Quasinormal Modes of MOdified Gravity (MOG) Black Holes

3rd Karl Schwarzschild Meeting
Gravity and the Gauge/Gravity Correspondence

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Coming to arXiv shortly!
Outline

• Introduction to Ringdown and Quasinormal Modes (QNMs)
• Asymptotic Iteration Method
• Scalar-Vector-Tensor MODified Gravity (MOG)
• MOG’s prediction for QNMs
• Comparison with General Relativity
Gravitational Waves from Compact Binaries

(LIGO)
Observation of Ringdown
Black Hole Ringdown

- After merger, the remnant black hole is strongly deformed.

- This can be viewed as a perturbation about the final state.

- This perturbation excites resonant modes of the black hole.

- These decay exponentially in a “ringdown”.

Lucho Manfredi – KSM 2017
Quasinormal modes: the math

- Scalar wave equation:
  \[ \Box_g \psi = 0 \]

- Separation of variables:
  \[ \psi_{\omega l} \sim \frac{1}{r} e^{-i\omega t} u_{\omega l}(r) Y_{lm}(\theta, \phi) \]

- Radial equation:
  \[ \frac{d^2 u_{\omega l}}{dr_*^2} + \left( \omega^2 - V \right) u_{\omega l} = 0 \]

- Only certain frequencies work for purely outgoing boundary conditions:
  \[ \omega_{lm} \]

- Same story for gravitational waves
  \[ s\psi_{\omega l} \rightarrow h \]
Asymptotic Iteration Method

Consider a second order differential equation of the form

\[ \chi'' = \lambda_0(x) \chi' + s_0(x) \chi \]

Differentiating the above equation \( n \) times yields

\[ \chi^{(n+2)} = \lambda_n(x) \chi' + s_n(x) \chi \]

where the coefficients satisfy the relations

\[ \lambda_n(x) = \lambda'_{n-1} + s_{n-1} + \lambda_0 \lambda_{n-1}, \quad s_n(x) = s'_{n-1} + s_0 \lambda_{n-1} \]

For sufficiently large values of \( n \), the asymptotic feature of the AIM is introduced requiring

\[ \frac{s_n(x)}{\lambda_n(x)} = \frac{s_{n-1}(x)}{\lambda_{n-1}(x)} \equiv \beta(x) \]

where the QNMs are obtained from the “quantization condition”

\[ \delta_n = s_n \lambda_{n-1} - s_{n-1} \lambda_n = 0 \]
Scalar-Tensor-Vector MOfified Gravity (MOG)

An exact generalized Schwarzschild-MOG solution of the STVG fields equations is obtained by requiring $G = G_N (1 + \alpha) \sim \text{constant}$ and $Q_g = \sqrt{\alpha G_N M} \sim \text{constant}$, and ignoring the small $\phi_\mu$ vector field particle mass $m_\phi \sim 10^{-28}$ eV in the present universe.

The field equations are given by

\[ R_{\mu \nu} = -8\pi G T^\phi_{\mu \nu}, \]
\[ \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} B^{\mu \nu}) = 0, \]
\[ \partial_\sigma B_{\mu \nu} + \partial_\mu B_{\nu \sigma} + \partial_\nu B_{\sigma \mu} = 0. \]

The energy-momentum tensor is

\[ T^\phi_{\mu \nu} = -\frac{1}{4\pi} (B_{\mu \alpha} B^{\nu \alpha} - \frac{1}{4} \delta^\nu_{\mu} B^{\alpha \beta} B_{\alpha \beta}) \]

The gravitational field metric is given by

\[ ds^2 = \left(1 - \frac{2GM}{r} + \frac{GQ_g^2}{r^2}\right) dt^2 - \left(1 - \frac{2GM}{r} + \frac{GQ_g^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2. \]
Cosmological Observations

STVG/MOG has been applied successfully to a range of astronomical, astrophysical, and cosmological phenomena.

Solar system and star clusters containing few million solar masses
→ No deviation from Newton or Einstein

• Theory accounts for the rotation curves of spiral galaxies, correctly reproducing the Tully-Fisher law.

• STVG is in good agreement with the mass profiles of galaxy clusters.

• STVG can also account for key cosmological observations, including:
  • The acoustic peaks in the cosmic microwave background radiation;
  • The accelerating expansion of the universe
  • The matter power spectrum of the universe

J. W. Moffat et al.: MNRAS (2013) [1439-1451], [arXiv:1510.07037v2], [arXiv:1611.05382]
| l | n | \( \alpha = 0 \) | \( \alpha = 1 \) | \( \alpha = 4 \) | \( \alpha = 9 \) |
|---|---|---|---|---|---|
| 1 | 0 | 0.2483 - 0.09249i | 0.1448 - 0.04805i | 0.06343 - 0.01881i | 0.03268 - 0.009084i |
|    | 1 | 0.2145 - 0.2937i | 0.1308 - 0.1506i | 0.05882 - 0.05828i | 0.03038 - 0.02796i |
|    | 2 | 0.1748 - 0.5252i | 0.1135 - 0.2654i | 0.05258 - 0.1014i | 0.02675 - 0.04833i |
|    | 3 | 0.1462 - 0.7719i | 0.1090 - 0.3866i | 0.05036 - 0.1494i | 0.02434 - 0.07008i |
| 2 | 0 | 0.4576 - 0.09500i | 0.2651 - 0.04917i | 0.1164 - 0.01930i | 0.06000 - 0.009351i |
|    | 1 | 0.4365 - 0.2907i | 0.2565 - 0.1498i | 0.1136 - 0.05854i | 0.05861 - 0.02830i |
|    | 2 | 0.4012 - 0.5016i | 0.2420 - 0.2563i | 0.1087 - 0.09948i | 0.05607 - 0.04791i |
|    | 3 | 0.3626 - 0.7302i | 0.2257 - 0.3699i | 0.1028 - 0.1425i | 0.05272 - 0.06840i |
| 3 | 0 | 0.6569 - 0.09562i | 0.3771 - 0.04933i | 0.1648 - 0.01936i | 0.08493 - 0.009395i |
|    | 1 | 0.6417 - 0.2897i | 0.3709 - 0.1492i | 0.1627 - 0.05842i | 0.08391 - 0.02831i |
|    | 2 | 0.6138 - 0.4921i | 0.3594 - 0.2522i | 0.1589 - 0.09841i | 0.08198 - 0.04760i |
|    | 3 | 0.5779 - 0.7063i | 0.3446 - 0.3600i | 0.1537 - 0.1398i | 0.07927 - 0.06743i |

Table 1: QNMs accurate to 4 decimal places for \( M = 1 \) scaled MOG electromagnetic perturbations \( V_{i=1}^{(-)} \) for \( l = 1 \), \( l = 2 \) and \( l = 3 \) modes.
|   | n   | **\( \alpha = 0 \)**                  | **\( \alpha = 1 \)**                  | **\( \alpha = 4 \)**                  | **\( \alpha = 9 \)**                  |
|---|-----|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| 2 | 0   | 0.3737 - 0.0890i                      | 0.2220 - 0.04650i                    | 0.1021 - 0.01867i                    | 0.05431 - 0.009171i                  |
|   | 1   | 0.3467 - 0.2739i                      | 0.2115 - 0.1423i                     | 0.09872 - 0.05678i                   | 0.05270 - 0.02781i                   |
|   | 2   | 0.3011 - 0.4783i                      | 0.1937 - 0.2457i                     | 0.09283 - 0.09696i                   | 0.04974 - 0.04725i                   |
|   | 3   | 0.2515 - 0.7051i                      | 0.1742 - 0.3579i                     | 0.08582 - 0.1397i                    | 0.04584 - 0.06776i                   |
| 3 | 0   | 0.5994 - 0.0927i                      | 0.3353 - 0.04758i                    | 0.1496 - 0.0189i                     | 0.07875 - 0.009267i                  |
|   | 1   | 0.5826 - 0.2813i                      | 0.3281 - 0.1441i                     | 0.1472 - 0.0571i                     | 0.07761 - 0.02795i                   |
|   | 2   | 0.5517 - 0.4791i                      | 0.3149 - 0.2444i                     | 0.1428 - 0.0964i                     | 0.07543 - 0.04706i                   |
|   | 3   | 0.5120 - 0.6903i                      | 0.2979 - 0.3503i                     | 0.1368 - 0.1373i                     | 0.07238 - 0.06680i                   |
| 4 | 0   | 0.8092 - 0.0942i                      | 0.4452 - 0.04804i                    | 0.1965 - 0.01903i                    | 0.1030 - 0.009311i                   |
|   | 1   | 0.7966 - 0.2843i                      | 0.4398 - 0.1449i                     | 0.1947 - 0.05731i                    | 0.1021 - 0.02802i                    |
|   | 2   | 0.7727 - 0.4799i                      | 0.4294 - 0.2441i                     | 0.1912 - 0.09625i                    | 0.1004 - 0.04699i                    |
|   | 3   | 0.7398 - 0.6839i                      | 0.4151 - 0.3468i                     | 0.1863 - 0.1362i                     | 0.09796 - 0.06636i                   |

Table 2: QNMs accurate to 4 decimal places for \( M = 1 \) scaled MOG gravitational perturbations \( V_{i=2}^{(-)} \) for \( l = 2, l = 3 \) and \( l = 4 \) modes.
|    | n  | GR          |          | MOG          |          |
|----|-----|-------------|----------|--------------|----------|
|    |     | α = 0       | α = 1    | α = 4        | α = 9    |
| 1  | 0   | 0.2483 - 0.09249i | 0.2896 - 0.09611i | 0.3171 - 0.09403i | 0.3268 - 0.09084i |
|    | 1   | 0.2145 - 0.2937i  | 0.2616 - 0.3012i  | 0.2941 - 0.2914i  | 0.3038 - 0.2796i  |
|    | 2   | 0.1748 - 0.5252i  | 0.2271 - 0.5309i  | 0.2629 - 0.5072i  | 0.2675 - 0.4833i  |
|    | 3   | 0.1462 - 0.7719i  | 0.2179 - 0.7733i  | 0.2518 - 0.7470i  | 0.2434 - 0.7008i  |
| 2  | 0   | 0.4576 - 0.09500i | 0.5302 - 0.09833i | 0.5821 - 0.09650i | 0.6000 - 0.09351i |
|    | 1   | 0.4365 - 0.2907i  | 0.5131 - 0.2995i  | 0.5680 - 0.2927i  | 0.5861 - 0.2830i  |
|    | 2   | 0.4012 - 0.5016i  | 0.4840 - 0.5126i  | 0.5435 - 0.4974i  | 0.5607 - 0.4791i  |
|    | 3   | 0.3626 - 0.7302i  | 0.4514 - 0.7397i  | 0.5139 - 0.7125i  | 0.5272 - 0.6840i  |
| 3  | 0   | 0.6569 - 0.09562i | 0.7542 - 0.09867i | 0.8239 - 0.09681i | 0.8493 - 0.09395i |
|    | 1   | 0.6417 - 0.2897i  | 0.7418 - 0.2983i  | 0.8136 - 0.2921i  | 0.8391 - 0.2831i  |
|    | 2   | 0.6138 - 0.4921i  | 0.7189 - 0.5045i  | 0.7944 - 0.4921i  | 0.8198 - 0.4760i  |
|    | 3   | 0.5779 - 0.7063i  | 0.6891 - 0.7200i  | 0.7687 - 0.6988i  | 0.7927 - 0.6743i  |

Table 3: QNMs accurate to 4 decimal places for $M = 1/(1 + \alpha)$ scaled MOG electromagnetic perturbations $V^{(-)}_{i=1}$ for $l = 1$, $l = 2$ and $l = 3$ modes.
| l  | n  | GR   | MOG   |
|----|----|------|-------|
|    |    | **α = 0** | **α = 1** | **α = 4** | **α = 9** |
| 2  | 0  | 0.3737 - 0.0890i | 0.4441 - 0.09300i | 0.5105 - 0.09333i | 0.5431 - 0.09171i |
|    | 1  | 0.3467 - 0.2739i | 0.4229 - 0.2847i | 0.4936 - 0.2839i | 0.5270 - 0.2781i |
|    | 2  | 0.3011 - 0.4783i | 0.3874 - 0.4914i | 0.4642 - 0.4848i | 0.4974 - 0.4725i |
|    | 3  | 0.2515 - 0.7051i | 0.3484 - 0.7158i | 0.4291 - 0.6985i | 0.4584 - 0.6776i |
| 3  | 0  | 0.5994 - 0.0927i | 0.6706 - 0.09516i | 0.7479 - 0.09456i | 0.7875 - 0.09267i |
|    | 1  | 0.5826 - 0.2813i | 0.6563 - 0.2882i | 0.7360 - 0.2856i | 0.7761 - 0.2795i |
|    | 2  | 0.5517 - 0.4791i | 0.6298 - 0.4888i | 0.7138 - 0.4821i | 0.7543 - 0.4706i |
|    | 3  | 0.5120 - 0.6903i | 0.5957 - 0.7006i | 0.6842 - 0.6865i | 0.7238 - 0.6680i |
| 4  | 0  | 0.8092 - 0.0942i | 0.8904 - 0.09608i | 0.9826 - 0.09513i | 1.030 - 0.09311i |
|    | 1  | 0.7966 - 0.2843i | 0.8795 - 0.2898i | 0.9735 - 0.2865i | 1.021 - 0.2802i |
|    | 2  | 0.7727 - 0.4799i | 0.8588 - 0.4881i | 0.9560 - 0.4812i | 1.004 - 0.4699i |
|    | 3  | 0.7398 - 0.6839i | 0.8301 - 0.6936i | 0.9315 - 0.6810i | 0.9796 - 0.6636i |

Table 4: QNMs accurate to 4 decimal places for $M = 1/(1 + \alpha)$ scaled MOG gravitational perturbations $V^{(-)}_{i=2}$ for $l = 2$, $l = 3$ and $l = 4$ modes.
Graph shows $n = 0, l = 2$ QNMs for gravitational perturbations for increasing $\alpha$ and $1/(1 + \alpha)$ scaled MOG black hole.
Acknowledgments

Dr. Jonas Mureika
Dr. John W. Moffat
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THANKS!

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