Neutrino mean free paths in spin-polarized neutron Fermi liquids

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Abstract. Neutrino mean free paths in magnetized neutron matter are calculated using the Hartree-Fock approximation with effective Skyrme and Gogny forces in the framework of the Landau Fermi liquid theory. It is shown that describing nuclear interaction with Skyrme forces and for magnetic field strengths \( \log_{10} B(G) \gtrsim 17 \), the neutrino mean free paths stay almost unchanged at intermediate densities but they largely increase at high densities when they are compared to the field-free case results. However, the description with Gogny forces differs from the previous and mean free paths stay almost unchanged or decrease at densities \([1-2]n_0\). This different behaviour can be explained due to the combination of the common mild variation of the Landau parameters with both types of forces and the values of the nucleon effective mass and induced magnetization of matter under the presence of a strong magnetic field as described with the two parametrizations of the nuclear interaction.

1 Introduction

The study of the behaviour of hadronic matter in the density temperature, \((\rho, T)\), diagram allows to have a deeper understanding of matter under extreme conditions. In this context, the high-density, low-temperature limit can be addressed for a fermion system using the Landau Fermi liquid theory (FLT) [1]. From a theoretical point of view the properties of this type of normal quantum systems can be studied calculating the interaction matrix element of quasiparticle (qp) excitations close to the Fermi surface. The inclusion of an additional component in the problem, a magnetic field, \(B\), allows further testing the properties of magnetized Fermi liquids. The role of magnetic fields in bulk properties and equation of state has been partially analyzed in the past for nuclear matter [2,3] and quark matter [4,5]. Due to the tiny value of the neutron magnetic moment \(\mu_n = -1.9130427(5)\mu_N\) \((\mu_N = 3.1524512326(45) \times 10^{-8}\text{MeV}\text{G}^{-1})\) [6] and in order to provide a sizable magnetization, huge magnetic fields are needed.

The only scenarios where we have indication of such intense fields are, first, from estimates of the background magnetic fields created in heavy-ion collisions like those at RHIC [7] and, second, in a subgroup of pulsars called magnetars. For these astrophysical objects surface magnetic field strengths are of the order \(B \approx 10^{15}\text{G}\) [9,8]. Recent numerical simulations [10] of the formation of proto-neutron stars show that the field configuration plays a significant role in the dynamics of the core if the initial magnetic field is large enough. In particular, in the rapid cooling of the newly formed neutron-rich object neutrino transport is an important ingredient [11]. However, some of these simulations lack from accurate and consistent neutrino transport, missing the impact of magnetic fields in the microphysics input that affects the dynamics of the collapsing dense objects.

In most of the existing calculations of nuclear matter (either symmetric, pure neutron or beta-equilibrated) the effect due to the presence of strong magnetic fields and the consistently induced spin polarization are discarded in a first approximation. Either relativistic [12,13] or effective approaches [14] have been used to obtain some insight into the equation of state (EOS) or some structure properties [2] in the presence of magnetic fields. These include a possible transition to a ferromagnetic state, although simulations using realistic potentials seem to prevent it [15]. In general, a non-vanishing magnetization in a low-temperature nuclear plasma [16] produces a resolution of some degenerated observables as obtained in the context of the FLT [17,18].

2 Formalism

In this work we are interested in the response of a spin-polarized pure neutron plasma to a weak neutrino probe. It can be seen [12] that for the density range \(\rho \leq 4\rho_0\), where the quark deconfinement is not expected to take place, and for magnetic field strengths of maximum strength \(B \approx 10^{18}\text{G}\), allowed in principle by the scalar virial theorem, the neutral system is mostly neutrons. The maximum magnetic field strength we will consider is \(B^* \approx 2 \times 10^4\) (as measured in units of the electron
critical field $B^* = B/B_0^*$ with $B_0^* = 4.4 \times 10^{13} \text{G}$) and the neutron fraction is $Y_n > 0.98$ [12]. So the neutral plasma is mostly neutrons but leptons and additional baryons are also present in a tiny fraction that should be considered for full application in an astrophysical scenario where $\beta$ equilibrium holds.

We are interested in exploring the effect of a strong magnetic field and the spin polarization of a pure neutron plasma through the structure functions, which provide information on density and spin density in-medium correlations. The homogeneous system under study is under the presence of an internal magnetic field, $B = Bk$ populated by species with paricle density $\rho_{\sigma}$, where $\sigma = \pm 1$ is the spin z-projection. $\Delta = \rho_{+} - \rho_{-}$ is the spin excess and $\rho = \rho_{+} + \rho_{-}$ is the total particle density. For given thermodynamical conditions $\Delta$ is obtained by minimizing the Helmholtz free energy per particle, $f(\rho, T, B, \Delta) = \epsilon - \mu_{\sigma}\Delta\rho_{B}$, where $\epsilon$ is the energy per particle. Note that parallel (antiparallel) aligned magnetic moments (spins) are energetically favoured. We have considered an effective approach to describe the nuclear interaction using zero-range Skyrme forces [19] with two of the most widely used parametrizations given by the Lyon group SLy4 and SLy7 [20, 21] and finite-range Gogny with D1P [22] and D1S [23] forces. All of them provide good values for the binding of nuclei and also for neutron matter EOS.

In the context of the FLT the properties of nonmagnetized systems at low temperature have been evaluated [24] by calculating the qp matrix element around the Fermi surface where the only dependence is on fermionic densities and the qp scattering angle, $\theta$, involved. In the usual formalism, for the nonmagnetic case the qp matrix element is written as a multipolar expansion in Legendre polynomials,

$$V_{ph} = \sum_{l=0}^{\infty} [f_l + g_l \sigma_1 \cdot \sigma_2] P_l(\cos \theta),$$

with the auxiliar definitions, $\Gamma^{(\sigma, \sigma')}_l = \int (\frac{d^3k}{(2\pi)^3}) \cos(\theta) G^{(\sigma, \sigma')}_l = \int (\frac{d^3k}{(2\pi)^3}) \cos(\theta) G^{(\sigma)}_l G^{(\sigma')}_0$. Notice that the qp propagators $G^{(\sigma)}_l$ have been given in [26] and the expressions for the coefficients $\gamma^{(\sigma)}_l$ can be written [25] in the Landau limit as $\gamma^{(\sigma)}_1 = \nu^{(\sigma)}(\chi_0)^2$ and $\gamma^{(\sigma)}_2 = \nu^{(\sigma)}(\chi_0)^2 - \frac{k_{F,\sigma}^2}{m_0}$ where $\nu^{(\sigma)} = m_0 \Delta - \frac{\epsilon}{\sigma}$. The qp effective mass in a magnetized system depends on the polarized dipolar coefficients [1],

$$m^*_l = m + \frac{1}{3} N_{0\sigma} \left[ f_l^{(\sigma, \sigma)} + \left( \frac{k_{F,\sigma}^2}{k_{F}^2} \right) f_l^{(\sigma, -\sigma)} \right], \quad (4)$$

where $N_{0\sigma} = \frac{m^*_l k_{F,\sigma}^2}{2 \pi^2}$ is the quasiparticle level density at each polarized Fermi surface with momentum $k_{F,\sigma}$.

The generalized parameters $f_l^{(\sigma, \sigma')} = \nu^{(\sigma)}(\chi_0)^2$ are obtained by deriving the Helmholtz free energy with respect to the polarized density component, $f_{k, \sigma, \sigma'} = \frac{\partial E}{\partial n_{k, \sigma, \sigma'}}$ [17], setting momenta on the polarized Fermi surfaces and expanding the resulting expression as a series in Legendre polynomials of multipolarity $l$. These generalized parameters fulfill the following relations recovering the usual ones in FLT in the limit $\Delta \to 0$ [17]:

$$f_l = f_l^{(\sigma, \sigma)} + f_l^{(\sigma, -\sigma)} \frac{2}{l(l+1)}, \quad (5)$$

$$g_l = f_l^{(\sigma, \sigma)} - f_l^{(\sigma, -\sigma)} \frac{2}{l(l+1)}. \quad (6)$$

With the generalized parameters and using the expressions in eq. (2) the corresponding Lindhard function for the isovector ($S = 0$) response of the plasma can be written as

$$\chi^{(S=0)}(\omega, q) = \chi^{(++)} + \chi^{(+-)} + \chi^{(++)} + \chi^{(-+)}, \quad (7)$$

and for the vector-axial ($S = 1$) response as

$$\chi^{(S=1)}(\omega, q) = \chi^{(++)} + \chi^{(+-)} - \chi^{(++)} - \chi^{(-+)}. \quad (8)$$

Then the previous expression of the Lindhard function in RPA approximtion [27] includes in-medium correlations at zero temperature. From them, one can obtain the structure functions given by

$$S^{S=0,1}(\omega, q) = -\frac{1}{\pi} \text{Im} \chi^{S=0,1}(\omega, q). \quad (9)$$

The structure function allows to calculate the nonrelativistic differential cross-section of neutrinos scattering off matter via neutral currents from [28]

$$\frac{1}{V} \frac{d\sigma}{d\Omega d\omega} = \frac{G_F^2}{8\pi^2} E^2 [C_V^2 (1 + \cos \theta) S^0(\omega, q) + C_A^2 (3 - \cos \theta) S^1(\omega, q)], \quad (10)$$

where $E$ ($E'$) is the incoming (outgoing) neutrino energy and $k$ ($k'$) is the neutrino incoming (outgoing) three-momentum. The transferred energy is $\omega = E - E'$ and