QUANTUM EFFECTS FOR EXTRINSIC GEOMETRY OF STRINGS VIA THE GENERALIZED WEIERSTRASS REPRESENTATION

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Abstract

The generalized Weierstrass representation for surfaces in $\mathbb{R}^3$ is used to study quantum effects for strings governed by Polyakov-Nambu-Goto action. Correlators of primary fields are calculated exactly in one-loop approximation for the pure extrinsic Polyakov action. Geometrical meaning of infrared singularity is discussed. The Nambu-Goto and spontaneous curvature actions are treated perturbatively.

1 Introduction

Since the Polyakov’s suggestion [1] to add new rigidity term (Polyakov extrinsic action) to the old Nambu-Goto action, the string theory based on such an extended action has been intensively studied (see e.g. [2]-[4]). Effects generated by an extrinsic geometry term have been analyzed by several authors [5]-[13]. A canonical description of the strings world-sheets by means of coordinates $\vec{X}$ of the target space in which strings are assumed to evolve was not very successful. This is mainly due to the presence of fourth order derivatives of $\vec{X}$ with respect to local coordinates on world-sheet. In last years, the discretization of surfaces and associated matrix models have been the most favorite tool to treat the problem (see e.g. [14]). A different approach has been developed in the papers [12]-[13]. It is based on the description of the strings world-sheets via the Gauss map for surfaces conformally immersed into the Euclidean spaces. Within this approach both the Nambu-Goto and Polyakov actions can be written in terms of constrained Kähler $\sigma$-model action. The use of the Gauss map makes easier the calculations of quantum effects induced by extrinsic geometry [12]-
Unfortunately nonlinear constraints associated with the Gauss map for nonminimal surfaces give rise to serious computational difficulties.

In this paper we present an approach based on the generalized Weierstrass representation introduced in [15]. Any surface in $\mathbb{R}^3$ can be generated via this representation provided a system of two linear equations is solved. Within the generalized Weierstrass representation the Nambu-Goto action and, particularly, the Polyakov action have a very simple form. This allows us to calculate the one loop correction to the background for the full Polyakov action exactly. The calculations in the momentum space occur to be convenient. The propagators of fields are found and their infrared behavior is analyzed. Quantum correction to the classical Nambu-Goto and spontaneous curvature actions are evaluated perturbatively.

Note that the generalized Weierstrass representation has been used recently within a different technique [16]-[17] to evaluate quantum effects in the string theory.

The paper is organized as follows. In section 2 we briefly summarize the generalized Weierstrass representation for surfaces in $\mathbb{R}^3$. The one-loop quantum effects of strings are studied in section 3. The Nambu-Goto and spontaneous curvature actions are discussed in section 4. Section 5 is devoted to discussion.

2 Generalized Weierstrass representation

The generalized Weierstrass representation for a surface conformally immersed in $\mathbb{R}^3$ ($\vec{X}(z, \bar{z}) : C \rightarrow \mathbb{R}^3$) is given by the formulae [15]

\[
\begin{align*}
X^1 + iX^2 &= i \int_{\Gamma} \left( \overline{\psi'} dz' - \overline{\varphi'} d\bar{z} \right) \\
X^3 &= - \int_{\Gamma} \left( \varphi' \overline{\psi'} dz' + \psi' \overline{\varphi'} d\bar{z} \right)
\end{align*}
\]

(1)

where $z, \bar{z} \in C$, bar means complex conjugation, $\Gamma$ is a contour in $C$ and complex-valued functions $\psi, \varphi$ obey the system of equations

\[
\begin{align*}
\partial \psi &= p \varphi \\
\overline{\partial} \varphi &= -p \psi
\end{align*}
\]

(2)

where $p = p(z, \bar{z})$ a real-valued function. The formulae (1)-(2) define a conformal immersion of a surface into $\mathbb{R}^3$ with the induced metric

\[
ds^2 = \left( |\psi|^2 + |\varphi|^2 \right)^2 dz d\bar{z}
\]

(3)

The Gaussian and the mean curvature are

\[
\begin{align*}
K &= - \frac{4}{\left( |\psi|^2 + |\varphi|^2 \right)^2} \left[ \log \left( |\psi|^2 + |\varphi|^2 \right) \right]_{z, \bar{z}} \\
H &= 2 \frac{p}{\left( |\psi|^2 + |\varphi|^2 \right)}
\end{align*}
\]

(4)
Any surface in $R^3$ can be represented in the form (1)-(2) \cite{15}, \cite{18}. At $p = 0$ one gets a minimal surface ($H = 0$) and the formulae (1)-(2) are reduced to the classical Weierstrass formulae for minimal surfaces.

The generalized Weierstrass representation (1)-(2) is equivalent to another Weierstrass type representation which was proposed in \cite{19} and has been used within the Gauss map approach in the papers \cite{12}-\cite{13}. The equivalence is established by simple formulae \cite{20}

$$f = i \frac{\psi}{\varphi}, \quad \eta = i \frac{\varphi^2}{2}$$

which however convert the linear system (2) into a nonlinear one which appeared in \cite{19} and, consequently, in \cite{12}-\cite{13}. The fact that in the generalized Weierstrass representation (1)-(2) the functions $\psi$ and $\varphi$ are constrained by the linear equation (3) (instead of a nonlinear one of [18]) is one of its advantages. One more advantage is that the extrinsic Polyakov action $S_P = \int H^2 [dS]$, where $[dS]$ is the area element, takes a very simple form \cite{20}. Indeed, the use of (3) gives

$$S_P = 4 \int p^2 [d^2 z]$$

where $[d^2 z] = (i/2) dz d\bar{z}$. The Nambu-Goto action $S_{NG} = \alpha_0 \int [dS]$ becomes

$$S_{NG} = \alpha_0 \int \left( \left| \psi \right|^2 + \left| \varphi \right|^2 \right)^2 [d^2 z]$$

The generalized Weierstrass representation (1)-(2) has allowed already to obtain several interesting results in differential geometry of surfaces where the extrinsic Polyakov action is known for a long time as the Willmore functional \cite{21} (see \cite{22}-\cite{24}), in the theory of liquid membranes \cite{25} and in the string theory \cite{26}, \cite{16}-\cite{17}. The representation (1)-(2) gives also the possibility to define an infinite class of integrable deformations of surfaces generated by the modified Veselov-Novikov hierarchy \cite{15}. A characteristic feature of these deformations is that they preserve the extrinsic Polyakov action \cite{20}, \cite{22}. This circumstance has been used in \cite{16}-\cite{17} to quantize the Willmore surface (surfaces which provide extremum to the Willmore functional (Polyakov action)).

The generalized Weierstrass representation (1)-(2) can be viewed as a parametrization of a surface in $R^3$ in terms of $p, \psi, \varphi$. We will see that this parametrization is quite convenient.

### 3 One loop effects

One-loop corrections for the Polyakov action have been already studied in \cite{1}, \cite{12}-\cite{13}. However nonlinear constraints associated with the Gauss map did not allow to calculate one-loop corrections for the full Polyakov action. In this
section we will show that the generalized Weierstrass representation enables to overcome this difficulty and provides us a deeper geometrical understanding of results.

We will follow to the method of calculations proposed in [1] and then used in [12]-[13]. So we start with the classical action

\[ S = \alpha_0 \int \left( |\psi|^2 + |\varphi|^2 \right)^2 [d^2z] + \beta_0' \int p^2 [d^2z] \]  

(8)

where \( \alpha_0 \) and \( \beta_0' \) are the tension and extrinsic coupling constant respectively.

The first step is to take into account equation (2) which relates the primary fields \( \psi, \varphi \) and \( p \). Introducing complex Lagrange multiplier fields and requiring the action to be real, one arrives at the following constraint term

\[ S_c = \int \left[ d^2z \right] \left[ \lambda (\partial \psi - p\varphi) + \sigma (\bar{\varphi} p + p\psi) + c.c. \right] \]  

(9)

which has to be added to the action (8) (here and below c.c. means complex conjugation). It is well known that, once constraints are introduced into the generating functional of Green functions \( Z = \int [\mathcal{D} \Pi] \exp(-S) \), then the correct definition of measure \([\mathcal{D} \Pi]\) requires the evaluation of the Faddeev-Popov determinant. In our case fields are constrained by the Dirac equation (2), i.e.

\[ L \left( \begin{array}{c} \psi \\ \varphi \end{array} \right) = \left( \begin{array}{cc} \partial & -p \\ p & \bar{\varphi} \end{array} \right) \left( \begin{array}{c} \psi \\ \varphi \end{array} \right) = 0 \]  

(10)

To evaluate \( \det(L) \) we will follow the heat kernel procedure (see [27], [16]). The Faddeev-Popov action term is defined via

\[ S_{FP}' = - \log \left[ \det \left( \frac{d}{ds} \zeta' (s|L) \right) \right] = \left[ \frac{1}{2} \frac{d}{ds} \zeta' (s|A) \right]_{s=0} \]  

(11)

where \( A = L^\dagger L = LL^\dagger \) and

\[ \zeta' (s|A) = tr' \left[ (A)^{-s} \right] = \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} tr' [\exp(-tA)] \]  

Here the Riemann function \( \xi \) is constructed from eigenvalues of the operator \( A \) and the prime means that the contribution of zero modes is omitted. The heat kernel of \( A \) is defined as [27]:

\[ K_t (z, z'|A) = \exp(-tA) (z, z') \]  

\[ \partial_t K_t (z, z'|A) + AK_t (z, z'|A) = 0 \]  

\[ \lim_{t \to 0^+} K_t (z, z'|A) = \delta (z, z') \]  

(12)

The small \( t \) expansion of \( A \) looks like

\[ K_t (z, z'|A) \big|_{z=z'} \cong \frac{1}{8\pi t} \sum k_n (z) t^n \]  

(13)
where the $k_n (z)$ are matrix-valued functions. As a result, one gets
\[
\zeta' (0)A = \frac{1}{4\pi} \int [d^2 z] \; \text{tr}' [k_1 (z)] \quad , \quad \left[ \frac{d}{ds} \zeta' (s)A \right]_{s=0} = \gamma \zeta' (0)A \quad (14)
\]
where $\gamma$ is the Euler-Mascheroni constant. Since for operator $A$ one has $\text{tr}' [k_1] = -2p^2 [27]$, we finally get
\[
S_{FP} = -\frac{\gamma}{4\pi} \int [d^2 z] \; p^2 \quad (15)
\]
Thus the effect of Faddeev-Popov determinant is reduced into a redefinition of the extrinsic coupling constant $\beta'$. Such a new coupling constant will be denoted as $\beta$ and we will assume $\beta > 0$. Therefore, the total action turns out to be
\[
S_T = S_{NG} + S_c + S_{ext} \quad (16)
\]
where are $S_{NG}$, and $S_c$ are defined by (8), (9) and $S_{ext} = \beta \int [d^2 z] \; p^2$.
First, let us concentrate on the pure extrinsic action: $\alpha_0 = 0$. i.e.
\[
S_T = S_c + S_{ext} \quad . \quad (17)
\]
The corresponding action in one-loop approximation can be derived by employing the background field method. So splitting all fields into slow and fast components ($p = p_0 + p_1$ etc.) and keeping only terms quadratic in fast variables, we get the following one-loop action
\[
\bar{S}^{(2)} = \int [d^2 z] \left\{ \lambda_1 (\partial \psi_1 - p_0 \varphi_1 - p_1 \varphi_0) + \sigma_1 (\overline{\partial} \varphi_1 + p_0 \psi_1 + p_1 \psi_0) \\
+ p_1 (\psi_1 \sigma_0 - \varphi_1 \lambda_0) + c.c. \right\} + \beta \int [d^2 z] \; p_1^2 \quad . \quad (18)
\]
In Fourier space we have $\bar{S}^{(2)} = \int [d^2 k] \; \bar{S}^{(2)} (k)$ where
\[
\bar{S}^{(2)} (k) = \frac{\beta}{4} \overline{\psi} (k) \; p (k) + \\
\left\{ \overline{\chi} R (k) \left[ \frac{(k+\bar{k})}{4i} \psi_R (k) - \frac{(k-\bar{k})}{4i} \psi_I (k) - p_0 \varphi_R (k) - p (k) \varphi_R \right] + \\
- \overline{\lambda}_I (k) \left[ \frac{(k+\bar{k})}{4i} \psi_R (k) - \frac{(k-\bar{k})}{4i} \psi_I (k) - p_0 \varphi_I (k) - p (k) \varphi_I \right] + \\
\sigma R (k) \left[ \frac{(k+\bar{k})}{4i} \varphi_R (k) + \frac{(k-\bar{k})}{4i} \varphi_I (k) + p_0 \psi_R (k) + p (k) \psi_R \right] + \\
- \overline{\sigma} I (k) \left[ \frac{(k+\bar{k})}{4i} \varphi_R (k) - \frac{(k-\bar{k})}{4i} \varphi_I (k) - p_0 \psi_R (k) - p (k) \psi_R \right] + \\
\overline{\psi} (k) \left[ \sigma R \psi_R (k) - \sigma I \psi_I (k) - \lambda_0, R \varphi_R (k) + \lambda_0, I \varphi_I (k) \right] + c.c. \right\}
\]
and $[d^2 k] = (i/2) \; dk \; d\bar{k}$. Here the subscripts $R$ and $I$ mean the real and imaginary parts of complex fields involved in (18) respectively and the fast field
index 1 is suppressed for simplicity. In deriving (19) we used the symmetrization procedure so that \( S^{(2)}(k) = S^{(2)}(-k) \). As usual, Fourier components \( f(k) = f(k, \bar{k}) \) of fast fields are defined via

\[
f(z, \bar{z}) = \frac{1}{2\pi} \int [d^2k] f(k) \exp \left[ -\frac{i}{2} (k \bar{z} + \bar{k} z) \right]
\]

where the integration is performed over the domain \( \tilde{\Lambda} < |k| < \Lambda \).

Exact propagators can be calculated using standard methods. They are presented in the Appendix. In particular, in the ultraviolet regime the two-point functions behave like

\[
\langle p_R(k) p_R(k) \rangle \approx 2 \beta^{-1}, \quad \langle \psi_R(k), \lambda_R(k) \rangle, \quad \langle \psi_I(k), \lambda_I(k) \rangle, \quad \langle \sigma_R(k), \sigma_R(k) \rangle \propto |k|^{-1}
\]

\[
\langle \psi_R(k), \lambda_R(k) \rangle, \quad \langle \psi_I(k), \lambda_R(k) \rangle, \quad \langle \sigma_R(k), \sigma_R(k) \rangle, \quad \langle \sigma_I(k), \sigma_R(k) \rangle \propto |k|^{-1}
\]

and like \( |k|^{-2} \) in all the other cases.

4 Perturbative evaluation of Nambu-Goto and spontaneous curvature action

Results obtained in previous section can be used for a perturbative analysis of the intrinsic geometry term in the action (8). At the one-loop level the Nambu-Goto action reads

\[
S^{(2)}_{NG} = \int [d^2z] \mathcal{L}^{(2)}_{NG} \equiv 2\alpha_0 \int [d^2z] \left[ \left( |\psi_0|^2 + |\varphi_0|^2 \right) \left( \psi_R^2 + \psi_I^2 + \varphi_R^2 + \varphi_I^2 \right) + 2 (\psi_{0,R} \psi_R + \psi_{0,I} \psi_I + \varphi_{0,R} \varphi_R + \varphi_{0,R} \varphi_I)^2 \right].
\]

The contribution to the classical Lagrangian from the intrinsic geometry term treated perturbatively is

\[
\Delta S^{(2)}_{NG} = \int [d^2z] < \mathcal{L}^{(2)}_{NG} >. \]

Using the propagators following from (13), we get

\[
< \mathcal{L}^{(2)}_{NG} > = \frac{\alpha_0}{\pi^2} \left( |\psi_0|^2 + |\varphi_0|^2 \right)^2 \int [d^2k] \left( |k|^2 + 12 \rho_0^2 \right) D_p^{-1}. \]

Therefore, the counterterm to the classical Nambu-Goto action turns out to be

\[
\Delta S^{(2)}_{NG} = \frac{\alpha_0}{\beta (\xi - 1)} \tilde{I} \left( \Lambda, \tilde{\Lambda}, \rho_0^2, \xi, 3 \right) \int [d^2z] \left( |\psi_0|^2 + |\varphi_0|^2 \right)^2.
\]
where

$$I \left( \Lambda, \bar{\Lambda}, p_0^2, \xi, a \right) = \log \left( \frac{\left( |k|^2 - 4\xi p_0^2 \right)^{\xi + a}}{\left( |k|^2 - 4p_0^2 \right)^{1+a}} \right)_{|k|=\bar{\Lambda}}.$$ 

and $\xi = (1 + 2\beta''/\beta)$. This counterterm gives rise to the renormalized Nambu-Goto action

$$\bar{S}_{NG} = \bar{\alpha} \int [d^2 z] \left( |\psi|_0^2 + |\varphi|_0^2 \right)^2,$$

where $\bar{\alpha} = \alpha_0 \left[ 1 + (\pi\beta)^{-1} (\xi - 1)^{-1} I \left( \Lambda, \bar{\Lambda}, p_0^2, \xi, 3 \right) \right]$ is the renormalized string tension.

Since the generalized Weierstrass representation allows to express other action terms in simple forms, above perturbative approach can be successfully pursued in more general cases. In particular, we focus on a term which is peculiar of surfaces in $R^4$. It is the spontaneous curvature action defined as (see F. David in [3]):

$$S_H = \eta_0 \int [dS] H.$$

In terms of the generalized Weierstrass representation, it results (see eq. (4))

$$S_H = \eta_0 \int [d^2 z] p u \quad .$$

(20)

The one-loop spontaneous curvature action following from (20) reads

$$S_{H}^{(2)} = \eta_0 \int [d^2 z] \left( |\psi|_0^2 + |\varphi|_0^2 \right) + 2p (\psi_{R,0} \psi_R + \psi_{I,0} \psi_I + \varphi_{R,0} \varphi_R + \varphi_{I,0} \varphi_I) \right) .$$

(21)

The counterterm to the classical spontaneous curvature action looks like

$$\Delta S_{H}^{(2)} = \int [d^2 z] < L_{H}^{(2)} >$$

where

$$< L_{H}^{(2)} > = \eta_0 \int [d^2 z] \left( |\psi|_0^2 + |\varphi|_0^2 \right) \int [d^2 k] \left( 3 |k|^2 - 4p_0^2 \right) D_p^{-1}$$

Therefore, the renormalized spontaneous curvature action reads

$$\bar{S}_H = \eta \int [d^2 z] \left( |\psi|_0^2 + |\varphi|_0^2 \right)$$

where $\eta = \eta_0 \left[ 1 + 3 (2\pi\beta)^{-1} (\xi - 1)^{-1} I \left( \Lambda, \bar{\Lambda}, p_0^2, \xi, -\frac{1}{4} \right) \right]$.
5 Discussion

In this paper we have calculated one-loop effects in the theory of a string world-sheet conformally immersed in $R^3$. We have considered the complete Nambu-Goto-Polyakov action. Propagators for primary fields have been calculated in the case of a pure extrinsic action (extrinsic Polyakov action). In general, all of them have the following structure (see the Appendix)

$$\frac{N_P}{D_P}$$

where $N_P$’s are functions of slow fields and momentum and

$$D_P = \beta \left( |k|^2 - 4p_0^4 \right) \left( |k|^2 - 4\xi p_0^4 \right)$$

where $\xi = (1 + 2\beta_0' / \beta)$. The Faddeev-Popov determinant contributes to (22) by mean of the $\beta$. In our approach, we have a simple geometrical interpretation of the results since, in virtue of (4), the $p_0$ field acts as a link between extrinsic and intrinsic geometry of strings world-sheets.

The appearance of singularities which depend on background geometry, as in (22), seems to be completely new. This is due to the fact that we have treated the full extrinsic Polyakov action while the other authors dealt principally with the kinetic part of one-loop action [12] or by fixing minimal surface as background [13]. As a result, these singularities did not show up in their formulae.

The singular behavior of propagators is naturally removed once an infrared cut-off $\tilde{\Lambda}$ is introduced. An infrared cut-off constrained by the background geometry is commonly used in theory of random surfaces. Nevertheless, in our formalism the role of intrinsic geometry is more transparent even in the pure extrinsic action case. The key point relies on the one-loop approximation: quantum fluctuations of fields must have large momenta with respect to the classical background ones. In our language this means that infrared cut-off of the model $\tilde{\Lambda}$ must be sufficiently large with respect to $2p_0$, as it can be easily understood by rewriting equation (4) in the Fourier space. Therefore, inspite of the fact that the intrinsic geometry does not play a role explicitly, it enters into the constraint for $\tilde{\Lambda}$ by mean of (4). In view of this, it would be interesting to study how the Nambu-Goto and spontaneous curvature actions contribute to propagators. This will be done in a separate paper.

Coming back to (22) we note that, when $\xi$ is of the order of unity, the points $|k_1| = 2p_0$ and $|k_2| = \sqrt{4\xi p_0^4}$ do not belong to the allowed spectrum of momenta and hence the two-points functions are well-defined. If $\xi \gg 1$, we can set $\tilde{\Lambda} = \sqrt{4\xi p_0^4}$.

In the case of minimal surfaces $p_0 \to 0$ and, consequently, the formulae obtained for propagators are drastically simplified and hold into a wide range of $|k|$. The infrared singularity is absent provided that $|k| \geq \tilde{\Lambda} > 0$. The perturbative analysis of the Nambu-Goto and the spontaneous curvature actions
leads to a logarithmic dependence on $|k|$ of the effective tension and spontaneous curvature couplings according to

$$\alpha (|k|) \simeq \alpha_0 \left[ 1 + \frac{2}{\beta \pi} \log \left(\frac{\Lambda}{|k|}\right)\right], \quad \eta (|k|) \simeq \eta_0 \left[ 1 + \frac{3}{\pi \beta} \log \left(\frac{\Lambda}{|k|}\right)\right].$$

The short wavelength regime takes place for such $k$ that $|k|^2 \gg 4\xi p_0^2$. General formulae look similar to the minimal surface case, the only difference is the value of the infrared cut-off $\tilde{\Lambda}$.

We would like also to mention that the propagators between the Lagrange multiplier fields have the right sign, in contrast to the results obtained in [12].

**APPENDIX**

$$\langle \bar{p}(k) p(k) \rangle = \left(|k|^2 - 4p_0^2\right) D_p^{-1}$$

$$\langle \bar{\psi}_R(k) \psi_R(k) \rangle = 2 \left| 2p_0 \text{Re}(\psi_0) + i \text{Re}(k\varphi_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\psi}_I(k) \psi_I(k) \rangle = 2 \left| 2p_0 \text{Im}(\psi_0) + i \text{Im}(k\varphi_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\varphi}_R(k) \varphi_R(k) \rangle = 2 \left| 2p_0 \text{Re}(\varphi_0) + i \text{Re}(k\varphi_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\varphi}_I(k) \varphi_I(k) \rangle = 2 \left| 2p_0 \text{Im}(\varphi_0) + i \text{Im}(k\varphi_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\lambda}_R(k) \lambda_R(k) \rangle = 2 \left| 2p_0 \text{Re}(\lambda_0) + i \text{Re}(k\sigma_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\lambda}_I(k) \lambda_I(k) \rangle = 2 \left| 2p_0 \text{Im}(\lambda_0) + i \text{Im}(k\sigma_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\sigma}_R(k) \sigma_R(k) \rangle = 2 \left| 2p_0 \text{Re}(\sigma_0) + i \text{Re}(k\lambda_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\sigma}_I(k) \sigma_I(k) \rangle = 2 \left| 2p_0 \text{Im}(\sigma_0) + i \text{Im}(k\lambda_0) \right|^2 D_p^{-1}$$

$$\langle \bar{\psi}_R(k), \psi_R(k) \rangle = 2 \left| 2p_0 \text{Re}(\psi_0) + i \text{Re}(k\varphi_0) \right| D_p^{-1}$$

$$\langle \bar{\psi}_R(k), \psi_I(k) \rangle = 2 \left| 2p_0 \text{Im}(\psi_0) + i \text{Im}(k\varphi_0) \right| D_p^{-1}$$

$$\langle \bar{\psi}_I(k), \varphi_R(k) \rangle = 2 \left[ 2p_0 \text{Re}(\varphi_0) - i \text{Re}(k\varphi_0) \right] D_p^{-1}$$
\[ \langle \bar{\psi} (k) , \varphi_I (k) \rangle = 2 \left[ 2p_0 Im (\varphi_0) + i Im (k \bar{\psi}_0) \right] \bar{D}_P^{-1} \]

\[ \langle \bar{\psi} (k) , \lambda_R (k) \rangle = 2 \left[ 2p_0 Re (\lambda_0) + i Re (k \sigma_0) \right] \bar{D}_P^{-1} \]

\[ \langle \bar{\psi} (k) , \lambda_I (k) \rangle = 2 \left[ 2p_0 Im (\lambda_0) + i Im (k \sigma_0) \right] \bar{D}_P^{-1} \]

\[ \langle \bar{\psi} (k) , \sigma_R (k) \rangle = 2 \left[ 2p_0 Re (\sigma_0) - i Re (k \bar{\sigma}_0) \right] \bar{D}_P^{-1} \]

\[ \langle \bar{\psi} (k) , \sigma_I (k) \rangle = 2 \left[ 2p_0 Im (\sigma_0) + i Im (k \bar{\sigma}_0) \right] \bar{D}_P^{-1} \]

\[ \langle \bar{\psi}_R (k) , \psi_I (k) \rangle = 2 \left( 2p_0 \psi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \psi_{oI} + i Im [k \varphi_0] \right) D_P^{-1} \]

\[ \langle \bar{\psi}_R (k) , \phi_R (k) \rangle = 2 \left( 2p_0 \phi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \phi_{oI} - i Re [k \bar{\psi}_0] \right) D_P^{-1} \]

\[ \langle \bar{\psi}_R (k) , \varphi_I (k) \rangle = 2 \left( 2p_0 \psi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \psi_{oI} + i Im [k \bar{\psi}_0] \right) D_P^{-1} \]

\[ \langle \bar{\psi}_R (k) , \lambda_R (k) \rangle = \left[ \frac{(k + \bar{k})}{4i} \bar{D}_P + 2 \left( 2p_0 \psi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \lambda_{oR} + i Re [k \sigma_0] \right) \right] D_P^{-1} \]

\[ \langle \bar{\psi}_R (k) , \lambda_I (k) \rangle = \left[ -\frac{(k - \bar{k})}{4} \bar{D}_P + 2 \left( 2p_0 \psi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \lambda_{oI} + i Im [k \sigma_0] \right) \right] D_P^{-1} \]

\[ \langle \bar{\psi}_R (k) , \sigma_R (k) \rangle = \left[ -p_0 \bar{D}_P + 4 \left( 2p_0 \psi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \sigma_{oR} - i Re [k \bar{\sigma}_0] \right) \right] D_P^{-1} \]

\[ \langle \bar{\psi}_I (k) , \sigma_I (k) \rangle = 2 \left( 2p_0 \psi_{oR} - i Re [k \varphi_0] \right) \left( 2p_0 \sigma_{oI} + i Re [k \bar{\sigma}_0] \right) D_P^{-1} \]

\[ \langle \bar{\psi}_I (k) , \phi_R (k) \rangle = 2 \left( 2p_0 \phi_{oR} - i Im [k \varphi_0] \right) \left( 2p_0 \phi_{oI} - i Re [k \bar{\psi}_0] \right) D_P^{-1} \]

\[ \langle \bar{\psi}_I (k) , \varphi_I (k) \rangle = 2 \left( 2p_0 \psi_{oR} - i Im [k \varphi_0] \right) \left( 2p_0 \psi_{oI} + i Im [k \bar{\psi}_0] \right) D_P^{-1} \]

\[ \langle \bar{\psi}_I (k) , \lambda_R (k) \rangle = \left[ -\frac{(k - \bar{k})}{4} \bar{D}_P + 4 \left( 2p_0 \psi_{oR} - i Im [k \varphi_0] \right) \left( 2p_0 \lambda_{oR} + i Re [k \sigma_0] \right) \right] D_P^{-1} \]

\[ \langle \bar{\psi}_I (k) , \lambda_I (k) \rangle = \left[ \frac{(k + \bar{k})}{4} \bar{D}_P + 4 \left( 2p_0 \psi_{oR} - i Im [k \varphi_0] \right) \left( 2p_0 \lambda_{oI} + i Im [k \sigma_0] \right) \right] D_P^{-1} \]

\[ \langle \bar{\psi}_I (k) , \sigma_R (k) \rangle = 2 \left( 2p_0 \psi_{oR} - i Im [k \varphi_0] \right) \left( 2p_0 \sigma_{oR} - i Re [k \bar{\sigma}_0] \right) D_P^{-1} \]
\[ \langle \bar{\psi}_I(k), \sigma_I(k) \rangle = \left[ p_0 \tilde{D}_P \pm 4(2p_0\psi_{0I} - iRe[k\bar{\psi}_0]) (2p_0\sigma_{0I} + iIm[k\bar{\lambda}_0]) \right] D_P^{-1} \]
\[ \langle \bar{\varphi}_R(k), \varphi_I(k) \rangle = 2 \left( 2p_0\varphi_R + iRe[k\bar{\varphi}_0] \right) (2p_0\varphi_I + iIm[k\bar{\varphi}_0]) D_P^{-1} \]
\[ \langle \bar{\varphi}_R(k), \lambda_R(k) \rangle = \left[ p_0 \tilde{D}_P + 4(2p_0\varphi_R + iRe[k\bar{\varphi}_0]) (2p_0\lambda_R + iRe[k\sigma_0]) \right] D_P^{-1} \]
\[ \langle \bar{\varphi}_R(k), \lambda_I(k) \rangle = 2 \left( 2p_0\varphi_R + iRe[k\bar{\varphi}_0] \right) (2p_0\lambda_I + iIm[k\sigma_0]) D_P^{-1} \]
\[ \langle \bar{\varphi}_R(k), \sigma_R(k) \rangle = \left[ \frac{k + \bar{k}}{4i} D_P + 4(2p_0\varphi_R + iRe[k\bar{\varphi}_0]) (2p_0\sigma_R - iRe[k\bar{\lambda}_0]) \right] D_P^{-1} \]
\[ \langle \bar{\varphi}_R(k), \sigma_I(k) \rangle = \left[ \frac{k - \bar{k}}{4} D_P + 4(2p_0\varphi_R + iRe[k\bar{\varphi}_0]) (2p_0\sigma_I + iIm[k\bar{\lambda}_0]) \right] D_P^{-1} \]
\[ \langle \bar{\varphi}_I(k), \lambda_R(k) \rangle = 2 \left( 2p_0\varphi_I - iIm[k\bar{\varphi}_0] \right) (2p_0\lambda_R + iRe[k\sigma_0]) D_P^{-1} \]
\[ \langle \bar{\varphi}_I(k), \lambda_I(k) \rangle = \left[ -p_0 \tilde{D}_P + 4(2p_0\varphi_I - iIm[k\bar{\varphi}_0]) (2p_0\lambda_I + iIm[k\sigma_0]) \right] D_P^{-1} \]
\[ \langle \bar{\varphi}_I(k), \sigma_R(k) \rangle = \left[ \frac{k - \bar{k}}{4} D_P + 4(2p_0\varphi_I - iIm[k\bar{\varphi}_0]) (2p_0\sigma_R - iRe[k\bar{\lambda}_0]) \right] D_P^{-1} \]
\[ \langle \bar{\varphi}_I(k), \sigma_I(k) \rangle = \left[ \frac{k + \bar{k}}{4i} D_P + 4(2p_0\varphi_I - iIm[k\bar{\varphi}_0]) (2p_0\sigma_I + iIm[k\bar{\lambda}_0]) \right] D_P^{-1} \]
\[ \langle \bar{\lambda}_R(k), \lambda_I(k) \rangle = 2 \left( 2p_0\lambda_R - iRe[k\sigma_0] \right) (2p_0\lambda_I + Im[k\sigma_0]) D_P^{-1} \]
\[ \langle \bar{\lambda}_R(k), \sigma_R(k) \rangle = 2 \left( 2p_0\lambda_R - iRe[k\sigma_0] \right) (2p_0\sigma_R - iRe[k\bar{\lambda}_0]) D_P^{-1} \]
\[ \langle \bar{\lambda}_R(k), \sigma_I(k) \rangle = 2 \left( 2p_0\lambda_R - iRe[k\sigma_0] \right) (2p_0\sigma_I + iIm[k\bar{\lambda}_0]) D_P^{-1} \]
\[ \langle \bar{\lambda}_I(k), \sigma_R(k) \rangle = 2 \left( 2p_0\lambda_I - iIm[k\sigma_0] \right) (2p_0\sigma_R - iRe[k\bar{\lambda}_0]) D_P^{-1} \]
\[ \langle \bar{\lambda}_I(k), \sigma_I(k) \rangle = 2 \left( 2p_0\lambda_I + iIm[k\sigma_0] \right) (2p_0\sigma_I + iIm[k\bar{\lambda}_0]) D_P^{-1} \]
\[ \langle \sigma_R(k), \sigma_I(k) \rangle = 2 \left( 2p_0\sigma_R + iRe[k\bar{\lambda}_0] \right) (2p_0\sigma_I + iIm[k\bar{\lambda}_0]) D_P^{-1} \]

where

\[ D_P = \left( |k|^2 - 4p_0^2 \right) \left[ \beta \left( |k|^2 - 4p_0^2 \right) - 8p_0Re(\lambda_0\varphi_0 - \sigma_0\psi_0) \right] \]

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\[ \tilde{D}_P = 2 \left( |k|^2 - 4p_0^2 \right)^{-1} D_P \]

By using the classical equation of motion
\[ 2 \beta'_0 p_0 = 2 Re(\lambda_0 \varphi_0 - \sigma_0 \psi_0), \]
\( D_P \) and \( \tilde{D}_P \) can be expressed in terms of the only momentum and \( p_0 \):
\[ D_P = \beta \left( |k|^2 - 4p_0^2 \right) \left( |k|^2 - 4\xi p_0^2 \right), \quad \tilde{D}_P = 2\beta \left( |k|^2 - 4\xi p_0^2 \right) \]

where \( \xi = (1 + 2\beta'_0/\beta) \).

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