Laser lever method in the application of young's modulus measurement

Yingyuan Fan*, Zhaoyun Qiu and Renhong Zhao
Weifang Medical University, Weifang 261053, China

*Corresponding author e-mail: yyfan@126.com

Abstract. Young's modulus of solid material is usually measured by tensile test. We adopt a new laser lever to instead of the traditional optical lever in this article. Contrast experiments with two different methods are designed to measure the steel-wire’s Young's modulus. The results show that the new method is more accurate and efficient than the old one.

1. Introduction
Young's modulus is an important physical quantity in describing the ability of solid material to resist elastic deflection. It is necessary to measure the Young's Modulus if you want to get insight into some kind of material, such as metal, semiconductor, optical fiber, ceramic rubber and rubber material.

2. The traditional optical lever method of Young's modulus measurement
Young's modulus is the modulus of longitudinal elasticity. The traditional measuring device is shown in figure 1: One plane mirror with which deflection surface parallel to the axis of the telescope and connected with the steel wire is installed on three pivot. If the length of the steel wire changed by different hook weights, the plane mirror will revolve. And so the light reflected by the plane mirror will revolve. Of course, the scale value in telescopic views will also change.

Figure 1. The traditional device to measure Young's modulus by optical lever

* Corresponding author e-mail: yyfan@126.com
When the steel wire stretched by hook weights, the forces (\(F\)) is caused by the interaction of the adjacent material. According to Hooke's Law, the force of unit cross-sectional area (\(\frac{F}{S}\)) is proportional to the elongation of unit length (\(\frac{\Delta L}{L}\)). The relationship between them is shown as formula 1.

\[
\frac{F}{S} = E \frac{\Delta L}{L}
\]  

(1)

In formula 1, \(E\) is Young's modulus and \(S\) is the cross-sectional area of the steel wire. Assuming the diameter of the steel wire is \(d\), then the cross-sectional area \(S = \frac{1}{4}\pi d^2\), and formula 2 was deduced from formula 1 as flowing.

\[
E = \frac{4FL}{\pi d^2 \Delta L}
\]  

(2)

\(F\), \(L\) and \(S\) are easy to measure, yet the small variation of the steel-wire's length (\(\Delta L\)) is relatively difficult. Optical lever method shown in figure 2 is usually used to measure \(\Delta L\).

\[\text{Figure 2. Magnification of the optical lever method}\]

In figure 2, the deflection angle of the plane mirror normal is \(\theta\), and then the angle between the incident and reflected ray is \(2\theta\). Usually numerical value of \(\theta\) is tiny enough that \(\tan \theta \approx \theta\) and \(\tan 2\theta \approx 2\theta\). From the geometrical relationship in figure 2, we can easily point out the equation as formula 3:

\[
\Delta L = \frac{b}{2D} \Delta X
\]  

(3)

It can be seen from formula 3: The small variation of the steel-wire's length (\(\Delta L\)) correspond to the length of two intersections (\(\Delta X\)) in figure 2. So the magnifying multiple of \(\Delta L\) is \(\frac{2D}{B}\) and formula 3 can be rewritten as formula 4:
In formula 4, \( F, D, L, d, b \) respectively represent the tensile force, the distance between the plane mirror and scale, the steel-wire's length, the steel-wire's diameter and the optical lever’s length. Naturally, \( E \) represent the Young's modulus of the steel wire.

3. Laser lever method of measuring Young's modulus by the improved device

The light of the semiconductor laser is better in monochromatic and directionality. In figure 1: The telescope is replaced by a small cross laser connected to the scale. The laser beam produced by the cross laser vertically shots out through a tiny hole. And then the laser beam will be reflected by the plane mirror on the scale. It is easy to judge the angle of the laser reflection by the improved measurement device in figure 3. Because the principle of measurement between the instrument in figure 1 and in figure 3 is selfsame, the way to calculate \( E \) by laser lever is identical to formula 4.

\[
E = \frac{8FLD}{\pi d^2 b\Delta X}
\]  

(4)

4. Contrast experiment with two different measuring devices

A contrast experiment is designed to bear out the instrument precision of the improved measuring device to measure Young's modulus by laser lever. A piece of steel wire is measured separately by optical lever method and laser lever method. In the process of measurement, the mount of hook weights is firstly increased and secondly decreased to adapt to steel-wire’s delayed effect.

| Number of weights | 0  | 1  | 2  | 3  | 4  | 5  | 4  | 3  | 2  | 1  | 0  |
|-------------------|----|----|----|----|----|----|----|----|----|----|----|
| \( X / \text{cm} \) | 9.51 | 9.81 | 10.10 | 10.40 | 10.70 | 10.98 | 10.69 | 10.40 | 10.12 | 9.81 | 9.52 |
| \( \bar{X} / \text{cm} \) | 9.515 | 9.81 | 10.11 | 10.40 | 10.695 | 10.98 |

Table 1. Measured result by optical lever method

Note: \( X \) is the reading of the telescope. \( \bar{X} \) is the average of \( X \) with same number of weights

\[ L = 43.10 \text{cm}, \quad d = 2.90 \text{mm}, \quad b = 9.50 \text{cm}, \quad D = 112.80 \text{cm} \]

Gravitational acceleration: \( g = 9.80 \text{m/s}^2 \)

Each weight quality: \( m = 360 \text{g} \)

The gravity of the weight caused the tensile force \( F \) during the measurement, so \( F = mg \). And so the formula 4 means:
The length of two intersections ($\Delta X$) is obtained by gradual deduction method on the basis of above data: $\Delta X = 0.293\text{cm}$.

Result was obtained through the calculation according to formula (5):

$$E = \frac{8mgLD}{\pi d^2 b \Delta X}$$

Table 2. Measured result by laser lever method

| Number of weights | 0  | 1  | 2  | 3  | 4  | 5  | 4  | 3  | 2  | 1  | 0  |
|------------------|----|----|----|----|----|----|----|----|----|----|----|
| $X/\text{cm}$    | 7.18 | 7.45 | 7.72 | 8.01 | 8.29 | 8.60 | 8.32 | 8.02 | 7.73 | 7.46 | 7.20 |
| $(D = 105.11\text{cm})$ | 7.19 | 7.455 | 7.725 | 8.015 | 8.305 | 8.60 |
| $\bar{X}/\text{cm}$ | 4.13 | 4.61 | 5.06 | 5.53 | 6.02 | 6.47 | 5.99 | 5.53 | 5.08 | 4.65 | 4.16 |
| $(D = 174.38\text{cm})$ | 4.145 | 4.63 | 5.07 | 5.53 | 6.005 | 6.47 |
| $X/\text{cm}$    | 6.09 | 6.89 | 7.71 | 8.50 | 9.25 | 10.06 | 9.26 | 8.49 | 7.72 | 6.91 | 6.12 |
| $(D = 297.22\text{cm})$ | 6.105 | 6.90 | 7.715 | 8.495 | 9.255 | 10.06 |

Note: The parameters in this research are the same as the first experiment except the parameter $D$.

$X$ is the reading of the laser reflection point on the scale. $\bar{X}$ is the average of $X$ with same number of weights.

When $D = 105.11\text{cm}$, Then $\Delta X = 0.282\text{cm}$, and then $E = 1.806 \times 10^{11}\text{N/m}^2$.

When $D = 174.38\text{cm}$, Then $\Delta X = 0.465\text{cm}$, and then $E = 1.817 \times 10^{11}\text{N/m}^2$.

When $D = 297.22\text{cm}$, Then $\Delta X = 0.791\text{cm}$, and then $E = 1.821 \times 10^{11}\text{N/m}^2$.

5. Conclusion

The theoretical value of the steel-wire’s Young's modulus is given that $E = 1.820 \times 10^{11}\text{N/m}^2$. The results obtained in two different methods are in close proximity to the theoretical value. Furthermore the result obtained by laser lever method is more accurately than the one by optical lever method. Another conclusion is that the result obtained by laser lever method is even more accurately with a longer $D$.

Conclusions could be drawn from this paper: Firstly, the improved measurement device is a precise, simple and easily operated method to measure Young's modulus. Secondly, the method by laser lever is more accurately and predominantly than by optical lever. Lastly, a longer distance between the plane mirror and scale should be designed during the measuring process.

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