The influence of the vertical cantledge mass on the vibratory drum vibrations during the material compaction

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Abstract. The road roller vibratory drum is considered as a three-mass vibration system consisting of the eccentric weight mass $m_1$, vibratory drum mass $m_2$, additional mass $m_3$, which is transmitted to the vibratory drum axis through the roller frame. Differential equations of vertical vibrations were formulated, they were solved, analytical expressions were obtained of the amplitude $A_1$ of the vibrator drum vertical vibrations and of the amplitude $A_2$ of the additional mass $m_3$ vibrations, rigidly connected with the road roller frame. The dependences of the vertical vibration amplitudes $A_1$ and $A_2$ on the vibration frequency, mass ratio $(m_1 + m_2)$ and $m_3$, stiffness factors and viscous friction factors of the soil and rubber-metal shock absorbers have been established. The additional mass $m_3$ is considered as an additional shock absorber that protects the roller frame from vibration. The differential equation of the additional mass $m_3$ vertical vibrations has been obtained, which are excited by the harmonic inertia force. The analytical dependence conclusion for the determination of the dynamic factor $K_d$ as the ratio of the dynamic and static vibration amplitudes of the additional mass $m_3$ of the vibratory drum have been specified. The specified dependence of the driving force transfer factor $K_f$ determination on mass $m_3$ to the roller frame has been obtained. The diagrams of the dependence of $K_d$ and $K_f$ factors on the frequency ratio of forced and natural vibrations of the mass $m_3$ are constructed.

Key-words: vibratory drum, dynamic processes, differential equations, amplitudes of vertical vibrations, vibration protection.

1. Introduction

A modern vibration roller is an example of the beneficial use of vibration in technological processes of the material and soil compaction. The vibratory drum vertical vibration is combined with complex and energy-consuming dynamic processes of the mechanical energy transmission from the vibratory drum to the compacted material, in this case from the vibration from the vibratory drum is transmitted through rubber-metal shock absorbers to the roller frame and power units. The vibration roller studies take place in areas of improving the design, increasing quality and productivity of the compaction process, creating the intelligent control computer systems of the material and soil compaction modes by vibratory rollers etc. Fundamental studies of the vibratory roller interaction with deformable soil were performed by Russian scientists in the 1930’s-60’s.

In modern studies [1] the authors consider the influence of the soil mass, which is involved in the vibration process during its compaction, the vibratory drum and vibratory drum frame masses are taken into account.

In paper [2] the authors propose the compaction optimization method from the point of view of the process energy efficiency and productivity increase. The idea of the work consists in the taking into account the interaction of the compacted material layer with the vibratory drum.
A large number of studies is devoted to the compaction mode control, creation of computer intelligent control systems based on the analysis of a large number of factors characterizing dynamic processes [3–5].

Despite the huge number of publications devoted to vibration roller [6-11], there are no studies relative to the influence of the vibratory drum on the material compaction working process and roller frame protection from vibration.

In the reviewed papers the relationships of the vibratory drum vibration amplitudes and cantledge additional mass with the vibratory drum frame are not studied. There are no studies of the vibration roller frame vibration from the vibratory roller vibrations.

2. Problem statement

At present the vibratory drum vibration theory cannot explain many of the dynamic processes accompanying the vibratory drum working process.

The road roller vibratory drum of a medium type possessing the mass of 5-10 tons, performs vertical vibrations of the limited amplitude $A_1 = 1÷3$ mm with a frequency of up to $f = 80$ Hz. In the technical documentation companies – manufacturer indicate the total mass of the road roller and distribution of this mass along the axes. However, the mass falling on the front vibratory drum includes the own mass of the vibratory drum, consisting of the shell mass, mass of shafts and hydraulic motors located inside the vibratory drum and additional mass transmitted vertically from the roller frame, where mechanisms and power unit are installed. The additional mass $m_3$ transmitted to the vibratory drum is connected by rubber-metal shock absorbers with the roller frame. In this paper the task of studying the influence of the additional mass, which is transmitted through the roller frame to the vibratory drum during the material and soil compaction.

3. Theory

The vibratory drum shown in Figure 1 consists of vibration exciter $I$ mounted on the shaft inside vibratory drum $2$. The front frame is supported at the vibratory drum axis through which an additional vertical cantledge is transmitted on the vibratory drum from the roller mass in the form of conditional body $3$ with the mass $m_3$, which is connected to the drum by rubber-metal shock absorbers.

![Figure 1. Simulation model of the vibratory drum interaction with the soil and roller frame.](image)

The compacted medium is also an element of the mechanical system, which is characterized by the stiffness and viscous friction factor. The masses considered in Figure 1 perform vertical vibrations and ensure the material compaction over which the vibratory drum rolls.

With a discrepancy of less than 1% the sum of the masses $(m_1 + m_2)$ can be considered as a single mass with the generalized coordinate $z_1$ and mass $m_3$ having the generalized coordinate $z_2$ in the coordinate system $Oyz$ (see Figure 1).
The mechanical properties of the material to be compacted are characterized by the stiffness factor $c_1$ and viscous friction factor $b_1$, rubber-metal shock absorbers are characterized by parameters $c_2$ and $b_2$ respectively.

The mechanical system is affected by the radial harmonic force of the vibration exciter, which creates the vertical driving harmonic force on the vibratory drum determined according to the formula

$$P_{d0} = P_0 \sin pt = m_1 r_1 p^2 \sin pt,$$

where $m_1$ – is unbalanced mass of the eccentric weight; $r_1$ – is the eccentricity of the eccentric weight; $p$ – is the angular velocity of the vibration exciter rotation.

The equations of the linear mechanical system movement with harmonic forcing effects are known in the vibration theory [12, 13, 14].

The movement of the mechanical system in the coordinate system $Oyz$ for the vibratory drum can be described by two differential equations

$$m_1 \ddot{z}_1 + (b_1 + b_2) \dot{z}_1 - b_2 \dot{z}_2 + (c_1 + c_2) z_1 - c_2 \dot{z}_2 = P_0 \sin pt, \quad (2)$$

$$m_2 \ddot{z}_2 + b_2 \dot{z}_2 - \dot{z}_1 + c_2 (z_2 - z_1) = 0. \quad (3)$$

The solution of the system of differential equations (2), (3) can be performed in a complex form, i.e. to be presented as [15]

$$z_1 = A_1 e^{ipt}; \quad z_2 = A_2 e^{ipt}. \quad (4)$$

In equations (4) $i$ – is an imaginary unit.

Inserting solution (4) into the differential equations (2), (3), we obtain the algebraic equations of the vibration amplitudes, containing the real and imaginary components

$$-(m_1 + m_2) p^2 A_1 + (b_1 + b_2) ip A_1 - b_2 ip A_2 + (c_1 + c_2) A_1 - c_2 A_2 = P_0, \quad (5)$$

$$-m_1 p^2 A_2 + b_2 (ip A_2 - ip A_1) + c_2 (A_2 - A_1) = 0. \quad (6)$$

In equations (5), (6) the components containing imaginary values $i$ can be neglected, as for real technical systems $b_1$ and $b_2$ are small values in these equations.

Therefore, equations (5), (6) with taking into account expression (1) allow to determine expressions for the amplitude $A_1$ of the vibratory drum vertical vibrations and amplitude $A_2$ of the additional mass by means of using the real components in equations (5), (6)

$$A_1 = \frac{m_1 r_1 p^2 (c_2 - m_3 p^2)}{((c_1 + c_2) - (m_1 + m_2) p^2)(c_2 - m_3 p^2) - c_2^2}. \quad (7)$$

$$A_2 = \frac{m_1 r_1 p^2 c_2}{((c_1 + c_2) - (m_1 + m_2) p^2)(c_2 - m_3 p^2) - c_2^2}. \quad (8)$$

4. Results discussion

Numerical experiments have been performed for the road roller with the total mass $m=10320$ kg, with the mass distribution on the vibratory drum $(m_1 + m_2 + m_3)=5050$ kg; eccentricity of the vibration exciter eccentric weight $r_1 = 0.0655$ m; eccentric weight mass $m_1 = 64.89$ kg; declared eccentric weight rotation frequency $p = 36.7$ Hz, $p = 230.59$ rad/s.

In Figure 2 for the soil with the stiffness factor $c_1 = 8890$ kN/m the dependences of the vibratory drum vertical vibration amplitude $A_1$ on the frequency $p$ of the forced vibrations with different mass ratios $(m_1 + m_2)$ and $m_3$ are presented. For four dependencies the vibratory drum total mass constant value has been maintained, i.e. $(m_1 + m_2 + m_3) = 5050$ kg, at the same time the mass $m_3$ of the vertical eccentric weight and own mass $(m_1 + m_2)$ of the vibratory drum was changing.
Figure 2. Dependence of the amplitude $A_1$ of the vibratory drum vertical vibrations on the forced vibration frequency $p$ for different ratios of the masses of the vertical eccentric weight $m_3$ and vibratory drum mass $(m_1 + m_2)$:

1 - $(m_1 + m_2) = 2500 \text{ kg, } m_3 = 2550 \text{ kg};$
2 - $(m_1 + m_2) = 3550 \text{ kg, } m_3 = 1500 \text{ kg};$
3 - $(m_1 + m_2) = 4000 \text{ kg, } m_3 = 1050 \text{ kg};$
4 - $(m_1 + m_2) = 4500 \text{ kg, } m_3 = 550 \text{ kg}.$

The amplitude $A_1$ of the vibratory drum vertical vibrations depends on the driving force $R_d$, according to formula (1) raised to the second power on the eccentric weight rotation frequency. For different soils (see Figure 2) the amplitude $A_1$ of the vibratory drum vertical vibrations varies within the limits $A_1 = 0 \div 0.0052 \text{ m}$. For Figure 2 we can make the conclusion that the amplitude $A_1$ of the vibratory drum vertical vibrations increases with increasing the frequency $p$ and with increasing the vibratory drum mass $(m_1 + m_2)$.

In Figure 3 for the soil with the stiffness factor $c_1 = 8890 \text{ kN/m}$, the dependences of the amplitude $A_1$ of the vibratory drum vibrations on the frequency of forced vibrations for the constant mass of the vibratory drum $(m_1 + m_2) = 3000 \text{ kg}$ are presented. A numerical experiment has been performed to vary the mass values $m_3$ of the eccentric weight from the roller frame: $m_3 = 150 \text{ kg}; m_3 = 1000 \text{ kg}; m_3 = 1500 \text{ kg}; m_3 = 2050 \text{ kg}.$

Figure 3. Dependence of the amplitude $A_1$ of the vibratory drum vertical vibrations on the the forced vibration frequency $p$.

At the same time it has been established that for different values of the masses $m_3$ these four dependences coincidence is observed. Consequently, the amplitude $A_1$ in the specified range of the mass changes doesn’t depend on the mass $m_3$ of the vertical eccentric weight.
Figure 4 shows the dependences of the amplitudes $A_1$ and $A_2$ on the soil stiffness factor $c_1$, which varies within the range $c_1 = 8890 \div 97222$ kN/m. Initial data: frequency of forced vibrations $p = 36.7$ Hz; vibratory drum mass $(m_1 + m_2) = 4000$ kg; vertical cantledge mass $m_3 = 1050$ kg.

The amplitude $A_1$ of the vibratory drum vertical vibrations during this experiment varies within the range $A_1 = 0.0017 \div 0.0052$ m, at the same time the amplitude $A_2$ of the additional mass vertical vibrations remains a small value for the entire range of the soil strength change.

For two types of the soil Figure 5 shows the dependences of the vibration amplitudes $A_1$ and $A_2$ on the stiffness factor $c_2$ of rubber-metal shock absorbers. During the determination of the amplitude $A_1$, curve 1 corresponds to heavy soil with the stiffness factor $c_1 = 97222$ kN/m and curve 2 for light soil with the factor $c_1 = 8890$ kN/m.

The amplitude $A_2$ of the roller frame mass $m_3$ vibrations are of a small value for the considered range (see Figure 5, curves 3, 4).

The considered method has allowed to establish the dependence of the $A_1$ amplitude of the vibratory drum vertical vibrations on the vibratory drum mass and its parameters. The independence has been established of the amplitude $A_1$ of vibrations from the mass $m_3$ of the vibratory drum additional eccentric weigh. However, mass $m_3$ in Figure 1 for reasons of design cannot be equal to zero, as it depends on the vibration roller mass distribution across the vibratory drum axles and own mass.

In Figure 1 the vibratory drum is considered as a mechanical system performing vertical vibrations. However, the mass $m_3$ on this scheme as a structural element plays the role of a damper of mechanical vibrations of the vibratory drum frame. The mass $m_3$ acts as the roller frame shock absorber, i.e. protects it from the vibratory drum vibration.
With an inertial disturbing effect on the mass $m_3$ the following forces act: restoring force $c_3z_2$, viscous friction force $b_3\dot{z}_2$, and d’Alembert inertia force of this mass, brought into the order of target forces.

The equation of the mass $m_3$ vertical vibrations can be written as the basic equation of Newton dynamics for the material point [12, 13]

$$m_3\ddot{z}_2 = -c_3z_2 - b_3\dot{z}_2 + F(t),$$  \hspace{1cm} (9)

where $F(t)$ is the d’Alembert inertia force converted to the category of active Newtonian forces.

In this case, the inertia force of the additional mass $m_3$ is the harmonic force

$$F(t) = H\sin pt = -m_3\ddot{z}_2\sin pt.$$  \hspace{1cm} (10)

Equation (9) can be represented as an equation of forced vibrations for a material point.

$$\ddot{z}_2 + 2n_2\dot{z}_2 + \omega_2^2z_2 = \frac{H}{m_3}\sin pt,$$  \hspace{1cm} (11)

where $H/m_3$ – is the mass unit acceleration.

The values of equation (11) are determined by the formulas

$$n_2 = \frac{b_3}{2m_3}, \quad \omega_2^2 = \frac{c_3}{m_3}.$$  

The solution of the differential equation of forced vibrations (11) with a harmonic forcing effect has the following form [12-16]

$$z_2 = \frac{H}{m_3\sqrt{(\omega_2^2 - p^2)^2 + 4n_2^2p^2}}\sin(pt - \delta_2),$$  \hspace{1cm} (12)

where $\delta_2$ – is the phase shift of the driving force and displacement

$$\tan \delta_2 = \frac{2n_2p}{\omega_2^2 - p^2}.$$  \hspace{1cm} (13)

Let’s determine the speed of body 2 as the time derivative of expression (12)

$$\dot{z}_2 = \frac{Hp}{m_3\sqrt{(\omega_2^2 - p^2)^2 + 4n_2^2p^2}}\cos(pt - \delta_2).$$  \hspace{1cm} (14)

The reaction force $R$, transmitted by the mass $m_3$ with the help of rubber-metal connections to the frame is determined by the formula

$$R = -c_3z_2 - b_3\dot{z}_2.$$  \hspace{1cm} (15)

Inserting (12) and (14) into equation (15) we’ll obtain

$$R = -\frac{c_3H(\sin pt - \delta_2) + (pb_3/c_3)\cos(pt - \delta_2))}{m_3\sqrt{(\omega_2^2 - p^2)^2 + 4n_2^2p^2}}.$$  \hspace{1cm} (16)

From equation (16) we can distinguish the constant

$$K'_{d} = \frac{H}{m_3\sqrt{(\omega_2^2 - p^2)^2 + 4n_2^2p^2}}.$$  \hspace{1cm} (17)

As per its physical essence the factor $K'_{d}$ – is nothing but the dynamic amplitude $A_2$ of the $m_3$ mass vertical vibrations.
In the vibration theory it is customary to use the concept – the magnification factor, by which we understand the dynamic amplitude ratio to static amplitude, which is determined by the formula of paper [13]

\[ A_{2cm} = \frac{H}{c_2}. \]  

Using equation (17) we obtain the magnification factor in the following form

\[ K_d = \frac{A_2}{A_{2cm}} = \frac{H}{m_3 \sqrt{(\omega_2^2 - p^2)^2 + 4n_2^2 p^2}} \cdot \frac{c_2^2}{H}. \]  

Considering that \( c_2 = m_3 \omega_2^2 \), after the conversion of formula (19) we obtain an expression for determining the magnification factor

\[ K_d = \frac{1}{\sqrt{\left(1 - \frac{p^2}{\omega_2^2}\right)^2 + \frac{4n_2^2 p^2}{\omega_2^4}}}. \]  

By replacing in expression (16) \( \frac{pb_2}{c_2} \) to \( \frac{2n_2 p}{\omega_2} \) we determine the reaction force \( R \) in the following form

\[ R = -K_d H \left( \sin(pt - \delta_2) + \frac{2n_2 p}{\omega_2} \cos(pt - \delta_2) \right). \]  

Taking into account that \( R \) and \( H \) are of the same dimensionality, from equation (21) by appropriate transformations we can obtain the factor \( K \) of the transfer of the inertial force \( H \) transfer to the roller frame

\[ K = \frac{R}{H} = K_d \sqrt{1 + \frac{4n_2^2 p^2}{\omega_2^4}} \sin(pt - \delta_2 + \varepsilon_2), \]  

where \( \tan \varepsilon_2 = \frac{2n_2 p}{\omega_2^2} \).

The maximum value of the transfer coefficient \( K \) is determined by the formula

\[ K = K_d \sqrt{1 + \frac{4n_2^2 p^2}{\omega_2^4}}. \]  

Formula (23) can be given a final form as a result of the transformation

\[ K = \frac{1}{\sqrt{\left(1 - \frac{p^2}{\omega_2^2}\right)^2 + \frac{4n_2^2 p^2}{\omega_2^4}}}. \]  

5. Consideration of the results

According to expression (20) Figure 6 shows the obtained dependences of the \( K_d \) factor on \( p/\omega_2 \) for different parameters of the vibration roller.
Figure 6. Dependences of the magnification factor $K_d$ on $p/\omega_2$ for different values of the viscous friction factor of rubber-metal shock absorbers.

When performing numerical experiments, the parameters of a medium-type vibration roller are used, the total mass of which is $m = 10320$ kg; vibratory drum has the mass $(m_1 + m_2 + m_3) = 5050$ kg; eccentricity of the eccentric weight $r_1 = 0.0655$ m; additional mass of the cantledge $m_3 = 2050$ kg; frequency of forced vibration $p = 36.7$ Hz. Studies have been carried out for the following values of the viscous friction factor $b_2$: $b_2 = 0$; $b_2 = 6$ kNs/m; $b_2 = 344$ kNs/m; $b_2 = 690$ kNs/m; $b_2 = 1370$ kNs/m.

Using Figure 6 there is a visual transformation of the vibration theory for which the magnification factor $K_d > 1$, to the theory of vibration protection for which the dynamic factor $K_d < 1$. The magnification factor $K_d$ when solving problems of the vibration protection has a value less than the unity $K_d << 1$. For the medium-type roller, the magnification factor changes within the range $K_d = 0.001 \div 0.005$, which correspond to $p/\omega_2 = 27 \div 13.5$. These values of $K_d$ and $p/\omega_2$ are outside the graph, but they explain the physical nature of the emergence of the vibration theory from the theory of oscillations (see Figure 6).

Figure 7 shows the dependences of the factor $K$ of the dynamic transmission of the effect on the additional mass $m_3$ of the roller according to expression (24).

Figure 7. Dependence of the transfer factor $K$ on the $p/\omega_2$ ratio for the vibration roller different parameters.

Similarly Figure 6 and Figure 7 show the dependences of the factor $K$ for different values of the viscous friction factor $b_2$. In the above resonance zone $p/\omega_2 > 1$ the curves intersect at one point with increasing $p/\omega_2$. Curve 1 on this graph corresponds to the value $b_2 = 0$ and it is some kind of reference against which other options are compared.
In Figure 6 for the magnification factor \( K_d \) all the curves of the working area are located in the range of \( K_d = 0.001 \div 0.004 \) with the frequency \( p/\omega^2 = 15 \div 30 \) Hz.

In Figure 7 the transfer factor curves \( K \) have uniformly occupied the area of possible values \( K=0.01 \div 1.0 \) for the corresponding values of the \( b_2 \) values of the viscous friction factor.

For a medium-type roller with the viscous friction factor of rubber-metal shock absorbers \( b_2 = 6 \) kNs/m the magnification factor \( K_d = 0.00333 \), transfer factor \( K \) of forcing effects from the mass \( m_3 \) to the roller frame is equal to \( K = 0.01316 \). This means that the mass \( m_3 \) is the vibration damper from the vibratory drum to the roller frame.

6. Conclusion
For the first time the studies of the vibratory drum three-mass mechanical system, containing the vibration exciter mass, vibratory drum mass and additional mass of the cantledge of the roller mass have been performed. As a result of solving the vibratory drum differential equation the dependence of the amplitude \( A_1 \) of the vibratory drum vertical vibrations on the soil characteristics and vibratory drum parameters has been established. New knowledge was obtained and expressions were improved for the determination of the magnification factor of an additional mass and transfer factor of the reaction driving force from the additional mass \( m_1 \).

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