Spectral anisotropies and intermittency of plasma turbulence at ion kinetic scales

Simone Landi
Dipartimento di Fisica e Astronomia, Università di Firenze, Firenze, Italy, and
Osservatorio Astrofisico di Arcetri, INAF, Firenze, Italy

Luca Franci
School of Physics and Astronomy, Queen Mary University of London, London, UK

Emanuele Papini and Andrea Verdini
Dipartimento di Fisica e Astronomia, Università di Firenze, Firenze, Italy.

Lorenzo Matteini
LESIA, Observatoire de Paris,
Université PSL, CNRS, Sorbonne Université,
Univ. Paris-Diderot, Sorbonne Paris Cité
place J. Janssen 5, F-92192 Meudon, France

Petr Hellinger
Astronomical Institute, CAS, Bocni II/1401, CZ-14100 Prague, Czech Republic and
Institute of Atmospheric Physics, CAS, Bocni II/1401, CZ-14100 Prague, Czech Republic

(Dated: April 9, 2019)

By means of three dimensional high-resolution hybrid simulations we study the properties of the magnetic field spectral anisotropies near and beyond ion kinetic scales. By using both a Fourier analysis and a local analysis based on multi-point 2nd-order structure function techniques, we show that the anisotropy observed is less than what expected by standard wave normal modes turbulence theories although the non linear energy transfer is still in the perpendicular direction, only advected in the parallel direction as expected balancing the non-linear energy transfer time and the decorrelation time. Such result can be explained by a phenomenological model based on the formation of strong intermittent two-dimensional structures in the plane perpendicular to the local mean field that have some prescribed aspect ratio eventually depending on the scale. This model support the idea that small scales structures, such as reconnecting current sheets, contribute significantly to the formation of the turbulent cascade at kinetic scales.

I. INTRODUCTION

In-situ measurements of solar wind plasma and electromagnetic fields show spectra with power-laws scaling spanning several decades in frequency, f [e.g. 1–3]. Power-laws support an interpretation in terms of turbulent fluctuations, although the rich variety of spectral features is not easily explained in the framework of known turbulent theories and phenomenologies.

For frequencies corresponding to the magneto-hydrodynamic (MHD) range, $10^{-2} \text{ Hz} \lesssim f \lesssim 10^{-1} \text{ Hz}$ at 1 AU, spectra of the magnetic field, the ion bulk velocity, and the electric field are dominated by the transverse components with respect to the ambient magnetic field $B_0$. Magnetic fluctuations typically show a Kolmogorov-like spectrum (i.e., having the $-5/3$ slope), while the velocity field power is slightly shallower (typically close to $-3/2$) [e.g. 4–8] and strongly coupled which electric field fluctuations [9] as expected in the ideal MHD regime.

At higher frequencies, $f \gtrsim 10^{-1} \text{ Hz}$, corresponding to length scales approaching typical ion kinetic scales, a change in the nature of the self-similar spectra of the fluctuations is observed. Near these scales, a spectral transition (break) appears in the magnetic field fluctuations, separating the MHD inertial range from a second, steeper, power-law interval at kinetic scales. The spectral index of magnetic fluctuations after the break varies between $(−4,−2)$ [e.g. 10,12], although it tends to cluster around a slope of $−2.8$ for higher frequencies [13,14]. At those scales, the ion bulk velocity decouples from magnetic field fluctuations and its spectrum shows (if any) an even steeper power-law slope [e.g. 15]; on the contrary, the electric field spectrum is shallower at sub-ion scales [16,17] and becomes dominant over the magnetic field power, with a typical spectral index around $−0.8$ [e.g. 18,19], consistent with the scaling predicted by the generalized Ohm’s law [20]. What ion characteristic kinetic scale is the most relevant in determining the position of such transition has not been clearly identified, although observational evidences [21,22] and numerical simulations [23] suggest that it is likely related to the larger between the ion inertial length and the ion gyroradius for extreme values of the the plasma beta and to a combination of both in the intermediate-beta regime. At ion kinetic scales, also the nature of the compressible fluctuations changes: an increase of the magnetic compressibility [24,25] and a reduced variance anisotropy [26] are observed; the density, while it follows a Kolmogorov spectrum at MHD scales, shows a flattening near the break [27], to then be strongly cou-
plied to the magnetic field fluctuations at sub-ion scales\cite{28}.

An important aspect is how the cascade toward small scales proceeds with respect to the parallel and perpendicular component of a DC magnetic field. At fluid scales, three-dimensional (3D) numerical simulations \cite[e. g. 29-40]} have highlighted the role of spectral anisotropy in the formation of the MHD turbulent spectra, including the case when the role of the radial expansion is taken into account \cite{41,42}. In particular several numerical simulations \cite[e. g. 31,32] have shown that spectra tend to organize themselves in the so called critical balance state \cite{43,45} in which the cascade proceeds by transferring energy only to scales where the non-linear interactions of eddies are faster than the linear propagating time of the normal modes of the fluid system. At such scales this balance predicts that energy is transferred mostly in direction perpendicular to the (local) mean field following a Kolmogorov cascade, while the parallel spectrum should decrease as $k_\parallel^{-2}$ and, as a consequence, the two-dimensional (2D) spectral anisotropy should scale as $k_\parallel \propto k_\perp^{1/3}$. Observations seem to confirm to some extent that such energy transfer occurs in the solar wind plasma \cite{7,46-48}.

The critical balance argument can be extended to the kinetic (sub-ion) scales, provided that one uses the electron-velocity eddy turn-over time as the relevant non-linear time and takes into account the dispersive nature of the fluctuations in this regime. Regardless of the normal mode used, either low-frequency strongly-oblique propagating Kinetic Alfvén Waves (KAW) \cite{49} or quasi-parallel high-frequency whistler modes \cite{50}, standard wave-wave interaction models of turbulence predict a $k_\perp^{-7/3}$ scaling at sub-ion scales, and a further increase of the spectral anisotropy as $k_\parallel \propto k_\perp^{1/3}$ with a steeper (than MHD scales) parallel spectrum $\propto k_\parallel^{-5}$. A magnetic power decreasing as $k_\perp^{-7/3}$ has indeed observed in several simulations using electron MHD \cite[e. g. 50-54]. Hall MHD \cite{55} and girokinetic models \cite{56} and numerical evidences of the critically balanced scaling $k_\parallel \propto k_\perp^{1/3}$ have been reported by Cho and Lazarian \cite{53}.

Steeper magnetic field spectra, $\propto k_\perp^{-2.8}$, in good agreement with solar wind observations, have been reproduced in girokinetic \cite{57} simulations where the steepening (with respect to the $-7/3$ prediction) was attributed to electron Landau damping. Using a frequency analysis TenBarge and Howes \cite{58} found that these simulations are in agreement with a $k_\parallel \propto k_\perp^{1/3}$ anisotropy scaling, although a perpendicular spectrum steeper than $k_\perp^{-7/3}$ should lead to a significant stronger anisotropy. Indeed, the critical balance condition both at MHD and kinetic scales can be written as $k_\parallel \propto k_\perp b(k_\perp)$, with $b(k_\perp)$ being the magnetic fluctuations amplitude at the scale $k_\perp$; as the spectrum is steeper, $b(k_\perp)$ is steeper and the anisotropy increases. For a spectral slope of the magnetic power $P_B = b^2(k_\perp)/k_\perp \propto k_\perp^{-2.8/3}$, $b(k_\perp) \propto k_\perp^{-0.9}$ and the critical balance predicts $k_\parallel \propto k_\perp^{-1}$. In this Kolmogorov-like phenomenological model only a slope $-7/3$ is consistent with the scaling $k_\parallel \propto k_\perp^{1/3}$.

Retaining the effects of linear ion and electron Landau damping, models \cite{59} and numerical simulations \cite{60,61} predict a strong variability of the slope in the perpendicular spectrum together with an increase in the spectral anisotropy in the sub-ion range. Such variability is correlated with the ratio between the non-linear energy transfer time and the propagation/damping properties of low-frequency highly-oblique electromagnetic waves (the Kinetic Alfvén waves). Some variability of the magnetic energy slope has been reported also in 2D \cite{62} and 3D full pic simulations \cite{63}. However Hellinger et al. \cite{64}, analysing the properties of the 3rd order structure functions in 2D high resolution numerical simulations at low and moderate values of the plasma $\beta$, have shown that the sub-ion scales electromagnetic field power can be reasonably well described by an inertial (i. e. dissipativeless) Hall-MHD range. Moreover, fluid 2D Hall-MHD simulations, where kinetic damping effects are not taken into account, has been able to produce spectra steeper than $-7/3$ in extremely good agreement with analogous hybrid pic simulations and observations \cite{65}.

Several 3D numerical simulations investigating sub-ion scales have highlighted the role of coherent structures and intermittency in characterizing the turbulent cascade and its dissipation \cite[e. g. 66,68]. Boldyrev and Perez \cite{69} have shown that, if at small scales, energy is concentrated in spatially localized structures, the spectrum of magnetic fluctuations steepens following a $-8/3$ power law \cite[see also 70], the anisotropy should follow a $k_\parallel \propto k_\perp^{1/3}$, and the parallel spectrum a $-7/2$ power-law. Meyrand and Galtier \cite{71}, in the framework of EMHD, report a similar perpendicular scaling but a steeper parallel spectrum, $\propto k_\parallel^{-5}$, consistent with a turbulence cascade driven merely by a 2D dynamic. More recently, \cite{72} reported 3D hybrid Vlasov simulations where parallel and perpendicular spectra are qualitatively consistent with the model proposed by \cite{69} at $\beta_p = 1$ while steeper slopes are observed for a lower value of the proton plasma beta ($\beta_p = 0.2$). Alternatively, Franci et al. \cite{73}, by analyzing the spectral properties of a high-resolution hybrid PIC simulation able to cover almost two decades in k-vectors, found that the spectral anisotropy, though large in the inertial range, becomes frozen in the sub-ion range following a $k_\parallel \propto k_\perp$ scaling. A similar observation have been reported by Arzamasskiy et al. \cite{74}.

In this work we present results from a high-resolution hybrid (fluid electrons, kinetics protons) 3D Direct Numerical Simulation (DNS) of freely-decaying turbulence in presence of a mean magnetic field. This simulation is very similar to that described in Franci et al. \cite{73}, but uses a very high spatial grid resolution to focus on the spectral anisotropies at sub-ion scales. Fourier spectra show a transition near the ion scales and the simulation is able to produce a well defined spectrum in the kinetic range over more than one decade. A closer inspection of the magnetic and the density spectra shows that the spectral anisotropy with respect to the mean field stops to increase near the ion scales but the turbulence cascade still proceeds essentially in the perpendicular direction. A local analysis, using the multi-point 2nd-order structure function technique confirms that the anisotropy is frozen also with respect to the local mean field. Such result can be understood assuming that at ion scales energy is mostly contained in intermittent coherent structures whose characteristic filling factor.
follows a prescribed scaling, generalizing the model proposed by [79]. This seems to be consistent with models [75, 78] and numerical simulations [79] which show that current sheets actively participate in the formation of the turbulent cascade.

II. NUMERICAL SETUP

We employ the hybrid particle-in-cell (HPIC) code CAMELIA (Current Advance Method Et cycliLe lAprefro), where the electrons are considered as a massless, charge neutralizing fluid, whereas the ions (protons) are described by a particle-in-cell model and are advanced by the Boris scheme (see Matthews [80] for detailed model equations). Units of mass, length, and time are the ion (proton) mass $m_i$, its inertial length $d_i$ and the inverse of its gyroradius $1/\Omega_i$. We use $512^3$ collocation points in a spatial cubic grid with resolution $\Delta x = \Delta y = \Delta z = 0.0625 d_i$ and 2048 particle per cell (ppc) representing ions. Accumulation of energy at small scales is prevented adopting a resistivity coefficient $\eta = 1.5 \times 10^{-3} 4\pi n_i e^2 \Omega_i^{-1}$. The ions are advanced with a time step $\Delta t = 0.00625 \Omega_i^{-1}$, while the magnetic field $B$ is advanced with a smaller time step $\Delta t_i = \Delta t/10$. The initial condition consists of a uniform and neutral density plasma $n = n_i = n_e$, a uniform magnetic field directed along $z$, $B_0 = B_0 \hat{z}$, where species have isotropic and equal temperatures, $T_i = T_e$. The relative strength of the thermal and field pressure is measured in term of the ion (electron) plasma beta, $\beta_e = 8\pi n_i k_B T_i / B_0^2$, $k_B$ being the Boltzmann’s constant. The setup of the simulation is similar to that shown in [73]: Decorrelated, Alfvén-like fluctuations, initially isotropic in the $k$-vector range $0.20 < kd_i < 0.80$, perturb the initial condition and are left to decay. The energy is equally partitioned between velocity and magnetic field with $B^\text{rms} / B_0 \sim 0.38$, the electron and proton temperatures are the same $\beta_e = \beta_i = 0.5$. The analysis here reported has been performed at the maximum of the turbulent activity, $t = t_{\text{max}} \equiv 36 \Omega_i^{-1}$, when the current density rms reaches its peak [73]. The maximum of turbulent activity time here is less than the simulation shown in [73], simply because the energy is injected from the beginning at smaller scales. Anyways rms and physical quantities evolve in a very similar manner.

III. RESULTS

An overview of the spectral properties of the simulation is shown in Fig. 1 where we report the magnetic (top) and density (bottom) power spectra as a function of the perpendicular (solid) and parallel (dashed) $k$-vectors. The original spectra, shown as dotted lines in each panel are post-processed to clean them from numerical noise accumulated at small scales in a way similar to that shown in [73]: we set to zero the amplitude of each 3D Fourier mode which is below a given noise threshold and all the modes whose wave-number modulus is larger than a defined cut-off $k_{\text{c}}$. Here the noise level has been chosen as $10^{-11}$ and the cut-off is $k_{\text{c}} = 2k_{\text{max}}/3 \approx 170 k_0$ with $k_0 = 2\pi/L_\chi$ and $k_{\text{max}}$ the Nyquist wave-vector. For both fields, the spectra reduced in the perpendicular direction show a well defined $-3$ power law in almost one-decade for $kd_i < 1$. At large scales the filtered and unfiltered are superimposed while the filtering procedure allow to extend the slopes significantly in the high $k$-vector region. As expected, for both fields, at all scales the parallel reduced spectrum is subdominant with respect to the perpendicular counterpart, meaning that a spectral anisotropy is formed; however its power-law is the same of the perpendicular reduced spectrum, meaning that the spectral anisotropy produced at large scales $k_{\perp} d_i > 1$ is then frozen at sub-ion scales $k_{\perp} d_i < 1$.

To understand the reduced 1D power spectra it is useful to analyse the axisymmetric 2D spectra. In Fig 2 the 2D reduced spectrum of the magnetic fluctuations in log-log scale is reported. At large scales a spectral anisotropy is developed with the power mostly confined in the region $k_{\perp} > k_\|$. At higher wave-vectors, $k_{\perp} d_i \gtrsim 1$, this anisotropy appears to remain almost constant and most of the power seems confined in a region $k_{\parallel} \propto k_{\perp}$ (highlighted with the dashed white-line here used only as reference). Such behaviour is different from the spectral anisotropy ($k_\parallel \propto k_{\perp}^{1/3}$) predicted by the critical balance conditions in KAW or Whistler mediated turbulence [49, 53] and also to that proposed by [69] where a spectral anisotropy $k_\parallel \propto k_{\perp}^{2/3}$ was obtained assuming intermittency and critical balance condition. A closer inspection to the 2D spectrum shows that in the region where the spectral anisotropy is
At the lowest level of approximation, the 2D magnetic power spectrum at small scales can be described as a spectrum that is almost independent on \( k_\parallel \) and bounded by the condition \( k_\perp \leq C k_\parallel^3 \) and where each perpendicular spectrum at constant frozen (constant aspect ratio) the iso-levels are almost independent of \( k_\parallel \) meaning that the energy transfer occurs essentially in the perpendicular direction. To better elucidate this, cuts with \( k_\parallel = \text{const} \) of the 2D reduced spectra are shown in the bottom panel of Fig. 2. For small value of \( k_\parallel \), they have a significant level of energy at small value of \( k_\perp \) with different slopes. On the other hand, at approximately \( k_\perp d_i = 2 \) they all collapse and follow the same power-law which is well approximated by \( P_{2D}^{B}(k_\perp, k_\parallel = \text{const}) \propto k_\perp^{-4} \). Cuts with larger values of \( k_\parallel \) are almost deprived of energy for small values of \( k_\perp \) while then increase near \( k_\perp = C k_\parallel \), following approximately the same power law, \( \propto k_\perp^{-4} \) (until small-scale numerical effects become dominant).

At the lowest level of approximation, the 2D magnetic power spectrum at small scales can be described as a spectrum that is almost independent on \( k_\parallel \) and bounded by the condition \( k_\perp \leq C k_\parallel^3 \) and where each perpendicular spectrum at constant

\[
\begin{align*}
k_\parallel &\text{ follows a } k_\perp^{-\alpha} \text{ power-law:} \\
p^{B}_{2D} = P_0 k_\perp^{-\alpha} &\quad \text{if } |k_\parallel| < k_\parallel^3 \\
0 &\quad \text{Otherwise}
\end{align*}
\]

Such model produces a reduced perpendicular spectrum \( k_\perp^{-\alpha} \) while the parallel spectrum is \( k_\parallel^{-\beta} \). If we now assume \( \alpha = 1 \) and \( \beta = 4 \) we obtain the same power-law \(-3\) for both perpendicular and parallel spectra. Within this a model one should observe \( \beta = 10/3 \) if the spectral anisotropy was the one proposed by \( \{69\} \) (\( \alpha = 2/3 \)) while \( \beta = 6/3 \) is the power-law required to obtained the reduced spectra in the self-similar model of turbulence (\( \alpha = 1/3 \)). Both values are significantly shallower than the behavior observed in our simulation.

One important question that arises here is whether the \( z \)-direction maps correctly the parallel direction of the (local) mean field that is the relevant framework with respect to which the spectral anisotropy properties are supposed to be valid \( \{7, 30, 47, 53, 81, 82\} \). To this aim, the scale-dependent anisotropy can be computed using a multi-point second order structure function \( (SF) \) calculated in the local frame. Since the slope of the fluctuations is near \(-3\) in the sub-ion regime, the usual 2-points second-order structure function is not suitable to study spectral anisotropy at such scales and multi-points second-order structure functions must be used \( \{82, 83\} \). Here we use the 3- and 5-points, defined respectively as \( SF^{(3)}(A; r) = \langle (A(x+r) - 2A(x) + A(x-r))^2 \rangle \) and \( SF^{(5)}(r; A) = \langle (A(x+2r) - 4A(x+r) + 6A(x) - 4A(x-r) + A(x-2r))^2 \rangle \), which are roughly proportional to \( r^{m-1} \) for a spectral slope \( \propto k^{-m} \) if \( m < 5 \) and \( m < 9 \) respectively. Fig. 3 reports the structure functions for the magnetic and the density fields measured in the local frame defined by the parallel to the local mean \( \vec{B}_t \) direction \( \vec{l} = \vec{B}_t/|\vec{B}_t| \) (dash-dotted lines), \( \hat{\lambda} \) (solid) the normal to the local mean and the fluctuation \( \vec{B}_t \) direction, and \( \hat{\xi} = \hat{\lambda} \times \hat{l} \) (dashed) which completes the reference frame. As expected the 2nd-order structure functions are more energetic in the perpendicular component: for separations smaller than about \( d_i \) they follow approximately a power-law whose index \( m - 1 \) corresponds to a spectral index \( m = 2.8 \), not far to what observed in Fourier spectra. The important result here is however related to behaviour of the direction parallel to the local field, which follows the same power-law as the perpendicular counterpart, shifted roughly by a factor of five to larger scales. The results is quite robust being almost the same for both the 3-point and 5-point computed SFs and confirm that, in this simulation where a relatively strong mean field is present, global spectra reproduce the spectral anisotropies at local scales.

The SFs here reported are computed using the fields once the filtering procedure has been applied in Fourier space: In the same figure (dotted-lines) are also reported the SFs calculated without the filtering to show how the noise affects the computation of the SF. Although strongly localized in Fourier space, i.e. at small scale, the noise impacts significantly SFs over a much larger interval in the separation scales and it is stronger as the statistic used for the SFs increases. The reason lies in the fact that the high-\( k \) noisy modes behave as...
FIG. 3. Local second-order three (left) and five (right) point structure functions for the magnetic (top panel), and density (bottom) fields. Solid and dashed lines are the two perpendicular components \( \hat{\lambda} \) and \( \hat{\xi} \) while the dash-dotted is for the parallel one \( \hat{l} \). Power laws are drawn in thin solid black lines as a reference and are expressed in terms of the corresponding slopes in Fourier spectra. Dotted lines are the same quantities computed without using the filtering procedure outlined in the text.

white noise which in each point \( i \) introduces a random fluctuations \( \pm \delta f \) in the computation of the \( SF_2(f) \) of the quantity \( f \): the last, being \( \propto \sum_i f^2/N \) (\( N \) is the number of points used for the statistics), is polluted by the noise roughly as \( \langle \delta f^2 \rangle / \sqrt{\langle \delta f^2 \rangle} \propto N \). We have verified that it is indeed the case in this simulation. It should be noticed also, that such result is consistent with Arzamasskiy et al. [74] in which a spectral anisotropy scaling as \( k_\parallel \propto k_\perp \) with respect to the local mean field was observed in analogous hybrid kinetic simulations by using a different techniques based on Fourier space filtering.

IV. DISCUSSION

Both the global analysis with Fourier spectra and that based on 2nd-order structure functions in the local magnetic field frame show that the scaling of the spectral anisotropy is significantly less than what expected from current theories of turbulence at sub-ion scales. In particular the anisotropy tends to become frozen once ion scales are reached. It is worth underling that although the growth of the spectral anisotropy is reduced, the 2D Fourier spectrum of the magnetic field shows that the non-linear energy transfer happens in the perpendicular direction and is only advected along the parallel direction, thus suggesting that, like in the standard model of the critical balance [43] and in RMHD models of MHD turbulence [84, 85], the non-linear energy transfer time-scale is bounded by the decorrelation time which turns out to be that of the linear wave-propagation time.

A possible interpretation of the results follows and extends the arguments of Boldyrev and Perez [69] based on the presence of intermittent features [86] localized in a 2D space normal to the local mean field. Building on that argument, let us assume that at the scale \( \lambda \), the energy density \( B^2(k_\perp) \) fills only a fraction \( \lambda^\alpha \propto k_\perp^{-\alpha} \) of the space. In this case the energy density scales as \( E(k_\perp) \propto B^2(k_\perp)k_\perp^{-\alpha} \); then, using the standard expression for the non linear time \( \tau_{NL} \propto 1/(k_\perp^2 B(k_\perp)) \) at sub-ion scales [e.g., 50] and requiring that the energy flux is conserved through the scales, \( E(k_\perp)/\tau_{NL} \propto k_\perp^{\alpha} \), the power spectrum follows:

\[
P_B \propto E(k_\perp)/k_\perp \propto k_\perp^{-4(7+\alpha)} , \tag{2}
\]

and the magnetic fluctuations follow a power law given by

\[
B(k_\perp) \propto k_\perp^{-4(2-\alpha)} . \tag{3}
\]

The requirement that the non linear time is bounded by the decorrelation time, assuming the latter given by dispersive modes [e.g., 82], requires \( l \propto \lambda/b_\lambda \) or \( k_\parallel \propto k_\perp b(k_\perp) \) from which the spectral anisotropy is

\[
k_\parallel \propto k_\perp^{4(\alpha+1)} . \tag{4}
\]

For \( \alpha = 0 \) one recovers the standard \(-7/3\) and \( k_\parallel \propto k_\perp^{1/3} \), while taking the filling factor proportional to \( \lambda \) (\( \alpha = 1 \)) Boldyrev
We have analyzed the spectral anisotropy properties of turbulence in a high-resolution hybrid 3D simulation. The setup of the simulation is close to that of the simulation discussed in [73] except for the resolution at small scales: here the grid step has been chosen 4 times smaller than previously to better cover the sub-ion scales where our analysis is focussed. The global spectral properties at sub-ion scales are very similar to what already observed: a slope steeper than −7/3 and −8/3 predicted by wave and intermittent based models, consistent with what observed in the solar wind and planetary magnetospheres and to what already observed both in 2D and 3D simulations [20, 23, 65, 72, 74, 87, 88].

The main aspect investigated in this work is that in the global frame, both the parallel and perpendicular spectra show the same scaling after the ion break, with $k^{-5/3}$. This picture has been confirmed further by performing a local analysis, by means of multi-point SFs, suggesting that in the simulation the aspect ratio of the turbulent fluctuations at sub-ion scales is maintained approximately constant. We have then proposed a model, based on a generalization of the one by [69], in which the spectral slopes (and thus the spectral anisotropies) are related to the nature of the intermittent structures that populate the turbulence at small scales and how they fill the total volume.

This result is in favour of a model where intermittency driven by 2D structures in the plane perpendicular to the main field plays a dominant role in determining the spectral properties at kinetic scales; such structures should have two characteristic lengths that both reduce with decreasing the scale and in a way that satisfies a specific aspect ratio, which can also be scale-dependent. One possibility is that the turbulent cascade is strongly mediated by magnetic reconnection events where the aspect ratio of the current sheets is the important parameter that sets the efficiency of the reconnection process [e.g. 89–95]. We have already shown in Franci et al. [79] and Papini et al. [65] that the formation of a sub-ion range in 2D simulations is strongly correlated with the emergence of reconnecting events between larger scale MHD vortices. Moreover it has been shown that current sheet disruption can have consequences on the spectral properties both at MHD [76, 77, 96, 97] and kinetic scales [75, 78]. Our present findings seem to support this scenario.

Solar wind observations [e.g. 13, 98] show that the magnetic (and density) fluctuations converge toward a $-2.8$ scaling at smaller scales, a behaviour also well captured by high resolution 2D simulations [20, 23, 87]. In the framework of the model presented in this work, this corresponds to $\alpha = 1.4$, implying that the aspect ratio of the structure is not self-similar but depends on the scales. In this case it is expected that the spectral anisotropy in the sub-ion range will moderately continue to increase as $k_0 \propto k_\perp^{0.8}$ and the parallel spectrum will follow a $-3.1$ power law.

Although measurements of the spectral anisotropy are possible in the solar wind and have been performed in the inertial range [7, 47, 68], this aspect has been more scarcely addressed at kinetic scales in part because of observational constraints.
Spectral anisotropy in the sub-ion range has been discussed by Chen et al. [99] using Cluster measurements and using two-points II order structure functions. They found that the perpendicular spectrum was steeper than the theoretical $-7/3$, but that the spectral index in the parallel direction was less steep than that expected by standard wave-mediated turbulence and close to $-3$. We note that these values are consistent with the predictions from our model discussed above. However, as discussed by the authors, one should be careful in interpreting the slope of parallel spectrum, since when using 2-point scales. For example a close to $-8/3$ scaling will give $k_\parallel \propto k_\parallel^{-1/6}$ and a parallel spectrum $\propto k_\parallel^{-11}$. In the near future, to disentangle from such models will require multi-point, multi-scale analysis space missions as it has been proposed in the Plasma 2020 Decadal Survey [100-102].

VI. ACKNOWLEDGMENTS

The work has been funded by Fondazione Cassa di Risparmio di Firenze through the projects Giovani Ricercatori Protagonisti’ and the project HYPERCRHEL. PH acknowledges GACR grant 15-10057S. Numerical simulations and data reduction was performed at the CINECA facilities under the programs Accordo Quadro INAF-CINECA (2017-2019) (grant C3A22a) and ISCAR-B (grant HP10BP6XYP). This work was supported also by the Programme National PNST of CNRS/INSU co-funded by CNES. We thanks Tullio for stimulating discussions and Dr. Ku Fu from Nancy’s Tibet University to point out us useful improvements.

[1] R. Bruno and V. Carbone, Living Reviews in Solar Physics 10, 2 (2013)
[2] K. H. Kiyani, K. T. Osman, and S. C. Chapman, Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 373 (2015), 10.1098/rsta.2014.0155
[3] C. H. K. Chen, Journal of Plasma Physics 82, 535820602 (2016), arXiv:1611.03386 [physics.plasm-ph]
[4] J. J. Podesta, D. A. Roberts, and M. L. Goldstein, Astrophys. J. 664, 543 (2007)
[5] C. Salem, A. Mangeney, S. D. Bale, and P. Veltri, Astrophys. J. 702, 537 (2009)
[6] D. A. Roberts, Journal of Geophysical Research (Space Physics) 115, A12101 (2010)
[7] R. T. Wicks, T. S. Horbury, C. H. K. Chen, and A. A. Schekochihin, Physical Review Letters 106, 045001 (2011) arXiv:1009.2427 [physics.plasm-ph]
[8] J. A. Tessein, C. W. Smith, B. T. MacBride, W. H. Matthaeus, M. A. Forman, and J. E. Borovsky, Astrophys. J. 692, 684 (2009)
[9] C. H. K. Chen, S. D. Bale, C. Salem, and F. S. Mozer, Astrophys. J. Let. 737, L41 (2011), arXiv:1105.2390 [physics.space-ph]
[10] R. J. Leamon, C. W. Smith, N. F. Ness, W. H. Matthaeus, and H. K. Wong, J. Geophys. Res. 103, 4775 (1998)
[11] C. W. Smith, K. Hamilton, B. J. Vasquez, and R. J. Leamon, Astrophys. J. Let. 645, L85 (2006)
[12] F. Sahraoui, M. L. Goldstein, G. Belmont, P. Canu, and L. Rezeau, Physical Review Letters 105, 131001 (2010)
[13] O. Alexandria, J. Saur, C. Lacombe, A. Mangeney, J. Mitchell, S. J. Schwartz, and P. Robert, Physical Review Letters 103, 165003 (2009) arXiv:0906.3236 [physics.plasm-ph]
[14] O. Alexandria, C. Lacombe, A. Mangeney, R. Grappin, and M. Maksimovic, Astrophys. J. 760, 121 (2012) arXiv:1212.0412 [astro-ph.SR]
[15] J. Safaránková, Z. Němeček, F. Němec, L. Pfech, C. H. K. Chen, and G. N. Zastenker, Astrophys. J. 825, 121 (2016)
[16] S. D. Bale, P. J. Kellogg, F. S. Mozer, T. S. Horbury, and H. Reme, Phys. Rev. Lett. 94, 215002 (2005)
[17] P. J. Kellogg, S. D. Bale, F. S. Mozer, T. S. Horbury, and H. Reme, Astrophys. J. 645, 704 (2006) physics/0602179
[18] J. E. Stawarz, S. Eriksson, F. D. Wilder, R. E. Ergun, S. J. Schwartz, A. Pouquet, J. L. Burch, B. L. Giles, Y. Khotyaintsev, O. Le Contel, P. A. Lindqvist, W. Magnes, C. J. Pollock, C. T. Russell, R. J. Strangeway, R. B. Torbert, L. A. Avanov, J. C. Dorelli, J. P. Eastwood, D. J. Gershman, K. A. Goodrich, D. M. Malaspina, G. T. Marklund, L. Mirioni, and A. P. Sturman, Journal of Geophysical Research (Space Physics) 121, 11 (2016)
[19] L. Matteini, O. Alexandra, C. H. K. Chen, and C. Lacombe, MNRAS 466, 945 (2017)
[20] L. Franci, S. Landi, L. Matteini, A. Verdini, and P. Hellinger, Astrophys. J. 812, 21 (2015) arXiv:1506.05999 [astroph.SR]
[21] C. H. K. Chen, L. Leung, S. Boldyrev, B. A. Maruca, and S. D. Bale, Geophys. Res. Let. 41, 8081 (2014)
[22] R. Bruno and L. Trenche, Astrophys. J. Let. 787, L24 (2014) arXiv:1404.2191 [astroph.SR]
[23] L. Franci, S. Landi, L. Matteini, A. Verdini, and P. Hellinger, Astrophys. J. 833, 91 (2016) arXiv:1610.05158 [physics.space-ph]
[24] C. S. Salem, G. G. Howes, D. Sundkvist, S. D. Bale, C. C. Chaston, C. H. K. Chen, and F. S. Mozer, Astrophys. J. Let. 745, L9 (2012)
[25] K. H. Kiyani, S. C. Chapman, F. Sahraoui, B. Hnat, O. Fauvaquie, and Y. V. Khotyaintsev, Astrophys. J. 763, 10 (2013) arXiv:1008.0525 [physics.space-ph]
[26] J. J. Podesta and J. M. TenBarge, Journal of Geophysical Research (Space Physics) 117, A10106 (2012)
[27] J. Safaránková, Z. Němeček, F. Němec, L. Pfech, A. Pitá, C. H. K. Chen, and G. N. Zastenker, Astrophys. J. 803, 107 (2015)
[28] C. H. K. Chen, S. Boldyrev, Q. Xia, and J. C. Perez, Physical Review Letters 110, 225002 (2013) arXiv:1305.2958 [physics.space-ph]
[29] S. Oughton, E. R. Priest, and W. H. Matthaeus, Journal of Fluid Mechanics 280, 95 (1994)
