Quasi-3D TEM inversion based on lateral constraint

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Abstract. Quasi-2D inversion based on a lateral constraint is an effective technique for the interpretation of the survey data of transient electromagnetic method (TEM). In this paper, the principle of the lateral constraint is introduced to construct a quasi-3D TEM inversion. Considering that the 3D measuring grid is often not strictly regular, the quasi-3D TEM inversion is constructed using the distance-weighted adjacent stations’ lateral constraint, which can suppress the irrational model mutation generated by single-station 1D inversion, and has no requirement concerning the regularity of the 3D measuring grid. The application in the inversion of field TEM data had verified the effectiveness of this method.

1. Introduction
The transient electromagnetic method (TEM) is an important method of electrical prospecting that widely used in engineering, environmental research, hydrology, and mineral exploration. A number of achievements have been made in researching 2D&3D forward modeling and inversion [1-5]. However, 2D and 3D inversions remain time consuming, and require a large-scale computing platform [6-11]. At present, 1D inversion is an important technique in the processing and interpretation of TEM field data. However it encounters the problem of poor continuity along the inversion section. In addition, the late stage of TEM data is weak, and the deep part of the 1D inversion model is vulnerable to noise. In order to improve the continuity along the 1D inversion section, Auken et al. [12] used the lateral constraint to inversion of DC data. Vallée [13] and Cai [9] inverted the airborne electromagnetic data using the lateral constraint. Wang Ying [14] applied the lateral constraint to the ground TEM data inversion. The continuity of the inversion sections and accuracy of the resulting interpretation were significantly improved. In 2016, Yin elaborated the quasi-2D inversion based on the lateral constraint to invert airborne TEM data. However, for applications of TEM survey, a 3D measuring grid is often deployed, and the distribution of the stations is often not strictly regular. Thus, effectively applying the lateral constraint method to the inversion of a general 3D grid of TEM data remains a challenge. In this paper, the distance-weighted adjacent stations’ lateral constraint is used to construct the quasi-3D inversion, which can effectively suppress the irrational model mutation generated by single-station 1D inversion, and does not impose a requirement on the regularity of the 3D measuring grid.

2. Quasi-3D inversion method based on lateral constraint
The classical quasi-2D inversion method based on the lateral constraint [10] integrates the data measured along a line and simultaneously inverts the geo-electric models of all stations along the line under the lateral constraint. The essence of quasi-2D inversion based on the lateral constraint is to add the differences in the geo-electric model between adjacent stations as a constraint to the objective inversion function. This ensures the continuity of the model between adjacent stations, and suppress model mutation caused by single station noise. Based on the principles, we constructed a quasi-3D
TEM inversion with a distance-weighted adjacent stations’ lateral constraint method, which is suitable for regular as well as irregular 3D grids.

It is assumed that there are l stations in a general 3D measuring grid and k time points for the TEM data at each station. The time points of the stations may be inconsistent. For convenience of expression, all of them are set as k. Then, the integrated data of the l stations can be written as
\[ \mathbf{d} = (d_{11}, d_{12}, ..., d_{1k}, d_{21}, d_{22}, ..., d_{2k}, ..., d_{l1}, d_{l2}, ..., d_{lk})^T \] (1)

It is assumed that the inversion model of each station has n layers, and only the resistivity of the model is considered. Thus, the overall model of l stations in the logarithmic domain is
\[ \mathbf{m} = (\lg(\rho_{11}), \lg(\rho_{12}), ..., \lg(\rho_{1n}), ..., \lg(\rho_{l1}), \lg(\rho_{l2}), ..., \lg(\rho_{ln}))^T \] (2)

For any station in the 3D grid, models of the p stations closest to it within radius r are applied to constrain the inversion by using distance as weight. Then, the integrated objective function of the quasi-3D inversion based on the lateral and roughness constraints can be written as:
\[ \varphi = (\mathbf{F}(\mathbf{m}) - \mathbf{d})^T \mathbf{C}^{-1}(\mathbf{F}(\mathbf{m}) - \mathbf{d}) + u_p (\mathbf{E}_p \mathbf{m})^T + u_h (\mathbf{E}_h \mathbf{m})^T \mathbf{C}_1^{-1} (\mathbf{E}_h \mathbf{m}) + ... + u_h (\mathbf{E}_h \mathbf{m})^T \mathbf{C}_i^{-1} (\mathbf{E}_h \mathbf{m}) + ... + u_h (\mathbf{E}_h \mathbf{m})^T \mathbf{C}_p^{-1} (\mathbf{E}_h \mathbf{m}) \] (3)

In equation (3), \( \mathbf{F} \) is the 1D forward operator of all stations, \( \mathbf{C} \) is the covariance matrix of the data, which is used to weigh the fitting of the observed data based on data error; \( \mathbf{E}_v \) is the first-order difference matrix in the direction of the depth of all station models, \( u_v \) is the constraint factor in the direction of depth, \( \mathbf{E}_h \) is the lateral constraint matrix of the first adjacent station of all stations, \( \mathbf{C}_1 \) is the distance-weighted matrix of the first adjacent station of all stations, and \( u_h \) is the lateral constraint factor; \( \mathbf{E}_h \) and \( \mathbf{C}_i \) are, respectively, the lateral constraint matrix of the i-th adjacent station of all stations and the corresponding distance-weighted matrix; \( \mathbf{E}_{hp} \) and \( \mathbf{C}_p \) are, respectively, the lateral constraint matrix of the p-th adjacent station of all stations and the corresponding distance-weighted matrix. The matrices of \( \mathbf{E}_v \), \( \mathbf{E}_h \), and \( \mathbf{C}_i \) are shown in equations (4), (5), and (6). \( \mathbf{E}_h \) and \( \mathbf{E}_{hp} \) have the same form as \( \mathbf{E}_i \) and \( \mathbf{C}_1 \) and \( \mathbf{C}_p \) have the same form as \( \mathbf{C}_i \), and thus are not listed again.

\[
\mathbf{E}_v = \begin{bmatrix}
-1 & 1 & ... & 0 & 0 & 0 & ... & 0 \\
0 & -1 & 1 & ... & 0 & 0 & 0 & ... & 0 \\
0 & ... & 0 & 0 & 0 & ... & -1 & 1 \\
\end{bmatrix}_{n \times 1 / n \times n}
\] (4)

\[
\mathbf{E}_h = \begin{bmatrix}
-1 & 0 & ... & 0 & 1 & 0 & ... & 0 \\
0 & -1 & 0 & ... & 0 & 1 & 0 & ... & 0 \\
0 & ... & 0 & 1 & 0 & ... & 0 & ... & -1 \\
\end{bmatrix}_{n \times 1 / n \times n}
\] (5)
\[
C_i^{-1} = \begin{bmatrix}
\frac{1}{r_i} & 0 & \ldots & 0 & \ldots & \ldots & 0 \\
0 & \frac{1}{r_i} & 0 & \ldots & 0 & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\
\end{bmatrix}
\]

where the \( r_i \) in \( C_i^{-1} \) denotes the distance between the first station and the ith adjacent station to it, and \( r_i \) denotes the distance between the ith station and the ith adjacent station to it.

Ev is the first-order difference matrix in the direction of the depths of all models; then, (Evm) is the integration of the roughness constraint of all stations. Ehi is essentially the lateral first-order difference operator between each station and the ith station adjacent to it; then, (Ehim) is the lateral model difference between each station and the ith station adjacent to it. In equation (3), the differences in models between each station and the p closest stations adjacent to it are added to the objective function. Then, when the objective function is minimized, the least-square solution with lateral constraint can be obtained. As we use the p closest stations to constrain the inversion of each station without considering the distribution of the stations, the quasi-3D inversion based on the lateral constraint is suitable for the general 3D grid.

3. **Solution of the quasi-3D inversion objective function**

The Gauss–Newton solution features stable iterations and fast convergence. Therefore, this method is used here to solve the quasi-3D objective inversion function with the distance-weighted adjacent stations’ lateral constraint (equation (3)).

Let \( m_0 \) be the initial model and \( \Delta m \) the amendment to it. By linearizing \( F \) at \( m_0 \) in equation (3), the iterative formula of the Gauss–Newton method can be derived as

\[
(J^T C^{-1} J + \sum E_r^T E_r + \sum E_{hi}^T C_i E_{hi} + \ldots + \sum E_{hp}^T C_p E_{hp}) \Delta m = J^T C^{-1} [d - F(m_0)] - \sum E_r^T E_r m_0 - \sum E_{hi}^T C_i E_{hi} m_0 - \ldots - \sum E_{hp}^T C_p E_{hp} m_0
\]

In equation (7), \( J \) is the Jacobian matrix of the forward function \( F \) for model \( m_1 \), which is the block diagonal matrix of the integration of the Jacobian matrix of the TEM’s 1D forwarding function to the corresponding model at each station, and contains only 1xkxn non-zero elements.

The amendment \( \Delta m \) is obtained by solving equation (7), and the amended \( m \) is obtained as follows:

\[
m = m_0 + \Delta m
\]

The final result of the inversion is obtained by multiple iterations until the data misfit reaches the preset fitting error or the maximum number of iterations. Each matrix in equation (7) is a sparse matrix, of which only the non-zero elements need to be stored to save memory and improve computational efficiency.

The most commonly used TEM configuration is discussed here, of which a horizontal rectangular loop is laid on the ground as the source of transmission, and the induced electromotive force of the vertical magnetic field is measured. In this paper, the classical forward method is used, of which the response in the frequency domain is first calculated and converted into the time domain by using the sine transform. The Jacobian matrix is also quickly obtained by using the forward method.
The four sides of the rectangular loop can be regarded as four wire sources. Then, the response of rectangular loop source can be superposed by the responses of the four wire sources. Based on the formula for the calculation of the wire source [15], the vertical magnetic field at a point A on the ground, excited by the rectangular loop source in the frequency domain, can be calculated by equation (9).

\[ H_z(\omega) = \sum_{i=1}^{4} \frac{I_i}{4\pi} \left[ \frac{y}{R} \int_0^\infty (1 + r_{TE}) \frac{\lambda^2}{\mu_0} \frac{J_1(\lambda R)}{\lambda} d\lambda dx \right] \]

\[ \sum_{i=1}^{4} \frac{I_i}{4\pi} \left[ \frac{y}{R} \int_0^\infty (1 + r_{TE}) \frac{\lambda^2}{\mu_0} \frac{J_1(\lambda R)}{\lambda} d\lambda dx \right] \tag{9} \]

In the above, \( H_z(\omega) \) is the vertical component of the magnetic field, \( J_1(\lambda R) \) is the first-order Bessel function, \( \lambda \) is the inner integral variable, \( L_i \) is the length of the \( i \)th side of the transmitting loop, \( y \) is the distance between point A and the \( i \)th side, \( x \) is the distance between point A and the midpoint of the \( i \)th side, \( \lambda \) is the integral variable along the \( i \)th side, \( R = \sqrt{(x - x')^2 + y^2} \), \( r_{TE} \) is the reflection coefficient of the TE mode on the ground, and \( I \) is the transmitting current.

The time derivative of the magnetic field in the time domain can be obtained by sine transformation (equation (10))

\[ \dot{H}_z = \frac{2}{\pi} \int_0^\infty \text{Im}[H_z(\omega)] \sin(\omega t) d\omega \]

\[ \dot{H}_z = \frac{2}{\pi} \int_0^\infty \text{Im}[H_z(\omega)] \sin(\omega t) d\omega \tag{10} \]

According to equations (9) and (10), the TEM responses of the vertical magnetic field at point A on the ground of the rectangular loop source can be calculated. From equations (9) and (10), it is clear that only the reflection coefficient \( r_{TE} \) contains the model parameters. Thus, the partial derivatives of the TEM response to the model can be obtained by calculating only the derivation of \( r_{TE} \) to the model parameters and executing a forward calculation.

4. Application in TEM data inversion

In an oil field in the Middle East, the seismic data showed that there might be a local depression in a shallow part (400-m deep). However, this could not be confirmed only using these data. Such a shallow depression can significantly affect the deep seismic images. To know more about the shallow depression and improve seismic imaging, a TEM survey was carried out to investigate the structure of the stratum within a depth of 400 m. A central loop configuration of 200 m × 200 m was used in the field, and the vertical magnetic field component was observed. Even though the planned survey line was strictly regular, the line spacing was 400 m and the station spacing was 250 m. However, the distribution of the survey points was very irregular to avoid interference by oil field facilities (Fig. 1).

![Figure 1. Map of distribution of TEM stations (using two color symbols to distinguish lines)](image)

As the distribution of the TEM stations was irregular, the quasi-3D inversion described above was used to invert the TEM data. To analyze the effectiveness of quasi-3D inversion, line N1 was also
inverted by using conventional single-station 1D inversion, and the inversion result is shown in Fig. 2a. Figs. 2b, c, and d are, respectively, the resistivity sections of quasi-3D inversion along lines N1, N2, and N3, and Fig. 3 shows the resistivity maps at elevations of 200 m and 0 m of the results of quasi-3D inversion.

Fig. 2a (conventional 1D inversion section) basically shows the characteristics of the shallower structure along the survey line, indicating some unreliable, prominent, and isolated anomalies. The electrical characteristics of the formations in Fig. 2a were thus unclear. However, in Fig. 2b (results of quasi-3D inversion), the irrational mutation was greatly suppressed, and the electrical characteristics of the formation were clear. Figs. 2b, c, and d show that the electrical layers were all laterally continuous which can be tracked among multiple lines, and the shallow depression was clearly delineated (from a distance of 2500 m to that of 4200 m). The resistivity maps in Fig. 3 also clearly show that the shallow depression in the regional center stretches north–south.

In Fig. 2c, the resistivity section of quasi-3D inversion of line N2 is superimposed with the resistivity logging curves. The inversion resistivity was consistent with the resistivity logging curves, which further shows that the proposed quasi-3D inversion was effective.

![Figure 2. TEM Inversion sections. (a. The conventional 1D inversion resistivity section of line N1; b. The quasi-3D inversion resistivity section along line N1; c. The quasi-3D inversion resistivity section along line N2 superimposed with the resistivity logging curves; d. The quasi-3D inversion resistivity section along line N3)](image-url)
Figure 3. The resistivity maps of quasi-3D inversion results at different elevations (a. at 200 m; b. at 0 m)

5. Conclusions
Based on the principle of the lateral constraint, a distance-weighted adjacent stations’ lateral constraint method was applied to construct a quasi-3D TEM inversion. The quasi-3D TEM inversion was used to interpret the field TEM data, the results show that the quasi-3D TEM inversion can suppress the irrational mutation caused by single-station 1D inversion and the random noise in TEM data and it does not impose a requirement on the regularity of the 3D measuring grid.

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