Design and Analysis of a Novel Planar Translational Parallel Robotic Mechanism with Three Limbs

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Abstract. A novel planar translational parallel robotic mechanism with three limbs is designed. The degree of freedom (DOF) and output characteristics of mechanism are studied based on the screw theory. The mathematic models including position, velocity and acceleration are derived according to different forms of the actuated inputs. When the linear displacements of two cylindrical joints are selected as the actuated inputs, the velocity Jacobian is an identity matrix and the mechanism has fully isotropic kinematics characteristics. Singularity of the mechanism is discussed and all singular configurations are explored. Finally, kinematics simulation of the mechanism is carried out by ADMAS and MATLAB software, and the kinematic curves are drawn.

1. Introduction

Compared with the traditional serial mechanism, parallel mechanism has the advantages of compact structure, strong bearing capacity and small synthetic error, and becomes one of the hot topics in the field of mechanism and robotics [1-2]. Because of its low manufacturing cost and relatively simple motion control, less degree of freedom (DOF) parallel mechanism has received more attention and research in recent years [3-4]. Several typical low DOF parallel mechanisms have been successfully applied in the world, such as Delta mechanism [5], Hunt mechanism [6], Diamond mechanism [7], and so on.

As a common mechanism in the mechanical processing industry, planar translational parallel mechanism has attracted wide attention from scholars at home and abroad [8-9]. Most of the planar translational parallel mechanisms have strong kinematic coupling. It is undeniable that strong motion coupling can improve the stiffness and load-carrying capacity of the mechanism. However, it also brings difficulties in solving kinematics and dynamics of the mechanism, complex control design and reduced workspace. In some applications for requiring high accuracy and low bearing capacity, decoupled parallel mechanisms show their superior performance. Carricato [10] studied the type synthesis method of decoupled translational parallel mechanism and designed many new mechanisms. Zhang [11] established the type synthesis method of uncoupled parallel mechanism based on the actuation screw theory and designed many new mechanisms.

In this paper, a novel planar translational robotic parallel mechanism with three limbs is proposed. Mobility of the mechanism is analyzed based on the position and orientation characteristic method. Kinematic equations, including position, velocity and acceleration, are derived. Singularities of the mechanism are discussed in detail. Finally, kinematic simulations are performed as well.
2. Structural design and mobility analysis

The diagram of the novel planar translational parallel robotic mechanism is shown in Figure 1. It consists of a moving platform (MP) connected to a fixed base (FB) and three kinematic chains, including two active chains L1 and L2 and a pure constrained chain L3. Two active chains have the same structure composed of a cylindrical joint (C), a prismatic joint (P) and a revolute joint (R) from the platform to the base in serial. Axes of C joint and R joint are parallel to each other and perpendicular to P joint. So this chain can be recorded as $\{SOC \{C||P\}/|R\}$. The chain L3 only consists of three R joints with parallel axes and is recorded as $\{SOC \{R_{31}||R_{32}||R_{33}\}\}$. Axes of three joints mounted on the base are orthogonal. Similarly, the axes of three R joints installed on the platform are also perpendicular to each other.

![Figure 1. Diagram of the novel parallel robotic mechanism](image)

According to the configuration relationship of mechanism joints, the motion characteristic matrix of three chains can be written as follows

$$M_1 = \begin{bmatrix} t^3 \\ r^1 (// R) \end{bmatrix}$$

(1)

$$M_2 = \begin{bmatrix} t^2 \\ r^1 (// R) \end{bmatrix}$$

(2)

$$M_3 = \begin{bmatrix} r^2 (\perp R_{31}) \\ r^1 (// R_{31}) \end{bmatrix}$$

(3)

where, the right superscript number denotes the DOF of the limb, $t$ the translational DOF and $r$ the rotary DOF.

From equations (1), (2) and (3), the motion characteristic matrix of the mechanism can be obtained, and have

$$M_p = M_1 \cap M_2 \cap M_3 = \begin{bmatrix} t^2 (\perp R_{31}) \\ r^0 (// R_{31}) \end{bmatrix}$$

(4)

Equation (4) shows the limb only has two translational degrees of freedom.

Therefore, DOF of parallel mechanism can be calculated by the following formula

$$F = \sum_{i=1} f_i - \min \left\{ \sum_{i=1} \lambda_i \right\} + \Omega$$

(5)

where, $F$ is DOF of the mechanism, $g$ the total number of the joints in the mechanism, $f_i$ DOF of the $i$th joint, $\lambda_i$ the number of independent displacement equations in the $i$th loop, $\delta$ the number of independent loop, $\min \{\bullet\}$ the minimum number of independent displacement equations in a limb,
\[ \Omega \] the virtual constraint number of mechanism. Therefore, DOF of the mechanism is \( F = 10 - 10 + 2 = 2 \). This result is consistent with the previous analysis.

3. Kinematics analysis of mechanism

The static coordinate \( O-XYZ \) is attached on the base and its origin point \( O \) falls at the intersection of two \( C \) joint axes. \( X \)-axis and \( Y \)-axis coincide with the corresponding \( C \) joint axis, respectively. \( Z \)-axis conforms to the right-hand principle. \( C \) joints in two active limbs are selected as the active joints of the mechanism. In Figure 1, point \( A_i \) is the intersection of \( C_i \) joint and \( P_i \) joint axes, point \( B_i \) is the intersection of \( R_i \) joint and \( P_i \) joint axes, point \( P \) is the centroid of the platform. If the coordinate of point \( P \) is \((x, y, n)\), so the vectors \( OP = r = (x, y, n) \) and \( PB_i = e_i = (0, w, 0) \).

In terms of the screw theory, the kinematic screw of each joint in chain \( L_i \) can be written as

\[
\begin{align*}
\mathbf{s}_{11} &= (0 \ 0 \ 0; \ 1 \ 0 \ 0) \\
\mathbf{s}_{12} &= (1 \ 0 \ 0; \ 0 \ 0 \ 0) \\
\mathbf{s}_{13} &= (0 \ 0 \ 0; \ 0 \ \cos \theta_i \ \sin \theta_i) \\
\mathbf{s}_{14} &= (1 \ 0 \ 0; \ 0 \ \sin \theta_i \ -\cos \theta_i)
\end{align*}
\]

(6)

Where, \( d_i \) is distance between joints \( C_i \) and \( R_i \), \( \theta_i \) is angular displacement of joint \( C_i \).

According to the reciprocal screw theory, we can get the reciprocal screw of limb \( L_i \), and yields

\[
\begin{align*}
\mathbf{s}'_{11} &= (0 \ 0 \ 0; \ 0 \ 1 \ 0) \\
\mathbf{s}'_{12} &= (0 \ 0 \ 0; \ 0 \ 0 \ 1)
\end{align*}
\]

(7)

According to the selection principle for the actuated joints of parallel mechanism, it can be proved that the linear displacements or angular displacements of two cylindrical joints can be used as the actuated inputs. Next, the kinematics of the mechanism will be analyzed in terms of two different input forms.

3.1 Angular displacements as the actuated inputs

3.1.1 Position analysis. Based on the spatial polygon enclosed by points \( O \), \( A_i \), \( B_i \) and \( P \) (seeing Fig.1), the following vector equation can be written, and have

\[
\mathbf{OP} + \mathbf{PB}_i = \mathbf{OA}_i + \mathbf{A}_i \mathbf{B}_i
\]

(8)

Substituting the relative parameters into equation (9), we get

\[
\mathbf{r} + \mathbf{e}_i = m_i \mathbf{s}_{12} + d_i \mathbf{s}_{13}
\]

(9)

Where, \( s_{12} \) is the direction vector of the actuated screw \( s_{12} \) in chain \( L_i \), \( s_{13} \) is the direction vector of \( P_i \) joint, \( m_i \) is the distance from points \( O \) to \( A_i \).

Based on equations (6) and (7), the actuation screw \( \mathbf{s}'_{ai} \) applied to the moving platform by the actuated joint \( C_i \) through chain \( L_i \) can be calculated, yields

\[
\begin{align*}
\mathbf{s}'_{ai} &= (0 \ -\sin \theta_i \ \cos \theta_i; \ d_i \ 0 \ 0)
\end{align*}
\]

(10)

Equation (10) shows that the actuation screw \( \mathbf{s}'_{ai} \) is a line vector and it direction vector is \( \mathbf{s}'_{ai} = (0 \ -\sin \theta_i \ \cos \theta_i) \).

Making the dot products with vector \( \mathbf{s}'_{ai} \) on both sides of equation (9), and have

\[
(\mathbf{r} + \mathbf{e}_i)\mathbf{s}'_{ai} = (m_i \mathbf{s}_{12} + d_i \mathbf{s}_{13})\mathbf{s}'_{ai}
\]

(11)

Because the actuation screw \( \mathbf{s}'_{ai} \) is perpendicular to the axes of \( C_i \) and \( P_i \) joints, it does not work in either direction. Then equation (11) can be simplified as follows

\[
-(y + w)\sin \theta_i + n\cos \theta_i = 0
\]

(12)
Where, \( n \) is the distance between the platform and \( XOY \) plane and it is constant, \( w \) is the distance between points \( P \) to \( B_1 \).

Similarly, by using the vector polygon enclosed by limb \( L_2 \), points \( O \) and \( P \), the following equation can be obtained, and
\[
-(x + f) \sin \theta_2 + n \cos \theta_2 = 0 \quad (13)
\]
where, \( f \) is the distance between points \( P \) to \( B_2 \).
Then, the coordinate of the point \( P \) can be derived from equations (12) and (13), i.e.,
\[
y = n \cot \theta_2 - w \quad (14)
\]
\[
x = n \cot \theta_2 - f \quad (15)
\]
Equations (14) and (15) are the forward kinematics of the mechanism when the angular displacements of \( C \) joints are selected as the actuated inputs.

3.1.2 Velocity analysis. According to the position equations, the velocity equation is achieved by the first-order kinematic influence coefficient method, and
\[
\dot{V} = J \dot{\theta} \quad (16)
\]
Where, \( V = [v_x, v_y]^T \) is the output velocity vector of the moving platform, \( \dot{\theta} = [\dot{\theta}_2, \dot{\theta}_1]^T \) is the input velocity vector of the actuated joints, \( J \) is the velocity Jacobian matrix, and
\[
J = \begin{bmatrix} -n \csc^2 \theta_2 & 0 \\ 0 & -n \csc^2 \theta_1 \end{bmatrix} \quad (17)
\]
Equation (17) shows that the Jacobian is a diagonal matrix, which implies that each output motion of the platform is only controlled by one actuated input. Therefore, the mechanism has uncoupled kinematic characteristics under this kind of input forms.

3.1.3 Acceleration analysis. Acceleration equation of the mechanism can be obtained by using the second-order kinematic influence coefficient method, and have
\[
\ddot{a} = H \ddot{\theta} + J \ddot{\theta} \quad (18)
\]
Where, \( a = [a_x, a_y]^T \) is the output acceleration vector of the platform, \( \ddot{\theta} = [\ddot{\theta}_2, \ddot{\theta}_1]^T \) is the input acceleration vector of actuated joints, \( H \) is
\[
H = \begin{bmatrix} 2n \csc \theta_2 \cot \theta_2 & 0 \\ 0 & 2n \csc \theta_1 \cot \theta_1 \end{bmatrix} \quad (19)
\]

3.2 Linear displacements as the actuated inputs

3.2.1 Position analysis. When the linear displacement of the joint \( C_i \) is taken as the actuated input, the actuation screw \( s_{ai} \) of limb \( L_i \) can be calculated, and
\[
s_{ai} = (1 \ 0 \ 0; \ 0 \ 0 \ 0) \quad (20)
\]
Making dot products with the direction vector \( s_{ai} = (1 \ 0 \ 0) \) of \( s_{ai} \) on both sides of equation (8), yields
\[
(r + e_1)s_{ai} = (m_1 s_{i1} + d_1 s_{i3})s_{ai} \quad (21)
\]
Equation (20) shows that the actuation screw \( s_{ai} \) applied to the platform by joint \( C_i \) is a linear force vector parallel to \( X \)-axis and perpendicular to the direction of the kinematic screw \( s_{i3} \). Consequently, the screw \( s_{ai} \) does not work in the direction of the joint \( P_1 \). Then, equation (21) can be simplified as follows
\[
x = m_i \quad (22)
\]
Similarly, based on the chain $L_2$ we get
\[ y = m_2 \]  
(23)

Equations (22) and (23) are the forward kinematics when the linear displacements of $C$ joints are selected as the actuated inputs.

### 3.2.2 Velocity analysis.
Differentiating equations (22) and (23) with respect to time and rewriting them in matrix form, we have
\[ V = Jm \]  
(24)

where, $m = [\dot{m}_1, \dot{m}_2]^T$ is the output velocity vector of the platform, $J$ is the velocity Jacobian matrix of mechanisms and it is a 2x2 identity matrix. Therefore, when the linear displacements of two $C$ joints are used as the inputs, not only it is a one-to-one control relationship between the inputs and the outputs, but also the determinant of the Jacobian is equal to one. So the mechanism shows fully isotropic characteristics in the whole workspace.

### 3.2.3 Acceleration analysis.
The acceleration equation of the mechanism can be obtained by differentiating equation (24) with respect to time, and have
\[ a = J\ddot{m} \]  
(25)

where, $\ddot{m} = [\ddot{m}_1, \ddot{m}_2]^T$ is the input acceleration vector of actuated joint.

### 4. Singularity analysis
In this paper, the singular configurations of the mechanism are analyzed by the singularity of the Jacobian matrix $J$. When $|J| = 0$ is occurrence, the mechanism is stuck. If $|J| \rightarrow \infty$ is occurrence, the mechanism will lose its bearing capacity.

When the linear displacements of two $C$ joints are taken as the actuated inputs, the Jacobian matrix is an unit one and its determinant is always equal to 1. Therefore, there is no any singular configuration in the whole workspace for the mechanism.

When the angular displacements of two $C$ joints are selected as the inputs, the value of the Jacobian matrix determinant by means of equation (18), and have
\[ |J| = n^2 \csc^2 \theta_1 \csc^2 \theta_2 \]  
(26)

According to equation (26), we know that when $\theta_1 = k \pi$ or $\theta_2 = k \pi (k = 0, 1)$, the value of $|J|$ will attend infinite. At these configurations, the mechanism is out of control, in other words, it shows singular. According to the geometric form of the mechanism, we can get $\theta_i = \arcsin(d_i/n)$. If structural size satisfies $d_i \neq 0$, $\theta_i$ will be always not zero, which means these singular configurations can be avoided.

In addition, if the axes of three revolute joints in the pure constrained chain are located in the same plane, the kinematic screws corresponding to these revolute joints will be linear dependent. In a result, mechanism will be boundary singularity. However, the singular configuration can be avoided by adjusting the structural length of the chain $L_3$.

In summary, whatever the angular displacements or linear displacements of the cylindrical joints are chosen as the actuated inputs, the Jacobian matrix keeps diagonal, which means there is an one-to-one mapping relationship between the inputs and the outputs. So the mechanism has well kinematic decoupling. Especially, when the linear displacements are used as the inputs, the Jacobian of the mechanism is an identity matrix. The mechanism has the same kinematics transferring performance along all directions. Therefore, the linear driving mode is more suitable for practical application.

### 5. Example simulation
Three-dimensional model prototype of the mechanism is set up by Solidworks software, as shown in Figure 1. Then it is imported into ADAMS software to redefine the constraints of each joint. With the linear displacement of the joint in the limb as input, the input displacement equations of the limb $L_1$ and $L_2$ are $X = 10\cos(\pi t/10)$ and $Y = 5\cos(\pi t/5 + \pi /2)$. The kinematic curves including position, velocity and acceleration are plotted based on the virtual prototype, seeing Figure 2. Furthermore, simulation curves are also given by using MATLAB software in terms of the theoretical kinematic equations established here, shown in Figure 3. Comparing two simulation results, it is obvious that the corresponding curves are identical, which verifies the correctness of the kinematical models.

![Position curves](image1)
![Velocity curves](image2)
![Acceleration curves](image3)

**Figure 2.** Virtual prototype simulation curves

![Position curves](image4)
![Velocity curves](image5)
![Acceleration curves](image6)

**Figure 3.** Numerical simulation curves

6. Conclusions
A novel planar translational parallel robotic mechanism with three limbs is proposed in this paper. The output characteristics of the mechanism are analyzed and the kinematic equations of the mechanism under two different input modes are derived. Then, kinematical simulations of the mechanism are completed by ADAMS and MATLAB. The results verify the correctness of the theoretical analysis. It is more interesting that the mechanism has the fully-isotropic characteristics when the linear displacements of two cylindrical joints are selected as the actuated inputs.

Acknowledge
The authors would like to thank the financial support from the Fundamental Project of Key Scientific Research of Henan Advanced Education (18A460001), and Scientific and Technological Project of Henan Province(192102210221).

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