Gravitational Collapse in Generalized Vaidya Space-Time for Lovelock Gravity Theory

Prabir Rudra *
Ritabrata Biswas †
Ujjal Debnath *

*Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.
[prabirrudra92@gmail.com, ujjal@iucaa.ernet.in, ujjaldebnath@yahoo.com]

† Department of Mathematics, Jadavpur University, Kolkata-700 032, India.
[biswas.ritabrata@gmail.com]

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Abstract

In this work, we have assumed the generalized Vaidya solution in Lovelock theory of gravity in \((n + 2)\)-dimensions. It has been shown that Gauss-Bonnet gravity, dimensionally continued Lovelock gravity and pure Lovelock gravity can be constructed by suitable choice of parameters. We have investigated the occurrence of singularities formed by the gravitational collapse in above three particular forms of Lovelock theory of gravity. The dependence of the nature of singularity on the existence of radial null geodesic for Vaidya space-time has been specially considered. In all the three models, we have shown that the nature of singularities (naked singularity or black hole) completely depend on the parameters. Choices of various parameters are shown in tabular form. In Gauss-Bonnet gravity theory, it can be concluded that the possibility of naked singularity increases with increase in dimensions. In dimensionally continued Lovelock gravity, the naked singularity is possible for odd dimensions for several values of parameters. In pure Lovelock gravity, only black hole forms due to the gravitational collapse for any values of parameters. It has been shown that when accretion is taking place on a collapsing object, it is highly unlikely to get a black hole. Finally on considering the phantom era in the expanding universe it is observed that there is no possibility of formation of a black hole if we are in the Gauss-Bonnet gravity considering the accreting procedure upon a collapsing object.

1 Introduction

Gravitational collapse is one of the most important problem in classical general relativity. The study of gravitational collapse was started by Oppenheimer and Snyder (1939). They studied collapse of dust with a static Schwarzschild exterior while interior space-time is represented by Friedman like solution. One would like to know whether, and under what initial conditions, gravitational collapse results in black hole (BH) formation. In particular, one would like to know if there are physical collapse solutions that lead to naked singularities (NS). In last few years, there have been extensive studies on gravitational collapse in order to investigate the nature of the singularities. The study of gravitational collapse of spherically symmetric space-times led to many examples (Eardly et al, 1979; Christodoulou, 1984; Neuman, 1986; Waugh et al, 1986, Dwivedi et al, 1989) of NSs. There is no general theory of the nature and visibility of singularities. There do exist a number of exact solutions of the Einstein equation which admit, depending upon the initial data, BHs or NSs (Joshi, 1993, 2000; Clarke, 1993; Wald, 1997; Jhingan, 1999; Singh, 1999). In particular the Vaidya solution (Vaidya, 1951) (contains outgoing radiation) is extensively used to show that the end state of collapse for regular initial data results in a NS.

Harko et al (2000) have studied the gravitational collapse of strange matter and analyzed the condition for formation of a NS in the spherically symmetric Vaidya space-time. It has been shown that depending on the
initial distribution of density and velocity and on the constitutive nature of the collapsing matter, either a BH or a NS is formed. Santos and collaborators (Santos, 1984, 1985; de Oliveira et al, 1985, 1986, 1987, 1988) included the dissipation in the source by allowing radial heat flow (while the body undergoes radiating collapse). Ghosh and Deskar (2000, 2003) have considered collapse of a radiating star with a plane symmetric boundary and have concluded with some general remarks. Ghosh et al (2002) have discussed the study of collapse of radiating star in Vaidya space-time in \((n + 2)\)-dimensions. Wang et al (1999) has generalized the Vaidya solution which include most of the known solutions to the Einstein equation such as anti-de-Sitter charged Vaidya solution. Husain solution has been used to study the formation of a BH with short hair (Brown et al, 1997) and can be considered as a generalization of Vaidya solution (Wang et al, 1999). Recently, Patil et al (2005, 2006) have studied the gravitational collapse of the Husain solution in four and five dimensional space-times and Debnath et al (2008) have studied the gravitational collapse of the Husain solution in \((n + 2)\)-dimensional space-times for various types of equation of state.

The Lovelock gravity theory (Lovelock, 1971) is a generalization of Einstein gravity theory in higher dimensional space-times, but the Lovelock theory is not a higher derivative gravity theory. The Lagrangian of the Lovelock gravity consists of the dimensionally extended Euler densities

\[
\mathcal{L} = \sum_{i=0}^{p} c_i \mathcal{L}_i
\]  

where \(p \leq \left(\frac{n+1}{2}\right)\) (where, \([x]\) denotes the integer part of the number \(x\), \(c_i\) are arbitrary constants with dimension of \([length]^{2i-2}\), \((n + 2)\) is the space-time dimension and \(\mathcal{L}_i\) are the Euler densities of a \((2i)\)-dimensional manifold (Cai et al, 2008)

\[
\mathcal{L}_i = \frac{1}{2i} \sqrt{-g} e_{a_1...a_id_1...d_i}^{c_1...c_i b_1...b_i} R_{a_1b_1}^{c_1} ... R_{a_ib_i}^{c_i}
\]

Here, the generalized delta function is totally antisymmetric in both sets of indices. \(\mathcal{L}_0\) is set to one, therefore the constant \(c_0\) is just the cosmological constant. \(\mathcal{L}_1\) gives us the usual scalar curvature term. If we set \(c_1 = 1\), \(\mathcal{L}_2\) gives just the Gauss-Bonnet (GB) term. So the Einstein-Gauss-Bonnet (EGB) gravity is a special case of Lovelock’s theory of gravitation, whose Lagrangian just contains the first three terms in (1).

Static spherically symmetric BH solutions can be found in Lovelock theory in the sense that a metric function is determined by solving for a real root of a polynomial equation (Myers et al, 1988; Wheeler, 1986). More recently, static, non-spherically symmetric BH solutions have been also found in the Lovelock gravity (Cai, 2004). Spherically symmetric BH solutions in the GB gravity have been found and discussed in (Myers et al, 1988; Whiler, 1986; Boulware et al, 1985) and rotating GB BHs have been discussed in (Kim et al, 2008). Some exact solutions for a Vaidya-like solution in the EGB gravity have been found in (Kobayashi, 2005; Maeda, 2006; Dominguez et al, 2006; Ghosh et al, 2008). Gravitational collapse and study of NS formation in dimensionally continued Lovelock gravity have been investigated in ref. (Nozawa et al, 2006; Ilha et al, 1997, 1999).

In this paper, we are mainly studying the nature of singularities (BH or NS) formed by the gravitational collapse in Lovelock theory of gravity. In section 2, we present the brief overview of generalized Vaidya solution in general Lovelock theory of gravity. Next we investigate the gravitational collapse in GB gravity, dimensionally continued Lovelock gravity and pure Lovelock gravity in sections 3-5. We have discussed an accretion phenomena upon the collapsing object in these gravity theories in section 6. Finally, the paper ends with a short discussions in section 7.

2 Brief Overview of Generalized Vaidya Solution in Lovelock Gravity Theory

The metric ansatz in \((n + 2)\)-dimensional spherically symmetric Vaidya space-time can be written as

\[
ds^2 = -f(v, r)dv^2 + 2dvdr + r^2d\Omega_n^2
\]
where \( r \) is the radial coordinate \((0 < r < \infty)\) and the null coordinate \( v (\infty < v < \infty)\) stands for advanced Eddington time coordinate and \( f(v, r) = 1 - \frac{m(v, r)}{r^{n-\sigma}} \) where \( m(v, r) \) gives the gravitational mass inside the sphere of radius \( r \) and \( d\Omega_{\text{n}}^2 \) is the line element of a \( n \)-dimensional unit sphere. The energy-momentum tensor for the Vaidya null radiation in the space-time is given by \( T_{ab} = \mu \dot{l}_a l_b \), where \( l_a = (1, 0, 0, \ldots, 0) \) and \( \mu \) is the energy density of the null radiation. Recently, Cai et al (2008) have calculated the field equations for Vaidya metric and their solutions in Lovelock gravity. Here we write briefly the field equations and solutions from their work.

The Einstein’s tensors for Vaidya metric in Lovelock gravity are given by [Cai et al, 2008]

\[
G^v_v = -\frac{1}{2} \sum_{i=0}^{p} c_i \left( \frac{n!}{(n-2i+1)!} \right) \left( \frac{1-f}{r^2} \right)^i \left( (n-2i+1) - \frac{i rf'}{1-f} \right)
\]  
(4)

\[
G^r_v = \frac{1}{2} \sum_{i=0}^{p} c_i \left( \frac{n!}{(n-2i+1)!} \right) \left( \frac{1-f}{r^2} \right)^i \left( \frac{i' f}{r^2} \right)
\]  
(5)

\[
G^i_k = \frac{1}{2} \sum_{i=0}^{p} c_i \left( \frac{n!}{(n-2i+1)!} \right) \left( \frac{1-f}{r^2} \right)^{i-1} \left( 2(n-2i)(n-2i+1) - 2(n-2i+1) \sum_{j=0}^{i} \frac{f'(r')}{r} - 4n f' \right)
\]  
\[+ \frac{i(i+1)}{2} \left( \frac{f''}{1-f} \right)^2 \]  
(6)

The Einstein’s field equation is given by

\[
G_{ab} = 8\pi GT_{ab}
\]  
(7)

Since we have \( G^r_v = G^v_v \), so from above equation (7) we must have \( T^r_r = T^v_v \). Now we assume that the spherical part of the energy-momentum tensor has the form \( T^i_i = \sigma T^v_v \), where \( \sigma \) is a constant.

Now from conservation equation \( \nabla_a T^a_v = 0 \), we get the following two equations as [Cai et al, 2008]

\[
\partial_v T^v_v + \partial_r T^r_v + \frac{n}{r} T^r_v = 0
\]  
(8)

and

\[
\partial_r T^r_v + \frac{n(n-\sigma)}{r} T^r_v = 0
\]  
(9)

For pure null radiation, we get \( T^r_r = T^v_v = 0 \). Here we consider \( T^r_r = T^v_v \neq 0 \). So from equation (9), we get [Cai et al, 2008]

\[
T^r_r = T^v_v = C(v)r^{-n(1-\sigma)}
\]  
(10)

where \( C(v) \) is a function of \( v \). Now define a function

\[
F(v, r) = \frac{1-f(v, r)}{r^2}
\]  
(11)

From (4), (7), (10) and (11), we get the equation

\[
\frac{1}{2} \sum_{i=0}^{p} c_i \left( \frac{n!}{(n-2i+1)!} \right) \partial_i (r^{n+1} F^i) = 8\pi G C(v) r^{-n\sigma}
\]  
(12)

which integrates to yield

\[
\sum_{i=0}^{p} c_i \left( \frac{n!}{(n-2i+1)!} \right) F^i = 16\pi G \left( \frac{m(v)}{\Omega_{\text{n}} r^{n+1}} + \frac{C(v)\Theta(r)}{r^{n+1}} \right)
\]  
(13)

where \( m(v) \) is arbitrary function of \( v \), \( \Omega_{\text{n}} = \frac{n^2}{\Gamma(n+3)} \) and \( \Theta(r) = \int r^{n\sigma} dr \) i.e.,
\[
\Theta(r) = \begin{cases} 
\frac{r^{n+1}}{m^{n+1}} & \text{when } \sigma \neq -\frac{1}{n} \\
\ln r & \text{when } \sigma = -\frac{1}{n}
\end{cases}
\]  

(14)

Now from equations (5), (7) and (11), we get

\[
\frac{1}{2} \sum_{i=0}^{p} c_i \frac{n! r}{(n-2i+1)!} \partial_{r} F^i = 8\pi GT_v
\]

(15)

Now using (13) and (15), we obtain

\[
T_v^r = \mu = \frac{\dot{m}(v)}{\Omega_{n} r^{n+1}} + \frac{\dot{C}(v) \Theta(r)}{r^{n+1}}
\]

(16)

We see that (10) and (16) satisfy (8). From (6), (7) and (11), we get (using \( T_j^k = 0 \))

\[
\delta^i_j \sum_{i=0}^{p} c_i \frac{(n-1)!}{(n-2i+1)!} \frac{1}{r^{n-1}} \partial_{rr}(r^{n+1} F^i) = 0
\]

(17)

So the energy-momentum tensor can be written as

\[
T_{ab} = \mu l^a l^b + (\rho + p)(n^a n^b + n_a l_b) + pg_{ab}
\]

(18)

In the comoving co-ordinates \((v, r, \theta_1, \theta_2, ..., \theta_n)\), the two eigen vectors of energy-momentum tensor namely \( l_a \) and \( n_a \) are linearly independent future pointing light-like vectors (null vectors) having components \( l_a = (1, 0, 0, ..., 0) \) and \( n_a = (f/2, -1, 0, ..., 0) \) and they satisfy the relations

\[
l_a l^a = n_a n^a = 0, \quad l_a n^a = -1
\]

(19)

Here, \( \mu \) is the energy density of Vaidya null radiation, \( \rho \) is the energy density and \( p \) is the radial pressure satisfying \( p = -\sigma \rho \) with \( \rho = C(v) r^{-n(1-\sigma)} \). The density \( \rho \) and pressure \( p \) come from the Lovelock gravity. The solution (13) is called the generalized Vaidya solution in Lovelock theory. Clearly this equation denotes the linear barotropic equation of state. We must follow that when \( \sigma = -\frac{1}{3} \) the radiation era is signified whereas for \( \sigma = 0 \) pressureless dust filled model is realised. When \( \sigma > \frac{1}{3} \), it denotes dark energy, to be very particular quintessence era. At last when \( \sigma = 1 \) it implies \( \Lambda \)CDM whereas \( \sigma > 1 \) means phantom era.

### 3 Gravitational Collapse in GB Gravity

Setting \( p = 2, c_0 = 0, c_1 = 1 \) and \( c_2 = \alpha \), the generalized Vaidya solution (13) in Lovelock gravity reduces to generalized Vaidya solution in GB gravity. Here the parameter \( \alpha \) is called the GB coupling parameter having dimension \((\text{length})^2\) and is related to string tension as \( \alpha^{-1} \). In the GB theory the function \( f(v, r) \) can be expressed as [Cai et al, 2008]

\[
f(v, r) = 1 + \frac{1}{2(n-1)(n-2)\alpha} \left[ r^2 \pm \sqrt{r^4 + \frac{64\pi G (n-1)(n-2)}{n^2} \frac{m(v)}{\Omega_{n} r^{n-3}} + \frac{C(v) \Theta(r)}{(r^{n-3})}} \right]
\]

(20)

We shall discuss the existence of NS in generalized Vaidya space-time by studying radial null geodesics. In fact, we shall examine whether it is possible to have outgoing radial null geodesics which were terminated in the past at the central singularity \( r = 0 \). The nature of the singularity (NS or BH) can be characterized by the existence of radial null geodesics emerging from the singularity. The singularity is at least locally naked if there exist such geodesics and if no such geodesics exist it is a BH.

The equation for outgoing radial null geodesics can be obtained from equation (19) by putting \( ds^2 = 0 \) and \( d\Omega_n^2 = 0 \) as
\[
\frac{dv}{dr} = \frac{2}{f(v, r)}.
\]  

(21)

It can be seen easily that \( r = 0, \ v = 0 \) corresponds to a singularity of the above differential equation. Suppose \( X = \frac{v}{r} \) then we shall study the limiting behaviour of the function \( X \) as we approach the singularity at \( r = 0, \ v = 0 \) along the radial null geodesic. If we denote the limiting value by \( X_0 \) then

\[
X_0 = \lim_{v \to 0} X = \lim_{v \to 0} \frac{v}{r} = \lim_{v \to 0} \frac{dv}{dr} = \lim_{r \to 0} \frac{2}{f(v, r)}
\]

(22)

Using (20) and (22), we have

\[
X_0 = \lim_{v \to 0} \frac{2}{r} \left\{ 1 + \frac{1}{2(n-1)(n-2)\alpha} \left[ r^2 \pm \sqrt{r^4 + \frac{64\pi G(n-1)(n-2)\alpha}{n} \left( \frac{m(v)}{\pi} + \frac{C(V)\Theta(r)}{r^4} \right)} \right] \right\}
\]

(23)

Now choosing \( m(v) = m_0 v^{n-3} \) and \( C(v) = C_0 v^{n(1-\sigma)-4} \), the equation (23) yields

\[
X_0 = \frac{2}{1 \pm \frac{1}{2(n-1)(n-2)\alpha} \left[ \sqrt{\frac{64\pi G(n-1)(n-2)\alpha}{n} \left( \frac{m_0 X_0^{n-3}(1+\frac{v}{r})}{\pi} + \frac{C_0 X_0^{n(1-\sigma)-4}}{n\sigma+1} \right)} \right]}
\]

(24)

On simplifying we have the algebraic equation of \( X_0 \) as

\[
\frac{16\pi^{1-\frac{2}{n}} G m_0 \Gamma \left( \frac{n+1}{2} \right) n^{n-1}}{n \alpha (n-1)(n-2)} X_0^{n-1} + \frac{16\pi G C_0}{n \alpha (n-1)(n-2)(n\sigma+1)} X_0^{n(1-\sigma)-2} - X_0^2 + 4X_0 - 4 = 0
\]

(25)

This is a very complicated algebraic equation. So as it will be bit difficult to express the value of the root \( X_0 \) in terms of all the parameters, we will vary the value of these parameters to check whether the concerned equation gives positive real root for these set of parametric values. Below we have prepared a table I where some particular sets of values of the parameters have been considered. At first we have to note that as we increase the dimension the tendency of having positive root increases. Besides as the value of \( m_0 \) is increased for lower dimension we may not have positive root. So in this case we will have a BH singularity. The parameter \( \alpha \) takes a leading role in having positive roots. If we take smaller values of \( \alpha \) the tendency of ‘not having’ positive roots increases even if for larger dimensions too. \( \alpha \) plays a role of multiplier to the GB term \( R_{GB}(= R^{\alpha \beta \gamma \delta} R_{\alpha \beta \gamma \delta} - 4 R_{\alpha \beta} R_{\alpha \beta} + R^2) \) in the action [Boulware et al, 1985]. When \( \alpha \) is very low it is very obvious that the impact of the GB term in the gravity will be lower. As \( \alpha \to 0 \) we get back the Einstein gravity. So we can predict that tendency of formation of BH reduces due to expanding nature of universe. Hence physically we can interpret this phenomenon as : in Einstein gravity circumstances were ready to give a BH or to wrap a singularity by as event horizon. On the contrary when we introduce the effect of GB term, i.e., we take the expanding universe in account it is observed that the event horizon is not been formed under the same circumstances. The negative pressure here is forcing the singularity to be naked. \( C_0 \) has no such impact on getting positive roots.

At last we will discuss the impact of the most important parameter \( \sigma \) which is actually determining the nature of the fluid present inside the universe. When \( \sigma > \frac{1}{3} \) we know it denotes the dark energy (DE) era (both quintessence and phantom) as this will not obey the strong energy condition (\( \rho + 3p > 0 \)). For this case we have seen that the chance of getting a BH is less in the higher dimensions. In some previous works when Debnath et al (2003, 2004), Banerjee et al (2003) have studied the collapse in higher dimensional Tolman-Bondi model they have shown that in Einstein gravity the possibility of getting a BH increases in higher dimensions. So we have reached an opposite result in our current work i.e., when GB gravity is considered. This may be a strong
impact of negative pressure of the fluid.

So ultimately we may tell that the possibility of the formation of BH in expanding universe is bleak. In literature, Babichev et al (2004) and Biswas et al (2011) had shown that the accretion of dark fluid either reduces the mass of the BH or weakens its accretion procedure. Similarly, here we can conclude that while forming a singularity by collapsing, DE/expanding universe opposes the formation of the BH and rather it increases the preferences of growing a NS.

| $m_0$  | $\sigma$ | $\alpha$ | $C_0$    | Positive roots ($X_0$) |
|--------|----------|----------|----------|------------------|
|        |          |          |          | 5D   | 6D   | 7D   | 8D   | 9D   | 10D  |
| 0.01   | 2(phantom) | 0.2      | 0.5      | 2.93572| 2.61882| 0.73852| 0.68112| 0.65449| 0.64221|
| 0.01   | 2(phantom) | 0.002    | 0.002    | –     | 0.84339| 1.03722| 1.13178| 1.17769| 1.19622|
| 0.01   | 2(phantom) | 0.00002  | 0.2      | –     | –     | –     | –     | –     | –     |
| 0.25   | 1(ΛCDM)   | 0.0003   | 0.05     | –     | –     | –     | –     | –     | –     |
| 0.5    | 1(ΛCDM)   | 0.3      | 0.5      | –     | 0.78632| 1.17709| 1.27789| 1.31614| 1.32483|
| 0.5    | 1(ΛCDM)   | 0.3      | 0.5      | –     | –     | –     | –     | 0.93492| 1.01816|
| 0.75   | 1(ΛCDM)   | 0.3      | 0.05     | –     | 0.73861| 1.05256| 1.18076| 1.23866| 1.26091|
| 0.75   | 1(ΛCDM)   | 0.0003   | 0.05     | –     | –     | –     | –     | –     | –     |
| 0.25   | 0.5(phantom) | 0.3     | 0.5      | –     | 0.83101| 1.17329| 1.30484| 1.36071| 1.37884|
| 0.25   | 0.5(phantom) | 0.003  | 0.5      | –     | –     | 0.009856| 0.18802| 0.42496| 0.60799|
| 0.5    | 0.5(phantom) | 0.01    | 0.5      | –     | –     | 0.09242| 0.44689| 0.68008| 0.81153|
| 0.75   | 0.5(phantom) | 0.3     | 0.5      | –     | 0.73861| 1.05256| 1.18076| 1.23866| 1.26091|
| 0.2    | 0.3      | 0.2      | 0.5      | 0.00003| 0.6537 | 1.00079| 1.16498| 1.266  | 2.1094 |
| 0.2    | 0.1      | 0.2      | 0.5      | 0.11429| 0.6280 | 0.9057 | 1.05539| 1.099  | 1.1453 |
| 0.2    | 0(dust)  | 0.2      | 0.5      | 0.159342| 0.598256| 0.843498| 0.980193| 1.0618 | 1.11264|
| 0.2    | −\frac{1}{3}(radiation) | 0.0002  | 0.5      | –     | –     | –     | –     | –     | –     |

**Table I:** Nature of the roots ($X_0$) of the equation (25) for various values of parameters involved.

4 Gravitational Collapse in Dimensionally Continued Lovelock Gravity

If we consider dimensionally continued Lovelock gravity, the solution of the equation (13) yields [Cai et al, 2008; Ilha et al, 1997, 1999; Nozawa, 2006]

$$f(v, r) = \begin{cases} 
1 - \left[\frac{m(v) + \Omega_\alpha C(v) \Theta(r)}{r} + \frac{16\pi G_l^2}{n! m}\right]^\frac{\alpha}{\sqrt{n}} + \frac{3m^2 G_l^2 r^2}{16(\alpha)!}, & \text{for even values of } n. \\
1 - [m(v) + \Omega_\alpha C(v) \Theta(r)]^{\frac{\alpha}{r\sqrt{n}}} + \frac{3m^2 G_l^2 r^2}{16(\alpha)!}, & \text{for odd values of } n.
\end{cases} \quad (26)$$

In the first solution of equation (26), put $m(v) = m_0 v$ and $C(v) = C_0 v^{-n\sigma}$ and using (22), we have
\[ X_0 = \frac{2}{1 - \left[ X_0 m_0 + \frac{n_0}{\Gamma(n_0 + \frac{1}{2})} C_0 \right]^{\frac{2}{n_0}}} \]  

(27)

On simplifying we have the algebraic equation of \( X_0 \) for even values of \( n \) as

\[ X_0 - 2 - X_0 \left[ m_0 X_0 + \frac{\pi^2}{\Gamma(1 + \frac{n}{2}) (n_0 + 1) X_0^{n_0}} \right]^{\frac{2}{n_0}} = 0 \]  

(28)

In the second solution of equation (28), put \( m(v) = m_0 v \) and \( C(v) = C_0 v^{-n_0-1} \) and using (22) and applying the limits, we get the algebraic equation of \( X_0 \) for odd values of \( n \) as

\[ X_0 - 2 - X_0 \left[ \frac{\pi^2}{\Gamma\left(1 + \frac{n}{2}\right) (n_0 + 1) X_0^{n_0+1}} \right]^{\frac{2}{n_0+1}} = 0 \]  

(29)

Like the previous section here also it is tough to determine the explicit form of the solution in terms of the different governing parameters. So here also we have varied the values of the parameters and dimension to observe their impact on forming positive or non-positive/non-real solution. Like the previous section here also we have listed all the data in a table format (table II). Now analyzing the table we can say that \( \sigma \) has no big impact on the nature of root. But if \( m_0 \) and \( C_0 \) are both small then irrespective of the dimension we will get positive roots. But as either of these two parameters are increased we will have real positive solution only if the dimension is odd. So in this case, the singularity will be naked. For even dimension we will have black holes.

It is to be noted that when \( \sigma = 2 \), i.e., in phantom era no BH forms. As like the previous section we interpret this phenomenon as follows: the negative pressure of DE is opposing a singularity to be a BH and forcing it to be a NS.

| \( m_0 \) | \( \sigma \) | \( C_0 \) | Positive roots (\( X_0 \)) | 4D | 5D | 6D | 7D | 8D | 9D |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 2(phantom) | 0.001 | 2.04176 | 2.0043 | 2.36332 | 2.01212 | 2.88472 | 2.01929 |
| 0.25 | 1(ΛCDM) | 0.000002 | - | 2.00072 | - | 2.00601 | - | 2.01635 |
| 0.75 | 1(ΛCDM) | 0.000002 | - | 2.00072 | - | 2.00601 | - | 2.01635 |
| 0.25 | 0.5(quintessence) | 0.0002 | - | 2.01536 | - | 2.0594 | - | 2.10965 |
| 0.01 | 0.5(quintessence) | 0.1 | 2.20573 | 2.33127 | 2.57982 | 2.45778 | 2.96694 | 2.50744 |
| 0.75 | 0.5(quintessence) | 0.0002 | - | 2.01536 | - | 2.0594 | - | 2.10965 |
| 0.01 | 0.25 | 2 | - | 4.65149 | 9.3907 | 4.42596 | 6.3532 | 4.11906 |
| 0.25 | 0.25 | 2 | - | 4.65149 | - | 4.42596 | - | 4.11906 |
| 0.75 | 0.25 | 2 | - | 4.65149 | - | 4.42596 | - | 4.11906 |
| 0.75 | 0(dust) | 2 | - | 2.04135 | - | 2.17054 | - | 2.33073 |
| 0.75 | -\( \frac{1}{3} \)(radiation) | 2 | - | - | - | - | - | - |

**Table II:** Nature of the roots (\( X_0 \)) of the equations (28) and (29) for various values of parameters involved.
5 Gravitational Collapse in Pure Lovelock Gravity

In pure Lovelock gravity, only two coefficients $c_0$ and $c_k$ are non-vanishing with $1 \leq k \leq \left[ \frac{n+1}{2} \right]$. Choosing $c_k = \frac{(n-2k+1)!}{(n+1)!} \alpha^{2k-2}$, the solution of $f(v, r)$ can be found from (13) and is given by [Cai et al, 2008]

$$\alpha^{2k-2} (1 - f(v, r))^k = -\frac{c_0 r^{2k}}{n(n+1)} + \frac{16\pi G n}{\Omega_n r^{n-2k+1}} + \frac{C(v)\Theta(r)}{r^{n-2k+1}}$$

(30)

where $\alpha$ is a constant length scale.

Assuming $m(v) = m_0 v^{n-2k+1}$ and $C(v) = C_0 v^{n-2k-n\sigma}$, using (22) and applying the limits we have,

$$\left\{1 - \frac{2}{X_0}\right\}^k \alpha^{2k-2} = \frac{16\pi G n}{\pi^2} \left\{ m_0 \Gamma \left(1 + \frac{n}{2}\right) X_0^{n-2k+1} + \frac{C_0 X_0^{n-2k-n\sigma}}{n\sigma + 1} \right\}$$

(31)

and simplifying, we get the algebraic equation of $X_0$ as

$$16\pi^{1-\frac{n}{2}} G \Gamma \left(1 + \frac{n}{2}\right) m_0 X_0^{n-2k+1} + \frac{16\pi G C_0}{n\sigma + 1} X_0^{n-2k-n\sigma} - \left(1 - \frac{2}{X_0}\right)^k n\alpha^{2k-2} = 0$$

(32)

As this is also a complicated equation to give an explicit solution of $X_0$ as a function of the parameters and dimension, we will again check the values of roots extracted from the equation for constant values of the parameters. All the parametric values, dimensions and the roots found are given below in a tabular form (table III). Here we can see unlike the previous cases irrespective of whatever parametric values used we are having non-positive roots in each cases. It implies that in pure Lovelock gravity we will always have a black hole.

| $m_0$ | $\sigma$ | $C_0$ | Positive roots ($X_0$) |
|-------|----------|-------|------------------------|
|       |          |       | 4D | 5D | 6D | 7D | 8D | 9D |
| 0.11  | 2(phantom) | 0.001 | - | - | - | - | - | - |
| 1     | 2(phantom) | 0.001 | - | - | - | - | - | - |
| 0.3   | 2(phantom) | 0.001 | - | - | - | - | - | - |
| 0.3   | .75(quintessence) | 1 | - | - | - | - | - | - |
| 0.11  | .75(quintessence) | 1 | - | - | - | - | - | - |
| 1     | .75(quintessence) | 1 | - | - | - | - | - | - |
| 0.3   | 0.2 | 2 | - | - | - | - | - | - |
| 0.11  | 0.2 | 2 | - | - | - | - | - | - |
| 1     | 0.2 | 2 | - | - | - | - | - | - |
| 1     | 0(dust) | 2 | - | - | - | - | - | - |
| 1     | $\frac{1}{3}$(radiation) | 2 | - | - | - | - | - | - |

Table III: Nature of the roots ($X_0$) of the equation (32) for various values of parameters involved.

6 Gravitational collapse when some fluid is also accreting upon the collapsing object

It is an obvious fact that a highly massive star accretes fluid around it mainly from the dust cloud in which the star is present or from any companion star which is present in a binary system with the super massive star as a companion. As the star absorbs mass the gravitational pull increases irrespective of the increment of the electronic force at its surface. So when the system collapses under its own gravity there may be an impact of the accreting fluid. Even when the accreting fluid is of DE type this may cause a chance of evolving a NS whereas
the fluid like normal matter may increase the chance of forming an event horizon. In this section we will check whether our speculation is true or not.

It is well known that the rate of change of mass of the central object for accreting phenomena is \( \Omega_n r^n T_0^1 \) (generalizing the result of Babichev et al (2004)). So the total mass will be

\[
M - \dot{M} dv = M - \Omega_n r^n T_0^1 dv
\]

Here, \( T_0^1 \) is non-diagonal stress energy tensor component evolved due to the accreting given by (16). So equation (20) will be changed to

\[
\bar{f}(v, r) = f(v, r) + \frac{\Omega_n}{r^n} \left[ \frac{\dot{m}(v)}{\Omega_n} + \dot{C}(v) \Theta(r) \right]
\]

and equation (22) becomes

\[
X_0 = \frac{2}{\lim_{v \to 0} \lim_{r \to 0} \left[ f(v, r) + \frac{\Omega_n}{r^n} \left\{ \frac{\dot{m}(v)}{\Omega_n} + \dot{C}(v) \Theta(r) \right\} \right]}
\]

where \( f(v, r) \) is given in eq.(20).

For all gravity, we take \( m(v) = \bar{m}_0 v^{n+1} \) and \( C(v) = \bar{C}_0 v^{n(1-\sigma)} \) we get

\[
\bar{m}_0(n + 1) X_0^{n+1} + \frac{\bar{C}_0 n^{\frac{\pi}{2}} n(1-\sigma)}{(n\sigma + 1) \Gamma(1 + \frac{n}{2})} X_0^{n(1-\sigma)} + X_0 - 2 = 0
\]

Clearly in the equation (35), \( X_0 \) is not depending at all upon the GB parameter \( \alpha \) or any other information of the Vaidya metric given by eq.(20). So if we try to modify eqs.(28), (29) and (32) we will have the same equation (35). While the fluid is accreting upon a collapsing object, the nature of singularity (NS or BH) will be same for above mentioned three types of gravities.

Like the previous sections it is really tough to get the explicit solution of \( X_0 \) as a function of parameters and dimension. So we will again check the values of roots extracted from the equation for constant values of parameters. All the parametric values, dimensions and the roots found are given in a tabular form (table IV).

Here we have seen for all the ranges of different parameters, irrespective of dimensions we get NS. As we have stated before in 2004, Babichev et al have shown that while our universe is expanding the existence of highly compact objects like BH at Big Rip is quite impossible. They have shown that the fluid (DE) which is responsible for creating such a negative pressure and responsible for the accelerated expansion of present day universe decreases the mass of BH while it is been accreted upon the BH (such that before arriving the Big Rip point all the BHs will be evaporated completely). Likewise when we have assumed the fluid is accreted upon a collapsing object it will be very obvious for the fluid that it will oppose the collapsing object to have an event horizon (i.e., to form a BH) and will have a tendency to produce a NS.
Table IV: Nature of the roots ($X_0$) of the equation (35) for various values of parameters involved.

7 Conclusions

The generalized Vaidya solution in Lovelock theory of gravity in $(n + 2)$-dimensions have been thoroughly assumed in this work. Gauss-Bonnet gravity, dimensionally continued Lovelock gravity and pure Lovelock gravity have been considered and it has been successfully shown that these three particular forms of Lovelock theory of gravity can be constructed by suitable choice of parameters. We have studied the occurrence of singularities formed by the gravitational collapse in the above three particular forms of Lovelock theory of gravity.

In GB gravity three major remarkable results have been observed: (i) Controlling the parameter $\alpha$ when we go back to Einstein gravity we find that the possibility of forming a NS increases. (ii) When we increase dimensions possibility of finding NS increases while for Einstein gravity contrary to the increase in chances of forming NS, the BH is formed for higher dimensions. (iii) The equation of state parameter ($\sigma$ here), when denotes negative pressure, tendency of forming NS increases. From the above three observations we can infer that for modified gravity theory chances of forming naked singularity is much much higher than forming a BH. Dimensionally continued Lovelock gravity theory resembles with the GB gravity in the matter of collapse whereas pure Lovelock gravity allows only BH to be formed. We can interpret this as, the gravity in case of pure Lovelock gravity theory is more stronger than the expanding force (dark force). At last in section 6 we have considered collapse under accretion where we have seen it is almost impossible to form a BH while the accretion is going upon the collapsing object in the expanding universe (presented by the modified gravity theory). Babichev et al (2004) have shown that under the accretion of phantom fluid the BH mass gets evaporated and the concerned BH will never be able to face the Big Rip. But here in the current work we can speculate that in expanding universe (specially in phantom era) no BH at all will be formed if we are in Gauss-Bonnet gravity or considering accretion procedure upon the collapsing object.

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