A Galois-Connection between Myers-Briggs’ Type Indicators and Szondi’s Personality Profiles

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Abstract
We propose a computable Galois-connection between Myers-Briggs’ Type Indicators (MBTIs), the most widely-used personality measure for non-psychiatric populations (based on C.G. Jung’s personality types), and Szondi’s personality profiles (SPPs), a less well-known but, as we show, finer personality measure for psychiatric as well as non-psychiatric populations (conceived as a unification of the depth psychology of S. Freud, C.G. Jung, and A. Adler). The practical significance of our result is that our Galois-connection provides a pair of computable, interpreting translations between the two personality spaces of MBTIs and SPPs: one concrete from MBTI-space to SPP-space (because SPPs are finer) and one abstract from SPP-space to MBTI-space (because MBTIs are coarser). Thus Myers-Briggs’ and Szondi’s personality-test results are mutually interpretable and inter-translatable, even automatically by computers.

Keywords: applied order theory, computational and mathematical psychology, depth psychology, machine translation, MBTI, personality tests.

1 Introduction
According to [8, Page xxi and 210], the Myers-Briggs Type Indicator (MBTI) [7], based on C.G. Jung’s personality types [3], has become “the most widely-used personality measure for non-psychiatric populations” and “the most extensively used personality instrument in history” with over two million tests taken per year. In this paper, we propose a computable Galois-connection [2] between MBTIs and Szondi’s personality profiles (SPPs) [9], a less well-known but, as we show, finer personality measure for psychiatric as well as non-psychiatric populations, and conceived as a unification [10] of the depth psychology of S. Freud, C.G. Jung, and A. Adler.

Our result is a contribution to mathematical psychology in the area of depth psychology, which does not yet seem to have been explored with mathematical means besides those of statistics (often not part of mathematics departments). It is also meant as a contribution towards practicing psychological research with
the methods of the exact sciences, for obvious ethical reasons. The practical significance of our result is that our Galois-connection provides a pair of efficiently computable, interpreting translations between the two personality spaces of MBTIs and SPPs (and thus hopefully also between their respective academic and non-academic communities): one concrete translation from MBTI-space to SPP-space (because SPPs are finer than MBTIs) and one abstract translation from SPP-space to MBTI-space (because MBTIs are coarser than SPPs). Thus Myers-Briggs’ and Szondi’s personality-test results are mutually interpretable and inter-translatable, even automatically by computers. The only restriction to this mutuality is the subjective interpretation of the faithfulness of these translations. In our interpretation, we intentionally restrict the translation from SPP-space to MBTI-space, and only that one, in order to preserve (our perception of) its faithfulness. More precisely, we choose to map some SPPs to the empty set in MBTI-space (but every MBTI to a non-empty set in SPP-space). Our readers can experiment with their own interpretations, as we explain below.

We stress that our Galois-connection between the spaces of MBTIs and SPPs is independent of their respective test, which evaluate their testees in terms of structured result values—the MBTIs and SPPs—in the respective space. Both tests are preference-based, more precisely, test evaluation is based on choices of preferred questions in the case of the MBTI-test [7] and on choices of preferred portraits in the case of the Szondi-test [9, 6]. Due to the independence of our Galois-connection from these tests, their exact nature need not concern us here. All what we need to be concerned about is the nature of the structured result values that these tests generate. (Other test forms can generate the same form of result values, e.g. [3].) We also stress that our proposed Galois-connection is what we believe to be an interesting candidate brain child for adoption by the community, but that there are other possible candidates, which our readers are empowered to explore themselves. In fact, not only do we propose a candidate Galois-connection between MBTI-space and SPP-space, but also do we propose a whole methodology for generating such candidates. All what readers interested in generating such connections themselves need to do is map their own intuition about the meaning of MBTIs to a standard interlingua, called Logical Pivot Language (LPL) here, and check that their mapping has a single simple property, namely the one stated as Fact 2.1 about our mapping i in Figure 1. Their desired Galois-connection is then automatically induced jointly by their chosen mapping and a mapping, called p here, from SPP-space to LPL that we choose once and for all possible Galois-connections of interest. What is more, our methodology is applicable even more generally to the generation of Galois-connections between pairs of result spaces of other personality tests. SPPs just happen to have a finer structure than other personality-test values that we are aware of, and so are perhaps best suited to play the distinguished role of explanatory semantics for result values of other personality tests. Of course our readers are still free to choose their own preferred semantic space.

An SPP can be conceived as a tuple of eight, so-called signed factors whose signatures can in turn take twelve values. So SPPs live in an eight-dimensional space. On the other hand, an MBTI can be conceived as a quadruple of two-
valued components, namely, first, extro-/introversion, second, perception, being either sensing or intuition, third, judgment, being either thinking or feeling, and fourth, a dominance flag, indicating either a dominance of perception or judgment. So MBTIs live in a coarser, four dimensional space. Hence the translation from SPPs to MBTIs must be a projection (and thus surjection) of SPP-space onto MBTI-space. An insight gained in the finer referential system of SPPs is that MBTIs turn actually out to be non-orthogonal or not independent, contrary to common belief [7, 8]. Of course our readers are still free to disagree on the value of this insight by giving a convincing argument for why SPP-space would be an inappropriate semantics for MBTI-space. After all, Szondi conceived his theory of human personality as a unifying theory that also includes Jung’s theory, on which MBTI-theory is based. We now put forward our own argument for why we believe SPP-space is indeed an appropriate—though surely not the only—semantics for MBTI-space. In Section 2.1, we present the defining mathematical structures for each space, and in Section 2.2, the defining mathematical mappings for their translation. No prior knowledge of either MBTIs or SPPs is required to appreciate the results of this paper.

2 The connection

In this section, we present the defining mathematical structures for MBTI-space, the interlingua LPL, and SPP-space, as well as the defining mathematical mappings for the concrete translation of MBTI-space to SPP-space and the abstract translation of SPP-space back to MBTI-space, both via LPL, see Figure 1.

2.1 Structures

In this section, we present the defining mathematical structures for MBTI-space, the interlingua LPL, and SPP-space. We start with defining MBTI-space.

**Definition 1** (The Myers-Briggs Type Indicator Space). Let

\[ MB = \{E, I, F, T, N, S, J, P\} \]

be the set of basic type indicators, with E meaning “extroversion,” I “introversion,” F “feeling,” T “thinking,” N “intuition,” S “sensing,” J “judging,” and P
be the set of Myers-Briggs Type Indicators (MBTIs) \[7, 8\].

Then,
\[\text{MBTI} = \langle 2^{\text{MBTI}}, \emptyset, \cap, \cup, \text{MBTI}, \subseteq \rangle\]
defines our Myers-Briggs Type Indicator Space, that is, the (inclusion-ordered, Boolean) powerset algebra \([2]\) on MBTI (the set of all subsets of MBTI).

Note that we do need to define \(\text{MBTI}\) as the set of all subsets of MBTI and not simply as the set of all elements of MBTI. The reason is the aforementioned fact that in the finer referential system of SPP-space (see Definition 2), MBTIs turn out to be non-orthogonal or not independent, and thus an MBTI may have to be mapped to a proper set of SPPs (see Table 2). So the proper setting for SPP-space is a set of subsets of SPPs, which in turn, via the backward translation from SPP-space to \(\text{MBTI}\), means that the proper setting for \(\text{MBTI}\), as the target of a mapping of subsets, is also a set of subsets. Further, notice that the MBTI-test \([2]\), which as previously mentioned requires answering to questions, actually requires the \(P\)-faculty (perception of the question), the \(N\)-faculty (intuition in the sense of textual, and thus symbolic understanding), and the \(J\)-faculty (judgment in the sense of choice of and decision about an answer) as its own prerequisites. Incidentally, the concept of choice is the key concept in Szondi’s depth-psychological fate analysis \([11, 10]\), which is the background theory for his test \([9]\) and the SPPs that it generates.

We continue to define SPP-space.

**Definition 2 (The Szondi Personality Profile Space).** Let us consider the Hasse-diagram \([2]\) in Figure 2 of the partially ordered set of Szondi’s twelve signatures \([9]\) of human reactions, which are:

- approval: from strong +!!!, +!, and + to weak +;
- indifference/neutrality: 0;
- rejection: from weak −, −!, and −!! to strong −!!!; and
- ambivalence: ± (approval bias), ± (no bias), and ± (rejection bias).

(Szondi calls the exclamation marks in his signatures *quanta.*)

Further let us call this set of signatures \(\mathcal{S}\), that is,
\[ \mathcal{S} = \{-!!!, -!!, -!, -, 0, +, +!, +!!, \pm, \pm, \pm!\}. \]

Now let us consider Szondi’s eight factors and four vectors of human personality \([9]\) as summarised in Table 1 (Their names are of clinical origin and need not concern us here.) And let us call the set of factors \(\mathcal{F}\), that is,
\[ \mathcal{F} = \{h, s, e, hy, k, p, d, m\}. \]
Then,

\[ SPP = \{ ((h, s_1), (s, s_2), (e, s_3), (hy, s_4), (k, s_5), (p, s_6), (d, s_7), (m, s_8)) \mid s_1, \ldots, s_8 \in S \} \]

is the set of Szondi’s personality profiles, and

\[ SPP = (2^{SPP}, \emptyset, \cup, SPP, \cap, \subseteq) \]

defines our Szondi Personality Profile Space, that is, the (inclusion-ordered, Boolean) powerset algebra on SPP (the set of all subsets of SPP).

As an example of an SPP, consider the norm profile for the Szondi-test:

\[ ((h, +), (s, +), (e, -), (hy, -), (k, -), (p, -), (d, +), (m, +)) \]

Spelled out, this norm profile describes the personality of a human being who approves of physical love, has a proactive attitude, has unethical but moral behaviour, wants to have and be less, and is unfaithful and dependent.

We conclude this subsection with the definition of our interlingua LPL.

**Definition 3 (The Logical Pivot Language).** Let

\[ \mathcal{A} = \{ hs_1, ss_2, es_3, hy s_4, ks_5, ps_6, ds_7, ms_8 \mid s_1, \ldots, s_8 \in S \} \]
Table 1: Szondi’s factors and vectors

| Vector | Factor | Signature       |
|--------|--------|-----------------|
|        | h (love) | physical love   |
|        | s (attitude) | (proactive) activity   |
|        | e (ethics) | ethical behaviour |
|        | hy (morality) | immoral behaviour |
|        | k (having) | having more |
|        | p (being) | being more |
|        | d (relations) | unfaithfulness |
|        | m (bindings) | dependence |

be our set of atomic logical formulas, and \( \text{LPL}(\mathbb{A}) \) the classical propositional language over \( \mathbb{A} \), that is, the set of sentences constructed from the elements in \( \mathbb{A} \) and the classical propositional connectives \( \neg \) (negation, pronounced “not”), \( \wedge \) (conjunction, pronounced “and”), \( \vee \) (disjunction, pronounced “or”), etc.

Then, \( \mathcal{LPL} = \langle \text{LPL}(\mathbb{A}), \Rightarrow \rangle \) defines our logical pivot language, with \( \Rightarrow \) being logical consequence.

Logical equivalence \( \equiv \) is defined in terms of \( \Rightarrow \) such that for every \( \phi, \varphi \in \text{LPL}(\mathbb{A}) \), \( \phi \equiv \varphi \) by definition if and only if \( \phi \Rightarrow \varphi \) and \( \varphi \Rightarrow \phi \).

### 2.2 Mappings between structures

In this section, we present the defining mathematical mappings for the concrete translation \( \flat \) of \( \text{MBTI} \) to \( \text{SPP} \) via \( \mathcal{LPL} \) and the abstract translation \( \flat \) of \( \text{SPP} \) back to \( \text{MBTI} \) again via \( \mathcal{LPL} \) by means of the auxiliary mappings \( i \) and \( p \). We also prove that the ordered pair \( (\flat, \flat) \) is a Galois-connection, as promised.

**Definition 4** (Mappings). Let the mapping (total function)

- \( i \) be defined in the function space \( (\text{MBTI} \rightarrow \text{LPL}(\mathbb{A})) \) as in Table 2 and in the function space \( (2^{\text{MBTI}} \rightarrow \text{LPL}(\mathbb{A})) \) such that for every \( I \in 2^{\text{MBTI}} \),

\[
i(I) = \bigwedge \{ i(i) \mid i \in I \};
\]

- \( p \) be defined in the function space \( (\text{SPP} \rightarrow \text{LPL}(\mathbb{A})) \) such that

\[
p((h, s_1), (s, s_2), (e, s_3), (hy, s_4), (k, s_5), (p, s_6), (d, s_7), (m, s_8)) =
hs_1 \wedge ss_2 \wedge es_3 \wedge hys_4 \wedge ks_5 \wedge ps_6 \wedge ds_7 \wedge ms_8
\]

and in the function space \( (2^{\text{SPP}} \rightarrow \text{LPL}(\mathbb{A})) \) such that for every \( P \in 2^{\text{SPP}} \),

\[
p(P) = \bigvee \{ p(p) \mid p \in P \}.
\]
Table 2: Translating MB and MBTI to LPL(\(A\))

| \(i(E)\) | \(hy+ \lor hy+! \lor hy+!! \lor hy+!!! \lor hy^1\) |
| \(i(I)\) | \(hy- \lor hy-! \lor hy-!! \lor hy-!!! \lor hy^1\) |
| \(i(F)\) | \((h+ \lor h\pm \lor h\pm^1) \land (p- \lor p\pm \lor p^1)\) |
| \(i(T)\) | \(k- \lor k\pm \lor k^1\) |
| \(i(N)\) | \((k+ \lor k\pm \lor k\pm^1) \land (p+ \lor p\pm \lor p^1)\) |

\[
i(E) = i(I) \land i(S) \land i(T)
i(ISFJ) = i(I) \land i(N) \land i(F)
i(INTJ) = i(I) \land i(T) \land i(T)
i(ISTP) = i(I) \land i(S) \land i(S)
i(ISFP) = i(I) \land i(S) \land i(F)
i(INFP) = i(I) \land i(N) \land i(F)
i(INTP) = i(I) \land i(N) \land i(T)
i(ESTP) = i(E) \land i(S) \land i(S)
i(ESFP) = i(E) \land i(S) \land i(F)
i(ENFP) = i(E) \land i(N) \land i(F)
i(ENTP) = i(E) \land i(N) \land i(T)
i(ESTJ) = i(E) \land i(S) \land i(T)
i(ESFJ) = i(E) \land i(S) \land i(F)
i(ENFJ) = i(E) \land i(N) \land i(F)
i(ENTJ) = i(E) \land i(N) \land i(T)\]
Then, the mapping

- $\circ : MBTI \to SPP$ defined such that for every $I \in 2^{MBTI}$,

$$I^\circ = \{ p \in SPP \mid p(p) \Rightarrow i(I) \}$$

is the so-called right polarity and

- $\triangleleft : SPP \to MBTI$ defined such that for every $P \in 2^{SPP}$,

$$P^\triangleleft = \{ i \in MBTI \mid p(P) \Rightarrow i(i) \}$$

is the so-called left polarity of the ordered pair ($\circ, \triangleleft$).

Spelled out, (1) the result of applying the mapping $i$ to a set $I$ of MBTIs $i$ as defined in Definition 4 is the conjunction of the results of applying $i$ to each one of these $i$ as defined in Table 2; (2) the result of applying the mapping $p$ to a set $P$ of SPPs $p$ as defined in Definition 4 is the disjunction of the results of applying $p$ to each one of these $p$, which simply is the conjunction of all signed factors in $p$ taken each one as an atomic proposition; (3) the result of applying the mapping $\circ$ to a set $I$ of MBTIs is the set of all those SPPs $p$ whose mapping under $i$ implies the mapping of $I$ under $i$; (4) the result of applying the mapping $\triangleleft$ to a set $P$ of SPPs is the set of all those MBTIs $i$ whose mapping under $i$ is implied by the mapping of $P$ under $p$. Thus from a computer science perspective [2, Section 7.35], MBTIs are specifications of SPPs and SPPs are implementations or refinements of MBTIs. The Galois-connection then connects correct implementations to their respective specification by stipulating that a correct implementation imply its specification. By convention, $\bigwedge \emptyset = \top$ and $\bigvee \emptyset = \bot$, that is, the conjunction over the empty set $\emptyset$ is tautological truth $\top$, and the disjunction over $\emptyset$ is tautological falsehood $\bot$, respectively.

Note that an example of an SPP that maps to the empty set under $\triangleleft$ happens to be the Szondi norm profile mentioned before, because its mapping under $p$ does not contain any (individually) dominant factor (all factors are simply either positive or negative, and thus are without quanta). So it does not imply the mapping of any MBTI under $i$, which requires a dominant factor, as we have alluded to in the introduction with the dominance flag, and are going to explain now by spelling out the translation of MBTIs to SPPs, see Table 2. There, dominance is indicated by a quantum subscript of the translation $i$. As can be seen, $J$ and $P$ merely act as flag values, which indicate which one of the judgment or perception faculties is the dominant faculty for dealing with the outer world, (that is, the observably dominant faculty) and this as a function of extro-/introversion. The rule is that extroverts do show their dominant faculty for dealing with the outer world, whereas introverts do not [8 Page 14]. So with judging ($J$) introverts (I), a perceptive faculty shows up as dominant, with perceiving ($P$) introverts (I), a judging faculty, with perceiving ($P$) extroverts (E), a
perceptive faculty, and with judging (J) extroverts (E), a judging faculty. As can be seen in Table 2 our interpretation of extroversion is a positive tendency of Szondi’s factor hy (immoral behaviour: being seen, showing off). Whereas our interpretation of introversion is a negative tendency thereof (moral behaviour: seeing, hiding). Our readers might want to stipulate additional constraints for their own interpretation, but must take care not to create conflicts. Consistency is a necessary condition for the faithfulness of the translation i! (It is also one for the faithfulness of the translation p, but there it is obvious.)

**Fact 1** (Consistency of the translation i).

1. For every \( b \in \{E, I\} \) and \( b' \in MEB \setminus \{E, I\} \), \( i(b) \land i(b') \not\equiv \bot \).

2. For every \( b \in \{F, T\} \),
   - \( i(b) \land i(N) \not\equiv \bot \) and \( i(b) \land i(S) \not\equiv \bot \);
   - \( i(b) \land i(N) \not\equiv \bot \) and \( i(b) \land i(S) \not\equiv \bot \).

**Proof.** By inspection of Table 2.

Our interpretations of the remaining faculties follow a simple generating pattern:

- for a non-dominant positive factor \( f \), the pattern is \( f \lor f^+ \lor f^+! \lor f^{!!} \lor f^{!!!} \);
- for a non-dominant negative factor \( f \), it is \( f^\lor f^+ \lor f^+! \lor f^{!!} \lor f^{!!!} \);
- for a dominant positive factor \( f \), it is \( f^\lor f^+ \lor f^+! \lor f^{!!} \lor f^{!!!} \);
- for a dominant negative factor \( f \), it is \( f^\lor f^+ \lor f^+! \lor f^{!!} \lor f^{!!!} \).

As can be seen in Table 2 our interpretation of feeling is the conjunction of personal warmth (h+) and empathy (p−), which [7] stipulate as the characteristic properties of this faculty. Our interpretation of thinking is simply its corresponding factor (k−) in Szondi’s system. Our interpretation of the two perceptive faculties contains as a conjunct the factor (k+), which corresponds to perception in Szondi’s system. Our interpretation of intuition then further contains its corresponding factor (p+) in Szondi’s system. Finally, our interpretation of sensing further contains the disjunction of all those factors that correspond to the human senses in Szondi’s system, namely: touching (h+), hearing (e−), seeing (hy−), smelling (d+), and tasting (m+).

Our readers might be interested in comparing our interpreting mapping of MBTIs with D.W. Keirsey’s [4]: for him, ISTJ maps to the Inspector, ISFJ to the Counselor, INFJ to the Mastermind, ISTP to the Crafter, ISFP to the Composer, INFP to the Healer, INTP to the Architect, ESTP to the Promoter, ESFP to the Performer, ENFP to the Champion, ENTJ to the Inventor, ESTJ to the Supervisor, ESFJ to the Provider, ENFJ to the Teacher, and ENTP to the Fieldmarshal.

We now prove in two intermediate steps that the ordered pair (\( \hat{\circ}, \hat{\circ} \)) is indeed a Galois-connection. The first step is the following announced fact, from
which the second step, Lemma 1, follows, from which in turn the desired result, Theorem 1, then follows—easily. As announced, all that our readers need to check on their own analog of our mapping \( i \) is that it has the property stated as Fact 2.1. Their own Galois-connection is then automatically induced.

**Fact 2** (Some facts about \( i \) and \( p \)).

1. if \( I \subseteq I' \) then \( i(I') \Rightarrow i(I) \)
2. if \( P \subseteq P' \) then \( p(P) \Rightarrow p(P') \)
3. The functions \( i \) and \( p \) are injective but not surjective.

**Proof.** By inspection of Definition 4 and Table 2.

We need Fact 2.1 and 2.2 but not Fact 2.3 in the following development. Therefore, note the two macro-definitions \( \triangleleft \triangleleft := \triangleleft \circ \triangleleft \) and \( \triangleleft \triangleleft := \triangleleft \circ \triangleleft \) with \( \circ \) being function composition, as usual (from right to left, as usual too).

**Lemma 1** (Some useful properties of \( \triangleright \triangleright \) and \( \triangleright \triangleright \)).

1. if \( I \subseteq I' \) then \( I'^{\triangleright \triangleright} \subseteq I^{\triangleright \triangleright} \) (\( \triangleright \triangleright \) is antitone)
2. if \( P \subseteq P' \) then \( P^{\triangleright \triangleright} \subseteq P^{\triangleright \triangleright} \) (\( \triangleright \triangleright \) is antitone)
3. \( P \subseteq (P^{\triangleright \triangleright})^{\triangleright \triangleright} \) (\( \triangleright \triangleright \) is inflationary)
4. \( I \subseteq (I^{\triangleright \triangleright})^{\triangleright \triangleright} \) (\( \triangleright \triangleright \) is inflationary)

**Proof.** For (1), let \( I, I' \in 2^{\text{MBTI}} \) and suppose that \( I \subseteq I' \). Hence \( i(I') \Rightarrow i(I) \) by Fact 2.1. Further suppose that \( p \in I^{\triangleright \triangleright} \). By definition, \( I^{\triangleright \triangleright} = \{ p \in \text{SPP} \mid p(p) \Rightarrow i(I) \} \). Hence \( p(p) \Rightarrow i(I') \). Hence \( p(p) \Rightarrow i(I) \) by transitivity. By definition, \( I^{\triangleright \triangleright} = \{ p \in \text{SPP} \mid p(p) \Rightarrow i(I) \} \). Hence \( p \in I^{\triangleright \triangleright} \). Thus \( I^{\triangleright \triangleright} \subseteq I^{\triangleright \triangleright} \).

For (2), let \( P, P' \in 2^{\text{SPP}} \) and suppose that \( P \subseteq P' \). Hence \( p(P) \Rightarrow p(P') \) by Fact 2.2. Further suppose that \( i \in P^{\triangleright \triangleright} \). By definition, \( P^{\triangleright \triangleright} = \{ i \in \text{MBTI} \mid p(P') \Rightarrow i(i) \} \). Hence \( p(P') \Rightarrow i(i) \). Hence \( p(P) \Rightarrow i(i) \) by transitivity. By definition, \( P^{\triangleright \triangleright} = \{ i \in \text{MBTI} \mid p(P) \Rightarrow i(i) \} \). Hence \( i \in P^{\triangleright \triangleright} \). Thus \( P^{\triangleright \triangleright} \subseteq P^{\triangleright \triangleright} \).

For (3), consider:

1. \( p \in P \) hypothesis
2. \( \{p\} \subseteq P \) 1
3. \( p(p) \Rightarrow p(P) \) 2, Fact 2.2
4. \( p(p) \) is true hypothesis
5. \( p(P) \) is true 3, 4
6. \( \phi \in \{ i(i) \mid p(P) \Rightarrow i(i) \} \) hypothesis
7. there is \( i \) s.t. \( \phi = i(i) \) and \( p(P) \Rightarrow i(i) \) 6
8. \( \phi = i(i) \) and \( p(P) \Rightarrow i(i) \) hypothesis
9. \( p(P) \Rightarrow i(i) \) 8
10. \( i(i) \) is true 5, 9
11. \( \phi = i(i) \)
12. \( \phi \) is true 10, 11
13. \( \phi \) is true 7, 8–12
14. for every \( \phi \in \{i(i) \mid p(P) \Rightarrow i(i)\}, \phi \) is true 6–13
15. \( \forall \{i(i) \mid p(P) \Rightarrow i(i)\} \) is true 14
16. \( \forall \{i(i) \mid i \in \text{MBTI} \mid p(P) \Rightarrow i(i)\} \) is true 15
17. \( \forall \{i(i) \mid i \in P^c\} \) is true 16
18. \( i(P^c) \) is true 17
19. \( p(p) \Rightarrow i(P^c) \)
20. \( p \in \{p \in \text{SPP} \mid p(p) \Rightarrow i(P^c)\} \)
21. \( p \in (P^c)^p \)
22. \( P \subseteq (P^c)^p \) 1–21.

For (4), consider:

1. \( i \in I \) hypothesis
2. \( \{i\} \subseteq I \) 1
3. \( i(I) \Rightarrow i(i) \) 2, Fact 2.1
4. \( p(P^c) \) is true hypothesis
5. \( \forall \{p(p) \mid p \in P^c\} \) is true 4
6. \( \forall \{p(p) \mid p \in \{p \in \text{SPP} \mid p(p) \Rightarrow i(I)\}\} \) is true 5
7. \( \forall \{p(p) \mid p(p) \Rightarrow i(I)\} \) is true 6
8. there is \( p \) s.t. \( p(p) \Rightarrow i(I) \) and \( p(p) \) is true 7
9. \( p(p) \Rightarrow i(I) \) and \( p(p) \) is true hypothesis
10. \( i(I) \) is true 9
11. \( i(i) \) is true 3, 10
12. \( i(i) \) is true 8, 9–11
13. \( p(P^c) \Rightarrow i(i) \) 4–12
14. \( i \in \{i \in \text{MBTI} \mid p(P^c) \Rightarrow i(i)\} \)
15. \( i \in (P^c)^a \) 13
16. \( I \subseteq (P^c)^a \) 1–15.

\[ \square \]

We are ready for making the final step.
Theorem 1 (The Galois-connection property of \((\triangledown, \triangleleft)\)). The ordered pair \((\triangledown, \triangleleft)\) is an antitone or order-reversing Galois-connection between \textsc{MBTI} and \textsc{SPP}. That is, for every \(I \in 2^{\textsc{MBTI}}\) and \(P \in 2^{\textsc{SPP}}\),
\[
P \subseteq P^\circ \text{ if and only if } I \subseteq P^\triangledown.
\]

Proof. Let \(I \in 2^{\textsc{MBTI}}\) and \(P \in 2^{\textsc{SPP}}\). Suppose that \(P \subseteq I^\circ\). Hence \((I^\circ)^\circ \subseteq P^\circ\) by Lemma 1.2. Further, \(I \subseteq (I^\circ)^\circ\) by Lemma 1.4. Hence \(I \subseteq P^\circ\) by transitivity. Conversely suppose that \(I \subseteq P^\circ\). Hence \((P^\circ)^\circ \subseteq I^\circ\) by Lemma 1.1. Further, \(P \subseteq (P^\circ)^\circ\) by Lemma 1.3. Hence \(P \subseteq I^\circ\). \(\square\)

Thus from a computer science perspective [2, Section 7.35], smaller (larger) sets of MBTIs and thus less (more) restrictive specifications correspond to larger (smaller) sets of SPPs and thus more (fewer) possible implementations.

Note that Galois-connections are connected to residuated mappings [1]. Further, natural notions of equivalence on \textsc{MBTI} and \textsc{SPP} are given by the kernels of \(\triangledown\) and \(\triangleleft\), respectively, which are, by definition:
\[
I \equiv I' \text{ if and only if } I^\triangledown = I'^\triangledown;
\]
\[
P \equiv P' \text{ if and only if } P^\circ = P'^\circ.
\]

Proposition 1 (The efficient computability of \((\triangledown, \triangleleft)\)).

1. Given \(I \in 2^{\textsc{MBTI}}\), \(I^\circ\) is efficiently computable.
2. Given \(P \in 2^{\textsc{SPP}}\), \(P^\circ\) is efficiently computable.

Proof. Even a relatively brute-force approach is feasible with today’s laptop computers, which have a (high-speed) random access memory of several giga (\(10^9\)) bytes and a processor power of several giga instructions per second. Moreover in psychological practice, the sets \(I\) and \(P\) will usually be singletons of either only one type indicator or only one personality profile, respectively.

- Given \(I \in 2^{\textsc{MBTI}}\) (thus \(0 \leq |I| \leq 16\)), we precompute the list of all SPPs, which contains \(16^8\) entries, once and for all \(I\), and store the list on a computer hard disk and then load it into the (fast) random access memory of the computer. Then we model-check the formula \(i(I)\), which can also be precomputed, against each entry \(p\) in the list. That is, we check whether a given model \(p\) satisfies (makes true) the given \(i(I)\), and collect up into the result set all those \(p\) that do satisfy \(i(I)\). It is well-known that model-checking a propositional formula, such as \(i(I)\), takes only a polynomial number of computation steps in the size of the formula, and thus only a polynomial number in the size of \(I\) (being at most 16).

Of course, this computation can be done once and for all of the \(2^{16}\) possible sets \(I\), and the results stored on a hard disk for faster, later look-up.

- Given \(P \in 2^{\textsc{SPP}}\) (thus \(0 \leq |P| \leq 16^8\)), we model-check each \(p\) in \(P\) against the (pre-computable) mapping \(i(i)\) of each \(i\) of the 16 MBTIs, and collect up into the result set all those \(i\) whose mapping \(i(i)\) satisfies \(p\).
Of course, optimisations of the computation procedure given in the previous proof are possible, but we consider them as not sufficiently interesting implementation details.

3 Conclusion

We have proposed a computable Galois-connection between Myers-Briggs Type Indicators and Szondi’s personality profiles as promised in the abstract. In addition, we have proposed a simple methodology for generating other such Galois-connections, including Galois-connections not only between this pair of spaces of personality-test result values but also between other such pairs.

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