Lepton flavor violation at linear collider experiments in supersymmetric grand unified theories

Masahide Hirouchi and Minoru Tanaka*

Department of Physics,
Graduate School of Science, Osaka University,
Toyonaka, Osaka 560, Japan
(April 22, 2021)

Abstract

Lepton flavor violation at linear collider experiments is discussed. We show that detectable lepton flavor violation could occur through scalar lepton pair production and decay in the supersymmetric SU(5) grand unified theory in spite of the stringent present experimental constraints by rare process searches. Possible cross sections about 40fb for an $e^+e^-$ collider and 280fb for an $e^-e^-$ collider are illustrated.

12.60.Jv, 14.80.Ly

*E-mail: tanaka@phys.wani.osaka-u.ac.jp
I. INTRODUCTION

Search for lepton flavor violation (LFV) is one of the most important ways to explore physics beyond the standard model, because lepton flavors are conserved individually in the standard model. In the standard model with supersymmetry (SUSY), which is one of the most attractive extensions of the standard model, LFV is allowed. The soft SUSY breaking masses of the scalar leptons (sleptons) do not have to conserve the lepton flavors in general. However, the resulting LFV exceeds much the present experimental bounds such as the one from $\mu \rightarrow e\gamma$ search if an arbitrary, but consistent with the naturalness argument, set of slepton soft masses is allowed. The universal soft mass scenario lead from supergravity [1] is often assumed to avoid this large LFV (and the same problem in the scalar quark sector). In this scenario, all the sleptons degenerate and we have no flavor mixing in the lepton and slepton sector.

However, this is not the whole story when grand unified theories (GUTs) are considered at the same time [2]. In SUSY GUTs with the universal soft breakings at Planck scale, the large top Yukawa coupling affects the third generation slepton mass through renomalization group evolution from Planck scale to GUT scale, since quark and lepton supermultiplets are unified into larger GUT multiplets. As a result, the sleptons no longer degenerate and LFV is expected to take place.

Along this line, rates of $\mu \rightarrow e\gamma$ decay, $\mu \rightarrow 3e$ decay, $\tau \rightarrow \mu\gamma$ decay and $\mu \rightarrow e$ conversion in nuclei have been estimated in the literature [3,4]. In the minimal SU(5) model, the $\mu \rightarrow e\gamma$ branching ratio has been found to be typically one or two order below the present experimental upper bound ($4.9 \times 10^{-11}$ [5]). The other modes tend to give two or more order smaller values than experimental bounds. In the SO(10) model, the $\mu \rightarrow e\gamma$ amplitude is enhanced by a factor $m_\tau/m_\mu$ due to its chiral structure different from the SU(5) model, and thus the decay rate could be the same order as or even larger than the experimental bound [3].

Besides these rare processes that are caused by virtual slepton exchange, it is possible to search for LFV through real slepton production and its successive decay [6–9]. The most prominent qualitative difference between the virtual and the real processes can be seen in behaviors of their amplitudes as the sleptons are getting to degenerate. The virtual process behaves as $\Delta m^2/\bar{m}^2$, while the real process does as $\Delta m^2/(\bar{m}\Gamma)$ [6], where $\Delta m^2$ is a slepton mass squared difference, $\bar{m}$ is the average mass of the sleptons and $\Gamma$ is the average slepton width. Because $\bar{m} \gg \Gamma$, we expect a good chance to observe LFV even for relatively degenerate sleptons once their real production at collider experiments becomes possible. The advantage of real production is maximized if $\Delta m^2/\bar{m}^2 \ll 1 \ll \Delta m^2/(\bar{m}\Gamma)$ is realized. In the following, we show that it happens in the minimal SUSY SU(5) model.

Another important point to have a realistic LFV cross section of the real production and decay is necessity of relatively large flavor mixing in the lepton-slepton sector. In the minimal SUSY SU(5) model, the leptons and the down-type quarks have the same Yukawa couplings at the GUT scale. This means that LFV is essentially controlled by Cabibbo-Kobayashi-Maskawa (CKM) matrix which describes the quark mixing. Since CKM matrix is almost diagonal [10], LFV cross sections are suppressed by the small off-diagonal elements of CKM matrix. It is shown below that LFV cross sections at linear collider experiments are hopelessly small in the minimal SUSY SU(5) model.

2
However, the minimal SUSY SU(5) model is no more than a calculable example. In fact, it cannot describe the whole known fermion masses and mixing. The above-mentioned equality of the down-type quark Yukawa couplings and the leptonic ones leads to inconsistent mass relation for the first and second generations, although it gives the celebrated bottom to tau mass ratio. Once we extend the model to overcome this insufficiency, the leptonic Yukawa couplings are no longer necessary to be the same as the down-type quark Yukawa couplings. Thus, the leptonic mixing is independent of CKM matrix in general.

In this paper, we show that LFV cross sections of charged slepton pair production and decay at linear collider experiments are sizable if the lepton mixing has some appropriate structures. In Sec. II, we describe our framework based on SUSY SU(5) GUT more explicitly. Our numerical results on present experimental constraints and LFV cross sections are presented in Sec. III. Sec. IV is devoted to concluding remarks.

II. FRAMEWORK

Apart from Yukawa sector, our working model is the minimal SUSY SU(5) GUT with the universal soft SUSY breaking terms at Planck scale, which detail is discussed in Ref. [3] in the context of LFV. The quarks and leptons are unified into three pairs of SU(5) chiral supermultiplets, 10 (\(T_i\)) and \(\bar{5}\) (\(\bar{F}_i\)), with their superpartners, where \(i = 1, 2, 3\) is a generation index. As for Higgs sector, we assume the minimal one, i.e. 24 (\(\Sigma\)), 5 (\(H\)) and \(\bar{5}\) (\(\bar{H}\)).

In the minimal model, Yukawa superpotential is given by

\[
W_0 = T_i f^T_{ij} T_j H + T_i f^F_{ij} \bar{F}_j \bar{H},
\]

where \(f^T\) is the Yukawa coupling matrix which gives up-type quark masses, and \(f^F\) is the one giving down-type quark and charged lepton masses. We can take \(f^T\) to be diagonal without loss of generality. Thus, the flavor mixing in the lepton sector as well as the quark sector is governed by \(f^F\) in the minimal model. As will be shown later, LFV cross sections of charged slepton pair production and decay are too small to be measured in this case.

However, as is mentioned in Sec. I, the above minimal model is known to predict incorrect mass relation for the first and the second generations. To be realistic, it is natural to extend Eq. (1). As a result, we expect that the leptonic Yukawa matrix is different from the down-type one. For instance, this is realized by introducing the following higher dimensional term that might be induced by gravity:

\[
W_1 = \frac{f'_{ij}}{M_{\text{Planck}}} F_i^{\alpha} \Sigma_{\alpha\beta} T_{j,\beta\gamma} \bar{H}^\gamma,
\]

where the Greek indices are SU(5) ones. Note that the effective Yukawa coupling \(f'\langle\Sigma\rangle/M_{\text{Planck}} \sim f'M_{\text{GUT}}/M_{\text{Planck}}\) could have the same order of magnitude as \(f^F\) due to the small masses of bottom and tau provided that \(f' \sim O(1)\) and \(\tan \beta = \langle H \rangle/\langle \bar{H} \rangle\) is not too large.

In the following, we do not discuss specific extensions like Eq. (2). Instead, we simply regard the leptonic mixing as independent from the quark mixing.

Lepton mass matrix at the weak scale, \(M_\ell\), is diagonalized as \(\bar{\ell}_L M_\ell \ell_R = \bar{\ell}_L D_\ell \ell_R\) by unitary transformations \(\ell_R = V_\ell \ell_R\) and \(\ell_L = U_\ell \ell_L\), where \(M_\ell = U_\ell D_\ell V_\ell^\dagger, D_\ell = \)
\( \text{diag}(m_e, m_\mu, m_\tau), \) \( e_{R,L} \) denote the gauge eigen states, \( \ell_{R,L} \) are the mass eigen states, and generation indices are suppressed. Making the same unitary transformations for corresponding sleptons, we obtain the following \( 6 \times 6 \) charged slepton mass matrix:

\[
\left( \tilde{\ell}^\dagger_L, \tilde{\ell}^\dagger_R \right) \left( \begin{array}{cc} m^2_L & m^2_{LR} \\ m^2_{LR} & m^2_R \end{array} \right) \left( \begin{array}{c} \tilde{\ell}_L \\ \tilde{\ell}_R \end{array} \right),
\]

(3)

where \( I = \text{diag.}(0,0,1), \) \( I' = \text{diag.}(0,0,1') \),

\[ m^2_L = \bar{m}^2_L1, \ m^2_R = \bar{m}^2_R1 - V^\dagger_e I V_e, \ m^2_{LR} = -D_e(A_e1 - \frac{1}{3}V^\dagger_e I' V_e + \mu \tan \beta 1), \]

Because of the degeneracy of the left-handed slepton soft masses, \( U_e \) does not appear in Eq. (3). Thus, LFV is controlled by a \( 3 \times 3 \) unitary matrix \( V_e \). It turns out that \( V_e \) contains only two parameters since, as seen in Eq. (3), the first and second generation right-handed slepton soft breaking parameters are the same. In addition, apparently no CP violating complex phase exists in \( V_e \). In the following analysis, we take absolute values of (3,1) and (3,2) elements of \( V_e \) as the parameters in \( V_e \). We denote them as \( |V_{31}| \) and \( |V_{32}| \).

### III. NUMERICAL RESULTS ON LFV

By diagonalizing Eq. (3) numerically, we calculate rates of several LFV processes as functions of \( |V_{31}| \) and \( |V_{32}| \). Masses and couplings of SUSY particles at the weak scale are determined through the renormalization group equations by giving the universal scalar mass \( (m_0) \), the GUT gaugino mass \( (M_0) \) and the universal \( A \) parameter \( (A_0) \) at Planck scale, in addition to sign of the supersymmetric Higgsino mass \( (\mu) \), \( \tan \beta \), \( |V_{31}| \) and \( |V_{32}| \) at the weak scale. For an illustrative purpose, we take \( m_0 = 100 \text{GeV}, \ M_0 = 150 \text{GeV} \ A_0 = 0 \), and \( \text{sign}(\mu) = +1 \). As for \( \tan \beta \), results for \( \tan \beta = 3 \) and 10 are shown. Top quark mass is assumed to be 175GeV. These input parameters are consistent with present experiments \[13\] other than LFV experiments discussed below for all possible values of \( |V_{31}| \) and \( |V_{32}| \). The lightest SUSY particle (LSP) is the lightest neutralino, which is almost Bino, with mass around 63GeV depending on \( \tan \beta \). Mass spectrum of the six charged sleptons is given as \((150,163,163,182,182,183)\text{GeV} \) for \( \tan \beta = 3 \) and \((149,164,164,183,183,190)\text{GeV} \) for \( \tan \beta = 10 \). The precise values of the charged slepton masses depend on \( V_e \) and the above values are obtained for \( V_e = 1 \). The lightest charged slepton, which production and decay with LFV is discussed below, decays mostly into a LSP and a charged lepton. A typical
considering a large tan β where we neglect terms other than the one proportional to $m$. Eq. (3): is 3 that these processes are complimentary to $\mu$ of scalar tau leptons. To see this, it is enough to consider the following processes. In Fig. 2, we find that the LFV cross sections are smaller for larger tan β ratios of $\tau$ are about 150fb, 80fb, and 280fb respectively. These values are realized in the case that $V_{33}$ is stronger because the $\tau \rightarrow e\gamma$ cross section in spite of the strong constraint because of different dependence on $\nu$. The allowed maximal LFV is of order 0.1fb if $V_3$ and beam polarization are assumed as Fig. 2(a). The maximal values of cross sections are about 150fb, 80fb, and 280fb respectively. These values are realized in the case that $|V_{32}| \ll |V_{31}| \sim |V_{33}|$. Note that the LFV cross section is of order 0.1fb if $V_e$ has a similar structure to CKM matrix as in the minimal SU(5) model.

Other lepton flavor violating combinations in the final state charged lepton pair give less interesting cross sections for both $e^+\nu$ and $e^-\nu$ collisions.

Fig. 3 shows the same quantities as Fig. 2 but for tan β = 10. Comparing Fig. 3 with Fig. 4, we find that the LFV cross sections are smaller for larger tan β. This means that these processes are complimentary to $\tau \rightarrow e\gamma$ which is enhanced for larger tan β.

The reduction of the LFV cross sections for larger tan β due to large left-right mixing of scalar tau leptons. To see this, it is enough to consider the following 3 × 3 submatrix of Eq. (3):

$$M^2 = \begin{pmatrix} m^2_L & (m^2_{LR})_{33} & (m^2_{LR})_{23} \\ (m^2_{LR})_{33} & m^2_R & (m^2_{LR})_{32} \\ (m^2_{LR})_{23} & (m^2_{LR})_{32} & m^2_R \end{pmatrix} \simeq \begin{pmatrix} m^2_L & 0 & -m_r \mu \tan \beta \\ 0 & m^2_R - IV^2_{32} & -IV^2_{32} + V^2_{33} \\ -m_r \mu \tan \beta & -IV^2_{32} + V^2_{33} & m^2_R - IV^2_{33} \end{pmatrix},$$

where we neglect terms other than the one proportional to $m_r \tan \beta$ in $m^2_{LR}$. Since we are considering a large tan β case, the left-right mixing angle of the scalar tau leptons is almost...
By making the 45 degree rotation in the 1–3 plane of $M^2$, we obtain a matrix closer to a diagonal form:

$$O_0^T M^2 O_0 = \begin{pmatrix}
\frac{m_1^2 + m_R^2 - IV_{32}^2}{2} + m_\tau |\mu| \tan \beta & -\frac{\mu}{\sqrt{2} |\mu|} IV_{32} V_{33} & \frac{\mu}{\sqrt{2} |\mu|} (m_L^2 - m_R^2 + IV_{33}^2) \\
-\frac{\mu}{\sqrt{2} |\mu|} IV_{32} V_{33} & \frac{m_R^2 - IV_{32}^2}{2} & IV_{32} V_{33} / \sqrt{2} \\
\frac{m_\tau^2 + m_R^2 - IV_{33}^2}{2} - m_\tau |\mu| \tan \beta & -\frac{\mu}{\sqrt{2} |\mu|} IV_{32} V_{33} & \frac{m_L^2 - m_R^2 + IV_{33}^2}{2}
\end{pmatrix},$$

(5)

where $O_0$ is the 45 degree rotation matrix:

$$O_0 = \begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & \frac{\mu}{|\mu| \sqrt{2}} \\
0 & 1 & 0 \\
-\frac{\mu}{|\mu| \sqrt{2}} & 0 & \frac{1}{\sqrt{2}}
\end{pmatrix}.$$

(6)

It is legitimate to diagonalize Eq. (5) perturbatively in order to see qualitative behavior of the slepton flavor mixing. As a result, we find that LFV related off-diagonal elements of the slepton mixing matrix are approximately proportional to

$$\sim \frac{I}{m_\tau |\mu| \tan \beta}$$

(7)

in the large $\tan \beta$ case. Thus, the LFV cross sections are suppressed as $\tan \beta$ becomes large.

IV. CONCLUDING REMARKS

Before concluding we discuss some background issues. Possible extensions of our calculation are also discussed here.

Our LFV signals are $\tau^\pm \ell^\mp + \text{missing}$ where $\ell$ denotes $e$ or $\mu$ for the $e^+e^-$ collision, and $\tau^- \ell^- + \text{missing}$ with $\ell = e$ or $\tau$ for the $e^-e^-$ collision. The tau leptons are identified with their hadronic decays. The pure leptonic decay modes can also be used with impact parameter analysis in principle. A CCD pixel vertex detector proposed for linear collider experiments has a typical resolution better than 10$\mu$m [15], while $c\tau$ of tau lepton is about 90$\mu$m [10].

The most serious standard model background in $e^+e^-$ collision is the one from $W$ boson pair production. A leptonic $W$ pair decay $WW \rightarrow \tau \ell \nu\nu$ ($\ell = e, \mu$) is a background event. $WW \rightarrow \tau \tau \nu\nu$ followed by a pure leptonic decay of one of the $\tau$’s can also be a background if the above-mentioned impact parameter analysis is not effective. However, these $WW$ backgrounds are reduced by employing a right-handed electron beam as we did in the above calculation. Appropriate kinematical cuts are also useful. With these procedures, the background cross section is reduced to less than $O(1)$fb with a reasonable efficiency of about 30 $\sim$ 50% for the signal [16,17]. $e\bar{\nu}W$ and $e\bar{e}WW$ could also be backgrounds of about 1fb [16].

$ZZ$, $Zh$, and $e\ell\tau\tau$ could also be backgrounds. They can be reduced to $O(1)$fb or less by appropriate selection cuts [16,17]. Moreover, the impact parameter analysis could reduce these modes to negligible levels.
As for the $e^-e^-$ collision, our LFV signals are essentially free from backgrounds. In particular, the right-handed beams reduce $e^-\nu W^-$ mode to a negligible level without any selection cuts [18].

Production of other superparticles could also be background. In particular, heavier slepton production makes LFV signals more complicated. To avoid it, we can choose such a $\sqrt{s}$ that only a pair of the lightest charged sleptons is created. Although the cross sections decrease, especially for the s-channel contribution in the $e^+e^-$ collision, we still have sizable cross sections both for the $e^+e^-$ and $e^-e^-$ collisions.

Our observations in the present work can also be applied to other SUSY GUTs qualitatively. For instance, the LFV cross sections are expected to be sizable in the SO(10) model. In this model, the left-handed sleptons, as well as the right-handed ones, cause LFV since the left-handed lepton supermultiplet of the third generation is unified into the same GUT multiplet as the top quark. Then, as is mentioned in Sec. I, the $\mu \rightarrow e\gamma$ rate is enhanced by a factor $\sim m_\tau/m_\mu$ because of the chirality-flip nature of this process. While $\tau \rightarrow \mu(e)\gamma$ is not enhanced. Although the detail depends on the model, especially its Yukawa superpotential, we expect a stronger constraint from $\mu \rightarrow e\gamma$ and similar constraints from $\tau \rightarrow \mu(e)\gamma$ compared with the SU(5) model. As can be seen in Fig. 2 and 3, a stronger $\mu \rightarrow e\gamma$ constraint alone does not exclude sizable LFV cross sections.

We also expect sizable LFV cross sections at a muon collider experiment [19]. The s-channel amplitude in $\mu^+\mu^-$ collision is the same as $e^+e^-$ case. While the t-channel one has different dependence on $V_e$. The lower initial state radiation of muon collider would make the threshold operation mentioned above more effective.

In conclusion, we have shown that the LFV phenomena through slepton pair production and decay could be detectable at linear collider experiments in the SUSY SU(5) GUT. Treating the lepton mixing matrix as a set of parameters independent of CKM matrix, we have discussed the constraints on it from the present experiments and calculated LFV cross sections. In spite of the stringent constraint from $\mu \rightarrow e\gamma$, some of LFV processes which have tau lepton(s) in the final state could have sizable cross sections at future linear collider experiments.
REFERENCES

[1] For a review, see e.g. H.P. Nilles, Phys. Rep. 110, 1 (1984).
[2] L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986).
[3] R. Barbieri and L.J. Hall, Phys. Lett. B 338, 212 (1994); R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B 445, 219 (1995).
[4] J. Hisano et al., Phys. Rev. D 53, 2442 (1996); Phys. Lett. B 391, 341 (1997), erratum ibid. 397, 357 (1997).
[5] R.D. Bolton et al., Phys. Rev. D 38, 2077 (1988).
[6] N. Arkani-Hamed et al., Phys. Rev. Lett. 77, 1937 (1996).
[7] N. Arkani-Hamed et al., Nucl. Phys. B 505, 3 (1997).
[8] N.V. Krasnikov, Mod. Phys. Lett. A 9, 791 (1994); Phys. Lett. B 388, 783 (1996).
[9] N.V. Krasnikov, JETP Lett. 65, 148 (1997); S.I. Bityukov and N.V. Krasnikov, preprint IFVE-97-67, hep-ph/9712358.
[10] Particle Data Group (R.M. Barnett et al.), Phys. Rev. D 54, 1 (1996).
[11] For a recent review on fermion masses and mixing in SUSY GUT, see e.g. Z. Berezhiani, in Proceedings of ICTP Summer School in High-energy Physics and Cosmology, Trieste, Italy, 1995, edited by E. Gava et al. (preprint INFN-FE 21/95, hep-ph/9602325).
[12] J. Ellis and M.K. Gaillard, Phys. Lett. 88B, 315 (1979).
[13] P. Janot, talk given at “International Europhysics Conference on High Energy Physics”, 19-26 August 1997, Jerusalem, Israel.
[14] CLEO collaboration, K.W. Edwards et al., Phys. Rev. D 55, 3919 (1997).
[15] C.J.S. Damerell and D.J. Jackson, in Proceedings of Workshop on Physics and Experiments with Linear Colliders, Morioka-Appi, Iwate, Japan (1995), edited by A. Miyamoto et al.
[16] R. Becker and C. Vander Velde, in Proceedings of the European Meeting of the Working Groups on Physics and Experiments at Linear $e^+ e^-$ Colliders, edited by P.M. Zerwas (Report No. DESY-93-123C).
[17] M.M. Nojiri, Phys. Rev. D 51, 6281 (1995).
[18] F. Cuypers, G. Jan van Oldenborgh and R. Rückl, Nucl. Phys. B 409, 128 (1993).
[19] H.-C. Cheng, preprint FERMILAB-CONF-97/418-T, hep-ph/9712427.
FIGURES

FIG. 1. Present experimental constraint on $|V_{31}|$ and $|V_{32}|$ from $\mu \to e\gamma$ search: (a) for $\tan\beta = 3$ and (b) for $\tan\beta = 10$. Br($\tau \to \mu(e)\gamma$) is also shown.

FIG. 2. LFV cross sections at $\sqrt{s} = 500\text{GeV}$ for $\tan\beta = 3$. The $\mu \to e\gamma$ constraint is also shown for comparison.

FIG. 3. The same as Fig. 2 except for $\tan\beta = 10$. 

FIG. 1
FIG. 2
FIG. 3