Intrinsic Morphology Evolution of Ultra-diffuse Galaxies

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(Received July 29, 2019; Revised tomorrow; Accepted the day after tomorrow)

Submitted to ApJ

**ABSTRACT**

With the published data of apparent axis ratios for 1109 ultra-diffuse galaxies (UDGs) located in 17 low-redshift (\(z \sim 0.020 - 0.063\)) galaxy clusters and 84 UDGs in 2 intermediate-redshift (\(z \sim 0.308 - 0.348\)) clusters, we take advantage of a Markov Chain Monte Carlo approach and assume a ubiquitous triaxial model to investigate the intrinsic morphologies of UDGs. In contrast to the conclusion of Burkert (2017), i.e., the underlying shapes of UDGs are purely prolate (\(C = B < A\)), we find that the data favor the oblate-triaxial models (\(C < B \lesssim A\)) over the nearly prolate ones. We also find that the intrinsic morphologies of UDGs are relevant to their stellar masses/luminosities, environments, and redshifts. First, for the low-redshift UDGs in the same environment, the more-luminous ones are always thicker than the less-luminous counterparts, possibly due to the more violent internal supernovae feedback or external tidal interactions for the progenitors of the more-luminous UDGs. The UDG thickness dependence on luminosity is distinct from that of the typical quiescent dwarf ellipticals (dEs) and dwarf spheriodals (dSphs) in the local clusters and groups, but resembles that of luminous UDGs. The UDG thickness dependence on luminosity is distinct from that of the typical quiescent dwarf ellipticals (dEs) and dwarf spheroidals (dSphs) in the local clusters and groups, but resembles that of luminous UDGs. The UDG thickness dependence on luminosity is distinct from that of the typical quiescent dwarf ellipticals (dEs) and dwarf spheroidals (dSphs) in the local clusters and groups, but resembles that of luminous UDGs.

**Keywords:** galaxies: dwarf — galaxies: stellar content — galaxies: photometry — galaxies: evolution — galaxies: interactions — methods: statistical

1. **INTRODUCTION**

The investigation of the properties and formation of ultra-diffuse galaxies (UDGs), which have Milky-Way sizes but are as faint as typical dwarfs (van Dokkum et al. 2015), has not reached a clear consensus. The high-redshift strong feedback model agrees that UDGs may be failed \(L^*\) galaxies embedded in massive halos but ceased their in-situ star formation in the early Universe (e.g., Yozin & Bekki 2015);
the observations for the largest UDG in the Coma cluster, DF 44, also support UDGs to be hosted in massive halos with total masses of \( M_\odot \sim 10^{12} \). (van Dokkum et al. 2016). However, the semi-analytic galaxy formation models as well as gravitational lensing technique (Sifón et al. 2018), Lim 2016; Beasley et al. 2016; Beasley & Trujillo 2016) as a system and their parent halo (e.g. Prole et al. 2019; Peng & Lim 2016; Beasley et al. 2016; Beasley & Trujillo 2016) as well as HDGs located in galaxy clusters, estimated from the empirical relation between masses of member globular cluster system and their parent halo (e.g. Prole et al. 2019; Peng & Lim 2016; Beasley et al. 2016; Beasley & Trujillo 2016) as well as gravitational lensing technique (Sifón et al. 2018), support that UDGs may be genuine dwarfs; the further HI detections for the UDGs located in the low-density field environments also agree with the relatively higher specific angular momenta of UDGs, compared with the typical dwarfs (Leisman et al. 2017). Other possible origins, including the supernova-energy injection, tidal interactions, etc. (Carleton et al. 2019; Ogiya 2018; Jiang et al. 2018; Martin et al. 2019; Liao et al. 2019), are also plausibly supported by the controversial photometric tidal/interaction evidence (Greco et al. 2018; Bennet et al. 2018; Müller et al. 2019) as well as spectroscopic results (Chilingarian et al. 2019; Martín-Navarro et al. 2019; Struble 2018); particularly, the recent studies for two member UDGs in the NGC 1052 group, NGC 1052-DF2 and -DF4, possibly suggest a ubiquitous lack of dark matter in UDGs, which may unveil an outstanding role of tidal stripping in UDG evolution (van Dokkum et al. 2018, 2019; Danieli et al. 2019; Ogiya 2018).

For the member UDGs in the low-density and high-density environments, they primarily populate the blue cloud and red sequence in the color-magnitude diagram, respectively (e.g., van der Burg et al. 2016; Janssens et al. 2017; Román & Trujillo 2017a,b; Venhola et al. 2017; Spekkens & Karunakaran 2018; Miños et al. 2015; Lee et al. 2017; Trujillo et al. 2017), and their morphologies are also distinct, with the former having mostly irregular, and the latter mostly elliptical, appearance (e.g., Leisman et al. 2017; Trujillo et al. 2017; Yagi et al. 2016; Román & Trujillo 2017a; Eigenthaler et al. 2018; Conselice 2018). A large fraction of UDGs in galaxy clusters exhibit unresolved nuclear star clusters (e.g., Yagi et al. 2016; Eigenthaler et al. 2018; Miños et al. 2015). Several UDGs show clear evidence for association with tidal material and interaction with a larger galaxy halo (e.g., Toloba et al. 2016; Bennet et al. 2017). These photometric evidences suggest the diverse morphologies of UDG populations and plausibly imply the evolution of UDG intrinsic morphologies with redshifts and environments, which further provide a clue to the formation and evolution of UDGs.

According to the distribution of the apparent axis ratios \( q = b/a \) of the Coma UDGs, in particular, the absence of UDGs with \( q > 0.9 \), Burkert (2017) claimed that the on-average intrinsic shapes of the cluster UDGs are more likely to be purely-prolate (i.e., the three intrinsic axes of UDGs satisfy \( C = B < A \)), compared with a purely-oblolute disk model (i.e., \( C < B = A \)); the strong radial alignment signals of cluster UDGs (e.g., van der Burg et al. 2017; Rong et al. 2019; Yagi et al. 2016) may also prefer a prolate model. However, it is worth to note that a more reasonable diagnostic for the underlying morphologies of UDGs is to assume a prevalent triaxial (\( C \leq B \leq A \)) model, rather than to simply choose between the purely-prolate and purely-oblolute models. Specifically, the sharply reduced number of Coma UDGs with \( q > 0.9 \) in the \( q \) distribution can also be well explained by an oblate-triaxial model (e.g., Binney & Merrifield 1998, see section 4.3.3 and Fig. 4.36), except for the purely-prolate model.

Therefore, the three-dimensional (3D) morphologies of UDGs should be carefully studied again with a ubiquitous triaxial model. We aim to analyze the possible evolution of UDG morphologies from, e.g., low-density to high-density environments, high-redshifts to low-redshifts, and low-mass to high-mass, etc. In section 2, we introduce the UDG samples studied in this work, and show the distributions of their apparent axis ratios. In section 3, we investigate the intrinsic shapes of UDGs by assuming a triaxial model, and study the possible morphology evolution of UDGs. We summarize our results in section 4.1.2. UDG DATA

2.1. UDG samples

The UDG samples used in this work are gathered from the previous literatures, located in 17 low-redshift (low-\( z \); \( z \sim 0.200 - 0.063 \)) clusters/groups and 2 intermediate-redshift (intermediate-\( z \); \( z \sim 0.308 - 0.348 \)) clusters, as listed in Table 1. Almost all of these UDGs follow the red sequence in the color-magnitude diagram.

Sample 1: the publicly available \(^1\) Coma UDG sample reported by Yagi et al. (2016). These UDGs are distributed within \( R_{200} \) (\( R_{200} \) is the radius within which the mean cluster density is 200 times the critical density) of Coma, with \( r\text{-}band \) absolute magnitudes of \(-17 < M_r < -9 \) mag, effective radii \( r_e > 1.5 \) kpc, as well as mean surface brightness within \( r_e \), \( \langle \mu_e(r) \rangle \), between 24 and 27 mag arcsec\(^{-2} \). Only 1\% UDGs show Sérsic indices of \( n > 2 \).

Sample 2: UDGs selected by Mancera Piña et al. (2019), located in both the inside and outside regions of eight low-\( z \) clusters. UDGs were selected with \( r > 1.5 \) kpc. \( \langle \mu_e(r) \rangle \) \( > 24 \) mag arcsec\(^{-2} \), and \( n < 4 \) (only < 3\% UDGs have \( n > 2 \)); these UDGs have the absolute magnitudes of \(-18 < M_r < -12.5 \) mag.

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\(^1\) http://vizier.cfa.harvard.edu/viz-bin/VizieR?-source=J/ApJS/225/11

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2. The original magnitude and surface brightness values of UDGs in Yagi et al. (2016) are in the \( R\text{-}band \); in order to compare the surface brightness and magnitudes of the Coma UDGs with those of the other UDG samples, we convert the \( R\text{-}band \) surface brightness and magnitudes to \( r\text{-}band \) properties with \( R - r = 2.5 \log (r/i) - 0.08 \) mag (Yagi et al. 2016), where the colors of the cluster UDGs approximately are \( r - i \sim 0.25 \) (e.g., Rong et al. 2017a), and constants \( c_R \) are obtained from Table 2 in Yagi et al. (2016). Indeed, the \( R \) and \( r\text{-}band \) magnitudes/surface brightness levels only show a marginal difference.
UDGs Morphology

Table 1. Information of the parent galaxy clusters/groups where the UDG samples located. Col. (1): cluster name; Col. (2): redshift; Col. (3): virial radius $R_{200}$ (Mpc); Col. (4)-(6): numbers of UDGs in $R \leq 0.5R_{200}$, $0.5R_{200} < R \leq R_{200}$, and $R > R_{200}$, respectively. a Yagi et al. (2016); b Mancera Piña et al. (2019); c van der Burg et al. (2016); d Lee et al. (2017); * Brilenkov et al. (2015).

| Clusters   | $z$  | $R_{200}$ (Mpc) | $N_{\text{inner}}$ | $N_{\text{median}}$ | $N_{\text{outer}}$ |
|------------|------|----------------|--------------------|---------------------|-------------------|
| Coma$^a$   | 0.023| 2.6$^*$         | 204                | 124                 | 0                 |
| R1204$^b$  | 0.020| 0.6            | 7                  | 15                  | 17                |
| A779$^b$   | 0.023| 0.7            | 7                  | 17                  | 7                 |
| R1223$^b$  | 0.026| 0.6            | 10                 | 7                   | 20                |
| MKW48$^b$  | 0.027| 0.6            | 5                  | 9                   | 28                |
| R1714$^b$  | 0.028| 0.4            | 2                  | 6                   | 35                |
| A2634$^b$  | 0.031| 1.3            | 51                 | 61                  | 8                 |
| A1177$^b$  | 0.032| 0.7            | 5                  | 9                   | 25                |
| A1314$^b$  | 0.033| 0.9            | 16                 | 20                  | 55                |
| A119$^c$   | 0.044| 1.9            | 38                 | 18                  | 0                 |
| MKW3S$^c$  | 0.044| 1.2            | 8                  | 10                  | 0                 |
| A85$^c$    | 0.055| 2.0            | 37                 | 30                  | 0                 |
| A780$^c$   | 0.055| 1.7            | 11                 | 23                  | 0                 |
| A133$^c$   | 0.056| 1.7            | 27                 | 27                  | 0                 |
| A1991$^c$  | 0.059| 1.2            | 17                 | 8                   | 0                 |
| A1781$^c$  | 0.062| 0.9            | 4                  | 12                  | 0                 |
| A1795$^c$  | 0.063| 1.6            | 22                 | 47                  | 0                 |
| A2744$^d$  | 0.308| 2.4            | 26                 | 13                  | 0                 |
| AS1063$^d$ | 0.348| 2.5            | 33                 | 6                   | 6                 |

Sample 3: UDGs in eight low-$z$ clusters selected by van der Burg et al. (2016). Only the UDG candidates with circular effective radii of $r_{\text{e},c} = r_{\text{e}} \sqrt{b/a} \in (1.5, 7.0)$ kpc ($b/a$ denotes the elongation of a galaxy), $\langle \mu_e(r) \rangle \in (24.0, 26.5)$ mag arcsec$^{-2}$, and $n < 2$ are included. These UDGs are distributed in $R_{200}$.

Sample 4: the publicly available 1 UDG sample in the two intermediate-$z$ clusters, Abell 2744 and Abell S1063 (Lee et al. 2017). UDGs were selected with $r_{\text{e},c} > 1.5$ kpc and $\langle \mu_e(r) \rangle > 23.8$ mag arcsec$^{-2}$ ($\langle \mu_e(r) \rangle = \langle \mu_{\text{e,z}}(r) \rangle - 10 \log(1 + z) - E(z) - K(z)$ (Graham & Driver 2005), where $\langle \mu_{\text{e,z}}(r) \rangle$ and $\langle \mu_{\text{e,x}}(r) \rangle$ are the mean surface brightness at $z = 0$ and $z$, respectively, and the values of $E(z)$ and $K(z)$ are -0.36 and +0.11 for Abell S1063, and -0.32 and +0.09 for Abell 2744, respectively; see Lee et al. 2017) which corresponds to the surface brightness criterion of sample 3, i.e., $\langle \mu_{\text{e,z}}(0.055(r)) \rangle > 24.0$ mag arcsec$^{-2}$. 90% UDGs show $n < 2$.

The apparent axis ratio $q = b/a$ and its error for the spheroid of each UDG in the four samples were obtained from GALFIT (Peng et al. 2002, 2010) fitting with a Sersic profile$^2$.

2.2. Apparent axis ratios of UDGs

Note that the four UDG samples were detected by the different telescopes, and thus the faint-ends (for the four samples, the UDG faint-ends of $\langle \mu_{\text{e,abs}}(r) \rangle$ are $\sim 27.0$, 26.5, 26.5, and 26.6 mag arcsec$^{-2}$, respectively) and detection completeness of the four UDG samples are slightly different. If we assume that the detection completeness of the faintest UDGs is related to their apparent axis ratios (i.e., the edge-on (face-on) oblate (prolate) galaxies perhaps are easier to be detected due to their brighter surface brightness, compared with the face-on (edge-on) ones with the same intrinsic 3D light distribution), the incompleteness of the faint-end UDGs may therefore introduce a bias in the following studies of intrinsic morphologies. Analogously, the four UDG samples adopt a similar UDG bright-end definition of $\langle \mu_{\text{e,abs}}(r) \rangle > 24$ mag arcsec$^{-2}$, which may also cause an absence of the edge-on (face-on) oblate (prolate) UDGs and lead to a bias in this work. Therefore, for each UDG sample, we change the surface brightness faint-end and bright-end of selecting UDGs and test whether the distribution of the apparent axis ratios $q = b/a$ changes with the different criteria. As explored in Fig. 1, for each UDG sample, the $q$ distributions for the different faint-ends and bright-ends are consistent with each other within the 1σ Poisson errors, and the Kuiper tests between the subsamples (colored) with the different criteria and the entire sample (black) also return large $p$ values, suggesting that the faint- and bright-ends for the four UDG samples will not affect our following studies of UDG intrinsic morphologies.

In Fig. 2, we compare the $q$ distributions of UDGs in the four different samples. We find that the three low-$z$ UDG samples show vastly similar $q$ distributions, i.e., flat in the range of $q \in [0.4, 0.9]$ but decrease drastically in the ranges $q < 0.4$ and $q > 0.95$. Therefore, hereafter the three samples will be combined and treated as one low-$z$ UDG sample in the following studies. We also find that, compared with the $q$ distributions of the low-$z$ samples (the blue, magenta, and green histograms), the intermediate-$z$ UDG sample (the red histogram) exhibits a plausibly less-flat distribution, which resembles a ‘double-peak’ distribution peaking at $q \approx 0.53$ and 0.77, respectively. It may imply a UDG morphology evolution with redshifts.

It has been known that the properties of UDGs are relevant to environment and stellar mass (Román & Trujillo 2017a; Gu et al. 2018). For the low-$z$ UDGs, in order to investigate whether their morphologies evolve from the outside to the inner regions of galaxy clusters, we split the low-$z$ UDGs into three groups, i.e., the inner sample within $R/R_{200} \leq 0.5$ ($R$ denotes the projected distance from a galaxy to the center of

1 http://vizier.cfa.harvard.edu/viz-bin/VizieR?-source=IApJ/844/157

2 These UDGs have been visually inspected to verify whether GALFIT provides a proper fit to the data; the bad ones were abandoned by those authors.
Figure 1. The four panels show the distributions of $q$ for the four different UDG samples, described in section 2.1. In each panel, the black, blue, red, and green histograms highlight the distributions of $q$ for the whole sample and UDGs with $\langle \mu_{e,abs}(r) \rangle < 26.0 \, \text{mag arcsec}^{-2}$, $\langle \mu_{e,abs}(r) \rangle < 25.0 \, \text{mag arcsec}^{-2}$, and $\langle \mu_{e,abs}(r) \rangle > 24.5 \, \text{mag arcsec}^{-2}$, respectively. The values $N$ and $p(K)$ in the brackets indicate the numbers of UDGs and $p$ values returned from the Kuiper tests with the entire sample (black) for the subsamples (colored) with the different criteria, respectively.

Figure 2. The apparent axis ratio $q = b/a$ distributions of the four UDG samples studied in this work. Hereafter, the error bars in all of the $q$ distributions assume Poisson statistics.
In this section, we will take advantage of a Markov Chain Monte Carlo (MCMC) approach to investigate the intrinsic morphologies of UDGs in the different samples. The method described in Sánchez-Janssen et al. (2016) is used to analyze the intrinsic morphologies of UDGs. Here we briefly outline the procedure.

1. **UDGs Morphology**

   **3.1. Modelling**

   In this section, we will take advantage of a Markov Chain Monte Carlo (MCMC) approach to investigate the intrinsic morphologies of UDGs in the different samples. The method described in Sánchez-Janssen et al. (2016) is used to analyze the intrinsic morphologies of UDGs. Here we briefly outline the procedure.

   **Figure 3.** Upper: $q$ distributions for the low-$z$ UDGs located in the inner ($R \leq 0.5R_{200}$; red), median ($0.5R_{200} \leq R < R_{200}$; green), outer ($R > R_{200}$; blue), and innermost ($R < 0.2R_{200}$) regions, respectively. Lower: $q$ distributions for the low-$z$ more-luminous ($M_r < -15.2$ mag; red) and less-luminous ($M_r > -15.2$ mag; blue) UDGs, respectively.

   **Figure 4.** Panel A: the distributions of $R/R_{200}$ for the low-$z$ high-mass ($M_r < -15.2$ mag; red) and low-mass ($M_r > -15.2$ mag; blue) UDGs. Panel B: the distributions of $q$ for the low-$z$ high-mass (red) and low-mass (blue) UDGs located in $R < R_{200}$ (solid) and $R > R_{200}$ (dashed), respectively. Panel C: the colored points show $q$ versus $R/R_{200}$ for the entire low-$z$ UDG sample; their colors denote $M_r$, as shown by the inset color bar; the error bars indicate the median values and 1σ scatters of $q$ and $R/R_{200}$ for the low-$z$ high-mass (red) and low-mass (blue) UDGs located in $R \leq 0.5R_{200}, 0.5R_{200} < R \leq R_{200},$ and $R > R_{200}$.

   **El-Badry et al. 2016**, etc. For the intermediate-$z$ UDG sample, because of the small galaxy number, we cannot split them into the different $R/R_{200}$ ranges and study their possible morphology evolution with the weak statistical power.

   We also divide the low-$z$ UDG sample into two subsamples with the relatively high ($M_r < -15.2$ mag) and low ($M_r > -15.2$ mag) luminosities (the low-$z$ UDG luminosities range in $M_r \in (-18, -12)$ mag, and $M_r = -15.2$ mag is the median luminosity; see also Fig. 11), and compare their distributions in the lower panel of Fig. 3. UDGs with the lower luminosities seem to be more elliptical compared with the ones with the relatively higher luminosities. Since the low-$z$ cluster UDGs show similar colors (e.g., Rong et al. 2017a), we can roughly assume a uniform stellar mass-to-light ratio\(^1\) for these UDGs; therefore, the results also suggest that the high-mass UDGs are rounder than the low-mass ones.

   Note that, if the spatial distribution preferences of the high-mass and low-mass UDGs are different, the dependence of $q$ on luminosity/stellar mass may be actually caused by the dependence on environment, or vice versa. Therefore, we compare the distributions of $R/R_{200}$ for the high-mass and low-mass UDG samples, as shown in panel A of Fig. 4; we also divide both of the high-mass and low-mass UDG samples into the inner ($R < R_{200}$) and outer ($R > R_{200}$) subsamples, and compare their axis ratios in panels B and C of Fig. 4. Apparently, the spatial distributions of the high-mass and low-mass UDGs are barely different (panel A), whereas the $q$ distributions for the four subsamples with the different luminosities and locations are quite different (panel B). We find that, both the high-mass and low-mass UDG samples located in $R > R_{200}$ are more elliptical compared with their counterparts located in $R < R_{200}$; both of the inner-region and outer-region low-mass UDGs are more elliptical compared with the high-mass counterparts. The results indicate that the morphologies of low-$z$ UDGs depend on both of the luminosity and environment.

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\(^1\) $M_r/L_r \sim 1.96$ (Carleton et al. 2019); $M_r \in (-18, -12)$ mag roughly corresponding to $(10^7, 10^8) M_{\odot}$; $M_r = -15.2$ mag roughly corresponding to $10^8.2 M_{\odot}$. 
Figure 5. Posterior probability density functions of $\bar{E}$, $\sigma_E$, $\bar{T}$, and $\sigma_T$ for sample 1 of low-z UDGs in the Coma cluster. The panels in the diagonal show the posterior pdfs for each of the parameters, marginalized over all the other ones. The grey scale in the non-diagonal panels shows the corresponding joint posterior pdfs. Contours enclose the regions that contain 68% of the cumulative posterior probability. The dotted and dashed lines in the diagonal panels indicate the 50% and 16% and 84% of the corresponding marginalized posteriors, respectively.

the key points of the method. In this method, the galaxies in each sample are modelled as a family of optically-thin triaxial ellipsoids. The 3D galaxy density is structured as a set of coaligned ellipsoids characterized by a common ellipticity $E = 1 - C/A$, and a triaxiality $T = (A^2 - B^2)/(A^2 - C^2)$, where $A \geq B \geq C$ are the intrinsic major, intermediate, and minor axes of the ellipsoid, respectively (Franx et al. 1991). The purely prolate (oblate) model corresponds to $T \simeq 1$ ($T \simeq 0$).

For each UDG sample, their $E$ and $T$ are proposed to follow Gaussian distributions, with mean values and standard deviations of $\bar{E}$, $\sigma_E$, $\bar{T}$, and $\sigma_T$. Given the distribution of intrinsic axis ratios and random viewing angles for the model galaxies, the distribution of apparent axis ratios $q$ can be derived via projecting these ellipsoids (Rong et al. 2015a, see their Appendix for the projecting method details). Therefore conversely, the posterior probability density function (pdf) of the model parameters $\bar{E}$, $\sigma_E$, $\bar{T}$, and $\sigma_T$, representing the intrinsic shapes of each UDG population can also be inferred by applying a Bayesian framework (see Sánchez-Janssen et al. 2016), and assuming the prior probabilities of $\bar{E}$ and $\bar{T}$ to follow uniform distributions in $[0, 1]$, as well as $\sigma_E$ and $\sigma_T$ to follow $p(\sigma) \propto \sigma^{-1}$. We implement the EMCEE code (Foreman-Mackey et al. 2013) to sample the posterior distribution of the model parameters with 200 ‘walkers’ and 1500
steps (the steps are sufficient for the MCMC chains to reach equilibrium).

The modelling results for the different UDG samples are summarized in Table 2. In Figs. 5, 6, 7, and 8, we plot the posterior distributions of $\bar{E}$, $\sigma_E$, $\bar{T}$, and $\sigma_T$ for the low-$z$ and intermediate-$z$ UDG samples, respectively. The posterior distributions for $\bar{E}$ and $\sigma_E$ approximately resemble the single Gaussian distributions. However, for the low-$z$ UDG samples, the posterior distributions of $\bar{T}$ always show double-peaks (see also Appendix A), i.e., one pronounced peak at $\bar{T} \sim 0.3 - 0.4$ and one weak peak at $\bar{T} \sim 1.0$; for the intermediate-$z$ sample, the posterior distribution of $\bar{T}$ exhibits no significant $\bar{T} \sim 1.0$ peak. The two $\bar{T}$ peaks indicate two different probable underlying shapes for the low-$z$ UDGs: the left $\bar{T} \sim 0.3 - 0.4$ peak corresponds to a triaxial model, and right $\bar{T} \sim 1.0$ peak corresponds to a purely-prolate model. We note that, although the precise triaxiality distribution can only be well derived with a synergy of structural and kinematical data (e.g., Franx et al. 1991; van den Bosch et al. 2009; Rong et al. 2018a; Chilingarian et al. 2019), rather than with the photometric data alone, yet the data clearly favor triaxial models over nearly prolate ones.\(^1\)

The discovery of the triaxial UDG morphologies is in consistent with the hydrodynamical simulation results (e.g., Jiang et al. 2018) but in conflict with the conclusion of Burkert (2017). This conflict is due to the fact that Burkert (2017) only compared the purely-oblate and purely-prolate models.

\(^1\)We also note that, in theory, one can alternatively assume that one UDG sample is made up of two divergent populations with $\bar{T} \sim 0.3 - 0.4$ and $\bar{T} \sim 1.0$, respectively; however, we prefer a simpler model with only one set of ellipticity and triaxiality parameters to describe the intrinsic morphology of cluster UDGs in one sample, particularly in the case that the triaxial model alone can well recover the $q$ distributions of UDGs.
Figure 7. Analogous to Fig. 5, posterior probability density functions of $\bar{E}$, $\sigma_E$, $\bar{T}$, and $\sigma_T$ for sample 3 of low-$z$ UDGs in 8 nearby clusters.

3.2. UDG morphology evolution with masses

As listed in Table 2, the intrinsic morphologies of the high-mass and low-mass low-$z$ UDGs are significantly different, regardless of their environments. In the upper panel of Fig. 9, we compare the median intrinsic thickness, $C/A$, for the high-mass (red) and low-mass (blue) UDGs located in $R \leq 0.5 R_{200}$ (solid error bars), $0.5 R_{200} < R \leq R_{200}$ (dotted), and $R > R_{200}$ (dashed), respectively. We find that, 1) for the UDGs located in the same environment, the high-mass ones are always thicker, puffier, than the low-mass ones; 2) the morphology difference between the high-mass and low-mass ones is present in both of $R < R_{200}$ and $R > R_{200}$.

Since the thickness difference between the high-mass and low-mass UDGs always exists in the different environments, it may be originated from internal processes, e.g., supernovae feedback. Star formation in dwarf galaxies is expected to oc-
Figure 8. Analogous to Fig. 5, posterior probability density functions of $\bar{E}$, $\sigma_E$, $\bar{T}$, and $\sigma_T$ for sample 4 of intermediate-$z$ UDGs in Abell 2744 and Abell S1063.

Occur in episodic bursts at almost all redshifts (Muratov et al. 2015), and the associated supernovae-driven outflows pressurize gas and heat\(^1\) the stellar orbits (Pontzen & Governato 2012; El-Badry et al. 2016; Teyssier et al. 2013; Governato et al. 2010). If UDGs are proposed to be produced by outflows (e.g., Di Cintio et al. 2017), as a consequence of more starbursts or a larger amount of star formation, the higher-mass UDGs should have delivered more energy to heat up the stellar random motions and thus become less flattened.

Galaxy merging can also lead to morphological transformation (e.g., Starkenburg et al. 2016a,b), and simultaneously, starbursts, which can remarkably boost stellar mass assembly. However, many simulations and observational results have excluded mergers to play any significant role in determining the morphologies of dwarf galaxies (e.g., Rodriguez-Gomez et al. 2017, 2015; Stewart et al. 2008), as the mergers of dwarf galaxies are very rare.

Although direct mergers are extremely rare, the rate of tidal encounters with more massive galaxies during flybys (including ‘harassment’ and tidal stirring; Moore et al. 1996; Mayer et al. 2001, 2007) is considerable, particularly in the massive clusters comprising many massive satellites. The tidal interactions can also efficiently puff up UDGs, transform the kinematic and stellar distributions to resemble the present-day dwarf ellipticals (dEs) and dwarf spheroidals (dSphs) (e.g., Carleton et al. 2019; Moore et al. 1996; Mayer et al. 2001; Errani et al. 2015). In simulations, the stellar disper-

\(^1\) Multiple supernovae explosions induce strong and repeated fluctuations in the dwarf gravitational potential, which result in energy transfer to the collisionless components (dark matter and stars).
### Table 2. The MCMC results of intrinsic morphology analysis for the different samples of low-z and intermediate-z UDGs. Col. (1): low-z or intermediate-z UDG sample; Col. (2): the UDG sample with the different properties; Col. (3): number of UDGs in each sample used for MCMC; Col. (4)-(7): the mean values and standard deviations of ellipticity and triaxiality distributions $E$, $\sigma_E$, $T$, and $\sigma_T$; Col. (8)-(9): the median ratios of three intrinsic axes $A : B : C$. 

| Redshift   | UDG samples | $N$   | $E$        | $\sigma_E$ | $T$        | $\sigma_T$ | $A : B : C$ |
|------------|-------------|-------|------------|------------|------------|------------|-------------|
| All (Sample 1+2+3) | 1109 | 0.51$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.34$^{+0.22}_{-0.05}$ | 0.21$^{+0.18}_{-0.08}$ | 1 : 0.86 : 0.49 |
| Sample 1   | 328 | 0.49$^{+0.01}_{-0.01}$ | 0.12$^{+0.02}_{-0.01}$ | 0.34$^{+0.20}_{-0.21}$ | 0.22$^{+0.37}_{-0.12}$ | 1 : 0.86 : 0.51 |
| Sample 2   | 442 | 0.52$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.35$^{+0.41}_{-0.13}$ | 0.29$^{+0.38}_{-0.17}$ | 1 : 0.85 : 0.48 |
| Sample 3   | 339 | 0.49$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.36$^{+0.41}_{-0.20}$ | 0.50$^{+0.31}_{-0.27}$ | 1 : 0.86 : 0.51 |
| $R \leq 0.5R_{200}$ | 471 | 0.48$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.37$^{+0.29}_{-0.12}$ | 0.36$^{+0.37}_{-0.19}$ | 1 : 0.85 : 0.52 |
| $0.5R_{200} < R \leq R_{200}$ | 443 | 0.50$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.35$^{+0.38}_{-0.15}$ | 0.38$^{+0.36}_{-0.20}$ | 1 : 0.86 : 0.50 |
| $R > R_{200}$ | 195 | 0.57$^{+0.01}_{-0.02}$ | 0.22$^{+0.02}_{-0.01}$ | 0.37$^{+0.55}_{-0.18}$ | 0.39$^{+0.38}_{-0.44}$ | 1 : 0.84 : 0.43 |
| $R \leq 0.2R_{200}$ | 96 | 0.47$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.29$^{+0.09}_{-0.07}$ | 0.06$^{+0.05}_{-0.05}$ | 1 : 0.89 : 0.53 |
| $M_r < -15.2$ | 543 | 0.49$^{+0.01}_{-0.02}$ | 0.12$^{+0.02}_{-0.01}$ | 0.35$^{+0.14}_{-0.08}$ | 0.24$^{+0.36}_{-0.10}$ | 1 : 0.86 : 0.51 |
| $M_r > -15.2$ | 566 | 0.52$^{+0.01}_{-0.01}$ | 0.11$^{+0.01}_{-0.01}$ | 0.33$^{+0.21}_{-0.15}$ | 0.37$^{+0.34}_{-0.18}$ | 1 : 0.86 : 0.48 |
| $M_r < -15.2 \& R \leq 0.5R_{200}$ | 230 | 0.45$^{+0.02}_{-0.02}$ | 0.13$^{+0.02}_{-0.02}$ | 0.38$^{+0.51}_{-0.18}$ | 0.34$^{+0.38}_{-0.29}$ | 1 : 0.86 : 0.55 |
| $M_r < -15.2 \& 0.5R_{200} < R \leq R_{200}$ | 216 | 0.45$^{+0.02}_{-0.02}$ | 0.14$^{+0.03}_{-0.02}$ | 0.67$^{+0.22}_{-0.29}$ | 0.34$^{+0.36}_{-0.26}$ | 1 : 0.73 : 0.55 |
| $M_r < -15.2 \& R > R_{200}$ | 97 | 0.51$^{+0.03}_{-0.02}$ | 0.12$^{+0.03}_{-0.02}$ | 0.36$^{+0.16}_{-0.16}$ | 0.17$^{+0.47}_{-0.15}$ | 1 : 0.85 : 0.49 |
| $M_r > -15.2 \& 0.5R_{200} < R \leq R_{200}$ | 241 | 0.51$^{+0.02}_{-0.02}$ | 0.11$^{+0.02}_{-0.02}$ | 0.44$^{+0.32}_{-0.10}$ | 0.28$^{+0.38}_{-0.15}$ | 1 : 0.82 : 0.49 |
| $M_r > -15.2 \& 0.5R_{200} < R \leq R_{200}$ | 227 | 0.54$^{+0.02}_{-0.02}$ | 0.10$^{+0.02}_{-0.02}$ | 0.27$^{+0.14}_{-0.14}$ | 0.26$^{+0.20}_{-0.20}$ | 1 : 0.89 : 0.46 |
| $M_r > -15.2 \& R > R_{200}$ | 98 | 0.65$^{+0.02}_{-0.02}$ | 0.04$^{+0.02}_{-0.02}$ | 0.56$^{+0.36}_{-0.21}$ | 0.34$^{+0.37}_{-0.26}$ | 1 : 0.71 : 0.35 |
| intermediate-z UDGs | All (Sample 4) | 84 | 0.54$^{+0.02}_{-0.02}$ | 0.07$^{+0.03}_{-0.02}$ | 0.40$^{+0.05}_{-0.05}$ | 0.02$^{+0.07}_{-0.02}$ | 1 : 0.83 : 0.46 |

1 There are also literatures discussing that the fainter LTGs, with absolute magnitudes of $M_g > -18$ mag, again show thicker morphologies as they get fainter (e.g., Sánchez-Janssen et al. 2010; Roychowdhury et al. 2013); however in this work, we only focus on the morphology trend for the massive LTGs with $M_g < -18$ mag.
Figure 9. The median intrinsic thickness $\bar{C}/A$ as a function of luminosities (upper panel) and $R/R_{200}$ (lower panel) for our low-$z$ UDG samples. In the two panels, the black, red, and blue points denote the all, high-mass, and low-mass UDGs, respectively. In the upper panel, the solid, dotted, and dashed error bars denote the UDGs located in $R \leq 0.5 R_{200}$, $0.5 R_{200} < R \leq R_{200}$, and $R > R_{200}$, respectively. In the lower panel, the vertical dotted line denotes the virial radius of $R = R_{200}$. For comparison, we also plot the thickness of quiescent dE/dSphs following RS in nearby clusters (Virgo+Fornax; magenta diamond; Sánchez-Janssen et al. 2019) and groups (Local Group, orange; Centaurus A, cyan; McNamachie 2012; Taylor et al. 2017, 2018; Sánchez-Janssen et al. 2019), as well as massive ETGs (dark-green triangles) and LTGs (light-green triangles) selected from SDSS catalog by Padilla & Strauss (2008) and Rodríguez et al. (2013). The gray shaded regions show the thickness trends for the UDG samples with the different luminosities and located in the different environments; the dark- and light-green shaded regions highlight the corresponding thickness trends for the massive ETGs and LTGs within the different luminosity ranges, respectively; the yellow shaded region reveals the thickness trend for the dE/dSphs with the different luminosities in nearby clusters and groups shown by Sánchez-Janssen et al. (2019). The stripping process for the UDGs in cored halos (see Carleton et al. 2019). Since tidal influences are more prominent in higher-density environments (i.e., smaller $R/R_{200}$), we should therefore expect the sizes of stellar components to change with $R/R_{200}$, but luminosities to remain roughly constant. As shown by the $M_r$ and $r_e$ (the median value and $1\sigma$ scatter are shown by the solid and dashed components, respectively) as functions of $R/R_{200}$ in Fig. 10, the sizes $r_e$ for both of the high-mass and low-mass UDGs indeed slightly increase with decreasing $R/R_{200}$, while there is no obvious radial gradient in luminosity among our low-$z$ UDG samples.

We note that a large/dominant fraction of the present-day cluster UDGs might not be accreted as UDGs (Ferré-Mateu et al. 2018; Alabi et al. 2018) but be transformed from typical dwarf progenitors under tidal interactions in clusters (e.g., Jiang et al. 2018; Liao et al. 2019); these ‘in-situ-

\[ r_{e,c} = r_e \sqrt{q} > 1.5 \text{ kpc} \] for Samples 1 and 2, and Sample 3 UDGs are only located in $R < R_{200}$; therefore, in order to show the unbiased trends, we only show the radial trends for the low-$z$ UDGs with $r_{e,c} > 1.5$ kpc.
transferred’ UDGs may weaken the averaged radial trends of \( r_e \) and morphological transformation of UDGs.

Apart from the contamination of the ‘in-situ-transformed’ cluster UDGs, the less efficient ‘environmental quenching’ (e.g., ram-pressure/tidal stripping or ‘strangulation’; Moore et al. 1996; Larson et al. 1980; Kawata & Mulchaey 2008; Bekki 2009; Mayer et al. 2007; Read & Gilmore 2005; Arraki et al. 2014) may also reconcile the pronounced thickening of UDGs from \( R > R_{200} \) to \( R < R_{200} \), but mild thickening from \( R < R_{200} \) to \( R < 0 \). In this scenario, the environmental quenching timescale may be relatively long (Wheeler et al. 2014), so that the recently-accreted UDGs, primarily occupy the peripheries of clusters, may continue to retain their gas reservoirs, which cause galaxies to respond more impulsively to tides, significantly augmenting their morphological transformation (Kazantzidis et al. 2017). However, most of the inner-region UDGs are devoid of gas, and thus their morphological transformation becomes less efficient, compared with the outer-region counterparts.

Another possibility is the contamination of interlopers. The UDG samples are selected based on their locations in the color-magnitude diagrams, and may include a fraction of reddened background massive star-forming interlopers. The contamination is more remarkable in the projected ‘outer-regions’ of clusters where the clusters become less over-dense and comprising member galaxies with larger color scatters (e.g., Lee & Jang 2016; Lee et al. 2017; van der Burg et al. 2016); therefore, these disky interlopers with small thickness, primarily located in the outer-regions of clusters, can introduce a more significant thickening of “UDGs” from \( R > R_{200} \) to \( R < R_{200} \). In addition, the mild radial trend in \( R_{200} \) may be partly attributed to projection effects, i.e., the so-called ‘inner-region’ (\( R < 0.5 R_{200} \)) UDG population may actually contains many projected median/outer-region UDGs.

### 3.4. UDG morphology evolution with redshifts

The morphological transformation of UDGs from the intermediate to low redshift is also investigated. Since only

![Figure 11](image1.png)

**Figure 11.** The distributions of \( M_r \) for the low-z samples located in \( R < R_{200} \) and intermediate-z sample.

\(~7\%~\) of the intermediate-z UDGs are located out of \( R_{200} \), the intermediate-z UDGs can be roughly treated as a cluster-UDG sample; besides, as shown in Fig. 11, the luminosities of the intermediate-z UDG sample and low-z \( R \leq R_{200} \) sample are approximately in the same \( M_r \) range, with a median \( M_r \) value of \(-15.3\), and \(-15.2\) mag, respectively. Therefore, the intermediate-z UDG sample can be directly compared with the low-z \( R \leq R_{200} \) UDG sample.

As explored in Fig. 12, the median intrinsic thickness of cluster UDGs slightly increases from intermediate to low redshift, probably suggesting that the cluster UDGs are marginally puffed up from \( z \sim 0.35 \) to 0. Therefore naively, we can suspect that the high-redshift, initial UDGs may be more flattened, and plausibly have a ‘disky’ morphology. If we treat these intermediate-z UDGs as the progenitors of the present-day cluster UDGs, it may imply that UDGs were originated from a formation mechanism (e.g., high-spins of halos; Amorisco & Loeb 2016; Rong et al. 2017a) which can produce the ‘disky’ morphologies in the first place.

Here, we also point out that the morphology difference between the high- and low-redshift UDGs may be driven by the small UDG sample from two intermediate-z clusters, and thus the morphological transformation trend has large uncertainties and requires further confirmation with larger intermediate-z UDG samples.

### 4. SUMMARY AND DISCUSSION

With the data of apparent axis ratios for 1109 UDGs located in 17 low-z (\( z \sim 0.020 \) – 0.063) galaxy clusters and 84 UDGs in 2 intermediate-z (\( z \sim 0.308 \) – 0.348) clusters, we implement a Markov Chain Monte Carlo technology and assume a ubiquitous triaxial model to study the intrinsic mor-
phologies of UDGs. In contrast to the conclusion of Burkert (2017), we emphasize that the UDG data favor the oblate-triaxial models over purely-prolate models.

The morphologies of UDGs are relevant to luminosity, environment, and redshift. The high-mass (more-luminous) low-$z$ UDGs are always thicker than the low-mass (less-luminous) counterparts, regardless of their environments. It is possibly due to the more violent internal supernovae feedback or external tidal interactions for the progenitors of high-mass UDGs. The UDG thickness dependence on luminosity is distinct from that of the typical quiescent dE/dSphs in nearby clusters and groups, but resembles that of massive galaxies, which probably implies that UDGs may not be simply treated as an extension of the dE/dSph class with similar evolutionary histories.

Both of the high-mass and low-mass UDGs are remarkably puffed up from the outside to inside of the virial radii of clusters, but show relatively mild dependence on cluster-centric distance in clusters, which may be attributed to tidal interactions; the ‘in-situ’ formed UDG population (i.e., UDGs transformed from typical dwarfs in clusters by tides), environmental quenching, contamination of background interlopers, as well as projection effects, may be crucial for explaining the remarkable morphological transformation of UDGs from $R > R_{200}$ to $R < R_{200}$ but mild morphological transformation within clusters.

From intermediate to low redshift, the morphologies of cluster UDGs become marginally puffier, and have broader ranges of ellipticity and triaxiality, plausibly suggesting a formation mechanism producing ‘disky’ morphologies for the high-redshift, initial UDG progenitors, e.g., being formed in the high-specific-angular-momentum halos (Amorisco & Loeb 2016; Rong et al. 2017a). However, the number of UDGs at relatively high redshifts is small; we strongly encourage further high-spatial-resolution, deep surveys for high-redshift UDGs to examine this conclusion.

Note that, the supernovae-feedback heating and tidal interactions can also alleviate the tension between the detected high-specific-angular-momenta of the field UDGs (Leisman et al. 2017) and large dispersion or no signs for rotation in cluster-UDGs (e.g., van Dokkum et al. 2016; Danieli et al. 2019; Chilingarian et al. 2019): the high-redshift, rotationally-supported, field UDGs (resembling present-day dwarf irregulars) might be originated in the halos with high specific angular momenta, and have initial shapes with relatively small thickness; when UDGs were accreted to high-density environments and perturbed, the tidal interactions and stellar-feedback heating can transform the kinematic and stellar distributions to resemble dEs, dS0s, and dSphs (Ferré-Mateu et al. 2018; Alabi et al. 2018; Vijayaraghavan & Ricker 2013; Vijayaraghavan et al. 2015; De Lucia et al. 2012; Rong et al. 2015b, 2016; Cortese et al. 2006; Zabludoff & Mulchaey 1998; McGee et al. 2009; Peng et al. 2012).

We acknowledge the publicly available UDG catalogs in Yagi et al. (2016) and Lee et al. (2017). Y.R. acknowledges the helpful comments and suggestions from M. G. Lee, R. Sánchez-Janssen, L. Gao, Q. Guo, S.-H. Liao, and J. Wang, as well as funding supports from FONDECYT Postdoctoral Fellowship Project No. 3190354 and NSFC grant No.11703037. T.H.P. acknowledges support through FONDECYT Regular project 1161817 and CONICYT project Basal AFB-170002. This work is also supported by CAS South America Center for Astronomy (CASSACA), Chinese Academy of Sciences (CAS).

This research has made use of the NASA Astrophysics Data System Bibliographic Services, the NASA Extragalactic Database, PYTHON/EMCEE v.2.2.1 (Foreman-Mackey et al. 2013, https://emcee.readthedocs.io/en/v2.2.1/) package, GALFIT (Peng et al. 2002, 2010), as well as ds9 (a tool for data visualization supported by the Chandra X-ray Science Center (CXC) and the High Energy Astrophysics Science Archive Center (HEASARC) with support from the JWST Mission office at the Space Telescope Science Institute for 3D visualization). We also acknowledge the related literatures of Goodman & Weare (2010); Jones et al. (2001); VanderPlas et al. (2012); Rong et al. (2018b, 2017b); Astropy Collaboration et al. (2013); Hunter (2007).

A. MCMC RESULTS VERSUS OBSERVATIONS

In this section, we test whether the MCMC results are robust and can well recover the observed $q$ distributions of UDGs. Particularly for the low-$z$ samples, the $\bar{T}$ posterior pdfs actually contain two peaks (local maximum likelihood) overlapping at about $T \simeq 0.8$ (see Figs. 5, 6, and 7; the primary peak at $\bar{T} \sim 0.3$ – 0.4 and secondary peak at $\bar{T} \sim 1.0$), and the posterior pdfs of $\bar{T} < 0.8$ resembling Gaussian distribution correspond to the triaxial models, while posterior pdfs of $\bar{T} > 0.8$ suggest the nearly-prolate models.

It is worth to note that, in MCMC, these $\bar{T} > 0.8$ steps are not ‘burn-in’ steps which should be discarded. Indeed, the MCMC chain converges to an equilibrium distribution after approximately 200 ‘burn-in’ steps (Sánchez-Janssen et al. 2016) and we actually apply ‘nburn’=400 in the affine-invariant MCMC algorithm implemented in the Python EMCEE package (Foreman-Mackey et al. 2013). We also set ‘nburn’=500, 1000, 2000, and 4000 (remain the other parameters unchanged), and find that the $\bar{T}$ posterior pdfs of the low-$z$ samples always show double-peaks.
Figure A.1. The comparison between the observational $q$ distributions (solid histograms) and recovered distributions by the triaxial models (with the sets of $(\bar{E}, \sigma_E, \bar{T}, \sigma_T)$ corresponding to $\bar{T} < 0.8$; dotted) and purely-prolate models (with the sets of $(\bar{E}, \sigma_E, \bar{T}, \sigma_T)$ corresponding to $\bar{T} > 0.8$; dashed), derived from MCMC, for the four UDG samples.

Further, as shown in Fig. A.1, we find that both of the triaxial models and nearly prolate models generated from MCMC can well recover the observed $q$ distributions. In summary, the MCMC results are robust and the $\bar{T} > 0.8$ steps are not ‘burn-in’ steps.

Actually, compared with the nearly prolate models, the triaxial models can reproduce the marginally flatter $q$ distributions in the range of $q \sim 0.6 - 0.9$, more closely resembling the observed $q$ distributions, as shown in Fig. A.1
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