Comparison of s- and d-wave gap symmetry on nonequilibrium superconductivity

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Recent application of ultrafast pump/probe optical techniques to superconductors has renewed interest in nonequilibrium superconductivity and the predictions that would be available for novel superconductors, such as the high $T_c$ cuprates. We have re-examined two of the classical models which have been used in the past to interpret nonequilibrium experiments with some success: the $\mu^*$ model of Owen and Scalapino and the $T^*$ model of Parker. Predictions depend on pairing symmetry. For instance, the gap suppression due to the excess quasiparticle density $n$ in the $\mu^*$ model, varies as $n^{3/2}$ in d-wave as opposed to $n$ for s-wave. Finally, we consider these models in the context of SIN tunneling and optical excitation experiments. While we confirm that recent pump/probe experiments in YBCO, as presently interpreted, are in conflict with d-wave pairing, we refute the further claim that they agree with s-wave.

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I. INTRODUCTION

The field of nonequilibrium superconductivity was very active throughout the late 1970’s to mid 1980’s when it was realized that novel effects in the superconducting state could be induced by converting the electron distribution function into a nonequilibrium one.[1] Different experimental techniques were used to prepare such a nonequilibrium state, for example, tunnel injection and optical irradiation, and a body of work arose from both experimental and theoretical efforts in this area. A useful summary of this work near the end of this period of time can be found in a book edited by Langenberg and Larkin[1] and other broad-based texts have also appeared more recently.[2,3].

The advent of high $T_c$ cuprate superconductivity in 1986 interrupted work in this and other areas as the community turned its attention to this new challenge and, consequently, extensive work in the area of nonequilibrium superconductivity has languished until more recently. However, during the period following the original burst of activity, new state-of-the-art experimental probes have been developed which provide excellent opportunities for renewed interest in this field, not to mention the potential for new insights provided by the new generation of materials exhibiting novel superconductivity, such as the cuprates. Some of these probes which can be turned to this problem are: STM, ultrafast lasers, spin-polarized tunneling injection, terahertz spectroscopy, etc.

As early as the mid-eighties, the pump/probe femtosecond spectroscopy was exhibiting its potential as a technique for investigating nonequilibrium phenomena in metals and superconductors. In these experiments, an ultrafast laser pulse ($\sim 100$ fs) incident on a sample as a high energy “pump” quickly excites the electrons out of equilibrium which then relax back to thermal equilibrium with the lattice via the electron-phonon interaction. Another laser pulse delayed in time “probes” the system of electrons by reflection or transmission spectroscopy. As the system of electrons relaxes, the transient reflectivity or transmissivity decays with time over a scale of picoseconds or less allowing this experiment to probe carrier dynamics in a time-resolved fashion. A theory was proposed by P.B. Allen[4] for the relaxation of quasiparticles in the normal state, which could be measured in these experiments, resulting in the extraction of the electron-phonon renormalization parameter $\lambda$ (as the quasiparticles relax through interactions with the system of phonons). Experiments were performed which measured this parameter using Allen’s theory and excellent agreement was found with other values in the literature for both ordinary metals and superconductors in the normal state.[5] Indeed, this parameter was measured for the first time in Cr by this technique.[5] This extraordinary success has led experimentalists to use the femtosecond laser as a probe of high temperature superconductivity[6,7,8,9,10] and in general several groups have been developing ultrafast techniques of similar sort for measuring nonequilibrium phenomena in superconductors.[11,12].

Here, we are interested in the state that arises when the nonequilibrium excitations, created by a laser pulse or by tunneling injection, have fallen to the gap edge but have not yet recombined into the condensate (bottleneck effect). In the first case, there is some debate amongst experimentalists as to whether the high energy laser used for pumping and probing can truly measure the distribution of quasiparticles at low energy and several groups are developing techniques to probe at lower energy of order of the gap to address this issue.

The main thrust of our work has involved the use of two models employed in the past to describe a nonequilibrium distribution of quasiparticles: the $T^*$ model of Parker[13] which uses an equilibrium distribution function at an ef-
effective temperature $T^*$ relative to the bath temperature $T$ and the $\mu^*$ model, originally proposed by Owen and Scalapino\cite{14}, where the system is described in terms of a new chemical potential for the excited quasiparticles. The former approach has been used by Kabanov et al.\cite{7} to analyze their optical data, whereas the latter approach has been used for systems where excess particles are injected into tunnel junctions\cite{15}. While these two models are somewhat simplified, they appear to have been effective in capturing some of the experimental results on low $T_c$ superconductors.

In section II, we calculate how the superconductivity is modified as a function of the nonequilibrium excess quasiparticle number density $n$. This leads to modifications in the gap which we calculate numerically for various values of temperature $T$ characterizing the sample before irradiation as a function of $n$ in both $\mu^*$ and $T^*$ models and for s- and d-wave. For $T = 0$ and in the limit of $n \rightarrow 0$, we also obtain analytic results for the gap reduction versus $n$, for the chemical potential in the $\mu^*$ model and for the nonequilibrium effective temperature for the $T^*$ model, as well as for the free energy difference between the nonequilibrium superconducting state and the corresponding equilibrium normal state. The analytic limits are tested against the numerical work and found to be close to the exact results even as $n$ increases towards its critical value where superconductivity is destroyed. Results for d-wave are compared with s-wave and important differences are established. In section III, as an explicit example of an application of our results, we consider a S-I-N tunneling junction with a nonequilibrium state on the superconducting side which we assume can be described by a $\mu^*$ model. We show that the current voltage characteristics are modified in two major ways. First the amplitude of the gap is reduced because of the presence of a nonequilibrium number of excess quasiparticles $n$ and secondly the entire characteristic is shifted upward by a factor of $n$ in appropriate units. Also the voltage at which the current is zero can be used to measure the chemical potential $\mu^*$. Separate measurements of the gap reduction, the chemical potential, and the upward shift in I-V characteristic would allow a consistency test of the model. In section IV, we consider the specific case of pump/probe experiments and agree with previous theoretical work\cite{8} that the existing data, as currently interpreted, is not consistent with d-wave gap symmetry, but disagree that it is consistent with s-wave. In a final section V, we draw conclusions and give a summary of our results.

II. THEORY

We consider two models used in the past for the treatment of non-equilibrium superconductivity. For an s-wave BCS superconductor, Owen and Scalapino considered a state in which there exists a finite distribution of excess quasiparticles at the gap energy in addition to a condensate. In their $\mu^*$ model,\cite{14} thermal equilibrium is assumed although chemical equilibrium is not for the paired and unpaired electrons. This is mimicked through the introduction of a chemical potential $\mu^*$ in the Fermi function which represents a constraint on the quasiparticle excitation number. With this chemical potential the Fermi function is:

$$f(E_k - \mu^*) = [1 + \exp(\beta(E_k - \mu^*))]^{-1} \quad (1)$$

with the BCS gap equation modified to be:

$$\frac{1}{N(0)V} = \int_0^{\infty} \frac{de_k}{\epsilon_k^2 + \Delta^2(n)} \tanh(\beta(E_k - \mu^*)/2) \quad (2)$$

where $V$ is the pairing potential, $N(0)$ is the electronic density of states at the Fermi surface in the normal state and the excess quasiparticle density $n$ is given as:

$$n = \frac{1}{\Delta(0)} \int_0^{\infty} \left[ f(E_k - \mu^*) - f(E_k) \right] de_k \quad (3)$$

where $\beta = 1/(k_BT)$, $E_k = \sqrt{\epsilon_k^2 + \Delta^2(n)}$, and $k_B$ is the Boltzmann constant. Here $n$ is measured in units of $4N(0)\Delta(0)$. The 4 is introduced for spin and for particle-hole parts of the excitation spectrum. $\Delta(0) \equiv \Delta(n = 0)$ is the superconducting gap in the equilibrium state, finite and isotropic over the entire Fermi surface for s-wave gap symmetry. This model will be applied later to discuss tunneling.

Alternatively, Parker\cite{13} considered a $T^*$ model where instead of a $\mu^*$ in the Fermi function, a $T^*$ is used:

$$f(E_k, T^*) = [1 + \exp(E_k/k_BT^*)]^{-1} \quad (4)$$

with the other equations modified accordingly. This model is the one used by Kabanov et al.\cite{7} in their analysis of the pump/probe data.

We consider first, the $\mu^*$ model for an s-wave superconductor. At zero temperature the existence of the excess quasiparticles perturb the condensate by blocking states which would otherwise be available to form the condensate in a variational sense, and this lowers the value of the gap. The exact gap equation and relationship between chemical potential and $n$ are respectively,

$$\frac{\Delta(n)}{\Delta(0)} = \left( \frac{\mu^*}{\Delta(0)} + n \right)^2 \quad \text{and} \quad n\Delta(0) = \sqrt{\mu^{*2} - \Delta^2(n)} \quad (5)$$

The first equation in (5) comes directly from the gap equation (2) evaluated at zero temperature with reference made to the equilibrium case which allows us to eliminate the pairing potential in favour of $\Delta(0)$. The second follows from Eqn. (3). The grand potential $\Omega^S(n)$ (the familiar formula is given later for the anisotropic case in Eqn. (13)) in the isotropic case (at $T = 0$) is

$$\frac{\Delta\Omega(n)}{N(0)} \equiv \frac{\Omega^S(n) - \Omega^N(0)}{N(0)} = -\frac{1}{2} \Delta^2(n) - 2\mu^*\sqrt{\mu^{*2} - \Delta^2(n)}, \quad (6)$$
TABLE I: Analytical forms for $n \to 0$ at $T = 0$ in the $\mu^*$ model. Note $n$ is in units of $4N(0)\Delta(0)$, where $N(0)$ is the single spin density states and $\Delta(0)$ is the $T = 0$ and $n = 0$ gap (maximum in d-wave).

| $\mu^*$ model | s-wave | d-wave |
|---------------|--------|--------|
| $\Delta(n)/\Delta(0)$ | $1 - 2n$ | $1 - \frac{2\Delta^2}{\Delta(0)}$ |
| $2\Delta F(n)/N(0)\Delta^2(0)$ | $-1 + 8n$ | $-\frac{1}{2} + \frac{16\Delta^2}{\Delta(0)^3/2}$ |
| $\mu^*/\Delta(0)$ | $1 - 2n$ | $\sqrt{2n}^{1/2}$ |

where this is the difference between the nonequilibrium superconducting state and its normal equilibrium counterpart (i.e. with no excess quasiparticles). The difference normalized to the equilibrium superconducting state condensation energy is

$$\frac{2\Delta\Omega(n)}{N(0)\Delta^2(0)} = \frac{2[\Omega^{S}(n) - \Omega^{N}(n = 0)]}{N(0)\Delta^2(0)} \approx -1 + 8n^2 \quad (7)$$

to lowest order in $n$. To obtain Eqn. (7) we have used expressions for $\Delta(n)/\Delta(0)$ and for $\mu^*/\Delta(0)$ valid to second order in $n$. They are $\Delta(n)/\Delta(0) = 1 - 2n - 2n^2$ and $\mu^*/\Delta(0) = 1 - 2n - 3n^2/2$ (entered in Table I to lowest order). If we add to the grand potential, $\Delta\Omega(n)$, the number of excess quasiparticles multiplied by the chemical potential, i.e. $\mu^*\bar{n}$ where $\bar{n}$ is the first term of Eqn. (3), normalized in the same way as Eqn. (7), we get the normalized free energy difference at zero temperature which we denote by $2\Delta F(n)/N(0)\Delta^2(0)$. This is evaluated to be

$$\frac{2\Delta F(n)}{N(0)\Delta^2(0)} \simeq -1 + 8n \quad (8)$$

(entered in Table I).

In the top frame of Fig. 1, we present our numerical results for the ratio $\Delta(n)/\Delta(0)$ as a function of excess quasiparticles $n$ (solid curve) and compare with the approximate result $\Delta(n)/\Delta(0) = 1 - 2n$ (dashed curve). We see excellent agreement at small $n$. As $n$ is increased, the continuation of the solid curve is denoted by the dots. It is terminated at the point where the free energy for the nonequilibrium state becomes equal to its normal state value and a first order transition occurs. This can be seen more clearly in the bottom frame which shows the normalized free energy difference of the nonequilibrium ($n \neq 0$) state, $2\Delta F(n)/N(0)\Delta^2(0)$ as a function of $n$. The solid curve applies to the exact result at $T = 0$ while the dashed is the approximate result (Eqn. (7)), which fits well the exact result at small $n$ and is semiquantitative in the entire physical region. The first order phase transition to the normal state occurs at $n_c \sim 0.15$. The continuation of the solid line for the free energy difference to values of excess quasiparticles $n$ beyond the critical value is indicated by a dotted curve just as in the top frame for the gap. We note that both the gap $\Delta(n)$ and the free energy difference $\Delta F(n)$ as a function of $n$ fold back on themselves beyond a certain value of $n$, but that the free energy remains positive for the entire dotted region $i.e.$ the nonequilibrium state has higher free energy than does the normal state ($\Delta F(n) > 0$) in this region.

Now we treat the d-wave case. The equation relating $\mu^*$ to $n$ is $n\Delta(0) = \int_0^{\mu^*} N(E)dE$ where for small $E$, $\bar{N}(E) \approx \bar{E}/\Delta(n)$. Here, $\Delta(n)$ is the maximum d-wave gap where the gap $\Delta(\phi)$ at any point $\phi$ (the polar angle for momentum) on the 2-dimensional Fermi circle in the CuO$_2$ Brillouin zone is $\Delta(\phi) = \Delta(n)\cos(2\phi)$ with zeros in the $(\pi,\pi)$ direction and other symmetry related points. The small $n$ limit gives $\mu^*/\Delta(0) = \sqrt{2n}^{1/2}$ which differs radically from the s-wave case and reflects the gap symmetry with nodes (see Table I). Numerical results for $\mu^*/\Delta(0)$ versus $n$ are given in Fig. 2. The top frame applies to the s-wave case and is for comparison with the bottom frame for d-wave. The dashed curves in both frames are our approximate analytical results which are seen to match well the exact results (solid curve for $T = 0$) in the small $n$ limit. The remaining curves are at finite temperature $T$ as indicated in the caption, namely $T/T_c = 0.3, 0.5, 0.7$ and 0.9. Several features are worth noting. For s-wave, the zero temperature behaviour of the chemical potential as a function of $n$ is qualitatively different from the case for finite temperature. In the
The parameter $\mu^*$ versus $n$ for several temperatures shown for the s-wave (top frame) and d-wave (bottom frame) gaps. The dashed curve is the small $n$ limit (see Table I). From top to bottom, the solid curves are for $T/T_c = 0.3, 0.5, 0.7, 0.9$. Here only the physical part of the curves are shown.

Inequality ∆(0). Note that the chemical potential becomes small as $\mu \rightarrow 0$ as $n \rightarrow 0$. For the s-wave case at finite $T$ a different argument holds. In this case the thermal factor $f(E_{k} - \mu^*)$ gives the probability that the state $E_{k}$ is occupied at finite $T$. This probability can be increased over its value for $\mu^* = 0$ simply by having $\mu^*$ take on a small finite value to accommodate the excess quasiparticles. At low temperature, however, the thermal tails of the occupation factor are small in the region of the gap and $\mu^*$ must increase fairly rapidly as $n$ increases. This is seen most clearly in the second highest curve in the top frame of Fig. 2 which corresponds to $T/T_c = 0.3$. Also as the temperature is increased $\mu^*$ decreases as expected. In the d-wave case shown in the lower frame of Fig. 2, $\mu^*$ starts from zero at $n = 0$ even at zero temperature because, as we have already indicated, there are states available at any energy above $\omega = 0$. Comparing top and bottom frame we note that the chemical potential for $t = 0.3$ (to be specific) rises more rapidly in the s-wave case and becomes bigger than for d-wave. This can be traced to the fact that for d-wave the part of the density of states that is occupied by the excess quasiparticles is in the range 0 to $\mu^*$ while in the s-wave case it is the region just about the gap $\Delta(n)$ which is relevant. As the temperature is increased towards $T_c$, the differences in the quasiparticle density of states between s- and d-wave become smaller and the chemical potentials start to become very similar. A second feature to be noticed is that at finite $T$ the curves for $\mu^*$ extend to higher values of $n$ for the s-wave case than they do in the d-wave case although the reverse is true at zero temperature. In all cases the curves terminate when the free energy difference between normal and nonequilibrium superconducting state becomes zero or there are two solutions and the one with the lowest free energy is chosen. This occurs at smaller values of $n$ for the d-wave case as compared with s-wave for the given temperature $T \neq 0$ shown. We will return to this issue later on in our discussion of Fig. 4.

The gap equation with a pairing potential of the form

$$V_{kk'} = V \cos(2\phi') \cos(2\phi),$$

where $k$ is momentum on the Fermi surface, with a distribution of excess quasiparticles included through the introduction of a chemical potential takes the form

$$\frac{1}{N(0)V} = \left\langle \int_{0}^{\omega_c} \frac{2 \cos^2(2\phi)de_k}{\sqrt{\epsilon_k^2 + \Delta^2(n)\cos^2(2\phi)}} \tanh\left(\frac{\beta}{2}(E_k - \mu^*)\right) \right\rangle$$

with $E_k = \sqrt{\epsilon_k^2 + \Delta^2(n)\cos^2(2\phi)}$. The bracket $\langle \cdots \rangle$ indicates the angular average and $\epsilon_k$ is energy integrated in a rim of width $\omega_c$ about the Fermi energy. With reference to the $n = 0$ case (i.e., $\mu^* = 0$) we can rewrite Eqn. (2) to read at $T = 0$

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = \left\langle \int_{0}^{\omega_c} \frac{-4 \cos^2(2\phi)de_k}{\sqrt{\epsilon_k^2 + \Delta^2(n)\cos^2(2\phi)}} E_k \leq \mu^* \right\rangle$$

The situation is very different in the d-wave case and in s-wave at finite temperature. In these two cases the chemical potential becomes small as $n \rightarrow 0$. For the d-wave case this is easily understood because there is a small but finite density of states at any nonzero value of energy $\omega \neq 0$. The excess quasiparticles can occupy these states and hence $\mu^* \rightarrow 0$ as $n \rightarrow 0$. For the s-wave case at finite $T$ a different argument holds. In this case the thermal factor $f(E_{k} - \mu^*)$ gives the probability that the state $E_{k}$ is occupied at finite $T$. This probability can be increased over its value for $\mu^* = 0$ simply by having $\mu^*$ take on a small finite value to accommodate the excess quasiparticles.
where the integration over energy and φ must duly take account of the restriction \( E \leq \mu^* \). For small \( n \to 0 \) the leading order gives \( \Delta(n) = \Delta(0) + \delta \Delta(n) \)

\[
\frac{\delta \Delta(n)}{\Delta(0)} = -\frac{8}{\pi} \int_0^{\pi/2} \cos^2 \phi' d\phi' \\
\times \int_{\Delta(n) \cos \phi'}^{\mu^*} \frac{dE}{\sqrt{E^2 - \Delta^2(n) \cos^2 \phi'}}
\]

(11)

where we have changed from \( \phi \) to \( \phi' = 2\phi \). But the lower limit in the \( \phi' \) integration in Eqn. (11) restricts the integration to the nodal region which corresponds to \( \phi' = \pi/2 \). We find

\[
\frac{\delta \Delta(n)}{\Delta(0)} = -\frac{8}{\pi} \left( \frac{\mu^*}{\Delta(n)} \right)^3 \int_0^1 x^2 dx \ln \frac{1 + \sqrt{1 - x^2}}{x} \\
\simeq -\frac{4\sqrt{2}}{3} n^{3/2}
\]

(12)

(entered in Table I) where we have used the relationship \( \mu^*/\Delta(n) = \sqrt{2}n^{1/2} \) to lowest order. In Fig. 3 we show exact numerical results for the normalized gap \( \Delta(n)/\Delta(0) \) as a function of \( n \) for the d-wave case (solid curve) and compare with our approximate result (dashed curve) which applies only at small \( n \). The agreement is excellent even up to the point where the first order transition to the normal state occurs. This is where the solid curve is extended into the dotted curve. The gap function as a function of \( n \) is reduced less in d-wave (Fig. 3) as compared to s-wave (Fig. 1) all the way to \( n = n_c \). The free energy difference \( \Delta F(n) \) becomes zero at \( n = n_c \simeq 0.17 \) which is to be compared with \( n_c \simeq 0.15 \) in the s-wave case. At the critical \( n_c \), \( \Delta(n)/\Delta(0) \) is almost 0.6 for s-wave while in the d-wave case it has not yet reached 0.8. The blocking of states by the excess quasiparticles has much less effect on the condensate wavefunction as reflected in the change in the value of the gap in d-wave than in s-wave because now the excess quasiparticles accumulate in the nodal region. Since the gap is zero or near zero in that region, it is clear that these states do not contribute much to the lowering of energy brought about by the formation of Cooper pairs.

To establish where this first order transition occurs, we need the free energy. The formula for the grand potential for the superconducting state with \( n \) excess quasiparticles is

\[
\Omega_S(n) = 2k_B T \sum_k \ln(1 - f(E_k - \mu^*)) \\
+ \sum_k \epsilon_k - E_k + \frac{\Delta^2}{2E_k}(1 - 2f(E_k - \mu^*))(13)
\]

and for the normal state with \( n = 0 \) it is

\[
\Omega_D(n) = 2k_B T \sum_k \ln(1 - f(|\epsilon_k|)) + \sum_k (\epsilon_k - |\epsilon_k|)
\]

(14)

The sum over \( k \) can be converted to energy and the constant two-dimensional electron density of states factor

\[
\frac{\Delta \Omega(n)}{N(0) \Delta^2(n)} = \frac{\Omega_S(n) - \Omega_D(n)}{N(0) \Delta^2(n)} = \frac{1}{4} \Delta^2(n) \\
+ 4 \int_0^{\mu^*} \tilde{N}(E)(E - \mu^*) dE - \frac{1}{2} I \Delta^2(n)
\]

(15)

where \( I \) is the same integral as appears on the right-hand side of Eqn. (11). The first term in (15) is the usual expression for the condensation energy of a d-wave superconductor but with \( \Delta(n) = \Delta(0)(1 - 4\sqrt{2}n^{3/2}/3) \) replacing the gap amplitude \( \Delta(0) \) which applies to \( n = 0 \). In \( \Delta \Omega(n) = N(0)/N(0) \) only \( \Delta^2(n)/4 \) enters. The two extra terms in Eqn. (15) can be worked out analytically as \( n \to 0 \) and lead to

\[
\frac{\Delta \Omega(n)}{N(0) \Delta^2(n)} = -\frac{1}{4} \Delta^2(n) - \frac{2}{3} \frac{\mu^*}{\Delta(n)} - \frac{1}{3}(\frac{\mu^*}{\Delta(n)})^3
\]

(16)

only in the first term on the right-hand side of the equation must we retain the \( n \) dependence in \( \Delta(n) \). The difference in grand potential \( \Delta \Omega(n) \) normalized to \( \Delta^2(n)/4 \) is easily worked out to be

\[
\frac{4 \Delta \Omega(n)}{N(0) \Delta^2(n)} = -1 - \frac{16\sqrt{2}}{3} n^{3/2}
\]

(17)
The normalized free energy $\Delta F$ is obtained by adding $\mu^* \bar{n}$ to Eqn. (13) and after normalization we get

$$\frac{4\Delta F(n)}{N(0)\Delta^2(0)} = -1 + \frac{32\sqrt{2}}{3} n^{3/2}$$

which is entered in Table I. Numerical results at any value of $n$ are shown in the bottom frame of Fig. 3. The solid line is our numerical result for $4\Delta F(n)/N(0)\Delta^2(0)$ at $T=0$ and the dashed curve our approximate result (Eqn. (18)). The analytic result agrees well with the full numerical solution at small $n$ and differs slightly near the critical value of $n = n_c$ where the first order transition to the normal state occurs at $n_c \approx 0.17$.

At finite temperature the blocking is less effective because it is the states closest to zero energy that are the most effective in forming the condensed pairs while the thermal factor depopulates these states. By contrast, for d-wave, the $T = 0$ curve is above the $t = 0.3$ (dotted curve) as we have already noted. In this instance the blocking at $T = 0$ is much less effective and consequently temperature is not as important an effect. We note again that, at $T = 0$, the d-wave gap is reduced less than in s-wave for the same value of $n$ and that the critical value of $n$, at which a first order transition to the normal state takes place, is larger. At the higher temperatures shown, however, the reverse holds. Also, note that as the temperature rises towards $T_c$, the difference between s- and d-wave get less pronounced as the differences between the two quasiparticle density of states become small and also more states are involved.

The nonthermal quasiparticle distribution used in the $\mu^*$-model has an interesting aspect in that it allows for the system to become unstable to quasiparticle density fluctuations. Essentially, if the quasiparticles are injected uniformly in the sample, the density fluctuations will act to draw off quasiparticles from some regions thereby increasing the superconducting gap locally and flowing those quasiparticles to other regions, causing an accumulation which lowers the local gap, possibly even driving the local region normal. This phase separation could be either a static or a temporal structure. Such a state has been studied initially by Chang and Scalapino[16] and Scalapino and Huberman[17] for the s-wave superconductor and experimental verification of a density instability leading to an inhomogeneous multigap state has been done by several groups[18] using tunnel injection in thin film nonequilibrium superconductors. The theoretical signature of such an inhomogeneous state in the $\mu^*$-model is that $\partial \mu^*/\partial n|_T < 0$.[16, 17] From Fig. 2, we find that the variation of $\mu^*$ with $n$ differs in s- and d-wave and by examining the slopes of these curves, in particular, the point where the slope goes negative, we can reproduce the s-wave phase diagram of Chang and Scalapino[16], shown in the upper frame of Fig. 5, and provide the equivalent prediction for d-wave in the bottom frame. The dashed curve in these phase diagrams marks the boundary between the normal state (NS) and the superconducting state (either homogeneous or inhomogeneous). This boundary is entirely a first-order transition. The area labelled IN, is the region of $n$ and $T$, where the slope of the chemical potential curve is negative and an inhomogeneous state is predicted to exist. The solid line marks the boundary between it and the homogeneous superconducting state (SC). There are quantitative differences between the s- and d-wave cases. The region of the inhomogeneous phase is quite large in the s-wave case and almost non-existent in d-wave and at low temperature the s-wave superconductor would likely be phase separated whereas, the d-wave one would not be. While the inhomogeneous state may be of interest to study in itself, in the d-wave case it may be encouraging
to note that attempts at experimental verification of our predictions for power law dependences, and other results presented in this paper, are unlikely to be hampered by the presence of an inhomogeneous phase.

Next we consider briefly the case of the $T^\ast$ model which is just a simple heating model if only the electronic system is considered. Similar approximate analytic calculations can be done to get various relationships in the limit $n \to 0$ for the case when the sample before irradiation is assumed to be zero. These analytic derivations are supplemented with full numerical work in which we also consider the case when the sample is initially at finite temperature $T$.

We begin with the s-wave case and return to the gap equation shown in Eqn. (2), now modified according to Eqn. (4) rather than Eqn. (1). In the limit of $T \to 0$, the result for the lowest order correction to the gap is well known:

$$\delta \Delta(n)/\Delta(0) = -\sqrt{\frac{2\pi k_B T^\ast}{\Delta(0)}} e^{-\Delta(0)/k_B T^\ast} \quad (19)$$

The relation between $n$ and $T^\ast$ can be trivially obtained as $n = \sqrt{\pi T^\ast/2\Delta(0)} e^{-\Delta(0)/k_B T^\ast}$ and so $\delta \Delta(n)/\Delta(0) = -2n$.

The d-wave case is not as well known and we include the critical steps here

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = -4 \int_0^{\pi/2} \frac{d\phi'}{\pi} \cos^2 \phi' \times \int_{\omega_c}^{\omega_n} \frac{dE e^{-E/k_B T^\ast}}{\sqrt{k^2 - \Delta^2(n) \cos^2 \phi'}} \quad (20)$$

which can be manipulated into

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = -8 \int_0^{\pi/2} d\phi' \cos^2 \phi' e^{-\Delta(n) \cos \phi'/k_B T^\ast} \times \int_{0}^{\omega_n} \sqrt{x(x + 2\Delta(n) \cos \phi')} \quad (21)$$

The integral over $\phi'$ is peaked around $\cos \phi' = 0$, i.e. $\phi'$ near $\pi/2$ which allows us to approximate it by

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = -8 \int_0^{\pi/2} d\phi' \cos^2 \phi' e^{-\Delta(n) \cos \phi'/k_B T^\ast} \times \int_0^{\omega_n} \sqrt{x(x + 2\Delta(n) y)} \quad (22)$$

from which we get

$$\frac{\delta \Delta(n)}{\Delta(0)} = -4 \left(\frac{T^\ast}{\Delta(0)}\right)^3 \quad (24)$$

Also from the definition of $n$ we get immediately

$$n = \frac{\pi^2}{12} \left(\frac{T^\ast}{\Delta(0)}\right)^2 \quad (25)$$

The d-wave case is not as well known and we include the critical steps here

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = -4 \int_0^{\pi/2} \frac{d\phi'}{\pi} \cos^2 \phi' \times \int_{\omega_c}^{\omega_n} \frac{dE e^{-E/k_B T^\ast}}{\sqrt{k^2 - \Delta^2(n) \cos^2 \phi'}} \quad (20)$$

which can be manipulated into

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = -8 \int_0^{\pi/2} d\phi' \cos^2 \phi' e^{-\Delta(n) \cos \phi'/k_B T^\ast} \times \int_{0}^{\omega_n} \sqrt{x(x + 2\Delta(n) \cos \phi')} \quad (21)$$

The integral over $\phi'$ is peaked around $\cos \phi' = 0$, i.e. $\phi'$ near $\pi/2$ which allows us to approximate it by

$$\ln\left(\frac{\Delta(n)}{\Delta(0)}\right) = -8 \int_0^{\pi/2} d\phi' \cos^2 \phi' e^{-\Delta(n) \cos \phi'/k_B T^\ast} \times \int_0^{\omega_n} \sqrt{x(x + 2\Delta(n) y)} \quad (22)$$

from which we get

$$\frac{\delta \Delta(n)}{\Delta(0)} = -4 \left(\frac{T^\ast}{\Delta(0)}\right)^3 \quad (24)$$

Also from the definition of $n$ we get immediately

$$n = \frac{\pi^2}{12} \left(\frac{T^\ast}{\Delta(0)}\right)^2 \quad (25)$$

Exact numerical results agree well with these approximate $n \to 0$ expressions which we summarize in Table II. In Fig. 6 we show numerical results for $\Delta(n, T)/\Delta(0, 0)$ versus $n$ where we have normalized the maximum gap $\Delta(n, T)$ to the zero temperature equilibrium case. The top frame is for s-wave while the bottom is d-wave. In each frame the short dashed curve is the approximate
result at \( T = 0 \) derived above. We see that it compares well with the exact result (solid curve). The other curves apply to \( T/T_c = 0.3 \) (dotted), 0.5 (short-dashed), 0.7 (long-dashed) and 0.9 (dot-dashed). In this case the \( \Delta(n, T)/\Delta(0, 0) \) curves do not cross and are all constructed from BCS curves for the temperature dependence of the gap. The temperature \( T \) refers to the sample temperature before the injection of excess quasiparticles \( n \). The intersection of the various curves with the vertical axis simply gives the temperature variation of the gap in BCS. At finite \( n \), the extra quasiparticles are accommodated into the system by assuming a higher temperature thermal distribution \( T' \), with \( T' \) made sufficiently larger than \( T \) to have \( n \) extra thermal quasiparticles.

Note that in contrast to the \( \mu^* \) model, the differences between s- and d-wave are much less pronounced at \( T = 0 \). This reflects the fact that in a thermal distribution, blocking effects are not an important consideration. In fact now the gap in the d-wave case terminates at a value of \( n \) which is smaller than in the s-wave case. This is opposite to what is found for the \( \mu^* \) model. Also the curves show no first order transition to the normal state which now occurs only when the gap is zero.

In Fig. 7 we show the value of \( T' \) as a function of the nonequilibrium distribution \( n \) for various values of \( T \). The temperatures used are \( T/T_c = 0.01, 0.3, 0.5, 0.7, 0.9 \). Note that the curve with the lowest sample temperature (solid curve) at small \( n \) agrees well with our analytic expressions for the same quantity shown as the dashed lines. These follow from the transcendental equation \( n = 0.94 \sqrt{T'/T_c} e^{-1.76T'/T'} \) for s-wave and the explicit equation \( T'/T_c = 2.36n^{1/2} \) for d-wave. These results are also entered in the final line of Table II.

### III. S-I-N TUNNELING JUNCTION

Now we consider a specific application of our results to the case of a superconducting-insulator-normal metal tunneling junction. Denote the current in a S-I-N junction with nonequilibrium distribution on the superconducting side, described by the \( \mu^* \) model, by \( I_{\mu^*}^{SN}(V) \) where \( V \) is the voltage across the junction. It is given by a straightforward modification of the usual tunneling formula:

\[
I_{\mu^*}^{SN}(V) = \int_{-\infty}^{\infty} d\epsilon \tilde{N}_S(\epsilon)\left[f(\epsilon - \mu^*) - f(\epsilon + V)\right] \tag{26}
\]
where \( \tilde{N}_S(\epsilon) \) is the normalized density of states given by

\[
\tilde{N}_S(\epsilon) = \text{Re} \left( \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta_k^2}} \right)
\]  

(27)

with \(< ... > \) the average over angles as before.

We have seen in the previous section that the introduction of a nonequilibrium \( \mu^* \) modifies the gap but does not change its symmetry and Eqn. (26) still holds for the density of states in Eqn. (25) although the new gap amplitude is reduced by a factor of \((1-2n)\) and \((1-4\sqrt{2}n^{3/2}/3)\) for s- and d-wave, respectively, at zero temperature and \( n \) small. Besides the change in \( \bar{N}(\epsilon) \) just described, one of the thermal factors in Eqn. (25) is also displaced by the new chemical potential \( \mu^* \). The structure of Eqn. (25) makes it useful to separate these two factors, and it is convenient to rewrite \( I_{\mu^*}^{SN}(V) \) in the form

\[
I_{\mu^*}^{SN}(V) = \int_{-\infty}^{\infty} d\epsilon \tilde{N}_S(\epsilon)[f(\epsilon - \mu^*) - f(\epsilon)]
+ \int_{-\infty}^{\infty} d\epsilon \tilde{N}_S(\epsilon)[f(\epsilon) - f(\epsilon + V)]
\]

(28)

The second term in Eqn. (27) has the identical form that applies to an ordinary S-I-N junction in equilibrium at temperature \( T \). We denote the current in this case by \( \mathcal{I}(V) \)

\[
\mathcal{I}(V) \equiv \int_{-\infty}^{\infty} d\epsilon \tilde{N}_S(\epsilon)[f(\epsilon) - f(\epsilon + V)]
\]

(29)

where the gap amplitude defining \( \tilde{N}_S(\epsilon) \) is that appropriate to the nonequilibrium superconductor. The first term in Eqn. (27) is simply a number, independent of voltage. Reference to the defining Eqn. (3) for \( n \) shows that this number is equal to \( n\Delta(0) \). Thus we find

\[
I_{\mu^*}^{SN}(V) = \mathcal{I}(V) + n\Delta(0)
\]

(30)

We see from Eqn. (29) that the current voltage characteristics are modified in two ways by the nonequilibrium distribution. The entire equivalent equilibrium distribution is shifted up by an amount \( n\Delta(0) \). This allows one to measure \( n \) once the gap is known. Secondly, the “equivalent equilibrium” current voltage characteristics are those of an equilibrium junction with the smaller nonequilibrium gap used instead of its equilibrium value. This knowledge allows one to fully characterize the nonequilibrium current voltage characteristic and to apply checks to see how well the \( \mu^* \) model works. For example, the derivative of \( I_{\mu^*}^{SN}(V) \) with \( V \) at zero temperature simply gives

\[
\frac{dI_{\mu^*}^{SN}}{dV} = \tilde{N}^S(V),
\]

(31)

the quasiparticle density of states with the nonequilibrium gap but otherwise it is the same as for an equilibrium distribution. For s-wave it will have an inverse square root singularity at \( \Delta(n) \) and for d-wave it will go like \( \ln(8\Delta(n)/\Delta(n^2 - \omega^2)/\pi) \) instead (see Abanov-Chubukov [21]). In both cases, \( \Delta(n) \) can be determined from these singularities. Comparison with its equilibrium value gives a measure of \( n \) in both s- and d-wave. Next it should be possible to check if this is consistent with the value of the chemical potential related to \( n \) by \( \mu^*/\Delta(0) = 1-2n \) and \( \mu^*/\Delta(0) = \sqrt{2}n^{1/2} \), respectively, for s- and d-wave, at \( T = 0 \) and in the limit of small \( n \). The chemical potential is measured directly by noting that in Eqn. (27) for \( V = -\mu^* \), first and second terms on the right-hand side are equal but of opposite sign, giving a sum of zero. In Fig. 8, we show numerical results for \( I_{\mu^*}^{SN}(V) \) versus \( V \) at a low temperature \( T/T_c = 0.1 \). The top frame applies to s-wave while the bottom frame is for d-wave. It is verified that these curves obey the expected rules mentioned above. For the s-wave case \( \Delta(n)/\Delta(0) \) is set equal to 0.8 while for the d-wave case we have used \( \Delta(n)/\Delta(0) = 0.9 \) instead. Reference to Fig. 1 for s-wave and to Fig. 3 for d-wave shows that these choices correspond to an excess quasiparticle number of approximately 0.09 and 0.12, respectively. The excess quasiparticle number is greater in the d-wave case than in s-wave even though the gap is only reduced by 10% as compared with 20% for s-wave.

IV. PUMP/PROBE OPTICAL MEASUREMENTS OF \( n(T) \)

In the following, we wish to discuss recent experimental pump/probe laser experiments which have been used to infer information about the excess quasiparticle density. In particular, we wish to address a claim that these experiments provide evidence for s-wave pairing in the high \( T_c \) cuprates. To address this issue, following Kabanov et al. [3], we use the \( T^* \) model. While we do not report explicitly on this here, we have also examined these properties within the \( \mu^* \)-model and have found similar results.

To calculate the excess quasiparticle density \( n(T) \), we have used Eqn. (3) modified for the \( T^* \) model via Eqn. (4) such that

\[
n(T) = \frac{1}{\Delta(0)} \left\langle \int_{\Delta(0)}^{\infty} [f(E_k^*, T^*) - f(E_k, T)] dE_k \right\rangle
\]

(32)

where \( E_k^* = \sqrt{\epsilon_k^* + \Delta_k^2(T^*)} \) (the asterisk always referring to quantities depending on \( T^* \) instead of \( T \)) and an average over the angle \( \phi \) is done in the case of d-wave.

To evaluate \( n(T) \) at a temperature \( T \), it is necessary to know \( T^* \) and \( \Delta(T^*) \) and this is determined from the amount of laser energy \( E_l \) deposited in the system. In this work, the laser energy will be assumed to go into both electron and phonon systems

\[
E_l = \Delta E_{\text{electron}} + \Delta E_{\text{phonon}}.
\]

(33)

To begin with, however, we examine the case where the energy is assumed to go only into the electronic system:
the quasiparticles and a modification of the superconducting condensate due to a $\Delta(T^*)$. The energy going into the quasiparticles relative to the reference nonequilibrium state at temperature $T$ is

$$\Delta E_{qp} = 4N(0) \left[ \int_0^\infty \left| E_k f(E_k^*, T^*) - E_k f(E_k, T) \right| d\epsilon_k \right].$$  

(34)

Kabanov et al.\[7\] treated this piece as $|n(T^*) - n(T)|(\Delta(T) + k_BT/2)$ which is not completely correct near $T_c$.

We find that the average quasiparticle energy per particle calculated as

$$\frac{E_{qp}}{N} = \frac{\int_0^\infty E_k f(E_k, T) d\epsilon_k}{\int_0^\infty f(E_k, T) d\epsilon_k}$$  

(35)

gives a constant equal to $\Delta(0)$ at zero temperature for $s$-wave, since excitations can only exist at the gap edge because the density of states is zero below this energy. This behaviour is seen in Fig. 9 for the dashed curve which gives Eqn. (34) normalized to $\Delta(0)$. As $T$ increases, the energy per particle increases slightly and then decreases near $T_c$ to a value of $\pi^2 k_B T_c/12 \ln(2) \Delta(0) = 1.19(k_B T_c/\Delta(0)) \approx 0.67$ as now the gap in the quasiparticle density of states has shrunk to zero and the energy of the quasiparticles is controlled by $k_BT$ which is less than $\Delta(0)$. Similar physics is found for a $d$-wave order parameter with the essential difference that excitations can now occur at zero energy and therefore the average quasiparticle energy per particle starts from zero at $T = 0$ and rises linearly reflecting the linear increase in energy of the density of states. It can be shown analytically that $E_{qp}/N\Delta(0) \approx 1.03 T_c/T_c$ for $T \ll \Delta(0)$, the regime where a nodal approximation is valid. At $T_c$, the quasiparticle energy per particle is once again controlled by $k_BT$ and so the limiting number is given by the same formula as above but with the BCS $d$-wave gap ratio of $\Delta(0)/k_BT_c = 2.14$ instead of 1.76 for $s$-wave. The number at $T_c$ is approximately 0.55 These results are shown in Fig. 9 (solid curve) and we will refer back to them at a later point.

The reaction of the condensate is simply given as

$$\Delta E_{cond} = 2N(0) \left( \int_0^\infty [E_k - E_k^* + \frac{\Delta_k^2}{2E_k^*}(1 - 2f(E_k^*, T^*)) - \frac{\Delta_k^2}{2E_k}(1 - 2f(E_k, T))] d\epsilon_k \right).$$  

(36)

This term reflects the fact that the presence of excess quasiparticles causes a readjustment to the superconducting condensate through a change in the gap $\Delta^* \equiv \Delta(T^*)$. This term was not included by Parker\[13\] and neither was it included in the work of Kabanov et al.\[7\].

FIG. 8: SIN tunneling I-V characteristics for $T = 0$ shown for the $\mu^*$ model with $s$-wave gap symmetry (upper frame) and $d$-wave (lower frame). The current $I$ is normalized by $N(0)$ and by the maximum zero-temperature gap in the standard way and the voltage $V$ is normalized to the maximum gap at $T = 0$. An excess quasiparticle density $n$ leads to a reduction in the gap by $\Delta(n)/\Delta(0)$ and a vertical shift by $n$ in the I-V characteristic. $I = 0$ at $V = -\mu^*$. The dotted curve is the normal state $n = 0$, the solid curve is superconducting state with $n = 0$ and the dashed curve is for a reduced gap $\Delta(n)/\Delta(0) = 0.8$ in the $s$-wave case and 0.9 in the $d$-wave case.

FIG. 9: The average quasiparticle energy per particle normalized to the maximum zero temperature equilibrium gap, $E_{qp}/N\Delta(0)$, versus $T/T_c$ for $s$-wave (dashed curve) and $d$-wave (solid curve). The $T = 0$ value is set by the lowest available energy state in the quasiparticle density of states, whereas near $T_c$, the energy scale is set by $k_BT$. 

$\mu^*$-model
For our purposes, we used the BCS temperature dependence of the condensation energy in the equilibrium state. Note that the curves shown here are normalized to the zero temperature condensation energy in units of the condensation energy)

\[ n(T)/n_0 = \frac{1 + 2.9t^3}{E_I} - \frac{(2.9t^3)^{2/3}}{E_I^{2/3}} \]  

(37)

for small reduced temperature \( t = T/T_c \) with \( T \ll \Delta(0) \). For \( t \ll E_I \), \( n(T)/n(0) \approx 1 - (2.9/E_I)^{2/3}t^2 \). For \( t \gg E_I \) but still with \( T \ll \Delta(0) \), \( n(T)/n(0) \approx (2/3)t(E_I/2.9)^{1/3} \) (inverse \( t \) law). Our numerical results confirm to these limits. Also note that \( n(0) = 0.18(E_I/2.9)^{2/3} \) so that \( n(T) \) unnormalized to \( n(0) \) will go like \( E_I \) in the region of \( 1/t \) law applies. Once again as the energy scale reverts to \( k_BT \) near \( T_c \), the slight upturn in \( n(T)/n(0) \) is reflecting the smaller energy required to create the quasiparticles. Kabanov et al.\[7\] do not find this result due to their approximations and the details of their curve would differ as they have only included an approximate linear form of the d-wave quasiparticle density of states rather than the full form with temperature dependence as is done here. In fact, if their data did not go so low in temperature and given that \( n(0) \) is not known experimentally, the flatness of the d-wave curve with a slight upturn near \( T_c \) placed on an arbitrary scale, would probably make as viable a comparison with their data as the s-wave case. However, we note that they do show data at lower temperature and so this interpretation does not hold, also the 2/3 dependence on \( E_I \) at \( T = 0 \) is not verified. On the face of things, it may appear that their data agree best with s-wave. However, we argue, as they did, that it is necessary to include phonons in this picture.

In their analysis, to obtain agreement with their data, Kabanov et al.\[7\] did include the fact that some of the laser energy would be distributed to phonons in the system. In this case, we partition the laser energy with the phonons as well:

\[ E_I = \Delta E_{qp} + \Delta E_{cond} + \Delta E_{phonon} \]  

(38)

The phonon piece is calculated assuming that only phonons with energy \( \hbar \omega \) above \( 2\Delta \) can be considered to be out of thermal equilibrium with the lattice and therefore at a temperature \( T^* \). It is in this way that a bottleneck at \( 2\Delta \) is introduced into the model, which then deviates from pure heating. Such was the same consideration of Kabanov et al.\[7\]. As such we calculate the amount of energy going into the phonon system as:

\[ \Delta E_{phonon} = \int_{2\Delta}^{\infty} \omega F(\omega)(n(\omega, T^*) - n(w, T))d\omega, \]  

(39)
where the usual Bose-Einstein factor \( n(\omega, T) = 1/(\exp(\hbar \omega/k_B T) - 1) \) and \( F(\omega) \) is the phonon frequency distribution function measured by Renker et al.\[22\] from neutron scattering experiments on YBCO. In our calculation, we effectively fix \( N(0) \) to get the correct ratio of phononic specific heat at \( T_0 \) relative to the electronic part via comparison with the specific heat data of Loram et al.\[23\]. This is to ensure that, the phononic and electronic portions are balanced in accordance with experiment. As the phonon energy increases typically as \( T^4 \), one sees that this term, as long as \( T^* > T \), will take more and more of the fixed laser energy away from the electronic system and hence, there are fewer excess quasiparticles that can be created at higher temperature and the curve for \( n(T)/n(0) \) must go down. This is illustrated in Fig. 11, where curves decay rapidly as \( T \) increases and are further reduced for lower \( E_I \). Also shown on the same figure are the experimental results of Kabanov et al.\[7\] (solid squares). We conclude that their data does not support an interpretation of s-wave gap symmetry in the high \( T_c \) cuprates. Nor does it agree with d-wave (Fig. 10, bottom frame).

We have also done this calculation with the Pb phonon spectrum\[24\] (adjusted to the specific heat in Pb and using a BCS gap ratio) and we find similar curves to those shown here. The excess quasiparticle density in Pb has been measured by Carr et al.\[11\] and compared successfully to rate equation calculations\[25\] used for determining the nonequilibrium distribution. We show the Pb data (solid circles) on our curves to emphasize that Pb, as an s-wave superconductor, does follow the trend of showing a suppressed excess quasiparticle density as the temperature increases and agrees well with our calculations. This comparison also serves to show that our simple procedure of introducing the bottleneck at \( 2\Delta \) and sharing the laser energy between phonons and electrons agrees qualitatively and even semiquantitatively with the more sophisticated and accurate rate equation calculations used by Carr et al.\[11\] and validates our simpler method.

Here, we have done the calculation using a BCS gap ratio of 3.53. A full strong coupling Eliashberg calculation\[26\] would have to be done to include a larger ratio, as from our experience, simply inserting a larger ratio in a BCS calculation can give incorrect, and therefore misleading, results. Aside from the inherent complexity of such a calculation, we would need to commit to some specific mechanism since phonons are not believed to be the source of the high \( T_c \). But there is no consensus on mechanism. To fit experiment, however, Kabanov et al.\[7\] phenomenologically increase the value of the ratio \( 2\Delta(0)/k_B T_c \) to about 9. There is, however, no rigorous justification for such a procedure and this is our main objection to such a fit. To increase the gap ratio, it is necessary to increase the ratio of \( T_c/\omega_m \) in Eliashberg theory\[26\], where \( \omega_m \) is a particular moment of the electron-phonon spectral function which gives the appropriate measure of the average phonon energy involved. When this is done, damping effects, entirely left out of BCS, become dominant and superconducting properties acquire behaviours that are qualitatively different from straightforward extrapolations of BCS behaviour (see, for instance, many properties calculated in Ref. \[26\] in the limit of large \( T_c/\omega_m \) ratio). For YBCO, the gap ratio is closer to 3\[27\] and is certainly not 9. Further, for a gap ratio ratio of 9-10, the cutoff of \( 2\Delta \) applied to the phonons falls at 70-80 meV which is at the very top of the measured phonon spectrum\[22\]. This large value of the cutoff has the effect of greatly reducing the ability of the phonons to share in the laser energy and this partially accounts for why the curve for \( n(T) \) in this case stays flat to much higher temperature than for the BCS curve.

V. CONCLUSIONS

In summary, we have examined the differences between an s-wave order parameter versus a d-wave in a nonequilibrium superconductor using two prominent models in the literature, the \( T^* \) model of Parker\[18\] and the \( \mu^* \) model of Owen and Scalapino\[14\]. While these models may be considered to be somewhat crude, they have the virtue of being simple, and accessible in terms of both calculation and physical intuition. As a result, one finds them still being used by experimental groups to aid in the interpretation of their data. With the advent of high \( T_c \)
cuprates and the deeper examination of the issue of order parameter symmetry, a re-examination of these models allows for the prediction of different power law dependences on excess quasiparticle density expected between s- and d-wave gaps. Tables I and II summarize such predictions. These predictions are grounded in interesting physics such as the blocking of states and how the condensate readjusts and as such, they should remain relevant even within more complicated models. It is hoped that the simplicity of our results may inspire further experimental and theoretical efforts to examine nonequilibrium phenomena in the presence of an order parameter symmetry, a re-examination of these models including phonons, do not produce an excess quasiparticle distribution \( n(T) \) which is nearly constant in temperature with a peak near \( T_c \). Rather a quick decay with increasing \( T \) is found as more of the laser energy is taken up by the phonon system. When the explicit case of Pb is considered rather than YBCO, the same rapidly decaying characteristic is found and this is in good agreement with the recent data of Carr et al.\(^{[12]}\) in this classic s-wave superconductor. Our final conclusion is that present pump/probe experiments in YBCO cannot be accounted for by either s- or d-wave gap symmetry and it may be necessary to re-examine the interpretation of the data in terms of the excess quasiparticle density. In this regard, a next step might be to calculate the optical conductivity itself in the nonequilibrium state so as to make a more direct contact with what is measured.

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