Edge-enhanced dual discriminator generative adversarial network for fast MRI with parallel imaging using multi-view information

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Abstract
In clinical medicine, magnetic resonance imaging (MRI) is one of the most important tools for diagnosis, triage, prognosis, and treatment planning. However, MRI suffers from an inherent slow data acquisition process because data is collected sequentially in k-space. In recent years, most MRI reconstruction methods proposed in the literature focus on holistic image reconstruction rather than enhancing the edge information. This work steps aside this general trend by elaborating on the enhancement of edge information. Specifically, we introduce a novel parallel imaging coupled dual discriminator generative adversarial network (PIDD-GAN) for fast multi-channel MRI reconstruction by incorporating multi-view information. The dual discriminator design aims to improve the edge information in MRI reconstruction. One discriminator is used for holistic image reconstruction, whereas the other one is responsible for enhancing edge information. An improved U-Net with local and global residual learning is proposed for the generator. Frequency channel attention blocks (FCA Blocks) are embedded in the generator for incorporating attention mechanisms. Content loss is introduced to train the generator for better reconstruction quality. We performed comprehensive experiments on Calgary-Campinas public brain MR dataset and compared our method with state-of-the-art MRI reconstruction methods. Ablation studies of residual learning were conducted on the MICCAI13 dataset to validate the proposed modules. Results show that our PIDD-GAN provides high-quality reconstructed MR images, with well-preserved edge information. The time of single-image reconstruction is below 5ms, which meets the demand of faster processing.

Keywords Fast MRI · Parallel imaging · Multi-view learning · Generative adversarial networks · Edge enhancement
1 Introduction

Magnetic resonance imaging (MRI) is one of the most important clinical tools for diagnosis, triage, prognosis, and treatment planning. MRI produces accurate, potentially high-resolution, and reproducible images with various contrast and functional information. Furthermore, it is non-invasive and harmless to the human body. However, MRI has an inherently slow data acquisition process since data is collected sequentially in k-space, where speed is limited by physiological and hardware constraints, rather than directly in image space \[1\]. Prolonged acquisition time can lead to severe motion artefacts due to patient movement and physiological motion. Early approaches to accelerate acquisition followed Nyquist-Shannon sampling criteria, e.g. implementing multiple radio frequency \[2\] or gradient refocusing \[3\], with limited speed improvement.

Undersampling in k-space can improve acquisition speed, but causes aliasing artefacts and blur. Several studies have attempted to reduce aliasing artefacts, such as parallel imaging (PI) and compressed sensing (CS). Parallel imaging was first introduced in 1997 as the simultaneous acquisition of spatial harmonic (SMASH) \[4\] to reduce scan time using multi-channel k-space data and includes sensitivity encoding (SENSE) \[5\] and generalized auto-calibrating partially parallel acquisition (GRAPPA) \[6\]. The acceleration factor and geometry factor influence signal noise ratio for the reconstructed image, where geometry factor depends on the receiver coil distribution. Parallel imaging requires phased array coils where each coil receives data at the same time (i.e., parallel). If the local sensitivity for each coil is already known, then the field of view can be made arbitrarily small in the phase-encoding direction, enabling aliasing to be unwrapped using this information.

Compressed sensing \[7\] reconstructs signals from significantly fewer measurements with higher sampling efficiency than traditional Nyquist-Shannon sampling. There are three requirements for successful CS \[8\], which MRI naturally meets except for the third one:

1. **Transform sparsity**: MRI is compressible by transform coding and has a sparse representation in an appropriate transform domain.
2. **Incoherence**: It can be achieved by random undersampling MRI data in k-space.
3. **Nonlinear reconstruction**: MRI should be reconstructed using a nonlinear method. The main task of CS is to find an appropriate nonlinear method to reconstruct MRI.

Several methods applying fixed sparsifying transforms for reconstruction have been proposed, e.g., total variation (TV) \[9\], curvelets \[10\], and double-density complex wavelet \[11\], while adaptive sparse model represented by dictionary learning \[12\] was also developed as an extension. In addition, CS-MRI worked well with parallel imaging data jointly. Aelterman et al. \[13\] proposed a joint algorithm, i.e., COMPASS, of parallel imaging and compressed sensing for MRI reconstruction. Trzasko et al. \[14\] designed an offline, sparsity-driven reconstruction framework for Cartesian sampling time-series acquisitions. However, the iterative computation significantly limits the speed of the reconstruction, leading to the fact that the traditional CS-MRI is hard to be applied in the clinical environment.

Deep learning has been developed enormously recently. Convolutional neural networks (CNNs) have made a significant impact on many computer vision learning tasks, such as classification \[15\], segmentation \[16\], object detection \[17\], and image reconstruction \[18\]. CNNs offer better feature extraction performance compared with traditional machine learning algorithms due to their deeper structure of hierarchically stacked neural layers. Consequently, several CNNs have been proposed for medical imaging to solve traditional limitations. CNN was first introduced for MRI reconstruction in \[19\], where it was used to identify mapping relationships between MR images obtained from zero-filled and fully-sampled k-space data. Yang et al. \[20\] proposed a deep architecture, i.e., ADMM-Net, inspired by the alternating direction method of multipliers (ADMM) algorithm to optimizing CS based MRI models. Schlemper et al. \[21\] developed a deep cascade CNN to reconstruct dynamic sequences for 2D cardiac MR images from undersampled data. Zhu et al. \[22\] proposed a novel framework for MRI reconstruction, i.e., automated transform by manifold approximation (AUTOMAP), allowing a mapping between the sensor and the image domain to emerge from an appropriate corpus of training data. Transfer learning has been also proposed to solve the data scarcity problem when training deep networks for accelerated MRI \[23\].

In 2014, Goodfellow et al. \[24\] designed generative adversarial networks (GANs). Then, various improved GAN models were presented. Wasserstein GAN (WGAN) \[25\] was proposed for improving the training stability of GAN and optimise the learning curves. Radford et al. \[26\] introduced DCGAN, first applied CNNs to GANs, bridging the gap between supervised learning and unsupervised learning. GAN-based models has been widely used in image-to-image translation \[27–30\], super-resolution \[31–33\], as well as MRI reconstruction \[34–36\].

Several studies have reported that the deep learning based GAN models perform well in MRI reconstruction. DAGAN \[34, 37\] applied the modified U-Net \[38\] architecture as the generator and incorporated additional perceptual loss by pre-trained VGG \[39\] networks. Shaul et al. \[35, 37\]
introduced a dual-generator GAN, i.e., KIGAN, for both estimating the missing $k$-space samples and refinement of the image space data. Quan et al. [36, 37] presented RefineGAN, which was a variant of a fully-residual convolutional autoencoder and GAN with cyclic data consistency loss. Lv et al. [40], proposed a deep GAN model, i.e., PICGAN, with parallel imaging for accelerated multi-channel MRI reconstruction, where data fidelity and regularisation terms were integrated into the generator. Nader et al. [41] combined the traditional MR reconstruction algorithm GRAPPA with a conditional GAN to build the GRATTA-GAN, which was developed and tested on the multi-coil brain data. Guo et al. [42] proposed DAWAGAN, which coupled WGAN with Recurrent Neural Networks to adopt the relationship among MRI slices for fast MRI reconstruction. Hu et al. [43] designed a general texture-based GAN for MR image synthesis, where a multi-scale mechanism for the texture transfer between source and target domain was adopted. Ma et al. [44] introduced a novel GAN-based model, i.e., CSI-GAN, for medical image enhancement, where illumination regularisation and structure loss were used as constraints of training. Zhang et al. [45] combined the reverse edge attention module into the generator for a global understanding of images and improved the discriminator for the utilisation of reverse edge attention module significantly improved the performance of segmentation. Yuan et al. [46] incorporated the self-attention mechanism into the generator for a global understanding of images and improved the discriminator for the utilisation of prior knowledge. Li et al. [47] proposed RSCA-GAN for fast MRI reconstruction, where both spatial and channel-wise attention mechanisms were applied in the generator. This team also applied the GAN-based model with attention mechanisms in the super-resolution task [48]. Chen et al. [49], incorporated wavelet packet decomposition into the de-aliasing GAN [34] for the texture feature. Lv et al. [50] applied transfer learning to a parallel image coupled GAN model, improving the generalisability of networks based on small samples. Jiang et al. [51] proposed FA-GAN for the super-resolution task, where both global and local feature fusion were utilised in the generator for better performance. Zhou et al. [52] designed a GAN-based model, i.e., ESSGAN, with a structurally strengthened generator, which consisted of the strengthened connection and the residual in the residual block. This team also introduced a novel spatial orthogonal attention GAN model [53], where the computational complexity was significantly decreased.

Most proposed MRI reconstruction approaches focused on integral MR image property. The design of GAN-approaches relied on loss function definitions that do not consider structural characteristics of practical value that are present in the image such as edge information. However, edge information can be crucially conclusive for clinical diagnosis. Accordingly, to solve this problem, studies related to edge information preservation in MRI reconstruction has been reported. Yu et al. [54] proposed Ea-GANs, in which the edge information was utilised via the Sobel operator. Ea-GANs contain a generator-induced gEa-GAN, and a discriminator-induced dEa-GAN, for enriching the reconstruction images with more details. Chai et al. [55] designed an edge-guided GAN (EG-GAN), to restore brain MRI images which decoupled reconstruction into edge connection and contrast completion. Li et al. [56], proposed a dual-discriminator GAN, i.e., EDDGAN, of which one discriminator was used for holistic image reconstruction and the other one was for edge information preservation.

Recently, a large number of multi-view data based methods by considering the diversity of various views have been proposed [57, 58]. The main task of multi-view learning is to find a function to model each view and jointly optimises all the functions to improve the generalisation performance [59, 60]. In this work, the idea of multi-view learning was adopted. The multi-channel MR data we used in training was the multi-view information of the MRI raw data, by multi-coil parallel imaging. In addition, information from different views including image-space information, $k$-space information and edge details were all included and used as constraints on the training process.

The methods mentioned above focused on the preservation of edge information from various aspects. However, all were based on single-channel MR data. In fact, complementary multi-view information can be provided by multi-channel MR data. This work takes advantage of parallel imaging technology by using multi-channel data rather than single-channel data. In particular, we propose a novel parallel imaging coupled dual discriminator generative adversarial network (PIDD-GAN) for fast multi-channel MRI reconstruction. The main contributions can be summarised as follows.

- We introduce a dual discriminator GAN architecture, where two discriminators are used for holistic image reconstruction and edge information enhancement respectively.
- The GAN generator is an improved U-Net with local and global residual learning that stabilises and accelerates the training process. Frequency channel attention blocks (FCA Blocks) are embedded in the improved U-Net to incorporate attention mechanisms.
- Although the proposed network is designed for multi-channel MR image reconstruction, single-channel image reconstruction can also be accomplished.
- Comprehensive experiments on the Calgary-Campinas public brain MR dataset\(^1\) (359 subjects) compared the proposed method with current state-of-the-art MR recon-

\(^1\)https://sites.google.com/view/calgary-campinas-dataset/
struction methods. Ablation studies for residual learning were conducted on the MICCAI13 dataset\(^2\) to validate the proposed modules. Experimental results confirm that the proposed PIDD-GAN achieves high reconstruction quality with faster processing time.

## 2 Method

### 2.1 Traditional MRI

The problem of traditional CS-MRI is to recover a vector \(x \in \mathbb{C}^N\) in image space from an undersampled vector \(y \in \mathbb{C}^M\) \((M \ll N)\) in k-space, which can be expressed as follows

\[
y = \mathcal{M}Fx + n,
\]

where \(\mathcal{M}\) denotes the undersampling trajectory, \(\mathcal{F}\) denotes the Fourier transform, and \(n\) denotes noise.

For parallel imaging, coil sensitivity encoding is incorporated in the reconstruction, i.e.,

\[
y = \mathcal{M}\mathcal{C}x + n,
\]

where \(\mathcal{C}\) is the coil sensitivity.

Let \(\mathcal{E}\) be an operator including undersampling trajectory \(\mathcal{M}\), Fourier transform \(\mathcal{F}\) and coil sensitivity \(\mathcal{C}\), the reconstruction problem can now be expressed as follows

\[
\min_x \frac{1}{2} \| y - \mathcal{E}x \|^2_2 + \lambda R(x),
\]

where \(R(x)\) denotes regularisation terms on \(x\), and \(\lambda\) is a regularisation coefficient, controlling the degree of regularisation. \(\| \cdot \|_2\) denotes the \(l_2\) norm.

### 2.2 Deep learning-based fast MRI

#### 2.2.1 CNN-based fast MRI

A deep network can be incorporated into fast MRI reconstruction to generate image \(f_{\text{CNN}}(x_u \mid \theta)\) from the zero-filled image \(x_u\), where \(\theta\) are the optimised parameters of the deep network. The problem can be expressed as follows

\[
\min_x \frac{1}{2} \| y - \mathcal{E}x \|^2_2 + \lambda R(x) + \zeta \| x - f_{\text{CNN}}(x_u \mid \theta) \|^2_2,
\]

where \(\lambda\) and \(\zeta\) are coefficients that balance each term.

#### 2.2.2 GAN-based fast MRI

In general, a GAN model consists of two parts: a generator and a discriminator. The generator aims to produce fake data \(G_{\theta_g}(z)\) that is as real as possible by modelling and sampling the distribution of the ground truth \(x\), so that samples drawn from the modelled distribution succeed at deceiving the discriminator. The goal of the discriminator is to distinguish the fake data generated by the generator from the ground truth.

Ideally, the best discriminator can be represented as

\[
D_{\theta_d}(x) = 1, \quad D_{\theta_d}(G_{\theta_g}(z)) = 0.
\]

In this way, the generator and the discriminator form a min-max game. The training process of GAN can be described as follows

\[
\min_{\theta_g} \max_{\theta_d} \mathcal{L}(\theta_g, \theta_d) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_d}(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D_{\theta_d}(G_{\theta_g}(z)))].
\]

where \(p_{\text{data}}(x)\) is the distribution of real data, and \(p_z(z)\) is the latent variables distribution. During the training process, both sides constantly optimise themselves until the balance is reached — neither side can get better, that is, the fake samples are completely indistinguishable from the true samples.

However, the discriminator may be very confident and almost always output 0 initially. To circumvent this problem practically, the loss function is replaced by:

\[
\min_{\theta_g} \max_{\theta_d} \mathcal{L}(\theta_g, \theta_d) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_d}(x)] - \mathbb{E}_{z \sim p_z(z)}[\log D_{\theta_d}(G_{\theta_g}(z))].
\]

When the GAN model is used for the reconstruction task, the generator is trained to generate reconstructed MR images \(G_{\theta_g}(x_u)\) from zero-filled undersampled MR images \(x_u\). The discriminator is trained to distinguish \(G_{\theta_g}(x_u)\) from the ground truth MR image \(x\) by maximizing the log-likelihood for estimating the conditional probability, which can be represented as

\[
D_{\theta_d}(x) = 1, \quad D_{\theta_d}(G_{\theta_g}(x_u)) = 0.
\]

The adversarial loss \(\mathcal{L}_{\text{adv}}\) that drives the training process can be parameterised by \(\theta_g\) and \(\theta_d\) as follows

\[
\min_{\theta_g} \max_{\theta_d} \mathcal{L}_{\text{adv}}(\theta_g, \theta_d) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_d}(x)] - \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_d}(G_{\theta_g}(x_u))].
\]

### 2.3 Proposed dual discriminator GAN for fast MRI reconstruction

#### 2.3.1 Formulation

In this work, a first discriminator \(D_1\) is used to distinguish the reconstructed MR images \(\hat{x}_u\) from the ground truth MR images \(x_u\), whereas an additional discriminator \(D_2\) is designed to assist the reconstruction of the edge information. Edge information is usually extracted by means of a Sobel operator \(S(\cdot)\). The edge information of the reconstructed MR image \(S(\hat{x}_u)\) and the edge information of the MR ground truth \(S(x)\) are fed to \(D_2\), so that the result is counted into the adversarial loss \(\mathcal{L}_{\text{adv}}\).

The new adversarial loss \(\mathcal{L}_{\text{adv}}\) can be defined as follows

\[ \mathcal{L}_{\text{adv}}(\theta_g, \theta_d, \theta_{D_2}) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_d}(x)] - \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_d}(G_{\theta_g}(x_u))] + \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_{D_2}}(S(x))] - \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_{\theta_{D_2}}(S(\hat{x}_u))]. \]
\[
\min_{\theta_G} \max_{\theta_{D_1}, \theta_{D_2}} \mathcal{L}_{adv}(\theta_G, \theta_{D_1}, \theta_{D_2}) = \mu \left[ E_{x_t \sim p_{\text{train}}(x_t)}[\log D_{\theta_{D_1}}(x_t)] - E_{x_u \sim p_G(x_u)}[\log D_{\theta_{D_1}}(\hat{x}_u)] \right] \\
+ \nu \left[ E_{x_t \sim p_{\text{train}}(x_t)}[\log D_{\theta_{D_2}}(S(x_t))] - E_{x_u \sim p_G(x_u)}[\log D_{\theta_{D_2}}(S(\hat{x}_u))] \right],
\]

where \( \mu \) and \( \nu \) denote the weights of the discriminator for the holistic image and the edge information correspondingly, and \( \hat{x}_u = G_{\theta_G}(x_u) + x_u \) as it will be later discussed.

### 2.4 Network architecture

The overall architecture of the proposed PIDD-GAN for MR image reconstruction is shown in Fig. 1. Sensitivity-weighted ground truth \( x_t \) is derived from the multi-channel ground truth \( x^q_t \), corresponding sensitivity map \( C^q \) (\( q \) denotes the coil number) and sensitivity-weighted zero-filled image \( x_u \) by undersampling \( x_t \). The generator produces the reconstructed MR image \( \hat{x}_u \) from \( x_u \). Two discriminators are used for the holistic image and the edge information, respectively.

Ronneberger et al. [38] proposed U-Net for semantic segmentation of medical images. This deep architecture comprises an encoder path to capture context and a symmetric decoder path that enables precise localisation, i.e., a U-shaped model. Skip connections are applied between corresponding layers in encoder and decoder paths, passing features directly from undersampling path to upsampling path. U-Net can be trained in an end-to-end manner and performs well in medical image segmentation [34, 36, 56].

An improved U-Net is proposed as the generator in our proposed PIDD-GAN for higher reconstruction performance, where the generator input is sensitivity-weighted zero-filled image \( x_u \). As shown in Fig. 2, in our improved U-Net, cascaded downsampling blocks are placed in the encoder path (left), and corresponding cascaded upsampling blocks are placed in the decoder path (right). Skip connection and concatenation are applied between the downsampling blocks and the symmetrical upsampling blocks with...
the same scale to preserve the feature from different levels and, ultimately, yield better reconstruction details.

Figure 2 shows the \(i\)th downsampling block structure. First, a \(3 \times 3\) convolution layer with stride = 2 is applied to downsample the \((i-1)\)th output. Then, a residual block extracts further features and avoids gradient vanishing and exploding problems [61]. There are two \(3 \times 3\) convolution layers in the backbone and a \(1 \times 1\) convolution layer adjusting the channel and fusing feature maps at different scales. Leaky ReLU layers and Batch normalisation (BN) are applied after each convolution layer except the final one. Finally, a FCA Block [62] learns the different channel weights with attention.

The structure of upsampling blocks and downsampling blocks are similar. A deconvolution layer with stride = 2 is applied to upsample the \((i-1)\)th output. The shortcut is removed to reduce computation cost but maintain high reconstruction quality here.

Traditional CNNs treat all channels in a feature map with the same importance, ignoring the importance differences. Therefore, the attention mechanism is adopted to make use of importance difference information by learning the different channel weights, i.e., effective channels have high weights and ineffective channels have small weights, which helps to train the model and enhance the results.

FCA Block [62] is a novel attention mechanism based on the squeeze and excitation block (SE Block) [63], as shown in Fig. 3. First, an \(H \times W \times C\) feature map is squeezed into a \(1 \times C\) vector. Then channel weights are extracted by two fully-connected layers, and channel weights multiply the original feature maps. A two-dimensional discrete cosine transform (DCT) is applied in FCA Blocks to squeeze the feature map, rather than global average pooling employed in SE Blocks, since this latter operation is equivalent to the lowest DCT frequency. Hence using only GAP leads to loss of other frequency components in the feature channel.
containing useful information. In the squeezing step, the $H \times W \times C$ feature map is divided evenly into $n$ parts (each size is $H \times W \times C'$). The squeezing step can be represented as

$$\text{Freq}_i = \text{DCT}_i(\text{Imag}_i), \quad (i = 0, 1...n - 1),$$

(11)

where $\text{DCT}_i$ denotes the $i^{th}$ preset DCT template and $\text{Freq}_i$ denotes the corresponding frequency component. In this work, FCA Blocks are utilised after every residual blocks in the generator.

As the network architecture goes deeper, more granular information can be extracted, but leading to gradient vanishing and exploding problems, making the network converge slowly. The introduction of residual learning [61] solves this problem effectively. The main idea of residual learning is the utilisation of shortcut connections between the convolution layer. It makes the deep network easier to be trained and converge faster. Global residual learning (GR) is applied in our improved U-Net. The output of the generator adopts $\hat{x}_u = G_{\theta}(x_u) + x_u$ instead of $\hat{x}_u = G_{\theta}(x_u)$ in the original U-Net. This change transfers the generator from a conditional generative function to a refinement function. This work also applies local residual learning (LR) by the shortcut connection in each residual block in the downsampling path. The utilisation of LR aimed to stabilise the training and accelerate the model convergence.

Traditional GAN trains a single discriminator to compete against the generator. Although it improves reconstruction quality compared with other methods, only integral MR image properties are considered, without enhancing edge details. The current study proposes a dual discriminator GAN for edge information enhancement. The generated MR image $\hat{x}_u$ and sensitivity-weighted MR ground truth $x_t$ are fed into discriminator $D_1$ for holistic image reconstruction. We use the Sobel operator $\mathcal{S}(\cdot)$ to extract the edge information from MR images, and input edge information for the reconstructed image $\mathcal{S}(\hat{x}_u)$ and ground truth $\mathcal{S}(x_t)$ into $D_2$. Thus, both holistic image information and edge details can be simultaneously reconstructed.

Figure 4 shows the common network structure used for both discriminators. First, a cascade of $3 \times 3$ convolution layers with stride = 2 downsamples and extracts MR image features. Two $1 \times 1$ convolution layers and a residual block follow the final $3 \times 3$ convolution layer. The residual block consists of three $1 \times 1$ convolution layers, and input and output are connected by a shortcut. All convolution layers above are followed by a BN layer and Leaky ReLU layer. Finally a full connection layer and a Sigmoid layer output the prediction results. Results of both discriminators are incorporated into the adversarial loss $\mathcal{L}_{adv}$.

### 2.5 Loss function

This study introduces content loss to train the generator for better reconstruction quality. Content loss comprises
3 Experiments and results

3.1 Datasets

This work used the Calgary Campinas multi-channel dataset [64] and the MICCAI 2013 grand challenge single-channel dataset [65], which are both publicly available for the experiment section.

The Calgary Campinas dataset was used to train and validate our proposed method and compare with other methods. The MR images were acquired with a 12-channel coil. We randomly chose 15360 12-channel T1-weighted brain MR images. The dataset was divided into training, validation, and testing sets (7680, 3072, and 4608 slices respectively), according to the ratio of 5:2:3.

The MICCAI 2013 grand challenge dataset was used for ablation studies. We randomly chose 18850 single-channel T1-weighted brain MRI images, and divided these into training, validation, and testing sets (9935, 3974, and 5961 slices, i.e., ratio of 5:2:3, respectively).

3.2 Implementation detail

The proposed PIDD-GAN was implemented using PyTorch, and trained and tested on an NVIDIA TITAN RTX GPU with 24GB GPU memory. We set the same hyperparameters and trained and tested on an NVIDIA TITAN RTX GPU with 24GB GPU memory. We set the same hyperparameters and trained and tested on an NVIDIA TITAN RTX GPU with 24GB GPU memory.

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To evaluate the proposed method, we compared the following variations: (1) PIDD: dual discriminator GAN model trained with parallel imaging dataset; (2) PISD: single discriminator GAN trained with parallel imaging dataset; (3) nPIDD: dual discriminator GAN model trained with single-channel imaging dataset. Thus, the role for each component in the proposed method can be compared more fairly and clearly.

3.3 Evaluation methods

Various assessment methods were applied to evaluate reconstruction quality. Normalised mean square error measures average squared difference between generated images and ground truth. Structural similarity (SSIM) was used to...
Table 1  Testing results of different method. Results are obtained by Gaussian 2D masks with 30%. (Bold values indicate the best performed method)

| Method        | Gaussian 30% | MOS     | Time             |
|---------------|--------------|---------|------------------|
|               | NMSE         | PSNR    | SSIM             | FID     | Holistic | Edge | GPU Time (ms) | CPU Time (s) |
| GT            | 3.83±0.52    | 3.70±0.46|                  |         |          |      |               |              |
| ZF            | 0.0247±0.0013| 28.4701±0.3806| 0.8756±0.0060| 103.33 | 3.80±0.54 | 3.40±0.66 | 4.95±0.01 | 0.47±0.00   |
| TV            | 0.0254±0.0023| 28.5259±0.6997| 0.8840±0.0117| 106.04 | 3.80±0.54 | 3.40±0.66 | 4.95±0.01 | 0.47±0.00   |
| L1-ESPIRiT    | **0.0071±0.0008**| 32.4709±0.8978| 0.9056±0.0122| 21.21  | 3.70±0.59 | 3.26±0.51 | 4.95±0.01 | 0.47±0.00   |
| ADMM-Net      | 0.0129±0.0022| 31.5171±0.9744| 0.9368±0.0115| 67.97  | 3.80±0.54 | 3.40±0.66 | 4.95±0.01 | 0.47±0.00   |
| DAGAN         | 0.0169±0.0011| 30.1123±0.3710| 0.9072±0.0053| 53.67  | 3.70±0.59 | 3.26±0.51 | 4.95±0.01 | 0.47±0.00   |
| nPIDD         | 0.0099±0.0009| **32.4809±0.4680**| **0.9438±0.0044**| 29.42  | 3.80±0.54 | 3.40±0.66 | 4.95±0.01 | 0.47±0.00   |
| PISD          | 0.0103±0.0009| 32.2685±0.4808| 0.9398±0.0050| 24.12  | 3.70±0.59 | 3.26±0.51 | 4.95±0.01 | 0.47±0.00   |
| PIDD          | 0.0101±0.0009| 32.3694±0.4443| 0.9425±0.0046| **15.38**| 3.80±0.54 | 3.40±0.66 | 4.95±0.01 | 0.47±0.00   |

Fig. 5  Testing examples of different methods. Results are obtained by Gaussian 2D masks with 30%. Line1: MR images; Line2: Differences of MR images (×15); Line3: Edge Information of MR images; Line4: Differences of Edge Information (×15)

Fig. 6  Horizontal line profiles of different methods. Results are obtained by Gaussian 2D masks with 30%

measure similarity between two images and hence predict perceived quality. Peak signal-to-noise ratio (PSNR) is the ratio between maximum signal power and corrupting noise power, which quantifies the representations fidelity.

However, PSNR and SSIM do not necessarily correspond with visual quality for human observers. Therefore, we adopted two different metrics to evaluate reconstruction quality. Frchet inception distance (FID) [66] measures similarity between two sets of images, calculated by computing the Frchet distance between two Gaussian fitted feature representations for the inception network. FID correlates well with human derived visual quality and it is widely used to evaluate GAN sample quality.

Meanwhile, the judgement of domain experts is considered. Mean opinion score (MOS) from expert observers was used to evaluate the holistic image and edge information for the reconstructed images. Likert scales [67] from 1 (poor), 2 (fair), 3 (good) to 4 (very good) were used that were based on the holistic image quality and edge information
quality, the visibility of the atrial scar and occurrence of the artefacts. All compared reconstruction results of different methods were randomly shuffled and blinded for the expert observers scrutinisation.

### 3.4 Comparisons with other methods

To better evaluate the reconstruction performance of the PIDD-GAN, we compare it with other traditional and MR reconstruction algorithms, including TV [9], L1-ESPIRiT [68], ADMM-Net [20], DAGAN [34], as well as PISD and nPIDD. Among them, TV, ADMM-Net, DAGAN, nPIDD were implemented based on the single-channel MRI data, whereas L1-ESPIRiT, PIDD, PISD were implemented based on the multi-channel MRI data. The Calgary Campinas multi-channel MRI data was used in this experiment.

The testing results are shown in Table 1. Zero-filled image (ZF) is undersampled by a Gaussian 30% downsampling trajectory. According to the comparison study results, the proposed method shows significant improvement for all test indicators compared with other methods. The FID of PIDD is significantly lower compared to PISD and nPIDD.

Reconstruction samples of six methods, including PIDD, PISD, DAGAN, ADMM-Net, together with GT and ZF, were chosen for unbiased rating by domain experts. PIDD achieved the highest MOS (except GT) both in holistic image and edge information reconstruction.

Testing examples of different methods are shown in Fig. 5. Our method is superior to other methods in terms of overall reconstruction quality and edge information reconstruction. We can see that ZF is quite blurred, mixed with many artefacts. Compared with traditional TV and L1-ESPIRiT...
methods, the noise reduction of ADMM and DAGAN are greatly improved, but the detailed information is still lost significantly. The reconstruction result of L1-ESPIRiT shows rich details, but it is poor in de-noising and time-consuming. Compared with PIDD and PISD, nPIDD lacks multi-channel information. Although the basic structure can be completely restored, and most noise is reduced, the excessive smoothing phenomenon is severe compared to PIDD and PISD. From the zoom-in area, it can be seen that PIDD clearly reconstructs the edge information of the brain, but this structure is very shallow in the results of PISD.

Horizontal line profiles of the samples in Fig. 5 are shown in Fig. 6. ZF and DAGAN still contain lots of noise, while our proposed methods preserve more detail information. Zoom-in areas clearly show that the line profile of PIDD-GAN achieve more accurate than other methods.

### 3.5 Experiments on mask

In this experiment, different downsampling trajectories were adopted to evaluate the robustness of the proposed method. G2D10%, G2D20%, G2D30%, G2D40%, G2D50%, G1D30% and P2D30% indicate Gaussian 2D 10%, Gaussian 2D 20%, Gaussian 2D 30%, Gaussian 2D 40%, Gaussian 2D 50%, Gaussian 1D 30% and Poisson 2D 30% downsampling trajectories, respectively. This experiment was tested on the Calgary Campinas multi-channel MRI dataset.

The testing result are shown in Figs. 7 and 8. Experimental results exhibit the same trend under all the different downsampling trajectories. The proposed method provides significant advantages for low downsampling percentage (high acceleration factor), with correspondingly significantly improved reconstruction quality.

The testing examples of the reconstruction are shown in Fig. 9. It can be seen from the results that PIDD has a significant recovery effect on edge information in the case of a low downsampling percentage (10–30%). The edge information of PIDD, particularly in sulci and cerebellum areas, is greatly preserved, compared to PISD. The texture details of PIDD are also richer than nPIDD. In the case of a high downsampling percentage (40–50%), the
reconstruction problem becomes simpler, and the advantages of PIDD with enhanced reconstruction of edge information and the advantages of multi-channel data are less obvious. In the experiment of Gaussian 2D 50% undersampling, PIDD, PISD and nPIDD basically have the same quality of the reconstruction.

3.6 Experiments on noise

In this experiment, the same downsampling trajectory and different noise levels were used to test the reconstruction performance of the model under the influence of noise. This experiment was tested on the Calgary Campinas multi-channel MRI dataset. Here the noise level (NL) is defined as follows

\[ NL = \frac{N}{N + S}, \]  

(18)

where \( S \) and \( N \) denotes the power of signal and noise respectively.

The testing results are shown in Fig. 10, and testing samples are shown in the Fig. 11.

All considered methods can restore image structure and edge information for low and medium noise levels (20–50%), with PIDD having strong advantages over PISD and nPIDD. This advantage weakens as noise level increases, and PISD, which focuses on overall information recovery, performance slightly surpasses PIDD, which focuses more on edge information preservation, for high noise (70–80%).

3.7 Ablation experiments on residual learning

In this experiment, the effect of residual learning in the network was discussed. Our proposed model was tested on MICCAI 2013 grand challenge dataset, using Gaussian 1D 30% downsampling. The experiment was divided into four groups: (1) GRLR (model with GR and LR), (2) GRnLR (model with GR without LR), (3) nGRLR (model with LR without GR), (4) nGRnLR (model without LR and GR). Early stopping strategy was turned off in this experiment to prolong the training process for a better and more distinguishable comparison for the training step.

Figure 12 shows NMSE, SSIM, PSNR and generator loss (G Loss) of the four groups changing with the training process, and Fig. 13 shows testing examples with respect to different epoch weights.
Models with GR (GRLR, GRnLR) have faster convergence and better final results compared with those without GR (nGRLR, nGRnLR). If the model applies GR, then using LR has little further impact effect on the results. For non-GR models, nGRLR converges significantly slower than nGRnLR but final results are superior. Therefore, we chose GRLR as the generator for subsequent study.

4 Discussion

In this work, we introduce the PIDD-GAN for multi-channel MRI reconstruction using multi-view parallel imaging information, focusing on the enhancement of edge information and the utilisation of the multi-channel MR data.

Experimental results verified that the proposed dual discriminator design does greatly improve reconstruction quality of edge information, particularly for sulci and cerebellum areas with rich edge information. The utilisation of multi-channel data also reduces the excessive smoothing phenomenon to a certain extent, and the texture of the tissue can be also better preserved.

During experiments, we found that SSIM and PSNR do not reflect the reconstruction quality very well, since these two indicators have better tolerance for over-smoothing and are relatively insensitive to the edge information details. Therefore, we adopted FID using a deep network.
and MOS based on subjective scoring by experts to assess our results more comprehensively.

Meanwhile, we did a series of experiments to test the role of the attention mechanism (FCA Block and SE Block) in the entire model. In most cases, the utilisation of attention mechanisms slightly improved the convergence speed and final results of the model. However, incorporating the attention mechanism degraded the final result for a few cases. The specific mechanism requires further study.

The proposed model still has some remaining limitations. Dual discriminator structure superiority reduces for high downsampling percentage, and single discriminator structure, focusing on overall recovery, may offer better performance. Noise can affect edge information extraction, particularly for higher noise levels. This effect is very marked when using the Sobel operator, and hence may jeopardise $D_2$ performance, reducing image reconstruction quality.

Further studies will continue to focus using multi-channel information to help edge information reconstruction and restoration. We will also consider how to reduce network calculations and improve network efficiency.

5 Conclusion

This work proposed a parallel imaging based dual discriminator generative adversarial network for multi-channel MRI reconstruction, enhancing edge information and multi-channel MR data utilisation using multi-view information. Experiment results verified that the proposed method offers significantly better performance preserving edge information for MRI reconstruction.
Fig. 12  Testing results of the ablation studies with residual learning. GRLR: model with GR and LR; GRnLR: model with GR without LR; nGRLR: model with LR without GR; nGRnLR: model without LR and GR

Fig. 13  Testing examples of the ablation studies with residual learning. GRLR: model with GR and LR; GRnLR: model with GR without LR; nGRLR: model with LR without GR; nGRnLR: model without LR and GR
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