Conformal to harmonic gauge for bosonic strings

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Abstract – We consider the Polyakov theory of bosonic strings in conformal gauge which are used to study the conformal anomaly. However, it exhibits ghost number anomaly. We show how this anomaly can be avoided by connecting this theory to that of in background covariant harmonic gauge which is known to be free from conformal and ghost number current anomaly, by using suitably constructed finite-field-dependent BRST transformation.

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Introduction. – Bosonic strings in path integral formulation \([1–3]\) have been studied in various gauges. The simplest choice among such gauges is the conformal gauge \([1,3–5]\) which has been used to study many important properties of bosonic strings. In conformal gauge, upon renormalization the effective action which is defined as a whole by a one-loop Feynman diagram, exhibits conformal anomaly \([6]\). Besides the conformal anomaly, there is another important anomaly associated with conformal gauge which is the ghost number current anomaly on curved worldsheet \([5]\). Bosonic strings have also been investigated in detail in a bit complicated harmonic gauge \([7]\) (a choice similar to the Lorenz gauge in QED). In harmonic gauge the standard \((D−26)\) factor appears naturally \([1]\). Even though the harmonic gauge is a bit difficult to use for quantization, it does not require the ghost field interaction at the vertex. Further, the ghost number current anomaly does not appear in the background covariant harmonic gauge \([8]\) but the absence of a ghost number anomaly is achieved at the expense of a new anomaly in the sector involving the Nakanishi-Lautrup field \([9]\). The BRST analysis of the bosonic string in this perspective have been discussed in \([10]\).

BRST quantization \([11]\) is an important and powerful technique to deal with a system with constraints \([12]\). It enlarges the phase space of a gauge theory and restores the symmetry of the gauge fixed action in the extended phase space keeping the physical contents of the theory unchanged. Recently BRST quantization has been used to study some new properties in various theories like string theories \([13–17]\), superstring theories \([18–27]\), \(M\) theory \([28,29]\), Chern-Simons theory \([30–32]\) and ABJM theory \([33–35]\). We indicate how various BRST invariant effective theories are interlinked by considering the finite-field-dependent version of the BRST (FFBRST) transformation, introduced by Joglekar and Mandal \([36]\) about twenty-five years ago. FFBRST transformations are the generalization of the usual BRST transformation where the usual infinitesimal, anti-commuting, global transformation parameter is replaced by a field-dependent but global and anti-commuting parameter. Such generalized transformation protects the nilpotency and retains the symmetry of the gauge fixed effective actions. The remarkable property of such transformations is that they relate the generating functionals corresponding to different effective actions. The non-trivial Jacobian of the path integral measure under such a finite transformation is responsible for all the new results. In virtue of this remarkable property, FFBRST transformations have been investigated extensively and have found many applications in various gauge field theoretic systems \([37–50]\). A similar generalization of the BRST transformation with the same motivation and goal has also been carried out more recently in a slightly different manner \([51]\) where a Jacobian contribution for such transformation is calculated without using any ansatz. Recently FFBRST transformation has been used successfully in some models of the string theory \([52–55]\).

In the present work we consider the Polyakov action in the path integral formulation in the conformal gauge where ghost number current anomaly in curved worldsheet is present. Constructing appropriate finite-field-dependent BRST transformation we connect the generating functional in background covariant (bc),

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in order to calculate the gauge-fixing and ghost part of the action we split the metric $g_{ab}$ into a classical background field $\hat{g}_{ab}$ and a (quantum) metric perturbation $h_{ab}$ as

$$g_{ab} = \hat{g}_{ab} + h_{ab}. \quad (6)$$

We need to fix the gauge condition only for the quantum field $h_{ab}$,

$$\delta h_{ab} = \nabla_a \xi_b + \nabla_b \xi_a + h_{bc} \nabla_a \xi^c + h_{ac} \nabla_b \xi^c + \xi \nabla_c h_{ab}, \quad (7)$$

where the classical field $\hat{g}_{ab}$ is invariant under a general coordinate transformation together with standard tensor transformation rules for the other fields.

The linearized form of the gauge-fixing condition in harmonic gauge is

$$\frac{1}{2} \nabla_a h - \nabla^b h_{ab} = 0, \quad (8)$$

where $h = \hat{g}^{ab} h_{ab}$. The gauge conditions in eq. (8) as well as in eq. (5) are not quantum Weyl covariant. To make them Weyl covariant we need a special gauge-fixing condition. The gauge-fixing condition is given by

$$\hat{g}^{ab} h_{ab} = 0. \quad (9)$$

Now the gauge-fixing and ghost part of the action can be written in BRST-invariant manner as

$$L_{gf} + L_{gh} = \delta B \left[ \bar{C}^a \left( \frac{1}{2} \nabla_a h - \nabla^b h_{ab} \right) + \bar{\tau} \hat{g}^{ab} h_{ab} \right], \quad (10)$$

where $\bar{\tau}$ is Weyl antiquost field.

This action can be further simplified using the technique in ref. [8]. The simplified gauge-fixing and ghost term can be written in BRST-invariant manner as

$$L_{hm} \equiv \lambda (L_{gf} + L_{gh})$$

$$= -i \delta B \left[ C^a \left( \frac{1}{2} \nabla_a h - \nabla^b h_{ab} \right) \right]. \quad (11)$$

The total Lagrangian density in background covariant harmonic gauge is then written in extended form as

$$L_{hm} = \frac{1}{2} \sqrt{-g^{ab} \partial_a X^\mu \partial_b X_\mu} - \sqrt{-\hat{g}^{ab} \hat{\nabla}^b \hat{h}_{ab}}$$

$$+ i \sqrt{-\hat{g}} \bar{\nabla}^b \bar{C}^a \nabla_b C_a - \bar{C}^a \hat{\nabla}_{ab} C^b$$

$$+ (\nabla^b C^a + \nabla^a C^b - \hat{g}^{ab} \nabla_c C^c) h_{bc} \nabla_a C^c$$

$$+ \nabla^b C^a \bar{C}^c \nabla_{ab} C_{cd}$$

$$- h_{ab} \nabla^a C^b (\nabla \cdot C + h_{ij} \nabla^i C^j), \quad (12)$$

where $\hat{g}^{ab}$ is Nakaniishi-Lautrup-type auxiliary field.

**BRST symmetry.** The total action is defined as

$$S_0' = \int d^2 x (\mathcal{L}_0 + L_{gf} + L_{gh}), \quad (13)$$

where $\mathcal{L}_0$ is the kinetic part of the total Lagrangian density. This total action in eqs. (4), (12) is invariant under
the following BRST transformation [8]:

$$\delta_B h_{ab} = i\lambda [\nabla_a C_b + \nabla_b C_a + h_{ab} \nabla_c C^c + h_{ac} \nabla_b C^c - (g^{-1} \cdot C + h_{ij} \nabla^i C^j)(h_{ab} + g_{ab})],$$

$$\delta_B X^a = i\lambda C^c \partial_a X^c, \quad \delta_B C^a = i\lambda \delta^a_{cb} \nabla_b C^c,$$

$$\delta_B \tilde{C}^a = \lambda b^a, \quad \delta_B b^a = 0,$$

(14)

where $\lambda$ is the infinitesimal, anti-commuting, global BRST parameter. One can verify that these transformations are nilpotent.

**Connection between generating functionals in background covariant harmonic and conformal gauges.** – Before going to show the connection between the two effective theories we briefly discuss the ideas of FFBRST developed in ref. [13]. The BRST transformations are generated from the BRST charge using the relation $\delta_B \phi = -[Q, \phi] \lambda$, where $\lambda$ is an infinitesimal anti-commuting global parameter. Following the technique in ref. [13] the anti-commuting BRST parameter $\lambda$ is generalized to be a finite-field-dependent but space-time-independent parameter $\Theta[\phi]$. Since the parameter is finite in nature unlike the usual case, the path integral measure is not invariant under such finite transformation. The Jacobian for these transformations for a certain $\Theta[\phi]$ can be calculated by summing the Jacobian contribution of the two effective theories we briefly discuss the ideas of FFBRST parameter in such a way that Jacobian contribution accounts for the differences of the two FP effective actions.

To express the Jacobian contribution in terms of a local functional of fields, we make an ansatz for $S_{ef}$ by considering successively infinitesimal BRST transformations ($\phi(k) \rightarrow \phi(k + dk)$). The non-trivial Jacobian $J(k)$ is then written as a local functional of the fields and will be replaced as $e^{i\Theta(t);[\phi(k), k]}$ if the condition

$$\int D\phi(k) \left[ \frac{1}{J(k)} \frac{dJ(k)}{dk} - i \frac{dS_1}{dk} \right] e^{i(S_1 + S_{eff})} = 0$$

(15)

holds [13]. Here $\frac{dS_1}{dk}$ is a total derivative of $S_1$ with respect to $k$ in which the dependence on $\phi(k)$ is also differentiated. The change in the Jacobian is calculated as

$$\frac{J(k)}{J(k + dk)} = \Sigma_\phi \frac{\delta \phi(x, k + dk)}{\delta \phi(x, k)} = \frac{1}{J(k)} \frac{dJ(k)}{dk}.$$  

(17)

$\pm$ is for bosonic and fermionic fields, respectively. Equation (16) further can be simplified as

$$\frac{1}{J(k)} \frac{dJ(k)}{dk} = \Sigma_\phi \pm \delta_B \phi \frac{\partial \Theta'}{\partial \phi},$$

(18)

where $\Theta'[\phi]$ is used to construct the finite parameter $\Theta = \int_0^1 \Theta'dk$. In this section, we construct the FFBRST transformation with an appropriate finite parameter to obtain the generating functional corresponding to $\mathcal{L}_{1m}$ from that of corresponding to $\mathcal{L}'_f$. We calculate the Jacobian corresponding to such a FFBRST transformation following the method outlined in ref. [13] and show that it is a local functional of fields. We construct out the FFBRST parameter in such a way that Jacobian contribution accounts for the differences of the two FP effective actions.

Now we start with the generating functional corresponding to the FP effective action in conformal gauge which is written as

$$Z_{cf} = \int D\phi \exp(iS_{ef}[\phi]),$$

(19)

where $S_{ef}$ is given by

$$S_{ef} = \int d^2 x [\mathcal{L}_0 + \mathcal{L}_{cf}].$$

(20)

Now, to obtain the generating functional corresponding $S_{bfm}$, we apply the FFBRST transformation with a finite parameter $\Theta[\phi]$ which is obtained from the infinitesimal but field-dependent parameter, $\Theta'[\phi(k)]$ through $\int_0^1 \Theta'[\phi(k)]dk$, we construct $\Theta'[\phi(k)]$ as

$$\Theta'[C, h] = i \int d^2 x \gamma [\hat{C}^a - \left(\frac{1}{2} \hat{\nabla}_a h - \hat{\nabla}_b h_{ab}\right)] \right].$$

(21)

Here $\gamma$ is an arbitrary constant parameter and all the fields depend on the parameter $k$. The infinitesimal change in the Jacobian corresponding to this FFBRST transformation is calculated using eq. (18)

$$\frac{1}{J(k)} \frac{dJ(k)}{dk} =$$

$$-i \int d^2 x \gamma \left[ \left(\frac{1}{2} \nabla_a h - \nabla_b h_{ab}\right) \right]$$

$$+ \frac{1}{2} \delta(A_a) \hat{C}^a - \frac{1}{2} \hat{\nabla}_a h \delta(C^a) + \hat{\nabla}_b \delta(h_{ab}) \hat{C}^a \].$$

(22)

To express the Jacobian contribution in terms of a local functional of fields, we make an ansatz for $S_1$ by considering all possible terms that could arise from such a transformation as

$$S_1[\phi(k), k] = \int d^2 x \left[ \xi_k \delta(C^a) A_a + \xi_k \delta(C^a) \nabla_a h +$$

$$\xi_k \delta(C^a) \hat{\nabla}_b h_{ab} + \frac{\xi_k}{2} \delta(A_a) \hat{C}^a +$$

$$\frac{\xi_k}{2} \nabla_a h \delta(C^a) + \xi_k \nabla_b \delta(h_{ab}) \hat{C}^a \right].$$

(23)

where all the fields are considered to be $k$ dependent and we have introduced arbitrary $k$-dependent parameters.
\[\xi_n = \xi_n(k) \quad (n = 1, 2, \ldots, 6)\] with initial condition \(\xi_n(k = 0) = 0\). It is straightforward to calculate

\[\frac{dS_1}{dk} = \int d^2x \left[ \frac{\xi_1}{2}(C^a)A_a + \frac{\xi_2}{2}(\bar{C}^a)\bar{A}_a + \frac{\xi_3}{2}\delta(C^a)\bar{A}_a h + \frac{\xi_4}{2}\delta(\bar{C}^a)A_a + \frac{\xi_5}{2}\delta(C^a)\bar{A}_a \delta h(C^a) + \frac{\xi_6}{2}\delta(\bar{C}^a)A_a \delta h(C^a) \right] + \Theta \left( \frac{\xi_1}{2} \right) \delta(A_a) \delta(C^a) + \Theta \left( \frac{\xi_2}{2} \right) \delta(\bar{C}^a) \bar{A}_a \delta h(C^a) + \Theta \left( \frac{\xi_3}{2} \right) \delta(C^a)\bar{A}_a \delta h(C^a) + \Theta \left( \frac{\xi_4}{2} \right) \delta(\bar{C}^a)A_a \delta h(C^a) + \Theta \left( \frac{\xi_5}{2} \right) \delta(C^a)\bar{A}_a \delta h(C^a) + \Theta \left( \frac{\xi_6}{2} \right) \delta(\bar{C}^a)A_a \delta h(C^a) \right], \quad (24)

where \(\xi_n \equiv \frac{d\xi_n}{dk}\). Now to satisfy the condition in eq. (16)

\[\int \frac{d^2x}{\sqrt{-g}} \left\{ 2\xi_1 \delta(C^a)A_a + 2\xi_2 \delta(\bar{C}^a)\bar{A}_a + 2\xi_3 \delta(C^a)\bar{A}_a \delta h(C^a) + 2\xi_4 \delta(\bar{C}^a)A_a \delta h(C^a) + 2\xi_5 \delta(C^a)\bar{A}_a \delta h(C^a) + 2\xi_6 \delta(\bar{C}^a)A_a \delta h(C^a) \right\} = 0. \quad (25)

The terms proportional to \(\Theta'\), which are non-local due to \(\Theta'\), vanish independently if

\[\xi_1 - \xi_4 = 0, \quad \xi_2 - \xi_5 = 0, \quad \xi_3 - \xi_6 = 0. \quad (26)

To make the remaining local terms in Eq. (25) vanish, we need the following conditions:

\[\gamma + \xi_1 = 0, \quad -\gamma + \xi_2 = 0, \quad \gamma + \xi_3 = 0, \quad -\gamma + \xi_4 = 0, \quad \gamma + \xi_5 = 0, \quad -\gamma + \xi_6 = 0. \quad (27)

The differential equations for \(\xi_n(k)\) can be solved with the initial conditions \(\xi_n(0) = 0\) to obtain the solutions

\[\xi_1 = -\gamma k, \quad \xi_2 = \gamma k, \quad \xi_3 = -\gamma k, \quad \xi_4 = -\gamma k, \quad \xi_5 = \gamma k, \quad \xi_6 = -\gamma k. \quad (28)

Putting values of these parameters in the expression of \(S_1\), and choosing the arbitrary parameter \(\gamma = 1\)

Thus, the FFBRST transformation with the finite parameter \(\Theta\) that is defined by eq. (21) changes the generating functional \(Z_{CF}\) as

\[Z_{CF}' = \int D\phi \exp[i(S_{CF}'[\phi]) + S_1[\phi(k), 1]] \quad (29)

which can be written as

\[Z_{CF}' = \int D\phi \exp\left[i \int d^2x \left\{ \frac{1}{2} \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_{\mu} - \frac{1}{2} \hat{g} A_a - \frac{1}{2} \hat{g} \delta h(A_a)C^a + \frac{1}{2} b^a h_{ab} \right\} \right] \quad (30)

After simplification we will get

\[Z_{CF}' = \int D\phi \left[ \int d^2x \left\{ \exp \left[ \frac{1}{2} \hat{g} A_a - \frac{1}{2} \hat{g} \delta h(A_a)C^a + \frac{1}{2} b^a h_{ab} \right] \right] \right] \quad (31)

\[\gamma + \xi_1 = 0, \quad -\gamma + \xi_2 = 0, \quad \gamma + \xi_3 = 0, \quad -\gamma + \xi_4 = 0, \quad \gamma + \xi_5 = 0, \quad -\gamma + \xi_6 = 0. \quad (27)

The differential equations for \(\xi_n(k)\) can be solved with the initial conditions \(\xi_n(0) = 0\) to obtain the solutions

\[\xi_1 = -\gamma k, \quad \xi_2 = \gamma k, \quad \xi_3 = -\gamma k, \quad \xi_4 = -\gamma k, \quad \xi_5 = \gamma k, \quad \xi_6 = -\gamma k. \quad (28)

Putting values of these parameters in the expression of \(S_1\), and choosing the arbitrary parameter \(\gamma = 1\)
or

$$Z_{cf}^{t} = \int D\phi \exp(iS_{hm}^{t}[\phi]) \equiv Z_{hm}^{t}. \quad (33)$$

Here $S_{hm}^{t}$ is defined as

$$S_{hm}^{t} = \int d^{2}x(L_{x} + L_{hm}). \quad (34)$$

In this way FFBRST transformation with the finite-field-dependent parameter in eq. (21) connects the generating functional for the Polyakov action in the conformal gauge to that of in background covariant harmonic gauge, where ghost number anomaly does not appear. Thus, we can avoid the ghost number anomaly by using suitably constructed FFBRST transformation.

**Conclusion.** – In this present work we have shown how the ghost number current anomaly present in conformal gauge in curved worldsheet is removed using field transformation. By constructing an appropriate FFBRST transformation we obtain the generating functional in conformal gauge from that of in harmonic gauge. This provides a convenient way to go from a theory with ghost number anomaly to the theory where there is no ghost number current anomaly. Further, the harmonic gauge which is complicated is directly connected through the constructed field transformation to the conformal gauge theory which is simpler to use. It will be interesting to generalize this formulation for superstring theories.

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