PIV measurement study of fully developed superfluid turbulent thermal counterflow jet

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Abstract. The particle image velocimetry (PIV) technique was successfully applied for measuring the normal component velocity of a He II thermal counterflow jet. Neutrally buoyant hydrogen-deuterium solid particles were used as tracer particles for the PIV measurement. The jet velocity profiles and the decay properties were investigated for the cases of small and large normal component velocities, and were compared with those of fully developed turbulent round jets of ordinary viscous fluids. The velocity measured near the jet nozzle exit was compared with the theoretical prediction for the normal component velocity, $U_{n,tho}$. It is found the ratio of the PIV velocity to $U_{n,tho}$ is smaller than unity at any temperatures irrespectively of $U_{n,tho}$.

1. Introduction
The fluid dynamic behavior of He II, in particular that in the superfluid turbulent state, have not fully been studied so far. As for the studies on thermal counterflow jet that were based on the direct measurement of the velocity, only a few researches have been conducted to which flow visualization by using solid particle tracers [1] and the laser Doppler velocimetry [2] were applied. Recently, particle image velocimetry (PIV) has come to be used for He II flow measurements [3-6]. In reality, these applications were attended with a certain difficulty. There have been fundamental issues of the adequate selection of tracer particles with desirable size and number density in He II. In addition, it is recognized to arise another problem that tracer particles may directly interact with quantized vortices and do not simply follow the normal component flow in the superfluid turbulent state.

In the present study, we applied PIV to the measurement of He II thermal counterflow jet, in particular for the cases of fully developed superfluid turbulent jets. The jet velocity profiles and the decay properties were compared with those of fully developed turbulent round jets of ordinary viscous fluids. The velocity measured near the jet nozzle exit was compared with the theoretical prediction.

2. Experimental Set-up
The cryostat has three sets of optical windows with an aperture of 60 mm in the directions of 0, 90 and 180 degrees. The thermal counterflow jet is generated by heating He II in a small jet chamber. The normal flow formed in the jet chamber is injected from the circular nozzle into quiescent He II forming a jet as shown in figure 1. The theoretical value of the averaged normal component jet velocity injected from the nozzle is given by

$$U_{n,tho} = q / \rho sT ,$$

where the heat flux $q$ is given by the heat generation rate $Q$ divided by the nozzle cross sectional area, $\pi d^2/4$. The jet chamber has the circular nozzle with an inner diameter of 3.8 mm. In this figure, the
PIV measurement region (35 mm x 17.5 mm) is also shown. The jet axis is taken as the x-coordinate, and the y-axis in the perpendicular direction with the coordinate origin taken at the center of the nozzle exit. All the experiments were conducted under the nearly saturated vapor pressure condition.

In the PIV application, the velocity is calculated from the average displacement of tracer particles $\Delta X$ in two images that were successively taken with a prescribed time interval of $\Delta t$. It is, in fact, the velocity of tracer particles that is measured by PIV. The value of $\Delta X$ is computed through the principle of the maximum mutual correlation coefficient for the light intensity distributions of the two corresponding interrogation regions, instead of pursuing each individual particle. For this purpose each image with a pixel size of 1024 in the x-direction by 512 in the y-direction is subdivided into narrower interrogation regions with a pixel size of 16 by 16. The average displacement of a group of particles in each interrogation region is computed instead of pursuing each individual particle. Thus the two-dimensional velocity vector is computed as follows;

$$\mathbf{v}(x, y, t) = \frac{\Delta X}{\Delta t} .$$

(2)

In this PIV application, each interrogation region overlaps by 50%, and thus the total 128 (in the x-direction) by 64 (in the y-direction) velocity vectors are calculated. The time interval $\Delta t$ that is decided depending on the flow velocity was 20 ms for low velocity and 4.9 ms for the highest velocity case. For the PIV camera, a general-purpose high-speed video camera was used (FASTCAM-1024PCI100K, Photron). The light source was a 5W YAG laser (CW532-5W, Kanomax), of which laser beam was converted into a light sheet with a thickness of less than 2 mm by a cylindrical lens.

It is expected that the velocity of the normal component is measured in He II because the particles follow the normal component motion due to the viscosity. Therefore, the right selection of tracer particles is of crucial importance in PIV application. In the present PIV application, hydrogen-deuterium solid particles with an average diameter of several ten $\mu$m were used, which had been used for LDV measurement in our laboratory for more than 25 years. The pre-mixed gas of hydrogen and deuterium (3 kPa : 3 kPa mixture) pressurized by helium gas up to 0.4 MPa was repeatedly injected onto the He II free surface with an injection time of about 30 ms [3,4]. The number of particles remaining in the measurement zone decreased with time, and thus the gas injection must be repeated several times to renew particles in the measurement zone during a run of the experiment.

3. Results and Discussion

There was an insufficiency in snapshot PIV data [8]. In some regions of a PIV velocity map the velocity values are not properly assigned due to very low particle number density, the indistinctness of the particle image or the lack of particles in the jet chamber. In order to circumvent this defect the conditionally ensemble-average was introduced, where extraordinary low velocity values, lower than 4% of $U_{e,th}$, are excluded in averaging several hundred PIV output results. In the following, the fluid dynamics discussion will be developed on the basis of these averaged data.
3.1 Decay of jet velocity and spatial distribution of averaged velocity
The discussion on the decay of jet velocity as it flows and the spatial distribution of the averaged velocity was developed on the basis of the comparison of them with those of the fully developed turbulent circular jets of viscous fluids [8]. They are summarized as follows: It is seen from the conditionally ensemble-averaged PIV velocity data that some superfluid turbulent flow properties of the thermal counterflow jet are similar to those of a fully developed turbulent circular jet of viscous fluids in the following aspects. The $x$-component velocity measured on the jet axis decays in proportion to $x^{-1}$. The spatial distribution of averaged $x$-component velocity agrees with the Goertler profile that is typically observed in fully developed turbulent circular jets of ordinary viscous fluids. On the other hand, the following superfluid turbulent flow properties are different from those of turbulent viscous fluid jets [8]. The $x^{-1}$ jet decay and the approach to the Goertler profile of superfluid turbulent thermal counterflow jet start more upstream, mostly around $x/d = 4$ than those of turbulent circular jets of viscous fluids, of which transition typically occurs around $x/d = 6$.

3.2 Comparison of PIV data with theoretical prediction
The PIV data of the averaged $x$-component velocity measured on the jet axis near $x = 0$, $U_{\text{max}}$, are compared with the theoretical prediction by equation (1), $U_{n,\text{theo}}$. Shown in figure 2 is $U_{\text{max}}$ plotted against the heat flux $q$ together with $U_{n,\text{theo}}$ taking the temperature as a parameter. In all the cases, $q$ is above the critical heat flux for the onset of turbulence. It is seen $U_{\text{max}}$ is in proportion to $q$ up to $1.5 \times 10^4$ W/m² within the range of the accuracy of the present experiment. This means that $U_{\text{max}}/q$ and
$U_{n,\text{theo}} / q = 1/(\rho sT)$ are given uniquely as a function of temperature. It should be noted in Fig. 2 that $U_{\text{max}}$ data for $T = 2.10$ K are mostly larger than the theoretical prediction. However, the average value over a jet cross section is not larger than the prediction as seen in Fig. 3 even though the maximum value in a cross section is larger than the prediction. In figure 3, the ratio $U_{\text{ave}} / U_{n,\text{theo}}$ is plotted against the temperature, where $U_{\text{ave}}$ is a mean value of the averaged x-component velocity over a cross section measured near $x = 0$. It is evident that the ratio is considerably smaller than unity at the temperatures below 2.0 K, though it approaches to unity as $T$ does to $T_1$. The feature that the ratio is below unity had been also observed in the LDV result [2]. It was indicated in the references [3,9] that small particles are trapped by quantized vortex line and consequently the velocity of the tracer particles was reduced by the interaction with vortices resulting in some velocity slip between the normal component and the particles.

4. Summary

PIV was successfully applied to the measurement of the He II thermal counterflow jet and the following conclusions were drawn:

1. Neutrally buoyant solid particles suitable for PIV measurement can be produced from premixed hydrogen-deuterium gas with a one to one concentration pressurized by helium gas by pulsed injection onto He II free surface.
2. There is a problem in the snap-shot PIV result as a result of the lack of data output due to low particle number density somewhere in the jet flow field, but conditionally ensemble averaging of snap-shot data nearly solves this problem and leads to satisfactory result.
3. The ratio of the x-component velocity measured at the nozzle exit to the theoretical value $U_{n,\text{theo}}$ is less than unity below 2.0 K though its magnitude depends on temperature.

5. References

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