Gravitational scattering of spinning neutrinos by a rotating black hole with a slim magnetized accretion disk

Maxim Dvornikov

Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation (IZMIRAN), Troitsk, 108840 Moscow, Russia

E-mail: maxdvo@izmiran.ru

Received 26 July 2022; revised 13 October 2022
Accepted for publication 18 November 2022
Published 30 November 2022

Abstract
We study neutrinos gravitationally scattered off a rotating supermassive black hole which is surrounded by a thin accretion disk with a realistic magnetic field. Neutrinos are supposed to be Dirac particles having a nonzero magnetic moment. Neutrinos move along arbitrary trajectories, with the incoming flux being parallel to the equatorial plane. We exactly account for the influence of both gravity and the magnetic field on the neutrino motion and its spin evolution. The general statement that the helicity of an ultrarelativistic neutrino is constant in the particle scattering in an arbitrary gravitational field is proven within the quasiclassical approach. We find the measurable fluxes of outgoing neutrinos taking into account the neutrino spin precession in the external field in curved spacetime. These fluxes turn out to be significantly suppressed for some parameters of the system. Finally, we discuss the possibility to observe the predicted phenomena for core-collapsing supernova neutrinos in our Galaxy.

Keywords: Kerr metric, neutrino gravitational scattering, neutrino spin oscillations, magnetized accretion disk, locally Minkowskian frame

(Some figures may appear in colour only in the online journal)

1. Introduction
Neutrinos are experimentally confirmed to be massive and mixed particles (see, e.g. [1]). It results in transitions between different neutrino types named neutrino flavor oscillations. Standard model neutrinos are left particles, i.e. their spin is opposite to the particle momentum. However,
the neutrino polarization can change under the influence of an external field. This process is called neutrino spin oscillations. The combination of these two phenomena is also possible. In this situation, we deal with neutrino spin-flavor oscillations.

External fields, i.e. the neutrino interaction with matter \([2]\) and with an electromagnetic field \([3]\), are known to affect neutrino oscillations. The gravitational interaction, despite it is quite weak, can also induce neutrino oscillations. Neutrino flavor \([4]\), spin \([5]\), and spin-flavor \([6]\) oscillations in a gravitational field were previously studied. In the present work, we deal with the evolution of a neutrino spin in a curved spacetime under the influence of a magnetic field.

For the first time, the behavior of a spinning particle in a curved spacetime was studied in \([7]\). The dynamics of the fermion spin in a gravitational field was analyzed in \([8]\) basing on the Dirac equation in a curved spacetime. The quasiclassical equation for a particle spin in a gravitational field was derived in \([9]\). The method of \([9]\) was applied in \([5]\) to describe neutrino oscillations in the vicinity of a nonrotating black hole (BH) in frames of the general relativity (GR). Neutrino spin oscillations in various extensions of GR were considered in \([10–14]\). The evolution of the relic neutrinos spin in stochastic gravitational fields was studied in \([15]\). The recent studies of the fermion spin evolution in external gravitational fields were reviewed in \([16]\).

Using the quasiclassical approach, here, we study neutrino spin oscillations in the particle scattering off a rotating BH. In this situation, neutrinos are in the flat spacetime asymptotically. Hence, we can control their ‘in’ and ‘out’ spin states. The gravitational scattering of fermions, including neutrinos, was studied in \([17–19]\). We analyzed this problem in \([20–22]\) accounting for only the equatorial neutrino motion.

Now, for the first time, we discuss this problem in a quite general form. The incoming flux of neutrinos is parallel to the equatorial plane. However we do not restrict ourselves to the equatorial motion. Particles can propagate both above and below the equatorial plane. We take into account the change of the neutrino latitude in the scattering. Moreover, the realistic magnetic field is accounted for in our work. The neutrino interaction with a magnetic field is owing to the nonzero neutrino magnetic moment \([3]\). We suppose that a neutrino is a Dirac particle.

The motivation for this work was the direct observation of the event horizon silhouette of the supermassive BHs (SMBHs) in the centers of M87 \([23]\) and our Galaxy \([24]\). These observations are the first direct tests of GR in the strong field limit. The images in \([23, 24]\) are formed by photons emitted by the hot gas in the accretion disks around these SMBHs \([25]\). The review of the analytical studies of BH shadows is provided in \([26]\). Accretion disks in some active galactic nuclei can be hot and dense enough to emit both photons, protons and secondary neutrinos \([27]\). The observation of these objects in a neutrino telescope (see, e.g. \([28]\)) should account for the neutrino spin precession in strong external fields including gravity. Moreover, we can imagine a hypothetical situation when a flux of neutrinos, e.g. from a core-collapsing supernova (SN), is gravitationally lensed by BH \([29]\).

This work is organized in the following way. First, in section 2, we formulate the main equations for the description of the general motion of ultrarelativistic neutrinos in the Kerr metric, as well as the spin evolution equation in the curved spacetime under the influence of an electromagnetic field. Then, we fix the parameters of the system and represent electromagnetic and gravi-electromagnetic fields in the chosen geometry of the spacetime in section 3. In section 4, we find the measurable fluxes of scattered neutrinos accounting for their spin precession in the given external fields. Finally, in section 5, we summarize and discuss the possibility
to detect the predicted effects for SN neutrinos. The evolution of the helicity of an ultrarelativistic neutrino in its scattering in an arbitrary gravitational field is studied in appendix.

2. Motion of a neutrino in Kerr metric and the particle spin evolution

The spacetime outside a rotating BH is described by the Kerr metric. In Boyer–Lindquist coordinates $x^\mu = (t, r, \theta, \phi)$, this metric has the form,

$$\begin{align*}
    ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \left(1 - \frac{rr_g}{\Sigma}\right)dt^2 + 2\frac{r r_g a \sin^2 \theta}{\Sigma} dtd\phi - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2,
\end{align*}$$

where

$$\begin{align*}
    \Delta &= r^2 - r r_g + a^2, \quad \Sigma = r^2 + a^2 \cos^2 \theta, \quad \Xi = (r^2 + a^2) \Sigma + r r_g a^2 \sin^2 \theta.
\end{align*}$$

Here we use units where the gravitational constant is equal to one. In this situation, the mass of BH is $M = r_g/2$ and its angular momentum is $J = Ma$, where $r_g$ is the Schwarzschild radius. The BH spin is directed upward with respect to the equatorial plane $\theta = \pi/2$.

A test particle in the Kerr metric has three integrals of motion: the energy, $E$, the angular momentum, $L$, and the Carter constant, $Q$. If we study the scattering problem, $Q > 0$. The law of motion of a test particle can be found in quadratures [30],

$$\begin{align*}
    t &= E \left[ \int \frac{r^2 dr}{\sqrt{R}} + a^2 \int \frac{d\theta}{\sqrt{\Theta}} \cos^2 \theta \right] + r_g \int \frac{rdr}{\sqrt{R}\Delta} \left[ r^2 E - a(L - aE) \right],
\end{align*}$$

where $R(r) = [(r^2 + a^2)E - aL]^2 - \Delta[Q + (L - aE)^2]$ and $\Theta(\theta) = Q + \cos^2 \theta \left[ a^2 E^2 - L^2 \right] / \sin^2 \theta$. Here we consider an ultrarelativistic neutrino. The form of a trajectory can be also determined in quadratures [30],

$$\begin{align*}
    \phi &= a \int \frac{dr}{\sqrt{R}\Delta} [(r^2 + a^2)E - aL] + \int \frac{d\theta}{\sqrt{\Theta}} \left( \frac{L}{\sin^2 \theta} - aE \right),
\end{align*}$$

and

$$\begin{align*}
    \int \frac{dr}{\sqrt{R}} = \pm \int \frac{d\theta}{\sqrt{\Theta}}.
\end{align*}$$

One should choose the sign in equation (2.5), e.g. for an incoming particle at $r \to \infty$ and then keep the choice for the whole trajectory. The signs in radial integrals in equations (2.3) and (2.4) depend whether a neutrino approaches or moves away from BH. The description of the motion of a test particle near a Kerr BH can be made in two ways. One can either solve the system of the geodesics equations as in [31]. Alternatively, we can analyze the integrals in equations (2.3)–(2.5).

The scattering of a test particle has three main difficulties. First, some neutrinos in the incoming beam can fall into BH. Hence, we should take only the specific values of $Q$ and $L$ for incoming neutrinos (see, e.g. [32]). Second, a neutrino can make multiple revolutions around BH, i.e. the polar angle $|\phi|$ in equation (2.4) can be greater than $2\pi$. Third, the latitude of an incoming neutrino does not coincide with that for an outgoing particle. Moreover, the
\( \theta \)-dependence in equation (2.5) can be oscillating. One should account for these facts in the analysis of the particle trajectories.

Now, we can describe the general dynamics of the neutrino spin in the Kerr metric. Following [5, 9], we define the invariant neutrino spin \( \zeta \) in rest frame in the locally Minkowskian coordinates \( x_\mu = e_\mu^a x_a \), where

\[
e_0^\mu = \left( \frac{1}{\sqrt{1 - \Delta}}, 0, 0, \frac{\mathbb{R} \sigma}{\sqrt{\Delta \Sigma}} \right), \quad e_1^\mu = \left( 0, \frac{\sqrt{\Delta}}{\Sigma}, 0, 0 \right),
\]

\[
e_2^\mu = \left( 0, 0, \frac{1}{\sqrt{\Sigma}}, 0 \right), \quad e_3^\mu = \left( 0, 0, 0, \frac{1}{\sin \theta} \sqrt{\frac{\Sigma}{\Sigma}} \right),
\]

(2.6)

are the vierbein vectors, which satisfy the relation \( e^\mu_a e^\nu_b g_{\mu \nu} = \delta_{ab} \), where \( \delta_{ab} = \text{diag}(1, -1, -1, -1) \) is the Minkowski metric tensor.

In our problem, we consider the neutrino gravitational scattering off a rotating BH surrounded by a thin magnetized accretion disk. In this situation, only the interaction with gravity and with the poloidal component of the magnetic field contribute to the neutrino spin evolution (see section 3 below). The vector \( \zeta \) obeys the equation

\[
\frac{d \zeta}{dt} = 2(\zeta \times \Omega),
\]

(2.7)

where

\[
\Omega = \frac{1}{U} \left\{ \frac{1}{2} \left[ \mathbf{b} + \frac{1}{1 + u^\theta} (\mathbf{e}_\theta \times \mathbf{u}) \right] + \mu \left[ u^0 \mathbf{b} - \frac{\mathbf{u}(u^b \mathbf{b})}{1 + u^\theta} + (\mathbf{e} \times \mathbf{u}) \right] \right\}.
\]

(2.8)

Here \( u^\mu = (u^0, \mathbf{u}) = e^\mu_a U^a, U^\mu = (U^\theta, U^\varphi, U^\phi, U^\phi) \) is the four velocity of a neutrino in the world coordinates, \( \mathbf{e}_\theta \) and \( \mathbf{b}_\phi \) are the components of the tensor \( G_{ab} = (\mathbf{e}_a, \mathbf{b}_b) = \gamma_{abc} u^c, \gamma_{abc} = \eta_{abc} e^\mu_a e^\nu_b e^\rho_c \) are the Ricci rotation coefficients, the semicolon stays for the covariant derivative, \( f_{ab} = e^\mu_b e^\nu_a F_{\mu \nu} = (\mathbf{e}, \mathbf{b}) \) is the electromagnetic field tensor in the locally Minkowskian frame, with \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) being the electromagnetic field in the world coordinates. We suppose that a neutrino is a Dirac particle having the magnetic moment \( \mu \). The details of the derivation of equations (2.7) and (2.8) can be found in [33].

3. Parameters of the system

We consider SMBH with the mass \( M = 10^8 M_\odot \) surrounded by a thin magnetized accretion disk. The incoming flux of neutrinos is taken to be parallel to the equatorial plane of BH. The consideration of a thin disk makes it possible to neglect the effect of the neutrino electroweak interaction with matter (see, e.g. [34]) of a disk since only a small fraction of neutrinos moves in the equatorial plane. We mentioned in section 2 that the latitude of a neutrino can oscillate in its motion, i.e. a particle can cross the equatorial plane multiple times. However, the path inside a thin disk for such neutrinos is short. Hence, we neglect the electroweak interaction with plasma of a disk. Moreover, the consideration of a thick disk makes the problem more complex. Indeed, a plasma cannot rotate on circular orbits in such a disk. Slim accretion disks were mentioned in [35] to be possible around SMBHs.

The plasma rotation in a disk generates the magnetic field. Both poloidal and toroidal components are created. However, if the disk is thin, we can neglect the toroidal component since
it is inside a disk, even despite the toroidal field can be quite strong. The poloidal field is taken to correspond to the following vector potential in the world coordinates [36]:

\[
A_r = aB \left[ 1 - \frac{rr_b}{2\Sigma} (1 + \cos^2 \theta) \right], \\
A_\phi = -\frac{B}{2} \left[ r^2 + a^2 - \frac{\alpha^2 rr_b}{2\Sigma} (1 + \cos^2 \theta) \right] \sin^2 \theta, 
\]

(3.1)

where \( B \) is the magnetic field strength, which is uniform and is along the BH spin at \( r \to \infty \).

However, the assumption that \( B \) is finite at \( r \to \infty \) is unphysical since the magnetic field is created by the plasma motion in a disk, which has the finite size. Thus, we should suppose that \( B \to 0 \) at \( r \to \infty \). For example, we can take that \( B \propto B_0 r^{-5/4} \) [37], where \( B_0 \) is the strength of the magnetic field in the vicinity of BH at \( r \sim r_g \). We suppose that \( B_0 = 10^{-2} \)

\( B_{\text{Edd}} = 3.2 \times 10^2 \) G for \( M = 10^8 M_\odot \), where \( B_{\text{Edd}} \) is the Eddington limit for the magnetic field which arrests the accretion [38].

A Dirac neutrino is taken to have the magnetic moment in the range \( \mu = (10^{-14} - 10^{-13}) \mu_B \), where \( \mu_B \) is the Bohr magneton. The smaller value of \( \mu = 10^{-13} \mu_B \) is consistent with the model independent upper bound on the Dirac neutrino magnetic moment established in [39]. The greater \( \mu = 10^{-13} \mu_B \) considered is below the best astrophysical upper limit on the neutrino magnetic moment in [40].

We consider the incoming flux of neutrinos moving from the point with the coordinates \((r, \theta, \phi) = (\infty, \pi/2, 0)\). In this case, the asymptotic neutrino velocity is \( u_{\pm \infty} = (\pm |v_1|, 0, 0) \), i.e. incoming and outgoing neutrinos move oppositely and along the first axis in the locally Minkowskian frame. Instead of equation (2.7), we can deal with an effective Schrödinger equation \( i \psi = H \psi \), where \( H = -\mathcal{U}_{\Sigma} (\sigma \cdot \Omega) \mathcal{U}^{\dagger}_{\Sigma} \), \( \mathcal{U}_{\Sigma} = \exp(i\pi \sigma_2/4) \), \( \sigma = (\sigma_1, \sigma_2, \sigma_3) \) are the Pauli matrices, and \( \Omega \) is given in equation (2.8). We suppose that initially all neutrinos are left polarized, i.e. their initial helicity is \( h = (\zeta_{-\infty} \mathbf{u}_{-\infty}) / |\mathbf{u}_{-\infty}| = -1 \). It corresponds to the initial effective wavefunction \( \psi_{-\infty}^T = (1, 0) \). We are interested in the survival probability \( P_{LL} \), which indicates how many neutrinos remain left polarized after the scattering. If the wavefunction of outgoing neutrinos is \( \psi_{+\infty}^T = (\psi_{+\infty,1}, \psi_{+\infty,2}) \), then \( P_{LL} = |\psi_{+\infty,1}|^2 \).

It is convenient to introduce the dimensionless variables, \( r = x r_g, L = y r_g E, a = z r_g, Q = w r_g^2 E^2 \). The components of gravi-electromagnetic field \((\mathbf{e}_g, \mathbf{b}_g) = \frac{\partial}{\partial r} (\mathbf{e}_g, \mathbf{b}_g) \) in equation (2.8) have the form,

\[
\hat{e}_{g1} = \frac{1}{2 \sqrt{z^2 \cos^2 \theta (z^2 - x + z^2) + z^2 x^2 + x^2 (x^2 + z^2 \cos^2 \theta)^2}} \times \left\{ \frac{d\phi}{dx} \left[ \frac{z}{2} \cos^2 \theta (z^2 - x^2) - z \cos^2 \theta (x^2 + 3z) + z^3 x^2 + 3z x^4 \right] \right. \\
\frac{d}{dx} \left[ z^2 \cos^2 \theta (z^2 - x^2) - z^2 x^2 + x^4 \right] \right\}, \\
\hat{e}_{g2} = \frac{2 \sqrt{z^2 \cos^2 \theta (z^2 - x + z^2) + z^2 x^2 + x^2 (x^2 + z^2 \cos^2 \theta)^2}}{z x \sin 2\theta \sqrt{x^2 - x + z^2}} \left[ \frac{d}{dx} - \frac{d}{dz} \frac{dz}{dx} \sin \theta \right], \\
\hat{e}_{g3} = \frac{2 \sqrt{z^2 \cos^2 \theta (z^2 - x + z^2) + z^2 x^2 + x^2 (x^2 + z^2 \cos^2 \theta)^2}}{z \sin \theta \sqrt{x^2 - x + z^2}} \times \left( \frac{x z \sin \theta}{\sqrt{x^2 - x + z^2}} \right)
\]
The electromagnetic field vectors for a massless neutrino moving along null geodesics are given by

\[ \tilde{e}_1 = \frac{\cos \theta}{\sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4 + (x^2 + z^2 \cos^2 \theta)^2}} \times \left\{ r_s \frac{d\phi}{dr} \left[ z^2 \cos^2 \theta (x^2 - x + z^2) + 2z^2 \sin^2 \theta (x^2 - x + z^2) + z^2 x + z^2 x^4 \right] + z \frac{dr}{dr} (z^2 + x^2) \right\}, \]

\[ \tilde{e}_2 = \frac{\sin \theta \sqrt{x^2 - x + z^2}}{2 \sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4 + (x^2 + z^2 \cos^2 \theta)^2}} \times \left\{ r_s \frac{d\phi}{dr} \left[ z^2 \cos^2 \theta (2x - 1) + z^2 \cos^2 \theta (z^2 + 4x^2 + x^4) + 2z^2 x^2 \right] + z \frac{dr}{dr} (x^2 - z^2 \cos^2 \theta) \right\}, \]

\[ \tilde{e}_3 = \frac{\sin \theta \cos \theta + r_s \frac{dr}{dr} (x^2 - x + z^2)}{\sqrt{x^2 - x + z^2} (x^2 + z^2 \cos^2 \theta)} \],

where the derivatives with respect to \( r \) can be obtained on the basis of equations (2.3)–(2.5). Despite the expressions for \( \tilde{e}_s \) and \( \tilde{b}_s \) in equation (3.2) are valid for particles with arbitrary masses, we consider mainly massless neutrinos moving along null geodesics. Analogously, we find the electromagnetic field (\( e, b \)) in the locally Minkowskian frame,

\[ e_1 = \frac{\mu Bz [z^2 \cos^2 \theta (z^2 - x^2) + \cos^2 \theta (z^4 + 2z^2 x^2 - 3x^4)] - z^2 x^2 + x^4]}{2 \sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4 + (x^2 + z^2 \cos^2 \theta)^2}}, \]

\[ e_2 = \frac{\mu Bz^3 \sin \theta \sqrt{x^2 - x + z^2} (1 + \cos^2 \theta)}{2 \sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4 + (x^2 + z^2 \cos^2 \theta)^2}}, \]

\[ b_1 = \frac{\mu B \cos \theta \left[ (x^2 + z^2 \cos^2 \theta)^2 (x^2 - x + z^2) + x (x^2 - z^2) \right]}{\sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4 + (x^2 + z^2 \cos^2 \theta)^2}}, \]

\[ b_2 = \frac{\mu B \sin \theta \sqrt{x^2 - x + z^2} \times [z^2 \cos^2 \theta (1 - 2x) + z^2 \cos^2 \theta (z^2 - x^2 - 4x^3) - z^2 x^2 - 2x^2]}{2 \sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4 + (x^2 + z^2 \cos^2 \theta)^2}}, \]

and \( e_3 = b_3 = 0 \). These fields correspond to the vector potential in equation (3.1).

It should be noted that equation (2.8) was derived for a massive particle. However, it has the finite limit for a massless neutrino. For example, the components of the vector \( \mathbf{u} = \frac{\mathbf{u}}{\gamma \tau' \nu} \) are

\[ \tilde{u}_1 = \frac{\sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4} \frac{dr}{dr}}{z^2 + x^2 - x} \]

\[ \tilde{u}_2 = \frac{\sqrt{z^2 \cos^2 \theta (x^2 - x + z^2) + z^2 x^2 + x^4}}{\sqrt{z^2 + x^2 - x}} \times r_s \frac{d\theta}{dr}, \]

\[ \tilde{u}_3 = \frac{(x^2 + z^2 \cos^2 \theta) \sqrt{(x^2 - x + z^2)}}{(x^2 + z^2 \cos^2 \theta) \sqrt{(x^2 - x + z^2)}} \times \left\{ r_s \frac{d\phi}{dr} \left[ z^2 \cos^2 \theta (z^2 + x^2) + z^2 x^2 + x^4 \right] - z x \right\}, \]
where the time derivatives can be obtained again using equations (2.3)–(2.5). The ratio
\[
\frac{u^0}{U^0} = \frac{\sqrt{z^2 + x^2} + x}{\sqrt{z^2 + x^2} + x^2 + x^2},
\]
(3.5)
is also finite.

Finally, we rewrite the effective Schrödinger equation in the form,
\[
\frac{d\psi}{dx} = H_x \psi, \quad H_x = r_x H \frac{dt}{dr}.
\]
(3.6)
Equations (2.3)–(2.5) and (3.6) completely define the evolution of spinning ultrarelativistic neutrinos in their scattering off a rotating BH.

4. Results

Before we discuss the results, some description of the computational details should be present since they are not so trivial. The initial beam of neutrinos has a circular form with the radius \(r_m = 10^3 r_g\) from BH. This beam is taken to be denser towards its center to probe trajectories close to the BH surface. Initially it has \(3.2 \times 10^3\) neutrinos. After eliminating particles which fall to BH, we deal with \(\geq 10^5\) neutrinos. The partition of a trajectory from \(r_m\) to the turn point is made with \(4 \times 10^3\) nodes.

To reconstruct the trajectory, we start with equation (2.5). The \(r\)-integral is computed numerically, whereas the \(\theta\)-integral is expressed in terms of the elliptic integrals. Then, we find the number of extrema for the latitude and \(\theta\) for each point of the trajectory using the Jacobi elliptic functions. Having the coordinates \((r, \theta)\) in any point of the trajectory, we compute \(\phi\) using equation (2.4) and express the angular integral again in terms of the elliptic integrals. In principle, the spin evolution does not depend on \(\phi\), as one can see in equations (3.2)–(3.4).

Nevertheless, the \(\phi\)-dependence of the trajectory is required for the representation of the results. Eventually, we obtain the angles \(\theta_{\text{obs}}\) and \(\phi_{\text{obs}}\) which correspond to an outgoing neutrino. The adopted calculation procedure guarantees that \(0 < \theta_{\text{obs}} < \pi\). However, as we mentioned above, a particle can make multiple revolutions around BH. Thus, we have to project \(\phi_{\text{obs}}\) to the interval \((0, 2\pi)\).

One situates an observer in the point with the coordinates \((r, \theta, \phi)_o = (\infty, \theta_{\text{obs}}, \phi_{\text{obs}})\). In our work, we vary both \(0 < \theta_{\text{obs}} < \pi\) and \(0 < \phi_{\text{obs}} < 2\pi\), whereas the coordinates of the source of the neutrino beam are fixed, \((r, \theta, \phi)_i = (\infty, \pi/2, 0)\). Thus, e.g. the point with \(\theta_{\text{obs}} = \pi/2\) and \(\phi_{\text{obs}} = \pi\) corresponds to the forward neutrino scattering, and that with \(\theta_{\text{obs}} = \pi/2\) and \(\phi_{\text{obs}} = 0\) (or \(2\pi\)) to the backward one. Any pair of the final angular coordinates \((\theta_{\text{obs}}, \phi_{\text{obs}})\) correspond to the specific \(L\) and \(Q\) in the incoming beam.

It should be noted that we have to reconstruct the whole trajectory since we are interested in the neutrino spin evolution rather than in the calculation of a differential cross section for scalar particles. Thus we have to build the trajectory from \(r_m\) to the turn point for incoming particles and from the turn point to \(r_m\) for outgoing ones. Since a rotating BH does not correspond a central field, the reconstruction of both branches of the trajectory consumes computational resources. The details for the finding of a trajectory in a general gravitational scattering of an ultrarelativistic spinless particle can be found, e.g. in [41].

Then, we solve equation (3.6) along the trajectory. Since the dependence \(\theta(r)\) is known only in certain nodes, we cannot use a precise Runge–Kutta solver with an adaptive stepsize. Instead, we apply the Euler method to integrate equation (3.6). It significantly reduces the accuracy of computations. Analogously to the reconstruction of the trajectory, equation (3.6)
is solved separately for both branches of the trajectory. We use the result at the turn point as the initial condition for the second branch of the trajectory. Finally, we find $P_{LL}$ and plot it for any $(\theta_{obs}, \phi_{obs})$ point. We use the 2D cubic interpolation to get the smooth surface which is represented as a contour plot.

In some cases, contour plots have white gaps meaning the insufficient number of neutrinos scattered to these areas. It happens especially for a rapidly rotating BH. The gaps can be eliminated by increasing initial number of particles. However, in this case the computational time increases significantly. Since we have the limited access to the computer facilities, this problem will be tackled in a future work.

Standard model neutrinos are created as left polarized particles. If their spin is flipped because of the interaction with an external field, we observe the effective reduction of the neutrino flux since a detector can also register left neutrinos only. By definition, the flux of neutrinos is reduced by up to a factor of 2. We recall that we deal with ultrarelativistic neutrinos. The statement that the gravitational interaction only does not lead to the spin-flip of ultrarelativistic neutrinos is valid in flat spacetime. In curved space-time, the statement that the gravitational interaction only does not lead to the spin-flip of ultrarelativistic neutrinos can change its polarization in the gravitational scattering. Earlier, we established that the polarization of ultrarelativistic neutrinos is conserved in their gravitational scattering off non-rotating BHs. However, these results were obtained only for the neutrino interaction with a magnetic field in an accretion disk. Thus, if we deal with spinning neutrinos, the observed flux in the wake of their scattering is $F_\nu = P_{LL}F_0$, where $F_0$ is the flux of scalar particles. It is the consequence of the fact that, in the quasiclassical approximation, used in our work (see also [9]), the spin of a particle does not influence its motion.

The flux of scalar ultrarelativistic particles, $F_0$, corresponds to the situation when only their propagation along null geodesics is accounted for. The very detailed study of $F_0$, which includes not only ultrarelativistic particles, is provided in [42]. Our goal is to study the ratio $P_{LL} = F_\nu/F_0$ for neutrinos gravitationally scattered off a rotating BH accounting for the neutrino interaction with a magnetic field in an accretion disk. If we have the map of $P_{LL}(\phi_{obs}, \theta_{obs})$ for all scattered particles, we can reconstruct the flux of spinning neutrinos by combining our results with those in [42].

First, we turn off the magnetic field and consider only the contribution of gravity to the neutrino spin-flip. We recall that we deal with ultrarelativistic neutrinos. The statement that ultrarelativistic fermions conserve their polarization is valid in flat spacetime. In curved space-time, it may be not the case. For example, it was claimed in [43, 44] that a massless neutrino can change its polarization in the gravitational scattering. Earlier, we established that the polarization of ultrarelativistic neutrinos is conserved in their gravitational scattering off non-rotating BHs and rotating BHs. However, these results were obtained only for the neutrino motion in the equatorial plane.

To examine the issue of the neutrino polarization for arbitrary trajectories, we plot $F_\nu/F_0$ for BHs with different spins in figure 1. We present the cases of an almost non-rotating BH with $a = 2 \times 10^{-2}M$ in figure 1(a) and an almost maximally rotating BH with $a = 0.98M$ in figure 1(b). We can see that, in both situations, $P_{LL} \sim 1$ with the accuracy $\sim 1\%$. Thus, we get that the gravitational interaction only does not lead to the spin-flip of ultrarelativistic neutrinos. This result generalizes our findings in [21, 22]. In appendix, we prove the general statement that the helicity of an ultrarelativistic particle is constant when it scatters in an arbitrary gravitational field.

Now we can account for the neutrino interaction with the magnetic field in an accretion disk. In this case, $F_\nu/F_0$ is depicted in figure 2 for the different BH spins and the different values of the magnetic parameter $V_B = \mu B_0 r_g$. If we consider SMBH with $M = 10^8M_\odot$, fix the magnetic field $B_0 = 3.2 \times 10^2$ G, and vary the neutrino magnetic moment in the range $10^{-14} \mu_B \leq \mu < 10^{-13} \mu_B$ (see section 3), we get that $2.7 \times 10^{-2} < V_B < 2.7 \times 10^{-1}$. As in figure 1, we consider two cases. Figures 2(a) and (c) correspond to $a = 2 \times 10^{-2}M$, whereas figures 2(b) and (d) to $a = 0.98M$.

Taking the very conservative value of $\mu = 10^{-14}\mu_B$, we can see in figures 2(a) and (b) that the flux of neutrinos is reduced by up to $(5 \pm 1)\%$ compared to that of scalar particles.
Figure 1. The ratio of the fluxes of spinning neutrinos $F_{\nu}$ and the scalar particles $F_0$ gravitationally scattered off BHs with different angular momenta. (a) $z = 10^{-2} (a = 2 \times 10^{-2}M)$; (b) $z = 0.49 (a = 0.98M)$.

Figure 2. The same as in figure 1 accounting for the neutrino magnetic interaction. Panels (a) and (b): $V_B = 2.7 \times 10^{-2}$; panels (c) and (d): $V_B = 2.7 \times 10^{-1}$. Panels (a) and (c): $z = 10^{-2} (a = 2 \times 10^{-2}M)$; panels (b) and (d): $z = 0.49 (a = 0.98M)$. 
This result is in agreement with [22], where the similar reduction factor was obtained while considering the equatorial neutrino motion. However, if we increase the neutrino magnetic moment by one order of magnitude to $\mu = 10^{-13} \mu_B$, we can observe in figures 2(c) and (d) that the neutrino flux is almost suppressed in certain directions. We mentioned in section 3 that such neutrino magnetic moments are not excluded by the astrophysical observations.

To check the conservation of the probability in our simulations, in figure 3, we plot the quantity $P_{LL} + P_{LR}$ for neutrinos scattered off BH with a magnetized accretion disk. The transition probability $P_{LR}$ is computed directly basing on the solution of equation (3.6) as $P_{LR} = |\psi_{+\infty}|^2$. One can see in figure 3 that the unitarity condition $P_{LL} + P_{LR} = 1$ is fulfilled for any neutrino trajectory with the accuracy $\sim 1\%$. The validity of the same condition can be checked for the purely gravitational scattering shown in figure 1. It means that our simulations are reliable.

Finally, we notice that the plots in figures 1 and 2 are symmetric with respect to equator $\theta_{\text{obs}} = \pi/2$. It is the consequence of the symmetry of the metric in equation (2.1) to the reflection $\theta \rightarrow \pi - \theta$. 
5. Discussion

In the present work, we have studied spin effects in the neutrino gravitational scattering off BH. Particles were supposed to move on general trajectories unlike previous works [21, 22], where only the motion in the equatorial plane was considered. The effects of gravity were accounted for exactly in the reconstruction of the neutrino trajectory, i.e. we have considered the strong gravitational lensing. The neutrino spin evolution was studied in the locally Minkowskian frame by solving the effective Schrödinger equation. Neutrinos were supposed to be left polarized before scattering. If the neutrino helicity changes, we would observe the effective reduction of the outgoing neutrino flux.

We have found that the gravitational interaction only does not result in the change of the neutrino polarization. The flux of outgoing spinning neutrinos is seen in figure 1 to be identical to that of scalar particles. It generalizes our findings in [21, 22], where we studied neutrinos moving in the equatorial plane only. This result also corrects the claims in [43, 44] that gravity can change the helicity of an ultrarelativistic fermion. The general theorem that the helicity of an ultrarelativistic neutrino remains constant in the particle scattering by an arbitrary gravitational field has been proven in appendix.

To produce the neutrino spin-flip we have added the neutrino interaction with a poloidal magnetic field which is generated in an accretion disk surrounding BH. We have assumed that the disk is slim. Thus one takes into account neither matter effects nor a toroidal magnetic field. Considering the strength of the magnetic field which is allowed in realistic SMBHs and the moderate value of the Dirac neutrino magnetic moment \( \mu = 10^{-14} \mu_B \), we have obtained that the observed neutrino flux can be reduced by \( \sim 5\% \) in certain directions; cf figures 2(a) and (b). This result is in agreement with [22]. If we take greater magnetic moment \( \mu = 10^{-13} \mu_B \), that still does not violate astrophysical constrains, the neutrino flux turns out to be vanishing for some scattering directions; see figures 2(c) and (d). These results are valid for both non-rotating, with \( a \ll M \), and maximally rotating, with \( a \lesssim M \), BHs. For example, SMBH in the center of M87 has a quite great \( a \approx 0.9 \) [45].

Described neutrino spin oscillations in the particle gravitational scattering can be potentially observed if a core-collapsing SN, which emits huge amount of neutrinos, explodes in our Galaxy. The event is schematically depicted in figure 4. Suppose that SN explodes somewhere in the Galaxy. Using current or future neutrino telescopes, we detect the direct flux of neutrinos \( F_1 \) along the path 1 shown in figure 4 by the blue arrow. Then, one starts to look for a neutrino signal in the direction to the galactic center where SMBH is situated. These particles propagate along the path 2 depicted in figure 4 by the red arrows. Such neutrinos are gravitationally lensed and their polarizations are affected by the magnetic field in the vicinity of SMBH. The flux \( F_2 \) is related to \( F_1 \) by \( F_2 = \frac{d}{4 \pi l_1 l_2} \left( \frac{d\sigma}{d\Omega} \right) F_1 \), where \( l_1, l_2 \), and \( l_1', l_2' \) are the distances between objects in figure 4. The differential cross-section \( d\sigma/d\Omega \) corresponds to the scattering angles fixed by the positions of the objects.

One has that \( d\sigma/d\Omega = r_\theta^2 f \), where \( r_\theta^2 = 1.4 \times 10^{34} \text{cm}^2 \) for the SMBH in Sgr A* and the function \( f(\theta_{\text{obs}}, \phi_{\text{obs}}) \) has great values for forward and backward scatterings (see, e.g. [21, 46]), as well as in caustics [41, 47]. If \( l_1 \sim l_2' \sim l_2' \sim 10 \text{kpc} \), then \( F_2 = 8.2 \times 10^{-23} F_1 \). The observed flux of neutrinos, if a core-collapsing SN takes place in our Galaxy, is estimated by \( F_1 \sim 7 \times 10^3 \) events for the JUNO detector [48] and \( F_1 \sim 7 \times 10^4 \) events for the Hyper-Kamiokande detector [49].

To get the sizable flux of lensed neutrinos, \( F_2 \), these particles should be observed, e.g. very close to the SMBH surface. Thus, SN, SMBH and the Earth should be on one line practically, with \( \theta_{\text{obs}} \approx \pi/2 \) and \( \phi_{\text{obs}} \approx \pi \). If this case, the function \( f \) can become great enough to exceed
Figure 4. The schematic illustration of the neutrino gravitational scattering off SMBH, which is depicted by the black blob, in the center of our Galaxy. Neutrinos are emitted in a core-collapsing SN shown by the yellow asterisk. Then, they travel along the paths 1 and 2. The path 1, represented by the blue arrow, goes directly to the Earth depicted by the symbol $\oplus$. The path 2, shown by red arrows, accounts for the gravitational lensing of neutrinos by SMBH. The distances $l_1$, $l'_2$, and $l''_2$ are between SN and the Earth, SN and SMBH, SMBH and the Earth, respectively.

The factor $8.2 \times 10^{-23} F_1 = 5.7 \times 10^{-18}$ for the Hyper-Kamiokande detector. Using figure 2(c) or figure 2(d), we obtain that the observed neutrino flux can be 50% reduced because of spin oscillations provided that the neutrino magnetic interaction is strong enough.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

I am thankful to A F Zakharov for the useful discussion.

Appendix. Conservation of helicity in the gravitational scattering of utrarelativistic neutrinos

In this appendix, we examine the evolution of the helicity of utrarelativistic neutrinos in their scattering in an arbitrary gravitational field. We prove that the helicity is conserved.

The general quasiclassical spin evolution in a gravitational field was studied in [5, 9]. The four vector of a fermion spin $s^a$, defined in a locally Minkowskian frame, evolves as

$$\frac{ds^a}{dt} = \frac{1}{U'} G^{ab} s_b,$$

(A.1)

where $U'$ and $G_{ab}$ are given in section 2. Besides equation (A.1), we should take into account the evolution of the particle velocity $u^a$ which has the form,

$$\frac{du^a}{dt} = \frac{1}{U'} G^{ab} u_b.$$

(A.2)
We rewrite equations (A.1) and (A.2) in the three dimensional form,
\[
\frac{d\zeta}{dt} = \frac{1}{U^\mu} \left[ \zeta \times \left\{ b_g + \frac{1}{1-u_0^\mu} (e_g \times u) \right\} \right], \\
\frac{du}{dt} = \frac{1}{U^\mu} (u \times b_g), \\
\frac{du_0}{dt} = \frac{1}{U^\mu} (u \cdot e_g),
\]
(A.3)
where the vectors \( \zeta, e_g, \) and \( b_g \) are also given in section 2.

If a neutrino is ultrarelativistic, then both \( u_0 \to \infty \) and \( u \to \infty \), whereas the velocity in the locally Minkowskian frame \( v = u/u_0 \) is a unit vector, \( |v| \to 1 \). Thus, the helicity of such neutrinos is \( h \equiv (\zeta \cdot v) \). Its time evolution, measured by a distant observer, reads
\[
\frac{dh}{dt} = (\dot{\zeta} \cdot v) + (\zeta \cdot \dot{v}) \to -\frac{(\zeta \cdot e_g)}{U^\mu}.
\]
(A.4)
where we use equation (A.3) and the fact that a neutrino is ultrarelativistic.

In the gravitational scattering, a neutrino propagates in the region outside the BH surface, i.e. \( e_\mu a \) in equation (2.6) are nonzero and finite. In fact, in the case of the Kerr metric, the allowed values of \( L \) and \( Q \) are outside the BH shadow which is greater than the BH horizon. Thus, \( U^\mu = e_\mu a \to \infty \) for an ultrarelativistic neutrino. Therefore, using equation (A.4), we get that \( h = \text{const.} \).

It is important that we consider the scattering problem, where a source and a detector of neutrinos are in asymptotically flat spacetime, i.e. we measure the neutrino helicity with respect to the world time \( t \). In this case, the neutrino helicity is conserved. In other situations, e.g. when the helicity of ultrarelativistic neutrinos is measured by a comoving observer, it can change in a gravitational field.

ORCID iD
Maxim Dvornikov  https://orcid.org/0000-0003-4028-0660

References

[1] Acero M A et al (NOvA Collaboration) 2022 An improved measurement of neutrino oscillation parameters by the NOvA experiment Phys. Rev. D 106 032004
[2] Smirnov A Y 2005 The MSW effect and matter effects in neutrino oscillations Phys. Scr. T121 57
[3] Giunti C, Kouzakov K A, Li Y-F, Lokhov A V, Studenikin A I and Zhou S 2016 Electromagnetic neutrinos in laboratory experiments and astrophysics Ann. Phys., Lpz. 528 198–215
[4] Cardall C Y and Fuller G M 1997 Neutrino oscillations in curved spacetime: a heuristic treatment Phys. Rev. D 55 7960–6
[5] Dvornikov M 2006 Neutrino spin oscillations in gravitational fields Int. J. Mod. Phys. D 15 1017–34
[6] Piriz D, Roy M and Wudka J 1996 Neutrino oscillations in strong gravitational fields Phys. Rev. D 54 1587–99
[7] Papapetrou A 1951 Spinning test-particles in general relativity. I Proc. R. Soc. A 209 248–58
[8] Obukhov Y N, Silenko A J and Teryaev O V 2017 General treatment of quantum and classical spinning particles in external fields Phys. Rev. D 96 105005
[9] Pomeranski A A and Khriplovich I B 1998 Equations of motion of spinning relativistic particle in external fields J. Exp. Theor. Phys. 86 839–49
[10] Alavi S A and Nodeh S 2015 Neutrino spin oscillations in gravitational fields in noncommutative spaces Phys. Scr. 90 035301
[11] Chakraborty S 2015 Aspects of neutrino oscillation in alternative gravity theories J. Cosmol. Astropart. Phys. JCAP10(2015)019
[12] Mastrototoaro L and Lambiase G 2021 Neutrino spin oscillations in conformally gravity coupling models and quintessence surrounding a black hole Phys. Rev. D 104 024021
[13] Alavi SA and Serish TF 2022 Neutrino spin oscillations in gravitational fields in higher dimensions (arXiv:2206.01940)
[14] Pantig RC, Mastrototoaro L, Lambiasi G and Övgün A 2022 Shadow, lensing and neutrino propagation by dyonic ModMax black holes (arXiv:2208.06664)
[15] Baym G and Peng J-C 2021 Evolution of primordial neutrino helicities in cosmic gravitational inhomogeneities Phys. Rev. D 103 123019
[16] Vogeles S N, Nikolaev N N, Yu N O, Silenko A Y and Teryaev O V General relativity effects in precision spin experimental tests of fundamental symmetries Phys.-Usp. (accepted (https://doi.org/10.3367/UFNe.2021.09.039074))
[17] Lambiasi G, Papini G, Punzi R and Scarpetta G 2005 Neutrino optics and oscillations in gravitational fields Phys. Rev. D 71 073011
[18] Dolan S, Doran C and Lasenby A 2006 Fermion scattering by a Schwarzschild black hole Phys. Rev. D 74 064005
[19] Sorge F 2012 Ultra-relativistic fermion scattering by slowly rotating gravitational sources Class. Quantum Grav. 29 045002
[20] Dvornikov M 2020 Spin effects in neutrino gravitational scattering Phys. Rev. D 101 056018
[21] Dvornikov M 2020 Spin oscillations of neutrinos scattered off a rotating black hole Eur. Phys. J. C 80 474
[22] Dvornikov M 2021 Neutrino scattering off a black hole surrounded by a magnetized accretion disk J. Cosmol. Astropart. Phys. ICAP04(2021)005
[23] Akiyama K et al (Event Horizon Telescope Collaboration) 2019 First M87 Event Horizon Telescope results. I. The shadow of the supermassive black hole Astrophys. J. Lett. 875 L1
[24] Akiyama K et al (Event Horizon Telescope Collaboration) 2022 First Sagittarius A* Event Horizon Telescope results. I. The shadow of the supermassive black hole in the center of the Milky Way Astrophy. J. Lett. 930 L12
[25] Dokuchaev V I and Nazarova N O 2020 Silhouettes of invisible black holes Phys.-Usp. 63 583
[26] Perlick V and Tsupko O Y 2022 Calculating black hole shadows: review of analytical studies Phys. Rep. 947 1–39
[27] Kimura S S, Murase K and Meszaros P 2021 Soft gamma rays from low accreting supermassive black holes and connection to energetic neutrinos Nat. Commun. 12 5615
[28] Aartsen MG et al (IceCube Collaboration) 2018 Neutrino emission from the direction of the blazar TXS 0506+056 prior to the IceCube-170922A alert Science 361 147–51
[29] Eiroa E F and Romero G E 2008 Gravitational lensing of transient neutrino sources by black holes Phys. Lett. B 663 377–81
[30] Chandrasekhar S 1983 The Mathematical Theory of Black Holes (Oxford: Clarendon)
[31] Zakharov A F 1991 Orbits of photons and ultrarelativistic particles in the gravitational field of a rotating black hole Sov. Astron. 35 30 (available at: https://ui.adsabs.harvard.edu/abs/1991SvA...35...30Z/abstract)
[32] Gralla S E, Lupasca A and Strominger A 2018 Observational signature of high spin at the Event Horizon Telescope Mon. Not. R. Astron. Soc. 475 3829–53
[33] Dvornikov M 2013 Neutrino spin oscillations in matter under the influence of gravitational and electromagnetic fields J. Cosmol. Astropart. Phys. ICAP06(2013)015
[34] Dvornikov M and Studenikin A 2002 Neutrino spin evolution in presence of general external fields J. High. Energy Phys. JHEP09(2002)016
[35] Abramowicz M A, Czerny B, Lasota J P and Suszczewicz E 1988 Slim accretion disks Astrophys. J. 332 646–58
[36] Wald R M 1974 Black hole in a uniform magnetic field Phys. Rev. D 10 1680
[37] Blandford R D and Payne D G 1982 Hydromagnetic flows from accretion disks and the production of radio jets Mon. Not. R. Astron. Soc. 199 883
[38] Beskin V S 2010 MHD Flows in Compact Astrophysical Objects: Accretion, Winds and Jets (Heidelberg: Springer) (https://doi.org/10.1007/978-3-642-01290-7)
[39] Bell N F, Cirigliano V, Ramsey-Musolf M J, Vogel P and Wise M B 2005 How magnetic is the Dirac neutrino? Phys. Rev. Lett. 95 151802
[40] Viaux N, Catelan M, Stetson P B, Raffelt G G, Redondo J, Valcarce A A R and Weiss A 2013 Particle-physics constraints from the globular cluster M5: neutrino dipole moments Astron. Astrophys. 558 A12
[41] Bozza V 2008 Optical caustics of Kerr spacetime: the full structure Phys. Rev. D 78 063014
[42] Grudich M Y 2014 Classical gravitational scattering in the relativistic Kepler problem (arXiv:1405.2919)

[43] Mergulhão Jr C 1995 Neutrino helicity flip in a curved space-time Gen. Relativ. Gravit. 27 657–67

[44] Singh D, Mobed N and Papini G 2004 Helicity precession of spin-1/2 particles in weak inertial and gravitational fields J. Phys. A: Math. Gen. 37 8329–47

[45] Tamburini F, Thidé B and Della Valle M 2020 Measurement of the spin of the M87 black hole from its observed twisted light Mon. Not. R. Astron. Soc. 492 L22–L27

[46] Collins P A, Delbourgo R and Williams R M 1973 On the elastic Schwarzschild scattering cross section J. Phys. A: Math. Gen. 6 161–9

[47] Rauch K P and Blandford R D 1994 Optical caustics in a Kerr spacetime and the origin of rapid X-ray variability in active galactic nuclei Astrophys. J. 421 46–68

[48] An F et al 2016 Neutrino physics with JUNO J. Phys. G: Nucl. Part. Phys. 43 030401

[49] Abe K et al (Hyper-Kamiokande Proto-Collaboration) 2018 Physics potentials with the second Hyper-Kamiokande detector in Korea Prog. Theor. Exp. Phys. 2018 063C01