Non-scattering Systems for Field Localization, Enhancement, and Suppression

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We propose and analyse invisible (non-scattering) partially open cavities where the field distribution inside can be engineered. It is demonstrated both theoretically and experimentally that the cavities exhibit unidirectional invisibility at the operating frequency with localized field enhancement and suppression at different positions inside the cavity volume. Several examples of applications of the designed cavities are proposed and analyzed, in particular, cloaking sensors and obstacles, enhancement of emission, and “invisible waveguides”. The non-scattering mode excited in the proposed cavity is driven by the incident wave and resembles an ideal bound state in continuum of electromagnetic frequency spectrum. In contrast to known bound states in the continuum, the mode can stay localized in the cavity infinitely long, provided that the incident wave illuminates the cavity.

Localization of electromagnetic waves plays central role for applications in various fields of technology such as in lasers, filters, optical fibers, nonlinear devices, etc. During last decades, it became also a rapidly growing subject of nanophotonics where achieving high field concentration in optically compact systems is of paramount importance\textsuperscript{[1,2]}. A simple example of wave localization is the echo inside a cave. Sound waves created by an object inside the cave cannot escape outside and experience multiple reflections from its hard walls before getting eventually absorbed. Meanwhile, the strength of sound can be strongly amplified in this cavity due to the wave interference. A pertinent question arises whether it is possible to create wave localization in a space region without isolating it by walls and, most importantly, without disturbing waves coming into it from outside. Such space region would appear for an external observer as free space (fully imperceivable), while it would operate as a resonant cavity for an observer inside it.

An apparent solution for realizing strong field localization in an open system is based on Fabry-Perot etalons or interferometers\textsuperscript{[3,5]}. Their structure includes two partially transparent mirrors separated by a specific distance. At discrete wavelengths, lossless Fabry-Perot resonators pass through all power carried by incident waves due to the destructive interference of reflected waves from the mirrors. However, these resonators are still perceivable for an external observer since they alter the phase of the transmitted waves, and therefore, cause electromagnetic scattering. In contrast, the solution for open non-scattering cavities implies that the field outside of the cavity is not modified at all. Theory of non-scattering and non-radiating bodies has a long history dating back to the work by Ehrenfest, where he recognized that radiationless current distributions are possible\textsuperscript{[6,9]}. In fact, this discovery was the starting point for new and innovative concepts and applications\textsuperscript{[10]}. Nevertheless, such systems do not necessary allow to achieve strong fields localizations, as is evident from a simple example of a non-scattering configuration which includes two Huygens’ planar interfaces\textsuperscript{[11,12]}: Each interface does not produce back scattering, while in the forward direction their scattering is mutually compensated. Obviously, in this case, the field between the interfaces is identical to that of the incident wave and cannot reach extreme values.

Thus, realization of invisible cavities implies combination of field enhancement properties of Fabry-Perot resonators and invisibility features of non-scattering electromagnetic systems. This scenario to some extent resembles bound states in the continuous spectrum of frequencies (BICs), i.e. non-radiating eigenmodes captured into a localized volume\textsuperscript{[13]} (see Fig.\textsuperscript{1}(a)). The first studies of this concept were made by von Neumann and Wigner\textsuperscript{[14,15]} in quantum mechanics, and expanded through the years to other physics branches, like photonics\textsuperscript{[16–29]}, acoustics\textsuperscript{[30,31]}, electronics\textsuperscript{[32,33]}, and others\textsuperscript{[34]}. Ideal BICs with an infinite quality factor are impossible in practice due to dissipation loss present in all physical systems, therefore, it is more relevant to speak about quasi BICs with a large but finite lifetime (see Fig.\textsuperscript{1}(b)). Importantly, one should clearly distinguish between quasi BICs and the non-scattering eigenmodes of invisible cavities discussed above. The former ones are eigenmodes of the system in the absence of incident waves, which slowly leak away from the localization region. The latter modes (see Fig.\textsuperscript{1}(c)), in contrast, appear in the presence of the incident wave and do not leak outside the localization region even in systems with a finite quality factor. These states, “driven” by the incident wave, resemble ideal BICs. Their distinctive features make invisible cavities good candidates for various applications, while practical implementation of quasi BICs have been scarce to date and limited to very recent works\textsuperscript{[35,37]}.

In this paper, we propose, design, and study invisible cavities formed by two parallel thin sheets. The paper considers the 1D scenario where the cavity is formed by two planar metasurfaces\textsuperscript{[38,39]} (or frequency-selective
surfaces in special cases [38, 39] separated by a distance. We demonstrate both numerically and experimentally unidirectional invisibility of the cavity at the operating frequency with localized field enhancement and suppression at different positions inside the cavity bounded by the metasurfaces. The amplitude of the field within the cavity depends on the transparency of the metasurfaces, and, in the limiting case, the non-scattering modes (so-called driven BICs) reach ideal BICs. Several examples of applications of the designed invisible cavities are proposed and analyzed, in particular, cloaking sensors and obstacles, and cavities for Purcell’s factor enhancement. Furthermore, we demonstrate that in the limiting transparency case, the proposed cavities have essentially different convergence to the ideal BICs compared to conventional cavities, which provides efficient route towards finite-time excitation of BICs with theoretically infinite lifetime.

RESULTS

Design of invisible cavities

Let us consider a one-dimensional scenario, where a dielectric slab (with the wave impedance η and wavenumber k) of thickness d is bounded by two metasurfaces infinite in the xy plane, as shown in Fig. 2. The metasurfaces represent artificial periodic structures with negligible thickness, designed for electromagnetic wave manipulations [38, 39]. When such a cavity system is excited by a normally incident plane wave, electric currents (characterized by the surface current densities J_{e1} and J_{e2}) are excited in the metasurfaces. These currents produce scattered waves inside and outside the cavity. Each metasurface is characterized by their grid impedance Z_e, which is defined as the ratio of the tangential component of the surface-averaged surrounding electric field at the metasurface position and the induced electric current density.

We design the cavity so that it produces no scattered waves outside it, meaning that the reflected wave should be eliminated (E_R = 0) and the transmitted wave should be eliminated (E_T = E_I) without any phase lag. Solutions of the boundary conditions at both interfaces (see Supplementary Methods) provide the required values of the grid impedances Z_{e1} and Z_{e2} for particular dielectric slab material and thickness:

\[
Z_{e1} = \left[ e^{2jkd} \left( 2\eta_0 + e^{j(k_0+k)d} (\eta - \eta_0) \right) - e^{j(k_0+k)d} (\eta + \eta_0) \right]^{-1} e^{j(k_0+k)d} (e^{2jkd} - 1) \eta_0 \eta, \tag{1a}
\]

\[
Z_{e2} = \left[ e^{-2jkd} \left( 2\eta_0 + e^{-j(k_0+k)d} (\eta - \eta_0) \right) - e^{-j(k_0+k)d} (\eta + \eta_0) \right]^{-1} e^{-j(k_0+k)d} (1 - e^{-2jkd}) \eta_0 \eta. \tag{1b}
\]

where k_0 and \eta_0 correspond to the free-space wavenumber and characteristic impedance, respectively. To understand the behaviour of both impedances, Figure 3 plots them as functions of the slab thickness for a particular scenario where the relative permittivity ε_r of the dielectric slab is equal to 2. First of all, it is seen that both impedances are generally complex \(Z_e = R_e + jX_e\) with real and imaginary parts corresponding to resistance and reactance, respectively (here, time dependence \(e^{j\omega t}\) is adopted). Second, the imaginary parts of both grid
immediately shown by considering radiation from an arbitrary metasurface which supports electric and (possible) magnetic polarization currents. Since an electric current sheet \( J_e \) scatters plane waves symmetrically to both sides (\( E_{e,\text{back}} = E_{e,\text{forward}} \)) and a magnetic current sheet \( J_m \) scatters waves anti-symmetrically (\( E_{m,\text{back}} = -E_{m,\text{forward}} \)), their combined scattered field cannot be zero simultaneously at the two sides of the metasurface.

Although the scattering is completely suppressed from a metasurface cavity whose properties are given by Eqs. (3), the fields inside it are still to be analyzed. The field inside the cavity is the combination of the forward and backward propagating waves (in the realistic implementation both propagating and evanescent waves will be excited due to the discrete structure of the metasurface). Imposing Eqs. (2) and assuming that evanescent fields of currents on one metasurface do not extend to the other one, we can obtain the total electric field (see derivations in Supplementary Methods):

\[
E(z) = \frac{E_1}{2Z_{e1}} \left[ (2Z_{e1} - \eta_0) e^{-jk_0z} - \eta_0 e^{jk_0z} \right].
\]

Figure 4(a) illustrates the electric field at different locations inside a lossless cavity \((Z_{e1,2} = jX_{e1,2}, d = \lambda_{\text{op}})\) as a function of the reactance value \(X_{e1}\). It is seen that the field distribution has a repeating pattern along the \( z \) direction with the periodicity of \( \lambda_{\text{op}}/2 \). Each period includes points where the electric field is minimum and maximum. As the impedance values \(|X_{e1,2}|\) decrease (metasurfaces behave more similar to a perfect electric conductor sheet), the total field inside the cavity increases. Such property is analogous to that of conventional Fabry-Perot resonators. Nevertheless, in contrast to the latter ones which transmit waves with additional phase change \((E_T = -E_1)\), this cavity remains always non-scattering for an external observer for arbitrary designed pair \(|X_{e1,2}|\). The ratio between the electric field in the maximum and minimum of the cavity is conventionally characterized by the standing wave ratio (SWR) which is defined for the invisible cavity as

\[
\text{SWR} = \frac{|E_{\text{max}}|}{|E_{\text{min}}|} = \frac{|2Z_{e1} - \eta_0| + \eta_0}{|2Z_{e1} - \eta_0| - \eta_0}.
\]

This ratio is applicable for cavities with and without dissipation loss (see derivation in Supplementary Methods). Figure 4(b) illustrates the dependence of the standing wave ratio for lossless invisible cavities versus the grid reactance. As expected, the higher ratio corresponds to the low-reactance cavities.

There are other alternative characteristics for description of cavity properties which are applied in different
The cavity thickness is $d = \lambda_{\text{op}}$. (b) Standing wave ratio, quality factor $Q$, and number of reflection cycles for convergence to the steady-state regime $m_{90\%}$ for an invisible cavity with thickness $d = \lambda_{\text{op}}$.

The quality factor does not only depend inversely proportionally on the grid impedances, but is also proportional to the distance between the metasurfaces (represented by the integer $n$). Thus, maximization of the quality factor, desired for many applications, is achieved when the cavity thickness is large and the grid impedance is small, as shown in Fig. 4(b). It should be noted that this definition of the quality factor implies lossless and dispersionless unit cells of the metasurfaces (additional frequency dispersion of lossless metasurfaces generally results in increasing the quality factor due to additional storage of reactive energy). Next, we analyze the time duration required for the cavity to converge to the steady state regime. This time can be estimated by the use of the well-known circulating-field approach [42, pp. 413–428]. This approach assumes a steady state and calculates the transmitted and reflected fields as a combination of fields from waves multiple times reflected from the two interfaces. Applying it to our invisible cavity (see Supplementary Methods for details), we can find the number of reflection cycles after which the wave interference reaches to that of the steady-state regime by 90% (in all cavities ideal convergence occurs after an infinite number of cycles):

$$m_{90\%} = \text{ceil} \left( \frac{\ln (1 - 0.9)}{\ln |\Gamma_1 \Gamma_2|} - 1 \right),$$

where “ceil” denotes the ceiling function which maps its argument to the least integer greater than or equal to it, and $\Gamma_{1,2} = -\eta_0/(2Z_{e1,2} + \eta_0)$ are the reflection coefficients from each metasurface. It should be mentioned that this circulating-field analysis is applied only under assumption of non-dispersive metasurfaces. The evaluation of the convergence factor (as shown in Fig. 4(b)) reveals that cavities with low grid impedances store stronger fields but they require more reflection cycles to become invisible. It should be mentioned that all three curves plotted in Fig. 4(b) follow a similar quadratic dependence on the metasurface reactance (bear in mind the logarithmic axes in the plot).

Finally, it is important to analyze how the presence of dissipation loss in metasurfaces will affect the performance of the invisible cavity. Naturally, loss can be compensated in the cavity via additional energy pumping which ensures that condition (2) holds. However, in most applications, external energy pumping is not desired due to the additional complexities it results in. Therefore, we analyze the performance degradation of cavities when the impedances of both metasurfaces have additional and equal positive real parts $R_{\text{loss}}$ corresponding to grid resistances:

$$Z_{e1} = Z_{e2}^* = R_{\text{loss}} + jX_{e1}.$$ (7)

Taking into account the dissipation loss, the total electric field across the cavity is modified, as shown in Fig. 4(a). If the grid losses are much greater than the reactances (by absolute values), both metasurfaces are weakly excited, resulting in low scattering and low field amplitude inside the cavity. When the metasurface resistance is comparable to the reactance, the cavity becomes reflective and also its absorption increases, as is seen in Fig. 4(b) which depicts the reflectance $|\Gamma|^2$, transmittance $|\tau|^2$, and absorbance $|A|^2$ through the cavity. The structure remains nearly non-scattering only when the ratio of the grid reactance and resistance is very small. Therefore, it is important to determine how small ratios $R_{\text{loss}}/|X_{e1}|$ can be achieved with realistic materials in different frequency ranges. Here, we analyze realizations in microwave (wavelength from 1 to 20 GHz) and infrared (wavelength from 1.4 to 1.7 μm) regions. We evaluate grid resistance $R_{\text{loss}}$ under assumption that each metasurface represents a continuous layer of metal (gold, silver, copper, or aluminum) with a specific thickness. Naturally, metasurface patterning will result in higher (for inductive metasurfaces) or lower (for capacitive metasurfaces) values of grid resistance compared to the estimated ones. Nevertheless, such estimation gives a straightforward way to predict the influence of dissipation loss in invisible cavities.
FIG. 5: Analysis of the cavity performance in the presence of dissipation loss. The imaginary parts of the metasurface grid impedances was chosen $X_{e1} = -X_{e2} = 38.08\,\Omega$. (a) Effect of loss in metasurfaces on the total electric field across the cavity. (b) Transmittance $|\tau|^2$, reflectance $|\Gamma|^2$, and absorbance $|A|^2$ through a cavity versus the normalized resistance of the metasurfaces. (c) Grid resistance evaluation of a continuous metal layer of 17 $\mu$m thickness in the microwave frequency range. Data for different metals (silver, gold, copper, and aluminum) is shown. (d) Same as (c) but for the near infrared region. Metal layer thickness is 30 nm.

For the microwave analysis, we choose the typical metal cladding thickness of 17 $\mu$m. The details of the resistance evaluation can be found in Supplementary Methods. The analytical results in Fig. 5(c) demonstrate that in the microwave region, the dissipation loss in metasurfaces is very small and can be neglected unless the ratio $R_{\text{loss}}/|X_{e1}|$ is not very small. Using Fig. 5(b), one can estimate the minimum reactance $|X_{e1}|$ which one can achieve without compromising the cavity operation. For example, at a frequency of 11 GHz, one can design copper invisible cavity with reactance $|X_{e1}| \approx 14\,\Omega$, providing that the transmittance will be not less than 90%. Such small reactance ensures cavity Q-factor of the order of $10^3$, as is seen from Fig. 4(b).

Driven bound states in the continuum

As it was mentioned above, the non-scattering eigenmode (eigenstate), distinguishing the invisible cavities from conventional Fabry-Perot resonators, is a so-called driven bound state in the continuous spectrum of frequencies. At the frequency of the mode, the scattered field is completely localized inside the cavity (the cavity appears invisible as in Fig. 1(c)), while at all higher and lower frequencies, the cavity produces scattered fields, yielding a continuous spectrum of unbounded states. The name “driven” comes from the fact that this bound state can exist only when the cavity is illuminated by a plane wave. If the illumination is switched off, the excited mode energy will eventually leak away, as in the case of usual quasi BICs.

A distinctive feature of driven BICs is that they have infinite lifetime (limited only by duration of the external illumination) in cavities with a finite Q-factor. According to Fig. 4(b), as the grid impedances of the metasurfaces approach zero, the Q-factor tends to infinity, and the driven BIC becomes ideal BIC. Figure 6 shows the transmittance through the cavity as a function of the metasurface grid reactance $X_{e1}$ and the wavelength of incident waves $\lambda$. One can clearly recognize sharp resonances of the cavity at $\lambda = 2d/n$ ($n$ is an positive integer).
to zero from the opposite sides so that condition $X \to 0$. Nevertheless, it is important that the grid reactances of both metasurfaces become identical. Nevertheless, it is important that the grid reactances of both metasurfaces become identical. Nevertheless, it is important that the grid reactances of both metasurfaces become identical.

FIG. 6: Transmittance through the cavity as a function of the metasurface grid reactance and the wavelength of incident waves. At wavelengths $\lambda = 2\lambda_{op}/n$, the cavity approaches ideal BIC in the limit of $X_{e1} \to 0$. In this scenario, the distance between metasurfaces is $d = \lambda_{op}$.

![Graph showing transmittance through the cavity](image)

FIG. 7: (a) Reflectance and transmittance through the cavity in two scenarios: $X_{e1} = -X_{e2}$ and $X_{e1} = X_{e2}$. In the limit of zero reactances, the two scenarios provide drastically different performances. (b) Comparison of the maximum electric field inside the cavity for the two scenarios as in (a). In both scenarios, the distance between the cavity walls is fixed to $d = \lambda_{op}$.

![Graph showing reflectance and transmittance](image)

corresponding in the limit of $X_{e1} \to 0$ to ideal BICs.

Interestingly, in the limit of $X_{e1} \to 0$ and $X_{e2} \to 0$, both metasurfaces become identical. Nonetheless, it is important that the grid reactances $X_{e1}$ and $X_{e2}$ tend to zero from the opposite sides so that condition $X_{e1} = -X_{e2}$ holds. Figure 7(a) illustrates the drastically different reflectance and transmittance through the cavity for two convergence scenarios: When $X_{e1} = -X_{e2} \to 0$ and when $X_{e1} = X_{e2} \to 0$. In the former case, the cavity is invisible even for extreme low values of reactances, while in the latter case, the cavity becomes impenetrable even for moderate reactances. The maximum value of the electric field inside the cavity for these two cases is plotted in Fig. 7(b). The dramatic difference of the two curves is evident. In contrast, the standing wave ratio remains the same in the two cases.

Thus, the contrasting polarization properties of the two metasurfaces forming a cavity are essential for achieving driven (non-scattering) BICs. Analyzing Fig. 7(a), one can see that if the reactances of both metasurfaces could be dynamically tuned to extremely low values keeping the balance $X_{e1} = -X_{e2}$, this would be an efficient route to excite BICs with theoretically infinite lifetime. This locking (or releasing) energy would not require infinitely long excitation.

Realization of invisible cavities and experimental verification

In order to verify the theoretically predicted behavior of invisible cavities, a set of waveguide experiments was carried out. It is well known that the fundamental TE$_{10}$ mode propagating in a waveguide with the rectangular cross-section of the dimensions $a \times b$ with electric field magnitude $E_0$ and the wavenumber $k_z = \sqrt{k_0^2 - (\alpha/a)^2}$ can be described as a superposition of two TEM plane waves as follows:

$$E_y(x, z) = A e^{jk_0 \cos \theta z} (e^{jk_0 \sin \theta x} - e^{-jk_0 \sin \theta x}). \quad (8)$$

In the last expression $\theta = \cot^{-1} \sqrt{(\lambda_{op} \alpha)^2 - 1}$ is the frequency-dependent angle of incidence of the plane waves with respect to $z$-axis of the waveguide and $A = -jE_0/2$ is their magnitude. Therefore, the problem of scattering of the fundamental mode by a thin obstacle placed in the cross-section of the waveguide is electromagnetically equivalent to that of plane-wave scattering by a periodic array of such obstacles with the periodicities $a$ and $b$ along the $x$ and $y$ axes, respectively. This approach has been widely used in experimental characterization of various periodic structures for reflection/transmission coefficient and effective material properties extraction. Although this method does not provide an arbitrary choice of the incidence angle at an operational frequency and does not support normal incidence, it is still applicable to analyze the proposed invisible cavities. As the wavelength of the TE$_{10}$ mode equal to $\Lambda = 2\pi/k_z$ differs from the wavelength in free space, in our approach the separation of metasurfaces in the cavity (multiple of half-wavelengths) should be modified accordingly. It is, however, possible to demonstrate that the requirement of mutually complex-conjugate grid impedances still holds for the metasurfaces providing theoretically perfect non-scattering regime in the case of oblique incidence. We designed such capacitive and inductive metasurfaces to provide the required grid impedances at the operational frequency of $f_1 = 11.11$ GHz, as described in the Methods section. The designed metasurfaces (or frequency-selective surfaces) are depicted in Fig. 8 (in this case, the space between the metasurfaces is filled...
with vacuum). The distance between them is three half-wavelengths, i.e. \( L_1 = 3\lambda/2 \) (third Fabry-Perot resonance).

The aim of the first experiment was to demonstrate the operation of the invisible cavity or, in other words, to show that properly choosing properties of both metasurfaces, high transmission at the resonance is achieved along with a zero transmission phase. Figure 8 shows the measured transmission (left) and reflection (right) coefficients through the cavity in the frequency range from 10 to 12 GHz. The insets demonstrate the same plots but in the extended frequency range from 7 to 12 GHz. As one can see from the plots, there are three separate Fabry-Perot resonances in the extended range. At each resonant frequency, the transmission coefficient magnitude is high, however it exceeds 0.9 only at the third resonance (11.11 GHz), i.e. at the frequency where the impedances of the inserted metasurfaces were designed to be complex conjugate of each other. The experimentally achieved transmission coefficient magnitude at the maximum was 0.9 instead of 0.97 predicted by the simulation due to some misalignment of the capacitive and inductive unit cells with respect to the cross section plane. These results are in good agreement with theoretical predictions for lossy metasurfaces [see Fig. 5(b)]. Indeed, the theoretical data provide estimated transmission coefficient of \( \sqrt{0.9} \approx 0.95 \) and nearly zero reflection coefficient for the designed metasurfaces with \( R_{\text{loss}}/|X_1| \approx 5 \times 10^{-3} \) (see Methods for more details). Furthermore, as one can check from the measured phase plots in Fig. 8(b), the transmission coefficient phase crosses through zero at the same resonance frequency. The measured quality factor is \( Q = 414 \). Therefore, it is experimentally observed that the cavity is indeed practically invisible (non-scattering) for the chosen excitation. It is worth noting that the experimental curves are in good correspondence with numerically calculated ones.

Next, to demonstrate the effect of resonant field enhancement in the invisible cavity, the second experiment was carried out, in which a movable probe was used inside a special measurement waveguide section to measure the electric field distribution inside the cavity at its resonance frequency (see Methods section for further details). For comparison, the realized metasurface unit cells were numerically simulated inside a waveguide section and the calculated field patterns were compared with the measured ones. The measured and simulated electric field distributions at the resonant frequency of 11.11 GHz are compared in Fig. 9, where the field in the empty waveguide was taken as 1 V/m. Despite that the measurement section with the probe was precisely three times longer than the uniform section used in the first experiment, in Fig. 9 only the coordinate range from \(-0.6\lambda\) to \(0.6\lambda\) mm is shown. This is due to the fact that the slot in which the probe could move was shorter than the whole section length. However, we note that in this case at 11.11 GHz we operate with the ninth-order Fabry-Perot resonance with nine lobes in total, as schematically depicted in Fig. 11(c). The measured field profile is in good agreement with the simulated one. The difference of the measured resonant field enhancement from the numerically predicted value (10 versus 11.6) can be explained by radiation leakage losses due to the measurement slot and the probe effect. Therefore, it was observed that the designed non-scattering cavity at 11.11 GHz causes an order-of-magnitude field enhancement and suppression. Likewise, these results are in good agreement with theoretical predictions for lossy metasurfaces [see Fig. 5(a)] under assumption of \( R_{\text{loss}}/|X_1| \approx 5 \times 10^{-3} \).

**Manipulation of scattering from an object placed inside the cavity**

As was discussed above, the proposed cavities provide enhancement and suppression of fields inside, keeping fields outside undisturbed. Such functionality provides unique opportunities for manipulating scattered fields from objects placed inside the cavity. Indeed, placing an object in the field minima (maxima), one can suppress (enhance) wave scattering from it since the local field at the object position is decreased (increased) compared to the incident field. The best effect is achieved when the object is planar (since we consider 1D invisible cavities). The object can be of negligible thickness (e.g., conducting mesh or screen) or arbitrary large thickness (e.g., dielectric slabs). The latter case corresponds to the problem shown in Fig. 2. Generally, the metasurfaces of the cavity must be active and lossy (see Fig. 9), however, in the case of a lossless object of negligible thickness placed inside the cavity, the corresponding metasurfaces can be lossless. Here, we consider this scenario.

We model the object whose scattering should be enhanced or suppressed as a surface with the grid impedance \( Z_{\text{obj}} = R_{\text{obj}} + jX_{\text{obj}} \) (only electric polarization response is assumed). Positioning the object at a distance \( z = \delta_d \), we can determine reflected and transmitted fields from it in the presence and absence of the cavity (see Supplementary Methods). Figure 10 shows the reflectance, transmittance, and absorbance of incident waves in three scenarios: When the object is in free-space (without the cavity), and when it is placed in the field maximum and minimum inside the cavity (here, we chose \( X_{e1} = X_{e2} = 38.08\Omega \). Comparison of the plots in the first and second rows (free-space and field-minimum scenarios) reveals that by positioning an arbitrary object inside the field minimum of the cavity, one can drastically decrease reflection from the object. Absorption can be decreased or increased depending on the grid impedance of the object. The transmittance in this case is more than 0.9 when \( |R_{\text{obj}} + jX_{\text{obj}}| > 0.1\eta_0 \). The phase of transmission is close to zero, which implies strongly suppressed scattering from the object (note that the object alone transmits less than 3% when \( |R_{\text{obj}} + jX_{\text{obj}}| = 0.1\eta_0 \)). Thus, the invisible cavity can be exploited for scattering suppression from planar layers. The smaller grid reac-
FIG. 8: Simulated and measured magnitudes and phases of the transmission (left column) and reflection (right column) coefficients in the frequency range from 10 to 12 GHz (not power coefficients). The insets show the transmission and reflection coefficient magnitudes versus frequency in the extended range from 7 to 12 GHz.

FIG. 9: Simulated and measured distributions of electric field magnitude across the central part of an invisible cavity composed of capacitive and inductive unit cells places in a waveguide (the ninth order Fabry-Perot resonance at 11.11 GHz). Here $L_2 = 150$ mm is the distance between the metasurfaces in the second experiment.

tances of the metasurfaces, the greater suppression effect can be achieved (higher Q-factor). On the contrary, the smaller grid impedance of the object to be hidden, the lower the effect. In the limit of the object made from perfect electric conductor, full reflection occurs even in the presence of the cavity.

By comparing the first and third rows in Fig. 10 (the free-space and field-maximum scenarios), one can see that depending on the grid impedance of the object, the cavity can increase (up to 50%) or decrease absorption in the object. At the same time, reflection is always increased and transmission decreased. Such functionality can be used for enhancement of the energy captured by sensors.

Next, in our third experiment, we verify scattering suppression from a planar object placed inside the cavity with the waveguide setup. As the planar object, we used a capacitive metasurface unit cell to be placed at an arbitrary position inside the previously studied invisible cavity (see details in Methods). The object has the grid impedance of around $-j31 \, \Omega$, which, in the absence of the cavity, results in the simulated and measured transmission and reflection coefficients shown in Fig. 12(a). As can be seen, the low grid capacitance of the object leads to high but not unitary reflection (0.97 at 11.11 GHz). However, when the same object is placed in the electric field maximum inside the invisible cavity, the reflection coefficient becomes very close to unity in the whole frequency range from 10 to 12 GHz. The latter effect was further confirmed by numerical simulations. Therefore, in this case one observes broadband enhancement of reflectivity or scattering from an object placed inside the cavity.

On the contrary, when the object is placed in the minimum of the electric field distribution inside the cavity, the simulated and measured transmission and reflection coefficients shown in Fig. 13(a) look drastically different. Indeed, as is seen from Fig. 13(a), the measured transmission coefficient magnitude reaches 0.79 at the reso-
FIG. 10: First, second, and third rows represent scenarios of an object placed in free space, when it is placed in the field minimum and field maximum of the cavity, respectively. The plots on the right depict transmittance $|\tau|^2$, reflectance $|\Gamma|^2$, and absorbance $|A|^2$ of incident waves for these scenarios. The cavity metasurfaces have reactances $X_{e1} = X_{e2} = 38.08 \, \Omega$. On the left side, $E_{w/o}$ denotes the total electric field without the object.

nance frequency of the cavity (11.11 GHz) despite that the inserted metasurface itself was highly reflective. Interestingly, the transmission coefficient phase is still zero at this frequency. The experimentally achieved transmission coefficient magnitude of 0.79 differs from the simulated value of 0.95 due to little misalignments of the capacitive and inductive unit cells as well as the object with respect to the cross section plane. Nevertheless, it is possible to conclude that when the reflective metasurface is properly inserted into the invisible cavity, the whole system becomes nearly zero scattering (some scattering occurs due to absorption). The other noticeable effect is the second resonance appeared at 11.32 GHz, which can be explained by the fact that the object paired with the capacitive metasurface of the cavity forms a new Fabry-Perot cavity. In this new cavity both walls have the same reactance of $-j31 \, \Omega$, so this new cavity is not transparent, as one can observe from the transmission and reflection magnitudes at 11.32 GHz in Fig. 13(a,c). Moreover, this new cavity does not provide a zero transmission phase at its resonance frequency as can be clearly seen from Fig. 13(b).

The above mentioned method to manipulate scattering is applicable for thin sheets with arbitrary grid impedances. However, if the object impedance is known, the scattering can be suppressed by cascading the object and one metasurface (with the opposite grid impedance). In this way, the object plays the role of the second metasurface in an invisible cavity and the total scattering from this system becomes zero.

Another interesting application of invisible cavities is creating transparent waveguides. While the structure is invisible under normal incidence, it can concurrently operate as a waveguide. Indeed, it is analogous to a parallel-plate waveguide. We studied the guided modes which can propagate within the cavity parallel to its plane and calculated the corresponding attenuation constants. At a frequency $f = 11.2$ GHz, at which the transmission coefficient is unity for normal incidence (assuming $X_{e1} = X_{e2} = 38.08 \, \Omega, d = \lambda_{op}/2$), it is simultaneously possible to excite two different modes at the input port of the waveguide as shown in Fig. 14. These two modes are transverse electric mode TE (whose longitudinal component of electric field is zero) and the transverse magnetic mode TM (whose longitudinal component of magnetic field is zero), respectively. For both modes, the energy is confined inside the cavity and attenuates outside in free space. However, the attenuation constant is quite different for the modes. The TE mode has a higher attenuation constant ($\alpha = 1150 \, \text{m}^{-1}$) compared to the TM mode ($\alpha = 472 \, \text{m}^{-1}$). Knowing the attenuation constants, we can find the distance $\delta$ away from the metasurface at
FIG. 11: Experimental setups used to demonstrate the properties of the proposed invisible cavities. The cavity is composed of unit cells (printed on a circuit board) of the capacitive and inductive metasurfaces placed into a WR-90 waveguide section. The subplots illustrate the three experiments performed: (a,b) schematic and photograph of the setup for measurements of transmission and reflection coefficients of the invisible cavity (insets show manufactured metasurface unit cells of the cavity); (c,d) schematic and photograph of the setup for studying the field distribution in the invisible cavity; (e,f) schematic and photograph of the setup for analyzing scattering suppression (Position A) and enhancement (Position B) from the object inserted as illustrated in (f).

which the field reaches $1/e$ of its magnitude at the metasurface boundary. For the TE mode $\delta \approx 0.87 \text{ mm}$, while $\delta \approx 2.12 \text{ mm}$ for the TM mode (less than 1/12 of the wavelength). It is worth noting that increasing the distance between the metasurfaces from $\lambda/2$ to $\lambda$ does not change the phase and attenuation constants.

**Emission enhancement and suppression using invisible cavities**

Above we considered illumination of the cavity externally by a plane-wave source. The total field inside the cavity was found to be smaller or larger than that of the incident wave. It means, due to reciprocity, that if a homogeneous current sheet (a source of plane waves) is positioned inside the cavity, then the field which it radiates outside the cavity will be smaller or larger than if it were located in free space (without cavity). Such behaviour is explained by poorer or better matching of the source to free space. We define the surface current density of the source positioned at $z = \delta_S$ as $\mathbf{J}_s = -2\mathbf{E}_i/\eta_0$, so that the electric field produced by it equals to $\mathbf{E}_i$ (the same amplitude as that of the incident waves in the previous sections). The position of the current sheet will also determine the magnitude of the radiated fields as shown in Fig. 15(a). Nevertheless, the locations of the current source $\delta_S$ for which the radiation is maximized (minimized) are identical with the locations of the maxima (minima) of the standing wave shown in Fig. 4(a).

Thus, the invisible cavities provide emission enhancement or suppression of radiation from a planar source (taking into account the 1D geometry of the considered problem) placed inside. It is interesting to analyze this effect for sources and cavities of finite sizes. We consider a two-dimensional scenario where a cavity is infinite only along the $y$-axis and has a limited height along the $x$-axis. A strip current source, infinite along the $y$-direction and with height of $\lambda_{op}/2$ along the $x$-direction, has surface current density $J_s(x) = 0.021\cos(2\pi x/\lambda_{op})$ A/m with current flowing along the $x$-direction. The radiated linear power density from the strip source was numerically calculated (see Methods) in the case when the source
was located in the field maximum inside the cavity (here, \(d = \lambda_{op}/2\) and \(X_{c1} = -X_{c2} = 38.08\ \Omega\) were chosen). The radiated linear power density is plotted in Fig. 15(b) versus the height of the cavity in the \(y\)-direction. The point of zero cavity height corresponds to the scenario when the strip source is in free space.

This result was used as normalization to calculate the radiated power gain due to the cavity. As is seen from the figure, the power gain linearly increases when the cavity height increases from 2\(\lambda_{op}\) to 4\(\lambda_{op}\), then reaches maximum for the height of 6\(\lambda_{op}\) (19.04), and converges to the constant value of 17.63 times for larger heights. Figure 16 depicts the linear power density distribution for the strip source in free space and inside the cavity of height 6\(\lambda_{op}\). Strong power enhancement is observed as a consequence of the Purcell effect [44].

**DISCUSSION**

In this work, we have proposed and designed one-dimensional cavities which are imperceivable (non-scattering) for an external observer and still provide strong field localization inside their volumes. The non-scattering regime occurs in a cavity consisting of two metasurfaces due to destructive interference of waves scattered by each metasurface. We have demonstrated that the requirements for metasurface grid impedances are drastically different when the cavity is empty or filled with a dielectric material. In the former case, the distance between metasurfaces must be equal to an integer of half-wavelengths, while their grid impedances must have opposite signs. The non-scattering mode excited in such a cavity is driven by the incident wave and resembles an ideal bound state in continuum of electromagnetic frequency spectrum. In contrast to known bound states in the continuum, the mode can stay localized in the cavity infinitely long, provided that the incident wave illuminates it.

Although the described theory analyzes a one-dimensional scenario of invisible cavities, it can be extended in a similar way to the two- and three-dimensional scenarios. In fact, with the proper modifications, the fabricated cavity was proved to be functional inside a rectangular waveguide, preserving all its unique properties. The proposed invisible cavities can find applications for scattering modulation and emission control. With the proper positioning of a thin planar object inside the cavity, one can increase or reduce electromagnetic scattering from it, with the potential applications in cloaking or sensing enhancement. Moreover, the proposed structures can be used to guide transverse electromagnetic waves along them, remaining invisible for normally incident radiation (invisible parallel plate waveguides). Finally, the cavity offers new possibilities for emission control of active sources located inside it.

**METHODS**

**A. Design of metasurfaces of a non-scattering cavity**

To reach the non-scattering regime, both the capacitive and inductive metasurfaces represented by their single rectangular unit-cells were numerically optimized to provide the grid impedance of \(\pm j31\ \Omega\). The capacitive metasurface unit cell comprised a rectangular copper patch of the length \(L_{cx} = 20.7\ \text{mm}\) and height \(b-W_{cy} = 9.7\ \text{mm}\) (here \(W_{cy} = 0.3\ \text{mm}\) is the gap width) printed over a 0.5-mm-thick Arlon AD250 substrate with the relative permittivity of 2.5 and the dielectric loss tangent 0.0018 [see illustration in Fig. 11(a)]. To ensure the grid impedance of \(1/(j\omega C) = -j31\ \Omega\) at \(f = f_1\), the width of the gap between adjacent patches in the corresponding periodic metasurface \(W_{cx}\) was parametrically tuned to 0.3 mm. The unit cell of the inductive metasurface was represented by three rectangular apertures of the dimensions \(L_{cx} \times L_{cy} = 4.6 \times 5\ \text{mm}^2\) each separated by copper strips of the widths \(W_{cx} = 3\ \text{mm}\) printed on a similar substrate. The latter value was chosen to provide the required value of the grid impedance of \(j\omega L = j31\ \Omega\). The parameters of \(W_{cx}\) for the capacitive unit cell and \(W_{cx}\) for the inductive one were determined separately by placing the corresponding structure into a waveguide sec-
FIG. 13: Simulated and measured magnitudes and phases of the transmission (left column) and reflection (right column) coefficients in the frequency range from 10 to 12 GHz. The additional capacitive metasurface with the grid impedance of around $-j31 \, \Omega$ is placed in the minimum of the electric field distribution of the designed cavity.

B. First experiment: Reflection and transmission coefficients from the cavity

In the waveguide setup, each of the two unit cells was surrounded by a continuous metalized area on both layers of the PCB with multiple vias at the periphery of the waveguide cross-section to ensure no current discontinuity on the waveguide walls. The protographs of the manufactured unit cells to be inserted before and after the waveguide section are shown in insets of Fig. 11(a). The assembled waveguide with the resonant section and the capacitive and inductive grid unit cells attached are shown in Fig. 11(b). The complex transmission and reflection coefficients were retrieved from S-parameters measured by a vector network analyzer (VNA) using additional calibration data corresponding to an empty waveguide and a metal plate connected to the waveguide instead of each metasurface unit cell. The experimental setup used for characterization of the reflection and transmission coefficients of the invisible cavity is schematically shown in Fig. 11(a). It is based on a uniform WG90 waveguide section fixed between two coax-to-waveguide transitions from Agilent X11644A WR-90 X-band kit connected to ports of a vector network analyzer Agilent E8362C. The length $L_1$ of the section was equal to 50 mm, which for the waveguide cross-section of $a \times b = 23 \times 10 \, \text{mm}^2$ resulted in the third Fabry-Perot resonance condition at $f_1 = 11.11 \, \text{GHz}$. In other words, $L_1$ equals to three half-wavelengths in the waveguide exactly at $f = f_1$, while the wavelength $\Lambda$ equals to $33.35 \, \text{mm}$. 

FIG. 14: An invisible cavity operates as a waveguide for transverse electric and magnetic waves which is completely transparent for normal-incidence illuminations at the operating frequency.
FIG. 15: (a) Total electric field across the invisible cavity with metasurface reactances $X_{e1} = -X_{e2} = 38.08 \, \Omega$ as a function of the position of the current sheet source placed inside the cavity $\delta_S$. (b) Radiated linear power density by a current strip inside a cavity as a function of the cavity height. The distance between the metasurfaces is equal to $d = \lambda_{op}/2$ and the strip source is placed in the maximum of the standing field.

C. Second experiment: Field amplitude across the cavity

In the second experiment, the same manufactured unit cells were employed, but the uniform section was replaced with another, three times longer calibrated measurement waveguide section P1-20 with the length $L_2 = 150$ mm. The latter had a longitudinal thin slot in the middle of the broad side of the waveguide and was equipped with a movable thin-wire probe to measure the electric field distribution with minimum insertion and reflection losses. The probe was connected to the digital oscilloscope Rohde & Schwarz HMO2022 through a diode detector. The waveguide was fed from one side by connecting the signal generator Rohde & Schwarz SMB100A, while loaded with a matched load from the other side. The generator produced a wave at the carrier frequency of 11.11 GHz modulated with a harmonic signal with the frequency of 100 kHz. The probe was mechanically moved to plot the electric field magnitude versus the longitudinal coordinate by checking the amplitude of the detected signal at the modulation frequency and applying the calibration characteristic of the detector. The scheme of the experiment is given in Fig. 11(c), and the photograph of the setup is shown in Fig. 11(d). Normalized to the field magnitude in the waveguide section without the cavity, the field can be determined quantitatively.

D. Third experiment: Suppression and enhancement of scattering from a planar object inside the cavity

In the third experiment, we studied scattering from a planar object (with given grid impedance) when it is placed either at the minimum or at the maximum of the electric field distribution inside the cavity. From the theoretical considerations, one may expect almost total transmission and negligible reflection of the incident wave when the object having the grid impedance of $-j31 \, \Omega$ is positioned in the plane of minimum electric field [Position A indicated in Fig. 11(e)]. However, as the same object is located in the plane of the maximum of the electric field distribution [i.e. in the lobe of a standing wave marked as Position B in Fig. 11(e)], one can expect total reflection. For the experiment schematically depicted in Fig. 11(e), we used the same capacitive and inductive grids as shown in Fig. 11(a) to form the cavity. The planar object was also produced as a circuit board with a carefully designed copper shape printed on one side of Arlon AD250 0.5-mm-thick substrate. However, to place the object into an arbitrary position in the previously used uniform section, the dimensions of the circuit board were chosen almost equal to those of the waveguide cross-section. At the same time, the metal pattern was designed to provide the grid impedance of $(1 - j31)$ $\Omega$ at $f_1 = 11.11$ GHz with no electric contact to the waveguide walls required. The photograph of the object and the scheme of its insertion into the waveguide are illustrated on the insets in Fig. 11(f). The shape was a rectangular patch of the dimensions $L_{ix} \times L_{iy} = 14.2 \times 7.6$ mm$^2$, in which two symmetrical notches of the width of 0.2 mm and the length of 4.1 mm were made to reach the desired grid capacitance. To fix the object inside the waveguide, thin foam holders were employed.

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FIG. 16: Comparison of the linear power density produced by a half-wavelength dipole source (a) located in free-space and (b) located in a field maximum inside the cavity of height $6\lambda_{op}$ along the $y$-direction. Here, the metasurface grid reactances were chosen $X_{e1} = X_{e2} = 38.08\, \Omega$, and $d = \lambda_{op}/2$. The surface current density in the source follows a cosine distribution $J_S(x) = 0.021\cos\left[\frac{2\pi x}{\lambda_{op}}\right] \, A/m$. 

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