Spin waves in semiconductor microcavities

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We show theoretically that a weakly interacting gas of spin-polarized exciton-polaritons in a semiconductor microcavity supports propagation of spin waves. The spin waves are characterised by a parabolic dispersion at small wavevectors which is governed by the polariton-polariton interaction constant. Due to spin-anisotropy of polariton-polariton interactions the dispersion of spin waves depends on the orientation of the total polariton spin. For the same reason, the frequency of homogeneous spin precession/polariton spin resonance depends on their polarization degree.

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Introduction. Spin waves in nonmagnetic Fermi liquids predicted over 50 years ago [1] and discovered in the end of 1960s in alkali metals [2] are among the most fascinating manifestations of collective effects in interacting system. Later it was understood theoretically that the spin waves can exist in a non-degenerate electron gas [3] as well as in atomic gases [4, 5], and the spin waves were indeed observed in a number of interacting gases such as Hydrogen and Helium [6–8], see Ref. [9] for review. Spin waves were also observed in atomic Bose gases, namely, in ultracold 87Rb vapors at temperatures of about 850 nK which exceeded the Bose-Einstein condensation temperature [10], see also Refs. [11–13] where this experiment was interpreted in detail.

Recently, semiconductor microcavities with quantum wells sandwiched between highly reflective mirrors have attracted a lot of interest in the solid state and photonics communities [14]. In these artificial structures the strong coupling is achieved between excitons, being material excitations in quantum wells, and photons trapped between the mirrors [15]. Resulting mixed light-matter particles, exciton-polaritons, demonstrate the Bose-Einstein statistics and may condense at critical temperatures ranging from tens Kelvin [16] till several hundreds Kelvin [17, 18], which exceeds by many orders of magnitude the Bose-Einstein condensation temperature in atomic gases. Due to the high transition temperatures and the strong coupling with light, semiconductor microcavities are perfectly suited for benchtop studies of collective effects of Bosons.

In characteristic GaAs based microcavities, exciton-polaritons may have two spin projections onto the structure growth axis, ±1, corresponding to right- and left-circular polarizations of photons (and spin moment of excitons) forming polaritons. Owing to the composite nature of exciton-polaritons, the interactions between them are strongly spin-dependent [14, 15, 19]. A number of prominent spin-related phenomena both in interacting and in noninteracting polariton systems have already been predicted and observed in the microcavities, such as, e.g., polarization multistability [20, 21] and optical spin Hall effect [22, 23], see Refs. [14, 15, 19] for reviews.

Here we predict the existence of weakly-damped spin waves for a non-degenerate or weakly degenerate polariton gas in a microcavity with embedded quantum wells. We calculate the frequencies of spatially homogeneous spin excitations and find the dispersion and damping of weakly inhomogeneous spin distributions, i.e. spin waves. The stability of spin oscillations is analyzed, it is shown that under certain excitation condition the interacting system of exciton-polaritons may become unstable and spin oscillations may be self-induced. The experimental manifestations of spin waves in the photoluminescence spectroscopy and spin noise studies are discussed.

Model. We consider non-degenerate or weakly degenerate polariton gas at a temperature higher than the Berezinskii-Kosterlitz-Thouless transition temperature. The polariton single-particle spin density matrix is parametrized as \( \hat{\rho}_k = (N_k/2) \hat{I} + \hat{S}_k \cdot \hat{\sigma} \), where \( N_k \) is the occupancy of the orbital state with the wavevector \( k \), \( \hat{S}_k \equiv \hat{S}_k(\mathbf{r}) \) is the coordinate \( r \)-dependent spin distribution function, \( \hat{I} \) and \( \hat{\sigma} \) are \( 2 \times 2 \) unit and Pauli matrices, respectively [24]. The spin distribution function satisfies the kinetic equation [24, 25]

\[
\frac{\partial \hat{S}_k}{\partial t} + \mathbf{v}_k \cdot \frac{\partial \hat{S}_k}{\partial r} + \mathbf{S}_k \times \Omega^{(\text{eff})}_k = Q\{\hat{S}_k\},
\]

where \( \mathbf{v}_k \) is the polariton group velocity in the state \( k \), \( \Omega^{(\text{eff})}_k \) is the total effective field acting on the polariton pseudospin

\[
\Omega^{(\text{eff})}_k = \Omega_L + \alpha_1 \sum_{k'} S_{k'z} e_z.
\]

Here \( \Omega_L \) is the spin precession frequency which is dependent on the external magnetic field \( B \), on the splitting of linearly polarised polariton states due to the structure anisotropy and TE-TM splitting of the cavity modes: \( \Omega_L = g_\mu_B \mathbf{B} \cdot \mathbf{e}_z + \Omega_\alpha \mathbf{e}_z + \Omega(\mathbf{k}) \), where \( \mathbf{e}_i \) are unit vectors of Cartesian axes, \( i = x, y, z \); \( g \) is the exciton-polariton g-factor [26], we assumed that the anisotropy field is parallel to \( x \)-axis, \( \Omega_\alpha \) is the anisotropic splitting, and \( \Omega(\mathbf{k}) \) is the TE-TM splitting neglected hereafter. The constant \( \alpha_1 \) describes interaction of polaritons with parallel spins. We recall that the polariton-polariton interactions are strongly anisotropic [27–31]. Here we neglect for simplicity the interactions of polaritons having opposite signs of circular polarization, which are usually weak compared to the interactions of polaritons with parallel spins [33]. In the
relevant range of low polariton wavevectors (much smaller than inverse excitonic Bohr radius) the dependence of $\alpha_1$ on the wavevector can be neglected, see e.g. Ref. [29]. The operator $Q\{S_k\}$ in the right hand side of Eq. (1) is the collision integral which accounts for the polariton generation, scattering and decay processes,

$$Q\{S_k\} = -\frac{S_k}{\tau_0} + g_k + \sum_{k'} |W_{kk'} S_{k'} - W_{k'k} S_k|.$$  

Here $\tau_0$ is the lifetime of polaritons, $g_k$ is the polariton generation rate accounting for the in-coming flow of quasiparticles from the reservoir, and $W_{kk'}$ is the scattering rate from the state $k$ to the state $k'$ which accounts for both elastic and inelastic scattering processes. Due to the bosonic nature of exciton-polaritons and polariton-polariton interactions $g_k$ and $W_{kk'}$ depend, generally, on the occupancies and spin polarizations in the states $k$, $k'$ [24, 27, 32]. The dynamics of polaritons can be described by Eq. (1) which is valid provided that the renormalization of spectrum due to polariton-polariton interactions is negligible, otherwise the excitations spectrum should be found from the spin-dependent Gross-Pitaevskii equation [34].

Under the steady-state excitation, the quasi-equilibrium distribution $S_k^{(0)}$ of exciton-polaritons is formed. This function satisfies Eq. (1) with derivatives $\partial/\partial t$, $\partial/\partial r$ being equal to zero. For simplicity we assume that the pumping is isotropic, hence $S_k^{(0)}$ depends only on the absolute value of the polariton wavevector $k = |k|$. Such a distribution is caused by the quasi-equilibrium spin polarization in the field $\Omega_L$ and by the polarized pumping. For instance, in the case of excitation of polaritons on the elastic circle with the energy $\varepsilon_0$ by a polarized light, the particles are described by the distribution function $S_k^{(0)} \propto \delta(\varepsilon - \varepsilon_0) e_i$, where $i = x$ or $y$ for the linearly polarized excitation and $i = z$ for the circularly polarized one, $\varepsilon \equiv \varepsilon_k$ is the polariton dispersion, see scheme in Fig. 1(a). In order to analyze the spin excitations, the total spin distribution function is presented as a sum of its quasi-equilibrium part $S_k^{(0)}$ and the fluctuating correction $\delta S_k \ll S_k^{(0)}$. A standard linearization of Eq. (1) and substitution of $\delta S_k = \exp(iq r - i\omega t) s_k$ with $q$ being the wavevector and $\omega$ being the frequency of the fluctuation yields, cf. [1, 9]:

$$(\tau_c^{-1} - i\omega) s_k + \alpha_1 S_k^{(0)} \times e_z \sum_{k'} s_{k',z} + s_k \times \left(\Omega_L + \alpha_1 e_z \sum_{k'} S_k^{(0)}\right) = -\frac{s_k - \bar{s}_k}{\tau},$$  

where we introduced the lifetime of the fluctuation $\tau_c$ and the isotropization time $\tau$. Bar symbolizes averaging over possible orientations of $k$. In the simplest approximation, polaritons are assumed to be supplied directly by the polarized pump or from the incoherent but spin-polarized reservoir, the polariton-polariton scattering is neglected as well as inelastic processes, and the elastic scattering is assumed to be isotropic, in which case $W_{kk'} = W(\varepsilon_k) \delta(\varepsilon_k - \varepsilon_{k'})$,

$$\frac{1}{\tau} = \sum_{k'} W(\varepsilon_k) \delta(\varepsilon_k - \varepsilon_{k'}).$$

The lifetime of a fluctuation is governed by an interplay of polariton decay processes accounted for by the lifetime $\tau_0$ in our formalism, and by the bosonic stimulation effect, which increases the lifetime of the fluctuations, $\tau_c = \tau_0(1+N_k)$ [24]. Equation (4) determines the dynamics of spin fluctuations in the system. Its eigenmodes represent the spin waves in the interacting polariton ensemble.

**Results.** The solution of Eq. (4) can be expressed by decomposing the function $s_k$ in the angular harmonics of the polariton wavevector $k$ as $s_k = \sum_m \exp(i m \varphi) s_m(\varepsilon)$, with $\varphi$ being the azimuthal angle of $k$, and reducing Eq. (4) to a system of equations for the energy dependent functions $s_m(\varepsilon)$. The condition of compatibility for this system of equations yields dispersions of the waves. Below we analyze the spectrum of excitations and eigenmodes for different particular cases.

**Homogeneous excitations.** We start the analysis from the homogeneous case, $q = 0$. In this case the angular harmonics $\exp(i m \varphi) s_m(\varepsilon)$ are the eigensolutions of Eq. (4). For all $m \neq 0$ one eigenmode corresponds to the damped solution with $s_m$ parallel to the total field $\Omega^{(tot)} = \Omega_L + \alpha_1 S_0^{(0)} e_z$, $S_0 = \sum_k S_k^{(0)}$, whose damping rate is $\nu = \tau_c^{-1} + \tau^{-1}$, and two other eigenmodes precessing in the plane perpendicular to $\Omega^{(tot)}$ with frequencies $\Omega^{(tot)}$ and the damping rate $\nu$.

The harmonic with $m = 0$ is isotropic in $k$-space, its eigenfrequency corresponds to the spin resonance frequency. We introduce $\bar{s}_0 = \sum_k s_0(\varepsilon)$ and perform the summation of Eq. (4) over $k$ which yields

$$(\tau_c^{-1} - i\omega) \bar{s}_0 - \alpha_1 \bar{s}_0 e_z \times S_0 + \alpha_1 \bar{s}_0 e_z S_0 + \bar{s}_0 \times \Omega_L = 0.$$

We recall that in the case of spin-anisotropic interactions the Larmor theorem [35] is not applicable, and the homogeneous spin excitation frequency can be renormalized by
the interactions. Indeed, in the particular case of $\mathbf{S}_0 \parallel \Omega_L$ the complex eigenfrequencies of Eq. (5) can be recast in a compact way [33]

$$\omega_0 = -\frac{i}{\tau_c},$$

$$\omega_{\pm} = -\frac{i}{\tau_c} \pm \sqrt{\Omega_L^2 + \alpha_1^2 S_0^2 \cos^2 \theta + \alpha_1 \Omega_L S_0 (3 \cos^2 \theta - 1)},$$

where $\theta$ is the angle between $\mathbf{S}_0 \parallel \Omega_L$ and z axis. For instance, if $\Omega_L$ and $\mathbf{S}_0$ are parallel to z-axis, $\theta = 0$, the frequencies of the precessing modes are $\pm |\Omega_L| + \alpha_1 S_0|$, and the damping rate is $1/\tau_c$.

Real part of $\omega_+$ and imaginary parts of $\omega_{\pm}$ are shown in Fig. 2 as a function of angle between the field and the $z$-axis. Note, that for large enough $\alpha_1 S_0$ the expression under the square root in Eq. (6) becomes negative, hence $\omega_{\pm}$ become imaginary as shown by red/solid curve in Fig. 2. Moreover, the imaginary part of one of the frequencies can be positive which manifests the instability of the system, see inset in Fig. 2. To analyze it in more detail we put $\theta = \pi/2$ ($S_0$ and $\Omega_L$ are in the structure plane). In this case $\omega_{\pm} = -i/\tau_c \pm \sqrt{\Omega_L^2 - \alpha_1^2 S_0^2 \Omega_L}$. The system becomes unstable for

$$\alpha_1 S_0 \Omega_L > \Omega_L^2 + \frac{1}{\tau_c^2},$$

and small fluctuations of $s_y$ and $s_z$ grow exponentially. This is because the anisotropic interactions between polaritons favor in-plane orientation of the pseudospin [20, 36–39]. The instability of small spin fluctuations can result in the nonlinear oscillations of spin polarization similar to those discussed in Refs. [40, 41] or in changes in the polarisation of the ground state accompanied by the decrease of absolute value/change of sign of $S_0$ where the condition (7) no longer holds.

**Spin waves.** Let us consider the spatially inhomogeneous solutions of Eq. (4) which describe the propagation of spin fluctuations and spin waves. To be specific, we consider the case where $\mathbf{S}_0^0$ and $\Omega_L$ are parallel to x-axis, and to simplify the treatment we assume $\tau \gg \tau_c$ [33]. Moreover, we assume that the system is stable at $q = 0$, i.e. the condition (7) is not fulfilled. We seek the solution, which corresponds to the precessing mode at $q = 0$, where $s_{k,\pm} = 0$. From Eq. (4) we arrive to the set of linear homogeneous integral equations for $s_{k,y}$ and $s_{k,z}$, whose self-consistency requirement yields

$$\sum_k \frac{\alpha_1 \Omega_L^2 + \Omega_L^2}{[1 - i \omega_{\tau_c} - i(qv_k^2)/\tau_c^2 + (\Omega_L \tau_c)^2] = 1.}$$

This equation describes the dispersion of spin waves. It has a more complex form compared with the dispersion equation for the spin waves in the systems with spin-isotropic interactions [1, 3, 9].

To solve Eq. (8) one has to specify the function $S_k^{(0)}$ whose form is determined by the excitation conditions. It is instructive to consider the case of resonant excitation of polaritons at the elastic circle where a monoeenergetic distribution of particles is generated. For the isotropic dispersion the product $qv_k = qv_0 \cos \varphi$, $v_0 = \hbar^{-1} d \delta k / dk$ the polariton dispersion on the elastic circle. Here the summation over $k$ reduces to the averaging over the azimuthal angle $\varphi$ and Eq. (8) is reduced to the transcendental equation

$$\frac{1}{\sqrt{\omega_+^2 - (qv_0)^2}} - \frac{1}{\sqrt{\omega_-^2 - (qv_0)^2}} = \frac{2}{\alpha_1 S_0},$$

where $\omega_{\pm} = \omega \mp \Omega_L - i/\tau_c$. Equation (9) determines the dispersion of the spin waves. For $q = 0$ it passes to $\omega_{\mp}$ in Eq. (6). For small $qv_0 \ll |\alpha_1 S_0|$ and $|\alpha_1 S_0| \ll |\Omega_L|$ the dispersion of the spin waves reads

$$\omega(q) = \Omega_L - \alpha_1 S_0/2 - (qv_0)^2/(\alpha_1 S_0) - i/\tau_c,$$

where we took the solution which passes to $\omega_{\pm}$ in Eq. (6). As follows from (10) the dispersion is parabolic for small $qv_0$ and the “effective mass” is proportional to $\alpha_1$. Similarly to the previously studied electronic and atomic
systems [1, 3, 9, 42] the dispersion of spin wave results from an interplay of the gradient, $\propto \mathbf{q}\mathbf{v}$, and interaction, $\propto \alpha_1$, terms in the kinetic equation (4). Indeed, owing to the gradient contribution, the spin density in $\mathbf{k}$-space acquires $\propto (\mathbf{q}\mathbf{v})^2$ correction, yielding gain or loss of energy depending on the sign of $\alpha_1 S_0$. The spin wave frequency increases with the increase of the wavevector for $\alpha_1 S_0 < 0$ and decrease for $\alpha_1 S_0 > 0$. The behavior of $\omega(q)$ is illustrated in Fig. 3. Noteworthy, for the solutions with $\alpha_1 S_0 > 0$, the real part of the frequency vanishes at some $q$, in which case the solutions of Eq. (10) may become unstable. For arbitrary direction of $\mathbf{Q}_L$ and $S_0$ the dispersions of waves has a similar form, but its parameters depend on the orientation of the magnetic field and the total spin due to anisotropy of polariton-polariton interactions.

It is instructive to compare the parabolic dispersion of spin waves in a weakly interacting polariton gas with the dispersion of excitations of an interacting polariton condensate [34]. In the case of a condensate of polaritons (in the absence of TE-TM splitting) the dispersion of excitations is linear. By contrast, the dispersion of spin waves for non-condensed polaritons is parabolic at small wavevectors.

In the case of a non-resonant excitation where a continuous distribution $S_k^{(0)}$ is formed, the analysis of the dispersion of spin waves is more complex, particularly because the additional channel of damping caused by the spatial dispersion appears [3, 9], but the basic physics remains the same. To illustrate this we consider the case of a thermalized non-degenerate gas where $S_k^{(0)}$ is described by the Boltzmann function characterised by an effective temperature $T$. The evaluation of the sum in Eq. (8) under the assumptions $|\alpha_1 S_0| \ll |\Omega_L|$, $|\alpha_1 S_0|\tau_\gamma > 1$, and $q v_T \ll |\alpha_1 S_0|$, where $v_T = \sqrt{k_B T/m}$ is the thermal velocity with $m$ being the polariton effective mass and $k_B$ being the Boltzmann constant, and the solution of the resulting equation yield the dispersion in the form

$$\omega(q) = \Omega_L - \alpha_1 S_0/2 - \frac{2(q v_T)^2}{\alpha_1 S_0} - i\gamma_L, \quad (11)$$

where the Landau damping can be estimated as

$$\gamma_L = \sqrt{\frac{\pi}{2}} \frac{(\alpha_1 S_0)^2}{4 q v_T} \exp \left( - \frac{(\alpha_1 S_0)^2}{8(q v_T)^2} - 1 \right), \quad (12)$$

and it is exponentially small for small wavevectors, in agreement with Refs. [3, 9]. The allowance for Landau damping in Eq. (11) is correct only if the damping is large enough compared with $1/\tau_\gamma$ but small compared to $|\alpha_1 S_0|$.

Conclusions. To conclude, we predicted the existence of exciton-polariton spin waves in semiconductor microcavities with embedded quantum wells. The dispersion and damping of spin waves were calculated in two important particular cases: (i) resonant excitation of a quasimonoenergetic distribution of polaritons at an elastic circle and (ii) nonresonant excitation, where the Boltzmann distribution of quasi-particles is formed. In the state-of-the-art microcavities the polariton polarisation splittings induced by the cavity anisotropy, $\hbar \Omega_L$, and interaction-induced effective field $\hbar S_0$ are of the order of 100 $\mu$eV, usually $43-45$. For the polariton lifetime $\gtrsim 10$ ps the spin waves can be readily detectable even at the relatively weak pump. The spin waves can be excited, e.g., in two-beam photoluminescence experiments where the cw beam creates the desired steady distribution of polaritons with a given spin polarization $S_k^{(0)}$ and the probe beam injects a small non-equilibrium portion of polaritons with the spin polarization different from $S_k^{(0)}$. The time-resolved microphotoluminescence spectroscopy as used e.g. in Refs. [46, 47] would be a suitable tool for detection of the spin waves. Another possibility to observe the spin waves is to use the spin noise spectroscopy [48, 49] and measure temporal and spatial correlations of spin fluctuations in the presence of the pump only.

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