Raman scattering from phonons and magnons in RFe$_3$(BO$_3$)$_4$

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Inelastic light scattering spectra of several members of the RFe$_3$(BO$_3$)$_4$ family reveal a cascade of phase transitions as a function of temperature, starting with a structural, weakly first order, phase transition followed by two magnetic phase transitions. Those consist of the ordering of the Fe-spin sublattice revealed by all the compound, and a subsequent spin-reorientational transition for GdFe$_3$(BO$_3$)$_4$. The Raman data evidence a strong coupling between the lattice and magnetic degrees of freedom in these borates. The Fe-sublattice ordering leads to a strong suppression of the low energy magnetic scattering, and a multiple peaked two-magnon scattering continuum is observed. Evidence for short-range correlations is found in the ‘paramagnetic’ phase by the observation of a broad magnetic continuum in the Raman data, which persists up to surprisingly high temperatures.

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I. INTRODUCTION

The family of RM$_3$(BO$_3$)$_4$ crystals (R=Y or a rare earth, M=Al, Ga, Sc, Cr, Fe) has attracted considerable attention during the last years. Different combinations of R and M lead to a large variety of physical properties that, together with their excellent physical characteristics and chemical stability, make these crystals extremely interesting both from application and fundamental points of view. The lack of inversion symmetry (they crystallize in a trigonal space group) has stimulated different applications in the field of optical and optoelectronic devices. Crystals of YAl$_3$(BO$_3$)$_4$ and GdAl$_3$(BO$_3$)$_4$ doped with Nd have been widely studied and used in optical devices, for self-frequency doubling and self-frequency summing lasers.$^{1,2,3}$ Concentrated NdAl$_3$(BO$_3$)$_4$ crystals are efficient media for mini-lasers.$^3$ Rare-earth borates with magnetic ions (M = Cr, Fe) are much less studied. Recently, some iron borates were reported to possess multiferofer features, demonstrating the coexistence of magnetic and ferroelectric order parameters.$^4$ They may therefore be considered for optoelectronic applications. These magnetic borates are also expected to exhibit interesting magnetic properties because of the presence of two different kinds of magnetic ions (3d and 4f elements), and in particular because their structure features isolated helicoidal chains of magnetic 3d atoms.

The crystal structure of RM$_3$(BO$_3$)$_4$ borates belongs to the structural type of the natural mineral hunteite CaMg$_3$(CO$_3$)$_4$ that crystallizes in the space group $R32$ of the trigonal system.$^{4,5,6,7,8}$ The primitive unit cell contains one formula unit. Three kinds of coordination polyhedra are present, trigonal prisms for RO$_6$, octahedra for MO$_6$, and two types of planar triangular BO$_3$ groups: equilateral B1O$_3$ and isosceles B2O$_3$ (the numbering of inequivalent ions position is the same as in Ref.8). The MO$_6$ octahedra share edges forming mutually independent helicoidal chains which run parallel to the c-axis (see Fig.1). The RO$_6$ prisms are isolated polyhedra, each of them connects three helicoidal MO$_6$ chains, while there are no direct R-O-R links between different RO$_6$ prisms. This structure can also be seen as a succession of alternating BO$_3$ and R+M layers perpendicular to the c axis.

Recently, a systematic study of rare-earth iron borates RFe$_3$(BO$_3$)$_4$ (R = Y, La-Nd, Sm-Ho) was undertaken for polycrystalline samples.$^9$ The measurements of magnetization, specific heat, and $^{57}$Fe Mössbauer spectra revealed an antiferromagnetic ordering at low temperatures. The magnetic ordering temperature increases with decreasing R$^{3+}$ ionic radius, from 22 K for LaFe$_3$(BO$_3$)$_4$ to 40 K for TbFe$_3$(BO$_3$)$_4$. In addition to this, X-ray diffraction, specific heat

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FIG. 1: (Color online) The structure of RM$_3$(BO$_3$)$_4$ incorporates helicoidal chains of M$^{3+}$ ions. The picture shows the structure of iron borates (M=Fe) viewed from different angles a) perpendicular to the $C_3$ axis and b) parallel to it. The possible exchange paths are shown.
measurements, and differential thermal analysis have indicated a structural phase transition for RFe$_3$(BO$_3$)$_4$ compounds with R = Eu-Ho, Y. Its temperature increases with decreasing $R^{3+}$ ionic radius, from 88 K for EuFe$_3$(BO$_3$)$_4$ to 445 K for YFe$_3$(BO$_3$)$_4$. Magnetic ordering of the La- and Nd-iron borates has been confirmed by spectroscopic and magnetic measurements on non-oriented single crystals. Studies on oriented single crystals have only been carried out for GdFe$_3$(BO$_3$)$_4$, using a variety of methods. Recent specific heat, Raman scattering, and Nd$^{3+}$ probe absorption measurements revealed a cascade of three spontaneous phase transitions in GdFe$_3$(BO$_3$)$_4$. A structural, weakly first-order, phase transition into a less symmetric low-temperature phase was found at $T_S$=156 K (this in contrast to the powder result $T_S$=174 K). At lower temperature, two magnetic transitions have been observed. A second-order phase transition occurs at 37 K, resulting in an antiferromagnetic ordering of the Fe spin sublattice. This transition is accompanied by a polarization of the Gd spin subsystem. The second magnetic transition is observed at 9 K, and has been identified as a first-order Fe spin-reorientation transition. A detailed picture of magnetic phase transitions in GdFe$_3$(BO$_3$)$_4$ as a function of temperature and magnetic field has recently been suggested in Ref. on the basis of antiferromagnetic resonance studies.

Quite recently, the low-temperature structural phase of GdFe$_3$(BO$_3$)$_4$ has been identified as a P3$_1$21 trigonal structure. To our knowledge, no information exists on the low-temperature structure of other rare-earth iron borates. Such information can be obtained from the comparison of low-temperature Raman spectra of GdFe$_3$(BO$_3$)$_4$ and other compounds from the RFe$_3$(BO$_3$)$_4$ family. Raman studies of RFe$_3$(BO$_3$)$_4$ were performed for R=La and Nd at room temperature. Preliminary temperature dependent results on the R=Gd compound have also been reported recently.

The present work investigates two different aspects of the rare-earth iron borates. First, inelastic light scattering studies of the vibrational and structural properties of different RFe$_3$(BO$_3$)$_4$ are presented. Amongst others they reveal the weak first-order phase transition from the high-temperature $R32$ structure to the low-temperature $P3_1$21 one. The second part of this paper focuses on the magnetic properties. Apart from observing changes in phonon frequencies at the antiferromagnetic ordering transition arising from magneto-elastic couplings, the spectra also exhibit relatively strong two-magnon scattering features. The magnetically ordered phases exhibit well defined magnon excitations resulting in a multiple peaked scattering continuum at finite energy. In the paramagnetic phase a strong low energy continuum is found, which results from the persistence of short range correlations in the helicoidal Fe spin chains up to quite high temperatures.

II. EXPERIMENTAL DETAILS

Different samples of RFe$_3$(BO$_3$)$_4$, with R = Gd, Nd, Tb, Er, and Y were grown using a Bi$_2$Mo$_3$O$_{12}$ - based flux, as described in Ref. Unlike the Bi$_2$O$_3$ based fluxes, in Bi$_2$Mo$_3$O$_{12}$ - based flux Bi$_2$O$_3$ is strongly bonded to MoO$_3$ which excludes a partial substitution of bismuth for a rare earth during crystallization. Spontaneous nucleation from the flux resulted in small single crystals (1x1x1 mm). We used these crystals as seeds to grow large (10x5x5 mm) single crystals of Gd, Nd, and Tb iron borates. As for ErFe$_3$(BO$_3$)$_4$ and YFe$_3$(BO$_3$)$_4$, only small single crystals of these compounds were studied in this work. All the crystals were green in color. The samples were oriented either by X-ray diffraction technique or by using their morphology and optical polarization methods.

The Raman measurements were performed in a backscattering configuration, using a three-grating micro-Raman spectrometer (T64000 Jobin Yvon) equipped with a liquid nitrogen cooled charged coupled device (CCD) detector. The frequency resolution was better than 1 cm$^{-1}$ for the frequency region considered. The samples were placed in a optical microscope cryostat. The temperature was varied from 2.7 to 500 K, with a stability of $\pm 0.02$ K. The scattering was excited by the second harmonic light of a Nd:YVO$_4$ laser (532 nm), focused down to $\sim 50 \, \mu$m$^2$ with the power density on the sample kept below 0.1 mW/µm$^2$. The polarization was controlled both on the incoming and outgoing beams giving access to all relevant polarization combinations.

Most of the experiments were performed with light incident perpendicular to the C$_3$-axis of the crystal, $\vec{k}\perp c$ (as a rule, precise orientation of $\vec{k}$ in the (ab)-plane was not known), however, when the shape of the samples allowed, also the $\vec{k}\parallel c$ configuration has been employed. Spectra are marked using the Porto notation. For $\vec{k}\perp c$, only the polarization of incident and scattered light are indicated. The conventions $x||C_3$, $x||C_2$ are used to describe the geometry of the Raman experiments. We were able to obtain only partially polarized Raman spectra of ErFe$_3$(BO$_3$)$_4$ and YFe$_3$(BO$_3$)$_4$.

Preliminary dielectric measurements were performed for RFe$_3$(BO$_3$)$_4$ on non oriented crystals of typical size 1x1x0.2 mm$^3$. The surfaces were polished and covered with Ag paste and the electrical contacts were made using Pt wires connected to the surface by additional Ag paste. The samples were mounted on a home-made insert for a Quantum Design PPMS system. The capacitance was measured using an Andeen-Hagerling AH2500A capacitance bridge operating at a fixed frequency of 1kHz.
light propagating along the polarizability tensor (Raman activity). Doubly degenerated E modes are polar and both IR and Raman active. The 3
FIG. 4: Raman spectra of GdFe$_3$(BO$_3$)$_4$ at room temperature. The bars at the top indicate the frequency regions for the internal vibrations of BO$_3$ groups.

FIG. 2: Polarized Raman spectra of GdFe$_3$(BO$_3$)$_4$ at room temperature. The regions where the internal BO$_3$ vibrations are expected are indicated by the horizontal bars in the top of the panel. The spectra show a strong dependence on the polarization of the incoming and scattered light, but also show a distinct difference between (xx+yy+xy) polarized spectra for $\vec{k}$ parallel to c and for $\vec{k}$ perpendicular to c. The spectra for different rare earth iron borates are, as expected, qualitatively quite similar, as is shown in Fig. 3.

The phonon lines narrow progressively with lowering the temperature and several new modes appear abruptly in GdFe$_3$(BO$_3$)$_4$, TbFe$_3$(BO$_3$)$_4$, ErFe$_3$(BO$_3$)$_4$, and YFe$_3$(BO$_3$)$_4$, at $T_s$=155 K, 198 K, 340 K, and 350 K, respectively. Figures 4 and 5 and Tab. III give a detailed comparative picture of the phonon modes observed above and below $T_s$ in RFe$_3$(BO$_3$)$_4$. The intensities of the new modes show a hysteresis as a function of temperature (see Fig. 6). The lowest-frequency intense new mode appears abruptly below the phase transition, but then exhibits a soft-mode-like behavior with further lowering of the temperature; i.e. for GdFe$_3$(BO$_3$)$_4$ and TbFe$_3$(BO$_3$)$_4$ the low frequency mode shifts from 26 cm$^{-1}$ at $T_s$ to respectively 56 cm$^{-1}$ and 57 cm$^{-1}$ at 3 K, and for ErFe$_3$(BO$_3$)$_4$ and YFe$_3$(BO$_3$)$_4$ it goes respectively from 25 to 72 cm$^{-1}$ and from 27 to 76 cm$^{-1}$. At the same time its linewidth decreases from 12 to 3 cm$^{-1}$ (see Figs. 7a and 4b). The spectra in (xz+yz) polarization of all the compounds demonstrate a low-frequency scattering continuum that gradually changes its shape with lowering the temperature and exhibits the opening of a “gap” at $T_N$ (see Fig. 8a). For GdFe$_3$(BO$_3$)$_4$ at the lowest temperatures a well-defined narrow peak at about 10 cm$^{-1}$ is observed within the gap. This mode shifts to zero frequency and strongly broadens upon approaching $T_N$ from below (Fig. 8c). For the Gd compound a similar mode is observed below $T_N$ at about 18 cm$^{-1}$.

III. EXPERIMENTAL RESULTS

The room-temperature (RT) spectra of GdFe$_3$(BO$_3$)$_4$ for all studied experimental configurations are presented in Fig. 2. The regions where the internal BO$_3$ vibrations are expected are indicated by the horizontal bars in the top of the panel. The spectra show a strong dependence on the polarization of the incoming and scattered light, but also show a distinct difference between (xx+yy+xy) polarized spectra for $\vec{k}$ parallel to c and for $\vec{k}$ perpendicular to c. The spectra for different rare earth iron borates are, as expected, qualitatively quite similar, as is shown in Fig. 3.

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IV. GROUP-THEORETICAL ANALYSIS

A. High-temperature structure $R32$ ($D_3^2$)

The primitive cell of the high-temperature structure $R32$ of RFe$_3$(BO$_3$)$_4$ contains 20 atoms which results in 57 vibrational normal modes. Knowing the local symmetry of all the atomic positions, one can perform the factor-group analysis to find the symmetries of these modes$^{18,19}$. R and B1 atoms reside in highly symmetric $D_3$ positions$^8$ and generate $A_2 \oplus E$ modes each. Fe, B2, O1, and O2 atoms are in $C_2$ symmetry positions, which results in $A_1 \oplus 2A_2 \oplus 3E$ modes for each of them. O3 atoms occupy general $C_1$ positions and give $3A_1 \oplus 3A_2 \oplus 6E$ modes. Summing all these modes and subtracting the $A_2 \oplus E$ acoustic modes one gets the following optical vibrational modes of the crystal, in accordance with the results of Ref.$^{15}$.

\[
\Gamma_{vibr} = 7A_1(xx,yy,zz) \oplus 12A_2(E//z) \\
\oplus 19E(E//x,E//y,xz,yz,xy)
\] (1)

Notations in parentheses refer to the allowed components of electric dipole moment (infrared (IR) activity) and the polarizability tensor (Raman activity). Doubly degenerated E modes are polar and both IR and Raman active. The light propagating along the c-axis ($\vec{k}||c$) probes TO modes, while the $\vec{k} \perp c$ configuration gives pure LO modes$^{15}$.

FIG. 4: Raman spectra of GdFe$_3$(BO$_3$)$_4$ above and below $T_s$=156 K, namely, at 161 K (the lowest traces in each panel), 150 K (middle traces), and 30 K (upper traces). (a) (zx+zy) polarization; (b) (zz) polarization. New modes that appear below $T_s$ are indicated by arrows. Asterisks mark the lines from another polarization.
FIG. 5: Comparison of several low-temperature new modes in different RFe$_3$(BO$_3$)$_4$. Spectra in (xx+xy)-polarization for R=Gd and Tb and partially polarized spectra for R=Y and Er.

FIG. 6: The lowest-frequency $A_1$-mode appearing in low-temperature $P3_121$ structure demonstrates hysteresis in a plot of its intensity vs temperature. The data for Gd(Tb)-compound are presented by open (black) triangles. Inset shows this mode during cooling down for temperatures 154.91, 155.03, 155.08, 155.11, 155.16, 155.20, 155.45 K in Gd-compound. The lower temperature axis refer to R=Gd, the upper one - to Tb.

The strongest bonds in the structure of RFe$_3$(BO$_3$)$_4$ borates are B-O bonds within the planar BO$_3$ triangles. Considering them as molecular units that occupy D$_3$ (for B$_1$O$_3$) and C$_2$ (for B$_2$O$_3$) positions, and adding the modes of translational character coming from R and Fe ions (respectively in D$_3$ and C$_2$ position), we obtain the following optical modes:

$$\Gamma_{\text{tr}}^{\text{vib}} = 2A_1 \oplus 5A_2 \oplus 7E; \quad \Gamma_{\text{lib}}^{\text{vib}} = A_1 \oplus 3A_2 \oplus 4E,$$

(2)

where the librational ones (right equation) come from the BO$_3$ units. The total number of external optical vibrations is, thus,

$$\Gamma_{\text{ext}}^{\text{vib}} = 3A_1 \oplus 8A_2 \oplus 11E$$

(3)

Subtracting (2) from (1), one gets the internal vibrational modes related to the vibrations of BO$_3$ groups:

$$\Gamma_{\text{int}}^{\text{vib}} = 4A_1 \oplus 4A_2 \oplus 8E$$

(4)

The correlation scheme of Fig.9 helps to get a deeper insight into the origin of the vibrational modes of RFe$_3$(BO$_3$)$_4$. It follows from this scheme that two of the seven $A_1$ modes originate from the fully symmetric vibration of the B1O$_3$ and B2O$_3$ groups, two correspond to the B2O$_3$ $\nu_3$ and $\nu_4$ vibrations, and two to the rotational and translational modes of the B2O$_3$ group. The remaining $A_1$ mode results from displacements of the iron atom. Table I indicates the origin of crystal modes in the frequency ranges of internal vibrations of the BO$_3$ molecules.

**B. Low-temperature structure $P3_121(D_3^1)$**

The primitive cell of the low temperature structure contains three formula units (60 atoms) which gives 177 vibrational modes. Unique positions for the R and B1 atoms remain but their symmetry lowers from $D_3$ in the R32 structure to $C_2$ in the $P3_121$ one. On the contrary, instead of single threefold $C_2$ symmetry positions for Fe, B2, O1, and O2 there are now two different positions, namely, a threefold $C_2$ and a sixfold $C_1$ positions for each of them. O3 oxygen atoms occupy now three different general ($C_1$) positions. The factor-group analysis of the low-temperature structure $P3_121$ gives the following optical vibrational modes:

$$\Gamma_{\text{vib}}(P3_121) = 27A_1 \oplus 32A_2 \oplus 59E$$

(5)

Carrying out the same procedure as in the previous subsection one finds what modes correspond to the external (translational and librational) motions of the R and Fe atoms and of the BO$_3$ entities and to the internal vibrations of the BO$_3$ "molecules". The results are summarized in Table II.

**V. DISCUSSION**

**A. Phonons of RFe$_3$(BO$_3$)$_4$ in the R32 and P3$_1$21 phases**

First of all, we consider the polarized room-temperature Raman spectra of RFe$_3$(BO$_3$)$_4$, R=Nd, Gd, and Tb (Fig.5). Table III summarizes the observed Raman modes of these iron borates at RT. It has been constructed taking...
into account the results of the group-theoretical analysis for the $R32$ structure and the frequency ranges for normal modes of the regular planar free ion $(BO_3)^{3-}$ (they are indicated in Fig. 2, see also Tables I and III). The spectra of ErFe$_3$(BO$_3$)$_4$ and YFe$_3$(BO$_3$)$_4$ were only partially polarized. Comparing the 390 K spectra of ErFe$_3$(BO$_3$)$_4$ and the 420 K spectra of YFe$_3$(BO$_3$)$_4$ with the RT spectra of other iron borates we complete Table III for these compounds as well (at room temperature, ErFe$_3$(BO$_3$)$_4$ and YFe$_3$(BO$_3$)$_4$ are already in the low-temperature phase).

The low-frequency part of the Raman spectrum is dominated by external modes arising from translational motions of the R and Fe atoms and from librational and translational modes of BO$_3$ groups. The number and polarization properties of the modes observed below 500 cm$^{-1}$ agree with the group-theoretical predictions. All the modes observed in the frequency ranges of the $\nu_4$, $\nu_2$, and $\nu_1$ vibrations of BO$_3$ correspond well to those derived using the correlation analysis and summarized in Fig. 3. In contrast, there is one extra mode in the region of the $\nu_3$ vibration. Possibly, the strong doublet at about 1220 cm$^{-1}$ in the (zx+zy)-polarization arises due to the Fermi resonance between the $\nu_3$ vibrations and an overtone of the $\nu_4$ vibration.

The highest vibrational frequency (originating from the $\nu_3$ vibration of the BO$_3$ group) observed in ErFe$_3$(BO$_3$)$_4$ differs markedly from the highest frequencies of both TO ($\vec{k} \parallel c$) and LO ($\vec{k} \perp c$) phonons (1280 and 1414 cm$^{-1}$, respectively, in GdFe$_3$(BO$_3$)$_4$). This is because in our particular non-oriented ErFe$_3$(BO$_3$)$_4$ sample the direction of $\vec{k}$ was arbitrary, while the frequencies of polar E modes depend on the direction of $\vec{k}$.

Summarizing the RT results, it is concluded that (i) the number and symmetries of the Raman modes observed for the RFe$_3$(BO$_3$)$_4$ in the high-temperature phase are fully consistent with the predictions of group-theoretical analysis based on the $R32$ space group; (ii) the modes above $\sim$550 cm$^{-1}$ originate from the internal vibrations of the BO$_3$ groups, while those below $\sim$550 cm$^{-1}$ can be considered as external modes.

In the low-temperature $P3_121$ phase, 20 $A_1$ (12 external and 8 internal) and 40 $E$ (24 external an 16 internal) new modes should appear, according to the results of the group-theoretical analysis (see Table III). These new modes arise due to two reasons. First, the symmetries of the local positions for some of the atoms reduce. The intensity of these new modes should be proportional to the deviation from the former symmetric position. Second, the primitive cell contains now not one but three formula units, leading to an additional Davydov (factor-group) splitting proportional to the strength of interaction between equivalent atoms inside the new primitive cell. Both these effects are, as a rule, small. That is why the number of the observed new modes (10$A_1$ $\oplus$ 18$E$ in GdFe$_3$(BO$_3$)$_4$) is lower than the number predicted by group-theoretical analysis. The same new modes appear below $T_s$ in all the compounds RFe$_3$(BO$_3$)$_4$, with R=Gd, Tb, Er, and Y (see Fig. 5 and the bottom of Table III), which strongly suggests the same $P3_121$ low-temperature structure in all of them.

**B. Weak first-order structural phase transition**

The most characteristic Raman feature that announces the phase transition from the high-temperature $R32$ structure to the low-temperature $P3_121$ one is the sudden appearance of a strong new low-frequency mode (see Figs. 4 and 5) in the Raman spectra. As the intensities of new modes are proportional to the squares of atomic displacements, $I \sim \delta^2$, the strongest new mode is believed to be associated with the biggest displacements. A detailed analysis, according to Ref. 4, of the structural changes shows that the biggest displacements are those associated with the BO$_3$ "molecules". In particular, BO$_3$ triangles, perpendicular to the $C_3$ axis in the $R32$ structure, tilt by $\sim 7^\circ$ in the $P3_121$ phase; the B1 atoms shift by $\sim 0.03$ Å from the centers of regular triangles. The shifts of boron ions relative to neighboring oxygen ions create local dipole moments; their triangular arrangement corresponds to an antiferroelectric ordering below the temperature of the structural phase transition. This ordering manifests itself via a strong dielectric anomaly at $T_s$ observed in our preliminary dielectric measurements (Fig. 11).

The structural changes considered above give rise to many new Raman active vibrational modes connected with the BO$_3$ groups (see Table III), in particular, to 4$A_1$ additional librational modes. The energy value of the intense excitation measured is in the typical range of the molecular librations. At 3 K, it is 56, 57, 72, and 76 cm$^{-1}$ for Gd(157), Tb(159), Er(167), and Y(89) compounds, respectively. As the values for the Er and Y compounds are very close, notwithstanding a big difference in atomic masses of Er and Y, we conclude that the rare earth does not take part in this lowest-frequency vibration. A difference between the values for the compounds with relatively big
(Gd, Tb) and small (Er, Y) ions comes from the difference in interatomic distances and, hence, force constants. The ensemble of these observations strongly suggests that the phase transition in RFe$_3$(BO$_3$)$_4$ is correlated to a rotational mode of the BO$_3$ groups.

The abrupt appearance of new phonon modes and the hysteresis observed for their intensities when approaching $T_S$ from different sides indicate the first-order character of the phase transition. The energy of the low frequency mode strongly increases upon lowering the temperature below $T_S$ (Fig. 7). At the lower temperature (3K) the frequency of the mode has more than doubled. Such an anomalously large shift is typical for soft modes that announce a second-order structural phase transition. Apparently, we deal with a so-called "weak first-order" phase transition. It is worth noting that the lowering of the symmetry from R32 ($D_3^2$) to P3121 ($D_3^4$) is indeed also compatible with a second-order phase transition. Thus, the observed first-order character of the phase transition does not depend on symmetry changes but rather can arise from the third order therm in the Landau expansion of the crystal free energy $h$, allowed by the crystal symmetry, or from the negative sign of the fourth order coefficient $u$:

$$G = G_0 + rQ^2 + hQ^3 + uQ^4 + ...,$$

(6)

where $Q$ is the order parameter.

Figure [11] shows the temperatures of the structural phase transition in RFe$_3$(BO$_3$)$_4$ plotted versus ionic radii of R$^{3+}$, as determined in Ref.[3] for powder samples and in the present study for single crystals. A mismatch between our data and those of Ref.[3] could arise from a rather poor quality of the Er and Y iron borate single crystals. A further study is necessary to clarify this question.

Finally we note that NdFe$_3$(BO$_3$)$_4$ preserves the R32 structure down to the lowest temperatures, while RFe$_3$(BO$_3$)$_4$ with R$^{3+}$ smaller than Tb$^{3+}$ have the low-temperature P3121 structure already at room temperature.

C. Magnetoelastic coupling

The inset of Fig.[11] shows the derivative of the peak frequency of the strongest new $A_1$ mode as a function of temperature in the vicinity of the magnetic ordering temperature $T_N$. The coupling of vibrational and magnetic degrees of freedom is evident. The discontinuity in the phonon frequency observed at $T = T_N$ is ascribed to magneto-elastic coupling. The magnetic ordering causes spontaneous magnetostriction, that is, atomic displacements. The latter, evidently, influence vibrational frequencies. Recently, Ref.[4] reported on the study of magnetoelectric and magnetoelastic properties of GdFe$_3$(BO$_3$)$_4$. A strict correlation between both has been shown experimentally and discussed theoretically. The authors of Ref.[4] assumed that, below $T_N$, the crystal symmetry lowers, due to magnetoelastic coupling with spontaneous magnetic moments lying in the ab-plane. This breaks the symmetry of the antiferroelectric arrangement of the electric dipole moments in the P3121 structure and leads to the appearance of an electric polarization which could be responsible for the weak growth of the dielectric constant in the temperature region $T_R < T < T_N$ (see also Fig. [11]). The proposed lowering of crystal symmetry below $T_N$, however, should result in the appearance of new vibrational modes. No indications for additional modes below $T_N$ have been found. This does not necessarily mean that the model by Zvezdin et al. is in error. It might very well be that the structural changes are simply too small to be detected by Raman scattering.

As Reference [5] suggests, the spin-reorientation first-order phase transition at $T_R$ into the antiferromagnetic configuration of spins parallel to the c-axis recovers the P3121 structure (that does not allow the presence of the electric
moments order antiferromagnetically in the direction perpendicular to the c-axis and polarize the Gd subsystem. Below spectra and of the specific heat data, that the Gd subsystem does not undergo a magnetic phase transition at $J$ same chain (distance between different chains. The main exchange interaction is therefore expected to be between Fe ions on the are arranged in helicoidal chains. As a result, the Fe-Fe distance inside the chain is substantially smaller than the $\langle W \rangle$ has been drawn in Ref. $\langle W \rangle$ to a first-order spin-reorientation magnetic phase transition

FIG. 10: Dielectric measurements of GdFe$_3$(BO$_3$)$_4$. The structural phase transition is made evident by a step in the capacitance at $T_B$. No evidence is found of the Fe ordering phase transition at $T_N$, while the capacitance shows a discontinuity at the spin-reorientation phase transition at $T_R$ (displayed in more detail in the Inset).

FIG. 11: (Color online) Temperatures of the structural phase transitions for different RFe$_3$(BO$_3$)$_4$ as a function of the ionic radius. Our data on single crystals are shown by circles, those of Ref$^a$ on powder samples are presented by squares.

dipole moment) and, therefore, is expected to be accompanied by a jump of the dielectric constant. As is clear from Fig. 10 the dielectric function indeed exhibits such a jump, thereby strongly supporting the suggested scenario.

D. Magnetic scattering

As discussed in the introduction, due to the structural properties of the RFe$_3$(BO$_3$)$_4$ family, Fe atoms (S=5/2) are arranged in helicoidal chains. As a result, the Fe-Fe distance inside the chain is substantially smaller than the distance between different chains. The main exchange interaction is therefore expected to be between Fe ions on the same chain ($J_{||}$/$J_{\perp}$), suggesting that low dimensionality could play a crucial role in the magnetic properties of the family. This, together with the presence of two coupled magnetic sublattices makes the compounds with magnetic R$^{3+}$ ions good candidates for exotic magnetic ground states.

A complex magnetic behavior has indeed been found and widely discussed in GdFe$_3$(BO$_3$)$_4$. The first magnetization measurements on oriented single crystals were interpreted under the assumption that both Fe and Gd subsystems order antiferromagnetically at $T_N=40$ K$^a$. In this model, below 10 K, the total magnetic moment of the three Fe sublattices (S = 5/2) oriented at polar angles $\sim 60^\circ$ to the c axis is compensated by the Gd moment (S = 7/2) oriented along the c axis. Sharp singularities in magnetization in the magnetic field $H||c$ observed below 10 K were attributed to spin-flop transitions in the Fe subsystem. It was assumed further that in the range of temperatures between 10 and 40 K the magnetic moments of every Fe sublattice fall into the ab plane, perpendicular to the c axis, preserving the $120^\circ$ azimuthal orientation.

This model was later reconsidered in Ref$^a$. It has been suggested, on the basis of the analysis of the Nd-probe spectra and of the specific heat data, that the Gd subsystem does not undergo a magnetic phase transition at $T_N$ but only gets polarized by the Fe subsystem. The following model has been put forward. At $T_N=37$ K, the iron magnetic moments order antiferromagnetically in the direction perpendicular to the c-axis and polarize the Gd subsystem. Below $T_R=9$ K, the magnetic moments order antiferromagnetically along the c-axis. The temperature $T_R=9$ K corresponds to a first-order spin-reorientation magnetic phase transition$^{12}$. Simultaneously and independently, the same conclusion has been drawn in Ref$^{13}$ where magnetic structures and magnetic phase transitions in GdFe$_3$(BO$_3$)$_4$ were studied using antiferromagnetic resonance experiments. This study revealed also a detailed picture of the magnetic structure and anisotropy for different magnetic phases in GdFe$_3$(BO$_3$)$_4$ as a function of both temperature and magnetic field. At $T_N=38$ K, the Fe magnetic subsystem orders into a two-sublattice collinear easy-plane antiferromagnet and polarizes the Gd spins, which also form a two-sublattice antiferromagnetic subsystem. The anisotropy constant of the Gd subsystem has an opposite sign to that of the Fe subsystem. The Gd contribution to the total anisotropy energy grows with lowering the temperature, in conjunction with the growing polarization of the Gd subsystem, and becomes appreciable below $\sim 20$ K. At $T_R=10$ K the total energy of anisotropy changes its sign which results in the spontaneous spin reorientation transition.

The long-range order of the spins on the iron sublattice manifests itself in the inelastic light scattering experiments with two main features. The first one, as reported in Fig$^b$ is the arising of a low-frequency peak (LFP) at $\sim 10$ cm$^{-1}$ in NdFe$_3$(BO$_3$)$_4$ and at $\sim 18$ cm$^{-1}$ in GdFe$_3$(BO$_3$)$_4$. These peaks soften and broaden upon approaching $T_N$. The energy of 10 cm$^{-1}$ in NdFe$_3$(BO$_3$)$_4$ has been identified, by absorption spectroscopy of Nd$^{3+}$ crystal-field levels, as the exchange splitting energy of the Nd$^{3+}$ ground Kramers doublet at 5 K arising from the interaction with the ordered Fe sublattice$^{10}$. The temperature dependences of the frequency and the linewidth of the Raman LFP are the same as those of the ground-state splitting and, respectively, the ground-level width found from optical spectroscopy$^{10}$. Therefore the 10 cm$^{-1}$ Raman mode observed in NdFe$_3$(BO$_3$)$_4$ is assigned to spin flip scattering on a single Nd$^{3+}$ moment in the effective field created by the Fe sublattice. Most likely, the 18 cm$^{-1}$ excitation observed in GdFe$_3$(BO$_3$)$_4$ has the same origin. The absence of LFP in Raman scattering of YFe$_3$(BO$_3$)$_4$ with nonmagnetic Y$^{3+}$ (see Fig. 9) is in favor of such interpretation. The crystal field of RFe$_3$(BO$_3$)$_4$ splits the ground-state multiplet of an ion R$^{3+}$ with odd number of electrons into $\Gamma_4$ and $\Gamma_{56}$ Kramers doublets within the D$_3$ site symmetry. The analysis of Raman selection

\[ \text{[width=0mm]fig10.eps} \]

\[ \text{[width=0mm]fig11.eps} \]
rules for the ground-state spin flip scattering shows that in the case of the Γ₄ ground state all the components of the Raman tensor are allowed while in the case of the Γ₆ ground state only the antisymmetric xy-yy component is Raman active. Thus, a relatively strong (xz) Raman LFP of NdFe₃(BO₃)₄ points to the Γ₄ symmetry of the Nd³⁺ ground state.

The second major feature of the iron spin ordering is the arising of a broad structured scattering band around 60 cm⁻¹ ascribed to two-magnon Raman scattering involving the creation of a pair of magnons with wave vectors $\vec{k}$ and $-\vec{k}$. Below the Néel temperature this is a characteristic feature for all the different compound investigated of the RFe₃(BO₃)₄ family (R=Y, Er, Tb, Gd and Nd). Figure 8 shows the temperature evolution of this broad signature of magnetic scattering. The spectra reported in Fig. 8 are found in the (xz+yz) polarization only. No evidence of magnetic scattering is found when the $\vec{k}$ vector of light is parallel to the c-axis (z(xx+yy+xy)z).

As discussed in the previous section, NdFe₃(BO₃)₄ does not undergo a structural phase transition and the crystal space group remains the high temperature one (R32), while the symmetry of all the other compounds investigated (for R=Gd, Tb, Y, and Er) is reduced to the space group P3121. It is evident from Fig. 8 that the magnetic scattering spectra of all iron borates are quite similar. So, the presence of inequivalent Fe chains in the low-temperature structure of RFe₃(BO₃)₄, R = Gd, Tb, Y, and Er does not strongly affect the magnetic excitation spectra, and the spectra can be analyzed taking into account only the four different magnetic ions present in the R32 unit cell (three Fe³⁺ ions and one R³⁺ ion). Still, a rather complicated two-magnon spectrum is expected, strongly depending on the anisotropy parameters. At the lowest temperature reached ($T=2.7$ K) the two-magnon spectra indeed show a complex structure exhibiting at least three main peaks (Fig. 8). As the most efficient mechanism of two-magnon scattering in antiferromagnets is usually the exchange-scattering mechanism, and the strongest exchange interaction is between the iron atoms along the helicoidal chains, these peaks presumably arise from three magnon branches representing spin excitations on the iron chains.

The broad magnetic scattering feature of Gd, Tb, Er and Y compounds is approximately centered around $\approx 70$ cm⁻¹, while the one of Nd is centered at lower frequency ($\approx 60$ cm⁻¹). This suggests that the bigger ionic radius of Nd³⁺, causes the interatomic Fe-Fe distances to be larger, and therefore the exchange interaction to be smaller, this would be consistent with the scaling of $T_N$ with the rare earth’s ionic radii. Unfortunately, due to the presence of phonon modes at the same frequency of the magnetic excitation, it is not possible to determine the frequency scaling of the magnetic scattering among the other compounds (Gd, Tb, Er and Y) to confirm it.

At the temperature of the spin-reorientational transition ($T_R$), no drastic changes are observed in the magnon spectra. This observation confirms that the observed scattering is mainly due to magnetic excitations on the Fe³⁺ chains, and sheds light on the proposed exotic magnetic ordering of Fe spins for the GdFe₃(BO₃)₄ compound. The configuration with $120^\circ$ angle between nearest neighbor Fe³⁺ spins for $T_N > T > T_R$ which then reorient at $T_R$ into an easy axis antiferromagnet along the c-axis as proposed in the extension of $T_R$ to lower frequency ($\approx 10$% of the "0 temperature value") at $T_N$ and a subsequent smooth decrease to zero for $T > T_N$, in excellent qualitative agreement with the theory prediction for the moments. Also in the case of RFe₃(BO₃)₄ the two-magnon spectrum softens continuously its frequency approaching the Néel temperature (Fig. 8). Yet, surprisingly, the paramagnon scattering survives at all measured temperatures above $T_N$ (i.e. up to about 10 times $T_N$). As reported in Fig. 12 the first moment of the excitation decreases at $T_N$ less than expected and approaches zero only very slowly above $T_N$. Considering NdFe₃(BO₃)₄, the minimum at $T_N$ and subsequent increases with temperature up to 2.5 $T_N$ of the first moment is possibly due to the presence of luminescence lines overlapping with features of the magnetic scattering. This is likely also the origin of the temperature shift to high frequency (lower absolute energy) of the excitation.

FIG. 12: (Color online) First moment $< \omega >$ of the magnetic excitation vs temperature calculated from 5 to 85 cm⁻¹ for GdFe₃(BO₃)₄ (circles) and NdFe₃(BO₃)₄ (squares). The inset shows the integrated intensity of the magnetic signal at different temperature, compared to the magnetic heat capacity of YFe₃(BO₃)₄ (solid line).
FIG. 13: (Color online) Low frequency Raman scattering above the Néel temperature for GdFe$_3$(BO$_3$)$_4$. The different temperature are depicted with an offset for clarity. The paramagnon scattering is present up to almost $4T_N$. The fit is obtained with an phenomenological curve $f(\omega) = \frac{G(\omega)}{1+\alpha\omega} + L$, where $G$ is a Gaussian distribution centered at 0 frequency, and $L$ is a Lorentzian distribution to take into account the presence of the vibrational mode (see text for discussion).

observed at around 80cm$^{-1}$.

From a theoretical point of view the presence of quasi-elastic scattering well into the paramagnetic phase of three-dimensional antiferromagnets has been ascribed to two possible origins, namely, spin diffusion\textsuperscript{25,26} and spin density fluctuations or energy diffusion\textsuperscript{25,26}. In the scenario of spin diffusion, absent in a perfect one-dimensional system, the four-spin correlation function (describing the Raman response) is well approximated by a Gaussian shape centered at zero energy. In contrast, for the energy diffusion scenario one expects a Lorentzian lineshape in the absence of strong spin-lattice interactions. The integrated intensity $I_m$ in the energy diffusion scenario is expected to follow the magnetic contribution to the heat capacity $C_m$ as $I_m \propto C_m T_{225,26}^2$. In the presence of strong spin-lattice coupling this later relation is expected to remain valid, even though the low energy scattering is expected to broaden significantly due to the rapid relaxation of the magnetic excitation energy into the lattice.

The shape of the low-energy scattering feature in the paramagnetic phase is different from any previously reported. Neither a Lorentzian nor a Gaussian distribution can fit our data. This anomalous shape of the paramagnon scattering continuum is a common feature of all the compounds investigated; Fig. 13 depicts the observed spectra of GdFe$_3$(BO$_3$)$_4$ as an example. The observation that the spectra can not be modelled with either a Gaussian or a Lorentzian response might evidence the presence of important spin-lattice interactions, in line with the observations discussed in the previous section. A comparison of the integrated intensity for GdFe$_3$(BO$_3$)$_4$ and NdFe$_3$(BO$_3$)$_4$ to the magnetic heat capacity determined by Hinatsu \textit{et al.}\textsuperscript{2} (see inset fig. 12) for YFe$_3$(BO$_3$)$_4$ does not show a fair agreement in the ‘paramagnetic’ phase, even though there is a good correlation below the phase transition. Apparently energy diffusion, combined with spin-lattice coupling does not explain the observed scattering. On the other hand, as shown in Figure 13 the spectra for GdFe$_3$(BO$_3$)$_4$ can be fitted nicely at all temperature with a broad phenomenological function consisting of a renormalized Gaussian distribution $I(\omega) = \frac{G(\omega)}{1+\alpha G(\omega)} + L$, where $G(\omega)$ is a pure Gaussian, and $L(\omega)$ is a Lorentzian to account for the phonon line observed in the spectra. This equation is reminiscent of the response function of interacting magnons in low dimensional quantum spin systems, although it would be unclear why this would still hold for a $S = 5/2$ system at high temperatures. At present one can not draw a definite conclusion, but the results do strongly suggest that short range order correlations in the spin system survives up to quite high temperatures. Most likely an interplay between the low dimensionality and strong spin-lattice interactions are responsible for the observed unique behavior of the magnetic scattering up to several times $T_N$, and deserves further studies.

VI. SUMMARY

In summary, structural, magnetic, and magneto-elastic properties of various members of the RFe$_3$(BO$_3$)$_4$ family (R = Gd, Tb, Nd, Er, and Y) have been studied using primarily inelastic light scattering. The compounds show a weak first-order structural phase transition which results in the activation of a strong mode of BO$_3$ librational nature, and its subsequent temperature evolution. The scattering spectra observed in the low-temperature phase are fully consistent with the low-temperature structure reported earlier. A detailed analysis of the vibrational spectra at the temperature of the magnetic ordering transition showed a strong magneto-elastic coupling for all the compounds. The analysis of the two-magnon Raman scattering in the magnetic phases showed qualitatively similar scattering spectra for the five compounds investigated, evidencing that the structural differences between them do not strongly affect the magnetism and the magnetic excitation spectra. Finally an unprecedented an intriguing paramagnon scattering continuum up to quite high temperatures. The origin of this scattering does not seem to be the usual energy diffusion observed in a variety of other magnetic system, and seems to indicate the presence of short-range order spin-spin correlations arising from the low-dimensional nature of these compounds.

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**TABLE I:** Frequency range and assignment of the vibrations of the BO$_3$-group. The second column lists the assignments for the free ions. The third column shows the expected internal modes and their symmetries in the high temperature phase of the rare-earth iron borates (the superscripts 1 and 2 refer to the different borate groups in the rare-earth iron borates).

| Frequency range (cm$^{-1}$) | BO$_3$ vibrations (D$_{3h}$) | Crystal vibrations |
|-----------------------------|-----------------------------|--------------------|
| 600                         | $\nu_4(E')$: in-plane bending | $\nu_4^{(2)}(A_1) + \nu_4^{(1)}(E) + 2\nu_4^{(2)}(E)$ |
| 700-800                     | $\nu_2(A''_2)$             | $\nu_2^{(2)}(E)$   |
| 950                         | $\nu_1(A'_1)$: symmetric breathing | $\nu_1^{(1)}(A_1) + \nu_1^{(2)}(A_1) + \nu_1^{(2)}(E)$ |
| 1250-1400                   | $\nu_3(E')$: asymmetric breathing | $\nu_3^{(2)}(A_1) + \nu_3^{(1)}(E) + \nu_3^{(2)}(E)$ |

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TABLE II: The origin of vibrations for two structural modifications of $\text{RFe}_3(\text{BO}_3)_4$ and the new modes that should appear below the temperature of the structural phase transition $T_s$.

| species          | $T > T_s$: $R32$ ($D_3^2$) | $T < T_s$: $P3_121$ ($D_3^2$) | New modes                        |
|------------------|-----------------------------|--------------------------------|----------------------------------|
| R                | $A_2 \oplus E$              | $A_1 \oplus 2A_2 \oplus 3E$  | $A_1 \oplus 2E \oplus A_2$      |
| Fe               | $A_1 \oplus 2A_2 \oplus 3E$ | $4A_1 \oplus 5A_2 \oplus 9E$ | $3A_1 \oplus 6E \oplus 3A_2$    |
| external BO$_3$  |                              |                                |                                  |
| translational    | $A_1 \oplus 3A_2 \oplus 4E$ | $5A_1 \oplus 7A_2 \oplus 12E$ | $4A_1 \oplus 8E \oplus 4A_2$    |
| librational      | $A_1 \oplus 3A_2 \oplus 4E$ | $5A_1 \oplus 7A_2 \oplus 12E$ | $4A_1 \oplus 8E \oplus 4A_2$    |
| internal BO$_3$  | $4A_1 \oplus 4A_2 \oplus 8E$ | $12A_1 \oplus 12A_2 \oplus 24E$ | $8A_1 \oplus 16E \oplus 8A_2$   |
| Total            | $7A_1 \oplus 13A_2 \oplus 20E$ | $27A_1 \oplus 33A_2 \oplus 60E$ | $20A_1 \oplus 40E \oplus 20A_2$ |
### TABLE III: Observed vibrational modes for the high-temperature phase of several rare-earth iron borates.

|                | A₁  | E₁  | E₂₁ | A₁  | E₁  | E₂₁ | A₁  | E₁  | E₂₁ | A₁  | E₁  | E₂₁ | A₁  | E₁  | E₂₁ |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| NdFe₃(BO₃)₄   | 180 | 89  | 180 | 93  | 93  | 89  | 180 | 93  | 93  | 180 | 89  | 180 | 93  | 93  | 89  |
| GdFe₃(BO₃)₄   | 298 | 159 | 160 | 160 | 160 | 160 | 159 | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 160 |
| TbFe₃(BO₃)₄   | 473 | 193 | 195 | 198 | 198 | 195 | 193 | 198 | 198 | 193 | 195 | 198 | 198 | 195 | 193 |
| ErFe₃(BO₃)₄   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| YFe₃(BO₃)₄    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|                |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| Internal modes (4A₁ ⊕ 8E) |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| NdFe₃(BO₃)₄   | 636 | 579 | 580 | 635 | 579 | 632 | 579 | 632 | 579 | 631 | 579 | 632 | 579 | 632 | 579 |
| GdFe₃(BO₃)₄   | 950 | 625 | 631 | 633 | 635 | 632 | 632 | 632 | 632 | 631 | 631 | 632 | 632 | 632 | 632 |
| TbFe₃(BO₃)₄   | 990 | 669 | 670 | 676 | 676 | 676 | 676 | 676 | 676 | 676 | 676 | 676 | 676 | 676 | 676 |
| ErFe₃(BO₃)₄   | 1220| 734 | 735 | 734 | 734 | 734 | 734 | 734 | 734 | 734 | 734 | 734 | 734 | 734 | 734 |
| YFe₃(BO₃)₄    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|                |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| Modes appearing in the low T phase: 20A₁ (12 ext. and 8 int.) ⊕ 40E (24 ext. an 16 int.) |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| NdFe₃(BO₃)₄   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| GdFe₃(BO₃)₄   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| TbFe₃(BO₃)₄   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ErFe₃(BO₃)₄   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| YFe₃(BO₃)₄    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
