Least squares support vector machine method for load identification of nonlinear system

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Abstract. In order to eliminate the dependence of load identification problem on the prior knowledge of current mechanical system, least squares support vector machine was applied to identify the inverse model of nonlinear system, and then based on this inverse model operational responses were adopted to determine real time excitation force. A nonlinear system was applied to conduct the simulation and calculate the steady and unsteady input force in this paper to verify the validity of the proposed method. Simulation results reveal that least squares support vector machine is able to identify reliable inverse model of nonlinear system and then reconstruct accurate real time excitation force. According to the present approach, a small quantity of training samples is needed rather than complete knowledge of the mathematical model and parameters of nonlinear system, so this approach can be extended to engineering application.

1. Introduction

Due to the fact that most engineering structures are nonlinear systems, the problem of load identification of nonlinear systems is increasingly concerned by many scholars. At present, most of the researches on load identification of nonlinear systems are based on modern control theories. In 1998, Ma et al. used Kalman Filter (KF) and Recursive least Square (RLS) method to identify impact loads of linear lumped mass system[1]. In 2004, Ma and Ho extended to nonlinear systems by adopting Extended Kalman Filter (EKF) and RLS based on previous studies of linear systems[2]. The identification method based on EKF and RLS can be applied to simple and complex loads, as well as single and multiple excitations. Because this method is based on the state space model of the system, it requires a clear understanding of the mathematical model and system parameters, which limits its application and popularization in practical engineering. In 2013, Cao et al. proposed flight load identification model based on improved support vector machine regression (SVM-R), which used flight parameters to identify bending moment of a key component in semi-roll flight maneuver[3].

In this paper, Least squares support vector machine (LS-SVM) is proposed to identify the time-domain load of nonlinear system. LS-SVM is firstly used to identify the inverse model of nonlinear system with given input and output data. The response of the nonlinear system under working state is taken as the input of the inverse model, and the output is the excitation to be identified. The method is no longer dependent on the specific mathematical model and system parameters, and the inverse model identification can be completed with a small amount of input and output sample data to prepare for load identification.
2. Least squares support vector machines

Support vector machine (SVM) is a new learning machine based on statistical learning theory proposed by Vapnik et al., which automatically learns the structure of the model based on the principle of structural risk minimization\(^4\). Compared with the traditional neural network algorithm, SVM has simple structure and excellent performance, especially generalization ability of which is obviously improved. It is suitable for high-dimensional data and dimension disaster is avoided. The problem is finally converted to quadratic optimization problem, and the solution is the global optimal solution, which solves the local extremum problem in neural network\(^5\). This is an important reason why SVM algorithm is adopted in this paper instead of neural network algorithm.

In 1999, Suykens proposed least squares support vector machine (LS-SVM), in which least squares linear system was adopted as loss function, instead of the quadratic programming algorithm adopted by SVM, with simple operation, fast convergence and high precision\(^6\).

The identification principle of LS-SVM is introduced below\(^7\). Supposing \( L \) groups of training sample sets \((x_i, y_i)\), \( i = 1, 2, \ldots, L \) are given with input data \( x_i \in \mathbb{R}^n \) and output data \( y_i \in \mathbb{R} \), nonlinear function estimation is:

\[
h(x) = w^T \phi(x) + b \tag{1}\]

The function estimation problem can be described as follows:

\[
\begin{align*}
\min J(w, e) &= \frac{1}{2} w^T w + \frac{1}{2} \gamma \sum_{i=1}^{L} e_i^2 \\
\text{s.t. } y_i &= w^T \phi(x_i) + b + e_i
\end{align*}
\tag{2}
\]

where \( \gamma \) is penalty factor (regularization parameter), and \( e_i \) is relaxation factor of insensitive loss function.

Lagrange method is used to solve the optimization problem:

\[
L(w, \alpha, b, e_i) = J + \sum_{i=1}^{L} \alpha_i \left[ y_i - w^T \phi(x_i) - b - e_i \right] \tag{3}
\]

where \( \alpha_i \) is Lagrange multiplier. According to Karush-Kuhn-Tucher (KKT) optimal condition, the following can be obtained:

\[
\begin{align*}
\frac{\partial L}{\partial w} &= 0 \Rightarrow w = \sum_{i=1}^{L} \alpha_i \phi(x_i) \\
\frac{\partial L}{\partial b} &= 0 \Rightarrow \sum_{i=1}^{L} \alpha_i = 0 \\
\frac{\partial L}{\partial e_i} &= 0 \Rightarrow \alpha_i = \gamma e_i \\
\frac{\partial L}{\partial \alpha_i} &= 0 \Rightarrow y_i - w^T \phi(x_i) - b - e_i = 0
\end{align*}
\tag{4}
\]

The above equation can be expressed as following matrix form:

\[
\begin{bmatrix}
0 & I_L^\top \\
I_L & \Omega + 1/\gamma I_L
\end{bmatrix}
\begin{bmatrix}
b \\
\alpha
\end{bmatrix}
= \begin{bmatrix}
0 \\
y
\end{bmatrix}
\tag{5}
\]

where \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_L]^\top \), \( y = [y_1, y_2, \ldots, y_L]^\top \), \( I_L = [1, \ldots, 1]^\top \), \( I_L \) is identity matrix.

\( \Omega = K(x, x) = \phi(x)^\top \phi(x_i) \) is a kernel function satisfying Mercer condition. Linear, polynomial and Gaussian kernel functions are commonly used at present. Gaussian kernel function (or radial basis kernel function) is linear feedforward network learning algorithm with global convergence properties, which
has fast learning speed and wide application. Gaussian kernel function presented below is adopted in this paper:

\[ K(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{(2\sigma^2)}\right) \]  

(6)

where \( \sigma \) is kernel width.

The input vector coefficient and threshold can be calculated through L sets of training sample data. And the least-squares support vector machine regression estimation is:

\[ h(x) = \sum_{i=1}^{L} \alpha_i K(x_i, x) + b \]  

(7)

3. Inverse model identification for nonlinear continuous systems

The dynamic equation of SISO nonlinear continuous system can be expressed simply as:

\[ f(\ddot{y}, \dot{y}, y, u) = 0 \]  

(8)

\( \ddot{y}, \dot{y}, y, u \) are respectively dynamic acceleration, velocity, displacement and external excitation, and \( f(\cdot) \) represents nonlinear continuous function.

For the SISO nonlinear continuous system mentioned above, if it satisfies \( \partial f / \partial u \neq 0 \) in an open set \( D \) and is continuous everywhere in \( D \), the system is invertible \([8]\). The inverse system can be expressed as:

\[ u = g(\ddot{y}, \dot{y}, y) \]  

(9)

The inverse system can be represented as the following form because of \( Ty = \ddot{y}(l+1) - \ddot{y}(l) \) and \( Ty = y(l+1) - y(l) \):

\[ u(l) = G(y(l+1), \dot{y}(l+1), y(l), \dot{y}(l)) \]  

(10)

The input data for inverse model identification is \( x(l) = [y(l+1), \dot{y}(l+1), y(l), \dot{y}(l)] \), and output data is corresponding excitation \( u(l) \). According to equation (10), the velocity and displacement at this moment and the next moment are used to estimate the excitation at this moment. Through LS-SVM learning, the input vector coefficient and threshold value are calculated by equation (5), so as to project the data in the input space to the feature space for linear fitting using the nonlinear mapping function.

4. Simulation results and analysis

4.1. Inverse model identification for nonlinear system

Air-damped isolator model shown below is adopted for numerical simulations in referring to literature \([9]\):

\[ m\ddot{y}(t) + c\ddot{y}(t) + ky(t) = u(t) \]  

(11)

where \( m=1 \text{kg} \), \( c=1 \text{Ns/m} \), \( k=1 \text{N/s} \). The initial displacement and velocity are \( y(0) = 0 \text{m} \) and \( \dot{y}(0) = 0 \text{ms}^{-1} \), respectively.

Sufficient excitation signals should be selected for inverse system identification to reflect the dynamic characteristics of inverse system. Random excitation signals with mean value of 0 and variance of 1 and corresponding velocity and displacement responses are selected as training samples for inverse model identification. The sampling time is 20s and time resolution is 0.1s. The time history of random excitation is shown in Figure 1. The classical Runge-Kutta method is used to calculate the velocity and displacement response of the nonlinear system under random excitation, as shown in Figure 2.
With the velocity and displacement response as input data to the training sample, random excitation as output data, inverse model identification for nonlinear system is completed. The parameters of LS-SVM are selected as penalty factor $\gamma=1.5e10$ and radial basis kernel width $\sigma=10$.

The identification results of the inverse model are shown in Figure 3. Desired Force is the output data of the training sample, namely the random excitation, and Estimated Force is the output of the inverse model of the nonlinear system. The simulation results show that the expected excitation and the estimated excitation are in good agreement, which indicates that the inverse model identification of nonlinear system is accurate. It is worth mentioning that the excitation at 20s is not identified and given as 0 because the excitation at this moment is identified through the velocity and displacement at this moment and next moment.

4.2. Identification results of time domain excitation

The velocity and displacement responses of the nonlinear system are obtained by Runge-Kutta method under steady and unsteady excitations respectively, which are used as the input data of the inverse model, and the output is estimated excitation.

4.2.1. Identification results of steady excitation. The following dual frequency sinusoidal excitation is applied to the nonlinear system:
The corresponding velocity and displacement obtained by Runge-Kutta method are presented in Figure 4. The estimated excitation based on inverse model of nonlinear system is shown in Figure 5, which is approximately consistent with the actual excitation.

\[
    u(t) = 0.4\sin\left(\frac{\pi t}{5}\right) + 0.1\cos(\pi t) \quad 0 \leq t \leq 20 \text{ (s)}
\]  \hspace{1cm} (12)

The estimation results of sinusoidal excitation and dual frequency sinusoidal excitation show that the force identification method based LS-SVM inverse model can accurately reconstruct the steady excitations of nonlinear system.

4.2.2. Identification results of unsteady excitation. The following semi sinusoidal impulse excitation is applied to the nonlinear system:

\[
    \begin{cases}
    u(t) = 0 \text{ (N)} & 0 \leq t < 5 \text{ (s)} \\
    u(t) = -\sin\left(\frac{\pi t}{5}\right) \text{ (N)} & 5 \leq t \leq 10 \text{ (s)} \\
    u(t) = 0 \text{ (N)} & 10 < t \leq 20 \text{ (s)}
    \end{cases}
\]  \hspace{1cm} (13)

The corresponding velocity and displacement are presented in Figure 6. And Figure 7 shows that the estimated force is almost consistent with the actual impact excitation.
The random excitation acting on the nonlinear system is calculated finally. The corresponding velocity and displacement of the system are shown in Figure 8, and the identification results of random excitation are shown in Figure 9. Similarly, the identification results of random excitation are satisfactory.

The estimation results show that inverse system method based on LS-SVM algorithm can not only accurately identify the steady excitations in time domain, but also be applicable to the unsteady excitations.

It should be noted that for all excitations listed above, the value at 20s is not identified, but set to 0, which is caused by the inverse model identification of nonlinear system. The reason has been described before and will not be repeated.

5. Summaries
Least square support vector regression algorithm was introduced into time domain load identification of nonlinear system in this paper, and satisfied results were obtained. Excitation forces in operational conditions are reconstructed using corresponding velocity and displacement responses based on inverse model of nonlinear system identified by LS-SVM algorithm. Therefore, the complicate inverse problem of load identification is converted to optimization and forward problem to deal with. Simulation results show that the learning ability and generalization ability of inverse model can be guaranteed by selecting the appropriate LS-SVM parameters. LS-SVM method can be used to effectively reconstruct the steady and unsteady loads of nonlinear system. In addition, the method does not rely on prior knowledge of the
system. The inverse model of nonlinear system can be established with a small number of input and output data, therefore this method can be extended to practical engineering applications.

The time domain load identification method based on least square support vector machine (LSSVM) algorithm is a new way for load identification technology and has a certain use value. In this paper, only the single excitation of the nonlinear system is studied. It is theoretically feasible to identify multiple excitation sources using LS-SVM algorithm, which needs in-depth study.

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