Experimental Investigation of the Warm Rotation in the Order-to-Chaos Region by Gamma-Spectroscopy in the Quasi-Continuum

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Abstract. The collective nuclear rotation, at high spins and as a function of temperature T, is investigated in the transition region between the regular ordered motion at T=0 and the chaotic compound nucleus regime around the particle binding energy. The experimental analysis, based on statistical techniques, is carried out in nuclear systems characterized by specific quantum numbers (such as K) and deformation (as for example superdeformed nuclei). Data from high statistics EUROBALL experiments are discussed, focusing on nuclear structure effects at the onset of the chaotic regime. The experimental findings are compared to prediction from a Montecarlo simulation of the \( \gamma \)-decay flow based on microscopic cranked shell model calculations at finite temperature.

1. Introduction
The properties of the atomic nucleus at the limits of angular momentum, temperature and neutron/proton number are currently investigated in great details with selective \( \gamma \)-spectroscopy measurements. In this contribution we focus on the collective response of the nuclear system, in particular on the warm rotation which allows to obtain nuclear structure information in the transition region between the ordered system at temperature T = 0, and the chaotic systems at the compound nucleus level. In fact, the \( \gamma \)-decay of excited rotational nuclei, produced at high spins and moderate excitation energy by fusion reactions between heavy ions, provides information on nuclear structure properties as a function of angular momentum and excitation energy: it probes the region of high level density where a strong mixing between states takes place, leading to the formation of compound nucleus levels [1].

Experimentally, the \( \gamma \)-cascades from the excited nucleus can be measured, with great selectivity, by the use of high-efficiency HpGe-array spectrometers (such as EUROBALL or AGATA in the near future), consisting of more than 100 Ge crystals in 4\( \pi \) geometry around the reaction center, usually combined with other types of ancillary detectors.

After a brief review of the experimental techniques used to extract quantitative nuclear structure information from \( \gamma \)-coincidence spectra (presented in section 2), we will discuss some of the basic results obtained in the study of the warm rotation, such as i) the fundamental role played by the two-body residual interaction in the description of nuclear structure properties already at few MeV of excitation energy above the ground state (section 3); ii) the onset of the chaotic regime, as a weakening of selection rules on specific quantum number such as K (section 4); the evidence for a
persistence of nuclear structure effects in the order-to-chaos region, in connection with the warm rotational motion of superdeformed nuclei (section 5).

2. Statistical Analysis of Rotational Spectra

By constructing $\gamma$-$\gamma$ spectra from the measured $E_\gamma$ energies of high-fold $\gamma$ cascades (produced by the de-excitation of warm nuclei at high spins), one can distinguish the contribution from different regimes: the $\gamma$-decay from the COLD region of discrete bands at $T=0$ forms ridges in the two-dimensional spectrum, while the excited rotational motion from the WARM region where the decay becomes more irregular/fragmented over many final states gives rise to a rather uniform distribution, filling also the central valley region. This is shown in panel a) of Figure 1, in the case of the normal deformed nucleus $^{168}$Yb: the ridge-valley landscape is a direct consequence of the constant spacing between $E^2$ transitions in a regular rotational band, which can not produce counts along the $E^2_\gamma = E^2_{\gamma 2}$ valley region, if the moment of inertia of the band stays constant.

The very existence of many, closely spaced, excited bands results in a quasi-continuum (QC) ridge-valley spectrum, i.e. in a distribution of $\gamma$-transitions which can not be resolved experimentally due to the finite detector resolution, although generated by a discrete energy spectrum. As a consequence, the fluctuations of the events collected in the $\gamma$-$\gamma$ spectrum acquire a statistical meaning, providing a powerful tool to estimate the number of bands populating the ridge-valley structures [2]. In particular, the effective number of bands (named paths, $N_{\text{path}}$) both in the ridge and valley region, can be estimated through the relation

\[ N_{\text{path}} = \frac{N_{\text{nev}}}{\frac{\mu_2}{\mu_1} - 1} \times P^{(2)} \]

being $N_{\text{nev}}$, $\mu_1$ and $\mu_2$ the number of events and the first and second moment of the distribution of counts in a given sector [2]. In the previous expression $P^{(2)}$ is a correction factor taking into account the finite energy resolution of the detection system. The analysis is typically performed in a two-dimensional energy region where a rotational band contributes in average with one transition, namely $\Delta E_\gamma \times \Delta E_\gamma = 4h^2/\mathcal{I}(2) \times 4h^2/\mathcal{I}(2)$, being $\mathcal{I}(2)$ the effective dynamic moment of inertia of the rotational bands.

Figure 1, panels b) and c), shows the average spectrum “Fit” (obtained as a smooth polynomial interpolation of the two-dimensional ridge-valley distribution shown in panel a)) and the resulting $\mu_2/\mu_1$ fluctuation spectrum, which strongly emphasizes the original ridge-valley landscape: large fluctuations are observed along the ridges, pointing to a limited number of discrete regular bands similar to the yrast band, while the small fluctuations observed along the valley suggest the existence of a large number of bands in the warm region of fragmented decay.

The fluctuations of the spectrum events can also provide information on the similarities/correlations between two different event distributions. This can obtained by studying the spectrum covariance [3], defined as

\[ \mu_{2,\text{cov}}(A,B) = \frac{1}{N_{\text{nev}}} \sum_j (M_j(A) - <M>_j(A)) \times (M_j(B) - <M>_j(B)) \]

where $M(A)$ and $M(B)$ represent the two event distributions, while $<M>_j$ denotes, again, an average “Fit” spectrum. To normalize the covariance and determine the degree of correlations between spectrum A and B, the correlation coefficient $r(A,B)$ is defined as

\[ r(A,B) = \frac{\mu_{2,\text{cov}}(A,B)}{\sqrt{(\mu_2(A) - <\mu_2>_j(A))(\mu_2(B) - <\mu_2>_j(B))}} \]
The subtraction of the average value of the first moment (over the region $N_{ch}$) in the denominator corrects for the contribution to $\mu_2$ from counting statistics. From the previous definition one finds that the $r$ correlation coefficient becomes 1 for identical distribution, 0 for completely different spectra. Illustration of the basic idea behind the covariance technique is shown in the right column of Figure 1, in the case of a limited region of a $\gamma-\gamma$ spectrum.

The fluctuations and covariance techniques have been employed in the study of quasi-continuum ridge-valley spectra in different region of mass and deformation, as discussed in the next sections.

**Figure 1**: Example of spectral distributions employed in the statistical analysis of $\gamma-\gamma$ correlated matrices. The left column refers to the fluctuation analysis, from which the effective number of decay paths, $N^{(2)}_{path}$, is extracted, starting from the events distribution (panel a)) and the second moment fluctuation spectrum $\mu_2/\mu_1$ (panel c)). The last one is calculated relative to the average spectrum (Fit), shown in the middle panel b). The right column illustrates the basic idea behind the covariance technique. The top and middle panels show two sections of $\gamma-\gamma$ spectra (spectrum A and B) with the red and blue arrows indicating the positions where similar/different structures are present. In the bottom panel the corresponding covariance spectrum $\mu_{2,cov}$ is given, with emphasized/suppressed structures in correspondence with similar/different distributions (see text for details).
3. Evidence for a band mixing regime
One of the first, basic results obtained from the analysis of quasi-continuum spectra with statistical techniques, is the evidence for the fundamental role played by the two-body residual interaction in the description of the warm rotation [4]. As shown in Figure 2, the number of decay paths extracted from the analysis of the ridge and valley structure for a typical deformed nucleus of mass A=160, such as $^{168}$Yb, is of the order of $30$ and $10^5-10^6$, respectively. It is found that these experimental results can be well explained by microscopic cranked shell model calculations, only including a two-body residual interaction $V_{res}$ of surface delta type (red solid lines in each panel). This gives rise to a fragmentation of the rotational decay above $\sim 1$ MeV of excitation energy above yrast, which largely enhances the number of decay paths the nucleus can follow in its warm decay. Moreover, it significantly reduces the number of discrete regular bands in the cold regime. On the contrary, a pure cranked shell model (mean field, MF) description of the excited energy spectrum would provide too many discrete regular bands populating the ridges, and too few ($\sim 10^3$) decay paths populating the valley region, in agreement with simple estimates based on pure level density calculations.

![Figure 2](image)

**Figure 2.** The effective number of path $N^{(2)}_{\text{path}}$ extracted from the analysis of the first ridge (bottom) and central valley (top) of the measured and calculated $\gamma-\gamma$ coincidence spectra of $^{168}$Yb. Symbols refer to data, dashed lines to simulations based on non-interacting bands (mean field description, MF), solid lines to simulations based on bands mixed by the two-body residual interaction (adapted from ref. [4]).

4. Onset of chaos in the atomic nucleus
The statistical analysis of the ridge-valley landscape can also be used to investigate the transition between order and chaos in the atomic nucleus. This can be done by looking at the gradual vanishing of the selection rules associated with the quantum numbers at $T = 0$, such as $K$ (the projection of the angular momentum on the symmetry axis). The deformed nucleus $^{163}$Er is an ideal case, being characterized by a number of rotational bands having low-K ($K=5/2$) and high-K ($K=19/2$) values [5]. Following a high statistics EUROBALL experiment, $\gamma-\gamma$ matrices in coincidence with low-K and high-K structures have been constructed and analyzed by fluctuation and covariance techniques. It is found that the fluctuation analysis give evidence for a total number of $\sim 40$ discrete excited bands, populating the ridge structures of $^{163}$Er, half of which of high-K nature. On the contrary, many more
bands, of the order of $10^3$-$10^5$, are found to populate the valley region, with large differences between low-K and high-K states, being the latter $\sim$10 times fewer. This suggests that the K-quantum number is at least partially conserved up to moderate excitation energies, of the order of $\sim$1.5 MeV above yrast, where the rotational motion is fragmented. At higher excitation energies more similar number of bands are obtained for low-K and high-K gated spectra, pointing to a vanishing of selection rules on K and to the onset of a chaotic regime, in which quantum numbers and selection rules lose their meaning. The previous results are strongly supported by the covariance analysis of the spectrum fluctuations [5], which provides specific information on the correlations between spectra gated by different K-states, as shown in Figure 3. In particular, the ridge and valley covariance between spectra gated by low-K states leads to a correlation of the order of 20%, as a consequence of the cross talk by E1 transitions (panel a) and c). On the contrary, in the case of high-K and low-K covariances, no correlations are found from the analysis of the ridge structures (panel b)), while from the valley region one obtains strong similarities/correlation between high-K and low-K gated distributions only at the highest transition energy values (panel d)). This suggests a gradual transition to a chaotic regime around 1.5 MeV of internal energy. The experimental results shown in Figure 3 are well reproduced by a shell model which combines a cranked mean-field and a residual two-body interaction, together with terms taking into account the angular momentum carried by the K-quantum number [6]. According to the model, K-mixing is induced by the interplay of the Coriolis and residual interaction, and it is found to gradually increase until a complete violation of the K-quantum number is reached above 2-2.5 MeV of internal energy.

![Figure 3](image.png)

**Figure 3.** Results of the covariance analysis of ridge/valley structures of $^{163}$Er (bottom/top panels). Symbols refer to data, lines to theoretical predictions, as obtained from the analysis of simulated spectra. Panels a) and c) give the correlation coefficient $r$ obtained by averaging over pairs of $\gamma-\gamma$ spectra gated by low-K configurations, while results for the low-K versus high-K matrices are shown in panels b) and d) [5].
5. Nuclear structure effects at the onset of chaos in superdeformed nuclei

The study of the thermally excited, rapidly rotating atomic nucleus becomes particularly challenging both experimentally and theoretically, in the case of very elongated systems. The study of superdeformed (SD) nuclei requires, in fact, high selectivity and high statistics, in order to focus on the small fraction of the decay (few %) which is finally trapped into the SD well and populates discrete rotational bands. A step forward in the understanding of the population and decay of warm SD nuclei has been recently obtained by a comparative study of the nuclei $^{151}$Tb and $^{196}$Pb, populated with high statistics in two EUROBALL IV experiments [7-9]. The analysis is performed in terms of intensities and fluctuations of several independent observables extracted from quasi-continuum (QC) $\gamma$-coincidence spectra, providing quantitative information on the dynamics of the $\gamma$ decay flow and of the tunneling through the potential barriers between SD and normal deformed (ND) excited states, over a wide spin range. A key point is the comparison of the data with predictions based on a “microscopic” Monte Carlo simulation of the $\gamma$-decay in the SD well, i.e. on simulations based on discrete levels calculated microscopically and on microscopic potential barriers between the ND and SD well. It is found that such a detailed study of the order-to-chaos region reveals the influence of nuclear structure effects. In particular, an enhancement in the E1 strength around 1–2 MeV, which could be related to octupole vibrations, is found to be needed to reproduce the intensity of the $\gamma$-decay flow directly feeding into the SD band. This is shown in Figure 4, in connection with the population of the SD yrast (panel a)) and of the first ridge/E2 QC in coincidence with the SD yrast (panel b) and c)).

Figure 4: Influence of the low-energy component of the E1 strength (associated with octupole vibrations) on the population of the SD yrast (panel a)), SD-gated 1$^{\text{st}}$ ridge (panel b)) and SD-gated E2-QC (panel c)) of $^{151}$Tb. In each panel, symbols refer to data, while dashed lines correspond to simulations with standard GDR strength. Thick (thin) solid lines are obtained from calculation including an extra component in the total E1 strength $f_{\text{E1}}$, as shown in the top panel [9].
Furthermore, the simulated intensities and fluctuations can be used to probe the tunneling probability between the SD and ND well, which strongly depends on the shape and height of the microscopically calculated potential barriers, and on the collective mass of the system. This latter quantity is one of the most uncertain parameter, being strongly driven by pairing. It is found that the decay-out properties of nuclei of mass $A=190$ are particularly sensitive to the magnitude of the collective mass. This is illustrated in Figure 5 where we show the results of the simulation of $^{196}$Pb for different value of the parameter $C_m$, which is here used to scale the collective mass. As one can see, both the intensity of the yrast and of excited bands (panels a) and b)) and the number of discrete excited bands obtained by the fluctuation analysis of the ridge structure (panel c)) are very sensitive to the $C_m$ value. In particular, the analysis shows the need for a strong renormalization of the collective mass ($C_m \sim 2.5$) in order to obtain a reasonable agreement with the data, both for the yrast and excited bands.

![Figure 5: Dependence of the results of the simulation of $^{196}$Pb on the scaling factor $C_m$ of the mass parameter. Panels a) and b) refer to the intensity of the SD yrast and of the 1st ridge, panel c) to the number of paths populating the 1st ridge. Symbols refer to the corresponding experimental data (see text for details).](image)

6. Conclusions
We have presented selected examples of nuclear structure studies at high spins and temperature, focusing on the $\gamma$-decay of the warm rotating nucleus. It is shown how $\gamma$-spectroscopy in the quasi-continuum provides a unique tool for investigating in details the transition between order and chaos, which takes place $\sim 1.5$ MeV of excitation energy above yrast. The experimental study is based on statistical analysis techniques and the comparison with theory requires a simulation of the $\gamma$-flow based on microscopic quantities (e.g. energy levels and potential barriers). These studies provide a stringent test to the conservation of selection rules, configuration changes and pairing with temperature.

7. References
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