Elimination of unwanted qubit interactions for parametric exchange two-qubit gates in a tunable coupling circuit

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We experimentally demonstrate a simple-design tunable coupler, achieving a continuous tunability for eliminating unwanted qubit interactions. We implement two-qubit iSWAP gate by applying a fast-flux bias modulation pulse on the coupler to turn on parametric exchange interaction between computational qubits. Aiming to fully investigate error sources on the two-qubit gates, we perform quantum process tomography measurements and numerical simulations as varying static ZZ coupling strength. Our results reveal that the change in the two-qubit gate error is mainly attributed to unwanted high-frequency oscillation error terms, while the dynamic ZZ coupling parasitising in two-qubit gate operation may also contribute to the dependency of the gate fidelity. This approach, which has not yet been previously explored, provides a guiding principle to improve gate fidelity of parametric iSWAP gate by the elimination of unwanted qubit interactions. This controllable interaction, together with the parametric architecture by using modulation techniques, is desirable for crosstalk free multiqubit quantum circuits and quantum simulation applications.

Building large superconducting circuits requires highly coherent and strongly interacting physical qubits to achieve high-fidelity gates. As the circuits become more complex, however, the fidelity of quantum algorithms will begin to be dominated by unwanted qubit interactions, increased decoherence and frequency-crowding, all inherent to traditional frequency-tuned architectures [1]. Spurious unwanted qubit interactions can degrade gate performance. It thus becomes increasingly crucial to develop robust protocols for multiqubit control [2–4]. Much effort has been devoted to eliminating the unwanted coupling and achieving controllable interactions [5, 6]. Parametric schemes based on tunable couplers can help mitigate the problem of unwanted coupling and frequency crowding [3, 7]. In this scheme, the effective interaction between two qubits is mediated via a frequency-tunable transmon bus, which dispersively couples to both computational qubits. To turn on the interaction between two qubits, the tunable bus is modulated by an external magnetic flux, at the qubit frequency detuning, which causes a parametric oscillating of the qubit-qubit exchange coupling and activates a resonant XX+YY interaction [7–9]. Such a flux-modulation scheme with microwave-only control provides frequency-selectivity and allows to use fixed-frequency computational qubits, thereby minimizing the sensitivity of the qubits with respect to the sources of possible noise. The microwave-only control gates have been proposed and realized for fixed-frequency qubits by applying one or more microwave drives [10, 11]. In particular, the leading two-qubit gate for fixed-frequency qubits, the cross-resonance (CR) gate [6, 12], has demonstrated fidelities greater than 99% [6]. The coupling, however, is only effective when the qubit-qubit detuning is closely spaced compared to the anharmonicity of the qubits. Unlike drive-activated gates, the parametric exchange interaction does not decrease as the frequency detuning of qubits is larger than the anharmonicity. Therefore, it is promising for implementing entangling gate in larger circuits where a range of qubit frequencies are needed to avoid crosstalk.

Although two-qubit interactions with parametric modula-

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FIG. 1: (a) Optical micrograph of two qubits coupled via the coupler. Q1 and Q2 and coupler are shown in grey, yellow and white, respectively. Each qubit has an independent reaout cavity for qubit readout, XY control line to apply a microwave pulse, Z line to tune the qubit frequency. (b) A zoomed-in area in (a) highlights the tunable coupler. (c) Schematic circuit of the coupler system. Parametric gate can be realized by applying a fast-flux bias \( \phi(t) \) to the coupler SQUID loop. (d) Sketch of the coupler system. Two qubits are coupled directly by a coupling strength \( g_{12} \) and couple to the coupler qubit with a coupling strength \( g_1 \) and \( g_2 \), respectively.

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Building large superconducting circuits requires highly coherent and strongly interacting physical qubits to achieve high-fidelity gates. As the circuits become more complex, however, the fidelity of quantum algorithms will begin to be dominated by unwanted qubit interactions, increased decoherence and frequency-crowding, all inherent to traditional frequency-tuned architectures [1]. Spurious unwanted qubit interactions can degrade gate performance. It thus becomes increasingly crucial to develop robust protocols for multiqubit control [2–4]. Much effort has been devoted to eliminating the unwanted coupling and achieving controllable interactions [5, 6]. Parametric schemes based on tunable couplers can help mitigate the problem of unwanted coupling and frequency crowding [3, 7]. In this scheme, the effective interaction between two qubits is mediated via a frequency-tunable transmon bus, which dispersively couples to both computational qubits. To turn on the interaction between two qubits, the tunable bus is modulated by an external magnetic flux, at the qubit frequency detuning, which causes a parametric oscillating of the qubit-qubit exchange coupling and activates a resonant XX+YY interaction [7–9]. Such a flux-modulation scheme with microwave-only control provides frequency-selectivity and allows to use fixed-frequency computational qubits, thereby minimizing the sensitivity of the qubits with respect to the sources of possible noise. The microwave-only control gates have been proposed and realized for fixed-frequency qubits by applying one or more microwave drives [10, 11]. In particular, the leading two-qubit gate for fixed-frequency qubits, the cross-resonance (CR) gate [6, 12], has demonstrated fidelities greater than 99% [6]. The coupling, however, is only effective when the qubit-qubit detuning is closely spaced compared to the anharmonicity of the qubits. Unlike drive-activated gates, the parametric exchange interaction does not decrease as the frequency detuning of qubits is larger than the anharmonicity. Therefore, it is promising for implementing entangling gate in larger circuits where a range of qubit frequencies are needed to avoid crosstalk.

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coupler by utilizing the unique competition between the positive direct and negative indirect coupling to achieve a continuous tunability [13]. It allows a direct control of qubit interactions, with large 'on' coupling consistent with small to zero 'off' coupling, without introducing nonidealities that limit the gate performance [14–16]. It can thus be used to more efficiently realize surface code implementations requiring iSWAP gates [17, 18]. We employ the parametric modulation on the coupler to turn on the interaction of qubits, and aim to fully investigate the effect of the unwanted high-order oscillation terms and parasitic ZZ coupling on the two-qubit gates, which have not yet been previously explored. This controllable interaction, together with the parametric architecture by using modulation techniques, paves a way for crosstalk free multi-qubit quantum circuits, and is desirable for scalability architectures and quantum simulation applications [2, 19, 20].

Our sample consists of two transmon qubits (Q1, Q2) coupled via a frequency-tunable transmon bus (coupler). Q1, Q2 and coupler are shown in grey, yellow and white in Fig. 1(a)(b), respectively. A false-color circuit image is illustrated in Fig. 1(c). The coupler, consisting of a superconducting loop with two Josephson junctions, couples to a flux line, which is used to tune the coupler frequency by a magnetic flux through the loop. As depicted in Fig. 1(d), the tunable coupler, together with the capacitive coupling from the cross-shaped capacitor, contributes to the total coupling between Q1 and Q2. Each computational qubit has a dedicated flux bias line to tune the qubit frequency. The qubit, Q1, Q2, with a transition frequency 4.9607, 4.9265 GHz at each sweet spot, has an energy relaxation time T1 ≈ 14.0, 13.7 μs, a pure dephasing time T2 ≈ 8.4, 4.0 μs and an anharmonicity of -206, -202 MHz, respectively. Each computational qubit is coupled to an individual readout cavity, with a frequency ω1,2/2π = 6.825, 6.864 GHz, in a strong dispersive coupling regime with a coupling strength g1,2/2π ≈ 86.6, 90.6 MHz to the qubit. The readout cavity is coupled to a transmission line, which connects to a Josephson parametric amplifier, allowing for a high-fidelity simultaneous single-shot readout for the both qubits. Derivative removal adiabatic gate (DRAG) pulse is used for single qubit rotation and pulse correction to reduce phase error and leakage to higher transmon levels [21]. Measurements are performed in a dilution refrigerator at a base temperature about 20 mK. The measurement setup and device parameters are presented in Supplementary Materials [22].

Without loss of generality, the system Hamiltonian can be presented, in the eigenbasis of the corresponding mode, as,

$$H/\hbar = \sum_{i=1,2} \frac{1}{2} \omega_i \sigma_i^x + \frac{1}{2} \omega_c \sigma_c^z + \sum_{i=1,2} g_i (\sigma_i^+ \sigma_c^- + \sigma_i^- \sigma_c^+) + g_{12} (\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-),$$

where ωi, ωc is the frequency of Q1, Q2 and coupler respectively, σαi, σαc (α = 1, 2, c) is the Pauli Z, raising and lowering operators for Q1, Q2 and coupler respectively. Both qubits (ω1,2/2π = 4.9607, 4.9265 GHz at each sweet spot) are negatively detuned from the coupler (ωc/2π = 5.976 GHz at sweet spot), Δi(φ) = ωc - ωi < 0. The two qubits each couple to the coupler with a coupling strength gi (i = 1, 2), as well as to each other with a direct capacitive coupling strength g12. The two qubits interact through two channels, the direct capacitive coupling and the indirect virtual exchange coupling via the tunable coupler. Assuming that the coupler mode remains in its ground state, the virtual exchange interaction can be approximated by Schrieffer-Wolff transformation, $e^{-i\frac{\hbar}{\Delta(\phi)}(\sigma_1^- \sigma_2^+ + \sigma_2^- \sigma_1^+)}$ [23, 24], to decouple the coupler from the system up to the second order in $\frac{g_{12}}{\Delta(\phi)}$, resulting in an effective two-qubit Hamiltonian for each mode [25],

$$\tilde{H}/\hbar = \sum_{i=1,2} \frac{1}{2} \delta_i \sigma_i^x + J_{12} (\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-),$$

where $\delta_i = \omega_i + \frac{g_{12}^2}{\Delta(\phi)}$ is the Lamb-shifted qubit frequency, $J_{12} = g_{12} + \frac{g_{12}^2}{\Delta(\phi)}$, $\Delta(\phi) = 2/(1/\Delta_1(\phi) + 1/\Delta_2(\phi))$. The combination of two terms, $g_{12} + \frac{g_{12}^2}{\Delta(\phi)}$, gives the total effective qubit-qubit coupling $J_{12}$, which can be adjusted by the coupler frequency through $\Delta(\phi)$. Since the tunability is continuous, one can always find a critical value $\omega_c,_{off}$ to turn off the effective coupling ($J_{12}(\omega_c,_{off}) = 0$), as well as the static ZZ coupling $\xi_{ZZ}$ when two qubits are detuned in dispersive regime [13]. Now we apply a flux to the tunable coupler, φ(t) = φdc + Ωcos(ωdt + φ), where φdc is the DC flux bias, Ωcos(ωdt + φ) is a sinusoidal fast-flux bias modulation with an amplitude $\Omega$, frequency $\omega_0$ and phase $\phi$. When $\omega_0$ is in resonant with the effective detuning $\Delta_2 = \Delta_1 + \Omega^2/4 (\frac{\delta_1}{\sigma_0} - \frac{\delta_2}{\sigma_0}),$ in a rotating frame at the qubit frequencies $\delta_i$, the Hamiltonian becomes $H/\hbar = J_{eff}(\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-)$ (see Supplementary Materials for details [22]), where $\Delta_2 = \delta_1 - \delta_2$. $J_{eff}$ is the effective exchange coupling between the two qubits, $J_{eff} = \frac{\Omega}{2} \frac{\delta_2}{\delta_0}$. This effective parametric modulation brings the computational qubits into resonance and can be used to implement two-qubit gates.

We can construct the two-qubit swap operation by using the effective exchange coupling $J_{eff}$ through the parametric modulation pulse on the coupler. Accordingly, in order to realize the parametric isWAP gate, we should get a full control over the effective coupling $J_{12}$, where $J_{12} = g_{12} + \frac{g_{12}^2}{\Delta(\phi)}$. We determine the direct coupling strength between the qubit and coupler, $g_i$ (i=1,2), by an energy-swap experiment of tuning the coupler frequency into resonance with each qubit, as shown in Fig.S4(a)(b) in Supplementary Materials [22]. The extracted coupling strength $g_{1,2}/2\pi$ is about 76.9 MHz. Also, the qubit-qubit direct coupling strength $g_{12}$, can be extracted through a qubit-qubit energy-swap experiment [14, 26], as illustrated in Fig.S4(c)(f). We estimate the corresponding direct capacitive coupling strength $g_{12}/2\pi$ to be around 6.74 MHz. Furthermore, extracting the coupler frequency is also critical for controlling the total coupling strength of $J_{12}$ since $g_{12}$ can be varied by tuning the coupler frequency. In our experiment, the coupler frequency is measured by probing the
FIG. 2: (a) Pulse sequence for realizing parametric iSWAP gate. (b) The exchange oscillation between two-qubit state |00⟩ and |01⟩ as a function of the gate length and drive frequency ω0 (with respect to the detuning between two qubits when Ω = 0, ΔJ12/Ω = 0 = 34.2 MHz). (c) The simultaneous quantum-state population for both qubits, indicating that the excitation oscillates between the two qubits. The iSWAP and √SWAP gate can be implemented at the special evolution time τ ~ 204 ns and τ ~ 102 ns, respectively. (d) Simulation curves of ∂J12/∂Φ vs coupler flux amplitude (coupler frequency). We show four different qubit frequency detuning, as an example, to indicate the dependency of ∂J12/∂Φ on the coupler flux amplitude. The blue curve is corresponding to the qubit frequency detuning of 34.2 MHz at which we acquire data in our experiment. The black, purple, orange and red star superposed on the blue curve present the operation points we choose. The right panel shows the change of the static ZZ coupling strength as varying the coupler flux amplitude (coupler frequency).

The exchange oscillation between two-qubit state |00⟩ and |01⟩ as a function of the gate length and drive frequency ω0 (with respect to the detuning between two qubits when Ω = 0, ΔJ12/Ω = 0 = 34.2 MHz). The static ZZ coupling strength is ξZZ = J12/ΔJ12. To turn on the exchange interaction between computational qubits, a fast-flux bias modulation pulse is then applied on Q1. The fast modulation pulse is held for a certain time to perform the two-qubit gate. The effective coupling strength, J_{eff} = ∂J12/∂Φ, depends on the derivative of J12 with respect to φ. The theoretical calculation of ∂J12/∂Φ versus DC flux on the coupler is illustrated as a blue, yellow, red and green curve in Fig. 2(d) at four qubit frequency detunings of 34.2, 121.9, 233.1, 390.6 MHz, respectively. The effective coupling can thus be tuned from zero to a few MHz for a moderate modulation amplitude Ω. By applying the fast-flux bias modulation pulse of Ωcos(ωt + φ) on the coupler, we measure the simultaneous quantum-state population of both qubits, as shown in Fig. 2(c), indicating the excitation oscillations between the two qubits. Specific locations in Fig. 2(c), J_{eff}t = nπ/2, rep-
resent primitive two-qubit gate, iSWAP gate, which can be used to construct a universal gate set for quantum computing. We implement the iSWAP gate at the specific evolution time, indicated by a dark arrow in the figure, where $t \sim 204$ ns. At the evolution time marked by a red arrow, where $t \sim 102$ ns, the excitation is equally shared between both qubits, and a maximally entangled Bell state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ can be generated. Fig. 2(b) shows a chevron pattern on $Q_1$ population, as a function of the coupler flux pulse length and the parametric modulation frequency $\phi_p$ with respect to the detuning between the qubits when $\Omega = 0, \Delta_{12}, \Omega = 0 = 34.2$ MHz. The parametric flux modulation induces a DC shift of the tunable coupler, and thus the resonance frequency of the exchange oscillating is shifted down from $\Delta_{12}, \Omega = 0$ by approximately 0.25 MHz. One can expect a faster two-qubit gate by increasing the modulation strength or setting the DC flux bias to yield a lower coupler frequency but a larger value of $\frac{\partial J_2}{\partial \phi}$, but this will result in the increase of the static ZZ coupling. In addition, the large modulation strength may lead to leakage out of the computational basis into the higher transmon levels or into the coupler; also, decreasing the coupler frequency to move closer into resonance with the qubit will lead to a reduction of protection of the qubit to flux noise. Therefore, we take the data using a moderate modulation amplitude $\Omega \approx 0.115\Phi_0$ ($\Phi_0$ is the flux quantum) at the DC flux bias with the zero static ZZ coupling.

We perform quantum process tomography (QPT) by implementing 16 independent two-qubit input states and construct the $\chi$ matrix for iSWAP, as shown in the upper insert of Fig. 3(b). The gate fidelity can be determined from the $\chi$ matrix through the expression $F = \text{tr}(\chi_{\text{exp}} \chi_{\text{ideal}})$, where $\chi_{\text{exp}}$ and $\chi_{\text{ideal}}$ are the experimental and ideal $\chi$ matrices. QPT gives a full description of the gate, and it is susceptible to state preparation and measurement errors [32]. The fidelity from QPT is 93.5% from maximum-likelihood estimation. The gate error could be attributed to decoherence, state preparation, unwanted qubit interactions and population leakage out of the computational subspace. To measure the intrinsic gate error, we concatenate a series of gates and determine the fidelity decay as the number of gates (N) increases, using the pulse sequence indicated in Fig. 3(a). The QPT fidelity, shown as red dots in Fig. 3(b) with error bars, decays with concatenating the iSWAP gates. A representative $\chi$ matrix ($\chi_{\text{exp}}$ and $\chi_{\text{ideal}}$) of QPT measurements at N=21 is shown in the lower insert of Fig. 3(b). By fitting the data (black curve) under the assumption of independent error for each gate, we obtain an average gate fidelity of 94.0%.

To characterize error source for the parametric iSWAP gate, we perform quantum process tomography measurements as varying the DC flux bias on the coupler (static ZZ coupling strength). We measure the QPT by setting the coupler at four different DC flux biases, as indicated by the black, purple, orange and red star superposed on the blue simulation curve of $\frac{\partial J_2}{\partial \phi}$ in Fig. 2(d), with respect to the static ZZ coupling at 0, 0.23, 0.28 and 0.34 MHz, respectively. For comparison sake, we set a same gate time of 204 ns for all two-qubit gates at four different static ZZ couplings by adjusting the parametric modulation amplitude, to compensate the change in the $\frac{\partial J_2}{\partial \phi}$ as varying the DC flux bias of the coupler. To eliminate the measurement uncertainty, the QPT measurement is repeated five times at each operation point of the static ZZ coupling strength. A representative experimental matrix $\chi_{\text{exp}}$ and ideal matrix $\chi_{\text{ideal}}$ is shown as shaded bars in upper insert of Fig. 3(b) and Fig. 4(b)(c)(d), yielding a gate fidelity of 93.5%, 89.7%, 88.8% and 87.9%, for the QPT data acquired at the static ZZ coupling of 0, 0.23, 0.28 and 0.34 MHz, respectively. Apparently, the corresponding average value of fidelity drops quickly as the static ZZ coupling strength increases from the zero to 0.34 MHz. As the DC flux bias varies, except for the disturbance variation of unwanted ZZ coupling, the degradation of gate fidelity may also be attributed to the change in the decoherence and population leakage out of the computational subspace. The decoherence error, however, should be identical because of the same gate time used. The error variation due to the real excitation of coupler can also be ruled out. Indeed, when the coupler frequency is reduced to be close to the computational qubit frequency, virtue excitation of coupler will become real excitation, exchanging energy between the qubit and coupler.
The four DC flux biases used here to perform the QPT measurements, however, are all in the non-leakage regime (see Fig.S5 in Supplementary Materials [22]). The change in the parametric modulation amplitude may attribute to the gate fidelity variation, since the large signal amplitude could lead to leakage out of the computational basis into the higher transmon levels or into the coupler. This is, however, contrary to our case. We use the moderate drive amplitude of modulation pulses, 0.115φ0, 0.021φ0, 0.019φ0, 0.017φ0, at the four DC flux biases of the coupler. We yield, instead, the highest gate fidelity at the largest drive amplitude of 0.115φ0. For other three cases, the drive strength is similar. The variation of gate error, again, has no explicit dependence on the drive amplitude. To confirm the above hypothesis, we extract a fidelity difference of the QPT measurements, at each operation point of the DC flux bias, between the cases with (experimental QPT fidelity) and without (reference QPT fidelity) the parametric modulation drive on the coupler. The corresponding pulse sequence used for the measurements is illustrated in Fig. 4(a). In this way, the effect of the single-qubit gate error and measurement error, as well as the decoherence and population leakage, on the variation of gate fidelity can be ruled out at the different DC flux biases of the coupler. Accordingly, the change in the QPT fidelity difference is only attributed to the error source parasitising in the two-qubit gate operation.

In fact, we observe that the QPT infidelity difference gradually increases to 4.7%, 5.3% and 6.9% when taking the first operation point as reference.

We further verify the two-qubit gate error source by performing numerical simulations using the generalized Hamiltonian in Eq. (1), restricting the Hilbert space to the lowest two states of each transmon [23, 24]. We plot the simulation result in consideration of the qubit decoherence (purple-dotted line marked as SFD3) in Fig. 4(e) as a function of the static ZZ coupling strength. The calculated value is slightly higher than the corresponding experimental fidelity at the zero static ZZ coupling, while the full data set demonstrates a reduction trend as the static ZZ coupling increases. This dependency of the gate fidelity can be explained by the fact that, except for the effective parametric iSWAP term \( J_{\text{eff}} (\sigma^x_1 \sigma^x_2 + \sigma^y_1 \sigma^y_2) \), the simulation Hamiltonian also involves some unwanted high-frequency oscillation terms at frequency of \( \omega_0, 2\omega_0 \) and higher, which vary as changing the DC flux bias of the coupler and, thus, degrade the gate fidelity by the increase of the static ZZ interaction strength (see Supplementary Materials for details [22]). At other three operation points, rapid decline of the simulation values below the experimental fidelities reflects that the model of two-level system is too simplistic and more levels need to be included for a quantitative accurate description, though the result qualitatively predicts the degradation of the gate fidelity by these unwanted high-frequency oscillation terms. Consequently, we carry out the simulation based on a Hamiltonian including an anharmonicity term for each qubit (see Supplementary Materials for details [22]). The calculated two-qubit gate fidelity is plotted in Fig. 4(e) as a black (SFD3) and blue-dotted (SF3) line with and without consideration of the qubit decoherence, respectively. At the zero static ZZ coupling, the SFD3 simulation value is consistent with the experimental QPT fidelity (red-dotted line marked as EF). However, deviation appears in presence of the static ZZ coupling for the rest of operation points. The higher SF3 simulation fidelity (difference between SFD3 and SF3 varies from 2.1% to 3.3%) reveals that the gate error is mainly limited by the decoherence of the computational qubits. The SFD3 simulation fidelity, setting an upper bound of the intrinsic two-qubit gate fidelity, again, decreases as the static ZZ coupling arises, which is in agreement with our experimental observation. It is

FIG. 4: (a) Pulse sequence for observing the fidelity difference between the experimental QPT and reference QPT measurements. The fidelity difference is extracted between the cases with and without the parametric modulation drive on the coupler. (b)-(d) Experimental \( \chi \) matrix of the QPT measurements at the operating points with respect to the static ZZ coupling strength of 0.23, 0.28 and 0.34 MHz. (e) Simulation and Experimental QPT fidelity vs static ZZ coupling strength. The black-dotted line (SFD3) and blue-dotted line (SF3) represent the simulation fidelity with and without consideration of the qubit decoherence based on the Hamiltonian including the anharmonicity term; The purple-dotted line (SF3) shows the simulation fidelity in consideration of the qubit decoherence based on the Hamiltonian with the Hilbert space restricted to the lowest two states; The red-dotted line (EF) illustrates the experimental fidelity. The first data point of EF is extracted from the \( \chi \) matrix shown in the upper inset of Fig. 3(b), while other three points are calculated from the \( \chi \) matrix (b)-(d). Through the comparison between the simulation fidelity and experimental fidelity, we confirm that the change in the parametric gate error is mainly attributed to the unwanted high-frequency oscillation terms, as well as the dynamic ZZ coupling parasitising in two-qubit gate operation.
noteworthy that, in addition to the high-frequency oscillation terms, the dynamic ZZ coupling parasitising in the two-qubit gate operation may also contribute to the dependency of the gate fidelity. The effect of dynamic ZZ coupling can not be ruled out only based on the simulations and measurements we done here, and a further investigate is needed to clarify the explicit attribution of the parasitic coupling to the parametric iSWAP gate.

The gate fidelity we acquired, 94.0% (intrinsic average gate fidelity), is not very high, mainly due to the decoherence limit, but we do not think that it will become a constraint of application of our coupler scheme. In fact, we can improve the gate fidelity by fabricating the coupler with a higher frequency to acquire a larger derivative of $J_{12}$ in an appropriate operation regime, and thus significantly reduce the gate time, meanwhile maintaining a zero or near zero static ZZ coupling.

In summary, we experimentally demonstrate a simply-designed tunable coupler capacitively coupled to two computational Xmon qubits and, thus, allows continuously varying the adjacent superconducting qubit coupling from positive to negative values. We measure the static ZZ interaction by the Ramsey-type measurement. By utilizing the simultaneous RB protocol, we observe that the static ZZ coupling degrades the single-qubit gate performance. We conduct the two-qubit iSWAP gate by applying the fast-flux bias modulation pulse on the coupler to turn on the parametric exchange interaction between the computational qubits. As varying the static ZZ coupling strength, we perform quantum process tomography measurements and numerical simulations to fully investigate the error sources on the parametric two-qubit gates. Our results reveal that the variation of two-qubit gate error is mainly attributed to the unwanted high-frequency oscillation terms, but the dynamic ZZ coupling parasitising in two-qubit gate operation may also contribute to the dependency of the gate fidelity. This controllable interaction, together with the parametric architecture by using modulation techniques, is desirable for crosstalk free multiqubit quantum circuits and quantum simulation applications.

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Supplementary Materials for “Elimination of unwanted qubit interactions for parametric exchange two-qubit gates in a tunable coupling circuit”

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THEORY

Theory of parametric iSWAP gate

We first briefly summarize the theory of parametric iSWAP gate. In dispersive regime, the system Hamiltonian can be written as,

\[
\frac{\hat{H}}{\hbar} = \sum_{i=1,2} \left( -\frac{1}{2} \hat{\omega}_i \sigma_i^z + J_{12}(\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) \right), \tag{S1}
\]

where \( \hat{\omega}_i = \omega_i + \frac{\hat{g}^2}{2} \Delta_j(\phi) \) is the dispersive qubit frequency, \( J_{12} \) is the effective coupling strength between two qubits, \( J_{12} = g_{12} + \frac{\bar{g}_{12}}{\Delta_j(\phi)} \), \( \Delta_j(\phi) = 2 / (\frac{1}{\Delta_1(\phi)} + \frac{1}{\Delta_2(\phi)}) \) [1]. Apparently, given qubit frequency, there always exists a coupler frequency \( \hat{\omega}_{c,off} \) that can satisfy \( J_{12} = 0 \) [2–4].

According to the qubit parameters, the relationship between coupler frequency and its flux is nonlinear [5]. Thus, there is a second-order DC shift and an oscillating term at \( 2\hat{\omega}_c \) when expanding \( J_{12} \) in the parameter \( \Omega \cos(\omega t) \) to second-order (ignoring higher-order terms) [1]. Substituting Eq. (S1), the Hamiltonian in a frame rotating at the qubit frequencies (including the drive-induced shift) becomes:

\[
\frac{\hat{H}}{\hbar} = \left[ \left( J_{12} + \frac{\Omega^2}{4} \frac{\partial^2 J_{12}}{\partial \phi^2} \right) + \Omega \frac{\partial J_{12}}{\partial \phi} \cos(\omega_\phi t) \right] + \frac{\Omega^2}{4} \frac{\partial^2 J_{12}}{\partial \phi^2} \cos(2\omega_\phi t) \left( e^{i\Delta_{12} t} \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ e^{-i\Delta_{12} t} \right), \tag{S2}
\]

where \( \Delta_{12,\Omega} = (\hat{\omega}_1 - \hat{\omega}_2) + \Omega^2 \left( \frac{\partial^2 \hat{\omega}_2}{\partial \phi^2} - \frac{\partial^2 \hat{\omega}_1}{\partial \phi^2} \right) \). In our experiment, we apply a sinusoidal fast-flux bias modulation pulse at the frequency \( \omega_\phi = \Delta_{12,\Omega} \) on the tunable coupler to realize parametric iSWAP gate between qubits [1]. Replacing \( \Delta_{12,\Omega} \) with \( \omega_\phi \) and Using Euler’s formula we get:

\[
\frac{\hat{H}}{\hbar} = \frac{\Omega}{2} \frac{\partial J_{12}}{\partial \phi} \left( \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ \right) + \left( J_{12} + \frac{\Omega^2}{4} \frac{\partial^2 J_{12}}{\partial \phi^2} \right) \left( e^{i\omega_\phi t} \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ e^{-i\omega_\phi t} \right) + \frac{\Omega^2}{8} \frac{\partial^2 J_{12}}{\partial \phi^2} \left( e^{i2\omega_\phi t} \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ e^{-i2\omega_\phi t} \right) + \frac{\Omega}{2} \frac{\partial J_{12}}{\partial \phi} \left( e^{i2\omega_\phi t} \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ e^{-i2\omega_\phi t} \right) + \frac{\Omega^2}{8} \frac{\partial^2 J_{12}}{\partial \phi^2} \left( e^{i3\omega_\phi t} \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ e^{-i3\omega_\phi t} \right), \tag{S3}
\]

where the first term represents the resonant exchange interaction between \( Q_1 \) and \( Q_2 \) and the coefficients determine the exchange rate to realize the parametric iSWAP gate. The other terms represent unwanted high-frequency oscillation errors generated from high-order expansion of \( J_{12} \). Actually, both coefficients before each error term and frequency detuning between \( Q_1 \) and \( Q_2 \) directly decide the effects of errors [1, 6]. However, the mechanism that causes errors is very complicated, and each high-frequency oscillation error term may have different effects, such as the accumulation or cancellation of errors.

Here, we numerically calculate the high-frequency oscillation error terms by using our device parameters. We first calculate and simulate \( J_{12}, \frac{\partial J_{12}}{\partial \phi}, \frac{\partial^2 J_{12}}{\partial \phi^2} \) when changing coupler flux bias, shown in Fig. S1. Then according to the data, we calculate the coefficients of terms in Eq. (S3) for four different coupler flux biases discussed in the main text (\( Q_1 \) and \( Q_2 \) are biased at each sweet spot), see Table S1.

### TABLE S1: Coefficients of oscillation terms.

| Coupler flux bias | \( J_{12} \) | \( \frac{\partial J_{12}}{\partial \phi} \) | \( \frac{\partial^2 J_{12}}{\partial \phi^2} \) | \( \frac{\partial^3 J_{12}}{\partial \phi^3} \) |
|-------------------|-------------|-----------------|-----------------|-----------------|
| −6.31 mV          | 0.610       | 0.305           | 0.645           |
| −103.76 mV        | −4.678      | 0.230           | 0.982           |
| −106.81 mV        | −5.076      | 0.221           | 0.988           |
| −109.86 mV        | −5.513      | 0.209           | 0.989           |

1. Coefficients only consider the second-order expansion of \( J_{12} \).
2. All data are divided by \( 2\pi \) and in MHz.

![FIG. S1: Calculated datas for \( J_{12}, \frac{\partial J_{12}}{\partial \phi}, \frac{\partial^2 J_{12}}{\partial \phi^2} \) vs coupler flux pulse amplitude. The calculation uses the device and coupling parameters given in Table S2 and Table S3.](image-url)
EXPERIMENTAL DEVICE

Fabrication

The tunable coupling device consists of two Xmon qubits \((Q_1, Q_2)\) coupled via a coupler. Fabrication of the sample includes five main following steps: (1) A 100 nm aluminum film is deposited directly onto a c-plane sapphire wafer in Plassys MEB 550S. (2) Photolithography followed by inductively-coupled-plasma etching is used to define all the base wiring, readout resonators and large pads of the Xmon qubits on the wafer. (3) Two photolithography processes, aluminum deposition and wet etching are used to construct airbridges \([7]\). (4) Josephson junctions are fabricated by electron-beam lithography and double-angle evaporation of aluminum. (5) The completed chip is selected to wire bonded in an aluminum box for final packaging before measurement. In the third step, airbridges are fabricated to connect segments of grounding planes in order to reduce the impact of parasitic slotline modes. For purpose of reducing magnetic vortices loss, we also apply flux trapping holes (square holes of side length 2 \(\mu m\) and an edge to edge separation of 10 \(\mu m\)) \([8]\).

Diagram of the experimental setup

Our sample is measured in a dilution refrigerator with a base temperature of about 20 mK. Details of our measurement circuitry are shown in Fig. S2. For full manipulation of the device, we use three AWGs (two Tek5014C and one Tek70002A). One AWG (Tek5014C) is connected to input-output line for reading out qubits simultaneously, meanwhile another one channel is used to realize individual Z control of \(Q_1\). The second AWG (Tek5014C), synchronized with the first one, provides two pairs of sideband modulations for \(Q_1\) control. The \(XY\) control signals are generated from a single microwave signal generator modulated with different sideband frequencies. This method of control guarantees stable phase differences during the quantum tomography experiments. The third AWG (Tek70002A) directly generates flux pulse to realize individual Z control of \(Q_2\) and coupler.

Finally, a broadband Josephson Parametric Amplifier (JPA) \([9–12]\) is used for high-fidelity simultaneous single-shot readout. A JPA, which is pumped and biased by a signal generator and a voltage source (yoko) respectively, has a gain of more than 20 dB and a bandwidth of about 313MHz at 20 mK, see gain profile in Fig. S3(a). It is used as the first stage of amplification followed by a HEMT amplifier at 4K and room-temperature amplifiers, allowing for high-fidelity single-shot measurements of the qubits. In the JPA circuit design, 50\(\Omega\) impedance matching is applied without any other specific impedance engineering.

Quantum Process Tomography

Quantum process tomography (QPT) offers us a way to reconstruct quantum channel \(\varepsilon\). For a standard theory of QPT, we need a fixed set of operators \(\{E_i\}\) to form a basis for the set of operators on the state space and a fixed, linearly independent initial state basis \(\rho_j\), \(1 \leq j \leq d^2\), to construct any \(d \times d\) matrix. A quantum channel can be expressed as:

$$\varepsilon(\rho) = \sum_{mn} \chi_{mn} E_m \rho E_n^\dagger, \quad (S4)$$

where \(\chi_{mn}\) is a positive Hermitian by definition \([13]\). This expression is the \(\chi\) matrix representation. In order to construct quantum channel, we just need to calculate \(\chi\) matrix. Here, we first deal with \(E_m \rho E_n^\dagger\) term for each initial state \(\rho_j\), it can be reconstructed as:

$$E_m \rho_j E_n^\dagger = \sum_k \beta_{jk}^{mn} \rho_k, \quad (S5)$$

where \(\beta\) matrix is a four-dimensional matrix if considering all the initial state \(\rho_j\). It’s northworthy that the components of \(\beta\) matrix are complex numbers. By ensuring the operators set \(\{E_i\}\) and initial state set \(\rho\), we can calculate this \(\beta\) matrix. Besides, the final state term \(\varepsilon(\rho)\) can also be reconstructed for each \(\rho_j\) as:

$$\varepsilon(\rho_j) = \sum_k \lambda_{jk} \rho_k, \quad (S6)$$

where \(\lambda\) matrix can be easily calculated once we know the final state \(\varepsilon(\rho)\) combined with initial state set \(\rho\). If we get both \(\beta\) matrix and \(\lambda\) matrix, we can finally calculate \(\chi\) matrix by standard algorithms from linear algebra.

In our experiments, considering thermal problem in the system leading to a relatively low fidelity of the initial state preparation, we use the measured initial state set \(\rho^{meas}\) instead of the ideal initial state set \(\rho^{ideal}\) to reconstruct \(\beta\) matrix and \(\lambda\) matrix. We prepare standard initial states \(\rho_j\) followed by measuring them first using quantum state tomography (QST). The measurement process is the same as other experiments by using Bayes’s rule as mentioned in the expriment setup. Actually, the measured initial states \(\rho^{meas}\) are still linearily independent and, thus, we can rewrite the Eq. \((S5)\) for initial state \(\rho_j\) as:

$$E_m \rho_j E_n^\dagger = \sum_k \beta_{jk}^{mn} \rho_k^{meas}, \quad (S7)$$

Here, we can get \(\beta\) matrix by using measured initial state set \(\rho^{meas}\). Now we perform QPT to measure the quantum channel we are interested in. By performing QST for the final state, we get \(\varepsilon(\rho)\) and again we use the measured initial state set \(\rho^{meas}\) to calculate the \(\lambda\) matrix for each initial state \(\rho_j\) as:

$$\varepsilon(\rho_j) = \sum_k \lambda_{jk} \rho_j^{meas}, \quad (S8)$$

Finally, we use \(\beta\) matrix and \(\lambda\) matrix to construct \(\chi\) matrix.
FIG. S2: Diagram of the control wiring and circuit components. Our chip is packaged in an aluminum box and the whole sample is protected by magnetic and infrared shieldings, shown in grey shaded part. Three AWGs (two Tek5014C and one Tek70002A) are used to modulate XY control signals and readout signals generated from two signal generators respectively and generate DC bias signals to tune flux ($\Phi_1$ flux is generated from Tek5014C while $\Phi_2$ and coupler flux is generated from Tek70002A). Other devices such as one 10 MHz clock, one data analyzer card, one voltage source (yoko) and some microwave elements are all shown in the diagram. Josephson Parametric Amplifier (JPA), pumped and biased by one signal generator and one voltage source (yoko) respectively, followed by a HEMT amplifier and room temperature amplifiers, is used to support single-shot measurement by amplifying readout signal to improve Signal-to-noise Ratio (SNR). Signal is finally demodulated by an analog to digital converter (ADC).
MEASUREMENT RESULTS

Readout properties

With the help of the JPA, the single qubit gates and readout measurements are performed at the sweet spot of each qubit individually with high fidelities. The coupler frequency is tuned to 5.905 GHz to turn off the coupling between two qubits. Fig. S3(b) shows the I-Q data for single-shot qubit-state differentiation of each qubit when another one is in its thermal steady state. For each picture, the dot in the left represents the qubit prepared in a ground state \( |0\rangle \) while the dot on the right identifies the qubit prepared in an excited state \( |1\rangle \). Mismatch between the dispersive shift and decay of the readout resonator accounts for a non-perfect distinction between the ground state and excite state on each qubit. Due to the non-perfect distinction between the qubit states and thermal population of qubits and coupler, we use a calibration matrix to reconstruct the readout results based on Bayes rule [14]. For the \( j \)th qubit, we have the readout-calibration matrix:

\[
M_{Bj} = \begin{pmatrix}
F_{gj} & 1 - F_{ej}
\end{pmatrix},
\]

where \( F_{gj} \) is the readout fidelity for each qubit in \( |0\rangle \) and \( F_{ej} \) is the readout fidelity for each qubit in \( |1\rangle \).

Qubit parameters

Readout frequencies, qubit frequencies, qubit coherence times, qubit anharmonicities, and dispersive shift between qubits and readout cavity are all presented in Table. S2. The dispersive shift between each qubit and the coupler can be calculated through quantization of the qubit-coupler-qubit system. The readout frequencies of the qubits span a frequency range of about 50 MHz, well falling within the bandwidth of the JPA.

Although the coupler has no readout cavity, we can extract the coupler frequency by means of the dispersive shift between the qubit and coupler. The pulse sequence scheme is shown in Fig. S4(e). We scan the coupler frequency with a 500 ns rectangular pulse through the XY control line of \( Q_1 \), and then excite the \( Q_2 \) by a wide time-domain Gaussian pulse at its bare frequency. Once the frequency scan towards right to the coupler frequency, the coupler will be excited and dispersively shifts the frequency of \( Q_2 \), leading to a condition that \( Q_2 \) is unable to be excited with a drive pulse at its bare frequency. Thus we can get the coupler frequency spectrum as varying the flux bias amplitude of the coupler, as shown in the bottom panel in Fig. S4(f).

Deconvolution and Z-crosstalk calibration matrix

In the experiment, we use a fast-flux control to manipulate the qubits and coupler by driving a current into its correspond-ing SQUID loop. Nevertheless, the return path of current on each line is not explicitly controlled and, accordingly, there always exists a DC crosstalk between the each flux line. That is, varying the bias on any individual flux line actually changes all frequencies of the qubit and coupler. Fortunately, the frequency dependency is approximately linear for a small voltage, so the crosstalk can be corrected by the orthogonalization of the flux bias lines through multiplying the correction matrix [15]. The flux bias line orthogonalization is shown in Fig. S3(c). We measure the frequency response from both the two qubits and the coupler, and the Z-crosstalk calibration matrix \( (Q_1, Q_2, \text{Coupler}) \) is presented as follows:
FIG. S4: (a) Pulse sequence for measuring the coupling strength $g_2$ between $Q_2$ and the coupler. (b) Qubit-coupler energy swap. We tune the coupler frequency into resonance with one qubit, for instance, $Q_2$. At the max resonance point, we can extract the qubit-coupler direct coupling strength $g_{i}/2\pi = 76.9$ MHz (i=1,2). (c)-(e) Pulse sequence for measuring and extracting the qubit-qubit direct coupling strength $g_{12}/2\pi$, the qubit-qubit ZZ coupling strength $\xi_{ZZ}/2\pi$ and the coupler frequency, respectively. (f) Top panel: Qubit-qubit effective coupling strength $J_{12}/2\pi$ vs coupler flux pulse amplitude. Black dots are raw data extracted from Fig. S5, while yellow line is fitting result to extract $g_{12}$. Middle panel: Qubit-qubit ZZ coupling strength $\xi_{ZZ}/2\pi$ vs coupler flux pulse amplitude. The ZZ coupling strength is measured when the two qubits are biased at each sweet spot. Bottom panel: Coupler frequency vs coupler flux pulse amplitude. We measure the coupler frequency by probing the dispersive shift of the qubit frequency when the coupler is pulsed into the excited state.

### TABLE S2: Device parameters.

| Qubit Parameter | $Q_1$ | $Q_2$ | Coupler |
|-----------------|-------|-------|---------|
| Readout frequency (GHz) | 6.825 | 6.864 | |
| Qubit frequency (GHz) | 4.961 | 4.926 | 5.977 |
| $T_1$ (sweet point) (μs) | 14 | 13.7 | |
| $T_2$ (sweet point) (μs) | 8.4 | 4 | |
| $T_{2E}$ (sweet point) (μs) | 8.7 | 4.4 | |
| $E_c/2\pi$ (MHz) | 206 | 202 | 254 (Simulation) |
| $\chi_{Q}/2\pi$ (MHz) | 0.4 | 0.4 | |

### TABLE S3: Coupling parameters.

| Coupling Parameter | Simulation | Experiment |
|--------------------|------------|------------|
| $g_{1}/2\pi$ (MHz) | 86.6 | 90.6 |
| $g_{2}/2\pi$ (MHz) | | |
| $C_{ie}(i = 1, 2)$ (fF) | 2.4 | |
| $C_{12}$ (fF) | 0.13 | |
| $g_{i}/2\pi(i = 1, 2)$ (MHz) | 81.3 | 76.9 |
| $g_{12}/2\pi$ (MHz) | 3.8 | 6.74 |

$$\tilde{M}_z = M_z^{-1} = \begin{pmatrix} 0.9963 & 0.0096 & 0.0264 \\ -0.0798 & 0.9997 & 0.0094 \\ -0.0116 & 0.0384 & 0.9974 \end{pmatrix}.$$ 

Where $M_z$ is the qubit frequency response matrix. Actually the $Z$ crosstalk is very small owing to a well ground-plane connection by using airbridges even if the coupler is designed geometrically close to the two qubits.

### Qubit coupling strength

To simulate and optimize the gate fidelity for the single-qubit gates and the parametric iSWAP gate, we should efficiently get the qubit-qubit effective coupling strength $J_{12}$. For a full understanding and control of $J_{12}$, we need to measure and extract the qubit-coupler direct coupling strength $g_i$ (i=1,2) and qubit-qubit direct coupling strength $g_{12}$. First, we measure the qubit-coupler interaction by executing qubit-coupler energy-swap experiments. The corresponding pulse sequence used for the measurement, $g_2$ as an example, is shown in Fig. S4(a). We tune the coupler frequency into resonance with $Q_2$, meanwhile keeping $Q_1$ away by modifying the $Q_1$ frequency. The energy-swap pattern is shown in Fig. S4(b). Then, we fit and extract the qubit-qubit interaction from a qubit-qubit energy-swap experiment. The pulse sequence for extracting $g_{12}$ is shown in Fig. S4(c). The two qubits are initialized in the ground state at each sweet spot with a detuning of 34.2 MHz. The coupler is originally set to the critical point where the coupling between the two qubits is nearly cancelled out. A $\pi$ pulse is applied on $Q_2$, followed by flux pulses on $Q_1$ and the coupler to bring $Q_1$ into resonance with $Q_2$ and turn on the qubit interaction. In this way, we can directly measure $J_{12}$ as varying the coupler flux bias amplitude when the two qubits are in resonance according to the Hamiltonian in this scheme. We extract the corresponding qubit-qubit direct coupling strength $g_{12}$ by fitting $J_{12}$ as a function of the flux bias on the coupler, as shown in the top panel of Fig. S4(f). All relevant parameters to the coupling
strength are shown in Table. S3.

The single-qubit gate fidelity is affected by the qubit-qubit static ZZ-crosstalk coupling. To perform a high-fidelity single-qubit gate, the static ZZ-crosstalk should be reduced or eliminated through optimization of circuit parameters and experimental sequences. We extract the static ZZ-crosstalk coupling strength at each sweet spot by varying the coupler frequency using Ramsey-type measurements, which involve probing the frequency of \( Q_1 \) with initializing \( Q_2 \) in either its ground or excited state [15]. The pulse sequence is shown in Fig. S4(d). At the critical coupler frequency \( \omega_{c,off} = 5.905 \) GHz, as shown in the middle panel of Fig. S4(f), the measured static ZZ coupling strength \( \xi_{ZZ} \approx 1 \) KHz, limited by the detection resolution.

To perform a fast and high fidelity two-qubit parametric iSWAP gate operation, we need to balance the effective coupling strength \( \frac{1}{\hbar} \frac{\partial^2 H_{ij}}{\partial \phi^2} \) and the unwanted error terms. Apparently, a fast gate operation can be achieved by modifying the coupler frequency closer to the frequency of computational qubit, or increasing the amplitude of the coupler flux modulation pulse; however, unwanted real excitations between the qubit and coupler, or more serious high-frequency oscillation errors may occur. We characterize the tunability of the coupler and estimate the threshold of operating coupler by the pulse sequence shown in Fig. S4(c). The results are plotted in Fig. S5. In Fig. S5, we can easily find that the qubit-qubit swap interaction is completely off in the regime marked by a dark dashed line, while, a threshold, indicated by a red dashed line, defines a regime of non-negligible excitation of the coupler. When the coupler frequency is lower than the threshold, small ripples in the population oscillation of \( Q_2 \) start to show up, indicating the leakage out of the computational space due to the real excitations between the qubits and coupler. Besides, the ZZ-crosstalk error may also affect the gate operation if the coupler frequency is tuned to be close to the qubit frequency. Optimization of the parameters and sequences can be achieved to reduce the gate error and improve the gate fidelity by the simulation and parameter modulation using the data shown in Fig. S1 and Fig. S4(f).

**SIMULATION**

**Numerical Simulations of the Parametric iSWAP Gate**

We simulate the process of parametric iSWAP gate under different conditions to verify the two qubit gate error by means of Qutip [16, 17]. Above all, the numerical simulations should be strictly consistent with the real experiments, so we set all the qubit parameters according to measurement results. We completely follow the experiment procedures in the simulation including pulse shape calibration, phase modulation and quantum tomography.

At first, we carry out the simulation based on the Hamiltonian which restricts the Hilbert space to the lowest two states of each transmon qubit. [18, 19]. The static Hamiltonian can be written as:

\[
H / \hbar = \sum_{i=1,2} -\frac{1}{2} \omega_i \sigma_i^z - \frac{1}{2} \omega_c \sigma_c^z + \sum_{i=1,2} g_i (\sigma_i^+ \sigma_c^- + \sigma_c^+ \sigma_i^-) + g_{12} (\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-),
\]

(S9)

where \( \omega_{1,2}, \omega_c \) is the frequency of \( Q_1, Q_2 \) and coupler respectively, \( g_i \) (i=1,2) is the coupling of each qubit to the coupler and \( g_{12} \) is the direct coupling strength between two qubits. In this model, we conduct the parametric iSWAP gate by applying a sinusoidal fast-flux bias modulation pulse on the coupler. The derivation terms of Eq. (S9) in the theory part illustrate that the unwanted interactions in the two-qubit gate operation process come from these high-frequency oscillation terms. We choose four different coupler flux bias points, as mentioned in the main text, to analyze the error source. After the calibration at these four operation points (the coupler frequency is 5.905, 5.491, 5.472, 5.452 GHz with respect to the static ZZ coupling at 0, 0.23, 0.28 and 0.34 MHz, respectively), we perform quantum process tomography to characterize the gate quality. The gate fidelities at each points are 94.3%, 87.0%, 85.6% and 84.0%. A distinct decreasing trend in fidelity with increasing the static ZZ coupling accords with the numerical calculation of the high-frequency oscillation terms shown in Table. S1. It predicts that the unwanted high-frequency oscillation terms account for degradation of the parametric iSWAP gate, while rapid decline of the last three points reveals that, for accurate description of the gate, we should take more energy levels into account.

Therefore, further simulation based on Hamiltonian which includes anharmonicity terms for each qubit has to be carried.

![Fig. S5: Population of \( Q_2 \) as a function of the coupler frequency and the swap time. The swap pattern clearly demonstrates the tunability of the coupling strength. The dark dashed line indicates the condition where the coupling is off, while the red dashed line marks the threshold for non-negligible excitation of the coupler. To perform a parametric iSWAP gate, we should choose an appropriate coupler frequency and amplitude of parametric modulation pulse to keep a non-excitation under this threshold.](image-url)
The static Hamiltonian now can be written as:

\[
\frac{H}{\hbar} = \sum_{i=1,2} \omega_i a_i^\dagger a_i + \omega_c a_c^\dagger a_c - \sum_{i=1,2} \frac{E_{ci}}{2} a_i^\dagger a_i a_i^\dagger a_i - \frac{E_{cc}}{2} a_c^\dagger a_c a_c^\dagger a_c + \sum_{i=1,2} g_i (a_i a_c^\dagger + a_c a_i^\dagger) + g_{12} (a_1 a_2^\dagger + a_2 a_1^\dagger),
\]

(S10)

Where \(E_{ci}\) (i=1,2) and \(E_{cc}\) are the anharmonicity of \(Q_1, Q_2\) and the coupler. This model takes both the nonlinear of superconducting qubit and more relevant energy levels into account so that it is closer to the practice. All the steps remain unchanged in the simulation except for the operators when solving differential equations. We simulate two sets of iSWAP gate with and without qubit decoherence to explore the effect of coherence limit on the gate fidelity. The difference between the two sets varies from 2.1% to 3.3% at four operation points, which reveals that the decoherence error generally affects the gate performance to a great extent. The simulation fidelity with decoherence sets an upper bound of the intrinsic two-qubit gate fidelity. As shown in Fig.4(e) in the main text, the simulation results and experiment data are plotted together. It is apparent that the simulation fidelity decreases as the static ZZ coupling arises, which is in agreement with our experimental observation. We attribute this dependency to the previously mentioned high-frequency oscillation terms as well as the dynamic ZZ-type coupling parasitising in the two-qubit gate process. Even if we could not accurately extract the effect of parasitic ZZ coupling, it provides a guiding principle to improve gate fidelity of the parametric iSWAP gate.

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