Bounds on R–parity Violating Couplings at the Grand Unification Scale from Neutrino Masses

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We consider the embedding of the supersymmetric Standard Model with broken R–parity in the minimal supergravity (mSUGRA) model. We restrict ourselves to the case of broken lepton number, the B3 mSUGRA model. We first study in detail how the tree–level neutrino mass depends on the mSUGRA parameters. We find in particular a strong dependence on the trilinear supersymmetry breaking A–parameter, even in the vicinity of the mSUGRA SPS1a point. We then reinvestigate the bounds on the trilinear R-parity violating couplings at the unification scale from the low–energy neutrino masses including dominant one–loop contributions. These bounds were previously shown to be very strict, as low as $O(10^{-6})$ for SPS1a. We show that these bounds are significantly weakened when considering the full mSUGRA parameter space. In particular the ratio between the tree–level and 1–loop neutrino masses is reduced such that it may agree with the observed neutrino mass hierarchy. We discuss in detail how and in which parameter regions this effect arises.

I. INTRODUCTION

The experimental observation of neutrino oscillations, and thus of neutrino masses, is an experimental indication that the Standard Model of particle physics (SM) is incomplete [1–3].

Experimentally, neutrinos must be relatively light. Direct laboratory measurements restrict their masses to be below $O(10\text{MeV}–1\text{eV})$ [7–12], depending on the flavor. Cosmological observations even give upper bounds of $O(0.1\text{eV})$ on the sum of the neutrino masses [7,12,14]. Furthermore, the atmospheric and solar neutrino oscillation data are best fit if the squared neutrino mass differences are $O(10^{-3}\text{eV}^2)$ and $O(10^{-5}\text{eV}^2)$, respectively [7,12]. This allows for one massless neutrino.

In principle, it is easy to extend the SM Lagrangian by a Dirac neutrino mass term [3]. However, right–handed neutrinos and new Yukawa couplings of $O(\lesssim10^{-12})$ are in this case needed. Such tiny couplings seem to be very unnatural and might point towards a dynamical mechanism, that explains the small neutrino masses. Furthermore, the right–handed neutrinos can have an unspecified Majorana neutrino mass.

Most prominently discussed are extensions of the SM involving the see–saw mechanism, by introducing right–handed neutrinos and fixing the new Majorana neutrino mass scale to be large, cf. Refs. [7,10,21]. The see–saw mechanism is also naturally incorporated into supersymmetry (SUSY) [22,23].

Supersymmetry is one of the most promising extensions of the SM. It is the unique extension of the Lorentz spacetime symmetry, when allowing for graded Lie algebras [22,23]. Furthermore, it provides a solution to the hierarchy problem of the SM [24,30]. More importantly here: neutrino masses can be generated without introducing right–handed neutrinos if lepton number is violated, cf. for example Refs. [31,41].

The most general gauge invariant and renormalizable superpotential of the supersymmetric extension of the SM with minimal particle content (SSM) possesses lepton number conserving (LNC) terms [12,43]

$$W_{\text{LNC}} = \epsilon_{ab}[(Y_E)_{ij}L_i^aH_u^bE_j + (Y_D)_{ij}Q_i^{ax}H_u^bD_jx + (Y_U)_{ij}Q_i^{bx}H_u^bU_{jx} - \mu H_u^bH_u^b]$$

(1)

and also lepton number violating (LNV) terms

$$W_{\text{LNV}} = \epsilon_{ab} \left[ \frac{1}{2} \lambda_{ijk}L_i^aL_j^bL_k^c + \chi_{ijk}L_i^aQ_j^{bx}D_kx \right. - \left. \epsilon_{abk}L_i^aH_u^b \right],$$

(2)

where $i,j,k = 1,2,3$ are generation indices. We have employed the standard notation of Ref. [44].

The LNV interactions violate the discrete symmetries R–parity and proton–hexality ($P_6$), however, they conserve baryon triality ($B_3$) [45,48]. Note that $B_3$ stabilizes the proton because it suppresses the baryon number violating interactions. R–parity, $P_6$ and $B_3$ are the only discrete symmetries, which can be written as a remnant of a broken anomaly free gauge symmetry [45,48]. In the following, we assume that
B$_3$ is conserved and thus R–parity and P$_6$ are violated. Eq. (1) and Eq. (2) constitute the full renormalizable superpotential allowed by this symmetry. For reviews of such theories see for example Refs. [49–51].

Beside the superpotential, also the soft-breaking Lagrangian of the B$_3$ conserving SSM exhibits lepton number violating operators [52]

$$-L^{LNV}_{soft} = \epsilon_{ab} \left[ \frac{1}{2} h_{ijk} \tilde{L}^a_i \tilde{L}^b_j E_k + h_{ijk} \tilde{Q}^a_i \tilde{D}^b_j + \text{h.c.} \right] - \epsilon_{ab} \tilde{D}_i \tilde{D}^b_i + \text{h.c.} + (h_u^a)^a m_{h_u L_i} \tilde{L}^a_i,$$

(3)

where again $i, j, k = 1, 2, 3$ are generation indices. $\tilde{L}$, $\tilde{E}$, $\tilde{Q}$ and $\tilde{D}$ are the scalar components of the lepton doublet, lepton singlet, quark doublet and down quark singlet superfield, respectively. Furthermore, $h_u$ and $h_d$ denotes the up–type (down–type) scalar Higgs field. Beside the term proportional to $m_{h_u L_i}^2$, the operators in Eq. (3) are the soft-breaking analog of the terms in Eq. (2).

The LNV terms in Eq. (2) and Eq. (3) lead to the dynamical generation of neutrino masses, for example, the bilinear terms in Eq. (2) mix the Higgsinos, the supersymmetric partners of the Higgs bosons, with the neutrino fields and thus generate one non–vanishing neutrino mass at tree–level [31–33, 37–40].

In this paper, we derive bounds on the trilinear LNV couplings of the superpotential, Eq. (2), from the upper cosmological bound on the sum of neutrino masses [13, 14], i.e.

$$\sum m_{\nu_i} < 0.40 \text{ eV},$$

(4)

at 99.9% confidence level. The bound was determined by a combination of the Wilkinson Microwave Anisotropy Probe (WMAP) and Large Scale Structure (LSS) data.

In order to perform a systematic study, we restrict ourselves to the well motivated framework of the B$_3$ minimal supergravity model (mSUGRA) [52], which provides simple boundary conditions for the SSM parameters at the grand unification scale ($M_{\text{GUT}}$). We describe the model in the next section in detail. We employ the full set of renormalization group equations (RGEs) at one loop [52, 55] in order to obtain the B$_3$ SSM spectrum and the neutrino masses at the electroweak scale ($M_{\text{EW}}$). We then derive bounds on the LNV trilinear couplings at $M_{\text{GUT}}$.

Bounds on trilinear LNV couplings within this model were also derived in Ref. [52] from the generation of neutrino masses at tree–level. It was claimed that neutrino masses put an upper bound of $\mathcal{O}(10^{-3} – 10^{-6})$ on most of the trilinear couplings in Eq. (2). However, it was shown in Ref. [54] that the tree–level neutrino mass can vanish in certain regions of the B$_3$ mSUGRA parameter space. In our analysis, we especially focus on these regions of parameter space. We show that the bounds on the trilinear couplings can be weakened up to $\mathcal{O}(10^{-1})$, depending on the boundary conditions.

We go beyond the former work in several aspects. Beside the tree–level neutrino mass, we also include the dominant contributions to the neutrino mass matrix at one–loop. These contributions were neither included in the calculation of the bounds in Ref. [52] nor in Ref. [54]. However, as we show in Sec. IV, the loops dominate in the regions of parameter space where the tree–level mass vanishes. They must thus be included when determining the bounds.

In Ref. [54] there is only a brief explanation of the dominant effect that leads to a vanishing tree–level mass in B$_3$ mSUGRA. We give for the first time a detailed and complete explanation of how different configurations of the B$_3$ mSUGRA parameters at $M_{\text{GUT}}$ can affect the tree–level and loop contributions to the neutrino masses at $M_{\text{EW}}$. Although we restrict ourselves to the framework of B$_3$ mSUGRA, the mechanisms described in this publication also work in more general models. Furthermore, we calculate bounds for all trilinear LNV couplings, whereas Ref. [54] focused only on the couplings $\lambda_{333}$ and $\lambda'_{333}$. We also update the bounds given in Ref. [52] according to the more recent and stronger bound on the sum of neutrino masses, cf. Eq. (4).

Going beyond the work presented here, we believe our results can help find LNV SUSY scenarios that explain the observed neutrino masses and mixing angles. Within the framework of B$_3$ mSUGRA, Ref. [41] searched for a minimal set of LNV parameters which can explain the measured neutrino parameters. They found sets of five parameters [two trilinear LNV couplings together with the three mixing angles that describe the lepton Yukawa matrix, cf. Eq. (1)] that give the right masses and mixing angles. Ref. [41] claimed that the tree–level mass is always much larger than the loop induced masses. But we show in the following, that the loops can exceed the tree–level masses in B$_3$ mSUGRA. Therefore, it should be possible to find a smaller set of LNV parameters that lies in this region of parameter space and thus posses much larger LNV couplings than those found in Ref. [41]. However, an investigation of the complete neutrino sector is beyond the scope of this paper and will be postponed to a future publication.

We finally note that (large) trilinear LNV couplings can lead to distinct collider signatures at the Large Hadron Collider (LHC), e.g.

- Supersymmetric particles (sparticles) can be produced singly at a collider, possibly on resonance [51]. For example, single resonant slepton production at the LHC via $\lambda'_{ijk}$, Eq. (2). An excess over the SM backgrounds is visible if $\lambda'_{ijk}>\mathcal{O}(10^{-3})$, depending also on the sparticle masses [52, 55, 56, 61].
- A LNV coupling $\lambda_{ijk}$ ($\lambda'_{ijk}$) of $\gtrsim \mathcal{O}(10^{-2})$ at
**II. THE B₃ MSUGRA MODEL**

The general B₃ SSM has more than 200 free parameters [52]. This large number is intractable for detailed phenomenological studies. For that purpose the simplifying B₃ mSUGRA model was proposed in Ref. [52], which we now discuss.

**A. Free Parameters**

In the B₃ mSUGRA model the boundary conditions at $M_{\text{GUT}}$ are described by the six parameters

$$M₀, M_{1/2}, A₀, \tan \beta, \text{sgn}(\mu), \Lambda,$$

with

$$\Lambda \in \{\lambda_{ijk}, \lambda_{ijk}'\}.$$  

Here $M₀$, $M_{1/2}$ and $A₀$ are the universal scalar mass, the universal gaugino mass and the universal trilinear scalar coupling at the grand unification scale ($M_{\text{GUT}}$), respectively. $\tan \beta$ denotes the ratio of the Higgs vacuum expectation values (vevs) $v_u$ and $v_d$, and $\text{sgn}(\mu)$ fixes the sign of the bilinear Higgs mixing parameter $\mu$. The magnitude of $\mu$ is determined dynamically by radiative electroweak symmetry breaking (REWSB) [83]. These five parameters are the conventional free parameters of the R-parity or proton-hexality conserving mSUGRA model [84].

In order to incorporate the effects of the LNV interactions in Eq. (2) and Eq. (3) exactly one additional non–vanishing trilinear coupling $\Lambda \in \{\lambda_{ijk}, \lambda_{ijk}'\}$ is assumed at $M_{\text{GUT}}$. Further LNV couplings are generated via the RGEs at the lower scale. Note, that the bilinear couplings $\kappa_{ij}$ and $\tilde{D}_i$ are both set to zero at $M_{\text{GUT}}$ via a basis transformation of the lepton and Higgs superfields [31]. (For the most general case of a complex rotation see Ref. [85].) This is natural for universal SUSY breaking [52]. However, at lower scales $\kappa_{ij}$ and $\tilde{D}_i$ are generated via the RGEs [37]; see Sec. [37].

The complete low energy spectrum is obtained by running the RGEs down from $M_{\text{GUT}}$ to $M_{\text{EW}}$. For that purpose we employ the program SOFTSUSY-3.0.12 [84, 57]. We calculate the neutrino masses with our own program. Note that we work in the CP-conserving limit throughout this paper.

**B. Benchmark Scenarios for Parameter Scans**

We center our analysis around the following $B₃$ mSUGRA parameter points

**Point I:** $M_{1/2} = 500$ GeV, $M₀ = 100$ GeV, $\tan \beta = 20$, $\text{sgn}(\mu) = +1$, $A₀ = 900$ GeV, $\Lambda = \lambda_{233}$

**Point II:** $M_{1/2} = 500$ GeV, $M₀ = 100$ GeV, $\tan \beta = 20$, $\text{sgn}(\mu) = +1$, $A₀ = 200$ GeV, $\Lambda = \lambda_{233}$

Point II differs from Point I only by the choice of the LNV coupling and the size of $A₀$. We have chosen these points as examples because the tree–level contribution to the neutrino mass is small around Point I and II and therefore one–loop contributions are important. Both points lead to squark masses of $\mathcal{O}(1)$ TeV and slepton masses of around 300 GeV, with a scalar tau (stau) as the LSP.

Note that in the LNV SSM a stau LSP is as well motivated as a neutralino LSP [52, 58, 65, 88–90]. Either will decay via the LNV interactions and cosmological constraints do not apply [91].

In addition, we ensured that both points lie in regions of parameter space where various other experimental constraints are fulfilled, such as the lower bound on the lightest Higgs mass from LEP2 [92, 93] and constraints from the anomalous magnetic moment of the muon [94], from $b \to s \gamma$ [95], and from $B_s \to \mu^+ \mu^-$ [95]; see Sec. [95] for details.

**C. Renormalization Group Equations and Radiative Electroweak Symmetry Breaking**

An important feature of the $B₃$ mSUGRA model is that lepton number violation leads to mixing between the lepton superfields $L_i$ and the Higgs superfield $H_d$. Furthermore, sneutrinos, the superpartners of the neutrinos, can acquire vevs $\nu_i$ ($i = 1, 2, 3$). Note that it is possible to rotate away the $\kappa_{ij}$ terms in the superpotential at any given energy scale by an orthogonal rotation of the fields $L_{\alpha} = (H_d, L_i)$ [31, 52, 83].
The corresponding bilinear soft-breaking terms proportional to $\tilde{D}_i$, Eq. (3), can be rotated away in conjunction with $\kappa_i$ if $\tilde{D}_i$ and $\kappa_i$ are aligned. This condition is fulfilled at $M_{\text{GUT}}$ in the $B_3$ mSUGRA model if the underlying supergravity superpotential satisfies the quite natural condition [52]

$$f(z_i; y_\alpha) = f_1(z_i) + f_2(y_\alpha),$$

(7)

where the superfields $z_i$ belong to the observable sector and the superfields $y_\alpha$ to the hidden sector.

However, when evolving the parameters down to the weak scale, $\kappa_i$, $\tilde{D}_i \neq 0$ are generated via the RGEs. The leading terms for $\Lambda \in \{\lambda'_{ijk}\}$ are given by [52]

$$16\pi^2 \frac{d\lambda'_{ijk}}{dt} = -3\kappa_i \left[ \frac{g_1^2}{5} + g_2^2 - \left( Y_U \right)_{33}^2 - \frac{\left( Y_E \right)_{33}^2}{3} \delta_{3i} \right] - 3\mu \lambda'_{ijk} (\tilde{Y}_D)_{jk} + ...$$

(8)

and

$$16\pi^2 \frac{d\tilde{D}_i}{dt} = -3\tilde{D}_i \left[ \frac{g_1^2}{5} + g_2^2 - \left( Y_U \right)_{33}^2 - \frac{\left( Y_E \right)_{33}^2}{3} \delta_{3i} \right] + 6\kappa_i \left[ \frac{g_1^2}{5} M_1 + g_2^2 M_2 \right] + 6\kappa_i \left[ \left( Y_U \right)_{33}^2 h_U \right]_{33}^2 + \frac{\left( Y_E \right)_{33}^2 (h_E)_{33}^2}{3} \delta_{3i} \right] - 3(\tilde{Y}_D)_{jk} (2\mu h'_{ijk} + \tilde{B} \lambda'_{ijk}) + ...$$

(9)

Here $t \equiv \ln(Q/\mu_0)$ with $Q$ the renormalization scale and $\mu_0$ an arbitrary reference scale. $h'_{ijk} \equiv A_0 \times \lambda'_{ijk}$ at $M_{\text{GUT}}$, cf. Eq. (3). $B$ is the soft supersymmetry breaking analog of the Higgs mixing parameter $\mu$ and $(h_U)_{33}^2 [ (h_E)_{33}^2 ]$ is the soft-breaking analog of the Yukawa coupling $Y_{ij}^33$ $[ (Y_E)_{33}^3 ]$ [52]. $g_1$ and $g_2$ ($M_1$ and $M_2$) are the $\text{U}(1)_Y$ and $\text{SU}(2)$ gauge couplings (gaugino masses), respectively. We see in Eqs. (8) and (9) that the RGEs differ, and therefore $\kappa_i$ and $\tilde{D}_i$ will no longer be aligned at the weak scale [33]. The case $\Lambda \in \{\lambda_{ijk}\}$ is analogous up to the color factor 3.

The sneutrino vevs $v_\nu$, the bilinear Higgs parameter $|\mu|$ and the corresponding soft breaking term $B$ are determined by REWSB, which has been discussed in detail in Ref. [52] for the LNV case.

Neglecting higher order corrections [96, 98], which are not important for the following qualitative discussion [116], the sneutrino vevs can be written as [52]

$$(M^2_\nu)_{ij} v_\nu = -m^2_{h_d} L_i^* + \mu \kappa_i \nu_d + \tilde{D}_i v_u,$$

(10)

where $(m^2_\nu)$ is the squared soft-breaking lepton doublet mass matrix and $g = \sqrt{3/5} g_1$. $m^2_{h_d L_i}$ originates from the LNV soft-breaking Lagrangian, Eq. (3). It mixes the down-type Higgs fields, $h_d$, with the lepton doublet scalars, $L_i$, and is zero at $M_{\text{GUT}}$. That is, because we take within mSUGRA the mass matrix of the fields $\mathcal{L}_a = (h_d, L_i)$ to be diagonal and proportional to $M_0$ at $M_{\text{GUT}}$. However, $m^2_{h_d L_i} \neq 0$ is subsequently generated via the RGEs, cf. Eq. (33).

As we will see in Sec. [116] neutrino vevs and non–zero bilinears $\kappa_i$ lead to neutrino masses at tree-level because they mix neutrinos and neutralinos.

D. Quark Mixing

The RGE evolution of the parameters in the $B_3$ mSUGRA model from $M_{\text{GUT}}$ to $M_{\text{EW}}$ depends on the Higgs–Yukawa coupling matrices $Y_E$, $Y_D$, and $Y_U$, cf. Eqs. (8) and (9). In particular, the RGEs of the LNV violating parameters are coupled via the non–diagonal matrix elements of the Higgs–Yukawa couplings. Therefore a knowledge of the latter is crucial for the analysis of bounds on the LNV parameters.

The initial parameter set of the $B_3$ mSUGRA model at $M_{\text{GUT}}$ is given in the electroweak basis so that for the RGE evolution the Higgs–Yukawa couplings (or the quark– and lepton–mass matrices) are also needed in the electroweak basis. However, from experiment we only know the masses and the CKM matrix

$$V_{\text{CKM}} = U^*_L D_L$$

(12)

at $M_{\text{EW}}$. Here $U^*_L$ and $D_L$ rotate the left–handed up–(down–) quark fields from the mass eigenstate basis to the electroweak basis. For simplicity, we take $Y_U$ and $Y_D$ to be real and symmetric and thus the rotation matrices for the right–handed quark fields are identical to the ones for left–handed quark fields, $U_R = U_L$ and $D_R = D_L$. Because of the uncertainty about the neutrino masses and mixings we will assume a diagonal $Y_E$ in the weak basis.

When determining the neutrino masses, we will consider two limiting cases at $M_{\text{EW}}$, following Ref. [52, 55]:

- **“up–type mixing”** the quark mixing is only in the up–quark sector,

$$U_{LR} = V_{\text{CKM}}, \quad D_{LR} = 1,$$

(13)

$$Y_D \times v_d = \text{diag}(m_d, m_s, m_b),$$

$$Y_U \times v_u = V_{\text{CKM}} \times \text{diag}(m_u, m_c, m_t) \times V_{\text{CKM}}^T.$$  

- **“down–type mixing”** the mixing is only in the down–quark sector,

$$D_{LR} = V_{\text{CKM}}, \quad U_{LR} = 1,$$

(14)

$$Y_D \times v_d = V_{\text{CKM}} \times \text{diag}(m_d, m_s, m_b) \times V_{\text{CKM}}^T,$$

$$Y_U \times v_u = \text{diag}(m_u, m_c, m_t).$$
Here $m_d, m_s, m_b$ ($m_u, m_c, m_t$) denote the masses of the down--type (up--type) quarks.

The choice between up- and down--type mixing has a strong effect on the final results for the LNV couplings $A \in \{\lambda_{ijk}\}$ with $j \neq k$, as we will show in Sec. III 3. (see Tab. I). The reason is that the generated tree level neutrino mass is proportional to the off--diagonal matrix element $(Y_D)_{jk}$, cf. the discussion in Sec. III 2 and Sec. IV. Our results (for the tree--level neutrino mass) in Sec. IV can be easily translated to scenarios which lie between the limiting cases of Eqs. (13) and (14).

One only needs to know the respective Yukawa matrix elements $(Y_D)_{jk}$.

III. NEUTRINO MASSES

In this paper, we investigate bounds on lepton--number violating couplings at $M_{GUT}$ within the $B_3$ mSUGRA model, which arise from the generation of too large neutrino masses at $M_{EW}$. We therefore need to identify the dominant contributions to the neutrino masses.

It was stated in Ref. [52] that the main contribution stems from mixing between neutralinos and sneutrinos, which leads to one non-vanishing neutrino mass at tree--level, cf. Sec. III 3. However, as we will show in the next two sections, this is only true in parts of the $B_3$ mSUGRA parameter space. It is possible that the different terms in the tree--level mass formula cancel each other. We then need to identify the dominant contributions, which arise at one--loop.

A complete list of all one--loop contributions is given in Ref. [52], where they are formulated in a basis--independent manner. Most of the one--loop contributions are proportional to the mass insertions that mix the neutrinos with the neutralinos. They thus also vanish when the tree--level neutrino mass vanishes and are negligible in the region we are interested in.

The remaining dominant one--loop contributions are on the one hand due to loops involving two R--parity violating vertices and are thus either proportional to $\lambda^2$ or to $\lambda'^2$, cf. Fig. 1 [117]. We will review these contributions in Sec. III 2. On the other hand, loops with virtual neutral scalars (i.e. Higgses and sneutrinos) and neutralinos, which are shown in Fig. 2, can also give large contributions to neutrino masses. These loops are proportional to the mass difference between CP-even and CP-odd neutral scalars, cf. Sec. III 3.

According to Ref. [55], there is in principle also a contribution which is proportional to $\lambda \times D_i$. However, this contribution is suppressed by two or more orders of magnitude in the regions of parameter space where the loops dominate over the tree--level mass. Note that $D_i$ vanishes near the tree--level mass minimum as we will show in Sec. IV. We therefore neglect it in the following.

Further one--loop contributions are only present in a lepton- and Higgs-superfield basis with non-vanishing sneutrino vevs. This is the case in the $B_3$ mSUGRA model. However, we have checked that in our parameter scans, these contributions are at least one order of magnitude smaller than the dominant one and thus negligible for calculating the bounds. Note, that they are also aligned with the tree--level mass, because the sneutrino vevs vanish near the tree--level mass minimum, cf. Sec. IV 3.

We conclude, that the contributions to the neutrino masses which we review in the following, are sufficient to calculate the correct bounds on the LNV couplings $\lambda$ and $\lambda'$. However, in order to calculate the correct

![Fig. 1: Loop contributions to the neutrino mass matrix via a non-vanishing product of $B_3$ couplings $\lambda_{ikn} \times \lambda'_{jnk}$ (upper figure) and $\lambda_{ikn} \times \lambda'_{jnk}$ (lower figure). See Sec. III 2 for more details.](image)

![Fig. 2: Loop contributions to the neutrino mass matrix via a non-exact cancellation of loops with CP-even and CP-odd neutral scalars. Note, that there is a relative minus sign between the two diagrams. See Sec. III 3 for more details.](image)
neutrino mass spectrum and mixing angles, all one–loop contributions given in Ref. \[35\] must be taken into account. This lies beyond the scope of this paper.

1. Tree–Level Contributions

In the context of the B\(_3\) mSUGRA Model, neutrino masses are generated at tree–level through mixing between neutrinos and neutralinos. Analogously to the standard see-saw mechanism \[7\] \[16\] \[21\] (with the neutralinos taking over the role of the right-handed neutrinos), an effective 3 \times 3 neutrino mass matrix is generated \[32\] \[33\].

\[
\mathcal{M}_{\nu}^{\text{eff}} = \frac{\mu (M_1 \delta_{1}^2 + M_2 g^2)}{2 v_u v_d (M_1 g_2^2 + M_2 g^2) - 2 \mu M_1 M_2} \begin{pmatrix}
\Delta_1 \Delta_1 & \Delta_1 \Delta_2 & \Delta_1 \Delta_3 \\
\Delta_2 \Delta_1 & \Delta_2 \Delta_2 & \Delta_2 \Delta_3 \\
\Delta_3 \Delta_1 & \Delta_3 \Delta_2 & \Delta_3 \Delta_3
\end{pmatrix},
\]

(15)

where \(M_1\) (\(M_2\)) is the bino (wino) soft-breaking mass and

\[
\Delta_i = v_i - v_d \frac{\kappa_i}{\mu}, \quad i = 1, 2, 3.
\]

(16)

This matrix has one non–zero eigenvalue which can at \(M_{\text{EW}}\) be simplified to \[52\].

\[
m_{\nu}^{\text{tree}} \approx -\frac{16 \pi \alpha_{GUT}}{5} \sum_{i=1}^{3} \frac{\Delta_i^2}{M_{1/2}}.
\]

(17)

if we take into account the gaugino universality assumption at \(M_{\text{GUT}}\), leading to \(M_2 = \frac{3}{2} \frac{\alpha_2^2}{\alpha_1^2} M_1 = \frac{\alpha_2^2}{\alpha_1^2} M_{1/2}\) at \(M_{\text{EW}}\) \[52\]. Here \(\alpha_{GUT} = g_{\text{GUT}}/4\pi \approx 0.041\) is the grand unified gauge coupling constant \[52\].

2. Contributions from \(\lambda\lambda\)– and \(\chi\chi\)–Loops

In the region of parameter space where the tree-level neutrino mass, Eq. \[14\], vanishes, loop induced neutrino masses give the dominant contributions. As we will show in Sec. \[V\] the most important loops are those proportional to the product of two LNV trilinear couplings. The corresponding squark-quark and slepton-lepton loops are shown in Fig. \[1\]. The resulting \(m_{\nu}^{\lambda\lambda}\) mass contributions are \[33\].

\[
(m_{\nu}^{\lambda\lambda})_{ij} = \frac{1}{32 \pi^2} \sum_{k,n} \lambda_{jkn} \lambda_{jnk} m_{d_k} \sin 2 \phi_n \ln \left( \frac{m_{d_{1n}}^2}{m_{d_{2n}}^2} \right) + \frac{3}{32 \pi^2} \sum_{k,n} \lambda_{jkn} \lambda_{jnk} m_{d_k} \sin 2 \phi_n \ln \left( \frac{m_{d_{3n}}^2}{m_{d_{4n}}^2} \right),
\]

(18)

where \(m_{d_k}\) (\(m_{d_k}\)) are the lepton (down-quark) masses of generation \(k\), and \(\phi_n\) (\(\phi_n\)) the mixing angles that describe the rotation of the left– and right–handed slepton (down-quark) current eigenstates of generation \(n\) to the two mass eigenstates, \(m_{\tilde{\nu}_1}\) and \(m_{\tilde{\nu}_2}\) (\(m_{\tilde{d}_1}\) and \(m_{\tilde{d}_2}\)), respectively. Note that the squared sfermion masses are linear functions of the mSUGRA parameters \(M_0^2\) and \(M_{1/2}^2\), see for example Ref. \[104\]. For the calculation of Eq. \[18\] and all following calculations, we have used the two-component spinor formalism as described in Ref. \[101\].

For the first two sfermion generations, the sfermion mixing angles are small and we approximate Eq. \[18\] by using the mass insertion approximation (MIA) as described in Ref. \[11\]. The slepton (and down-quark) mass eigenstates are replaced by the respective left– and right–handed eigenstates with mass \(m_{\tilde{\nu}_{1n}}\) and \(m_{\tilde{\nu}_{2n}}\). The mixing angle can be approximated by

\[
\sin 2 \phi_n = \frac{2 (M_{\tilde{\nu}}^{LR})_{n}^2}{m_{\tilde{\nu}_{1n}}^2 - m_{\tilde{\nu}_{2n}}^2},
\]

(19)

where

\[
(M_{\tilde{\nu}}^{LR})_{n}^2 = m_{\tilde{\nu}_n} \left( \frac{(h_E)_{nn}}{(Y_E)_{nn}} - \mu \tan \beta \right)
\]

(20)

denotes the left–right mixing matrix element of the charged sleptons of generation \(n\). \((h_E)_{nn}\) is the trilinear soft-breaking analog of the lepton Yukawa matrix element \((Y_E)_{nn}\) \[52\].

A similar formula is obtained for \(\sin 2 \phi_n^d\). One only needs to replace in Eq. \[19\] and Eq. \[20\] \(\ell \leftrightarrow \tilde{\ell}\), \(d \leftrightarrow \bar{d}\), \((Y_E)_{nn} \leftrightarrow (Y_D)_{nn}\), and \((h_E)_{nn} \leftrightarrow (h_D)_{nn}\), where \((h_D)_{nn}\) is the soft-breaking analog of the down-quark Yukawa matrix element \((Y_D)_{nn}\).

3. Contributions from Neutral Scalar–Neutralino–Loops

As the final source of neutrino masses, we consider contributions arising from loops with neutral scalars and neutralinos, cf. Refs. \[34\] \[36\]. Most important is...
If lepton number is conserved, the ˜\nu as we will see in Eq. (26).

If lepton number is violated, the ˜\nu masses are degenerate and the CP-even (CPE) and CP-odd (CPO) contributions to the neutrino mass from neutral scalar–neutralino–loops cancel, cf. Fig. 2. In contrast, if lepton number is violated, the ˜\nu fields. They can be obtained with 4th scalar sectors. The cancellation is thus induced by

\[
\begin{align*}
\tilde{\nu}_i^+ & = \frac{1}{\sqrt{2}} (\tilde{\nu}_i + \tilde{\nu}_i^*) , \\
\tilde{\nu}_i^- & = \frac{1}{i \sqrt{2}} (\tilde{\nu}_i - \tilde{\nu}_i^*) .
\end{align*}
\]

If lepton number is conserved, the ˜\nu masses are degenerate and the CP-even (CPE) and CP-odd (CPO) contributions to the neutrino mass from neutral scalar–neutralino–loops cancel, cf. Fig. 2.

In contrast, if lepton number is violated, the ˜\nu masses are in general different, so the cancellation is no longer exact. This is due to the fact that the CPE and CPO neutrinos mix differently with the CPE and CPO Higgs fields, respectively. The size of this contribution to the neutrino masses is roughly proportional to the mass splitting \(\Delta m^2_{\nu_i} = m^2_{\tilde{\nu}_i} - m^2_{\tilde{\nu}_i}\), cf. Eq. (26) and Refs. 34, 36.

The neutral scalar–neutralino-loops, shown in Fig. 2, lead to the following contributions to the neutrino mass matrix:

\[
(m_{\nu}^{\tilde{\nu}})_{ij} = \frac{1}{32\pi^2} \sum_{k=1}^{4} \sum_{L=1}^{5} m_{\chi_k^0} (g N_{1k} - g_2 N_{2k})^2 \left[ Z^{(2+i)L}_{(2+j)L} B_0(0, m_{\tilde{\nu}_i}^2, m_{\chi_k^0}^2) - Z^{(2+i)L}_{(2+j)L} B_0(0, m_{\tilde{\nu}_j}^2, m_{\chi_k^0}^2) \right] ,
\]

where \(m_{\chi_k^0}(k = 1 \ldots 4)\) are the neutralino masses and \(N\) is the 4 \times 4 neutralino mixing matrix in the bino, wino, Higgsino basis. The two-point Passarino-Veltman function is conventionally denoted \(B_0\) with \(L = 1 \ldots 5\) are the mass eigenvalues of the CPE (CPO) neutral Higgs bosons and CPE (CPO) sneutrinos. They can be obtained with the help of the unitary matrix \(Z^+ (Z^-)\), which diagonalizes the mass matrices of the CPE (CPO) neutral scalars, i.e.

\[
(Z^+)^T M_{\text{CPE}} Z^+ = \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2, m_{\tilde{\nu}_4}^2) \equiv \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2) .
\]

and

\[
(Z^-)^T M_{\text{CPO}} Z^- = \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2, m_{\tilde{\nu}_4}^2) \equiv \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2) .
\]

see Ref. 41 for additional details.

In order to analyze the dependence of this contribution on the mSUGRA parameters, we make use of the fact that in the B3 mSUGRA model, Eq. (26) can be approximated by:

\[
(m_{\nu}^{\tilde{\nu}})_{ij} \approx \frac{1}{32\pi^2} \sum_{k=1}^{4} m_{\chi_k^0}^2 (g N_{1k} - g_2 N_{2k})^2 \times \frac{\Delta m^2_{\nu_i}}{(m_{\tilde{\nu}_i}^2 - m_{\tilde{\nu}_j}^2)^2} \ln \left( \frac{m_{\tilde{\nu}_i}^2}{m_{\tilde{\nu}_j}^2} \right) \delta_{ij} .
\]

by expanding around \(m_{H_0}^2\) and \(m_{A_0}^2\). The mass splitting, \(\Delta m^2_{\nu_i}\), in Eq. (26) between CPE and CPO neutrinos of generation i is then given by:

\[
\Delta m^2_{\nu_i} = -\frac{4B^2 M_2^2 m_{\tilde{\nu}_i}^2 \sin^2 \beta}{(m_{H_0}^2 - m_{\tilde{\nu}_i}^2)(m_{\tilde{\nu}_i}^2 - m_{\tilde{\nu}_i}^2)(m_{A_0}^2 - m_{\tilde{\nu}_i}^2)} \times \left( \frac{B_{\tilde{\nu}_i} - \tilde{D}_{\tilde{\nu}_i}}{\sqrt{v_d^2 + v_d^2}} \right)^2 .
\]

4. Numerical Implementation

The numerical calculation of the neutrino mass matrix was done in the following way. We first employed SOFTSUSY-3.0.12 to obtain the low energy mass spectrum. Then we used our own program to calculate the neutrino mass matrix. The tree–level contribution was derived from Eq. (15). For the \(\lambda\)– and \(\lambda\)'–loops, we employed Eq. (16), if third generation sfermions were involved. However, for sfermions of the first two generations we used the MIA as given in Eqs. (19) and (20).

For the neutral scalar–neutralino–loops, we in principle employed Eq. (24). However, instead of performing the large numerical cancellation between CPE and CPO neutral scalars directly [square bracket in Eq. (24)], we used an MIA to calculate the deviation from exact cancellation in the R-parity conserving (RPC) limit, following Ref. 44. The resulting formula is quite lengthy and we refer the interested reader to Ref. 41 for details. We have cross checked our program with the help of Eq. (26) and Eq. (27). All our calculations are performed in the CP-conserving limit.

IV. \(\nu\)-MASSES: DEPENDENCE ON MSUGRA PARAMETERS

In the literature it has frequently been assumed that the tree–level contribution to the neutrino mass, Eq. (17), in the B3 mSUGRA model dominates over the loop contributions, cf. for example Refs. 41, 42. However, as has been noted in Ref. 44, in certain regions of B3 mSUGRA parameter space, the tree–level neutrino mass vanishes even when \(\kappa_i \neq 0\).

We demonstrate this effect in Fig. 3 where we display the tree–level neutrino mass (solid red line) as a function of \(A_0\). The other B3 mSUGRA parameters are given by Point I with \(\lambda_{233}^{\text{GUT}} = 10^{-5}\), cf.
We see that the tree–level mass, \( m_\nu^{\text{tree}} \), vanishes around \( A_0 \approx 910 \text{ GeV} \). In the vicinity of this minimum, \( m_\nu^{\text{tree}} \) drops by several orders of magnitude over a wide range of \( A_0 \), and it is therefore not a (large) fine–tuning effect. In this case the loop contributions will dominate the neutrino mass matrix, resulting in much weaker bounds on the involved \( A \) coupling, cf. Sec. \[11\]. Thus the bound crucially depends on the choice of \( A_0 \).

We emphasize that the range of \( A_0 \) for which weaker bounds may be obtained is quite large. In an interval of \( \Delta A_0 \approx 100 \text{ GeV} \) around the minimum, we obtain bounds on \( \lambda'_{ij3} \) that are at least one order of magnitude smaller than the bound derived at for example \( A_0 = 0 \text{ GeV} \). Much weaker bounds can therefore be obtained without a lot of fine tuning.

In this section, we aim to explain in detail the origin of this cancellation, considering as an explicit example the case \( A \in \{ \lambda'_{ijk} \} \). We focus on the dependence of \( m_\nu^{\text{tree}} \) on the mSUGRA parameter \( A_0 \), because it is always possible to find a value of \( A_0 \) [for a given set of parameters \( \tan \beta, M_1/2, M_0, \) and \( \sgn(\mu) \)] such that the tree–level neutrino mass vanishes. All arguments can analogously be applied to a \( \lambda_{ijk} \) coupling, as discussed in App. \[A5\]. Note for the further discussion that we can always obtain a positive \( A \) by absorbing a possible sign of \( A \) via a re–definition \( L \rightarrow -L \) and \( E \rightarrow -E \) of the lepton doublet and lepton singlet superfields, respectively. We also note that the generated neutrino masses scale roughly with \( A^2 \), cf. the following discussion.

### A. \( A_0 \) Dependence of the Tree–Level Neutrino Mass

We now discuss the dependence of the tree–level neutrino mass at \( M_{\text{EW}} \) as a function of \( A_0 \) at \( M_{\text{GUT}} \). Recall from Sec. \[11\] that

\[
m_\nu^{\text{tree}} \propto \Delta_i^2 = \left( v_i - v_d \frac{\kappa_i}{\mu} \right)^2.
\]  

From the RGE of \( \kappa_i \), Eq. \[8\], we obtain as the dominant contribution

\[
\kappa_i \propto \mu \lambda'_{ijk} (Y_D)_{jk} \equiv \mu \lambda'_{ijk} \frac{(m_d)_{jk}}{v_d}
\]

at all energy scales, where \( (m_d)_{jk} \) denotes a matrix element of the down quark mass matrix. Therefore,

\[
v_d \frac{\kappa_i}{\mu} \propto \lambda'_{ijk} \cdot (m_d)_{jk},
\]

without further dependence on mSUGRA parameters.

Thus, the dependence of the tree–level neutrino mass, Eq. \[28\], on the mSUGRA parameters is solely through the sneutrino vev \( v_i \). In Fig. \[3\] the dashed green line explicitly shows the dependence of \( |v_i|, i = 2 \) on \( A_0 \). It possesses a clear minimum which is close to the minimum of \( m_\nu^{\text{tree}} \).

This behavior can be understood by taking a look at the (tree–level) formula for the vev \( v_i \), Eq. \[10\]. For \( A \in \{ \lambda'_{ijk} \} \) it can be written as

\[
v_i = \frac{1}{(M^2)_{ii}} \left[ \bar{D}_i v_u - (m^2_{h_dL_i} + \mu \kappa_i v_d) \right],
\]

with

\[
(M^2)_{ii} = (m^2_L)_{ii} + \frac{1}{2} M^2_\tilde{2} \cos 2\beta.
\]

Here, we have neglected terms proportional to \( \kappa_i^2 \) and \( v_d^2 \), because they are much smaller than \( (m_L^2)_{ii} \) and \( M^2 \). Note that we only obtain one non–zero sneutrino vev because \( \lambda'_{ijk} \) violates only one lepton flavor.

In many regions of parameter space the sneutrino vev in Eq. \[28\] is at least two orders of magnitude larger than the term \( v_d \kappa_i / \mu \). Thus the minimum of the neutrino mass can only occur when the sneutrino vev is drastically reduced. As we shall see, the sneutrino vev becomes very small, when there is a cancellation between the two terms in Eq. \[31\].

The second term of \( v_i \) in Eq. \[31\], \( (m^2_{h_dL_i} + \mu \kappa_i v_d) \), and the prefactor \( 1/(M^2)_{ii} \) are always positive and depend only weakly on \( A_0 \). This can be seen in Fig. \[3\] for \( (m^2_{h_dL_i} + \mu \kappa_i v_d) \) (dotted–dashed blue line) and also
for $1/(M^2_{\text{GUT}})$ (solid turquoise line). This behavior can be easily understood:

The soft breaking parameter, $m^2_{h_a L_i}$, Eq. (4), is zero at $M_{\text{GUT}}$ and is generated at lower scales via

$$16\pi^2 \frac{dm^2_{h_a L_i}}{dt} = -\lambda_{ijk}(Y_D)_{jk} F(\tilde{m}^2) - 6\lambda_{ijk}(h_D)_{jk},$$

where $F(\tilde{m}^2)$ is a linear function of the soft-breaking scalar masses squared and of the down-type Higgs mass parameter squared. $h_{ijk}(Y_D)_{jk}$ is the soft-breaking analog of $\lambda_{ijk}([Y_D]_{jk})$ with $h_{ijk} = \lambda_{ijk} \times A_0$ at $M_{\text{GUT}}$. The second term in Eq. (33) thus depends on $A_0^2$. However, $F(\tilde{m}^2)$ is in general much larger than $A_0^2$ due to several contributions from soft breaking masses. Therefore, varying $A_0$ does not significantly change the magnitude of $m^2_{h_a L_i}$ as long as $A_0$ is not much larger than the sfermion masses.

Concerning the term $\mu \kappa_i$ in $(m^2_{h_a L_i} + \mu \kappa_i)v_d$, we note from the RGE for $\kappa_i$, Eq. (5), that the only $A_0$ dependence of $\kappa_i$ stems from its proportionality to $\mu$. $\mu$ at $M_{\text{EW}}$ can be approximated by:

$$\mu^2 = c_1 M^2_0 + c_2 M_{1/2}^2 + c_3 A_0^2 + c_4 A_0 M_{1/2} - \frac{M^2_{\text{GUT}}}{2}. \quad (34)$$

Here $c_1$ and $c_2$ are numbers of $\mathcal{O}(1)$ whereas $c_3$ and $c_4$ are only of $\mathcal{O}(10^{-1} - 10^{-2}$). Therefore, except for $A_0 \gg M_0, M_{1/2}$, the order of magnitude of $\mu$ remains constant when varying $A_0$.

We conclude that $(m^2_{h_a L_i} + \mu \kappa_i)v_d$ depends only weakly on $A_0$ and therefore, $\hat{D}_i$ is decisive for the $A_0$ dependence of the vev $v_i$ and thus of $m^2_{\nu\text{tree}}$. If the first term in Eq. (31), $\hat{D}_i v_u$, is positive and only slightly larger than the (nearly constant) second term, $(m^2_{h_a L_i} + \mu \kappa_i)v_d$, $v_i$ can equal $v_d \kappa_i/\mu$ and we get $m^2_{\nu\text{tree}} = 0$, cf. Eq. (26).

The strong $A_0$ dependence of the magnitude of $\hat{D}_i v_u$ is also displayed in Fig. 3 (dotted magenta line). We observe that $|\hat{D}_i v_u|$ is often larger than $(m^2_{h_a L_i} + \mu \kappa_i)v_d$ (dotted–dashed blue line). However, near the tree–level neutrino mass minimum (solid red line), it drops below $(m^2_{h_a L_i} + \mu \kappa_i)v_d$ and $v_i$ can equal $v_d \kappa_i/\mu$. In this case $m^2_{\nu\text{tree}}$, Eq. (26), vanishes.

In order to understand this behavior of $\hat{D}_i$, we need to understand how $\hat{D}_i$ is generated via the RGEs. Recall that $\hat{D}_i = 0$ at $M_{\text{GUT}}$ within the B3 mSUGRA model. The generation of $\hat{D}_i$ primarily depends on the running of the trilinear soft breaking mass $h_{ijk}$.

$$16\pi^2 \frac{dh_{ijk}}{dt} = -6\mu(Y_D)_{jk} h_{ijk} + \ldots. \quad (35)$$

We find the contribution in Eq. (41) proportional to $\hat{B}$ is typically much smaller and we here focus on the effects due to $h_{ijk}$. The dominant terms of the corresponding RGE are given by:

$$16\pi^2 \frac{dh_{ijk}}{dt} = \frac{16}{3} g_3^2 (2 M_3 \lambda_{ijk} - h_{ijk}) + \ldots. \quad (36)$$

where $g_3$ ($M_3$) denotes the SU(3) gauge coupling (gaugino mass). At $M_{\text{GUT}}$ this equation simplifies to

$$16\pi^2 \frac{dh_{ijk}}{dt} = \frac{16}{3} g_3^2 (2 M_{1/2} - A_0) \lambda_{ijk} + \ldots. \quad (37)$$

Keeping for now all parameters except $A_0$ fixed (with $\text{sgn}(\mu) = +1$ and $\lambda_{ijk} > 0$), we can classify the running of $h_{ijk}$, Eq. (36) and Eq. (37), in the following way (see also Ref. [66] for a detailed discussion):

(a) $A_0 \ll 2 M_{1/2}$ (including negative values of $A_0$): Since the right hand side (RHS) of the RGE for $h_{ijk}$, Eq. (36), is always positive and large, $h_{ijk}$ is quickly reduced from its initial value of $A_0 \times \lambda_{ijk}$ and even becomes negative when running to lower energies. This behavior is displayed in Fig. 4 (dashed green line), where the running of $h_{233}$ is shown for different boundary conditions at $M_{\text{GUT}}$.

(b) $A_0 \approx 2 M_{1/2}$: If the size of $A_0$ is comparable to $2 M_{1/2}$, $h_{ijk}$ will be fairly constant at high energies, cf. the dotted magenta line in Fig. 4. However, when running to lower energies it will still start decreasing, but more slowly than in case (a). This is due to the fact that $M_3$ and

![Image](https://via.placeholder.com/150)

FIG. 4: Running of $h_{233}$ for various values of $A_0$. The other B3 mSUGRA parameters are that of Point I, Sec. II B, with $\lambda_{233|\text{GUT}} = 10^{-5}$ and $M_{1/2} = 500$ GeV.
\[ \lambda_{ijk} \text{ themselves increase significantly (by factors of approx. 2.5 and 3, respectively; see Ref. [64]) when running to lower energies. Thus the term} \]
\[ 2 M_3 \lambda_{ijk} \text{ eventually dominates in Eq. [60] even if initially} A_0 \gtrsim 2 M_{1/2}. \text{ This leads to a small, negative} h_{ijk}' \text{ at low energies.} \]

\[ (c) \ A_0 \gg 2 M_{1/2}: \text{ This is due to the negative RHS of the RGE for} h_{ijk}' \text{; Eq. [60]; see also the dotted–dashed blue line in Fig. 4.} \]

**Caveat:** Since the term \( 2 M_3 \lambda_{ijk} \) in Eq. [60] increases by a factor of approximately \( 8 \approx 3 \cdot 2.5 \) when running from \( M_{\text{GUT}} \) to \( M_{\text{EW}} \) [as mentioned in (b)], \( h_{ijk}' \) only strictly displays the behavior of case (c) when \( A_0 \gtrsim 20 M_{1/2} \). Otherwise, \( h_{ijk}' \) will decrease once the term \( 2 M_3 \lambda_{ijk} \) dominates.

Because \( \tilde{D}_i \) is zero at \( M_{\text{GUT}} \) and, according to Eq. [55], also proportional to the integral of \( h_{ijk}' \) over \( \ln(Q) \), points (a) - (c) have the following consequences for \( \tilde{D}_i \):

(a) \( A_0 \ll 2 M_{1/2} \): Since \( h_{ijk}' \) always becomes negative below some energy scale close to \( M_{\text{GUT}} \), the RHS of Eq. [55] is positive. This leads to a large negative \( \tilde{D}_i \) at \( M_Z \) as can be seen in Fig. 4 (dashed green line). Consequently, all terms except \( \tilde{D}_i v_u \) become negligible in \( v_i \), Eq. [31], and thus \( |v_i| \) at \( M_{\text{EW}} \) is large, dominating the tree–level neutrino mass, Eq. [28].

(b) \( A_0 \approx 2 M_{1/2} \): Due to the initially negative RHS of Eq. [35] at energies close to \( M_{\text{GUT}} \) (where \( h_{ijk}' \approx A_0 \times \lambda_{ijk} \)), \( \tilde{D}_i \) first increases when running to lower energies but then starts decreasing once \( h_{ijk}' \) becomes negative, cf. the dotted magenta lines in Fig. 4 and Fig. 5. At some energy scale \( Q \), \( \tilde{D}_i \) becomes small such that \( v_i \), Eq. [31], can equal \( v_i^{\text{tree}} \). A cancellation between these two terms in \( m_{\nu}^{\text{tree}} \), Eq. [28], at the scale \( Q \) will then occur. This corresponds to a vanishing tree–level neutrino mass if \( Q = M_{\text{EW}} \).

(c) \( A_0 \gg 2 M_{1/2} \): The RHS of Eq. [35] is always negative with a large magnitude such that we get a large positive \( \tilde{D}_i \) at the weak scale, cf. the dotted–dashed blue line in Fig. 5. As in case (a), \( \tilde{D}_i v_u \) provides the main contribution to \( |v_i| \), Eq. [31]. Therefore, \( |v_i| \) is large and dominates \( m_{\nu}^{\text{tree}} \), Eq. [28].

Summarizing, the tree–level neutrino mass has a minimum in the parameter region where the size of \( A_0 \) is comparable to \( 2 M_{1/2} \). This is mainly due to the running of the parameters \( \tilde{D}_i \) and \( h_{ijk}' \) that affect the sneutrino vevs; in particular due to a partial cancellation in Eq. [30]. Note that in Fig. 3 the tree–level neutrino mass vanishes at \( A_0 \approx 910 \text{ GeV} \), which is indeed close to \( 2 M_{1/2} \).

As we see in the following section, the position of the minimum is shifted towards higher values of \( A_0 \) for small \( \tan \beta \). In this case, a change of the sign of the bilinear \( \beta \) provides the main contribution to \( \tilde{D}_i \), cf. Fig. 4.

**B. Dependence of the Tree–Level Neutrino Mass on the other mSUGRA Parameters**

In App. A we discuss in detail how the neutrino mass matrix depends on the other mSUGRA parameter besides \( A_0 \). Here we summarize the most important effects and illustrate them in Fig. 6.

In Fig. 4, we show two dimensional mSUGRA parameter scans of the tree–level neutrino mass. The other mSUGRA parameters are those of Point I, Sec. 4.12 with \( \chi^2_{233|\text{GUT}} = 10^{-5} \). One scan parameter is always \( A_0 \) in order to show how the position of the minimum, which was described in the last section, changes with the other mSUGRA parameters.

Fig. 6(a) shows the \( A_0-M_{1/2} \) plane. We can clearly see that the position of the neutrino mass minimum is at \( A_0 \approx 2 M_{1/2} \) as was concluded above. This illustrates that varying \( M_{1/2} \) has a similar effect on the running of \( h_{ijk}' \), Eq. [36] and Eq. [37], as varying \( A_0 \). This is clear from the arguments (a)-(c) in Sec. 4.13.

We could just rephrase the case differentiation as

(a) \( M_{1/2} \gg A_0/2 \).

(b) \( M_{1/2} \approx A_0/2 \).

(c) \( M_{1/2} \ll A_0/2 \).
For $\mathbf{A} \in \{\lambda_{ijk}\}$ the relation is altered to $A_0 \approx M_{1/2}/2$. The change of the prefactor is due to the fact that $\lambda_{ijk}$ couples only leptonic fields to each other. Consequently, only superfields carrying SU(2) and U(1) charges, but not SU(3) charges, contribute to the relevant RGEs; see App. A5 for more details.

In Fig. 6(b) we present the tree–level neutrino mass as a function of $A_0$ and $M_0$. We observe that the position of the neutrino mass minimum is fairly insensitive to $M_0$, compared to $A_0$, $M_{1/2}$ and $\tan \beta$ (see below). The minimum is shifted to slightly higher values of $A_0$ for large $M_0$. However at large $M_0$, the interval around the minimum in the $A_0$ direction where the tree–level neutrino mass is considerably reduced (and therefore the bounds on $\lambda_{ijk}$ are substantially weakened) is significantly broadened.

Finally we show in Fig. 6(c) and Fig. 6(d) the $A_0$–$\tan \beta$ plane for $\text{sgn}(\mu) = +1$ and $\text{sgn}(\mu) = -1$, respectively. In Fig. 6(c) we can see that for low $\tan \beta$, the neutrino mass minimum shifts to higher values of $A_0$.

This is due to a decrease of the down–type Yukawa coupling for low $\tan \beta$ leading to a decrease of the RHS of Eq. (35). This decrease needs to be balanced by increasing $A_0$; recall that $h'_{ijk} = \alpha_{ijk} \times A_0$ at $M_{\text{GUT}}$ in Eq. (35).

A comparison of Fig. 6(c) and Fig. 6(d) also shows that there is a mirror effect around $A_0 = 800$ GeV ($\approx 2M_{1/2}$) when we change the sign of the bilinear Higgs parameter $\mu$. This happens because of a reversal of the sign of the RGE for $\tilde{D}_i$, cf. App. A3. Therefore the shift of the minimum for low $\tan \beta$ now appears towards lower values of $A_0$.

C. The Dependence of the Loop Contributions to the Neutrino Mass on the mSUGRA Parameters

The loop contributions to the neutrino mass matrix are usually several orders of magnitude smaller than
Therefore, we now briefly discuss the dependence of the loop contributions on the mSUGRA parameters.

- **$\lambda\lambda$- and $\lambda'\lambda'$-loops:** This contribution to the neutrino mass, $m_\nu^{\lambda\lambda}$, depends only weakly on the mSUGRA parameters, in particular it depends logarithmically on the relevant sfermion mass. For example, varying $A_0$ from 0 to 1400 GeV (−200 GeV to 1000 GeV) around Point I (Point II) leaves the magnitude of $m_\nu^{\lambda\lambda}$ nearly unchanged [123]; cf. the dotted–dashed blue line in Fig. 7 (Fig. 8). However, increasing $M_0$ or $M_{1/2}$ results in a decreasing $m_\nu^{\lambda\lambda}$: as the SUSY spectrum gets heavier the sfermions in the loops decouple.

- **Neutral scalar–neutralino–loops:** This contribution to the neutrino mass, $m_\nu^{\tilde{\nu}\tilde{\nu}}$, as a function of $A_0$ possesses a minimum which lies in the vicinity of the $m_\nu^{\text{tree}}$ minimum. However, there is no exact alignment. This behavior can be understood by noting that the minima of $m_\nu^{\tilde{\nu}\tilde{\nu}}$ arise due to the vanishing of $D_i$, because roughly

$$m_\nu^{\tilde{\nu}\tilde{\nu}} \propto D_i^2,$$

cf. App. B. This can be seen in Fig. 7 as well as in Fig. 8 (dotted magenta line). Again, increasing $M_0$ or $M_{1/2}$ will in general decrease $m_\nu^{\tilde{\nu}\tilde{\nu}}$, because the SUSY mass spectrum gets heavier.

- **NLO corrections to the sneutrino vevs are typically at least one order of magnitude smaller than the tree level quantities determining the sneutrino vevs, $m_{\tilde{\nu}}^2 = v_d / (M_{\tilde{\nu}}^2)_{ii}$ and $D_i \times v_u / (M_{\tilde{\nu}}^2)_{ii}$, in Eq. 10 [96]. For illustration, one could consider this as an $O(10\%)$ correction to $m_\nu^{\tilde{\nu}\tilde{\nu}}$. This shift upwards of the dotted–dashed blue line in Fig. 8 slightly changes the position of the tree–level neutrino mass minimum, but does not alter any of the conclusions drawn in this section. Since the effects that we investigate in this paper arise mainly from the contribution $D_i v_u$ to the sneutrino vevs (see Sec. IV A), these corrections are not important for the qualitative analysis.

For parameter Points I and II, Sec. 11B the $A_0$ interval, $\Delta A_0$, where the loops dominate is relatively small, cf. Fig. 7 and Fig. 8. However, there are other parameter regions where the loops dominate in intervals of $\Delta A_0 = O(100\text{GeV})$. This is for example the case if one varies $A_0$ around the benchmark point SPS1a [104]. We investigate the resulting bounds on the LNV trilinear couplings in the following section.

**V. Bounds on Trilinear B3 Couplings from $\nu$–Masses**

In this section, we calculate upper bounds on all trilinear LNV couplings $\Lambda \in \{\lambda_{ijk}, \lambda'_{ijk}\}$ at $M_{\text{GUT}}$ from...
the cosmological upper bound on the sum of neutrino masses as given in Eq. (41). Note that in good approximation

\[ m_{\nu|\text{EW}} \propto A^2|_{\text{GUT}}, \]  

(39)
as explained in Sec. [LV A] [124], Eq. (18) and App. A6. Based on this approximation we employ an iterative procedure to account for effects beyond Eq. (39).

In Sec. [LV A] we first compare our bounds with those given in Ref. [52], where the mSUGRA parameters of the benchmark point SPS1a [104] (in addition to A) were used. We choose the same mSUGRA parameters beside \( A_0 \) in order to show how the bounds change in the vicinity of the tree–level neutrino mass minimum, cf. Sec. [LV A]. We then perform in Sec. [VC] two dimensional parameter scans around the benchmark scenarios Point I and Point II (cf. Sec. [II B]) to show more generally how the bounds depend on the B3 mSUGRA parameters.

In our parameter scans we exclude parameter regions where a tachyon occurs [53] or where the LEP2 exclusion bound on the light SSM Higgs mass is not fulfilled [92, 93]. However, we reduce the LEP2 bound by 3 GeV in order to account for numerical uncertainties of SOFTSUSY [102, 103]. For instance, in the decoupling limit (where the light Higgs, \( h^0 \), is SM-like) a lower bound of

\[ m_{h^0} > 111.4 \text{ GeV} \]  

(40)
is imposed. In the figures, we also show contour lines for the 2\( \sigma \) window of the SUSY contribution to the anomalous magnetic moment of the muon [94, 108, 110].

\[ 8.6 \times 10^{-10} < \delta a_\mu^{\text{SUSY}} < 40.6 \times 10^{-10}. \]  

(41)
For more details see Ref. [60] and references therein.

We also note that the complete parameter space which we investigate in the following (having rejected parameter regions which contain tachyons or violate the LEP2 Higgs bound) is consistent with the experimental upper bound on the branching ratio of \( B_s \to \mu^+\mu^- \), i.e.

\[ \text{BR}(B_s \to \mu^+\mu^-) < 4.7 \times 10^{-8}, \]  

(42)
and with the 2\( \sigma \) window for the branching ratio of \( b \to s\gamma \), [92, 111],

\[ 2.74 \times 10^{-4} < \text{BR}(b \to s\gamma) < 4.30 \times 10^{-4}. \]  

(43)
We employed micrOMEGAs2.2 [112], for the evaluation of \( \delta a_\mu^{\text{SUSY}} \), BR\( (B_s \to \mu^+\mu^-) \), and BR\( (b \to s\gamma) \). Note that there is a significant correlation in mSUGRA models between the muon anomalous magnetic moment and \( B_s \to \mu^+\mu^- \) [113]. Furthermore, we are well above the standard supersymmetric mass bounds, as for example on the charginos.

A. Comparison with Previous Results

In Ref. [52], bounds on single couplings \( \Lambda \) at \( M_{\text{GUT}} \) in the B3 mSUGRA model were determined for the mSUGRA parameters of SPS1a, in particular \( A_0 = -100 \text{ GeV} \). However, the possibility of obtaining much weaker bounds on the coupling \( \Lambda \) in the region of the tree–level neutrino mass minimum was not exploited. Note that the bounds in Ref. [52] were also obtained for a less restrictive cosmological bound of \( \sum m_{\nu} < 0.71 \text{ eV} \) [123]. We present here an update of these results by using Eq. (39). We then explore the mSUGRA parameter dependence of the bounds.

In Tab. II and Tab. III (\( \Lambda \in \{\lambda_{ijk}\} \) and \( \Lambda \in \{\lambda_{ijk}\} \), respectively), we compare the previous results with bounds (at \( M_{\text{GUT}} \)) that we obtain for identical B3 mSUGRA parameter points, where only the choice of \( A_0 \) differs. In order to obtain corresponding bounds at \( M_{\text{GUT}} \) one needs to take into account the RG evolution of the couplings. Quantitatively this results in multiplying the bounds in Tab. II (Tab. III) by roughly a factor of 3.5 (1.5), cf. Ref. [14, 52, 53, 65, 67].

In addition to \( A_0 = -100 \text{ GeV} \) (SPS1a), we choose two parameter points which lie \( \Delta A_0 \approx 10 \text{ GeV} \) and \( \Delta A_0 \approx 50 - 70 \text{ GeV} \), away from the neutrino mass minimum. In Tab. II (\( \Lambda \in \{\lambda_{ijk}\} \)), we choose \( A_0 = 500 \text{ GeV} \) (column 3 and 6) and \( A_0 = 550 \text{ GeV} \) (column 4 and 7). In Tab. III (\( \Lambda \in \{\lambda_{ijk}\} \)), we choose \( A_0 = 200 \text{ GeV} \) (column 3) and \( A_0 = 120 \text{ GeV} \) (column 4). This enables us to examine the dependence of the bounds on \( A_0 \) around the tree–level mass minimum.

Note that at SPS1a and when varying \( A_0 \), the neutrino mass minimum for \( \lambda_{ijk} \neq 0 \) lies at \( A_0 = 563 \text{ GeV} \). This value is mostly independent of the choice of the indices \( i, j, k \). This is clear because the condition for the minimum to occur, \( A_0 \approx 2M_{1/2} \), does not depend on \( i, j, k \), cf. Sec. [IV]. Similarly, for \( \lambda_{ijk}|_{\text{GUT}} \neq 0 \) the minimum is expected at \( A_0 \approx M_{1/2}/2 \). For the SPS1a parameters we thus obtain \( A_0 \approx 127 \text{ GeV} \) [120].

We first concentrate on Tab. II. Comparing the columns for \( A_0 = -100 \text{ GeV} \) and then for \( A_0 = 500 \text{ GeV} \), i.e. approaching the minimum up to \( \Delta A_0 = 63 \text{ GeV} \), the bounds from too large neutrino masses are weakened by a factor of 13–15. When we go even closer, i.e. \( A_0 = 550 \text{ GeV} \) and \( \Delta A_0 = 13 \text{ GeV} \), the bounds are weakened by a factor of 40–64 compared to \( A_0 = -100 \text{ GeV} \). As we discuss below, in the case of up–mixing, some couplings in Tab. II (column 2–4) can not be restricted at all by too large neutrino masses. In this case we show the bounds at \( M_{\text{GUT}} \) [marked by \( \ddagger \)], that one obtains from the absence of tachyons; see also Ref. [52].

We differentiate in Tab. III between up– and down–type quark mixing, cf. Sec. [I ID]. Different quark mixing has important consequences for the bounds on the couplings \( \lambda_{ijk} \) if \( j \neq k \). As is clear from Sec. [IV A] the tree–level neutrino mass is generated proportional to \( \lambda_{ijk} \times (Y_D)_{jk} \). Thus, no tree–level mass is gener-
TABLE I: Upper bounds on the trilinear couplings $\lambda_{ijk}$, Eq. (2), at $M_{\text{GUT}}$ for several values of $A_0$ (second row). The other mSUGRA parameters are those of SPS1a [104]. We assume up-mixing (down-mixing) in column 2-4 (5-7), cf. Sec. [11] [12] [13] [14] Bounds arising from the absence of tachyons are in parentheses and marked by a superscript $t$: ($\chi$).

TABLE II: Upper bounds on the trilinear couplings $\lambda_{ijk}$, at $M_{\text{GUT}}$ for different values of $A_0$ (first row). The other mSUGRA parameters are those of SPS1a [104]. Bounds arising from the absence of tachyons are marked by $t$: ($\chi$).

- **Table I:**

| $A_0$ (GeV) | Up mixing | Down mixing |
|------------|-----------|-------------|
|            | -100      | 500         | 550         | -100      | 500         | 550         |
| $\chi'_{111}$ | $2.0 \times 10^{-3}$ | $2.7 \times 10^{-2}$ | $8.3 \times 10^{-2}$ | $9.7 \times 10^{-4}$ | $1.3 \times 10^{-2}$ | $5.3 \times 10^{-2}$ |
| $\chi'_{221}$ | $2.8 \times 10^{-3}$ | $2.7 \times 10^{-2}$ | $8.3 \times 10^{-2}$ | $9.7 \times 10^{-4}$ | $1.3 \times 10^{-2}$ | $5.3 \times 10^{-2}$ |
| $\chi'_{331}$ | $2.8 \times 10^{-3}$ | $2.7 \times 10^{-2}$ | $8.3 \times 10^{-2}$ | $9.7 \times 10^{-4}$ | $1.3 \times 10^{-2}$ | $5.3 \times 10^{-2}$ |
| $\chi'_{121}$, $\chi'_{132}$ | $(1.3 \times 10^{-1})^t$ | $(1.7 \times 10^{-1})^t$ | $(1.7 \times 10^{-1})^t$ | $4.3 \times 10^{-4}$ | $6.0 \times 10^{-3}$ | $2.7 \times 10^{-2}$ |
| $\chi'_{221}$, $\chi'_{232}$ | $(1.3 \times 10^{-1})^t$ | $(1.7 \times 10^{-1})^t$ | $(1.7 \times 10^{-1})^t$ | $4.3 \times 10^{-4}$ | $6.0 \times 10^{-3}$ | $2.7 \times 10^{-2}$ |
| $\chi'_{331}$, $\chi'_{312}$ | $(1.3 \times 10^{-1})^t$ | $(1.7 \times 10^{-1})^t$ | $(1.7 \times 10^{-1})^t$ | $4.3 \times 10^{-4}$ | $6.0 \times 10^{-3}$ | $2.7 \times 10^{-2}$ |
| $\lambda_{111}$ | $2.4 \times 10^{-2}$ | $(1.9 \times 10^{-1})^t$ | $(1.9 \times 10^{-1})^t$ | $6.9 \times 10^{-4}$ | $9.5 \times 10^{-3}$ | $4.2 \times 10^{-2}$ |
| $\lambda_{221}$ | $2.4 \times 10^{-2}$ | $(1.9 \times 10^{-1})^t$ | $(1.9 \times 10^{-1})^t$ | $6.9 \times 10^{-4}$ | $9.5 \times 10^{-3}$ | $4.2 \times 10^{-2}$ |
| $\lambda_{331}$ | $2.4 \times 10^{-2}$ | $(1.9 \times 10^{-1})^t$ | $(1.9 \times 10^{-1})^t$ | $6.9 \times 10^{-4}$ | $9.5 \times 10^{-3}$ | $4.2 \times 10^{-2}$ |
| $\lambda_{122}$ | $9.1 \times 10^{-5}$ | $1.3 \times 10^{-3}$ | $5.3 \times 10^{-3}$ | $8.9 \times 10^{-5}$ | $1.2 \times 10^{-3}$ | $5.2 \times 10^{-3}$ |
| $\lambda_{222}$ | $9.1 \times 10^{-5}$ | $1.3 \times 10^{-3}$ | $5.3 \times 10^{-3}$ | $8.9 \times 10^{-5}$ | $1.2 \times 10^{-3}$ | $5.2 \times 10^{-3}$ |
| $\lambda_{332}$ | $9.0 \times 10^{-5}$ | $1.3 \times 10^{-3}$ | $5.3 \times 10^{-3}$ | $8.8 \times 10^{-5}$ | $1.2 \times 10^{-3}$ | $5.2 \times 10^{-3}$ |
| $\lambda_{132}$ | $2.4 \times 10^{-2}$ | $(1.9 \times 10^{-1})^t$ | $(1.9 \times 10^{-1})^t$ | $5.8 \times 10^{-5}$ | $8.0 \times 10^{-4}$ | $3.9 \times 10^{-3}$ |
| $\lambda_{232}$ | $2.4 \times 10^{-2}$ | $(1.9 \times 10^{-1})^t$ | $(1.9 \times 10^{-1})^t$ | $5.8 \times 10^{-5}$ | $8.0 \times 10^{-4}$ | $3.9 \times 10^{-3}$ |
| $\lambda_{332}$ | $2.4 \times 10^{-2}$ | $(1.9 \times 10^{-1})^t$ | $(1.9 \times 10^{-1})^t$ | $5.8 \times 10^{-5}$ | $8.0 \times 10^{-4}$ | $3.9 \times 10^{-3}$ |
| $\lambda_{113}$ | $4.2 \times 10^{-3}$ | $5.5 \times 10^{-2}$ | $1.9 \times 10^{-1}$ | $6.3 \times 10^{-4}$ | $8.7 \times 10^{-3}$ | $3.8 \times 10^{-2}$ |
| $\lambda_{213}$ | $4.2 \times 10^{-3}$ | $5.5 \times 10^{-2}$ | $1.9 \times 10^{-1}$ | $6.3 \times 10^{-4}$ | $8.7 \times 10^{-3}$ | $3.8 \times 10^{-2}$ |
| $\lambda_{313}$ | $4.2 \times 10^{-3}$ | $5.4 \times 10^{-2}$ | $1.7 \times 10^{-1}$ | $6.2 \times 10^{-4}$ | $8.6 \times 10^{-3}$ | $3.7 \times 10^{-2}$ |
| $\lambda_{123}$ | $5.9 \times 10^{-4}$ | $8.7 \times 10^{-3}$ | $2.4 \times 10^{-2}$ | $5.3 \times 10^{-5}$ | $7.4 \times 10^{-4}$ | $3.4 \times 10^{-3}$ |
| $\lambda_{223}$ | $5.9 \times 10^{-4}$ | $8.7 \times 10^{-3}$ | $2.4 \times 10^{-2}$ | $5.3 \times 10^{-5}$ | $7.4 \times 10^{-4}$ | $3.4 \times 10^{-3}$ |
| $\lambda_{323}$ | $5.8 \times 10^{-4}$ | $8.5 \times 10^{-3}$ | $2.4 \times 10^{-2}$ | $5.3 \times 10^{-5}$ | $7.2 \times 10^{-4}$ | $3.4 \times 10^{-3}$ |
| $\lambda_{133}$ | $2.3 \times 10^{-6}$ | $3.2 \times 10^{-5}$ | $1.3 \times 10^{-4}$ | $2.3 \times 10^{-6}$ | $3.2 \times 10^{-5}$ | $1.3 \times 10^{-4}$ |
| $\lambda_{233}$ | $2.3 \times 10^{-6}$ | $3.2 \times 10^{-5}$ | $1.3 \times 10^{-4}$ | $2.3 \times 10^{-6}$ | $3.2 \times 10^{-5}$ | $1.3 \times 10^{-4}$ |
| $\lambda_{333}$ | $2.3 \times 10^{-6}$ | $3.1 \times 10^{-5}$ | $1.3 \times 10^{-4}$ | $2.3 \times 10^{-6}$ | $3.1 \times 10^{-5}$ | $1.3 \times 10^{-4}$ |

At this level when we consider $j \neq k$ and up-type mixing (which implies a diagonal $Y_D$). But, an additional $\lambda'_{ijk}$ coupling will be generated via RGE running at lower scales, cf. Ref. [52]. This coupling will still generate a tree-level neutrino mass, which is however suppressed by the additional one–loop effect [127].

This effect can be seen in Tab. I if we compare for example the upper bounds on $\lambda'_{223}$ and $\lambda'_{233}$ for up– and down–type quark mixing. The ratio between these bounds is roughly 200 in the case of up–type mixing whereas there is only one order of magnitude difference for down–type mixing.

In the latter case, the ratio between the $\lambda'_{223}$ and
\( \lambda_{233} \) bounds originate mainly from the ratio
\[
\frac{(Y_D)_{23}}{(Y_D)_{33}} = \frac{(V_{CKM})_{23}}{(V_{CKM})_{33}}, \tag{44}
\]
since the tree–level mass is generated via \( \lambda'_{233} \times (Y_D)_{23} \) and \( \lambda'_{233} \times (Y_D)_{33} \), respectively.

To conclude, the bounds from the generation of neutrino masses (at least in the case of down–type mixing) are usually the strongest bounds on the couplings \( \lambda'_{ijk} \) at \( M_{GUT} \). As considered in Ref. \[52\], they range from \( O(10^{-4}) \) to \( O(10^{-6}) \) for the parameter point SPS1a (column 5 in Tab. II). However, there is a large window around the tree-level neutrino mass minimum, where bounds may be obtained that are between one and two orders of magnitude weaker than those in Ref. \[52\]. Around the minimum, the couplings are only bounded from above by \( O(10^{-2}) \) to \( O(10^{-4}) \) (cf. column 7 in Tab. II). Thus, other low energy bounds become competitive.

We now discuss in Tab. III the case of a non-vanishing coupling \( \lambda_{ijk} \) at \( M_{GUT} \). Contrary to Tab. II in the case considered in Tab. II the quark mixing assumption does not affect the bounds since \( \lambda_{ijk} \) couples only to lepton superfields. Due to the antisymmetry \( \lambda_{ijk} = -\lambda_{jki} \) there are only 9 independent couplings.

We observe in Tab. III that if \( i \neq j \neq k \neq i \) there are no bounds from too large neutrino masses. The only bound we obtain stems from the absence of tachyons. This is because we assume a diagonal lepton Yukawa matrix \( Y_L \) as stated in Sec. II.D and therefore, only couplings of the form \( \lambda_{ikk} \) can generate a neutrino mass \[126\].

For these couplings, the bounds at \( M_{GUT} \) for \( A_0 = -100 \text{ GeV} \) (column 2) range from \( 1.1 \times 10^{-1} \) (\( \lambda_{211} \) and \( \lambda_{311} \)) to \( 2.7 \times 10^{-5} \) (\( \lambda_{133} \) and \( \lambda_{233} \)). If we approach the tree–level mass minimum, \( \text{i.e.} \) going from column 2 to column 4 with \( A_0 = 120 \text{ GeV} \), the bound is weaker than the tachyon bound (\( \lambda_{211} \) and \( \lambda_{311} \)) or it is weakened to \( 2.8 \times 10^{-4} \) (\( \lambda_{133} \) and \( \lambda_{233} \)). The bounds from neutrino masses are thus decreased by roughly a factor of 10.

Comparing the bounds on \( \lambda_{ikk} \) at \( M_{GUT} \), one can see nicely how the choice of \( k \) influences the strength of the bound. The bounds resemble the hierarchy between the lepton Yukawa couplings \( (Y_L)_{ikk} \) analogously to Eq. \[14\]. Therefore, the bounds are strongest for \( k = 3 \).

In contrast to Tab. II, the bounds are only reduced by one order of magnitude when we approach the tree-level mass minimum. This is because the loop contributions play an important role for the bounds in Tab. III as we discuss in the following section.

### B. Influence of Loop Contributions

We now shortly discuss the influence of the neutrino mass loop contributions on the bounds. Typically, one expects that the closer we approach the tree–level neutrino mass minimum the more important the loop contributions become. This is because the loops are not aligned to the tree–level mass, \textit{cf.} Sec. IV.C.

However, in the case of the neutral scalar loops there is still partial alignment, because both the tree–level mass minimum and the minima of the neutral scalar loops crucially depend on the vanishing of the bilinear LNV parameter \( D_i \), \textit{cf.} Sect. IV.C. Therefore, it is the \( \lambda' \)-loops and \( \lambda \)-loops, Sec. III.C, that are relevant whenever the loop contributions become dominant over the tree–level contributions.

We now give a few examples. For \( \lambda \in \{ \lambda_{ijk} \} \), Tab. III the loop contributions dominate over the tree–level mass in a range of \( \Delta A_0 \approx \pm 50 \text{ GeV} \) around the tree–level mass minimum at \( A_0 = 127 \text{ GeV} \). Therefore, the bounds in this region are much more restrictive \( \text{i.e.} \) the value of the bounds \textit{decreases} when taking into account the loop contributions. For example,
\[
\lambda_{233}^{\text{tot}} \approx 0.3, \tag{45}
\]
for \( A_0 = 120 \text{ GeV} \); column 4 in Tab. III. Here, \( \lambda_{233}^{\text{tot}} \) is the bound on \( \lambda_{233} \) at \( M_{GUT} \) if we take into account both tree–level and loop–contributions to the neutrino mass. In contrast, \( \lambda_{233}^{\text{tree}} \) would be the bound if we only employ the tree-level mass.

Further away from the minimum, the influence of the loop contributions is weaker. The bounds are strengthened by approximately 5% for \( A_0 = 200 \text{ GeV} \) (column 3 of Tab. II) and < 1% for \( A_0 = -100 \text{ GeV} \) (column 2 of Tab. III).

The loop contributions are less important for the bounds in Tab. II \textit{i.e.} \( \lambda \in \{ \lambda'_{ijk} \} \). For example, even near the tree-level mass minimum (column 4 and 7 with \( A_0 = 550 \text{ GeV} \)), the bounds become only stronger by up to 20% if we take the loop induced neutrino masses in addition to the tree–level mass into account.

### C. Dependence of Bounds on B3 mSUGRA Parameters

In this section, we discuss the dependence of the bounds on \( \lambda \in \{ \lambda_{ijk}, \lambda'_{ijk} \} \) at \( M_{GUT} \) on the B3 mSUGRA parameters. For that purpose we perform two-dimensional parameter scans around the benchmark scenarios, Point I and Point II, of Sec. III.B. For the calculation of the bounds all contributions to the neutrino mass considered in Sec. III are included. We will focus here on the couplings \( \lambda'_{233} \) and \( \lambda_{233} \), because these couplings have the strongest constraints from neutrino masses, \textit{cf.} Tab. II and Tab. III.

We have analyzed in Sec. IV how the neutrino mass changes with the mSUGRA parameters. Due to its approximate proportionality to \( \Lambda^2 \), \textit{cf.} Eq. \[39\], the analysis in Sec. IV is directly transferable to the mSUGRA dependence of bounds on the LNV trilinear.
FIG. 9: Upper bounds on $\lambda'_{233}$ at $M_{\text{GUT}}$ from the cosmological bound on the sum of neutrino masses, Eq. (4), as a function of mSUGRA parameters. The parameter scans are centered around the benchmark Point I, cf. Sec. II B. The blackened-out region denotes parameter points where tachyons occur or where the LEP2 Higgs bound is violated.

We show in Fig. 9 how the bounds on $\lambda'_{233}$ [$\lambda_{233}$] at $M_{\text{GUT}}$ vary with mSUGRA parameters. We present in Figs. 9(a), 9(c) [Figs. 10(a), 10(c)] the $A_0$-$M_{1/2}$, $A_0$-$\tan\beta$, and $A_0$-$M_0$ planes, respectively. The bounds are shown on a logarithmic scale. The blackened out regions designate areas of parameter space which are rejected due to tachyons in the model or violation of the LEP2 bound on the lightest Higgs mass, cf. Eq. (40). Furthermore, we include contour lines of the $2\sigma$ window for the SUSY contribution to the anomalous magnetic moment of the muon, Eq. (41). Imposing Eq. (41) disfavors the parameter space below [above] the green contour line in Figs. 9(a), 9(b), 10(a) and 10(b) [Fig. 9(c) and Fig. 9(d)]. We observe in Fig. 9 that the strictest bounds on $\lambda'_{233}$ from too large neutrino masses are of $O(10^{-6})$. However, there are sizable regions of parameter space where the bounds are considerably weakened. For example, in the $A_0$-$M_{1/2}$ plane, Fig. 9(a) the bounds are of $O(10^{-6})$ only in approximately half of the parameter space whereas in the other half, the bounds are $O(10^{-5})$ or weaker. In roughly 10% of the allowed region in Fig. 9, the bounds even lie at or above $O(10^{-4})$! In this region, the loop contributions to the heaviest neutrino mass are essential for determining the bounds since the corresponding tree–level neutrino mass vanishes, cf. also the discussion in Sec. V B.

We can see in Fig. 10 a similar behavior for the parameter dependence of the bounds on $\lambda_{233}$. Here, the strongest bounds are now of $O(10^{-5})$. However, for example in the $A_0$-$M_0$ plane, Fig. 10(c) the bounds are as strong as $O(10^{-5})$ in only about 25% of the
parameter plane. The remaining 75% have bounds of $\mathcal{O}(10^{-4})$ (50%) or even $\mathcal{O}(10^{-3})$ (25%)!

Up to now, we have analyzed how the bounds on the trilinear LNV couplings $\lambda'_{233}$ and $\lambda_{233}$ vary with the mSUGRA parameters. However, from the analysis in Sec. V A, we can easily deduce how most of these bounds change for different couplings $\lambda'_{ijk}$ and $\lambda_{ijk}$, i.e. for different indices $i, j, k$. For $\lambda'_{ijk}$, the index $i$ does not significantly influence the bound, because the employed Yukawa coupling, $(Y_D)_{jk}$, via which the tree-level mass is generated, does not depend on $i$. But, the situation is totally different when we change the indices $j, k$. In general, for $\lambda'_{ijk}$ (and down-mixing) the bounds will display the hierarchy of the down-type Yukawa couplings. Therefore, bounds for couplings $\lambda'_{111}$ are about three orders of magnitude weaker than bounds for the couplings $\lambda'_{233}$ as long as the other B3 mSUGRA parameter are the same.

To conclude, one can use the Yukawa matrix $Y_D$ ($Y_E$) to easily translate the bounds in Fig. 9 (Fig. 10) to bounds on couplings other than $\lambda'_{233}$ ($\lambda_{233}$).

VI. SUMMARY AND CONCLUSION

We have calculated upper bounds on all trilinear lepton number violating couplings at the grand unification scale within the B3 (i.e. lepton number violating) minimal supergravity (mSUGRA) model, which result from the cosmological bound on the sum of neutrino masses. We have shown that these bounds on the couplings can be weaker by one to two orders of magnitude compared to the ones which were previously presented in the literature for the benchmark scenario SPS1a; cf. Sec. V. In general, the bounds can be as weak as $\mathcal{O}(10^{-1})$. Thus other low energy bounds become competitive.

The reason for these large effects is that the tree–level neutrino mass depends strongly on the trilinear soft-breaking $A_0$–parameter (and also similarly on the gaugino masses). We concluded in Sec. V that in regions of parameter space with $A_0 \approx 2M_1/2$ ($A_0 \approx M_1/2$) for $\lambda'_{ijk}|_{\text{GUT}} \neq 0$ ($\lambda_{ijk}|_{\text{GUT}} \neq 0$), a
cancellation between the different contributions to the tree–level mass can occur. We have explained this effect in detail and have shown that such a cancellation is significant in large regions of the mSUGRA parameter space. For example, the bounds can be weakened by one order of magnitude in $A_0$ intervals of up to $O(100 \text{ GeV})$, see Figs. 9 and 10. Therefore, much weaker bounds (compared to previous ones) can be obtained without significant fine–tuning.

In order to obtain the correct bounds in the vicinity of the tree–level neutrino mass minimum, we included the main loop contributions to the neutrino mass matrix, cf. Sec. IX. We also described in Sec. VI and App. A for the first time the dependence of the tree–level and loop induced neutrino mass on all mSUGRA parameters. Although we concentrated in this work on the $B_3$ mSUGRA model, the mechanisms described will also work in more general R–parity violating models.

Our work can help to find new supersymmetric scenarios that are consistent with the observed neutrino masses and mixings. We have shown in this publication how the (typically large) hierarchy between the tree–level and 1–loop neutrino masses can systematically be reduced. Together with at least one additional lepton number violating coupling, one can use this mechanism to match the ratio between tree–level and 1–loop induced masses to the observed neutrino mass hierarchy, both for hierarchical neutrino masses and for a degenerate spectrum.

We also note, as described in the introduction, that large lepton number violating couplings can lead to distinct collider signatures. We will address these topics in future publications.

Acknowledgments

We thank Ben Allanach, Howie Haber, Jong Soo Kim and Steve C. H. Kom for discussions. The work of H. Dreiner was partially financed by the SFB–TR 33 ‘The Dark Universe’ and partially by DOE grant DE-FG02-04ER41286. S. Grab’s work was financed by the DOE grant DE-FG02-04ER41286. The work of M. Hamussek was funded by the Konrad–Adenauer–Stiftung, the Bonn Cologne Graduate School and the Deutsche Telekom Stiftung.

Appendix A: $\nu$–Masses: Dependence on Further $B_3$ mSUGRA Parameters

In Sec. VI A we described in detail the dependence of the tree–level neutrino mass, Eq. (17), on the $B_3$ mSUGRA parameter $A_0$. We also reviewed some further effects in Sec. VI B. In this appendix, we explain now in more detail the dependence of the tree–level neutrino mass and the loop induced masses on the remaining $B_3$ mSUGRA parameters.

1. $M_{1/2}$ Dependence

The tree–level neutrino mass minimum can be explained equivalently in terms of its dependence on $M_{1/2}$ instead of its dependence on $A_0$. This is because varying $M_{1/2}$ has a similar effect on the sneutrino vev $v_1$, Eq. (31), as varying $A_0$, cf. Sec. IV A and Sec. IV B. However, when varying $M_{1/2}$ there are additional effects coming on the one hand from the dependence of $\mu^2$, $(M^2_\nu)_{ii}$ and $m^2_{H_dL_i}$ on $M_{1/2}$. These quantities are linear functions of $M^2_{1/2}$. For $\mu^2$ this can be seen from Eq. (44). For $(M^2_\nu)_{ii}$ and $m^2_{H_dL_i}$ this follows because the respective RGEs are functions of the squared sfermion masses. One obtains for example

$$ (M^2_\nu)_{ii} \approx M^2_0 + 0.52M^2_{1/2} + \frac{1}{2}M^2_Z \cos 2\beta. \quad (A1) $$

On the other hand, there is also a direct proportionality of $m^2_{H_dL_i}$ to $M^{-1/2}_{1/2}$, cf. Eq. (17). All these additional effects do not significantly influence the position of the tree–level neutrino mass minimum, i.e. $A_0 \approx 2M_{1/2}$ still holds for $A \in \{\lambda^i_{ijk}\}$; see Sec. IV B. However, the effects add a global slope to the terms (as a function of $M_{1/2}$), which contribute to the tree level mass. This behavior can be seen in Fig. 11.

We show in Fig. 11 the same contributions as in Fig. 3 but now for the mSUGRA parameter $M_{1/2}$ instead of $A_0$. Here $A_0$ has been fixed to 900 GeV. On the one hand, we observe that the quantities $\tilde{D}_2v_1$ (dotted magenta line) and $(m^2_{H_dL_i} + \mu_{1i})v_1$ (dotted–dashed blue line) are nearly constant for low values of $M_{1/2}$, but they have a positive slope for large values of $M_{1/2}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11}
\caption{Same as Fig. 3 but now for the mSUGRA parameter $M_{1/2}$ instead of $A_0$.}
\end{figure}
This (weak) tan $\beta$ dependence of $|\tilde{D}_t v_u|$ is illustrated in Fig. 12 for our $B_3$ mSUGRA parameter set Point I; see Sec. 11. One observes that the dotted magenta line ($|\tilde{D}_t v_u|$) increases between tan $\beta = 2$ and tan $\beta = 40$. Here, $\text{sgn}(c_2) > 0$. Above tan $\beta \approx 40$, $|\tilde{D}_t v_u|$ starts decreasing, i.e. $\text{sgn}(c_2) < 0$. This is due to the enhancement of the down–type Yukawa coupling when increasing tan $\beta$, since this reduces $h'_{ijk}$ further and further until it becomes negative. This decrease of $|\tilde{D}_t v_u|$ is only partially visible in Fig. 12, since the parameter region with high tan $\beta$ is excluded due to tachyons.

One can also see in Fig. 12 that the other term determining the sneutrino vev, $(m^2_{\nu_L\nu_L} + \mu \kappa_i)v_d$, which is displayed as a dotted-dashed blue line, is fairly constant regarding tan $\beta$. This contribution to the sneutrino vev is subtracted from the first term, $\tilde{D}_t v_u$ (dotted magenta line), so that the sneutrino vev becomes zero when the two lines intersect; see Eq. 31.

We observe this intersection in Fig. 12 at tan $\beta \approx 22$, thus yielding the tree–level neutrino mass minimum in this region. In theory, there could even arise two minima because above tan $\beta \approx 40 \tilde{D}_t v_u$ starts decreasing again, leading to another intersection with $(m^2_{\nu_L\nu_L} + \mu \kappa_i)v_d$. However, as mentioned before, this usually happens in an excluded region of parameter space.

As is also illustrated in Fig. 12 there is quite a sizable difference between the two terms which determine the sneutrino vev, i.e. $(m^2_{\nu_L\nu_L} + \mu \kappa_i)v_d$ (dotted–dashed blue line) and $\tilde{D}_t v_u$ (dotted magenta line) in the region of low tan $\beta$. If we are looking for a neutrino mass minimum in this region of parameter space, we need to adjust $A_0$ towards higher values, which will increase $h'_{ijk}$ [cf. Eq. 30]. Therefore, increasing $A_0$ will shift the dotted magenta line upwards until it intersects with the dotted-dashed blue line at the desired low tan $\beta$ value. This shift of the tree–level neutrino mass minimum to higher $A_0$ is clearly visible in Fig. 8 for tan $\beta = 20$, the minimum lies at $A_0 \approx 900$ GeV whereas for tan $\beta = 5$, it has shifted to $A_0 \approx 1300$ GeV.

2. tan $\beta$ Dependence

Varying tan $\beta$ most importantly affects the tree–level neutrino mass via the term $\tilde{D}_t v_d$ in Eq. 31. The RGE for $\tilde{D}_t$, Eq. 33, is proportional to the down–type Yukawa coupling $(Y_D)_{jk} \equiv (m_d)_{jk}/v_d$. Therefore,

$$\tilde{D}_t v_u \propto c_1 + c_2 \frac{v_u}{v_d} \equiv c_1 + c_2 \tan \beta , \quad (A2)$$

at $M_{\text{EW}}$. The factors $c_1$ and $c_2$ depend on the other mSUGRA parameters but their magnitude is approximately independent of tan $\beta$. However, there is a dependence of $\text{sgn}(c_2)$ on tan $\beta$ via the RGE of $h'_{ijk}$. Especially in case (b) of Sec. 1VA i.e. in the region around the tree–level neutrino mass minimum, this becomes relevant 129.

3. $\text{sgn}(\mu)$ Dependence

A change of $\text{sgn}(\mu)$ notably affects the tree–level neutrino mass via the RGE running of $\tilde{D}_t$ [Eq. 33], in which the overall sign of the RGE is changed. Therefore, the sign of $\tilde{D}_t$ itself is reversed at any energy scale but its magnitude is mostly unaffected. Consequently, the $A_0$ value where $\tilde{D}_t = 0$ is still mostly the same after a sign change.

However, at the position of the tree–level neutrino mass minimum, $\tilde{D}_t$ needs to be slightly larger than zero in order to cancel the other terms contributing to the tree–level mass, cf. Sec. 1VA and Sec. 1A2. When we are at a parameter point where the tree–level neu-
the dependence of several parameters on \( \mu \) (i.e. \( \tilde{D}_i \) is small and positive), a sign change to \( \text{sgn}(\mu) = -1 \) will yield a \( \tilde{D}_i \) which is small and negative. The other contributing terms undergo no overall sign change. If we would like to obtain a neutrino mass minimum now, \( \tilde{D}_i \) needs to be increased in order to become slightly larger than zero again. This can be achieved by decreasing \( A_0 \), Sec. [IV.A] (or, equivalently, increasing \( M_{1/2} \), Sec. [A.1] since this increases \( \tilde{D}_i \) via \( h_{ijk}' \) in its RGE, Eq. (35), when \( \mu \) is negative. Therefore, the tree-level minimum will occur at smaller values of \( A_0 \) (or equivalently larger values of \( M_{1/2} \)) when we change \( \text{sgn}(\mu) = +1 \) to \( \text{sgn}(\mu) = -1 \).

This effect becomes more important when we go to regions of low \( \tan \beta \). Here the influence of \( h_{ijk}' \) on \( \tilde{D}_i \), Eq. (35), becomes weaker due to the decrease of the down-type Yukawa coupling, as we discussed in Sec. [A.2]. In order to still obtain a positive \( \tilde{D}_i \) after reversing \( \text{sgn}(\mu) \), \( h_{ijk}' \) has to decrease in a more substantial fashion than for large \( \tan \beta \). Therefore, the parameter point where the tree-level neutrino mass minimum is located will shift to smaller \( A_0 \) when changing \( \text{sgn}(\mu) = +1 \) to \( \text{sgn}(\mu) = -1 \), especially for \( \tan \beta \lesssim 10 \).

Overall, this leads to a “mirroring” of the tree-level mass minimum curve in the \( A_0-\tan \beta \) plane around \( A_0 = 800 \text{ GeV} (\approx 2M_{1/2}) \). This can be seen in Fig. 6(c) and Fig. 6(d) for \( \text{sgn}(\mu) = +1 \) the minimum shifts to higher values of \( A_0 \) with decreasing \( \tan \beta \), whereas for \( \text{sgn}(\mu) = -1 \) the minimum shifts to lower values of \( A_0 \).

4. \( M_0 \) Dependence

Varying \( M_0 \) does not greatly affect the tree-level neutrino mass. However, similar effects as those described in Sec. [A.4] as additional effects, arise due to the dependence of several parameters on \( M_0^2 \), cf. for example Eq. (34) and Eq. (A1). This can be seen in Fig. 13 where we again show the terms, which enter the tree-level neutrino mass formula, Eq. (17). We can see that most of the quantities depend only weakly on \( M_0 \). This results in a nearly constant tree-level neutrino mass, cf. solid red line in Fig. 13.

However, the above mentioned \( M_0^2 \) dependences lead to a moderate shift of the tree-level neutrino mass minimum towards higher values of \( A_0 \) when increasing \( M_0 \). Explaining this in detail is fairly lengthy because the \( M_0 \) dependence of the parameters determining the tree-level neutrino mass is not as straightforward as the dependence on other mSUGRA parameters. However, the effect is shown numerically in Fig. 6(b).

It should be noted that there is a similar mirror effect when changing \( \text{sgn}(\mu) \) as for \( \tan \beta \). For \( \text{sgn}(\mu) = -1 \), the minimum shifts towards lower values of \( A_0 \) when increasing \( M_0 \).

FIG. 13: Same as Fig. 8 but now for the mSUGRA parameter \( M_0 \) instead of \( A_0 \).

5. Changes for \( \Lambda \in \lambda_{ijk} \)

We now consider the case of \( \Lambda \in \{ \lambda_{ijk} \} \) instead of \( \Lambda \in \{ \lambda_{ijk}' \} \). Since \( \lambda_{ijk} \) only couples lepton superfields to each other (as opposed to the \( \lambda_{ijk}' \) operator which also involves quark superfields), the RGEs in Sec. [IV.A] are reduced by a (color) factor of 3 \([52, 54]\). In addition, the down quark Yukawa matrix elements,
(Y_D)_{jk}, need to be replaced by the respective lepton Yukawa matrix elements, (Y_E)_{jk}. Otherwise, the structure of the RGEs remains the same.

The only RGE where there are more extensive relevant changes is that for h_{ijk} (which replaces h'_{ijk}); cf. Eq. (3). Eq. (30) must be replaced by

\[ 16\pi^2 \frac{d h_{ijk}}{dt} = \frac{9}{5} g_1^2 (2M_1 \lambda_{ijk} - h_{ijk}) + 3g_2^2 (2M_2 \lambda_{ijk} - h_{ijk}) + \ldots \]  

with \( h_{ijk} = A_0 \times \lambda_{ijk} \) at \( M_{\text{GUT}} \). This looks exactly the same as the RGE for \( h'_{ijk} \), Eq. (30), only with \( g_3 \) and \( M_3 \) replaced by \( g_\alpha \) and \( M_\alpha \) (\( \alpha = 1, 2 \)). However, it is important to realize that the running of \( g_\alpha \) and \( M_\alpha \) is different from the running of \( g_3 \) and \( M_3 \). As was mentioned in Sec. [IV A] the latter quantities increase when running to lower energy scales whereas the former decrease [23].

This has important consequences for the position of the tree–level neutrino mass minimum. The terms \( g_3^2 M_3 \lambda_{ijk} \) of Eq. (30) now decrease as opposed to \( g_3^2 M_3 \lambda'_{ijk} \) in Eq. (30). It is thus necessary to choose \( A_0 \) smaller in order to have a smaller \( h_{ijk} \) at \( M_{\text{GUT}} \) and at lower scales to compensate for this. Quantitatively, we checked numerically that we now need \( A_0 \approx \frac{M_{1/2}}{2} \) (\( \Lambda \in \{ \lambda_{ijk} \} \)) to achieve a vanishing tree–level neutrino mass rather than \( A_0 \approx 2M_{1/2} \) (\( \Lambda \in \{ \lambda'_{ijk} \} \)) as was the case in Sec. [IV A].

For illustrative purpose, we show in Fig. [14] the \( A_0 \) dependence of the tree–level neutrino mass (solid red line) and of the terms determining the sneutrino vev \( \tilde{v}_2 \) for a non–vanishing coupling \( \lambda_{333} \) at \( M_{\text{GUT}} \). Fig. [14] is equivalent to Fig. [8] beside the fact that we now employ the parameter Point II with \( \lambda_{333}[\text{GUT}] = 10^{-4} \) instead of the parameter Point I with \( \lambda_{333}[\text{GUT}] = 10^{-5} \), cf. Sec. [II B]. The qualitative behavior of all terms is the same in both figures. However, in Fig. [14] the minima are shifted to lower values of \( A_0 \) compared to Fig. [8].

We conclude that the line of argument explaining the minimum of the tree–level neutrino mass in the case of \( \Lambda \in \{ \lambda_{ijk} \} \) still holds for \( \Lambda \in \{ \lambda'_{ijk} \} \). However, the position of the minimum now shifts to \( A_0 \approx \frac{M_{1/2}}{2} \).

6. \( A_0 \) Dependence of the Neutral Scalar–Neutralino–Loops

According to Eqs. (26) and (27), the dominant loop contribution from neutral scalar–neutralino–loops to the neutrino mass matrix, \( (m_{\nu}^{\tilde{D}})_{ii} \), is proportional to

\[ (m_{\nu}^{\tilde{D}})_{ii} \propto (\tilde{D}_i \tilde{v}_d - \tilde{D}_j \tilde{v}_i)^2 \times f(m_{\chi^0_k}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2) \]  

where \( f \) is a function of the neutralino, sneutrino and Higgs masses squared, respectively.

The \( A_0 \) dependence of Eq. (33) is mainly determined by \( \tilde{D}_i \), since the \( A_0 \) dependence of \( \tilde{v}_i \) is governed by \( \tilde{D}_i(A_0) \),

\[ \tilde{v}_i(A_0) \propto \tilde{D}_i(A_0) + c \]  

where the term \( c \) depends mainly on the other mSUGRA parameters but barely on \( A_0 \), as discussed in Sec. [IV A]. Therefore \( (m_{\nu}^{\tilde{D}})_{ii} \) is roughly proportional to \( \tilde{D}_i^2 \). The behavior of \( \tilde{D}_i \) has been discussed in detail in Sec. [IV A] in the context of the tree–level neutrino mass. We have shown that there is always a value of \( A_0 \) where \( \tilde{D}_i \) becomes zero. Thus the neutral scalar–neutralino loops display a similar minimum as the tree–level neutrino mass. The position of the minimum is close to the tree–level one, but not exactly aligned. This can be seen by comparing the dotted magenta line and dashed green line in Fig. [7] and Fig. [8]. However, since Eq. (33) is only an approximate formula (for the exact formula, cf. Eq. (23)), the real curve is slightly shifted downwards such that its minimum reaches negative values. Therefore \(|(m_{\nu}^{\tilde{D}})_{ii}| \) in Fig. [7] and Fig. [8] appears to have two minima.

It is also immediately obvious from Eq. (33) that the scalar–neutralino–loops are roughly proportional to \(|A \times (Y_D)_{jk}|^2 \) like the tree–level mass.

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Note that there is one exception, namely the direct proportionality $m^\nu_{\text{tree}} \propto 1/M_{1/2}$, cf. Eq. (17). However, compared to $v_i$, the impact of this term on $m^\nu_{\text{tree}}$ and thus on the bounds of the trilinear LNV couplings is much weaker.

All $c_i$ depend also weakly on $\tan\beta$. However, this becomes only relevant for very small $\tan\beta$.

Only in parameter regions with small $\tan\beta$ and small $M_{1/2}$, a term proportional to $\tilde{B}$, Eq. (9), becomes equally important. This is because $\tilde{B}$ increases with decreasing $\tan\beta$, whereas $\mu \times h^i_{ijk}$ decreases with decreasing $M_{1/2}$, cf. Eq. (31) and Eq. (30). The term proportional to $\tilde{B}$ in Eq. (9) is then enhanced with respect to the term proportional to $h^i_{ijk}$. However, in this parameter region $v_i$ will typically end up being negative because $D_i$ is further reduced than the other term in $v_i$, such that the latter dominates. Then there can be no cancellation in the tree–level neutrino mass, Eq. (28).

From Eq. (8) it is easy to see that this implies $\text{sgn}(\kappa_i) = +1$ below $M_{\text{GUT}}$.

In principle, there is an $A_0$ dependence that stems from left-right mixing of the sfermions inside the loop, cf. the first term in Eq. (20). However, in most regions of parameter space we have $\mu \tan\beta \gg A_0$. In this case only the second term in Eq. (20) plays a role.

This value is smaller than would be expected by estimating $A_0 \approx M_{1/2}/2 = 250 \text{ GeV}$, because we are considering a parameter point with relatively low $\tan\beta$ ($\tan\beta = 10$ for SPS1a). As discussed in Sec. IVB this leads to a shift of the tree–level neutrino mass minimum towards lower values of $A_0$, cf. Fig. 6(c).

Note that also the loop contributions are strongly suppressed, because the $\lambda X'$–loops are proportional to $\lambda^i_{ijk} \times \lambda^i_{ijk}$, Eq. (13), and the neutral scalar loops are aligned with the tree–level mass, cf. Sec. A6.

This would change drastically if the $Y_E$ were strongly mixed.

In case (a), $c_2$ remains always negative and in case (c), $c_2$ is positive.