Production, storage and release of spin currents in quantum circuits

Michele Cini
Dipartimento di Fisica, Università di Roma Tor Vergata,
Via della Ricerca Scientifica 1, I-00133 Rome, Italy,
and Istituto Nazionale di Fisica Nucleare - Laboratori Nazionali di Frascati, Via E. Fermi 40, 00044 Frascati, Italy.

Quantum rings connected to ballistic circuits couple strongly to external magnetic fields if the connection is not symmetric. By analytical theory and computer simulation I show that properly connected rings can be used to pump currents in the wires giving raise to a number of interesting new phenomena. One can pump spin polarized currents into the wires by using rotating magnetic fields or letting the ring rotate around the wire. This method works without any need for the spin-orbit interaction, and without stringent requirements about the conduction band filling. On the other hand, another method works at half filling using a time-dependent magnetic field in the plane of the (fixed) ring. This can be used to pump a pure spin current, excited by the the spin-orbit interaction in the ring. One can use magnetizable bodies as storage units to concentrate and save the magnetization in much the same way as capacitors store electric charge. The polarization obtained in this way can then be used on command to produce spin currents in a wire. These currents show interesting oscillations while the storage units exchange their polarizations and the intensity and amplitude of the oscillations can controlled by tuning the conductance of the wire used to connect the units.

PACS numbers: 05.60.Gg, Quantum, transport

I. INTRODUCTION

In the past 15 years or so, there has been in the literature a growing interest in the relation between magnetic and transport properties of materials1; on the other hand, there have been several works on the persistent as well as transient currents in quantum rings threaded by a magnetic flux, with a promising outlook in the quest for new device applications in spintronics, memory devices, optoelectronics, quantum pumping, and quantum information processing2–5. Aharonov-Bohm-type thermopower oscillations of a quantum dot embedded in a ring for the case when the interaction between electrons can be neglected, were investigated in the literature, showing it to be strongly flux- and experimental geometry- dependent.6 Spin interference modulations of the conductances and Berry phase effects on the magnetoresistance have been studied.

The present paper is devoted to the magnetic properties of quantum rings linked to ballistic circuits. If the connection is asymmetric, a current in the external circuit selects a chirality in the ring and produces a magnetic moment; as we shall see, the reverse is also true, namely, a chiral current in the ring can pump charge in the wire. The current excited in the wires by a local ac-field that produces a current in the external circuit and is itself a purely quantum phenomenon. By the same token, a symmetrically connected quantum ring inserted in a circuit cannot choose a chirality and has zero magnetic moment when a current flows through it. A symmetric connection is unfavorable for quantum pumping. This is why a maximally asymmetrical connection is relevant in this respect. I call this geometry a laterally connected ring (see Figure 1 a)).

Compelling physical arguments based on thought experiments suggest that the magnetic moment is not obtained by substituting the quantum current in the classical formula. The basic reason is that any measurement of the magnetic moment of the ring requires the measurement of a force exerted on the ring by a probe flux which according to Quantum Mechanics has an influence on the current. Actually, rephrasing the conclusions of Ref2, the magnetic moment is given by the operator

\[
\hat{M}(\phi) = \frac{\partial H^G_{\text{ring}}}{\partial \phi} + \Delta_1 H^G_{\text{ring}} + H^G_{\text{ring}} \Delta,
\]

where \( \phi \) is the probe magnetic flux through the ring, \( H^G_{\text{ring}} \) is the grand-canonical ring Hamiltonian referenced to the equilibrium chemical potential of the system and \( \Delta = \frac{\partial \phi}{\partial G} \). As a consequence, when the circuit is biased by a small potential difference \( V_{\text{bias}} \) and a small current flows, \( \langle M \rangle \) is found to go with \( V_{\text{bias}} \) while of course classically one expects \( M \sim V_{\text{bias}} \) in a linear circuit. Indeed at small \( V_{\text{bias}} \) the quantum effects favor a laminar current which is not coupled to the magnetic field.

A complementary way to study those quantum effects is to consider the reversed situation when \( V_{\text{bias}} = 0 \) and it is the interaction of the ring with a magnetic field that produces a current in the external circuit. Romeo and Citro, in a very interesting paper, have discussed memory and pumping effects in rings coupled to parasitic nonlinear dynamics. Laterally connected rings have an intrinsic non-linear dynamics; in it was shown that they have peculiar properties for quantum pumping; besides, they can be used to pump spin, rather than just charge. In principle Quantum Mechanics allows us to build a device to achieve that in more than one way. In the present paper I present new data to further clarify these purely quantum properties.
These findings raise new questions. Can we build a ring device that can produce a pure spin current, without any charge current associated to it? Can we transfer magnetization between two distant bodies directly through a lead, without moving any charges? Can we store a spin current as we do with charge currents in batteries? In the following I show that these questions have a positive answer.

Such theoretical ideas, once realized in a practical device, would offer new strategies to attack the problems connected to Spintronic applications.

II. GEOMETRY AND DYNAMICS OF SPIN CURRENT GENERATION

In this Section I introduce a master Hamiltonian $H_{prod}$ which, by a proper choice of parameters, allows to study 3 different mechanisms for pumping (partially or totally polarized) current in the absence of an external bias. Below, $H_{prod}$ will be complemented with other terms to allow for spin current storage and release. All three methods use a laterally connected ring. I focus on maximally asymmetric geometries, i.e. laterally connected rings, such that the external circuit is tangential to the ring. Fig.1 a) shows the geometry for a ring of N=8 sites; the side length is of order of 1 Angstrom.

The time dependent external magnetic field $\vec{B}$ has a component $B_{i}(t)$ in the plane of the ring and a component $B_{i}(t)$ which produces a flux $\phi(t) = B_{i}S$. Let me write the Hamiltonian for the spin current generation that for various choices of the parameters leads to 3 different operating schemes. Each bond in the ring is modified by the field in that it acquires a phase. In the time-dependent case, different distributions of the phase having the same sum over the sides of the ring are not equivalent in principle. Here for simplicity I assume the symmetric prescription

$$t_{ring} \rightarrow t_{ring} \exp[i\alpha(t)/N_{ring}].$$

The dynamics of the spin-current production is given by the Hamiltonian of Ref.16, namely:

$$H_{prod} = H_{D} + H_{B}'$$

where $H_{D}$ is the device Hamiltonian and $H_{B}'$ the in plane magnetic term. Here,

$$H_{D} = H_{wires} + H_{ring} + H_{ring-wires}. \quad (3)$$

The polygonal ring, with an even number $N_{ring}$ of sides, is represented by

$$H_{ring} = \sum_{i \in ring} H_{ring}(i) \quad (4)$$

where, with the identification of ring site $N_{ring} + 1$ with site 1, we may write

$$H_{ring}(i) = (t_{ring} \exp[i\alpha(i)/N_{ring}] \sum_{\sigma} \exp[i\sigma \sigma_{SO}]c_{i+1,\sigma}^{\dagger}c_{i,\sigma} + h.c.) + \mu_{B}B_{i}\sigma_{z}(i), \quad (5)$$

with the Bohr magneton $\mu_{B} = 5.7375 \times 10^{-5}$ tesla and $\sigma_{z}(i) = \hat{n}_{i+} - \hat{n}_{i-}$. Here $\sigma_{SO}$ is a phase due to the spin-orbit interaction, and can be of order unity or smaller. The magnetic interaction due to $B_{i}$ acts exclusively on spin and is:

$$H_{B}' = \mu_{B}B_{i}(t) \sum_{i \in ring} (c_{i,\uparrow}^{\dagger}c_{i,\downarrow} + c_{i,\downarrow}^{\dagger}c_{i,\uparrow}). \quad (6)$$

FIG. 1: Lateral connection of $N_{ring} = 6$ ring to wires. (Any even $N_{ring} > 2$ can be considered.) The circles represent the sites; those in the hexagon feel the spin-orbit interaction and the magnetic field. In panel a) the geometry of the device is shown. The alternating colors visualize the bipartite property of the associated graph. The time dependent external magnetic field has a component $B_{i}(t)$ in the plane of the ring which couples to spin and a component $B_{i}(t)$ which produces a flux $\phi(t) = B_{i}S$ and also interacts with spins in the octagon; all sites are connected horizontally to the first neighbors by spin-diagonal matrix elements. In b) the adjacency graph of the Hamiltonian (on the basis of local spin-orbitals) is depicted. The star between the rings represents a magnetic monopole which produces a central magnetic field with equal fluxes in the up and down-spin sublattices; however the field has opposite chiralities for opposite spins, and thus represents the effects of the spin-orbit interaction. The vertical bonds represent the effects of the in-plane field $B_{i}$ acting on the spin degrees of freedom. The alternating colors again visualize the bipartite property of the adjacency graph in the spin-orbital basis, which is used for proving the absence of charge current.
The Hamiltonian for the left and right wires is a standard tight-binding model
\[ H_{\text{wires}} = H_L + H_R = \hbar c \sum_{n,\sigma} c_{n,\sigma}^\dagger \! c_{n+1,\sigma} + h.c. \quad (7) \]

Finally, the ring-wires contacts are modeled in \( H_{\text{ring-wires}} \) whereby the ring has a nearest neighbor connection to the leads via a tunneling Hamiltonian with hopping matrix elements \( t_f \) in the obvious way. In the numerical calculations below I take \( t_f = t_{\text{ring}} = t_f = 1 \) eV as a reasonable order of magnitude.

A. Numerical evolution and calculation of the currents

The system with a spin-independent chemical potential \( E_F \) is in equilibrium for \( t < 0 \) in the presence of no field. The natural time unit for this problem is \( \tau = \frac{\hbar}{E_F} \). In the code the Hamiltonian is constant during time slices of 0.2\( \tau \) and jumps to the next value at the end. In this way the many-electron Schrödinger equation is integrated by a succession of sudden approximations.

Taking the spin quantization axis along \( z \) (orthogonal to the plane of the ring), the number current operator may be written:
\[ J_{m,\sigma} = \frac{i\hbar t_f}{\hbar} (c_{m+1,\sigma}^\dagger c_{m,\sigma} - c_{m,\sigma}^\dagger c_{m+1,\sigma}) \quad (8) \]

I calculate the time-dependent number current by my own partition-free approach\(^{18}\), namely,
\[ J_{n,\sigma}(t) = -2t_f \text{Im}(G_{n,\sigma,n-1,\sigma}^-) \quad (9) \]

where, in terms of the retarded function \( g_{i,j}^r \),
\[ G_{i,j}^r(t) = \sum_q n_q^0 g_{i,q}(t,0) g_{j,q}^*(t,0) \quad (10) \]

where \( q \) runs over the ground state spin-orbitals for \( B = 0 \) and \( n_q^0 \) is the Fermi function. More generally, we need the current \( J_m(\overrightarrow{n}) \) at site \( m \) with the spin quantization axis along \( \overrightarrow{n} = (\sin(\theta), 0, \cos(\theta)) \) in the \( xz \) plane (\( \varphi = 0 \)). The up-spin creation operator becomes
\[ c_{+,\overrightarrow{n}}^\dagger = \cos \left( \frac{\theta}{2} \right) c_{+ \overrightarrow{n}}^\dagger + \sin \left( \frac{\theta}{2} \right) c_{- \overrightarrow{n}}^\dagger \]

and the down-spin operator becomes
\[ c_{-,\overrightarrow{n}}^\dagger = -\sin \left( \frac{\theta}{2} \right) c_{+ \overrightarrow{n}}^\dagger + \cos \left( \frac{\theta}{2} \right) c_{- \overrightarrow{n}}^\dagger \]

Therefore:
\[ J_m(\overrightarrow{n}) = \cos \left( \frac{\theta}{2} \right) J_{m,+} + \sin \left( \frac{\theta}{2} \right) J_{m,-} + \sin \left( \frac{\theta}{2} \right) \cos \left( \frac{\theta}{2} \right) J_{m,0} \quad (11) \]

where the spin-flip number current at site \( m \) is:
\[ J_m^f = \frac{i\hbar t_f}{\hbar} \sum_{\sigma} (c_{m+1,\sigma}^\dagger c_{m,\sigma} - c_{m,\sigma}^\dagger c_{m+1,\sigma}^-) \quad (12) \]

Consequently, we are interested in the the spin current at site \( m \) polarized along \( \overrightarrow{n} \)
\[ J_m^{\text{spin}}(\overrightarrow{n}) = J_m(\overrightarrow{n}) - J_m(-\overrightarrow{n}) \quad (13) \]

given in terms of Equation (8) by
\[ J_m^{\text{spin}}(\overrightarrow{n}) = (J_{m,+} - J_{m,-}) \cos(\theta) + J_m^f \sin(\theta) \quad (14) \]

My codes calculate number currents taking \( t_f = 1 \). If this is interpreted to mean that \( t_f = 1 \) eV, which corresponds to the frequency \( 2.42 \times 10^{11} \text{ s}^{-1} \), a current \( J = 1 \) from the code means \( 2.42 \times 10^{14} \text{ electrons per second} \), which corresponds to a charge current of \( 3.87 \times 10^{-5} \text{ Ampere} \). We also need a characteristic magnetic field. Recalling that \( \phi_0 = \frac{\hbar}{e} \sim 4.13410^{-15} \text{ in MKSA units} \), we introduce the magnetic field \( B_\phi \) such that 
\[ B_\phi S(N) = \phi_0 \text{ where } S(N) = \frac{\mu_B}{a} \text{ is the ring area.} \]

Thus, \( B_\phi = \frac{1}{a^2} \tan(\frac{\pi}{N}) \text{ tesla} \), with \( a \) in Angstrom.

III. MAGNETIC INTERACTIONS OF SPINLESS MODELS

The limiting case in which we neglect spin in the Hamiltonian (therefore \( \alpha_{SO} = 0 \), and \( \mu_B = 0 \) in Equations (5,6)) is already very remarkable\(^{12}\). One method for pumping current while leaving the ring neutral is based on the introduction of integer numbers of fluxons, another method consisting in connecting the ring to a junction. As a direct consequence of the above mentioned nonlinearity of the magnetic moment, one can achieve, by employing suitable flux protocols, single-parameter nonadiabatic pumping, where an arbitrary amount of charge can be transferred from one side to the other, a phenomenon which, for a linear system, would be readily ruled out by the Brower theorem\(^{12}\). In this way, a current is excited in the wires and charge is pumped in the external circuit without changing the neutrality of the ring. The total energy of the ring after inserting the flux is the same. This happens because inserting an integer number of fluxons is a gauge and the process is slow. In Figure 2 I exemplify the results for a hexagonal ring, showing the charge transferred from a wire to the other as a function of time. Curve a) is obtained at half filling and shows that each fluxon shifts the same amount of charge, producing a staircase. Curve b) is taken at a higher filling, and is not very different. Curve c) is again at half filling, but the stairs are wider because the flux is inserted more slowly. Moreover the stairs are slightly lower. The amount of charge transferred for each cycle decreases, because the process is not adiabatic. In the adiabatic limit there is no pumping, as shown in a beautiful analysis by Avron, Raveh and Zur\(^{22}\).
FIG. 2: Charge pumped by a laterally connected 6-site ring excited by a time-dependent magnetic flux which grows linearly in time. In this example, spin is not considered. Time is measured in units of $\tau = \frac{\hbar}{t}$. In all cases, there is no charging of the ring but charge is transferred from left to right. In a) the system is half filled ($E_F = 0$) and 6 fluxons are inserted in the (0, 100 $\tau$) and each fluxon produces a stair in the pumped charge curve; b) the same but with $E_F = 0.5\mu_F$. In c) $E_F = 0$ as in a) but the rate of insertion of the flux is reduced, as one can judge from the wider stairs. One can also see a slight reduction of the height of the stairs.

The present treatment neglects interactions. However, the above phenomena were studied within the Luttinger liquid approach. Within the pumping context, a distinct place was attributed to the pumping properties of a Luttinger liquid and, in particular, it was found that the pumping phenomena persist unhindered in the presence of the interactions.

A. Rotating field or rotating ring

In the presence of spin it is no longer true that inserting an integer number of fluxons is a gauge; this complicates matters but also produces interesting possibilities. The magnetic moment of the laterally connected bond can pump a partially spin-polarized current even in the absence of a spin-orbit coupling. An example in which a rotating ring produces a partially polarized current is shown in Figure 3. For more data in these arrangements see Ref. (16). When a charge current is present the storage of the spin polarization is complicated by the problem of charging and the theoretical analysis cannot avoid to deal with correlation. The present paper is devoted to the next arrangement in which purely spin currents are pumped.

1. Field in plane of ring: pure spin current

Now I consider a geometry such that both spin directions are treated on the same footing. This is obtained by setting the magnetic field in the plane of the ring. In the master Hamiltonian $H_{prod}$ we set $B(t) = 0$ and $\alpha(t) = 0$, since in such an arrangement there no flux piercing the ring. The spin symmetry is broken by $\alpha_{SO}$ which provides a driving force. For any time dependence of $V(t) = \mu_B B(t)$ the charge current vanishes identically at half filling and a pure spin current (Figure 4) obtains. The discrete spin-space symmetry of the model is $P \Sigma$ where $P$ is the parity and $\Sigma$ is the spin reflection. A good analytic understanding of the purely spin current and of its non-adiabatic character was achieved in (14). Since the adjacency graph for the Hamiltonian in the spin-orbital basis (Fig.1 b) is bipartite for even $N_{ring}$, the absence of a charge current can be deduced from the invariance of the problem under a canonical transformation that exchanges electrons with holes, spin up with spin down and changes sign to one of the sublattices. The importance of even-odd effects in the physics of quantum rings has been stressed elsewhere.

Numerical results (not shown here) obtained with an odd $N_{ring}$ show a partial spin polarization, in line with the fact that they do not correspond to bipartite lattices. The extra atom produces a charge current that becomes small when the ring gets large. Below we consider even $N_{ring}$. It was shown (14) that finite temperatures do not change significantly the results up to $K_B T \sim 0.025$ eV.

FIG. 3: Pumping from a 50-site ring laterally connected to wires and rotating with $\omega = \frac{0.044}{\tau}$ in a magnetic field $B = 578$ Tesla if $a = 1$ Angstrom. Spin current polarized in the direction of the magnetic field (marked S) and charge current (marked C) at the first bond of the right wire. The Fermi energy is $E_F = 0$ (half filling), but at $E_F = \pm 0.5$ the charge and spin currents are similar to those shown here. No spin-orbit interaction is included in this example.

FIG. 4: A pure spin current does not shift charges but only polarization. It is a novel, fascinating object to study.
the absence of the spin-orbit interaction. In low atomic number materials the polarization could be long lived enough to make interesting experiments on it and maybe technological applications.

To this end, I wish to present the computer experiment outlined in Figure 7. Let the total Hamiltonian become:

\[ H = H_{\text{prod}} + H_{\text{ext}} + H_{L,\text{store}}(t) + H_{R,\text{store}}(t). \]  

This is the sum of the above described \( H_{\text{prod}} \) plus the terms describing the left and right storage units \( H_{L,\text{store}}(t), H_{R,\text{store}}(t) \) and the external wire \( H_{\text{ext}} \) of the same form as Equation (6) connecting the storage units. Here \( H_{L,\text{store}}(t), H_{R,\text{store}}(t) \) also contain the connections to the external wire and to the respective wires to the ring, and these depend on time according to the pattern of Figure 7. One must choose \( H_{L,\text{store}}(t), H_{R,\text{store}}(t) \) such that the whole graph is bipartite, and the storage units will be half filled initially and for all times. In Figure 7a), the whole system is in equilibrium in the absence of external fields. Then the direct connection is removed and in 7b) the external field in the plane of the ring excites a spin current that will tend to polarize the 8 site cubes in opposite way. In 7c) the cubes are isolated and the spin current is stored in them as magnetization. Finally, the direct connection is established in 7d) and a spin current is excited.

### 2. Storage cubes

I performed a numerical experiment in which both storage units where 8-atom cubes. The field along the \( x \) axis is taken to be \( B(t) = B_{\text{max}} \sin(\frac{2\pi t}{T}) \) where \( B_{\text{max}} = \frac{B_{\text{max}}}{10^9} \sim 58 \) Tesla for \( a = 10^{-10} \) in and a 60 atom ring. The spin-orbit constant is taken to be \( \alpha_{SO} = 1 \). The leads from the ring to the cubes are 70 sites long while the two cubes are at direct contact with each other (that is, \( H_{\text{ext}} \) does not exist and there is direct hopping between the polarization reservoirs.). A pure spin current flows...
in the wires from the ring to the cubes, and its polarization axis is somewhere in the x-z plane; it is more intense when computed with the spin axis along x than along z. The contacts to the ring are broken at $t_1 = 60\tau$ while the hopping between cubes is allowed after $t_2 = 70\tau$; here I recall that times are measured in units of $\tau = \hbar/4t_h$.

In Figure 8 the populations of the two cubes are shown, and it is obvious that they are opposite for up and down spins and also opposite for the two cubes. This realizes the polarization transfer without charging, and is quite substantial. The populations start from half filling and begin to evolve after a time 40 which is needed for the perturbation to cover the 70 site distance from the ring. The speed of the disturbance is about 2 sites in time $\tau$. Then the left cube receives a up polarization and the right ring polarization is opposite.

While this behavior had been expected in the design of the thought experiment, there are surprises, too. After a (quite large) maximum polarization is achieved, most of it goes away in a rather short time. One could engineer the timing of the process and the size of the storage units if the aim is to maximize the polarization. From $t_1$ to $t_2$ the rings are isolated and polarization is constant, as it should. The cubes remain strictly neutral like the rest of the system. At $t_2$ the cubes are set in direct contact (in this example $H_{ext}$ is replaced by a direct hopping between the cubes) and the populations start to oscillate. In figure 9 one can observe the immediate, rapidly oscillating purely spin current which is obtained in this way. These oscillations are interesting by their own right and it is likely that one can engineer the process, varying the number of sites in the storage units and many available parameters, in order to modify both frequency and amplitude of the oscillations. The oscillations are complex and fast, and will require further investigations. It should be interesting to observe the spectrum and polarization of electromagnetic waves emitted by these oscillating magnetic currents.

3. Storage rings

In the next computer experiment the storage cubes are replaced by 4-atom storage rings; the connection $H_{ext}$ between them at the beginning and at the end of the experiment is obtained with a wire composed by $N_{ext} = 200$ sites, with hopping $t_h$, while the ring is $N_{ring} = 30$ sites large (See Figure 10). The field is supposed to grow linearly up to $t = 50\tau$. The spin current in Figure 10 b) is largely reduced, compared to the previous figure, and this is due to the smaller object being magnetized. In Figure 12 c) the length of the wire is shortened, and this produces a shorter delay before the spin current arrives. Moreover the wire connecting the storage rings now has matrix elements $t_h^3$ and this produces a decrease of the overall intensity of the spin current and a slowing down of the oscillations.

V. CONCLUSIONS

I presented the theoretical study of tight-binding model devices consisting of a ring laterally connected to a wire and designed to produce spin polarized currents. Using rotating magnetic fields or letting the ring rotate around the wire one can produce a strong spin polarization even without using the spin-orbit interaction, and this operates in a wide range of fillings. In the case of fast and large rotating rings the centrifugal force must be included and this work is under way.

In another experiment at half filling a tangent time-dependent magnetic field in the plane of the (fixed) ring
FIG. 9: The current in the bond connecting the cubes during the thought experiment. Top: spin up Bottom: spin down.

can be used to pump a purely spin current, excited by the spin-orbit interaction in the ring. This behavior is understood analytically and is found to be robust with respect to temperature and small deviations from half filling. Above, I presented new numerical results illustrating the above arrangements and the relation to previous findings with spinless models that show charge pumping.

However the main results of the present paper concern possible schemes to store, gather, and then release magnetization. Suitable reservoirs or storage units have been shown to work with spin currents in analogy with condensers for the usual charge currents. The polarization can be stored and then used on command to produce spin currents in a wire. These currents show oscillations while the storage units exchange their polarizations and the intensity and amplitude of the oscillations can be modified by changing the conductivity of the ring between the polarized units.

The present model neglects electron-electron interactions, but it is physically reasonable that adding to the Hamiltonian a correlation term like \(U(\hat{n}_{\uparrow} + \hat{n}_{\downarrow} - 1)^2\) would tend to reinforce the charge confining effects described here; in the Hartree approximation, however, it would change nothing since its average at half filling vanishes strictly during the evolution of the system.

The above theoretical effort leads to several intriguing possibilities from the viewpoint of basic research and possible applications and I hope that this will stimulate experimentalists to work on the magnetic properties of laterally connected quantum rings.

FIG. 10: A thought experiment in which the storage cubes are replaced by storage rings. a): time dependent magnetic field. b): spin-up current in the 200 atoms long external (storage ring-storage ring) connection, where the hopping matrix element is \(t_{h}\); the leads from the 30 atom central ring to the storage rings are 50 sites long. The down spin current is opposite. c) spin-up current in the 100 atoms long external (storage ring-storage ring) connection, where the hopping matrix element is \(\frac{3}{5}t_{h}\); the leads from the 30 atom central ring to the storage rings are 50 sites long. The down spin current is opposite.

REFERENCES

1. L. Amico, R. Fazio, A. Osterloh, V. Vedral, Reviews of Modern Physics 80, 517 (2008).
2. S. Souma and B. K. Nikolic, Phys. Rev. B 70, 195346 (2004).
3. Z. Barticevic, M. Pacheco, and A. Latge, Phys. Rev. B 62, 6963 (2000).
4. P. Foldi, O. Kalman, M. G. Benedict, and F. M. Peeters, Nano Lett. 8, 2556 (2008).
5. O. Kalman, P. Foldi, M. G. Benedict, and F. M. Peeters, Phys. Rev. B 78, 125306 (2008).
6 Y.M. Blanter, C. Bruder, R. Fazio, H. Schoeller, Phys. Rev. B 55, 4069 (1997).
7 Diego Frustaglia and Klaus Richter, Phys. Rev. B69, 235310 (2004)
8 Fumiya Nagasawa, Diego Frustaglia, Henry Saarikoski, Klaus Richter and Junasaku Nitta, Nature Commun. 4, 2526 (2013)
9 M. Cini, E. Perfetto and G. Stefanucci, Phys. Rev. B 81, 165202 (2010).
10 John David Jackson, Classical Electrodynamics, John Wiley and Sons Ltd (1962) Chapter 5.
11 M. Cini and E. Perfetto, Phys. Rev. B 84, 245201 (2011).
12 F. Romeo and R. Citro, Phys. Rev. B 82, 085317 (2010).
13 M. Cini and S. Bellucci, ICEAA-IEEE APWC-EMS.
14 Michele Cini and Stefano Bellucci, J. Phys.: Condens. Matter 26 145301 (2014)
15 Michele Cini, Enrico Perfetto, Chiara Ciccarelli, Gianluca Stefanucci, and Stefano Bellucci, Phys. Rev. B80, 125427 (2009)
16 Michele Cini and Stefano Bellucci, Eur. Phys. B 14 87 106 (2014)
17 A. A. Zvyagin, Phys. Rev. B 86, 085126 (2012).
18 M. Cini, Phys. Rev. B22, 5887 (1980); Phys. Rev. B89, 239902 (2014).
19 P. W. Brouwer, Phys. Rev. B 58, R10135 (1998).
20 J. E. Avron, A. Raveh and B. Zur, Rev. Modern Phys. 60 873 (1988).
21 D. M. Haldane, J. Phys. C: Solid State Phys. 14, 2585 (1981).
22 D. E. Feldman and Yuval Gefen, Phys. Rev. B 67, 115337 (2003).
23 A. Agarwal and D. Sen, Phys. Rev. B 76, 035308 (2007).
24 M. J. Salvay, Phys. Rev. B 79, 235405 (2009).
25 E. Perfetto, M. Cini, S. Bellucci, Phys. Rev. B 87, 035412 (2013).