The pre-breakdown characteristics of weakly ionized media in the high non-uniform electric field

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Abstract. The theoretical model of pre-breakdown ions formation in a liquid dielectric and their flows, caused by high non-uniform electric field is represented. The 3D system of macroscopic pre-breakdown electrohydrodynamic equations is written. The electric field influence on the molecule dissociation rate is taken into account. The system are included the Poison equation for electric field potential, equation of ions formation and Navier-Stokes equations with electric force. The our up-steady analytical electrodynamic solution of these equations for potential of electric field distribution at spherical high voltage capacitor with liquid transformer oil type dielectric is described deviations from Ohms law, observed in many experiments. The analytical non-stationary solution for velocities distributions calculations of considerable liquid dielectric flows are obtained. The theoretical and experimental micro-breakdown radius zone dependences on applied voltage at air corona discharge are obtained for edge high voltage electrode.

1. Introduction
Deviations from Ohms law for weakly ionized solid media in pre-breakdown uniform electric fields experimentally discovered Poole about 100 years ago [1]. In a weakly conductive liquid media the same experimental effect was obtained by Wien about 100 years ago too [2]. For gaseous deviations from Ohms law were also obtained in high electric field many years ago.

Theoretically exponential effect for considerable media was explained by Frenkel for solid dielectrics and Onsager [2] for liquid weak electrolytes and for weakly conductive liquid dielectrics. The space charge and electrohydrodynamic flows observed in these dielectrics at a pre-breakdown condition [3].

The space charge formation, according [4], may be in pre-breakdown fields by up steady all electrohydrodynamic characteristics. And steady conduction may be as unipolar (corona discharge type) or as quasi-neutral (plasmas or electrolyte type). The last case considered us early in [4]. The pre-breakdown current-voltage theoretical and experimental characteristics of considerable media in non-uniform electric fields are described us in [5]. The purpose of present work is researches of the electrohydrodynamic flows, caused by these high non-uniform electric field. These intense flows are observed in transformer oil type liquids [6, 7] with complex molecular structure [8]. The hydrodynamic transfer of high voltage space charge, appeared in considerable liquids, is described in [9]. In review [7] the surface high voltage electrode effects influence on considerable pre-breakdown electrohydrodynamic flows is researched. This influence must be taken into account at plan high voltage
electrodes. And when applied high voltage field is non-uniform the volume effects influence on considerable flows is dominated.

2. Theoretical model

The following inequalities are valid for bulk charge concentrations $n_\pm$, impurities concentrations $n_p$ and neutrals concentrations $n_a$ in a low-conducting medium:

$$\begin{align*}
n_\pm & \ll n_a, \\
n_p & \ll n_a.
\end{align*}$$

The upper relation can be considered as the condition of weak ionization (dissociation) or low conductivity in considered medium. The rates of volume ionization (or dissociation) and recombination are supposed to be known thermodynamics functions of the above mentioned concentrations, temperature and field intensity $E$ (up to breakdown values). They can be represented in following form:

$$\begin{align*}
W_i = W_i(n_a, n_p, T, |E|) = W_i = W_i(n_a, n_p, T, 0) f(|E|) \\
W_e = K_n n_\pm \\
K_r = \frac{(b+b_\pm)(|E|)}{\varepsilon \varepsilon_0} Z.
\end{align*}$$

Here $W_i$ denotes the volume rate of ionization (dissociation), $W_e$ denotes the volume rate of ion recombination, $b$ is the mobility of the ions, $e$ is the elementary charge and $T$ is the absolute temperature. The expression for $K_r$ (recombination constant) was obtained by Langevin in 1903 and by Onsager in 1934 for particular case of weak electrolytes. It is two-particle ion-ion recombination, when the energy excess is absorbed by a medium. Onsager had shown as well the weak influence of sub-breakdown electrical fields on $K_r$ when direct and reverse processes are ionization and recombination correspondingly. But in case when these processes are the dissociation and reverse dissociation the influence of $E$ on $K_r$ can not be disregarded. Besides weak electrolytes the Langevin formula is valid for dense gases with chemical reactions, where ion conductivity is much greater than electronic one. Corresponding conductivity can be due to various processes. Among them are the neutral molecules (with ionic chemical bonds) dissociation and the neutral molecules (with covalent chemical bonds) ionization. The latter goes through the ionization of the electron from some neutral particle with relatively low ionization potential and attaching it to another with relatively high electron affinity.

For the ion diffusion coefficient we have used more known Einstein-Nernst relation:

$$ZD_\pm = \frac{k_B T}{|e|} n_\pm,$$

where $Z$ is the ion valence, $D$ is the diffusion coefficient, $k_B$ is the Boltzmann constant.

The function $f(|E|)$ describes the dependence of the ionization (dissociation) rate on the electric field intensity. The expression for it was obtained by Frenkel for the solid dielectric and generalized by Ostroumov in [3] for the liquid weakly conductive dielectric case ($Z > 1$) with ion chemical bonds in molecules. It was used in present research and has form:

$$f(|E|) = \exp(\beta |E|^{0.5}); \quad \beta = \frac{|Ze|^{1.5}}{\sqrt{\varepsilon \varepsilon_0 k_B T}}.$$  

(5)

Corresponding equations, describing creation and annihilation of the space charge, high voltage conductivity and electric field distributions, can be written as in [4]:

$$\begin{align*}
\frac{\partial q}{\partial t} + (V, \nabla q) - \frac{k_B T}{|e|} \Delta q + (E, \nabla \sigma) &= \sigma_0 \frac{q}{\varepsilon \varepsilon_0}, \\
\Delta \phi &= -\frac{q}{\varepsilon \varepsilon_0} + \sigma_0 \frac{q}{\varepsilon \varepsilon_0} \exp(\beta |E|^{0.5}) + \frac{q}{\varepsilon \varepsilon_0} = 0,
\end{align*}$$

(6a)

$$\begin{align*}
\frac{\partial q}{\partial t} = (V, \nabla \sigma) + b^2 (E, \nabla q) - \frac{k_B T}{|e|} \Delta \sigma - \frac{\sigma_0}{Z} \frac{q}{\varepsilon \varepsilon_0} \exp(\beta |E|^{0.5}) + \frac{q}{\varepsilon \varepsilon_0} = 0,
\end{align*}$$

(6b)

where $V$ is the velocity, $q$ is the volume charge density, $\sigma$ is the conductivity, $\phi$ is the electric potential.

The well-known hydrodynamics equations should be added to Eqs. (6) to construct the closed system of equations. The first of them is the continuity equation for incompressible media:

$$\text{div} V = 0.$$  

(7)
We used equation (7) for obtaining electrodynamic equations (6). Due to incompressibility it is enough to consider the law of momentum conservation (without the law of energy conservation). The momentum conservation equation can be written as:

$$\rho \frac{dx}{dt} = \rho(\chi \nabla)V - \rho v \Delta \chi = 2[\nabla q, E],$$

(8)

where $\chi$ is the vortex, $\rho$ is the density, $v$ is the kinematic viscosity.

The initial conditions for system (6) - (9) are

$q(t = 0) = V(t = 0) = 0, \sigma(t = 0) = \sigma_0$.

(10)

The quasi-stationary analytical electrodynamic solution of Eqs. (6) for the spherical symmetry electrical potential distribution $\phi$ in quasi-neutral medium can be obtained from charge conservation law (this well-known law may be obtained from equations (6) too [10]). This solution looks like

$$\phi(r) = \left(\frac{1}{4\pi \sigma_0}\right)^0 \frac{8}{\beta} \left(\frac{\epsilon_0}{\epsilon_0 + \sigma_0} \left(|E|^0 + \frac{2}{\beta}\right)\right) \text{sign}(\phi(r_0));$$

(11a)

$$|E| \exp\left(\frac{\beta}{2} |E|^0\right) = \frac{1}{4\pi \sigma_0 r^2}; \quad |\phi(r_0)| = U,$$

where $I$ is the electric current, $\sigma_0$ is the low voltage conductivity and $U$ is the applied voltage.

For the flat geometry one can obtain nonstationary solutions of Eqs. (6) [10]:

$$I = \frac{\sigma SU}{d}, \quad q = 0, \quad |E| = \frac{U}{d},$$

$$\sigma = \frac{\epsilon_0 \sigma_0 t + \epsilon_0 e^{-(\sigma_0 - \sigma_0) e^{-2z/t}}} \tau \left(\frac{\epsilon_0 + \epsilon_0 e^{(\epsilon_0 - \sigma_0) e^{-2z/t}}} \right),$$

$$\tau = \frac{\sigma_0 \exp(0.5 \beta |E|^{0.5})}{\epsilon_0}.$$

(11b)

One can see from (11b) that relaxation time $\tau = \epsilon_0/\sigma_0$.

And quasi-neutral solution (11) is the zero approaching of hydrodynamic space charge transfer differential operator serious [9]:

$$q = \sum_{i=0}^{\infty} (\tau v^{1+i}) \alpha E \nabla \tau,$$

(12)

where $\alpha$ is the charge relaxation time.

The mathematical space of differential operators, obtained in [9], is not Banach space. The quasi-exponential dependence as dependence [5] for volt-ampere pre-breakdown characteristics calculations of high voltage spherical capacitors may be obtained from (11). These deviations were explained early in [2] for plan capacitors. The Laplace condition of pre-breakdown electric field, obtained in [6], may be obtained from (11) too.

For cylindrical symmetry the analytical solution of equations (6) as (11) cannot be obtained. For 2D and 3D high voltage electrodes configurations geometry the solutions of equations (6) may be obtained only numerically. Graphic of volt-ampere characteristics (11) for different distances between spherical electrodes is shown in figure 1. As the distance between electrodes decreased the pre-breakdown current with same high voltage increased.

And in contrary for the unipolar conduction solutions of our equations (6) the analytical formulae for pre-breakdown volt-ampere characteristics calculations may be obtained analytically only for cylindrical symmetry case. This solution was obtained early for weakly ionized gaseous in [11]. This solution more often used for weakly ionized gaseous media. Due to conductivity of the air is much smaller that conductivity of the transformer oil one should use empirical constant of corona ignition voltage in the solutions of equations (6). And the simplest formulae for such characteristics was obtained by Townsend about 100 years ago:

$$I = \frac{2BU(u - U_c) L}{(r_0 + d)^2 \ln(1 + d/r_0)},$$

(13)

where $U_c$ is the corona ignition voltage, $L$ is the electrodes length, $d$ is the distance between electrodes, $r_0$ is the radius of the corona electrode. According [12] formula (13) can be used for point-plate electrode.

Based on the law of conservation of energy one can obtain expression for the size $r_d$ of the corona region where elementary processes occur:
\[ r_d = \left( \frac{2b U^2(U-U_0)t_i L}{4\pi(r_0+d)^2 \ln(1+d/r_0) \delta} \right)^{0.5}, \tag{14} \]

where \( t_i \) is the corona ignition time, \( \delta \) is the surface tension coefficient.

**Figure 1.** Quasi-exponential volt-ampere characteristics of the spherical capacitors with transformer oil according to (11). \((Z = 2, \text{ according to [8]})\). Distance between electrodes is given in cm.

The radius of high voltage inner electrode is 1.5 mm.

Our non-stationary hydrodynamic solution of equations (6) – (9) with using (10), (11) and [13] for development of weakly conductive liquid jet flows from high voltage point electrode has the following form:

\[ \psi = \frac{\varepsilon \varepsilon_0 U^2 (t-\tau) \sin^2 \theta}{32 \pi \rho r}, \tag{15} \]

where \( \psi \) is the stream function.

**Figure 2.** Graphic of the constant transformer oil jets stream lines (13) for \( U = 4 \text{ kV} \) and debit 0.1 L/s for different times moment. The sizes of Cartesian coordinates are given in mm.
3. Experimental results of prebreakdown characteristic in the case of the corona discharge in air

Stationary corona discharge without additional ionization source was investigated in this work. The experiments were carried out in air at atmospheric pressure. The needle with an edge radius equal to 70 µm was placed above the center of a flat round electrode with a diameter of 100 mm at a distance of 13 mm. In this case, the finite size of the flat electrode can be neglected. High dc voltage was applied to the needle. The flat electrode was grounded. The dc voltage was measured using voltage divider with the voltage ratio of 1:1000 and multimeter APPA 505. The error of measuring the dc voltage at the limit of 10 V was equal to ±0.17 V. The current was measured using ammeter with the accuracy class of 0.5.

We believe that the corona discharge region where the ionization processes occur corresponds to the region emitting in the visible range. Photography was used to determine the size of this area. Nikon 1 V2 (4608×3072) camera with Nikon 60mm f/2.8D AF Micro-Nikkor lens was used for taking pictures. The photography was carried out with a fully open aperture, the exposure was equal to 5 s, and the photosensitivity was equal to 400 ISO units. A pixel of the matrix corresponded to a 0.0044×0.0044 mm². The position of the camera during the experiment did not changes.

![Figure 3. Photos of the discharge with the following parameters: a) I = 6 µA, U = 5.01 kV; b) I = 27 µA, U = 8.01 kV; c) I = 68 µA, U = 11.04 kV.](image)

![Figure 4. The distance from the end of the needle to the boundary of the ionization region along the vertical axis (circles) in comparison with the square root of the cross-sectional area (crosses) of the ionization region and theoretical calculations (14) in the case of the corona discharge in air.](image)
Figure 2 shows photographs of the discharge of negative polarity for various parameters. At the discharge voltage less than 7 kV the glow area is stationary and vertically symmetrical and expands with increasing discharge current. At higher discharge currents, the glow area loses symmetry and begins to fluctuate. The breakdown occurred at the discharge voltage more than 12 kV.

It can be seen that the size of the ionization region increases monotonically with increasing discharge voltage.

Figure 4 shows size of the ionization region in the case of the corona discharge in air.

4. Conclusion
This study presents in the form of graphs and formulas the monotonously growing dependences of the prebreakdown stationary volt–ampere characteristics of weakly conductive liquid dielectrics as well as weakly ionized gases. Obtained characteristics differ from the traditional volt-ampere characteristics for point electrodes by a sharper current increase with increasing applied voltage. In the case of the interelectrode gap of the high-voltage spherical capacitor being filled with viscous heat-conducting liquid dielectrics such as transformer oil independent of the change in polarity of the electrodes, the calculation formula testifies to the growth of the prebreakdown current with a decrease in the interelectrode gap.

It was found that in the case of the corona discharge in weakly ionized gases like air volt-ampere characteristics for positive and negative discharge are significantly different in contrast to weakly conductive liquid dielectrics. The size of the ionization region in the case of corona discharge in air increases monotonically with increasing discharge voltage.

An analytical solution for the evolution of flows of viscous incompressible fluid (transformer oil) for the point-plane geometry was obtained.

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