Simulation of radiation generation in a multistage laser

E A Sharandin¹ and V O Gladyshev²
Bauman Moscow State Technical University, Moscow, Russia
E-mail: ¹shar@bmstu.ru, ²gladyshev@bmstu.ru

Abstract. The mathematical model of a solid-state multi-stage laser is described. The model can be used to analyze the radiation generating process in the laser in the presence of interstage couplings, complex optimization and determining the requirements for interstage cross-coupling nodes.

1. Introduction
Developing multistage pulsed lasers featuring maximum achievable emission parameters is in demand in contemporary problems of gravitation, spectroscopy, laser ranging, etc. [1,2]. This requires creating mathematical models of laser emitters taking into account complex physical processes going on in active laser media, radiation interacting with optical elements, as well as the interaction between laser stages.

This mutual influence of laser stages leads to changes in the kinetics of emission pulse shaping and lowers the threshold of parasitic oscillation [6]. Parasitic oscillation, in its turn, may lead to a dramatic decline in the energy stored in gain elements or even destruction of those in the moment of pulse generation in Q-switched lasers. The mathematical model should also take into account the finite lifetime of the lower laser level [7].

Introducing co-activator ions along with activator ions during gain medium creation makes it possible to improve population inversion generation efficiency due to expansion of absorption lines and an increase in their number, as well as due to raising the pumping radiation absorption coefficient. In a number of cases, such active laser media as scandium garnets doped with active Cr³⁺ and Nd³⁺ ions may be adequately described by a traditional four-level model. In other cases, more complex mathematical models are required to describe the interaction between the gain medium and optical radiation, for example, for media doped with active Cr³⁺, Tm³⁺ and Ho³⁺ ions, which allow lasing to occur in the 2...3 µm wavelength range [8].

As a result, it becomes necessary to develop a mathematical model for a multi-stage single-pulse solid-state laser. This model must take into account the following: the two-way interaction of the radiation flows under amplification; noise generation in laser elements; mutual effects of laser stages; the degree to which the pumping radiation source spectrum is aligned to the absorption spectrum of the gain elements (GE); the lifetime of the lower operational laser level in four-level media, and the rates of population variation in gain media doped with active Cr³⁺, Tm³⁺ and Ho³⁺ trivalent ions.
2. Methods

In order to describe the physical processes taking place in multi-stage pulsed lasers, it is necessary to obtain a self-consistent system of differential equations for electromagnetic field strength and intensity, medium polarisation, energy level populations and the gain factor of the active laser medium. Subsequently, when computing time, energy and polarisation parameters of multi-stage lasers, we used equations for laser radiation intensity and active laser medium gain factor.

In a perfect four-level active laser medium, laser radiation intensity and gain factor may be described by equations of the following form [3]:

\[ (-1)^n \frac{\partial I_n(t, z)}{\partial z} + \frac{1}{V} \frac{\partial I_n(t, z)}{\partial t} = (k(t, z) - \beta) I_n(t, z), \]

\[ \frac{dk(t, z)}{dt} = (\aleph - k(t, z)) W_{03}(t, z) - \frac{k(t, z) \sum_n I_n(t, z)}{Q_s} - \frac{k(t, z)}{\tau_{21}}. \]

Here \( I_n(t, z) \) is the laser radiation intensity; \( t \) is the current time; \( z \) is the longitudinal coordinate along the optical axis of the gain element; \( n = 1, 2 \) is the number assigned to the laser radiation flow along the 0Z axis and in reverse; \( k(t, z) \) is the gain factor of the active laser medium; \( \beta \) is the non-resonant loss factor of the laser radiation; \( V \) is the speed of light in the medium; \( \aleph \) is the maximum gain factor; \( W_{03}(t, z) \) is the pumping rate; \( Q_s \) is the saturation energy density; \( \tau_{21} \) is the average lifetime of the upper laser level.

By considering energy level diagrams it is possible to derive population equations for four-level active laser media that take into account the finite life-time of the lower laser level, as well as level population equations in gain media doped with Cr, Tm and Ho atoms.

In the plane wave approximation, a mathematical model of a pulsed multistage solid-state laser may be based on transfer equations [9]. The model takes into account the interaction of all radiation flows under amplification, interstage connections, development of noise radiation, alignment between pumping radiation spectrum and absorption spectra of the gain elements, and polarisation characteristics of the radiation.

When developing the mathematical model for a multi-stage single-pulse laser, we assumed it to consist of a set of independent optical elements. During simulation, we represented these optical elements as optical quadrupoles (Figure 1), where \( I_i^j \) are the intensities of incident laser radiation flows, \( I_o^j \) are the intensities of flows exiting the optical element, \( j \in (1, 2, 3, 4) \) is the number assigned to the radiation flow. Each flow, in its turn, is a superposition of two mutually

![Optical quadrupole](image-url)
orthogonally polarised electromagnetic waves, designated as subscript “ν” or “h”. Each optical quadrupole transforms input flows into output flows according to its algorithm.

We should note that, when describing the interaction between laser radiation and a moving medium, we must account for the fact that the radiation polarisation may vary with the motion velocity in the medium [4].

In order to describe the gain elements, we must take into account the electromagnetic radiation propagation processes taking place inside. To do this, we introduce additional intensities for radiation passing through the gain elements along the 0Z axis and in reverse (Figure 2). To compute the intensities of the laser radiation flows propagating through the gain element $I_{νr}$, $I_{rh}$, $I_{νl}$, $I_{lh}$ and the gain factor, we used a system of first-order partial differential equations:

$$
\frac{\partial I_{hr}(z,t)}{\partial z} + \frac{1}{V} \frac{\partial I_{hr}(z,t)}{\partial t} = (k(z,t,\lambda,\theta) - \beta) (I_{hr}(z,t) + D_{ae} (I_{νr}(z,t) - I_{hr}(z,t))) + \frac{k(z,\lambda,\theta)Q_{s}(\lambda,\theta)}{\tau_{ef}},
$$

(3)

$$
\frac{\partial I_{νr}(z,t)}{\partial z} + \frac{1}{V} \frac{\partial I_{νr}(z,t)}{\partial t} = (k(z,t,\lambda,\theta) - \beta) (I_{νr}(z,t) + D_{ae} (I_{hr}(z,t) - I_{νr}(z,t))) + \frac{k(z,\lambda,\theta + 90^\circ)Q_{s}(\lambda,\theta + 90^\circ)}{\tau_{ef}},
$$

(4)

$$
\frac{\partial I_{hl}(z,t)}{\partial z} + \frac{1}{V} \frac{\partial I_{hl}(z,t)}{\partial t} = (k(z,t,\lambda,\theta) - \beta) (I_{hl}(z,t) + D_{ae} (I_{νl}(z,t) - I_{hl}(z,t))) + \frac{k(z,\lambda,\theta)Q_{s}(\lambda,\theta)}{\tau_{ef}},
$$

(5)

$$
\frac{\partial I_{νl}(z,t)}{\partial z} + \frac{1}{V} \frac{\partial I_{νl}(z,t)}{\partial t} = (k(z,t,\lambda,\theta) - \beta) (I_{νl}(z,t) + D_{ae} (I_{hl}(z,t) - I_{νl}(z,t))) + \frac{k(z,\lambda,\theta + 90^\circ)Q_{s}(\lambda,\theta + 90^\circ)}{\tau_{ef}},
$$

(6)

$$
\frac{\partial k(z,t,\lambda,\theta)}{\partial t} = - \frac{k(z,t,\lambda,\theta) (I_{hr}(z,t) + I_{νr}(z,t) + I_{hl}(z,t) + I_{νl}(z,t))}{Q_{s}(\lambda,\theta)} + P_{p}(t)G(\pi N(\lambda,\theta) - k(z,t,\lambda,\theta)) - \frac{k(z,t,\lambda,\theta)}{\tau_{ef} (k(z,t,\lambda,\theta))}.
$$

(7)
Here $\lambda$ is the laser radiation wavelength; $\theta$ is the angle between the crystal-optics coordinate system of the gain element and the radiation polarisation plane; $D_{ae}$ is the gain element depolarisation per unit of length; $\zeta$ is the angular fraction of the gain element luminescence radiation contributing to lasing radiation; $\tau_{ef}$ is the effective lifetime of the upper laser level; $P_e(t)$ is the pumping radiation power; $G$ is the factor connecting pumping rate and power; $\eta(P_e(t))$ is the pumping efficiency in the spectral absorption range of the gain element.

We solved the system of equations above with the following initial and boundary conditions:

$$
I_{rh}(z, 0) = 0; \quad I_{rv}(z, 0) = 0; \quad I_{lh}(z, 0) = 0; \quad I_{lu}(z, 0) = 0; \quad k(z, 0) = 0.
$$

where $R_e$ is the reflection index of the coating at the end faces of the gain element, $l_{ae}$ is the length of the gain element. The boundary conditions set the intensity variations for all laser radiation flows at the gain element borders, considering the reflection indices at their end faces.

To solve the transfer equations (3)-(6) in the system of equations (3)-(9), we used an explicit first-order accuracy difference method with the Courant timestep. Equations (3) and (4) used backward difference, equations (5) and (6) used forward difference. The Euler method was used to solve the gain factor equation (7).

We added the Kirchhoff diffraction integral to our mathematical model in order to solve the problem of shaping spatial characteristics of light beams in multi-stage lasers featuring non-homogeneous population inversion distribution over the gain element cross-section. Solving the integral makes it possible to compute the gain-phase distribution of the electromagnetic field in a laser beam while transiting from one spatial layer to the next:

$$
\hat{E}_2(x_2, y_2, z_2) = \frac{1}{i\lambda \Delta z} \int \int_A \hat{E}_1(x_1, y_1, z_1) \hat{G}(x_2, y_2, x_1, y_1, \Delta z) dx_1 dy_1,
$$

where $\hat{E}_1$ and $\hat{E}_2$ are gain-phase electromagnetic field distributions in the planes at the coordinates $z_1$ and $z_2$ respectively; $\hat{G}(x_2, y_2, x_1, y_1, \Delta z)$ is the Green’s function for a layer of a thickness $\Delta z$; $\Delta z = z_2 - z_1$.

Numerical simulation results are in excellent agreement with the experimental data obtained. Figure 3 plots the output energy as a function of pumping energy; the functions were obtained in a numerical simulation and actual experiments, for an actively $Q$-switched laser using a shutter, based on the effect of in-terrupting total internal reflection [10]. A YSGG:Cr$^{3+}$:Nd$^{3+}$ $\phi 6 \times 100$ mm crystal was used as the active medium.

We used a dual-stage laser as an example for investigating the effects of inter-stage connections and interaction of opposite flows (the one undergoing amplification and the one containing noise) on the lasing evolution kinetics as well as the self-excitation and lasing thresholds. The laser included one-, two- or four-pass amplifiers with polarisation decoupling of the flows under amplification. Simulation results demonstrated that two-pass amplifiers with polarisation decoupling will affect lasing evolution kinetics especially strongly, and lasers comprising them have a low self-excitation threshold.

Theoretical and experimental studies of spatial characteristics of optical emission in multi-stage lasers showed that a population inversion profile that is non-uniform across the gain
medium, that is, axially symmetrical and overpumped in the centre, ensures a smoother laser beam distribution in the Fresnel zone, less scattered emission and, at the same time, better power characteristics [11].

3. Conclusion
Varying the population inversion profile over the gain element cross section while varying the semiconductor pumping emitter temperature may become the foundation of a method for controlling spatial properties of optical laser emission during highly accurate measurements of electromagnetic radiation properties in moving media and so on.

We should also note that the model proposed may be developed further, accounting for laser motion in an accelerating frame of reference [5]. This is especially important when using laser emitters in equipment installed, for instance, on board spacecraft or when maximum possible measurement accuracy is required in a situation when the Earth rotation affects the result (the Sagnac effect).

References
[1] Gorelik V S, Gladyshev V O and Kauts V L 2018 On the Generation and Detection of High-Frequency Gravitational Waves Optically Excited in Dielectric Media *Bulletin of the Lebedev Physics Institute* 45 2 pp 39–45
[2] Gladyshev V O, Strunin A G, Kauts V L, Kayutenko A V and Bazleva D D 2018 Effects of moving media optics in GLONASS optical segment of new generation *Journal of Physics Conference Series* 1051 012031
[3] Sharandin E A and Gladyshev V O 2019 Mathematical model of generation and amplification of radiation in multistage laser *Mathematical Models and Computer Simulations* 11 2 pp 277-86
[4] Gladyshev V O and Portnov D I 2016 Dependence of Laser Radiation Intensity on the Elastic Deformation of a Revolving Optical Disk with a Reflective Coating *Optics and Spectroscopy* 121 6 pp 901–6
[5] Gladyshev V O, Gorelik V S, Portnov D I and Filatov V V 2016 Entrainment of Polaritons in Rotating Ruby *JETP Letters* 104 3 pp 152–6
[6] Grechin S G, Raevsky E V, Rojdestvin V N and Sharandin E A 1999 Kinetics of laser processes in the multi-cascade optical schemes *Proceedings of SPIE* 3682 pp 163-9
[7] Mak A A, Somes L N, Fromzel V A and Yashin V E 1990 Nd-glass lasers (Moscow, Nauka) in Russian
[8] Alpat’ev A N, Smirnov V A and Shcherbakov I A 1993 Model for an active medium based on a (Cr,Tm,Ho):YSGG crystal *Quantum Electronics* 23 11 pp 962–6

![Figure 3. Output laser emission energy of a YSGG:Nd:Cr laser as obtained in an experiment and simulation.](image-url)
[9] Dmitriev V G, Grechin S G, Sharandin E A and Kiiko V V 1998 Methods and Program Sets for Investigation and Design of Laser and Nonlinear Optical Devices - LID Program Sets IX Conference on Laser Optics June 22-26 1998

[10] Laptev V V, Mikhailov V A, Nikolaev D A, Pak S K, Raevskii E V, Fefelov A P, Khomenko S I and Shcherbakov I A 1991 Single-pulse YSGG:Cr:Nd laser with a 4% efficiency Soviet Journal of Quantum Electronics 21 5 pp 525-7

[11] Sharandin E A, Kauts V L, Gladysheva T M, Kautienko A V and Portnov D I 2019 Mathematical modeling of the beam spatial distribution for the laser with ceramic active elements Journal of Physics: Conference Series 1348 012105