Radio Emission from Pulsars due to Relativistic Plasma

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Received 2018 November 2; revised 2019 April 11; accepted 2019 May 3; published 2019 June 24

Abstract

Pulsar radio emission is not well understood because of a lack of knowledge about the emission geometry and the plasma process involved. We develop a mechanism for pulsar radio emission that takes into account the detailed viewing geometry of pulsars and the dipolar magnetic field configuration. Using a suitably chosen geometry and plasma parameters, we derive analytical expressions for the Stokes parameters of the radiation field in the frame centered on the neutron star. We have simulated pulse profiles based on our analytical formulation. It seems that we can explain the enhanced radiation and most of the diverse polarization properties of radio pulsars. We have estimated the brightness temperature, which seems to agree with the observations. The polarization angle predicted by the model is in good agreement with the rotating vector model.

Key words: acceleration of particles – polarization – pulsars: general – radiation mechanisms: non-thermal

1. Introduction

Pulsars are highly magnetized ($B \approx 10^8 – 10^{12}$ G), highly gravitating, and highly compact (with interior density comparable to nuclear density or even higher) rotating neutron stars (NSs), which are born in supernovae and found among their remnants (e.g., Crab and Vela pulsars). Most pulsars emit electromagnetic radiation in the form of periodic radio signals with a periodicity exactly equal to their rotation period. Pulsar radiation is coherent, highly polarized, and variable in single pulses. There are at least two valid reasons to believe this: (i) the brightness temperature in the radio band of the pulsar ranges from $10^{25}$ to $10^{32}$ K, and (ii) the signal shows an erratic nature on a timescale far shorter than the pulse width (Ginzburg & Zhelezniakov 1975). In addition, more convincing models of polar gaps, sparks, and microwave radiation have emerged, explaining the observed radio luminosity and the various properties of the integrated pulse profile shape, polarization, and so on (Sturrock 1971; Ruderman & Sutherland 1975).

Coherent synchrotron radiation has been proposed as one possible high-energy emission mechanism in pulsars. The lifetime of a synchrotron photon is shown to be $t_{\text{ph}} \approx E_{\text{ph}}/E_{\text{ph}} = (\gamma_{p} m_{e} c^{2})/P$, where $\gamma_{p} \approx 10^5$ is the Lorentz factor of the primary plasma, $m_{e}$ and $q$ are electron mass and charge, $c$ is the velocity of light, and $P = (2/3)\gamma_{p}^{2} m_{e} c^{3} B^{2}$ is the power radiated by the primary plasma. The classical electron radius $r_{0} = q^{2}/(m_{e} c^{2}) \sim 10^{-13}$ cm, and $\beta$ is the velocity factor of primary plasma particles (Rybicki & Lightman 2004). In the huge ambient magnetic field, the synchrotron photons ($\gamma$-ray) decay into electrons and positrons, and the further pair cascade leads to secondary plasma pulses. For $B \approx 10^{15}$ G and the Lorentz factor of primary plasma $\gamma_{p}$, which ranges from $10^{3}$ to $10^{6}$, we get a lifetime for the synchrotron photon of $\sim 10^{-22}$ s, which is too short. Hence the plasma particles dissipate their perpendicular component of momentum very quickly, and hence they hardly continue gyrating around the magnetic field lines. Therefore the motion of plasma particles almost becomes one-dimensional and curved, which may be approximated as bead-on-wire motion.

Most emission models claim that the source of radio emission is due to the secondary pair plasma, placing the origin in the inner part of the magnetosphere. Based on observations, several authors (e.g., Cordes 1979) believe that the coherent curvature radiation is a promising radio emission mechanism. However, the emission by bunches has been criticized by Melrose et al. (1981) for its failure to explain the formation mechanism and stability of bunches. Benford & Buschauer (1977) have shown that the two-stream instability is the most favorable plasma process, which dominates over other plasma mechanisms in pulsars. They have also indicated that other types of instabilities, such as beam-plasma instability, do not grow rapidly enough to produce sufficient bunching to reach the observed radio luminosity of pulsars. According to Benford & Buschauer (1977), the plasma is perturbed by plane waves, and simple electrostatic streaming instability drives the electron–positron plasma into a phased array of antennas. They have also conjectured that there is a high-frequency steepening of the luminosity curve as a function of frequency arising from linear growth near the pulsar, while a low-frequency turnover appears when the relativistic plasma energy density exceeds the dipolar field energy density, rather than via self-absorption.

Pulsars are known to be the most highly polarized of all known radio sources (Gould & Lyne 1998), and individual pulses sometimes show 100% polarization. Using a well-calibrated polarimeter, one can construct a full set of Stokes parameters. The percentage of polarization can vary greatly depending upon the sampling time as well as the observing frequency (e.g., Gangadhara et al. 1999). High-frequency (>5 GHz) profiles usually show a diminishing degree of linear polarization (Manchester 1971; Xilouris & Kramer 1996). Generally, the polarization angle (PA) exhibits a smooth swing with respect to the rotation phase, like an “S” shaped curve. This behavior was first interpreted by Radhakrishnan & Cooke (1969) in their rotating vector model (RVM). We normally fit the observed PA data with the RVM model to obtain the inclination angle $\alpha$ of the magnetic axis and the line-of-sight impact parameter $\sigma$; however, this method does not produce unique values in most cases. Hence constraining the geometry with the RVM leads to large uncertainties at times and does not
really help one obtain confident estimates about the angles involved.

The circular polarization in pulsar radio profiles is an important property of radio emission mechanisms. We basically observe two types of circular polarization: antisymmetric and symmetric. Antisymmetric circular polarization is found to be correlated with the PA swing (Radhakrishnan & Rankin 1990; Han et al. 1998; Gangadhara 2010). Another intriguing phenomenon in pulsar polarization is orthogonal mode switching. In an observational study of single pulses, it was shown that several sources exhibit rapid jumps (≈90°) in their PA swing (Manchester et al. 1975; Backer & Rankin 1980). Such discontinuities in PA are believed to arise due to the presence of two orthogonal polarization modes. The jump from one “S” curve to another happens when one polarization mode dominates over the other (Gangadhara 1997). Circular polarization also shows similar types of jumps between left-handed and right-handed senses (or vice versa) (Cordes 1978).

For pulsars with a well-ordered PA swing and periods between 0.06 and 3.7 s, it is possible to estimate the height of radio emission, and this is found to be not more than 2000 km in the frequency range 0.43–1.4 GHz (e.g., Blaskiewicz et al. 1991; Gangadhara & Gupta 2001; Gupta & Gangadhara 2003). These authors have also shown that higher frequency radio emission comes from a lower altitude than that at lower frequency. By developing a relativistic phase shift method for estimating the altitude of pulsar radio emission, Gangadhara (2005) has confirmed the above results and has also shown that emission at any given frequency does not come from the same altitude across the pulse window.

In the literature, several mechanisms have been suggested for pulsar radio emission, which can be broadly classified into three groups: (i) plasma antenna mechanisms (Benford & Buschauer 1977), (ii) maser amplification (Melrose 1978; Kazbegi et al. 1991; Luo & Melrose 1992), and (iii) relativistic plasma instabilities (Asseo et al. 1990; Asseo & Porzio 2006). A maser emission mechanism has been proposed as an alternative source of pulsar radiation, which is interpreted as negative absorption. Suppose a small-amplitude electromagnetic (EM) wave is launched very near the polar cap and starts propagating along our line of sight. Now if several discrete sets of columns, filled with materials that have a negative absorption coefficient, are arranged along the direction of propagation, the EM wave amplifies itself continuously while passing through each column, finally producing enhanced radiation. Again, different types of maser mechanisms have been suggested: among them, the two most prominent ones are (i) a free electron maser (Melrose 1989; Rowe 1995) and (ii) emission driven by curvature drift instability (Luo & Melrose 1992). An important advantage of the maser mechanism over relativistic plasma emission is that it can radiate directly escape from the magnetosphere.

The rotationally induced electric field \( \mathbf{E} \) does have some nonzero components along open magnetic field lines (i.e., \( \mathbf{E} \times \mathbf{B} \neq 0 \)). Therefore, the induced electric field accelerates particles and produces curvature photons. Radio emission from the pulsar is generated from secondary plasma, with a detailed description provided by Gurevich et al. (1993). The basic curvature radiation model was developed by Sturrock (1971) and Ruderman & Sutherland (1975), with relativistic effects included by Blaskiewicz et al. (1991). More recently, Gangadhara (2010) developed the incoherent curvature radiation model by including the detailed viewing geometry and the modulation effect, and simulated pulse profiles by integrating over the emission region. Further, perturbation effects such as aberration–retardation and effects of polar cap current were successfully introduced in the theoretical model to explain the observed asymmetry in the phase location of trailing and leading side components relative to the core (Kumar & Gangadhara 2012a, 2012b, 2013; Wang et al. 2012). In their single-particle model, the full set of Stokes polarization profiles were simulated for a given set of geometrical parameters: \( \alpha \) (inclination of the magnetic axis to the rotation axis), \( \sigma \) (line-of-sight impact parameter), and \( r_e \) (dipolar field line constant of the magnetic field). But the incoherent curvature radiation is not sufficient to explain the high brightness temperature of pulsars of \( 10^{20}–10^{25} \) K (Cordes 1979). A model for the coherent radio emission from a relativistic plasma bunch accelerated along a curved trajectory is developed by Sturrock (1971) and Buschauer & Benford (1976). However, their model has not taken into account the viewing geometry in the dipolar magnetic field line topology and lacks the polarization details: linear and circular polarizations. Hence there is a great need to develop the model of coherent curvature radiation in order to deduce the polarization and explain the observations.

In this paper, we develop a model for pulsar radio emission due to collective plasma emission, and estimate the polarization and brightness temperature. We consider the actual dipolar magnetic field lines in a slowly rotating (non-rotating) magnetosphere such that the rotation effects can be ignored. In Section 2 we discuss the basic idea of the proposed model and provide the relevant details of viewing geometry, with detailed mathematical descriptions for the simulation of the pulse profile with respect to pulse longitude. The transformation of electric field to the frame of the NS origin is given in Section 3. The frame of the NS origin is the stationary laboratory frame (i.e., the observer frame \( xyz \); see Figure 18) that has its origin at the center of the NS. The NS spins about its rotation axis \( \Omega \), which is parallel to the \( z \)-axis. In Section 4 the computation of the emission region is shown, and in Section 5 we depict the simulated pulse profiles. In Section 6 we estimate the typical brightness temperature of pulsars. Finally, we provide a discussion in Section 7 and conclusion in Section 8. In Appendix A we give the derivation of coordinates of the emission region in rotation axis frame. In Appendix B we derive the electric field of radio waves emitted by plasma bunch, while in Appendix C we give the radio wave emission geometry in a neutron star origin’s frame. Appendix D provides a method for the estimation of the subpulse width of a pulsar for a collective plasma system. Finally, Appendix E lists the main notations we have used and includes brief descriptions.

## 2. Collective Plasma Radio Emission

Observations suggest that the radio emission of pulsars is coherent and highly polarized. The typical brightness temperature of pulsars ranges from \( 10^{25} \) to \( 10^{27} \) K. Their radio emission is believed to originate from the polar cap region, the boundary of which is demarcated by the footprint of the last open field lines. The very strong magnetic field of pulsars induces large electric fields of the order of \( 10^{11}–10^{14} \) V cm\(^{-1}\) based on rotation. Such a large electric field pulls out charged particles from the NS surface, initiating primary particle motion along open curved magnetic field lines, which results in acceleration and hence produces curvature radiation. The (\( \gamma \)-ray) photons of curvature radiation produce secondary pair plasma by a cascade process. The typical Lorentz factor \( \gamma \) of the secondary plasma is
expected to be around 10–1000 (e.g., Ruderman & Sutherland 1975; Gil 1983; Lyubarskii & Petrova 2000; Gédelin et al. 2002; Gangadhara 2004). Physicists believe that the generation of pulsar radio emission originates mainly from the huge charged cloud of secondary pair plasma or soliton-like charge clumps with smaller transverse dimension, whose power spectrum corresponds to very high brightness temperatures in the radio band. Some literature suggests that modulation instability can be a promising mechanism for bunch formation in the pulsar magnetosphere, but this remains a paradox, because there is no fully understandable theory that can explain both the mechanism of bunch formation and its stability. Collective radio emission due to relativistic plasma constrained to move in dipolar magnetosphere, but this remains a paradox, because there is no promising mechanism for bunch formation in the pulsar band. Some literature suggests that modulation instability can be utilized to explain the mechanism of bunch formation in the pulsar band. Plasma bunches and waves are expected to build up due to an instability process (e.g., two-stream instability). We basically assume that plasma bunches can sustain plane waves, which arise because of the collective motion of the constituent plasma particles (e’ , e’’). Owing to this building-up of waves via wave–wave interaction, the plane waves will attain a steady nonlinear state and finally be partially converted into large-amplitude transverse EM waves and small-amplitude longitudinal waves.

We make the following assumptions:

1. The mean free path of constituent particles of a plasma bunch is much less than the wavelength \( \lambda_0 \) of the emitted radio waves.

2. The plasma bunch can support longitudinal waves. We assume that the transverse dimensions of the bunch \( \xi_0 \) and \( \eta_0 \) are much less than \( \lambda_0 \) (see Figure 18). The plasma waves are confined within the bunch even after traversing a distance of one wavelength or an integer number of wavelengths of longitudinal waves, and find themselves in phase. In other words, they behave like a phased array antenna, correlated in phase, and emit EM waves coherently (Benford & Buschauer 1977). But there is no restriction on \( \xi_0 \) (the bunch length) in principle (i.e., it could be either smaller or larger than the emitted wavelength). A longer plasma bunch intuitively helps accommodate an integer number of plasma waves and hence builds up waves to a large amplitude via wave–wave interaction. The visibility of the angular span of the bunch gets restricted by a relativistic beaming effect; we can see up to an angular span of \( 2/\gamma \) only.

3. Plasma is charge-neutral and non-relativistic in its rest frame, but in the laboratory frame it is highly relativistic.

4. Changes in the radius of curvature of the field lines are negligible during the period of beamed radiation received by the observer.

5. Plasma motion is described by perturbations of current density and charge density, which take the form of sinusoidal functions of space and time. Actually, this assumption has significance in the following limit: the whole bunch is behaving like a fluid and flowing with some average velocity. The sinusoidal oscillation of plasma makes the particles radiate collectively in phase.

6. We represent the plasma motion as plane waves. Realistically this has implications in two limits: (a) the linear growth rate of the wave is very low, and (b) due to wave–wave or wave–particle interaction, linear growth of the phase attains a steady nonlinear state. Finally, the nonlinear wave dissipates and gets partially converted into electromagnetic waves.

7. We assume that plasma bunches possess cylindrical symmetry for the simplicity of calculation.

The derivation of the electric field of radio waves emitted collectively by bunched plasma is given in Appendix B.

3. Transformation of Electric Field to the Laboratory Frame

By assuming that the pulsar or NS magnetic field is dipolar, as is evident from observations, Gangadhara (2004) has derived expressions for the tangent and curvature vectors of magnetic field lines at the emission point by adopting the following conditions. (i) Curvature radiation is emitted along the tangent to the magnetic field line. (ii) To receive radiation, the observer’s line of sight has to be aligned along the tangent to the field line. In the real case of pulsars, the magnetic axis is tilted by an angle \( \alpha \) with respect to the rotation axis and rotated by an angle \( \phi' \). Expressions for the tangent vector and curvature vector are derived in the frame centered on the magnetic axis (the “magnetic axis frame”) by using spherical polar coordinates and Cartesian geometry. Then, by operating tilt and rotation matrices successively on the aforementioned vector quantities, they are transformed to the frame of the rotation axis (lab frame). The details of the geometry of radio emission from pulsars are given in Gangadhara (2004, 2010). For the sake of completeness, we only give the expressions for the curvature vector and binormal vectors in a frame centered on the rotation axis (the “rotation axis frame”). The respective symbols used to represent the unit tangent vector, unit curvature vector, and unit binormal vector in the rotation axis frame are \( \hat{b}_t, \hat{Y}_\kappa, \hat{e} = \hat{b}_n = \hat{b}_t \times \hat{\kappa}, \) respectively. The expressions for the unit curvature vector and unit binormal vector, appearing in Equations (42) and (43), are provided as follows:

\[
\hat{Y}_\kappa = \{k_{t1}, k_{t2}, k_{t3}\}, \quad \hat{e} = \{b_{n1}, b_{n2}, b_{n3}\}, \quad \text{(1)}
\]

where

\[
k_{t1} = \frac{\cos \alpha \cos \phi \cos \phi'(1 + 3 \cos(2\theta)) - \cos \theta \sin \alpha \sin \theta \cos \phi' - (1 + 3 \cos(2\theta)) \sin \phi \sin \phi'}{\sqrt{10 + 6 \cos(2\theta)}},
\]

\[
k_{t2} = \frac{\sin \phi \cos \phi'(1 + 3 \cos(2\theta)) + \sin \phi'(\cos \alpha \cos \phi(1 + 3 \cos(2\theta)) - 6 \cos \theta \sin \theta \sin \alpha)}{\sqrt{10 + 6 \cos(2\theta)}},
\]

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Here, $\phi'$ is the rotation phase or pulse longitude.

The radius of curvature of dipolar magnetic field lines, $r_{\text{rot}} = (1/12)r_s\sin^2\theta\sqrt{(10 + 6\cos(2\theta))^2/(3 + \cos(2\theta))^2}$, and the $(\theta, \phi)$ expressions from Gangadhara (2004) are used to simulate the pulse profiles. Now the binormal vector and curvature vector have three components in the $xyz$ frame; therefore the electric field in the frame of the NS origin can be expressed as

$$
E_{\text{NS}} = \{E_{\text{el}}|k_{r1} + E_{\text{el}b}|b_{m1}, E_{\text{el}b}|k_{r2} + E_{\text{el}b}|b_{m2},
E_{\text{el}b}|k_{r3} + E_{\text{el}b}|b_{m3}\}. 
$$

(2)

Then we have to find the components of $E_{\text{NS}}$ in the plane of the sky. The projected spin axis on the plane of the sky is \(\hat{e}_\parallel = \hat{\Omega}_p = \{-\cos \zeta, 0, \sin \zeta\},\) and \(\hat{e}_\perp = \hat{e}_\parallel \times \hat{n}\) is perpendicular to \(\hat{e}_\parallel\) in the plane of the sky. Therefore we have

$$
E_\parallel = \hat{e}_\parallel \cdot E_{\text{NS}} 
$$

(3)

$$
E_\perp = \hat{e}_\perp \cdot E_{\text{NS}}. 
$$

(4)

The Stokes parameters are constructed as follows by using Equations (3) and (4):

$$
I = E_\parallel E_\parallel^*, \quad Q = E_\parallel E_\perp^* - E_\perp E_\parallel^*, \quad U = 2 \text{Re}[E_\parallel^* E_\perp], \quad V = 2 \text{Im}[E_\parallel^* E_\perp]. \tag{5}
$$

The linear polarization and PA are defined as $L = \sqrt{Q^2 + U^2}$ and $\psi = (1/2)\tan^{-1}(U/Q)$, respectively.

4. Computation of the Emission Region

The relativistically beamed emissions due to collective emission from plasma bunches add up to form the pulsar’s main beam of radiation, which sweeps past the observer in each cycle of rotation. The half beam opening angle of the emission from individual sources is $\theta_b = 1/\gamma$. In Figure 1 for $\alpha = 10^\circ$ and $\sigma = 5^\circ$ we present the emission region within $1/\gamma$ at rotation phase $\phi' = 0^\circ$ of the magnetic axis. Using Equations (8) and (28) from Gangadhara (2010), we have estimated the emission region—Figure 1(a) for emission in the magnetic axis frame and Figure 1(b) for the corresponding region in the rotation axis frame—and hence estimated the resultant or net contributed intensity by integrating over the emission region. By choosing a grid of size $100 \times 100$ on the $2/\gamma$ region, we have estimated the emission region presented in panels (a) and (b) of Figure 1. We observe that, due to geometric effects, the emission regions in the rotation axis frame are smaller in the $(\Theta, \Phi)$ plane than the corresponding

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.pdf}
\caption{Emission regions having angular radius $\sim 1/\gamma$ centered on the line of sight, which is at $\phi' = 0^\circ$: (a) for the magnetic axis frame and (b) for the rotation axis frame (lab frame).}
\end{figure}
regions in the \((\theta, \phi)\) plane of the magnetic axis frame. In Figure 2 we show the distribution of points representing the center of the emission region over the entire range of pulse longitude for both magnetic axis and rotation axis frames. From Figure 2 we can see that this distribution pattern in the rotation axis frame is reversed with respect to that in the magnetic axis frame, due to geometrical mapping. Owing to the compatibility of notation according to Equation (1) of Gangadhara (2005), we have replaced \(\Theta\) with \(\theta'\) while plotting the boundary of the emission regions in Figure 1(b). For the sake of completeness,
Figure 4. Model pulse profile for a pulsar with period 1 s. (a) Intensity $I$ (black continuous line), linear polarization $L$ (blue dashed line), and circular polarization $V$ (red line) plotted against rotation phase $\phi'$. (b) Polarization curve (superposed with RC curve, i.e., the model of Radhakrishnan & Cooke 1969) plotted against rotation phase (deg). The chosen bunch length is $S_0 \approx 0.3r_{\text{LC}}$ and the transverse dimensions are $\xi_0 = \eta_0 = 12$ cm, with $\mu = 0^\circ$. The current density is $10^{14}$ esu cm$^{-2}$ s$^{-1}$, the plasma density in the comoving frame is $n' = 1.24 \times 10^{10}$ cm$^{-3}$, and the corresponding density in the lab frame is $n_e = \gamma n' = 6.94 \times 10^{12}$ cm$^{-3}$ for a Lorentz factor $\gamma = 560$. The radius of the light cylinder $r_{\text{LC}} = 4.7 \times 10^9$ cm for a pulsar with period 1 s at a distance $R = 1$ kpc.

Figure 5. (a) $I, L, V$ vs. rotation phase $\phi'$; (b) polarization angle (superposed with the RC curve). Geometrical parameters are mentioned in (a), and the rest of the parameters are the same as in Figure 4.
we have given the formulation for the transformation of the emission region from the magnetic axis frame to the rotation axis frame in Appendix A. We observe that when $\alpha$ is small the emission regions in the two frames look nearly same, whereas when $\alpha$ is large the emission region in the rotation axis frame (lab frame) becomes smaller than that in the magnetic axis frame, and this is evident from Equations (14) and (15). It is an effect of geometric mapping on the emission region in the $(\theta, \phi)$ plane of the magnetic axis frame to the corresponding region in the $(\Theta, \Phi)$ plane of the rotation axis frame. The rotation axis is always perpendicular to the equatorial plane of the pulsar, whereas the magnetic axis is inclined.

The radii of curvature of field lines at the emission point are plotted in Figure 3 for different $\alpha$: panel (a) is for positive $\sigma$ and panel (b) is for negative $\sigma$. Though the difference between the panels is minimal at $\sigma = \pm 2^\circ$, it increases at $|\sigma| \geq 5^\circ$. According to the polar cap model of Ruderman & Sutherland (1975), the electric field due to curvature radiation is inversely proportional to the radius of curvature of magnetic field lines. Hence the profiles for negative $\sigma$ become broad compared to those for positive $\sigma$.

5. Simulation of Pulse Profiles

We have simulated pulse profiles with the help of Equations (3) and (4) and the formulae for the geometry of pulsar radio emission given by Gangadhara (2004, 2010). For simulation, the chosen bunch length is $S_0 \approx 0.3\eta_{LC}$ with transverse dimensions $\xi_0 = \eta_0 \approx 12$ cm, and $\mu = 0^\circ$. The current density is $10^{14} \text{ esu cm}^{-2} \text{ s}^{-1}$, the plasma density in the comoving frame is $n' = 1.24 \times 10^{10} \text{ cm}^{-3}$, and the corresponding density in the lab frame is $n_\rho = \gamma n' = 6.94 \times 10^{12} \text{ cm}^{-3}$ for a Lorentz factor $\gamma = 560$. The radius of the light cylinder is $r_{LC} = 4.7 \times 10^9 \text{ cm}$.

For a pulsar with period 1 s at a distance $R = 1 \text{ kpc}$. The plasma wave frequency is $\omega_0 = 6.2 \times 10^9 \text{ rad s}^{-1}$, the wavenumber is $k_0 = 2\pi/\lambda_0$, and $\lambda_0 = 30$ cm. In the comoving frame the plasma wave frequency is $\omega'_0 = 4\pi \times 10^4 \text{ rad s}^{-1}$ and the phase velocity is $\omega'_0/k_0 = 3.43 \times 10^4 \text{ cm s}^{-1}$.

We have simulated pulse profiles for different geometrical parameters $\alpha$, $\sigma$, and field line constant $r_\rho$, as shown in Figures 4–7. For smaller $\alpha$, pulse profiles are usually broad compared with those at larger $\alpha$ because the radius of curvature encountered by the line of sight is smaller at smaller $\alpha$, as is evident from Figure 3. The intensity of radio emission is roughly inversely proportional to the square of $\rho$ (Ruderman & Sutherland 1975). Another significant feature of the pulse profiles is that they show broadening for negative $\sigma$ compared to positive $\sigma$ for the same $\alpha$ and $r_\rho$ combinations. This is evident from Figure 3(b), as the curves become flatter than in Figure 3(a) for the same $\alpha$ and $r_\rho$ combinations. Since the parameters $I$ and $L$ are inversely proportional to the square of the radius of curvature, the Stokes profiles corresponding to negative $\sigma$ do not fall quickly. For negative $\sigma$, line of sight gets aligned with the tangent vector at smaller polar angle $\theta$, which corresponds to a smaller height and hence smaller radius of curvature. Again, for smaller $\alpha$ with same set of $\sigma$ and $r_\rho$ (see Figure 3(a)), the slopes of profiles of radii of curvature are smaller; hence smaller $\alpha$ generates sporadically broadened pulse profiles, because $I$ and $L$ are inversely proportional to the square of the radius of curvature. When the field lines are viewed edge-on, the circular polarization goes to zero.

By treating $\mu$, the angle between the plane of the field line and the line of sight, as a free parameter and choosing bunch length $S_0 = 0.2\eta_{LC}$, with the transverse dimensions $\xi_0 = \eta_0 = 3$ cm, phase $\phi' = 0^\circ$, and the rest of the parameters...
the same as mentioned above, we have generated pulse profiles and plot them in Figures 8 and 9. We observe that the change in sign of circular polarization when the line of sight cuts the plane of magnetic field lines is determined by the sign of $\sigma$, i.e., whether the line-of-sight impact angle is negative or positive.

By considering multiple bunches at different rotation phases we simulated multicomponent pulses profiles and plot them in Figures 10–13. Using $\alpha = 10^\circ$, $\sigma = 5^\circ$, $\mu = -0.007$, $r_v = 20r_{LC}$, bunch length $S_0 = 0.3r_{LC}$, and transverse dimensions $\xi_0 = \eta_0 = 3$ cm, a three-component profile is simulated and presented in Figure 10. The bunch emitting the core component is chosen at $\phi' = 0^\circ$ and the conal outriders at $\phi' = \pm 75^\circ$; the field line constant is $r_v = 18r_{LC}$. The bunch length chosen for the subpulse component is 0.3 times the bunch length corresponding to the core component. The transverse dimensions chosen for the subpulse are the same as for the main pulse component. Other parameters are chosen to be same as in Figure 4. These profiles are similar to single pulses in pulsar observations when the line of

![Figure 7](image_url)  
**Figure 7.** (a) $I, L, V$ versus rotation phase $\phi'$; (b) polarization angle (superposed with the RC curve). Geometrical parameters are mentioned in (a), and the rest of the parameters are the same as in Figure 4.

![Figure 8](image_url)  
**Figure 8.** Plot of $I, L, V$ vs. angle $\mu$ in degrees. The bunch length chosen here is $S_0 = 0.2r_{LC}$ (the radius of the light cylinder is $4.77 \times 10^9$ cm for a pulsar with rotation period 1 s) and the transverse dimensions are $\xi_0 = \eta_0 = 3$ cm. Others parameters are the same as mentioned in Figure 4.
sight encounters field lines at any arbitrary values of $\mu$. Similarly in Figure 11 a three-component profile is presented for $\alpha = 10^\circ$, $\sigma = -5^\circ$, and $\mu = 00.007$. Similarly simulated five-component profiles are plotted in Figures 12 and 13.

The pulse profiles in Figures 14–17 are simulated by integrating over the emission region as well as over $\mu$ within the $\pm 1/\gamma$ region to include the net emission coming from all those field lines from the polar cap that contribute to the beaming effect. Figures 14 and 15 show one-component pulse profiles and Figures 16–17 show three-component pulse profiles. To produce three-component profiles we have used three individual coherent sources separated in phase by $\pm 12^\circ$
with respect to the meridional plane. It is also interesting to see that our simulated pulse profiles show a low degree of symmetric circular polarization, which is induced intrinsically by the modulation effect of the bunch. In the coherent case this scenario might be expected because a cylindrically shaped bunch is encompassed by series of parallel magnetic field lines to maintain the coherence condition. As the field lines are parallel and spaced compactly, the electric fields align in the direction of the curvature vector and add up in phase. In other words, magnetic field lines adjacent to the center of momentum of the bunch are behaving like a single entity for the coherent case although they are collective in nature. So an observer will find maximum circular polarization near the center of the bunch, and this will slowly decrease along the side of bunch boundary as field lines start to deviate from strict parallelism. However, in the regime of weak coherence each individual field line will behave like a single entity, and circular polarization profiles will resemble those in Figures 8 and 9. In the region where magnetic field lines are not parallel to each other emission will be in the weak coherence limit. So it is expected that the circular polarization profile will be the same as in the case of incoherent curvature radiation. In the coherent radiation limit the observer will not be able to separate two adjacent planes of magnetic field lines individually, but for the incoherent case the observer can distinguish this. We know that in the case of incoherent curvature radiation, circular polarization does show a mostly antisymmetric nature (Gangadhara 2010), and in some cases it switches to a single sense depending upon perturbation effects such as aberration–retardation (Kumar & Gangadhara 2013).

6. Brightness Temperature of Radio Pulsars

The brightness temperature or radiance temperature is the temperature that a blackbody in thermal equilibrium with its surroundings would have to reproduce the observed intensity of a graybody object at a frequency \( \nu \). This concept is used in radio astronomy and sometimes in planetary science too. The brightness temperature is not a temperature as ordinarily understood. It characterizes radiation, and depending upon the mechanism of radiation, it can differ significantly from the physical temperature of a radiating body. Non-thermal radiation can have a high brightness temperature: for example, observed pulsar luminosity corresponds a high brightness temperature that ranges from \( 10^{25} \) to \( 10^{32} \) K roughly. The brightness temperature can be derived from Planck’s law in the limit of long wavelengths.

The relation between the Stokes parameter \( I \) and the flux density \( S_\nu \) is given by (see Rybicki & Lightman 2004, Equation (2.34))

\[
S_\nu = \frac{c}{T} |E(\omega)|^2 = \frac{c}{T} I,
\]

where \( T \) is defined as the average timescale over which electric field shows a significant variation, which is determined by analyzing the spectrum. In other words, one can infer the correct value of \( T \) by analyzing a portion of the signal so that a suitable frequency resolution \( \Delta \nu \approx 1/T \) can be obtained. We
estimate the brightness temperature using the formula given by Cordes (1979):

\[
T_b = \frac{1}{2k_B} \frac{\lambda^2 S_v}{(c \Delta t/R)^2},
\]

(7)

where \(\lambda\) is the wavelength of the emitted radio signal, \(\Delta t\) is the same as \(T\), and \(R\) is the distance between observer and pulsar. Literature (e.g., Cordes 1979) suggests the value of \(\Delta t\) is of the order of a few milliseconds for subpulses and 10 \(\mu\)s to 1 ms for micropulses. For main pulses we have taken the average duty

Table 1

| Pulsar Name | Period (s) | Distance (kpc) | \(S_{900\ MHz}\) (mJy) | \(S_{1.4\ GHz}\) (mJy) | \(W_{\beta}\) (ms) | \(T^{\Delta t=W_{\beta}}_{900\ MHz}\) (K) | \(T^{\Delta t=W_{\beta}}_{1.4\ GHz}\) (K) |
|-------------|------------|----------------|-------------------------|-------------------------|----------------|--------------------------------|--------------------------------|
| B0138+59    | 1.223      | 2.30           | 13                      | 4.5                     | 104.2          | 3.84 \times 10^{17}              | 2.69 \times 10^{18}              |
| B0809+74    | 1.292      | 0.43           | ...                     | 10                      | 89.5           | 4.05 \times 10^{16}              | ...                              |
| B0818-13    | 1.238      | 1.90           | 14                      | 6                       | 35.6           | 3.00 \times 10^{18}              | 1.69 \times 10^{19}              |
| B2044+15    | 1.138      | 3.34           | ...                     | 1.7                     | 50.1           | 1.32 \times 10^{18}              | ...                              |
| B2154+40    | 1.525      | 2.90           | ...                     | 17                      | 114.2          | 1.92 \times 10^{18}              | ...                              |
| B0628-28    | 1.244      | 0.32           | 57                      | 31.9                    | 119.7          | 4.00 \times 10^{16}              | 1.73 \times 10^{17}              |
| B0834+06    | 1.275      | 0.19           | 5                       | 5                       | 33.9           | 2.75 \times 10^{16}              | 6.66 \times 10^{16}              |
| B1133+16    | 1.187      | 0.35           | 37                      | 20                      | 41.8           | 2.46 \times 10^{17}              | 1.10 \times 10^{18}              |
| B1237+25    | 1.381      | 0.84           | 26                      | 23.2                    | 60.6           | 7.82 \times 10^{17}              | 2.12 \times 10^{18}              |
| B2111+46    | 1.014      | 4.00           | 44                      | 19                      | 152.8          | 1.47 \times 10^{17}              | 8.22 \times 10^{17}              |

Note.

* Estimated based on the data from the ATNF catalog.
cycle of a pulse $\Delta t \approx T$, which ranges from 0.1 to 0.5 s in the case of normal pulsars.

In Table 1, we have estimated the brightness temperature of a few pulsars (10) based on data from the ATNF catalog. Column 1 gives the pulsar's name, column 2 its period, column 3 the distance between pulsar and observer, columns 4 and 5 the flux density at 900 MHz and 1.4 GHz respectively, column 6 the pulse width at 10% intensity level, and columns 7 and 8 the estimate of brightness temperature.

In Table 2, we have estimated the brightness temperature for multicomponent profiles purely from theory. If we compare Tables 1 and 2, we can infer that theoretical estimates of brightness temperature are roughly comparable with the observational data. However, some differences do exist, which

![Figure 13](attachment:image.png)

**Figure 13.** (a) $I, L, V$ plotted vs. rotation phase; (b) polarization angle superposed with RC curve plotted vs. rotation phase. Here everything is the same as in Figure 12 except $\sigma = -6^\circ$. As $\sigma$ is negative, the measurement direction of $\mu$ is counterclockwise. The chosen value of $\mu$ is 0°007.

**Table 2**

| Figure Number and Pulse Component | $\phi'_{\deg}$ | $\delta t = (P/360)\phi'$ | $l_{\text{max}}$ ($10^{-34}$ erg cm$^{-3}$ Hz$^{-1}$ s) | $S_{\nu} = (c/\delta t)l_{\text{max}}$ (Jy) | $T_{b}$ (K) |
|----------------------------------|---------------|------------------|-----------------------------|------------------------|----------------|
| Figure 10, core                  | 80            | 0.220            | 4.68                        | 6.31                   | $4.45 \times 10^{19}$ |
| Figure 10, conal component       | 50            | 0.140            | 0.69                        | 1.49                   | $2.68 \times 10^{19}$ |
| Figure 12, core                  | 20            | 0.056            | 0.99                        | 5.34                   | $6.01 \times 10^{20}$ |
| Figure 12, 1st conal component   | 10            | 0.028            | 0.45                        | 4.86                   | $2.19 \times 10^{21}$ |
| Figure 12, 2nd conal component   | 8             | 0.022            | 0.29                        | 3.91                   | $2.75 \times 10^{21}$ |
| Figure 14, core                  | 80            | 0.220            | $1.68 \times 10^{-4}$      | $2.16 \times 10^{-4}$  | $1.50 \times 10^{15}$ |
| Figure 16, core                  | 10            | 0.028            | $4.85 \times 10^{-3}$      | $5.23 \times 10^{-2}$  | $2.34 \times 10^{19}$ |
| Figure 16, conal component       | 5             | 0.014            | $1.58 \times 10^{-3}$      | $3.41 \times 10^{-2}$  | $6.10 \times 10^{19}$ |

**Note.**

We have used Cordes’ formula to estimate brightness temperature. We chose pulse period $P = 1$ s, distance $R = 1$ kpc, and emitted frequency $\nu = 1$ GHz for simulating pulse profiles. The second column gives the pulse width in the rotation phase, the third column gives pulse width in time, the fourth column gives the maximum value of profile $I$, the fifth column gives the flux density, and the sixth column gives the brightness temperature.
could be due to different parameters. For example, the peak frequency of radio emission chosen in our model is 1 GHz, whereas observational flux data are at 900 MHz and 1.4 GHz. Apart from that, the pulsar period is also slightly different, which can affect the radius of the light cylinder, and hence the dipolar field line constant as well. Another important discrepancy between observation and theory is the difference in pulse width, which can vary significantly depending upon geometrical parameters such as \( \alpha \), \( \sigma \), and \( r_p \), and bunch dimensions. So without knowing the details of pulse period, distance, inclination angle of the magnetic axis, line-of-sight impact angle, \( r_z \), and possibly plasma parameters also, any

**Figure 14.** A single-component profile: (a) \( I, L, V \) vs. rotation phase; (b) polarization angle \( \psi \) vs. rotation phase. For simulating profiles we integrated over the emission region over \( \mu \) from \(-1/\gamma \) to \( 1/\gamma \), where Lorentz factor \( \gamma = 560 \). The parameter chosen here is \( r_e = 20r_{LC} \). The bunch-related parameters are \( S_0 = 0.3r_{LC} \), \( \xi_0 = \eta_0 = 3 \) cm and the frequency of emitted radiation chosen here is \( \nu_{em} = 1 \) GHz.
fine-tune adjustment between theory and observation seems to be difficult.

In Table 3 we have estimated brightness temperature for fully linearly polarized profiles from the theoretical point of view. We see that brightness temperatures in Table 3 are a bit higher than those reported in Table 2. This is because the dipolar field line constant \( r_e \) chosen for profiles in Table 2 is a bit larger than for Table 3.

For our calculations we have assumed the distance to the pulsar as \( R_0 = 1 \) kpc, radio wavelength \( \lambda = 30 \) cm. The peak value of \( I \) in Figure 4 is around \( 5.91 \times 10^{-28} \) erg cm\(^{-3}\) Hz\(^{-1}\) s. If we plug in all these values in Equation (7), then we get \( T_b \sim 5.61 \times 10^{25} \) K. Similarly the Figure 6 gives a peak value of \( 1.86 \times 10^{-29} \) erg cm\(^{-3}\) Hz\(^{-1}\) s, which corresponds to \( T_b \) around \( 3.35 \times 10^{25} \) K. We observe that our calculations of brightness temperature based on \( I \) from Figures 10, 12, 14, and 16 are quite close to the values obtained in Table 1.

We have estimated the brightness temperature for subpulses (conal components) also in Table 2, which shows that their brightness temperature is comparatively greater than that of the

---

**Table 3**

| Figure  | \( \delta \phi' \) (deg) | \( \delta t = (P/360)\delta \phi' \) (s) | \( I_{\text{max}} \) (10\(^{-24}\) erg cm\(^{-3}\) Hz\(^{-1}\) s) | \( S_{\nu} = (c/\delta t)I_{\text{max}} \) (Jy) | \( T_b \) (K) |
|---------|-----------------|-------------------------------|---------------------|-----------------------|-----------|
| Figure 4 | 80              | 0.22                          | \( 5.91 \times 10^{-4} \) | \( 7.97 \times 10^{5} \) | \( 5.61 \times 10^{25} \) |
| Figure 6 | 30              | 0.08                          | \( 1.86 \times 10^{-5} \) | \( 6.69 \times 10^{5} \) | \( 3.35 \times 10^{25} \) |

Note.  
\(^a\) We used Cordes’ formula to estimate brightness temperature. We chose pulse period \( P = 1 \) s, distance \( R = 1 \) kpc, and emitted frequency \( \nu = 1 \) GHz for simulating the pulse profiles. We have used the maximum value of parameter \( I \) appearing in Figures 4 and 6 to compute the flux density and hence the brightness temperature. The second column gives the pulse width in the rotation phase, the third column gives the pulse width in time, the fourth column gives the maximum value of profile \( I \), the fifth column gives the flux density, and the sixth column gives the brightness temperature.
core component, as their duty cycles are shorter. We have not simulated micropulses but we can guess that they can show even higher $T_b$ as their duty cycles are of the order of microseconds, and $T_b$ is inversely proportional to the square of $\Delta t$. So the micropulses are expected to show even higher $T_b$ than the subpulses.

7. Discussion

Our model includes the actual dipolar field lines and viewing geometry, which use the accurate coordinates of the emission region and also estimate the polarization of the emitted radiation. The model of Buschauer & Benford (1976) considers an arbitrary trajectory of the source and does not deduce the polarization of emitted radiation, but it is an important property of pulsar radio emission. We have simulated the pulse profiles (see Figures 4–17), and estimated the brightness temperature, which is found to be comparable to the observed values of $\sim 10^{25}$ K.

The PA swing predicted by our model exactly matches that of the RVM (Radhakrishnan & Cooke 1969; Komesaroff 1970), implying that radio emission is generated from the dipolar magnetic field. The linear polarization is comparable with the total intensity, and the circular polarization is nearly zero because we have estimated the radiation electric field in the plane of magnetic field lines ($\hat{n} \cdot \hat{v} = 1$) for most cases. In actual observations we find some portion of unpolarized emission, which could be due to the superposition of radiation fields emitted from sources that are out of phase. The simulated pulse profiles (shown in Figures 4–7) show the profile broadening and narrowing features due to geometric effects.

From our analysis (especially for simulating profiles in Figures 4–7) we have chosen the longitudinal dimension of the collective system to be $\leq 0.3 r_{LC}$ and the transverse dimensions $\zeta_0 = \eta_0 = 12$ cm. Transverse dimensions are carefully chosen to be less than half the emitted radio wavelength ($\lambda_0 = 30$ cm) to satisfy the conditions of coherency. It is evident from our theoretical analysis that pulsars with smaller $\alpha \sim 2^\circ$ are capable of producing highly energetic electromagnetic radio emission, so as to reach very high brightness temperatures of the order of $10^{30}$ K. But for $\alpha = 10^\circ$ and $45^\circ$ the brightness temperature reaches of the order of $10^{26}$ and $10^{25}$ K.
respectively for the same dimensions of the collective system. This analysis clearly indicates that pulsars with larger $\alpha$ are not efficient enough to produce highly energetic radio emission, unlike those with smaller $\alpha$. We have found that if we set the length of the collective system at least to $0.3r_{LC}$, then it is sufficient to reach the observed brightness temperature for all possible $\alpha$, ranging from $1^\circ$ to $80^\circ$. Second, we have simulated subpulse component profiles (see Figures 11–13) by choosing a different bunch length ($S_0 \sim 0.2r_{LC}$ and transverse dimensions $\xi_0 = \eta_0 \sim 3$ cm). We have produced multicomponent profiles by using the collective emission property of source and pulsar emission-beam geometry. Although many reports have suggested that source modulation is responsible for subpulse generation in the pulsar magnetosphere, they have not implemented this concept to simulate a multicomponent pulse profile so far, but rather have used a Gaussian function in both polar and azimuthal directions around the magnetic axis as a source of perturbation.

Our estimates of brightness temperature from multicomponent profiles range from $10^{19}$ to $10^{21}$ K, which is quite close to the values existing in the literature. But for fully linearly polarized profiles, we are able to reach a brightness temperature of the order of $10^{26}$ K. Although a lot of work has been done on the mechanism of coherent radio emission from pulsars, the brightness temperature has not been estimated from a purely theoretical perspective. Researchers have rather tried to generate a multi-segment broken power law for pulsar radio spectra and radio luminosity by assuming either a different possible scenario of the family of magnetic field trajectories associated with curvature radiation or a different plasma dispersion relation associated with the nonlinear plasma process. Though the chosen parameters ($P = 1$ s, $\nu = 1$ GHz, and $R = 1$ kpc) for our theoretical estimates are slightly different from those reported in Table 1 we can see that theoretical and observational estimates of brightness temperature are roughly matching. We explored polarization properties of radio pulsars without introducing any perturbation effect (such as aberration–retardation, polar cap current etc.) from our formulation, and the PA swing agrees well with the famous RVM.

The propagation effects due to magnetized pair plasma on the electromagnetic waves are not considered here, but for the
sake of completeness we include some illuminating discussion to interpret the effect of plasma wave propagation in the pulsar magnetosphere. In our model the propagation vector of an electromagnetic wave is parallel to the magnetic field $B$. For this case two types of electromagnetic waves exist: $R$ waves and $L$ waves (see Chen 1984, Figure 4.39). The dispersion relations for the $R$ waves and $L$ waves are, respectively, (see Chen 1984, Equations (4.116) and (4.117))

$$n_R^2 = \frac{c^2}{v_\phi^2} - \frac{\omega_p^2/\omega^2}{1 - \omega_c/\omega}, \quad (8)$$

$$n_L^2 = \frac{c^2}{v_\phi^2} - \frac{\omega_p^2/\omega^2}{1 + \omega_c/\omega}, \quad (9)$$

where $n_{R,L}$ are the refractive indices, $v_\phi$ the phase velocity of emitted electromagnetic waves, $\omega_p$ the plasma frequency, and $\omega_c$ the cyclotron frequency. We discuss four cases below.

Case I: $R$ wave $(\omega < \omega_p < \omega_c)$

Consider the radius of the NS $R_{NS} = 10 \text{ km}$, magnetic field $B_0 = 10^{12} \text{ G}$, emission height $r_{em} = 400 \text{ km}$, Lorentz factor $\gamma = 560$, particle density in comoving frame $n' = 1.2 \times 10^{10} \text{ cm}^{-3}$. Then the particle density in the lab frame $n_p = \gamma n' = 6.72 \times 10^{12} \text{ cm}^{-3}$, magnetic field at emission height $B_{em} = B_0(R_{NS}/R)^3 = 1.6 \times 10^6 \text{ G}$, synchrotron radiation frequency $\omega_s = qB_{em}/(\gamma m_e c) \sim 4.9 \times 10^{11} \text{ rad s}^{-1}$, plasma frequency $\omega_p = \sqrt{4\pi n_p q^2/m_e} \sim 1.5 \times 10^{11} \text{ rad s}^{-1}$. For the emitted radiation frequency $\omega = 6.2 \times 10^9 \text{ rad s}^{-1}$ Equation (8) reduces to

$$n_R = \sqrt{1 + \omega_p^2/\omega \omega_s} \sim 2.9$$

and the corresponding magnitude of the propagation vector of the $R$ wave is $k_R = n_R \omega/c \sim 0.6 \text{ cm}^{-1}$. The propagation vector is positive so the wave can propagate. For this case the phase velocity is $v_R = c/n_R \sim 10^{10} \text{ cm s}^{-1}$.

Case II: $L$ wave $(\omega < \omega_p < \omega_c)$

Equation (9) can be rewritten in approximate form as

$$n_L = \sqrt{1 - \omega_p^2/\omega \omega_s} \sim 2.5i$$

and the corresponding magnitude of the propagation vector is $k_L \sim n_L \omega/c \sim 0.5i \text{ cm}^{-1}$. The propagation vector is complex so the wave will slowly attenuate and lose all of its energy.

Case III: $R$ wave $(\omega_p < \omega < \omega_c)$

For particle density $n_e = 10^{10} \text{ cm}^{-3}$ in the lab frame, the corresponding plasma frequency will be $\omega_p = 5.6 \times 10^9 \text{ rad s}^{-1}$ and refractive index $n_R = \sqrt{1 + \omega_p^2/\omega \omega_s} \sim 1.002$, and the corresponding magnitude of the propagation vector is $k_R = n_R \omega/c \sim 0.2$. The propagation vector is a real number so the wave can propagate. The phase velocity of the $R$ wave for this case will be $v_R = 2.985 \times 10^{10} \text{ cm s}^{-1}$, which is approximately equal to the velocity of light $c$.

Case IV: $L$ wave $(\omega_p < \omega < \omega_c)$

For particle density $n_e = 10^{10} \text{ cm}^{-3}$ the refractive index for the $L$ wave is $\sim 0.9948$. The magnitude of the propagation vector becomes $\sim 0.21 \text{ cm}^{-1}$ and the phase velocity is $\sim 3.02 \times 10^{10} \text{ cm s}^{-1}$. We can clearly see three distinct regions (see Chen 1984, Figure 4.39). $\omega_c$ and $\omega_p$ are the cutoff frequencies of $L$ and $R$ waves, whose expression again can be found (see Chen 1984, Equation (4-108)). The $R$ wave has a stop band at frequencies $\omega_c$ and $\omega_R$, whereas the $L$ wave does not have any stop band. The $L$ wave can propagate beyond the frequency $\omega_c$. From case II above it is evident that the $L$ wave cannot propagate below the frequency $\omega_c$, which is pictorially manifested in a figure (see Chen 1984, Figure 4.39). We strongly believe that this will not affect the pulse profile structure and the peak magnitude of intensity by a significant amount as long as the emitted angular frequency remains fixed, but the propagation effect can play a significant role in changing the sense of circular polarization of pulse profiles.

### 8. Conclusion

We summarize some of the salient features of our models below.

1. The PA swing predicted by our model is consistent with the RVM.
2. The pulse profiles for small inclination angle $\alpha$ are usually broader than those for larger values of $\alpha$. Similarly the line-of-sight impact angle $\alpha$ makes the profile become broad when it is negative and narrow when it is positive.
3. In collective radio emission due to the plasma process the broadening and narrowing effects are much more sensitive to inclination angle than in the incoherent model.
4. Our model predicts a brightness temperature of pulsars $\sim 10^{25} \text{ K}$, comparable to the values deduced from observation.
5. Our model also predicts the symmetric type of circular polarization profiles, which we believe to be induced intrinsically by the nature of bunches. But this vanishes when it is computed exactly in the plane of magnetic field lines.
6. Our model also predicts that the circular polarization corresponding to the core component is greater than that of the conal components, in agreement with the literature.

We thank the anonymous referee for many useful comments and suggestions.

### Appendix A

#### Formulation of the Emission Region in the Rotation Axis Frame

Here we show the transformation of magnetic colatitude ($\theta$) and magnetic azimuth ($\phi$) from the magnetic axis frame to the rotation axis frame. It is well known that magnetic dipole field lines are governed by the field line equation:

$$r = r_e \sin^2 \theta, \quad (10)$$

where $\theta$ is the polar angle and $r$ the distance from the origin.

The parameter $r_e$ is the dipolar field line constant, which is a distance from origin to the point where the field line intersects the magnetic equatorial plane, which is at $\theta = \pi/2$ (Alfvén & Falthammar 1963).

The position vector of an arbitrary point on the magnetic field line in the magnetic axis frame is

$$r = r \{ \sin \theta \cos \phi, \ \sin \theta \sin \phi, \ \cos \theta \}. \quad (11)$$

Now, when the magnetic axis is rotated and inclined with respect to the rotation axis by angles $\phi'$ and $\alpha$, respectively, we
construct a matrix Λ, which is the product of the rotation matrix \( M_{\text{rot}} \) and the inclination matrix \( M_{\text{inc}} \). The details of rotation and inclination matrices can again be found in Gangadhara (2004). In order to get an expression for the position vector in the rotation axis frame, we have to multiply Equation (11) by 
\[
\Lambda = M_{\text{rot}} \cdot M_{\text{inc}}.
\]
So we get
\[
r_b = \Lambda \cdot r = r \{ r_{bx}, r_{by}, r_{bc} \},
\]
where
\[
\begin{align*}
    r_{bx} &= \cos \theta \sin \alpha \cos \phi' + \sin \theta (\cos \alpha \cos \phi \cos \phi' - \sin \phi \sin \phi'), \\
    r_{by} &= \sin \theta \sin \phi \cos \phi' + \sin \phi' (\cos \theta \sin \alpha + \cos \alpha \cos \phi \sin \theta), \\
    r_{bc} &= (\cos \alpha \cos \theta - \cos \phi \sin \alpha \sin \theta).
\end{align*}
\]
In analogy we can define the position vector in the rotation axis frame as
\[
r_{\text{rot}} = r \{ \sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta \},
\]
where \((\Theta, \Phi)\) are the colatitude and azimuth in the rotation axis frame. Now Equations (12) and (13) both represent the same quantity, i.e., \( r_b = r_{\text{rot}} \). So by equating the vector components of Equations (12) and (13), and doing some algebraic manipulations, we get
\[
\cos \Theta = \cos \theta' = \cos \alpha \cos \theta - \cos \phi \sin \alpha \sin \theta \tag{14}
\]
and
\[
\tan \Phi = \frac{\cos \phi' \sin \theta \sin \phi + \cos \theta \sin \alpha \sin \phi' + \cos \alpha \cos \phi \sin \theta \sin \phi'}{\cos \theta \cos \phi' \sin \alpha + \cos \alpha \cos \phi \cos \phi' \sin \theta - \sin \theta \sin \phi \sin \phi'}. \tag{15}
\]
Equations (14) and (15) are used for simulating emission regions in the rotation axis frame.

Appendix B
Derivation of Electric Field of Radio Waves Emitted by Plasma

Consider a relativistic plasma bunch accelerated along dipolar magnetic field lines of a NS that is emitting radio waves due to a collective plasma process. To model the radio emission we consider three Cartesian coordinate systems: \( xyz, XYZ, \) and \( x'y'z' \). The coordinate system \( xyz \) is the stationary laboratory frame of the observer, whose origin is assumed to be at the center of the NS. The \( z' \)-axis is aligned along the rotation axis \((\hat{\Omega})\) and the magnetic axis \((\hat{n})\) is inclined with respect to \( \hat{\Omega} \) by an angle \( \alpha \). The line of sight \( \hat{n} = \{ \sin \zeta, 0, \cos \zeta \} \) lies in the \( x-z \) plane, where \( \zeta = \alpha + \sigma \) is the angle between \((\hat{\Omega})\) and \( \hat{n} \), and \( \sigma \) is the impact angle of \( \hat{n} \) with respect to \( \hat{\Omega} \), as shown in Figure 18.

The frame \( XYZ \) is chosen in such a way that the \( x' \)-axis is parallel \( \hat{b} \), and the \( XY \)-plane defines the plane of a magnetic field line at any given rotation phase \( \phi' \). The origin of the frame \( XYZ \) is at the local center of curvature of the magnetic field line, whose tangent \( \hat{b} \) aligns with \( \hat{n} \). Note that the analytical formulae for magnetic colatitude \( \theta \) and magnetic azimuth \( \phi \) (see Gangadhara 2004, Equations (9) and (11)) uniquely identify the specific tangent \( \hat{b} \) that is parallel to \( \hat{n} \) at any given rotation phase of the pulsar. The \( Y \)-axis is chosen to be parallel to the corresponding curvature vector \( \hat{k}_t \). Then the \( Z \)-axis will be parallel to the binormal \( \hat{b}_m = \hat{b} \times \hat{k}_t \). Hence as the NS spins the frame \( XYZ \) reorients in such a way that the \( XY \)-plane lies parallel to the plane of the field line whose tangent aligns with \( \hat{n} \).

In the systems \( XYZ \) and \( XYZ \) the motion of the plasma bunch is relativistic with center-of-momentum velocity \( \mathbf{v} \).

Next, the system \( x'y'z' \) is a comoving frame (rest frame) of the bunch in which the plasma motion is non-relativistic, and its coordinate axes are chosen in such a way that \( x' \parallel X, y' \parallel Y, \) and \( z' \parallel Z \) at time \( t = 0 \). The frame \( x'y'z' \) is comoving with velocity \( \mathbf{v} \) along a curved magnetic field line as time progresses. As the line of sight is a constant vector, we keep it fixed in both the frames \( xyz \) and \( XYZ \). But in order to receive radiation, we make the \( XYZ \) frame rotate in such a way that the tangent vector corresponding to each rotation phase subtends a maximum angle of \( \pm 1/\gamma \) with respect to the line of sight. Here \( \gamma \) is the Lorentz factor corresponding to the bulk velocity of collective plasma. To make this more general we invoke a very small angle \( \mu \) (in the range from \(-1/\gamma \) to \(1/\gamma \)), which denotes the angle between the line of sight and the plane of the magnetic field line. So according to our geometry the line of sight in the \( XYZ \) frame is confined in the \( X-Z \) plane. So we express the line-of-sight vector in the \( XYZ \) frame as \( \hat{b}_{\text{cf}} = [\cos \mu, 0, \sin \mu] \). Note that \( \mu = 0 \) is the special case, called edge-on position, where the specific tangent vector is exactly aligned with the line-of-sight vector, and therefore circular polarization vanishes. So while computing electric field in the \( XYZ \) frame, we substituted \( \hat{n} = \hat{b}_{\text{cf}} \) in order to obtain a generalized expression for circular polarization for any orientation.

After transforming the electric field back to the \( xyz \) frame one is always free to use the conventional expression for the line-of-sight vector as mentioned in the first paragraph of this appendix. Note that the Equation (16) below is applied in the \( XYZ \) frame first.

Consider the electric field of radiation due to an accelerated charge given by Jackson (1975):
\[
E(r, t) = \frac{q}{c} \left[ \hat{n} \times \left( \hat{n} - \beta \times \beta \right) \right] \left( \frac{\xi^3}{R_0} \right)_{\text{ret}},
\]
where \( q \) is the charge on the particle and \( c \) the velocity of light. The velocity and acceleration of a relativistic charge are \( \beta \) and \( \dot{\beta} \) as fractions of light speed, \( \xi = 1 - \hat{n} \cdot \beta \) is the beaming factor, and \( R_0 \) is the distance between the center of the \( XYZ \) frame and the observation point. We find the Fourier transform of \( E(r, t) \) at the retarded time \( t' \) using the relation \( t = t' + (R_0 - \hat{n} \cdot \hat{r}_N)/c \). Then we have the following expression for the radiation field due to a collective system of \( N \) particles (bunch):
\[
E_{\text{cf}}(\omega) = -i \omega \frac{e^{i\omega R_0/c}}{\sqrt{2\pi} R_0 c} \sum_{j=1}^{N} \int_{-\infty}^{\infty} \left( \hat{n} \times (\hat{n} \times \beta) \right) \times \exp \left\{ i \omega \left( t - \frac{\hat{n} \cdot \hat{r}_j}{c} \right) \right\} dt.
\]
\( \omega \) is the frequency, \( \hat{r}_j \) the path of \( j \)-th particle. The calculation in this appendix is only done analytically for a single particle.
If the mean free path of radiating entities in the bunch is much less than the wavelength of the emitted radiation, then the discrete sum can be replaced by continuous integration. For mathematical convenience of defining the bunch dimensions we adopt cylindrical coordinates \((\rho_{\text{cf}}, \phi_{\text{c}}, \eta_{\text{c}})\), where \(\rho_{\text{cf}}\) is the radial coordinate of an arbitrary point of the bunch, \(\phi_{\text{c}}\) is the angle subtended by the position vector of an arbitrary point in the bunch and the negative \(Y\)-axes (see Figure 18), and \(\eta_{\text{c}}\) the vertical height of the cylindrical bunch. The cylindrical symmetry axis is the \(Z\)-axis, which is parallel to the binormal \(\hat{b}_{\text{nt}}\) and perpendicular to the plane of the magnetic field line, i.e., the \(XY\)-plane. In terms of cylindrical coordinates the position vector \(\mathbf{r}_c\) of an arbitrary point inside the bunch with respect to the origin \(O\), and the local direction of motion \(\hat{e}_{\phi_{\text{c}}}\) are given by

\[
\mathbf{r}_c = \{\rho_{\text{cf}} \sin(\phi_{\text{c}}), -\rho_{\text{cf}} \cos(\phi_{\text{c}}), \eta_{\text{c}}\},
\]

\[
\hat{e}_{\phi_{\text{c}}} = \{\cos(\phi_{\text{c}}), \sin(\phi_{\text{c}}), 0\}.
\]

By making the substitutions (i) \(\sum_{i=1}^{N} q_i = \int \int \sigma_{\text{cf}} \rho_{\text{cf}} d\rho_{\text{cf}} d\phi_{\text{c}} d\eta\) and (ii) \(\sigma_{\text{cf}} \beta = (J/c) \hat{e}_{\phi_{\text{c}}}\), where \(\sigma_{\text{cf}}\) is the charge density, \(J\) is the current density, and \(\hat{e}_{\phi_{\text{c}}}\) is the local direction of motion of plasma particles, in Equation (17) we get

\[
E_{\text{cf}}(\omega) = -i\omega \frac{e^{i\omega R/c}}{\sqrt{2\pi} R_{\text{cf}}^2} \int \int \rho_{\text{cf}} d\rho_{\text{cf}} d\phi_{\text{c}} d\eta \\
\times \int_{-\infty}^{\infty} (\hat{n} \times (\hat{n} \times \mathbf{J})) \exp\left\{i\omega \left(t - \frac{\mathbf{n} \cdot \mathbf{r}_c}{c}\right)\right\} dt,
\]

(20)

where \(\mathbf{J}\) is the current density in the coordinate system \(XYZ\). We assume that the current density and the charge density in the comoving frame \((x'y'z')\) possess the following sinusoidal forms:

\[
J' = J'_0 \sin(k'_0 x' - \omega'_0 t'), \quad \sigma' = \sigma'_0 \sin(k'_0 x' - \omega'_0 t').
\]

(21)

The phase of the longitudinal wave \(k'_0 x' - \omega'_0 t'\) is a Lorentz invariant quantity. Therefore using Lorentz transformation and the continuity equation \(\nabla' \cdot \mathbf{J'} + (d\sigma'/dt') = 0\), we get the
expression for the current density in the frame XYZ:

\[ J = \gamma_0 J_0 \left( \omega_0 \over k_0^2 \right) \sin(k_0 X - \omega_0 t) \hat{e}_{oc} \]

\[ = J_0 \sin(k_0 \rho_0 \phi_x - \omega_0 t) \hat{e}_{oc}, \]  \hfill (22)

where \( \gamma \) is the Lorentz factor corresponding to the center-of-momentum velocity (\( v_{cm} \)) of the plasma bunch with respect to the rest frame of the observer (laboratory frame). The angular frequency and wavenumber of longitudinal waves (\( \omega_0, k_0 \)) in the laboratory frame are connected to (\( \omega_0', k_0' \)) in the comoving frame by the Doppler effect (Buschauer & Benford 1976). \( J_0 = \gamma_0 J_0 \left( \omega_0 \over k_0^2 \right) \) is the magnitude of current density, \( \omega_0'/k_0' \) the phase velocity of longitudinal waves in the comoving frame of plasma, and \( \rho_0 \) the radius of curvature of the magnetic field line tied to the plasma bunch. Here \( \sigma_0' = n'q \) is the charge density in the comoving frame and \( n' \) is the charge number density. By substituting the current density from Equation (22) into (20) we get

\[ E_{ct}(\omega) = -i \omega J_0 e^{i \omega R/c \over 2 \pi R_0^2 c} \int \int \rho_{ct} d \rho_{ct} d \phi_x d \eta \]

\[ \times \int_{-\infty}^{\infty} \left( (\hat{n} \times (\hat{n} \times \hat{e}_{oc})) \sin(k_0 \rho_0 \phi_x - \omega_0 t) \right) \]

\[ \times \exp \left( i \omega t - \frac{\hat{n} \cdot R_b}{c} \right) dt. \]  \hfill (23)

It can be shown that the vector quantity \( \hat{n} \times (\hat{n} \times \hat{e}_{oc}) = \hat{e} \sin \mu \cos \phi_x - \hat{y} \sin \phi_x \), where \( \hat{e} = \hat{n} \times \hat{y} \) is the binoormal vector, which lies perpendicular to the plane of the magnetic field line, and the curvature vector \( \hat{y} \) lies in that plane. \( \mu \) is the angle between the plane of the magnetic field line and the line of sight. Due to the relativistic effect we get radiation in a narrow cone with beaming angle \( \sim 1/\gamma \) around the tangent vector, i.e., along \( \hat{X} \). We resolve the electric field in the frame XYZ into components in the plane of the magnetic field line and perpendicular to it:

\[ E_{ct}(r, \omega) = \frac{i \omega J_0 e^{i \omega R/c \over 2 \pi R_0^2 c} \int \gamma_0^2 \eta_0^2 d \eta_0 \sin \mu \over c} \exp \left[ -i \omega \eta_0 \sin \mu \over c \right] d \eta \]

\[ \times \int_{\rho_0 - \eta_0 \rho_0 / \omega_0 c}^{\rho_0 + \eta_0 \rho_0 / \omega_0 c} \exp \left[ -i \gamma_0^2 c \mu \sin \phi_x \right] d \rho_{ct}, \]  \hfill (24)

\[ E_{ct \perp}(r, \omega) = \frac{-i \omega J_0 e^{i \omega R/c \over 2 \pi R_0^2 c} \int \gamma_0^2 \eta_0^2 d \eta_0 \sin \mu \over c} \exp \left[ -i \omega \eta_0 \sin \mu \over c \right] d \eta \]

\[ \times \int_{\rho_0 - \eta_0 \rho_0 / \omega_0 c}^{\rho_0 + \eta_0 \rho_0 / \omega_0 c} \exp \left[ -i \gamma_0^2 c \mu \sin \phi_x \right] d \rho_{ct}, \]  \hfill (25)

where \( \xi_0 \) is the radial width of the bunch, \( \phi_0 \) is the angular coordinate of the center of momentum of the bunch, \( \alpha_m = S_0 / \rho_0 \) is the angular span of the bunch, \( S_0 \) the length of the bunch, and \( \eta_0 \) the vertical dimension or height of the bunch. The integration with respect to \( \eta \) is straightforward, and evaluated to be \( \sin(\omega_0 \eta_0 \mu / 2c) / (\omega_0 \mu / 2c) \). At the center of momentum we can assume \( J = 0 \), which readily translates to \( \phi_0 = v_{cm} t / \rho_0 \) and \( v_{cm} \approx c \). So \( \phi_x \) is a time-dependent quantity, which can be expressed as

\[ \phi_x = \frac{ct}{\rho_0} + \alpha_m. \]  \hfill (26)

The length of the bunch in the frame XYZ can be written as \( S = \rho_0 \phi_x \) and hence \( ds = \rho_0 d \phi_x \). We make the following approximations for evaluating the time integral and \( \phi_x \) integral:

(i) \( \cos \mu \approx 1 - \mu^2 / 2 \),

(ii) \( \sin \phi_x \approx \phi_x - \phi_x^3 / 6 \),

(iii) \( \mu \rho_0 \approx \rho_0(1 + \xi_0 / \rho_0) \).

We introduce a function \( F_n \) for representing the electric field in a more simplistic and compact way:

\[ F_n = e^{i \omega t} \sin(k_0 \rho_0 \phi_x - \omega_0 t) \]

\[ \times \exp \left[ -i \omega \over c \rho_0 \left( 1 + \xi_0 / \rho_0 \right) \left( 1 - \mu^2 / 2 \right) \sin \phi_x \right]. \]  \hfill (27)

After doing some algebraic manipulation we get

\[ \int \int F_n d \phi_x dt = (2i)^{-1}(\Delta T_+ - \Delta T_-. \]  \hfill (28)

where

\[ \Delta_1 = \frac{1}{\rho_0} \int_{S_0 / 2}^{S_0 / 2} \exp[i S(k_0 - k M)] d S, \]

\[ \Delta_2 = \frac{1}{\rho_0} \int_{S_0 / 2}^{S_0 / 2} \exp[-i S(k_0 + k M)] d S. \]

Here \( M = 1 - \mu^2 / 2 \) \((1 + \xi_0 / \rho_0)(1 - \alpha_m^2 / 6)\) and \( T_\pm \) (time integrations) are given by

\[ T_- = \int_{-\infty}^{\infty} \exp[i (C_1 t + C_2 t^2 + C_3 t^3)] dt, \]

\[ T_+ = \int_{-\infty}^{\infty} \exp[i (C_1 t + C_2 t^2 + C_3 t^3)] dt. \]

The coefficients are

\[ C_1 \approx \omega \left( \frac{1}{2 \gamma^2} - \frac{\gamma_0 \omega_0}{\gamma_0 \omega} + \frac{\alpha_m^2}{2} + \frac{\mu^2}{2} - \frac{\xi_0}{\rho_0} \right), \]

\[ C_{1a} \approx \omega \left( \frac{1}{2 \gamma^2} + \frac{\gamma_0 \omega_0}{\gamma_0 \omega} + \frac{\alpha_m^2}{2} + \frac{\mu^2}{2} - \frac{\xi_0}{\rho_0} \right), \]

\[ C_2 = -k \rho_0 A_2 \left( 1 - \frac{\mu^2}{2} \right) \left( 1 + \frac{\xi_0}{\rho_0} \right) \approx \frac{\omega \alpha_m}{2 \rho_0}, \]

\[ C_3 = -k \rho_0 \left( 1 - \frac{\mu^2}{2} \right) \left( 1 + \frac{\xi_0}{\rho_0} \right) A_3 \approx \frac{\omega \alpha_m}{6 \rho_0}. \]

We avoid the byproduct term arising from the product of \( \xi_0 \) with \( \mu^2 \) and \( \alpha_m^2 \), while estimating \( C_1, C_{1a}, C_2, C_3 \), as they are small entities. The maximum length of a coherently emitting column is \( S_0 \leq \rho_0 / \gamma_0^2 \) (Buschauer & Benford 1976). Moreover
the transverse dimensions \( \xi_0 \) and \( \eta_0 \) should both be less than the emitted wavelength to satisfy the coherent condition. But the length of the bunch can pervade up to 10\%-35\% of the radius of the light cylinder in normal radio pulsars.

Due to the factor \( k_0 + k \mathcal{M} \) in \( \Delta_2 \), the resonance condition is not satisfied, and hence we drop \( \Delta_2 \). So we deal with the \( \Delta_1 T_- \) term:

\[
\Delta_1 = \int_{-\infty}^{\infty} \exp \left[ i S (k_0 - k \mathcal{M}) \right] dS = \frac{2 \sin \left( \frac{(k_0 - k \mathcal{M}) S_0}{2} \right)}{k_0 - k \mathcal{M}}.
\]

This term is called the selection factor. We can see that resonance condition is satisfied in the limit of \( k_0 \to k \mathcal{M} \). Here \( k \) and \( \omega \) are the wavenumber and frequency of emitted electromagnetic waves. Perfect coherence occurs when \( \mathcal{M} \) attains its maximum value. This occurs at \( \mu = 0^\circ \), i.e., in the plane of magnetic field lines for any combination of geometrical parameters allowed by observations. This happens because at \( \mu = 0^\circ \) both \( \xi_0 \) and \( \alpha_m \) become zero.

The solution to the integral along the radial direction is straightforward and it is given by

\[
\int_{\rho_0 - \xi_0 / \mathcal{C}}^{\rho_0 + \xi_0 / \mathcal{C}} \exp \left[ i C t (t + \xi_0 / \mathcal{C}) \right] d\rho = \rho_0 \xi_0 \mathcal{C}
\]

This integral solution is important only in the aspect of boosting the peak magnitude of intensity. Apart from that, another significant role played by transverse dimensions is that the smaller the transverse dimensions \( (\xi_0, \eta_0) \), compared to the radio wavelength, the more pronounced will be the proportion of circular polarization.

The solution to time integrals in Equations (24) and (25) carries all the information regarding pulse profile structure and polarization. The time integrals appearing in the parallel and perpendicular components of electric field are

\[
T_{||} = \int_{-\infty}^{\infty} \left( A_0 + A_1 t + A_2 t^2 + A_3 t^3 \right) \exp \left[ i (C t + C_2 t^2 + C_3 t^3) \right] dt
\]

\[
T_{\perp} = \int_{-\infty}^{\infty} \left( B_0 + B_1 t + B_2 t^2 + B_3 t^3 \right) \exp \left[ i (C t + C_2 t^2 + C_3 t^3) \right] dt
\]

where

\[
A_i = \alpha_m (1 - \alpha_m^2 / 6), \quad A_1 = c / \rho_0 (1 - \alpha_m^2 / 2), \quad A_2 = -c^2 \alpha_m^3 / (2 \rho_0^2), \quad A_3 = -c^3 \left( 6 \rho_0^3 \right), \quad B_0 = 1 - \alpha_m^2 / 2, \quad B_1 = -\alpha m c / \rho_0, \quad B_2 = -c^2 / (2 \rho_0^2), \quad B_3 = 0.
\]

The limits of the time integrals are deliberately pushed to \( \pm \infty \) because the emissions are received over a short interval of time during which \( \mathbf{t} \) and the velocity vector \( \mathbf{r} \) are within the angle \( 1/\gamma \). Outside this interval, the time integrand oscillates very rapidly and contributes a negligibly small amount to the radiation.

Let \( \mathbf{r} = \mathbf{x} + \mathbf{m} \), then \( dt = dx / l \), where \( l = C_3^{1/3} \) and \( m = -C_2 / (3 C_3) \). Then the solutions to the integrals in Equations (35)–(38) are given by

\[
L_0 = U_{\mathbf{j}_0}, \quad L_1 = \frac{U}{C_3} \left( j_1 - \frac{C_2}{3 C_3^{1/3} \rho_0} \right),
\]

\[
L_2 = U \left( j_0 \left( \frac{2 C_2^2}{3 C_3} - C_3 \right) - 2 j_1 \frac{C_2}{3 C_3^{1/3}} \right) \text{ and }
\]

\[
L_3 = \frac{U}{9 C_3^{1/3}} \left( j_1 \left( 4 C_2^2 - 3 C_3 \right) + j_0 \left( 9 C_2 C_3 - 4 C_3^2 + 9 i c_3^2 \right) \right) \frac{3 C_3}{C_3^{1/3}}
\]

where

\[
U = \frac{1}{C_3^{1/3}} \exp \left[ -i \frac{C_2}{3 C_3} \frac{2 C_2^2}{3 C_3} - C_3 \right],
\]

\[
j_0 = \frac{2}{3} z^{-1/2} K_{1/3} \left( \frac{2}{3} z^{3/2} \right),
\]

\[
j_1 = \frac{2}{3} z^{-1/2} K_{2/3} \left( \frac{2}{3} z^{3/2} \right) \text{ and }
\]

\[
z = \frac{1}{C_3^{1/3}} \left( C_3 - \frac{C_2^2}{3 C_3} \right).
\]

The functions \( K_{1/3} \) and \( K_{2/3} \) are the modified Bessel functions of order \( 1/3 \) and \( 2/3 \), respectively.

If we plug in the values of \( C_1, C_2, C_3 \) into the expression for \( z \) and simplify, we get

\[
z = \left( \frac{6 \omega^2 \rho_0^2}{c^2} \right)^{1/3} \left( \frac{1}{2 \gamma_0^2} - \frac{\omega^2}{\gamma^2} + \frac{\mu^2}{2} - \frac{\xi_0^2}{\rho_0^2} \right).
\]

Finally the components of the radiation electric field in the frame XYZ are given by

\[
E_{\mathbf{r}_1}(r, \omega) = A_m E_0 \sin \left[ \frac{(k_0 - k \mathcal{M}) S_0}{2} \right] \sin \left( \kappa \eta_0 \mu / 2 \right) \times (A_0 L_0 + A_1 L_1 + A_2 L_2 + A_3 L_3) \mathbf{\hat{Y}},
\]

\[
E_{\mathbf{r}_2}(r, \omega) = -A_m E_0 \sin \left[ \frac{(k_0 - k \mathcal{M}) S_0}{2} \right] \sin \left( \kappa \eta_0 \mu / 2 \right) \times (B_0 L_0 + B_1 L_1 + B_2 L_2) \mathbf{\hat{Z}},
\]

where

\[
A_m = \frac{\omega \gamma L (\omega_c + \omega_0 k_0 / \omega)}{c} \quad \text{and} \quad E_0 = \frac{q}{2 \sqrt{2 \pi} R_0 c} \exp(i \omega R_0 / c).
\]
Appendix C
Brief Formulation of Radio Emission Geometry in the Frame of the NS Origin and Behavior of Angle $\mu$ with Respect to the Rotation Phase

In this appendix, we briefly elaborate about the emission-beam geometry in the frame of the NS origin by recasting the dipolar magnetic field line topology. Then we show expressions for the magnetic colatitude and magnetic azimuth at the emission spot and the behavior of angle $\mu$. Although the geometry related to radio emission was given in Gangadhara (2004), for the sake of completeness we briefly discuss the emission geometry along with some extra theory. Consider a magnetic dipole situated at the origin with magnetic dipole moment $\hat{m}$ oriented at an angle $\alpha$ to the rotation axis ($\hat{\Omega}$). The line-of-sight vector ($\hat{n}$) is confined in the $x$-$z$ plane, making an angle $\zeta$ with respect to the rotation axis $\hat{\Omega}$. In a Cartesian coordinate system with the $z$-axis parallel to the spin axis $\Omega$, the position vector of an arbitrary point on a field line is given by

$$r_c = r_c\{\sin^3 \theta \cos \phi, \sin^3 \theta \sin \phi, \sin^2 \theta \cos \theta\}, \quad (44)$$

where $\phi$ is the magnetic azimuth. Now consider the situation where the dipole ($\hat{m}$) is inclined at an angle $\alpha$ with respect to $\hat{\Omega}$, and rotated by phase $\phi'$ around the $z$-axis. The position vector of the point on the magnetic field line, which is tilted and rotated, is given by

$$r_{ct} = \Lambda \cdot r_c, \quad (45)$$

where the transformation matrix $\Lambda = M_{rot} \cdot M_{inc}$. The matrices for tilt (inclination) $M_{inc}$ and rotation $M_{rot}$ are given by

$$M_{inc} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}, \quad M_{rot} = \begin{bmatrix} \cos \phi' & -\sin \phi' & 0 \\ \sin \phi' & \cos \phi' & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (50)$$

The matrix $M_{inc}$ produces clockwise rotation of the dipole around the $y$-axis, and $M_{rot}$ counterclockwise rotation around the $z$-axis.

For a particular point located on the field line, we find the tangent to the field line by evaluating $\mathbf{b}_t = \partial r_c / \partial \theta$, and the curvature vector by $\mathbf{k}_t = \mathbf{b}_t / ds = (1/|b_t|) \partial \mathbf{b}_t / \partial \theta$, where $|b_t| = r_c \sqrt{5 + 3 \cos 2\theta \sin \theta / \sqrt{2}}$. Therefore, the field line curvature radius is given by

$$\rho = \frac{1}{|k_t|} = r_c \sin \theta (5 + 3 \cos 2\theta)^{3/2} / \sqrt{3 \times 2 (3 + \cos 2\theta)}. \quad (46)$$

Since $\hat{m}$ is chosen to be parallel to $\hat{z}$, the transformed magnetic dipole moment is given by

$$\hat{m}_t = \Lambda \cdot \hat{z} = \{\sin \alpha \cos \phi', \sin \alpha \sin \phi', \cos \alpha\}. \quad (47)$$

C1. Magnetic Colatitude and Azimuth of Emission Spot

Consider the sight line $\hat{n} = (\sin \zeta, 0, \cos \zeta)$, which lies in the $x$-$z$ plane and makes an angle $\zeta$ with respect to $\hat{\Omega}$, where $\zeta = \alpha + \sigma$, and $\sigma$ is the angle of closest approach of the sight line with respect to the magnetic axis. The half opening angle $\Gamma$ of the emission beam is given by

$$\cos \Gamma = \hat{n} \cdot \hat{m}_t = \cos \alpha \cos \zeta + \sin \alpha \sin \zeta \cos \phi'. \quad (48)$$

If $\tau$ is the angle between $\hat{b}_t$ and $\hat{m}_t$, then we have

$$\cos \tau = \hat{b}_t \cdot \hat{m}_t = \frac{1 + 3 \cos 2\theta}{\sqrt{10 + 6 \cos 2\theta}}. \quad (49)$$

In relativistic flow, radiation is beamed in the direction of the field line tangent. So, at any instant, observable radiation comes from a spot in the magnetosphere where the tangent vector $\hat{b}_t$ points in the direction of $\hat{n}$. That is $\Gamma \approx \pi$ at the emission spot. Therefore, using Equations (48) and (49), we get relation for magnetic colatitude $\theta$ as a function of $\Gamma$:

$$\cos 2\theta = \frac{1}{3}(\cos \Gamma \sqrt{8 + \cos^2 \Gamma} - \sin^2 \Gamma). \quad (50)$$

For $\Gamma \ll 1$, Equation (50) reduces to the well known approximate form $\theta \approx (2/3)\Gamma$. Next we introduce another angle in the emission geometry, $\kappa$, the angle between $\hat{n}$ and $\hat{b}_t$. Then we have

$$\cos \kappa = \hat{n} \cdot \hat{b}_t = \cos^2 \Gamma + \sin \cos (\cos \alpha \sin \zeta \cos \phi' - \sin \alpha \cos \zeta) - \sin \zeta \sin \Gamma \sin \phi', \quad (51)$$

Again, to receive radiation $\kappa \approx 0$; therefore, we find the magnetic azimuth $\phi$ of the emission spot by solving $\hat{n} \cdot \hat{b}_t = 1$ and $\hat{n} \times \hat{b}_t = 0$. The first and third components of the equation $\hat{n} \times \hat{b}_t = 0$ give

$$\sin \phi = -\sin \zeta \csc \Gamma \sin \phi', \quad (52)$$

and the second component gives

$$\cos \phi = (\cos \alpha \sin \zeta \cos \phi' - \sin \cos \alpha \sin \zeta \cos \phi'). \quad (53)$$

Therefore combining Equations (52) and (53) we get

$$\phi = \arctan \left( \frac{\sin \zeta \sin \phi'}{\cos \zeta \sin \alpha - \cos \alpha \sin \zeta \cos \phi'} \right). \quad (54)$$

C2. Behavior of Angle $\mu$

Let $\mu$ be the angle between the line of sight and the plane of magnetic field lines. In Section 3, we have given the expression for the binormal vector. So

$$\mu = \arcsin (\hat{n} \cdot \hat{\tau}) = \arcsin (-\sin \zeta (\cos \alpha \sin \phi \cos \phi' + \cos \phi \sin \phi') + \cos \phi \sin \phi' + \cos \phi \sin \phi). \quad (55)$$

Now we substitute the expression for $\phi$ from Equation (54) and thereafter $\phi' = \phi' + \delta \phi'$ to get the span of the phase pulse around the peak point of the main pulse and subpulses. The span of subpulses in $\mu$ space corresponds to some spread around the peak point in the domain of the rotation phase.

Appendix D
Calculation of Subpulse Width of a Pulsar for a Collective Plasma System

We calculate the width of subpulses from the expression for the radius of curvature associated with a bunch (see Equation (46)). If we project the transverse width of the source in the meridional plane, we can estimate the width of the main pulse as well as subpulses. From Equation (46) we see that $\rho$ is
a function of $r_e$ and $\theta$. So we can write
\begin{equation}
\rho = f(r_e, \theta).
\end{equation}

Differentiating the above equation we get
\begin{equation}
\xi_0 = d\rho = \frac{\partial f}{\partial r_e} \delta r_e + \frac{\partial f}{\partial \theta} \delta \theta,
\end{equation}

where $\xi_0$ (radial width of the bunch) is nothing but the variation of the radius of curvature. So from Equation (46) we have
\begin{equation}
\frac{\partial f}{\partial r_e} = \frac{\sin \theta (5 + 3 \cos 2\theta)^{3/2}}{3\sqrt{2}(3 + \cos 2\theta)}
\end{equation}

and
\begin{equation}
\frac{\partial f}{\partial \theta} = \frac{r_e (5 + 3 \cos 2\theta)^{3/2}(30 \cos \theta + 31 \cos 3\theta + 3 \cos 5\theta)}{6\sqrt{2}(3 + \cos 2\theta)^2}.
\end{equation}

Again for a particular pulsar, the colatitude at the emission spot is a function of rotation phase only (see Equation (50)). So by differentiating Equation (50) with respect to rotation phase we get
\begin{equation}
\delta \theta = \frac{\sin \alpha \sin \zeta \sin \phi' (\cos \Gamma + (8 + \cos^2 \Gamma)^{1/2})^2}{6(8 + \cos^2 \Gamma)^{1/2}\left(1 - \frac{1}{9}(-1 + \cos \Gamma(8 + \cos^2 \Gamma)^{1/2} + \cos^2 \Gamma)^2\right)^{1/2}} \delta \phi'.
\end{equation}

So once we know the half beam opening angle ($\Gamma$), $\alpha$, $\sigma$, the peak location of subpulses, the radial width, the span of the dipolar field line constant as well as the dipolar field line constant of a bunch at the particular phase location of the subpulse, we can easily get the width of the subpulse with the help of Equation (58).

**Appendix E**

**List of Important Notation with Brief Description**

1. $c$ Velocity of light.
2. $q$ Charge of single particle.
3. $R_{NS}$ Radius of NS.
4. $\hat{b}_d$ Specific tangent vector in $XYZ$ frame.
5. $\hat{n}$ Line-of-sight vector in $xyz$ frame.
6. $E_d$ Electric field due to radiation in $XYZ$ frame (lab frame).
7. $E_{NS}$ Electric field due to radiation in $xyz$ frame (lab frame, origin is centered on NS).
8. $J'$ Current density in comoving frame associated with Langmuir waves.
9. $J$ Current density in $XYZ$ frame associated with Langmuir waves.
10. $R$ Distance between emission region in pulsar magnetosphere and observer.
11. $\gamma$ Lorentz factor associated with secondary plasma particles in pulsar plasma.
12. $\gamma_p$ Lorentz factor associated with primary plasma particles in pulsar plasma.
13. $B_0$ Surface magnetic field in NS.
14. $\omega_0$ Angular frequency associated with perturbed Langmuir waves in comoving frame.
15. $k_0$ Wavenumber associated with perturbed Langmuir waves in comoving frame.
16. $\omega_0$ Angular frequency associated with perturbed Langmuir waves in $XYZ$ frame.
17. $k_0$ Wavenumber associated with perturbed Langmuir waves in $XYZ$ frame.
18. $\omega$ Angular frequency of electromagnetic radiation emitted by pulsar.
19. $k$ Emitted wavenumber associated with electromagnetic radiation in pulsar.
20. $r_b$ Position vector of an arbitrary point inside the bunch in $XYZ$ frame.
21. $\phi$ Dynamical angular span of charge column (see Figure 18) in $XYZ$ frame.
22. $\sigma_0$ Charge density in lab frame.
23. $n'$ Charge number density in comoving frame.
24. $J_0$ Magnitude of current density in lab frame.
25. $N$ Number of cooperating particles in bunch.
26. $\xi$ Beaming factor.
27. $\xi_0$ Radial width of the bunch (see Figure 18).
28. $\rho_0$ Vertical height of the bunch, measured with respect to symmetry axis in $XYZ$ frame (see Figure 18).
29. $S_0$ Length of the bunch.
30. $\alpha_m$ Angular width of the charge column.
31. $\alpha$ Inclination angle of magnetic axis with respect to rotation axis.
32. $\sigma$ Minimum impact angle of line of sight with magnetic axis.
33. $\Gamma$ Half beam opening angle of pulsar.
34. $\theta$ Magnetic colatitude, which is the angle subtended between the magnetic axis and the radius vector of an arbitrary point located on magnetic field lines.
35. $\phi$ Magnetic azimuthal angle, which is measured in a plane perpendicular to the magnetic axis. Sense of measurement is counterclockwise usually.
36. $\Theta = \theta'$ Colatitude in the frame centered on the rotation axis.
37. $\Phi$ Azimuthal angle in the frame centered on the rotation axis, which is measured in a plane perpendicular to the rotation axis.
38. $\rho_0$ Radius of curvature of a magnetic field line passing through the center of momentum of a bunch.
39. $\zeta$ Angle between line of sight and rotation axis.
40. $r_b$ Dipolar field line constant of magnetic field line.
41. $\mu$ Angle between line of sight and plane of magnetic field line.
42. $\phi'$ Rotation phase or pulse longitude.
43. $\hat{b}_d$ Tangent vector in emission spot.
44. $k_b$ Curvature vector in emission spot.
45. $\hat{e}$ Binormal vector in emission spot.
46. $\epsilon\parallel$ Projected spin axis vector lying in the plane of the sky.
47. $\epsilon_{\perp}$ Orthogonal vector to line of sight, lying parallel to the $y$-axis.

15.
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