Integrating cardinal direction relations and other orientation relations in Qualitative Spatial Reasoning* **

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THE WORK IS ABOUT COMBINING, AND HANDLING THE INTERACTION OF, EXISTING QUALITATIVE SPATIAL LANGUAGES. THE WEAKNESS OF A QUALITATIVE (SPATIAL) LANGUAGE IS THAT IT CAN MAKE ONLY A FINITE NUMBER OF KNOWN-IN-ADVANCE DISTINCTIONS. THE STRENGTH IS THAT, (1) SUCH A LANGUAGE IS COGNITIVELY ADEQUATE, IN THAT THE DISTINCTIONS IT CAN MAKE CORRESPOND TO THOSE THAT A SPECIFIC CLASS OF APPLICATIONS DO NEED, AND (2) REASONING ABOUT KNOWLEDGE EXPRESSED IN THE LANGUAGE COSTS MUCH LESS THAN REASONING ABOUT KNOWLEDGE EXPRESSED IN THE (QUALITATIVELY) ABSTRACTED QUANTITATIVE LANGUAGE. YET, MANY NOWADAYS APPLICATIONS, WHILE HAVING NO NEED OF THE WHOLE EXPRESSIVENESS OF A PURELY QUANTITATIVE LANGUAGE, ARE NOT FULLY SATISFIED BY JUST ONE SINGLE QUALITATIVE LANGUAGE. THE SOLUTION PROPOSED BY THE WORK IS THE USE OF THE INTEGRATION OF EXISTING QUALITATIVE LANGUAGES, SO THAT THE INTEGRATED QUALITATIVE LANGUAGES COMPENSATE EACH OTHER’S DEFICIENCIES. THE MOST IMPORTANT ISSUE IS TO PROVIDE EFFECTIVE PROCEDURES FOR THE HANDLING OF THE INTERACTION BETWEEN THE INTEGRATED QUALITATIVE LANGUAGES. THE PAPER PROVIDES SUCH A PROCEDURE, AND SHOWS THAT IT TERMINATES AND DETECTS INCONSISTENCIES WHICH CANNOT BE DETECTED BY REASONING SEPARATELY ON EACH OF THE PROJECTIONS OF THE KNOWLEDGE ONTO THE INTEGRATED QUALITATIVE LANGUAGES. ONE POTENTIAL, HIGHLY 21ST-CENTURY, APPLICATION OF THE WORK IS THE PROCESSING OF MULTI-SATELLITE KNOWLEDGE, EACH SATELLITE SENDING ITS CONTRIBUTION TO THE GLOBAL KNOWLEDGE IN ITS OWN LANGUAGE. THE IMPORTANCE OF PROCESSING SUCH HETEROGENOUS, MULTI-SOURCE KNOWLEDGE IS EASILY SEEN BY IMAGINING WHAT COULD HAVE BEEN THE SAVING IN HUMAN LIFES IN THE STILL-ONGOING DISASTROUS WAR IN IRAQ, HAD THE INVOLVED, “HIGHLY” OBJECTIVE, INTELLIGENCE AGENCIES OBJECTIVELY GONE THROUGH THE PROCESSING OF THE INTERACTION BETWEEN THE DIFFERENT PIECES OF THE MULTI-SOURCE GATHERED KNOWLEDGE ON WMDs (Weapons of Mass Destruction).

This work was partly supported by the EU project “Cognitive Vision systems” (CogVis), under grant CogVis IST 2000-29375.

A preliminary version of this work has appeared in the Proceedings of the Eighth International Symposium of Artificial Intelligence and Mathematics [1].
THE VERY FIRST VERSION OF THE WORK, WHICH I THOUGHT, AND STILL THINK, WAS A HUGE NOVELTY TO GIS\textsuperscript{1}, I SUBMITTED IT, IN 2001, TO A CONFERENCE WHOSE MAIN TOPIC IS GIS, THE COSIT CONFERENCE.\textsuperscript{2}

- The actual paper is the version of the work exactly as rejected at the special volume of the journal of the Annals of Mathematics and Artificial Intelligence, AMAI, dedicated to the 2004 symposium on Artificial Intelligence and Mathematics, AIM’2004.

- The reviews of the AMAI journal are added after the references, for potential people interested in objectivity of journals’ reviewing processes.

- The reviews of the very first version of the work, as rejected at the COSIT’2001 conference, are also added after the references, for potential people interested in objectivity of conferences’ reviewing processes.

**Abstract.** Integrating different knowledge representation languages is one of the main topics in Qualitative Spatial Reasoning (QSR). Existing languages are generally incomparable in terms of expressive power; as such, their integration compensates each other’s representational deficiencies, and is seen by real applications, such as Geographic Information Systems (GIS), or robot navigation, as an answer to the well-known poverty conjecture of qualitative languages in general, and of QSR languages in particular. Knowledge expressed in such an integrating language decomposes then into parts, or components, each expressed in one of the integrated languages. Reasoning internally within each component of such knowledge involves only the language the component is expressed in, which is not new. The challenging question is to come with methods for the interaction of the different components of such knowledge. With these considerations in mind, we propose a calculus, \( c\text{COA} \), integrating two calculi well-known in QSR: Frank’s projection-based cardinal direction calculus, \( CDA \), and a coarser version, \( ROA \), of Freksa’s relative orientation calculus. An original constraint propagation procedure, \( PcS4c+() \), for \( c\text{COA} \)- CSPs is presented, which aims at (1) achieving path consistency (\( Pc \)) for the \( CDA \) projection; (2) achieving strong 4-consistency (\( S4c \)) for the \( ROA \) projection; and (3) more (+) —the “+” consists of the implementation of the interaction between the two integrated calculi. Dealing with the first two points is not new, and involves mainly the \( CDA \) composition table and the \( ROA \) composition table, which can be found in, or derived from, the literature. The originality of the propagation algorithm comes from the last point. Two tables, one for each of the two directions \( CDA\text{-to-}ROA \) and \( ROA\text{-to-}CDA \), capturing the interaction between the two kinds of knowledge, are defined, and used by the algorithm. The importance of taking into account the interaction is shown with a real example providing an inconsistent knowledge base, whose inconsistency (a) cannot be detected by reasoning separately about each of the two components of the knowledge, just because, taken separately, each is consistent, but (b) is detected by the proposed algorithm, thanks to the interaction knowledge propagated from each of the two components to the other.

**Key words:** Qualitative spatial reasoning, Cardinal directions, Relative orientation, GIS, Constraint satisfaction, Path consistency, Strong 4-consistency.

\textsuperscript{1} Geographic Information Systems.

\textsuperscript{2} COSIT is an (international) COnference on Spatial Information Theory.
1 Introduction

Reasoning about orientation has been, for more than a decade now, one of the main aspects focussed on in Qualitative Spatial Reasoning (QSR). A possible explanation stems from the large number of real applications in need for a qualitative formalism for representing and reasoning about orientation; among these, we have Geographic Information Systems (GIS), and robot navigation. The reader is referred to [4] for a survey article on the different representation techniques, and the different aspects dealt with, in QSR.

Two important, and widely known, calculi for the representation and processing of orientation are the projection-based calculus of cardinal directions, $\text{CDA}$, in [8], and the relative orientation calculus in [9]. The former uses a global, west-east/south-north reference frame, and represents knowledge as binary relations on (pairs of) 2D points. The latter allows for the representation of relative knowledge as ternary relations on (triples of) 2D points. Both kinds of knowledge are of particular importance, especially in large-scale GIS for the former, and in robot navigation for the latter. An example on high-level satellite-like surveillance of a geographic area will illustrate that the integration of the two calculi is much better suited for large-scale GIS reasoning, than $\text{CDA}$ alone.

Research in constraint-based QSR has reached a point where the need for combining, on the one hand, different kinds of existing relations, such as, in the present work, binary relative orientation relations based on a global frame of reference (pseudo ternary relations) [8] and (purely) ternary relative orientation relations [9], and, on the other hand, different levels of local consistency, such as, also in the present work, path consistency and strong 4-consistency, is necessary in order to face the increasing and often challenging demand coming from real applications.

The aim of this work is to look at the importance of integrating the two orientation calculi mentioned above. Considered separately, the projection-based calculus in [8], $\text{CDA}$, represents knowledge such as “Hamburg is north-west of Berlin”, whereas the relative orientation calculus in [9] represents knowledge such as “You see the main train station on your left when you walk down to the cinema from the university”. We propose a calculus, $\text{cCOA}$, integrating $\text{CDA}$ and a coarser version, $\text{ROA}$, of the calculus in [9]. $\text{cCOA}$ allows for more expressiveness than each of the integrated calculi, and represents, within the same base, knowledge such as the one in the following example.

Example 1 Consider the following knowledge on four cities, Berlin, Hamburg, London and Paris:

1. viewed from Hamburg, Berlin is to the left of Paris, Paris is to the left of London, and Berlin is to the left of London;
2. viewed from London, Berlin is to the left of Paris;
3. Hamburg is to the north of Paris, and north-west of Berlin; and
4. Paris is to the south of London.

The first two sentences express the $\text{ROA}$ component of the knowledge (relative orientation relations on triples of the four cities), whereas the other two express the $\text{CDA}$ component of the knowledge.
Fig. 1. A model for the ROA component (left), and a model for the CDA component (right), of the knowledge in Example 1.

**component of the knowledge (cardinal direction relations on pairs of the four cities)**. Considered separately, each of the two components is consistent, in the sense that one can find an assignment of physical locations to the cities that satisfies all the constraints of the component — see the illustration in Figure 1. However, considered globally, the knowledge is clearly inconsistent (from “viewed from Hamburg, Paris is to the left of London”, we infer that Hamburg, London and Paris are not collinear -they form a triangle-, whereas from the conjunction “Hamburg is to the north of Paris” and “Paris is to the south of London”, we infer that Hamburg, London and Paris are collinear). Example 1 clearly shows that reasoning about combined knowledge consisting of an ROA component and a CDA component, e.g., checking its consistency, does not reduce to a matter of reasoning about each component separately — reasoning separately about each component in the case of Example 1 shows two components that are both consistent, whereas the conjunction of the knowledge in the two components is inconsistent. As a consequence, the interaction between the two kinds of knowledge has to be handled. With this in mind, we propose a constraint propagation procedure, \(PC_{S4c+}\), for \(CA\)-CSPs, which aims at:

1. achieving path consistency \((PC)\) for the CDA projection;
2. achieving strong 4-consistency \((S4c)\) for the ROA projection; and
3. more (+).

The procedure does more than just achieving path consistency for the CDA projection, and strong 4-consistency for the ROA projection. It implements as well the interaction between the two integrated calculi. For this purpose:

1. The procedure makes use, on the one hand, of an augmented composition table of the CDA calculus:
   a. the table records, for each pair \((r, s)\) of CDA atoms, the standard composition, \(r \circ s\), of \(r\) and \(s\), which is not new, and can be found in the literature [8,18];

3. Two cardinal direction calculi, to be explained later, are known in the literature [8]: a cone-shaped and a projection-based (see illustration in Figure 2). We assume in this example the latter.
(b) more importantly, the table records the \textit{CDA}-to-ROA interaction, by providing, for each pair \((r, s)\) of CDA atoms, the most specific ROA relation, \(r \otimes s\), such that, for all \(x, y, z\), the conjunction \(r(x, y) \land s(y, z)\) logically implies \((r \otimes s)(x, y, z)\).

2. On the other hand, the procedure makes use of a table for the \(\text{ROA}-\text{to-CDA}\) interaction, providing, for each ROA atom \(t\), the CDA constraints it imposes on the different pairs of its three arguments.

The procedure is, to the best of our knowledge, original.

The rest of the paper is organised as follows. Section 2 provides some background on constraint satisfaction problems (CSPs), on constraint matrices and on relational algebras. Section 3 presents a quick overview of the cardinal direction calculi in [8], and of the relative orientation calculus in [9]. Section 4 defines a relative orientation calculus, ROA, which is a coarser version of the one in [9]. Reasoning in the integrating language of CDA relations and ROA relations is dealt with in detail in Section 5; in particular, the section presents the CDA-to-ROA and the ROA-to-CDA interaction tables, as well as the constraint propagation algorithm \(PcS^4c+()\), both alluded to above. Section 6 provides a short discussion relating the work to current research on spatio-temporalising the well-known \(\text{ALC}(D)\) family of description logics (DLs) with a concrete domain [2]: the discussion shows that if two (spatial) ontologies operate on the same universe of objects (in this work, the universe of 2D points), while using different languages for their knowledge representation, then integrating the two ontologies needs an inference mechanism for the interaction of the two languages, so that, given knowledge expressed in the integrating ontology, consisting of two components (one for each of the integrated ontologies), each of the two components can infer knowledge from the other. Section 7 summarises the work.

2 Constraint satisfaction problems

A constraint satisfaction problem (CSP) of order \(n\) consists of:

1. a finite set of \(n\) variables, \(x_1, \ldots, x_n\);
2. a set \(U\) (called the universe of the problem); and
3. a set of constraints on values from \(U\) which may be assigned to the variables.

An \(m\)-ary constraint is of the form \(R(x_{i1}, \ldots, x_{im})\), and asserts that the \(m\)-tuple of values assigned to the variables \(x_{i1}, \ldots, x_{im}\) must lie in the \(m\)-ary relation \(R\) (an \(m\)-ary relation over the universe \(U\) is any subset of \(U^m\)). An \(m\)-ary CSP is one of which the constraints are \(m\)-ary constraints. We will be concerned exclusively with binary CSPs and ternary CSPs.

For any two binary relations \(R\) and \(S\), \(R \cap S\) is the intersection of \(R\) and \(S\), \(R \cup S\) is the union of \(R\) and \(S\), \(R \circ S\) is the composition of \(R\) and \(S\), and \(R^{-}\) is the converse of \(R\); these are defined as follows:

\[
\begin{align*}
R \cap S &= \{(a, b) : (a, b) \in R \text{ and } (a, b) \in S\}, \\
R \cup S &= \{(a, b) : (a, b) \in R \text{ or } (a, b) \in S\}, \\
R \circ S &= \{(a, b) : \text{for some } c, (a, c) \in R \text{ and } (c, b) \in S\}, \\
R^{-} &= \{(a, b) : (b, a) \in R\}.
\end{align*}
\]
A ternary CSP

Three special binary relations over a universe $U$ are the empty relation $\emptyset$ which contains no pairs at all, the identity relation $I_U = \{(a, a) : a \in U\}$, and the universal relation $\top_U = U \times U$.

Composition and converse for binary relations were introduced by De Morgan [5,6]. In [15], the authors extended the two operations to ternary relations; furthermore, they introduced for ternary relations the operation of rotation, which is not needed for binary relations. For any two ternary relations $R$ and $S$, $R \cap S$ is the intersection of $R$ and $S$, $R \cup S$ is the union of $R$ and $S$, $R \circ S$ is the composition of $R$ and $S$, $R^\sim$ is the converse of $R$, and $R^\triangle$ is the rotation of $R$; these are defined as follows:

$$R \cap S = \{(a, b, c) : (a, b, c) \in R \text{ and } (a, b, c) \in S\},$$

$$R \cup S = \{(a, b, c) : (a, b, c) \in R \text{ or } (a, b, c) \in S\},$$

$$R \circ S = \{(a, b, c) : \text{for some } d, (a, b, d) \in R \text{ and } (a, d, c) \in S\},$$

$$R^\sim = \{(a, b, c) : (a, c, b) \in R\},$$

$$R^\triangle = \{(a, b, c) : (c, a, b) \in R\}.$$

Three special ternary relations over a universe $U$ are the empty relation $\emptyset$ which contains no triples at all, the identity relation $I_U = \{(a, a, a) : a \in U\}$, and the universal relation $\top_U = U \times U \times U$.

### 2.1 Constraint matrices

A binary constraint matrix of order $n$ over $U$ is an $n \times n$-matrix, say $B$, of binary relations over $U$ verifying the following:

$$\forall i \leq n \forall i \leq n (B_{ii} \subseteq T_U^B)$$ (the diagonal property),

$$\forall i, j \leq n (B_{ij} = (B_{ji})^\sim)$$ (the converse property).

A binary CSP $P$ of order $n$ over a universe $U$ can be associated with the following binary constraint matrix, denoted $B^P$:

1. Initialise all entries to the universal relation: $(\forall i, j \leq n)((B^P)_{ij} \leftarrow \top_U^B)$
2. Initialise the diagonal elements to the identity relation:
   $(\forall i \leq n)((B^P)_{ii} \leftarrow T_U^B)$
3. For all pairs $(x_i, x_j)$ of variables on which a constraint $(x_i, x_j) \in R$ is specified:
   $(B^P)_{ij} \leftarrow (B^P)_{ij} \cap R, (B^P)_{ji} \leftarrow ((B^P)_{ji})^\sim$.

A ternary constraint matrix of order $n$ over $U$ is an $n \times n \times n$-matrix, say $T$, of ternary relations over $U$ verifying the following:

$$\forall i \leq n \forall i \leq n (T_{iii} \subseteq T^T_U)$$ (the identity property),

$$\forall i, j, k \leq n (T_{ijk} = (T_{ikj})^\sim)$$ (the converse property),

$$\forall i, j, k \leq n (T_{ijk} = (T_{kij})^\triangle)$$ (the rotation property).

A ternary CSP $P$ of order $n$ over a universe $U$ can be associated with the following ternary constraint matrix, denoted $T^P$:

1. Initialise all entries to the universal relation:
   $(\forall i, j, k \leq n)((T^P)_{ijk} \leftarrow \top_U^T)$

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2. Initialise the diagonal elements to the identity relation:
\[(\forall i \leq n)((T^P)_{ii} \leftarrow I^P_i)\]
3. For all triples \((x_i, x_j, x_k)\) of variables on which a constraint \((x_i, x_j, x_k) \in R\) is specified:
\[
\begin{align*}
(T^P)_{ijk} &\leftarrow (T^P)_{ijk} \cap R, (T^P)_{ikj} \leftarrow ((T^P)_{ijk})^\sim, \\
(T^P)_{jki} &\leftarrow ((T^P)_{ijk})^\sim, (T^P)_{jik} \leftarrow ((T^P)_{jki})^\sim, \\
(T^P)_{kij} &\leftarrow ((T^P)_{jki})^\sim, (T^P)_{kji} \leftarrow ((T^P)_{kij})^\sim.
\end{align*}
\]
We make the assumption that, unless explicitly specified otherwise, a CSP is given as a constraint matrix.

2.2 Strong \(k\)-consistency, refinement
Let \(P\) be a CSP of order \(n\), \(V\) its set of variables and \(U\) its universe. An instantiation of \(P\) is any \(n\)-tuple \((a_1, a_2, \ldots, a_n)\) of \(U^n\), representing an assignment of a value to each variable. A consistent instantiation is an instantiation \((a_1, a_2, \ldots, a_n)\) which is a solution:

- If \(P\) is a binary CSP: \((\forall i, j \leq n)(a_i, a_j) \in (B^P)_{ij}\)
- If \(P\) is a ternary CSP: \((\forall i, j, k \leq n)(a_i, a_j, a_k) \in (T^P)_{ijk}\)

\(P\) is consistent if it has at least one solution; it is inconsistent otherwise. The consistency problem of \(P\) is the problem of verifying whether \(P\) is consistent.

Let \(V' = \{x_{i_1}, \ldots, x_{i_k}\}\) be a subset of \(V\). The sub-CSP of \(P\) generated by \(V'\), denoted \(P|_{V'}\), is the CSP with \(V'\) as the set of variables, and whose constraint matrix is obtained by projecting the constraint matrix of \(P\) onto \(V'\):

- If \(P\) is a binary CSP then: \((\forall k, l \leq j)((B^P|_{V'})_{kl} = (B^P)_{ii})\)
- If \(P\) is a ternary CSP then: \((\forall k, l, m \leq j)((T^P|_{V'})_{klm} = (T^P)_{ii})\)

\(P\) is \(k\)-consistent \([10,11]\) if for any subset \(V'\) of \(V\) containing \(k - 1\) variables, and for any variable \(X \in V\), every solution to \(P|_{V'}\) can be extended to a solution to \(P|_{V'\cup\{X\}}\). 
\(P\) is strongly \(k\)-consistent if it is \(j\)-consistent, for all \(j \leq k\).

1-consistency, 2-consistency and 3-consistency correspond to node-consistency, arc-consistency and path-consistency, respectively \([20,21]\). Strong \(n\)-consistency of \(P\) corresponds to what is called global consistency in \([7]\). Global consistency facilitates the important task of searching for a solution, which can be done, when the property is met, without backtracking \([11]\).

A refinement of \(P\) is a CSP \(P'\) with the same set of variables, and such that

- \((\forall i, j)((B^{P'})_{ij} \subseteq (B^P)_{ij})\), in the case of binary CSPs.
- \((\forall i, j, k)((T^{P'})_{ijk} \subseteq (T^P)_{ijk})\), in the case of ternary CSPs.

2.3 Relation algebras
The reader is referred to \([23,16]\) for the definition of a binary Relation Algebra (RA), and to \([15]\) for the definition of a ternary RA. Of particular interest to this work are:
1. binary RAs of the form \( \langle A, \oplus, \circ, -, \perp, \circlearrowleft, \circlearrowright, \I \rangle \), where \( A \) is a non empty finite set, and \( \circ \) and \( \circlearrowleft \) are the operations of composition and converse, respectively; and

2. ternary RAs of the form \( \langle A, \oplus, \circ, -, \perp, \circlearrowleft, \circlearrowright, \lrcorner, \I \rangle \), where \( A \) is a non empty finite set, and \( \circ, \circlearrowleft \) and \( \lrcorner \) are the operations of composition, converse and rotation, respectively.

3 Existing orientation calculi

Some background on existing orientation calculi is in order.

3.1 The cardinal direction calculi in [8]

The models of cardinal directions in 2D developed in [8] are illustrated in Figure 2. They use a partition of the plane into regions determined by lines passing through a reference object, say \( S \). Depending on the region a point \( P \) belongs to, we have \( \text{No}(P, S), \text{NE}(P, S), \text{Ea}(P, S), \text{SE}(P, S), \text{So}(P, S), \text{SW}(P, S), \text{We}(P, S), \text{NW}(P, S), \text{Eq}(P, S) \), corresponding, respectively, to the position of \( P \) relative to \( S \) being north, north-east, east, south-east, south, south-west, west, north-west, or equal. Each of the two models can thus be seen as a binary RA, with nine atoms. Both use a global, west-east/south-north, reference frame. We focus our attention on the projection-based model (Figure 2(right)), which has been assessed as being cognitively more adequate [8] (cognitive adequacy of spatial orientation models is discussed in [9]).

3.2 The relative orientation calculus in [9]

A well-known model of relative orientation of 2D points is the calculus in [9]. It is derived from a specific partition, into 15 regions, of the plane, determined by a parent object, say \( A \), and a reference object, say \( B \) (Figure 3(d)). The partition is based on the following:

Fig. 2. The cone-shaped (left) and projection-based (right) models of cardinal directions in [8].
Fig. 3. The partition of the universe of 2D positions on which is based the relative orientation calculus in [9].

1. the left/straight/right partition of the plane determined by an observer placed at the parent object and looking in the direction of the reference object (Figure 3(a));
2. the front/neutral/back partition of the plane determined by the same observer (Figure 3(b)); and
3. the similar front/neutral/back partition of the plane obtained when we swap the roles of the parent object and the reference object (Figure 3(c)).

Combining the three partitions (a), (b) and (c) of Figure 3 leads to the partition of the universe of 2D positions on which is based the calculus in [9] (Figure 3(d)).

4 A new relative orientation calculus

The projection-based model of cardinal directions in [8] uses a global, west-east/south-north, reference frame; its use and importance in GIS are well-known. The calculus in [9] is more suited for the description of a configuration of 2D points (a spatial scene) relative to one another. Integrating the two kinds of relations would lead to more expressiveness than allowed by each of the integrated calculi, so that one would then be able to represent, within the same base, knowledge such as the one in the 4-sentence example provided in the introduction.

The coarser relative orientation calculus can be obtained from the one in [9] by ignoring, in the construction of the partition of the plane determined by a parent object and a reference object (Figure 3(d)), the two front/neutral/back partitions (Figure 3(b-c)). In other words, we consider only the left/straight/right partition (Figure 3(a)) — we also keep the 5-element partitioning of the line joining the parent object to the reference object. The final situation is depicted in Figure 4 where A and B are the parent object and the reference object, respectively:

1. Figure 4(b-c) depicts the general case, corresponding to the parent object and the reference object being distinct from each other: this general-case partition leads
to 7 regions (Figure 4(c)), numbered from 2 to 8, corresponding to 7 of the nine atoms of the calculus, which we refer to as \( lr \) (to the left of the reference object), \( bp \) (behind the parent object), \( cp \) (coincides with the parent object), \( bw \) (between the parent object and the reference object), \( cr \) (coincides with the reference object), \( br \) (behind the reference object), and \( rr \) (to the right of the reference object).

2. Figure 4(a) illustrates the degenerate case, corresponding to equality of the parent object and the reference object. The two regions, corresponding, respectively, to the primary object coinciding with the parent object and the reference object, and to the primary object distinct from the parent object and the reference object, are numbered 0 and 1. The corresponding atoms of the calculus will be referred to as \( de \) (degenerate equal) and \( dd \) (degenerate distinct).

![Fig. 4. The partition of the universe of 2D positions on which is based the ROA calculus.](image)

From now on, we refer to the calculus in [8] as \( CDA \) (Cardinal Direction Algebra), and to the coarser version of the calculus in [9] as \( ROA \) (Relative Orientation Algebra). A \( CDA \) (resp. \( ROA \)) relation is any subset of the set of all \( CDA \) (resp. \( ROA \)) atoms. A \( CDA \) (resp. \( ROA \)) relation is said to be atomic if it contains one single atom (a singleton set); it is said to be the \( CDA \) (resp. \( ROA \)) universal relation if it contains all the \( CDA \) (resp. \( ROA \)) atoms. When no confusion raises, we may omit the brackets in the representation of an atomic relation.

5 **Reasoning about combined knowledge of \( CDA \) relations and \( ROA \) relations**

We start now the main part of the paper, i.e., the representation of knowledge about 2D points as a combined conjunction of:

1. \( CDA \) relations on (pairs of) the objects, on the one hand; and
2. \( ROA \) relations on (triples of) the objects, on the other hand.

More importantly, we deal with the issue of reasoning about such a combined knowledge. We first present for each of the integrated calculi, \( CDA \) and \( ROA \):
1. tables recording the internal reasoning: the tables of converse and composition for \( CDA \), which can be found in the literature \cite{13}, and the tables of converse, rotation and composition for \( \text{ROA} \), which can be derived from the work in \cite{15}; and

2. a table for the interaction with the other calculus: a \( \text{CDA-to-ROA} \) interaction table, recording the \( \text{ROA} \) knowledge inferred from \( \text{CDA} \) knowledge; and an \( \text{ROA-to-CDA} \) interaction table, recording the \( \text{CDA} \) knowledge inferred from \( \text{ROA} \) knowledge.

We then give a quick presentation of what is already known in the literature: CSPs of \( \text{CDA} \) relations \cite{13}, and the way to solve them \cite{13}. Then come the definition of CSPs of \( \text{ROA} \) relations, and a discussion on how to adapt a known propagation algorithm \cite{15} to such CSPs. We finish the section with the presentation of CSPs combining both kinds of knowledge (CSPs of \( \text{CDA} \) relations and \( \text{ROA} \) relations on 2D points): most importantly, this last part will present in detail the propagation algorithm \( \text{PcS}4c+/ \) we have already alluded to.

### 5.1 Reasoning within \( \text{CDA} \) and the \( \text{CDA-to-ROA} \) interaction: the tables

The table in Figure 5 presents the augmented \( \text{CDA} \) composition table; for each pair \((r_1, r_2)\) of \( \text{CDA} \) atoms, the table provides:

1. the standard composition, \( r_1 \circ r_2 \), of \( r_1 \) and \( r_2 \) \cite{13}; and
2. the most specific \( \text{ROA} \) relation \( r_1 \otimes r_2 \) such that, for all 2D points \( x, y, z \), the conjunction \( r_1(x, y) \land r_2(y, z) \) logically implies \( (r_1 \otimes r_2)(x, y, z) \).

| \( \text{ROA} \) | \( \text{CDA} \) | \( \text{ROA} \) | \( \text{CDA} \) | \( \text{ROA} \) | \( \text{CDA} \) | \( \text{CDA} \) |
|---|---|---|---|---|---|---|
| No | NW | We | SW | SE | EA | NE |
| NW | SE | No | NW | SE | EA | NE |
| We | SW | SE | No | NW | SE | EA |
| SW | SE | No | NW | SE | EA | NE |
| SE | EA | No | NW | SE | EA | NE |
| EA | NE | No | NW | SE | EA | NE |

**Fig. 5.** The augmented composition table of the projection-based cardinal direction calculus in \cite{13}: for each pair \((r_1, r_2)\) of \( \text{CDA} \) atoms, the table provides the composition, \( r_1 \circ r_2 \), of \( r_1 \) and \( r_2 \), as well as the most specific \( \text{ROA} \) relation \( r_1 \otimes r_2 \) such that, for all 2D points \( x, y, z \), the conjunction \( r_1(x, y) \land r_2(y, z) \) logically implies \( (r_1 \otimes r_2)(x, y, z) \). The question mark symbol ? represents the \( \text{CDA} \) universal relation \{No, NW, We, SW, So, SE, Ea, NE, Eq\}. 

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The operation $\circ$ is just the normal composition: it is internal to $\mathcal{CD}A$, in the sense that it takes as input two $\mathcal{CD}A$ atoms, and outputs a $\mathcal{CD}A$ relation. The operation $\otimes$, however, is not internal to $\mathcal{CD}A$, in the sense that it takes as input two $\mathcal{CD}A$ atoms, but outputs an $\mathcal{ROA}$ relation; $\otimes$ captures the interaction between $\mathcal{CD}A$ knowledge and $\mathcal{ROA}$ knowledge, in the direction $\mathcal{CD}A$-to-$\mathcal{ROA}$, by inferring $\mathcal{ROA}$ knowledge from given $\mathcal{CD}A$ knowledge. As an example for the new operation $\otimes$, from

$$SE(Berlin, London) \land No(London, Paris),$$

saying that Berlin is south-east of London, and that London is north of Paris, we infer the $\mathcal{ROA}$ relation $l_{r}$ on the triple $(Berlin, London, Paris)$:

$$l_{r}(Berlin, London, Paris),$$

saying that, viewed from Berlin, Paris is to the left of London. As another example, from

$$No(Paris, Rome) \land So(Rome, London),$$

the most specific $\mathcal{ROA}$ relation we can infer on the triple $(Paris, Rome, London)$ is $\{bp, cp, bw\}$:

$$\{bp, cp, bw\}(Paris, Rome, London).$$

The reader is referred to [8, 18] for the $\mathcal{CD}A$ converse table, providing the converse $r^{-}$ for each $\mathcal{CD}A$ atom $r$.

### 5.2 Reasoning within $\mathcal{ROA}$ and the $\mathcal{ROA}$-to-$\mathcal{CD}A$ interaction: the tables

Figure 8 provides for each of the $\mathcal{ROA}$ atoms, say $t$, the converse $t^{-}$ and the rotation $t^{\circ}$ of $t$. Figure 9 provides the $\mathcal{ROA}$ composition tables, which are computed in the following way. Given four 2D points $x, y, z, w$ and two $\mathcal{ROA}$ atoms $t_{1}$ and $t_{2}$, the conjunction $t_{1}(x, y, z) \land t_{2}(x, z, w)$ is inconsistent if the most specific relation $b_{1}(x, z)$, one can infer from $t_{1}(x, y, z)$ on the pair $(x, z)$, is different from the most specific relation $b_{2}(x, z)$, one can infer from $t_{2}(x, z, w)$ on the same pair $(x, z)$. The $\mathcal{ROA}$ composition splits therefore into two composition tables, one for each of the following two cases:

1. Case 1: $x = z$ (i.e., each of $b_{1}$ and $b_{2}$ is the relation $=$). This corresponds to $t_{1} \in \{de, cp\}$ and $t_{2} \in \{de, dd\}$.
2. Case 2: $x \neq z$ (i.e., each of $b_{1}$ and $b_{2}$ is the relation $\neq$). This corresponds to $t_{1} \in \{dd, lr, bp, cp, bw, br, rr\}$ and $t_{2} \in \{lr, bp, cp, bw, cr, br, rr\}$.

The $\mathcal{CD}A$ knowledge one can infer from $\mathcal{ROA}$ relations is presented in the table of Figure 8 which makes use of the following two functions, $L_{r}$ and $R_{r}$:

$$L_{r}(r) = \begin{cases} 
{SE, Ea, NE} & \text{if } r = So, \\
{SE, Ea, NE, No, NW} & \text{if } r = SE, \\
{NE, No, NW} & \text{if } r = Ea, \\
{NE, No, NW, We, SW} & \text{if } r = NE, \\
{NW, We, SW} & \text{if } r = No, \\
{NW, We, SW, So, SE} & \text{if } r = NW, \\
{SW, So, SE} & \text{if } r = We, \\
{SW, So, SE, Ea, NE} & \text{if } r = SW.
\end{cases}$$

$$R_{r}(r) = \begin{cases} 
{NW, We, SW} & \text{if } r = So, \\
{NW, We, SW, So, SE} & \text{if } r = SE, \\
{SW, So, SE} & \text{if } r = Ea, \\
{SW, So, SE, Ea, NE} & \text{if } r = NE, \\
{SE, Ea, NE} & \text{if } r = No, \\
{SE, Ea, NE, No, SW} & \text{if } r = NW, \\
{NE, No, NW} & \text{if } r = We, \\
{NE, No, NW, We, SW} & \text{if } r = SW.
\end{cases}$$

4 A similar way of splitting the composition into more than one table has been followed for the ternary RA, $\mathcal{CYC}$, presented in [15].
The function $\mathcal{L}r$ (Left inferred relation) provides for its argument, say $r$ (a CDA atom), the most specific CDA relation $R$ such that for all $x, y, z$, the conjunction $r(x, y) \land \mathcal{L}r(x, y, z)$ logically implies $R(x, z)$. For instance, if $r$ is $\mathcal{S}o$ then $R = \mathcal{L}r(\mathcal{S}o) = \{\mathcal{S}e, \mathcal{E}a, \mathcal{N}E\} —$ from $\mathcal{S}o(\mathcal{P}aris, \mathcal{L}ondon)$ and $\mathcal{L}r(\mathcal{P}aris, \mathcal{L}ondon, \mathcal{M}adrid)$, we get $\{\mathcal{S}e, \mathcal{E}a, \mathcal{N}E\}(\mathcal{P}aris, \mathcal{M}adrid)$. As another example, if $r$ is $\mathcal{S}e$ then $R = \mathcal{L}r(\mathcal{S}e) = \{\mathcal{S}e, \mathcal{E}a, \mathcal{N}E, \mathcal{N}o, \mathcal{N}W\} —$ see the illustration of Figure 8 from $\mathcal{S}e(\mathcal{B}erl\in, \mathcal{H}amburg)$ and $\mathcal{L}r(\mathcal{B}erl\in, \mathcal{H}amburg, \mathcal{P}aris)$, we get $\{\mathcal{S}e, \mathcal{E}a, \mathcal{N}E, \mathcal{N}o, \mathcal{N}W\}(\mathcal{B}erl\in, \mathcal{P}aris)$. The function $\mathcal{R}r$ (Right inferred relation) is defined in a similar way, with $\mathcal{L}r$ replaced with $\mathcal{R}r$.

Given a $c\mathcal{C}DA$-CSP $P$, the table in Figure 8 illustrates how the $\mathcal{R}OA$ constraint $(T^P)_{ijk}$ on the triple $(X_i, X_j, X_k)$ of variables interacts with each of the three CDA constraints $(B^P)_{ij}, (B^P)_{ik}$ and $(B^P)_{jk}$ on the pairs $(X_i, X_j), (X_i, X_k)$ and $(X_j, X_k)$. If $(T^P)_{ijk}$ is an atomic relation, say $r$, then the interaction is given by the three functions roa-to-cda12, roa-to-cda13 and roa-to-cda23 of Figure 8 namely:

1. $(B^P)_{ij} \leftarrow \text{roa-to-cda12}(r, P, i, j, k)$;
2. $(B^P)_{ji} \leftarrow (B^P)_{ij}^-;
3. (B^P)_{ik} \leftarrow \text{roa-to-cda13}(r, P, i, j, k);
4. (B^P)_{ki} \leftarrow (B^P)_{ik}^-;
5. (B^P)_{jk} \leftarrow \text{roa-to-cda23}(r, P, i, j, k);
6. (B^P)_{kj} \leftarrow (B^P)_{jk}^-;
achieve path consistency (hence strong 3-consistency) for CDA. A simple adaptation of Allen’s constraint propagation algorithm [1] can be used to solving a CDA, which indicates a trivial inconsistency, then a CDA. If we make the assumption that a CDA matrix of P is a scenario of a CDA.

We define a CDA-CSP as a CSP of which the constraints are CDA relations on pairs of the variables. The universe of a CDA-CSP is the set $\mathbb{R}^2$ of 2D points.

A CDA-matrix of order $n$ is a binary constraint matrix of order $n$ of which the entries are CDA relations. The constraint matrix associated with a CDA-CSP is a CDA-matrix.

A scenario of a CDA-CSP is a refinement $P'$ such that all entries of the constraint matrix of $P'$ are atomic relations.

If we make the assumption that a CDA-CSP does not include the empty constraint, which indicates a trivial inconsistency, then a CDA-CSP is strongly 2-consistent.

### 5.3 CSPs of cardinal direction relations on 2D points

We define a CDA-CSP as a CSP of which the constraints are CDA relations on pairs of the variables. The universe of a CDA-CSP is the set $\mathbb{R}^2$ of 2D points.

A CDA-matrix of order $n$ is a binary constraint matrix of order $n$ of which the entries are CDA relations. The constraint matrix associated with a CDA-CSP is a CDA-matrix.

A scenario of a CDA-CSP is a refinement $P'$ such that all entries of the constraint matrix of $P'$ are atomic relations.

If we make the assumption that a CDA-CSP does not include the empty constraint, which indicates a trivial inconsistency, then a CDA-CSP is strongly 2-consistent.

#### Solving a CDA-CSP

A simple adaptation of Allen’s constraint propagation algorithm [1] can be used to achieve path consistency (hence strong 3-consistency) for CDA-CSPs. Applied to a
Fig. 9. From “Berlin is south-east of Hamburg” and “viewed from Berlin, Paris is to the left of Hamburg”, we infer that “Berlin is south-east, east, north-east, north, or north-west of, Paris”.

$\mathcal{CDA}$-CSP $P$, such an adaptation would repeat the following steps until either stability is reached or the empty relation is detected (indicating inconsistency):

1. Consider a triple $(X_i, X_j, X_k)$ of variables verifying $(\mathcal{B}^P)_{ij} \not\subseteq (\mathcal{B}^P)_{ik} \circ (\mathcal{B}^P)_{kj}$

2. $(\mathcal{B}^P)_{ij} \leftarrow (\mathcal{B}^P)_{ij} \cap (\mathcal{B}^P)_{ik} \circ (\mathcal{B}^P)_{kj}$

3. If $((\mathcal{B}^P)_{ij} = \emptyset)$ then exit (the CSP is inconsistent).

Path consistency is complete for atomic $\mathcal{CDA}$-CSPs [18]. Given this, Ladkin and Reinefeld’s solution search algorithm [17] can be used to search for a solution, if any, or otherwise report inconsistency, of a general $\mathcal{CDA}$-CSP.

5.4 CSPs on relative orientation of 2D points

We define an $\mathcal{ROA}$-CSP as a CSP of which the constraints are $\mathcal{ROA}$ relations on triples of the variables. The universe of an $\mathcal{ROA}$-CSP is the set $\mathbb{R}^2$ of 2D points.

An $\mathcal{ROA}$-matrix of order $n$ is a ternary constraint matrix of order $n$ of which the entries are $\mathcal{ROA}$ relations. The constraint matrix associated with an $\mathcal{ROA}$-CSP is an $\mathcal{ROA}$-matrix.

A scenario of an $\mathcal{ROA}$-CSP is a refinement $P'$ such that all entries of the constraint matrix of $P'$ are atomic relations.

If we make the assumption that an $\mathcal{ROA}$-CSP does not include the empty constraint, which indicates a trivial inconsistency, then an $\mathcal{ROA}$-CSP is strongly 3-consistent.

Searching for a strongly 4-consistent scenario of an $\mathcal{ROA}$-CSP

A simple adaptation of the constraint propagation algorithm in [15] can be used to achieve strong 4-consistency for $\mathcal{ROA}$-CSPs. Applied to an $\mathcal{ROA}$-CSP $P$, such an
adaptation would repeat the following steps until either stability is reached or the empty relation is detected (indicating inconsistency):

1. Consider a quadruple \((X_i, X_j, X_k, X_l)\) of variables verifying \((T^P)_{ijl} \not\subseteq (T^P)_{ijk} \circ (T^P)_{ikl}\)
2. \((T^P)_{ijl} \leftarrow (T^P)_{ijl} \cap (T^P)_{ijk} \circ (B^P)_{ikl}\)
3. If \(((T^P)_{ijl} = \emptyset)\) then exit (the CSP is inconsistent).

In [15], the authors have proposed a complete solution search algorithm for CSPs expressed in their CYC calculus. The algorithm is similar to the one in [17] for temporal interval networks, except that:

1. it refines the relation on a triple of variables at each node of the search tree, instead of the relation on a pair of variables; and
2. it makes use of a constraint propagation procedure achieving strong 4-consistency, in the preprocessing step and as the filtering method during the search, instead of a procedure achieving path consistency.

Unless we can prove that the strong 4-consistency procedure in [15] is complete for the ROA atomic relations, we cannot claim completeness of the solution search procedure for general ROA-CSPs. But we can still use the procedure to search for a strongly 4-consistent scenario of the input CSP. For more details on the algorithm, and on its binary counterpart, the reader is referred to [15,17].

5.5 CSPs of cardinal direction relations and relative orientation relations on 2D points

We define a cCOA-CSP as a CSP of which the constraints consist of a conjunction of CDA relations on pairs of the variables, and ROA relations on triples of the variables. The universe of a cCOA-CSP is the set \(\mathbb{R}^2\) of 2D points.

Matrix representation of a cCOA-CSP

A cCOA-CSP \(P\) can, in an obvious way, be represented as two constraint matrices:

1. a binary constraint matrix, \(B^P\), representing the CDA part of \(P\), i.e., the subconjunction consisting of CDA relations on pairs of the variables; and
2. a ternary constraint matrix, \(T^P\), representing the ROA part of \(P\), i.e., the rest of the conjunction, consisting of ROA relations on triples of the variables.

We refer to the representation as \((B^P, T^P)\). The \(B^P\) entry \((B^P)_{ij}\) consists of the CDA relation on the pair \((X_i, X_j)\) of variables. Similarly, the \(T^P\) entry \((T^P)_{ijk}\) consists of the ROA relation on the triple \((X_i, X_j, X_k)\) of variables.
A constraint propagation procedure for $c\text{COA}$-CSPs

A path consistency algorithm, such as the one in [1], applied to a binary CSP such as a $\text{CODA}$-CSP, uses a queue $\text{Queue}$, which can be supposed, for simplicity, to have been initialised to all pairs $(x, y)$ of the CSP variables verifying $x \leq y$ (the variables are supposed to be ordered). The algorithm removes one pair of variables from $\text{Queue}$ at a time; a removed pair is used to eventually update the relations on the neighbouring pairs of variables (pairs sharing at least one variable). Whenever such a pair is successfully updated, it is entered into $\text{Queue}$, if it is not already there, in order to be considered at a future stage for propagation. The algorithm terminates if the empty relation, indicating inconsistency, is detected, or if $\text{Queue}$ becomes empty, indicating that a fixed point has been reached and the input CSP is made path consistent.

A strong 4-consistency algorithm, such as the one in [15], applied to a ternary CSP such as an $\text{ROA}$-CSP, is, somehow, an adaptation to ternary relations of a path consistency algorithm. It uses a queue $\text{Queue}$, which can be supposed, for simplicity, to have been initialised to all triples $(x, y, z)$ of the CSP variables such that $x \leq y \leq z$. The algorithm removes one triple from $\text{Queue}$ at a time; a removed triple is used to eventually update the relations on the neighbouring triples (sharing at least two variables). Whenever such a triple is successfully updated, it is entered into $\text{Queue}$, if it is not already there, in order to be considered at a future stage for propagation. The algorithm terminates if the empty relation, indicating inconsistency, is detected, or if $\text{Queue}$ becomes empty, indicating that a fixed point has been reached and the input CSP is made strongly 4-consistent.

In Figure 10, we propose a constraint propagation procedure, $\text{PcS4c+()}$, for $c\text{COA}$-CSPs, which aims at:

1. achieving path consistency ($\text{Pc}$) for the $\text{CODA}$ projection, using, for instance, the algorithm in [1];
2. achieving strong 4-consistency ($\text{S4c}$) for the $\text{ROA}$ projection, using, for instance, the algorithm in [15]; and
3. more (+).

The procedure does more than just achieving path consistency for the $\text{CODA}$ projection, and strong 4-consistency for the $\text{ROA}$ projection. It implements as well the interaction between the two combined calculi; namely:

1. The path consistency operation, $(\text{BP})_{ik} \leftarrow (\text{BP})_{ik} \cap (\text{BP})_{ij} \circ (\text{BP})_{jk}$, which, under normal circumstances, operates internally, within a same CSP, should now be, and is, augmented so that it can send information from the $\text{CODA}$ component into the $\text{ROA}$ component; this is achieved by a call to the procedure $\text{pair-propagation()}$. Specifically, whenever a pair $(X_i, X_j)$ of variables is taken from $\text{Queue}$ for propagation, the following is performed for all variables $X_k$:
   - the procedure $\text{pair-propagation()}$ of Figure 10 checks whether the relation on the pair $(X_i, X_k)$ —see lines 14— or the relation on the pair $(X_k, X_j)$ —see lines 9— can be successfully updated. If this happens, the corresponding pairs of variables are entered into $\text{Queue}$ in order to be considered for propagation at a later point of the process. This part of the propagation is not new, and is widely known in the literature on propagation algorithms, such as
path consistency (see [1] for the case of constraint-based qualitative temporal reasoning). What is new in the procedure pair-propagation() is the call to the procedure CDA-to-ROA() —see lines [b] and [c]— which aims at checking, whenever a pair \((X_i, X_j)\) is taken from Queue, whether the CDA relation on \((X_i, X_j)\) can update the ROA relation on the triple \((X_i, X_j, X_k)\) or that on the triple \((X_k, X_i, X_j)\). If either of the two ROA relations gets successfully updated, the corresponding triple of variables is entered into Queue in order to be considered for propagation at a later point of the process. The procedure CDA-to-ROA() is the implementation of the CDA-to-ROA interaction operation, \(\otimes\), defined in the table of Figure 5 which outputs the ROA relation, \(r \otimes s\), logically implied by the conjunction of two CDA atoms, \(r\) and \(s\).

2. The strong 4-consistency operation, \((T^{P})_{ijk} \leftarrow (T^{P})_{ijk} \cap (T^{P})_{ikl} \circ (T^{P})_{ikl}\), which also operates internally under normal circumstances, is augmented so that it can send information from the ROA component into the CDA component: this is achieved by a call to the procedure triple-propagation(). Specifically, whenever a triple \((X_i, X_j, X_k)\) is taken from Queue for propagation, the following is performed for all variables \(X_m\):

- the procedure triple-propagation() of Figure 10 checks whether the relation on the triple \((X_i, X_j, X_m)\) —see lines 14— or the relation on the triple \((X_i, X_k, X_m)\) —see lines 8— or the relation on the triple \((X_j, X_k, X_m)\) —see lines 9— can be successfully updated. If this happens, the corresponding triples of variables are entered into Queue in order to be considered for propagation at a later point of the process. This part of the propagation is taken from the strong 4-consistency algorithm in [15]. What is new in the procedure triple-propagation() is the call to the procedure ROA-to-CDA() —see line 13— which aims at checking, whenever a triple \((X_i, X_j, X_k)\) is taken from Queue, whether the ROA relation on \((X_i, X_j, X_k)\) can update the CDA relations on the different pairs of the three arguments: the pairs \((X_i, X_j)\), \((X_i, X_k)\) and \((X_j, X_k)\). If any of the three CDA relations gets successfully updated, the corresponding pair of variables is entered into Queue in order to be considered for propagation at a later point of the process. The procedure ROA-to-CDA() is the implementation of the ROA-to-CDA interaction table of Figure 5.

**Theorem 1** The constraint propagation procedure PcS4c+() runs into completion in \(O(n^4)\) time, where \(n\) is the number of variables of the input cCOA-CSP.

**Proof.** The number of variable pairs is \(O(n^2)\), whereas the number of variable triples is \(O(n^3)\). A pair as well as a triple may be placed in Queue at most a constant number of times (9 for a pair, which is the total number of CDA atoms; and also 9 for a triple, which is the total number of ROA atoms). Every time a pair or a triple is removed from Queue for propagation, the procedure performs \(O(n)\) operations.

**Example 2** Consider again the description of Example 7. We can represent the situation as a cCOA-CSP with variables \(X_h\), \(X_k\), \(X_l\) and \(X_p\), standing for the cities of Berlin, Hamburg, London and Paris, respectively.

1. The knowledge "viewed from Hamburg, Berlin is to the left of Paris" translates into the ROA constraint \(lr(X_h, X_p, X_b)\): \((T^P)_{hpb} = \{lr\}\).
Input: the matrix representation \( \langle B^P, T^P \rangle \) of a \( cCOA \)-CSP \( P \) with set of variables \( V \).

Output: the CSP \( P \) made strongly 4-consistent.

\begin{verbatim}
procedure PcS4c+();
1. initialise Queue: Queue ← \{\( (x, y) \in V^2 : x \leq y \) \} \cup \{\( (x, y, z) \in V^3 : x \leq y \leq z \)\};
2. repeat{
3. get (and remove) next element \( Q \) from Queue;
4. if \( Q \) is a pair, say \( (X_i, X_j)\)\{\n5. for \( k ← 1 \) to \( n \)\{pair-propagation\( P, i, j, k; \)\} \}
6. \}
7. else \( Q \) is a triple, say \( (X_i, X_j, X_k)\)\{\n8. for \( m ← 1 \) to \( n \)\{triple-propagation\( P, i, j, k, m; \)\} \} \}
9. \}
10. until Queue is empty:

procedure pair-propagation\( P, i, j, k; \);
1. \( Temp ← (B^P)_{ik} \land (B^P)_{ij} \lor (B^P)_{jk}; \)
2. if \( Temp = \emptyset \) then exit (the CSP is inconsistent);
3. if \( Temp ≠ (B^P)_{ik} \)
4. \{add-to-queue\( (X_i, X_k); (B^P)_{ik} ← Temp; \( B^P)_{ik} ← Temp \}; \}
5. \( cDA-to-ROA(P, i, j, k); \)
6. \( Temp ← (B^P)_{kj} \land (B^P)_{kj} \lor (B^P)_{kj}; \)
7. if \( Temp = \emptyset \) then exit (the CSP is inconsistent);
8. if \( Temp ≠ (B^P)_{kj} \)
9. \{add-to-queue\( (X_j, X_k); (B^P)_{kj} ← Temp; \( B^P)_{kj} ← Temp \}; \}
10. \( cDA-to-ROA(P, k, i, j); \)

procedure triple-propagation\( P, i, j, k, m; \);
1. \( Temp ← (T^P)_{ijm} \land (T^P)_{ikm} \lor (T^P)_{ikm}; \)
2. if \( Temp = \emptyset \) then exit (the CSP is inconsistent);
3. if \( Temp ≠ (T^P)_{ijm} \)
4. \{add-to-queue\( (X_i, X_j, X_m); update(P, i, j, m, Temp); \}
5. \( Temp ← (T^P)_{ikm} \land (T^P)_{ikm} \lor (T^P)_{ikm}; \)
6. if \( Temp = \emptyset \) then exit (the CSP is inconsistent);
7. if \( Temp ≠ (T^P)_{ikm} \)
8. \{add-to-queue\( (X_j, X_k, X_m); update(P, i, j, m, Temp); \}
9. \( Temp ← (T^P)_{ijk} \land (T^P)_{ijk} \lor (T^P)_{ijk}; \)
10. if \( Temp = \emptyset \) then exit (the CSP is inconsistent);
11. if \( Temp ≠ (T^P)_{ijk} \)
12. \{add-to-queue\( (X_j, X_k, X_m); update(P, i, j, k, m, Temp); \}
13. \( ROA-to-cDA(P, i, j, k); \)

procedure update\( P, i, j, k, T; \);
1. \( (T^P)_{ijk} ← (T^P)_{ijk} \lor (T^P)_{ijk} \lor (T^P)_{ijk}; \)
2. \( (T^P)_{ijk} ← ((T^P)_{ijk})^\neg; (T^P)_{ijk} ← ((T^P)_{ijk})^\neg; (T^P)_{ijk} ← ((T^P)_{ijk})^\neg; \)
\end{verbatim}

Fig. 10. A constraint propagation procedure, \( Pcs4c+() \), for \( cCOA \)-CSPs. The procedures \( cDA-to-ROA \) and \( ROA-to-cDA \) used by the algorithm are defined in Figure 34.
procedure CDA-to-ROA(A, i, j, k);
1. roa-ir ← \( r_1 \cap r_2 \);
2. Temp ← \((T^p)^{i,j,k} \cap \text{roa-ir}\);
3. if Temp = \(\emptyset\) then exit (the CSP is inconsistent);
4. if Temp \(\neq (T^p)^{i,j,k}\) then exit (the CSP is inconsistent);
5. \{add-to-queue(X_i, X_j, X_k); update(A, i, j, k, Temp);\}

procedure ROA-to-CDA(A, i, j, k);
1. Temp ← \(\bigcup_{r \in R} \text{roa-to-cda12}(r, P, i, j, k)\);
2. if Temp = \(\emptyset\) then exit (the CSP is inconsistent);
3. if Temp \(\neq (T^p)^{i,j,k}\) then exit (the CSP is inconsistent);
4. \{add-to-queue(X_i, X_j); (B^p)_{ij} ← Temp; (B^p)_{ij} \leftarrow Temp^-; \}
5. Temp ← \(\bigcup_{r \in R} \text{roa-to-cda13}(r, P, i, j, k)\);
6. if Temp = \(\emptyset\) then exit (the CSP is inconsistent);
7. if Temp \(\neq (T^p)^{i,j,k}\) then exit (the CSP is inconsistent);
8. \{add-to-queue(X_i, X_k); (B^p)_{ik} ← Temp; (B^p)_{ik} \leftarrow Temp^-; \}
9. Temp ← \(\bigcup_{r \in R} \text{roa-to-cda23}(r, P, i, j, k)\);
10. if Temp = \(\emptyset\) then exit (the CSP is inconsistent);
11. if Temp \(\neq (T^p)^{i,j,k}\) then exit (the CSP is inconsistent);
12. \{add-to-queue(X_j, X_k); (B^p)_{jk} ← Temp; (B^p)_{jk} \leftarrow Temp^-; \}

Fig. 11. The procedures CDA-to-ROA and ROA-to-CDA used by the constraint propagation algorithm PcS4c+() of Figure 10.

2. The other ROA knowledge translates as follows: \((T^p)_{hp} = \{lr\}, (T^p)_{hb} = \{lr\}, (T^p)_{pb} = \{lr\}.
3. The CDA part of the knowledge translates as follows: \((B^p)_{hp} = \{No\}, (B^p)_{hb} = \{NW\}, (B^p)_{pl} = \{So\}.

As discussed in Example 4, reasoning separately about the two components of the knowledge shows two consistent components, whereas the combined knowledge is clearly inconsistent. Using the procedure PcS4c+, we can detect the inconsistency in the following way. From the CDA constraints \((B^p)_{hp} = \{No\} and (B^p)_{pl} = \{So\}, the algorithm infers, using the augmented CDA composition table of Figure 2—specifically, the CDA-to-ROA interaction \(\otimes\)—the ROA relation \{bp, cp, bw\} on the triple \((X_h, X_p, X_l)\). The conjunction of the inferred knowledge \{bp, cp, bw\}(X_h, X_p, X_l) and the already existing knowledge \{lr\}(X_h, X_l) gives the empty relation, indicating the inconsistency of the knowledge.

6 Discussion

Current research shows clearly the importance of developing spatial RAs: specialising an ALC(D)-like Description Logic (DL) [2], so that the roles are temporal immediate-successor (accessibility) relations, and the concrete domain is generated by a decidable spatial RA in the style of the well-known Region-Connection Calculus RCC-8 [22], leads to a computationally well-behaving family of languages for spatial change in general, and for motion of spatial scenes in particular:
1. Deciding satisfiability of an $\mathcal{ALC}(D)$ concept w.r.t. to a cyclic TBox is, in general, undecidable (see, for instance, \cite{19}).

2. In the case of the spatio-temporalisation, however, if we use what is called weakly cyclic TBoxes in \cite{13}, then satisfiability of a concept w.r.t. such a TBox is decidable. The axioms of a weakly cyclic TBox capture the properties of modal temporal operators. The reader is referred to \cite{13} for details.

Spatio-temporal theories such as the ones defined in \cite{13} can be seen as single-ontology spatio-temporal theories, in the sense that the concrete domain represents only one type of spatial knowledge (e.g., RCC-8 relations if the concrete domain is generated by RCC-8). We could extend such theories to handle more than just one concrete domain: for instance, two concrete domains, one generated by $CDA$, the other by $ROA$. This would lead to what could be called multi-ontology spatio-temporal theories. The presented work clearly shows that the reasoning issue in such multi-ontology theories does not reduce to reasoning about the projections onto the different concrete domains.

Before we provide an example, we adapt a definition from \cite{13}.

$\mathcal{MTALC}_{0,1}(D_{cCOA})$ is obtained from $\mathcal{ALC}(D)$ by temporalising the roles, and spatialising the concrete domain: $\mathcal{MTALC}_{0,1}(D_{cCOA})$ has exactly one role which is functional, and which we refer to in the following as $f$ (the subscript 0 indicates the number of general, not necessarily functional roles, and the subscript 1 the number of functional roles). The roles in $\mathcal{ALC}$, as well as the roles other than the abstract features in $\mathcal{ALC}(D)$, are interpreted in a similar way as the modal operators of the multi-modal logic $K_{(m)}$ \cite{12}. A functional role is also referred to as an abstract feature. $f$ plays the role of the $\text{NEXT}$ operator in linear time temporal logic: $f$ is antisymmetric, serial and linear.

**Definition 1 ($\mathcal{MTALC}_{0,1}(D_{cCOA})$ concepts).** Let $N_C$ and $N_{cF}$ be mutually disjoint and countably infinite sets of concept names and concrete features, respectively. A (concrete) feature chain is any finite composition $f_1 \ldots f_n g$ of $n \geq 0$ abstract features $f_1, \ldots, f_n$ and one concrete feature $g$. The set of $\mathcal{MTALC}_{0,1}(D_{cCOA})$ concepts is the smallest set such that:

1. $\top$ and $\bot$ are $\mathcal{MTALC}_{0,1}(D_{cCOA})$ concepts
2. an $\mathcal{MTALC}_{0,1}(D_{cCOA})$ concept name is an $\mathcal{MTALC}_{0,1}(D_{cCOA})$ (atomic) concept
3. if $C$ and $D$ are $\mathcal{MTALC}_{0,1}(D_{cCOA})$ concepts; $g$ is a concrete feature; $u_1$ and $u_2$ are feature chains; and $P$ is an $\mathcal{MTALC}_{0,1}(D_{cCOA})$ predicate,\(^5\) then the following expressions are also $\mathcal{MTALC}_{0,1}(D_{cCOA})$ concepts:
   (a) $\neg C, C \sqcap D, C \sqcup D, \exists f.C, \forall f.C$; and
   (b) $\exists (u_1)(u_2).P$.

**Example 1 (illustration of $\mathcal{MTALC}_{0,1}(D_{cCOA})$).** Consider a satellite-like high-level surveillance system, aimed at the surveillance of flying aeroplanes within a three-landmark environment. The basic task of the system is to situate qualitatively an aeroplane relative to the different landmarks, as well as to relate qualitatively the different positions of an aeroplane while in flight. If the system is used for the surveillance of the European sky, the landmarks could be capitals of European countries, such as Berlin, London and Paris. For the purpose, the system uses a high-level spatial description language, such as a QSR language, which we suppose in this example\(^5\) A predicate is any $cCOA$ relation.
One might want as well the system to provide plane’s different positions during the flight relate to each other. For example, that the subflight to take place next says that the aeroplane is northeast landmark \( L_1 \) immediately follows \( f_B \), which have the task of “referring”, respectively, to the actual positions of the aeroplane \( O \) while in Region \( X \), and gives rise to a defined concept \( B_X \) describing the panorama of the aeroplane \( O \) while in Region \( X \), and saying which subflight takes place next, i.e., which Region is flown over next. We make use of the concrete features \( g_{11}, g_{12}, g_3 \) and \( g_o \), which have the task of “referring”, respectively, to the actual positions of landmarks \( l_1, l_2, l_3 \), and of the aeroplane \( O \). As roles, the unique (functional) role of \( \mathcal{MTALC}_{0,1}(D_{cCOA}) \), referred to as \( f \), and denoting the linear-time immediate successor function. The acyclic TBox composed of the following axioms describes the flight:

\[
\begin{align*}
B_A \doteq & \exists(g_o)(g_{11}).NE \cap \exists(g_o)(g_{12}).SE \cap \exists(g_o)(g_{13}).SE \cap \exists f.B_B \\
B_B \doteq & \exists(g_o)(g_{11}).No \cap \exists(g_o)(g_{12}).So \cap \exists(g_o)(g_{13}).SE \cap \exists f.B_C \\
B_C \doteq & \exists(g_o)(g_{11}).NW \cap \exists(g_o)(g_{12}).SW \cap \exists(g_o)(g_{13}).SE \cap \exists f.B_D \\
B_D \doteq & \exists(g_o)(g_{11}).NW \cap \exists(g_o)(g_{12}).SW \cap \exists(g_o)(g_{13}).Ew \cap \exists f.B_E \\
B_E \doteq & \exists(g_o)(g_{11}).NW \cap \exists(g_o)(g_{12}).SW \cap \exists(g_o)(g_{13}).NW \cap \exists f.B_F \\
B_F \doteq & \exists(g_o)(g_{11}).NW \cap \exists(g_o)(g_{12}).NW \cap \exists(g_o)(g_{13}).NW \\
B_G \doteq & \exists(g_o)(g_{11}).NW \cap \exists(g_o)(g_{12}).NW \cap \exists(g_o)(g_{13}).NW
\end{align*}
\]

Fig. 12. Illustration of \( \mathcal{MTALC}_{0,1}(D_{cCOA}) \): the upward arrow pointing at N indicates North.

The concept \( B_A \), for instance, describes the snapshot of the plane while in Region \( A \). It says that the aeroplane is northeast landmark \( L_1 \) (\( \exists(g_o)(g_{11}).NE \)); southeast landmark \( L_2 \) (\( \exists(g_o)(g_{12}).SE \)); and southeast landmark \( L_3 \) (\( \exists(g_o)(g_{13}).SE \)). The concept also says that the subflight to take place next is \( f_B \) (\( \exists f.B_B \)).

One might want as well the system to provide \( CD_A \) knowledge on how the aeroplane’s different positions during the flight relate to each other. For example, that the
aeroplane, while in region C, remains northwest of its position while in region B; or, that the position, while in the goal region G, remains northwest of the position while in region E. These two constraints can be injected into the TBox by modifying the axioms $B_B$ and $B_E$ as follows:

\[
B_B \equiv \exists (g_o)(g_{11}).No \cap \exists (g_o)(g_{12}).So \cap \exists (g_o)(g_{13}).SE \cap \exists (g_o)(f g_o).SE \cap \exists f.B_C
\]

\[
B_E \equiv \exists (g_o)(g_{11}).NW \cap \exists (g_o)(g_{12}).SW \cap \exists (g_o)(g_{13}).NW \cap \exists (g_o)(f f g_o).SE \cap \exists f.B_F
\]

So far, the example has made use of CDA relations only as predicates. One might want to represent knowledge such as, the flight from Region B until Region D had a clockwise curvature (i.e., the aeroplane, while in region C, kept to the left of the directed line joining the position while in Region B to the position while in Region D). Another kind of knowledge one might want to represent is that, the flight was rectilinear in Region E. These can be added to the existing knowledge by modifying defined concepts $B_B$ and $B_D$ as follows:

\[
B_B \equiv \exists (g_o)(g_{11}).No \cap \exists (g_o)(g_{12}).So \cap \exists (g_o)(g_{13}).SE \cap \exists (g_o)(f f g_o)(f g_o).br \cap \exists f.B_C
\]

\[
B_D \equiv \exists (g_o)(g_{11}).NW \cap \exists (g_o)(g_{12}).SW \cap \exists (g_o)(g_{13}).Eq \cap \exists (g_o)(f f g_o)(f g_o).bw \cap \exists f.B_E
\]

7 Summary

We have presented the integration of two calculi of spatial relations well-known in Qualitative Spatial Reasoning (QSR): the projection-based cardinal direction calculus in [8], and a coarser version of the relative orientation calculus in [9]. With a GIS example, we have shown that reducing the issue of reasoning about knowledge expressed in the integrating language to a simple matter of reasoning separately about each of the two components was not sufficient. In other words, the interaction between the two kinds of knowledge has to be handled: we have provided a constraint propagation algorithm for such a purpose, which:

1. achieves path consistency for the cardinal direction component;
2. achieves strong 4-consistency for the relative orientation component; and
3. implements the interaction between the two kinds of knowledge.

Integrating different kinds of knowledge is an emerging and challenging issue in QSR. Similar work could be carried out for other aspects of knowledge in QSR, such as qualitative distance [3] and relative orientation [9], an integration known to be highly important for GIS and robot navigation applications, and on which not much has been achieved so far.

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Dear Amar,

We are sorry to inform you that the paper you submitted to the special volume of the Annals of Mathematics and Artificial Intelligence dedicated to the 2004 AI+Math Symposium, "Integrating cardinal direction relations in QSR", has not been accepted for publication.

We have attached the reviews of the paper below.

We thank you for your efforts in preparing this submission, and hope that you will find an appropriate forum for this work.

Sincerely,

Special Issue Co-Editors

REVIEWER 1

The work presented in this paper concerns the qualitative spatial reasoning (QSR). A formalism (called cCOA) combining the cardinal direction calculus (CDA) and the relative orientation calculus (ROA) is studied. In this formalism the spatial information about relative positions between the objects is represented by two CSPs: a CSP whose constraints are defined with relations of CDA (a CDA CSP) and a second CSP defined with relations of ROA (a ROA CSP). Such a pair is called a cCOA CSP. The main contribution of the paper consists in the definition of a constraint propagation algorithm allowing to test the consistency of a cCOA CSP. A section of the paper is devoted to the using of description logics with a concrete domain (ALC(D)) to handle spatio-temporal information with concrete domains generated from CDA and ROA.

Comments and recommendation

Main comments:

1. Personally, I think that the paper is not very well structured. In particular, I would displace the background on CSP after the presentation of the calculi and before the definition of the cCOA CSP. Moreover, the cCOA CSP are used in Section 5.2 and Section 5.3 but defined in Section 5.5.

2. I found that numerous parts and sections could be enriched with relevant and necessary details. For example, consider Section 2.3 which concerns Relation Algebras. The content of this section is empty and does bring anything to the reader. As another example, consider Section 3 where the CDA calculus and the ROA calculus are presented. In this section, we have just the names of the relations of these calculus and the intuitive definitions of them. The minimum thing would be to formally define these relations. There is other examples through the paper.

3. To me, the fundamental problem of this paper is its contribution. Actually, the main contribution of the paper consists in the definition of a constraint propagation procedure allowing to test the consistency of a cCOA CSP. This algorithm is based on the usual path-consistency method which is improved by operations making the translation of CDA relations to ROA relations and the translation of ROA relations to CDA relations. This type of interaction between two kinds of relations is not new, it has been already used for CSP containing both qualitative and quantitative constraints. To be published in a journal the paper must be enriched by a fundamental
and relevant result such as the proof that the proposed algorithm is complete when the constraints are atomic relations or own another property.

4. Section 6 should be removed, I don’t see the direct connection between this section and the other sections.

Other comments:

Page 6, Section 2.2: the 3-consistency and the path-consistency are the same thing in the case where the constraints are binary constraints.

Page 7, line +1, the author must give some explanations about the different symbols, what is I for example? Actually, the section 2.3 could be deleted.

Page 9, Section 4. What is the motivation to define a coarser relative orientation calculus? In the sequel, what is the calculus used: the initial ROA calculus or the coarser ROA calculus?

The author must give the order of the arguments for the relations lr, ll, bw, ... For example, take the case where A is between B and C, what is the notation: bw(A,B,C) or bw(A,C,B) or ...

Page 11, the given definition of the composition is not clear for me. It seems that Lir(r)=Rlr(r). Is it exact?

The functions roa-to-cdaxx use the CDA and ROA constraints to compute some new CDA constraints. Consequently, the interaction is from CDA+ROA to CDA and not from ROA to CDA.

Page 15, just before Section 5.5. : you must give the details (the proofs for example) of these claims.

Page 19, Section 6: the title of this section is not appropriated with its content. Moreover, the topic of Section 6 is not directly connected with the rest of the paper. I would remove this section.

Recommendation:

I cannot recommend to accept the paper in this form. I suggest to the author to improve the paper in taking into account the previous comments before submitting again.

Title: Integrating cardinal direction relations and other orientation relations in Qualitative Spatial Reasoning

Author: Amar Isli

Overall evaluation: reject with encouragement to resubmit

GENERAL COMMENTS

This paper presents a very interesting (and potentially useful) model of qualitative spatial reasoning. This model expresses directional relations on points by combining the binary relations of Frank and the ternary relations of Freksa. Then, it presents a constraint propagation algorithm for the combined model.

My major comments are as follows:

- Related work.
The author did not present related work at all. They only presented briefly and very unclearly the adopted models. This must be fixed. The author should present and discuss the models of Goyal and Egenhofer [Goyal00] and Billen and Clementini [Billen04]. These model defined binary and ternary relations on points and extended regions. Inference algorithms for the above models appear in [Skiadopoulos04], [Skiadopoulos02] and [Billen04b]. References to the above work should be also included.

- Presentation

The language of this paper is satisfactory (see also detailed comments). But the presentation of this paper must be improved. The presentation of the model is not complete. There are a lot of questions that are not answered. I believe that the author should present formal definitions for this model. Moreover, some examples would definitely help the reader.

- Contribution

The model is interesting but by itself it does not constitute a major contribution. It is a simple combination of two existing models. On the other hand, inference procedures are generally interesting and useful. In the present paper the inference procedure is based on a transitivity table. There is no discussion on whether this mechanism is complete or not. I can see that it finds inconsistencies that the inference procedures in the binary and ternary model (alone) can not but does it work in EVERY case?

To conclude, I believe that Section 3 should be a section discussing related work and maybe moved earlier. Then Section 4, should be extended, so that it would describe and exemplify in DETAIL the proposed model. Finally the authors should discuss (or better prove) whether their method is complete or not.

DETAILED COMMENTS

Abstract, Line 6, "to the well-known poverty conjecture": Which conjecture? Please explain and give references.

Page 2, Line 5, "the reader is referred to [4]": This is a 1997 survey. Since then, there have been proposed several interesting models (both for points and extended regions) for directional information (see discussion above). The author should discuss these models and give the appropriate references.

Page 2, Paragraph 2, "An example on ...": I could not spot this example in this paper. Moreover, I could not see the interaction and the integration of the two calculi.

Page 2, Paragraph 3: This is an one-paragraph sentence. It is very difficult to read and understand. Please rephrase.

Page 2, Paragraph 4: Please explain the initials cCOA, CDA and ROA.

Page 2, Example 1: Please give a pointer to Figure 1 in the first two lines of the example.

Page 4, middle of the page, "the well-known ALC": The author should be very careful when he uses expressions like this one. It is well known to whom? Please consider a reader that does not know ALC.

Page 4, Section 2, "A constraint satisfaction ...": Please give references.

Page 4, Section 2, Paragraph 2, "An m-ary constraint ...": Please replace "cdots" with "ldots" everywhere in the text.
Page 5: What the superscript b and t in expressions \( I^b_U \) and \( I^t_U \) stand for in your case. Please explain in advance. The notation used is very "crowded". A simplification should be considered. This also applies for Section 2.1-2.2 as well.

Page 5, Section 2.1, Paragraph 1: Please replace "verifying" with satisfying.

Page 6, Section 2.2, "P is k-consistent ...": Give examples.

Page 6, Section 2.2, "1-consistency, 2-consistency ...": What n in n-consistency stands for. Please explain.

Page 6, Section 2.2, "A refinement": Please delete ", and".

Page 6, Section 2.3: This is a journal paper. It should contain a more thorough presentation of related work. See also previous comments

Page 7, Items 1 and 2: You have explained only the symbols that appear in Section 2. What are the other symbols (e.g., \( \otimes \) and \( \oplus \)).

Section 3.1 and 3.2: I think that you should give representative names for the calculi of [8] and [9] to distinguish between binary and ternary relations. Then, you should use that name and the citation (e.g., the ternary relations [9]) to refer to them in the text (and not just the citation). Using just the citation number is confusing.

Section 3.1, last sentence: I thing that most people working on spatial modelling would disagree with this statement. There many arguments that you can say in favor of the projection-based model (for instance, it can be defined using simply order constraint on the projections of the point on the x and y axis) but I do not believe that it is more cognitive plausible that the cone based model, on the contrary.

Section 4: Give more details. Give formal definitions. Give examples. Also explain why you have adopted this model and not for instance the region based models of [Goyal00, Billen04]

Section 4, Items 1 and 2: The model is defined very badly. Terms and relations are not uniquely defined. For instance, relation ls is is to the left of the reference object but with respect to which reference frame. Also please mention relation names instead of numbers in Fig. 4.

Section 4, last paragraph: Which are the relations of CDA and ROA. Please also give examples and illustrations. "we omit brackets" this is the first time you mention brackets.

Section 5.2, middle of page: What \( \otimes \) stands for. Define and exemplify.

Section 5.2, Fig. 5: Use bigger font size. Does r1 runs in rows and r2 runs in columns or vise versa? Please explain.

Page 11, Section 5.1: Please help the reader and present an illustration.

Section 5.2, Paragraph 1: The definition you have presented in this paragraph is not the standard (existential/set-theoretic) composition but, it is something weaker called consistency-based composition. Set-theoretic/existential composition also requires that for two given regions a and c such that \( ar_1 \circ r_2 b \) holds we can always find a region b such that a \( r_1 b \) and b \( r_2 c \). (For definitions please consider the following references: [Bennett97, Ligozat01, Duentsch2000arcc, Duentsch99a, Skiadopoulos04]). For some models of spatial information composition and consistency-based coincide. For other models they are different [Duentsch2000arcc, Duentsch99a, Skiadopoulos04]. Is consistency-based composition that you discuss in this paper or
(existential/set-theoretic) composition? I believe that since you talk about algebras and calculi then the answer should be (existential/set-theoretic) composition. Please clarify.

Section 5.2, Items 1 and 2. What $b_1$ and $b_2$ stand for? Please explain.

Page 12, Paragraph 1: Please present an illustration for every example.

Page 12, Paragraph 2: What is cCOA-CSP. The definition appears in Page 15. Move it earlier.

Page 13, Figure 8: What $EQ$ stands for?

Items 1 to 3 of Pages 12 and 13. I would like to see proof of these statements.

Page 15, Item 3: more? Explain.

Page 17, Theorem 1. This is not really a theorem. One can say that it is actually a lemma. It only measures the computational complexity of the procedure. The algorithm achieves 4-consistency and it runs in $O(n^4)$. No surprise. I believe that the real theorem would have been to prove that procedure PcS4c+ is complete. Can you prove that? This would be the major contribution of the paper.

References. If you wish to abbreviate first names you should use a dot. Thus, it is A. Isli and A. C. Cohn and not A Isli and A C Cohn respectively.

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[Billen04b] Billen, B. and Clementini, E., "Introducing a reasoning system based on ternary projective relations", Proceedings of SDH’04, 2004.
Dear Dr. Isli,

Due to the large number of high-quality papers submitted to COSIT'01, the Program Committee had a difficult task in selecting contributions for the conference program. In addition, issues of accessibility and topical balance prevented us from accepting several papers of high quality.

I regret to inform you that your paper "Combining cardinal direction relations with relative orientation relations in Qualitative Spatial Reasoning" has not been selected for presentation at the Conference on Spatial Information Theory (COSIT'01). Below, please find reviewers' comments you may find informative.

We still enthusiastically invite you to consider attending COSIT'01, to be held at the Inn at Morro Bay, near San Luis Obispo, September 19-23.

The COSIT'01 Web site is continually being updated. You will soon find information there for registering for the conference. You will also find information on the Tutorial Program and the Doctoral Colloquium. The site is:

http://de.f125.mail.yahoo.com/ym/Compose?To=cosit01@geog.ucsb.edu&YY=54185&order=down&sort=date&pos=2&view=a&head=b

The Program Committee and I thank you for your interest in COSIT'01 and regret that the outcome was not more positive on this occasion.

Sincerely,

COSIT'01 Program Committee Chair

———PAPER REVIEWS———

Paper Number: [32]
Paper Title: [Combining cardinal direction relations with relative orientation relations in Qualitative Spatial Reasoning]

1. Topic Appropriateness: How appropriate is the paper to the themes of COSIT?
[a] a. extremely appropriate
[ ] b. very appropriate
[ ] c. appropriate
[ ] d. only a little appropriate
[ ] e. not appropriate at all

[ The paper deals with combining two kinds of qualitative directional information about the locations of points in 2D space: one is absolute information (related to a fixed frame of reference), and the other relative information (where is a target object situated with respect to another, relatively to a given point of view. Integrating various types of qualitative spatial calculi is a valuable and "hot" topic (in particular because each calculus, taken separately, is very weak). This work fits nicely in this general context by integrating two well-known calculi: the cardinal direction calculus and (a coarse version of) Freksa’s 15-relation calculus. ]

2. Scientific or technical quality of research:
Comments: [The research is good research, the authors have an excellent knowledge of the field and deal with it in a very competent way.]

3. How novel or innovative is the paper?
[ ] a. extremely innovative
[X] b. innovative
[ ] c. similar to other work but still somewhat innovative
[ ] d. not innovative

Comments: [The paper is innovative: To my knowledge, the subject has not been treated before, and the authors do quite a good job. However, for the time being, they seem to have attained only modest results. The main contribution of the paper consists in an algorithm which exploits some of the interaction between relative and absolute orientations. But we do not know much about what it achieves apart from 4-consistency and from the fact that it runs in quartic time, which is fairly obvious.

This is not to imply that this work is of little worth. On the contrary, I think that it is quite valuable and should be pursued. But it would be quite good to have some more substantial results in the direction explored by the authors. They should be encouraged to get them.]

4. Presentation (structure of the paper, language, graphical presentation, etc.):
[ ] a. excellent
[X] b. good
[ ] c. acceptable with minor improvement [please detail]
[ ] d. needs major improvement [please detail]

Comments: [The presentation is basically quite good. Some typos:
Page 7, l. -7 and -17: "knowledge" instead of "konwledge".
Page 14, references 14 and 15: "2D" rather than "2d" (You may need "D in the Latex source).]

5. Disciplinary Breadth (content and presentation):
[X] a. will definitely appeal to more than one discipline
[ ] b. may appeal to more than one discipline
[ ] c. probably will appeal to only one discipline
[ ] d. will definitely appeal only to one discipline

Comments: [This is technical work, of course. But the elaboration of good qualitative spatial calculi is an important issue which has a definite import at least for AI, GIS theory, linguistics, as well as an interest for computer science at large (Constraint Satisfaction Problems). The authors make a reasonable effort for justification.]
6. Overall judgment: Do you believe that the paper should be included in the program?
[ ] a. I STRONGLY recommend that the paper be included
[ ] b. I recommend that the paper be included
[X] c. I could go either way
[ ] d. I recommend that the paper NOT be included
[ ] e. I STRONGLY recommend that the paper NOT be included

7. How well do you know the topical area of the paper?
[ ] a. Extremely well, I consider myself an expert.
[X] b. Pretty well.
[ ] c. Moderately well, I am somewhat familiar with the area.
[ ] d. Not well, I’m really just guessing.

8. Other comments for the author(s)?
[ ] a. Any dose of extra theoretical knowledge on the subject would make the paper definitively QUITE acceptable. ]

END OF REVIEW

Paper Number: [ 32 ]

Paper Title: [Combining cardinal direction relations with relative orientation relations in Qualitative Spatial Reasoning ]

1. Topic Appropriateness: How appropriate is the paper to the themes of COSIT?
[ ] a. extremely appropriate
[X] b. very appropriate
[ ] c. appropriate
[ ] d. only a little appropriate
[ ] e. not appropriate at all

Comments: The subject of the paper is spatial reasoning about direction and orientation relations.

2. Scientific or technical quality of research:
[ ] a. Excellent
[ ] b. Very good (upper 1/3)
[ ] c. Good (middle 1/3)
[X] d. Fair (bottom 1/3)
[ ] e. Poor

Comments: The paper is overly complex and lacks any expansion of how the rather complex algebra has any relation to reality. Especially confusing is the extension of binary relation calculus to ternary relation calculus. Basically, a formal system in introduced and no rational description of how it may model reality is given.

3. How novel or innovative is the paper?
[ ] a. extremely innovative
[ ] b. innovative
[ ] c. similar to other work but still somewhat innovative
[x] d. not innovative

Comments: All I see is complexity.

4. Presentation (structure of the paper, language, graphical presentation, etc.):
[ ] a. excellent
[ ] b. good
[x] c. acceptable with minor improvement [please detail]
[ ] d. needs major improvement [please detail]

Comments: English is ill-structured and confusing at times. I suspect that it sounds better in the authors native language. For example, Section 1 the first sentence is: "Reasoning about orientation is one of the main aspects research in Qualitative Spatial Reasoning (QSR) has focused on for about a decade now." This is arguably correct English, but the argument would take a while. Splitting prepositions is not considered good form. The structure is complex and no punctuation is used that might help to parse the sentence. A more understandable version might be: In the last decade, as one of its major topics, research in Qualitative Spatial Reasoning (QSR) has focused on reasoning about orientation. Mixed with the complexity of the topic, the complexity of the language used makes the paper almost impossible to read.

5. Disciplinary Breadth (content and presentation):
[ ] a. will definitely appeal to more than one discipline
[ ] b. may appeal to more than one discipline
[ ] c. probably will appeal to only one discipline
[x] d. will definitely appeal only to one discipline

Comments: To be read by a general audience, this paper needs more information on its connection to reality. There is not sufficient support to show that solving a problem in the combined calculus would be logically equivalent to solving a problem in the real world.

6. Overall judgment: Do you believe that the paper should be included in the program?
[ ] a. I STRONGLY recommend that the paper be included
[ ] b. I recommend that the paper be included
[ ] c. I could go either way
[ ] d. I recommend that the paper NOT be included
[x] e. I STRONGLY recommend that the paper NOT be included

7. How well do you know the topical area of the paper?
[ ] a. Extremely well, I consider myself an expert.
[ ] b. Pretty well.
[x] c. Moderately well, I am somewhat familiar with the area.
[ ] d. Not well, I’m really just guessing.
8. Other comments for the author(s)? My reaction to this article is based mainly on its presentation. The paper is presented in such a complex fashion that it limits its own audience. Published to a general audience, the paper will be read by only a few, and understood by only a small portion of those.