Baseline for the cumulants of net-proton distributions at STAR

Xiaofeng Luo\textsuperscript{a}\textsuperscript{,1}, Bedangadas Mohanty\textsuperscript{b}, Nu Xu\textsuperscript{a}\textsuperscript{,c}

\textsuperscript{a}Institute of Particle Physics and Key Laboratory of Quarks \& Lepton Physics (MOE), Central China Normal University, Wuhan, 430079, China.
\textsuperscript{b}School of Physical Sciences, National Institute of Science Education and Research, Bhubaneswar 751005, India
\textsuperscript{c}Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA.

Abstract

We present a systematic comparison between the recently measured cumulants of the net-proton distributions by STAR for 0-5\% central Au+Au collisions at $\sqrt{s_{\text{NN}}}=7.7-200$ GeV and two kinds of possible baseline measures, the Poisson and Binomial baselines. These baseline measures are assuming that the proton and anti-proton distributions independently follow Poisson statistics or Binomial statistics. The higher order cumulant net-proton data are observed to deviate from all the baseline measures studied at 19.6 and 27 GeV. We also compare the net-proton with net-baryon fluctuations in UrQMD and AMPT model, and convert the net-proton fluctuations to net-baryon fluctuations in AMPT model by using a set of formula.

Keywords: QCD Critical Point, Higher Moments, Net-proton, Heavy-ion Collision, Quantum Chromodynamics

1. Introduction

In the phase diagram of Quantum Chromodynamics (QCD-theory of strong interactions), it is conjectured on the basis of theoretical calculations that there will be a QCD Critical Point (CP) at high temperature ($T$) and non-zero baryonic chemical potential region ($\mu_B$) \cite{1}. Since the ab initio Lattice QCD calculation meet notorious sign problem, there are still large uncertainties in in theoretically determining the location of the CP in the QCD phase diagram \cite{2, 3}. Different QCD based models also give very different results \cite{4}. Finding the existence of CP experimentally will be an excellent test of QCD theory in the non-perturbative region and a milestone of exploring the QCD phase diagram \cite{5}. This is one of the main goals of the Beam Energy Scan (BES) Program at the Relativistic Heavy Ion Collider (RHIC). By tuning the colliding energy of gold nucleus from $\sqrt{s_{\text{NN}}}=200$ GeV down to 7.7 GeV, one can access a board region of the QCD Phase Diagram (20 < $\mu_B$ < 420 MeV) \cite{6}. Due to the high sensitivity to the correlation length ($\xi$) of the dynamical system \cite{7, 8, 9} and direct connection to the susceptibilities in theoretical calculations, for example, the Lattice QCD calculations \cite{5, 10}, higher moments of multiplicity distributions of conserved quantities, such as net-baryon, net-charge and net-strangeness, have been applied to search for the QCD critical point in the heavy-ion collision experiment \cite{11}. As the correlation length will diverge near the CP, the non-monotonic variation of the moments of multiplicity distribution with respected to the colliding energy is the golden signature of the CP. To extract the CP induced experimental signal, it is crucial to understand the non-CP physics effects in heavy-ion collisions, such as the effects of conservations for charges (electric, baryon number and strangeness number), finite size, resonance decay and hadronic scattering, on the experimental observable. On the other hand, proper baseline needs to be constructed for experimental observables to search for the CP.

\textsuperscript{1}xluo@mail.ccnu.edu.cn
2. Results and Discussion

In this paper, we will make the comparison between the baselines (Poisson, Binomial) and recently measured cumulants of net-proton distribution published by the STAR Collaboration and discuss the deviations of the data from the baselines. In addition, the results from AMPT [12] and UrQMD [13] model will be discussed.

1. Poisson: If the protons and anti-protons are independently distributed as Poissonian distributions. Then the net-proton multiplicity follows the Skellam distribution, which is expressed as:

\[ P(N) = \binom{M_p}{N} \binom{M_\bar{p}}{N} \exp[-(M_p + M_\bar{p})] \]

where \( I_N(x) \) is a modified Bessel function, \( M_p \) and \( M_{\bar{p}} \) are the mean protons and anti-protons, as shown in Fig. 1. The various order cumulants \( C_n \) are closely connected with the moments, e.g., \( C_1 = \langle N > = M, C_2 = \langle (\Delta N)^2 \rangle = \sigma^2, C_3 = \langle (\Delta N)^3 \rangle = S \sigma^3, C_4 = \langle (\Delta N)^4 \rangle = -3 < (\Delta N)^2 >^2 = S \sigma^4 \), where the \( \sigma^2 \), \( S \) and \( \kappa \) are variance, skewness and kurtosis, respectively. Then we construct, \( S \sigma = C_3/C_2 = (M_p - M_{\bar{p}})/(M_p + M_{\bar{p}}) \) and \( \kappa \sigma^2 = C_4/C_2 = 1 \), which provides the Poisson expectations for the various order cumulants/moments of net-proton distributions. The only input parameters of the Poisson baseline for cumulants of net-proton distributions are the mean values of the protons and anti-protons.

2. Binomial/Negative Binomial: If the protons and anti-protons are independently distributed as Binomial or Negative Binomial distributions (BD/NBD). Then various order cumulants of the net-proton distributions can be expressed in term of cumulants of the proton and anti-proton distributions: \( C_n^{net-p} = C_n^p + (-1)^n C_n^{\bar{p}} \).

The first four order cumulants of proton/anti-proton can be written as:

\[ C_2 = \sigma_x^2 = \varepsilon_x \mu_x, C_3 = S_x \sigma_x^3 = \varepsilon_x \mu_x(2\varepsilon_x - 1), C_4 = \kappa_x \sigma_x^4 = \varepsilon_x \mu_x(6\varepsilon_x^2 - 6\varepsilon_x + 1) \], where \( \varepsilon_x = \sigma_x^2 / \mu_x, \mu_x = M_x, \kappa_x \) is the mean values of the protons or anti-protons. \( x=p \) or \( \bar{p} \). \( \varepsilon_x > 1 \) means the underlying distributions of proton or anti-proton are

![Figure 1. (Color Online) Efficiency corrected mean net-proton, proton, anti-proton as a function of average number of participant nucleon \(<N_{part}>\) in Au+Au collisions at \( \sqrt{s_{NN}}=7.7-200\,\text{GeV} \). The dashed lines are used to guide eyes.](image)
Negative Binomial distributions, while \( \varepsilon < 1 \) gives Binomial distributions. The input parameters for BD/NBD expectations are the measured mean and variance of the protons and anti-protons.

3. **Independent Production:** If the protons and anti-protons multiplicity distribution are independent, the various order cumulants of the net-proton distributions can be expressed in term of cumulants of the proton and anti-proton distributions: \( C_{n}^{\text{net-p}} = C_{n}^{p} + (-1)^{n} C_{n}^{\bar{p}} \). The \( C_{n}^{p} \) and \( C_{n}^{\bar{p}} \) are the measured cumulants of proton and anti-proton distributions, respectively.

Figure 2 shows the comparison between the cumulants of net-proton distributions and the Poisson, Binomial baselines. The Negative Binomial distribution is ruled out, as the variance over mean ratio less than one for proton and anti-proton distributions:

\[
C_{n}^{\text{net-p}} = 2C_{1}^{\text{net-p}} - 2C_{1}^{\text{tot-p}},
C_{2}^{\text{net-B}} = 4C_{2}^{\text{net-p}} - 2C_{1}^{\text{tot-p}},
C_{3}^{\text{net-B}} = 8C_{3}^{\text{net-p}} - 12(C_{2}^{p} - C_{2}^{\bar{p}}) + 6C_{1}^{\text{net-p}},
C_{4}^{\text{net-B}} = 16C_{4}^{\text{net-p}} + 16C_{3}^{\text{tot-p}} - 64(C_{3}^{p} + C_{3}^{\bar{p}}) + 48C_{2}^{\text{net-p}} + 12C_{2}^{\text{tot-p}} - 26C_{1}^{\text{tot-p}},
\]

where the "tot-p" means proton number plus anti-proton number. In the right side of the Fig. 3 shows the net-baryon \( \kappa \sigma^{2} (C_{4}/C_{2}) \) results converting from the net-protons fluctuations are consistent with the net-baryon results directly calculated from AMPT model, but with large uncertainty.
Figure 3. (Color Online) Energy dependence of $\kappa\sigma^2$ of net-proton and net-baryon distributions for 0-5% Au+Au collisions from UrQMD (left) and AMPT string melting model (right). The results marked as solid back star is theoretical calculations based on Asakawa and Kitazawa’s formula and the error calculation are based on Bootstrap method.

3. Summary

We have compared the energy dependence of cumulants of net-proton distributions in 0-5% central Au+Au collisions with Poisson and Binomial baselines. The Binomial baseline has better description of the data than Poisson baseline, which only use the mean value as parameter. Largest deviation is observed for the fourth order net-proton cumulant ($C_4$) from Poisson and Binomial baselines at 19.6 and 27 GeV. A proper comparison of the experimental measurements to QCD calculations is needed to extract the exact physics process that leads to deviation of the data from the baselines presented. Difference between net-proton and net-baryon fluctuations are observed in the AMPT and UrQMD model without CP physics. The A&K’s formula are used to convert the net-proton fluctuations to net-baryon fluctuations in AMPT model, which are consistent with the net-baryon results calculated from the model.

Acknowledgement

The work was supported in part by the NSFC under grant No. 11205067, 11221504 and 11228513. CCNU-QLPL Innovation Fund(QLPL2011P01) and China Postdoctoral Science Foundation (2012M511237, 2013T60732).

References

[1] K. Rajagopal and F. Wilczek, “At the Frontier of Particle Physics / Handbook of QCD”, volume 3. World Scientific.
[2] Z. Fodor, S. Katz, JHEP 0203, 014 (2002), 0404, 050 (2004); R. V. Gavai, S. Gupta, Phys. Rev. D 71, 114014 (2005); 78, 114503 (2008).
[3] R. V. Gavai, arXiv:1404.6615.
[4] M. A. Stephanov, Int. J. Mod. Phys. A 20, 4387 (2005).
[5] S. Gupta, X. Luo, B. Mohanty, H. G. Ritter, N. Xu, Science 332, 1525 (2011).
[6] M. M. Aggarwal et al. (STAR Collaboration), arXiv: 1007.2613.
[7] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
[8] C. Athanasiou et al., Phys. Rev. D 82, 074008 (2010).
[9] M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).
[10] R. V. Gavai and S. Gupta, Phys. Lett. B 696, 459 (2011).
[11] L. Adamczyk et al., (STAR Collaboration), Phys. Rev. Lett. 105, 022302 (2010); Phys. Rev. Lett. 112, 032302 (2014); arXiv:1402.1558.
[12] Z.W. Lin et al., Phys. Rev. C 72, 064901 (2005).
[13] M. Bleicher et al., J. Phys. G: Nucl. Part. Phys. 25, 1859 (1999).
[14] Y. Hatta and M. A. Stephanov, Phys. Rev. Lett. 91, 102003 (2003).
[15] L. Adamczyk, et al. (STAR Collaboration), Phys. Rev. Lett. 112, 032302 (2014).
[16] X. Luo, J. Xu, B. Mohanty, N. Xu, J. Phys. G 40, 105104 (2013); X. Luo, J. Phys.: Conf. Ser. 316, 012003 (2011).
[17] X. Luo, J. Phys. G: Nucl. Part. Phys. 39, 025008 (2012).
[18] STAR Public Webpage for Data Table: https://drupal.star.bnl.gov/STAR/files/starpublications/205/data.html.
[19] P. K. Netrakanti et al., arXiv: 1405.4617.
[20] M. Kitazawa and M. Asakawa, Phys. Rev. C 85, 021901 (2012); Phys. Rev. C 86, 024904 (2012) [Erratum-ibid. C 86 (2012) 069902].