Axion Interferometry

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Strong CP Problem

- Axions are arguably the simplest and most minimal solution to the Strong CP problem
  - Closest competitor is the minimal LR symmetric models
- Solves a problem and can be dark matter
- If it is dark matter, how can we look for it?
Axion dark matter

- Axion dark matter obtains its number abundance through the misalignment mechanism
  - Produces cold dark matter regardless how light the axion is
- The axion is a classical field due to large number abundance
  \[ a(t) \sim a_0 \cos m_\alpha t \]
- Non-relativistic, \( v \sim 10^{-3} \)
Axion dark matter

Axion is localized to a distance

\[ L \sim \frac{1}{m_a v} \]

So it takes a time

\[ \tau \sim \frac{1}{m_a v^2} \]

Until whole axion wave passes by and replaced by a new axion wave
Axion dark matter

- Axion dark matter is a wave with
  \[ \omega \sim m_a \pm 10^{-6} m_a \]
- It has a quality factor of
  \[ Q = \frac{1}{\omega \tau} \sim 10^6 \]
Looking for the axion

\[ \mathcal{L} \supset \frac{a}{4f} F \tilde{F} \]

- Looking for the axion through the coupling to gluons is HARD
  - Very few experiments can reach the QCD axion line
- Instead look for the axion through its coupling with the photon
DISCLAIMER

• QCD Axion
  • Solves the Strong CP problem
  • Couples to photons and gluons and fermion spin

• ALP (Axion like particles)
  • Does NOT solve the Strong CP problem
  • Couples to photons and/or fermion spin

• Axions
  • Can be either
  • Figure it out from context
Effect of photon coupling

\[ \mathcal{L} \supset \frac{a}{4f} F \tilde{F} \]

- For circularly polarized light

\[-\omega^2 + k^2 = \frac{da}{dt} \frac{k}{f} = 0 \quad \Rightarrow \quad \nu_{\text{phase}} \approx 1 \pm \frac{\dot{a}}{2kf} \]
Effect of photon coupling

\[ v_{\text{phase}} \approx 1 \pm \frac{\dot{a}}{2kf} \]

- Phase velocity of circularly polarized light is different depending on which polarization it is
- Device most sensitive to differences in phase velocities is an interferometer

Axion interferometry

There are many proposals for experiments to look for axions and ALPs. See Refs. [7–13] for a small subset of these proposals.

As the effect of axion dark matter is to change the phase of light, we propose using interferometry of circularly polarized light as a mechanism by which to test for axion dark matter. These interferometers differ from standard interferometers only by the addition of a few quarter waveplates to preserve the polarization of light upon reflection. We show that using an interferometer is most sensitive to differences in phase velocities is an interferometer.
Axion interferometry

- One-to-one mapping between axion interferometry and gravity wave interferometry
- An axion interferometer can double as a gravity wave detector
- Axion dark matter appears in the same manner as a continuous gravity wave signal with a quality factor of $10^6$
Gravity wave interferometry

Consider a plus polarized gravity wave incident perpendicular to the interferometer
No Gravity Wave

\[ E = \frac{1}{2} E_0 e^{-i\omega_L(t-2L_x)} - \frac{1}{2} E_0 e^{-i\omega_L(t-2L_y)} \]

Relative minus sign due to reflection at beam splitter

\[ E^2 = E_0 \sin^2 \omega (L_x - L_y) \]

Typically adjust \( L_x \) and \( L_y \) so that the electric field at the detector is small
Gravity wave

Time it takes for the light to go to the mirror and back

\[ 2L_x = (1 - \frac{1}{2}h_+(t)) \int_{t_0}^{t_0+\tau} dt \]
\[ h_+ = h_0 \cos \omega_g t \]

\[ \tau = 2L_x + h_0 L_x \frac{\sin w_g L_x}{w_g L_x} \cos w_g (t_0 + L_x) = 2L_x + L_x h(t_0 + L_x) \frac{\sin w_g L_x}{w_g L_x} \]

Can get the y result by \( L_x \) to \( L_y \) and \( h_0 \) to \(-h_0\)
Gravity wave

Look for the oscillations in the amplitude of the electric field

\[ E = -i E_0 e^{-i\omega_L t + i\omega^2_L} \sin(\omega_L(L_x - L_y) + \omega_L L h(t - L_x) \frac{\sin \omega_g L}{\omega_g L}) \]

\[ \Delta \phi = \frac{\omega_L h_0}{\omega_g} \sin (\omega_g L) \cos (\omega_g t + \alpha) \]
Gravity wave

\[ \Delta \phi = \frac{\omega_L h_0}{\omega_g} \sin (\omega_g L) \cos (\omega_g t + \alpha) \]

1. Optimal Length is as expected \( L = \lambda_g / 4 \)
2. Broadband detector
Axion wave interferometry

Equivalent Axion interferometer involves adding 4 waveplates
Axion interferometry

Equivalent Axion interferometer involves adding 4 waveplates

Left Polarized Light

Detector

Right Polarized Light

Laser
Axion wave

- Only difference is the presence of wave plates
- Needed to maintain polarization
Axion wave
No Axion DM

Exactly the same as a gravity wave interferometer

Experiment doubles as a gravity wave detector

No need to send the legs in different directions otherwise
Axion interferometry

Time it takes for the light to go to the mirror and back

\[ 2L_x = (1 - \frac{1}{2} \frac{\dot{a}(t)}{\omega_L f}) \int_{t_0}^{t_0+\tau} dt \]

\[ \dot{a}(t) = i m_a a(t) = \sqrt{2\rho_{DM}} \cos m_a t \]

\[ \tau = 2L_x + L_x \frac{m_a a(t)}{\omega_L f} \frac{\sin m_a L_x}{m_a L_x} \]

Can get the y result by \( L_x \) to \( L_y \) and \( a(t) \) to \(-a(t)\)
Axion interferometry

\[ \tau = 2L_x + L_x \frac{m_a a(t)}{\omega_L f} \frac{\sin m_a L_x}{m_a L_x} \]

\[ \tau = 2L_x + h_0 L_x \frac{\sin w_g L_x}{w_g L_x} \cos w_g(t_0 + L_x) = 2L_x + L_x h(t_0 + L_x) \frac{\sin w_g L_x}{w_g L_x} \]

Axion interferometer equivalent to gravity wave interferometer!

\[ h_0 \rightarrow \frac{m_a a_0}{f \omega} = \sqrt{2 \rho_{DM}} \]

\[ \omega_g \rightarrow m_a \]
Resonant interferometry

What happens if you don’t add in the extra wave plates?
Resonant interferometry

Resonant Detector instead!

Not to scale
Resonant interferometry

1. Optimal Length is as expected \( L = \lambda_g / 2 \)
2. Resonant detector
Fabry-Perot

Laser

Detector

Mirror

Mirror

Mirror
Fabry-Perot

Fabry-Perot Cavity
The phase accumulated over a single round trip is

\[ \Delta \phi = \frac{w_L h_0}{w_g} \sin w_g L \]

\[ E = E_0 e^{-i w_L t + i \Delta \phi \cos (w_g t + \alpha)} \]
Fabry-Perot

The phase accumulated over a single round trip is

$$\Delta \phi = \frac{w_L h_0}{w_g} \sin w_g L$$

$$E = E_0 e^{-i w_L t + i \Delta \phi \cos(w_g t + \alpha)}$$

$$\approx E_0 \left( e^{-i w_L t} + \frac{i}{2} \Delta \phi e^{i \alpha} e^{-i (w_L - w_g) t} + \frac{i}{2} \Delta \phi e^{-i \alpha} e^{-i (w_L + w_g) t} \right)$$

Effect of gravity waves is to create side bands (light with slightly different frequencies)
Fabry-Perot

What comes out of a Fabry-Perot Cavity is an infinite sum of light that has bounced around many times.

$$\Delta \phi_x = h_0 \omega_L L \frac{2F}{\pi} \frac{1}{\sqrt{1 + \left(\frac{f_g}{f_p}\right)^2}}$$

An enhanced sensitivity over the standard interferometer by Finesse ~ number of times light bounces around before escaping.
For low frequencies Fabry Perot Cavity better by a factor of Finesse

\[ \Delta \phi = h_0 \omega_L L \frac{2F}{\pi} \quad \Delta \phi = h_0 \omega_L L \]

- Get an interferometer whose arm length is effectively longer
Fabry-Perot

Better sensitivity at low frequency but not as broadband as before

\[ \Delta \phi \]

\[ \omega_g \]
Axion Interferometer

The axion equivalent of a standard interferometer (still acts like a gravity wave detector)

Add 4 wave plates
Axion Interferometer

Same Mapping as before
Otherwise identical to Gravity wave detector
Noise

• An interferometer counts the number of photons arriving at the detector a second
• How the number of photons a second changes tells us about a time varying phase
• Main sources of noise
  • Shot Noise
  • Radiation Pressure
Shot Noise

In some time $T$, there are an average number of photons that arrive

$$N_\gamma = \frac{PT}{w_L}$$

The number is given by Poisson statistics

$$\Delta P = \frac{\sqrt{N_\gamma w}}{T} = \sqrt{\frac{Pw_L}{T}}$$
Shot Noise

\[ P = E_0^2 \sin^2 \phi_0 + \Delta \phi_x \quad \phi_0 = w_L(L_x - L_y) \]

This is so that when a signal arrives

\[ \Delta P = P_0 \Delta \phi_x \sin 2\phi_0 \]

Linear piece would vanish is sitting on dark spot

aLIGO sits slightly off the dark spot
Shot Noise

\[
\frac{S}{N} = \frac{\Delta P_{GW}}{\Delta P} = \frac{P_0 \Delta \phi_x \sin 2\phi_0}{\sin \phi_0} \sqrt{\frac{T}{P_0 w_L}} = \frac{h_0}{S_n^{1/2}} \sqrt{T}
\]

\[
S_n^{1/2} = \frac{1}{4L \mathcal{F}} \sqrt{\frac{\pi \lambda}{P_0}} \frac{1}{\sqrt{1 + \left(\frac{f_g}{f_p}\right)^2}}
\]

Shot Noise is constant at low frequencies

Shot Noise increases at high frequencies
Radiation pressure

• When a photon hits beam splitter 50/50 chance of going up or down
  • Sometimes more photons go up than down

• Thus the force on the mirrors are not always the same
  • Position of the mirrors will fluctuate
  • Frequency of restoring force small compared to frequency of gravity wave so mirror is effectively a freely falling object

• The fact that $L_x$ and $L_y$ vary in time induces a background for gravity wave detection
Radiation pressure

Via similar calculation to before

\[ S_{\text{radiation}}^{1/2} = \frac{16F}{MLm_a^2} \sqrt{\frac{P}{\pi \lambda \sin m_a L}} \frac{m_a L}{m_a L} \]

Radiation pressure relevant at low frequencies
Thus the final SNR is

\[
\text{SNR} = \frac{h_0}{S_{SQL}^{1/2} \left( T \tau \right)^{1/4}}
\]

Errors added in quadrature \( S_{SQL} = S_{\text{shot}} + S_{\text{radiation}} \)

SNR only grows like \( T^{1/2} \) until approximation that signal is a sin wave breaks down
SNR

\[ \text{SNR} = \frac{h_0}{S^{1/2}} (T \tau)^{1/4} \]  

Qualitatively: Add these units of time in quadrature to get $T^{1/4}$ growth
Signal Processing

• There is an optimal way to look for a signal called matched filtering
• Given a hypothetical data stream
  \[ s(t) = h(t) + n(t) \]
• The noise obeys
  \[ \langle n(t) \rangle = 0 \quad \langle n(f)n(f') \rangle = \delta(f - f') \frac{1}{2} S_n(f) \]
Signal Processing

- Define a signal we are interested in

\[ \hat{s} = \frac{1}{T} \int_{0}^{T} dt s(t) K(t) \]

- Average signal and background are

\[ S = \int_{-\infty}^{\infty} dt \langle s(t) \rangle K(t) = \int_{-\infty}^{\infty} df h(f) K(f) \]

\[ N^2 = \langle \hat{n}^2 \rangle = \int_{-\infty}^{\infty} dt dt' K(t) K(t') \langle n(t) n(t') \rangle = \int_{-\infty}^{\infty} df \frac{1}{2} S_n(f) K(f)^2 \]
Define a dot product
\[ A \cdot B = \int_{-\infty}^{\infty} df \frac{A(f)B(f)}{1/2S_n(f)} = 4 \int_{0}^{\infty} df \frac{A(f)B(f)}{S_n(f)} \]

Maximizing SNR corresponds to choosing an optimal vector
\[ u = \frac{1}{2} S_n(f) K(f) \quad SNR = \frac{u \cdot h}{\sqrt{u \cdot u}} \]
Signal Processing

• Clearly the best way to maximize the signal is to choose $u$ proportional to $h$

$$SNR^2 = h \cdot h = 4 \int_0^\infty df \frac{h(f)^2}{S_n(f)}$$

• Gives the general formula for calculating SNR called waveform matching
Axion Interferometer

Seismic Noise becomes an issue

- 40 m arm Length
- 10 kg mirror
- Red : 1 MW power
- Black : 1 kW power
- Dotted : $F = 10^6$
- Solid : $F = 10^2$
Axion Interferometer

- If detector is dedicated to an axion search and not gravity wave search, can do better!
- Radiation pressure can be mitigated if same mirror is used for both arms!
Axion Interferometer

Radiation Pressure replaced by Radiation Torque
Radiation Torque

Via similar calculation to before

\[ S_{\text{torque}}^{1/2} = \frac{Mr^2}{I} S_{\text{rad}}^{1/2} = \frac{16r^2 \mathcal{F}}{ILm_a^2} \sqrt{\frac{P}{\pi \lambda}} \frac{m_a L}{\sin m_a L} \]

Mirrors can only be made so heavy

Geometry is much easier to manage
Axion Interferometer

10 kg mirror
10 cm diameter
1 cm between beams
Red : 1 MW power
Black : 1 kW power
Dotted : F = 10^6
Solid : F = 10^2
Conclusion

- Axion dark matter changes the phase velocity of circularly polarized light
- Can look for this effect in an interferometer
- Can extend bounds by up to 2-3 orders of magnitude over some range of parameters
- Do not need the newest fanciest technology
  - Need to make sure that birefringent backgrounds are under control!