Practical Channel Estimation and Phase Shift Design for Intelligent Reflecting Surface Empowered MIMO Systems

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Abstract—In this paper, channel estimation techniques and phase shift design for intelligent reflecting surface (IRS)-empowered single-user multiple-input multiple-output (SU-MIMO) systems are proposed. The two novel channel estimation techniques proposed in the paper, single-path approximated channel (SPAC) and selective emphasis on rank-one matrices (SEROM), have low training overhead to enable practical IRS-empowered SU-MIMO systems. SPAC is mainly based on parameter estimation by approximating IRS-related channels as dominant single-path channels. SEROM exploits IRS phase shifts as well as training signals for channel estimation and easily adjusts its training overhead. A closed-form solution for IRS phase shift design is also developed to maximize spectral efficiency where the solution only requires basic linear operations. Numerical results show that SPAC and SEROM combined with the proposed IRS phase shift design achieve high spectral efficiency even with low training overhead compared to existing methods.

Index Terms—Intelligent reflecting surface (IRS), channel estimation, training overhead, phase shift design, spectral efficiency, single-user multiple-input multiple-output (SU-MIMO).

I. INTRODUCTION

INTELLIGENT reflecting surface (IRS), also well known as reconfigurable intelligent surface, is drawing great interest in recent years as a way to tackle the energy consumption problem of future wireless communication systems [1]–[6]. The IRS is a 2D surface consisting of low-cost passive scattering elements that can be deployed in an energy-efficient way and can present the benefit of array and beamforming gain as multiple antennas do in the multiple-input multiple-output (MIMO) systems. While there are no active elements in general, the IRS can manage magnitude (by turning on/off the passive elements) and phase shift of the incoming signals in order to strengthen the reflected signals achieving high spectral efficiency and overcoming large path-loss due to blockage [1], [2].

To fully exploit the advantages of the IRS-empowered communication systems, acquiring proper channel information on the IRS-related channels is essential at the base station (BS) or user equipment (UE). This is difficult in general since the IRS is not capable of transmitting or receiving training signals [7], [8]. Several works were conducted to estimate the IRS-related channels. In [9], an estimation technique was developed to minimize the Cramér-Rao lower bound of IRS-related channel estimation. A least squares approach was adopted in [10], and the estimation error was analyzed with regard to the error offset on the IRS setting caused by the imperfect implementation. In [11], channel estimation under finite bit phase quantization of IRS elements was examined to afford a large number of IRS elements. These estimation techniques, though, are limited to multiple-input single-output or single-input single-output systems.

To realize high spectral efficiency that comes from spatial multiplexing, channel estimation for the IRS-empowered MIMO systems is necessary. In [12], the sparsity of MIMO channels was assumed, and the sparse matrix factorization and matrix completion were alternately repeated to construct estimated channels. The channel estimation in [13] utilized parallel factorization by reformulating the concatenation of received signals. In [14], minimum mean squared error (MMSE) estimation was developed based on the Rayleigh fading channel structure. In [15], alternating least squares based channel estimation was proposed, and the Cramér-Rao lower bound was computed for the Rayleigh fading channel. The above techniques, however, did not consider channel training overhead. The required training sequence length of the IRS-empowered communication systems could be much larger than the systems without the IRS due to a huge number of IRS elements [16], [17].

Another important issue of using the IRS is to properly set the phases of IRS elements. A min-rate maximization problem was formulated in [14], and an iterative technique was proposed to obtain a sub-optimal solution. To maximize downlink energy efficiency, low complexity phase shift design and power allocation technique were proposed in [18]. The IRS element design algorithm in [4] was developed to solve the proposed capacity characterization problem. Though, the IRS element designs in [14] and [18] are hard to be applied for data rate maximization, and the design in [4] does not guarantee its performance for imperfect channel information.
that is obtained by practical channel estimators.

In this paper, we configure a realistic IRS-empowered single-user MIMO (SU-MIMO) system. Considering the passive operation of IRS, we first express the cascaded channel through the IRS not in terms of two separate IRS-related channels, i.e., the UE-IRS link and IRS-BS link, but as a weighted sum of rank-one matrices. Based on the representation, we handle the two practical issues of this system: cascaded UE-IRS-BS channel estimation and IRS phase shift design.

We develop several estimation techniques for the cascaded channel through the IRS. The two simple techniques are first introduced as baselines while the two novel techniques, single-path approximated channel (SPAC) and selective emphasis on rank-one matrices (SEROM), have low training overhead. SPAC is developed by approximating the UE-IRS and IRS-BS links into dominant single-path channels. The BS estimates effective channel parameters to reconstruct the cascaded UE-IRS-BS channel, which largely reduces the training overhead compared to full channel matrix estimation. SEROM efficiently estimates the cascaded UE-IRS-BS channel by designing IRS reflection-coefficient matrices for training, which enables SEROM to easily adjust its training overhead. Low-complexity IRS phase shift design with a closed-form solution is also proposed to maximize spectral efficiency.

We verify through simulations that SPAC and SEROM combined with the proposed IRS phase shift design achieve high spectral efficiency even with low training overhead. SPAC is specialized for the situation where the channel consists of a few dominant paths and is not affected from quantization of IRS phase shifts. SEROM achieves high spectral efficiency when the number of IRS elements is large because it can benefit from designing IRS reflection-coefficient matrices for training. The performance of proposed IRS phase shift design is comparable to that of exhaustive search with low computation complexity in the IRS-empowered SU-MIMO system. In terms of effective data transmissions, it is shown that the BS does not have to know the information of IRS-related channels separately to achieve high spectral efficiency.

The paper is organized as follows. In Section II, we explain the system model of SU-MIMO with the IRS. The basic channel estimation techniques for direct channel and cascaded UE-IRS-BS channel are explained as preliminaries in Section III. The novel cascaded UE-IRS-BS channel estimation techniques to have low training overhead are proposed in Section IV, and the low-complexity IRS phase shift design is proposed in Section V. After presenting numerical results for channel estimation and IRS phase shift design in Section VI, we conclude the paper in Section VII.

Notations: We use lower and upper boldface letters to represent column vectors and matrices. The element-wise conjugate, transpose, and conjugate transpose of a matrix $A$ are denoted by $A^*$, $A^T$, and $A^H$, respectively. For a square matrix $A$, $\text{det}(A)$, $\text{Tr}(A)$, and $A^{-1}$ are the determinant, trace, and inverse of $A$. $A(:,m:n)$ implies the submatrix that consists of the $m$-th column to the $n$-th column of the matrix $A$, and the $m$-th element of a vector $a$ is denoted by $a_m$. $\lfloor a \rfloor$ stands for the vector whose elements are phases of each element of a vector $a$. The diagonal matrix with the entries of a vector $a$ on its main diagonal is expressed as $\text{diag}(a)$. The Kronecker product is denoted by $\otimes$, and $\odot$ implies the Hadamard product. $\Omega_m$ and $I_m$ represent the $m \times 1$ all-zero vector and all-one vector, and $I_m$ represents the $m \times m$ identity matrix. $CN(\mu, Q)$ is used for the complex Gaussian distribution with mean vector $\mu$ and covariance matrix $Q$. Notations $|a|$ and $\Re(a)$ stand for the magnitude and real part of a complex number $a$. $|a|$ represents the largest integer that is not smaller than a real number $a$. Similarly, $|a|$ represents the largest integer that is not larger than a real number $a$. $\|a\|_F$ is the Frobenius-norm of a matrix $A$.

II. System Model

We consider an IRS-empowered time division duplexing (TDD) SU-MIMO system as shown in Fig. 1. The BS deploys $N$ antennas and serves the UE equipped with $M$ antennas. The IRS, which consists of $L$ low-cost passive elements, is assumed to be connected to the BS via a controller where the BS is able to control the IRS elements for favorable signal reflection. For a practical setup, we consider a uniform planar array (UPA) for the BS and UE antennas and IRS elements.

During a channel coherence time block, the uplink received signal at the $t$-th time slot is [19]

$$y_{UL}[t] = (H_{UB} + H_{IB}\Phi[t]H_{UI})f[t]s_{UL}[t] + n_{UL}[t], \quad (1)$$

where $s_{UL}[t] \in \mathbb{C}$ is the transmit signal from the UE satisfying $\mathbb{E}\{|s_{UL}[t]|^2\} \leq P_{UL}$ with the uplink transmit power $P_{UL}$. The transmit beamformer $f[t] \in \mathbb{C}^{N \times 1}$ satisfies $\|f[t]\|^2 = 1$, and $n_{UL}[t] \sim CN(0_N, N_0 I_N)$ is the thermal noise at the BS with the noise variance $N_0$. The uplink channels of the UE-BS direct link, IRS-BS link, and UE-IRS link are denoted by $H_{UB} \in \mathbb{C}^{N \times M}$, $H_{IB} \in \mathbb{C}^{N \times L}$, and $H_{UI} \in \mathbb{C}^{L \times M}$, respectively. The $L \times L$ IRS reflection-coefficient matrix $\Phi[t]$ is defined by $\text{diag}\left([\beta_1[t]e^{j\phi_1[t]}, \ldots, \beta_L[t]e^{j\phi_L[t]}]^T\right)$ where $\beta[t]$ and $\phi[t]$ are the magnitude and phase shift of the $t$-th IRS element. Considering practical passive operation of the
IRS elements, we assume on/off magnitude\(^1\) \(\beta_\ell[t] \in \{0,1\}\) and \(B\)-bit uniform quantization for each phase shift such that \(\phi_\ell[t] \in \left\{0, \frac{2\pi}{2^B}, \ldots, \frac{(2^B-1)2\pi}{2^B}\right\}\). Since it is already shown in [23] that \(B \geq 4\) is enough to achieve almost the same performance of \(B = \infty\), we first assume \(B = \infty\) for conceptual explanation from Section III to Section V. Then, the numerical results in Section VI are based on \(B = 2\) and \(B = 4\) for practically.

We adopt the Rician fading with one line-of-sight (LoS) path and multiple non-line-of-sight (NLoS) paths for all channels [24], [25]. As an example, the uplink channel of the UE-BS direct link channel \(\mathbf{H}_{UB}\) is given by

\[
\mathbf{H}_{UB} = \sqrt{\mu_0 (d_{UB}/d_0)^{-\eta_{UB}}} \sqrt{NM} 1 + K_{UB}^{-1} \times \left( K_{UB} \mathbf{a}_{UB} \mathbf{a}_{BS} \nu_{UB,0}^{\mathbf{x}} \nu_{UB,0}^{\mathbf{e}} \right) + \frac{1}{\sqrt{G_{UB}}} \sum_{g=1}^{G_{UB}} \alpha_{UB,g} \mathbf{a}_{BS} (\nu_{UB,g}^{\mathbf{x}} \nu_{UB,g}^{\mathbf{e}}) \times \mathbf{a}_{UE}^{\mathbf{x}} \nu_{UE,0}^{\mathbf{x}} \nu_{UE,0}^{\mathbf{e}} 
\]

(2)

where \(\mu_0\) is the path-loss at the distance \(d_0\), and the distance and path-loss exponent between the UE and BS are denoted by \(d_{UB}\) and \(\eta_{UB}\) [17], [26], [27]. The Rician K-factor is denoted by \(K_{UB}\), and \(G_{UB}\) is the total number of NLoS paths. For the \(g\)-th path, \(\alpha_{UB,g} \sim \mathcal{C}\mathcal{N}(0,1)\) is the complex path gain, and the vertical and horizontal arrival spatial frequencies at the BS are defined by \(\nu_{UB,0}^{\mathbf{x}} \triangleq \pi \sin(\theta_{UB,0}^{\mathbf{x}})\) and \(\nu_{UB,0}^{\mathbf{e}} \triangleq \pi \sin(\psi_{UB,0}^{\mathbf{e}})\) with the vertical and horizontal arrival angles \(\theta_{UB,0}^{\mathbf{x}}\) and \(\psi_{UB,0}^{\mathbf{e}}\). Similarly, the vertical and horizontal departure spatial frequencies at the UE are defined by \(\nu_{UB,g}^{\mathbf{x}} \triangleq \pi \sin(\theta_{UB,g}^{\mathbf{x}})\) and \(\nu_{UB,g}^{\mathbf{e}} \triangleq \pi \sin(\psi_{UB,g}^{\mathbf{e}})\) with the vertical and horizontal departure angles \(\theta_{UB,g}^{\mathbf{x}}\) and \(\psi_{UB,g}^{\mathbf{e}}\). Assuming half wavelength spacing, the array response vectors at the BS and UE, i.e., \(\mathbf{a}_{BS}(\cdot)\) and \(\mathbf{a}_{UE}(\cdot)\), are given as

\[
\mathbf{a}_{BS} (\nu_{UB,g}^{\mathbf{x}}, \nu_{UB,g}^{\mathbf{e}}) = \frac{1}{\sqrt{N}} \left[ 1, \ldots, e^{j(N_{v}-1)\nu_{UB,g}^{\mathbf{x}}} \ldots, e^{j(N_{h}-1)\nu_{UB,g}^{\mathbf{e}}} \right]^T,
\]

(3)

\[
\mathbf{a}_{UE} (\nu_{UB,g}^{\mathbf{x}}, \nu_{UB,g}^{\mathbf{e}}) = \frac{1}{\sqrt{M}} \left[ 1, \ldots, e^{j(M_{v}-1)\nu_{UB,g}^{\mathbf{x}}} \ldots, e^{j(M_{h}-1)\nu_{UB,g}^{\mathbf{e}}} \right]^T,
\]

(4)

where \(N = N_v N_h\) with \(N_v\) vertical and \(N_h\) horizontal antennas at the BS, and \(M = M_v M_h\) with \(M_v\) vertical and \(M_h\) horizontal antennas at the UE. Note that \(\mathbf{H}_{UB}\) and \(\mathbf{H}_{UI}\) are modeled in the same way as in (2) with proper adjustments on the distance, path-loss exponent, Rician K-factor, number of NLoS paths, number of antennas, array response vectors, and spatial frequencies.

\(^1\)In practice, each IRS element can be controlled so that the power of reflected signal can be arbitrarily small through multi-layer surface design or locally adjustable integrated circuits [20]–[22].

Considering the reflection of incident signals at each IRS element, we can express the cascaded channel through the IRS as a weighted sum of rank-one matrices, which is given as

\[
\mathbf{H}_{IB} \mathbf{F}[t] \mathbf{H}_{UI} = \sum_{\ell=1}^{L} \beta_\ell[t] e^{j\phi_\ell[t]} \mathbf{H}_{IB,\ell} \mathbf{H}_{UL,\ell}^H.
\]

(5)

where \(\mathbf{H}_{IB} = [h_{IB,1}, \ldots, h_{IB,L}]^\mathsf{H}\) and \(\mathbf{H}_{UI} = [h_{UI,1}, \ldots, h_{UI,L}]^\mathsf{H}\). In (5), the \(\ell\)-th rank-one matrix \(h_{IB,\ell} h_{UL,\ell}^H\) is weighted one by \(\beta_\ell[t] e^{j\phi_\ell[t]}\). For simplicity, we denote the \(\ell\)-th rank-one matrix as \(\mathbf{R}_\ell = h_{IB,\ell} h_{UL,\ell}^H\), which gives

\[
\mathbf{H}_{IB} \mathbf{F}[t] \mathbf{H}_{UI} = \sum_{\ell=1}^{L} \beta_\ell[t] e^{j\phi_\ell[t]} \mathbf{R}_\ell.
\]

(6)

The equality (6) implies that it is sufficient to estimate the rank-one matrices \(\mathbf{R}_\ell\) instead of separately estimating \(\mathbf{H}_{IB}\) and \(\mathbf{H}_{UI}\). Hence, we will consider uplink channel estimation techniques for the direct channel \(\mathbf{H}_{UB}\) and the rank-one matrices \(\mathbf{R}_\ell\) in Sections III and IV.

III. PRELIMINARIES

We first explain the estimation of the direct link channel \(\mathbf{H}_{UB}\) and then elaborate on two rudimentary and straightforward techniques to estimate the \(L\) rank-one matrices \(\mathbf{R}_\ell\). The two simple estimation techniques for \(\mathbf{R}_\ell\) are introduced as baselines for the two novel ones proposed in the next section.

A. UE-BS direct link channel estimation

To estimate the direct link channel \(\mathbf{H}_{UB}\), the BS turns off all the IRS elements as \(\mathbf{F}[t] = \text{diag}(0_L)\). The UE transmits the length \(\tau_d\) training sequence using the training beamformer \(\mathbf{f}[t]\) for \(1 \leq t \leq \tau_d\) with the training signal \(s_{UL}[t] = \sqrt{P_{UL}}\). By stacking the \(\tau_d\) received signals, we have

\[
\mathbf{y}_{UL} = \begin{bmatrix} y_{UL}[1], \ldots, y_{UL}[\tau_d] \end{bmatrix}
\]

\[
= \sqrt{P_{UL}} \mathbf{H}_{UB} \mathbf{F}_{UB} + \mathbf{N}_{UB},
\]

(7)

where \(\mathbf{F}_{UB} = [\mathbf{f}[1], \ldots, \mathbf{f}[\tau_d]]\) is the training beamformer, and \(\mathbf{N}_{UB} = [n_{UL}[1], \ldots, n_{UL}[\tau_d]]\) is the noise. The training beamformer \(\mathbf{F}_{UB}\) can be composed of \(M\) rows of \(\tau_d \times \tau_d\) discrete Fourier transform (DFT) matrix with proper normalization, and we set \(\tau_d = M\) to take the minimum sequence length such that \(\mathbf{F}_{UB} \mathbf{F}_{UB}^\mathsf{H} = \mathbf{I}_M\). Then, the channel estimate for the direct link between the UE and BS is computed as

\[
\hat{\mathbf{H}}_{UB} = \frac{1}{\sqrt{P_{UL}}} \mathbf{Y}_{UL} \mathbf{F}_{UB}^\mathsf{H}
\]

\[
= \mathbf{H}_{UB} + \frac{1}{\sqrt{P_{UL}}} \mathbf{N}_{UB} \mathbf{F}_{UB}^\mathsf{H}.
\]

(8)

We define the additional training sequence length as \(\tau_c\), which varies with estimation techniques, to estimate the cascaded UE-IRS-BS channel represented with \(\mathbf{R}_\ell\). For \(\tau_d + 1 \leq t \leq \tau_d + \tau_c\), the BS eliminates the effect of direct link channel as

\[
\tilde{y}_{UL}[t] = y_{UL}[t] - \sqrt{P_{UL}} \hat{\mathbf{H}}_{UB} \mathbf{f}[t]
\]
where \( \tilde{n}_{UL}[t] \) is the effective uplink noise. We adopt the received signal \( \tilde{y}_{UL}[t] \) to explain the rank-one matrix estimation in the following subsections.

### B. One-by-one (OBO) channel estimation

The OBO estimation is to simply estimate a rank-one matrix by one by one. The BS can estimate the \( \ell \)-th rank-one matrix \( \tilde{R}_\ell \) by turning on only the \( \ell \)-th IRS element while keeping the others off and conduct this process in turn for each \( \ell \). Specifically, the IRS reflection-coefficient matrix \( \Phi(\ell) \) to estimate \( \tilde{R}_\ell \) is defined as

\[
\Phi(\ell) = \text{diag}\left( [0^T, \ldots, e^{j\phi_\ell}, 0^T_{L-\ell}] \right),
\]

and \( \Phi[t] \) is fixed as \( \Phi(\ell) \) during the \( \ell \)-th training period \( \tau_d + (\ell - 1)M + 1 \leq t \leq \tau_d + \ell M \). The UE transmits the length \( M \) training sequence with \( s_{UL}[t] = \sqrt{P_{UL}} \) using \( f[t] \) during the \( \ell \)-th training period. The BS stacks the \( M \) received signals in (9) to estimate the \( \ell \)-th rank-one matrix \( \tilde{R}_\ell \), written as

\[
\tilde{Y}_{UL,\ell} = [\tilde{y}_{UL}[\tau_d + (\ell - 1)M + 1], \ldots, \tilde{y}_{UL}[\tau_d + \ell M]],
\]

where \( F_{UL,\ell} \triangleq [f[\tau_d + (\ell - 1)M + 1], \ldots, f[\tau_d + \ell M]] \) and \( \tilde{N}_{UL,\ell} \triangleq [\tilde{n}_{UL}[\tau_d + (\ell - 1)M + 1], \ldots, \tilde{n}_{UL}[\tau_d + \ell M]] \) are respectively the training beamformer and noise. As in Section III-A, the normalized \( M \times M \) DFT matrix can be used as the training beamformer \( F_{UL,\ell} \).

With the \( M \) received signals in (11), the BS estimates the rank-one matrix for the \( \ell \)-th IRS element as

\[
\tilde{R}_\ell = \frac{e^{-j\phi_\ell}}{\sqrt{P_{UL}}} \tilde{Y}_{UL,\ell} F_{UL,\ell}^H,
\]

Conducting this process for all \( L \) rank-one matrices, the additional training sequence length for the OBO estimation becomes \( \tau_c = 7_{\text{OBO}} = LM \). It is obvious that the OBO estimation is inefficient since only one IRS element is turned on during each training period, resulting in significantly high training overhead.

### C. Cooperative one-by-one (Co-OBO) channel estimation

With cooperative uplink and downlink signalings from the UE and BS, the training sequence length can be made significantly small, compared to that of the OBO estimation in Section III-B. The IRS reflection-coefficient matrix in the Co-OBO estimation is the same as in (10) but is employed only for two time slots for each \( \ell \). In other words, in order to estimate \( \tilde{R}_\ell \), \( \Phi[t] \) is fixed as \( \Phi(\ell) \) during the \( \ell \)-th training period \( \tau_d + 2(\ell - 1) + 1 \leq t \leq \tau_d + 2\ell \). The uplink and downlink signalings are sequentially conducted for the first and second time slots of each training period, i.e., the UE transmits \( s_{UL}[\tau_d + 2(\ell - 1) + 1] = \sqrt{P_{UL}} \), and the BS transmits \( s_{DL}[\tau_d + 2\ell] = \sqrt{P_{DL}} \) with the downlink transmit power \( P_{DL} \). For the first time slot, the \( \ell \)-th uplink signal \( \tilde{y}_{UL,\ell} \triangleq \tilde{y}_{UL}[\tau_d + 2(\ell - 1) + 1] \) is expressed as

\[
\tilde{y}_{UL,\ell} = e^{j\phi_\ell} h_{IB,\ell} \sqrt{P_{UL}} h_{UL,\ell}^H + \tilde{n}_{UL,\ell},
\]

where we define \( f_{UL,\ell} = [f[\tau_d + 2(\ell - 1) + 1] \) and \( \tilde{n}_{UL,\ell} = \tilde{n}_{UL}[\tau_d + 2(\ell - 1) + 1] \) for simplicity. The product \( \sqrt{P_{UL}} h_{UL,\ell}^H f_{UL,\ell} \) in (13) can be regarded as the effective scalar-valued signal \( \tilde{s}_{UL,\ell} \).

Applying the channel reciprocity from TDD [28], the downlink received signal is

\[
y_{DL}[t] = \left( H_{UL}^H + H_{IB,\ell}^H \Phi(\ell) H_{IB,\ell}^H \right) w[t] s_{DL}[t] + n_{DL}[t],
\]

where \( w[t] \in \mathbb{C}^{N_1 \times 1} \) is the training beamformer at the BS satisfying \( ||w[t]||^2 = 1 \), and \( n_{DL}[t] \sim \mathcal{CN}(0, M, N_2 I) \) is the thermal noise at the UE. Assuming perfect analog feedback, the UE feeds the received downlink signal \( y_{DL}[\tau_d + 2\ell] \) back to the BS. Then, similar to (9), the BS can compute \( \tilde{y}_{DL,\ell} \) as

\[
\tilde{y}_{DL,\ell} = y_{DL}[\tau_d + 2\ell] - \sqrt{P_{DL}} H_{UL}^H w_{DL,\ell} + \tilde{n}_{DL,\ell}
\]

\[
= \sqrt{P_{DL}} \left( H_{UL}^H - H_{UL}^* \right) + e^{-j\phi_\ell} h_{UL,\ell} \Phi(\ell) H_{UL,\ell} w_{DL,\ell} + n_{DL,\ell}
\]

\[
= e^{-j\phi_\ell} h_{UL,\ell} \left( \sqrt{P_{DL}} h_{UL,\ell}^H w_{DL,\ell} \right) + \tilde{n}_{DL,\ell},
\]

where \( w_{DL,\ell} = w[\tau_d + 2\ell] \) and \( n_{DL,\ell} = n_{DL}[\tau_d + 2\ell] \), and \( \tilde{n}_{DL,\ell} \) is the effective downlink noise. Again, the product \( \sqrt{P_{DL}} h_{UL,\ell}^H w_{DL,\ell} \) in (15) can be regarded as the effective scalar-valued signal \( \tilde{s}_{DL,\ell} \).

Using \( \tilde{y}_{UL,\ell} \) and \( \tilde{y}_{DL,\ell} \) in (13) and (15), the BS finally estimates the rank-one matrix as

\[
\tilde{R}_\ell = \frac{\tilde{y}_{UL,\ell} \tilde{y}_{DL,\ell}^*}{\sqrt{P_{DL}} e^{-j\phi_\ell} w_{DL,\ell} H_{UL}^H + \tilde{n}_{UL,\ell}}
\]

\[
= \frac{\tilde{y}_{UL,\ell} \tilde{y}_{DL,\ell}^* + \tilde{n}_{UL,\ell}}{\sqrt{P_{DL}} e^{-j\phi_\ell} w_{DL,\ell} H_{UL}^H + \tilde{n}_{UL,\ell}}
\]

\[
= \frac{\tilde{R}_\ell}{1 + \tilde{n}_{UL,\ell}},
\]
where $\tilde{n}_\ell$ and $\hat{N}_\ell$ are the noise terms, expressed as
\begin{equation}
\tilde{n}_\ell = \sqrt{P_{DL} e^{-j\phi_\ell} w_{DL,\ell}^H} \hat{a}_{UL,\ell},
\end{equation}
\begin{equation}
\hat{N}_\ell = \frac{1}{1 + \tilde{n}_\ell} \left( e^{-j2\phi_\ell} \hat{h}_{IB,\ell}^H \hat{a}_{DL,\ell} + e^{-j\phi_\ell} \hat{h}_{UL,\ell}^H \hat{a}_{UL,\ell}^* + e^{-j\phi_\ell} \hat{h}_{UL,\ell}^H \hat{a}_{DL,\ell}^* + e^{-j2\phi_\ell} \hat{h}_{UL,\ell}^H \hat{a}_{DL,\ell}^* \right),
\end{equation}
respectively. The Co-OBO estimation requires only two time slots to estimate $\mathbf{R}_\ell$ for each $\ell$ through sequential uplink and downlink signalings. Conducting this for all $\ell$, the additional training sequence length for the Co-OBO estimation becomes $\tau_c = \tau_{Co-OBO} = 2L$. For $M \gg 2$, which is valid for typical MIMO systems, it is obvious that $\tau_{Co-OBO} \ll \tau_{OBO} = LM$.

However, employing only two training signals to estimate each rank-one matrix makes the Co-OBO estimation vulnerable to burst noise, and perfect analog feedback is difficult to achieve in practice as well.

IV. CHANNEL ESTIMATION FOR IRS-EMPOWERED MIMO SYSTEM

In this section, we propose the two novel channel estimation techniques, called SPAC and SEROM, to estimate the $L$ rank-one matrices $\mathbf{R}_\ell$. The two techniques are designed to overcome the high training overhead of OBO estimation, which resolve the burst noise and imperfect feedback issues of Co-OBO estimation as well.

A. Single-path approximated channel (SPAC)

With partially applying the OBO estimation only for a small number of IRS elements, SPAC is developed to estimate the rank-one matrices for the remaining IRS elements without additional training overhead. SPAC considers the structural property of IRS-empowered system and extracts the necessary channel parameters by focusing on a dominant path. For example, with the vanishing effect of non-dominant paths, the UE-BS direct link channel $\mathbf{H}_{UB}$ in (2) can be approximated as
\begin{equation}
\mathbf{H}_{UB} \approx \sqrt{\rho_0 (d_{UB}/d_0)^{-\eta/mH}} \sqrt{NMK_{UB}} \frac{\hat{a}_{UB,0}^H}{1 + K_{UB}} \alpha_{UB,0} \times \mathbf{a}_{BS} (\nu_{UB,0}^{rx}, \xi_{UB,0}^{tx}) \mathbf{a}_{UE}^H (\nu_{UB,0}^{tx}, \xi_{UB,0}^{rx})
\end{equation}

where the last term is the channel for the dominant path with the overall gain $\gamma_{UB}$. This approximation is valid as the Rician K-factor increases and thereby effective in channel estimation or beamforming especially when a channel is dominated by the strongest path component.

To handle a dominant path of each IRS-related channel in a similar way, the single-path approximations for $\mathbf{H}_{IB}$ and $\mathbf{H}_{UI}$ can be expressed as
\begin{equation}
\mathbf{H}_{IB} \approx \tilde{\mathbf{H}}_{IB} = \gamma_{IB} \mathbf{a}_{BS} (\nu_{IB}^{rx}, \xi_{IB}) \mathbf{a}_{RS}^H (\nu_{IB}^{tx}, \xi_{IB}^{rx}),
\end{equation}
\begin{equation}
\mathbf{H}_{UI} \approx \tilde{\mathbf{H}}_{UI} = \gamma_{UI} \mathbf{a}_{RS} (\nu_{UI}^{rx}, \xi_{UI}) \mathbf{a}_{UE}^H (\nu_{UI}^{tx}, \xi_{UI}^{rx}),
\end{equation}

Focusing on (20), $\gamma_{IB}$ is the effective complex-valued gain between the IRS and BS. Similar to (3) and (4), the array response vector at the IRS is given as
\begin{equation}
\mathbf{a}_{RS} (\nu_{IB}^{rx}, \xi_{IB}^{tx}) = \frac{1}{\sqrt{L}} \left[ 1, \ldots, e^{j(L_u-1)\nu_{IB}^{rx}} \right] \otimes \left[ 1, \ldots, e^{j(L_h-1)\xi_{IB}^{tx}} \right]^T,
\end{equation}
with the vertical and horizontal spatial frequencies $\nu_{IB}^{tx}$ and $\xi_{IB}^{tx}$. The numbers of vertical and horizontal IRS elements are denoted by $L_u$ and $L_h$ satisfying $L = L_u L_h$. The parameters in (21) are similarly defined.

We can estimate the two gains and eight spatial frequencies embedded on the rank-one matrix
\begin{equation}
\tilde{\mathbf{R}}_\ell = \tilde{\mathbf{H}}_{IB}(\cdot, \ell) \tilde{\mathbf{H}}_{UI}(\ell, :)
= \gamma_{IB} \mathbf{a}_{BS} (\nu_{IB}^{tx}, \xi_{IB}^{tx}) \mathbf{a}_{RS}^H (\nu_{IB}^{rx}, \xi_{IB}^{tx}) \ell
\times \gamma_{UI} \mathbf{a}_{RS} (\nu_{UI}^{rx}, \xi_{UI}^{tx}) \mathbf{a}_{UE}^H (\nu_{UI}^{tx}, \xi_{UI}^{rx})^T,
\end{equation}
for each $\ell$. The novel part of SPAC is that the BS does not estimate all the parameters in (23) separately but acquire the effective parameters concerned with them. The overall process of SPAC is summarized as follows:

Step 1: By sequentially turning on only a small number of IRS elements one by one, a few rank-one matrices $\mathbf{R}_\ell$ are estimated by the OBO estimation.

Step 2: The spatial frequencies $(\nu_{IB}^{tx}, \xi_{IB}^{tx})$ and $(\nu_{UI}^{tx}, \xi_{UI}^{tx})$ are estimated to reconstruct the array response vectors $\mathbf{a}_{BS} (\nu_{IB}^{tx}, \xi_{IB}^{tx})$ and $\mathbf{a}_{UE} (\nu_{UI}^{tx}, \xi_{UI}^{tx})$ at the BS and UE sides.

Step 3: The two effective IRS-side spatial frequencies are estimated to obtain $\left[ \mathbf{a}_{RS}^H (\nu_{IB}^{rx}, \xi_{IB}^{tx}) \ell \times \mathbf{a}_{RS} (\nu_{UI}^{rx}, \xi_{UI}^{tx}) \right]^T$ for all $\ell$.

Step 4: The overall gain $\gamma_{IB}$, $\gamma_{UI}$ common for the rank-one matrices is obtained.

Step 5: The remaining rank-one matrices not estimated in Step 1 are constructed by (23) using the parameters obtained from Steps 2-4.

For clear understanding of the estimation process, we first specify the IRS element index sets for the $x$-th column and
the $y$-th row as $\mathcal{S}_{\text{IRS},x}^y$ and $\mathcal{S}_{\text{IRS},y}^x$, respectively. The common sequential numbering is considered to index the IRS elements as in Fig. 2. With such indexing, the two index sets $\mathcal{S}_{\text{IRS},x}^y$ and $\mathcal{S}_{\text{IRS},y}^x$ are defined as

\[
\mathcal{S}_{\text{IRS},x}^y = \{x, L_h + x, \cdots, (L_v - 1)L_h + x\},
\]
\[
\mathcal{S}_{\text{IRS},y}^x = \{(y - 1)L_h + 1, (y - 1)L_h + 2, \cdots, yL_h\}.
\]

In terms of the BS and UE, the UPA antenna index sets are similarly defined.

In Step 1, the index set $\mathcal{S}_{\text{IRS}} \subset \{1, \cdots, L\}$ is defined, and the BS estimates the rank-one matrices $\mathbf{R}_\ell$ only for $\ell \in \mathcal{S}_{\text{IRS}}$ using the OBO estimation in Section III-B. Considering the UPA structure of IRS, we employ $\mathcal{S}_{\text{IRS}} \triangleq \mathcal{S}_{\text{IRS},1} \cup \mathcal{S}_{\text{IRS},1}$ in order that the set $\mathcal{S}_{\text{IRS}}$ contains the information of both the vertical and horizontal spatial frequencies at the IRS side. To reduce the training overhead, we let $\mathcal{S}_{\text{IRS}}$ be a small set with $L_v + L_h - 1$ IRS elements, while the set can include multiple columns and rows of the IRS elements. Once the rank-one matrices for $\mathcal{S}_{\text{IRS}}$ are estimated by the OBO estimation, the BS extracts the effective parameters based on the estimates $\mathbf{R}_\ell$ for $\ell \notin \mathcal{S}_{\text{IRS}}$ to construct the remaining rank-one matrices for $\ell \notin \mathcal{S}_{\text{IRS}}$.

The spatial frequencies related to the BS and UE sides are estimated in Step 2 to reconstruct $\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^x)$ and $\mathbf{a}_{\text{UE}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^x)$. In (23), it can be seen that the column and row spaces of $\mathbf{R}_\ell$ are the same as those of $\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^x)$ and $\mathbf{a}_{\text{UE}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^x)$, respectively. Therefore, we treat the left and right singular vectors corresponding to the largest singular value of the rank-one matrix $\mathbf{R}_\ell$ as its representative column and row. Based on the left and right singular vectors for $\ell \in \mathcal{S}_{\text{IRS}}$, we extract the spatial frequencies $\nu_{\text{IB}}$ and $\xi_{\text{IB}}$ in (23). Focusing on the BS side and a specific $\ell \in \mathcal{S}_{\text{IRS}}$, the left singular vector $\mathbf{u}_\ell \in \mathbb{C}^{N_v \times N_h}$ can be rearranged into a matrix by arranging the elements of $\mathbf{u}_\ell$ to follow the BS antenna numbering, which is similarly defined to that of the IRS in Fig. 2. In other words, the rearranged matrix $\mathbf{U}_{\text{BS},\ell} \in \mathbb{C}^{N_v \times N_h}$ can be defined as

\[
\mathbf{U}_{\text{BS},\ell} = \begin{bmatrix}
    \mathbf{u}_{\ell,1} & \cdots & \mathbf{u}_{\ell,N_h} \\
    \mathbf{u}_{\ell,N_h+1} & \cdots & \mathbf{u}_{\ell,2N_h} \\
    \vdots & \ddots & \vdots \\
    \mathbf{u}_{\ell,(N_v-1)N_h+1} & \cdots & \mathbf{u}_{\ell,N_vN_h}
\end{bmatrix}.
\]

We denote the $x$-th column and $y$-th row vectors of $\mathbf{U}_{\text{BS},\ell}$ by $\mathbf{u}_{\text{BS},\ell}^x$ and $\mathbf{u}_{\text{BS},\ell}^y$, respectively.

As in (3), $\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^x)$ is composed of the vertical and horizontal array response vectors. Based on the structure, the vertical spatial frequency $\nu_{\text{IB}}^x$ is estimated as

\[
\hat{\nu}_{\text{IB}}^x = \frac{\sum_{\ell \in \mathcal{S}_{\text{IRS}}} \sum_{y=1}^{N_v} (\mathbf{u}_{\text{BS},\ell}^y \cdot \mathbf{u}_{\text{BS},\ell}^{y-1})}{(L_v + L_h - 1)(N_v - 1)N_h}.
\]

Similarly, we estimate the horizontal spatial frequency $\xi_{\text{IB}}^y$ as

\[
\hat{\xi}_{\text{IB}}^y = \frac{\sum_{\ell \in \mathcal{S}_{\text{IRS}}} \sum_{x=1}^{N_h} (\mathbf{u}_{\text{BS},\ell}^x \cdot \mathbf{u}_{\text{BS},\ell}^{x-1})}{(L_v + L_h - 1)(N_h - 1)N_v}.
\]

In words, the estimates in (27) and (28) are the sample averages of spatial frequencies based on (26). Now, the estimate of BS-side array response vector $\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^y)$ is reconstructed as in (3) with the two estimated spatial frequencies $\hat{\nu}_{\text{IB}}^x$ and $\hat{\xi}_{\text{IB}}^y$. Using the right singular vectors $\mathbf{v}_\ell$ for $\ell \in \mathcal{S}_{\text{IRS}}$, the UE-side array response vector is similarly estimated as

\[
\mathbf{a}_{\text{UE}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^y) = \mathbf{U}_{\text{BS},\ell} \mathbf{v}_\ell,
\]

where (a) is based on the property that $(\mathbf{A} \odot \mathbf{B}) \circ (\mathbf{C} \odot \mathbf{D}) = (\mathbf{A} \odot \mathbf{C}) \circ (\mathbf{B} \odot \mathbf{D})$. This implies that only the two effective spatial frequencies $\nu_{\text{BS}}$ and $\xi_{\text{BS}}$ are needed to construct $\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^y) \odot \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^y)$.

\[
\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^y) \odot \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}^y) = \frac{1}{\sqrt{L_v}} \begin{bmatrix}
    1 \\
    \vdots \\
    e^{-j(L_v-1)\nu_{\text{IB}}^x} \\
    e^{-j(L_h-1)\xi_{\text{IB}}^y}
\end{bmatrix} \odot \begin{bmatrix}
    1 \\
    \vdots \\
    e^{-j(L_v-1)\nu_{\text{IB}}^x} \\
    e^{-j(L_h-1)\xi_{\text{IB}}^y}
\end{bmatrix}
\]

\[
\times \frac{1}{\sqrt{L_h}} \begin{bmatrix}
    1 \\
    \vdots \\
    e^{-j(L_v-1)\nu_{\text{IB}}^x} \\
    e^{-j(L_h-1)\xi_{\text{IB}}^y}
\end{bmatrix} \odot \begin{bmatrix}
    1 \\
    \vdots \\
    e^{-j(L_v-1)\nu_{\text{IB}}^x} \\
    e^{-j(L_h-1)\xi_{\text{IB}}^y}
\end{bmatrix}
\]

\[
\times \frac{1}{\sqrt{L_v}} \begin{bmatrix}
    1 \\
    \vdots \\
    e^{-j(L_v-1)\nu_{\text{IB}}^x} \\
    e^{-j(L_h-1)\xi_{\text{IB}}^y}
\end{bmatrix} \odot \begin{bmatrix}
    1 \\
    \vdots \\
    e^{-j(L_v-1)\nu_{\text{IB}}^x} \\
    e^{-j(L_h-1)\xi_{\text{IB}}^y}
\end{bmatrix}
\]

\[
= \frac{1}{\sqrt{L_v}} [\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}) \odot \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}})]^T
\]

\[
\times \frac{1}{\sqrt{L_h}} [\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}) \odot \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}})]^T
\]

\[
= \frac{1}{\sqrt{L_v}} [\mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}) \odot \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}})]
\]

\[
= \frac{1}{\sqrt{L_v}} \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}}) \mathbf{a}_{\text{BS}}(\nu_{\text{IB}}^x, \xi_{\text{IB}})
\]

(31)
To estimate the two spatial frequencies $\nu_{IRS}$ and $\xi_{IRS}$, we can exploit the actual observation of $c_{\ell}$ in the form of $\hat{c}_{\ell} = a_{BS}^H (\hat{\nu}_{IRS}^x, \hat{\xi}_{IRS}^x) \tilde{R}_{\ell} a_{UE} (\hat{\nu}_{UE}^x, \hat{\xi}_{UE}^x)$ for $\ell \in S_{IRS}$ with the parameters obtained in Steps 1 and 2. For $L_v$ observations of $\hat{c}_{\ell}$ for $\ell \in S_{IRS,1}$, the estimated vertical spatial frequency $\hat{\nu}_{IRS}$ is

$$\hat{\nu}_{IRS} = \frac{1}{L_v} \sum_{\ell \in S_{IRS,1}} \ell \left( \frac{\hat{c}_{\ell}}{\hat{c}_{\ell - L_h}} \right). \quad (32)$$

For $L_h$ observations for $\ell \in S_{IRS,1}^h$, the horizontal spatial frequency is estimated as

$$\hat{\xi}_{IRS} = \frac{1}{L_h} \sum_{\ell \in S_{IRS,1}^h} \ell \left( \frac{\hat{c}_{\ell}}{\hat{c}_{\ell - 1}} \right). \quad (33)$$

With the estimated spatial frequencies, the BS constructs the IRS-side array response vector in (31).

The overall gain $\gamma_{IRS} = \gamma_{IB} \gamma_{UI}$ is estimated in Step 4 instead of each gain separately. Using (30) and (31) in Step 3, the overall gain can be directly given as

$$\gamma_{IRS} = \frac{1}{\sqrt{L}} \left| a_{IRS} (\nu_{IRS}, \xi_{IRS}) \right| \ell = \frac{\gamma_{IB} \gamma_{UI} [a_{IRS}^x (\nu_{IRS}^x, \xi_{IRS}^x) \odot a_{IRS} (\nu_{IRS}^x, \xi_{IRS}^x)] \ell}{\sqrt{L}} \left| a_{IRS} (\nu_{IRS}, \xi_{IRS}) \right| \ell$$

$$= \gamma_{IB} \gamma_{UI}, \quad (34)$$

which can be obtained for any $\ell \in S_{IRS}$. By utilizing $\hat{c}_{\ell}$, $\hat{\nu}_{IRS}$, and $\hat{\xi}_{IRS}$ for $\ell \in S_{IRS}$ obtained in Step 3, the overall gain can be computed as in (34). Based on the observations, the effective gain is estimated as

$$\hat{\gamma}_{IRS} = \frac{1}{L_v + L_h - 1} \sum_{\ell \in S_{IRS}} \frac{\hat{c}_{\ell}}{\sqrt{L}} a_{IRS} (\hat{\nu}_{IRS}, \hat{\xi}_{IRS}) \ell. \quad (35)$$

Now we can reconstruct the remaining rank-one matrices $\tilde{R}_{\ell}$ for $\ell \notin S_{IRS}$ by using all estimated parameters as

$$\tilde{R}_{\ell} = \frac{\hat{\gamma}_{IRS}}{\sqrt{L}} a_{BS} (\hat{\nu}_{IRS}^x, \hat{\xi}_{IRS}^x) \left[ a_{IRS} (\hat{\nu}_{IRS}, \hat{\xi}_{IRS}) \ell a_{UE} (\hat{\nu}_{UE}^x, \hat{\xi}_{UE}^x) \right]. \quad (36)$$

As the rank-one matrix estimation utilizing uplink signaling is conducted only for $\ell \in S_{IRS}$, the training overhead for SPAC is $\tau_c = \tau_{SPAC} = (L_v + L_h - 1)M$. Compared with the training overhead of the OBO estimation $\tau_{OBO} = LM = L_v L_h M$, the overhead of SPAC is remarkably low especially with large $L$.

With the single-path channel approximation, SPAC substitutes the problem of large dimensional channel estimation into that of a small number of parameter estimations, contributing to low training overhead.

**B. Selective emphasis on rank-one matrices (SEROM)**

SEROM is proposed to conduct efficient channel estimation with the design of IRS reflection-coefficient matrices. Different from the previous techniques, SEROM always turns on the entire IRS elements and utilizes both the IRS phase shifts and uplink signaling for channel estimation. We first denote the IRS reflection-coefficient matrix for the $q$-th training period by $\Phi(q)$, which is defined as

$$\Phi(q) = \text{diag} \left( [e^{j\phi_1(q)}, \ldots, e^{j\phi_L(q)}]^T \right), \quad (37)$$

with $q \in \{1, \ldots, Q\}$ where $Q$ is the total number of training periods. The IRS reflection-coefficient matrix $\Phi(q)$ is fixed as $\Phi(q)$ during the $q$-th training period $\tau_q + (q - 1)M + 1 \leq t \leq \tau_q + qM$. The length of each training period for SEROM is $M$, which is equal for the OBO estimation and SPAC. However, the number of training periods for the two previous techniques is $L$ and $L_v + L_h - 1$, and it implies that their training overhead depends on the number of IRS elements. SEROM can adapt the training overhead flexibly since $Q$ is the adjustable parameter independent of a system structure.

As in Section III-B, the UE transmits the length $M$ training sequence with $s_{UL}[t] = \sqrt{P_{UL}}$ and exploits the normalized $M \times M$ DFT matrix as the training beamformer $F_{UIB,q} = \sum_{\tau_q} [f(\tau_q + (q - 1)M + 1), \ldots, f(\tau_q + qM)]$ for each $q$. The BS processes the $M$ received signals as

$$\frac{1}{\sqrt{P_{UL}}} Y_{UIB,q} = H_{IB} \Phi(q) H_{UI} + \frac{1}{\sqrt{P_{UL}}} \tilde{N}_{UIB,q} F_{UIB,q}^H,$$

for the $q$-th training period. Recalling that $H_{IB} \Phi(q) H_{UI} = \sum_{\ell=1}^{L} e^{j\phi_{\ell}(q)} R_{\ell}$ as in (6), the cascaded UE-IRS-BS channel can be expressed as

$$H_{IB} \Phi(q) H_{UI} = \left[ e^{j\phi_1(q)} I_N \quad \ldots \quad e^{j\phi_L(q)} I_N \right] \left[ \begin{array}{c} R_1 \\ \vdots \\ R_L \end{array} \right]$$

$$= \left( \left[ e^{j\phi_1(q)} I_N \quad \ldots \quad e^{j\phi_L(q)} I_N \right] \otimes I_N \right) \left[ \begin{array}{c} R_1 \\ \vdots \\ R_L \end{array} \right]. \quad (39)$$

Then, we can stack the cascaded channel through the IRS $H_{IB} \Phi(q) H_{UI}$ as

$$
\begin{bmatrix}
H_{IB} \Phi^{(1)} H_{UI} \\
\vdots \\
H_{IB} \Phi^{(Q)} H_{UI}
\end{bmatrix} = \left( \left[ e^{j\phi_1^{(1)}} I_N \quad \ldots \quad e^{j\phi_L^{(1)}} I_N \right] \otimes I_N \right) \left[ \begin{array}{c} R_1 \\ \vdots \\ R_L \end{array} \right], \quad (40)
$$

where $\Omega \in C^{Q \times L}$ is the IRS training matrix, whose elements are unit modulus.

The IRS training matrix $\Omega$ in (40) can be designed to have mutually orthogonal columns for the product $(\Omega^H \otimes I_N)(\Omega \otimes I_N) = (\Omega^H \Omega \otimes I_N)$ to be a non-zero diagonal matrix. This
condition facilitates perfect extraction of the rank-one matrices from the stacked cascaded channel in (40). However, it is feasible only when the number of training periods $Q$ is larger than or equal to the number of the IRS elements $L$. For large $L$, which is typical for IRS-empowered systems, a number of training periods are required to satisfy such orthogonality, and this motivates us to design the IRS training matrix under the condition $Q < L$.

Since it is impossible to make the columns of $\Omega$ mutually orthogonal for $Q < L$, we design the IRS training matrix to have pseudo-orthogonal columns as

$$
(\Omega(:, \ell))^H \Omega(:, k) = \begin{cases} a_\ell, & \text{for } \ell = k, \\
 b_{\ell, k}, & \text{otherwise},
\end{cases}
$$

satisfying $|a_\ell| \gg |b_{\ell, k}|$ for all $\ell$ and $k$. To design such $\Omega$, we can employ a submatrix by choosing $Q$ rows for $Q < L$ or $L$ columns for $Q \geq L$ from the $M \times M$ DFT matrix where $M = \max\{Q, L\}$. The BS finally conducts the rank-one matrix estimation as

$$
\begin{bmatrix}
R_1 \\
\vdots \\
R_L
\end{bmatrix} = \frac{A}{\sqrt{P_{UL}}} \begin{bmatrix}
\hat{\Omega}_1^H,1 F^H_{UIB,1} \\
\vdots \\
\hat{\Omega}_L^H,1 F^H_{UIB,1}
\end{bmatrix}
$$

$$
= A \left( \begin{bmatrix}
a_1 & \cdots & b_{1,L} \\
\vdots & \ddots & \vdots \\
b_{L,1} & \cdots & a_L
\end{bmatrix} \otimes I_N \right) \begin{bmatrix}
R_1 \\
\vdots \\
R_L
\end{bmatrix}
$$

$$
+ \frac{A}{\sqrt{P_{UL}}} \begin{bmatrix}
\hat{\Omega}_1^H,1 F^H_{UIB,1} \\
\vdots \\
\hat{\Omega}_L^H,1 F^H_{UIB,1}
\end{bmatrix}.
$$

The normalization factor $A$ to cancel the amplification effect of $\Omega^H \Omega$ is defined as

$$
A = \frac{\sum_{\ell=1}^L (\Omega(:, \ell))^H \Omega(:, \ell)}{Q \sum_{\ell=1}^L \sum_{k=1}^L (\Omega(:, \ell))^H \Omega(:, k)}
$$

$$
= \frac{\sum_{\ell=1}^L a_\ell}{Q \sum_{\ell=1}^L |a_\ell + \sum_{k \neq \ell} b_{\ell, k}|}
$$

$$
\leq \frac{L}{\sum_{\ell=1}^L Q + \sum_{k \neq \ell} b_{\ell, k}},
$$

where $a_\ell = Q$ holds for all $\ell$ since the entire IRS elements are turned on with the unit modulus constraint. For $Q \geq L$, the $L$ columns of $Q \times Q$ DFT matrix can be chosen to give $b_{\ell, k} = 0$ and $A = 1/Q$. For $Q < L$, the $Q$-row submatrix from the $L \times L$ DFT matrix can be chosen to satisfy $|a_\ell| \gg |b_{\ell, k}|$ and $A \approx 1/Q$. It is important to point out that, by keeping the design of $\Omega$ to satisfy the pseudo-orthogonality, SEROM can also be adopted in multi-user MIMO systems where each row of $F_{UIB,q}$ can be taken into account as a training sequence of each UE.

The overall training overhead of SEROM is $r_{SE} = r_{SEROM} = QM$. Note that $r_{SEROM}$ is independent from the number of IRS elements $L$. For the small number of the IRS elements, we can take $Q \geq L$ with moderate training overhead, and the IRS training matrix with $A = 1/Q$ ensures perfect rank-one matrix estimation in (42) at noiseless circumstance. However, keeping the condition $Q \geq L$ makes the minimum length of training sequences proportional to $L$, which is undesirable for typical IRS-empowered systems adopting large $L$. In this case, we can set $Q < L$ or even $Q \ll L$ to suppress the training overhead in a moderate range.

V. IRS Phase Shift Design

The considered IRS-empowered SU-MIMO system is intended to serve the UE with high spectral efficiency through the support of the IRS. In this section, we propose a novel phase shift design for the IRS to achieve high spectral efficiency. It can be shown that the proposed design gives an optimal phase shift that maximizes the spectral efficiency for each IRS element while the phase shifts of other IRS elements are fixed. In addition, all the processes require only basic linear matrix operations making the proposed design practical.

Considering the channel estimation techniques of the previous sections, the proposed phase shift design operates even when the BS does not know $H_{UB}$ and $H_{UI}$ separately and only has channel knowledge in the form of rank-one matrices, i.e., $R_\ell$. We first assume perfect channel information at the BS for conceptual explanation. Then, for the numerical results in Section VI, we examine the proposed phase shift design with the perfect channel information and also with the estimated channels by the proposed techniques in Section IV.

A. Optimal phase shift for each IRS element

Relying on the downlink and uplink channel reciprocity in TDD [28], we take the conjugate transpose to represent the total downlink channel $H_{DL}$, where the total channel $H_{tot}$ is represented by

$$
H_{tot} = H_{UB} + H_{IB} \Phi[t] H_{UI} = H_{UB} + \sum_{\ell=1}^L e^{j \phi[t]} R_\ell.
$$

Then, the downlink spectral efficiency $R_{DL}$ is given as [29]

$$
R_{DL} = \log_2 \left( \det \left( I_r + \frac{P_{DL}}{r N_0} W^H H_{DL} H_{DL}^H W \right) \right),
$$

where $r$ is the rank of total downlink channel $H_{DL}$, and $W \in \mathbb{C}^{N \times r}$ is the downlink transmit beamformer at the BS. Since $\Phi[t]$ is designed based on given channels, $\Phi[t]$ is fixed during the data transmissions, omitting the time index $t$ as $\Phi$. We turn on all the IRS elements, i.e., $\Phi = \text{diag} \left( \left[ e^{j \phi_1}, \cdots, e^{j \phi_L} \right]^T \right)$, to maximize the reflected signal strengths.

With the given $\Phi$ and $H_{DL}$, the beamformer $W$ is given as the dominant $r$ right singular vectors of $H_{DL}^H$ as [30], [31]

$$
W = V_{tot}(:, 1 : r),
$$

$$
H_{tot}^H = U_{tot} S_{tot} V_{tot}^H.
$$

where (47) is the singular value decomposition of $H_{tot}^H$. On one hand, $H_{tot}^H$ contains $\Phi$ as in (44), which let $W$ depend on $\Phi$. On the other hand, the design of $\Phi$ that is to maximize $R_{DL}$ in (45) also depends on $W$. This entangled correlation of $\Phi$ and $W$ makes it difficult to jointly design the optimal $\Phi$ and $W$. Hence, we first reformulate $R_{DL}$ in (45) to decompose
the design of $\Phi$ and $W$ by exploiting the property between $H_{\text{tot}}$ and $W$ in (46) as
\[
R_{\text{DL}} = \log_2 \left( \det \left( I_r + \frac{P_{\text{DL}}}{rN_0} V_r H_{\text{tot}} \right) \times H_{\text{tot}}^H V_r \right)
\]
\[= \log_2 \left( \det \left( I_N + \frac{P_{\text{DL}}}{rN_0} H_{\text{tot}} H_{\text{tot}}^H V_r \right) \right) \tag{a}
\]
\[\leq \log_2 \left( \det \left( I_N + \frac{P_{\text{DL}}}{rN_0} \right) \right) \tag{b}
\]
where (a) holds since the rank of $H_{\text{tot}}^H$ is given by $r$, and (b) holds with the fact that $\det(AB) = \det(A) \det(B)$ for any square matrices $A$ and $B$ with the same dimension and that $V_r$ is a unitary matrix. The reformulated spectral efficiency $R_{\text{DL}}$ in (48) is independent from the specific value of $W$. This allows to design $\Phi$ first to maximize $R_{\text{DL}}$. Then, $W$ can be designed as in (46) with the designed $\Phi$ and downlink channel $H_{\text{tot}}$.

To get the optimal value of the $\ell$-th phase shift $\phi_\ell$ that maximizes $R_{\text{DL}}$ in (48) for given $\{\phi_k\}_{k=1, k \neq \ell}$, we set the optimization problem as
\[
\max_{\phi_\ell} \det \left( I_N + \lambda \left( H_{-\ell} + e^{j\phi_\ell} R_{\ell} \right) \left( H_{-\ell} + e^{j\phi_\ell} R_{\ell} \right)^H \right), \tag{49}
\]
where $\lambda = P_{\text{DL}}/(rN_0)$, and $H_{-\ell} = H_{\text{UB}} + \sum_{k=1, k \neq \ell}^{L} e^{j\phi_k} R_k$, which gives $H_{\text{tot}} = H_{-\ell} + e^{j\phi_\ell} R_\ell$. By substituting $R_\ell = h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H$, the objective function in (49) can be reformulated as
\[
\det \left( I_N + \lambda \left( H_{-\ell} + e^{j\phi_\ell} R_\ell \right) \left( H_{-\ell} + e^{j\phi_\ell} R_\ell \right)^H \right)
\]
\[= \det \left( I_N + \lambda \left( H_{-\ell} + e^{j\phi_\ell} h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right) \right)
\]
\[= e^{-j\phi_\ell} \left( H_{-\ell} h_{\text{UL}, \ell} \right) h_{\text{IB}, \ell}^H + h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \left( h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right)^H. \tag{50}
\]
For simplicity, let us define the following variables:
\[\kappa_\ell = e^{j\phi_\ell} \lambda, \tag{51}
\]
\[p_\ell = h_{\text{IB}, \ell}, \tag{52}
\]
\[q_\ell = H_{-\ell} h_{\text{UL}, \ell}, \tag{53}
\]
\[A_\ell = I_N + \lambda \left( H_{-\ell} H_{-\ell}^H + h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \left( h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right)^H \right). \tag{54}
\]
By using these variables, (50) can be represented as
\[
\det \left( A_\ell + \kappa_\ell p_\ell q_\ell^H + \kappa_\ell^* q_\ell^H p_\ell^H \right)
\]
\[= \det \left( A_\ell + [p_\ell, q_\ell] \text{diag} \left( [\kappa_\ell, \kappa_\ell^*]^T \right) [q_\ell, p_\ell]^H \right) \tag{a}
\]
\[= \det \left( \text{diag} \left( [\kappa_\ell, \kappa_\ell^*]^T \right) \right) \left( [q_\ell, p_\ell]^H A_\ell^{-1} [p_\ell, q_\ell] \right) \times \det \left( \text{diag} \left( [\kappa_\ell, \kappa_\ell^*]^T \right) \right) \det(A_\ell), \tag{55}
\]
where (a) can be derived using the Sylvester’s determinant theorem [32]. The existence of $A_\ell^{-1}$ in (55) can be proven by the following lemma using the structure of $A_\ell$ in (54).

**Lemma 1.** For any positive definite matrix $A$ and a matrix $B$ with a proper dimension, $A + BB^H$ is an invertible matrix.

**Proof.** Suppose that $x$ is any non-zero vector. Then, we have
\[
x^H (A + BB^H) x = x^H Ax + x^H BB^H x
\]
\[= x^H Ax + \|B^H x\|^2 > 0,
\]
where the inequality in (56) implies $A + BB^H$ is also a positive definite matrix. Since a positive definite matrix is invertible, $A + BB^H$ is an invertible matrix, which finishes the proof. ■

In (55), since $\det \left( \text{diag} \left( [\kappa_\ell, \kappa_\ell^*]^T \right) \right) = |e^{j\phi_\ell} \lambda|^2$ and $\det(A_\ell)$ are constants and independent of $\phi_\ell$, the optimization problem in (49) can be represented as
\[
\max_{\phi_\ell} \det \left( \begin{bmatrix} \frac{e^{-j\phi_\ell}}{\lambda} & 0 \\ 0 & \frac{e^{j\phi_\ell}}{\lambda} \end{bmatrix} + [q_\ell, p_\ell]^H A_\ell^{-1} [p_\ell, q_\ell] \right), \tag{57}
\]
and the optimal phase shift $\phi_\ell^*$ can be obtained as
\[
\phi_\ell^* = \arg \max_{\phi_\ell} \det \left( \left[ \begin{bmatrix} \frac{e^{-j\phi_\ell}}{\lambda} & q_\ell^H A_\ell^{-1} p_\ell \\ p_\ell^H A_\ell^{-1} p_\ell & \frac{e^{j\phi_\ell}}{\lambda} + q_\ell^H A_\ell^{-1} q_\ell \end{bmatrix} \right] \right) \tag{a}
\]
\[= \arg \max_{\phi_\ell} \text{Re} \left( \frac{e^{-j\phi_\ell}}{\lambda} p_\ell^H A_\ell^{-1} q_\ell \right)
\]
\[= \left( p_\ell^H A_\ell^{-1} q_\ell \right)
\]
\[= \left( h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right) \left( I_N + \lambda \left( H_{-\ell} H_{-\ell}^H + h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right) \right)^{-1} H_{-\ell} h_{\text{UL}, \ell}. \tag{58}
\]
where (a) can be derived by straightforward linear operations. Although the optimal value $\phi_\ell^*$ can be derived by (58), the solution requires the BS to know $h_{\text{IB}, \ell}$ and $h_{\text{UL}, \ell}$ to compute $\phi_\ell^*$. When the BS has the channel information in the form of the rank-one matrices $R_\ell$ instead of $H_{\text{IB}}$ and $H_{\text{UL}}$, the BS is able to get the optimal $\phi_\ell^*$ as
\[
\phi_\ell^* = \arg \max \left( \text{Tr} \left( h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \left( I_N + \lambda \left( H_{-\ell} H_{-\ell}^H + h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right) \right)^{-1} H_{-\ell} h_{\text{UL}, \ell} \right) \right) \tag{a}
\]
\[= \arg \max \left( \text{Tr} \left( h_{\text{UL}, \ell} h_{\text{IB}, \ell}^H \left( I_N + \lambda \left( H_{-\ell} H_{-\ell}^H + h_{\text{IB}, \ell} h_{\text{UL}, \ell}^H \right) \right)^{-1} \right) \right)
\]
\[= \arg \max \left( \text{Tr} \left( R_\ell^H \left( I_N + \lambda \left( H_{-\ell} H_{-\ell}^H + \lambda R_\ell R_\ell^H \right) \right)^{-1} H_{-\ell} \right) \right), \tag{59}
\]
Algorithm 1 Proposed phase shift design for the IRS

Initialize
1: Set \( \epsilon > 0 \)
2: for \( \ell = 1, \ldots, L \) do
3: \[ H_{\ell} = \begin{cases} H_{\text{UB}}, & \ell = 1 \\ H_{\text{UB}} + \sum_{k=1}^{L} e^{j\phi_{k}} R_k, & \text{else} \end{cases} \]
4: Update \( \phi_{\ell}^{*} \) by (59)
5: end for
6: Calculate \( R_{DL, \text{tmp}} \) by (48) with \( \{ \phi_{\ell}^{*} \}_{\ell=1}^{L} \)

Iterative update
7: for \( i = 1, \ldots, I \) do
8: for \( \ell = 1, \ldots, L \) do
9: \[ H_{\ell} = H_{\text{UB}} + \sum_{k=1, k\neq \ell}^{L} e^{j\phi_{k}} R_k \]
10: Update \( \phi_{\ell}^{*} \) by (59)
11: end for
12: Calculate \( R_{DL} \) by (48) with \( \{ \phi_{\ell}^{*} \}_{\ell=1}^{L} \)
13: if \( R_{DL} < R_{DL, \text{tmp}} + \epsilon \) then
14: Break
15: else
16: Set \( R_{DL, \text{tmp}} = R_{DL} \)
17: end if
18: end for
19: Return \( \phi_{\ell}^{*} \forall \ell \in \{1, \ldots, L\} \)

where the property \( \text{Tr}(AB) = \text{Tr}(BA) \) for any matrix \( A \) and \( B \) whose multiplication produces a square matrix is used in (a).

For given \( \{ \phi_{k} \}_{k=1, k\neq \ell} \), the optimal phase shift of the \( \ell \)-th IRS element \( \phi_{\ell}^{*} \) is given by (59). Then, we can derive the optimal values of all \( L \) phase shifts in an iterative way. The proposed IRS phase shift design algorithm is summarized in Algorithm 1. The algorithm can stop when the outer iteration index \( i \) reaches its maximum value \( I \) or when the increase of downlink spectral efficiency becomes less than a positive number \( \epsilon \). Note that the optimality in (59) ensures that every update of \( \phi_{\ell}^{*} \) in Algorithm 1 monotonically increases the spectral efficiency until the algorithm stops. Also, it is obvious that the spectral efficiency is upper bounded by the channel capacity. Following the aforementioned reasons, the convergence of Algorithm 1 is guaranteed. With the designed phase shifts \( \phi_{\ell}^{*} \), the transmit beamformer is obtained as \( W \) in (46).

With regard to the uplink data transmission, the phase shifts \( \phi_{\ell} \) and uplink transmit beamformer \( F \) can be similarly designed. The uplink spectral efficiency \( R_{UL} \) is given by

\[
R_{UL} = \log_{2} \left( \det \left( I_{M} + \frac{P_{UL}}{r N_0} H_{\text{tot}}^{H} H_{\text{tot}}^{H} F^{H} F \right) \right),
\]

where \( F \in \mathbb{C}^{M \times r} \) is given by \( F = U_{\text{tot}}(1: 1: r) \), and \( U_{\text{tot}} \) is given in (47). Exploiting the same property used in (48), we can represent \( R_{UL} \) as

\[
R_{UL} = \log_{2} \left( \det \left( I_{M} + \frac{P_{UL}}{r N_0} U_{\text{tot}}^{H} H_{\text{tot}}^{H} H_{\text{tot}}^{H} U_{\text{tot}} \right) \right)
= \log_{2} \left( \det \left( I_{M} + \frac{P_{UL}}{r N_0} H_{\text{tot}}^{H} H_{\text{tot}}^{H} U_{\text{tot}} \right) \right),
\]

where (a) can be derived using the Sylvester's determinant theorem [32]. For the same transmit power \( P_{UL} = P_{DL} \), the uplink spectral efficiency in (61) becomes the same as the downlink spectral efficiency in (48). This implies that the optimal phase shift in (59) also maximizes \( R_{UL} \) and the BS can use the same IRS phase shifts for both the uplink and downlink data transmissions.

VI. NUMERICAL RESULTS

In this section, we investigate the proposed IRS phase shift design and compare the channel estimation performance of proposed SPAC and SEROM with those of existing estimation techniques. Regarding the UPA structure, we consider \( N = N_{c} \times N_{h} \) antennas for the BS, \( M = M_{c} \times M_{h} \) antennas for the UE, and \( L = L_{c} \times L_{h} \) elements for the IRS. For the uplink and downlink training sequences, we exploit DFT matrices with proper sizes depending on channel estimation techniques. The uplink transmit power for channel estimation is set to be 10 dB less than the downlink transmit power. The \( B \)-bit quantization of each IRS phase shift is realized by rounding off to the nearest quantized value in \( \{0, \frac{2\pi}{2^B}, \ldots, \frac{2\pi(2^B-2)}{2^B} \} \). The Rician fading channel is established with \( K_{UL} = 5 \) dB, \( K_{IB} = 5 \) dB, and \( K_{UB} = 3 \) dB where \( d_{UL}, d_{IB}, \) and \( d_{UB} \) are uniformly distributed in \([5, 10], [90, 100],\) and \([d_{IB} - d_{UL}, d_{IB} + d_{UL}]\) in the meter scale. For each channel, the number of NLoS paths is set as \( G_{UL} = 4, G_{IB} = 4, \) and \( G_{UB} = 7. \) The path-loss exponents for the large scale fading are set as \( \eta_{UL} = 2.1, \eta_{IB} = 2.2, \) and \( \eta_{UB} = 3.5. \) and the path-loss is \( \mu_{UL} = -20 \) dB at the unit distance \( d_{0} = 1 \) m. With noise spectral density \(-174 \) dBm/Hz and bandwidth 1 MHz, the noise variance is set as \( N_{0} = -114 \) dBm.

The training sequence length of direct channel estimation is set as \( t_{d} = M, \) and those of the rank-one channel estimations are set as \( \tau_{OBO} = LM, \tau_{CO-OBO} = 2L, \tau_{SPAC} = (L_{c} + L_{h} - 1)M, \) and \( \tau_{SEROM} = QM. \) With a configurable training sequence length, that of SEROM is simply set as \( \tau_{SEROM} = \tau_{SPAC} \) by setting the parameter \( Q = L_{c} + L_{h} - 1. \)

A. Investigation of the proposed IRS phase shift design

We evaluate the spectral efficiency of proposed IRS phase shift design in Section V and compare the result to that of the algorithm in [4]. The design purpose of algorithm in [4] is the maximization of spectral efficiency where the transmit beamformer and IRS phase shifts are updated alternately until convergence. To solely compare the IRS element design performance, the proposed IRS phase shift design and the algorithm in [4] are operated with the perfect channel information, i.e., \( \tilde{H}_{\text{UB}} = H_{\text{UB}} \) and \( \tilde{R}_{\ell} = R_{\ell}. \) Without channel estimation, the spectral efficiency is computed as (45) in Section V. In Fig. 3, two-bit phase quantization \( B = 2 \) is considered, and the maximum spectral efficiency, which is found by the exhaustive search of all the possible quantized
IRS phase shifts, is demonstrated as a reference. The spectral efficiencies of proposed phase shift design and algorithm in [4] are very close to the result of exhaustive search. This means that the two techniques provide proper IRS phase shifts to maximize the spectral efficiency.

Note that Fig. 3 only considers a small number of IRS elements due to the complexity of exhaustive search. While the proposed design and the algorithm in [4] both can serve a large number of IRS elements, there is difference on the computation complexity. In Table I, the computation complexity of three IRS phase shift techniques is listed by counting the number of scalar multiplications, i.e., the notation $\mathcal{P}(x)$ means that the number of scalar multiplications is proportional to $x$. The complexity of exhaustive search is remarkably higher than the other two techniques; it increases exponentially with the phase quantization bits and the number of IRS elements. The computation complexity of algorithm in [4] contains the parameters $I_{\text{init}}$ and $I_{\text{outer}}$ that are the numbers of initial random generations and outer algorithm iterations. The random initialization of algorithm in [4] is to find good initial values that can reduce the number of outer algorithm iterations. On account of the interdependency of IRS phase shifts and transmit beamformer in the algorithm in [4], searching good initial values is not easy and results in the numbers of initial random generations and outer algorithm iterations, and it becomes significant when a large number of antennas, IRS elements, and algorithm iterations are deployed. Therefore, we use the proposed phase shift design, which gives the similar result to the exhaustive search but operates with the lowest complexity among the three, to compare the channel estimation techniques.

**TABLE I: Computation complexities of IRS phase shift techniques with $M \leq N \leq L$.**

| IRS phase shift techniques | Complexity |
|---------------------------|------------|
| Algorithm in [4]          | $\mathcal{P}(I_{\text{init}}LMN + I_{\text{outer}}L(4M^2N + 3M^3))$ |
| Proposed phase shift design | $\mathcal{P}(IL(3M^2N + 2M^3))$ |
| Exhaustive search         | $\mathcal{P}(2^{LB}(LMN))$ |

**B. Comparison of channel estimation techniques**

In this subsection, we compare the channel estimation performance of proposed SPAC and SEROM with those of existing estimation techniques in [13], [14], [16]. The results of elementary techniques in Sections III-B and III-C are also depicted as references. In [13], the least squares Khatri-Rao factorization (LSKRF) is proposed to estimate $\mathbf{H}_{\text{UB}}$ and $\mathbf{H}_{\text{UI1}}$. In [14], $\mathbf{H}_{\text{UB}}$ is assumed as a known LoS channel, and $\mathbf{H}_{\text{UB}}$ and $\mathbf{H}_{\text{UI}}$ are assumed as Rayleigh fading channels. Based on these assumptions, the MMSE-DFT is proposed to estimate $\mathbf{H}_{\text{UB}}$ and $\mathbf{H}_{\text{UI}}$. Without considering any training overhead, the two estimation techniques in [13] and [14] require the training sequence lengths $\tau_c = \tau_{\text{LSKRF}} = \tau_{\text{MMSE-DFT}} = LM$ that are clearly longer than those of SPAC $\tau_{\text{SPAC}} = (L_c + L_b - 1)M$ and SEROM $\tau_{\text{SEROM}} = Q.M$. In [16], the three-phase channel estimation is designed with the relatively short training sequence length $\tau_{\text{three-phase}} = M + L + \max \left\{ M - 1, \left[ \frac{(M-1)L}{N} \right] \right\}$, which is comparable to those of SPAC and SEROM depending on the numbers of antennas and IRS elements.

To analyze the performance of channel estimation techniques, we adopt various performance metrics: spectral efficiency per channel use, training sequence length, effective spectral efficiency, computation complexity, and normalized squared error. The first metric measures the effectiveness of estimated channels to design IRS phase shifts, and the second metric assesses the training overhead of estimation technique. The third metric jointly evaluates the estimated channels and the training overhead of estimation techniques. Since the principal challenges of IRS-empowered systems are the training overhead and spectral efficiency, we mainly focus on the first three metrics, and the last two metrics can be optionally considered depending on the design purposes.

1) **Spectral efficiency per channel use:** Based on estimated channels $\hat{\mathbf{H}}_{\text{UB}}$ and $\mathbf{R}_f$ for $f \in \{1, \ldots, L\}$, the spectral efficiency per channel use can be computed as (45) by replacing $r$ and $\mathbf{W}$ with $\hat{r}$ and $\hat{\mathbf{W}}$ where $\hat{r}$ is the rank of estimated channel $(\hat{\mathbf{H}}_{\text{UB}} + \sum_{f=1}^{L} e^{j\psi_f} \hat{\mathbf{R}}_f)^{\mathsf{H}}$ and $\hat{\mathbf{W}}$ is composed of the first $\hat{r}$ right singular vectors of $(\hat{\mathbf{H}}_{\text{UB}} + \sum_{f=1}^{L} e^{j\psi_f} \hat{\mathbf{R}}_f)^{\mathsf{H}}$ corresponding to the $\hat{r}$ dominant singular values. In Fig. 4, the spectral efficiencies per channel use are depicted for two cases: one with small numbers of antennas and IRS elements and the other with large numbers of antennas and IRS elements.
The complexity of SPAC is usually lower than that of SEROM, ties of OBO and Co-OBO estimations linearly increase with derive the full advantage of IRS. As a baseline, the complexity mainly depends that would be very large to

\[ \mathcal{P}(LM^2N) \]

\[ \mathcal{P}(2(LN + L_v - 1)M^2N + (L - L_h - L_v + 1)MN) \]

\[ \mathcal{P}(QM^2N + QLMN) \]

\[ \mathcal{P}(L(MN^2 + M^2N) + L^2N) \]

\[ \mathcal{P}(L^3N) \]

\[ \mathcal{P}(M^2N + N^3 \left( \frac{L}{N} \right)^{-1}) \]

With high training overhead, the LSKRF in [13] provides the highest spectral efficiency. The MMSE-DFT in [14] also requires high training overhead, but its spectral efficiency is lower than that of LSKRF. This is because the MMSE-DFT is based on the assumption of Rayleigh fading channel, which deteriorates the estimation accuracy for the Rician fading with the LoS path. With low training overhead, the three-phase channel estimation in [16] gives moderate spectral efficiency similar to those of SPAC and SEROM. The spectral efficiency of three-phase estimation, however, has more gentle slope. The three-phase estimation requires arbitrary \( M \) columns of \( \mathbf{H}_{10} \) to be linearly independent, and this condition is rarely satisfied without rich scattering environments. On the contrary, SPAC is designed based on the Rician fading and SEROM is designed for arbitrary channel structure. This makes the spectral efficiencies of SPAC and SEROM grow fast as the transmit power increases. The spectral efficiency of Co-OBO estimation increases with power most quickly due to the cooperative uplink and downlink signalings and the ideal feedback while other techniques exploit uplink signalings only. The OBO estimation shows the lowest spectral efficiency despite its long training sequence length. Different from the LSKRF, the MMSE-DFT, and SEROM, the OBO estimation does not average the noise at each estimation of rank-one matrices, and this can disturb finding proper IRS phase shifts with the estimated rank-one matrices.

2) Training sequence length and complexity: The total training sequence length and computation complexity of each technique are listed in Table II where \( \tau_d = M \) is considered for the estimation of direct channel \( \mathbf{H}_{10} \), and \( \tau_c \) is to estimate the \( L \) rank-one matrices \( \mathbf{R}_k \). The complexity mainly depends on the number of IRS elements \( L \) that would be very large to derive the full advantage of IRS. As a baseline, the complexities of OBO and Co-OBO estimations linearly increase with \( L \). The complexity of SPAC is usually lower than that of SEROM, which benefits from the parameter estimation and rank-one channel reconstruction with the single-path approximations in (20) and (21). SEROM can be applied for any channel model, while its utility requires higher complexity than that of SPAC. However, it is important that the complexities of SPAC and SEROM linearly increase with \( L \), which is much smaller than those of LSKRF and MMSE-DFT increasing in square or cube of \( L \). The complexity of three-phase estimation increases in the linear scale of \( L \) but increases in square of \( N \), which results in higher complexity than those of SPAC and SEROM. The advantage of SPAC and SEROM in terms of the complexity escalates as the number of IRS elements or BS antennas increases.

Meanwhile, the main challenge that we consider in the paper is the extremely large training sequence length with the increase of number of IRS elements. For the two cases in Fig. 4, the training sequence lengths of channel estimation techniques are compared in Fig. 5. The trend of training sequence lengths matches with that of spectral efficiencies per channel use in general. While the LSKRF and MMSE-DFT provide high spectral efficiencies per channel use, and their
training overhead is higher than that of other techniques. The three-phase estimation has shorter training sequence length and gives lower spectral efficiency per channel use than the LSKRF and MMSE-DFT. The training sequence lengths of SPAC and SEROM are the shortest, but their spectral efficiencies per channel use are similar to or higher than those of three-phase estimation and OBO estimation. Although the OBO and Co-OBO estimations employ long training sequence lengths, their vulnerability to noise decreases their spectral efficiencies per channel use.

In Fig. 5, it is shown that the overhead of training grows with the numbers of antennas and IRS elements. However, the coherence time block length of typical communication system is hard to be longer than 1,200 [33], [34], and the training sequence length longer than 1,200 would not be acceptable. For the second case with the large numbers of antennas and IRS elements, the training sequence lengths of OBO estimation, LSKRF, and MMSE-DFT are already over 1,000, which means the three estimation techniques have only a little time for data transmissions after channel estimation.

3) Effective spectral efficiency: Now, we jointly assess the estimated channel and the training overhead by measuring the effective spectral efficiency as [35]

\[
\frac{\Gamma - \tau_{tot}}{\Gamma} \log_2 \left( \det \left( I_B + \frac{P_{DL}}{r N_0} \hat{W}^H \left( H_{UB} + H_{IB} \Phi H_{UI} \right) \right. \\
\left. \times \left( H_{UB} + H_{IB} \Phi H_{UI} \right)^H \hat{W} \right) \right) = \frac{\Gamma - \tau_{tot}}{\Gamma} \log_2 \left( \det \left( I_B + \frac{P_{DL}}{r N_0} \hat{W}^H \left( H_{UB} + \sum_{\ell=1}^{L} e^{j\phi_{\ell}} R_{\ell} \right) \right) \right)
\]

where \( \Gamma \) is the coherence time block length. In Fig. 6a, with their short training sequence lengths and moderate spectral efficiencies per channel use, SPAC and SEROM provide the highest spectral efficiencies. With its longer training sequence length and gentle slope of spectral efficiency per channel use, the three-phase estimation provides lower effective spectral efficiency than SPAC and SEROM. Since the Co-OBO and OBO estimations, the LSKRF, and the MMSE-DFT require high training overhead as in Fig. 5a, their effective spectral efficiencies are deteriorated lower than those of SPAC, SEROM, and three-phase estimation.

In Fig. 6b, the effective spectral efficiencies with large numbers of antennas and IRS elements are depicted. SPAC, SEROM, and the three-phase estimation still provide high spectral efficiencies as in Fig. 6a. With little time for data transmissions, the spectral efficiencies of OBO estimation, LSKRF, and MMSE-DFT are significantly reduced. The spectral efficiency of Co-OBO estimation is relatively improved.
where its training sequence length benefits from a large number of antennas. As the numbers of antennas and IRS elements increase, the gap between two estimation groups, one with long training sequence lengths and the other with short training sequence lengths, becomes more noticeable.

In Fig. 7a, the effective spectral efficiencies of channel estimation techniques are compared over coherence time block length $\Gamma$. The spectral efficiencies of group with high training overhead, i.e., the OBO estimation, the LSKRF, and the MMSE-DFT, are zero due to their long training sequence lengths until $\Gamma = 900$. On the contrary, the spectral efficiencies of group with low training overhead, i.e., the Co-OBO estimation, SPAC, SEROM, and the three-phase estimation, are positive from low $\Gamma$ and quickly increase as $\Gamma$ grows. Especially, SPAC and SEROM provide the highest effective spectral efficiencies for all $\Gamma$. This is by virtue of the efficient channel estimation of SPAC and SEROM demanding only short lengths of training sequences.

To investigate the effect of single-path approximations in (20) and (21), the effective spectral efficiencies are measured over the Rician K-factors in Fig. 7b where we set $K_{UB}$ to 2 dB less than $K_{IB}$ for the direct link channel $H_{UB}$. As the Rician K-factors increase, the channel matrices become ill-conditioned, and the spectral efficiencies of most of estimation techniques decrease. With high Rician K-factors, on the contrary, the approximation of SPAC better matches to the true channel and provides constantly high spectral efficiency. For the practical situations, we can adopt SPAC for the channels with the strong LoS path and SEROM for other situations.

4) Squared error: Instead of the two channel matrices $H_{IB}$ and $H_{UI}$, we tried to estimate the rank-one matrices $R_\ell$ that contain the channel information in a different form. For the estimated rank-one matrices $\hat{R}_\ell$, $\ell \in \{1, \ldots, L\}$, we measured the normalized squared errors as

$$
\sum_{\ell=1}^{L} \frac{\|R_\ell - \hat{R}_\ell\|_F^2}{\|R_\ell\|_F^2} = \sum_{\ell=1}^{L} \frac{\|h_{IB,\ell}h_{UI,\ell}^H - \hat{R}_\ell\|_F^2}{\|h_{IB,\ell}h_{UI,\ell}^H\|_F^2},
$$

(63)

where $\|R_\ell\|_F$ is the normalization factor. In Fig. 8, the cumulative distribution function of normalized squared error is depicted. The order of squared error of each technique is similar to the spectral efficiency per channel use in Fig. 4b. With high training overhead, the LSKRF and the MMSE-DFT provide low squared errors, and the three-phase estimation has high squared error with low training overhead. Similarly, the Co-OBO estimation gives low squared error with its cooperative uplink and downlink signalings and ideal feedback condition.

In terms of the OBO estimation, SPAC, and SEROM, the trend of squared errors and that of spectral efficiencies per channel use are not consistent and even reversed. The spectral efficiencies per channel use of SPAC and SEROM are higher than that of OBO estimation at $P_{DL} = 30$ dBm in Fig. 4b. This means that SPAC and SEROM provide better channel estimation performance compared to the OBO estimation. However, this fact can not be observed in Fig. 8. This is because the squared error does not fully reveal how well a
matrix is described. As a simple example, we can consider two $N \times M$ matrices $\{0_N, \cdots, 0_N\}$ and $2R_R$ for the estimate of $R_R$. The squared errors of two estimated matrices are equal, but the latter estimate $2R_R$ would be more desirable with regard to the use of IRS.

With long training sequence length, the OBO estimation reduces the squared error, but its non-averaged noise induces the estimates $R_R$ to be well-conditioned, i.e., not rank-one, while the true matrices $R_R$ have rank one. Its low spectral efficiency states that the estimates $R_R$ of OBO estimation are not proper to design the IRS phase shifts. Contrastively, SEROM averages noise and provides high spectral efficiency, but imperfect pseudo-orthogonality condition in (60) results in high squared error. Hence, we mainly consider the effective spectral efficiency in (62), which examines the effectualities of training overhead as well as estimated channels, to evaluate the estimation techniques.

VII. CONCLUSION

We proposed two novel practical channel estimation techniques and an IRS phase shift design. The proposed SPAC and SEROM are designed to estimate channel information in SU-MIMO systems while consuming short training sequence lengths. The proposed IRS phase shift design is developed to maximize spectral efficiency while requiring only linear operations. Numerical results showed that the proposed phase shift design provides a spectral efficiency close to that of exhaustive search. When the proposed IRS phase shift design was utilized, the effective spectral efficiencies of SPAC and SEROM were higher than those of other estimation techniques. The results verified that the high spectral efficiency can be achieved by considering both the training overhead and the spectral efficiency per channel use. A possible future work is to develop a joint framework of channel estimation and IRS element design to have low training overhead while extracting only necessary information to design IRS elements, still achieving a high spectral efficiency.

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