Large primordial gravitational wave background in a class of BSI $\Lambda$CDM models

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ABSTRACT

We consider the primordial gravitational wave (GW) background in a class of spatially-flat inflationary cosmological models with cold dark matter (CDM), a cosmological constant, and a broken-scale-invariant (BSI) steplike primordial (initial) spectrum of adiabatic perturbations produced in an exactly solvable inflationary model where the inflaton potential has a rapid change of its first derivative at some point. In contrast to inflationary models with a scale-free initial spectrum, these models may have a GW power spectrum whose amplitude (though not its shape) is arbitrary for fixed amplitude and shape of the adiabatic perturbations power spectrum. In the presence of a positive cosmological constant, the models investigated here possess the striking property that a significant part of the large-angle CMB temperature anisotropy observed in the COBE experiment is due to primordial GW. Confronting them with existing observational data on CMB angular temperature fluctuations, galaxy clustering and peculiar velocities of galaxies, we find that for the best parameter values $\Omega_\Lambda \approx 0.7$ and $h \approx 0.7$, the GW contribution to the CMB anisotropy can be as large as that of the scalar fluctuations.

Key words: cosmology:theory - early Universe - cosmic background radiation - large-scale structure of Universe.

1 INTRODUCTION

There seems to be increasing observational evidence in support of the inflationary paradigm (see the review in Linde 1990; Kolb & Turner 1990; Lyth & Riotto 1998), which offers an elegant solution to some of the outstanding problems of standard Big Bang cosmology. During inflation, primordial quantum fluctuations (Hawking 1982, Starobinsky 1982, Guth & Pi 1982) of some scalar field(s) (inflaton(s)) are produced, which eventually form galaxies, clusters of galaxies and the large-scale structure of the Universe through gravitational instability.

However, the simplest inflationary CDM model in flat space ($\Omega = 1$) and with an approximately flat ($n_S \approx 1$) initial spectrum of adiabatic perturbations does not agree with observational data because it has too much power on small scales when normalized to the COBE data at large scales. One of the simplest and most elegant ways to reach agreement with observations is to add a positive cosmological constant to dustlike CDM. On one hand, this improves pure CDM models a great deal; on the other hand, the cosmological constant itself is viable in the presence of cold dark matter only (as was emphasized e.g. in Kofman & Starobinsky 1985). Now, the $\Lambda$CDM model is perhaps the most promising CDM variant (Bagla, Padmanabhan & Narlikar 1996; Ostriker & Steinhardt 1995). A recent argument in favour of $\Omega < 1$, ($\Omega \equiv 1 - \Omega_\Lambda$) (and $\Omega \approx (0.2 - 0.4)$) follows from the evolution of rich galaxy clusters (Bahcall, Fan & Cen 1997; Fan, Bahcall & Cen 1997; Eke et al., 1998). The most recent data on SNe Ia events at large redshifts also strongly favour $\Omega < 1$ (Perlmutter et al. 1998; Garnavich et al. 1998; Riess et al. 1998), while the latest CMB constraints seem inconsistent with these low $\Omega$ values in case $\Omega_\Lambda = 0$ (Lineweaver 1998). Finally, galaxy abundance at large redshifts also disfAVours open models (Peacock 1998), leaving the $\Lambda$CDM model with $n_S \approx 1$ as the most successful model for structure formation.

On the other hand, the observational fact that the smoothed slope of the initial spectrum $n_S$ cannot be sig-
significantly different from 1 does not exclude the possibility of local strong deviations from scale invariance, i.e., steps and/or spikes in the initial spectrum. Of course, such a behaviour is not a typical one, and may be expected to occur only in exceptional points in Fourier space (if at all). At present, one scale \( k \approx 0.05 h \text{Mpc}^{-1} \) is a candidate for this role, since there exists an evidence for a peculiar behaviour (in the form of a peak or a sharp change in \( n_S \)) in the Fourier power spectrum of rich Abell-ACO clusters of galaxies around this point (Einasto et al. 1997a, 1997b, 1997c; Retzlaff et al. 1998). This evidence is, certainly, inconclusive, especially since there is no peak at \( k \approx 0.05 h \text{Mpc}^{-1} \) in the power spectra of both APM clusters (which are generally less rich than Abell-ACO clusters) and APM galaxies (Tadros, Efstathiou & Dalton 1998), though a break in \( n_S \) may exist in the latter spectrum too (Gaztañaga & Baugh 1998).

Note that the attractive possibility to explain this feature by Sakharov (acoustic) oscillations fails (Arrigo-Barandela et al. 1997, Eisenstein et al. 1997). Hence, if confirmed in the future, this feature should originate in the initial perturbation spectrum itself.

Therefore, there exist serious reasons to continue the investigation of BSI CDM models. Then, it is natural to fit the arbitrary wavenumber of the singularity appearing in these models to \( k = 0.05 h \text{Mpc}^{-1} \). From the theoretical point of view, inflationary models with BSI spectra having such a characteristic scale naturally appear as a result of a phase transition approximately 60 e-folds before the end of inflation (Kofman & Linde 1986; Adams, Ross & Sarkar 1997; see also Starobinsky 1998). However, it is important that inflation cannot produce very sharp features in the primordial spectrum. In particular, an exact \( \theta \)-function-like step (which would result from a naive application of the slow-roll expressions to the case in which the first derivative of an inflaton potential \( V(\varphi) \) sharply changes near some point \( \varphi = \varphi_0 \)) is not permitted; what appears instead follows from the exact solution found in Starobinsky (1992).

Summarizing, if we believe both in the present cosmological constant and in the cluster data exhibiting a preferred scale in the perturbation spectrum, we have to consider the \( \Lambda \)CDM model with a BSI initial spectrum. Remarkably, a detailed comparison with the bulk of all existing observational data shows that the \( \Lambda \)CDM model plus the above mentioned BSI spectrum gains much from the cosmological constant too (Lesgourgues, Polarski & Starobinsky 1998, hereafter Paper I). Not only does the inclusion of \( \Lambda > 0 \) enlarges the allowed region of cosmological parameters \( \Omega \) and \( H_0 \), but it also permits the new possibility of an inverted step (i.e. more power on small scales) in the initial spectrum. Also, this model avoids the main problem found in a model of double inflation (Lesgourgues & Polarski 1997) for which the Doppler peak turns out to be low even for those values of the parameters with an acceptable matter power spectrum \( P(k) \). However, in Paper I, the free parameters of the model were chosen in such a way that the primordial GW background generated during inflation in addition to adiabatic (scalar) perturbations was too small to contribute significantly to the observed large-angle CMB temperature anisotropy. Now we want to investigate if it is possible to obtain a large GW contribution to the temperature anisotropy \( \Delta T/T \) in this model.

Historically, the primordial GW background generated from quantum vacuum metric fluctuations during inflation was the first observational prediction of inflation: its spectrum was first calculated in Starobinsky (1979) even before the first viable models of inflation were constructed, and the CMB temperature anisotropy for the multipoles \( l = 2, 3 \) produced by this background was found by Rubakov, Sazhin & Veryasikin (1982); see Lyth & Riotto (1998) for references on numerous subsequent papers. The GW and adiabatic contributions to \( \Delta T/T \) cannot be separated if only the latter quantity is measured, so the quantity of practical interest is actually \( \sqrt{1 + C_T^2} - 1 \), i.e., the relative excess of the observed rms value of \( \Delta T/T \) over the rms value calculated under the assumption that there is no GW contribution at all. However, measurement of the CMB polarization provides a unique opportunity to distinguish both these contributions and to prove the existence of a primordial GW background directly. In the case of the simplest inflationary models with scale-free spectra, \( C_T^{10}/C_S^{10} \) can be very small, e.g., in case of the "new" inflation or the \( R + R^2 \) inflation (where \( R \) is the Ricci scalar). On the other hand, for chaotic inflationary models with a power-law inflaton potential \( V(\varphi) \), \( C_T^{10}/C_S^{10} \) is comparable with unity, though still rather small, e.g., \( C_T^{10}/C_S^{10} \approx 0.2 \) for the quartic potential. Another example is provided by power-law inflation with an exponential potential \( V(\varphi) \) leading to a tilted CDM model. However, such models did not prove to be successful in explaining the height of the first acoustic (Doppler) peak for \( n_S < 0.9 \), while \( n_S > 0.9 \) results in \( (C_T^{10}/C_S^{10}) < 0.5 \). Note that inflationary models having non-negligible \( C_T^{10}/C_S^{10} \) may be also characterized by the condition that the total variation of the inflaton field during the period corresponding to present scales in the range \((10 - 10^4)h^{-1}\text{Mpc} \) is not negligible compared to the Planck mass (Lyth 1997).

The obstacle for having a sufficiently large \( C_T^{10}/C_S^{10} \) in the case of a scale-free initial perturbation spectrum lies in the observational fact that the bandpower \( l(l+1)C_l \) grows almost by an order of magnitude when \( l \) changes from 10 to 200 - 300 (see Table 1 below). This is even larger than what is expected in the standard \( n_S = 1 \) CDM model without GW and a cosmological constant, while the presence of a significant GW contribution to \( \Delta T/T \) on large angles should manifest itself in a decrease of the height of the first Doppler peak relative to the normalization at \( l = 10 \). In other words, a first high Doppler peak is an argument against significant primordial GW background in case of scale-free initial perturbation spectra. Note that even using old CMB data from the Saskatoon 94 and South Pole 94 experiments referring to \( l = (70 \pm 20) \), it was already possible to reach the conclusion that \( (C_T^{10}/C_S^{10}) < 0.7 \) with 97.5% probability in the cases of chaotic and power-law inflation with \( h = 0.5 \) and \( \Omega = 1 \) (Markevich & Starobinsky 1996). Now, with the whole set of data presented in Table 1, this upper limit becomes much lower. The existence of a positive cosmological constant only slightly relaxes this argument, so that we still get \( (C_T^{10}/C_S^{10}) \leq 0.15 \) in this case (see Sec. 3.2 below, the case \( p = 1 \)).

Thus, at present, the only possibility to have a significant primordial GW background is to require some kind of significantly non-scale-invariant initial density power spectrum. The first attempt in this direction was performed by Lukash & Mikheeva (1996, 1998) who considered the case of
the inflaton potential $V(\phi) = V_0 + \frac{m^2 \phi^2}{2}$ with $m^2$ not small as compared to $H_0^2 \equiv 8\pi G V_0/3$. However, this model has a “blue” initial spectrum with $n_S > 1$ at small scales, and faces serious problems regarding its excess of power at scales smaller than $8h^{-3}$Mpc if $n_S \geq 1.3$. On the other hand, if the parameters of the model are taken in such a way that the asymptotic slope satisfies $n_S < 1.3$, then $(C_{10}^T/C_{10}^S)^2 < 0.5$. So, it appears that in order to get $(C_{10}^T/C_{10}^S)^2 = 1$ or more, one has to take a BSI spectrum with a significantly sharper feature in it. This gives one more reason to investigate the model considered in Paper I with respect to the possible existence of a large primordial GW background.

Therefore, as in paper I, we suppose that the inflaton potential derivative, $V'(\phi)$, has a rapid change from $A_\pm$ to $A_\pm$, when $\phi$ decreases, in a neighbourhood of $\phi_0$. This generates a step-like spectrum of primordial adiabatic fluctuations, with a specific substructure. In a vicinity of the step, the amplitude of large-scale and small-scale plateaus is given by:

$$k^3\Phi^2(k) \equiv \frac{81}{50} H_0^2 A_\pm^2, \quad H_0^2 = \frac{8\pi G}{3} V(\phi_0),$$

(1)

where $\Phi$ is the (peculiar) gravitational potential at the matter dominated stage. So, the amplitude of the step is given by $p \equiv A_-/A_+$, and we call $k_0$ the characteristic scale of the step (for details, see paper I). In this model, it is still possible to fix freely the amount of primordial GW for given $p$ and normalization, since their initial spectrum near $k = k_0$ is given, for each polarization state, by the standard expression:

$$k^3(h_{mn}h_{mn})(k) = 16\pi GH_0^2$$

(2)

which does not depend on $A_\pm$ in contrast to $k^3\Phi^2(k)$ (here $m, n = 1, 2, 3$). Without the inclusion of a cosmological constant, we would be forced to consider the case $p > 1$ only, in order to increase power on large scales. Since $\Lambda > 0$ already produces a desired excess of large-scale power, we are now free to consider both cases $p > 1$ and $p < 1$.

However, in order to confront the model with observational data, we need the expression for the primordial spectra far from the point $k = k_0$, up to $|y| = |\ln(k/k_0)| \sim 5$. Since we adopt the natural assumption that any deviation from the slow-roll regime requires some special origin (e.g., some kind of phase transition during inflation) and, thus, that it should be an exceptional phenomenon, we assume that both slow-roll conditions given above are valid everywhere far from the point $k = k_0$ (before the end of inflation, of course). Then standard expressions for the initial spectra of adiabatic perturbations and GW are obtained in this region. So, the full initial spectra for these perturbations follow from (1) and (2), respectively, by the substitution:

$$H_0^2 \rightarrow H_k^2 \equiv \frac{8\pi GV_k}{3}, \quad A_\pm \rightarrow V'_k$$

(3)

where the index $k$ means that the quantity is taken at the moment of the first Hubble radius crossing ($k = aH$) during the inflationary stage.

Note that since the second derivatives of $V(\phi)$ are not fixed by Eq.(1), we may freely take $n_s(k)$ far from the break point $k = k_0$. However, $|n_s - 1|$ should be small in this region due to the two slow-roll conditions mentioned above. In paper I, we assumed that the smooth part of $|n_s - 1|$ is so small that it can be neglected at all for $|\ln(k/k_0)| \leq 5$, so the upper and lower plateaus of the scalar spectrum may be considered as flat ones. This assumption is self-consistent in the case of a negligible GW background. In our case, the situation is more complicated. In the slow-roll regime we have the well-known relation:

$$C_{10}^T/C_{10}^S \approx -5nt \approx \frac{5}{8\pi G} \left( \frac{V'}{V} \right)_k^2, \quad \frac{d\phi_k}{d\ln k} = -\frac{1}{8\pi G} \left( \frac{V'}{V} \right)_k, \quad (4)$$

where the latter expression implicitly defines $\varphi_k$ as a function of $k$. The coefficient $5$ appearing here is approximate, its exact value depends on both $h$ and $\Omega$ (see Bunn, Liddle & White 1996 for a fitting expression at $l = 14$, which can be easily transformed to $l = 10$). Further, small corrections proportional to $n_T$ and $(n_s - 1)$ should be also taken into account if one wants to make this coefficient even more accurate (see Lidsey et al. (1997) for a review). Note that if we neglect the existence of a radiation dominated stage in the past (which corresponds to the approximation $t_{eq} \ll t_{rec}$), it would be equal to 5.3 for $\Omega = 1$, $n_s = 1$, $n_T = 0$ (as was given in Polarski & Starobinsky 1995).

Since we are interested in the case when $C_{10}^T/C_{10}^S$ is not small, we cannot assume that $n_T \approx 0$. So, the amplitude of the GW background determines the slope of its power spectrum. Moreover, the values of $n_T$ on the right and on the left sides of the break point are different:

$$n_T(k > k_0) - n_T(k < k_0) = \frac{A_\pm^2 - A_\pm^2}{8\pi GV_0^2}.$$  

Strictly speaking, this equation refers to the smooth part of $n_T$ defined by differentiation of $H_k^2$ only; in addition, there is some substructure near $k = k_0$. However, for our choice of $k_0$, the only quantity of interest is $n_T(k)$ for $k = (0.01 - 0.5)k_0$.

Therefore, we have to specify more accurately our initial spectra (or, equivalently, $V(\phi)$). As a result, the BSI CDM model which we consider in this paper is actually different from the one with negligible GW studied in Paper I (though they have the same structure at $k \approx k_0$). We shall consider the two most representative cases.

1) $n_s \approx 1$ for both positive and negative large values of $\ln(k/k_0)$. In the slow-roll regime, this corresponds to an inflaton potential approximately proportional to $(\varphi - \varphi_\pm)^{-2}$ for $\varphi > \varphi_0$ and $\varphi < \varphi_0$ respectively, where $\varphi_\pm$ are two different constants. As discussed above, the choice of the initial scalar spectrum with flat plateaus at large and small $k$’s is very suitable for an explanation of both the small-scale observational data and the COBE results. Then, however, we cannot assume $n_T(k)$ to be constant: it must satisfy the equation (the “consistency relation” for single-field slow-roll inflation) which takes the following form for $n_S = 1$:

$$\frac{d\ln n_T}{d\ln k} = n_T^2$$

(6)

(see, e.g., Eq.(14) in Polarski & Starobinsky 1995 and the more general discussion in the review by Lidsey et al. 1997). As expected, this case leads to the largest possible values of $C_{10}^T/C_{10}^S$.

2) $n_s \approx 1$ for positive large values of $\ln(k/k_0)$, but $n_T = n_s - 1 = const < 0$ for negative large $\ln(k/k_0)$. This
corresponds to the exponential form of \( V(\varphi) \) above \( \varphi_0 \). Here we may assume constant slopes far from the break point \( k = k_0 \).

2 CONFRONTATION WITH OBSERVATIONS

We want now to compare our model with observations. We use experimental data which constrain the matter power spectrum \( P(k) \) on one hand and the radiation power spectrum on the other hand. As we have already noted earlier (Lesgourgues & Polarski 1997) when dealing with a model of double inflation, another model with broken scale-invariant primordial spectrum, the constraints on both types of fluctuations are essentially complementary, so that successfull confrontation of one spectrum of fluctuations does not preclude bad results for the other spectrum. One has to remember that the point of this analysis is to investigate how large fluctuations are essentially complementary, so that successfull confrontation of one spectrum of fluctuations does not preclude bad results for the other spectrum. One has to remember that the point of this analysis is to investigate how large the amplitude of the produced GW can be while leaving the model still in good agreement with observations. Hence, we must adopt a strategy that enables us to answer this question with sufficient accuracy while leaving the possibility to refine the analysis later on if required by new observational evidence.

The primordial scalar spectra under investigation have four free parameters: an overall normalization factor \( Q_{10} \), \( p \), \( k_0 \), and \( C_{10}^T/C_{10}^S \). Given these four parameters, and some cosmological parameters, the scalar and (running) tensor tilts are completely set, for each possibility \( n_S = 1 \) or \( n_S = 1 + n_T \). We choose the step location as well as the overall normalization fixed: \( k_0 = 0.016 h \) \( \text{Mpc}^{-1} \), which corresponds to a bump in the present matter power spectrum \( P(k) \) at \( k = 0.05 h \) \( \text{Mpc}^{-1} \), while we take \( Q_{10} = 15 \mu K \) (Bennett et al. 1996). The parameter \( p \) is found by requiring that \( \sigma_8 = 0.60 \Omega^{-0.56} \) (White, Efstathiou & Frenk 1993; see also Viana & Liddle 1998), where \( \sigma_8 \) is the variance of the total mass fluctuation in a sphere of radius \( 8 h^{-1} \) \( \text{Mpc} \). Then, all models are simultaneously normalized to COBE (large scales, scalar plus tensor components) and to \( \sigma_8 \) (small scales, scalar component only). Note that \( p \) is still a function of the remaining three free parameters: the two cosmological parameters \( h \) and \( \Omega_\Lambda \) on one hand, the inflationary parameter \( C_1^T/C_1^S \) on the other hand. In this way, the latter is singled out as the only remaining inflationary free parameter.

We then compute numerically, using the fast Boltzmann code cmbfast by Seljak & Zaldarriaga (1996), the matter power spectrum \( P(k) \) and the CMB power spectrum \( C(\theta) \) for various values of the parameters \( h \), \( \Omega_\Lambda \), \( C_1^T/C_1^S \). We will find it interesting to state the results in two-dimensional cuts of the parameter space for several given values of \( h \). This is particularly appropriate if we expect other observations to yield some refined a priori knowledge of the value of \( h \).

In order to constrain the matter power spectrum, we use:

- peculiar velocities taken from the MARK III catalog (Willick et al. 1997) and the POTENT reconstruction (Bertschinger & Dekel 1989; Kolatt & Dekel 1997) of the velocity field. Peculiar velocities have the advantage that they probe all mass (and not just galaxies), but they have rather large uncertainties. We use here the rms bulk velocity at \( R = 50 h^{-1} \text{Mpc} \), with Gaussian smoothing radius \( R_s = 12 h^{-1} \text{Mpc} \):
  \[
  290 \leq V_{50} \leq 460 \text{ km s}^{-1} \quad (1 - \sigma \text{ confidence level}) \tag{7}
  \]
  in the absence of cosmic variance, and:
  \[
  245 \leq V_{50} \leq 505 \text{ km s}^{-1} \tag{8}
  \]
  when cosmic variance is taken into account. As we will see, the lower bounds implied by these results yield stringent constraints on our models.

- the STROMLO-APM redshift survey which gives a count-in-cells analysis of large scale clustering. Like other redshift surveys, it probes the matter perturbations well into the linear regime. Results are given in Loveday et al. (1992) for cells of nine different sizes. We compare these data points with the power spectrum (normalized to \( \sigma_8 = 1.00 \), and convolved with the nine corresponding window functions) through a \( \chi^2 \) analysis. Since we can vary 3 parameters, there are 6 degrees of freedom.

- the power spectrum of rich Abell galaxy clusters, consisting of 36 points taken from Einaust et al. (1997a). Since a few points on the largest and smallest scales suffer from large uncertainties, we perform a \( \chi^2 \) analysis with only 30 points, corresponding to \( 0.023 \leq k \leq 0.232 h \) \( \text{Mpc}^{-1} \). As already said in the introduction, this spectrum exhibits a clear feature with a bump at \( k = 0.05 h \) \( \text{Mpc}^{-1} \) and \( k_0 \) is chosen so that the matter power spectrum also exhibits a feature at the right scale. Hence, we are testing here a theory with 4 free parameters, and the \( \chi^2 \) distribution has 26 degrees of freedom. Strictly speaking, the Einaust et al. (1997a) data are not independent. Taking into account their correlation will decrease the effective number of degrees of freedom, however this does not noticeably change our conclusions. We do not assume any specific value for the biasing factor for these data: for each set of parameters \( (h, \Omega_\Lambda, C_1^T/C_1^S) \), we calculate and adopt the biasing factor yielding the smallest \( \chi^2 \). In that sense, this test probes only the shape of the power spectrum.

In order to constrain the radiation power spectrum, we use:

- the CMB anisotropy data on large and small angular scales found in various experiments. We use the bandpower estimates \( \Delta T_l \pm \sigma \) given for an experiment characterized by a window function \( W_l \). More precisely, we have:
  \[
  \Delta T_l = \frac{\sigma_{o,b}(T)}{\sqrt{I(W_l)}} \tag{9}
  \]
  where \( \sigma_{o,b}(T) \), the observed rms temperature fluctuation (which differs from the theoretical value by the inclusion of the window function \( W_l \) characterizing the experiment), is given by:
  \[
  \frac{\sigma_{o,b}^2(T)}{T^2} = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l W_l \tag{10}
  \]
  and \( I(W_l) \) is defined as:
  \[
  I(W_l) = \sum_{l=2}^{\infty} (l + \frac{1}{2}) \frac{W_l}{l(l+1)} \tag{11}
  \]
  The bandpower estimate assumes a flat spectrum \( l(l+\)
1) $C_1 \propto C_2$ being a good approximation over the width of the window function $W_i$ centered around some “effective” multipole number $l_e$ defined through $l_e \equiv I(W_i)/I(W_i)$.

We compare the theoretical CMB power spectrum with 19 experimental points given in Table 1, again through a $\chi^2$ analysis, with 16 degrees of freedom. The Saskatoon data provide an important constraint on any candidate model, as they give a clue to the correct height of the first Doppler (or acoustic) peak. However these are given in Netterfield et al. (1997) with a calibration error of ±14%, which applies equally to all five points. This remaining uncertainty is treated as in Lineweaver & Barbosa (1998a, 1998b): for each set of free parameters $(h, \Omega_\Lambda, C_{10}^T/C_{10}^S)$, we make a preliminary $\chi^2$ analysis in order to find the best calibration.

Then, the Saskatoon data are treated on equal footing with other CMB data, and a second $\chi^2$ is computed with the 19 points.

We chose to perform three separate $\chi^2$ analysis for the Stromlo-APM, the CMB and the rich Abell cluster data. In this way, we see clearly what is implied by each set of data separately. We further avoid to put on the same footing observations of a very different kind.

3 RESULTS

The result of all the tests are given in Figure 1, in both case $n_S = 1$ ans $n_S = 1 + n_T$, for a few two-dimensional cuts of the parameter space corresponding to $h = 0.5, 0.6, 0.7, 0.8$. For peculiar velocities, we plot the curves corresponding to the lower limit $V_{50} = 245$ km s$^{-1}$ (the limit without cosmic variance, $V_{50} = 290$ km s$^{-1}$, is also given for completeness). For the three $\chi^2$ tests, we plot contours inside the preferred regions, in which $\chi^2$ is smaller or equal to the number of degrees of freedom. Let us comment these results step by step. Since the results for the $n_S = 1$ and $n_S = 1 + n_T$ are quite similar, we will discuss them simultaneously.

3.1 Peculiar velocities and STROMLO-APM

First, let us not consider the rich Abell galaxy clusters distribution. In this case, the matter power spectrum is constant CMB experiments have been published. They confirm the data presented in Table 1, and, therefore, strengthen our conclusions. The first experiment, QMAP (de Oliveira-Costa et al. 1998) confirms the Saskatoon normalization at $\ell = 90$, and the shape of the Saskatoon data up to $\ell = 150$. The second one, the NCP at OVRO (Leitch et al. 1998) confirms the second CAT 1 point at $\ell \approx 600$.

* After this paper was submitted, the results of two new important CMB experiments have been published. They confirm the data presented in Table 1, and, therefore, strengthen our conclusions. The first experiment, QMAP (de Oliveira-Costa et al. 1998) confirms the Saskatoon normalization at $\ell = 90$, and the shape of the Saskatoon data up to $\ell = 150$. The second one, the NCP at OVRO (Leitch et al. 1998) confirms the second CAT 1 point at $\ell \approx 600$.

On the other hand, the STROMLO-APM redshift survey probes the shape of $P(k)$ only at scales smaller than the step, i.e. those scales given by the high-$k$ plateau of the primordial spectrum. Hence, it only depends on the matter transfer function, not on $C_{10}^T/C_{10}^S$, and on Figure 1 it appears as a constraint on $\Omega_\Lambda$ only.

These two constraints define a wide preferred region in parameter space, and it is crucial to include CMB anisotropy measurements in order to obtain good predictions.

3.2 CMB anisotropies

The CMB $\chi^2$ test selects preferred elliptic regions in the $C_{10}^T/C_{10}^S - \Omega_\Lambda$ planes, which truly admit an extension up to $C_{10}^T/C_{10}^S = 1$ and even more. This is due to the ‘inverted’ step in the primordial spectrum, which compensates for the loss of power in small-scale anisotropy usually implied by a high tensor contribution. The GW contribution even improves the model: in all cases plotted in Figure 1, the best values are obtained for $0.4 < C_{10}^T/C_{10}^S < 0.8$.

For all $h$ considered here, intersections with the previously preferred regions are found, and enhanced in yellow (or gray) on Figure 2. As expected from what was just said, the major part of the preferred regions correspond to ‘inverted’ steps $0.45 < p < 1$. Those with $p = 1$ (Harrison-Zeldovich) and $1 < p < 1.1$ (‘usual’ step) correspond to the lower left corners of the preferred regions (not shown in Figure 2), with $C_{10}^T/C_{10}^S \lesssim 0.15$.

3.3 Clusters distribution

Let us now consider the rich Abell galaxy cluster distribution data which constitutes the main motivations for the characteristic scale appearing in our model, and which determines its location through the value of $k_0$. The elliptic preferred regions (see Figure 1) are narrow when compared with STROMLO-APM regions, because data error bars are smaller, and sensitive to larger scales (where $p$, and therefore $C_{10}^T/C_{10}^S$ values are crucial). The best values are obtained in the absence of gravitational waves, but values of $C_{10}^T/C_{10}^S \approx 1$ are still acceptable (except for $n_S = 1 + n_T$, $h < 0.6$). There is no intersection with the CMB preferred region when $h < 0.5$. The intersection of all preferred regions is coloured in bright yellow (or dark gray) on Figure 2 showing that:

- for $h \geq 0.7$, larger preferred region in the $C_{10}^T/C_{10}^S - \Omega_\Lambda$ plane are obtained, with $0.68 \leq \Omega_\Lambda \leq 0.77$ and $0 \leq C_{10}^T/C_{10}^S \leq 1$.
- for $h = 0.6$, a rather narrow region survives, for which the GW contribution is substantial, $\Omega_\Lambda \sim 0.63$ and $0.2 \leq C_{10}^T/C_{10}^S \leq 1$.
- for $h = 0.5$, a marginal intersection between the CMB and cluster favoured regions is found for $\Omega_\Lambda \sim 0.53$. This possibility is still acceptable, because we are rather severe in the definition of the preferred windows.

In the previous discussion, we made almost no distinction between $n_S = 1$ and $n_S = 1 + n_T$, since the results are quite similar in both case. This shows that our predictions
do not depend very much on the assumptions on large scale tilts, provided that the consistency relation is satisfied. In particular, the results would still hold for an arbitrary constant scalar tilt in the range $1 + n_T(k_b) < n_S < 1$. The only difference is that the preferred parameter windows and the lowest $\chi^2$ regions are systematically obtained for smaller $C_{10}^T/C_{10}^S$ when $n_S = 1 + n_T$, while the first case, with $n_S \approx 1$, admits slightly larger values of $C_{10}^T/C_{10}^S$.

4 CONCLUSION

The generation of a GW cosmological background is an essential prediction of all inflationary models. Amplitude and statistics of this background can be computed from first principles, as also their contribution to the CMB temperature (and polarization) anisotropies. However, only part of the inflationary models have a primordial GW background sufficiently large in order to be actually measured, using either CMB measurements, or else gravitational-wave antennas for their direct detection. An accurate measurement of the CMB angular temperature anisotropy and the CMB polarization (especially on angular scales $\theta \geq 1^\circ$) may result in the remarkable discovery of a primordial GW background on cosmological scales which, of course, would be of great importance by itself, and would also provide a very strong argument for the inflationary scenario of the early Universe.

The already measured height of the first Doppler peak for multipoles $l = 200 - 300$, in spite of all the indeterminacies, is so high that it precludes a significant GW contribution to the multipole dispersion $C_{10}$ (where $l = 10$ is the characteristic multipole for the COBE data) if the initial power spectrum of adiabatic perturbations is scale-free. Shifting to a BSI initial spectrum helps to avoid this obstacle and to keep the possibility of finding the GW background.

In this paper, as in Paper I, we have considered the BSI CDM model with a positive cosmological constant and we have confronted it with recent observational data on CMB temperature anisotropies, large-scale galaxy-galaxy correlations, peculiar velocities of galaxies, and spatial correlations of rich Abell clusters. Both new ingredients of this model, as compared to the standard CDM model with a flat initial perturbation spectrum, namely the cosmological constant and the particular form of the BSI initial spectrum, have been already introduced earlier to account for hitherto unexplained data, without reference to a primordial GW background. Now we have shown that this model, with a slightly different choice of its parameters as compared to the one considered in Paper I, admits a large GW background, too. A positive cosmological constant is essential for this since the initial power spectrum admits an inverted step, $p < 1$ (i.e., more power on small vs. large scales), without which the quantity $(C_T/C_S)_{10}$ characterizing the relative GW contribution to $C_{10}$ would be small.

We have considered two subclasses of this model, differing by the behaviour of the scalar (density) power spectrum in its approximately scale-free part far below the breakpoint at $k = k_b$. In the first case, the spectral slope is $n_S \approx 1$ in this region (while we may not assume neither $n_T \approx 0$, nor neglect its scale dependence). In the second case, $n_T = n_S - 1 = \text{const} < 0$. For each class, we have investigated how large this GW background can be while still leaving the model in good agreement with observations. The difference between the two cases is small, as expected. We find that the best models are located in essentially the same window of the cosmological parameters $h$ and $\Omega_\Lambda$: $h \geq 0.7$, $\Omega_\Lambda \approx 0.7$ while the GW contribution to the temperature fluctuations on large angular scales, measured here by the parameter $C_{10}^T/C_{10}^S$, can be as large as that of the scalar fluctuations. Note that this is still of course many orders of magnitude below a value allowing their possible direct detection by ground-based gravitational-wave antennas.

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Table 1. Bandpower estimates used for the CMB $\chi^2$ test.

| Experiment | $\Delta T_l \pm \sigma$ (\(\mu K\)) | $l_e$ | Reference |
|------------|-----------------------------------|------|-----------|
| Tenerife   | $32.5^{+10.1}_{-8.8}$            | 20   | Gutiérrez et al. (1997) |
| South Pole 91 | $30.2^{+8.9}_{-5.5}$             | 60   | Gunderson et al. (1995) |
| South Pole 94 | $36.3^{+14.6}_{-5.1}$            | 60   | Gunderson et al. (1995) |
| Python     | $37.3^{+10.1}_{-13.6}$           | 91   | Ruhl et al. (1995)     |
| ARGO 1     | $39.1^{+8.7}_{-8.7}$             | 95   | De Bernardis et al. (1994) |
| ARGO 2     | $46.8^{+12.1}_{-19.4}$           | 95   | Masi et al. (1996)     |
| MAX GUM    | $54.5^{+19.9}_{-12.8}$           | 138  | Tanaka et al. (1996)   |
| MAX ID     | $46.3^{+21.8}_{-23.8}$           | 138  | Tanaka et al. (1996)   |
| MAX SH     | $49.1^{+21.5}_{-16.4}$           | 138  | Tanaka et al. (1996)   |
| MAX HR     | $37.2^{+10.9}_{-8.2}$            | 138  | Tanaka et al. (1996)   |
| MAX PH     | $51.8^{+10.1}_{-10.9}$           | 138  | Tanaka et al. (1996)   |
| Saskatoon  | $49.1^{+8.2}_{-3.5}$             | 86   | Netterfield et al. (1997) |
| Saskatoon  | $69.1^{+3.5}_{-5.6}$             | 166  | Netterfield et al. (1997) |
| Saskatoon  | $85.1^{+10.0}_{-6.4}$            | 236  | Netterfield et al. (1997) |
| Saskatoon  | $86^{+12}_{-9.6}$                | 285  | Netterfield et al. (1997) |
| Saskatoon  | $60^{+19}_{-28}$                 | 348  | Netterfield et al. (1997) |
| CAT 1      | $51.8^{+11.6}_{-13.6}$           | 396  | Scott et al. (1996)    |
| CAT 2      | $57.3^{+10.9}_{-13.6}$           | 396  | Baker et al. (1998)    |
| CAT 1      | $49.1^{+19.1}_{-13.7}$           | 607  | Scott et al. (1996)    |
Large primordial GW background in a class of BSI $\Lambda$CDM models

Figure 1. Results of all the tests, in both case $n_S = 1$ ans $n_S = 1 + n_T$, for a few two-dimensional cuts of the parameters space corresponding to $h = 0.5, 0.6, 0.7, 0.8$. The preferred regions for each test are limited by dotted curves in blue (bulk velocities), red (STROMLO-APM), black (cluster distribution), and green (CMB anisotropy). Inside the former three regions, we plot the contours associated with integer values of $\chi^2$.

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Figure 2. On this plot, we coloured (yellow or gray) the intersection of velocity, STROMLO-APM and CMB tests. The intersection of all tests, including cluster distribution, is the bright yellow (or dark gray) region alone. High values of $C_{\Omega}^T / C_{\Omega}^S$ up to 1 are allowed, or even preferred.
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