Dynamics of a magnetic skyrmionium in an anisotropy gradient

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Received June 13, 2019; revised July 9, 2019; accepted July 10, 2019; published online July 23, 2019

A magnetic skyrmion is a novel magnetization configuration with a zero skyrmion number, which can be viewed as a combination of two skyrmions with opposite skyrmion numbers. Here, we analytically and numerically study the dynamics of a skyrmionium under an anisotropy gradient. We find that the skyrmionium moves straight and can be efficiently driven by an anisotropy gradient with higher speed. Furthermore, skyrmionium deformation in a larger anisotropy gradient can be efficiently suppressed by narrowing the width of the film. Our work shows an alternative driving method that may be promising for applications of skyrmionium-based racetrack memory.

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Recently, magnetization configurations, such as droplets,¹ domain walls,² skyrmions,³,⁴ and skyrmioniums,⁵–⁶ have attracted considerable attention in the application of spintronic devices as information carriers. As topologically protected magnetic structures, skyrmions are quite small in size and can be driven by spin transfer torque or spin Hall effect with lower current density.⁷–¹⁰ Since they have been theoretically predicted and experimentally discovered in bulk magnetic materials and films contacted with heavy metal layers,¹¹–¹³ skyrmions are suggested for use in next-generation memory and spintronic devices, such as racetrack memory,²²,²³ spin transfer nano oscillators,¹⁵,¹⁶ logic devices,¹⁷–¹⁹ neuromorphology devices²⁰,²¹ and so on.

The skyrmion Hall effect is a significant obstacle to the manipulation of skyrmions in confined magnetic geometries, where skyrmions experience a transverse shift induced by the Magnus force.²⁰,²¹ Several concepts have been proposed to overcome this limitation, such as modifying the magnetic parameters of the racetrack edge [magnetic anisotropy, Dzyaloshinskii–Moriya interaction (DMI)],²²,²³ and reducing the symmetry of the contacted heavy metal layer.²⁴ Another idea is the combination of two skyrmions carrying opposite Q, whose zero topological number induces a zero skyrmion Hall angle, like bilayer skyrmions,²⁵ antiferromagnetic skyrmions²⁶ and skyrmioniums.²⁷–²⁹ Since it is difficult to detect antiferromagnetic skyrmions as well as bilayer skyrmions, skyrmioniums, which are skyrmion-like magnetization configurations with a topological number of Q = 0, may be another good candidate to overcome this obstacle. The generation and stabilization of skyrmioniums have been theoretically proposed in systems with DMI²⁷,²⁸ or the gradient of curvature.²⁹ Early experimental results show skyrmioniums are stabilized in FePt nanodots with perpendicular anisotropy and without DMI.³⁰ They have also been experimentally verified in a thin ferromagnetic film and ferromagnetic–magnetic topological insulator heterostructure.³¹–³³ The electrical writing and moving of skyrmioniums have been investigated in a few works,³⁴–³⁶ where the skyrmionium moves in a racetrack with high velocity without the skyrmion Hall effect. Other manipulation methods are also proposed, like the use of magnetic fields,³⁴ and spin waves.³⁵ Meanwhile, a more energy-efficient driving method is needed and the dynamics of skyrmioniums under an anisotropy gradient remain unexplored.

In this letter, we investigate the dynamics of a skyrmionium under an anisotropy gradient. It is found that the skyrmionium can be efficiently driven along the direction of an anisotropy gradient without deflection, and the moving velocity is obviously higher than that of skyrmions. This driving method can be used in the insulator heterostructure. Moreover, skyrmionium deformation under a larger anisotropy gradient can be suppressed by narrowing the film width.

Micromagnetic simulations have been performed using the mumax³ code.³⁶ The time-dependent magnetization dynamics are given by the Landau–Lifshitz–Gilbert equation

\[
\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t},
\]

where \( \mathbf{m} = M/M_s \) is the unit vector of the magnetization, and \( M_s \) is the saturation magnetization. \( \gamma \) and \( \alpha \) are the gyromagnetic ratio and the Gilbert damping, respectively. \( \mathbf{H}_{\text{eff}} = 2/(\mu_0 M_s) [\nabla \times (\mathbf{m} + \Delta K \mathbf{m}) \mathbf{e}_z + D(\nabla \mathbf{m} - (\nabla \mathbf{m}) \mathbf{e}_z)] + \mathbf{H}_B \) is the effective field including the Heisenberg exchange field with the exchange stiffness \( A \), the perpendicular magnetic anisotropy field with the anisotropy constant \( K_u \), the interfacial DMI field characterized by the DMI constant \( D \) and the dipolar field \( \mathbf{H}_d \), where \( \Delta K \) is the anisotropy gradient and is equal to zero in the stabilization of the skyrmionium. The ferromagnetic film size is \( 512 \text{ nm} \times 512 \text{ nm} \times 0.6 \text{ nm} \), and the mesh size is \( 2 \text{ nm} \times 2 \text{ nm} \times 0.6 \text{ nm} \). The model parameters adopted are as follows:³⁷–³⁹ \( A = 15 \times 10^{-12} \text{ J m}^{-1} \), \( K_u = 0.8 \times 10^5 \text{ J m}^{-3} \), \( M_s = 580 \times 10^3 \text{ A m}^{-1} \). The Gilbert damping \( \alpha \) varies from 0.01 to 0.3 and the DMI strength is \( D = 3.5 \text{ mJ m}^{-2} \).

We consider an electrode layer and a wedged insulating dielectric layer fabricated on top of the ferromagnetic layer. A constant linear anisotropy gradient is generated along the \( x \)-axis of the system,³⁷–⁴¹ as shown in Fig. 1. The maximum \( (K_u_{\text{max}}) \) and minimum \( (K_u_{\text{min}}) \) magnetic anisotropies are shown at the edge of the film. The inset shows the magnetization configuration of a skyrmionium located in the ferromagnetic film, where the red and blue colors represent positive and negative \( \mathbf{m}_z \), respectively. The anisotropy distribution along the \( x \)-axis is \( K_u = K_u + (dK_u/dx)x \), in which \( dK_u/dx \)
represents the anisotropy gradient; its values with \( K_{ua\text{ max}} \) and \( K_{ua\text{ min}} \) are shown in Table I.

Figure 2 depicts the skyrmionium radius \( r_{in} \) and \( r_{out} \) as a function of \( K_u \) and \( D \). The results show that the size increases with increasing \( D \) and decreasing \( K_u \). Four different magnetization states appear, which are a single domain, skyrmion, skyrmionium and labyrinth domain. As shown in Table I and Fig. 2, the anisotropy variation along the \( x \)-axis is tiny in our case and has a weak influence on the skyrmionium size during the motion (dashed circles in Fig. 2).

To describe the skyrmionium dynamics, we use a model in the framework of the Thiele equation, which treats the skyrmionium as a rigid particle,

\[
G \times \mathbf{v}_i + D\alpha \mathbf{v}_i = \mathbf{F},
\]

where \( \mathbf{G} \) is the velocity of the skyrmionium, and \( \mathbf{G} = Ge = 4\pi Qe_z \) is the gyromagnetic coupling vector. \( Q = \frac{1}{4\pi} \int (\partial_i \mathbf{m} \times \partial_j \mathbf{m})\,dx\,dy \) is the skyrmion number. For a skyrmion with \( Q = 0 \), the corresponding \( \mathbf{G} = 0 \).\[ D = \begin{pmatrix} D_{ij} & 0 \\ 0 & D_{0} \end{pmatrix} \]
is the dissipative force tensor, where \( D_{ij} = \int dx\,dy \partial_i \mathbf{m} \cdot \partial_j \mathbf{m} = \int dx\,dy \partial_i \mathbf{m} \cdot \partial_j \mathbf{m} \) with \( i = j = x, y \). \( \mathbf{F} \) is the driving force from the anisotropy gradient along the \( x \)-axis, described as

\[
\mathbf{F} = \frac{\gamma}{\mu_0 M_s} \frac{dK_u}{dx} \int\!dx\,dy(1 - m_i^2).
\]

Force due to the boundary effect is not considered because the skyrmionium is far from the edge of the film. As the anisotropy gradient is only applied along the \( x \)-axis, \( \mathbf{F} = \mathbf{F}_{ex} \), we obtain

\[
\begin{align*}
G_{vx} + D\alpha v_x &= F_x, \\
-G_{vy} + D\alpha v_y &= 0
\end{align*}
\]

where \( v_x \) and \( v_y \) are the moving velocity in the \( x \) and \( y \) directions. Thus

\[
v_x = \frac{\gamma}{\mu_0 M_s} \frac{dK_u}{dx} \int dx\,dy(1 - m_x^2),
\]

\[
v_y = 0
\]

which show that the skyrmionium motion is along the direction of the anisotropy gradient (\( x \)-axis), and the velocity is proportional to \( dK_u/dx \) and inversely proportional to \( \alpha \).

For a skyrmion with \( Q = \pm 1 \), \( G \neq 0 \), the skyrmion velocities \( v_x \) and \( v_y \) are expressed as

\[
\begin{align*}
v_x &= \frac{dK_u}{dx} \frac{D_{ij}0\gamma}{\mu_0 M_s((4\pi Q)^2 + (\alpha D_{ij})^2)} \int dx\,dy(1 - m_i^2) \\
v_y &= \frac{dK_u}{dx} \frac{4\pi Q\gamma}{\mu_0 M_s((4\pi Q)^2 + (\alpha D_{ij})^2)} \int dx\,dy(1 - m_i^2)
\end{align*}
\]

thus the skymionium velocity

\[
v = \sqrt{v_x^2 + v_y^2} = \frac{\gamma}{\mu_0 M_s((4\pi Q)^2 + (\alpha D_{ij})^2)} \int dx\,dy(1 - m_i^2).
\]

The skyrmionium velocity is much higher than that of the skyrmion. The moving directions for them are quite different, which can be characterized as the skyrmion Hall angle

\[
\theta = \arctan(v_y/v_x),
\]

which is \( \theta = 0 \) for the skyrmionium, and \( \theta = \arctan(4\pi Q/\langle D_{ij}0\rangle) \) for the skyrmion. As the skyrmionium is considered as two skyrmions with opposite skyrmion numbers \( Q = 1 \) and \( Q = -1 \), the total effective force acting on the skyrmionium can be treated as the combined forces from two skyrmions, where the transverse force is zero.

Based on the analytical model, we carried out a simulation of the motion of the skyrmionium. The simulated and analytical skyrmionium velocities \( v_x \) and \( v_y \) as a function of the damping constant \( \alpha \) are shown in Fig. 3. An inverse function relationship of \( v_y \) to \( \alpha \) is observed as predicted in Eq. (5), and \( v_x \) is zero. The inset shows the simulated results of \( \theta \). It is worth noting that the skyrmionium size will slightly increase when moving from a higher \( K_u \) area to a lower \( K_u \).
area. In our system, however, when we set the DMI strength to \( D = 3.5 \text{ mJ m}^{-2} \) and \( dK/dx = -0.5 \times 10^{11} \text{ J m}^{-4} \), the skyrmion magnetization profile only exhibits small differences between the regions of higher \( K_u \) and lower \( K_u \), and the change of skyrmionium size is negligible, as previously shown in Fig. 2 and Table 1.

In order to further investigate the skyrmionium dynamics driven by an anisotropy gradient, we investigate the skyrmionium motion under different \( dK/dx \), and compare it with the dynamics of skyrmions with \( Q = \pm 1 \), as shown in Fig. 4. The results depicted in Fig. 4(a) show that a larger anisotropy gradient will create a stronger driving force which in turn leads to a higher velocity for the skyrmionium; the velocity \( v_x \) is linearly proportional to \( |dK/dx| \). However, \( v_y \) is close to zero. These results are consistent with the analytical results in Eq. (5). Figure 4(b) shows the trajectories of the skyrmionium \((Q = 0)\) and skyrmions \((Q = \pm 1)\) at \( \alpha = 0.3 \); the skyrmionium moves straight along the \( +x \) direction. The skyrmions’ motions exhibit a deflection towards the \( +y \) and \( -y \) directions for \( Q = -1 \) and \( Q = 1 \), respectively.

Figure 4(c) shows the skyrmion Hall angle \( \theta \) as a function of the anisotropy gradient for \( \alpha = 0.02 \) and \( \alpha = 0.3 \). The skyrmionium moves straight under anisotropy gradients and \( \theta = 0^\circ \). Moreover, the skyrmion Hall angle for the skyrmionium is independent of the damping constant. However, the skyrmions with \( Q = -1 \) and \( Q = 1 \) exhibit a large skyrmion Hall angle \( \theta \). For the skyrmion with \( Q = -1 \) (upper inset), \( \theta = 88.25^\circ \) with \( \alpha = 0.02 \), and it decreases to 65° with \( \alpha = 0.3 \). For the skyrmion with \( Q = 1 \) (lower inset), \( \theta = -88.25^\circ \) and \( -65^\circ \) for \( \alpha = 0.02 \) and 0.3, respectively. By comparing the simulated results of the skyrmionium and skyrmion velocities at certain values of \( |dK/dx| \) for \( \alpha = 0.02 \) [Fig. 4(d)], where \( |dK/dx| = 0.01, 0.5, 1.0 \times 10^{11} \text{ J m}^{-4} \), it is found that the skyrmionium velocity is much higher than that of the skyrmions. It is worth mentioning that only \( v_y \) of the skyrmionium and \( v_y \) of the skyrmions are depicted to demonstrate the moving directions of the skyrmions in Fig. 4, because \( v_x = 0 \) for the skyrmionium and \( v_x \) for the skyrmions are quite low.

A larger anisotropy gradient is necessary to increase the skyrmionium velocity. However, it will induce a significant change of the skyrmionium size and deformation, which limits application in racetrack memory. Skyrmionium deformation in magnetic field under a larger anisotropy at \( \alpha = 0.02 \) is indicated in Fig. 5. When the skyrmionium moves in the film with width \( d = 256 \text{ nm} \), its magnetization profile remains unchanged in a small anisotropy gradient with \( |dK/dx| = 0.5, 1 \times 10^{11} \text{ J m}^{-4} \). For \( |dK/dx| = 2 \times 10^{11} \text{ J m}^{-4} \), the transverse shift of the inner part and the elongation of the outer part for the skyrmionium are more significant. The two nested skyrmions experience the Magnus force when the skyrmionium is moving, which are in opposite directions \((+y)\) and \((−y)\). In a large anisotropy gradient, the Magnus force acting on the two skyrmions increases with increasing speed. Thus, the skyrmionium stabilization is

Fig. 3. (Color online) Skyrmionium velocity \( v_x \) (red points) and \( v_y \) (blue points) as a function of \( \alpha \), where \( D = 3.5 \text{ mJ m}^{-2} \) and \( dK/dx = -0.5 \times 10^{11} \text{ J m}^{-4} \). The inset shows the skyrmion Hall angle \( \theta \) for the skyrmionium with varying \( \alpha \).

Fig. 4. (Color online) (a) Velocities of the skyrmionium \((v_x \text{ and } v_y)\) as a function of the anisotropy gradient with \( \alpha = 0.02 \). (b) Trajectories of the skyrmionium and skyrmions \((Q = \pm 1)\) driven by the anisotropy gradient along the \( x \) direction with \( \alpha = 0.3 \). The dots denote the skyrmion and skyrmionium centers. The insets are snapshots of the skyrmion with \( Q = -1 \), the skyrmionium with \( Q = 0 \) and the skyrmion with \( Q = 1 \) (from top to bottom). (c) Skyrmion Hall angle \( (\theta = Q - 1, 0, 1) \) as a function of the anisotropy gradient with \( \alpha = 0.02 \) and \( \alpha = 0.3 \). (d) Comparison of velocities \( v_x \) of the skyrmions with \( Q = \pm 1 \) and \( v_y \) of the skyrmionium with \( Q = 0 \) at \( |dK/dx| = 0.01, 0.5, 1.0 \times 10^{11} \text{ J m}^{-4} \); the corresponding velocities are marked.

Fig. 5. (Color online) Comparison of skyrmion motion driven by anisotropy gradient in films with different widths, \( d = 256 \text{ nm} \) (a–c, b) and \( d = 100 \text{ nm} \) (d–g); the anisotropy gradient is \( |dK/dx| = 0.5, 1.0, 2.0 \times 10^{11} \text{ J m}^{-4} \). The white dashed lines indicate the center position along the \( y \) direction. (h) is the deformation of a skyrmionium with negative polarity.
perturbed with the inner and outer skyrmions moving toward the $-y$ and $y$ directions, respectively. The competition between these two effects induces skyrmion deformation in the large anisotropy. As depicted in Fig. 5(h), a skyrmionium with negative polarity moves with a deformation that exhibits a slight change with a small transverse shift along the $y$ direction for the inner part, which is the opposite of that shown in Fig. 5(c).

With the film width decreased to 100 nm, the skyrmionium size for the initial state reduces compared to the skyrmionium in the film with $d = 256$ nm [Figs. 5(d)–5(g)]. In different anisotropy gradients, the skyrmionium magnetization profiles remain stable, and only a small deformation appears in a larger anisotropy gradient $|dK_u/dx| = 2 \times 10^{11}$ J m$^{-2}$. This is because the Magnus force acting on the inner and outer parts of the skyrmionium is compensated by the edge of the film, and the deformation is limited compared to the case of $d = 256$ nm. By modifying the boundary effect of the skyrmionium-based racetrack, the stabilization and moving speed can be enhanced.

In summary, we have studied skyrmionium motion in an anisotropy gradient. We find that the skyrmionium moves straight along the direction of the anisotropy gradient, and the velocity increases linearly with the gradient, which is much higher than that of magnetic skyrmions in the same system. Moreover, a larger anisotropy gradient induces deformation of the skyrmionium, which can be limited by narrowing the magnetic film. Our results may be useful in applications of energy-efficient skyrmionium-based spintronic devices.

**Acknowledgments.** This work is supported by the National Natural Science Fund of China (Grants No. 11574121 and No. 51771086).}

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