Closed string tachyons in a smooth curved background

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Abstract
Closed string tachyon condensation has been studied in orbifolds \( \mathbb{C}^2/\mathbb{Z}_{N,p} \) of flat space, using the chiral ring of the underlying \( \mathcal{N} = 2 \) conformal field theory. Here we show that similar phenomena occur in the curved smooth background obtained by adding NS5-branes, such that chiral tachyons are localised on lens submanifolds \( SU(2)/\mathbb{Z}_{N,p} \). We find a level-independent subring which coincides with that of \( \mathbb{C}^2/\mathbb{Z}_{N,p} \), corresponding to condensation processes similar to those of hep-th/0111154. We also study level-dependent chiral tachyons.
1 Introduction and overview

Time evolution between two metastable configurations in string theory was related by A. Sen to a renormalisation group flow between the underlying worldsheet theories. The original proposal [1] concerned dynamic processes of D-branes in a given closed string background, driven by the better understood condensation of open string tachyons. In recent years, however, the condensation of closed string tachyons has been addressed ingeniously in a simple class of toy models, namely non-supersymmetric orbifolds of flat space $\mathbb{C}^2/\mathbb{Z}_{N,p}$ [2, 3, 4].

These toy models are examples of theories with extended worldsheet supersymmetry which nevertheless have space-time tachyons (and hence break space-time supersymmetry). The key idea is that this extended supersymmetry is preserved all along the renormalisation group flows driven by target space tachyons which are BPS states of the worldsheet theory. This property allows to describe some properties of the endpoint of the flow; thus, the flows studied for $\mathbb{C}^2/\mathbb{Z}_{N,p}$ were shown to describe a gradual opening of the orbifold singularity, eventually leading in the infrared to flat space.

One of the original difficulties of the topic – that closed string tachyon condensation would lead, according to the Zamolodchikov $c$-theorem, to a decrease of the central charge – has been turned into the so-called $g_{cl}$-conjecture [3]: that localised closed string tachyons in a non-compact space contribute at subleading order (as compared to untwisted states) to the free energy of the theory, and that it is this contribution $g_{cl}$ which decreases with their condensation. In the toy models above, the tachyons are localised at the orbifold singularity and everywhere else the theory is free.

The purpose of this note is to show how this strategy can be applied to study closed string
tachyons in certain curved, smooth backgrounds. These include orbifolds of the $SU(2)$ WZW model, the generalised lens spaces $L_{N,p} = SU(2)/\mathbb{Z}_{N,p}$ studied in [5].

The note is organised as follows: we first describe embeddings of generalised lens spaces in string theory in section 2 using NS5-branes. In section 3 we briefly describe the corresponding CFT constructions. We show how space-time supersymmetry is broken and then focus on the chiral ring of the $N = 2$ worldsheet superconformal algebra. It turns out there exists a level-independent subring which is defined by the pair $(N, p)$ and coincides with the chiral ring of the $\mathbb{C}^2/\mathbb{Z}_{N,p}$ theories. Since the methods used in [3] to analyse RG flows rely exclusively on the structure of the chiral ring, such tachyons in our backgrounds drive processes $(N, p) \rightarrow (N', p')$ just like those described for $\mathbb{C}^2/\mathbb{Z}_{N,p}$. The level-dependent subring, on the other hand, is identical to that of $\mathbb{C}/\mathbb{Z}_k$. We save the conclusions and outlook for section 4.

2 Smooth, curved, tachyonic backgrounds

Consider a stack of $k$ coincident NS5-branes in $\mathbb{R}^{1,9}$. It is well known that the 4-dimensional space transverse to the NS5-branes becomes curved due to backreaction and in the near horizon limit develops a throat of section $S^3$ whose radius is parametrized by the dilaton: the corresponding CFT is $SU(2)_k \times \mathbb{R}_{\text{dil}}$ [6]. If instead we put the NS5-branes along the $\mathbb{R}_{1,5}$ directions of the flat space orbifold $\mathbb{R}_{1,5} \times \mathbb{C}^2/\mathbb{Z}_{N,p}$, the transverse $SU(2)$ becomes a generalized lens space $L_{N,p}$. To see this, let us consider the $\mathbb{Z}_N$ actions involved.

If we parametrise $\mathbb{C}^2$ by the polar coordinates $(r_1 e^{i\phi}, r_2 e^{i\psi})$, then the $\mathbb{Z}_N$ acts as $\phi \rightarrow \phi + 2\pi/N$ and $\psi \rightarrow \psi + 2\pi p/N$. Adding the NS5-branes then curves the transverse space to $SU(2) \times \mathbb{R}$, where $SU(2)$ is spanned by $(\phi, \psi, \arctan(r_1/r_2))$, up to these identifications. If we now switch to the Euler coordinates $g = e^{i\psi/2} e^{i\sigma_3} e^{i\phi/2} e^{i\sigma_1} e^{i\phi/2} e^{i\sigma_3}$ for $SU(2)$, these identifications become

$$g \sim \omega^{p+1} g \omega^{p-1}$$

where we chose $\omega = e^{i\sigma_3/2}$ to be the $\mathbb{Z}_N$ generator. The quotient by (1) defines the lens space $L_{N,p}$. Notice that $p$ takes values in $\{1, \ldots, N\}$. This action is free as long as $p$ and $N$ are mutually prime, which we will assume in the following. The particular case $p = 1$ corresponds to the more familiar left quotient of $SU(2)$ (furthermore, $L_{N,p}$ and $L_{N,-p}$ are diffeomorphic, being related by a conjugation by $\omega$). The various lens spaces with a given $N$ are topologically very similar (they have the same cohomologies, but can be distinguished by certain knot invariants [2]).

Thus the near horizon limit of our configuration is $L_{N,p} \times \mathbb{R}_{\text{dil}}$ as advertised. Notice in particular that adding the NS5-branes smoothens out the orbifold singularity by replacing it
with the infinite throat. On the other hand, the $\mathbb{Z}_N$ quotient in general breaks all spacetime supersymmetry, since it acts differently on the right and on the left. In fact, analysing the spectrum in the next section, we will see that theories with $p \neq 1$ have spacetime tachyons.

3 CFT description and chiral rings

Let us be more precise in the conformal field theory description of our configuration of NS5-branes. Adding $k$ NS5 branes at the origin of flat space yields $SU(2)_k \times \mathbb{R}_{\text{dil}}$. The $SU(2)$ transverse to the NS5-branes can now only be orbifolded by an action whose order $N$ divides the level $k$, such that the exponential of the Wess-Zumino action is well defined \[5\]. Thus, we are in fact considering a system of $k = NN'$ NS5-branes spread out in a circle at a finite distance of the orbifold singularity. This singularity, which would describe the strong string coupling region where the CFT description breaks down, is thus naturally cut off from the theory. According to the analysis of \[8\], the chiral algebra describing the theory near the NS5-branes is then $SL(2, \mathbb{R})/U(1) \times SU(2)/U(1)$.

To construct the superconformal field theory appearing in this near horizon limit, let us recall the bosonic CFT describing string propagation in lens spaces $L_{N,p}$ \[3\]. As explained in the introduction, for our study of tachyon condensation, we need an $\mathcal{N} = 2$ supersymmetric and non-compact version of that. The simplest extension is to add one non-compact direction to the three-dimensional $L_{N,p}$.\(^1\) For that purpose, recall that the relevant conformal field theory has as chiral algebra $SU(2)/U(1)_k \times U(1)_k$, and that its spectrum is given by fields of the form $\Phi_{j,m}^p \Phi_{n}^{u(1)} \cdot \Phi_{j',m'}^{p} \bar{\Phi}_{n'}^{u(1)}$ (where the $pf$ stands for the parafermions $SU(2)/U(1)$) with selection rules

$$m - n = (p + 1) \frac{k}{N} r, \quad m + n = (p - 1) \left( \frac{k}{N} r + s \right) \mod 2N$$

$$m' = m + 2s, \quad n' = n + 2s + 2 \frac{k}{N} r, \quad j' = j$$

with $r = 0, \ldots, N - 1$ and $s = 0, \ldots, k - 1$ and $p \in \{1, \ldots, N\}$ (recall that $N$ divides $k$). In CFT terms the orbifold group is $\mathbb{Z}_k \times \mathbb{Z}_N$, where the $\mathbb{Z}_N$ factor acts only on the $u(1)$ chiral algebra, so there are $Nk - 1$ twisted (left-right asymmetric) sectors, corresponding to non-zero values of $r$ and $s$. Notice that different choices of $p$ only affect the set of asymmetric fields in the spectrum; in (rational) conformal field theory this choice is referred to as discrete torsion \[10\]\[9\].

\(^1\)Recall that $\mathcal{N} = 2$ worldsheet supersymmetry implies a target space with complex structure, and thus with even dimension.
In moving from the bosonic $L_{N,p}$ towards an $\mathcal{N} = 2$ superconformal field theory, it suffices to replace the $U(1)$ factor, since the parafermions $SU(2)_k/U(1)$ are naturally an $\mathcal{N} = 2$ minimal model. Given the analysis above, the most natural choice is to start from the chiral algebra

$$\mathcal{A} = \frac{SL(2, \mathbb{R})_{k+2}}{U(1)} \times \frac{SU(2)_{k-2}}{U(1)}$$

The level of the $\mathcal{N} = 2$ “cigar CFT” $SL(2, \mathbb{R})/U(1)$ has been chosen such that the chiral algebra has (bosonic) central charge $c = 4$. As the name ”cigar” suggests, the target space of the full $SL(2, \mathbb{R})/U(1)$ CFT is a semi-infinite cylinder whose compact direction shrinks to zero size at the origin. The natural interpretation of a CFT with symmetry $\mathcal{A}_L \times \mathcal{A}_R$ and spectrum \cite{23}, would be that of a lens space whose radius grows along the non-compact cigar dimension. Indeed, different orbifolds of $\mathcal{A}$ have already been used to describe $AdS_3 \times SU(2)$, namely \cite{11}

$$\frac{SL(2, \mathbb{R})}{U(1)} \times U(1) \times \frac{SU(2)}{U(1)}$$

and also to describe $\mathbb{R}_\phi \times SU(2)_k$, namely \cite{12}

$$\frac{SL(2, \mathbb{R})/U(1) \times SU(2)/U(1)}{Z_k}$$

In both cases, the orbifold acts at the level of the chiral algebra, ie. it is a symmetric orbifold. In contrast, generalising the $SU(2)$ factor in these models to lens spaces involves imposing the asymmetric orbifold constraints \cite{23} on the spectrum.

While the proof of modular invariance of general simple current partition functions is complicated \cite{10}, that for the $L_{N,p}$ case relies only in the modular properties of the $U(1)$ characters, much like Vafa’s original analysis of discrete torsion in the 2-torus \cite{3}. Nevertheless, this analysis does not carry straight away to our case here, because (unlike for the parafermions) the modular transformations of $SL(2, \mathbb{R})/U(1)$ extended characters do not factorize into $SL(2, \mathbb{R})$ and $U(1)$. Fortunately, in the following the explicit expression of the partition function is not essential, so we will just assume that our theory is defined by the triplet $(k, N, p)$ such that its bosonic sector is given by \cite{23}, as justified by the geometric arguments of the previous section.

We can now nevertheless see how space-time supersymmetry is broken. As mentioned in \cite{13}, the left-moving supercharges are mutually local with the fields of the theory if the latter obey

$$m - n \in \mathbb{Z}$$

since that is the conformal weight appearing in the most singular term of the OPE of the supercharges with a given state (and similarly for the right movers). This means that only
for \((N, p) = (1, 1)\) or \((k, 1)\) are all the left- and right-moving supercharges conserved (actually, the configurations \((N, 1)\) and \((k/N, 1)\) are T-dual to each other \([3]\)). In this case there is a GSO projection which leads to a consistent type II string theory \([11]\). For non-trivial quotients (both \(N\) and \(N'\) greater than 1) with \(p = 1\), only the left-moving supercharges are conserved \([13]\). Again there exists a GSO projection (the restriction of the one above to a chiral half of the Hilbert space) such that these supercharges still generate a consistent type II superstring theory\(^2\). Finally, when \(p \neq 1\), none of the supercharges is conserved and there is no space-time supersymmetry.

### 3.1 The chiral ring

Chiral fields \(\Phi\) of the \(\mathcal{N} = 2\) superconformal algebra saturate the BPS bound

\[
\Delta \Phi = \frac{1}{2} Q \Phi
\]

where \(\Delta \Phi\) is the conformal weight of \(\Phi\) and \(Q\) is its R-charge.\(^3\) Since \(Q\) is additive, the set of chiral fields forms a ring, called the chiral ring \([14]\). Following \([3]\), we are interested in the chiral ring as a way to characterize our theories \((k, N, p)\) which is manageable under RG flows. Indeed, one can see in a lagrangian formalism that this BPS property is preserved along the renormalization group flow driven by a chiral relevant perturbation. The chiral ring is deformed under such a flow, but can provide information about the infrared fixed point.

Chiral fields are always in the NS sector. The NS representations of our chiral algebra are specified by 4 indices, e.g. the left moving NS states are \(|j, m)^cg |l, n)^pf\). The parafermion indices are restricted to \(l = 0, \frac{1}{2}, \ldots, k/2 - 1\) and \(n\) is an integer defined modulo \(2k\) such that \(2l + n = 0 \mod 2\). The cigar representations (normalizable and unitary, with integer level \(k + 2\)) are similarly labelled by two parameters \((j, m)\) and fall into three classes: the positive discrete series, with \(j = 0, \frac{1}{2}, \ldots\) and \(m = j + t\) where \(t \in \mathbb{N}\); the negative discrete series, with \(j = 0, \frac{1}{2}, \ldots\) and \(m = -j - t\) where \(t \in \mathbb{N}\); and the continuous representations, with \(j = \frac{1}{2} + i \lambda\), where \(\lambda \in \mathbb{R}\), and \(m \in \mathbb{Z}\). Their weights and charges are

\[
\begin{align*}
\Delta_{jm}^{cg} &= \frac{m^2 - j(j - 1)}{k}, & Q_{jm}^{cg} &= -\frac{2m}{k} \mod 2 \\
\Delta_{ln}^{pf} &= -\frac{n^2}{4k} + \frac{l(l + 1)}{k}, & Q_{ln}^{pf} &= -\frac{n}{2k} \mod 2
\end{align*}
\]

\(^2\)However, in \([13]\), only the untwisted part of the spectrum of this theory was studied since the purpose was to match it to the supergravity description.

\(^3\)The worldsheet \(\mathcal{N} = 2\) symmetry factorises naturally in the algebra \(\mathcal{A}\) \([4]\). Here we choose fields which are chiral for both the cigar and the parafermions, and also for both the left and the right-moving algebras. Other choices, involving antichiral fields with \(\Delta \Phi = -\frac{1}{2} Q \Phi\) would lead to similar results.
The chiral fields are therefore of the form

$$|j, -j\rangle^c_L |l, l\rangle^f_L \otimes |j', -j'\rangle^c_R |l', l\rangle^f_R$$

(10)

Selecting the chiral fields (10) which obey the selection rule (2,3) will then yield the chiral fields in the theory with target space $L_{N,p} \times \mathbb{R}_\phi$, see below.

Before we go on to study the structure of the chiral ring, notice that only the discrete representations of $SL(2, \mathbb{R})/U(1)$ appear in the chiral fields. Since these states are localised at the tip of the cigar, the chiral states of (11) will be localised at the origin of the non-compact direction. In particular, and even though the target space is smooth, the chiral tachyons (those chiral fields with $Q < 1$, i.e. chiral relevant operators) are localised in the $L_{N,p}$ submanifold. Rolling down the potential of such tachyons will therefore not change the total central charge, which is dominated by large volume contributions [2, 3].

Let us now combine the $\mathcal{N} = 2$ chirality condition (7) with the lens space selection rules (2,3). The analysis can be made at the CFT chiral level; left-moving chiral fields satisfying (2) can be divided into three branches, specified by their cigar and parafermionic $U(1)$ charges (previously called $m, n$):

I : fields with $r \neq 0, s = 0$, of the form $W_r = | - r p \frac{k}{N} \rangle^c | r \frac{k}{N} \rangle^f$

II : fields with $r = 0, s \neq 0$, of the form $V_s = | (p - 1)s \rangle^c | (p - 1)s \rangle^f$

III : fields with $r = s = 0$, of the form $T_c = | Nc \rangle^c | Nc \rangle^f$

(11) (12)

where in branches I and II $s = 1, N - 1$ and $r = 1, k - 1$ and in branch III $c = 1, \ldots, 2k/N$. Recall that fields with $r, s \neq 0$ have different left and right $U(1)$ charges, via the selection rules (3). In particular, since for lens spaces the parafermionic number $l$ must be the same in the left-movers and the right-movers, $l = l'$, the chiral left-movers with a non-zero $s$ value (those in branch II) are coupled to non-chiral right movers and vice-versa. Similarly, in branch I the variable $r$ shifts the $U(1)$ charge $n'$ associated to the right-moving $SL(2, \mathbb{R})/U(1)$. So branch I and II do not give rise to chiral fields of the full conformal field theory.

Finally, the fields in branch III give rise to left-right symmetric (i.e. independent of $p$) chiral fields of the full CFT. This branch includes tachyonic fields for any $N > 1$, even in the supersymmetric cases reviewed in section 2 when $p = 1$. We are led to conclude that these fields are projected out in the supersymmetric $p = 1$ case, and perhaps also for general $p$. In any case, these fields are in the untwisted sector, and so we will not consider them in the following.
3.2 Twist fields

So we have no twisted chiral states in our spectrum. Nevertheless, the twisted sectors are just as well described by the twist fields which permute them [15] (even though they are not in the partition function, being non-local). Thus to branches I and II above correspond two branches of twisted fields. An analysis of the boundary conditions of the fields in branch I, for instance, is exactly similar to the one for \(\mathbb{C}^2/\mathbb{Z}_{N,p} \) [3] (modulo the change of \(U(1)\) variables mentioned before equation [1]). The branch I twist fields actually form a chiral ring isomorphic to that of \(\mathbb{C}^2/\mathbb{Z}_{N,p}\), and is thus independent of \(k\). In fact, these twist fields are characterized by the same (left-moving) R-charges as the fields in branch I, which are independent of \(k\). This means that the ring doesn’t change as we increase \(k\), when the target space \(L_{N,p} \times \mathbb{R}_\phi\) approaches its large volume limit – the flat space orbifold \(\mathbb{C}^2/\mathbb{Z}_{N,p}\). It is not surprising, therefore, that branch I of our chiral ring in fact coincides with the chiral fields of \(\mathbb{C}^2/\mathbb{Z}_{N,p}\). In fact, if the branch I fields are volume independent they should depend only on the asymptotic structure of the target space; the background curvature is then effectively replaced with a singularity in the same class of ALE spaces.

For branch II, which has only one generator, the analysis is similar to that for \(\mathbb{C}/\mathbb{Z}_k\) [2]. These twist fields are similarly characterized by the branch II (left-moving) R-charges, which do depend on \(k\). These fields thus probe the high-curvature region of \(L_{N,p} \times \mathbb{R}_{dil}\), if only the preserved diagonal \(U(1)\) symmetry of \(SL(2,\mathbb{R})/U(1) \times SU(2)/U(1)\).

To clarify the structure of the complete chiral ring, the diagram of the corresponding R-charges, for the theory with level \(k = 20\), orbifold order \(N = 10\) and discrete torsion parameter \(p = 3\), is presented in Figure 1. Here we have used the fact that the R-charges are defined modulo 1. The red line indicates the marginal fields, with \(Q_{\text{tot}} = Q_{\text{Pf}} + Q_{\text{cg}} = 1\). The fields below this line are tachyons. The diagram is in units of \(1/k\), and from it we can extract the structure of the ring – in particular its generators, as pictured. In the general case, the analysis then follows directly from [3]. For completeness, we briefly review it here: for the branch I subring, there are \([N,p]\) generators, where \([N,p]\) is the number of entries in the continued fraction

\[
\frac{N}{p} = a_1 - \frac{1}{a_2 - \frac{1}{... - \frac{1}{a_{[N,p]}}}} \equiv [a_1, \ldots, a_{[N,p]}] \quad (13)
\]

and the \(a_i\) encode the ring structure of branch I. In terms of the twisted sectors [11] this is

\[
W^{a_r}_{r} = W_{(r+1)} W_{(r-1)} \quad (14)
\]

In the example of Figure 1, equation (13) becomes \(10/3 = [4, 2, 2]\) and in particular there are three generators.
Figure 1: R-charges $Q^{pf}$ and $Q^{cg}$ of the twist fields corresponding to the twisted sectors $W_i, V_i$ under the $\mathcal{N} = 2$ superconformal algebras $SU(2)/U(1)_{k-2}$ and $SL(2,\mathbb{R})/U(1)_{k+2}$, respectively. Level $k = 20$, orbifold order $N = 10$ and discrete torsion parameter $p = 3$. Grid in steps of $1/k$.

On the other hand, branch II has only one generator, corresponding to the twisted sector $V_1$, of order $k$ and R-charge $(p - 1)/k$. This branch always includes tachyons, except in the supersymmetric case $p = 1$ where the whole branch collapses to the identity.

3.3 Tachyon condensation

Having perturbed the theory with a chiral tachyon, one can study by a variety of methods the chiral ring of the endpoint of the RG flow. For ALE spaces in the class of $\mathbb{C}^2/\mathbb{Z}_{N,p}$, such as $\mathbb{R}_{dil} \times L_{N,p}$, the analysis in [3][4] is very efficient, so we may use it to study RG flows under our level-independent tachyons $W_r$. It was shown there that the R-charge diagram provides an algebraic description of the ALE structure of our space in terms of the pair $(N, p)$, and that tachyon condensation here can be thought of as blowing up particular curves of that space. This method thus identifies the ALE structure of the target space of the endpoint of condensation under our tachyon $W_r$,

$$(N, p) \longrightarrow_{W_r} (N_{r_1}, p_{r_1}) \oplus \cdots \oplus (N_{r_t}, p_{r_t})$$

where the $r_i$ and $t$ depend on $W_r$. Even though the ALE structure on the rhs (ie. the chiral ring of the infrared theory) does not specify the worldsheet theory entirely, it is reasonable to assume that the endpoint of the flow is in fact the (set of) lens space with the corresponding ALE structure. In particular, all the condensations determined in [3] in this way verified the $g_{cl}$ conjecture.

The branch II chiral subring (11) coincides with the chiral ring of the flat space orbifold
Naively then (if one forgets about the branch II subring), one may be tempted to assume that condensation of a $V_n$ tachyon will similarly drive the theory to smaller deficit angles from the point of view of the twisted sectors:

$$\mathbb{C}/\mathbb{Z}_{2k} \rightarrow_{V_n} \mathbb{C}/\mathbb{Z}_{k_1(n)} \oplus \cdots \oplus \mathbb{C}/\mathbb{Z}_{k_t(n)}$$

where in particular $\sum_i k_i = k$. This would describe the gradual disengagement of the $SL(2, \mathbb{R})/U(1)$ and $SU(2)/U(1)$ factors, eventually leading to the direct product $SL(2, \mathbb{R})/U(1) \times SU(2)/U(1)$. However, a more detailed analysis would be needed to investigate how such flows affect the entire chiral ring.

4 Conclusion and Outlook

We have embedded generalized lens spaces $L_{N,p} = SU(2)/\mathbb{Z}_{N,p}$ in string theory by adding NS5-branes to the flat space orbifold $\mathbb{C}^2/\mathbb{Z}_{N,p}$ of [3]. This preserves the $\mathcal{N} = 2$ superconformal worldsheet symmetry but leads to spacetime tachyons. The worldsheet chiral ring can be divided into two parts: a level-independent part identical to that of $\mathbb{C}^2/\mathbb{Z}_{N,p}$, and another part identical to that of $\mathbb{C}/\mathbb{Z}_k$. Thus, chiral fields which are spacetime tachyons drive RG flows similar to those analysed in [3] for these two cases.

It would be important to explicitly write the partition functions of our backgrounds, to explore how modular invariant partition functions of non-rational theories exist in families parametrised by discrete parameters (such as the discrete torsion $p$) corresponding to different choices of twisted sectors.

It would also be interesting to have an effective description of the tachyons in terms of NS5-branes interactions. For instance, our branch I tachyons act only at the level of the orbifold, so here the question is shifted to how this orbifold is actually implemented in string theory. To circumvent this particularity of our construction, one could try to embed a generalized lens space in string theory by modifying a configuration which already includes a usual lens space $L_{N,1}$. Examples include the appearance of Taub-NUT spaces as the 4d transverse space to KK-monopoles [16]. For $N$ monopoles distributed at positions $(\psi_i, \vec{x}_i)$ in transverse space, we get the multicentered Taub-NUT metric

$$ds_{\perp}^2 = V(x)^{-1} (d\psi + \omega \cdot x)^2 + V(x) d\vec{x} \cdot \vec{x}$$

where $\omega$ is a one-form determined by $V$ and we take the ansatz $V(x) = \epsilon + \sum_{i=1}^{N} |\vec{x} - \vec{x}_i|$. Writing $\vec{x}$ in polar coordinates $(r, \theta, \phi)$ around an $\vec{x}_i$, we can take the near horizon limit $r \rightarrow 0$. The space spanned by $(\psi, \theta, \phi)$ is then a lens space $L_{M_i,1}$ where $M_i$ is the number of monopoles.
stacked at $\vec{x}_i$ (the non-trivial $S^1$ is along $\psi$, whose periodicity assures the space is smooth, turning the singularity at $x = x_i$ into a coordinate singularity). It would be very interesting to find a modification of this configuration such that the near horizon limit would include a generalized lens space $L_{N,p}$ instead.

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