Study on the Matching of Toe-in and Camber of the Double-Front-Axle Steering Automobile

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Abstract. Toe-in and camber of the wheel are important driving performance parameters in the four-wheel positioning parameters of automobile. In this paper, the mathematical geometrical models of toe-in and camber of the first and second bridges of the double-front-axle automobile are established, concluded that the relationship of toe-in and camber between the first bridge and the second bridge. At the same time, based on the side slip mechanism of the tire, the reasonable matching formula for toe-in and camber of the double-front-axle are concluded. Finally, according to the dynamic model established in the Adams/Car, the matching values of toe-in and camber are simulated and verified. The experimental results show that the matching value is reasonable and effective, for the matching of toe-in and camber of double-front-axle steering wheel has a certain guiding role.

1. Introduction
Toe-in and camber of wheels are important front-wheel positioning parameters to the front wheel of automobile. Reasonable matching between toe-in and camber of wheels can reduce the wear condition of tire [9] and improve the handling stability of automobile. At present, there is a lot of literature [1-5] about the matching of toe-in and camber of single-axle steering automobile. Considering the aspect of mechanics in the literature [1, 2], the matching relationship between toe-in and camber is derived by cancelling out the force which generated by toe-in and camber of the tire. In the literature [3, 5], according to the motion trail of wheels, the matching relationship between toe-in and camber can also be deduced. But none of these literatures mentioned above refer to the matching of toe-in and camber of steering wheels in the double-front-axle automobile. In addition, there are few studies on toe-in and camber of the double-front-axle steering wheels at present.

In order to achieve this goal, first, the geometrical relationship between toe-in and camber of the first and second bridges can be derived by establishing the mathematical geometry of toe-in and camber of the double-front-axle steering wheels. Then, the matching relationship between toe-in and camber of double-front-axle steering wheels can also be deduced according to the side slip mechanism of the tire. Finally, the result of simulation experiments can be verified by the dynamic model.

2. Matching study of toe-in and camber of double-front-axle

2.1. The relationship of camber between first and second bridges of double-front-axle
On account of the camber angle of wheels, wheels of automobile will always have the tendency to roll outward during driving. Taking the first bridge camber as an example, the geometry model of rolling wheel camber can be established, as shown in figure 1 [3, 5]:

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[Note: The figure 1 is not provided in the text.]

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Figure 1. Rolling schematic diagram of the first-bridge camber.

Where, $R_1$ means the outer rolling radius of the first bridge which are caused by the camber of the tire, $r_1$ represents the rolling radius under load, and $\gamma_1$ indicates the camber of the first bridge of steering wheels.

Therefore, according to the geometric relationship graph of the first bridge, we can get the following relation:

In $\triangle ABC$, $R_1 = r_1 / \sin \gamma_1$. Due to the small camber angle, $\sin \gamma_1 \approx \gamma_1$, the rolling radius of the first bridge can be expressed as

$$R_1 = r_1 / \gamma_1$$ (1)

In the same way, the rolling radius of the second bridge can be obtained:

$$R_2 = r_2 / \gamma_2$$ (2)

Where

$$r_1 = D / 2 - G_1 / 6K_z$$ (3)

$$r_2 = D / 2 - G_2 / 6K_z$$ [6] (4)

$D$ is typical of the diameter of wheels under no load, $G_1$ and $G_2$ represent the vertical load above the first and second bridges, and $K_z$ is the radial stiffness of the tire.

During steering process of the automobile, the camber angle will change with the change of the wheel angle, but in the process of this change, we can get such a regularity, that is, the turning radius of each steering wheel ($\rho_1$, $\rho_2$) and the rolling cone radius caused by the camber of wheels ($R_1$, $R_2$) are as close as possible.

Figure 2. Steering schematic diagram of the double-front-axle.
Where $\delta_1$, $\delta_2$ are the angles of the first and second bridges of steering wheels respectively, $\rho_1$, $\rho_2$ indicate the turning radius of the first and second bridges of steering wheels. According to the geometric model of figure 2, we can obtain the following geometric relationship:

$$\rho_1 = \frac{L}{\sin \delta_1}, \quad \rho_2 = \frac{L_2}{\sin \delta_2};$$

When the conditions $R_1 = \rho_1$, $R_2 = \rho_2$ have been met, the minimum wear of the tire can be guaranteed, we can get the following relationship at present:

$$\gamma_1 = r_1 \sin \delta_1 / L$$

$$\gamma_2 = r_2 \sin \delta_2 / L_2$$

When steering angles of automobile is small, that is, the condition $\cos \delta_1 / \cos \delta_2 \approx \delta_1 / \delta_2$ can be satisfied. So it can be approximated that the automobile is in straight-line driving state, and the camber angles of the first and second bridges are both initial camber angles.

According to (5) and (6), the relationship between the first bridge and the second bridge of camber can be obtained:

$$\gamma_1 / \gamma_2 = (L_2 / L) \times (\sin \delta_1 / \sin \delta_2) \times (r_1 / r_2)$$

$$= (L_2 / L) \times (\tan \delta_1 / \tan \delta_2) \times (\cos \delta_1 / \cos \delta_2) \times (r_1 / r_2)$$

$$= (L_2 / L) \times (L / L_2) \times (\cos \delta_1 / \cos \delta_2) \times (r_1 / r_2)$$

$$\approx (r_1 / r_2)$$

2.2. The relationship of toe-in between first and second bridges of double-front-axle

Compared with the single-bridge automobile, the double-front-axle automobile has two more steering wheels, so the rolling radius arose from toe-in of the double-front-axle automobile is slightly different from that of the single-bridge automobile. So it can be approximated that the toe-in angle is equivalent to a small corner of the tire. In order to ensure the tire wears minimally during the driving process, the steering wheel should roll around a same point [10].

![Figure 3. Toe-in rolling schematic diagram of the double-front-axle](image)

Where $\phi_1$, $\phi_2$ indicate toe-in angle of the first and second bridges, $R'_1, R'_2$ represent the rolling radius of the first and second bridges which caused by the toe-in. $\phi_1$, $\phi_2$ are very small among them, so it can be approximated that $\sin \phi_1 \approx \phi_1$, $\sin \phi_2 \approx \phi_2$.

According to the aforementioned toe-in rolling schematic diagram shown in figure 3:
In $\triangle OBC$:

$$R'_1 \times \sin \varphi_1 \approx R'_1 \times \varphi_1 = L = AC$$  \hspace{1cm} (8)

In $\triangle OAC$:

$$R'_2 \times \sin \varphi_2 \approx R'_2 \times \varphi_2 = L_2$$  \hspace{1cm} (9)

We can conclude that:

$$AC = L_1 + BC$$  \hspace{1cm} (10)

by observing the geometric relationship of the graph above.

According to (8), (9), (10) the relationship of toe-in of double-front-axle can be obtained as follow:

$$R'_1 \times \varphi_1 = R'_2 \times \varphi_2 + L_1$$  \hspace{1cm} (11)

In the above-mentioned formula, $\varphi_1, \varphi_2$ can be expressed as

$$\varphi_1 = \frac{T_1}{d}$$  \hspace{1cm} (12)

$$\varphi_2 = \frac{T_2}{d}$$  \hspace{1cm} (13)

Where $T_1, T_2$ represent toe-in of the first and second bridges of steering wheels respectively, and $d$ is the diameter of hub.

In the relation (11), the rolling radius of toe-in of the first and second bridges varies with the change of the toe-in angle of the first and second bridges of steering wheels. The toe-in value of the first and second bridges will be determined there-in-after by matching toe-in with that camber of the first and second bridges.

2.3. Study on the matching relationship between toe-in and camber of the first and second bridges of double-front-axle

In the following, based on the side slip mechanism of the tire, considered the minimum slip of the tire in the double-front-axle automobile during driving, with a view to the parameters of mechanism, combined with the relationship of toe-in and camber of the first and second bridges of double-front-axle above. The reasonable matching formula of toe-in and camber of the first and second bridges of double-front-axle has been determined.

Considering the toe-in and camber synthetically, we can take the first-bridge steering wheels of double-front-axle automobile as an example, the motion curve [3, 5] of tire shown in the following figure is established:

![Figure 4. Rolling schematic diagram of the first-bridge camber and toe-in.](image-url)
In figure 4, $\overline{AB}$ and $\overline{AE}$ indicate the grounding mark caused by toe-in and camber of wheels during $\Delta t$ (time) respectively. $\varphi_1$ represents the toe-in angle of the first-bridge steering wheels, $\phi_1, \theta_1$ respectively express the rolling angle of wheel during $\Delta t$ (time) under the action of toe-in and camber. $R, R' \, \forall$ represent the rolling radius of toe-in and camber respectively.

In order to ensure the minimum side slip of tires during driving, $EF=BC$ must be guaranteed. 

In $\triangle AEF$:

$$EF = AE \times \sin\left(\frac{\varphi_1}{2} - \varphi_1\right),$$

In $\triangle ABC$:

$$BC = AB \times \sin\left(\frac{\phi_1 + \varphi_1}{2}\right);$$

Available from $EF=BC$,

$$\theta_1 = \phi_1 + 4\varphi_1 \quad (14)$$

can be obtained.

At the same time, when the rolling radius is large enough, the following formulas are satisfied:

$$\overline{AB} = AB = \overline{AE} = AE = l_1,$$

where $l_1$ indicates the actual grounding mark of the first bridge [7].

Also, $\overline{AB} = R'_1 \times \phi_1 = l_1 \quad (15)$

$$\overline{AE} = R_1 \times \theta_1 = l_1 \quad (16)$$

From the formulas (1), (3), (12), (14), (15), (16), the matching formula of toe-in and camber of the first bridge can be obtained:

$$T_1 = \frac{12KzL_1\gamma_1d}{(4L + l_1)(3DK_1 - G_1)}$$

In the same way, we can also get the matching formula of the second bridge:

$$T_2 = \frac{12KzL_2\gamma_2d}{(4L + l_2)(3DK_2 - G_2)}$$

Combined with the relationship between toe-in and camber of the first and second bridges of the double-front-axle, we can get the matching relationship between the toe-in and the camber of the double-front-axle:

$$T_1 = \frac{12KzL_1\gamma_1d}{(4L + l_1)(3DK_1 - G_1)}, \quad T_2 = \frac{12KzL_2\gamma_2d}{(4L + l_2)(3DK_2 - G_2)},$$

$$\gamma_1 \approx \gamma_2 \approx r_1 / r_2$$

Where, $l_1, l_2$ represent the grounding mark of the first and second bridges of the tires respectively,

$$l_1 = 2\sqrt{(D - \Delta_1)\Delta_1},$$

$$l_2 = 2\sqrt{(D - \Delta_2)\Delta_2},$$

$$\Delta_1 = 19.1CK (0.5G_1)^{0.85} / B^{0.7} D^{0.45} P^{0.6},$$

$$\Delta_2 = 19.1CK (0.5G_2)^{0.85} / B^{0.7} D^{0.45} P^{0.6},$$
\[ \Delta z = 19.1 C K \left( \frac{G_1}{0.5 G_2} \right)^{0.85} / B^{0.7} D^{0.45} P^{0.6} \]

In the formula above, C and K are coefficients, K=0.0015B+0.42, B is the section width of tire, G1, G2 are the loads on the first and second bridges respectively, D represents the diameter of tire, and P is the tire pressure.

3. Practical calculation and simulated analysis

3.1. Practical calculation of toe-in value and camber angle of the steering wheel of the first and second bridges of the double-front-axle prototype vehicle

In order to verify the correctness of the matching formula of toe-in and camber of the double-front-axle mentioned above, the JAC GEFA Y4AP0 double-front-axle heavy truck will be used as the prototype vehicle. According to the data parameters of the prototype vehicle, the toe-in value and the camber angle of the first and second bridges of the double-front-axle will be calculated.

**Table 1.** Data parameters of the double-front-bridge prototype vehicle.

| d/mm | D/mm | G1/N | G2/N | B/mm | K2/(N/mm) |
|------|------|------|------|------|------------|
| 571.5| 1008 | 37500 | 19950 | 300  | 117.78     |
| 4300 | 1900 | 2400 | 7    | 1.5  |            |

According to the aforementioned vehicle data parameters, combining with the matching formulas of toe-in and camber of the double-front-axle. If a camber angle of the first bridge of a wheel can be given, the toe-in value of the first bridge, the camber angle and toe-in value of the second bridge can all be calculated by matching formula mentioned above.

In the general case, the camber angle of wheels are mostly about 1°, according to the actual situation of this automobile, the camber angle of the first bridge can be given \( \gamma_1 = 1 \degree \), then combining with the parameters and matching formulas of double-front-axle automobile, the toe-in value of the first bridge of wheels are \( T_1 = 2.47 \text{mm} \), and the toe-in angle is \( \phi_1 = 0.104 \degree \). Similarly, the camber angle of the second bridge of wheels are \( \gamma_2 = 1.05 \degree \), the toe-in value of the second bridge of wheels are \( T_2 = 1.42 \text{mm} \), and the toe-in angle is \( \phi_2 = 0.059 \degree \).

3.2. Simulation verification of vehicle dynamics model based on Adams/car

According to the vehicle dynamics model established in Adams/car, we conduct the relevant simulation experiments of straight-line driving and steering driving of whole vehicle. In order to verify the rationality of the matching formulas mentioned above, we should compare with the side slip of the four-steering wheels of the first and second bridges, before and after modifying toe-in angles (values) and camber angle of the first and second bridges of double-front-axle.

Simulation of straight-line driving: Straight-line Maintain driving simulation with the dynamic model in Adams/Car are performed, with the duration of 15s, keeping the steering wheel for free.

The following simulation contrast curves were made by the above-mentioned straight-line driving simulation which was carried out separately on the original double-front-axle dynamic model and that after modifying toe-in and camber.
Figure 5. Side-slip comparison of the first and second bridges of tires on the left side of the straight-line driving

Figure 6. Side-slip comparison of the first and second bridges of tires on the right side of straight-line driving

We can get some information from Figure 5 that the left side slip angle of the first bridge is reduced from 0.43° to about 0.02°, and the left side slip angle of the second bridge is reduced from 0.63° to about 0.18°. Again, as we can see from Figure 6, the right side slip angle of the first bridge is changed from -0.58° to about -0.16°, and the right side slip angle of the second bridge is changed from -0.38° to about 0.06°. Comparing with two simulation lines above, it can be found that the side slip of the tire is obviously improved after modifying the camber and toe-in angle of the first and second bridges of double-front-axle steering wheels.

Simulation of steering driving: Performing the dynamic model of Cornering steering Release driving simulation in Adams/Car, the lateral acceleration and the longitudinal driving speed are set to 0.2g and 28km/m respectively.

The following simulation contrast curves were made by the above-mentioned steering driving simulation which was carried out separately on the original double-front-axle dynamic model and that after modifying the toe-in and camber.

Figure 7. Side-slip comparison of the first and second bridges of tires on the left side of steering driving.

Figure 8. Side-slip comparison of the first and second bridges of tires on the right side of steering driving.

The conclusion about side-slip angle is obtained in figure 7: after modifying toe-in and camber of double-front-axle model, the left side-slip angle curve of the first and second bridges of steering wheels tends to be about 0° compared with the original model. Similar conclusions can also be drawn in Figure 8.

4. Conclusions

- Considering the minimum wear of tires, the geometric relationship between toe-in and camber of the first and second bridges of the double-front-axle is established. At the same time, based on the side
slip mechanism of tires, the matching formula of camber and toe-in of the double-front-axle is derived. In addition, the simulation experiment is verified by the dynamic model Y4AP0, which proves the rationality of the matching formula.

- The relationship between the first and second bridges of the double-front-axle was clarified, and exploring the relationship between the first and second bridges of the double-front-axle.
- The matching formula obtained in this paper has a certain theoretical guiding effect on the matching of the four-wheel positioning parameters of steering wheels of the double-front-axle automobile.

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References

[1] Wang B L, Zhou J S, Zhao B et al. 1995 Trans. Chinese Soc. Agric. Eng. 11(1) 89-94.
[2] Pan H D, Su J, Jia Z R et al. 1998 Chinese J. Highways, 11(suppl.) 116-21.
[3] Wei D G, Chen X Q, Hu N J et al. 2003 Trans. Chinese Soc. Agric. Eng. 19(6) 139-42.
[4] Wei D G 2004 J. Hefei Univ. Technol. (Natural Sci.) 27(6) 1594-8.
[5] Ma J, Qian L J 2012 J. Hefei Univ. Technol. (Natural Sci.) 35(1) 25-8.
[6] Dixon J C 1996 Tire, suspension and Handling 2nd ed. Warrendal, PA: SAE 81-109.
[7] Zhuang J 1995 Automotive Tire Science Beijing: Beijing Inst. Technol. Publishing 127-98.
[8] Xiao H Y 2012 Study on the influence of four wheel alignment parameters on vehicle frequency characteristics under serpentine conditions Hefei: Hefei Univ. Technol.
[9] Guo F 2008 Research and discussion on the relationship between tire wear and wheel alignment Shenyang: Northeastern Univ.
[10] Zhang Y S, Wen L Z, Liu X F 2009 Auto Parts, 4 74-8.
[11] Sun Y S, Wu Z C 2018 IOP Conf. Series: Mater. Sci. Eng.
[12] Ju K, Ye J Wang X H 2017 Dimensional Variation Simulation Analysis of Front Wheel Camber for Macpherson Suspension The 19th Asia-Pacific automotive eng. annual conf. and 2017 China automotive eng. soc. annual conf.
[13] Ma W L, Li F G, Liu R X 2006 Research on Reasonable Matching of Wheel Angle and Front Beam Tractor and Agric. Trans. Vehicle
[14] Zuo S N, Lu B, Wheel Angle Accuracy Improvement Scheme Analysis and Application Automobile Technol. 20