Mathematical model of radio-frequency ion thruster with an additional magnetostatic field

S Kanev, A Melnikov*, I Nazarenko and S Khartov
Department of Electric Propulsion and space power plants, Moscow Aviation Institute
(National Research University), 4 Volokolamskoe shosse, Moscow 125993, Russia

*E-mail: k208@mai.ru

Abstract. The paper presents an engineering simplified mathematical model for estimating integral characteristics of radio-frequency ion thrusters with an additional magnetostatic field. In addition to the integral characteristics calculation, the proposed model allows to obtain the distributions of some plasma local parameters in a gas-discharge chamber of thruster – atoms and charged particles concentrations, and temperature of electrons. In a calculation, axisymmetric geometry of main elements of a thruster structure is considered. Two approaches to taking into account the influence of an additional magnetic field on the distribution of charged particles concentration are implementing. Each of them allows to estimate integral characteristics of radio-frequency ion thrusters with acceptable accuracy. The comparison of the experimental data with simulation results showed that error of thruster's parameters estimation does not exceed 20-25%, which is sufficient for preliminary designing of new radio-frequency ion thrusters models with an additional source of magnetostatic field.

1. Introduction
The increasing of active lifetime of small low-orbiting spacecrafts is one of the relevant trends in the space technology development. For increasing of active lifetime of satellites of this type, it is necessary to take into account the influence of residual atmosphere of the Earth, which leads to aerodynamic drag. For compensation of this effect, the using of ion thrusters is most promising, since they provide highest values of specific impulse and, as a result, more economical consumption of the working medium [1].

Currently, of special interested are development and using radio-frequency ion thrusters (RIT) on board of the small spacecraft. As opposed classic Kaufman type ion thrusters, RIT has not problems associated with a decreasing of its life due sputtering of the cathode surface inside the volume of gas-discharge chamber (GDC). In addition, this type of ion thruster allows to regulate the thrust value by changing both the input RF power and the working medium flow rate. However, RIT has a significant deficiency - high energy consumption for ionization of the working medium. One of the reasons of it is the large loss of charged particles due to their recombination on the walls of the GDC. These losses can be reduced by using additional axial external magnetostatic field in the area of RF discharge.

The first experimental studies in this direction were carried out at radio-frequency technological ion sources at the Department of Physical Electronics of Moscow State University [2-4]. However, the design of technological units and their operating modes differed from the solutions which used in RIT. Therefore, independent experimental studies were carried out at the Moscow Aviation Institute [5]. It also showed a positive result - thruster performance improvement. The studies of radial distribution of plasma local parameters in one of cross-sections of the GDC of the thruster with using electrostatic
probe showed that a registered characteristic improvement in the presence of the additional magnetostatic field was related with an increasing of the electron’s concentration and redistribution of their temperature along the radius [6]. As a result of complex studies for the laboratory model of RIT with a beam diameter of 8 cm, optimal parameters of the additional magnetic field at which the maximum improvement of characteristics were observed.

For designing of other RIT models with an additional magnetic field source, it is expedient to have possibility of quick estimation of their integral characteristics. In doing so, it is necessary to take into account the geometry of main elements of the thruster structure and its operation parameters (frequency and amperage on the inductor, flow rate of working medium, amperage at an additional winding or properties of permanent magnet).

Currently, a number of mathematical models have been developed, which differ in the detailing and depth of the physical processes’ description in the plasma of radio-frequency plants [7-14]. Some of these models doesn't take into account the geometry of plants structure elements and, moreover, no one of them allows to estimate the influence of an additional magnetostatic field on the RIT integral characteristics or on the distribution of local plasma parameters in its GDC. The more appropriate mathematical models were developed at the Moscow State University [15, 16]. They were used to physically explanation the observed regularities during experimental studies of radio-frequency ion sources and plasma generators, as well as in the development of own RIT prototype with an additional magnetic field source [17, 18]. However, these models require a considerable time for calculation, what is not always appropriate it for preliminary design of thrusters, since most often it is necessary to consider and to compare a large number of different design solutions and operating modes.

Thus, the purpose of the presented work is creating the simplified mathematical model which will allow with low time and computational resources costs, but with acceptable accuracy, to estimate the RIT integral characteristics and the distribution of local plasma parameters in its GDC with taking into account the geometry of the main construction elements, the operating mode, and the presence of an additional magnetostatic field.

2. The calculation method of the engineering mathematical model

Calculations were implemented in the COMSOL Multiphysics software package [19]. The axisymmetric geometry of main elements of RIT structure is considering: the gas-discharge chamber, the inductor, a gas inlet and an ion-optical system (IOS). With the purpose reducing calculation time, a description of atoms and charged particles behavior in the model is carried out using a number of analytical expressions.

The influence of an additional magnetostatic field on the distribution of charged particles and integral characteristics of RIT was estimated in two methods. In the first case, a coefficient was used, which had been obtained by considering the motion of a monoenergetic electron flux across uniform magnetostatic field. The second approach assumes the using of a coefficient which approximately describes the Bohm diffusion of charged particles in a magnetic field.

2.1. Assumptions of the mathematical model

In assumptions of the model, characteristic features of an inductive RF discharge, which is realized in the RIT, were taken into account.

The particles mean free path in a RF plasma during electron-atom and ion-atom interactions often exceeds the size of the gas-discharge chamber. Therefore, processes of volumetric charge-exchange weren't taken into account in the model. In addition, volume recombination of charged particles also wasn't considered, since its intensity is much lower than on walls of the thruster gas-discharge chamber.

Based on a fact that a rate of doubly charged ions in the RIT plasma doesn't exceed 1% [20], we can ignore processes of working medium atoms double ionization. A viscous interaction of neutral particles with each other in the model is also neglected.
Since the Debye radius in the RF plasma is of an order of $10^{-5}–10^{-6}$ m, which is much smaller than the characteristic size of the gas-discharge chamber, in its volume with high precision a quasineutrality condition is satisfied — electrons and ions concentrations are equal ($n_e = n_i = n_{pl}$).

A possibility of transferring momentum and kinetic energy, when elastic collisions of electrons with ions and working medium atoms is taken into account, is neglected, since this effect will be minor due to a large difference in masses.

A close to magnetohydrodynamic approximation is used to describe a behavior of charged particles in plasma [21]. A possibility of applying this approach is due to the fact that a collisional energy transfer mechanism between charged particles prevails in the RIT plasma [22]. Based on this, it is also assumed in the model that the electron energy distribution function corresponds to the Maxwell function.

It is assumed in the model that all charged particles falling onto walls of the gas-discharge chamber recombine and returned into the plasma volume in form of neutral atoms. In doing so, the temperature of neutral particles is assumed constant throughout at the chamber volume and equal to the wall temperature.

The influence of material properties of main RIT structure elements on local plasma parameters and on its integral characteristics isn't considered in the model. Also, the presence of a capacitive component of the discharge, existing due to inter-turn capacitances formed by inductor, isn't taken into account.

To simplify the mathematical model a propagation of waves in the plasma isn't considered.

### 2.2. Expressions used in the calculation

Initial data required for the calculation are following parameters: frequency $f_{RF}$ and amperage $I_{ind}$ at the inductor, surface temperature of the inner wall of the gas-discharge chamber $T_w$, mass flow rate of working medium $\dot{m}$, amperage in the additional winding $I_{aw}$ (or magnetic field induction $B_{cm}$ when RIT with a permanent magnet is calculated) and a transparency of emission $\sigma_{ee}$ and accelerating $\sigma_{ae}$ electrodes of the ion-optical system. Also, before the calculation, initial values of the concentration $n_e$ and the temperature of electrons $T_e$, and the concentration of working medium atoms $n_a$ are additionally set.

The distribution of an electromagnetic field of the inductor is determined using the standard module “Magnetic fields” of COMSOL Multiphysics [19]. In doing so, the calculation was made on amplitude value of amperage on the inductor, which is expressed in terms of the current density $j_{ind}$. It allows with using the Ampere-Maxwell equation to obtain the distribution of amplitude value induction of magnetic field $B_{ind}$ in the volume of gas-discharge chamber:

$$\nabla \times B_{ind} = \mu_0 j_{ind},$$  \hspace{1cm} (1)

where $\mu_0$ — the vacuum permeability.

The distribution $B_{ind}$ is used to find a vector magnetic potential $A_{ind}$, through which the amplitude strength of vortex electric field $E_0$ is determined in the model:

$$B_{ind} = \nabla \times A_{ind};$$ \hspace{1cm} (2)

$$E_0 = -\omega A_{ind}.$$

where $\omega = 2\pi f_{RF}$ — circular frequency of RF field.

The amplitude velocity of electrons $u_{e0}$, which they acquire in the vortex electric field in the model is determined through the value $E_0$ [21]:

$$u_{e0} = \frac{e\omega A_{ind}}{m_e(\omega^2 + v_{ea}^2)^{1/2}},$$

where $e$ — elementary charge; $m_e$ — electron mass; $v_{ea}$ — the frequency of elastic collisions of electrons with working medium atoms, which is defined as:

$$v_{ea} = n_a \sigma_{ea} \left( \frac{3kT_e}{m_e} + \frac{u_{e0}^2}{2} \right)^{1/2},$$ \hspace{1cm} (5)
where $\sigma_{ea}$ – elastic scattering cross section of electrons by working medium atoms; $k$ – the Boltzmann constant.

In [21], it was shown that time-averaged energy of oscillating motion, taking into account expressions (4) and (5), would be determined as:

$$\frac{m_e u_0^2}{4} = \frac{e^2 \omega^2 A_{ind}^2}{4m_e (\omega^2 + \nu_{ea}^2)}. \quad (6)$$

Then the effective temperature $T_{eff}$, which determines the electrons total energy and includes a thermal component and accumulated in the vortex electric field kinetic energy, can be estimated by the following expression [21]:

$$\frac{3}{2} kT_{eff} = \frac{3}{2} kT_e + \frac{e^2 \omega^2 A_{ind}^2}{4m_e (\omega^2 + \nu_{ea}^2)}. \quad (7)$$

The values of the ionization rate coefficient $\langle \sigma_i \theta \rangle$ and the scattering cross section $\sigma_{ea}$ are calculated through the $T_{eff}$.

Also, the density of induced in the plasma electron current $j_e$, which reverse to the flowing through the inductor current is determined through the value $u_{e0}$:

$$j_e = -e u_{e0}. \quad (8)$$

Since this current influenced on the distribution of inductor’s electromagnetic field, in the model, when $B_{ind}$ is calculated, it is set and calculated together with $j_{ind}$. It makes it possible to relatively simple simulate the presence of skin-effect in the plasma.

At first, the distribution of neutral atoms concentration in the volume of the gas-discharge chamber is assumed to be uniform and determined with using the equation of ion balance:

$$n_{a0} = 0.7 S_{GDC} \left( \frac{kT_e}{M_i} \right)^{1/2} \langle \sigma_i \theta \rangle v_{GDC}^{-1}, \quad (9)$$

where $S_{GDC}$ – the inner surface area of the gas-discharge chamber; $M_i$ – mass of working medium ion; $V_{GDC}$ – the inner volume of the gas-discharge chamber. The ionization coefficient for used in the RIT working medium – xenon, $\langle \sigma_i \theta \rangle$ is determined by the following expression:

$$\langle \sigma_i \theta \rangle = 5 \cdot 10^{-14} \exp \left( -\frac{\varphi_{Xe}}{T_e} \right) \sqrt{T_e}, \quad (10)$$

where $\varphi_{Xe}$ – xenon atom single ionization energy, eV; electrons temperature $T_e$ is set in eV.

An approximate analytical expression, which was obtained by considering the one-dimensional task of gas flow in the tube, is used to describe the distribution of working medium atoms concentration:

$$\text{div} \left( \frac{u_{aT}}{4(u_{aT} + n_d \langle \sigma_i \theta \rangle l)} \right) \text{grad} n_a = n_a n_e \langle \sigma_i \theta \rangle, \quad (11)$$

where $u_{aT} = (kT_e / \pi M_{Xe})^{1/2}$ – xenon atoms thermal velocity ($M_{Xe}$ – xenon atom mass); $l$ – the characteristic size of the plasma formation (in the model it is equal to the gas-discharge chamber radius).

The distribution of the charged particles concentration $n_{pl}$ over the gas-discharge chamber volume is determined through the ionic component $n_i$ with using the following expression [21]:

$$\nabla^2 n_i = -2.5 \frac{M_i}{kT_e} (n_a \langle \sigma_i \theta \rangle)^2 n_i. \quad (12)$$

As noted above, the influence of an additional magnetostatic field on the distribution of charged particles concentration was realized in two ways. In the first case, for the plasma concentration the following correction coefficient was introduced:

$$n_{mf} = n_{pl} \exp \left( -\frac{e^2 A_{ind}^2}{18m_e kT_e} \right). \quad (13)$$
where $A_C$ – the vector magnetic potential of an additional magnetostatic field. It is based on the assumption that when electrons move across magnetic field, their momentum changes in proportion to the difference of vector magnetic potential that they pass, which is shown in [23].

In the second approach the coefficient that approximately describes the Bohm’s diffusion of charged particles as they move in magnetic field is used:

\[
\eta_{mf} = n_{pl} \exp \left( -\frac{16 A_C u_i}{T_e} \right),
\]

where $u_i = (kT_e m_i)^{-1/2}$ – Bohm’s ion velocity; the coefficient 0.1 before $u_i$ characterizes the ion velocity in the plasma volume, which is assumed to be 10% of the Bohm’s velocity.

The influence of an additional magnetic field on the effective temperature distribution is estimated using the empirical coefficient $K_{MF}$, which was obtained by analyzing the results of probe measurements of plasma local parameters in the gas-discharge chamber of RIT laboratory model [6]:

\[
K_{MF} = \left( a + B_{avw} - \frac{K_{AF} B_{ind}}{2} \right) \left( a + B_{av} + K_{AF} B_{ind} \right)^{-1},
\]

where $a = 10^{-8}$ – the dimensionless value which is necessary to prevent the coefficient $K_{MF}$ from zeroing in the absence of an additional magnetostatic field; $K_{AF}$ – the correction dimensionless coefficient, which characterizes the degree of influence of an additional magnetic field on the effective electron temperature distribution and which is determined as:

\[
K_{AF} = 0.06 \left( \frac{B_{av}}{B_{ind}} \right)^2 + 0.11 \frac{B_{av}}{B_{ind}},
\]

where $B_{avw}$ – the average induction of an additional magnetostatic field; $B_{ind}$ – the average amplitude value of the magnetic field induction which is generated by the inductor.

Taking into account the coefficient $K_{MF}$ the expression for the effective temperature will take the following form:

\[
T_{eff} = T_e + \frac{e^2 \omega^2 A_{ind}^2}{6 k m_e (\omega^2 + v_{ea}^2)} K_{MF}.
\]

To the electron temperature $T_e$ calculation in the model the energy balance equation is used, in which the right part determines the power supplied into the plasma, and the left part – the power consumptions for ionization of working medium atoms and the losses of charged particles on the walls of the gas-discharge chamber:

\[
\int_V (v_{ea} + v_{el}) n_e \frac{m_e u_{el}^2}{4} dV = \int_S n_i u_i \left( e \epsilon_{eff} + \frac{3}{2} kT_e + kT_e + e \Delta \varphi + \frac{5}{2} kT_w \right) dS,
\]

where $v_{el}$ – the frequency of electron-ion interactions, which is determined through the Coulomb’s logarithm $(\ln \Lambda)$; $\epsilon_{eff} = 12.1 \left( 1 + 0.09 (T_{eff} / 12.1)^{-1} \right)^2$ – the effective ionization energy for xenon [21]; $\Delta \varphi$ – the pre-sheath plasma potential drop.

The value of the extractable ion current is determined by the following expression:

\[
l_i = 2 \pi e \sigma_e \int_0^R n_{mf} u_i r dr.
\]

The following boundary conditions are used in the model:

- at the boundary of calculation area, the condition for the absence of a magnetic field is set:
  \[
x_n \times A = 0,
\]

- on the gas-discharge chamber walls, the condition for the absence pass through walls flow of atoms and ions of working medium is set:
\[ n_a u_{a\perp} + n_{mf} u_{i\perp} = 0, \]  
(21)

where \( u_{a\perp} \) and \( u_{i\perp} \) – perpendicular components of working medium atoms and ions velocity to the gas-discharge chamber wall; and the reverse flow of neutral particles, which is determined by falling ions onto the walls with a Bohm’s velocity, is set:

\[ \Gamma_{wa} = n_{mf} \left( \frac{kT_e}{M_i} \right)^{1/2}; \]  
(22)

on the IOS, the condition which determines the number of atoms passing through this thruster assembly, is set:

\[ n_x \frac{u_{aT}}{2} \sigma_{ae} = \left[ n_a \frac{u_{aT}}{2} - n_i u_i (1 - \sigma_{ee}) \right] \left( \frac{\sigma_{ae}}{2 - \sigma_{ae}} \right), \]  
(23)

where \( n_x \) – the concentration of atoms which move in the IOS direction, and the common condition for gas-discharge chamber walls and IOS, which takes into account the charged particles flow:

\[ \frac{\partial n_i}{\partial x_{ni}} = -2.5 \left( \frac{M_i}{kT_e} \right)^{1/2} n_{mf} n_a (\sigma_{fe} \sigma), \]  
(24)

where \( x_{ni} \) – normal to the wall surface and the IOS electrode.

The calculation according to the engineering model was carried out using the Nelder-Mead’s optimization method. With varying the values of \( T_e, n_i \) and \( n_a \) with certain accuracy the energy balance condition (18) was verified. With the absence of convergence, a jump to a new iteration is carried out.

3. Calculation results
Some operating modes of RIT laboratory model with a beam diameter of 8 cm (figure 1), were used to verify the mathematical model.

![Figure 1](image)

**Figure 1.** The schematic diagram of the RIT laboratory model with a beam diameter of 8 cm.

In the figure 2 the simplified geometry of the main elements of thruster was given, and in the figure 3 – the created for calculation finite element mesh.
Figure 2. The calculation area and geometry of the main elements of thruster.

Figure 3. The created in the calculation area mesh of finite elements.

Figure 4 shows the comparison of the calculated dependence of the extractable ion current on the RF power with the experimental data for xenon flow rate 3 and 6 sccm without additional magnetic field. Since the model allows to calculate only the RF power absorbed by the plasma, the efficiency coefficient of energy transfer from the inductor was introduced for comparison with the experiment.

Figure 4. The comparison of the calculated dependence of the extractable ion beam current on the RF power with experiment: xenon flow rate 3 sccm (a), xenon flow rate 6 sccm (b).
For comparing of two approaches to taking into account the presence of an additional magnetostatic field in the ionization area, similar dependencies were constructed, which are shown in figure 5. Calculation No. 1 was carried out with using coefficient (13), No. 2 – with using coefficient (14).

**Figure 5.** The comparison of calculated dependences of the extractable ion beam current on the RF power with the experiment in the presence of an additional magnetic field. Xenon flow rate is 3 sccm. The amperage in the additional winding is 9 A.

The presented dependences show that coefficient (14) in considered operating modes underestimates the value of the extractable ion beam current.

The comparison of calculated dependences of the extractable ion beam on the amperage in the additional winding with experimental data were carried out to estimate the correspondence of the change integral characteristics character while using an additional magnetostatic field. The comparison result is given in figure 6.

**Figure 6.** The comparison of calculated dependences of the extractable ion beam current on the amperage in the additional winding with experiment. Mode: No.1 – $N_{\text{RF}} = 60$ W, the xenon flow rate 3.07 sccm; No.2 – $N_{\text{RF}} = 90$ W, the xenon flow rate 3.18 sccm; No.3 – $N_{\text{RF}} = 90$ W, the xenon flow rate 5.14 sccm.
Since the model allows to estimate the plasma local parameters, calculated two-dimensional electron concentration distributions over the gas-discharge chamber volume in the absence and the presence of an additional magnetostatic field were also considered. The calculation result of charged particles concentration distribution for the operating mode of the RIT laboratory model with RF power of 60 W and xenon flow rate 2.24 sccm is given in the figure 7.

![Figure 7](image)

**Figure 7.** The calculated two-dimensional distribution of electron concentration over the RIT gas-discharge chamber. The RF power 60 W, the xenon flow rate 2.24 sccm.

According to calculation results, without additional magnetic field, the concentration maximum is localized near the gas-discharge chamber wall. However, as a magnetic field is applied, the maximum shifts closer to the axis of a thruster. The comparison of two approaches of the calculation of the electron concentration distribution with the same mode of the laboratory model operation, but in the presence of an additional magnetic field is given in the figure 8 and the figure 9.

![Figure 8](image)

**Figure 8.** The calculated two-dimensional electron concentration distribution over the RIT gas-discharge chamber volume: the approach No.1 (a), the approach No.2 (b). The RF power 60 W, the xenon flow rate 2.24 sccm, the amperage in the additional winding is 3 A.
Figure 9. The calculated two-dimensional electron concentration distribution over the RIT gas-discharge chamber volume: the approach No.1 (a), the approach No.2 (b). The RF power 60 W, the xenon flow rate 2.24 sccm, the amperage in the additional winding is 9 A.

At amperage of 3 A in the additional winding, the difference in the charged particles concentration distribution is minimal. At amperage of 9 A in the distribution, which was calculated using coefficient (13), the stronger influence of an additional magnetic field is observed. Using in calculations the coefficient based on the Bohm’s diffusion gives more uniform distribution. At the same time, it somewhat reduces the increasing of charged particles concentration under the influence of an additional magnetic field. Radial dependences of the electron concentration and the effective temperature in the cross-section of RIT laboratory model gas-discharge chamber in which probe measurements were made. were plotted for the comparing of two approaches with the experiment. The comparison of resulting distributions is presented in the figure 10 and the figure 11.

Figure 10. The comparison of calculated radial distributions of electron concentration with the result of probe studies. The RF power 60 W, the xenon flow rate 2.24 sccm.
Figure 11. The comparison of calculated radial distributions of electron effective temperature with the result of probe studies. The RF power 60 W, the xenon flow rate 2.24 sccm.

As one can see from the figures, both approaches allow an acceptable estimation for distribution of plasma local parameters in the presence of an additional magnetic field. However, it is worth noting that using the coefficient (14) understates the charged particles concentration.

4. Conclusions

The using of simplified relations in the engineering model allows significantly reduce the required time for estimation of the RIT integral characteristics and the distributions of local plasma parameters in its gas-discharge chamber. For calculation of one operating mode from 200 to 600 iterations are required and on average takes from 1 to 3 minutes. Both considered approaches, which take into account the influence of an additional magnetostatic field on the thruster parameters, with sufficient accuracy allows to simulate of working process. The verification of numerical results with the experimental data has shown that the calculation error doesn't exceed 20 – 25 %. Such accuracy is sufficient for preliminary design of RIT thrusters with the source of an additional magnetic field. However, further improvement of the proposed model is also possible. In particular, additional probe studies of local plasma parameters may allow the more detailed understanding process of changing the electron temperature distribution in the presence of an additional magnetostatic field in the area of the RF discharge.

References

[1] Goebel D M, Katz I 2008 Fundamentals of Electric Propulsion: Ion and Hall Thrusters (California Institute of Technology) 486 p

[2] Alexandrov A F, Bugrov G E, Kerimova I, Kondranin S G, Kralkina E A, Pavlov V B, Plaksin V Yu, Rukhadze A A, Vavilin K V 2003 Development of Low Power 13.56 MHz Plasma Source Family. Proc. of 30th International Conference on Plasma Science (Jeju, South Korea) DOI: 10.1109/PLASMA.2003.1228688

[3] Vavilin K V, Rukhadze A A, Ri M K, Plaksin V Yu 2004 Low-power inductive RF plasma sources for technological applications. Plasma Phys. Rep. 30 687 https://doi.org/10.1134/1.1788762

[4] Petrov A K, Vavilin K V, Kozlov G P, Kralkina E A, Nekliudova P A, Nikonov A M, Pavlov V B 2015 Plasma parameters in a dual-camera low-power inductive RF discharge with an external magnetic field. Moscow University Physics Bulletin 70 527 https://doi.org/10.3103/S0027134915060144

[5] Melnikov A V, Kharton S A 2018 Radio-Frequency Ion Thruster with Additional Magnetic Field: Experimental Investigation. Thermal Engineering 65 980 DOI: S0040601518130086
[6] Khartov S A, Melnikov A V, Kozhevnikov V V 2019 Characteristics of Radio-Frequency Ion Thruster with an Additional Magnetic Field in the Ionization Area. *Proc. of 36th International Electric Propulsion Conference* (Vienna, Austria) IEPC-2019-240

[7] Celik M, Turkoz E 2013 Optimization of radio-frequency ion thruster discharge chamber using an analytical model. *Proc. of 6th International Conference on Recent Advances in Space Technologies* (Istanbul, Turkey) DOI: 10.1109/RAST.2013.6581262

[8] Goebel D M 2008 Analytical discharge model for RF ion thrusters. *IEEE transactions on plasma science* 36 2111 DOI: 10.1109/TPlas.2008.2004232

[9] Chabert P, Monreal J A, Bredin J, Popelier L, Aanesland A 2012 Global model of a grided-ion thruster powered by a radiofrequency inductive coil. *Physics of Plasmas* 19 073512 DOI: 10.1063/1.4737114

[10] Tsay M M T 2010 Two-dimensional numerical modeling of radio-frequency ion engine discharge, PhD thesis, Massachusetts Institute of Technology

[11] Bilén S, Mistoco V 2008 Numerical Modeling of a Miniature Radio-Frequency Ion Thruster. *Proc. of 44th Joint Propulsion Conference* (Hartford, CT, USA) DOI: 10.2514/6.2008-5194

[12] Celik M, Turkoz E 2014 2-D electromagnetic and fluid models for inductively coupled plasma for RF ion thruster performance evaluation. *IEEE Transactions on Plasma Science* 42 235 DOI: 10.1109/TPS.2013.2287903

[13] Heiliger C, Henrich R 2013 Three Dimensional Simulation of Micro Newton RITs. *Proc. of 33rd International electric propulsion conference* (Washington, DC, USA) IEPC-2013-301

[14] Dobkevicius M, Feili D, Müller J 2015 Comprehensive Radio – Frequency Ion Thruster Electromagnetic and Thermal Modelling. *Proc. of Joint Conference of 30th International Symposium on Space Technology and Science 34th International Electric Propulsion Conference and 6th Nano-satellite Symposium* (Hyogo-Kobe, Japan) IEPC-2015-410/ISTS-2015-b-410

[15] Aleksandrov A F, Bugrov G E, Kerimova I F, Kondranin S G, Kralkina E A, Pavlov V B, Plaksin V Yu, Rukhadze A A, Vavilin K V 2003 Self-Consistent Model of an RF Inductive Plasma Source Located in an External Magnetic Field. *Journal of Russian Laser Research* 24 301 https://doi.org/10.1023/A:1024891824073

[16] Alexandrov A F, Vavilin K V, Kralkina E A, Pavlov V B, Petrov A K, Tarakanov V P 2015 KARAT modeling of the inductive RF discharge placed into an external magnetic field. *Applied Physics* 5 44

[17] Kralkina E, Zadiriev I, Kharlan A 2017 Exploratory Testing of a Radio Frequency Thruster for Small Satellites. *Proc. of 35th International Electric Propulsion Conference* (Atlanta, Georgia, USA) IEPC-2017-425

[18] Kralkina E A, Vavilin K V, Zadiriev I I, Nekliudova P A, Shvydkiy G V 2019 Optimization of discharge parameters in an inductive RF ion thruster prototype. *Vacuum* 167 136 https://doi.org/10.1016/j.vacuum.2019.05.041

[19] Official site of COMSOL Multiphysics developers, available at: https://www.comsol.com/

[20] Loeb H W 2010 *Principle of Radio-Frequency Ion Thrusters RIT. RIT-22 Demonstration Test of Astrum ST at University of Giessen* (University of Giessen)

[21] Kanev S V, Kozhevnikov V V, Khartov S A 2017 Physical and Mathematical Model of Processes in Ionization Chamber of Electric Propulsion Thruster with Atmospheric Gases as a Propellant Proceedings of the Russian Academy of Sciences. *Power Engineering* 3 21

[22] Mikellides I, Katz I, Goebel D, Polk J 2005 Hollow cathode theory and experiment. II. A two-dimensional theoretical model of the emitter region. *Journal of Applied Physics* 98 113303 https://doi.org/10.1063/1.2135409

[23] Landau L D, Lifshitz E M 1987 *The Classical Theory of Fields. 4th Edition* (Butterworth-Heinemann) 402 p