Numerical simulation of turbulent oscillating flow in porous media

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Abstract

Two macroscopic turbulent models, P-dL and N-K, have been proposed in recent years for simulating turbulent unidirectional flow in porous media. In this paper a modification on N-K model has been proposed for turbulent oscillating flow in porous media. To this purpose, Turbulent oscillating flow in porous media has been simulated in microscale employing a periodic array. The k-\(\varepsilon\) model was applied to solve turbulent oscillating flow in periodic array. Control volume approach has been used to discretize Navier-Stokes and k-\(\varepsilon\) equations and the well-established SIMPLE method has been conducted to deal with pressure and velocity coupling. To modify N-K model the effect of different parameters such as frequency and Reynolds number has been investigated and the constants in source terms of turbulent kinetic energy and its dissipation rate has been modified versus Re according to microscale results. In order to validate the new modified constants, the modified N-K model was applied to turbulent oscillating flow in porous media and results were compared to original N-K macroscopic model.

Keywords: Oscillating flow; Turbulence; Porous media; k-\(\varepsilon\) model; Volume averaging;

1. Introduction

Porous media and the corresponding heat transfer phenomena have attracted considerable attention, due to their relevance to a wide variety of applications in science and engineering, such as underground heat exchangers for energy saving, solar collectors, geothermal energy, oil extraction, cooling electronic devices, nuclear reactors, thermal insulations, etc [1]. Due to various applications of a porous media in different industries, the length scale of a pore in porous media can range from Angstrom to centimeters or even higher. Therefore, pore scale Reynolds \((Re_p = \rho_f u_p d_p/\mu)\) covers a wide range in these types of flows [2]. Different studies such a Jolls and Hanratty, Dybbs and Edwards, and Horton and Pokrajac confirm the existence of turbulence in the porous media [3,4,5]. Jolls and Hanratty stated that the transition from laminar flow to turbulent flow happens in Reynolds 110 to 150. Dybbs and Edwards experimental studies showed that the fluid flow can show turbulent characteristic in Reynolds little over a few hundreds. They emphasized the existence of four flow regimes including the Darcy regime \((Re_p<1)\), Forchheimer Regime \((1-10<Re_p<150)\), Transient Laminar \((150<Re_p<300)\), and turbulent regime \((300<Re_p)\). Since, the transition to the turbulent flow in the porous media occurs in low Reynolds number, the flow is in turbulent form in many different applications of porous media. Due to the inherent difficulty of measuring flow velocity and turbulent intensity in microscale, macroscale modeling of turbulence is used to investigate turbulence in porous media. To model a turbulent flow in a porous media, researchers use the same method of volume

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averaging used in the laminar flow. For this purpose, they use time averaging for handling turbulence and volume averaging for modeling flow in porous media simultaneously. The difference between the various models is in the order of time and volume averaging. However, regarding the definition of averaging, different definitions can be provided for turbulent kinetic energy and its dissipation rate [6]. Almost all of the models are based on the k-ε model in non-porous flow.

Nomenclature

| Symbol | Description | Greek letters |
|--------|-------------|---------------|
| \(c_p\) | Specific heat capacity | |
| \(d_p\) | Pore scale diameter | \(\mu\) Dynamic viscosity |
| \(D\) | Unit cell length | \(\epsilon\) Turbulent energy dissipation rate |
| \(f\) | Frequency | \(\varphi\) Porosity |
| \(k\) | Turbulent kinetic energy | \(\rho\) Density |
| \(H\) | Unit cell distance | |
| \(p\) | Pressure | |
| \(Re\) | Reynolds number | \(f\) Fluid |
| \(T\) | Temperature | \(\infty\) Variable in Fully developed region |
| \(u\) | \(x\)-velocity | \(s\) Solid |
| \(v\) | \(y\)-velocity | |

Masouka and Takatsu provided a zero equation turbulence model for flow in a porous media, in which eddy viscosity is a function of permeability, velocity (turbulent kinetic energy) of the fluid inside the porous media and the Forchheimer coefficient. They stated that momentum transfer in turbulent flow in a porous media is carried out by two type of vortexes; pseudo vortex, and void vortex. Therefore, the viscosity of the turbulent flow is the result of adding pseudo vortex and void vortex together \(\mu_t = \mu_{t,pseudo} + \mu_{t,void}\). Figure 1 shows pseudo vortex and void vortex [7].

Masouka and Takatsu model places transient conditions to the turbulent flow after the Darcy regime, though it is apparent that in the porous media, Forchheimer regime exists after the Darcy regime, and transient condition cannot start directly after the Darcy regime. They have considered \(Re_p=10\) as the onset of turbulent flow. They have also correlated the Forchheimer flow resistance to the turbulent flow characteristic, an assumption which none of their references has verified. Therefore, their model is based on an incorrect assumption and their result cannot be relied upon [8]. Alvarez et al. provided an equation for modeling the turbulent flow in a packed sphere beds. They calculated their constants based on experimental results. However, the
The k-ε model has been tailored specifically for planar shear layers and recirculating flows. This model is the most widely used and validated turbulence model with applications ranging from industrial to environmental flows. So many researchers based their model on k-ε turbulence model. Anthe and Lage used time averaging of volume averaged Navier-Stokes equations to provide a two equations model for turbulent flow in a porous media [10]. The kinetic energy in the equations they provided is considered zero for a unidirectional, fully developed and steady flow. Nakayama and Kuwahara provided a two equation model by volume averaging of time averaged equations [11]. Pedras and de Lemos, demonstrated that the order of averaging Navier-Stokes equation has no effect on the results, except for turbulent kinetic energy equation [12]. They showed that by volume averaging after time averaging, turbulence in pore scale can be considered in the model. Both of them provided an equation for modeling the terms created in turbulent kinetic energy equation and its dissipation rate after volume averaging [13]. However, it should be noted that the source terms they presented is not valid in all porous media. The constants of N-K and P-dL terms need to be obtained by experimental data, however, that’s not an easy task to be done. Therefore, they used a periodic array to obtain the constants of the proposed equations. Guo et al. researches [14] showed that P-dL proposed model suffers from a number of deficiencies. They demonstrated that Eddy viscosity is highly dependent to the pore size in the P-dL model, due to the presence of \( k/u_D^2 \) term in Eddy viscosity of P-dL model (Equation 2). Another deficiency of P-dL model lies in the source terms, in a way that both equations of this model will turn into one in case of a fully developed flow. In fact, all the terms of the transport equation except the source term are omitted.

\[
\varepsilon = \frac{c_k k \rho u}{\sqrt{K}} 
\]  

\[
\mu_\varphi = \frac{c_\mu \rho \varphi u_D d_p \varphi^{3/2}}{c_k \sqrt{150(1-\varphi)} u_D^2 k} 
\]

Therefore, the amount of Eddy viscosity in P-dL model is dependent of dimensionless turbulent energy \( \frac{k}{u_D^2} \) which itself is dependent of turbulence intensity in the porous media entrance. This is inconsistent with previous results on the independency of turbulent values in the developed area in the porous media to the inlet conditions. Guo et al. compared three different models for flow simulation in a porous media and deduced that Nakayam and Kuwahara model is superior to others and provides more realistic results [14]. Nakayama and Kuwahara showed dimensionless turbulent energy for \( \text{Re}_p > 3000 \) is independent of Reynolds number, and N-K model benefits from values independent of \( \text{Re}_p \) for calculating \( G_e^i \) and \( G_k^i \). Nakyama and Kuwahara model has been used extensively for different turbulent models in porous media like flow in channels, pipes, packed beds and etc. in recent years. Nouri-Borujerdi and Seyyed-Hashemi studied heat transfer of turbulent flow in porous media. They have used k-ε model to handle turbulence in porous media and investigate different porous thickness and Darcy number...
Kazerooni and Hannani simulated turbulent flow in porous media by using v2f model and applying finite element method. They concluded in the macroscopic models, the inherent advantages of using a v2f model are not obvious and should be proved mathematically [16].

Kim and Kang conducted a numerical simulation to study anomalous transport through free-flow-porous media interface [17]. They showed RANS models can affectively be applied to simulate turbulence flow in porous media at pore scale. De Lemos and Assato investigated turbulence structure and heat transfer in a sudden expansion with a porous insert using linear and non-linear turbulence models. Their results showed complete damping of turbulence kinetic energy generation along the channel for thick porous inserts [18]. Soulaine and Quintard proposed an approach to derive a macro-scale momentum equation that is free from the turbulence model chosen for the pore-scale simulations and that is able to capture large-scale anisotropy. This technique gives a macro-scale generalized Darcy–Forchheimer equation to which is associated a closure problem that can be used to evaluate the apparent permeability tensor including inertia effects [19]. Torabi et al. analyzed fluid flow, heat transfer and entropy generation of turbulent forced convection through isotropic porous media using RANS models [20]. They investigated different Re numbers, porosities and cross-sections for two turbulence models. They concluded with considering symmetric contours of thermophysical properties, the RNG k-ε model revealed a more successful illustration of contours when compared with the SST k-ω model. Kundu et al. investigated turbulent flow in isotropic porous media numerically. They compared v2f model and low Re-k-ε-Lam-Bremhorst (LB) with LES predictions and found low Re-k-ε-(LB) results are closer to LES Results [21]. Chu et al. conducted direct numerical simulation to study heat transfer and pressure drop in porous media. To this purpose, they used periodic square cylinder in a staggered array and deduced increasing Reynolds numbers lead to more pressure loss than improved heat transfer [22]. Four kinds of two equation turbulence models were compared in simulating DPF’s porous media and swirl type regenerator burner. It was depicted back flow features of the Realizable k-ε are clearer than that of RNG k-ε model [23]. Ahmad Kan and Straatman proposed two equation turbulent model for investigation non-equilibrium heat transfer for unidirectional flow in porous media [24]. Due to complexity of turbulent flow in porous media, Linsong et al. studied turbulent flow in randomly packed beds by pore-scale three dimensional simulation. It was demonstrated that complicated turbulent eddy structures exist in the pores of the packed bed [25].

Despite widespread effort of researchers to provide an accurate model to simulate turbulence in a porous media, macroscopic models developed so far are unsuitable for simulating unsteady and oscillating flows. Therefore, in this paper it has been attempted to provide a model better suited to simulate unsteady oscillating flow by correcting N-K model coefficients. To achieve this, the flow has been simulated in a porous media using periodic arrays in different Reynolds and frequencies, and then, N-K model coefficients has been corrected using the simulation results.
2. Flow equations in macroscale

Turbulence can be significant in high Reynolds numbers in the porous media. If the Reynolds number is high enough in the microscale and the length scale of turbulence is considerably lower than length scale of porous media pores, turbulent models provided for non-porous conditions can be used to solve the flow field in a porous media in microscale [11].

Time averaged equations including continuity, Navier-Stocks, energy, turbulent kinetic energy and rate of dissipation equations are as follows:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0
\]  
\[3\]

\[
\frac{\partial (\rho \bar{u})}{\partial t} + \frac{\partial (\rho \bar{u}_j \bar{u}_i)}{\partial x_j} = -\frac{\partial (p)}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right) - \rho \bar{u} \bar{u}_j
\]  
\[4\]

\[
\frac{\partial (\rho c_{pf} \bar{T})}{\partial t} + \frac{\partial (\rho c_{pf} \bar{u}_j \bar{T})}{\partial x_j} = \frac{\partial}{\partial x_j} \left( k_j \frac{\partial \bar{T}}{\partial x_j} - \rho \bar{u}_j \bar{u}_j \right)
\]  
\[5\]

\[
\frac{\partial (\rho \bar{k})}{\partial t} + \frac{\partial (\rho \bar{u}_j \bar{k})}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \mu + \frac{\mu_s}{k} \right) \frac{\partial \bar{k}}{\partial x_j} \right) - u_j u_j \frac{\partial \bar{u}_j}{\partial x_j} - \epsilon
\]  
\[6\]

\[
\frac{\partial (\rho \bar{\epsilon})}{\partial t} + \frac{\partial (\rho \bar{u}_j \bar{\epsilon})}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \mu + \frac{\mu_s}{k} \right) \frac{\partial \bar{\epsilon}}{\partial x_j} \right) + \left( -c_{\mu u_j} \frac{\partial \bar{u}_j}{\partial x_j} - c_{\mu u_j} \frac{\partial \bar{u}_j}{\partial x_j} \right) \frac{\epsilon}{k}
\]  
\[7\]

The term of Reynolds stress can be modeled using the Boussinesq model as the following:

\[
\rho \bar{u}_j u_j = \mu \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} \rho \bar{k} \delta_{ij}
\]  
\[8\]

And also:

\[
\rho_{pf} \bar{u}_j = \frac{\mu_s}{\sigma} \frac{\partial \bar{T}}{\partial x_j}
\]  
\[9\]

Turbulent viscosity can be obtained using equation (10).

\[
\mu_s = \frac{k^2}{\epsilon}
\]  
\[10\]

For the solid phase, the energy equation is as seen in equation (11).

\[
\frac{\partial (\rho_{pf} \bar{T})}{\partial \bar{T}} = \frac{\partial}{\partial x_j} \left( k_s \frac{\partial \bar{T}}{\partial x_j} \right)
\]  
\[11\]

Coefficients used in the above equations are as follows:
\[ c_\mu = 0.09, \quad c_1 = 1.44, \quad c_2 = 1.92 \]

\[ \sigma_k = 1.00, \quad \sigma_e = 1.30, \quad \sigma_f = 0.90 \]

Equation (12) uses intrinsic volume average values of turbulence to define turbulent viscosity.

\[ \mu_t = c_\mu \frac{(k')^2}{\langle \epsilon' \rangle} \] (12)

By integrating \( k \) and \( \varepsilon \) equations, in a representative elementary volume, the equations change into the following [11]:

\[ \frac{\partial (\rho_t \langle k' \rangle)}{\partial t} + \frac{\partial (\rho_t \langle \bar{u}_i \rangle \langle k' \rangle)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( (\mu + \mu_t) \frac{\partial \langle k' \rangle}{\partial x_j} \right) + 2\mu_t \langle s'_{ij} \rangle \langle s'_{ij} \rangle - \langle \epsilon' \rangle + 2\mu_t \langle s''_{ij} \rangle \langle s''_{ij} \rangle + \frac{\mu}{V_f} \int_{\lambda_m} \frac{\partial k}{\partial x_j} n_j dA - \frac{\partial}{\partial x_j} \langle \bar{u}_i \rangle^{-1} \langle k' \rangle \] (13)

\[ \frac{\partial (\rho_t \langle \epsilon' \rangle)}{\partial t} + \frac{\partial (\rho_t \langle \bar{u}_i \rangle \langle \epsilon' \rangle)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( (\mu + \mu_t) \frac{\partial \langle \epsilon' \rangle}{\partial x_j} \right) + 2c_1 \mu_t \langle s''_{ij} \rangle \langle s''_{ij} \rangle - \langle \epsilon' \rangle + 2c_1 \mu_t \langle s''_{ij} \rangle \langle s''_{ij} \rangle + \frac{\mu}{V_f} \int_{\lambda_m} \frac{\partial \epsilon}{\partial x_j} n_j dA - \frac{\partial}{\partial x_j} \langle \bar{u}_i \rangle^{-1} \langle \epsilon' \rangle \] (14)

\[ s_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \] (15)

In the above equations, the third and higher order terms are omitted. Due to the presence of the porous media, two extra terms are created which include the production term \( 2\mu_t \langle s''_{ij} \rangle \langle s''_{ij} \rangle \) and dissipation rate \( \frac{\mu}{V_f} \int_{\lambda_m} \frac{\partial k}{\partial x_j} n_j dA \). The \( 2\mu_t \langle s''_{ij} \rangle \langle s''_{ij} \rangle \) term represents the production of turbulent kinetic energy for a laminar uniform flow without shear stress, and the \( \int_{\lambda_m} \frac{\partial k}{\partial x_j} n_j dA \) part of the dissipation term is negative as result of the no-slippage condition. Sum of these two terms shows net production in the presence of porosity. Therefore, these two terms can be modeled as seen in (16).

\[ \varepsilon_{s} = 2\mu_t \langle s''_{ij} \rangle \langle s''_{ij} \rangle + \frac{\mu}{V_f} \int_{\lambda_m} \frac{\partial k}{\partial x_j} n_j dA \] (16)

These terms are modeled in the turbulent energy dissipation rate equation, seen in equation (17).
Experimental determination of unknown constants $k$ and $\varepsilon$ needs difficult accurate measurements in the pore scale. Therefore, they are obtained using numerical experiments. For a macroscopic flow without shear stress $k$ and $\varepsilon$ equations are simplified as seen below.

$$
\langle u \rangle_f \frac{\partial \langle k \rangle_f}{\partial x} = -\langle \varepsilon \rangle_f + \varepsilon_\infty
$$

(18)

$$
\langle u \rangle_f \frac{\partial \langle \varepsilon \rangle_f}{\partial x} = -c_2 \frac{(\langle \varepsilon \rangle_f)^2}{\langle k \rangle_f} + c_2 \frac{\varepsilon_\infty^2}{k_\infty}
$$

(19)

By neglecting the velocity gradient in the X direction, the equation is simplified as the following.

$$
\langle k \rangle_f = k_\infty, \quad \langle \varepsilon \rangle_f = \varepsilon_\infty
$$

(20)

Therefore, by simulating the flow in the microscale, constants in the equations above can be obtained. Kuwahara et al. in 1998 [26] and Kundu et. al. in 2016 [27] conducted several numerical experiments using periodic arrays in square, circular, cubic and spherical shapes in two dimensional and three dimensional forms and similar results in calculation of permeability in the laminar regime was achieved, by each array. Two arrangements of arrays are considered for numerical investigation in porous media. These two arrangements are inline and staggered arrangement. In most prior studies, staggered arrangements were selected due to its capability in capturing non-homogenous and random structure of porous media physics in real world condition. Thus, two-dimensional staggered square arrays are used to model the flow in microscale. Figure (2) shows these arrays used in simulating oscillating turbulent flow in microscale. The porosity of these arrays can be calculated using this equation.

$$
\varepsilon = 1 - \left( \frac{D}{H} \right)^2
$$

(21)

Prior studies show that inlet and outlet effects of the flow are eliminated by choosing a proper inlet and outlet length. Kim suggested $2H$ for the inlet length and $7H$ for the outlet length, to eliminate boundary condition in the outlet [28].

3. Numerical Method

Researchers use periodic boundary conditions to study the details of a flow in a unit cell. However, in an oscillating flow, a unit cell is insufficient to study the input effects and flow phase difference using periodic boundary conditions in the direction of the flow. Therefore, to the study an oscillating flow, 7 duplicated cells are used [29]. The symmetry boundary condition is used in the up and down of each cell for minimizing computational cost. This is acceptable due to the symmetry of the flow to the x-axis. Figure 3 shows the periodic array used to simulate turbulent oscillating flow in a porous environment. Boundary conditions are considered symmetrical above and below the periodic array (equation 22), and in the tail of this array, fully
developed boundary conditions are used (equation 23). Also, velocity in the inlet of the array is considered sinusoidal which is defined as seen in equation 24. The well-established SIMPLE method is used to deal with pressure and velocity coupling. Collocated arrangement of grid is adopted to discretize the aforementioned equations. Rhie and chow interpolation technique is utilized to handle checker board in pressure field [30,31].

\[
\frac{\partial u}{\partial y} = 0, \quad \frac{\partial v}{\partial y} = 0
\]

(22)

\[
\frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0
\]

(23)

\[
u_{in} = U_{in} \sin(2\pi f \times t)
\]

(24)

Based on the inlet velocity, Reynolds is considered at its maximum [2].

\[
Re_{max} = \frac{\rho_f U_{in} d_p}{\mu_f}
\]

(25)

Figure 4 shows the cell units for 0.305, 0.49, 0.61, 0.69, 0.75 and 0.826 porosity. The standard k-\(\varepsilon\) model is used for simulating the flow. The equations were discretized using finite volume technique. The SIMPLE method is adopted to handle pressure and velocity coupling. The algorithm starts with two momentum equations and then by solving the pressure correction equation, pressure is corrected. Afterwards, k-\(\varepsilon\) equations are solved to calculated turbulent viscosity. The TDMA algorithm is used to solve these algebraic set of equations [32].

To study the independency of the solution to the grid in macroscale, three grids each with 27600, 50400 and 141500 cells were used to solve the flow in \(Re_{p,\text{max}}=3200\), \(f=1\) Hz frequency, and porosity of \(\varepsilon=0.69\). Figures 5 and 6 depict averaged turbulent kinetic energy and dissipation rate of turbulent energy in a periodic array for these 3 solution grids. As it can be seen, the two finer grids produce similar results. Therefore, the 50400 grid is suitable for solving the flow. This method was used for other Reynolds and frequencies to check for grid independency, to obtain the optimum grid number.

To check for the independency of the solution to time step, three time steps were taken into account. Figures 7 and 8 demonstrate averaged turbulent kinetic energy and turbulent energy dissipation rate in a periodic array for an oscillating flow with \(Re_{p,\text{max}}=3200\), \(f=1\) Hz frequency, \(\varepsilon=0.69\) porosity. As shown, choosing a 0.001 s time step is enough for solving the flow. The time step has been chosen similarly for other frequencies. The results were compared with Tu and et al. work for validation [33]. They measured the velocity profile of oscillating flow in fully developed region of a pipe experimentally. The test section where velocity measurements were made is a Plexiglas tube, 0.3 m long and 50 mm in internal diameter. A rotating profiled sleeve driven by a regulated, geared D.C. motor controls the exit area for the water. The sleeve profile
is designed to give two complete cycles of sinusoidal oscillation in discharge in one revolution. As it is depicted in Fig. 9, there is a good agreement between present code results and experiment results. It shows this code can be used practically for oscillating flow simulation.

Figures 10 through 14 depict velocity, pressure and turbulent kinetic energy, dissipation rate and turbulent viscosity contours in T/4, T/2, 3T/4 and T times (T is the period time of a cycle). Except in the first and the last cells, the velocity contour is repeated in other cells at each time step, and thus, to calculate time varying turbulent values such as turbulent kinetic energy and its dissipation, a central cell can be chosen, and by surface averaging, its average amount in a cycle is obtained. Contours are shown as enlarged in figures 15 through 18. A vortex has formed at the corner of the cells due to the abrupt change in flow direction. The vortexes are formed due to the flow separation and adverse pressure gradient in abrupt expansion. In the first half-cycle, the vortex is placed symmetrically with the second half-cycle. In the piston dead zone, prolate vortexes have been formed in the entire flow field in the first and second half-cycle. This is due to the piston returning and drawing part of the flow with it. Other part of the flow moves opposite the movement of the piston due to inertia, and this has created two vortexes opposite each other. In the areas far from the walls, in the boundaries considered as symmetrical, the velocity is at its maximum which created boundary layers by the walls. In figure 11, fluid pressure drop can be seen in the length of the periodic array. Highest pressure drop was witnessed while the fluid velocity in the periodic array placed Reynolds in the Forchheimer regime. In lower Reynolds and Darcy regime, pressure drop is more affected by fluid viscosity and therefore pressure drop is lower compared to that of the Forchheimer regime. In the Forchheimer regime, fluid pressure drops by the square of fluid velocity, while in the Darcy regime, the relation between the two is linear.

Figures 19 and 20 demonstrate turbulent kinetic energy and its dissipation rate for \( Re_{p,max}=3200 \) and \( \varepsilon=0.69 \). As it can be seen, frequency has no effect on turbulent kinetic energy and its dissipation rate. Figure 21 depicts turbulent kinetic energy and its dissipation rate constant coefficients for \( Re_p=800 \) and \( Re_p=1600 \). It’s plain to see that Reynolds number affects turbulent energy and its dissipation rate. By increasing Reynolds number, turbulent kinetic energy correction coefficient increases in the N-K model, while dissipation rate of turbulent energy correction coefficient decreases.

In Figures 22 and 23 corrected coefficients are plotted versus Reynolds. By increasing Reynolds numbers, turbulent kinetic energy corrected coefficient increases, while its dissipation rate corrected coefficient decreases. The coefficients of the both equations behave asymptotically, and become independent of Reynolds number by increasing it. An equation has been suggested for each of the corrected coefficients. For turbulent kinetic energy and its dissipation rate, corrected coefficient equation is formed as equations 26 and 27.

\[
\text{coefficient} = 3.94 - \frac{16.34}{Re^{0.4}_{p,\text{max}}}
\]  

(26)
To validate the corrected model results, porous media channel flow is modeled by volumetrically averaged equations and then compared with microscale model and N-K model results. Figures 24 and 25 represent turbulent kinetic energy and its dissipation rate in one cycle. The corrected model provides better results compared to that of the N-K model. N-K model coefficients are provided for higher Reynolds numbers and in those conditions, those coefficients are independent of the Reynolds number. However, in many applications, flow Reynolds number is lower than what Kuwahara and Nakayama considered for their provided model. Therefore, these coefficients need correction for use in other Reynolds numbers. Figures 24 and 25 demonstrate that the corrected model has better correlation to the microscale model.

4. Conclusion

One of the most applied turbulent models in porous media was proposed by Nakayama and Kuwahara. N-K model was based on unidirectional and high Reynolds number flow in porous media and is not applicable in turbulent oscillating flow in porous media so that a modification to this model has been proposed in this paper. Hence, the turbulent oscillating flow in porous media in microscale and macroscale has been investigated numerically. Turbulent flow in microscale has been simulated by using standard k-ε model. The equations were discretized by using control volume approach and the well-established SIMPLE algorithm was used to deal with pressure and velocity coupling. The N-K model constants have been modified to solve turbulent oscillating flow in porous media via microscale results. The effect of different parameters in oscillating flow such as frequency and Re has been studied. It has been demonstrated that frequency has no effect on N-K model constants. But, these constants in turbulent kinetic energy and its dissipation rate have been modified according to the Reynolds number. Two equations for these constants have been proposed. The modified model for simulating turbulent oscillating flow in porous media has been verified by simulating flow with volume averaged equations. The modified model results had been compromised in comparison to N-K original model.

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Fig. 1 Schematics of void and pseudo vortexes in a packed bed [7]

Fig. 2 Periodic arrays for modeling an oscillating flow in a porous media in microscale

Fig. 3 Periodic array chosen to simulate oscillating turbulent flow in a porous media

Fig. 4 Periodic arrays for 0.305, 0.49, 0.61, 0.69, 0.75 and 0.826 porosity

Fig. 5 Dimensionless kinetic energy in periodic arrays versus time for three solution grids in a cycle, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$ where

Fig. 6 Dimensionless dissipation rate of kinetic energy in periodic arrays over time for three solution grids in a cycle, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 7 Dimensionless kinetic energy in periodic arrays versus time for three time steps in a cycle, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 8 Dimensionless dissipation rate of kinetic energy in periodic arrays versus time for three time steps in a cycle, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 9 Code validation: Comparison of velocity profile in fully developed turbulent oscillating flow in one period with tu et al. [33]

Fig. 10 Velocity contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$ show on figure

Fig. 11 Pressure (Pa) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 12 Turbulent kinetic energy ($m^2/s^2$) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 13 Dissipation rate of turbulent kinetic energy ($m^2/s^3$) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 14 Turbulent viscosity (kg/m.s) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$

Fig. 15 Velocity (m/s) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$ in one periodic cell

Fig. 16 Turbulent kinetic energy ($m^2/s^2$) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$ in one periodic cell

Fig. 17 Dissipation rate of turbulent kinetic energy ($m^2/s^3$) contours at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$ in one periodic cell

Fig. 18 Turbulent viscosity contours (kg/m.s) at three times, $T/4$, $T/2$, $3T/4$ and $T$, $Re_{p,max}=3200$, $f=1$ Hz, $\varepsilon=0.69$ in one periodic cell many contours

Fig. 19 Dimensionless turbulent kinetic energy versus porosity at $Re_p=3200$ in different frequencies

Fig. 20 Dimensionless dissipation rate of turbulent kinetic energy diagram over porosity at $Re_p=3200$ in different frequencies
**Fig. 21** Dimensionless turbulent kinetic energy versus porosity at a) $Re_p=800$ and $f=1$ Hz, b) $Re_p=1600$ and $f=1$ Hz

**Fig. 22** Turbulent kinetic energy correction coefficient versus $Re_{p,max}$

**Fig. 23** Dissipation rate of Turbulent kinetic energy correction coefficient versus $Re_{p,max}$

**Fig. 24** Dimensionless turbulent kinetic energy in one cycle versus time at $Re_p=800$, $f=1$ Hz and $\phi=0.305$

**Fig. 25** Dimensionless Dissipation rate of turbulent kinetic energy in one cycle versus time at $Re_p=800$, $f=1$ Hz and $\phi=0.305$
Fig. 4.

Fig. 5.
Fig. 6.

Fig. 7.
Fig. 8.

Fig. 9.
Fig. 12.

Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 19.

\[ y = 3.2766x - 0.01383 \]
\[ R^2 = 0.9989 \]

Fig. 20.

\[ y = 49.775x + 0.0902 \]
\[ R^2 = 0.9988 \]
Fig. 21.

Fig. 22.
Fig. 23.

\[ \text{Modified Coefficient} \]

\[ R_{e_{p,max}} \]

Fig. 24.

\[ k/U^2 \]

\[ t/T \]
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