Electroweak Gauge-Higgs Unification Scenario

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Abstract. In the gauge-Higgs unification scenario 4D Higgs fields are unified with gauge fields in higher dimensions. The electroweak model is constructed in the Randall-Sundrum warped space. The electroweak symmetry is dynamically broken by the Hosotani mechanism due to the top quark contribution. The Higgs mass is predicted to be around 50 GeV with the vanishing ZZH and WWH couplings so that the LEP2 bound for the Higgs mass is evaded.

Keywords: gauge-Higgs unification, Hosotani mechanism

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Introduction

We are in the hunt for the Higgs particle. It is expected to be discovered at LHC in the near future. In the standard model of the electroweak interactions the Higgs particle is necessary to induce the electroweak symmetry breaking, but it is not obvious if the Higgs particle appears as described in the standard model.

The Higgs sector in the standard model is not completely satisfactory. It lacks underlying principles. Is there a principle governing the Higgs field? What is the origin of the Higgs particle? After all, what is the mechanism of the electroweak gauge symmetry breaking?

The gauge-Higgs unification scenario tries to answer to these questions.

It identifies the Higgs field as a part of gauge fields in higher dimensional theory. The 4D Higgs field is unified with gauge fields, the Higgs interactions being controlled by the gauge principle, once the back-ground spacetime is specified. In this scenario the electroweak symmetry is dynamically broken, the Higgs mass being predicted at a finite value. The Higgs couplings to the W and Z bosons and fermions deviate from those in the standard model, which can be checked experimentally in the near future.

EW Gauge-Higgs unification

We consider a gauge theory defined in higher dimensions with not-simply-connected extra-dimensional space. There appear Aharonov-Bohm (AB) phases along the extra dimension, which, though giving vanishing field strengths, become physical degrees of freedom. 4D Higgs fields are nothing but 4D fluctuation modes of such AB phases. In terms of gauge potentials \( A_M(x,y) = (A_\mu, A_\pi) \), AB phases are generated by zero modes of the extra-dimensional components \( A_\pi(x,y) \). In non-Abelian gauge theory those AB phases can develop non-trivial vacuum expectation values at the quantum level, inducing dynamical gauge symmetry breaking.

This scenario has many attractive features. The Higgs particle is massless at the tree level, as AB phases \( \theta_H \) give vanishing field strengths. The effective potential for the AB phases \( V_{\text{eff}}(\theta_H) \), however, becomes nontrivial at the quantum level. The Higgs mass, which is proportional to the curvature of \( V_{\text{eff}} \) at its global minimum, is generated.

Thanks to the gauge invariance the mass is predicted at a finite value, irrespective of an ultra-violet cutoff introduced, which can be used for solving the gauge hierarchy problem.

Further the Higgs couplings are determined by the gauge principle.

There are several key ingredients in applying the scenario to the electroweak interactions.

1. Larger gauge group \( G \)

In the gauge-Higgs unification the Higgs field is a part of gauge fields which are in the adjoint representation of the gauge group \( G \). This implies that one needs to start with a larger gauge group \( G \) which contains \( SU(2)_L \times U(1)_Y \) as a subgroup. Examples are \( SU(3), SU(3) \times U(1) \times U(1), \) and \( SO(5) \times U(1) \).

2. Orbifolds

As an extra-dimensional space we take an orbifold. The simplest example is \( S^1/Z_2 \) in which the points \( y, y+2\pi R, \) and \( -y \) are identified. Physics must be the same at those points, but gauge potentials need not. Gauge potentials obey, around two fixed points \( y_0 = 0 \) and \( y_1 = \pi R \),

\[
\left( \begin{array}{c} A_u \\ A_\pi \end{array} \right) (x,y_j-y) = P_j \left( \begin{array}{c} A_u \\ -A_\pi \end{array} \right) (x,y_j+y) P_j^+, \quad (1)
\]

where \( P_j = P_j^{-1} \in G \). It follows that \( A_M(x,y+2\pi R) = U A_M(x,y) U^\dagger \) where \( U = P_1 P_0 \). The Lagrangian density remains invariant under the parity transformations. The
set \( \{P_0, P_1\} \) defines the orbifold boundary conditions (BC).

3. Four-dimensional Higgs

4D Higgs fields reside in the \( A_y \) components which are even under \( P_0 \) and \( P_1 \). Take \( G = SO(5) \) and \( P_0 = P_1 = \text{diag} \ (-1, -1, -1, -1, 1) \). With this orbifold BC \( SO(5) \) breaks down to \( SO(4) \). \( A_y \)'s have zero modes (4D gauge fields) in the diagonal \( SO(4) \simeq SU(2)_L \times SU(2)_R \). \( A_y \) on the other hand, has zero modes in the off-diagonal parts with * in

\[
A_y \sim \begin{pmatrix}
* & * & * \\
* & * & * \\
* & * & * \\
\end{pmatrix}.
\]

(2)

The zero mode multiplet is an \( SO(4) \) vector, or a \( (2,2) \) representation of \( SU(2)_L \times SU(2)_R \). It can be identified with the EW Higgs field.

The 4D Higgs field naturally appears from the orbifold BC. The AB phase, or the Wilson line phase, is given by

\[
ed^{i\theta_H/2} = P \exp \left \{ ig \int_{y_0}^{y_1} dy A_y \right \}.
\]

(3)

4. Chiral fermions

Another virtue of the orbifold structure is that it naturally gives rise to chiral fermions. Take a vector fermion multiplet \( \Psi \) in the \( SO(5) \) model above. The orbifold BC for \( \Psi \) is given by

\[
\Psi(x,y,y-y) = \pm P_i \gamma^5 \Psi(x,y,y+y).
\]

(4)

The factor \( \gamma^5 \) is necessary to assure the invariance of \( \Psi_i (\gamma^\mu D_\mu + \gamma^5 D_5) \Psi \). With + sign in (2), the first four components of \( \Psi \) have zero modes only for \( \gamma^5 = -1 \) (left-handed components), whereas the fifth component has a zero mode only for \( \gamma^5 = 1 \) (a right-handed component). All the massive Kaluza-Klein excited states appear vector-like, but the lowest, light modes appear chiral.

5. Flat v.s warped

With all virtues of orbifolds, many models of electroweak interactions have been constructed. [7, 11] The value of the Wilson line phase \( \theta_H \) is determined once the matter content is specified. With simple, minimal matter content the effective potential \( V_{\text{eff}}(\theta_H) \) is minimized, typically, either at \( \theta_H = 0 \) or at \( \theta_H = O(1) \). In the former case the EW symmetry is unbroken, whereas in the latter case the symmetry generally breaks.

In models in flat space, say, on \( M^4 \times (S^1/Z_2) \), the Kaluza-Klein scale is given by \( m_{KK} = 1/R \) where \( R \) is the radius of \( S^1 \). With \( \theta_H = O(1) \) the W boson acquires a mass \( m_W \sim (\theta_H/2\pi)m_{KK} \), which leads to a too low \( m_{KK} \). Similarly, the Higgs mass is generated at the one loop level so that \( m_H \sim \sqrt{\eta_m}(2\pi/\theta_H)m_W \) where \( \eta_m = g^2_\text{W}/4\pi \). This leads to a too small \( m_H \sim 10\text{GeV} \).

Thirdly, the WWZ coupling deviates from the value in the standard model as the wave functions of \( W \) and \( Z \) in the fifth dimension \( y \) acquire significant dependence on \( y \), which contradicts with the LEP2 experiment.

There are two approaches to solve these problems. One way is to stay in flat space and tune the matter content such that \( V_{\text{eff}}(\theta_H) \) is minimized at a small value for \( \theta_H \). For instance, one can introduce many matter multiplets, or even supersymmetry, to have cancellation among dominant parts of the contributions to \( V_{\text{eff}} \). Or, one can incorporate quarks in several representations of the gauge group to have small \( \theta_H \).

An alternative way is to consider models in the curved space, particularly in the Randall-Sundrum (RS) warped space. [12] It is remarkable that all the problems mentioned above are naturally solved in the RS space.

Models in the warped space

The metric in the Randall-Sundrum (RS) warped spacetime is given by

\[
ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,
\]

(5)

where \( \eta_{\mu\nu} = \text{diag} (-1, 1, 1, 1) \), \( \sigma(y) = \sigma(y+2L) \), and \( \sigma(y) \equiv k|y| \) for \( |y| \leq L \). The extra-dimensional space has the topology of \( S^1/Z_2 \). The fundamental region is given by \( 0 \leq y \leq L \). The bulk region \( 0 < y < L \), which is a sliced AdS spacetime with a negative cosmological constant \( \Lambda = -6k^2 \), is sandwiched by the Planck brane at \( y = 0 \) and the TeV brane at \( y = L \). A large warp factor \( z_L = e^{L} \gg 1 \) bridges the Planck scale and the weak scale. The KK mass scale for fields defined in the bulk is

\[
m_{KK} = \frac{\pi k}{e^{L} - \pi k} \sim \pi ke^{-kL}.
\]

(6)

1. Gauge group

At the moment the most promising model is a model based on \( SO(5) \times U(1)_X \) gauge symmetry. [13, 14, 15] The orbifold boundary condition \( \{P_0, P_1\} \) is given by \( P_0 = P_1 = \text{diag} (-1, -1, -1, -1, 1) \) in the SO(5) part, which reduces the residual symmetry to \( SO(4) \times U(1)_X \simeq SU(2)_L \times SU(2)_R \times U(1)_X \). On the Planck brane the symmetry \( SU(2)_R \times U(1)_X \) is further spontaneously broken by additional brane dynamics to \( U(1)_Y \). The residual symmetry is \( SU(2)_L \times U(1)_Y \).

2. Higgs field

There appear zero modes in \( A_y \), as shown in (2), which become the 4D Higgs fields \( \Phi_H(x) \) in the EW theory. The relation in \( 0 \leq y \leq L \) is given by

\[
A_y^\text{eff}(x,y) = \Phi_H(x) \sqrt{\frac{2k}{z_L} - 1} e^{2ky} + \cdots.
\]
\[ \Phi_H(x) = \frac{1}{\sqrt{2}} \left( \phi^2 + i\phi^3 \right). \]  

We note that the Higgs field is localized near the TeV brane in the warped space. When \( \phi^\mu = v \delta^{\mu4} \), the Wilson line phase is given by

\[ \theta_H = \frac{g_A v}{2} \sqrt{\frac{Z^2 - 1}{k}} \]  

where \( g_A \) is the gauge coupling of SO(5) related to the 4D SU(2)_L weak coupling by \( g_W = g_A/\sqrt{L} \).

3. Photon, W and Z

\( U(1)_\text{EM} \) remains as an exact symmetry. The photon wave function is constant in the y coordinate, being completely flat and independent of \( \theta_H \). With \( \theta_H \neq 0 \) the EW symmetry breaks and W and Z become massive. For large \( ZL \)

\[ m_W \sim \sqrt{\frac{k}{L}} e^{-kL} |\sin \theta_H| \sim \frac{m_{KK}}{\pi \sqrt{kl}} |\sin \theta_H|. \]  

With \( |\sin \theta_H| = O(1) \) one finds that for \( ZL \sim 10^{15}(10^{17}) \), \( k \sim 5 \times 10^{17} \) (5 \( \times 10^{19} \)) GeV and \( m_{KK} \sim 1.5 \) (1.6) TeV. The Z boson mass is given by

\[ m_Z \sim \frac{m_W \cos \theta_W}{\cos \theta_W}, \quad \sin \theta_W = \frac{g_B}{\sqrt{g_B^2 + 2g_B^2}} \]  

where \( g_B \) is the gauge coupling of \( U(1)_X \).

As the EW symmetry breaks down, the wave functions of W and Z acquire y-dependence. However, as \( \langle A_Y \rangle \) or the Higgs field is localized near the TeV brane, the W and Z wave functions remain almost flat in the bulk, nontrivial y-dependence appearing only near the TeV brane.

4. Gauge self-couplings

In the standard model the gauge coupling is universal. Gauge couplings of quarks, leptons and the Higgs field as well as WWZ, WWWW, WWZZ couplings are all specified with the two gauge coupling constants of SU(2)_L and U(1)_Y. In higher dimensional theory this is not the case anymore. The reason is that four-dimensional gauge couplings are determined by overlap integrals of wave functions of associated fields in the extra dimension. The only exactly-universal coupling is the electromagnetic coupling associated with the exact symmetry of \( U(1)_\text{EM} \).

This poses us a challenging test for higher dimensional theory. The WWZ coupling has been already measured indirectly at LEP2. The coupling agrees with that in the standard model within a few percent.

In the gauge-Higgs unification scenario the gauge self-couplings and gauge-Higgs couplings in four dimensions are determined from the \( Tr F_{\mu \nu} F_{\mu \nu} \) term. In particular the part \( Tr F_{\mu \nu} F_{\mu \nu} \) (\( \mu, \nu = 0, \cdots, 3 \)) gives the WWZ, WWWW, WWZZ couplings. At \( \theta_H = 0 \) these couplings reduce to those in the standard model.

In the Randall-Sundrum spacetime, as mentioned above, the wave functions of W and Z remain almost y-independent even at \( \theta_H \neq 0 \), though the weight in the group components have significant \( \theta_H \) dependence. Thanks to this property the WWZ, WWWW, WWZZ couplings remain almost universal. For instance, the deviation of the WWZ coupling from that in the standard model is about \( 4 \times 10^{-5} \) or \( 2 \times 10^{-4} \) at \( \theta_H = \pi/4 \) or \( \pi/2 \), respectively.

We remark that in the flat spacetime limit \( k \to 0 \) the deviation becomes substantial as the wave functions acquire significant y-dependence. The deviation becomes as large as 7% at \( \theta_H = \pi/2 \), which already contradicts with the LEP2 data.

5. Gauge-Higgs couplings

The 4D Higgs field (H) is contained in \( A_Y \). Overlap integrals of the \( Tr F_{\mu \nu} F_{\mu \nu} \) term give the WW, ZZ, WWWH and ZZHH couplings. The Higgs wave function is localized near the TeV brane so that these couplings sensitively depend on the behavior of the W and Z wave functions in the vicinity of the TeV brane, and therefore on \( \theta_H \).

The result is robust. These couplings are given by

\[ \lambda_{WWH} \simeq \frac{g_W m_W \cos \theta_H}{\cos \theta_W}, \quad \lambda_{ZZH} \simeq \frac{g_W m_Z \cos \theta_H}{\cos \theta_W}, \quad \lambda_{W_{WWHH}} \simeq \frac{g_W^2}{3} \left( 1 - 2 \sin^2 \theta_H \right), \quad \lambda_{ZZHH} \simeq \frac{g_W^2}{3 \cos^2 \theta_W} \left( 1 - 2 \sin^2 \theta_H \right). \]  

The factor \( \cos \theta_H \) in \( \lambda_{WWH} \) and \( \lambda_{ZZH} \) gives significant suppression compared with the couplings in the standard model. The suppression can be measured at LHC, once the Higgs particle is found.

All of the W, Z and H bosons have their Kaluza-Klein (KK) towers. The KK excited states also have nontrivial gauge couplings. It is shown that the \( WWH^{(n)} \) and \( ZZH^{(n)} \) couplings identically vanish. The \( WW^{(n)} H \) and \( ZZ^{(n)} H \) couplings are substantial, however. In the \( WWHH \) coupling, for instance, \( W^{(n)} \) can appear as an intermediate state (\( WH \to W^{(n)} \to WH \)). It gives an important contribution to the low energy effective \( \lambda_{WWHH} \) coupling as described below.

6. Effective Lagrangian

It is convenient to write down the effective Lagrangian at low energies in terms of low-energy fields, integrating over heavy fields (KK excited states). This can be consistently carried out in the RS warped space, by utilizing the holographic property as shown by Panico and Wurzer \[16] and by Sakamura \[17].
The effective Lagrangian describing couplings of $H$ to $W, Z$ is given by
\[ \mathcal{L}_{\text{eff}} \sim \frac{g_2^2}{2} \sin^2 \left( \theta_H + \frac{H}{\sqrt{2}f_H} \right) \times \left\{ W^\mu \nabla^\mu W + \frac{Z \mu Z^\mu}{2\cos^2 \theta_W} \right\}; \]
\[ f_H = \frac{1}{g_W} \sqrt{\frac{2kL}{e^{-kL}}} = \sqrt{\frac{2}{kL}} \frac{m_{\text{KK}}}{\pi g_W}. \] (12)

Notice that $\mathcal{L}_{\text{eff}}$ is periodic in $\theta_H + (H/\sqrt{2}f_H)$.

Expanded in a Taylor series in $H$, $\mathcal{L}_{\text{eff}}$ yields the mass terms for $W$ and $Z$ and the Higgs couplings to $W$ and $Z$. $m_W, m_Z$, and $\lambda_{WWH}$ and $\lambda_{ZZH}$ in (11) are reproduced. The couplings $\lambda_{WWHH}$ and $\lambda_{ZZHH}$, however, deviate from those in (11) as they incorporate contributions from intermediate $W(n)$ and $Z(n)$. They are found to be
\[ \lambda_{WWHH}^{\text{eff}} \simeq g_2^2 \cos 2\theta_H, \]
\[ \lambda_{ZZHH}^{\text{eff}} \simeq \frac{g_2^2}{\cos^2 \theta_W} \cos 2\theta_H. \] (13)

The suppression factor, compared with the values in the standard model, is given by $\cos 2\theta_H$.

7. Tree unitarity in $W_i W_i$ scattering

In the standard model the Higgs field plays an important role to restore the unitarity in the elastic scattering of the longitudinal components of $W$ and $Z$. In the gauge-Higgs unification scenario, however, the $\lambda_{WWH}$ and $\lambda_{ZZH}$ couplings are suppressed by a factor $\cos \theta_H$. If this is the case, one might wonder if the unitarity is destroyed as the contribution from the Higgs field exchange is diminished.

This problem is analyzed in ref. [18]. It is shown that the decrease in the Higgs contribution is compensated by contributions from $W^{(n)}$ or $Z^{(n)}$ exchange so that the tree unitarity is maintained.

Quarks and leptons

Let us adopt the viewpoint that the observed quarks and leptons live in the bulk five-dimensional spacetime. A fermion in the bulk is described by
\[ \overline{\Psi} \left\{ \Gamma^A e_A^M \left( \partial_M - \frac{1}{8} \omega_{MBC} [\Gamma^B, \Gamma^C] \right) - ig_A A_M - ig_B B_M - c \sigma^I (y) \right\} \Psi \] (14)

where $e_A^M$ and $\omega_{MBC}$ are tetrads and spin connections. $c$ is a dimensionless bulk mass parameter, which controls wave functions of quarks and leptons. [19] Implementing quarks and leptons in the $SO(5) \times U(1)_Y$ model is not trivial as one has, in general, additional light exotic fermions. They have to be made heavy by some means.

1. Medina-Shar-Wagner (MSW) model

In the quark sector three $SO(5)$ multiplets per generation are introduced. [20] For the third generation one has
\[ 5_1 = \begin{pmatrix} t_L \cr b_L \cr \end{pmatrix}, \quad 5_2 = \begin{pmatrix} t_R \cr b_R \cr \end{pmatrix}, \quad 10 = \begin{pmatrix} \cdot \cr \cdot \cr \end{pmatrix} \] (15)

where light modes are denoted in the parentheses. With the orbifold boundary condition $\{ P_1, P_1 \}$ alone there appear 20 light modes, 16 unwanted modes are made heavy by assigning flipped boundary conditions. Further brane masses are introduced at the TeV brane.

This model is consistent with the electroweak precision measurements. It is shown that the model leads to dynamical electroweak symmetry breaking in a wide range in the parameter space.

2. HOOS model

A model with simpler matter content has been proposed. [21] For the third generation one has two 5 multiplets in the bulk with bulk mass parameters $c_1$ and $c_2$

\[ \begin{pmatrix} T \\ B \\ t \\ b \\ \end{pmatrix} \Rightarrow Q_{1L} = \begin{pmatrix} T_L \\ B_L \\ \end{pmatrix}, \quad q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \quad t'_L, \quad \]
\[ \begin{pmatrix} U \\ D \\ X \\ Y \end{pmatrix} \Rightarrow Q_{2L} = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad Q_{3L} = \begin{pmatrix} X_L \\ Y_L \end{pmatrix}, \quad b'_R, \quad \] (16)

and fermions living on the Planck brane which come in three right-handed multiplets belonging to $(2, 1)$ representation of $SU(2)_L \times SU(2)_R$

\[ \hat{\chi}_{1R} = \begin{pmatrix} T_R \\ B_R \cr \end{pmatrix}, \quad \hat{\chi}_{2R} = \begin{pmatrix} U_R \\ D_R \cr \end{pmatrix}, \quad \hat{\chi}_{3R} = \begin{pmatrix} X_R \\ Y_R \cr \end{pmatrix}. \] (17)

The bulk fermions obey the normal orbifold boundary conditions. On the right side of each 5 multiplet in (16), light modes allowed by the orbifold BC are written. Among them, $Q_{1L}$ $(\alpha = 1, 2, 3)$ are made heavy by coupling with $\hat{\chi}_{3R}$ on the Planck brane. The most general $SU(2)_L \times U(1)_Y$ invariant brane mass term is given by

\[ -i \delta (y) \left\{ \sum_{\alpha=1}^3 \mu_{\alpha}(\hat{T}_{\alpha R} Q_{\alpha L} - Q_{\alpha L} \hat{T}_{\alpha R}) + \bar{\mu}(\hat{X}_{3R} q_L - q_L^\dagger \hat{X}_{3R}) \right\}. \] (18)

We demand only that the scale of the brane masses is much larger than the KK scale. With this modest ansatz
the equations of motion are solved in the background of non-vanishing $\theta_H$. It turns out that the low energy spectrum is determined in terms of $c_1$, $c_2$, $\theta_H$, and the ratio $\tilde{\mu}/\mu_2$. The value of each $\mu_\alpha$ or $\tilde{\mu}$ is irrelevant, provided that $\mu_\alpha^2, \tilde{\mu}^2 \gg m_{KK}$. With these brane mass interactions the lightest modes in $Q_{\text{tol.}}$ ($\alpha = 1, 2, 3$) acquire masses of $O(m_{KK})$.

The top quark mass is generated mainly by the Hosotani mechanism in the first $5$ multiplet in (16). The bottom quark mass is generated by the combination of the Hosotani mechanism in the second $5$ multiplet and the brane mass terms involving $\mu_2$ and $\tilde{\mu}$ in (18). For $c_1 = c_2 \equiv c < \frac{1}{2}$ one finds

$$m_t \sim \frac{m_{KK}}{\sqrt{2}\pi} \sqrt{1 - 4c^2} |\sin \theta_H|,$$
$$m_b \sim \frac{\tilde{\mu}}{\mu_2} m_t.$$  \hspace{1cm} (19)

Given the values of $\theta_H$ and $z_L$, $m_{KK}$ is fixed from $m_W$. Hence $c$ is determined from $m_t$ to be $\sim 0.43$ for $\theta_H = \frac{1}{2}\pi$ and $z_L = 10^{15}$.

For the first and second generations similar construction can be done. For fermions with masses smaller than $m_W$, $c$ becomes larger than $\frac{1}{2}$. Indeed, $c \sim 0.65$ and $0.85$ for $c$ and $u$ quarks, respectively.

**EW symmetry breaking**

The effective potential $V_{\text{eff}}(\theta_H)$ can be evaluated from the spectra of the fields in the background $\theta_H$. Its evaluation in the RS warped spacetime was first done by Oda and Weiler [22]. A powerful method of evaluating $V_{\text{eff}}$ has been developed by Falkowski [23]. Concrete evaluation in the gauge-Higgs unification models of electroweak interactions in the RS spacetime has been given in refs. [20, 21, 24].

The effective potential in the HOOS model is depicted in Figure 1. In the pure gauge theory without fermions (see the curve denoted as “gauge”), $V_{\text{eff}}(\theta_H)$ is minimized at $\theta_H = 0, \pi$ so that the electroweak symmetry is unbroken. The contribution from the top-bottom multiplets with $c \sim 0.43$ denoted as “top” in the figure dominates over the contribution from the gauge fields. Contributions from other quarks and leptons are numerically negligible. The total effective potential, the curve denoted as “total”, has the global minima at $\theta_H = \pm \frac{1}{2}\pi$, where the EW symmetry is spontaneously broken.

The top quark triggers the EW symmetry breaking by the Hosotani mechanism in the RS warped space. One might wonder what would happen in flat space. The effective potential depends on the warp factor $z_L$. As $z_L$ decreases, $c$ decreases. At $z_L \sim 9.4 \times 10^3$, $c$ becomes $0$ to reproduce $m_t$. One can further decrease $z_L$ with $c = 0$ kept fixed. The relative weight of the top contribution to $V_{\text{eff}}$ decreases, and at $z_L \sim 900$ the global minima of $V_{\text{eff}}$ shift to $\theta_H = 0, \pi$ so that the EW symmetry is unbroken. In other words the EW symmetry is unbroken in flat space with the matter content in the HOOS model.

We also note that if all fermions in the bulk belong to the fundamental representation of $SO(5)$, then there would be no EW symmetry breaking. Their contribution to $V_{\text{eff}}(\theta_H)$ is periodic in $\theta_H$ with a period $2\pi$, not $\pi$, and has a minimum either at $\theta_H = 0$ or $\pi$. Consequently the total $V_{\text{eff}}(\theta_H)$ has the global minimum either at $\theta_H = 0$ or $\pi$ where the EW symmetry is unbroken. If fermions appear in various representations of $SO(5)$, then the global minimum can be located at $\theta_H$ other than $0, \pm \frac{1}{2}\pi$ and $\pi$ as in the MSW model.

**Higgs mass and couplings**

The Higgs mass $m_H$ is generated at the one loop level. It is related to the curvature of $V_{\text{eff}}$ at the minimum;

$$m_H^2 = \frac{\pi^2 g_W^2 k L}{4 m_{KK}^2} \frac{d^2 V_{\text{eff}}}{d\theta_H^2} |_{\theta_H}.$$  \hspace{1cm} (20)

In the gauge-Higgs unification scenario $m_H$ is determined, in essence, from the top quark mass $m_t$. The numerical values are tabulated in Table 1 with various values of $z_L$.

![FIGURE 1. Effective potential in the HOOS model. $U(\theta_H/\pi) = (4\pi)^2 (kz_L^{-1})^{-4} V_{\text{eff}}(\theta_H)$ is plotted at $z_L = 10^{15}$.](image)

| $z_L$ | $k$ (GeV) | $m_{KK}$ (TeV) | $c$ | $m_H$ (GeV) |
|-------|-----------|----------------|-----|-------------|
| $10^{17}$ | $5.0 \times 10^{19}$ | 1.58 | 0.438 | 53.5 |
| $10^{15}$ | $4.7 \times 10^{17}$ | 1.48 | 0.429 | 49.9 |
| $10^{13}$ | $4.4 \times 10^{15}$ | 1.38 | 0.417 | 46.1 |

It is seen that the Higgs mass is predicted around 50 GeV for $z_L = 10^{13} \sim 10^{17}$. We stress that this is in no
conflict with the LEP2 bound for \( m_H \) which states that \( m_H < 114 \text{GeV} \) is excluded. The crucial observation is that the \( ZZH \) coupling vanishes at \( \theta_H = \pm \frac{1}{2} \pi \) as shown in (11). The process \( e^+ e^- \rightarrow Z \rightarrow ZH \) does not take place at \( \theta_H = \pm \frac{1}{2} \pi \) so that the LEP2 bound is not applicable. The \( ZZHH \) coupling, on the other hand, is multiplied by a factor \( \cos 2\theta_H \) to the coupling in the standard model as in (13) so that \( e^+ e^- \rightarrow ZHH \) can proceed. Light Higgs particles might have been already produced. It is of great interest that a similar scenario emerges in a version of MSSM where the lightest Higgs boson has a different coupling to \( Z \) from that of the Higgs boson in the standard model (25)–(28). We remark that in the gauge-Higgs unification scenario the light Higgs particle with vanishing \( WWH \) and \( ZZH \) couplings follows from the dynamics in the theory, but not by tuning parameters.

The Yukawa couplings are also expected to be suppressed compared with the values in the standard model (29). The dominant decay modes of the Higgs particle at \( \theta_H = \pm \frac{1}{2} \pi \) would be two \( \gamma \) decay through a top loop and \( b\bar{b} \) decay.

**Summary**

At the onset of the LHC experiments we have, for the first time in history, the opportunity for directly seeing the origin and structure of the EW symmetry breaking. It could well force us to go beyond the standard model. Exciting scenarios include supersymmetry, the little Higgs theory, the Higgsless theory, and the gauge-Higgs unification theory.

The gauge-Higgs unification scenario predicts large deviation from the standard model in the Higgs sector. The \( WWH \) and \( ZZH \) couplings are substantially suppressed. The Yukawa couplings are also expected to be suppressed. In the HOOS model the Higgs mass is predicted rather low with vanishing \( WWH \) and \( ZZH \) couplings. Further tiny violation of the weak universality is predicted (29) though experimental detection is difficult. Small deviation in the \( WWZ \) coupling can be measured in the future ILC experiments.

The gauge-Higgs unification scenario needs elaboration and refinement. We need a model with quarks and leptons which reproduces the observed Kobayashi-Maskawa matrix and is consistent with the electroweak precision measurements. The forthcoming experiments at LHC certainly give us clues in understanding the structure of the symmetry breaking and the origin of the Higgs particle.

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