Symbolic Dynamics of Music from Europe and Japan

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Abstract

After a brief introduction to the theory underlying block-entropy, and its relation to the dynamics of complex systems as well as certain information theory aspects, we study musical texts coming from two distinct musical traditions (Japanese and Western European) encoded via symbolic dynamics. We quantify their information content or also known as the degree of “non-randomness” which essentially defines the complexity of the text. We analyze the departure of “total randomness” to the constrains underlying the dynamics of the symbol generating process. Following Shannon on his attribution to these constraints as the emergence of complexity, we observe that it can be accurately assessed by the texts’ block-entropy versus block-length scaling laws.
1 Introduction

One of the first accounts of a learned westerner \cite{1} when encountering the rich Japanese musical tradition highlighted the great differences in genre, tonality and methodological ethos between western and eastern music, expressing a great admiration toward it. As it is well known, the Chinese and Japanese tuning also differs from the established western equal-tempered scale. In this work we put forth a first, to our knowledge, comparative analysis of two classic music texts from the European and the Japanese tradition, using the tools and concepts stemming from complex system’s science \cite{2,3,4}.

Many natural or man-made processes can be recorded in the form of a sequence of symbols, which we refer to as “text”. This text is information-rich and as such can be studied in a way similar to that of complex systems in the framework of Symbolic Dynamics. Already from the early studies in the area of information dynamics of complex nonlinear systems the analysis of musical texts using the conceptual and computational tools of Statistical Mechanics were pursued \cite{5,7,6}. The same tools have been used in a variety of information rich texts, either natural or man-made; for characteristic publication on the subject one can see for example \cite{8,9} for literature texts and music, \cite{10} for sequences generated by automata and \cite{11} for DNA coding and non-coding sequences.

In this study, after a proper definition of information and useful concepts such as Shannon block-entropy, we compare music texts coming from the two distant musical traditions, the Japanese and the Western European ones: (a) From the Western European we have chosen a representative and well studied piece of classical western music, Beethoven’s Sonata Op.31 No.2 \cite{12} and (b) from the Japanese musical tradition two traditional equally classic and well studied music pieces, Yatsuhashi Kengyoo’s Rokudan no Shirabe \cite{13} and Yoshizawa Kengyoo II’s Chidori no kyoku \cite{14}. The latter author’s life and work (circa 1808-1872) overlaps with the period of Beethoven’s life and work (1770-1827).

A naturally arising issue is whether or not these different musical compositional genres and their syntax and semantics would show any quantifiable differences under the scrutiny of the tools and methods already in use in complex systems. Since the musical text, or for that matter any text, can be viewed as a coded, one-dimensional, symmetry-broken, unfolding trace of the complex dynamical process that produced it.

We ask ourselves the question “Do the audible differences of the music pieces reflect in the block-entropy analysis?”, or in a more simple way “Does one observe differences between music pieces in the block-entropy analysis?”. As the answer to this question turns to be affirmative we conclude this work.
by looking for the specific power laws that govern the scaling of the block-
entropy of these music texts. For references and comparison we also study
some well known dynamical maps and present a few selected results relevant
to our study.

The structure of the paper is as follows: Section 2 provides the key concep-
tual and computational tools from complex systems and symbolic dynamics
utilized herein. These include the methods of encoding and informational
processing of musical pieces, introducing the concept of discrete Shannon, or
Block, Entropy, and the measure of ‘uncertainty per word’ as well and the
finite length effects present in any realistic implementation of these mathe-
matical concepts. Subsequently, in Section 3 we investigate dynamical maps
on the interval with well defined entropic characteristics in order to review
the entropic concepts introduced before. Then the analysis concerning the
musical texts is presented and their characteristic times and scaling laws are
identified and discussed. In the final section we conclude and discuss the
possible outlook and relevance of this work.
2 Materials and methods

The present work being mostly made of computations, the materials used are of course not of the physical, tangible type. A major part of these materials is software. In this section we could describe the detailed functioning of the programs used but that would only bring tiredness to the reader. We will only mention those programs while exposing the method of work applied, and highlight their main functions. Links and credits to the programs can be found in the References at the end of this work. The last part of the current section is dedicated to a short introduction to the block-entropy analysis and related theory, followed by illustration example.

2.1 Encoding and processing musical pieces

A most important, and tangible - at the very least audible -, material of this study are the scores of the chosen musical pieces. The scores used were in the form of .midi files. This format cannot be directly used for our analysis and needed first to be converted in human-readable format. This was done by the software MIDIFile2Text [15] which converted .midi files into .text files containing various information such as the time the note was played, its duration and its tone which were the variables used for our study. The file conversion process MIDIFile2Text also separated the tracks/instruments of the .midi files which proved quite helpful given that we are mainly looking for monotonic samples.

The method used to analyse the music samples was based on the three letters encoding approach of Ebeling & Nicolis on Beethoven’s Sonata Op. 31 No.2 [16], slightly adapted in an attempt to better conserve the dynamics of the music. The three symbols or ‘letters’ used here for the encoding of the musical text are:

- \( u \) for up, when the tone of the note rises in comparison to the previous one
- \( d \) for down, when the tone of the note drops in comparison to the previous one
- \( s \) for sustains, when the tone of the note remains the same in comparison to the previous one.

In the study of the dynamic of the music the duration of notes is possibly an important variable. For example, this is one of the main audible differences between occidental and Japanese traditional music [17]. The duration of
notes in Japanese pieces tends to be much longer than in occidental pieces [18]. Will this difference show itself in the block-entropy analysis? This is one of the questions we asked in the introduction and will try to answer in the following.

The challenging step for the block-entropy procedure was how to describe the duration of a single note by means of the three lettered symbolic representation. The natural solution is to repeat the letter “s” a number of times proportional to the duration of the note. This requires to chose a time unit, a time scale, a choice which will probably impact strongly on the dynamics of the music. The unit should be common to all of the pieces: we choose the smallest measure of time present among our pieces. Another way of understanding this method is to see the music as a succession of notes with a constant duration equal to the chosen time unit. In this view, a sustained note is in fact a succession of shorter notes of the same tone, the number of which is given by the division of the global duration by the time unit; and thus a natural time-scale is defined.

Once the encoding was constructed the resulting text is processed by a program in C++ provided by one of the authors and freely available in a public code repository [19]. In the course of this study we calculated all the probabilities (i.e. relative frequencies), the Shannon block-entropy and the Shannon entropy of words ranging from 2 letters (the minimum block-length) to 32 letters (the maximum block-length due to our computing time limitation) letters.

### 2.2 Methods of comparison and analysis

The second part of the current section concerns the way we analyzed and compared our musical pieces to well-known discrete dynamical systems on the interval, i.e. dynamical maps such as the Logistic Map.

Each of these maps, via a suitable coarse-graining of its state-space [2, 20, 21] produces a “text” in a binary alphabet which is then processed by the program computing the entropies [19], in the exact same way as for the musical “texts”. The comparison is then made on basis of the graphs of the block-entropy against the length of the words. Here is a list of the maps with a short description for each of them:

- Manneville’s intermittent systems:

\[ x_{n+1} = x_n + x_n^z \pmod{1}. \] (1)

Those systems are known to be intermittent for \( z < 2 \) and sporadic when \( z \geq 2 \) [22]. We studied them with values \( z = 0.5 \) and \( z = 2 \).
A map used in [20]:

\[
\begin{align*}
  f(x) &= \begin{cases} 
  f_L(x) = \frac{a - \sqrt{a^2 - (4a-2)x}}{2a-1} & \text{if } 0 \leq x < \frac{1}{2} \\
  f_R(x) = f_L(1 - x) & \text{if } \frac{1}{2} \leq x < 1 
  \end{cases}
\end{align*}
\]

(2)

This map is known to be intermittent in the case \(a = 1\), which is the case studied here.

The study of Eqs. (1) and (2) is presented in Section 3 for \(z = \{0.5, 2\}\) and \(a = 1.0\), respectively, and their results of the Block Entropy analysis are recorded in Fig. 4.

2.3 About block-entropy

It is no original approach to give here a brief introduction to the theory underlying block-entropy and its relation to dynamical systems. As a matter of fact this is a pattern tirelessly repeated over the numerous works, articles, books... which can be found on the subjects of complex systems, information theory or block-entropy analysis itself. Therefore the reader could find all he needs (and wishes) to know about block-entropy by consulting the references given at the end of this work (in particular, see [3, 20, 21, 23] for a comprehensive review). However, on behalf of the self-consistence of the present work, it cannot be avoided mentioning it.

Many natural (physical, biological) or man-made processes (mathematical sequences, social processes, written texts and/or musical scores) can be recorded, or “written down”, in a symbol sequence. This symbol sequence -or “text”- is essentially one-dimensional, information-rich and uni-directional, in space as well as in time. The information content, or else the degree of “non-randomness”, is essentially what defines the complexity of the text. After the seminal work of Shannon in the 1950’s a plethora of methods have been proposed in order to quantify this complexity. Shannon himself, attributed the emergence of complexity, i.e. the departure of “total randomness” to the constrains underlying the dynamics of the symbol generating process [23].

A measure of the dependence of the number of sequences of a given length (to which we will refer hereafter as “words”) with respect to the length is a Shannon-like entropy measure, the so called “block-entropy”. Let us define this notion explicitly [2, 3, 4]. We consider a text composed on a certain alphabet \(\{A_1, \ldots, A_m\}\) and which can be divided in words of length \(n\). The block-entropy is given by

\[
H_n = - \sum p^{(n)}(A_1 \cdots A_n) \log p^{(n)}(A_1 \cdots A_n) ,
\]

(3)
where the $p^{(n)}(A_1\cdots A_n)$ are the probabilities (relative frequencies) associated with the words $(A_1\cdots A_n)$. The summation is made over all the words “$A_1\cdots A_n$” and it is understood that $0\log 0 = 0$. Note that for uniformly distributed probability, block-entropy is exactly the logarithm of the number of words of a given length $n$.

The following notions are also essential for the course of this study. The uncertainty of the next letter appearing after a given block of a given length $n \geq 1$:

$$h_n = H_{n+1} - H_n$$

with $H_0 = 0$. Then, the entropy of the source which is a discrete analog of the Kolmogorov-Sinai entropy and is defined as:

$$h = \lim_{n \to \infty} h_n.$$  \hfill (5)

The entropy of the source can be shown, for a given alphabet, to take the largest value for a Bernoulli processor [3, 25]. Evidently, for $m$th order Markov sources [1] the limit in Eq. (5) is reached for $n = m$ i.e. $h_n = h$ if $n \geq m$.

For sequences generated by Bernoulli or Markov processes the Shannon-McMillan theorem [26] asserts that the number of words of length $n$ in Eq. (3) scales exponentially with $h$. Recalling from before that for uniformly distributed probability the block-entropy is exactly the logarithm of the number of words of length $n$, this implies that:

(i) block-entropy scales linearly with the word length (Eq. (8)),

(ii) long words are extremely improbable, being exponentially penalized (with a maximal penalization for the largest value of $h$, that is for a Bernoulli processor).

This being said, many real world systems are not Bernoulli or Markovian processors and we do not observe exponential rates of growth of the number of words of length $n$. Therefore there must exist a procedure of selection of realizable sequences [20, 24]. In this context we can assume [10] that for a large class of systems the following scaling behavior is valid for the block-entropy:

$$H_n = nh + gn^\mu (\log n)^\nu + e$$  \hfill (6)

with $0 < \mu < 1$ and $\nu < 0$, where $h$ is the entropy of the source defined above. Ebeling and Nicolis showed that for classical literature texts or music the block-entropy parameter values are $h = 0$, $\nu = 0$ and $\mu < 1$. Dynamical systems showing weak chaos in the form of intermittency, and sporadic systems have also been shown to give rise to a sub-linear scaling of their block-entropy [2, 25].
2.4 Illustration on an example

The illustration example chosen comes from \[16\]. The purpose here is not only to illustrate our definitions above, it is also to demonstrate our ability to generate and process “texts” from a dynamical source, therefore demonstrating the good functioning and use of the programs mentioned above. We will not interpret the results on this example (the interpretation can be found in the work of Ebeling & Nicolis \[16\]).

We study the tent map with \(1 < r < 2\):

\[
f(x) = \begin{cases} \text{rx} & \text{if} \ 0 < x < \frac{1}{2} \\ \text{r(1 - x)} & \text{if} \ \frac{1}{2} < x < 1 \end{cases}
\]

(7)

This map generates a text (we chose a length of \(10^4\) letters\(^{1}\) on the alphabet \(O\) and \(L\), given the partition: \(O \leftarrow x \in (0, 1/2), L \leftarrow x \in (1/2, 1)\). In our study we consider the four cases \(r = \frac{1}{2}(1 + \sqrt{5}) \approx 1.618\) and \(r = 2^{1/2k}\) with \(k = 1, 2, 3\).

As we can see on Fig. 1, we do obtain the same results of Nicolis and Ebeling \[16\]. We have deliberately pushed the study of the system to values of \(n\) on a wider range than Nicolis and Ebeling (\(0 \rightarrow 30\) in place of \(0 \rightarrow 10\)) so as to show the effects of finite length of the text on the block-entropy. One can see on Figure 1 that the uncertainty per letter \(h_n\) goes to 0 for \(n > 12\). This will be the subject of the following section.

\(^{1}\)The length of the texts in this study will always be \(10^4\) letters.
Figure 1: Uncertainty $h_n$ for the tent map vs. the length $n$ of the words for several values of the parameter $r$.

2.5 Finite length effects on block-entropy

As it has been exposed precedently, when no analytic derivation of the probabilities can be made, as it is the case for texts generated by language-like processes, those probabilities can be estimated by the frequencies of words. It appears quite clearly that this estimation will gain in precision with the total length (in terms of number of letters) of the text. In particular, for words with large length $n$ which may not be given the chance to appear if the text is too short. The required length to obtain a good approximation of the probabilities depends thus on the length of the words, but also on the selectivity in words of the dynamics generating the text. Were all words of length $n$ equiprobable, the risk of them not appearing frequently enough would be greater. To realize this, consider first the extreme case where only one word is allowed. It is clear that this will give the right probability (equal to 1). Now if one word is allowed with great probability (say 0.80) while several others are allowed with smaller probabilities, the chance is low that the most probable word does not appear frequently enough as to give a correct estimate. The probabilities of the other being small, their contribution to the block-entropy will be small as well and thus the estimate of the block-entropy will be correct. We can express the condition on the total length of the text in a more mathematical way. We take $N_n$ as the total number of allowed words of length $n$, $L$ the total length of the text. The condition is:

$$N_n \ll L.$$ (8)
In the case of equiprobable words of length $n$ (that is a generator of Bernoulli type) on an alphabet of $\alpha$ letters:

$$N_n = \alpha^n$$

which leads us to the condition $n \ll \log_\alpha L$. If we consider more selective dynamics, \[16, 20, 23\], we may have:

$$N_n = \alpha^{\sqrt{n}}$$

which gives the more permissive condition $\sqrt{n} \ll L$.

The example of the tent map given in the previous section, through its comparison with the study of Nicolis and Ebeling, demonstrates perfectly the effects of the finite length $L$ on the block-entropy at large values of $n$. This system, for $r = 1.618$, being the generator of a first order Markov chain, the uncertainty per letter, Eq. (4), of the source is analytically known to be $h = 0.694$, which is verified numerically for $n \leq 10$ as shown on figure 1. However, it goes to zero for $n > 12$ when it should remain constant. This indicates that, due to the finite length $L$, no more information is added for large $n$. One can also see on figure 1 that those effects of finite length appears for greater $n$ for the other values of the parameter $r$. This is an illustration of the fact that the equiprobable distribution is the most penalized in terms of precision of the estimation.

Another example of the latter statement is presented in Fig. 2 this time with a 3-letters Bernoulli Generator and a text of length $L = 10^4$. We see that the estimate is correct (cf. Eq. (5)) only for $n < \log_3(10^4)$, i.e. approximately $n < 8$. 


3 Results and discussion

In this section we compare musical pieces and/or dynamical maps, including of course a discussion on the scaling laws. We begin by exposing the results for the dynamical maps described by Eqs. (1) and (2).

3.1 Dynamical maps

Both maps in Eqs. (1) and (2) are studied on the interval $[0, 1]$ on a 2-letter (binary) alphabet defined by the usual partition: $O \leftarrow x \in (0, 1/2)$, $L \leftarrow x \in (1/2, 0)$. The texts generated were of $10^4$ letters long.

One can see on Figs. 3 and 4 that the two intermittent systems behave very similarly, while the sporadic system adopts a quite different aspect. The curves of the latter, both for $H_n$ and $h_n$, are indeed way smoother than those of the intermittent systems and take on overall smaller values. This can be explained by the fact that a sporadic system spends more time around a specific point where its derivative is null, thus favouring the dominance of one letter of the alphabet on the other and therefore reducing the gain of information (that is of entropy, in the sense of Shannon) by addition of a letter. Another way to see this is that for a sporadic system, the Lyapunov exponent vanishes $[22]$, giving a greater stability to the system.

Figure 2: Uncertainty $h_n$ for a 3-letters Bernoulli Generator vs. the word length $n$. 

![Figure 2: Uncertainty $h_n$ for a 3-letters Bernoulli Generator vs. the word length $n$.]
3.2 Musical analysis

The method used to analyze the musical pieces has been largely exposed in the Section 2. With our method of transcription, we obtained four texts of the following lengths: “Rokudan no Shirabe” $L = 6644$, “Chidori no kyoku” $L = 7909$, “Beethoven track 1” $L = 11616$, “Beethoven track 2” $L = 11968$. We can concentrate ourself on the interpretation of the Figs. 5 and 6. They
resume the results of the four pieces for the block-entropy and the uncertainty per letter, respectively.

![Figure 5: Block-entropy $H_n$ (in bits) vs. word length $n$. Blue squares “Chidori no kyoku”, purple dots “Rokudan no Shirabe”, green crosses “Sonata for pianoforte Op. 31 No.2, track 1”, red triangles “Sonata for pianoforte Op. 31 No.2, track 2”.](image)

The first thing to be noticed on these two graphs is the encouraging similarity between the curves of the two tracks from the Sonata of Beethoven, as well as the dissimilarity of the letters towards the Japanese pieces. This is easier seen on the Fig. 6. Beethoven’s tracks follow exactly the same “staircase” pattern, with a small shift towards the y-axis. They even coincide with great precision for $18 \leq n \leq 22$. This result in itself is correct and encouraging because it shows that our block-entropy analysis is consistent with the fact that both tracks came from the same piece of music and are therefore generated by the same dynamics, despite their audible small differences. That is something we expected as a reliable verification and we are glad to have been observed. The two Japanese pieces are quite shifted from Beethoven’s tracks, close to each other for $n < 11$ and far apart for $11 \leq n < 21$. Again it is reassuring to see the differences between text from distinct origins and it can give us faith in our method. However, this leads to small values of $H_n$ and $h_n$, which is observed in our figures. Those small values are a first point of analogy with the sporadic system given by the Manneville’s systems for $z = 2$. Of course, this is subject to discussion as the analogy seems to be the direct consequence of our method of encoding, but it is well known that block-entropy analysis depends on the chosen
alphabet/encoding.

Let us now comment the “staircase” structure appearing on figure 6, in particular for the uncertainty per letter $h_n$ of both Beethoven’s tracks. This is related to the time unit we chose to generate our text. The values of $n$ at which a new step occurs correspond indeed to multiples of this time unit. The fact that our analysis allows the extraction of information such as characteristic times of the musical pieces is quite satisfactory in itself!

We illustrate this in Fig. 7 which shows the characteristic times (and their frequency) of the three musical pieces in units given by the .MIDI files. The time unit we have chosen is the smallest appearing time, i.e. 12. Among the characteristic times in Fig. 7 some are (close to) multiple of this time unit, the ratios being given by: $n = 5$ for $T = 60$, $n = 7, 8$ for $T = 90$, $n = 10$ for $T = 120$ and $n = 20$ for $T = 240$. When comparing Figs. 6 and 7 one sees that the steps for a given piece of music coincide indeed with the characteristic times of that piece shown in Fig. 7.

Fig. 7 thus reflects the fact that Beethoven’s Sonata has shorter characteristic times and less varied while Japanese music is characterized by longer and more diverse times. The answer to the question “Will the difference in duration of notes show itself in the block-entropy analysis?” asked in Section 2 is then yes, it does!
3.3 Scaling laws

In this section, we discuss the scaling laws of the pieces of music; cf. Eqs. (8, 9, 10). Our objective is to show that the scaling behavior of Eq. (6) is valid with $h = 0$ and some value of $\mu$ and $\nu$.

For what the pieces of music are concerned, Fig. 8 (where the entropy is represented against $n^{(1/4)}$ and where a linear fit has been made) shows that the entropy of Beethoven’s Sonata follows a power law with $\mu = 1/4$, which is the same result as in Ref. [16]. The Japanese pieces however do not follow the same law, though it may be difficult to easily see it on Fig. 8.

The deviations from the linear fit were represented on Fig. 9 where it is easier seen that only Beethoven’s Sonata follows a power law with $\mu = 1/4$, $\nu = 0$. The track 2 of Beethoven’s Sonata is not represented in order to avoid redundancy. Again, the observation of this difference between the Japanese pieces and the Sonata is in agreement with our expectations that these pieces of music correspond to distinct dynamics.

Similarly, we can repeat the process for *Rokudan no Shirabe*. We find that its block-entropy scales in a power law with $\mu = 0.9$. This is illustrated on Fig. 10. The residues are represented on Fig. 11 for easier visualisation. We see that block-entropy of the other pieces do not fit to the same power law.
4 Conclusion

In the course of this study, after having properly defined the concepts and tools in use (information, block-entropy, entropy of the source, . . .), we have been able to apply them on various systems: dynamical, mathematical maps, western music text and Japanese music texts.

The study of the dynamical maps defined by Eqs. (1, 2) and (7) allowed

Figure 8: Power law in $n^{(1/4)}$ of the block-entropy $H_n$ (in units of $\log_3$) for Beethoven’s Sonata.

Figure 9: Distance of the curves to the linear fit in Fig. 8.
Figure 10: Power law in $n^{(0.9)}$ of the block-entropy $H_n$ (in units of $\log_3$) for “Rokudan no Shirabe”.

Figure 11: Distance of the curves to the linear fit in Fig. 10.

us to observe the differences between fully chaotic, intermittent and sporadic systems. We noted that sporadic systems present smoother block-entropy $H_n$ and uncertainty per letter $h_n$ curves with significantly lower values than intermittent systems.

This result, in regard of the curves of the block-entropy and uncertainty per letter for the music texts gave us insights on the nature of the dynamics generating those music pieces. Furthermore, we showed that both Japanese and occidental chosen pieces have their block-entropy scaling in a sub-linear
way (Eq. (9)) with the words length. In particular Yatsuhashi Kengyo’s *Rokudan no Shirabe* and Beethoven’s *Piano Sonata Op. 31 No. 2* were shown to have a power-law block-entropy scaling with distinct exponents which, as we saw, were $\mu = 1/4$ for *Piano Sonata Op. 31 No. 2* (a new result yet consistent with the general theory developed in the first study of this piece by Ebeling and Nicolis) and $\mu = 0.9$ for *Rokudan no Shirabe*. We therefore obtained a first favorable answer to the introductory question “Does one observe differences between music pieces in the block-entropy analysis?”.

Before studying the block-entropy scaling we conducted a first analysis of curves of the block-entropy and uncertainty per letter of the music pieces, carefully indexing the differences and similarities between them. We observed a great similarity between the two tracks of Beethoven’s *Piano Sonata Op. 31 No. 2* and a relative similarity between the Japanese pieces for words of length inferior to $n = 11$, thus validating our method of analysis. Moreover, we noted “staircase-like” structure in the uncertainty per letter curves and identified them with the presence (or absence for some music pieces) of characteristic times, giving rise to the differences between the curves. We therefore answered the introductory question “Do the audible differences of the music pieces reflect in the block-entropy analysis?” . The answer is yes, they do.

In later work, we might address the question of the dependence in the time-scale (or in the choice of the time unit) of the block-entropy and entropy per letter of the music pieces. We expect this dependence to be important due to the link between characteristic times of the music and entropy per letter. It would be also very interesting to look for the presence of palindromes in the generated music texts, which can directly be accomplished using [19]. This question has recently attracted some interest [27] as it is related to both to recurrence quantification analysis and also to entropic analysis. Any future connection between these two approaches will prove to be an excellent step toward a more general method that will enable us to reveal the fine interplay of selection rules at different levels of word length scales and time scales not only in musical texts but also in the broader area of symbolic dynamics in complex system studies.

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