Photon Starvation Artifact Reduction by Shift-Variant Processing

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ABSTRACT The x-ray computed tomography (CT) images with low dose are noisy and may contain photon starvation artifacts. The artifacts are location and direction dependent. Therefore, the common shift-invariant denoising filters do not work well. The state-of-the-art methods to process the low-dose CT images are image reconstruction based; they require the raw projection data. In many situations, the raw CT projections are not accessible. This paper suggests a method to denoise the low-dose CT image using the pseudo projections generated by the application of a forward projector on the low-dose CT image. The feasibility of the proposed method is demonstrated by real clinical data.

INDEX TERMS Image processing, Image reconstruction, Biomedical imaging, Computed Tomography, Filters

I. INTRODUCTION
An immediate negative effect of using a low dose in CT imaging is that the images become noisy. The conventional denoising methods are based on the shift-invariant assumption. They can be implemented either in the spatial-domain as convolution methods or in the Fourier-domain as multiplication methods.

Shift-invariant filters can also be nonlinear. The nonlinear filters may outperform the linear filters in terms of sharp edge preservation.

Convolutional neural network (CNN) based methods can be very effective in removing noise from the images provided a large amount of noisy/noiseless image pairs are available to train the neural network.

This paper presents an effective nonlinear shift-variant procedure that does not need any image pairs to train. This proposed procedure blends the concepts of linear filtering, shift-variant filtering, and tomography. The feasibility and effectiveness of the proposed procedure are illustrated by its application to real clinical data.

II. METHODS
In this paper, we assume that the image $x$ is already somehow reconstructed, for example, by the analytical filtered backprojection (FBP) algorithm. The image is noisy and contains photon starvation artifacts. The original measured projections are NOT available anymore.

A. The proposed algorithm
The proposed artifact reduction algorithm is introduced as follows.

Step 1. For a given image $x_{old}$, generate simulated pseudo projections as

$$ p_i = a_i^T x_{old} $$ \hfill (1) 

for all $i$.

Step 2. Select a threshold value $T$.

Step 3. Loop through all projections $p_i$. 

If \( p_i < T \), do nothing.
If \( p_i \geq T \), replace \( p_i \) by its filtered version using a one-dimensional moving-average filter along the detector direction.

Step 4. Apply the filtered backprojection (FBP) algorithm to the processed pseudo projections, to obtain the final image \( x_{\text{new}} \).

The threshold value \( T \) is a user-selected parameter, and we used \( T \) as the 75\% of the maximum projection value in our study in this paper.

We now explain what motivates this algorithm. We do not choose any shift-invariant filters, because the artifacts are location and direction dependent. Since the state-of-the-art denoising algorithms are image reconstruction based, we choose an image reconstruction-based algorithm.

Our biggest obstacle is that we do not have an access of the original measurements in the projection domain. We only have a noisy reconstruction \( x_{\text{old}} \). The simulated pseudo forward projection \( a_i^T x_{\text{old}} \) is not the same as the originally measured projection.

The original projections due to noise are inconsistent. The inconsistency carries the noise information. The inconsistency information is lost in the forward projection \( a_i^T x_{\text{old}} \). The objective function is already at its minimum with the pseudo projections because \( p_i = a_i^T x_{\text{old}} \). Therefore, the strategy of selecting a set of weights to minimize the objective function does not help.

Realizing that the re-projected pseudo measurements do not carry the same information and do not have the same values as the original raw measurements, our novel strategy of this paper is to use the transmission data noise model to estimate the noise variance in the re-projected pseudo measurements. The FBP algorithm is selected to reconstruct the final image, because it is fast and easy to implement.

B. Image evaluation

The most common way to determine the effectiveness of artifact removal algorithms is by visual inspection or human observer studies. A quantitative evaluation metric adopted in this paper is the Sum Square Difference (SSD), defined as

\[
SSD = \frac{\sum_{i,j} [X_{\text{gold}}(i,j) - X(i,j)]^2}{\sqrt{\sum_{i,j} [X_{\text{gold}}(i,j)]^2 \sum_{i,j} [X(i,j)]^2}}
\]

where \( X_{\text{old}} \) is the gold standard image, which is the FBP reconstructed from the regular-dose projections, and \( X \) is another image to compare with. The SSD essentially is the normalized distance between two images \( X_{\text{old}} \) and \( X \).

III. RESULTS

In this section, the methods are labeled with A – G. We point out that methods A and F use the ‘unavailable’ projections. In Figs. 1, the following labels are used for the images: (A) the gold standard image FBP reconstructed from the regular-dose x-ray projections; (B) the raw FBP reconstruction image reconstructed from the measured low-dose x-ray data; (C) the processed image using the proposed algorithm in the paper using the pseudo data; (D) the image is FBP reconstructed with a linear Hanning filter applied to the pseudo data; (E) the image is FBP reconstructed with a nonlinear bilateral filter applied to the pseudo data; (F) the image is post-processing result of image from (B) with a BM3D filter in the image domain; (G) almost the same as (C) except that the ‘unavailable’ low-dose x-ray data is used instead of the pseudo data.

The numerical results of the Sum Square Difference (SSD) values are listed in Table 1. The SSD is a non-negative quantity, the smaller value the better. The ideal SSD value is 0. In all these cases, the proposed method gives the smallest SSD values, indicating the best performance.

The projection-domain images (also known as the sinograms) are displayed in Fig. 2. The images are (a) the ‘unavailable’ regular-dose projections, (b) the difference between the raw ‘unavailable’ low-dose projections and the ‘unavailable’ regular-dose projections, (c) the difference between the pseudo forward projections from the low-dose FBP reconstruction and the ‘unavailable’ low-dose projections, and (d) the difference between the processed version of the pseudo forward projections from the low-dose FBP reconstruction and the unprocessed version, respectively. It is observed from Fig. 2d that the proposed method only alters a very small portion of the projections.
The proposed algorithm is effective in reducing the streaking artifacts and keeping the image resolution. As a comparison, the images produced by a linear Hanning filter, a nonlinear bilateral filter, or a BM3D filter are unable to keep small details while the streaking artifacts are still severe.

(1A) Standard dose image. The yellow line segment indicates the path that the line profiles are taken along in Fig. 14.

(1B) Low-dose image

(1C) Low-dose image processed by proposed algorithm
(1D) Low-dose image processed by FBP-Hann

(1E) Low-dose image processed by a bilateral filter

(1F) Low-dose image processed by a BM3D filter
Low-dose image processed by proposed algorithm using the ‘unavailable’ measured projections

Figure 1. Processed and unprocessed images. The standard-dose image in (1A) is the gold standard. The image with the proposed method (1C) gives the best result among all images using the low-dose raw image (1B).

Table 1. Full width at half maximum value comparison

| Method                                                                 | FWHM (pixels) | Severe Artifacts? |
|------------------------------------------------------------------------|---------------|-------------------|
| A. Regular-dose FBP                                                   | 2.54          | No                |
| B. Low-dose FBP, using the ‘unavailable’ low-dose measurements        | 2.34          | Yes               |
| C. Low-dose FBP using proposed method                                 | 2.96          | No                |
| D. Low-dose FBP using linear Hann filter                              | 3.76          | Yes               |
| E. Low-dose FBP using nonlinear bilateral filter                      | 7.00          | Yes               |
| F. Measured Low-dose using BM3D filter                                 | 2.55          | Yes               |
| G. Low-dose FBP using proposed method, but using the ‘unavailable’ low-dose measurements | 2.34 | No               |
| H. Low-dose FBP, using the pseudo-data                                 | 2.98          | No                |

a. The sinogram for the regular-dose projections
b. The difference between the low-dose projections and the regular-dose projections

c. The difference between the low-dose projections and the forward projections of the FBP reconstruction from the low-dose projections

d. The difference between the forward projections of the FBP reconstruction from the low-dose projections and the processed projections by using the proposed algorithm

Figure 2. Sinogram domain images.

Figure 3. A sensitivity study of the SSD with respect to the threshold value $T$.

**IV. DISCUSSION**

When a noisy reconstructed image is available while the original projection measurements are no longer available, the pseudo re-projected line integrals are not helpful to reduce noise if a conventional iterative image reconstruction algorithm is to be used. The conventional iterative image reconstruction algorithms work in the principle of reducing the data fidelity term. By using the pseudo re-projected line integral data, this data fidelity term is already at its minimum value, which is zero.
One way of denoising is to stop the iterations early. This approach is equivalent to lowpass filtering, which is almost shift invariant. As we demonstrated in the Results section, shift-invariant denoising smooths the images but still cannot reduce the streaking artifacts.

A filter is referred to as shift-invariant if the filter operation is the same everywhere. In our proposed filter, the filter operation is only applied to a small amount of selected pseudo projections. Therefore, our proposed filter is shift variant.

Our proposed algorithm is NOT an iterative image reconstruction algorithm; it is an analytic FBP algorithm with a nonlinear pre-filter. In the FBP algorithm, a ramp filter (which is a high-pass filter) must be used to cancel the backprojection blurring. The purpose the low-pass filter is to reduce the noise in the image. The application of a low-pass filter is optional in FBP, only when image denoising is necessary. The main goal of this paper is photon-starvation artifact reduction, we do not apply a linear low-pass filter in the FBP algorithm. In the proposed algorithm, there is a threshold value \( T \); any pseudo projection data value that is less than this threshold value is not affected. Most of the pseudo projections are less than this threshold. Thus, the image resolution degradation is kept to its minimum.

The proposed algorithm contains a user-determined hyper parameter \( T \). This hyper parameter \( T \) is determined by trial and error. In fact, parameter \( T \) is not very sensitive. As shown in Fig. 3, the SSD vs \( T \) curve has a flat valley, which means that a wide range of the parameter \( T \) can give the optimal solution.

Three noise-reducing filters have been used to compare with the proposed shift variant filter in the task of photon starvation artifact reduction. Those three noise-reducing filters do not perform well for this task. If the filters are adjusted to remove the artifacts, many image details are removed as the price to pay. The message of our paper is that the noise reduction task is different from the artifact reduction task. For artifact reduction, where to filter (or equivalently, where not to filter) is far more important than what filter to use. Once the region to be filtered is identified, many filters are effective as long as the filters have enough smoothing power. We choose the simplest linear moving-average filter with a large enough kernel size. Other noise reduction filters such as bilateral and BM3D filters will work just fine when applied only in the specified region. The critical point is that we do not apply the lowpass filter to the entire image or the entire sinogram.

V. CONCLUSIONS
We have developed an effective method to reduce the photon starvation streaking artifacts in low-dose x-ray CT images. The proposed method is shift-variant; it only applies lowpass filtration for some pre-determined measurement values in the sinogram domain.

We assume that the raw, low-dose measurements are not available, and the noisy reconstruction is available. A set of pseudo re-projections are generated from noisy reconstruction. A threshold value \( T \) is selected by the user using a trial-and-error method. The pre-determined measurements are selected if the pseudo measurement value is greater than \( T \). The pre-selected pseudo measurements are filtered in the sinogram domain by a simple moving-average lowpass filter along the detector direction. The FBP algorithm is performed to generate a final image using the selectively filtered pseudo measurements.

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