Undistorted lensed images in galaxy clusters

L. L. R. Williams and G. F. Lewis
1Institute of Astronomy, Madingley Road, Cambridge CB3 0HA
2Astronomy Group, Department of Earth and Space Sciences, SUNY at Stony Brook, NY 11794-2100, USA

Accepted 1997 September 22. Received 1997 August 19; in original form 1997 April 17

ABSTRACT
To date, the study of high-magnification gravitational lensing effects of galaxy clusters has focused upon the grossly distorted, luminous arc-like features formed in massive, centrally condensed clusters. We investigate the formation of a different type of image, highly magnified yet undistorted, in two widely employed cluster mass density profiles, namely an isothermal sphere with a core, and a universal dark matter halo profile derived from the numerical simulations of Navarro et al. We examine the properties of images of extended sources produced by these two cluster profiles, paying particular attention to the undistorted images. Using simple assumptions about the source and lens population, we estimate the relative frequency of the occurrence of highly magnified, undistorted images and the more commonly known giant arcs.

Key words: galaxies: clusters: general - gravitational lensing.

1 INTRODUCTION AND MOTIVATION

Gravitational lensing by rich galaxy clusters produces highly magnified, grossly distorted images of background galaxies (Fort & Mellier 1994; Schindler et al. 1995; Lavery 1996). The mechanism of production of these ‘giant arcs’ is well understood; in fact, the observed arcs are commonly used to determine the mass distribution in the lensing clusters (Lavery & Henry 1988; Fort & Mellier 1994; Kneib et al. 1995). Despite their low surface brightness, the giant arcs are highly visible and easily detected because of their unique morphology.

However, it is possible that galaxy clusters are capable of producing highly magnified yet undistorted images of background sources. Whether such images are produced in reasonable numbers depends on the cluster mass profiles: steep density gradients produce thin arc-like images (Hammer 1991; Wu & Hammer 1993), whereas flat central cores can easily give rise to undistorted images. Because of their regular morphology these latter images would not be as prominent as arcs, and so would not be readily recognized and studied.

Undistorted magnified images are the subject of the present paper. Our aim is two-fold. First, we ask whether realistic cluster profiles can produce highly magnified undistorted images (HMUs) and, if so, under what conditions. We consider two types of clusters: an isothermal model with a core, and a universal dark matter cluster model derived from the numerical simulations of Navarro et al. (1995, 1996). The major difference between these two profiles occurs in their central regions: isothermal models have a flat density core, whereas those drawn from numerical simulations are characterized by a gradually flattening logarithmic slope, with decreasing radius, and are singular at the centre. We examine the image properties produced by these two profiles, paying particular attention to HMUs. Secondly, we use simple assumptions about cluster and source properties to estimate how common such HMUs might be. As a way to avoid some parameter dependencies, we do not calculate the actual frequency of occurrence of such images; instead, we estimate the ratio of the frequency of HMUs to giant arcs for the two different cluster mass profiles. In the present paper we calculate this ratio assuming circularly symmetric clusters, whereas real clusters exhibit substructure which can greatly increase the cross-sections of the clusters for generating arcs (Bartelmann, Steinmetz & Weiss 1995). However, our results can be readily converted to apply to substructural clusters, as outlined in Section 6.

Our results indicate that the statistics of HMUs are quite dependent on the overall form of the lensing potential,
suggesting a diagnostic tool of cluster cores. Such a probe of cluster centres is welcome, as current observational techniques have yet to resolve the matter distribution in the inner regions of clusters. Dynamical methods are generally inadequate in addressing the issue because of their reliance on assumptions (cluster is relaxed, mass follows light), as well as the small numbers of galaxy radial velocities (e.g. Sadat 1997). X-ray methods are more promising, because the emitting gas is in hydrostatic equilibrium with the cluster gravitational potential. X-ray-derived cores tend to be large; based on the whole sample of the EM S S clusters, H enry et al. (1992) estimate that the average cluster core radius is 125 Mpc. These results are in apparent contradiction with strong lensing analysis of clusters. Le Fèvre et al. (1994), on the basis of a sample of 16 brightest high-z EM S S clusters, conclude that arc statistics are incompatible with X-ray core radii, and much smaller cores are required. However, W axman & M iraida-E scudé (1995) and M iraida-E scudé & Babul (1995) pointed out that X-ray and lensing observations can be reconciled, and both are compatible with a singular dark matter potential like the universal dark matter profile, since multiphase cooling flow gas in this type of potential tends to be isothermal, and naturally produces cores. In any case, X-ray observations do not constrain cluster dark matter cores. That leaves us with lensing. Lensing is favoured above other techniques, because it directly probes the distribution of all mass, regardless of its baryonic/non-baryonic nature, dynamical and physical state. Strong lensing observations place a tight upper limit on cluster cores; however, it is possible that lensing clusters with giant arcs are a biased sample of clusters, because steeper-than-average density profiles are more likely to produce thin long arcs. Consequently, cores of ~50 Mpc are even larger, may still prove to be a common feature of clusters. In fact, there is tentative observational evidence for the existence of cluster cores: (i) Cl 0024+1654 is a spectacular cluster-lens with five HST-resolved images of a high-z galaxy. The presence of the central demagnified image E requires the cluster to have a finite core (Colley, Tyson & Turner 1996; Smail et al. 1996); (ii) an extended highly luminous z = 2.7 galaxy was recently discovered 6 arcsec from the centre of an intervening cluster (Y ee et al. 1996). Even though its true ‘lensing status’ is still uncertain, its properties can be explained if the lensing cluster had a core (Williams & Lewis 1996; but see Seitz et al. 1997). Thus the existence and sizes of flat cluster cores is a matter of debate.

If found, HMU images themselves would prove useful for the detailed study of galaxies at high redshift. M any highly distorted arcs have already been studied in order to deduce the properties of the high-z sources. For example, the star formation rate can be derived, because spectra of magnified sources can be obtained with a reasonable telescope integration time (E bbels et al. 1997). High-resolution HST observations of lensed galaxies are used to infer galaxy morphologies (Smail et al. 1996). However, the distorted appearance of arcs, coupled with the complicated nature of the lens model, implies that lensing inversions, which are needed in order to derive the kinematic and structural properties of the source galaxy, are difficult. Ideally, one would like to observe the (overall) simpler case of highly magnified, undistorted images.

2 CLUSTER MASS PROFILE MODELS

We consider two circularly symmetric cluster mass profiles: the isothermal sphere with a core (ISC), and the N avarro et al. (1996) universal dark matter profile (NFW).

The 2D projected mass density profile of ISC model is given by

\[ \Sigma(r) = \frac{\rho_c r_c}{(1 + r/r_c)^p}, \]

We adopt \( p = 0.5 \) to obtain an isothermal sphere at large radii, the ISC model. However, any value between 0 and 0.5 results in a realistic mass model; \( p = 0 \) represents a Plummer model (see Schneider, Ehlers & Falco 1992 for a detailed description of the profile). In equation (1), \( r \) is the core radius, and \( \Sigma_0 \) is the central surface mass density.

The 3D mass density profile of the NFW model is

\[ \rho(r) = \frac{\rho_c r_c}{r(r + c)^2}, \]

where \( r \) is the scale radius, and \( \rho_c \) is the mass density at \( r = 0.466 r_0 \). This model is derived from N-body simulations of large-scale structure, and is found to be applicable on scales of \( 3 \times 10^4 \lesssim M_{200}/M_\odot \lesssim 3 \times 10^{15} \), where \( M_{200} \) is the mass within a radius \( r_{200} \). Within this radius the mean over-density is a factor of 200.

We use normalized lengths, \( x = r/r_c \), and \( x = r/r_c \), respectively, and express the surface mass density in terms of the critical surface density for lensing:

\[ \kappa(x) = \frac{\Sigma(x)}{\Sigma_{crit}} = \frac{c^2}{4\pi G} \frac{D_{ls}}{D_{ls} D_{ol}}, \]

where \( D_{ls} \) are the relative angular diameter distances between the observer (o), lens (l) and source (s).

For the ISC model, the normalized surface mass density is

\[ \kappa(x) = \kappa_0 \left( \frac{1 + px^2}{(1 + x^2)^{2+p}} \right) \]

For the NFW model, the corresponding equation was calculated by Bartelmann (1996), who showed that the NFW model is compatible with the existence of radial arcs in clusters. We reproduce the relevant equations here for completeness:

\[ \kappa(x) = 2\kappa_0 \frac{f(x)}{x^2 - 1}, \quad \text{where} \quad \kappa_0 = \rho_c r_c / \Sigma_{crit}, \]

and

\[ f(x) = \begin{cases} \frac{1 - \frac{2}{\sqrt{x^2 - 1}} \arctan \frac{x - 1}{x + 1}}{\sqrt{x^2 - 1}} & (x > 1) \\ \frac{1 - \frac{2}{\sqrt{1 - x^2}} \arctanh \frac{1 - x}{1 + x}}{\sqrt{1 - x^2}} & (x < 1) \\ 0 & (x = 1) \end{cases} \]

For a circularly symmetric lens, the lens equation, which
is the relation between the source position $y$, the image position $x$, and the deflection angle, is given by

$$y = \frac{\int_x^y 2x' x(x') \, dx'}{x} = \frac{m(x)}{x},$$  

(7)

where $m(x)$ is the dimensionless mass interior to $x$, in units of $(c^2/4G)(D_L D_s D_o/D_s)$. A lensed image is distorted both radially and tangentially with respect to the centre of the lensing potential. The tangential distortion is the ratio of the tangential size of the image to that of the source; for a sufficiently small source it is given by the ratio of their respective distances from the lens centre, $x/y$. Similarly, the radial distortion is $d_x/d_y$, if the radial extent of the source is $d_y$. Since magnification is just the ratio of the size of the image to that of the source, it is given by

$$\mu = \frac{dx}{dy}.$$  

(8)

The length-to-width ratio of an image whose source is circular is

$$L/W = \frac{dy}{dx},$$  

(9)

and is the most commonly used measure of image distortion. Note that an isothermal density profile, $\kappa(x)\propto x^{-1}$, implies that $dy/dx = 1$ (equation 7), i.e. images suffer no radial (de)magnification. Steeper profiles always result in radially demagnified images, while shallower profiles produce radially magnified images.

Using equations (4)–(9), we can calculate all the image properties needed in this paper.

### 3 Image Properties

#### 3.1 ISC model

The top panel of Fig. 1 shows the relation between the image and source positions, for two values of $\kappa_0$, of an ISC model. Supercritical clusters, represented here by the $\kappa_0 = 1.1$ case (solid lines), produce three images if the source impact parameter is smaller than the radial caustic, $y_0$ (see equation 8.42 of Schneider et al. 1992). Subcritical clusters, such as the $\kappa_0 = 0.9$ case (dashed line) always produce one image, and the $x$–$y$ relation is one-to-one and monotonically increasing.

Fig. 2 shows the relation between image magnification and distortion. Two supercritical cases are shown: solid lines are the three images of a $\kappa_0 = 3.0$ lens, and dot–dashed lines are for a $\kappa_0 = 1.1$ lens. The long-dashed line is the single image of a critical lens, $\kappa_0 = 1.0$, while the short-dashed line is the single image of a $\kappa_0 = 0.9$ lens. Each lens with $\kappa_0 > 1$ has three branches corresponding to three images. The primary image, which is formed at the minimum of the lensing potential (see Schneider 1985 and Blandford & Narayan 1986), is labelled I. This image appears on the same side of the lens centre as the unlensed source, and is tangentially extended into an arc, with $L/W > 1$. It is always magnified with respect to the source. The image formed at the

![Undistorted lensed images in galaxy clusters](https://academic.oup.com/mnras/article-abstract/294/2/299/1101399/fig1)
saddle-point of the lensing potential, sometimes called the ‘counter arc’, possesses reversed parity and is labelled II. The central image, labelled III, is formed at the maximum of the lensing potential, and is radially extended, i.e. has L/W < 1. Images II and III are on the side of the lens opposite to the location of the source. Note that for very centrally condensed lenses, i.e. those with \( k_0 > 2 \), image III can be demagnified, \( \mu < 1 \), if the source is sufficiently close to the lens centre.

The ‘hidden’ parameter in this plot is the source position, \( y \). To illustrate its influence on the location of the images, we have plotted the three images of a source at \( y = 0.010 \) (open circles), and the three images of a source at \( y = 0.016 \) (filled circles), for a \( k_0 = 1.1 \) lens. As the source approaches the lens centre in projection, images I and II become very elongated and tend to merge along the tangential critical line, while image III moves close towards the lens centre.

Notice that the single image branch of a \( k_0 < 1 \) lens joins to and continues as the branch of the central image, labelled III, of the corresponding \( 2 - k_0 \) lens. As an example, the image of a \( k_0 = 0.9 \) lens (short-dashed line) continues as the central image branch of a \( k_0 = 1.1 \) lens. This can be shown as follows. For an image position \( x \) very close to the lens centre, the image magnification, \( \mu \approx (1 - k_0)^{-1} \), has the same numerical value for a \( k_0 \) lens as for a \( 2 - k_0 \) lens. The distortion L/W of the image located at the centre is \( 1 \), from symmetry. Therefore, for small \( x \), the branches of central images of a \( k_0 \) lens and a \( 2 - k_0 \) lens meet at the same point in the log(L/W) versus log(\( \mu \)) diagram. The slope of both the branches at this point can be shown to be d log(L/W)/d log(\( \mu \)) = \( -\frac{1}{2} \), independent of \( k_0 \) and \( p \); these two branches are therefore continuous.

The most visible feature in a lensing cluster is the primary arc, since it is always well displaced from the cluster centre, highly elongated, and always magnified. For this image, the magnification is an increasing function of distortion. This is why high magnification is associated with high distortion of lensed images in galaxy clusters. However, high magnification need not always imply high distortion.

It is apparent from Fig. 2 that subcritical clusters, represented here by a \( k_0 = 0.9 \) case, can produce highly magnified, undistorted images.\(^1\) For example, if the source is located close to the centre of a \( k_0 = 0.9 \) lens, its magnification is \( \sim 100 \), while its distortion is negligible. In fact, the largest L/W ratio attained by an image of any subcritical lens is not greater than L/W = 3, as we now show.

The length-to-width ratio of any image in the ISC cluster model can be derived from equations (9), (7) and (4):

\[
\text{L/W} = \frac{1 - k_0(1 + x^2)^{p - 1/2}[1 + x^2(2p - 3)]}{1 - k_0(1 + x^2)^{p-1}}.
\]

(10)

Comparing this expression for \( 0 < k_0 < 1 \) to the corresponding expression for \( k_0 = 1 \), one can show that the latter is always greater or equal to the former, for any values of \( x \) and \( p \). Therefore the maximum value of the image distortion in subcritical lenses of equation (1) can be obtained from equation (10) with \( k_0 = 1 \) and small \( x \). This maximum value, L/W = 3, is independent of \( \mu \) and \( p \). Thus the single image of any subcritical lens is rather undistorted, regardless of its impact parameter and magnification.

From Fig. 2 it appears that supercritical clusters with \( k_0 \leq 2 \) can produce undistorted images with \( \mu \geq 10 \). We will now show that in a realistic supercritical cluster the probability of detecting undistorted images is negligible. To address this, we need to reintroduce the projected source position, \( y \). The bottom panel of Fig. 1 shows the distortion and image position as a function of source position for images of a \( k_0 = 1.1 \) lens (solid lines). It may seem that images II and III are undistorted. Image III tends to fall very close to the centre of the cluster, as is seen in the upper panel. Since a core radius, i.e. \( x = 1 \), typically corresponds to \( \sim 50 \) kpc, an image at \( x \leq 0.1 \) will probably be hidden within the central cluster galaxy. That leaves us with a small range of impact parameters in the range log(\( y \)) \(- 2 \) to \(- 1.8 \) which could produce detectable undistorted magnified images of type II. For typical cluster parameters, that range translates to roughly 0.1 arcsec and is thus smaller than a typical source, i.e. a galaxy several kpc across at \( z \sim 0.2 \) – 2. Additionally, as the L/W ratio changes very rapidly at these \( y \)s, an extended image will appear distorted.

### 3.2 NFW model

The NFW lens model is singular at the centre for all values of \( k_0 \) (equations 5 and 6), and so is formally supercritical. This profile will always produce three images if the source impact parameter is sufficiently small.

Fig. 3 shows the relation between the image position, \( x \), and source position, \( y \). Note that the values of \( k_0 \) were picked such that the total mass within 2.5 Mpc of these clusters is the same as that of ISC clusters with \( k_0 = 1.1 \) and \( L/W = 3 \).
0.5 (see Section 4.1). The images are labelled as in Figs 1 and 2, and have similar properties to those described in Section 3.1. The major difference when compared to the ISC model is that there is no NFW case that would correspond to a subcritical, i.e. single-image-only, ISC cluster. In fact, relations between image properties, y versus x (Fig. 3), and L/W versus μ (Fig. 4), are qualitatively the same regardless of the value of κ. In particular, the distortion of the primary images of all NFW clusters is an increasing function of magnification; therefore giant arcs would be a common type of image in these clusters.

Clusters with κ ≥ 0.3 cannot have magnified undistorted images at all (Fig. 4), but those with smaller values of κ have parts of all three image branches in the HMU region. Since we are interested in HMUs, let us derive equations describing image properties for small κ cases. We will take advantage of the fact that for small κ, models the triple image region moves towards small x; see top panel of Fig. 3. Equation (1.10) of Bartelmann (1996) gives function g(x), which is proportional to the mass interior to x; m(x) = 4kσs g(x). For small x, this simplifies to

$$g(x) \approx \frac{x^2}{2} \left[ 2 \ln \left( \frac{x}{2} \right) + 1 \right],$$

and the lens equation (7) becomes

$$y \approx x \left[ 1 + 2\kappa_1 \ln \left( \frac{x}{2} \right) + \kappa_1 \right].$$

These demonstrate that a line of constant L/W in Fig. 4 intersects the primary image branches of various κ models at magnifications such that μ ∝ κs. Therefore, as one goes to less massive clusters, the magnification of primary images of a given fixed distortion increases. Highly magnified undistorted images of infinitesimally small sources are therefore possible with low-mass NFW clusters.

To determine if these images will stay undistorted for sources of finite size, one needs to look at how L/W changes with y, i.e., the bottom panel of Fig. 3. For κ = 0.109, images of types II and III arise only for sources at y ≤ 0.001. For a typical r, of 300 h⁻¹ kpc, this corresponds to ~ 0.1 arcsec, which is smaller than the expected source size. Therefore images of types II and III would not produce HMUs.

4 Model Assumptions

In the last section we have shown that both ISC and NFW models can produce HMUs under certain conditions. Next, we need to estimate the relative frequency of occurrence of HMUs with the two cluster models. Assuming that the cluster and the sources are at fixed redshifts, this involves two steps: first, for a given cluster one has to integrate over source impact parameters, taking into account finite source size; secondly, one must integrate over the distribution of cluster properties.

4.1 Galaxy Clusters

We assume that galaxy clusters form a one-parameter family, with the core radius r and scale radius r of NFW being fixed, but possessing a range of masses. The cluster mass function, dN(M)/dM, is derived from two observed relations of X-ray-selected clusters. Based on a sample of E M55 clusters, Henry et al. (1992) derived the cluster luminosity function to be dN/dL ∝ L⁻α, where α varies with redshift, but is approximately 0.3 between the redshifts of 0.15 and 0.6. A relation between cluster bolometric luminosity, derived from EXOSAT data, and velocity dispersion is given by Edge & Stewart (1991): L ∝ σ⁻α. Combining
these two relations with the assumption that $M \propto \sigma^2$, we arrive at the cluster mass function,
\[
\frac{dn(M)}{dM} \propto M^{-3.9}.
\]
(16)

The absolute normalization is irrelevant, since we will only be dealing with ratios. We need not assume upper or lower cluster mass limits: the lower mass cut-off is effectively achieved because low-mass clusters have a negligible lensing cross-section, while at the upper mass end the lensing cross-section increases more slowly than the rate at which the numbers of clusters decrease as a result of the steep slope of their mass function.

We fix the core radius $r_c$ at 50 h\(^{-1}\) kpc, which is substantially smaller than the derived X-ray 'core' sizes, but is consistent with lensing observations (Fort & Mellier 1994). For the NFW clusters, we fix $r_c$ at 300 h\(^{-1}\) kpc, which is close to a typical value obtained in the Navarro et al. (1996) simulations.

With the physical length-scale of the two cluster models fixed, we can now derive the correspondence between the one-parameter ISC and NFW models, i.e. we ask what is the relation between $\kappa_\text{c}$ and $\kappa_\text{f}$ of clusters that have the mass within 2.5 h\(^{-1}\) Mpc. The latter is roughly equal to $r_{200}$, and corresponds to 50 h\(^{-1}\) kpc. The numbers of giant arcs and HMUs per cluster, weighted by the cluster mass function. The numbers plotted along the vertical axis are proportional to $n_i \times dN(M)/dM$, where $n_i$ is given by equation (18), and $i = \text{HMU} \text{ or } \text{arcs}$.

4.3 Image selection criteria

Images are selected on the basis of their size or morphology. To be selected as either a giant arc or an HMU, an image must have an undistorted nature. For an arc to be detected, its L/W ratio across the image should not exceed 50 per cent.

4.2 Sources

We assume that the unlensed parent population of HMUs is the same as that of arcs in clusters. The half-light radii of the sources of arc images are almost the same as the observed widths of the arcs, because the cluster potential is not expected to distort tangential arc images in the radial direction. Smail et al. (1996) measure half-light radii for a sample of eight HST arcs (see their fig. 5). The average half-light radius is about 0.5 arcsec, which is what we will adopt in the present paper. We further assume that all the sources are circular with a uniform surface brightness.

At any given redshift the luminosity function (LF) of sources is assumed to be a power law, with the slope corresponding to that of the Schechter LF, $a$. The value of $a$ is estimated to be about 1.1 locally, but it may have been steeper in the past, $a \approx 1.5$ (Ellis et al. 1996). We use both values below, to account for the possible range of $a$, depending on the type of galaxies and their evolution. The results are not very sensitive to $a$. We neglect the sources brighter than $L_*$, the characteristic luminosity of Schechter LF, because of their small numbers.

We need not make any further assumptions about the source LF, as we explain below. The number of images of type $i$, where $i = \text{HMU} \text{ or } \text{arcs}$, for a given cluster characterized by $\kappa_\text{c}$ or $\kappa_\text{f}$ (or cluster mass $M$), is given by
\[
n_i \propto \int y \, dL \, \frac{dL}{L_n} \propto L_n^{3-a},
\]
(17)

where $H_i$ is the Heaviside step function, which is 1 if the image selection criteria are satisfied (Section 4.3), and 0 otherwise; $L_{\text{lim}}$ is the faintest observable luminosity. The outer integral is over the source impact parameter, $y$, in the source plane. The integral can be written as
\[
n_i \propto \int y \left( \frac{L_{\text{lim}}}{L_n} \right)^{3-a} \, dy \times H_i.
\]
(18)

The function $\mu(y)$ is determined by the ISC or NFW model. The last step in the above equation is justified, because the minimum magnification $\mu$ required for a detectable image is large (Section 4.3) and $\mu$ is always $> 1$. Both $L_{\text{lim}}$ and $L_n$ are functions of source redshift, but since we are only interested in the ratio of $n_\text{HMU}$ to $n_\text{arcs}$, the dependency on these quantities cancels out.

As the results are quite insensitive to $z$, we make no assumptions about the source redshift distribution.

5 Results

Fig. 5 shows the numbers of giant arcs and HMUs per cluster, weighted by the cluster mass function. The numbers plotted along the vertical axis are proportional to $n_i \times dN(M)/dM$, where $n_i$ is given by equation (18), and $i = \text{HMU}$ or $\text{arcs}$.

Figure 5. Number of HMU and arc images convolved with the cluster mass function, as a function of cluster mass. The vertical axis has arbitrary normalization. Solid lines are for the ISC model, while dashed lines are for the NFW model. Source and lens redshifts were fixed at $z_s = 0.3$ and $z_l = 1.0$; the Schechter LF slope was assumed to be $a = 1.5$. 

© 1998 RAS, MNRAS 294, 299–306

Downloaded from https://academic.oup.com/mnras/article-abstract/294/2/299/1101399 by guest on 24 March 2020
Table 1. The ratio of the number of HMUs images to arcs in the ISC model, i.e., \( f_{\text{ISC}} \), for a range of source, \( z_s \), and lens, \( z_l \), redshifts. Two values of the Schechter LF slope, \( \alpha \), were tried: 1.5 and 1.1 (in parentheses).

| \( z_l \) | \( z_s \) | \( f_{\text{ISC}} \) |
|---|---|---|
| 0.2 | 0.5 | 1.28 (1.53) |
| 0.3 | 1.0 | 1.22 (1.46) |
| 0.4 | 1.5 | 1.21 (1.44) |

Table 2. Same as Table 1, but for the NFW model.

| \( z_l \) | \( z_s \) | \( f_{\text{NFW}} \) |
|---|---|---|
| 0.2 | 0.5 | 0.35 (0.47) |
| 0.3 | 1.0 | 0.34 (0.44) |
| 0.4 | 1.5 | 0.33 (0.44) |

\( \frac{dn(M)/dM}{dn(M)/dM} \) by equation (16). The solid and dashed lines represent ISC and NFW models, respectively, and arcs and HMUs are labelled. Here we assumed \( z_s = 0.3, z_l = 1.0, \) and \( \alpha = 1.5 \). Notice that for both mass profiles, HMU images are produced by low-mass clusters only, as was explained in Sections 3.1 and 3.2. The relative numbers of HMUs and arc images produced by ISC and NFW profiles can be understood intuitively. The shapes of images depend on the mass density gradient at the location of the images (Section 2). The core region of ISC is flat, and so it produces roughly equal radial and tangential magnifications, and is therefore ideal for generating HMUs. The projected density of the NFW profile, on the other hand, goes as \( \ln(1+x^2) \) at small \( x \), and so does not have a flat core. It is also steeper than isothermal at large radii. Overall, the NFW profile is steeper than ISC, and thus is better than ISC at making giant arcs, and worse than ISC at making HMUs.

We are interested in the ratio of HMUs to giant arcs for a given cluster model, i.e.,

\[
\frac{f_{\text{model}}}{f_{\text{arc}}} = \frac{\int n_{\text{arc}}(M) \cdot dn(M)/dM \cdot dM}{\int n_{\text{arc}}(M) \cdot dn(M)/dM \cdot dM}
\]

where the model is either ISC or NFW. This is just the ratio of the areas under the HMU and arc curves in Fig. 5. For the parameters of the figure, \( f_{\text{arc}} = 1.16 \), and \( f_{\text{NFW}} = 0.23 \).

Table 1 shows how the \( f_{\text{arc}} \) ratio changes with source and lens redshift. The numbers (in parentheses) are for the Schechter LF slope of 1.5 (1.1). Table 2 contains the corresponding values for the NFW model. It is seen from these two tables that the predictions are virtually independent of source redshift, but depend on lens redshift. The source and lens redshifts come in only in \( \Sigma_{\text{crit}} \) (equation 3), and both of these trends arise because of the way \( \Sigma_{\text{crit}} \), which determines the lensing strength of the cluster, varies with \( z_l \) and \( z_s \). When \( z_s \) is held constant, \( \Sigma_{\text{crit}} \) attains its asymptotic value at redshifts just beyond \( z_s \), and thus the source redshift has very little effect on \( f \). On the other hand, when \( z_l \) is constant, \( \Sigma_{\text{crit}} \) changes quite rapidly at low-to-moderate lens redshifts. The sense of the trend is also understood in terms of \( \Sigma_{\text{crit}} \): when the lensing strength of a cluster of a certain mass increases as a result of the decrease in \( \Sigma_{\text{crit}} \), more arcs are produced, and hence \( f \) goes down. The weak dependency on \( z_s \) is a useful feature, because source redshift distribution is arguably the most uncertain of the relevant model parameters.

The \( f \) ratios depend somewhat on the source LF slope, \( \alpha \). The dependency is not very strong, because most of the images are magnified just by the minimum required \( \mu \) (because the cluster cross-section declines rapidly with \( \mu \)), and so both HMUs and arcs are drawn from a rather narrow portion of the LF. The decline in cluster cross-section with \( \mu \) is more severe for HMUs because of the additional selection restrictions placed on undistorted images. Therefore, when \( \alpha \) is steeper and there are more faint sources, the number of HMUs does not increase as much as the number of arcs; hence \( f \) is smaller for steeper LFs.

Dependence on cosmology is weak. Tables 1 and 2 assume flat-universe model, dominated by a cosmological constant; \( \Omega = 0.2, \lambda = 0.8 \). A adopting a matter-dominated, flat universe, \( \Omega = 1.0 \), produces at most a 40 per cent increase in \( f \). Since \( f \) is the ratio of the numbers of images, it does not depend on the Hubble constant.

Note that the predictions for relative numbers of HMUs to giant arcs are a function of chosen image selection criteria. For example, if minimum HMU magnification is increased from 10 to 30, then \( f_{\text{arc}} \) drops by about a factor of 10 compared to the tabulated values, but virtually no HMUs are expected with the universal dark matter profile.

6 CONCLUSIONS

In this paper we considered two cluster mass density profiles: an isothermal sphere with a core, and a universal dark matter profile. We studied the properties of gravitationally lensed images of extended sources produced by these models. We were particularly interested in highly magnified, undistorted images, and we have shown that both profiles can, in principle, produce such images; however, when realistic cluster properties and mass functions were folded in, ISC models proved to be much more efficient than NFW at making HMUs images. Using simplified assumptions, we calculated \( f \), the ratio of HMUs to giant arcs for the two models, as a function of source and lens parameters. To account for cluster asymmetries and substructure, which were not considered in this paper, the derived \( f \) ratios should be multiplied by the ratio of the frequency of arcs in substructured clusters to that in smooth symmetric clusters. The latter ratio can be obtained from numerical simulations of the type described in Bartelmann et al. (1995). We have shown that \( f \) is not very sensitive to cosmology, source luminosity function and redshift distribution, and lens redshift distribution.

In fact, the strongest dependency by far is on the cluster model: with the ISC model HMUs are on average 1.2 times as abundant as giant arcs, whereas with the NFW model HMUs are only 0.2 times as common as arcs (see Tables 1 and 2). Thus relative frequency of highly magnified undistorted images to giant arcs can be used to discriminate between isothermal clusters with cores and universal dark matter halo.
profiles, if a complete sample of clusters is examined for HMUs/arc images. Two points need to be stressed here. First, since HMUs are produced by clusters at the lower end of the mass function, $M(< 2.5 \times 10^{15} M_\odot) \leq 10^{15} M_\odot$, a cluster sample should extend down to such masses, or corresponding X-ray luminosities; alternatively, model predictions should make an allowance for a high-mass cluster cut-off. Secondly, HMUs would not be as easily detected as giant arcs, since they would have regular morphology. If they are at high redshift, their surface brightness will be correspondingly low. However, they should be appreciably larger than typical high-z galaxies, with half-light radii of about 1.5–2 arcsec. To find HMUs, one would need to do spectroscopy on extended, low-surface-brightness galaxies with regular morphology, located within 10–20 arcsec of cluster centres, in clusters with no giant arcs. Extensive searches with such observational constraints have not been undertaken; it is therefore not surprising that highly magnified undistorted images, whether their number density on the sky is comparable to or much smaller than that of giant arcs, have so far escaped detection.

ACKNOWLEDGMENTS

We thank Alastair Edge, Richard Ellis and Mike Irwin for helpful suggestions regarding an early version of the paper. LLRW acknowledges the support of PPARC Fellowship at the Institute of Astronomy, Cambridge.

REFERENCES

Bartelmann M., 1996, A&A, 313, 697
Bartelmann M., Steinmetz M., Weiss A., 1995, A&A, 297, 1
Blandford R. D., Narayan R., 1986, ApJ, 310, 568
Colley W. N., Tyson J. A., Turner E. L., 1996, ApJ, 461, L83
Ebbels T. M. D., Le Borgne J.-F., Pello R., Ellis R. S., Kneib J.-P., Smail I., Sanahuja B., 1996, MNRAS, 281, L75
Edge A. C., Stewart G. C., 1991, A&AR, 5, 239
Fort B., Mellier Y., 1994, A&AR, 5, 239
Hammer F., 1991, ApJ, 383, 66
Henry J. P., Gioia I. M., McCaughrean M., Morris S. L., Stocke J. T., Wolter A., 1992, ApJ, 386, 409
Kneib J.-P., Mellier Y., Pello R., Miralda-Escude J., Le Borgne J.-F., Buhlinger H., Picat J.-P., 1995, A&A, 303, 27
Lavery R. J., 1996, AJ, 112, 1812
Lavery R. J., Henry J. P., 1988, ApJ, 239, L21
LeBorgne J.-F., Hamner F., Angonin M. C., Gioia I. M., Luppino G. A., 1994, ApJ, 422, L5
Miralda-Escude J., Babul A., 1995, ApJ, 449, 18
Navarro J. F., Frenk C. S., White S. D. M., 1995, MNRAS, 275, 720
Navarro J. F., Frenk C. S., White S. D. M., 1996, ApJ, 462, 563
Sadat R., 1997, preprint, also available as astro-ph/9702050
Schindler S. et al., 1995, A&A, 299, L9
Schneider P., 1985, A&A, 143, 413
Schneider P., Ehlers J., Falco E. E., 1992, Gravitational Lenses. Springer-Verlag Press, Berlin
Seitz S., Saglia R. P., Bender R., Hopp U., Belloni P., Ziegler B., 1997, preprint, also available as astro-ph/9706023
Smail I., Dressler A., Kneib J.-P., Ellis R. S., Couch W. J., Sharples R. M., Oemler A., 1996, ApJ, 469, 508
Waxman E., Miralda-Escudé J., 1995, ApJ, 451, 451
Williams L. L. R., Lewis G. F., 1996, MNRAS, 281, L35
Wu X.-P., Hammer F., 1993, MNRAS, 262, 187
Yee H. K. C., Ellington E., Bechtold J., Carlberg R. G., Cuillandre J. C., 1996, AJ, 111, 1783