On the Temperature Dependence of the Casimir Force for Bulk Lossy Media

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Abstract

We discuss the limitations of the applicability of the Lifshitz formula to describe the temperature dependence of the Casimir force between two bulk lossy metals. These limitations follow from the finite sizes of the interacting bodies. Namely, Lifshitz’s theory is not applicable when the characteristic wavelengths of the fluctuating fields, responsible for the temperature-dependent terms in the Casimir force, is longer than the sizes of the samples. As a result of this, the widely discussed linearly decreasing temperature dependence of the Casimir force can be observed only for dirty and/or large metal samples at high enough temperatures. This solves the problem of the inconsistency between the Nernst theorem and the “linearly decreasing temperature dependence” of the Casimir free energy, because this linear dependence is not valid when $T \rightarrow 0$.

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INTRODUCTION

The Casimir effect is one of the most interesting macroscopic manifestation of the zero-point vacuum oscillations of the quantum electromagnetic field. This effect manifests itself as an attractive force arising between two uncharged bodies due to the difference of the zero-point oscillation spectrum in the absence and in the presence of these bodies (see, e.g., Refs. [1, 2, 3, 4]).

The Casimir effect has attracted considerable attention because of its numerous applications in quantum field theory, atomic physics, condensed matter physics, gravitation and cosmology [1, 2, 3, 4, 5]. The noticeable progress in the measurements of the Casimir force [6] has opened the way for various potential applications in nanoscience [7], particularly in the development of nano-mechanical systems [2, 4, 7].

Problems linked to Lifshitz’s theory for the Casimir force

In spite of intensive studies on the Casimir effect, it is surprising that such an important problem as the temperature dependence of this effect is still unclear and is still an issue of lively discussion (see, e.g., Refs. [8, 9, 10, 11, 12, 13, 14]). The zero-temperature contribution to the force, originating from quantum fluctuations of the electromagnetic field, is well understood. However, the contribution $F_{\text{rad}}(T)$ to the Casimir force originating from thermal fluctuations is a source of numerous controversies.

First, within the Lifshitz theory [15], there is no continuous transition for the forces between ideal metals and real metals [9]. The Lifshitz formula predicts an increase of $F_{\text{rad}}(T)$ when increasing $T$ only for ideal metals without relaxation. At the same time, for lossy media with relaxation frequency $\nu \neq 0$, this formula gives a decrease of $F_{\text{rad}}(T)$ in a wide region of temperatures. This decreasing term is related to the transparency of real metals for $s$-polarized (transverse electric) low-frequency fields. In other words, the behavior of $F_{\text{rad}}(T)$ changes abruptly, in a jump-like manner, for infinitesimal $\nu$, in comparison to the case when $\nu = 0$. This discontinuous jump is not physical.

Second, the Casimir-Lifshitz entropy does not go to zero when $T \to 0$. This is unphysical, because it violates the Nernst theorem. This problem is still the focus of discussions (see, e.g., Ref. [11]).
These obvious contradictions to common sense show that some important physics is missing. Recently, Ref. [14] indicated that the problems mentioned above can be solved if one takes into account the spatial dispersion of the low-frequency metal conductivity.

Summary

In this work, we demonstrate that there exist simple limitations for the applicability of Lifshitz’s theory. We show that the Casimir force for good metals, where the plasma frequency \( \omega_p \) is much higher than other characteristic frequencies of the system, increases monotonically with temperature \( T \) if the following inequality is satisfied:

\[
\nu \ll \frac{2\pi c}{L}.
\]

(1)

Here \( L \) is the width of a sample. This condition (1) is typically satisfied for pure metals. For instance, it is valid for metal discs with area \( \sim 10^{-2} \) cm\(^2\), if the relaxation frequency is less than \( 10^{12} \) s\(^{-1}\).

ANALYSIS OF THE TEMPERATURE DEPENDENCE OF THE LIFSHITZ FORMULA

The purpose of this section is to show that the main contribution to the “linearly decreasing with temperature term” in the Casimir force between infinite plates of lossy metals comes from the fluctuating fields with small frequencies

\[
\omega \lesssim \nu.
\]

(2)

We analyze the Lifshitz expression for the Casimir force taken from Ref. [15] in the form of an integral over real frequencies \( \omega \). We use the Drude model for the permittivity \( \varepsilon \),

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}.
\]

(3)

In this case, the thermal term \( F_{rad} \) in the Casimir force per unit area can be written in the following form:

\[
F_{rad} = \frac{\hbar}{\pi^2 c^3} \Re \int_0^\infty d\omega \int dp p^2 \omega^3 \frac{1}{\exp(2\hbar\omega/kT) - 1} \times \left\{ \left[ \frac{(s + p)}{s - p} \right]^2 \exp(-2ip\omega l/c) - 1 \right\}^{-1} + \left[ \frac{(s + \varepsilon p)}{s - \varepsilon p} \right]^2 \exp(-2ip\omega l/c) - 1 \right\}^{-1}
\]

(4)
where
\[ s = \sqrt{\varepsilon(\omega) - 1 + p^2}, \] (5)
l is the separation between the interacting bodies, and the symbol \( \Re \) denotes the real part.
The integration trajectory over \( p \) consists of two parts: from 1 to 0 over the real axis, and from \( i0 \) to \(+i\infty\) over the imaginary axis.

We examine the difference \( \Delta F_{\text{rad}} \) between the contributions to the Casimir force from thermal fluctuations for a dissipationless metal \((\nu = 0)\) and for a metal with weak dissipation \((\nu \to 0)\),
\[ \Delta F_{\text{rad}} = F_{\text{rad}}\bigg|_{\nu \to 0} - F_{\text{rad}}\bigg|_{\nu = 0}. \] (6)
Namely, \( \Delta F_{\text{rad}} \) describes the “linearly decreasing with \( T \)” part of the Casimir force \( F_{\text{rad}}(T) \) that appears in a jump-like manner at \( \nu \neq 0 \). It is important to note that only the first term in the curly brackets in Eq. (4) [integrated over \( p \) from \( i0 \) to \(+i\infty\), and over \( \omega \) from 0 to \(+\infty\)] produces this discontinuity. So, the difference \( \Delta F_{\text{rad}} \) can be written as
\[ \Delta F_{\text{rad}} = \frac{\hbar}{\pi^2 c^3} \Re \int_0^\infty d\omega \int_{i0}^{+i\infty} dp p^2 \omega^3 \frac{1}{\exp(2\hbar \omega/kT) - 1} \quad \times \quad \left\{ \left[ \left( \frac{s + p}{s - p} \right)^2 \exp(-2ip\omega l/c) - 1 \right] - \left[ \left( \frac{s|_{\nu=0} + p}{s|_{\nu=0} - p} \right)^2 \exp(-2ip\omega l/c) - 1 \right] \right\}^{-1}. \] (7)
Introducing the notation,
\[ t = \frac{\omega}{\nu}, \quad x = -\frac{2ip\omega l}{c}, \quad \alpha = \frac{c}{2l\omega_p}, \] (8)
and assuming that
\[ \hbar \nu \ll kT, \] (9)
we obtain
\[ \Delta F_{\text{rad}} = -\frac{kT}{8\pi^2 l^3} \Im \int_0^\infty dt \int_0^{+\infty} \frac{dx x^2}{(\alpha x + \sqrt{\alpha^2 x^2 + \frac{t}{t+i}})^4 \left( \frac{t+i}{t} \right)^2 e^x - 1} \] (10)
where the symbol \( \Im \) denotes the imaginary part.

It is seen from this equation that the difference \( \Delta F_{\text{rad}} \) does not depend on \( \nu \), and that the main contribution to this integral comes from \( x \sim 1 \) and \( t \lesssim 1 \). Thus, according to Eq. (8), the characteristic values of \( \omega \) are either of the order or less than \( \nu \).
For good metals with $\alpha \lesssim 1$, the value of the integral in Eq. (10) is of the order of 1, and
\[
\Delta F_{\text{rad}} \sim \frac{kT}{l^3} \quad \text{(for good metals with } \alpha \lesssim 1). \tag{11}
\]
In the opposite case $\alpha \gg 1$, the characteristic frequencies are even smaller than $\nu$, $\omega \sim \nu/\alpha \ll \nu$. For such small distances $l$, the value of the integral in Eq. (10) is of the order of $1/\alpha^3$, and
\[
\Delta F_{\text{rad}} \sim \frac{kT\omega_p^3}{c^3} \left( \text{when } \alpha \lesssim 1; \text{ i.e., when } l \ll \frac{c}{\omega_p} \right). \tag{12}
\]

LIMITATIONS FOR THE APPLICABILITY OF THE LIFSHITZ FORMULA AND DISCUSSIONS

Thus, the main contribution to the “linearly decreasing with $T$ term” $F_{\text{rad}}(T)$ in the Lifshitz theory comes from small frequencies satisfying two inequalities,
\[
\omega \ll kT/h, \quad \omega \lesssim \nu. \tag{13}
\]
Obviously, the wavelengths of the fluctuating fields with such frequencies should be much smaller than the size of the sample. Otherwise, the sample cannot be considered as semi-infinite. In other words, the Lifshitz theory gives a (physically correct) $F_{\text{rad}}(T)$ decreasing with temperature if
\[
\nu \gg 2\pi c/L \tag{14}
\]
and
\[
kT \gg 2\pi hc/L. \tag{15}
\]
Moreover, it is clear that a metal with plasma frequency much higher than other characteristic frequencies $\omega_i$, and with $\nu \ll \omega_i$ (the frequency $2\pi c/L$ is among them), should possess properties close to the ones of an ideal metal. This means, that the inequality (14) ensures the increase of the Casimir force with temperature, similarly to the case of ideal metals.

Note that the condition (15) did not hold in the experiment in Ref. [16] (with a tiny metal sphere of radius $R = 151.3 \, \mu m$). For all temperatures used in that experiment, the wavelengths of the fluctuating fields responsible for the temperature decrease of the Casimir force (expected within the Lifshitz theory with the Drude model for the permittivity) were of the order of $R$ and longer than the radius $r \sim (RL)^{1/2}$ of the effective interacting region of
the tiny sphere. Therefore, it is not surprising that the temperature decrease of the Casimir force was not observed in Ref. [16].

Note also that the condition in Eq. (15) solves the problem of the inconsistency between the Nernst theorem and the linearly decreasing temperature dependence of the Casimir free energy. Indeed, the linear asymptotic dependence of $F_{\text{rad}}$ on $T$ is not applicable when $T \to 0$.

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