Dynamic compliances of rigid foundation on layered poroelastic soils

Suraparb Keawsawasvong¹, Teerapong Senjuntichai¹*, Rawiphas Plangmal¹

¹Applied Mechanics and Structures Research Unit, Department of Civil Engineering, Faculty of Engineering, Chulalongkorn University, Bangkok 10330

Email: Teerapong.S@chula.ac.th

Abstract. Dynamic interaction between a rigid foundation and layered poroelastic soils is studied in this paper. The foundation is subjected to time-harmonic vertical horizontal and moment loading, and bonded to a layered poroelastic half-space. The interaction problem is formulated by employing a discretization technique based on the rigid body displacement boundary conditions at the foundation-soil contact surface and the displacement influence functions of a layered poroelastic half-space. The discretization method used in this paper yields a flexibility equation system for the contact traction at the foundation-soil contact area. Comparison with existing solution for a rigid square foundation on an elastic half-space is presented to verify the accuracy of the present scheme. Selected numerical results for compliances of rigid square foundations are presented to portray the influence of poroelastic effects on the dynamic interaction problem.

1. Introduction

The classical problem concerned with dynamic interaction between foundations and supporting soils has a rich history in geomechanics due to its close relevance to various practical problems in geotechnical engineering such as the analysis and design of pavement, mat and raft foundations, anchor plates and theoretical modelling of some in-situ tests. In the past, several researchers have employed a variety of techniques to study dynamic interaction between rigid foundations and elastic soils. For example, Wong and Luco [1] presented time-harmonic response of a rigid rectangular foundation resting on an elastic half-space under vertical, rocking and horizontal motions. Vibrations of rigid rectangular foundations bonded to an elastic medium subjected to excitations from horizontal waves were considered by Wong and Luco [2]. In addition, Shahi and Noorzad [3] presented dynamic impedances of rigid foundations of arbitrary shape resting on a semi-infinite elastic medium.

Geomaterials often consist of two phases, i.e. solid and voids filled with water, and they are commonly known as poroelastic materials. These materials are considered to be more suitable representation of soils and rocks than ideal elastic materials. The theory of elastic wave propagations in a poroelastic medium was presented by Biot [4, 5], and it has been widely employed by several researchers to study a variety of practical problems related to poroelastic materials including dynamic interaction between foundations and poroelastic soils (e.g., Halpern and Christiano [6], Kassir and Xu [7], Rajapakse and Senjuntichai [8], Zeng and Rajapakse [9], Senjuntichai and Kaewjuea [10], Senjuntichai et al. [11]). In this paper, dynamic compliances of a square foundation bonded to the surface of layered poroelastic soils are determined. The foundation is assumed to be massless and rigid, and subjected to time-harmonic vertical, horizontal, and moment loading. The supporting soils under consideration consist...
of a poroelastic layer perfectly bonded to an underlying poroelastic half-space, and are governed by Biot’s poroelastodynamics theory. A computer program based on the proposed solution scheme has been developed, and its accuracy is confirmed by comparing with existing solutions. Selected numerical results are presented to demonstrate the influence of poroelastic effects on dynamic compliances of rigid square foundation on different layered poroelastic soils.

2. Problem description
Consider a layered poroelastic half-space with a Cartesian coordinate system as shown in Fig. 1. Let $u_i$ and $w_i$ denote the average displacement of the solid matrix and the fluid displacement relative to the solid matrix in the $i$-direction ($i = x,y,z$) respectively. The constitutive relation of a homogeneous poroelastic medium can then be expressed according to Biot [12] by using the standard indicial notation as,

\begin{align}
\sigma_{ij} &= 2\mu \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk} - \alpha \delta_{ij} p , \quad i, j = x, y, z \\
p &= -M \left( \alpha \varepsilon_{kk} + w_i \right)
\end{align}

(1a)
(1b)

In the above equations, $\sigma_{ij}$ is the total stress component of the bulk material; $\varepsilon_{ij}$ is the strain component, which is related to the displacement $u_i$ as in ideal elasticity; $\mu$ and $\lambda$ are Lame’ constants of the bulk material; $\delta_{ij}$ is the Kronecker delta; and $p$ is the excess pore fluid pressure (suction is considered negative). In addition, $\alpha$ and $M$ are Biot’s parameters accounting for compressibility of the two-phased material.

![Figure 1. A square foundation on a layered poroelastic half-space.](image1)

![Figure 2. Applied time-harmonic loading on a square foundation.](image2)

A square foundation perfectly bonded to a layered poroelastic half-space with fully permeable contact surface is shown in Fig 1. The foundation is assumed to be massless and rigid, and subjected to time-harmonic vertical, horizontal, and moment loading with the factor of $e^{i\omega t}$, where $\omega$ is the frequency of excitation. The amplitudes of loads and displacements, which are also assumed to be time-harmonic with the factor of $e^{i\omega t}$, are shown in Fig. 2. The discretization technique is employed in the formulation of this dynamic interaction problem. The proposed technique requires a set of influence functions, which are the displacements on the surface of a layered poroelastic half-space under uniform surface traction of unit intensity applied in the $i$-direction ($i = x,y,z$). These influence functions are determined in the Fourier transform space by employing an exact stiffness matrix scheme presented by Senjuntichai et al. [13] and the details of this scheme are omitted here for brevity. The obtained influence functions are then employed in the formulation of dynamic interaction between a rigid rectangular foundation and a layered poroelastic half-space outlined in the next section.
3. Formulation of interaction problem
Consider a rigid square foundation of size $2H \times 2H$ on the surface of a layered poroelastic half-space illustrated in Fig. 1 subjected to the time-harmonic loading as shown in Fig. 2. In addition, the contact condition between the foundation and the supporting medium is assumed to be perfectly bonded and fully permeable. Under this condition, the traction in the $i$-direction ($i = x, y, z$), denoted by $T_i$, on the contact area is unknown. The displacements of rigid foundation $U_i$ ($i = x, y, z$) can be expressed in terms of three translation and three rotation components as,

$$U_x = \Delta_x - \theta_y, U_y = \Delta_x - \theta_z, U_z = \Delta_x + \theta_z - \theta_x$$

(2)

On $|x| \leq H$ and $|y| \leq H$. In addition, $\Delta_x$ represents the displacement amplitude in the $i$-direction ($i = x, y, z$) at the centre of the foundation, and $\theta_i$ denotes the amplitude of the rotation about the $i$ axis ($i = x, y, z$) respectively. These equations can be represented by the following matrix equation:

$$U = \Omega \Delta$$

(3)

where $\mathbf{U} = \{U_x, U_y, U_z\}^T, \Delta = \{\Delta_x, \Delta_y, \Delta_z, \theta_x, \theta_y, \theta_z\}^T$ and $\Omega$ is a $3 \times 6$ matrix, defined as,

$$\Omega = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -y \\ 0 & 1 & 0 & 0 & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}$$

(4)

Let $A$ be the square contact area between the foundation and the layered half-space. The equilibrium equations of forces applied to the rigid foundation shown in Fig. 2 can then be expressed as,

$$F_i = -\int_A T_i dA \quad (i = x, y, z), M_x = -\int_A T_y dA$$

(5a)

$$M_y = -\int_A T_x dA, M_z = -\int [T_x - T_y] dA$$

(5b)

In the present study, the discretization technique [1] is adopted to solve for the unknown contact traction $T_i$ $(i = x, y, z)$. The contact area is divided into a total number of $N_e$ square elements, in which $N_e = N_x \times N_y$, and $N_x$ and $N_y$ represent the number of elements in the $x$ and $y$ directions respectively. Nodal points are located at the centre of each discretized element. It is assumed that the unknown contact traction $T_i$ $(i = x, y, z)$ is constant within each element. The foundation displacement $U_i$ $(i = x, y, z)$ at the discretized element with the nodal point at $(x_n, y_n, 0)$ can then be expressed as,

$$U_i \left(x_n, y_n, 0\right) = \sum_{n=1}^{N_e} \sum_{m=1}^{N_e} G_{ij} \left(x_n, y_n, x_m, y_m, 0\right) T_j \left(x_m, y_m\right)$$

(6)

where $G_{ij} \left(x_n, y_n, x_m, y_m\right)$ denote the displacement in the $i$-direction ($i = x, y, z$) at the nodal point with the coordinate $(x_n, y_n, 0)$ of the layered half-space due to uniform traction of unit intensity applied in the $j$-direction ($j = x, y, z$) over the discretized element with the nodal location at $(x_m, y_m, 0)$. The influence function $G_{ij}$ are obtained from the exact stiffness matrix scheme [13]. Eq. (6) can then be expressed in the following matrix form:

$$G T = U$$

(7)

where the vector $U$ is defined in Eq. (3), and the elements of the matrices $G$ and $T$ correspond to $G_{ij}$ and the contact traction at the discretized elements of the contact area respectively. Finally, the dynamic response of a rigid square foundation is characterized by the following relationship:

$$\Delta = CF$$

(8)

where
The matrix $C$ given by Eq. (9) is a $6 \times 6$ symmetric complex frequency-dependent compliance matrix and its elements are defined as follows: $C_v = \mu^{(1)} H \Delta_z / F$ for vertical compliance; $C_h = \mu^{(1)} H \Delta_z / F$ for horizontal compliance; $C_m = \mu^{(1)} \rho \theta / M$ for rocking compliance; $C_{hm} = \mu^{(1)} H^2 \Delta_z / M$ for coupling compliance; $C_c = \mu^{(1)} H \theta / M$ for torsional compliance.

4. Numerical results and discussion

In this study, an adaptive numerical quadrature scheme with a 21-point Gauss–Kronrod rule is employed to evaluate the semi-infinite integral for the inversion of Fourier integral transform. A non-dimensional frequency $\delta = \omega H \sqrt{\rho^{(1)} / \mu^{(1)}}$ is used in the numerical results presented in this paper, where $\rho^{(1)}$ and $\mu^{(1)}$ are mass density and shear modulus of the upper layer respectively. Fig. 3 shows the comparison of dynamic compliances of a rigid square foundation ($2H \times 2H$) resting on a homogenous elastic half-space with Poisson’s ratio of 0.33 between the present study with $N_x = 64$ elements ($N_x = N_y = 8$) and the solution by Wong and Luco [2]. It is evident from Fig. 3 that all compliances obtained from the present study agree very closely with the existing solutions [2].

![Figure 3](image)

**Figure 3.** Comparison of compliances of a square foundation on an elastic half-space

The dynamic interaction between a rigid square foundation of size $2H \times 2H$ and a layered poroelastic half-space is investigated in the remaining of this paper. Non-dimensional dynamic compliances are presented for the square foundation bonded to a layered poroelastic half-space consisting of an upper layer of finite thickness $h = H$ and an underlying half-space. Three systems are considered in this set of numerical results, namely System A, System B and System C. The material properties for both upper layer and underlying half-space of all three systems are given as follows: $\lambda = \mu = 2 \times 10^8$ N/m$^2$; $M = 24.4 \times 10^8$ N/m$^2$; $\rho = 2 \times 10^3$ N/m$^2$; $\rho_f = 1.06 \times 10^3$ N/m$^2$; $m = 2.2 \times 10^3$ N/m$^2$; and $\alpha = 0.97$, where $\rho$ and $\rho_f$ are the mass densities of the bulk material and the pore fluid respectively; and $m$ is a density-like parameter that depends on $\rho_f$ and the geometry of the pores. In addition, the parameter $b$, which takes into account the internal friction due to the relative motion between the solid matrix and the pore fluid, is set to be different in the upper layers of the three systems. This parameter is found to have the most significant influence on compliances of rigid foundation in poroelastic media when compared to other poroelastic material parameters [9]. The values of $b$ in the upper layers of System A,
System B and System C are equal to $6.32 \times 10^2$ N/m$^2$, $1.45 \times 10^6$ N/m$^2$, and $6.32 \times 10^6$ N/m$^2$ respectively, whereas the values of $b$ for the underlying half-spaces of all three systems are identical, and they are equal to $1.45 \times 10^6$ N/m$^2$. Thus, System B is a homogeneous poroelastic half-space.

Figs. 4 and 5 display the vertical ($C_v$) and horizontal ($C_h$) compliances of the rigid square foundation resting on the three systems. The rocking ($C_m$) and torsional ($C_t$) compliances of the square footing are shown in Figs. 6 and 7 respectively. In addition, the coupling compliance ($C_{hm}$) is presented in Fig. 8.

Numerical results shown in Figs. 4 to 8 indicate that all compliance components of the surface square footing vary smoothly with $\delta$ over the frequency range considered in the numerical study. The real part of $C_v$ and $C_h$ decreases with frequency over the range $0 < \delta < 4$ whereas those of $C_m$, $C_t$, and $C_{hm}$ increase with increasing $\delta$ at low frequencies. In addition, the imaginary parts of all compliance components increase with the frequency in the low frequency range, and remain negative over the range $0 < \delta < 4$.

Comparison of the compliances for Systems A, B and C indicates substantial difference in the obtained values. Note that the difference between the three systems under consideration is in the value of $b$ in the upper layer. The parameter $b$ is related to the internal friction between solid matrix and pore fluid, and it is inversely proportional to permeability. The material with lower value of $b$ is then more permeable with the one with higher $b$. Numerical results presented in Figs. 4 to 8 reveal that both real and imaginary parts of all compliance components in System A are higher than the corresponding values in System B and System C indicating that the system with more permeable upper layer, i.e. System A, is less stiff and less damped than the other two systems.

**Figure 4.** Vertical compliances of a square foundation on different poroelastic soils

**Figure 5.** Horizontal compliances of a square foundation on different poroelastic soils

**Figure 6.** Rocking compliances of a square foundation on different poroelastic soils
5. Conclusions

The dynamic interaction between a rigid foundation and a layered poroelastic medium is investigated in this paper by employing a semi-analytical discretization technique and the exact stiffness matrix scheme. The rigid foundation is subjected to the time-harmonic vertical, horizontal and moment loading, and the contact surface between the foundation and the layered medium is assumed to be perfectly bonded and fully permeable. The accuracy of the present solution is confirmed by comparing with an existing solution for compliances of a rigid foundation on an elastic half-space. Numerical results presented in this paper indicate that the compliances of rigid square foundations are significantly influenced by the parameter $b$, which takes into account the internal friction of the layered medium due to the relative motion between the solid matrix and the pore fluid. The frequency of excitation also shows a significant influence on the foundation response. The numerical solution scheme developed in this paper can be extended to investigate more complicated problems involving foundations with other shapes and foundations with small flexural rigidity.

References

[1] H. Wong, J.E. Luco, Earthquake Eng. Struct. Dyn. 4, 579-587. (1976)
[2] H. Wong, J.E. Luco, Earthquake Eng. Struct. Dyn. 6, 3-16. (1978)
[3] R. Shahi, A. Noorzad, Int. J. Geomech. 11, 391-398. (2011).
[4] M.A. Biot, J. Acoust. Soc. Am. 28, 168-179 (1956)
[5] M.A. Biot, J. Appl. Phys. 334, 1482-1498 (1962)
[6] M.R. Halpern, P. Christiano, Earthquake Eng. Struct. Dyn. 14, 439-454 (1986)
[7] M.K. Kassir, J. Xu, Int. J. Solid Struct. 24, 915-936 (1988)
[8] R.K.N.D. Rajapakse, T. Senjuntichai, Int. J. Numer. Anal. Meth. Geomech. 19, 587-614 (1995)
[9] X. Zeng, R.K.N.D. Rajapakse, Int. J. Numer. Anal. Meth. Geomech. 23, 2075–2095 (1999)
[10] T. Senjuntichai, W. Kaewjuea, J. Mech. Mater. Struct. 3, 1885-1901. (2008)
[11] T. Senjuntichai, S. Keawwasavong, R. Plangmal, Comput. Geotech. 100, 121-134. (2018)
[12] M.A. Biot, J. Appl. Phys. 122, 155-164 (1941)
[13] T. Senjuntichai, S. Keawwasavong, R. Plangmal, Mar. Georesour. Geotec. (2019) doi: 10.1080/1064119X.2018.1446200