Stability of $R$ parity in supersymmetric models extended by $U(1)_{B-L}$

J. E. Camargo-Molina, B. O’Leary, and W. Porod

Institut für Theoretische Physik und Astronomie, Universität Würzburg
Am Hubland, 97074 Würzburg

F. Staub

Bethe Center for Theoretical Physics & Physikalisches Institut der Universität Bonn,
53115 Bonn, Germany

Abstract

We perform a study of the stability of $R$-parity-conserving vacua of a constrained version of the minimal supersymmetric model with a gauged $U(1)_{B-L}$ which can conserve $R$-parity, using homotopy continuation to find all the extrema of the tree-level potential, for which we also calculated the one-loop corrections. While we find that a majority of the points in the parameter space preserve $R$-parity, we find that a significant portion of points which naively have phenomenologically acceptable vacua which conserve $R$-parity actually have deeper vacua which break $R$-parity through sneutrino VEVs. We investigate under what conditions the deeper $R$-parity-violating vacua appear. We find that while previous exploratory work was broadly correct in some of its qualitative conclusions, we disagree in detail.

Keywords: supersymmetry, $R$-parity violation, vacuum stability, extended gauge sector
I. INTRODUCTION

With the discovery of a resonance at 126 GeV at the LHC [1, 2], the interpretation that it is the Higgs boson of the standard model (SM) raises the issue of the stability of the SM vacuum [3–7]. Since there is only one field in the SM that can possibly have a non-zero vacuum expectation value (VEV) (assuming Lorentz invariance), finding the minima of the potential energy is straightforward, though of course evaluating it to the accuracy required is quite involved [5–7].

Many extensions of the SM introduce extra scalar fields, and thus could in principle have many more VEVs. In particular, the minimal supersymmetric extension of the SM (MSSM) not only introduces complex scalar partners for each SM fermion, it also introduces a second Higgs $SU(2)_L$ doublet. Even only allowing for the neutral components of the Higgs doublets to gain non-zero VEVs, the vacuum structure is non-trivial. Fortunately, as a particular case of a two-Higgs-doublet model, there is a proof that at tree level, there are at most two minima that conserve electric charge [8], and there are analytic formulae to determine which vacuum has a lower energy at tree level [9].

However, the possibility of VEVs for the scalar partners of the SM fermions, the sfermions, is a troubling prospect, as all but the sneutrinos are charged under $SU(3)_c$ and/or $U(1)_{em}$, and thus any VEVs for these scalars are ruled out by these being good symmetries of the vacuum. Unfortunately, the potential as a function of several scalar fields is so complicated that even at tree level, finding the global minimum is highly non-trivial, in general [10–14]. The only tractable approaches thus far have been to investigate the “most dangerous” possibilities of stop or stau VEVs [12, 15, 16].

Sneutrino VEVs, on the other hand, are not necessarily a problem. Of course, they would break $R$-parity, leading to mixings between SM and SUSY particles carrying the same spin and $SU(3)_c \times U(1)_{em}$ quantum numbers, see e.g. [18] and refs. therein. However, in the absence of baryon-number-violating trilinear couplings in the superpotential, proton decay would still be perturbatively forbidden. If $R$-parity is conserved, the lightest supersymmetric partner (LSP), if uncharged under $SU(3)_c$ and $U(1)_{em}$, could be a perturbatively stable dark matter candidate [19, 20]. If $R$-parity is not conserved, the LSP is not absolutely stable,

\footnote{For example, in the case of bilinear $R$-parity violation, the sneutrino VEVs have to be small to explain neutrino data implying that in this model essentially the conclusions concerning the minima of the potential can be taken over from the MSSM [17].}
but, depending on the size of the sneutrino VEVs, a small amount of $R$-parity violation still allows for a metastable LSP to make up the observed dark matter. However, even if the neutralino is not sufficiently long-lived, another particle like the gravitino [21–25] or the axion [25–27] could be long-lived enough to serve as dark matter.

The conservation of baryon number $B$ and of lepton number $L$ is only accidental in the SM, and this conservation through the imposition of $R$-parity may appear to be rather ad hoc in the MSSM, since the only difference between a lepton $SU(2)_L$ doublet and the Higgs $SU(2)_L$ doublet responsible for mass for the down-type quarks $H_d$ is that $H_d$ is assigned $L = 0$ instead of 1. Still, noting that $R$-parity is defined as $(-1)^{3B+L−2s} = (-1)^{3(B−L)−2s}$ if lepton number is integer or half-integer for every field, one may consider a symmetry based on $B − L$ as a motivated extension.

Adding $U(1)_{B−L}$ as an extension to the SM [28–32] or the MSSM [33–35] has been considered quite a lot in the literature, as in addition to motivating $R$-parity, it leads to a rich phenomenology, and, in some models, naturally leads to type I see-saw mechanisms for neutrino masses.

The most minimal of such extensions of the MSSM requires superfields with right-handed neutrinos as fermionic components to cancel gauge anomalies, and their scalar components provide the VEVs necessary to break $U(1)_{B−L}$ as required by phenomenology. Already such a model has eight complex scalar fields that in principle could have non-zero VEVs without breaking $SU(3)_c$ or $U(1)_{em}$, with many possible minima.

Conserved $R$-parity is still attractive from a phenomenological point of view, since there are tight limits on many $R$-parity-violating parameters [36–38]. One can extend the neutral sector with further superfields with VEVs which break $U(1)_{B−L}$ without breaking $R$-parity [33–35]. However, given the complicated structure of the potential, it is prudent to be concerned about whether such extensions really do conserve $R$-parity at their global minima over relevant parameter ranges.

In the following sections, we investigate how stable $R$-parity-conserving vacua are in such a minimal $U(1)_{B−L}$-gauged extension of the MSSM. In sec. II we define the model. In sec. III we describe how we approach the difficult problem of minimizing such a complicated potential, including going to the full one-loop potential, since it is well-known that there can be rather large corrections in the Higgs sector of the MSSM [39–62]. In sec. IV we present the results of two scans over phenomenologically interesting regions of the parameter space,
II. THE MODEL

A. Particle content and superpotential

There are several ways to extend the MSSM by $U(1)_{B-L}$. Here we choose the minimal extension which allows for a spontaneously broken $U(1)_{B-L}$ without necessarily breaking $R$-parity. This requires the addition of two SM gauge-singlet chiral superfields carrying $B - L$ which may develop VEVs, as well as the addition of three generations of superfields containing right-handed neutrinos. We refer to this model as the BLSSM.

The BLSSM appears to be a relatively straightforward extension of the MSSM, yet it has a rich phenomenology, with a $Z'$ boson (with prospects for the LHC discussed in \cite{63}), Majorana neutrinos with see-saw masses, several qualitatively new dark matter candidates with respect to the MSSM \cite{64}, and a rich Higgs boson sector \cite{65}. It also has many interesting technicalities \cite{66}, which are fully explained in \cite{67}.

However, all the above works assume that $R$-parity is conserved. Indeed, the model was constructed as the minimal extension of the MSSM that includes $U(1)_{B-L}$ as an extra gauge symmetry, while allowing it to break spontaneously without necessarily violating the conservation of $R$-parity. As mentioned in the introduction, VEVs for the scalar partners of the right-handed neutrinos could break $U(1)_{B-L}$ at the cost of losing $R$-parity, and aspects of the LHC phenomenology in the case of broken $R$-parity are discussed in \cite{68,69}. The purpose of this work is to investigate how robustly $R$-parity is conserved for parameters of phenomenological interest.

While we refer the reader to Ref. \cite{67} for a full description of the BLSSM and its implementation in SARAH \cite{70,73} and SPheno \cite{74,75}, for reference we recap the gauge symmetries and particle content of the model here.

The model consists of three generations of matter particles including right-handed neutrinos which can, for example, be embedded in $SO(10)$ 16-plets. For convenience, we refer to their scalar components as R-sneutrinos. Moreover, below the GUT scale the usual MSSM Higgs doublets are present, as well as two fields $\eta$ and $\bar{\eta}$ responsible for the breaking of

and in sec. [IV A], we compare our results to previous investigations in the literature. Finally, we sum up in sec. [V].
TABLE I. Chiral superfields and their quantum numbers.

| Superfield | Spin 0 | Spin $\frac{1}{2}$ | Generations | $U(1)_Y \otimes SU(2)_L \otimes SU(3)_C \otimes U(1)_{B-L}$ |
|------------|--------|--------------------|-------------|--------------------------------------------------|
| $\hat{Q}$  | $\tilde{Q}$ | $Q$              | 3           | $(\frac{1}{6}, 2, 3, \frac{1}{6})$               |
| $\tilde{d}^c$ | $\tilde{d}^c$ | $d^c$           | 3           | $(\frac{1}{7}, 1, 3, -\frac{1}{6})$             |
| $\tilde{u}^c$ | $\tilde{u}^c$ | $u^c$           | 3           | $(\frac{2}{3}, 1, 3, -\frac{1}{6})$             |
| $\hat{L}$  | $\tilde{L}$ | $L$              | 3           | $(-\frac{1}{2}, 2, 1, -\frac{1}{2})$           |
| $\tilde{e}^c$ | $\tilde{e}^c$ | $e^c$           | 3           | $(1, 1, 1, \frac{1}{2})$                        |
| $\tilde{\nu}^c$ | $\tilde{\nu}^c$ | $\nu^c$     | 3           | $(0, 1, 1, \frac{1}{2})$                        |
| $\hat{H}_d$ | $H_d$ | $\tilde{H}_d$       | 1           | $(-\frac{1}{2}, 2, 1, 0)$                       |
| $\hat{H}_u$ | $H_u$ | $\tilde{H}_u$       | 1           | $(\frac{1}{7}, 2, 1, 0)$                        |
| $\tilde{\eta}$ | $\eta$ | $\tilde{\eta}$    | 1           | $(0, 1, 1, -1)$                                 |
| $\tilde{\bar{\eta}}$ | $\bar{\eta}$ | $\tilde{\bar{\eta}}$ | 1          | $(0, 1, 1, 1)$                                 |

The superpotential is given by

$$W = Y_u^{ij} \hat{u}_i^c \hat{Q}_j \hat{H}_u - Y_d^{ij} \hat{d}_i^c \hat{Q}_j \hat{H}_d - Y_e^{ij} \hat{e}_i^c \hat{L}_j \hat{H}_d + \mu \hat{H}_u \hat{H}_d$$

$$+ Y_{\nu}^{ij} \hat{\nu}_i^c \hat{L}_j \hat{H}_u - \mu' \hat{\eta} \hat{\bar{\eta}} + Y_x^{ij} \hat{\nu}_i^c \hat{\nu}_j^c,$$

and we have the additional soft SUSY-breaking terms:

$$\mathcal{L}_{SB} = \mathcal{L}_{\text{MSSM}} - m_{\tilde{\eta}}^2 |\tilde{\eta}|^2 - m_{\tilde{\bar{\eta}}}^2 |\tilde{\bar{\eta}}|^2 - m_{\tilde{\nu}^c, ij}^2 (\tilde{\nu}_i^c)^* \tilde{\nu}_j^c$$

$$+ \left( T_{\nu}^{ij} H_u \tilde{\nu}_i^c \tilde{L}_j + T_{\nu}^{ij} \eta \tilde{\nu}_i^c \tilde{\nu}_j^c - \lambda_{B'} \lambda_{B'} M_{BB'} - \frac{1}{2} \lambda_{B'} \lambda_{B'} M_{BB'} - \eta \bar{\eta} B_{\nu} + c.c \right),$$

with $i, j$ being the generation indices. Without loss of generality one can take $B_\nu$ and $B'_{\nu'}$ to be real. The extended gauge group breaks to $SU(3)_C \otimes U(1)_{em}$ as the Higgs fields and
bileptons receive vacuum expectation values (VEVs):

\[ H_d^0 = \frac{1}{\sqrt{2}} (v_d + \sigma_d + i\phi_d), \quad H_u^0 = \frac{1}{\sqrt{2}} (v_u + \sigma_u + i\phi_u), \quad (3) \]

\[ \eta = \frac{1}{\sqrt{2}} (v_\eta + \sigma_\eta + i\phi_\eta), \quad \bar{\eta} = \frac{1}{\sqrt{2}} (v_{\bar{\eta}} + \sigma_{\bar{\eta}} + i\phi_{\bar{\eta}}). \quad (4) \]

We define \( \tan \beta' = v_\eta / v_{\bar{\eta}} \) in analogy to the ratio of the MSSM VEVs \( \tan \beta = v_u / v_d \). For certain parameter combinations a spontaneous breakdown of \( R \)-parity can occur as also some or all sneutrinos can obtain non-vanishing VEVs \[35\]. We denote the VEVs for the sneutrinos of the \( SU(2)_L \) doublets \( \tilde{L}_i \) by \( v_{L,i} \) and those of the \( SU(2)_L \) singlet sneutrinos \( \tilde{\nu}_i^c \) by \( v_{R,i} \), with \( i = 1, 2, 3 \).

**B. Constrained model**

We will consider in the following a scenario motivated by minimal supergravity assuming a GUT unification of all soft SUSY-breaking scalar mass parameters as well as a unification of all gaugino mass parameters:

\[ m_0^2 = m_{H_d}^2 = m_{H_u}^2 = m_\eta^2 = m_{\bar{\eta}}^2, \quad (5) \]

\[ m_0^2 = m_D^2 = m_U^2 = m_Q^2 = m_E^2 = m_L^2 = m_{\tilde{\nu}}^2, \quad (6) \]

\[ M_{1/2} = M_1 = M_2 = M_3 = M_{\tilde{B}}. \quad (7) \]

Also, for the trilinear soft SUSY-breaking couplings, we assume the ordinary mSUGRA-inspired conditions

\[ T_i = A_0 Y_i, \quad i = e, d, u, x, \nu. \quad (8) \]

Furthermore, we assume that there are no off-diagonal gauge couplings or off-diagonal gaugino mass parameters present at the GUT scale, motivated by the possibility that the two Abelian groups are a remnant of a larger product group which gets broken at the GUT scale, as discussed in \[67\], to which we refer the reader for details of the gauge coupling structure.

In addition, we consider the mass of the \( Z' \) and \( \tan \beta' \) as inputs and use the following set of free parameters:

\[ m_0, M_{1/2}, A_0, \tan \beta, \tan \beta', \text{sign}(\mu), \text{sign}(\mu'), m_{Z'}, Y_x \text{ and } Y_\nu. \quad (9) \]
$Y_\nu$ is constrained by neutrino data and must therefore be very small in comparison to the other couplings in this model, as required by the embedded TeV-scale type-I seesaw mechanism, so we take $Y^{ij}_\nu = 10^{-5} \delta^{ij}$ as its precise structure does not affect any of our conclusions. $Y_x$ can always be taken diagonal and thus effectively we have 9 free parameters and 2 signs. We denote this constrained version of the BLSSM as the CBLSSM.

III. MINIMIZING THE POTENTIAL

The primary question that we wish to answer is: within the CBLSSM, what portion of the parameter points which have a phenomenologically acceptable, $R$-parity-conserving local minimum of the potential turn out to have a global minimum with different VEVs, and are there any patterns in the parameters of such points? By “phenomenologically acceptable”, we restrict ourselves to parameter points which have a local minimum with the expectation values of the Higgs doublets leading to the correct values for $m_W$ and $m_Z$, there are no tachyons, and no charged or colored scalar has a VEV. Hence any minima that are “phenomenologically acceptable” and $R$-parity-conserving have expectation values for the Higgs doublet fields $H_d, H_u$ and the bilepton fields $\eta, \bar{\eta}$, and no other field has a non-zero expectation value.

Our method to address this question was as follows: we performed a random scan over a range of input parameters, constrained to having phenomenologically acceptable, $R$-parity-conserving local minima at expectation values for the Higgs doublet and bilepton fields given as input; we then found all the extrema of the tree-level potential for each parameter point allowing non-zero sneutrino VEVs; we evaluated the loop-corrected potential at these points (ensuring that the corrections did not move the minima significantly) and selected the deepest, and thus can say if the minimum chosen as input was not the global minimum.

A. Generation of parameter points

The package SARAH was used to create a SPhe no executable specific to the CBLSSM, which was used with the SSP package [76] to perform random scans over two different parameter regions. The first scan, which we refer to as the “democratic” scan, took random values for each diagonal entry of the R-sneutrino – bilepton Yukawa coupling $Y_x$ independently over
its range. The other, which we refer to as the “hierarchical” scan, kept the (1, 1) and (2, 2) entries as $10^{-3}$ and $10^{-2}$ respectively. The hierarchical scan was motivated by the hierarchy of the quark and charged lepton Yukawa couplings, but is not favored for any other reason. The ranges of the CBLSSM parameters for each scan are shown in Tab. II.

The mass parameter ranges were chosen to be consistent with non-observation of sparticles at the LEP and LHC experiments\(^2\) while remaining in a region where the LHC may be able to see some phenomena of the model, while the couplings were chosen to cover the perturbative range. Thus we generated sets of parameter points over the full region of phenomenological interest where $R$-parity is possibly conserved.

In general, picking a set of GUT-scale parameters is very unlikely to result in a potential which has a phenomenologically acceptable minimum. Instead of wasting time searching the parameter space for such points, the strategy adopted in SPheno (and others, such as ISAJET \cite{77}, SOFTSUSY \cite{78} and SUSPECT \cite{79}) is to instead fix some parameters (in our case, $\mu, B_\mu, \mu', B_{\mu'}$) at the SUSY scale, defined as $Q_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, by demanding that the tadpole equations are satisfied for VEVs given as input (indirectly, by specifying $m_Z, \tan \beta, m_Z', \tan \beta'$). This is sufficient to ensure that the chosen point is an extremum, but does not guarantee that the extremum is not a saddle point or maximum. Any parameter points that were found to have such engineered extrema at their input VEVs which were saddle points or maxima (by the presence of tachyons) were discarded.

**B. Finding all the tree-level extrema**

As mentioned in the introduction, finding the global minimum of even the tree-level potential as a function of several scalar fields is highly non-trivial. The Gröbner basis method can be used to find all the minima of a polynomial function, as has recently seen some discussion in the literature \cite{80, 81}. A less well-known method is that of homotopy continuation \cite{82, 83}, which has found use in several areas of physics \cite{84–86}, in particular finding string theory vacua \cite{87, 88} and extrema of extended Higgs sectors \cite{89}, where the authors investigated a system of two Higgs doublets with up to five singlet scalars in a general tree-level potential. In contrast, the Gröbner basis method is deemed prohibitively

\(^2\) As discussed in \cite{63} the bounds on $m_{Z'}$ are about 300 GeV lower than claimed by the LHC experiments once gauge kinetic mixing is taken into account properly. For squarks and gluinos we required them to be above one TeV.
| Parameter | Common to both |
|-----------|----------------|
| $M_{1/2}$ | 100 – 1000 |
| $M_0$     | 100 – 3000 |
| $A_0$     | -3000 – 3000 |
| $\tan \beta$ | 3 – 45 |
| $m_{Z'}$  | 1500 – 3000 |
| $\tan \beta'$ | 1.0 – 1.5 |

| Parameter | Democratic | Hierarchical |
|-----------|------------|--------------|
| $Y^{11}_x$ | 0.05 – 0.6 | fixed $10^{-3}$ |
| $Y^{22}_x$ | 0.05 – 0.6 | fixed $10^{-2}$ |
| $Y^{33}_x$ | 0.05 – 0.6 | 0.1 – 0.6 |

**TABLE II.** Ranges of parameters used in generating the samples. All dimensionful quantities are to be read as in units of GeV. The signs of $\mu$ and $\mu'$ were fixed to both be positive. The democratic scan consisted of 2330 points, the hierarchical of 1640 points.

Computationally expensive for systems involving more than five or six degrees of freedom [80], while for our purposes eight to ten degrees of freedom are necessary, and the solutions for thousands of parameter points needed to be calculated.

The numerical polyhedral homotopy continuation method is a powerful way to find all the roots of a system of polynomial equations quickly [90]. Essentially it works by continuously deforming a simple system of polynomial equations with known roots, with as many roots as the classical Bézout bound of the system that is to be solved (i.e. the maximum number of roots it could have). The simple system with known roots is continuously deformed into the target system, with the position of the roots updated with each step. We made use of the publicly-available program HOM4PS2 [90] to find all the extrema of the tree-level potential via the polynomial tree-level tadpole equations. The tadpole equations for the simple case of one generation of sneutrinos with non-zero VEVs are given by eqs. (10)-(15) below for illustration. For each parameter point, the complete set of tadpole equations for the appropriate number of allowed non-zero sneutrino VEVs were passed to HOM4PS2, which provided the VEV configurations of all the solutions of the system of equations.
1. Tadpole equations

If we consider $R$-parity violation in the case where only one generation of sneutrino acquires a VEV and restrict ourselves to real parameters, the tree-level potential minimization conditions, or tadpole equations, are:

\[
t_d = \frac{1}{8} v_d \left( g g_{BL} \left( -2 v_\eta^2 + 2 v_\eta^2 - v_R^2 + v_L^2 \right) + \left( g_1^2 + g_2^2 \right) \left( -v_u^2 + v_a^2 + v_L^2 \right) \right) - \frac{1}{\sqrt{2}} v_L v_R Y_\nu \mu + v_d \left( m_{H_u}^2 + \mu^2 \right) - v_u B_\mu \tag{10}
\]

\[
t_u = \frac{1}{8} v_u \left( \left( -g_1^2 + g_2^2 \right) \left( -v_u^2 + v_a^2 + v_L^2 \right) + g g_{BL} \left( 2 v_\eta^2 - 2 v_\eta^2 - v_L^2 + v_R^2 \right) \right) + \frac{1}{2} \left( -2 v_d B_\mu + v_L v_R \left( 2 v_\eta Y_\nu \sqrt{2} T_\nu \right) + v_u \left( 2 \left( m_{H_u}^2 + \mu^2 \right) + \left( v_L^2 + v_R^2 \right) Y_\nu^2 \right) \right) \tag{11}
\]

\[
t_L = \frac{1}{8} v_L \left( \left( g_1^2 + g_2^2 \right) \left( -v_u^2 + v_a^2 + v_L^2 \right) + g g_{BL} \left( -2 v_\eta^2 + 2 v_\eta^2 - v_R^2 + v_L^2 \right) \right) + g g_{BL} \left( -2 v_\eta^2 + 2 v_\eta^2 - v_R^2 + v_L^2 \right) \tag{12}
\]

\[
t_R = \frac{1}{8} g g_{BL} v_R \left( -g \left( -v_u^2 + v_a^2 + v_L^2 \right) + g g_{BL} \left( 2 v_\eta^2 - 2 v_\eta^2 - v_L^2 + v_R^2 \right) \right) + \frac{1}{2} \left( 2 v_\eta^2 Y_x^2 + v_R \left( -2 \sqrt{2} \mu' v_\eta Y_x + 2 \left( \sqrt{2} v_\eta T_x + m_{\eta}^2 \right) + 4 v_\eta^2 Y_x^2 + \left( v_L^2 + v_u^2 \right) Y_\nu^2 \right) \right)
+ v_L \left( \sqrt{2} v_\eta T_\nu + Y_\nu \left( 2 v_\eta v_\eta Y_x + \sqrt{2} v_d \mu \right) \right) \tag{13}
\]

\[
t_\eta = \frac{1}{4} g g_{BL} v_\eta \left( g \left( -v_u^2 + v_a^2 + v_L^2 \right) + g g_{BL} \left( -2 v_\eta^2 + 2 v_\eta^2 - v_R^2 + v_L^2 \right) \right) + v_\eta \left( 2 v_\eta^2 Y_x^2 + m_{\eta}^2 + \mu'^2 \right)
+ v_L v_R Y_x Y_\nu - v_\eta B_\mu + \frac{1}{\sqrt{2}} v_R T_x \tag{14}
\]

\[
t_\bar{\eta} = \frac{1}{4} g g_{BL} v_\bar{\eta} \left( -g \left( -v_u^2 + v_a^2 + v_L^2 \right) + g g_{BL} \left( 2 v_\eta^2 - 2 v_\eta^2 - v_L^2 + v_R^2 \right) \right)
- \frac{1}{\sqrt{2}} \mu' v_R^2 Y_x + \left( m_{\bar{\eta}}^2 + \mu'^2 \right) v_\bar{\eta} - v_\eta B_\mu \tag{15}
\]

Obviously this is a highly coupled system of cubic equations in the VEVs and can only be solved by numerical methods. One might be tempted to simplify these equations by using $|v_L/v_d|, (v_u^2 + v_a^2)/(v_\eta^2 + v_R^2) \ll 1$ to satisfy the phenomenological constraints such as the required hierarchy of the vector boson masses or neutrino masses. However, for our purposes we are only allowed to do so to engineer the starting minimum, but have to use the general formulas to check if the phenomenologically allowed minimum is indeed the deepest one.
2. Vacuum expectation value phase rotations

Degenerate lines of minima, corresponding to unconstrained phases of some of the VEVs, cause problems since the homotopy continuation method maps discrete numbers of solutions of a simple system to discrete numbers of solutions of the target system. From the physics point of view this is equivalent to the statement that only phase differences are relevant but not the phase of a single parameter itself. Already at the level of the superpotential in eq. (1) it is easy to see that two VEVs in the extended Higgs sector can be chosen real without changing the phase of a single parameter. We chose to take the VEVs of $H_u$ and $\bar{\eta}$ as purely real, and all other phases to be relative to these. As we are neglecting the flavour structure of $Y_\nu$, we can also choose all $v_{R,i}$ to be real (or equivalently we could take all $v_{L,i}$ real). If this flavour structure is taken into account, only one of the six sneutrino VEVs can be taken real in general. However, even assuming that we can neglect flavour mixing in $Y_\nu$, we have five real and five complex VEVs, resulting in fifteen real parameters which have to be calculated by HOM4PS2. Even though it is relatively fast, solving a system with ten degrees of freedom in typically twenty minutes on a single 3.4 GHz Intel i7 core, it takes about 10 days for the 15 degrees of freedom. For this reason we decided that for the democratic scan, all the VEVs would be taken to be real. Three points out of the 2330 were found which had one or more mass-squared that was negative at the deepest extremum found, and these are taken to have deeper minima where at least one of the VEVs is complex. In all three cases, at least one sneutrino VEV was non-zero. The hierarchical case can effectively be treated like a one generation model and here we allowed for complex VEVs. We found in this case that every minimum with a non-zero imaginary part for any VEV was degenerate with another minimum of the same parameter point with purely real VEVs, with the same magnitudes but some different signs. Hence our conclusions would be unchanged if we had restricted ourselves to purely real VEVs in the hierarchical scan.

---

3 One could in principle also take in addition the VEVs of of $H_d$ and $\eta$ to be real by redefining the phases of $\mu$ and $\mu'$. However, as this cannot be done consistently within HOM4PS2 we take them to be complex and keep the two parameters real.
C. Loop corrections

Before reaching any conclusion we have to consider the loop corrections to the scalar potential which are known to be important. The homotopy continuation method allows us to find all the solutions to the tree-level minimization conditions. However the effective potential at the one-loop level already has a more complicated structure including logarithmic contributions.

Loop corrections might modify our previous discussion by changing the relative depth between $R$-parity-conserving and $R$-parity-violating minima or promoting a gauge-conserving minimum to be the global minimum at loop level. It might also happen (although less likely) that that the nature of the extremum is changed, e.g. a tree-level saddle point becomes a lower-lying minimum of the effective potential.

In order to assess the effect of loop corrections, we evaluated the one-loop effective potential at all the extrema found by HOM4PS2 for every parameter point. In its more general form, the expression for the effective potential contains field-dependent masses. As our study centers around minima for the potential, the field-dependent masses are just the tree-level masses as function of the VEVs. We chose to remain in the Landau gauge and $\overline{\text{DR}^\prime}$ scheme and follow the results in [91].

Specifically, to evaluate the effective potential for a given set of VEVs we used the expression:

$$V^1 = V^0 + \frac{1}{16\pi^2} \sum_n (-1)^{2S_n} (2S_n + 1) \left( \frac{m_n^4}{4} \ln \left( \frac{m_n^2}{Q^2} \right) - \frac{3}{2} \right),$$

where $V^0$ is the tree-level scalar potential with corrections of the same polynomial form from the finite parts of the counterterms and $n$ goes over all the real scalar ($S_n = 0$), Weyl fermion ($S_n = \frac{1}{2}$) and vector ($S_n = 1$) degrees of freedom. The $m_n$ are the tree-level masses for a given set of parameters and VEVs at the renormalization scale $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. The renormalization scale is fixed by the stop masses for the VEV configuration given as input, and is held fixed for comparison with other VEV configurations, as changing the scale in evaluating the one-loop potential for different VEV configurations is inconsistent for a fixed-order calculation [92]. A more detailed formula is given in appendix A.

The masses were evaluated by a routine taking the tree-level expressions from SARAH, including the $R$-parity-violating mixings of neutrinos with neutralinos, charginos with charged leptons, charged Higgs bosons with charged sleptons and sneutrinos with neutral Higgs to-
gether with bilepton scalars/pseudoscalars in the presence of sneutrino VEVs. The corresponding formulas are given in appendix II. These masses were then used to calculate the full effective one-loop potential according to eq. (16).

When the potential is non-convex the conventional loop expansion becomes complex 93. Specifically, the value of the potential on a saddle point can be interpreted as an amplitude with a corresponding decay width (its complex part) 94. It was sufficient to use the real part of the effective potential to estimate whether a saddle point might have been promoted to a lower-lying minimum by comparing it with the value of the effective potential in all of the other minima.

By doing this it was possible to determine whether the hierarchy for the minima found at tree level was respected once loop corrections are included. We found that loop corrections have a non-negligible impact to the tree-level conclusions.

When this analysis indicated that a parameter point had a deeper minimum than the input minimum, we ran the code Vevacious 95 with a model file generated by SARAH to i) check that the one-loop potential did indeed have a minimum near this VEV configuration and that it was indeed deeper, using MINUIT 96, and ii) estimate the tunneling time to the deeper minimum, using CosmoTransitions 97.

IV. RESULTS

In general, each parameter point of our scans had several minima, both R-parity-conserving and R-parity-violating. We categorized the points by the nature of the lowest of their minima as in Tab. III and we present a breakdown of the number of points in each category in Tab. IV. Every point fell into one of these three categories: e.g. there were no points which had a global minimum with no unbroken symmetries, and there were no points which broke R-parity without breaking both SU(2)\textsubscript{L} and U(1)\textsubscript{B−L}. As it happens, there were no “RPC” parameter points in either scan at either tree or one-loop level which had global minima deeper than the input minima, by which we mean the set of VEVs chosen as input for SPheno.

The majority of “RPV” points broke R-parity through non-zero sneutrino VEVs, but there were six points in the hierarchical scan which had one-loop global minima with negative stop mass-squared, hence we assume that such parameter points have non-zero stop VEVs.
| Categorization | Description |
|----------------|-------------|
| “RPC”          | $SU(2)_L$, $U(1)_{B-L}$ both broken, $R$-parity conserved. |
| “RPV”          | $SU(2)_L$, $U(1)_{B-L}$ both broken, $R$-parity broken. |
| “unbroken”     | Either $SU(2)_L$ or $U(1)_{B-L}$ broken but not both, $R$-parity conserved. |

TABLE III. Categorization of parameter points according to the symmetries broken by their global minima.

| Categorization | Hierarchical scan | Democratic scan |
|----------------|-------------------|-----------------|
|                | tree level | one-loop level | tree level | one-loop level |
| “RPC”          | 1422       | 1275           | 2236       | 2167           |
| “RPV”          | 218        | 212            | 94         | 86             |
| “unbroken”     | 0          | 153            | 0          | 77             |

TABLE IV. Number of parameter points in the various categories. All of the parameter points from both scans categorized as “unbroken” broke $SU(2)_L$ without breaking $U(1)_{B-L}$. Not all parameter points that are “RPC” at the one-loop level were “RPC” at tree level, and likewise for the “RPV” category.

We plot them along with the other “RPV” points since they do break $R$-parity, in addition to $SU(3)_c$ and $U(1)_{em}$.

The third category, “unbroken”, only appears in the one-loop-level analysis. It is known that particularly precarious arrangements of parameters that break symmetries spontaneously at tree level may not break these symmetries when evaluated at one-loop level [98]. In such cases, the (not obvious) combination of Lagrangian parameters that parameterizes $U(1)_{B-L}$-breaking for example may be of the same magnitude as the loop corrections. We note that all such points that we found had zero sneutrino VEVs.

As can be seen in Figs. [14], there are $R$-parity-conserving points all over the parameter space. Based purely on a tree-level analysis, one might conclude that there are clear regions where the BLSSM has a stable, $R$-parity-conserving vacuum with the correct broken and unbroken gauge groups. These are, as one might expect, in regions where the $R$-sneutrino-bilepton Yukawa coupling $Y_x$ is not so large, and the trilinear soft SUSY-breaking parameter
FIG. 1. Projections into various parameter planes of the 1640 hierarchical scan parameter points, categorized by the nature of their global minima (see Tab. III) at tree level. “RPC” points are plotted in green (light grey), and “RPV” points are plotted in red (medium grey). In the plots on the left, the “RPC” points are plotted on top of the “RPV” points, which are faded (very light grey), while in the plots on the right the “RPV” points are plotted on top of the “RPC” points, which are faded (very light grey). (\(m_{\tilde{\nu}}^2\) is the lowest or most negative of the three soft SUSY-breaking mass-squared parameters for the R-sneutrinos, evaluated at the SUSY scale.)
FIG. 2. Projections into various parameter planes of the 1640 hierarchical scan parameter points, categorized by the nature of their global minima (see Tab. III) at one-loop level. As in Fig. 1 “RPC” points are plotted in green (light grey), but now the “RPV” points are in two groups based on the tunneling time from the “RPC” input minimum to the deeper “RPV” minimum: lower than one tenth of the age of the Universe as orange (medium-light grey) circles, greater than a tenth of the age of the Universe as red (medium grey) triangles. “Gauge conserving” points are in blue (dark grey).
FIG. 3. Projections into various parameter planes of the 2330 democratic scan parameter points, categorized by the nature of their global minima (see Tab. III at tree level. The scheme is the same as in Fig. I.)
FIG. 4. Projections into various parameter planes of the 2330 democratic scan parameter points, categorized by the nature of their global minima (see Tab. III) at one-loop level. The scheme is the same as in Fig. [2]
$A_0$ is not large compared to the soft SUSY-breaking scalar mass parameter $m_0$, as, intuitively, large $Y_x$ and $A_0$ can lead to large negative contributions to the potential energy for large values of $v_R$ and $v_{\eta/\bar{\eta}}$, as well as reducing the effective R-sneutrino mass term. Counter to this, a higher $m_0$ leads to a higher $m^2_{\tilde{\nu}}$ which, if positive, tends to penalize high $v_R$ values. This pattern is seen in both the hierarchical and democratic sets. However, these conclusions run into trouble when loop corrections are taken into account: the regions where $R$-parity appears to be safe at tree level apparently have very finely-tuned breaking of $SU(2)_L$ and $U(1)_{B-L}$ which often does not survive loop corrections. Hence, while parameter points often preserved $R$-parity, there are points all over the region where the global vacuum is not the phenomenologically-acceptable vacuum given as input, and this other vacuum is not trivial to find. It turns out that besides the known large contributions of third generation sfermions and fermions, the additional new particles of the $B-L$ sector also play an important role. The main reason for this is that the experimental bounds require $m_{Z'}$ to be in the multi-TeV range, implying that $v_{\eta}$ and $v_{\bar{\eta}}$ are also in this range. For $\tan\beta' \neq 1$ these VEVs give SUSY-breaking D-term contributions to the masses and, as they are much larger than the MSSM sector, this results in the observed importance of the corresponding loop contributions. These contributions are also responsible for the observed restoration of $U(1)_{B-L}$ at the one-loop level. Moreover, as discussed in [35, 67] at least one entry of $Y_x$ has to be large to achieve the breaking of $U(1)_{B-L}$. Here we have found that one-loop contributions due to the additional $B-L$ sector are more important in the hierarchical case, driving a larger set of points from the global tree-level $R$-parity-conserving minimum to the global $R$-parity-violating minimum at one-loop level. The main reason for this is that now a single entry has to play the role of the trace and thus is correspondingly larger than the average of the democratic scan.

A. Comparison with previous work

Previous studies on how much of the BLSSM parameter space conserves $R$-parity [35] considered the positivity of all the soft SUSY-breaking mass-squareds of the R-sneutrinos to be the necessary and sufficient condition. However, as can be seen in Figs. 2 and 4, this is neither necessary nor sufficient. There are, of course, obvious trends, and parameter points with very low or negative $m^2_{\tilde{\nu}}$ are as likely to break $R$-parity as conserve it, but there are
clearly parameter points which conserve $R$-parity despite having a negative $m_{\tilde{\nu}_c}^2$. This can be understood by the fact that one has to consider the eigenvalues of the mass matrix which can be all positive because of contributions of the F-terms, D-terms and other soft SUSY-breaking terms for non-zero bilepton VEVs despite negative $m_{\tilde{\nu}_c}^2$ entries. For instance, the $(\tilde{\nu}_c^3, \tilde{\nu}_c^3)$ element of the tree-level scalar mass-squared matrix is given by

$$m_{(\tilde{\nu}_c^3, \tilde{\nu}_c^3)}^2 = m_{\tilde{\nu}_c}^2 + \frac{1}{2} (v_L^2 + v_u^2) |Y_\nu|^2 - \sqrt{2} v_\eta R (Y_x \mu^* + \sqrt{2} v_\eta R (T_x) + (2v_\eta^2 + 3v_R^2) |Y_x|^2$$

$$+ \frac{1}{8} (g g_{BL} (v_u^2 - v_d^2 - v_L^2) + g_{BL}^2 (2(v_\eta^2 - v_\eta^2) + 3v_R^2 - v_L^2)). \quad (17)$$

Parameter points that have all positive $m_{\tilde{\nu}_c}^2$ yet still have $R$-parity-violating global minima are less surprising, as the presence of trilinear terms obviously could dominate for reasonably large values of sneutrino and bilepton VEVs.

Furthermore, Ref. [35] concluded that a hierarchical $Y_x$ would always break $R$-parity, because of the way the couplings enter the RGEs and drive the soft SUSY-breaking sneutrino mass-squared parameters negative before the bilepton mass-squared parameters, unless all three generations contribute substantially. The large fraction of “RPC” points in our hierarchical scan refutes this claim. Such points are harder to find (as evidenced by the smaller size of the hierarchical scan to the democratic scan), and in some sense rather finely-tuned, as evidenced by the large fraction of “unbroken” points where loop corrections to the differences in potential are as large as the differences leading to the breaking of the symmetries at tree level. Still, almost half of the hierarchical points conserve $R$-parity even at the one-loop level.

One should note, however, the different emphasis: we generated points that were na"ively $R$-parity-conserving, and explored whether they had global minima elsewhere, which would break $R$-parity or not break $U(1)_{B-L}$ or $SU(2)_L$, scanning over both the soft SUSY-breaking parameters and the additional Yukawa couplings; Ref. [35] explored various GUT-scale parameter configurations in the Yukawa sector for two points in the soft SUSY-breaking parameters to see whether RGE running would lead to a negative $m_\nu^2$. Since we started with a set of parameters engineered to have an $R$-parity-conserving local minimum, perhaps it is not so surprising to find that we did not find as strong a tendency for $R$-parity violation. However, we also feel that the criterion used in Ref. [35] is not in perfect correspondance with whether or not the parameter point breaks or preserves $R$-parity at its global minimum at the SUSY scale. Moreover, we have shown that loop effects do play an important role
and should not be neglected.

Finally, we examined whether the “RPC” minima of the “RPV” points were long-lived with respect to the age of the Universe. Unfortunately there seem to be no clear correlations, as can be seen in Figs. 1 to 4; we note that the SUSY-scale sign of $m_\nu^2$ appears to have little bearing even on the tunneling time from the “RPC” vacuum to the “RPV” vacuum.

V. CONCLUSION

We have investigated the stability of $R$-parity in a phenomenologically interesting region of the parameter space of the BLSSM by considering all the extrema of the tree-level potential and using the full one-loop evaluation of the potential at its extrema near those of the tree-level potential. We found that a significant fraction of points that were chosen to conserve $R$-parity while having the correct mass for the weak vector bosons actually have global minima which violate $R$-parity through non-zero sneutrino VEVs. However, we also found that more parameter points conserve $R$-parity than violate it.

We also found that both the conservation and violation of $R$-parity were possible all over the parameter space, in contradiction to previous studies, and while trends in the parameters leading either case are visible, it is difficult to identify in advance what values of which parameters will lead definitely to the conservation or violation of $R$-parity.

ACKNOWLEDGMENTS

This work has been supported by the DFG, project No. PO-1337/2-1, and partly by the Helmholtz alliance ‘Physics at the Terascale’. BO’L has been supported by DFG research training group GRK 1147.
Appendix A: The one-loop effective potential

The effective potential at one-loop is calculated by using eq. (16) and reads in its full form:

$$16\pi^2 (V^1 - V^0) = -3 \left( \sum_{i=1}^{3} \left( \frac{m_{d_i}^4}{4} \left[ \ln \left( \frac{m_{d_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) + \sum_{i=1}^{3} \left( \frac{m_{u_i}^4}{4} \left[ \ln \left( \frac{m_{u_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) \right)$$

$$+ \frac{3}{2} \left( \sum_{i=1}^{6} \left( \frac{m_{d_i}^4}{4} \left[ \ln \left( \frac{m_{d_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) + \sum_{i=1}^{6} \left( \frac{m_{u_i}^4}{4} \left[ \ln \left( \frac{m_{u_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) \right)$$

$$+ \frac{3}{4} \left( \frac{m_W^4}{2} \left[ \ln \left( \frac{m_W^2}{Q^2} \right) + \frac{3}{2} \right] + \sum_{i=1}^{3} \left( \frac{m_{d_i}^4}{4} \left[ \ln \left( \frac{m_{d_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) + \frac{m_Z^4}{4} \left[ \ln \left( \frac{m_Z^2}{Q^2} \right) - \frac{3}{2} \right] \right)$$

$$- \sum_{i=1}^{5} \left( \frac{m_{\tilde{d}_i}^4}{4} \left[ \ln \left( \frac{m_{\tilde{d}_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) - \frac{1}{2} \sum_{i=1}^{13} \left( \frac{m_{\tilde{u}_i}^4}{4} \left[ \ln \left( \frac{m_{\tilde{u}_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right)$$

$$+ \frac{1}{4} \left( \sum_{i=1}^{10} \left( \frac{m_{h_i}^4}{4} \left[ \ln \left( \frac{m_{h_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right) + \sum_{i=1}^{10} \left( \frac{m_{A_0}^4}{4} \left[ \ln \left( \frac{m_{A_0}^2}{Q^2} \right) - \frac{3}{2} \right] \right) \right)$$

$$+ \frac{1}{2} \sum_{i=1}^{8} \left( \frac{m_{\tilde{e}_i}^4}{4} \left[ \ln \left( \frac{m_{\tilde{e}_i}^2}{Q^2} \right) - \frac{3}{2} \right] \right). \quad (A1)$$

The first two lines are the contributions from quarks and squarks which are the same as for the MSSM. In the third line the loops including the three gauge bosons in the given model are counted. The fourth line contains the contributions due to the charged lepton-chargino as well as neutrino-neutralino mixed states. The fifth and sixth lines show the contributions from charged slepton-charged Higgsino states as well as sneutrino-Higgs states. We have included formally in the sums the would-be Goldstone-bosons to account for the fact that potentially the gauge group is broken only partially. This does not result in a double-counting because, in the Landau gauge, would-be Goldstone-bosons are massless and thus give a zero contribution to the one-loop effective potential. The tree-level mass matrices where $R$-parity violation induces a mixing between SM and SUSY particles are listed in appendix B. For completeness we note that in addition the sneutrino VEVs give contributions to the mass matrices of vector bosons and squarks.

Appendix B: Mass matrices

Here we give the tree-level masses suppressing the generation indices of (s)neutrinos and (s)leptons.
• Neutrino-Neutralino states

The mass matrix \( m_{\tilde{\chi}_0}^2 \) is given, in the basis

\[
\left( \lambda_B, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \nu_L, \nu_R, \tilde{\eta}, \tilde{\bar{\eta}}, \lambda_B' \right),
\]

by

\[
\begin{pmatrix}
M_{1} & 0 & -\frac{1}{2}g_1v_d & \frac{1}{2}g_1v_u & -\frac{1}{2}g_1v_L & 0 & 0 & 0 & M_{BB'} \\
0 & M_{2} & \frac{1}{2}g_2v_d & -\frac{1}{2}g_2v_u & \frac{1}{2}g_2v_L & 0 & 0 & 0 & 0 \\
-\frac{1}{2}g_1v_d & \frac{1}{2}g_2v_d & 0 & -\mu & 0 & 0 & 0 & 0 & -\frac{1}{2}\bar{g}v_d \\
\frac{1}{2}g_1v_u & -\frac{1}{2}g_2v_u & -\mu & 0 & \frac{1}{\sqrt{2}}v_RY_\nu & \frac{1}{\sqrt{2}}v_LY_\nu & 0 & 0 & \frac{1}{2}\bar{g}v_u \\
-\frac{1}{2}g_1v_L & \frac{1}{2}g_2v_L & 0 & \frac{1}{\sqrt{2}}v_RY_\nu & 0 & \frac{1}{\sqrt{2}}v_LY_\nu & 0 & 0 & -\frac{1}{2}(\bar{g} + g_B)v_L \\
0 & 0 & 0 & \frac{1}{\sqrt{2}}v_RY_\nu & \frac{1}{\sqrt{2}}v_uY_\nu & 0 & \sqrt{2}v_RY_x & 0 & \frac{1}{2}g_Bv_R \\
0 & 0 & 0 & 0 & \sqrt{2}v_RY_x & 0 & -\mu' & -g_Bv_\eta & 0 \\
0 & 0 & 0 & 0 & -\mu' & 0 & g_Bv_\eta & 0 & 0 \\
M_{BB'} & 0 & -\frac{1}{2}\bar{g}v_d & \frac{1}{2}\bar{g}v_u & -\frac{1}{2}(\bar{g} + g_B)v_L & \frac{1}{2}g_Bv_R & -g_Bv_\eta & g_Bv_\bar{\eta} & M_{B'}
\end{pmatrix}
\]

(B1)

• Charged lepton-Charginos

The mass matrix is given, in the basis

\[
\left( e_L, \tilde{W}^-, \tilde{H}_d^-, \nu_L, \nu_R, \tilde{\eta}, \tilde{\bar{\eta}}, \lambda_B' \right),
\]

by

\[
m_{\tilde{\chi}^-}^2 = \begin{pmatrix}
\frac{1}{\sqrt{2}}v_dY_e & \frac{1}{\sqrt{2}}g_2v_L & -\frac{1}{\sqrt{2}}v_RY_\nu \\
0 & M_2 & \frac{1}{\sqrt{2}}g_2v_u \\
-\frac{1}{\sqrt{2}}v_LY_e & \frac{1}{\sqrt{2}}g_2v_d & \mu
\end{pmatrix}
\]

(B2)

• CP even sneutrino-Higgs

In the basis

\[
\left( \sigma_d, \sigma_u, \sigma_L, \sigma_R, \sigma_\eta, \sigma_\bar{\eta} \right),
\]

the entries of the mass matrix read:

\[
m_{\sigma_d\sigma_d} = \frac{1}{8}(\bar{g}g_{BL} \left( 2v_\eta^2 - 2v_\bar{\eta}^2 - v_R^2 + v_L^2 \right) + \left( g_1^2 + \bar{g}^2 + g_2^2 \right) \left( 3v_d^2 - v_u^2 + v_L^2 \right)) + m_{\tilde{H}_d}^2 + \mu^2
\]

(B3)
\[ m_{\sigma d \sigma u} = - B_\mu - \frac{1}{4} \left( g_1^2 + g_2^2 + g_2^2 \right) v_d v_u \]  
(B4)

\[ m_{\sigma u \sigma u} = m_{\sigma u}^2 + \frac{1}{8} \left( \left( - g_1^2 - g_2^2 \right) \left( v_{\eta}^2 - 3 v_{\eta}^2 + v_{L}^2 \right) + \bar{g} g_{BL} \left( 2 v_{\eta}^2 - 2 v_{\eta}^2 - v_{L}^2 + v_{R}^2 \right) \right) + \frac{1}{2} \left( v_{L}^2 + v_{R}^2 \right) Y_\nu^2 + \mu^2 \]  
(B5)

\[ m_{\sigma d \sigma L} = \frac{1}{8} \left( \left( - g_1^2 - g_2^2 \right) v_d v_L - \frac{1}{\sqrt{2}} v_R Y_\nu \mu \right) \]  
(B6)

\[ m_{\sigma u \sigma L} = - \frac{1}{4} \left( \bar{g} \left( \bar{g} + g_{BL} \right) + g_1^2 + g_2^2 \right) v_d v_u + \frac{1}{\sqrt{2}} v_R T_\nu + Y_\nu \left( v_L v_\eta Y_\nu + v_R v_\eta Y_\nu \right) \]  
(B7)

\[ m_{\sigma L \sigma L} = m_{\sigma L}^2 + \frac{1}{8} \left( \left( - g_1^2 + g_2^2 + 2 v_{\eta}^2 \right) + g_{BL} \left( - 2 v_{\eta}^2 + 2 v_{\eta}^2 + 3 v_{L}^2 - v_{R}^2 \right) + \bar{g} g_{BL} \left( - 2 v_{\eta}^2 + 2 v_{\eta}^2 + 6 v_{L}^2 - v_{R}^2 + v_{u}^2 + v_{R}^2 \right) \right) + \frac{1}{2} \left( v_{R}^2 + v_{u}^2 \right) Y_\nu^2 \]  
(B8)

\[ m_{\sigma d \sigma R} = \frac{1}{4} \bar{g} g_{BL} v_d v_R - \frac{1}{\sqrt{2}} v_L Y_\nu \mu \]  
(B9)

\[ m_{\sigma u \sigma R} = \frac{1}{4} \bar{g} g_{BL} v_R v_u + \frac{1}{\sqrt{2}} v_L T_\nu + Y_\nu \left( v_L v_\eta Y_\nu + v_R v_\eta Y_\nu \right) \]  
(B10)

\[ m_{\sigma L \sigma R} = - \frac{1}{4} g_{BL} \left( \bar{g} + g_{BL} \right) v_L v_R + \frac{1}{\sqrt{2}} v_u T_\nu + v_L v_R Y_\nu^2 + Y_\nu \left( - \frac{1}{\sqrt{2}} v_d \mu + v_u v_\eta Y_\nu \right) \]  
(B11)

\[ m_{\sigma R \sigma R} = m_{\sigma R}^2 - \frac{1}{8} g_{BL} \left( \bar{g} \left( - v_{\eta}^2 + 2 v_{\eta}^2 + v_{L}^2 \right) + g_{BL} \left( - 2 v_{\eta}^2 + 2 v_{\eta}^2 + 3 v_{L}^2 + v_{R}^2 \right) \right) \]  
(B12)

\[ m_{\sigma d \sigma \eta} = \frac{1}{2} \bar{g} g_{BL} v_d v_\eta \]  
(B13)

\[ m_{\sigma u \sigma \eta} = - \frac{1}{2} \bar{g} g_{BL} v_u v_\eta + v_L v_R Y_\eta^2 \]  
(B14)

\[ m_{\sigma L \sigma \eta} = \frac{1}{2} g_{BL} \left( \bar{g} + g_{BL} \right) v_L v_\eta + v_R v_u Y_\eta \]  
(B15)

\[ m_{\sigma R \sigma \eta} = - \frac{1}{2} g_{BL} v_R v_\eta + \sqrt{2} v_R T_\eta + Y_\eta \left( 4 v_R v_\eta Y_\eta + v_L v_u Y_\eta \right) \]  
(B16)

\[ m_{\sigma \eta \sigma \eta} = 2 v_R Y_\eta^2 + \frac{1}{4} g_{BL} \left( \bar{g} \left( v_{\eta}^2 - v_{u}^2 + v_{L}^2 \right) + g_{BL} \left( 6 v_{\eta}^2 - 2 v_{\eta}^2 - v_{R}^2 + v_{u}^2 \right) \right) + m_{\sigma}^2 + \mu^2 \]  
(B17)

\[ m_{\sigma d \sigma \bar{\eta}} = \frac{1}{2} \bar{g} g_{BL} v_d v_{\bar{\eta}} \]  
(B18)

\[ m_{\sigma u \sigma \bar{\eta}} = \frac{1}{2} \bar{g} g_{BL} v_u v_{\bar{\eta}} \]  
(B19)

\[ m_{\sigma L \sigma \bar{\eta}} = - \frac{1}{2} g_{BL} \left( \bar{g} + g_{BL} \right) v_L v_{\bar{\eta}} \]  
(B20)

\[ m_{\sigma R \sigma \bar{\eta}} = \frac{1}{2} g_{BL} v_R v_{\bar{\eta}} - \sqrt{2} \mu v_R Y_\eta \]  
(B21)

\[ m_{\sigma \eta \sigma \bar{\eta}} = - B_\mu - g_{BL}^2 v_\eta v_{\bar{\eta}} \]  
(B22)
\[ m_{\sigma_0 \sigma_0} = -\frac{1}{4} g_{BL} \left( \bar{g} \left( - v_u^2 + v_d^2 + v_L^2 \right) + g_{BL} \left( 2v_\eta^2 - 6v_\eta^2 - v_R^2 + v_L^2 \right) \right) + m_0^2 + \mu^2 \] (B23)

- **CP odd sneutrino-Higgs**

In the basis \((\phi_d, \phi_u, \phi_L, \phi_R, \phi_\eta, \phi_\eta)\) and using Landau gauge, the nonzero entries of the mass matrix read:

\[ m_{\phi_d \phi_d} = \frac{1}{8} \left( g_{BL} \left( 2 v_\eta^2 - v_R^2 - v_L^2 \right) + \left( g_1^2 + g_2^2 + g_3^2 \right) \left( v_u^2 - v_d^2 + v_L^2 \right) \right) + m_{H_d}^2 + \mu^2 \] (B24)

\[ m_{\phi_d \phi_u} = B_\mu \] (B25)

\[ m_{\phi_u \phi_u} = m_{H_u}^2 + \frac{1}{8} \left( - g_2^2 - 2 g_1^2 - g_3^2 \right) \left( - v_u^2 + v_d^2 + v_L^2 \right) + g_{BL} \left( 2v_\eta^2 - 2v_\eta^2 - v_R^2 + v_L^2 \right) \]

\[ + \frac{1}{2} \left( v_L^2 + v_R^2 \right) Y_\nu^2 + \mu^2 \] (B26)

\[ m_{\phi_d \phi_L} = -\frac{1}{\sqrt{2}} v_R Y_\nu \mu \] (B27)

\[ m_{\phi_u \phi_L} = -\frac{1}{2} v_R \left( 2v_\eta Y_x Y_\nu + \sqrt{2} T_\nu \right) \] (B28)

\[ m_{\phi_L \phi_L} = m_L^2 + \frac{1}{8} \left( g_1^2 + g_2^2 + g_3^2 \right) \left( v_u^2 - v_d^2 + v_L^2 \right) + g_{BL} \left( 2 v_L^2 + v_\eta^2 - v_\eta^2 - v_R^2 - v_u^2 + v_d^2 \right) \]

\[ + g_{BL} \left( 2 \left( - v_\eta^2 + v_\eta^2 \right) - v_R^2 + v_L^2 \right) + \frac{1}{2} \left( v_R^2 + v_u^2 \right) Y_\nu^2 \] (B29)

\[ m_{\phi_d \phi_R} = -\frac{1}{\sqrt{2}} v_L Y_\nu \mu \] (B30)

\[ m_{\phi_u \phi_R} = v_L \left( \frac{1}{\sqrt{2}} T_\nu - v_\eta Y_x Y_\nu \right) \] (B31)

\[ m_{\phi_L \phi_R} = \frac{1}{\sqrt{2}} v_u T_\nu + Y_\nu \left( -\frac{1}{\sqrt{2}} v_d \mu - v_u v_\eta Y_x \right) \] (B32)

\[ m_{\phi_R \phi_R} = m_{\nu e} + \frac{1}{8} g_{BL} \left( - g_2 \left( - v_u^2 + v_d^2 + v_L^2 \right) + g_{BL} \left( 2v_\eta^2 - 2v_\eta^2 - v_L^2 + v_R^2 \right) \right) \]

\[ + \frac{1}{2} \left( -2\sqrt{2} v_\eta T_x + 2Y_x \left( 2v_\eta^2 + v_R^2 \right) Y_x + \sqrt{2} \mu' v_\eta \right) + \left( v_L^2 + v_u^2 \right) Y_\nu^2 \] (B33)

\[ m_{\phi_\nu \phi_\eta} = v_L v_R Y_x Y_\nu \] (B34)

\[ m_{\phi_L \phi_\eta} = v_R v_u Y_x Y_\nu \] (B35)

\[ m_{\phi_R \phi_\eta} = \sqrt{2} v_R T_x + v_L v_u Y_x Y_\nu \] (B36)

\[ m_{\phi_R \phi_\eta} = \sqrt{2} \mu' v_R Y_x \] (B37)
\[
m_{\phi \phi} = 2v_R^2Y_x^2 + \frac{1}{4}g_{BL}(\bar{g}(v_d^2 - v_u^2 + v_L^2) + g_{BL}(2(v_\eta^2 - v_\eta^2) - v_R^2 + v_L^2)) + m_\eta^2 + \mu^2
\]

(B38)

\[
m_{\phi g \phi} = \frac{1}{4}g_{BL}(-\bar{g}(-v_u^2 + v_d^2 + v_L^2) + g_{BL}(2v_\eta^2 - 2v_\eta^2 - v_R^2 + v_L^2)) + m_\eta^2 + \mu^2
\]

(B39)

\section*{Charged slepton - charged Higgs}

In the basis

\[
(H_d^{-}, H_u^{+}, \bar{e}_L, \bar{e}_R),
\]

the entries of the mass matrix read:

\[
m_{H_d^{-}H_d^{-}} = m_{H_d}^2 + \frac{1}{8}(\bar{g}g_{BL}(2(-v_\eta^2 + v_\eta^2) - v_R^2 + v_L^2) + (g_1^2 + g_2^2)(-v_u^2 + v_d^2 + v_L^2)
\]

\[
+ g_2^2(v_d^2 + v_L^2 + v_u^2)) + \frac{1}{2}v_L^2Y_e^2 + \mu^2
\]

(B40)

\[
m_{H_u^{+}H_u^{+}} = \frac{1}{4}g_2^2v_dv_u + B_\mu
\]

(B41)

\[
m_{H_d^{-}H_u^{+}} = m_{H_u}^2 + \frac{1}{8}((-\bar{g}^2 - g_1^2)(-v_u^2 + v_d^2 + v_L^2) + \bar{g}g_{BL}(2v_\eta^2 - 2v_\eta^2 - v_R^2 + v_L^2)
\]

\[
+ g_2^2(v_d^2 + v_L^2 + v_u^2)) + \frac{1}{2}v_R^2Y_\nu^2 + \mu^2
\]

(B42)

\[
m_{H_d^{-}\bar{e}_L} = \frac{1}{4}v_dY_e^2 + \frac{1}{4}g_2^2v_dv_L - \frac{1}{\sqrt{2}}v_RY_\nu
\]

(B43)

\[
m_{H_u^{+}\bar{e}_L} = \frac{1}{2}v_d^2Y_x^2 + \frac{1}{8}g_2^2v_Lv_u - \frac{1}{\sqrt{2}}v_RT_\nu
\]

(B44)

\[
m_{H_u^{+}\bar{e}_L} = m_\phi^2 + \frac{1}{8}(g_1^2 + g_2^2)(v_d^2 - v_u^2 + v_L^2) + g_2^2(v_L^2 - v_u^2 + v_d^2) + \frac{1}{2}(v_d^2Y_e^2 + v_R^2Y_\nu^2)
\]

\[
+ \bar{g}g_{BL}(-2v_\eta^2 + 2(v_L^2 + v_\eta^2) - v_R^2 - v_u^2 + v_\eta^2) + g_{BL}^2(2(-v_\eta^2 + v_\eta^2) - v_R^2 + v_L^2))
\]

(B45)

\[
m_{H_d^{-}\bar{e}_R} = \frac{1}{2}v_dY_e^2 - \frac{1}{\sqrt{2}}v_LT_e
\]

(B46)

\[
m_{H_u^{+}\bar{e}_R} = -\frac{1}{8}Y_e^2(\sqrt{2}v_L\mu + v_dY_R^2Y_e^2)
\]

(B47)

\[
m_{\bar{e}_L\bar{e}_R} = \frac{1}{\sqrt{2}}(v_dT_e - v_uY_e\mu)
\]

(B48)

\[
m_{\bar{e}_R\bar{e}_R} = m_\phi^2 + \frac{1}{8}(-2\bar{g} + g_{BL})((\bar{g}(-v_u^2 + v_d^2 + v_L^2) + g_{BL}(-2v_\eta^2 + 2v_\eta^2 - v_R^2 + v_L^2))
\]

\[
- 2g_1^2(-v_u^2 + v_d^2 + v_L^2)) + \frac{1}{2}(v_d^2 + v_L^2)Y_e^2
\]

(B49)

We write only independent entries as the matrix is hermitian thus \(m_{ij}^2 = (m_{ji}^2)^*\).
[1] ATLAS Collaboration, G. Aad et al., Phys.Lett. B716, 1 (2012), arXiv:1207.7214.
[2] CMS Collaboration, S. Chatrchyan et al., Phys.Lett. B716, 30 (2012), arXiv:1207.7235.
[3] G. Isidori, G. Ridolfi, and A. Strumia, Nucl.Phys. B609, 387 (2001), arXiv:hep-ph/0104016.
[4] J. Ellis, J. Espinosa, G. Giudice, A. Hoecker, and A. Riotto, Phys.Lett. B679, 369 (2009), arXiv:0906.0954.
[5] J. Elias-Miro et al., Phys.Lett. B709, 222 (2012), arXiv:1112.3022.
[6] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl, and M. Shaposhnikov, JHEP 1210, 140 (2012), arXiv:1205.2893.
[7] G. Degrassi et al., JHEP 1208, 098 (2012), arXiv:1205.6497.
[8] I.P. Ivanov, Phys.Rev. D75, 035001 (2007), arXiv:hep-ph/0609018.
[9] A. Barroso, P. Ferreira, I. Ivanov, R. Santos, and J. P. Silva, (2012), arXiv:1211.6119.
[10] M. Claudson, L. J. Hall, and I. Hinchliffe, Nucl.Phys. B228, 501 (1983).
[11] M. Drees, M. Glück, and K. Grassie, Phys.Lett. B157, 164 (1985).
[12] J. Gunion, H. Haber, and M. Sher, Nucl.Phys. B306, 1 (1988).
[13] R. Kuchimanchi and R.N. Mohapatra, Phys.Rev. D48, 4352 (1993), arXiv:hep-ph/9306290.
[14] J. Casas, A. Lleyda, and C. Munoz, Nucl.Phys. B471, 3 (1996), arXiv:hep-ph/9507294.
[15] T. Kitahara, JHEP 1211, 021 (2012), arXiv:1208.4792.
[16] M. Carena, S. Gori, I. Low, N. R. Shah, and C. E. Wagner, (2012), arXiv:1211.6136.
[17] M. Hirsch, C. Hugonie, J. Romao, and J. Valle, JHEP 0503, 020 (2005), arXiv:hep-ph/0411129.
[18] M. Hirsch, M.A. Diaz, W. Porod, J.C. Romao, and J.W.F Valle, Phys.Rev. D62, 113008 (2000), arXiv:hep-ph/0004115.
[19] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Nucl. Phys. B238, 453 (1984).
[20] J. R. Ellis, K. A. Olive, Y. Santos, and V. C. Spanos, Phys. Lett. B565, 176 (2003), arXiv:hep-ph/0303043.
[21] S. Borgani, A. Masiero, and M. Yamaguchi, Phys.Lett. B386, 189 (1996), arXiv:hep-ph/9605222.
[22] F. Takayama and M. Yamaguchi, Phys.Lett. B485, 388 (2000), arXiv:hep-ph/0005214.
[23] M. Hirsch, W. Porod, and D. Restrepo, JHEP 0503, 062 (2005), arXiv:hep-ph/0503059.
[24] L. Covi, M. Grefe, A. Ibarra, and D. Tran, JCAP 0901, 029 (2009), arXiv:0809.5030.
[25] F. D. Steffen, Eur.Phys.J. C59, 557 (2009), arXiv:0811.3347.
[26] K. Rajagopal, M. S. Turner, and F. Wilczek, Nucl.Phys. B358, 447 (1991).
[27] T. Goto and M. Yamaguchi, Phys.Lett. B276, 103 (1992).
[28] S. Khalil, J.Phys. G35, 055001 (2008), arXiv:hep-ph/0611205.
[29] W. Emam and S. Khalil, Eur. Phys. J. C55, 625 (2007), arXiv:0704.1395.
[30] L. Basso, A. Belyaev, S. Moretti, and C. H. Shepherd-Themistocleous, Phys. Rev. D80, 055030 (2009), arXiv:0812.4313.
[31] L. Basso, S. Moretti, and G. M. Pruna, Phys. Rev. D83, 055014 (2011), arXiv:1011.2612.
[32] L. Basso, S. Moretti, and G. M. Pruna, Eur. Phys. J. C71, 1724 (2011), arXiv:1012.0167.
[33] S. Khalil and A. Masiero, Phys. Lett. B665, 374 (2008), arXiv:0710.3525.
[34] V. Barger, P. Fileviez Perez, and S. Spinner, Phys. Rev. Lett. 102, 181802 (2009), arXiv:0812.3661.
[35] P. F. Perez and S. Spinner, Phys. Rev. D83, 035004 (2011), arXiv:1005.4930.
[36] H.K. Dreiner, M. Kramer, and B. O’Leary, Phys.Rev. D75, 114016 (2007), arXiv:hep-ph/0612278.
[37] H.K. Dreiner, M. Hanussek, and S. Grab, Phys.Rev. D82, 055027 (2010), arXiv:1005.3309.
[38] H.K. Dreiner, K. Nickel, F. Staub, and A. Vicente, Phys.Rev. D86, 015003 (2012), arXiv:1204.5925.
[39] Y. Okada, M. Yamaguchi, and T. Yanagida, Prog.Theor.Phys. 85, 1 (1991).
[40] Y. Okada, M. Yamaguchi, and T. Yanagida, Phys.Lett. B262, 54 (1991).
[41] J. R. Ellis, G. Ridolfi, and F. Zwirner, Phys.Lett. B257, 83 (1991).
[42] H. E. Haber and R. Hempfling, Phys.Rev.Lett. 66, 1815 (1991).
[43] J. R. Ellis, G. Ridolfi, and F. Zwirner, Phys.Lett. B262, 477 (1991).
[44] P. H. Chankowski, S. Pokorski, and J. Rosiek, Phys.Lett. B274, 191 (1992).
[45] A. Brignole, Phys.Lett. B281, 284 (1992).
[46] A. Dabelstein, Z.Phys. C67, 495 (1995), arXiv:hep-ph/9409375.
[47] R. Hempfling and A. H. Hoang, Phys.Lett. B331, 99 (1994), arXiv:hep-ph/9401219.
[48] H. E. Haber, R. Hempfling, and A. H. Hoang, Z.Phys. C75, 539 (1997), arXiv:hep-ph/9609331.
[49] S. Heinemeyer, W. Hollik, and G. Weiglein, Phys.Rev. D58, 091701 (1998), arXiv:hep-ph/9803277.

[50] S. Heinemeyer, W. Hollik, and G. Weiglein, Phys.Lett. B440, 296 (1998), arXiv:hep-ph/9807423.

[51] S. Heinemeyer, W. Hollik, and G. Weiglein, Eur.Phys.J. C9, 343 (1999), arXiv:hep-ph/9812472.

[52] R.-J. Zhang, Phys.Lett. B447, 89 (1999), arXiv:hep-ph/9808299.

[53] J. R. Espinosa and R.-J. Zhang, JHEP 0003, 026 (2000), arXiv:hep-ph/9912236.

[54] J. Espinosa and I. Navarro, Nucl.Phys. B615, 82 (2001), arXiv:hep-ph/0104047.

[55] G. Degrassi, P. Slavich, and F. Zwirner, Nucl.Phys. B611, 403 (2001), arXiv:hep-ph/0105096.

[56] A. Brignole, G. Degrassi, P. Slavich, and F. Zwirner, Nucl.Phys. B643, 79 (2002), arXiv:hep-ph/0206101.

[57] A. Brignole, G. Degrassi, P. Slavich, and F. Zwirner, Nucl.Phys. B631, 195 (2002), arXiv:hep-ph/0112177.

[58] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, and G. Weiglein, Eur.Phys.J. C28, 133 (2003), arXiv:hep-ph/0212020.

[59] S. P. Martin, Phys.Rev. D67, 095012 (2003), arXiv:hep-ph/0211366.

[60] B. Allanach, A. Djouadi, J. Kneur, W. Porod, and P. Slavich, JHEP 0409, 044 (2004), arXiv:hep-ph/0406166.

[61] R.V. Harlander, P. Kant, L. Mihaila, and M. Steinhauser, Phys.Rev.Lett. 100, 191602 (2008), arXiv:0803.0672.

[62] P. Kant, R. Harlander, L. Mihaila, and M. Steinhauser, JHEP 1008, 104 (2010), arXiv:1005.5709.

[63] M. E. Krauss, B. O’Leary, W. Porod, and F. Staub, Phys.Rev. D86, 055017 (2012), arXiv:1206.3513.

[64] L. Basso, B. O’Leary, W. Porod, and F. Staub, JHEP 1209, 054 (2012), arXiv:1207.0507.

[65] L. Basso and F. Staub, (2012), arXiv:1210.7946.

[66] R. M. Fonseca, M. Malinsky, W. Porod, and F. Staub, Nucl. Phys. B854, 28 (2012), arXiv:1107.2670.

[67] B. O’Leary, W. Porod, and F. Staub, JHEP 1205, 042 (2012), 1112.4600.
[68] P. Fileviez Perez, S. Spinner, and M. K. Trenkel, Phys.Rev. D84, 095028 (2011), arXiv:1103.5504.

[69] P. Fileviez Perez and S. Spinner, JHEP 1204, 118 (2012), arXiv:1201.5923.

[70] F. Staub, (2008), arXiv:0806.0538.

[71] F. Staub, Comput. Phys. Commun. 181, 1077 (2010), arXiv:0909.2863.

[72] F. Staub, Comput. Phys. Commun. 182, 808 (2011), arXiv:1002.0840.

[73] F. Staub, (2012), arXiv:1207.0906.

[74] W. Porod, Comput. Phys. Commun. 153, 275 (2003), arXiv:hep-ph/0301101.

[75] W. Porod and F. Staub, Comput.Phys.Commun. 183, 2458 (2012), arXiv:1104.1573.

[76] F. Staub, T. Ohl, W. Porod, and C. Speckner, Comput.Phys.Commun. 183, 2165 (2012), arXiv:1109.5147.

[77] F. E. Paige, S. D. Protopopescu, H. Baer, and X. Tata, (2003), arXiv:hep-ph/0312045.

[78] B. Allanach, Comput.Phys.Commun. 143, 305 (2002), arXiv:hep-ph/0104145.

[79] A. Djouadi, J.-L. Kneur, and G. Moultaka, Comput.Phys.Commun. 176, 426 (2007), arXiv:hep-ph/0211331.

[80] M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur.Phys.J. C49, 1067 (2007), arXiv:hep-ph/0608314.

[81] J. Gray, Y.-H. He, A. Ilderton, and A. Lukas, Comput.Phys.Commun. 180, 107 (2009), arXiv:0801.1508.

[82] A. Sommese and C. Wampler, The numerical solution of systems of polynomials arising in engineering and science. 2005.

[83] T. Li, Handbook of numerical analysis 11, 209 (2003).

[84] D. Mehta, A. Sternbeck, L. von Smekal, and A. G. Williams, PoS QCD-TNT09, 025 (2009), arXiv:0912.0450.

[85] D. Mehta, Phys.Rev. E84, 025702 (2011), arXiv:1104.5497.

[86] D. Mehta, J. D. Hauenstein, and M. Kastner, Phys.Rev. E85, 061103 (2012), arXiv:1202.3320.

[87] D. Mehta, Adv.High Energy Phys. 2011, 263937 (2011), arXiv:1108.1201.

[88] D. Mehta, Y.-H. He, and J. D. Hauenstein, JHEP 1207, 018 (2012), arXiv:1203.4235.

[89] M. Maniatis and D. Mehta, Eur.Phys.J.Plus 127, 91 (2012), arXiv:1203.0409.

[90] T. Lee, T. Li, and C. Tsai, Computing 83, 109 (2008).

[91] S. P. Martin, Phys.Rev. D65, 116003 (2002), arXiv:hep-ph/0111209.
[92] P. Ferreira, Phys.Lett. B509, 120 (2001), arXiv:hep-ph/0008115.

[93] Y. Fujimoto, L. O’Raifeartaigh, and G. Parravicini, Nucl.Phys. B212, 268 (1983).

[94] E. J. Weinberg and A. Wu, Phys. Rev. D 36, 2474 (1987).

[95] J. E. Camargo-Molina, B. O’Leary, W. Porod, and F. Staub, (2013), arXiv:1307.1477

[96] F. James and M. Roos, Comput.Phys.Commun. 10, 343 (1975).

[97] C. L. Wainwright, Comput.Phys.Commun. 183, 2006 (2012), arXiv:1109.4189.

[98] B. de Carlos and J. Casas, Physics Letters B 309, 320 (1993).