Instanton method for the electron propagator

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A nonperturbative theory of the electron propagator is developed and used to calculate the one-particle Green's function and tunneling density-of-states in strongly correlated electron systems. The method, which is based on a Hubbard-Stratonovich decoupling of the electron-electron interaction combined with a cumulant expansion of the resulting noninteracting propagator, provides a possible generalization of the instanton technique to the calculation of the electron propagator in a many-body system. Application to the one-dimensional electron gas with short-range interaction is discussed.

I. INTRODUCTION

In a large class of one-dimensional systems, electron-electron interaction leads to the breakdown of Landau's Fermi liquid theory and to the formation of a highly correlated Tomonaga-Luttinger liquid phase. In 1990, Wen [1] proposed that an edge state in the fractional quantum Hall effect (FQHE) regime of the two-dimensional electron gas is a chiral Luttinger liquid (CLL), a chiral counterpart to this non-Fermi-liquid state. Theoretical work by Wen [2] and by Kane and Fisher [3] showed that the transport and spectral properties of edge states in the FQHE regime should be strikingly different than edge states in the integral regime, and several of these properties have been observed experimentally [4].

Recently, however, an important experiment by Grayson et al. [5] on the tunneling spectra of quantum Hall edges found three surprising results: First, power-law tunneling characteristics were observed over a range of filling factors \( \nu \), irrespective of whether a quantized Hall state existed at that filling factor. Second, within a given FQHE plateau, the tunneling exponent was found to vary with the filling factor and was not fixed when the Hall conductance was quantized. Measurements on some samples, however, do show a density-of-states (DOS) plateau [6]. And third, the experiment shows that the tunneling DOS predicted by the multicomponent CLL theory for the hierarchy states is incorrect.

This experiment was motivated by an earlier experiment by Chang et al. [7], who observed CLL-like tunneling characteristics at \( \nu = 1/2 \). Although some aspects of Ref. [3] are beginning to be understood [8], especially in connection with the third point above, the most fundamental aspects are not.

In this paper I will outline a new theoretical method to calculate the one-particle Green's function and tunneling DOS in a strongly correlated electron system. The method, discussed in detail in Ref. [9], is based on a Hubbard-Stratonovich decoupling
of the electron-electron interaction combined with a cumulant expansion of the resulting noninteracting propagator. It provides a potential generalization of the instanton technique \cite{10} to the calculation of the electron propagator in a many-body system \cite{11}, and a variant of the intuitive but phenomenological “charge spreading” picture \cite{12} emerges automatically. After describing the general formalism I will apply it to spinless fermions with short-range interaction. In future work the method will be applied to the sharp and smooth edges of FQHE systems.

II. OUTLINE OF THE METHOD

I consider a $D$-dimensional interacting electron system, possibly in an external magnetic field. For simplicity I will suppress spin indices and assume the system to be translationally invariant with density $n_0$. The grand-canonical Hamiltonian is $H = H_0 + V$, where

\begin{equation}
V \equiv \frac{1}{2} \int d^D r \, d^D r' \, \delta n(r) \, U(r - r') \, \delta n(r'), \quad \delta n(r) \equiv \bar{\psi}(r) \psi(r) - n_0.
\end{equation}

$H_0$ is the noninteracting Hamiltonian. We want to calculate the Euclidean propagator

\begin{equation}
G(r_f, r_i, \tau_0) \equiv -\langle T \bar{\psi}(r_f, \tau_0) \psi(r_i, 0) \rangle_{H_0},
\end{equation}

which can be written (in the interaction representation with respect to $H_0$) as

\begin{equation}
G(r_f, r_i, \tau_0) = -\frac{\langle T \bar{\psi}(r_f, \tau_0) \psi(r_i, 0) e^{-\int_0^\beta V(\tau) \, d\tau} \rangle_0}{\langle T e^{-\int_0^\beta V(\tau) \, d\tau} \rangle_0}.
\end{equation}

A Hubbard-Stratonovich transformation then leads to

\begin{equation}
G(r_f, r_i, \tau_0) = \mathcal{N} \int D\phi \, e^{-\frac{1}{2} \int d\tau \, U^{-1}(\phi) \, g(r_f, r_i, \tau_0) \phi} \, \int D\phi \, e^{-\frac{i}{2} \int d\tau \, U^{-1}(\phi) \, \phi},
\end{equation}

where

\begin{equation}
g(r_f, r_i, \tau_0) \equiv -\langle T \bar{\psi}(r_f, \tau_0) \psi(r_i, 0) e^{i \int_0^\beta \phi(r, \tau) \delta n(r, \tau)} \rangle_0
\end{equation}

is a noninteracting correlation function, and $\mathcal{N} \equiv \langle T \exp(-\int_0^\beta V) \rangle_0^{-1}$ is a constant. So far everything is exact. What remains is to find an appropriate approximation for $g(r, r', \tau|\phi)$ and to do the resulting functional integral.

I evaluate \cite{5} with a second-order cumulant expansion,

\begin{equation}
g(r_f, r_i, \tau_0) = G_0(r_f, r_i, \tau_0) e^{\int C_1(r, \tau) \phi(r, \tau) + \int C_2(r, r', \tau) \phi(r, \tau) \phi(r', \tau')}.
\end{equation}

The $C_n$ are known in terms of noninteracting Green’s functions. For example,

\begin{equation}
C_1(r, \tau) = -i \frac{G_0(r_f, r, \tau_0 - \tau) G_0(r, r_i, \tau)}{G_0(r_f, r_i, \tau_0)}.
\end{equation}
The functional integral in (4) can be done exactly, leading to
\[ G(\mathbf{r}_f, \mathbf{r}_i, \tau_0) = A(\tau_0) G_0(\mathbf{r}_f, \mathbf{r}_i, \tau_0) e^{-S(\tau_0)}, \] (8)
where \( A \equiv \mathcal{N} \left[ \text{Det} \left( 1 - 2C_2 U \right) \right]^{-\frac{1}{2}} \) is a fluctuation determinant and
\[ S \equiv \frac{1}{2} \int_0^\beta d\tau \int d\tau' \int d^D r d^D r' \rho(\mathbf{r}, \tau) U_{\text{eff}}(\mathbf{r}, \tau, \mathbf{r}') \rho(\mathbf{r}', \tau'). \] (9)

Here \( \rho(\mathbf{r}, \tau) \equiv -i C_1(\mathbf{r} \tau) \) and \( U_{\text{eff}}(\mathbf{r}, \tau, \mathbf{r}') \equiv (U^{-1} - 2C_2)^{-1}_{\mathbf{r}, \tau'; \mathbf{r}'}. \)

I interpret (8) as follows: \( S \) is the Euclidean action for a time-dependent charge distribution \( \rho(\mathbf{r}, \tau) \) whose dynamics, governed by \( H_0 \), describes the charge density associated with an electron inserted into position \( \mathbf{r} = \mathbf{r}_i \) at time \( \tau = 0 \) and removed from \( \mathbf{r}_f \) at \( \tau_0 \). The charge interacts via an effective interaction \( U_{\text{eff}} \) that accounts for the modification of the electron-electron interaction by dynamic screening. My interpretation of \( -i C_1(\mathbf{r} \tau) \) as the charge density associated with a tunneling electron follows from extensive numerical studies and from the exact (at \( T = 0 \)) identity \( \int d\mathbf{D} r \rho(\mathbf{r}, \tau) = \Theta(\tau) \Theta(\tau_0 - \tau). \)

III. APPLICATION TO ONE-DIMENSIONAL SPINLESS FERMIONS

Here I assume a short-range interaction of the form \( U(x - x') = U_0 \lambda \Delta(x - x') \), where \( \Delta(x) \) is a broadened delta function with range \( \lambda \), and show that (8) correctly predicts a power-law DOS. First consider the “tree-level” instanton approximation, obtained by keeping the first cumulant \( C_1 \) only. In this case the action is
\[ S = \frac{U_0 \lambda}{2} \int_0^\beta d\tau \int_{-\infty}^\infty dx \rho(x, \tau)^2, \] (10)
and we can choose \( x_i = x_f = 0 \). In polar coordinates \( x = \mathbf{R} \cos \theta \) and \( v_F \tau = \mathbf{R} \sin \theta \), the low-energy noninteracting propagator can be written as
\[ G_0(R, \theta) = \frac{\sin(k_F R \cos \theta - \theta)}{\pi R}, \] (11)
which shows that \( G_0 \) falls off as \( 1/R \) in the Euclidean plane \( \mathbf{R} = (x, v_F \tau) \). Because \( \rho(x, \tau) \) has \( 1/R \) “singularities” at \( (0, 0) \) and \( (0, v_F \tau_0) \), the action diverges logarithmically, the infrared divergence from the tail of one singularity being cut off by the position of the other, a distance \( v_F \tau_0 \) away. \( S \) therefore diverges logarithmically in \( \tau_0 \), leading to the well-known power-law DOS in one dimension. In the limit \( \lambda \ll k_F^{-1} \) it can be shown that
\[ S = \frac{3}{8\pi} \frac{U_0 \lambda}{v_F} \ln \left( \frac{\tau_0}{a} \right), \] (12)
where \( a \) is a microscopic cutoff length, leading to a DOS
\[ N(\epsilon) = \text{const} \times \epsilon^\delta \quad \text{with} \quad \delta = \frac{3}{8\pi} \frac{U_0 \lambda}{v_F}. \] (13)
Including the second cumulant \( C_2 \) in does not qualitatively change this picture. However, it affects the value of the DOS exponent \( \delta \). Although the ultimate limitations of this method are not understood at present, application of the second-cumulant (or “one-loop”) analysis to the spinless Tomonaga-Luttinger model leads to an exponent \( \delta \) in exact agreement with the bosonization result [9].

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[13] These singularities arise from using the low-energy approximation [11] to the noninteracting propagator; they are not present in the exact expression. However, the \( \tau_0 \)-dependent part of \( S \) comes from the \( 1/R \) tail of \( G_0 \), which is correctly described by the low-energy form.