Obtaining the diffusion coefficient for cosmic ray propagation in the Galactic Centre Ridge through time-dependent simulations of their γ-ray emission

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Abstract

Recent observations by the H.E.S.S. collaboration of the Galactic Centre region have revealed what appears to be γ-ray emission from the decay of pions produced by interactions of recently accelerated cosmic rays with local molecular hydrogen clouds. Synthesizing a 3-D hydrogen cloud map from the available data and assuming a diffusion coefficient of the form \( \kappa(E) = \kappa_0(E/E_0)^\delta \), we performed Monte Carlo simulations of cosmic ray diffusion for various propagation times and values of \( \kappa_0 \) and \( \delta \). By fitting the model γ-ray spectra to the observed one we were able to infer the value of the diffusion coefficient in that environment (\( \kappa = 3.0 \pm 0.2 \text{kpc}^2\text{Myr}^{-1} \) for \( E = 10^{12.5}\text{eV} \) and for total propagation time \( 10^4\text{yr} \)) as well as the source spectrum (\( 2.1 \leq \gamma \leq 2.3 \)). Also, we found that proton losses can be substantial, which justifies our approach to the problem.

1. Introduction

The High Energy Stereoscopic System (H.E.S.S.) γ-ray telescope began operations in 2004 and among its first targets was the Galactic Centre region. Images obtained from the H.E.S.S. observations had an angular resolution 0.1°, giving us a map of γ-ray emission of unprecedented detail. A few point sources were first detected, which were compatible with the positions of known objects, like the black hole Sagittarius A*, the supernova remnant Sgr A East and the supernova remnant/pulsar wind nebula G0.9+0.1. By subtracting those known sources, the H.E.S.S. collaboration was able to produce a mapping of fainter γ-ray emission, stretching over an area of approximately 300pc by 100pc around the Galactic Centre. This emission seems to follow the contours of molecular gas density, as measured by its carbon monosulfide distribution, up to a distance of approximately 1.3° in galactic longitude from the Galactic Centre. Beyond that distance the γ-ray emission diminishes to background levels.

This apparent emission correlation points towards possible models, for which two theories were initially proposed. The first postulated that a population of electron accelerators produces the observed emission via inverse Compton scattering. The objects that would make up such a population, such as pulsar wind nebulae, would thrive in regions of high-density molecular gas; however the large number of sources needed to reproduce the observed emission renders this possibility rather unlikely.

The other theory claims that the γ-rays are produced by neutral pion decay, resulting from the interaction of locally accelerated cosmic rays with the ambient molecular gas. The lower energy threshold of the H.E.S.S. survey was 380 GeV, so cosmic ray protons of higher energies would be needed to produce the observed γ-rays. Furthermore, these cosmic rays would have to have been accelerated near the Galactic Centre at some point in the past, yet not diffused significantly beyond 1.3° from it. Assuming the validity of this theory, one may use this data to infer the diffusion properties of cosmic rays in the Galactic Centre region for various possible sources of cosmic rays. Considering as a source of the cosmic rays either the supernova remnant Sgr A East, with an estimated age of 10 kyr, or the black hole Sgr A* with a more remote age of activity, Aharonian et al. inferred that the diffusion coefficient should be no more than 3.5 kpc\(^2\) Myr\(^{-1}\) in that area. Busching et al. went one step further with an analytical reproduction of the H.E.S.S. observations by calculating the emission for different diffusion coefficients. Using relativistic protons of mean energy \( \sim 3 T\text{eV} \) to represent the cosmic rays and a small number of Gaussian functions to represent the molecular clouds they arrived at a diffusion coefficient of \( \kappa = 1.3\text{kpc}^2\text{Myr}^{-1} \). Büsching and de Jager subsequently expanded their results for different ages and on-times for the source. More recently, Wommer et al. employed a different approach, in which the motion of individual test particles is computed by solving the Lorentz force equation for short time periods, and the resulting distributions provide the coefficients for the diffusion equation. From their initial conditions for the turbulent magnetic field, it is derived that no single source can account for the observed emission, but that a more continuous source of cosmic ray protons produces a better correlation.

In this paper we present a series of time-dependent sim-
ulations of proton propagation in the Galactic Centre in a detailed 3D distribution of molecular hydrogen concentrations for different diffusion parameters. By utilizing a wide array of data on molecular clouds in the Galactic Centre from various observations we construct a fairly accurate density grid of the area of diffusion, as shown in §2.1. We can then inject cosmic rays, assumed to be protons, from an origin of our choice, which propagate according to the diffusion model and its free parameters described in §2.2. In §2.3 we deal with the interactions of these protons with hydrogen molecules and with the production of photons from the resulting neutral pion decays. In §2.4 we briefly discuss the likely sources of cosmic ray origin, and the implications on their age. In §2.5 we describe the more technical aspects of the simulation program, such as the initial choice of free parameters and the methods used to raise the efficiency of the numerical code. Finally, in §3 we present the results of the simulations and in §4 we summarize and discuss the effectiveness of our approach and the likely significance of our results. The present paper expands upon the initial results of Dimitrakoudis et al. [3].

2. Simulations

2.1. Synthesizing a hydrogen cloud map

The area of the Galactic Centre in which we simulated the diffusion of cosmic rays is rich in H$_2$ gas, contained in a complex setup of high-density clouds, ridges and streams that comprise about 10% of our galaxy’s interstellar molecular gas, i.e. about 2 to 5 ×10$^7$ solar masses [10]. Due to the high densities involved, tracer molecules were used to determine the mass of each gas cloud. To create a realistic 3-D map of that environment we first obtained the data for the 159 distinct molecular cloud clumps, as compiled by Miyazaki & Tsuboi [12], i.e. we assigned to each one a galactic longitude, latitude, radius and density. Their radial distances are unknown, so we assumed a random function to simulate them, within reasonable limits. To these data we added the locations and densities of the radio-sources Sgr A [14], Sgr B1, Sgr B2 and Sgr C [9], which are rich in atomic hydrogen. We then broke down the entire CS map of the Galactic Centre by Tsuboi et al. [10] into blocks of uniform density, which we treated as clouds of equal radius and of random radial distances. Finally we added the larger CO clouds by Oka et al. [13] in order to expand the spatial distribution of our map. Every time we added a new set of clouds we checked the possibility that clouds from previous sets were sharing their projected locations with the new ones, and we subtracted their masses accordingly. The result was 584 clouds of hydrogen gas with a high uncertainty as to their radial distances. These clouds were then turned into a 3-D grid of 120 × 60 × 60 boxes of uniform density, which form the volume of our diffusion model, 5.4 × 10$^7$pc$^3$. The projection of that grid constitutes an area far greater than the extent of the observed γ-ray emission by [1], comprising a total mass of 1.2 · 10$^8$M$_⊙$, so particle interactions with clouds well outside the observed area of emission are accounted for.

2.2. Diffusion model

We used the diffusive model of CR propagation, assuming a diffusion coefficient of the form:

$$\kappa = \kappa_0 (E/E_0)^\delta$$

where E is the energy of the CR protons, $E_0 = 1$GeV, while $\kappa_0$ and $\delta$ are free parameters, whose values we will try to infer through our simulations. The index $\delta$ is a measure of the turbulence of the magnetic field and is assumed to be $0.3 \leq \delta \leq 0.6$ [15], while $\kappa_0$ is a constant whose value we will attempt to estimate once $\delta$ has been obtained. The diffusion coefficient in our simulations is represented by the mean free path $\ell$, which is given by the usual expression

$$\ell = 3\kappa/c$$

where c is the velocity of light.

Thus, the test protons move in straight lines of length equal to $\ell$, after which their directions change randomly. At the end of each such free walk, the box number of the hydrogen density grid is calculated and a check is made for collisions with hydrogen protons. If such a collision occurs, the diffusion continues with a lower proton energy, as shown in the next section.

2.3. Production of γ-rays

We have assumed that all γ-rays forming the observed emission are produced from neutral pion decay. Moreover, we have assumed that pions are produced in proton-proton collisions through two main channels

a) $p + p \rightarrow p + p + \pi^+ + \pi^- + \pi^0$

b) $p + p \rightarrow n + p + \pi^+$

In case (a) the initial proton loses part of its energy and a multiplicity of pions is created that take equal amount of the energy lost from the proton. The positive and negative pions break down into muons, and ultimately into electrons, positrons and neutrinos, while the neutral pions decay into γ-rays. Assuming an inelasticity $k_{pp} = 0.45$ [11], 15% of the initial proton energy will go to the produced γ-rays, while the original proton will continue its diffusion with 55% of its initial energy. This approach, while simplified, gives results which are in good agreement with the more detailed spectra produced by Büsching et al. [8].

Channel (b) produces no photons and thus does not contribute directly to our observed emission. However, the initial proton is turned into a neutron, having lost some of its energy in the collision, and will thus continue its propagation in a straight line, unaffected by the magnetic fields.
that regulated its trajectory as a proton. It will revert, though, back into a proton after half life \( \tau = 886.7 \pm 0.8 \text{sec} \) \([19]\), and then it will continue its diffusion as before. Accounting for time dilation, the distances traveled by neutrons are always much smaller than the mean free path for each proton energy (0.028 pc for \( E = 10^{12.5} \text{eV} \); 0.92 pc for \( E = 10^{14} \text{eV} \)); the radii of molecular gas clouds range between 2 pc and 40 pc), so we can safely assume that this case will not affect the diffusion process.

Using the above we calculate the optical depth for each random walk. This is calculated as the product of the interaction cross section with the mean free path \( \ell \) and the density \( n \) of hydrogen molecules. That density is retrieved from the grid box where each step ends, a fairly effective approximation as most random walks are contained within single grid boxes. We have assumed that the cross section is \( \sigma_{pp} = 4 \cdot 10^{-26} \text{cm}^2 \) \([2]\), while an increase by a factor of 1.3 is sufficient to account for the known chemical composition of the Interstellar Medium \([10]\). The reaction probability would then be \( P = 1 - e^{-\tau} \), where \( \tau \) is the optical depth. In practice though, the optical depth is a very good approximation of the interaction cross section with the mean free path \( \ell \) and the density \( n \) of hydrogen molecules. That density is retrieved from the grid box where each step ends, a fairly effective approximation as most random walks are contained within single grid boxes. We have assumed that the cross section is \( \sigma_{pp} = 4 \cdot 10^{-26} \text{cm}^2 \) \([2]\), while an increase by a factor of 1.3 is sufficient to account for the known chemical composition of the Interstellar Medium \([10]\). The reaction probability would then be \( P = 1 - e^{-\tau} \), where \( \tau \) is the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth. In practice though, the optical depth is a very good approximation of the optical depth.
tional time and the mean propagation distance inferred from the observations.

Once each simulation is completed and the results normalised, the resulting $\gamma$-rays are compared against the results from H.E.S.S. using the reduced $\chi^2$ criterion.

3. Results

The resulting reduced $\chi^2$ values for cosmic rays originating from the supernova remnant Sgr A East, assumed to be produced $10^5 yr$ ago, are shown in Fig.1 for various values of the diffusion coefficient. The minimum value of $\chi^2$ calculated is 1.8, and corresponds to $\delta = 0.3$ and $\kappa_0 = 0.25kpc^2Myr^{-1}$. However we can find minima which are less than 2 for all values of $\delta$. If we use those values of $\delta$ and $\kappa_0$ to calculate the diffusion coefficient for protons of energy $10^{12.5}eV$ (the lowest energy in our sample and also the most important due to their relative abundance over those at higher energies), we arrive at the results illustrated in Fig.2. We can see that, for each value of $\delta$, the diffusion coefficient displays the same minimum at $\kappa = 3.0 \pm 0.2kpc^2Myr^{-1}$. This value is higher than that calculated by B"usching et al. [3] but close to that suggested by Aharonian et al. [1] (less than 3.5$kpc^2Myr^{-1}$).

For higher values of $\kappa$, we also see a disparity between the way the reduced $\chi^2$ values increase in our results and those of B"usching et al. The reason for this is that for these values the mean free path becomes very large and comparable to the dimensions of the area of simulation, therefore the simulation becomes inefficient, resulting in poor statistics even with increased numbers of test particles.

Fig.3 shows the reduced $\chi^2$ values that correspond to a total propagation time of $10^5 yr$ a possible past activity of Sgr A*. If we once again use the values of $\delta$ and $\kappa_0$ which correspond to a minimum to calculate the diffusion coefficient for protons of energy $10^{12.5}eV$ we arrive at the results illustrated in Fig. 4. The resulting best choices for the diffusion coefficients appear in Table 1.

| $\delta$ | $\kappa[kpc^2Myr^{-1}]$ |
|----------|-------------------------|
| 0.3      | 0.45                    |
| 0.4      | 0.4                     |
| 0.5      | 0.56                    |
| 0.6      | 0.32                    |

Table 1: Diffusion coefficients for different values of $\delta$ for total propagation time $10^5 yr$.

This variation for different values of $\delta$ is more pronounced than in the case of propagation for $10^4 yr$, but if we were to derive a mean value of $\kappa$ like before, we would find $\kappa = 0.43 \pm 0.05kpc^2Myr^{-1}$. The higher propagation time requires the cosmic rays to diffuse more slowly than in the case of $10^4 yr$, which is why the resulting diffusion coefficient is considerably smaller.

Fig.5 and 6 plot the proton spectral indices $\gamma$, as these were inferred from the simulations, for the two total propagation times versus $\kappa_0$ for different values of $\delta$.

In Fig.5 we can see a fluctuation of $\gamma$ values, most likely due to irregularities in the interactions of the different energy populations with the complex setup of hydrogen clouds in the small propagation time. A progression to lower $\gamma$ values as we raise the value of $\kappa_0$ is, however, evident. The values of $\gamma$ for the minimum reduced $\chi^2$ values are in the range $2.1 \leq \gamma \leq 2.3$. For the larger propagation time, we can see in Fig.6 a similar progression to lower $\gamma$ values as a function of $\kappa_0$. In the range of our best results, the proton indices are in the range $2.0 \leq \gamma \leq 2.1$, so the proton spectrum is clearly slightly steeper than the resulting photon spectrum. These results are in general agreement with leaky box model predictions [1].

Furthermore, it is possible to make an estimate of the
Figure 3: Reduced $\chi^2$ values for different values of $\kappa_0$ and $\delta$ for total propagation time $10^5$ yr.

Figure 4: Plot of the reduced $\chi^2$ as a function of the diffusion coefficient $\kappa$ for different values of $\delta$. The total propagation time was assumed to be $10^5$ yr. Higher values of $\delta$ tend to favor lower values of $\kappa$.

Figure 5: Plot of spectral index $\gamma$ inferred from simulations as a function of the diffusion normalization $\kappa_0$ for different values of $\delta$. The total propagation time was taken to be $10^5$ yr.

Figure 6: Plot of spectral index $\gamma$ inferred from simulations as a function of the diffusion normalization $\kappa_0$ for different values of $\delta$. The total propagation time was taken to be $10^5$ yr.
total energy of the protons accelerated at their source, by comparing the number of observed photons to the number of diffusing protons. Doing so for the lowest observable proton energy and extrapolating for the energy range $10^9 - 10^{15}$ eV yields a total energy of $(8 \pm 1) \times 10^{49}$ erg, for our different time and optimal diffusion parameters. That represents approximately 10% of the energy output of a typical supernova explosion.

The Fermi Gamma-ray Space Telescope should be able to observe that area of space in a lower energy range (from 20 MeV up to 300 GeV). We have repeated our simulations for the optimal diffusion coefficients we have found, for a total propagation time $10^4$ yr, with the proton injection spectrum extended to $10^{16}$ eV. In these cases, the emission is dominated by the lower energy protons, and thus extends from about $-0.3^\circ$ to $0.2^\circ$ in galactic longitude.

4. Summary/Discussion

We have presented the results of a series of time-dependent simulations of the diffusion of cosmic rays in the Galactic Centre region. In the first scenario it was assumed that a burst of cosmic rays occurred $10^3$ yrs ago, while in the second $10^5$ yrs ago. Likely candidates could have been the SNR Sgr A East or the black hole candidate Sgr A*. Their $\gamma$-ray emission was compared with observations from the H.E.S.S. collaboration [1], in order to determine the diffusion coefficient in that region. For that purpose, a detailed 3-D map of hydrogen concentrations was synthesized from various observations [2, 12, 13, 14, 16], and two scenarios for the origin of the cosmic rays were taken into account. For the SNR Sgr A East, $10^4$ yrs is the estimated upper limit on its age, so the resulting diffusion coefficient should be taken as a lower limit. There is much more uncertainty concerning the possible age of activity of Sgr A*, but it is clear that should that have been in the order of $10^5$ yrs ago or earlier, the random component of the interstellar magnetic field would have to be considerably more pronounced than in the first case.

The need for such elaborate simulations as were described in the preceding chapters arises from the considerable losses of protons during their propagation, losses which are inextricably connected to the local densities of hydrogen gas. In Fig.7 one can compare the resulting reduced $\chi^2$ values from a series of test runs where proton energy losses are not taken into account, as opposed to the same values from our actual simulations. An underestimation of the diffusion coefficient is evident in the simulations without losses. Furthermore, this underestimation explains the discrepancy between our results and those of B"usching et al. [3], as the diffusion coefficient calculated without losses is about two times smaller than when losses are taken into account in the propagation.

The above results were derived under the assumption that the acceleration of the cosmic rays responsible for the observed $\gamma$-rays occurred $10^4$ or $10^5$ yrs ago at a single source, with no subsequent periods of activity at that source. Recent papers [8] have noted that this may be a very simplified approach, as there have been many supernovae in the Galactic Centre region in the past millennia. Also, the uncertainty in the radial distances of hydrogen concentrations may have had a significant impact on the final results. Finally, these simulations assumed that the diffusion coefficient remains constant throughout the whole region of propagation, and local orderings of magnetic fields (including their correlation with molecular density within the gas clouds) were not taken into account.

Those limitations notwithstanding, the results of these simulations are useful in providing an estimate of the diffusion coefficient in the Galactic Centre, taking the different magnetic turbulence theories (that become manifest in the different values of $\delta$) into account.

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