Dark Matter interpretation of the neutron decay anomaly

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**ABSTRACT:** We add to the Standard Model a new fermion $\chi$ with minimal baryon number $1/3$. Neutron decay $n \to \chi\chi\chi$ into non-relativistic $\chi$ can account for the neutron decay anomaly, compatibly with bounds from neutron stars. $\chi$ can be Dark Matter, and its cosmological abundance can be generated by freeze-in dominated at $T \sim m_n$. The associated processes $n \to \chi\chi\chi\gamma$, hydrogen decay $H \to \chi\chi\nu(\gamma)$ and DM-induced neutron disappearance $\bar{\chi}n \to \chi\chi(\gamma)$ have rates below experimental bounds and can be of interest for future experiments.

**KEYWORDS:** Beyond Standard Model, Cosmology of Theories beyond the SM

**ArXiv ePrint:** 2112.09111
1 Introduction

The neutron life-time has been measured with two different experimental techniques, which consistently give different results:

- The ‘bottle method’ measures the total neutron decay width, with the result \( \Gamma_{n}^{\text{tot}} = 1/(878.3 \pm 0.3) \) s, by storing ultra-cold neutrons in a magnetic bottle and counting their remaining number after some time [1–8].

- The ‘beam method’ measures the \( n \rightarrow p e\bar{\nu}_e \beta \)-decay rate, with the result \( \Gamma_{n}^{\beta} = 1/(888 \pm 2) \) s, by counting the protons produced from a beam of cold neutrons [9–11].

These measurements contradict at about 4.6\( \sigma \) level the Standard Model (SM) that predicts \( \Gamma_{n}^{\text{tot}} = \Gamma_{n}^{\beta} \) up to sub-leading channels that can be neglected. If confirmed, they indicate new physics. It has been proposed that

\[
\Delta \Gamma = \Gamma_{n}^{\text{tot}} - \Gamma_{n}^{\beta} \approx 1.2 \times 10^{-5} \text{ sec} (1 \pm 0.21)
\]

(1.1)

could be due to an extra decay channel of the neutron that does not produce protons. The new decay mode needs to be nearly invisible with a branching ratio around 1%. Various authors proposed a \( n \rightarrow \chi \gamma \) decay into a new neutral fermion \( \chi \) with mass \( M \) slightly below the neutron mass so that \( E_{\gamma} \approx m_n - M \) is small [12, 13] (see also [14] and [15, 16]). However, this interpretation has the following problems:

1. First, if \( M < m_p + m_e \) the new particle \( \chi \) does not promptly decay back to SM charged particles and can be Dark Matter; but in this range \( E_{\gamma} > m_n - m_p - m_e \) is large enough that the extra decay is visible enough that it has been tested and disfavoured [13, 17, 18]. An alternative that can explain both the neutron anomaly and Dark Matter (DM) would be welcome.
2. Second, \( n \leftrightarrow \chi \gamma \) would lead to \( \chi \) thermalization inside neutron stars, as they are older than a Myr, while the neutron decay anomaly needs the much short time-scale of eq. (1.1). Thermalized \( \chi \) soften the equation of state \( \varphi(\rho) \) of neutron stars. Free \( \chi \) add more energy density \( \rho \) than pressure \( \varphi \) reducing the maximal mass of neutron stars below \( 0.7M_{\text{sun}} \), like in the 1939 Oppenheimer and Volkoff’s calculation that assumed free neutrons, neglecting their nuclear self-repulsion [19].

3. Third, the precise SM prediction of the neutron decay rate, possibly \( \Gamma_{n}^{\text{SM}} = 1/(878.7 \pm 0.6) \) s [20, 21], agrees with the bottle experiments disfavouring new physics contributions that increase the neutron decay rate. This might indicate that the neutron decay anomaly could just be due to some under-estimated systematic uncertainty of ‘beam’ experiments, or that it needs alternative interpretations where the neutron oscillates (with a resonant enhancement thanks to magnetic fields in the apparatus) into a composite twin of the neutron [21–23].\(^1\) However, the quoted precise SM prediction for \( \Gamma_{n}^{\text{SM}} \) employs determinations of \( V_{ud} \) and measurements of the neutron axial coupling constant \( g_{A} \) that are potentially problematic at the claimed level of precision. In particular, the pre-2002 measurements of \( g_{A} \) gave instead a \( \Gamma_{n}^{\text{SM}} \) value close to the beam experiments [20]. We thereby proceed ignoring this possible third problem.

A proposed solution to the second problem is adding some new light mediator such that \( \chi \) undergoes repulsive interactions (with itself or with neutrons) stronger than the QCD repulsion among neutrons: this would make energetically favourable to avoid producing \( \chi \) in thermal equilibrium inside a neutron star, stiffening the equation of state [34–38]. However this requires QCD-size scattering cross sections that, if \( \chi \) is DM, risk conflicting with the bullet-cluster bound on DM interactions. Again, an alternative that can explain both the neutron anomaly and Dark Matter would be welcome.

In section 2 we discuss how more general models where new physics opens different neutron decay channels affect neutron stars depending on the conserved baryon number \( B_{\chi} \) assigned to \( \chi \). In section 3 we show that the minimal choice \( B_{\chi} = 1/3 \) leads to \( n \to \chi\chi\chi \) decays into free DM \( \chi \) that give a milder modification of neutron stars, compatible with observations. In section 4 we elaborate on the theory behind and compute related processes. In section 5 we show that \( \chi \) can be DM, with abundance possibly generated via freeze-in. In section 6 we show that \( \chi \) DM has unusual signals compatible with current bounds. Conclusions are presented in section 7.

2 Overview of possible models

To motivate the model we will propose, we first classify models where SM particles couple to one new particle, by describing how baryon number \( B \) and lepton number \( L \) can be assigned to the new particle such that they are conserved, in order to satisfy the strong phenomenological bounds on \( B, L \) violations.

\(^1\)Among these models, those where the dark sector that is a mirror copy of the SM also explain the near coincidence in mass needed to fit the anomaly, and were studied earlier because of their intrinsic theoretical interest [24–33].
Table 1. Possible models of a new DM particle that carries baryon and lepton number $B, L$. The left columns lists possible $B, L$ assignments, and the consequent minimal DM spin (a scalar $\phi$ or a fermion $\chi$). The right columns show representative examples of lowest-dimension effective operators that couple DM to the SM conserving $B$ and $L$: here $f$ denotes a generic SM fermion, either a generic quark $q$, or a charged lepton $\ell$ or a neutrino $\nu$. $X$ denotes a derivative or a Higgs doublet $H$ (both have dimension 1). $L$ is the SM lepton doublet that contains a neutrino, so that $LH$ contains $\nu\nu$.

The $L$ and $B$ charges of the new particle determine its spin needed to couple to SM particles via operators with lowest dimension. Since in the SM only fermions carry lepton and baryon number, the new particle must have charge $(-1)^{B+L}$ under the $\mathbb{Z}_2$ symmetry that flips signs of all fermions and leaves bosons unchanged. Table 1 lists the main possibilities, focusing on the minimal spin: either a fermion $\chi$ with spin $1/2$, or a boson $\phi$ with spin $0$ (or, equivalently, two fermions $\chi\chi$). See [39–43] for lists of SM operators, partially relevant for our study.

The apparently minimal choice (upper row of table 1) arises when a fermion $\chi$ carries baryon number $B_\chi = 1$ and vanishing lepton number $L_\chi = 0$. This is the model of [12, 13] where the neutron decays as $n \rightarrow \chi \gamma$. Models that involve a boson $\phi$ not protected by Fermi repulsion worsen the neutron star problem. Models where a fermion $\chi$ carries both lepton and baryon number do not avoid the neutron star problem, as neutrinos freely escape from neutron stars. Independently of the specific particle-physics interactions, the $\chi$ chemical potential $\mu_\chi$ in thermal equilibrium in a neutron star is fixed in terms of the chemical potential of conserved charges (baryon number and vanishing electric charge) as

$$\mu_\chi = B_\chi \mu_n. \quad (2.1)$$

Eq. (2.1) shows that all models affect neutron stars in a qualitatively similar way. Eq. (2.1) also shows how important quantitative differences can arise: reducing $\mu_\chi$ allows to substantially reduce the impact on neutron stars. This is achieved noticing that a more minimal choice of quantum numbers exists, given that baryon number can be fractional, like the
electric charge. In the more minimal model $\chi$ carries baryon number $B = 1/3$ and $L = 0$ (second row of table 1). So neutrons decay as $n \to \chi\chi\chi$ if the new particle is light enough, $M < m_n/3$. This neutron decay can have a small enough impact on neutron stars for the same reason why ordinary neutron decay $n \to pe\bar{\nu}_e$ has a small impact: the neutron decays to particles light enough that their Fermi repulsion is big enough, without the need of introducing extra repulsive interactions.

We thereby focus on this model. The DM mass $M$ is strongly restricted as illustrated in figure 1, where we plot the following key kinematical thresholds:

- One needs $M < m_n/3 \approx 313.19\text{ MeV}$ so that $n \to \chi\chi\chi$ is kinematically open.
- One needs $M > (m_n - E_{\text{Be}})/3 \approx 312.63\text{ MeV}$ so that nuclear decays into $\chi\chi\chi$ are kinematically closed, where $E_{\text{Be}} = 1.664\text{ MeV}$ since the strongest bound comes from $^8\text{Be}$ [12].
- Proton stability gives the weaker bound $M > (m_p - m_e)/3 = 312.59\text{ MeV}$.
- Hydrogen decay $H \to \chi\chi\nu_e$ is kinematically open for $M < (m_p + m_e)/3 \approx 312.93\text{ MeV}$. This part of the allowed parameter space leads to the possible signals discussed later.

The neutron decay anomaly can be explained in the mass range highlighted in green in figure 1, while it cannot be explained in the red shaded range.

### 3 Bounds from neutron stars

Neutron stars are described by the Tolman-Oppenheimer-Volkoff (TOV) equations [19, 44]

\[
\frac{d\varphi}{dr} = \frac{G}{r^2} \frac{(M + 4\pi r^3 \varphi)(\rho + \varphi)}{1 - 2GM/r}, \quad \frac{dM}{dr} = 4\pi r^2 \rho
\]

for the pressure $\varphi(r)$ at radius $r$ and for the mass $M(r)$ inside radius $r$. The TOV equations describe spherical hydrostatic equilibrium in general relativity and can be solved for any equation of state that gives the density $\rho$ in terms of $\varphi$, by starting with an arbitrary pressure at the center at $r = 0$ where $M(0) = 0$, and by evolving outwards until finding the neutron star radius $r = R$ at which $\rho(R) = 0$. Repeating this procedure predicts the relation between the radius $R$ and the total mass $M$, in particular giving a maximal mass $M$ above which neutron stars become unstable and collapse into black holes. In the SM this maximal mass is around two solar masses, compatibly with observations of neutron stars around this mass.
Neutron star equation of state

Figure 2. Left: equations of state for neutron stars in the SM (black curves, considering two different computations of nuclear repulsion: the continuous curve is BSk24 from [45], the dotted curve is $\rho_n = \kappa_0 \rho_n^2$); adding $n \leftrightarrow \chi\chi\chi$ (blue curves); adding $n \leftrightarrow \chi\gamma$ (red curves). Right: the corresponding relation between the radius and mass of neutron stars. This shows that the observed neutron stars with mass around two solar masses are compatible with $n \leftrightarrow \chi\chi\chi$, but not with $n \leftrightarrow \chi\gamma$. The solutions below the peaks at smaller radii are unstable.

Adding new stable particles $\chi$ to neutrons and to the other sub-dominant SM particles, the total energy density and pressure become $\rho = \rho_n + \rho_\chi$ and $\varphi = \varphi_n + \varphi_\chi$. The equation of state with the two components in equilibrium is computed as follows. We assume that the $\chi$ particles are free fermions with mass $M$ and $g = 2$ degrees of freedom, so that their levels are occupied up to their Fermi momentum $p_\chi$. Their number density and pressure are

$$n_\chi = g \int \frac{d^3p}{(2\pi)^3} = \frac{gp_\chi^3}{6\pi^2}, \quad \rho_\chi = g \int \frac{d^3p}{(2\pi)^3} E, \quad \varphi_\chi = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E}. \quad (3.2)$$

Thermal equilibrium of $n \leftrightarrow \chi\chi\chi$ relates the $\chi$ chemical potential $\mu_\chi = \sqrt{M^2 + p_\chi^2}$ to the chemical potential of neutrons as in eq. (2.1), i.e. $\mu_\chi = \mu_n/3.2$

Concerning the SM particles (mostly neutrons), their equation of state is significantly affected by nuclear repulsion, and different approximations give somehow different results [45–50]. We adopt the following computations:

While $n \to \chi\gamma$ gives a larger $\mu_\chi = \mu_n$. Indeed a reaction $i + j \leftrightarrow i' + j'$ fast enough to be in thermal equilibrium implies the relation $\mu_i + \mu_j = \mu_{i'} + \mu_{j'}$ among the chemical potentials, independently of particle-physics details. We recall the thermodynamic relations at $T = 0$ used later. Particles are added with energy $E = \mu$ at the Fermi sphere, so $d\rho = \mu \, dn$. The pressure $\varphi$ is then given by

$$\varphi = -\frac{\partial U}{\partial V} \bigg|_T = n^2 \frac{\partial \rho}{\partial n} \frac{\partial n}{n} = n\mu - \rho \quad (3.3)$$

having used $U = N(E)$ and $N = nV$. 

– 5 –
1. A first, rough but simple, equation of state that includes nuclear effects is \( \rho_n = \kappa_0 \rho_n^2 \) with \( \kappa_0 \approx 52/\text{GeV}^4 \) [46]. The thermodynamic relation \( dn_n/n_n = d\rho_n/(\rho_n + \varphi_n) \) of eq. (3.3) determines \( n_n \) up to a constant, fixed imposing \( \mu_n = d\rho_n/dn_n \rightarrow m_n \) as \( n_n \rightarrow 0 \). The result is

\[
\rho_n = \frac{m_n n_n}{1 - m_n n_n \kappa_0}, \quad \mu_n = \frac{m_n}{(1 - m_n n_n \kappa_0)^2}, \quad \varphi_n = \kappa_0 \rho_n^2. \tag{3.4}
\]

This equation of state is shown by the black dashed curve in figure 2a.

2. The possibly precise BSk24 equation of state from [45], shown by the black curve in figure 2a, agrees well with neutron star observations. We use its numerical form, and present a rough analytic Taylor approximation:

\[ \rho_n = a_1 n_n + a_2 n_n^2 + a_3 n_n^3 \]

with \( a_1 \approx m_n, a_2 \approx 16/\text{GeV}^2 \) and \( a_3 \approx 4800/\text{GeV}^5 \). Then thermodynamic relations imply

\[ \mu_n = m_n + 2a_2 n_n + 3a_3 n_n^2 \quad \text{and} \quad \varphi_n = a_2 n_n^2 + 2a_3 n_n^3. \]

The red curves in figure 2a show how the equations of state computed in the SM get significantly softened if \( n \leftrightarrow \chi \chi \) is in thermal equilibrium (i.e. \( \mu_n = \mu_\chi \)) with \( M = m_n \). The blue curves in figure 2a show how thermal equilibrium of \( n \leftrightarrow \chi \chi \chi \) (i.e. \( \mu_n = 3\mu_\chi \)) with \( M = m_n/3 \) leads to a milder modification of the equation of state, that remains almost as hard as in the SM.

As a result, figure 2b shows that the relation between the neutron star mass \( M \) and radius \( R \) in the presence of \( n \leftrightarrow \chi \chi \chi \) remains close enough to the SM limit (in particular allowing for observed neutron stars with mass \( M \approx 2M_{\text{sun}} \)), unlike what happens if \( n \leftrightarrow \chi \gamma \) is in thermal equilibrium. In particular, while \( n \leftrightarrow \chi \gamma \) reduces the maximal mass of neutron stars in contradiction with data, \( n \leftrightarrow \chi \chi \chi \) leads to a mild reduction comparable to current SM uncertainties. The neutron star radius is reduced in a similarly mild way compatible with data. Different SM computations lead to maximal neutron star masses between \( 1.8M_{\text{sun}} \) and \( 2.6M_{\text{sun}} \) and minimal radii between 10 km and 14 km [47] in apparent

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**Figure 3.** Maximal neutron star mass allowing for multiple \( \chi \) particles with mass \( M \approx m_n/3 \) such that \( n \rightarrow \chi \chi \chi \) is in thermal equilibrium (blue) or mass \( M \approx m_n \) such that \( n \rightarrow \gamma \chi \) is in thermal equilibrium (red). The continuous and dashed curves again refer to two different computations of the neutron equation of state.
agreement with data. Thereby, if the SM can account for observed neutron stars, its $n \to \chi\chi\chi$ extension can too. More precise future computations and observations might be able to test the mild difference.

So far we considered $N = 1$ generation of $\chi$ particles. Figure 3 shows the maximal neutron star mass as function of the number $N$ of $\chi$ generations, assuming for simplicity they all have the same mass. We see that $n \leftrightarrow \chi\chi\chi$ leads to such a small reduction that also $N > 1$ is allowed. The SM corresponds to $N = 0$.

4 Possible theories

Having established that $n \leftrightarrow \chi\chi\chi$ is compatible with neutron star bounds, we study its possible theory. Since conserved $\chi$ number is needed, $\chi$ must be a complex fermion(s) described by Dirac spinor(s) $\Psi = (\chi_L, \bar{\chi}_R)$ containing two Weyl spinors $\chi_L$ and $\chi_R$. The $n \to \chi\chi\chi$ decay arises from 4-fermion effective operators of the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{\Psi} (i\gamma - M) \Psi + \frac{(\bar{\Psi} \Gamma \Psi) (\bar{n} \Gamma \Psi)}{3! A^2_{\chi n}} + h.c. \quad (4.1)$$

at nucleon level. This unusual operator involves the charge-conjugated field $\Psi_c = C \bar{\Psi}^T$ such that the neutron decays into three $\chi$ particles and no $\bar{\chi}$ anti-particles. In the relevant non-relativistic limit $M \lesssim m_n/3$ the decay rate is given by

$$\Gamma_{n\to\chi\chi\chi} \simeq |\mathcal{A}|^2_{n\to\chi\chi\chi} m_n \left(1 - \frac{3M}{m_n}\right)^2 = \frac{m_n^5}{27\pi^3 A^4_{\chi n}} \left\{ \begin{array}{ll}
g_L^2 g_R^2 (1 - 3M/m_n)^3/16 & \text{if } \Gamma = g_L P_L + g_R P_R \\
g_A^2 (1 - 3M/m_n)^3 & \text{if } \Gamma = \gamma_{\mu}(g_V + g_A \gamma_5) \\
O(1 - 3M/m_n)^2 & \text{if } N > 1 \end{array} \right. \quad (4.2)$$

where $\mathcal{A}$ is the $n \leftrightarrow \chi\chi\chi$ decay amplitude. A minimal non-relativistic suppression $|\mathcal{A}|^2_{n\to\chi\chi\chi} \equiv (m_n/A_{\chi n})^4 \epsilon$ with $\epsilon \sim 1 - 3M/m_n$ necessarily arises if the decay is into $N = 1$ $\chi$ generation. This can be seen from non-relativistic quantum mechanics: if the $\chi$ momenta vanish, the Pauli principle demands the fully anti-symmetric product of three $\chi$ spinors (heavy-quark effective theory can be adapted to obtain a systematic expansion [51]). On the other hand, decays involving $N > 1$ generations of $\chi$ particles allow for $\epsilon \sim 1$. We thereby estimate

$$\Gamma_{n\to\chi\chi\chi} \approx \epsilon \frac{m_n^5 (1 - 3M/m_n)^2}{128\pi^3 A^4_{\chi n}} \sim \epsilon \Delta \Gamma \left(\frac{100 \text{ TeV}}{A_{\chi n}}\right)^4 \left(\frac{m_n - 3M}{E_{\rm Be}}\right)^2. \quad (4.3)$$

A lower $A_{\chi n} \approx 30 \text{ TeV}$ is needed if $\epsilon \sim 1 - 3M/m_n$.

The rate of the related visible decay channel with an extra $\gamma$ produced from the neutron magnetic moment interaction is estimated as

$$\Gamma_{n\to\chi\chi\chi\gamma} \approx \frac{4\pi E_{\gamma}^2}{4m_n^2} \Gamma_{n\to\chi\chi\chi} \sim 10^{-9} \Gamma_{n\to\chi\chi\chi}, \quad E_{\gamma} \approx m_n - 3M \quad (4.4)$$

and is safely below the experimental bounds [17].

In the present model $\chi$ can be DM while preserving hydrogen stability: hydrogen decay is kinematically open only in the part of the parameter space with lower $M$, as shown in
In such a case, the hydrogen decay rate can be estimated taking into account weak interactions of neutrons,

\[ \Gamma_{H \to \chi \chi \nu_e} \sim |\psi(0)|^2 G_F^2 E_\nu^2 \frac{\Gamma_{n \to \chi \chi \nu_e}}{m_n} \approx 10^{30} \text{yr} \]  

(4.5)

where \( |\psi(0)|^2 \approx \alpha^3 m_e^3 / \pi \) is the inverse atomic volume, and \( E_\nu \lesssim m_p + m_e - 3M \). Since this dominant hydrogen decay mode is fully invisible, the experimental bound on its rate is comparable to the inverse universe age and largely satisfied. A mildly stronger bound \( \Gamma_{H \to \chi \chi \nu_e} \lesssim 1/(10^{14} \text{yr}) \) arises considering electron \( e p \) capture in the sun [52]. Adding an extra photon gives a sub-dominant but visible hydrogen decay mode. Its rate

\[ \Gamma_{H \to \chi \chi \nu_e \gamma} \approx \frac{\alpha E_\gamma^2}{4 \pi m_n^2} \Gamma_{H \to \chi \chi \nu_e} \approx 10^{39} \text{yr} \]  

(4.6)

is compatible with the bound from BOREXINO, \( \Gamma_{H \to \chi \chi \nu_e \gamma} \lesssim 1/(10^{28} \text{yr}) \) [18, 53].

Coming back to theory, the nucleon-level operator of eq. (4.1) can arise from 6-fermion quark-level operators invariant under SM gauge interactions, such as

\[ \chi \chi \chi d_R \bar{d}_R \bar{u}_R / 3! \Lambda_{QCD}^5 \]

(4.7)

where \( Q = (u, d)_L, d_R, u_R \) are the SM quarks in Weyl notation and we omitted \( \chi \) chiralities, \( \chi \sim \chi_L \sim \bar{\chi}_R \) and Lorentz indices that can be contracted in multiple ways. Lattice computations of the nuclear matrix element \( \langle 0 | (ud)_L R d_L R | n \rangle = \beta_{L,R} n \) find \( |\beta_{L,R}| \approx 0.014 \text{GeV}^3 \sim \Lambda_{QCD}^3 \) [54], and chiral perturbation theory allows to compute interactions with extra pions or photons [55].

In view of their large dimension 9, the operators in eq. (4.7) are strongly suppressed by \( \Lambda_{QCD}^5 \). The neutron-level operator of eq. (4.1) is obtained with coefficient \( 1/\Lambda_{QCD}^2 \), so that the neutron decay anomaly is reproduced for low \( \Lambda_{QCD} \sim 30 \text{GeV} \). Since \( \Lambda_{QCD} \sim 30 \text{GeV} \) is below the weak scale, a mediator below the weak scale is needed. This can be achieved, compatibly with collider bounds, adding for example a neutral fermion \( n' \), singlet under all SM gauge interactions, coupled via dimension-6 operators as

\[ \bar{n}' \chi \chi \chi / \Lambda_{QCD}^2 + \bar{n}' d d u / \Lambda_{\text{dark}}^2 \]  

(4.8)

Baryon number is conserved assigning \( B_{n'} = 1 \) to \( n' \). In turn, the 4-fermion operators in eq. (4.8) can be mediated by renormalizable couplings of bosons, such as a \( W_R \) or a lepto-quark, and by their dark-sector analogous. One gets \( \Lambda_{\text{dark}}^2 = M_{n'} \Lambda_{\chi n'}^2 \Lambda_{\bar{\chi} n'}^2 \), so that the 4-fermion operator \( \bar{n}' d d u \) involving SM particles can be suppressed by a \( \Lambda_{\text{dark}}^2 \) above the weak scale, up to a few TeV, while keeping \( M_{n'} > 1.5 m_n \) in order to avoid conflicting with neutron star bounds [36], and keeping the dark interaction \( \Lambda_{\chi n'} \) above the QCD scale in order to avoid conflicting with bounds on DM self-interactions, \( \sigma / M \lesssim 10^4 / \text{GeV}^3 \) [56, 57].

5 Dark Matter cosmological abundance

To match the observed cosmological DM abundance, the \( \chi \) particles must have abundance \( Y \equiv n / s = 0.44 \text{eV} / M = 1.4 \times 10^{-9} \), where \( n \) is the \( \chi + \bar{\chi} \) number density and
\[ s = 2\pi^3 d_{SM} T^3 / 45 \] is the entropy density of the thermal bath with \( d_{SM}(T) \) degrees of freedom at temperature \( T \).

The neutron decay anomaly is reproduced for \( \Lambda_{\chi n} \gg v \), so the interaction rates of \( \chi \) particles at \( T \sim m_n \) are below electroweak rates and thereby below the Hubble rate: thermal freeze-out is not possible.\(^3\) For the same reason, no equilibration of asymmetries happens at \( T \lesssim m_n \).

We thereby consider freeze-in, assuming vanishing initial \( \chi \) abundance and computing the multiple contributions it receives from particle-physics processes. We avoid entering into model-dependent mediator issues, and explore the effect of the interaction with neutrons in eq. (4.1), for example by assuming that the reheating temperature is below the mediator masses. At leading order, neutrons and anti-neutrons that decay at \( T \sim m_n \) close to thermal equilibrium\(^4\) contribute to the DM abundance as

\[ Y = \frac{405 \sqrt{5} M_{Pl}}{4\pi^{3/2} d_{SM}^{3/2} m_n^2} 3 \Gamma_{n \rightarrow \chi \chi \chi} R \approx 10^{-13} \] (5.1)

where \( R \sim e^{-\Lambda_{QCD}/m_n} \) corrects the freeze-in formula taking into account that neutrons only form after the QCD phase transition at \( T \lesssim \Lambda_{QCD} \). This \( Y \) is about \( 10^4 \) smaller than the cosmological DM abundance. Three different effects provide the needed enhancement.

1. First, thermal scatterings such as \( \pi^0 n \leftrightarrow \chi \chi \chi \) and \( \gamma n \leftrightarrow \chi \chi \chi \) arise at higher orders in the QCD or QED coupling \( g \), and avoid the non-relativistic suppression of \( n \leftrightarrow \chi \chi \chi \). This suppression was computed in eq. (4.3): two powers of \( 1 - 3M/m_n \approx \text{few} \times 10^{-3} \) arise from the phase space, and possibly one extra power arises from the squared amplitude. Thereby, at the relevant temperature \( T \sim m_n \), the scattering rates are enhanced by a factor of order \( g^2 (T/E_{Be})^2 \sim 10^{4-6} \) compared to the \( \Gamma_{n \rightarrow \chi \chi \chi} \) rate. Indeed these finite-temperature scatterings can be partially accounted by a \( n \leftrightarrow \chi \chi \chi \) rate enhanced by taking into account the thermal contribution to the neutron squared mass, \( m_n^2 + O(g^2 T^2) \).

2. Second, \( n^* \rightarrow \chi \chi \chi \) decays of the excited neutron \( n^* \) with mass \( m_{n^*} \approx 1.44 \text{ GeV} \) and of other QCD resonances similarly avoid the non-relativistic suppression of \( n \rightarrow \chi \chi \chi \). Dedicated lattice computations are needed to confirm that these resonances have matrix elements similar to those of the neutron. We expect that their effect is numerically similar to thermal scatterings at point 1.

3. Third, the processes \( \bar{\chi} n \leftrightarrow \chi \chi \) and \( \chi \bar{n} \leftrightarrow \bar{\chi} \bar{\chi} \) have similar phase-space unsuppressed rates, and increase the number of DM particles that have already been produced, adding \( \bar{\chi} \) anti-particles. Their effect is numerically comparable to those at previous points.

\(^3\)Ignoring the neutron decay anomaly, one could consider the regime \( m_n/3 < M < m_n \) where neutron decay is kinematically closed, allowing for larger DM couplings.

\(^4\)We can neglect the later out-of-equilibrium decays at BBN of neutrons that remained thanks to the baryon asymmetry: as the neutron BR into DM is about \( 1\% \), their freeze-in contribution to the DM abundance is too small.
A precise computation is not possible since non-perturbative QCD effects determine the rates at point 2. Furthermore, larger but model-dependent contributions to freeze-in can arise at higher temperature, if the reheating temperature is higher than $\Lambda_{QCD}$.

6 Dark Matter signals

The $\tilde{\chi} n \leftrightarrow \chi \chi$ process at point 3 leads to unusual DM direct detection signals: it is kinematically open today and the non-relativistic $\tilde{\chi}$ and $n$ in its initial state produce relativistic $\chi$ with energy $E = 2m_n/3$. The $\tilde{\chi} n \leftrightarrow \chi \chi$ cross section avoids the non-relativistic suppression of $n \rightarrow \chi \chi \chi$ and is thereby estimated as

$$\sigma_{\tilde{\chi} n \rightarrow \chi \chi} \approx \frac{m_n^2}{4\pi\Lambda_{\chi n}^4} \sim 10^{-46} \text{ cm}^2 \left(\frac{20 \text{ TeV}}{\Lambda_{n}}\right)^4.$$  \hspace{1cm} (6.1)

This is below current bounds from direct detection experiments for the value of $\Lambda_{\chi n}$ motivated by the neutron decay anomaly. No nuclear enhancement arises, since the energies involved in $\tilde{\chi} n \leftrightarrow \chi \chi$ are higher than the tens of keV produced by usual DM-induced nuclear recoils. For the same reason, bigger experiments dedicated to more energetic signals produced by neutrinos and proton decay have better sensitivity. Following their practice, we convert eq. (6.1) into the event rate per neutron as

$$\tau = \frac{\rho_{\odot}}{2M} v\sigma_{\tilde{\chi} n \rightarrow \chi \chi} = \frac{1}{2.5 \times 10^{31} \text{ yr}} \frac{\sigma_{\tilde{\chi} n \rightarrow \chi \chi}}{10^{-46} \text{ cm}^2} \frac{\rho_{\odot}}{0.4 \text{ GeV/cm}^3} \frac{v}{200 \text{ km/s}} \frac{m_n/3}{M} \quad (6.2)$$

having assumed an equal number of $\chi$ and $\tilde{\chi}$. We thereby have the following signals:

1. **DM scatterings that lead to the disappearance of a neutron into invisible DM.** These scatterings effectively make ordinary matter radioactive with life-time given by eq. (6.2), since a neutron that disappears within a nucleus leaves a hole, triggering nuclear de-excitations and decays. For example DM scatterings convert water $^{16}\text{O}_8$ into $^{15}\text{O}_8$, that de-excites emitting $\gamma$ rays with tens of MeV, and decays a few minutes later into $^{15}\text{N}_7 e^+\bar{\nu}_e$ emitting a $\sim$ MeV positron. Similar processes happen with $^{12}\text{C}_6$. The experimental bound on DM-induced neutron disappearance is approximatively obtained recasting the bounds on invisible neutron decays from experiments done with $^{16}\text{O}_8$ or $^{12}\text{C}_6$:

$$\tau(n \rightarrow \text{invisible}) > \begin{cases} 4.9 \times 10^{26} \text{ yr from Kamiokande} \ [58] \\ 2.5 \times 10^{29} \text{ yr from SNO} \ [59] \\ 5.8 \times 10^{29} \text{ yr from KamLand} \ [60] \end{cases} \quad (6.3)$$

The stronger SuperKamiokande [61] bound on $np \rightarrow e^+\nu$ cannot be recasted into a bound on neutron disappearance, as the experimental search triggered on more energetic positrons with $p > 100 \text{ MeV}$.

2. **DM scatterings that lead to the disappearance of a neutron into invisible DM plus visible SM particles.** Processes like $\tilde{\chi} n \rightarrow \chi \chi \gamma$ and $\tilde{\chi} n \rightarrow \chi \chi \pi^0 \rightarrow \chi \chi \gamma \gamma$ arise at higher
orders and thereby have cross sections (and consequently effective neutron decay rates) smaller by \(\alpha/4\pi \sim 10^{-2\sim 3}\) compared to the fully invisible neutron disappearance. The extra photons have energy around 100 MeV. While a precise recasting needs computing the photon spectra, these processes are loosely similar to \(n \to \nu\gamma\), subject to the bound

\[\tau(n \to \nu\gamma) > 5.5 \times 10^{32} \text{ yr} \quad \text{from SuperKamiokande} \quad [61] \quad (6.4)\]

that is thereby satisfied.

Concerning indirect detection signals, extra interactions not needed by the model might lead to \(\chi\bar{\chi}\) annihilations into pairs of SM particles.

7 Conclusions

Past literature showed that the interpretation of the neutron decay anomaly in terms of \(n \to \chi\gamma\) decays into new free invisible particles \(\chi\) with baryon number \(B_\chi = 1\) implies their thermalization inside neutron stars, and that the modified equation of state is in contradiction with observed neutron stars.

We found that the neutron star issue is more general, affecting all possible models listed in table 1 where we considered generic \(\chi\) charges under conserved baryon and lepton numbers, \(B_\chi\) and \(L_\chi\). Independently of particle-physics details, the \(\chi\) chemical potential in thermal equilibrium is \(\mu_\chi = B_\chi\mu_n\). Neutron star bounds are found to be compatible with the minimal possibility that \(\chi\) carries the smallest baryon number \(B_\chi = 1/3\), so that the neutron decay anomaly can be accounted by \(n \to \chi\chi\chi\) decays. The resulting modification of neutron stars, shown in figure 2, is comparable with SM uncertainties. Figure 3 shows that neutron stars get modified in a specific and mild enough way even in the presence of multiple generations of \(\chi\) particles, thereby providing a signal for improved future observations and computations.

In section 4 we presented possible theories for \(n \to \chi\chi\chi\), showing that associated processes are compatible with bounds from colliders, on \(n \to \chi\chi\gamma\) decays, and on hydrogen decays into \(\chi\chi\nu_e\) or \(\chi\chi\nu_e\gamma\) (kinematically open only in a part of parameter space shown in figure 1).

We next found that \(\chi\) can be Dark Matter. In section 5 we showed that the cosmological DM abundance can be possibly matched by freeze-in production of \(\chi\) after the QCD phase transition at \(T \sim m_n\). Three processes contribute: \(\pi^0 n \leftrightarrow \chi\chi\chi\) and \(\gamma n \leftrightarrow \chi\chi\chi\) thermal scatterings; decays of excited neutrons; \(\chi\bar{n} \leftrightarrow \bar{\chi}\bar{\chi}\) and \(\bar{\chi}n \leftrightarrow \chi\bar{\chi}\) scatterings. In section 6 we found that \(\bar{\chi}n \leftrightarrow \chi\chi\) provide unusual DM signals with interesting rates below present bounds. Such DM scatterings lead to neutron disappearance, that effectively makes matter (such as water, carbon, etc) radioactive, giving signatures similar to those of neutron invisible decays. Predictions are compatible with bounds from DM detection experiments and from neutrino experiments such as SNO and KAMLAND. Furthermore, the higher order related processes with extra visible photons, \(\bar{\chi}n \to \chi\chi\gamma\) and \(\bar{\chi}n \to \chi\chi\pi^0 \to \chi\chi\gamma\gamma\), are compatible with Super-Kamiokande bounds.
Acknowledgments

This work was supported by MIUR under PRIN 2017FNJFMW and by the ERC grant 669668 NEO-NAT. We thank Zurab Berezhiani, Raghuveer Garani, Benjamin Grinstein, Masayuki Nakahata, Michele Redi and Daniele Teresi for discussions.

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References

[1] A. Serebrov et al., Measurement of the neutron lifetime using a gravitational trap and a low-temperature Fomblin coating, Phys. Lett. B 605 (2005) 72 [nucl-ex/0408009] [inSPIRE].

[2] A. Pichlmair, V. Varlamov, K. Schreckenbach and P. Geltenbort, Neutron lifetime measurement with the UCN trap-in-trap MAMBO II, Phys. Lett. B 693 (2010) 221 [inSPIRE].

[3] A. Steyerl, J.M. Pendlebury, C. Kaufman, S.S. Malik and A.M. Desai, Quasielastic scattering in the interaction of ultracold neutrons with a liquid wall and application in a reanalysis of the Mambo I neutron-lifetime experiment, Phys. Rev. C 85 (2012) 065503 [inSPIRE].

[4] V.F. Ezhov et al., Measurement of the neutron lifetime with ultra-cold neutrons stored in a magneto-gravitational trap, JETP Lett. 107 (2018) 671 [Pisma Zh. Eksp. Teor. Fiz. 107 (2018) 707] [arXiv:1412.7434] [inSPIRE].

[5] S. Arzumanov et al., A measurement of the neutron lifetime using the method of storage of ultracold neutrons and detection of inelastically up-scattered neutrons, Phys. Lett. B 745 (2015) 79 [inSPIRE].

[6] R.W. Pattie, Jr. et al., Measurement of the neutron lifetime using a magneto-gravitational trap and in situ detection, Science 360 (2018) 627 [arXiv:1707.01817] [inSPIRE].

[7] A.P. Serebrov et al., Neutron lifetime measurements with a large gravitational trap for ultracold neutrons, Phys. Rev. C 97 (2018) 055503 [arXiv:1712.05663] [inSPIRE].

[8] UCNr collaboration, Improved neutron lifetime measurement with UCNr, Phys. Rev. Lett. 127 (2021) 162501 [arXiv:2106.10375] [inSPIRE].

[9] J. Byrne and P.G. Dawber, A revised value for the neutron lifetime measured using a Penning trap, Europhys. Lett. 33 (1996) 187 [inSPIRE].

[10] J.S. Nico et al., Measurement of the neutron lifetime by counting trapped protons in a cold neutron beam, Phys. Rev. C 71 (2005) 055502 [nucl-ex/0411041] [inSPIRE].

[11] A.T. Yue et al., Improved determination of the neutron lifetime, Phys. Rev. Lett. 111 (2013) 222501 [arXiv:1309.2623] [inSPIRE].

[12] B. Fornal and B. Grinstein, Dark matter interpretation of the neutron decay anomaly, Phys. Rev. Lett. 120 (2018) 191801 [Erratum ibid. 124 (2020) 219901] [arXiv:1801.01124] [inSPIRE].

[13] B. Fornal and B. Grinstein, Neutrons' dark secret, Mod. Phys. Lett. A 35 (2020) 2030019 [arXiv:2007.13931] [inSPIRE].
[14] Z. Berezhiani, *Unusual effects in n-n' conversion. Sci-fi in two parts*, talk at Institute for Nuclear Theory, Seattle, WA, U.S.A., 22–27 October 2017.

[15] A.P. Serebrov and O.M. Zherebtsov, *Trap with ultracold neutrons as a detector of dark matter particles with long-range forces*, Astron. Lett. 37 (2011) 181 [arXiv:1004.2981] [INSPIRE].

[16] S. Rajendran and H. Ramani, *Composite solution to the neutron lifetime anomaly*, Phys. Rev. D 103 (2021) 035014 [arXiv:2008.06061] [INSPIRE].

[17] Z. Tang et al., *Search for the neutron decay n → X + γ where X is a dark matter particle*, Phys. Rev. Lett. 121 (2018) 022505 [arXiv:1802.01595] [INSPIRE].

[18] Borexino collaboration, *A test of electric charge conservation with Borexino*, Phys. Rev. Lett. 115 (2015) 231802 [arXiv:1509.01223] [INSPIRE].

[19] J.R. Oppenheimer and G.M. Volkoff, *On massive neutron cores*, Phys. Rev. 55 (1939) 374 [INSPIRE].

[20] A. Czarnecki, W.J. Marciano and A. Sirlin, *Neutron lifetime and axial coupling connection*, Phys. Rev. Lett. 120 (2018) 202002 [arXiv:1802.01804] [INSPIRE].

[21] Z. Berezhiani, *Neutron lifetime puzzle and neutron-mirror neutron oscillation*, Eur. Phys. J. C 79 (2019) 484 [arXiv:1807.07906] [INSPIRE].

[22] Z. Berezhiani, R. Biondi, M. Mannarelli and F. Tonelli, *Neutron-mirror neutron mixing and neutron stars*, Eur. Phys. J. C 81 (2021) 1036 [arXiv:2012.15233] [INSPIRE].

[23] Z. Berezhiani, D. Comelli and F.L. Villante, *The early mirror universe: inflation, baryogenesis, nucleosynthesis and dark matter*, Phys. Lett. B 503 (2001) 362 [hep-ph/0008105] [INSPIRE].

[24] T.D. Lee and C.-N. Yang, *Question of parity conservation in weak interactions*, Phys. Rev. 104 (1956) 254 [INSPIRE].

[25] I.Y. Kobzarev, L.B. Okun and I.Y. Pomeranchuk, *On the possibility of experimental observation of mirror particles*, Sov. J. Nucl. Phys. 3 (1966) 837 [Yad. Fiz. 3 (1966) 1154] [INSPIRE].

[26] S.I. Blinnikov and M. Khlopov, *Possible astronomical effects of mirror particles*, Sov. Astron. 27 (1983) 371 [Astron. Zh. 60 (1983) 632] [INSPIRE].

[27] E.W. Kolb, D. Seckel and M.S. Turner, *The shadow world*, Nature 314 (1985) 415 [INSPIRE].

[28] R. Foot, H. Lew and R.R. Volkas, *Possible consequences of parity conservation*, Mod. Phys. Lett. A 7 (1992) 2567 [INSPIRE].

[29] H.M. Hodges, *Mirror baryons as the dark matter*, Phys. Rev. D 47 (1993) 456 [INSPIRE].

[30] Z.G. Berezhiani, A.D. Dolgov and R.N. Mohapatra, *Asymmetric inflationary reheating and the nature of mirror universe*, Phys. Lett. B 375 (1996) 26 [hep-ph/9511221] [INSPIRE].

[31] Z.G. Berezhiani, *Astrophysical implications of the mirror world with broken mirror parity*, Acta Phys. Polon. B 27 (1996) 1503 [hep-ph/9602326] [INSPIRE].

[32] R. Foot, *A dark matter scaling relation from mirror dark matter*, Phys. Dark Univ. 5-6 (2014) 236 [arXiv:1303.1727] [INSPIRE].

[33] R. Foot, *Mirror dark matter: cosmology, galaxy structure and direct detection*, Int. J. Mod. Phys. A 29 (2014) 1430013 [arXiv:1401.3965] [INSPIRE].
[34] Z. Berezhiani and L. Bento, Neutron-mirror neutron oscillations: how fast might they be?, *Phys. Rev. Lett.* 96 (2006) 081801 [hep-ph/0507031] [insPIRE].

[35] D. McKeen, A.E. Nelson, S. Reddy and D. Zhou, Neutron stars exclude light dark baryons, *Phys. Rev. Lett.* 121 (2018) 061802 [arXiv:1802.08244] [insPIRE].

[36] G. Baym, D.H. Beck, P. Geltenbort and J. Shelton, Testing dark decays of baryons in neutron stars, *Phys. Rev. Lett.* 121 (2018) 061801 [arXiv:1802.08282] [insPIRE].

[37] T.F. Motta, P.A.M. Guichon and A.W. Thomas, Implications of neutron star properties for the existence of light dark matter, *J. Phys. G* 45 (2018) 05LT01 [arXiv:1802.08427] [insPIRE].

[38] B. Grinstein, C. Kouvaris and N.G. Nielsen, Neutron star stability in light of the neutron decay anomaly, *Phys. Rev. Lett.* 123 (2019) 091601 [arXiv:1811.06546] [insPIRE].

[39] F. del Aguila, S. Bar-Shalom, A. Soni and J. Wudka, Heavy Majorana neutrinos in the effective Lagrangian description: application to hadron colliders, *Phys. Lett. B* 670 (2009) 399 [arXiv:0806.0876] [insPIRE].

[40] L. Lehman, Extending the Standard Model effective field theory with the complete set of dimension-7 operators, *Phys. Rev. D* 90 (2014) 125023 [arXiv:1410.4193] [insPIRE].

[41] H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu and Y.-H. Zheng, Complete set of dimension-eight operators in the Standard Model effective field theory, *Phys. Rev. D* 104 (2021) 015026 [arXiv:2005.00008] [insPIRE].

[42] H.-L. Li, Z. Ren, M.-L. Xiao, J.-H. Yu and Y.-H. Zheng, Complete set of dimension-nine operators in the Standard Model effective field theory, *Phys. Rev. D* 104 (2021) 015025 [arXiv:2007.07899] [insPIRE].

[43] Y. Liao and X.-D. Ma, An explicit construction of the dimension-9 operator basis in the Standard Model effective field theory, *JHEP* 11 (2020) 152 [arXiv:2007.08125] [insPIRE].

[44] M.M. Forbes, S. Bose, S. Reddy, D. Zhou, A. Mukherjee and S. De, Constraining the neutron-matter equation of state with gravitational waves, *Phys. Rev. D* 100 (2019) 083010 [arXiv:1904.04233] [insPIRE].

[45] D. Logoteta, A. Perego and I. Bombaci, Microscopic equation of state of hot nuclear matter for numerical relativity simulations, *Astron. Astrophys.* 646 (2021) A55 [arXiv:2012.03599] [insPIRE].
[51] M. Neubert, Heavy quark symmetry, Phys. Rept. 245 (1994) 259 [hep-ph/9306320] [inSPIRE].

[52] Z. Berezhiani, Neutron lifetime and dark decay of the neutron and hydrogen, LHEP 2 (2019) 118 [arXiv:1812.11089] [inSPIRE].

[53] D. McKeen and M. Pospelov, How long does the hydrogen atom live?, arXiv:2003.02270 [inSPIRE].

[54] Y. Aoki, T. Izubuchi, E. Shintani and A. Soni, Improved lattice computation of proton decay matrix elements, Phys. Rev. D 96 (2017) 014506 [arXiv:1705.01338] [inSPIRE].

[55] M. Claudson, M.B. Wise and L.J. Hall, Chiral Lagrangian for deep mine physics, Nucl. Phys. B 195 (1982) 297 [inSPIRE].

[56] D. Clowe et al., A direct empirical proof of the existence of dark matter, Astrophys. J. Lett. 648 (2006) L109 [astro-ph/0608407] [inSPIRE].

[57] F. Kahlhoefer, K. Schmidt-Hoberg, M.T. Frandsen and S. Sarkar, Colliding clusters and dark matter self-interactions, Mon. Not. Roy. Astron. Soc. 437 (2014) 2865 [arXiv:1308.3419] [inSPIRE].

[58] Kamiokande collaboration, Study of invisible nucleon decay, $n \rightarrow \nu\nu\bar{\nu}$, and a forbidden nuclear transition in the Kamiokande detector, Phys. Lett. B 311 (1993) 357 [inSPIRE].

[59] SNO+ collaboration, Search for invisible modes of nucleon decay in water with the SNO+ detector, Phys. Rev. D 99 (2019) 032008 [arXiv:1812.05552] [inSPIRE].

[60] KamLAND collaboration, Search for the invisible decay of neutrons with KamLAND, Phys. Rev. Lett. 96 (2006) 101802 [hep-ex/0512059] [inSPIRE].

[61] Super-Kamiokande collaboration, Search for nucleon and dimuon decays with an invisible particle and a charged lepton in the final state at the Super-Kamiokande experiment, Phys. Rev. Lett. 115 (2015) 121803 [arXiv:1508.05530] [inSPIRE].