Regionalized location obfuscation mechanism with personalized privacy levels

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Abstract—Global Positioning Systems are now a standard module in mobile devices, and their ubiquity is fueling the rapid growth of location-based services (LBSs). This poses the risk of location privacy disclosure. Effective location privacy preservation is foremost for various mobile applications. Recently two strong privacy notions, geo-indistinguishability and expected inference error, are proposed based on statistical quantification. They are shown to be complementary for limiting the leakage of location information. In this paper, we argue that personalization means regionalization for geo-indistinguishability, and we propose a regionalized location obfuscation mechanism, DPIVE, with personalized utility sensitivities. This substantially corrects the differential privacy problem of PIVE framework proposed by Yu, Liu and Pu on ISOC Network and Distributed System Security Symposium (NDSS) in 2017. Since PIVE fails to provide differential privacy guarantees on adaptive protection location set (PLS) as pointed in our previous work, we develop DPIVE with two phases. In Phase I, we determine disjoint sets by partitioning all possible positions such that different locations in the same set share the common PLS. In Phase II, we construct a probability distribution matrix by exponential mechanism in which the rows corresponding to the same PLS have their own sensitivity of utility (diameter of PLS). Moreover, we improve DPIVE with refined location partition and fine-grained personalization, in which each location has its own privacy level on two privacy control knobs, minimum inference error and differential privacy parameter. Experiments with two public datasets demonstrate that our mechanisms have the superior performance typically on skewed locations.

Index Terms—Differential privacy, geo-indistinguishability, inference attack.

1 INTRODUCTION

WITH the rapid development of mobile Internet and digital communications, many mobile devices are equipped with Global Positioning Systems (GPSs) and other location-sensing technology. With GPS, mobile users can sense their location and get some location-based services (LBSs), such as picking up express delivery, finding nearby restaurants, and so on. In recent years, LBSs have been accepted by a large number of users and play an indispensable role in people’s livings. However, with the wide applications of LBSs, severe privacy concerns are raised for most users. With benefiting from LBSs, users’ locations are continuously collected by untrusted service providers, which leads to the disclosure of location privacy, such as working place and habitation. Then the adversary can attack more sensitive information of the users based on their background knowledge. Therefore, how to protect user’s location privacy in LBSs is an urgent problem to be solved.

Geo-indistinguishability and expected inference error are two privacy notions recently used for location privacy protection. Geo-indistinguishability deriving from differential privacy ensures that for two arbitrary locations within a certain distance, their produced pseudo-locations are similarly distributed. Then, an adversary with any prior knowledge can not infer the true location by observing the pseudo-location. The expected inference error reflects the accuracy of the adversary to guess the true location by observing the pseudo-location and using available prior knowledge.

Since 2015 some authors [1], [2] have proposed that expected inference error and geo-indistinguishability can be combined to protect location privacy. Later Yu et al. [3] formally study the relationship between the two privacy notions and verify that they are complementary. Indeed, geo-indistinguishability only limits the adversary’s posterior knowledge after observing the pseudo-location, but does not consider the adversary’s inference attack based on prior knowledge, such as the distance between the inferred and true location, while the expected inference error does not consider the constraint on the posterior information derived from the release of pseudo-locations. For this, they propose PIVE, a two-phase dynamic differential location privacy framework. In Phase I, it searches for the protection location set (PLS) satisfying the privacy requirements on each (true) location, and in Phase II, it publishes the pseudo-location through the differential privacy mechanism. However, the PLS of each location depends on its local situation. Then, the PLSs generally have different diameters and even intersect with each other. Thus, the proof of differential privacy for PIVE is problematic with respect to geo-indistinguishability. Our recent paper [4] confirms this differential privacy problem and proposes a couple of correction approaches with analyzing theoretically their satisfied privacy characteristics. The constructive privacy framework is still left open.

To finish the problems in PIVE pointed above, we should...
ensure that all PLSs have the same diameter if any two of them have the possibility of intersecting with each other, or all PLSs can have different diameters if any two of them do not intersect with each other (which implies regionalization of PLSs). Following the latter, we should address three challenges as follows: 1) satisfying the personalization of sensitivity and improving the data utility, 2) achieving the differential privacy inside each PLS and also between PLSs, and 3) allowing for the scenario with skewed locations. For this, we propose DPIVE a regionalized mechanism in this paper. Given the relevant privacy parameters, the set of entire locations involved is divided into multiple disjoint PLSs, and the locations in the same PLS share the same diameter. Each PLS ensures the lower bound of the inference error for every apriori location inside, and the locations within the same PLS are strongly geo-indistinguishable, while locations across different PLSs satisfy weak differential privacy. We first propose QK-means, a 2-D method replacing the former approach based on 1-D Hilbert curve for region partitioning, which is much helpful to reduce the service quality loss. Besides, we consider the more general scenario that allows users to personalize two privacy control knobs on each location and we develop PDPIVE mechanism that meets the personalized requirements of location privacy.

In this paper, we introduce regionalization to the task of location obfuscation. Our proposed regionalized framework DPIVE achieves differential privacy protection and its personalization PDPIVE satisfies user’s specified privacy level on each apriori location. The main contributions are as follows.

(1) We consider the scenario where the user wants to protect the privacy of her/his true location by reporting a pseudo-location in a set of discretized locations and may have potential requirements of geo-indistinguishability and expected inference error for all positions. The goal here is to ensure that the privacy mechanism designed satisfies differential privacy and inference error threshold as required. Our proposal DPIVE utilizes regionalization for PLSs to implement differentially private location obfuscation to achieve this goal.

(2) We then extend the above proposal to the scenario where each location has its own privacy level on two privacy control knobs. For this purpose, we develop PDPIVE a personalized privacy framework by constructing the QK-means algorithm that has much potential for more PLSs with smaller diameters in the 2-D space.

(3) We carry out a series of experiments on two public datasets. The results demonstrate that our DPIVE approach is more efficient than the existing mechanisms in particular on the privacy protection of skewed locations and PDPIVE also exhibits high quality of obfuscation.

The remainder of this paper is structured as follows. In Section 2, we conduct a survey of related work. Section 3 introduces some necessary backgrounds. Section 4 describes the proposed privacy framework. Section 5 provides the QK-mean clustering technique. Section 6 designs the personalized privacy framework. Experimental results are presented in Section 7. Finally, we conclude this paper in Section 8.

2 RELATED WORK

The location privacy issue has been extensively studied in the past decade [5]. Many techniques are proposed, such as cloak-region, dummy location, and cryptographic solutions. The notion $k$-anonymity is the most widely used anonymous method for protecting location privacy in the literatures. This technique produces $k - 1$ dummy locations properly selected such that the attacker can not infer which is the real location among the set of $k$ locations [6]. Wang et al. [7] formalize optimization problem for cloaking area generation, which utilizes users’ footprints to decide the cloaking areas with privacy requirements expressed through both $k$-anonymity and entropy based metrics. However, only using anonymous method can not achieve good protection to a wide range of data and is vulnerable to background knowledge attack [8]. Cryptographic is suitable for multiple parties but induces extra computational cost, and the availability of data decreases greatly [8], [9].

Expected inference error is a stronger privacy notion first proposed by Shokri et al. [10], which is a natural way to measure the location privacy by the expected distance between the guessed location by the adversary and the real location. Then a number of location obfuscation mechanisms have been developed relying on this notion. In [11], an optimal obfuscation mechanism for achieving maximum level of privacy is designed by solving a linear program with constraint on the service quality loss. The expected inference error can resist against the Bayesian attack to some extent, however, it does not take into account the constraint on the posterior information gain obtained by the reported pseudo-locations [6].

Andres et al. [9] introduce geo-indistinguishability, a strong concept based on differential privacy, which ensures that any two geographically close locations have similar probability distributions on any pseudo-location so that the adversary can not infer the true location by observing the pseudo-locations. Due to this, several location privacy protection mechanisms have been proposed recently [12], [13], [14]. Xu et al. [12] propose a geo-indistinguishability based framework to preserve the privacy of individuals on ridesharing platforms. Tao et al. [13] investigate privacy protection for online task assignment with the objective of minimizing the total travel distance.

The scheme in [14] uses linear programming to minimize global expected service quality loss averaged over all locations, with a uniform privacy parameter for geo-indistinguishability. Later, Some authors [11], [2] propose to combine the two privacy notions using linear programming. Particularly, a joint mechanism [11] is applied in the mobile crowdsourcing for optimal task allocation [15]. Recently, Yu et al. [3] point out that the formulation above [11], [14], [15] uses uniform differential privacy parameter and emphasizes the globally average performance on privacy/quality metrics over all locations. For this, they formally examine the relationship between the two privacy notions and propose PIVE mechanism with adding user-defined lower bound of inference error. PIVE is a two-phase dynamic differential location privacy framework that focuses on local performance of privacy protection. In phase I, it searches for the protection location set (PLS) satisfying...
user’s privacy requirements for the true location, and in phase II, it publishes the pseudo-location through the exponential mechanism. However, we find that PIVE fails to provide provable privacy guarantee on adaptive protection location sets as claimed, and we discuss this problematic framework in detail in [4]. In short, the diameter of the PLS obtained in PIVE by adaptive search around each apriori location is generally different and there exist intersection cases for PLSs, which leads to that PIVE cannot theoretically preserve differential privacy on the PLSs. We also propose a pair of possible correction approaches and analyze their respective privacy characteristics. Particularly, the results on geo-indistinguishability (or differential privacy) within each region and between the points of different regions are presented therein.

In this paper, we are intended to correct the problematic construction of PIVE. Given the relevant privacy parameters and conditions, the entire location set is partitioned into multiple disjoint parts. Each part is assigned as the PLS for all apriori locations inside and ensures the lower bound of the inference error. Thus, the locations within the same PLS are protected with strong differential privacy while those across different PLSs protected with weak differential privacy. Our proposed DPIVE mechanism allows users to define their own privacy level for both phases. Besides, for the personalization of two privacy notions at each location, we implement the location obfuscation mechanism PDPIVE theoretically and practically.

3 OVERVIEW

3.1 Differential Privacy

Differential privacy (DP) is a strict privacy concept that provides provable privacy protection for users. Regardless of the adversary’s prior knowledge, it ensures that any adversary can not determine the presence of a particular individual from the processed data set. Formally,

Definition 1 ($\epsilon$-DP [16]). Given any two neighboring datasets $D$ and $D'$ in a universe $D$ that differ on one element, for any set of outcomes $\Omega$, a randomized mechanism $\mathcal{M}$ gives $\epsilon$-DP if the probability distribution of the mechanism output on $D$ and $D'$ is bounded by:

$$
\frac{Pr(\mathcal{M}(D) \in \Omega)}{Pr(\mathcal{M}(D') \in \Omega)} \leq e^\epsilon,
$$

where the privacy parameter $\epsilon$ represents the privacy level to be achieved.

Definition 2 (Sensitivity [17]). Let $D$, $D' \in D$ be any pair of neighboring datasets that differ on one element. The sensitivity of a function $f : D \rightarrow \mathbb{R}^d$ is given by

$$
\Delta f = \max_{D, D' \in D} \| f(D) - f(D') \|_1,
$$

which means the $L_1$ norm of the maximal change on the output of $f$ when altering any record to $D \in D$.

For functions where the output space is non-numeric, the exponential mechanism is widely used to achieve differential privacy.

Definition 3 (Exponential Mechanism [18]). Given a scoring function $f : D \times R \rightarrow \mathbb{R}$ with $R$ representing the collection of output range of a query function, the exponential mechanism $\mathcal{M}(D, f)$ outputs $r \in R$ with probability proportional to $\exp \left( \frac{f(D, r)}{2\Delta f} \right)$.

3.2 Bayesian Adversary Model

As all the location-based service (LBS) providers require the access permission to users’ location data, the location privacy is potentially disclosed to untrusted entities. Knowing user’s locations, an adversary can perform a broad spectrum of attacks. Thus, ensuring location privacy is foremost for LBS applications.

In LBS, users usually send their true locations to the service provider to get services. However, the service provider is often an untrusted entity and may disclose users’ location privacy. For this, a common method is location perturbation, which generates a pseudo-location based on the true location and the user sends it to the server.

Following [14], [19], we suppose that the discretized location set $\mathcal{X}$ represents the user’s possible locations. An obfuscation mechanism takes the user’s real location $x$ from $A$ as input and randomly chooses a pseudo-location $x'$ from $\mathcal{X}$ with the probability distribution $f(x'|x)$:

$$
f(x'|x) = \Pr(O = x'|A = x), \quad x, x' \in \mathcal{X}.
$$

In general, the objective of obfuscation mechanisms is mainly to design suitable probability distribution $f(\cdot|\cdot)$ in the sense of some metrics.

As before [20], [11], [3], we assume that the adversary has prior knowledge about user’s location, which can be regarded as background knowledge to perform inference attacks. The adversary usually collects background knowledge by building a prior probability distribution $\pi$ on $\mathcal{X}$. The prior probability $\pi$ can be obtained via population density, historical locations, and so on. The adversary is also informed of the location obfuscation mechanism $f$. Assuming more information known by the adversary implies the higher privacy security of the required framework.

In the current scenario, the adversary infers the user’s real location $x$ under the Bayesian adversary model. After the user reports her/his pseudo-location $x' \in \mathcal{X}$, the adversary computes the probability that each apriori location $x \in \mathcal{X}$ is the true location in the condition of generating $x'$, i.e., the posterior probability distribution $Pr(x|x')$, by

$$
Pr(x|x') = \frac{Pr(x,x')}{Pr(x')} = \frac{\pi(x)f(x'|x)}{\sum_{x \in \mathcal{X}} \pi(x)f(x'|x)}.
$$

Afterwards, a Bayesian adversary can launch optimal inference attack to get the estimated location $\hat{x}$ which has the minimal expected inference error, i.e.,

$$
\hat{x} = \arg \min_{y \in \mathcal{X}} \sum_{x \in \mathcal{X}} Pr(x|x')d_p(y, x),
$$

where $d_p$ is usually Euclidean distance $d$. When $d_p$ denotes Hamming distance $d_h$, that is, $d_h(x,x') = 0$ if $x = x'$, and $d_h(x,x') = 1$ otherwise, this attack is called Bayesian inference attack and simply

$$
\hat{x} = \arg \max_{x \in \mathcal{X}} Pr(x|x').
$$


3.3 Location Privacy Notions

Geo-indistinguishability based on differential privacy is a statistical notion of location privacy, which has been widely used in the field of location privacy protection.

**Definition 4** (Geo-indistinguishability [9]). Suppose that a location obfuscation mechanism satisfies, for any locations \( x, y \in \mathcal{X} \),

\[
\frac{f(x'|x)}{f(x'|y)} \leq e^{\epsilon d(x,y)}, \quad x' \in \mathcal{X},
\]

then the mechanism achieves \( \epsilon \)-geo-indistinguishability, where \( d(x, y) \) is the Euclidean distance between \( x \) and \( y \).

This means that two geographically close locations have similar probability distributions, which theoretically achieves that they are indistinguishable to each other for the adversary. Here, \( \epsilon \) represents the geo-indistinguishability parameter that is determined by the privacy budget and the circular region usually centered at the user’s location. All locations in the region have similar release distribution \( f \) so that the true location can be hidden in this region, and the whole locations in this region are called the protection location set (PLS). Generally, we can define \( \epsilon = \epsilon_g \cdot D \), where \( D \) denotes the diameter of the protection region. Then by Definition 4, the mechanism \( f \) satisfies \( \epsilon \)-DP on PLS as follows.

**Definition 5** (\( \epsilon \)-DP on PLS [3]). A randomized location obfuscation mechanism \( f(\cdot) \) achieves \( \epsilon \)-differential privacy on protection location set \( \Phi \), if for any locations \( x, y \in \Phi \), and any output \( x' \in \mathcal{X} \), we have

\[
\frac{f(x'|x)}{f(x'|y)} \leq e^{\epsilon}.
\]

As for the inference attack of Bayesian adversary, the location privacy of a scheme can be measured by unconditional expected inference error [10], [11], which is the expected inference error of adversary averaged on \( \mathcal{X} \),

\[
\text{ExpErr} = \sum_{x' \in \mathcal{X}} \Pr(x') \min_{\hat{x} \in \mathcal{X}} \sum_{x \in \mathcal{X}} \Pr(x|x') d(\hat{x}, x) = \sum_{x' \in \mathcal{X}} \min_{\hat{x} \in \mathcal{X}} \sum_{x \in \mathcal{X}} \pi(x) f(x'|x) d(\hat{x}, x).
\]

The service quality loss is usually defined by the unconditional expected distance between true and perturbed locations,

\[
\text{QLoss} = \sum_{x \in \mathcal{X}} \sum_{x' \in \mathcal{X}} \pi(x) f(x'|x) d(x', x),
\]

where the quality metric \( d \) denotes the Euclidean distance as [14], [1].

3.4 Problem Statement

Expected inference error and geo-indistinguishability are two statistical quantification based privacy notions. They can be integrated for globally optimizing utility subject to their joint guarantee [11], [2]. Later, they are argued to be complementary for location privacy and are combined effectively by developing PIVE, a two-phase dynamic differential location privacy framework [3]. Pseudo-locations (i.e., perturbed locations) are generated by exponential mechanism for achieving differential privacy over the PLS. However, the privacy framework turns out to be theoretically problematic. That is, in the given scenario the PLSs adaptively determined usually intersect with each other and each apriori location may have different diameters of PLSs, which directly harms the differential privacy preservation of the whole PIVE. For this, we are intended to correct the location privacy model.

That is, under the same assumption as before that the user wants to protect the privacy of her/his true location by reporting a pseudo-location in a set \( \mathcal{X} \) of nearby discrete locations. It is desirable to develop a location obfuscation mechanism that combines the two privacy notions and generates perturbed locations with effective local performance. The mechanism should allow that the informed adversary has prior knowledge of probability distribution \( \pi \) over a discretized set \( \mathcal{X} \) with the true location included and knows the location obfuscation distribution \( f \). Specifically, given the user’s location, construct PLSs to make different apriori locations inside the same PLS share the same sensitivity (diameter) in the public mechanism, with preserving differential privacy. This motivates the presentation of DIVE, a regionalized location privacy framework integrating both notions of location privacy.

Besides, realizing the personalization on both user-controlled privacy knobs, minimum inference error and differential privacy parameter, enables mobile users to define both knobs simultaneously and freely on all locations with different levels. How to optimize obfuscation mechanism from various perspectives (particularly to achieve smaller service quality loss) with respect to region partitioning is also a meaningful problem. To solve this, we develop PDIVE a personalized framework with quasi \( k \)-means clustering algorithm.

4 Our Regionalization Approach

In this section we introduce DIVE, a two-phase dynamic regionalization mechanism to protect location privacy including both geo-indistinguishability and expected inference error. We first propose the framework and then describe its two phases, partitioning protection location sets (PLSs) and applying exponential mechanism with regionalized sensitivity, in detail. In the first phase, the core of our scheme, the set of discretized locations is partitioned into disjoint subsets (i.e., private PLSs) to protect user’s true location, with preserving the expected location inference errors exceeding the user-defined lower bound against adversary’s attacks via prior knowledge on the user’s location. We develop a partitioning method of location set over a Hilbert curve selected optimally for determining disjoint PLSs. In the second phase, we utilize an exponential mechanism to generate pseudo-locations with small service quality loss, which produces a distribution matrix satisfying 1) independence of the input of true location, and 2) user’s location privacy preferences on \( \epsilon \) and \( E_m \). Then, we prove the differential privacy for locations both within each PLS and across all PLSs.
4.1 DPIVE Regionalization Framework

Yu et al. [3] verify that geo-indistinguishability and expected inference error are two complementary notions and derive a sufficient condition [15] to ensure the lower bound on expected inference error.

As before [3], the conditional expected inference error is

\[ \text{ExpEr}(x') = \min_{\tilde{x} \in \mathcal{X}} \sum_{x \in \mathcal{X}} \Pr(x|x') d(\tilde{x}, x), \quad \text{for } x' \in \mathcal{X}. \]  

(11)

Assume that the adversary narrows possible guesses to the range of PLS that includes the user’s true location. By normalization in PLS we conclude the lower bound shortly,

\[
\begin{align*}
&\min_{x \in \mathcal{X}} \sum_{x \in \Phi} \frac{\Pr(x|x')}{\sum_{y \in \Phi} \Pr(y|x')} d(\tilde{x}, x) \\
&= \min_{x \in \mathcal{X}} \sum_{x \in \Phi} \frac{\pi(x) f(x'|x)}{\sum_{y \in \Phi} \pi(y) f(x'|y)} d(\tilde{x}, x) \quad \text{(due to [3])} \\
&\geq \min_{x \in \mathcal{X}} \sum_{x \in \Phi} \frac{\pi(x) e^{-\epsilon} f(x'|y)}{\sum_{y \in \Phi} \pi(y) f(x'|y)} d(\tilde{x}, x) \quad \text{(due to [3])} \\
&= e^{-\epsilon} \min_{x \in \mathcal{X}} \sum_{x \in \Phi} \frac{\pi(x)}{\sum_{y \in \Phi} \pi(y)} d(\tilde{x}, x).
\end{align*}
\]

To guarantee the expected inference error, we define

\[ E'(\Phi) = \min_{x \in \mathcal{X}} \sum_{x \in \Phi} \frac{\pi(x)}{\sum_{y \in \Phi} \pi(y)} d(\tilde{x}, x). \]

(13)

Then the lower bound for \( \text{ExpEr}(x') \) is indeed achieved in the worst case as

\[ \text{ExpEr}(x') \geq e^{-\epsilon} E'(\Phi). \]

(14)

This yields a sufficient condition for each local PLS, similar to [3] (Theorem 1),

\[ E'(\Phi) \geq e^{\epsilon} E_m, \]

(15)

to satisfy the user-defined threshold, \( \text{ExpEr}(x') \geq E_m \), for the optimal inference attack using any observed pseudo-location \( x' \).

**Theorem 1.** Suppose that a location obfuscation mechanism satisfies the \( \epsilon \)-differential privacy on each PLS \( \Phi \). If \( E'(\Phi) \geq e^{\epsilon} E_m \), then for the optimal inference attack using any observed pseudo-location \( x' \), we have \( \text{ExpEr}(x') \geq E_m \).

We mention that a slightly different assertion is given in [3] (Theorem 1). That is, the sufficient condition [15] is replaced by \( E(\Phi) \geq e^{\epsilon} E_m \) in [3], where

\[ E(\Phi) = \min_{x \in \mathcal{X}} \sum_{x \in \Phi} \frac{\pi(x)}{\sum_{y \in \Phi} \pi(y)} d(\tilde{x}, x). \]

(16)

It is claimed in [3] that, given \( \Phi \) is convex in the discrete set \( \mathcal{X} \), the authors obtain \( E(\Phi) = E'(\Phi) \). However, this is not true in general and we present a counterexample as follows.

Suppose that, the prior distribution \( \pi \) is uniformly distributed on \( \mathcal{X} = \{A, B, C, F\} \), and \( \Phi = \{A, B, C\} \), see Fig. 1. Obviously, \( \Phi \) is convex in \( \mathcal{X} \), that is, on the plane the convex hull of \( \Phi \), the triangular range \( \Delta ABC \) (the lengths of edges are 130, 130, 100), does not include any point from \( \mathcal{X} \). Then \( E(\Phi) = 76.7 \) is larger than \( E'(\Phi) = 74.3 \) since the minimal point for \( E'(\Phi) \) is \( F \) out of the range \( \Delta ABC \).

![Fig. 1: Counterexample for convex PLS.](image-url)

The adaptive PLS for each apriori location is constructed based on Theorem 1 in the phase I of PIVE. Since \( E'(\Phi) = 0 \) for any single-point set \( \Phi \), each PLS certainly includes at least two locations. For each apriori location \( x \), PIVE first searches in a large range for all possible sets of locations neighboring on Hilbert curve ranking that satisfy [15] and PIVE chooses the set having the smallest diameter as PLS. Then in phase II the diameter is assigned as the sensitivity of the exponential mechanism to generate pseudo-locations.

Unfortunately, the PLS obtained by PIVE depends locally on the true location adaptively and is usually different for each apriori location. Different PLSs may intersect with each other. Then in the location obfuscation distribution matrix \( \{f(x_i| x_i')\} \), each apriori location \( x_i \)'s row may have different sensitivities depending on the true location. Such a problematic approach affects the differential privacy preservation on each PLS. We will review the PIVE Framework in Section 4.4, see our paper [4] for detailed analysis.

To solve this, we propose DPIVE, a regionalized location obfuscation mechanism. Given the privacy parameters without the input of true position, we first partition the entire discrete location set into \( k \) parts as many as possible each of which satisfies [15]. Then in the second phase each apriori location (row \( i \)) in the same part shares the same sensitivity in the exponential mechanism while all parts are regarded as possible PLSs symmetrically in the public location obfuscation distribution matrix. This means that any two apriori locations from different parts have no intersection on their PLSs and their corresponding rows usually have different diameters (sensitivities) in the matrix, which does not affect differential privacy preservation on each PLS indeed. Finally, the true position is not input to produce a pseudo-location before the generation of distribution matrix. Such a procedure theoretically guarantees the privacy protection of the true location. The framework of DPIVE is shown in Fig.

DPIVE is mainly composed of two components: the partitioning algorithm \( F \) to determine disjoint PLSs and the differential privacy mechanism \( K \) to generate a pseudo-location. \( F \) has four inputs, prior distribution \( \pi \), inference error threshold \( E_m \), privacy parameter \( \epsilon \) and location sets \( \mathcal{X} = \{x_i\} \). For the two privacy parameters specified by users, \( \epsilon \) allows users to control the posterior information
leakage via the provisioning of differential privacy and $E_m$ aims to locally bound the expected inference error in the worst case. Each PLS contains obviously at least two locations and ensures the lower bound of inference error.

Obviously, the result of our Algorithm $F$ does not depend on the true location due to its no input. For minimization of the quality loss, $F$ globally partitions the entire location domain into (as many as possible) disjoint PLSs satisfying (15). Then the mechanism $K$ uses the diameter of each PLS as the sensitivity of the exponential mechanism in corresponding $x_i$’s rows to calculate the probability distribution $f = \{f(x_j|x_i)\}$. Afterwards, with the input of user’s true location, DPIVE produces a pseudo-location via the public matrix $f$.

We mention that given the prior probability $\pi$ and the parameters $\epsilon$ and $E_m$, the PLSs partitioned in the dataset are determined by Algorithm 1 and then the public matrix $f$ is computed and fixed. Moreover, the true location is $\epsilon_o$-geo-indistinguishable among the locations within PLS even in the worst case that the adversary knows the PLS. That is, DPIVE can provide users with location privacy protection satisfying their privacy requirements on $\epsilon$ and $E_m$ while the prior distribution $\pi$, Algorithm $F$, differential privacy mechanism $K$ and obfuscation probability matrix $\{f(x_j|x_i)\}$ are all public to the adversary. Besides, while in DPIVE any user has to employ unified privacy parameters of $\epsilon$ and $E_m$ for all regions, and in Section 4 we will consider the personalization of both knobs. In the next two subsections, we present the details of Algorithm $F$ and differential privacy mechanism $K$, respectively.

### 4.2 Partitioning Protection Location Sets

Hilbert curve [21] is a common space-filling curve, which can map points in 2-D space to one dimensional space and has the clustering properties with preserving the proximity of points. Fig. 3 shows the Hilbert curves for $4 \times 4$ and $8 \times 8$ grids. Specifically, the curve maps a location point $x$ to a 1-D value denoted by $H(x)$ called the Hilbert value of $x$, for example, Hilbert values 1-16 of all cell centers in Fig. 3(a). Following this, we connect the locations in the GeoLife dataset in the order of $H(x)$ and sort all locations in $X$ with the rank denoted by $R(x)$, like 50 points numbered in Fig. 4(a). It should be noted that the Hilbert curve generated in a 2-D space is not unique. Rotating one Hilbert curve 90, 180, 270 degrees clockwise around the center can generate other three Hilbert curves. For our regionalized location obfuscation mechanism, a region partition can only be performed on one Hilbert curve. In order to improve the performance of our mechanism, we execute Algorithm 1 independently on multiple (four) rotated Hilbert curves to perform region partitions and then choose the result with the smallest average diameter.

Since we partition regions from a global perspective, the search range used in [13] can be omitted in Algorithm 1. Given a location set sorted according to the Hilbert curve, protection regions are constructed from the two sides of the curve to the middle and the initialized two alternatives are at the two ends, $\Phi_L$ and $\Phi_R$, respectively (Line 1). Then supply $\Phi_L$ with neighboring locations on the right side along the curve one by one (Line 2) until $\Phi_L$ is qualified for the condition (15) and similarly supply for $\Phi_R$ (Line 3).

If both $\Phi_L$ and $\Phi_R$ satisfy (15), assign the set with the larger diameter between $\Phi_L$ and $\Phi_R$ as a PLS to be removed into $\Phi_{pls}$ (Line 4, isolated locations would be relatively preferred) and initialize new $\Phi_L$ or $\Phi_R$ if removed. Process the steps by iterations (Lines 2-4) until $|Q| \leq 1$, and afterwards we have to combine the remainder elements (Lines 5-8). If $\Phi_{RL}$ can not satisfy (15) (Line 9), remove the locations with continuous rankings in $\Phi_{RL}$ to the two-sided $\Phi_j$’s on the Hilbert curve, and keep the new protection region satisfying (15) and with smallest diameter in the average sense of

$$\frac{\Phi_1 \cdot D(\Phi_1) + \Phi_2 \cdot D(\Phi_2)}{|\Phi_1| + |\Phi_2|}.$$ (17)

There exists a situation with low probability, that is, no matter how the locations in $\Phi_{RL}$ are split for being allocated
Algorithm 1 Partitioning Algorithm for disjoint PLSs

Input: sorted user’s locations \( X = \{x_0, x_1, \ldots, x_n\} \), prior probability \( \pi \), inference error bound \( E_m \), user privacy parameter \( \epsilon \)

1. Initialize \( \Phi_L = \{x_0, x_1\}, \Phi_R = \{x_{n-2}, x_{n-1}\}, Q = \{x_2, \ldots, x_{n-3}\} \)
2. Remove \( x_i \)'s with the smallest subscript in \( Q \) to \( \Phi_L \) until satisfying (15)
3. Remove \( x_i \)'s with the largest subscript in \( Q \) to \( \Phi_R \) until satisfying (15)
4. if \( |Q| \geq 2 \) then Add the set with the larger diameter between \( \Phi_L \) and \( \Phi_R \) to \( \Phi_{\text{pls}} \), initialize new \( \Phi_j \), if selected above) using points with the smallest subscript from \( Q \), or new \( \Phi_R \) similarly, and go to Line 2
5. if \( |Q| = 1 \), then Remove the only element to the nearer \( \Phi_L \) or \( \Phi_R \)
6. if (15) holds for \( \Phi_L \) and \( \Phi_R \), then Add both \( \Phi_L \) and \( \Phi_R \) to \( \Phi_{\text{pls}} \) and go to Line 10
7. else \( \Phi_{RL} \leftarrow \Phi_L \cup \Phi_R \)
8. if (15) holds for \( \Phi_{RL} \) then Add \( \Phi_{RL} \) to \( \Phi_{\text{pls}} \)
9. else Bisect the curve \( \Phi_{RL} \) and allocate the two parts to two-sided neighbors from \( \Phi_{\text{pls}} \) with traversing for the smallest average diameter in the sense of (17)
10. return disjoint PLSs \( \Phi_{\text{pls}} \)

4.3 Exponential Mechanisms with Regionalized Sensitivity

Given disjoint PLSs \( \{\Phi_j\} \), DPIVE realizes differential privacy on each PLS \( \Phi_j \) via the exponential mechanism [17]. The set \( X \) is regarded as both input and output range of DPIVE. Since smaller distance produces higher utility, the utility of output location \( x' \) can be measured by the Euclidean distance between perturbed and true locations, \( d(x, x') \). The sensitivity of \( u \) for each PLS \( \Phi_j \) is

\[
\Delta u(\Phi_j) = \max_{x' \in X} \max_{y \in \Phi_j} |d(x, x') - d(y, x')|.
\]

Then from triangle inequality, we have \( \Delta u(\Phi_j) = D(\Phi_j) \), i.e., the diameter of \( \Phi_j \).

Since the disjoint \( \Phi_j \)'s are determined by the given privacy parameters instead of the true location, then each input location (true location) can not determine simply the sensitivity of \( u \) and all locations in the same PLS \( \Phi_j \) share the same sensitivity \( D(\Phi_j) \).

Exponential Mechanism \( \mathcal{K} \): Given the disjoint sets \( \{\Phi_j\} \) determined by privacy parameters \( \epsilon \) and \( E_m \) with satisfying (15), for each apriori location \( x \in X \) and its corresponding PLS \( \Phi_j \) derived from the given family \( \{\Phi_j\} \), the mechanism \( \mathcal{K} \) computes the probability distribution \( f(x'|x) = w_x \exp(-\frac{\epsilon d(x, x')}{2D(\Phi_j)}) \) for any possible pseudo-location \( x' \), where

\[
w_x = \left( \sum_{x' \in X} \exp \left( -\frac{\epsilon d(x, x')}{2D(\Phi_j)} \right) \right)^{-1}.
\]

Following the public matrix \( \{f(x'|x_i)\} \), DPIVE mechanism generates a pseudo-location \( x' \in X \), which deploys user’s true location information (to be protected with differential privacy) for the first time in the whole procedure.

We achieve \( \epsilon \)-differential privacy on each PLS and also between different PLSs as follows.

Theorem 2 (H). Assume disjoint PLSs \( \{\Phi_j\} \), then the exponential mechanism \( \mathcal{K} \) in DPIVE satisfies \( \epsilon \)-differential privacy on each \( \Phi_j \).

Proof. For each set \( \Phi_j \) and any \( x, y \in \Phi_j \), we know in DPIVE that \( x \) and \( y \) share the same PLS \( \Phi_j \) and have the same sensitivity of utility, i.e., the diameter \( D(\Phi_j) \). Further,

\[
\frac{f(x'|x)}{f(x'|y)} = w_x \exp \left( -\frac{\epsilon d(x, x')}{2D(\Phi_j)} \right) \frac{w_x}{w_y} \exp \left( -\frac{\epsilon d(y, x')}{2D(\Phi_j)} \right) \leq \frac{w_x}{w_y} e^{\epsilon/2} \leq \frac{\epsilon}{2} \exp \left( -\frac{\epsilon d(x, y)}{2D(\Phi_j)} \right) \frac{\epsilon}{2} \leq \frac{\sum_{x' \in X} \exp \left( -\frac{\epsilon d(x, x')}{2D(\Phi_j)} \right)}{\sum_{x' \in X} \exp \left( -\frac{\epsilon d(x, x')}{2D(\Phi_j)} \right)} \frac{\epsilon}{2} e^{\epsilon/2} = e'.
\]

We mention that the DPIVE framework satisfies \( \epsilon \)-geo-indistinguishability with \( \epsilon_g = \epsilon/(2D(\Phi_j)) \) for each PLS \( \Phi_j \). Indeed, we could obtain easily from (20) that for any \( x' \in X \),

\[
\frac{f(x'|x)}{f(x'|y)} \leq \frac{\epsilon}{2} e^{\epsilon/2} e^{d(x, y)/(2D(\Phi_j))}, \quad x, y \in \Phi_j,
\]

which produces a deviation \( \epsilon/2 \) on the coefficient.

To be general, for the privacy preservation on whole \( X \), we have a weak assertion.

Theorem 3 (H). Assume disjoint PLSs, \( \Phi_i \) and \( \Phi_j (i \neq j) \), in the domain \( X \), then the exponential mechanism \( \mathcal{K} \) in DPIVE satisfies \( \left( \frac{D(\mathcal{X})}{D(\Phi_i)} + \frac{D(\mathcal{X})}{D(\Phi_j)} \right) \frac{\epsilon}{2} \) differential privacy between \( \Phi_i \) and \( \Phi_j \): For any \( x \in \Phi_i, y \in \Phi_j \) and \( x' \in \mathcal{X} \), we have

\[
\frac{f(x'|x)}{f(x'|y)} \leq \left( \frac{D(\mathcal{X})}{D(\Phi_i)} + \frac{D(\mathcal{X})}{D(\Phi_j)} \right) \frac{\epsilon}{2}.
\]
Proof.

\[
f(x'|x) = \frac{\exp\left(-\frac{\varepsilon d(x,x')}{2D(\Phi_j)}\right)}{\sum_{s \in \mathcal{X}} \exp\left(-\frac{\varepsilon d(x,s)}{2D(\Phi_j)}\right)} = \frac{\sum_{t \in \mathcal{X}} \exp\left(-\frac{\varepsilon d(x,t)}{2D(\Phi_j)}\right)}{\sum_{s \in \mathcal{X}} \exp\left(-\frac{\varepsilon d(x,s)}{2D(\Phi_j)}\right)} = \exp\left(\frac{\varepsilon d(x,t)}{2D(\Phi_j)}\right)
\]

(23)

Theorem 3 shows that any two locations from different PLSs are protected with weaker differential privacy. This gives us a relatively complete result on the differential privacy preservation for the whole \( \mathcal{X} \) no matter whether the two apriori locations are in the same PLS.

### 4.4 Review of PIVE Framework

In this subsection, we mainly recall the privacy problem of PIVE framework proposed in Yu et al. [3], which is analyzed in detail in our previous work [4]. Since our current DPIVE framework is a constructive correction of PIVE under the same assumption on parameter setting and Bayesian adversary model, it is enough for us to recall firstly their differences on the procedure and the privacy problem of PIVE. Indeed, PIVE includes also two phases as follows.

**Phase I: Determining Protection Location Set.** The PLS for each location is generated adaptively and optimally. PIVE regards \( \Phi \) as a variable and dynamically searches region \( \Phi \) satisfying (24) with diameter as small as possible.

To be specific, for each input location \( x \) denoted by \( x_0 \), the search algorithm returns a set having the smallest diameter satisfying

\[
E(\Phi) \geq e^\varepsilon E_m.
\]

(24)

The locations in the output set are with consecutive rankings in \( \mathcal{X} \) with respect to their mappings on a Hilbert curve. Then each (true) location \( x \) has its own PLS \( \Phi_x \) and diameter \( D(\Phi_x) \), and different (even neighboring) locations have different PLSs with different diameters. Even PLSs intersect with each other.

**Phase II: Differentially Private Mechanism.** The exponential mechanism is devised as above to generate pseudo-locations, which is desired (but failed) to achieve differential privacy on the PLS. This is mainly due to the fact that different locations in the same PLS may have different diameters for applying the exponential mechanism.

For each PLS \( \Phi_j \) determined by a true location \( t \) and any \( x, y \in \Phi_j \), we know in PIVE that \( x \) and \( y \) have their own PLS \( \Phi_x \) and \( \Phi_y \), respectively, and in general they have different sensitivities, i.e., the diameters \( D(\Phi_x) \neq D(\Phi_y) \). Further, in the initial proof of differential privacy,

\[
\frac{f(x'|x)}{f(x'|y)} = \frac{w_x \exp(-\varepsilon d(x,x')/(2D(\Phi_x)))}{w_y \exp(-\varepsilon d(x,y')/(2D(\Phi_y)))},
\]

(25)

we cannot use the triangular inequality, \(|d(x,x') - d(y,x')| \leq d(x,y)\), in (25) as before. Thus, PIVE fails to achieve the guarantee of differential privacy as desired.

In conclusion, the main mistake of PIVE is derived from the adaptive search of PLSs. Besides, the sufficient condition (24) is wrong, which is explained by a counterexample, cf. Fig. 1. The corrected condition (15) is given in our Theorem 1.

### 5 Region Partitioning by QK-means Clustering

In this section, we partition the region back in the 2-D space to achieve a more efficient privacy mechanism. Although the Hilbert curve method can well represent the proximity of locations in 2-D space, it can only search the adjacent locations on the curve along a single direction, while the adjacent locations in 2-D space may be far away from each other on the Hilbert curve (e.g., locations 2 and 15 in Fig. 3(a)). Even multiple Hilbert curves can not significantly improve the performance of the scheme. To overcome the limitations of the selection space on Hilbert curves, we design quasi \( k \)-means clustering (QK-means) algorithm via the popular \( k \)-means algorithm in machine learning. Basically we focus on constructing the protection location set (PLS) including the true location and satisfying (15). When adding adjacent locations to the cluster, the QK-means method in 2-D space has much more selections in clustering unlike the Hilbert curve method in 1-D space.

Moreover, it is expected to achieve a suitable tradeoff between privacy protection and quality loss. Some PLSs may be composed of only two locations for small privacy knobs, which will inevitably leak location privacy in the worst case that the adversary narrows the guesses within the PLS. For this, we can make a restriction on the smallest number of locations covered in every PLS, which is assigned as 2 currently. Then we construct a partition for disjoint PLSs as many as possible for small diameter in the average sense.

The QK-means method determines the final disjoint parts by adaptively searching for the optimal number of clusters \( k \) as shown in Algorithm 2. For each \( k \), the clustering centers are initialized on Lines 5-6. The first center is randomly selected in \( \mathcal{X} \), and each subsequent center depends adaptively on those selected ahead with sampling probability proportional to distance between each remainder location and its nearest center. This means that the longer the distance, the larger probability to be the new center, to make centers relatively sparse. On selecting locations to join the cluster, we search for the location each time that has the minimum distance to the centers (Line 11). Once a cluster satisfies (15), close it temporarily. If all clusters are closed, the remaining locations are added directly to their nearest clusters in order (Line 12). Then, improve the center by the mean vector in each cluster and carry out the next iteration until the mean vectors varies within a small range or the upper iteration times Max Iter is achieved (Line
The personalization of DPIVE mechanism is called PDPIVE. This allows users to set their privacy levels to (15). In order to satisfy the privacy requirements of all locations within the same PLS \( \Phi_j \), DPIVE has to achieve the user’s highest privacy level in \( \Phi_{j+1} \), that is, the region’s privacy budget \( \epsilon_j = \min_{x \in \Phi_j} \epsilon_x \) due to (14) theoretically.

In this scenario, the privacy parameter \( \epsilon \) has to be considered on partitioning the region. Adding each location to a PLS may affect the privacy level of PLS. However, current QK-means considers only the distance while ignoring the differences on \( \epsilon \) among locations. For this, the Euclidean distance \( d_{ji} \) between \( x_i \) and \( \Phi_j \) used on Line 11 of Algorithm 2 is replaced by \( d_{ji} \cdot w_e \) with weight \( w_e \), emphasizing the influence of \( \epsilon \) on \( \sigma_{ji} \).

\[
w_e = 1 + \lambda \cdot \min(\epsilon, \epsilon_{ji}) / \max(\epsilon, \epsilon_{ji}),
\]

where \( \epsilon_j \) represents the current privacy budget of the PLS \( \Phi_j \) that is to be updated once a new location with privacy \( \epsilon \) is added, \( \lambda \) is a parameter to control the range of \( w_e \) and the default value of \( \lambda \) is 0.5. Such a setting prefers those locations with \( \epsilon \) value more than and closed to current \( \epsilon_j \), see Fig. 5(a). Indeed, the newly added location with smaller \( \epsilon_j \) will certainly modify the current \( \epsilon_j \) which probably produces larger quality loss, while the added location with larger \( \epsilon_j \) will not change the \( \epsilon_j \). The parameter \( \lambda \) aims mainly to avoid the case of \( w_e = 0 \) that totally ignores the effect of distance.

The personalization of \( E_m \). Similar to the above, the \( E_j \) of the PLS \( \Phi_j \) represents the largest \( E_m \) of all locations inside. Similarly, we define \( \sigma_{ji} = d_{ji} \cdot w_{Em} \) with weight

\[
w_{Em} = 1 + \lambda \cdot \min(2E_j - E_m - E_j, 0) / \max(2E_j - E_m - E_j, 0),
\]

which emphasizes the influence of \( E_m \) on \( \sigma_{ji} \). This setting prefers those locations with \( E_m \) value smaller than and closed to current \( E_j \), see Fig. 6(b). Similarly, the newly added location with larger \( E_m \) will certainly modify the current \( E_j \) which produces larger quality loss, while the added location with larger \( E_m \) will not change the \( E_j \). In general, a previously constructed set may accept locations with very large \( \epsilon \) but not with relatively large \( E_m \).

Taking personalization of \( E_m \) as an example, we test the effect of \( w_{Em} \) on the GeoLife dataset. The \( E_m \) of each

\[
9.\text{To eliminate the randomness of cluster center selection, we repeat sampling Max_Samp times on each } k \text{ (Line 4), for finding efficient partitioning that results in PLSs with minimum average diameter and satisfying} (15). \text{Increasing } k \text{ continues to find the next family of disjoint PLSs } \Phi_{k+1}. \text{If } \Phi_{k+1} \text{ cannot be found or its average diameter is larger than } \Phi_k, \text{then } \Phi_k \text{ gives the final PLSs as required.}

Fig. 5 compares the average diameter of the PLSs between Hilbert curve based method and QK-means method under different \( \epsilon \) and \( E_m \), in the sense of (17). We sample three values of \( \epsilon \) and \( E_m \) separately to obtain 9 groups of experiments. The results show that on using QK-means, the globally average diameter is 21.8% smaller than that for Hilbert curve. In particular, it decreases 27.2% in the setting, \( \epsilon = 2.0 \) and \( E_m = 0.4 \). More experiments will be evaluated in Section 7.3.

6 PERSONALIZING \( \epsilon \) AND \( E_m \)

Now we consider the personalization of user’s privacy parameters. This allows users to set their privacy levels by customizing the privacy parameters \( \epsilon \) and \( E_m \). The personalization of DPIVE mechanism is called PDPIVE.

The personalization of \( \epsilon \). Different privacy levels of users generate different \( \epsilon \), which brings some challenges to the search of PLSs. As we know, the PLSs constructed in DPIVE result in the same privacy level for users due
location is uniformly and randomly sampled in $[0.1, 0.3]$ to simulate the $E_m$ of user personalization and $\epsilon = 1.7$ is fixed. The quality loss (average on 20 times of experiments) of PDPIVE using $w_{E_m}$-weight strategy is shown in Fig. 7 with comparison to that of general DPIVE using QK-means.

$partition_wE_m$ means that the weight $w_{E_m}$ is involved in partitioning while $partition_{general}$ denotes general DPIVE using maximal $E_m$ in local PLS with ignoring $w_{E_m}$. The experimental results show that the average quality loss decreases from 4.24 to 4.04 after taking weight into accounts. The quality loss at about half of locations has been improved much, especially reduced by 31.3% at location 30. This demonstrates that considering the factor of weights associated to the personalized parameters could make more locations of closer privacy levels on $E_m$ join in the same PLS, which effectively reduces the service quality loss.

Fig. 8: 50 regions distributed in two datasets.

(a) Geolife  
(b) Gowalla

Fig. 7: Effect of adding weights on PDPIVE.

The personalization of both knobs. We also study the personalization of both $\epsilon$ and $E_m$. The weight in $\sigma_{ji}$ for partitioning can be given by $(w_e + w_{E_m})/2$. In each disjoint PLS to be constructed, we use the minimum $\epsilon$ and the maximum $E_m$ inside to achieve the requirement of all locations in the PLS, in particular, the basic condition [15]. This realizes the overall personalization of our DPIVE regionalization framework and such a personalized framework of DPIVE can be called PDPIVE. In this case, we still obtain that the exponential mechanism $K$ for PDPIVE satisfies $\epsilon_j$-differential privacy on each PLS $\Phi_j$. More performance analysis of PDPIVE is detailed in Section 7.3.

Next, on the real-world location-based service applications, as mentioned in Fig. 2 both control knobs, minimum inference error and differential privacy parameter, are assumed to be private for each user. Algorithm $F_j$, differential privacy mechanism $K$ and obfuscation probability matrix $\{f(x_j|x_j)\}$ are all public to adversaries, and they are used locally by the user to produce a pseudo-location. Each user can define both control knobs personally on each location in the following two provided ways: 1) detailed operation instruction with some prime examples; and 2) default setting for different privacy levels, like conservative (small value), moderate (middle value) and liberal (great value) levels, in which the concrete knob values for each level can be adjusted appropriately.

7 Performance Valuation

We first compare our DPIVE approach with some previous mechanisms on the metrics of location privacy and service quality, then present an experimental evaluation of PDPIVE scheme. The results show that our mechanisms effectively combine both privacy notions and efficiently address privacy protection issues on isolated locations.

7.1 Experimental Methodology

Datasets. Two location sets are used in the experiment, which are extracted from GeoLife and Gowalla datasets, respectively. The location distribution in GeoLife is relatively dense while sparse in Gowalla. For GeoLife, we use the same distribution as [3], and for convenience we assign the grid size as $1\text{km} \times 1\text{km}$. Gowalla is a social network check-in dataset containing 224 days of check-in data for California in 2010. We divide the main area of Gowalla into also $1\text{km} \times 1\text{km}$ cells and make random selections for 50 relatively sparse cells. The distributions of both datasets are shown in Fig. 8 in which most isolated regions are numbered behind.

We simulate a prior distribution uniformly on both datasets, in which each value is sampled randomly and uniformly in $[0.01, 0.03]$ with normalization, see Table 1.

Parameters setting. The lower bound of inference error $E_m \in \{0.05, 0.1, ..., 0.5\}$. The privacy budget $\epsilon \in \{0.1, 0.3, ..., 1.9, 2.0\}$ in GeoLife and $\epsilon \in \{0.1, 0.3, ..., 2.5\}$ in Gowalla. The reason for the difference on budget range is that large $\epsilon$ would imply large PLS for satisfying the condition [15] and particularly the whole (relatively dense)
TABLE 1: Values of prior probability ($\times 10^{-2}$).

| Interval | Dataset       | GeoLife | Gowalla |
|----------|---------------|---------|---------|
|          | Schemes       |         |         |
|          | DPIVE         | EM      | Opt-Geo | Joint  |
|          |               |         |         |
| 1-10     | 1.53          | 2.41    | 1.11    | 1.23   | 2.29   | 2.00   | 2.13   | 2.06   | 1.87   | 1.43   |
| 11-20    | 1.84          | 2.24    | 1.54    | 1.50   | 2.53   | 2.15   | 2.59   | 2.46   | 1.90   |
| 21-30    | 2.43          | 2.10    | 2.46    | 1.62   | 1.50   | 2.35   | 1.97   | 2.61   | 2.82   |
| 31-40    | 2.69          | 2.27    | 1.81    | 1.79   | 2.78   | 2.84   | 1.66   | 2.69   | 1.07   | 1.99   |
| 41-50    | 1.99          | 1.92    | 1.06    | 2.49   | 1.09   | 2.68   | 1.93   | 2.40   | 1.84   | 1.64   |

TABLE 2: The percentage of locations exceeding given success probability, and quality loss.

| Metrics | Dataset       | GeoLife | Gowalla |
|---------|---------------|---------|---------|
|         | Schemes       |         |         |
|         | DPIVE         | EM      | Opt-Geo | Joint  |
|         |               |         |         |
| $X > 50\%$ |               |         |         |
|          | 2%            | 2%      | 6%      | 12%    | 4%     | 8%     | 8%     | 26%    |
| $X > 70\%$ |               |         |         |
|          | 0%            | 0%      | 4%      | 8%     | 0%     | 2%     | 6%     | 18%    |
| $X > 90\%$ |               |         |         |
|          | 0%            | 0%      | 2%      | 6%     | 0%     | 0%     | 0%     | 12%    |
| Quality Loss | 3.22          | 3.27    | 3.12    |
|             | 3.9           | 9.88    | 9.93    | 9.46   | 9.98   |

50-point dataset GeoLife cannot satisfy [15] as a PLS with $\epsilon = 2.1$ for some $E_m$.

On the aspect of personalization, randomly and uniformly sampling parameters is restricted in the middle of the above ranges, $\epsilon \in [0.5, 1.5]$ and $E_m \in [0.1, 0.3]$. In order to measure the performance improvement brought by personalized mechanism, we assign DPIVE scheme as baseline that uses unified privacy parameters for the whole $X$. Specifically, in order to meet the highest privacy requirements of all PLSs, $\epsilon = 0.5$ if personalized and $E_m = 0.3$ if so.

7.2 Performance Analysis of DPIVE

In this section, we compare DPIVE (using Hilbert curve based method) with previous typical mechanisms, EM [3], Joint [1] and Opt-Geo [14], especially to verify the advantages of DPIVE on protecting isolated regions as in [3]. Rather than the globally average performance of privacy protection emphasized in previous work, DPIVE pays more attention to the local performance. Then we also check the detailed privacy protection performance on each region. In order to make a fair comparison between different schemes, we specify the parameters of DPIVE ($\epsilon = 1.0$, $E_m = 0.05$) and adjust the parameters of other schemes to ensure the same location privacy, that is, the same unconditional expected inference error.

The EM mechanism is similar to the exponential mechanism proposed in PIVE, except that a constant diameter is used for the protection region of each location. EM adopts the same $\epsilon$ as DPIVE and adjusts the constant diameter (1.66km) so that their expected inference errors achieve the same (their difference within 0.005 is acceptable).

Opt-Geo is an efficient privacy mechanism that minimizes quality loss through linear programming while satisfying geo-indistinguishability. We use $\delta = 0.05$ commonly as in [14] and determine $\epsilon_g = 0.3$ to reach the same expected inference error.

Joint is the first mechanism that uses linear programming to combine two privacy notions of expected inference error and geo-indistinguishability. We use the same $\epsilon = 1.0$, and then use DPIVE’s global expected inference error as the minimum desired distortion privacy level $d_m$, via adjusting $\epsilon_g = 0.3$ to obtain the same expected inference error.

The scheme privacy is measured by the average inference error $\text{AvgErr}$ of the optimal inference attack and success probability $p_s$ of Bayesian inference attack [3]. Define

$$\text{AvgErr}(x) = \sum_{x' \in X} f(x'|x)d(\hat{x}, x), \quad (28)$$

$$p_s(x) = \sum_{x' \in X} f(x'|x)d_h(\hat{x}, x), \quad (29)$$

where $\hat{x}$ (determined by $x'$) is obtained by (5) for $\text{AvgErr}$ with $d(\hat{x}, x)$ representing Euclidean distance while obtained by (6) for $p_s$ with $d_h(\hat{x}, x)$ denoting Hamming distance.

Fig. 9 shows the comparisons of the average inference error and expected success probability of Bayesian inference attack (using Hamming distance) on each region among four mechanisms. Due to the above adjustments for reaching the same unconditional expected inference error for four schemes, DPIVE has a lower average inference error $\text{AvgErr}$ in most regions while it has higher $\text{AvgErr}$ on isolated regions than the other schemes. It does not mean that DPIVE is easier to be attacked, and the analysis is as follows.

In some isolated regions (such as 48-50 in GeoLife and 46-50 in Gowalla, marked in red in Fig. 9), the schemes EM, Opt-Geo and Joint have a significant increase in the expected success probability, even the Joint reaches 100% (accurate attack), while DPIVE has less than 20%. Indeed, DPIVE partitions the local protection region according to the privacy parameters $\epsilon$ and $E_m$ to ensure the lower bound of inference error in the worst case, thus it effectively and locally protects the isolated regions.

Moreover, under the premise of the same location privacy requirements, we count the percentage of regions whose attack success probability exceeds $X\%$ for each scheme as shown in Table 2. It demonstrates that DPIVE has always the lowest attack success probability when $X$
Fig. 9: Comparison of DPIVE with typical mechanisms.

Fig. 10: Performance of each scheme with personalized \( \epsilon \).

Fig. 11: Performance of each scheme with personalized \( E_m \).

7.3 Performance Analysis of PDPIVE

In this section, we mainly evaluate the impact of \( \epsilon \) and \( E_m \)'s personalization on the performance of PDPIVE. We focus on \( Q\elloss \) for comparisons among four approaches, two PDPIVE schemes (PDPIVE_QK and PDPIVE_Hilbert) and two DPIVE baselines (DPIVE_QK and DPIVE_Hilbert). To be specific, the personalized schemes, PDPIVE_QK and PDPIVE_Hilbert, search for optimal disjoint PLSs along respective lines as before, and each PLS meets the highest privacy requirements among the locations included while the baselines use the highest requirements in the whole \( \mathcal{X} \). Besides, PDPIVE_QK constructs disjoint PLSs with considering the impact of weights (26) and (27). The results of personalizing \( \epsilon \) and \( E_m \) are shown in Figs. 10 and 11 respectively. Our analysis is given from two perspectives.

Compared with the baselines, both personalized schemes can effectively reduce quality loss. With personalized \( \epsilon \), the schemes, PDPIVE_Hilbert and PDPIVE_QK, reduce quality loss by 4.1% and 4.9% on GeoLife, and 9.1% and 11.8% on Gowalla, respectively (Fig. 10). With personalized \( E_m \), PDPIVE_Hilbert and PDPIVE_QK reduce by 2.1% and 3.0% on GeoLife, and 4.6% and 4.9% on Gowalla, respectively (Fig. 11). Since the baseline schemes adopt globally unified privacy parameters that meet the highest privacy requirements, many regions are protected...
with privacy levels much higher than their requirements, which results in greater quality losses.

In terms of region partitioning strategy, compared with Hilbert methods, QK-means method has lower quality loss at any fixed $E_m$ or $\epsilon$. With personalizing $\epsilon$, the quality loss are reduced at an average of 2.9% and 9.6% on two datasets, respectively (Fig. 10), while with personalizing $E_m$, 3.6% and 10.5%, respectively (Fig. 11). Obviously, QK-means method has more advantages on Gowalla, which is mainly due to the fact that Gowalla locations are sparser than those in GeoLife and has more selection space in clustering. Although the privacy level of QK-means method declines to some extent, it satisfies privacy requirements in each region.

Besides, we observe from Figs. 10 and 11 that for the four schemes location privacy and quality loss are usually monotonic with varied $E_m$. However, location privacy and quality loss first decrease with $\epsilon$ and then increase. The reason for the increase is that the diameter of each PLS increases exponentially with $\epsilon$ due to (13) and large diameter becomes a major influence on privacy and quality loss.

Fig. 12: Comparing PDPIVE with DPIVE on two strategies.

Finally, we analyze the situation where both privacy parameters are personalized. The experimental results are shown in Fig. 12. Compared with using the Hilbert curve, on GeoLife and Gowalla, the PDPIVE using QK-means reduces the quality loss by 3.6% and 9.8%, respectively, while the DPIVE using QK-means reduces by 1.7% and 9.7%, respectively. Moreover, the PDPIVE saves much quality loss compared to DPIVE: 7.4% in Geolife and 14.0% in Gowalla using QK-means, and 5.5% in Geolife and 13.8% in Gowalla using Hilbert curve.

8 CONCLUSION

This paper investigates the differential privacy preservation of location obfuscation mechanism based on PIVE framework. Since PIVE fails to offer differential privacy guarantees on adaptive protection location set (PLS), we develop PDPIVE, a regionalized location obfuscation mechanism. According to the relevant privacy parameters and their relationship, the entire location set is partitioned into multiple disjoint PLSs, and the locations in the same PLS share the same sensitivity of utility. Each PLS satisfies the lower bound of the inference error for the locations inside. The priori locations within the same PLS are strongly geo-indistinguishable to each other while those locations across different PLSs satisfy weak differential privacy. As a generalization that allows users to personalize their own privacy levels on both privacy notions, we first design quasi-$k$-means clustering algorithm and implement the location obfuscation mechanism PDPIVE theoretically and practically. Experiments with two public datasets demonstrate that our mechanisms improve significantly the performance particularly on skewed locations.

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