One-dimensional magnonic circuits with size-tunable band gaps and selective transmission

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Abstract. We present a review of our theoretical calculations about magnon transport in quasi-one-dimensional (1D) magnonic circuits constituted by waveguides coupled to side resonators. Phenomena such as the existence of band gaps, rejective and selective transmissions and Fano resonances will be discussed as well as the applications of these structures in filtering and demultiplexing devices. The calculations are performed based on two types of models and in the frame of the Green’s function method. First, the continuum long-wavelength Heisenberg model is studied in 1D monomode waveguide containing symmetric and asymmetric loops or coupled with grafted stubs. Then, we use the discrete dipole approximation in structures composed of a chain of nanometric magnetic clusters coupled to finite clusters on its vicinity. All such circuits exhibit a variety of interference effects in their transport properties which should have important consequences for designing integrated devices such as microwave filters.

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1. Introduction

The propagation of waves in periodically structured materials is of fundamental interest in modern physics and technology. In particular, spin systems which have a regular distribution of scattering centers have been seen to possess a distinct and interesting array of magnonic properties, perhaps most strikingly frequency band gaps within which magnons cannot propagate through the structure-a so-called magnonic band gap [1, 2, 3, 4, 5, 6, 7, 8, 9]. The interest in these band gaps is related to a number of advantages where the magnonic crystals has in comparison with photonic crystals [1]. The wavelength of a spin wave and, hence, the properties of such crystals depend on the external magnetic field and can be controlled by this field. The wavelength of propagating spin waves for a wide class of ferromagnetic materials in the microwave range is on the order of tens or even hundreds of micrometers. The phase and group velocities of spin waves are also functions of the structure size and the applied external field and may vary over a wide range [1].

After a great deal of works devoted to the lamellar materials and superlattices, the first attempt to study the spin waves in 2D magnonic crystals was presented in [3]. Nowadays, the number of studies on this topic has surged and continues to grow at a fast pace [10]. On the other hand, studies of lower dimensional systems such as 1D periodic layered media [2, 11, 12, 13] and periodic waveguide systems with different geometries [14, 15, 16, 17] are conducted as analogues...
of 2D and 3D systems and for applications in their own right. These structures are attractive since their production is more feasible and they require only simple analytical and numerical calculations. Spin waves are governed by different interactions dominating on different length scales, e.g., by the exchange and dipolar interactions dominating on nanoscopic and microscopic length scales, respectively. Spin waves propagation in micro- and nanopatterned magnonic waveguides have been discussed in some recent papers [18, 19, 20].

In this paper, we briefly review some of our theoretical calculations about magnon transport in two types of quasi-one-dimensional (1D) magnonic circuits constituted by waveguides coupled to side resonators, in analogy to the case of other excitations such as phonons, photons or electrons [21]. In the first case, the continuum long-wavelength Heisenberg model is studied in 1D monomode waveguide containing symmetric and asymmetric loops or coupled with grafted stubs. Such structures can exhibit band gaps resulting both from their periodicity or from the existence of zeros of transmission associated with the resonant modes of each loop or grafted stub. By introducing a defect loop or a defect stub along the periodic waveguide, one can expect the existence of localized modes in the band gaps that can contribute to a selective filtering transmission. In this short paper, we emphasize (section 2) only some results [22, 23] about the so-called Fano resonances [24] which have been also reported in the electronic transport in mesoscopic systems using interferometric structures such as the Aharonov-Bohm systems [25, 26, 27, 28].

Then, in section 3, we use the discrete dipole approximation in structures composed of a chain of nanometric magnetic clusters coupled to finite clusters on its vicinity [29, 30, 31, 32]. Such circuits exhibit a variety of interference effects in their transport properties that may be useful in microwave filtering and multiplexing devices.

2. Magnon mono-mode circuits: Transmission gaps and Fano resonances
The simple magnonic filter with which we are dealing in this section is depicted in Fig. 1. It consists of \( N \) \((N')\) dangling side branches (DSB), which play the role of resonators, grafted at two sites on an infinite 1D mono-mode waveguide. The length \( d_2 \) of the finite segment and the lengths \( d_1 \) and \( d_3 \) of the grafted resonators characterize the geometry of the structure. The media are assumed to be Heisenberg ferromagnets, which means that we are neglecting the dipole-dipole interactions as compared with the exchange contribution to the Hamiltonian. Moreover, we are dealing with long wavelength magnetic excitations and therefore take use of the continuum approximation of the Heisenberg model (see for details Refs. [14, 33]). The backbone and the grafted branches are assumed to be monomode waveguides for the propagation of magnons. With these ingredients, one can derive analytically the dispersion relation as well as the transmission and reflection coefficients through the waveguide structure. The transmission gaps have been established through an analysis of the transmission function (amplitude and phase) calculated within the framework of the Green’s function method [22, 14]. We show that this simple structure can exhibit transmission gaps (their width depend on the number of dangling resonators) and Fano-like resonances. In particular, we show that the transmission amplitude through such a system can be written following the Fano-like shape around these resonances.

Although in our analytical calculation, the backbone and the DSB can be constituted by different magnetic materials, we shall assume, for the sake of simplicity, that all materials are constituted by the same ferromagnet. Indeed, the occurrence of magnonic gaps and Fano resonances in our structure does not require the use of two different materials, in contrast to the case of usual multilayered structures.

In Fig. 2(a) we show the transmission amplitude versus the reduced frequency \( \Omega \left(= \omega d_2^2 / D \right) \) for a simple structure consisting of one resonator grafted on an infinite guide (see Fig. 1(a)). Here
Figure 1. Schematic illustration of the one-dimensional waveguide with dangling resonators studied in the present section. The one-dimensional media constituting the infinite monomode guide and the finite segments are assumed to be of the same material. (a) An infinite line with one grafted segment of length $d_1$. (b) The same as in (a) but for an infinite line with two segments of lengths $d_1$ and $d_3$ grafted at two sites separated by a segment of length $d_2$. (c) The same as in (b) but for two side resonators grafted at two sites.

$$D = (2Ja^2M)/\gamma h^2$$

where $M$, $J$, and $\gamma$ stand, respectively, for the spontaneous magnetization, the exchange interaction between neighboring magnetic sites in the simple cubic lattice of lattice parameter $a$ constituting the ferromagnetic medium, and the gyromagnetic ratio. One can notice that for this composite system there exist an infinite set of forbidden frequencies $\Omega_g$ corresponding to the eigenmodes of the grafted finite branch. This grafted branch behaves as a resonator and this simple composite system filters out these modes. This phenomenon is related to the resonances associated with the finite additional path offered to the spin-wave propagation. The variation in the phase versus the reduced frequency $\Omega_1$ shown in Fig. 2(b) shows an abrupt change in $\pi$ at the transmission zeros induced by the grafted resonator.

Figure 2(c) gives the transmission coefficient versus the reduced frequency $\Omega_2(=\omega d_2^2/D)$ in presence of two identical dangling resonators, namely, $N = N' = 1$ and $d_1 = d_3 = 0.5d_2$ (see the structure shown in Fig.1(b)). One can notice that the transmission coefficient presents well-defined dips induced by the grafted branches. These dips appears at $\Omega_2 = H + ((l + 0.5)\pi)^2$, where $H = \gamma H_0 d_2^2/D$, $l = 0, 1, 2, \ldots$, and $H_0$ is the static external field. These dips transform into large transmission gap when the number of branches increase as it is illustrated in Fig. 2(e) for $N = N' = 2$. It is worth mentioning that because of the existence of two resonators, one can expect two phase drops of $\pi$ (i.e., $2\pi$) at the transmission zeros induced by the grafted branches. However, one can see in Figs. 2(d) and 2(f) that the phase presents only a phase drop of $\pi$. This is due to existence of a resonant state with zero width at these values of $\Omega_2$ which induce a phase jump of $\pi$. These resonances collapse when $d_1 = d_3$ is taken exactly equal to $0.5d_2$. To enlarge these resonances, we have to take $d_1$ and $d_3$ slightly different from $0.5d_2$.

An example corresponding to the case where $d_1$ and $d_3$ is different from $0.5d_2$ is given in Fig.
The width of the resonance falling at \( \Omega \) (i.e., \( \Delta = \Omega - \Omega_0 \)) is characterized by the interference between the bound states and the propagating continuum states \([24, 25, 26]\). where \( A \) is the coupling parameter, it gives qualitatively the interference between the bound states and the propagating continuum states \([24, 25]\). \( \epsilon \) is a reduced frequency, \( \lambda \) is a positive integer and \( \Omega_g = \omega_g d_1^2 / D \) is a reduced frequency, \( H_0 \) is the static external field, \( l \) is a positive integer and \( H' = \gamma H_0 d_1^2 / D \). For convenience \( H' \) is considered to be 1. (c) Transmission coefficient vs. the reduced frequency \( \Omega_2 \) for the structure depicted in Fig. 1(b) with \( d_1 = d_3 = 0.5d_2 \) and \( N = N' = 1 \). (d) (c) and (f) are respectively the same as (a), (c) and (e) but for the variation of the phase.

The resonance in Fig. 3(a) shows the same characteristics as a Fano resonance but with two zeros of transmission around the resonance instead of one as it is usually the case \([24, 25]\). Indeed, one can obtain an approximate analytical expression for the transmission function in the vicinity of the resonance. A Taylor expansion around the resonance enables us to write the transmission coefficient \( T \) (following the Fano line shape \([24, 25]\)) in the form

\[
T = A \frac{(\epsilon + q_1 \Gamma)^2(\epsilon - q_2 \Gamma)^2}{\epsilon^2 + \Gamma^2},
\]

where \( A = (1-4\Delta^2/\pi^2)^2/4[N(N+1)+\Delta^2(2-N^2)]^2 \) and \( \Delta \) is the detuning of \( d_1 \) and \( d_3 \) from 0.5\( d_2 \) (i.e., \( \Delta = \pi(0.5 - d_1/d_2) = \pi(-0.5 + d_3/d_2) \)). \( \Gamma = 2\Delta^2/[N(N+1)+\Delta^2(2-N^2)]/\Delta(1+2\Delta/\pi) \) characterizes the width of the resonance falling at \( \epsilon = 0 \). \( q_1 = [N(N+1)+\Delta^2(2-N^2)]/\Delta(1+2\Delta/\pi) \) and \( q_2 = [N(N+1)+\Delta^2(2-N^2)]/\Delta(1-2\Delta/\pi) \) are the coupling parameters, they give qualitatively the interference between the bound states and the propagating continuum states \([24, 25, 26]\).

One can notice that when increasing \( \Delta \), \( \Gamma \) increases and \( q_1(q_2) \) decreases. The results of the approximate expression (Eq. (1)) are shown in Fig. 3(b) by open circles. These results are in accordance with the exact ones (solid lines) and clearly show that the resonance is of Fano type.
Figure 3. (a) The same as in Fig.2(c) but the lengths of the resonators are taken such that $d_1 = 0.46d_2$ and $d_3 = 0.54d_2$ and $N = N' = 1$. Solid circles on the abscissa axis indicate the positions of the transmission zeros induced by the dangling resonators on both sides of the resonance. (c) The same as in (a) but for the variation of the phase. (b) and (d) give the approximate results (open circles) around the resonance.

with $q_1 \simeq 14.85$ and $q_2 \simeq 17.43$ and width $2\Gamma \simeq 0.03$. The commonly studied Fano resonances are asymmetric because of the presence of only one transmission zero near the resonance (see below). In addition, in the electronic counterparts studies, a perturbation is often introduced to the system in order to create the resonance state [24, 25, 26, 27]. However, the above results shows that, without introducing any perturbation in the structure, one can find a well-defined symmetric Fano resonance with width $2\Gamma$ and coupling parameters $q_1$ and $q_2$ that can be adjusted by tailoring the lengths of the resonators (i.e., $\Delta$). A Taylor expansion around the resonance enables us also to obtain an approximate expression for the phase [22]. The approximate result for the phase is plotted by open circles in Fig. 3(d) and clearly shows [in accordance with the exact results (solid line)] two abrupt phase changes in $\pi$ at the transmission zeros around the resonance.

One can also create an asymmetric Fano resonance by adjusting the transmission zeros on only one side of the resonance, this can be obtained by considering a structure where the resonators are supposed to be identical with lengths slightly different from $0.5d_2$. This is shown in Fig.4(a) for $d_1 = d_3 = 0.46d_2$ and $N = N' = 1$. Indeed, an analytical Taylor expansion around the resonance enables us to write the following Fano line shape transmission coefficient

$$T \simeq \frac{B}{N^2} \frac{(\varepsilon - \varepsilon_R + q\Gamma)^4}{(\varepsilon - \varepsilon_R)^2 + \Gamma^2} \quad (2)$$

where $B = (1 + 2\Delta/\pi)^4/4(N + 1 + 2\Delta/\pi)^2$, $q = (N + 1 + 2\Delta/\pi)^2/\Delta(1 + 2\Delta/\pi)$ is the Fano parameter. $\Gamma = 2\Delta^2/N^2[1 + \frac{1}{N}(1 + 2\Delta/\pi)^3$ and $\varepsilon_R = -2\Delta/(N + 1 + 2\Delta/\pi)$ characterize the width and the shift of the resonance respectively.

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One can notice that the width of the resonance is small as compared to the preceding case, this is in accordance with the numerical results of Figs. 3(a) and 4(a). Also $q$ increases when $\Delta$ decreases and tends to infinity when $\Delta$ vanishes. In this case the resonance falls at $\varepsilon_R = 0$ and, as expected, its width $2\Gamma$ reduces to zero (see Fig.2(c)). The results of the approximate expression (Eq.(2)) are sketched (open circles) in Fig.4(b) for $\Delta = \pi(d_1/d_2 - 0.5) = -0.04\pi$ (i.e., $d_1/d_2 = 0.46$) and $N = N' = 1$. These results are in accordance with the exact ones (solid lines) and clearly show that the resonance is of Fano type with $|q| \simeq 32$ and width $2\Gamma \simeq 0.0089$.

Concerning the evolution of the phase of the spin waves in this structure, one can notice that (at $\Omega_2 = 12.66$, indicated by a filled circle on the abscissa of Fig.4(a)) the transmission zeros induced by the two identical resonators fall at the same frequency, therefore the phase (Figs. 4(c), 4(d)) shows a phase drop of $2\pi$ at these frequencies. Indeed, as the phase is defined modulo $2\pi$, the $2\pi$ phase change can be observed if we take into account the absorption in the system [34, 35].

Finally, we would like to mention that Fano resonances and the transmission spectra can be significantly changed by using the pinning field at the end of resonators as a tuning parameter (see for details Ref.[23]). Indeed, the pinning field can vary over a wide range of values, depending on the nature of the ends of the resonators and the properties of the magnetic bearing ions[36].

3. Magnon in nanometric cluster chains
3.1. Coupling to side resonators
The object of this section is to investigate the magnon propagation in a nanometric magnetic cluster chain and focus on the effects of a few additional clusters near the chain. We take into account the dipole-dipole interactions between the nearest-neighbor cluster local magnetic moments. We show that an appropriate choice of the geometrical or magnetic parameters of the additional clusters can lead either to narrow peaks or to narrow dips in the transmission spectrum of magnons along the cluster chain. Such a device can be useful as a selecting or rejecting magnon filter. We consider such a linear chain of nanoparticles bearing a permanent magnetic moment and study the effect on the magnon transmission spectrum of three additional clusters coupled to the chain, as depicted in Fig.5(a). The effect of coupling the infinite wire to such a local resonator is to induce peaks and dips (or zeros) in the transmission coefficient. Our purpose is to discuss the possibility of narrow peaks or narrow dips in the magnon transmission by selecting appropriately

Figure 4. (a) The same as in Fig.3(a) but the resonators are taken to be of identical lengths $d_1 = d_3 = 0.46\ d_2$. (c) The same as in (a) but for the variation of the phase. (b) and (d) give the approximate results (open circles) around the resonance.
Figure 5. (a) Sketch of the geometry of the considered nanometric device. It consists of one cluster chain and three other clusters. The distances between these clusters are respectively \(d\) and \(d_1\) as indicated on the figure. The static magnetic moments are \(M_s\) (for the chain clusters) and \(M'_s\) (for the three additional clusters) and the external magnetic field is \(H_0\), along the z-axis). (b) Transmission through the structure of Fig. ?? as a function of \(\omega/\gamma\) for \(H_0 = 5\), \(M_s = M'_s = 1\), \(d = 1\) and \(d_1 = 1.5\). (c) Transmission in function of \(\omega/\gamma\) for \(H_0 = 5\), \(M_s = M'_s = 1\), \(d = 1\) and \(d_1 = 0.8\).

The geometrical or magnetic parameters of the problem. The mathematical derivation of the transmission coefficients, based on a Green’s function method is given in Ref. [29].

Fig. 5(b) presents the intensity transmission coefficient \(T\) in function of \(\omega/\gamma\), for \(d_1 = 1.5\) and \(M'_s = 1\). Note the sharp zero of transmission a little above the middle of the bulk band. Such a narrow stop band could be useful to construct a rejecting signal device. When one decreases \(d_1\) to 0.8, leaving all the other parameters as in Fig. 5(b), we obtain Fig. 5(c) which presents now an interesting transmission peak in between two zeros of transmission. This feature could be used as a transmission filter for a magnon signal.

It is interesting to underline that the sharp zeros of transmission are obtained when the resonator clusters are weakly coupled to the chain and can be related to the eigenfrequencies of the isolated three cluster resonator. The sharp transmission peaks appear when the resonator clusters are strongly coupled to the chain. In that case the transmission features can be
Figure 6. Sketch of the geometry of the considered nanometric device. It consists of three semi-infinite chains made out of a periodic sequence of equidistant nanometric magnetic clusters and three additional clusters attached to the upper (lower) output chain. The distances between these clusters are respectively $d$, $d_1$, $d_2$, $d_3$, and $d_4$ as indicated on the figure. The permanent static magnetic moments $M_s$, $M'_s$ ($M''_s$) (for the three semi-infinite cluster chains, and for the additional clusters attached to the upper (lower) output chain respectively), and the external field $H_0$ are assumed to be along the $z$ axis.

interpreted as mainly due to the widening of the frequencies falling inside the bulk chain band of the eigenmodes of the ensemble of the six perturbed clusters.

Let us finally mention that the device presented in this section can be tuned in order to be a good magnon filter as well by transmission as by reflection. In order to achieve this the resonant structure must provide zeros and ones of transmission inside the bulk band. This was realized with the three adsorbed clusters device reported here. In order to get sharp transmission peaks one must tune the system parameters such that the transmission ones fall in between two zeros of transmission close one to the other.

3.2. Magnon multiplexers

A system exhibiting the spin wave transfer effect can be used as a multiplexer. A multiplexer is a system enabling to transfer one magnon frequency from one guide to another one. It is selective when only one magnon frequency is transferred, leaving the propagation of all the other incident magnon frequencies contained in a given energy window unperturbed. Such transfer processes are particularly important in wavelength multiplexing and in telecommunication routing devices.

In this section we describe a simple system, which under certain conditions, realizes the directional transfer of magnons with a good selectivity. The system is depicted in Fig.6. In order to illustrate the signal multiplexing results we present in Fig.7, the variations of the intensity transmission coefficients $T_{15}$ and $T_{15'}$ versus $\omega/\gamma$ for the structure shown in Fig.6. Fig.7 shows together the direct transmission $T_{15}$ from site 1 to site 5 (solid line) and $T_{15'}$ from site 1 to site 5’ (dashed line). For the parameters $d = d_1 = 1$, $d_2 = 1.35$, $d_3 = 1.25$, $d_4 = 1.1$, $M_s = 1$, $M'_s = 4.3$, $M''_s = 3.6$, $H_0 = 5$, we obtain (Fig.7(a)) an interesting narrow dip (centered
at $\omega/\gamma \simeq 7.84$) squeezed between two transmission one in the upper output channel together with a peak with weak amplitude, at the same frequency, in the lower output channel. To increase the transmission amplitude in the lower channel we increased the coupling between the lower cluster chain and the resonators attached to it. This is done by shortening the distance $d_4$ (see Fig.6). In addition, a very slight decrease of the distance $d_2$ was also useful in enhancing the transmission probability. Fig.7(b) shows the $\omega/\gamma$ dependence of the transmission factor for the set of parameters $d_2 = 1.27, d_4 = 0.8$ (The rest of the parameters have the same values as in Fig.7(a)). In this plot one can notice that the transmission peak in the lower channel reaches almost 80% of the input signal. One can also notice a slight shift in the frequency position of the resonance peak (dip). For other values of the parameters it is possible to obtain in the bulk transmission band two transmission peaks (one in each channel) where the amplitude in one channel goes to one, at a certain frequency, while the amplitude of transmission in the other channel goes to zero at the same frequency (and vice versa). This is effect, which could be useful in constructing a transmission filter for a magnon signal, is illustrated in Fig. 7(c) for $d_2 = 1.2, d_3 = 1, d_4 = 1.2 , M_s^f = 4.35, M_s^\prime = 2.85$. The rest of the parameters have the same values as in (a).

Figure 7. (a) Transmission in function of $\omega/\gamma$ for the structure shown in Fig. 6. The solid (dashed) line corresponds to the direct transmission $T_{15}$ ($T_{15}'$) from site 1 to site 5 (from site 1 to site 5'). The parameters are $d = d_1 = 1, d_2 = 1.35, d_3 = 1.25, d_4 = 1.1, M_s = 1, M_s^f = 4.3, M_s^\prime = 3.6, H_0 = 5$. (b) The same as in (a) but for the parameters $d_2 = 1.27, d_4 = 0.8$. The rest of the parameters have the same values as in (a). (c) The same as in (a) but for the parameters $d_2 = 1.27, d_4 = 0.8$. The rest of the parameters have the same values as in (a). Before ending this paragraph it is worth mentioning that a first step in the choice of the parameters is done on the basis of our previous calculation where a resonator (made of several clusters) is attached to a linear chain [29, 30]. Then, in a second step, the parameters have to be adjusted around these values in order to increase the transmission in one or both channels.
Acknowledgments
H. Al-W. gratefully acknowledge the hospitality of the Université des Sciences et Technologies de Lille 1. The work of B. D.-R., A.A. and L.D was supported by Ministry of Higher Education and Research, Nord-Pas de Calais Regional Council and FEDER through the 'Contrat de Projets Etat Region (CPER) 2007-2013’

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