D–term Inflation at the TeV Scale and Large Internal Dimensions

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ABSTRACT

We show that D–term inflation at the TeV scale is possible in the presence of large internal dimensions. This requires very small couplings and masses which arise from the large size of the compact dimensions. We show that acceptable number of e-folds, magnitude and spectrum of density perturbations $\delta \rho/\rho$ and $\eta$ can be obtained in this scenario at the price of a reheating temperature which is too high. Demanding an acceptable $T_R$ results in very small density perturbations.

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1. Introduction

The existence of large (e.g. from micron to milimeter) internal dimensions is a very exciting possibility that was raised recently[1,2]. This requires that the higher dimensional gravitational scale be around the TeV scale. In particular it means there must be string theories with $M_s \sim TeV$. In this scenario, our world lives on a brane and gravity propagates only in the bulk. The weakness of gravity with respect to the gauge interactions is a result of the large internal dimensions. This scenario which is drastically different than previously considered cases requires a complete revision of cosmology above the nucleosynthesis scale. In particular, inflation which is so successful in solving the horizon, flatness, monopole etc. problems of the Big Bang cosmology must be realized in a novel way in this framework.

The possibility of inflation around the TeV scale in the presence of large internal dimensions was first examined in [3,4]. There it was shown that chaotic inflation at the TeV scale is almost impossible whereas hybrid inflation around the TeV scale requires very small couplings and/or masses. For example, hybrid inflation with the potential[5]

$$V(\phi, \sigma) = g^2 (M^2 - \sigma^2)^2 + m^2 \phi^2 + \lambda^2 \phi^2 \sigma^2$$ (1)

gives the correct density perturbations for $g \sim \lambda \sim 1$ and $M \sim 1 TeV$ only if the mass of the inflaton is very small, i.e. $m^2 \sim 10^{-10} eV$. This mass is six orders of magnitude smaller than the Hubble constant $H \sim 10^{-4} eV$ and is in general difficult to obtain. A possible inflationary scenario was obtained in [6] by asymmetric inflation in the bulk and on the brane which occurs before the internal dimensions are stabilized. Recent work on related subjects appeared in [7-14].

In this letter, we study D–term inflation around the TeV scale in the presence of large internal dimensions. We assume that the internal dimensions are stabilized before inflation. In fact this assumption is crucial because the large dimensions
result in very small couplings and masses which make this scenario possible. We show that we can obtain the acceptable number of e-folds and magnitude and spectrum of density fluctuations quite naturally. We comment on the prospects for obtaining an acceptable reheating temperature.

2. D-term Inflation at the TeV Scale

We now consider D-term inflation around the TeV scale in the presence of large internal dimensions. For concreteness we study the case with only two large dimensions but our results can be easily generalized to the cases with more than two large dimensions. Also since models with large internal dimensions are naturally realized as type I string orbifolds or type IIB orientifolds[15,16], we consider D-term inflation in these string models[19]. D-term inflation is a specific realization of hybrid inflation in which the vacuum energy during inflation is dominated by a D-term rather than an F-term[17]. For example, consider a type I string orbifold with an anomalous D-term contribution to the potential

$$V_D = g^2 |\sigma|^2 + M^2 |\sigma|^2 + M^2$$

and a tree level superpotential

$$W = \lambda \phi \sigma^2$$

Here \(\sigma\) is the trigger field which carries \(-1\) charge under the anomalous \(U(1)_A\) symmetry whereas \(\phi\) is the inflaton which is neutral. The gauge coupling \(g\) and the Yukawa coupling \(\lambda\) are not fixed; in particular they can be very small. As we will see there is a natural reason for the smallness of these couplings. \(M\) is a twisted modulus field of the type I orbifold model and its VEV gives the scale of the anomalous D-term. In particular this VEV can be larger than the string scale which is about 50 TeV[18] for two large dimensions. The superpotential gives rise
to an F–term contribution in the potential

\[ V_F = \lambda^2 \phi^2 \sigma^2 + \lambda^2 \sigma^4 \]  

(3)

We see that \( V = V_F + V_D \) contains two of the three terms required for hybrid inflation. Both of the above anomalous D–term and the superpotential arise generically in type I orbifold models.

The above potential \( V = V_F + V_D \) has two minima; one at \( \sigma = M \) and \( \phi = 0 \) and the other at \( \sigma = 0 \) and \( \phi \) free. Inflation occurs for the initial conditions of large \( \phi \) and small \( \sigma \). Then the vacuum energy is dominated by the D–term and is given by

\[ V_0 \sim H^2 M^2_p \sim g^2 M^4 \]  

(4)

The masses for the two scalars are given by

\[ m^2_{\sigma} = \lambda^2 \phi^2 - 2g^2 M^2 \]  

(5)

and

\[ m^2_{\phi} = \lambda^2 \sigma^2 \]  

(6)

We see that \( m^2_{\sigma} \) is positive for \( \phi > \sqrt{2gM/\lambda} = \phi_{cr} \). For \( \phi > \phi_{cr} \) we find \( m^2_{\sigma} > H \). As a result, \( \sigma \) settles to its minimum at \( \sigma = 0 \) very fast. At the minimum the tree level inflaton mass vanishes, \( m_{\phi} = 0 \). However, the third term required for hybrid inflation which is the mass term for \( \phi \) arises from the one–loop contribution to the potential due to SUSY breaking during inflation.

\[ V_1 = g^2 M^4 \left( 1 + \frac{g^2}{16\pi^2 \log \frac{\lambda^2 \phi^2}{\mu^2}} \right) \]  

(7)

Here \( \mu \) is the renormalization scale which does not affect the physics. \( V_1 \) leads to a mass for \( \phi \) given by

\[ m^2_{\phi} = \frac{g^4 M^4}{16\pi^2 \phi^2} \]  

(8)

Now that we have all the terms required for D–term inflation we need to fix
the parameters of the model, e.g. $g, \lambda$ and $M$. The first requirement for inflation is the slow–roll condition

$$m_{\phi}^2 < H^2 \sim \frac{g^2 M^4}{M_P^2} \quad (9)$$

Since $M \ll M_P$ in this TeV scale scenario one needs an extremely small inflaton mass for inflation. From the form of the inflaton mass in eq. (8) we see that this is possible only if the gauge coupling is extremely small. This is not uncommon in models with very large internal dimensions. For example consider a gauge theory in six dimensions with a large internal torus. Then the relation between the four and six dimensional gauge couplings is

$$g_4^2 = \frac{g_6^2}{R^2 M_s^2} \quad (10)$$

The same holds for the Yukawa coupling $\lambda$. Thus we see that for a large enough compact two torus the four dimensional gauge and Yukawa couplings will be extremely small.

For example consider a model in which there are two D5 branes overlapping over three dimensions. The type I string theory is compactified over four small dimensions of size $M_s^{-1}$ and two large dimensions given by

$$R^2 \sim \frac{M_P^2}{M_s^2} \sim 2 \times 10^{17} \text{ GeV}^{-2} \quad (11)$$

The three dimensions common to both branes are noncompact and describe the world. We assume that one D5 brane is wrapped around a large two torus whereas the other one is wrapped around a string size torus. The Standard Model degrees of freedom come from the D5 brane wrapped over the small two torus with gauge couplings of $O(1)$. Now if $\sigma$ lives on the D5 brane wrapped around the large dimensions and also the anomalous $U(1)_A$ comes from this brane, then the four dimensional gauge coupling, i.e. $g^2$ and will be suppressed by the factor $R^2 M_s^2 \sim 4 \times 10^{26}$. Assuming that the six dimensional gauge coupling is $O(1)$ we get $g^2 \sim
On the other hand, if the inflaton $\phi$ lives on the D5 brane wrapped around the small torus, then the Yukawa coupling will be of $O(1)$, $\lambda \sim 1$, since there is no suppression factor due to the volume of the two torus. In fact this is exactly what happens in orbifolds of type I strings (or orientifolds of type IIB strings) for gauge and Yukawa couplings. We see that the extreme smallness of the gauge coupling is a direct result of the large internal dimensions whereas the large value of the Yukawa coupling is a result of the small torus. We also need to know the value for $M$. The VEV of $M$ is a free parameter ($M$ is a twisted modulus[21]) before SUSY breaking due to the nonzero vacuum energy and it is perfectly reasonable for it to be larger than the string scale as long as this does not lead to an energy density higher than $M_s^4$. For the moment we leave $M$ free but note that it will be much larger than $M_s \sim 50$ $TeV$. Similarly $\phi$ can be much larger than $M_s$. We take for the large initial value of the inflaton $\phi \sim 6 \times 10^6$ $GeV$. Later we will see that this is more than enough for 60 e–foldings.

Initially using eq. (8) we get for the inflaton mass $m_\phi \sim 7 \times 10^{-5}$ $eV$ whereas the Hubble constant is $H \sim 10^{-1}$ $eV$. The inflaton mass is four orders of magnitude smaller than $H$ and therefore inflation happens in this scenario. Inflation continues as long as $m_\phi < H$ and stops when $m_\phi \sim H$. Then $\phi$ rolls to its minimum at $\phi = 0$. On the other hand, $m_\sigma^2$ becomes negative and $\sigma$ rolls from the (now) maximum at $\sigma = 0$ to the new minimum at $\sigma = M$. The vacuum energy becomes zero, inflation ends and SUSY is restored. The number of e-folds during inflation is given by

$$N \sim \left(\frac{\phi_i}{M_P}\right)^2 \frac{4\pi^2}{g^2}$$

(12)

where $\phi_i$ is the initial value of the inflaton. We obtain $N \sim 4 \times 10^5$ which is certainly enough inflation to solve the horizon and flatness problems. Now for inflation we are interested in quantities corresponding to the latest 60 e–folds. From eq. (8) we find that the magnitude of the inflaton $\phi$ corresponding to $N \sim 60$ is $\phi \sim 7 \times 10^4$ $GeV$ slightly above the string scale $M_s$. Note that this value is much smaller than $\phi_i$ we assumed above but any $\phi_i > 7 \times 10^4$ $GeV$ will give acceptable inflation.
Substituting the value of $\phi$ from eq. (12) into eq. (8) we find that the inflaton mass is smaller than the Hubble constant by a factor of $\sqrt{4N} \sim 12$ independently of $g^2$ and $M$ at the time of the latest 60 e–foldings. Later once we fix $M$ we will find the values for the inflaton mass and Hubble constant at 60 e–folds.

The magnitude of density perturbations is given by

$$\frac{\delta \rho}{\rho} \sim \frac{\lambda g^2 M^5}{M_P m_{\phi}^2}$$

(13)

Using the values for $g$, $\lambda$, $m_{\phi}$ from eq. (8) and $\phi$ from eq. (12) we obtain $\delta \rho/\rho \sim 4\lambda NM/M_P$. Until now $M$ was not fixed and we see that it has to be $M \sim 4 \times 10^{10}$ GeV in order to get the correct density perturbations of $\delta \rho/\rho \sim 10^{-5}$ at 60 e–foldings obtained from the COBE data. We see that $M$ is many orders of magnitude larger than $M_s$ but this is acceptable since the vacuum energy density $g^2 M^4 \sim 10^{-3} M_s^4$. Note that the magnitude of the density perturbations is independent of the very small gauge coupling constant $g^2$ but depends on the Yukawa coupling $\lambda$.

Now that we have fixed $M$ we can calculate the $m_{\phi}^2$ and $H^2$ from eqs. (8) and (9). We find that at 60 e–foldings $m_{\phi} \sim 10^{-2}$ eV whereas $H \sim 10^{-1}$ eV.

A second requirement for inflation is the condition on the spectrum of density perturbations

$$\eta \sim \sqrt{\frac{g^2}{16\pi^2}} \frac{M_P}{M} < 0.2$$

(14)

This has to be satisfied at the time of 60 e–foldings, i.e. for $\phi \sim 7 \times 10^4$ GeV. Substituting $g^2$ and $\phi$ we find $\eta \sim 0.06$. Note that the extreme smallness of $g^2$ is crucial for this result.

We find that the D–term inflation with large internal dimensions satisfies all the requirements of acceptable inflation. We stress that the smallness of the gauge coupling plays a crucial role in this scenario. The flatness of the inflaton potential and satisfying the constraints on $\delta \rho/\rho$ and $\eta$ is a direct consequence of the very small gauge coupling and the large value of $M$ (compared to $M_s$). On the other
hand, extremely small gauge couplings are generic to models with large internal dimensions. Therefore, if inflation occurs after the internal dimensions are stabilized at large values the D–term inflation scenario becomes very natural.

Finally we would like to find out the reheating temperature $T_R$. In [20] it was shown that if one does not want to heat up the bulk by graviton production one needs $T_R \sim 10\ MeV$. At the end of inflation the inflaton mass becomes

$$m_{\phi}^2 \sim \lambda^2 \sigma^2 \sim \lambda^2 M^2 \sim 10^{21} \ GeV^2$$

(15)

which gives $m_{\phi} \sim 4 \times 10^{10} \ GeV$. The main decay channel for the inflaton is to two fermionic $\sigma$’s due to the tree level superpotential in eq. (3). (Note that after inflation the fermionic $\sigma$’s are massless since $\phi = 0$.) The rate of this decay is given by $\Gamma \sim \lambda^2 m_{\phi}/25 \sim 10^9 \ GeV$ which is much larger than the Hubble constant. Therefore, after inflation ends all the vacuum energy is converted into heat very efficiently. As a result,

$$T_R^4 \sim g^2 M^4$$

(16)

This gives $T_R \sim \sqrt{g}M \sim 10^4 \ GeV$ which is six orders of magnitude larger than what is needed. We see that the high $T_R$ is a result of the very large value for $M$ which was required for the correct amount of density perturbations. If we take $M \sim 10^5 \ GeV$ then we get $T_R \sim 10^{-2} \ GeV$ which is about the normalcy temperature but then the density perturbations are too small, i.e. $\delta \rho/\rho \sim 10^{-10}$.

3. Discussion

In this letter we considered D–term inflation around the TeV scale with large internal dimensions. We showed that if the internal dimensions are stabilized before inflation they lead to very small couplings and masses which are crucial for the success of the scenario. We find the fact that we obtained realistic values for the magnitude and spectrum of density perturbations, quite naturally very
promising. It is highly interesting that the main requirement of the scenario which is small couplings are in fact a consequence of the large compact dimensions. Thus once the internal dimensions are stabilized at their large values there is not a new naturalness problem for the couplings. We also found that an acceptable reheating temperature is hard to obtain in this scenario.

The main drawback of this scenario is the fact that we need the large internal dimensions to be stabilized before inflation. In fact in ref. [4] it was shown that even in that case there is a naturalness problem. Inflation occurs at times of order $H^{-1}$ whereas the natural time scale is $M^{-1}_s$ which is many orders of magnitude smaller. The requirement for the homogeniety of the universe between these two times is very hard to explain. Nevertheless, we think that the success of D-term inflation at the TeV scale merits further study of this problem.

For simplicity we only considered the case of two large dimensions in this letter. However, all of our results can be generalized easily to cases with more than two large dimensions. For example if there are three large dimensions one can consider the anisotropic case where two large dimensions are of the size given above while the third is very small. Assuming the same values for $M$ and the coupling constants we trivially get the same results. Note that in this case the lower bound on the string scale is much smaller but the value of $M$ is not necessarily related to $M_s$.

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