NON-SINGLETS IN
SEMI-INCLUSIVE DIS AND INCLUSIVE $E^+E^-$ ANNIHILATION

Ekaterina Christova†, Elliot Leader††
† Institute for Nuclear Research and Nuclear Energy, Sofia, echristo@inrne.bas.bg
†† Imperial College, London, e.leader@imperial.ac.uk

Abstract

We show that non-singlets in semi-inclusive DIS with $\pi^\pm$ determine without assumptions $\Delta u_\nu$, $\Delta d_\nu$ and $D_{u_\nu}^{x+\nu}$. Non-singlets in SIDIS and inclusive $e^+e^-$-annihilation with $K^\pm$ determine $s - \bar{s}$ and $\Delta s - \Delta \bar{s}$, but an assumption is necessary – we choose as most natural $D_{K^0}^{K^+K^-} = 0$. Measurements with charged $K^\pm$ and neutral $K^0 + \bar{K}^0$ in semi-inclusive DIS and $e^+e^-$-annihilation determine $D_{u,d,s}^{K^+K^-}$ without any assumptions and allow tests of LO approximation in QCD.

1. The difference between the spin-dependent structure functions on proton and neutron, $g_1^p$ and $g_1^n$, measures the non-singlet (NS) flavour combination $\Delta q_3 = (\Delta u + \Delta \bar{u}) - (\Delta u + \Delta \bar{u})$: $g_1^p - g_1^n = \frac{1}{6} \Delta q_3 \left( 1 + \frac{\alpha_s}{2\pi} + \ldots \right)$. (1)

This measurable quantity is free of any uncertainties and, being a NS, this holds in all orders of QCD. Actually, $\Delta q_3$ is the only quantity determined in inclusive DIS without any assumptions.

In general, for all quantities that measure NS combinations we have that first, in all orders in QCD and second, in their $Q^2$-evolution, no new parton densities appear. That last one is especially useful as it allows to compare directly measurable quantities at different $Q^2$ and different experiments.

We consider measurable quantities that single out NS’s in semi-inclusive DIS (SIDIS), both polarized and unpolarized, and in inclusive $e^+e^-$-annihilation:

$$e + N \to e + h + X, \quad e^+ + e^- \to h + X$$ (2)

and discuss the non-perturbative information about the parton distribution (pdf) and fragmentation (ff) functions one can obtain.

2. Recently we pointed out [1] that the difference cross section in SIDIS measures a NS both in pdf's and in ff's. This follows immediately from $C$-inv. of the ff's:

$$D_{\tilde{h}} = 0, \quad D_{h}^{\tilde{h}} = -D_{\tilde{h}}^{\tilde{h}}, \quad D_{i}^{\tilde{h}} = D_{i}^{\tilde{h}} - D_{i}^{\tilde{h}}.$$ (3)

Here $\tilde{h}$ is the $C$-conjugate of the hadron $h$. Then for the difference cross section $\sigma_{N}^{h-k} \equiv \sigma_{N}^{h} - \sigma_{N}^{k}$, we obtain that in all orders in QCD gluons cancel, i.e. no $g(x)$ and no $D_{g}^{h}(z)$:

$$\tilde{\sigma}_{N}^{h-k}(x, z) = \frac{1}{9} \left[ 4uV \otimes D_{u}^{h-k} + dV \otimes D_{d}^{h-k} + (s - \bar{s}) \otimes D_{s}^{h-k} \right] \otimes \tilde{\sigma}_{qg}(\gamma q \to qX).$$ (4)
Here $\hat{\sigma}_{qq}$ is the perturbatively QCD-calculable partonic cross section $q\gamma^* \to q + X$:

$$\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}_{qq}^{(1)} + ...$$ (5)

Thus, $\hat{\sigma}_{N}^{h-\bar{h}}$ is a NS in both the pdf’s and ff’s, and in addition – each term is such a NS. Eq. (4) shows that it is sensitive only to the valence quark pdf’s and $(s - \bar{s})$. Depending on the properties of the final measured hadrons $h - \bar{h}$, one can single out different pieces of $\hat{\sigma}_{N}^{h-\bar{h}}$.

3. It is clear that the above arguments hold for polarized SIDIS as well, except that the unpolarized pdf’s and unpolarized partonic cross sections are replaced by the polarized ones. Then, measuring the difference polarization asymmetry $A_{N}^{h-\bar{h}}$ [1]:

$$A_{N}^{h-\bar{h}}(x, z) = \frac{1 + (1 - y^2)}{2y(2 - y)} \frac{\Delta \sigma_{N}^{h} - \Delta \sigma_{N}^{\bar{h}}}{\sigma_{N}^{h} - \sigma_{N}^{\bar{h}}}$$ (6)

one obtains information about $\Delta u_V$, $\Delta d_V$ and $(\Delta s - \Delta \bar{s})$ without requiring any knowledge about $\Delta g$, $g$ and $D_{g}^{h}$. If $h = \pi^\pm$ one determines $\Delta u_V$ and $\Delta d_V$ without knowledge even about the stange quarks $s$ and $\Delta s$ (due to SU(2) symmetry of the pions $D_{s}^{\pi^+ - \pi^-} = 0$). In JLab this asymmetry will be measured with enough accuracy [2]. If $h = K^\pm$ measuring $A_{N}^{K^+ - K^-}$ one can obtain information about $(\Delta s - \Delta \bar{s})$, provided we have determined $\Delta u_V$ and $\Delta d_V$ in $A_{N}^{K^+ - \pi^-}$.

In (4) the ff’s $D_{q}^{h-\bar{h}}$ enter. Usually these quantities are simulated by the LUND model. As pointed out in [3], and very recently also in [4], the result of the analysis for the polarized pdf’s (especially for the sea-quarks) depend on the precision of our knowledge of the ff’s. Our goal is to discuss the information that one can obtain about ff’s using only measurable quantities from unpolarized SIDIS and/or inclusive $e^+e^-$-annihilation. We shall show that when analyzing $A_{N}^{K^+ - K^-}$ an additional assumption is needed. We choose as most reasonable one $D_{d}^{K^+} = D_{d}^{K^-}$.

4. Following (4), measurements of the difference ratio $R_{N}^{h-\bar{h}}$ of unpolarized SIDIS to inclusive DIS:

$$R_{N}^{h-\bar{h}} = \frac{\sigma_{N}^{h} - \sigma_{N}^{\bar{h}}}{\sigma_{DIS}^{h}}$$ (7)

can be used to determine, in any order in QCD, the difference of the ff’s, $D_{q}^{h-\bar{h}}$, and to obtain information about $s - \bar{s}$.

If $h = \pi^\pm$, SIDIS on protons and neutrons provide two independent measurements to determine $D_{u}^{\pi^+ - \pi^-}$ (SU(2) inv. implies $D_{u}^{\pi^+ - \pi^-} = -D_{d}^{\pi^+ - \pi^-}$).

It is more complicated if $h = K^\pm$, because we cannot use SU(2) inv. to reduce the number of ff’s. If we assume $D_{d}^{K^+ - K^-} = 0$, then measurements on $p$ and $n$ provide two measurements for $D_{u}^{K^+ - K^-}$ and $(s - \bar{s}) \otimes D_{s}^{K^+ - K^-}$. Note that because $D_{s}^{K^+ - K^-}$ is presumably not a small quantity (s is a valence quark for $K^-$ and non-valence quark for $K^+$), the product $(s - \bar{s}) \otimes D_{s}^{K^+ - K^-}$ can be zero (non-zero) only if $s - \bar{s} = 0$ ($s - \bar{s} \neq 0$).

From the quark content of $K^\pm$, the assumption $D_{d}^{K^+ - K^-} = 0$ seems very reasonable if $K^\pm$ are directly produced. However, if resonances have to be taken into account, and
\(K^\pm\) appear also as decay products of the resonances (which is very often the real situation in experiment) this assumption is not undisputed. It’s a bad luck that, as we shall show, even inclusive \(e^+e^- \to K^\pm X\) does not help to determine \(D_d^{K^+ - K^-}\). However it helps to determine other NS’s.

5. In general, the cross section for inclusive \(e^+e^- \to hX\) is [5]:

\[
\frac{d\sigma^h}{dz\,d\cos\theta} = \frac{3}{8}(1 + \cos^2\theta)\,d\sigma_T^h(z) + \frac{3}{4}(1 - \cos^2\theta)\,d\sigma_L^h(z) + \frac{3}{4}\cos\theta\,d\sigma_A^h(z) \tag{8}
\]

where \(d\sigma_{T,L,A}^h\) are the transverse, longitudinal and asymmetric cross sections: \(d\sigma_T^h, d\sigma_L^h\) are measured through the total (integrated over \(\cos\theta\)) cross section, \(d\sigma_A^h\) is singled out through the forward-backward asymmetry. In LO we have \((d\sigma_L^h\) does not contribute in LO):

\[
d\sigma_T^h(z) \ = \ \int_{-1}^{1} \! d\cos\vartheta \left( \frac{d\sigma_T^h}{dz\,d\cos\theta} \right) = 3\sigma_0 \sum_q \hat{e}_q^2 D_q^{h+h}, \quad \sigma_0 = \frac{4\pi\alpha^2_{\text{em}}}{3s} \tag{9}
\]

\[
A_{FB}^h(z) \ = \ \left[ \int_{-1}^{0} - \int_{0}^{1} \right] \! d\cos\vartheta \left( \frac{d\sigma_T^h}{dz\,d\cos\theta} \right) = 3\sigma_0 \sum_q \frac{3}{2} \hat{a}_q D_q^{h-h} \tag{10}
\]

i.e. \(d\sigma_T^h(z)\) measures \(D_q^{h+h}\) while \(A_{FB}^h(z)\) measures the NS’s \(D_q^{h-h}\) we are interested in. Assuming that the photon and the neutral \(Z^0\)-boson are exchanged we have:

\[
\hat{e}_q^2(s) = \hat{e}_q^2 - 2e_q v_e v_q \Re Z + (v_e^2 + a_e^2) \left[ (v_q)^2 + (a_q)^2 \right] |h_Z|^2 \\
\hat{a}_q = 2a_e a_q \left( -e_q \Re h_Z + 2v_e v_q |h_Z|^2 \right). \tag{11}
\]

Here \(e_q\) is the electric charge of the quark \(q\) and

\[
v_e = -1/2 + 2\sin^2\theta_W, \quad a_e = -1/2,
\]

\[
v_q = I_3^q - 2e_q \sin^2\theta_W, \quad a_q = I_3^q, \quad I_3^u = 1/2, \quad I_3^d = -1/2.
\]

\[h_Z = [s/(s - m_Z^2 + im_Z\Gamma_Z)]/\sin^22\theta_W. \tag{12}\]

If \(h = K^\pm\) we have

\[
\frac{A_{FB}^{K^+ - K^-}}{3\sigma_0} = \frac{3}{2} \left\{ \hat{a}_u D_u^{K^+ - K^-} + \hat{a}_d (D_d^{K^+ - K^-} + D_s^{K^+ - K^-}) \right\}. \tag{13}\]

If we combine this measurement with the SIDIS measurements \(R_N^{K^+ - K^-}\) on unpolarized \(p\) and \(n\) (see (eq.(4), \(h = K^\pm\)) we obtain 3 measurements for the 4 unknown quantities \(D_{u,d,s}^{K^+ - K^-}\) and \((s - \bar{s})\). We need an assumption: either \(s = \bar{s}\) or \(D_d^{K^+ - K^-} = 0\). We consider \(D_d^{K^+ - K^-} = 0\) as a reasonable one. Note that up to now all analysis of the experimental data have been performed assuming \(s = \bar{s}\) and measurements of \(R_N^{K^+ - K^-}\) will offer a real possibility to justify this assumption.

6. If in unpolarized SIDIS and \(e^+e^-\)-inclusive processes, in addition to the charged \(K^\pm\) also the neutral \(K^0 + \bar{K}^0\) are observed, we can use SU(2) invariance to relate the neutral to the charged Kaon ff’s:

\[
D_u^{K^+ + K^-} = D_d^{K^0 + \bar{K}^0}, \quad D_d^{K^+ + K^-} = D_u^{K^0 + \bar{K}^0}, \quad D_s^{K^+ + K^-} = D_s^{K^0 + \bar{K}^0}. \tag{14}
\]
and no new ff’s appear in the cross sections. In LO we have:

\begin{align}
  d\sigma_T^{K^+K^-(K^0\bar{K}^0)}(z) &= 3\sigma_0 \left[ (e_u^2 - \bar{e}_d^2) m_Z^2 (D_u - D_d)^{K^+K^-} \right] \\
  d\sigma_p^{K^+K^-(K^0\bar{K}^0)}(x, z) &= \frac{1}{9} \left[ 4(u + \bar{u}) - (d + \bar{d}) \right](D_u - D_d)^{K^+K^-} \\
  d\sigma_n^{K^+K^-(K^0\bar{K}^0)}(x, z) &= \frac{1}{9} \left[ 4(d + \bar{d}) - (u + \bar{u}) \right](D_u - D_d)^{K^+K^-} 
\end{align}

i.e. we have three measurements – $R^{K^+K^-(K^0\bar{K}^0)}_{p,n}$ and $d\sigma_T^{K^+K^-(K^0\bar{K}^0)}$ to determine $(D_u - D_d)^{K^+K^-}$, but presumably at different $Q^2$. As $(D_u - D_d)^{K^+K^-}$ is a NS we can easily evolve it in $Q^2$ and form possible tests of the LO approximation. For example:

\begin{equation}
  \frac{9 d\sigma_p^{K^+K^-(K^0\bar{K}^0)}(x, z, Q^2)}{d\sigma_T^{K^+K^-(K^0\bar{K}^0)}(z, m_Z^2)_{\perp Q^2}} = \left[ \frac{4(u + \bar{u}) - (d + \bar{d})}{3\sigma_0 (e_u^2 - \bar{e}_d^2) m_Z^2} \right](x, Q^2)^2
\end{equation}

Here $d\sigma_T^{K^+K^-(K^0\bar{K}^0)}(z, m_Z^2)_{\perp Q^2}$ denotes that the data is measured at $m_Z^2$ and then evolved to $Q^2$ according to the DGLAP equations. As this is a NS, the evolution does not introduce any new quantities. Combined with measurements of any of the two quantities $d\sigma_T^{K^+K^+(K^0\bar{K}^0)}(z)$ or $R_{p,n}^{K^+K^+(K^0\bar{K}^0)}(x, z)$, we obtain enough measurements to determine $D^{K^+K^-}_{u,d,s}$ without any assumptions (recall that $(s + \bar{s})$ is known from DIS measurements). Contrary to $d\sigma_T^{K^+K^-(K^0\bar{K}^0)}(z)$, though the total cross section $d\sigma_T^{K^+K^+(K^0\bar{K}^0)}(z)$ is very precisely measured at $s = m_Z^2$, its $Q^2$-evolution involves the poorly known $D^\rho_s$ and large errors are introduced when combined with the SIDIS measurements performed at much lower $Q^2$ [3]. Note that when forming the cross sections for the difference $K^+K^- - (K^0 + \bar{K}^0)$, measurement of the neutral Kaons is essential in order to cancel the $s$-quark contributions, while for the sum $K^+K^- + (K^0 + \bar{K}^0)$ the neutral Kaons will only improve the statistics.

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