Compensation and control of two-inputs systems with hysteresis

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Abstract. The paper discuss the most assessed issues concerning the control of systems with memory and their application to realistic smart device, with particular focus to those applications where two-inputs fields (i.e. magnetic field and stress) are simultaneously concerned.

1. Introduction
In the last decade, the design of smart devices, based both on newly developed and known materials, has shown an enormous increase. Without any claim of exhaustivity we can recall the well known piezo-electric materials (PZT), magnetostrictives (GMM), or shape memory alloys (SMA), which are the oldest in the set of the so-called smart materials. As well known they can ‘couple’ electric, magnetic fields or temperature to mechanical stress or strain. Further, more recently, other solution have been proposed. For the sake of example we can recall ferro-fluids, where viscosity of the material is related to an applied magnetic field; Ni-Mn-Ga alloys (also called ferromagnetic shape memory), alloys (FSMA)) where a similar behavior, driven by the magnetic field, of classic SMAs is observed; other materials based on known phenomena, such as magneto-caloric effect, Peltier effect, etc. could also be mentioned and the list could be continued. All these alloys share the property that the constitutive relationship links together variable of different physical nature. A suitable name to describe this set could be that of Multi-Functional Materials, (MFM). Among them PZTs, SMAs, GMMs have been widely applied in actuators design in last years.

Mini-actuators based on SMA share the simplicity of mechanisms, cleanliness, noiseless actuation, remotability, sensing ability, low driving voltage. SMAs also show some weakness, such as low energy efficiency (< 10%), quite a low response speed, nonlinear and irreversible behavior. They found wide applications in medical robotics, but also in other fields. The design of such devices as actuators should face with constructive and practical problems (clamping difficulties, low electrical resistance, etc.) and with very low efficiency, generally in the order of 4-5 per cent, [1], [2]. The recently developed FSMAs experience a huge deformation when subjected to a mechanical stress or to a magnetic field. Due to their fast response time (less than a millisecond) and to the high achievable strains (over 10%), FSMAs represent a promising candidate in the development of smart actuators, [3]. Electro-Rheological (ER) fluids are a further example of MF materials. Defined as a dispersion of particles in a insulated oil, they rapidly modifies the internal stress if driven by an external electric field. For this reason they
could be a promising tool for the vibrations active control, [4].

Unlike the former, Magneto-Rheological (MR) fluids found successful applications in the development of new generation of car brakes, dampers, because of their accuracy, dynamics, fast responses and quite simple implementation. They can also find interesting applications in car or train suspension or vibration control of marine diesel engines, [5],[6]. Other interesting materials, controlled by the application of magnetic fields are MR foams and MR elastomers, where the controllable fluid, in the first case is contained into an absorbive matrix, while in the latter, the magnetic field controls the stiffness of the material (which unlike foams is a rubber-like solid material). Their possible application span from active vibration control to suspension devices, [7].

In the field of smart device, Piezo-materials are certainly the most known and widely applied. They show good efficiency, fast responses and quite high forces per unit volume. Applications range from mini-actuators for micro-positioning tasks, to ultrasonic sources and to linear drives. Moreover, due to their high integration capabilities, and again to their high force densities they also found an ever growing interest in Microelectromechanical systems (MEMS) where the successful integration of piezo-materials could yield to the innovation of inertial sensors for navigation, high-frequency crystal oscillators and filters, microactuators for RF applications, etc., [8],[9]. Even if more recently developed, Giant Magnetostrictive Materials (GMM) due to their promising (and to some extent complementary to Piezos) physical properties have focused a large interest for their potentialities. Applications could span from actuation, linear and rotational motors, sonar transducers, etc.,[10]. In particular, they found also a great interest in the design of sensors, such as load cells, torque meters, pressure gauges, tensile stress sensors, thinfilm thickness sensors, and dynamic stress sensors, strain sensing in civil engineering, [11],[12]. A review on the applications as sensor or actuators of the smart materials, mainly focused on piezoelectrics and SMAs is provided in [13]. There also GMM and Electrostrictive materials are considered as potential candidates for these applications. In this case however the need of an accurate modeling and a wide experimental characterization is outlined. In fact, almost all MFM share nonlinearities and rate independent memory effects (hysteresis) which has proved to be a drawback in several tasks. This undesired behavior could strongly limits the performances of smart actuator unless a suitable and accurate modelling and control strategy is adopted. In the paper, with the aim to improve the performances of smart devices, all the basic progresses in modelling and compensation of these phenomena are presented. A particular emphasis is given to the modeling of magnetostrictive materials.

2. Multi-variate systems with hysteresis
Multi-variate systems, such as magnetostrictive, PZTs or any device employing multifunctional materials share the property of linking together more than two physical quantities. As well known, these relationships show rate independent memory phenomena [14], and therefore a generalization of the standard hysteresis operators is required. So, accordingly to the definition of the Classical Preisach model, we can recall the Preisach magnetostrictive operator, as introduced in [14].

Let us consider the continous functions, $u_k$, with $k = 1, 2, \ u_k \in C^1[t_0, T]$ of the kind

$$u_k : [t_0, T] \rightarrow \mathbb{R},$$

**Definition 1** Let be $\hat{\gamma}_{\alpha, \beta}$ the relay operator, and $\mu_j(\alpha, \beta, x)$ and $\nu_j(\alpha, \beta, x)$ with $j = 1, 2$ bounded measurable functions over the half-plane, $T_\infty = \{(\alpha, \beta) \in \mathbb{R}^2 \mid \alpha \geq \beta, \forall x \in \mathbb{R}\}$. The following correspondence:

$$y_j(t) = \int_{\alpha \geq \beta} \mu_j(\alpha, \beta; u_1(t)) \hat{\gamma}_{\alpha, \beta} u_2(t) \, d\alpha \, d\beta + \int_{\alpha \geq \beta} \nu_j(\alpha, \beta; u_2(t)) \hat{\gamma}_{\alpha, \beta} u_1(t) \, d\alpha \, d\beta \quad (1)$$
maps the functions $u_k(t) \in C^1[t_0, T], k = 1, 2$, into the functions $y_j(t) \in C^1[t_0, T], j = 1, 2$, and can be referred as Multi-variate Preisach operator.

It should be noted that such definition slightly generalizes the definition provided in [14]. The two inputs could represent a mechanic and an electric or magnetic variable; in a similar way can be interpreted the output functions. The model defined above has been proposed in Mayergoyz’s book and in [15], with only one output, since the attention was focused on magnetostrictive hysteresis. In that case it shows several important properties, such as *wiping-out* property, *equal vertical chords* and *path-independent* properties, [14]. The equation is an operator of Preisach-type linking a couple of input functions into a couple of output functions. An alternative approach, proposed by [16], exploits the structure of the vector Preisach models, proposed in [14], to describe the magnetostrictive coupling between mechanical and magnetic fields. The class of models defined in equation (1) has properties which make it quite interesting for the modelling of multi-functional (MF) devices. It can easily be employed to describe the behavior of giant magnetostrictive materials, based on Terfenol-D alloys, characterised by the following link between strain $\varepsilon$ and flux density $B$ on one side, and magnetic field $H$ and mechanical stress $\sigma$ on the other:

$$
\varepsilon = G_1(H, \sigma) \quad B = G_2(H, \sigma).
$$

The same machinery could also be employed to model the static constitutive characteristic of Piezo-electric materials, which could be stated as follows:

$$
\varepsilon = F_1(E, \sigma) \quad D = F_2(E, \sigma),
$$

being $D$ and $E$ the electric displacement and the electric field respectively, while $\varepsilon$ and $\sigma$ are the strain and the stress, respectively. A typical magnetostrictive behavior is shown in figures 1, and 2.

**Figure 1.** Typical measured magnetostrictive loops. It is shown the strong dependence of the magnetostrictive response on the applied stress.

Then, it should be stressed that these materials are employed for the design of smart devices where a robust control algorithms is required. The effects of rate independent memory yield the classical control strategies unsuitable and require different approaches to take hysteresis into account. To this end, the model-based control strategy [17]-[20] where a suitable inverse hysteresis operator is able to compensate memory effects, is the most applied. In fact, the availability of a compensator strongly simplifies the design of the controller. For these applications only the first component of the output field shown in equation (1) is considered.
Figure 2. Typical experimental magnetic loops. It is evident the dependence on the applied stress.

Further, it should also be mentioned that almost all the proposed approaches considered the device operating under a constant applied stress. Hence, out of few exceptions, the model based control strategy employed the classical one-input one-output hysteresis operators, linking the input (magnetic or electric field) to the material strain. It should also be observed that, the compensation of Piezo-devices would be an easier task because of less evident saturation effects (cfr. figure 4.2 in [21]). Further, in this case, compensators in a closed form could maybe be exploited [22]. In summary, even if the model described in equation (1) could represent a suitable and complete approach to the multi-variable hysteresis modelling, when control applications are of concern simpler models should be applied. They can span from the classical one-input to one-output Preisach model, to a two-input/one-output operator, when, for example, also the applied stress has to be taken into account. The latter case is of interest, since it allows to extend the limits of applicability of the approach. Unfortunately, the operator described in (1) has not an inverse in a closed form nor could be inverted with high computational efficiency, as the classical Preisach model, [23]. It is therefore such motivation which pushed to propose a new operator able to fully model the device but still holding the property to be inverted with a low computational cost [24]. This would guarantee applicability to real time control tasks.

The model we aim to discuss in the sequel, is a simplified version of the one proposed by Bergqvist and Engdahl in [25], and defined as follows:

**Definition 2** Let be $\mu(\alpha, \beta)$ a bounded measurable function over the half-plane, $T_\infty = \{(\alpha, \beta) \in \mathbb{R}^2 \mid \alpha \geq \beta\}$ while $u_k \in C^1[t_0, T]$, with $k = 1, 2$, and $\Phi_e$ continuous functions. The following equation defines a Preisach-type operator with two inputs.

$$y(t) = \int \int_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta} \Phi_e(u_1, u_2, \alpha, \beta) d\alpha d\beta. \quad (4)$$

It is quite clear that the function $\Phi_e$ plays the role of what is commonly called mean field contribution. Here, the mean field depends on each point $(\alpha, \beta)$ in the Preisach plane. However, a general way to identify and model the material by this approach seemed to be not completely suitable for the defined control task. To this aim, therefore, a simplification to this model was proposed, with the aim to guarantee at the same time an easier model definition and implementation and the availability of a well suited compensation algorithm with a low computational cost.

**Definition 3** Let be $f : \Omega \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$, a continuous function in $\Omega$, while $u_k \in C^1[t_0, T]$, with $k = 1, 2$, $\mu(\alpha, \beta)$, a bounded measurable functions over the half-plane $T_\infty$, and $\lambda \in C^1(\Theta)$, with
\( \Theta \subseteq \mathbb{R} \). The two-inputs operator of Preisach-type is defined as follows:

\[
y(t) = \int \int_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha \beta} f(u_1, u_2) d\alpha d\beta + \lambda(u_2).
\]  

(5)

The operator relates the input fields \( u_1 \) and \( u_2 \) to the variable of interest. It is important to observe that, unlike other attempts, [26], such a simplified approach guarantees the dependence of the state to both input fields, still admitting a compensator operator with low computational cost, as will be specified later. For the sake of example, and in order to particularize the functions introduced so far, let us recall some phenomena which can be observed in magnetostriction hysteresis, and reported in the figures 3 and 4, which clearly show the strong interaction of magnetic field and stress in affecting the global macroscopic behavior of the material, with particular emphasis to its deformation.

![Figure 3](image_url)

**Figure 3.** Experimental magnetic field, stress and strain vs. time. For the sake of clarity, the strain measurement has been normalized to zero by subtracting the sensor offset.

Both figures use experimental data over a magnetostrictive actuator and show the dependence of the output (strain) on both the magnetic field and stress and their linked action in affecting the internal state of the active material. In particular, figure 3 shows the time behavior of measured stress, strain and magnetic field of the sample; moreover, figure 4 displays the further dependence of the system internal state on the stress. In particular, observing the smaller loops, corresponding to the same stress variation but at different magnetic fields, we can infer that the lower the magnetic field applied to the sample, the smaller the loop width. This conclusion would be of help in specifying the parameters of the model.

When a magnetostrictive material is concerned, the model could be specified by assuming \( u_1 \) and \( u_2 \) as the magnetic field \( H \) and the stress \( \sigma \), respectively, while the output, represents the displacement of the sample. The last term in equation, conversely, would represent the pure mechanical response, with zero applied magnetic field. In particular, assuming \( \sigma^* \) as the reference stress value (i.e. the pre-stress normally applied to the sample), the following conditions can be assumed:

\[
\lambda(\sigma^*) = 0, \quad f(H, \sigma = \sigma^*) = H, \quad f(H = 0, \sigma) = 0.
\]
The latter condition underline that in the working range stress values, no pure mechanical hysteresis is possible, without any applied magnetic field.

**Figure 4.** Measured mechanical loops. For the sake of clarity, the strain measurement has been normalized to zero by subtracting the sensor offset.

### 2.1. Identification of the model

The model defined so far, allows to exploit several useful results proposed for the classical Preisach operator. In particular, let us assume that the Preisach operator is invertible [27], [28]; moreover, let us assume the Preisach operator in the following fashion:

\[
\Gamma[x] = \int_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha \beta} x d\alpha d\beta.
\]

(6)

This allow to rewrite equation 5 as follows:

\[
\varepsilon(t) = \Gamma[f(H, \sigma)] + \lambda(\sigma).
\]

(7)

Preliminary, it should be stressed that the \(\lambda(\cdot)\) function represents the pure elastic response of the sample and therefore it can be determined by a series of pure elastic measurements at zero magnetic field. Now recalling that if \(\sigma = \sigma^*\) then \(f(H, \sigma^*) = H\) and \(\lambda(\sigma^*) = 0\), the operator takes the following classical form at \(\sigma = \sigma^*\):

\[
\varepsilon(t) = \Gamma[H],
\]

(8)

which could be easily identified [14]. Once the Preisach distribution function is known, the procedure to identify the \(f(\cdot, \cdot)\) function can start. Recalling now that in suitable conditions \(\Gamma\) admits an inverse and that an efficient inversion algorithm is available, the equation (7) can easily be re-arranged as:

\[
\Gamma^{-1}[\varepsilon - \lambda(\sigma)] = f(H, \sigma).
\]

(9)

Let us preliminary define the interval of variation of the stress, let’s say \([\sigma_{\text{min}}, \sigma_{\text{max}}]\). For the sake of example, in figure 5 the anhysteretic magnetostrictive responses at different mechanical
load are shown. In that case we observe a monotonic decrease of the magnetostrictive response for increasing compressive stresses. Assume now that $\sigma^*$ is the stress value corresponding to the maximum strain variation (which with reference to the figure, corresponds to a 1.05 MPa applied mechanical stress). Moreover, let be $\sigma_{\text{min}} = \sigma^*$.

The interval $[\sigma_{\text{min}}, \sigma_{\text{max}}]$ can now be splitted into $M$ elements, each of length $\Delta\sigma = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{M}$, from $\sigma_0 = \sigma_{\text{min}} = \sigma^*$ to $\sigma_M = \sigma_{\text{max}}$. Now the following procedure for each sampled stress $\sigma_k$ and $k = 1, \ldots, M$ is applied.

(i) Apply an AC demagnetization procedure to the sample;

(ii) Apply N times a Reduced Memory Sequence (RMS) for the $H$-field such that to obtain $N$ points of the anhysteretic curve; let us call the output strain on such a curve $\varepsilon_i$, $i = 1, \ldots, N$;

(iii) the following equation allows therefore to build up $M \times N$ samples of the unknown function $f$:

$$\Gamma^{-1}[\varepsilon_i - \lambda(\sigma_k)] = f(H_i, \sigma_k).$$

Of course, any other value of the $f(\cdot, \cdot)$ can easily be computed by interpolation. Figure 6 shows the $f(\cdot, \cdot)$ identified for a sample of magnetostrictive material.

![Figure 6](image-url)

**Figure 5.** Anhysteretic magnetostrictive responses for varying applied stresses.

3. **Model-based control of systems with hysteresis**

The control of system with hysteresis requires an accurate updating of past input history. The most easy way to do this is to employ models of hysteresis which take memory into account in a natural way. This could be done by exploiting the state updating algorithm which characterizes any well-behaved hysteresis operator. In detail, starting from the device to be controlled, and defining the hysteresis operator able to describe the system, it is possible to build up the inverse of such operator (i.e. compensator) which allows the control system to operate as the device.

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1 For the definition of Reduced Memory Sequence cfr. [29]
were linear. In figure 7 a typical model-based feedback control system is sketched. This of course strongly simplifies the controller design and is the reason why, in the last decade, a wide spread of contribution in the analysis and design of model-based control systems for smart devices (mostly based on Piezo-, SMA or magnetostrictive materials) were proposed, [30]-[35].

![Figure 6. Identified f-function.](image)

![Figure 7. Scheme of the model-based control system.](image)

Basically, the attention is focused on the definition of a reliable model of the device which allows the derivation of a suitable compensation algorithm with, hopefully, a low computational weight. The basic theory concerning the inversion of operators with hysteresis can be found in [36]-[39].

Several approaches have been proposed, to implement suitable compensators in control algorithms, spanning from the control of electromagnetic actuators, [40] or SMA-based devices, [41], [42], to magnetostrictive actuators [43]-[48]. A further contribution was also to take into account dynamic phenomena [20], [49] taking place in the device. For the sake of example, in [20] the compensator has been exploited to identify the dynamic component of the process (supposed as linear), which allowed to improve the model performances. The design of the control device yielded to an improvement of the performances of the closed loop control system, both in terms of error tracking and stability, [45], [47]. The figures 8 and 9 show the improvements in the tracking of a reference signal when a compensator is employed. It is
evident that, with respect to the absence of a compensator, the tracking error is largely reduced, even with a complex reference signal. Moreover, in figure 10 it is shown how compensation could improve the stability of the system. In particular, it refers to a reference sine wave where the linear controller gain has been set to $k = 1000$ for both cases. The uncompensated system becomes unstable, while the model based strategy is still assuring stability to the system. It should be noted that this results have been achieved with a minimum increase in the system complexity.

The model-based control strategy adopted until now found its applicability on the assumption that only one of the input variable is varying, while the other (commonly the applied stress) is kept constant. This assumption works well as soon as micro-positioning tasks are concerned. Unfortunately, when both input variable are free to vary during the process, we assist to an evident worsening of the global performances of the device. Therefore, in any application where the stress is no longer a constant variable, a completely new approach has to be conceived. In the next section we will describe the new approach based on a two-variate modelling of the hysteretic device.

![Figure 8](image_url)

**Figure 8.** Tracking of a reference signal both for a uncompensated and compensated actuator.

### 4. Compensation algorithm for multi-variate systems with hysteresis

Before describing the procedure of model-based control for two-variate systems with hysteresis, it is useful to recall all those concepts regarding compensation procedures of classical (e.g. 1-input/1-output) systems with hysteresis. Let us recall the definition [29], [36], [38] as reported in [28]:

**Definition 4** A hysteresis operator $\mathcal{U}$ with initial state $\phi_{-1} \in \Psi_0$, is called a compensator (or inverse) of the operator $\mathcal{W}$, with initial state $\psi_{-1}$ if, for any state $\phi \in \Psi_0$, there is a state $\psi \in \Psi_0$, such that $\mathcal{W}_\psi \circ \mathcal{U}_\phi x(t) = \mathcal{U}_\phi \circ \mathcal{W}_\psi x(t) = x(t)$, for every input function $x(t)$.

Within the set of Preisach-kind operators, a sufficiently wide number of compensators for 1-input to 1-output operators with hysteresis is outlined, [28]. It is interesting to recall those which, starting from a result given in [39] proposed a generalized version of a compensator in a closed form, [20]. Moreover, other attempt were carried out, which driven to define a simple algorithm which is able to give the Preisach compensator with the same computational effort of
the classical Preisach operator, [23].

![Graph](image)

**Figure 9.** Tracking error of a reference signal both for a uncompensated and compensated actuator.

![Graph](image)

**Figure 10.** Example of tracking of a sinusoidal reference. The linear controller gain has been set to $k = 1000$ for both compensated and uncompensated control strategy.

Let us now consider a system with hysteresis (as those introduced above) having two input variables and one output variable of interest\(^2\). Let us assume that it is representable by the model given in equation (5) or, in the case example of magnetostrictive actuators, by equation (7). Let us also now assume that the operator starts from the initial state $\psi_{-1}$, [28], so for a

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\(^2\) It should be mentioned that all real systems have also two output variables. In the present case the interest is focused on only one of them
given output value \( y \) (or \( \varepsilon \)) and given input \( u_2 \) (or \( \sigma \)), the first input field is required, e.g. \( u_1 \) (or \( H \)). Let us state the following definition.

**Definition 5** Let be \( u_k : [t_0, T] \to S' \subseteq \mathbb{R} \), with \( k = 1, 2 \) continuous functions, \( \lambda \) and \( f \) suitable continuous functions, and \( z \equiv \Gamma^{-1}[y(t) - \lambda(u_2)] \), the equation

\[
f(u_1, u_2) = z \tag{10}
\]

identifies the set

\[
S \equiv \{(z, u_2) \subseteq \mathbb{R}^2 : \exists u_1 : f(u_1, u_2) \equiv z\}. \tag{11}
\]

**Definition 6** Let be \( u_k : [t_0, T] \to S' \subseteq \mathbb{R} \), with \( k = 1, 2 \) continuous functions, the function is defined:

\[
\Phi : (z, u_2) \in S \to u_1 \in S' \subseteq \mathbb{R}. \tag{12}
\]

It is quite obvious that the following property holds:

**Theorem 1** The function \( \Phi \) associates to every \((z, u_2) \in S \) only one value of \( u_1 \in S' \), and vice-versa (e.g. \( \Phi \) is invertible) if and only if equation (10) admits only one solution.

This result guarantees that once the conditions of invertibility are stated, the definition of a compensation algorithm, for a two-inputs system with hysteresis, is well-posed.

**Definition 7** Let us now consider the following mapping \( g : \mathbb{R}^2 \to \mathbb{R}^2 \)

\[
\begin{align*}
v_1 &= f(u_1, u_2); \\
v_2 &= u_2
\end{align*} \tag{13}
\]

the following domain \( W \subseteq \Omega \) can be defined:

\[
W = \{P_0 \in \Omega | J_g(P_0) \neq 0\}, \tag{14}
\]

where

\[
J_g(P_0) = \begin{vmatrix} \frac{\partial f}{\partial u_1} & \frac{\partial f}{\partial u_2} \\ 0 & 1 \end{vmatrix} = \frac{\partial f}{\partial u_1} \tag{15}
\]

is the Jacobian of \( g \) in \( P_0 \).

Of course it is equal to the partial derivative of \( f \) with respect to \( u_1 \).

**Theorem 2** Given the mapping \( g \) defined above, for every \( P_0 \in W \subseteq \Omega \), the function \( f \) is locally invertible in \( P_0 \).

This theorem, according to the well-known theorem on implicit functions [50], [51], guarantees that it is possible to express for every point in \( W \) the variable \( u_1 \) as a function of \( z \) and \( u_2 \), i.e. \( u_1 = \Phi(z, u_2) \). For the sake of example, figure 11 shows a case where the conditions specified above are fulfilled, and so it is possible to build up the function \( \Phi \), providing \( u_1 \) once \( z \) and \( u_2 \) are specified. In particular, for a given \( z \) it is possible to find the value of \( \varepsilon - \lambda \) on the magnetostrictive anhysteretic response on the unperturbed curve (i.e. \( u_2 = \sigma^* \)). Now, once the actual value of \( u_2 \) is specified, it is possible to find the value of \( u_1 \) which guarantees the value of \( z \) for the new \( u_2 \). The procedure also gives the domain \( W \) within \( \Phi \) is defined. In fact the same figure also shows \( z \) values having no \( u_1 \) image. Further, figure 12 give an example of the \( \Phi \) function built up from suitable experimental data, picked from a magnetostrictive sample.
5. Real time control algorithms and experimental verification

As mentioned above, in control systems applications where real time operations are of concern, the most important feature is the computation efficiency of the control algorithm. Indeed, the computation capabilities of the microcontrollers are usually limited by costs considerations. The two-inputs compensation algorithm presented above needs a computation effort that is comparable with the one of a one-input Preisach operator. At each computation time, $t_k$, the steps are the following:

- acquire the actual reference $y(t_k)$ and stress $u_2(t_k)$ values;
- compute the elastic displacement $\lambda[u_2(t_k)]$;
- compute the $z(t_k)$ as output of the one-input Preisach compensator, $z(t_k) = \Gamma^{-1}[y(t_k) - \lambda[u_2(t_k)]]$;
- compute the output as $u_1(t_k) = \Phi(z(t_k), u_2(t_k))$ as a double interpolation on a 2D look-up table;
- as reported in [23], the one-input Preisach compensator uses the same machinery of a standard Preisach operator plus a double interpolation.

Once the algorithm has been specified, it is important to use a versatile tool to implement and check the performances of the compensated control system in a real-time environment. The Matlab package xPCTarget has been chosen for this aim. Indeed, this toolbox allows to implement the control strategies in Simulink, a user-friendly environment, and then to compile and download the generated C-code to a real-time capable controller. A minimum hardware needed to implement the system should be composed by the following items:

- desktop PC: on this PC the simulink models are implemented. It allows to compile them in C-code and then to download onto the Target desktop PC by means of a LAN connection;
- target desktop PC: this PC runs a specific operating system and allows to run any algorithm in a real time environment. This PC has been equipped with a data acquisition/generation card to acquire and to generate the external signals;
- bipolar amplifier: this unit can operate as a voltage-controlled current generator, it is driven by the Target desktop PC and feed the actuator;
- eddy current proximity sensor: this sensor is used to measure the actuator displacement;
- load cell: it is used to measure the load applied to the actuator;
- traction-compression test machine: it is used to apply a time-varying mechanical load to the actuator.

Figure 11. Sketch of the procedure to build up the $\Phi$ function, once the conditions of invertibility are specified.
In order to test the performance of the two-inputs compensator, two control systems have been implemented: a open loop one and a closed loop one. Both systems made use of a custom magnetostrictive actuator with a Terfenol-D active elements, preloaded with washer springs and with a permanent magnet. Typical values of the actuator in the performed tests were a current of ±6 A and a maximum strain of 1350 ppm, equivalent to a displacement of 135 μm.

In the figure 13 it is presented the sketch of the implemented open loop control scheme. The inputs to the system are the desired displacement $\epsilon_d$ (generated by a reference signal generator) and the actual measured compressive stress $\sigma_m$. The pure elastic contribution $\epsilon_{el}$ is obtained from the memoryless function $\lambda$ and is subtracted to the input. The displacement measurement, the load cell and the current amplifier are represented and modeled as constant gains $k_l$, $k_m$ and $k_a$ respectively. The gain $k_g$ can be used to convert the scale of the generated signals to the maximum allowable input range of the compensator.

In the figure 14, it is reported the experimental results of a comparison between the stress-dependent compensator of figure 13 with a standard one-input compensator, i.e. without any stress-dependence. The reference signal was a triangular periodic wave (middle-plot) and the mechanical stress was increasing over time (upper-plot). Then, from the analysis of the relative error presented in the lower-plot, it is apparent that the stress-dependent compensator greatly improved the tracking of the reference strain.

In the figure 15, it is presented the sketch of the implemented closed loop control scheme. In this case, the feedback system is completed by a proportional-integral (PI) controller. The PI controller, if carefully tuned, helps to reduce the tracking error of the control system. The figure 16 shows the experimental result of a comparison between the system with the two-inputs compensator plus the PI controller with a closed loop system without the compensator. Both systems shared the same PI parameters. The reference signal was a sinc-like periodic wave (middle-plot) and the mechanical stress was increasing over time (upper-plot). Then, from the relative error presented in the lower-plot, it is apparent that both systems have a lower relative
error with respect to the open loop system, because of the feedback effect. But, the system with the two-inputs compensator further reduce the tracking error of a 50% with respect to the closed loop system without compensation.

Figure 13. Sketch of the open loop control scheme.

Figure 14. Comparison between the stress-dependent compensator and a standard one-input compensator with a triangular wave reference strain under a time increasing compressive stress.

Figure 15. Sketch of the closed loop control scheme.
Figure 16. Comparison between the two closed loop control system with a sinc-like reference displacement under a time increasing compressive stress. One system has a stress-dependent compensator plus a PI controller while the other has only the PI controller.

6. Conclusion
In this paper it has been presented a review of the issues concerning the control of systems with memory, with particular emphasis to realistic smart device where, in many applications, two-inputs should be considered. In the latter case, a suitable two-inputs hysteresis model can be defined together with its inverse. The range of invertibility of this model is not restrictive. The computational effort of this model is comparable with the one of a standard Preisach operator and, then, a real-time feedback control system can be built. Experimental results over a magnetostrictive actuator show that the improvement in the tracking error makes it suitable for an implementation in realistic feedback systems of smart devices.

References
[1] Bellouard Y 2008 Materials Science and Engineering A 481-482 582
[2] Sreekumar M, Nagarajan T, Singaperumal M, Zoppi M and Molino R 2007 Industrial Robot: An International Journal 34 285
[3] Nespoli A, Bessegghi S, Pittaccio S, Villa E and Viscuso S 2010 Sensors and Actuators A 158 149
[4] Stanwayy R, Sprostonz J L and El-Wahedz A K 1996 Smart Mater. Struct. 5 464
[5] Olabi A G and Grunwald A 2007 Materials and Design 28 2658
[6] Muhammad A, Yao X and Deng Z 2006 Journal of Marine Science and Application 5 17
[7] Carlson J D, Jolly M R, 2000 Mechatronics 10 555
[8] Hensel T and Wallaschek J 2000 Ultrasonics 38 (2000) 37
[9] Tadigadapa S and Mateti K 2009 Meas. Sci. Technol. 20 (2009) 092001.
[10] Olabi A G and Grunwald A 2008 Materials and Design 29 (2008) 469
[11] Hristoforou E and Ktena A 2007 Journal of Magnetism and Magnetic Materials 316 372
[12] Pasquale M 2003 Sensors and Actuators A 106 142
[13] Chopra I 2002 AIAA Journal 40 2145
[14] Mayorga J D 1991 Mathematical Models of Hysteresis (Springer)
[15] Adly A A, Bergqvist A and Mayorga J D J. Appl. Phys. 69 5777
[16] Adly A A and Mayorga J D 1996 IEEE Trans. on Magnetics 32 4773
[17] Kuhnen K 2003 Eur. J. of Control 9 407
[18] Tan X, Baras J S and Krishnaprasad P S 2005 Syst. and Control Lett. 54 483
[19] Iyer R V, Tan X and Krishnaprasad P S 2005 IEEE Trans. on Automatic Control 50 798
[20] Davino D, Natale C, Pirozzi S, Visone C 2004 Physica-B 343 112
[21] Damjanovic D 2006 *Hysteresis in Piezoelectric and Ferroelectric Materials, The Science of Hysteresis* 3 I. Mayergoyz and G.Bertotti (Eds.) (Elsevier)
[22] Visone C and Sjöstrom M 2004 *Physica-B* 343 148
[23] Davino D, Natale C, Pirozzi S and Visone C 2005 *J. of Mag. and Mag. Mat.* 290-291 1351
[24] Davino D, Giustiniani A and Visone C 2009 *J. of Applied Physics*, 105 07D512.
[25] Bergqvist A and Engdahl G 1991 *IEEE Trans. on Magn.* 27 4796
[26] Cavallo A, Davino D et. al 2008 *Physica B* 403 261
[27] Brokate M 1989 *IEEE Trans. on Magn.* 25 2922
[28] Visone C 2008 *J. Phys.: Conf. Series* (2008) 012028
[29] Visintin A 1991 *Differential Models of Hysteresis* (Springer)
[30] Ge P and Jouaneh M 1996 *IEEE Trans. on Contr. Sys. Tech.* 4 209
[31] Schäfer J and Janocha H 1995 *Sensors And Actuators A* 49 97
[32] Webb G V, Lagoudas D C and Kurdila J 1998 *J. Of Int. Mat. Syst. And Struct.* 9 432
[33] Krejci P and Kuhnen K 2001 *IEEE Proc. on Control Theory and Appl.* 148 185
[34] Tan X, Venkataraman R and Krishnaprasad P S 2001 SPIE Modeling, Signal Processing, and Control in Smart Structures 4326 101
[35] Natale C, Velardi F and Visone C 2001 *Proc. of IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics, Como, Italy* 744
[36] Krasnosel'skii M A and Pokrovskii A V 1989 *Systems with Hysteresis* (Springer-Verlag)
[37] Krejci P 1996 *Gakuto Int. Series Math. Sci. & Appl.* 8
[38] Brokate M and Sprekels J 1996 *Hysteresis and Phase Transitions* (Springer)
[39] Krejci P 1986 *Mat. Zeit.* 193 247
[40] Mittal S and Menq C H 2000 *IEEE/ASME Trans. on Mechatronics* 5 394
[41] Majima S, Kodama K and Hasegawa T 2001 *IEEE Trans. on Control Sys. Tech.* 9
[42] Hasagawa T and Majima S 1998 *Proc. IEEE Int. Symp. on Micromech. and Human Sci.* 171
[43] Kuhnen K and Janocha H 2001 *Proc. of the 20th IASTED-Conf. on Modeling, Identification and Control* 375
[44] Natale C, Velardi C and Visone C 2001 *Physica-B* 306 161
[45] Cavallo A, Natale C, Pirozzi S and Visone C 2003 *IEEE Trans. on Magn.* 39 1389
[46] Tan X and Baras J S 2005 *IEEE Trans. on Automatic Control* 50 827
[47] Cavallo A, Natale C, Pirozzi S and Visone C 2004 *IEEE Trans. on Magn.* 40 876
[48] Davino D, Giustiniani A, Vacca V and Visone C 2006 *IEEE Trans. on Magn.* 42 3443
[49] Ekanayake D B, Iyer R V and Dayawansa W P 2007 *Proc. of American Control Conference* 4321
[50] Munkres J R 1991 *Analysis on Manifolds* (Addison-Wesley)
[51] Weisstein E W “Implicit Function Theorem.” From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/ImplicitFunctionTheorem.html