Multilevel Coding over Two-Hop Single-User Networks

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Abstract

In this paper, a two-hop network in which information is transmitted from a source via a relay to a destination is considered. It is assumed that the channels are static fading with additive white Gaussian noise. All nodes are equipped with a single antenna and the Channel State Information (CSI) of each hop is not available at the corresponding transmitter. The relay is assumed to be simple, i.e., not capable of data buffering over multiple coding blocks, water-filling over time, or rescheduling. A commonly used design criterion in such configurations is the maximization of the average received rate at the destination. We show that using a continuum of multilevel codes at both the source and the relay, in conjunction with decode and forward strategy at the relay, performs optimum in this setup. In addition, we present a scheme to optimally allocate the available source and relay powers to different levels of their corresponding codes. The performance of this scheme is evaluated assuming Rayleigh fading and compared with the previously known strategies.

I. INTRODUCTION

In recent years, relay-assisted transmission has gained significant attention as a powerful technique to enhance the performance of wireless networks. The main idea is to employ some extra nodes (relay nodes) in the network to facilitate the communication between the terminal nodes. The concept of relaying was first introduced by Van der Meulen in [1] and is defined as a scheme to improve the coverage/reliability of a wireless network. For instance, relays are usually deployed in networks when the direct link between the source and the destination is either blocked or has a very poor quality. The term two-hop network usually refers to such a network configuration in which there is no direct link between the source and

Financial supports provided by Nortel, and the corresponding matching funds by the Federal government: Natural Sciences and Engineering Research Council of Canada (NSERC) and Province of Ontario: Ontario Centres of Excellence (OCE) are gratefully acknowledged.
the destination, and one relay node assists the transmission of data between the end terminals, see Fig. 1. Two-hop networks have been implemented widely in different applications, including TV broadcasting and satellite communications.

Following the introduction of relay channel in [1], Cover and El Gamal introduce two different coding strategies for single relay networks [2]. In the first strategy, known as “Decode and Forward” (DF), the relay decodes the transmitted message and cooperates with the source to send the message in the next block. Instead of decoding, in the second strategy, the relay compresses the received signal and forwards it to the destination in the next block. The terms “Compress and Forward” (CF) or “Quantize and Forward” (QF) usually refer to this transmission scheme. Besides DF and CF, in some recent results, [3]–[6], the authors investigate another transmission scheme called “Amplify and Forward” (AF) for the Gaussian relay network. In this strategy, without decoding the information, the relay amplifies the received signal and retransmits it to the destination.

Knowing these schemes, the performance of relaying is analyzed for different network topologies. For instance, considering a single-relay network, authors of [5] and [6] derive a single-letter expression for the maximum achievable rate of AF relaying using a simple linear scheme (assuming frequency division and AWGN channel). As another example, [4] shows that AF relaying achieves the network capacity of Gaussian parallel single-antenna relay network. The extension of [4] to the case of multiple-antenna Rayleigh fading networks is presented in [7] and [8]. The first capacity result for relay networks is obtained in [2], where the authors prove the optimality of DF strategy in a single-relay network when the received signal at the destination is a degraded version of the relay received signal. Clearly, the degradedness condition holds in the two-hop setting. Thus, DF would be the optimal relaying scheme for two-hop networks. Indeed, most of the results in the literature on relay networks either assume static channels between the nodes or perfect knowledge of the Channel State Information (CSI) at both end nodes of each link, for the case of fading channels.

Recently, some papers discuss different transmission schemes over relay networks when the CSI is not available at the transmitting nodes, where most of them focus on Diversity-Multiplexing Trade-off (DMT) [9]–[14]. Obviously, for such settings, the ergodic capacity is not defined, however, the outage capacity is defined as the maximum rate decodable with a given probability [15]. From the throughput maximization point of view, the goal is to propose a scheme to maximize the average data rate received at the destination. The simplest form of such a problem is to find the optimal transmission strategy for a one-hop single-user network when CSI is available only at the receiver and the channel has quasi-static fading characteristic. In a pioneering work, Shamai has addressed this problem by substituting the receiver by a continuum of virtual receivers, each corresponding to a specific realization of the channel gain [16].
Relying on the resulting degraded broadcast channel, [16] shows that an infinite-level coding scheme with a proper power allocation among the different levels of code maximizes the destination’s average data rate.

There are several extensions for this work; in [17]–[19] the authors try to find the optimal transmission strategy when partial information is available at the source. [20] suggests the application of multilevel coding in a multicast network with some QoS constraints and derives the optimum throughput-coverage trade-off for such a network. [21] combines multilevel coding scheme with Hybrid Automatic Retransmission Request (HARQ) and shows that this approach results in high throughput and low latency in a point-to-point link. The Multiple Input Multiple Output (MIMO) extension of [16] is also discussed in [22], [23], and [24].

Considering the two-hop network with no CSI at the transmitter side of each link, Steiner et al in [25] propose different transmission schemes, assuming that the relay node has a power constraint and is simple, i.e., is not capable of data buffering over multiple coding blocks or rescheduling. To find the optimal transmission scheme, they study different multilevel coding schemes when the relay operates in different modes, including DF, AF, and CF. As discussed in [25], due to the high complexity of the infinite-level DF coding scheme, they have only considered a finite level code in their proposed DF strategies. Comparing the results of [25], it turn out that AF strategy outperforms all other investigated transmission schemes, specifically in the high SNR regime. However, as concluded in [25], although AF has the best performance among the other strategies, the optimality of AF scheme is not implied. In fact, since the general infinite-level DF coding scheme remains unsolved, there is always a question of whether or not DF relaying achieves a higher expected rate as compared to AF. The main motivation of this work is to answer this question.

In this work, we study the performance of DF relaying scheme which uses infinite-level codes at the source and the relay in the same setup as in [25]. To this end, we first prove that the infinite-level code and DF relaying is indeed the optimal two-hop transmission strategy for maximizing the destination’s average data rate. We further propose an algorithm to determine the optimum power allocation for each indefinite-level code. Numerical results are presented to verify that the proposed scheme outperforms the AF strategy discussed in [25].

The organization of this paper is as follows: First, section II describes the two-hop system model. A brief review of some previous results on single-hop links are presented in section III. Formulating the DF multilevel coding scheme in subsection IV-A we prove the optimality of this scheme in subsection IV-B. Afterwards, in section IV-C we present a procedure to actually determine the optimal infinite-level code parameters at the source and the relay. Numerical results and comparison with other schemes for the
special case where both links are Rayleigh fading is presented in section V. Finally, section VI concludes the paper.

Throughout this paper, we represent the expectation operation by $E[\cdot]$. The notation $\log(\cdot)$ is used for the natural logarithm and rates are expressed in nats. We denote $f_y(\cdot)$ and $F_y(\cdot)$ as the probability density function (pdf) and the cumulative distribution function (CDF) of random variable $y$.

II. System Model

In this paper, we investigate the performance of a two-hop network, in which a relay assists the transmission of data between a source and a destination. As Fig. 1 shows, in a two-hop network, the destination can solely receive data via the relay. All nodes are assumed to have single antennas. It is assumed that the source has no information about either of the channel gains, the relay knows only the channel gain between itself and the source, and the destination knows both channel gains. This CSI assumptions are indeed practical, since the receiver of each hop can evaluate its immediate channel gain by measuring the pilot signal sent from the corresponding transmitter. In addition, the receiver can measure the equivalent channel (the source to the destination) if the relay forwards the pilot signal of the source towards the destination. Having the equivalent channel gain and the relay to destination channel gain, the destination can find the source to relay channel gain as well.

It is also assumed that the source and relay both know the fading power distribution of both links. By $f_T(\gamma)$ and $f_T(\nu)$ we denote the probability density function of fading power in the first and second hop, respectively. The channels of both links are assumed to be quasi-static, i.e., they are chosen randomly (based on their corresponding pdf) at the start of the transmission and remain fixed for the whole transmission.

Source and relay are assumed to have power constraints $P_s$ and $P_r$, respectively. Furthermore, they are assumed to be capable of constructing infinite-level codes. However, the relay is assumed to be simple, i.e., is not capable of buffering any unsent data, rescheduling the untransmitted data from the previous blocks. It is also assumed that the relay can not perform water-filling of its power over multiple blocks and has a maximum power constraint in each block. Therefore, it can only retransmit the data which it has just received from the first hop. Successive decoding is used as the decoding procedure at both destination and relay (in case the relay wants to decode the data).
III. MULTILEVEL CODING SCHEME FOR SINGLE-HOP NETWORKS

In this section, we review some previous studies on the application of the multilevel codes for a single-hop network in two scenarios: 1) No CSI is available at the transmitter, which is known as “broadcasting approach” [16] and 2) no CSI is available at the transmitter and in addition, the transmission data rate can not exceed a certain value [25].

A. Single-Hop Broadcasting Strategy

The optimum scheme for a single-hop link, in case that the transmitter knows the fading power for each block is to design a single-level code with the rate of \( \log(1 + lP) \) for that block. Here, \( P \) denotes the normalized transmission power, i.e, the equivalent transmission power if the noise power is equal to one and \( l \) is the fading power for that specific block. Therefore, the average achievable rate for this setup would be \( R_{\text{erg}} = E_l[\log(1 + lP)] \) [26].

Although designing a single-level code is optimum in the above scenario, it can not be applied for a transmitter which does not have access to the channel state information. For such a scenario, Shamai has introduced a technique called broadcasting strategy [16]. In this technique, the transmitter sends the data through infinite levels of a superposition code. Then, conditioned on the channel state, i.e., the fading power, the receiver decodes up to a certain level of the code. Therefore, the total receiving rate for each channel realization, say \( l \), can be evaluated as:

\[
R(l) = \int_0^l dR(a),
\]

where \( dR(a) \) represents the differential rate transmitted over level ‘a’ of the code. Defining \( \rho(l)dl \) as the power assigned for the \( l^{th} \) level, \( dR(l) \) is given by:

\[
dR(l) = \log\left(1 + \frac{l\rho(l)dl}{1 + lI(l)}\right)
\]

\[
\approx \frac{l\rho(l)dl}{1 + lI(l)},
\]

where \( I(l) = \int_l^\infty \rho(a)da \) and the second line follows from the assumption of infinitesimal rate assignment which is shown to be optimal in [20] . The aim is to find a \( \rho(.) \) such that the average data rate at the destination is maximized while the source power constraint \( (P) \) holds. It means that the power should be assigned to different code levels such that:

\[
\max_{\rho(.)} \quad R_{\text{av}} = \int_0^\infty dl f(l)R(l)
\]

\[
s.t. \quad \int_0^\infty \rho(l)dl = P,
\]
where \( f(.) \) shows the pdf of the channel fading power.

This maximization problem has been studied and solved using calculus of variations technique (see [16] for details of the proof). Here, we only mention the final solution as:

\[
I^*(l) = \begin{cases} 
  P & l < l_0 \\
  \frac{1-F(l)-f(l)}{l f(l)} & l_0 < l < l_1 \\
  0 & l_1 < l
\end{cases}
\]  \tag{4}

where \( F(l) = \int_{-\infty}^{l} f(a)da \) is the CDF of the fading power. \( l_0 \) and \( l_1 \) are determined such that they satisfy \( I^*(l_0) = P \) and \( I^*(l_1) = 0 \), respectively. Clearly, the optimum power assignment can be determined by \( \rho^*(l) = -\frac{dI^*(l)}{dl} \). The maximum destination’s average data rate will be:

\[
R^*_{av} = \int_{l=0}^{\infty} dl f(l) R^*(l),
\]  \tag{5}

where \( R^*(l) \) is obtained by setting \( \rho(l) = \rho^*(l) \) in (1) and (2). The total rate of the final superimposed code can be evaluated by:

\[
R^*_F = \int_{0}^{\infty} dR^*(l).
\]  \tag{6}

Note that this value only depends on the fading power distribution and the transmitter power.

Finally, it is important to mention that the above scheme is indeed the optimal transmission scheme for a fading link when the source does not have the CSI. It is due to the result of [27] which proves that the multilevel coding maximizes any weighted sum-rate of a degraded broadcast channel. Therefore, considering the equivalent broadcast model for the single-user fading channel, the multilevel coding achieves the optimal destination’s average data rate.

\section*{B. Single-Hop Rate-Limited Broadcast Strategy}

An interesting extension of the above broadcast strategy is designing a multilevel code for a source with available data rate limited to \( R_{in} \), i.e., the transmission rate should be less than \( R_{in} \) [25].

The problem formulation is similar to subsection \[\text{III-A}\] except it has one more constraint on the transmission rate. The modified optimization problem will be, [25]:

\[
\max_{\rho(.)} R_{av} = \int_{0}^{\infty} dl f(l) R(l)
\]  \tag{7}

\[
\text{s.t.} \quad \int_{0}^{\infty} \rho(l)dl = P, \quad \text{and} \quad \int_{0}^{\infty} dR(l) \leq R_{in},
\]

where the second condition ensures that the transmission rate remains less than the available source data rate.
Reference [25] uses constrained calculus of variations to solve this problem. More precisely, initially the authors in [25] substitute the first constraint by two end-point conditions of \( I(0) = P \) and \( I(\infty) = 0 \). Then, using \( \rho^*(l) = -\frac{dI^*(l)}{dl} \) and Lagrangian multiplier, (7) is reconstituted as a variations problem (See [25] for more details). It turns out that the optimum distribution function is as follows:

\[
I^*(l) = \begin{cases} 
    P & l < l_0 \\
    \frac{1-F(l)+\lambda-lf(l)}{f(l)l^2} & l_0 < l < l_1 \\
    0 & l_1 < l
\end{cases}, \tag{8}
\]

where \( F(l) = \int_{-\infty}^{l} f(a)da \). \( l_0 \) and \( l_1 \) are determined as a function of \( \lambda \) and satisfy \( I^*(l_0) = P \) and \( I^*(l_1) = 0 \), respectively. Finally, \( \lambda \) is computed such that the transmission rate constraint holds.

As an example, for the case of Rayleigh fading channel, i.e., \( F(l) = 1 - e^{-l} \), the optimum distribution is as follows [25]:

\[
I^*(l) = \begin{cases} 
    P & l < l_0 \\
    \frac{\lambda}{e^{-x^2}} + \frac{1}{x^2} - \frac{1}{x} & l_0 < l < l_1 \\
    0 & l_1 < l
\end{cases}, \tag{9}
\]

\[
l_1 = 1 - W_L(-\lambda e) \tag{10}
\]

where \( W_L(x) \) is the Lambert W-function and finds \( w \) such that \( we^w = x \). The values for \( \lambda \) and \( l_0 \) are also determined by solving the following system of equations:

\[
\begin{cases} 
    R_{in} &= 2 \log(l_1) - l_1 - (2 \log(l_0) - l_0) \\
    I^*(l_0) &= P
\end{cases} \tag{11}
\]

One important observation is that in the case of \( R_{in} \geq R_F^* \) (\( R_F^* \) is defined in equation (6)) the above rate-limited problem will be simplified to the original problem in subsection III-A. Although this statement can be verified mathematically, its intuitive explanation would be insightful. It is obvious that if \( R_{in} \) tends to infinity, the rate condition is always satisfied (will not be an active constraint). Therefore, maximization problem of (7) relaxes to the form of equation (3). Moreover, III-A shows that the source, in the optimal transmission of the original problem (without rate constraint), feeds the channel with a rate equal to \( R_F^* \). Hence, it turns out that even though the available rate at the source is more than \( R_F^* \), the source needs to transmit only \( R_F^* \) bits of information in each block. Therefore, if \( R_{in} \geq R_F^* \), the solutions of the two optimization problems of (3) and (7) are equal.

At the end, note that the resulted multilevel code is also the optimal transmission scheme for the rate-limited one-hop set-up. To prove, we should use the broadcast equivalent structure of the network. Indeed, the capacity region of the rate-limited degraded broadcast network is the intersection of the
original degraded broadcast channel capacity region and the region below the surface associated with the transmission rate constraint. For illustration, the solid line in Fig. 2 shows a typical capacity region of a rate-limited two-user degraded broadcast channel. In this figure, dotted and dashed lines depict the capacity region of the original degraded broadcast channel (without rate limitation) and the line representing the rate constraint, respectively. The destination’s average data rate can be evaluated as the sum of the received data rate for each channel state \( l \) \((R(l))\) times the probability of occurring that specific channel state \((f(l))\). Clearly, this value is maximized on the boundary of the resulted capacity region (the solid line). Therefore, the optimal point would be either over the capacity region without rate limitation (Arc \( \overline{AB} \)) or the end point of the rate limitation line (Point C). Since \( \overline{AB} \) is a part of the original capacity region, multilevel coding is the optimal scheme to achieve any point on \( \overline{AB} \). Furthermore, point C is archived using a single level code which is again a special case of multilevel codes. This proves the optimality of multilevel coding scheme for rate limited scenarios.

IV. MULTILEVEL CODING SCHEME FOR TWO-HOP NETWORKS

As an extension of the one-hop set-up, [25] addresses the problem of maximizing the destination’s average data rate in a two-hop network, where there is no direct link between the source and the destination. In reference [25], several schemes have been studied, including broadcasting strategy with AF relaying, and DF relaying with finite level broadcasting at the source and the relay. Infinite level codes with DF relaying is also addressed in [25]. However, the performance of this method remains as an open problem there.

In this section, we will first describe the infinite level DF strategy in details. Then, in subsection IV-B...
we prove the optimality of this scheme. Finally, subsection [IV-C] presents an algorithm to optimally design such an infinite level code.

A. DF Infinite-Level Codes for Two-hop Networks

Based on the system model, each transmission block of the infinite level DF strategy consists of the following two steps:

1) In the first phase, the source allocates its power among different code levels with the power distribution function $\rho_s(.)$. Of course, $\rho_s(.)$ should satisfy the power constraint $\int_0^{\infty} \rho_s(a)da = P_s$. Then, based on the source-relay channel fading power, say $\gamma$, the relay is able to decode up to the level $\gamma$ of the transmitted data. Thus, the relay received rate is:

$$R_r(\gamma) = \int_0^{\gamma} \log \left(1 + \frac{a\rho_s(a)da}{1 + aI_s(a)}\right) \simeq \int_0^{\gamma} \frac{a\rho_s(a)da}{1 + aI_s(a)},$$

(12)

where $I_s(a) = \int_a^{\infty} \rho_s(a)da$.

2) In the second phase, the relay should transmit the data to the destination. As noted earlier, in this work, we only focus on simple relays which can neither buffer any of the previously received data nor do any scheduling tasks. As a results, these relays have two features which seem obvious but have important effects on the code design. To illustrate, consider a case in which the relay has decoded $R_r(\gamma)$ bits of the transmitted data. It turns out that, firstly, the relay can not transmit with the rate greater than $R_r(\gamma)$. Secondly, if the relay transmits with the rate $R_2$, $R_2 < R_r(\gamma)$, the rest of the data ($R_r(\gamma) - R_2$) can not be stored and should be discarded. Consequently, the relay, in each transmission block, should choose the optimal power distribution of the multilevel code such that it satisfies the relay total power constraint ($P_r$). Meanwhile, the relay should keep the transmission rate below its received data rate in that block ($R_r(\gamma)$).

Defining $\rho_r(.|R_r(\gamma))$ as the power distribution of each code level at the relay conditioned on the input rate of $R_r(\gamma)$, we can summarize these conditions as:

a) Power constraint at the relay: $\forall R_r(\gamma)$:

$$\int_0^{\infty} \rho_r(a|R_r(\gamma))da = P_r.$$

b) Available rate constraint at the relay: $\forall R_r(\gamma)$:

$$\int_0^{\infty} \frac{a\rho_r(a|R_r(\gamma))da}{1 + aI_r(a|R_r(\gamma))} \leq R_r(\gamma),$$

where $R_r(\gamma)$ is defined by (12).

Clearly, the relay requires to know $\rho_r(a|R_r(\gamma))$ for all possible values of $R_r(\gamma)$.

Transmitting a multilevel code on the relay-destination link, the destination is able to decode up to a certain level ‘$\upsilon$’. Here, ‘$\upsilon$’ denotes the fading power of the second link. Therefore, for each
Given these, we are now able to formulate the two-hop optimization problem. Similar to the single-hop scenario, we want to maximize the average data rate received at the destination. Assuming \( f_\Gamma(\gamma) \) and \( f_T(v) \) as the probability density functions of the fading power in the source-relay and relay-destination links, respectively, the destination’s average data rate can be written as:

\[
E[R_d] = E_T \{ E_\Gamma [R_d(v|R_r(\gamma))] \}
\]

\[
= \int_0^\infty \int_0^\infty f_\Gamma(\gamma) f_T(v) \int_0^v \frac{a \rho_r(a|R_r(\gamma))}{1 + a I_r(a|R_r(\gamma))} dv \, d\gamma.
\]

Therefore, we obtain the final optimization problem as follows:

\[
\max_{\rho_s(\cdot), \rho_r(\cdot)} \int_0^\infty \int_0^\infty f_\Gamma(\gamma) f_T(v) R_d(v|R_r(\gamma)) dv \, d\gamma
\]

\[
s.t. \quad \int_0^\infty \rho_s(a) da = P_s,
\]

\[
\forall R_r(\gamma) : \int_0^\infty \rho_r(a|R_r(\gamma)) da = P_r,
\]

\[
\forall R_r(\gamma) : \int_0^\infty \frac{a \rho_r(a|R_r(\gamma))}{1 + a I_r(a|R_r(\gamma))} da \leq R_r(\gamma).
\]

Note that the above optimization problem is similar to the one derived in [25]. However, in [25], the last constraint (relay rate limitation) is stated as an equality. In fact, it is more accurate to formulate the rate limitation by an inequality constraint instead of equality. It is due to the fact that the relay may not have to send all information it receives from the first hop to achieve the optimal performance. In other words, the last constraint of equation (15) lets the relay to discard some of its received data if it wants to do so. For instance, this may happen when the relay receives data rate higher than its corresponding \( R_r^* \) (\( R_r^* \) is defined in equation (6)). In such a scenario, the relay only uses \( R_r^* \) bits of the received information and ignores the rest.
B. Optimality of Two-Hop DF Multilevel Coding

The main focus of this section is to show that the multilevel coding approach combined with decode and forward (DF) relaying maximizes the average data rate at the destination of a two-hop network.

To start, let us emphasize that according to the two-hop structure of the network, all information received by the destination should be first passed through the relay and there is not any direct link between the source and the destination. As a result, the destination received signal is always a degraded version of what has been received at the relay. In other words, no information can be decoded by the destination unless it has been decodable at the relay. Thus, it can be concluded that decode and forward is the optimal relaying scheme for two-hop settings.

Knowing the optimality of DF, similar to the single-hop network (See [22]), we model both first and second fading hops by infinite number of virtual relays and virtual users, respectively. More precisely, we substitute the relay with infinite relays, each has a constant channel gain which corresponds to a specific realization of the first hop channel. These virtual relays constitute a degraded set. If we assume that the channel fading power is selected from a set of discrete values \( \{a_1, a_2, \cdots, a_\kappa\} \), there would be \( \kappa \) virtual relays in the network. Without loss of generality, we assume \( a_1 < a_2 < \cdots < a_\kappa \). Of course, this model is accurate only if \( \kappa \) tends to infinity. In a similar way, the second hop can be modeled with \( \kappa \) virtual users for each of the virtual relays; thus, in total there would be \( \kappa^2 \) virtual users in the network. \( U_{ij} \) denotes the virtual user which is associated to the virtual relay \( R_i \), and its channel gain is \( a_j \). Moreover, \( B_{ij} \) represents the decodable rate at this node. For illustration purpose, Fig. 3 depicts the network model for the case that \( \kappa = 3 \), i.e., \( A = \{a_1, a_2, a_3\} \).

The aim is to find the optimum transmission scheme for the source and the relay. We start from the second

\footnote{Note that here, for simplicity, we assume that the fading levels of both links are selected from the same set. However, the statements of optimality holds for the general case. Moreover, this assumption is valid for the case of \( \kappa \to \infty \).}
hop and assume the source uses an arbitrary transmission scheme. Furthermore, let \( \{ B_1, B_2, \ldots, B_\kappa \} \) be the rate that each of the virtual relays, i.e., \( \{ R_1, R_2, \ldots, R_\kappa \} \), can decode under this transmission scheme. Having the data rate \( B_i \) at relay \( R_i \), the optimal second-hop strategy is to maximize the destination’s average data rate for each of the virtual relays. As will be described in subsection IV-C, the solution to this problem is similar to the result of the subsection III-B and the optimal power distribution, \( \rho_{B_i}^*(u) \), for each of the virtual relays can be determined by (8), in which \( R_{in} = B_i \). In other words, multilevel coding is the optimal strategy for the second hop.

Note that the successful decoding is only possible if the destination knows the applied \( \rho_{B_i}^*(u) \) (or equivalently the value of \( B_i \)) at the relay. In our two hop model, this information can be obtained by the knowledge of the source to the relay channel gain at the destination. The optimal received data rate for each of the virtual users, say \( U_{il} \), can be evaluated by:

\[
R^*(B_i, \Upsilon = a_l) = \int_0^{a_l} \frac{u \rho_{B_i}^*(u) du}{1 + u I_{B_i}^*(u)}
\]  
(16)

where \( I_{B_i}^*(u) = \int_u^{\infty} \rho_{B_i}^*(a) da \). Let \( \psi_T(j) \) be probability that the second hop gain is equal to \( a_j \). Hence, the optimal average rate of the second hop under rate condition \( B_i \) would be:

\[
D^*(B_i) = \sum_{l=1}^{\kappa} \psi_T(l) R^*(B_i, \Upsilon = a_l).
\]
(17)

Given \( D^*(B_i) \), the two-hop network average data rate can be written as:

\[
\overline{R} = \sum_{l=1}^{\kappa} \psi_T(l) D^*(B_i),
\]
(18)

where \( \psi_T(l) \) represents the probability that the first hop channel is in state \( a_l \). The goal of the code design is to maximize \( \overline{R} \). As (18) shows, the destination’s average data rate is the weighted sum of a non-linear function of \( B_i \). The domain of acceptable \( B_i \) for \( i \in \{1, 2, \ldots, \kappa\} \) is a convex set which is known as the capacity region of the underlying broadcast channel. Moreover, due to the degradedness of the virtual relays, all points on the boundary of this capacity region can be achieved by a multilevel code [2]. Therefore, to prove the optimality of the multilevel code for the first hop, it is enough to show that \( \overline{R} \) is maximized on the boundary of this capacity region. This argument can also be justified if we can show that \( \frac{\partial \overline{R}}{\partial B_i} \) is positive \( \forall i \in \{1, 2, \ldots, \kappa\} \). To show this, we write:

\[
\frac{\partial \overline{R}}{\partial B_i} = \psi_T(i) D^*(B_i)
\]
(19)

where \( D^*(B_i) \) is defined in equation (17) and shows the destination’s average data rate when the relay has the rate constraint of \( B_i \). By definition, \( \psi_T(i) \) is a non-negative value. Moreover, from the results of subsection III-B it can be concluded that \( D^*(B_i) \) is non-negative. In fact, if \( B_i \geq R_F^* \), where \( R_F^* \)
is defined in equation (6), then \( D^{*''}(B_i) = 0 \) and \( D^{*'}(B_i) > 0 \), otherwise. Therefore, \( \frac{\partial \mathcal{R}}{\partial B_i} \geq 0 \) for \( \forall i \in \{1, 2, ..., \kappa\} \). Consequently, \( \mathcal{R} \) is maximized for the values of \( \{B_i\}_{i=1}^{\kappa} \) which are on the boundary of the capacity region of the first hop broadcast channel. In other words, the multilevel coding scheme is the optimal transmission strategy for the first hop. This completes the proof for the optimality of multilevel coding for the two-hop network.

C. Optimal Design of Two-Hop DF Multilevel Coding

Having the optimality of two-hop multilevel coding scheme, in this section, we present a procedure in order to solve the two-hop optimization problem introduced in equation (15). The main difficulty of this problem is that, unlike single-hop scenarios (the original and the rate-limited broadcasting cases), equation (15) cannot be directly solved by variations methods. It is due to the fact that the constraint on the second hop rate does not have a fixed value on the right hand side, i.e., it does not have a form of isoperimetric problem. For a complete discussion on isoperimetric problem, refer to [28]. To solve this problem, we use the following lemma.

**Lemma 4.1:** The following maximization problems on two non-negative functions \( f_1(.) \) and \( f_2(.) \):

\[
\max_{f_1(.), f_2(\cdot)} \int_{x=a}^{b} \int_{y=c}^{d} dx dy \mathcal{H}(x) \mathcal{K}(f_1(\cdot), f_2(\cdot), x, y),
\]

(20)

and

\[
\max_{f_1(.)} \int_{x=a}^{b} dx \mathcal{H}(x) \max_{\{f_2(.)|f_1(.)\}} \int_{y=c}^{d} \mathcal{K}(f_1(\cdot), f_2(\cdot), x, y) dy,
\]

(21)

where \( \mathcal{H}(\cdot) \) and \( \mathcal{K}(\cdot) \) are two known non-negative functions, are equivalent.

**Proof:** Let us denote the solution of (20) by \( (f_1^*, f_2^*) \) and the solution of (21) by \( (f_1^+, f_2^+) \).

We can write

\[
\int_{x=a}^{b} \int_{y=c}^{d} dx dy \mathcal{H}(x) \mathcal{K}(f_1^+, f_2^+, x, y) = \int_{x=a}^{b} dx \mathcal{H}(x) \int_{y=c}^{d} \mathcal{K}(f_1^+, f_2^+, x, y) dy \\
\leq \int_{a}^{b} dx \mathcal{H}(x) \max_{\{f_2(\cdot)|f_1(\cdot)\}} \int_{y=c}^{d} \mathcal{K}(f_1^+, f_2(\cdot), x, y) dy \\
\leq \max_{f_1(.)} \int_{a}^{b} dx \mathcal{H}(x) \max_{\{f_2(.)|f_1(.)\}} \int_{y=c}^{d} \mathcal{K}(f_1, f_2(\cdot), x, y) dy.
\]

(22)

On the other hand,

\[
\max_{f_1(.)} \int_{a}^{b} dx \mathcal{H}(x) \max_{\{f_2(.)|f_1(.)\}} \int_{y=c}^{d} \mathcal{K}(f_1, f_2(\cdot), x, y) dy = \int_{x=a}^{b} dx \mathcal{H}(x) \int_{y=c}^{d} \mathcal{K}(f_1^+, f_2^+, x, y) dy \\
= \int_{x=a}^{b} dx \int_{y=c}^{d} dx dy \mathcal{H}(x) \mathcal{K}(f_1^+, f_2^+, x, y) \\
\leq \max_{f_1(.)} \int_{a}^{b} \int_{y=c}^{d} dx dy \mathcal{H}(x) \mathcal{K}(f_1, f_2(\cdot), x, y).
\]

(23)
Combining (22) and (23), lemma is proved.

Using this lemma and noting that \( f_\Gamma(\gamma) \geq 0, \forall \gamma \), we can reform \( E[R_d] \) in (15) as follows:

\[
\max_{\rho_r(\cdot),\rho_s(\cdot)} E[R_d] = \max_{\rho_s(\cdot)} \int_0^\infty d\gamma \ f_\Gamma(\gamma) \ \max_{\rho_r(\cdot)|\rho_s(\cdot),\gamma} \int_0^\infty dv \ f_\Gamma(v) \ \int_0^v \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da,
\]

\[
\equiv \max_{\rho_s(\cdot)} \int_0^\infty d\gamma \ f_\Gamma(\gamma) \ \max_{\rho_r(\cdot)|\rho_s(\cdot),\gamma} \int_0^\infty dv \ f_\Gamma(v) \ \int_0^v \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da,
\]

(24)

where the outer maximization is subject to:

\[
\int_0^\infty \rho_s(a) da = P_s,
\]

(25)

and the constraints of the inner problem are as follows:

\[
\forall R_r(\gamma) : \int_0^\infty \rho_r(a|R_r(\gamma)) da = P_r,
\]

(26)

\[
\forall R_r(\gamma) : \int_0^\infty \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da \leq R_r(\gamma).
\]

(27)

In (24), \( a \) follows from the fact that \( R_r(\gamma) \) can be determined with the knowledge of \( \gamma \) and \( \rho_s(\cdot) \) and the dependence of the term \( \int_0^\infty dv \ f_\Gamma(v) \ \int_0^v \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da \) on \( \rho_s(\cdot), \gamma \) is only through \( R_r(\gamma) \).

Given (24)-(27), in the following two subsections, we will discuss how this two-step maximization problem can be solved using Euler’s equations [29].

1) Relay-Destination Link Optimization Problem

Receiving \( R_r(\gamma) \) bits from the first hop, the aim of the relay is to maximize the average data rate received at the destination. In fact, if the input rate changes, the relay should modify its power distribution, accordingly. However, the knowledge of the input rate \( (R_r(\gamma)) \), the relay total power, and the pdf of the second hop fading power is sufficient for determining the optimum power distribution function, \( \rho^*_r(.|R_r(\gamma)) \). It is evident that the optimum power distribution function, \( \rho^*_r(\cdot|R_r(\gamma)) \), can be completely determined by evaluating \( \rho^*_r(\cdot|R_r(\gamma)) \) for all values of \( R_r(\gamma) \). \( \rho^*_r(\cdot|R_r(\gamma)) \), itself, is the solution of the following problem:

\[
h(R_r(\gamma)) \triangleq \max_{\rho_r(\cdot)|R_r(\gamma)} \int_0^\infty dv \ f_\Gamma(v) \ \int_0^v \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da
\]

(28)

\[\text{s.t.} \ \ \int_0^\infty \rho_r(a|R_r(\gamma)) da = P_r, \]

\[\int_0^\infty \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da \leq R_r(\gamma).\]
Note that, in (28), \( R_r(\gamma) \) is a constant; hence, the problem takes the form of the rate-limited broadcast strategy problem, subsection III-B. Therefore, the optimum solution is:

\[
I^*_r(l|R_r(\gamma)) = \begin{cases} 
P_r & l < l_0 \\
\frac{1-F_T(l) + \lambda - lF_T(l)}{f_T(l)^2} & l_0 < l < l_1 \\
0 & l_1 < l 
\end{cases},
\]

\[(29)\]

where \( F_T(l) = \int_{-\infty}^{l} f_T(a)da \). \( l_0 \) and \( l_1 \) are determined as a function of \( \lambda \) to satisfy \( I^*_r(l_0) = P_r \) and \( I^*_r(l_1) = 0 \), respectively. The optimum multilevel power distribution at the relay can be found by \( \rho^*_r(l|R_r(\gamma)) = -\frac{dI^*_r(l|R_r(\gamma))}{dl} \). Finally, \( \lambda \) is computed to satisfy:

\[
\int_{0}^{\infty} \frac{a\rho_r(a|R_r(\gamma))}{1 + aI_r(a|R_r(\gamma))} da = \min(R_r(\gamma), R^*_F),
\]

\[(30)\]

where \( R^*_F \) is defined by (6). This condition comes from the fact that achieving the maximum average rate at the destination requires the relay not to transmit more than \( R^*_F \) bits (refer to the discussion in subsection III-B).

As an example, we have solved (28) for a network in which the second hop can be modeled as a Rayleigh fading channel, i.e., \( F_T(\nu) = 1 - e^{-\nu} \). Fig. 4 shows the maximum destination’s average data rate, \( h^*(R_r(\gamma)) \), for different relay input rates \( (R_r(\gamma)) \) and different relay powers \( (P_r) \).
2) Source-Relay Link Optimization Problem

Knowing the optimum value for the inner integration, \(h^*(R_r(\gamma))\), (24) can be written as follows:

\[
E[R_d] = \max_{\rho_s(.)} \int_0^\infty d\gamma \ f_r(\gamma) h^*(R_r(\gamma)) \quad \text{subject to} \quad \int_0^\infty \rho_s(a) da = P_s, \tag{31}
\]

where:

\[
R_r(\gamma) = \begin{cases} 
0 & \gamma < l_0 \\
\int_0^\gamma \frac{a \rho_s(a)}{1 + a I_s(a)} da & l_0 \leq \gamma < l_1 \\
\int_0^{l_1} \frac{a \rho_s(a)}{1 + a I_s(a)} da & l_1 \leq \gamma
\end{cases}
\tag{32}
\]

and \(I_s(l) = \int l \rho_s(a) da\), \(l_0\) and \(l_1\) satisfy \(I_s(l_0) = P_s\) and \(I_s(l_1) = 0\), respectively. As (32) suggests, \(R_r(\gamma)\) only depends on \(\gamma\), \(\rho_s(.)\), and \(I_s(.)\). Remembering \(\rho_s(l) = \frac{-dI_s(l)}{dl}\), we can write the integrand of (31) as

\[
G(l, I_s, I_s') = f_r(l)h^*(R_r(l, I_s, I_s')).
\]

With this notation, equation (31) takes the form of a fixed end-point Calculus of Variations problem and can be solved using Euler’s equation, [29],

\[
\zeta(l, I_s, I_s') = G_{I_s} - \frac{dG}{dl} = 0, \tag{33}
\]

where \(G_{I_s} = \frac{\partial G}{\partial R_r} \frac{\partial R_r}{\partial I_s}\), \(G_{I_s'} = \frac{\partial G}{\partial R_r} \frac{\partial R_r}{\partial I_s'}\), and \(\frac{dG}{dl}\) is the derivative of \(G_{I_s'}\) with respect to \(l\). Thus, we have:

\[
G_{I_s} = \frac{\partial G}{\partial R_r} \frac{\partial R_r}{\partial I_s} = \begin{cases} 
 f_r(l)h^*(R_r(l, I_s, I_s')) \int_0^l \frac{-a I_s(a)}{(1 + a I_s(a))^2} da & l_0 < l < l_1 \\
0 & \text{otherwise}
\end{cases}, \tag{34}
\]

\[
G_{I_s'} = \frac{\partial G}{\partial R_r} \frac{\partial R_r}{\partial I_s'} = \begin{cases} 
 f_r(l)h^*(R_r(l, I_s, I_s')) \int_0^l \frac{-a}{1 + a I_s(a)} da & l_0 < l < l_1 \\
0 & \text{otherwise}
\end{cases}, \tag{35}
\]

\[
\frac{dG_{I_s'}}{dl} = \begin{cases} 
 f_r(l)h^*(R_r(l, I_s, I_s')) \int_0^l \frac{-a}{1 + a I_s(a)} da + f_r(l)h^*(R_r(l, I_s, I_s')) \frac{l l_s'}{1 + l I_s(l)} & l_0 < l < l_1 \\
0 & \text{otherwise}
\end{cases}, \tag{36}
\]

where \(h^*(.)\) and \(h^{**}(.)\) denote the first order and the second order derivative of \(h^*(.)\), respectively. Substituting (34)-(36) in (33), the optimal \(I_s^*(l)\) is derived.
As an example, in the scenario where both source-relay and relay-destination links are modeled with a Rayleigh fading channel, i.e., \( F(t) = 1 - e^{-l} \), \((33)\) can be simplified to:

\[
\zeta(l, I_s, I_s') = h'(i) \left[ \int_0^l \frac{1 - a - a^2 I_s(a)}{(1 + a I_s(a))^2} da \right] - h''(i) \left[ \frac{-II'(l)}{1 + II(l)} \int_0^l \frac{-a}{1 + a I_s(a)} da \right] = 0,
\]

where \( i = R_r(l, I_s, I_s') \). To solve \((37)\), we first need to have \( h'(i) \) and \( h''(i) \). Indeed, these functions can be numerically evaluated using the results of subsection IV-C1. In the next step, we replace \( I_s \) by \([I_s(1), I_s(2), ..., I_s(N)]\), corresponding to the amount of interference in each level, \( I_s(m) \)'s are in descending order, such that \( I_s(1) = P_s \) and \( I_s(N) = 0 \). As a result, we have a nonlinear system of \( N \) equations, i.e., \( \zeta(m, I_s, I_s') = 0, m = \{1, 2, ..., N\} \) which can be solved numerically. The final solution for these \( N \) variables shows the optimal interference function, \( I_s^*(l) \). As an example, Fig. 5 presents \( I_s^*(l) \) in the case of Rayleigh fading model for both hops and \( P_s = P_r = 20 \)dB. Having \( I_s^*(l) \), the amount of power associated for each code level can be determined by \( \rho_s^*(l) = -\frac{dI_s^*(l)}{dl} \).

\footnote{In fact, we have approximated a continuous variable \( I_s(l) \) with a discrete \( N \)-level function, which becomes precise as \( N \) tends to infinity.}
V. NUMERICAL RESULTS AND COMPARISON WITH OTHER SCHEMES

In the previous sections, we have proposed a DF multilevel coding scheme, which is shown to be optimum in the underlying network setup, and derived the optimum source and relay power distribution through different levels of code. In this section, we compare the performance of the proposed scheme (which is the optimal scheme) with the cut-set upper-bound and two other sub-optimal schemes proposed in [25].

A. Broadcasting Cutset Bound, $C_{\text{cutset}}$:

This bound simply says that the achievable average data rate of a two-hop network can not exceed the achievable average rate of any of the single-hop links, i.e., the source-relay and the relay-destination links. This is independent of the relay structure and its operation. In other words, the cutset bound is an upper-bound on the network throughput when we put no limitation on the relay, i.e., the relay is capable of buffering the unsent data or rescheduling the buffered data. Therefore, the gap between the performance of the proposed scheme and the cutset upper-bound shows the maximum possible gain of having a “complicated” relay instead of a simple one. The cutset bound can be written as:

$$C_{\text{cutset}} = \min \left[ \int_{0}^{\infty} da \ f_{\Gamma}(a) R_1(a), \int_{0}^{\infty} da \ f_{\Gamma}(a) R_2(a) \right],$$

where $R_1(a)$ ($R_2(a)$) denotes the rate that the relay (destination) can successfully decode when the source (relay) transmits over a channel with fading power equal to ‘$a$’.

B. Amplify and Forward, AF:

This is the achievable rate of a two-hop network in which the relay performs the amplify and forward (AF) on the received source signal. To design the optimum multilevel power distribution, first, the total equivalent channel should be evaluated. In other words, the source-relay and relay-destination channels combined with AF relaying can be modeled as one channel with a new probability density function. Having this new pdf, the optimum power distribution can be evaluated. Details of the proof can be found in [25]. The final result can be written as:

$$R_{\text{AF}} = \int_{l_0}^{l_1} da \left[ \frac{2(1 - F_{sb}(a))}{a} + \frac{(1 - F_{sb}(a)) f'_{sb}(a)}{f_{sb}(a)} \right],$$

where:

$$F_{sb}(x) = 1 - \int_{\frac{a}{l_0} + \ln \frac{I_{\text{opt}}}{\frac{a}{l_1}} + \ln I_{\text{opt}}}^{\infty} dl \ e^{-l} \frac{a(1 + P_{sb}l)}{I_{\text{opt}} - a I_{\text{opt}}},$$

and $f_{sb}(a) = \frac{da}{dF_{sb}(a)}$. $l_0$ and $l_1$ are defined such that $I_{\text{opt}}(l_0) = P_s$ and $I_{\text{opt}}(l_1) = 0$, respectively. $I_{\text{opt}}(l)$ is presented in [25].
C. Outage at the Source, Broadcasting at the Relay, $\text{DF}_{1-\text{bs}}$:

This scheme is another suboptimal strategy that has been studied in [25]. In this case, the source uses a one level code, known as the outage approach, and the relay uses the optimal multilevel code. The subscript “$1-\text{bs}$” represents the one-level coding and the broadcast scheme at the source and relay, respectively. Clearly, this approach is a special case of the optimum $\text{DF}$ broadcast strategy, i.e., the proposed scheme. The achievable average rate of this scheme can be computed by:

\[
R_{\text{DF},1-\text{bs}} = \max_{\gamma_s, \rho_r} \left( 1 - F_T(\gamma_s) \right) \int_0^\infty da \left( 1 - F_T(a) \right) \frac{a \rho_r(a)}{1 + a I_r(a)}
\]

\[
\text{s.t. } \int_0^\infty \frac{u \rho_r(u) du}{1 + u I_r(u)} = \log(1 + P_s \gamma_s).
\]

Figures (6) and (7) represent the destination’s average data rate at the destination versus the relay power $P_r$ for the proposed scheme, as well as the $\text{AF}$ and $\text{DF}_{1-\text{bs}}$ schemes, where $P_s = 20\, \text{dB}$ and $P_s = 30\, \text{dB}$, respectively. The upper-bound $C_{\text{cutset}}$ is also depicted in both figures.

As expected, the proposed $\text{DF}$ strategy (the optimal scheme) outperforms the $\text{AF}$ and $\text{DF}_{1-\text{bs}}$ schemes. Note that, the superiority of the proposed scheme over $\text{DF}_{1-\text{bs}}$ is obvious since $\text{DF}_{1-\text{bs}}$ is a special case of the proposed scheme. The important observation in these figures is that the infinite multilevel $\text{DF}$ strategy is strictly superior to the $\text{AF}$ strategy, which was previously the best known scheme for this setup at the high SNR [25]. However, as the SNR at the relay side $P_r$ increases, the performance of $\text{AF}$ approaches the optimal performance. Furthermore, as $P_r$ increases, the proposed scheme approaches the
Fig. 7. Destination’s Average Data Rate $P_s=30\text{ dB}$, $P_r=30 - 42\text{ dB}$

cutset bound which means that for high values of $P_r$ the relay does not need to be “complicated”. Another observation from these figures is that, as $P_r$ decreases, $\text{DF}_{1-\text{bs}}$ approaches the optimal performance. This can be explained as follows: when the power of the relay is much smaller than the source power, the relay-destination link limits the performance. Therefore, even using a one-level code at the source is sufficient to deliver an average rate of $R_F^*$ to the relay, which is the maximum rate that relay can transmit to the destination.

VI. CONCLUSION

In this paper, a two-hop network in which the data is transmitted from the source node via a single relay to a destination node was considered. It was assumed that the knowledge of the channel for each hop is not available at the corresponding transmitter. The relay was assumed to be simple, i.e., not capable of data buffering over multiple coding blocks, water-filling over time, or rescheduling. For this network setup, we proposed an infinite-level coding scheme at the source and the relay. It is shown that this scheme in conjunction with the Decode and Forward (DF) relaying is indeed the optimal strategy for maximizing the average data rate received at the destination. We also proposed an algorithm to find the optimum amount of power which should be assigned to each code level at the source and relay. The optimality of the DF multilevel coding strategy is also verified through numerical results by showing its superiority over the Amplify and Forward (AF) scheme, which was previously the best known scheme for the high SNR regime.

Note that even if relay receives a rate more than $R_F^*$, the extra rate should be discarded.
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