Cavity-assisted enhanced and dephasing immune squeezing in the resonance fluorescence of a single quantum dot
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We theoretically demonstrate the enhanced and dephasing immune squeezing in the resonance fluorescence of a single quantum dot (QD) confined to a pillar-microcavity and driven by a continuous wave laser. We employ the formalism based on Polaron master equation theory for incorporating the influence of exciton-phonon coupling quite accurately in dot-cavity system. We find a significant enhancement of squeezing due to the cavity coupling of the dot in the appropriate regime in contrast with that maximum possible squeezing from an ideal single two-level system in free space. We also show the persistence of squeezing even when the pure dephasing becomes greater than the radiative decay. These novel features may be attributed to arise due to the cavity-induced backaction effects in the form of asymmetric stimulated emission and absorption rates. As a consequence, the resulting cavity-induced strong modifications in the spectral profile of resonance fluorescence are also demonstrated. The effects of phonon-bath temperature on the squeezing and spectral profile are also investigated.

I. INTRODUCTION

The system of a single quantum emitter coupled to the tailored electromagnetic vacuum modes is of both fundamental and practical interest due to its novel spectral features. Tailoring of the ambient electromagnetic modes of a quantum emitter, by placing it inside a cavity, give rise to several practically useful effects such as inhibition and acceleration of natural decay rate of the quantum emitter, vacuum Rabi splitting, and Mollow triplet [1-4]. Further, fundamental understanding of such systems have led to the observation of several intriguing phenomena such as photon antibunching, entangled photons, and squeezed light. These are now serving as the key building blocks in the development of quantum technologies ranging from quantum information to quantum metrology [5-10]. Particularly, squeezed light is quite useful and essential in quantum metrology for carrying out the measurements beyond the standard quantum limit. Squeezing refers to a possible reduction in fluctuations of one of the canonically conjugate variables below the minimum at the expense of enhancing that of the other within the Heisenberg’s uncertainty principle. Squeezed light generation have been demonstrated through quadratic process and Kerr effect in nonlinear systems [11–12], four-wave mixing in atomic and solid states systems [13–16], cavity-QED [17, 18], Bose–Einstein condensation [19], and optomechanical systems [20–23]. All these processes depend upon the quadratic form of the bosonic operators and multi-photon processes.

It was predicted in 1981 and later realized experimentally that squeezing can be obtained via a radically different approach that does not require quadratic form of the bosonic operators [24]. It involves the interaction of two-level emitter with a resonant light field. This unique form of squeezing stems from a build-up and survival of steady state coherence in the weak excitation regime. Recently such form of squeezing was observed both experimentally and theoretically in the resonance fluorescence from a resonantly excited quantum dot (QD) [25, 26]. The achievable degree of squeezing was found to be quite low because of the limited coherence generation in the weak excitation regime. Additionally, unlike the real atoms, QDs are unavoidably coupled to the phonon bath of lattice vibrations, which generally introduces the phonon-induced dephasing and
incoherent scattering of the exciton states of the QDs [27-33]. These phonon-induced incoherent processes were also shown to greatly limit the achievable degree of squeezing by hindering the building-up of exciton coherence. Furthermore, there was no squeezing obtained whenever the pure dephasing rate of QD exciton state becomes comparable or greater than the natural decay rate, which is a quite common scenario in QDs [25]. However, there exist few proposals for enhancing the squeezing either by advantageously harnessing the exciton-phonon coupling in a very strongly driven single QD [34] or by emitter-cavity coupling [35].

In this paper, we show that the cavity coupling to an appropriately driven single QD can facilitate the enhanced and dephasing immune squeezing even in the generally deteriorating regime of exciton-phonon coupling. The present numerically calculated maximum value of squeezing is found to be quite greater than the maximum obtainable squeezing from an ideal single two-level system in free space. However, we report a four-fold enhanced squeezing in a cavity coupled QD as compared to a similarly driven single QD without cavity coupling. This enhancement in squeezing primarily could be attributed to be facilitated by the reduction in exciton state population and sufficient build-up of exciton coherence, resulting due to the cavity-induced strong asymmetry in the stimulated emission and absorption rates of the QD for an appropriate value of cavity detuning. It is also shown that the detrimental effect of pure dephasing can also be minimized. In fact, the significant amount of squeezing persists even when the pure dephasing rate exceeds the natural decay rate. The effect of phonon bath temperature on the squeezing is also investigated. Furthermore, we also demonstrate the strong modifications in fluorescence spectral profile when the cavity mode is resonant with the right sideband of the Mollow triplet. The impact of phonon-bath temperature on the squeezing and spectral profile is shown to be quite appreciable. It is worthy to mention that we also utilize the cavity coupling advantageously for obtaining the enhanced squeezing similar to the Ref. [35]. However, in contrast, we employ a rigorous theoretical formalism for incorporating the crucial Rabi frequency and temperature dependent exciton-phonon coupling in a realistic dot-cavity system at typical cryogenic temperatures. This formalism paves the way for investigating the appropriate value of Rabi frequency for obtaining the greater value of squeezing while minimizing the detrimental effect of exciton-phonon coupling.

II. THEORETICAL MODEL

A. Model Hamiltonian and Polaron Transformation

We consider a single self-assembled InGaAs/GaAs QD embedded in a single mode pillar-microcavity. Single QD can be modeled as a two-level system with ground and exciton states interacting with single cavity mode driven by a continuous wave (CW) laser field. The unavoidable coupling of exciton state to the longitudinal acoustic (LA) phonons is included in our theoretical model for simulating realistic CW laser driven QD cavity system quite accurately. The Hamiltonian of the model system describing the interaction of a QD with LA phonons with the cavity mode excited by CW laser field can be written as,

\[ H = \hbar \Delta_{xl} \sigma^+ \sigma^- + \hbar \Delta_{cl} a^+ a + \frac{\hbar \Omega}{2} (\sigma^+ + \sigma^-) + \hbar g (\sigma^+ a + a^+ \sigma^-) + \sum_q \hbar \omega_q b_q^+ b_q + \sigma^+ \sigma^- \sum_q \hbar \lambda_q (b_q^+ + b_q), \]  

(1)

where \( \Delta_{xl} = \omega_x - \omega_l \) represents the detuning of CW laser with respect of exciton state, \( \Delta_{cl} = \omega_c - \omega_l \) is the detuning of cavity mode with respect of exciton state, \( \sigma^+ \) and \( \sigma^- \) describe the creation and annihilation of the exciton state, \( \Omega = \mu E / \hbar \) is the Rabi frequency which quantifies the coupling strength of CW laser with exciton, \( g \) is the dot-cavity coupling strength, \( b_q^+ \) and \( b_q \) represent the creation and annihilation operators for mode \( q \) of phonon bath, \( \lambda_q \) represents the exciton-phonon coupling by means of deformation potential.
The exciton-phonon interaction in terms of effective polaron renormalized QD-laser and QD-cavity couplings can be realized by transforming the Hamiltonian (Eq. (1)) to polaron frame as \( H' = e^P H e^{-P} \), where \( P = \sigma^+ \sigma^- \sum_q \lambda_q / \omega_q (b_q^\dagger - b_q) \) [36, 37]. The polaron-transformed Hamiltonian in terms of modified QD-laser system, phonon bath, and interaction part respectively are now given by,
\[
H'_{\text{sys}} = \hbar (\Delta_{\text{xl}} - \Delta_P) \sigma^+ \sigma^- + \hbar \Delta_c \sigma^+ a + \langle B \rangle X_g, \tag{2a}
\]
\[
H'_B = \sum_q \hbar \omega_q b_q^\dagger b_q, \tag{2b}
\]
\[
H'_I = X_g \zeta_g + X_u \zeta_u, \tag{2c}
\]
where \( X_g = \frac{\hbar \Omega}{2} (\sigma^+ + \sigma^-) + \hbar g (\sigma^+ a + a^\dagger \sigma^-) \), \( X_u = \frac{i \hbar \omega_x}{2} (\sigma^+ - \sigma^-) + \hbar g (\sigma^+ a - a^\dagger \sigma^-) \). Further, \( \zeta_g \) and \( \zeta_u \) represent the fluctuations operators defined as: \( \zeta_g = \frac{1}{2} (B_+ + B_- - 2 B) \) and \( \zeta_u = \frac{1}{2} (B_+ - B_-) \). The coherent displacement operators of phonon modes, \( B_{\pm} \) will then be \( B_{\pm} = \exp \left[ \pm \sum_q \frac{\lambda_q}{\omega_q} (b_q^\dagger - b_q) \right] \). The Hamiltonian, \( H'_I \), describes the interaction of phonon-induced fluctuations with the CW laser driven quantum dot. The thermally averaged phonon displacement operator \( \langle B \rangle \) has the form \( \langle B \rangle = \exp \left[ - \frac{1}{2} \int_0^\infty d\omega j(\omega) \coth \left( \frac{\hbar \omega}{2 K_B T} \right) \right] \), where \( T \) represents the temperature of phonon-bath. The polaron shift \( \Delta_P \) is given by \( \Delta_P = \int_0^\infty d\omega j(\omega) \). We further simplify the formulation by including the polaron shift in the definition of \( \omega_x \) itself by redefining the detuning as \( \Delta_{\text{xl}} = \Delta_{\text{xl}} - \Delta_P \). The phonon spectral function, \( j(\omega) \), characterizing the exciton-phonon coupling can be expressed as \( j(\omega) = \alpha_p \omega^3 \exp \left( - \frac{\omega^2}{2 \omega_b^2} \right) \), where \( \alpha_p \) represents the strength of exciton-phonon coupling and \( \omega_b \) represents the phonon cutoff frequency.

**B. Polaron Master Equation Theory**

Several distinct theoretical approaches such as variational formulation, quasi-adiabatic path integral, and polaron transformation techniques have been employed for incorporating the influence of exciton-phonon interaction on the dynamics of driven QDs [38-40]. However, master equation based on the polaron transformation technique is quite well-known for facilitating the accurate results and much faster computation under the suitable parameters regime: \( (\Omega/\omega_b)^2 (1 - \langle B \rangle^4) \ll 1 \) [41]. Therefore in this work, we employ the polaron master equation theory for investigating the influence of exciton-phonon interaction on squeezing in the driven QD-cavity system. Following the procedural details given in Refs. [39-42], we derive time-local master equation (ME) of reduced density operator, \( \rho(t) \), of the QD-cavity laser-phonon system specifically considered in this paper. The Polaron ME obtained under second-order Born approximation of QD-phonon coupling reads as
\[
\frac{d\rho(t)}{dt} = -\frac{i}{\hbar} [H'_{\text{sys}}, \rho(t)] - \frac{1}{\hbar^2} \int_0^\infty d\tau \sum_{m=g,u} \{ G_m(\tau) [X_m, e^{-iH'_{\text{sys}}^T} X_m e^{iH'_{\text{sys}}^T} \rho(t)] + H.C. \} + \mathcal{L}[\rho(t)], \tag{3}
\]
where polaron Green functions, \( G_m(\tau) = \langle \zeta_m(\tau) \zeta_m(0) \rangle \), are calculated to be \( G_g(\tau) = \langle B \rangle^2 \{ \cosh[\phi(\tau)] - 1 \} \) and \( G_g(\tau) = \langle B \rangle^2 \sinh[\phi(\tau)] \) [36]. Polaron correlation function, \( \phi(\tau) \), reads as \( \phi(\tau) = \int_0^\infty d\omega j(\omega) \frac{\coth \left( \frac{\hbar \omega}{2 K_B T} \right)}{\omega^2} \cos(\omega \tau) - isinh(\omega \tau) \). The second term represents the phonon-induced incoherent processes. This term explicitly depends on the Phonon-bath temperature and Rabi frequency via \( G_m(\tau) \) and \( X_m \), respectively. The superoperator
term, $L[\rho(t)] = \frac{\gamma}{2} E[\sigma^-] \rho(t) + \frac{\gamma'}{2} E[\sigma^+] \rho(t) + \frac{\kappa}{2} E[a] \rho(t)$, is added phenomenologically for incorporating the radiative ($\gamma$) and pure dephasing ($\gamma'$) rates of exciton state along with the cavity decay ($\kappa$) rate.

C. Squeezing in Resonance Fluorescence

Following Ref. [25, 35], the amplitude quadrature of fluorescence field is defined as: $E_\theta = (E^+e^{i\theta} + E^-e^{-i\theta})$, where $E^+$ and $E^-$ represent the positive and negative frequency components of the electric field, and $\theta$ represents the quadrature phase. The squeezing properties can be investigated by analyzing the normally ordered variance, $\langle \Delta E^2: \rangle = \langle E^2_\theta: \rangle - \langle E_\theta \rangle^2$, of the electric field quadrature as defined above. We use the correspondence between atomic and field operators, $E^+ = |e\sigma^- and E^- = |e\sigma^+$, to calculate the normally ordered variance of electric field quadrature. The normally ordered variance is given by $\langle \Delta E^2: \rangle = |e|^2 (2(\langle \sigma^+ \sigma^- \rangle - [\langle \sigma^+ \rangle]^2) - 2[\langle \sigma^+ \rangle]^2 \cos(2\theta))$ [26]. Assuming the proportionality constant, $|e|$, to be unity, we find that the quadrature variance is minimum for quadrature phase, $\theta = 0$ and is given by

$$\langle \Delta E^2: \rangle = 2(\langle \sigma^+ \sigma^- \rangle - 2[\langle \sigma^- \rangle]^2)$$  \hspace{1cm} (4)

The fluorescence field is squeezed if normally ordered variance is negative, corresponding to a noise reduction below the vacuum level. Theoretically, the minimum value of -0.125 of normally ordered field variance represents the maximum squeezing obtainable in the fluorescence of an ideal single two-level system in free space [24, 35]. Using relation, $\langle \hat{\mu} \rangle = Tr(\hat{\rho} \hat{O})$, exciton state population, $\langle \sigma^+ \sigma^- \rangle$, and exciton coherence, $\langle \sigma^- \rangle$, are calculated by numerically solving the Polaron ME [Eq. (3)] using quantum optics toolbox [43].

III. RESULTS AND DISCUSSIONS

For investigating the squeezing in resonance fluorescence, we use the relevant parameters of InGaAs self-assembled QDs for the simulation. The typical values of the chosen parameters are: $\gamma = 2\mu eV$, $\alpha_p/(2\pi)^2 = 0.06 ps^2$, $\gamma' = 0.5 \mu eV$ and $\omega_b = 1 meV$ [32, 33]. However, the values of decay and pure dephasing rates can vary in the range of $1-3 \mu eV$ and $0.5-3 \mu eV$ respectively for different dots depending on their varying size and shapes. Therefore, for the greater generality of our results, we also investigate the effects of variation in decay and dephasing rates by treating them as variables. The other simulation parameters relating to the laser field and cavity are mentioned at appropriate places either in text or in the figure captions.

In this work, we consider a strongly driven QD with $\Omega_R \gg \gamma, \gamma'$ along with the laser detuning $\Delta_{lx} = \Omega_R$ for restricting the significant flow of population into the exciton state, otherwise it could reduce the magnitude of squeezing significantly [see Eq. (4)]. The phonon-renormalized Rabi frequency, $\Omega_R$, is defined as: $\Omega_R = \langle B \rangle \Omega$, with $\langle B \rangle = 0.91$ at temperature, $T = 4 K$. In Fig. 1, we show the evolution of variance, $\langle \Delta E^2: \rangle$, as a function of cavity detuning, $\Delta_{ct}$, and phonon-renormalized Rabi frequency, $\Omega_R = \langle B \rangle \Omega$ under the optimized values of cavity coupling strength, $g_R = \langle B \rangle g = 11.90 \mu eV$, and decay rate, $\kappa = 17.85 \mu eV$. The values of $g_R$ and $K$ were optimized separately by numerically solving the Polaron ME [Eq. (3)] with the analyses of the variance $\langle \Delta E^2: \rangle$, although the exact analytical solution may be highly desirable which is beyond the scope of this work. It is clear from Fig. 1 that the of variance of -0.16 is achieved for $\Omega_R = 19.83 \mu eV$ and $\Delta_{ct} \approx \sqrt{\Omega_R^2 + \Delta_{ct}^2}$. The present minimum value of variance is clearly less than the minimum achievable value of variance (-0.125) in the fluorescence of an ideal two-level emitter in free space. Furthermore, the corresponding value of
Fig. 1 (color online) Evolution of the normally ordered variance, $\langle : \Delta E^2 : \rangle$, as a function of normalized cavity detuning, $\Delta_{cl}/\sqrt{\Omega_R^2 + \Delta_x^2}$, and phonon-renormalized Rabi frequency, $\Omega_R = \langle B \rangle \Omega$ for the fixed value of phonon-bath temperature, $T = 4$ K, optimized cavity coupling strength, $g_R = \langle B \rangle g = 11.90 \mu eV$, and optimized decay rate, $\kappa = 17.85 \mu eV$.

Squeezing (4.4 dB) is four-fold greater than the maximally obtained squeezing (1.1 dB) for a single QD without cavity coupling as reported in Ref. [25, 26]. The possible reasons for the enhanced squeezing at a particular value of cavity detuning $\Delta_{cl} \approx \sqrt{\Omega_R^2 + \Delta_x^2}$ are explained next as can be comprehended more clearly and precisely with the help of figure 2.

We demonstrate the evolution of exciton state population $\langle \sigma^+ \sigma^- \rangle$ and coherence $2|\langle \sigma^- \rangle|^2$ as a function of cavity detuning, $\Delta_{cl}$ as shown in Fig. 2. It can be observed quite clearly the difference $[\langle \sigma^+ \sigma^- \rangle - 2|\langle \sigma^- \rangle|^2]$ showing the maximum negative value around $\Delta_{cl} \approx \sqrt{\Omega_R^2 + \Delta_x^2}$ as indicated in the figure by the black dotted line. The significance of this difference is that it is directly proportional to the normally ordered variance $\langle : \Delta E^2 : \rangle$ (see eqn. 4) which is the indicative of the degree of squeezing. Therefore, the enhancement in the squeezing can clearly be attributed to the reduced exciton state population with an almost maximum exciton coherence at cavity detuning, $\Delta_{cl} \approx \sqrt{\Omega_R^2 + \Delta_x^2}$. This can further be supported and justified by the explanation at a more fundamental level following the arguments and results reported in Ref. [44]. The cavity coupling introduces the asymmetry between stimulated emission rate, $R_{xg}$, and stimulated absorption rate, $R_{gx}$. For the self-completeness, the analytical expressions of $R_{xg}$ and $R_{gx}$ are given in the Appendix. The evolution of these stimulated rates as a function of cavity detuning and with the fixed values of optimized parameters is shown in the inset of Fig. 2. Therein it is quite clear that the stimulated emission rate, $R_{xg}$, is maximum and is almost four times greater than the stimulated absorption rate, $R_{gx}$, at the cavity detuning, $\Delta_{cl} \approx \sqrt{\Omega_R^2 + \Delta_x^2}$. Thus for an
Fig. 2 (color online) Evolution of exciton state population, $\langle \sigma^+ \sigma^- \rangle$, and coherence, $2|\langle \sigma^- \rangle|^2$, as a function of normalized cavity detuning, $\Delta_{ct}/\sqrt{\Omega_R^2 + \Delta_x^2}$, with a fixed value of phonon-bath temperature, $T = 4$ K, optimized phonon-renormalized Rabi frequency, $\Omega_R = 19.83$ $\mu eV$, optimized cavity coupling strength, $g_R = \langle B \rangle g = 11.90$ $\mu eV$, and optimized decay rate, $\kappa = 17.85$ $\mu eV$. Inset Fig. depicts the evolution of stimulated emission rate, $R_{xg}$, and stimulated absorption rate, $R_{gx}$, as a function of normalized cavity detuning with other parameters same as those used in main Fig. 2.

appropriate value of cavity detuning, the cavity coupling to the QD restricts the accumulation of population into the exciton state, which is a desirable and favorable situation for squeezing. Therefore, the enhancement of squeezing in resonance fluorescence can be attributed to the pronounced cavity backaction effects in the form of cavity-induced asymmetry between stimulated emission rate, $R_{xg}$, and stimulated absorption rate, $R_{gx}$, at cavity detuning, $\Delta_{ct} = \sqrt{\Omega_R^2 + \Delta_x^2}$.

We show the evolution of the normally ordered variance $\langle :\Delta E^2: \rangle$ as a function of exciton decay ($\gamma$) and dephasing rate($\gamma'$) as in Fig. 3. It can clearly be observed that the persistence of squeezing depicts quite weak dependence on radiative decay rate $\gamma$ and a stronger dependence on pure dephasing rate $\gamma'$. The persistence of squeezing decreases faster with pure dephasing rate $\gamma'$. This is clearly in sharp contrast and advantageous compared to the results reported in a bare QD wherein no squeezing persists at all in this regime [25]. More precisely, the squeezing persists in the regime $\gamma' < 4$ $\mu eV$ only and beyond this it dies out quickly. We anticipate that this occurs due to the inadequacy of cavity-induced backaction effects to tackle the increased dephasing beyond $\gamma' \geq 4$ $\mu eV$. 


Fig. 3 (color online) Evolution of the normally ordered variance, $\langle \Delta E^2 \rangle$, as a function of exciton state decay ($\gamma$) and dephasing ($\gamma'$) rates, with a fixed value of phonon-bath temperature, $T = 4$ K, cavity detuning, $\Delta_{cl} = \sqrt{\frac{\Omega_R^2}{\kappa} + \frac{\Omega_{\delta}^2}{\kappa}}$, phonon-renormalized Rabi frequency, $\Omega_R = 19.83$ $\mu eV$, cavity coupling strength, $g_R = \langle B \rangle g = 11.90$ $\mu eV$, and cavity decay rate, $\kappa = 17.85$ $\mu eV$. Rest of the parameters are same as those mentioned in the text.

We further investigate the dependence of variance on the phonon-induced incoherent processes (by the second term in Eq. (3)) by the two examples of different Rabi frequencies at different phonon-bath temperatures. The evolution of the variance corresponding to the first case for which we choose the ratio of Rabi frequency to radiative decay rate to be $\frac{\Omega}{\gamma} \sim 10^1$, is presented in Fig. 4 (a). Similarly, the second case, with the choice of the ratio of Rabi frequency to radiative decay rate greater by an order of magnitude $\left(\frac{\Omega}{\gamma} \sim 10^2\right)$ is presented in Fig. 4 (b). The values of variance $\langle \Delta E^2 \rangle$ are found to be -0.17 and -0.21 for $T = 0$ K at cavity coupling strength $g_R = 0.6 \Omega_R$, respectively from Fig. 4(a) and 4(b). However, for the typical and practical values of phonon-bath temperature for QDs (4-20 K), this behavior is reversed. That is, for example, the values of variance $\langle \Delta E^2 \rangle$ are found to be -0.15 and -0.08 for $T = 10$ K at cavity coupling strength $g_R = 0.6 \Omega_R$, for the first and second cases respectively. This reversed behavior is maintained for all the non-zero phonon-bath temperatures of the above range. This is precisely why we have chosen the first case for the investigation in this paper for achieving the greater possible squeezing with a set of experimentally feasible simulation parameters. Furthermore, the value of variance increases with phonon-bath temperature due to the increased amount of exciton state population. The observed phonon-bath temperature dependency is consistent with the recently reported works [25, 32].
Fig. 4 (color online) Evolution of the normally ordered variance, $\langle \Delta E^2 \rangle$, as a function of cavity coupling strength, $g_R = \langle B \rangle g$, at the different values of phonon-bath temperatures, $T = 0$ K (dashed-dotted red line), $T = 4$ K (dashed blue line), $T = 10$ K (solid green line), and $T = 10$ K (solid cyan line). The other parameters are given as: (a) Rabi frequency, $\Omega = 21.79 \mu eV$, detuning, $\Delta_{xl} = \Omega_R$, cavity detuning, $\Delta_{cl} = \sqrt{\Omega_R^2 + \Delta_{xl}^2}$, and cavity decay rate, $\kappa = 0.9 \Omega_R \mu eV$; (b) Rabi frequency, $\Omega = 175 \mu eV$, detuning, $\Delta_{xl} = \Omega_R$, cavity detuning, $\Delta_{cl} = \sqrt{\Omega_R^2 + \Delta_{xl}^2}$, and cavity decay rate, $\kappa = 0.9 \Omega_R \mu eV$.

Next in Fig. 5, we depict the spectral profile of resonance fluorescence for cavity detuning $\Delta_{cl} = 0$ and $\Delta_{cl} = \sqrt{\Omega_R^2 + \Delta_{xl}^2} = 28.05 \mu eV$ for the fixed value of phonon-bath temperature $T = 4$ K in Fig. 5. The spectrum of fluorescence field is defined as $S(\omega - \omega_t) = \int_0^\infty \langle \sigma^+ (\tau) \sigma^- (0) \rangle_{ss} e^{i(\omega - \omega_t)\tau} d\tau$, where $\langle \sigma^+ (\tau) \sigma^- (0) \rangle_{ss}$ is a two-time correlation function. The dramatically changing spectral profile is quite evident from the figure for cavity detuning $\Delta_{cl} = 28.05 \mu eV$ in comparison with that of $\Delta_{cl} = 0$. This can be explained in terms of dressed states. Normally, under strongly driven conditions $\Omega_R \gg g_R, \gamma, \kappa$ the well-known Mollow triplet becomes clearly visible in the fluorescence spectrum. The Mollow triplet consist right and left sidebands at frequencies $\omega_{\pm} = \omega_t \pm \sqrt{\Omega_R^2 + \Delta_{xl}^2}$ respectively on the either side of the central peak at frequency $\omega_0 = \omega_t$. It arises due to the transitions from the upper doublet to the lower doublet of dressed states $\left| + \right\rangle \left( - \right\rangle \rightarrow \left( - \right\rangle \left( + \right\rangle\right)$ and $\left| + \right\rangle \left( - \right\rangle \rightarrow \left( + \right\rangle \left( - \right\rangle\right)$ respectively [4]. However, a triplet like structure also appears for the cavity detuning $\Delta_{cl} = 0$,
with the usual three peaks (quite similar to those observed in free space excitation of a QD) at 
\( \omega - \omega_1 \approx -\sqrt{\Omega_R^2 + \Delta_{xl}^2} = -28.50 \mu eV \), \( \omega - \omega_1 \approx 0 \mu eV \), and \( \omega - \omega_1 \approx \sqrt{\Omega_R^2 + \Delta_{xl}^2} = 28.90 \mu eV \). The enhanced right sideband of Mollow triplet at 28.90 \( \mu eV \) occurs due to the positive detuning of exciton-laser \( \Delta_{lx} \). It can be further noted that for cavity detuning \( \Delta_{cl} = 28.05 \ \mu eV \), the spectral profile gets strongly modified with smaller multiple peaks along with an enhanced and quite dominating peak at \( \omega - \omega_1 \approx -\Omega_R = -19.63 \ \mu eV \). This occurs due to the fact that at detuning, \( \Delta_{cl} \approx \sqrt{\Omega_R^2 + \Delta_{xl}^2} \), cavity mode becomes resonant to the right sideband of

**Fig. 5 (color online)** Spectral profile of resonance fluorescence for two different values of cavity detuning, \( \Delta_{cl} = 0 \) and \( \Delta_{cl} = 28.05 \ \mu eV \) with a fixed value of phonon-bath temperature, \( T = 4 \) K, phonon-renormalized Rabi frequency, \( \Omega_R = 19.83 \ \mu eV \), cavity coupling strength, \( g_R = 11.90 \ \mu eV \), and cavity decay rate, \( \kappa = 17.85 \ \mu eV \). Rest of the parameters are same as those mentioned in the text. Inset Fig. depict the Mollow triplet frequencies, \( \omega_- \), \( \omega_0 \), and \( \omega_+ \), from different transitions between dressed states \( |+\rangle \) and \( \langle -| \).

the Mollow triplet \([|+\rangle \rightarrow |\rangle \)]. Under this detuning, the cavity-assisted population transition rates, \( R_{+-} \) and \( R_{-+} \), read as: \( R_{+-} = \frac{\gamma_+-\gamma_c}{2} \), \( R_{-+} = \frac{\gamma_y}{2} \gamma_c = \frac{g_R^2}{\kappa} \) [44], where \( R_{+-} (R_{-+}) \) represent the population transition rate from state \( |+\rangle (|\rangle \) to \( \langle -| (|+\rangle \). For the chosen values of relevant parameters in this work, the transition rates are connected as \( R_{+-} \approx 5 R_{-+} \), resulting the more population in state, \( |\rangle \). The emission of this state predominately takes place into the free space vacuum modes due to its off-resonant coupling with cavity mode. This seems to manifest itself in the form of an enhanced and dominating peak at \( \omega - \omega_1 \approx -\Omega_R = -19.63 \ \mu eV \) as shown in Fig. 5.

Finally, we show the evolution of the spectral profile of resonance fluorescence, phonon-renormalized Rabi frequency and phonon-renormalized cavity coupling strength as a function of
phonon-bath temperature in Fig. 6. It can be observed from Fig. 6(a) that the Mollow triplet sidebands appear at \( \pm \Omega_R \) and their splitting do decrease for the increased values of phonon-bath temperature, \( T \). This simply can be attributed to the phonon-bath induced renormalization of the Rabi frequency, \( \Omega \). For illustration, we show the evolution of the phonon-renormalized Rabi frequency, \( \Omega_R = \langle B \rangle \Omega \), as a function of phonon-bath temperature, \( T \) with the utilized value of \( \Omega = 21.79 \mu eV \) at \( T = 4K \). It is clear from Fig. 6(b) that \( \Omega_R \) decreases as the temperature, \( T \), increases. Further in addition to the decrease in sideband splitting, the yield of resonance fluorescence increases with the increased temperature, \( T \), as can be observed from Fig. 6(a). This is due to the increased population in the exciton state, which mainly emits the photons into the free space vacuum modes owing to decreased coupling strength, \( g_R \), at higher temperature, \( T \) [see Fig. 6(b)].

IV. CONCLUSION

We have demonstrated the cavity-enhanced and decoherence immune squeezing in the resonance fluorescence of a single quantum dot using a rigorous theoretical formalism based on Polaron master equation theory. We have found a four-fold enhancement in squeezing for a
cavity coupled single QD in contrast with that an isolated one. The squeezing is found to be quite robust for the increment in pure dephasing and even persists till the pure dephasing rate remains less than the twice of the radiative decay rate. We have shown that the enhancement in squeezing is mainly facilitated by the reduction in exciton state population along with the build-up of nearly maximum exciton coherence for an appropriate amount of cavity detuning. It is argued and explained that at more fundamental level this occurs due to the cavity-induced pronounced asymmetry in the stimulated emission and absorption rates of the QD exciton. The phonon-bath temperature dependent incoherent effects are shown to be detrimental and do reduce the squeezing. Furthermore, the cavity-induced large modifications in the spectral profile of resonance fluorescence are also demonstrated.

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APPENDIX
Exactly following the procedural details given in Ref. [44], the obtained stimulated emission rate, $R_{xg}$, and stimulated absorption rate, $R_{gx}$, are given below. For simplicity, we have not included the phonon-bath coupling and pure dephasing in our calculations.

\[
R_{xg} = 2Re \left( \frac{(\gamma + i\Delta_{xt} + \gamma_c A_2)}{|(\gamma + i\Delta_{xt} + \gamma_c A_2)|^2 - |(\gamma_c A_1)|^2} \right), \quad (A1)
\]

\[
R_{gx} = 2\gamma + 2\gamma_c Re(A_2) - 2Re \left( \frac{(\gamma_c A_0 - \frac{i\Omega_R}{2})[(\gamma + i\Delta_{xt} + \gamma_c A_2)(\gamma_c A_0 - \frac{i\Omega_R}{2}) + \gamma_c Conj(A_0)(\gamma_c A_0 + \frac{i\Omega_R}{2})]}{|(\gamma + i\Delta_{xt} + \gamma_c A_2)|^2 - |(\gamma_c A_1)|^2} \right), \quad (A2)
\]

where, \( A_0 = \frac{\kappa \Omega_R}{4(\Omega_R^2 + \Delta_{xt}^2)} \left( \frac{2\Delta_{xt}}{(\kappa + i\Delta_{cl})} - \frac{\sqrt{\Omega_R^2 + \Delta_{xt}^2 + \Delta_{xt}}}{\kappa + i\left(\Delta_{cl} - \sqrt{\Omega_R^2 + \Delta_{xt}^2}\right)} - \frac{\sqrt{\Omega_R^2 + \Delta_{xt}^2 - \Delta_{xt}}}{\kappa + i\left(\Delta_{cl} + \sqrt{\Omega_R^2 + \Delta_{xt}^2}\right)} \right) \), \quad (A3)

\[
A_1 = \frac{\kappa \Omega_R^2}{4(\Omega_R^2 + \Delta_{xt}^2)} \left( \frac{2}{(\kappa + i\Delta_{cl})} - \frac{1}{\kappa + i\left(\Delta_{cl} - \sqrt{\Omega_R^2 + \Delta_{xt}^2}\right)} - \frac{1}{\kappa + i\left(\Delta_{cl} + \sqrt{\Omega_R^2 + \Delta_{xt}^2}\right)} \right), \quad (A4)
\]

\[
A_2 = \frac{\kappa \Omega_R^2}{4(\Omega_R^2 + \Delta_{xt}^2)} \left( \frac{2\Omega_R^2}{(\kappa + i\Delta_{cl})} - \frac{\left(\sqrt{\Omega_R^2 + \Delta_{xt}^2 + \Delta_{xt}}\right)^2}{\kappa + i\left(\Delta_{cl} - \sqrt{\Omega_R^2 + \Delta_{xt}^2}\right)} + \frac{\left(\sqrt{\Omega_R^2 + \Delta_{xt}^2 - \Delta_{xt}}\right)^2}{\kappa + i\left(\Delta_{cl} + \sqrt{\Omega_R^2 + \Delta_{xt}^2}\right)} \right), \quad (A5)
\]

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