The phase between the three gluon and one photon amplitudes in quarkonium decays

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Abstract

The phase between three-gluon and one-photon amplitudes in $\psi(2S)$ and $\psi(3770)$ decays is analyzed.

1 Motivations

It has been known that in $J/\psi$ decays, the three gluon amplitude $a_{3g}$ and one-photon amplitude $a_{\gamma}$ are orthogonal for the decay modes $1^+0^- (90^\circ)$ [1], $1^-0^- (106 \pm 10)^\circ$ [2], $0^-0^- (89.6 \pm 9.9)^\circ$ [3], $1^+1^- (138 \pm 37)^\circ$ [4] and $NN$ $(89 \pm 15)^\circ$ [5].

J. M. Gérard and J. Weyers [6] augued that this large phase follows from the orthogonality of three-gluon and one-photon virtual processes. The question arises: is this phase universal for quarkonium decays? How about $\psi(2S)$, $\psi(3770)$ and $\Upsilon(nS)$ decays?

2 Quarkonium produced in electron-positron colliding experiments

Recently, more $\psi(2S)$ data has been available. Most of the branching ratios are measured in $e^+e^-$ colliding experiments. For these experiments, there are three diagrams [7, 8], as shown in Fig. 1, which contribute to the processes. Although such formulas were written in the early years after $J/\psi$ was discovered, but

- (a) three-gluon annihilation
- (b) one-photon annihilation
- (c) one-photon continuum

Figure 1: The Feynman diagrams of $e^+e^- \rightarrow light\ hadrons$ at charmonium resonance.

the diagram in Fig. 1(c) is usually neglected. This reflects a big gap between theory and the actual experiments.

How important is this amplitude? For $\psi(2S)$, at first glance, $\sigma_{Born} = 7887\text{nb}$; while $\sigma_e \approx 14\text{nb}$. But for $e^+e^-$ processes, initial state radiation modifies the Breit-Wigner cross section. With radiative correction, $\sigma_{r.e.} = 4046\text{nb}$; more important, the $e^+e^-$ colliders have finite beam energy resolution, with $\Delta$ at the order of magnitude of MeV; while the width of $\psi(2S)$ is only 300KeV. Here $\Delta$ is the standard deviation of the guassian function which describes the C.M. energy distribution of the electron-positron. This reduces the observed cross section by an order of magnitude. For example, with $\Delta = 1.3\text{MeV}$

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(parameter of BES/BEPC at the energy of $\psi(2S)$ mass), $\sigma_{obs} = 640\text{nb}$. If $\Delta = 2.0\text{MeV}$ (paramters of DM2/DCI experiment at the same energy), $\sigma_{obs} = 442\text{nb}$. The contribution from direct one-photon annihilation is most important for pure electromagnetic process, like $\mu^+\mu^-$, where the continuum cross section is as large as the resonance itself and the interference is apparent. This is seen in the $\mu^+\mu^-$ cross section curve in the experimental scan of $\psi(2S)$ resonance, as shown in Fig. 2.

![Figure 2: $\mu^+\mu^-$ curve at $\psi(2S)$ resonance scanned by BES](image)

3 Pure electromagnetic decay

BES reports $B(\psi(2S) \rightarrow \omega\pi^0) = (3.8 \pm 1.7 \pm 1.1) \times 10^{-5}$, What it means is the cross section of $e^+e^- \rightarrow \omega\pi^0$ at $\psi(2S)$ mass is measured to be $(2.4 \pm 1.3) \times 10^{-2} \text{nb}$. About 60% of this cross section is due to continuum $\mathrm{lO}$. This gives the form factor $F_{\omega\pi}(m^2_{\psi(2S)})/F_{\omega\pi}(0) = (1.6 \pm 0.4) \times 10^{-2}$. It agrees well with the calculation by J.-M. Gérard and G. López Castro $\mathrm{lI}$ which predicts it to be $(2\pi f_\pi^2)/3s = 1.66 \times 10^{-2}$ with $f_\pi$ the pion decay constant. Similarly $\pi$ form factor at $\psi(2S)$ is revised $\mathrm{lI}$.

4 $\psi(2S) \rightarrow 1^{−0}$ and $0^{−0}$ decays

The $\psi(2S) \rightarrow 1^{−0}$ decays are due to three-gluon amplitude $a_{3g}$ and one-photon amplitude $a_\gamma$. With these two amplitudes, a previous analysis $\mathrm{12}$ yielded $a_{3g} \approx -a_\gamma$, i.e. the phase $\phi$ between $a_{3g}$ and $a_\gamma$ is 180° and $\phi = 90°$ is ruled out. Here the SU(3) breaking amplitude $\epsilon$ is small compared with $a_{3g}$. But these branching ratios so far are all measured by $e^+e^-$ experiments. So actually we have three diagrams and three amplitudes. The analysis should be based on Table $\mathrm{l}$

| modes            | amplitude           | B.R.(in 10^{-5}) |
|------------------|---------------------|------------------|
| $\rho^0\pi^+$   | $a_{3g} + a_\gamma + a_c$ | < 0.09          |
| ($\rho^0\pi^0$) |                     |                  |
| $K^{*+}K^-$     | $a_{3g} + \epsilon + a_\gamma + a_c$ | < 0.15          |
| $K^{*0}K^0$     | $a_{3g} + \epsilon - 2(a_\gamma + a_c)$ | 0.41 ± 0.12 ± 0.08 |
| $\omega\pi^0$   | $3(a_\gamma + a_c)$ | 0.38 ± 0.17 ± 0.11 |

Table 1: $e^+e^- \rightarrow \psi(2S) \rightarrow 1^{−0}$ process

In Table $\mathrm{l}$ $a_{3g}$ interferes with $a_\gamma + a_c$, destructively for $\rho\pi$ and $K^{*+}K^-$, but constructively for $K^{*0}K^0$ ($\epsilon$ is a fraction of $a_{3g}$). Fitting measured $K^{*+}K^-$ and $\rho\pi$ modes with different $\phi$’s are listed in Table $\mathrm{2}$.

It shows that a $−90°$ phase between $a_{3g}$ and $a_\gamma$ is still consistant with the data within one standard deviation of the experimental errors $\mathrm{l3}$.
Table 2: Calculated results for $\psi(2S) \rightarrow K^{*+}K^-$ and $\rho^0\pi^0$ with different $\phi$.

$$
\begin{array}{|c|c|c|c|c|}
\hline
\phi & C = \frac{a_{3g}}{a_\gamma} & \sigma_{pre}(K^{*+}K^-)(pb) & B^{0}_{K^{*+}K^-}(\times 10^{-5}) & \sigma_{pre}(\rho^0\pi^0)(pb) & B^{0}_{\rho^0\pi^0}(\times 10^{-5}) \\
\hline
+76.8^\circ & 7.0^{+3.1}_{-2.2} & 37^{+24}_{-23} & 5.0^{+3.2}_{-3.1} & 64^{+43}_{-41} & 9.0^{+6.1}_{-6.0} \\
-72.0^\circ & 5.3^{+3.1}_{-2.6} & 19^{+14}_{-14} & 3.1^{+2.3}_{-2.3} & 33^{+25}_{-24} & 5.5^{+4.1}_{-4.0} \\
-90^\circ & 4.5^{+1.1}_{-0.7} & 12^{+9}_{-9} & 2.0^{+1.5}_{-1.5} & 22^{+17}_{-17} & 3.7^{+2.9}_{-2.9} \\
180^\circ & 3.4^{+4.0}_{-2.2} & 4.0^{+4.3}_{-3.2} & 0.39^{+0.42}_{-0.31} & 7.8^{+8.6}_{-6.7} & 1.0^{+1.1}_{-0.8} \\
\hline
\end{array}
$$

The newly measured $\psi(2S) \rightarrow K_S K_L$ from BES-II \cite{15}, together with previous results on $\pi^+\pi^-$ and $K^+K^-$, is also consistent with a $-90^\circ$ phase between $a_{3g}$ and $a_\gamma$ \cite{13}. This is discussed in more detail by X.H. Mo in this conference.

5 $\psi(3770) \rightarrow \rho\pi$

J.L. Rosner \cite{16} proposed that the $\rho\pi$ puzzle is due to the mixing of $\psi(2S)$ and $\psi(1D)$ states, with the mixing angle $\theta = 12^\circ$. In this scenario, the missing $\rho\pi$ decay mode of $\psi(2S)$ shows up instead as decay mode of $\psi(3770)$, enhanced by the factor $1/sin^2\theta$. He predicts $B_{\psi(3770) \rightarrow \rho\pi} = (4.1 \pm 1.4) \times 10^{-4}$. With the total cross section of $\psi(3770)$ at Born order to be $(11.6 \pm 1.8)$ nb, $\sigma_{\text{Born}}^{\psi(3770) \rightarrow \rho\pi} = (4.8 \pm 1.9)$ pb.

But one should be reminded that for $\psi(3770)$, the resonance cross section, with radiative correction is only 8.17 nb, while the continuum is 13 nb. So to measure it in $e^+e^-$ experiments, we must know the cross section $e^+e^- \rightarrow \gamma^* \rightarrow \rho\pi$. The cross section $\sigma_{e^+e^- \rightarrow \gamma^* \rightarrow \rho\pi}(s)$ can be estimated by the electromagnetic form factor of $\omega\pi^0$, since from SU(3) symmetry, the coupling of $\omega\pi^0$ to $\gamma^*$ is three times of $\rho\pi$. The $\omega\pi^0$ form factor measured at $\psi(2S)$ is extrapolated to $\sqrt{s} = M_{\psi(3770)}$ by $|F_{\omega\pi^0}(s)| = 0.531$ GeV/s. With this, the continuum cross section of $\rho\pi$ production at $\psi(3770)$ $\sigma_{\text{Born}}^{\psi(3770) \rightarrow \rho\pi} = 4.4$ pb. Compare the two cross sections, the problem arises: how do these two interfere with each other?

If the phase between $a_{3g}$ and $a_\gamma$ is $-90^\circ$, then as in the case of $\psi(2S)$, the interference between $a_{3g}$ and $a_\gamma$ is destructive in $\rho\pi$ and $K^{*+}K^-$ modes, but constructive in $K^{*0}\overline{K}^0$ mode \cite{18}. The $e^+e^- \rightarrow \rho\pi$ cross section at $\psi(3770)$, as a function of $B_{\psi(3770) \rightarrow \rho\pi}$ for different $\phi$'s are shown in Fig. 3(a); while the $e^+e^- \rightarrow \rho\pi, K^{*+}K^-, K^{*0}\overline{K}^0$ cross sections as functions of $B_{\psi(3770) \rightarrow \rho\pi}$ for $\phi = -90^\circ$ are shown in Fig. 3(b).

To measure $\psi(3770) \rightarrow \rho\pi$ in $e^+e^-$ collision, we must scan the $\psi(3770)$ peak (as we measure $\Gamma_{ee}$, $\Gamma_{\text{total}}$ and $M_{\psi(3770)}$).

**Figure 3:** (a) $e^+e^- \rightarrow \rho\pi$ cross section as a function of $B_{\psi(3770) \rightarrow \rho\pi}$ for different phases, and (b) $e^+e^- \rightarrow K^{*0}\overline{K}^0 + \text{c.c.}, K^{*+}K^- + \text{c.c.}$, and $\rho\pi$ cross sections as functions of $B_{\psi(3770) \rightarrow \rho\pi}$.

Fig. 3(a) shows the $e^+e^- \rightarrow \rho\pi$ cross section vs C.M. energy for different $\phi$'s. Fig. 3(b) shows the $e^+e^- \rightarrow K^{*0}\overline{K}^0$ cross section with $\phi = -90^\circ$.

MARK-III gives $\sigma_{e^+e^- \rightarrow \rho\pi}(\sqrt{s} = M_{\psi(3770)}) < 6.3$ pb, at 90% C.L. \cite{19}. It favors $-90^\circ$. 


Figure 4: (a) $e^+e^- \rightarrow \rho\pi$ cross section vs C.M. energy for different phases: $\phi = -90^\circ$, $+90^\circ$, $0^\circ$, and $180^\circ$ respectively. (b) $e^+e^- \rightarrow K^{*0}\bar{K}^0$ cross section vs C.M. energy with $\phi = -90^\circ$.

$\psi(3770) \rightarrow 1^-0^-$ modes test the universal orthogonal phase between $a_{3g}$ and $a_\gamma$ in quarkonium decays as well as Rosner’s scenario. A small cross section of $e^+e^- \rightarrow \rho\pi$ at $\psi(3770)$ peak means $B(\psi(3770) \rightarrow \rho\pi) \approx 4 \times 10^{-4}$. (With radiative correction, the cancellation between $a_{3g}$ and $a_\gamma$ cannot be complete. With a practical cut on the $\rho\pi$ invariant mass, the cross section is a fraction of 1pb. ) It also implies the phase of the three gluon amplitude relative to one-photon decay amplitude is around $-90^\circ$. These will be tested by the 20pb$^{-1}$ of $\psi(3770)$ data by BES-II, or 5pb$^{-1}$ of $\psi(3770)$ data by CLEO-c.

6 Summary

- The universal orthogonality between $a_{3g}$ and $a_\gamma$ found in various decay modes of $J/\psi$ can be generalized to $\psi(2S)$ and $\psi(3770)$ decays. A $-90^\circ$ phase between $a_{3g}$ and $a_\gamma$ is consistent with the data on $\psi(2S) \rightarrow 1^-0^-$ and $0^-0^-$ modes.

- The $\psi(3770) \rightarrow \rho\pi, K^{*+}K^-, K^{*0}\bar{K}^0$ test the universal $-90^\circ$ phase, as well as Rosner’s scenario on $\rho\pi$ puzzle. This should be pursued by BES-II and CLEO-c.

- The existing $\Upsilon(nS)$ data should be used to test the phase in bottomonium states.

References

[1] M. Suzuki, Phys. Rev. D63, 054021 (2001).
[2] J. Jousset et al., Phys. Rev. D41, 1389 (1990); D. Coffman et al., Phys. Rev. D38, 2695 (1988); N N Achasov, Talk at Hadron2001.
[3] M. Suzuki, Phys. Rev. D60, 051501 (1999).
[4] L. Köpke and N. Wermes, Phys. Rep. 74, 67 (1989).
[5] R. Baldini, et al., Phys. Lett. B444, 111 (1998).
[6] J. M. Gérard and J. Weyers, Phys. Lett. B462, 324 (1999).
[7] S. Rudaz, Phys. Rev. D14, 298 (1976).
[8] P. Wang, C. Z. Yuan, X. H. Mo and D. H. Zhang, Phys. Lett. B593, 89 (2004).
[9] E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. 41, 466 (1985); G. Altarelli and G. Martinelli, CERN 86-02, 47 (1986); O. Nicrosini and L. Trentadue, Phys. Lett. B196, 551 (1987); F. A. Berends, G. Burgers and W. L. Neerven, Nucl. Phys. B297, 429 (1988); ibid. 304, 921 (1988).

[10] P. Wang, X. H. Mo and C. Z. Yuan, Phys. Lett. B557, 192 (2003).

[11] J.-M. Gérard and G. López Castro, Phys. Lett. B425, 365 (1998).

[12] M. Suzuki, Phys. Rev. D63, 054021 (2001).

[13] P. Wang, C. Z. Yuan and X. H. Mo, Phys. Rev. D69, 057502 (2004).

[14] C. Z. Yuan, P. Wang and X. H. Mo, Phys. Lett. B567, 73 (2003).

[15] X. H. Mo’s talk in this proceeding.

[16] J. L. Rosner, Phys. Rev. D64, 094002 (2001).

[17] H. E. Haber and J. Perrier, Phys. Rev. D32, 2961 (1985).

[18] P. Wang, C. Z. Yuan and X. H. Mo, Phys. Lett. B574, 41 (2003).

[19] Yanong Zhu, Ph. D. thesis California Institute of Technology, 1988, Caltech Report No. CALT-68-1513; W. A. Majid, Ph. D. thesis) University of Illinois, 1993, UMI-94-11071-mc.