Crossing the phantom divide in an interacting generalized Chaplygin gas

H García-Compeán\textsuperscript{1,4}, G García-Jiménez\textsuperscript{2}, O Obregón\textsuperscript{3} and C Ramírez\textsuperscript{2}

\textsuperscript{1} Centro de Investigación y de Estudios Avanzados del IPN, Unidad Monterrey, Autopista al Aeropuerto km 9.5, CP 66600, Apodaca, NL, Mexico
\textsuperscript{2} Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Puebla, PO Box 1364, 72000 Puebla, Mexico
\textsuperscript{3} Instituto de Física de la Universidad de Guanajuato, PO Box E-143, 37150 León Gto., Mexico

E-mail: compean@fis.cinvestav.mx, ggarcia@fcfm.buap.mx, ggarcia@fisica.ugto.mx, octavio@fisica.ugto.mx and cramirez@fcfm.buap.mx

Received 12 April 2008
Accepted 3 July 2008
Published 23 July 2008

Online at stacks.iop.org/JCAP/2008/i=07/a=016
doi:10.1088/1475-7516/2008/07/016

Abstract. Unified generalized Chaplygin gas models assuming an interaction between dark energy and dark matter fluids have been previously proposed. Following these ideas, we consider a particular relation between dark densities, which allows the possibility of a time varying equation of state for dark energy that crosses the phantom divide at a recent epoch. Moreover, these densities decay throughout the evolution of the Universe, avoiding a big rip. We find also a scaling solution, i.e. these densities are asymptotically proportional in the future, which contributes to the solution of the coincidence problem.

Keywords: dark matter, dark energy theory, classical tests of cosmology

ArXiv ePrint: 0710.4283
Contents

1. Introduction 2
2. The generalized Chaplygin gas 3
3. The model 4
4. Dark energy equation of state 5
   4.1. Scaling solution ................................. 6
   4.2. Numerical results ................................ 8
5. Conclusions 8
Acknowledgments 10
References 11

1. Introduction

Today we confront the challenge in cosmology of explaining the observation that, in the recent past, our Universe has experienced a phase of accelerated expansion [1]. This acceleration is attributed to the mysterious dark energy (for a review see [2]), whose understanding is nowadays a major problem. Although the simplest candidate for playing the role of dark energy is the cosmological constant $\Lambda$ allowed by Einstein equations, its smallness $\Lambda_{\text{observed}} \sim 10^{-122} M_p^2$ is very problematic as it requires an extreme fine-tuning. Besides, recent observations indicate that a time dependent equation of state is possible. In this case, the simplest approach is to consider a scalar field to play the role of dark energy. There are many types of such models and an extended literature; we refer only to a few works on four of these approaches, namely quintessence [3], k-essence [4], tachyons [5] and the quintom [6].

Moreover, the SNIa data admit an equation of state $\omega_{\text{de}} < -1$, which is attributed to the so-called phantom dark energy [7]. This has the striking feature that its density grows without limit with the expansion of the Universe. Usually, this behavior leads to a violation of the weak energy condition and then to the so-called big rip in a finite time [8]. This is true if dark energy satisfies a conservation equation which corresponds to a non-interacting fluid. Now, if dark energy interacts, for example with dark matter, then the energy conservation equation is modified, and it is possible to circumvent the blowing up of the dark energy density. Further, there are proposals that can encode the crossing $\omega_{\text{de}} = -1$ (dubbed the phantom divide) without violating the weak energy condition [9]. This behavior can be exhibited in dark energy–dark matter unified models. An example of such a model is the Chaplygin gas [10] and its generalization [11]. The Chaplygin gas has a connection with string theory and can be obtained from the light-cone parameterization of the Nambu–Goto action, associated with a D-brane [12]. In this model, a single self-interacting scalar field is responsible for both dark energy and dark matter, giving also the observed accelerated expansion. In a recent work [13], it has been claimed that the galaxy cluster Abell A586 exhibits evidence that dark energy and
dark matter are interacting. The authors trace back the coupling to a departure from the equilibrium settled by the virial theorem and show that it could be explained by the generalized Chaplygin gas (GCG) with dark energy equation of state $\omega_{\text{de}} = -1$.

In the light of these last results and the considerations mentioned above, in this paper we will consider the generalized Chaplygin gas as a source of dark matter and dark energy which are interacting, assuming a time dependent dark energy equation of state with the property of encoding the phantom-like behavior. For simplicity, we will not consider baryonic or other matter. A particular and simple ansatz for the separation of the dark energy and dark matter densities is assumed. We find a phantom-like equation of state for dark energy that crosses the phantom divide $\omega_{\text{de}} = -1$ in a recent epoch. Its associated density smoothly decays with the expansion of the Universe (and thus, is phantom-like). It is also shown that the normalized densities $\Omega_i$ are in the past in agreement with the $\Lambda$CDM model, but in the future they enter in a phase of mutual equilibrium, where their final values are practically the same as today. This indicates a scaling solution, similar to that obtained in [14], where there was assumed a stationary dark energy which could solve the coincidence problem. Also, we find a coupling that it is in agreement with what one would expect in relation to the coincidence problem.

We organize the present paper as follows. In section 2, we briefly revisit the unified generalized Chaplygin gas. In section 3, we construct the model by splitting the total density into dark energy and dark matter components, by means of our assumption. In section 4, we find a dark energy equation of state and we analyze its general behavior. We obtain also the densities involved explicitly as functions of the scale factor, and show that the model has a scaling solution. At the end of this section, we plot the relevant quantities for best fitted values of the parameters. Finally, we present the conclusions in section 5.

2. The generalized Chaplygin gas

In the GCG model the pressure $p$ of the fluid has the following form [11]:

$$ p = -\frac{A}{\rho^\alpha}, $$

(1)

where $\rho$ is the total density and $A > 0$ and $\alpha \geq 0$ are parameters. For $\alpha = 1$ we recover the standard Chaplygin gas and $\alpha = 0$ corresponds to the $\Lambda$CDM model. Following the definition $p \equiv \omega \rho$, we read from (1) the GCG equation of state (EoS)

$$ \omega = -A_s \left( \frac{\rho_0}{p} \right)^{1+\alpha}, $$

(2)

where $A_s \equiv A/\rho_0^{1+\alpha}$ and $\rho_0$ is the total density today, given by $\rho_0 \equiv (3/8\pi G) H_0^2$ in terms of the Hubble constant $H_0$ in a flat universe. In [15] the ranges of the values of $A_s$ and $\alpha$ have been analyzed, and it is argued that the observations favor $0 < A_s < 1$ with $\alpha > 1$, although $0 < A_s < 1$ with $0 < \alpha \leq 1$ is still possible. In this work, we will apply our results in both regions. The density satisfies the conservation equation

$$ d\rho + 3\frac{da}{a}(\rho + p) = 0, $$

(3)
Using (1) in this last equation, we find the density in terms of the scale factor

$$\rho = \left[A_s + \frac{1 - A_s}{a^{3(1+\alpha)}}\right]^{1/(1+\alpha)} \rho_0,$$

(4)

where we normalized the scale factor as \(a_0 = 1\) today. The GCG EoS (2) in terms of the scale factor is

$$\omega = -\frac{A_s a^{3(1+\alpha)}}{1 - A_s + A_s a^{3(1+\alpha)}}.$$

(5)

It is easy to see that with the restriction \(A_s > 0\), this EoS is constrained to the interval \(-1 \leq \omega \leq 0\). Therefore, the model is in general of quintessence and excludes the phantom region \(\omega < -1\), with a phase of acceleration for \(\omega < -1/3\). Furthermore, it allows us to interpret the fluid as cold matter \(\omega \simeq 0\) for \(a \to 0\), and as dark energy \(\omega \simeq -1\) for \(a \to \infty\), and it can be considered as an unified model of matter and dark energy. If we assume in the simplest case an EoS for dark energy \(\omega_{de} = -1\), the resulting model gives slight deviations from the ΛCDM model [11]. However, due to the fact that the model is a mixture of dark matter and dark energy, we cannot exclude a priori the possibility that the dark energy EoS \(\omega_{de}\), has a phantom phase. As the observations tend to support, it is possible that the EoS for dark energy evolves in time and eventually crosses the boundary \(\omega_{de} = -1\), in a recent epoch. In fact, as we will see below, this is the case in the approach considered in this work.

### 3. The model

Let us consider the Friedmann equation,

$$H^2 = \frac{8\pi G}{3} \rho,$$

(6)

where \(H \equiv \dot{a}/a\) is the Hubble parameter and \(\rho\) is the total density. We assume that this density is decomposed as \(\rho = \rho_{de} + \rho_{dm}\), where \(\rho_{de}\) and \(\rho_{dm}\) are the densities of dark energy and dark matter respectively. Thus for cold dark matter, the conservation equation (3) becomes

$$d\rho_{de} + 3\frac{da}{a}(\rho_{de} + p_{de}) = -\left(d\rho_{dm} + 3\frac{da}{a}\rho_{dm}\right).$$

(7)

The problem now is to find a relationship between dark energy and dark matter in order to solve (7). To do this, in this work we will assume a particular and simple ansatz which gives results in agreement with the observations, as follows:

$$\rho_{de}^2 = \lambda \rho_{dm},$$

(8)

where \(\lambda\) is a constant. We will show that this is consistent with the usual behavior of dark energy, suppressed at early times, and then increasing and triggering the acceleration at late times. We should note that equation (7) has also been used in connection with the generalized Chaplygin gas in holographic models [16]. In contrast with our proposal (8), in these kind of models a holographic dark energy density is assumed, to be able to relate
it with the dark matter content. Under ansatz (8), we find the normalized densities
\[
\Omega_{\text{de}} \equiv \frac{\rho_{\text{de}}}{\rho} = \frac{\lambda}{\lambda + \rho_{\text{de}}} \quad \text{and} \quad \Omega_{\text{dm}} \equiv \frac{\rho_{\text{dm}}}{\rho} = \frac{\rho_{\text{de}}}{\lambda + \rho_{\text{de}}},
\]
which clearly satisfy the flat universe constraint \( \Omega_{\text{dm}} + \Omega_{\text{de}} = 1 \). If we impose the restriction \( \rho_{\text{de}} \geq 0 \), then we have from (9)
\[
\rho_{\text{de}} = \frac{\lambda}{2} \left[ \sqrt{1 + \frac{4\rho}{\lambda}} - 1 \right].
\]

4. Dark energy equation of state

Under the assumption of cold dark matter, we can identify the pressures \( p = p_{\text{de}} = \omega_{\text{de}} \rho_{\text{de}} \).

Therefore,
\[
\omega \rho = \omega_{\text{de}} \rho_{\text{de}},
\]
and the dark energy EoS is
\[
\omega_{\text{de}} = \frac{\omega \rho}{\rho_{\text{de}}} = \frac{\omega}{\Omega_{\text{de}}}. \tag{12}
\]

After using (2), (9) and (10) in the last equation, we obtain
\[
\omega_{\text{de}} = -\frac{A_s}{2} \left[ 1 + \sqrt{1 + \frac{4\rho_0}{\lambda}} \left( \frac{\rho_0}{\rho} \right)^{1+\alpha} \right]. \tag{13}
\]

Substituting (4) in this equation, we get the dark energy EoS
\[
\omega_{\text{de}} = -\frac{A_s a^{3(1+\alpha)}}{2X} \left[ 1 + \sqrt{1 + \frac{4\rho_0 X^{1/(1+\alpha)}}{\lambda a^3 \omega}} \right], \tag{14}
\]
where
\[
X \equiv 1 + A_s \left[ a^{3(1+\alpha)} - 1 \right] > 0. \tag{15}
\]

We can see from (13) that \( \omega_{\text{de}} \) decreases as \( a \) increases. Moreover, for large enough values of \( a \), we find in general that \( \omega_{\text{de}} < -1 \). If we consider today’s value \( a = 1 \), we get
\[
\omega_{\text{de0}} = -\frac{A_s}{2} \left[ 1 + \sqrt{1 + \frac{4\rho_0}{\lambda}} \right], \tag{16}
\]
which, by considering the most probable values of \( A_s \) given in [15], we show below to fulfill as well the phantom divide condition \( \omega_{\text{de0}} < -1 \). Notice that once the boundary \( \omega_{\text{de}} = -1 \) is crossed, dark energy is phantom-like for all the rest of the evolution of the Universe and it never returns to the quintessence region \( \omega_{\text{de}} > -1 \). Moreover, in the limit \( a \sim 0 \), the dark energy EoS consistently approaches \( \omega_{\text{de}} \sim 0 \).
4.1. Scaling solution

Scaling solutions are interesting, as they could solve the coincidence problem [14]. In the scaling regime, the ratio $\rho_{\text{dm}}/\rho_{\text{de}}$ is a non-zero constant. Thus, dark energy and dark matter remain of the same order. We will see that our model has an asymptotic scaling region for $a$ large. In order to see this, we compute the dark energy density by means of (4) and (10):

$$\rho_{\text{de}} = \frac{\lambda}{2} \left[ \sqrt{1 + \frac{4\rho_0 X^{1/(1+\alpha)}}{\lambda a^3}} - 1 \right]. \tag{17}$$

Thus, taking into account (8) we get

$$\rho_{\text{dm}} = \frac{\lambda}{4} \left[ \sqrt{1 + \frac{4\rho_0 X^{1/(1+\alpha)}}{\lambda a^3}} - 1 \right]^2. \tag{18}$$

These are solutions of equation (7), taking into account (14). Indeed, a simple calculation reproduces the GCG density (4). Substituting (4) into (9), we get the dark energy normalized density in terms of the scale factor

$$\Omega_{\text{de}} = \frac{2}{\sqrt{1 + (4\rho_0 X^{1/(1+\alpha)}/\lambda a^3)} + 1}, \tag{19}$$

as well as the dark matter normalized density

$$\Omega_{\text{dm}} = \frac{\sqrt{1 + (4\rho_0 X^{1/(1+\alpha)}/\lambda a^3)} - 1}{\sqrt{1 + (4\rho_0 X^{1/(1+\alpha)}/\lambda a^3)} + 1}. \tag{20}$$

Then, a ratio is obtained:

$$\frac{\rho_{\text{dm}}}{\rho_{\text{de}}} = \frac{1}{2} \left[ \sqrt{1 + \frac{4\rho_0 X^{1/(1+\alpha)}}{\lambda a^3}} - 1 \right], \tag{21}$$

which clearly differs from the non-scaling models $\Lambda$CDM and GCG, with $\rho_{\text{dm}}/\rho_{\text{de}} \propto a^{-3}$ and $\rho_{\text{dm}}/\rho_{\text{de}} \propto a^{-(1+\alpha)}$ (see [11]) respectively. For both models $\omega_{\text{de}} = -1$.

In our case, we find that this ratio is today

$$\frac{\rho_{\text{dm}0}}{\rho_{\text{de}0}} = \frac{1}{2} \left[ \sqrt{1 + \frac{4\rho_0}{\lambda}} - 1 \right], \tag{22}$$

and for large values of $a$, we have a scaling solution

$$\frac{\rho_{\text{dm}}}{\rho_{\text{de}}} \sim \frac{1}{2} \left[ \sqrt{1 + \frac{4\rho_0 A_x^{1/(1+\alpha)}}{\lambda}} - 1 \right]. \tag{23}$$

We can see that (22) and (23) are practically the same for the allowed values of $A_x \sim 1$. Therefore, dark energy and dark matter are of the same order today and in the future, which alleviates the coincidence problem. Further, the usual ‘small’ time interval that takes the transition between dark matter and dark energy (present for instance in the
ACDM model) in our case stretches to infinity in such a way that the dark energy and the dark matter densities tend asymptotically to constant values
\[
\rho_{\text{de}} \simeq \frac{\lambda}{2} \left[ \sqrt{1 + (4\rho_0 A_s^{1/(1+\alpha)}/\lambda) - 1} \right], \quad \rho_{\text{dm}} \simeq \frac{\lambda}{4} \left[ \sqrt{1 + (4\rho_0 A_s^{1/(1+\alpha)}/\lambda) - 1} \right]^2.
\] (24)

Let us now consider the coupled equation for the interacting model (7) in the scaling regime, when \( \rho_{\text{dm}} \) and \( \rho_{\text{de}} \) are practically constant. Then, \( d\rho_{\text{dm}} \simeq 0 \) and \( d\rho_{\text{de}} \simeq 0 \), and therefore \( p_{\text{de}} \simeq -(\rho_{\text{dm}} + p_{\text{de}}) \). Thus, considering the EoS \( p_{\text{de}} = \omega_{\text{de}} \rho_{\text{de}} \), we get in general for this model
\[
\frac{\dot{\rho}_{\text{dm}}}{\rho_{\text{de}}} \simeq -(1 + \omega_{\text{de}}).
\] (25)

Hence,
\[
\omega_{\text{de}} \simeq -\left(1 + \frac{\rho_{\text{dm}}}{\rho_{\text{de}}} \right) < -1.
\] (26)

Thus, in the scaling region dark energy EoS is phantom-like. Because the ratio \( \rho_{\text{dm}}/\rho_{\text{de}} \) in the future is practically the same as today, we can put \( \rho_{\text{dm}}(0)/\rho_{\text{de}}(0) = \Omega_{\text{dm0}}/\Omega_{\text{de0}} \simeq 0.3/0.7 \simeq 0.43 \) in (26). Then we get the lower bound \( \omega_{\text{de}} \simeq -1.43 \). In this limit, the total density of the GCG model (4) is \( \rho \simeq A_s^{1/(1+\alpha)} \rho_0 \), and the Universe enters the de Sitter phase \( a(t) \simeq \exp(H_f t) \), where \( H_f^2 \simeq (8\pi G A_s^{1/(1+\alpha)} \rho_0)/3 \). Therefore, the Universe expands, accelerating forever. Now, we can analyze the coupling between dark energy and dark matter in more detail. We will follow the definition of the coupling given in [13]
\[
\dot{\rho}_{\text{dm}} + 3H \rho_{\text{dm}} = \zeta H \rho_{\text{dm}}, \quad \dot{\rho}_{\text{de}} + 3H (1 + \omega_{\text{de}}) \rho_{\text{de}} = -\zeta H \rho_{\text{dm}},
\] (27)

where \( \zeta \) is the coupling. Making the change \( \dot{\rho}_{\text{dm}} = \dot{a} \rho_{\text{dm}}/da \), \( \dot{\rho}_{\text{de}} = \dot{a} \rho_{\text{de}}/da \) and using (8) we find
\[
\zeta = -3\frac{(1 + 2\omega_{\text{de}})}{1 + 2r}.
\] (28)

where we have introduced the ratio \( r \equiv \rho_{\text{dm}}/\rho_{\text{de}} \). We note that both fluids always interact \( (\zeta \neq 0) \); only when \( \omega_{\text{de}} \) passes through \(-1/2\) do they uncouple. Also notice that the coupling evolves from a negative value through a positive value, once the condition \( \omega_{\text{de}} < -1/2 \) is satisfied. Eventually when the approximation (25) is valid, this coupling approaches a non-vanishing constant \( \zeta \leq 3 \) for large values of \( a \). This is consistent with the fact that the fluid enters the scaling regime. It is interesting to compare our model with the work in [13] where \( \omega_{\text{de}} \) is considered constant. In that case the coupling \( \zeta' \) (we denote with primes the quantities in that work) results in
\[
\zeta' = -\frac{(\eta + 3\omega_{\text{de}}')}{1 + r'},
\] (29)

where for the GCG model with \( \omega_{\text{de}}' = -1 \), one has \( \eta = 3(1 + \alpha) \) and \( r' = \rho_{\text{dm}}'/\rho_{\text{de}}' = \Omega_{\text{dm0}}/\Omega_{\text{de0}} a^{-\eta} \). Now, using (27) for our case and a similar set of equations for the model in [13] we find the relation between \( \zeta \) and \( \zeta' \):
\[
\zeta = \frac{\rho_{\text{dm}}'}{\rho_{\text{dm}}} \frac{d\rho_{\text{de}}}{\rho_{\text{de}}'} \zeta' - 3\frac{\rho_{\text{de}}'}{\rho_{\text{dm}}}(1 + \omega_{\text{de}}'),
\] (30)

which shows that these couplings are different. Note, however, that the two couplings coincide if \( \omega_{\text{de}} = -1 \) and the densities with primes and those without primes coincide as well.
Crossing the phantom divide in an interacting generalized Chaplygin gas

Figure 1. Dark energy EoS versus log $a$. The solid line corresponds to $A_s = 0.79$ and $\alpha = 0.999$. In this case the dark energy EoS has a value $\omega_{de} \approx -1.13$ today. The dashed line corresponds to $A_s = 0.936$ and $\alpha = 3.75$, and $\omega_{de} \approx -1.34$ today. In both cases the phantom divide is crossed recently.

4.2. Numerical results

For practical purposes and in order to be specific, we shall take numerical values for the parameters involved in the model. Taking the present values for the fractional densities as $\Omega_{de0} = 0.7$ and $\Omega_{dm0} = 0.3$, we obtain from (9) $\lambda = (\Omega_{de0}^{2}/\Omega_{dm0})\rho_0 \approx 1.63$.$\rho_0$. The dark energy EoS (16) is today approximately $\omega_{de0} \approx -1.43A_s$, independent of $\alpha$, and is phantom-like for $1 > A_s \geq 0.7$. This falls quite well in the region of confidence given by the constraints on the observations for a flat Universe [15]. We will take the best fitted values in the two parameterizations: $\alpha = 0.999$, $A_s = 0.79$ for the range $0 \leq \alpha \leq 1$, and $\alpha = 3.75$, $A_s = 0.936$ for the range $\alpha > 1$ [15]. In figure 1, we show the phantom-like behavior of $\omega_{de}$. We see that it crosses the phantom divide at $a \approx 0.9$ for $A_s = 0.79$ and has a value $\omega_{de} \approx -1.13$ today. In the case $A_s = 0.936$, it crosses at $a \approx 0.95$ and has a value $\omega_{de} \approx -1.34$ today. In figure 2, we plot the quotient of the dark matter over dark energy densities for the same values as in figure 1. We see that it approaches a non-vanishing constant limit as the Universe expands, and it remains constant in this region. Further, from (23) we have in this region, $\rho_{de} \approx 2.55\rho_{dm}$ for $A_s = 0.79$ and $\rho_{de} \approx 2.35\rho_{dm}$ for $A_s = 0.936$. In figure 3, we plot the coupling for the same choice of the parameters. As we can see, the coupling is constrained to $\zeta < 3$. Finally, in figure 4 we plot $\Omega_{dm}$ and $\Omega_{de}$. In the past they behave as in the $\Lambda$CDM model, but in the future for $A_s = 0.79$ they approach the limits $\Omega_{dm} \approx 0.282$ and $\Omega_{de} \approx 0.718$ whereas for $A_s = 0.936$ they approach $\Omega_{dm} \approx 0.298$ and $\Omega_{de} \approx 0.702$. These are almost the same as the present values.

5. Conclusions

In this work we have studied the generalized Chaplygin gas as a unified self-interacting fluid. By assuming a simple relation between dark matter and dark energy densities (8)
we obtain a dark energy EoS which exhibits a phantom-like behavior. In the regime of the most probable values of $A_s$ and $\alpha$ [15], this identification leads to a dark energy EoS that crosses the phantom regime $\omega_{de} = -1$ in recent time. In fact, the numerical values $\omega_{de0} \simeq -1.13$ (for $A_s = 0.79$) and $\omega_{de0} \simeq -1.34$ (for $A_s = 0.936$) are consistent with the

**Figure 2.** Dark matter density over dark energy density $r$ versus log $a$. The solid line corresponds to $A_s = 0.79$ and $\alpha = 0.999$. The dashed line corresponds to $A_s = 0.936$ and $\alpha = 3.75$.

**Figure 3.** Behavior of the coupling $\zeta$ versus log $a$. The solid line corresponds to $\alpha = 0.999$ and $A_s = 0.79$ and the dashed line corresponds to $A_s = 0.936$ and $\alpha = 3.75$. 
Crossing the phantom divide in an interacting generalized Chaplygin gas

**Figure 4.** Normalized dark matter $\Omega_{dm}$ (decaying curves) and dark energy $\Omega_{de}$ (growing curves) versus log $a$. The solid lines correspond to $A_s = 0.79$ and $\alpha = 0.999$. The dashed lines correspond to $A_s = 0.936$ and $\alpha = 3.75$. We see that they behave like the $\Lambda$CDM model in the past.

observations. Furthermore, the general behavior of the dark energy EoS in our model, is in complete agreement with the best fit in [17] for $\Omega_{dm0} = 0.3$. Once the dark energy EoS crosses the phantom barrier, it never returns to quintessence, giving a phantom-like sector for all the future evolution of the Universe. The densities smoothly decay for all values of the scale factor, thus avoiding a big rip. We find also that in contrast with the $\Lambda$CDM and GCG with $\omega_{de} = -1$ models [13], our model exhibits a scaling solution when $a$ is large, as can be seen from the numerical results. This regime is achieved once the dark energy EoS has crossed the phantom divide, and remains so for all the rest of the evolution. We show also that the coupling between dark energy and dark matter tends to a positive constant in this region and the Universe tends to be de Sitter. This type of solution could provide a clue to how to solve the coincidence problem [14]. We also find that the present model with time varying EoS and with the assumptions made could be considered at least as a phenomenological model which could encode the interaction conjectured in [13], inferred from the Abell cluster A586. The relation between the two interacting models is given through (30). It will be of interest to find scalar fields and their corresponding potentials that could reproduce our results. This will be studied in a future work.

**Acknowledgments**

This work was supported in part by CONACyT Grants Nos 45713-F, 51306, VIEP-BUAP grant No. 30/exc/07, PROMEP UGTO-CA-3 and IAC. GGJ thanks CONACyT for a postdoctoral grant under the program Apoyos Integrales para la Formación de Doctores en Ciencias 2006.
Crossing the phantom divide in an interacting generalized Chaplygin gas

References

[1] Riess A G et al, 1998 Astron. J. 116 1009 [SPIRES]
[2] Perlmutter S et al, 1999 Astrophys. J. 517 565 [SPIRES]
[3] Copeland E J, Sami M and Tsujikawa S, 2006 Int. J. Mod. Phys. D 15 1753 [SPIRES]
[4] Peebles P J and Ratra B, 1988 Astrophys. J. 325 L17 [SPIRES]
[5] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[6] Urena-Lopez L A and Matos T, 2000 Phys. Rev. D 62 081302 [SPIRES]
[7] Perlmutter S et al, 1999 Astrophys. J. 517 565 [SPIRES]
[8] Peebles P J and Ratra B, 2003 Rev. Mod. Phys. 75 559 [SPIRES]
[9] Peebles P J and Ratra B, 1998 Phys. Rev. D 37 3406 [SPIRES]
[10] Peebles P J and Ratra B, 2003 Int. J. Mod. Phys. D 12 189 [SPIRES]
[11] Peebles P J and Ratra B, 2003 Phys. Rev. D 68 023512 [SPIRES]
[12] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[13] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[14] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[15] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[16] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[17] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[18] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[19] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[20] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]
[21] Peebles P J and Ratra B, 1988 Phys. Rev. D 37 3406 [SPIRES]