Holographic QCD & Perfection

Nick Evans

Department of Physics and Astronomy, Southampton University, Southampton, S017 1BJ, UK

ABSTRACT

A holographic description of chiral symmetry breaking in the pattern of QCD is reviewed. D7 brane probes are used to include quark fields in a simple non-supersymmetric deformation of the AdS/CFT Correspondence. The axial symmetry breaking is realized geometrically and the quark condensate and meson masses are computable. Surprisingly, treating the model as a description of QCD works quantitatively at the 15% level. Models of this AdS/QCD type typically have a strongly coupled, conformal UV regime that is far from QCD. To systematically move closer to QCD, we propose cutting out the large radius gravitational description and matching operators and couplings at a finite UV cut off in the spirit of a perfect lattice action. A simple example is discussed. Based on a talk presented at SCGT06 in Nagoya, Japan.

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1 Introduction

Recently the first attempts have been made to bring the holographic techniques of the, string theory derived, AdS/CFT Correspondence\[1\] to bare on QCD. The hope is that there is some weakly coupled gravitational theory in five or more dimensions that describes the strong coupling regime of QCD. Here we will review a holographic description of chiral symmetry breaking\[2\] starting from the AdS/CFT Correspondence and discuss to what extent it can be used as a phenomenological tool for real QCD.

2 A Non-Supersymmetric Gravity Dual

The AdS/CFT Correspondence\[1\] is a duality between the conformal, large \(N_c\), \(N=4\) super Yang Mills theory and IIB strings (supergravity) on 5d Anti-de-Sitter space cross a five sphere. The field theory’s global symmetries (an SO(2,4) superconformal symmetry and an SU(4)\(_R\) symmetry) match to space-time symmetries of the \(AdS\) space and the five sphere respectively. The supergravity fields enter the field theory in symmetry invariant ways and so appear as sources for field theory operators. The radial direction in \(AdS\) has the conformal symmetry properties of an energy scale and corresponds to the renormalization group scale. Thus the radial behaviour of the supergravity fields describes the RG flow of the field theory sources.

Let us consider a very simple example of AdS with a scalar field, the dilaton, switched on, due to Constable and Myers\[3\]

\[
 ds^2 = H^{-1/2} \left( \frac{u^4 + b^4}{u^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left( \frac{u^4 + b^4}{u^4 - b^4} \right)^{(2-\delta)/4} \frac{u^4 - b^4}{u^4} du_6^2 
\]

\[
 H = \left( \frac{u^4 + b^4}{u^4 - b^4} \right)^{\delta} - 1, \quad e^\Phi = \left( \frac{u^4 + b^4}{u^4 - b^4} \right)^{\Delta/2}, \quad C_4 = H^{-1} 
\]

\[
 \delta = \frac{R^4}{2b^4}, \quad \Delta^2 = 10 - \delta^2 
\]

The \(x_4\) directions correspond to the field theory’s 4d space and \(u\) is the radial direction in the 6d transverse space. At large \(u\) the space becomes \(AdS_5 \times S^5\) with radius \(R\). Here \(b\) is a parameter that controls the size of the deformation from AdS - note it enters along with \(u\) and so has energy dimension one. The SO(6) isometry of the transverse plane survives at all \(u\) and thus the \(R\) symmetry of the field theory is not broken. From these facts we can deduce that \(b^4\) corresponds in the field theory to a vacuum expectation value for the dimension four, \(R\)-chargeless operator \(TrF^2\).

It is worth stressing that the vacuum of the \(N = 4\) gauge theory has \(TrF^2 = 0\) (this quantity is the D-term of a superfield and supersymmetry would be broken were it generated) and so the
above geometry describes a non-vacuum state of the field theory. The geometry does though describe some non-supersymmetric, strongly coupled gauge configuration and is relatively simple - for these virtues we will use it below. A consequence of the supersymmetry breaking is that the dilaton (the gauge theory coupling) changes with \(u\) ie the gauge coupling runs with energy scale. It has a pole at \(u = b\) which we interpret as playing the role of the pole in the QCD coupling at the scale \(\Lambda_{QCD}\).

3 D7 Branes and Quarks

The \(N = 4\) gauge theory only has adjoint matter fields - the original construction realized the gauge theory through open string modes with both ends tied to a D3 brane (they transform as \(N_c, \bar{N}_c\)). To generate fundamental representation quarks one must detach one of the string’s ends from the D3 - it is useful to tie it to a D7 brane\(^4\) as shown in Fig. 1.

![D3/D7 configuration](image)

Figure 1: D3/D7 configuration that introduces quarks into the AdS/CFT Correspondence.

The D3 and the D7 share the 0-3 directions, the D7 are in addition extended in the 4-8 directions (we will call the radial coordinate in this space \(\rho\)), and finally the D3 and D7 can be separated in the 8-9 directions (\(w_5\) and \(w_6\) below). This configuration which preserves \(N = 2\) supersymmetry corresponds to the \(N = 4\) gauge theory with an added fundamental representation quark hypermultiplet.

The minimum length D7-D3 string indicates (length \(\times\) tension) the mass of the quark. If the D7 brane lies along the \(\rho\) axis then the quarks are massless and there is an SO(2) symmetry in the \(w_5 - w_6\) plane. If the D7 lies off axis there is a non-zero quark mass and the SO(2) symmetry is explicitly broken. This indicates that the SO(2) symmetry is a geometric realization of the U(1) axial symmetry of the gauge theory (in the supersymmetric case that symmetry is part of a U(1)\(_R\) symmetry). Note that at large \(N_c\) we neglect anomalies.

Using these techniques we can next include quarks into the dilaton deformed geometry above\(^2\). We will work in the approximation where the D7 brane is a probe (so there is no backreaction on the geometry) - this is the quenched limit where the number of flavours \(N_f \ll\)
One simply embeds the D7 brane so as to minimize its world volume via its Dirac Born Infeld action

\[ S_{D7} = -T_7 \int d^8\xi \sqrt{P[G_{ab}]}, \quad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b} \]

where \( T_7 \) is the tension, \( \xi \) the coordinates on the D7, \( x^M \) are the spacetime coordinates and \( G_{MN} \) the background metric.

In the Constable Myers geometry one finds that the D7 brane is repelled by the singular core of the geometry and the regular embeddings of interest are those shown in Fig. 2. At large \( \rho \) the solutions become flat as the gauge theory returns to AdS. The solution is of the form \( w_6 = m + c/\rho^2 + \ldots \) Here \( m \) corresponds to the quark mass and \( c \) to the \( \bar{q}q \) condensate - we can read off the condensate as a function of the quark mass in this theory. A more intuitive understanding of the embedding results from interpreting the separation of the D7 brane from the \( \rho \) axis as the effective quark mass. As one moves in \( \rho \) one is changing RG scale - at large \( \rho \) one sees a small bare quark mass but in the IR (small \( \rho \)) a dynamical mass is generated.

![Figure 2: Embedding solutions for a D7 probe in the Constable Myers geometry and a plot of the meson mass vs quark mass in that model. We chose \( b = R \) here as an example.](image)

In particular we can see that the solution exhibits chiral symmetry breaking. If we try to lie a D7 along the \( \rho \) axis, so \( m = 0 \), it is repelled from the origin and there is a non-zero value of the quark condensate. In fact the D7 may be deflected to any point on a circle in the \( w_5 - w_6 \) plane. We thus explicitly see the breaking of the SO(2) symmetry in that plane and the circle is the vacuum manifold. There should be a Goldstone boson associated with fluctuations of the D7 along the vacuum manifold. One can seek solutions to the equations of motion from the DBI action for those angular fluctuations of the form

\[ \theta(\rho, x) = f(\rho)e^{-ikx}, \quad k^2 = -M^2 \]

Only for particular values of \( M \) is \( f(\rho) \) regular and hence the meson bound state masses are picked out. In Fig. 2 the meson masses as a function of quark mass are shown. There is a
massless Goldstone at \( m = 0 \) and it’s mass grows as \( \sqrt{m} \) as in chiral perturbation theory. The mass of the meson associated with radial fluctuations is also shown - it always has a mass gap.

Note this simple model is sometimes criticized for the presence of a singularity in the metric. It is possible a source is present at \( w = b \) that explains the singularity - one escapes addressing this problem because the D7 never penetrates the singularity. There are alternative D4-D6 descriptions of chiral symmetry breaking\([5]\) that use completely smooth metrics and yet show the same generic structure. The UV of that theory is six dimensional though. See also other constructions in \([6]\).

4 AdS/QCD

The holographic description of chiral symmetry breaking above provides the pion spectrum. In addition a vector field on the D7 world volume describes the vector mesons. Solutions for these fields with non-trivial harmonics on the \( S^3 \) of the D7 brane also exist and describe R-charged mesons, reflecting the supersymmetric origin of the theory. There is no significant decoupling of these R-charged states since the theory is strongly coupled at the scale of the supersymmetry breaking parameter, \( b^4 \) or \( TrF^2 \).

In spite of the differences from QCD one can boldly move to a toy model of QCD in the spirit of the work in \([7]\) (and \([8]\)). In that work a five dimensional theory consisting of axial and vector gauge fields and \( N_c^2 - 1 \) pions in an AdS space with a hard IR (small \( r \)) cut off is studied as a model of the QCD pion, \( \rho \) and \( a \) mesons. We can now repeat that model but using the D7 world volume metric from the theory above\([9]\). This has the advantage that the conformal symmetry breaking is smoothly included in the metric which exists down to \( r = 0 \) rather than through an adhoc cut off. The condensate is also a prediction of the gauge dynamics in this model whereas it was included by hand in the pure AdS model.

The original AdS/CFT Correspondence was for a large \( N_c \) theory. \( N_c \) enters through the prediction for the relative coefficients of the scalar and vector fields’ kinetic terms - in the phenomenological approach this is instead set by requiring that one reproduces the perturbative QCD result for the vector vector correlator \([7]\). Here one is hoping that the conformal nature of the UV asymptotics of AdS in someway mimics the conformal behaviour of weakly coupled QCD. The remaining parameters in the model are then the conformal symmetry breaking scale \( b \) (\( \Lambda_{QCD} \)) and the quark mass (position of the D7 at large \( r \)). Performing a global fit to meson data one finds the results below\([9]\) - the fit is rather good (rms error 12.8%).

|          | holography | expt   |          | holography | expt   |
|----------|------------|--------|----------|------------|--------|
| \( m_\pi \) | 139.0 MeV  | 139.6 MeV | \( f_\pi \) | 83.9 MeV  | 92.4 MeV |
| \( m_\rho \) | 742.7 MeV  | 775.8 MeV | \( f_\rho \) | 297.0 MeV  | 345 MeV   |
| \( m_a \)  | 1337 MeV   | 1230 MeV  | \( f_a \)  | 491.4 MeV  | 433 MeV   |
5 Perfection

The success of the AdS/QCD approach is rather shocking - we used a quenched, large $N_c$ gauge theory with superpartners present! Gauge gravity dualities are also a strong weak coupling duality and so by assuming the gravity dual is weakly coupled out to large radius we lost QCD’s asymptotic freedom. Was the success of the fit just luck then? To answer this one must address systematic errors - this appears hard since the theory is a model and is not derived from QCD. Let us attempt to understand how, at least in principle, one could make a perfect holographic description of QCD [10].

It is clear that a weakly coupled gravity description should only exist below the scale where QCD becomes strongly coupled. We should therefore impose a UV cut off, to represent where QCD undergoes this transition, and work in the gravity theory only at values of the radius below this. This is analogous to working in lattice QCD but with a rather coarse lattice. In fact it has been understood that one can simulate QCD on a coarse lattice and nevertheless precisely reproduce QCD [11]. The crucial point is that as one blocks from a fine lattice to a coarse lattice one must include higher dimension operator couplings. By analogy one should be careful to make sure all the couplings needed to reproduce QCD are present in the gravity dual with a UV cut off. One also needs to ensure all operators take their appropriate vacuum value and have the correct anomalous dimension.

In principle this is straight forward but there are an infinite number of possible operators and couplings and all could be large. One might worry about whether these couplings will be sufficiently small to keep the gravity theory perturbative - there is no guarantee but let us hope they will. In practice our only method to fix these values is phenomenological. One might pick on a small number of couplings and fix their values using a fit to measured hadron data. The hope is then that those are the significant changes needed and that the remainder of the physical spectrum will be predicted more accurately (here the analogy is to improving lattice actions).

As a toy example consider the AdS/QCD model in [12] of the $\rho$ meson and its excited states. The theory is just a gauge field in AdS$_5$ with a non-zero dilaton that blows up in the IR, $\Phi \sim r^{-2}$. In the usual approach one would fix the large $r$ behaviour of the gauge field to enforce the $\bar{q}\gamma^\mu q$ operator to be dimension 3 in the UV. We now though impose that boundary condition not at infinity but at some finite UV cut off[10]. In other words we ensure the scaling dimension of the operator is three down to the scale where QCD becomes non-perturbative. In Fig. 3 we plot the $\rho$ meson masses for the low excitation numbers and compare to the experimental values. Lowering the cut off improves the fit (rms error of 2% for the best fit). Thus changing the anomalous dimension of the quark bilinear operators seems to be an example of an improvement of the holographic dual. One should caution that the importance of many other operators and couplings should be checked although it is far from clear how to include some of these in the
gravity dual. Hopefully though one has understood how to be more systematic in the approach.

Figure 3: The $\rho$ meson and its excited states’ masses with varying UV cut off in the model in [12]. The dots are the QCD data.

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