Spectrum analysis of optical signals using sequential heterodyning

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Abstract. The possibility of transferring the idea of radio-frequency sequential heterodyne spectrum analysis to the optical range is considered. The question of a panoramic optical local oscillator in the form of a tunable laser is considered, where the consideration of stochastic phenomena is presented in the form of the sum of oscillations with a linear change in the instantaneous frequency and the stochastic component as interference. In this regard, it is proposed to use the technique of compression of radar signals to reduce the influence of the stochastic component of the oscillations of the laser panoramic local oscillator.

1. Introduction
In physics and technology, spectral methods and spectral devices are among the most common, and at present there are no visible reasons that would lead to a change in this position. Harmonic spectrum analysis is one of the most important physical and technical measurements. In spectroscopic measurements, spectral instruments examine electromagnetic radiation as a signal sent by matter and carrying spectroscopic information not only about the chemical composition of a substance, but also about its aggregate state, temperature, physical and chemical processes occurring in it, and also about the physical properties of the medium, through which radiation propagates. Thus, spectral instruments are in many ways tools for studying the microworld.

Spectroscopic methods of obtaining information are the only ones possible when studying very remote or inaccessible objects; these methods are widely used in various fields of science and technology: in physics, chemistry, biology, geology, medicine, etc., in educational and factory laboratories. The progress of spectral instrumentation is associated with the general progress of science and technology and, at the same time, itself actively influences it. Among instruments for scientific research, harmonic analysis equipment occupies a special place: the spectroscopy technique has been developing for many years at a very fast pace, faster than in other areas of physical experiment and analysis, and further progress in science and technology requires the development of new methods and principles for measuring spectra, as well as the creation of increasingly sophisticated spectral measurement equipment.

One of these areas is the transfer of the idea of sequential heterodyne spectrum analysis (SHSA) of radio signals, i.e. vibrational phenomena in the optical range. This direction is characterized by the fact that it opens up the possibility of creating optical spectral instruments with a very high resolution, unattainable for all optical spectral instruments built on the principles currently known.
2. Sequential heterodyne spectrum analysis in the radio range
The block diagram of a sequential heterodyne analysis of the radio spectrum is shown in figure 1.

Figure 1. Block diagram of sequential heterodyne spectrum analysis: 1 - preselector, 2 - mixer (signal multiplier), 3 - panoramic local oscillator, 4 - narrow-band path, 5 - amplitude detector, 6 - spectrometric information processing unit.

Essentially, a serial heterodyne spectrum analyzer [1], called a panoramic radio receiver [2], is a superheterodyne radio receiver periodically tuned in automatic mode and includes the following components characteristic of the radio reception technique, which are electronic circuits: preselector (broadband path [2]) 1, which sets band of analyzed frequencies $\Delta f_a$; a mixer 2, performing the multiplication of the analyzed signal and oscillations of the panoramic local oscillator; panoramic local oscillator 3, forming a periodic sequence of radio pulses with a linear change in instantaneous frequency; narrow-band path 4, which is understood as an intermediate-frequency path with a special transfer function that is close to a bell function [2]; amplitude detector 5; Spectrometric information processing unit 6. The following is a summary of these basic components:

1. Preselector, i.e. the broadband path is a conventional radio engineering device that sets the analyzed frequency band (filtering) and the signal level necessary for successful multiplication of the analyzed signal $s_a(t)$ and oscillations $s_0(t)$ of the panoramic local oscillator in the mixer.

2. The mixer is a nonlinear (less often parametric) element, the current-voltage characteristic of which allows the multiplication of oscillations $s_a(t)$ and $s_0(t)$, or, following the terminology of radio reception, carries out frequency conversion.

3. The panoramic local oscillator forms a periodic sequence of radio pulses $s_h(t)$ of duration $T_a$ with a linear change in the instantaneous oscillation frequency:

$$s_h(t) = S_0 \cos(\Omega_0 t + M \nu^2 / 2),$$

(1)

where $S_0$ – is the amplitude of the oscillations, $\Omega_0$ – is the angular average frequency; $M$ – rate of change of frequency $t \in (kT_a, (k + 1)T_a)$ $k = 0, 1, 2, ..., 4$.

4. The narrow-band path is a resonant system with a transfer function $K_m(\omega)$ and a resonant circular frequency $\omega_0$ and carries out frequency filtering of the vibrations $s_m(t)$.

5. The amplitude detector provides a preliminary result of spectral measurements in the form of an envelope of oscillations at the output of a narrow-band path.

6. The spectrometric information processing unit is intended for issuing spectrometric information in a form convenient for the recipient.

The main characteristic of any spectral device is its spread function $(\text{SF}) K(\cdot)$, which, according to the theory of linear systems [4], is the response of a linear system to the $\delta$–effect.
\[
K(\xi, \xi') = \hat{L} \delta(\xi - \xi')
\]  
(2)

\[\hat{L} \] is a linear operator describing the action of a linear system; \(\xi, \xi'\) - variables.

When performing spectral measurements, such an \(\delta\) - effect is a spectral function \(\delta(\omega - \omega')\) that has no physical interpretation, however

\[
\hat{F}^{-1} \delta(\omega - \omega') = \exp(i\omega' t),
\]

(3)

where \(\hat{F}^{-1}\) is the inverse Fourier transform operator; \(\omega\) - circular temporal spectral frequency.

If the action of a spectral device that measures the spectra of vibrational phenomena is described by a limited linear operator \(\hat{A}\), then the result of the transformation of harmonic oscillations \(s_a(t) = \exp(i\omega' t)\) in operator form is given by the relation:

\[
K_s(\omega, \omega') = \hat{A}\exp(i\omega' t) = \hat{A}\hat{F}^{-1} \delta(\omega - \omega') = \hat{L} \delta(\omega - \omega')
\]

(4)

Relation (4) shows that \(K_s(\omega, \omega')\) is the spread function of a spectral instrument that measures the spectrum of vibrational phenomena.

The theory of operation of a SHSA basically comes down to the theory of dynamic frequency characteristics of linear narrow-band systems [2,4] i.e. envelopes of vibrations in these systems under the influence of vibrations of the form (1), where the frequency deviation is much greater than the system passband. Namely, this oscillations take place when a harmonic oscillation spectrum (3) is exposed at the input of a SHSA. These oscillations are pulses with a linear change in the instantaneous frequency, and their envelope is determined by the shape of the transfer function \(K_s(\omega)\) of the narrow-band path and reaches its maximum value at the time when, \(|\omega' - \Omega(t)| \approx \omega_0\) i.e. various harmonic spectral components are displayed sequentially in time, which defines the name of this type of spectral measurements as a SHSA.

The dynamic frequency response is an envelope of the SF of a SHSA and the ultimate goal of the theory of dynamic frequency characteristics is to determine the resolution of a SHSA.

The main correlation of the theory of SHSA is the relationship between the band \(\Delta f_d\) of the dynamic frequency response and rate of change of the frequency of the panoramic local oscillator [2]:

\[
\Delta f_d = \Delta f \approx \sqrt{M / 2\pi}
\]

(5)

In the framework of the theory of dynamic frequency characteristics [2,4] it was found that the optimal transfer function \(K_o(\omega)\) of a narrow-band path is a bell-frequency response:

\[
K_o(\omega) = \exp\left\{-\left[\frac{\omega - \omega_0}{2\beta}\right]^2 + j(\omega - \omega_0)t_1\right\}
\]

(6)

\(\omega_0\) is the resonant frequency; \(4\beta\) - bandwidth at the level of (-1) neper; \(t_1\) slope of the phase response.

3. Serial heterodyne spectrum analysis in the optical range

As noted earlier, the SHSA is a periodically tunable auto-heterodyne radio receiver, i.e. its construction is based on the principles of radio reception, the difference is only in the periodic automatic rearrangement of the local oscillator. Similarly, the principles of creating a SHSA in the optical range, using the idea of SHSA of radio range signals, should be based on the principles of heterodyne reception of optical signals [5], i.e. a SHSA of optical signals should be considered as an automatically tunable optical heterodyne receiver, where a tunable laser is used as a panoramic local oscillator. Therefore, the analogue of the preselector in the optical SHSA is the input devices of the optical receivers, and as a multiplier, i.e. optical mixer applies a photo detector. Further, the result of
frequency conversion by a photodetector (multiplication of tunable laser signals and a received optical signal), i.e., the signal \( s_m(t) \) is thought in the form of oscillatory phenomena, i.e. the narrow-band path of a SHSA of the optical range is a radio engineering device, and its description is conceived in the framework of the theory of electrical circuits.

The block diagram of a SHSA of the optical range is shown in figure 2.

![Generalized block diagram of SHSA of the optical range](image)

**Figure 2.** Generalized block diagram of SHSA of the optical range: 1 - input device; 2 - photodetector (signal multiplier); 3 - tunable laser; 4 - narrowband system; 5 - amplitude detector; 6 - spectrometric information processing unit.

According to [6], the narrow-band path of an optical SHSA is thought of as a microwave radio receiver, its block diagram is shown in figure 3, and the microwave superheterodyne radio receiver functions as blocks 4, 5, 6, presented in the generalized block diagram of figure 2.

![Block diagram of a microwave superheterodyne radio receiver](image)

**Figure 3.** Block diagram of a microwave superheterodyne radio receiver: 1 - microwave input device; 2 - microwave mixer; 3 - microwave local oscillator; \( S_{m2}(t) \) - oscillation of the second intermediate frequency \( 4 - \) the path of the second intermediate frequency with the transfer function; 5 - amplitude detector.

In terms of radio reception, a SHSA of the optical range is a device with double frequency conversion, the first intermediate frequency is the frequency of microwave oscillations, the actual path of the intermediate frequency of the microwave part provides the desired transfer function \( K_m(\omega) \). In the optical range, wave processes are subject to processing, and in this case, the question of the spread function of the optical spectral device, as a response to the spectral function \( \delta(\omega - \omega') \), is solved as follows: a homogeneous plane monochromatic wave can be represented as
\[
\exp(i(\omega' t - k' z)) = \int_{-\infty}^{\infty} \exp(i\omega' t') \delta(t - t' - z / c_0) dt' = \hat{\nabla} \hat{F}^{-1} \delta(\omega - \omega'), \tag{7}
\]

where \( k' = \omega' / c_0 \) is the wave number; \( \hat{\nabla} \) - linear operator of transition from harmonic oscillation to a uniform plane monochromatic wave.

If the transformation of a uniform plane monochromatic wave by an optical spectral device is described by the action of a linear bounded operator \( \hat{B} \), then the result of the action of the spectral device on wave (7) is given by the expression

\[
\hat{B} \exp(i(\omega' t - k' z)) = \hat{B} \hat{\nabla} \hat{F}^{-1} \delta(\omega - \omega') = \hat{L}_e \delta(\omega - \omega') = K_e(\omega, \omega') \tag{8}
\]

where \( K_e(\omega, \omega') \) - is the complex spread function of the optical spectral instrument, satisfying the definition of the theory of linear systems [3]

\[
S_e(\omega) = \hat{F} S(t) = \int_{-\infty}^{\infty} u(t) \exp(-i\omega t) dt, \tag{10}
\]

where \( \hat{F} \) - is the direct Fourier transform operator; \( u(t) \) - source of vibrations.

When performing spectral measurements, the aim is to obtain information on temporary changes, i.e. on the dynamics of source functions, or temporary frequency spectra. Therefore, in the framework of the SHSA of optical range, it is not optical radiation that is meant, but optical oscillatory processes \( e(t) = u(t) \) in a scalar form.

In the radio range, the study of the SHSA assumes an idealized description of the oscillations of the panoramic local oscillator (1), i.e., without stochastic phenomena. Transferring the SHSA idea to the optical range requires taking account of stochastic phenomena caused by spontaneous transitions in tunable lasers [7], then oscillations of the optical panoramic local oscillator can be written in the form:

\[
s_o(t) = E_o(t) \cos(\omega_o t + (\mu t^2 / 2) + \phi(t)) \tag{11}
\]

\( E_o(t) \) - envelope of the oscillations of the optical panoramic local oscillator; \( \omega_o \) - average oscillation frequency of the optical panoramic local oscillator, \( \phi(t) \) - phase term. Oscillation (11) has stochastic components in the form of amplitude modulation \( E_o(t) \neq const \) and instantaneous frequency:

\[
\omega_o(t) = \omega_o + \mu t + \frac{d\phi(t)}{dt} \tag{12}
\]

Which is not a linear function of time. Envelope \( E_o(t) \) and additional frequency modulation in the form \( d\phi(t) / dt \) determine the deviation of the HF of the SHSA of the optical range from the similar spread function of the corresponding radiofrequency device.

The oscillation of the optical panoramic local oscillator (11) can be written in the form

\[
s_o(t) = S_o \cos(\omega_o t + \mu t^2 / 2) + n(t), \tag{13}
\]

Where \( S_o, \omega_o, \mu \) - the amplitude of the oscillations, the average circular frequency and the rate of change of the instantaneous circular frequency of the deterministic component, respectively; \( n(t) \) - stochastic component.
The stochastic component \( n(t) \) can be considered as generating an interference, against which the SHSA is performed. This makes it advisable to apply the technique of compression of radar pulse signals in the task of improving parameters of panoramic radio reception [8] to the SHSA of optical range. Taking into account the deterministic component of the oscillation (13) when determining the oscillation at the output of a narrow-band path with an auxiliary function (6) and its subsequent processing by the compression device for radar pulse signals were studied in detail in [9] and are not here. Taking into account the influence of the stochastic component \( n(t) \) requires further research.

4. Conclusion
The possibility of transferring the idea of the most important type of analogue method for measuring the spectra of the radio range - SHSA of electrical oscillations into the optical range is considered. Currently, radio monitoring methods based on the use of traditional panoramic radios [2] have largely outlived themselves due to the rapid development of computer technology and its implementation in radio signal processing systems. On the other hand, in the optical range, analog methods for measuring optical spectra are the only possible ones and require further development in connection with the role of spectral measurements in the optical range noted above.

When solving spectroscopic, it is possible to provide a very high resolution of the optical SHSA, unattainable in other spectral instruments in the optical range. Increasing the resolution of spectral instruments in the optical range is of particular relevance in the study of so-called spectral lines, which are narrow-band radiation that carries very valuable spectroscopic information.

The main task of the spread implementation of the proposed idea of transferring the SHSA principle to the optical range is seen in the creation of a panoramic panoramic local oscillator with frequency tuning in a given frequency band and with a minimal presence of a stochastic component of laser radiation generation. It seems that the use of a dispersing system as an optimal filter in the narrow-band path of a serial heterodyne spectrum analyzer of the optical range will not only increase its resolution, but also reduce the influence of the stochastic component on the results of spectral measurements of optical signals.

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