Secure Information Flow Connections

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Abstract

Denning’s lattice model provided secure information flow analyses with an intuitive mathematical foundation: the lattice ordering determines permitted flows. We examine how this framework may be extended to support the flow of information between autonomous organisations, each employing possibly quite different security lattices and information flow policies. We propose a connection framework that permits different organisations to exchange information while maintaining both security of information flow as well as their autonomy in formulating and maintaining security policies. Our prescriptive framework is based on the rigorous mathematical framework of Lagois connections proposed by Melton, together with a simple operational model for transferring object data between domains. The merit of this formulation is that it is simple, minimal, adaptable and intuitive. We show that our framework is semantically sound, by proving that the connections proposed preserve standard correctness notions such as non-interference. We then illustrate how Lagois theory also provides a robust framework and methodology for negotiating and maintaining secure agreements on information flow between autonomous organisations, even when either or both organisations change their security lattices. Composition and decomposition properties indicate support for a modular approach to secure flow frameworks in complex organisations. We next show that this framework extends naturally and conservatively to the Decentralised Labels Model of Myers et al. — a Lagois connection between the hierarchies of principals in two organisations naturally induces a Lagois connection between the corresponding security label lattices, thus extending the security guarantees ensured by the decentralised model to encompass bidirectional inter-organisational flows.

Keywords: Security Information Flow, Lagois Connection, Security Types, Non-interference, Composition and Decomposition, Decentralised Label Model

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1. Introduction

Denning’s seminal work [1] established complete lattices as the mathematical basis for a variety of analyses regarding secure information flow (SIF), i.e., showing only authorised flows of information are possible. An information flow model (IFM) \( \langle N, P, SC, \sqcup, \sqsubseteq \rangle \) consists of storage objects \( N \), which are assigned security classes drawn from a (finite) complete lattice \( SC \). The partial ordering \( \sqsubseteq \) represents permitted flows between classes. Reflexivity and transitivity capture intuitive aspects of information flow; antisymmetry helps avoid redundancies in the framework. The join operation \( \sqcup \) succinctly captures the combination of information belonging to different security classes in arithmetic, logical and computational operations. \( P \) is a set of processes, which are assigned security classes as “clearances”.

The ensuing decades have seen a plethora of static and dynamic analysis techniques using that framework for programming languages [2, 3, 4, 5, 6, 7], operating systems [8, 9, 10, 11, 6], databases [12], and hardware architectures [13, 14], etc. However, the question on how information can flow securely between independent organisations each with possible quite different policies has not been adequately addressed. An answer needs to indicate how security classes from one lattice are mapped to those in another. In doing so, we wish to abjure ad hoc approaches to reclassifying information.

We revisit and expand on our previous work [15], where we proposed a simple and versatile mathematical framework involving monotone increasing functions between lattices, which guaranteed secure and modular inter-organisational flows of information. Our work is based on the observation that large information systems are not monolithic, and are often constructed by autonomous organisations, each with its own security lattice, policies and mechanisms, negotiating agreements (MoUs) that promise respecting the security policies of others. The MoUs typically mention only a small set of security classes, called transfer classes, between which all information exchange is managed. Modularity and autonomy are important: each organisation would wish to retain control over its own security policies and the ability to redefine them.

We identified the elegant theory of Lagois Connections [16] of order-preserving functions between security lattices as the appropriate framework, showing that they guarantee SIF, without the need for re-verifying the security of the application procedures in either of the domains, and confining the analysis to only the transfer classes involved in potential exchange of data. This paper substantiates this by showing (i) how language-based techniques such as security type systems given by Volpano et al. [17] can be adapted to a setting with different systems having distinct secure flow policies communicating data objects between each other; (ii) how results from Lagois theory provide a robust methodological framework for negotiating and maintaining secure agreements on information flow between autonomous organisations, even when either or both organisations change their security policies. (iii) how decentralised secure flow systems such as that proposed by Myers [18] can be smoothly and conservatively extended to cross-organisational delegation and decentralisation. Indeed,
one way to view this work is that Lagois connections support a conservative extension of SIF analysis techniques developed on complete lattices to embrace structures involving lattices and morphisms between them.

In §2 we identify intuitive requirements for secure bidirectional flow, present the definition of Lagois connections [16], and show that Lagois connections between the security lattices satisfy security and other requirements. We include a brief account of how lattices and Lagois connections can be very succinctly represented to support efficient algorithms.

We present in §3 a minimal operational language consisting of a small set of atomic primitives for effecting the transfer of data between domains. The framework is simple and can be adapted for establishing secure connections between distributed systems at any level of abstraction (language, system, database, ...). We assume each domain uses atomic transactional operations for object manipulation and intra-domain computation. The primitives of our model include reliable and atomic communication between two systems, transferring object data in designated output variables of one domain to designated input variables of a specified security class in the other domain. To avoid interference between inter-domain communication and the computations within the domains, we assume that the sets of designated input and output variables are all mutually exclusive of one another, and also with the program/system variables used in the computations within each domain. Thus by design we avoid the usual suspects that cause interference and insecure transfer of data. The language should be seen as notation for execution sequences of atomic actions performed by concurrent communicating systems. So it does not include conditional or iterative constructs, assuming these are absorbed within atomic intradomain transactions. Thus, we do not have to concern ourselves with issues of implicit flows that arise due to branching structures (e.g., conditionals and loops in programming language level security, pipeline mispredictions at the architectural level, etc.)

The operational description of the language consists of the primitives together with their execution rules (§3.1). The correctness of our framework is demonstrated by expressing soundness (with respect to the operational semantics) of a type system (§3.2), stated in terms of the security lattices and their connecting functions. In particular, Theorem 6 shows the standard semantic property of non-interference [15] in both domains holds of all operational behaviours. We adapt and extend the approach taken by Volpano et al. [17] to encompass systems coupled using the Lagois connection conditions, and (assuming atomicity of the data transfer operations) show that security is conserved.

Since our language is a minimal imperative model with atomic operations, and security types are exactly the security classes, our proof pares down the techniques of Volpano et al. to the bare essentials.

We revisit in §4 several results of Melton et al. [16] that help us develop a methodical approach to finding and defining suitable MoUs for secure bidirectional flow, pictorially illustrating the systematic development on an example. In §4.1 we use the fact that the morphisms of a Lagois connection uniquely determine each other to complete a MoU when given one side of a proposed
mapping, by finding its Lagois adjoint. §1.2 discusses a methodical approach to negotiating a viable secure MoU ab initio. In §1.3 we use a compositionality result on Lagois connections to chain secure flows through a sequence of organisations. §4 tackles the issue of renegotiating and re-establishing a secure MoU when either party changes its security lattice. This is made possible by an decomposition result on Lagois connections discussed in §5.1. §5.2 details the various techniques for rebuilding a secure MoU when changes are made to the security lattice, using appropriate results from [16].

We then show that our Lagois connection framework readily accommodates decentralised flow control mechanisms within and across organisations, conservatively extending the model of Myers [20, 18]. After a brief summary of the DLM framework in §6.1 we show in Theorem 16 (§6.2) that a Lagois connection between the principals hierarchies of two domains induces a Lagois connection between their corresponding lattices of labels. A simple corollary, stated for static principals hierarchies that are connected by a Lagois connection, is that the declassification rule remains safe even with bidirectional information exchange between the domains.

In §7 we briefly review some related work. We conclude in §8 with a discussion on our approach and directions for future work.

Note: This paper substantially expands on our earlier work [15]. Contents of that paper included here appear in the preliminaries in §2, the technical results of §3 and the discussion on related work in §7.

2. Lagois Connections and All That

Motivating Example. Consider a university $U$ in which students study in semi-autonomously administered colleges (e.g., $C$) affiliated to the university. A college has students, faculty members, deans, all of whom work under a College Principal. The university has its own university professors, a dean of colleges, and a vice-chancellor working under a chancellor. Students can take classes with both college faculty members and university professors. Assume that each institution has established its secure information flow mechanisms and policies, and information flows from $C$ to $U$, as shown by the blue arrows in Figure 1.

Monotonicity (or order-preservation) of a function mapping security classes in $C$ to $U$ suffices for $U$ to respect $C$’s security policies. However, as we showed earlier [15], when flow is bidirectional, composing order-preserving functions between $C$ and $U$ is insufficient. Indeed, even a Galois connection between the two domains does not ensure security (see Figure 2). While Galois insertions ensure security, they require one of the functions to be surjective, whereas in many situations, an organisation may not wish to expose its entire security class lattice. Further, we do not wish information flowing between two domains to be reclassified in an overly restrictive manner that makes it inaccessible (we call these “precision” and “convergence” requirements).

Requirements. Accordingly, we identified the following requirements [15] for viable secure information flow between two lattices $(L, \sqsubseteq)$ and $(M, \sqsubseteq')$ with
Figure 1: Unidirectional flow: If the solid blue arrows denote identified flows connecting important classes, then the dashed green arrows are constrained by monotonicity to lie between them.

Figure 2: The arrows between the domains define a Galois Connection. However, the red dash-dotted arrows highlight flow security violations when information can flow in both directions.

order-preserving functions $\alpha : L \rightarrow M$ and $\gamma : M \rightarrow L$.

- **Security:**
  \[
  \text{SC1 } \lambda l. l \sqsubseteq \gamma \circ \alpha \\
  \text{SC2 } \lambda m'. m' \sqsubseteq' \alpha \circ \gamma
  \]

- **Precision:** Let $\alpha[L]$ and $\gamma[M]$ denote the images of set $L$ under mapping $\alpha$ and set $M$ under mapping $\gamma$.
  \[
  \text{PC1 } \alpha(l_1) = \bigsqcup \{m_1 \mid \gamma(m_1) = l_1\}, \quad \forall l_1 \in \gamma[M] \\
  \text{PC2 } \gamma(m_1) = \bigsqcup \{l_1 \mid \alpha(l_1) = m_1\}, \quad \forall m_1 \in \alpha[L]
  \]

- **Convergence:** (CC1 and CC2) Fixed points for the compositions $\gamma \circ \alpha$ and $\alpha \circ \gamma$ are reached as low in the orderings $\sqsubseteq$ and $\sqsubseteq'$ as possible.

Lagois Connections. We identified the elegant formulation of Lagois Connections\cite{16} as an appropriate structure that satisfies these requirements:

**Definition 1** (Lagois Connection\cite{16}). If $L = (L, \sqsubseteq)$ and $M = (M, \sqsubseteq')$ are two partially ordered sets, and $\alpha : L \rightarrow M$ and $\gamma : M \rightarrow L$ are order-preserving functions, then we call the quadruple $(L, \alpha, \gamma, M)$ an increasing Lagois connection, if it satisfies the following properties:

\[
\begin{align*}
\text{LC1 } & \lambda l. l \sqsubseteq \gamma \circ \alpha \\
\text{LC2 } & \lambda m'. m' \sqsubseteq' \alpha \circ \gamma \\
\text{LC3 } & \alpha \circ \gamma \circ \alpha = \alpha \\
\text{LC4 } & \gamma \circ \alpha \circ \gamma = \gamma
\end{align*}
\]
LC3 ensures that \( \gamma(\alpha(c_1)) \) is the least upper bound of all security classes in \( C \) that are mapped to the same security class, say \( u_1 = \alpha(c_1) \) in \( U \).

Observe that Lagois connections are transposable: if \((L, \alpha, \gamma, M)\) is a Lagois connection, then so is \((M, \gamma, \alpha, L)\). Lagois connections are fundamentally different from Galois connections in that they relate two linked closure operators in two posets, as opposed to a linking a closure and an interior operator.

We showed that Lagois connections satisfy the desired requirements:

**Theorem 1** (Theorem in [15]). Let \( L = (L, \subseteq, \cup, \cap) \) and \( M = (M, \subseteq', \cup', \cap') \) be two complete security class lattices, and let \( \alpha : L \rightarrow M \) and \( \gamma : M \rightarrow L \) be order-preserving functions. Then the flow of information permitted by \( \alpha, \gamma \) satisfies conditions SC1, SC2, PC1, PC2, CC1 and CC2 if \((L, \alpha, \gamma, M)\) is an increasing Lagois connection.

In the following discussion, let \((L, \alpha, \gamma, M)\) be a Lagois connection. Let \( \alpha[L] \) and \( \gamma[M] \) refer to the images of the order-preserving functions \( \alpha \) and \( \gamma \), respectively. The images \( \gamma[M] \) and \( \alpha[L] \) are in fact isomorphic lattices. For all \( m \in \alpha[L] \) and \( l \in \gamma[M] \), \( \gamma(m) \) and \( \alpha(l) \) exist.

**Proposition 2** (Proposition 3.7 in [10]). Let \( m \in \alpha[L] \) and \( l \in \gamma[M] \). Then \( \alpha^{-1}(m) \) has a largest member, which is \( \gamma(m) \), and \( \gamma^{-1}(l) \) has a largest member, which is \( \alpha(l) \).

We call these dominating members of the pre-images of \( \alpha \) and \( \gamma \) *budpoints*. Indeed:

\[
\gamma(\alpha(l)) = \cap\{l^* \in \gamma[M] \mid l \subseteq l^*\},
\]

\[
\alpha(\gamma(m)) = \cap\{m^* \in \alpha[L] \mid m \subseteq' m^*\}.
\]

Let \( \sim_M \) and \( \sim_L \) be the equivalence relations induced by the functions \( \gamma \) and \( \alpha \). \( L^* = \gamma[\alpha[L]] = \gamma[M] \) and \( M^* = \alpha[\gamma[M]] = \alpha[L] \) define a system of representatives for \( \sim_L \) and \( \sim_M \). Element \( m^* = \alpha(\gamma(m')) \) in \( M^* \), which is a *budpoint*, acts as the representative of the equivalence class \( [m'] \) in the following sense:

\[
\text{if } m \in M \text{ and } m^* \in M^* \text{ with } m \sim_M m^* \text{ then } m \subseteq' m^*
\]

Symmetrically, \( L^* = \gamma[\alpha[L]] = \gamma[M] \) defines a system of representatives for \( \sim_L \). These budpoints play a significant role in delineating the connection between the transfer classes in the two lattices. Further, Proposition 3 shows that these budpoints are closed under meets. This property enables us to confine our analysis to just these security classes when reasoning about bidirectional flows.

**Proposition 3** (Proposition 3.11 in [16]). If \( A \subseteq \gamma[M] \), then

1. the meet of \( A \) in \( \gamma[M] \) exists if and only if the meet of \( A \) in \( L \) exists, and whenever either exists, they are equal.

2. the join \( \hat{a} \) of \( A \) in \( \gamma[M] \) exists if the join \( \hat{a} \) of \( A \) in \( L \) exists, and in this case \( \hat{a} = \gamma(\alpha(\hat{a})) \).
2.1. Algorithmic Issues

We propose using a recently proposed succinct representation for lattices \cite{21}, in which order-testing comparisons can be answered in \( O(1) \) time, and the meet or join of two elements in \( O(n^{3/4}) \), where \( n \) is the number of elements in the lattice. The data structure occupies \( O(n^{3/2} \log n) \) bits of space, with pre-processing time \( O(n^2) \). The functions \( \alpha, \gamma \) and their inverses are represented using hash-based dictionaries, and equivalence classes induced by \( \alpha, \gamma \) are represented using the union-find data structure. Thus, the total space for a Lagois connection representation is \( O(n^{3/2}) \), where \( n \) is \( \max(n_1, n_2) \), with \( n_1 \) and \( n_2 \) being the number of elements in \( L \) and \( M \) respectively.

Order-comparisons within a domain are checked using the succinct data structure. Cross-domain flows, e.g., between \( x \in L \) and \( y \in M \), are permitted if and only if \( (\exists z \in M. \alpha(x) = z \text{ and } z \subseteq y) \). Since looking up \( \alpha(x) \) or \( \gamma(y) \) in a hashing-based dictionary can be done in \( O(1) \) time, all order-comparisons can be performed in \( O(1) \).

Testing \( \text{LC1 and LC3} \) takes \( O(n_1) \) time, and \( \text{LC2 and LC3} \) \( O(n_2) \) time. Thus checking whether \( (L, \alpha, \gamma, M) \) is a Lagois connection takes \( O(n) \) time, where \( n = \max(n_1, n_2) \).

If the transfer classes have been identified and these are far fewer than the number of lattice points, we can further optimise the pre-processing and the data structure (since the two sets of transfer classes are order-isomorphic to each other as we shall in Theorem \( \text{T3} \) of \( \S5.1 \)).

3. An Operational Model

We present an operational model consisting of a language and its operational semantics, and then a security type system, which we show to be sound in that well-typed programs exhibit non-interference \cite{19}. The objective of this section is to show that under reasonable assumptions of atomicity and isolation, the Lagois connection framework allows SIF analyses performed within systems to be lifted systematically to concurrent systems exchanging information.

3.1. Computational Model.

Assume two organisations \( L \) and \( M \) which have their own IFMs want to share data with each other. The two domains comprise storage objects \( Z \) and \( Z' \) respectively, which are manipulated using their own sets of atomic transactional operations, ranged over by \( t \) and \( t' \) respectively. We further assume that these transactions within each domain are internally secure with respect to their flow models, and have no insecure or interfering interactions with the environment. Thus, we are agnostic to the level of abstraction of the systems we aim to connect securely, and since our approach treats the application domains as “black boxes”, it is readily adaptable to any level of discourse (language, system, OS, database) found in the security literature.

We extend these operations with a minimal set of operations to transfer data between the two domains. To avoid any concurrency effects, interference
or race conditions arising from inter-domain transfer, we augment the storage objects of both domains with a fresh set of export and import variables into/from which the data of the domain objects can be copied atomically. We designate these sets \( X, X' \) as the respective export variables, and \( Y, Y' \) as the respective import variables, with the corresponding variable instances written as \( x_i, x'_i \) and \( y_i, y'_i \). These export and import variables form mutually disjoint sets, and are distinct from any extant domain objects manipulated by the applications within a domain. These variables are used exclusively for transfer, and are manipulated atomically. We let \( w_i \) range over all variables in \( N = Z \cup X \cup Y \) (respectively \( w'_i \) over \( N' = Z' \cup X' \cup Y' \)). Domain objects are copied to export variables and from import variables by special operations \( rd(z, y) \) and \( wr(x, z) \) (and \( rd'(z', y') \) and \( wr'(x', z') \) in the other domain). We assume atomic transfer operations (trusted by both domains) \( T_{RL}, T_{LR} \) that copy data from the export variables of one domain to the import variables of the other domain as the only mechanism for inter-domain flow of data. Let “phrase” \( p \) denote a command in either domain or a transfer operation, and let \( s \) be any (empty or non-empty) sequence of phrases. Note: This language should be understood as a notation for describing a distributed system’s execution sequences involving computation, communication, input and output, rather than as a programming language. Thus, we only need to consider sequences of atomic actions. Hence the absence of constructs such as conditionals, iteration or repetition. Further, the importance of atomicity of the computational steps and communication-related operations should be evident. We will later see that the “types” are exactly the security classes of the IFMs, and so there are no constructions such as cartesian product, records and function types.

\[
\text{A store (typically } \mu, \nu, \mu', \nu') \text{ is a finite-domain function from variables to a set of values (not further specified). We write, e.g., } \mu(w) \text{ for the contents of the store } \mu \text{ at variable } w, \text{ and } \mu[w := \mu'(w')] \text{ for the store that is the same as } \mu \text{ everywhere except at variable } w, \text{ where it now takes value } \mu'(w').
\]

The rules specifying execution of commands are given in Fig. 3. Assuming the specification of intradomain transactions \( (t, t') \) of the form \( \mu \vdash t \implies \nu \) and \( \mu' \vdash t' \implies \nu' \), our rules allow us to specify judgments of the form \( \langle \mu, \mu' \rangle \vdash p \implies \langle \nu, \nu' \rangle \) for phrases, and the reflexive-transitive closure for sequences of phrases. Note that phrase execution occurs atomically, and the intra-domain transactions, as well as copying to and from the export/import variables affect the store in only one domain, whereas the atomic transfer is only between export variables of one domain and the import variables of the other.

### 3.2. Typing Rules

Let the two domains have the respective different IFMs:

\[
FM_L = \langle N, P, SC, \sqcup, \sqsubseteq \rangle \quad FM_M = \langle N', P', SC', \sqcup', \sqsubseteq' \rangle,
\]
Figure 3: Execution Rules

such that the flow policies in both are defined over different sets of security classes \( SC \) and \( SC' \).

The (security) types of the core language are as follows.

Metavariables \( l \) and \( m' \) range over the sets of security classes, \( SC \) and \( SC' \) respectively, which are partially ordered by \( \sqsubseteq \) and \( \sqsubseteq' \). Note that, with the language being minimal, there are no other type constructions. A type assignment \( \lambda \) is a finite-domain function from variables \( N \) to \( SC \) (respectively, \( \lambda' \) from \( N' \) to \( SC' \)). The important restriction we place on \( \lambda \) and \( \lambda' \) is that they map export and import variables \( X, Y, X', Y' \) only to points in the security lattices \( SC \) and \( SC' \) respectively which are in the domains of \( \gamma \) and \( \alpha \), i.e., these points participate in the Lagois connection. Intuitively, a variable \( w \) mapped to security class \( l \) can store information of security class \( l \) or lower. The type system works with respect to a given type assignment. Given a security level, e.g., \( l \), the typing rules track for each command within that domain whether all written-to variables in that domain are of security classes “above” \( l \), and additionally for transactions within a domain, they ensure “simple security”, i.e., that all variables which may have been read belong to security classes “below” \( l \). We assume for the transactions within a domain, e.g., \( L \), we already have a security type system that will give us judgments of the form \( \lambda \vdash t : l \). The novelty of our approach is to extend this framework to work over two connected domains, i.e., given implicit security levels of the contexts in the respective domains. Cross-

\[ \text{SEQS} \quad \frac{\langle \mu, \mu' \rangle \vdash s_1 \Rightarrow^* \langle \mu_1, \mu_1' \rangle, \quad \langle \mu_2, \mu_2' \rangle \vdash p \Rightarrow \langle \mu_2, \mu_2' \rangle}{\langle \mu, \mu' \rangle \vdash s_1; p \Rightarrow^* \langle \mu_2, \mu_2' \rangle} \]

\[ \text{SEQ} \quad \frac{\langle \mu, \mu' \rangle \vdash \epsilon \Rightarrow^* \langle \mu, \mu' \rangle} {\langle \mu, \mu' \rangle \vdash \epsilon} \]

\[ \text{TRL} \quad \frac{\langle \mu, \mu' \rangle \vdash T_{RL}(y', x) \Rightarrow \langle \mu[y := \mu'(x')], \mu' \rangle} {\langle \mu, \mu' \rangle \vdash T_{RL}(y', x)} \]

\[ \text{WR} \quad \frac{\langle \mu, \mu' \rangle \vdash wr(x, z) \Rightarrow \langle \mu[x := \mu(z)], \mu' \rangle} {\langle \mu, \mu' \rangle \vdash wr} \]

\[ \text{WR'} \quad \frac{\langle \mu, \mu' \rangle \vdash wr'(x', z') \Rightarrow \langle \mu[x' := \mu'(z')], \mu' \rangle} {\langle \mu, \mu' \rangle \vdash wr'} \]

\[ \text{RD} \quad \frac{\langle \mu, \mu' \rangle \vdash rd(z, y) \Rightarrow \langle \mu[z := \mu(y)], \mu' \rangle} {\langle \mu, \mu' \rangle \vdash rd} \]

\[ \text{RD'} \quad \frac{\langle \mu, \mu' \rangle \vdash rd'(z', y') \Rightarrow \langle \mu[z' := \mu'(y')], \mu' \rangle} {\langle \mu, \mu' \rangle \vdash rd'} \]

\[ \text{T} \quad \frac{\mu \vdash t \Rightarrow \nu} {\langle \mu, \mu' \rangle \vdash \langle \nu, \mu' \rangle} \]

\[ \text{T'} \quad \frac{\mu' \vdash t' \Rightarrow \nu'} {\langle \mu, \mu' \rangle \vdash \langle \mu, \nu' \rangle} \]

\[ \text{SEQ} \quad \frac{\langle \mu, \mu' \rangle \vdash \epsilon \Rightarrow^* \langle \mu, \mu' \rangle} {\langle \mu, \mu' \rangle \vdash \epsilon} \]

\[ \text{SEQS} \quad \frac{\langle \mu, \mu' \rangle \vdash s_1 \Rightarrow^* \langle \mu_1, \mu_1' \rangle, \quad \langle \mu_2, \mu_2' \rangle \vdash p \Rightarrow \langle \mu_2, \mu_2' \rangle}{\langle \mu, \mu' \rangle \vdash s_1; p \Rightarrow^* \langle \mu_2, \mu_2' \rangle} \]

\[ \text{SEQ} \quad \frac{\langle \mu, \mu' \rangle \vdash \epsilon \Rightarrow^* \langle \mu, \mu' \rangle} {\langle \mu, \mu' \rangle \vdash \epsilon} \]
domain transfers will require pairing such judgments, and thus our type system will have judgments of the form

\[ \langle \lambda, \lambda' \rangle \vdash p : \langle l, m' \rangle \]

We introduce a set of typing rules for the core language, given in Fig. 4. In many of the rules, the type for one of the domains is not constrained by the rule, and so any suitable type may be chosen as determined by the context, e.g., \( m' \) in the rules \( TT, TRD, TWR \) and \( TTRL \), and both \( l \) and \( m' \) in \( \text{Com0} \).

\[
\begin{align*}
\text{Tt} & \quad \langle \lambda, \lambda' \rangle \vdash t : \langle l, m' \rangle \quad \text{if for all } z \text{ assigned in } t, \ l \subseteq \lambda(z) \\
& \quad \quad \quad \quad \quad \quad \text{and for all } z_1 \text{ read in } t, \ \lambda(z_1) \subseteq l \\
\text{Tt}^+ & \quad \langle \lambda, \lambda' \rangle \vdash t : \langle l, m' \rangle \quad \text{if for all } \ z' \text{ assigned in } t', \ m' \subseteq l' \quad \lambda(z') \\
& \quad \quad \quad \quad \quad \quad \text{and for all } z'_1 \text{ read in } t', \ \lambda(z'_1) \subseteq m' \\
\text{TRD} & \quad \lambda(y) \subseteq \lambda(z) \\
\langle \lambda, \lambda' \rangle \vdash rd(z, y) : \langle \lambda(z), m' \rangle \\
\text{TRD}^+ & \quad \lambda(y') \subseteq \lambda(z') \\
\langle \lambda, \lambda' \rangle \vdash rd(z', y') : \langle l, \lambda(z') \rangle \\
\text{TWR} & \quad \lambda(z) \subseteq \lambda(x) \\
\langle \lambda, \lambda' \rangle \vdash wr(x, z) : \langle \lambda(x), m' \rangle \\
\text{TWR}^+ & \quad \lambda(z') \subseteq l' \lambda(x') \\
\langle \lambda, \lambda' \rangle \vdash wr'(x', z') : \langle l, \lambda(x') \rangle \\
\text{TT}_{RL} & \quad \gamma(X(x')) \subseteq \lambda(y) \\
\langle \lambda, \lambda' \rangle \vdash T_{RL}(y', x) : \langle \lambda(y), \lambda(x') \rangle \\
\text{TT}_{RL}^+ & \quad \alpha(\lambda(x)) \subseteq l' \lambda(y') \\
\langle \lambda, \lambda' \rangle \vdash T_{LR}(y', x) : \langle \lambda(y), \lambda(x') \rangle \\
\text{Com0} & \quad \langle \lambda, \lambda' \rangle \vdash e : \langle l, m' \rangle \\
\langle \lambda, \lambda' \rangle \vdash s : \langle l, m' \rangle \\
\langle \lambda, \lambda' \rangle \vdash p : \langle l_1, m'_1 \rangle \\
\langle \lambda, \lambda' \rangle \vdash s ; p : \langle l_1 \cap l, m'_1 \cap m' \rangle
\end{align*}
\]

Figure 4: Typing Rules

For transactions e.g., \( t \) entirely within domain \( L \), the typing rule \( \text{Tt} \) constrains the type in the left domain to be at a level \( l \) that dominates all variables read in \( t \), and which is dominated by all variables written to in \( t \), but places no constraints on the type \( m' \) in the other domain \( M \). In the rule \( \text{TRD} \), since a value in import variable \( y \) is copied to the variable \( z \), we have \( \lambda(y) \subseteq \lambda(z) \), and the type in the domain \( L \) is \( \lambda(z) \) with no constraint on the type \( m' \) in the other domain. Conversely, in the rule \( \text{TWR} \), since a value in variable \( z \) is copied...
to the export variable \(x\), we have \(\lambda(z) \subseteq \lambda(x)\), and the type in the domain \(L\) is \(\lambda(pzq)\). In the rule \(TT_{RL}\), since the contents of a variable \(x'\) in domain \(M\) are copied into a variable \(y\) in domain \(L\), we require \(\gamma(\lambda'(x')) \subseteq \lambda(y)\), and constrain the type in domain \(L\) to \(\lambda(y)\). The constraint in the other domain is unimportant (but for the sake of convenience, we peg it at \(\lambda'(x')\)). Finally, for the types of sequences of phrases, we take the meets of the collected types in each domain respectively, so that we can guarantee that no variable of type lower than these meets has been written into during the sequence. Note that Proposition 3 ensures that these types have the desired properties for participating in the Lagois connection.

3.3. Soundness

We now establish soundness of our scheme by showing a non-interference theorem with respect to the operational semantics and the type system built on the security lattices. This theorem may be viewed as a conservative adaptation (to a minimal secure data transfer framework in a Lagois-connected pair of domains) of the main result of Volpano et al. [17].

We assume that underlying base transactional languages in each of the domains have the following simple property (stated for \(L\), but an analogous property is assumed for \(M\)). Within each transaction \(t\), for each assignment of an expression \(e\) to any variable \(z\), the following holds: If \(\mu, \nu\) are two stores such that for all \(w \in \text{vars}(e)\), we have \(\mu(w) = \nu(w)\), then after executing the assignment, we will get \(\mu(z) = \nu(z)\). That is, if two stores are equal for all variables appearing in the expression \(e\), then the value assigned to the variable \(z\) will be the same. This assumption plays the role of “Simple Security” of expressions in [17] in the proof of the main theorem. The type system plays the role of “Confinement”. We start with two obvious lemmas about the operational semantics, namely preservation of domains, and a “frame” lemma:

**Lemma 4** (Domain preservation). If \(\langle\mu, \mu'\rangle \vdash s \Rightarrow^* \langle\mu_1, \mu'_1\rangle\), then \(\text{dom}(\mu) = \text{dom}(\mu_1)\), and \(\text{dom}(\mu') = \text{dom}(\mu'_1)\).

*Proof.* By induction on the length of the derivation of \(\langle\mu, \mu'\rangle \vdash s \Rightarrow^* \langle\mu_1, \mu'_1\rangle\).

**Lemma 5** (Frame). If \(\langle\mu, \mu'\rangle \vdash s \Rightarrow^* \langle\mu_1, \mu'_1\rangle\), \(w \in \text{dom}(\mu) \cup \text{dom}(\mu')\), and \(w\) is not assigned to in \(s\), then \(\mu(w) = \mu_1(w)\) and \(\mu'(w) = \mu'_1(w)\).

*Proof.* By induction on the length of the derivation of \(\langle\mu, \mu'\rangle \vdash s \Rightarrow^* \langle\mu_1, \mu'_1\rangle\).

The main result assumes an “adversary” that operates at a security level \(l\) in domain \(L\) and at security level \(m'\) in domain \(M\). Note however, that these two levels are interconnected by the monotone functions \(\alpha : L \rightarrow M\) and \(\gamma : M \rightarrow L\), since these levels are connected by the ability of information at one level in one domain to flow to the other level in the other domain. The following theorem says that if (a) a sequence of phrases is well-typed, and (b,c) we start its execution in two store configurations that are (e) indistinguishable with respect to...
all objects having security class below \( l \) and \( m' \) in the respective domains, then the corresponding resulting stores after execution continue to remain indistinguishable on all variables with security classes below these adversarial levels.

**Theorem 6 (Type Soundness).** Suppose \( l, m' \) are the “adversarial” type levels in the respective domains, which satisfy the condition \( l = \gamma(m') \) and \( m' = \alpha(l) \). Let

\[
(a) \quad \langle \lambda, \lambda' \rangle \vdash s : \langle l_0, m'_0 \rangle; \quad (s \text{ has security type } \langle l_0, m'_0 \rangle)
\]

\[
(b) \quad \langle \mu, \mu' \rangle \vdash s \Rightarrow \langle \mu_f, \mu'_f \rangle; \quad (\text{execution of } s \text{ starting from } \langle \mu, \mu' \rangle)
\]

\[
(c) \quad \langle \nu, \nu' \rangle \vdash s \Rightarrow \langle \nu_f, \nu'_f \rangle; \quad (\text{execution of } s \text{ starting from } \langle \nu, \nu' \rangle)
\]

\[
(d) \quad \text{dom}(\mu) = \text{dom}(\nu) = \text{dom}(\lambda) \text{ and } \text{dom}(\mu') = \text{dom}(\nu') = \text{dom}(\lambda');
\]

\[
(e) \quad \mu(w) = \nu(w) \text{ for all } w \text{ such that } \lambda(w) \sqsubseteq l, \text{ and } \mu'(w') = \nu'(w') \text{ for all } w' \text{ such that } \lambda'(w') \sqsubseteq m'.
\]

Then \( \mu_f(w) = \nu_f(w) \) for all \( w \) such that \( \lambda(w) \sqsubseteq l \), and \( \mu'_f(w') = \nu'_f(w') \) for all \( w' \) such that \( \lambda'(w') \sqsubseteq m' \).

**Proof.** By induction on the length of sequence \( s \). The base case is vacuously true. We now consider a sequence \( s_1; p. \quad \langle \mu, \mu' \rangle \vdash s_1 \Rightarrow \langle \mu_1, \mu'_1 \rangle \) and \( \langle \mu_1, \mu'_1 \rangle \vdash p \Rightarrow \langle \mu_f, \mu'_f \rangle \) and \( \langle \nu, \nu' \rangle \vdash s_1 \Rightarrow \langle \nu_1, \nu'_1 \rangle \) and \( \langle \nu_1, \nu'_1 \rangle \vdash p \Rightarrow \langle \nu_f, \nu'_f \rangle \). By induction hypothesis applied to \( s_1 \), we have \( \mu_1(w) = \nu_1(w) \) for all \( w \) such that \( \lambda(w) \sqsubseteq l \), and \( \mu'_1(w') = \nu'_1(w') \) for all \( w' \) such that \( \lambda'(w') \sqsubseteq m' \).

Let \( \langle \lambda, \lambda' \rangle \vdash s_1 : \langle l_1, m'_1 \rangle \), and \( \langle \lambda, \lambda' \rangle \vdash p : \langle l_p, m'_p \rangle \). We examine four cases for \( p \) (the remaining cases are symmetrical).

**Case p is t:** Consider any \( w \) such that \( \lambda(w) \sqsubseteq l \). If \( w \in X \cup Y \) (i.e., it doesn’t appear in \( t \)), or if \( w \in Z \) but is not assigned to in \( t \), then by Lemma \ref{lem:security} and the induction hypothesis, \( \mu_f(w) = \nu_1(w) = \nu_f(w) \).

Now suppose \( z \) is assigned to in \( t \). From the condition \( \langle \lambda, \lambda' \rangle \vdash p : \langle l_p, m'_p \rangle \), we know that for all \( z_1 \) assigned in \( t \), \( l_p \sqsubseteq \lambda(z_1) \) and for all \( z_1 \) read in \( t \), \( \lambda(z_1) \sqsubseteq l_p \).

Now if \( l \sqsubseteq l_p \), then since in \( t \) no variables \( z_2 \) such that \( \lambda(z_2) \sqsubseteq l \) are assigned to. Therefore by Lemma \ref{lem:security}, \( \mu_f(w) = \mu_1(w) = \nu_1(w) = \nu_f(w) \), for all \( w \) such that \( \lambda(w) \sqsubseteq l \).

If \( l_p \sqsubseteq l \), then for all \( z_1 \) read in \( t \), \( \lambda(z_1) \sqsubseteq l_p \). Therefore, by assumption on transaction \( t \), if any variable \( z \) is assigned an expression \( e \), since \( \mu_1, \nu_1 \) are two stores such that for all \( z_1 \in Z_e = \text{vars}(e) \), \( \mu_1(z_1) = \nu_1(z_1) \), the value of \( e \) will be the same. By this simple security argument, after the transaction \( t \), we have \( \mu_f(z) = \nu_f(z) \). Since the transaction happened entirely and atomically in domain \( M \), we do not have to worry ourselves with changes in the other domain \( M \), and do not need to concern ourselves with the adversarial level \( m' \).

**Case p is rd(z, y):** Thus \( \langle \lambda, \lambda' \rangle \vdash \text{rd(z, y)} : \langle \lambda(z), m' \rangle \), which means \( \lambda(y) \sqsubseteq \lambda(z) \).

If \( l \sqsubseteq \lambda(z) \), there is nothing to prove (Lemma \ref{lem:security} again). If \( \lambda(z) \sqsubseteq l \), then since by I.H., \( \mu_1(y) = \nu_1(y) \), we have \( \mu_f(z) = \mu_1(z) = \mu_1(z := \mu_1(y))[z] = \nu_1(z := \nu_1(y))[z] = \nu_f(z) \).
**Case** \(p\) is \(\text{wr}(x, z)\): Thus \(\langle \lambda, \lambda' \rangle \vdash \text{wr}(x, z) : \langle \lambda(x), m' \rangle\), which means \(\lambda(z) \sqsubseteq \lambda(x)\). If \(l \sqsubseteq \lambda(x)\), there is nothing to prove (Lemma\[\text{5} \text{ again}) If \(\lambda(x) \sqsubseteq l\), then since by I.H., \(\mu_1(z) = \nu_1(z)\), we have \(\mu_f(x) = \mu_1[x := \mu_1(z)](x) = \nu_1[x := \nu_1(z)](x) = \nu_f(x)\).

**Case** \(p\) is \(T_{RL}(y, x')\): So \(\langle \lambda, \lambda' \rangle \vdash T_{RL}(y, x') : \langle \lambda(y), \lambda'(x') \rangle\), and \(\gamma(\lambda'(x')) \sqsubseteq \lambda(y)\). If \(l \sqsubseteq \lambda(y)\), there is nothing to prove (Lemma\[\text{5} \text{ again). If \(\lambda(y) \sqsubseteq l\), then by transitivity, \(\gamma(\lambda'(x')) \sqsubseteq l\). By monotonicity of \(\alpha\): \(\alpha(\gamma(\lambda'(x'))) \sqsubseteq' \alpha(l) = m'\) (By our assumption on \(l\) and \(m'\)). But by \(LC2\), \(\lambda'(x') \sqsubseteq' \alpha(\gamma(\lambda'(x')))\). So by transitivity, \(\lambda'(x') \sqsubseteq' m'\). Now, by I.H., since \(\mu_1(x') = \nu_1(x')\), we have \(\mu_f(y) = \mu_1[y := \nu_1(x')] (y) = \nu_1[y := \nu_1(x')] (y) = \nu_f(y)\).

\[\square\]

4. Finding Lagois Connections

Lagois connections go well beyond providing a simple and elegant framework for secure connections between independent security lattices. They exhibit several properties that support the creation of secure connections, negotiating secure MoUs, and maintaining secure connections when organisational changes occur in the two connected lattices.

Let \(\alpha[L]\) and \(\gamma[M]\) refer to the images of the order-preserving functions \(\alpha\) and \(\gamma\), respectively. Further, let us define the upward closures \(\uparrow m = \{z \in M \mid m \sqsubseteq z\}\) and \(\uparrow l = \{z \in L \mid l \sqsubseteq z\}\).

The two monotone functions \(\alpha : L \to M\) and \(\gamma : M \to L\) in a Lagois connection \((L, \alpha, \gamma, M)\) uniquely determine each other.

**Proposition 7** (Proposition 3.9 in \[16\]). If \((L, \alpha, \gamma, M)\) is a Lagois connection, then the functions \(\alpha\) and \(\gamma\) uniquely determine each other, in fact:

\[
\gamma(m) = \bigsqcup \alpha^{-1}\bigsqcap \{m^* \in \alpha[L] \mid m \sqsubseteq m^*\} \tag{4}
\]

\[
= \bigsqcup \alpha^{-1}\bigsqcap (\uparrow m \cap \alpha[L]) \tag{5}
\]

and

\[
\alpha(l) = \bigsqcup \gamma^{-1}\bigsqcap \{l^* \in \gamma[M] \mid l \sqsubseteq l^*\} \tag{6}
\]

\[
= \bigsqcup \gamma^{-1}\bigsqcap (\uparrow l \cap \gamma[M]) \tag{7}
\]

4.1. Negotiating an MoU when given one order-preserving map

Suppose we are given two security lattices \(L\) and \(M\) and an order-preserving function \(\alpha : L \to M\), we can find a Lagois adjoint \(\gamma : M \to L\), i.e., an order-preserving function such that \((L, \alpha, \gamma, M)\) is a Lagois connection, and thus secure information flow between \(L\) and \(M\) is ensured.

**Proposition 8** (Proposition 3.10 in \[16\]). Let \(L\) and \(M\) be posets. Then an order-preserving function \(\alpha : L \to M\) has a Lagois adjoint \(\gamma : M \to L\) iff:

1. \(\alpha^{-1}(m)\) has a largest member, for all \(m \in \alpha[L]\).
2. \( \uparrow m \cap \alpha[L] \) has a smallest member, for all \( m \in M \).

3. The restriction of \( \alpha \) from \( \bigcup \{ \alpha^{-1}(m) | m \in \alpha[L] \} \) to its image \( \alpha[L] \) is an order isomorphism.

Consider the example of the order-preserving function \( \alpha \) in Figure 1. First we check whether the given function \( \alpha \) has a Lagois adjoint or not, using Proposition 8. The conditions on \( \alpha \) are obviously satisfied in our example. We then use the third condition of Proposition 8 to identify the order-isomorphic substructures of the participating security lattices. The security classes which form such an order-isomorphic structure for our example are shown in blue/red in Figure 5.

We allow the information to flow back to domain \( L \) by mapping first only the budpoints in \( M \) to the budpoints in \( L \), while respecting the conditions for Lagois connections, i.e., LC3 and LC4. These mappings are shown as dotted brown arrows in Figure 5. Then we invoke Proposition 7 to complete the mappings of function \( \gamma \). The unmapped security classes in domain \( M \), say \( m_1 \), are connected to the greatest of those elements of \( L \) which are mapped by \( \alpha \) to the least budpoint in \( M \) greater than \( m_1 \), as shown by solid brown arrows in Figure 6.

4.2. Negotiating an MoU ab initio

In the absence of any constraints on \( \alpha \) and \( \gamma \), it is always possible to define a Lagois connection between two finite security class lattices, e.g., by mapping all elements to the topmost security class of the other lattice. But such a Lagois connection is of little use to the participating organisations as the shared information, being in the topmost security class, is inaccessible to most principals.
Theorem 9 (Theorem 3.20 in [16]). Let \((L, \subseteq)\) and \((M, \subseteq')\) be posets. There is a Lagois connection between \((L, \subseteq)\) and \((M, \subseteq')\) if and only if the following four conditions hold:

1. There exist order-isomorphic subsets \(L^* \subseteq L\) and \(M^* \subseteq M\).
2. There exists equivalence relations \(\sim_L\) on \(L\) and \(\sim'_M\) on \(M\) such that \(L^*\) is a system of representatives for \(\sim_L\) and \(M^*\) is a system of representatives for \(\sim'_M\) respectively.
3. if \(l_1 \subseteq l_2\) in \(L\) and \(l^*_1, l^*_2 \in L^*\) with \(l_1 \sim_L l^*_1\), then \(l_1 \subseteq l^*_2\); and if \(m_1 \in M\) and \(m^*_1 \in M^*\) with \(m_1 \sim'_M m^*_1\), then \(m_1 \subseteq m^*_2\).
4. If \(l_1 \subseteq l_2\) in \(L\) and \(l^*_1, l^*_2 \in L^*\) with \(l_1 \sim_L l^*_1\) and \(l_2 \sim'_L l^*_2\), then \(l_1 \subseteq l^*_2\); and if \(m_1 \subseteq m_2\) and \(m^*_1, m^*_2 \in M^*\) with \(m_1 \sim'_M m^*_1\) and \(m_2 \sim'_M m^*_2\), then \(m_1 \subseteq m^*_2\).

Corollary 10 (Corollary 3.21 in [16]). Let \(L\) and \(M\) be posets, and \(c : L \rightarrow L\) and \(i : M \rightarrow M\) be closure operators such that \(c[L]\) and \(i[M]\) are isomorphic (with their inherited orders). If \(h : c[L] \rightarrow i[M]\) is such an isomorphism, then \((L, c, h^{-1}, i, M)\) is a Lagois connection.

Theorem 9 suggests the following method, which we illustrate using an example in Figures 7-11 where an agreement is negotiated between Dorm-Life and College.

1. Find the maximal order-isomorphic substructures \(L^* \subseteq L\) and \(M^* \subseteq M\) (which include the transfer classes [Student-Student, HouseMaster-Dean(S)], the bottom-most and the topmost class) in the given security class lattices \((L, \subseteq)\) and \((M, \subseteq')\). These classes in the order-isomorphic structures of the two domains are indicated in bold in Figure 7.

2. Identify an equivalence relation \(\sim_L\) such that \(L^*\) is a system of representatives for \(\sim_L\). Similarly, identify an equivalence relation \(\sim'_M\) such that \(M^*\) is a system of representatives for \(\sim'_M\). The equivalence relations should be such that the two conditions (3) and (4) of Theorem 9 hold, which essentially ensure that one is allowed to reason about information flows from all the security classes in the given security class lattices.

The members of \(L^*\) (resp. \(M^*\)) are called budpoints, play a significant role in delineating the connection between the transfer classes in the two lattices. We show the equivalence classes for our example in Figure 8 where the

\[\text{We use term “viable” informally to mean that the natural constraints of the application are taken into account, and that the security level of information is escalated only to the extent required, thus not making its access overly restricted.}\]

\[\text{The members of } L^* \text{ and } M^* \text{ are called budpoints and the equivalence classes are called blossoms.}\]
budpoints are the security classes coloured blue and red in the respective domains.

3. Now interconnect the budpoints of both domains to each other while respecting the inherited order, forming an isomorphic structure, as done in Figure 9 for our example.

4. Now keeping in mind Corollary 10 we define closure operators $c : L \rightarrow L$ and $i' : M \rightarrow M$, using the equivalence relations $\sim_L$ and $\sim_M$, such that $c[L]$ and $i'[M]$ are isomorphic (with their inherited orders). Figure 11 shows the closure operators for our example.

5. Thereafter, define an increasing Lagois connection using Corollary 10 as $(L, h_c, h^{-1}_i, M)$. For our example, the Lagois connection is defined as shown in Figure 10 with green arrows completing the mapping from Dorm-Life and brown arrows completing the mapping from College.

![Figure 7: Security lattices for two autonomous organisations that want to negotiate a secure MoU ab initio.](image)

![Figure 8: Identifying equivalence relations in given security lattices for discovering a Lagois connection ab initio.](image)

4.3. MoUs involving several administrative domains

Suppose there is a sequence of administrative domains such that each adjacent pair of domains has negotiated a secure Lagois connection to ensure bidirectional SIF between them. Theorem 11 and Corollary 12 allow the composition of these Lagois connections using simple functional composition.
Theorem 11 (Theorem 3.22 in [16]). If $(L_1, \alpha_1, \gamma_1, M_1)$ and $(M_1, \alpha_2, \gamma_2, Q)$ are increasing Lagois connections, then the flow defined by the increasing Lagois connection $(L_1, \alpha_2 \circ \alpha_1, \gamma_1 \circ \gamma_2, Q)$ is secure iff

$$\gamma_2 \circ \alpha_2 \circ \alpha_1 [L_1] \subseteq \alpha_1 [L_1]$$

and

$$\alpha_1 \circ \gamma_1 \circ \gamma_2 [Q] \subseteq \gamma_2 [Q]$$
Corollary 12 (Corollary 3.23 in [16]). If $(L_1, \alpha_1, \gamma_1, M_1)$ and $(M_1, \alpha_2, \gamma_2, Q)$ are Lagois connections and if either $\gamma_2[Q] \subseteq \alpha_1(L_1)$ or $\gamma_2[Q] \supseteq \alpha_1(L_1)$, then $(L_1, \alpha_2 \circ \alpha_1, \gamma_1 \circ \gamma_2, Q)$ is a Lagois connection.

We illustrate how an MoU negotiated between Dorm-Life (which has security classes HouseMaster, DiningManager, Caretaker, Assistant, and Student) and College can be composed with the MoU which has been negotiated between the College and University in Figure 12 to come up with an MoU which allows secure bidirectional information flow between all three domains.

![Diagram of Lagois connections](image)

Figure 12: Composing Lagois connections. Here an MoU negotiated between Dorm-Life and College is composed with another MoU which has been negotiated between College and University.

5. Maintaining MoUs When Security Lattices Change

5.1. Analysing a Lagois Connection

Before we discuss how to update an existing MoU, we present Theorem 13 which provides us a decomposed, analytical view of the anatomy of an increasing Lagois connection. For example, the Lagois connection given in Figure 6 can be alternatively viewed as shown in Figure 13.

Let us assume that $(L, \alpha, \gamma, M)$ is a Lagois connection. Then, let $L^* = \gamma[M]$ and $M^* = \alpha[L]$. Also, let $r_1 = (\gamma \circ \alpha)|_{L^*}$, $r_2 = (\alpha \circ \gamma)|_{M^*}$, $i_1 = \alpha|_{L^*}$, $i_2 = \gamma|_{M^*}$ and let $e_1$ be the embedding (inclusion) from $L^*$ to $L$ and $e_2$ be the embedding (inclusion) from $M^*$ to $M$. Then, $(L^*, i_1, i_2, M^*)$ is a Lagois isomorphism. This isomorphic substructure is very helpful in reducing the computational effort
involved in maintaining the negotiated MoUs based on Lagois connections, as
detailed in the sequel.

The key insight is that as long as the inter-domain mappings, i.e., \( i_1 \) and
\( i_2 \) are not changed, and new security lattices can be connected to old lattices
with an increasing Lagois insertion\(^4\), we will not need to re-negotiate the inter-
domain mappings. For example, Figure 14 shows two such Lagois insertions
between old and new security lattices.

**Theorem 13** (Theorem 3.24 in [16]). *Every increasing Lagois connection \((L, \alpha, \gamma, M)\)*
is a composite \( (L, r_1, e_1, L^*) \odot (L^*, i_1, i_2, M^*) \odot (M^*, e_2, r_2, M) \) where

1. \( (L, r_1, e_1, L^*) \) is an increasing Lagois insertion,
2. \( i_1 \) and \( i_2 \) are isomorphisms that are inverses to each other, and
3. \( (M^*, e_2, r_2, M) \) is an increasing Lagois insertion.

---

A Lagois connection is an insertion if one of the two mappings is injective.

\(^5\)For easy left-to-right readability, we have used a composition operator \( \odot \) and reversed the
order of the components from how they appear in Theorem 3.24 in [16].
3. Adding new edges to the existing security lattices;
4. Removing edges from existing security lattices.

We examine how to re-establish a secure Lagois connection when changes in the lattice structures occur. The key observation is that we only need to monitor if the original isomorphic substructure mediating the old Lagois connection between the participating security lattices changes or not.

If the participating isomorphic substructure remains unchanged, and one can find an increasing Lagois insertion between the changed security lattice and the original isomorphic substructure then the MoU need not be re-negotiated. The necessary updates of the monotone function can be done independently of the other organisation.

1. **No change in the isomorphic sub-structure:**
   
   (a) The number of equivalence classes remains the same. This is a simple case. Use Theorem 13 to first find a Lagois insertion between the new security lattice and the isomorphic substructure, and then update the functions $r$, mentioned in Theorem 13, which can be done independently of the other parts of the diagram. We illustrate this with an example where both security lattices are updated by adding Teaching Assistants (TAs) and student Mentors in College and department heads (HOD) in University, as shown in Figure 14. Using Theorem 13, we get the updated Lagois connection shown in Figure 15 — adding 2 new green arrows on the left, and a new brown arrow on the right.

2. **Change in the isomorphic sub-structure:**
   
   (a) **Increase in number of equivalence classes:** Check the four conditions of Lagois connections (LC1, LC2, LC3 and LC4) for the affected equivalence classes in the security lattices. This is similar to re-negotiating a Lagois connection for those parts of security lattices, as discussed earlier. Use Theorem 9 for finding a new Lagois connection.
   
   (b) **Decrease in number of equivalence classes:** Use Corollary 15 to check if the new mappings (of which the initial mappings are a refinement) form a Lagois connection (discussed in more detail below).

5.2.1. **Coarsening of inter-domain mappings.**

When the isomorphic structure is changed, the MoU has to be renegotiated. When deleting a security class, we can always “up-classify” information pertaining to that class, to cause a stricter flow of information to a higher security class in the other lattice. When coarsening a lattice, we would like to re-establish a Lagois connection that respects the previous Lagois connection as much as possible (maintaining legacy) while making the necessary up-classifications where necessary.
Figure 14: Organisations can add security classes to their lattice structures autonomously as long as they are able to connect the new lattice structures with the old lattice structures (participating in the MoU) via a Lagois insertion. Dashed black arrows define permissible flows between budpoints.

Let \((L, \alpha, \gamma, M)\) be an existing secure Lagois connection. Suppose we need to replace \(\alpha\) by another order-preserving function \(\alpha' : L \rightarrow M\). We look for a suitable map \(\gamma' : M \rightarrow L\) such that \((L, \alpha', \gamma', M)\) is a Lagois connection using
the following result:

**Proposition 14** (Proposition 2.4 in [22]). If \( \alpha_1 : L \to M \) is an order-preserving map with semi-inverse \( \gamma_1 : M \to L \), i.e., \( \alpha_1 \circ \gamma_1 \circ \alpha_1 = \alpha_1 \), then \((L, \alpha_1, \gamma_1 \circ \alpha_1 \circ \gamma_1, M)\) is a (Lagois) connection.

As mentioned above, when the isomorphic structure is disturbed due to coarsening a lattice, we would like to retain as much of original functions as possible. We can re-establish a Lagois connection making minimal changes to the old functions \( \alpha \) and \( \gamma \) Lagois by considering an (order-preserving) \( \alpha' \) which satisfies the assumptions:

1. \( \ker(\alpha) \subseteq \ker(\alpha') \)
2. for each \( l \in L \), \( \alpha'(l) = \alpha(l^*) \) when \( l^* \) is the largest element in \( \{ l_1 \in L | \alpha'(l_1) = \alpha'(l) \} \).

Typically we would consider as a candidate an \( \alpha' \) that induces as fine a coarsening of the kernel of \( \alpha \) as possible while ensuring the second condition (i.e., mapping elements of equivalence classes induced by \( \alpha' \) (which should be closed with respect to joins) according to how \( \alpha \) mapped the largest element in that class).

**Corollary 15** (Corollary 3.16 in [16]). Let \((L, \alpha, \gamma, M)\) be a Lagois connection, and let \( \alpha' : L \to M \) be an order-preserving map such that

1. \( \equiv_{\alpha} \) is a refinement of \( \equiv_{\alpha'} \) (i.e., \( \ker(\alpha) \subseteq \ker(\alpha') \)) and
2. for each \( l \in L \), the equivalence class \([l]_{\alpha'} \) has a largest element – call it \( l^* \) – with \( \alpha'(l) = \alpha(l^*) \).

Then \((L, \alpha', \gamma \circ \alpha' \circ \gamma, M)\) is a Lagois connection.

### 6. Securely Connecting Decentralised Label Models

In this section, we show how Lagois Connections can be used to ensure secure information flow between two organisations, both of which have employed the decentralized label model (DLM) of Myers [20] for SIF. This illustrates how the Lagois framework can extend autonomy in IFC within an organisation to secure cross-organisational decentralised control. We show that if a Lagois Connection \( LC \) has been established between the Principals hierarchies of both organisations, we can establish a Lagois connection \( \overline{LC} \) between the security lattices formed by the labels derived from the respective Principals hierarchies. The second result we show is that the declassification rule [20] does not introduce any insecure flows, even when exchanging information between domains.

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\(^6\)\( \ker(\alpha) \) is the equivalence relation on \( L \) given by \((a, b) \in \ker(\alpha) \) iff \( \alpha(a) = \alpha(b) \). For \( b \in L \), we denote the equivalence class containing \( b \) by \([b]_{\alpha} \), as the equivalence relation on \( L \) is defined by \( \alpha \).

\(^7\)We prefer the term “principals hierarchy” to “principal hierarchy” for grammatical reasons.
6.1. The Decentralised Label Model

Principals Hierarchy. DLM based systems \cite{2, 23, 5} use abstract principals to represent entities that can trust or be trusted, e.g., users, roles, groups, organizations, privileges, etc. Principals express trust via acts-for relations \cite{20}. If a principal $p$ acts-for a principal $q$, then $q$ trusts $p$ completely and $p$ may perform any action (read/declassify) that $q$ may perform (written $p \geq q$). The acts-for relation is a pre-order and the principals hierarchy refers to the set of principals under the acts-for ordering. The operators $\wedge$ and $\vee$ can be used to form conjunction and disjunction of principals in more elaborate principals hierarchies. The conjunctive principal $p \wedge q$ represents the joint authority of $p$ and $q$, and acts for both: $p \wedge q \geq p$ and $p \wedge q \geq q$. The disjunctive principal $p \vee q$ represents the disjoint authority of $p$ and $q$, and is acted for by both: $p \geq p \vee q$ and $q \geq p \vee q$. For convenience we include the most restrictive and least restrictive principals, denoted as $\top$ and $\bot$ respectively.

Label Model. The security policies in DLM are expressed using labels: each label is the conjunction of a set of policies each of which expresses privacy requirements in terms of principals \cite{18}. In the DLM framework, a privacy policy has two parts: an owner, and a set of readers, and is written in the form “owner: readers”. The owner of a policy is a principal whose data has contributed to constructing the value that is labeled by this policy. The readers of a policy are a set of principals who are permitted by the owner to read the value so labelled.

For example, in the label $L = \{o_1 : r_3, r_4; o_2 : r_4, r_5\}$, there are two policies (semicolons are separators) – one owned by owner $o_1$, which permits the set of readers $\{r_3, r_4\}$, and the second owned by owner $o_2$ that permits the set of readers $\{r_4, r_5\}$. A principal wishing to access an object is required to satisfy all confidentiality policy components in the object’s label to be able to learn that object’s value. Thus each policy can be viewed as a constraint placed by the owner on what flows are permitted between principals, and a principal who is not the owner of any policy labelling a value places no constraints on allowed flows for that value.

If a policy $K$ is part of the label $L$ (i.e., $K \in L$), then $o(K) : policy \rightarrow principal$ denotes the owner of that policy, and $r(K) : policy \rightarrow (principal \ set)$ denotes the set of readers specified by that policy. The functions $o$ and $r$ completely characterize a label.

These policies then can be modified safely by the individual owners – a form of safe decentralised declassification. An owner may add readers to the reader set of its policy in a label, or remove the entire policy, effectively allowing all readers. Arbitrary declassification is not possible because flow policies of other principals remain in force.

\footnote{As observed by Denning, the analysis for the integrity of written values is dual to the analysis for privacy of values read.}
Derived Information Flow Lattice. A pre-ordering relation on labels is derived from the acts-for relation $\geq$.

We write $P \vdash L_1 \subseteq L_2$, when $L_1$ is less or equal to $L_2$, given a principals hierarchy $P$. The relation $P \vdash L_1 \subseteq L_2$ is defined formally in [18], as shown in Figure 16. The definition says that given the principals hierarchy $P$, it is safe to relabel information tagged with label $L_1$ to $L_2$, if each policy $I \in L_1$ is subsumed by a policy $J \in L_2$. Policy $J$ subsumes $I$ when the owner in $J$ “acts-for” the owner of $I$, and for every reader $r_j$ (effectively) permitted by policy $J$, there is a reader $r_i$ (effectively) permitted by $I$ such that $r_j$ can act for $r_i$. Note that each policy component of a label (e.g., $I, J$) can also be considered a label, so writing $P \vdash I \subseteq J$ is only mild abuse of notation. The relation $\subseteq$ is a pre-order, i.e, reflexive and transitive, but not necessarily anti-symmetric. However, since the acts-for pre-order $\geq$ supports join and meet operations, and because the semantics of labels is given in terms of sets of permitted flows, we can construct an information flow lattice (IFL) by defining an equivalence relation $L_1 \equiv L_2 \iff (L_1 \subseteq L_2 \land L_2 \subseteq L_1)$ and taking equivalence classes to be lattice elements. The join and meet on this information flow lattice are called label join and label meet (written as $\sqcup$ and $\sqcap$ respectively). Since labels are sets of policies that must be together satisfied, we get the so-called “Join rule”, namely $L_1 \sqcup L_2 = L_1 \sqcup L_2$. The least label and greatest label are $\{\}$ and $\{\top\}$ respectively.

Myers presents a rule for safe relabelling by declassification [18]: Let $A$ be a set of principals in the current authority. Let $L_A = \bigcup_{p \in A} \{p:\}$. Then $L_1$ can be safely declassified to $L_2$ if $L_1 \subseteq L_2 \sqcup L_A$.

6.2. Lagois Connections on Principals Hierarchies and Derived IFLs

Assume that two organizations with their own principals hierarchies $P_L$ and $P_R$ negotiate an increasing Lagois Connection $LC = (P_L, \alpha, \gamma, P_R)$ between their principals hierarchies, $\alpha : P_L \rightarrow P_R$ and $\gamma : P_R \rightarrow P_L$. This Lagois connection may be considered as a coupling between the input and outputs channels of the respective individual domains, together with certain strong information flow guarantees.

We show that there exists an increasing Lagois connection, $\widehat{LC}$, between the information flow lattices of the two organisations which is derived from the Lagois Connection between their principals hierarchies (see Figure 17).
**Definition 2 (LC on IFLs).** Define $\widehat{LC} = (IFL_L, \hat{\alpha}, \hat{\gamma}, IFL_R)$, where $IFL_L$ and $IFL_R$ are the corresponding information flow lattices, and $\hat{\alpha} : IFL_L \rightarrow IFL_R$ and $\hat{\gamma} : IFL_R \rightarrow IFL_L$ are specified as:

$$\hat{\alpha}(I_l) = \bigcup_{l \in L_l} \hat{\alpha}(I_l)$$

$$\hat{\alpha}(I_l) = \{ \langle \alpha(o(I_l)) : \alpha(r_l) | r_l \in r(I_l) \rangle \}$$

$$\hat{\gamma}(I_r) = \bigcup_{r \in L_r} \hat{\gamma}(I_r)$$

$$\hat{\gamma}(I_r) = \{ \langle \gamma(o(I_r)) : \gamma(r_r) | r_r \in r(I_r) \rangle \}$$

Observe that $\hat{\alpha}$ and $\hat{\gamma}$ distribute homomorphically over joins:

$$\hat{\alpha}(L_1 \sqcup L_2) = \hat{\alpha}(L_1) \sqcup \hat{\alpha}(L_2)$$

$$\hat{\gamma}(L'_1 \sqcup L'_2) = \hat{\gamma}(L'_1) \sqcup \hat{\gamma}(L'_2)$$

---

**Theorem 16.** Let $LC = (P_L, \alpha, \gamma, P_R)$ be an increasing Lagois connection and $IFL_L$ be derived from $P_L$ and $IFL_R$ be derived from $P_R$, as described earlier. Then $\widehat{LC} = (IFL_L, \hat{\alpha}, \hat{\gamma}, IFL_R)$ is also an increasing Lagois connection.

**Proof.** We prove the following properties for $\hat{\alpha}$ and $\hat{\gamma}$ using Definition 2:

1. Monotonicity
   
   (a) If $P_L \vdash L_1 \sqsubseteq L_2$ then $P_R \vdash \hat{\alpha}(L_1) \sqsubseteq \hat{\alpha}(L_2)$
   
   (b) If $P_R \vdash L'_1 \sqsubseteq L'_2$ then $P_L \vdash \hat{\gamma}(L'_1) \sqsubseteq \hat{\gamma}(L'_2)$

---

Note $\alpha : P_L \rightarrow P_R$ and $\gamma : P_R \rightarrow P_L$ are total functions by Theorem 7 and Corollary 10.
2. Increasing

(a) \( P_L \vdash L_{i1} \sqsubseteq \hat{\alpha}(\hat{\alpha}(L_{i1})), \quad \forall L_{i1} \in IFL_L \)
(b) \( P_R \vdash L_{r1} \sqsubseteq \hat{\alpha}(\hat{\alpha}(L_{r1})), \forall L_{r1} \in IFL_R \)

3. Identity/ Equality/ Fixed Points

(a) \( P_R \vdash \hat{\alpha}(L_{i1}) \equiv \sqsubseteq \hat{\alpha}(\hat{\alpha}(L_{i1})), \quad \forall L_{i1} \in IFL_L \)
(b) \( P_L \vdash \hat{\alpha}(L_{r1}) \equiv \sqsubseteq \hat{\alpha}(\hat{\alpha}(L_{r1})), \forall L_{r1} \in IFL_R \)

We show proofs for properties [1a], [2a] and [3a] as others are similar. We use notation \( r^+(I) = r(I) \cup o(I) \).

[1a] Given \( LC = (P_L, \alpha, \gamma, P_R) \) is an increasing Lagois connection, we know that \( \alpha \) is an order-preserving function. So, if \( p_i \preceq p_j \) then \( \alpha(p_i) \preceq \alpha(p_j) \), for all principals \( p_i, p_j \in P_L \).

\[
P_L \vdash L_{i1} \sqsubseteq L_{i2} \iff \forall (I \in L_{i1}) \exists (J \in L_{i2}) P_L \vdash I \sqsubseteq J \quad \text{(Def of \( \sqsubseteq \))} \tag{13}
\]
\[
P_L \vdash I \sqsubseteq J \equiv P_L \vdash o(I) \leq o(J) \quad \land \quad \forall (r_j \in r^+(J)) \exists (r_i \in r^+(I)) P_L \vdash r_i \preceq r_j \quad \text{(Def of \( \sqsubseteq \))} \tag{14}
\]

As \( \alpha \) is order-preserving, we get

\[
P_R \vdash o(o(I)) \preceq o(o(J)) \quad \land \quad \forall (r_j \in r^+(J)) \exists (r_i \in r^+(I)) P_R \vdash \alpha(r_i) \preceq \alpha(r_j) \tag{16}
\]
\[
P_R \vdash o(\hat{\alpha}(J)) \preceq o(\hat{\alpha}(J)) \quad \land \quad \forall (r_j' \in r^+(\hat{\alpha}(J))) \exists (r_i' \in r^+(\hat{\alpha}(J))) P_R \vdash r_i' \preceq r_j' \tag{19}
\]
\[
\Rightarrow \quad P_R \vdash \hat{\alpha}(J) \equiv \hat{\alpha}(J) \tag{19}
\]
\[
\forall (I \in L_{i1}) \exists (J \in L_{i2}) P_R \vdash \hat{\alpha}(I) \equiv \hat{\alpha}(J) \tag{18}
\]
\[
P_R \vdash \hat{\alpha}(L_{i1}) \equiv \hat{\alpha}(L_{i2}) \tag{20}
\]

Hence we have proved Monotonicity.

[2a] Assuming \( \gamma \circ \alpha \) is increasing we have for any \( p \in P_L \):

\[
P_L \vdash p \preceq \gamma(o(p)) \quad \forall p \in L \tag{23}
\]
\[
\forall (I \in L_{i1}) P_L \vdash o(I) \preceq \gamma(o(o(I))) \quad \land \quad \forall (r_i \in r^+(I)) P_L \vdash r_i \preceq \gamma(o(r_i)) \tag{24}
\]
\[
\forall (I \in L_{i1}) P_L \vdash o(I) \preceq \gamma(\hat{\alpha}(I)) \quad \land \quad \forall (r_j \in r^+(\hat{\alpha}(I))) \exists (r_i \in r^+(I)) P_L \vdash r_i \preceq r_j \tag{27}
\]
\[
\forall (I \in L_{i1}) P_L \vdash J \sqsubseteq \hat{\alpha}(I) \quad \text{(Def of \( \sqsubseteq \))} \tag{28}
\]
\[
\forall (I \in L_{i1}) \exists (J \in \hat{\alpha}(L_{i1})) P_L \vdash I \sqsubseteq J \quad \text{(Def of \( \sqsubseteq \))} \tag{29}
\]
\[
P_L \vdash L_{i1} \sqsubseteq \hat{\alpha}(L_{i1}) \quad \text{(Def of \( \sqsubseteq \))} \tag{30}
\]

Hence we have proved \( \hat{\gamma} \circ \hat{\alpha} \) is increasing.
We have by Definition 2, \( \forall I_1 \in L_1 
abla \)
\[
\dot{a}(I_1) = \{a(o(I_1)) : \{a(r) \mid r \in r(I_1)\}\},
\]
\[
\dot{a}(\dot{\gamma}(\dot{a}(I_1))) = \{a(\gamma(a(o(I_1)))) : \{a(\gamma(a(r))) \mid r \in r(I_1)\}\}.
\]

As \( LC = (P_L, \alpha, \gamma, P_R) \) is an increasing Lagois connection, we know that \( \alpha \circ \gamma \circ \alpha = \alpha \). Therefore, \( \alpha(L_{11}) \equiv \downarrow \gamma(L_{11}) \), \( \forall L_{11} \in IFL_L \).

Hence proved.

Assume the principals hierarchies in two domains are static and Laogis-connected. Let \( A \) and \( A' \) represent two Lagois-connected sets of principals under which authority two communicating processes are operating in their respective domains. Then the safe declassification rule continues to apply even with bidirectional communication between the two domains.

**Corollary 17.** Let \( LC = (P_L, \alpha, \gamma, P_R) \) be an increasing Lagois connection and let \( \dot{L}C = (IFL_L, \dot{\alpha}, \dot{\gamma}, IFL_R) \) be the derived Lagois connection between the corresponding IFLs. Let \( A \) and \( A' \) be sets of principals such that \( \dot{\alpha}[A] = A' \) and \( \dot{\gamma}[A'] = A \). Then (a) \( L_1 \) can be safely declassified to \( \dot{\gamma}(L_2') \) iff \( \dot{\alpha}(L_1) \) can be safely declassified to \( L_2' \), and (b) \( L_1' \) can be safely declassified to \( \dot{\gamma}(L_2) \) iff \( \dot{\alpha}(L_1') \) can be safely declassified to \( L_2 \).

**Proof.** Straightforward from monotonicity, expansiveness, homomorphic distribution over joins, and the assumed Lagois connection between \( A \) and \( A' \).

7. Related Work

The only cited use of the notion of Lagois connections [16] in computer science of which we are aware is the work of Huth [24] in establishing the correctness of programming language implementations. To our knowledge, our work is the first to propose their use in secure information flow control.

Abstract Interpretation and type systems [25] have been used in secure flow analyses, e.g., [26, 27] and [28], where security types are defined using Galois connections employing, for instance, a standard collecting semantics. Their use of two domains, concrete and abstract, with a Galois connection between them, for performing static analyses within a single domain should not be confused with our idea of secure connections between independently-defined security lattices of two organisations.

At the systems level, there has been quite some work on SIF in a distributed setting. An exemplar is DStar [29], which uses sets of opaque identifiers to define security classes. The DStar framework takes a particular Decentralized Information Flow Control (DIFC) model [8, 9] for operating systems and extends it to a distributed network. Subset inclusion is the (only) partial order considered in DStar’s security lattice. Thus it is not clear if DStar can work on general
IFC mechanisms such as FlowCaml\cite{4}, which can employ any partial ordering. Nor can the DStar model express the labels of JiF\cite{3} or Fabric\cite{5} completely. DStar allows bidirectional communication between processes $R$ and $S$ only if $L_R \sqsubseteq_{O_R} L_S$ and $L_S \sqsubseteq_{O_S} L_R$, i.e., when there is an order-isomorphism between the labels. We have argued that such a requirement is far too restrictive for most practical arrangements for data sharing between organisations.

Fabric\cite{23,5} adds trust relationships directly derived from a principals hierarchy to support systems with mutually distrustful nodes and allows dynamic delegation of authority. It is not immediately clear whether that framework supports modular decomposition and analysis, a topic for future investigation. Most of the previous DIFC mechanisms\cite{3,9,8,11,6,10} including Fabric are susceptible to the vulnerabilities mentioned in the motivating examples of our previous work\cite{15}.

8. Conclusions and Future Work

Our work follows Denning’s proposal of lattices as the mathematical basis for analysis about secure information flows. We segue to order-preserving morphisms between lattices as a natural framework for a scalable and modular analysis for secure inter-domain flows. From the basic secure flow requirements that preserved the autonomy of the individual organisations, we identified the simple and elegant theory of Lagois connections as an appropriate formulation. Lagois connections provide us a way to connect the security lattices of two (secure) systems in a manner that does not expose their entire internal structure and allows us to reason only in terms of the interfaced security classes.

We have also illustrated that the theory of Lagois connections provides a versatile framework for supporting the discovery, decomposition, update and maintenance of secure MoUs for exchanging information between administrative domains. Compositionality of Lagois connections provides the necessary modularity when chaining connections across several domains, while the canonical decomposition results provide methodological rules within which we can re-establish secure connections when the security lattices are updated.

Moreover, as illustrated here, we have shown this framework is also applicable in more intricate information flow control formulations such as decentralised IFC and models with declassification\cite{18}. Ongoing work indicates that the framework works smoothly in formulations with data-dependent security classes\cite{7} as well.

Note that the secure Lagois connection between two domains, especially in the decentralised model, introduces new flows from principals in one domain to those in another, and conversely. It can be argued that the sound and complete relabelling rule in the Decentralised Label Model

$$P \vdash L_1 \sqsubseteq L_2 \text{ iff } (\forall P' \supseteq P) X(L_1, P') \supseteq X(L_2, P')$$

(where $X(L, P')$ denotes the set of flows permitted by label $L$ given principals hierarchy $P'$), in a sense already accounts for these new flows. From the
viewpoint of one domain, the permitted flows to and within the other domain can be viewed as an extension $P'$ of its principals hierarchy $P$ that now incorporates principals from the other domain. The Lagois conditions, however, guarantee that when data flow to another domain and back, no new flows are created within each individual domain. Thus, our connections-based framework provides a modular approach to the static analysis of permitted flows, by partitioning the analysis to flows within each domain and inter-domain flows. Indeed, the analysis is confined to the syntactic framework of the principals hierarchies, and the associated policies and labels. The proofs of correctness with respect to the semantics do not have to be reworked to consider the slew of new flows.

We believe that it is important to have a framework in which secure flows should be treated in a modular and autonomous manner for the following reason. The notion of a principal delegating to others the capacity to act on its behalf (e.g., in the DIFC model of Myers [18]) does not scale well to large, networked systems since a principal may repose different levels of trust in principals on various hosts in the network. For this reason, we believe that frameworks such as Fabric [23, 5] may provide more power than mandated by a principle of least privilege. In general, since a principal rarely vests unqualified trust in another in all contexts and situations, one should confine the influence of the principals possessing delegated authority to only specific domains. A mathematical framework that can deal with localising trust and delegation of authority in different domains and controlling the manner in which information flow can be secured deserves a deeper study. We believe that mathematical theories such as Lagois connections provide the necessary structure for articulating these concepts.

We conclude by noting that it is surprising that Lagois connections have not seen greater use in computer science and particularly in static analysis. Most applications of Galois connections in fact employ Galois insertions, which also happen to be special cases of Lagois connections [16]. While the duality between closure and interior operators in Galois connections provides them an elegance, the quite different symmetries exhibited by Lagois connections also seem to be natural and useful in many settings.

References

[1] D. E. Denning, A Lattice Model of Secure Information Flow, Commun. ACM 19 (5) (1976) 236–243.
[2] A. Sabelfeld, A. C. Myers, Language-based information-flow security, IEEE Journal on Selected Areas in Communications 21 (1) (2003) 5–19.
[3] A. C. Myers, Jflow: Practical mostly-static information flow control, in: POPL ‘99, Proceedings of the 26th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, San Antonio, TX, USA, January 20-22, 1999, 1999, pp. 228–241.
[4] F. Pottier, V. Simonet, Information Flow Inference for ML, ACM Trans. Program. Lang. Syst. 25 (1) (2003) 117–158.
[5] J. Liu, O. Arden, M. D. George, A. C. Myers, Fabric: Building open distributed systems securely by construction, Journal of Computer Security 25 (4-5) (2017) 367–426.

[6] I. Roy, D. E. Porter, M. D. Bond, K. S. McKinley, E. Witchel, Laminar: practical fine-grained decentralized information flow control, in: Proceedings of the 2009 ACM SIGPLAN Conference on Programming Language Design and Implementation, PLDI 2009, Dublin, Ireland, June 15-21, 2009, 2009, pp. 63–74.

[7] L. Lourenço, L. Caires, Dependent information flow types, in: Proceedings of the 42nd Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, POPL 2015, Mumbai, India, January 15-17, 2015, 2015, pp. 317–328.

[8] M. N. Krohn, A. Yip, M. Z. Brodsky, N. Cliffer, M. F. Kaashoek, E. Kohler, R. T. Morris, Information flow control for standard OS abstractions, in: Proceedings of the 21st ACM Symposium on Operating Systems Principles 2007, SOSP 2007, Stevenson, Washington, USA, October 14-17, 2007, 2007, pp. 321–334.

[9] N. Zeldovich, S. Boyd-Wickizer, E. Kohler, D. Mazieres, Making Information Flow Explicit in Histar, in: 7th Symposium on Operating Systems Design and Implementation (OSDI’06), November 6-8, Seattle, WA, USA, 2006, pp. 263–278.

[10] W. Cheng, D. R. K. Ports, D. A. Schultz, V. Popic, A. Blankstein, J. A. Cowling, D. Curtis, L. Shirira, B. Liskov, Abstractions for Usable Information Flow Control in Aeolus, in: 2012 USENIX Annual Technical Conference, Boston, MA, USA, June 13-15, 2012, 2012, pp. 139–151.

[11] P. Efthathopoulos, M. N. Krohn, S. VandenBogart, C. Frey, D. Ziegler, E. Kohler, D. Mazieres, M. F. Kaashoek, R. T. Morris, Labels and event processes in the Asbestos operating system, in: Proceedings of the 20th ACM Symposium on Operating Systems Principles 2005, SOSP 2005, Brighton, UK, October 23-26, 2005, 2005, pp. 17–30.

[12] D. A. Schultz, B. Liskov, IFDB: Decentralized Information Flow Control for Databases, in: Eighth Eurosys Conference 2013, EuroSys ’13, Prague, Czech Republic, April 14-17, 2013, 2013, pp. 43–56.

[13] A. Ferraiuolo, M. Zhao, A. C. Myers, G. E. Suh, Hyperflow: A Processor Architecture for Nonmalleable, Timing-Safe Information Flow Security, in: Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, CCS 2018, Toronto, ON, Canada, October 15-19, 2018, 2018, pp. 1583–1600.

[14] D. Zhang, Y. Wang, G. E. Suh, A. C. Myers, A Hardware Design Language for Timing-Sensitive Information-Flow Security, in: Proceedings of the
[15] C. Bhardwaj, S. Prasad, Only connect, securely, in: Formal Techniques for Distributed Objects, Components, and Systems - 39th IFIP WG 6.1 International Conference, FORTE 2019, Held as Part of the 14th International Federated Conference on Distributed Computing Techniques, DISCoTec 2019, Kongens Lyngby, Denmark, June 17-21, 2019, Proceedings, 2019, pp. 75–92. doi:10.1007/978-3-030-21759-4_5

[16] A. Melton, B. S. W. Schröder, G. E. Strecker, Lagois Connections - a Counterpart to Galois Connections, Theor. Comput. Sci. 136 (1) (1994) 79–107.

[17] D. M. Volpano, C. E. Irvine, G. Smith, A Sound Type System for Secure Flow Analysis, Journal of Computer Security 4 (2/3) (1996) 167–188.

[18] A. C. Myers, [Mostly-static decentralized information flow control], Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, MA, USA (1999). URL http://hdl.handle.net/1721.1/16717

[19] J. A. Goguen, J. Meseguer, Security Policies and Security Models, in: 1982 IEEE Symposium on Security and Privacy, Oakland, CA, USA, April 26-28, 1982, 1982, pp. 11–20.

[20] A. C. Myers, B. Liskov, A Decentralized Model for Information Flow Control, in: Proceedings of the Sixteenth ACM Symposium on Operating System Principles, SOSP 1997, St. Malo, France, October 5-8, 1997, 1997, pp. 129–142.

[21] J. I. Munro, B. Sandlund, C. Sinnamon, Space-efficient data structures for lattices, in: 17th Scandinavian Symposium and Workshops on Algorithm Theory, 2020.

[22] A. Melton, B. S. W. Schröder, G. E. Strecker, Connections, in: Mathematical Foundations of Programming Semantics, 7th International Conference, Pittsburgh, PA, USA, March 25-28, 1991, Proceedings, 1991, pp. 492–506.

[23] J. Liu, M. D. George, K. Vikram, X. Qi, L. Waye, A. C. Myers, Fabric: a platform for secure distributed computation and storage, in: Proceedings of the 22nd ACM Symposium on Operating Systems Principles 2009, SOSP 2009, Big Sky, Montana, USA, October 11-14, 2009, 2009, pp. 321–334.

[24] M. Huth, On the equivalence of state-transition systems, in: Theory and Formal Methods 1993, Springer, 1993, pp. 171–182.
[25] P. Cousot, Types as Abstract Interpretations, in: Conference Record of POPL’97: The 24th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, Papers Presented at the Symposium, Paris, France, 15-17 January 1997, 1997, pp. 316–331.

[26] A. Cortesi, P. Ferrara, M. Pistoia, O. Tripp, Datacentric Semantics for Verification of Privacy Policy Compliance by Mobile Applications, in: Verification, Model Checking, and Abstract Interpretation - 16th International Conference, VMCAI 2015, Mumbai, India, January 12-14, 2015. Proceedings, 2015, pp. 61–79.

[27] A. Cortesi, P. Ferrara, R. Halder, M. Zanioli, Combining symbolic and numerical domains for information leakage analysis, Trans. Computational Science 31 (2018) 98–135.

[28] M. Zanotti, Security Typings by Abstract Interpretation, in: Static Analysis, 9th International Symposium, SAS 2002, Madrid, Spain, September 17-20, 2002, Proceedings, 2002, pp. 360–375.

[29] N. Zeldovich, S. Boyd-Wickizer, D. Mazieres, Securing Distributed Systems with Information Flow Control, in: 5th USENIX Symposium on Networked Systems Design & Implementation, NSDI 2008, April 16-18, 2008, San Francisco, CA, USA, Proceedings, 2008, pp. 293–308.