Verification of new identity for the Green functions in 
\( N = 1 \) supersymmetric non-Abelian Yang–Mills 
theory with the matter fields.

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Abstract

We investigate a new identity for Green functions using the higher covariant 
derivative regularization. It relates some coefficients in the vertex function of the 
matter superfield, in which one of external matter lines is not chiral. The calculation 
in the first nontrivial order (for the two-loop vertex function) reveals that the new 
identity is also valid for the non-Abelian Yang–Mills theory with matter fields. The 
new identity is shown to appear because three-loop integrals, defining the Gell-
Mann–Low function are factorized into integrals of total derivatives.

1 Introduction.

Investigation of quantum corrections in supersymmetric theories is an interesting and 
sometimes nontrivial problem. For example, in theories with the \( N = 1 \) supersymmetry 
it is possible to suggest \[1\] a form of the \( \beta \)-function exactly to all orders. One way 
of obtaining the exact \( \beta \)-function, proposed in Ref. \[2, 3\], is substituting the solution of 
Slavnov–Taylor identities into the Schwinger–Dyson equations. Then the exact \( \beta \)-function 
is obtained if we propose existence of a new identity, relating the Green functions \[2, 3\]. 
Due to this identity some contributions to the Gell-Mann-Low function disappear starting 
from the three-loop approximation.

The existence of the new identity is related with the interesting observation \[4, 5\], 
which was made using the higher derivative regularization \[6, 7\] in supersymmetric theories. 
All contributions to the Gell-Mann–Low function appear to be integrals of total 
derivatives. Partially this can be explained substituting solutions of Ward identities into 
the Schwinger–Dyson equations. In the Abelian case a straightforward summation of
diagrams is also possible \[^8\]. Nevertheless, there are new types of diagrams in the non-Abelian case and a method, used in Ref. \[^8\], is not already working. Therefore, in the non-Abelian case it is necessary to verify the new identity again. Such a verification is made in this paper.

This paper is organized as follows. In Sec. 2 we recall basic information about the \(N = 1\) supersymmetric Yang-Mills theory, the background field method, and the higher derivatives regularization. A verification of the new identity is made in Sec. 3. A brief discussion of the results is given in the conclusion. Some technical details are presented in the Appendix.

\section*{2 \(N = 1\) supersymmetric Yang-Mills theory, background field method and higher derivative regularization}

We will consider the \(N = 1\) supersymmetric Yang-Mills theory with massless matter superfields, which in the superspace is described by the action

\[ S = \frac{1}{4e^2} \text{Re} \text{tr} \int d^4x \, d^2\theta \, W_a C^{ab} W_b + \frac{1}{4} \int d^4x \, d^4\theta \left( \phi^+ e^{2V} \phi + \tilde{\phi}^+ e^{-2V'} \tilde{\phi} \right), \quad (1) \]

Here \(\phi\) and \(\tilde{\phi}\) are chiral matter superfields, and \(V\) is a real scalar superfield, which contains the gauge field \(A_\mu\) as a component. The superfield \(W_a\) is a supersymmetric analogue of the gauge field stress tensor. It is defined by

\[ W_a = \frac{1}{32} \bar{D}(1 - \gamma_5) D \left[ e^{-2V}(1 + \gamma_5) D_a e^{2V} \right]. \quad (2) \]

In our notation, the gauge superfield \(V\) is expanded over the generators of the gauge group \(T^a\) as \(V = e V^a T^a\), where \(e\) is a coupling constant. Generators of the fundamental representation we normalize by the condition

\[ \text{tr}(t^a t^b) = \frac{1}{2} \delta^{ab}. \quad (3) \]

Action (1) is invariant under the gauge transformations

\[ \phi \to e^{i\Lambda} \phi; \quad \tilde{\phi} \to e^{-i\Lambda} \tilde{\phi}; \quad e^{2V} \to e^{i\Lambda} e^{2V} e^{-i\Lambda}, \quad (4) \]

where \(\Lambda\) is an arbitrary chiral superfield.

For quantization of this model it is convenient to use the background field method. The matter is that the background field method allows constructing the effective action, which is invariant under some background gauge transformations. In the supersymmetric case it can formulated as follows \[^9\] [\(^{10}\)]: Let us make the substitution

\[ e^{2V} \to e^{2V'} \equiv e^{\Omega^+} e^{2V} e^{\Omega} \quad (5) \]

in action (1), where \(\Omega\) is a background scalar superfield. The obtained theory will be invariant under the background gauge transformations.
\[ V \rightarrow e^{iK}V^{-iK}; \quad e^\Omega \rightarrow e^{iK}e^\Omega e^{-i\Lambda}; \quad e^{\Omega^+} \rightarrow e^{i\Lambda^+}e^{\Omega^+}e^{-iK}, \quad (6) \]

where \( K \) is a real superfield and \( \Lambda \) is a chiral superfield.

Let us construct the chiral covariant derivatives

\[ D \equiv -\frac{1}{2}(1 + \gamma_5)De^{\Omega^+}; \quad \bar{D} \equiv \frac{1}{2}(1 - \gamma_5)De^{-\Omega}. \quad (7) \]

Acting on some field \( X \), which is transformed as \( X \rightarrow e^{iK}X \), these covariant derivatives are transformed in the same way. It is also possible to define the background covariant derivative with a Lorentz index

\[ D_\mu \equiv -\frac{i}{4}(C\gamma^\mu)^{ab}\{D_a, \bar{D}_b\}, \quad (8) \]

which will have the same property. It is easy to see that after substitution (5) action (1)

\[ S = \frac{1}{2e^2}\text{tr} \text{Re} \int d^4x d^2\theta W^a W_a - \frac{1}{64e^2}\text{tr} \text{Re} \int d^4x d^4\theta \left[ 16(e^{-2V}D^n e^{2V})W_a + \left( e^{-2V}D^n e^{2V}\right)\bar{D}^2 \left( e^{-2V}D_a e^{2V}\right) \right], \quad (9) \]

where

\[ W_a = \frac{1}{32}e^\Omega \bar{D}(1 - \gamma_5)D \left( e^{-\Omega}e^{-\Omega^+}(1 + \gamma_5)D_a e^{\Omega^+}e^\Omega \right)e^{-\Omega}, \quad (10) \]

and the notation

\[ D^2 \equiv \frac{1}{2}\bar{D}(1 + \gamma_5)D; \quad \bar{D}^2 \equiv \frac{1}{2}\bar{D}(1 - \gamma_5)D; \]

\[ D^a \equiv \left[ \frac{1}{2}\bar{D}(1 + \gamma_5) \right]^a; \quad D_a \equiv \left[ \frac{1}{2}(1 + \gamma_5)D \right]_a; \]

\[ D^a \equiv \left[ \frac{1}{2}\bar{D}(1 - \gamma_5) \right]^a; \quad D_a \equiv \left[ \frac{1}{2}(1 - \gamma_5)D \right]_a \quad (11) \]

is used. Action of the covariant derivatives on the field \( V \) in the adjoint representation is defined by the standard way.

We note that action (3) is also invariant under the quantum transformations

\[ e^{2V} \rightarrow e^{-\lambda^+}e^{2V}e^{-\lambda}; \quad \Omega \rightarrow \Omega; \quad \Omega^+ \rightarrow \Omega^+ \quad (12) \]

where \( \lambda \) is a background chiral superfield, which satisfies the condition

\[ \bar{D}\lambda = 0. \quad (13) \]

Such a superfield can be presented in the form \( \lambda = e^\Omega \Lambda e^{-\Omega} \), where \( \Lambda \) is a usual chiral superfield.

It is convenient to choose a regularization and gauge fixing so that invariance (6) will be unbroken. We fix the gauge by adding
\[ S_{gf} = -\frac{1}{32e^2} \text{tr} \int d^4x d^4\theta \left( V D^2 \bar{D}^2 V + V \bar{D}^2 D^2 V \right) \] (14)

to the action. In this case terms quadratic in the superfield \( V \) will have the simplest form:

\[ \frac{1}{2e^2} \text{tr} \text{Re} \int d^4x d^4\theta V D^2 \mu V. \] (15)

Also it is necessary to add an action for the Faddeev–Popov ghosts \( S_c \) and an action for the Nielsen–Kallosh ghosts. Because in this paper we will calculate a contribution of the matter superfields, we are not interested in the concrete form of these terms. The gauge fixing breaks the invariance of the action under quantum gauge transformations \( \text{(12)} \), but there is a remaining invariance under the BRST-transformations. The BRST-invariance leads to the Slavnov–Taylor identities, which relate vertex functions of the quantum gauge field and ghosts.

However, in order to simplify the calculations it is convenient to choose a regularization so that it breaks the invariance under the BRST-transformations. We will add the following term with the higher covariant derivatives

\[ S_\Lambda = \frac{1}{2e^2} \text{tr} \text{Re} \int d^4x d^4\theta V \left( D^2 \mu \right)^{n+1} \Lambda_{2n} V \] (16)

to the action. (A method, used here, is a slightly different from the one, proposed in Ref. \([9]\).) Because the regularization is not invariant under the BRST-transformations, it is necessary to use a special renormalization scheme, which ensures that the Slavnov–Taylor identities are satisfied in each order of the perturbation theory due to some additional subtractions. Such a renormalization scheme was proposed in Refs. \([12, 13]\), and generalized to the supersymmetric case in Refs. \([14, 15]\).

It is important to note that the Gell-Mann–Low function is scheme independent and does not depend on a regularization and a renormalization prescription.

The generating functional is written as

\[ Z[J, \Omega] = \int D\mu \exp \left\{ iS + iS_\Lambda + iS_{gf} + iS_{gh} + iS_{\phi_0} + i \int d^4x d^4\theta \left( J + J[\Omega] \right) \left( V'[V, \Omega] - V \right) \right\}, \] (17)

where the superfield \( V \) is defined by

\[ e^{2V} \equiv e^{\Omega^+} e^\Omega, \] (18)

and \( J[\Omega] \) is an arbitrary functional. \( S_{gf} \) is gauge fixing action \( \text{(14)} \) and \( S_{gh} = S_c + S_B \) is the corresponding action for the Faddeev–Popov and Nielsen–Kallosh ghosts. Moreover, we added terms with additional sources \( \phi_0 \), which are written as

\[ S_{\phi_0} = \frac{1}{4} \int d^4x d^4\theta \left( \phi_0^+ e^{\Omega^+} e^{2V} e^\Omega \phi + \phi^+ e^{\Omega^+} e^{2V} e^\Omega \phi_0^+ + \phi_0^+ e^{-\Omega^+} e^{-2V^t} e^{-\Omega} \phi + \phi^+ e^{-\Omega^+} e^{-2V^t} e^{-\Omega} \phi_0^+ \right). \] (19)
Unlike the fields $\phi$ and $\tilde{\phi}$, the fields $\phi_0$ and $\tilde{\phi}_0$ are not chiral. In principle, adding such terms is not quite necessary, but it is convenient to formulate the new identity in terms of variational derivatives with respect to these sources.

Using the functional $Z[J, \Omega]$ it is possible to construct the generating functional for connected Green functions

$$W[J, \Omega] = -i \ln Z[J, \Omega] = - \int d^4x d^4\theta \left( J + J[\Omega] \right) V + W_0 \left[ J + J[\Omega], \Omega \right]$$

and the corresponding effective action

$$\Gamma[V, \Omega] = - \int d^4x d^4\theta \left( JV + J[\Omega]V \right) + W_0 \left[ J + J[\Omega], \Omega \right] - \int d^4x d^4\theta JV,$$

where the sources should be expressed in terms of fields using the equation

$$V = \frac{\delta}{\delta J} W[J, \Omega] = -V + \frac{\delta}{\delta J} W_0 \left[ J + J[\Omega], \Omega \right].$$

In order to understand how $\Gamma[V, \Omega]$ is related with the ordinary effective action, we perform the substitution $V \to V'$ in the generating functional $Z$. Then we obtain

$$Z[J, \Omega] = \exp \left\{ -i \int d^4x d^4\theta \left( J + J[\Omega] \right) V \right\} Z_0 \left[ J + J[\Omega], \Omega \right],$$

where

$$Z_0[J, \Omega] = \int D\mu \exp \left\{ iS + iS_\lambda + iS_{gf} + iS_{gh} + i \int d^4x d^4\theta JV \right\}.$$ 

Therefore,

$$\Gamma[V, \Omega] = W_0 \left[ J + J[\Omega], \Omega \right] - \int d^4x d^4\theta \left( J[\Omega]V + J \frac{\delta}{\delta J} W_0 \left[ J + J[\Omega], \Omega \right] \right).$$

Let us now set $V = 0$, so that

$$V = \frac{\delta}{\delta J} W_0 \left[ J + J[\Omega], \Omega \right].$$

and take into account that in this case the superfield $K$ is nontrivially present only in gauge transformation (6) for the fields $\Omega$ and $\Omega^+$, and the only invariant combination is expression (18). (It is invariant in a sense, that the corresponding transformation law does not contain the superfield $K$.) Therefore, if $V = 0$, then we can set

$$\Omega = \Omega^+ = V.$$

In this case the effective action is

$$\Gamma[0, V] = W_0 \left[ J + J[V], V \right] - \int d^4x d^4\theta \left( J + J[V] \right) \frac{\delta}{\delta J} W_0 \left[ J + J[V], V \right].$$
and does not depend on the form of the functional $J[\Omega]$.

If the gauge fixing terms, ghosts, and the terms with higher derivatives depended only on $V'$, expression (28) would coincide with the ordinary effective action. However, the dependence on $V$, $\Omega$, and $\Omega'$ is not factorized into the dependence on $V'$ in the proposed method of renormalization and gauge fixing. According to Ref. [16, 17] the invariant charge (and, therefore, the Gell-Mann-Low function) is gauge independent, and the dependence of the effective action on gauge can be eliminated by renormalization of the wave functions of the gauge field, ghosts, and matter fields. Therefore, for calculating the Gell-Mann–Low function we may use the background gauge described above.

Nevertheless, generating functional (17) is not yet completely constructed. The matter is that adding the term with higher derivatives does not remove divergences from one-loop diagrams. To regularize them, it is necessary to insert the Pauli-Villars determinants in the generating functional [7]:

$$\prod_i \left( \det PV(V, V, M_i) \right)^{c_i},$$

(29)

where the coefficients $c_i$ satisfy conditions

$$\sum_i c_i = 1; \quad \sum_i c_i M_i^2 = 0.$$

(30)

The Pauli–Villars fields are constructed for the quantum gauge field, ghosts, and matter fields. Because in this paper we consider only a contribution of the matter superfields, we present explicit expression only for them:

$$\left( \det PV(V, M) \right)^{-1} = \int D\Phi D\bar{\Phi} \exp \left( iS_{PV} \right),$$

(31)

where (taking into account condition (27))

$$S_{PV} \equiv Z(e, \Lambda/\mu) \frac{1}{4} \int d^4x d^4\theta \left( \Phi^+ e^V e^{2V} e^V \Phi + \bar{\Phi}^+ e^{-V^t} e^{-2V^t} e^{-V^t} \bar{\Phi} \right) +$$

$$+ \frac{1}{2} \int d^4x d^2\theta M \Phi^t \Phi + \frac{1}{2} \int d^4x d^2\bar{\theta} M \bar{\Phi}^t \bar{\Phi}^*.$$

(32)

We assume that $M_i = a_i \Lambda$, where $a_i$ are some constants. Inserting the Pauli-Villars determinants allows cancelling the remaining divergences in all one-loop diagrams.

3 New identity for Green functions and its verification in the non-Abelian theory

In the massless case the new identity for Green functions can be formulated as follows [2, 3]:

It is easy to see that the two-point Green function for the matter superfield is written as
\[
\frac{\delta^2 \Gamma}{\delta \phi^+_x \delta \phi_y} = \frac{D_x^2 D_y^2}{16} G(\partial^2) \delta_{xy},
\]

(33)

where \( G \) is a function. Then, setting the momentum of the gauge field to 0, from the Slavnov–Taylor identities it is possible to find the vertex

\[
\frac{\delta^3 \Gamma}{\delta V^a_y \delta \phi_0 \delta \phi_x} \bigg|_{p=0} = E \left[ -2 \partial^2 \Pi_{1/2y} \left( D^2 y_{xy} \delta^8_{yz} \right) F(q^2) + \frac{1}{8} D^b c_b D^2 y_{xy} \left( D^2 y_{xy} D^c y_{yz} \right) f(q^2) - \frac{1}{16} q^\mu G'(q^2) D^\gamma \gamma_5 D_y \left( D^2 y_{xy} \delta^8_{yz} \right) - \frac{1}{4} D^2 y_{xy} \delta^8_{yz} G(q^2) \right] T^a,
\]

(34)

where \( T^a \) denotes the generators of the gauge group in a representation, in which the matter superfields are. The functions \( f \) and \( F \) cannot be found from the Slavnov–Taylor identities. The new identity can be written in the form

\[
\int d^4 q \Lambda \frac{d}{d \Lambda} \frac{f(q^2)}{q^2 G(q^2)} = 0.
\]

(35)

The derivative with respect to \( \ln \Lambda \), appearing in this expression, is introduced in order to obtain well defined integrals. In the end of this section we explain this by a concrete example.

In the Abelian case such an identity can be verified by the straightforward summation of Feynman diagrams [8]. However, the Feynman rules are different in a non-Abelian theory mostly due to vertexes with the selfaction of the gauge field. This essentially complicates applying this method. For diagrams, which do not contain such vertexes the calculations are similar to the Abelian case. But for diagrams with the triple vertex of the gauge field a proof, made in Ref. [8] is not applicable or, at least, should be essentially modified. So, there is a problem, whether the new identity is valid in this case also. In order to answer it, it is not necessary to calculate all Feynman diagrams in one or another order of the perturbation theory. According to Refs. [4, 18], if we fix an arbitrary diagram with a loop of the matter superfields and without external lines, then the new identity should be valid for the sum of diagrams, which are obtained by cutting a loop of the matter superfields by all possible ways. (In order to obtain the vertex function we should attach to them one more line of the background gauge field by all possible ways.)

In this paper we consider a diagram, presented in Fig. 1, as a starting point.

Figure 1: Diagram, generating the considered contribution to the new identity

From the topological point of view there is the only way to cut a loop of the matter superfield, presented in Fig. 2. Therefore, it is necessary to calculate a set of diagrams,
In all these diagrams the chiral field $\phi$ is at the first external line, and the non-chiral field $\phi^*_0$ is at the second one. Therefore, all presented diagrams are not topologically equivalent.

Calculating these diagrams we can find the function $f$. The function $G$ in the lowest approximation should be set to 1. Really, in the tree approximation $G = 1$. Therefore, in the given order for the considered class of diagrams we have:

$$G(q^2) = 1 + O(\alpha^2); \quad f(q^2) = \alpha^2 f^{(2)}(q^2) + O(\alpha^3).$$ (36)

Therefore,

$$\int d^4 \Lambda \frac{d}{d\Lambda} \frac{f(q^2)}{q^2 G(q^2)} = \int d^4 \Lambda \frac{d}{d\Lambda} \frac{\alpha^2 f^{(2)}(q)}{q^2} + O(\alpha^3).$$ (37)

So, we see that the considered contribution is actually determined by the two-loop value of the single function $f^{(2)}$.

In order to find the two-loop value of the function $f^{(2)}$, it is necessary to make an explicit calculation of Feynman diagrams, presented in Fig. 3 using the standard supergraph technique. The result is (in Euclidean space, after the Weak rotation)

$$f^{(2)}(q) = -2\pi^2 C_2 \left( C_2(R) - \frac{1}{2} C_2 \right) \int \frac{d^4 k \, d^4 l}{(2\pi)^8} \left( \frac{l^\mu}{(k + q + l)^2} + \frac{(k + q)^\mu}{(k + q)^2} \right) \times$$ (38)

$$\times \frac{1 + k + l}{(k + q)^2 (k + q + l)^2 k^2} \left( 1 + k^2 n / \Lambda^2 n \right) \left( 1 + l^2 n / \Lambda^2 n \right) \left( 1 + (k + l)^2 n / \Lambda^2 n \right).$$
where \(C_2(R)\) and \(C_2\) are defined by

\[
T^a T^a = C_2(R),
\]

\[
f^{amn} f^{bmn} = C_2 \delta^{ab}.
\]

Substituting this expression into Eq. (37), we obtain (technical details are presented in Appendix A) that in the considered approximation for the considered diagrams

\[
\int \frac{d^4q}{(2\pi)^4} \Lambda \frac{d}{d\Lambda} q^2 G(q^2) = \alpha^2 \pi^2 \Lambda \frac{d}{d\ln \Lambda} \left[ \frac{(k + q + l)^\mu}{q^2 (k + q)^2 (k + q + l)^2} \right] \times
\]

\[
\frac{1}{k^2 \left( 1 + k^{2n}/\Lambda^{2n} \right) l^2 \left( 1 + l^{2n}/\Lambda^{2n} \right) (k + l)^2 \left( 1 + (k + l)^{2n}/\Lambda^{2n} \right)} = 0.
\]

Therefore, the new identity for Green functions seems to be valid in the non-Abelian theory.

Using the considered example it is convenient to explain why we introduce the derivative

\[
\Lambda \frac{d}{d\Lambda} = \frac{d}{d\ln \Lambda}
\]

in the integrand. Let us first propose that this derivative is absent. Then, after taking the well defined integrals with respect to \(d^4k\) and \(d^4l\) from the dimensional considerations we obtain the integral

\[
\int \frac{d^4q}{(2\pi)^4} a(q^2/\Lambda^2) q^4,
\]

where \(a\) is a dimensionless function, which is rapidly decreasing at \(q \to \infty\). In general, it is possible that \(a(0) \neq 0\). (It is easy to see that the value \(a(0)\) is a finite constant.) But if \(a(0) \neq 0\), then the integral in Eq. (43) is not well defined: it is divergent in the infrared region. In order to avoid this we introduce the additional differentiation with respect to \(\ln \Lambda\). Due to its presence the term \(a(0)\), which does not depend on \(\Lambda\), disappears, and the integral becomes finite in the infrared region.

According to Refs. [2, 3] the left hand side of Eq. (35) is actually a contribution to the two-point Green function of the gauge field, all other contributions being integrals of total derivatives. Therefore, appearing of a total derivative in Eq. (41) confirms a proposal that in supersymmetric theories all contributions to the Gell-Mann–Low function are integrals of total derivatives if the higher derivatives are used for the regularization.

4 Conclusion

In this paper we showed that new identity (35) was also valid in the non-Abelian theory. Similar to the case of the electrodynamics, it follows from the fact that all integrals...
defining the Gell-Mann–Low function are factorized to the total derivatives. The consid-
ered identity seems to be a consequence of a rather nontrivial symmetry. Deriving this
identity from the first principles is a rather interesting and complicated problem.
Moreover, the calculations, performed in this paper, confirm a hypothesis that all
contributions to the Gell-Mann–Low function in supersymmetric theories are integrals of
total derivatives. The reason of this fact is also so far unclear.

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A Obtaining the integral of total derivative

Here we present a detailed derivation of Eq. \( \text{(41)} \) from Eq. \( \text{(38)} \), because it is not
quite trivial.

After substituting the function \( f \) the left hand side of the new identity is written as

\[
\int d^4q \frac{d}{d\Lambda} f^{(2)}(q) \frac{1}{q^2} = -2\pi^2 C_2 \left( C_2(R) - \frac{1}{2} C_2 \right) \int \frac{d^4q \, d^4k \, d^4l}{(2\pi)^{12}} \Lambda \frac{d}{d\Lambda} \times
\]

\[
\times \left( \frac{l^\mu}{(k + q + l)^2} + \frac{(k + q)^\mu}{(k + q)^2} \right) \frac{(k + q + l)_\mu}{q^2(k + q)^2(k + q + l)^2k^2 \left( 1 + k^{2n}/\Lambda^{2n} \right)} \times
\]

\[
\times \frac{1}{l^2 \left( 1 + l^{2n}/\Lambda^{2n} \right)} (k + l)^2 \left( 1 + (k + l)^{2n}/\Lambda^{2n} \right).
\]

In the first term we perform the following sequence of substitutions: \( q \rightarrow q - k - l; \)
\( k \rightarrow -k; l \rightarrow -l \). As a result we obtain

\[
\frac{(k + q + l)_\mu l^\mu}{q^2(k + q)^2(k + q + l)^4} \rightarrow -\frac{q_\mu l^\mu}{q^4(q + k)^2(q + k + l)^2};
\]

all other multipliers being the same. Then we perform the substitutions \( l \rightarrow l - k; k \rightarrow -k; \)
\( k \rightarrow l \), after which this factor becomes

\[
-\frac{q_\mu (k + l)^\mu}{q^4(q + k)^2(q + k + l)^2}.
\]

And, finally, we add to the expression in the round brackets in Eq. \( \text{(44)} \)

\[
0 = -2 + \frac{q^\mu q_\mu}{q^2} + \frac{(k + q + l)^\mu(k + q + l)_\mu}{(k + q + l)^2}.
\]

Finally, the contribution to the two-point Green function of the gauge field, we are inter-
ested in, can be rewritten in the form
\[
\int d^4q \Lambda \frac{d}{d\Lambda} \frac{f^{(2)}(q)}{q^2} = -2\pi^2 C_2 \left( C_2(R) - \frac{1}{2} C_2 \right) \int d^4q d^4k d^4l \Lambda \frac{d}{d\Lambda} \left\{ -2 + \frac{q^\mu (k + q + l)_\mu}{q^2} + \frac{(k + q)^\mu (k + q + l)_\mu}{(k + q)^2} + \frac{(k + q + l)^\mu (k + q + l)_\mu}{(k + q + l)^2} \right\} \frac{1}{(k + q)^2} \times
\frac{1}{q^2(k + q + l)^2} \left( 1 + k^{2n}/\Lambda^{2n} \right) \left( 1 + l^{2n}/\Lambda^{2n} \right) \left( 1 + (k + l)^{2n}/\Lambda^{2n} \right),
\]

Derivation of Eq. (41) from this expression is evident.

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