First Accurate Normalization of the $\beta$-delayed $\alpha$ Decay of $^{16}$N and Implications for the $^{12}$C($\alpha$, $\gamma$)$^{16}$O Astrophysical Reaction Rate

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The $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction plays a central role in astrophysics, but its cross section at energies relevant for astrophysical applications is only poorly constrained by laboratory data. The reduced $\alpha$ width, $\gamma_{11}$, of the bound $1^-$ level in $^{16}$O is particularly important to determine the cross section. The magnitude of $\gamma_{11}$ is determined via sub-Coulomb $\alpha$-transfer reactions or the $\beta$-delayed $\alpha$ decay of $^{16}$N, but the latter approach is presently hampered by the lack of sufficiently precise data on the $\beta$-decay branching ratios. Here we report improved branching ratios for the bound $1^-$ level and for $\beta$-delayed $\alpha$ emission. In the case of the $\beta$-delayed $\alpha$ branch, we find a 5$\sigma$ deviation from the literature value. With our new branching ratios, the constraints imposed on $\gamma_{11}$ by the $\beta$-decay and $\alpha$-transfer data are of similar precision and, for the first time, in good agreement. The weighted average of the two gives a robust and precise determination of $\gamma_{11}$, which may permit the $^{12}$C($\alpha$, $\gamma$) cross section to be constrained within 10% in the energy range relevant to hydrostatic He burning.

In the hot and dense interior of stars, helium is burned into carbon and oxygen by means of the triple-$\alpha$ reaction and the $^{12}$C($\alpha$, $\gamma$) reaction. The rates of the two reactions regulate the relative production of carbon and oxygen—a quantity of paramount importance in astrophysics affecting everything from grain formation in stellar winds to the late evolution of massive stars and the composition of type-Ia supernova progenitors [1]. At the temperatures characteristic of hydrostatic He burning, the triple-$\alpha$ reaction is dominated by a single, narrow resonance—the so-called Hoyle resonance—and hence it has been possible to constrain the reaction rate through measurements of the properties of the Hoyle resonance. In contrast, the $^{12}$C($\alpha$, $\gamma$) reaction receives contributions from several levels in $^{16}$O which, as it happens, all lie outside the energy window where thermal fusion of $\alpha + ^{12}$C in the stellar environment is efficient—the so-called Gamow window. This makes the task of determining the $^{12}$C($\alpha$, $\gamma$) rate rather complex. While the triple-$\alpha$ rate is now considered known within 10% in the energy range relevant to hydrostatic He burning [2], efforts underway to reduce the uncertainty to 5% [3, 4], the uncertainty on the $^{12}$C($\alpha$, $\gamma$) rate was recently estimated to be at least 20% which is insufficient for several astrophysical applications [1].

The $^{12}$C($\alpha$, $\gamma$) cross section has been measured down to center-of-mass energies of $E_{c.m.} \approx 1.0$ MeV, but the rapidly decreasing tunneling probability makes it challenging to extend the measurements to lower energies and practically impossible to reach the Gamow energy of 0.3 MeV. According to current understanding [1], the capture cross section at 0.3 MeV receives its largest single contribution from the high-energy tail of the bound $1^-$
level in $^{16}\text{O}$, situated 45 keV below the $\alpha + ^{12}\text{C}$ threshold at an excitation energy of $E_\alpha = 7.12$ MeV. The reduced $\alpha$ width of this level, $\gamma_{11}$, provides a measure of how strongly the level couples to the $\alpha + ^{12}\text{C}$ channel. Therefore, $\gamma_{11}$ is a critical quantity in determining the level’s contribution to the capture cross section at 0.3 MeV and, more generally, in constraining the extrapolation of the $^{12}\text{C}(\alpha, \gamma)$ cross section to the energy range relevant for stellar helium burning.

It has long been known [3] that the shape of the $\beta$-delayed $\alpha$ spectrum ($\beta\alpha$ spectrum) of $^{16}\text{N}$ is highly sensitive to $\gamma_{11}$, but currently this approach to determining $\gamma_{11}$ is hindered by uncertainties in the normalization of the spectrum and small but significant discrepancies in the spectral shape inferred from two existing high-precision measurements [4, 5]. In this Letter, we report on an experimental study of the $\beta\alpha$ decay of $^{16}\text{N}$ in which the unique radioactive-isotope production capabilities of the ISOLDE facility [8] are exploited to provide the first accurate and precise normalization of the $\beta\alpha$ spectrum. We also present a novel $R$-matrix analysis of the $\beta\alpha$ spectra of Refs. [6, 7] and extract an improved value for $\gamma_{11}$ which, for the first time, is in good agreement with the value inferred from sub-Coulomb $\alpha$-transfer reactions. Finally, we comment on the implications of our findings for the determination of the $^{12}\text{C}(\alpha, \gamma)$ cross section at 0.3 MeV. A detailed account of the experimental work and the $R$-matrix analysis will be published separately [6].

The experiment was performed at the ISOLDE radioactive-beam facility of CERN [8]. Radioactive isotopes were produced by the impact of a 1.4-GeV proton beam on a nano-structured CaO target [10], before being ionized in a cooled plasma ion source and accelerated through an electrostatic potential difference of 30 kV. Ions with the desired mass-to-charge ($A/q$) ratio were selected in the High-Resolution Separator and guided to the ISOLDE Decay Station [11] where their decay was studied. The ions were stopped in a thin (30 $\mu$g/cm$^2$) carbon foil surrounded by five double-sided silicon strip detectors (DSSD) and four high-purity germanium (HPGe) covers, allowing for the simultaneous detection of charged particles and $\gamma$ rays. Meanwhile, auxiliary detectors were used to check that the beam was being fully transmitted to the center of the setup and stopped in the foil. During five days of data taking, the $\beta\alpha$ decay of $^{16}\text{N}$ was studied mainly on $A/q = 30$ ($^{16}\text{N}^{14}\text{N}^+$) but also on $A/q = 31$ ($^{16}\text{N}^{14}\text{N}^+\text{H}^+$). Additionally, the decays of $^{17}\text{Ne} (\beta\gamma, \beta p, \beta\alpha)$, $^{18}\text{N} (\beta\gamma, \beta\alpha)$, $^{34}\text{Ar} (\beta\gamma)$ were studied on $A/q = 17$, 32, and 34, providing crucial data for the efficiency calibration of the HPGe array and the energy calibration of the DSSD array.

Three of the DSSDs were sufficiently thin (40 $\mu$m and 60 $\mu$m) to allow the $\alpha$ spectrum of $^{16}\text{N}$ to be clearly separated from the $\beta$ background. The other two DSSDs were much thicker (300 $\mu$m and 1 mm) and served primarily to detect the $\beta$ particles. The distortions of the $\alpha$ spectrum due to $\beta$ summing was negligible due to the high granularity of the DSSDs [12]. Fig. 1 (a) shows the $\alpha$ spectrum obtained in one of the thin DSSDs on $A/q = 30$ during 32 hours of measurement at an average $^{16}\text{N}$ implantation rate of $2 \times 10^4$ ions/s. The two narrow $\alpha$ lines from the $\beta\alpha$ decay of $^{16}\text{N}$ feature prominently in the spectrum obtained on $A/q = 32$, while the spectrum obtained on $A/q = 30$ is due almost entirely to the $\beta\alpha$ decay of $^{16}\text{N}$ except for a $\sim 2\%$ contamination from the $\beta\alpha$ decay of $^{17}\text{N}$ which has been subtracted. The $R$-matrix fit to the spectrum of Ref. [6] (downscaled and properly corrected for experimental resolution) has been superimposed on the data.

![FIG. 1. $\beta$-Delayed $\alpha$ spectra obtained in one of the 60-µm thick DSSDs on $A/q = 30$ (a) and $A/q = 32$ (b). The two narrow $\alpha$ lines from the $\beta\alpha$ decay of $^{16}\text{N}$ feature prominently in the spectrum obtained on $A/q = 32$, while the spectrum obtained on $A/q = 30$ is due almost entirely to the $\beta\alpha$ decay of $^{16}\text{N}$ except for a $\sim 2\%$ contamination from the $\beta\alpha$ decay of $^{17}\text{N}$ which has been subtracted. The $R$-matrix fit to the spectrum of Ref. [6] (downscaled and properly corrected for experimental resolution) has been superimposed on the data.](image-url)
calibrated $^{152}$Eu source was used to determine the detection efficiency at low energies ($E_x < 1.5$ MeV), while $\beta\gamma$, $\gamma\gamma$, and $\gamma\gamma\gamma$ coincidence-data were used to extend the efficiency calibration to higher energies, achieving a relative uncertainty of only 1.4% at 6.13 MeV thanks in particular to the $8.87 \rightarrow 6.13 \rightarrow \text{g.s.}$ cascade, shown in Fig. 2(b). A GEANT4 simulation $^{15}$, normalized only to the $^{152}$Eu data, was used to validate the efficiency calibration. As seen in Fig. 2(c), there is excellent agreement across the entire energy range. Based on the relative $\gamma$-ray yields, we determine the $\beta$-decay branching ratio to the 7.12-MeV level in $^{16}$O to be

$$b_{\beta,11} = (5.02 \pm 0.10) \times 10^{-2},$$

in agreement with Refs. $^{6,13}$, but with a reduced uncertainty. Based on the number of detected $\alpha$ particles, the measured 6.13-MeV $\gamma$-ray yield, and the known relative intensity of the 6.13-MeV $\gamma$-ray line of $0.670 \pm 0.006$ $^{13}$, we determine the branching ratio for $\alpha$ emission to be

$$b_{\beta\alpha} = (1.59 \pm 0.06) \times 10^{-5},$$

where the error estimate includes the statistical uncertainty on the $\alpha$-particle yield (1.3%) and the uncertainties on the $\alpha$-particle and $\gamma$-ray detection efficiencies (2.6% and 1.4%, respectively), the relative intensity of the 6.13-MeV $\gamma$-ray line (0.9%), and the subtraction of the $^{17}$N contamination (0.4%), all added in quadrature. Our value for $b_{\beta\alpha}$ is significantly larger than the literature value of $(1.20 \pm 0.05) \times 10^{-5}$ $^{13}$, but is consistent with the less precise value of $(1.49 \pm 0.05 \text{(stat)} \pm 0.10 \text{sys}) \times 10^{-5}$ obtained by us in a previous study using a different experimental technique $^{15}$.

In order to parameterize the shape of the $\alpha$ spectrum, we adopt an $R$-matrix model similar to that of Refs. $^{6,7}$, consisting of two physical $p$-wave levels at $E_x = 7.12$ and 9.59 MeV, two physical $f$-wave levels at $E_x = 6.13$ and 11.60 MeV, and a $p$-wave background pole at higher energy. The $R$-matrix model of Refs. $^{6,7}$ additionally includes an $f$-wave background pole with zero feeding, but we find that the inclusion of such a pole only gives a marginal improvement of $\chi^2/N$ and a slightly worse $\chi^2/N$ and hence we do not include it. On the other hand, we allow the feeding of the 11.60-MeV level, which was also set to zero in Refs. $^{6,7}$, to vary freely. Our analysis differs from those of Refs. $^{6,7}$ in a few significant respects. First and most importantly, the analyses of Refs. $^{6,7}$ were aimed at determining the capture cross section at 0.3 MeV and therefore involved the simultaneous fitting of $\beta\alpha$-decay data, $\alpha$-scattering data, and $\alpha$-capture data. Our analysis, on the other hand, is aimed at determining the constraints imposed on $\gamma_{11}$ by the $\beta\alpha$-decay data alone and at resolving the discrepancies between Refs. $^{6,7}$, and hence we restrict our attention to the $\beta\alpha$-decay data. We also adopt our improved values for $b_{\beta,11}$ and $b_{\beta\alpha}$, and we fix the asymptotic normalization coefficient (ANC) of the 6.13-MeV level to the rather precise value of $C = 139 \pm 9 \text{ fm}^{-1/2}$ inferred from sub-Coulomb transfer reactions. $^{19}$ All $R$-matrix calculations have been performed with the code ORM $^{20}$. Following Refs. $^{6,7}$ we fix the channel radius to 6.5 fm and ignore the four data points in the vicinity of the narrow $2^+$ level at $E_x = 9.68$ MeV. As shown in the left panel of Fig. 3 we obtain a very good fit to the spectrum of Ref. $^{6}$ ($\chi^2/N = 94.6/80 = 1.18$, $P_{\chi^2>94.6} = 0.127$), yielding a best-fit value of

$$\gamma_{11} = 0.0979 \pm 0.0023 \text{(stat)} \pm 0.0051 \text{(sys)} \text{ MeV}^{1/2} \ (3)$$

for the reduced $\alpha$ width of the 7.12-MeV level. We follow the standard practice in cases of good fit quality and determine the statistical uncertainty as the change in $\gamma_{11}$ required to produce a $\chi^2$-increase of unity. The largest contribution to the systematic uncertainty comes from the energy calibration (3.8%) with smaller contributions from $b_{\beta\alpha}$ (2.7%) and $b_{\beta,11}$ (2.0%) and even smaller contributions from the subtraction of $^{17}$N and $^{18}$N impurities (1.0%), the ANC of the 6.13-MeV level (0.4%), and the energy resolution (0.3%).

As shown in the right panel of Fig. 3 we obtain a worse fit to the spectrum of Ref. $^{6}$ ($\chi^2/N = 122.5/80 = 1.53$, $P_{\chi^2>122.5} = 0.0016$), yielding a best-fit value of $\gamma_{11} =$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(a) $\gamma$-ray spectrum from the $\beta$ decay of $^{16}$N with main transitions indicated. (b) $\gamma\gamma$ coincidence spectrum zoomed in on the $8.87 \rightarrow 6.13 \rightarrow \text{g.s.}$ cascade. (c) Experimentally determined and simulated $\gamma$-ray detection efficiency.}
\end{figure}
0.0928 ± 0.0076(stat) MeV\(^{1/2}\) in good agreement with the value inferred from the spectrum of Ref. [6], but with significantly larger statistical uncertainty. Given the discrepancies between the two spectra [21], the good agreement between the inferred values for \(\gamma_{11}\) is initially surprising. As seen in Fig. 4, the dip around \(E_\alpha = 1.0\) MeV is less pronounced in the spectrum of Ref. [6], and the main peak is slightly wider and shifted by \(-6\) keV relative to the spectrum of Ref. [6]. However, a detailed analysis reveals the agreement to be little more than a lucky coincidence: The less pronounced dip favours a larger \(\gamma_{11}\) value, but the downward energy shift has the opposite effect on \(\gamma_{11}\) so the two differences cancel out.

The spectrum obtained in the present work contains significantly fewer counts (\(1.07 \times 10^4\)) than the spectra of Refs. [6, 7] (\(1.03 \times 10^6\) and \(2.75 \times 10^5\)) and hence does not impose any useful constraints on \(\gamma_{11}\). Our spectrum does, however, impose useful constraints on the position of the maximum of the \(R\)-matrix distribution. Taking into account the uncertainty on the energy calibration, the maximum is found to be consistent with Ref. [6], but shifted by \(6 \pm 3\) keV relative to Ref. [7]. Apart from this small shift, our spectrum is consistent with both previous spectra as the level of statistics is insufficient to reveal the small discrepancies in the region around \(E_\alpha = 1.0\) MeV.

Thus, our analysis shows that the spectrum of Ref. [6] is supported by the better fit quality, is in better agreement with the present spectrum, and provides the more precise determination of \(\gamma_{11}\).

In Table I we compare the ANC deduced from our \(\gamma_{11}\) value given in Eq. (3) to the ANCs obtained in \(\alpha\)-transfer experiments. The precision of the present result is similar to that of the most precise \(\alpha\)-transfer result and good agreement is found between the two methods. We note that if the old branching ratio of \(b_{\beta\alpha} = 1.20 \times 10^{-5}\) [15] is used, the ANC inferred from the analysis of the \(\beta\alpha\)-

### Table I. Experimental values for the square of the ANC of the 7.12-MeV level and weighted average.

| \(C^2\) \(10^{28}\) fm\(^{-1}\) | Ref. |
|-----------------|------|
| 4.00 ± 1.38     | [22] |
| 4.33 ± 0.84     | [23] |
| 3.48 ± 2.00     | [24] |
| 4.39 ± 0.59     | [19] |
| 4.80 ± 0.56     | This work |
| 4.49 ± 0.35     | Weighted average |

FIG. 3. (a), (c): \(R\)-matrix fits to the \(\beta \alpha\) spectra of Refs. [6, 7]. (b), (d): Normalized residuals.

FIG. 4. (a) Comparison of the \(R\)-matrix distributions determined from the \(\beta \alpha\) spectra of Refs. [6, 7]. (b) Zoom-in on the maximum of the distribution.
decay data is reduced to $C^2 = 3.53 \times 10^{-28} \text{ fm}^{-1}$ (with no change in fit quality) in slight disagreement with the $\alpha$-transfer data.

In conclusion, we have obtained the first accurate normalization of the $\beta$-delayed $\alpha$ spectrum of $^{16}\text{N}$ and shown that existing high-precision measurements of the spectral shape now constrain the reduced $\alpha$ width, $\gamma_{11}$, of the bound $1^-$ level in $^{16}\text{O}$ within 5.7%. Our present value for $\gamma_{11}$ is in good agreement with the value inferred from sub-Coulomb $\alpha$ transfer studies, and the weighted average has an uncertainty of only 3.9%. Since the high-energy tail of the bound $1^-$ level dominates the $E1$ component of the $^{12}\text{C}(\alpha, \gamma)$ cross section at 0.3 MeV and $\gamma_{11}$ enters quadratically in the expression for the cross section, the uncertainty on the $E1$ component could now be as low as $\sim 8\%$, though a detailed analysis, beyond the scope of the present Letter, is needed to demonstrate this. Considering the progress made in recent years in constraining the other components of the $^{12}\text{C}(\alpha, \gamma)$ cross section, it may finally be possible to bring the uncertainty on the total cross section at 0.3 MeV below 10%.

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