On the width of the $K^-D$ atomic ground state

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Abstract

Experiments at DAΦNE-Frascati and at J-PARC are scheduled to produce $K^-D$ atoms and observe their X-ray cascade down to the $1S$ ground state (g.s.), thereby measuring its strong-interaction width and shift away from a purely Coulomb state. A width $\Gamma_{1S} \lesssim 1$ keV will ensure good resolution of the X-ray transitions feeding the $1S$ g.s. Here we study the expected $K^-D$ $1S$ g.s. width from the perspective of global fits to level shifts and widths in heavier kaonic atoms across the periodic table, using $K^-$ nuclear optical potentials constructed from $\bar{K}N$ chiral interaction models. Special attention is paid to the subthreshold energy at which the $\bar{K}N$ subsystem interacts in the $K^-D$ atomic g.s. Within this approach we predict strong-interaction upward level shift of close to 700 eV and width of about 1.2 to 1.3 keV for the $K^-D$ atom $1S$ g.s., in fair agreement with genuinely three-body $K^-D$ atom calculations. Comparison is made with $\pi^-D$ atom phenomenology.

Keywords: $K^-$-nucleon and $K^-$-nucleus interactions near threshold; Kaonic atoms; Kaonic deuterium.

1. Introduction

Strong-interaction level shifts and widths observed in hydrogen and deuterium hadronic atoms provide valuable information on hadron-nucleon scattering lengths, as accomplished for pions and antiprotons [1]. For $\bar{K}$ mesons, following several high-resolution experiments [2], the SIDDHARTA-1 experiment at DAΦNE-Frascati [3, 4] provides a fairly accurate determination of the $K^-p$ complex scattering length by measuring the strong-interaction level shift and width in the $K^-H$ atomic $1S$ g.s. To extract the $K^-n$ complex scat-

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tering length one needs to form $K^-D$ atoms and observe their X-ray cascade down to the $1S$ g.s. Two experiments, SIDDHARTA-2 at DAΦNE-Frascati and E57 at J-PARC are running or scheduled to run, aiming to observe this X-ray cascade [2]. It is tacitly assumed, based on past theoretical calculations, that the broadening of $K^-D$ atom cascade lines leading to the $1S$ g.s. is of order 1 keV. Several genuinely three-body $K^-D$ atom calculations are listed in Table 1 in support of this working assumption. In these calculations the input $\bar{K}N$ potentials reproduce the SIDDHARTA-1 $K^-H$ 1$S$ level shift and width and, furthermore, the $K^-D$ 1$S$ level shift and width are derived directly from the calculated $K^-D$ atom complex energy g.s. eigenvalue, independent of the Deser formula or its improved versions [9, 10]. The Deser formula links the $K^-D$ 1$S$ atomic level shift and width to the purely nuclear $K^-D$ scattering length and is known to involve errors of up to 30% for the width $\Gamma$, see e.g. Ref. [5].

Table 1: $K^-D$ 1$S$ level shift ($\epsilon_{1S}$) and width ($\Gamma_{1S}$) in eV from three-body calculations. Note that $\epsilon_{1S} < 0$ stands for upward shift with respect to the purely Coulomb case.

| $-\epsilon_{1S}$ | $\Gamma_{1S}$ | Method | Reference |
|------------------|---------------|--------|-----------|
| 828              | 1055          | Faddeev + $V_{opt}$ | Shevchenko-Révai 2014 [5] |
| 800±30           | 960±40        | Faddeev | Révai 2016 [6]; see also [7] |
| 670              | 1016          | 3-body Schrödinger | Hoshino et al. 2017 [8] |

Given the model dependence of the input subthreshold $\bar{K}N$ interaction, in the presence of a quasi-free spectator nucleon in three-body calculations, the good agreement of $\lesssim 10\%$ between all three values listed in Table 1 for the $K^-D$ 1$S$ g.s. width, $\Gamma_{1S} \approx 1$ keV, is striking. These and other $K^-D$ calculations were implicitly questioned recently by Liu et al. [11] in the context of multiple-scattering expansions of the $K^-D$ scattering length in terms of $K^-p$ and $K^-n$ subthreshold scattering amplitudes. Shifting the $\bar{K}N$ energy argument from threshold towards the $\Lambda(1405)$ resonance, in order to accommodate the recoil energy of the spectator nucleon, is claimed there to increase $\Gamma_{1S}$ from $\approx 1.1$ keV up to $\approx 2.3$ keV. However, recalling that a recoil energy shift is implemented standardly in the Faddeev calculations listed in Table 1 the meaning of such a claim is questionable. Here we consider this issue differently, guided by lessons from optical model studies of kaonic atoms strong-interaction level shifts and widths across the periodic table [12, 13, 14].
The $K^{-}D$ optical model calculations with subthreshold $K^{-}N$ input reported in the present work do not support Liu et al. [11], yielding $\Gamma_{1S} \approx 1.2 - 1.3$ keV, in fair agreement with the values listed in Table 1.

The paper is organized as follows. In Sect. 2 we review the issue of subthreshold energy used in $K^{-}$ atoms calculations, particularly in few-body systems, applying it in Sect. 3 to optical model calculations of the $K^{-}D$ scattering length and the $K^{-}D$ atom 1S g.s. shift and width. A similar calculation of the $\pi^{-}D$ atom for which the 1S g.s. level is known experimentally is reported in Sect. 4, thereby testing the optical model methodology of the present work. In Sect. 5 we comment on multiple-scattering expansions of the $K^{-}D$ scattering length in terms of $K^{-}p$ and $K^{-}n$ scattering amplitudes input, demonstrating the extent to which the results of Liu et al. [11] differ from those of a genuinely three-body Faddeev calculation [15]. Finally, conclusions drawn from the present work are summarized in Sect. 6.

2. Subthreshold energy considerations in $K^{-}$ atoms

The $K^{-}$-nucleus optical potential used in global fits to strong-interaction data in kaonic atoms from $^7$Li to $^{238}$U is of the form $V_{K^-} = V_{K^-}^{(1)} + V_{K^-}^{(2)}$ [14]. Its single-nucleon term $V_{K^-}^{(1)}$ is given by

$$2\mu_K V_{K^-}^{(1)}(\rho) = -4\pi (\tilde{f}_{K^-p}(\rho)\rho_p + \tilde{f}_{K^-n}(\rho)\rho_n),$$

where $\mu_K$ is the $K^{-}$-nucleus reduced mass. The in-medium $K^{-}N$ scattering amplitudes in the $K^{-}$-nuclear center-of-mass (cm) frame, $\tilde{f}_{K^-N}(\rho)$, are kinematically related to the in-medium $K^{-}N$ cm amplitudes $f_{K^-N}(\rho)$,

$$\tilde{f}_{K^-N}(\rho) = (1 + \frac{A - 1}{A} \frac{\mu_K}{m_N}) f_{K^-N}(\rho), \quad N = p, n.$$  

In Eqs. (1,2), the density dependence of $f_{K^-N}$ arises partly by introducing a density-dependent subthreshold energy shift $\delta\sqrt{s(\rho)}$, defined below, and partly from Pauli correlations. The latter do not enter s-shell nuclear optical potentials and are suppressed here for $K^{-}D$.

The $V_{K^-}^{(2)}$ term of the optical potential is a phenomenological density-dependent term representing $K^{-}$ multinucleon processes:

$$2\mu_K V_{K^-}^{(2)}(\rho) = -4\pi B \left(\frac{\rho}{\rho_0}\right)^\alpha \rho, \quad \rho_0 = 0.17 \text{ fm}^{-3},$$
with a complex strength $B$ and a positive exponent $\alpha$, the three of which serve as fit parameters. For $\alpha = 1$, as used here, the $\rho^2$ dependence of $V_{K-}^{(2)}(\rho)$ agrees with the traditional $\rho^2$ dependence motivated in mesic atoms by absorption on two nucleons [16]. This term accounts for more than 20% of the $K^-$ absorption width measured in kaonic atoms, and it is essential in reaching good agreement between calculations and experiment [14].

Given that the $K^-N$ scattering amplitude (2) is strongly energy dependent, owing to the $\Lambda(1405)$ resonance, one needs to determine $K^-N$ sub-threshold energy values at which $f_{K^-N}$ should enter $V_{K-}^{(1)}$. The Mandelstam variable $\sqrt{s} = \sqrt{(E_{K^-} + E_N)^2 - (\vec{p}_{K^-} + \vec{p}_N)^2}$ which reduces to $(E_{K^-} + E_N)$ in the $K^-N$ two-body cm system is an acceptable choice, although it is not a conserved quantity in the $K^-N$ nuclear problem. Since spectator nucleons move the interacting $K^-N$ two-body subsystem outside of its cm, $(\vec{p}_{K^-} + \vec{p}_N)$ no longer vanishes, leaving $\vec{p}_{K^-}$ and $\vec{p}_N$ little correlated and making $f_{K^-N}$ and thereby $V_{K-}^{(1)}$ momentum dependent, or equivalently density dependent as discussed and practised in past $K^-$ atoms studies [12, 13, 14].

In $K^-N$ nuclear few-body problems, $A \leq 4$, where density is not introduced explicitly, one requires instead that a $\delta \sqrt{s} = \sqrt{s} - \sqrt{s_{th}}$ input choice is reproduced self consistently by the expectation value $\langle \delta \sqrt{s} \rangle$ generated in solving the respective hadron-nuclear few-body problem [17, 18, 19], viz.

$$\langle \delta \sqrt{s} \rangle = - \frac{B_A}{A} - \xi_N \frac{1}{A} \langle T_A \rangle + A - 1 \frac{1}{A} \xi_A \xi_K^- \frac{A}{A} \left( \frac{A - 1}{A} \right)^2 \langle T_K^- \rangle.$$  (4)

Here $\xi_{N(K^-)} = m_{N(K^-)}/(m_N + m_{K^-})$, $\xi_A = A m_N / (A m_N + m_{K^-})$, $T_A$ and $T_K^-$ denote the nuclear and $K^-$ kinetic energy operators in appropriate Jacobi coordinates, $B_A$ is the total binding energy (including $K^-$ if bound) and $\xi = \langle H - H_A \rangle$ with each Hamiltonian defined in its own cm frame.

### 3. Application to $K^-D$ atoms

Applying Eq. (4) directly to $K^-D$ atoms, with $A = 2$, $B_D=2.2$ MeV and $\langle T_D \rangle \approx 15 - 20$ MeV, we disregard the two $K^-$ terms which are negligibly small in light $K^-$ atoms. The resulting $K^-D$ atom subthreshold downward energy shift is rather small:

$$- \langle \delta \sqrt{s} \rangle_{K^-D} \approx 6.0 - 7.7 \text{ MeV},$$  (5)
considerably smaller than the several tens of MeV range of values encountered in heavier kaonic atoms analyses \cite{20}. Below we use a representative value of $\delta\sqrt{s} = -7$ MeV for demonstration.

![Diagram](image_url)

**Figure 1:** Calculated $K^-D$ 1S level shift $\epsilon_{1S}$ and width $\Gamma_{1S}$ as a function of the subthreshold energy $\delta\sqrt{s}$ at which the KM \cite{21} and BCN \cite{23} $K^-N$ amplitudes, input to $V_K^{(1)}$, are evaluated. The calculations use an optical potential $V_K = V_K^{(1)} + V_K^{(2)}$, with $V_K^{(2)}$ from global fits to kaonic atoms level shifts and widths across the periodic table \cite{14}.

The $K^-D$ atom 1S g.s. level shift and width are derived from the complex binding energy obtained by solving the Klein-Gordon (KG) equation, using an optical potential of the form Eqs. (1-3) with Gaussian deuteron densities discussed in Appendix A. The $V_K^{(1)}$, input $K^-N$ subthreshold amplitudes $f_{K^-N}$ are taken from the Kyoto-Munich (KM) chiral model \cite{21}, and from the Barcelona (BCN) chiral model \cite{22} (augmented recently by $K^-N$ absorption terms evaluated in nuclear matter \cite{23}). The $V_K^{(2)}$ complex parameter $B$, for $\alpha = 1$, was derived in a global fit to $K^-$ atom strong-interaction data across the periodic table \cite{14}. Calculated 1S level shift ($\epsilon_{1S}$) and width ($\Gamma_{1S}$) are shown in Fig. 1 as a function of the subthreshold energy shift $\delta\sqrt{s}$ at which the $K^-N$ amplitudes $f_{K^-N}$ were evaluated. The plotted values of the total width $\Gamma_{1S}$ are seen to increase steadily with $-\delta\sqrt{s}$, reflecting the increased Im $f_{K^-N}^{I=0}$ as one approaches the peak of the $\Lambda(1405)$ subthreshold resonance. However, the increase from $\Gamma_{1S}(\delta\sqrt{s} = 0) \approx 1.1 - 1.2$ keV to
\(\Gamma_{1S}(\delta\sqrt{s} = -7 \text{ MeV}) \approx 1.2 - 1.3 \text{ keV}\) is rather slow. As for absolute values we note that \(\Gamma_{1S}(\delta\sqrt{s} = 0) = 1091 \text{ eV}\), calculated here using the KM chiral model input, fares well with the value \(\Gamma_{1S} = 1016 \text{ eV}\) from Table 1 derived by solving the three-body \(K^-D\) Schrödinger equation, also with \(K^-N\) threshold amplitudes and using essentially the same KM chiral model input.

Regarding the multinucleon absorption partial width contributed by \(\text{Im } B\), it comes out relatively small in these optical-model calculations, about 80 eV in model KM and somewhat larger in model BCN. Old bubble chamber measurements constrain this partial width to 1.2±0.1% of the total \(K^-D\) 1S width [24], about 15 eV here, much smaller than our derived value of about 80 eV. This suggests that multinucleon absorption in heavier \(K^-\) atoms involves \(I = 1\) NN pairs rather than the \(I = 0\) pn pair of which the deuteron consists, quite differently from the way multinucleon absorption in pionic atoms is perceived, as discussed briefly in the next section. Interestingly, the widths calculated upon setting \(B = 0\) exceed by about 100 eV those for \(B \neq 0\) when taken from global \(K^-\) atoms fits. This somewhat unexpected behavior is representative of the saturation property of atomic widths caused by the strong imaginary part of the \(K^-\) nuclear optical potential which suppresses the overlap of the \(K^-\) atomic wavefunction with the nuclear density, thereby reducing its absorptive effect [25, 26]. A strong \(\text{Im } V_{K^-}\) is also responsible for the increased repulsive level shift \(\epsilon_{1S}\) exhibited in Fig. 1 in spite of \(\text{Re } V_{\text{opt}}\) becoming more attractive as the \(\Lambda(1405)\) is approached.

Further support for the saturation property of \(K^-\) atomic widths is provided by the approximate \(A\)-independence found for the imaginary part of the \(K^-\) nuclear scattering length of the \(K^-\) nuclear optical potential \(V_{K^-}\), Eqs. (1-3), in a global fit to \(K^-\) atoms data across the periodic table. This is demonstrated in Fig. 2 by the black points calculated using the KM chiral model \(K^-N\) amplitudes [21] with energy argument shifted in a self consistent procedure to density dependent subthreshold energies \(\delta\sqrt{s}(\rho)\) [12, 13, 14]. Applying the fitted \(K^-\) optical potential to \(K^-D\), at the \(K^-N\) threshold and at 7 MeV below it, we get the following values for the \(K^-D\) scattering length:

\[
a_{K^-D} : \quad -0.978 + i 1.193 \ (\delta\sqrt{s} = 0), \quad -1.260 + i 1.414 \ (\delta\sqrt{s} = -7),
\]

(6)

with \(\delta\sqrt{s}\) in MeV and \(a_{K^-D}\) in fm. Both imaginary values, added in red in Fig. 2, compare well with \(\text{Im } a_{K^-D} = 1.32 \text{ fm}\) from the \(K^-D\) Faddeev calculation using chiral interactions input reported in Ref. [3]. Clearly they are far off the value \(\text{Im } a_{K^-D} = 2.70 \text{ fm}\) claimed by Liu et al. [11].
Imag. scatt. length (fm)

kaonic atoms, KM+phen.

Figure 2: $\text{Im } a_{K^{-} A}$ from a global $K^{-}$ nuclear optical potential $V_{K^{-}} = V_{K^{-}}^{(1)} + V_{K^{-}}^{(2)}$ fit to kaonic atoms level shifts and widths [14] plotted (in black) as a function of the atomic number $A$. The input to $V_{K^{-}}^{(1)}$ are density dependent $\bar{K}N$ subthreshold amplitudes based on the KM chiral model [21] whereas $V_{K^{-}}^{(2)}$ is determined by the fit. The $A = 2$ points (in red) are from the present $K^{-} D$ calculation (lower: $\delta \sqrt{s} = 0$, upper: $\delta \sqrt{s} = -7$ MeV).

4. Comparison with $\pi^{-} D$ atoms

To make sure that applying a fitted $K^{-}$ nucleus optical potential to as light kaonic atom as $K^{-} D$ makes sense, we followed this same methodology in the $\pi^{-} D$ atom. The $\pi^{-}$ nuclear optical potential parameters are well fitted to pionic atoms data across the periodic table as demonstrated in Ref. [27]. We used this optical potential in a calculation of the strong-interaction $\pi^{-} D 1S$ g.s. level shift and width. No $\pi^{-} N$ subthreshold energy shift was applied since the energy dependence of the input $\pi^{-} N$ scattering amplitudes near threshold is known to be negligible [28]. Furthermore, the $p$-wave part of the $\pi^{-}$ optical potential proves to be completely ineffective in the $\pi^{-} D$ atom, so the structure of $V_{\pi^{-}}$ effectively used here agrees with that of $V_{K^{-}}$, Eqs. [13]. Our calculated (calc) $\pi^{-} D 1S$ g.s. level shift and width are

$$\epsilon_{1S}^{\text{calc}}(\pi^{-} D) = -2.56 \text{ eV}, \quad \Gamma_{1S}^{\text{calc}}(\pi^{-} D \rightarrow nn) = 0.62 \text{ eV},$$

(7)
disregarding the $\pi^{-}D \rightarrow nn\pi^{0}$ partial width which is known to be suppressed by several orders of magnitude. Experimentally [29, 30],

$$\epsilon_{1S}^{\exp}(\pi^{-}D) = -2.46 \pm 0.05 \text{ eV}, \quad \Gamma_{1S}^{\exp}(\pi^{-}D \rightarrow nn) = 0.86^{+0.03}_{-0.05} \text{ eV},$$

where the observed ratio $\Gamma_{\pi^{-}D}(nn)/\Gamma_{\pi^{-}D}(nn\gamma + nn\pi^{+}) = 2.76 \pm 0.04$ [30] was used. It is seen that, whereas the $\pi^{-}D$ $1S$ g.s. measured level shift is reproduced to within 4% (or within $2\sigma_{\text{exp}}$), the calculated hadronic width amounts to only 72% of the experimentally determined hadronic width. One may hardly expect more from such a simple model.

5. Remarks on $K^{-}D$ multiple scattering expansions

The energy dependence of the $K^{-}N$ scattering amplitudes below threshold was shown in Sect. 3 using two EFT chiral model inputs, to increase the $K^{-}D$ $1S$ g.s. width $\Gamma_{1S}$ by about 10%, from 1.1−1.2 keV to 1.2−1.3 keV. A much larger increase, by about 100% from 1.1 keV to 2.3 keV, was claimed recently by Liu et al. [11] using a similar EFT $\bar{K}N$ amplitudes input. These authors applied the Deser formula [10] to derive the $K^{-}D$ $1S$ g.s. level shift and width from the $K^{-}D$ scattering length $a_{K^{-}D}$. The disagreement noted above is traced back to a similar disagreement between respective values of $\text{Im}a_{K^{-}D}$ considered in these two approaches. From Eq. (6) here one deduces about 20% increase of $\text{Im}a_{K^{-}D}$, from $\approx 1.2 \text{ fm}$ to $\approx 1.4 \text{ fm}$, whereas Liu et al. [11] find about 70% increase, from $\approx 1.6 \text{ fm}$ to $\approx 2.7 \text{ fm}$.

To figure out the source of such a large increase of $\text{Im}a_{K^{-}D}$ in Ref. [11] we note that these authors rely on extension of the fixed-center multiple scattering (MS) expansion of $a_{K^{-}D}$ in terms of $\bar{K}N$ scattering lengths [31],

$$a_{K^{-}D} = \frac{\mu_{K^{-}D}}{m_{K^{-}}} \int d^{3}\vec{r}|\psi_{D}(\vec{r})|^{2} \hat{A}_{K^{-}D}(r),$$

where $\psi_{D}(\vec{r})$ is the wave function for nucleons in the deuteron, and

$$\hat{A}_{K^{-}D}(r) = \frac{\tilde{a}_{K^{-}p} + \tilde{a}_{K^{-}n} + (2\tilde{a}_{K^{-}p}\tilde{a}_{K^{-}n} - b_{x}^{2})/r - 2b_{x}^{2}\tilde{a}_{K^{-}n}/r^{2}}{1 - \tilde{a}_{K^{-}p}\tilde{a}_{K^{-}n}/r^{2} + 2b_{x}^{2}\tilde{a}_{K^{-}n}/r^{3}},$$

with $\tilde{a}_{K^{-}N} = a_{\bar{K}N}(1 + m_{K}/m_{N})$, and $b_{x} = \tilde{a}_{K^{-}p} + \tilde{a}_{K^{-}n}/\sqrt{1 + \tilde{a}_{\bar{K}n}/r}$. This expression can be somewhat simplified if desired [32]. In the first stage Liu et al. [11] considered the single-scattering (SS) term

$$a_{K^{-}D}^{SS} = \frac{\mu_{K^{-}D}}{\mu_{K^{-}N}} (a_{K^{-}p} + a_{K^{-}n}),$$

where $\mu_{K^{-}N} = a_{\bar{K}N}(1 + m_{K}/m_{N})$, and $b_{x} = \tilde{a}_{K^{-}p} + \tilde{a}_{K^{-}n}/\sqrt{1 + \tilde{a}_{\bar{K}n}/r}$. This expression can be somewhat simplified if desired [32].
replacing the threshold scattering lengths $a_{K-N}^{th}$ by appropriately constructed subthreshold scattering amplitudes $a_{K-N}^{sub}$ that account for the spectator-nucleon recoil in a standard Faddeev approach. This increases the threshold SS value $\text{Im} a_{K-D}^{SS}(th)=1.59 \text{ fm}$ to $\text{Im} a_{K-D}^{SS}(sub)=2.55 \text{ fm}$ as listed in the line denoted MS in Table 2. A similar increase is observed also in the two other calculations listed in the table, a Faddeev calculation by Toker et al. [15] and the present optical-potential calculation. Given that the input threshold values of $a_{K-p}$ and $a_{K-n}$ in all three calculations are similar, the near agreement between their $a_{K-D}^{SS}(sub)$ values is gratifying.

Table 2: $K^-D$ scattering lengths $a_{K-D}^{full}$ (in fm) calculated in three methods: (i) summing up a fixed-scatterer multiple-scattering (MS) series [10] with $a_{K-N}^{sub}$ input values [11]; (ii) solving exact $KNN$ Faddeev equations without introducing additional subthreshold dependence [15]; and (iii) solving the $K^-D$ two-body problem using $V_{opt}$ from a global fit to kaonic atoms data, taken here at $\delta \sqrt{s} = -7 \text{ MeV}$. Single-scattering contributions $a_{K-D}^{SS}$ [11], using $a_{th}^{th}$ and $a_{sub}^{sub}$ input amplitudes, are also listed.

| Method    | Ref. | $a_{K-D}^{SS}(th)$ | $a_{K-D}^{SS}(sub)$ | $a_{K-D}^{full}$ |
|-----------|------|--------------------|---------------------|-----------------|
| MS [11]   |      | $-0.58+i1.59$      | $-0.06+i2.55$       | $-0.59+i2.70$   |
| Faddeev   | [15] | $-0.37+i1.65$      | $-0.16+i2.44$       | $-1.47+i1.08$   |
| $V_{opt}$ | present | $-0.08+i1.86$  | $0.18+i2.49$       | $-1.26+i1.41$   |

In the second stage, Liu et al. [11] substituted in the MS series [10] the subthreshold amplitudes $a_{K-N}^{sub}$ used in the first stage to construct $a_{K-D}^{SS}(sub)$. This increases slightly $\text{Im} a_{K-D}^{SS}(sub)=2.55 \text{ fm}$ to $\text{Im} a_{K-D}^{full}=2.70 \text{ fm}$, whereas in the full Faddeev calculation [15] $\text{Im} a_{K-D}^{SS}(sub)=2.44 \text{ fm}$ decreases substantially to $\text{Im} a_{K-D}^{full}=1.08 \text{ fm}$. A similar, although somewhat weaker decrease to $\text{Im} a_{K-D}^{full}=1.41 \text{ fm}$ is observed in our $V_{opt}$ calculation, Eq. (6).

6. Conclusion

The present note was motivated by a recent claim that the width of the $K^-D$ atomic $1S$ g.s. might be larger than 2 keV [11], twice as much as values of $\Gamma_{1S} \approx 1 \text{ keV}$ obtained in genuinely three-body $K^-D$ atom calculations listed in Table [11]. The larger $\Gamma_{1S}$ is, the more ambiguous the identification of the $1S$ g.s. in forthcoming $K^-D$ atom cascade measurements might be [2]. A width of $\Gamma_{1S} \approx 1 \text{ keV}$ is expected to ensure the success of these experiments. The present calculation of the strong-interaction $1S$ level shift and width
in kaonic deuterium uses a $K^-$ nuclear optical potential, constructed from chiral-model subthreshold $K^-N$ scattering amplitudes $f_{K^-N}$, with added multinucleon dispersion and absorption parameters fitted to kaonic atoms data across the periodic table, from $^7$Li on. A key element in securing a good fit to the data is a self-consistent implementation of a density dependent $K^-N$ subthreshold energy argument $\delta \sqrt{s}$ for $f_{K^-N}$. Here we applied this optical potential methodology to the $K^-D$ atom using the right-hand side of Eq. (4) for $\delta \sqrt{s}$, as appropriate to few-body $K^-$ atomic and nuclear systems. This gives about 7 MeV downward subthreshold shift, considerably less than practised in heavier and denser $K^-$ atoms [20]. We then calculated the $1S$ g.s. level shift and width, finding a rather small increase of the width from $\Gamma_{1S}(\delta \sqrt{s} = 0) \approx 1.1 - 1.2$ keV to $\Gamma_{1S}(\delta \sqrt{s} = -7$ MeV) $\approx 1.2 - 1.3$ keV. We note that this range of values for $\Gamma_{1S}$ exceeds by merely 10-20% the range of values reached in genuinely three-body calculations [3, 6, 8], see Table 1. However, it is much smaller than the extremely large value $\approx 2.3$ keV reached by Liu et al. [11] by applying the Deser formula [10] to an equally large value of $\text{Im } a_{K^-D} \sim 2.7$ fm.

Finally, we compared in Table 2 the MS calculation [11] of $\text{Im } a_{K^-D}$ to our optical potential calculation, and more significantly to a genuinely three-body Faddeev calculation [15]. The extremely large value of $\text{Im } a_{K^-D}^{\text{full}}$ in the MS approach cannot be reconciled with the moderate values derived in the other two, more conservative approaches. We therefore question the validity of using subthreshold amplitudes in the fixed-center MS series [10] to simulate spectator-nucleon recoil effects, as done by Liu et al. [11]. Recoil corrections were considered by Baru et al. [10, 33, 34] who concluded that the leading recoil effect at threshold contributes less than 10% to $a_{K^-D}$. Forthcoming $K^-D$ experiments will hopefully help resolve this issue.

Appendix A. Choice of deuteron density

Optical model global analyses of hadronic atoms normally exclude atoms lighter than Li. Phenomenological nuclear densities are employed in terms of experimentally deduced r.m.s. radii. A natural choice for very light atoms is harmonic oscillator single-particle (s.p.) nuclear densities which for $A \leq 4$, namely $s$-shell nuclei, are simply one-parameter Gaussian functions:

$$\rho(\vec{r}_j; a) = (\sqrt{\pi}a)^{-3} \exp \left(-r_j^2/a^2\right), \quad \langle r_j^2 \rangle = \frac{3}{2} a^2, \quad j = 1, \ldots, A. \quad (A.1)$$
The s.p. density product $\rho(\vec{r}_1; a) \ldots \rho(\vec{r}_A; a)$ may be rewritten in terms of an overcomplete product of a center-of-mass (cm) Gaussian $\rho(\vec{R}; a/\sqrt{A})$, with $\vec{R} = (1/A) \sum_{j=1}^{A} \vec{r}_j$, and the same $A$ s.p. Gaussians (A.1) with arguments $\vec{r}_j$ replaced by $\vec{r}_j - \vec{R}$. This proves useful for $A \gg 4$ when the r.m.s. radius of $\vec{r}_j$ with respect to a fixed $\vec{R}$ is directly related to the nuclear matter radius.

For $A \leq 4$, it is useful to replace the overcomplete relative-coordinate density product $\rho(\vec{r}_1 - \vec{R}; a) \ldots \rho(\vec{r}_A - \vec{R}; a)$ by a product of $A - 1$ Gaussians in terms of Jacobi relative coordinates and their related size parameters $a_i$:

$$\vec{R}_i = \vec{r}_i - \frac{1}{i-1} \sum_{j=1}^{i-1} \vec{r}_j, \quad a_i = \sqrt{\frac{i-1}{i}} a, \quad i = 2 \ldots A. \quad (A.2)$$

Here, the last Jacobi coordinates $\vec{R}_A$ is proportional to the s.p. coordinate $\vec{r}_A$ with respect to the cm coordinate $\vec{R}$:

$$\vec{R}_A = \frac{A}{A-1} (\vec{r}_A - \vec{R}). \quad (A.3)$$

To relate the desired $\langle \vec{R}_A^2 \rangle$ to $\langle (\vec{r}_A - \vec{R})^2 \rangle$ we recall the $\frac{A}{A-1}$ factor in (A.3) and the $\sqrt{\frac{A-1}{A}}$ factor for $i = A$ in (A.2), yielding

$$\langle \vec{R}_A^2 \rangle = \frac{A}{A-1} \langle (\vec{r}_A - \vec{R})^2 \rangle, \quad (A.4)$$

in agreement with Elton’s book [35]. For the deuteron, $A=2$, subtracting the proton charge radius squared from the deuteron charge radius squared as given in Ref. [36] we obtain a matter r.m.s. radius of 1.954 fm, which upon multiplying by $\sqrt{2}$ from (A.4) gives $\langle \vec{R}_{A=2}^2 \rangle^{1/2} = 2.763$ fm, as used in the present $K^- D$ and $\pi^- D$ calculations.

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