Back to (Mass-)Square(d) One: 
The Neutrino Mass Ordering in Light of Recent Data

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We inspect recently updated neutrino oscillation data – specifically coming from the Tokai to Kamioka and NuMi Off-axis $\nu_e$ Appearance experiments – and how they are analyzed to determine whether the neutrino mass ordering is normal ($m_1 < m_2 < m_3$) or inverted ($m_3 < m_1 < m_2$). We show that, despite previous results giving a strong preference for the normal ordering, with the newest data from T2K and NOvA, this preference has all but vanished. Additionally, we highlight the importance of this result for non-oscillation probes of neutrinos, including neutrinoless double beta decay and cosmology. Future experiments, including JUNO, DUNE, and T2HK will provide valuable information and determine the mass ordering at a high confidence level.

Introduction. — By observing the phenomenon of neutrino oscillations, we have determined a number of their properties fairly precisely. This information has come from a wide variety of regimes, including atmospheric neutrinos, solar neutrinos, reactor antineutrinos, and long-baseline neutrino oscillation experiments. Current data allow us to understand, to a reasonable degree, how the neutrinos mix and that there are two non-zero mass scales. Because neutrinos in any oscillation environment are highly relativistic, these experiments are only sensitive to differences of masses squared, the so-called mass-squared-splittings $\Delta m^2_{ij}$, with masses $m_i$. We label the mass eigenstates by defining $\nu_1$ and $\nu_3$ as the mass eigenstates with the largest and smallest admixture of $\nu_e$, respectively.

Among a combination of solar and reactor neutrino experiments, it has been determined that $\Delta m^2_{31} \approx +7.5 \times 10^{-5}$ eV$^2 > 0$ [4, 5]. Accelerator and atmospheric neutrinos have determined that $|\Delta m^2_{31}| \approx 2.5 \times 10^{-3}$ eV$^2 \gg \Delta m^2_{21}$, but, in general, are not sensitive to the sign of $\Delta m^2_{31}$ – this is the crux of the neutrino mass ordering (MO) problem – whether nature prefers $m_1 < m_2 < m_3$, the normal mass ordering (NO), or $m_3 < m_1 < m_2$, the inverted mass ordering (IO) [6].

There are two straightforward ways to determine the MO, utilizing either interference or matter effects in neutrino oscillations. The first relies on measuring neutrino oscillations in a regime where both mass-squared-splittings $\Delta m^2_{21}$ and $\Delta m^2_{31}$ are relevant and an experiment that can measure $\Delta m^2_{31}$ to a precision smaller than the magnitude of $\Delta m^2_{21}$ – this is the strategy of the upcoming Jiangmen Underground Neutrino Observatory (JUNO) [7], a reactor antineutrino experiment operating at $L \approx 50$ km and $E \approx 4$ MeV. In contrast, accelerator neutrino experiments, operating where effects due to $\Delta m^2_{31}$ are dominant and matter effects (coming from coherent neutrino interactions with rock along the path of propagation) are relevant, are also sensitive to the MO. A combination of measuring oscillation probabilities for muon-neutrino disappearance $P(\nu_\mu \rightarrow \nu_\mu)$ and electron-neutrino appearance $P(\nu_\mu \rightarrow \nu_e)$ (as well as the corresponding probabilities for antineutrinos) allows for long-baseline experiments to measure the MO. However, challenging degeneracies exist between determining the MO, the atmospheric octant (whether $\sin^2 \theta_{23}$ is smaller or larger than $1/2$), and the degree of CP violation in the leptonic sector, parameterized by the phase $\delta_{CP}$.

The latter strategy, where matter effects allow for sensitivity to the MO, octant, and $\delta_{CP}$ is employed by the currently-operating Tokai to Kamioka (T2K) [8, 10] and NuMi Off-axis $\nu_e$ Appearance (NOvA) [11, 13] experiments, which measure $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ at long distances. Their $\nu_e$ appearance measurements can be well-approximated as measurements at a fixed length and energy – T2K operates at $L = 295$ km and $E \approx 0.6$ GeV, while NOvA has $L = 810$ km and $E \approx 1.9$ GeV. Super-Kamiokande (SK) [14], which shares a common detector with T2K, also has modest sensitivity to the MO by studying atmospheric neutrino oscillations, where matter effects are also important.

As of mid-2020, existing experimental data, driven largely by these three experiments, exhibited a fairly strong preference for the NO over the IO: $\Delta \chi^2_{\nu_{LO}} \equiv \chi^2_{\nu_{LO}} - \chi^2_{\nu_{IO}} \approx 10$ as consistently determined by a variety of efforts to fit the global neutrino experimental data [15, 17]. However, T2K, NOvA, and SK have each recently provided preliminary updated data [18, 20]. We will demonstrate that this NO preference has all but vanished due to interesting correlations between the data, as well as the degeneracies between MO, octant, and $\delta_{CP}$.
This letter is organized as follows. First, we explain how the long-baseline experiments are sensitive to the MO, as well as the degeneracies with the atmospheric octant and $\delta_{\text{CP}}$. We show how previous data, driven likely by fortuitous statistical fluctuations, provided the previous strong preference for NO over IO, as well as how the updated data drive this preference back to being marginal at best. Finally, we discuss the ramifications of this result and provide some outlook for the future.

Mass ordering sensitivity at Long-Baseline Oscillation Experiments. — In long-baseline experiments like T2K and NOvA (and the planned T2HK [21] and DUNE [22, 23] experiments), oscillations due to the smaller mass-squared splitting $\Delta m_{21}^2$ have yet to develop, so the expansion parameter $\Delta m_{21}^2 L/4E$ can be considered to be perturbatively small. Assuming neutrinos propagate through constant-density matter, the oscillation probability of a muon neutrino into an electron neutrino with energy $E$ and after travelling a distance $L$ can be approximated as [24]

$$
\begin{align*}
P_{\mu e} &\equiv P(\nu_{\mu} \to \nu_e) 
\approx 4s_{23}s_{13}c_{13}^2 \frac{\sin^2 (\Delta_{31} - aL)}{(\Delta_{31} - aL)^2} \Delta_{31}^2 \\
+ 8 \frac{J}{\sin \delta_{\text{CP}}} \frac{\sin (\Delta_{31} - aL)}{(\Delta_{31} - aL)} &\Delta_{31} \Delta_{21} \cos (\Delta_{31} + \delta_{\text{CP}}) \\
+ 4s_{12}^2c_{12}^2 \frac{\sin^2 (aL)}{(aL)^2} &\Delta_{21}^2,
\end{align*}
$$

where $\Delta j_{1} \equiv \Delta m_{j1}^2 L/4E$, $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ and $J \equiv s_{23}c_{23}s_{13}c_{13}^2 s_{12}c_{12} \sin \delta_{\text{CP}}$ is the Jarlskog invariant [25]. Effects of propagation through matter are given by the matter potential [26] as follows:

$$
a = \frac{G_F n_\rho}{\sqrt{2}} \approx \frac{1}{3500 \text{ km}} \left( \frac{\rho}{3.0 \text{ g/cm}^3} \right),$$

where $\rho$ is the assumed-constant density along the path of propagation. For current and planned $\nu_e$ appearance oscillation experiments, the parameter $(aL)$ can be determined to be

$$
aL = \begin{cases} 
0 & \text{Vacuum, any } L \\
0.065 & \text{T2K/T2HK} \ [21] \\
0.22 & \text{NOvA} \ [13] \\
0.29 & \text{T2HK} \ [27] \\
0.35 & \text{DUNE} \ [22]
\end{cases}
$$

while, by design, $|\Delta_{31}| \approx \pi/2$ so that oscillations due to the atmospheric mass-squared splitting are maximized.

For antineutrinos traveling through matter, $P_{\bar{\nu} e} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$ can be determined by taking Eq. (1) and replacing $\delta_{\text{CP}} \to -\delta_{\text{CP}}$ as well as $(aL) \to -(aL)$. In Fig. 1 we display how the oscillation probabilities $P_{\mu e}$ and $P_{\nu e}$ vary at NOvA (left panel) and T2K (right) baselines/energies. We assumed fixed $L = 810 \text{ km}$ (left) and 295 km (right), as well as $E = 1.9 \text{ GeV}$ (left) and 0.6 GeV (right). For both panels, we fix the oscillation parameters $\sin^2 \theta_{12} = 0.310, \sin^2 \theta_{13} = 0.022,$ and $\Delta m_{21}^2 = 7.39 \times 10^{-5} \text{ eV}^2$ [15]. The colored ellipses are generated by varying $\delta_{\text{CP}}$ for different combinations of $(\sin^2 \theta_{23}, \Delta m_{31}^2)$. These combinations are determined by obtaining the best-fit parameters according to a fit to NOvA (blue ellipses) or T2K (red), assuming the MO is normal (solid) or inverted (dashed) – we discuss how these points are obtained in the “results” section. Fig. 1 also displays measured oscillation probabilities (with statistical uncertainty) as black (current data [18, 19]) and grey (pre-2020 data [8, 13]) crosses. Comparing older results to the current ones, we immediately see that the measured oscillation probabilities are trending toward the “IO” region of this space, where $P_{\bar{\nu} e} > P_{\nu e}$.

We find it instructive to analyze the sums and differences of the neutrino and antineutrino oscillation probabilities, $\Sigma P_{\mu e} \equiv P_{\mu e} + P_{\bar{\nu} e}$ and $\Delta P_{\mu e} \equiv P_{\mu e} - P_{\bar{\nu} e}$. Near

**FIG. 1.** Bi-probability plots depicting the oscillation probability for neutrinos (x-axes) and antineutrinos (y-axes) at the baseline length/neutrino energy for NOvA (left panel) and T2K (right panel) while varying $\delta_{\text{CP}}$. Black (grey) crosses indicate extracted measurements with statistical uncertainty only for the two experiments using their 2020 (pre-2020) results. Different ellipses correspond to best-fit points according to NOvA (blue) and T2K (red) fits under the assumption of the Normal (solid) or Inverted (dashed) mass ordering. The dots denote probabilities for $\delta_{\mu e} = 0$, with the arrow indicating increasing $\delta_{\text{CP}}$ values. See text for more detail.
the first oscillation maximum, $|\Delta_{31}| \approx \pi/2$, and under

$$
\Sigma P_{\mu e} \rightarrow 8 s^2_{13} c^2_{13} s^2_{23} - 16 s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin \delta_{\text{CP}} (a L) \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \sin \left( \Delta m^2_{31} \right),
$$

$$
\Sigma P_{\mu e} \approx 0.17 s^2_{23} - 0.03 (a L) s_{23} c_{23} \sin \delta_{\text{CP}} \sin \left( \Delta m^2_{31} \right),
$$

$$
\Delta P_{\mu e} \approx \frac{32 (a L)}{\pi} s^2_{13} c^2_{13} s^2_{23} \sin \left( \Delta m^2_{31} \right) - 8 \pi s_{12} c_{12} s_{13} c_{13} s_{23} c_{23} \sin \delta_{\text{CP}} \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|},
$$

$$
\Delta P_{\mu e} \approx 0.22 (a L) s^2_{23} \sin \left( \Delta m^2_{31} \right) - 0.05 s_{23} c_{23} \sin \delta_{\text{CP}}.
$$

(4a)

where in Eqs. (4a) and (4b) we have used the current best-fit measurements of $\theta_{12}$, $\theta_{13}$, $\Delta m^2_{21}$, and $|\Delta m^2_{31}|$. Analyzing Eq. (4a), it is apparent that measurements of the sum of oscillation probabilities are beneficial for extracting $s^2_{23}$ (giving the atmospheric octant), while the effects of CP violation and the MO have an impact at a small level. On the other hand, according to Eq. (4b), measurements of $\Delta P_{\mu e}$ can allow for extraction of the MO, octant, and CP violation, however these are all comparable and competing effects.

We show the sums and differences of oscillation probabilities at NOvA and T2K in Fig. 2, presenting $\Sigma P_{\mu e}$ ($\Delta P_{\mu e}$) in the top (bottom) panel. We show the extracted measurements of these sums/differences as black (current) and grey (pre-2020) crosses, assuming statistically-independent measurements of $P_{\mu e}$ and $P_{\mu \mu}$ at each experiment, adding uncertainties in quadrature. The red and blue ellipses are again generated fixing all parameters except $\delta_{\text{CP}}$ to the same combinations as in Fig. 1. Here, specifically in the bottom panel, the impact of the mass ordering is abundantly clear – even while varying $\delta_{\text{CP}}$, NOvA requires $\Delta P_{\mu e} > 0$ for NO and $\Delta P_{\mu e} < 0$ for IO. While the separation is not as powerful for T2K (where $(a L)$ is a factor of $\sim 3$ smaller), the normal ordering prefers larger $\Delta P_{\mu e}$. We also note that, in the top panel, NOvA’s NO best-fit point predicts a much smaller value of $\Sigma P_{\mu e}$ at T2K than what is observed, so this combination of parameters is slightly disfavored by T2K data. As with Fig. 1 we see that current data have moved in a direction that begins to favor IO for both T2K and NOvA. In what follows, we quantify all of these effects, performing fits to T2K and NOvA individually, as well as a joint fit, to determine their individual and joint preferences for the neutrino mass ordering.

**Analysis.** — In the case of T2K, we consider the latest results from data collection equivalent to $1.97 (1.63) \times 10^{21}$ protons-on-target (POT) in neutrino (antineutrino) mode [15]. T2K observes a total of 108 (16) $\nu_e$ ($\bar{\nu}_e$) like events, and 318 (137) $\nu_\mu$ ($\bar{\nu}_\mu$) like events. We classify the data in the same five categories as the collaboration, muon-ring (1$R\mu$) and electron-ring (1$Re$) events in both neutrino and antineutrino modes, plus $\nu_e - CC1\tau$ events in neutrino mode. We perform our simulation by defining a loglikelihood function comparing the expected and observed events, including pull parameters related to

![FIG. 2. Sums (top) and differences (bottom) of oscillation probabilities at NOvA (x-axes) and T2K (y-axes) at fixed baseline lengths and energies as described in the text. Ellipses are generated by varying $\delta_{\text{CP}}$ while fixing the other oscillation parameters. The dots denote $\Sigma P_{\mu e}$ (top) and $\Delta P_{\mu e}$ (bottom) for $\delta_{\text{CP}} = 0$, with arrows indicating increasing $\delta_{\text{CP}}$ values. Crosses display extracted sums/differences of oscillation probabilities assuming statistically independent measurements at each experiment for current (black) and pre-2020 (grey) results.](image-url)
find that we are best able to reproduce their results if we include priors on the effective oscillation probability \( P_{ee} = c_{13}^2 s_{13}^2 + s_{13}^2 \), which is consistent with 0.2893 ± 0.00134 and \( \Delta m_{21}^2 = (6.11 ± 1.21) \times 10^{-5} \text{ eV}^2 \). From reactor antineutrino experiments, we include priors on \( \Delta m_{23}^2 \) = (7.53 ± 0.18) \times 10^{-3} \text{ eV}^2 \) from KamLAND [5] and \( \sin^2 (2\theta_{13}) = 0.0856 ± 0.0029 \) and \( \Delta m_{32}^2 = (2.471 ± 0.070) \times 10^{-3} \text{ eV}^2 \) from Daya Bay [28]. Additionally, we include the \( \Delta \chi^2 \) map from Ref. [14] which we refer to as “SK18” henceforth.

For NOvA, we include information from the muon neutrino disappearance channels in the following way. We find that we are best able to reproduce their results if we include Gaussian priors on the parameters \( |\Delta m_{23}^2| = (2.41 ± 0.07) \times 10^{-3} \text{ eV}^2 \) and \( 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) = 4c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) = 0.99 ± 0.02 \). For electron (anti)neutrino appearance, we assume that NOvA measures an event rate of \( \nu_e \) and \( \bar{\nu}_e \) at a fixed \( L = 810 \text{ km} \) and \( E = 1.9 \text{ GeV} \), with a constant matter density along the path of propagation of \( \rho = 2.84 \text{ g/cm}^3 \). Under this assumption, we treat NOvA as a counting experiment and approximate the number of events observed as:

\[
\begin{align*}
n_{\nu_e}^{\text{NOvA}} &= 1202.7 \times P(\nu_\mu \rightarrow \nu_e) + 29.1, \\
n_{\bar{\nu}_e}^{\text{NOvA}} &= 438.4 \times P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) + 16.6,
\end{align*}
\]

where the factors 1202.7 and 438.4 represent weighted flux-times-cross-section and detector mass, and the numbers 29.1 and 16.6 are the expected background rates, which we assume to be independent of oscillations. NOvA observes 82 (33) \( \nu_e \) (\( \bar{\nu}_e \)) events, and we include these factors using a Poissonian likelihood function, incorporating only statistical uncertainties (due to the relatively small number of events). See Refs. [29, 30] for further details.

**Results.** — We perform three different analyses and compare their results. First, we perform a joint analysis of T2K/NOvA/SK18, including the priors on other parameters discussed above. As we have observed before, cf. Fig. 1 without SK18, this fit results in a mild preference for the IO (\( \Delta \chi^2_{(\text{NO,IO})} = -1.83 \)) and a strong preference for \( \delta_{CP} \approx -\pi/2 \), maximal CP violation. When SK18 is included, this preference changes to \( \Delta \chi^2_{(\text{NO,IO})} = 2.2 \) and \( \delta_{CP} \approx -3 \) is favored. Fig. 3 presents the results of this fit in black, where the top panel displays the one-dimensional \( \Delta \chi^2 \) as a function of \( \delta_{CP} \) (after marginalizing over the other five oscillation parameters) when we fix ourselves to be in the NO (solid black line) or IO (dashed black line). The middle (bottom) panel presents two-dimensional measurement contours (at 68.3% CL, dashed, and 90% CL, solid) of \( \delta_{CP}^2 \) vs. \( \sin^2 \theta_{23} \), assuming NO (IO). The best-fit point, \( \sin^2 \theta_{23} \approx 0.55 \) and \( \delta_{CP} \approx -3 \), NO, is shown as a star in the middle panel. Note that we find that the combined results are consistent with the hypothesis that CP is conserved (\( \delta_{CP} = 0 \) or \( \pm \pi \)) at \( < 1 \sigma \) CL.

The other two fits we perform are with only T2K or only NOvA data. In each case, information from solar/reactor neutrino experiments are included independently. The results of these two fits are shown in Fig. 3 for NOvA (blue) and T2K (red). Both of these fits result in a small preference for NO over IO, again, as discussed cf. Fig. 1 with T2K giving \( \Delta \chi^2_{(\text{NO,IO})} = 1.2 \) and NOvA giving \( \Delta \chi^2_{(\text{NO,IO})} = 0.13 \). The red and blue
stars in the middle panel of Fig. 3 represent the best-fit points of these two fits – T2K prefers \( \delta_{\text{CP}} \approx -\pi/2 \) and \( \sin^2 \theta_{23} \approx 0.55 \). On the other hand, NOvA prefers \( \delta_{\text{CP}} \approx 0.47 \) and \( \sin^2 \theta_{23} \approx 0.46 \). The two best-fit regions are somewhat in tension, leading to a joint T2K/NOvA fit preferring maximal CP violation, but inverted mass ordering. We show a version of Fig. 3 without SK18 in Appendix A.

Table I summarizes the MO preference by each experiment or combination of experiments we consider, as well as the updated SK 2020 result presented in Ref. [20] – as these results are not yet published, we do not have a \( \Delta \chi^2 \) map for this to perform a complete fit with T2K and NOvA. We comment on what may happen with a joint T2K/NOvA/SK20 fit in the following section.

**Discussion & Conclusions.** — The neutrino mass ordering remains one of the largest outstanding mysteries in the Standard Model of particle physics. Prior to this Summer, experimental data seemed to be preferring the normal mass ordering, \( m_1 < m_2 < m_3 \), corresponding to the same ordering that the charged fermions of the Standard Model obey. However, as we have shown, this evidence is waning given the updated results from the long-baseline oscillation experiments T2K and NOvA, specifically when the two are combined in a joint fit. With SK18, a mild preference for the normal ordering is obtained. However, preliminary updated results from SK are likely to reduce this preference even further – Ref. [20] showed that with updated data, SK no longer has as strong of a preference for NO as it did with Ref. [14]. Additionally, the updated results prefer the lower octant, \( s_{23}^2 < 1/2 \). In combination with T2K and NOvA, this will likely result in the IO, upper or lower octant, and maximal CP violation \( \delta_{\text{CP}} \) being the overall favored solution.

The importance of this result cannot be understated. If neutrinos do follow the inverted ordering, this can have far-reaching consequences. If in addition neutrinos are Majorana fermions, there exists some minimum mass relevant for neutrino-less double beta decay. If the inverted ordering is true and neutrino-less double beta decay remains unobserved by upgraded experiments, then we can determine that neutrinos are Dirac fermions. Moreover, measurements of the cosmic microwave background and the matter power spectrum allow us to infer the sum of the neutrino masses. If neutrinos follow the inverted ordering, their sum is at least \( \sim 100 \text{ meV} \), while for normal ordering \( \sum m_i \geq 60 \text{ meV} \). The lower limit for the inverted ordering is attainable by next-generation experiments. Furthermore, experiments that measure neutrino masses via kinematic effects, such as KATRIN [31], could also be impacted by the mass ordering, as the minimum effective electron neutrino mass is about 50 meV for the inverted ordering as opposed to 9 meV for normal ordering. Finally, the mass ordering may also play an important role in the potential observation of relic neutrinos from the early universe by the proposed PTOLEMY experiment [32].

It could well be that statistical fluctuations in the data, moving in the same direction in the bi-probability planes of Fig. 1, are the cause for this vanishing preference for normal ordering. This highlights the importance of two things. First, the accumulation of more data. As T2K and NOvA continue to run, their statistical uncertainties will decrease, and thus will become more robust against statistical fluctuations. Second, this shows the need for the future experiments that will definitively pin down the neutrino mass ordering. Between JUNO’s long-baseline reactor antineutrino measurements, and DUNE and T2HK’s long-baseline high-energy oscillation and atmospheric oscillation measurements (which take different approaches to determine the ordering, see, e.g., [33]), we will be able to determine the neutrino mass ordering absolutely.

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2 In generating Figs. 1 and 2 we used the preferred points of the NOvA-only and T2K-only fits in each MO. In NO (IO), T2K prefers \( s_{23}^2 = 0.55 \), \( \Delta m_{21}^2 = +2.56 \times 10^{-3} \text{ eV}^2 \), \( \delta_{\text{CP}} = -1.98 \) (0.55, \(-2.46 \times 10^{-3} \text{ eV}^2\), -1.49). NOvA prefers \( s_{23}^2 = 0.46 \), \( \Delta m_{21}^2 = 2.52 \times 10^{-3} \text{ eV}^2 \) and \( \delta_{\text{CP}} = 0.473 \) (0.56, \(-2.36 \times 10^{-3} \text{ eV}^2\), -1.38) for the NO (IO).
A. Results of T2K/NOvA Fits without Super-Kamiokande

In Fig. 3 we presented the results of fits to NOvA and T2K’s recently-updated data, as well as a combined fit to T2K, NOvA, and Super-Kamiokande’s published result from Ref. 11, using the $\Delta \chi^2$ map from that publication. In this appendix, we repeat the exercise of Fig. 3 with a joint T2K/NOvA fit without Super-Kamiokande. This is shown in Fig. 4.

As discussed in the main text, the combination of T2K and NOvA prefer the inverted mass ordering over the normal at the $\Delta \chi^2_{\text{NO,IO}} = -1.8$ level. Their combination prefers maximal CP-violation, $\delta_{CP} \approx -\pi/2$, and the upper octant $s_{12}^2 > 1/2$. The marginalized one-dimensional $\Delta \chi^2$ lines in the top panel of Fig. 4 allow us to determine T2K on its own (with the included priors) can exclude a small interval of $\delta_{CP} \approx \pi/2$ at $> 3 \sigma$ CL. However, once NOvA is included, the interval shrinks (note that near $\delta_{CP} \approx \pi/2$, the exclusion of the red solid line is higher than that of the black solid line). According to our results, the combination of T2K and NOvA can only exclude the hypothesis that CP is conserved ($\delta = 0$ or $\delta = \pm \pi$) at roughly $1 - 2 \sigma$ CL.
FIG. 4. Results of our fit of the oscillation parameters $\delta_{CP}$ and $\sin^2 \theta_{23}$. In the top panel we show $\Delta \chi^2$ as a function of $\delta_{CP}$ for a fixed MO. In the middle (bottom) panel we show $\delta_{CP}$ versus $\sin^2 \theta_{23}$ for the NO(IO). Except for the top panel, all contours are 68.3% CL (dashed) and 90% CL (solid) – black lines indicate a joint fit of T2K/NOvA, where blue (red) indicate a fit to NOvA (T2K) alone. The corresponding stars indicate the best-fit point of each fit, and the text indicates the relative preference for mass ordering by each fit ($\Delta \chi^2_{(NO, IO)} \equiv \chi^2_{(\text{min, IO})} - \chi^2_{(\text{min, NO})}$).