Modeling and Analysis of sub–Terahertz Communication Channel via Mixture of Gamma Distribution

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Abstract—Transceivers operating in the range of 0.1 terahertz (THz)–10THz, which are known as THz bands, will enable ultra–high throughput wireless communications. However, actual implementation of the high–speed and reliable THz band communication systems should start with providing extensive knowledge in regards to the propagation channel characteristics. Considering a huge bandwidth and the rapid changes in the characteristics of THz wireless channels, ray tracing and one–shot statistical modeling are not adequate to define a proper channel model. In this work, we propose Gamma mixture based channel modeling for the THz band via expectation–maximization (EM) algorithm. First, maximum likelihood estimation (MLE) is applied to characterize the Gamma mixture model parameters, and then EM algorithm is used to compute MLEs of the unknown parameters of the measurement data. The performance is investigated by using the Weighted relative mean difference (WMRD) error metrics, Kullback–Leibler (KL)–divergence, and Kolmogorov–Smirnov (KS) test to show the difference between the proposed model and the actual probability density functions (PDFs) that are obtained via the designed test environment. To efficiently evaluate the performance of the proposed method in more realistic scenarios, all the analysis is done by examining measurement data from a measurement campaign in the 240GHz to 300GHz frequency range, using a well–isolated anechoic chamber. According to WMRD error metrics, KL–divergence, and KS test results, PDFs generated by the mixture of Gamma distributions fit to the actual histogram of the measurement data. It is shown that instead of taking pseudo–average characteristics of sub–bands in the wide band, using the mixture models allows determining the channel parameters more precisely.

I. INTRODUCTION

The ability of wireless communication technology to meet consumer needs requires that next generation wireless networks' data rates reach terabits per second (Tbps) levels at a higher link density [1][2]. Although free space optical (FSO) and millimeter wave (mmWave) communications are used for high data rates, the THz band differs from the currently used bands though the THz band will provide a way to achieve Tbps data rates, the THz band differs from the currently used bands in the channel characteristics that change rapidly and sharply across the spectrum [2][4]. Therefore, all elements of the system should be rethought and designed to develop a proper communication system. For example, the propagation channel is required to be analyzed on the aspects of materials in the medium, and the operating frequency. Wireless communication in wide band around 60GHz requires a channel model considering characteristics of sub–bands which are windows such that propagation characteristics can be assumed to be static throughout the window.

A. Related Work

In the studies on channel modeling, various approaches can be encountered for frequency, time and spatial analysis. As known, the wireless communication channel can be modeled by using deterministic or statistical methods. Although deterministic models like ray–tracing enable to completely model channel characteristics in a given propagation environment [5], they cannot fulfill their performance in the presence of the slightest change in the propagation environment. They require an environment completely defined with sizes, shapes, and materials of objects. As a result, considering that even molecular changes affect the propagation characteristics of THz waves, it can be said that ray–tracing method is not suitable for this band. Another reason that makes this method useless is the exponential increase in the complexity of the method as the size of the medium to be modeled increases. Recently, [6][7] analyze channels in a data center in terms of temporal and spatial channel characteristics. Furthermore, [8] recommends that the channel parameters in time, frequency,
and spatial domains can be modeled for each ray type for THz kiosk communications. Our previous work [9] proposes two-slope path loss model for short-range THz communication links.

The statistical approaches get the average of the environmental effects, unlike a deterministic model. Some stochastic models have been recently proposed in [10][12]. For example, the use of statistical methods to model in-vivo channels is inevitable. Since human tissue consists of materials with different electromagnetic transmissivity, the path loss and noise power fluctuate for in-vivo applications. Lately, [13][14] focus on the channel models for nano device THz communication in the human body.

Another important consideration in channel modeling is the careful selection of signal processing methods to be used for modeling the wide-band channel. To set an example, in [15], the frequency sweeping method, which is not safe due to artifacts created when the post-processing of the smaller chunks of bandwidth, is employed to model the spectrum between 260GHz and 400GHz. Another problem in channel modeling is to make the assumption that the derived impulse response has a linear phase. This assumption implies that the impulse response is symmetrical to line-of-sight (LOS) propagation delay. However, the real physical environments do not allow this phenomenon because it contradicts causality. Therefore, Kazuhiro et al. propose a causal channel model for THz band [16].

In addition to single-input single-output (SISO) channel models, channels for THz communication systems with spatial diversity are investigated by using multi-input multi-output (MIMO) systems. The studies in [17][18] detail the channel characteristics for $2 \times 2$ MIMO system. Due to small antenna size in THz band, ultra-massive MIMO systems can be utilized in communication systems. THz communication channels for ultra-massive MIMO systems are studied in [19][21]. Furthermore, application-specific scenarios are available in the literature. Some application-specific studies focus on channel modeling at 300GHz [22][23].

Studies up to this point assume that the THz band of interest has a single statistical distribution. In this case, it can be concluded that channel modeling with a single probability density function (PDF) is not sufficient considering the presence of windows that behave differently in the THz band due to the effect of molecules in the medium. The THz band contains changes across the spectrum, so it may not be sufficient to express this extremely wide-band with a single statistical model. For example, suppose that the three sub-bands behave differently from each other as demonstrated in Fig. 1. Hence, the use of mixtures to add the characteristics of each sub-band into the model provides better convergence to the actual histogram.

Although mixture models have been used in many different fields, in this study, we are confined to mentioning only the studies on wireless communication channel models. In [24], it is stated that mixture Gamma is able to model $\alpha - \kappa - \mu$ shadowed fading channels, even though they consist of intractable statistical properties. [25] and [26] employ the mixture of Gaussian distributions to construct a generalized shadowing model. By the way, they utilize expectation-maximization (EM) algorithm to find the mixture parameters. The error probability and ergodic capacity can be analyzed by using Gamma mixtures for diversity reception schemes over generalized-$K$ fading channels [27]. Moreover, the physical layer security analysis can be performed by utilizing mixture models in generalized-$K$ fading channels [28][29], which is one of the most important studies in this research area, proposes a mixture of Gamma distributions for the signal-to-noise ratio (SNR) of fading channels; thereby, it allows to derive the outage probability, the average channel capacity, and the symbol error rate.

**B. Contributions**

In this study, we utilize mixture models to investigate channel models for sub-THz band between 240GHz and 300GHz. With inspiration from the studies such as [30][31], which adopt the mixture models to characterize the wireless propagation channel, we propose mixture models which we think are very suitable for the nature of the THz band to model the distribution of the received power for the sub-THz band between 240GHz and 300GHz. We believe that a model based on a single distribution cannot provide a good enough representation, as the THz band allows for very broadband communication and there is a significant change in channel characteristics throughout this wide band. The contributions of this study can be categorized under two main points:

- For the THz band, measurement based channel model study is performed. Using measurement data, it is shown that Gamma mixtures can be used effectively in channel modeling for THz band. Thus, the characteristics of the channel can be expressed in a realistic manner.
- Weighted relative mean difference (WMRD), Kolmogorov–Smirnov (KS) and Kullback-Leibler (KL)–divergence approaches are studied to investigate how well Gamma mixture models fit into measurement data.

Moreover, considering that the measurement data used in this study is a very valuable source of information and the necessity of making serious investments to reach such data, it is offered as a public dataset [32]. We believe that the sharing of this measurement data will foster new studies.

**C. Organization of the Paper**

The rest of this manuscript is organized as follows. Section II details the signal model and gives mathematical preliminaries. The measurement setup is introduced in Section III. In Section IV Gamma mixture modeling results are given and discussed. Finally, Section V concludes the paper.

**II. BACKGROUND**

**A. Signal Model**

The received signal is represented as:

$$r(t) = \text{Re}\{x(t) + jx_Q(t)e^{j2\pi f_c t}\},$$ (1)
where \( j \) denotes the unit imaginary number and \( \text{Re}\{\cdot\} \) is the real part of the complex number. \( x_I(t) \) and \( x_Q(t) \) represent in–phase and quadrature (IQ) parts of complex baseband equivalent of the signal. \( f_c \) stands for the carrier frequency of the signal.

The multipath channel at passband with different delays and attenuation levels can be given as:

\[
h(t) = \sum_{l=0}^{L-1} a_l \delta(t-t_l),
\]

where \( L \) is the number of multipath components. \( a_l \) and \( t_l \) denote the attenuation and delay factors for the \( l \)th path, respectively. The complex baseband representation of (2) is

\[
h(t) = \sum_{l=0}^{L-1} a_l \delta(t-t_l)e^{-j2\pi f_c t_l}.
\]

If the channel consists of only LOS component, \( L \) in (3) is equal to 1. Then, LOS channel is given as:

\[
h(t) = a_0 \delta(t-t_0)e^{-j2\pi f_c t_0},
\]

where \( a_0 \) and \( 2\pi f_c t_0 \) denote amplitude and phase of channel, respectively. \( t_0 \) is propagation delay given with

\[
t_0 = \frac{d}{c},
\]

where \( d \) is the distance between transmitter and receiver and \( c \) is the speed of light.

As anechoic chambers, as used in our measurements, do not allow non–line–of–sight (NLOS) propagation. The losses are limited to path loss, antenna misalignment, and imperfections created by hardware. Thus, the signal model can be reduced to a direct path which is comprised of distant dependent path loss and antenna misalignment. The contribution of path loss to the channel amplitude \( a_0 \) is given as:

\[
P_{RX} = P_{TX} - 10n\log(d) + M. \tag{6}
\]

The received power \( P_{RX} \) including antenna gain considering misalignment, \( M \), is calculated as the difference between transmitted power \( P_{TX} \) and path loss with exponent \( n \).

B. Gamma Distribution

The Gamma function, \( \Gamma(a) \), is defined as \[33\]:

\[
\Gamma(a) = \int_0^\infty e^{-x}x^{a-1}dx, \ \ a > 0. \tag{7}
\]

By using integration by parts, \( \Gamma(a) = (a-1)! \) when \( a \) is a positive integer. Consider the random variable \( G \) which is a mixture of \( m \) Gamma distributions and defined as:

\[
f_G(x) = \sum_{l=1}^{m} \rho_l f_l(x; \alpha_l, \beta_l), \ \ l = 1,2,\ldots,m, \ \ x > 0, \ \ \rho_l > 0 \tag{8}
\]

where \( f_l(x; \alpha_l, \beta_l) = \frac{1}{\beta_l^{\alpha_l} \Gamma(\alpha_l)} x^{\alpha_l-1}e^{-x/\beta_l}; \ \ \alpha_l > 0 \) and \( \beta_l > 0 \) are the shape and scale parameters of the \( l \)th component of the mixture distribution; \( \rho_l \) denotes mixture proportions or weights that satisfy the conditions (a) \( 0 < \rho_l < 1, \ \forall l = 1,2,\ldots,m \) and (b) \( \sum_{l=1}^{m} \rho_l = 1 \). Here, \( m \)
denotes the number of components in the mixture. The main reasons for using a mixture of Gamma distributions in the paper are: (i) the tractability of its moment generating function (MGF) and cumulative distribution function (CDF), (ii) giving an approximation for small-scale fading channels [29], and (iii) high accuracy by properly adjusting parameters.

C. Maximum Likelihood Estimation

Here, we provide the maximum likelihood estimation (MLE) technique that can be used to obtain the parameters of the gamma mixture from the actual channel PDF. Let assume that \(X_1,\ldots,X_n\) are random variables with Gamma distribution (with unknown parameters \(\alpha > 0\) and \(\beta > 0\)). The likelihood function is given as:

\[
L(x; \alpha, \beta) = \prod_{i=1}^{n} \frac{x_i^{\alpha-1} e^{-\frac{x_i}{\beta}}}{\Gamma(\alpha) \beta^\alpha}
\]

\[
= \left\{ \prod_{i=1}^{n} x_i \right\}^{-1} \left\{ \prod_{i=1}^{n} x_i \right\} e^{-\frac{n}{\beta} \sum_{i=1}^{n} x_i} \beta^{n\alpha} \Gamma^{-n}(\alpha)
\]

The uninformative factor, \(\left\{ \prod_{i=1}^{n} x_i \right\}^{-1}\), is discarded

\[
= \left\{ \prod_{i=1}^{n} x_i \right\} e^{-\frac{\sum_{i=1}^{n} x_i}{\beta}} \beta^{-n\alpha} \Gamma^{-n}(\alpha)
\]

\[
= \beta^{-n\alpha} \Gamma^{-n}(\alpha) \left\{ \prod_{i=1}^{n} x_i \right\} e^{-\frac{\sum_{i=1}^{n} x_i}{\beta}}
\]

The corresponding log likelihood function of (9) leads to:

\[
\ln(L) = -n\alpha \ln(\beta) - n \ln(\Gamma(\alpha)) + \alpha \sum_{i=1}^{n} \ln(x_i) - \sum_{i=1}^{n} \frac{x_i}{\beta}
\]

After that point, maximum likelihood estimates can be found for \(\alpha\) and \(\beta\) by taking partial derivatives with respect to \(\alpha\) and \(\beta\), then we obtain:

\[
\frac{\partial \ln(L)}{\partial \alpha} = -n \ln(\beta) - n \frac{\partial \ln(\Gamma(\alpha))}{\partial \alpha} + \sum_{i=1}^{n} \ln(x_i)
\]

\[
\frac{\partial \ln(L)}{\partial \beta} = -n \alpha \frac{1}{\beta} + \sum_{i=1}^{n} \frac{x_i}{\beta^2}
\]

Because of the diGamma and logarithm functions in (11), a closed-form solution could not be provided [33]. Numerical methods such as Newton–Raphson can be applied to find the values for \(\alpha\) and \(\beta\) which is not the scope of this study.

D. Expectation Maximization

We have a training set \(r = (r_1, r_2, \ldots, r_m)\) consisting of \(m\) independent observations captured by considering each measurement data at different transmitter–receiver separation distances such as \(d=20\text{cm}, 40\text{cm}, 60\text{cm}, 80\text{cm}\). Our goal is to fit the Gamma distribution parameters by utilizing the EM algorithm. EM algorithm, which is a machine learning technique [34], provides a simplification to MLE problems, which are mostly seen in mixture models [31]. The EM algorithm consists of two steps, namely, the expectation (E–step) and the maximization (M–step). The reader is referred to [35] for more detailed explanations about the EM algorithm.

The EM algorithm requires number of mixtures as a priori. Initially, the parameters are randomly chosen for the mixture model parameters \(\theta_{1,M} = (\theta_1, \ldots, \theta_M)\). Then, the parameters are updated in each iteration until the convergence criteria hold. E–step calculates membership coefficients for all data point \((i = 1, \ldots, L)\) and mixture components \((k = 1, \ldots, M)\) by utilizing the current parameters \(\theta_{1,M}\) [31] [36]

\[
\phi_{ik} = \frac{\pi_k p_k(x_i | \theta_k)}{\sum_{k=1}^{M} \pi_k p_k(x_i | \theta_k)},
\]

where \(x_i\) is the data in the \(k\)th mixture; \(\pi_k\) denotes the mixing proportion. It is obvious that \(\sum_{k=1}^{M} \phi_{ik} = 1\). Then, the parameter values and the mixing proportions for each mixture component are updated to maximize the likelihood probability in the M–step. In the M–step, the membership coefficients calculated in E–step are used to find parameters and mixing proportions as:

\[
\pi_k^{new} = \frac{\sum_{i=1}^{L} \phi_{ik}}{L}
\]

\[
E[X_k]^{new} = \frac{\sum_{i=1}^{L} \phi_{ik} x_i}{\sum_{i=1}^{L} \phi_{ik}} = \alpha \beta
\]

\[
Var[X_k]^{new} = \frac{\sum_{i=1}^{L} \phi_{ik} (x_i - E[X_k]^{new})^2}{\sum_{i=1}^{L} \phi_{ik}} = \alpha \beta^2.
\]

The parameters \((\alpha, \beta)\) for each Gamma mixture can be found by using (14).

E. Error Metrics

In this subsection, we provide an overview of the possible error metrics to determine the goodness–of–fit for the proposed model.

1) Weighted Mean Relative Difference: The proposed models are quantified by using WMRD, which gives a measurement for the difference between the model and actual PDFs. It is defined as [31]:

\[
WMRD = \frac{\sum_{\rho} |y_\rho - \hat{y}_\rho|}{\sum_{\rho} (y_\rho + \hat{y}_\rho) \times 0.5}
\]

where \(\rho\) represents the received power and \(y_\rho\) is the number of \(\rho\) value observations in the received power set. As well as, \(\hat{y}_\rho\) is related to the estimated model.

2) Kolmogorov–Smirnov Test: KS test is a non–parametric goodness–of–fit test, namely it does not make an assumption of any distribution. In addition to vector norm based error technique, the KS test is employed as goodness–of–fit test with the confidence level \(p = 0.05\) to compare the actual PDF with the estimated mixture models.

3) Kullback–Leibler Divergence: KL distance or divergence is interpreted as the distance between the actual probability distribution, \(P_{act}\) and the estimated probability distribution, \(P_{est}\). Let \(P_{act} = \{p_1, p_2, \ldots, p_n\}\) and \(P_{est} = \{q_1, q_2, \ldots, q_n\}\), then KL–divergence is defined as

\[
D_{KL}(P_{act} \parallel P_{est}) = - \sum_{x \in \mathcal{X}} P_{act}(x) \log \left( \frac{P_{est}(x)}{P_{act}(x)} \right).
\]
In this paper, KL–divergence is utilized to compare the actual distribution and the estimated models via the EM algorithm. KL–divergence gets a higher value when two distributions have less similarities.

III. CHANNEL MEASUREMENT CAMPAIGN AND DATA PROCESSING

The experimental measurement setup is performed in an anechoic chamber to secure the LOS transmission at the Millimeter Wave and Terahertz Technologies Research Laboratories (MiLTAL), Scientific and Technological Research Council of Turkey (TÜBİTAK), Kocaeli. The measurement setup is shown in Fig. 2 [12]. The dimensions of anechoic chamber are $7m \times 4m \times 3m$.

The measurement setup consists of four major parts: Agilent performance network analyzer (PNA) vector network analyzer (VNA) E8361A, Oleson Microwave Labs (OML) V03VNA2–T and V03VNA2–T/R–A millimeter wave extender modules and N5260A extender controller. Since the upper limit of the VNA is 67GHz, the 240GHz to 300GHz extender modules are attached to the VNA using the controller to measure channel characteristics at THz frequencies. V03VNA2–T/R–A extender module contains multipliers ($\times 18$) that extends 12.2GHz to 18.1GHz radio frequency (RF) input signal to 220GHz to 325GHz frequency range. Test intermediate frequency (IF) and reference IF signals for VNA are obtained by using down–conversion mixers before transmitting. After passing through the channel, the received signal is down–converted at the receiver module, V03VNA2–T, by using down–conversion mixers and the resulting test IF (5MHz to 300MHz) is fed back to the VNA. The difference in the transmitted and received signal is analyzed to find channel characteristics. The corresponding block diagram of our setup is shown in Fig. 3.

The 220GHz to 325GHz source modules include balanced multipliers, which are driven by an extended band WR–10 multiplier chain with a WR–03 waveguide output interface. The output source match of the modules is typically 9dB. The RF drive signal may be either continuous wave (CW) or swept frequency. The required RF or local oscillator (LO) signal power to operate OML modules is $+10$dBm. The dynamic range is typically 75dB (min. 60dB) and the WR–03 waveguide output power of the V03VNA2–T/R–A is around $-23$dBm. The magnitude and phase stability of the extender modules are $\pm 0.4$dB and $\pm 8^\circ$, respectively.

In this study, 240GHz to 300GHz is specially used to get better performance from the extender modules, in terms of magnitude and phase stability. Channel transfer function is acquired by recording scattering parameters (s–parameter) using the measurement setup shown in Fig. 2 and Fig. 3. For channel modeling, calibration is performed with direct interconnection of the transmitting and receiving extender module’s waveguide ports. All the later measurements are taken with standard gain horn antenna, with the gain of 24.8dBi at the center frequency, attached at both the transmitter and the receiver. Full 60GHz band measurements are recorded with 4096 points averaging and 100Hz IF bandwidth (BW). These parameters significantly reduce noise floor and improves dynamic range.

A. Measurement Methodology

VNA measurements are performed for the frequency interval of 240GHz to 300GHz. This interval is measured using standard gain horn antennas which are attached to OML extenders. Each spectral measurement is represented with 4096 equally spaced frequency points (data points) within the interval specified by the VNA. Therefore, a spectral resolution of 14.648MHz is obtained. The measurement system including connectors and cables is calibrated to remove the impairment caused by the components. The calibration data is saved both into the internal memory of the Agilent PNA VNA E8361A and an external universal serial bus (USB) storage device. Then, the calibration data is removed from the measurement by using its internal memory and provided $S_{21}$ parameter in complex number format with real and imaginary parts for each data point. Each set of captured IQ samples are transferred into a laptop computer. After applying the necessary conversions, all the analyses are done on MATLAB R2018b software to carry out the baseband operations for each transmitter–receiver separation distance given in Table 1.

IV. GAMMA MIXTURE MODEL FOR TERAHertz WIRELESS CHANNELS

In this section, Gamma mixture models are employed to model received power distribution for five measurement described in Section III. The received power, $P_{rx}$, is calculated in the linear scale as

$$P_{rx} = |S_{21}|^2 P_{tx}$$

(17)
where $P_{tx}$ is the transmitted signal power and it is constant during the transmission time. $|S_{21}|$ denotes the amplitude response of the propagation channel. It is known that the instantaneous SNR for a signal with bandwidth of $W$ is...
under the additive white Gaussian noise (AWGN) with power spectral density $N_0/2$. Therefore, SNR is related to the fading channel parameters, as well as the received power. By utilizing the instantaneous SNR, it is possible to derive the channel outage probability and the channel capacity.

In order to model the received signal power, both MLE and the EM algorithm are used. EM algorithm enables to determine the parameters of the estimated mixture components for the measurements. As stated before, the Gamma distributions are utilized because of the facts that its MGF is tractable and there is an approximation for small-scale fading channels.

In Fig. 4, it can be clearly seen that MLE estimation is not a good fit for the measured histogram; however, the mixture models fit better. For example, it can be said that three mixtures of Gamma distribution are sufficient to estimate the actual histogram for the distance of 20cm, but MLE gets hampered to fit since it assumes that there is no serious change in the channel behavior through the transmission band. It is seen in Fig. 4 that the mean of the received signal power decreases for increasing distance, as expected. Furthermore, the figure clearly exhibits different clusters in the histograms especially for the distances longer than 30cm, which is consistent with [2,4].

Moreover, WMRD results and KL–divergence also confirm that mixture models converge to the actual histogram better than MLEs. WMRD results, KL distance, and KS test results are presented in Table 1. WMRD results are not accurate enough to show the difference between MLE and mixture models. WMRD can demonstrate that the mixture model is more successful than MLE with only a small variation in its value. However, KL–divergence creates metrics more sensitive to differences between mixture models and MLEs. For instance, KL–divergence is found as 4.635 for MLE at 20cm, whereas it is 0.651 for two mixtures. KL–divergences show that the models consisting four mixtures are more similar to the actual PDFs for all measurements except 20cm. Surprisingly, KL–divergence of the model with two mixtures is the smallest for the measurement with the distance of 20cm. Furthermore, the results obtained from goodness–of–fit test with the confidence level $p = 0.05$ imply the suitability of the mixture models to actual PDFs.

V. CONCLUDING REMARKS AND FUTURE WORKS

In this paper, we investigate the channel model for the THz band in between 240GHz–300GHz by using Gamma mixture models. To find the mixture parameters, EM algorithm is utilized. It is clearly seen that the mixture models are better to fit the measurement histogram for all measurements compared to MLEs. The comparison between the mixture models and the actual PDFs is carried out by WMRD, KS and KL–divergence metrics. The metrics administrate that the mixture of Gamma distributions can accurately model the THz channels.

Since the outage probability, the average channel capacity, and the symbol error rate are derived for mixture Gamma wireless channels, the analytical analyse can be carried by using mixture parameters given in this study. As known, the EM algorithm requires the number of mixtures as a priori information. However, to determine the number of mixtures, the Dirichlet process mixture model and Bayesian information criterion can be utilized. In–vivo channel characteristics are heavily dependent on the density of the materials in the tissue; therefore, the in–vivo channels have more and more changes in their behaviors. It can be claimed that mixture models are reasonable for also in–vivo channels. This claim can be investigated in future studies.

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| Distance | Mixture | Parameters | WMRD ($\times 10^{-2}$) | KL Divergence | KS Test ($p = 0.05$) |
|----------|---------|------------|--------------------------|---------------|---------------------|
| 20cm     | MLE     | 1.00 30.084 0.227 | 1.580 | 4.635 | Passed |
|          | N = 2   | 0.540 72.285 0.0824 | 1.573 | 0.651 | Passed |
|          | N = 3   | 0.463 116.797 0.0539 | 1.572 | 0.813 | Passed |
|          | N = 4   | 0.538 100.310 0.063 | 1.572 | 0.797 | Passed |
| 30cm     | MLE     | 1.00 39.060 0.079 | 1.541 | 3.881 | Passed |
|          | N = 2   | 0.752 67.765 0.048 | 1.536 | 0.822 | Passed |
|          | N = 3   | 0.388 123.224 0.024 | 1.536 | 0.906 | Passed |
|          | N = 4   | 0.303 144.100 0.020 | 1.536 | 0.903 | Passed |
| 40cm     | MLE     | 1.00 49.334 0.034 | 1.497 | 3.349 | Passed |
|          | N = 2   | 0.626 172.946 0.010 | 1.486 | 1.118 | Passed |
|          | N = 3   | 0.550 295.648 0.006 | 1.484 | 1.067 | Passed |
|          | N = 4   | 0.532 322.175 0.005 | 1.483 | 1.016 | Passed |
| 60cm     | MLE     | 1.00 48.192 0.008 | 1.392 | 2.227 | Passed |
|          | N = 2   | 0.634 158.089 0.003 | 1.388 | 1.725 | Passed |
|          | N = 3   | 0.412 119.250 0.003 | 1.384 | 1.590 | Passed |
|          | N = 4   | 0.432 281.987 0.0016 | 1.381 | 1.331 | Passed |
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