Twisted Superfields

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A model is presented that could lead to an interesting extension of the Standard Model. Like a supersymmetric gauge theory, the model is holomorphic and invariant to local superspace gauge transformations. However, the model is not invariant to superspace translations, so it is not super-symmetric. It is proposed that this combination allows the model to have many of the attractive features of supersymmetric theories, while at the same time predicting fewer particles that have not yet been seen experimentally. For example, the “superpartners” of the gauge bosons in the model are quarks. The model is able to generate the symmetries and particles of the Standard Model, but with some significant differences that have observable consequences. These consequences provide possible explanations for a number of 3-7 sigma deviations from Standard Model calculations that have been found in recent experiments.

INTRODUCTION

Supersymmetric gauge theories have many attractive properties. For example, they provide a natural mechanism for cancellation of quadratic divergences and a resolution of the Hierarchy Problem. They do this by being (i) holomorphic, (ii) invariant to local gauge transformations in superspace, and (iii) invariant to global translations in superspace. A difficulty with supersymmetric theories, however, is that they predict that for every particle that has been observed, there is another partner particle that has not yet been seen. As experiments probe higher and higher energies, the fact that no partner particle has been found becomes more problematic. Early on, supersymmetry practitioners asked whether some of the existing observed particles could actually be supersymmetric partners with each other. The HLS theorem \cite{1} mostly rules out this possibility with some minor exceptions (like the Higgs boson being a slepton \cite{2}). But even those exceptions are generally not accepted for other reasons.

The model presented in this paper incorporates the first two features of supersymmetry listed above, but not the third. Since the model is not invariant to superspace translations, it is not supersymmetric. Consequently, there is no a priori guarantee that quadratic divergences cancel. That being said, many quadratic divergences are cancelled for supersymmetric gauge theories primarily due to the fact that the theories are holomorphic and invariant to superspace gauge transformations. Also, local superspace gauge transformations turn scalar bosons into fermions, and fermions do not have quadratic divergences. Divergences may cancel in this theory for similar reasons. To that point, for nonsupersymmetric gauge theories similar to the one presented in this paper, it has been shown explicitly that quadratic divergences cancel to at least the two-loop order \cite{3}.

The superspace gauge transformations of the model are built on the group U(3)×U(3). The field content of the model includes constructions built in N=1 superspace that are not N=1 superfields but are nonetheless called “twisted superfields” by way of analogy. Specifically, the model includes a real “twisted superfield”, an adjoint-representation chiral “twisted superfield”, and three flavors of fundamental and anti-fundamental chiral “twisted superfields”.

An advantage to this theory not being supersymmetric is that it can allow existing observed particles to be “superpartners” with each other (in the sense that superspace gauge transformations change them into each other). For example, in this model the partners of the gauge bosons (within the real “twisted superfield”) are quarks. Also, due to the fact that the theory is holomorphic and supergauge invariant, it is argued that some of the nonperturbative phenomena of supersymmetric gauge theories may apply to the present theory.

After presenting the theory in the first two sections (and the Appendix), the third section shows how the theory can reproduce the existing forces and particles of the Standard Model, including neutrinos with their observed masses and mixing. The fourth section shows how the coupling constants of the theory converge at a unification scale. The fifth section of this paper shows how the model has the correct structure to reproduce many of the anomalies presented in \cite{4}, where experimental results differ from Standard Model predictions by 3-7\sigma.

Many of the ideas of this paper were originally published by the author in \cite{5}. However, this paper has heavily revised the structure of the theory, the parameter values, and the mapping to experimental results. This paper replaces that original paper.

1. U(3) X U(3) SYMMETRIES AND FIELDS

The theory is constructed in N=1 superspace, extending four-dimensional spacetime by including four additional anticommuting coordinates $\theta_\alpha, \bar{\theta}_\dot{\alpha}$. For reviews of superspace, see \cite{6–12}; the notational conventions of \cite{6} are used throughout. The gauge group of the model is

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U(3)×U(3), and it is described using 6×6 matrices with the gauge fields in the 3×3 diagonal blocks. The model includes a construction called a real “twisted superfield” that has the following attributes: field components in the 3×3 diagonal blocks have an even number of θα, θβ factors, while those in the 3×3 off-diagonal blocks have an odd number of θα, θβ factors. An adjoint-representation chiral “twisted superfield” has this same structure, and fundamental chiral “twisted superfields” have structures consistent with those.

\[ V = \left( \begin{array}{c}
C_1 + N_1 \theta^2 + \bar{\theta}^2 N_1^\dagger - \bar{\theta} \bar{\sigma}^\mu A_1 \theta + \frac{1}{2} \bar{\theta}^2 d_1 \theta^2 \\
\bar{\eta} \theta + \bar{\eta} \bar{\eta}^\dagger + i \bar{\theta} \lambda \bar{\theta}^2 - i \theta \lambda \theta \\
C_2 + N_2 \theta^2 + \bar{\theta}^2 N_2^\dagger - \bar{\theta} \bar{\sigma}^\mu A_2 \theta + \frac{1}{2} \bar{\theta}^2 d_2 \theta^2 
\end{array} \right), \quad (1.1) \]

where each component field above is a U(3) matrix function of spacetime coordinates \( x^\mu \). For example, \( A_\mu = A^\dagger_\mu (x) t^A \), where \( t^A \) are 3×3 U(3) matrices normalized by \( \text{tr} (t^A t^B) = \frac{2}{3} \delta^{AB} \). Lower case letters are used to denote SU(3) adjoint indices \( a, b \in \{1, 2, 3\} \). Upper case letters are used to denote U(3) adjoint indices \( A, B \in \{0, 1, 2, 3\} \) that include the Abelian matrix \( t^0 = \frac{1}{\sqrt{6}} \text{diag} (1, 1, 1) \). The \( \theta_\alpha \) are 2-component anti-commuting Grassman coordinates, and \( \bar{\theta}_\alpha \) are their Hermitian conjugates. As a result of their \( \theta_\alpha, \bar{\theta}_\beta \) factors, the fields in the diagonal blocks of \( V \) are bosons, while the fields in the off-diagonal blocks are fermions.

As mentioned previously, a theory built using the above real twisted superfield is not supersymmetric, since the fermion fields in the superfield are in a different representation of the U(3)×U(3) group than the boson fields. Despite not being supersymmetric, the real twisted superfield is assumed to transform as follows under a local “twisted supergauge transformation”:

\[ e^V \rightarrow e^{i \Lambda^\dagger} e^V e^{-i \Lambda}. \quad (1.2) \]

In the above expression,

\[ \Lambda = \begin{pmatrix}
\alpha_1 (y) + \theta^2 n_1 (y) \\
\theta_2 (y) \\
\theta_2 (y) + \theta^2 n_2 (y)
\end{pmatrix} \quad (1.3) \]

is a chiral “twisted superfield” whose component fields are U(3) matrix functions (e.g. \( \alpha_1 = \alpha_1^A t^A \)) of \( y^\mu = x^\mu + i \theta \sigma^\mu \theta \). The twisted supergauge transformation of eq (1.2) maintains the boson-fermion structure of the real twisted superfield as well as its group structure. To the latter point, if the group was SU(3)×SU(3) instead of U(3)×U(3), the supergauge transformation would not be consistent, since a general supergauge transformation would generate terms in each block proportional to \( t^0 \).

On the other hand, a U(3)×U(3) twisted supergauge transformation is consistent.

Like normal real superfields, the real twisted superfield supports conjugate representations. To see this, it is helpful to follow the presentation of [7] and re-express an infinitesimal twisted supergauge transformation as:

\[ V \rightarrow V + i \Lambda^\dagger - i \Lambda - \frac{i}{2} \left[ V, (\Lambda^\dagger + \Lambda) \right] + i \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!} \left[ V, [V, \ldots \left[ V, (\Lambda^\dagger + \Lambda) \right], \ldots] \right], \quad (1.4) \]

where \( B_{2k} \) are Bernoulli numbers. Both \( V \) and \( \Lambda \) can be expanded in terms of component fields multiplied by U(6) matrices \( T^X \), where the index \( X \) runs over the 36 adjoint indices of U(6). As with any unitary group, the same structure functions \( f_{XYZ} \) satisfy both \( [T^X, T^Y] = i f_{XYZ} T^Z \) and \( [-T^{XT}, -T^{YT}] = -i f_{XYZ} T^{ZT} \). Since products of matrices in eq (1.4) only enter by way of commutators, a conjugate representation is available by replacing each \( T^X \) in eq (1.4) with its negative transpose \( (-T^X)^T \). In other words, a twisted real superfield that transforms by eq (1.2) also transforms as follows:

\[ e^{-V} \rightarrow e^{i \Lambda^\dagger} e^{-V} e^{-i \Lambda}. \quad (1.5) \]

Despite the fact that (1.1) and (1.3) are not N=1 superfields and (1.2) is not a normal supergauge transformation, the word “twisted” will be dropped for brevity in much of the rest of the paper.

A consequence of eq (1.4) is the fact that one component of the real superfield has a supergauge transformation independent of the other components. Taking the trace of eq (1.4), one finds

\[ \sqrt{3} \text{Tr} (V) = V_+^0 \rightarrow V_+^0 + i \Lambda_+^0 - i \Lambda_+^0 = \frac{1}{\sqrt{3}} \text{Tr} (\Lambda), \quad (1.6) \]

where Tr is the 6×6 trace. The reason that this supergauge transformation is independent is because all of the commutators in eq (1.4) are proportional to some 6×6 traceless matrix, so none of them can contribute to eq (1.6). Since by definition, the fermions of the real superfield are all in off-diagonal blocks, the field \( V_+^0 \) does not
include any fermions, only bosons. Inside of $V$, the field $V^0_+^\dagger$ is multiplied the 6x6 matrix $T^0_+$ defined via

$$T^0_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} T^A_2 & T^A_1 \\ 0 & 0 \end{pmatrix}$$

This definition provides another way of saying that $T^0_+ = \frac{1}{\sqrt{2}}$ of the 6x6 unit matrix.

As is often done in superspace gauge theories, the real superfield will be rescaled to explicitly show the coupling constant. In this case, the following rescaling is performed:

$$V = 2gV' + 2g_\alpha V^0_+ T^0_+$$

Since $V^0_+$ has its own, independent supergauge transformation, it also has its own coupling constant.

Now that the gauge transformation properties of the real superfield have been identified, gauge invariant action terms can be defined. Just as with normal superfields, the following chiral twisted superfields can be defined:

$$W'^0_\alpha = -\frac{1}{3g^2} \bar{D}^2 \left(e^{-2gV'} D_\alpha e^{2gV'}\right)$$

$$W'^0_\alpha = -\frac{1}{3g^2} \bar{D}^2 D_\alpha V^0_+,$$

where $D_\alpha = \partial_\alpha + i\sigma^\mu_\alpha \bar{\theta}^\mu \partial_\mu$. Under a supergauge transformation, these fields transform as follows:

$$W'^0\alpha \to e^i\Lambda W'^0\alpha e^{-i\Lambda}$$

$$W'^0\alpha \to W'^0\alpha.$$  \hspace{1cm} \text{(1.10)}

As a result, the following terms in the action are supergauge invariant:

$$S_V = -\frac{1}{2} \int d^4xd^2\bar{\theta} \left(1 + 4m_\Lambda \theta^2\right) \text{Tr} \left(W'^0\alpha W'^\alpha\right) \right.$$  \hspace{1cm} \text{(1.11)}

$$-\frac{1}{4} \int d^4xd^2\theta W'^0\alpha W'^\alpha + h.c.,$$

where $h.c.$ stands for Hermitian conjugate and $m_\Lambda$ is a "gaugino mass".

The action may also include the following gauge-invariant Fayet Iliopoulos term:

$$S_\xi = \frac{1}{\sqrt{2}} \xi_+ \int d^4xd^2\theta \bar{d}^2\bar{\theta} g_+ V^0_+.$$  \hspace{1cm} \text{(1.12)}

In addition to the real superfield, the theory includes the following chiral twisted superfield in an adjoint representation of twisted U(3)xU(3):

$$\Phi = \begin{pmatrix} \varphi_1(y) + \theta^2 f_1(y) + \sqrt{2}\theta \chi(y) \\ \sqrt{2}\bar{\theta} \bar{\chi}(y) \\ \varphi_2(y) + \theta^2 f_2(y) \end{pmatrix}.$$  \hspace{1cm} \text{(1.13)}

The adjoint superfield transforms as follows:

$$\Phi \to e^{i\Lambda} \Phi e^{-i\Lambda}.$$  \hspace{1cm} \text{(1.14)}

The following action terms involving this field are supergauge invariant:

$$S_\Phi = 2 \int d^4xd^2\theta d^2\bar{\theta} \times$$

$$\times \text{Tr} \left(\Phi^1 e^{2gV'} \Phi e^{-2gV'} (1 - \theta^2 \bar{\theta}^2 \sum_m m_\Phi^2 \sqrt{2T^0_m}) \right)$$

$$-2 \int d^4xd^2\theta \text{Tr} \left(\frac{1}{2} \Phi^1 \Phi^2 + \frac{1}{2} \Gamma^{(3)} \Phi^3 \right) + h.c.$$  \hspace{1cm} \text{(1.15)}

The $m_\Phi^2$ terms are scalar mass terms. Despite the explicit group matrices $T^0_m$, the terms are supergauge invariant since the factor of $\theta^2 \bar{\theta}^2$ limits gauge transformations to ones that remain within the same 3x3 diagonal block. Just like the gaugino mass term (and like analogous soft supersymmetry breaking terms), the scalar mass terms break superspace translation invariance (which is not imposed in this model anyway), but do not break superspace gauge invariance.

In addition to adjacent-representation chiral fields, the theory also includes three flavors of 6-vector chiral twisted superfields in the fundamental and anti-fundamental representations of twisted U(3)xU(3):

$$Q_{1F} = \frac{\phi_{1F} + \theta^2 f_{1F}}{\sqrt{2}\theta \psi_{1F}}$$

$$Q_{2F} = \frac{-\sqrt{2}\theta \psi_{2F}}{\phi_{2F} + \theta^2 f_{2F}}$$

$$\tilde{Q}_{1F} = (\tilde{\phi}_{1F} + \theta^2 \tilde{f}_{1F}, \sqrt{2}\theta \tilde{\psi}_{2F})$$

$$\tilde{Q}_{2F} = (\sqrt{2}\theta \tilde{\psi}_{1F}, \tilde{\phi}_{2F} + \theta^2 \tilde{f}_{2F})$$

where $F \in \{1, 2, 3\}$ is a flavor index, and each component field is a chiral 3-vector (or covector). The supergauge transformation for each of these superfields depends upon their flavor in the following way:

$$\tilde{Q}_{2F} \to \tilde{Q}_{2F} e^{-i\Lambda}, \hspace{0.5cm} Q_{2F} \to e^{i\Lambda} Q_{2F}, \hspace{0.5cm} F = 2, 3$$

$$\tilde{Q}_{21} \to \tilde{Q}_{21} e^{-i(\Lambda - \Lambda^0_{21} T^0_2)}, \hspace{0.5cm} Q_{21} \to e^{i(\Lambda - \Lambda^0_{21} T^0_2)} Q_{21}, \hspace{0.5cm} F = 2, 3$$

$$\tilde{Q}_{1F} \to \tilde{Q}_{1F} e^{i(\Lambda - \Lambda^0_{1F} T^0_1)}, \hspace{0.5cm} Q_{1F} \to e^{i(\Lambda - \Lambda^0_{1F} T^0_1)} Q_{1F}.$$  \hspace{1cm} \text{(1.16)}

As a result, the following terms in the action are supergauge invariant:

$$S_Q = \sum_{mF} \int d^4x d^2\theta d^2\bar{\theta} \times$$

$$\times \left( (1 - m^2_{mF} \theta^2 \bar{\theta}^2) Q_{mF}^1 e^{2(\bar{\theta}V^\dagger + q_{mF} g_+ V^0_+ T^0_2)} Q_{mF}^1 \right)$$

$$+ (1 - m^2_{mF} \theta^2 \bar{\theta}^2) Q_{mF}^2 e^{-2(\bar{\theta}V^\dagger + q_{mF} g_+ V^0_+ T^0_2)} \tilde{Q}_{mF}^1 \times$$

$$\times \left( \tilde{Q}_{mF} \left( m_{mF} \phi + \sqrt{2} \Gamma_{mFF} \Phi \right) Q_{mF} \right)$$

$$+ h.c.$$  \hspace{1cm} \text{(1.17)}

$q_{1F} = q_{21} = -2, \hspace{0.5cm} q_{22} = q_{23} = 1.$  \hspace{1cm} \text{(1.18)}
where $m \in \{1,2\}$ and $[m FF']$ means to only sum over combinations where $\tilde{Q}_{mF}$ and $Q_{mF'}$ have the same $m_F$ charge (and corresponding gauge transformations from eq (1.17)). The $m_{mF}^2$ and $\tilde{m}_{mF}^2$ terms generate additional mass terms for the fundamental and conjugate scalars.

The theory presented above is free of gauge anomalies. There is a simple reason: for every fermion in the theory, there is another fermion in a conjugate representation with opposite Abelian charges. Since the theory is a gauge theory and is free of gauge anomalies, the theory is renormalizable.

Since all of the action terms presented above are invariant to twisted supergauge transformations, it is possible to restrict the real superfield to a Wess-Zumino-like gauge. In that gauge, the real superfield takes the form:

$$V = \left( -\bar{\theta} \sigma^\mu A_{\mu} \theta + \frac{i}{2} \bar{\theta} \lambda^2 d_1 \theta^2 - i \bar{\theta} \lambda^1 \bar{\lambda}^2 \theta - \bar{\theta} \sigma^\mu A_{2\mu} \theta + \frac{i}{2} \bar{\theta} \lambda^2 d_2 \theta^2 \right).$$

(1.19)

In [3], it was shown that a Wess-Zumino-like gauge is accessible for a theory with this kind of twisted supergauge invariance. After imposition of this Wess-Zumino gauge, the residual gauge invariance is just local spacetime gauge invariance. In the following, the fermions $\lambda$ and $\bar{\lambda}$ will be referred to as “gauginos” despite the fact that they are in the $(3,3)$ and $(\bar{3},\bar{3})$ representations of the gauge group, rather than the adjoint representation.

Although the Abelian field $V^0$, or $\tilde{V}$, has its own, independent gauge transformation, the second Abelian field $V_F$ (group structure $T^0$) does not decouple from the gauge transformations described above. That is why the same designation $g$ was used for both the $V_F^0$ and non-Abelian couplings above. Nonetheless, in the Wess-Zumino gauge, since $V^0$ is an Abelian field, it can accommodate different charges multiplying its coupling constant when acting on different chiral fields. This flexibility is used to make the charges of $V^0_F$ (labelled as $q_{mF}'$) the same as those for $V^O_F$ defined in eq (1.18):

$$q_{mF}' = q_{mF}. \quad (1.20)$$

The charge of $V^0_F$ is also 1 when interacting with gauginos and the fermions in $\Phi$; it is only -2 when acting on fields with $m_F = 21$ or $1F$.

The classical theory has 50 parameters that can be adjusted classically: 2 coupling constants, 2 Abelian charges, 30 masses, 15 superpotential couplings, and a Fayet-Iliopoulos term. In the unification section of the paper, it is argued that both gauge couplings may be the same at the unification scale ($g_+ = g$). It is also assumed that the following 8 parameters are zero classically:

$$m_{23F} = m_{223} = \Gamma_{23F} = \Gamma_{223} = m_{23} = \tilde{m}_{23} = 0 \quad (1.21)$$

Many of the remaining parameters may also be zero classically but acquire values via quantum corrections.

### 2. DYNAMICAL SYMMETRY BREAKING

This section identifies a minimum of the scalar potential that breaks the gauge symmetry in stages from SU(3)×SU(3)×U(1)×U(1) down to SU(3)×U(1) and labels the fermions in the model based on their Standard Model symmetries.

The scalar potential for this model can be expressed in terms of its auxiliary fields and scalar mass terms:

$$V = \frac{1}{2} (d_+^2)^2 + \frac{1}{2} (d_-^2)^2 + \frac{1}{4} \sum_{m,a} (\phi_{mF}^0)^2$$

$$+ \sum_{mF} \left( f_m^F f_m^{F'} + \tilde{f}_m^F f_m^{F'} \right) + 2tr \left( f_1^F f_1 \right) + 2tr \left( f_2^F f_2 \right)$$

$$+ \sum_{mF} \left( m_{mF}^2 \phi_{mF}^0 \phi_{mF} + \tilde{m}_{mF}^2 \tilde{\phi}_{mF}^0 \tilde{\phi}_{mF} \right)$$

$$+ m_{\tilde{q}1} tr (\tilde{\phi}_1^F \tilde{\phi}_1) + m_{\tilde{q}2} tr (\tilde{\phi}_2^F \tilde{\phi}_2),$$

(2.1)

where $d_{\pm} = \frac{\sqrt{2}}{2} (d_1 \pm d_2)$, and lower-case tr defines a 3×3 trace. By their equations of motion, the auxiliary fields are equal to linear or quadratic functions of the scalar fields. For example, the equations of motion for the $d$ fields of $V$ result in:

$$-d_+^2 = \frac{g}{\sqrt{12}} \left( \xi + \sum_{mF} q_{mF} \left( \phi_{mF}^0 \phi_{mF} - \tilde{\phi}_{mF}^0 \tilde{\phi}_{mF}^0 \right) \right)$$

$$-d_-^2 = -\frac{g}{\sqrt{12}} \left( \sum_{mF} q_{mF}' \left( \phi_{mF}^{a0} \phi_{mF}^0 - \tilde{\phi}_{mF}^{a0} \tilde{\phi}_{mF}^{a0} \right) \right)$$

$$-d_{mF}^2 = 2gtr \left( t^a [\phi_m, \phi_m^a] \right)$$

$$+ g \sum_{F} \left( \phi_{mF}^{a0} \phi_{mF}^0 - \tilde{\phi}_{mF}^{a0} \tilde{\phi}_{mF}^{a0} \right).$$

(2.2)

The Abelian charges $q_{mF}$ and $q_{mF}'$ for $V^0_F$ (including $d_{\pm}^2$) were defined in eqs (1.18) and (1.20). Just as for the $d$ auxiliary fields, the equations of motion can also be used to derive expressions for the $f$ auxiliary fields in terms of scalar fields.

It is assumed that the masses $m_{1F}$ and $\tilde{m}_{1F}$ are large compared to $\xi$. In that case, the minimum of the scalar potential is achieved when the fundamental and conjugate scalars with an $m = 1$ subscript have no vacuum expectation value (vev):

$$\langle \phi_{1F}^0 \rangle = \langle \phi_{1F} \rangle = 0, \quad (2.3)$$

where $\langle \phi_{mF} \rangle$ denotes the vev of $\phi_{mF}$.

Due to eq (1.21) along with the assumption that $m_{21}$ and $m_{22}$ are small, it is assumed that the $m = 2$ fundamental and conjugate scalars acquire vevs. Following precedent from Supersymmetric QCD (SQCD) [12, 13], the vevs in the $m = 2$ sector are assumed to take the following form:

$$\langle \tilde{\phi}_{2F} \rangle = i \delta F \phi_{2F}$$

$$\langle \phi_{2F} \rangle = -i \delta F \phi_{2F},$$

(2.4)
where an overbar on a component of a scalar field (e.g. \( \bar{\phi}_{2F} \) above) is used to denote the magnitude (real, positive) of the vev of that component. In eq (2.4), the index \( n \) represents the SU(3) index of the 3-vectors \( \bar{\phi}_{2F} \) and \( \phi_{2F} \). For example, writing out the SU(3) “color” components: \( \langle \phi_{23} \rangle = i \begin{pmatrix} 0 & 0 & \tilde{\phi}_{23} \end{pmatrix} \). In other words, the vevs \( \langle \phi_{2F} \rangle \) and \( \langle \phi_{F} \rangle \) form 3×3 diagonal matrices in their flavor-“color” indices. The word “color” is being used here in order to make a connection with SQCD techniques, but in this model after symmetry breaking, the 3 “color” indices of the \( m = 2 \) scalars will actually correspond to 2 isospin doublet indices and 1 singlet index. The phases of the vevs are chosen to simplify fermion mass matrices in the next section.

Unlike \( m_{21} \) and \( m_{22} \), it is assumed that the scalar masses \( \tilde{m}_{21} \) and \( \tilde{m}_{22} \) are large compared to \( \xi_+ \). To accommodate a nontrivial minimum in the presence of these masses, the following is assumed classically:

\[
\bar{\phi}_{21} = \bar{\phi}_{22} = 0 \quad \text{classically.} \tag{2.5}
\]

In the appendix, it is argued that small vevs are generated for these fields quantum mechanically.

The following adjoint vevs are considered for the classical theory:

\[
\begin{align*}
\langle \varphi_1 \rangle &= 0 \quad \text{classically} \\
\langle \varphi_2 \rangle &= \frac{\bar{\phi}_2}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.6}
\end{align*}
\]

In the next section, in light of the quantum vacuum moduli space, different assumptions will be made for both \( \langle \varphi_1 \rangle \) and \( \langle \varphi_2 \rangle \) such that \( \text{tr} \langle \varphi \rangle \neq 0 \).

Assuming \( m_{211} \) and \( m_{222} \) are small classically, the vev of the classical scalar potential is equal to

\[
\begin{align*}
2 \langle V \rangle &= \frac{1}{16} g_+^2 \left( \xi_+ - 2 \bar{\phi}_{21} + \bar{\phi}_{22} + \Delta \bar{\phi}_3 \right)^2 \\
&+ \frac{1}{16} g_+^2 \left( 2 \bar{\phi}_{21} - \Delta \bar{\phi}_3 \right)^2 + \frac{1}{4} g_+^2 \bar{\phi}_{22}^4 \\
&+ \frac{1}{4} g_+^2 \left( \bar{\phi}_{22} - \Delta \bar{\phi}_3 + \bar{\phi}_2 \right)^2 + 2m_{21} \bar{\phi}_{21}^2 + 2m_{22} \bar{\phi}_{22}^2 + m_{3}^2 \bar{\phi}_3^2 \\
&- \Delta \bar{\phi}_3 = \bar{\phi}_{23} - \bar{\phi}_{23} + \bar{\phi}_3 \bar{\phi}_3 \\
&\tilde{m}_{3}^2 = m_{3}^2 + m_{32}^2 
\end{align*} \tag{2.7}
\]

It can be seen that if \( \bar{\phi}_{23} = \bar{\phi}_{23} \) but all other vevs, masses and \( \xi_+ \) vanish, then an absolute minimum of the potential is attained: \( \langle V \rangle = 0 \). In this model, it is assumed that \( \bar{\phi}_{23} \) and \( \bar{\phi}_{23} \) are at a unification scale of \( \sim 10^{16} \text{GeV} \), but all other vevs, masses and \( \xi_+ \) in eq (2.7) are at the electroweak scale (\( \sim 10^2 \text{GeV} \)) or smaller. In that case, deviations from the absolute minimum value \( \langle V \rangle = 0 \) only appear at the electroweak scale.

Well below the unification scale, the superpotential is more complicated since the couplings for different groups run differently. Nonetheless, by adjusting \( \xi_+, \Delta \bar{\phi}_3 \) and certain mass relations, classical minima can still be found where the electroweak scale is primarily defined by \( \bar{\phi}_{21} \), and smaller scales are defined by \( \bar{\phi}_{22} \) and \( \bar{\phi}_2 \).

In particular, a minimum exists where three of the vevs are at the following scales:

| vev          | scale          | Symmetry Breaking                                      |
|--------------|----------------|--------------------------------------------------------|
| \( g_{23} \) | \( \sim 10^{16} \text{GeV} \) | Unification: SU(3) × SU(3) × U(1) × U(1) → SU(3) × SU(2) × U(1) × U(1) |
| \( g_{22} \) | \( \sim 10^2 \text{GeV} \)     | Electroweak: SU(3) × SU(2) × U(1) → SU(3) × U(1) × U(1) |
| \( g_{Z} \bar{\phi}_{22} \) | \( \sim 10^{-2} \text{GeV} \) | \( Z' \) acquires mass: SU(3) × U(1) × U(1) → SU(3) × U(1) |

where \( g_2 \) is the Weak coupling and \( g_{Z'} \) is the \( Z' \) coupling that is driven to a very small value by an effective anomaly, as discussed in section 4. Since \( \Delta \bar{\phi}_3 \) is at the electroweak scale, but \( g \bar{\phi}_{23} \) is at the unification scale, the following vev must also be at the unification scale:

\[
\bar{\phi}_{23} \sim 10^{16} \text{GeV}. \tag{2.9}
\]

In Supersymmetric QCD (SQCD), the classical vacuum does not determine actual values of vevs, but just differences like \( \bar{\phi}_{23} - \bar{\phi}_{23} \) in eq (2.7). But for fewer flavors than colors, the SQCD quantum vacuum causes the vevs involved in those differences to get very large. In fact, the quantum vacuum drives them to infinity, so that for fewer flavors than colors, SQCD does not have a vacuum [12, 13]. In the Appendix, it is pointed out that similar forces are at work in this theory. However, they can be counterbalanced by other quantum effects or even a very small mass \( m_{233} \). If \( m_{233} \bar{\phi}_{23} \lesssim \xi_+ \), such a mass still allows large vevs for \( \bar{\phi}_{23} \) and \( \bar{\phi}_{23} \). But it provides a counterbalance, so that the vevs \( \bar{\phi}_{23} \) and \( \bar{\phi}_{23} \) do not become infinite. This is justification for why these two fields have very large unification scale vevs.

Section 4 of this paper determines the numerical value of the unification scale by starting at electroweak energies and running the SU(2) and SU(3) coupling constants up to the scale where they become the same. In that section, it is argued that the U(1) coupling \( g_+ \) may also unify with the nonAbelian couplings at that same scale.
A difference from the Standard Model in the above symmetry breaking is that there is an extra U(1) field (the $Z'$) that acquires a mass well below the electroweak scale. This will be discussed in more detail later in this section and in section 5.

In the above symmetry breaking, the SU(3) gluons of the Standard Model come from the $A_{1\mu}^0$ gauge bosons, while the SU(2) weak fields come from the $A_{2\mu}^0$ gauge bosons. The U(1) fields in the model are a mixture of $A_{1\mu}^0$ and $A_{2\mu}^0$ gauge bosons. The progression of the U(1) fields through the various stages of symmetry breaking is discussed in detail below.

Via the Brout-Englert-Higgs mechanism, the scalar vevs $\tilde{\phi}_{23}$ and $\tilde{\phi}_{23}$ impart unification-scale masses to $A_{2\mu}^0$, $A_{2\mu}^1$, $A_{2\mu}^2$, and $A_{2\mu}^3$ as well as to one diagonal gauge boson. Consequently, the gauge symmetry is broken down to SU(3)$\times$SU(2)$\times$U(1)$\times$U(1). To see the group structure of the remaining massless diagonal gauge fields, it is helpful to use the notation of eq (1.7) and re-expand the gauge fields into the linear combinations below:

$$
\begin{pmatrix}
A_Y^\mu \\
A_Y^{\mu'} \\
A_Y^U \\
A_Y^{U'}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi_U & \sin \phi_U \\
0 & -\sin \phi_U & \cos \phi_U
\end{pmatrix}
\begin{pmatrix}
\cos \theta_U & 0 & \sin \theta_U \\
0 & 1 & 0 \\
-\sin \theta_U & 0 & \cos \theta_U
\end{pmatrix}
\begin{pmatrix}
A_0^\mu \\
A_0^{\mu'} \\
A_0^U \\
A_0^{U'}
\end{pmatrix},
$$

(2.10)

where $A_Y^U$ acquires a unification-scale mass, but $A_Y^\mu$ and $A_Y^{\mu'}$ remain massless. In order to achieve the relation

$$q_{mF} g + A_{\nu\mu}^0 g_A_{-\mu}^0 T^0_+ + g A_{2\mu}^3 T^8_2 = g_Y A_{\mu}^0 T^Y_{mF} + g_Y A_{\mu}^{\mu'} T^{\mu'}_{mF} + g_Y A_Y^U T^U_{mF},
$$

(2.11)

appearing in the action (eqs (1.18) and (1.20)), the coupling constants and group matrices must satisfy:

$$
\begin{pmatrix}
g_Y T^Y_{mF} \\
g_Y' T^{\mu'}_{mF} \\
g_Y T^U_{mF}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi_U & \sin \phi_U \\
0 & -\sin \phi_U & \cos \phi_U
\end{pmatrix}
\begin{pmatrix}
\cos \theta_U & 0 & \sin \theta_U \\
0 & 1 & 0 \\
-\sin \theta_U & 0 & \cos \theta_U
\end{pmatrix}
\begin{pmatrix}
q_{mF} g + A_{\nu\mu}^0 g_A_{-\mu}^0 \\
q_Y A_{\mu}^0 T^0_+ \\
q_Y A_Y^8 T^8_2
\end{pmatrix}.
$$

(2.12)

This is just a generalization of a Weinberg angle rotation. A more complete generalization could involve a third angle mixing the two massless fields, but that is not needed here. The reason that the group matrices on the left have an $mF$ dependence is because eq (2.11) involves the Abelian charges $q_{mF}$ and $q_{mF}$. The angles $\theta_U$ and $\phi_U$ in eq (2.12) are chosen so that $T^Y_{23}$ and $T^{\mu'}_{23}$ have zeros in their sixth diagonal slot, so that they get no mass contribution from $\phi_{23}$ or $\phi_{23}$. Specifically, the angles are given by:

$$
\begin{align*}
\tan \theta_U &= g_+/g_2 = \frac{1}{2} \\
\tan \phi_U &= \frac{1}{2} \cos \theta_U = \frac{1}{\sqrt{5}}.
\end{align*}
$$

(2.13)

where the second equalities above assume that $g_+ = g$ at the unification scale. In that case $g_Y = g_Y' = g$, and the group matrices take the forms:

$$
\begin{align*}
T^Y_{23} &= T^{\mu'}_{23} = \frac{1}{2} \sqrt{\frac{2}{5}} \text{diag} (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 1, 1, 0) \\
T^Y_{21} &= T^{\mu'}_{1F} = -\frac{1}{2} \sqrt{\frac{2}{5}} \text{diag} (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 1, 1, 2)
\end{align*}
$$

(2.14)

$$
\begin{align*}
T^Y_{23} &= T^{\mu'}_{22} = \frac{1}{\sqrt{10}} \text{diag} (-1, -1, -1, 1, 1, 0) \\
T^Y_{21} &= T^{\mu'}_{11} = \frac{1}{\sqrt{10}} \text{diag} (2, 2, 2, -1, -1, -2)
\end{align*}
$$

(2.15)

Either version of $T^Y_{mF}$ can be used when acting on gaugino or adjoint fields since $T^0_+$ commutes with those. The $A_{\mu}^{\mu'}$ group structures when acting on gauginos and adjoint fermions is the same as $T^Y_{23}$.

It will be seen below that the $T^Y_{mF}$ matrices have the correct form for their gauge boson $A_{\mu}^Y$ to be identified as the U(1) weak hypercharge field of the Standard Model with $-\frac{1}{2} \sqrt{\frac{2}{3}} g_Y$ identified as the weak hypercharge coupling. The $A_{\mu}^{\mu'}$ gauge field with its coupling $g_Y'$ is a second U(1) gauge boson in this model that remains massless at the unification scale. Due to eq (2.10), there is no mixing between the $A_{\mu}^Y$ and $A_{\mu}^{\mu'}$ gauge fields. Below the unification scale, the couplings for the SU(3), SU(2), $Y$ and $Y'$ groups run differently, so they are denoted by $g_3$, $g_2$, $g_Y$ and $g_Y'$.

In this model according to eq (2.8), electroweak symmetry is primarily broken by $\phi_{23}$. This vev gives masses to the $W$ and $Z$ bosons, leaving only the SU(3) gluons, the photon and the $Z'$ boson massless. To see the structure of the diagonal fields, one may again make a Weinberg-angle rotation:
The angle $\theta_Z$ is chosen to make $(T_{21})_{44} = (T_{22})_{55} = 0$, so that the photon gets no mass from $\bar{\phi}_{21}$, $\bar{\phi}_{22}$ or $\phi_Z$.

The angle $\phi_Z$ is chosen to make $(T_{21})_{44} = 0$. The resulting angles are given by:

\[
\begin{align*}
\tan \theta_Z &= \sqrt{\frac{3}{2}} g_Y/g_2 \\
\tan \phi_Z &= \frac{1}{\sqrt{10}} \cos \theta_Z g_Y/g_2.
\end{align*}
\]  

(2.17)

In section 4, it will be argued that $g_Z$ will be driven to a very small value from an effective anomaly. As a result, the angle $\phi_Z$ is very small, and $\theta_Z$ is very close to the Weinberg angle $\theta_W$ of the Standard Model.

The photon group structure is given by

\[
\begin{align*}
e T_{22}' &= e T_{22} = -e \text{diag} \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1, 0, 0 \right) \\
e T_{21}' &= e T_{12}' = e \text{diag} \left( \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 1, 1 \right),
\end{align*}
\]  

(2.18)

where the following coupling constant normalization is used:

\[
e = -g_2 \sin \theta_Z.
\]  

(2.19)

For the case where $|\phi_Z| \ll 1$, the $Z$ boson group structure is approximately:

\[
\begin{align*}
g_Z T_{22}' &\simeq -(g_2/\cos \theta_W) \text{diag} \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2} + x, \frac{1}{2}, 0 \right) \\
g_Z T_{12}' &\simeq (g_2/\cos \theta_W) \text{diag} \left( \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{1}{2} - x, \frac{1}{2}, \frac{1}{2} + x, x \right) \\
x &= \sin^2 \theta_Z \simeq \sin^2 \theta_W.
\end{align*}
\]  

(2.20)

The group structure of the $Z'$ boson is shown in eq (5.2) of section 5.

Now that the weak hypercharge and electric charge have been established, it is possible to map the fermions in this model to fermions of the Standard Model. Based on their $SU(3) \times SU(2) \times U(1)_Y$ interactions, the fermions defined in eqs (1.16), (1.13), and (1.19) can be labelled:

\[
\begin{align*}
\lambda &= \frac{1}{\sqrt{2}} \begin{pmatrix}
\hat{u}_W^G & \hat{u}_W^{G_i} & \hat{u}_W^{G_3} \\
\hat{d}_W^G & \hat{d}_W^{G_i} & \hat{d}_W^{G_3}
\end{pmatrix} \\
\tilde{\lambda} &= \frac{1}{\sqrt{2}} \begin{pmatrix}
\hat{d}_W^A & \hat{d}_W^{A_i} & \hat{d}_W^{A_3} \\
\hat{u}_W^A & \hat{u}_W^{A_i} & \hat{u}_W^{A_3}
\end{pmatrix} \\
\psi_{11} &= \begin{pmatrix}
\psi_{11}^{(1)} \\
\psi_{12}^{(1)} \\
\psi_{13}^{(1)}
\end{pmatrix} \\
\tilde{\psi}_{11} &= \begin{pmatrix}
\tilde{\psi}_{11}^{(1)} \\
\tilde{\psi}_{12}^{(1)} \\
\tilde{\psi}_{13}^{(1)}
\end{pmatrix}
\end{align*}
\]  

(2.21)

In the above labelling, lower numerical indices are fundamental-representation indices for the unbroken $SU(3)$ group (the strong interaction). Fermions with a “$W$” index interact with the $W$ boson (as members of an isodoublet). Based on the magnitude of their electric charges, up-type quarks, down-type quarks, charged leptons, and neutrinos are labelled with $u, d, e, \nu$. All of the fermion fields are 2-component Weyl fermions with a lower, undotted spin index. In the convention of [6] (which is also the Wess/Bagger & Bilal convention of [14]), a Weyl fermion with a lower undotted index corresponds to a right-chiral fermion that vanishes when acted on by $1 - \gamma_5$ (see appendix A of [15]).

In that convention, the $u$ and $d$ fields are right-chiral fermions with electric charges of $\frac{2}{3}$ and $-\frac{1}{3}$, respectively, so they are mapped to right-chiral quarks. The fields $\bar{u}$ and $\bar{d}$ are right-chiral fermions with electric charges of $\frac{1}{3}$ and $-\frac{1}{3}$, respectively, so they are mapped to Hermitian conjugates of left-chiral quarks. One way that this model differs from the Standard Model is that some of the right-chiral quarks have a “$W$” index so they interact with the $W$ boson, while some of the left-chiral quarks lack that index so they do not interact with the $W$ boson. That difference is discussed in the next section.
3. \textbf{MASSES AND MIXING OF OBSERVED PARTICLES}

This section begins by discussing quantum-generated interactions and their effect on the vacuum. Given certain assumptions about those quantum interactions, it is shown how this model produces the observed spectrum of particle masses. To validate the assumptions made, detailed quantum calculations would be needed, and those calculations are not performed in this paper. Instead, a picture is sketched as to what those calculations would need to produce in order to generate measured masses and mixing.

\textbf{A. Confinement}

For a supersymmetric gauge theory involving a chiral superfield in the adjoint representation, it has been shown that a tree-level mass term \( m_{\Phi} \) for the adjoint superfield will lead to quark confinement [13, 14, 16, 17]. The duality inherent in these theories permits moving from a description in terms of strongly coupled scalars with color-electric charge to a description in terms of weakly coupled monopoles with color-magnetic charge. A tree-level mass term can cause the vacuum to settle on one of two configurations where the vev of the trace of the square of the adjoint superfield does not vanish \( \langle \text{tr} \left( \phi_1^2 \right) \rangle \neq 0 \). In one of those configurations, color-magnetic monopoles become massless, condense, and cause quark confinement through a dual Meissner effect [13, 14, 16, 17].

The theory of this paper is not supersymmetric, since its “superfields” are “twisted”. Nonetheless, this theory does have an adjoint-representation “twisted superfield” with a mass \( m_{\Phi} \). Also, the scalars in this theory are in the same representation as the scalars in the corresponding supersymmetric theory, so the vacuum moduli spaces of the two theories should be similar, particularly below the scale where the \( m = 1 \) fundamental and conjugate scalars of the theory (aka leptoquarks) get large masses (see section 5). That being the case, it is speculated that quantum effects similar to those in the supersymmetric theory cause the following vev to form:

\[ \langle \text{tr} \left( \phi_1^2 \right) \rangle \neq 0 \text{ quantum mechanically.} \quad (3.1) \]

It is further speculated that the similarity with the corresponding supersymmetric moduli space is sufficient so that the scalars in \( \phi_1 \) form color-magnetic monopoles that become massless, condense and cause quark confinement through the dual Meissner effect.

The vev \( \langle \phi_1 \rangle \) will also generate quark mass terms of the following form (connecting gaugino and adjoint quarks):

\[ -\sqrt{2}g_3 i \int d^4 x \text{tr} \left\{ \tilde{\chi} \left( \langle \phi_1 \rangle \lambda - \lambda \langle \phi_1 \rangle \chi \right) + \text{h.c.} \right\}, \quad (3.2) \]

where it has been assumed that the strong coupling \( g_3 \) is the appropriate coupling to use for these quark mass terms.

There are a couple of additional quantum effects pertinent to this section. For supersymmetric theories, instantons can generate nonperturbative low-energy effective superpotential terms. The Appendix proposes that a similar effect occurs for this theory. To accommodate these terms, the following vevs that are zero classically acquire small but nonzero vev quantum mechanically:

\[ \bar{\phi}_{21}, \bar{\phi}_{22} \neq 0 \text{ quantum mechanically.} \quad (3.3) \]

A quantum superpotential term proportional to \( 1/\bar{\phi}_{21} \) (see eq (A.13)) sets the scale for masses of the heavy right-handed neutrinos discussed later in this section.

As discussed in the Appendix, the quantum-generated superpotential terms also try to reduce the value of \( \langle \text{tr} \left( \phi_1^2 \right) \rangle + \langle \text{tr} \left( \phi_2^2 \right) \rangle \). To accomplish this in the presence of eq (3.1), the quantum vacuum modifies \( \langle \phi_2 \rangle \) of eq (2.6) to the following:

\[ \langle \phi_2 \rangle = \frac{\bar{\phi}_2}{\sqrt{2(1+c^2)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & c & 0 \end{pmatrix}, \quad (3.4) \]

where the quantum-generated \( c \) is much smaller than 1. The square of the above vev has a nonvanishing trace that can reduce \( \langle \text{tr} \left( \phi_1^2 \right) \rangle + \langle \text{tr} \left( \phi_2^2 \right) \rangle \).

\textbf{B. Observed boson masses}

From the symmetry breaking defined in eq (2.8), the mass of the \( W \) boson is primarily determined by \( \bar{\phi}_{21}^2 \). In other words, at tree level:

\[ M^2_W \approx \frac{1}{2} g_2^2 \bar{\phi}_{21}^2. \quad (3.5) \]

Phenomenologically, the mass of the \( W \) boson determines the vev \( \bar{\phi}_{21}^2 \), with the running coupling \( g_2^2 \) evaluated at the \( W \) boson mass scale.

The \( Z \) boson mass in this model is also primarily determined by \( \bar{\phi}_{21}^2 \). Due to the form of eq (2.16), the \( Z \) boson mass in this model differs slightly from the Standard Model expression. The mass of the \( Z \) boson is:

\[ M_Z \approx M_W / \left( \cos \phi_Z \cos \theta_Z \right), \quad (3.6) \]

where the angles are defined by coupling constants as in eq (2.17). In section 5 when discussing the \( Z' \) boson, \( \sin \phi_Z \) is estimated to be on the order of \( 10^{-3} \) at the scale of \( \sim 17 \) MeV. It could be a little larger at the mass of the \( Z \) boson, but should still be very small. For that reason, the following approximation can be used for most purposes in this paper:

\[ \cos \theta_Z \approx \cos \theta_W \]

\[ \cos \phi_Z \approx 1, \quad (3.7) \]
where $\theta_W$ is the Weinberg angle.

In this model, the scalar vev $\phi_2$ also generates most of the mass of the observed Brout-Englert-Higgs boson, through the d-term part of the scalar potential. The d-term has two parts: one part from terms like $\langle d_1^2 \rangle d_2^2$, and the other part where each $d_i^2$ in $\frac{1}{2}d_1^2d_2^4$ has one vev and one Higgs field. Those d-term contributions can be found by first re-expanding the d term part of the scalar potential of eq (2.1) using the following basis of diagonal $U(3) \times U(3)$ generators:

$$g_U T_U^i, g_Y T_Y^i, g_{3T_3}^i, g_5 T_5^i, g_3 T_1^i.$$  

(3.8)

The expansion of the complex scalar field $\phi_{21}$ into its component fields includes the following:

$$i\phi_{21} = \left( \phi_{21} + \frac{1}{\sqrt{2}} h_{21} \right) (0, 0)^T + ..., \quad (3.9)$$

where $h_{21}$ is the real scalar Higgs boson field and + includes the fields that get “eaten” by the W and Z gauge bosons.

In the basis of eq (3.8), the vevs of the auxiliary d fields at the minimum of the scalar potential are approximately:

$$-g_U \langle d_U \rangle \simeq \sqrt{2} m_{21}^2$$

$$-g_2 \langle d_2^2 \rangle = \frac{1}{2} g_2^2 \phi_{22}$$

$$-g_Y \langle d_Y \rangle = \frac{3}{8} \sqrt{2} g_2 \phi_{21}^2,$$

(3.10)

where it has been assumed that $g_+ = g$ at the unification scale and that $g_Y$ is negligible due to reasons discussed below. The approximation has also assumed that $\phi_{22}^2 \gg \phi_{21}^2, \phi_2$.

Assuming $m_{21}$ is small, the mass of the Higgs boson (before radiative corrections) is:

$$M_H^2 \simeq m_{21}^2 - \frac{1}{2} g_2 \langle d_2^2 \rangle + \frac{\sqrt{2}}{2} g_Y \langle d_Y \rangle + \frac{1}{3\sqrt{2}} g_U \langle d_U \rangle + 2 \phi_{21} \left[ g_U \left( T_U^1 \right)_{44}^2 + g_2 \left( T_2^1 \right)_{44}^2 + g_3 \left( T_3^1 \right)_{44}^2 \right]$$

$$\simeq M_0^2 + \left( \frac{1}{4} \phi_{21} \right)^2 m_{21}^2 \simeq (97 \text{ GeV})^2 + \frac{3}{2} m_{21}^2,$$

(3.11)

where the scalar mass $m_{21}$ is from eq (1.18). The third line above comes from the relations in eqs (2.17) and (3.6) along with the approximation that $g_V \approx g_x$. To get the correct mass squared, the mass parameter $m_{21}$ plus radiative corrections contribute $\sim 80$ GeV to the Higgs Boson mass.

Just as in the Standard Model, it is assumed that radiative corrections of the Higgs Boson are at a scale similar to the electroweak scale, not the unification scale. In the Standard Model, it is an open question as to how cancellation of unification-scale corrections are able to generate electroweak-scale corrections to the Higgs mass. By contrast, supersymmetric theories provide a simple reason for such cancellation: superspace gauge transformations and translations can change a scalar into a fermion, and fermions do not have quadratic divergences. Even if some part of the gauge symmetry is broken at a very large scale in a SUSY theory, as long as the superpotential remains equal to zero, the remaining superspace gauge invariance continues to ensure cancellation of quadratic divergences in the symmetry-broken theory.

Although the theory considered here is not invariant to superspace translations, it is invariant to superspace gauge transformations that change scalars into fermions. Also, the theory’s unification-scale symmetry breaking leaves the superpotential almost equal to zero, with variations from zero only showing up at the electroweak scale. Parallels with supersymmetry then open the door to the possibility that quadratic divergences naturally cancel in this model down to the electroweak scale. Further work would be required to establish this.

The $h_{21}$ real scalar field described above has the same interactions with the $W$ boson, $Z$ boson, and top quark as does the Standard Model Higgs boson. So this model is consistent with measurements of Higgs boson decays and interactions involving these particles, since those measurements are consistent with the Standard Model.

On the other hand, interactions of $h_{21}$ with other quarks or leptons are different in this model than in the Standard Model. As described later in this section, leptons and quarks (other than the top) acquire masses from different mechanisms, not from Yukawa interactions with the Higgs Boson. Those particles do have interactions with $h_{21}$ via the nonperturbative terms described in the Appendix. More work would need be done to determine whether those interactions are consistent with observations.

### C. Quark masses and mixing

Keeping in mind the particle designations of this model defined in eq (2.21), the up-type quarks can be arranged into the following 3x3 mass matrix (and its Hermitian conjugate):

$$\begin{pmatrix}
\tilde{u}^{(1)} & \tilde{u}_A^{(1)} & \tilde{u}_W^{(1)} \\
\tilde{u}_{i1}^{(1)} & \tilde{u}_{i2}^{(1)} & \tilde{u}_{i3}^{(1)} \\
\tilde{u}_{i4}^{(1)} & \tilde{u}_{i5}^{(1)} & \tilde{u}_{i6}^{(1)}
\end{pmatrix}$$

$$\begin{pmatrix}
u_A^{(1)} & \nu_W^{(1)} \\

\nu_{i1}^{(1)} & \nu_{i2}^{(1)} & \nu_{i3}^{(1)} \\

\nu_{i4}^{(1)} & \nu_{i5}^{(1)} & \nu_{i6}^{(1)}
\end{pmatrix}$$

Columns : $u_A^{(1)}, u_W^{(1)}$ Rows : $\tilde{u}_i^{(1)}$ (3.12)

$$M_U = \begin{pmatrix}
\tilde{\phi}_{21} & \tilde{\phi}_{22} & \tilde{\phi}_{23} \\

\tilde{\phi}_{24} & \tilde{\phi}_{25} & \tilde{\phi}_{26} \\

\tilde{\phi}_{27} & \tilde{\phi}_{28} & \tilde{\phi}_{29}
\end{pmatrix},$$

where the gaugino coupling $\tilde{g}$ is discussed below in eq (3.14). A tilde is put on tree-level masses and superpotential couplings to show that they include quantum modifications from the Appendix. For all of the fermion mass matrices in this paper (including the above matrix) the rows have an upper undotted spin index (using the convention of [6]) while the columns have a lower undotted spin index (e.g. $\tilde{u}_i^{(1)}$ or $\nu_i^{(1)}$). Those indices are summed over, and they are suppressed.

The parameter $\tilde{M}_{\bar{G}}^{(1)}$ is generated nonperturbatively from terms like eq (A.17) in the Appendix. The mass $\Delta$ is from eq (3.2).
The first, second, and third generation up-type quarks correspond to the first, second and third rows and columns of the up-type quark matrix. To a first approximation, the third-generation quark is just the top quark and its mass is approximately:

\[ m_t \simeq \hat{g} \bar{\phi}_{21} \text{ Top quark.} \quad (3.13) \]

Recalling the convention from eq (2.21) that fields with a tilde are Hermitian conjugates of left-chiral quarks while those without one are right-chiral quarks, it can be seen from eq (3.12) that for the third-generation quark, only its left-chiral component interacts with the W boson. That allows this model to be consistent with top-quark polarization measurements by ATLAS [18].

The first- and second-generation quark interactions with the W boson, however, differ from those of the Standard Model. For the second-generation up-type quark (mostly charm), both its left- and right-chiral components interact with the W boson. For the first-generation up-type quark (mostly up), only its right-chiral component interacts with the W boson. This chiral flipping of the first generation is not ruled out by experiment, since the spin of the proton (and other light hadrons) comes primarily from gluons and orbital angular momentum [19]. As a result, it is not possible to experimentally disentangle the spins of up and down quarks from other hadronic spins and thereby verify that only left-handed up and down quarks interact with the W boson. Of course, a requirement of this model is that it must be able to reproduce CKM data. It does this even better than the Standard Model, as described below.

Above in eq (3.5), it was shown that the magnitude of the vev \( \bar{\phi}_{21} \) is approximately determined by the W boson mass and the SU(2) weak coupling constant \( g_2 \). In this model, the top quark mass is also determined by \( \bar{\phi}_{21} \), but multiplied by the gaugino coupling \( \hat{g} \) rather than the weak coupling \( g_2 \) (at the unification scale, these are the same). To be consistent with observation, this model would need to show that the difference in the coupling constants \( g_2 \) vs. \( \hat{g} \) at the electroweak scale is the amount needed for the model to correctly reproduce both the W boson mass and the top quark mass.

To get the actual value for this model’s top quark mass, the calculation should be performed to determine how \( \hat{g} \) runs in this model as the scale is lowered from the unification scale. Such a calculation is outside the scope of this paper.

Instead, a phenomenological approach is used. Relations for the W boson and top quark masses require

\[ \hat{g} \simeq \frac{m_t}{\sqrt{2} M_W} g_2 \simeq 1.5 g_2 \simeq 0.8 g_3, \quad (3.14) \]

where \( g_2 \) and \( g_3 \) are evaluated at the Z boson mass scale. It is presumed that the gaugino coupling \( \hat{g} \) of this model takes the above value at the Z boson mass scale.

From eq (2.21), it can be seen that there are twice as many flavors of down-type quarks in this model (6L × 6R) as there are up-type quarks (3L × 3R). However, the unification scale vevs generate unification-scale masses for \( \tilde{d}^G \tilde{d}^3 \) and \( \tilde{d}^d \tilde{d}^G \), so those down-type quarks decouple. The remaining 4x4 down-type quark mass matrix has the following structure:

\[
M_D = \begin{pmatrix}
\bar{\phi}_{22} & \cdots & \bar{\phi}_{22} \\
\cdots & \frac{1}{\sqrt{2}} \bar{\Gamma}_c \bar{\phi}_2 & \cdots \\
\bar{\phi}_{22} & \cdots & \frac{1}{\sqrt{2}} \bar{\Gamma}_c \bar{\phi}_2
\end{pmatrix},
\quad (3.15)
\]

where only tree-level couplings are shown explicitly. Tildes again indicate that these couplings can be modified by quantum-generated couplings discussed in the appendix. Quantum couplings can also be generated for the components without tree-level contributions; those are denoted by “...”. Primes and double primes on \( \bar{m}_F \) and \( \bar{m}_L \) are used to show that these masses in the down-type matrix do not need to be the same as those in the up-type matrix, as discussed in the Appendix.

To a first approximation, the masses of the \( d, s, \) and \( b \) quarks as well as an additional \( f \) quark are given by the diagonal elements of eq (3.15). For example, \( m_b \simeq \hat{g} \bar{\phi}_{22} \). Comparison with the top quark mass sets the value for \( \bar{\phi}_{22} \). The down-type quark mass is primarily determined by the adjoint vev \( \varphi_2 \). In this model, none of the down-type quarks get their masses from the Higgs vev \( \bar{\phi}_{21} \).

For this model, the numerical values of the quark mass matrices (in GeV) are assumed to be:

\[
\begin{align*}
\text{Rows} & : \bar{u}'_L, \bar{c}'_L, \bar{t}'_L \quad \text{Columns} : u'_R, c'_R, t'_R \\
M_U & = \begin{pmatrix}
-0.0033 & 0.0000 & -0.0277 \\
0.1532 & 1.2316 & 0.9688 \\
7.2510 & -7.8110 & 171.59
\end{pmatrix},
\quad (3.16)
\end{align*}
\]

\[
\begin{align*}
\text{Rows} & : \bar{d}'_L, \bar{s}'_L, \bar{b}'_L, \bar{t}'_L \quad \text{Columns} : d'_R, s'_R, b'_R, t'_R \\
M_D & = \begin{pmatrix}
0.0039 & -0.0232 & 0.1192 & 0.0503 \\
-0.0330 & 0.0872 & 0.1953 & -0.1188 \\
0.0093 & -0.0589 & 4.1507 & 0.2934 \\
0.2497 & 0.0127 & -0.4219 & 2.8693
\end{pmatrix},
\quad (3.17)
\end{align*}
\]

The rows and columns of these matrices are the same gauge eigenstates as in eqs (3.12) and (3.15); they have just been renamed to reflect their dominant mass eigenstate. For example,

\[
\bar{d}^A_W = \bar{g}_{L} = 0.97 \bar{s}_L + 0.22 \bar{d}_L + 0.04 \bar{b}_L - 0.04 \bar{t}_L,
\quad (3.18)
\]

where \( \bar{s}_L \) is in the mass eigenbasis (see below) and \( \bar{q}_L \) denotes the right-handed field (\( q_L \)).
data. In section 5, there is a discussion about how the theory is able to reproduce the many experiments that would seemingly rule out the possibility of an additional light quark. As more data is collected, it may be that rotations of these quark matrices that still reproduce quark masses and CKM data may provide a better global fit. Future work should also verify that there are no inconsistencies between these matrices and the quantum calculations described in the Appendix.

Just as in the Standard Model, each quark mass matrix can be diagonalized via a unitary matrix $V$ on each side:

$$V_U^T M_U V_R = \text{diag}(-m_u, m_c, m_t)$$
$$V_L^T M_D V_R^\dagger = \text{diag}(-m_d, m_s, m_b),$$

(3.19)

where the subscript $f$ is used to denote the fourth down-type quark. The unitary matrices that diagonalize the quark mass matrices can be found by first multiplying each mass matrix on the left or on the right by its transpose, then finding the eigenvectors of those product matrices. The allowed flexibility to introduce complex phases was not used for the fits of this paper, so no attempt was made to fit the experimentally measured complex phases of the CKM matrix. The mass eigenvalues of the matrices in eqs (3.16) and (3.17) are:

$$m_u = 0.002, \quad m_c = 1.28, \quad m_t = 172$$
$$m_d = 0.005, \quad m_s = 0.095, \quad m_b = 4.18, \quad m_f = 2.9,$$

(3.20)

where all mass values are in GeV.

In the Standard Model, the CKM matrix displays the connections that the $W$ boson makes between up-type and down-type quark mass eigenstates. Since there are 3 quarks of each type in the Standard Model, the CKM matrix is a $3 \times 3$ matrix. In this model, there are 3 up-type and 4 down-type quarks, so the equivalent “CKM” matrix is a $3 \times 4$ matrix. Also, in the Standard Model only the left-handed quarks (and hence $V_U^T, V_L^T$) matrices have connections to the $W$ boson, but in this model, all four of the matrices have connections to the $W$.

From the diagonalizing matrices, one may construct the following 2 versions of CKM matrices:

$$V_{\text{CKM}}^\pm = V_R^T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} V_R^D$$

$$\pm V_L^T \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} V_L^D$$

(3.21)

The placement of the 1’s in the above matrices is based on which quark fields in eqs (3.12) and (3.15) have a $W$ subscript (signifying that they interact with the $W$ boson to change an up-type quark to a down-type quark and vice versa).

In this model, a different CKM matrix should be used depending on whether a vector current or axial vector current process is being considered. Specifically:

Vector current decays: $V_{\text{CKM}}^+$

Axial vector current decays: $V_{\text{CKM}}^-$. (3.22)

For CKM measurements involving vector current decays (e.g. an exclusive semi-leptonic decay from one spin-0 meson to a different spin-0 meson), $V_{\text{CKM}}^+$ should be used for comparison to this model. For CKM measurements involving axial vector current decays (e.g. the purely leptonic decay of a spin-0 pseudo-scalar meson), $V_{\text{CKM}}^-$ should be used.

Plugging in the unitary matrices $V$ found from eq (3.19) into eq (3.21), the following CKM matrices are obtained:

$$|V_{\text{CKM}}^+| = \begin{pmatrix} 0.9737 & 0.2230 & 0.0039 & 0.0013 \\ 0.2241 & 0.9718 & 0.0394 & 0.9569 \\ 0.0072 & 0.0393 & 0.9934 & 0.0619 \end{pmatrix}$$

(3.23)

$$|V_{\text{CKM}}^-| = \begin{pmatrix} 0.9732 & 0.2253 & 0.0043 & 0.0012 \\ 0.2183 & 0.9750 & 0.0430 & 1.0414 \\ 0.0824 & 0.0188 & 0.9936 & 0.1451 \end{pmatrix}$$

Comparing the first three columns of the above matrices to data presented in [20], it can be seen that despite having both left- and right-handed $W$ boson connections for quarks, the model does a good job of reproducing absolute values of most CKM data. In fact, comparing to data in [21], it can be seen that the model even does a good job of reproducing the perplexing $3\sigma$ difference seen between vector- and axial-vector-current data for $|V_{us}|$. As noted in [4], such a difference cannot arise in a model where all quark interactions with the $W$ boson are left-handed.

In this model, to be consistent with the $W$-boson interactions, the $Z$-boson interactions can be Flavor Changing Neutral Currents (FCNCs). The couplings to the $Z$ boson by quark generation are:

$$\bar{u}_i \gamma^\mu u_i Z_\mu$$

$$g_L \begin{pmatrix} -\frac{1}{3} x \\ \frac{1}{3} \beta - \frac{2}{3} x \end{pmatrix}$$

$$g_R \begin{pmatrix} \frac{1}{3} \alpha \beta - \frac{2}{3} x \\ -\frac{2}{3} x \end{pmatrix}$$

(3.24)

$$\bar{d}_i \gamma^\mu d_i Z_\mu$$

$$g_L \begin{pmatrix} -\frac{1}{3} + \frac{1}{3} x \\ \frac{1}{3} x \end{pmatrix}$$

$$g_R \begin{pmatrix} \frac{1}{3} \beta + \frac{2}{3} x \\ \frac{1}{3} \alpha \beta - \frac{2}{3} x \end{pmatrix}$$

(3.25)
where $x = \sin^2 \theta_W$ and the “SM” rows show the Standard Model couplings for up-type and down-type quarks. The primed quark variables are gauge eigenstates, just as in eqs (3.16) and (3.17).

Since the couplings of this model differ by generation, $Z$ boson connections mix mass eigenstates. For example, the $Z$ mixing matrix for left-chiral down-type quarks is:

$$V_{ZD}^L = V_L^D \begin{pmatrix} \frac{1}{\sqrt{2}} x & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} + \frac{1}{3} x & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} + \frac{1}{3} x & 0 \\ 0 & 0 & 0 & \frac{1}{3} x \end{pmatrix} V_L^{D\dagger}$$

where the diagonalizing matrix $V_L^D$ is the same as was used for CKM matrices, and $x = \sin^2 \theta_W = 0.2315$ was used. There are also $Z$ mixing matrices for right-chiral down-type quarks as well as for right- and left-chiral up-type quarks.

Most of the off-diagonal FCNC elements of these matrices are small, but a few are more significant. The only $Z$ mixing matrix elements with magnitudes greater than 0.006 are $V_{Zd}^A$, $V_{Zd}^L$, $V_{Zd}^\prime$, $V_{Zd}^{L\prime}$, $V_{Zd}^R$, $V_{Zd}^{R\prime}$, and $V_{Zd}^\prime R$. More detailed work would be needed to verify that the $Z$ boson FCNC mixing of this model is fully consistent with experimental data.

D. Leptons

In this model, charged leptons get their masses not from the vev of the Higgs boson but from vevs of the $\bar{\phi}_2$ adjoint scalars (see eq (3.4)). Through the superpotential couplings of eq (1.18), $m_{1FF'}$ and $\Gamma_{1FF'}$ generate charged lepton masses. Combining those with quantum-generated masses from (A.12) leads to the following lepton mass terms:

$$\begin{pmatrix} e^{(F)}_W \\ \bar{e}^{(F)}_W \\ \tilde{e}^{(F)}_W \end{pmatrix}^T \begin{pmatrix} \tilde{m}_{1FF'} & 0 & 0 \\ 0 & \tilde{m}_{1FF'} & \bar{\phi}_2 \Gamma_{1FF'} \\ 0 & c \bar{\phi}_2 \bar{\Gamma}_{1FF'} & \tilde{m}_{1FF'} \end{pmatrix} \begin{pmatrix} e^{(F)}_W \\ \bar{e}^{(F)}_W \\ \tilde{e}^{(F)}_W \end{pmatrix} .$$

As discussed in the Appendix, quantum effects can generate differences between $\tilde{m}_{1FF'}$ and $\tilde{m}_{1\prime FF'}$. The $\tilde{m}_{1FF'}$ terms are much smaller than the $\bar{\phi}_2 \bar{\Gamma}_{1FF'}$ terms.

In that case, since $c \ll 1$, the lepton mass terms for $\tilde{e}_W^{(F)} e^{(F)}_W$ are much larger than those for $\tilde{e}^{(F)}_W e^{(F)}_W$. For that reason, $\tilde{e}_W^{(F)}$ and $e^{(F)}_W$ are mapped to the right- and left-handed components of proposed heavy charged leptons, denoted “Omega” leptons. $\tilde{e}^{(F)}$ and $e^{(F)}_W$ are mapped to the right- and left-handed components of the known light charged leptons (electron, muon, and tau).

The masses of the Omega leptons should be between 103 GeV and ~30-90 TeV. The lower limit comes from the 95% confidence exclusion limit from heavy lepton searches [22]. The upper limit comes from the anomaly-based analysis surrounding eq (4.1).

From eq (2.18), the last three diagonal components of the photon field when acting on leptons are $\epsilon (0,1,1)$. As a result, if the left- and right-chiral components of the normal light charged leptons ($l$) are mapped to $l_L^{(F)} = \epsilon^{(F)}_l$ and $l_R^{(F)} = \bar{\epsilon}^{(F)}_l$, then those leptons have the correct electric charges of $+1$ and $-1$ as well as the correct isodoublet and isosinglet designations. The three flavors of heavy and light leptons ($\Omega$ and $l$) can therefore be identified as follows:

$$l^{(F)}_L = \epsilon^{(F)}_l, \quad \bar{e}^{(F)}_W = l^{(F)}_L, \quad \tilde{e}^{(F)}_W = \Omega^{(F)}_R, \quad \epsilon^{(F)}_W = l^{(F)}_R.$$  

From this identification, it can be seen that the negatively charged light leptons have the correct behavior that only their left-handed components interact with the $W$ boson, forming isodoublets with their neutrinos. On the other hand, the opposite is true for negatively charged Omega leptons; their $W$ boson interactions are right-handed. The light leptons not only have the same interactions with the photon and $W$ boson as in the Standard Model, they also have the same interactions with the $Z$ boson, as can be seen from eqs (2.19) and (2.20).

Appendix eq (A.14) shows how quantum effects generate a large Majorana mass $M_l$ for all three of the $l^{(F)}_L$ right-handed neutrinos. Again, for reasons discussed around eq (4.1), $M_l$ should be less than ~30-90 TeV. If $M_l$ is near that upper limit and the Dirac neutrino mass terms $\tilde{m}_{1FF'}$ are around the 1 MeV range, then a seesaw with $M_l$ will lead to very small masses for the left-handed neutrinos that are in the correct range to match observations. The parameters $\tilde{m}_{1FF'}$ may be chosen so that the eigenvalues of the seesaw-generated light neutrino mass matrix reproduce the observed mass differences [23]. For each choice of the nine parameters $\tilde{m}_{1FF'}$, diagonalization of the neutrino mass matrix determines the $3 \times 3$ left-handed and $3 \times 3$ right-handed rotation matrices that are required to transform from the flavor basis of this paper to the neutrino mass eigenbasis.

The masses of the charged Omega leptons are defined by the $3 \times 3$ mass matrix $\bar{\phi}_2 \bar{\Gamma}_{1FF'}$. The light charged lepton $3 \times 3$ mass matrix is partly generated by $c \bar{\phi}_2 \bar{\Gamma}_{1FF'}$ and partly by a seesaw between $\bar{\phi}_2 \bar{\Gamma}_{1FF'}$ and the $\tilde{m}_{1FF'}$ masses. The parameters $\Gamma_{1FF'}$, $\tilde{m}_{1FF'}$ and $c$ may be chosen so that the eigenvalues of the light charged lepton mass matrix are equal to the electron, muon and tau masses. For each choice of these parameters, diagonalization of the light charged lepton mass matrix determines the $3 \times 3$ left-handed and $3 \times 3$ right-handed rotation matrices that are required to transform from the flavor basis of this paper to the light charged lepton mass eigenbasis. In particular, these parameters may be chosen so that the combined left-handed rotation from the light neu-
trino mass eigenbasis to the light charged lepton mass eigenbasis is able to reproduce the observed Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [23].

4. UNIFICATION AND ANOMALIES

As mentioned in section 1, at a scale above any symmetry breaking, the model of this paper is free of anomalies. The reason is simple: for every fermion in every representation, there is another fermion in a conjugate representation of each of the SU(3) groups that also has opposite charges for the Abelian fields $A^0_{\nu_R}$ and $A^0_{\nu_L}$. Consequently, all of the gauge anomaly triangle diagrams cancel.

The unification scale $\Lambda_U$ of this model is defined as the scale where the vevs $\bar{u}_{23}$ and $\bar{\phi}_{23}$ break the original SU(3)×SU(3)×U(1)×U(1) symmetry down to SU(3)×SU(2)×U(1)×U(1). Below the unification scale, the four couplings $g_3, g_2, g_Y$ and $g_{Y'}$ for these four groups run differently. As mentioned in section 3, this symmetry breaking causes the quark pairs $d^c_2$, $d^c_3$ and $d^c_1$ to acquire unification-scale masses. These heavy fermions are in conjugate representations of the remaining SU(3) group, are SU(2) singlets, and have equal and opposite charges for the Abelian $Y$ and $Y'$ fields. Consequently, the effective theory below the unification scale that ignores these heavy quarks is also free of local gauge anomalies.

As can be seen from $T_{21}^Y$ and $T_{21}'$ of eqs (2.14) and (2.15), the coupling of right-handed top quarks to $Y'$ is $\sqrt{6}$ times larger than to $Y$. The $Y'$ charges of all of the other fermions and scalars are also larger than their $Y$ charges. The net result is that below the unification scale, $g_{Y'}$ drops to lower scales much more quickly than $g_Y$. This is a partial justification for the small value of $\sin \phi_Z$ assumed in sections 3 and 5.

Below the unification scale, the next lower scale of the theory is defined by the large Majorana mass $M_\nu$ of the three right-handed neutrinos $\bar{\nu}_W$ discussed in eq (A.14) of the Appendix. As a default model, it is assumed that the two of the charged Omega leptons predicted by the model also have masses at this scale, but the mass of the third is closer to 103 GeV, the experimental lower limit for an additional charged lepton [22]. An effective theory below the scale of $M_\nu$ that excluded those heavy leptons would have an anomaly (see [24, 25]).

Applying arguments from [26], this effective anomaly implies the following upper limit for the mass of those neutrinos:

$$\tilde{M}_\nu \lesssim \frac{64\pi^3 M_Z}{3 + 2 - 2 \times 2^3 |1/2 \sqrt{3} g_Y|^3} \simeq 32 \text{ TeV.} \quad (4.1)$$

When $\tilde{M}_\nu$ obeys the above relation, the Z boson mass is able to resolve the obstruction to renormalizability of the effective theory that is generated by anomalous diagrams involving the $Y'$ boson.

This upper limit depends on how many of the charged Omega leptons have masses similar to $M_\nu$. If all three of them do, then the limit is lower (~19 TeV). If none of them do (they all have masses nearer 103 GeV), then the limit is much higher since the effective anomaly is not as severe (~120 TeV). If only one has mass similar to $M_\nu$, then the upper limit is ~30 TeV.

After the $Z$ and $W$ bosons acquire masses at the electroweak scale, the remaining massless bosons are gluons, photons, and the $Z'$ boson. None of these couple to the heavy neutrinos, so they no longer generate an effective anomaly below the electroweak scale. Also, the right-handed Omega leptons $(\Omega_R)$ and the Hermitian conjugates of the left-handed Omega leptons $(\Omega_L)$ have equal and opposite charges for the photon and $Z'$, so the Omega leptons do not generate an effective anomaly below the electroweak scale. But another effect causes an effective anomaly for the $Z'$ boson.

The different right-handed components of the top quark $(t_R$ and $t_L$) have opposite electric charges, but they do not have opposite $Z'$ charges. Therefore, an effective theory without the top quark has an anomaly for the $Z'$ boson. Since the $Z'$ boson does not acquire mass at the electroweak scale, it is proposed that this effective anomaly drives $g_{Z'}$ to a very small value.

Said another way, since the underlying theory with the top quark is anomaly free, the $Z'$-induced anomaly of an effective theory without it must manifest itself by causing the $Z'$ coupling to run much more quickly than usual to very small values as the scale under consideration is lowered. In other words, the effective anomaly provides the remainder of the qualitative justification for the very small values of $g_{Z'}$ and $\sin \phi_Z$ of this model.

At a scale $\mu$ below the unification scale but above $\nu_3$, the running coupling constants for the SU(N) groups obey the following equation:

$$\frac{4\pi}{g^2_S(\mu)} = \frac{4\pi}{g^2_S(\Lambda_U)} - \frac{b_N}{2\pi} \ln \left( \frac{\mu}{\Lambda_U} \right)$$

$$b_N = \left( \frac{11}{3} N + \frac{1}{3} n_f + \frac{1}{6} n_s + \frac{2}{3} N n_{fA} + \frac{1}{3} N n_{sA} \right), \quad (4.2)$$

where $n_f$ and $n_s$, $n_{fA}$ and $n_{sA}$ are the numbers of fundamental fermion and scalar N-tuplets, and the numbers of fermion and scalar adjoint representations, respectively. In all cases, the fermions are 2-component Weyl fermions and the scalars are complex.

In this model, $n_{fA} = 0$ and $n_{sA} = 1$ for each SU(N) group. The particle content to use between $\Lambda_U$ and $M_\nu$ is the following:

1. Up quarks: 4 W triplets and 2 non-W triplets
2. Down quarks: 4 W triplets and 4 non-W triplets
3. Charged leptons: 6 W and 6 non-W
4. Neutrinos: 6 W
5. SU(3) scalars: 6 triplets
6. SU(2) scalars: 6 doublets (2x flavors 1 & 2, 2x adjoint)
7. Singlet scalars: 6 (2x flavors 1 & 2, 2x adjoint),
where in the notation of this paper, a “W” fermion is part of an isodoublet that interacts with the $W$ boson.

The scalars fall into these categories for the following reasons: The $m = 2$ flavor 3 triplets get eaten (or made massive) by the unification-scale symmetry breaking. Of the 9 components of the $m = 2$ U(3) adjoint scalars, 3 are an SU(2) adjoint multiplet, 4 form 2 doublets, and the remaining 2 form 2 singlets.

With the above particle content, the beta factors for running of couplings between $\Lambda_U$ and $\tilde{M}_\nu$ are:

$$b_3 = \left( -\frac{11}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right) = -\frac{13}{3}$$

$$b_2 = \left( -\frac{11}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right) = \frac{1}{3}. \quad (4.4)$$

Between the $\tilde{M}_\nu$ scale and the electroweak scale, $b_3$ remains the same, but $b_2$ is reduced due to decoupling of the three heavy neutrinos, two of the three heavy charged leptons (in the default model) and 1 scalar. For the SU(2) coupling, the 18 fermion doublets are reduced to 15 and the 6 scalar doublets are reduced to 5. So between $\Lambda_U$ and $\tilde{M}_\nu$, $b_2 = -5/6$.

The unification scale can be found by starting with the measured values of the SU(2) and SU(3) coupling constants at the scale of $M_Z$, using $b_3 = -\frac{13}{3}$ and $b_2 = -5/6$ to run the couplings up to the scale of $\tilde{M}_\nu$, and then using $b_3 = -\frac{13}{3}$ and $b_2 = 1/3$ to run them up further until they have the same value. Assuming that $\tilde{M}_\nu \sim 32$ TeV (the top of the allowed range from eq (4.1)), then the unification scale is:

$$\Lambda_U \simeq 6 \times 10^{16} \text{ GeV}. \quad (4.5)$$

The inverse of the nonAbelian coupling at the unification scale is:

$$\alpha_{\gamma}^{-1}(\Lambda_U) = \alpha_{3}^{-1}(\Lambda_U) \simeq 28, \quad (4.6)$$

where $\alpha_N = g_N^2/4\pi$.

In this model, the beta factor for the weak hypercharge coupling $g_Y$ between $\Lambda_U$ and $\tilde{M}_\nu$ is:

$$b_Y = \frac{3}{20} \left( 2 \left( \frac{1}{4} \right)^2 2 + \left( \frac{3}{4} \right)^2 2 + \left( \frac{5}{4} \right)^2 6 \right)$$

$$+ \frac{3}{20} \left( \frac{2}{5} \left( 1 \right)^2 12 + \left( 2 \right)^2 6 \right) + \frac{1}{3} \left( (1)^2 12 + (2)^2 2 \right)$$

$$b_Y = \frac{121}{15}. \quad (4.7)$$

where the $m = 2$ flavor 2.3 singlet scalars have zero hypercharge. Between $\tilde{M}_\nu$ and the electroweak scale, the 6 neutrinos are reduced to 3, the 6 charged lepton flavors are reduced to 4, and the 12 non-singlet scalars are reduced to 11. So between $\tilde{M}_\nu$ and $M_Z$, $b_Y = 403/60$.

Running the $Y$ coupling up to $\tilde{M}_\nu$ and then up to $\Lambda_U$, the hypercharge coupling at the unification scale is

$$\alpha_{\gamma}^{-1}(\Lambda_U) \simeq 27. \quad (4.8)$$

So in the simplified scenario presented here, the three couplings come close to unifying at the same scale. Different assumptions about masses of the heavy charged leptons could lead to $\alpha_{\gamma}^{-1}(\Lambda_U) \simeq 28$. In that case, $g_+$ and $g$ would take the same value at the unification scale (motivating that assumption in section 2).

As mentioned above, $g_{Y'}$ also runs more quickly below the scale of $m_1$ due to a combination of a larger beta function and the top-quark-induced effective anomaly. It is proposed that if the effect of the effective anomaly was fully taken into account, a $g_{Y'}$ that unified with the other couplings at the unification scale could potentially run down fast enough (transforming to $g_{Z'}$ at $M_Z$) in order to reach the extremely small values required to reproduce the measurements of $Z'$ boson interactions described in section 5. Future work would be required to determine whether the suggested scenario can be supported by detailed calculations.

5. EXPERIMENTAL IMPLICATIONS

The model proposed in this paper is very different than the Standard Model. To truly define this model, more detailed calculations would need to be performed that are outside the scope of this paper. But even in the absence of these calculations, a number of statements can be made about features (e.g. masses, couplings) the model would have to have in order to reproduce experimental data.

This section shows that the model has a structure that may allow it to reproduce well-established precision experiments while also providing new physics explanations for many of the anomalies described in [4]. These anomalies are instances where experimental observations disagree with Standard Model predictions by 3-7$\sigma$. The first subsection below describes the additional U(1) $Z'$ boson required by this model, and how it can reproduce measurements of the X17 anomaly. The next subsection describes implications of the fact that the model has a seventh flavor of quark as well as right-handed quark couplings. The latter provide an explanation for anomalous CKM data, while the former allow the model to re-interpret the observed exotic charmonium hadron spectrum as just mesons and baryons involving the additional quark. The next two subsections describe how the model can reproduce hadronic cross section data and precision measurements at the $Z$ pole. The section ends by discussing the model’s additional predicted leptons and scalars, relating some of them to hints of potential new particles seen at the LHC.

A. A Light $Z'$ Boson

A consequence of the structure of this model is the existence of an additional light U(1) gauge boson – a $Z'$. In order to be consistent with precision electroweak
experiments, the angle $\phi_Z$ (from eqs (2.16) and (2.17)) that determines the mass and coupling of the $Z'$ must be very small. In section 4, a mechanism was suggested for this very small value. The calculations in this paper are not detailed enough to predict the mass and coupling of the $Z'$ boson. Instead, a phenomenological approach is taken; an observed experimental anomaly is studied and mapped to the $Z'$ of this model.

A group at the Institute for Nuclear Research ATOMKI in Hungary has published evidence consistent with the existence of a neutral boson with a mass of ~17 MeV [27–31]. So far, only ATOMKI and a second group in Hanoi that included some ATOMKI personnel [32] have detected this “17 MeV anomaly” (aka the X17). That being said, the X17 anomaly has been observed in experiments involving three different nuclei ($^6$Be, $^4$He, $^{12}$C) and at a 7$\sigma$ level of significance. The analysis below assumes that the $Z'$ boson of this model is the X17 with a mass of ~17 MeV.

In section 4, it is mentioned that the top quark generates an anomaly for $Z'$. This is actually assumed to first affect $Y'$ from the top quark scale to the $Z$ boson mass scale, and then to affect $Z'$ below the $Z$ boson mass scale. That anomaly causes the coupling for $Y'$ interactions with $Q_{21}$ and gaugino fields (both of which have top-quark components) to run down more quickly than the coupling for $Y'$ interactions with $Q_{22}$ fields (which only have bottom and down quark components). That effect can be parametrized by a factor $\kappa_{22}$ by which the $g_{Y'}$ coupling to $Q_{22}$ exceeds that to $Q_{21}$ and gaugino fields at the scale of $m_Z$.

In that case, then the $T'_{22}$ group structure is derived by the following modification of the Weinberg rotation of eq (2.16):

$$g_Z T'_{22} = \frac{g_Z}{\sin \phi_Z / \sin \theta_W \cos \theta_W} \times (5.4)$$

$$g_Z T'_{22} = \kappa_{22} (\kappa_{22} \text{diag}(1,1,1,1,1,1,1,1,1) + \left( \frac{1}{3} x, \frac{2}{3} x, \frac{4}{3} x, 2x - 1, 1, 1, 0, 0 \right)) e \sin \phi_Z / \sin \theta_W \cos \theta_W \times 10^{-3}, (5.5)$$

where $T'_{22}^G$ is the group structure when acting on gauginos, and $\kappa_{22}$ is a secondary parameter that expresses the fact that below $m_Z$ but above $m_t$, the $g_{Z'}$ coupling to $Q_{22}$ runs down more slowly than the $g_{Z'}$ coupling to $Q_{21}$ and gaugino fields. The $Z'$ group structure when interacting with adjoint fields is assumed to be the same as $T'_{22}^G$, since adjoint fields also do not have any top-quark components.

Given those group structures along with the definitions of quarks and leptons earlier in the paper and the quark mass matrices of section 3, $Z'$ couplings to left- and right-handed up and down quarks, electrons and neutrinos are:

| $g_L$ | $g_R$ | $V$ | $A$ |
|-------|-------|-----|-----|
| $u$   | $-1.15$ | $-0.65$ | $e \sin \phi_Z / \sin \theta_W \cos \theta_W$ |
| $d$   | $3.0$   | $0.71$   | |
| $e$   | $-0.15$ | $-0.15$ | $|\varepsilon| \simeq 1.0$ |
| $\nu_e$ | $0$   | $0$   | $0$ |

It was assumed for the fit that $\frac{1}{3} \kappa_{22} (\kappa_{22} + \frac{2}{3} x) \simeq 3.0$.

If $\sin \phi_Z$ takes the following value

$$\frac{e \sin \phi_Z}{\sin \theta_W \cos \theta_W} \simeq 2.0 \times 10^{-4}, (5.4)$$

then the quark couplings can be translated into Vector (V) and Axial-vector (A) proton, neutron and electron couplings as follows:

| $V$ | $A$ |
|-----|-----|
| $p$ | $C_p \simeq 0.02$ | $a_p \simeq 0.25$ |
| $n$ | $C_n \simeq 1.1$ | $a_n \simeq -0.44$ |
| $e$ | $|\varepsilon| \simeq 1.0$ | $0$ |

where notation from [33] (Appendix C) is being used for protons and neutrons, and $\epsilon$ is the electron vector coupling divided by $\epsilon \simeq 0.3$.

With these choices, the $Z'$ boson of this model can reproduce X17 data while satisfying all applicable constraints. From fig. 5 of [33], it can be seen that the proton and neutron vector couplings can reproduce the X17 $^{12}$C data, while the axial vector couplings can reproduce the $^6$Be and $^4$He data. The vector coupling is protophobic enough to satisfy the NA48 constraint. The fact that there are no interactions with electron neutrinos allows the model to satisfy neutrino constraints [34]. The electron interaction strength is above the limit imposed by NA64 but below the level that would contradict electron $g - 2$ data [35]. Finally, as described in Appendix F of [33], the very stringent atomic parity constraints on mixed-parity models do not apply when the electron has only a vector interaction (no axial vector), as in this model.

The mass of the X17 in this model is generated by the
vev $\bar{\phi}_2$ and $\bar{\varphi}_2$. From the form of $g_2 T_{22}'$ in eq (5.2) and the expression $m_b = 1.5g_2 \bar{\phi}_2$ (from eqs (3.15) and (3.14)), the $\bar{\phi}_2$ part of the tree-level mass is:

$$m_{Z'} \simeq m_b \left( \frac{1}{2} \kappa_{22}' (\kappa_{22} - 1) \right) \left( \frac{1.5 \sin \phi_2}{\cos \theta_Z} \right) + \ldots . \quad (5.6)$$

This tree-level value from $\bar{\phi}_2$ can be as much as 5 MeV, or possibly less, depending on the individual values of $\kappa_{22}$ and $\kappa_{22}'$. The remainder of the 17 MeV mass of the $Z'$ in this model comes from its interaction with the adjoint vev $\bar{\varphi}_2$ and radiative corrections.

**B. A Seventh Quark**

This model predicts the existence of a seventh flavor of quark (denoted here by $f$). The model does not predict the mass of the quark, and by choosing a large value for the superpotential parameter $\Gamma_{22}$, it is possible to construct a version of this model where the $f$ quark has a mass larger than that of the top quark. However, for reasons discussed below, it is proposed that the mass of the $f$ quark is smaller than the mass of the bottom quark.

A natural question is how an additional low-mass quark could have evaded detection so far. A recent paper shows that if there is a fourth down-type quark with a mass of 2.9 GeV, then most of the exotic hadrons discovered over the last twenty years fit nicely into the quark model as normal mesons and baryons rather than as 4- or 5-quark hadrons [36]. In other words, as opposed to evading detection, it is proposed that the additional quark has been observed hundreds of times. In this model, the production and decay processes of hadrons involving the additional quark are mediated by adjoint scalar fields.

In the Scalars subsection below, it is suggested that the scalar $\varphi_5$ proportional to diag(1, 1, 1, 1, 1, -1) may be very light. This scalar does not interact with any of the gauge bosons or gaugino quarks below the unification scale, but it does interact with quarks in the adjoint twisted superfield. The interaction with those quarks is governed by the superpotential coupling $\Gamma_{f}$ from eq (1.15).

In particular, the model includes the following interactions of quarks with $\varphi_5$:

$$\Gamma_{f} \varphi_5 \left( \bar{u}^A_1 u^A_1 + \bar{d}^A_1 d^A_1 \right) + h.c. \simeq \Gamma_{f} \varphi_5 \left( \bar{c}_L c_R + (\bar{2d}^2 d_L + .97 \bar{s} s_L + .05 \bar{b} b_L - .04 \bar{f} f_L) f_R \right) + \ldots + h.c. , \quad (5.7)$$

where the second line comes from diagonalizing the mass matrices of section 3, and “...” in the last line includes much smaller interactions for the other quark combinations. Eq (5.7) shows that the $\varphi_5$ scalar interacts primarily with $c\bar{c}$ as well as with $f\bar{s}$, $f\bar{d}$ and their Hermitian conjugates.

For that reason, one way to produce $\bar{d} f$ and $\bar{s} f$ mesons is via $e^+ e^- \rightarrow c\bar{c} \rightarrow f\bar{d}$ or $\bar{s} f$, where the first process is mediated by a photon and the second by $\varphi_5$. Another production mechanism is via decay of a $b$ quark. For example: $\bar{u} b \rightarrow \bar{u} c\bar{c} s \rightarrow \bar{u} s + f\bar{d}$, where the first process is mediated by a $W$ boson and the second by $\varphi_5$.

The same scalar also mediates the decay of $f$-quark mesons. For example, $\varphi_5$ mediates the decay $f\bar{u} \rightarrow d\bar{u} + c\bar{c}$. Production and decay mechanisms like these are discussed in detail in [36], enabling this model to reproduce properties of most of the observed exotic hadrons.

An important feature of the model is that the $bs$ and $bd$ interactions with $\varphi_5$ are very small (<0.002). To be consistent with observations, they must be small enough so that scalar-mediated decays such as $b \rightarrow s c\bar{c}$ are rarer than weak decays for $B$ mesons and $A_0$. The strength of $bf$ is not a problem since $B^+ \rightarrow d\bar{u} + c\bar{c}$ does not have enough mass to decay to an $fu$ meson along with a $c\bar{c}$ or $f\bar{d}$ meson.

The strength of scalar interactions affecting each down-type quark is highly dependent on the details of the model. Eq (5.7) only shows scalar interactions mediated by $\varphi_5$. In the next subsection, other scalars are proposed to play a role in certain processes, but in this version of the model, the do not contribute significantly to hadron decay. Another model dependence is the mass matrices of eqs (3.16) and (3.17). There is flexibility to rotate, transform, or otherwise alter those matrices in ways that still reproduce the quark masses, CKM data, and $Z'$ data well. This flexibility can be used for example to modify the relative coupling of $\bar{s} f$ vs. $\bar{d} f$ or other details of the model.

If the proposed additional quark exists, one might expect it to generate predictions for CKM data that no longer agree with experimental data. The opposite is true.

Currently there are no CKM measurements that disagree with the Standard Model by 5$\sigma$. There are, however, some 3$\sigma$ hints of disagreement. For example, measurements of CKM matrix elements lead to a first-order unitarity calculation of 0.9985 ± 0.0005 [20], which is a 3$\sigma$ variation from the unitary value of 1 predicted by the Standard Model. In addition, the following 2.9$\sigma$ difference is measured:

- **Vector Current:** $|V_{us}| = 0.2231 ± 0.0006$
- **Axial Vector Current:** $|V_{us}| = 0.2254 ± 0.0006,$ \quad (5.8)

where the first result above is from semi-leptonic kaon decay and the second is from leptonic [21]. Also, $V_{cb}$ measured via inclusive decays is 2-3$\sigma$ larger than $V_{cb}$ measured via exclusive decays [37].

Since the Standard Model has only left-chiral $W$ boson interactions and six quarks, its CKM matrix should be unitary and there should be no difference between vector-current and axial-vector-current CKM matrices. If increased precision causes the above discrepancies to exceed 5$\sigma$, it will therefore be problematic for the Standard Model, as noted in [4].

The model presented here is able to reproduce the above 3$\sigma$ differences. In fact, the quark mass matrices of eqs (3.16) and (3.17) were chosen to reproduce them,
as can be seen from the $V_{ud}$ elements generated by those matrices (see eq (3.23)). Also, Z boson FCNC interactions enable $b$-hadron to $f$-hadron decays, where the $f$-hadron then decays to other hadrons involving $c\bar{c}$. Some of these decays will contribute to inclusive measurements of $V_{cb}$ but not to exclusive decay measurements. That could provide a qualitative explanation for the observed difference between inclusive and exclusive measurements of $V_{cb}$.

The additional quark may also provide an alternative explanation for the following: The LHCb collaboration measured from $pp$ collisions the number of events emerging at very forward rapidities that involved a $Z$ boson and a charm jet [38]. The number of measured events was significantly larger than the number predicted by Standard-Model event generators that incorporate Parton Distribution Functions (PDFs) that assume that the only charm quarks inside of a proton are those generated perturbatively (by gluons). On the other hand, the data were well reproduced by PDFs that assume that a proton has some “intrinsic charm” [39]. Valence quarks carry a much larger percentage of a proton’s momentum than do sea quarks. In this model a $d$ valence quark in a $pp$ collision can undergo a scalar-mediated transformation into an $f$ quark and a $c\bar{c}$ pair. Such a process would lead to an excess of charm quarks at very forward rapidities (relative to the Standard Model). The effect should be doubled for neutron anti-neutron collisions, so this model predicts that a much larger excess per nucleon of forward-rapidity charm should be seen in heavy ion collisions compared to $pp$ collisions. There have been dozens of direct searches for an additional down-type quark. Most of these searches do not rule out a quark of the type proposed here [40]. In particular, most model-independent searches do not exclude a new quark with a mass smaller than that of the bottom quark.

One notable exception is the fact that inclusive hadronic cross section data seem at first glance to rule out the possibility of a light additional quark. That is the topic of the next subsection.

C. Hadronic Cross Sections

Over the last fifty years, many experiments have measured $R$, the ratio of the cross section for $e^+e^-\rightarrow$ hadrons to the cross section for $e^+e^-\rightarrow \mu^+\mu^-$ [41]. As the center-of-mass energy $\sqrt{s}$ is increased above 3 GeV, $R$ data show narrow spikes associated with $J/\psi$ and $\psi(2S)$, the first two $J^{PC} = 1^{--}$ $c\bar{c}$ meson resonances. These mesons are not massive enough to decay to $D\bar{D}$, so they must decay by “OZI-suppressed” channels. That makes these mesons long-lived, generating the observed narrow spikes in $R$. For $\sqrt{s}$ greater than those spikes, more $1^{--}$ $c\bar{c}$ meson resonances are seen in $R$ data, but those resonances are much wider since they can decay to $D\bar{D}$.

As $\sqrt{s}$ is increased above 9 GeV, a similar pattern is seen for the lowest-lying $1^{--}$ $b\bar{b}$ meson resonances. The $R$ data show spikes for mesons not massive enough to decay to $B\bar{B}$ followed by wider resonances for mesons that can decay to $B\bar{B}$.

So if a quark existed that had a mass of 2.9 GeV, one might expect to see a similar pattern of spikes in $R$ data between the $c\bar{c}$ and $b\bar{b}$ spikes. No such spikes are seen. But in this model, it is proposed that the $f\bar{d}\rightarrow c\bar{c}$ scalar interactions experienced by the $f$ quark pull down the mass of the lowest lying $1^3S_0$ meson of $f\bar{d}+d\bar{f}$ so that it is closer to the mass of $\eta_c$, the $1^3S_0$ state of $c\bar{c}$. Isospin symmetry pulls the mass of $f\bar{u}$ and $u\bar{f}$ down to a similar scale. The result is that the least massive $1^{--}$ $f\bar{f}$ meson has enough mass to decay to a pair of $f\bar{d}+d\bar{f}$ or $f\bar{u}+u\bar{f}$ mesons. For that reason, in this model one would not expect to see any OZI-suppressed $f\bar{f}$ spikes, just wide bumps in $R$ between the charmonium and upsilon spikes.

From the detailed fit to the exotic hadron spectrum presented in [36], the $1^3S_1$ meson of $f\bar{f}$ in this model has a mass of around 6638 MeV, corresponding to the $X(6600)$ observed by CMS [42] and ATLAS [43]. The $1^3S_0, 1^3S_1$ mesons of $f\bar{d}$ and $f\bar{u}$ of this model are the $X^{0,0}(3250)$ observed many years ago [44, 45]. Given these mappings, one would expect to see a wide bump in the $R$ value at around 6600 MeV. Although there appear to be normalization inconsistencies between various data sets, the Mark I data in this range do indeed show the kind of bump expected by this model (see for example fig 10 of [46]). The data for slightly larger $\sqrt{s}$ also appear to be able to accommodate the additional $1^{--}$ $f\bar{f}$ resonances predicted by the model, such as the $2^3S_1$, $f\bar{f}$ resonance predicted at 7250 MeV [47]. Incidentally, $e^+e^-\rightarrow \mu^+\mu^-$ data also show a $3.5\sigma$ hint of a resonance at 7250 MeV [48]. It would be very helpful to have more up-to-date, precision $R$ value measurements in this CM energy range.

But the $R$ value presents another puzzle that must be solved by this model. For $\sqrt{s}$ much larger than quark masses but well below the $Z$ boson peak, the Standard Model predicts that

$$R = 3 \sum_F Q_F^2 + \text{quantum corrections},$$

where $Q_F$ is the electric charge of a quark of flavor $F$. Data in the $20 < \sqrt{s} < 45$ GeV range have been measured to have $R \sim 3.9$. Summing over the five quarks in the Standard Model (excluding top) generates $11/3$ for the first term of $R$ (tree-level). The quantum corrections then bring the total $R$ in this range up to $3.9$.

If another quark with charge $-1/3$ existed, its contribution (with SM quantum corrections) would add $\sim 9\%$ to the $R$ value. The total $R$ would then be around 4.25, too large to reproduce the data. In the context of the Standard Model, this rules out the possibility of an additional quark with mass less than $\sim 22$ GeV.

But the model proposed has additional scalar-field quantum corrections that are not present in the Standard Model. Through mechanisms described below, those cor-
corrections produce a negative contribution of \( \sim 9\% \), allowing this model to generate \( R \sim 3.9 \) and thereby reproduce the data despite having an additional quark.

At lowest order, gluon-mediated perturbative corrections increase \( R \) by \( \alpha_s / \pi \), where \( \alpha_s = g_3^2 / 4\pi \) and \( g_3 \) is the strong coupling constant. That first-order contribution comes from two sources: the square of a gluon-emission diagram and the interference between a tree-level diagram and a gluon-mediated vertex-correction diagram. The gluon emission contribution is positive (since it is the absolute square of matrix element), and the interference contribution is negative. Both terms have infrared logarithmic divergences, but they cancel in the sum, with the remainder being the \( \alpha_s / \pi \) positive contribution.

The topology of scalar-mediated perturbative diagrams is the same, so the light \( \varphi_5 \) scalar correction will give another positive contribution similar to that of the gluon (with cancelling divergences from scalar emission and vertex correction diagrams). So \( \varphi_5 \) does not provide the negative correction needed for this model to reproduce data. However, the model includes additional scalars beyond \( \varphi_5 \) that do contribute negative corrections.

The model includes an octet of scalars \( \varphi_i^a \) that interact with gluons but no other gauge bosons. It is argued in section 3 that in the vacuum, these scalars become massless, form monopoles, and condense, generating a confinement via a dual Meissner effect. It is assumed that in their condensed state, these scalars do not participate significantly in the decay of most hadrons. On the other hand, above a certain scale, there is a phase transition, and these scalar participate in processes such as perturbative corrections to \( e^+e^- \rightarrow q\bar{q} \).

\( R \) data may indicate the energy scale for this phase transition. Values of nonperturbative condensates with dimension 2, 4, 6, etc. can be extracted from low-energy \( R \) data by considering its moments. In the Standard Model, there are no gauge-invariant dimension-2 condensates, so one would expect the data to generate zero for its value. This model, however, has a gauge-invariant dimension-2 condensate: \( tr(\varphi_1 \varphi_1) \).

In [49], it was shown that \( R \) data imply a nonzero value for the dimension-2 condensate (although the value is consistent with zero within large uncertainties). The analysis shows that perturbative methods that ignore the condensate should only be used above \( \sqrt{s} \sim 4 \) GeV. If the phase transition energy of this model is at that scale, that would be consistent with the statement above that the \( \varphi_1^a \) scalars do not play a significant role in the decay of most hadrons. If the masses of the \( \varphi_1^a \) scalars in the perturbative regime are also at this scale, they can provide a negative quantum correction to the \( R \) value through the mechanism described below.

If the masses of the \( \varphi_1^a \) scalars are similar to that of the \( b \) quark, then their masses are too large to permit a scalar-emission diagram in the quantum correction. As described above, that is the diagram that provides the positive quantum correction. The interference term involving the vertex correction diagram, however, is negative. The \( \varphi_1^a \) scalars generate a negative contribution from this term, one with an infrared divergence regulated by the scalar mass. The model’s \( \Gamma_\varphi \) parameter can be chosen such that in the \( 20 < \sqrt{s} < 45 \) GeV range, the \( \varphi_1^a \) generated negative quantum correction across all quarks (other than top) offsets the positive contribution from the additional quark and the \( \varphi_5 \) quantum correction. This allows the model to reproduce the \( R \) data in that range. To see this in more detail, it is helpful to look at the specific interactions of the \( \varphi_1^a \) scalars.

For perturbative calculations of \( e^+e^- \rightarrow q\bar{q} \) above \( \sqrt{s} \sim 4 \) GeV, \( \varphi_1^a \) scalars interact with quarks via the following tree-level superpotential coupling:

\[
\Gamma_{\varphi_1^a} (\bar{u}_L^a t^a u_W^A + d_L^a t^a d_W^A + \bar{d}_W^A t^a d_W^A) + h.c. \tag{5.10}
\]

\[
= \Gamma_{\varphi_1^a} (\bar{c}_L^a c^a_R + s_L^a t^a f_R + \bar{f}_L^a s^a_R) + h.c.,
\]

where \( q_L = (q_L)^{1/2} \), and the primed variables in the second line just rename the variables in the first line based on the dominant quark mass eigenstate (using notation that was introduced in eqs (3.16) and (3.17)).

In Appendix eq (A.16), it is argued that the following superpotential coupling is generated nonperturbatively:

\[
i \bar{t}_L^a \varphi_1^a \bar{b}_L^a t^a c_R + h.c. \tag{5.11}
\]

The same mechanism that gives large masses to right-handed neutrinos also gives a large value to this coupling.

There are also contributions from the supergauge interaction 2\( g \Gamma (\Phi | V, \Phi) \):

\[
\frac{1}{\sqrt{2}} g_3 \varphi_1^a \left( \bar{u}_W^A t^a \bar{u}_W^A - \bar{d}_W^A t^a d_W^A + \bar{d}_W^A t^a d_W^A - \bar{d}_W^a t^a d_W^A \right) = \frac{1}{\sqrt{2}} g_3 \varphi_1^a (\bar{c}_L^a c^a_R - \bar{t}_L^a t^a f_R + s_L^a t^a d_R - \bar{f}_L^a s^a_R) + h.c. \tag{5.12}
\]

The \( \Gamma_{\varphi} \) and supergauge scalar interaction terms shown above generate negative quantum corrections for the quarks involved, whereas the \( \Gamma_{\varphi} \) interaction generates a positive correction due to the additional factor of \( i \) in the coupling.

At \( \sqrt{s} \sim 11.2 \) GeV, an overall negative correction of \( \sim 9\% \) is required in order to compensate for the additional quark. Due to the \( \bar{b}_L^a \varphi_1^a f_R \) term in eq (5.12), part of that correction comes from the \( R_b (e^+e^- \rightarrow b\bar{b}) \) part of \( R \). The model’s negative correction to \( R_b \) at this scale (relative to the Standard-Model prediction) is similar to the overall \( R \) correction: 9-10\%.

Recently, an analysis was performed to adjust \( \sqrt{s} \sim 11-11.3 \) GeV BABAR cross section data [50] for the effects of Initial State Radiation [51]. After this adjustment, these data can be compared with the Standard Model calculation of \( R_b \). It was found that the values of the adjusted BABAR data were 10\% (3\sigma) smaller than the Standard-Model predictions. The authors of [51] suggested that the BABAR data may have been normalized incorrectly. Another possibility is that the data show evidence of the scalar-mediated \( R_b \) reduction predicted by this model.
The $\bar{b}_L \nu_1 f_R$ term in eq (5.12) has another interesting consequence. This model predicts the following to be significant: $e^+e^- \to c\bar{c} \to \bar{b}b$, where the first part is mediated by a photon and the second by a scalar. Therefore, it may be possible to see $\bar{b}b$ meson resonances in $R$ data.

It indeed appears that the MD1 data for $R$ in the $7.5-10$ GeV range would be better fit by including resonances at around $8.1$ and $8.9$ GeV (see for example fig. 10 of [46]). These resonances would have the correct masses to be identified with the $1^3S_1$ and $1^3D_1$ mesons of $\sqrt{s}(f\bar{b}+b\bar{f})$.

Going to larger $\sqrt{s}$, the scalar-mediated quantum corrections should get larger. The vertex-correction diagram has an infrared divergence for zero scalar mass. As $\sqrt{s}$ gets larger, $m_{\nu_1}/\sqrt{s}$ gets closer to zero, so the correction gets larger. The scalar masses and superpotential couplings can be adjusted so that at $\sqrt{s} = m_Z$, $R$ gets a scalar-mediated negative quantum correction of $\sim 25\%$.

That allows this model to generate the same hadronic cross section $\sigma^0_{\text{had}}$ at the $Z$ pole as the Standard Model, despite the fact that this model’s tree-level coupling implies an $R$ value $\sim 33\%$ larger than that of the Standard Model (see eqs (3.24) and (3.25)). Since the leptons in this model have the same coupling to the $Z$ boson as in the Standard Model, this model is also then able to reproduce $R_{\tau}$, $R_{b}$ and $R_{\mu}$ at the $Z$ pole. Specific $b$ and $c$ quark $Z$ pole measurements are discussed in the next subsection.

D. Z decay to b and c quarks

The precision experiments at LEP and SLD measured partial width and asymmetry data for $e^+e^- \to Z \to b\bar{b}$ and $e^+e^- \to Z \to c\bar{c}$ events [52]. Based on those data, the experiments inferred the left-handed and right-handed couplings of the $Z$ boson to $b$ and $c$ quarks. The inferred couplings match those of the Standard Model. This section describes how this model can reproduce those data despite an additional quark and the fact that the charm quark has a different right-handed coupling to the $Z$ boson than in the Standard Model.

It was mentioned above that scalar-mediated vertex corrections generate a $\sim 25\%$ negative scalar correction to $R$ at $\sqrt{s} = m_Z$. But how this correction is spread among the various quark flavors depends on the relative strengths of the superpotential and supergauge couplings of eqs (5.10), (5.11) and (5.12). Just like the gluon gauge coupling, the supergauge coupling runs to smaller values at higher energies. By this mechanism, this model predicts that at $\sqrt{s} = m_Z$ the supergauge coupling is much smaller than either of the superpotential couplings mentioned above. In a first approximation, the supergauge contribution to scalar-mediated corrections can be ignored.

Within that approximation, the following scenario allows the model to reproduce the data: The $\Gamma_{\nu}$ vertex corrections reduce the amplitudes of the $Z \to q\bar{q}'$ diagrams by $\sim 19\%$ for each quark in eq (5.10). Also, the $\Gamma_{\nu}$ vertex corrections add $\sim 60\%$ of the $Z \to u_L' \bar{u}_L'$ coupling to the $Z \to c_R \bar{c}_R$ coupling value, and vice versa. Assuming $\sin^2 \theta_W \simeq 0.23$, the net result is that scalar-mediated vertex corrections modify the couplings of eqs (3.24) and (3.25) as follows:

$$
\begin{array}{cccc}
q\gamma^\mu q Z_\mu & g_L & g_R & g_L & g_R \\
\nu \nu & .154 & .346 & .054 & .346 \\
\nu \nu & .346 & .346 & .280 & .188 \\
\nu \nu & .423 & .077 & -.343 & .062 \\
c & .077 & -.423 & .062 & -.343
\end{array}
$$

By squaring these couplings and adding them to squares of the $d'$ and $b'$ couplings from eq (3.25), it can be seen that the overall $R$ value of this model (with its scalar-mediated vertex corrections) is within $1\%$ of the $R$ value in the Standard Model.

Since the light leptons of this model have the same coupling to the $Z$ boson as the leptons in the Standard Model, that means that this model is able to reproduce the hadronic cross section $\sigma^0_{\text{had}}$, the measured width of the $Z$ boson, and the conclusion that there are only 3 flavors of light neutrinos.

In the scenario presented above, there is no scalar-mediated correction to the $Z \to b'b'$ diagrams. Also, from eq (3.25), the $Z$ interactions with $b_L$ and $b_R$ in this model are the same as the $Z$ interactions with $b_L$ and $b_R$ in the Standard Model. Since these interactions are the same and $\sigma^0_{\text{had}}$ is the same, the $b$-quark partial width ($R_b$) and asymmetry predicted by this model at $\sqrt{s} = m_Z$ are the same as in the Standard Model.

From eqs (5.10) and (5.11), the $f_L$ quark produced in $Z \to f_L' f_R'$ decays subsequently mostly decay by $f_L' \to s_L' c_L' \bar{c}_L$, $s_L' c_L' \bar{u}_L$ (except for the $s_L$ and $d_L$ components of $f_L'$). Since $c_L' > .99 c_L$ (where primed quarks are gauge eigenstates and unprimed quarks are mass eigenstates), this means that this model’s effective $Z \to c_L \bar{c}_L$ coupling measured in charm-specific events is approximately the sum of the $c_L'$ and $f_L'$ couplings. Similarly, this model’s effective $Z \to c_R \bar{c}_R$ coupling measured is approximately the sum of the $c_R'$ and $f_R'$ couplings. In this approximation, this model’s effective Z to charm couplings are:

$$c_{L, \text{eff}} = .343 \quad c_{R, \text{eff}} = -.155 \quad . \ (5.14)$$

These are within $1\%$ of the Standard-Model $Z$ to charm couplings.

At this level of approximation, the model produces the same effective results as the Standard Model, and can therefore reproduce the $c$ quark (and $b$ quark) partial width and asymmetry measurements made at the $Z$ pole. Future work should verify that detailed quantum calculations as well as refinements to the above approximations do not introduce inconsistencies with the data.
E. Weak Radiative Decay of Hyperons

For several decades, there has been a debate about how to reproduce data from weak radiative decays of hyperons [53]. Hara’s theorem states on general grounds that in the limit of flavor SU(3) symmetry, these decays should have very small parity violation [54]. The large asymmetry observed in these decays can be interpreted in one of two ways: (a) flavor SU(3) symmetry is significantly broken for weak interactions or (b) Hara’s theorem is violated. Baryon magnetic moments show that flavor SU(3) symmetry provides a good approximation for electromagnetic interactions, so if Hara’s theorem is not violated, a reason must be provided for why flavor SU(3) is significantly broken just for weak interactions. On the other hand, violation of Hara’s theorem would require violation of electromagnetic gauge invariance, locality (at the hadron level), or CP symmetry.

This model assumes Hara’s theorem is not violated and provides an alternative explanation for the breaking of flavor SU(3) for Weak interactions but not electromagnetic interactions. As in the Standard Model, the $d$ and $s$ quarks of this model have the same electric charge, so SU(3) flavor symmetry provides a good approximation for electromagnetic interactions, so if Hara’s theorem is not violated, a reason must be provided for why flavor SU(3) is significantly broken just for weak interactions. On the other hand, violation of Hara’s theorem would require violation of electromagnetic gauge invariance, locality (at the hadron level), or CP symmetry.

This model assumes Hara’s theorem is not violated and provides an alternative explanation for the breaking of flavor SU(3) for Weak interactions but not electromagnetic interactions. As in the Standard Model, the $d$ and $s$ quarks of this model have the same electric charge, so SU(3) flavor symmetry provides a good approximation for electromagnetic interactions. Unlike the Standard Model, they have different weak interaction couplings, as can be seen from eq (3.25). Therefore, even in the limit that the $d$ and $s$ quark masses are the same, flavor SU(3) is significantly broken for hadronic matrix elements involving the Weak interaction.

F. Three Heavy Charged Leptons

This model predicts the existence of three additional heavy charged leptons (referred to in this paper as “Omega leptons”). As mentioned in section 3, direct searches have ruled out an additional charged lepton with a mass of less than 103 GeV [55]. As discussed in section 4, the upper limit for the Omega lepton masses is between 30 and 90 TeV. All three of the charged Omega leptons should have masses smaller than those of their partner right-handed neutrinos, so they do not have W-mediated decays. On the other hand, the large superpotential couplings $\Gamma_{ff'}$ cause these heavy leptons to decay very quickly to the three Standard-Model light leptons and one of the light scalars ($\varphi_5$ and $\varphi_6$) discussed in the Scalar subsection below.

G. Scalars

The model includes the following complex scalars: 6 triplets, 1 octet, and 1 singlet for each of the two original U(3) groups (before the symmetry of the $m = 2$ group is broken to SU(2) at the unification scale). This section describes a proposed scenario for masses of those 54 complex scalars.

The six $m = 1$ scalar triplets are leptoquarks since they connect leptons with both gaugino quarks and adjoint-representation quarks. The model requires the scalar mass parameters $m_{1F}$ and $m_{1F'}$ to be much larger than the electroweak scale. The masses are chosen to be larger than the lower limits from leptoquark searches. If the flavor-1 leptoquarks have masses of around 10 TeV, they could be at least partly responsible for the non-resonant di-electron anomaly discussed in [4].

The model requires the two $m = 2$ scalar triplets $\phi_{23}$ and $\phi_{23}$ to get masses at the unification scale as part of the SU(3)$\to$SU(2) gauge symmetry breaking.

The observed Higgs Boson accounts for the SU(2) doublet within the triplet $\phi_{23}$. As described in section 3, the Higgs gets its mass from d terms involving its vev, the mass parameter $m_{21}$ and radiative corrections. In this model, the Higgs has supergauge interactions with $WW$, $ZZ$, and $tt$ that match its interactions in the Standard Model. It also has a small superpotential interaction with $u\bar{c}$ via $\Gamma_{211}$, but it does not have classical interactions with any other quark or lepton pairs. However, there are quantum-generated interactions with these.

The third member of the $\phi_{21}$ triplet is an SU(2) singlet charged scalar ($\phi_{21}$). In section 3, it was mentioned that the $m_{21}$ scalar mass parameter and radiative corrections contribute $\sim80$ GeV to the Higgs Boson mass (the d terms involving the Higgs vev contribute the rest). Lacking a contribution from the Higgs vev, the mass of ($\phi_{21}$) is in its interactions in the Standard Model. It also has a small superpotential interaction with $u\bar{c}$ via $\Gamma_{211}$, but it does not have classical interactions with any other quark or lepton pairs. However, there are quantum-generated interactions with these.

The charged scalar ($\phi_{22}$), the neutral scalar ($\phi_{22}$) acquires its mass in a similar fashion to the Higgs boson, via a vev, a scalar mass $m_{22}$ and radiative corrections. Since the vev is much smaller than for the Higgs, it would be consistent with this model if ($\phi_{22}$) had a mass of $\sim95$ GeV. It has supergauge interactions with $WW$, $ZZ$ and $b\bar{b}$ as well as a small $\Gamma_{22}$ superpotential interaction with $uj$. ($\phi_{22}$) also has a di-photon decay primarily mediated by a W boson loop. The ($\phi_{22}$) scalar of this model has a structure that may allow it to reproduce the hints of signals in $\gamma\gamma$, $WW$, and $ZH$ (with $H \to b\bar{b}$) that are mentioned in [4].

The charged scalar ($\phi_{21}$) should have a similar mass to its SU(2) doublet partner mentioned above. It has supergauge interactions with $W$, $WZ$, $W\gamma$, and $tb$ as well as a superpotential interaction with $c\bar{d}$. Within the $m = 2$ octet, the ($\phi_{22}$) scalar also acquires its mass in the same way as the Higgs (via its vev in superpotential d terms, a scalar mass $m_{22}$ and radiative corrections). It would be consistent with this model if ($\phi_{22}$) had a mass of $\sim152$ GeV. It has supergauge interactions with $WW$, $ZZ$ and $b\bar{b}$ as well as a large $\Gamma_{22}$ superpotential interaction with $ff$ (which has a similar signature to $b\bar{b}$). Through large $\Gamma_{1FF'}$ superpotential couplings, it also interacts with the heavy Omega leptons predicted by this model (that quickly decay to
normal leptons). This scalar has a structure that may allow it to reproduce the hints of signals for a \( \sim 152 \) GeV Higgs that are mentioned in [4].

Together with the \( \tilde{\varphi}_3 \) vev, the large superpotential coupling \( \Gamma_6 \) imparts large masses to the oppositely charged scalars \( (\varphi_2)_{a3} \) and \( (\varphi_2)_{12} \) as well as to the neutral scalar \( (\varphi_2)_{23} \) and the \( (\varphi_2)_{22} \) component of \( \varphi_2^3 \). Through large \( \Gamma_{1FF'} \) superpotential couplings, each of the scalars can decay to a \( q\bar{q}' \) combination of quarks. It would be consistent with this model if the masses of the first three scalars were \( \sim 950 \) GeV, while that of the \( \varphi_2^3 \) scalar was \( \sim 950 \sim 670 \) GeV. The first three scalars may be able to provide an explanation for the di-jet excesses seen at \( \sim 950 \) GeV [4].

Through quark loops, \( \varphi_2^3 \) has decays to \( \gamma \gamma \) and \( ZZ \). Since SU(2) is a self-conjugate group, a quantum-generated interaction below the unification scale can also enable the decay \( \varphi_2^3 \rightarrow (\varphi_2)_{23}(\varphi_2)_{11} \), where \( (\varphi_2)_{11} \) is the observed Higgs Boson and \( (\varphi_2)_{23} \) has a mass of \( \sim 95 \) GeV (as mentioned above). So \( \varphi_2^3 \) may be able to reproduce the hints for a Higgs-like resonance at \( \sim 650-680 \) GeV [4].

The model requires the mass parameters giving masses to the scalar triplets \( \varphi_{21} \) and \( \varphi_{22} \) to be larger than the electroweak scale. It is possible to choose the mass of the neutral scalar \( (\varphi_2)_{23} \) to be \( \sim 3.5 \) TeV. Through a quantum-generated superpotential interaction and connection to the vev \( \varphi_2 \), the following decay to the 950 GeV oppositely charged scalars mentioned above is enabled: \( (\varphi_2)_{23} \rightarrow (\varphi_2)_{31}(\varphi_2)_{12} \). Similarly, the decay to neutral scalars \( (\varphi_2)_{23} \rightarrow (\varphi_2)_{32}(\varphi_2)_{22} \) is also enabled. In this way, \( (\varphi_2)_{23} \) may be able to reproduce the di-di-jet excess seen at a mass of \( \sim 3.5 \) TeV [4].

It would be consistent in this model if the charged scalar \( (\varphi_2)_{21} \) had a similar mass to that of the neutral \( (\varphi_2)_{22} \), \( \sim 3.5 \) TeV. \( \varphi_{21} \) does not interact with \( W \), but a small superpotential coupling \( \Gamma_{211} \) allows it to decay to \( t\bar{f} \). In [4], it is mentioned that ATLAS saw a slight excess in \( tb \) events [56] at a mass of \( \sim 3.5 \) TeV. Since an \( f \) quark would create a small-\( R \) jet (like a \( b \) quark), this model would predict a small excess in \( tb \)-like events from \( (\varphi_2)_{21} \) decay.

The SU(2) doublet partners of the above scalars should have somewhat similar masses to the ones mentioned above, but they could differ due to quantum corrections below the unification scale. Supergauge interactions involving the vevs \( \tilde{\varphi}_{22} \) and \( \tilde{\varphi}_{21} \) enable the decays \( (\varphi_2)_{21} \rightarrow WZ \) and \( (\varphi_2)_{22} \rightarrow WZ \). CMS has seen a 3.6\( \sigma \) local excess whose signal hypothesis is a \( W' \) boson with a mass of 2.1 or 2.9 TeV that decays to a \( W \) and a \( Z \) boson, each of which then decay to a jet [57]. If the masses of the \( (\varphi_2)_{21} \) and \( (\varphi_2)_{22} \) were \( \sim 2.1 \) or 2.9 TeV, they could be responsible for this hint.

This model predicts another neutral scalar with a mass similar to those above: \( (\varphi_2)_{22} \). Supergauge interactions involving the vev \( \tilde{\varphi}_{22} \) enable the decay of this scalar to \( WW, ZZ, dd \), and \( \gamma \gamma \).

In the Appendix, it is argued that the neutral scalar \( (\varphi_2)_{11} \) component of the \( \tilde{\varphi}_{21} \) has a mass similar to that of the heavy right-handed neutrinos – in the 30-90 TeV range.

Above it was suggested that four of the \( m = 2 \) scalar nonet masses were large (\( \sim 950 \) and \( \sim 670 \) GeV) due to the \( \varphi_2 \) vev and the large superpotential coupling \( \Gamma_6 \). One scalar mass was much smaller (\( \sim 152 \) GeV), due to \( \Gamma_5 \) terms involving the \( \varphi_2 \) vev, a scalar mass \( m_{\xi_2} \) and radiative corrections. The two remaining charged scalars \( (\varphi_2)_{13} \) and \( (\varphi_2)_{21} \) in the nonet should also have masses in a similar range (\( \sim 90-150 \) GeV). Through the \( \Gamma_6 \) superpotential coupling, they decay to \( c\bar{s}, s\bar{c} \). The remaining members of the \( \varphi_2 \) nonet \( \varphi_3 \) and \( \varphi_3 \) are discussed below.

As discussed in section 3, in an \( N = 2 \) Super Yang-Mills (SYM) theory where the adjoint superfield has a tree-level mass, the adjoint scalars become massless color monopoles. The monopoles condense and cause confinement [13, 14, 16, 17]. Since the \( m = 1 \) sector of this theory has a very similar moduli space, it was proposed in section 3 that the octet of \( m = 1 \) SU(3) adjoint scalars of this theory become monopoles, condense, and cause color confinement in the same way. Above the confinement scale, these scalars also participate in the processes mentioned in the Hadronic Cross Section subsection above.

The model’s remaining neutral scalars that have not yet been discussed are \( (\varphi_2)_{33}, \varphi_1^0, \varphi_2^0, \varphi_3^0 \). None of these interact with any gauge bosons below the unification scale. The last three of these can be re-expressed as scalars \( \varphi_5, \varphi_6, \) and \( \varphi_7 \) with the following group structures: \( T_5 = \frac{1}{\sqrt{12}}(1,1,1,1,1,1,1), T_6 = \frac{1}{\sqrt{60}}(1,1,1,1,1,5), \) and \( T_7 = \frac{3}{20}(\frac{1}{5}, \frac{2}{5}, -\frac{3}{5}, -1, -1, 0) \).

The scalar \( \varphi_5 \) has supergauge interactions (with gaugino quarks). For this reason, it is assumed that radiative corrections cause it to have a mass closer to the others in the \( \varphi_2 \) octet: \( \sim 90-150 \) GeV. It has supergauge and superpotential decays to \( qq \).

The scalars \( (\varphi_2)_{33}, \varphi_5, \) and \( \varphi_6 \) have no interactions with gaugino quarks, so they have no supergauge interactions at all. For this reason, radiative corrections for these scalars may be smaller, and consequently their masses may be smaller than the others. \( \varphi_5 \) has a stronger interaction with leptons than \( \varphi_5, \) and \( (\varphi_2)_{33} \) has no interaction with leptons. The strong superpotential coupling that leads to the heavy Omega lepton masses predicted by this model may cause \( \varphi_5 \) to have a larger mass than \( \varphi_5 \) and \( (\varphi_2)_{33} \). It is assumed that \( \varphi_6 \) has a mass of many tens of GeV, but \( \varphi_5 \) has a much smaller mass, and \( (\varphi_2)_{33} \) has the smallest mass.

In the Seventh Quark subsection above, it was suggested that \( \varphi_5 \) plays the dominant role in \( f \)-quark hadron decay. The primary production mechanism for these scalars is via the prompt decay of the heavy Omega leptons to the known three flavors of light leptons.

With their small masses and very few interactions, both \( \varphi_5 \) and \( (\varphi_2)_{33} \) could be stable particles and interesting dark matter candidates.

Some of the arguments above rely on quantum cor-
rections that have been qualitatively discussed, but not calculated in detail. Future work would be needed to ensure that the quantum corrections of this model are consistent with the picture painted above.

**DISCUSSION**

The theory presented in this paper is being proposed as an alternative to the Standard Model. The paper has taken a two-pronged approach: theoretical and phenomenological.

In sections 1, 2, 4 and the appendix, a model is presented that has a number of attractive theoretical features. For example, it is holomorphic, invariant to local superspace gauge transformations, supports coupling constant unification, and is similar at the unification scale to a theory that has been shown to be free of quadratic divergences to at least two loops [3].

In sections 3 and 5, detailed experimental data are considered, including data that differ by 3-7σ from the Standard Model. The results that nonperturbative calculations would have to generate in order to reproduce the data are identified.

The theory looks promising on the theoretical side and also on the phenomenological side. More work needs to be done to tie these two sides together. Would actual nonperturbative calculations support the parameter values required to reproduce data?

But even without that work, the model provides a couple of interesting explanations and makes a number of predictions. The model provides explanations for the mechanisms of confinement and neutrino oscillations. The model predicts three additional charged leptons, a seventh quark (without an eighth), a Z′ boson, right-handed quark interactions with the W boson, and dozens of additional scalar particles. So far, these predictions do not appear to be ruled out by existing data; in fact, they could provide new physics explanations for many of the anomalies discussed in [4].

**Appendix A: Quantum Effective potential**

This appendix proposes possible superpotential terms that may be generated by nonperturbative quantum effects.

In [13, 16, 17, 58–60], it is shown how holomorphy and symmetry arguments can be used to determine the exact superpotential terms that get generated nonperturbatively for Supersymmetric QCD (SQCD) at low energies. That analysis is anchored in the fact that SQCD is an asymptotically free theory with an ultraviolet renormalization group fixed point.

The U(3)xU(3) theory of this paper has Abelian groups, so it not entirely asymptotically free. However, in this appendix it is implicitly assumed that this theory is an effective theory of a more general asymptotically free theory, applicable above the unification scale. With that assumption in hand, this appendix makes arguments similar to those used for SQCD and proposes general features of nonperturbative superpotential terms.

It is possible that instanton calculations could be employed to determine the exact nonperturbative superpotential terms for this model, but that is outside the scope of this paper. Instead, allowed functional forms of terms are derived, and then the magnitude of the quantum-generated couplings that would be required for this model to fit experimental data are phenomenologically identified.

The first step in deriving the effective superpotential terms is to specify the beta function for the two SU(3) groups, evaluated for the case where all tree-level masses and superpotential couplings are zero. The beta function for an SU(N) theory is:

\[
\beta = g \left( \frac{g^2}{16\pi^2} \right) b_N
\]

\[
b_N = \left( -\frac{11}{3} N + \frac{1}{3} n_f + \frac{1}{6} n_s + \frac{2}{3} N n_fA + \frac{1}{3} N n_sA \right),
\]

(A.1)

where \(n_f\) and \(n_s, n_fA\) and \(n_sA\) are the numbers of fundamental fermion and scalar N-tuplets, and fermion and scalar adjoint-representation multiplets. In all cases, the fermions are 2-component Weyl fermions and the scalars are complex.

In the model presented in this paper, \(n_f = 18, n_s = 6, n_fA = 0\) and \(n_sA = 1\) for each SU(3) group. The fundamental scalar number comes from 3 flavors of both fundamental and anti-fundamental representations. The fundamental fermions have those plus another 6 fundamental fermion triplets from the gauginos and another 6 from the adjoint superfield. Putting that together, one finds:

\[
b_3 = -3. \quad (A.2)
\]

Following standard techniques of integrating the one-loop beta function, one finds:

\[
\ln \left( \frac{\Lambda^2}{\mu^2} \right) = -16\pi^2 / (g^2(\mu) (-b_3)), \quad (A.3)
\]

where \(\mu\) is the scale at which the SU(3) coupling is evaluated and \(\Lambda\) is the quantum-generated scale of each SU(3) gauge theory.

The next step is to determine which fermion representations generate an axial anomaly. In this model, and using notation similar to that of [12], the axial anomaly is proportional to:

\[
\sum_f \Tr_{R_i} (T^0_{A,R+} \{ (T^B_{V_{R+}}, T^C_{V_{R-}}) + (T^B_{V_{R-}}, T^C_{V_{R+}}) \})
\]

\[
+ \sum_f \Tr_{R_i} (2 T^0_{A,R-} \{ T^B_{V_{R+}}, T^C_{V_{R-}} \}). \quad (A.4)
\]

In the above expression, \(R\) represents the representation of fermion \(f\). A representation’s contribution to a local gauge current proportional to \(T^B_{L\pm}\) (as defined in eq
identifying the $m = 1$ flavor $F = 1$ and analogous terms will be constructed.

Identifying the $m = 1$ flavor $F = 1$ with the $m = 2$ flavor $F = 1$, the following combination chiral fields and meson operators can be constructed:

$$\begin{align*}
Q_F &= Q_{1F} + Q_{2F} \\
\tilde{Q}_F &= \tilde{Q}_{1F} + \tilde{Q}_{2F} \\
M^{(n)}_{FF'} &= \Lambda^{-1} \tilde{Q}_F \Phi^n Q_{F'}.
\end{align*}$$

(A.7)

In the meson factors, any terms that are not gauge invariant are set to zero. For example,

$$M_{12}^{(0)} = \Lambda^{-1}(\tilde{Q}_{11} Q_{12} + \tilde{Q}_{21} Q_{12}).$$

(A.8)

The terms $\tilde{Q}_{11} Q_{22} = \tilde{Q}_{21} Q_{22} = 0$ inside of $M_{12}^{(0)}$ since $q_{21} = q_{11} \neq q_{22}$.

The following determinant is then defined

$$\Delta^{(pqr)} = \det \begin{bmatrix}
M_{11}^{(p)} & M_{12}^{(q)} & M_{13}^{(r)} \\
M_{21}^{(p)} & M_{22}^{(q)} & M_{23}^{(r)} \\
M_{31}^{(p)} & M_{32}^{(q)} & M_{33}^{(r)}
\end{bmatrix}$$

(A.9)

Following symmetry arguments similar to those used for SQCD, quantum interactions will generate low energy effective superpotential terms with mass dimension 3 and positive powers of $\Lambda$ that are constructed from the above determinants as well as factors of $\text{Tr}(\Phi^2)$, and other gauge invariant factors such as those mentioned later in this Appendix.

An example of a term meeting the above criteria is the following:

$$O_0 = (\text{Tr}(\Phi^2))^2 \left(\Delta^{(000)}\right)^{-1/3}.$$  

(A.10)

The scalar potential can be derived from the superpotential terms like the one above by taking derivatives to extract “$f$ terms” proportional to $\theta^2 f$ and inserting them into the $f$-term part of eq (2.1). The quantum vacuum can be found by minimizing the vev of the scalar potential after including these quantum contributions.

In this process, vevs of scalar potential terms derived from $O_0$ will be proportional to

$$\tilde{O}_0 = \langle O_0 \rangle = \frac{\Lambda (\text{tr} \langle \varphi^2 \rangle + \text{tr} \langle \tilde{\varphi}^2 \rangle)^2}{\langle \tilde{\varphi}_{21} \tilde{\varphi}_{21} \tilde{\varphi}_{22} \tilde{\varphi}_{22} \tilde{\varphi}_{23} \tilde{\varphi}_{23} \rangle^{1/3}}.$$  

(A.11)

In section 2, it was noted that if the model includes large tree-level masses $\tilde{m}_{21}$ and $\tilde{m}_{22}$, then the vevs $\tilde{\varphi}_{21}$ and $\tilde{\varphi}_{22}$ vanish classically. However, in order to stabilize expressions such as the one above, it is assumed that $\varphi_{21}^2$ and $\tilde{\varphi}_{22}^2$ acquire small vevs quantum mechanically. This is mentioned in section 3. As shown below, those quantum-generated small vevs lead to a very large right-chiral neutrino Majorana mass term that allows the model to reproduce observed neutrino masses and mixing.

The classical scalar potential only restricts the difference $\varphi_{23}^2 - \tilde{\varphi}_{23}^2$. However, it can be seen from eq (A.11)
that the quantum scalar potential will try to make each of these vevs go to infinity while maintaining the difference. A mechanism like that is what causes Supersymmetric QCD with fewer flavors than colors to not have a vacuum solution. But as described in section 2, a nonzero tree-level or quantum-generated superpotential coupling provides a small counterbalancing effect that stops $\phi_{23}$ from becoming infinite.

Minimizing the quantum scalar potential generates the adjoint vevs in $\langle \phi_2 \rangle$ of eq (3.4) to reduce $\text{tr} \langle \phi_2^2 \rangle = -\text{tr} \langle \phi_2^2 \rangle$ in scalar potential contributions proportional to $\tilde{O}_0$. Since $\text{tr} \langle \phi_2^2 \rangle \neq 0$ (see the beginning of section 3), minimization makes $\text{tr} \langle \phi_2^2 \rangle \neq 0$ for the vacuum solution.

Once the scalar vevs have been adjusted and a minimum of the quantum potential has been achieved, quantum superpotential terms can also produce fermion and scalar mass terms. For example, the term $O_0$ generates the following terms that modify the tree-level masses $m_{m11}$ and $m_{m22}$:

$$-\frac{1}{3} \tilde{O}_0 \sum_m \left( \frac{\tilde{Q}_{m1} Q_{m1}}{\phi_{21} \phi_{21}} + \frac{\tilde{Q}_{m2} Q_{m2}}{\phi_{22} \phi_{22}} \right),$$

(A.12)

where terms involving $1/\phi_{21}$ and $1/\phi_{22}$ have been ignored due to their inverse unification-scale vevs.

The quantum potential also generates a Majorana mass term along with its scalar mass counterpart:

$$\frac{4\theta^2 \tilde{O}_0}{9\phi_{21}^2} \left( 2 \langle f_{21} \rangle_1 (\phi_{21})_1 - \tilde{\nu}_W^{(1)} \tilde{\nu}_W^{(1)} \right),$$

(A.13)

where terms involving $1/\phi_{21}$ have been ignored. Since the vev $\tilde{\phi}_{21}$ is very small, the second term generates a very large neutrino Majorana mass. The first term generates the corresponding scalar mass, where the notation $(\phi_{21})_1$ refers only to the first SU(3) component of that scalar.

The quantum-generated mass terms in eqs (A.12) and (A.13) were constructed in the context of the $m = 1$ flavor $F = 1$ being identified with the $m = 2$ flavor $F = 1$. Due to the $3 \times 3$ flavor symmetry in the $m = 1$ sector, there are also analogous terms where the $m = 1$ flavors $F = 2$ or $F = 3$ are identified with the $m = 2$ flavor $F = 1$. With those included, the neutrino Majorana mass term becomes

$$\frac{4\theta^2 \tilde{O}_0}{9\phi_{21}^2} \sum_F \nu_W^{(F)} \nu_W^{(F)},$$

(A.14)

In other words, all three right-handed neutrinos have the same very large Majorana mass, $M_\nu$. For $m = 2$ scalars, there is no analogous flavor rotation since the $m = 2$ sector only has $1 \oplus 2 \times 2$ flavor symmetry. So only $(\phi_{21})_1$ gets a very large mass by the above mechanism.

Under the above $m = 1$ flavor rotation, the $m = 1$ mass terms of eq (A.12) become

$$-\frac{1}{3} \tilde{O}_0 \left( \frac{1}{\phi_{21} \phi_{21}} + \frac{1}{\phi_{22} \phi_{22}} \right) \sum_F \tilde{Q}_{1F} Q_{1F}.$$  

(A.15)

The same mechanism that generates large Majorana neutrino masses can also generate an effective coupling important for scalar-mediated quark decays. Just as quantum effects generate $O_0$ in eq (A.10), they will also generate an $O_2$ that replaces $\Delta^{(000)}$ in $O_0$ with $\Delta^{(200)} + \Delta^{(020)} + \Delta^{(002)}$, while changing $(\text{Tr}(\Phi^2))^2 \to (\text{Tr}(\Phi^2))^{7/3}$. This generates the following quantum superpotential term:

$$-i \frac{\sqrt{2}\theta \tilde{O}_0}{3c\phi_{21}^2} \left( \tilde{\nu}_W^{(1)} \phi_{21} \tilde{\nu}_W^{(1)} \right).$$  

(A.16)

Just as for the Majorana neutrino masses, since $\tilde{\phi}_{21}$ is very small, this quantum-generated coupling is large. This coupling is used in section 5 when showing how the model can reproduce $Z$ pole observables.

Another expression that is supergauge invariant is

$$\text{Tr} \left( W_{\alpha \dot{\alpha}}^\alpha W_\alpha^\alpha \Phi \right)$$

(A.17)

The above expression has dimension 4 and no axial anomaly charge, so it can replace $(\text{Tr}(\Phi^2))^2$ in $O_0$. Since from eq (3.10), $(\langle W_{\alpha \dot{\alpha}} \rangle \neq 0$, eq (A.17) generates quark mass terms that mix gaugino quarks with adjoint quarks, such as $\tilde{M}_G^{(1)}$ in eq (3.12).

The quantum superpotential can also include expressions like $O_0$ of eq (A.10) where the “meson” factors in the denominator are generalized in the following way:

$$M_{m1m2FFr}^{(0)} \to M_{m1m2FFr}^{(n)}.$$  

(A.18)

These terms must have corresponding additional factors of $\Phi^2$ in the numerator. For example, a term where the flavor 2 and 3 mesons have a total of 3 additional factors of $\Phi^2$ in the denominator, there must be an additional factor of $\Phi^2$ in the numerator. One of these terms can have $\text{Tr} \left( W_{\alpha \dot{\alpha}}^\alpha W_\alpha^\alpha \Phi^3 \right)$ in the numerator. This term generates an adjoint-gaugino mass term for down-type quarks but not for up-type quarks (since the “11” component of $\langle \Phi^2 \rangle$ is zero). The presence of terms like these provide justification for not forcing these types of terms to be the same in the up-type and down-type quark matrices of section 3. Terms involving $\text{Tr} \left( \Phi^4 \right)$ in the numerator can similarly lead to differences in adjoint-adjoint quark mass terms for up-type and down-type quarks.

Superpotential terms with $\text{Tr}(\Phi^2)^3$ in the numerator can lead to mass terms like that of eq (A.12), but where $\tilde{Q}_{m1} Q_{m1} \to \tilde{Q}_{m1} \langle \Phi^2 \rangle Q_{m1}$. These terms only generate masses for the second two components of lepton triplets. This shows that the first component may have a different mass than the second two components. This freedom is used in the lepton mass matrices of section 3.

For superpotential terms in which the numerator has dimension 5 (such as $W_{\alpha \dot{\alpha}}^\alpha W_{\alpha \dot{\alpha}}^\alpha \text{Tr} (\Phi^2)$), there must be an odd number of $\Phi$ factors in the denominator. A term like this involving $M_{m111}^{(2)}$ generates a quark mass term for $\tilde{u}_W^A \tilde{u}^A \tilde{\nu}_W^{(1)}$ that supplements the tree level term involving $\phi_{211} \Gamma_{211}$. Similarly, many other quantum-generated couplings can supplement equivalent tree-level couplings.
Presumably, detailed nonperturbative calculations could be performed to determine exact forms of superpotential terms for this theory. In the absence of those calculations which would produce the exact parameter values and scalar vevs at the quantum minimum, this paper has phenomenologically determined the values those parameters and vevs would have to take in order to reproduce experimental data.

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