Compressive performance of circular hollow section brace with eccentrically installed rib-stiffened splice plates

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Abstract
Hollow section braces are often adopted in steel frame structures because of their superior performance. To connect tubular members with framing elements, field-bolted connections of the slotted-in single splice plate and gusset plate are majorly used because of their ease of construction. However, the eccentricity between the splice and gusset plate axes reduces the compressive strength of the brace and induces local connection failures under seismic excitations. To improve the compressive strength, this paper proposes a new connection configuration for circular hollow section braces in which cruciform rib-stiffened splice plates are eccentrically installed such that the gusset plate axis coincides with the brace axis. The concept of the proposed connection was first described with theoretical considerations. To demonstrate the effectiveness of the proposed connections, compressive loading tests and subsequent cyclic loading tests were conducted for four test specimens. Furthermore, finite element analyses with long-brace models were performed to demonstrate the good performance of the proposed connection for a wide range of slenderness ratios.

Keywords Hollow section brace · Compressive performance · Splice and gusset plates · Eccentric installation · Loading test · Finite element analysis

1 Introduction

Bracing is one of the most important elements in the design of steel frame structures for increasing the lateral resistance. Among the possible sections, hollow sections are often adopted as bracing because they exhibit superior performance under compressive loads. To connect tubular members with framing elements, field-bolted connections of the slotted-in single splice plate and gusset plate are often used because of their ease of construction. However, the eccentricity between the splice and gusset plate axes reduces the compressive strength of the brace. Furthermore, local connection failure may occur, as shown in Fig. 1 (Tanaka and Tagawa 2016).

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In the 2011 Great East Japan Earthquake, such damages occurred in some steel buildings (Asada et al. 2018). The local connection failure, which is sometimes referred to as sway mode connection failure, leads to an insufficient performance of the tubular brace. Thus, extensive theoretical, numerical, and experimental studies have revealed the eccentricity effect and local failure in single splice and gusset plate connections (Kitipornchai et al. 1993; Tada and Kasahara 2002; Khoo et al. 2010; Fang et al. 2015; Tshunza et al. 2015; Asada et al. 2018). The inelastic behaviors of gusset plates have also been intensively investigated (Yam and Cheng 2002; Lehman et al. 2008; Martinez et al. 2008; Nascimbene et al. 2012). To suppress the problems induced by such eccentricity, slotted-in double splice plates or dual gusset plates can be used. However, the attachment of these two plates is somewhat burdensome. Previous studies have proposed the application of cast steel connectors for circular hollow section brace ends (Oliveira et al. 2008; Iwashita et al. 2012). In that bracing, steel casting and fork-end-shaped connectors are welded to both ends. Laboratory tests have revealed the good performance of bracing with cast steel connectors under cyclic loads, and some finite element analyses (FEA) revealed detailed connection behaviors, including weld portions. For the buckling-restrained braced frames, a previous study (Chou et al. 2012) presented dual-gusset-plate connections in which a steel core plate was inserted into the gusset plates. Thus, this type of connection can eliminate the need for splice plates and reduce the connection size. Welded end connections were also investigated to simplify the brace end connections (Lin et al. 2012). Moreover, some mathematical considerations have been made to avoid connection failures in buckling-restrained braces (Takeuchi et al. 2014).

To improve the compressive strength of the hollow section braces, an earlier study (Tagawa et al. 2018) proposed the use of eccentrically installed splice plates such that the gusset plate axis coincides with the brace axis. In that study, four compressive loading tests revealed that the eccentric installation of splice plates was effective for improving the compressive strength. To avoid connection failure, thick plates were provided for the splice or gusset plates in the test specimens. The test results indicated that plastic hinge formation in the gusset plates under buckling deformation was essential when the proposed concept was adopted for the connection design.

This study investigates the compressive performance of circular hollow section braces in which slotted-in rib-stiffened splice plates are eccentrically installed such that the gusset plate axis coincides with the brace axis. The fundamental concept of the proposed connection is first described with theoretical considerations in this paper.
Afterward, the results of four compressive loading tests are presented to compare the buckling behavior of the braces with different joint configurations. Furthermore, FEA with long-brace models were used in this study to demonstrate the effectiveness of the proposed connection for a wide range of slenderness ratios. Finally, cyclic loading test results are presented to demonstrate the sufficient brace performance under repeated loads.

2 Concept of the proposed connection

A previous study (Tagawa et al. 2018) proposed a concept for the compressive strength improvement of hollow section braces using eccentrically installed single-sided splice plates such that the gusset plate axis coincides with the brace axis. In that study, thick splice plates, as shown in Fig. 2(a), were used to avoid connection failure. A certain strength increase attributed to the eccentric installation of the splice plate was observed by comparison with the non-eccentric installation model. Because a rib-plate is effective in increasing the flexural rigidity of a splice plate, a subsequent study (Tagawa et al. 2019) examined the use of a rib-plate welded on one side of the splice plate, as shown in Fig. 2(b). In this configuration, a rib-stiffened splice plate was T-shaped as a whole. These two types (Figs. 2a, b) obey the concept that the gusset plate axis coincides with the brace axis. However, the discrepancy between the brace-gusset plate axis (black dot-dash line) and the splice plate axis (red dot-dash line) remained, possibly reducing the brace compressive performance.

To match the splice plate axis with the brace-gusset plate axis, this study proposes the application of a cruciform rib-stiffened splice plate, as shown in Fig. 2c. In this configuration, in addition to rib-plate A, rib-plate B was welded on the opposite side of the splice plate. To insert rib-plate B, it is necessary to cut a groove in the gusset plate.

Figure 3 shows a cross-section of the proposed connection portion with some dimension notations. The heights of the lower and upper rib-plates are denoted as $h_1$ and $h_2$, respectively. It is herein assumed that the rib-plate and gusset plate thicknesses, $t_r$ and $t_g$, are the same as that of the splice plate, $t$, and the gusset plate width $w$ is the same as the vertical height, that is, $w = h_1 + h_2 + t$. The calculation of the cross-sectional centroid of the slotted-in plates, that is, the yellow-colored area, gives the distance $D_c$ from the bottom to the centroid as follows:

$$D_c = \frac{3h_1^2 + 4h_1h_2 + h_2^2 + 3th_1 + 3th_2 + t^2}{4h_1 + 4h_2 + 2t}. \quad (1)$$

Fig. 2 Suppression methods of local connection failure: a Use of thick splice plate (Tagawa et al. 2018), b Use of T-shaped rib-stiffened splice plate (Tagawa et al. 2019), c Use of cruciform rib-stiffened splice plate (proposal of this study)
The analysis of Eq. (1) as presented in Appendix A proved the following relationship:

\[ h_1 - \frac{t}{2} \leq D_c \leq h_1 \]  

(2)

This relationship indicates that the cross-sectional centroid of the slotted-in plates is definitely located within the gusset plate thickness, as indicated by the blue lines in Fig. 3. Consequently, the member axes of the brace, gusset, and splice plates approximately coincide with each other, as shown by the blue dot-dash line in Fig. 2c. This is a fundamental concept of the proposed connection in which local connection failure and compressive strength reduction in the hollow section brace can be avoided.

Figure 4 shows the comparison of the slotted-in splice plate-to-gusset plate bolted connections. The connection type shown in Fig. 4a is often used in practice for tubular braces with a large cross-section. In this connection, cruciform rib-stiffened splice plates and cruciform rib-stiffened gusset plates are connected using eight splice plates colored in red. No discrepancy in the member axes between the brace, gusset, and splice plates can avoid a reduction in the compressive strength. However, the number of connection elements, including additional splice plates and high-strength bolts, is much more than that of the proposed type shown in Fig. 4b. One benefit of the proposed connection is the simplification of the construction process.

3 Compressive loading tests

3.1 Test specimens

Four test specimens were fabricated. The configurations of test specimens C-1 to C-3 and C-4 are shown in Fig. 5a, b, respectively. A circular hollow steel section of 101.6 × 3.2 (diameter × thickness) was adopted for the braces. For C-1 to C-3, 9-mm-thick splice plates were welded with 9-mm-thick rib-plates on both sides to avoid connection failure. Splice and rib-plates were then slotted into the tubular brace and welded. A groove was made in each 9-mm-thick gusset plate to insert the rib-plate. The splice
Fig. 4  Slotted-in splice plate-to-gusset plate bolted connections: a existing type, b proposed type

Fig. 5  Configuration of test specimens a specimens C-1 to C-3, b specimen C-4
and gusset plates were mutually connected using six high-strength bolts (M20). As shown in Fig. 6, the splice plates for C-1 were slotted-in with no eccentricity from the brace axis. The discrepancy between the brace and gusset plate axes reduces the compressive strength of C-1. The splice plates for C-2 and C-3 were eccentrically installed such that the gusset plate axis coincided with the brace axis, as proposed in this study. The position of the right-side splice plates was different between specimens C-2 and C-3, as shown in Fig. 6. From the point of view of the construction, specimen C-2 is preferable because the brace member can be attached directly to the frame from one side. Meanwhile, specimen C-3 was tested for comparison. In specimen C-4 (Fig. 5b), the gusset plates were stiffened with rib-plates on both sides and connected to the cruciform rib-stiffened splice plates using eight additional 6-mm-thick splice plates with 16 high-strength bolts (M16). The design practices of steel buildings often adopt this type of C-4 connection with no eccentricity between the brace and gusset plate axes, as shown in Fig. 4a. Thus, among the test specimens, C-4 is expected to have the largest compressive strength. Table 1 summarizes the characteristics of the test specimens. The material properties of the members are listed in Table 2.

![Fig. 6 Connection details of specimens C-1 to C-3](image)

Table 1 Characteristics of test specimens

| Specimen | Installation of splice plates                  | Compressive strength* |
|----------|------------------------------------------------|-----------------------|
| C-1      | With no eccentricity from brace axis          | 166 kN                |
| C-2      | With symmetric eccentricity                   | 227 kN                |
| C-3      | With anti-symmetric eccentricity              | 223 kN                |
| C-4      | Using additional eight splice plates           | 227 kN                |

*Values obtained in the compressive loading tests
3.2 Loading conditions and measurements

Figure 7 shows a schematic of the test setup. The left end of the test specimen was fixed to a fixed support and its right end was fixed to a roller support which was expected to suppress the brace end rotation and twisting. Figure 8 shows the photographs of the connections for specimens C-2 and C-4. The lateral distance between the left and right fixed boundaries was 2360 mm, and the lateral displacement was measured using two displacement sensors d1 and d2. The average of these two sensor values is regarded as the axial displacement $u$. The brace compression side indicates a positive direction of the value of $u$.

| Component          | Steel grade | Yield stress (N/mm$^2$) | Ultimate stress (N/mm$^2$) | Elongation (%) |
|--------------------|-------------|-------------------------|----------------------------|---------------|
| 9 mm–thick steel plate | SN400B      | 334                     | 477                        | 23            |
| Steel tube 101.6 × 3.2 | STK400      | 340                     | 460                        | 34            |

Fig. 7 Test setup and measuring system (specimen C-3)

Fig. 8 Photographs of the connection details: a specimen C-2, b specimen C-4
The out-of-plane displacement of the brace center \( d_3 \) was measured using a displacement sensor \( d_3 \). The direction of the arrow of \( d_3 \), as depicted in Fig. 7, indicates the positive direction of the value of \( d_3 \).

Subsequently, strain gauges were mounted on both sides and designated as A and B, as shown in Figs. 5 and 7. Gauges 1 and 7 were located at the gusset plate portions, where plastic hinges were formed under buckling. Gauges 2 and 6 were located at the edges of the rib-plates. Gauges 3 and 5 were located at the tube ends, and gauges 4A and 4B were located at the middle of the brace, where a plastic hinge was formed under buckling.

A compressive axial load \( P \) was then applied incrementally from the right side of the test specimen. The loading was terminated when the axial displacement reached 11.5 mm, which corresponds to an average compressive strain of 0.5% (= 11.5/2360). As presented in Sect. 5, the loading was continuously applied to examine the cyclic behavior.

3.3 Test results

Figure 9 shows the specimen deformation at the final stage of compressive loading, in which an overall buckling with no local connection failure was observed for all the specimens. Figure 10a shows the relationship between the compressive axial load and compressive axial displacement, that is, \( P-u \) relationship. It is noteworthy that the curves of C-2, C-3, and C-4 were almost identical. Figure 10b presents the relationship between the compressive axial load and out-of-plane displacement of the brace center, that is, \( P-d_3 \) relationship. The value of \( d_3 \) for C-1 increased rapidly from the beginning, and it was induced by the eccentricity between the brace and gusset plate axes. The values of \( d_3 \) for C-2, C-3, and C-4 also began to increase from the beginning because of slightly eccentric loading owing

![Fig. 9 Overall bucking configurations at the final stage of compressive loading: a C-1, b C-2, c C-3, d C-4](image)
However, these rates of increase were much lower than that for C-1, revealing the effectiveness of the proposed connection concept for both C-2 and C-3.

The maximum loads, regarded as the compressive strengths, are listed in Table 1. The compressive strengths of C-2 and C-3 were approximately 35% higher than that of C-1. The reason for the almost equal strengths for C-2 and C-3 is that the cross-sectional centroids of their rib-stiffened splice plates are located within the gusset plate thickness. It is noteworthy that the compressive strengths of C-2 and C-3 were identical to that of C-4. In conclusion, the proposed connection performed well at the same level as the completely non-eccentric connection.

For further detailed investigations, the variations in strains for some portions are presented in Fig. 11. A positive value corresponds to a tensile strain, while negative is compressive. The upper diagrams present strains at the plastic hinge locations, that is, Nos.
1 and 7 for the gusset plates and No. 4 for the brace center. Because strain gauges 1 and 7 for C-4 were mounted close to the rib-plate, stress concentration was observed in the earlier stage. The middle diagrams present strains of the rib-plates, that is, Nos. 2 and 6. The lower diagrams present strains at both ends of the steel tube, that is, Nos. 3 and 5. For C-1, the compressive strains of the rib-plates (2A and 6A) and tube ends (3A and 5A) were much larger than their B-side values because brace bending deformation started to appear from the initial stage. In contrast, for C-2 and C-3, the compressive strains of the tube ends (3A, 3B, 5A and 5B) were similar to those of C-4 until buckling occurred, revealing the effectiveness of the proposed connection concept. Note that the discrepancy of the strains of the rib-plates between the A and B sides for C-2 and C-3 is partly attributed to the difference in the rib-plate height.

4 Numerical study

Finite element analysis was performed to further investigate the proposed connections under compressive loads. In addition to the models of the test specimens, long models were used to examine the brace performance within a wide range of slenderness ratios. This study used ANSYS 15.0 simulation software (ANSYS 2014). The Young’s modulus was $2.05 \times 10^5$ N/mm$^2$, and the Poisson’s ratio was 0.3. A bilinear stress–strain relationship with a hardening ratio of 0.01 was adopted for the steel material. A yielding condition obeying the von Mises law was also given, in which the yield stress of the elements was assigned the values listed in Table 2. In addition to the material nonlinearity, geometrical nonlinearity was also considered in the FEA.

4.1 Analysis of test specimen models

Figure 12 presents the finite element models with the same configuration as the test specimens. All the members used four-node shell elements with six degrees of freedom at each node (SHELL181). For models C-1 to C-3, under the assumption of non-slippage occurrence in the slip-critical bolted connection, the splice plates and gusset plates were modeled by separated shell elements, and the 9-mm eccentricity between these plates was faithfully represented by shell elements with a 9-mm thickness between them. For C-4, eight additional splice plates were not modeled. Instead, the gusset and slotted-in splice plates were integrally modeled as continuous plates.

Figure 12a shows the common boundary conditions for all models. All the nodes of the left and right sides of the model were fixed except for the $x$-directional displacement of the left-side nodes in which the compressive axial displacement of 11.5 mm was applied. In this analysis, the displacement control procedure was adopted rather than the load control one. The total length of the model between the two boundaries was 2.36 m, similar to the test setup presented in Fig. 7. The tubular members had an initial deflection in the shape of a half-sine curve with a central deflection of one-200th total length. One reason for the adoption of a rather large initial deflection was the eccentric loading observed in the test results, as explained in Sect. 3.3. A previous study (Wijesundara et al. 2009) adopted the one-350th total length as an initial imperfection at a midspan of the brace model.

Figure 13 shows the relationship between the compressive axial load and compressive axial displacement, that is, $P-u$ relationship. Figure 14 shows the relationship between the compressive axial load and out-of-plane displacement of the brace center, that is, $P-d_3$. 

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Fig. 12  Analysis model of test specimens: a C-1, b C-2, c C-3, d C-4

Fig. 13  Comparison of $P-u$ relationship: a C-1, b C-2, c C-3, d C-4
relationship. The good agreement between the test and FEA curves indicates the validity of the analysis models. Figure 15 shows the buckling deformation at the final stage of analysis for C-2, in which the von Mises stress contour was displayed. The deformed configuration was similar to that of the test specimen, as shown in Fig. 9b. The appearance of the three plastic hinges corresponds to the strain variations of gauges 1, 4, and 7, as shown in Fig. 11.

4.2 Analysis of long brace models

The slenderness ratio, defined as $\lambda = L/i$, was 68 for the test specimen model, where $L$ is the lateral length between the left and right boundaries of the model, and $i$ denotes the radius of gyration of the tubular section. In this definition, pin connections for both gusset plate ends were assumed considering low flexural rigidity of those portions. To examine the influence of the slenderness ratio on the effectiveness of the proposed connection, the models with total lengths of 3 m ($\lambda = 86$), 4 m ($\lambda = 115$), 5 m ($\lambda = 144$), and 6 m ($\lambda = 172$) were analyzed in addition to the original test specimen model with a total length of 2.36 m ($\lambda = 68$). The configuration of each length model was the same, except for the tube length. The tubular members had an initial deflection in the shape of a half-sine curve with a central deflection of one-1000th total length, which is often considered in the design of the compression members in steel building structures. It is noteworthy that a previous study (Uriz et al. 2008) investigated the selection of initial deflection, i.e., initial camber, considered for brace models in the finite element analysis. In that study, an initial camber displacement of 0.05–0.1% of the brace length was recommended at a midspan of the tubular brace model.
Figure 16 shows the FEA results regarding the relationship between compressive axial load and displacement, that is, $P$-$u$ relationship, for long brace models. The strength of C-1 was the smallest in all cases, which was induced by the eccentricity between the brace and gusset plate axes. The stiffness and strength of C-4 were the largest in all cases. The largest stiffness is attributed to the length of the tubular member for C-4 being slightly shorter than the others because its connection portion requires more length. The strengths of C-2 and C-3 were almost the same as that of C-4 for the model with smaller slenderness ratios ($\lambda = 86, 115$). It was observed from the $P$-$u$ curves of C-2 and C-3 for larger slenderness ratios ($\lambda = 144, 172$) that their curves began to separate from that of C-4 just below the maximum load points. As a result, the strengths of C-2 and C-3 were slightly smaller than that of C-4.

Figure 17 shows the relationship between the compressive strength and slenderness ratio for all the models. The compressive strength $P_c$, obtained as the maximum axial load of the FEA result, was normalized by the axial yield strength of the tubular section, $P_y = 336$ kN. The solid curve represents the compressive strength stipulated in Design standard for steel structures (2017) of AIJ, expressed as

$$\frac{P_c}{P_y} = \begin{cases} 
 1 - 0.4 \left( \frac{\Lambda}{\lambda} \right)^2 & \text{for } \lambda < \Lambda \\
 0.6 \left( \frac{\Lambda}{\lambda} \right)^2 & \text{for } \lambda \geq \Lambda 
\end{cases}$$

where $\Lambda$ denotes the limiting slenderness ratio dividing the inelastic and elastic buckling corresponding to the value of $P_c = 0.6P_y$. The former curve is parabolic, and the latter curve represents the Euler buckling strength. The dashed curve represents the allowable
compressive strength $P_{ca}$ for the temporary loads (snow, wind, or earthquake) presented in Design standard for steel structures (2017) of AIJ as

$$\frac{P_{ca}}{P_y} = \frac{P_c}{P_y} \frac{1.5}{\nu},$$

where $\nu$ denotes the safety factor of the compression members for the sustained loads as follows:

$$\nu = \begin{cases} \frac{3}{2} + \frac{2}{3} \left( \frac{\lambda}{\Lambda} \right)^2 & \text{for } \lambda < \Lambda \\ 2.17 & \text{for } \lambda \geq \Lambda \end{cases}.$$  

Note that the safety factor for the sustained loads is 1.5 times that for the temporary loads.

It can be observed from Fig. 17 that the compressive strengths of C-2 to C-4 match well with the solid curve of Eq. (3). The values of C-2 and C-3 were almost the same and slightly smaller than that of C-4. In contrast, the compressive strengths of C-1 were smaller than those of the solid curve, except for the largest slenderness ratio model. For the allowable compressive strength, the two lowest slenderness ratio models of C-1 had values smaller than or equal to the dashed curve. These results prove the efficacy of the proposed connection for braces with a wide range of slenderness ratios.

### 5 Cyclic performance

The previous sections revealed the performance of the proposed connection in improving the compressive strength of the braces. Furthermore, their cyclic performance is discussed in this section because plastic deformation capacity is important when bracing is used as a seismic resisting element. Cyclic loading tests for test specimens C-1 to C-4 were conducted continuously following the compressive loading tests presented in Sect. 3.
Figure 18 shows the loading protocol, and Fig. 19 presents the $P-u$ relationship, in which the red lines correspond to the compressive loading tests. The cyclic hysteresis curves of C-1 to C-3 were almost the same as those of C-4 which was expected to exhibit the best performance. These test results reveal that the proposed connection configuration has no undesirable influence on the cyclic performance of the brace. Figure 20 shows the deformation of C-3 at the final loading stage. It is observed that the brace center portion, rather than the connection area, suffered damage from brittle failure. The other specimens exhibited a similar final behavior.
6 Conclusion

To improve the compressive strength of circular hollow section braces, this study proposed brace end connections in which rib-stiffened splice plates are installed eccentrically such that the gusset plate axis coincides with the brace axis. Rib-plates were welded on both sides of the splice plate to avoid connection failure. A theoretical consideration of the centroid position of the cruciform rib-stiffened splice plates revealed that the member axes of the brace, gusset, and splice plates approximately coincided with each other. These characteristics enable the improvement of the compressive strength to the same level as that of the completely non-eccentric connection.

Compressive loading tests were conducted on four test specimens to assess the effectiveness of the proposed connections. Furthermore, finite element analysis was performed for long-brace models with a wide range of slenderness ratios. The results of the loading tests and numerical analyses revealed the compressive behavior characteristics and effectiveness of the proposed concept. Cyclic loading tests also revealed that the proposed connection configuration had no undesirable influence on the cyclic performance of the brace.

Fig. 20 Deformations of specimen C-3 at the final cyclic loading stage: a overall view (u = 34 mm under compression), b central portion (u = 34 mm under compression), c central portion (u = -50 mm after fracture)
Appendix A: Location of the cross-sectional centroid of the slotted-in plates

This appendix demonstrates that the cross-sectional centroid of the slotted-in plates is located within the gusset plates thickness, as described in Sect. 2. The discussion refers Fig. 21 in which, in addition to Fig. 3, the slotted-in plates are separated into two areas (S1 and S2), and the y-axis is defined in red on the cross section. As stated in Sect. 2, it is assumed that the rib-plate and gusset plate thicknesses are the same as that of the splice plate, and the gusset plate width is the same as the vertical height. One convenient approach from the physics perspective is described first, and then its mathematical proof is presented.

Physics perspective

As depicted in Fig. 21, the yellow area filled with diagonal lines is denoted by S1, which covers the cross sections of the two rib-plates and the central part of the splice plate. The yellow area filled with small grids is denoted by S2, which is the cross section of the splice plate excluding the central part. The centroids of S1 and S2 are located at their center points, that is, at $h_1 - t/2$ and $h_1 + t/2$ in the y-axis, respectively. The centroid of the entire yellow area is indeed located between the centroids of S1 and S2. Furthermore, because the area of S1 is greater than that of S2, the centroid of the entire yellow area is closer to the centroid of S1, that is, between $h_1 - t/2$ and $h_1$, as presented by Eq. (2).

Mathematical proof

The distance $D_c$ from the bottom to the centroid of the slotted-in plates, that is, the yellow area, is obtained using the conventional method as follows:

$$D_c = \frac{\sum (A_i \cdot Y_i)}{\sum A_i} = \frac{3h_1^2 + 4h_1h_2 + h_2^2 + 3th_1 + 3th_2 + t^2}{4h_1 + 4h_2 + 2t}.$$  (6)

Here, $A_i$ denotes the area of the $i$th divided portion, and $Y_i$ denotes the y-axis coordinate of its centroid.
Considering that the cross-sectional centroid of the gusset plate is located at \( h_1 - t/2 \) in the \( y \)-axis, the following relationship, Eq. (7), represents the necessary condition for the cross-sectional centroid of the slotted-in plates to be located within the gusset plate thickness.

\[
h_1 - \frac{t}{2} \leq \frac{3h_1^2 + 4h_1h_2 + h_2^2 + 3th_1 + 3th_2 + t^2}{4h_1 + 4h_2 + 2t} \leq h_1
\]  

(7)

The right inequality of Eq. (7) can be transformed as follows:

\[
h_1 - \frac{3h_1^2 + 4h_1h_2 + h_2^2 + 3th_1 + 3th_2 + t^2}{4h_1 + 4h_2 + 2t} \geq 0
\]  

(8)

\[
\rightarrow \frac{h_1^2 - th_1 - 3th_2 - h_2^2 - t^2}{4h_1 + 4h_2 + 2t} \geq 0
\]  

(9)

\[
\rightarrow \frac{(h_1 - h_2)^2}{5h_1 + 3h_2} \geq 0
\]  

(10)

where the final transformation considers the relationship of \( h_1 - h_2 = 2t \). Eq. (10) indicates that the inequality is invariably valid.

The left inequality of Eq. (7) can be transformed as follows:

\[
\frac{3h_1^2 + 4h_1h_2 + h_2^2 + 3th_1 + 3th_2 + t^2}{4h_1 + 4h_2 + 2t} - \frac{(h_1 - t/2)}{2} \geq 0
\]  

(11)

\[
\rightarrow \frac{-h_1^2 + 3th_1 + 5th_2 + h_2^2 + 2t^2}{4h_1 + 4h_2 + 2t} \geq 0
\]  

(12)

\[
\rightarrow \frac{(h_1 - h_2)(h_1 + h_2)}{5h_1 + 3h_2} \geq 0
\]  

(13)

Considering the relation of \( h_1 > h_2 \), Eq. (13) indicates that the inequality is invariably valid.

In conclusion, the relationship as expressed in Eq. (7) is certainly true, and thus, the cross-sectional centroid of the slotted-in plates is definitely located within the gusset plate thickness.

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