Analysis of the stress field in an annular sector using the method of fast expansions

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Abstract. The method of fast expansions is utilized to derive the solution for the elastic problem with mixed boundary conditions for an annular sector. The problem solution obtained is valid for any opening angle \(0 \leq \theta_0 \leq 2\pi\). For \(\theta_0 = 2\pi\) it turns into the solution for a ring with a cut. The computed profiles of displacements and stresses in an annular sector are provided. The influence of the sector’s dimensions on the maximum value of stress \(\sigma_{\text{max}}\) and its location is investigated.

1. Introduction
Annular sectors bounded by the angles \(\pi, \pi/2, \pi/4\) are considered in [1]. The particular solutions for the truncated circular sector are given in [2]. Plane deformations of a circular sector are discussed in [3-5]. The stress field for circular sectors’ anti-plane deformation is examined in [6-8]. In these studies, the results were obtained by applying the Mellin transforms supplemented by the Laplace transform [6], using functions of a complex variable [7], and by means of the finite Fourier cosine transform [8]. The finite element method was utilized in [9,10].

In this paper, the method of fast expansions [11] is applied which allows obtaining the high accuracy solution for the stress problem in an annular sector in an explicit analytical form. The method of fast expansions is applicable for a solution to the problems associated with partial differential [12,13], integro-differential [11], and ordinary differential [14] equations. The problem being considered is especially complicated by setting mixed boundary conditions.

2. Materials and methods
All the considerations will be conducted in a cylindrical coordinate system. The equilibrium equations for a plane stress state are given by

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma_r \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \sigma_{\theta\theta} \right) + \frac{\sigma_r - \sigma_{\theta\theta}}{r} = 0, \quad \frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma_{\theta\theta} \right) = 0. \quad (1)
\]

The outer circumferential edge \(r = R\) of an annular sector is assumed to be fixed

\[
U\bigg|_{r=R} = 0, \quad V\bigg|_{r=R} = 0. \quad (2)
\]
The radial edges $\theta = 0$, $\theta = \theta_0$ and the inner circumferential edge $r = r_0$ are loaded by normal and tangential external forces
\[
\sigma_{rr}|_{\theta=0} = \mu \Phi_1(r), \quad \sigma_{\theta \theta}|_{\theta=0} = \lambda' \Phi_3(r), \quad \sigma_{rr}|_{r=r_0} = \mu \Phi_2(r), \quad \sigma_{\theta \theta}|_{r=r_0} = \lambda' \Phi_4(r), \quad \sigma_{r\theta}|_{r=r_0} = \lambda' \Phi_1(\theta), \quad \sigma_{\theta r}|_{r=r_0} = \mu F_1(\theta).
\]
(3)

Relying on a physical meaning of the problem, the stresses and strains should be bounded and smooth throughout the sector domain, therefore the additional conditions must be imposed
\[
(U,V) \in \left\{ C^0(0 \leq \theta \leq \theta_0), \ C^0(r_0 \leq r \leq R) \right\}, \quad |U,V| < \infty.
\]
(4)

The necessity of the calculation of sixth order derivatives will be shown further. The stresses can be expressed through the strains using Hooke’s law, and the components of the stress tensor - using Cauchy’s formulæ
\[
\sigma_r = \lambda' e_r + \mu' e_\theta = \lambda' \frac{\partial U}{\partial r} + \mu' \left( \frac{1}{r} \frac{\partial U}{\partial \theta} + \frac{U}{r} \right), \quad \sigma_\theta = \lambda' e_\theta + \mu' e_r = \lambda' \left( \frac{1}{r} \frac{\partial V}{\partial \theta} + \frac{V}{r} \right) + \mu' \frac{\partial U}{\partial r},
\]
\[
\sigma_{r\theta} = 2\mu e_\theta = \mu \left( \frac{1}{r} \frac{\partial V}{\partial \theta} + \frac{V}{r} \right), \quad \lambda' = \frac{4\mu(\lambda + \mu)}{\lambda + 2\mu}, \quad \mu' = \frac{2\lambda\mu}{\lambda + 2\mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)} = \frac{\mu'}{\lambda'}.
\]
(5)

The deformation component $e_z$ is not equal to zero and it can be derived from the equation $\sigma_z = 0 \Rightarrow e_z = -(e_r + e_\theta) \lambda/(\lambda + 2\mu)$ in the case of a plane stress state. Taking (5) into account, the equilibrium equations (1) and boundary conditions (3) have form:
\[
\frac{2}{1-\nu} \left( \frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} \right) + \frac{1+\nu}{(1-\nu)r} \frac{\partial^2 U}{\partial \theta^2} - \frac{3-\nu}{(1-\nu)r^2} \frac{\partial V}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} = 0,
\]
\[
\frac{1+\nu}{(1-\nu)r} \frac{\partial^2 U}{\partial \theta^2} + \frac{3-\nu}{(1-\nu)r^2} \frac{\partial U}{\partial \theta} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V}{\partial r^2} = 0,
\]
\[
\left. \frac{1}{r} \frac{\partial U}{\partial \theta} + \frac{\partial V}{\partial r} - \frac{V}{r} \right|_{\theta=0} = \Phi_1(r), \quad \left( \frac{U}{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \right) + \nu \left. \frac{\partial U}{\partial \theta} \right|_{\theta=0} = \Phi_3(r),
\]
\[
\left. \frac{1}{r} \frac{\partial U}{\partial \theta} + \frac{\partial V}{\partial r} - \frac{V}{r} \right|_{r=r_0} = \Phi_2(r), \quad \left( \frac{U}{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \right) + \nu \left. \frac{\partial U}{\partial \theta} \right|_{r=r_0} = \Phi_4(r),
\]
\[
\left. \frac{\partial U}{\partial r} + \nu \left( \frac{1}{r} \frac{\partial V}{\partial \theta} + \frac{V}{r} \right) \right|_{\theta=\theta_0} = F_1(\theta), \quad \left( \frac{1}{r} \frac{\partial U}{\partial \theta} + \nu \frac{\partial V}{\partial r} - \frac{V}{r} \right) \right|_{r=r_0} = F_2(\theta).
\]
(6)

The statement of the elastic problem (1)-(8) is given in a classical form, but it is not complete. The requirement (4) to the solution of being smooth and bounded is essential since it imposes specific restrictions on the boundary conditions formulation. For the case of piecewise-defined boundary conditions, the relations (4) lead to the necessity of additional "consistency" conditions’ introduction. To illustrate this, the simple example when "consistency" conditions are not satisfied can be provided: it is the case of the rectangular domain $\Omega = \{0 \leq x \leq a, 0 \leq y \leq b\}$ with the displacements defined on
its boundary
\[
U_{x=0} = f_1(y), \quad U_{y=0} = f_2(x), \quad U_{x=b} = f_3(y), \quad U_{y=b} = f_4(x).
\] (9)

Similar conditions can be written for the second projection of the displacements vector. It is not sufficient to require only the smoothness of functions \( f_i, \quad i = 1 \div 4 \) so as to derive a smooth solution. This becomes obvious if one makes the limit transition towards the rectangle’s \( \Omega \) angular points. Indeed, by approaching to the point \( x = 0, \quad y = 0 \), first along the edge \( x = 0 \), and then, along the edge \( y = 0 \) gives
\[
U_{x=0, y=0} = f_1(0), \quad U_{y=0, x=0} = f_2(0).
\] In order to the displacements be continuous at angular points, it is required that they do not depend on the direction of approach to these points. This requirement leads to additional restrictions on the choice of functions \( f_i, \quad i = 1 \div 4 \), that are the consistency conditions for the boundary ones:
\[
f_1(0) = f_2(0), \quad f_1(b) = f_4(0), \quad f_2(a) = f_3(0), \quad f_4(a) = f_1(b).
\] (10)

If the boundary conditions for displacements derivatives had been set instead of (9), the consistency conditions (10) would have had the form of additional relations for these derivatives. If the mixed boundary conditions are set, the issue of the fulfillment of consistency conditions at angular points still remains, but the solution for the boundary value problem takes a more complex form. In the case of the stress problem for an annular sector with boundary conditions (2), (7), (8), we face the same issue at the angular points \( (r = R, \quad \theta = 0), \quad (r = R, \quad \theta = \theta_0), \quad (r = r_0, \quad \theta = 0), \quad (r = r_0, \quad \theta = \theta_0) \).

Here, consistency conditions can be obtained from the continuity condition for the stresses in the sector’s angular points and the symmetry property of the stress tensor \( \sigma_{\theta\theta} = \sigma_{\theta\theta} \). Therefore, the boundary conditions in (7) and (8) must satisfy the following relations:
\[
\Phi_1(r_0) = F_2(0), \quad \Phi_2(r_0) = F_1(\theta_0).
\] (11)

The radial edge \( \theta = 0 \) is assumed to be zero loaded, that is \( \Phi_1(r) = \Phi_1(r) = 0 \). Taking into account the relations (11), the loadings \( \Phi_2(r), \quad \Phi_2(r), \quad F_1(\theta), \quad F_2(\theta) \) can be set, for example, as follows
\[
\Phi_2(r) = \left[ \Phi_{01} + \Phi_{02} \left( 1 - \frac{r}{R} \right) + \Phi_{03} \left( 1 - \frac{r}{R} \right)^2 \right] \left( 1 - \frac{r}{R} \right)
\]
\[
\Phi_4(r) = \left[ \Phi_{04} + \Phi_{05} \left( 1 - \frac{r}{R} \right) + \Phi_{06} \left( 1 - \frac{r}{R} \right)^2 \right] \left( 1 - \frac{r}{R} \right)
\]
\[
F_1(\theta) = \left[ F_{01} + F_{02} \frac{\theta}{\theta_0} + F_{03} \frac{\theta^2}{\theta_0^2} \right] \frac{\theta}{\theta_0}, \quad F_2(\theta) = \left[ \Phi_{01} + \Phi_{02} \left( 1 - \frac{r_0}{R} \right) + \Phi_{03} \left( 1 - \frac{r_0}{R} \right)^2 \right] \left( 1 - \frac{r_0}{R} \right) \frac{\theta}{\theta_0}
\]

The constant coefficients \( F_{03}, \Phi_{03}, \Phi_{06} \) are determined experimentally, and the constants
\[
F_{01}, \quad F_{02}, \quad \Phi_{01}, \quad \Phi_{02}, \Phi_{04}, \quad \Phi_{05}
\] are unknown and they are to be determined throughout the solution process.

According to the method of fast expansions [11], the displacements \( U(r, \theta) \) and \( V(r, \theta) \) can be expressed as the superposition of boundary function and truncated Fourier sine series
\[ U = M^U_\theta (r, \theta) + \sum_{n=1}^{N_1} u_n (r) \sin m \pi \frac{\theta}{\theta_0}, \quad V = M^V_\theta (r, \theta) + \sum_{n=1}^{N_1} v_n (r) \sin m \pi \frac{\theta}{\theta_0}. \]  

where \( N_1 \) is the number of Fourier series terms retained, \( M^U_\theta (r, \theta) \) and \( M^V_\theta (r, \theta) \) are the boundary functions of sixth order.

\[
M^U_\theta (r, \theta) = A_1 (r) Q_1 (\theta) + A_2 (r) Q_2 (\theta) + A_1 (r) Q_3 (\theta) + A_2 (r) Q_4 (\theta) + A_1 (r) Q_5 (\theta) + A_2 (r) Q_6 (\theta),
\]

\[
M^V_\theta (r, \theta) = B_1 (r) Q_1 (\theta) + B_2 (r) Q_2 (\theta) + B_3 (r) Q_3 (\theta) + B_4 (r) Q_4 (\theta) + B_5 (r) Q_5 (\theta) + B_6 (r) Q_6 (\theta),
\]

\[
Q_i (\theta) = \left( 1 - \frac{\theta}{\theta_0} \right), \quad Q_2 (\theta) = \left( \frac{\theta^2}{2} - \frac{\theta^2}{6 \theta_0} - \frac{\theta_0 \theta}{3} \right), \quad Q_3 (\theta) = \left( \frac{\theta^3}{6 \theta_0} - \frac{\theta_0 \theta^3}{18} + \frac{\theta_0^3 \theta}{45} \right), \quad Q_4 (\theta) = \left( \frac{\theta^4}{120 \theta_0} - \frac{\theta_0 \theta^4}{36} + \frac{7 \theta_0^3 \theta}{360} \right),
\]

\[
Q_5 (\theta) = \left( \frac{\theta^5}{720} - \frac{\theta^5}{5040 \theta_0} - \frac{\theta_0 \theta^5}{360} + \frac{\theta_0^3 \theta}{270} - \frac{2 \theta_0^3 \theta}{945} \right), \quad Q_6 (\theta) = \left( \frac{\theta^6}{5040 \theta_0} - \frac{\theta_0 \theta^6}{720} + \frac{7 \theta_0^3 \theta}{2160} - \frac{31 \theta_0^5 \theta}{15120} \right).
\]

This representation of the displacement components involves \( 16 + 2N_1 \) unknown functions

\[
A_i (r) + A_0 (r), \quad B_i (r) + B_0 (r), \quad u_m (r), \quad v_m (r), \quad m = 1 + N_1.
\]

depending only on the radial coordinate \( r \). Following the same approach the unknown functions (14) can also be represented in a fast expansion form:

\[
A_j (r) = M^U_{6,j} (r) + \sum_{n=1}^{N_1} a_{6,n}^{(j)} \sin n \pi z, \quad B_j (r) = M^V_{6,j} (r) + \sum_{n=1}^{N_1} b_{6,n}^{(j)} \sin n \pi z, \quad z = \frac{r - r_0}{R - r_0} \in [0,1], \quad j = 1 \pm 8,
\]

\[
u_m (r) = M^U_{6,m} (r) + \sum_{n=1}^{N_1} u_{6,n}^{(m)} \sin n \pi z, \quad \nu_m (r) = M^V_{6,m} (r) + \sum_{n=1}^{N_1} v_{6,n}^{(m)} \sin n \pi z, \quad m = 1 + N_1,
\]

\[
M^U_{6,j} (r) = a_{1,j} P_1 (z) + a_{2,j} P_2 (z) + a_{3,j} P_3 (z) + a_{4,j} P_4 (z) + a_{5,j} P_5 (z) + a_{6,j} P_6 (z),
\]

\[
M^V_{6,j} (r) = b_{1,j} P_1 (z) + b_{2,j} P_2 (z) + b_{3,j} P_3 (z) + b_{4,j} P_4 (z) + b_{5,j} P_5 (z) + b_{6,j} P_6 (z),
\]

\[
M^U_{6,m} (r) = u_{1,m} P_1 (z) + u_{2,m} P_2 (z) + u_{3,m} P_3 (z) + u_{4,m} P_4 (z) + u_{5,m} P_5 (z) + u_{6,m} P_6 (z),
\]

\[
M^V_{6,m} (r) = v_{1,m} P_1 (z) + v_{2,m} P_2 (z) + v_{3,m} P_3 (z) + v_{4,m} P_4 (z) + v_{5,m} P_5 (z) + v_{6,m} P_6 (z),
\]

\[
P_1 (z) = (1 - z), \quad P_2 (z) = z, \quad P_3 (z) = \left( \frac{z^2}{2} - \frac{z^3}{6} + \frac{z}{3} \right), \quad P_4 (z) = \left( \frac{z^3}{6} - \frac{z}{6} \right).
\]
\[ P_3(z) = \left( \frac{z^4}{24} - \frac{z^8}{2} - \frac{z^9}{9} + \frac{z}{18} \right), \quad P_6(z) = \left( \frac{z^8}{120} - \frac{z^4}{36} + \frac{7z}{360} \right), \]
\[ P_7(z) = \left( \frac{z^6}{720} - \frac{z^5}{5040} - \frac{z^5}{360} + \frac{z^2}{270} + \frac{2z}{945} \right), \quad P_8(z) = \left( \frac{z^6}{5040} - \frac{z^6}{720} - \frac{7z^3}{2160} + \frac{31z}{15120} \right). \]

As a result, the displacements \( U(r, \theta) \) and \( V(r, \theta) \) are expressed as a double fast expansion containing 2\((N_1+8)\)(\(N_2+8\)) unknown constant coefficients

\[ a_n^{(j)}, b_n^{(j)}, u_n^{(m)}, v_n^{(m)}, \quad j = 1 + 8, \quad n = 1 + N_2 + 8, \quad m = 1 + N_1. \]  

In order to determine the unknown coefficients (15), the point-wise method previously developed and tested in [14,15] will be employed. Following this technique, it is necessary to discretize the annular sector domain \( \Omega \) in a uniform grid with \((N_1 + 6)(N_2 + 6)\) nodes \((r_k, \theta_k)\) so that the interval \([0, \theta_f]\) is divided by \(N_1 + 6\) inner points \( \theta_k = k\theta_0/(N_1 + 5), \quad k = 0,1,\ldots,N_1 + 5 \) and the interval \([r_0, R]\) by \(N_2 + 6\) inner points \( r_s = r_0 + s(R-r_0)/(N_2 + 5), \quad s = 0,1,\ldots,N_2 + 5 \). Substituting the values of each node \((r_s, \theta_k)\) into the double fast expansion of the governing differential equations (6) gives \(2(N_1+8)(N_2+8)\) linear algebraic equations in terms of unknown coefficients (15).

Further, it is required to discretize the boundary of the domain \( \Omega \) in the same way. Dividing the interval \([0, \theta_f]\) by \(N_1 + 7\) inner computational points \( \theta_k = k\theta_0/(N_1 + 6), \quad k = 0,1,\ldots,N_1 + 6 \) and substituting these values into the relations (2) and (8) gives \(4(N_1 + 7)\) linear equations. Similarly, dividing the interval \([r_0, R]\) by \(N_2 + 7\) inner computational points \( r_s = r_0 + s(R-r_0)/(N_2 + 6), \quad s = 0,1,\ldots,N_2 + 6 \) and substituting these values into the relations (7) gives \(4(N_2 + 7)\) more linear equations. Thus, from the boundary conditions (2), (7) and (8) we obtain \(4(N_1 + 7) + 4(N_2 + 7)\) linear equations in terms of unknown coefficients (15).

All in all, we derive the consistent system of \(2(N_1+8)(N_2+8)\) linear algebraic equations in the unknowns (15) and 6 additional equations for the constants (12) following from the consistency conditions. This system then has been solved numerically in Maple software.

3. Results and discussion

As an example, the numerical results for solution to the boundary value problem (2), (6)-(8) for the annular sector made of instrumental quick-cutting steel of grade R18 are provided. This steel has the following values of elastic moduli [16,17]

\[ \sigma_{0.2} = 5.1 \times 10^8 \text{ Pa}, \quad \nu = 0.33, \quad E = 2,28 \times 10^{11} \text{ Pa}, \quad \lambda = 1.66 \times 10^{11}, \quad \mu = 8.57 \times 10^{10}, \]

where \( \sigma_{0.2} \) is the 0.2\% offset yield stress.

The values of parameters \( F_{d1}, \Phi_{d1}, \Phi_{d6}, r_0, R, \theta_0 \) were selected in such a way as to ensure the stresses do not exceed \( \sigma_{0.2} \) [18]

\[ \sigma_{0.2} \geq \sqrt{\left( \sigma_r^* \right)^2 + \left( \sigma_\theta^* \right)^2 + \left( \sigma_z^* \right)^2 + 2(\sigma_{rz}^*)^2}/2 = \hat{\sigma}, \]

where \( \sigma_r^* = \sigma_r - (\sigma_r + \sigma_\theta)/3 \), \( \sigma_\theta^* = \sigma_\theta - (\sigma_r + \sigma_\theta)/3 \), \( \sigma_z^* = \sigma_z - (\sigma_r + \sigma_\theta)/3 \), \( \sigma_{rz}^* = 0 \).

The calculated displacements \( U(r, \theta) \) and \( V(r, \theta) \) approximately satisfy the differential equations.
(6) and boundary conditions (2), (7) and (8). The distributions of relative residual $\delta_D$ of the approximate solution (13) to the equations (6) retaining only three terms ($N_1 = 3$) in first expansion, and thirty terms ($N_2 = 30$) in the second one are shown in figure 1. These results were computed for the following values of parameters:

$$\Phi_{02} = 2 \cdot 10^{-5}, \Phi_{01} = 2 \cdot 10^{-5}, F_{01} = 2 \cdot 10^{-5}, r_0 = 10^{-2} \text{m}, R = 10^{-1} \text{m}, \theta_0 = 40\pi/180.$$  

(16)

It can be seen (refer to figure 1) that the maximal values of relative residual $\delta_D$ for any value of coordinates $r_0 \leq r \leq R, 0 \leq \theta \leq \theta_0$ do not exceed $8.8 \cdot 10^{-3}$ and $5.2 \cdot 10^{-3}$ for the first and second differential equations, respectively. The distribution of relative residual $\delta_D$ along the region's boundary as well as residual's magnitudes for all boundary conditions (2), (7), (8) are essentially the same. The typical behavior of relative residual $\delta_D$ at the edges $\theta = \theta_0$ and $r = r_0$ of the domain $\Omega$ is shown in figure 2.

![Figure 1](image1.png)

**Figure 1.** The relative residual of the differential equations: (a) (6)\textsubscript{1}; (b) (6)\textsubscript{2}.

![Figure 2](image2.png)

**Figure 2.** The relative residual of the boundary conditions: (a) $\delta_D\big|_{\theta=\theta_0}$; (b) $\delta_D\big|_{r=r_0}$. 

It is apparent that the accuracy achieved (refer to figures 1, 2) is acceptable for the most technical purposes. It should be noted that the similar high accuracy approximate solutions were accomplished in other studies [13-15], where the method of fast expansions has also been involved.

The computed profiles of the displacements $U(r, \theta)$, $V(r, \theta)$ and stress components $\sigma_r$, $\sigma_\theta$, $\sigma_{r\theta}$ in an annular sector for parameters’ values (16) are shown in figure 3 and figure 4, respectively.

Figure 3. The displacement components: (a) $U(r, \theta)$; (b) $V(r, \theta)$.

Figure 4. The stress components: (a) $\sigma_r$; (b) $\sigma_\theta$; (c) $\sigma_{r\theta}$.
The distributions of $\sigma$ in an annular sector for $r_0 = 10^{-2} \text{m}$ and $r_0 = 9.5 \cdot 10^{-2} \text{m}$ are shown in figure 5. It can be noted that the maximal stress $\sigma_{\text{max}}$ for relatively small radius ratios $\Delta r = R - r_0$ occurs approximately at the point $(r_0; \pi/4)$ (refer to figure 5b), otherwise, $\sigma_{\text{max}}$ is localized at the point $(r_0; \theta_0)$ (refer to figure 5a).

![Figure 5. The distribution of $\sigma$: a) $r_0 = 10^{-2} \text{m}$; b) $r_0 = 9.5 \cdot 10^{-2} \text{m}$.](image)

The computed values of $\sigma_{\text{max}}(\theta_0)$ for various radius ratios $\Delta r = R - r_0$ are presented in figure 6. The curves 1, 2 and 3 represent data obtained for $\Delta r = 2 \cdot 10^{-3} \text{m}$, $\Delta r = 3 \cdot 10^{-3} \text{m}$, and $\Delta r = 4 \cdot 10^{-3} \text{m}$, respectively. It can be concluded examining these data that the stresses in an annular sector decline by either increasing an opening angle $\theta_0$ or decreasing a radius ratio $\Delta r$.

![Figure 6. Maximal values of stress $\sigma_{\text{max}}(\theta_0)$ for various $\Delta r$.](image)

4. Conclusion

Thus, the application of the method of fast expansions allowed obtaining the solution to the problem in an explicit analytical form. The displacement and stress profiles derived can be utilized in the analysis of the stress state arising in an annular sector. It follows from the analysis that the stresses in an annular sector decline by either increasing an opening angle $\theta_0$ or decreasing a radius ratio $\Delta r$. 
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