A novel fast SVPWM method based on two-stage transformation for three-level inverter

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Abstract. Compared with the two-level inverter, the three-level inverter has excellent performance in pressure voltage switching loss and harmonic applied to the fields of the speed regulation and new energy generation. However, the computational complexity of the space vector pulse width modulation (SVPWM) method for multilevel inverter restricts the implementation on low cost microprocessor. This paper proposes a novel fast SVPWM method for three-level inverter to convert the space voltage vector in the three-level space to the two-level by vector subtraction with the central vector. The conduction time of each switch can be computed quickly so that the runtime is reduced significantly. Finally, the experimental results verify the effectiveness and practicability of the new space vector modulation algorithm based on two-stage transformation.

1. Introduction

In recent years, three-level inverters have been widely used in the field of middle and high voltage due to their advantages of low switching frequency, less switching loss and smaller output harmonics compared with two-level inverters. At present, the modulation algorithm applicated to three-level inverters mainly contain the following methods: carrier modulation method (SPWM), specific harmonic elimination method (SHEPWM), space voltage vector modulation method (SVPWM) and switching point prediction method. SVPWM, characterized by high voltage utilization and small voltage ripple, is widely used in three-level inverters. There are two common approaches to obtain the on-times. The first approach is to determine the triangle, and then solve three simultaneous equations for this triangle to obtain the on-times as in [1]. Whereas, the second approach is to determine the triangle, and then use the particular on-time equations stored in the lookup for this triangle, as in [2]. However, as the number of level increases, both of these approaches become computationally intensive. In the vector synthesis algorithm based on 60° coordinate system, a new variable is introduced for coordinate transformation, which increases computation, and multiple rules are needed for sector judgment, which is relatively complicated [3]. Literature [4] puts forward an algorithm to expand from two level to three level, and proves the feasibility of transformation through mathematical derivation. However, the transformation is relatively complex, and the transformation relationship that is intuitively suitable for the action time of online transformation vector is not given. Literature [5] analyzes the essential relationship between SVPWM and SPWM, and presents a simple SVPWM modulation strategy, which is only applied in two levels. In this paper, a three-level space vector modulation algorithm based on two-level space vector
modulation is proposed.

2. Three-level T-type inverter with Si-IGBT and SiC-MOSFET device combinations

The topological structure of the three-level T-type inverter composed of Si-IGBT base and SiC-MOSFET is shown in Figure 1 (a). Each phase has 4 power switch devices, and the output voltage of bridge arm has 3 level states. When $S_{a1}$ and $S_{a2}$ are turned on, the output level is $U_{dc}/2$, which is expressed as 1; when $S_{a2}$ and $S_{a3}$ are turned on, the output level is 0, which is state 0; when $S_{a3}$ and $S_{a4}$ are turned on, the output level is $-U_{dc}/2$, which is the state -1. The turn-on and turn-off alternating of the 4 power switches determines the value of the output phase voltage and the switch output state.

![Figure 1 (a) three-level T-type inverter with hybrid devices (b) Space voltage vector diagram](image)

According to above definition, the switching states of the power devices are 1, 0, and -1 respectively. Therefore, the switching symbols $S_1$, $S_2$, and $S_3$ can be introduced to represent the switching states of the three-phase bridge arms of A, B, and C respectively, and the three-phase voltage of the inverter. The relationship between output and switching function is expressed as:

$$
\begin{align*}
U_a &= \frac{1}{2} S_1 U_{dc} \\
U_b &= S_2 U_{dc} \\
U_c &= S_3 U_{dc}
\end{align*}
$$

(1)

Where: $S_1$, $S_2$, $S_3$ can take a value of 1, 0, and -1 respectively, so each phase can output voltage values in three states, and the three-level inverter outputs a total of $3^3=27$ voltage states. Corresponding to 27 different switch states. Therefore, the space voltage vector is represented as:

$$
\hat{U} = \frac{2}{3} U_{dc} \left[ S_1 + S_2 \left( -\frac{1}{2} + \frac{\sqrt{3}}{2} j \right) + S_3 \left( -\frac{1}{2} - \frac{\sqrt{3}}{2} j \right) \right]
$$

(2)

Where: $S_1, S_2, S_3 \in \{1,0,-1\}$.

The 27 switching states were respectively substituted into the space voltage vector expression (2), corresponding to 19 values, which were called the base voltage vector. The spatial distribution diagram of the base voltage vector is shown in Figure 1 (b). When the position of the space voltage vector is given, it can be obtained according to the nearest three vector principle (NTV) with a combination of known similar switch states. Therefore, the SVPWM algorithm needs to solve three key problems: the judgment of the large and small sectors of the space voltage vector and the calculation of the vector action time. The traditional SVPWM algorithm has more trigonometric function calculations, in order to simplify the calculation method and improve the efficiency of calculation simulation. Therefore, a fast SVPWM method based on two-stage transformation for three-level inverter is proposed in this paper to improve the operation speed of the controller and reduce the amount of computation.
3. Fast SVPWM algorithm based on two stage transformation

The traditional SVPWM for three-level inverter has a clear idea and is easy to be accepted by developer. However, it needs to calculate the action time of each base voltage in 24 small triangle areas, and a large amount of data needs to be stored in advance. Moreover, this method involves a large number of trigonometric functions and matrix operations, which requires too much execution time so that control system needs a high-performance processor. In order to solve this problem, this paper proposes a method of converting three-level voltage space vector into two-level voltage space vector, and then applying the two-level voltage space vector to solve the action time of space vector, which will greatly simplify the computation and improve the operation speed.

3.1 First stage: judgment of large sectors

Assuming that the reference voltage vector is located in the first large sector in Figure 1 (b), according to the NTV principle of SVPWM, the nearest $V_X$, $V_Y$ basic vector and zero vector $V_Z$ must be used to synthesize the reference voltage vector. Figure 1 (b) shows the spatial positional relationship between $V_{ref}$ and $V_X$, $V_Y$ and $V_Z$, so $V_{ref}$ can be defined by the basic vector $(1, -1, -1)$ ($V_X$), $(1,0, -1)$ ($V_Y$) and $(0,0 ,0)$ ($V_Z$) synthesis. In a control period $T_s$, the time corresponding to the action of $V_X$, $V_Y$, and $V_Z$ is $T_X$, $T_Y$, and $T_Z$, and formula (3) is satisfied. However, it is inaccurate to determine the basic vector of the composite vector only by the angle, because under the same angle, different amplitudes can correspond to the reference voltage vectors of different sectors. Therefore, when determining the specific position of the reference voltage vector, it is also necessary to know the amplitude of the reference voltage vector, which will greatly increase the complexity of the calculation. As shown in Figure 2, the entire space vector diagram is divided into 6 sub-hexagons, and the 6 sub-hexagons are continuously distributed with an angle difference of 60°. The division of this large sector has a common part between adjacent sectors. If the reference voltage vector is located in the common part, there will be ambiguities. Therefore, the partition method of non-sub-hexagonal large sector shown in dark color in Figure 2 can solve the problem of sector duplication.

$$\begin{align*}
V_X T_X + V_Y T_Y + V_Z T_Z &= V_{ref} T_s \\
T_X + T_Y + T_Z &= T_s
\end{align*}$$

Figure 2 Distribution of sub-hexagons and non-sub-hexagons

A large sector can be divided into 6 large sectors by the angle with the horizontal axis. As shown in Figure 2, the range of the $PS1$ sector is $\left[\frac{\pi}{6}, \frac{\pi}{6}\right]$, and the range of the $PS2$ sector is $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ can also be judged. The other 4 large sectors can be deduced by analogy. Each large sector has a certain angle and area.

3.2 Second stage: central vector selection

In order to improve the quality of the output waveform of the inverter, the three-level inverter generally adopts the NTV method. First determine which large sector the reference vector is located in, and then
determine which small sector it belongs to, select the basic space vector of the synthesized reference vector according to the NTV principle, and calculate the action time of each basic space vector. This method involves a large number of the trigonometric function calculation, which is not conducive to the realization of multi-level inverters. This section will derive in detail the three-level SVPWM conversion to the two-level SVPWM algorithm.

When the given reference voltage vector $V_{\text{ref}}$ is located in the PS1 large sector in Figure 3 (a), the reference voltage vector can be mapped to the two-level sectors. The mapping formula is shown as follows:

$$V' = V_{\text{ref}} - V_{\text{map}} \quad (4)$$

When the reference voltage vector is in the PS1 large sector, the basic vector is 000 (111, -1-1-1), 101 (0-10), 1-10, 1-1-1, 10-1, 110 (00-1), 100 (0-1-1). The PS1 large sector can be regarded as 6 sectors under two levels, 100 (0-1-1) as the $V_{\text{map}}$ pivot vector, $V_{\text{map}}=V_0$. After selecting the central vector, as shown in Figure 3 (b) and (c), 7 vectors similar to two levels form a sub-hexagon.

Figure 3 (a) PS1 large sector (b) A minor sub-hexagon (c) PS1 maps to two-stage sectors

The reference voltage vector $V'_{\text{ref}}$ is shown in Figure 3 (c) and can be synthesized by three basic vectors. The formula (3) based on the volt-second balance principle can be calculated as follows:

$$V_xT_x + V_yT_y + V_zT_z = V_{\text{ref}}T_s$$

$$T_x + T_y + T_z = T_s \quad (5)$$

However, from formula (4), we can derive:

$$(V_x - V_{\text{map}})T_x + (V_y - V_{\text{map}})T_y + (V_z - V_{\text{map}})T_z = (V_{\text{ref}} - V_{\text{map}})T_s \quad (6)$$

So, we can get:

$$V_xT_x + V_yT_y + V_zT_z = V_{\text{ref}}T_s \quad (7)$$
5

\[ U_\alpha > 0 ? \]
\[ U_\beta > 0 ? \]
\[ U_\alpha > U_\beta ? \]
\[ -U_\alpha > -U_\beta ? \]
\[ U_\alpha > U_\beta ? \]
\[ -U_\alpha > -U_\beta ? \]

\[ T_X = U_\alpha - U_\beta \]
\[ T_Y = U_\beta + U_\beta \]
\[ T_X = U_\alpha + U_\beta \]
\[ T_Y = -U_\alpha + U_\beta \]
\[ T_X = -U_\alpha + U_\beta \]
\[ T_Y = -U_\alpha + U_\beta \]

Figure 4 Switch time calculation based on two-stage

It can be seen from formula (7) that if the action time of the three vectors is obtained, according to Figure 3, \( U_{3L} \) in the three-level can be completely converted to \( U_{2L} \) in the two-level, and then the time calculation is shown in Figure 4. For different large sectors in the three levels, there are corresponding different central vectors \( V_{map} \) as shown in Figure 5.

3.3. Calculation of duty cycle time

In the two-level SVPWM algorithm, first determine the position of the reference voltage vector, and then calculate the action time of each composite vector. For the three-level SVPWM algorithm, first determine the large sector where the reference vector is located and map it to the two-level sector, and then complete the same steps according to the two-level SVPWM algorithm to determine the small sector and the action time. But the three-level SVPWM has six pairs of complementary power switches, even if the action time of each vector is determined according to the two-level SVPWM algorithm, it is more complicated than the duty cycle distribution of the two-level SVPWM algorithm. Therefore, in order to simplify the calculation of the duty cycle, this paper introduces a method of calculating duty cycle with relatively small amount of calculation.

Each space vector corresponds to the corresponding switch state. The three states of each phase of each space vector act on the four switches of each phase. Therefore, for the switch \( S_{a1} \), calculating the total time of the state 1 (P), and for the switch \( S_{a2} \), calculating the total time of the state 1 (P) and state 0 (O). Since the action time of switch \( S_{a1} \) and switch \( S_{a3} \) is added to \( T_S \) of one cycle, and the action time of switch \( S_{a2} \) and switch \( S_{a4} \) is added to \( T_S \) of one cycle, the action time of switch \( S_{a3} \) and \( S_{a4} \) can be obtained according to \( S_{a1} \) and \( S_{a2} \).
The position of the space voltage vector is shown in Figure 3, \( T_X \) is the action time of \( V'_1 \), \( T_Y \) is the action time of \( V'_2 \), and \( T_Z \) is the action time of \( V'_4 \). \( T_S \) is a control period. According to the derived duty cycle calculation method, the duty cycle of switches \( S_{a1} \) and \( S_{a2} \) that are mapped from the three-level \( PS1 \) large sector to the two-level sector can be calculated. The duty cycle of switch \( S_{a3} \) and switch \( S_{a4} \) can be reversed. As shown in Table 1.

| switch A, B, C three-phase | \( T_{Sa1} \) | \( T_{Sa2} \) | \( T_{Sa3} \) | \( T_{Sa4} \) |
|--------------------------|-------------|-------------|-------------|-------------|
| A                        | \( T_S-T_Z/2 \) | \( T_S \)   | \( T_Z/2 \) | 0           |
| B                        | 0           | \( T_X+T_Z/2 \) | \( T_S \)   | \( T_S-T_X-T_Z/2 \) |
| C                        | 0           | \( T_Z/2 \)  | \( T_S \)   | \( T_S-T_Z/2 \)  |

The flow chart of proposed fast SVPWM based on two-stage transformation is given as Figure 6. \( V_{map} \) is the central vector, \( U_{3L} \) is the reference voltage vector in the three-level, \( U_{2L} \) is the reference voltage vector converted from the three-level to the two-level, this vector can be controlled in Figure 6 (c) partial modulation. An implementation flowchart of the proposed algorithm with low cost processor is shown in Table 2.

**Figure 6** The flow chart of proposed fast SVPWM based on two-stage transformation

**Table 2** A implementation flowchart of the proposed algorithm with low cost processor

| Operating procedure | |
|---------------------|----------------|
| Step 1              | Input two values of Alpha and Beta in three level dq coordinate: dq. 3. Level(Alpha,Beta) |
| Step 2              | The three-phase voltage is converted to Alpha and Beta by the Parker transformation: \[ A.v, B.v, C.v \] = Park transform(Alpha,Beta) |
| Step 3              | The position of the large sector is determined by the space voltage vector: \[ MainSector=MainSector[Alpha,Beta,C.v] \] |
| Step 4              | The central vector is selected by the position of the large sector: \[ MainSector.Alpha, MainSector.Beta \] = MapVector[Alpha,Beta,Map.Alpha,Map.Beta] |
| Step 5              | Vector subtraction is performed to transform it into the two-level sector: \[ Carrier1, Carrier2, Carrier3 \] = dq.2. Level[MainSector.Alpha, MainSector.Beta] |
| Step 6              | Calculate the action time of each space vector in two levels: \[ Ts1, Ts2, Tsb1, Tsb2, Tsc1, Tsc2 \] = DutyAssign(Carrier1, Carrier2, Carrier3) |
4. Simulation results
The fast SVPWM method for three-level inverter based on two-stage transformation is mainly realized by s-function module and connecting with the hardware circuit in matlab/simulink. The modulation waveforms of the three phases A, B, and C are shown in Figure 7, the phase A voltage waveform is shown in Figure 8 (a), the line voltage waveform $U_{AB}$ is shown in Figure 8 (b), and three-phase voltage output waveform is shown in Figure 9 as a sinusoidal waveform, common mode. It can be seen from the simulation results that the quality of the output waveform is good. The simulation results show that the output waveform is of good quality, and the simulation results verify the correctness and effectiveness of the three-level inverter fast SVPWM method based on two-level transformation.
5. Experimental setup and results

In order to verify the effectiveness of the proposed algorithm in this paper, a T-type three-level inverter control platform is established, as shown in Figure 10. MCU is adopted as single-chip microcomputer STM32F407 and sampling frequency $T_s=50\,kHz$. Experimental parameters: DC voltage is set as 600 V, DC capacitor is 4700 μF, the values of resistance of three-phase symmetrical load can be set as $R = 10\,\Omega$, $L = 3\,mH$.

![Experimental setup](image)

Figure 10 Experimental platform for T-type three-level inverter with low-cost processor

![Waveforms](image)

Figure 11. Measured waveforms of Three-Level (T-type) Bridged-legs with Si- and SiC-MOSFET when $V_{dc}=600\,V$, $P_o=10\,kW$ and $f_s=50kHz$ ((a) oscilloscope measurements: grid voltage $v_{ab}$ and output current $i_o$, (b) oscilloscope measurements: output current $i_o$, (c) the output current THD and harmonics spectrum).

The experimental waveform is shown in Figure 11. The output waveform quality of PWM signal, voltage and current is good. As shown in Table 3 above, compared with the method in the 60° coordinate system, the implementation time of this method is shorter and the efficiency is higher. Obviously, compared with the traditional algorithm, the execution time of this algorithm is significantly shortened. This is mainly due to the elimination of coordinate transformation and complex triangular expressions in the proposed method for the calculation of duty ratio. It can be seen that the test waveform is of good quality, which can verify the correctness of the algorithm in this paper.

| Modulation method                        | Different load | Control partial execution time (μs) | THD (current) |
|------------------------------------------|----------------|------------------------------------|---------------|
| Traditional SVPWM in the 60° coordinate system | no-load        | 2.8                                | 3.66%         |
|                                          | 50% load       | 2.9                                | 2.89%         |
|                                          | Full load      | 2.7                                | 2.12%         |
| Proposed SVPWM                           | no-load        | 2.7                                | 3.64%         |
|                                          | 50% load       | 2.8                                | 2.84%         |
|                                          | Full load      | 2.6                                | 2.02%         |
6. Conclusion
A fast SVPWM method of three-level inverter based on two-stage transformation is proposed. Since the space vector action time can be calculated under two levels, the calculation amount will not increase with the increase of the level, and the operation speed of the controller will not be greatly affected by the level series. The large sector and the central vector are determined by spatial position in the three-level. The central vector is used as a bridge to map the voltage space vector from the three level to the two level. At two levels, the action time of each voltage space vector is calculated and PWM driving signal is output. The simulation results and experimental results both prove that this method is feasible. The model is simple and easy to understand, which also has strong universality, small calculation amount and fast running speed. Moreover, this method can be extended to multi-level inverters through derivation.

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