Charm multiplicity and the branching ratios of inclusive charmless b quark decays in the general two-Higgs-doublet models

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October 25, 2018

Abstract

In the framework of general two-Higgs-doublet models, we calculate the branching ratios of various inclusive charmless b decays by using the low energy effective Hamiltonian including next-to-leading order QCD corrections, and examine the current status and the new physics effects on the determination of the charm multiplicity $n_c$ and semileptonic branching ratio $B_{SL}$. Within the considered parameter space, the enhancement to the ratio $BR(b \to sg)$ due to the charged-Higgs penguins can be as large as a factor of 8 (3) in the model III (II), while the ratio $BR(b \to \text{no charm})$ can be increased from the standard model prediction of 2.49% to 4.91% (2.99%) in the model III (II). Consequently, the value of $B_{SL}$ and $n_c$ can be decreased simultaneously in the model III. The central value of $B_{SL}$ will be lowered slightly by about 0.003, but the ratio $n_c$ can be reduced significantly from the theoretical prediction of $n_c = 1.28 \pm 0.05$ in the SM to $n_c = 1.23 \pm 0.05$, $1.18 \pm 0.05$ for $m_{H^+} = 200, 100$ GeV, respectively. We find that the predicted $n_c$ and the measured $n_c$ now agree within roughly one standard deviation after taking into account the effects of gluonic charged Higgs penguins in the model III with a relatively light charged Higgs boson.

PACS numbers: 13.25.Hw, 12.15.Ji, 12.38.Bx, 12.60.Fr
I. Introduction

In the forthcoming years, experiments at SLAC and KEK B-factories, HERA-B and other high energy colliders will measure various branching ratios and CP-violating asymmetries of B decays \[1, 2\]. The expected large number of B decay events (say \(10^8 - 10^9\)) may allow us to explore the physics of CP violation, to determine the flavor parameters of the electroweak theory, and to probe for signals or evidences of new physics beyond the Standard Model (SM) \[1 - 6\].

Among various B meson decay modes, the decay \(b \rightarrow s\gamma\) and \(b \rightarrow sg\) have been, for example, the hot subject of many investigations \[7\], since these decay modes may be affected by loop contributions from various new physics models. Great progress in both the theoretical calculation \[8\] and the experimental measurement \[9\] enable one to constrain the new physics models, such as the two-Higgs-doublet model (2HDM) \[10\], the minimal supersymmetric standard model \[11\] and the Technicolor models \[12\].

For many years, it appeared that the SM prediction for the semileptonic branching ratio \(B_{SL}\) \[13\] is much larger than the values measured at \(\Upsilon\) resonance and \(Z^0\)-peak \[14, 15\]. More recently, the theoretical predictions have been refined by including full \(O(\alpha_s)\) QCD corrections \[16, 17\]. These progress, consequently, have lowered the predicted \(B_{SL}\) and now adequately reproduce the experimental results \[15\]. However, the measurements of \(B_{SL}\) obtained at the \(\Upsilon(4S)\) and \(Z^0\) resonance are still disagree slightly \[18\]. Besides the \(B_{SL}\) problem, there is another so-called "missing charm puzzle" \[15, 19\]: the charm multiplicity \(n_c\) measured at CLEO and LEP \[18, 20\] (especially at CLEO, the \(\Upsilon\) resonance) is smaller than the theoretical prediction. Among various possible explanations for the missing charm/\(B_{SL}\) problem, the most intriguing one would be an enhanced \(B \rightarrow X_{nocharm}\) rate due to new physics beyond the SM \[19\]. An enhanced \(b \rightarrow sg\) can decrease the values of both \(n_c\) and the \(B_{SL}\) simultaneously \[19\]. The large branching ratio \(BR(B \rightarrow \eta'X_s)\) reported recently by CLEO \[21\] provided a new hint for enhanced \(b \rightarrow sg\). Besides those explanations based on the SM \[22\], new physics interpretation for this large ratio is also plausible \[23\].

In a previous paper \[24\], we calculated, from the first principle, the new contributions to inclusive charmless b quark decays \(b \rightarrow sg, b \rightarrow sq\) from the gluonic charged-Higgs penguin diagrams in the so-called Model III: the two-Higgs-doublet model with flavor changing couplings \[24\]. In the considered parameter space, we found that the branching ratio \(BR(b \rightarrow sg) (q^2 = 0)\) can be increased by roughly an order of magnitude, which is much larger than that in the ordinary 2HDM’s \[27\]. In \[24\], however, we used the language of form factors \(F_1\) and \(F_2\) and took into account the QCD corrections partially by using the \(\alpha_s(m_b)\) directly to calculate the branching ratios.

In this paper, in the framework of general 2HDM’s, we will calculate the branching ratios of various inclusive charmless b decays by using the low energy effective Hamiltonian including next-to-leading order (NLO) QCD corrections \[8\], and investigate the new physics effects on the theoretical predictions for both \(B_{SL}\) and \(n_c\).

This paper is organized as follows. In Sec.II, we describe the basic structures of the model III, extract out the Wilson coefficients, draw the constraint on parameter space of the model III from currently available data. In Sec.III, we calculate the branching ratios.
$BR(b \rightarrow sg)$ and $BR(b \rightarrow q' q \bar{q})$ for $q' \in d, s$ and $q \in u, d, s$ in the model III and II with the inclusion of NLO QCD corrections. In Sec.IV, we examine the current status and new physics effects on the determination of $B_{SL}$ and $n_c$. The conclusions and discussions are included in the final section.

II. The general 2HDM’s and experimental constraint

The simplest extension of the SM is the so-called two-Higgs-doublet models\[10\]. In such models, the tree level flavor changing neutral currents(FCNC’s)are absent if one introduces an ad hoc discrete symmetry to constrain the 2HDM scalar potential and Yukawa Lagrangian. Lets consider a Yukawa Lagrangian of the form\[26\]

$$L_Y = \eta^U_{ij} \bar{Q}_{i,L} \tilde{\phi}_1 U_{j,R} + \eta^D_{ij} \bar{Q}_{i,L} \phi_1 D_{j,R} + \xi^U_{ij} \bar{Q}_{i,L} \tilde{\phi}_2 U_{j,R} + \xi^D_{ij} \bar{Q}_{i,L} \phi_2 D_{j,R} + h.c.,$$

where $\phi_i (i = 1, 2)$ are the two Higgs doublets of a two-Higgs-doublet model, $\tilde{\phi}_{1,2} = i\tau_2 \phi_{1,2}^*$, $Q_{i,L}$ ($U_{j,R}$) with $i = (1, 2, 3)$ are the left-handed isodoublet quarks (right-handed up-type quarks), $D_{j,R}$ are the right-handed isosinglet down-type quarks, while $\eta_{i,j}^{U,D}$ and $\xi_{i,j}^{U,D}$ ($i, j = 1, 2, 3$ are family index ) are generally the nondiagonal matrices of the Yukawa coupling. By imposing the discrete symmetry

$$\phi_1 \rightarrow -\phi_1, \phi_2 \rightarrow \phi_2, D_i \rightarrow -D_i, U_i \rightarrow \mp U_i$$

one obtains the so called Model I and Model II. In Model I the third and fourth term in eq.(2) will be dropped by the discrete symmetry, therefore, both the up- and down-type quarks get mass from Yukawa couplings to the same Higgs doublet $\phi_1$, while the $\phi_2$ has no Yukawa couplings to the quarks. For Model II, on the other hand, the first and fourth term in Eq.(2) will be dropped by imposing the discrete symmetry. Model II has, consequently the up- and down-type quarks getting mass from Yukawa couplings to two different scalar doublets $\phi_1$ and $\phi_2$.

During past years, the models I and II have been studied extensively in literature and tested experimentally, and the model II has been very popular since it is the building block of the minimal supersymmetric standard model. In this paper, we focus on the third type of 2HDM \[25\], usually known as the model III \[25, 26\]. In the model III, no discrete symmetry is imposed and both up- and down-type quarks then may have diagonal and/or flavor changing couplings with $\phi_1$ and $\phi_2$. As described in \[26\], one can choose a suitable basis ($H^0, H^1, H^2, H^\pm$) to express two Higgs doublets $\phi_1$ and $\phi_2$.

$$\phi_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v + \sqrt{2}\chi^+ \\ H^0 + i\chi^0 \end{array} \right), \quad \phi_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2}H^+ \\ H^1 + iH^2 \end{array} \right),$$

and take their vacuum expectation values as the form

$$\langle \phi_1 \rangle = \left( \begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right), \quad \langle \phi_2 \rangle = 0,$$
where $v = (\sqrt{2} G_F)^{-1/2} = 246 GeV$. The transformation relation between ($H^0, H^1, H^2$) and the mass eigenstates $(\tilde{T}^0, h^0, A^0)$ can be found in \[26\]. The $H^\pm$ are the physical charged Higgs boson, $H^0$ and $h^0$ are the physical CP-even neutral Higgs boson and the $A^0$ is the physical CP-odd neutral Higgs boson. After the rotation of quark fields, the Yukawa Lagrangian of quarks are of the form \[26\],

$$
L^{III}_{Y} = \eta^{U,D}_{ij} \bar{Q}_{i,L} \tilde{\phi}_1 U_{j,R} + \eta^{D}_{ij} \bar{Q}_{i,L} \phi_1 D_{j,R} + \hat{\xi}^{U,D}_{ij} \bar{Q}_{i,L} \tilde{\phi}_2 U_{j,R} + \hat{\xi}^{D}_{ij} \bar{Q}_{i,L} \phi_2 D_{j,R} + H.c., \tag{5}
$$

where $\eta^{U,D}_{ij}$ correspond to the diagonal mass matrices of up- and down-type quarks, while the neutral and charged flavor changing couplings will be \[26\]

$$
\xi^{U,D}_{ij} = \frac{\sqrt{m_i m_j}}{v} \lambda_{ij}, \quad \xi^{U,D}_{neutral} = \xi^{U,D}_{charged} = \xi^{U,D}_{charged} = V_{CKM} \xi^{D}, \tag{6}
$$

where $V_{CKM}$ is the Cabibbo-Kabayashi-Maskawa mixing matrix \[28\], $i, j = (1, 2, 3)$ are the generation index. The coupling constants $\lambda_{ij}$ are free parameters to be determined by experiments, and they may also be complex.

In the model II and assuming $\tan \beta = 1$, the constraint on the mass of charged Higgs boson due to CLEO data of $b \to s \gamma$ is $M_{H^+} \geq 350$ (200) GeV at the LO (NLO) level \[29, 30\]. For the model I, however, the limit can be much weaker due to the possible destructive interference with the SM amplitude.

For the model III, the situation is not as clear as the model II because there are more free parameters here. As pointed in \[26\], the data of $K^0 - \bar{K}^0$ and $B^0_d - \bar{B}^0_d$ mixing processes put severe constraint on the FC couplings involving the first generation of quarks. One therefore assume that,

$$
\lambda_{ij} = \hat{\lambda}_{dj} = 0, \quad for \quad j = 1, 2, 3 \tag{7}
$$

Imposing the limit in Eq.(7) and assuming all other $\lambda_{ij}$ parameters are of order 1, Atwood et al. \[31\] found a very strong constraint of $M_{H^+} > 600 GeV$ by using the CLEO data of $b \to s \gamma$ decay available in 1995. In Ref.\[32\], Aliev et al. studied the $b \to s \gamma$ decay in the model III by extending the NLO results of the model II \[30\] to the case of model III, and found some constraints on the FC couplings.

In a recent paper \[33\], Chao et al., studied the decay $b \to s \gamma$ by assuming that only the couplings $\lambda_{tt}$ and $\lambda_{bb}$ are non-zero. They found that the constraint on $M_{H^+}$ imposed by the CLEO data of $b \to s \gamma$ can be greatly relaxed by considering the phase effects of $\lambda_{tt}$ and $\lambda_{bb}$. The constraints by $B^0 - \bar{B}^0$ mixing, the neutron electric dipole moment(NEDM), the $Z^0$-pole parameter $\rho$ and $R_b$ give the following preferred scenario \[33\]:

$$
|\lambda_{tt}| \leq 0.3, \quad |\lambda_{bb}| \approx 50, \quad M_{A^0} \approx M_{H^0} = 80 - 120 GeV; \quad 80 GeV \leq M_{H^+} \leq 200 GeV \tag{8}
$$

In the following sections, we will calculate the new physics contributions to the inclusive charmless decays of b quark in the Chao-Cheung-Keung (CCK) scenario of model III \[33\]. Such model III has following advantages:

\[1\] We make the same ansatz on the $\xi^{U,D}_{ij}$ couplings as the Ref.\[26\]. For more details about the definition of $\xi^{U,D}$ one can see Ref.\[26\].
1. Since we keep only the couplings $\lambda_u$ and $\lambda_b$ none zero, the neutral Higgs bosons do not contribute at tree level or one-loop level. The new contributions therefore come only from the charged Higgs penguin diagrams with the heavy internal top quark.

2. The new operators $O_{b,10}$ and all flipped chirality partners of operators $O_{1,\ldots,10}$ as defined in [22] do not contribute to the decay $b \to s\gamma$ and other inclusive charmless decays under study in this paper.

3. The free parameters in this model III are greatly reduced to $\lambda_{tt}$, $\lambda_{bb}$ and $M_{H^+}$.

In order to find more details about the correlations between $M_{H^+}$ and couplings $\lambda_{tt,bb}$ by imposing the new CLEO data of $b \to s\gamma$, we recalculate the decay $b \to s\gamma$ in the model III. For the sake of simplicity, we do not consider the less interesting model I further in this paper.

The effective Hamiltonian for $b \to X_s\gamma$ at the scale $\mu = O(m_b)$ is given by

$$\mathcal{H}_{eff}(b \to s\gamma) = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{tb} \left[ \sum_{i=1}^{6} C_i(\mu) Q_i(\mu) + C_{\gamma\gamma}(\mu) Q_{\gamma\gamma} + C_{8G}(\mu) Q_{8G} \right]$$ (9)

The explicit expressions of operators $Q_{1-6}$, $Q_{\gamma\gamma}$ and $Q_{8G}$, as well as the corresponding Wilson coefficients $C_i(M_W)$ in the SM can be found for example in [4].

In the model III, the left-handed QED magnetic-penguin operator $Q_{\gamma\gamma}^L$ and the left-handed QCD magnetic-penguin operator $Q_{8G}^L$ may also play an important role,

$$Q_{\gamma\gamma}^L = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 - \gamma_5) b_\alpha F_{\mu\nu},$$ (10)

$$Q_{8G}^L = \frac{g_s}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 - \gamma_5) T^{a}_{\alpha\beta} b_\beta G^{a}_{\mu\nu},$$ (11)

In the SM and ordinary 2HDM’s, both operators $Q_{\gamma\gamma}^L$ and $Q_{8G}^L$ are absent because one usually assume that $m_s/m_b \sim 0$. In the model III, however, these two left-handed operators may contribute effectively because the Wilson coefficients $C_{\gamma\gamma}^L$ and $C_{8G}^L$ may be rather large to compensate for the suppression of $m_s/m_b$.

In Ref. [24], we calculated the $b \to sg$ decay in the model III from the first principle and obtained the corresponding form factors $F_1$ and $F_2$. Following the standard procedure and using the Feynman rules in the model III [20], we evaluate the Feynman diagrams for both $b \to s\gamma$ and $b \to sg$ decay as shown in Fig. 1, extract out the Wilson coefficients $C_i(M_W)$ at the energy scale $M_W$ by matching the full theory onto the effective theory,

$$C_i(M_W) = 0 \quad (i = 1, 3, 4, 5, 6),$$ (12)

$$C_2(M_W) = 1,$$ (13)

$$C_{\gamma\gamma}^L(M_W) = -\frac{m_s}{18m_b} D(y_t)|\lambda_t|^2,$$ (14)

$$C_{8G}^R(M_W) = C_{\gamma\gamma}(M_W)^{SM} - \frac{1}{12} A(y_t)|\lambda_t|^2 + \frac{1}{2} B(y_t)|\lambda_t| \lambda_{bb}|e^\theta,$$ (15)

$$C_{8G}^L(M_W) = -\frac{m_s}{12m_b} D(y_t)|\lambda_t|^2,$$ (16)

$$C_{8G}^R(M_W) = C_{8G}(M_W)^{SM} - \frac{1}{12} D(y_t)|\lambda_t|^2 + \frac{1}{2} E(y_t)|\lambda_t| \lambda_{bb}|e^\theta,$$ (17)
of NEDM [33]. For typical values of relevant parameters, say \( \theta = 0^\circ \) while \( C_W \) is much smaller than their right-handed counterparts and therefore will be neglected in the following calculations.

The Wilson coefficients given in Eqs.(12-17) contained the contributions from both the \( m_b \) suppressed by the ratio \( s_{\gamma} \approx 0 \) and \( s_{\eta} \approx 1 \) where \( s_{\gamma} \) is found in [4]. The Inami-Lim functions (A, B, D, E) are of the form,

\[
A(x) = \frac{7x - 5x^2 - 8x^3}{12(1 - x)^3} + \frac{2x^2 - 3x^3}{2(1 - x)^4} \log[x],
\]

\[
D(x) = \frac{2x + 5x^2 - x^3}{4(1 - x)^3} + \frac{3x^2}{2(1 - x)^4} \log[x],
\]

\[
B(y) = \frac{-3y + 5y^2}{12(1 - y)^2} - \frac{2y - 3y^2}{6(1 - y)3} \log[y],
\]

\[
E(y) = \frac{-3y + y^2}{4(1 - y)^2} \log[y] - \frac{y - y^2}{2(1 - y)3} \log[y]
\]

The Wilson coefficients given in Eqs.([12],[17]) contained the contributions from both the \( W^\pm \)-penguin and \( H^\pm \)-penguin diagrams.

It is easy to see that both \( C^L_{7\gamma}(M_W) \) and \( C^L_{8G}(M_W) \) in Eqs.([14]) and ([16]) will be doubly suppressed by the ratio \( m_s/m_b \) and \( |\lambda_t|^2 \) when \( |\lambda_t| \) is small as preferred by the data of NEDM [33]. For typical values of relevant parameters, say \( |\lambda_t| = 0.3, |\lambda_{tb}| = 40, \theta = 0^\circ \) and \( M_\mu = 200 \text{ GeV} \), one finds numerically that \( C^L_{7\gamma}(M_W) \approx C^L_{8G}(M_W) \approx 10^{-5} \), while \( C^R_{7\gamma}(M_W) \approx C^R_{8G}(M_W) \approx 0.8 \). Consequently, the left-handed Wilson coefficients are much smaller than their right-handed counterparts and therefore will be neglected in the following calculations.

At the lower energy scale \( \mu = O(m_b) \), the Wilson coefficients \( C_j(\mu) \) for the decay \( b \to s\gamma \) at the leading order are of the form

\[
C_j(\mu) = \sum_{i=1}^{6} k_{ji} \eta^{a_i} \quad (j = 1, \cdots, 6),
\]

\[
C_{7\gamma}(\mu)^{SM} = \eta^{H_8_7} C_{7\gamma}(M_W)^{SM} + \frac{8}{3} \left( \eta^{H_7_1} - \eta^{H_8_1} \right) C_{8G}(M_W)^{SM} + \sum_{i=1}^{8} h_i \eta^{a_i},
\]

\[
C_{7\gamma}(\mu)^{III} = \eta^{H_8_7} C_{7\gamma}(M_W)^{III} + \frac{8}{3} \left( \eta^{H_7_1} - \eta^{H_8_1} \right) C_{8G}(M_W)^{III} + \sum_{i=1}^{8} h_i \eta^{a_i},
\]

where \( \eta = \alpha_s(M_W)/\alpha_s(\mu) \), and the scheme-independent numbers \( a_i, k_{ji} \) and \( h_i \) can be found in [4].

Using the effective Hamiltonian, the branching ratio of \( b \to s\gamma \) at the leading order can be written as,

\[
BR(b \to s\gamma)^{(III)} = \frac{|V_{ts}V_{tb}|^2}{|V_{cb}|^2 \pi f(z)} |C_{7\gamma}(\mu)^{III}|^2 BR(b \to ce\bar{\nu}),
\]
where \( \mu = O(m_b) \), \( BR(b \rightarrow c\nu) = (10.7 \pm 0.4)\% \) is the measured semileptonic branching ratio of \( b \) decay, and \( f(z) \) is the phase space factor,

\[
f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \log[z],
\]

where \( z = m_c^{pole}/m_b^{pole} \). It is straightforward to write down the branching ratios \( BR(b \rightarrow s\gamma) \) for the SM and model II.

In the numerical calculations, the following input parameters \([15, 35]\) will be used implicitly:

\[
M_W = 80.41\text{GeV}, \quad M_Z = 91.187\text{GeV}, \quad \alpha_{em} = 1/137,
\]

\[
\alpha_s(M_Z) = 0.118, \quad G_F = 1.16639 \times 10^{-5}(\text{GeV})^{-2},
\]

\[
m_s = 0.13\text{GeV}, \quad m_c = 1.4\text{GeV}, \quad m_b = 4.8\text{GeV},
\]

\[
m_t = \overline{m}_t(m_t) = 168\text{GeV}, \quad \Lambda^{(5)}_{MS} = 0.225,
\]

\[
A = 0.84, \quad \lambda = 0.22, \quad \rho = 0.20, \quad \eta = 0.34,
\]

where \( A, \lambda, \rho \) and \( \eta \) are the Wolfenstein parameters of the CKM mixing matrix. \( \overline{m}_t(m_t) \) here refers to the running current top quark mass normalized at \( \mu = m_t \) and is obtained from the pole mass \( m_t^{pole} = 176 \text{GeV} \). For the running of \( \alpha_s \), the two-loop formulae \([4]\) will be used.

Fig.2 shows the branching ratios \( BR(b \rightarrow s\gamma) \) in the SM and models II and III, assuming \( \lambda_{tt} = 0.3, \lambda_{bb} = 35, \theta = 0^0, 30^0, \tan \beta = 1 \). The horizontal band between two dotted lines corresponds to the CLEO data \([9]\): \( 2 \times 10^{-4} \leq BR(b \rightarrow s\gamma) \leq 4.5 \times 10^{-4} \). The short-dashed line is the SM prediction, and the long-dashed and solid curve show the ratio in the model III for \( \theta = 0^0, 30^0 \), respectively. The dot-dashed curve shows the same ratio at the leading order in the model II. From the Fig.2, the lower and upper limit on \( M_{H^+} \) in the model III can be read out:

\[
185\text{GeV} \leq M_{H^+} \leq 238\text{GeV}, \quad \text{for} \quad \theta = 0^0,
\]

\[
215\text{GeV} \leq M_{H^+} \leq 287\text{GeV}, \quad \text{for} \quad \theta = 30^0
\]

These limits are consistent with those given in Eq.(8). If we take into account the errors of theoretical predictions in model III, the corresponding mass limit will be relaxed by about 20 GeV.

From above analysis, we get to know that for the model III the parameter space

\[
\lambda_{ij} = 0, \quad \text{for} \quad ij \neq tt, \text{ or } bb,
\]

\[
|\lambda_{tt}| = 0.3, \quad |\lambda_{bb}| = 35, \quad \theta = (0^0 - 30^0), \quad M_{H^+} = (200 \pm 100)\text{GeV},
\]

are allowed by the available data. For the mass \( M_{H^+} \), searches for pair production at LEP have excluded masses \( M_{H^+} \leq 77\text{GeV} \) \([36]\). Combining the direct and indirect limits together, we here conservatively consider a larger range of \( 100 \text{GeV} \leq M_{H^+} \leq 300 \text{GeV} \), while take \( M_{H^+} = 200 \text{GeV} \) as the typical value.
III. Inclusive charmless b quark decays

In this section, we will calculate the new physics contributions to the two-body and three-body inclusive charmless decays of b quark induced by the charged Higgs gluonic penguin diagrams in the models II and III.

A. \( b \to s \) gluon decay

The branching ratio of \( b \to sg \) at the leading order can be written as,

\[
BR(b \to sg) = \frac{|V_{ts}V_{tb}^*|^2 8\alpha_s(\mu)}{|V_{cb}|^2} \frac{1}{\pi f(z) \kappa(z)} |C_{SG}(\mu)|^2 BR(b \to ce\bar{\nu}), \tag{32}
\]

with

\[
C_{SG}(\mu)^{SM} = \eta \frac{14}{23} C_{SG}(M_W)^{SM} + \sum_{i=1}^{8} \bar{h}_i \eta^{a_i}, \tag{33}
\]

\[
C_{SG}(\mu)^{III} = \eta \frac{14}{23} C_{SG}^R(M_W) + \sum_{i=1}^{8} \bar{h}_i \eta^{a_i}, \tag{34}
\]

where \( \eta = \alpha_s(M_W)/\alpha_s(\mu) \) with \( \mu = O(m_b) \), and the numbers \( a_i \) and \( \bar{h}_i \) can be found in \[4\]. The factor \( \kappa(z) \) contains the QCD correction to the semileptonic decay rate \( BR(b \to ce\bar{\nu}) \) \[37, 38, 39\]. To a good approximation the \( \kappa(z) \) is given by \[39\]

\[
\kappa(z) = 1 - \frac{2\alpha_s(\mu)}{3\pi} \left[ \left( \frac{\pi^2 - 31}{4} \right) (1 - z)^2 + \frac{3}{2} \right]. \tag{35}
\]

And an exact analytic formula for \( \kappa(z) \) can be found in ref.\[38\].

For \( b \to dg \) decay, one simply substitues \( V_{ts}^* \) by \( V_{td}^* \) in Eq.(32). For the model II, one simply replaces \( C_{SG}(\mu) \) in Eq.(32) with \( C_{SG}^{II} \) as given in \[27\].

Fig.3 shows the branching ratios of \( BR(b \to sg) \) in the SM and the models II and III, assuming \( \lambda_{tt} = 0.3, \lambda_{bb} = 35, \) and \( \theta = 0^0, 30^0 \). The dots line in Fig.3 is the SM prediction \( BR(b \to sg) = 0.27\% \), while the short-dashed curve shows the branching ratio \( BR(b \to sg) = 0.81\% \) in the model II assuming \( \tan\beta = 2 \) and \( M_{H^+} = 200 \) GeV. In the model III, the enhancement to the ratio \( BR(b \to sg) \) can be as large as an order of magnitude: \( BR(b \to sg) \approx 2.34\%, 4.84\% \) for \( M_{H^+} = 200, 100 \) GeV respectively, as illustrated by the long-dashed and solid curves in Fig.3. The model III is clearly more promising than the model II to provide a large enhancement to the decay \( b \to sg \). Although the current enhancement is still smaller than \( \sim 10\% \) as expected, for example in Refs.\[19, 23\], such a significant increase is obviously very helpful for us to provide a reasonable solution for the problems such as the “missing charm puzzle” or the deficit \( B_{SL} \), as being discussed below.

B. Three-body charmless b quark decays

Within the SM, the three-body inclusive charmless b quark decays have been calculated at LO and NLO level for example in refs.\[3, 24, 40\]. In Ref.\[3\], Lenz et al. took into
account the NLO QCD corrections from the gluonic penguin diagrams with insertions of $Q_2$ and the diagrams involving the interference of the $Q_{8G}$ with $Q_1-6$.

The standard theoretical frame to calculate the decays $b \to s q \bar{q}$ for $q \in \{u, d, s\}$ is based on the effective Hamiltonian \[1\],

\[
\mathcal{H}_{\text{eff}}(|\Delta B| = 1) = \frac{G_F}{\sqrt{2}} \left\{ \sum_{j=1}^{2} C_j \left( v_c Q_j^c + v_u Q_j^u \right) - v_t \left\{ \sum_{j=3}^{6} C_j Q_j + C_8 Q_{8G} \right\} \right\} + \text{h.c.}, \tag{36}
\]

where $v_q = V_{qs}^* V_{qb}$ and the corresponding operator basis reads:

\[
Q_1 = (\bar{s}_\alpha q_\beta)_{V-A} (\bar{q}_\beta b_\alpha)_{V-A}, \tag{37}
\]

\[
Q_2 = (\bar{s}_\alpha q_\alpha)_{V-A} (\bar{q}_\beta b_\beta)_{V-A}, \tag{38}
\]

with $q = u$ and $q = c$, and

\[
Q_3 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}, \tag{39}
\]

\[
Q_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}, \tag{40}
\]

\[
Q_5 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}, \tag{41}
\]

\[
Q_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}, \tag{42}
\]

\[
Q_{8G} = -\frac{g_s}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a \tag{43}
\]

where the $Q_1$ and $Q_2$ are current-current operators, the $Q_3-Q_6$ are QCD penguin operators, while the $Q_{8G}$ is the chromo-magnetic dipole operator.

For the SM part, we will use the formulae presented in \[6\] directly. For the new physics part in the models II and III under study here, we take into account the new contributions from charged-Higgs gluonic penguins by using the Wilson coefficient $C_{8G}(\mu)^{III}$ as given in Eq.(34) in the calculation, this coefficient comprises both the SM and the new physics contributions. All other Wilson coefficients remain unmodified.

When the NLO QCD corrections are included, one usually expand the decay width to order $\alpha_s$,

\[
\Gamma(b \to sq\bar{q}) = \Gamma^{(0)} + \frac{\alpha_s(\mu)}{4\pi} \left( \Delta \Gamma_{cc} + \Delta \Gamma_{peng} + \Delta \Gamma_{W} + \Delta \Gamma_{8G} \right) + O(\alpha_s^2), \tag{44}
\]

where $\Gamma^{(0)}$ denotes the decay rate at the LO level, while the second part represents the NLO QCD corrections. We here use the renormalization-scheme(RS) independent terms $\Delta \Gamma_{cc}$, $\Delta \Gamma_{peng}$ and $\Delta \Gamma_{W}$. For the convenience of the reader, the explicit expressions of $\Delta \Gamma_{cc}$, $\Delta \Gamma_{peng}$ and $\Delta \Gamma_{W}$ will be given in Appendix. The term $\Delta \Gamma_{8G}$ in Eq.(44) (which will be defined below in Eq.(52)) is already RS independent \[6, 35\]. For the three-body decays $b \to dq\bar{q}$ one simply substitutes $s$ by $d$ in Eqs.(36-44).
At the NLO, the RS dependent Wilson coefficients $C_j(\mu)$ are given by

$$C_j(\mu) = C_j^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_j^{(1)}(\mu), \quad j = 1, \ldots, 6. \quad (45)$$

where $C_j^{(0)}$ are the RS independent LO Wilson coefficients, and $C_j^{(1)}$ are the RS dependent NLO corrections. \[35\]

$$C_j^{(0)}(\mu_b) = \sum_{i=3}^{8} k_{ji} \eta^{a_i}, \quad (46)$$

$$C_j^{(1)}(\mu_b) = \sum_{i=3}^{8} [e_{ji} \eta E_0(x_t) + f_{ji} + g_{ji} \eta] \eta^{a_i}, \quad (47)$$

where $\eta = \alpha_s(M_W) / \alpha_s(\mu_b)$, $x_t = m_t^2 / M_W^2$, the function $E_0(x_t)$ and all the numbers $a_i, k_{ji}, e_{ji}, f_{ji},$ and $g_{ji}$ can be found in \[35\]. The NLO QCD correction $C_j^{(1)}$ is RS dependent and can be split into two parts:

$$C_j(\mu)^{(1)} = \sum_{k=1}^{6} J_{jk} C_k^{(0)}(\mu) + \bar{C}_j(\mu)^{(1)}, \quad j = 1, \ldots, 6. \quad (48)$$

where parameters $J_{jk}$ are usually RS dependent, $\bar{C}_j(\mu)^{(1)}$ is RS independent, and the precise definitions of the terms in Eq.\((48)\) can be found for example in \[41\]. The terms involving $J_{jk}$ will be absorbed into $\Delta \Gamma_{cc}$ and $\Delta \Gamma_{peng}$ to make the latter scheme independent.

In the leading order the decays $b \to ss\bar{s}$, $sdd$, $s\bar{s}d$ and $dd\bar{d}$ are penguin-induced processes proceeding via $Q_{3-6}$ and $Q_{8G}$, while $b \to du\bar{u}$ and $b \to su\bar{u}$ also receive contributions from $Q_1$ and $Q_2$. Combining both cases, the decay width at the LO level can be written as \[1\]

$$\Gamma^{(0)} = \frac{G_F^2 m_b^5}{64\pi^3} \left\{ t \sum_{i,j=1}^{2} |v_u|^2 C_i^{(0)} C_j^{(0)} b_{ij} + \sum_{i,j=3}^{6} |v_t|^2 C_i^{(0)} C_j^{(0)} b_{ij} \right. \right.$$ 

$$\left. -2t \sum_{i=1,2}^{j=3,\ldots,6} C_i^{(0)} C_j^{(0)} \text{Re} (v_u v_t^*) b_{ij} \right\} \quad (49)$$

with $t = 1$ for $q = u$ and $t = 0$ for $q = d, s$. The coefficients $b_{ij}$ read

$$b_{ij} = \frac{16\pi^3}{m_b^6} \int d\Phi_3 (2\pi)^4 \langle Q_i \rangle^{(0)} \langle Q_j \rangle^{(0)*} = b_{ji} \quad (50)$$

with $Q_{1,2} = Q_{1,2}^u$ here. Setting the final state quark masses to zero one finds \[3\]

$$b_{ij} = \begin{cases} 
1 + r/3 & \text{for } i, j \leq 4, \text{ and } i + j \text{ even}, \\
1/3 + r & \text{for } i, j \leq 4, \text{ and } i + j \text{ odd}, 
\end{cases} \quad (51)$$

$$b_{55} = b_{66} = 1, \quad b_{56} = b_{65} = 1/3.$$
Here $r = 1$ for the decays $b \to d d \bar{d}$ and $b \to s s \bar{s}$, in which the final state contains two identical particles, and $r = 0$ otherwise. The remaining $b_{ij}$’s are zero.

Now we turn to study the contributions from the interference of the tree diagram with $Q_8$ with operators $Q_{1-6}$, as shown in Fig.3 of Ref.[6]. The tree-level correction $\Delta \Gamma_8$ is already at the order of $\alpha_s$ and is given by

$$\Delta \Gamma_8 = \frac{G_F^2 m_b^5}{32 \pi^3} \text{Re} \left[ -t v_u v_l C_{8G}(\mu)^{III} \sum_{j=1}^{2} C_j^{(0)} b_{j8} + |v_l|^2 C_{8G}(\mu)^{III} \sum_{j=3}^{6} C_j^{(0)} b_{j8} \right]. \quad (52)$$

in the model III, where $C_{8G}(\mu)^{III}$ has been given in Eq.(34) with $\mu = O(m_b)$. For the case of the SM and model II, simply replace $C_{8G}(\mu)^{III}$ with the appropriate $C_{8G}(\mu_b)$. The definitions and numerical values of coefficients $b_{j8}$ can be found in [6]. As mentioned previously, the Wilson coefficient $C_{8G}^{III}$ now comprises the contributions from both the W-penguin and the charged-Higgs penguin diagrams. In this way, the new physics contributions are taken into account.

For the $b$ quark decay rates one usually normalize them to the semileptonic decay rate of $b$ quark,

$$r_{q\ell} = \frac{\Gamma(b \to q \ell \nu_\ell)}{\Gamma(b \to c e \bar{\nu}_e)}, \quad r_{gg} = \frac{\Gamma(b \to g g)}{\Gamma(b \to c e \bar{\nu}_e)},$$

$$r_{q_1 q_2 \bar{q}_3} = \frac{\Gamma(b \to q_1 q_2 \bar{q}_3)}{\Gamma(b \to c e \bar{\nu}_e)}, \quad r_{sgg} = \frac{\Gamma(b \to s g g)}{\Gamma(b \to c e \bar{\nu}_e)}, \quad r_{sgg} = \frac{\Gamma(b \to s g g)}{\Gamma(b \to c e \bar{\nu}_e)} \quad (53)$$

for the sake of eliminating the factor of $m_b^6$ common to all $b$ decay rates. One also define the charmless decay rate of $b$ quark as

$$r_q = \sum_{q=u,d,s} (r_{dq\bar{q}} + r_{sqq}) + r_{sg} + r_{dg} + r_{sgg} + 2r_{ue} + r_{ut} \quad (54)$$

where rare radiative decays, for example $b \to s \gamma$, have been neglected. To order $\alpha_s$, the semileptonic decay rate takes the form

$$\Gamma(b \to c e \bar{\nu}_e) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 f(z) \kappa(z) \quad (55)$$

where the factors $f(z)$ and $\kappa(z)$ have been given in Eqs.(28) and (35).

To calculate $r_q$ we also need explicit expressions of $r_{ue}, r_{sg}, r_{dg}$ and $r_{sgg}$. For $r_{ue}$ one finds [12],

$$r_{ue} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \frac{1}{f(z)} \left\{ 1 + \kappa(z) - \kappa(0) + 6 \left[ \frac{(1 - z^2)^4}{f(z)} - 1 \right] \frac{\lambda_2}{m_b^6} \right\}, \quad (56)$$

where $\lambda_2 = 0.12 GeV^2$ encodes the chromomagnetic interaction of the $b$ quark with light degrees of freedom, and the factors of $f(z)$ and $\kappa(z)$ have been given in Eqs.(28) and (35).

From Eq.(32), we get

$$r_{sg} = \frac{|V_{us}^* V_{ub}|^2}{|V_{cb}|^2} \frac{8 \alpha_s(\mu)}{\pi f(z) \kappa(z)} |C_{8G}(\mu)|^2, \quad (57)$$

$$r_{dg} = \frac{|V_{us}^* V_{ub}|^2}{|V_{cb}|^2} \frac{8 \alpha_s(\mu)}{\pi f(z) \kappa(z)} |C_{8G}(\mu)|^2. \quad (58)$$
For $r_{sgg}$, we use the formulae as given in [40, 24],

$$r_{sgg} = \frac{1}{|V_{cb}|^2} \frac{3\alpha_s(\mu)^2}{16\pi^2 f(z) \kappa(z)} \sum_{i=u,c,t} V_{is}^* V_{ib} f_1(x_i, q^2)^2,$$

(59)

where $x_i = m_i^2/M_W^2$, the functions $f_1(x_i, q^2)$ can be found for example in [24]. In the numerical calculation, we assume that $q^2 = m_b^2/2$. Since the new contribution to the decay $b \to sgg$ due to the charged Higgs penguin is negligibly small [24], we do not consider the new physics corrections to this decay here. In Ref.[6], the authors did not include $r_{sgg}$ in the estimation of $r_c$. We here will include this mode, since its branching ratio is rather large [40, 24], as shown in the Table 1.

The corresponding branching ratios for two-body and three-body charmless $b$ decays are defined as

$$BR(b \to X) = r_X \cdot BR(b \to c e \bar{\nu}_e)^{exp},$$

(60)

where ratios $r_X$ have been defined previously. In the numerical calculations, $BR(b \to c \bar{\nu}_e)^{exp} = 10.70\%$ will be used.

By using the input parameters as given in Eq.(29) and assuming $|\lambda_{tt}| = 0.3, |\lambda_{tb}| = 35, M_{H^+} = 200 \text{GeV}$ and $\theta = 0^0$ or $30^0$, we find the numerical results of the decay rates and the branching ratios for various charmless $b$ quark decays and collect them in Table 1. We also show the corresponding results in the model II assuming $M_{H^+} = 200 \text{ GeV}$ and $\tan \beta = 2$. For larger $\tan \beta$ the new physics contributions in model II will become smaller. $\Delta BR$ in Table 1 is defined as

$$\Delta BR(b \to X) = \frac{[BR(b \to X) - BR(b \to X)^{SM}]}{BR(b \to SM)^{SM}}$$

(61)

Fig. 4 shows the mass dependence of the branching ratios $BR(b \to s q \bar{q})$ with $q \in \{u, d, s\}$ in the SM and model III, using the input parameters in Eq.(29) and assuming $|\lambda_{tt}| = 0.3, |\lambda_{tb}| = 35, \theta = 30^0$. In Fig. 4, the three curves ( horizontal lines) are the theoretical predictions in the model III ( SM ) for $q = u, d, s$, respectively. For $M_{H^+} = 200 \text{GeV}$, as listed in Table 1, the enhancement to the decay mode $b \to d u \bar{u}$ is only $4.7\%$, but the enhancements to other five-three-body $b$ quark decay modes are rather large: from $\sim 30\%$ to $\sim 70\%$. In the model II, however, the new contributions are negative and will decrease the branching ratios slightly, from $-0.3\%$ to $-13.5\%$ for different decay modes.

Fig. 5 shows the branching ratio $BR(b \to no \ charm)$ in the SM and models II and III, using the input parameters in Eq.(29) and assuming $|\lambda_{tt}| = 0.3, |\lambda_{tb}| = 35, \theta = 0^0, 30^0$. The dots line in Fig. 5 is the SM prediction $BR(b \to no \ charm) = 2.49\%$. The short-dashed curve shows the the ratio in the model II, $BR(b \to no \ charm) = 2.98\% (3.23\%)$ for $M_{H^+} = 200 (100) \text{ GeV}$ and $\tan \beta = 2$. The long-dashed and solid curve show the theoretical predictions in the model III: $BR(b \to no \ charm) = 4.67\% (4.91\%)$ for

\footnote{For more details, one can see the discussions about the semileptonic branching ratios of $b$ decay in next section.}
Table 1: The rates $r$ and branching ratios in the SM and models II and III, assuming $|\lambda_t|=0.3$, $|\lambda_{bb}|=35$, $M_{H^\pm}=200\text{GeV}$, $\tan \beta=2$, and $\theta=0^\circ$ or $30^\circ$ (the numbers in parenthesis). We also use $\text{BR}(B \to X_c e \bar{\nu}_e)^{\exp}=10.70\%$, as given in Eq.(63).

| decay mode | SM | Model III | Model II |
|-------------|----|-----------|----------|
| $b \to d u \bar{u}$ | $0.051$ | $0.052$ | $1.6$ |
| | $(0.053)$ | $(0.057)$ | $(4.7)$ |
| $b \to d d \bar{d}$ | $0.0005$ | $0.00078$ | $0.103$ |
| | $(0.0007)$ | $(0.010)$ | $(59.0)$ |
| $b \to d s \bar{s}$ | $0.0006$ | $0.0096$ | $0.008$ |
| | $(0.0009)$ | $(0.008)$ | $(59.5)$ |
| $b \to s u \bar{u}$ | $0.018$ | $0.027$ | $0.286$ |
| | $(0.0237)$ | $(0.255)$ | $(32.8)$ |
| $b \to s d \bar{d}$ | $0.019$ | $0.030$ | $0.322$ |
| | $(0.0285)$ | $(0.307)$ | $(48.5)$ |
| $b \to s s \bar{s}$ | $0.016$ | $0.024$ | $0.262$ |
| | $(0.0232)$ | $(0.250)$ | $(49.2)$ |
| $b \to s g$ | $0.025$ | $0.192$ | $2.065$ |
| | $(0.217)$ | $(2.339)$ | $(765.0)$ |
| $b \to d g$ | $0.00092$ | $0.007$ | $0.070$ |
| | $(0.008)$ | $(0.086)$ | $(765.0)$ |
| $b \to s g g$ | $0.070$ | $0.070$ | $0.757$ |
| | $-\quad$ | $-$ | $-$ |
| $b \to u e \bar{\nu}_e$ | $0.013$ | $0.144$ | $0.144$ |
| | $-\quad$ | $-$ | $-$ |
| $b \to u \mu \bar{\nu}_\mu$ | $0.013$ | $0.144$ | $0.144$ |
| | $-\quad$ | $-$ | $-$ |
| $b \to u \tau \bar{\nu}_\tau$ | $0.004$ | $0.004$ | $0.004$ |
| | $-\quad$ | $-$ | $-$ |
| $b \to no charm$ | $0.23$ | $0.43$ | $4.67$ |
| | $(0.46)$ | $(4.91)$ | $(97.3)$ |
\( M_{H^+} = 200 \text{ GeV} \) and \( \theta = 0^0, 30^0 \), respectively. For the model III with \( M_{H^+} = 100 \text{ GeV} \), one finds that \( BR(b \rightarrow \text{no charm}) = 7.27\% (7.60\%) \) for \( \theta = 0^0, 30^0 \), respectively.

It is easy to see from Fig.3 and Table 1 that the new physics enhancement to the branching ratios of three-body charmless b quark decays in the model III is much larger than that in model II within the parameter space considered.

IV. \( n_c \) and \( B_{SL} \)

The ratio \( B_{SL} \) is the average over weakly-decaying hadrons containing one b quark. For the CLEO experiments running on the \( \Upsilon(4S) \) resonance, the average is over \( B^+ \) and \( B^0 \) and their charge conjugate hadrons. For the experiments running on \( Z^0 \) resonance, however, the average is over \( B^+, B^0, B_s^0 \) and \( N_b \).

The charm multiplicity \( n_c \) is the average over the b-hadrons produced in the given environment. CLEO and LEP collaborations presented new measurements of inclusive \( b \rightarrow c \) transitions that can be used to extract \( n_c \). One naively expect \( n_c = 1.15 \) with the additional 15\% coming from the tree-level decay chain \( b \rightarrow uW^- \rightarrow u\bar{c}s \). This expectation can be verified experimentally by adding all inclusive \( b \rightarrow c \) branching ratios, and counting twice for the decay modes with 2 charm quarks in the final state.

In this section, we will investigate the new physics contributions, induced by the charged Higgs penguins in the models II and III, to the ratio \( B_{SL} \) and the charm multiplicity \( n_c \).

A. \( n_c \) and \( B_{SL} \): experimental measurements

The \( B_{SL} \) deficit was first point out in around 1994 \(^{13}\) when the theoretical prediction was considered to be difficult to produce \( B_{SL} \leq 12\% \) while the 1995 CLEO data on \( \Upsilon(4S) \) resonance was \( B_{SL} = (10.49 \pm 0.46)\% \). In the following, we use the 1998 Particle Data Group value \(^{13}\)

\[
B_{SL} = (10.45 \pm 0.21)\%
\] (62)

as the measured \( B_{SL} \) on \( \Upsilon(4S) \).

For the experiments on the \( Z^0 \)-peak, all the four LEP collaborations \(^{13, 14, 45, 46}\) reported their measured values of the ratio \( B_{SL} \) as listed in Table 2. The seventh row shows the averaged result of the ratio \( B_{SL} \) on \( Z^0 \)-peak \(^{4}\) \( B_{SL}^\nu = (10.66 \pm 0.17)\% \). This \( B_{SL}^\nu \) on the \( Z^0 \)-peak can be converted to \( \Upsilon(4S) \) value by multiplying a factor of \( \tau_B/\tau_b = 1.026 \): \( B_{SL} = (10.94 \pm 0.19)\% \) (\( Z^0 \) corrected). In fact, there is still a 2\( \sigma \) discrepancy in ratio \( B_{SL} \) between the high energy \( Z^0 \) value and the low energy \( \Upsilon(4S) \) value. The average of the \( Z^0 \) and \( \Upsilon(4S) \) values of \( B_{SL} \) is

\[
B_{SL} = (10.70 \pm 0.21)\% \quad \text{Overall average}
\] (63)

where we conservatively chose 0.21 as the overall error of the measured \( B_{SL} \).

\(^3\)\( N_b \) is in turn the mixture of \( \Lambda_b(udb) \), \( \Sigma_b(ushb) \), \( \Xi_b(dsb) \) and \( \Omega_b(ssb) \).

\(^4\) We here made an arithmetic average over four results as done in \(^{13}\), but the newest L3 data \(^{13}\) has been used here in the average.
Table 2: Recent CLEO and LEP measurements of the ratio $B_{SL}$.

| $B_{SL}$ (%) | Experiment                  |
|--------------|-----------------------------|
| 10.45 ± 0.21 | Υ(4S) PDG98 [15]            |
| 11.01 ± 0.10(stat.) ± 0.30(syst.) | ALEPH 95 [43]        |
| 10.65 ± 0.07(stat.) ± 0.25(syst.)$^{+0.28}_{-0.12}$(model) | DELPHI 99 [44]       |
| 10.16 ± 0.13(stat.) ± 0.30(syst.) | L3 99 [15]            |
| 10.83 ± 0.10(stat.) ± 0.20(syst.)$^{+0.20}_{-0.13}$(model) | OPAL 99 [46]         |
| 10.66 ± 0.17  | $Z^0$-peak                  |
| 10.94 ± 0.19  | $Z^0$ corrected             |
| 10.70 ± 0.21  | overall average             |

As for the charm counting, the value of $n_c$ measured at the Υ(4S) [20] is still smaller than that measured at $Z^0$-peak [15]:

$$n_c = \begin{cases} 1.10 \pm 0.05, & \text{Υ}(4S), \\ 1.20 \pm 0.07, & Z^0-\text{peak} \end{cases}$$ (64)

The average of the Υ(4S) and $Z^0$ result leads to

$$n_c = 1.14 \pm 0.04, (Z^0 + \Upsilon(4S))$$ (65)

B. $n_c$ and $B_{SL}$: theoretical predictions

Within the SM, the basis of the prediction for $B_{SL}$ and $n_c$ is the assumption of quark-hadron duality. The estimation for various inclusive decay rates is usually performed by using the heavy-quark expansion (HQE) [47] and the perturbative QCD in the framework of operator product expansion. The HQE allows to relate the inclusive decay rate of $B$ meson to that of the underlying $b$ quark decay process: $\Gamma(B \to X) = \Gamma(b \to x) + O(1/m_b^2)$.

The theoretical prediction for $B_{SL}$ with the inclusion of the $O(\alpha_s)$ QCD corrections and the hadronic corrections to the free quark decay of order $1/m_b^2$ is currently available [16, 17]. The $B_{SL}$ and $n_c$ can be defined as [16, 17]

$$B_{SL} = \frac{1}{\sum_l r_{cl} + r_{c\bar{u}d} + r_{c\bar{e}s} + r_{\bar{q}}}$$ (66)

$$n_c = 1 + \frac{r_{c\bar{e}s} - r_{\bar{q}}}{\sum_l r_{cl} + r_{c\bar{u}d} + r_{c\bar{e}s} + r_{\bar{q}}}$$ (67)

where $r_{ce} = r_{c\mu} = 1$, $r_{ce} = 0.25$, and $r_{c\bar{u}d}$ ($r_{c\bar{e}s}$) is the rate of the decay mode $b \to c\bar{u}d'$ ($b \to c\bar{e}s'$) where $d'$ ($s'$) is the appropriate Cabibbo mixture of $d$ and $s$ quarks.

The $r_{\bar{q}}$ has been defined and calculated in last section. In the SM, we have

$$r_{\bar{q}} = 0.23 \pm 0.08,$$ (68)
where the error mainly comes from the uncertainties of the scale \( \mu \) and the mass ratio \( m_c/m_b \).

As is well known, the main difficulty in calculating \( B_{SL} \) and \( n_c \) is in the non-leptonic branching ratios \( r_{c\bar{u}d} \) and \( r_{c\bar{s}s} \). For \( r_{c\bar{u}d} \), a complete NLO calculation has been performed \[16\] which gives

\[
    r_{c\bar{u}d} = 4.0 \pm 0.4, \tag{69}
\]

where the error mainly comes from the uncertainties of the scale \( \mu \), the quark masses \( m_c \) and \( m_b \), and the assumption of quark-hadron duality \[16\]. Furthermore, the error of the estimation for \( r_{c\bar{s}s} \) is generally considered to be larger than that for \( r_{c\bar{u}d} \). The enhancement of \( b \to c\bar{s}s \) due to large QCD corrections is about 30\% \[16\]. Such enhancement will decrease the value of \( B_{SL} \), but increase the size of \( n_c \).

Using the on-mass-shell scheme, the SM theoretical predictions for \( B_{SL} \) and \( n_c \) at the NLO level are

\[
    B_{SL} = (12.0 \pm 0.7 \pm 0.5 \pm 0.2^{+0.9}_{-1.2}) \%, \tag{70}
\]

\[
    n_c = 1.24 \mp 0.05 \pm 0.01, \tag{71}
\]

as given in Ref.\[16\]; and

\[
    B_{SL} = \begin{cases} 
    (12.0 \pm 1.0) \% & \mu = m_b, \\
    (10.9 \pm 1.0) \% & \mu = m_b/2,
    \end{cases} \tag{72}
\]

\[
    n_c = \begin{cases} 
    1.20 \mp 0.06 & \mu = m_b, \\
    1.21 \mp 0.06 & \mu = m_b/2,
    \end{cases} \tag{73}
\]

as given in Ref.\[17\] with the error mainly result from the variation of the scale \( \mu \) and \( m_c/m_b \).

Comparing the observed and predicted values of \( B_{SL} \) and \( n_c \), one can see that: (a) after considering all the corrections, the theoretical values of \( B_{SL} \) now come down and more or less consistent with the measurement, but unfortunately at the expense of boosting \( n_c \); (b) the central value of \( n_c \) in Ref.\[16\] is higher than that in Ref.\[17\], although two predictions are agree within errors; (c) there is still 2.8\% discrepancy between the \( n_c \) measured by CLEO and the theoretical prediction \[16\]: 1.10 \pm 0.05 against 1.24 \pm 0.05.

If we’d like to drop down the large uncertainty in the calculation for \( b \to c\bar{s}s' \) decay mode, we can eliminate the ratio \( r_{c\bar{s}s'} \) from the expression of \( B_{SL} \) and \( n_c \) and find,

\[
    n_c = 2 - \left( 2.25 + r_{c\bar{u}d} + 2r_{\bar{u}d} \right) B_{SL} \tag{74}
\]

which is a linear correlation between \( B_{SL} \) and \( n_c \). Using the values for \( B_{SL} \) \[63\], \( r_{c\bar{u}d} \) \[69\], and \( r_{\bar{u}d} \) \[68\], one finds

\[
    n_c = 1.28 \pm 0.05, \tag{75}
\]

\[5\]The last and largest error of \( B_{SL} \) comes from the uncertainty of the renormalization scale \( \mu \); while the main error of \( n_c \) is the the uncertainty in \( m_b \) \[13\].
for $B_{SL} = (10.70 \pm 0.21)\%$. The overall uncertainty of this prediction of $n_c$ should be smaller than that as given in Eqs.(71) and (73). The $2.6 \sigma$ discrepancy between the $n_c$ in Eq.(75) and $n_c$ measured at $\Upsilon(4S)$ motivated proposals of new physics which will enhance $r_\phi$ and in turn decrease $n_c$. That is what we try to do here.

As shown in Table I, the ratio $r_c/\bar{u}d$ will be increased significantly after taking the new physics effects into account, which will in turn decrease both $B_{SL}$ and $n_c$ accordingly. From Eq.(74) and using the values for $B_{SL}$ (63), $r_c/\bar{u}d$ (69), and $r_\phi$ (54), one finds

$$n_c = \begin{cases} 1.23 \pm 0.05 & \text{for } M_{H^+} = 200\text{GeV} \\ 1.18 \pm 0.05 & \text{for } M_{H^+} = 100\text{GeV} \end{cases} \quad (76)$$

for $\theta = 30^0$ and $\mu = m_b$. The $\mu$- and $\theta$-dependence of $n_c$ is rather weak: the central value of $n_c$ will go down (up) by only $\sim 0.01$ for $\mu = m_b/2$ ($\theta = 0^0$). For $B_{SL}$ in the model III, the agreement between the prediction and the data will be improved slightly by a decrease $0.003$ ($0.005$) for $M_{H^+} = 200$ ($100$) GeV due to the inclusion of new physics contributions. In the model II, the resulted decrease for $n_c$ ($B_{SL}$) is only $0.01$ ($0.001$) and plays no real role. Most importantly, one can see from Eqs.(64, 65, 75, 76) that the predicted $n_c$ and the measured $n_c$ now agree within roughly one standard deviation after taking into account the effects of gluonic charged Higgs penguins in the model III with a relatively light charged Higgs boson, as illustrated in Fig.4.

V. Summary and discussions

In the framework of the general two-Higgs doublet models, we calculated the charged-Higgs penguin contributions to (a) the rare radiative decay $b \to s\gamma$; (b) the inclusive charmless decays $b \to q'g$ and $b \to q'q\bar{q}$ with $q' \in \{d, s\}$ and $q \in \{u, d, s\}$; (c) the charm multiplicity $n_c$ and semileptonic branching ratio $B_{SL}$.

In section II, we studied the experimental constraint on the model III from the CLEO data of $b \to s\gamma$ decay. With the help of previous works [26, 31, 32, 33], we found the parameter space of the model III allowed by the available data, as shown in Eq.(31).

In section III, we firstly calculated the new physics contributions to the decay $b \to sg$ and found that the branching ratio $BR(b \to sg)$ can be greatly enhanced from the SM prediction of $0.27\%$ to $2.34\%$ ($4.84\%$) in the model III for $M_{H^+} = 200$ ($100$) GeV, as illustrated in Fig.3. Such a significant enhancement is clearly very helpful to resolve the missing charm/$B_{SL}$ problem appeared in B experiments.

Following the method of Ref.[6], we then calculated the new physics contributions to three-body inclusive charmless decays of b quark due to the interference between the operators $Q_{1-6}$ and $Q_{SG}$. The Wilson coefficient $C_{III}^{SG}$ in Eq.(74) now describe the contributions from both the $W^\pm$ and $H^\pm$ QCD penguins, the latter is the new physics part we focus in here. From numerical calculations, we found that: (a) the new physics enhancement to the decay $b \to du\bar{u}$ is only $\sim 1.6\%$ since this mode is dominated by the tree diagrams; (b) the branching ratios of other five three-body b decay modes are strongly enhanced by the new charged Higgs penguins: 30% to 70% increase can be achieved within the considered parameter space. The new contributions to the corresponding branching ratios in the
model II is, however, small in size and negative in sign against the theoretical predictions in the SM. As shown in Table 1 and Fig.5, the ratio $BR(b \rightarrow \text{no charm})$ can be increased from the SM prediction $BR(b \rightarrow \text{no charm}) = 2.49\%$ to $BR(b \rightarrow \text{no charm}) = 4.91\%$ (7.60\%) in the model III for $M_{H^+} = 200$ (100) GeV.

In section IV, we studied the current status about the theoretical predictions and experimental measurements for the semileptonic branching ratio of B meson decay $B_{SL}$ and the charm multiplicity $n_c$, and calculated the new physics contributions, induced by the charged Higgs penguins in the model III (II), to both $B_{SL}$ and $n_c$. With an enhanced ratio $BR(b \rightarrow \text{no charm})$, both the $B_{SL}$ and $n_c$ will be decreased accordingly: (a) the central value of $B_{SL}$ can be decreased slightly by 0.003 (0.005) for $M_{H^+} = 200$ (100) GeV; (b) the value of $n_c$ can be lowered significantly from the prediction $n_c = 1.28 \pm 0.05$ in the SM to $n_c = 1.23 \pm 0.05, 1.18 \pm 0.05$ for $M_{H^+} = 200, 100$ GeV, respectively.

In short, the predicted $n_c$ and the measured $n_c$ now agree within roughly one standard deviation after taking into account the effects of gluonic charged Higgs penguins in the model III with a relatively light charged Higgs boson, while the agreement between the theoretical prediction and the data for $B_{SL}$ can also be improved by inclusion of new physics effects.

ACKNOWLEDGMENTS

C.S. Li and K.T. Chao acknowledge the support by the National Natural Science Foundation of China, the State Commission of Science and technology of China and the Doctoral Program Foundation of Institution of Higher Education. Z.J. Xiao acknowledges the support by the National Natural Science Foundation of China under the Grant No.19575015 and the Outstanding Young Teacher Foundation of the Education Ministry of China.

Appendix: RS independent $\Delta \Gamma_{cc}$, $\Delta \Gamma_{peng}$ and $\Delta \Gamma_W$

For the convenience of the reader, we here present the explicit expressions of the RS independent NLO corrections $\Delta \Gamma_{cc}$, $\Delta \Gamma_{peng}$ and $\Delta \Gamma_W$. For more details one can see the original paper [6].

The term $\Delta \Gamma_{cc}$ in Eq.(44) describes the current-current type corrections proportional to $C_{1,2}^{(0)} \cdot C_{1,2}^{(0)}$:

$$\Delta \Gamma_{cc} = t \frac{G_F^2 M_b^5}{32 \pi^3} |v_u|^2 \sum_{i,j=1}^{2} C_i^{(0)} C_j^{(0)} \left[ h_{ij} + \sum_{k=1}^{2} J_{ki} b_{kj} \right]$$

(77)

with $t = 1$ for $q = u$ and $t = 0$ for $q = d, s$, and the coefficients $h_{ij}$ and $J_{ki}$ can be found in [6].

The term $\Delta \Gamma_{peng}$ in Eq.(44) describes the effect of penguin diagrams involving $Q_{1,2}$:

$$\Delta \Gamma_{peng} = \frac{G_F^2 M_b^5}{32 \pi^3} \text{Re} \left[ t \sum_{i,j=1,2} C_i^{(0)} C_j^{(0)} v_u \left[ v_u^* g_{ij}(x_c) + v_u^* g_{ij}(0) \right] \right]$$

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\[
- \sum_{i=1,2, j=3,...,6} C_i^{(0)} C_j^{(0)} v_t \left[ v_c^* g_{ij}(x_c) + v_u^* g_{ij}(0) \right] \\
+ \text{Re} \left[ -t v_u v_t^* \sum_{i,j=1,2 \atop k=3,...,6} C_i^{(0)} C_j^{(0)} J_{ki} b_{jk} + |\xi_t|^2 \sum_{i=1,2 \atop j,k=3,...,6} C_i^{(0)} C_j^{(0)} J_{ki} b_{jk} \right]. \tag{78}
\]

with \( t = 1 \) for \( q = u \) and \( t = 0 \) for \( q = d, s \). The explicit expressions of coefficients \( g_{ij} \) and \( J_{ki} \) can be found in [6].

Finally, \( \Delta \Gamma_W \) is given by

\[
\Delta \Gamma_W = \frac{G_F^2 m_h^5}{32 \pi^3} \left[ t \sum_{i,j=1}^2 |v_u|^2 \left[ C_i^{(0)} \overline{C}_j^{(1)} \right] b_{ij} + \sum_{i,j=3}^6 |v_t|^2 \left[ C_i^{(0)} \overline{C}_j^{(1)} \right] b_{ij} \\
- t \sum_{i=1,2 \atop j=3,...,6} \left[ C_i^{(0)} \overline{C}_j^{(1)} + \overline{C}_i^{(1)} C_j^{(0)} \right] \text{Re} (v_u^* v_t) b_{ij} \right]. \tag{79}
\]

where \( t = 1 \) for \( q = u \) and \( t = 0 \) for \( q = d, s \), the \( b_{ij} \) have been given in Eq.(51).
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Figure 1: The Feynman diagrams for the decays $b \to s\gamma$ and $b \to sg$ in the SM and 2HDM’s. The internal quarks are the upper type $u, c$ and $t$ quarks.

Figure 2: Plots of the branching ratio $BR(b \to s\gamma)$ versus $M_{H^+}$ in the SM and models II and III. The short-dashed line is the SM prediction, and the band between two dots lines refers to the CLEO data. The dot-dashed curve shows the ratio in the model II, while the long-dashed and solid curve show the ratios in the model III for $\theta = 0^0, 30^0$, respectively.
Figure 3: Plots of the branching ratio $BR(b \rightarrow sg)$ versus $M_{H^+}$ in the SM and models II and III. The dots line is the SM prediction, the short-dashed curve shows the ratio in the model II, and the long-dashed and solid curve show the ratios in the model III for $\theta = 0^0, 30^0$, respectively.

Figure 4: Plots of branching ratio $BR(b \rightarrow sq\bar{q})$ versus $M_{H^+}$ in model III. The three curves (horizontal lines) are the theoretical predictions in the model III (SM) for $q = u, d, s$, respectively.
Figure 5: Plots of the branching ratios $BR(b \to no\ charm)$ versus $M_{H^+}$ in the SM and models II and III. The dots line is the SM prediction, the short-dashed curve shows the the ratio in the model II, and the long-dashed and solid curve show the theoretical predictions in the model III for $\theta = 0^0, 30^0$, respectively.

Figure 6: Plots of Charm multiplicity $n_c$ versus $M_{H^+}$ in the SM and model III for $B_{SL} = 10.70\%$. The short-dashed line is the SM prediction, and the band refers to the data of $n_c = 1.14 \pm 0.04$. The solid curve, the upper and lower dot-dashed curves together show the central value and the 1$\sigma$ error of the theoretical prediction for $n_c$ in the model III.