Exploring the “Island of Inversion” by in-beam $\gamma$-ray spectroscopy of the neutron-rich sodium isotopes $^{31,32,33}\text{Na}$

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Abstract

The structure of the neutron-rich sodium isotopes $^{31,32,33}\text{Na}$ was investigated by means of in-beam $\gamma$-ray spectroscopy following one-neutron knockout and inelastic scattering of radioactive beams provided by the RIKEN Radioactive Ion Beam Factory. The secondary beams were selected and separated by the fragment separator BigRIPS and incident at $\approx 240$ MeV/nucleon on a natural carbon (secondary) target, which was surrounded by the DALI2 array to detect coincident de-excitation $\gamma$-ray rays. Scattered particles were identified by the spectrometer ZeroDegree. In $^{31}\text{Na}$, a new decay $\gamma$-ray was observed in coincidence with the known $(5/2^+ \rightarrow 3/2^+)^{(+)}$ transition, while for $^{32,33}\text{Na}$ excited states are reported for the first time. From a comparison to state-of-the-art shell-model calculations it is concluded that the newly observed excited state in $^{31}\text{Na}$ belongs to a rotational band formed by a $2p2h$ intruder configuration within the “Island of Inversion.”

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In the traditional perception of the shell model, the magic numbers, which were first reproduced correctly for stable nuclei by Mayer and Jensen \cite{1, 2} and define the shell closures, are recognized to be valid globally across the Segré chart. Evidence for a sudden, unanticipated change of shell structure in the neutron-rich sodium isotopes came from anomalously low masses measured for the isotopes $^{31}$Na and $^{32}$Na \cite{3}. These increased binding energies were seen as an indication for the quenching of the $N = 20$ magic number in this region of the Segré chart \cite{4}.

On the theoretical side, the shell-model study by Warburton et al. \cite{5} identified the nine nuclei with $Z = 10–12$ and $N = 20–22$ as a region in which $v(sd)^{-2}(fp)^2(2\hbar\omega)$ intruder configurations are more tightly bound than the normal $(0\hbar\omega)$ configurations, making them the ground states (g.s.). This region is since then referred to as the “Island of Inversion”. The inversion of the configurations can be attributed to a narrowing of the $N = 20$ neutron shell gap between the $d_{3/2}$ and $f_{7/2}$ orbitals from a lack of protons in the $d_{5/2}$ shell in neutron-rich nuclei \cite{6}.

Further experimental evidence for the erosion of the $N = 20$ shell closure is given by the small $E(2_1^+)$ energy and large $B(E2; 0_{g.s.}\rightarrow 2_1^+)$ value of $^{32}$Mg \cite{7, 8} and meanwhile a multitude of experiments have addressed the evolution of the $N = 20$ shell near the “Island of Inversion”. Despite this exceptional attraction, because of experimental restrictions, only for the even-$Z$ isotopes (Ne, Mg) information on excited states has been obtained from in-beam $\gamma$-ray spectroscopy beyond $N = 20$ \cite{9–15}.

Concerning the chain of the odd-$Z$ sodium isotopes, the experimental knowledge is very limited. While the ground-state spin is firmly established up to $N = 20$ \cite{16, 17}, for heavier isotopes, only for certain nuclei the parity and ranges of spin $J$ have been experimentally established. For instance for $^{32}$Na, $J \leq 4^−$ has been deduced from $\beta$-delayed neutron emission into $^{31}$Mg \cite{18}. Information on excited states has been merely extended to the $N = 20$ nucleus $^{31}$Na by means of intermediate-energy Coulomb excitation \cite{19} and inelastic scattering on a liquid hydrogen target \cite{13}. In both cases, the $(5/2_1^+ \rightarrow 3/2_{g.s.}^{(+)}$ transition was observed at energies of 350(20) and 370(12) keV, respectively.

Here, we report on the observation of excited states in the sodium isotopes with $N = 20–22$ ($^{31,32,33}$Na) following inelastic scattering and one-neutron removal reactions on a natural carbon target at $\approx 240$ MeV/nucleon. The experiment was performed at the Radioactive Ion Beam Factory (RIBF) \cite{20}, operated by the RIKEN Nishina Center and the Center for Nuclear Study, University of Tokyo. A high-intensity $^{48}$Ca beam at 345 MeV/nucleon was incident on a 20-mm thick rotating Be target \cite{21} at the focus F0 of the fragment separator BigRIPS \cite{22}. From the emerging
FIG. 1. (Color online) Particle identification before the secondary target for a BigRIPS setting in which $^{32}$Na was the main component of the cocktail beam.

Projectile fragmentation products the neutron-rich sodium isotopes with mass numbers $A = 31–34$ were selected and separated using the $B\rho - \Delta E - B\rho$ method in the first stage of BigRIPS with the aid of an achromatic aluminum energy degrader of 15-mm thickness located at the dispersive focus F1. In the second stage of BigRIPS (from focus F3 to focus F7), the transmitted fragmentation products were identified event-by-event employing the $\Delta E - B\rho -$velocity method. The energy loss $\Delta E$ was measured by means of an ionization chamber located at the focus F7 of BigRIPS. The magnetic rigidity $B\rho$ was obtained by position measurements with parallel plate avalanche counters [23] at the dispersive focus F5. Two thin plastic scintillators of 1- and 3-mm thickness were placed at F3 and F7 to measure the time-of-flight (TOF). The path length between both detectors was 47 m. Figure 1 presents an exemplary particle identification plot, showing clear separation between different nuclides in charge $Z$ and mass to charge ratio $A/Z$.

Three settings with different $B\rho$ values were applied to the BigRIPS fragment separator for the study of $^{31}$Na, $^{32}$Na, and $^{33}$Na by one-neutron knockout reactions and inelastic scattering. After the selection and identification with BigRIPS, which was operated in its full momentum acceptance mode of $\Delta p/p = \pm 3\%$, the secondary beams were incident on a carbon target with a 2.54 g/cm$^2$ thickness and a diameter of 30 mm at the focus F8. The energy at midtarget varied from $\sim 230$ MeV/nucleon to $\sim 250$ MeV/nucleon for the different sodium isotopes. The energy loss in the secondary target amounted to $\sim 14\%$ of the incident beam energy.

The DALI2 array [24], a NaI(Tl) based $\gamma$-ray spectrometer consisting of 180 individual crystals, surrounded the secondary target for $\gamma$-ray emission angles ranging from $\vartheta_\gamma = 11^\circ$ to $\vartheta_\gamma = 147^\circ$
in the laboratory system. A full energy peak efficiency of 15% at a γ-ray energy of 1332.5 keV was measured with a stationary source, in accordance with our GEANT4 [23] simulations. No add-back was performed.

Reaction products emerging from the secondary target were selected and identified using the ΔE–Bρ–TOF method on an event-by-event basis by the spectrometer ZeroDegree [26, 27] from focus F8 to focus F11. The angular and momentum acceptances were \(\sim 80 \times 60\) mrad\(^2\) and ±4%, respectively. The energy loss ΔE was obtained from an ionization chamber placed at the final focus F11, the magnetic rigidity Bρ was measured with parallel plate avalanche counters placed at the dispersive foci F9 and F10, and the TOF was detected between two thin plastic scintillators of 1-mm thickness mounted at F8 and F11 with a flight path length of 37 m. The difference in velocity Δβ before and after the secondary target was used for the selection of particles passing solely through the secondary target. Particles passing through the target frame or missing the target, in total less than 2 % of the secondary beam intensities, could be clearly identified.

In the present work, excited states in \(^{31-33}\)Na, populated in one-neutron knockout and inelastic scattering, are reported. As the results were obtained “parasitic” to main experiments [15, 28], the momentum distributions of the one-neutron knockout and scattered sodium isotopes were not centered in the spectrometer ZeroDegree and cropped to a large extent; the overall transmissions were much lower than 90%.

Gamma rays emitted from the fast moving reaction products (β \(\approx 0.6\)) were Doppler corrected taking into account the lifetimes of the excited states, as delineated in Ref. [29]. The uncertainty of the lifetime was included in the uncertainty of the reported γ-ray transition energies. Figure 2 displays the Doppler corrected γ-ray energy spectra for the one-neutron removal reactions from \(^{32}\)Na. Two distinct transitions are visible at 376(4) keV and at 787(8) keV. The former is in agreement with previous observations of 350(20) keV [19] and 370(12) keV [13] and is generally interpreted to be the \((5/2^+_1) \rightarrow 3/2^+_{g.s.}\) transition. Applying a gate ranging from 325 to 425 keV on this transition shows that both observed decays are in coincidence, as illustrated in the lower panel of Fig. 2. From the observed coincidence and from the later discussed comparison to shell-model calculations, we concluded that the observed transition at 787(8) keV is the \((7/2^+_1) \rightarrow (5/2^+_1)\) decay. No indication for a direct transition from the \((7/2^+_1)\) state to the ground state was found. This is consistent with the shell-model prediction of Ref. [19] for a strong \(B(M1;7/2^+_1 \rightarrow 5/2^+_1)\) causing the \((7/2^+_1)\) state to decay with an intensity of 95% into the \((5/2^+_1)\) state. From the intensity ratio of the two transitions it follows that 61(6) % of the \((5/2^+_1) \rightarrow 3/2^+_{g.s.}\) transition are because
FIG. 2. The top panel (a) displays the Doppler corrected γ-ray energy spectrum in coincidence (±5 ns) with the one-neutron removal from $^{32}$Na. In the bottom panel (b), a γ-ray energy cut between 325 and 425 keV on the $(5/2^+_1 \rightarrow 3/2^+_{g.s.})$ transition in $^{31}$Na was applied in addition for events of a γ-ray fold greater than one.

of feeding from the $(7/2^+_1)$ state.

For $^{32}$Na, a γ-ray decay was found at 569(12) keV after inelastic scattering. It is shown in Fig. 3.

Excited states of $^{33}$Na were detected from inelastic scattering and one-neutron knockout of $^{34}$Na. Gamma-ray transitions were observed at 476(12) keV for the former and at 447(13) keV for the latter case, respectively. As we assume that the two observed peaks belong to the same transition, it leads to a combined value of 467(13) keV. The Doppler corrected γ-ray spectra are shown in Fig. 4.

While the one-neutron removal reactions exhibited very low background, the Doppler corrected γ-ray energy spectra for inelastically scattered $^{32}$Na and $^{33}$Na, shown in Fig. 3 and Fig. 4(a), respectively, were dominated by an exponentially declining distribution that likely originated from
FIG. 3. Doppler corrected $\gamma$-ray energy spectrum in coincidence ($\pm 5$ ns) with inelastic scattering of $^{32}$Na. The $\gamma$-ray emission angle was restricted to $\vartheta_\gamma < 90^\circ$.

FIG. 4. Doppler corrected $\gamma$-ray energy spectra in coincidence ($\pm 5$ ns) with inelastic scattering of $^{33}$Na (a) and one-neutron removal (b). For the former the $\gamma$-ray emission angle was restricted to $\vartheta_\gamma < 55^\circ$. 
TABLE I. Summary of observed γ-ray transition energies in this work for the isotopes $^{31-33}$Na and proposed spin and parity assignments. For $^{31}$Na, the results of Refs. [13, 19] are shown for comparison.

| Isotope | Transition | Experimental Transition Energies (keV) |
|---------|------------|---------------------------------------|
|         | $J^\pi_i \rightarrow J^\pi_f$ | This work | Ref. [13] | Ref. [19] |
| $^{31}$Na | $(5/2_1^+ \rightarrow 3/2_\text{g.s.}^{(-)}$ | 376(4) | 370(12) | 350(20) |
| $^{31}$Na | $(7/2_1^+ \rightarrow (5/2_1^+)$ | 787(8) | | |
| $^{32}$Na | | 569(12) | | |
| $^{33}$Na | $(5/2_1^+, 3/2_1^+) \rightarrow (3/2_\text{g.s.}^{(+)} , 5/2_\text{g.s.}^{(-)}$ | 467(13) | | |

atomic processes during the slowing down of the fragments in the secondary target. The main component of this background originated from the stationary target and was thus not Doppler shifted in the laboratory system. Therefore, the observation limit for low-energy γ-ray transitions shifted as a function of the γ-ray emission angle $\vartheta_\gamma$ with respect to the velocity vector of the emitting nucleus. Gamma rays emitted toward backward (forward) angles are Doppler shifted to lower (higher) energies in the laboratory system. Thus, low energy γ-ray transitions are best separated from the background at forward $\vartheta_\gamma$ angles, when their energy is Doppler shifted to values above the atomic background. For scattered $^{32}$Na particles an angle cut of $\vartheta_\gamma < 90^\circ$ was applied; for $^{33}$Na the cut was set accordingly to $\vartheta_\gamma < 55^\circ$ because of the lower excitation energy at 476(12) keV. Gamma-ray transition energies below $\sim$400 keV could not be measured in the inelastic channels, even for the DALI2 detectors with the lowest $\vartheta_\gamma$ angles.

We will now turn to the discussion part of our experimental results, which are summarized in Table I. The low-lying levels of $^{31}$Na have been predicted in previous works via shell-model calculations [19, 30, 31]. In the case of Pritychenko et al., the neutron configuration was restricted to a pure 2$p$–2$h$ (2$h_\omega$) intruder configuration with allowed configurations of $(0d_{5/2})^6 (0d_{3/2}, s_{1/2})^4 (0f_{7/2}, 1p_{3/2})^2$ for neutrons and of $0d_{5/2}^2$ and $0d_{5/2}^2 (1s_{1/2}, 0d_{3/2})$ for protons, respectively [19]. Utsuno et al. performed Monte Carlo shell-model calculations, allowing for the use of a much wider model space (the entire sd shell and the $0f_{7/2}$ and the $1p_{3/2}$ orbits) and an unrestricted mixing of normal and intruder configurations [31, 32]. The valence space by Caurier et al. included the full sd shell for protons in a calculation of the normal configuration and in addition the full pf shell for neutrons in a 2$h_\omega$ intruder configuration calculation, but did not allow for any configuration...
FIG. 5. Comparison of experimental excitation energies of $^{31}$Na obtained in this work, by Elekes et al. [13], and by Pritychenko et al. [19], with shell-model results by the latter, by Utsuno et al. [31], and by Caurier et al. [30]. In the latter case, normal (N) and intruder (I) configurations are shown. The connecting lines between the $3/2^+$ (long-dashed line), $5/2^+$ (short-dashed line), and $7/2^+$ (dotted line) levels are drawn to guide the eye and the newly observed state is indicated by the arrow. The vertical, dash-dotted line separates the experimental results from the shell-model calculations.

Figure 5 compares the above-mentioned shell-model calculations with the experimental data. Good overall agreement is achieved only for the intruder calculations. On the other hand, the normal configuration by Caurier et al. not only predicts the wrong ground-state spin, but puts the $7/2^+$ level above 4 MeV, at variance with our observations.

For the odd-odd nucleus $^{32}$Na, several states very close in energy below 200 keV have been predicted [34]. As associated low-energy transitions were below our observation limit, no further conclusions could be drawn on the nature of the transition energy at 569(12) keV.

To investigate the nature of the $\gamma$-ray transition in $^{33}$Na, it can be compared to the known low-lying level systematics of particle-bound odd-$A$ sodium isotopes, displayed in Fig. 6. Shown are all known ground, first, and second excited states. The two lowest energy states have either spin $3/2^+$ or $5/2^+$ and all second excited states lie well above 1 MeV. Our experimental setup was insensitive to transition energies below 200 keV for the knockout channels. If the observed transition energy at 467(13) keV was a decay from the second to the first excited state, the level energy would be well below 700 keV, unlike all other Na isotopes. We therefore propose that the
is in contrast to our experimental results of 467(13) keV. Furthermore, transitions from the 7
excited state. However, the levels are almost degenerate in energy, being only 59 keV apart, which
right-hand side in Fig. 6, agree indeed with the spin-parity systematics for the ground and first
ground and first excited state in a 0p0h normal configuration. These calculations, shown on the
right-hand side in Fig. 6, agree indeed with the spin-parity systematics for the ground and first
excited state. However, the levels are almost degenerate in energy, being only 59 keV apart, which
is in contrast to our experimental results of 467(13) keV. Furthermore, transitions from the 7/2
or 3/2 levels to the 5/2 or 3/2 levels are predicted at around 1 MeV, which do not agree with
our observed transition energy as well.

In conclusion, we have observed a γ-ray transition in coincidence to the known (5/2) →
3/2 transition in Na. From a comparison to state-of-the-art shell-model calculations we con-
cluded that this (7/2) → (5/2) transition belongs to an intruder configuration within the “Island
of Inversion”. In addition, for Na and Na γ-ray transitions have been observed for the first time.
With the recent experimental progress owed to the commissioning of the new-generation facility
RIBF, in-beam γ-ray spectroscopy may now be applied to a much wider region of the “Island
of Inversion” and possibly even beyond its neutron-rich border lines. This in turn, necessitates
further theoretical fundamental examinations on their exact locations. Finally, we would like to

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\begin{array}{c|c|c|c|c|c|c|c}
\text{Mass A} & 21 & 23 & 25 & 27 & 29 & 31 & 33 \\
\text{Experiment} & 1/2^+ & 3/2^+ & 5/2^+ & 3/2^+ & 5/2^+ & 3/2^+ & 1/2^+ \\
\text{Nummela} & 7/2^+ & 5/2^+ & 3/2^+ & 3/2^+ & 3/2^+ & 3/2^+ & 7/2^+ \\
\end{array}
\]

**FIG. 6.** Excitation energy of the first and second excited states in particle-bound odd-A sodium isotopes. The data are from this work (indicated by the arrows) and Refs. [16, 35–38]. The 3/2+ (5/2+) states are connected by dashed (dotted) lines to guide the eye. In addition, the shell-model calculations for Na isotopes, suggest a 5/2+ level transition to the 3/2+ ground state or vice-versa.

Shell-model calculations have been performed by Nummela et al. [39] for Na that see the
ground and first excited state in a 0p0h normal configuration. These calculations, shown on the
right-hand side in Fig. 6, agree indeed with the spin-parity systematics for the ground and first
excited state.
remark that with the new-generation facility RIBF having gone online, a new-generation $\gamma$-ray spectrometer [40], currently under development, is required to optimally exploit the vast potential for in-beam $\gamma$-ray spectroscopy of fast radioactive nuclear beams.

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