MAXIMUM BRIGHTNESS TEMPERATURE OF AN INCOHERENT SYNCHROTRON SOURCE: INVERSE COMPTON LIMIT—A MISNOMER

ASHOK K. SINGAL
Astronomy & Astrophysics Division, Physical Research Laboratory, Navrangpura, Ahmedabad-380 009, India; asingal@prl.res.in
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ABSTRACT
We show that an upper limit of \(~10^{12}\) K on the peak brightness temperature for an incoherent synchrotron radio source, commonly referred to in the literature as an inverse Compton limit, really may not be due to inverse Compton effects. We show that a somewhat tighter limit \(T_{\text{eq}} \sim 10^{11}\) is actually obtained for the condition of equipartition of energy between radiating particles and magnetic fields, which happens to be a configuration of minimum energy for a self-absorbed synchrotron radio source. An order of magnitude change in brightness temperature from \(T_{\text{eq}}\) in either direction would require departures from equipartition of about 8 orders of magnitude, implying a change in total energy of the system up to \(\sim 10^4\) times the equipartition value. Constraints of such extreme energy variations imply that brightness temperatures may not depart much from \(T_{\text{eq}}\). This is supported by the fact that at the spectral turnover, brightness temperatures much lower than \(~10^{11}\) K are also not seen in VLBI observations. Higher brightness temperatures in particular would require in the source not only many orders of magnitude higher additional energy for the relativistic particles but also many orders of magnitude weaker magnetic fields. Diamagnetic effects do not allow such extreme conditions, keeping the brightness temperatures close to the equipartition value, which is well below the limit where inverse Compton effects become important.

Key words: galaxies: active – quasars: general – radiation mechanisms: non-thermal – radio continuum: general

1. INTRODUCTION
Kellermann & Pauliny-Toth (1969) first suggested that the observed upper limit on the maximum radio brightness temperatures of compact self-absorbed radio sources is an inverse Compton limit. They argued that at brightness temperature \(T_m \gtrsim 10^{12}\) K energy losses of radiating electrons due to inverse Compton effects become so large that these result in a rapid cooling of the system, thereby bringing the synchrotron brightness temperature quickly below this limit. Singal (1986), on the other hand, derived a somewhat tighter upper limit \(T_m \lesssim 10^{11.5}\) K, without taking recourse to any inverse Compton effects. He used the argument that due to the diamagnetic effects the energy in the magnetic fields cannot be less than a certain fraction of that in the relativistic particles and then an upper limit on brightness temperature close to the equipartition value follows naturally. However, Bodo et al. (1992) pointed out that this limit on the magnetic field energy changes when the drift currents due to magnetic field gradients at the boundary are considered. Equipartition brightness temperature values also have been used to show that much higher Doppler factors are needed to successfully explain the variability events (Singal & Gopal-Krishna 1985; Readhead 1994).

Here we first examine the dependence of the brightness temperature on the magnetic field and relativistic particle energies and calculate the equipartition value \(T_{\text{eq}}\). We then show how in a self-absorbed synchrotron case \(T_{\text{eq}}\) corresponds to a minimum energy configuration for the source. We further show why brightness temperatures of a source cannot rise much above this limit because of the diamagnetic effects, which keep the brightness temperatures close to the equipartition value, well below the limit where inverse Compton arguments become important.

Unless otherwise specified we use cgs system of units throughout.

2. EQUIPARTITION BRIGHTNESS TEMPERATURE
We want to examine an upper limit on the intrinsic brightness temperature achievable in a synchrotron source. Hence we will not consider here any effects of the cosmological redshift or of the relativistic beaming due to a bulk motion of the radio source, assuming that all quantities have been transformed to the rest frame of the source.

In radio sources, the specific intensity \(I_v\), defined as the flux density per unit solid angle at frequency \(v\), usually follows a power law in the optically thin part of the spectrum, i.e., \(I_v \propto v^{-\alpha}\), arising from a power-law energy distribution of radiating electrons \(N(E) = N_0 E^{-\gamma}\), with \(\gamma = 2\alpha + 1\). The source may become self-absorbed below a turnover frequency \(v_m\) (see Pacholczyk 1970), where the peak intensity \(I_m\) is related to the magnetic field \(B\) as

\[ B = 10^{-62} b(\alpha) v_m^5 I_m^{-2}. \]  

(1)

Values of various parameters of spectral index \(\alpha\), calculated from the tabulated functions in Pacholczyk (1977), are given in Table 1, where we have made the plausible assumption that the direction of the magnetic field vector, with respect to the line of sight, changes randomly over regions small compared to a unit optical depth. Using the Rayleigh–Jeans law,

\[ I_m = \frac{2 k T_m v_m^2}{c^2} = 3.07 \times 10^{-37} T_m v_m^2. \]

(2)

the magnetic field \(B\) can be expressed in terms of the peak brightness temperature \(T_m\),

\[ B = 1.05 \times 10^{11} b(\alpha) v_m T_m^{-2}. \]

(3)

The magnetic field energy density \(W_b = B^2/8\pi\) can then be written as

\[ W_b = 4.5 \times 10^{20} b^2(\alpha) v_m^2 T_m^{-4}. \]

(4)
The energy density of the relativistic electrons in a synchrotron radio source component is given by (Ginzburg & Syrovatskii 1965; Ginzburg 1979)

\[
W_k = \frac{1.48 \times 10^{12} I_\nu \nu^\alpha B^{-1.5}}{s a(\alpha)(\alpha - 0.5) b(\alpha)} \times \left[ \left( \frac{\nu_1}{\nu} \right)^{\alpha-0.5} - \left( \frac{\nu_2}{\nu} \right)^{\alpha-0.5} \right],
\]

where \(s\) is the characteristic depth of the component along the line of sight. Using Equations (2) and (3) we get

\[
W_k = \frac{1.3 \times 10^{-41} f(\alpha)}{s a(\alpha)(\alpha - 0.5) b(\alpha)^{1.5}} \nu_m^{\alpha+0.5} T_m^{1.5}
\times \left[ \left( \frac{\nu_1}{\nu} \right)^{\alpha-0.5} - \left( \frac{\nu_2}{\nu} \right)^{\alpha-0.5} \right].
\]

Writing \(\eta = W_k / W_0\) we get

\[
\eta = \left( \frac{s}{pc} \right)^{-1} \left( \frac{\nu_m}{GHz} \right)^{\alpha-1.5} \left( \frac{T_m}{10^{11}} \right)^{8}.
\]

For typical values of \(\alpha \sim 0.6\) (Table 1), the denominator in the last term amounts to \(\sim 10^{0.8}\). Any variations in \(\nu_m\) that we may consider would at most be an order of magnitude, around, say, 1 GHz (after all it is the brightness temperature limits seen in the radio-band that we are trying to explain), which will hardly affect \(T_m\) (Equation (7)). For example, for typical \(\alpha\) values of 0.75–1.0, a factor of 10 change in \(\nu_m\) will change \(T_m\) by a factor \(\lesssim 10^{0.1}\). Further, with the reasonable assumption that a self-absorbed radio source size may not be much larger than \(\sim 1\) pc, we see that at equipartition \((\eta = 1)\) the brightness temperature value is \(T_{eq} \lesssim 10^{11}\).

### Table 1

| \(\alpha\) | \(\gamma\) | \(a(\alpha)\) | \(b(\alpha)\) | \(f(\alpha)\) | \(\nu_1(\alpha)\) | \(\nu_2(\alpha)\) | \(r(\alpha)^a\) | \(\tau_m\) | \(\tau_0\) | \(\nu_0/\nu_m\) | \(T_\nu/\nu_m\) | \(\log T_{eq}\) |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.25      | 1.5  | 0.149| 0.61 | 1.10 | 1.3  | 0.011| 0.53 | 0.19 | 2.80 | 0.37 | 3.5  | 11.3 |
| 0.5       | 2.0  | 0.103| 0.85 | 1.19 | 1.8  | 0.032| 0.67 | 0.35 | 2.92 | 0.50 | 2.2  | 11.2 |
| 0.75      | 2.5  | 0.0831| 0.84 | 1.27 | 2.2  | 0.10 | 0.63 | 0.50 | 3.03 | 0.58 | 1.8  | 11.1 |
| 1.0       | 3.0  | 0.0741| 0.74 | 1.35 | 2.7  | 0.18 | 0.53 | 0.64 | 3.13 | 0.64 | 1.6  | 10.9 |
| 1.5       | 4.0  | 0.0726| 0.52 | 1.50 | 3.4  | 0.38 | 0.34 | 0.88 | 3.32 | 0.72 | 1.4  | 10.7 |

Note. \(^a\) For \(\nu_1\) and \(\nu_2\) taken as 0.01 and 100 GHz, respectively.

### 3. A CORRECTION TO THE DERIVED \(T_m\) VALUES

Actually the \(T_m\) values that have been considered hitherto (here as well as in the past literature) are for the turnover point in the synchrotron spectrum where the flux density peaks. However, a maxima of flux density is not necessarily a maxima for the brightness temperature as well since the definition of the brightness temperature also involves \(\nu^2\) (Equation (2)). In fact, a zero slope for the flux density with respect to \(\nu\) would imply for the brightness temperature a slope of \(-2\). Therefore the peak of the brightness temperature will be at a point where flux density \(\propto \nu^2\), so that \(T_0 \propto F \nu^{-2}\) has a zero slope with respect to \(\nu\). The peak of \(F_\nu \nu^{-2}\) can be determined in the following manner (Singal 2009). The specific intensity in a synchrotron self-absorbed source is given by

\[
I_\nu = \frac{c_2 \nu_\alpha}{c_0 \nu_\alpha} \left( \frac{\nu}{2c_1} \right)^{2.5} \left( \frac{1}{B_{-1.5}} \right)^{\nu/\nu} \left( \frac{T_m}{\nu} \right)^{10^{11}} \times \left[ 1 - \exp \left( -\left( \frac{\nu}{\nu_1} \right)^{(\alpha+2.5)} \right) \right],
\]

where \(c_1, c_2, c_0(\alpha)\) are tabulated in Pacholczyk (1970). The optical depth varies with frequency as \(\tau = (\nu/\nu_1)^{-(\alpha+2.5)}\), \(\nu_1\) being the frequency at which \(\tau\) is unity. The equivalent brightness temperature (in Rayleigh-Jeans limit) is then given by

\[
T_\nu = \frac{c_2^2 c_\alpha(\alpha)}{8 k c_1^2 c_\alpha(\alpha)} \left( \frac{\nu}{2c_1} \right)^{0.5} \left( \frac{1}{B_{-1.5}} \right)^{\nu/\nu} T_m^{10^{11}} \times \left[ 1 - \exp \left( -\left( \frac{\nu}{\nu_1} \right)^{(\alpha+2.5)} \right) \right] .
\]

We can maximize \(T_\nu\) by differentiating it with \(\nu\) and equating the result to zero. This way we get an equation for the optical depth \(\tau_0\), corresponding to the peak brightness temperature \(T_\nu\), which is different from the one that is available in the literature for the optical depth \(\tau_m\) at the peak of the spectrum. The equation that we get for \(\tau_0\) is

\[
\exp (\tau_0) = 1 + (2\alpha + 5) \tau_0.
\]

Solutions of this transcendental equation for various \(\alpha\) values are given in Table 1. It is interesting to note that while the peak of the spectrum for the typical \(\alpha\) values usually lies in the optically thin part of the spectrum \((\tau_m \lesssim 1; Table 1)\), the peak of the brightness temperature lies deep within the optically thick region \((\tau_m \sim 3)\). Both the frequency and the intensity have to be calculated for \(\tau_0\) to get the maximum brightness temperature values. The correction factors are then given by

\[
\frac{\nu_0}{\nu_m} = \left( \frac{T_m}{T_\nu} \right)^{1/\alpha+2.5}
\]

\[
\frac{T_\nu}{T_m} = \left( \frac{\nu_0}{\nu_m} \right)^{0.5} \left[ 1 - \exp \left( -\tau_0 \right) \right] / \left[ 1 - \exp \left( -\tau_m \right) \right].
\]

In Table 1 we have listed \(\nu_0/\nu_m\) for various \(\alpha\) values. The corresponding correction factors \((T_\nu/\nu_m)\) to the peak brightness temperature values, which need to be applied in both inverse Compton and equipartition cases before making a comparison with the observed values, are given in Table 1. Also listed are the accordingly corrected \(T_{eq}\) values. From observational data, the deduced values (Readhead 1994; Homan et al. 2006) of the intrinsic brightness temperature are \(T_\nu \lesssim 10^{11.3}\) K, quite consistent with the \(T_{eq}\) values from Table 1.
4. EQUIPARTITION AND MINIMUM ENERGY DENSITY

We notice that, for any increase in the brightness temperature at any given turnover frequency, while the energy density of radiating particles has to go up by a factor $\propto T_m^4$ (Equation (6)), that in the magnetic fields will have to go down by a similar factor (Equation (4)). We can then derive a minimum energy density of the system. The total energy density $W_k + W_b = c_1 T_m^4 + c_2 T_m^2$ can be minimized with respect to $T_m$ to get $W_k = W_b$ as the condition for the minimum total energy density. It should be noted that the relation between $W_k$ and $W_b$ here is somewhat different from the case of extended sources, where we get minimum energy density for an approximate equipartition condition $W_k = \frac{1}{2} W_b$ (see Pacholczyk 1970), the reason being that in extended sources the intensity was treated as an independent quantity, to be determined from observations, while in a self-absorbed case the maximum intensity and therefore $T_m$ are tied to the magnetic field value through Equations (1) and (3), yielding an exact equipartition condition. The equipartition brightness temperature $T_{eq} \sim 10^{11}$ thus corresponds to a minimum energy configuration of the system. It follows from Equation (7) that an order of magnitude higher $T_m$ values would require $\eta$ to increase by about a factor $\sim 10^2$, i.e., departure from equipartition will go up by about 8 orders of magnitude. Actually for a given $v_m$, the magnetic field energy density will go down by a factor $\sim 10^2$ (Equation (4)), while that in the relativistic particles will go up by a similar factor (Equation (6)). This implies that the total energy budget of the source will also need to be higher by about $10^2$ than from the minimum energy value.

Constraints of such extreme energy variations with brightness temperature imply that the latter may not depart much from $T_{eq}$. This is supported by the fact that brightness temperatures much lower than $10^{11}$ K are also not seen at the spectral peak (see, e.g., Kellermann & Pauliny-Toth 1969, Figure 4), since brightness temperatures much lower than $T_{eq}$ also require much larger total energies. (It is to be emphasized that the brightness temperatures being considered here are the peak values near the turnover in the synchrotron self-absorbed sources, and not the lower values which in any case occur in the optically thin regions.)

5. DIAMAGNETIC EFFECTS

How high could the brightness temperature rise above the equipartition value in a synchrotron source? Following Homan et al. (2006), we can envisage a scenario in which the particle energy density is increased by injecting a large number of additional relativistic particles into the system, e.g., by increasing $N_0$, as $W_k \propto N_0$ for any given energy index $\gamma$. $T_m$ remains below $T_{eq}$ as long as $W_k < W_b$. To increase $T_m$ further, the required change in $N_0$ can be more conveniently calculated from a proportionality expression that can be derived directly from synchrotron self-absorption (Pacholczyk 1970).

$$T_m \propto \left( \frac{v_m}{B} \right)^{0.5} \propto \left( \frac{N_0}{B} \right)^{1/(2\pi + 5)}.$$

Now an order of magnitude change in $T_m$ for a given magnetic field $B$ will require the particle energy density to increase $10^6 - 10^7$ times. However, any change in $W_k$ also brings a change in $B$ due to diamagnetic effects. Gyrating charged particles create their own magnetic field, in a direction opposite to the original field, thus giving rise to diamagnetic effects (Ginzburg & Syrovatskii 1969; Singal 1986). If $H$ is the original magnetic field (that is, the magnetic field value in the absence of diamagnetic effects) then the resultant field is given by (Bodo et al. 1992)

$$\frac{W_k}{3} + \frac{B^2}{8\pi} = \frac{H^2}{8\pi}. \quad (14)$$

This equation actually states that the pressure due to particles and magnetic field in the inner regions is balanced by the surrounding field pressure (Schmidt 1979). At equipartition, the energy density of radiating particles and magnetic fields is equal, $W_k = W_b = W_{eq}$ (say), which implies $W_{eq} = \frac{1}{4} H^2 / 8\pi$. We can then rewrite Equation (14) as

$$\frac{W_k}{3} + \frac{B^2}{8\pi} = \frac{4 W_{eq}}{3} \cdot \quad (15)$$

Due to diamagnetic effects an increase in $W_k$ not only lowers $W_b$, but it may also shift the turnover frequency somewhat since from Equations (4) and (6) $W_k W_b \propto v_m^{6+2.5}$. Apart from equipartition, another solution of Equation (15) exists for $W_k = 3 W_{eq}$ and $W_b = W_{eq}/3$ which does not change $v_m$. However, the resultant increase in $T_{eq}$ from Equation (7) is only a factor of $9^{1/8} \sim (10)^{0.1}$. The maximum $W_k$ that can be achieved is only $4 W_{eq}$. Larger $W_k$ may lead to a total screening of the field, instabilities or other circumstances like total disruption of the source as the external field pressure $H^2 / 8\pi$ may not be able to contain the inner pressure, or the source might expand adiabatically to find a situation which is not too far from equipartition (Ginzburg & Syrovatskii 1969; Bodo et al. 1992). Except under such non-equilibrium conditions, where a large amount of particle energy might have been injected (e.g., near the base of the radio jets) and the system has not yet relaxed to equilibrium, the results derived here should hold good.

6. DISCUSSION AND CONCLUSIONS

Kellermann & Pauliny-Toth (1969) explained the observed upper limit on the radio brightness temperatures of compact self-absorbed radio sources in terms of inverse Compton losses. They derived the ratio of inverse Compton to synchrotron radiation losses as

$$\frac{P_c}{P_s} = \frac{W_p}{W_0} \sim \left( \frac{v_m}{\text{GHz}} \right) \left( \frac{T_m}{10^{11.5}} \right)^5 \left[ 1 + \left( \frac{v_m}{\text{GHz}} \right) \left( \frac{T_m}{10^{11.5}} \right)^5 \right].$$

Here $W_p$ is the synchrotron photon energy density and the second term represents the effect of the second-order scattering. From this we gather that the two rates are comparable at $T_m \sim 10^{11.5}$ K. At higher brightness temperatures, say at $T_m \gtrsim 10^{12}$ K, energy losses of radiating electrons due to inverse Compton effects would become extremely large, resulting in a rapid cooling of the system and thereby bringing the synchrotron brightness temperature quickly below this value. However, these inverse Compton losses become important only if $T_m \gtrsim 10^{12}$ K. It should be noted that even though inverse Compton scattering increases the photon energy density, yet it does not increase the radio brightness as the scattered photons get boosted to much higher frequencies (in a range of infrared to X-rays). If anything, some photons get removed from the radio band, but the change in radio brightness due to that itself may not be very large. What could be important is the large energy losses by electrons which may cool the system rapidly. But can brightness temperatures ever rise to such high values for inverse Compton losses to come into play at a significant level?
The equipartition conditions may keep the temperatures well below this limit, as departures from minimum energy configuration may not grow very large. This is not to say that inverse Compton effects cannot occur; it is only that conditions in synchrotron radio sources may not arise for inverse Compton losses to become very effective. Here it may appear curious that two apparently different theoretical approaches lead to brightness temperature limits which are rather similar; inverse Compton value being only about three times higher than the equipartition one. The genesis of this similarity lies in the fact that for any given turnover frequency, \( T_m \) is exclusively determined by the magnetic field value (Equation (3)), and that in both cases a common factor \( T_4 \) arises due to \( W_b \) in the denominator.

But can magnetic fields get low enough for \( T_m \) to rise significantly where inverse Compton effects become appreciable? In the case where \( W_k \) approaches \( 4W_{eq} \) from Equation (15) \( W_b \) could become very small, thereby making \( \eta \) extremely large. However, in this case the turnover frequency \( \nu_m \) will also shift to a very low value. An order of magnitude increase in \( T_m \) from Equation (13) will imply magnetic field \( B \) falling by a factor \(~10^{-6}\) (i.e., \( W_b \) falling by \(~10^{-12}\) and \( \nu_m \) falling by \(~10^{-4}\). Even if such an unrealistic drop in magnetic field value to nano-Gauss and turnover frequency to kHz range were achievable, for one thing this will take \( \nu_m \) outside the radio-band of our interest where the brightness temperature limits have been seen and which we are trying to explain here. But, even more important, the consequential increase in brightness temperature will still not be contained by inverse Compton effects as even the inverse Compton limit will go up about an order of magnitude for such a large fall in the turnover frequency (Equation (16)). Hence, it is imperative that even in such a scenario inverse Compton effects do not get to play any significant role in maintaining the maximum brightness temperature limit in an incoherent synchrotron radio source.

What about the effects of inhomogeneities in the source on the brightness temperature limits? As mentioned in Section 2, we have assumed that the direction of the magnetic field vector, with respect to the line of sight, changes randomly over the source. This makes \( a \) in Equation (6) change by a factor \(~0.67\) (Ginzburg 1979), while \( b \) in Equation (3) changes by a factor \(~1.15\) (= \((c_29 c_{14})^2\) from the tables of Pacholczyk 1970, 1977). From Equations (4), (6), and (7) we see that \( T_m \propto [a b^{3.5}]^{1/8} \). Thus our derived \( T_m \) values in the case of a random magnetic field orientation are higher than those in the homogeneous case by a factor of \(~1.01\) \((\sim 10^{0.005})\), a negligible quantity. Another type of inhomogeneity one could consider is when the source may consist of a number of discrete components, each with its own cutoff frequency; such a scenario is suggested by the flat shape of the observed spectra as well as the VLBI observations (Kellermann & Pauliny-Toth 1981). However, in such a case the observed brightness temperature value, a sort of average over the various components, cannot exceed the highest value among the individual components, which will have an equipartition brightness temperature limit as derived above.

We can thus conclude that under a variety of conditions the diamagnetic effects will limit the brightness temperatures close to the equipartition value, well below the limit where inverse Compton effects become important.

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