Gauge Coupling Unification in the Standard Model

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The string landscape suggests that the supersymmetry breaking scale can be high, and then the simplest low energy effective theory is the Standard Model (SM). We show that gauge coupling unification can be achieved at about $10^{16-17}$ GeV in the SM with suitable normalizations of the $U(1)_{Y}$. Assuming grand unification scale supersymmetry breaking, we predict that the Higgs mass range is 127 GeV to 165 GeV, with the precise value strongly correlated with the top quark mass and $SU(3)_C$ gauge coupling. We also present 7-dimensional orbifold grand unified theories in which such normalizations for the $U(1)_{Y}$ and charge quantization can be realized.

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Introduction – There exists an enormous “landscape” for long-lived metastable string/M theory vacua. Applying the “weak anthropic principle”, the landscape proposal may be the first concrete explanation of the very tiny value of the cosmological constant, which can take only discrete values, and it may address the gauge hierarchy problem. Notably, the supersymmetry breaking scale can be high if there exist many supersymmetry breaking parameters or many hidden sectors. Although there is no definite conclusion that the string landscape predicts high-scale or TeV-scale supersymmetry breaking, it is interesting to consider models with high-scale supersymmetry breaking.

If the supersymmetry breaking scale is around the grand unification scale or the string scale, the minimal model at low energy is the Standard Model (SM). The SM explains the existing experimental data very well, including electroweak precision tests, and it is easy to incorporate aspects of physics beyond the SM through small variations. However, even if the gauge hierarchy problem can be solved by the string landscape proposal, there are still some limitations of the SM, for example, the lack of explanation of gauge coupling unification and charge quantization.

Charge quantization can be easily explained by embedding the SM into a grand unified theory (GUT). Should the Higgs particle be the only new physics observed at the Large Hadron Collider (LHC) and the SM is thus confirmed as a low energy effective theory, an important question will be: can we achieve gauge coupling unification in the SM without introducing any extra multiplets between the weak and GUT scales or having large threshold corrections? As is well known, gauge coupling unification cannot be achieved in the SM if we choose the canonical normalization for the $U(1)_{Y}$ hypercharge interaction, i.e., the Georgi-Glashow $SU(5)$ normalization. Also, to avoid proton decay induced by dimension-6 operators via heavy gauge boson exchanges, the gauge coupling unification scale is constrained to be higher than about $5 \times 10^{15}$ GeV.

In this Letter we reconsider gauge coupling unification in the SM. The gauge couplings for $SU(3)_C$ and $SU(2)_L$ are unified at about $10^{16-17}$ GeV, and the gauge coupling for the $U(1)_{Y}$ at that scale depends on its normalization. If we choose a suitable normalization of the $U(1)_{Y}$, the gauge couplings for the $SU(3)_C$, $SU(2)_L$, and $U(1)_{Y}$ can in fact be unified at about $10^{16-17}$ GeV, and there is no proton decay problem via dimension-6 operators. Therefore, the real question is: is the canonical normalization for $U(1)_{Y}$ unique?

For a 4-dimensional (4D) GUT with a simple group, the canonical normalization is the only possibility, assuming that the SM fermions form complete multiplets under the GUT group. However, the $U(1)_{Y}$ normalization need not be canonical in string model building, orbifold GUTs and their deconstruction, and in 4D GUTs with product gauge groups:

(1) In weakly coupled heterotic string theory, the gauge and gravitational couplings unify at tree level to form one dimensionless string coupling constant $g_{\text{string}}$

$$k_Y g_Y^2 = k_2 g_2^2 = k_3 g_3^2 = 8\pi G_N / \alpha' = g_{\text{string}}^2$$

(1) where $g_Y$, $g_2$, and $g_3$ are the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, $G_N$ is the gravitational coupling and $\alpha'$ is the string tension. Here, $k_Y$, $k_2$ and $k_3$ are the levels of the corresponding Kac-Moody algebras; $k_2$ and $k_3$ are positive integers while $k_Y$ is a rational number in general.

(2) In intersecting D-brane model building on Type II orientifolds, the normalization for the $U(1)_{Y}$ (and other gauge factors) is not canonical in general.

(3) In orbifold GUTs, the SM fermions need not form complete multiplets under the GUT group, so the $U(1)_{Y}$ normalization need not be canonical. This statement is also valid for the deconstruction of the orbifold GUTs and for 4D GUTs with product gauge groups.
We shall assume that at the GUT or string scale, the gauge couplings in the SM satisfy
\[ g_1 = g_2 = g_3 , \tag{2} \]
where \( g_1^2 \equiv k_Y g_Y^2 \), and \( k_Y = 5/3 \) for canonical normalization. We show that gauge coupling unification in the SM can be achieved at about \( 10^{16/17} \) GeV for \( k_Y = 4/3, 5/4, 32/25 \). Especially for \( k_Y = 4/3 \), gauge coupling unification in the SM is well satisfied at two loop order. Assuming GUT scale supersymmetry breaking, we predict that the Higgs mass is in the range 127 GeV to 165 GeV. In addition, we construct 7-dimensional (7D) orbifold GUTs in which such normalizations for the \( U(1)_Y \) and charge quantization can be realized. A more detailed discussion will be presented elsewhere \[15\].

**Gauge Coupling Unification** – We define \( \alpha_i = g_i^2/(4\pi) \) and denote the Z boson mass by \( M_Z \). In the following, we choose a top quark pole mass \( m_t = 178.0 \pm 4.3 \) GeV \[14\], \( \alpha_3(M_Z) = 0.1182 \pm 0.0027 \) \[17\], and the other gauge couplings, Yukawa couplings and the Higgs vacuum expectation value at \( M_Z \) from Ref. \[18\].

We first examine the one-loop running of the gauge couplings. The one-loop renormalization group equations (RGEs) in the SM are
\[ (4\pi)^2 \frac{d}{dt} g_i = b_i g_i^3 , \tag{3} \]
where \( t = \ln \mu, \mu \) is the renormalization scale, and
\[ b \equiv (b_1, b_2, b_3) = \left( \frac{41}{6k_Y}, \frac{19}{6}, -7 \right) . \tag{4} \]

We consider the SM with \( k_Y = 4/3, 5/4, 32/25 \) and 5/3. In addition, we consider the extension of the SM with two Higgs doublets (2HD) with \( b = (7/k_Y, -3, -7) \) and \( k_Y = 4/3, 5/3 \), as it has an excellent good unification. Introducing supersymmetry significantly improves the convergence. Meanwhile, the same level of convergences can be achieved in all the non-supersymmetric models. In particular, the SM with \( k_Y = 32/25 \) and the 2HD SM with \( k_Y = 4/3 \) have very good gauge coupling unification.

### TABLE I: Convergences of the gauge couplings at one loop.

| Model  | \( k_Y \) | \( M_{U-} \) | \( M_{U+} \) | \( \Delta_- \) | \( \Delta_+ \) |
|--------|---------|------------|------------|--------------|--------------|
| SM     | 4/3     | 1.9        | 1.0        | 4.3          | 3.5          |
| SM     | 5/4     |            |            | 2.1          | 3.0          |
| SM     | 32/25   | 0.32       | 0.60       | 0.32         | 1.5          |
| SM     | 5/3     |            |            | 23.4         | 22.8         |
| 2HD SM | 4/3     | 0.45       | 0.33       | 0.24         | 0.25         |
| MSSM I | 5/3     | 0.47       | 0.35       | 0.26         | 3.4          |
| MSSM II| 5/3     | 0.44       | 0.32       | 0.24         | 1.3          |

The two-loop running of the gauge couplings produces slightly different results. We perform the two-loop running for the SM with \( k_Y = 4/3 \), as it has an excellent unification. We use the two-loop RGE running for the gauge couplings and one-loop for the Yukawa couplings \[20\]. With the central value of \( \alpha_3 \), we show the gauge coupling unification in Fig. 1. At the unification scale of \( 4.3 \times 10^{16} \) GeV, the value of \( \alpha_1 \) precisely agrees with those of \( \alpha_2 \) and \( \alpha_3 \).

![FIG. 1: Two-loop gauge coupling unification for the SM with \( k_Y = 4/3 \).](image)

If the Higgs particle is the only new physics discovered at the LHC and the SM is thus confirmed as a low energy effective theory, the most interesting parameter is the Higgs mass. To be consistent with string theory or quantum gravity, it is natural to have supersymmetry in the fundamental theory. In the supersymmetry...
ric models, there generically exist one pair of the Higgs doublets $H_u$ and $H_d$. We define the SM Higgs doublet $H$, which is fine-tuned to have a small mass term, as $H \equiv -\cos \beta i \sigma_2 H_d^* + \sin \beta H_u$, where $\sigma_2$ is the second Pauli matrix and $\tan \beta$ is a mixing parameter. For simplicity, we assume that supersymmetry is broken at the GUT scale, i.e., the gauginos, squarks, sleptons, Higgsinos, and the other combination of the scalar Higgs doublets $(\sin \beta i \sigma_2 H_u^* + \cos \beta H_u)$ have a universal supersymmetry breaking soft mass around the GUT scale. We can calculate the Higgs boson quartic coupling $\lambda$ at the GUT scale:

$$\lambda(M_U) = \frac{k_Y g_2^2(M_U) + g_1^2(M_U)}{4k_Y} \cos^2 2\beta , \quad (5)$$

and then evolve it down to the weak scale. Using the one-loop effective Higgs potential with top quark radiative corrections, we calculate the Higgs boson mass by minimizing the effective potential. For the SM with $k_Y = 4/3$, the Higgs boson mass as a function of $\tan \beta$ for different $m_t$ and $\alpha_3$ is shown in Fig. 2. We see if we vary $\alpha_3$ within its 1$\sigma$ range, $m_t$ within its 1$\sigma$ and 2$\sigma$ ranges and $\tan \beta$ from 1.5 to 50, the predicted mass of the Higgs boson ranges from 127 GeV to 165 GeV. A large part of this uncertainty is due to the present uncertainty in the top quark mass. The top quark mass can be measured to about 1 GeV accuracy at the LHC. Assuming this accuracy and the central value of 178 GeV, the Higgs boson mass is predicted to be between 141 GeV and 154 GeV.

**Orbifold GUTs** – In string model building, the orbifold GUTs and their deconstruction, and 4D GUTs with product gauge groups, the normalization for the $U(1)_Y$ need not be canonical. As an explicit example, we show that $k_Y = 4/3$ can be obtained in the 7D orbifold $SU(6)$ model on the space-time $M^4 \times T^2 / Z_6 \times S^1 / Z_2$ where charge quantization can be realized simultaneously. Here, $M^4$ is the 4D Minkowski space-time. Similarly, $k_Y = 5/4$ and $k_Y = 32/25$ can be obtained in the 7D orbifold $SU(7)$ models with charge quantization.

We consider the 7D space-time $M^4 \times T^2 \times S^1$ with coordinates $x^\mu$, $z$ and $y$ where $z$ is the complex coordinate for the torus $T^2$ and $y$ is the coordinate for the circle $S^1$. The radii for $T^2$ and $S^1$ are $R$ and $R'$. The $T^2 / Z_6 \times S^1 / Z_2$ orbifold is obtained from $T^2 \times S^1$ by moduling the equivalent classes

$$\Gamma_T : \quad z \sim \omega z \quad ; \quad \Gamma_S : \quad y \sim -y , \quad (6)$$

where $\omega = e^{i \pi / 3}$. $(z, y) = (0, 0)$ and $(0, \pi R')$ are the fixed points under the $Z_6 \times Z_2$ symmetry.

$\mathcal{N} = 1$ supersymmetry in 7 dimensions has 16 supercharges and corresponds to $\mathcal{N} = 4$ supersymmetry in 4 dimensions; thus, only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under the 4D $\mathcal{N} = 1$ supersymmetry into a vector multiplet $V$ and three chiral multiplets $\Sigma_1$, $\Sigma_2$, and $\Sigma_3$ in the adjoint representation, where the fifth and sixth components of the gauge field $(A_5$ and $A_6$) are contained in the lowest component of $\Sigma_1$, and the seventh component of the gauge field $(A_T)$ is contained in the lowest component of $\Sigma_2$. The SM quarks, leptons and Higgs fields can be localized on 3-branes at the $Z_6 \times Z_2$ fixed points, or on 4-branes at the $Z_6$ fixed points.

Under the $Z_6 \times Z_2$ discrete symmetry, the bulk vector multiplet $V$ and the $\Sigma_i$ transform as [15]

$$\Phi(x^\mu, \omega z, \omega^{-1} z, y) = \eta_T^T R_T \Phi(x^\mu, z, \omega^{-1} z, y) R_T^{-1} , \quad (7)$$

$$\Phi(x^\mu, \omega z, \omega^{-1} z, -y) = \eta_S^S R_S \Phi(x^\mu, z, \omega^{-1} z, y) R_S^{-1} , \quad (8)$$

where $\Phi$ can be $V$ or $\Sigma_i$, and

$$\eta_T^T = \eta_{\Sigma_2}^T = 1 , \quad \eta_T^T = \omega^{-1} = \eta_{\Sigma_3}^T = \omega , \quad (9)$$

$$\eta_S^S = \eta_{\Sigma_2}^S = 1 , \quad \eta_S^S = \eta_{\Sigma_3}^S = -1 . \quad (10)$$

We also introduce non-trivial $R_T$ and $R_S$ to break the bulk gauge group.

Let us consider the $SU(6)$ model, which has $k_Y = 4/3$. We define the generator for the $U(1)_Y$ in $SU(6)$ as

$$T_{U(1)_Y} \equiv \text{diag} \left( \frac{1}{3} \cdot \frac{1}{3} , \frac{1}{3} , \frac{1}{3} , -\frac{1}{3} , -\frac{1}{3} , -\frac{1}{3} \right) . \quad (11)$$

Because $\text{tr}[T_{U(1)_Y}^2] = 2/3$, we obtain $k_Y = 4/3$.

To break the $SU(6)$ gauge symmetry, we choose the following $6 \times 6$ matrix representations for $R_T$ and $R_S$

$$R_T = \text{diag} \left( +1 , +1 , +1 , \omega^2 , \omega^2 , \omega^5 \right) , \quad (12)$$

$$R_S = \text{diag} \left( +1 , +1 , +1 , +1 , +1 , -1 \right) . \quad (13)$$

![FIG. 2: The predicted Higgs mass for the SM with $k_Y = 4/3$. The red (lower) curves are for $\alpha_3 + \delta \alpha_3$, the blue (upper) $\alpha_3 - \delta \alpha_3$, and the black $\alpha_3$. The dotted curves are for $m_t \pm \delta m_t$, the dash ones for $m_t \pm 2 \delta m_t$, and the solid ones for $m_t$.](image-url)
We obtain that, for the zero modes, the 7D $\mathcal{N} = 1$ supersymmetric $SU(6)$ gauge symmetry is broken down to the 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y} \times U(1)'$ [12]. Also, we have only one zero mode from $\Sigma_{i}$ with quantum number $(3, 1, -2/3)$ under the SM gauge symmetry, which can be considered as the right-handed top quark [13].

On the 3-brane at the $Z_{6} \times Z_{2}$ fixed point $(z, y) = (0, 0)$, the preserved gauge symmetry is $SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y} \times U(1)'$ [14]. Thus, on the observable 3-brane at $(z, y) = (0, 0)$, we can introduce one pair of Higgs doublets and three families of SM quarks and leptons except the right-handed top quark [15]. Because the $U(1)'_{Y}$ charge for the right-handed top quark is determined from the construction, charge quantization can be achieved from the anomaly free conditions and the gauge invariance of the Yukawa couplings on the observable 3-brane. Moreover, the $U(1)'_{Y}$ anomalies can be cancelled by assigning suitable $U(1)'_{Y}$ charges to the SM quarks and leptons, and the $U(1)'_{Y}$ gauge symmetry can be broken at the GUT scale by introducing one pair of the SM singlets with $U(1)'_{Y}$ charge $\pm 1$ on the observable 3-brane. Interestingly, this $U(1)'_{Y}$ gauge symmetry may be considered as a flavour symmetry, and then the fermion masses and mixings may be explained naturally. Furthermore, supersymmetry can be broken around the compactification scale, which can be considered as the GUT scale, for example, by Scherk–Schwarz mechanism [22].

We briefly comment on the 7D orbifold $SU(7)$ models which can have $k_{Y} = 5/4$ and $k_{Y} = 32/25$ [23]. The discussion is similar to that for the above $SU(6)$ model. The 7D $\mathcal{N} = 1$ supersymmetric $SU(7)$ gauge symmetry is broken down to the 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y} \times U(1)' \times U(1)''$ by orbifold projections. There is only one pair of zero modes from $\Sigma_{i}$ with quantum numbers $(1, 2, +1/2)$ and $(1, 2, -1/2)$ under the SM gauge symmetry, which can be considered as one pair of Higgs doublets. Also, charge quantization can be realized [15].

**Conclusions** – The string landscape suggests that the supersymmetry breaking scale can be high and then the simplest low energy effective theory is just the SM. We showed that gauge coupling unification in the SM with $k_{Y} = 4/3$, $5/4$, and $32/25$ can be achieved at about $10^{16} - 17$ GeV. Assuming GUT scale supersymmetry breaking, we predicted that the Higgs mass is in the range 127 GeV to 165 GeV. We also presented the 7D orbifold GUTs where such normalizations for the $U(1)_{Y}$ and charge quantization can be realized.

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