The distribution of wealth in the presence of altruism for simple economic models

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Abstract

We study the effect of altruism in two simple asset exchange models: the yard sale model (winner gets a random fraction of the poorer player’s wealth) and the theft and fraud model (winner gets a random fraction of the loser’s wealth). We also introduce in these models the concept of bargaining efficiency, which makes the poorer trader more aggressive in getting a favorable deal thus augmenting his winning probabilities. The altruistic behavior is controlled by varying the number of traders that behave altruistically and by the degree of altruism that they show. The resulting wealth distribution is characterized using the Gini index. We compare the resulting values of the Gini index at different levels of altruism in both models. It is found that altruistic behavior does lead to a more equitable wealth distribution but only for unreasonable high values of altruism that are difficult to expect in a real economic system.

Key words: economic models; econophysics; altruism

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1 Introduction

The study of wealth and income distributions in an economical system is a problem of interest from both the practical and theoretical points of view and, as expected, has a long history. Pareto did some of the first studies on the subject (1). He proposed that the wealth and income distributions obey universal power laws, but subsequent studies have shown that this is not the case for the whole range of wealth values. Mandelbrot (2) proposed that the Pareto conjecture only holds at the higher values of wealth and income. The initial part (low wealth or income) of the distribution has been recently identified with the Gibbs distribution (3; 4; 5), while the middle part, according to Gibrat (6), takes the form of a log–normal distribution.

Very recently, this and other aspects of the economy have been treated under the “econophysics” point of view, mainly applying the ideas and tools of statistical mechanics and Monte Carlo simulations with some degree of success and promising results (economists however, are still very skeptical about results obtained from these methods, see (7) for an interesting discussion). The wealth distribution of any country, as many other economic quantities, results from very complicated processes involving production, taxes, regulations and even fraud. Despite this complexity, very simple models that provide some insight into the whole process have been devised that qualitatively reproduce some of the features of real economies.

We can treat an economy in its simplest form as an interchange of wealth between pairs of people, or “agents” at successive instants of time (See Hayes

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for an interesting review). Every time two agents interact, wealth flows from one to the other according to some rule. In the so-called “yard sale” (YS) model, the winner takes a random fraction of the wealth of the poorer player, while in the “theft and fraud” (TF) model, the winner takes a random fraction of the loser’s wealth. There is no production or consumption of wealth in these models, nor taxes, savings etc. Under these circumstances, the yard sale model produces a collapse of the economy: all the wealth ends in the hands of a single agent, a phenomenon known as condensation. The theft and fraud model on the other hand does not collapse but leads to a wealth distribution given by the Gibbs distribution. See (9; 5; 4) for details.

The two models mentioned above are oversimplified, toy-model versions of a real economy, and several authors have made some refinements to introduce more realistic situations, for example, allowing the agents to go into debt (10), change in the agents’ probability of winning according to the relative wealth of the traders (11), constant and fractional savings (12), and altruistic behavior (13), among others. In particular, the introduction of altruism in these models has not been studied in depth, and therefore in this paper we investigate the effect that altruistic behavior has on the dynamics of the models and the changes that can produce in the distribution of wealth.

2 Models

In all models we use a fixed number $N$ of individuals with an identical initial amount of money $m$ to trade. The total wealth of the community, $Nm$, remains fixed in time. At each time step, two traders $i$ and $j$ are chosen at random. The winner (which is also randomly chosen) takes an amount $T$ from the loser.
The traders wealth $w$ at time $t+1$, assuming that $i$ is the winner, will be

$$w_i(t+1) = w_i(t) + T,$$  \hfill (1)  

$$w_j(t+1) = w_j(t) - T.$$  \hfill (2)

Then, another two traders interact, and the process is repeated $N$ times, which constitutes one Monte Carlo step (MCS). The amount $T$ of the transaction is defined as

$$T = \alpha \text{MIN}(w_i(t), w_j(t)), \hfill (3)$$

for the YS model and

$$T = \alpha w_j(t), \hfill (4)$$

for the TF model assuming that agent $j$ loses the transaction. The parameter $\alpha$ is a uniformly distributed random number in the interval $[0,1]$.

Altruistic behavior is introduced in the above models in the following way. First, a certain fraction $p$ of the $N$ traders is defined as altruists. An altruistic agent remains in that condition for the whole simulation. Second, a rate of altruism $r$ is defined and is the same for all of the $pN$ altruistic agents. Suppose that agents $i$ and $j$ trade at time $t$ and that $i$ is richer than $j$. If agent $i$ wins and is an altruist we will have

$$w_i(t+1) = w_i(t) + T - r(\Delta + T),$$  \hfill (5)  

$$w_j(t+1) = w_j(t) - T + r(\Delta + T),$$  \hfill (6)

where $\Delta = (w_i - w_j)/2$. With this definition, if an agent is not altruistic at all ($r = 0$), the transactions proceed as in the original YS and TF models. If the agent is totally altruistic ($r = 1$), the richer agent will give the other enough
of his money so that their fortunes become equal.

Since both of the previous models can be considered too simplistic to represent an economy, several attempts have been made to make these models more realistic, as mentioned in the introduction. Here we follow Sihna (11), who introduces the concept of “bargaining efficiency”: a rich agent who owns 1000 units and loses 1 unit during a deal is only losing a 0.1% of his wealth. However, an agent who loses the same 1 unit but whose wealth is of only 5 units is losing 20% of his fortune. Therefore it is expected that in a trade between a rich and a poor agent, the poorer will be more aggressive in getting a favorable deal, and that the aggressiveness will be a function of the relative wealths of the agents.

The implementation of the above concept is made via the following “Fermi function”: The probability that agent $i$ wins in a trade with agent $j$ is given by:

$$ p(i|i, j) = \frac{1}{1 + \exp(\beta \frac{x_i - x_j}{x_i} - 1)}, $$

where $\beta$ parametrizes the significance of the relative wealth of the agents. For any $\beta > 0$, the poorer agent has a greater probability of winning the trade.

3 Results for the YS and TF models with altruism

We first investigate the effect of altruism in the YS model. In order to quantify the inequality in the wealth distribution we use the Gini index (14) defined as

$$ G = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} |x_i - x_j|}{2N^2 \mu}, $$

(8)
where $\mu$ is the average wealth. A perfect distribution of wealth where everybody has the same amount of money will give a value of $G = 0$. The other extreme where one individual has all the money has a Gini value of 1.

It is known that in this model we have condensation: all the money ends up in the hands of a single trader, which represents the extreme case of wealth inequality. Altruism does not change this situation. In figure 1 we see the results of several simulations with 1000 traders that start with an initial fortune of 100 (these values will remain fixed for all the simulations in this paper). In the curve with open circles we have $p = r = 0$, that is, the pure YS model without altruism at all. Condensation takes place at about 1000 MCS. If we introduce altruism, condensation still takes place, the only difference is that it takes longer to reach. The curve with solid circles has values of $p = r = 0.95$, almost everybody is near totally altruistic, however, only at the beginning we see a difference in the Gini index compared to the pure YS model. As time goes by, we quickly arrive at the condensate phase. Only when we set $p = 1$, or everybody is altruistic, we get saturation of the Gini index. In the figure, the curve with crosses has $p = 1$ and $r = 0.1$ which gives a value of $G = 0.62$. This saturation is, however, uninteresting since the addition of a single non–altruist takes the system to the condensate phase.

In the TF model condensation does not take place. The effect of altruism has been studied by using several values for the fraction and rate of altruism in the model. For each set of values of $p$ and $r$ we let the system reach a stable distribution at about 300 MCS and obtain a value for the Gini index. As figure 2 shows, at low values of $p$ and $r$ the Gini index is high, resulting in an uneven distribution of wealth. Higher values of altruism result in a lower value of $G$, as expected.
4 Introducing bargaining efficiency in the transactions

We now introduce the bargaining efficiency concept in the models. Figures 3 and 4 show the results for the TF and YS models respectively. It is interesting that the condensation that occurs in the YS model disappears with the implementation of this scheme. This is illustrated in figure 4. Note that, comparing with the TF model (figure 2), the YS model with bargaining efficiency yields lower values of the Gini index for the same degree of altruism, that is, in order to attain a certain value of $G$, we need lower values of $r$ and $p$ in the YS model with bargaining efficiency (with $\beta = 1$) than in the stand–alone TF model.

If we compare the results for the TF model with and without bargaining efficiency (figures 2 and 3), we see that $G$ is smaller in the bargaining efficiency case, but only for small values of the altruism parameters. In fact, for some values of these parameters, the wealth is better distributed in the stand–alone TF model. This is shown in figure 5. What the data in this figure says is: if you take a TF economy without altruism, the addition of bargaining efficiency (giving the poor more chances to win) reduces the Gini index, that is, the wealth is more evenly distributed. However, if, in addition to the fact that the chances to win are biased in favor of the poor you also have altruism in your economy, then at a certain point, the pure TF economy performs better in terms of wealth distribution.

This behavior can be understood in the following way. Take the case of no altruists at all. In this situation the money is changing hands all the time, and at any point in time you can find extremely rich agents and very poor ones, which gives you a high value for $G$. If in these conditions you give the poor
more chances to win then you are leveling the field and $G$ diminishes. This is the behavior at low values of $r$ in figure 5. Now take the other extreme, almost everyone is altruistic at a high rate. Every time a rich wins a poor, he will give the poor money so that their fortunes will be almost the same. This situation gives you a low value for $G$, but if now you give the poor more chances to win in addition to the altruism which is already helping him, then they benefit in excess and $G$ increases.

We finally perform a set of simulations to study the effect of changing the aggressiveness of bargaining, which is controlled by the parameter $\beta$. The higher value of this parameter, the most chances has the poorer of the two traders to win the transaction. These simulations emphasize the behavior discussed above. By enhancing the bargaining efficiency, the Gini index decreases, but only when the altruism is low, for example when $r = p = 0.4$ (see the upper curve in figure 6). When altruism is high the behavior is interesting, since the Gini index first begins to decrease when $\beta$ increases, and then reaches a minimum value and starts to increase for higher values of $\beta$. This means that there is an optimum value for the bargaining parameter $\beta$ for which the wealth distribution reaches its more equitable form, at least under the Gini criteria. In figure 7 we present similar curves as in figure 6 but for the YS model. In this case the Gini index decreases monotonically as $\beta$ increases, except for the bottom curve where there is a very small increment in $G$ after the initial decrease.
5 Conclusions

We have investigated the effect of altruistic behavior in the YS and TF models with and without bargaining efficiency. We found that it is no easy to get rid of the condensate phase (when a single agent owns all the wealth) in the YS model. Only in the extreme case of 100% altruists condensation does not take place. When bargaining efficiency is introduced in the YS model, condensation is effectively avoided and a stable wealth distribution is achieved. The distribution of wealth becomes more equitable as the altruism is increased. In the stand-alone TF model, it is also observed that $G$ decreases when altruism is increased.

The introduction of bargaining efficiency gives interesting results since, for small values of altruism, it has the effect of decreasing the Gini index and thus leads to a better wealth distribution, however, at high values of altruism it can have the contrary effect and increase the value of $G$, and this behavior is more pronounced in the TF model. This implies that in these models, when high rates of altruism are present, there is no necessity of giving the poor more chances to win, because the wealth distribution will get worst.

An important point is that, despite the fact that we do observe a better distribution of wealth in both models when the altruism is increased, this effect is observed only at too high values of the altruism parameters. For example, in the stand-alone TF model, the value of the Gini index without altruism is about 0.65. From figure 2 we can see that in order to decrease it only to 0.55 we need to have approximately half of the population behaving as altruists, with an altruism rate of about 0.3. A value of $r = 0.3$ means that the rich agent
will give the poor one 30% of the difference in their fortunes, a value that is hard to expect in real life. Of course, we are dealing here with oversimplified models but, as other authors have found, they can be valuable to shed some light in a very complex issue, and our findings indicate that altruism cannot be expected to change the way wealth is distributed in a significant way.

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References

[1] V. Pareto, Cours d’Economie Politique, Lausanne, 1897.
[2] B. Mandelbrot, Int. Econom. Rev. 1 (1960) 79.
[3] B. K. Chakrabarti and S. Marjit, Ind. J. Phys. B 69 (1995) 681.
[4] A. Dragulescu and V. M. Yakovenko, Physica A 299 (2001) 213.
[5] A. Chakraborti and B. K. Chakrabarti, Eur. Phys. J. B 17 (2000) 167.
[6] R. Gibrat, Les inégalités économiques, Paris, 1931.
[7] R. Leombruni and M. Richiardi, Phys. A (2005), in press.
[8] B. Hayes, Am. Sci. 90 (2002) 400.
[9] S. Ispolatov, P. L. Krapivsky and S. Redner, Eur. Phys. J. B 2 (1998) 267.
[10] A. Dragulescu and V. M. Yakovenko, Eur. Phys. J. B 17 (2000) 723.
[11] S. Sihna, Phys. Scripta T106 (2003) 59.
[12] A. Chakraborti, Int. J. Mod. Phys. C 13 (2002) 1315.
[13] R. Trigaux, Phys. A 348 (2005) 453.
[14] Z. M. Berrebi and J. Silber, Quart. J. Econom. 100 (1985) 807.

FIGURE CAPTIONS

Figure 1. Time evolution of the Gini index in the YS model. Open circles are for a simulation without altruism at all. Solid circles are for 95% of altruists and 0.95 of altruism. Crosses are for 100% of altruists and 0.1 of altruism.

Figure 2. Contour plot of the Gini index as function of the fraction of altruists \( p \) and the rate of altruism \( r \) in the TF model. Results are averaged over at
least 5000 realizations.

Figure 3. Contour plot of the Gini index as function of the fraction of altruists \( p \) and the rate of altruism \( r \) for the TF model with bargaining efficiency with \( \beta = 1 \). Results are averaged over at least 5000 realizations.

Figure 4. Same as figure 3 but for the YS model with bargaining efficiency, and \( \beta = 1 \). Results are averaged over at least 5000 realizations.

Figure 5. Curves of \( G \) as function of \( r \) for a fixed value of \( p = 0.9 \). The curve with open circles is the stand–alone TF model, while the filled circles curve is for the TF model with bargaining efficiency and \( \beta = 1 \).

Figure 6. The Gini index as function of the bargaining parameter \( \beta \) under the TF dynamics. Each curve is for different altruism parameters, which, from top to bottom curves, are the following: \( r = p = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} \). Results are averaged over 1000 realizations.

Figure 7. The Gini index as function of the bargaining parameter \( \beta \) under the YS dynamics. Each curve is for different altruism parameters, which, from top to bottom curves, are the following: \( r = p = \{0.4, 0.6, 0.8, 0.95\} \). Results are averaged over at least 1000 realizations.
