Higher order fluctuations of strangeness
and flavour hierarchy

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Abstract. We present a preliminary analysis on the sensitivity to the chemical freeze-out
temperature of higher order moments of strange particle multiplicity distributions in heavy ion
collisions. Within the Hadron Resonance Gas (HRG) model we evaluate ratios of cumulants
for kaons ($K^{\pm}$) and hyperons ($\Lambda, \Sigma^{\pm}, \Xi^{-}, \Omega^{-}$) as a function of the temperature and compare
them to the sensitivity profiles obtained from ratios of particle yields. We show that ratios of
higher order fluctuations of strange baryons could provide a useful tool to extract the range of
freeze-out temperature, once experimental data are available. Finally, a connection to lattice
data through the fourth to second cumulant ratio is made. The deconfinement transition on the
lattice seems to indicate the possibility of a flavour hierarchy, namely strange quarks seem to
deconfine at a higher temperature. We would like to test the possibility for the same scenario
to occur at the chemical freeze-out and we show how the inclusion of multi-strange baryons in
the evaluation of higher order cumulants might provide a sensitive observable to extract the
freeze-out temperature.

1. Introduction

Strange particles have been a fundamental observable in the search for a possible state
of deconfined matter and as a signal for the formation of a Quark Gluon Plasma (QGP)
in relativistic heavy-ion collisions (HICs). The so-called strangeness enhancement has been
indicated as a signal of deconfinement since the early eighties [1].

The reason why strangeness is so easily produced in the presence of free quarks and gluons
is that processes as gluon fusion $g + g \rightarrow s + \bar{s}$ and quark-antiquark annihilation $q + \bar{q} \rightarrow s + \bar{s}$
are energetically favoured with respect to reactions involving confined hadrons. The $Q$-value
of gluon fusion and $q\bar{q}$ annihilation at temperatures close to the critical one, $T_c$, is roughly
$2m_s \approx 200$ MeV, while to produce strangeness in a hadronic environment, e.g. through
associated production $N + N \rightarrow N + \Lambda + K$, nearly $Q = m_\Lambda + m_K - m_N \approx 700$ MeV is
needed. The abundant production of strange quark pairs in the early stages of QGP leads to
the presence, in the subsequent hadronization process, of kaons and hyperons, which otherwise
would be rarely produced.

Experimental observations of such an enhancement are present at both RHIC and LHC
energies [2], where an increase of hyperon production ($\bar{\Lambda}, \Xi^+, \Omega$) at mid-rapidity as a function
of the number of participants has been seen.
The production of particles with strangeness content is also fundamental in the description of the hadrochemistry at chemical freeze-out. The particle abundances in HICs in a very wide range of energies are well reproduced by statistical hadronization models which assume a common freeze-out surface for all particle species [3]. Nevertheless, latest results on SHM fits to ratios of particle yields at LHC energies show a tension between light (protons) and strange particles (hyperons) [4][5]. These recent outcomes might represent a possible indication that in order to reproduce ratios or yields of strange hadrons a higher chemical freeze-out temperature is needed with respect to protons (see Fig.1). A different behaviour between light quarks and strange quarks is also present in lattice results [6]. Here the authors evaluate the ratio of cumulants $\chi_4/\chi_2$ for $(u, d)$ and $s$ quarks as a function of the temperature and compare the results to a HRG calculation. In the low temperature regime HRG results and lattice data show an agreement within the error bars in both light and strange sector. In Fig.2 it is evident that the non-monotic behaviour of the lattice curves indicates that the system undergoes a transition, from a confined phase at low $T$ to a deconfined one at higher $T$. This evolution is stressed by the separation of lattice data from HRG results as the temperature increases. Strange quarks start to show this separation at a higher temperature than light quarks: a flavour hierarchy seems to occur during the deconfinement transition.

Our analysis prepares the ground for an investigation of a similar flavour hierarchy in the chemical freeze-out process by studying higher order fluctuations of strange particles, in order to provide sizeable observables once experimental data are available [7][8].

2. The Hadron Resonance Gas model

The results presented here are obtained using a HRG model in partial chemical equilibrium, which means that contributions from strong decays from resonances with mass up to 2 GeV/c$^2$ are included. Consistently with what is done in the experiment, the contributions from weak decays have not been taken into account: the HRG results have been compared to feed-down corrected experimental data. The hadron spectrum of all resonant states is taken from the PDG list [9]. The observables to be compared to experimental data are the susceptibilities of conserved charges such as baryonic number $B$, electric charge $Q$ and strangeness $S$, defined as:

$$\chi_{BQS}^{l,m,n} = \frac{\partial^{l+m+n} P / T^4}{\partial(\mu_B/T)^l \partial(\mu_Q/T)^m \partial(\mu_S/T)^n}.$$
Susceptibilities are directly related to higher moments of particle multiplicity distributions through the definitions of mean ($M$), variance ($\sigma^2$), skewness ($S$) and kurtosis ($\kappa$) and the following volume-independent ratios can be extracted from experimental data and then evaluated in the HRG approach:

$$\frac{M}{\sigma^2} = \chi_1/\chi_2, \quad \frac{S\sigma^3}{M} = \chi_3/\chi_1, \quad S\sigma = \chi_3/\chi_2, \quad \kappa\sigma^2 = \chi_4/\chi_2.$$ 

Further details on the HRG model used can be found in [10], where we obtained the freeze-out parameters ($T, \mu_B$) from the comparison of HRG results to net-charge and net-proton distribution measurements from the STAR collaboration (see Fig. 3).

We obtain a lower freeze-out curve, approximately 20 MeV less than the value given by the statistical hadronization model fits to all available particle multiplicities ($T \approx 166$ MeV). This difference in our opinion is due to the fact that net-charge and net-proton moments are mainly driven by particles containing light quarks (protons and pions), while the fit of particle multiplicities contains also contributions from strange particles such as $K, \Lambda, \Sigma, \Xi, \Omega$. As already pointed out in the introduction, lattice data for $\chi_S^2/\chi_2^2$ seem to indicate a flavour hierarchy in the high $T$ region. If an analogous mechanism occurs at the chemical freeze-out, then strange particle multiplicities and moments might yield a higher freeze-out temperature. An indication of this decoupling between light particles ($\pi, K, p$) and multi-strange baryons has already been shown in [10]. Another possibility is that higher lying strange resonances, predicted by quark models but so far not detected experimentally, might lower the freeze-out temperature for strange hadrons and bring it closer to the one we find for light particles [11].

3. Results

At the moment the study of higher order fluctuations in experiments is available only for light particles, namely pions, protons and kaons.

A possible way to explore the flavour hierarchy hypothesis at chemical freeze-out is to perform an analysis on lower moments of net-kaons and to compare to data once available. Our preliminary studies show that the sensitivity to the temperature of $\chi_2/\chi_1$ for net-kaons is much higher with respect to ratios such as $K^\pm/\pi^\pm$ [12]. The analysis of ratios of lower moments might already provide a sensitive tool to extract the freeze-out parameters. In Fig. 4 ratios of moments for net-kaons and net-hyperons are plotted as a function of the temperature at baryochemical potential $\mu_B = 24.3$ MeV, corresponding to the $\sqrt{s} = 200$ GeV at STAR. The finite value of $\mu_B$ leads to a non-zero amount of net-densities for multi-strange baryons which explains the opposite behaviour of the ratio $\chi_2/\chi_1$ with respect to the net-kaon result. A detailed comparison with experimental data of lower moments ($\chi_2/\chi_1, \chi_3/\chi_2$) could already give some

Figure 3: Freeze-out curve in the $T - \mu_B$ plane obtained from the fit of $\sigma^2/M$ for net-electric charge and net-proton from [10].

Figure 4: Ratios of moments for net-kaons and net-hyperons as a function of the temperature at baryochemical potential $\mu_B = 24.3$ MeV.
A precise estimate of the freeze-out temperature in the strange sector. In particular, once efficiency corrected data on net-kaons lower moments from STAR collaboration will be available, we could perform a similar analysis to the one used for net-charge and net-proton moments in [10]. Since lower moments of net-kaons seem to be very sensitive to variations of temperature, as shown in Fig. 4, we expect that a study of kaons alone might be sufficient to extract the freeze-out parameters.

As expected the ratio $\chi_4/\chi_2$ looks quite flat for all particle species, so a future employment as a tool to investigate the chemical freeze-out stage might be excluded. Nevertheless this ratio can be directly linked to lattice calculations, since it involves even moments, which are the only non-zero ones for strangeness at $\mu_B = 0$. In Fig. 5 results for the ratio $\chi_4/\chi_2$ in our HRG model are presented for a few specific sets of strange particles, along with results from the lattice [6]. Since this ratio is proportional to the strangeness content of the particles involved, the inclusion of hyperons in the calculation of $\chi_4$ and $\chi_2$ is crucial in order to achieve a sensitivity to the temperature closer to the lattice one, which will allow the determination of the freeze-out temperature once compared to data. A high accuracy of experimental data on higher moments for multi-strange baryons represents a challenging issue at the moment.

**Figure 4:** Ratios of moments (normalized to the value at $T = 140$ MeV) for kaons and hyperons at $\mu_B = 24.3$ MeV as a function of $T$.

**Figure 5:** Comparison between lattice results for $\chi_4/\chi_2$ for net-strangeness and our HRG model curves including different sets of particles.
4. Conclusions

Several indications coming from both latest experimental data [5] and lattice results [6] suggest that particles with strangeness content might show a higher chemical freeze-out temperature with respect to light particles such as protons and pions. Higher moments for strange mesons and baryons, within a HRG model approach, have been shown to provide a complementary tool to explore the chemical freeze-out stage and to extract the corresponding $T$ and $\mu_B$, once data are available.

In our study, we show that a detailed analysis of lower moments of net-kaons could already show a discrepancy on the extracted freeze-out temperature with respect to the value obtained from the study of net-charge and net-proton multiplicity distributions at STAR [10].

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