Optomechanical Gigahertz Oscillator made of a Two Photon Absorption free piezoelectric III/V semiconductor

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(Dated: January 18, 2019)

Oscillators in the GHz frequency range are key building blocks for telecommunication, timing and positioning applications. Operating directly in the GHz and compactness while keeping high frequency stability, is still an up-to-date challenge. Recently, optomechanical crystals, compact by nature, have demonstrated GHz frequency modes, thus gathering prerequisite features for using them as oscillators. Here we report on the demonstration, in ambient atmospheric conditions, of an optomechanical oscillator designed with an original concept based on bichromatic one-dimensional optomechanical crystal. Self sustained oscillations directly at 3 GHz are routinely achieved with a low optical power threshold of 40 $\mu$W and short-term linewidth narrowed down to 100 Hz in agreement with phase noise measurements (-113 dBc/Hz at 1 MHz from the carrier) for free running optomechanical oscillators. This oscillator is made of InGaP, low loss and TPA-free piezoelectric material which makes it valuable for optomechanics.

I. INTRODUCTION

Optomechanical (OM) resonators, exploiting the interaction between light and a moving optical cavity [1], have been actively looked into in recent years with impressive fundamental demonstrations in the quantum regime [2–4]. Meanwhile, other important applications have also been found for ultra-compact sensors [5] or stable microwave oscillators [6]. Essential feature in modern navigation, communication and timing systems, microwave oscillators at high frequencies are compared in the light of their stability at their natural frequency and their form-factor. With their micrometric size and their mechanical resonance frequency already in the GHz range, OM crystals [7] (OMC) present a unique potential to reach ultra-compact stable microwave oscillators. OM oscillators have been investigated but still lie far from the microwave domain and spectral purity is, for the moment, an issue which has been scarcely addressed.

Besides, a shared limitation for every application is thermo-optical instabilities which limit the optical power injected inside the resonator. First OM resonators, and especially OMCs, made of silicon, suffer from two-photon absorption preventing quantum regime in cooling experiments to be achieved. Hence, different materials such as Silica [8], Silicon Nitride [9] and diamond [10,11] have been considered as materials of choice thanks to their large thermal conductivity and low optical absorption. Thus, a high number of photons has been reached with diamond OMC[10]. None of these materials shows piezoelectric properties which could efficiently bridge microwave to optics. Thus, they are unsuitable for microwave to optics transduction [12], radiofrequency signals amplification [13] or hybrid opto-electro-mechanical devices [14], particularly attractive in various contexts, from telecommunications to quantum information and from classical radar to quantum radar [15]. That is why non centro-symmetric crystals such as large electronic bandgap III-V semiconductors are appealing for optomechanics and have been recently investigated (Gallium Phosphide [16, 17] and Aluminium Nitride [18]) as they do not suffer from Two Photon Absorption (TPA) when operating in the practical telecom spectral range. Here we consider another material, Indium Gallium Phosphide (In$_{0.5}$Ga$_{0.5}$P) grown on GaAs. Owing to a large electronic forbidden gap ($\approx$ 1.9 $eV$) two-photon absorption is suppressed at telecom wavelengths [19], which allows reaching a very large optical energy density and trigger nonlinear effects such as soliton pulse compression [20]. For these reasons, InGaP has been introduced recently in optomechanics [21,23], but an OMC have not been realized yet. We introduce a new design concept, relying on bichromaticity [24], which presents the advantage of being robust to fabrication disorder [25] and thus achieved systematically functional devices with large optical Q factors and low mechanical losses. The self-sustained oscillation has been characterized in detail all the way to the measurement of the phase noise measurements, revealing that our OMC is comparable to much larger microtoroids made of Silicon Nitride but operated with about 2 orders of magnitude less optical power.
CAVITY DESIGN AND MODELING

The design of the OM cavity is based on the concept of the bichromatic photonic potential, whereby in two dimensional photonic crystals adjacent lattices with slightly different period \( a'/a \approx 1 \) induce an optimal confinement, as the intrinsic radiative cavity loss is minimized. Here we introduce a bichromatic lattice in a “nanobeam” structure. Periodically corrugated sidewalls, with period \( a' = 0.98a \) (Fig. 1c) and depth \( y_{th} = 0.27a \), are added to a line of holes with period \( a \) and radius \( r \). Thus, the design is described by only 4 parameters and requires no optimization as the radiative leakage limited Q, calculated by Finite Difference Time Domain method, is always above \( 10^5 \) as \( r \) and \( y_{th} \) are varied over a fairly broad range (see Supplementary Information), which also suggests robustness against fabrication tolerances.

The mechanical mode was computed using the Finite Element Method, implemented in the COMSOL. The confinement of the mechanical breathing mode oscillating at about 3 GHz is explained by the local increase of the stiffness in the structure induced by the increasing misalignment of holes and sidewalls as moving outwards from the center of the cavity. The calculated optical mode volume \( V_{opt} \), the effective oscillator mass \( m_{eff} \), the mechanical mode volume \( V_m \) and the vacuum optomechanical coupling constant \( g_0 \) depend on the parameter \( a'/a \approx 1 \), providing a simple “knob” for tuning the device properties. The calculated photoelastic and moving boundary contributions are: \( g_0, M_{mu}/2\pi = -117 kHz \) and \( g_{0, PE}/2\pi = 494 kHz \), hence \( g_0/2\pi = 377 kHz \). This design ensures the simultaneous localization of photons (Fig. 1b, \( V_{opt} = 0.97(\lambda/a)^3 \)) and phonons (Fig. 1c, \( V_m = 2.5 \times 10^{-19}m^3, m_{eff} = 1.01 fg \)).

The device is fabricated on a InGaP membrane grown by MOCVD lattice-matched to GaAs. The OM crystal is processed following the same recipe as for two dimensional photonic crystals, except that the resist is written by an EBPG 5000+ e-beam system.

OPTICAL CHARACTERIZATION

The optical resonances are probed in a reflection geometry using a high resolution optical heterodyne technique. This provides access to the complex spectrum of the cavity (see supplementary) which is shown in Fig. 2a. The loaded quality factor \( Q_L \) decreases by a factor 0.6 for each period removed, while the intrinsic quality factor \( Q_0 \), extracted from the fit of the measured complex amplitude, is \( 2.2 \pm 0.2 \times 10^5 \) (Fig. 2b). We measured an intrinsic quality factor over \( 10^5 \) in 9 out of 12 nominally identical cavities.

Absorption, at room temperature, is extracted from the normalized reflectivity as a function of the laser detuning \( \nu_L - \nu_0 \) swept from blue to red such that the resonance is thermally pulled until the bistable transition occurs (Fig. 2c). This, to a very good approximation, corresponds to the detuned resonance \( \nu' \). When plotted against the on-chip power, \( \nu' \) reveals a linear
PROBING OF THERMAL NOISE

The noise spectrum of the mechanical resonator reveals a peak at \( f_m = 2.92 \text{ GHz} \) (see inset 3a). The vacuum optomechanical coupling is measured at room temperature and standard pressure with the technique discussed in \[34\]. The reflected optical power is detected by a fast Avalanche Photodiode which is amplified by a 40dB low noise amplifier before going to an electric spectrum analyser (ESA). The electric power spectra corresponding to the mechanical motion of the resonator is compared to a calibration tone with spectrum \( S_{\text{mod}} \) generated by a phase modulator in the input optical path, allowing the measurement of the power spectrum of the frequency modulation \( S_{\nu \nu} \), as shown in the inset of Fig. 3a. The vacuum \( g_0 \) coupling constant is then: \( g_0^2 = \frac{\int S_{\nu \nu}(f) df}{n_{\text{abs}}} \). In our case, it was not possible to operate the OM resonator at low enough power to avoid dynamical backaction while maintaining the detected signal level well above noise. Thus, the measurement is carried out as a function of the laser-cavity detuning \( \nu_L - \nu' \) (which is corrected for the thermally induced spectral shift, see supplementary) and the measurement at zero detuning is retained (Fig. 3b). Considering the uncertainty on the photoelastic coefficients, the measured \( g_0/2\pi = 385 \text{ kHz} \) is very close to the calculations solely including the photoelastic and moving boundary contributions. This is consistent with the fact that the thermo-mechanical term \[23\] is negligible in our system (discussion in the Supplementary Information).

The corresponding mechanical linewidth (Fig. 3b) is measured and compared to theory \[35\] accounting for the narrowing due to the dynamical backaction \( \Gamma_{\text{om}} \), when \( \Delta = \nu_L - \nu' > 0 \):

\[
\Gamma_{\text{om}} = n_{\nu \nu} g_0^2 \times \frac{\kappa}{(\Delta + 2\pi f_m)^2 + \kappa^2/4} - \frac{\kappa}{(\Delta - 2\pi f_m)^2 + \kappa^2/4}
\]

with the number of photons in the cavity given by the usual coupled mode theory:

\[
n_{\nu \nu} = \frac{(\kappa - \Gamma_0) P_c}{\Delta^2 + \kappa^2/4} \frac{1}{\hbar \nu}
\]

The parameters used in the model are measured: \( \kappa/2\pi = 6.5 \text{ GHz} \), \( \Gamma_0/2\pi = 0.9 \text{ GHz} \), \( f_m = 2.92 \text{ GHz} \) and \( g_0/2\pi = 385 \text{ kHz} \). The on-chip laser power levels used in the model, \( P_c = 43.5, 47.9 \) and \( 51 \mu W \), have been adjusted within 20% of the experimental values indicated in Fig 3. From the Lorentzian fit in the inset of Fig 3b, the mechanical linewidth is equal to \( \Gamma_m = 1.2 \text{ MHz} \) and the mechanical Q factor \( Q_m = 2\pi f_m/\Gamma_m = 2300 \) corresponds to the measurement at zero detuning.

FIG. 3. a) Measured vacuum optomechanical coupling as a function of the normalized laser detuning for 3 different on-chip laser power; the calibrated power spectral density of the frequency fluctuation is represented in the inset along with the calibration tone and the Lorentzian fit; The on-chip laser power was estimated (see supplementary for details) b) corresponding measured mechanical linewidth compared with the theory.

dependence (Fig. 2b), hence suggesting linear absorption, likely due to defects at the surface. Following the same procedure as in [22], the dissipated power is extracted based on the calculated thermal resistance and the measured dependence of the resonance with temperature. This leads to an estimate of the absorption rate \( \Gamma_{\text{abs}}/2\pi = 8 \text{ MHz} \), which is much smaller than the total intrinsic losses \( \Gamma_0/2\pi \approx 1 \text{ GHz} \). Correspondingly, the fraction of the dissipated on-chip power is \( \alpha = 4 \Gamma_{\text{abs}}(\kappa - \Gamma_0)/\kappa^2 \approx 0.4\% \), with \( \kappa \) the photon cavity decay rate. Absorption could be interpreted in terms of an effective imaginary refractive index \[33\] through \( n'(\text{InGaP}) = n(\text{InGaP}) \Gamma_{\text{abs}}/2\pi \nu \approx 10^{-7} \), which is substantially lower than the estimate in [22] at \( \lambda =1064\text{ nm} \) and consistent with measurement of intrinsic \( Q > 10^6 \) still limited by elastic scattering[24].

The device is fabricated on an InGaP membrane grown by MOCVD lattice-matched to GaAs. The OM crystal is processed following the same recipe as for two dimensional photonic crystals[19, 24], except that the resist is written by an EBPG 5000+ e-beam system.
SELF SUSTAINED OSCILLATIONS

As the power is increased the resonator eventually undergoes regenerative oscillations. The threshold is predicted by the condition that the mechanical loss equates the optical anti-damping calculated above: \( \Gamma_m + \Gamma_{om} = 0 \). Using the measured parameters above yields \( F_{c,tr} = 47 \mu W \), which is again, within 20% of the measured value, \( 40 \mu W \).

The linewidth of the mechanical resonance in the RF spectrum (Fig. 4a) narrows and drifts by 700 kHz under the effect of the optical antidamping and spring effect [35]. The mechanical linewidth is very well fitted by a Voigt function (Fig. 4b) within a 95% confidence interval of 10%. The Lorentzian lineshape, corresponding to the short-term linewidth \( \Delta f_L \), is deconvolved from the random frequency fluctuations accumulated during the measurement of the spectra (see supplementary). The measurement is performed as the laser is swept towards the red across the resonance and repeated as the on-chip power is increased. \( \Delta f_L \) decreases from 1.2 MHz down below 100 Hz at a detuning of 0.3\( \kappa \) (Fig. 4b). This is consistent with the expected dependence on the number of phonons in the cavity [36] or, equivalently, the oscillator output RF power \( P_{osc} \), similarly to the Shawlow-Townes limit for lasers:

\[
2\pi \Delta f = \Gamma_m \frac{n_{th}}{2\pi} = \Gamma_m \frac{k_B T}{2 F_{osc}} \tag{1}
\]

This equation, valid above threshold, is used to predict the thermal noise limit to the measured short-term linewidth, which are indeed in good agreement (e.g. at \( P_c = 53 \mu W \) in Fig. 4d). As pointed out in Ref. [6], it can be concluded that the fundamental limit for the short-term linewidth is actually the thermal noise. The number of phonons in the cavity can be deduced from Fig. 4c, in particular a maximum of \( n \approx 10^7 \) is estimated, while the corresponding number of photons is \( n_{hv} = 3 \times 10^4 \).

Importantly, all the devices with comparable loaded Q factor, fixed by the design, start self-sustained oscillation with comparable amplitude at similar optical power levels.

A deeper insight in the noise properties of the oscillator [6] is gained by examining the spectral density of the phase noise \( \mathcal{L}(f) \) (Fig. 5), measured when the device is driven to the maximum amplitude. The cavity considered for this measurement has slightly different parameters, in particular a lower optical quality factor. In the range 50kHz to 2 MHz the phase noise spectral power density follows the slope \( PSD = \Delta f_L/f^2 \), which is associated to phase random walk. The Lorentzian linewidth \( \Delta f_L = 120 Hz \) is extracted, which is consistent with the direct measurement on the signal spectral power (Fig. 4e). While white phase noise, due to thermal noise in the photodetector, dominates at higher frequencies, technical noise \( (1/f^2) \) dominates below 50kHz, which is not surprising for a free running oscillator (no stabilization loop).

FIG. 4. a) Raw ESA trace of the detected signal as a function of the detuning for \( P_c = 53\mu W \); b) Fit of normalized RF spectrum with the Voigt function; c) fitted Lorentzian linewidth \( \Delta f_L \) as a function of the RF integrated power for different optical pump levels, the black line represents the estimated short term limit based on eq. 1; d) measured Lorentzian linewidth and short term limit and corresponding integrated spectral power \( P_{RF} \) (right axis); here \( P_c = 53\mu W \).

CONCLUSION

In conclusion, an optomechanical crystal based on InGaP, a III-V piezoelectric semiconductor alloy lattice-matched to GaAs, has been developed based on a novel design involving only 4 parameters and requiring no optimization. The typical intrinsic optical Q factor is about
FIG. 5. Measured phase noise spectrum (grey filled circles), reference (purple filled circle) and phase random walk noise corresponding to a Lorentzian linewidth $\Delta f_L = 120 \text{Hz}$ (red line) and $\Delta f_L = 50 \text{mHz}$ (blue line).

$2 \times 10^5$, whereas the loaded Q is controlled. While non-linear absorption is absent in the telecom spectral range, owing to the large electronic band-gap, the linear absorption is very small ($\Gamma_{\text{abs}}/2\pi = 8 \text{MHz}$), which, combined to a long thermal relaxation rate ($18 \text{µs}$) compared to the 3 GHz oscillation frequency, implies a negligible contribution of thermomechanical forces to the optomechanical damping $\Gamma_{\text{om}}$. The measured vacuum coupling constant is $g_0/2\pi \approx 380 \text{kHz}$, in good agreement with modeling. At room temperature and standard pressure, the mechanical damping is $Q_m = 2300$, with a corresponding figure of merit $Q \times f = 6 \times 10^{12}$, which is of the same order of magnitude as $[22]$. Self-sustained oscillation is achieved routinely with a loaded optical $Q_L > 2.5 \times 10^4$, with coupled optical power level of about $40 \mu W$. The measured mechanical short-term linewidth narrows down to 100 Hz, limited by classical Brownian noise and would decrease with temperature. Compared to other optomechanical oscillators, the $1/f^2$ term of the phase noise is basically the same as in a Silica$[6]$ and Silicon Nitride microtoroids$[37]$, which are also low loss materials, once corrected for the carrier frequency to allow a fair comparison$[38]$. We note that Micro-Electro-Mechanical Systems (MEMS) are about 10 dB below (after correction) and we highlight the fact that a better frequency stability than in $[39]$ was obtained with optical power levels lower by 2 orders of magnitude (again, after correction). As an optomechanical oscillator, our cavity is comparable with Silicon Nitride microtoroids and in contrast to MEMS but it also provides an optical output, convenient for the distribution of the signal on-chip. Completed with piezo-electric transducers and hybridized on a Silicon Photonic circuit $[40]$, this device could be used for microwave to optical conversion and more elaborate miniaturized optoelectronic oscillators. We note that self-stabilisation schemes have been proposed for OM resonators$[41]$. Further improvement could be achieved inducing tensile stress in the membrane $[21, 42]$. In perspective, this technology could be suitable for the investigation of complex non linear phenomena $[43]$, synchronization of several oscillators $[44]$ or quantum experiments.

ACKNOWLEDGEMENTS

This work was also partly supported by the RENATECH network - We acknowledge support by a public grant overseen by the French National Research Agency (ANR) as part of the Investissements dAvenir program: Labex GANEX (Grant No. ANR-11-LABX-0014) and Labex NanoSaclay (reference: ANR-10-LABX-0035) with Flagship CONDOR. Authors declare no competing interests.

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