Modified Friedman scenario from the Wheeler-DeWitt equation

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The standard approach to quantum cosmology consists of quantizing a minisuperspace model. The corresponding Wheeler-DeWitt (WDW) equation [1] contains an arbitrary operator ordering parameter. In one of our papers [2] it was noticed that the operator ordering term for large values of operator ordering parameter can affect the matching of false vacuum instanton with the Coleman-De Luccia bounce. Continuing in the spirit of that paper, in this brief note we would like to consider how the operator ordering term for large enough values of operator ordering parameter can affect the Friedman equation considered as a semiclassical limit of the WDW equation. (Throughout this paper we assume $c = 1$).

Let us consider a closed universe filled with a vacuum of constant energy density and the radiation, $\rho(a) = \rho_v + \frac{\epsilon}{a^4}$, where $\rho_v$ is the vacuum energy density, $a$ is the scale factor and $\epsilon$ is a constant characterizing the amount of radiation. The evolution equation for $a$ can be written as

$$-\frac{3\pi}{4G}a^2 - \frac{3\pi}{4G}a + 2\pi^2a^3\rho(a) = 0 .$$  

This equation is identical to that for a particle moving in a potential

$$U(a) = \frac{3\pi}{4G} - 2\pi^2a^2\rho(a) ,$$

with zero energy, Fig.1.

We tacitly assumed $256\pi G^2 \rho_v \epsilon / 9 < 1$. The universe can start at $a = 0$, expand to a maximum radius $a_0$ and then recollapse or tunnel through the potential barrier to the regime of unbounded expansion. Eq. (1) represents the zero energy condition for the Lagrangian

$$\mathcal{L} = -\frac{3\pi}{4G}a\dot{a}^2 + \frac{3\pi a}{4G}a - 2\pi^2a^3\rho(a) .$$

To quantize the model, we find the Hamiltonian

$$\mathcal{H} = -\frac{G}{3\pi} \frac{p^2}{a} - \frac{3\pi}{4G}a + 2\pi^2a^3\rho(a) ,$$

and replace the momentum in $\mathcal{H}$ by a differential operator $p \rightarrow -i\hbar d/da$. Then the WDW equation is written down as

$$\left( \frac{G\hbar^2}{3\pi} \frac{d^2}{da^2} + \frac{Gq\hbar^2}{3\pi a} \frac{d}{da} - \frac{3\pi}{4G}a^2 + 2\pi^2a^4\rho(a) \right) \psi(a) = 0 ,$$

where $q$ is an arbitrary parameter corresponding to the operator ordering term $p^2/a = a^{-q+1}\rho a^q p$. Throughout this paper $q$ denotes

$$q \equiv \frac{a^2_0}{\hbar G} ,$$

where $\alpha$ is an arbitrary dimensionless parameter. Disregarding the operator ordering term, the eq. (4) has the form of a one-dimensional Schrödinger equation for a particle described by a coordinate $a$, having zero energy and moving in the potential

$$V(a) = \frac{3\pi}{4G}a^2 - 2\pi^2a^4\rho(a) .$$

Assuming that $\alpha$ is sufficiently large, in the WKB approximation $\psi(a) = \exp \{-iW_0(a)/\hbar + W_1 + \cdots\} ,$

to the lowest order in $\hbar$ one obtains

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We have assumed that for large values of $\alpha$ the second term in eq. (4) may be of the order of the first one. Let us judge the validity of this assumption. From eq. (5) one obtains

$$
\frac{dW_0}{da} = -i\frac{\hbar}{2a} + i\frac{\sqrt{f(a)}}{2},
$$

where

$$f(a) = \frac{q^2\hbar}{a^2} + \frac{9\pi^2a^2}{G^2} - \frac{24\pi^3a^4\rho(a)}{G}.
$$

In what follows, in the region $a < a_0$, we assume the following boundary condition

$$
\frac{dW_0}{\hbar da} = 1 + \frac{1}{2} + 1 + \frac{9\pi^2a^4}{\alpha^2a_0^4} - \frac{24G\pi^3a^6\rho(a)}{\alpha^2a_0^4} \sim 1.
$$

(8)

So, under the boundary condition (8), in the region $a < a_0$, the second term in eq. (4) for large values of $\alpha$ can be of the order of the first one and must be kept. Notice that for the solution (9) with the positive sign the second term in eq. (4) becomes suppressed in comparison with the first one and should be omitted.

Let us now check the standard WKB validity condition (9). For large values of $\alpha$ in the region $a < a_0$ one obtains

$$
\frac{dW_0}{da} \sim -\frac{\alpha^2}{G\alpha}, \quad \frac{d^2W_0}{da^2} \sim \frac{\alpha^2}{G\alpha^2},
$$

and correspondingly

$$
\hbar \left| \frac{d^2W_0}{da^2} \left( \frac{dW_0}{da} \right)^2 \right| \sim q^{-1} \ll 1.
$$

(9)

Thus, for the solution (8) the approximation (9) is justified in the region $a < a_0$ for large values of $\alpha$.

From the Hamilton-Jacobi equation (5) one obtains the following Lagrangian (4):

$$
\mathcal{L} = -\frac{3\pi}{4G}\alpha a^2 - \frac{iq\hbar}{2} a + Gq^2h^2 - \frac{3\pi a}{4G} + 2\pi^2a^3\rho(a).
$$

In this Lagrangian one can omit the second term because it contains the total time derivative and thereby does not affect the equation of motion (3). Therefore one gets

$$
\mathcal{L} = -\frac{3\pi}{4G}\alpha a^2 + Gq^2h^2 - \frac{3\pi a}{4G} - 2\pi^2a^3\rho(a),
$$

for which the zero-energy condition takes the form

$$
-\frac{3\pi}{4G}\alpha a^2 - \frac{\alpha^2a_0^4}{12G\pi^4} - \frac{3\pi}{4G}\alpha a^2 + 2\pi^2a^3\rho(a) = 0.
$$

(10)

The modified Friedman equation (10) contains a new term, which decays very fast in the course of expansion of the universe and therefore cannot affect the late time cosmology. But, however, the appearance of this new term is quite important for it can avoid the collapse of the universe.

To summarize, we considered the influence of the operator ordering term on the semiclassical limit of the WDW equation for large enough values of operator ordering parameter. More precisely from the very beginning in eq. (4) we assume $q = \alpha a_0^2/\hbar G$, where $\alpha$ is an arbitrary dimensionless parameter and take the boundary condition (8) in the region $a < a_0$. For large values of $\alpha$, such that to ensure validity of eqs. (9, 10), the semiclassical limit of the WDW equation produces a modification of the Friedman equation eq. (10). The new term appearing in equation (10) decays very fast and therefore does not affect the cosmology in the course of expansion of the universe, but, however, can avoid the collapse of the universe.

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