SCALING RELATIONS IN CORONAL MASS EJECTIONS AND ENERGETIC PROTON EVENTS ASSOCIATED WITH SOLAR SUPERFLARES

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ABSTRACT

In order to discuss the potential impact of solar “superflares” on space weather, we investigated statistical relations among energetic proton peak flux with energy higher than 10 MeV ($F_p$), CME speed near the Sun ($V_{CME}$) obtained by Solar and Heliospheric Observatory/LASCO coronagraph, and flare soft X-ray peak flux in the 1–8 Å band ($F_{SXR}$) during 110 major solar proton events recorded from 1996 to 2014. The linear regression fit results in the scaling relations $V_{CME} \propto F_{SXR}^{\alpha}$, $F_p \propto F_{SXR}^{\beta}$, and $F_p \propto V_{CME}^{\gamma}$ with $\alpha = 0.30 \pm 0.04$, $\beta = 1.19 \pm 0.08$, and $\gamma = 4.35 \pm 0.50$, respectively. On the basis of simple physical assumptions, on the other hand, we derive scaling relations expressing CME mass ($M_{CME}$), CME speed, and energetic proton flux in terms of total flare energy ($E_{flare}$) as $M_{CME} \propto E_{flare}^{2/3}$, $V_{CME} \propto E_{flare}^{1/6}$, and $F_p \propto E_{flare}^{5/6} \times V_{CME}^{5/6}$, respectively. We then combine the derived scaling relations with observation and estimated the upper limit of $V_{CME}$ and $F_p$ to be associated with possible solar superflares.

Key words: solar–terrestrial relations – stars: activity – Sun: coronal mass ejections (CMEs) – Sun: flares

1. INTRODUCTION

Solar flares are the biggest explosion in the solar system where magnetic field energy stored in the active region (AR) corona is rapidly released through magnetic reconnection (Hudson 2011; Shibata & Magara 2011). During flares, coronal plasma is sometimes ejected out into interplanetary space (coronal mass ejections, CMEs; Illing & Hundhausen 1986; Gopalswamy 2009). A significant portion of AR magnetic field energy released during flares is converted to the kinetic energy of CMEs (Emseie et al. 2012). The resultant plasma and magnetic field structures detected at 1 au are called interplanetary CMEs (ICMEs; Zhang et al. 2007). When the helical magnetic field of ICME ejecta (magnetic cloud) or draped magnetic field in interplanetary sheath ahead of magnetic cloud have a strong southward component, geomagnetic storms occur (Klein & Burlaga 1982; Tsurutani et al. 1988).

Energetic protons are accelerated both at CME-driven shocks and flare sites (Reames et al. 1996; Tsurutani et al. 2009). In the case of the shock acceleration mechanism, the efficiency of particle acceleration depends on shock Mach number and its normal direction, and particles are known to be most efficiently accelerated when the shocks are quasi-parallel (Kennel et al. 1984; Tsurutani & Lin 1985). Accelerated particles arrive at Earth when Earth is magnetically well connected to the shock front (solar proton event, SPE). The shock acceleration mechanism is generally thought to be the dominant of the two, but extremely high energy protons (with energy ~GeV) accelerated at the flare site right after the flare onset are discussed as being responsible for the prompt component of ground level enhancement (GLE) events (Aschwanden 2012).

Large SPEs are often associated with large solar flares or fast CMEs (Gopalswamy et al. 2004). The largest SPE after 1970 in terms of $E > 10$ MeV proton peak number flux ($F_p$) occurred on 1972 August 4. The estimated $F_p$ is higher than $6 \times 10^8$ pfu (particle flux unit; particles cm$^{-2}$ s$^{-1}$; Kurt et al. 2004). Modern extreme events that occurred on 2012 July 23 recorded the $F_p$ value of $6.5 \times 10^8$ pfu observed with the STEREO-A spacecraft. In the 2012 July 23 event, $F_p$ had a peak when interplanetary shock wave passed the spacecraft (Russell et al. 2013; Gopalswamy et al. 2014). Such peaks of energetic particle flux, usually in lower-energy bands ($E < \sim 10$ MeV), associated with the passage of ICME-driven shocks are called energetic storm particle events (Bryant et al. 1962).

CMEs and SPEs are the two main drivers of hazardous space weather outcomes such as potential radiation hazards for space astronauts, geomagnetic storms, and telecommunication failures. The most intense geomagnetic storm in recorded history occurred in 1859 (Loomis 1861), which was almost certainly driven by fast ICME associated with huge solar flare (Carrington 1859; Tsurutani et al. 2003). On the other hand, fast CMEs and intense energetic protons might have had important influence on ancient terrestrial environment through chemical processes in the terrestrial atmosphere (Airapetian et al. 2016). Young stars (like our ancient Sun) that rotate very rapidly are known to frequently produce superflares (10–10$^7$ times more energetic flares than the largest solar flares ever observed) possibly due to its active dynamo. Recent observations by the Kepler satellite revealed that some solar-type stars with rotation periods longer than 10 days can also produce superflares, though not very frequently (Maehara et al. 2012).

The most widely used index of solar flare magnitude is the soft X-ray (SXR) peak flux monitored by GOES satellite in the 1–8 Å passband. The proportionality between hard X-ray fluence and SXR peak flux of individual flares is known as the Neupert effect, and it is thought to be a casual index of released magnetic field energy during flares (Neupert 1968). Solar flares are known to follow the frequency distribution of the power-law form in terms of SXR peak flux (Yashiro et al. 2006). The largest ever solar flare observed in X-ray that occurred on 2003 November 4 saturated the GOES X-ray detector in the 1–8 Å passband. The estimated flare class by linear extrapolation is X28, while X-ray class estimation based on ionospheric response resulted in X45 ± 5 (Thomson et al. 2004). Evidence of a spike of the carbon-14 isotope between 774 and 775 is discovered from tree rings indicative of

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huge solar energetic particle events driven by solar superflares (Miyake et al. 2012). Gopalswamy et al. (2010) estimated maximum flare energy to be of the order of 10^{33} erg (flare of the ∼X1000 class) based on observed maximum magnetic field strength in a sunspot and largest AR size. Based on this flare energy estimation, Gopalswamy (2011) presented estimated maximum CME speed, associated with the largest class of flares, to be 7200 km s⁻¹ assuming the CME mass to be of the order of 10¹⁸ g.

There are studies on statistical relations among flare SXR peak flux observed with GOES (Fₜ₟), CME speed near the Sun (Vₜ₟), and Fₚ from various data sets. Fₚ is known to be correlated both with Vₜ₟ and Fₜ₟ (Gopalswamy et al. 2003b; Gopalswamy 2011). From 19 SPEs occurring during the maximum to minimum of solar cycle 23, Gopalswamy et al. (2003a) reported statistical relations, Fₚ ∝ V̂ₜ₟₁⁷⁻ and Fₚ ∝ Fₜ₟₀.⁶³ respectively. Also, the correlation coefficients of Fₜ₟ and Vₜ₟ for events during 1996–2007 and GLE events are reported to be 0.37 and 0.50, respectively (Yashiro & Gopalswamy 2009; Gopalswamy et al. 2012). The correlation between Fₜ₟ and kinetic energy of CME (∆ = M_CMEV_CME²/2 with M_CME being CME mass estimated from coronagraph observations) are also reported (Gopalswamy 2009).

In this Letter, we analyzed the statistical relations among CME speed, peak energetic proton flux, and flare SXR peak flux from a single list of events, i.e., 110 SPEs recorded from 1996 to 2014 whose peak proton flux in the E > 10 MeV channel of GOES satellite exceeded 10 pufu. We derive scaling relations among CME speed, energetic proton flux, and flare SXR peak flux on the basis of simple assumptions and compared with observation.

Finally, we estimate how fast CMEs will be, and how intense proton flux will come during possible solar superflare events based on the combination of the scaling relations and observation.

2. DATA SET

A total of 143 major SPEs were recorded from 1996 to 2014, whose peak proton flux in the E > 10 MeV channel of GOES satellite exceeded 10 pufu. The events are listed in the CDAW major SEP event list page.¹ For 110 out of 143 SPEs, both flare SXR peak flux and CME speeds near the Sun accompanied with the SPEs are determined with X-ray Sensor on board the GOES satellite and Large Angle and Spectrometric Coronagraph (LASCO; Brueckner et al. 1995) on board the Solar and Heliospheric Observatory (SOHO; Domingo et al. 1995), respectively. For 80 out of these 110 SPEs, the mass of accompanied CMEs are also determined with LASCO. The estimated speed and mass of CMEs are obtained from the CDAW Data Center CME catalog (Yashiro et al. 2004; Gopalswamy et al. 2010), which is also available online.²

3. STATISTICAL RELATIONS AMONG FLARE SXR PEAK FLUX, CME SPEED, AND ENERGETIC PROTON FLUX

We study the statistical relation among Fₜ₟ and Fₚ during 110 major SPEs recorded between 1996 and 2014. Most CMEs, namely 92 out of 110, were “halo” ones. A “halo” CME is an expanding plasma of CME that appears to form a halo of enhanced brightness completely surrounding the occulting disk when observed with a coronagraph (Howard et al. 1982). The estimation of speed and mass of halo CMEs by coronagraph observations generally contains large uncertainty. We neglect the uncertainty of speed and mass estimation based on coronagraph observations throughout the analysis.

In Figure 1, we show a correlation plot of Fₚ and Fₜ₟. A regression line is drawn as a solid line to fit the log–log data plot. We use the ordinary least squares (OLS) bisector method for linear regressions hereafter, which is suitable for the discussion of underlying functional relation between two quantities (Isobe et al. 1990). The correlation coefficient is r = 0.41, and the linear regression fit gives Fₚ ∝ Fₜ₟ with β = 1.19 ± 0.08. Five out of 110 SPEs had Fₚ larger than 10⁴ pufu, and four out of the five were associated with X-class flares. Fₚ also correlates with Vₜ₟ in our data set (Figure 2). We note here that throughout the paper, the CME speed Vₜ₟ refers to the estimated speed of CME near the Sun based on observations with SOHO/LASCO. The average CME speed of all the 110 events is 1566 km s⁻¹, and the average CME speed of five of the most intense SPEs is 2016 km s⁻¹. The correlation coefficient between Fₚ and Vₜ₟ for our data set is r = 0.45, and the linear regression result gives Fₚ ∝ Vₜ₟ with γ = 4.35 ± 0.50. The event with Fₚ = 1860 pufu and Vₜ₟ = 882 km s⁻¹ shown as the unfilled circle in Figure 2 seems to be an outlier, and the analysis without the event makes the correlation coefficient a little bit higher and the slope a little steeper, namely, r = 0.48 and γ = 4.50 ± 0.48, respectively. The event shown as an unfilled circle was associated with an X7.1-class solar flare, and the CME was a halo one.

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¹ http://cdaw.gsfc.nasa.gov/CME_list/sepe/
² http://cdaw.gsfc.nasa.gov/CME_list/
Figure 2. $F_p$–$V_{\text{CME}}$ relation. Solid line is the linear regression fit $F_p \propto V_{\text{CME}}^\gamma$ with $\gamma = 4.35 \pm 0.50$. The dashed line is the upper limit of $F_p$ in terms of $V_{\text{CME}}$ whose spectral index is 5. The SPE represented by an unfilled circle in the figure seems to be an outlier, and the linear regression without it made the slope a bit steeper, namely, $\gamma = 4.50 \pm 0.48$.

Figure 3 shows a correlation plot between $V_{\text{CME}}$ and $F_{\text{SXR}}$. The correlation coefficient is 0.42, and the linear regression result gives $V_{\text{CME}} \propto F_{\text{SXR}}^{0.30} \pm 0.04$. In 80 SPEs out of 110 analyzed here, CME mass $M_{\text{CME}}$ is also estimated.\(^5\) Correlation between $M_{\text{CME}}$ and $F_{\text{SXR}}$ in the 80 SPEs was very poor with $r = -0.02$, possibly due to large uncertainty in the mass estimation of halo CMEs.

4. SCALING RELATIONS BETWEEN FLARE MAGNITUDE, CME SPEED, CME MASS, AND PEAK PROTON FLUX

We try to express CME mass ($M_{\text{CME}}$), CME speed ($V_{\text{CME}}$), and energetic proton peak flux ($F_p$) as a power-law form of the total released energy during flares ($E_{\text{flare}}$) based on three simple physical assumptions. First, we assume the CME mass is the sum of the mass within the gravitationally stratified AR corona,

$$M_{\text{CME}} = L^3 \int_0^L \rho_0 \exp \left( -\frac{z}{H} \right) dz \sim \rho_0 L^3 H$$  \hspace{1cm} (1)

where $\rho_0$, $L$, and $H$ are the density at the base of the AR corona, the length scale of flaring AR, and the pressure scale height, respectively. We implicitly assumed that AR corona size $L$ is much larger than the coronal scale height. For large AR, $L$ is typically of the order of $\sim 200$ Mm, while $H \sim 50$ Mm.

Next, we assume CME kinetic energy is proportional to the total energy released during the flare, which is also a constant fraction of the AR magnetic field energy (Emslie et al. 2012),

$$E_{\text{CME}} = \frac{1}{2} M_{\text{CME}} V_{\text{CME}}^2 \propto E_{\text{flare}} = f \frac{1}{8\pi} B_0^2 L^3$$  \hspace{1cm} (2)

where $E_{\text{flare}}$ and $E_{\text{CME}}$ are the total flare energy and CME the kinetic energy, respectively. $B_0$ is the AR magnetic field strength. Typical magnetic field strength of sunspots is of the order of 3000 G.

The first and second assumptions (Equations (1) and (2)) lead to the following relations:

$$M_{\text{CME}} \propto E_{\text{flare}}^{2/3}$$  \hspace{1cm} (3)

$$V_{\text{CME}} \propto E_{\text{flare}}^{1/6}$$  \hspace{1cm} (4)

Aarnio et al. (2011) studied CME flare pairs observed with LASCO and GOES occurring from 1996 to 2006 and found the statistical relationship $M_{\text{CME}} \propto F_{\text{SXR}}^{0.7}$. Aarnio et al. (2012) further discussed the statistical relation of CME mass $M_{\text{CME}}$ and energy released in the form of SXR during flares $E_{\text{SXR}}$ of the form $M_{\text{CME}} = K_M E_{\text{SXR}}^\delta$, where $K_M = (2.7 \pm 1.2) \times 10^{-3}$ in cgs units, and $\delta = 0.63 \pm 0.04$. Such observations seem to be consistent with our scaling relation of $M_{\text{CME}} \propto E_{\text{flare}}^{2/3}$. Very interestingly, such a scaling relation between $M_{\text{CME}}$ and $F_{\text{SXR}}$ obtained from solar flare statistics is consistent with mega-flare observations on a young T Tauri star, implying the scaling relation holds in a very wide energy range that is more than 10 orders of magnitude in flare energy (Aarnio et al. 2012).

We then try to relate energetic proton peak flux $F_p$ with flare energy $E_{\text{flare}}$. We assume that the total kinetic energy of solar energetic protons $E_p$ is proportional to flare energy, and the duration of proton flux enhancement is determined by the CME propagation timescale $t_{\text{CME}} \propto L/V_{\text{CME}}$:

$$E_p \propto F_p t_{\text{CME}} \propto E_{\text{flare}}.$$  \hspace{1cm} (5)

From Equations (4) and (5) we express $F_p$ as follows:

$$F_p \propto E_{\text{flare}}^{5/6} \propto V_{\text{CME}}^5.$$  \hspace{1cm} (6)

\(^5\) http://cdaw.gsfc.nasa.gov/CME_list/
In the derivation, we neglect the proton energy spectral variation depending on flare magnitude. The scaling relation (6) \( F_p \propto V_{\text{CME}} \) is plotted as a dashed line in Figure 2 and compared with the observed correlation. The linear regression fit in double logarithmic space was \( F_p \propto V_{\text{CME}}^\gamma \) with \( \gamma = 4.35 \pm 0.50 \), which has a slightly smaller slope compared to (6).

5. ESTIMATION OF CME SPEED AND PROTON FLUX ASSOCIATED WITH SOLAR SUPERFLARES

In this section, we compare the scaling relations derived above with observational statistical relations and try to estimate how fast CME and how intense proton flux will result in the case of solar superflares.

Emslie et al. (2012) discussed that the kinetic energy of a CME is comparable to flare energy, namely, \( E_{\text{CME}} \sim E_{\text{flare}} \).

Figure 4 shows the correlation between CME kinetic energy estimated from the LASCO observation by \( E_{\text{CME}} = 1/2M_{\text{CME}}v_{\text{CME}}^2 \) and flare SXR peak flux \( F_{\text{SXR}} \) associated with 80 major SPEs with CME mass estimation. The correlation coefficient is \( r = 0.28 \), and the linear regression with the OLS bisector method results in \( E_{\text{CME}} \propto F_{\text{SXR}}^{\beta} \) with \( \beta = 0.80 \pm 0.07 \).

In order to estimate the upper limit of the CME speed and energetic proton flux in response to the SXR class of solar flares, we try to relate \( V_{\text{CME}} \) and \( F_p \) with \( F_{\text{SXR}} \) by assuming \( F_{\text{SXR}} \) is roughly proportional to the total released energy during flares, namely, \( F_{\text{SXR}} \sim E_{\text{flare}} \).

Based on this assumption, \( F_p \) and \( V_{\text{CME}} \) are respectively scaled with \( F_{\text{SXR}} \) as

\[
V_{\text{CME}} \propto F_{\text{SXR}}^{1/6},
\]

\[
F_p \propto F_{\text{SXR}}^{5/6}.
\]

Scaling relations (7) and (8) are shown as dashed lines in Figures 3 and 1, respectively. The dashed lines are positioned in each plot so that they pass through the upper left-most SPE, in order that we can discuss the upper limit of CME speed (\( V_{\text{CME,upper limit}} \)) and proton flux (\( F_{p,upper limit} \)) in response to \( F_{\text{SXR}} \). Explicit formulas are as follows:

\[
V_{\text{CME,upper limit}} = V_0 F_{\text{SXR}}^{1/6},
\]

\[
F_{p,upper limit} = F_{p,0} F_{\text{SXR}}^{5/6},
\]

where \( V_0 = 1.3 \times 10^4 \text{ km s}^{-1} \), \( F_{p,0} = 10^{7.83} \text{ pfu} \), and \( F_{\text{SXR}} \) is normalized in units of \( 1 \text{ W m}^{-2} \).

Compared with linear regression fits, namely, \( V_{\text{CME}} \propto F_{\text{SXR}}^{\alpha} \) and \( F_p \propto F_{\text{SXR}}^{\beta} \) with \( \alpha = 0.30 \pm 0.04 \), \( \beta = 1.19 \pm 0.08 \), the derived scaling relations had gentler slopes of \( 1/6 \approx 0.17 \) and \( 5/6 \approx 0.83 \), respectively. We note that in Figures 3 and 1, the scaling relations (7) and (8) seem consistent with the line of the upper limit of observed \( V_{\text{CME}} \) and \( F_p \) with respect to \( F_{\text{SXR}} \).

From Equation (9), the upper limit of \( V_{\text{CME}} \) for X10, X100, and X1000 solar flares will be \( V_{\text{CME,X10}} = 4.2 \times 10^3 \text{ km s}^{-1} \), \( V_{\text{CME,X100}} = 6.2 \times 10^3 \text{ km s}^{-1} \), and \( V_{\text{CME,X1000}} = 9.1 \times 10^3 \text{ km s}^{-1} \), respectively (Figure 5(a)). From Equation (10), the upper limit of \( F_p \) for X10, X100, and X1000 solar flares will be \( F_{p,X10} = 2.0 \times 10^5 \text{ pfu} \), \( F_{p,X100} = 1.6 \times 10^5 \text{ pfu} \), and \( F_{p,X1000} = 1.0 \times 10^6 \text{ pfu} \), respectively (Figure 5(b)).

6. IMPACT OF SUPERFLARE-ASSOCIATED CMEs AND SPEs ON SPACE WEATHER AND TERRESTRIAL ENVIRONMENT

In this Letter, we studied CME properties and energetic proton flux associated with possible superflares on the Sun. The scaling relations expressing \( M_{\text{CME}}, \text{CME}, \) and \( F_p \) in terms of \( E_{\text{flare}} \) derived from simple assumptions are not inconsistent with statistical relations from solar observations. On the basis of the analysis above, we expect CMEs associated with superflares to be fast and heavy, which will have a huge impact on space weather (Tsurutani et al. 2003).

Huge geomagnetic storms are initiated by magnetic reconnection between injected southward magnetic field of ICMEs \( B_{\text{ICME}} \) and Earth’s northward magnetic field (Dungey 1961; Gonzalez et al. 1994). When ICME magnetic field is northward, no geomagnetic storms occur (Tsurutani & Gonzalez 1995). The magnitude of geomagnetic storms is mainly determined by solar wind westward electric field \( E_v \approx V_{\text{ICME,1 au}} B_{\text{ICME}} \), where \( V_{\text{ICME,1 au}} \) is the ICME speed near Earth (Burton et al. 1975; Gonzalez et al. 1994). The upper limit of the magnetic field strength of magnetic cloud \( B_{\text{ICM}} \) is estimated by the balance of magnetic pressure and dynamic pressure as \( B_{\text{ICM}}^2 /8\pi \approx 1/2 \rho_{\text{SW}} (V_{\text{ICME,1 au}} - V_{\text{SW}})^2 \approx 1/2 \rho_{\text{SW}} V_{\text{ICME,1 au}}^2 \), where \( \rho_{\text{SW}} \) and \( V_{\text{SW}} \) are the density and speed of the solar wind near Earth. If we assume a typical value range of solar wind proton number density at \( 1 \text{ au} \), namely, \( n_p = 3 - 8 \text{ cm}^{-3} \) (Schwenn 1990), \( B_{\text{ICM}} \) is estimated as \( B_{\text{ICM}} \approx (0.08-0.13) (V_{\text{ICME,1 au}}/1 \text{ km s}^{-1}) \text{ nT} \). This is consistent with the observationally known fact that magnetic clouds with higher peak speed \( (V_{\text{peak}}) \) also possess stronger core magnetic fields \( (B_{\text{peak}}) \), with the observational statistical relation \( B_{\text{peak}} = 0.047 (V_{\text{peak}}/1 \text{ km s}^{-1}) \text{ nT} \) (Gonzalez et al.
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Figure 5. (a) \( V_{\text{CME}} - F_{\text{SXR}} \) relation. The estimated upper limits of CME speed associated with X10-, X100-, and X1000-class flares are represented as black filled rectangles. (b) \( F_p - F_{\text{SXR}} \) relation. The estimated upper limits of energetic proton flux associated with X10-, X100-, and X1000-class flares are represented as black filled rectangles. The dashed lines in (a) and (b) are the upper limits of \( V_{\text{CME}} \) and \( F_p \) in terms of \( F_{\text{SXR}} \) given by Equations (4) and (6), respectively.

The upper limit of westward electric field \( E_x \) is estimated as
\[
E_x \approx \sqrt{4\pi/\rho_{\text{SW}}} V_{\text{CME},1\text{au}}^2.
\]
CMEs are decelerated during their propagation in interplanetary space (Gopalswamy et al. 2001), sweeping up interplanetary plasma on their path. We expect from conservation of momentum that if a CME ejecta is heavy enough (comparable to or heavier than the mass scraped up on its path), the CME will not be decelerated much. Fast and heavy ICMEs with southward magnetic fields associated with solar superflares would cause extreme geomagnetic storms.

Extreme increase of radiation levels in space associated with solar superflares also results in the increase of radiation at the flight altitude and sometimes at the sea level (GLE) through airshower formation in Earth’s atmosphere. The radiation levels in Earth atmosphere depend on the high-energy component of energetic proton flux in space. For example, GLEs are mainly caused by energetic protons injected into the top of the atmospheric layer whose energy is higher than \( \sim 1 \) GeV. The maximum energy of energetic protons associated with solar flares are known to be less than several GeV.

We estimate the maximum possible energy of energetic protons accelerated at CME-driven shocks. If we apply Hillas limit (Hillas 1984), we get the estimation of proton maximum energy as
\[
E_{\text{max}} \sim 2 \text{ GeV } B_{0.1} V_{3000} L_{1Rs},
\]
where \( B_{0.1} \), \( V_{3000} \), and \( L_{1Rs} \) are the upstream magnetic field strength in units of 0.1 G, shock propagation speed in units of 3000 km s\(^{-1}\), and length scale of acceleration site in units of the solar radius, respectively. If we assume \( L \) is independent of flare energy, we obtain \( E_{\text{max}} \propto V_{\text{CME}} \propto E_{\text{flare}}^{1/4} \). On the other hand, if we assume protons with highest energies are accelerated by the electric voltage generated by magnetic reconnection at the flare site, we obtain \( E_{\text{max}} \sim 7 \text{ GeV } B_{100} V_{1000} L_{0.1Rs} \), where \( B_{100} \), \( V_{1000} \), and \( L_{0.1Rs} \) are the AR magnetic field strength in units of 100 G, plasma “inflow” speed in units of 100 km s\(^{-1}\), and length scale of the flaring AR in units of 0.1 solar radius, respectively. Generally, the magnetic reconnection rate \( \sim V_{\text{in}}/V_{\text{CME}} \) is independent of flare magnitude (Shibata & Magara 2011). Applying Equations (2) and (4) we obtain \( E_{\text{max}} \propto E_{\text{flare}}^{1/2} \) for X1000-class flare, for example, may produce 10 GeV protons that result in a drastic increase of radiation level in the Earth’s atmospheric layer.

In the derivation of scaling relation (6), we assumed the relation \( L_{\text{CME}} \propto \sqrt{L_{\text{flare}}} \), where \( L \) is the AR size. If we use the constant length scale \( L_0 \) in place of \( L \), the scaling relation would change to \( E_\text{p} \propto E_\text{flare}^{7/6} \propto V_{\text{CME}}^{7/6} \). This might be the case if particles are accelerated at the shock front in the corona whose length scales do not depend on AR size.

The CME catalog we used in this study is generated and maintained at the CDAW Data Center by NASA and The Catholic University of America in cooperation with the Naval Research Laboratory. SOHO is a project of international cooperation between ESA and NASA. We studied SPEs from online SPE catalog provided by the CDAW Data Center.\(^\text{6}\) This work was supported by JSPS KAKENHI grant numbers 16H03955. The authors are grateful to Dr. Seiji Yashiro for providing valuable information and giving us fruitful comments on data handling. This work is motivated partly by Mr. Taira Hiraishi’s master thesis.

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\(^6\) http://cdaw.gsfc.nasa.gov/CME_list/sepe/
