The velocity of interfaces with short and long ranged elasticity under periodic subcritical driving

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(Dated: October 7, 2022)

A lot of research on elastic interfaces has been done on systems where the interface is pushed with a constant force. We studied the velocity of an interface under a periodic subcritical driving, which is relevant in for example magnetic hysteresis and fracturing of materials under repetitive loading. We obtained a modified version of the creep velocity that describes interfaces under small constant drivings and thermal noise. For short-range elastic systems, the velocity follows an approximate power law with a material dependent exponent. Long ranged systems have simpler behaviour without a material dependent exponent.

I. INTRODUCTION

For a large enough ferromagnet, the magnetization is not uniform, but rather the object divides into regions called magnetic domains. The spins of particles in each domain align roughly in the same direction, but neighboring domains will have different alignments, resulting in competing sub-magnets that together create the magnetization of the whole object.

Although the domains are affected by the structure of the magnet, they are not static, but rather change with an external magnetic field and thermal noise. Between the domains are thin strips called domain walls, where the spins point somewhere in between the directions of the spins inside the domains. The spins at these boundaries are easier to change than the whole domains, so the magnetization of a multi-domain magnet is changed by moving the domain walls, increasing the size of the domains that align with the applied external magnetic field.\textsuperscript{[1, 2]} The magnetization of a small single-domain magnet is harder to change, as the magnetization of the whole object has to be changed at once\textsuperscript{[3]}. Domain walls are an example of elastic interfaces. Elastic interfaces describe a variety of phenomena, ranging from wetting lines separating dry and wet regions in a material\textsuperscript{[4, 5]} to crack fronts in fracture mechanics\textsuperscript{[6, 7]}. Impurities and imperfections in realistic materials create interesting physics in these systems, as there is a competition between the local disorder, which makes the interface rough, and the elasticity, which tries to keep the interface smooth. For example, the fluctuating toughness of a material makes crack fronts irregular, as some parts of the crack advance before others. Neighboring segments try to keep up with each others’ movements, and as a result, a weak point in the material can trigger movement in its whole neighborhood. These collective movements that reinforce themselves are called avalanches.\textsuperscript{[8, 9]}

The behaviour can be divided into three categories depending on the strength of the applied field or force. Under very small driving, the avalanches are initiated by thermal noise. This region of slow intermittent movement is called creep, and it is the region of interest in this paper. When the driving exceeds a critical value, called the depinning point, the movement becomes continuous, and the average velocity grows as a power of the driving. Lastly, under a large external fields or forces the disorder becomes irrelevant, and the interface velocity grows linearly with the driving. A higher temperature blurs the transition points, causing for example depinning-like behaviour to start earlier.\textsuperscript{[10–12]}

Avalanches and the average movement of interfaces have been studied a lot under constant driving. A varying driving is however interesting in for example magnetic hysteresis, where the applied magnetic field can alternate between opposite directions. Nattermann, Pokrovsky, and others derived theory for domain wall motion under periodic driving in a series of papers for depinning\textsuperscript{[13, 14]} and for thermal motion\textsuperscript{[15, 16]}.

In this paper, we will continue from the previous work and calculate the average velocity of an interface under a sinusoidal external field in the creep region. We will consider both the case when the elasticity is short-ranged, such as magnetic domain walls, and long-ranged systems. Periodic driving in long-range elastic systems can be relevant in the case of fatigue fractures, as planar cracks are known to follow the long-range elastic interface model\textsuperscript{[17, 18]}. Fatigue can of course depend also on plastic deformations, but our results should apply for completely brittle materials and help connect fatigue to the study of creep.

II. INTERFACE VELOCITY

We will use the notation and terminology of magnetic domain walls in this section. The results still apply to elastic interfaces in other systems as well. In subsection II C we will however focus on long-range elastic interfaces and fracture mechanics and use corresponding terminology.

Several experiments have shown that thermally assisted domain wall velocity follows the creep behaviour for a large range of velocities until the driving field reaches its depinning value\textsuperscript{[19–23]}. The creep velocity
has been derived theoretically using energy arguments and an Arrhenius activation [24, 25] and later using the interface equation of motion and a functional renormalization group [26, 27]. The velocity under a magnetic field with strength $H$ is

$$v_c = v_0 e^{-CH^{-\mu}/T},$$

where $v_0$ and $C$ depend on the material and temperature and $\mu$ is a constant depending on the interface roughness. For magnetic domain walls $\mu \approx 0.25$ [19–23, 26]. $CH^{-\mu}$ describes the energy scale needed to move the domain wall in a disordered landscape, and it diverges as the driving field vanishes.

Jeudy et al. proposed to express the creep velocity in terms of effective experimental variables as

$$v_c = v(H_d, T) \exp \left(-\frac{T_d}{T} \left[\left(\frac{H}{H_d}\right)^{-\mu} - 1\right]\right),$$

where $H_d$ is the field strength at which the domain wall velocity starts to resemble depinning instead of creep. $\exp(-CH^{-\mu}/T)$ is the velocity at that field, $T$ is the temperature, and $T_d$ sets the material dependent energy scale [20, 21]. $H_d$ and $v(H_d, T)$ also depend on the material as well as the temperature, as higher temperatures cause the depinning transition to start at smaller drivings. With this definition, the exponent

$$-\frac{T_d}{T} \left[\left(\frac{H}{H_d}\right)^{-\mu} - 1\right],$$

vanishes at the effective depinning field $H_d$. Comparing Equations 1 and 2 we see that

$$v_0 = v(H_d, T) e^{T_d/T}$$

and

$$C = T_d H_d^\mu.$$

The creep formula is all the theoretical background that we will need for determining the interface velocity under a cyclic driving. We will mostly use the notation of Equation 1 in the following calculations, but for obtaining numerical results in subsection II B, we will use the notations of Jeudy et al. as we use material parameters from their publication [21].

### A. Periodic driving

If we replace the driving in the creep formula 1 with a periodic one $H_0 + \Delta H \sin(\omega t)$, we can study motion during a hysteresis loop. Integrating over the positive part of the cycle gives the expression

$$v_h = \frac{v_0}{\pi} \int_0^\pi \exp \left(-\frac{C}{T} \left[H_0 + \Delta H \sin x\right]^{-\mu}\right) dx$$

for the mean velocity of the interface. Here $x = \omega t$.

Using Laplace’s method for a saddle point approximation yields

$$v_h = \sqrt{\frac{2H_{\max}^{\mu+1} T}{\pi \mu \Delta H \mu c v_c}} v_c(H_{\max}).$$

where $H_{\max} = H_0 + \Delta H$ is the maximum field during the cycle and

$$v_c(H_{\max}) = v_0 e^{-C H_{\max}^{-\mu}/T}$$

is the creep velocity under the field $H_{\max}$.

The sinusoidally driven velocity or the velocity under some other variable magnetic field can of course be used as an alternative way to measure the energy parameter $C$ without doing creep experiments with a slower driving. If an experimentalist measures a creep velocity $v_c$ and a mean velocity $v_h$ under hysteresis, Equation 7 can be used to solve $C$ as

$$C = \frac{2H_{\max}^{\mu+1} T}{\pi \mu \Delta H} \frac{v_c^2(H_{\max})}{v_h^2}.$$

Even though the right side of the expression includes the magnetic field, $C$ does not depend on it. Instead, $H_{\max}^{\mu+1}/\Delta H$ cancels out the field dependence of $v_c$ and $v_h$.

Laplace’s method replaces the argument of the exponential function with an inverted parabola. We can also expand the argument further for a better approximation. Around its minimum at $x = \pi/2$ the expression

$$[H_0 + \Delta H \sin(x)]^{-\mu}$$

equals

$$H_{\max}^{-\mu} + \frac{\mu \Delta H}{H_{\max}^{\mu+1}} \left[\left(x - \frac{\pi}{2}\right)^2 + \left(x - \frac{\pi}{2}\right)^4\right] + \mathcal{O}(x^6).$$

where $a = [(3\mu + 2)\Delta H - H_0]/H_{\max}$.

The expression looks similar to the series representation of a hyperbolic cosine. Indeed, $\cosh(\sqrt{a} x)$ equals around its minimum at $x = 0$

$$1 + a \left(\frac{x^2}{2} + \frac{x^4}{24} a\right) + \mathcal{O}(x^6).$$

Thus, we can replace the expansion of $[H_0 + \Delta H \sin(x)]^{-\mu}$ around $x = \pi/2$ with the expression

$$\left[1 + \frac{\mu \Delta H}{a H_{\max}} \left(\cosh(\sqrt{a} x) - 1\right)\right] H_{\max}^{-\mu} + \mathcal{O}(x^6),$$

expanded around $x = 0$, if the integration boundaries are also modified. This is illustrated in Figure 1. There is however a divergence in $1/a$, so $\Delta H$ has to be larger than $H_0/(3\mu + 2)$. This is not a problem, since we already assume that the amplitude of the field $\Delta H$ is significantly larger than the background field $H_0$ by using a saddle-point expansion, and an amplitude of $H_0/(3\mu + 2)$ would mean an almost constant field.
Expanding both series further shows that the sixth order term in the series 10 is positive, and in the series 11 the term is also positive but smaller, so using a hyperbolic cosine is slightly better than just using a fourth order polynomial. In this case, a better approximation means that it works well also for smaller field amplitudes \( \Delta H \).

Replacing \([H_0 + \Delta H \sin(x)]^{-\mu}\) in the integral 6 with expression 12 and changing the integration region to the whole real axis as in Laplace’s method, we get

\[
\frac{v_c(H_{\max})}{\pi} e^b \int_{-\infty}^{\infty} \exp(b \cosh(\sqrt{a}x))dx, \quad (13)
\]

where \( b = (\mu C \Delta H)/(\alpha T H_{\max}^{\mu+1}) \) and again \( v_c(H_{\max}) = e^{-C H_{\max}^{-\mu}/T} \) is the creep velocity with field strength \( H_{\max} \). This integral is one of the definitions of the modified Bessel function of the second kind \( K_0 \). Therefore, we acquire the mean velocity

\[
v_h \approx \frac{2}{\pi \sqrt{a}} e^b K_0(b)v_c(H_{\max}) \quad (14)
\]

for an interface under periodic driving. Using the series representation of \( K_0 \) and writing \( a \) in terms of \( b \), we find that

\[
v_h \approx \sqrt{\frac{2H_{\max}^{\mu+1}}{\pi \mu C \Delta H}} \left( 1 - \frac{1}{8b} + \frac{3^2}{2!(8b)^2} - \frac{3^2 5^2}{3!(8b)^3} + ... \right) v_c(H_{\max}), \quad (15)
\]

where the first term is our previous approximation for \( v_h \) in Equation 7.

B. Numerical integral for short-range elasticity

To check our formula for the domain wall velocity during a hysteresis loop, we integrated numerically expression 6 and compared it to the approximation 14 using material parameters for Pt/Co/Pt films from [21]. Figure 2 shows the domain wall velocity under various sinusoidal fields. The continuous lines use our approximation and the points use the numerical integral. In Figure 2a the minimum field is \( H_0 = 0 \), whereas in Figure 2b \( H_0 = 0.3H_d \), i.e., 30\% of the effective depinning value at room temperature. Different lines represent samples with different thicknesses.

Let us look at the behaviour of the function multiplying the creep velocity in Equation 14. Figure 3 shows the ratio of the hysteresis velocity to the creep velocity as a function of the maximal field \( H_{\max} \). The ratio is roughly constant, especially when the minimum field is zero, so the hysteresis velocity follows largely the creep formula 1. The points use the previous numerical integration of Equation 6 and the continuous lines use Equation 14. When the amplitude \( \Delta H \) is smaller than the background field \( H_0 \), Equation 14 becomes less accurate. This is only natural, since for small amplitudes the field is closer to a constant, and we used a saddle point approximation.

The graphs for the hysteresis velocity, especially on Figure 2b, resemble very clean power laws, even though our approximation for the hysteresis velocity does not look like a power law. Indeed, Figure 4a shows the numerically integrated velocity of Figure 2b fitted with a simple power law \( \alpha x^\beta \). Different material parameters lead to different exponents for the fit, as they change the energy scale \( T_d \) and the effective depinning point \( H_d \), resulting in a different value for \( C \).

Since the hysteresis velocity is roughly proportional to the creep velocity in Figure 3, the apparent power-law behaviour arises mostly from the stretched exponential form of the creep velocity. The periodic driving and the minimum field \( H_0 \) however have a small effect as well, as Figure 2b with a positive minimum field resembles a cleaner power law than Figure 2a with \( H_0 = 0 \).

As \( \mu \approx 0.25 \) for magnetic domain walls, we can do a simple estimate for the fitted exponents using the small \( \mu \) approximation

\[
H^{-\mu} \approx 1 - \mu \ln H. \quad (16)
\]

Now the creep velocity \( v_c = v_0 e^{-C H^{-\mu}/T} \) turns it into the power law

\[
v_c \propto H^{C \mu/T}. \quad (17)
\]

In the units with \( H_d \) and \( T_d \) the relation is

\[
v_c \propto \left( \frac{H}{H_d} \right)^{\mu T_d/T}. \quad (18)
\]

Indeed the fitted exponents in Figure 4a grow roughly as \(-0.29 + 0.27 T_d/T \approx \mu T_d/T \), as is illustrated in Figure 4b.

\[\text{FIG. 1: Illustration for how the two approximations of } \sin^{-\mu} x \text{ change the velocity exp(} - \sin^{-\mu} x \text{), where the constants } v_0, C/T, \text{ and } \Delta H \text{ have been set to one and } H_0 \text{ to zero, during the cycle from } x = 0 \text{ to } x = \pi.\]
FIG. 2: The velocity of the domain wall with different minimum fields. On the left the minimum field is set to zero, and on the right the minimum field $H_0 = 0.3H_d$. The temperature is 293K in both panels. The points denote the velocity obtained from a numerical integration of Equation 6 and the continuous lines use the approximation 14. The data uses experimental values for Pt/Co/Pt films from [21]. The lines represent samples with different thicknesses. The values of $H_d$ are 28.5, 56, 76, and 72 in order of rising sample thickness. Notice that the axes are logarithmic.

FIG. 3: The deviation of a sinusoidally driven velocity from creep. The point use the numerically integrated velocities in Figure 2 divided by the creep velocity $v_c(H_{max}) = v_c(H_0 + \Delta H)$ and the continuous lines use Equation 14. In (a) the minimum field $H_0 = 0$ and in (b) $H_0 = 0.3H_d$. The error of Equation 14 for small fields in 3b results from using a saddle point approximation for a near constant field.

C. Long-range elasticity

Finally, we will consider long-range elastic systems such as fractures, contact line wetting [28] and low-angle grain boundaries [29]. The general form for the creep exponent is

$$\mu = \frac{d - \alpha + 2\zeta}{\alpha - \zeta},$$

where $d$ is the dimension of the moving interface, $\alpha$ is the range of the elasticity kernel, and $\zeta$ is the equilibrium roughness exponent [26]. For a long-range elastic one-dimensional interface $d = 1$ and $\alpha = 1$. Experiments where Plexiglas plates are attached by sintering and then detached mimicking a planar crack suggest that $\zeta \approx 0.35$ [30, 31], which results in $\mu \approx 1.08$. Similarly, a paper peeling experiment suggests that $\mu = 1$ [32]. Assuming that $\mu = 1$, the creep law turns into a pure exponential form, and our first approximation for the cyclic velocity
Reduced field ($H_0 + \Delta H/d^{1/2}$)

Average velocity (m/s)

Thickness=0.5nm

exponent=2.12

Thickness=0.6nm

exponent=3.58

Thickness=0.7nm

exponent=5.78

Thickness=0.8nm

exponent=8.83

FIG. 4: (a) shows the numerically integrated velocity in Figure 2b fitted with a power law. The continuous lines are power-law fits using the function $\alpha x^\beta$. (b) shows the fitted exponents as a function of the material parameter $T_d = C/H_d^{1/\mu}$. The values of $T_d$ are 2558, 4145, 6490, and 9720 in order of rising sample thickness.

7 leads to

$$v_h = \sqrt{\frac{2H_{\text{max}}^2 T}{\pi C \Delta H}} v_0 e^{-C/H_{\text{max}} T}.$$  \hspace{1cm} (20)

Now that $\mu = 1$, we cannot do the approximation for small $\mu$, so the approximate power-law behaviour with a material dependent exponent is lost. Instead, the velocity grows proportionally to $\sqrt{H_{\text{max}} T/C e^{-C/H_{\text{max}} T}}$ when $\Delta H \gg H_0$.

III. DISCUSSION ON FATIGUE CRACKS

Fractures in perfectly brittle materials should follow the behaviour in Equation 20. There is however a widely known power-law relation between crack velocity and the amplitude of an oscillating stress intensity factor known as Paris’ law, where the exponent is not universal [33, 34]. That contradicts our finding, which means that plastic deformations in the proximity of the crack line, which we ignored, must be more important for the periodically driven fatigue. On the other hand, fractures in practical applications are not planar, and the effective $\mu$-exponent may lead to qualitatively different behaviour for the crack velocity. In the case of an actually planar crack with no significant plastic deformations, such as the Plexiglas experiments, our results should apply. If fatigue was to follow the elastic interface model with just a small value for the $\mu$-exponent, like in the short-range elastic case, then we could use the previous approximation, yielding again a power law with a material dependent exponent. The exponent would decrease with increasing temperature. This would be a testable hypothesis. Interestingly, the research on how temperature affects the Paris’ law exponent and crack velocity is not quite unanimous. Some studies have found that a higher temperature indeed decreases the exponent [35, 36], some found that a higher temperature at least affects the crack velocity [37], and some found that temperature does not have a significant effect [38, 39].

IV. SUMMARY

In conclusion, we studied the average velocity of an elastic interface under periodic subcritical driving and thermal noise. We considered both short-range elastic systems such as magnetic domain walls during a hysteresis loop and long-ranged systems such as fractures.

We started with the creep velocity of an elastic interface and derived an approximation for the resulting average velocity under a sinusoidal driving. The acquired velocity is a modification of the creep law. We also compared the velocity to a numerical integral, finding our approximation to be good for practical applications.

For short-range elastic systems, the average velocity under sinusoidal driving grows approximately as a power of the driving field’s strength. The behaviour applies already for creep, as a creep velocity $e^{-C H^{1/\mu}/T}$ is roughly proportional to $H_{\text{max}} C e^{-C/H_{\text{max}} T}$ for small $\mu$, but a cleaner power law emerges in the sinusoidal case, especially if there is a positive minimum field. As the exponent for the creep velocity depends on a material specific energy scale, the exponent for the periodically driven velocity is also material dependent.

Our result for long-range elastic systems is simpler. Because of a different roughness exponent, the creep velocity is purely exponential, and as a result the average velocity under a sinusoidal driving grows as $\sqrt{\Delta H e^{-1/\Delta H}}$ with the amplitude $\Delta H$ of the driving field, when the
minimum field is small. Interestingly, planar cracks are known to follow the long-range elastic interface model, but fractures in general do not have a velocity of this form under periodic loading. Instead, periodically loaded cracks have power-law velocities with varying exponents. This resembles more our result for short-range elastic systems. This disagreement with our result is probably caused by plastic deformation in periodic loading, but on the other hand, realistic fractures are not necessarily planar and can therefore have different material parameters than the ones we used. At the very least, our work should help in connecting studies of creep and the periodically loaded fatigue, so that it is easier to tell how much of the propagation of a fatigue crack can be attributed to brittle creep, and what is caused by some other mechanism. Our results should especially be helpful in experiments with very brittle materials, since there is less plastic deformation, and in experiments that also connect creep and fatigue by using different loading amplitudes.

ACKNOWLEDGMENTS

J.S and M.J.A. acknowledge support from the Academy of Finland (Center of Excellence program, 278367 and 317464) and M.J.A. from the European Union Horizon 2020 research and innovation programme under grant agreement No 857470 and from European Regional Development Fund via Foundation for Polish Science International Research Agenda PLUS programme grant No MAB PLUS/2018/8. The authors acknowledge the computational resources provided by the Aalto University School of Science “Science-IT” project.

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