Orbital motion effects in astrometric microlensing

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ABSTRACT

We investigate lens orbital motion in astrometric microlensing and its detectability. In microlensing events, the light centroid shift in the source trajectory (the astrometric trajectory) falls off much more slowly than the light amplification as the source distance from the lens position increases. As a result, perturbations developed with time such as lens orbital motion can make considerable deviations in astrometric trajectories. The rotation of the source trajectory due to lens orbital motion produces a more detectable astrometric deviation because the astrometric cross-section is much larger than the photometric one. Among binary microlensing events with detectable astrometric trajectories, those with stellar-mass black holes have most likely detectable astrometric signatures of orbital motion. Detecting lens orbital motion in their astrometric trajectories helps to discover further secondary components around the primary even without any photometric binarity signature as well as resolve close/wide degeneracy. For these binary microlensing events, we evaluate the efficiency of detecting orbital motion in astrometric trajectories and photometric light curves by performing Monte Carlo simulation. We conclude that astrometric efficiency is 87.3 per cent whereas the photometric efficiency is 48.2 per cent.

1 INTRODUCTION

Gravitational field of an object acts as a gravitational lens which deviates light path of a collinear background source and produces distorted images (Einstein 1936). In Galactic scale, the angular separation of the images is of order of milli-arcsecond that is too small to be resolved even by the most modern telescopes. Instead, the combined light of images received by an observer is magnified in comparison to the un-lensed source. This phenomenon is called gravitational microlensing which was proposed as a method to probe dark objects in Galactic halo, extra solar planets, study stellar atmosphere, etc. (Liebes 1964; Chang & Refsdal 1979; Paczyński 1986a,b).

One of features of gravitational microlensing is the deviation of the light centroid of the source star images from the source star position. For the case of a point-mass lens, the centroid shift of source star images traces an ellipse in the lens plane (Walker 1995; Miyamoto & Yoshii 1995; Høg et al. 1995; Jeong et al. 1999). In contrast with the magnification factor which is a dimensionless scalar, the light centroid shift of the source star images is a dimensional vector and its size is proportional to angular Einstein radius, i.e. the angular radius of images ring when observer, source and lens are completely aligned. For a stellar-mass lens, the angular Einstein radius is a few hundreds microarcsecond, too small to be observed directly. However, its quantity enhances in two cases: (i) when the lens mass is high, e.g. a stellar-mass black hole as microlens and (ii) when the lens is very close to the observer. The light centroid shift of source star images in these microlensing events is measurable by high-precision interferometers e.g. Very Large Telescope Interferometry (VLTI) or the HST (Paczyński 1993). An important future project is GAIA, performing high precision astrometry, which will be operational in near future (Provost et al. 2011).

During a microlensing event by measuring the astrometric lensing and parallax effects, the mass of the deflector can be inferred without knowing the exact distances of the lens and the source from the observer (Paczynski 1997; Miralda-Escudé 1996). Also the degeneracy in close/wide caustic-crossing binary microlensing events (Dominik 1999) can be removed by astrometric measurements (Gould & Han 2000; Chune et al. 2009). The two configurations of the lens and source produce different astrometric trajectories even though they might present the same light curves. Hence, by measuring the astrometric trajectories this degeneracy can be removed (Han et al. 1999).

Generally, there are some anomalies in microlensing events which deviate the observed light curve and astrometric trajectory of source star from the standard model. Some deviations owing to these anomalies help obtain extra information about lens and source and thus resolve degeneracy. One of them is the effect of lens orbital motion in a binary microlensing event. In this case two components in a gravitationally bound binary system which act as lens rotate around their common center of mass. As a result, their orientation changes as a microlensing event progresses and the resulting light curve is different from that due to a static binary microlens. The ratio of the Einstein crossing time to the orbital period of lens system is considered as a criterion which indicates the probability of detecting the orbital motion in the microlensing light curve (Dominik 1998; Ioka et al. 1999). Measuring this effect gives information about the orbit of microlenses system which in turn helps to resolve the close/wide degeneracy (An et al. 2002; Gaudi et al. 2003; Dong et al. 2005; Shin et al. 2013). Detectability of orbital motion in light curves of binary microlensing events was investigated extensively by Penny et al. (2011) whereas the rel-
event deviation in the astrometric trajectory of source star and its
detectability are not clear yet.

In this work, we study lens orbital motion in astrometric mi-
crolensing and its detectability. In microlensing events, the astro-
metric trajectory of source star tends to zero much more slowly than
the light amplification as the source distance from the lens position
rises. As a result, perturbations enlarged with time such as lens or-
bital motion can produce considerable deviations in the astrometric
trajectories. Since the astrometric cross-section is much larger than
the photometric one, so the rotation in the source trajectory ow-
ing to the orbital motion is more probable to be detected in the
astrometric trajectory than the photometric light curve. However,
depending on the size of the angular Einstein radius this effect can
be detected. We find that, among binary microlensing events with
detectable astrometric trajectories those with rotating stellar-mass
black holes have most likely detectable astrometric signatures of
orbital motion. For these binary microlensing events, we evaluate
the efficiency of detecting orbital motion in astrometric trajectories
and a photometric light curves by performing Monte Carlo simula-
tion. Detecting lens orbital motion in their astrometric trajectories
helps to discover even secondary components with no photometric
binarity signatures as well as resolve close/wide degeneracy.

This paper is organized as follows: In section 2 the effect of
orbital motion in binary microlensing events are explained. We next
describe astrometric properties of microlensing events and finally
we investigate the orbital motion effect on the astrometric tra-
jectories. In the next section by preforming a Monte Carlo simulation
we study the astrometric and photometric efficiencies for detecting
lens orbital motion. In section 4 we explain the results.

2 ORBITAL MOTION IN ASTROMETRIC
MICROLENSING

In this section we first study the orbital motion effect in binary mi-
crolensing events. Having reviewed the astrometric microlensing,
we investigate the microlensing event with detectable astrometric
shifts in source trajectories. Finally, we study whether the orbital
motion effect in the astrometric trajectories is detectable.

2.1 Orbital motion effect of binary microlenses

Dominik (1998) first studied the effect of lens orbital motion in bi-
nary microlensing events and concluded that in the most microlens-
events this effect is ignorable, but in some long-duration mi-
crolensing events can probably be observed. Orbital motion effects
on the planetary signals in high-magnification microlensing events
was investigated by Rattenbury et al. (2002). Recently, Penny et al.
(2012) indicated how fraction of binary and planetary microlensing
events exhibit the orbital motion effects in their light curves by
performing Monte Carlo simulation and investigated those factors
which change this fraction. Until now, several observed microlens-
ing events have shown the signatures of lens orbital motion. In
some cases, these signatures have allowed to infer some of orbital
parameters and remove the close/wide degeneracy (An et al. 2003;
Gaudi et al. 2003; Dong et al. 2005; Shin et al. 2013). Sometimes,
there are several degenerate best-fitting solutions for orbital mo-
tion modelling. Three systems with complete orbital solutions
have been found, e.g. the microlensing event OGLE-2011-BLG-
0417 (Shin et al. 2011; Shin et al. 2012). However, one of these
solutions can actually be checked with radial velocity method
(Gould et al. 2013). In the following we study the orbital motion
effect in microlensing light curves in the same way as Dominik’s
approach.

Let us consider a gravitationally bound system as microlens.
Under the gravitational effect, the secondary component with re-
spect to the first one traces out an ellipse one of whose focus is at
the center of mass position. Two components of their relative distance
are given by:

\[ x(\xi) = a(\cos \xi - \varepsilon) \]
\[ y(\xi) = a\sqrt{1 - \varepsilon^2} \sin \xi, \]

(1)

where \( x \) and \( y \) axes are along the semi-major and semi-minor axes,
\( \varepsilon \) is the eccentricity, \( a \) is the semi-major axis and \( \xi \) is a periodic
function with time which for a small eccentricity and up to its first
order is estimated as:

\[ \xi(t) = 2\pi \frac{t - t_p}{P} + 2\varepsilon \sin(2\pi \frac{t - t_p}{P}), \]

(2)

where \( t_p \) is the time of arriving at the perihelion point of orbit, \( P \)
is the orbital period of the lenses motion. To study the orbital motion
of lenses on microlensing light curves, we should project the orbit plane of lenses into the sky plane. In that case, we need to
see two consecutive rotation angles: \( \beta \) around \( x \)-axis and \( \gamma \) around
\( y \)-axis. The projected components of the relative position vector of the
second one with respect to the first, in the sky plane normalized to the
Einstein radius are:

\[ x_1(t) = \rho \cos \gamma (\cos \xi(t) - \varepsilon) + \sin \beta \sin \gamma \sqrt{1 - \varepsilon^2} \sin \xi(t) \]
\[ x_2(t) = \rho \cos \beta \sqrt{1 - \varepsilon^2} \sin \xi(t), \]

(3)

where \( \rho \) is the normalized semi-major axis to the Einstein radius
(Dominik 1998)). Hence, lens orbital motion causes the projected
distance between two lenses in the lens plane changes with time
which is given by:

\[ d(t) = \sqrt{x_1(t)^2 + x_2(t)^2}. \]

(4)

On the other hand, the binary axis with respect to the projected
source trajectory rotates with angle \( \theta(t) = \tan^{-1}(x_2/x_1) \). So,
during a microlensing event with binary rotating lenses, the source
trajectory rotates with respect to the binary axis with angle \(-\theta\)
around the line of sight towards the observer. However, this rotation
is not always detectable in the observer reference frame.

The signature of lens orbital motion in the microlensing light
curves is not always observable and its Detectability depends
strongly on the ratio of the lensing characteristic time to the or-
bit period of lenses motion. Dominik (1998) considered the Ein-
stein crossing time as the characteristic time of a binary microlens-
ing event to evaluate the orbital motion detectability. However, the
orbital motion detection efficiency for binary microlensing events
without caustic-crossing features is too small and of order of one
per cent (Penny et al. 2011). As a result, we consider the effec-
tive time scale for detecting signatures of lens orbital motion as
\( t_E = \Delta(q,d) t_E \) instead of the Einstein crossing time, where \( \Delta(q,d) \)
refers to the average over the ratios of the lengths of the source
trajectories providing the sources are inside the caustic curve to the
total length of the source trajectories. We assume that there are several parallel source trajectories over the lens plane
with the same lengths i.e. \( L \) so that they cover the caustic curve. In
this case, \( \Delta(q,d) \) is given by:

\[ \Delta(q,d) = \frac{1}{L} \left( \sum_i N \right) \frac{l_i}{N} = \frac{1}{NL\delta} \sum_i N l_i \delta = S_c, \]

(5)
where $l_i$ is the length of the portion of $i$th source trajectory providing the source is inside the caustic curve, $N$ is the number of the source trajectories, $\delta = L/N$ is the normal distance between two consecutive source trajectories, $d$ is the projected distance between two lenses normalized to the Einstein radius and $q$ is the ratio of the lens masses. According to equation (5), it is clear that lens orbital motion can be characterized by the factor $f$ normalized to the Einstein radius and $\kappa$ which have the same caustic curve, is larger than those with less massive lenses. For this plot, we consider $\delta\kappa = 1.0$.

The map of detectability factor $f$ is shown in Figure 1 in the parameter space of lenses containing the total mass of lenses $M$ and semi-major axis $s$. In this figure the black dashed and solid lines represent the threshold values between the close-intermediate (or resonance) and intermediate-wide binaries respectively. For calculating the inner area of caustics, we first determine the caustic curve to an area equal to $L^2$ i.e., $S_c$. By considering this factor, the detectability of lens orbital motion can be characterized by the factor $f$:

$$ f = \frac{\delta\kappa}{\sqrt{D_l}} $$

The map of detectability factor $f$ is shown in Figure 1 in the parameter space of lenses containing the total mass of lenses $M$ and semi-major axis $s$. In this figure the black dashed and solid lines represent the threshold values between the close-intermediate (or resonance) and intermediate-wide binaries respectively. For calculating the inner area of caustics, we first determine the caustic points which have so small determinant of Jacobian matrix. Then, after sorting the consecutive points on the caustic line we calculate the interior area of caustics numerically. The detectability factor has the highest amount for the intermediate microlensing events which locate near to the close-intermediate threshold line (dashed line). However, Gaudi (2012) intuitively pointed out that the resonance microlensing events according to their large sizes and cross-sections are more sensitive to the small changes in $d$ owing to lens orbital motion. According to this figure, the orbital motion detectability in microlensing events with more massive lenses which have the same caustic curve, is larger than those with less massive lenses. For this plot, we consider $q = 1$, $v_i = 175\,\text{km/s}$, $D_l = 4\,\text{Kpc}$ and $D_s = 8\,\text{Kpc}$. However, the smaller amounts of $q$ just move two threshold lines towards each other so that they mostly form close and wide binaries while the detectability factor is maximum for intermediate binaries similar to the plotted case.

In the following, we explain the astrometric properties of microlensing and seek binary microlensing events which have detectable astrometric trajectories as well as very likely detectable orbital motion signatures.

### 2.2 Astrometric properties of microlensing

In microlensing events, the light centroid vector of source star images does not coincide with the source position. This astrometric shift in source star position changes with time as a microlensing event progresses. For a point-mass lens, the centroid shift vector of source star images is given by:

$$ \delta\theta = \frac{\mu_1\delta\theta_1 + \mu_2\delta\theta_2}{\mu_1 + \mu_2} - u\delta E = u\delta E + \frac{u^2}{2\kappa^2} \delta E, $$

where $\delta\theta_i$ and $\mu_i$ are the position and magnification factor of $i$th image, $u = px + uy$ is the vector of the projected angular position of the source star with respect to the lens normalized by the angular Einstein radius of the lens $\theta_E$ in which $p = (1 - t_0)/\theta_E$, $t_0$ is the time of the closest approach, $\theta_E$ is the Einstein crossing time, $x$ and $y$ are the unit vectors in the directions parallel with and normal to the direction of the lens-source transverse motion. The angular Einstein radius of lens is given by:

$$ \theta_E = \sqrt{\kappa} \frac{M_l}{\kappa \pi_{\text{rel}}} = 300\,\mu\text{as} \frac{M_l}{0.3M_\odot} \sqrt{\frac{\pi_{\text{rel}}(\text{mas})}{0.036}}. $$

where $M_l$ is the lens mass, $\kappa = \frac{4G}{c^2A_l}$ and $\pi_{\text{rel}} = 1.1\,\kappa\left(\frac{1}{D_l} - \frac{1}{D_s}\right)$ where $D_l$ and $D_s$ are the lens and source distances from the observer. Usually, a microlensing parallax is defined as $\pi_E = \pi_{\text{rel}}/\theta_E$ which can be measured from the photometric event.

The components of this centroid shift vector are given by:

$$ \delta\theta_{\text{c,}x}(u_0, p) = \frac{p}{u_0^2 + p^2 + 2}\theta_E, $$

$$ \delta\theta_{\text{c,}y}(u_0, p) = \frac{u_0}{u_0^2 + p^2 + 2}\theta_E. $$

The shift in the light centroid trajectory of the source star images owing to a point-mass lens traces an ellipse while the source is passing an straightforward line in the lens plane, plotted in Figure 2. By defining $X = \delta\theta_{\text{c,}x}$ and $Y = \delta\theta_{\text{c,}y} = \frac{u_0p\theta_E}{2(u_0^2 + 2)}$, these coordinates satisfy the following equation:

$$ X^2 + \frac{Y^2}{b^2} = a^2, $$

where $b = \frac{u_0}{\sqrt{u_0^2 + 2}}$ and $a = \frac{\theta_E}{2u_0}$. This ellipse is called the astrometric ellipse (Walker 1995; Jeong et al. 1999). The ratio of the...
axes of this ellipse is a function of impact parameter, so that for
the large impact parameter this ellipse converts to a circle whose
radius decreases by increasing impact parameter and for small
amounts, it becomes a straight line (see Figure 2) (Walker 1995).

Table 1. The possible ranges of physical parameters of two sets of binary microlensing events with measurable shifts in the astrometric light centroid of source
star images: \(\mathcal{A}\) gravitational microlensing events with binary stellar-mass black holes and \(\mathcal{B}\) those with high proper motion stars located in distances
smaller than 100 parsec from the sun.

\[
\begin{array}{cccccccccc}
\hline
M(M_\odot) & D_1(\text{Kpc}) & D_\odot(\text{Kpc}) & R_E(\text{Au}) & \theta_E(\text{millias}) & \mu_0(\text{/yr}) & V_\odot(\text{Km/s}) & t_E(\text{day}) & s(\text{Au}) & P(\text{day}) \\
\hline
(\mathcal{A}) & \geq 5 & \sim 6.5 & \sim 8.0 & \geq 7.0 & \geq 1.1 & \sim 0.006 & \sim 175 & \geq 70 & \sim 4 & \leq 1300 \\
(\mathcal{B}) & \sim 0.3 & \leq 0.1 & \sim 8.0 & \leq 0.5 & \geq 4.9 & \geq 0.4 & \sim 185 & \leq 5 & \sim 4 & \sim 5000 \\
\hline
\end{array}
\]

In contrast with the magnification factor which is a dimen-
sionless scalar, the light centroid shift of the source star images is
a dimensional vector and its size is proportional to the angular Ein-
stein radius given by equation (8). The angular Einstein radius of a
K- or M-dwarf star as the most probable lens with \(M_\odot \sim 0.3M_\odot\)
located in the Galactic disk \(D_1 \sim 6.5\text{Kpc}\), in the observations to-
wards the Galactic bulge i.e. \(D_\odot \sim 8.0\text{Kpc}\), is of order of a few
hundred micro-arcsecond (see equation 8) which is too small to be
observed. However, for two sets of microlensing events the angu-
lar Einstein radius enhances: \(\mathcal{A}\) when the lens mass is high and
\(\mathcal{B}\) when the lens is so close to the observer. These two classes of
microlensing events with detectable astrometric trajectories have
different properties owing to different amounts of the Einstein ra-
dius. In table (1) we briefly express the possible ranges of amounts
of their physical parameters.

The set \(\mathcal{A}\) of microlensing events which are characterized in
the second row of table (1) are long-duration microlensing events
with no light from the lens. These events with detectable astromet-
ric trajectories are best candidates to indicate the mass of stellar-
mass black holes located in the Galactic disk through the photomet-
ric and astrometric data as well as the parallax effect (Sahu 2011a).

Figure 3. Two typical binary microlensing events with rotating stellar-mass
black holes. In each subfigure, the light curves (left panels), astrometric tra-
njectories (right panels) and the source trajectories with respect to the caustic
curve (insets in the left-hand panels) without and with considering the ef-
fect of lens orbital motion are shown with green dashed and blue dotted
lines respectively. The point-mass lens models are shown by red solid lines.
The photometric and astrometric residuals with respect to the static binary
model are plotted with gray dotted lines. The relevant parameters can be
found in Table (3).

The second set \(\mathcal{B}\) which are characterized in the third row
of table (1) contains gravitational microlensing events in which the
lens is so close to the sun, e.g. closer than 100 parsec from the
sun position. These stars are best candidates for the astromet-
ric microlensing through GAIA mission for accurately indicating
their mass (Proft et al. 2011). Owing to the small distance of the
lens from the sun, their proper motion is so high and larger than
0.5 arsecond in year. These microlensing events are characterized
by the Einstein radius smaller than 0.5Au and Einstein crossing
time smaller than 5days. Let us assume that the lens is a binary sys-
tem. In that case, most of these binary microlensing events are wide
and a few of them are intermediate or close microlensing events in
which the orbital motion effect of lenses is ignorable due to the too small ratio of the Einstein crossing time to the orbital period and the finite-lens effect is probably considerable owing to the small Einstein radius. Note that, the finite-source effect is mostly ignorable because of the small amount of the relative distance of the lens to the source star from the observer i.e. $\delta \approx D_s/D_l \lesssim 0.0125$.

Hence, binary microlensing events with rotating stellar-mass black holes located in the Galactic disk which have detectable astrometric shifts in the source star trajectories have most likely detectable orbital motion signatures (case A). In these events, the ratio of the Einstein crossing time to the orbital period is high. If there is no caustic-crossing feature in their light curves, the astrometric centroid shift can be estimated as the case of point-mass lens, i.e. $\delta \theta_0 = \frac{3}{\pi} \frac{x^2 + y^2}{R_E}$.

In the subfigure (b) some oscillatory fashions can be seen in the astrometric trajectory which is owing to lens orbital motion. Indeed, whenever the source trajectory does not cross the caustic curve, the astrometric centroid shift can be estimated as the case of point-mass lens, i.e. $\delta \theta_0 = \frac{3}{\pi} \frac{x^2 + y^2}{R_E}$.

In Figure 4 we represent two examples of microlensing events with rotating stellar-mass black holes. In each subfigure, the light curves (left panels) and astrometric trajectories (right panels) without and with considering the effect of lens orbital motion are shown with green dashed and blue dotted lines respectively. The point-mass lens models are shown by red solid lines. The photometric and astrometric residuals with respect to the static binary model are plotted with gray dotted lines. The parameters of these microlensing events can be found in Table 2. We use the generalized version of the adaptive contouring algorithm (Dominik 2007) for plotting astrometric trajectories in binary microlensing events. Even though, the orbital motion effect is not obvious in the light curves, this effect can be detected in the astrometric trajectories.

### Table 2. The parameters used to make the microlensing light curves shown in Figure 3 and Figure 4.

| $M_1$ ($M_\odot$) | $q$ | $s$ (AU) | $d$ ($R_E$) | $t_E$ (day) | $P$ (day) | $\beta$ | $\gamma$ | $\alpha$ | $u_0$ | $v_t$ (km/s) |
|------------------|-----|---------|-------------|-------------|-----------|--------|--------|--------|------|-------------|
| (a)              | 8.5 | 0.06    | 2.5         | 0.33        | 48.6      | 478.9  | −5.3   | 85.7   | 101.7 | 0.43        | 271.6 |
| (b)              | 13.7| 0.09    | 3.9         | 0.28        | 276.7     | 734.8  | −23.1  | 69.3   | 71.6  | 0.45        | 88.6  |
| (c)              | 15.4| 0.12    | 9.5         | 0.64        | 233.1     | 2572.1 | 40.3   | −82.7  | 233.4 | 0.77        | 110.0 |

Figure 4. A simulated microlensing light curve and astrometric trajectory of source star. Data points taken by survey telescopes and the HST are shown with violet points and black stars. The parameters used to make this event can be found in table 2.
crossing feature, the probability of detecting the photometric signature of orbital motion is so small.

3 MONTE CARLO SIMULATION

In this section we perform a Monte Carlo simulation to obtain quantitatively detectability of lens orbital motion in astrometric and photometric microlensing with rotating stellar-mass black holes. In the first step, we produce an ensemble of binary microlensing events with stellar-mass black holes as microlenses according to the physical distribution of parameters. Then, corresponding light curves and astrometric trajectories are generated according to the real data points from an ensemble of observatories in the microlensing experiment. By considering a detectability criterion, we investigate whether the signatures of lens orbital motion can be seen in observations of microlensing light curves and astrometric trajectories. Our criterion for detectability of orbital motion is \( \Delta \chi^2 = \chi^2_{DM} - \chi^2_{SB} > \Delta \chi^2_{th} \) where \( \chi^2_{DM} \) and \( \chi^2_{SB} \) are the \( \chi^2 \)'s of the known rotating binary microlensing model and the static model with the same binary parameters respectively. We assume that from fitting process and searching all parameter space the best-fitted solution is the known binary solution. Our aim is to compare photometric efficiency of detecting lens orbital motion with the astrometric one. Hence, we calculate \( \Delta \chi^2 \) for simulated light curves and astrometric trajectories independently.

Here, we explain distribution functions of the lens and source parameters. For lens parameters we take the mass of the stellar-mass black hole as the primary from the following distribution function (Farr et al. 2011):

\[
P(M_l) \propto \exp\left(-\frac{M_l}{M_\odot}\right),
\]

where \( P(M_l) = dN/dM_l \) and \( M_\odot \) in the range of \( M_l \in [4.5, 25] M_\odot \). The mass ratio of the secondary to the primary is drawn from the distribution function (Duquennoy & Mayor 1991):

\[
\rho(q) \propto \exp\left(-\frac{(q - q_0)^2}{2\sigma_q^2}\right)
\]

where \( \rho(q) = dN/dq \) in the range of \( q \in [0.01, 1] \), \( q_0 = 0.23 \) and \( \sigma_q = 0.42 \). We choose the semi-major axis \( a \) of the binary orbit from the Opik law where the distribution function for the primary-secondary distance is proportional to \( \rho(s) = dN/ds \propto s^{-1} \) (Opik 1924) in the range of \( s \in [0.6, 30] \) AU. The location of lenses from the observer is calculated from the probability function of microlensing detection \( dN/dx \propto \rho(x)\sqrt{x(1-x)} \), where \( x = D_l/D_s \) changes in the range of \( x \in [0, 1] \) and \( \rho(x) \) is the stellar density of thin disk, chosen from Rahal et al. (2009). The time of arriving at the perihelion point of orbit \( t_p \) is chosen uniformly in the range of \( t_p \in [t_0-P, t_0+P] \). The projection angles to indicate the orientation of orbit of lenses with respect to the sky plane, i.e. \( \beta \) and \( \gamma \), are taken uniformly in the range of \([ -\frac{\pi}{2}, \frac{\pi}{2} ]\). We take the eccentricity of the lenses’ orbit uniformly in the range of \( e \in [0, 0.15] \).

For the source, we take the coordinate toward the Galactic bulge \((l, b)\), distribution of the matter in standard Galactic model and generate the distribution of the source stars according to the Besancon model (Dwek et al. 1995; Robin et al. 2003). We assign the the absolute, apparent color and magnitude of the source star in the same approach which is explained in (Sajadian et al. 2013). The mass of source star is taken from the Kroupa mass function, \( \xi = dN/dm \propto m_\odot^{-\alpha} \), in the range of \( m_\odot \in [0.3, 3] M_\odot \), where \( m_\odot \) is the mass of the solar star in the unit of the solar mass and \( \alpha = 0.3 \) for \( 0.01 \leq m_\odot \leq 0.08 \), \( \alpha = 1.3 \) for \( 0.08 \leq m_\odot \leq 0.5 \) and \( \alpha = 2.35 \) for \( m_\odot \geq 0.5 \) (Kroupa et al. 1993; Kroupa 2001). Another parameter we need in our simulation is the radius of source star. This parameter will be used in generating the light curve of microlensing events with the finite size effect. For the main-sequence stars, the relation between the mass and radius is given by \( R_s = m_s^{0.8} \) where all parameters normalized to the sun’s value.

The velocities of the lens and the source star are taken from the combination of the global and dispersion velocities of the Galactic disk and bulge (Rahal et al. 2009; Binney & Tremaine 1987). The relative velocities of the source-lens is determined by projecting the velocity of the source star into the lens plane.

We ignore the microlensing events in which the minimum distance between two lenses and also minimum impact parameter are not one order of magnitude larger than the Schwarzschild radius of the primary lens. Because they do not obey Keplerian laws.

Now, we generate data points over the light curves and astrometric trajectories. We assume that microlensing events are observed with the HST, OGLE and MOA surveys. The time interval between data points and the photometric uncertainties for each data point taken by survey telescopes are drawn from the archive of the microlensing light curves. The HST start observing these events after that the light curve reaches to a given threshold of magnification (i.e. \( \frac{\Delta m}{\Delta t} \)). We suppose that the HST is monitoring these microlensing events with cadence one day. The other factor in simulation of
the light curve is the exposure time for each data point taken by the HST. From the exposure time we can calculate the error bar for each data point. We can tune the exposure time in such a way that we have uniform error bars throughout the light curve. Here, we demand the photometric accuracy for each data point taken by the HST about 0.4 per cent which is chosen according to the recorded exposure times for those long-duration microlensing events being observed by the HST.

Astrometric observations of source star position are done by the HST with the astrometric accuracy of $\sigma_{\alpha} = 200$ microarcsecond. The simulated astrometric data points are shifted according to the astrometric accuracy of the HST by a Gaussian function. A sample of the simulated light curve and astrometric trajectory of source star is shown in Figure 4. In this figure the data taken by survey telescopes and the HST are shown with violet points and black stars. The parameters used to make this event are brought in the fourth row of Table 2. The time-variation rate of the astrometric centroid shift vectors is not constant. As a result, in some places over the astrometric trajectory the HST data points do not uniformly set despite similar cadences (see Figure 4). In this event, the orbital motion effect is obvious in the astrometric trajectory as well as the photometric light curve.

Our criterion for detectability of orbital motion is $\Delta \chi^2 = \chi_{OM}^2 - \chi_{SB}^2 > \Delta \chi_{th}^2$. We consider $\Delta \chi_{th}^2 = 250$ for both photometric and astrometric observations. From the Monte Carlo simulation we obtain that 80.2, 16.9 and 2.9 per cent of total simulated events are close, intermediate and wide binaries and the average detection efficiency $\epsilon_{OM}$ for the orbital motion detection in the astrometric trajectories and the amplification light curves of binary microlensing events with stellar-mass black holes are 57.3 and 48.2 per cent, respectively. As a result, in these events lens orbital motion can be seen most likely in astrometric trajectories whereas photometric signatures of orbital motion are often too small to be detected. Detecting lens orbital motion in their astrometric trajectories helps to discover further secondary components around the primary lens as well as resolve close/wide degeneracy.

In Figure 5 we plot the distribution functions of the time of the maximum photometric (green histogram) and astrometric (red histogram) deviations due to lens orbital motion, measured with respect to the time of the closest approach and in the unit of the Einstein crossing time $t_{E}$. For lens microlensing events with detectable effects of lens orbital motion in their light curves and astrometric trajectories respectively. According to this figure, the orbital motion signature in the astrometric trajectory can be seen at sometime often after one Einstein crossing time with respect to the time of the closest approach whereas this signal in the light curve can be detected at sometime almost before it. In wide or intermediate binary events with detectable astrometric signatures of orbital motion, the maximum deviation happens at very late time, e.g. $t - t_0 > 6t_E$, which have made some so small peaks in Figure 5.

In order to study the sensitivity of detecting orbital motion in astrometric trajectories of source star and photometric light curves on the parameters of the model, we plot the astrometric (solid line) and photometric (dashed line) detection efficiencies in terms of the relevant parameters of the binary lens and source in Figures 6 and 7. We ignore the irrelevant parameters that do not change the efficiency function. The detection efficiency function in terms of the binary lens and source parameters is given as follows.

(i) The first parameter is the primary lens mass, $M_1$. Here, the astrometric efficiency of detecting lens orbital motion rises with increasing the primary lens mass. The physical interpretation of this feature is that the angular Einstein radius and as a result of it the astrometric signal rise with increasing the primary lens mass. On the photometric detection efficiency there are two factors that effect inversely. With increasing the primary lens mass the Einstein radius rises which decreases the normalized distance between two lens components and the caustic size. On the other hand, the ratio of the Einstein crossing time to the orbital period of lenses motion.
increases. However, the first effect is dominant. Because, the photometric signature of orbital motion can more likely be detected while the source is crossing the caustic.

(ii) The second parameter is the ratio of the lens masses, $q$. The ratio of the lens masses has a geometrical effect on the shape of the caustic lines, where increasing it towards the symmetric shape maximizes the detection efficiency. Indeed, the size of the central caustic scales as $q$ [Gaudi 2012].

(iii) The third parameter is the projected Semi-major axis of lens orbit normalized to the Einstein radius. The photometric detection efficiency has the maximum amount around $1 R_E$ in which the caustic curves have the maximum size. However, the astrometric detection efficiency has the highest amount for close binaries with the large ratio of the Einstein crossing time to the orbital period.

(iv) Different topologies of caustic curve: The astrometric and photometric detection efficiencies are maximized in close and intermediate binaries respectively. Indeed, the size of the central caustic which indicates the probability of caustic crossing and the detectability of lens orbital motion photometrically has the highest amount for the intermediate microlensing events (see Figure 7). Note that most simulated events are close binaries.

(v) Impact parameter, $b_0$, shown in Figure 7. Decreasing the impact parameter increases the photometric detection efficiency. A smaller impact parameter from the center of the lens configuration rises the probability of the caustic crossing. However, the astrometric detection efficiency does not depend on the impact parameter.

(vi) The next parameter is $x = D_l/D_s$ the relative distance of the lens to the source star from the observer. The angular Einstein radius and astrometric signals increase by decreasing $x$. On the other hand, the Einstein radius has the highest amount around $x = 0.5$. When $x$ tends to 0.5, by increasing the Einstein radius the normalized distance between two lens components and the probability of the caustic crossing decrease.

(vii) and (viii) Inclination angles of orbital plane of lenses with respect to the sky plane, $\beta$ and $\gamma$. The astrometric detection efficiency is maximized in $\beta \sim 0$ and $\gamma \sim \pm \pi/2$ in which the rotation angle of the source trajectory with respect to the binary axis owing to lens orbital motion, $\theta_b$, has the maximum amount.

4 CONCLUSIONS

In this work we investigated lens orbital motion in astrometric microlensing and its detectability. In microlensing events, the astrometric centroid shift in source trajectories falls off much more slowly than the light amplification as the source distance from the lens position increases. Hence, perturbations developed with time e.g. the effect of lens orbital motion make considerable deviations in astrometric trajectories whereas the photometric signal of orbital motion can mostly be detected when the source is around the caustic curve.

The orbital motion of lenses changes the magnification pattern by changing the projected distance between two lenses as well as rotates the straightforward source star trajectory with respect to the binary axis. These effects can be seen in the microlensing light curve if lenses rotate each other considerably when the source star is around the caustic curve. At the same time, the orbital motion of lenses rotates the astrometric trajectory in the same was as the source trajectory. Depending on the size of the angular Einstein radius this effect can be seen at sometime often after one Einstein crossing time with respect to the time of the closest approach while this signal in the light curve can be detected at sometime almost before it.

Binary microlensing events with rotating stellar-mass black holes which have detectable astrometric shifts in the source star trajectories have most likely detectable orbital motion signatures. In these events, the ratio of the Einstein crossing time to the orbital period is high. Although, orbital motion effect in their light curves is too small especially when there is no caustic-crossing feature, but the astrometric signature owing to the orbital motion can be detected at the end parts of astrometric trajectories. In addition, the Hubble Space Telescope (HST) is observing some long-duration microlensing events which are likely to be owing to stellar-mass black holes, following the approved proposal by Sahu et al. (2010).

Hence, we expect that if there is a secondary lens, the astrometric signature of its orbital motion can much probably be observed.

By performing Monte Carlo simulation we evaluated the efficiency of detecting orbital motion in astrometric and photometric microlensing with binary stellar-mass black holes. We considered $\Delta \chi^2 > 250$ as the detectability criterion and concluded that astrometric efficiency is $87.3$ per cent whereas the photometric efficiency is $48.2$ per cent. From total simulated events $80.2$, $16.9$ and $2.9$ per cent were close, intermediate and wide binaries respectively. Also, the more massive lenses, the more detectable signatures of orbital motion. Detecting orbital motion in binary microlensing events helps not only to detect further secondary components even without any photometric binary signature but also to resolve close-wide degeneracy as well.

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