Single-step implementation of a multiple-target-qubit controlled phase gate without need of classical pulses

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We propose a simple method for realizing a multiqubit phase gate of one qubit simultaneously controlling $n$ target qubits, by using three-level quantum systems (i.e., qutrits) coupled to a cavity or resonator. The gate can be implemented using one operational step and without need of classical pulses, and no photon is populated during the operation. Thus, the gate operation is greatly simplified and decoherence from the cavity decay is much reduced, when compared with the previous proposals. In addition, the operation time is independent of the number of qubits and no adjustment of the qutrit level spacings or the cavity frequency is needed during the operation.

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Multiple qubit gates have many applications in quantum information processing (QIP). A multiqubit gate can be decomposed into two-qubit and one-qubit gates and thus can in principle be constructed using these basic gates. However, the number of basic gates increases dramatically as the number of qubits increases. Thus, it becomes difficult to build a multiqubit gate by using the conventional gate-decomposition protocol. Over the past years, many efficient schemes have been proposed for the direct implementation of a multiqubit controlled-phase or controlled-NOT gate with multiple-control qubits acting on one target qubit (e.g., [1-5]). This type of multiqubit gate plays significant roles in QIP, such as quantum algorithms and error corrections.

We here focus on another type of multiqubit gates, that is, a multiqubit phase gate with one control qubit simultaneously controlling multiple target qubits. This multiqubit phase gate is described by

$$
|0_1 \rangle |i_2 \rangle |i_3 \rangle \ldots |i_n \rangle \rightarrow |0_1 \rangle |i_1 \rangle |i_2 \rangle \ldots |i_n \rangle,
$$

$$
|1_1 \rangle |i_1 \rangle |i_2 \rangle \ldots |i_n \rangle \rightarrow |1_1 \rangle (-1)^{i_1} (-1)^{i_2} \ldots (-1)^{i_n} |i_1 \rangle |i_2 \rangle \ldots |i_n \rangle,
$$

where the subscript 1 represents the control qubit while subscripts 2, 3, ..., and $n$ represent the $n-1$ target qubits, and $i_2, i_3, \ldots, i_n \in \{0, 1\}$. The transformation (1) implies that when the control qubit is in the state |0\rangle, nothing happens to the states of each target qubit; however, when the control qubit is in the state |1\rangle, a phase flip (from the + sign to the − sign) happens to the state |1\rangle of each target qubit. This multiqubit gate is useful in QIP such as entanglement preparation, error correction, and quantum algorithms.

Several methods have been proposed for the direct implementation of this multiqubit phase gate, by employing two-level or four-level quantum systems coupled to a single cavity or resonator [6-8]. However, these methods require several steps of operation and application of classical pulses so that the gate operation is complex. Moreover, in these schemes cavity photons are populated during the operation and thus decoherence caused by the cavity decay may pose a problem. In addition, for the methods proposed in [7,8], a higher-energy fourth level was employed, which is experimentally challenging.

In the following, we present a new approach for implementing this multiqubit phase gate using three-level quantum systems (i.e., qutrits) coupled to a single cavity or resonator. Compared with the previous proposals, the proposal has these features: (i) only a single step of operation is needed and no classical pulse is used, thus the operation is greatly simplified; (ii) no photon is populated, thus decoherence caused by the cavity photon decay is much suppressed; and (iii) it is unnecessary to employ a fourth level. In addition, the proposal has the following additional advantages: the operation time is independent of the number of qubits and no adjustment of the qutrit level spacings or the cavity frequency is required during the operation.

Consider $n$ qutrits labeled by 1, 2, ..., and $n$. Each qutrit has three levels |g\rangle, |e\rangle, and |f\rangle (Fig. 1). Assume that qutrits 2, 3, ..., and $n$ are identical, whose levels spacings are different from those of qutrit 1. The cavity mode is coupled to the |e\rangle ↔ |f\rangle transition of each qutrit, but decoupled (highly detuned) from the transition between any other two levels (Fig. 1). These requirements can in principle be met by choosing or designing the qutrits (e.g., the level spacings of artificial atoms, such as superconducting quantum devices, can be readily adjusted by varying the device parameters appropriately). The interaction Hamiltonian in the interaction picture and under

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The qutrits could have a Λ-type, ladder-type or ∆-type three-level structure. For the Λ-type, the transition between the two lowest levels is forbidden or weak. For the ladder-type, the {g} ↔ {f} transition is forbidden or weak. For the Δ-type, there exists a transition between any two levels. The left is for qutrit 1, while the right is for qutrit k (k = 2, 3, ..., n). Note that the level spacing between the two lowest levels can be greater than that between the two upper levels (not drawn).

\[ H_1 = \hbar \sum_{k=2}^{n} \mu_k \left( e^{i \Delta t} a \sigma_k^+ + e^{-i \Delta t} a^+ \sigma_k \right) + \hbar \mu_1 \left( e^{i \Delta t} a \sigma_1^+ + e^{-i \Delta t} a^+ \sigma_1 \right), \quad (2) \]

where the subscript \( k \) represents the \( k \)th qutrit, \( a^+ (a) \) is the photon creation (annihilation) operator of the cavity mode with frequency \( \omega_c \), \( \mu \) is the coupling constant between the cavity mode and the \{e\} ↔ \{f\} transition of qutrits \((2, 3, ..., n)\), while \( \mu_1 \) is the coupling constant between the cavity mode and the \{e\} ↔ \{f\} transition of qutrit 1. In addition, \( \sigma_k^+ = |f_k\rangle \langle e_k|, \sigma_k^- = |e_k\rangle \langle f_k|, \Delta = \omega_{fe} - \omega_c \) and \( \Delta = \omega_{fe,1} - \omega_c \), with the \{e\} ↔ \{f\} transition frequency \( \omega_{fe} \) of qutrits \((2, 3, ..., n)\) and the \{e\} ↔ \{f\} transition frequency \( \omega_{fe,1} \) of qutrit 1.

For \( \Delta \gg \mu \) and \( \Delta_1 \gg \mu_1 \), there is no energy exchange between the qutrits and the cavity mode. Then the system dynamics described by the Hamiltonian of Eq. (2) is approximately equivalent to that determined by the following Hamiltonian \([9,10]\)

\[ H'_1 = -\hbar \sum_{k=2}^{n} \left[ \frac{\hbar^2}{\Delta} (a^+ a | e_k \rangle \langle e_k| - a a^+ | f_k \rangle \langle f_k|) \right] - \hbar \sum_{k=2, k'=2}^{n} \left( \sigma_k^+ \sigma_{k'}^- + \sigma_k^- \sigma_{k'}^+ \right) + \hbar \lambda \sum_{k=2}^{n} \left( e^{i \delta t} \sigma_k^+ \sigma_k^- + e^{-i \delta t} \sigma_k^- \sigma_k^+ \right). \quad (3) \]

where \( \lambda = \frac{\mu_1 \mu}{2 \Delta_1} \left( \frac{1}{\Delta_1} + \frac{1}{\Delta} \right) \) and \( \delta = \Delta_1 - \Delta \). The photon number is conserved during the interaction. When the cavity mode is initially in the vacuums state, it will remain in this state. Under this condition the photon number operator \( a^+ a \) in Eq. (3) can be set to be 0. Furthermore, when the condition \( \delta \gg \lambda, \frac{\mu^2}{\Delta_1}, \frac{\mu_1^2}{\Delta_1} \) is satisfied, qutrit 1 does not exchange energy with the other qutrits. Under these conditions Hamiltonian (3) can be replaced by the effective Hamiltonian \([11]\)

\[ H_{\text{eff}} = \hbar \sum_{k=2}^{n} \left( \frac{\mu^2}{\Delta} |f_k\rangle \langle f_k| + \frac{\mu_1^2}{\Delta_1} |f_1\rangle \langle f_1| \right) + \hbar \sum_{k=2}^{n} \left( \sigma_k^+ \sigma_{e_k}^--|e_1\rangle \langle e_1| \sigma_k^+ \sigma_{e_k}^- \right). \quad (4) \]

The last term of Eq. (4) describes the effective coupling of qutrits \((2, 3, ..., n)\) arising from the far-off-resonant coupling to qutrit 1 described by the last term of Eq. (3). When the level \{f\} of each of qutrits \((2, 3, ..., n)\) is not populated, it will remain unpopulated since the total number of qutrits \((2, 3, ..., n)\) being in the level \{f\} remains unchanged under the effective Hamiltonian \( H_{\text{eff}} \). In this case, \( H_{\text{eff}} \) reduces to

\[ H'_{\text{eff}} = \hbar \sum_{j,k=2}^{n} \left( |f_j\rangle \langle f_j| + \hbar \frac{\lambda^2}{\delta} \times \sum_{j,k=2}^{n} (|f_j| \langle f_j| \sigma_j^- \sigma_{e_k}^- - |e_1\rangle \langle e_1| \sigma_j^+ \sigma_{e_k}^- \right), \quad (5) \]

which can be further expressed as

\[ H_{\text{eff}} = \hbar \sum_{j,k=2}^{n} \left( |f_j\rangle \langle f_j| + \hbar \frac{\lambda^2}{\delta} \times \sum_{j,k=2}^{n} (|e_1\rangle \langle e_1| \sigma_j^+ \sigma_{e_k}^- \right). \quad (6) \]

We use the asymmetric encoding scheme \([5]\). Namely, for the gate of Eq. (1), the logic state \(|0\rangle\) of each qubit is represented by the level \{g\}, while \(|1\rangle\) is represented by the level \{f\} for qutrit 1 but by \{e\} for qutrit \(j\) \((j = 2, 3, ..., n)\). Since \{e\} is not involved for qutrit 1, the last term in Eq. (6) can be dropped due to \(|e_1| g_1\rangle \equiv 0\). In addition, because \{f\} is not involved for qutrit \(j\) \((j = 2, 3, ..., n)\), the third term in Eq. (6) can be discarded owing to \(|f_j| g_j\rangle \equiv 0\) \(j = 2, 3, ..., n\). Hence, the Hamiltonian (6) becomes

\[ H_{\text{eff}} = \hbar \sum_{j,k=2}^{n} \left( |f_j\rangle \langle f_j| + \hbar \frac{\lambda^2}{\delta} \sum_{j=2}^{n} |e_j\rangle \langle e_j| \right). \quad (7) \]
for which the time-evolution unitary operator is

$$ U = e^{-iH_{zt}t/\hbar} = U_1 \otimes \prod_{j=2}^{n} U_{1j}, \quad (8) $$

where $U_1$ is an unitary operator acting on qutrit 1 while $U_{1j}$ is a joint unitary operator acting on qutrits 1 and $j$, which are given by

$$ U_1 = \exp \left( -\frac{i\mu^2}{\Delta_1} |f\rangle \langle f| \right), \quad (9) $$

$$ U_{1j} = \exp \left( -\frac{i\mu^2}{\delta} |f\rangle \langle f| \otimes |e_j\rangle \langle e_j| \right), \quad (10) $$

Note that $U_1 |g_1\rangle = 0$, $U_1 |f_1\rangle = \exp(-i\mu^2 t/\Delta_1) |f_1\rangle$, $U_{1j} |g_1\rangle |l_j\rangle = |g_1\rangle |l_j\rangle$, and $U_{1j} |f_1\rangle |l_j\rangle = \exp(-i\langle e_j | l_j \rangle \lambda^2 t/\delta) |f_1\rangle |l_j\rangle$, where $|l_j\rangle \in \{|g_j\rangle, |e_j\rangle\}$ ($j = 2, 3, ..., n$). Thus, the operator $U$ leads to the transformation

$$ |g_1\rangle |l_2\rangle |l_3\rangle ... |l_n\rangle \rightarrow |g_1\rangle |l_1\rangle |l_2\rangle ... |l_n\rangle, $$

$$ |f_1\rangle |l_2\rangle |l_3\rangle ... |l_n\rangle \rightarrow e^{-i\mu^2 t/\Delta_1} |f_1\rangle e^{-i\langle e_2 | l_2 \rangle \lambda^2 t/\delta} |l_2\rangle $$

$$ e^{-i\langle e_3 | l_3 \rangle \lambda^2 t/\delta} |l_3\rangle ... e^{-i\langle e_n | l_n \rangle \lambda^2 t/\delta} |l_n\rangle. \quad (11) $$

For $t = \delta t/\lambda^2 = \Delta_1 2\pi/\mu^2$, i.e., setting $\mu^2/\Delta_1 = 2\lambda^2/\delta$, the transformation (11) can be further written as

$$ |g_1\rangle |l_2\rangle |l_3\rangle ... |l_n\rangle \rightarrow |g_1\rangle |l_1\rangle |l_2\rangle ... |l_n\rangle, $$

$$ |f_1\rangle |l_2\rangle |l_3\rangle ... |l_n\rangle \rightarrow (-1)^{\langle e_2 | l_2 \rangle} (-1)^{\langle e_3 | l_3 \rangle} ... (-1)^{\langle e_n | l_n \rangle} |f_1\rangle |l_2\rangle |l_3\rangle ... |l_n\rangle. \quad (12) $$

which shows that when qutrit 1 is in the state $|g\rangle$, nothing happens to the states of each of qutrits $(2, 3, ..., n)$; however, when qutrit 1 is in the state $|f\rangle$, a phase flip (from the + sign to the − sign) happens to the state $|e\rangle$ of each of qutrits $(2, 3, ..., n)$. Hence, a multi-target phase gate described by Eq. (1) is realized with $n$ qutrits, i.e., the control qutrit 1 and the $(n-1)$ target qutrits $(2, 3, ..., n)$.

To see the above more clearly, let us consider three qutrits for implementing a three-qubit phase gate. The three-qubit computational basis corresponds to $\{|ggg, |gge, |geg, |gge, |fgg, |fge, |feg, |feg\rangle\}}$. Based on Eq. (12), one can find that the four states $|fgg\rangle$, $|fge\rangle$, $|feg\rangle$, and $|fge\rangle$ of the qutrits become $|fgg\rangle$, $- |fge\rangle$, $- |feg\rangle$, and $- (-) |fge\rangle$, respectively; while the other four states $|ggg\rangle$, $|gge\rangle$, $|geg\rangle$, and $|gee\rangle$ remain unchanged.

We now give a general discussion on the fidelity of the operation. After taking the dissipation and dephasing into account, the dynamics of the lossy system is determined by the master equation

$$ \frac{d\rho}{dt} = -i[H_I, \rho] + \kappa \mathcal{L}[a] $$

$$ + \sum_{j=1}^{n} (\gamma_j \mathcal{L}[\sigma_j] + \gamma_{j,ff} \mathcal{L}[\sigma_{j,ff}] + \gamma_{j,eg} \mathcal{L}[\sigma_{j,eg}]) $$

$$ + \sum_{j=1}^{n} \gamma_{j,ff} \mathcal{L}[\sigma_{j,ff} | \sigma_{j,ff} - |\rho \sigma_{j,ff} / 2 \rho \sigma_{j,ff} / 2 \rho \sigma_{j,ff} / 2 \rho \sigma_{j,ff} / 2 \rho ], $$

$$ + \sum_{j=1}^{n} \gamma_{j,e,e} \mathcal{L}[\sigma_{j,e,e} | \sigma_{j,e,e} - |\rho \sigma_{j,e,e} / 2 \rho \sigma_{j,e,e} / 2 \rho ], \quad (13) $$

where $H_I$ is the Hamiltonian in Eq. (2), $\gamma_{j,eg} = |g\rangle \langle g| \langle f|$, $\gamma_{j,eg} = |g\rangle \langle g| \langle e|$, $\gamma_{j,eg} = |f\rangle \langle f| \langle e|$, $\gamma_{j,eg} = |e\rangle \langle e| \langle e|$, and $\mathcal{L}[\Lambda] = \Lambda \rho \Lambda^+ - \Lambda^+ \Lambda \rho / 2 - \rho \Lambda \Lambda / 2$, with $\Lambda = \alpha, \gamma_j, \gamma_{j,ff}$, and $\gamma_{j,eg}, \kappa$ is the photon decay rate of the cavity, $\gamma_{j,eg}$ is the relaxation rate of the level $|e\rangle$ of qutrit $j$ for the decay path $|e\rangle \rightarrow |g\rangle, |f\rangle$, $\gamma_{j,ff}$ is the relaxation rate of the level $|f\rangle$ of qutrit $j$ for the decay path $|f\rangle \rightarrow |g\rangle, |e\rangle$, and $\gamma_{j,e,e}$ is the dephasing rate of the level $|e\rangle$. The fidelity of the operation is given by $F = \langle \psi_{id} | \tilde{\rho} | \psi_{id} \rangle$, where $| \psi_{id} \rangle$ is the output state for an ideal system (i.e., without dissipation and dephasing) after the entire operation, while $\tilde{\rho}$ is the final density operator of the whole system when the operation is performed in a realistic physical system.

For the sake of definitiveness, let us consider the experimental feasibility of realizing a three-qubit phase gate. Assume that qutrit 1 is in the state $(|g\rangle + |f\rangle) / \sqrt{2}$, qutrits 2 and 3 are in $(|g\rangle + |e\rangle) / \sqrt{2}$, and the cavity mode is in the vacuum state before the gate operation.

Fig. 2 is plotted for the fidelity versus $\Delta_1/\mu_1$ and $\delta/\mu_1$, without considering the dissipation and dephasing of the whole system. The middle red-color convex surface in Fig. 2 shows that a high fidelity $\geq 99\%$ can be achieved for a wide range of $\Delta_1/\mu_1$ and $\delta/\mu_1$. Without loss of generality, choose $\Delta_1 = 10.7\mu_1$, $\Delta = 8.4\mu_1$, and $\mu = 3.08\mu_1$. 

FIG. 2: (Color online) Fidelity versus $\Delta_1/\mu_1$ and $\delta/\mu_1$, without considering the system dissipation and dephasing.
With these parameters and the dissipation and dephasing considered, we solve the master equation numerically. As an example, consider qutrits with a ladder-type level structure (available in natural atoms, quantum dots, superconducting phase qutrits, and transmon qutrits). We set \( \kappa = 0.01 \mu_1 \), \( \gamma_j_{\text{eff}} = \gamma_j \), \( j_{\text{eg}} = \gamma_j \), and plot the fidelity as a function of \( \gamma/\mu_1 \) in Fig. 3. The result shows that when \( \gamma/\mu_1 \leq 2 \times 10^{-4} \), a fidelity higher than 96.8\% can be obtained. The fidelity can be further increased by optimizing the system parameters. For \( \gamma/\mu_1 = 2 \times 10^{-4} \) and \( \mu_1 = 2 \pi \times 85 \text{ MHz} \), the qutrit decoherence time is 9.36 µs and \( \mu = 2 \pi \times 261.8 \text{ MHz} \), which are readily available for superconducting transmon qutrits. Decoherence time can be made to be on the order of 20 – 60 µs for state-of-the-art superconducting transmon devices [12-14], and a coupling constant \( \sim 2 \pi \times 360 \text{ MHz} \) has been reported for a superconducting transmon device coupled to a resonator [15]. For superconducting qutrits, the typical transition frequency between two neighbor levels is between 5 and 10 GHz. As an example, we take \( \omega_{\text{res}}/2\pi = 6.0 \text{ GHz} \), \( \omega_c/2\pi = 5.09 \text{ GHz} \), \( \mu_1 = 2 \pi \times 85 \text{ MHz} \), and \( \kappa = 2 \pi \times 0.85 \text{ MHz} \). The corresponding quality factor of the resonator is \( Q = 5.97 \times 10^5 \) (a value much lower than \( Q \sim 10^5 \) required by [6,7]). Note that superconducting resonators with a loaded quality factor \( Q \sim 10^6 \) have been experimentally demonstrated [16,17].

In conclusion, we have presented an approach for implementing the multiqubit phase gate. As shown above, the operation is greatly simplified and decoherence caused by the cavity photon decay is much reduced when compared with the previous proposals. This work is quite general, and can be applied to a wide range of physical implementation with natural atoms or artificial atoms (e.g., quantum dots, NV centers, or various superconducting qutrits such as flux, phase, charge, and transmon qutrits) coupled to a cavity or resonator.

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