Meson coupling constants at high mass and large $N_c$

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Quark-hadron duality implies that a process described in terms of quark loops should be the hadronic amplitude when averaged over a sufficient number of states. Ambiguities associated with the notion of quark hadron duality can be made arbitrarily small for highly excited mesons at large $N_c$. QCD is expected to form a string like description at large $N_c$ yielding an exponentially increasing Hagedorn spectrum for high mass. It is shown that in order to reconcile quantum-hadron duality with a Hagedorn spectrum, the magnitude of individual coupling constants between high-lying mesons in a typical decay process must be characteristically larger than the average of the coupling constants to mesons with nearby masses. The ratio of the square of the average coupling to the average of the coupling squared (where the average is over mesons with nearby masses) drops exponentially with the mass of the meson. Scenarios are discussed by which such a high precision cancellation can occur.

It is well known that QCD in the limit of an infinite number of colors has an infinite number of infinitely narrow meson states. The narrowness arises from the fact that meson-meson interactions are suppressed by powers of $1/N_c$. Clearly at finite $N_c$, the widths are finite due to decays. At large but finite $N_c$, one expects that mesons of increasing mass become increasingly broad—eventually to the point where they are no longer discernable. It is not immediately clear what, if anything, one can deduce about multi-meson couplings from general considerations of large $N_c$ QCD. In this paper we show that we can infer important qualitative information about couplings of highly excited mesons at large but finite $N_c$. In particular, one can show that the magnitude of the coupling constant for a typical three meson vertex involved in the decay process is much larger than the average of such coupling over a large number of mesons with the same quantum numbers and nearby masses. Indeed, at large $N_c$ the ratio of the square of the average coupling to the average of the coupling squared (where the average is over mesons with nearby masses) drops exponentially with the mass of the meson.

This rather striking result can be derived from two well-founded pieces of physics which are thought to become exact at large $N_c$. The first of these is quark-hadron duality—the general principle that the amplitude for a hadronic process smeared over a sufficient number of states will be equal to the amplitude as calculated from a (perturbative) quark process. The second is the widely accepted view that at large $N_c$ and high excitations mesons can be represented by QCD strings. We should note here that analogous arguments can be formulated for glueballs and one expects the same qualitative features.

We wish to stress, that result here is by no means obvious. Clearly, a Hagedorn spectrum implies that any two-point correlation function of QCD composite operators will have couplings of the external couplings falling exponentially with the mass: the spectral strength is simply shared by many states. For similarly pedestrian reasons, the three-point coupling averaged over many nearby hadrons can also be shown to fall exponentially rapidly with mass. What is remarkable here, is not that average value of the three-point coupling is small, but rather that the small average value does not imply that the typical three-point coupling for any given decay is also small. In fact the typical coupling is radically larger than its average. Thus, there must be extraordinary correlations between the individual couplings to yield such a small average coupling.

Before proceeding with a detailed description of the present problem, a few general comments about the nature of the large $N_c$ limit, quark hadron duality, and the QCD string are in order. One central issue is that of ordering; as is common in large $N_c$ QCD, ordering of limits can play a critical role. In the present case, the key point is that if one first considers the limit of high mass followed by the large $N_c$ limit, the spectrum for some correlation function is described by a QCD continuum without discernable resonances. In contrast, if one first takes the large $N_c$ limit with the large mass limit, then the spectrum will contain infinitely narrow resonances. The difference between these implies that the double limit will not converge uniformly for the spectrum. Here we will be focusing on a regime in which $N_c$ is large enough so that the mesons under consideration are weakly interacting and narrow enough to be treated as well isolated resonances. In practice this means that widths have to be narrow enough compared to the spacing between mesons of fixed quantum number. At the same time the masses under consideration must be high enough so that a) the mass scale is perturbative in the sense that it is large compared with $\Lambda_{QCD}$, and b) the density of states for mesons contains states well into the Hagedorn region of exponential growth. The tension between the two limits is implicit here. It is easy to see that as a problem in mathematical physics one ought to be able to satisfy these conditions with arbitrary accuracy; the extent to which such a regime will turn out to be of relevance for the $N_c = 3$ world is a bit more problematic.
The notion of quark hadron duality can be a bit elusive; formulating it in a precise mathematical way may be quite difficult when modelling a specific process. While the underlying idea is clear—that the two descriptions should become equivalent when averaged over some number of states—it is often unclear how many states need to be included. However, these ambiguities should disappear when going to highly excited states at large $N_c$. To make things concrete we will focus on the correlation function of local currents which are expressible in terms of quark (and/or gluon) operators. General arguments based on asymptotic freedom and the operator product expansion imply that the perturbative quark loop will dominate correlation functions in the exact QCD expression provided that one studies them at large virtuality. It is generally believed that this is consistent only if “semi-local” duality holds in the sense that averages over the full spectral function are over mass scales large compared with $\Lambda_{QCD}$. This is potentially problematic for the physical world in that this might force one to average over a region where discrete mesons have already “melted” into the QCD continuum. By considering a world with sufficiently large $N_c$ we can ensure that this problem does not occur. Moreover, at large $N_c$ there is no difficulty in considering sufficiently high-lying states so that the mass scale is unambiguously perturbative while at the same time having clearly discernable hadronic states.

There are two critical aspects of the QCD string which play a role in the present analysis. The first is the existence of the Hagedorn spectrum for the density of meson states at high excitation:

$$\rho(m) = A(M/T_H)^{-2B} e^{M/T_H} \quad (A, B \text{ constant}) . \quad (1)$$

The second is that the decay width for a meson as represented by a string should be proportional to the mass of the meson (corresponding to the length of the string). This is essentially a uniform probability for the string to break per unit length. Combining this with the fact that the decay amplitude for a meson breaking into two mesons goes as $N_c^{-1/2}$ implies that

$$\Gamma_s(m) \sim \frac{1}{N_c} \frac{\Lambda}{\sigma} m , \quad (2)$$

where $\Lambda$ is the QCD scale and $\sigma$, the string tension, is independent of $N_c$.

The strategy employed here is straightforward. We consider a three-point correlation function for currents with mesonic quantum numbers. Working in the regime discussed above, we evaluate the correlation function two ways: i) as a quark loop, and ii) as the sum over narrow mesons. We extract the spectral strength of the correlation function averaged over a range of masses for the two descriptions. By standard OPE type arguments the first description should be valid at sufficiently large masses and a sufficiently large averaging region. The meson-based description should be valid at sufficiently large $N_c$ and depends on unknown coupling constants. Since there is a common region of validity of the two descriptions, they should match. To proceed further we parameterize the coupling constants in the meson description by an average value over a large number of states times a factor which is state specific. The average value is determined by the matching on to the quark description.

Next we consider the width of a decaying meson based on the assumption that a typical coupling constant is approximately given by the average. The width so computed depends only on known quantities and the density of states. If one takes the density of states to be Hagedorn-like as one expects in a string theory, then one finds that width decreases exponentially with the mass of the meson. However, this violates the expectation that in a string description of hadrons the decay width grows linearly with the mass as given in Eq. (2). Thus we conclude that the assumption that a typical coupling contributing to the decay is equal to the average coupling is incorrect.

To achieve consistency between the string description and quark hadron duality at large $N_c$, there must be very large cancellations of some sort: the typical coupling relevant in the decay will be exponentially larger than the average. To make this more precise, the ratio of the square of the average coupling to the average of the coupling squared drops exponentially with the mass of the meson. In defining this the phases of the coupling are fixed by requiring that the meson field has the same phase as the current acting on the vacuum.

There are some illuminating observations we can make about the structure of a three-point meson interaction. Our concern is the impact that the Hagedorn spectrum has on its form.

We consider a process with the physical interpretation that the current $A$ creates a meson of type $a$ which then decays into mesons of type $b$ and $c$ which are destroyed by the analogous $B$ and $C$ currents (see eq. (14) in the Appendix for a precise mathematical formulation of the amplitude).

The first thing we observe is that the amplitudes for $A$, $B$, and $C$ to create mesons will also be part of the two-point function, and as such their averages can be determined by comparing the two-point function to the analogous quark loop.

For the comparison with the quark loop, we can exploit dimensional analysis to simplify the issues. When we calculate both descriptions of the scattering amplitudes over a small region of momentum and invoke duality to calculate the decay width for the meson, the scale dependence has two sources: the quark loop and the density of meson states. When the mass is very large compared to $\Lambda_{QCD}$ the only scales playing a role in the spectral function for the quark loop arise from the four-momenta of the
mesons. When one integrates over final states to find the total decay width for a meson state, then the only scales left are \( m \), the mass of the initial meson in its rest frame, and any scales which come into play from the density of mesonic states.

Keeping all this in mind, we find for the two point function from the quark loop with two insertions:

\[
\overline{a}_\lambda(s) = \sqrt{\frac{Q_{2\alpha}(s, \lambda)}{\rho_{a\lambda}(s)}}
\]  

(3)

where \( \overline{a}_\lambda(s) \) is the average value for a particle of a state \( \lambda \) to be created with a momentum \( s \), \( Q_{2\alpha}(s, \lambda) \) is the amplitude of the quark loop with two insertions, and \( \rho_{a\lambda}(s) \) is the density of states (See eq. (19)-(20) in the Appendix for precise definitions).

Thus, the amplitude to create any particular state must decrease as the density increases, as the quark loop is not a function of this density, but only of the kinematic parameters and the strong coupling.

We can do the same thing in the case of the three-point function, and find:

\[
Q'_3(s_0, \lambda_0, \ldots) = \\
\overline{F}(s_0, \lambda_0, \ldots) (\rho_{c\lambda_1}(s_1) \rho_{c\lambda_2}(s_2)) \\
\times \bigg( \frac{\overline{a}_{\lambda_1}(s_1) \overline{a}_{\lambda_2}(s_2)}{\overline{a}_\lambda(s_0)} \bigg) \bigg( \rho_{a\lambda_1}(s_1) \rho_{a\lambda_2}(s_2) \bigg)
\]  

(4)

where \( Q'_3 \) is the spectral strength of the quark three point function as defined in eq. (22) in the Appendix. \( F \) is an unknown function proportional to the three-point coupling which we can use to compute a meson decay width, and \( \overline{F} \) is its average (as defined in eq. (21) in the Appendix). This equation, along with what we know from the two-point function, gives us its form as:

\[
\overline{F} = \\
\frac{1}{\sqrt{\rho_{a\lambda_1}(s_0) \rho_{c\lambda_1}(s_1) \rho_{c\lambda_2}(s_2)}} \\
\times \frac{Q'_3(s_0, s_1, s_2)}{\sqrt{Q_{2\alpha}(s_0) Q_{2\beta}(s_1) Q_{2\gamma}(s_2)}}
\]  

(5)

The next step is to relate \( f \) to the decay width for a specific meson.

We have determined \( \overline{F} \) from a relationship with the quark loop. However, the \( f \) in each meson’s decay width is specific to the individual meson, and is not guaranteed to be the same as its average. Anticipating that it will not be, we quantify this difference by defining:

\[
f = R \overline{F}
\]  

(6)

\( R \) quantifies the unknown behavior of the individual meson coupling constants, and is thus far wholly unconstrained.

To find the total cross section for the decay of a particle of mass \( m_0 \), we integrate over all mass states and sum over all spin states for the outgoing particles. The density of states in this integration exactly cancels the density dependence from the coupling constant for the final states, so the full decay width is then:

\[
\Gamma(m_0, \lambda_0) = \frac{1}{\rho_{a\lambda_0}(m_0)} \sum_{\lambda_1, \lambda_2} \int dm_1 dm_2 |R|^2 \\
\times Q(m_0, \lambda_0, \ldots)
\]  

(7)

Here, \( Q \) includes factors from the quark loop and kinematic information. Its exact form (eq. (26) in the Appendix) is unimportant–The key point is its mass dependence.

We now make the observation that, aside from \( \rho_{a\lambda_0}(m_0) \) and \( R \), the only mass scale in the problem is \( m_0 \). As for \( N_c \), the quark loops yield the correct dependence of \( \frac{1}{\sqrt{\rho_{a\lambda_0}(m_0)}} \).

If it were the case that \( R = 1 \), meaning the average coupling constant was equal to each individual coupling constant, the decay width would be:

\[
\Gamma(m_0, \lambda_0) = \frac{1}{N_c \rho_{a\lambda_0}(m_0)} F(m_0)
\]  

(8)

\[
F(m_0) = \sum_{\lambda_1, \lambda_2} \int dm_1 dm_2 Q(m_0, \lambda_0, \ldots)
\]  

(9)

The density function has units of \( m^{-1} \), so \( \frac{1}{\rho_{a\lambda_0}(m_0)} \) has units of \( m \)—the same units as the decay width. Therefore, in this case, \( F(m_0) \) would be dimensionless, and, since it depends only on one scale, completely independent of this scale (but only at an energy much larger than \( \Lambda_{QCD} \) and the quark masses).

If, as is generally believed, the density of states follows the Hagedorn spectrum in Eq. (1), this width must exponentially decrease with mass. In contradiction, string theory predicts a decay width linearly increasing with mass as given in Eq. (2).

It is clear, then, that \( R \) must have a highly nontrivial mass dependence. We cannot derive the full form of \( R \) using these general methods, as it can depend on \( m_0 \), \( m_1 \) and \( m_2 \). For illustration of the effect we will set \( R = R(m_0) \), ignoring any possible dependence on the other two masses. In this case,

\[
\Gamma(m_0, \lambda_0) = \frac{1}{N_c \rho_{a\lambda_0}(m_0)} F(m_0)
\]  

(10)

\( F \) is still independent of \( m_0 \) by the above argument, which means:

\[
\frac{\Gamma}{\Gamma_s} \sim \frac{|R(m_0)|^2}{m_0 \rho_{a\lambda_0}(m_0)}\frac{F \sigma}{\Lambda}
\]  

(11)

Therefore, if \( R \) were only a function of \( m_0 \), its form would
be:

\[ |R(m_0)|^2 \sim m_0 \rho_{\alpha \lambda_0}(m_0) \]  

(12)

Using the Hagedorn spectrum in Eq. [11], \( \rho_{\alpha \lambda_0}(m_0) \) is exponentially increasing and \( |R(m_0)|^2 \) must also be exponentially increasing.

This simple calculation demonstrates that individual meson coupling constants must differ substantially from their average over some small momentum-squared region. The integration range is large enough to contain sufficient states for duality to be valid, but small enough that the amplitude for the quark loop (which we know to be smooth) can be taken to be constant.

In essence, then, the coupling constants within any momentum-squared range large enough for duality to be valid must have exponentially large rapid fluctuations of some type to yield an enormous amount of cancellation.

It is important to understand how this might come about. There are two obvious scenarios: One is that there are nonzero couplings to essentially all of the mesons but there are very strong cancellations. The variable \( f \), which summarizes the coupling information, might have a rapidly oscillating phase, which would serve to make it very small in the average. Another possibility is that the couplings are zero to all except an exponentially small fraction of the energetically allowed final mesons. It can then have “natural” size couplings to this tiny fraction of states. At first sight this second scenario might seem far fetched. However, it is worth recalling how the Hagedorn spectrum emerges in an idealized string theory. It does not do so with an essentially smooth density of states—indeed, for such bands emerging in the real world of string theory.

In summary, we have shown that the decay widths predicted in a string description require individual meson coupling constants to be much larger than the average coupling required by the quark loop through duality. The arguments we have formulated for both requirements are based on fairly general properties of the large-\( N_c \) limit. It remains unclear how relevant these cancellations are for the \( N_c = 3 \) world. We do not with certainty know how this cancellation occurs, but one interesting possibility is that it may be related to the division of the string spectrum into discrete degenerate bands.

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APPENDIX

This appendix gives some of the technical details for the general arguments presented in the paper.

First we present a specific expression for the amplitude for the meson process we are comparing the the quark loop with three insertions, the definition of the spectral strength we use to compare the meson regime to the quark regime, definitions of the averages we take, and an exact expression for the meson decay width. To make the the argument concrete we focus here on some particular class of meson decays: a process in which a particle of type \( a \) decays into two particles of types \( b \) and \( c \). The quantum number of the various types of mesons are left general in this; we will have to make appropriate contractions over the various quantum numbers of the states. These mesons have the quantum number associated with some type of local quark bilinear current. The currents for these mesons are represented by \( A^{(\alpha)} \), \( B^{(\beta)} \), and \( C^{(\gamma)} \). Indices in parentheses indicate that these particles are tensors of arbitrary rank. To connect the decay amplitudes to the currents we consider the three point function for the currents \( A, B \) and \( C \):

\[
iM(s_0, s_1, s_2, \lambda_0, \lambda_1, \lambda_2) = \\
\sum_{N,M,K} \sum_N d_M d_N d_K \langle 0 | A^{(\alpha)}(\lambda_0, N) | a(\lambda_0, N) \rangle \frac{D_a(\alpha)(\mu)}{s_0 - m(N)^2} \frac{D_b(\beta)(\nu)}{s_1 - m(M)^2} \frac{D_c(\gamma)(\xi)}{s_2 - m(K)^2} \langle b(\lambda_1, M) | B^{(\beta)} | 0 \rangle \langle c(\lambda_2, K) | C^{(\gamma)} | 0 \rangle
\]  

(14)
Here, $D_{(\alpha)[\mu]}$ is the tensor form of the propagator (for some general type of propagator), while we have written explicitly the pole structure. $\Delta$ will depend on the mass of the meson, and its form will be determined by the type of meson. $\Lambda^{(\mu)[\nu]}(\xi)$ is the 3-point vertex whose behavior we will analyze. $\lambda_0, \lambda_1,$ and $\lambda_2$ represent the particular state of the particle, including the spin state. $d_M$, $d_N$, and $d_K$ are the degeneracies of the states.

We make the following definitions:

\begin{align*}
\langle 0 | A^{(2)}(s) a(\lambda_0, N) \rangle &\equiv a_{\lambda_0 N} e^{i\phi e^{(\alpha)}} \\
\langle 0 | B^{(2)}(s) b(\lambda_1, M) \rangle &\equiv b_{\lambda_1 M} e^{i\phi e^{(\beta)}} \\
\langle 0 | C^{(2)}(s) c(\lambda_2, K) \rangle &\equiv c_{\lambda_2 K} e^{i\phi e^{(\gamma)}}
\end{align*}

(15)

\[ i \Delta (s_0, \lambda_0, \ldots) = (2\pi i)^3 \left( \sum_N a_{\lambda_0 N} d_N \delta(s_0 - m(N)^2) \right) \left( \sum_M b_{\lambda_1 M} d_M \delta(s_1 - m(M)^2) \right) \left( \sum_K c_{\lambda_2 K} d_K \delta(s_2 - m(K)^2) \right) f(s_0, \lambda_0, \ldots) \]

(17)

To do this, we define pole prescriptions for the propagators by making the replacement $s \rightarrow s \pm i\epsilon$.

We then see that the spectral strength is an asymmetric combination of amplitudes made with these replacements:

\[ i \Delta (s_0, s_1, s_2, \lambda_0, \ldots) = i \left[ M (s_0 + i\epsilon, s_1 + i\epsilon, s_2 + i\epsilon, \lambda_0, \ldots) \right. \]

\[ \left. - M (s_0 + i\epsilon, s_1 + i\epsilon, s_2 - i\epsilon, \lambda_0, \ldots) \right] \]

(18)

The quark loop amplitude will also be a function of these same $s$'s, with the precise dependance determined by the choice of frame, so we should be able to make the same replacements and therefore extract an expression corresponding to $\Delta (s_0, s_1, s_2)$. We will define $Q_3 (s_0, s_1, s_2)$ as the amplitude of a quark loop with three insertions, $Q_2 (s)$ is similarly defined as the amplitude of a quark loop with two insertions. $i \Delta_Q$ will then depend on $Q_3$ in the same way as $i \Delta$ depends on $M$.

We now return to the $\Delta$ function defined above, and switch from the individual meson values to the average values for the functions $f$, $a_{\lambda N}$, $b_{\lambda M}$, and $c_{\lambda K}$, defined as the values they would each take if all four were assumed to be constant over the integration range.

With $\rho_{a\lambda_0} (s_0)$, $\rho_{b\lambda_1} (s_1)$, and $\rho_{c\lambda_2} (s_2)$ as the densities of hadron states, we define these average functions as follows, with $R_0 \equiv (s_0, s_0 + \Delta s_0), R_1 \equiv (s_1, s_1 + \Delta s_1)$, and $R_2 \equiv (s_2, s_2 + \Delta s_2)$ defining the small momentum region over which we are comparing the quark regime to the hadronic regime:

\[ \rho (s) \Delta s = \sum_{N \in R} d_N \]

(19)

\[ \overline{a_{\lambda_0}} (s_0) \equiv \left( \sum_{N, s(N) \in R_0} a_{\lambda_0 N} \frac{d_N}{\rho_{a\lambda_0} (s_0) \Delta s_0} \right), etc... \]

(20)

Note here that $\frac{d_N}{\rho (s) \Delta s} \rightarrow \frac{d_N}{\sum_{N' \in R} d_{N'}}$, which, for a smooth $d_N$, is just equal to one over the number of states in our small integration range.

The integral over a range of states allows us to equate
the two descriptions as follows:

\[ Q'_3(2\pi i)^3 \equiv i\Delta_Q (s_0, \lambda_0, \ldots) = \]
\[ (2\pi i)^3 \int (s_0, \lambda_0, \ldots) (\rho_{\lambda\lambda_0}(s_0) \delta_{\lambda_0}(s_0)) \]
\[ \times (\rho_{\lambda\lambda_1}(s_1) \delta_{\lambda_1}(s_1)) (\rho_{\lambda\lambda_2}(s_2) \delta_{\lambda_2}(s_2)) \]  
(22)

We can trivially repeat the above calculation for the two-point function to find the \( \overline{\pi}_X, \overline{\lambda}, \) and \( \overline{\pi}_X \) functions:

\[ \overline{\pi}_X(s) = \sqrt{\frac{Q_{2\alpha}(s, \lambda)}{\rho_{\alpha\lambda}(s)}}, \text{etc...} \]  
(23)

\[ \Gamma(m_0, \lambda_0 \to m_1, \lambda_1; m_2, \lambda_2) = |R|^2 \frac{Q}{\rho_{\alpha\lambda_0}(m_0) \rho_{\alpha\lambda_1}(m_1) \rho_{\alpha\lambda_2}(m_2)} \]  
(25)

\[ Q(s_0, s_1, s_2, \lambda_0, \lambda_1, \lambda_2) \equiv \frac{2m_0m_1m_2}{2\pi m_0^2} \left( \frac{|Q'_3(s_0, s_1, s_2)|^2}{Q_{2\alpha}(s_0)Q_{2\beta}(s_1)Q_{2\gamma}(s_2)} \right) \left| \epsilon^{(\mu)} \epsilon^{(\nu)} \epsilon^{(\xi)} \right|^2 \]
(26)

where \( p \) is the magnitude of the momentum of either particle relative to the CM.

\[ \epsilon^{(\mu)} \epsilon^{(\nu)} \epsilon^{(\xi)} \]

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The amplitude of the decay for an individual meson is:

\[ iM(m_0, \lambda_0 \to \{m_f, \lambda_f\}) = \epsilon^{(\alpha)} \epsilon^{(\beta)} \epsilon^{(\gamma)} \Lambda^{(\alpha)(\beta)(\gamma)} \]  
(24)

with \( \Lambda^{(\alpha)(\beta)(\gamma)} \) related to the \( f \) function as in [14].

We encode the difference between \( f \) and its average by defining \( \int \equiv Rf \), and using the fact that \( \rho(s) = \frac{1}{\rho(m)} \), the decay width for a particle of mass \( m_0 \) to split into particles of masses \( m_1 \) and \( m_2 \) in the CM frame is just:

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