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Geometrical and computer modeling of the technical workpieces main objects shaping

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Abstract. Geometrical and computer modeling of the main objects of technical workpieces shaping is considered: envelope, real envelope and removable layers based on the common methodology. The carried out research of two-dimensional surfaces and three-dimensional hyper-surfaces discriminants allowed to offer a definition of the envelope and real envelope families of curvatures and surfaces from a single viewpoint of analytical and numerical methods, respectively. This methodology is developed to perform 3D-modeling by means of CAD tools of both an envelope curvature and removable layers. The obtained surface models are used for analyzing the influence on the envelope shape of the of the shape parameters profile family of the shaping curvature, as well as the parameter of setting a product relatively to the tool.

1. Introduction
The shaping of a detail surfaces is an important engineering and technological challenge in the production of various workpieces in several mechanical engineering branches [1–4]. In this task, a significant place is given to issues related to the interaction of workpiece and a tool [5]. The surface obtained after the processing by the tool will consist of the surfaces family envelope [6] and, rather often, of the transition curves. Therefore, along with the development of a mathematical model of the envelope, in the process of shaping the issues of defining a model of a real surface – envelope, are considered [7]. For obtaining models of the envelope and real envelope families of curvatures and surfaces a lot of methods based on different approaches to obtaining these shaping objects are developed. Most of them apply the methods of differential geometry or the kinematic method. Recently, the possibilities of modern computer technologies are successfully used for simulation of the surfaces shaping modeling [8–16].

In addition to the envelope and real envelope, in practical applications the removed layers models are often required [17]. For the study of removable layers the methods different from those used for modeling the envelope and the real envelope surface are used. However, as a rule, in known publications, the study of only one of the objects of shaping by different methods is discussed. Thus, the main shaping objects are an envelope, real envelope and removable layers. During their modeling, in many tasks there are a number of unsolved or not properly solved problems. This is resulted from the lack of a common methodology that would allow, if necessary, to determine the discriminant and real envelope families of lines and surfaces, as well as removable layers using the capabilities of modern geometric, polygon, and solid computer modeling in their full.

2. Problem Statement
The studies of a family of lines or surfaces envelope are carried out in many papers. Mostly, they are based on the methods of differential geometry, or on the kinematic method. In these methods,
obtaining an equation that establishes the relationship of the parameters of the line or surface and their family is carried out. As a rule, in applications this equation is transcendental, which is associated with significant difficulties in both its solution and defining the envelope as a whole. Another area of defining the envelope of a family of lines or surfaces is based on the study of the surface or hyper-surface discriminants. These surfaces and hyper-surface are formed by displaying the collections of lines and surfaces obtained in the process of shaping, into a space of higher dimensionality. Its dimensionality one greater than the dimensionality of the space where the lines and surfaces are [18–22]. In these works, basically, a number of differential characteristics of the discriminants of two-dimensional surfaces or algebraic surfaces of higher dimensionality is determined.

Publications [20, 21] and some others contain the study of peculiarities of displaying the orthogonal projection of different dimensions algebraic surfaces. For example, in [20] the examples of obtaining hyper-surface discriminant defined by a polynomial are given. Displaying the orthogonal projection of some two-dimensional and three-dimensional surfaces defined by the equation in implicit or parametric equations for a plane and a hyper-plane is considered in [23–25]. The analysis of these works shows that the study of peculiarities of displaying the surfaces and hyper-surfaces orthogonal projection on the plane and the hyper-plane allows to solve the problem of determining the envelope of a family of lines or surfaces, by both analytical and numerical methods that in some cases seems to be most efficient. Following are the research results of the displaying by orthogonal projection of two-dimensional surface and three-dimensional hyper-surface, in various forms of their setting, on the coordinate plane and hyper-plane analytical methods and using the capabilities of CAD systems, which in many cases allows us to obtain the best possible results.

3. Theory

The envelope of the family of lines and surfaces as the discriminant of a surface or hyper-surface.

Let the investigated two-dimensional surface is given by the equation in implicit form

\[ F(x, y, z) = 0 \] (1)

and it is displayed by the orthogonal projection on the coordinate \( XY \)-plane. A characteristic feature of this display is a discriminant or a contour of the surface. The corresponding line on the surface is a criminant or a contour line. At criminant points tangent plane to the surface is parallel to the coordinate axis \( Z \), denoted by equation in the following form

\[ F_z(x, y, z) = 0 \] (2)

It is suggested to consider equation (1) as the equation of a new surface. Then, equations (1) and (2) determine their line of intersection, which is the contour line of the surface (1). The studies show that on the curves obtained as the intersection of the surface with planes parallel to the coordinate planes \( XZ \) and \( YZ \), there are extreme points relative to the coordinate plane \( XY \). Cutting planes are parallel to the coordinate axes that defines the direction of projection that is \( Z \) axis (fig.1). Such points may include the point of folds of the assemblage point and the singular points of the surface and they define the contour line of the surface.

On this basis, discriminant \( D \) of surface (1) is the union of sets of extreme points of view

\[ D = \sum_{i=1}^{n} \min_{a_i} f(x, z) \cup \max_{x=a_i} f(x, z) \]

if equation (1) is written down explicitly \( y = f(x, z) \), and the variable is the \( z \)-coordinate in its region of definition.

The result of analyzing the differential characteristics of surface criminant allows us to propose the methodology for calculating the coordinates of the considered points using numerical methods to define the conditional extremum of one of the coordinates, e.g. \( y \), with constraining to the other coordinate \( x \). The independent variable in this case is \( z \)-coordinate.

The illustration of the obtained result can be a model of the workpiece with a helical groove and tool surface with a cylindrical surface (fig. 2). The surfaces of these workpieces are conjugated. Fig. 2 shows that a cross section of a screw surface of the workpiece by the coordinate plane \( YZ \) contains the
extreme point $A$ with respect to coordinate plane $XY$. This point defines one of the points of the profile of the tool cylindrical surface mating with a given workpiece screw surface. The coordinates of the point can be determined by both analytical and numerical methods.

**Figure 1.** Quasi-screw surface, its cross sections $m$ and $n$ by planes parallel to the coordinate $ZY$ and $ZX$, respectively, and tangent $t$ to these cross sections at point $A$.

**Figure 2.** Solid models of the compartments of the workpiece 1 and tool 2, their sections 3 by the coordinate plane $YZ$ and tangency point $A$ to these sections.
Thus, if equation (1) is considered as the equation of the surface obtained by displaying congruent plane curves in space $R^2$, the envelope of this family can be commonly identified by both analytical and numerical methods. By analytical methods - by solving equation (1) and (2), while numerically - by imposing the conditions of one of coordinates and calculating the extreme value of another coordinate. The third coordinate takes the discrete values.

Let us now consider the mapping of the orthogonal projection of a two-dimensional surface defined by parametric equations:

$$x = f_1(u, v), y = f_2(u, v), z = f_3(u, v).$$

The condition under which the plane tangental to surface (3) is parallel to axis $Z$ – to the direction of projection is written as:

$$F(u, v) = f_{1u} \cdot f_2_{u} - f_{2u} \cdot f_1_{v} = 0$$

(4)

This equation establishes a link between the parameters $u, v$. It, together with equations (3) determine the discriminant of the surface. In addition, this equation can be regarded as the equation of some curve $\mu$ in the Cartesian coordinate system $U$ and $V$. Criminant $\mu'$ of surface (3) is obtained by mapping curve $\mu$ on the given surface. This mapping can be written in the form:

$$(\mu \subset F) \subset R^2 \rightarrow (\mu' \subset \Phi) \subset R^3,$$ where $F$ is given by equations (4).

The conducted research of curves $\mu$ and $\mu'$ allowed to establish the location of surface (3) criminant points relative to the respective coordinate planes. This result is similar to the one obtained above for the surface specified in implicit form.

In this case, discriminant $D$ of surface (3) is the union of the set of extreme points, namely:

$$D = \sum_{i=1}^{n} \min_{f_i} f_i(u, v) \lor \max_{f_i} f_i(u, v)$$

The variable in this relationship is one of the parameters of the studied surface, and the surface itself is dissected by a family of planes parallel to the coordinate plane $ZY$.

In a number of applied tasks there is a need to define the envelope of the two-parameter family of surfaces. So, if a family of surfaces is defined by two independent parameters, the study of the orthogonal mapping to the hyper-plane is a subject to a four-dimensional hyper-surface $\Sigma_1$, having the following form:

$$F(x, y, z, u, v) = 0,$$

(5)

where $u$ and $v$ are the independent variable parameters.

In this case, the hyper-surface display is performed in the directions of axes $u$ and $v$ on the corresponding coordinate hyper-plane. For such a hyper-surface, the equation of the tangent to the hyper-plane is written in the form:

$$F_u \cdot (x - x_0) + F_y \cdot (y - y_0) + F_z \cdot (z - z_0) + F_u \cdot (u - u_0) + F_v \cdot (v - v_0) = 0,$$

(6)

where $x_0$, $y_0$, $z_0$, $u_0$, $v_0$ are the coordinates of point $N$ of the surface.

At the points of hyper-surface, where the tangent points to this hyper-surface of a hyper-plane are parallel to axis $0U$, the following condition is met:

$$F_u(x, y, z, u, v) = 0,$$

(7)

If equation (7) is considered as the equation of additional four-dimensional hyper-surface $\Sigma_1$, the intersection of hyper-surfaces (5) and (7) defines a three-dimensional hyper-surface $\Sigma_2$. It is a criminant of hyper-surface $\Sigma_1$ at its orthogonal mapping along axis $u$.

The equation of the tangent hyper-plane to hyper-surface (7) has the form

$$F_{ux} \cdot (x - x_0) + F_{uy} \cdot (y - y_0) + F_{uz} \cdot (z - z_0) + F_{uu} \cdot (u - u_0) + F_{uv} \cdot (v - v_0) = 0,$$

(8)

where $x_0$, $y_0$, $z_0$, $u_0$, $v_0$ are the coordinates of a point $K$ of this hyper-surface.

The obtained hyper-planes (6) and (8) intersect in three-dimensional hyper-planes tangent to hyper-surface $\Sigma_2$.

At hyper-surface (5) criminant points the following condition is met
As a result, equation (9) defines one more additional four-dimensional hyper-surface \( \Sigma^2 \). Then hyper-surface \( \Sigma \), criminant \( \Sigma \) when it is mapped orthogonally along axis \( 0V \) is determined by the intersection of four-dimensional hyper-surfaces (5) and (9) and is a three-dimensional hyper-surface \( \Sigma_3 \). The equation of a hyper-plane relating to hyper-surface (9), is written in the form:

\[
F_{xu}(x-x_0) + F_{yv}(y-y_0) + F_{uz}(z-z_0) + F_{uv}(u-u_0) + F_{vu}(v-v_0) = 0.
\]

(10)

where \( x_0, y_0, z_0, u_0, v_0 \) are the coordinates of a point \( L \) of the surface.

The intersection of three-dimensional hyper-surfaces \( \Sigma_2 \) and \( \Sigma_3 \) sets a two-dimensional surface \( \Sigma_4 \), which is a hyper-surface (5) criminant when it is orthogonally mapped along axes \( u \) and \( v \) on hyper-plane \( XYZ \).

Since the points \( N, K \) and \( L \) belong not only to the hyper-surfaces, but also to a two-dimensional surface \( \Sigma_4 \), the tangent plane to the surface \( \Sigma_4 \) is determined by the intersection of hyper-planes (6), (8), and (10). From these equations we get:

\[
u - v_0 = \frac{-A \cdot F_{uv} + A_1 \cdot F_{v}}{\Delta}, \quad v - v_0 = \frac{A_1 \cdot F_u + A \cdot F_{uu}}{\Delta},
\]

where:

\[
A = F_{x}(x-x_0) + F_{y}(y-y_0) + F_{z}(z-z_0), \quad A_1 = F_{xu}(x-x_0) + F_{yu}(y-y_0) + F_{zu}(z-z_0),
\]

\[
\Delta = F_{uu} \cdot F_{vu} + F_{vu} \cdot F_{uv},
\]

and the equation of the plane tangent to the surface \( \Sigma_4 \), will have the following form:

\[
(x - x_0) \cdot \begin{vmatrix}
F_x & F_u & F_v \\
F_{ux} & F_{uu} & F_{uv} \\
F_{vx} & F_{uv} & F_{vv}
\end{vmatrix} + (y - y_0) \cdot \begin{vmatrix}
F_y & F_u & F_v \\
F_{uy} & F_{uu} & F_{uv} \\
F_{vy} & F_{uv} & F_{vv}
\end{vmatrix} + (z - z_0) \cdot \begin{vmatrix}
F_z & F_u & F_v \\
F_{uz} & F_{uu} & F_{uv} \\
F_{vz} & F_{uv} & F_{vv}
\end{vmatrix} = 0.
\]

(11)

As a result, the envelope of the two-parameter family of surfaces (5) is determined by the system of equations (5), (7) and (9), under conditions:

\[
|F_x| + |F_y| + |F_z| \neq 0 \quad \text{and} \quad |F_{uu}| + |F_{uv}| + |F_{vv}| \neq 0.
\]

Thus, the obtained results allow to define the envelope and a real envelope for families of lines and surfaces by both analytical methods, by obtaining the constraint equations of line or surface parameters, and the parameter of their family (which is often time-consuming), and numerical methods not requiring such equations. These results are used for computing polygonal and solid modeling not only of the envelope and real envelope, but also of the third object of shaping that are removable volumes. The accuracy and efficiency of the above stated is confirmed by the experiments.

4. Experimental results

4.1. Polygon modeling of congruent curves families

In applied problems, solving a design study of a cutting tool, there is a need to define the envelope of the family of congruent curves associated with the centroid of the tool rolls without slipping on the centroid of the workpiece. When mapping families of congruent curves in space \( R^3 \) some auxiliary surfaces are obtained [22]. The analysis of the geometry of these surfaces by means of CAD-systems allows to identify the effect of the tool settings relative to the workpiece, as well as the shape of the curve family on the envelope shape. As an example, consider the family of curves associated with centroid – a circle rolling on another one, the centroid of the circle. Let the radii of these centroid be \( R_x \) and \( R_y \), respectively, and the curve of the family is defined by the parametric equations:
After the mapping of this family into space $\mathbb{R}^3$ we get a surface which equation has the following form

$$
\begin{align*}
x_s &= x(t) \cdot \cos k\varphi - y(t) \cdot \sin k\varphi - A \cdot \sin \varphi, \\
y_s &= x(t) \cdot \sin k\varphi + y(t) \cdot \cos k\varphi + A \cdot \cos \varphi, \\
z_s &= p \cdot \varphi, 
\end{align*}
$$

(12)

where $\varphi$ is the parameter of family of curves $k = \frac{R_1 + R_2}{R_1}$, and $p$ is a large zero constant.

Comparing the system of equations (12) with the equation of cylindrical screw surface, it can be stated that surface (12) is obtained by affine transformation of the cylindrical screw surface. Consequently, the resulting surface is quasi-winding. A computer model of one of these surfaces is shown in fig.3a. To obtain the envelope of the considered family of lines, mapping the orthogonal projection of the surface onto the coordinate plane (fig. 3b) is made. For the model presented in fig. 3a, the horizontal outline of the surface is the envelope of family of curves related to the circle (fig. 3b). This model is used in the visualization mode to study the effect of setting the parameters of the workpiece relative to the tool on the envelope shape.

![Figure 3](image)

**Figure 3.** Model of a quasi-screw surface (a) and its orthogonal projection (b); 1 – the discriminant of the surface (envelope of a family of profiles)

Another example of computer modeling is determining the envelope of the family of curves (tool profiles) associated with the direct rolling without slipping on a circle. The mapping of curves into space is not applied here. Fig. 4 shows two families of curves for two positions of the tool relative to the workpiece. In fig.4a, the tool has a positive displacement (convergence) relative to the workpiece, while fig. 4b – negative (receding). As a result, in the first case, the tooth profile is outlined not only by the involute, but also by a significant value of the transition curve. In the second case (fig. 4b) there is no transition curve, the tooth profile increases, which increases the strength of the tooth when bending; still, its vertex is pointed.

The carried out experiments on computer modeling of families of lines on the plane or their mapping into space with subsequent projection onto a plane, allow to obtain a qualitative picture of the shape of the family of curves envelope depending not only on the shape of the source profile, but on the relative positions of a workpiece and a tool.
4.2. Computer solid modeling of shaping
Another direction of studying the main objects of shaping is solid-state computer modeling of the interaction models of the tool and the workpiece in accordance with the selected kinematic scheme. In the process of designing a cutting tool along with a profiling of the forming part, an important role is played by the study of cutting process. The developed computer programs in the CAD environment perform the shaping of the workpiece surface by the tool on the basis of solid modeling in an automated mode. During modeling, the interaction of solid models of the tool and workpiece is performed using Boolean operations. Fig. 5 shows the simulation of the shaping by a helical groove disc mill on a cylindrical workpiece. The modeling result is real envelope surface of this groove. In addition to the real envelope surface formed by the workpiece of the modeled process to create that surface, and, therefore, set of possible transitions, the configuration of the removed layer and the load on the cutting edges of the tool.

Some of the possibilities of computer modeling of the spatial schema of the gear are illustrated by fig. 6. It shows a model of a parts cylindrical billet and a model of layered, removed from it by an end mill. Based on the solid models, it is possible to determine the qualitative characteristics of the form, removable layers, and quantitative ones – of their volumes. So, from fig. 6 it follows that for the case under consideration, the configuration and volumes of the removable layers do not change during processing. If, after shaping, to remove the models of the removable layers from the model of the workpiece, it is possible to investigate the existence of possible transition curves on the real envelope surface, obtained by lateral surface of the cutter and its face plane.

Shaping solid modeling algorithms and programs in the tasks of flat gear scheme along with getting paired profile, and conjugated spatial pattern allow us to observe the process of sequential cutting of the hollows between the wheel teeth. The opportunity to obtain quantitative parameters is also provided: the volume of the layers, removed in one double stroke of the tool; the volume of the layer, removed by the side cutting edges of the tool and its peripheral edges. These parameters are used to establish the relationship of the removed volume from the value of the cutting parameter and their analysis allows to assign the optimal parameter values of cut, number of passes and depth of cut for each pass.

**Figure 4.** Collection of tool profiles related to direct rolling circumference: (a) if positive and (b) if negative displacement of the tool relative to a workpiece
Figure 5. The model of the disk cutter 1 in the process of shaping screw grooves in a cylindrical workpiece 2.

Figure 6. The model of the workpiece 1 after its shaping, and the 2 removable layers.

The described possibilities are illustrated by fig. 7. It shows the model of the tool – shaping cutter and a gear workpiece in the process of shaping, and the models of the layers removed by the tool. Thus, the computer solid modeling allows to model automatically the two main objects of shaping – real envelope surface and removable layers. In some tasks this approach may be a key for solving the described problem or it can add, if necessary, to geometric modeling.
5. Results and discussion
The experiments showed that the proposed methodology allows to commonly determine the discriminant and real envelope of the family of lines and surfaces by analytical and numerical methods, as well as removable layers successfully using the possibilities of modern geometric, polygon, and solid computer modeling. Further improvement of this methodology will allow to model the shaping of workpieces with non-circular centroid, too, which is a complex task even by using the known methods.

6. Conclusion
The proposed methodology of modeling the shaping of technical products surfaces allows to solve the following tasks:
- to develop mathematical models of surfaces and hyper-surfaces, obtained from the display of families of lines and surfaces into the space of dimensionality one greater than the dimensionality of the space, where these families are;
- to perform the mapping of the obtained surfaces and hyper-surfaces by orthogonal projection on the corresponding plane and hyper-plane; as a result, the envelope and envelopes of lines or surfaces families can be determined by both analytical and numerical methods, without obtaining complex rationalized equations of line or surface forms parameters and the parameter of the family;
- to obtain a model of new surfaces formed by the family of plane curves associated with the centroid of the tool, rolling without slipping on the centroid of the product; the computer visualization of such surfaces allows to track the change in the envelope of the profiles family depending on the profile shape and the relative positions of a workpiece and a tool;
- to create solid models of the removed volumes, based on the analysis which provides the ability to assign the optimal supply parameter value and the number of passes while shaping.

Figure 7. Models of the tool 1, the workpiece 2, in the process of its shaping, and removable layers 3
Thus, the proposed methodology allows, based on a common theoretical framework, to model the envelope, real envelope and removable layers in the process of shaping parts with a tool by both analytical and numerical methods and also with the use of modern computer technology, which provides the necessary quality of designed products.

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