Precision electroweak constraints on Universal Extra Dimensions revisited

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Abstract

We reconsider the constraints on Universal Extra Dimensions (UED) models arising from precision electroweak data. We take into account the subleading contributions from new physics (expressed in terms of the $X, Y \ldots$ variables), as well as two loop corrections to the Standard Model $\rho$ parameter. For the case of one extra dimension, we obtain a lower bound on the inverse compactification scale $M = R^{-1}$ of 600 GeV (at 90% confidence level), with a Higgs mass of 115 GeV. However, in contradiction to recent claims, we find that this constraint is significantly relaxed with increasing Higgs mass, allowing for compactification scales as low as 300 GeV. LEP II data does not affect significantly these results.

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I. INTRODUCTION

Theories where the Standard Model fields propagate in large extra dimensions may lead to testable predictions for the direct observation of new particles (the Kaluza Klein excitations of the SM fields) at high energy colliders. However, these new particles may already make their presence known through loop corrections to low energy observables.

The predictions of the Standard Model agree extremely well with the wealth of data accumulated over the years in collider experiments at energies of order 100 GeV and below. This agreement leads to tight constraints on any new physics. One of the more important set of constraints is obtained by the measurements of fundamental parameters of the electroweak theory (like the gauge bosons masses, the $Z$ boson decay partial and total decay widths, or the effective Weinberg mixing angle) by the experiments taking place at the Large Electron Positron (LEP) collider. In particular, for models where only gauge bosons propagate in the bulk [1], the LEP data requires that the masses of the new KK excitations be so large (a few TeV) that is impossible to see them at the Tevatron, and one has to wait for the Large Hadron Collider.

However, for a particular class of models, where all the SM fields propagate in extra dimensions [2], the precision constraints are relatively weak. Such universal extra dimensions (UED) models enjoy the property of KK number conservation (which arises as a consequence of conservation of momentum in the extra dimensions). As a consequence, KK excitations must be produced in pairs (at least at tree level), and their contribution to processes taking place at energies below 100 GeV is relatively suppressed.

The strongest constraints on the compactification scale of UED models still comes from observables measured at the $Z$ pole, in particular, the relation between the $Z$ and $W$ gauge bosons masses. Since a single KK excitation does not couple directly to the SM fermions, the heavy KK states only contribute to the self energies of the gauge bosons, and their contribution can be parametrized in terms of the $S, T, U$ [3] variables. In particular, the constraint on the $T$ parameter is the more stringent one, and taking it into account requires that the mass $M$ of the first level KK excitations be
higher than about 550 GeV \[^1\] (for a Higgs mass of order 100 GeV). For comparison, constraints from different physics (for example, flavor violating processes like \(b \rightarrow s\gamma\) \[^3\], or the muon \(g-2\) \[^4\]) are of the order \(M \gtrsim 280\) GeV or weaker. Also, an important feature of the \(Z\) pole constraints on the compactification scale \(M\) is that it depends on the Higgs mass (as noted by \[^4\]). It turns out that the contributions of a heavy Higgs to the \(T\) parameter have the opposite sign to the leading contribution from KK states, and as such increasing the Higgs mass relaxes the bound on \(M\) considerably; for a Higgs mass of order 500 GeV, one can have values of the compactification scale as low as 300 GeV.

A more recent paper \[^7\] performs the analysis of the LEP data constraints on \(M\) by taking into account data taken above the \(Z\) pole, as well as the two loop electroweak corrections (involving the Higgs boson and the top quark) to the \(\Delta\rho\) parameter in the Standard Model. The conclusion reached in this paper is quite striking: they find that the constraint on the compactification scale increases strongly with the Higgs mass (in contradiction to the result derived in \[^4\]) thus setting a lower limit on \(M\) of about 800 GeV (at 95% CL). This has important consequences for phenomenology, as well as for the cosmological implications of the model. First, it would make the observation of KK excitations impossible at the Tevatron collider. (In certain scenarios, it would be possible to test at Tevatron compactification scales up to 500 GeV \[^8\]). Second, UED models provide a natural candidate for dark matter in the lightest KK excitation (LKP). However, agreement with the experimentally determined value of the current dark matter density requires that the mass of the LKP be in the 500 - 800 GeV range \[^9\] (although this constraint might be relaxed if coannihilation effects are important \[^10\]), which almost brings it into conflict with the precision constraints results.

For these reasons, we consider worthwhile to revisit in this paper the analysis performed in \[^7\]. Our results contradicts the conclusions of \[^7\], while being in rough agreement with \[^4\]). The outline of the paper is as follows. In section II we will re-

\[^1\] Note that the original paper \[^2\] has a factor of 2 missing in the evaluation of \(T\), and the constraint they cite is significantly weaker: \(M \gtrsim 350\) GeV. Also, the above result applies for the case of one extra dimension, which is the scenario discussed in this paper.
view the definitions of the parameters relevant to our analysis, as well as the SM contributions and the experimental constraints on these parameters. In section III, we consider the magnitude of new physics contributions, and derive the constraints on the compactification scale. We find a result in agreement with [4]; that is, for low Higgs mass (115 GeV), we find a slightly stronger constraint $M \gtrsim 600$ GeV (for a top mass value $m_t = 173$ GeV) which gets weaker with increasing Higgs mass, so that values of the compactification scale as low as 300 GeV are still allowed for heavy SM Higgs. We end with conclusions.

II. OBSERVABLES AND STANDARD MODEL PREDICTIONS

We start our discussion by defining the relevant parameters and reviewing the experimental constraints on these obtained from measurements at the LEP collider. We will use the epsilon parameter analysis introduced in [11]. The data on the basic physical observables measured at the Z pole (the ratio of the gauge boson masses, the Z boson decay widths and the forward/backward asymmetries) can be interpreted as constraints on three parameters $\epsilon_1, \epsilon_2, \epsilon_3$. These parameters, while being zero at tree level, get contributions from loop diagrams, due to SM as well as to new physics. Note that the light SM fermions (as well as photon) loops contribute to vertex corrections as well as box diagrams, while heavy particles (like the Higgs boson or the top quark) contribute mainly to the gauge bosons self energies. One can express the contribution of new physics to the $\epsilon_i$ parameters in terms of the vacuum polarization functions of the gauge bosons $\Pi_{XY}$ (with $X, Y$ standing for $B$ and $W_i$) and their derivatives at zero momentum transfer:

\begin{align}
\epsilon_1 &= \epsilon_{1,SM} + \hat{T} - W + 2X \frac{\sin \theta_W \cos \theta_W}{\cos^2 \theta_W} - Y \frac{\sin^2 \theta_W}{\cos^2 \theta_W} \\
\epsilon_2 &= \epsilon_{2,SM} + \hat{U} - W + 2X \frac{\sin \theta_W \cos \theta_W}{\cos \theta_W} - V \\
\epsilon_3 &= \epsilon_{3,SM} + \hat{S} - W + \frac{X}{\sin \theta_W \cos \theta_W} - Y.
\end{align}
with the SM contributions encapsulated in $\epsilon_{i,SM}$, and only new physics contributing to the parameters

\[
\hat{T} = \frac{1}{m_W^2} (\Pi_{W_3W_3}(0) - \Pi_{W_3W_3}^W(0)) \\
\hat{S} = \frac{g'}{g'} \Pi_{W_3B}(0) \\
\hat{U} = \Pi_{W+W-}(0) - \Pi_{W_3W_3}(0) \\
X = \frac{m_W^2}{2} \Pi_{W_3B}(0) \\
Y = \frac{m_W^2}{2} \Pi_{BB}(0) \\
W = \frac{m_W^2}{2} \Pi_{W_3W_3}(0) \\
V = \frac{m_W^2}{2} \left( \Pi_{W_3W_3}(0) - \Pi_{W_3W_3}^W(0) \right)
\]

We use here the notations in [12]; the $\hat{S}, \hat{T}, \hat{U}$ parameters are related to the usual $S, T, U$ defined in [3] by $S = 4 \sin \theta_W \hat{S}/\alpha, T = \hat{T}/\alpha, U = -4 \sin \theta_W \hat{U}/\alpha$.

For the experimental constraints on the parameters $\epsilon_i$ we use the following values [12]:

\[
\epsilon_1 = +(5.0 \pm 1.1) \times 10^{-3} \\
\epsilon_2 = -(8.8 \pm 1.2) \times 10^{-3} \quad \text{with correlation matrix } \rho = \begin{pmatrix} 1 & 0.66 & 0.88 \\ 0.66 & 1 & 0.46 \\ 0.88 & 0.46 & 1 \end{pmatrix}, \\
\epsilon_3 = +(4.8 \pm 1.0) \times 10^{-3}
\]

which are the same as those used in [7]. In order to translate these into constraints on the $\hat{S}, \hat{T}, \hat{U}$... parameters, we need first to evaluate the SM contributions $\epsilon_{i,SM}$. The main contributions to these parameters are due to loops involving the heavy top quark and Higgs boson. At first order, and keeping only the leading terms in $m_t, m_H$, one obtains:

\[
\epsilon_{1,SM} \approx \frac{3G_F m_t^2}{8\pi^2\sqrt{2}} - \frac{3G_F m_W^2}{4\pi^2\sqrt{2}} \tan^2 \theta_W \ln \frac{m_H}{m_Z} \\
\epsilon_{2,SM} \approx -\frac{G_F m_W^2}{2\pi^2\sqrt{2}} \ln \frac{m_t}{m_Z} \\
\epsilon_{3,SM} \approx \frac{G_F m_H^2}{12\pi^2\sqrt{2}} \ln \frac{m_H}{m_Z} - \frac{3G_F m_W^2}{4\pi^2\sqrt{2}} \ln \frac{m_t}{m_Z}
\]

(4)
Note that only $\epsilon_{1,SM}$ has quadratic dependence on the top quark mass. The leading order estimates acquire corrections from vertex and box diagrams due to light fermions, as well as two loop contributions to the gauge boson self energies. The results can parametrized by the following expressions [12]:

$$\epsilon_{1,SM} = \left( +6.0 - 0.86 \ln \frac{m_H}{m_Z} \right) 10^{-3}$$

$$\epsilon_{2,SM} = \left( -7.5 + 0.17 \ln \frac{m_H}{m_Z} \right) 10^{-3}$$

$$\epsilon_{3,SM} = \left( +5.2 + 0.54 \ln \frac{m_H}{m_Z} \right) 10^{-3},$$

evaluated at $m_t = 178$ GeV, and where the dependence on $m_H$ is explicit.

We make a short comment on the first parameter $\epsilon_{1,SM}$. Using the definitions for $\epsilon_i$ in [11, 13], one can write

$$\epsilon_{1,SM} = \delta \rho + M_Z^2 F_{ZZ} (M_Z^2) - \frac{\delta G_{V,B}^V}{G_F} - 4 \delta g_A^{V,B},$$

where for the $A$ gauge boson $F_{AA}(q^2) = (\Pi_{AA}(q^2) - \Pi_{AA}(0))/q^2$, and $\delta G_{F}^{V,B}, \delta g_{A}^{V,B}$ are the vertex and box corrections to the low energy value of the Fermi constant and to the value of the axial couplings at $M_Z$. The important thing about the above expression is that the strong dependence of $\epsilon_{1,SM}$ on the heavy top mass is confined to the $\delta \rho$ parameter, defined as:

$$\delta \rho = \frac{\Pi_{SM}^{ZZ}(0)}{M_Z^2} - \frac{\Pi_{SM}^{WW}(0)}{M_W^2}.$$  

The leading top quark contributions to $\delta \rho$ have been evaluated up to two-loops; one can write

$$\delta \rho = \delta \rho^0 + \delta \rho^{\alpha s} + \delta \rho^{\alpha^2},$$

with the leading contribution $\delta \rho^0 = 3x_t$, $x_t = G_F M_t^2 / 8\pi^2 \sqrt{2}$, appearing in Eq. (4). The most important correction to the leading term comes from the QCD gluon loops [14]

$$\delta \rho^{\alpha s} = -3x_t \frac{2}{9} (\pi^2 + 3) \frac{\alpha_s}{\pi} \approx 3x_t \delta_{QCD}$$

which amounts to about -11% of the leading order contribution ($\alpha_s^2$ corrections have also been computed, and they are small). By contrast, the two-loop order $\alpha^2$ contributions
vary from about -1.2% (for a low Higgs mass) to -1.6% (for a high Higgs mass) of the leading contribution [15]. We then rewrite the first equation in (4):

$$\epsilon_{1,SM} = 3x_t(1 + \delta_{QCD}) + \left(-2.86 - 0.86 \ln \frac{m_H}{m_Z}\right)10^{-3} ,$$

such that the dependence of the leading terms on the top mass is kept explicit.

### III. CONSTRAINTS ON NEW PHYSICS

The massive KK excitations contribute to the oblique parameters $S, T, U$ terms proportional to the mass splittings between the heavy particles at each KK level. Thus, the top quark excitations will contribute terms of order $m_t^2/M^2$, the gauge boson excitations will contribute terms of order $(m_W^2, m_Z^2)/M^2$, while the Higgs excitations will give contributions $\sim (m_H^2, m_W^2, m_Z^2)/M^2$. The top quark contributions are moreover enhanced by a term $m_t^2/m_W^2$. We therefore can expect that for small Higgs mass (of order 100 GeV), the terms associated with the top quark excitations will be dominant, while for larger values of the Higgs mass (which could be of the same order of magnitude as $M$), the terms associated with the Higgs boson excitations may give significant contribution.

Let us consider first the constraints on the scale of new physics $M$ in the approximation where the subleading terms $X, Y, W, Z$ in Eq. (11) are neglected, and the Higgs mass is small. Moreover, let us neglect the $\hat{U}$ parameter; as has been argued in previous analyses (and as it can be seen below), its magnitude is negligible. Then Eqs. (11),(10), evaluated for a value $m_H = 115$ GeV, give the following constraints on the $\hat{S}$ and $\hat{T}$ parameters:

$$\begin{align*}
\hat{T} &= (0.5 \pm 0.8) \times 10^{-3} \\
\hat{S} &= (-0.01 \pm 0.9) \times 10^{-3} ,
\end{align*}$$

(we neglect the theoretical errors on $\epsilon_{1,SM}, \epsilon_{3,SM}$). We use here and in the following a value $m_t = 173$ GeV, (the latest Tevatron analysis indicates $m_t = 172.5 \pm 2.3$ GeV [16]). Note also that with $\hat{U} = 0$, the SM value for $\epsilon_{2,SM}$ is about one sigma away from the experimental average. Due to the correlations between the three parameters, this
leads to a displacement of the mean values for $\hat{S}, \hat{T}$ from (-0.5, -0.2) (derived from Eqs. (1), (3)) to the values given in (11) above.

For purposes of clarity, we comment briefly on the methodology we use for the multiparameter fit. The goodness of the fit is obtained by evaluating the $\chi^2$ function

$$\chi^2 = \sum_{i,j} (\epsilon_i - \mu_i)(\sigma^2_{ij})^{-1}(\epsilon_j - \mu_j),$$

with $(\sigma^2_{ij}) = \sigma_i \rho_{ij} \sigma_j$, the $\mu_i$ and $\sigma_i$ are the mean experimental values and the errors for the observables $\epsilon_i$, and $\rho$ is the correlation matrix. For an analysis taking into account only a subset $A$ of observables, with values of the other observables set to their predicted SM values $\epsilon^0_k$, one derives and uses a new function $\chi^2_A$ such that:

$$\chi^2 = \chi^2_A + \chi^2_{\text{min}}, \quad \chi^2_A = \sum_{i,j \in A} (\epsilon_i - \mu'_i)(\sigma'^2_{ij})^{-1}(\epsilon_j - \mu'_j).$$

Note that the new mean values and errors $\mu'_i$ and $\sigma'_i$ are equal to the old ones only if $\epsilon^0_k = \mu_k$, for $k \notin A$. Moreover, the confidence level limits are defined in terms of the number of parameters involved in the fit; so, for example, for a two parameter fit, a 90%CL limit corresponds to $\chi^2 = 4.61$, while a 99%CL limit corresponds to $\chi^2 = 9.21$ (see [19]).

The leading order contributions of new physics to the $\hat{S}, \hat{T}, \hat{U}$ parameters are given by

$$\begin{align*}
\hat{T} &= \frac{3g^2}{2(4\pi)^2} \frac{m_t^2}{m_W^2} \left(\frac{2}{3} \frac{m_t^2}{M^2}\right) \zeta(2) + \frac{g^2 s_w^2}{(4\pi)^2} c_w^2 \left(\frac{5}{12} \frac{m_H^2}{M^2}\right) \zeta(2) \\
\hat{S} &= \frac{3g^2}{4(4\pi)^2} \left(\frac{2}{9} \frac{m_t^2}{M^2}\right) \zeta(2) + \frac{g^2}{(4\pi)^2} \left(\frac{1}{6} \frac{m_H^2}{M^2}\right) \zeta(2) \\
\hat{U} &= \frac{g^2 s_w^2 m_W^2}{(4\pi)^2} \left(\frac{1}{6} \zeta(2) - \frac{1}{15} \frac{m_H^2}{M^2}\right),
\end{align*}$$

(12)

where $s_w, c_w$ are the sine and cosine of the Weinberg angle, and the $\zeta(2) = \sum 1/n^2 = \pi^2/6$ factor accounts for the sum over KK levels in one extra dimension\(^2\). Note that for two or more extra dimensions, the sum over KK states becomes divergent, and one

\(^2\) The full expressions used for the gauge boson vacuum polarization functions are given in [4]. We have independently evaluated the leading parameters, and we find complete agreement with [4].
FIG. 1: The straight line represents the prediction of the UED model in $\hat{S}, \hat{T}$ plane, for values of the parameter $M$ from 400 GeV to 600 GeV. The $*$ symbols correspond to values $M = 400$ GeV (the upper one), 500 GeV (the middle one) and 600 GeV (the lower one). The ellipses represent the 90% CL (solid line) and 99% CL (dashed line) constraints on the $\hat{S}, \hat{T}$ parameters from Eq. (11).

has to parametrize the contributions from higher energies through effective operators. (However, although such models have interesting physical consequences [17], we do not discuss them here). Even for one extra dimension, unitarity arguments suggest that one should restrict the sum to only the first ten KK levels or less [18]; then the $\zeta(2) \simeq 1.645$ factor should be replaced by 1.55. Also note that the top quark contribution to the $\hat{T}$ parameter is a factor of 2 larger than the result given in the original paper [2], which partly explains the increase in the lower limit on the compactification scale from the $\sim 300$ GeV value given in [2] to about 600 GeV in subsequent works.

We show in Fig. 1 the predictions of the UED model as a straight solid line in the $(\hat{S}, \hat{T})$ plane, evaluated for $m_t = 173$ GeV and $m_H = 115$ GeV. (From the above expressions, we see that the $\hat{U}$ parameter is suppressed by a factor $s_w^2 (m_W/m_t)^2$ with respect to $\hat{S}$; hence we can neglect its variation). The lower end corresponds to a value $M = 600$ GeV, while the upper end corresponds to a value $M = 400$ GeV. The ellipses
are the 90% CL (solid) and 99% CL (dashed) contour lines, evaluated according to Eq. (11). We see that in this approximation, one can exclude values of $M$ lower than 550 GeV, for a Higgs mass $m_H = 115$ GeV. This result is consistent with the one obtained by Appelquist and Yee [4], for the same value of the top quark mass.

One can also use Fig. 1 to estimate what the effect of increasing the Higgs mass will be. Since the Higgs contribution to the $\hat{T}$ parameter appears with opposite sign with respect to the top contribution (and with the same sign for the $\hat{S}$ parameter), the solid line corresponding to the UED prediction will move downward and to the right. Also, from Eqs. (1), (3), one can see that increasing the Higgs mass will increase the mean value of $\hat{T}$ and decrease the mean value of $\hat{S}$; hence the ellipses in Fig. 1 will move upward and to the left. The conclusion is that the constraints on the compactification scale will soften with increased Higgs boson mass.

The expressions for the subleading parameters in the small Higgs mass limit are:

\[
\begin{align*}
V &= \frac{g^2 s_w}{(4\pi)^2} \frac{1}{120} \left( \frac{m_W^2}{M^2} \right)^2 \zeta(4) \\
W &= \frac{3g^2}{2(4\pi)^2} \frac{m_W^2}{M^2} \left( \frac{4}{15} \zeta(2) - \frac{1}{10} \frac{m_t^2}{M^2} \zeta(4) \right) + \frac{g^2}{(4\pi)^2} \frac{m_W^2}{60M^2} \left( -\zeta(2) + \frac{1}{4} \frac{m_H^2}{M^2} \zeta(4) \right) \\
X &= \frac{3gg'}{2(4\pi)^2} \frac{m_W^2}{M^2} \left( \frac{m_t^2}{180M^2} \right) \zeta(4) + \frac{gg'}{(4\pi)^2} \frac{m_W^2}{2M^2} \left( \frac{-m_H^2 + 2m_W^2 + 3m_Z^2}{240M^2} \right) \zeta(4) \\
Y &= \frac{3g^2}{2(4\pi)^2} \frac{m_W^2}{M^2} \left( \frac{34}{135} \zeta(2) + \frac{77}{540} \frac{m_t^2}{M^2} \zeta(4) \right) + \frac{g^2}{(4\pi)^2} \frac{m_W^2}{120M^2} \left( -\zeta(2) - \frac{1}{4} \frac{m_H^2}{M^2} \zeta(4) \right).
\end{align*}
\]

We have kept in these expressions terms up to $(m_{T,Z,H}/M)^4$. Note that in this limit, the third family quark loops do not contribute to the $\hat{U}$ and $V$ parameters. $X$ is also strongly suppressed. As a consequence, these three parameters do not contribute significantly to the constraints on $M$ scale.

The most important subleading parameter is $W$ (the $Y$ parameter is suppressed by a $(s_w/c_w)^2$ factor with respect to $W$). Numerically its magnitude is about half that of the $\hat{S}$ parameter (at $m_H = 115$ GeV). As to its effect on the constraints on the $\hat{T},\hat{S}$ parameters coming from Eqs. (1), note that it contributes with opposite sign from $\hat{T},\hat{S}$ to $\epsilon_1,\epsilon_3$. In consequence, taking $W$ into account would have the effect of moving a point on the solid line in Fig. 1 towards smaller values of $\hat{T},\hat{S}$, that is, roughly parallel.
to the contour lines corresponding to a given $\chi^2$. Then one would expect that it will not have a significant impact on the constraints obtained in the approximation when only the leading oblique parameters are taken into account.

We show in Fig. 2 the resulting constraints (at 90% CL and 99% CL) on the compactification scale $M^{-1}$ as a function of the Higgs mass. The left plot is obtained using the ST analysis from Eq. (11) (the $\hat{U}, X, Y, ...$ parameters are set to zero). The result is similar to the one reported in [4]; the somewhat tighter constraints we obtain may be attributed to the modifications to the SM constraints on the $S, T$ parameters (one notes that the preferred values for $S, T$ have moved significantly towards negative values for the 2004 Particle Data Group numbers as compared with the 2002 PDG numbers, even after taking into account variations due to modifications in the top mass). Also, we found that the effect of taking into account the two loop electroweak corrections to $\delta\rho$ is not significant. This can be understood by noting the fact that the term $\delta\rho^{\alpha^2}$ is of order $10^{-4}$ itself, and moreover that its dependence on the Higgs mass is not
very strong (the parametrization in $^{[15]}$ indicates a roughly logarithmic dependence). Therefore, the two loop term cannot affect the cancellation between the one loop Higgs contribution to $\epsilon_{1,SM}$ and the KK contribution to the $T$ parameter, contrary to the argument given in $^{[7]}$.

In the right panel in Figure 2 we show the constraints on the compactification scale obtained by a $\chi^2$ fit for the three $\epsilon_i$ parameters in Eq. $^{[11]}$, and taking into account the contributions from the $\hat{U}$ as well as the subleading $X, Y, ...$ parameters. (The $90\%$ CL limit corresponds to a $\chi^2 = 6.25$, while $99\%$ CL limit corresponds to a $\chi^2 = 11.34$). As expected, we find very small changes (the constraints are somewhat weakened at small Higgs mass and slightly tightened for large Higgs mass) compared with the result of the $S, T$ analysis.

Independent constraints on the $X, Y, W$ parameters can be obtained by studying $e^+e^-$ collisions at energies higher than the $Z$ pole (the LEP-II data, see, for example, $^{[20]}$). With the parametrization of new physics we use, this data gives the following constraints $^{[12]}$ (independent of the top and Higgs masses):

\[
\begin{align*}
X &= (-2.3 \pm 3.5) \times 10^{-3} \\
Y &= (+4.2 \pm 4.9) \times 10^{-3} \\
W &= (-2.7 \pm 2.0) \times 10^{-3}
\end{align*}
\]

with correlations $\rho = \begin{pmatrix}
1 & -0.96 & +0.84 \\
-0.96 & 1 & -0.92 \\
+0.84 & -0.92 & 1
\end{pmatrix}$. (14)

Since they come from different measurements, there are no correlations between these constraints and the ones on the $\epsilon_i$ parameters $^{[3]}$. It is then straightforward to see that taking (14) into account has a minimal effect on the results discussed above, since the errors on these parameters are significantly larger than the errors on the $\epsilon_i$'s, and the mean values are consistent with very small (or zero) $X, Y, W$.

Finally, we show in Fig. 3 the $90\%$ CL limits on the compactification scale for three different values of $m_t$. Note that these constraints tighten considerably with increased $m_t$. This is due to the increase in the magnitude of the UED prediction for the $\hat{T}$ parameter (which behaves like $m_t^4$), as well as to the change in the SM value for the $\epsilon_{1,SM}$ parameter $^{[10]}$, which increase with $m_t$, thus pushing the experimental constraints on $\hat{T}$ to lower values. Thus, for a top mass $m_t = 178$ GeV, the low Higgs mass constraint on the compactification scale increases to $M \gtrsim 800$ GeV, which is in
FIG. 3: Constraint on inverse compactification scale $M$ as a function of $m_H$, for top mass values $m_t =$ 173 GeV (solid line) 175 GeV (dashed line) and 178 GeV (dotted line). Results are obtained by including the contributions from the subleading terms.

rough agreement with the result of [7] at $m_H = 115$ GeV. However, unlike [7], we find that for increased Higgs mass, the constraint on the compactification scale weakens, such as that for Higgs mass of order 600 GeV, $M$ can be as low as $\sim 400$ GeV.

IV. CONCLUSIONS

We have recalculated the constraints on UED models arising from precision electroweak data, taking into account two loop electroweak correction involving the top quark and Higgs boson to the SM parameters, as well as subleading terms due to new physics. We consider the case with one UED. In this case we found that taking the SM Higgs mass 115 GeV and top quark mass 173 GeV, the lower bound on the inverse compactification scale should be $M \gtrsim 600$ GeV, in agreement with previous results. Also we find that this bound is weakened for increased values of the Higgs mass, in agreement with the results of [4], but in contradiction with those in [7]. As a consequence, the values for $1/R$ preferred by the models explaining dark matter as stable
KK excitations are still compatible with the electroweak precision constraints.

We also find that the constraint on the compactification scale depends strongly on the top quark mass. Part of this dependence is due to the $m_t^4$ behaviour of the $\mathcal{T}$ parameter; an equally important part is due to the $m_t^2$ dependence of the SM $\delta \rho$ parameter. Thus, if the the preferred value for the top mass increases by around 3%, one obtains a roughly 30% strengthening of the bound on the compactification scale $1/R$. However, even with a value of 178 GeV for the top mass, one can have a compactification scale as low as 400 GeV (with a large Higgs mass), thus potentially allowing for the observation of KK excitations at the Tevatron Run II.

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