Study of $B_c \to KK$ decay with perturbative QCD approach

Yueling Yang, Junfeng Sun and Na Wang

College of Physics and Information Engineering, Henan Normal University, Xinxiang 453007, China

Abstract

In the framework of the perturbative QCD approach, we study the charmless pure weak annihilation $B_c^- \to K^- K^0$ decay and find that the branching ratio $\mathcal{BR}(B_c \to KK) \sim O(10^{-7})$. This prediction is so tiny that the $B_c \to KK$ decay might be unmeasurable at the Large Hadron Collider.

PACS numbers: 12.39.St 13.25.Hw

*corresponding author
I. INTRODUCTION

The study of the decays of $B$ mesons is important and interesting for the determination of the flavor parameters of the Standard Model (SM), the exploration of $CP$ violation, the search of new physics beyond SM, etc. In recent years, theoretical studies of $B_{u,d}$ mesons have been investigated widely in the literatures. They are tested and supported by the experimental data collected by the detectors at the $e^+e^-$ colliders, such as the CLEO, Babar, and Belle. With the bright hope arising from the startup of the CERN Large Hadron Collider (LHC) [1], the heavier $B_s$ and $B_c$ mesons could be produced abundantly and studied in detail at the hadron colliders. It is estimated that one could expect around $5 \times 10^{10}$ “self-tagging” $B_c$ events per year at the LHC [2] due to a relatively large production cross section [3] plus the huge luminosity $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$ [4]. There seems to exist a real possibility to study not only some $B_c$ rare decays, but also $CP$ violation and polarization asymmetries. The study of the $B_c$ mesons will highlight the advantages of $B$ physics.

The $B_c$ mesons are the “double heavy-flavored” binding systems and share many features with the heavy quarkonia. The first observation of the $B_c$ mesons at the Tevatron [5] provokes the physicist’s particular interest in them. Many studies and investigation of the properties of the $B_c$ mesons have been made, and will be further scrutinized by the LHC experiments. Because the $B_c$ mesons lie below the $BD$ threshold (here we only discuss the lightest $1^1S_0$ ground state pseudoscalar $B_c$ mesons, excluding their excited states) and carry flavors, they cannot annihilate into gluon and/or photon so are stable for the strong and/or electromagnetic interaction. Because of the flavor quantum numbers $B = -C = \pm 1$, the $B_c$ mesons can decay through the weak interaction only. The $B_c$ mesons have more decay modes than the $B_{u,d,s}$ mesons due to several reasons. One is that many decay modes, such as $B_c \to B_{u,d,s} + X$, are only accessible by $B_c$ mesons because of their sufficiently large masses. Another is that the $B_c$ mesons carry open flavors, so either $b$ or $c$ quarks can decay individually. The potential decays of the $B_c$ mesons permit us to over-constrain quantities determined by the $B_{u,d,s}$ meson decays.

The decays of $B_c$ mesons can be divided into three classes: (1) the $b$-quark decay (i.e. $b \to q_u$, where the up-type quark $q_u = u, c$) accompanied with the spectator $c$-quark, (2) the $c$-quark decay (i.e. $c \to q_D$, where the down-type quark $q_D = d, s$) accompanied with the spectator $b$-quark, and (3) the annihilation channel (i.e. $B_c^- \to \ell^-\nubar_{\ell}, q_D\bar{q}_U$, where
the lepton $\ell^- = e^-, \mu^-, \tau^-$). Among the multitudinous $B_c$ decays, the weak annihilation channels are expected to take $\sim 10\%$ shares according to the estimates in [2, 3] for which a major part comes from the tree weak annihilation process $B_c^{-} \to s\bar{c}$ which is not helicity-suppressed because of the large charm quark mass and produces a large weak annihilation branching ratio with charm in the final state, while the charmless pure weak annihilation decay $B_c \to KK$ is helicity-suppressed like the $B_d \to KK$ decay and would have a very small branching ratio. It is highly expected that the LHC experiments might shed light on a better understanding of weak annihilation processes for $B_c$ mesons.

In recent years, several attractive methods have been proposed to study the nonleptonic $B$ decays, such as the QCD factorization (QCDF) [7], perturbative QCD method (pQCD) [8,10], soft and collinear effective theory [11,12], etc. Here, we would like to investigate the charmless pure weak annihilation $B_c \to KK$ decay with the pQCD approach due to several reasons. (1) One reason is that fits of nonleptonic charmless decays $B_{u,d} \to PP, PV$ without taking into account weak annihilation contributions are generally of poor quality [13] (here $P$ and $V$ denote the lightest ground pseudoscalar and vector mesons, respectively). Our present understanding of the weak annihilation contributions remains limited and unclear. So the pure weak annihilation processes, such as $B_c \to KK$ decays, are interesting and worthy of study, which will certainly help us to improve our understanding of the weak annihilation contributions. (2) Another is that due to both kinematic improvement from the large phase spaces and dynamic enhancement of the CKM factor $|V_{cb}V_{ud}^*|$, the $B_c \to KK$ decay is expected to have a large branching ratio among two-body nonleptonic charmless $W$-annihilation $B_c \to PP$ processes. In addition to the absence of penguin operators for the tree annihilation process $B_c \to KK$, final state interactions arising from soft gluon exchanges are expected to be extremely small because of the large momenta of the final $K$ mesons. Therefore a relatively accurate estimation of annihilation contributions could be obtained effectively from the charmless $B_c \to KK$ decay. (3) Still another is that Ref.[14] obtains a very large $B_c \to KK$ branching ratio, about 1.6%, at 4 orders of magnitude bigger than the estimate $O(10^{-6})$ of Ref.[15], but this estimate is not valid because Ref.[14] in their calculation incorrectly uses the measured penguin-dominated $B_u^{\pm} \to \pi^{\pm}K$ branching ratio while the decay $B_c \to KK$ is a pure tree weak annihilation and should be related to $B_d \to KK$. In addition, the branching ratio of the charmless decay $B_c \to KK$ is estimated to be $O(10^{-8})$ with the QCDF approach [15]. Recently, this charmless decay is also studied
with the pQCD approach and its branching ratio is \( \mathcal{O}(10^{-7}) \) with the off-mass-shell final states \[16\], which is the same order of magnitude as ours obtained in this paper with the on-shell final states.

This paper is organized as follows: In Section II we will discuss the theoretical framework and give the decay amplitudes for \( B_c \to KK \) with the perturbative QCD approach. In our calculation, we shall ignore the final state interactions because the final states have very large momenta and move far away before soft gluon exchange. Section III is devoted to the numerical result of the branching ratio. Finally, we summarize in Section IV.

II. THEORETICAL FRAMEWORK AND THE DECAY AMPLITUDES

A. The effective Hamiltonian

Using the Operator Product Expansion approach and renormalization group (RG) equation, the low-energy effective Hamiltonian for \( B_c \to KK \) decay can be written as

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^\ast \left\{ C_1(\mu)Q_1 + C_2(\mu)Q_2 \right\} + \text{H.c.},
\]

where \( G_F \) is the Fermi coupling constant for electroweak interactions. \( V_{cb} V_{ud}^\ast \) is the CKM factor accounting for the strengths of the nonleptonic \( B_c \) decays. \( C_i(\mu) \) are Wilson coefficients at the renormalization scale \( \mu \) which have been evaluated to the next-to-leading order with the perturbation theory. The local tree operators are process dependent. Their expressions are defined as

\[
Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha][\bar{d}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta][\bar{d}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha],
\]

where \( \alpha, \beta \) are \( SU(3) \) color indices. The most difficult problem in theoretical calculation of nonleptonic charmless decay \( B_c \to KK \) is how to evaluate the hadronic matrix elements \( \langle KK|Q_{1,2}|B_c \rangle \) properly and accurately.

B. Hadronic matrix elements

For convenience, the kinematics variables are described in the terms of the light cone coordinate. The momenta of the valence quarks and hadrons in the rest frame of the \( B_c \)
meson are defined by

\[ p_{B^-} = p_1 = \frac{m_{B}}{\sqrt{2}}(1, 1, 0), \quad k_1 = x_1 p_1 + (0, 0, \bar{k}_{1\perp}), \]

\[ p_{K^-} = p_2 = \frac{m_{B}}{\sqrt{2}}(1, 0, 0), \quad k_2 = x_2 p_2 + (0, 0, \bar{k}_{2\perp}), \quad n_2 = (1, 0, 0), \]

\[ p_{K^0} = p_3 = \frac{m_{B}}{\sqrt{2}}(0, 1, 0), \quad k_3 = x_3 p_3 + (0, 0, \bar{k}_{3\perp}), \quad n_3 = (0, 1, 0), \]

where \( n_2 \cdot n_3 = 1 \). The null vectors \( n_2 \) and \( n_3 \) are the plus and minus directions, respectively. \( k_1 \) is the momentum of \( c \) quark in the \( B_c \) meson. \( k_2 \) and \( k_3 \) are the momenta of the light non-strange quark in the \( K^- \) and \( K^0 \) mesons, respectively. \( \bar{k}_{i\perp} \) denotes the transverse momentum. \( x_i \) denotes the longitudinal momentum fraction of the valence quark.

The calculation of the hadronic matrix elements is difficult due to the nonperturbative effects arising from the strong interactions. Phenomenologically, using the Brodsky-Lepage approach [17], a modified perturbative QCD formalism has been proposed recently under the \( k_T \) factorization framework [8–10]. Taking into account the transverse momentum of the valence quarks in the hadrons, the Sudakov factors are introduced to modify the endpoint behavior of the hadronic matrix elements. The amplitudes are factorized into three convolution parts: the “harder” functions, the heavy quark decay subamplitudes, and the nonperturbative meson wave functions, which are characterized by the \( W^\pm \) boson mass \( m_W \), the typical scale \( t \) of the decay processes, and the hadronic scale \( \Lambda_{QCD} \), respectively. The pQCD approach has been extensively applied to study semileptonic and nonleptonic \( B \) decays with phenomenological results. More information about pQCD approach can be found in [8–10]. The final decay amplitudes can be expressed as

\[ A(B_c^- \to K^- K^0) \propto C(t) \otimes H(t) \otimes \Phi_{B_c^-}(x_1, b_1) \otimes \Phi_{K^-}(x_2, b_2) \otimes \Phi_{K^0}(x_3, b_3), \quad (3) \]

where the Wilson coefficient \( C(t) \) is calculated in perturbative theory at the scale of \( m_W \) and evolved down to the typical scale \( t \) using the RG equations. \( \otimes \) denotes the convolution over parton kinematic variables. \( H(t) \) is the hard-scattering subamplitude which is dominated by hard gluon exchange and can be factorized. The universal wave functions \( \Phi(x, b) \) absorb nonperturbative long-distance dynamics, which can be extracted from experiments or constrained by lattice calculation and QCD sum rules. \( b \) is the conjugate variable of the transverse momentum of the valence quark of the meson. According to the arguments in [8–10], the amplitude of Eq. (3) is free from the renormalization scale dependence.
C. Bilinear operator matrix elements

Within the pQCD framework, the long-distance hadronic information is contained by the so-called light-cone distribution amplitudes (LCDAs) which are defined from hadron-to-vacuum matrix elements of nonlocal bilinear operators. Although LCDAs are not calculable in QCD perturbation theory, some of their properties are well understood for both light and heavy mesons. For example, the LCDAs for the $K$ meson including higher-twist contributions are systematically presented in [18]. In our calculation, we only consider two-particle (valence quarks) twist-2 and twist-3 LCDAs for $K$ mesons, and neglect contributions from higher Fock states. The LCDAs for $K$ mesons are written as

$$
\langle K(p) | \bar{s}_\alpha(0) q_\beta(z) | 0 \rangle = \frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} \left\{ \gamma_5 \not{p} \phi^a_K(x) + \gamma_5 \mu_K \left[ \phi^p_K(x) - (\not{n}_+ + \not{n}_- - 1) \phi^t_K(x) \right] \right\}_{\beta \alpha},
$$

where $N_c$ is the color number. The parameter $\mu_K$ is the chiral factor $\mu_K = m^2_K/(m_s + m_q)$. The null vector $n_+$ and $n_-$ are parallel to $p$ and $z$, respectively. The expressions of the twist-2 LCDAs $\phi^a_K$ and the twist-3 LCDAs $\phi^p_K, \phi^t_K$ are collected in Appendix A.

Unlike the $\pi$ and $K$ mesons, our knowledge of the LCDAs for $B_c$ mesons has been relatively poor until recently (for a recent view, see [19]), but we know that the $B_c$ mesons are composed of heavy valence quark both $b$ and $c$. Given $m_{B_c} \approx m_b + m_c$, the $B_c$ mesons can be described approximately by nonrelativistic dynamics. In this paper, we will take

$$
\langle 0 | \bar{c}_\alpha(z) b_\beta(0) | B^{-}_c(p_1) \rangle = \frac{if_{B_c}}{4N_c} \int dx_1 e^{-ix_1p_1 \cdot z} \left[ (p'_1 + m_{B_c}) \gamma_5 \phi_{B_c}(x_1) \right]_{\beta \alpha},
$$

where $f_{B_c}$ is the decay constant of the $B_c$ meson. As the arguments in [19], this simplest form, $\phi_{B_c}(x) = \delta(x - m_c/m_{B_c})$, is the two-particle nonrelativistic LCDAs at the tree level where both heavy valence quarks just share the total momentum of the $B_c$ mesons according to their masses. For a rough estimation of the branching ratio for $B_c \rightarrow KK$ decay, we will take the simplest form as an approximation, and neglect the relativistic corrections and contributions from higher Fock states.

D. The decay amplitudes

The $B_c \rightarrow KK$ decay is the pure annihilation process. According to the effective Hamiltonian Eq. (11), the lowest order Feynman diagrams are shown in FIG 1 where (a) and (b)
are nonfactorizable topologies, (c) and (d) are factorizable topologies. After a straightforward calculation using the modified perturbative QCD formalism Eq. (3), we find that the contributions of factorizable topologies are zero, which is a result of exact isospin symmetry. The decay amplitude comes only from the nonfactorizable topologies, and can be written as

\[
\mathcal{A}(B_c^- \to K^- K^0) = -i \frac{G_F 8 \pi C_F m_{B_c}^2}{\sqrt{2} N_c} V_{cb} V_{ud}^* \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 \phi_{B_c}(x_1) \times \left\{ \alpha_s(t_a) C_2(t_a) E(t_a) H(\Delta, \alpha, b_1, b_2) \left[ \phi_{K^-}^a \phi_{K^0}^a (r_b + x_1 - x_3) + r_K - r_{K^0} \phi_{K^-}^b \phi_{K^0}^b (4r_b + 2x_1 - x_2 - x_3) + r_K - r_{K^0} \left( \phi_{K^-}^t \phi_{K^0}^t + \phi_{K^-}^a \phi_{K^0}^a \right) (x_3 - x_2) + r_K - r_{K^0} \phi_{K^-}^t \phi_{K^0}^t (2x_1 - x_2 - x_3) \right] \right\} b_2 = b_3 \]

where the CKM matrix elements \( V_{cb} V_{ud}^* = A \lambda^2 (1 - \lambda^2/2 - \lambda^4/8) + \mathcal{O}(\lambda^8) \) with the phenomenological Wolfenstein parameterization. \( r_b = m_b/m_{B_c} \) and \( r_c = m_c/m_{B_c} \) are the ratios of the mass of \( b \) and \( c \) quark to the mass of \( B_c \) mesons, respectively. \( r_K = \mu_K/m_{B_c} = m_K^2/[m_{B_c}(m_s + m_q)] \). \( C_F = 4/3 \) is the \( SU(3) \) color factor. \( t_{a(b)} \) is the characteristic scale. \( \Delta \) is the virtualities of internal gluons, which is a timelike variable for the pure annihilation \( B_c \to KK \) decay concerned. \( \alpha \) and \( \beta \) are the virtualities of internal quarks. \( E \) and \( H \) are the Sudakov factor and the hard kernel functions, respectively. Their expressions are listed in Appendix B.

### III. NUMERICAL RESULTS AND DISCUSSIONS

The branching ratio in the \( B_c \) meson rest frame can be written as:

\[
\mathcal{BR}(B_c \to KK) = \frac{\tau_{B_c}}{8 \pi \frac{P}{m_{B_c}^2}} |\mathcal{A}(B_c \to KK)|^2,
\]

where \( p \) is the center-of-mass momentum of \( K \) mesons. The lifetime and mass of the \( B_c \) meson are \( m_{B_c} = 6.276 \pm 0.004 \) GeV and \( \tau_{B_c} = 0.46 \pm 0.07 \) ps \(^2\), respectively. Other input
parameters are

\[ m_c = 1.27^{+0.07}_{-0.11} \text{ GeV} \quad \text{(22)}, \quad \lambda = 0.2257^{+0.0009}_{-0.0010} \quad \text{(22)}, \quad f_{B_c} = 489\pm4 \text{ MeV} \quad \text{(23)}, \]
\[ m_b = 4.20^{+0.17}_{-0.07} \text{ GeV} \quad \text{(22)}, \quad A = 0.814^{+0.021}_{-0.022} \quad \text{(22)}, \quad f_K = 159.8\pm1.4\pm0.44 \text{ MeV} \quad \text{(24)}.

If not specified explicitly, we shall take their central values as the default input. The numerical result of the branching ratio is

\[ \mathcal{BR}(B_c^- \to K^0 K^-) \approx [1.63^{+0.67}_{-0.17}(m_b)^{+0.35}_{-0.10}(m_c)] \times [1\pm0.3\%(\text{CKM})\pm1.6\%(f_{B_c})\pm3.7\%(f_K)] \times 10^{-7}, \]

where the errors come from the uncertainties of quark masses \( m_b \) and \( m_c \), the CKM factor \( V_{cb}V_{ud}^\ast \), and the decay constants \( f_{B_c} \) and \( f_K \). The largest error arises from the parameter of \( m_b \), which can reach 40%. The errors arising from both the CKM factor and the decay constants are relatively small. Of course, there are some other uncertainties not considered here, such as the radiative corrections to the LCDAs of \( B_c \) mesons, the final states interactions, etc. So the results might just be an estimation of the pQCD approach.

Our estimation of the branching ratio \( \mathcal{BR}(B_c \to K K) \) is slightly different with the result in [16], although they are calculated with the same pQCD approach resulting in the same order of magnitude \( O(10^{-7}) \). Besides the input parameters, the reasons may be (1) whether the final states are on-mass-shell or not, and (2) whether the contributions of factorizable topologies are zero or not. With appropriate input parameters, the results in [16] and ours are in agreement with each other within an error range.

As the arguments in [15], the inconsistencies among various estimations of the branching ratio \( \mathcal{BR}(B_c \to K K) \), such as \( O(10^{-6}) \) based on \( B_d \) annihilation by using the relations among the charmless weak annihilation \( B_c \) decay channels relying on the \( SU(3) \) flavor symmetry [15], \( O(10^{-7}) \) (or \( O(10^{-8}) \) [15]) based on perturbative one-gluon exchange with the pQCD (or QCDF) approach, arise from conceptually different methods. Anyway, for weak annihilation to light quarks in the final state, the tree annihilation \( B_c^- \to d\bar{u} \) process is helicity suppressed because of small light quark masses, so that gluon emission either from the initial or final state must occur in this annihilation and the decay amplitude is then \( O(\alpha_s) \) as given in pQCD. Both the estimations in [15, 16] and our result are in accordance with

\[ ^c \text{Because of almost equal masses of the final states, the same two-particle LCDAs for the charged and neutral } K \text{ mesons are taken in our calculation. With this approximation, a similar conclusion, that the contributions of factorizable topologies cancel each other because of the isospin symmetry, can be found in } B_s \to \pi \pi \text{ decays with the pQCD approach [25].} \]
an intuitive expectation for nonleptonic charmless $W$-annihilation of heavy meson decays which are usually suppressed. There are some additional factors for the tiny estimation of \( \mathcal{BR}(B_c \to K K) \). One the is that although the $B_c \to K K$ decay is a tree weak annihilation process, its amplitude is color suppressed and associated with $C_2/N_c$. Another is that there is a large destructive interference between the nonfactorizable topologies due to the near equal final state particle masses. This can be clearly found in Eq. (6). The numerical results also confirm the cancellation between the nonfactorizable topologies, and give the strong phases $\sim -31^\circ$ and $\sim +127^\circ$ for FIG. 1(a) and (b), respectively.

If the pQCD prediction is right, then there should be some $10^3$ events for $B_c \to K K$ decay per year at the LHC. Considering the detection efficiency and selection efficiency, there would be just a few events per year. The signal of the pure weak annihilation $B_c \to K K$ decay would be very tiny at the LHC. As $B$ nonleptonic charmless decays, the charmless pure weak annihilation is expected to be small in $B_c$ nonleptonic decays, so the LHC measurement could confirm our understanding of the annihilation terms in weak decays based on perturbative QCD.

IV. SUMMARY

In this paper, we study the $B_c \to K K$ decay with the pQCD approach, which would call for another reassessment of the weak annihilation processes and might provide some valuable hints of our understanding on perturbative QCD and long-distance contributions. It is found that the contributions of factorizable annihilation topologies are zero, and that there is a large cancellation between the nonfactorizable topologies, which result in the branching ratio $\mathcal{BR}(B_c \to K K) \sim \mathcal{O}(10^{-7})$. The branching ratio with the pQCD approach is so tiny that the $B_c \to K K$ decay might not be measured at the LHC experiments.

Acknowledgments

This work is supported by both National Natural Science Foundation of China (under Grant No. 10805014) and the program for Science & Technology Innovation Talents in Universities of Henan Province, China (under Grant No. 2010HASTIT001). We would like to thank the referees for their helpful comments.
Appendix A: Distribution amplitude of the $K$ meson

The expression of the LCDAs of the $K$ meson including higher-twist contributions can be found in [18]. In our calculation, the twist-2 distribution amplitude $\phi^a_K$ and the twist-3 distribution amplitude $\phi^p_K$ and $\phi^t_K$ are [20]

\[
\begin{align*}
\text{twist-2} & \quad \phi^a_K(x) = \frac{f_K}{2\sqrt{2}N_c} 6x\bar{x}\{1 + 0.17C_1^{3/2}(t) + 0.115C_2^{3/2}(t)\} \\
\text{twist-3} & \quad \phi^p_K(x) = \frac{f_K}{2\sqrt{2}N_c}\{1 + 0.24C_2^{3/2}(t) - 0.12C_4^{1/2}(t)\}, \\
\text{twist-3} & \quad \phi^t_K(x) = -\frac{f_K}{2\sqrt{2}N_c}\{C_1^{1/2}(t) + 0.35C_3^{1/2}(t)\}
\end{align*}
\]

where the decay constant $f_K = 160$ MeV. $t = x - \bar{x} = 2x - 1$. The Gegenbauer polynomials are

\[
\begin{align*}
C_1^{3/2}(z) &= 3z, & C_2^{3/2}(z) &= \frac{3}{2}(5z^2 - 1), \\
C_1^{1/2}(z) &= z, & C_2^{1/2}(z) &= \frac{1}{2}(3z^2 - 1), \\
C_3^{1/2}(z) &= \frac{1}{2}(5z^3 - 3z), & C_4^{1/2}(z) &= \frac{1}{8}(35z^4 - 30z^2 + 3)
\end{align*}
\]

Appendix B: Some parameters and formulas

The expression of the Sudakov factors $E$ is

\[
E(t) = \exp \left( -S_{B_c}(t) - S_{K^-}(t) - S_{K^0}(t) \right) \quad \text{(B1)}
\]

where

\[
\begin{align*}
S_{B_c}(t) &= s(x_1p_1^+, b_1) + 2\int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q \\
S_{K^-}(t) &= s(x_2p_2^+, b_2) + s(\bar{x}_2p_2^+, b_2) + 2\int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q \\
S_{K^0}(t) &= s(x_3p_3^-, b_3) + s(\bar{x}_3p_3^-, b_3) + 2\int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q
\end{align*}
\]

The anomalous dimension of the quark is $\gamma_q = -\alpha_s/\pi$. The explicit expression of $s(Q, b)$ can be found in [21].

The hard kernel function $H$ is defined as follows

\[
H(\Delta, Z, b_1, b_2) = \left\{ \theta(b_1 - b_2)\frac{i\pi}{2} H_0^{(1)}(\sqrt{\Delta}b_1)J_0(\sqrt{\Delta}b_2) + (b_1 \leftrightarrow b_2) \right\} \\
\times \left\{ \theta(Z)K_0(\sqrt{Z}b_1) + \theta(-Z)\frac{i\pi}{2} H_0^{(1)}(\sqrt{|Z|}b_1) \right\}
\]

\[
\text{(B5)}
\]
where the hard scales are

\[ \Delta = m^2(1-x_2)(1-x_3) \]  \hspace{1cm} (B6)

\[ -\alpha = m^2(x_1-x_2)(x_1-x_3) - m_b^2 \]  \hspace{1cm} (B7)

\[ -\beta = m^2(x_1+x_2-1)(x_1+x_3-1) - m_c^2 \]  \hspace{1cm} (B8)

\[ t_a = \max \left( \sqrt{\Delta}, \sqrt{\alpha}, 1/b_1, 1/b_2 \right) \]  \hspace{1cm} (B9)

\[ t_b = \max \left( \sqrt{\Delta}, \sqrt{\beta}, 1/b_1, 1/b_2 \right) \]  \hspace{1cm} (B10)

[1] K. Aamodt, et al. (The ALICE Collaboration), Eur. Phys. J. C65, 111 (2009).

[2] N. Brambilla, et al. (Quarkonium Working Group), CERN-2005-005, hep-ph/0412158.

[3] C. H. Chang, C. F. Qiao, J. X. Wang, X. G. Wu, Phys. Rev. D71, 074012 (2005); Phys. Rev. D72, 114009 (2005); C. H. Chang, J. X. Wang, X. G. Wu, Phys. Rev. D77, 014022 (2008); V. A. Saleev, D. V. Vasin, Phys. Lett. B605 311, (2005); A. K. Likhoded, V. A. Saleev, D. V. Vasin, Phys. Atom. Nucl. 69 94, (2006).

[4] http://ab-div.web.cern.ch/ab-div/Publications/LHC-DesignReport.html

[5] F. Abe, et al. (CDF Collaboration), Phys. Rev. D58, 112004, (1998); Phys. Rev. Lett. 81, 2432, (1998).

[6] V. V. Kiselev, hep-ph/0211021; Phys. Atom. Nucl. 56, 643 (1993); Int. J. Mod. Phys. A9, 4987 (1994); Mod. Phys. Lett. A10, 1049 (1995); V. V. Kiselev, A. K. Likhoded, A. I. Onishchenko, Nucl. Phys. B569, 473, (2000); V. V. Kiselev, A. E. Kovalsky, A. K. Likhoded, Nucl. Phys. B585, 353, (2000); hep-ph/0006104.

[7] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B591, 313 (2000).

[8] C. H. Chang, and H. N. Li, Phys. Rev. D55, 5577 (1997).

[9] T. W. Yeh, and H. N. Li, Phys. Rev. D56, 1615 (1997).

[10] Y. Y. Keum, H. N. Li, and A. I. Sanda, Phys. Lett. B504, 6 (2001); Phys. Rev. D63, 054008 (2001).

[11] C. W. Bauer, S. Fleming, D. Pirjol, I. W. Stewart, Phys. Rev. D63, 114020 (2001).

[12] C. W. Bauer, D. Pirjol, I. W. Stewart, Phys. Rev. D65, 054022 (2002).

[13] M. Beneke, M. Neubert, Nucl. Phys. B675, 333 (2003).

[14] R. C. Verma, A. Sharma, Phys. Rev. D65, 114007 (2002).
FIG. 1: The lower order Feynman diagrams contributing to the $B_c \to KK$ decay, with (a) and (b) for nonfactorizable annihilation, (c) and (d) for factorizable annihilation.