Moored offshore structures – evaluation of forces in elastic mooring lines

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Abstract. In most situations, the high frequency motions of the floating structure induce important effects in the mooring lines which affect also the motions of the structure. The experience accumulated during systematic experimental tests and calculations, carried out for different moored floating structures, showed a complex influence of various parameters on the dynamic effects. Therefore, it was considered that a systematic investigation is necessary. Due to the complexity of hydrodynamics aspects of offshore structures behaviour, experimental tests are practically compulsory in order to be able to properly evaluate and then to validate their behaviour in real sea. Moreover the necessity to carry out hydrodynamic tests is often required by customers, classification societies and other regulatory bodies. Consequently, the correct simulation of physical properties of the complex scaled models becomes a very important issue. The paper is investigating such kind of problems identifying the possible simplification, generating different approaches. One of the bases of the evaluation has been found considering the results of systematic experimental tests on the dynamic behaviour of a mooring chain reproduced at five different scales. Dynamic effects as well as the influences of the elasticity simulation for 5 different scales are evaluated together. The paper presents systematic diagrams and practical results for a typical moored floating structure operating as pipe layer based on motion evaluations and accelerations in waves.

1. Introduction
The first important parameter in the design of an adequate station keeping system of a defined floating structure is strongly dependent on the physical characteristics of the location (water depth, structure of the bottom, geometry of the bottom, water salinity, temperature etc.) and the environmental conditions due to the waves, current, wind, tide and multiannual fluctuation of the sea level. The collection of the last category of data is commonly named as “metocean” criteria and provides statistical results of systematic measurements on a designated location.

The second important aspect is related to the behaviour of the floating structure which is depending on the body lines and mass distribution on board (loading case). The form parameter is dictating the method to be used for the prediction of the motions and accelerations depending on the geometry of the hull. Generally, for large LWL/B ratios 2D techniques, based on the “slender body theory”, can be used. A good example is the theory developed by Solvesen, Tuck and Faltinsen approach using the Frank “close fit method” distribution of sources [1]. For low LWL/B ratios the utilization of 3D approaches [2] becomes mandatory and most of the applications are using the Green function method using the well-known formulation given by Wehausen and Laitone. LWL is the waterline length and B is the breadth of the floating structure. The weight distribution parameter is practically defining the
inertial characteristics of the floating structure for any loading case, which have an important impact on its natural frequencies and on the behaviour of the floating structure in waves defined by the response amplitude operators (RAO’s) as a function of the wave frequency.

The third important aspect to be taken into consideration is linked to the type of operation to be performed by the floating structure. This parameter could lead to significant limitations and are acting as criteria in the definition and selection of the appropriate station keeping system. On one hand motions limitation in the horizontal plane are imposed by the type of operation and are mandatory in order to avoid the failure of the specific equipment. This is the case of floating structures like drill ships, pipe layers, cable layers, etc. On the other hand, severe limits of the accelerations have to be taken into account in connection with comfort on board based on human exposure limitations provided by the international standards [3]. As a matter of fact, these two limitations are defining the operational limit which, practically, represents the performance index of the floating system, a higher affordable sea state level leading to a better efficiency and profitability.

The way to perform the evaluation of the forces in the mooring lines of a moored floating system is in connection with the level of the “dynamicity” of its behaviour. The assumption that the natural periods of motions of the floating body are far from those of the predominant exciting waves on the location could lead to a static approach. However, such a condition is difficult to be practically reached in practice. That is why a step in the evaluation of the forces in the mooring lines could be the so-called quasi-static method. Finally, the way to approach this problem has to be in line with the customer requirements and the provisions of the Classification Societies. Due to the complexity of the problem it is very often happening that experimental hydrodynamic tests become mandatory using scaled models in combination with specific computer codes used during the preliminary design stages.

Consequently, the consideration of the elasticity of the mooring lines has to be considered from two major points of view:
- Firstly, due to the necessity to reach an accurate evaluation of the real dynamic forces in the mooring line and in the anchors in order to have a correct choice of their characteristics (diameter, mass per unit length in water, length of lines, etc., and type and mass of anchor, its holding power etc.).
- Secondly, when experimental tests are required and the elasticity of the model of the mooring lines has to be properly reproduced based on the prototype characteristics, using appropriate laws of similitude.

2. The static analysis

The static evaluation of the mooring forces is based on the well-known catenary equations of Bernoulli[4], [5] or using the formulation given by Wilson [6] according to figure 1,

![Figure 1. General mooring line geometry.](image)

where:
- $h$ – water depth;
- $T_o$ – tension in anchor chain at upper end;
$T_{0v}$ – vertical tension at upper end;
$T_{0h}$ – horizontal tension at upper end;
$T_i$ – tension in lower end (anchor);
$T_{iv}$ – vertical tension in lower end (anchor);
$T_{ih}$ – horizontal tension in lower end (anchor);
$s$ – chain length;
$w$ – weight of chain per unit length of anchor chain;
$l$ – horizontal projection of chain;
$\phi_h$ – chain angle with the horizontal at the upper end (between $T_0$ and $T_{0h}$);
$\phi_i$ – chain angle with the horizontal at the anchor point (between $T_i$ and $T_{ih}$).

The main particularities of the catenary form consist in:
- The horizontal component of the tension along the line is constant;
- The tension at a given point on the line can be linearly expressed related to the current vertical co-ordinate at the same point and can be written, in vectorial form:

$$
\begin{align*}
\begin{pmatrix} T_0 \\ z \end{pmatrix} &= \begin{pmatrix} T_{0h} \\ wz \end{pmatrix}
\end{align*}
$$

(1)

The general case presented in the figure 1 refers to the taut case when $\phi_i \neq 0$.

$$
\begin{align*}
\frac{T_{0h}}{wh} &= \frac{1}{\sec \phi_h - \sec \phi_i} \\
\cos \alpha_{1A} &= \frac{T_{14} - 1}{2T_{14}} \\
\frac{s}{h} &= \frac{\tan \phi_h - \tan \phi_i}{\sec \phi_h - \sec \phi_i} \\
l &= \frac{1}{\sec \phi_h - \sec \phi_i} \sqrt{-6 + 36 + 12\left(\frac{\tan \phi_h - \tan \phi_i}{\sec \phi_h - \sec \phi_i}\right)^2 - (\sec \phi_h - \sec \phi_i)^2}
\end{align*}
$$

(2)

(3)

(4)

When the slope with respect to the horizontal at anchor becomes $\Phi_i = 0$, the equations will define the slack configuration:

$$
\begin{align*}
\frac{T_0}{wh} &= \frac{1}{1 - \cos \phi_s} \\
\frac{T_{0h}}{wh} &= \frac{\cos \phi_s}{1 - \cos \phi_s} \\
\cos \phi_s &= \frac{1 - wh}{T_0} \\
\frac{s}{h} &= \cot \frac{1}{2} \phi_s
\end{align*}
$$

(6)

(7)

(8)

(9)
$l = \frac{\cos \phi_s}{1 - \cos \phi_s} \sqrt{-6 + \sqrt{36 + 12 \left[ \tan^2 \phi_s - \left( \frac{1 - \cos \phi_s}{\cos \phi_s} \right)^2 \right]}}$ (10)

Having in mind that, in principle, the ways to validate theoretical models are the full scale trials and/or the experimental tests results, an extensive program based on scaled models has been conceived. The basic chain, chosen as a prototype, has $\phi_s = 0.1$ m calibre and an air mass per linear meter of 219 kg/m. The water depth at full scale was 52.5 m, representing the depth of water where the SBM (Single Buoy Mooring) system was located in the Black Sea.

For these data, considered as input data, the Froude similarity law was used in order to extrapolate the results at full scale, considering that the predominant nature of the forces is the gravitational one. The characteristics of the chain models are presented in table 1.

| Table 1. Main properties of chain models. |
|----------------------------------------|
| **Scale** | **Weight per linear meter** | **Diameter of chain wire, $q$ [10^{-3}m]** | **Water depth, $h$ [m]** |
| $\lambda$ | **Mass in air (calculation) [kg/m]** | **Scaled model [%]** | **In water [kg/m]** | |
| 52.50 | $q_1=0.079$ | 0.0785 | -0.6 | 0.071 | 1.9 | $h_1=1.00$ |
| 45.00 | $q_2=0.108$ | 0.1050 | -2.8 | 0.097 | 2.2 | $h_2=1.17$ |
| 35.00 | $q_3=0.178$ | 0.1740 | -2.9 | 0.160 | 2.9 | $h_3=1.50$ |
| 26.25 | $q_4=0.318$ | 0.3090 | -2.3 | 0.283 | 3.8 | $h_4=2.00$ |
| 17.50 | $q_5=0.715$ | 0.7120 | -0.4 | 0.641 | 5.7 | $h_5=3.00$ |

For the generation of a systematic experimental program [7],[8] the following parameters were considered: water depth, chain calibre, chain pretension, amplitude of the oscillation at the upper end, frequency of the oscillation at the upper end and the scale of the models. The experimental program is concisely presented in figure 2. Over 1,500 individual tests have been performed. Forced oscillation tests have been carried out using a special “sinus mechanism” which imposes a pure sinusoidal motion at the upper end (fairlead) of the mooring lines scaled models. The experimental arrangement is presented in figure 3.

![Figure 2. The experimental program.](image)
Based both, on the experimental and the theoretical results respectively, a first comparison was possible evaluating the so-called static characteristics (load – excursion curve) of the mooring lines. A very good agreement was found as presented, as examples, in figure 4 for the chain $q_1$ and figure 5 for the chain $q_4$, respectively.

Mention should be made that the influences of the differences between the model and full scale elongations shouldn’t have a decisive importance when a slack mooring is considered but will become very important for taut mooring or higher depth mooring conditions. That is why the consideration of the hydro-elastic models became compulsory and in addition to Froude similitude criterion, the Cauchy similitude one is necessary to be taken into account. These aspects have been carefully evaluated in a dedicated paper when it was clearly underlined the necessity to reproduce the prototype elongation of the mooring line at the model scale [8], [10]. In figure 6 are presented the results of the experimental pull tests for the five chain models. $\Delta L [10^{-3} \text{m}]$ is the experimentally measured elongation, $\varepsilon_R$ is the relative elongation, $F_{m} [\text{Kg}]$ are the pulling forces at model scales, $F_{t F} [\text{t}]$ is the full scale pulling force and $L_r [\text{m}]$ the reference length of each model chain.

Figure 3. General arrangement of the experimental tests.

Figure 4. Load excursion curve, $q_1$.

Figure 5. Load excursion curve, $q_4$.

Figure 6. Results of pull tests for scaled chains.
In order to obtain reliable results, the extrapolations of the above values of the elongations at model scales have to reproduce the same value at the prototype scale. To this purpose, as far as the models are made using the same material (steel), as previously mentioned the utilization of elastic compensators become mandatory.

An additional problem which could arise is in connection with the scale effects when an experimental approach is to be envisaged. However, no significant scale effects have been reported [6],[7]. On the other hand, the migration from deep water to very deep water resources, over 1,000 m, has created new experimental approaches and techniques leading to the so-called “hybrid tests”. Consequently, the elements which cannot be reproduced as part of the physical scaled model are replaced by their effects[11].

3. The quasi-static analysis
The static analysis is seldom used due to the quite few applications when such an approached is accepted. However, there are exceptions for some specific cases when this manner of solving the problem is considered but, it has to be anyway performed, being a first stage for any further steps to be followed. A very good example is related to the quasi-static analysis when the previous evaluations, i.e. static analysis, are already performed. A general view of the quasi-static approach is presented in figure 7.

![Figure 7. The principle of the quasi-static approach.](image)

The meaning of “quasi-static” refers to the way to consider the contribution of the motions of the floating body using the static characteristics of the moored system, when, practically, only the amplitude of the motions taken into consideration and no induced dynamic effects are considered due the accelerations. However, such an analysis needs to have as input data the evaluation of the environmental forces. The main sources of excitation are briefly presented in figure 8.
The availability of at least preliminary calculation modules or experimental tests results are necessary in order to obtain enough accurate evaluations to be used in the early design stages [11]. The example below refers to a moored pipe layer barge, operating in shallow water, when the quasi-static approach is applicable [12].

The first step was the evaluation of the environmental forces due to the current, wind and waves for two distinct cases: operational and extreme conditions using the statistic information (metocean). The calculations have been carried out for a three significant heading angles as presented in table 2 for the operational case.

Table 2. Summary of the results of external forces for operation condition.

| Operation condition | Head sea | Beam sea | Quartering sea |
|---------------------|----------|----------|----------------|
| T =1.36m            | T =2.12 m| T =1.36m | T =2.12 m      | T = 1.36m | T = 2.12 m      |
| Wind forces, $F_W$ [t] | 4.9      | 4.6      | 10.0           | 8.9      | 7.0             | 6.0             |
| Current forces, $F_C$ [t] | 5.1      | 8.0      | 14.8           | 22.8     | 10.1            | 18.0            |
| Wave drift forces, $F_D$ [t] | 7.8      | 14.5     | 18.0           | 31.6     | 12.6            | 23.0            |
| Total external forces [t] | 17.8     | 27.1     | 42.8           | 63.3     | 29.7            | 47.0            |

The configuration of the moored system is presented in figure 9.
The calculation of the forces in the mooring lines have been performed for 5 different operation condition, 2 loading cases (2 draughts), 2 pretensions and 2 water depths using an in-house dedicated computer code which is taking into account the elasticity of the lines. The amplitude of motions to be used as inputs for the load – excursion curve has been evaluated by means of seakeeping calculations according to the method suggested in [2]. The output data are:

- $S$ – suspended line span in inelastic conditions [m];
- $SE$ – suspended line span when the elasticity is taken into account [m];
- $X$ and $Z$ – current coordinates on the line where the forces is calculated [m];
- $\phi$ – current line angle from horizontal [°];
- $T_0$ – total tension in the line [t].

An example of calculations is presented in figure 10, where the influence of the elasticity of the line can be identified by comparing the values of $S$ and $SE$.

**Figure 9.** General description of the system.

**Figure 10.** Calculation example.
The calculations are further used in order to verify the mooring lines, the forces in winches and the selection of the anchor. Note that, in the Tab. 4, MBL means the maximum breaking load and MT means metric tons.

Case 1.1b Operating condition, draft 2.12 m, intact condition, pulling force 60 [t]
Total external force, \( T_{Ext} = 27.0 \) [t]
\( F_H(P_1) = F_H(S_1) = 34.0 \) [t], including pretension
\( F_H(P_2) = F_H(S_2) = 25.5 \) [t], including pretension

Case 1.3b Operating condition, draft 1.36 m, intact condition, pulling force 60 [t]
Total external force, \( T_{Ext} = 18.0 \) [t]
\( F_H(P_1) = F_H(S_1) = 31.0 \) [t], including pretension
\( F_H(P_2) = F_H(S_2) = 23.3 \) [t], including pretension

Table 3. Practical utilization of results.

| Required conditions | Case 1.1b | Case 1.3b |
|---------------------|-----------|-----------|
| Synthetic Rope      | Intact    | 60% of MBL = 80.4 MT | YES | YES |
| Mooring Line, \( \Phi = 0.6 \) m | Damage    | 80% of MBL = 107.2 MT | YES | YES |
| MBL = 134 MT        | Minimum Winch | 84 MT (Layer 3) | YES | YES |
| \( E = 9000 \) N/mm\(^2\) | Minimum* Anchor | 2.5 MT | 38.6 MT | YES | YES |
| \( W_{air} = 1.75 \) kg/m | Horizontal Holding | 3.0 MT | 45.6 MT |
| \( W_{water} = 1.52 \) kg/m | Capacity for different masses | 3.5 MT | 52.4 MT |
|                     |           | 4.0 MT | 59.2 MT |
|                     |           | 4.5 MT | 65.8 MT |
|                     |           | 5.0 MT | 72.4 MT |

The typical static characteristics for synthetic rope mooring lines and mooring line (synthetic rope and steel cable) are presented in the figure 11 and figure 12.

Figure 11. Load – excursion curve for a synthetic rope mooring line.

Figure 12. Load – excursion curve for a composite mooring system.

4. The dynamic approach
As previously stated, the above mentioned method is applicable when the behaviour of the structure is close to a quasi-static behaviour but this hypothesis can’t be any more acceptable for most of practical application. An additional step forward can be the evaluation of the acceleration in the connection point of the mooring line to the structure (fairlead) and to use this value like a correction playing the role of a “dynamic amplification factor”, procedure which is commonly used based on practical experience. However, this possibility, which is an important step, is linked to the assumption that the floating structure, due to its large mass, is imposing the motion to the upper end of the mooring line.
Unfortunately, even this hypothesis is not always acceptable and it was found that important differences could be observed due to the influences of the mooring lines. Figure 13 is showing the important differences of the surge motions for a semisubmersible model, in free and respective moored conditions, using one of the above mentioned chain model for a pre-established pretension. This an important conclusion based on the systematic experimental evaluations using the model of a semisubmersible model (scale 1:64) which was kept on location using, by turn, the 5 chain scaled models for a range of pretension [8].

![Figure 13](image)

**Figure 13.** Influence of initial pretension on surge motions (chain $q_3$).

As a result of the systematic evaluations, according to the experimental program (see figure 2), the influences of the mooring line parameters have been identified, providing important inputs for the design. The figure 15, figure 16 and figure 17 are just three examples showing the influences of pretensions, amplitude of motion of the upper end of the line (fairlead) and chain calibre respectively on the dynamic contribution which is evaluated using the dimensionless coefficient $K_d$

$$K_d = \frac{T_{dn}}{T_{st}}$$  \hspace{1cm} (11)

and graphically explained in figure 14. The dynamic factor is allowing the direct evaluation of the “dynamicity” of the mooring line for the same displacement of the upper end.
The specific computer codes which are used in order to calculate the dynamic forces in mooring lines are using the finite element method (FEM) or the lumped mass method (LMM) which is much more flexible and can be applied for a wide types of composite lines [7], [13].

5. Conclusions
The results are clearly putting in light the necessity to consider the elasticity of the mooring lines in order to have an accurate evaluation of the input data for design purposes. The influence of the elasticity is also depending on the “dynamicity” of the process to be investigated.

A special care has to be allocated when experimental tests have to be performed, the hydro elastic similitude being compulsory. This means that all criteria used for the scaled model definition have to be taken into considered. The different simplifications which can be used for the mooring forces evaluations are strictly linked to the type of phenomena to be analysed in connection with the dynamics of the floating bodies. Consequently, the utilization of certain simplifications during the evaluating process has to be handled with much care in both directions: theoretical calculations and experimental measurements on scaled models as well.

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