Thermal and non-thermal signatures of the Unruh effect in Casimir-Polder forces

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Introduction. Recent significant experimental progresses in measuring fluctuation induced forces with unprecedented level of accuracy [1–4], from the microscopic to the macroscopic level, have strongly renewed the interest in Casimir interactions [5–8], even at finite temperature [9, 10] and in out of equilibrium configurations [11]. Related effects are the dynamical Casimir effect, consisting in the emission of real photons from neutral bodies moving with nonuniform acceleration in the vacuum space [12, 13], and dissipative effects induced by quantum vacuum fluctuations such as quantum friction [14, 15].

A peculiar aspect of the quantum vacuum is that its particle content is observer dependent. The Unruh effect is a striking manifestation of this fact: a uniformly accelerated observer in flat spacetime associates a thermal bath to the power spectrum of quantum vacuum fluctuations, with a temperature proportional to its proper acceleration [16–19]. Despite its conceptual importance, also for connections with Hawking radiation and black holes evaporation [20], experimental detection of the Unruh effect remains elusive, since an acceleration of the order of $10^2 \text{m/s}^2$ would be required in order to measure a temperature of 1K, triggering in the last years experimental proposals for an indirect signature of the Unruh effect in analogous models [21], such as Bose-Einstein condensates [22]. On the other hand, it has been recently shown that the van der Waals interaction between two accelerated atoms could allow to detect the Unruh effect with reasonable values of the acceleration [23].

Even if it is a well established fact that an accelerated observer perceives the vacuum field as a thermal state [17], the tantalizing possibility to find non-thermal features associated to relativistic uniform acceleration would constitute a sharp representative signature of accelerated motion, beyond the Unruh thermal analogy [24–27].

In this Letter, we show that both thermal and non-thermal features associated to a relativistic uniformly accelerated motion can be probed through the Casimir-Polder force between two accelerating atoms. In particular, we consider the interaction energy, arising from quantum fluctuations and radiation reaction field to the Casimir-Polder interaction between two atoms moving in two generic stationary trajectories separated by a constant distance, and linearly coupled to a scalar field. The field can be assumed in its vacuum state or at finite temperature, resulting in a general method for the computation of Casimir-Polder forces in stationary regimes.
in terms of a local inertial frame, and it indicates that the Casimir-Polder interaction is strongly reshaped by the presence of the non-inertial space-time background, associated to the relativistic accelerated motion of the two atoms. This phenomenology is a simple non-trivial extension of the Unruh thermal response detected by a single accelerated observer, to a system of two accelerated particles. It shows that new physical features arise as a consequence of an accelerated motion when spatially separated objects are considered.

We derive these results by generalizing to the fourth order in perturbation theory a method originally introduced by Dalibard et al. [28] to separate the contributions of vacuum fluctuations and radiation reaction to second-order radiative corrections, specifically the atomic Lamb shift. We derive a formal expression for the scalar Casimir-Polder energy, separated in its contributions of vacuum fluctuations and radiation reaction contributions, between two atoms moving parallel each other on arbitrary stationary trajectories, and coupled to a scalar field. Our method can be easily generalized to a generic quantum field linearly coupled to the atoms and with generic boundary conditions.

**Casimir-Polder interactions.** We consider the Hamiltonian of a pair of two-level atoms $(A,B)$, characterized by the same transition frequency $\omega_0$ and linearly coupled to a massless scalar field $\phi(x)$ by the coupling constant $\lambda$. The Hamiltonian can be written in the Dirac notation [29] and in natural units ($\hbar = c = 1$), as

$$H = \omega_0 \sigma_A^1(\tau) + \omega_0 \sigma_B^1(\tau) + \int d^3k \omega_k a_k^\dagger a_k \frac{d}{d\tau} + \lambda \sigma_A^2(\tau) \phi(x_A(\tau)) + \lambda \sigma_B^2(\tau) \phi(x_B(\tau)),$$

(1)

where $\sigma_2 = \frac{i}{2}(|g\rangle\langle e| - |e\rangle\langle g|)$ and $\sigma_3 = \frac{i}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$ are Pauli matrices expressed in terms of the ground and excited state of the two atoms, $|g\rangle$ and $|e\rangle$ respectively; $a_k$, $a_k^\dagger$ are the annihilation and creation operators of the massless scalar field $\phi(x)$ with the linear dispersion relation $\omega_k = |k|$. The Hamiltonian (1) is expressed in terms of the same proper time $\tau$ of the two atoms (assuming a background flat spacetime), and the interaction term is evaluated on a generic stationary trajectory $x(\tau)$ of the two atoms. The distance $z$ between the atoms, perpendicular to their acceleration, is constant. Quantum fluctuations of the field, as well as radiation source fields, can induce an effective interaction among the two atoms at fourth order in the atom-field interaction. Following the procedure proposed in [28], we can split the rate of variation $\frac{dO^A}{d\tau}$ of an atomic observable $O^A$ in the sum of two contributions, $vf$ and $rr$,

$$\left(\frac{dO^A}{d\tau}\right)_{vf/rr} = \frac{i\lambda}{2} \left(\phi^f(x_A(\tau))|\sigma_2(\tau), O^A\rangle + |\sigma_2(\tau), O^A\rangle \phi^s(x_A(\tau))\right),$$

(2)

where the free term $\phi^f$ is the contribution present even in the absence of interaction, while the source term $\phi^s$ is the part due to the atom-field coupling and containing the coupling constant $\lambda$. The first contribution, $(dO^A/d\tau)_{vf}$ describes the change in $O^A$ caused by the fluctuations of the field that are present even in the vacuum ($\text{vacuum fluctuations}$), while the second term represents the influence of the atom on the field, which in turn can act back on the atom ($\text{radiation reaction}$). The method consists in rewriting these contributions at a given order in perturbation theory as quantum evolutions given by two effective Hamiltonians, $H_{vf/rr}$, and then compute the $vf$ and $rr$ contributions to the atomic energy level shift (a second-order calculation is sufficient for the Lamb shift). In order to derive the Casimir-Polder interaction for two atoms $(A,B)$, in the quantum states $|\alpha\rangle$ and $|\beta\rangle$ respectively, and moving with two arbitrary stationary trajectories $x_A(\tau)$ and $x_B(\tau)$ (the trajectories of the two atoms differ by a space translation only), we derive the effective Hamiltonians $H_{vf/rr}$ at fourth order in $\lambda$ for one of the two atoms, as we shall report in detail elsewhere [30]. We disregard the energy shifts independent from the atomic separation, because they do not contribute to the interatomic force. We find the following expression of the $\text{vacuum fluctuations}$ contribution to the energy level shift of atom $A$ in the state $|\alpha\rangle$

$$\delta E^A_{\alpha, vf} = 4i\lambda^4 \lim_{(\tau - \tau_0) \to +\infty} \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau'} d\tau'' \int_{\tau_0}^{\tau''} d\tau''' C^F(\phi^f(x_A(\tau')), \phi^f(x_B(\tau'''))) \chi^F(\phi^f(x_A(\tau')), \phi^f(x_B(\tau'''))) \times \chi^A_{\alpha}(\tau, \tau') \chi^B_{\beta}(\tau'', \tau'''),$$

(3)

where we have introduced respectively the field symmetric correlation function and the field susceptibility,
Analogously, we have defined the atomic susceptibility of atom $A$ ($B$) in state $\alpha$ ($\beta$),

$$
\chi^{A/B}_{\alpha/\beta}(\tau, \tau') = \frac{1}{2} \langle \alpha/\beta | [\sigma^f_{2,A/B}(\tau), \sigma^f_{2,A/B}(\tau')] | \alpha/\beta \rangle.
$$

In Eqs. (3-5) the superscript $f$ stands for the free evolution of the operators. Although in (4) we have assumed the field in its ground state $|0\rangle$, Eq. (3) is valid also for a quantum field at finite temperature, using the appropriate statistical functions: also, in the following we shall assume the two atoms prepared in their ground state $|g\rangle$.

The result (3) has a sharp physical interpretation. The vacuum fluctuations contribution to the interatomic energy originates from a non-local field correlation (expressed by $C^F$), present even in the vacuum state, which induces and correlates dipole moments in the two atoms ($\chi^{A/B}$), that eventually polarizes the field ($\chi^F$). This physical picture is also consistent with the paradigmatic interpretation of Casimir-Polder forces by Power and Thirunamachandran in [31], and often used to calculate also many-body Casimir-Polder forces [32–34]. The radiation reaction contribution can be obtained similarly [30], but for brevity we do not report here its explicit expression. It describes the complementary physical mechanism, in which the atom $A$ has a fluctuating dipole moment ($C^A$) and it polarizes the field ($\chi^F$): a dipole moment is thus induced in the second atom ($\chi^B$) and it polarizes the field ($\chi^F$), which eventually acts back on atom $A$. It is possible to show that the radiation reaction contribution vanishes for two static atoms at zero temperature and is negligible compared to the vacuum fluctuation contribution, at small temperatures, i.e. for $T \ll \omega_0$, or in the case of two uniformly accelerating atoms provided that their acceleration is much smaller than $\omega_0$ [30]. Thus we concentrate on the vacuum fluctuations contribution only.

As a non-trivial test for our results, we can first calculate the scalar Casimir-Polder interaction energy $E_{CP}$ between two stationary atoms. Similarly to the electromagnetic Casimir-Polder case [6], we find a transition from a near zone regime, defined by $\omega_0 z \ll 1$, where $E_{CP} \approx -\frac{1}{1024\pi^3} \frac{\lambda^4}{\omega_0^2} \frac{1}{z^5}$, to a far zone regime, $E_{CP} \approx -\frac{1}{512\pi^3} \frac{\lambda^4}{\omega_0^2} \frac{1}{z^5}$, defined for distances $\omega_0 z \gg 1$, where retardation effects in the propagation of the field are relevant. A generalization of Eq. (3) allows to obtain also the scalar Casimir-Polder force at finite temperature $T$, in terms of the thermal correlation function and susceptibility for a scalar field

$$
C^{F}_{\text{th}}(\phi^f(x_A(\tau)), \phi^f(x_B(\tau'))) = \frac{1}{8\pi^2} \frac{1}{z} \int_{0}^{\infty} d\omega \sin(\omega z) \coth \left( \frac{\omega}{2T} \right) \left( e^{-i\omega(\tau-\tau')} + e^{i\omega(\tau-\tau')} \right),
$$

$$
\chi^{F}_{\text{th}}(\phi^f(x_A(\tau)), \phi^f(x_B(\tau'))) = \frac{1}{8\pi^2} \frac{1}{z} \int_{0}^{\infty} d\omega \omega \sin(\omega z) \left( e^{-i\omega(\tau-\tau')} - e^{i\omega(\tau-\tau')} \right).
$$

The explicit computation is performed in the limit of small temperatures, $T \ll \omega_0$, following a general method originally introduced by Lifshitz [35–37]. In view of the comparison with the Casimir-Polder force between two accelerated atoms, which is the main point of this Letter, it is important to stress that at finite temperatures, the massless thermal wavelength $\lambda^\text{th} \sim 1/T$ separates a quantum regime from a classical thermal regime. Indeed, for distances $z \ll \lambda^\text{th}$, we find the expression for the static scalar Casimir-Polder force in near and far zone plus subleading thermal corrections proportional to $-\lambda^4 \left( \frac{T}{\omega_0} \right)^2$; on the other hand, for distances larger than the typical length scales associated to quantum effects, i.e. for $z \gg \lambda^\text{th}$, the Casimir-Polder force manifests again a classical thermal behaviour similar to that in the near zone

$$
E_{CP}^\text{th} = -\frac{1}{512\pi^3} \frac{\lambda^4}{\omega_0^2} \frac{T}{z^5},
$$

as it has been already noticed for the electromagnetic case [9, 37].

Unruh corrections to Casimir-Polder interactions. We now apply our result (3) to the case of two atoms moving with the same uniform acceleration, perpendicular to their separation. In this case, a modification of their Casimir-Polder interaction is expected, because the two
atoms perceive modified vacuum fluctuations, as the Unruh effect would suggest. We have already obtained hints of such modification [27] and discussed observability of this new phenomenon, that could be a way to confirm experimentally the evidence of the Unruh effect [23]. An atom moving with uniform relativistic acceleration \( a \) in the \( \hat{x} \) direction follows the worldline
\[
t(\tau) = \frac{1}{a} \sinh(a\tau) \quad \text{and} \quad \omega(z) = \frac{1}{a} \cosh(a\tau) \quad y(\tau) = z(\tau) = 0.
\] 

We are now going to show how interatomic Casimir-Polder interactions allow to distinguish the effect of a relativistic acceleration from a thermal behavior. Even if such a thermal character have been envisaged in a large number of situations [17, 38–40], departures from thermal predictions for accelerating atoms have been shown in the Lamb shift and in the spontaneous excitation of the 2p state. A close comparison between (9) and (6) shows that the correction
\[
d\omega f(\omega, z, a)(e^{-i\omega(\tau-\tau')} + e^{i\omega(\tau-\tau')})
\]
and
\[
d\omega f(\omega, z, a)(e^{-i\omega(\tau-\tau')} - e^{i\omega(\tau-\tau')})
\]
is significantly larger than the correction due to a finite temperature. We have already obtained hints [27] and discussed observability of such modification [27] and discussed observability of this new phenomenon, that could be a way to confirm experimentally the evidence of the Unruh effect [23].

In such a situation it is convenient to introduce a new set of coordinates, necessary to cover the Minkowski spacetime \((t, x)\) accessible to accelerated observers. They are defined in two regions, the Rindler wedges, which are causally disconnected, and where a Rindler metric can be defined accordingly [17, 44].

We consider two uniformly accelerating atoms, moving along the worldlines (8) with the same uniform acceleration \( a \ll \omega_0 \), and separated by a distance \( z \) orthogonal to the acceleration direction \( \hat{x} \). We show that, at short distances, Casimir-Polder interactions can probe thermal Unruh-like effects, while at larger distances they reveal a non-thermal behavior due to the intrinsically non-inertial nature of the Rindler metric. As done in (6) for the thermal Casimir-Polder force, we first obtain the correlation function and susceptibility of the scalar field in the accelerated background

\[
C_{\text{acc.}}(\phi^f(x_A(\tau)), \phi^f(x_B(\tau'))) = \frac{1}{8\pi^2 N(z, a)} \int_0^\infty d\omega f(\omega, z, a) \coth \left( \frac{\pi \omega}{a} \right) (e^{-i\omega(\tau-\tau')} + e^{i\omega(\tau-\tau')}),
\]

and

\[
\chi_{\text{acc.}}(\phi^f(x_A(\tau)), \phi^f(x_B(\tau'))) = \frac{1}{8\pi^2 N(z, a)} \int_0^\infty d\omega f(\omega, z, a)(e^{-i\omega(\tau-\tau')} - e^{i\omega(\tau-\tau')}),
\]

where \( f(\omega, z, a) = \sin(\frac{\omega}{a} \sinh^{-1}(\frac{az}{2})) \) and \( N(z, a) = z \sqrt{1 + (az/2)^2} \). A close comparison between (9) and (6) shows that for \( az \ll 1 \) the correlation function (9) has a thermal-like behavior set by the Unruh temperature, \( T_\text{U} \).

Hence, the vacuum fluctuations contribute (3) to the Casimir-Polder interaction exhibits, at the lowest order in \( az \), the same thermal-like correction \( \sim -\frac{\lambda^4}{2}(\frac{T_\text{U}}{\omega_0})^2 \), found for the Casimir-Polder interaction at finite temperature. At higher orders in \( az \), Eq. (9) shows that the correction due to the accelerated atomic motion starts to differ significantly from the correction due to a finite temperature (from a mathematical point of view, this occurs because of higher order terms in \( az \) coming from the series expansions of \( f(\omega, z, a) \) and \( N(z, a) \) in (9)). This discrepancy suggests a strong breakdown of the usual analogy between acceleration and finite temperature effects for the Casimir-Polder potential at distances \( z \gg za \sim 1/a \), resulting in a novel power law behavior of the Casimir-Polder interaction.

\[
E_{\text{C-P}}^{\text{acc.}} = -\frac{1}{512\pi^4 \omega_0^6} z a^4.
\]

Our result (10) shows that the Casimir-Polder interaction energy between two accelerated atoms decreases faster with the distance than in both near and far zones.
sharp contrast with the classical effect outlined above for the Casimir-Polder interaction at finite temperature (see Eq. (7)) and it is summarized in Fig. 1. It should be noted that such an effect cannot be detected by a single uniformly accelerated point-like detector, as in [17, 39], since in that case it is always possible to find a local set of Minkowski coordinates in the neighborhood of a point-like detector. With this respect, our result can be seen as a simple non-trivial extension of the Unruh thermal response detected by a single accelerating observer, to a system of two relativistic accelerated systems.

![Figure 1](https://via.placeholder.com/150)

**FIG. 1:** (Color online) This picture shows a comparison in logarithmic scale between the Casimir-Polder interaction among two atoms moving with relativistic uniform acceleration $a$, orthogonal to their constant separation $z$ (red continuous line), and the static interaction for atoms at rest at temperature $T = a/2\pi$ and separated by the same distance (blue dashed line), in far zone, $z \gg 1/\omega_0$. While for short distances, $z \ll 1/a$, both potentials display the same thermal-like behavior, at distances larger than the characteristic length scale for the breakdown of a local inertial description of the system, $z \gg 1/a$, the thermal and the accelerated Casimir-Polder potentials exhibit a sharply different power law decay with the interatomic distance.

**Conclusions.** In this Letter we have shown how Casimir-Polder forces among two uniformly accelerating atoms can probe non-thermal effects beyond the Unruh analogy between uniform acceleration and finite temperature. We have shown that for interatomic distances above the characteristic length scale associated to a local inertial description of the system, the Casimir-Polder energy shows a different power law dependence with the distance, compared to the corresponding potential at finite temperature. Even if this new effect would be relevant at the same extremely high accelerations required to measure the thermal response of an accelerating detector predicted by the Unruh effect, the possibility to realize experimentally similar setups in analogue models, BE condensates [22] for example, could pave the way to explore the new physical effect discussed in this Letter. Also, our new expressions for the fourth-order vacuum fluctuations and radiation reaction contributions to energy shifts have a general validity, and they could be easily applied to other physical systems, for example to electromagnetic dispersion interactions between accelerating atoms, where the strict validity of the Unruh thermal analogy is still matter of debate [24, 25, 42, 46], or to forces between atoms in circular motion, which could be relevant to detect the Unruh effect [47].

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