Circulation in Blowdown Flows

J. I. Katz

MITRE Corp., McLean, Va. 22102

(Dated: October 25, 2018)

Abstract

The blowdown of high pressure gas in a pressure vessel produces rapid adiabatic cooling of the gas remaining in the vessel. The gas near the wall is warmed by conduction from the wall, producing radial temperature and density gradients that affect the flow, the mass efflux rate and the thermodynamic states of both the outflowing and the contained gas. The resulting buoyancy-driven flow circulates gas through the vessel and reduces, but does not eliminate, these gradients. The purpose of this note is to estimate when blowdown cooling is rapid enough that the gas in the pressure vessel is neither isothermal nor isopycnic, though it remains isobaric. I define a dimensionless number, the buoyancy circulation number $BC$, that parametrizes these effects.

PACS numbers: 44.20.+b,47.15.Cb

Keywords: blowdown, buoyancy, circulation, pressure vessel

*Dept. Physics and McDonnell Center for the Space Sciences

Washington University, St. Louis, Mo. 63130; Electronic address: katz@wuphys.wustl.edu
I. INTRODUCTION

The process by which a pressure vessel containing high pressure gas is vented to the outside through a narrow (compared to its diameter) orifice is called blowdown [1]. The rapid reduction in pressure and density within the vessel produces adiabatic cooling of the gas. If cooling is sufficient, gases of high condensation temperature may condense as liquid droplets, leading to a complex multi-phase flow. These droplets tend to fall to the bottom of the vessel, where contact with the warm wall will evaporate them, but if small enough may instead be entrained in the flowing gas. The blowdown rate will then depend on the position of the venting orifice and the nucleation process that controls the droplet size.

In general, the gas inside the vessel may be regarded as isobaric because the blowdown time (for an orifice narrow compared to the vessel size) is much longer than the sound transit time across the vessel. Near the wall the gas is warmed by conduction from the wall. Typically, the vessel itself has a much greater mass and heat content than the contained gas, and may be regarded as a constant temperature heat bath. The thermal diffusion time across the gas is usually much longer than the blowdown time, so the temperature variation within the gas may be significant.

Under isobaric conditions the temperature gradient implies a density gradient. In the presence of a gravitational field this density gradient drives a circulatory flow, with gas acquiring heat in a boundary layer at the wall and gradually mixing with the bulk of the gas. There is some similarity to Ekman pumping, but in the blowdown problem there is an additional imposed time scale, the blowdown time, that has no analogue in Ekman pumping, while Ekman pumping has a characteristic rotational velocity that has no analogue in the blowdown problem.

II. TIME SCALES

Here we estimate the relevant time scales and other parameters. The circulation time $t_{circ}$ is the characteristic time for gas to flow through the boundary layer in a (nominally spherical) vessel of radius $r$ at speed $v$:

$$ t_{circ} \approx \frac{r}{v}. $$  \hspace{1cm} (1)
The boundary layer thickness is given by

\[ \delta \approx \sqrt{Dt_{\text{circ}}}, \]  

(2)

where \( D \) is the thermal diffusivity. The circulatory speed \( v \) in a gravitational field \( g \) (or equivalent acceleration) may be estimated, approximating the flow as freely accelerated (approximately valid if the Prandtl number is of order unity, as it is for dilute gases):

\[ v \approx \sqrt{grAt}, \]  

(3)

where \( At \) is an Atwood’s number:

\[ At \approx \frac{t_{\text{circ}}}{t_{\text{cool}}}, \]  

(4)

where \( t_{\text{cool}} \) is the adiabatic cooling time.

For unchoked flow of an ideal gas with adiabatic exponent \( \gamma \) and sound speed \( c_s \) from a volume \( V \) through an orifice of area \( A \) the adiabatic cooling time

\[ t_{\text{cool}} = \frac{V}{Ac_s(\gamma - 1)}. \]  

(5)

If flow through the boundary layer is fast compared to this time (\( t_{\text{circ}} \ll t_{\text{cool}} \)), as will usually be the case, \( At \) will be small because the gas in the interior of the vessel will only cool by a small fraction of its (absolute) temperature in the time required for gas in the boundary layer to flow through that layer.

Combining the previous results we estimate the circulation time:

\[ t_{\text{circ}} \approx \left( \frac{rt_{\text{cool}}}{g} \right)^{1/3}. \]  

(6)

The time required for the entire volume of contained gas to flow through the boundary layer is

\[ t_{\text{mix}} \approx t_{\text{circ}} \frac{r}{\delta} \approx \frac{r^{7/6}t_{\text{cool}}^{1/6}}{g^{1/6}D^{1/2}}. \]  

(7)
If $t_{\text{mix}} > t_{\text{cool}}$ then the temperature gradients are large and it is not valid to treat the gas as isothermal or isopycnic (constant density).

### III. THE BLOWDOWN CIRCULATION NUMBER

To facilitate dimensional insight, we rewrite the preceding equation:

$$t_{\text{mix}} \approx t_{\text{cool}}^{1/6} t_{\text{char}}^{5/6},$$  \hspace{1cm} (8)

where the characteristic time, a quantity defined only by the properties of the vessel, gas, and gravity, is

$$t_{\text{char}} \equiv \frac{r^{7/5}}{g^{1/5} D^{3/5}}.$$  \hspace{1cm} (9)

This permits defining a (possibly novel) dimensionless parameter, the blowdown circulation number:

$$BC \equiv \frac{r^{7/6}}{g^{1/6} D^{1/2} t_{\text{cool}}^{5/6}} \approx \frac{t_{\text{mix}}}{t_{\text{cool}}} \approx \left(\frac{t_{\text{char}}}{t_{\text{cool}}}\right)^{5/6}.$$  \hspace{1cm} (10)

If $BC \gg 1$ the gas is neither isothermal nor isopycnic and its circulation and density and temperature distributions must be considered in order to calculate the blowdown correctly, even if droplets do not condense. The escaping gas is drawn from near the surface of the pressure vessel, where it is warmed by thermal conduction from the wall, while the temperature in the central regions of the vessel drops nearly adiabatically. Because $t_{\text{mix}}$ is much greater than the actual blowdown time $t_{\text{cool}}(\gamma - 1)$, the gas issuing from the orifice late in the blowdown process will be much colder than its initial temperature. A quantitative calculation would, in general, require a three-dimensional numerical solution of the Navier-Stokes equations with heat flow. In the special case of an orifice at the top or bottom of the pressure vessel the flow would be two-dimensional with axial symmetry.
IV. PARAMETERS OF CIRCULATORY FLOW

Explicitly, the flow parameters $v$, $\delta$ and boundary layer Reynolds number $Re$ are (for Prandtl number of order unity):

\[
v \approx \frac{g^{1/3}r^{2/3}}{t_{cool}^{1/3}}; \quad (11)
\]
\[
\delta \approx \frac{D^{1/2}r^{1/6}t_{cool}^{1/6}}{g^{1/6}}; \quad (12)
\]
\[
Re \approx \frac{g^{1/6}r^{5/6}}{t_{cool}^{1/6}D^{1/2}}. \quad (13)
\]

For $Re > 10^3$–$10^4$ the boundary layer flow will be turbulent and $D$ should be replaced by a turbulent diffusion coefficient. Its value may be such as to reduce $Re$ to the threshold of turbulent breakdown. Such large values of $Re$ are only found for very large pressure vessels.

As a numerical example, consider a spherical pressure vessel with $r = 1$ m with orifice $A = 10^{-3}$ m$^2$ ($10$ cm$^2$) containing air at a pressure of $10^7$ Pa (100 bar) and $20^\circ$C. Then $D = 2.12 \times 10^{-7}$ m$^2$/sec ($0.00212$ cm$^2$/sec), $t_{cool} = 31$ sec, $v = 0.68$ m/sec (68 cm/sec), $\delta = 5.6 \times 10^{-4}$ m (0.056 cm) and $BC = 85$. The time for gas to mix throughout the vessel $t_{mix} = BTC_{cool} = 2600$ sec, much longer than the venting time $V/(Ac_s) = 12$ sec. The temperature and density distributions within the vessel will be strongly inhomogeneous, even though the pressure remains uniform to high accuracy everywhere except in the immediate vicinity of the orifice, and, in general, three-dimensional computational fluid dynamics would be required for a quantitative calculation of the efflux rate and thermodynamic parameters.

[1] This paper concerns pressure vessels that initially contain only gas. Blowdown of vessels that initially contain liquid or solid that flashes into vapor upon pressure release involves different phenomena.