Secure quantum bit commitment against empty promises. II. The density matrix

Guang Ping He

School of Physics and Engineering, Sun Yat-sen University, Guangzhou 510275, China

We further study the security of the quantum bit commitment (QBC) protocol we previously proposed [Phys. Rev. A 74, 022332 (2006).], by analyzing the reduced density matrix $\rho^B_b$ which describes the quantum state at Bob’s side corresponding to Alice’s committed bit $b$. It is shown that Alice will find $\rho^B_0 \perp \rho^B_1$ while the protocol remains concealing to Bob. On the contrary, the existing no-go theorem of unconditionally secure QBC is based on the condition $\rho^B_0 = \rho^B_1$. Thus the specific cheating strategy proposed in the no-go theorem does not necessarily applies to our protocol.

PACS numbers: 03.67.Dd, 03.67.Hk, 03.67.Mn, 89.70.+c

I. INTRODUCTION

Quantum bit commitment (QBC) is a two-party cryptography including two phases. In the commit phase, Alice (the sender of the commitment) decides the value of the bit $b$ ($b = 0$ or $1$) that she wants to commit, and sends Bob (the receiver of the commitment) a piece of evidence, e.g., some quantum states. Later, in the unveil phase, Alice announces the value of $b$, and Bob checks it with the evidence. An unconditionally secure QBC protocol needs to be both binding (i.e., Alice cannot change the value of $b$ after the commit phase) and concealing (Bob cannot know $b$ before the unveil phase) without relying on any computational assumption.

It is widely accepted that unconditionally secure QBC is impossible [1]-[24], despite of some attempts towards secure ones (a detailed list and brief history can be found in the introduction of [23]). This result, known as the Mayers-Lo-Chau (MLC) no-go theorem, was considered as putting a serious drawback on quantum cryptography.

Nevertheless, we must note that the correctness of the conclusion a theorem should not be confused with that of its proof. While a correct proof will surely lead to a correct conclusion, there could also be cases where someone may draw a correct conclusion despite that the existing proof is not sufficiently general. In quantum cryptography, though there are brilliant proofs (e.g., [26]) for the security of quantum key distribution, for other cryptographic tasks it could be hard to find a general proof showing that a protocol is unconditionally secure, since there could potentially exist numerous cheating strategies. Similarly, it is also hard to find a real general proof showing that a cryptographic task can never be accomplished securely (unless the definition of the task contains self-inconsistent goals), because the protocols potentially existed could also be numerous, some of which may even beyond our current imagination. As for QBC, it is important to notice that all the existing no-go proofs [1]-[24] are actually based on a specific cheating strategy of Alice, as it will be summarized below. No matter unconditionally secure QBC is possible or not, we could question whether this specific cheating strategy can be evaded. If there is a protocol which is secure against the specific cheating strategy in the no-go proofs while insecure against other cheating strategies, then it reveals that the existing proofs of the MLC no-go theorem should not be considered sufficiently general, despite that the conclusion of the theorem may remain valid.

In our previous work [27], we proposed a QBC protocol and proved that it is secure against some known attacks, while an attack strategy that can break our protocol successfully has yet to be found. Thus the exact boundary of the security of the protocol remains unclear. In this paper, we will further show that the density matrix in the protocol displays a distinct feature comparing with that of the QBC model studied in existing no-go proofs [1]-[24]. This makes it possible for our protocol to evade at least the specific cheating strategy that led to these proofs.

In the next section, we will briefly review the existing no-go proofs of QBC, and pinpoint out that the cheating strategies in all these proofs have the same requirement on the density matrix. Our previous QBC protocol [27] will be illustrated in section III. Then in section IV, we will analyze the density matrix in this protocol, and show that they does not satisfy a requirement on which the no-go proofs hold. In section V, we will elaborate why security can maintain in the absence of this requirement.

II. THE DENSITY MATRIX IN THE NO-GO PROOFS

Although there are many no-go proofs [1]-[24], they all have the following common features.

(1) The reduced model. According to the no-go proofs, any QBC protocol can be reduced to the following model. Alice and Bob together own a quantum state in a given Hilbert space. Each of them performs unitary transformations on the state in turns. All measurements are performed at the very end.

(2) The coding method. The quantum state corre-
sponding to the committed bit $b$ has the form

$$|\psi_b\rangle = \sum_j \lambda_j^{(b)} |e_j\rangle_A \otimes |f_j\rangle_B,$$

and it is known to both Alice and Bob. Here the systems $A$ and $B$ are owned by Alice and Bob respectively.

(3) The concealing condition. To ensure that Bob’s information on the committed bit is trivial before the unveil phase, any QBC protocol secure against Bob should satisfy

$$\rho_b^B \simeq \rho_1^B,$$

where $\rho_b^B = Tr_A |\psi_b\rangle \langle \psi_b|$ is the reduced density matrix of the state at Bob’s side corresponding to Alice’s committed bit $b$.

(4) The cheating strategy. Once Eq. (2) is satisfied, according to the Hughston-Jozsa-Wootters (HJW) theorem (which also appeared in many different names in literature, e.g., the Uhlmann theorem, etc.) [24,22,23], there exists a local unitary transformation for Alice to map $\{ |e_j\rangle_A \}$ into $\{ |f_j\rangle_A \}$ successfully with a high probability. Thus a dishonest Alice can unveil the state as either $|\psi_0\rangle$ or $|\psi_1\rangle$ at her will with a high probability to escape Bob’s detection. For this reason, a concealing QBC protocol cannot be binding.

The most important point for our discussion here is feature (3). We would like to emphasize again that it appears in all existing no-go proofs. Note that in some references (e.g., [10,13,22,24]), this feature was expressed using the trace distance or the fidelity instead of the reduced density matrices, while the meaning remains the same. On the other hand, it will be shown below that the density matrix in our previous QBC protocol displays an intriguing feature. Though the protocol remains concealing against Bob, at Alice’s point of view there will be $\rho_b^B \perp \rho_1^B$ (i.e., they are orthogonal) instead of $\rho_b^B \simeq \rho_1^B$. As Eq. (2) is necessary for constructing Alice’s cheating transformation in the above feature (4), our protocol is thus immune to this specific cheating strategy.

### III. OUR PROTOCOL

#### A. The rigorous description

In Ref. [27], we proposed the following QBC protocol. The commit protocol: $[\text{commit}(b)]$

(C1) Alice and Bob first agree on a security parameter $s$, then $DO_{i=1}^s$. Alice picks $\theta_i \in (0, \pi/2)$ ($\theta_i$ needs not to be different for each $i$. For example, Alice can fix $\theta_i = \pi/4$ throughout the whole protocol) and randomly picks $q_i \in \{0, 1\}$, and prepares an entangled state

$$|\psi_i\rangle = |\alpha_i \otimes \beta_i\rangle = \cos \theta_i |x_i\rangle \otimes |0, q_i\rangle + \sin \theta_i |y_i\rangle \otimes |1, q_i\rangle,$$

Then she sends the quantum register $\beta_i$ to Bob and stores $\alpha_i$. Here $|x_i\rangle$ and $|y_i\rangle$ are two orthogonal states of the quantum register $\alpha$, while we use $|p_i, q_i\rangle_\beta$ to denote the state of $\beta_i$, with $p_i$ denoting the basis and $q_i$ labelling the different states in the same basis. The state $|0, 0\rangle$ and $|0, 1\rangle$ are orthogonal to each other, and $|1, 0\rangle \equiv (|0, 0\rangle + |0, 1\rangle)/\sqrt{2}$, $|1, 1\rangle \equiv (|0, 0\rangle - |0, 1\rangle)/\sqrt{2}$

(C2) Bob chooses a number $s'$ ($0 \leq s' < s$) and randomly divides $S \equiv \{1, ..., s\}$ into two subsets $S'$ and $S''$ such that $|S'| = s'$, $|S''| = S - S'$. Then for $\forall i \in S'$ Bob stores $\beta_i$ unmeasured. And for $\forall i \in S''$ Bob randomly picks a basis $p_i' \in \{0, 1\}$ and measures $\beta_i$. The outcome is denoted as $|p_i', q_i'\rangle_\beta$.

(C3) Bob chooses $f_a, f_b, f_c$ ($f_a + f_c < 1/2$ and $f_b > f_c$) and announces to Alice the “false” result $\{ |p_i', q_i'\rangle_\beta \mid i \in S \}$ such that $f_a = \langle (|L_a| + s'/4)/s, f_b = \langle (|L_b| + s'/4)/s$ and $f_c = \langle (|L_c| + s'/4)/s, where $L_a = \{ i \in S'' \mid |p_i', q_i'\rangle_\beta = |p_i', q_i'\rangle_\beta \}, L_b = \{ i \in S'' \mid |p_i', q_i'\rangle_\beta = |p_i', q_i'\rangle_\beta \}$, and $L_c = \{ i \in S'' \mid |p_i', q_i'\rangle_\beta = |p_i', q_i'\rangle_\beta \}$.

(C4) Alice divides $S$ into two subsets: $M = \{ i \in S | q_i' = q_i \}$ and $U = \{ i \in S | q_i' = q_i \}$. For $\forall i \in M$, she measures $\alpha_i$ in the basis $\langle |x_i\rangle \rangle \otimes |y_i\rangle \rangle$. She sets $p_i = 0$ if she finds $|x_i\rangle$ or $p_i = 1$ if she finds $|y_i\rangle$. Then she sets $L = \{ i \in M | p_i = p_i' \}$ and announces it to Bob: (Since it could be shown that $|M| \simeq (1/2 + (f_a + f_c)/2)/s$, by checking whether $|M| < s/2$ Alice can test whether Bob has indeed chosen $f_a + f_c < 1/2$. Also, since $|L| \simeq (f_a/2 + f_b/4 + f_c)/s$, we have $|M| - |L| \simeq (1/4 - (f_a - f_b)/4)/s$. Thus by checking whether $|M| - |L| < s/4$ Alice can test whether Bob has indeed chosen $f_b > f_c$.)

(C5) Bob sets $L'_s = L \cap S'$. Then he measures $\beta_i (\forall i \in L'_s)$ in the basis $p_i' = p_i''$ and denotes the outcome as $|p_i', q_i''\rangle_\beta$. He agrees to continue only if $\{ i \in L'' | |p_i', q_i''\rangle = |p_i', q_i''\rangle_\beta \} = \phi$, $L \subset L_a \cup L_b \cup L_c \cup S'$ and $|L| \simeq (f_a/2 + f_b/4 + f_c)/s$.

(C6) Alice sets $c_0 = 0$ if $i \in U$ or $c_0 = 1$ if $i \in M - L$. Thus she obtains a binary string $c^0 = (c_1^0 c_2^0 ... c_n^0)$ ($n = |S - L|$).

(C7) Alice and Bob complete the commitment with the CPTP method similar to that of the BCJL protocol by using $c^0$ to encode the codeword ($c^0$ itself is not announced to Bob). That is:

(C7.1) Bob chooses a binary linear $(n, k, d)$-code $C$ and announces it to Alice, where the ratios $d/n$ and $k/n$ are agreed on by both Alice and Bob;

(C7.2) Alice chooses a nonzero random $n$-bit string $r = (r_1 r_2 ... r_n) \in \{0, 1\}^n$ and announces it to Bob;

(C7.3) Now Alice has in mind the value of the bit $b$ that she wants to commit. Then she chooses a random $n$-bit codeword $c = (c_1 c_2 ... c_n)$ from $C$ such that $c \oplus r = b$

Here $c \oplus r \equiv \bigoplus_{i=1}^n c_i \wedge r_i$;

(C7.4) Alice announces to Bob $c' = c \oplus c^0$.

The unveil protocol: $[\text{unveil}(b, c, c^0, |\psi_i\rangle)]$

(U1) Alice announces $b, c, c^0, \{ q_i, \theta_i \mid i \in S \}$ and $\{ p_i | i \in M \}$ to Bob;
(U2) Alice sends the quantum registers \( \{\alpha_i|i \in U\} \) to Bob;
(U3) Bob finishes the measurement on \( \{\alpha_i|i \in U\} \) and \( \{\beta_i|i \in S'\} \) to check Alice’s announcement;
(U4) Bob checks \( |M| \simeq [1/4 + (f_a + f_c)/2]|s \) and \( (M-L) \cap L_b = \phi \);
(U5) Bob checks \( b = c \cap r \) and \( (c \text{ is a codeword}) \).

B. Notes

Since it is an important theoretical problem whether secure QBC exists, here the feasibility of the protocol is not what we care of. Thus we do not consider the presence of detection error, channel noise, or any other implementation issue.

In Ref. [27] we used to require Bob to choose \( 0 < f_a, f_b, f_c < 1/4 \) in step (C3). The purpose is to prevent Bob from delaying his measurement too often, because if he announces the “fake” result \( |p''_i, q''_i\rangle \) before he actually performs the measurement and obtains the real outcome \( |p_i, q_i\rangle \), then there will be no specific relationship between \( |p''_i, q''_i\rangle \) and \( |p_i, q_i\rangle \), which is equivalent to choosing \( f_a = f_b = f_c = 1/4 \). By the time Ref. [27] was written we did not know whether Bob will be benefited if he delays the measurement, so we introduced the requirement \( 0 < f_a, f_b, f_c < 1/4 \). But now we know that Bob cannot cheat even if the measurement was delayed, as it will be elaborated later in this paper. Thus we can remove this requirement from now on.

C. An easy understanding

As the no-go proofs has been widely accepted for more than a decade and a half, if there is a loophole, it must be lying somewhere subtle. Thus it is not surprising that a counter-example would look very complicated. To fully understand how the above protocol works, it is strongly recommend to read Ref. [27] in detail. For easier comprehension, some main ideas will be outlined below. But for any security debate in the future, it is important to always get back to the above rigorous mathematical description, as the security of a protocol will depend heavily on its details.

The main part of the above commit protocol is to force Alice to accomplish a lie-detecting task. That is, Alice sends Bob \( s \) quantum registers \( \beta_i \) \((i = 1, ..., s)\) in step (C1). Bob measures them in (C2) and announces the results in (C3). But it is important to note that the protocol allows Bob to lie when announcing the results. Then in (C4), Alice is required to detect Bob’s lies and announces the label \( i \) whenever she finds that Bob’s announced result for \( \beta_i \) is a lie. The total number of lies \( l \) is required to detect is

\[ l \equiv |L| \simeq (f_a/2 + f_b/4 + f_c/4)s. \]  (4)

Here \( f_a, f_b, \) and \( f_c \) are the lying frequencies with which Bob announces different types of lies. A type \( a \) lie means that Bob announces his actual measurement basis \( p'_i \) honestly as \( p''_i \), while lies about the state he found in this basis. That is, when his actual measurement result is \( |p'_i, q'_i\rangle \), he takes \( |p''_i, q''_i\rangle = |p'_i, q'_i\rangle \) and announces \( |p''_i, q''_i\rangle \). On the contrary, a type \( b \) lie means that Bob lies about the basis, while announcing \( q'_i \) honestly, i.e., the actual result \( |p'_i, q'_i\rangle \) is announced as \( |p'_i, q'\rangle \). Instead a type \( c \) lie means that Bob lies about both the basis \( p'_i \) and the state \( q'_i \), i.e., the actual result \( |p'_i, q'_i\rangle \) is announced as \( |p''_i, q''_i\rangle = |p'_i, q'\rangle \) instead. Note that if a “fake” result \( |p''_i, q''_i\rangle \) Bob announced is not a lie at all, it will be called an honest result when we need to distinguish it from other lies. But in general, for simplicity we will still call everything (either lies or honest ones) denoted by \( |p''_i, q''_i\rangle \) as fake results.

The commit protocol not only require Alice to detect \( l \) lies, but also force her to use the optimal strategy. Here “optimal” means that while the total number of detected lies must reach \( l \), Alice should try her best to keep the number of the unmeasured quantum registers \( \alpha_i \) as large as possible, so that most \( \alpha_i \otimes \beta_i \) pairs remain entangled \( ||\beta|| = ||\alpha|| \otimes \beta_i \) while the rest \( ||\alpha|| - ||\beta|| \) pairs remain entangled into non-entangled product states. As shown in Ref. [27], when \( f_a + f_c < 1/2 \) and \( f_b > f_c \), the optimal strategy for Alice is to prepare the initial states of \( \alpha_i \otimes \beta_i \) in a non-maximally entangled form as Eq. (5). Then to detect \( l \) lies, the number of \( \alpha_i \) she needs to measure is as small as

\[ m \equiv |M| \simeq [1/4 + (f_a + f_c)/2]|s. \]  (5)

Therefore, after \( l \) lies were detected and the corresponding quantum registers were discarded, the remaining \( n = s - l \) pairs of quantum registers contain \( m - l \) pairs of measured ones, while the rest \( n - (m - l) = s - m \) pairs remain entangled from Alice’s point of view as she has not measured the corresponding \( \alpha_i \). By assigning a “0” to each of the unmeasured ones and “1” to each of the measured ones, respectively, Alice obtains an \( n \)-bit string \( c^0 \) in step (C6). As it is a basic law that entanglement cannot be created locally, Alice cannot change the “1” to “0” in \( c^0 \) freely. Step (C7) further connects \( c^0 \) with Alice’s commit bit \( b \). Thus Alice is forced to commit once she accomplishes the lie-detecting task.

IV. THE DENSITY MATRIX IN OUR PROTOCOL

A. Important hints

When calculating the density matrix \( \rho_B^B \), two things should be kept in mind.

(i) We only need to study the value when the participants act honestly. This may look weird at the first
glance. But we should note that the conclusion of the no-go proofs is: if $\rho_0^B \simeq \rho_1^B$ is satisfied when both participants execute the protocol honestly, then Alice can cheat. That is, the density matrix $\rho_0^B$ studied in the no-go proofs is the one that describes the state obtained in the honest protocol, before taking cheating into consideration. In fact, even if Alice cheats, $\rho_0^B$ should remain unchanged. Otherwise Bob can simply perform a measurement to distinguish the density matrices, thus reveal Alice’s cheating. On the other hand, suppose that Bob cheats by introducing ancillary systems and/or performing transformations to alter $\rho_0^B$. Then we can always treat all these ancillary systems and transformations as a part of his operations on distinguishing $\rho_0^B$, instead of a part of $\rho_0^B$ itself. Therefore, no cheating of either participant need to be considered when calculating $\rho_0^B$.

(ii) $\rho_0^B$ should not only describe the quantum system Alice sent to Bob (e.g., the registers $\beta_i$’s in our protocol), but also reflect the influence of classical communication. The latter includes the classical information Alice announces to Bob, as well as what Bob announces to Alice while she accepts without questioning (i.e., Bob can assume by default that his classical information has reached Alice successfully so that she knows the content). This is because the original MLC no-go theorem worked on a scenario without involving classical communication directly. But it is by no means indicating that classical communication can be simply ignored. Instead, they used an “indirect” approach (as named in Ref. [7]). That is, they treated classical communication as a special case of quantum communication, and replaced them with a quantum channel [6]. Consequently, any protocol using classical information are replaced with a full quantum protocol without classical information. As pinpointed out in section 2 of Ref. [2], the advantage is that the attack on the new protocol is easy to describe, while the disadvantage is that the attack obtained against the new protocol is not the one that applies on the original protocol. Therefore, to make our presentation consistent with the above description of our QBC protocol (which includes classical communication) so that it could be easier for the reader to understand, here we avoid using the indirect approach, and calculate $\rho_0^B$ with classical communication taken into account in its original form.

B. The constraint from $|p_i, q_i\rangle_\beta$

With the above considerations, let us study the quantum states at the end of our commit protocol. The informations corresponding to the quantum registers $\beta_i$’s ($i \in I$) were already detected as lies in step (C4) and were publicly known to both participants, so that they are no longer useful and can be discarded. Thus we are interested in the remaining $\beta_i$’s ($i \in S - I$) at Bob’s side. To each of them, Alice has assigned a bit $c_i^0$ in step (C6). Since Alice has not announced Bob’s corresponding fake result $|p''_i, q''_i\rangle_\beta$ as a lie, it indicates two possibilities by default.

(a) $c_i^0 = 1$, i.e., Alice has measured the corresponding $\alpha_i$ in step (C4) but detected no lie.

(b) $c_i^0 = 0$, i.e., Alice has chosen not to measure $\alpha_i$ in step (C4).

In step (C4) Alice measures all $\alpha_i$’s that satisfy $q''_i = \gamma q_i$, and announces these satisfying $p_i = p''_i$ as lies. Therefore according to Eq. (4), in case (a) Alice’s measurement will collapse $|\psi_i\rangle = |\alpha_i \otimes \beta_i\rangle$ into

$$|\psi_i\rangle = \begin{cases} |x\rangle_\alpha \otimes |p''_i, q''_i\rangle_\beta, & (p_i = 1) \\ |y\rangle_\alpha \otimes |p''_i, q''_i\rangle_\beta, & (p_i = 0) \end{cases}$$

(6)

In case (b), $|\psi_i\rangle$ can be written as

$$|\psi_i\rangle = \cos \theta_i |x\rangle_\alpha \otimes |0, q''_i\rangle_\beta + \sin \theta_i |y\rangle_\alpha \otimes |1, q''_i\rangle_\beta.$$  

(7)

But these are merely the forms of the states under the constraint of the relationship between the values of $|p''_i, q''_i\rangle_\beta$ and $|p_i, q_i\rangle_\beta$. We must further consider the constraints brought by the relationship between $|p''_i, q''_i\rangle_\beta$ and Bob’s actual measurement result.

C. Type $a$ lies

Suppose that Bob’s fake result $|p''_i, q''_i\rangle_\beta$ turns out to be a type $a$ lie, i.e., Bob’s actual result is $|p_i, q_i\rangle_\beta = |p''_i, \gamma q''_i\rangle_\beta$. Then we can see that from Alice’s point of view, in case (b) among the two components in the superposition in Eq. (7), the one corresponding to $|p''_i, q''_i\rangle_\beta$ will conflict with Bob’s actual result as they are orthogonal. Therefore, though $c_i^0 = 0$ means that Alice should keep the entangled state Eq. (7) unmeasured, this component must vanish when Bob’s measurement makes the state collapse. So the only component that takes effect should be $|x\rangle_\alpha \otimes |p''_i, q''_i\rangle_\beta$ (if $p_i = 1$) or $|y\rangle_\alpha \otimes |p''_i, q''_i\rangle_\beta$ (if $p_i = 0$). That is, if Alice wants to take $c_i^0 = 0$, then from her point of view (as she does not know Bob’s actual result $|p_i, q_i\rangle_\beta$), the state of the corresponding $\beta_i$ she sent to Bob has to take the form $|p''_i, q''_i\rangle_\beta$ at the end of the commit phase. On the other hand, as we showed above, in case (a) (i.e., if Alice wants to take $c_i^0 = 1$) the state of $\beta_i$ should be $|p''_i, q''_i\rangle_\beta$. In brief, when there is a type $a$ lies, the two states of $\beta_i$ corresponding to $c_i^0 = 0$ and $c_i^0 = 1$, respectively, are orthogonal to each other.

Some might wonder why the state at Bob’s side is not simply Bob’s actual result $|p_i, q_i\rangle_\beta$ itself. This is because, as we mentioned in the above point (ii), the classical information exchanged in the protocol should also be taken into consideration. That is, the state at Bob’s side that Alice can unveil successfully later must show no conflict not only with Bob’s actual result $|p_i, q_i\rangle_\beta$, but also with the type of lies that Bob’s announced fake result $|p''_i, q''_i\rangle_\beta$ belongs to, i.e., it should explain why Alice has not detected this lie. If in the univale phase Alice said that the state she sent was $|p_i, q_i\rangle_\beta = |p''_i, q''_i\rangle_\beta$, then in the case of type $a$ lies, it equals exactly to Bob’s actual result.
be later unveiled as $p_i = p_i'$. But it will conflict with $|p_i', q_i''\rangle_\beta$, because if there is $q_i = q_i''$, in step (C4) Alice should have categorized it into the measured set $M$. Then as there is also $p_i = p_i''$, she should have announced it as a detected lie in set $L$, which should be discarded in later steps without being assigned a $c_i^0$ value at all. Thus we see that Bob’s actual result $|p_i', q_i''\rangle_\beta$ cannot be taken as the state encoding Alice’s committed codeword. Instead, as shown in the previous paragraph, the state $|p_i'', q_i''\rangle_\beta$ (or $|p_i'', \gamma_i''\rangle_\beta$) will not conflict with Bob’s actual result $|p_i', q_i''\rangle_\beta$, because they are nonorthogonal so that Bob’s measurement can indeed find the result $|p_i', q_i''\rangle_\beta$ with a nonvanishing probability. Meanwhile, it also agrees with the fact that in step (C4) Alice chose not to measure the state (or she measured but did not detect it as a lie). Therefore, it is the correct state at Bob’s side at the end of the commit phase that corresponds to $c_i^0 = 0$ (or $c_i^0 = 1$).

**D. Type $b$ lies**

Now suppose that Bob’s fake result $|p_i', q_i''\rangle_\beta$ is a type $b$ lie, i.e., Bob’s actual result is $|p_i', q_i''\rangle_\beta = |p_i'', q_i''\rangle_\beta$. Since in step (C4) Alice only measures the $\alpha_i$’s that satisfy $q_i'' = q_i$, for a measured $\alpha_i$ there will be $q_i'' = q_i' = q_i$. Rewriting Eq. (9) as

$$|\psi_i\rangle = \frac{1}{\sqrt{2}} (|\beta\rangle_\alpha \otimes |0, q_i\rangle_\beta + |\gamma\rangle_\alpha \otimes |1, q_i\rangle_\beta)$$

we can see that in the current case, Bob’s measurement in the $p_i'$ basis collapses $|\psi_i\rangle$ into either $|\beta\rangle_\alpha \otimes |1, q_i\rangle_\beta$ (if $p_i' = p_i'' = 0$) or $|\gamma\rangle_\alpha \otimes |0, q_i\rangle_\beta$ (if $p_i' = p_i'' = 1$). When Alice measures this $\alpha_i$, in step (C4), her result will always be $p_i = p_i''$ so that she would detect it as a lie. That is, all the type $b$ lies in set $M$ will be detected. After Alice announced set $L$ in step (C4), there will be no more type $b$ lie left in the set $M - L$. (This is why Bob needs to check $(M - L) \cap L_b = \phi$ in step (U4).) Therefore at the end of the commit phase, for any $\beta_i$ corresponding to a bit in the string $c_i^0 = (c_i^0, c_{i-1}^0, \ldots, c_1^0)$, Bob’s fake result must not be a type $b$ lie if $c_i^0 = 1$. Any type $b$ lie has to indicate $c_i^0 = 0$. In other words, for a specific $i$ if Bob’s fake result is a type $b$ lie, then the $\beta_i$ at Bob’s side at this stage has to be in the state corresponding to $c_i^0 = 0$. There does not exist any legitimate state of $\beta_i$ that can be later unveiled as $c_i^0 = 1$. In this sense, the states of $\beta_i$ corresponding to $c_i^0 = 0$ and $c_i^0 = 1$ are also orthogonal to each other when there is a type $b$ lie.

The state of $\beta_i$ corresponding to $c_i^0 = 0$ in this case can be a mixture of $|p_i', q_i''\rangle_\beta$ and $|\gamma_i'', q_i''\rangle_\beta$, as none of these components in Eq. (9) conflicts with Bob’s actual result $|p_i', q_i''\rangle_\beta = |\gamma_i'', q_i''\rangle_\beta$.

**E. Type $c$ lies**

The above results on types $a$ and $b$ lies are already sufficient for our discussion on $\rho_B^c$ in this section. But for completeness, we further study the type $c$ lie, i.e., Bob’s actual result is $|p_i', q_i''\rangle_\beta = |\gamma_i'', q_i''\rangle_\beta$. Then from Alice’s point of view, in the above case (b) among the two components in the superposition in Eq. (7), the one corresponding to $|\gamma_i'', q_i''\rangle_\beta$ will conflict with Bob’s actual result. Therefore the only component that takes effect should be $|\beta\rangle_\alpha \otimes |p_i', q_i''\rangle_\beta$ (if $p_i' = 0$) or $|\gamma\rangle_\alpha \otimes |p_i', q_i''\rangle_\beta$ (if $p_i' = 1$). That is, if Alice wants to take $c_i^0 = 0$, then the state of $\beta_i$ should take the form $|\beta_i\rangle_\alpha \otimes |p_i', q_i''\rangle_\beta$ at the end of the commit phase. On the other hand, as showed above in case (a), if Alice wants to take $c_i^0 = 1$, the state of $\beta_i$ should be $|\beta_i\rangle_\alpha \otimes |\gamma_i'', q_i''\rangle_\beta$. Thus we see that, unlike types $a$ and $b$ lies, when there is a type $c$ lie, the two states of $\beta_i$ corresponding to $c_i^0 = 0$ and $c_i^0 = 1$, respectively, are nonorthogonal to each other.

**F. Honest results**

Similarly, it can be shown that if Bob announced $|p_i', q_i''\rangle_\beta = |\beta_i\rangle_\alpha \otimes |p_i', q_i''\rangle_\beta$ as an honest result without lying, then the state of $\beta_i$ corresponding to $c_i^0 = 0$ can be a mixture of $|p_i', q_i''\rangle_\beta$ and $|\gamma_i'', q_i''\rangle_\beta$, like that of the type $b$ lies. Meanwhile, there exists a legitimate state corresponding to $c_i^0 = 1$, which is $|\beta_i\rangle_\alpha \otimes |p_i', q_i''\rangle_\beta$, like those of the types $a$ and $c$ lies. Again, the two states are nonorthogonal.

For explicitness, we briefly summarized the above results in Table I.

**TABLE I:** The state of $\beta_i$ corresponding to different values of $c_i^0$ and the type of lies that the fake result $|p_i', q_i''\rangle_\beta$ belongs to. See section IV for details.

| $|p_i', q_i''\rangle_\beta$ | $\beta_i(c_i^0 = 0)$ | $\beta_i(c_i^0 = 1)$ |
|---------------------------|-----------------|-----------------|
| Type $a$ lie              | $|\gamma_i'', q_i''\rangle_\beta$ | $|\gamma_i'', q_i''\rangle_\beta$ |
| Type $b$ lie              | Mixture of $|p_i', q_i''\rangle_\beta$ and $|\gamma_i'', q_i''\rangle_\beta$ | Not available |
| Type $c$ lie              | $|p_i', q_i''\rangle_\beta$ | $|\gamma_i'', q_i''\rangle_\beta$ |
| Honest result             | Mixture of $|p_i', q_i''\rangle_\beta$ and $|\gamma_i'', q_i''\rangle_\beta$ | $|\gamma_i'', q_i''\rangle_\beta$ |
G. Bob’s required $d$

The above discussion is about the state of a single $\beta_i$. Now let us turn to the entire system $B \equiv \bigotimes_i \beta_i$ ($i \in S-L$) corresponding to the density matrix $\rho_B^{(d)}$. Note that all the $\beta_i$’s in $B$ are not completely independent from each other. Together they should represent a codeword. In brief, the binary linear $(n, k, d)$-code $C$ is a set of classical $n$-bit strings. It contains about $2^k$ strings in total. Each string is called a codeword, carefully selected from all the $2^n$ possible choices of $n$-bit strings, so that the distance (i.e., the number of different bits) between any two codewords is not less than $d$. Let $|B(c)| \equiv \bigotimes_i |\beta_i(c_i^d)|$ denote the state of system $B$ at the end of the commit phase that corresponds to a specific codeword $c$, i.e., the relationship $c' = c \oplus c^0$ is satisfied, where $c^0 = (c_0^0, c_2^0, \ldots, c_n^0)$ is the string that the state $|B(c)|$ represents according to the coding method in step (C6), and $c'$ is what Alice announces in step (C7.4). Let $|B(c^*)| \equiv \bigotimes_i |\beta_i(c_i^{d^*})|$ denote such a state of system $B$ that corresponds to another codeword $c^*$, with $c' = c^* \oplus c^{d^*}$. Note that $c$ and $c^*$ have at least $d$ different bits. According to the analysis above, if Bob’s fake result on one of these $d$ bits (denote it as the $i$-th bit) is a type $a$ or $b$ lie, the states $|\beta_i(c_i^d)|$ and $|\beta_i(c_i^{d^*})|$ are orthogonal to each other since $c_i \neq c_i^*$. In this case $\langle B(c) | B(c^*) \rangle = 0$ will be rigorously satisfied regardless the state of other $\beta_i$’s ($i \neq i$).

It is easy to ensure that the $d$ different bits between any two codewords contain at least one bit which is corresponding to a type $a$ or $b$ lie. According to step (C3) of our protocol, the numbers of the types $a$, $b$ and $c$ lies are about $f_a, f_b, s$ and $f_s$, respectively. Eq. (11) shows that the numbers of each type of the lies that Alice detected in step (C4) are about $f_a s / 2, f_b s / 4$ and $f_s s / 4$, respectively. Therefore after the commit phase, the numbers of these three types of lies that remain undetected are about

$$l'_a \simeq f_a s / 2, l'_b \simeq f_b s / 4, l'_c \simeq f_s s / 4.$$  

(9)

Meanwhile, the total number of honest results are about

$$h \simeq (1 - f_a - f_b - f_c)s.$$  

(10)

Note that the above numbers are all evaluated statistically. Some fluctuation around these statistical values must be allowed. But the tolerable range of fluctuation (that can ensure the protocol work properly with a very high probability) can be estimated using classical statistical theory, so we are not going into detail here. Obviously, the above numbers satisfy the following relationship regardless of statistical fluctuation

$$l'_a + l'_b + l'_c + h = n.$$  

(11)

Given that Bob’s choice of the type of lies is fixed for each and every $\beta_i$ ($i \in S-L$), then if $d$ is larger than the total numbers of honest results (i.e., $h$) and the type $c$ lies left undetected (i.e., $l'_c$), and the difference is significantly larger than the tolerable range of statistical fluctuation, we can be sure that among the $d$ different bits between any two codewords, there is at least one bit that corresponding to a type $a$ or $b$ lie. Therefore, in step (C7.1) Bob tends to accept a value of $d$ that satisfies

$$d > d_{\text{min}} \equiv h + l'_c \simeq (1 - f_a - f_b - f_c) / 4 s.$$  

(12)

With this $d$, $\langle B(c) | B(c^*) \rangle = 0$ will always be satisfied for any two codewords $c$ and $c^*$. Thus the two Hilbert spaces $H_B$ and $H_1$ are rigorously orthogonal to each other, where $H_B$ ($b = 0, 1$) denotes the space supported by all the states $|B(c)|$’s that corresponding to those codewords $c$’s which can unveil the value of the committed bit as $b$ successfully, i.e., $\{c \in C | c \oplus r = b\}$. Using $\lambda_c$ denote the probability for a codeword $c$ to be chosen when Alice’s committed value is $b$ ($b = 0, 1$), we have $\rho_B^b = \sum_{c \in C | c \oplus r = b} \lambda_c |B(c)| \langle B(c) | B(c)\rangle$. Then we reach one of the main conclusion of the current paper, that there will be $\rho_B^b \perp \rho_B^1$ as long as $d$ satisfies Eq. (12). In this case, Alice cannot cheat using the specific strategy proposed in the existing no-go theorem of QBC, because it has to rely on the condition $\rho_B^b \perp \rho_B^1$.

V. SECURITY AGAINST BOB’S CHEATING

A. Bob’s dilemma: which basis to measure

It remains to show that our protocol is still concealing even though $\rho_B^b \perp \rho_B^1$. At the first glance it seems impossible, because Bob can simple perform a collective measurement that distinguishes $\rho_B^b$ from $\rho_B^1$ and learn the committed bit $b$. More rigorously, Bob finds out all the codewords $c \in C$ that satisfy $c \oplus r = 0$, then constructs the projection operator

$$P_0 \equiv \sum_{c \in C | c \oplus r = 0} |B(c)| \langle B(c) |,$$  

(13)

and applies it on his quantum system $B = \bigotimes_i \beta_i$ ($i \in S-L$) after the commit phase. If the projection is successful, he knows that $b = 0$. Else if the projection fails, he knows that $b = 1$.

However, we must note that $\rho_B^b \perp \rho_B^1$ is conditional. It requires Eq. (12), which is derived under the assumption that Bob’s choice of the type of lies for every $\beta_i$ ($i \in S-L$) is already fixed. To calculate the operator $P_0$ in Eq. (13), Bob must know the form of the state $|B(c)| = \bigotimes_i |\beta_i(c_i^0)|$ corresponding to each codewords $c$ satisfying $c \oplus r = 0$. According to Table I, $|\beta_i(c_i^0)|$ has different forms for different types of lies. Without knowing the choice of the type of lies for each $\beta_i$, $P_0$ cannot be obtained.

On the other hand, suppose that Bob tries to calculate $P_0$ without fixing the type of lies. That is, he exhausts all possible combinations of the types of lies, finds the form of $|B(c)|$ corresponding to each of these combinations,
and includes all these $|B(c)\rangle$’s into the sum in Eq. (13). Then the resultant $P_0$ will be useless for the reason below. As the choice of the type of lies is not fixed, suppose that we first calculate the form of $|B(c)\rangle$ by assuming that the fake result $|p_i', q_i'\rangle_\beta$ for $\beta_i$ is a type $a$ lie when $i = 1$, and it is a type $c$ lie when $i = 2$, ..., then we calculate the form of $|B(c^*)\rangle$ corresponding to a different codeword $c^*$ by assuming that the fake result for $\beta_i$ is a type $b$ lie when $i = 1$, and it is an honest result when $i = 2$, .... In this case, for any given $i$, we cannot guarantee that $c_i$ and $c_i'$ are corresponding to the same type of lies. So we can no longer make the assertion above Eq. (12), that “among the $d$ different bits between any two codewords, there is at least one bit that corresponding to a type $a$ or $b$ lie”, even if we take $d > h + l'_c$. Consequently, $\langle B(c) | B(c^*)\rangle = 0$ will not necessarily hold, especially when we exhausts all possible combinations of the types of lies. In turns, $\rho^B_0 \perp \rho^B_1$ will become invalid. Also, there will be some codewords that lead to $\langle B(c) | B(c^*)\rangle \neq 0$, even if $c \cap r = 0$ while $c^* \cap r = 1$. Then although Eq. (14) sums over all $c$ satisfying $c \cap r = 0$ only, the operator $P_0$ thus obtained will actually contains components like $|B(c^*)\rangle \langle B(c^*)|$ where $c^* \cap r = 1$. Applying such a $P_0$ will no longer provide the value of $b$, no matter the projection is successful or not.

Thus we show that to construct the projection operator for the measurement to distinguish $\rho^B_0$, Bob needs to know the type of lies corresponding to every $\beta_i$ first. But as we know, the types of lies are defined according to the basis to use for his measurement. To know how to choose the type of lies and honest results, because there does not exist a single basis that can distinguish the type of lies and the value of $c_i^0$ simultaneously.

In short, though there is $\rho^B_0 \perp \rho^B_1$ when Eq. (12) is satisfied, constructing the measurement operator for distinguishing $\rho^B_0$ requires the knowledge on how $\rho^B_0$ are defined. But when there are types $a$ and $c$ lies and honest results, the definition of $\rho^B_0$ is unknown to Bob unless he performs another measurement to identify the types of lies. As the two measurements are not commutative, Bob cannot have the best of both worlds. The only exception, however, is type $b$ lies, which will be studied below.

### B. Alice’s required $d$

Unlike other types of lies and honest results, once a type $b$ lie is identified, the value of $c_i^0$ will be known to Bob automatically without requiring another measurement. As shown in Table I, there is no legitimate state of $\beta_i$ that can be unveiled as $c_i^0 = 1$. If Bob announced a fake result $|p_i', q_i'\rangle_\beta$ as a type $b$ lie, i.e., his measurement was performed in the $\gamma_{p_i'}$ basis and the actual result is $|p_i', q_i'\rangle_\beta = |\tilde{p}_i', \tilde{q}_i'\rangle_\beta$, Bob will know that there must be $c_i^0 = 0$ once Alice has not announced this $|p_i', q_i'\rangle_\beta$ as a detected lie in step (C4). No need for a further measurement in a different basis. Therefore Bob would like to maximize the number of the type $b$ lies, to save himself from the dilemma brought by types $a$ and $c$ lies and honest results. When the type $b$ lies occur with a sufficiently high frequency $f_b$ so that there is $l'_b > n - d$, then he knows more than $n - d$ bits of the codeword Alice chose. Here $l'_b$ is the number of the type $b$ lies remained undetected in the commit phase, as presented in Eq. (9). Since the distance between any two codewords is not less than $d$, there will be only one codeword in $C$ which contains these bits known to Bob. Thus he can deduce the rest unknown bits, and learn the complete codeword so that the value of Alice’s committed bit $b$ can be deduced.

However, this can be avoided by setting an upperbound for $d$. Although Alice does not know the respective values of $f_a$, $f_b$, and $f_c$ that Bob chose, in step (C4) she knows $|M|$, i.e., Eq. (5). Suppose that in step (C7.1) Alice accepts a value of $d$ that satisfies

$$d < d_{\text{max}} = m - s/4 \geq (f_a + f_c)s/2 \leq f_a s/2 + 3 f_c s/4 = l'_a + l'_c. \quad (14)$$

That is, $d$ is smaller than the total of types $a$ and $c$ lies that remain undetected after the commit phase. In this case, among any $n - d$ bits of the codeword, there will always be at least one bit that corresponds to a type $a$ or $c$ lie, i.e., $l'_b > n - d$ will never be satisfied. Then Bob will not have enough type $b$ lies to deduce the complete
codeword, because with more than \( d \) bits remained unknown, the possible choices for codewords will increase exponentially with the value of \( k \) of the \((n, k, d)\)-code \( C \). As a result, once Eq. (14) is met, the protocol is concealing no matter how Bob chooses his lying frequencies \( f_a, f_b, \) and \( f_c \).

C. The existence of \( d \)

So we can see that in step (C7.1) of the commit protocol, for their own benefit, Alice will try to lower the value of \( d \) so that Eq. (14) can be satisfied, while Bob will try to increase \( d \) to meet Eq. (12). Luckily we can have the best of both worlds. Since the above values are estimated statistically and subjected to fluctuations, when \( s \) is very large there will be enough gap between \( d_{\text{max}} \) and \( d_{\text{min}} \) for Alice and Bob to choose a proper \( d \).

The condition Eq. (16) can be met easily. For example, a simple choice is that Bob takes \( f_b = 1 - 3f_a/2 \) and \( f_c = 0 \) (in fact any \( f_c \) satisfying \( 0 \leq f_c < 1 - f_a - f_b \) will do) in step (C3). Note that even if Bob does not choose these values honestly, Alice does not need to worry. All she needs is to insist on choosing a value of \( d \) that satisfies Eq. (16), which is a legitimate action for an honest Alice that Bob cannot refuse. In this case she can still be sure that Bob does not have enough type \( b \) lies to deduce the codeword. Meanwhile, if the dishonest Bob accepts such a value of \( d \) in order to avoid Alice feeling suspicious, then Eq. (12) may not be satisfied. In this case \( \rho_0^B \perp \rho_1^B \) will no longer be ensured, which may allow room for Alice’s potential cheating. Thus we see that if Bob does not choose \( f_a, f_b, \) and \( f_c \) following Eq. (16) honestly, then he can only hurt his own benefit.

VI. SUMMARY AND REMARKS

We show above that our protocol satisfies \( \rho_0^B \perp \rho_1^B \) when Eq. (12) is met. Therefore Alice cannot cheat with the specific strategy presented in existing no-go proofs of unconditionally secure QBC. The key reason is that all these proofs are based on the HJW theorem, which requires \( \rho_0^B \simeq \rho_1^B \).

On the other hand, our protocol remains secure against Bob’s cheating when Eq. (14) is met, because the measurement basis for the discrimination between \( \rho_0^B \) and \( \rho_1^B \) is different from the basis for learning the definition of \( \rho_1^B \) and \( \rho_1^B \). Thus we see that \( \rho_0^B \simeq \rho_1^B \) is not a necessary condition for a QBC protocol to be concealing.

Also, Eq. (16) ensures that Eqs. (12) and (14) can be satisfied simultaneously, so that the security against both sides can be guaranteed.

It is worth noting that \( \rho_0^B \perp \rho_1^B \) actually can also be found in some unconditionally secure relativistic bit commitment protocols [25, 30], where the committed values are encoded with classical data instead of quantum states. As pointed out in the 3rd paragraph of the introduction of [15], “Kent’s relativistic bit commitment protocol does not rely on the existence of alternative decompositions of a density operator, and so its security is not challenged by the Mayers-Lo-Chau result.” They make use of relativistic constraints to achieve the security against Bob’s cheating. Another previous QBC proposal of ours [25] have the feature \( \rho_0^B \perp \rho_1^B \) too, as it is built on top of a quantum key distribution scheme based on orthogonal states [37]. Whether relativity is the key of its security is arguable [38, 39]. Our current protocol is completely free from the need of relativity. Its security against Bob is provided by keeping the definition of \( \rho_0^B \) secret from him at the beginning.

The fact that \( \rho_0^B \) is unknown without measurement is also an interesting feature that sets our protocol apart from the QBC model studied in many no-go proofs. In the own words of [3] (as stated below its Eq. (2)), “both Alice and Bob are supposed to know the states \( |0 \rangle \) and \( |1 \rangle \). This implies, in particular, that both of them know the states \( |\phi_i\rangle_B \) and \( |\phi_j\rangle_B \).” Here \( |0 \rangle \) (|1\rangle) and \( |\phi_i\rangle_B \) (|\phi_j\rangle_B) have the same meanings as these of \( |\psi_i\rangle \) and \( |f^{(b)}_j\rangle_B \) (\( b = 0,1 \)) in our Eq. (1). In many other no-go proofs, though it is not explicitly stated, we can still see from the details of their proofs that they hold the same viewpoint. But as already pinpointed out in another no-go proof [17, 18], previously “the no-go theorem asserts (this) without proof”, and “this assertion is actually not meaningful”. (An amendment to this problem was made in [17, 18]). But it only considered the case where the states are unknown to Alice, instead of Bob, and the proof is still based on \( \rho_0^B \simeq \rho_1^B \).)

These findings reveal that the existing impossibility proofs are not sufficiently general. If unconditionally secure QBC is indeed impossible, then it is necessary to show that there exists a more universal cheating strategy which does not rely on the condition \( \rho_0^B \simeq \rho_1^B \).

Finally, it is worth noting that entanglement plays an essential role in our protocol. In many previous QBC protocols proven insecure by the no-go theorem, the honest participants can execute the protocol successfully by exchanging pure states unentangled with any system at their sides. Entanglement is needed only when cheating. On the contrary, in our protocol even an honest Alice must make use of entangled states to accomplish the optimal strategy to detect Bob’s lies. If she prepares every \( \beta_i \) as a pure state \( |p_i, q_i\rangle_\beta \) instead, and sends it to Bob without entangling it with any system at her side, then
she cannot detect the lies with the efficiency required in the protocol. Therefore such an Alice will be caught as cheating instead of honest. That is, our protocol cannot be accomplished without entanglement. Since entanglement is a typical example of nonlocality, this result is consistent with the claim that nonlocality is necessary for secure QBC, as shown in section 7 of [26], as well as in [40].

The work was supported in part by the NSF of China under grant No. 10975198, the NSF of Guangdong province, and the Foundation of Zhongshan University Advanced Research Center.

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