Lepton Flavor Violating Processes and Muon $g - 2$
in Minimal Supersymmetric SO(10) Model

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Abstract

In the recently proposed minimal supersymmetric SO(10) model, the neutrino Dirac Yukawa coupling matrix, together with all the other fermion mass matrices, is completely determined once free parameters in the model are appropriately fixed so as to accommodate the recent neutrino oscillation data. Using this unambiguous neutrino Dirac Yukawa couplings, we calculate the lepton flavor violating (LFV) processes and the muon $g - 2$ assuming the minimal supergravity scenario. The resultant rates of the LFV processes are found to be large enough to well exceed the proposed future experimental bound, while the magnitude of the muon $g - 2$ can be within the recent result by Brookhaven E821 experiment. Furthermore, we find that there exists a parameter region which can simultaneously realize the neutralino cold dark matter abundance consistent with the recent WMAP data.

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1 Introduction

The problems of the observed solar neutrino deficit and the atmospheric neutrino anomaly can be naturally solved through the interpretation of the neutrino oscillations among the different flavor neutrinos. Now the evidence of the neutrino masses and flavor mixings are almost established [1], and this is also the evidence of new physics beyond the standard model. Interestingly, the neutrino mass and mixing properties have been revealed to be very different from those of the other fermions, namely, neutrino masses are very small and the flavor mixing angles are very large. A new physics must explain them.

The supersymmetric (SUSY) SO(10) grand unified model is one of the attractive candidates for new physics. The experimental data support the unification of the three gauge couplings at the grand unified theory (GUT) scale $M_G \sim 2 \times 10^{16}$ GeV with the particle contents of the minimal supersymmetric standard model (MSSM) [2]. This model incorporates the right-handed neutrinos as the member of the $16$ representation together with other standard model fermions, and provides the natural explanation of the smallness of the neutrino masses through the see-saw mechanism [3].

However, it is a non-trivial problem whether a SO(10) model can simultaneously accommodate all the observed quark-lepton mass matrix data, since the quark and lepton mass matrices are related with each other and severely constrained by virtue of the quark-lepton unification into the $16$ representation. There are lots of works on this subject. Recently it has been shown [4] that the so-called minimal supersymmetric SO(10) model can simultaneously reproduce all the observed quark-lepton mass matrix data involving the neutrino oscillation data by fixing free parameters in the model appropriately. In this model, the neutrino Dirac Yukawa coupling matrix, together with all the other fermion mass matrices, is completely determined.

The evidence of the neutrino flavor mixing implies that the lepton flavor of each generation is not individually conserved. Therefore the lepton flavor violating (LFV) processes in the charged-lepton sector such as $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ are allowed. In simply extended models so as to incorporate massive neutrinos into the standard model, the rate of the LFV processes is accompanied by a highly suppression factor, the ratio of neutrino mass to the weak boson mass, because of the GIM mechanism, and is far out of the reach of the experimental detection. However, in supersymmetric models, the situation is quite different. In this case, soft SUSY breaking parameters can be new LFV sources, and the rate of the LFV processes are suppressed by only the scale of the soft SUSY breaking parameters which is assumed to be the electroweak scale. Thus the huge enhancement occurs compared to the previous case. In fact, the LFV processes can be one of the most important processes as the low-energy SUSY search [5].

Any mechanism of the SUSY breaking mediation are normally considered so as to induce the soft SUSY breaking parameters being almost flavor blind and real because of sever constraints on the SUSY flavor and CP violating processes [6]. Therefore the rate of the LFV processes is negligible at the SUSY breaking mediation scale. However, in a
model with the LFV interactions, the sizable LFV sources at low energies may be induced through the renormalization effects. As well-motivated candidates for such models, the SUSY GUTs [7] and the models with see-saw mechanism [8] have been considered.

The magnitude of the LFV rate depends on the SUSY breaking mediation scenario and the LFV interactions. In this paper, we evaluate the rate of the LFV processes in the minimal SUSY SO(10) model, where the neutrino Dirac Yukawa couplings are the primary LFV sources. The minimal supergravity (mSUGRA) scenario is assumed as the SUSY breaking mediation mechanism. As mentioned above, all the Yukawa couplings are determined in the minimal SUSY SO(10) model, we can give the definite predictions for the rate of the LFV processes.

Recent result of the muon $g - 2$ from Brookhaven E821 experiment [9] reported the deviation from the standard model prediction. Although the uncertainty of the standard model prediction has not been settled [10], we would expect that the deviation lies in the range

$$\delta a_\mu = a_\mu(E821) - a_\mu(SM) = (10 - 40) \times 10^{-10},$$

where $a_\mu = (g_\mu - 2)/2$. This may be the first evidence of the low-energy SUSY [11].

Except for the flavor indeces, both of the LFV processes such as $\mu \rightarrow e\gamma$ and the muon $g - 2$ originate from the same dipole-moment operators, and there is a correlation between their magnitudes [12]. In the following, we calculate the SUSY contribution to the muon $g - 2$ with the same input parameters as in the calculations of the LFV processes, and show their correlations.

This paper is organized as follows: in the next section, we give a brief review of the minimal SUSY SO(10) model developed in [4], and summarize its predictions. Even when we concentrate our discussion on the issue how to reproduce the realistic fermion mass matrices in the SO(10) model, there are lots of possibilities for introduction of Higgs multiplets. The minimal supersymmetric

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2 Minimal SO(10) model and its predictions

Here we give a brief review of the minimal SUSY SO(10) model$^1$ recently reconsidered in Ref. [4], and summarize its predictions. Even when we concentrate our discussion on the issue how to reproduce the realistic fermion mass matrices in the SO(10) model, there are lots of possibilities for introduction of Higgs multiplets. The minimal supersymmetric

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$^1$There are other models which are also called “minimal SUSY SO(10) model” in literatures [13]. Such models include only $\mathbf{10}$ Higgs multiplet and realize the Yukawa unification between bottom quark and tau lepton. In our “minimal” model, a $\mathbf{126}$ Higgs multiplet is additionally introduced, and Yukawa unification is not necessarily satisfied.
SO(10) model is the one where only one $10$ and one $\overline{126}$ Higgs multiplets have Yukawa couplings (superpotential) with $16$ matter multiplets such as

$$W_Y = Y^{ij}_{10} 16_i H_{10} 16_j + Y^{ij}_{126} 16_i H_{126} 16_j , \quad \text{(2)}$$

where $16_i$ is the matter multiplet of the $i$-th generation, $H_{10}$ and $H_{126}$ are the Higgs multiplet of $10$ and $\overline{126}$ representations under SO(10), respectively. Note that, by virtue of the gauge symmetry, the Yukawa couplings, $Y_{10}$ and $Y_{126}$, are, in general, complex symmetric $3 \times 3$ matrices.

We assume some appropriate Higgs multiplets, whose vacuum expectation values (VEVs) correctly break the SO(10) GUT gauge symmetry into the standard model one at the GUT scale, $M_G \sim 2 \times 10^{16}$ GeV. Suppose the Pati-Salam subgroup $[14]$, $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$, at the intermediate breaking stage. Under this symmetry, the above Higgs multiplets are decomposed as $10 \rightarrow (6, 1, 1) + (1, 2, 2)$ and $\overline{126} \rightarrow (6, 1, 1) + (10, 3, 1) + (\overline{10}, 1, 3) + (15, 2, 2)$, while $16 \rightarrow (4, 2, 1) + (\overline{4}, 1, 2)$. Breaking down to the standard model gauge group, $SU(4)_c \times SU(2)_R \rightarrow SU(3)_c \times U(1)_Y$, is accomplished by non-zero VEV of the $(\overline{10}, 1, 3)$ Higgs multiplet. Note that the Majorana masses for the right-handed neutrinos are also generated by this VEV through the Yukawa coupling $Y_{126}$ in Eq. (2).

After this symmetry breaking, we find two pairs of Higgs doublets in the same representation as the pair in the MSSM. One pair comes from $(1, 2, 2) \subset 10$ and the other comes from $(\overline{15}, 2, 2) \subset \overline{126}$. Using these two pairs of the Higgs doublets, the Yukawa couplings of Eq. (2) are rewritten as

$$W_Y = (u_R^c)_i \left( Y^{ij}_{10} H_{10}^u + Y^{ij}_{126} H_{126}^u \right) q_j + (d_R^c)_i \left( Y^{ij}_{10} H_{10}^d + Y^{ij}_{126} H_{126}^d \right) q_j + (\nu_R^c)_i \left( Y^{ij}_{10} H_{10}^c + 3Y^{ij}_{126} H_{126}^c \right) \ell_j + (e_R^c)_i \left( Y^{ij}_{10} H_{10}^e + 3Y^{ij}_{126} H_{126}^e \right) \ell_j + (\nu_R^c)_i \left( Y^{ij}_{126} v_R \right) (\nu_R^c)_j , \quad \text{(3)}$$

where $u_R$, $d_R$, $\nu_R$ and $e_R$ are the right-handed $SU(2)_L$ singlet quark and lepton superfields, $q$ and $\ell$ are the left-handed $SU(2)_L$ doublet quark and lepton superfields, $H_{10}^u$ and $H_{126}^u$ are up-type and down-type Higgs doublet superfields originated from $H_{10}$ and $H_{126}$, respectively, and the last term is the Majorana mass term of the right-handed neutrinos developed by the VEV of the $(\overline{10}, 1, 3)$ Higgs, $v_R$. The factor $-3$ in the lepton sector is the Clebsch-Gordan coefficient.

In order to preserve the successful gauge coupling unification, suppose that one pair of Higgs doublets given by a linear combination $H_{10}^{u,d}$ and $H_{126}^{u,d}$ is light while the other pair is heavy ($\geq M_G$). The light Higgs doublets are identified as the MSSM Higgs doublets ($H_u$ and $H_d$) and given by

$$H_u = \tilde{\alpha}_u H_{10}^u + \tilde{\beta}_u H_{126}^u$$

$$H_d = \tilde{\alpha}_d H_{10}^d + \tilde{\beta}_d H_{126}^d , \quad \text{(4)}$$
where \( \tilde{\alpha}_{u,d} \) and \( \tilde{\beta}_{u,d} \) denote elements of the unitary matrix which rotate the flavor basis in the original model into the (SUSY) mass eigenstates. Omitting the heavy Higgs mass eigenstates, the low energy superpotential is described by only the light Higgs doublets \( H_u \) and \( H_d \) such that

\[
W_Y = (u_R^c)_i \left( \alpha^u Y_{10}^{ij} + \beta^u Y_{126}^{ij} \right) H_u q_j + (d_R^c)_i \left( \alpha^d Y_{10}^{ij} + \beta^d Y_{126}^{ij} \right) H_d q_j
+ (e_R^c)_i \left( \alpha^e Y_{10}^{ij} - 3 \beta^e Y_{126}^{ij} \right) H_u \ell_j + (\bar{e}_R^c)_i \left( \alpha^d Y_{10}^{ij} - 3 \beta^d Y_{126}^{ij} \right) H_d \ell_j
+ (\nu_R^c)_i \left( Y_{126}^{ij} v_R \right) (\nu_R^c)_j ,
\]

where the formulas of the inverse unitary transformation of Eq. (4), \( H_{10}^{u,d} = \alpha^{u,d} H_{u,d} + \cdots \) and \( H_{126}^{u,d} = \beta^{u,d} H_{u,d} + \cdots \), have been used. Note that the elements of the unitary matrix, \( \alpha^{u,d} \) and \( \beta^{u,d} \), are in general complex parameters, through which CP-violating phases are introduced to the fermion mass matrices.

Providing the Higgs VEVs, \( H_u = v \sin \beta \) and \( H_d = v \cos \beta \) with \( v = 174 \text{GeV} \), the quark and lepton mass matrices can be read off as\(^2\)

\[
M_u = c_{10} M_{10} + c_{126} M_{126} \\
M_d = M_{10} + M_{126} \\
M_D = c_{10} M_{10} - 3 c_{126} M_{126} \\
M_e = M_{10} - 3 M_{126} \\
M_R = c_R M_{126} ,
\]

where \( M_u, M_d, M_D, M_e, \) and \( M_R \) denote the up-type quark, down-type quark, neutrino Dirac, charged-lepton, and right-handed neutrino Majorana mass matrices, respectively. Note that all the quark and lepton mass matrices are characterized by only two basic mass matrices, \( M_{10} \) and \( M_{126} \), and three complex coefficients \( c_{10}, c_{126} \) and \( c_R \), which are defined as \( M_{10} = Y_{100} \alpha^d v \cos \beta \), \( M_{126} = Y_{1263} \beta^d v \cos \beta \), \( c_{10} = (\alpha^u/\alpha^d) \tan \beta \), \( c_{126} = (\beta^u/\beta^d) \tan \beta \) and \( c_R = v_R/(\beta^d v \cos \beta) \), respectively. Except for \( c_R \), which is used to determine the overall neutrino mass scale, this system has fourteen free parameters in total\(^1\), and the strong predictability to the fermion mass matrices.

Thirteen electroweak data of six quark masses, three mixing angles and one phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and three charged-lepton masses are extrapolated to the GUT scale according to the renormalization group equations (RGEs) with given tan \( \beta \), and are used as inputs at the GUT scale.\(^3\) Solving the GUT mass

\(^2\)In general, the \( SU(2)_L \) triplet Higgs in \((10,3,1) \subset 126 \) can obtain the VEV induced through the electroweak symmetry breaking and may play a crucial role of the light Majorana neutrino mass matrix. This possibility, so-called type II see-saw model, has been discussed in detail in Ref. \(^1\).

\(^3\)In our analysis, we have neglected SUSY threshold corrections to the quark Yukawa coupling whose importance in the case of large tan \( \beta \) has been pointed out\(^1\). In our following analysis with tan \( \beta = 45 \), we find the corrections about 10\%, which would affect our final results. However we can check that this effects can be compensated away by slight changes of other input values, for example, mixing angles in CKM matrix within the experimental errors, and our final results is found to be almost unchanged.
matrix relation among quarks and charged-leptons obtained by Eq. (6), we can describe the neutrino Dirac mass matrix $M_D$ and $M_{126}$ as functions of only one free parameter $\sigma$, the phase of the combination $(3_{10} + c_{126})/(-c_{10} + c_{126})$. The light Majorana neutrino mass matrix, $M_\nu = -M_D^T M_{R}^{-1} M_D = -c_R^{-1} M_D^T M_{126}^1 M_D$, given through the see-saw mechanism is extrapolated down to the electroweak scale according to the RGE for the effective dimension-five operator [18], and is compared to the neutrino oscillation data for various $\sigma$. It has been shown [4] that the appropriate value of $\sigma$ can reproduce three neutrino flavor mixing angles in the Maki-Nakagawa-Sakata (MNS) matrix and the ratio of mass-squared differences ($\Delta m^2_\odot/\Delta m^2_\odot$) consistent with the current neutrino oscillation data. Here $\Delta m^2_\odot$ and $\Delta m^2_\odot$ are the mass-squared differences relevant for the solar and the atmospheric neutrino oscillations, respectively. The parameter $c_R$ is used to determine the overall scale of the right-handed Majorana neutrino mass, and thus the scale of $\Delta m^2_\odot$. Once the appropriate values of $\sigma$ and $c_R$ are chosen, all the fermion mass matrix (or fermion Yukawa couplings) are completely determined. Therefore the model has definite predictions to physics related to the fermion Yukawa couplings.

Although in Ref. [4] various cases with given $\tan \beta = 40 - 55$ have been analyzed, we consider only the case $\tan \beta = 45$ in the following. Our final result in the next section is almost insensitive to $\tan \beta$ values in the above range. The predictions of the minimal SUSY SO(10) model that we need in our calculation in the next section are as follows [4]: with $\sigma = 3.198$ fixed, the right-handed Majorana neutrino mass eigenvalues are found to be (in GeV) $M_{R_1} = 1.64 \times 10^{11}$, $M_{R_2} = 2.50 \times 10^{12}$ and $M_{R_3} = 8.22 \times 10^{12}$, where $c_R$ is fixed so that $\Delta m^2_\odot = 2 \times 10^{-3}\text{eV}^2$. In the basis where both of the charged-lepton and right-handed Majorana neutrino mass matrices are diagonal with real and positive eigenvalues, the neutrino Dirac Yukawa coupling matrix at the GUT scale is found to be

$$Y_\nu = \begin{pmatrix} -0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\ 0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\ -0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i \end{pmatrix}. \quad (7)$$

3 LFV processes and muon $g - 2$

Even though the soft SUSY breaking parameters are flavor blind at the scale of the SUSY breaking mediation, the LFV interactions in the model can induce the LFV source at low energies through the renormalization effects. In the following analysis, we assume the mSUGRA scenario [19] as the SUSY breaking mediation mechanism. At the original scale of the SUSY breaking mediation, we impose the following boundary conditions characterized by only five parameters (all of them are assumed to be real in this paper), $m_0$, $M_1/2$, $A_0$, $B$ and $\mu$. Here, $m_0$ is the universal scalar mass, $M_1/2$ is the universal gaugino mass, and $A_0$ is the universal coefficient of the trilinear couplings. The parameters in the Higgs potential, $B$ and $\mu$, are determined at the electroweak scale so that the Higgs doublets obtain the correct electroweak symmetry breaking VEVs through the radiative
breaking scenario [20]. The soft SUSY breaking parameters at low energies are obtained through the RGE evolutions with the boundary conditions.

Although the SUSY breaking mediation scale is normally taken to be the (reduced) Planck scale or the string scale (∼ \(10^{18}\) GeV), in the following calculations we impose the boundary conditions at the GUT scale, and analyze the RGE evolutions from the GUT scale to the electroweak scale. This ansatz is the same as the one in the so-called constrained MSSM (CMSSM). Note that, even with only particle contents introduced in the previous section, the minimal SUSY SO(10) model is not the asymptotic free gauge theory. When a complete set of the Higgs multiplets is taken into account, the gauge coupling would quickly blow up just above the GUT scale. If this is the case, field theory description of the model will be no longer meaningful above the GUT scale, and thus it is reasonable to introduce the GUT scale cutoff into the model. If there exists new physics like M-theory [21], where the string scale comes down to the GUT scale, this treatment would be justified.

The effective theory which we analyze below the GUT scale is the MSSM with the right-handed neutrinos. The superpotential in the leptonic sector is given by

\[
W_Y = Y^{ij}_\nu (\nu^c_R)_i \ell_j H_u + Y^{ij}_e (e^c_R)_i \ell_j H_d + \frac{1}{2} M_{Rij} (\nu^c_R)_i (\nu^c_R)_j + \mu H_u H_d ,
\]

where the indeces \(i, j\) run over three generations, \(H_u\) and \(H_d\) denote the up-type and down-type MSSM Higgs doublets, respectively, and \(M_{Rij}\) is the heavy right-handed Majorana neutrino mass matrix. We work in the basis where the charged-lepton Yukawa matrix \(Y_e\) and the mass matrix \(M_{Rij}\) are real-positive and diagonal matrices:

\[
Y^{ij}_e = Y_e \delta_{ij} \quad \text{and} \quad M_{Rij} = \text{diag}(M_{R1}, M_{R2}, M_{R3}).
\]

Thus the LFV is originated from the off-diagonal components of the neutrino Dirac Yukawa coupling matrix \(Y_\nu\). The soft SUSY breaking terms in the leptonic sector is described as

\[
-L_{\text{soft}} = \tilde{\ell}_i^\dagger (m^2_{\tilde{\ell}})_{ij} \tilde{\ell}_j + \tilde{\nu}_R^{i\dagger} (m^2_{\tilde{\nu}_R})_{ij} \tilde{\nu}_{Rj} + \tilde{e}_R^{i \dagger} (m^2_{\tilde{e}_R})_{ij} \tilde{e}_{Rj} + m^2_{H_u} H_u^\dagger H_u + m^2_{H_d} H_d^\dagger H_d + \left( B_{\mu} H_d H_u + \frac{1}{2} B_{\nu} M_{Rij} \tilde{\nu}_{Ri} \tilde{\nu}_{Rj} + h.c. \right)
+ \left( A^{ij}_\nu \tilde{\nu}_R^{i \dagger} \tilde{\ell}_j H_u + A^{ij}_e \tilde{e}_R^{i \dagger} \tilde{\ell}_j H_d + h.c. \right)
+ \left( \frac{1}{2} M_1 \tilde{B} \tilde{B} + \frac{1}{2} M_2 \tilde{W}_a \tilde{W}^a + \frac{1}{2} M_3 \tilde{G}_a \tilde{G}^a + h.c. \right).
\]

As discussed above, we impose the universal boundary conditions at the GUT scale such that

\[
(m^2_{\tilde{\ell}})_{ij} = (m^2_{\tilde{\nu}_R})_{ij} = (m^2_{\tilde{e}_R})_{ij} = m_0^2 \delta_{ij} ,
\]

\[
m^2_{H_u} = m^2_{H_d} = m_0^2 ,
\]

\[
A^{ij}_\nu = A_0 Y^{ij}_\nu \quad \text{and} \quad A^{ij}_e = A_0 Y^{ij}_e ,
\]

\[
M_1 = M_2 = M_3 = M_{1/2} ,
\]

(10)
and extrapolate the soft SUSY breaking parameters to the electroweak scale according to their RGEs. The LFV sources in the soft SUSY breaking parameters such as the off-diagonal components of \( (m_{\tilde{\ell}}^2)^{ij} \) and \( A^{ij}_e \) are induced by the LFV interactions through the neutrino Dirac Yukawa couplings. For example, the LFV effect most directly emerges in the left-handed slepton mass matrix through the RGEs such as

\[
\mu \frac{d}{d\mu} (m_{\tilde{\ell}}^2)^{ij} = \mu \frac{d}{d\mu} (m_{\tilde{\ell}}^2)^{ij} \bigg|_{\text{MSSM}} + \frac{1}{16\pi^2} \left( m_{\tilde{\ell}}^2 Y_{\nu}^\dagger Y_{\nu} + Y_{\nu}^\dagger Y_{\nu} m_{\tilde{\ell}}^2 + 2Y_{\nu}^\dagger m_{Y}^2 Y_{\nu} + 2m_{R_u}^2 Y_{\nu} Y_{\nu} + 2A_y^4 A_y \right)^{ij},
\]

(11)

where the first term in the right hand side denotes the normal MSSM term with no LFV. In the leading-logarithmic approximation, the off-diagonal components \( (i \neq j) \) of the left-handed slepton mass matrix are estimated as

\[
(\Delta m_{\tilde{\ell}}^2)^{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2} \left( Y_{\nu}^\dagger L Y_{\nu} \right)^{ij},
\]

(12)

where the distinct thresholds of the right-handed Majorana neutrinos are taken into account by the matrix \( L = \log[M_G/M_{R_i}]\delta_{ij} \).

The effective Lagrangian relevant for the LFV processes \( (\ell_i \rightarrow \ell_j \gamma) \) and the muon \( g-2 \) is described as

\[
L_{\text{eff}} = -\frac{e^2}{2} m_{\ell_i} \ell_j \sigma_{\mu\nu} F^{\mu\nu} \left( A^{ij}_L P_L + A^{ij}_R P_R \right) \ell_i,
\]

(13)

where \( P_{R,L} = (1 \pm \gamma_5)/2 \) is the chirality projection operator, and \( A_{L,R} \) are the photon-penguin couplings of 1-loop diagrams in which chargino-sneutrino and neutralino-charged slepton are running. The explicit formulas of \( A_{L,R} \) etc. used in our analysis are summarized in [22] [23]. The rate of the LFV decay of charged-leptons is given by

\[
\Gamma(\ell_i \rightarrow \ell_j \gamma) = \frac{e^2}{16\pi} m_{\ell_i}^5 \left( |A^{ij}_L|^2 + |A^{ij}_R|^2 \right),
\]

(14)

while the real diagonal components of \( A_{L,R} \) contribute to the anomalous magnetic moments of the charged-leptons such as

\[
\delta a_{\ell_i}^{\text{SUSY}} = \frac{g_{\ell_i} - 2}{2} = -m_{\ell_i}^2 \text{Re} \left[ A^{ii}_L + A^{ii}_R \right].
\]

(15)

In order to clarify the parameter dependence of the decay amplitude, we give here an approximate formula of the LFV decay rate [22],

\[
\Gamma(\ell_i \rightarrow \ell_j \gamma) \sim \frac{e^2}{16\pi} m_{\ell_i}^5 \times \frac{\alpha_2}{16\pi^2} \frac{\left( |\Delta m_{\ell_i}^2| \right)^2}{M_S^8} \tan^2 \beta,
\]

(16)
where $M_S$ is the average slepton mass at the electroweak scale, and $\left(\Delta m^2_{ij}\right)$ is the slepton mass estimated in Eq. (12). We can see that the neutrino Dirac Yukawa coupling matrix plays the crucial role in calculations of the LFV processes. We use the neutrino Dirac Yukawa coupling matrix of Eq. (11) in our numerical calculations.

Now we present our results for the rate of the LFV processes and the muon $g-2$. In Fig. 1, the branching ratio of the muon LFV decay, $\text{Br}(\mu \to e\gamma)$, is plotted as a function of the universal scalar mass $m_0$ for the various universal gaugino masses, $M_{1/2} = 400, 600, 800, 1000$ GeV, and $A_0 = 0$. The horizontal line denotes the present experimental upper bound, $\text{Br}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [24]. We always take $\mu > 0$, which leads to the SUSY contribution to the muon $g-2$ being positive. On the other hand, for various $m_0 = 400, 600, 800, 1000$ GeV the branching ratio as a function of $M_{1/2}$ is depicted in Fig. 2. The predicted branching ratios are large so that a parameter space with small $m_0$ and $M_{1/2}$ has already been excluded. Note that even for $m_0, M_{1/2} \sim 1$ TeV the branching ratios well exceed the future planed upper bound, $\text{Br}(\mu \to e\gamma) \leq 10^{-14}$ [25]. The LFV process may be observed in the near future. The branching ratios for the process $\tau \to \mu\gamma$ are plotted in Fig. 3 and 4 for the same input parameters in Fig. 1 and 2, respectively, together with the current upper bound, $\text{Br}(\tau \to \mu\gamma) \leq 0.6 \times 10^{-6}$ [26]. In this case, the resultant branching ratio is too small to be accessible to the future sensitivity, $\text{Br}(\tau \to \mu\gamma) \leq 10^{-9}$ [26], for input parameters consistent with the current bound on the muon LFV decay (see Fig. 1, 2). As can be understood from Eqs. (12) and (16), the branching ratio depends on $A_0$. For fixed $m_0 = 600$ GeV and $M_{1/2} = 800$ GeV, the branching ratios of both $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are plotted in Fig. 5 as functions of $A_0$. The other LFV processes such as $\mu \to 3e$ and $\mu-e$ conversion rate in nuclei are also interesting, since they are strongly constrained by experiments. These processes consist of the 1-loop penguin-type and the box-type diagrams. In our parameter space, contributions from penguin-type diagrams involving the above $A_{L,R}$ dominates, and we can find the approximation formulas, $\text{Br}(\mu \to 3e)/\text{Br}(\mu \to e\gamma) \sim 7 \times 10^{-3}$ [22] and $R(\mu \to e; Ti(Al))/\text{Br}(\mu \to e\gamma) \sim 5(3) \times 10^{-3}$ (see, for example, Ref. [27] for detailed calculations) for the $\mu-e$ conversion with the Ti (Al) target. These are close to the present experimental upper bounds, $\text{Br}(\mu \to 3e) \leq 1.0 \times 10^{-12}$ [24] and $R(\mu \to e; N) \leq 6.1 \times 10^{-13}$ [28], when $\text{Br}(\mu \to e\gamma)$ is close to the present upper bound as seen in Fig. 1 and 2.

Next let us discuss the muon $g-2$. Numerical results are depicted in Fig. 6 and 7 for the same input parameters in Fig. 1 and 2, respectively. Although these results are almost insensitive to the neutrino Dirac Yukawa couplings and the universal trilinear coupling $A_0$, they are correlated with the LFV branching ratios through the dependences on $m_0$ and $M_{1/2}$. Note that the input parameters providing $\text{Br}(\mu \to e\gamma)$ close to the present upper bound predict the suitable magnitude for the observed muon $g-2$ in Eq. (11).

If the R-parity is conserved in SUSY models, the lightest superpartner (LSP) is stable. The neutralino, if it is the LSP, is the plausible candidate for the cold dark matter (CDM) in the present universe. The recent Wilkinson Microwave Anisotropy Probe (WMAP) satellite data [29] provide estimations of various cosmological parameters with greater
The current density of the universe is composed of about 73% of dark energy and 27% of matter. Most of the matter density is in the form of the CDM, and its density is estimated to be (in 2σ range)

\[ \Omega_{CDM}h^2 = 0.1126^{+0.0161}_{-0.0081}. \]

The parameter space of the CMSSM which allows the neutralino relic density suitable for the cold dark matter has been recently re-examined in the light of the WMAP data [30]. It has been shown that the resultant parameter space is dramatically reduced into the narrow stripe due to the great accuracy of the WMAP data. It is interesting to combine this result with our analysis of the LFV processes and the muon \( g - 2 \). In the case relevant for our analysis, \( \tan \beta = 45, \mu > 0 \) and \( A_0 = 0 \), we can read off the approximate relation between \( m_0 \) and \( M_{1/2} \) such as (see Figure 1 in the second paper of Ref. [30])

\[ m_0(\text{GeV}) = \frac{9}{28}M_{1/2}(\text{GeV}) + 150(\text{GeV}), \]

along which the neutralino CDM is realized.\(^4\) \( M_{1/2} \) parameter space is constrained within the range \( 300 \text{GeV} \leq M_{1/2} \leq 1000 \text{GeV} \) due to the experimental bound on the SUSY contribution to the \( b \rightarrow s\gamma \) branching ratio and the unwanted stau LSP parameter region. We show \( \text{Br}(\mu \rightarrow e\gamma) \) and the muon \( g - 2 \) as functions of \( M_{1/2} \) in Fig. 8 and 9, respectively, along the neutralino CDM condition of Eq. (18). We find the parameter region, \( 560 \text{GeV} \leq M_{1/2} \leq 800 \text{GeV} \), being consistent with all the experimental data.

Before closing this section, we give a comment on the electric dipole moments (EDMs) of the charged-leptons. If the diagonal components of \( A_{L,R} \) have imaginary parts, CP is violated and the EDMs of the charged-leptons are given by

\[ d_{\ell_i}/e = -m_{\ell_i}\text{Im}[A_{\ell_i}^{\ell_i} - A_{\ell_i}^{R\ell_i}]. \]

These complex \( A_{L,R} \) are induced through the renormalization effects in the same manner as for the LFV processes. Here the primary source of the CP-violation is the CP-phases in the complex neutrino Dirac Yukawa coupling matrix of Eq. (7). For the same input parameter region analyzed above, we obtain the results of the electron and muon EDMs such as (we find \( d_e, d_\mu < 0 \) ) \( |d_e| = 10^{-34} - 10^{-33} \text{ [e cm]} \) and \( |d_\mu| = 10^{-31} - 10^{-30} \text{ [e cm]} \), respectively, which are far below the present experimental upper bounds [23]. \( d_e \leq 1.6 \times 10^{-27} \text{ [e cm]} \) and \( d_\mu \leq 3.7 \times 10^{-19} \text{ [e cm]} \). As an example of our results, we present the electron EDM in Fig. 10 as a function of \( M_{1/2} \) along the cosmological constraint of Eq. (18). If the measurement of the electron EDM could be improved by six orders of magnitude, \( d_e \leq 10^{-33} \text{ [e cm]} \), as proposed in [31], there might be a chance to detect the electron EDM.

\(^4\)We can see that our resultant soft SUSY breaking mass spectrum at the electroweak scale is almost the same as those in the CMSSM. Thus we use the CMSSM result of Eq. (18) as the neutralino CDM constraint.
4 Summary

The evidence of the neutrino masses and flavor mixings implies the non-conservation of the lepton flavor symmetry in each generations. Thus the LFV processes in the charged-lepton sector are allowed. In SUSY theory, there is the possibility that the rate of the LFV processes will be accessible to the current or future experimental bounds. The concrete information of the LFV interactions is necessary to evaluate the rate of the LFV processes.

It has recently been shown [4] that the minimal SUSY SO(10) model can simultaneously accommodate all the observed quark-lepton mass matrix data involving the neutrino oscillation data with appropriately fixed free parameters. In this model, the neutrino Yukawa coupling matrix are completely determined, whose off-diagonal components are the primary source of the lepton flavor violation in the basis where the charged-lepton and the right-handed neutrino mass matrices are real and diagonal. Using this Yukawa coupling matrix, we have calculated the rate of the LFV processes assuming the mSUGRA scenario. The branching ratio of the $\mu \to e\gamma$ process is found to be large so that a parameter space with small universal scalar and gaugino masses has been already excluded by the present experimental upper bound. Even for 1 TeV input parameters, we found that the branching ratio well exceeds the future planned experimental upper bound.

We also have calculated the SUSY contribution to the muon $g - 2$, which is correlated with the rate of the LFV processes through the input parameters in mSUGRA scenario. The resultant magnitude of the muon $g - 2$ is found to be within the range of the recent result of Brookhaven E821 experiment for the input parameters providing the $\mu \to e\gamma$ branching ratio close to the current experimental bound.

In CMSSM the parameter region realizing the neutralino dark matter scenario has been dramatically reduced by the recent WMAP data. We have performed the same analysis for the LFV processes and the muon $g - 2$ by restricting the input parameters within the cosmologically allowed region. We have found that there exists a parameter region consistent with all the data.

References

[1] For a recent review, see S. Pakvasa and J. W. Valle, arXiv:hep-ph/0301061 and references therein.

[2] C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. A 6, 1745 (1991); P. Langacker and M. x. Luo, Phys. Rev. D 44, 817 (1991); U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260, 447 (1991).

[3] T. Yanagida, in Proceedings of the workshop on the Unified Theory and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by D. Freedman
and P. van Niewenhuizen (north-Holland, Amsterdam 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[4] T. Fukuyama and N. Okada, JHEP 0211, 011 (2002). Also see arXiv:hep-ph/0205066 v2, where some numerical errors in the version published in JHEP have been corrected: the mass eigenvalues $M_{R_i}$, and $\langle m_\nu \rangle_{ee}$ and $\epsilon$ in Table 3.

[5] For a recent review, see, for example, J. Hisano, arXiv:hep-ph/0204100 and references therein.

[6] For a general analysis see, for example, F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996) arXiv:hep-ph/9604387.

[7] For early works, see R. Barbieri and L. J. Hall, Phys. Lett. B 338, 212 (1994) arXiv:hep-ph/9408406; R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B 445, 219 (1995) arXiv:hep-ph/9501334; Some of other relevant references are found in Ref. [5].

[8] For the early work, see F. Borzumati and A. Masiero, Phys. Rev. Lett. 57, 961 (1986); Some of other relevant references are found in Ref. [5].

[9] G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 89, 101804 (2002) [Erratum-ibid. 89, 129903 (2002)] arXiv:hep-ex/0208001.

[10] M. Knecht, A. Nyffeler, M. Perrottet and E. De Rafael, Phys. Rev. Lett. 88, 071802 (2002) arXiv:hep-ph/0111059; M. Knecht and A. Nyffeler, Phys. Rev. D 65, 073034 (2002) arXiv:hep-ph/0111058; M. Hayakawa and T. Kinoshita, arXiv:hep-ph/0112102; J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B 626, 410 (2002) arXiv:hep-ph/0112255; I. Blokland, A. Czarnecki and K. Melnikov, Phys. Rev. Lett. 88, 071803 (2002) arXiv:hep-ph/0112117.

[11] There are numbers of papers on this subject. See, for example, the references listed in Ref. [5].

[12] J. Hisano and K. Tobe, Phys. Lett. B 510, 197 (2001) arXiv:hep-ph/0102315.

[13] T. Blazek, R. Dermisek and S. Raby, Phys. Rev. Lett. 88, 111804 (2002) arXiv:hep-ph/0107097; T. Blazek, R. Dermisek and S. Raby, Phys. Rev. D 65, 115004 (2002) arXiv:hep-ph/0201081.

[14] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).

[15] B. Bajc, G. Senjanovic and F. Vissani, Phys. Rev. Lett. 90, 051802 (2003) arXiv:hep-ph/0210207; H. S. Goh, R. N. Mohapatra and S. P. Ng, arXiv:hep-ph/0303055.
[16] K. Matsuda, Y. Koide and T. Fukuyama, Phys. Rev. D 64, 053015 (2001) [arXiv:hep-ph/0010026]; K. Matsuda, Y. Koide, T. Fukuyama and H. Nishiura, Phys. Rev. D 65, 033008 (2002) [Erratum-ibid. D 65, 079904 (2002)] [arXiv:hep-ph/0108202].

[17] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D 50, 7048 (1994) [arXiv:hep-ph/9306309]; M. Carena, M. Olechowski, S. Pokorski and C. E. Wagner, Nucl. Phys. B 426, 269 (1994) [arXiv:hep-ph/9402253]; R. Hempfling, Z. Phys. C 63, 309 (1994) [arXiv:hep-ph/9404226]; T. Blazek, S. Raby and S. Pokorski, Phys. Rev. D 52, 4151 (1995) [arXiv:hep-ph/9504364].

[18] P. H. Chankowski and Z. Pluciennik, Phys. Lett. B 316, 312 (1993) [arXiv:hep-ph/9306333]; K. S. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. B 319, 191 (1993) [arXiv:hep-ph/9309223]; S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B 519, 238 (2001) [arXiv:hep-ph/0108005]; S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B 525, 130 (2002) [arXiv:hep-ph/0110366].

[19] R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B 119, 343 (1982); A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982); L. J. Hall, J. Lykken and S. Weinberg, Phys. Rev. D 27 (1983) 2359.

[20] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. 68, 927 (1982); L. Iñariz and G.G. Ross, Phys. Lett. 110B, 215 (1982); L. Alvarez-Gaume, M. Claudson and M.B. Wise, Nucl. Phys. B207, 96 (1982).

[21] E. Witten, Nucl. Phys. B 471, 135 (1996) [arXiv:hep-th/9602070]; P. Horava and E. Witten, Nucl. Phys. B 460, 506 (1996) [arXiv:hep-th/9510209]; P. Horava and E. Witten, Nucl. Phys. B 475, 94 (1996) [arXiv:hep-th/9603142].

[22] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53, 2442 (1996) [arXiv:hep-ph/9510309].

[23] Y. Okada, K. i. Okumura and Y. Shimizu, Phys. Rev. D 61, 094001 (2000) [arXiv:hep-ph/9906446].

[24] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66, 010001 (2002).

[25] Y. Kuno and Y. Okada, Rev. Mod. Phys. 73, 151 (2001) [arXiv:hep-ph/9909265].

[26] K. Inami, T. Hokuue and T. Ohshima [BELLE Collaboration], arXiv:hep-ex/0210036.
[27] R. Kitano, M. Koike and Y. Okada, Phys. Rev. D 66, 096002 (2002) arXiv:hep-ph/0203110.

[28] P. Wintz, in Proceedings of the First International Symposium on Lepton and Baryon Number Violation, edited by H. V. Klapdor-Kleingrothaus and I. V. Krivosheina (Institute of Physics, Bristol, 1998), p. 534.

[29] C. L. Bennett et al., arXiv:astro-ph/0302207, D. N. Spergel et al., arXiv:astro-ph/0302209.

[30] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, arXiv:hep-ph/0303043, A. B. Lahanas and D. V. Nanopoulos, arXiv:hep-ph/0303130.

[31] S. K. Lamoreaux, arXiv:nucl-ex/0109014.
Figure 1: The branching ratio, $\log_{10}[\text{Br}(\mu \rightarrow e\gamma)]$, as a function of $m_0$ (GeV) for $M_{1/2} = 400, 600, 800, 1000$ GeV (from top to bottom) with $A_0 = 0$ and $\mu > 0$.

Figure 2: The branching ratio, $\log_{10}[\text{Br}(\mu \rightarrow e\gamma)]$, as a function of $M_{1/2}$ (GeV) for $m_0 = 400, 600, 800, 1000$ GeV (from top to bottom) with $A_0 = 0$ and $\mu > 0$. 

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Figure 3: The branching ratio, $\log_{10}[\text{Br}(\tau \rightarrow \mu \gamma)]$, as a function of $m_0$ (GeV) for $M_{1/2} = 400, 600, 800, 1000$ GeV (from top to bottom) with $A_0 = 0$ and $\mu > 0$.

Figure 4: The branching ratio, $\log_{10}[\text{Br}(\tau \rightarrow \mu \gamma)]$, as a function of $M_{1/2}$ (GeV) for $m_0 = 400, 600, 800, 1000$ GeV (from top to bottom) with $A_0 = 0$ and $\mu > 0$. 
Figure 5: The branching ratios, $\log_{10}[\text{Br}(\tau \rightarrow \mu\gamma)]$ (top) and $\log_{10}[\text{Br}(\mu \rightarrow e\gamma)]$ (bottom) as functions of $A_0$ (GeV) for $m_0 = 600$ GeV and $M_{1/2} = 800$ GeV.

Figure 6: The SUSY contribution to the muon $g-2$ in units of $10^{-10}$ as a function of $m_0$ (GeV) for $M_{1/2} = 400, 600, 800, 1000$ GeV (from top to bottom) with $A_0 = 0$ and $\mu > 0$. 
Figure 7: The SUSY contribution to the muon $g - 2$ in units of $10^{-10}$ as a function of $M_{1/2}$ (GeV) for $m_0 = 400, 600, 800, 1000$ GeV (from top to bottom) with $A_0 = 0$ and $\mu > 0$.

Figure 8: The branching ratio, $\log_{10}[\text{Br}(\mu \rightarrow e\gamma)]$, as a function of $M_{1/2}$ (GeV) along the cosmological constraint of Eq. (18).
Figure 9: The SUSY contribution to the muon $g - 2$ in units of $10^{-10}$ as a function of $M_{1/2}$ (GeV) along the cosmological constraint of Eq. (18).

Figure 10: The electron EDM, $\log_{10}[|d_e|[e\ cm]]$, as a function of $M_{1/2}$ (GeV) along the cosmological constraint of Eq. (18).