Realization of GHZ-like and W-like Third-order Spatial Correlation with Classical Light

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The spatial correlation of chaotic light has been an interesting and fundamentally important topic [G. Scarcelli, V. Berardi, and Y. Shih, Phys. Rev. Lett. 96, 063602 (2006)]. Here, we provide a unified model for the third-order spatial correlation effect with three chaotic lights. With the model we present a scheme to produce Greenberger-Horne-Zeilinger-like (GHZ-like) spatial correlation with classical light, and show that the third-order spatial correlation of the thermal light has the aspect similar to the W states.

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As one research area of light, high-order spatial correlation effect attracts much attention [1]. The investigation on high-order spatial correlation effect not only can help us to understand the nature of light effectively [2], but also is highly advantageous to bring it into practice [3, 4].

In 1995, the second-order spatial correlation phenomenon was observed experimentally by entangled photon pairs from spontaneous parametric down-conversion (SPDC) [5]. This spatial correlation effect was attributed to the nature of entanglement. This led to the researches about the theories and applications in the second-order spatial correlation effect with entanglement [6]. Several years later, it was discovered that the spatial correlation effect can also be realized with some classical light sources, and these classical light sources can reproduce many of the features obtained with entangled photon pairs, such as ghost imaging, subwavelength interference and so on [5].

As one aspect of high-order spatial correlation effect, third-order spatial correlation effect has been extensively studied both in theories and experiments [6–13]. In three-photon entangled states, it has been shown that there are two inequivalent classes of states: Greenberger-Horne-Zeilinger (GHZ) class [14] and W class [15]. The features of the spatial correlation of the two classes of the three-photon states have been investigated. It shows that the difference between the two classes of the states is the second-order spatial correlation: There is no second-order spatial correlation between two subsystems in the tripartite GHZ class state, however, there is second-order spatial correlation in the W class state [6]. The third-order spatial correlation of thermal light also has been discussed by many papers [6–13]. But what are the features of high-order spatial correlation of thermal light? How to implement the spatial correlation effect similar to GHZ state and W state? To answer these questions is important, not only for the foundations of high-order correlation function, but also for many potential applications based on high-order correlation function. In this paper, we present a unified model to discuss the third-order spatial correlation effect with three classical light sources. The three classical light sources are generated by applying three sets of time-variable phase masks onto three laser beams, respectively. It is shown that when the three sets of phase masks are different but correlated the behavior of third-order spatial correlation effect is similar to that of the GHZ state; when the three sets of phase masks are same, which is the case of thermal light, the behavior of the spatial correlation function is similar to that of the W state.

Fig. 1 presents the proof-of-principle setup to observe the third-order spatial correlation effect using three classical light sources. Firstly, three sets of phase masks \( \varphi_1(\xi, k) \), \( \varphi_2(\xi, k) \) and \( \varphi_3(\xi, k) \) are produced, where \( \xi \) denotes the coordinate on the phase masks, and \( k = 1, \cdots, K \) is the number of the phase modulation samples. Then, three light sources are obtained by applying the controllable phase masks \( \varphi_1(\xi, k) \), \( \varphi_2(\xi, k) \) and \( \varphi_3(\xi, k) \) onto three laser beams using spatial light modulators (SLM) with size \( D \), respectively. Here, we consider the quantum mechanical model in the photon-counting regime: the three-photon density matrix to describe the three light sources can be written as [16, 17]

\[
\rho_3 = \int d\xi_1 d\xi_2 d\xi_3 d\xi_4 d\xi_5 d\xi_6 \langle 1^{(1)}_{\xi_1} \rangle \langle 1^{(2)}_{\xi_2} \rangle \langle 1^{(3)}_{\xi_3} \rangle \\
\cdot \langle 1^{(1)}_{\xi_4} \rangle \langle 1^{(2)}_{\xi_5} \rangle \langle 1^{(3)}_{\xi_6} \rangle \cdot \exp\{i[\varphi_1(\xi_1, k) + \varphi_2(\xi_2, k) + \varphi_3(\xi_3, k) - \varphi_3(\xi_4, k) - \varphi_2(\xi_5, k) - \varphi_1(\xi_6, k)]\},
\]

(1)

where \( \langle \cdot \rangle = \frac{1}{K} \sum_{k=1}^{K} \cdot \) is defined to be the sample average over the \( K \) modulations, and the superscripts \( (n) \), \( (n = 1, 2, 3) \) denote the photon in the \( n \)-th light source, respectively. The two-photon state describing two of the three light sources can be obtained by tracing away one photon of the three-photon state. With loss of generality, when the third photon in the three-photon density matrix is traced out, the two-photon density matrix describing the two other of the three light sources is [16, 17]

\[
\rho_2 = \int d\xi \langle 1^{(3)}_{\xi} \rangle \rho_3 \langle 1^{(3)}_{\xi} \rangle.
\]

(2)
In this way, the density matrix to describe the three-photon state of the three light sources can be obtained:

\[\rho_3 \propto \int d\xi_1 d\xi_2 |\psi_3(\xi_1, \xi_2)\rangle\langle \psi_3(\xi_1, \xi_2)| + \prod_{n=1}^{3} \int d\xi_n |1_{\xi_n}^{(n)}\rangle\langle 1_{\xi_n}^{(n)}| \]

\[- \int d\xi_1 |1_{\xi_1}^{(1)}\rangle|1_{\xi_2}^{(2)}\rangle|1_{\xi_3}^{(3)}\rangle \langle 1_{\xi_1}^{(1)}|\langle 1_{\xi_2}^{(2)}|\langle 1_{\xi_3}^{(3)}| \]

\[- \int d\xi_1 |1_{\xi_1}^{(1)}\rangle \langle 1_{\xi_1}^{(1)}| \]

\[ \cdot \int d\xi_2 |1_{\xi_2}^{(2)}\rangle \langle 1_{\xi_2}^{(2)}| \]

\[ \cdot \int d\xi_3 |1_{\xi_3}^{(3)}\rangle \langle 1_{\xi_3}^{(3)}| ,\]

where the term \( |\psi_3(\xi_1, \xi_2)\rangle\) in Eq. (7) can be seen as GHZ-like or product states [14]:

\[|\psi_3(\xi_1, \xi_2)\rangle = \frac{1}{\sqrt{2}} \left[ |1_{\xi_1}^{(1)}\rangle |1_{\xi_2}^{(2)}\rangle |1_{\xi_3}^{(3)}\rangle + |1_{\xi_1}^{(1)}\rangle |1_{\xi_2}^{(2)}\rangle |1_{\xi_3}^{(3)}\rangle \right].\]

In the case that \(\xi_1 \neq \xi_2\), the state \( |\psi_3(\xi_1, \xi_2)\rangle\) can be regarded as a GHZ-like state. Meanwhile, when \(\xi_1 = \xi_2\), the state \( |\psi_3(\xi_1, \xi_2)\rangle\) can be considered as a product state. As a result, the third-order spatial correlation function is

\[G_3(x_1, x_2, x_3) = \frac{1}{2} \left[ \int d\xi_1 d\xi_2 |h_1(x_1, \xi_1) h_2(x_2, \xi_2) h_3(x_3, \xi_2)|^2 \right],\]

\[+ h_1(x_1, \xi_2) h_2(x_2, \xi_1) h_3(x_3, \xi_1)|^2 \right]\]

\[+ \prod_{n=1}^{3} \int d\xi_n |h_n(x_n, \xi_n)|^2 - \int d\xi_1 \prod_{n=1}^{3} |h_n(x_n, \xi_n)|^2 \]

\[- \int d\xi_1 d\xi_2 |h_1(x_1, \xi_1) h_2(x_2, \xi_2) h_3(x_3, \xi_2)|^2.\]

Here, we consider that the three light sources are placed at one same coordinate system. The first term on the right hand of Eq. (9) can be understood as follows [10, 13]: There is one photon in every light source, respectively. On the first hand, \(|h_1(x_1, \xi_1) h_2(x_2, \xi_2) h_3(x_3, \xi_2)|^2\) is the probability amplitude that a photon in position \(\xi_1\) of field 1 goes to detector \(D_1\), a photon in position \(\xi_2\) of field 2 and 3 goes to detector \(D_2\) and \(D_3\), respectively. On the other hand, \(h_1(x_1, \xi_2) h_2(x_2, \xi_1) h_3(x_3, \xi_1)|^2\) is the probability amplitude that a photon in position \(\xi_2\) of field 1 goes to detector \(D_1\), a photon in position \(\xi_1\) of field 2 and 3 goes to detector \(D_2\) and \(D_3\), respectively. Hence, the quantity in the first term on the right hand of Eq. (9) can be seen as the three-photon interference between these two different but indistinguishable alternatives: \(h_1(x_1, \xi_1) h_2(x_2, \xi_2) h_3(x_3, \xi_2)\) and \(h_1(x_1, \xi_2) h_2(x_2, \xi_1) h_3(x_3, \xi_1)\), which is shown in Fig. 2. Furthermore, when the distances \(d_1, d_2\) and \(d_3\) satisfy the condition:

\[\frac{1}{d_1} - \frac{1}{d_2} - \frac{1}{d_3} = 0,\]
the first term on the right hand of the third-order spatial correlation function in Eq. (9) can be simplified to

$$\int d\xi_1 d\xi_2 |h_1(x_1, \xi_1)h_2(x_2, \xi_2)h_3(x_3, \xi_3) + h_1(x_1, \xi_2)h_2(x_2, \xi_1)h_3(x_3, \xi_1)|^2 \right] \alpha 1 + \frac{\sin^2 \left( \frac{\pi D}{k} \left( \frac{x_1}{d_1} - \frac{x_2}{d_2} - \frac{x_3}{d_3} \right) \right)^2 }{ \left( \frac{\pi D}{k} \left( \frac{x_1}{d_1} - \frac{x_2}{d_2} - \frac{x_3}{d_3} \right) \right)^2 } \right] .$$  \tag{11}

There is a perfect spatial correlation among the three detection planes \((x_1, x_2, x_3)\) in the first term on the right hand of Eq. (9), and the spatial correlation effect is due to the superposition of these two three-photon amplitudes: \(h_1(x_1, \xi_1)h_2(x_2, \xi_2)h_3(x_3, \xi_3)\) and \(h_1(x_1, \xi_2)h_2(x_2, \xi_1)h_3(x_3, \xi_1)\), which is caused by the incoherent superposition of a set of GHZ-like and product states \(|\psi_3(\xi_1, \xi_2)\rangle\) in Eq. (7). Furthermore, the other terms on the right hand of the third-order spatial correlation function in Eq. (9) are constant, which are the background term of the third-order spatial correlation function. By tracing out the third photon in the three-photon density matrix in Eq. (8), the two-photon density matrix describing two of the three light sources is obtained:

$$\rho_2 = \int d\xi |1^{(3)}_\xi \rangle \langle 1^{(3)}_\xi |,$$

$$= \prod_{n=1}^2 \int d\xi_n |1^{(n)}_{\xi_n} \rangle \langle 1^{(n)}_{\xi_n}| .$$  \tag{12}

From Eq. (12), it is found that the two-photon density matrix is a product of the two single-photon density matrices \(\int d\xi_n |1^{(n)}_{\xi_n} \rangle \langle 1^{(n)}_{\xi_n}|,\ (n = 1, 2)\). As a result, there is no second-order spatial correlation. Therefore, with the three different but correlated classical light sources, the GHZ-like spatial correlation effect can be achieved.

In order to verify the GHZ-like third-order spatial correlation effect with the three different but correlated classical light sources, Fig. 4 presents the numerical simulation results using the schematic shown in Fig. 1. Three laser light sources with wavelength \(\lambda = 532\text{nm}\) illuminate three SLMs with size \(D = 2\text{mm}\), respectively. Then, one light beam transmitting one of the three SLMs illuminates an three-point object shown in Fig. 4a, and a single-pixel detector collects the light transmitting from the object. The other two light beams from the other two SLMs are measured by two multipixel detectors, respectively. The distances are as follows: \(d_1 = 10\text{cm},\ d_2 = d_3 = 20\text{cm}\), which guarantees the spatial correlation in the three detection planes. From Fig. 4b and Fig. 4c, it is found that the information of the object can be obtained by measuring the third-order spatial correlation functions. In addition, there are two ways of measurement operations to reconstruct the object. The first way is to move one multipixel detector \((x_3 = 0)\) fixed, and the magnification of the image is \(d_3/d_1 = 2\) shown in Fig. 4b. The second way is to move both multipixel detectors \((x_2 = x_3)\) together shown in Fig. 4c, and the magnification of the image is 1. However, the information of the object can not be reconstructed by measuring second-order correlation functions shown in Fig. 4d, e. The conclusion can be drawn from the simulated results above that with the three generated classical light sources, the GHZ-like third-order spatial correlation effect can be attained.

In this section, we will consider the case that the three sets of phase masks are same, leading to the W-like third-order spatial correlation. When the three sets of phase masks loaded on the three SLMs satisfy the condition: the three sets of phase masks are same: \(\varphi_1(\xi, k) = \varphi_2(\xi, k) = \varphi_3(\xi, k) \equiv \varphi(\xi, k)\), and they are statistically independent, uniformly distributed in the range \([0, 2\pi]\), which is equivalent to the setup of observing the third-order spatial correlation function with thermal light in Ref. 9, 10, 11, 12. As these phase masks obey the relationships: \(\langle \exp[i\varphi(\xi, k)] \rangle = 0\) and \(\langle \exp[i(\varphi(\xi, k) - \varphi(\xi_2, k))] \rangle = \delta(\xi - \xi_2)\), the three-photon density matrix to describe the three light sources is simplified to

$$\rho_3 \propto \int d\xi_1 d\xi_2 d\xi_3 |\psi_3(\xi_1, \xi_2, \xi_3)\rangle \langle \psi_3(\xi_1, \xi_2, \xi_3)| .$$  \tag{13}

Here, the term \(|\psi_3(\xi_1, \xi_2, \xi_3)\rangle\) can be seen as W-like or product states \(14\):

$$|\psi_3(\xi_1, \xi_2, \xi_3)\rangle = \frac{1}{\sqrt{6}} \sum P \prod_{n=1}^3 |1^{(n)}_{\xi_p(n)}\rangle ,$$  \tag{14}

where the sum \(\sum_P\) runs over the all \(3!\) possible permutations \(P\) of the set of integers 1, 2, 3. In this situation, the three-photon density matrix describing the three light
Bell-like or product states:

ent superposition of a set of W-like and product states.

As a result, the spatial correlation in the third-order and second-order spatial correlation functions with the three light sources are similar to that with the W entangled state, which has been studied theoretically and experimentally in Ref. 4, 11, 13.

In the analytical results above, it shows that the GHZ-like and W-like third-order spatial correlation can be realized with three classical light sources. Without loss of generality, this scheme can be expand to achieve GHZ-like and W-like N-order spatial correlation with N classical light sources. Furthermore, in the GHZ-like N-order spatial correlation scheme, the N sets of phase masks satisfy the condition: \( \varphi_1(\xi, k) = \sum_{n=2}^{N} \varphi_n(\xi, k) \). Moreover, the phase masks \( \varphi_n(\xi, k) \) (\( n = 2, 3, \ldots, N \)) are statistically independent, uniformly distributed in the range \([0, 2\pi]\). Meanwhile, in the W-like N-order spatial correlation scheme, the N sets of phase masks loaded on the N laser beams are same, and they are statistically independent, uniformly distributed in the range \([0, 2\pi]\).

With the two types of generated light sources, the GHZ-like and W-like N-order spatial correlation effect can be achieved, respectively.

In conclusion, the third-order spatial correlation effect with three classical light sources is investigated. In the scheme to observe the third-order spatial correlation effect with three classical light sources, three sets of light patterns are generated by modulating the wavefronts of three laser beams randomly in time. It is found that when the three sets of phase masks are different but correlated, the three-photon density matrix to describe the three light source can be seen as the incoherent superposition of a set of GHZ-like and product states, leading to the spatial correlation exists only in the third-order spatial correlation, and does not exists in the second-order spatial correlation. Meanwhile, when the three sets of phase masks are same, the three-photon density matrix to describe the three light source shows behavior similar to the W entangled state, which leads to spatial correlation in the third-order and second-order spatial correlation functions. Moreover, this scheme can be expanded to achieve GHZ-like and W-like N-order spatial correlation effect. These results can deepen the basic understanding of the high-order spatial correlation function effectively which is fundamentally important. Furthermore, it may lead to novel high-order coherence effects and applications, such as realizing the novel effect of entanglement.

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