Comparison of Fat7 and HYP fat links

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We study various methods of constructing fat links based upon the HYP (by Hasenfratz & Knechtli) and Fat7 (by W. Lee) algorithms. We present the minimum plaquette distribution for these fat links. This enables us to determine which algorithm is most effective at reducing the spread of plaquette values.

1. INTRODUCTION

Despite its prominent role in the non-perturbative study of QCD, lattice field theory has long struggled with the discretization errors inherent in its formulation, with the consequent development of many improvement schemes. One popular method of improvement has been the use of “fat links”, in which each ordinary or “thin” link of the lattice, $U_\mu(x)$, is replaced by a linear combination of the thin link and links adjacent to it. Two fattening algorithms which will be of interest to us in this report are HYP or hypercubic blocking [1] and SU(3)-projected Fat7 blocking [2] (Fat7). These two algorithms have been shown to be perturbatively equivalent at one-loop level [2]. Here we present the results of a preliminary numerical study to determine if HYP and Fat7 significantly differ non-perturbatively.

2. HYPERCUBIC BLOCKING

The HYP algorithm is a modification of standard APE smearing [4]. The APE-smeared link $V_\mu(x)$ is formed by adding the sum of staples, weighted by some factor $\alpha$, to the thin link,

$$V_\mu(x) = (1 - \alpha) U_\mu(x) + \alpha \sum_{\nu=0}^{3} W_\nu(x)$$

followed by projection back to SU(3) ($W_\nu(x)$ refers to the sum of three-link staples in the positive and negative $\nu$ directions, with central link parallel to $U_\mu(x)$, and $\nu \neq \mu$). To localize the smearing within the smallest possible volume, HYP blocking was defined as being equivalent to three iterations of APE smearing with the caveat that the staples at each stage could not be constructed from links which are fattened in the same plane as the staple itself. Due to space constraints in this report, we direct the reader to reference [1] for a more detailed description.

Since its introduction, HYP blocking has shown itself to be extremely effective at reducing taste-symmetry breaking effects, and has gained widespread attention and use.

3. FAT7 AND FAT7 BLOCKING

Fat7 blocking [5] incorporates not only three-link staples but also five-link and seven-link staples, making it possible to traverse all three directions orthogonal to the original thin link, as in HYP blocking. Unlike HYP, the original proposal for Fat7 blocking did not incorporate SU(3) projection. Fat7 is a modification of Fat7 which incorporates SU(3) projection after fattening [2].

We can use the second covariant derivative [6] operator to define a fattened link

$$L_\nu(\alpha) \cdot U_\mu(x) = (1 - 2\alpha) \cdot U_\mu(x) + \alpha W_\nu(x).$$

($W_\nu(x)$ defined as in Eq. (1)). With the parameter $\alpha$ taking the value one-quarter this operator can be interpreted as suppressing flavour-changing gluon interactions by vanishing in the limit as gluon momentum approaches the lattice cut-off. The Fat7 link can easily be constructed by repeated application of this operator as

$$V_\mu = \frac{1}{6} \sum_{\text{perm}(\nu,\rho,\lambda)} L_\nu(\alpha) \cdot (L_\rho(\alpha) \cdot (L_\lambda(\alpha) \cdot U_\mu))$$
where we note explicitly that the sum over permutations of the directions is taken. This repeated use of the same operator means we use a single weighting parameter $\alpha$ at each stage, whereas HYP uses three parameters $\alpha_1$, $\alpha_2$, $\alpha_3$. For Fat7 we may construct four different combinations of SU(3) projection options (i.e. projection at the final, initial+final, middle+final, and initial+middle+final stages).

4. COMPARISON OF FATTENING APPROACHES

The process of fattening tends to spread the minimum plaquette values of an ensemble of configurations over quite a wide range (compare Figs. 1 and 2-3). In [1], minimum plaquette values were used as a probe of the most severe short-range link fluctuations, which can lead to taste-symmetry violating effects and other problems. We expect that for certain types of measurements it will be desirable to reduce link fluctuations by increasing minimum plaquette values, while maintaining a narrow spread of minimum plaquette values across our ensemble to reduce statistical uncertainty. However it is worth noting that this is a speculative point. In this study we wish to determine whether the Fat7 blocking algorithm is capable of producing a narrower spread of minimum plaquette values than HYP, making it worthy of a more detailed investigation.

5. RESULTS

We present the minimum plaquette values for an ensemble of 194 quenched configurations generated on an $8^3 \times 32$ lattice at $\beta = 5.7$. We also present the minimum plaquette values for the ensembles obtained after blocking each configuration with the HYP algorithm (Fig. 2-3), and Fat7 with $\alpha = 0.25$ (Figs. 4-6). For HYP we are interested in two choices of parameters, those found non-perturbatively in [1], namely $\alpha_1 = 0.75$, $\alpha_2 = 0.6$, $\alpha_3 = 0.3$ (HYP-I), and those found perturbatively in [3], namely $\alpha_1 = 7/8$, $\alpha_2 = 4/7$, $\alpha_3 = 1/4$ (HYP-II), although we note the caveat that choices of $\alpha_1$ larger than 0.75 may tend to destabilize the smearing algorithm [7]. For the Fat7-blocked configurations we have analysed the four different SU(3) projection options described in section 3. The Fat7 configurations show less...
spread than HYP. Fat7 with projection at the final stage, and at all three stages are interesting from an analytical basis [3]. Although the number of configurations used in this study is small, the narrow, well-defined peak obtained after blocking with initial+final stage SU(3) projection seems significant enough to also warrant further investigation.

6. CONCLUSIONS

In this preliminary work Fat7 shows signs of producing results which are as good as HYP, and possibly better for some calculations. Both HYP and Fat7 can be constructed very efficiently by pre-calculating and storing the staples across the entire lattice, and using these to make the links at the next level of fattening. Using this approach Fat7 requires less memory than HYP. In future work we hope to determine whether Fat7-blocked ensembles can produce results which are superior to HYP for certain calculations.

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