{\nu}^{\mu} \nabla_{\sigma} \phi g_{\rho \sigma} \nabla_{\rho} \phi$$

$$+ C(\phi)g^{\mu \sigma} \nabla_{\sigma} \nabla_{\nu} \phi + D(\phi)\delta_{\nu}^{\mu} \Box \phi$$

and covariantly differentiating this produces

$$\nabla_{\mu} T_{\phi \nu} = \left[ A'(\phi) + B'(\phi) \right] g^{\mu \nu} \nabla_{\nu} \phi \nabla_{\nu} \phi \nabla_{\mu} \phi$$

$$+ \left[ A(\phi) + D'(\phi) \right] \nabla_{\nu} \phi \Box \phi$$

$$+ \left[ A(\phi) + 2B(\phi) + C'(\phi) \right] g^{\mu \sigma} \nabla_{\sigma} \nabla_{\nu} \phi \nabla_{\mu} \phi$$

$$+ D(\phi)\nabla_{\nu} (\Box \phi) + C(\phi)\Box (\nabla_{\nu} \phi) ,$$

where a prime (’) is differentiation w.r.t $\phi$. In order to examine the violation of the equivalence principle use is made of the Bianchi identities and the identity (observing the sign convention)

$$\nabla_{\sigma} \phi R_{\nu}^{\sigma} = \nabla_{\nu} (\Box \phi) - \Box (\nabla_{\nu} \phi) .$$

Covariantly differentiating equation 50, and remembering the Bianchi identities, yields

$$\nabla_{\mu} T_{M \nu}^{\mu} = \frac{\nabla_{\mu} \phi}{8\pi} \left( R_{\nu}^{\mu} - \frac{1}{2} \delta_{\nu}^{\mu} R \right) - \nabla_{\mu} T_{\phi \nu}^{\mu} .$$

Now if we take the trace of equation 4, we obtain

$$R = -\frac{8\pi}{\phi} \left[ T_{M \sigma}^{\sigma} + T_{\phi \sigma}^{\sigma} \right] ,$$

with

$$T_{\phi \sigma}^{\sigma} = [A(\phi) + 4B(\phi)] g^{\sigma \rho} \nabla_{\rho} \phi \nabla_{\sigma} \phi + [C(\phi) + 4D(\phi)] \Box \phi ,$$

14
and from equation 1

\[ T_{\sigma \sigma}^\sigma = \frac{1}{4\pi \lambda} \Box \phi . \]  

(59)

Substituting equations 58 and 59 in equation 57 yields

\[ R = -\frac{8\pi}{\phi} \left\{ \left[ A(\phi) + 4B(\phi) \right] g^{\sigma \rho} \nabla_\rho \nabla_\sigma \phi + \left[ C(\phi) + 4D(\phi) + \frac{1}{4\pi \lambda} \right] \Box \phi \right\} . \]  

(60)

While equations 53, 55 and 60 substituted in 56 produce

\[ \nabla_\mu T_{\lambda \nu}^{\mu} = -\frac{1}{8\pi} \nabla_\nu (\Box \phi) + \frac{1}{8\pi} \Box (\nabla_\nu \phi) + \frac{1}{2\phi} \left[ A(\phi) + 4B(\phi) \right] g^{\nu \sigma} \nabla_\sigma \phi \nabla_\mu \phi \Box \phi + \nabla_\nu \phi \nabla_\mu \phi - \left[ A'(\phi) + 4B'(\phi) \right] g^{\nu \sigma} \nabla_\sigma \phi \nabla_\mu \phi \Box \phi - \left[ A(\phi) + 2B(\phi) + C'(\phi) \right] g^{\nu \sigma} \nabla_\sigma \phi \nabla_\mu \phi \Box \phi - D(\phi) \nabla_\nu (\Box \phi) - C(\phi) \Box (\nabla_\nu \phi) . \]  

(61)

If the Principle of Mutual Interaction, equation 6, is applied

\[ \nabla_\mu T_{\lambda \nu}^{\mu} = f_\nu (\phi) \Box \phi \]

So the coefficients of: \((\Box \phi)_\nu\), \(\Box (\phi, \nu)\), \(\phi_\nu \phi_\nu \phi_\nu \phi_\nu\), and \(\phi_\nu \phi_\nu \phi_\nu \phi_\nu\), must vanish in equation 61, but those of \(\phi_\nu \Box \phi\) must satisfy equation 6. This yields five equations to solve for the five functions; \(A(\phi), B(\phi), C(\phi), D(\phi)\) and \(f_\nu(\phi)\).

| Term | Coefficients \((= 0)\) | Solution |
|------|-----------------|---------|
| \((\Box \phi)_\nu\) | \(-\frac{1}{8\pi} - D(\phi) = 0\) | \(D(\phi) = -\frac{1}{8\pi}\) \((i)\) |
| \(\Box (\phi, \nu)\) | \(+\frac{1}{8\pi} - C(\phi) = 0\) | \(C(\phi) = +\frac{1}{8\pi}\) \((ii)\) |
| \(\phi_\nu \phi_\nu \phi_\nu \phi_\nu\) | \(A(\phi) + 2B(\phi) + C'(\phi) = 0\) | \(A(\phi) = -2B(\phi)\) \((iii)\) |
| \(\phi_\nu \phi_\nu \phi_\nu \phi_\nu\) | \(\frac{1}{2\phi} \left[ A(\phi) + 4B(\phi) \right] - [A'(\phi) + 4B'(\phi)] = 0\) | (iv) |

Substituting equation \((iii)\) into \((iv)\)

\[ \frac{B'(\phi)}{B(\phi)} = -\frac{1}{\phi}, \]  

(63)

which has the solution

\[ B(\phi) = \frac{k}{\phi}, \]  

(64)
where $k$ is a constant, and therefore by equation (iii)

$$ A(\phi) = -\frac{2k}{\phi}. $$  \hspace{1cm} (65)

If $\kappa$ is now written as

$$ k = -\frac{\omega}{16\pi} $$

the non-unique solution is obtained

$$ A(\phi) = \frac{\omega}{8\pi\phi}, \quad B(\phi) = -\frac{\omega}{16\pi}\phi, $$
$$ C(\phi) = \frac{1}{8\pi}, \quad D(\phi) = -\frac{1}{8\pi}. $$ \hspace{1cm} (66)

This solution looks the same as the BD solution except that $\omega$ is as yet undetermined. In BD there is a fifth redundant equation, which may be used here in SCC to determine $\omega$. A solution for $\nabla_{\mu}T^{\mu}_{\nu}$ is obtained by substituting equation (66) into equation (61) and examining the coefficients of $\nabla_{\nu}\phi\Box\phi$. This results in a fifth equation that determines $\omega$:

$$ \nabla_{\mu}T^{\mu}_{\nu} = \left( \frac{1}{16\pi\phi} - \frac{1}{4\pi\phi} + \frac{1}{8\pi\lambda\phi} - \frac{\omega}{8\pi\phi} \right) \nabla_{\nu}\phi\Box\phi, $$ \hspace{1cm} (67)

which can be written as

$$ \nabla_{\mu}T^{\mu}_{\nu} = \frac{\kappa}{8\pi\phi} \nabla_{\nu}\phi\Box\phi, $$ \hspace{1cm} (68)

so from equation (6)

$$ f_{\nu}(\phi) = \frac{\kappa}{8\pi\phi} \nabla_{\nu}\phi $$ \hspace{1cm} (69)

where

$$ \kappa = \frac{1}{\lambda} - \frac{3}{2} - \omega. $$ \hspace{1cm} (70)

$\kappa$ can be thought of as an undetermined "creation coefficient". Note however that if $\kappa = 0$ then

$$ \omega = \frac{1}{\lambda} - \frac{3}{2}, $$ \hspace{1cm} (71)

and the BD field equations have been recovered as to be expected. However, in general, if $\kappa \neq 0$ then we have a modified version of BD. In particular if the condition

$$ \kappa = \frac{1}{\lambda} $$ \hspace{1cm} (72)
holds then we will have

$$\omega = -\frac{3}{2}$$

(73)

without $\lambda \to \infty$. It will be shown that applying the PMI determines both $\kappa$ and $\lambda$ to be unity and therefore condition equation (72) actually does hold. Furthermore with these values for $\lambda$ and $\kappa$ equation (68) becomes

$$\nabla_\mu T^\mu_{\ M\nu} = \frac{1}{2} \frac{1}{\phi} \nabla_\nu \phi T_M,$$

(74)

which is the same dynamical equation as in the JF of GR. (Quiros, 2001 equation 3.3)

Furthermore when $\nabla_\mu \phi = 0$ equation (74) reduces to equation (3) and in this immediate locality where $g_{\mu\nu} \to \eta_{\mu\nu}$ and $\phi = \phi_c$, a minima, SCC reduces to SR and the theory admits a ground state solution.

As with any scalar-tensor theory the question must be asked: "Which is the physical frame?" In SCC not only is the equivalence principle preserved in the EF, but also, if $\omega = -\frac{3}{2}$, the scalar field energy density is non-negative in the JF (see analysis by Santiago & Silbergleit, 2000). Therefore both EF and JF can be considered to be physical frames for the theory and which is appropriate for any particular situation depends on the choice of the invariant standard of measurement.

2.3 The choice of conformal frame

In addressing the question as to which conformal frame is appropriate it must be noted that a further form of the equations must be considered. There has been discussion in the literature about the correct form of the BD field equation motivated by the scalar field energy density not being positive definite in BD. This problem does not arise when $\omega = -\frac{3}{2}$ as in SCC, (Santiago & Silbergleit, 2000), however the subsequent analysis is pertinent to this theory. In the standard formulation of BD the energy-momentum tensor of the scalar field contains the second derivatives of $\phi$, which are necessarily convoluted with the gravitational terms of the affine connection. The "corrected" version is formulated so that this is not the case and this version is called the "true" stress-energy tensor" while the original version is called the "effective" stress-energy tensor." (Quiros, 2001) Whether the need for this correction exists in the $\omega = -\frac{3}{2}$ case is obscure but cast into its 'corrected' form the left hand side of the gravitational field equation, the
Einstein tensor $G_{\mu\nu}$ becomes the 'affine' Einstein tensor $\gamma G_{\mu\nu}$ and in this case the whole JF equation becomes

$$\gamma G_{\mu\nu} = \frac{8\pi}{\phi} T_{\mu\nu} \left( \frac{\omega + \frac{3}{2}}{\phi^2} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \right) \right)$$  \hspace{1cm} (75)

so in the SCC3 case with $\omega = -\frac{3}{2}$ this reduces down to

$$\gamma G_{\mu\nu} = \frac{8\pi}{\phi} T_{\mu\nu}. \hspace{1cm} (76)$$

In other words, in this representation of the theory the field equation reduces to that of the original SCC2, equation \[\Box\] with its attendant problems of the definition of the metric. In an earlier paper (Barber, 2002) the original, 'effective' scalar field energy-momentum tensor was treated as the physical representation of the theory in the JF and used to make the prediction for the Gravity Probe B geodetic precession. It proved to be false. Here the effective stress energy tensor shall be used consistently to determine the value of $\omega$ and explore the action of the scalar field force but predictions of the theory concerning the motion of massive test-particles must be calculated using the true stress-energy tensor in which case the field equations in vacuo reduce to that of GR ($R_{\mu\nu} = 0 = 0$). For example the static Schwarzschild solution is identical to GR and the Robertson parameters, which define the metric experienced in gravitational experiments, are therefore:

$$\alpha_{\text{true}} = 1, \hspace{0.5cm} \beta_{\text{true}} = 1 \hspace{0.5cm} \text{and} \hspace{0.5cm} \gamma_{\text{true}} = 1$$  \hspace{1cm} (77)

Using these values the theory is consistent with all the standard tests including the geodetic and Lens Thirring precession measurements of the Gravity Probe B experiment.

The GP-B experiment has demonstrated that (contrary to the paper by this author, 2002) the 'true' stress energy tensor, hereafter described as the 'true JF', must be used in predictions of the behaviour of massive particles. However massless particles such as photons, do not travel on geodesics of the metric in the true JF (as noted by Brans, 1987), therefore the path of light has to be evaluated using the effective stress-energy scalar field energy-momentum tensor in the 'effective JF'. Furthermore, as the scalar field is disconnected from the gravitational field and thereby becomes a 'ghost' field in the true JF as well as in the EF, then the effective JF has to be used...
when calculating scalar and gravitational fields. The EF can be used when 'atomic' units (with time measured by an atomic clock) have been chosen to interpret observations.

There are two questions to ask in order that a Weyl metric may be set up spanning extended space-time; "What is the invariant standard by which objects are to be measured?" and "How is that standard to be transmitted from event to event in order that the comparison can be made?" In both GR and the EF of SCC the invariant is defined by the principle of the conservation of energy-momentum to be invariant 'rest' mass. The standard of measurement therefore becomes that of 'fixed' (atomic i.e. steel or platinum) rulers and 'regular' atomic clocks. In the JF of SCC, on the other hand, the invariant is defined by the principle of the local conservation of energy, and therefore the standard of measurement is defined to be that of a carefully defined 'standard photon', with its frequency (inverse) determining the standard of time and space measurement, and its energy determining the standard of mass, all defined in the CoM, Machian, frame of reference. The concept of a standard photon is refined as the theory unfolds, however note that cosmological time in the JF is defined by the radiation of the CMB.

2.4 The field equations

The form of the scalar field is now developed in the effective JF in which the complete set of field equations are now:

1. The scalar field equation

\[ \Box \phi = 4\pi \lambda T_M \]

2. The gravitational field equation

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{M\mu\nu} + \frac{\omega}{\phi^2} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\sigma \phi g^{\sigma\rho} \nabla_\rho \phi \right) \]
\[ + \frac{1}{\phi} \left( \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right), \]

where \( \omega = \frac{1}{\lambda} - \frac{3}{2} - \kappa \) is a constant and \( \lambda \) and \( \kappa \) are coefficients yet to be determined.

3. The "creation" field equation which together with equation becomes,
\[ \nabla_\mu T^\mu_{M\nu} = \frac{\kappa}{8\pi} \nabla_\nu \phi \Box \phi = \frac{\kappa \lambda}{2} \frac{\nabla_\nu \phi}{\phi} T_M. \]  

(78)

3 Determining \( \lambda \) and \( \kappa \) to derive \( \omega \)

The source of curvature \( S_{\mu\nu} \) is defined by

\[ R_{\mu\nu} = \frac{8\pi}{\phi} S_{\mu\nu} \]  

(79)

where \( S_{\mu\nu} \) is derived from equation 34 to be

\[ S_{\mu\nu} = T_{M\mu\nu} - \frac{1}{2} \left( 1 - \frac{1}{2} \lambda \right) g_{\mu\nu} T_M^\sigma + \frac{\omega}{8\pi \phi} \phi_i^\mu \phi_i^\nu \]  

(80)

+ \frac{1}{8\pi} \phi_i^\mu ;^\nu .

The gravitational field equation can be written

\[ R_{\mu\nu} = \frac{8\pi}{\phi} \left[ T_{M\mu\nu} - \frac{1}{2} \left( 1 - \frac{1}{2} \lambda \right) g_{\mu\nu} T_M^\sigma \right] + \frac{\omega}{\phi^2} \phi_i^\mu \phi_i^\nu \]  

(81)

+ \frac{1}{\phi} \phi_i^\mu ;^\nu

so \( R_{\mu\nu} \) can be written in terms of the BD parameter, where \( \varpi = \frac{1}{\lambda} - \frac{3}{2} \) as follows:

\[ R_{\mu\nu} = \frac{8\pi}{\phi} \left[ T_{M\mu\nu} - \left( \frac{1 + \varpi}{3 + 2\varpi} \right) g_{\mu\nu} T_M^\sigma \right] \]  

(82)

+ \frac{\varpi}{\phi^2} \phi_i^\mu \phi_i^\nu + \frac{1}{\phi} \phi_i^\mu ;^\nu - \frac{\kappa}{\phi^2} \phi_i^\mu \phi_i^\nu

which is the same as the equivalent equation in the BD theory except with the addition of the last term which includes the ”creation coefficient” \( \kappa \). This expression was used in the 2002 paper to enable comparison of the solution with the standard BD theory.
3.1 The Post-Newtonian Approximation of the One-Body Problem

The development of the One Body Problem in SCC is the same as in the earlier paper (Barber, 2002), where a full treatment may be found, so here a summary is given. The equations in the One Body Problem differ from BD only insignificantly, as the extra SCC term involving $\kappa$ is time dependent and drops out in the static case.

Consider the stationary gravitational and scalar fields around a static, spherically symmetric, mass embedded in a cosmological space-time. In such an embedding the value of the scalar $\phi$ defining inertial mass is assumed to asymptotically approach a "cosmological" value $G_0^{-1}$ which holds "at great distance" from any large masses. $\phi$ is determined in the inertial, Lorentz frame of reference of the Centre of Mass using electromagnetic methods and this is the origin of our coordinate system.

\[
\phi = G_0^{-1} (1 + \epsilon) \tag{83}
\]

where $G_0$ is a constant of dimension and order $G_N$, and $\epsilon$ a scalar field defined by

\[
\Box \epsilon = \epsilon_{;\sigma}^{\sigma} = \frac{8\pi}{3 + 2\omega} G_0 T^{\sigma}_{\sigma} \tag{84}
\]

in which $\epsilon \to 0$ as $r \to \infty$ and we note $\omega$ is the BD parameter $\omega = \frac{1}{\lambda} - \frac{3}{2}$.

$T_{\mu\nu}$ is the energy-momentum tensor of ordinary matter and energy excluding the energy of the $\phi$ field.

The gravitational field equation now becomes

\[
R_{\mu\nu} = 8\pi G_0 (1 + \epsilon)^{-1} \left[ T_{M\mu\nu} - \frac{1 + \omega}{3 + 2\omega} g_{\mu\nu} T^{\sigma}_{M\sigma} \right] \tag{85}
\]

\[
+ \frac{\omega}{(1 + \epsilon)^2} \epsilon_{;\mu}^{\nu} \epsilon_{;\nu}^{\mu} + \frac{1}{(1 + \epsilon)^2} \epsilon_{;\mu}^{\nu} \epsilon_{;\nu}^{\mu} - \frac{\kappa}{(1 + \epsilon)^2} \epsilon_{;\mu}^{\nu} \epsilon_{;\nu}^{\mu}
\]

which again is the same as the equivalent BD equation except with the addition of the last term which includes $\kappa$.

It is this last term that drops out when we consider the stationary fields of the Post-Newtonian Approximation (PNA) and in which only slowly moving particles are considered. If $r$ and $\theta$ are typical distances and velocities of the system then the components of the metric and the Ricci tensor are expressed
in powers of the parameters $\frac{GM}{r}$ and $\nu^2$ and the PNA requires an expansion of these parameters to one order beyond Newtonian mechanics.

From equation 84 $\epsilon$ has the expansion

$$\epsilon = \frac{2}{\epsilon} + \epsilon + \ldots$$

(86)

where $\epsilon$ is of the order $\nu^N$ and in particular

$$\nabla^2 \epsilon = -\frac{8\pi G_0}{3 + 2\nu} T^{00}$$

(87)

Substituting the PNA formulas into the Ricci tensor and applying the result to the field equation 84 we obtain

$$\nabla^2 g^{00} = -8\pi G_0 \left(\frac{2\nu + 4}{2\nu + 3}\right) T^{00}$$

(88)

From this equation it follows that the usual relation between $g^{00}$ and the purely gravitational Newtonian potential $\Phi_m$ holds by defining $\Phi_m$ as

$$\nabla^2 \Phi_m = 4\pi G_m T^{00}$$

(89)

so normalized that

$$\Phi_m (\infty) = 0$$

where $G_m$ is the metric gravitational "constant" associated with the curvature of space-time measured in the limit $r \to \infty$. Then $G_m$ is related to $G_0$ by the relationship

$$G_m = \left(\frac{2\nu + 4}{2\nu + 3}\right) G_0$$

(90)

Consequently it is important to note that in the BD theory, where $G_m \equiv G_N$, the definitions of $G_0$, $\phi$, and $\psi$ in equations 2 and 83 give the result

$$\psi = \left(\frac{2\nu + 4}{2\nu + 3}\right)$$

(91)

Thus $\psi$ is not necessarily unity and has to be determined in this calculation. The relationship between $\epsilon$ and $\Phi_m$ is derived from the equations 87 and 89 to be

$$\frac{2}{\epsilon} = -\frac{1}{\nu + 2\Phi_m}$$

(92)
Note the solution for $\Phi_m$ around a spherically symmetric mass in vacuo is given by

$$\Phi_m = -\frac{G_m M}{r}$$

therefore

$$\frac{2}{\epsilon} = +\frac{G_m M}{(2 + \omega) r}$$

In the static PNA solution there is no difference between this theory and BD. Hence, as with BD, the gravitational field outside a static, spherically symmetric mass depends on $M$ alone but not any other property of the mass. Also the effective (but not the true) Robertson parameters for this theory are also given by the same formulas as in BD

$$\alpha_{eff} = 1, \, \beta_{eff} = 1, \, \gamma_{eff} = \frac{\omega + 1}{\omega + 2} = \frac{2 - \lambda}{2 + \lambda}. \quad (94)$$

### 3.2 At the centre of mass

The violation of the equivalence principle in the JF introduces two potentially serious complications; the definition of the metric, and the provision of a suitable frame of reference that admits a ground state solution in which consistent measurements of proper time, and therefore distance, can be made. It will be shown that both these complications are resolved by the PMI.

It has been shown that under the PMI photons travel along null-geodesics in the SCC JF. Another way of stating this is to note that conformal transformations from the EF canonical GR do not affect the trajectories of trace-free entities. Hence there is no problem defining the metric.

In order to examine the second complication consider the origin of our coordinate system in the static, spherically symmetric, case, which is the centre of mass of the system. In Relativity theory the centroid of an isolated system with energy-momentum tensor $T^{\mu\nu}$ and total 4-momentum $P^\alpha$, when observed by an observer $O$ with a 4-velocity $U^\alpha$ at his Lorentz time $x^0 = t$ and in his own Lorentz frame, is defined by

$$X^j_\mu (t) = \left( \frac{1}{P^0} \right) \int_{x^0=t} x^j T^{00} d^3x \quad (95)$$

and the co-moving centroid associated with the rest frame of the system is defined to be its Centre of Mass (CoM). At the CoM the resultant of all
gravitational forces vanishes hence so does $\nabla \Phi_N$. Furthermore $\phi = \phi (\Phi_N)$, therefore with $\nabla \Phi_N = 0$ at the CoM,

$$\nabla \phi = 0 \ .$$  \hspace{1cm} (96)

As $\phi (x_\nu)$ is static and not dependent on time we have for all four $\nu$

$$\nabla_\nu \phi = 0 \ ,$$  \hspace{1cm} (97)

thus at the CoM, by Equation [68] the PMI yields

$$\nabla_\mu T_{\mu \nu} = \frac{\kappa}{8\pi} \frac{1}{\phi} \nabla_\nu \phi \Box \phi = 0 \ .$$  \hspace{1cm} (98)

Hence at the unique location of the centre of mass of the system the energy-momentum tensor of matter is conserved with respect to covariant differentiation. Here the theory admits a ground state solution, $g_{\mu \nu} \rightarrow \eta_{\mu \nu}$ and $\nabla_\mu \phi = 0$ , here the equivalence principle holds, even for a massive particle, and here a free falling physical clock, remaining at rest, records proper time. Distances can be measured by timing the echo of light rays (radar) using that clock and the metric properly defined. Also Special Relativity is recovered as here the metric is Minkowskian and standards of mass, length, time and the physical constants defined for atoms, together with potential energy, retain their classical meaning. Such a standard defined atom emits a ‘reference’ photon mentioned earlier, which in the JF is transmitted across space-time with invariant energy and frequency.

There may be objections to the acceptance of a preferred frame of reference because this means giving up the principle of relativity. Note, however, that even in GR the CoM is a unique, ”preferred”, frame in that although all inertial frames are equivalent as far as the conservation of the four momentum vector, $P^\mu$, is concerned, it is the only frame of reference in which the total mass-energy, $P^0$, of the system is also conserved over time. In SCC the CoM preferred frame of reference may be selected if and only if energy is to be locally conserved, otherwise the equations are manifestly covariant.

In order that the theory may be fully determined and consequently tested, it is necessary to evaluate $\lambda$, $\kappa$ and $\varphi$. This is possible by requiring consistency in the above stated ’preferred’ frame of reference, the frame which is ’selected’ by Mach’s Principle, that of the CoM of the system.
3.3 Solving for $\phi$

The effect of allowing $T_{\mu\nu\mu} \neq 0$ in the effective JF has now to be calculated and its effect included in the modelling of experiments of slowly moving particles. That is, the violation of the equivalence principle will produce a force $G_{\nu}$ that will perturb particles, but not photons, from their geodesic world lines. The force density is given by

$$G_{\nu} = T_{\mu\nu\mu}.$$  \hfill (99)

In order to calculate this effect $\phi$ has to be determined to the third order of accuracy, ($^4\epsilon$), and this is possible both in BD and SCC. In the PNA solution to the One-Body problem the solution for $\phi$ obtained from equations 83, 86 and 93 is

$$\phi = G_{0}^{-1} \left[ 1 + \frac{G_{m}M}{(2 + \varpi) r} + \ldots \right] \hfill (100)$$

and when the metric takes the standard form of the Robertson expansion

$$d\tau^2 = \left( 1 - \frac{2G_{m}M}{r} + \ldots \right) dt^2 - \left( 1 + \frac{2\gamma G_{m}M}{r} + \ldots \right) dr^2 \hfill (101)$$

$$+ r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 ,$$

where $\gamma = \frac{\gamma}{(\varpi+2)}$. Then, as $\Box^2 \phi = 0$ in vacuo,

$$\Box^2 \phi = \frac{d^2 \phi}{dx^{\lambda} 2} + \Gamma_{\mu\lambda}^{\mu} \frac{d\phi}{dx^{\lambda}} = 0 , \hfill (102)$$

where the affine connection $\Gamma_{\mu\lambda}^{\mu}$ is given by

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\lambda\rho} \left( \frac{\partial g_{\mu\mu}}{\partial x^{\nu}} + \frac{\partial g_{\mu\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}} \right) . \hfill (103)$$

As $g_{\mu\nu}$ is diagonal and $\phi = \phi (r)$, the only non-vanishing components of the affine connection are

$$\Gamma_{tt}^{t} = \frac{1}{2} g_{00}^{0} \frac{dg_{00}}{dr} = + \frac{G_{m}M}{r^2} \left[ 1 + O \left( \frac{G_{m}M}{r} \right) \ldots \right] , \hfill (104)$$

$$\Gamma_{rr}^{r} = \frac{1}{2} g_{rr}^{r} \frac{dg_{rr}}{dr} = - \frac{(\varpi + 1) G_{m}M}{(\varpi + 2)} \frac{r^2}{r^2} \left[ 1 + O \left( \frac{G_{m}M}{r} \right) \ldots \right] ,$$

$$\Gamma_{\phi r}^{\phi} = \frac{1}{2} g_{\phi\phi}^{\phi} \frac{dg_{\phi\phi}}{dr} = 1 , \text{ and } \Gamma_{\theta r}^{\theta} = \frac{1}{2} g_{\theta\theta}^{\theta} \frac{dg_{\theta\theta}}{dr} = \frac{1}{r} ,$$

25
and equation \[102\] becomes
\[
\square \phi = \frac{d^2 \phi}{dr^2} + \left\{ 1 - \frac{(\varpi + 1)}{(\varpi + 2)} \right\} \frac{G_m M}{r^2} + \frac{2}{r} + \ldots \right\} \frac{d\phi}{dr} = 0 . \tag{105}
\]
Integrating twice w.r.t. \( r \), and expanding the exponential with \( \frac{G_m M}{r} \ll 1 \), produces a solution with two integration constants, \( k_1 \) and \( k_2 \);
\[
\phi = k_1 + \frac{k_2}{r} + \frac{1}{2(\varpi + 2)} \frac{G_m M}{r^2} + \ldots . \tag{106}
\]
Comparing coefficients with equation \[100\] evaluates \( k_1 \) and \( k_2 \) so
\[
\phi = G_0^{-1} \left\{ 1 + \frac{G_m M}{(2 + \varpi)} r + \frac{1}{2} \left[ \frac{G_m M}{(2 + \varpi)} r \right]^2 + \ldots \right\} , \tag{107}
\]
and therefore, to the accuracy of the post-post Newtonian approximation,
\[
\phi = \phi_0 \exp \left[ \frac{G_m M}{(2 + \varpi)} r \right] . \tag{108}
\]
Therefore
\[
\frac{1}{\phi} \frac{d\phi}{dr} = -\frac{G_m M}{(2 + \varpi) r^2} . \tag{109}
\]

### 3.4 The scalar field acceleration

The expression for \( T_{\mu \nu}^\mu \) for a system of \( n \) particles of rest mass \( m_n \) is given by
\[
T_{\mu \nu}^\mu = G_{\nu} = \sum_n \delta^3 \{ x - x_n (t) \} g_{\nu \alpha} \frac{d\tau}{dt} \frac{d}{dt} \left[ m_n \frac{dx^\alpha}{d\tau} \right] , \tag{110}
\]
where \( \delta^3 \{ x - x_n (t) \} \) is the Dirac delta function, \( d\tau \) the proper time defined by
\[
d\tau^2 = -g_{\mu \nu} dx^\mu dx^\nu \tag{111}
\]
and \( G_{\nu} \) is the force density.

Over the elemental volume of an individual test particle with density of inertial rest mass \( \rho (r, t) \) becomes
\[
T_{\mu \nu}^\mu = g_{\alpha \nu} \frac{d\tau}{dt} \frac{d}{dt} \left[ \rho \frac{dx^\alpha}{d\tau} \right] ,
\]
26
that is,

\[ T_{\mu \nu} = g_{\alpha \nu} \left[ \frac{d \rho}{d t} \frac{d x^\alpha}{d t} + \rho \frac{d^2 x^\alpha}{d t^2} \right] \]  

(112)

But using equation (78) together with the equation of state for a perfect fluid

\[ T_{\mu \sigma} = 3p - \rho = -\rho \]  

(113)

in the rest frame when the pressure is negligible, we obtain the PMI solution for the non-conservation of the energy-momentum tensor,

\[ T_{\mu \nu} = -\frac{\kappa \lambda}{2} \frac{\phi_{\nu}}{\phi} \rho \]  

(114)

Therefore the effect of the scalar field force is given by

\[ g_{\alpha \nu} \left[ \frac{d \rho}{d t} \frac{d x^\alpha}{d t} + \rho \frac{d^2 x^\alpha}{d t^2} \right] = -\frac{\kappa \lambda}{2} \frac{\phi_{\nu}}{\phi} \rho \]  

(115)

Now consider the effect of this force on a mass particle momentarily at rest in the frame of reference of the Centre of Mass, that is: \( \frac{d x^\alpha}{d t} = 0 \), equation (115) becomes

\[ \frac{d^2 x^\alpha}{d t^2} = -g_{\alpha \nu} \frac{\kappa \lambda}{2} \frac{\phi_{\nu}}{\phi} \]  

(116)

3.5 Equations of motion

It is now possible to examine the equations of motion in this theory. At every space-time event in an arbitrary gravitational field we can specify a set of coordinates \( \xi^i \) in which the local description of space-time is Minkowskian, with a Special Relativity metric \( \eta_{\alpha \beta} \) and in which a photon has an equation of motion

\[ \frac{d^2 \xi^\alpha}{d \sigma^2} = 0 , \]  

(117)

\[ 0 = -\eta_{\alpha \beta} \frac{d \xi^\alpha}{d \sigma} \frac{d \xi^\beta}{d \sigma} , \]  

(118)

where \( \sigma \equiv \xi^0 \) is a suitable parameter describing the null-geodesic. We now consider the equation of motion of a distant particle, momentarily stationary, in the coordinate system \( x^\mu \) of the frame of reference of the Centre of
Mass. Transforming coordinates into this system the particle would also ex-
perience the scalar field acceleration described in Equation 116 and as the
affine connection is defined by
\[ \Gamma^\alpha_{\mu\nu} = \frac{\partial x^\alpha}{\partial \xi^\beta} \frac{\partial^2 \xi^\beta}{\partial x^\mu \partial x^\nu}, \]
then, if the pressure is negligible,
\[ \frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -g^{\alpha\nu} \frac{\kappa \lambda \phi_{;\nu}}{2 \phi}. \] (119)
Therefore for a slow particle
\[ \frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{00} \left( \frac{dt}{d\tau} \right)^2 = -g^{\alpha\nu} \frac{\kappa \lambda \phi_{;\nu}}{2 \phi}, \] (120)
also
\[ \frac{d^2 t}{d\tau^2} = 0, \quad \text{so} \quad \frac{dt}{d\tau} = \sqrt{-g_{00}^{-1}} \quad \text{which is a constant at } r = r_1. \]
So multiplying through by \( \left( \frac{dt}{d\tau} \right)^2 \) produces
\[ \frac{d^2 x^\alpha}{dt^2} + \Gamma^\alpha_{00} = +g_{00} g^{\alpha\nu} \frac{\kappa \lambda \phi_{;\nu}}{2 \phi}. \] (121)
Now for a stationary field
\[ \Gamma^\alpha_{00} = -\frac{1}{2} g^{\alpha\nu} \frac{\partial g_{00}}{\partial x^\nu}, \] (122)
and for a weak field, \( g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \), where \( |h_{\alpha\beta}| \ll 1 \) and \( \eta_{\alpha\beta} \). The re-
sulting affine connection, linearized in the metric perturbation \( h_{\alpha\beta} \), becomes in the spherically symmetric case, (as in GR)
\[ \Gamma^r_{00} = -\frac{1}{2} \eta^{rr} \frac{dh_{00}}{dr}, \] (123)
so the only non zero component of Equation [121] is
\[ \frac{d^2 r}{dt^2} = \frac{1}{2} \frac{dh_{00}}{dr} + g_{00} g^{rr} \frac{\kappa \lambda \phi}{2 \phi} \frac{d\phi}{dr}. \] (124)
The general standard form of the metric in both the BD and SCC theories is
\[ d\tau^2 = \left[ 1 - \frac{2G_m M}{r} + 2(1 - \gamma) \left( \frac{G_m M}{r} \right)^2 + \ldots \right] dt^2 \]
\[ - \left( 1 + \frac{2\gamma G_m M}{r} + \ldots \right) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 , \] (125)
where in both theories \( \gamma = \frac{(2-\lambda)}{(2+\lambda)} \), therefore
\[ g_{00} = - \left[ 1 - \frac{2G_m M}{r} + \frac{4\lambda}{(2+\lambda)} \left( \frac{G_m M}{r} \right)^2 + \ldots \right] , \] (126)
\[ h_{00} = \frac{2G_m M}{r} - \frac{4\lambda}{(2+\lambda)} \left( \frac{G_m M}{r} \right)^2 + \ldots \] (127)
and
\[ g_{rr} = 1 + \frac{2(2 - \lambda) G_m M}{(2 + \lambda) r} + \ldots . \] (128)
Substituting equations 109, 126, 127 and 128 in Equation 124 yields
\[ \frac{d^2 r}{dt^2} = - \left[ 1 - \frac{\kappa \lambda^2}{(2 + \lambda)} \right] \frac{G_m M}{r^2} \]
\[ + \left\{ \frac{4\lambda(2 + \lambda - 2\kappa \lambda)}{(2 + \lambda)^2} \right\} \left( \frac{G_m M}{r} \right)^2 \] (129)
and therefore to first order the total acceleration experienced by a particle is
\[ \frac{d^2 r}{dt^2} = - \left[ 1 - \frac{\kappa \lambda^2}{2 + \lambda} \right] \frac{G_m M}{r^2} + \ldots . \] (130)
However Newtonian gravitational theory defines \( G_N \) by
\[ \frac{d^2 r}{dt^2} = - \frac{G_N M}{r^2} , \] (131)
therefore the effect of violating the equivalence principle in accordance with the PMI is that every mass experiences an extra acceleration similar to Newtonian gravitation and which therefore is confused with it. According to SCC the Newtonian gravitational constant \( G_N \), as measured in a Cavendish type
experiment, is a compilation of the effect of the curvature of space time, with its corresponding \( G_m \), and the action of the scalar field. The total Newtonian 'constant' experienced by a mass particle is therefore

\[
G_N = \left[ 1 - \frac{\kappa \lambda^2}{(2 + \lambda)} \right] G_m .
\]  

(132)

Note that \( G_N \) and \( G_m \) refer to the total "gravitational" accelerations experienced in physical experiments by atomic particles and photons respectively.

### 3.5.1 The relationship between \( \phi \) and \( m \).

Consider the general Gauss Divergence theorem applied to the gradient of the Newtonian potential \( \Phi_N \)

\[
\int\int\int_V \nabla \Theta \circ dV = \int\int_S \Theta \circ dS ,
\]

put \( \Theta = \nabla \Phi_N \) and define \( \Phi_N \) by \( \nabla^2 \Phi_N = 4\pi G_N \rho \) with \( \lim_{r \to \infty} \Phi_N (r) = 0 \),

\[
\int\int\int_V \nabla^2 \Phi_N dV = \int\int_S \nabla \Phi_N .dS .
\]

(133)

In the spherically symmetric One Body case the volume integral on the left hand side is simply \( 4\pi GM \) where \( M \) is the remote determination of the total mass of the central body radius \( R \). Consider several concentric external spheres of radius \( r_1, r_2 \) etc. \( \geq R \) centered on the mass \( M \). As the contributions from the vacuum are zero the volume integrals over each sphere are equal.

\[
\int\int\int_{V_1} \nabla^2 \Phi_N .dV = \int\int\int_{V_2} \nabla^2 \Phi_N .dV = 4\pi GM .
\]

(134)

Therefore observers on the surface of each sphere will have different determinations of the central mass, which will vary \( M \propto m_i^{-1} \) in the JF, that is when comparing \( M \) to their locally determined atomic masses \( m_i \) by observing the red shift of photons that are emitted from the central mass with invariant energy. As \( GM \) is constant for all \( r \geq R \) they will conclude

\[
G (r) \propto M^{-1} (r) \propto m_i (r) .
\]
But $G(r) = \frac{\psi}{\phi}$ therefore consistency demands

$$\phi(r) \propto m_i(r)^{-1}.$$  (135)

Integrating the surface integral on the right hand side of equation 133 over the sphere at constant $r$ gives, of course, $4\pi r^2 \nabla \Phi_N(r)$. (In the standard general form of the metric the surface area of a sphere is $4\pi r^2$. This fact is used both here and below). The Newtonian potential is defined by the measurement of acceleration of a 'free falling' test particle in a local experiment:

$$\frac{d^2r}{dt^2} = -\nabla \Phi_N(r)$$

Now consider a fixed observer at the CoM in a proper laboratory, where $M$ is constant, who would conclude from equations 133 and 134 and remembering that $G(r) = \frac{\psi}{\phi}$ that

$$\frac{d^2r}{dt^2} = -\frac{\psi M}{\phi r^2}.$$  (136)

3.6 Evaluating $\lambda$, $\kappa$ and $\psi$.

The parameters $\lambda$, $\kappa$, and $\psi$, will be calculated. There are two methods of calculating the combined gravitational and scalar field acceleration, one derived from the equations of motion, equation 129 and the other derived from the definition of the Newtonian potential applied to Gauss Divergence theorem: equation 136. Consistency between these two methods places constraints on the three parameters. Substituting for $\bar{\omega} = \frac{1}{\lambda} - \frac{3}{2}$ into equation 90 we obtain the relationship between $G_m$ and $G_0$:

$$G_m = \left(\frac{2 + \lambda}{2}\right) G_0,$$  (137)

and using this to substitute for $G_0$ in equation 129 the combined gravitational and scalar field acceleration of a free falling massive body is given by

$$\frac{d^2r}{dt^2} = -\left\{\frac{1}{2} (2 + \lambda - \kappa \lambda^2) - (2 + \lambda - 2\kappa \lambda) \frac{\lambda G_0 M}{r} + ... \right\} \frac{G_0 M}{r^2}.$$  (138)

But we also have an expression for this combined acceleration from equation 136 together with the solution for $\phi$ in equation 108 expanded for small
\( \frac{G_m M}{r} \). Using equation 137 to replace \( G_m \) with \( G_0 \) this becomes

\[
\frac{d^2 r}{d\tau^2} = -\psi \left[ 1 - \frac{\lambda G_0 M}{r} + \ldots \right] \frac{G_0 M}{r^2} .
\]  \hspace{1cm} (139)

Comparing coefficients between equations 138 and 139 sets two conditions on \( \lambda \), \( \kappa \), and \( \psi \). Consistency between the coefficients of \( \frac{G_0 M}{r^2} \) requires

\[
2 + \lambda - \kappa \lambda^2 = 2 \psi
\]  \hspace{1cm} (140)

and consistency between the coefficients of \( \frac{(G_0 M)^2}{r^4} \) requires

\[
2 + \lambda - 2 \kappa \lambda = \psi
\]  \hspace{1cm} (141)

Furthermore we have two solutions for \( \phi \); one from the solution to the scalar field equation in the One Body Problem, and the other from the local conservation of energy. The first solution, derived from equation 108 is

\[
\phi = \phi_0 \exp \left[ \frac{2\lambda}{2 + \lambda - \kappa \lambda^2} \frac{G_N M}{r} \right]
\]  \hspace{1cm} (142)

the second solution from equations 10 and 135 is

\[
\phi = \phi_0 \exp \left[ \frac{G_N M}{r} \right]
\]  \hspace{1cm} (143)

so consistency between these last two solutions, equations 142 and 143 sets a third condition on the three parameters

\[
2 - \lambda - \kappa \lambda^2 = 0
\]  \hspace{1cm} (144)

There are three simultaneous equations 140, 141 and 144 for \( \psi \), \( \lambda \) and \( \kappa \). Their unique solution is

\[
\psi = 1 \hspace{1cm} \lambda = 1 \hspace{1cm} \text{and} \hspace{1cm} \kappa = 1
\]  \hspace{1cm} (145)

Furthermore Equations 135 and 137 give the result

\[
G_N = \frac{1}{2} \left( 2 + \lambda - \kappa \lambda^2 \right) G_0 = G_0 = \lim_{r \to \infty} \frac{1}{\phi(r)}
\]  \hspace{1cm} (146)

Thus \( G_N \) is the proper value of \( \phi^{-1} \) as measured by atomic apparatus at infinity, and will be that value determined by physical apparatus in "Cavendish" type experiments elsewhere.
3.6.1 The evaluation of \( \omega \) and basic relationships

In conclusion, we have confirmed the findings published earlier (Barber, 2002). Following through carefully the consequences of introducing the principles of mutual interaction and the local conservation of energy we have determined that the three parameters introduced into the equations: \( \lambda \), \( \kappa \) and \( \psi \) are all unity. The original BD coupling constant becomes

\[
\omega = \frac{1}{\lambda} - \frac{3}{2} = -\frac{1}{2}, \tag{147}
\]

and finally the values of \( \lambda \) and \( \kappa \) yields from equation (70) the SCC coupling constant:

\[
\omega = \frac{1}{\lambda} - \frac{3}{2} - \kappa = -\frac{3}{2}. \tag{148}
\]

Hence the value \( \omega = -\frac{3}{2} \) required in SCC in order to make the EF of the theory canonical GR is just that value required for consistency between the two underlying principles of the theory.

With these values the field equations become:

the scalar field equation,

\[
\Box \phi = 4\pi T_M, \tag{149}
\]

the gravitational field equation,

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi}{\phi} T_{M\mu\nu} - \frac{3}{2\phi^2} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \right) + \frac{1}{\phi} \left( \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right), \tag{150}
\]

and the creation equation,

\[
\nabla_\mu T^\mu_{M\nu} = \frac{1}{8\pi} \frac{\nabla_\nu \phi}{\phi} \Box \phi = \frac{1}{2} \frac{\nabla_\nu \phi}{\phi} T_M. \tag{151}
\]

From equation (94) the effective Robertson parameters are

\[
\alpha_{\text{eff}} = 1 \quad \beta_{\text{eff}} = 1 \quad \gamma_{\text{eff}} = \frac{1}{3}, \tag{152}
\]
therefore the standard form of the Schwarzschild metric becomes
\[ d\tau^2 = \left(1 - \frac{3G_NM}{r} + ..\right) dt^2 - \left(1 + \frac{G_NM}{r} + ..\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 . \] (153)

The formula for \( m_p(x^\mu) \) is given by equation\(^{10}\)
\[ m_p(x^\mu) = m_0 \exp[\Phi_N(x^\mu)] , \] (154)
in addition from equation\(^{92}\) we obtain
\[ \phi(x^\mu) = G_N^{-1} \exp[-\Phi_N(x^\mu)]. \] (155)

From equation\(^{137}\) we have
\[ G_N = \frac{2}{3} G_m , \] (156)
so the acceleration of a massive body caused by the curvature of space-time is \( \frac{3}{2} \) the Newtonian gravitational acceleration actually experienced. However this is compensated by an acceleration caused by the scalar field of \( \frac{1}{2} \) Newtonian gravity in the opposite direction. This may form the basis for testing the theory in an experiment that asks the question; "Do photons fall at \( \frac{3}{2} \) the rate of freely falling test particles?"

Finally we have verified the earlier result (Barber, 2002) that in both equation\(^{138}\) and \(^{139}\) if we substitute the values \( \lambda = \kappa = \psi = 1 \) then we obtain:
\[ \frac{d^2r}{dt^2} = - \left\{ 1 - \frac{G_NM}{r} + .. \right\} \frac{G_NM}{r^2} . \] (157)
The effect of this non-Newtonian perturbation, adapting the Newtonian potential to allow for changes in potential energy, was examined in the 2002 paper and was shown to exactly compensate for the effect of the scalar field on the metric, as indeed the SCC JF conformal equivalence with canonical GR in its EF would lead us to expect.

### 3.7 Experiment and observation

#### 3.7.1 The Definitive experiment

Predictions for the standard experiments that test the trajectories of test particles and radiation in vacuo are all identical to GR, this is because of the
exact compensating nature of the scalar field with its non-minimal connection to matter. [Note: because of the second order term, $(\frac{GM}{r})^2$, in equation 129 the equivalence principle is experimentally violated in Eötvös type experiments to about one part in $10^{-17}$ or three orders of magnitude smaller than present day experimental sensitivity.(Barber, 2005)] Physical experiments on the curvature of space-time around a gravitating body, such as the Gravity Probe B experiment have to be evaluated using the 'true' stress energy tensor and its field equations, (SCC2) which again are identical to GR in vacuo. $(\alpha_{true} = \beta_{true} = \gamma_{true} = 1)$ However in this formulation of the field equations photons do not travel on geodesics of the metric and therefore the effective JF has to be used $(\alpha_{eff} = 1, \beta_{eff} = 1, \gamma_{eff} = \frac{1}{3}$ with $G_m = \frac{3}{2}G_N$) when interpreting observations of light crossing space-time, such as the gravitational deflection of light.

An experiment will now be briefly described to show the difference between SCC and GR. Compare the behaviour of light with that of matter in free fall, the question is whether they fall at the same rate or not. Although the predictions of the deflection of light by massive bodies are equal in both theories this does not answer this question as the effects of space curvature and time dilation are convoluted together in this situation and in SCC exactly compensate for each other. In order to extract the 'free fall' acceleration on its own the effects of space curvature and time dilation have to be separated. To achieve this we note that in SCC, according to equation 156, a photon in free fall should descend at $\frac{3}{2}$ the acceleration of matter. Therefore in free fall a beam of light travelling a distance $l$ across a solid apparatus is deflected downwards relative to the apparatus by an amount

$$\delta = \frac{1}{4}g \left( \frac{l}{c} \right)^2 .$$ (158)

As an experiment I have suggested launching into earth orbit an annulus, two meters in diameter, supporting $N$ (where $N \sim 1,000$) carefully aligned small mirrors. A laser beam is then split, one half reflected $N$ times to be returned and recombined with the other half beam, reflected just once, to form an interferometer at source. If the experiment is in earth orbit and the annulus orientated on a fixed star, initially orthogonal to the orbital plane then the gravitational or acceleration stresses on the frame, would vanish whereas they would predominate on earth. In such a space, or free fall, experiment SCC predicts a 2 Angstrom interference pattern shift with orbital periodicity whereas GR predicts a null result.
3.7.2 Intriguing observations

A possible explanation of the Pioneer Anomaly

The Pioneer Anomaly (PA) has been well documented, (Anderson et al, 1998, 2002) and may be a non-Newtonian real effect that is not explainable by conventional physics. It is measured as a residual blue Doppler shift on signals returned back to Earth from the two Pioneer spacecraft and the effect has been constant and equal for both spacecraft from 10AU - 90AU, outside Saturn’s orbit. The value of the frequency change or clock drift is equal to: \( a_D = (2.92 \pm 0.44) \times 10^{-18} \text{s}^{-1} \). This can be interpreted as an acceleration (either towards the Sun or the Earth) equal to \( a_P = (8.74 \pm 1.33) \times 10^{-10} \text{m.s}^{-2} \).

It does not show up in the orbital dynamics of the outer planets, which suggests that it cannot be modelled by a modification in the gravitational field of the Sun. (Iorio, 2007) Furthermore ‘normal physics’ from on-board systematics can so far only explain a maximum of the following possible sources:

1. Radio Beam Reaction Force \( a_{RB} = (1.10 \pm 0.10) \times 10^{-10} \text{m.s}^{-2} \).
2. Anisotropic Heat Reflection \( a_{AH} = (-0.55 \pm 0.55) \times 10^{-10} \text{m.s}^{-2} \).
3. Differential Change of the RTG’s Radiant Emissivity \( a_{RE} = 0.85 \times 10^{-10} \text{m.s}^{-2} \).
4. Constant Electrical Heat Radiation as the Source they was not a viable explanation.
5. Helium Expulsion from the RTGs \( a_{HE} = (0.15 \pm 0.16) \times 10^{-10} \text{m.s}^{-2} \).
6. Propulsive Mass Expulsion \( a_{PME} = \pm 0.56 \times 10^{-10} \text{m.s}^{-2} \). (Turyshiev et al. 2010),

In total these various possible sources make a maximum total of \( a_N = (2.1 \pm 0.8) \times 10^{-10} \text{m.s}^{-2} \) that can be caused by normal physics leaving at least a minimum anomalous acceleration of \( a_X = (6.6 \pm 2.1) \times 10^{-10} \text{m.s}^{-2} \) yet to be explained.

This is equivalent to a minimum Doppler shift or clock drift of \( a_{D\text{ residual}} = (2.20 \pm 0.70) \times 10^{-18} \text{s}^{-1} \).

It may be pertinent to note the Hubble parameter in similar units is equal to: \( H = (2.4 \pm 0.2) \times 10^{-18} \text{s}^{-1} \) (using \( h = 0.73 \) with \( \pm 10\% \) error bars), which is consistent with that unexplained residual.

In the cosmological solution of the SCC JF atomic masses increase secularly with

\[
m_p = m_0 \exp(H_0 t) \tag{159}
\]
thus speeding up atomic processes and clocks when compared to orbital periods, which remain constant in the theory. Therefore intriguingly the theory predicts a clock drift between ephemeris and atomic clocks of precisely $H = (2.4 \pm 0.2) \times 10^{-18} \text{s}^{-1}$ and this may be the explanation for the anomaly.

A secular increase of the Earth’s rotation rate A second anomaly as reviewed by Leslie Morrison and Richard Stephenson [(Morrison and Stephenson, 1998), (Stephenson, 2003)] arises from the analysis of the length of the day from ancient eclipse records. It is that in addition to the tidal contribution there is a long-term component acting to decrease the length of the day which equals

$$\triangle T/\text{day/cy} = -6 \times 10^{-4} \text{ sec/day/cy}.$$  

This component, which is consistent with recent measurements made by artificial satellites, is thought to result from the decrease of the Earth’s oblateness following the last ice age. Although this explanation certainly merits careful consideration, and it is difficult to separate the various components of the Earth’s rotation, it is remarkable that this value $\triangle T/\text{day/cy}$ is equal to $H_0$ if $H_0 = 67 \text{ km.sec}^{-1}/\text{Mpc}$. The question is why should this spinning up of the Earth’s rotation have a natural time scale of the order of the age of the universe rather than the natural relaxation time of the Earth’s crust or the periodicity of the ice ages? This anomaly may be cosmological rather than geophysical in nature and possibly explained by SCC in which dynamical problems are to be analysed in the JF. In its cosmological solution atomic masses increase secularly according to equation 159, consequently their radii will shrink (as the Bohr radius is inversely proportional to the mass,) and if angular momentum $mr^2\omega$ is conserved then we have:

$$m(t) = m_0 \exp (H_0 t) \quad \text{and} \quad r(t) = r_0 \exp (-H_0 t) \quad (160)$$

and if $\frac{d}{dt} (mr^2\omega) = 0$

Then

$$\frac{\dot{\omega}}{\omega_0} = - \left( \frac{\dot{m}}{m_0} + 2 \frac{\dot{r}}{r_0} \right) = +H_0 \quad , \quad (161)$$

therefore solid bodies such as the Earth should spin up when measured in ephemeris time at a rate equal to the Hubble parameter as indeed may have already been observed.
A comparison of the temporal and spatial Newtonian potentials of the metric

We may compare SCC against GR by casting the Schwarzschild metric as:

\[ d\tau^2 = [1 - 2\Psi(r)] dt^2 - [1 + 2\Theta(r)] dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \]  

(162)

where \( \Psi(r) \) and \( \Theta(r) \) are respectively the temporal and spatial Newtonian potentials of the theory. They may be compared by defining

\[ \eta = \frac{\Theta(r)}{\Psi(r)} \]  

(163)

While it is obvious that for GR \( \eta = 1 \), we have from equation 153 for SCC \( \eta = \frac{1}{3} \). The temporal gravitational potential is therefore three times larger than the spatial one. It might be possible to detect such a deviation from GR in appropriate surveys, for example in the analysis of Large Scale Structure growth.

4 Cosmological solutions to the field equations

The solution to the field equations in the cosmological case cast in the JF have been published earlier. (Barber, 2002) In the JF the cosmological model is static and eternal with exponentially 'increasing atomic particle masses', 'shrinking' rulers and 'accelerating' clocks. When transformed into the EF, with constant particle masses, 'fixed' rulers and 'regular' atomic clocks, the cosmological model is that of a linearly expanding universe. Such a model has been described as a 'freely coasting' or a 'Milne' universe, (but in SCC spatially closed). From Kolb’s initial paper onwards there have been attempts to show that coasting cosmology models could be concordant with observations and Big Bang nucleosynthesis. [(Kolb, 1989), (Batra, A. et al, 2000), (Gehlaut, S. et al, 2003), (Sethi, G. et al., 2005 a & b)]

On the other hand alternative cosmological solutions exist, using the 'true' stress energy tensor. In this case the field equations become that of the second SCC theory (Barber, 1982) and their ramifications have been developed by many authors (see abbreviated reference list below). It is shown below that with the addition of the principle of the local conservation of energy the effective JF equations yield a total density parameter \( \Omega = 1 \).
With the mysteries of Dark Matter and Dark Energy unresolved, and continuing problems with Inflation and quantum gravity, it may be that alternative models such as SCC should be examined more carefully.

4.1 The JF cosmological equations

The 'true JF' is to be used when dealing with massive particles and the 'effective JF' is to be used to deal with massless particles such as light or gravitons. Hence, the 'effective JF' is used to work out cosmological evolution and then the 'true JF' will be used to calculate the density parameter.

From the earlier paper, (Barber, 2002), we have the following cosmological equations for the Robertson-Walker metric and a perfect fluid:

the scalar field equation

\[ \ddot{\phi} + 3 \frac{\dot{\phi} \dot{R}}{R} = 4\pi (\rho - 3p) , \] (164)

two gravitational equations

\[ \left( \frac{\ddot{R}}{R} \right)^2 + \frac{k}{R^2} + \frac{8\pi \rho}{3\phi} - \frac{\dot{\phi} \dot{R}}{\phi R} - \frac{1}{4} \left( \frac{\dot{\phi}}{\phi} \right)^2 = 0 , \] (165)

and

\[ \frac{\ddot{R}}{R} + \left( \frac{\dot{R}}{R} \right)^2 + \frac{k}{R^2} = -\frac{1}{6} \left( \frac{\ddot{\phi}}{\phi} + 3 \frac{\dot{\phi} \dot{R}}{\phi R} \right) + \frac{1}{4} \left( \frac{\dot{\phi}}{\phi} \right)^2 , \] (166)

the creation equation (replacing the conservation equation of GR)

\[ \dot{\rho} = -3 \frac{\dot{R}}{R} (\rho + p) + \frac{1}{8\pi} \frac{\dot{\phi}}{\phi} \left( \ddot{\phi} + 3 \frac{\dot{\phi} \dot{R}}{R} \right) , \] (167)

and the equation of state

\[ p = \sigma \rho . \] (168)

In the coordinate system of the JF (with time based on the frequency of the CMB) the universe is static with \( R = R_0 \) a constant, therefore these equations become:

\[ \ddot{\phi} = 4\pi \rho (1 - 3\sigma) , \] (169)
\[ \frac{k}{R_0^2} = \frac{8\pi \rho}{3\phi} - \frac{1}{4} \left( \frac{\dot{\phi}}{\phi} \right)^2, \quad (170) \]

\[ \frac{k}{R_0^2} = -\frac{1}{6} \left( \frac{\ddot{\phi}}{\phi} \right) + \frac{1}{4} \left( \frac{\dot{\phi}}{\phi} \right)^2, \quad (171) \]

and

\[ \dot{\rho} = \frac{1}{8\pi} \frac{\dot{\phi} \ddot{\phi}}{\phi}. \quad (172) \]

Add equations [170] and [171] and use equation [169] to eliminate \( \rho \) we obtain

\[ \frac{k}{R_0^2} = \frac{(1 + \sigma)}{4(1 - 3\sigma)} \left( \frac{\ddot{\phi}}{\phi} \right), \quad (173) \]

The set of equations in the \( k = 0 \) case leads to the solution

\[ \phi = \phi_0 \left( \frac{t_0}{t} \right)^2 \text{ with } \sigma = -1. \quad (174) \]

However, if we consider a universe with matter (baryonic and dark), and false energy (\( \sigma_{fe} = -1 \)) but, in the present epoch, negligible electro-magnetic radiation and matter pressure, then the resultant equation of state

\[ p_{fe} = -\left( \rho_m + \rho_{fe} \right), \]

leads to \( \rho_m = 0 \), i.e. an empty universe. A realistic solution of equation [173] has to have a non-zero \( k \), therefore, with its left hand side being a non-zero constant, the solution is

\[ \phi = \phi_0 \exp \left[ H(t - t_0) \right], \]

where \( H \) is some arbitrary constant - calculated in the 2002 paper to be the Hubble parameter as measured in the present epoch, \( H_0 \) thus

\[ \phi = \phi_0 \exp (H_0 t) \quad (175) \]

Now eliminate \( \ddot{\phi} \) from equations [169] and [172]

\[ \frac{\dot{\rho}}{\rho} = \frac{1}{2} \frac{\dot{\phi}}{\phi} (1 - 3\sigma) \]
Integrating w.r.t. $t$ between the limits $t_0$ and $t$ we obtain
\[ \rho = \rho_0 \left( \frac{\phi}{\phi_0} \right)^{\frac{1}{2}(1-3\sigma)} , \] (176)

and eliminating $\frac{k}{R^2}$ between equations [170] and [171] and then using equation [169] to eliminate $\rho$ results in
\[ (5 - 3\sigma) \frac{\ddot{\phi}}{\phi} = 3 (1 - 3\sigma) \left( \frac{\dot{\phi}}{\phi} \right)^2 \]

substituting the solution from equation [175] we are left with
\[ (5 - 3\sigma) H_0^2 = 3 (1 - 3\sigma) H_0^2 , \]

so, in the effective JF, the cosmological equation of state is determined by the scalar, gravitational and creation field equations to be
\[ \sigma = -\frac{1}{3} . \] (177)

Equation [173] becomes
\[ \frac{k}{R_0^2} = \frac{1}{12} H_0^2 , \]

as $H_0^2$ and $R_0^2$ are positive definite, therefore $k = +1$ so the universe is closed with a scale length
\[ R_0 = +\sqrt{12H_0^{-1}} \simeq 47 G.lys. \] (178)

Substituting this equation of state in the 'effective JF' the following cosmological relationships were calculated in the 2002 paper:

\[ \phi = \phi_0 \exp (H_0 t) , \text{ where } \phi_0 = G_N^{-1} \text{ and } t = 0 \text{ is the present,} \]
and with the caveat that the 'effective JF' is not appropriate for massive particles we also have
\[ \rho_{eff} = \rho_0 \exp (H_0 t) , \]
from which, with equation [169] we derive
\[ \Omega_{eff} = \frac{1}{3} , \]
and finally
\[ m_{eff} = m_0 \exp (H_0 t) , \text{ where } m_0 \text{ is a particle mass in the present.} \] (179)
4.2 The EF Cosmological Equations

The dynamical evolution of the universe, determined by gravitational and scalar fields, has to be calculated in the 'effective JF', however the density parameter of the universe has to be calculated in the 'true JF' in which the scalar field becomes 'invisible'. In an early paper, (Soleng, 1987), following (Pimentel, 1985), the gravitational equations for the Robertson-Walker metric and a perfect fluid and with $\lambda = 1$ are given as:

the density gravitational equation

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = + \frac{8\pi \rho}{3\phi} \quad (180)$$

and the pressure gravitational equation

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = -\frac{8\pi p}{\phi}, \quad (181)$$

with the equation of state

$$p = \sigma \rho. \quad (182)$$

If we subtract equation (180) from (181) to obtain

$$2\frac{\ddot{R}}{R} = -\frac{8\pi (\rho + 3p)}{3\phi}, \quad (183)$$

and use the above equation of state with $\sigma = -\frac{1}{3}$ then $\ddot{R} = 0$ and the gravitational field equations are consistent with either a linearly expanding universe or, a static one.

$$R(t) = t, \text{ or } R(t) = t_0. \quad (\text{note } c = 1) \quad (184)$$

In this final version of the theory the JF, in either 'effective' or 'true' forms, includes the local conservation of energy as an additional principle to be added to the 1982 theory. With time therefore measured by a photon sampled from the CMB as the standard unit of measurement so 'light-rulers' expand with the universe, the solution by definition must be the static $R(t) = t_0$. 

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As \( k = +1 \) equation 180 becomes
\[
\frac{1}{t_0^2} = + \frac{8\pi \rho}{3\phi},
\]
(185)
with \( t_0^{-1} = H_0 \) and \( \phi = G_N^{-1} \) [185] is simply the equation for critical density,
\[
\frac{8\pi G_N \rho}{3H_0^2} = 1.
\]

In this ‘true JF’, used to measure massive, but not massless, particles the density of the universe is the critical density and
\[
\Omega_{\text{true}} = 1
\]
(186)

5 Conclusions

The 2002 version of the theory has been corrected to use the ‘true’ form of the stress-energy tensor to evaluate experiments and observations dealing with matter and the ‘effective’ form of the stress-energy tensor to interpret those dealing with light. With this correction the theory correctly predicts the geodetic precession measurement of the Gravity Probe B experiment, which the 2002 version did not. However that theory, and the present version, are concordant with all other tests of GR and two further experiments may resolve this degeneracy. Furthermore the theory offers an explanation for a real Pioneer Anomaly and also for hints of some other non-GR anomalies.

On the one hand, in the EF the universe is seen to expand linearly from a Big Bang thus resolving the smoothness and density problems without the need for Inflation, furthermore papers and eprints examining primordial nucleosynthesis in such a coasting cosmology suggest the baryon density would be much higher and might explain Dark Matter as being baryonic in nature, however what form this baryonic dark matter takes remains an unanswered question. On the other hand, in the ‘effective’ Jordan conformal frame, in which the unit of time is measured by a photon sampled from the peak of the CMB, the universe is closed and static with masses increasing exponentially with time, causing solid rulers to shrink and atomic clocks to accelerate in the same manner. The moment of the Big Bang itself is projected into the infinite past, thereby avoiding philosophical problems concerned with the concept of ‘an origin’. By using the true form of the JF the total density parameter \( \Omega \) is determined to be unity.
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