A theory for the conformal factor in quantum $R^2$-gravity

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Abstract

A new theory for the conformal factor in $R^2$-gravity is developed. The infrared phase of this theory, which follows from the one-loop renormalization group equations for the whole quantum $R^2$-gravity theory is described. The one-loop effective potential for the conformal factor is found explicitly and a mechanism for inducing Einstein gravity at the minimum of the effective potential for the conformal factor is suggested. A comparison with the effective theory of the conformal factor induced by the conformal anomaly, and also aiming to describe quantum gravity at large distances, is done.

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1. Introduction. It seems very unlikely nowadays that the fundamental problems which
afflict all theories of the early Universe —such as the cosmological constant problem— may
be solved by simply using some exact symmetry. Instead, it looks rather more reasonable
the idea that those fundamental problems should be treated dynamically, on the basis of
some approximate symmetries. In particular, it appears that low-energy (i.e. large distances
or infrared) dynamics of quantum gravity [1, 2, 3] should be relevant for the resolution of
the cosmological constant problem.

Some indications in favor of this viewpoint come from the study of quantum gravity on
the De Sitter background [1, 4, 5, 6], where it was shown that the graviton propagator may
grow without bound in the infrared (with a dominant contribution from the conformal part).
Then, as it was argued in [2], the De Sitter space is not the vacuum of quantum gravity, and
conformal factor theory should be developed as an effective theory for quantum gravity. It
is the purpose of the present letter to develop a theory of the conformal factor in quantum
$R^2$-gravity in a completely different fashion, as compared with the anomaly induced model
of ref. [2]. We will show that there exists an infrared asymptotically free solution for the
running coupling constants. Then, the effective potential for the conformal factor aiming to
describe the IR phase of the theory will be calculated. Applications of the theory to inducing
Einstein gravity with zero cosmological constant will be also discussed.

2. Theory for the conformal factor. Let us start from the action of the theory of
quantum $R^2$-gravity

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{\lambda} W - \frac{u}{3\lambda} R^2 - \frac{1}{\kappa^2} (R - 2\Lambda) \right], \quad (1)$$

where $W = R^2_{\mu\nu} - \frac{1}{3} R^2$, $\lambda$, $u$, $\kappa^2$ and $\Lambda$ are coupling constants. It is well-known that a theory
with the action (1) is multiplicatively renormalizable [7] and asymptotically free [8] (for a
general review of quantum $R^2$-gravity, see [9]). The only problem that prevents us from
considering the theory (1) as a very serious and consistent candidate for quantum gravity is
the well-known unitarity problem, that so often shows up in the context of higher-derivative
theories. However, it is very possible that the unitarity problem can be solved here by
simply using non-perturbative methods. But so far only perturbative techniques (actually,
to one-loop) have been developed in quantum $R^2$-gravity [4].

Our first interest will be to discuss the theory (1) on the following background

$$g_{\mu\nu} = e^{2\sigma} \eta_{\mu\nu}, \quad (2)$$
where $\sigma$ is the so-called conformal factor. Notice that, generally speaking, $\sigma$ is not a constant field. Substituting the background (2) into the classical action (1), one obtains the classical theory for the conformal factor

$$S = \int d^4x \left\{ \frac{12u}{\lambda} \left[ \sigma \Box^2 \sigma + 2(\partial \sigma)^2 \Box \sigma + (\partial \sigma)^2 (\partial \sigma)^2 \right] - \frac{6e^{2\sigma}(\partial \sigma)^2}{\kappa^2} + \frac{2\Lambda e^{4\sigma}}{\kappa^2} \right\}. \quad (3)$$

Here $(\partial \sigma)^2 = \eta^{\mu \nu} \partial_\mu \sigma \partial_\nu \sigma$. Now, starting from this point, one can use the approximation to the total quantum theory where all spin-two modes are frozen and only the $\sigma$-field is quantized, and gives the dominant contribution to the dynamics of the theory. Then, we immediately see that the infrared phase of quantum gravity in the sense of ref. [3] is impossible in such a model (see also the discussion in the next section), just because all the coefficients of the three higher-derivative terms in (3) are equal. Hence, we cannot set any of these coefficients equal to zero, because if we do so any higher-derivative dynamics disappears and we are left merely with effective theory of the conformal factor in Einsteinian gravity.

Furthermore, there is no need at all to use such approximation in the above theory. In fact, the $R^2$-gravity theory (1) is multiplicatively renormalizable and the one-loop $\beta$-functions of this theory are known [8, 9]. The conformal parametrization (2) in such a theory is simply a specific choice of background and all the one-loop calculations can be carried out in that background, taking into account quantum corrections coming from all the degrees of freedom. We will show this below.

First of all, let us write the one-loop renormalization group equations for the coupling constants of the theory (1) [8, 9]

$$\frac{d\lambda^{-1}(t)}{dt} = \frac{133}{10}, \quad \lambda(0) = \lambda,$n
$$\frac{du(t)}{dt} = -\lambda(t) \left[ \frac{10}{3} u^2(t) + \frac{183}{10} u(t) + \frac{5}{12} \right], \quad u(0) = u,$n
$$\frac{d\kappa^{-2}(t)}{dt} = -\frac{\lambda(t)}{\kappa^2(t)} \left[ \frac{10}{3} u(t) - \frac{13}{6} - \frac{1}{4u(t)} \right], \quad \kappa^2(0) = \kappa^2,$n
$$\frac{d\bar{\Lambda}(t)}{dt} = \frac{\lambda(t)}{\kappa^4(t)} \left[ \frac{5}{2} + \frac{1}{8u^2(t)} \right] + \frac{\lambda(t)\bar{\Lambda}(t)}{2} \left[ \frac{56}{3} + \frac{2}{3u(t)} \right], \quad \bar{\Lambda}(0) = \bar{\Lambda}, \quad (4)$$

where $\bar{\Lambda} \equiv 2\Lambda/\kappa^2$ (the RG equations (4) were first correctly obtained in the harmonic gauge in the paper by Avramidi [8]). Notice that, as usually, the last two equations (4) are gauge dependent, because here we do not construct the essential coupling constant as a combination of $\kappa^2$ and $\Lambda$ [8].
The solution of the first of the RG equations (4) is
\[ \lambda(t) = \frac{\lambda}{1 + 13.3\lambda t}. \] (5)

For \( \lambda > 0 \) (what corresponds to the classical positive action) we get asymptotic freedom in the ultraviolet limit [8]. In principle, however, considering \( R^2 \)-gravity as an effective, consistent theory of quantum gravity (let aside the well-known unitarity problem mentioned above) and hoping that the \( \beta \)-functions of quantum \( R^2 \)-gravity (4) —which were obtained in the ultraviolet domain— will (at least qualitatively) describe in fact the infrared phase of quantum gravity (i.e., large distances), we are then allowed to choose the initial value \( \lambda \) to be negative in (3), what leads to a theory that is asymptotically free in the IR limit.

We shall now obtain the IR asymptotic behavior of the solutions of eq. (4). Actually, these equations can be solved explicitly (as it is easy to see), however the most interesting point for us is their asymptotic behavior [8].

Calling \( a_1 \) and \( a_2 \) (with \( a_1 < a_2 \)) the roots of the polynomial in the second of eqs. (4), we have
\[ a_1 = -5.46714, \quad a_2 = -0.02289, \] (6)
and the solution to this equation can be written as
\[ u(t) = a_2 + (a_1 - a_2) \left[ 1 - \frac{u - a_1}{u - a_2} (1 + 13.3\lambda t)^{(a_2-a_1)/3.99} \right]^{-1} \to a_2, \quad t \to -\infty. \] (7)

That is, \( u(t) \) tends asymptotically to the fixed point \( a_2 \)—which is the biggest of the two roots of the second of eqs. (4) (notice that it is still negative). From the two last eqs. (4)), we get the asymptotic behavior
\[ \kappa^{-2}(t) \sim \frac{\kappa^{-2}}{13.3} \left( -\frac{10}{3} a_1 + \frac{13}{6} + \frac{1}{4a_1} \right) \frac{1}{1 + 13.3\lambda t} \to 0, \quad t \to -\infty \] (8)
and
\[ \bar{\Lambda}(t) \sim -\frac{\bar{\Lambda}}{13.3} \left( \frac{28}{3} + \frac{1}{3a_1} \right) \frac{1}{1 + 13.3\lambda t} \to 0, \quad t \to -\infty, \] (9)
respectively. Again, one may argue that the choice of negative initial values for \( \lambda \) and \( u \) is not very convenient for the action (4). Such a choice makes the classical action become negative at the Euclidean region and this indicates explicitly the presence of a tachyon. However, if \( R^2 \)-gravity is considered to be just an effective theory (and not a fundamental one) this may not be a big drawback and the case deserves consideration. For the asymptotic behaviors above, we see that we obtain quite reasonable results, that indicate the existence of an infrared phase coming from the RG.
Now we are going to consider the effective potential for the conformal factor (i.e., choosing $\sigma$ to be a constant) — a concept that was introduced in refs. [10, 11] in a different context. Notice that this effective potential of the conformal factor should presumably describe quantum gravity in the infrared phase given above by the RG equations. Introducing the notation $e^\sigma = \Phi$ and using the RG improved effective potential (see for example [12]), we get

$$V_{RG}(\Phi) = \tilde{\Lambda}(t) \Phi^4,$$

(10)

where $\tilde{\Lambda}(t)$ is defined as the solution of the RG equations (15) and $t = [2(4\pi)^2]^{-1} \ln(\Phi^2/\mu^2)$, $\mu^2 \equiv e^{2\sigma_0}$ (observe that $\mu^2$ is dimensionless). However, due to the fact that we need a closed, analytic solution in order to study $V(\Phi)$ (10) — and that we get explicitly only approximate asymptotic solutions — in our specific analysis we will have to restrict ourselves to a one-loop (non-improved) potential only. This can be easily obtained from (10). For example, using the Coleman-Weinberg normalization conditions, it becomes

$$V^{(1)}(\Phi) = \tilde{\Lambda} \Phi^4 + \frac{1}{2(4\pi)^2} \left[ \frac{\lambda}{\kappa^4} \left( \frac{5}{2} + \frac{1}{8u^2} \right) + \lambda \tilde{\Lambda} \left( \frac{28}{3} + \frac{1}{3u} \right) \right] \Phi^4 \left( \ln \frac{\Phi^2}{\mu^2} - \frac{25}{6} \right)$$

$$\equiv \tilde{\Lambda} \Phi^4 + A \Phi^4 \left( \ln \frac{\Phi^2}{\mu^2} - \frac{25}{6} \right).$$

(11)

In the above effective potential for the conformal factor, all degrees of freedom of the consistent theory of $R^2$-gravity are taken into account, and not just the ones induced by the quantum conformal mode, as was done in [11], or either just part of the contribution from $R^2$-gravity, as was done in [11]. It is not difficult to analyze the one-loop effective potential (11) for different choices of the coupling constants. The v.e.v. for the conformal factor is given by

$$\frac{\sigma_{vev}}{\sigma_0} = \frac{11}{3} - \frac{\tilde{\Lambda}}{A}, \quad A > 0,$$

(12)

which depends, of course, on the choice of the parameters of the theory. Let us recall the fact that the potential is defined only on the positive half-axis of $\Phi$. Further, we observe that the classical (as well as the one-loop) potential is defined for the value $\Phi = 0$ only asymptotically (since this is obtained for $\sigma \to -\infty$). This means that the classical potential grows monotonically and yields no preferred value for the conformal factor. Such a value, given by (12), appears only as a result of quantum corrections.

One can extend the above picture to a more dynamical situation, by choosing the background metric in the following form:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} e^{2\sigma},$$

(13)
where $\tilde{g}_{\mu\nu}$ is some curved metric (with constant curvature) and the conformal factor $\sigma$ is a constant one. Using then the technique of ref. [13] and the explicit form of the $\beta$-functions, one can find the one-loop effective potential for the conformal factor as follows (in the linear curvature approximation and using the normalization prescriptions of ref. [13])

\[
V^{(1)}(\Phi) = \tilde{\Lambda}\Phi^4 - \frac{1}{\kappa^2}\tilde{\Phi}^2 + \frac{\tilde{\Phi}^2}{2(4\pi)^2}\frac{\lambda}{\kappa^2} \left( \frac{10}{3} u - \frac{13}{6} - \frac{1}{4u} \right) \left( \ln \frac{\Phi^2}{\mu^2} - 3 \right) + \frac{\Phi^4}{2(4\pi)^2} \left[ \frac{\lambda}{\kappa^2} \left( \frac{5}{2} + \frac{1}{8u^2} \right) + \lambda\tilde{\Lambda} \left( \frac{28}{3} + \frac{1}{3u} \right) \right] \left( \ln \frac{\Phi^2}{\mu^2} - \frac{25}{6} \right) \equiv \tilde{\Lambda}\Phi^4 - \frac{1}{\kappa^2}\tilde{\Phi}^2 + A\Phi^4 \left( \ln \frac{\Phi^2}{\mu^2} - \frac{25}{6} \right) - B\tilde{\Phi}^2 \left( \ln \frac{\Phi^2}{\mu^2} - 3 \right). \tag{14}
\]

Here we suppose that the conformal factor in the metric (13) gives the dominant contribution to the effective potential. Such hypothesis is very reasonable for the early universe (before the GUT epoch) in an infrared phase for quantum gravity of the type (7)-(9). Then, in the effective action for quantum $R^2$-gravity it is reasonable to expand this action in powers of the constant curvature $\tilde{R}$, as was done above.

We are now ready to consider the conformal factor as being dynamical, in the same frame of an approximate description of quantum gravity as an effective theory. Then, there appears the very interesting possibility of inducing Einstein gravity in the infrared phase, as a result of a first-order $\tilde{R}$-curvature induced phase transition in the effective potential for the conformal factor.

Indeed, let us make the potential (14) be dimensionless, by considering, as usually, $\kappa^4V/\mu^4$ and by further introducing the dimensionless variables

\[
x \equiv \Phi^2 / \mu^2, \quad y \equiv \tilde{R} \kappa^2 / \mu^2, \quad \epsilon = \text{sgn} \, \tilde{R}, \tag{15}
\]

where $\mu^2 = e^{2\sigma_0}$, as above. The standard conditions for a first-order curvature-induced phase transition are [14] (a first example of a curvature-induced phase transition in QED in curved space has been given in [14])

\[
V(x_c, y_c) = 0, \quad \frac{\partial V}{\partial x} \bigg|_{x=x_c, y=y_c} = 0, \quad \frac{\partial^2 V}{\partial x^2} \bigg|_{x=x_c, y=y_c} > 0. \tag{16}
\]

After the phase transition has taken place, the effective action in the minimum has the form of classical Einstein gravity

\[
\Gamma = -\frac{1}{16\pi G_{\text{ind}}} \int d^4x \sqrt{-\tilde{g}} \left( \tilde{R} - 2\Lambda_{\text{ind}} \right), \tag{17}
\]
where the general expressions for $\Lambda_{\text{ind}}$ and $G_{\text{ind}}$ may be found in analogy with ref. [13] (see also [9]). Instead of doing this, we will present some examples which show the possibility of inducing (17).

Let us choose $u << 1, 1/u > \bar{\Lambda} \kappa^4, \lambda/u < 10^{-3}, \bar{\Lambda} \kappa^4 << 1$, and $\lambda << 1$. Then, by analysing eqs. (16) we get

$$\varphi^2_c \simeq e^{19/6} \mu^2 (\sigma_c \simeq 19/12 + \sigma_0), \quad \hat{R}_c \simeq -e^{19/6} \frac{\epsilon \lambda \mu^2}{16(4\pi)^2 u^2 \kappa^2}. \quad (18)$$

In particular, for $u \simeq 10^{-2}$ and $\lambda \simeq 10^{-4}$ we obtain a reasonable estimation that does not violate our approximation, what proves that the phase transition is possible. Then,

$$\frac{1}{16\pi G_{\text{ind}}} \simeq \frac{\varphi^2_c}{\kappa^2} \left(1 + \frac{\lambda}{48(4\pi)^2 u} \right),$$

$$\frac{\Lambda_{\text{ind}}}{8\pi G_{\text{ind}}} \simeq \varphi^4_c \left(\bar{\Lambda} - \frac{\lambda}{16(4\pi)^2 u^2 \kappa^4} \right). \quad (19)$$

As is easy to see, after convenient fine-tuning of the parameters of the theory (always within the allowed range of our approximation) we can get indeed $\Lambda_{\text{ind}} = 0$. In principle, one could suggest other choices for the parameters of the theory, for which the mechanism described above to induce Einstein gravity could also work. Of course, taking this mechanism as a serious possibility, we ought eventually to apply the point of view of ref. [11], where the metric $\hat{g}_{\mu\nu}$ was considered to be the physical metric corresponding to the real low-energy world.

3. **Anomaly induced theory for the conformal factor.** Let us now compare our theory with the effective theory of the conformal factor suggested in ref. [2]. Starting from the conformal anomaly [14] (for a general discussion see [16])

$$T_{\mu \nu} = b \left(F + \frac{2}{3} \Box R\right) + b'G + b'' \Box R, \quad (20)$$

where the contributions of the fields of spin 0, 1/2, and 1 to the coefficients of (20) are known, one can integrate over the conformal anomaly and thus get an anomaly-induced action [17].

Choosing, for simplicity, the conformal parametrization as in (4), we get the following [2, 17]

$$S_0 = \int d^4x \left\{ - \frac{Q^2}{(4\pi)^2} (\Box \sigma)^2 - \zeta \left[2(\partial \sigma)^2 \Box \sigma + (\partial \sigma)^2 (\partial \sigma)^2\right] - \frac{6e^{2\sigma}}{\kappa^2} (\partial \sigma)^2 + \frac{2\Lambda e^{4\sigma}}{\kappa^2} \right\}, \quad (21)$$

where $Q^2/(4\pi)^2 = 2b + 3b'$ and $\zeta = 2b + 2b' + 3b''$. As in expression (3), the last two terms give the contribution of the classical Einsteinian action.
It was suggested in [2] that spin-2 gravitational modes do not change the structure of the action that was considered as an effective action for quantum gravity in the infrared phase \((\zeta = 0)\). This proposal may be justifiable partially by the fact that inclusion of spin-2 gravitational modes in different models may be done by means of finite renormalization of the conformal anomaly coefficients. In other words, spin-2 quantum gravity modes give an additional contribution to \(T^{\mu}_{\mu}[[8, 19]]\).

However, as one can see if one starts from \(R^2\)-gravity without matter, the infrared phase in the sense of paper [2] cannot be realized, just because in (3) all the coefficients of the higher-derivative terms are equal. Of course, if one adds to the system conformal invariant matter, then the anomaly-induced action corresponding to matter gives a finite renormalization of the coefficients in (3) (working in the spirit of [2]), and then the IR phase may be realized for \(\zeta = 0\). But in this case we actually loose the direct interpretation (as a background) developed above, and are obliged to invent some kind of synthesis of the two approaches (what looks rather artificial).

4. Concluding remarks. In this letter we have discussed a consistent theory of the conformal factor in quantum \(R^2\)-gravity. In particular, we have calculated the effective potential for the conformal factor at the one-loop level and we have shown how it can be applied to inducing Einstein gravity. We have compared our theory with an alternative one in which the conformal factor is anomaly induced. It is remarkable that through our simple formulation the coupling of the conformal sector with interacting matter can be so naturally discussed. For, notice that if one would try to do the same using the anomaly-induced conformal factor theory, one would readily fall into difficulties, owing to the fact that the matter interaction terms (as \(\lambda \phi^4\) itself) induce an additional \(R^2\)-term in the conformal anomaly at the multiloop level. This term breaks the consistency conditions and, also, it cannot be simply integrated in the conformal anomaly in a unique way—in order to give the corresponding part in the anomaly-induced action [17].

In order to see this point with an explicit example, let us consider the \(\lambda \phi^4\)-theory with the action
\[
S_m = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{2} \xi R \phi^2 - \frac{1}{4!} f \phi^4 \right),
\]
interacting with the theory (1), where for simplicity we set \(\kappa^{-2} = \Lambda = 0\) (no dimensional coupling constants). Such theory is multiplicatively renormalizable (see the last of refs. [8] and [9]). Working in the background metric (13) and in the linear curvature approximation (i.e., \(\varphi^2 >> \tilde{R}, \sigma = \text{const.}\)), and using the results of refs. [8] (last reference) and [20] (where
the one-loop $\beta$-functions have been given explicitly), one finds

\[ V(\varphi) = -\frac{1}{2}\xi e^{2\sigma} \hat{R} \varphi^2 + \frac{1}{4!} f e^{4\sigma} \varphi^4 + \frac{1}{32(4\pi)^2} \left\{ 3f^2 + \lambda^2 \xi^2 \left[ 15 + \frac{3}{4u^2} + \frac{9\xi}{u^2} (3\xi - 1) \right] \right. \]

\[ + \lambda f \left[ -\frac{28}{3} + 8\xi + \frac{1}{u} \left( -18\xi^2 + 8\xi - \frac{1}{3} \right) \right] \right\} e^{4\sigma} \varphi^4 \left( \ln \frac{e^{2\sigma} \varphi^2}{m^2} - \frac{25}{6} \right) \]

\[ - \frac{1}{4(4\pi)^2} \left\{ f \left( \xi - \frac{1}{6} \right) + \lambda \xi \left[ 8\xi + \frac{5}{6} u + \frac{10}{3} u + \frac{1}{u} \left( -3\xi^2 + 6\xi + \frac{13}{12} \right) \right] \right\} \]

\[ \times e^{2\sigma} \hat{R} \varphi^2 \left( \ln \frac{e^{2\sigma} \varphi^2}{m^2} - 3 \right) . \]  

(23)

Here $m^2$ is a dimensional parameter. Using this potential one can estimate the possibility of inducing Einstein’s gravity as a result of a curvature-induced phase transition with order parameter $e^{2\sigma} \varphi^2$. The result is that such possibility exists indeed for a variety of theory parameters. Notice also that, in principle, one can generalize the above potential and calculate it for the theory [1] with the Einsteinian part of the action, and also develop an approach where $e^{2\sigma} > \varphi^2$ and find the effective potential for the conformal factor in this approach.

As a final remark let us note that the recently proposed average effective action [21] seems to be a most natural one, to be used for the study of the effective potential in the infrared sector. It would be very interesting (albeit not so easy) to try to apply this action to the theory of quantum gravity, in particular to the conformal-f actor sector (see also [11]).

**Acknowledgments**

SDO would like to thank I. Antoniadis and R. Percacci for helpful discussions, and the members of the Dept. ECM, Barcelona University, for their kind hospitality. This work has been supported by DGICYT (Spain), project No. PB90-0022, by CIRIT (Generalitat de Catalunya), and by the ISF Project RI1000 (Russia).
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