2-D Scanning Galvanometer Error Analysis and Its Correction

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Abstract: The 2-D Scanning Galvanometer is an optical scanner with high precision, high reliability and high repeatability. It is also widely used. In order to eliminate the nonlinear scanning error of the 2-D Scanning Galvanometer system, in order to improve the scanning precision of the system, the theoretical distortion is derived, and the geometric distortion and various errors in the 2-D Scanning Galvanometer system are analyzed in detail. The targeted coordinate transformation method is used to correct the distortion error. The experiment proves that the method can significantly improve the scanning precision of the rapid prototyping machine.

1. Introduction
The 2-D Scanning Galvanometer is an optical scanner with high precision, high reliability and high repeatability. Therefore, it is widely used in aerospace, laser material processing, biomedical detection, laser projection and display, and graphics. Important areas such as image processing and semiconductor processing. However, there are nonlinear distortions of the scanned image during the 2-D galvanometer scanning, especially when the scanning area is large, which will affect the positioning accuracy of the laser scanning, and also bring some difficulty to the further data analysis and processing. Therefore, a coordinate transformation method is proposed to correct the distortion error.

2. Working Principle
The principle of 2-D galvanometer laser scanning is shown in Figure 1. It is mainly composed of galvanometer motor (special oscillating motor), X-axis rotating mirror, Y-axis rotating mirror, focusing device and control system. The principle of the system is to use a high-density, high-power laser source as the processing body, and to deflect the angle of the mirror fixed on the motor by a servo circuit controlled by the computer output position, thereby converting the motion of the galvanometer into a stationary laser. The movement ultimately results in a laser scanning back and forth on the work surface.
Suppose the deflection angle of the \( X \)-axis galvanometer is \( \theta_x \), the deflection angle of the \( Y \)-axis galvanometer is \( \theta_y \), \( e \) is the vertical distance between the \( X \)-axis galvanometer and the \( Y \)-axis galvanometer, and \( d \) is the straight line from the \( Y \)-axis galvanometer to the measured area. distance. Then the relationship between the deflection angle of the \( X \)-axis galvanometer and the \( Y \)-axis galvanometer and the coordinates of the \( P \) point can be expressed as:

\[
\begin{align*}
  x &= e \tan 2 \theta_x + \tan 2 \theta_x \sqrt{d^2 + y^2} \\
  y &= d \tan 2 \theta_y
\end{align*}
\]  

(1)

When the laser beam passes through two galvanometers and is incident on the plane to be measured, the optical path \( L \) of the beam can be expressed as:

\[
L = \sqrt{(\sqrt{d^2 + y^2} + e)^2 + x^2}
\]  

(2)

It can be seen from Figure 3 that in \( \triangle AOB \), the relationship between the yaw deflection angle and the \( P \) point coordinates in the measured area can be expressed as:

\[
\tan 2 \theta_y = \frac{y}{d}
\]  

(3)

\[
AB = \sqrt{d^2 + y^2}
\]  

(4)

Where: \( \theta_y \) is the angle between the beam and the \( Y \)-axis; \( d \) is the distance between the galvanometer and the target being measured.

In \( \triangle DAP \), the geometric relationship can be expressed as:

\[
DA = AB + BD = \sqrt{d^2 + y^2} + e
\]  

(5)

\[
\tan 2 \theta_x = \frac{x}{\sqrt{d^2 + y^2} + e}
\]  

(6)

Where: \( \theta_x \) is the angle between the beam and the \( X \) axis; \( e \) is the distance between the two galvanometers.

According to formulas (3) and (4), the relationship between the angle of rotation of the galvanometer
and the coordinate $P(x, y)$ of any point in the measured area $OXY$ can be expressed as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tan 2\theta_x \\ \tan 2\theta_y \end{bmatrix} + \begin{bmatrix} \sqrt{d^2 + y^2} + e \\ 0 \end{bmatrix}$$

(7)

The scan trajectory is hyperbolic as shown in the equation. Simulation with Matlab, when the scanning distance is very close, the simulation results are shown in Figure 3;

![Figure 3. Small Area Matlab Simulation.](image)

However, when the scanning distance is long, the scanning area is large, and the simulation result is shown in Figure 4;

![Figure 4. Large Area Matlab Simulation.](image)

It can be seen from Figure 4 that there is a certain distortion area at the edge of the scanning area, which is the theoretical scanning pincushion distortion of the 2-D Scanning Galvanometer, and is also a nonlinear distortion that cannot be eliminated by the optical element.

3. 2-D Scanning Galvanometer scanning distortion correction model

Suppose the scan has a point $(x', y')$ in the distortion map. In the actual graph, the coordinates of the corresponding point are $(x, y)$, then there is a mapping relationship between $(x', y')$ and $(x, y)$.

$$\begin{cases} x' = f(x, y) \\ y' = g(x, y) \end{cases}$$

(8)

To determine the functional relationship between the distortion point and the actual point, find $n$ actual points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ in the scan plane. These points are called constraint points. The corresponding distortion point coordinates in the distortion map are $(x'_1, y'_1), (x'_2, y'_2), \ldots, (x'_n, y'_n)$, respectively.

Based on the $n$ pairs of constraint points, the least square method is used to determine the parameters in equation (8) to determine the mapping relationship between the actual point and the distortion point coordinates. This is the principle of the plane coordinate transformation geometric correction algorithm.

Therefore, as long as the function mapping relationship between the pincushion distortion point and the actual point is obtained, a corresponding mathematical correction model can be established. The relationship between the distortion point and the actual point is analyzed by taking the distortion on the right side of the $y$-axis as an example.

In Fig. 2, the laser reaches the measurement plane after two reflections, and the position of the ideal
point of the beam in the x direction can be obtained according to the optical reflection relationship, that is,

\[ x = (e + \sqrt{d^2 + y_{\text{max}}^2}) \tan \theta_y \]  

(9)

Let \( k = e + \sqrt{d^2 + y_{\text{max}}^2} \), there is

\[ x = k \cdot \tan \theta_y \]  

(10)

It can be seen that the reflected light is not distorted in the Y direction, and the correction function of the pincushion distortion can be obtained from the equations (8) and (10).

\[
\begin{aligned}
\left\{ & x' = \left( \frac{e}{k} \right) \left( e + \sqrt{y^2 + d^2} \right) \\
& y' = y
\end{aligned}
\]  

(11)

For the correction of the linear error, it can be implemented by a linear transformation method. If the ideal straight line function is \( y = \alpha_1 x + \alpha_2 \), the linear function is \( y' = \alpha_3 x + \alpha_4 \) after distortion, and the ideal straight line that is distorted is parallel to the distortion line, and then translated, there are

\[
n_1 y + n_2 = \alpha_3 x (m_1 x + m_2) + \alpha_4.
\]

Therefore, it can be concluded that the address mapping relationship between the ideal point \((x, y)\) and the distortion point \((x', y')\) occurs when a linear error occurs:

\[
\begin{aligned}
x' &= m_1 x + m_2 \\
y' &= n_1 y + n_2
\end{aligned}
\]  

(12)

In summary, from Equation 11 and Equation 12, the relationship between the actual point of linear distortion and the distortion point is obtained:

\[
\begin{aligned}
x' &= \left( \frac{e}{k} \right) \left( e + \sqrt{y^2 + d^2} \right) + m_1 x + m_2 \\
y' &= (n_1 + 1) y + n_2
\end{aligned}
\]  

(13)

Similarly, the mapping relationship after the left side distortion of the Y-axis can also be obtained:

\[
\begin{aligned}
x' &= \left( \frac{e}{k} \right) \left( e - \sqrt{y^2 + d^2} \right) + m_3 x + m_4 \\
y' &= (n_3 + 1) y + n_4
\end{aligned}
\]  

(14)

Where \( k = e - \sqrt{y_{\text{max}}^2 + d^2} \)

4. Experimental Verification

Through the following experiment:

Get 20 points of data

|   | 1    | 2    | 3    | 4    | …    | 17   | 18   | 19   | 20   |
|---|------|------|------|------|------|------|------|------|------|
| X | 0.09 | 0.1  | 0.19 | 0.2  | …    | 0.89 | 0.9  | 0.91 | 0.92 |
| Y | -0.08565 | -0.08648 | -0.09792 | -0.09963 | …    | -0.42911 | -0.43696 | -0.44489 | -0.4529 |

Obtained by coordinate transformation
The comparison between the two groups of data shows that the error is small, which proves that the method is feasible.

5. Conclusions
Coordinate transformation technology has been continuously improved and improved as a geometric correction technique. Although the algebraic equations used are relatively high and the amount of calculation is relatively large for the current complex nonlinear distortions, the computational complexity is not the primary consideration in engineering applications due to the increasing speed of computers. In this paper, the main error in the 2-D Scanning Galvanometer process is analyzed and studied, and a relatively simple plane coordinate transformation relation is established. The experiment proves that the algorithm does not generate computational rounding error in the real-time system operation, which can significantly improve the laser scanning precision of the system, and can obtain satisfactory correction effect for both pincushion distortion and general linear distortion.

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Table 2. Actual point data after coordinate transformation.

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| X | 0.09 | 0.1 | 0.19 | 0.2 | ... | 0.89 | 0.9 | 0.91 | 0.92 |
| Y | -0.08566 | -0.08656 | -0.09781 | -0.09975 | ... | -0.42921 | -0.43699 | -0.44492 | -0.4539 |