The Prediction of New Particles in Contemporary Physics

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This paper proves that a particle with mass different of zero cannot be its own antiparticle. Hence, particles such as \( \pi^0 \) and \( Z^0 \) must have their correspondent antiparticle.

**Introduction**

In a previous paper [1] is questioned the concept of inversion of coordinates symmetry, from which the violation of parity in weak interaction was a consequence. After a lengthy review, it was found the cause of this misleading interpretation. The stringent requirement that imposes the theory of relativity on the equivalence of the four-space-temporal coordinates was ignored in the formulation of the principle of parity conservation. To be more precise the four coordinates, where an event take place are on the same footing. Time coordinate is equivalent to any space coordinate and must be bound to these in the physical description of any event. Therefore, the “full” Poincare group, which includes the simultaneous inversion of the space-time coordinates, was not considered on the formulation of the reflection symmetry. In previous paper this misunderstanding was discussed and a solution suggested [1].

The prediction of new particles by this new theory it is the point of departure of these two inversion concepts. CP, CT, PT are violated in a theory were the group of symmetry is not the “full” Poincare group [2]. The “full Poincare group” preserves CPT invariance. However, the same statement could not be extrapolated to theories based on higher symmetry groups, Such as Superstrings [3], Quantum Gravity, etc.; where CPT could be violated. Another larger symmetry in these theories, it would lead to new invariants principles such as CPTX conservation.

In section 2 we presented the current physical interpretation of an antiparticle [4]. Section 3 is devoted to construct the unitary irreducible representations of the “full Poincare” where the existence of new particles is suggested.

**2. Antiparticles**

In the one particle scheme Feynman and Stückelberg interpret antiparticles as particles moving backward in time [5]. This argument is reinforced by S. Weinberg who realizes
that antiparticles existence is a consequence of the violation of the principle of causality in quantum mechanics [4]. The temporal order of the events is distorted when a particle wanders in the neighborhood of the light cone. How is the antimatter generated from matter? According to Heisenberg’s uncertainty principle a particle wandering in the neighborhood of the light-cone suddenly tunnels from the timelike region to the spacelike region; in this region the relation of cause and effect collapses. Since if an event, at \( x_2 \) is observed by an observer A, to occur later than one at \( x_1 \), in other words \( x_2^0 > x_1^0 \). An observer B moving with a velocity \( \mathbf{v} \) respect to observer A, will see the events separated by a time interval given by

\[
x_2^0 - x_1^0 = L_{\alpha}^0 (\mathbf{v}) (x_2^\alpha - x_1^\alpha),
\]

(1)

where \( L_{\alpha}^0 (\mathbf{v}) \) is a Lorentz boost. From equation (1), it is found that if the order of the events is exchanged for the observer B, that is, \( x_2^0 < x_1^0 \) (the event at \( x_1 \) is observed later than the event at \( x_2 \)), then a particle that is emitted at \( x_1 \) and absorbed at \( x_2 \) as observed by A, it is observed by B as if it were absorbed at \( x_2 \) before the particle were emitted at \( x_1 \). The temporal order of the particle is inverted. This event is completely feasible in the neighborhood of the light-cone, since the uncertainty principle allows a particle tunnel from time-like to space-like cone regions. That is, the uncertainty principle consents to a particle to reach the space-like region. Since values greater than zero are allowed in next equation

\[
(x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 \leq \left( \frac{\hbar}{mc} \right)^2,
\]

(2)

where \( \left( \frac{\hbar}{mc} \right)^2 > 0 \), and \( \frac{\hbar}{mc} \) is the Compton wave-length of the particle. The left hand side of the equation (2) can be positive or space-like for distances less or equal than the square of the Compton wavelength of the particle. Therefore, causality is violated. The only way of interpreting this phenomenon is assuming that the particle absorbed at \( x_2 \), before it is emitted at \( x_1 \) as it is observed by B, is actually a particle with negative mass and energy that it moves backward in time; that is \( t_2 < t_1 \) [4]. This event is equivalent to see an antiparticle moving forward in time with positive mass and energy that it is emitted at \( x_1 \) and it is absorbed at \( x_2 \). With this reinterpretation the causality is recovered.

The square of the mass for the observers A and B is a Poincare invariant. In other words, the mass of a particle itself is not an invariant, but its square [6]. There is nothing to prevent that the rest mass could have different sign in distinct reference frame, as happen with its energy, electric charge and spin. In a former paper, it is conjectured that a particle moving backward in time possesses a negative mass [7]. When it is observed as
an antiparticle moving forward in time its mass is positive. Still its mass square is a Poincare invariant. According to the interpretation of antiparticle that it is given in this article, bosons and fermions cannot collapse toward states of negative energy. The reason is that a particle (boson or fermion) has to overcome the speed of light to acquire a negative energy. This event is classically impossible but quantum mechanically feasible.

3. Unitary Irreducible Representations of Poincare Group with Simultaneous Space-Time Reflections

The unitary irreducible representations of the extended group of Poincare have been constructed by constraining from the onset the energy values to be positive as a consequence of this assumption time reversal operator is antilinear. Later the antilinear property of the time reversal operator is utilized to prevent the existence of negative energy states for elementary particles [8][9]. In this paper, however the opposite approximation is proposed. The quantum negative energy states are accepted on the basis that the creation of an antiparticle (particle of negative energy) is briefly allowed by an interval of time ruled by the uncertainty principle; while the particle moves around the light cone boundaries that separate time-like from space-like regions. On the neighborhood of this boundary the probability of tunneling is greater than zero. A collapse of particles toward negative energy states is classically prohibited by the impossibility that a particle can exceed the speed of light. However, in quantum mechanics is briefly permitted by the Heinsenberg uncertainty principle (see eq. (2)); as it is discussed above. If the pass of particles of positive energy to particles of negative were a downhill event then the propagation speed of the information between two objects would be infinite. Furthermore, tunneling has a low probability of occurrence, and the mean lifetime of an antiparticle is in general short, since this is immediately absorbed by its particle to transform into gamma rays.

The classical way to construct the unitary irreducible representations of the Poincare group is by using the technique of the little group. Let us consider the orbits or regions where the magnitude of the four vector $\hat{\mathbf{s}}^2 = x^2 + y^2 + z^2 - c^2 t^2$ is: zero, with all its components equal to zero; zero with its components different of zero; greater than zero, and less than zero. To obtain these unitary irreducible representations we will use the two dimensions space-time t-z of Figure 1, in the momentum representation. Therefore the six regions to be discussed are

1. $\hat{s}. \hat{s} < 0$, space-like
2. $\hat{s}. \hat{s} > 0, t > 0$, future time-like
3. $\hat{s}. \hat{s} > 0, t < 0$, past time-like
4. $\hat{s}. \hat{s} = 0, \hat{s} \neq 0, t > 0$, future null
5. $\hat{s}. \hat{s} = 0, \hat{s} \neq 0, t < 0$, past null
6. $\mathbf{s} = 0$, the zero vector

The representation for the Poincare group with simultaneous space-time inversions will be constructed following any of the references [9].

Let us choose the vectors $k^\lambda$ as the basis vectors of an irreducible representation $T^{(k)}$, of the subgroup of translations of the Poincare group. Then the basis vectors $\hat{k}^\lambda$, with $\hat{k}' = W\hat{k}$, and $W$ a Lorentz transformation, lie in the same representation. That is, $k'$ and $k$ are in the same region of the space-time; these vectors have the same “length”, $(k',k') = (k,k)$.

$$
T(\hat{u})T(W)\hat{k} = T(W)\hat{k}\exp(W\hat{k},\hat{u})
$$

(3)

The vectors $T(W)\hat{k}$ transform according to the irreducible representation $T^{(Wk)}$, of the subgroup of translations.

Let us start our analysis with the future time-like cone or region 1. By choosing the time-like four-vector $\hat{k}_0 = (0,0,0,k)$ with $k > 0$, it is found that the little group, corresponding to
the orbit of the point \((0,0,0,k)\), is the group of rotations in three dimensions SU(2). Therefore, the representation is labeled by \(\hat{k}_0\) and by an irreducible representation label of the three dimensional rotational group SU(2). Starting from the \(2s + 1\) basis vectors of the little group SU(2) the bases vectors \(\hat{k}_0 sm_s\), of an irreducible representation of the Poincare group, are generated by a pure Lorentz transformation (a boost) which carries \(\hat{k}_0\) into \(\hat{k}\), that is \(W_{\hat{k}} \hat{k}_0 = \hat{k}\). The basis vectors of the representation are given by

\[ \left| \hat{k}_0 s m_s \right> = T(W_{\hat{k}}) \left| \hat{k}_0 s m_s \right> \]  

(4)

This group operation preserves the “length” of the vector \(\hat{k}_0\), that is \((k,k) = (k_0,k_0)\). By applying a Lorentz transformation followed by a translation to equation (4) can be proved that the vectors \(\hat{k}_0 sm_s\) furnish a unitary irreducible representation of the Poincare group

\[ T(\hat{u}) T(W) \left| \hat{k}_0 s m_s \right> = \left| \hat{k}' s m'_s \right> D_{m,m}(W_{k_0}) \exp(\hat{k}', \hat{u}) \]. 

(5)

Where \(\hat{k}' = W_{\hat{k}}\), and \((W_{\hat{k}} W_{\hat{k}'}) = (\hat{k}, \hat{k})\). \(D_{m,m}(W_{k_0})\) are the unitary irreducible representations of the little group SU(2), \(W\) is an arbitrary Lorentz transformation. The representations are unitary because the generators of the group are unitary, and irreducible because the bases vectors of the representations are generated from a single vector \(\left| \hat{k}_0 s m_s \right>\), with linear momentum equal to zero. Their equivalent unitary irreducible representations are denoted by \(P^{(k_0,s)}\). These representations have the same length of the four-vector \(\hat{k}\) than that of \(\hat{k}_0\). The set of inequivalent representations with different magnitude of the four-vector \(\hat{k}\) are denoted by \(P^{(k,s)}\).

Let us construct the unitary irreducible representations of the Poincare group with simultaneous space-time reflections. First note that if the vectors \(\hat{k}\) are the bases of an irreducible representation of the translation group, and \(I\) is the space-time reflection operator, then

\[ T(\hat{u}) T(I) \left| \hat{k} \right> = T(I) T(\hat{u}) \left| \hat{k} \right> = T(I) \left| \hat{k} \right> \exp(\hat{k}, \hat{u}) \]  

(6)

Therefore, the vectors \(T(I) \left| \hat{k} \right>\) transform according to the \(T^{(k)}\) representation of the translation subgroup of the Poincare group. But, the operator \(I\) reverses, the space and time components of \(\hat{k}\) that is, the energy \(E = \hbar c k\) becomes negative. On the rest frame
of the $P^{(k,s)}$ representation, the rest mass, $m_0 = \frac{\hbar k_0}{c} = -\frac{\hbar k_0}{c}$, of such a particle would also be negative. There is nothing to prevent this from happening, since $m_0^2$ is a Poincare invariant [6], but not $m_0$. That is, if one observer sees a particle with rest mass positive another observer on an inertial frame could see the same particle with negative rest mass. An electron with negative rest mass and energy moving backward in space and time could be interpreted as its antiparticle moving forward in space and time with positive rest mass and energy. This phenomenon could be explained by constructing the unitary irreducible representations of the Poincare group with simultaneous space-time inversions.

We have found that on the rest frame of the $P^{(k,s)}$ representation, the rest mass, $m_0 = \frac{\hbar k_0}{c} = -\frac{\hbar k_0}{c}$, can have two possible signs, positive or negative. The fact that elementary particles traveling backward in time have being identified as antiparticles [5] leads one to the conclusion that particles with mass can not be identified with its own antiparticles, as it is the case with $\pi^0$ and $Z^0$.

To construct the unitary irreducible representations of the full Poincare group one applies the space inversion followed by time inversion to the basis vectors $\left| \hat{k}_s m_0 \right\rangle$ of $P^{(k,s)}$.

These basis vectors were generated from the special basis vector $\left| \hat{k}_0^0 \right\rangle$, where $\hat{k}_0 = (0,0,0,k)$. The space inversion $I_s$ leaves $\hat{k}_0$ invariant, and commutes with the generators of the little group SU(2). Therefore, the irreducible representations of the group, $Z_\times \times SU(2)$, can be labeled by the labels of the rotations, and reflection groups, namely the spin $s$, and the parity $\eta$. If one applies space inversions to the basis vector $\left| \hat{k}_0^s m_0 \eta \right\rangle$ with definite parity label $\eta = \pm$, one obtains

$$T(I_s) \left| \hat{k}_0^s m_0 \eta \right\rangle = \left| \hat{k}_0^s m_0 \eta \right\rangle \eta \tag{7}$$

On the rest frame the parity, $\eta$, is an eigenvalue of the basis vector $\left| \hat{k}_0^s m_0 \eta \right\rangle$.

Now, Let us start to construct the inequivalent unitary irreducible representations of the Poincare group with space inversions $P^{(k,s,\eta)}$. By applying these inversions to the basis vectors $\left| \hat{k}s m_0 \eta \right\rangle$, given by equation (4), and including the parity label $\eta$ one obtains

$$T(I_s) \left| \hat{k}sm_0 \eta \right\rangle = T(I_s)T(W_k) \left| \hat{k}_0^s m_0 \eta \right\rangle = T(W_{\pm}) T(I_s) \left| \hat{k}_0^s m_0 \eta \right\rangle = \left| \hat{k}sm_0 \eta \right\rangle \eta \tag{8}$$

$$T(W_{\pm}) \left| \hat{k}_0^s m_0 \eta \right\rangle \eta = \left| I_s \hat{k}sm_0 \eta \right\rangle \eta$$
The Lorentz boost $W_{\hat{k}}$ changes the direction of space components of $\hat{k}$. Equation (8) shows that the basis vector $|\hat{\text{sm}}, \eta\rangle$ is an invariant space under space reflections; the action of the space inversion operator on a general basis vector leads to another basis vector. Hence, these bases vectors yield a representation for the Poincare group with space inversions.

If one considers the simultaneous action of the time-space inversions on a general basis vector one obtains

$$
T(I)|\hat{\text{sm}}, \eta\rangle = T(I)T(W_{\hat{k}})|\hat{\text{sm}}, \eta\rangle = T(I_s)T(W_{\hat{k}})T(I)|\hat{\text{sm}}, \eta\rangle = T(I_s)T(W_{\hat{k}})|\hat{\text{sm}}, \eta\rangle = |\hat{\text{sm}}, \eta\rangle
$$

The action of the full inversion operator on a general basis vector $|\hat{\text{sm}}, \eta\rangle$, which belongs to the unitary irreducible representation $P^{(k,s,\eta)^+}$ generated from the vector $\hat{k}_0 = (0,0,0,k)$, leads to a basis vector in the $P^{(k,s,\eta)^-}$ representation generated from the vector $I\hat{k}_0 = (0,0,0,-k)$. But, these two representations of the full Poincare group are equivalents. Therefore, the action of time inversion generates negative energy states, and also induces charge conjugation, as we will show it below.

### 3.1 Time Inversion and Charge Conjugation

If one applies linear time reflections to the basis vector $|\hat{\text{sm}}, \eta\rangle$, one finds that

$$
T(I_s)|\hat{\text{sm}}, \eta\rangle = |I_s\hat{\text{sm}}, \eta\rangle
$$

Since, from equation (6) $T(I_s)|\hat{\text{sm}}, \eta\rangle$ transforms according to the $T^{(I_s,\hat{k}_0)}$ representation of the subgroup of translations. That is, the basis vector $|\hat{\text{sm}}, \eta\rangle$ with $\hat{k}_0 = (0,0,0,k)$ from the $P^{(k_0,s,\eta)^+}$ representation is taken into the basis vector $|I_s\hat{\text{sm}}, \eta\rangle$ with $I_s\hat{k}_0 = (0,0,0,-k)$ of the $P^{(k_0,s,\eta)^-}$ representation. We will show that in the $P^{(k_0,s,\eta)^-}$ representation, time reflections prevent that SU(2) commutes with $Z_2$. In spite of SU(2) does not commute with $Z_2$, in the $P^{(k_0,s,\eta)^-}$ representation, still $\eta$ is a label for the full Poincare group representation. Since the Casimir operators of the full Poincare groups are $P^2$, $W^2$, and $I^2$.

If the vector $|\hat{k}\rangle$ is any vector that transforms according to a representation of the group of translations, then
\[ \hat{P} |\hat{k}\rangle = -i\hat{k} |\hat{k}\rangle \]  
(11)

Hence, in the \( P^{(k_0, s, \eta)+} \) representation

\[ \hat{P} |\hat{k}_0, sm, \eta\rangle = -ik |\hat{k}_0, sm, \eta\rangle \]  
(12)

While in the \( P^{(k_0, s, \eta)^-} \) representation

\[ \hat{P} |I, \hat{k}_0, sm, \eta\rangle = +ik |I, \hat{k}_0, sm, \eta\rangle \]  
(13)

Then, time reflection induces a change on the energy sign, that is

\[ I, P_o I_t^{-1} = -P_o \]  
(14)

That is, time inversions do not commute with \( P_0 \).

The Pauli-Lubansky four vector components on the rest frame in the \( P^{(k_0, s, \eta)+} \) representation is

\[ W_q |\hat{k}_0, sm, \eta\rangle = -kJ_q |\hat{k}_0, sm, \eta\rangle, \text{and} \]  
(15)

\[ W_r = 0, q = x, y, z \]

Now, in the \( P^{(k_0, s, \eta)^-} \) representation, on the orbit of the vector \( I, \hat{k}_0 = (0,0,0,-k) \) yields

\[ W_q |I, \hat{k}_0, sm, \eta\rangle = +kJ_q |I, \hat{k}_0, sm, \eta\rangle, \text{and} \]  
(16)

\[ W_r = 0, q = x, y, z \]

So that, time inversions do not commute with the Pauli-Lubansky four-vector in the larger space composed by \( P^{(k_0, s, \eta)+} \) and \( P^{(k_0, s, \eta)^-} \).

\[ I, W_q I_t^{-1} = -W_q, \]  
(17)
Therefore, from equation (15) one obtains

\[ I_i J_q I_i^{-1} = -J_q \quad (18) \]

That is, time reversal induces an inversion on the direction of a rotation and changes the sign of the rest energy (rest mass) of the particle. Then, by using equation (18) one gets

\[ J_z I \left| \hat{k}_o s m \eta \right\rangle = -I, J_z \left| \hat{k}_o s m \eta \right\rangle = I, \left| \hat{k}_o s m \eta \right\rangle (-m_z) \quad (19) \]

The vector \( I, \left| \hat{k}_o s m \eta \right\rangle \) transforms like \(-m_z\), under rotations about the z-axis. Therefore from equation (18) for a general rotation, we get

\[ T(R(\theta))T(I) \left| \hat{k}_o s m \eta \right\rangle = T(I)T(R(-\theta)) \left| \hat{k}_o s m \eta \right\rangle = T(I) \sum_{m_s} \hat{k}_o s m \eta D_m^{(s)l} (\theta) = T(I) \sum_{m_s} \hat{k}_o s m \eta \overline{D_m^{(s)l} (\theta)} \quad (20) \]

Hence, the vectors \( T(I) \left| \hat{k}_o s m \eta \right\rangle \) transform like the transpose conjugate complex, \( D_{m, m_s}^{(s)l} \), of the representation \( D_{m, m_s}^{(s)l} \) of the unitary irreducible representation of SU(2). This fact explains why it is necessary to conjugate and transpose the wave equation of a particle to describe its antiparticle. Thus, time reflection induces negative energy states and these states induce charge conjugation. If a mirror reflection is a symmetry transformation, then this reflection must be accompanied by simultaneous space-time inversions, since the intrinsic parity label is generated by space reflection.

Due to the fact that one has to conjugate and transpose, time reversal acquires the properties of an antilinear operator. And since the character of the representations of the three-dimensional rotation group is real, the representations, \( D_{m, m_s}^{(s)l} \) and \( D_{m, m_s}^{(s)l} \), are equivalent and those representations can be reached one from the other by a change of basis. We apply this transform to transpose of complex conjugated representation. On the new basis the basis vector is given by

\[ \left| \hat{k}_o s m \eta \right\rangle_{\text{new basis}} = \sum_{m_s} \delta_{m, m_s} (-)^{s - m_s} \left| \hat{k}_o s m \eta \right\rangle \quad (21) \]

Since the matrix \( D^{(s)} \) transforms into the matrix \( D^{(s)l} \) according to a set of similarity transformations. As a matter of fact we are mapping a vector over its dual. Then, one has to define
\[
\overline{D^{(s)}_{m,m_i}} = F^{-1} D^{(s)}_{m,m_i} F^1
\]  

where the transformation matrix \( F \) is given by

\[
F = \delta^{m_i}_{-m_i} (-)^{s_{-m_i}}
\]

By applying the linear time reversal operator, on the new basis vector, to the general basis vector \( |k_{sm}, \eta\rangle \) which, is defined by equation (4), one gets

\[
T(I_s) |\hat{k}_{sm}, \eta\rangle = \sum_{m_i} T(I_s) |\hat{k}_{sm_i} \rangle \delta^{m_i}_{-m_i} (-)^{s_{-m_i}} = |I_s \hat{k}s - m, \eta\rangle (-)^{s+m_i}
\]

Under the action of the time reversal operator a particle on the rest frame reverses its energy, and spin.

We define the scalar product according to reference [9] by

\[
|\psi\rangle = \sum_{m_i} \int \psi_{m_i}(\hat{k}) |\hat{k}_{sm_i} \rangle k_i^{-1} dk
\]

Therefore applying the bra-vector of (24) to equation (25) one gets

\[
\psi'_{m_i}(\hat{k}) = (-1)^{s_{-m_i}} \psi_{-m_i}^* (I_s \hat{k})
\]

Here, the conjugation is a consequence of the bra-vector. Applying the space reversal operator to the general basis vector \( |k_{sm}, \eta\rangle \), one obtains

\[
T(I_s) |\hat{k}_{sm}, \eta\rangle = T(W_{-k}) T(I_s) |\hat{k}_{o sm}, \eta\rangle = T(W_{-k}) |\hat{k}_{o sm}, \eta\rangle \eta = |I_s \hat{k}_{sm} \eta, \eta\rangle
\]

The momentum is reversed and the particle acquires a definite parity. In order to generate the basis vectors of the representation for the Poincare group with simultaneous space-time reflections, we apply the time reversal operator to equation (8)

\[
T(I_s) |\hat{k}_{sm}, \eta\rangle = T(I_s) T(I_s) T(W_{-k}) |\hat{k}_{o sm}, \eta\rangle = T(I_s) T(W_{-k}) |\hat{k}_{o sm}, \eta\rangle \eta = |I_s \hat{k}_{sm} \eta, \eta\rangle (-)^{s+m_i}
\]
Therefore, in this formulation the simultaneous action of space-time inversions on a general basis vector (particle) of the representation reverses the energy, momentum, and spin (antiparticle). The space inversion furnishes a definite parity, $\eta$, to the elementary particle described by the unitary irreducible representation. The elementary particle violates the causality principle because it is represented in the negative energy sector of the light cone. If it has enough energy it can be observed as a particle moving backwards in space and time (antiparticle). Therefore, the $C$, $T$, $P$, $CT$, $CP$, $PT$ cannot conserve separately, but $CPT$. In my opinion not all the symmetries that would enhance the full Poincare group are known. Then, it is quite possible that exists a violation of $CPT$ for physical phenomena that requires more large symmetries. The full group of diffeomorphisms (space-time reflections included) is a larger symmetry than that of the full Poincare group; therefore $CPT$ could be violated by quantum gravity. By the same token supersymmetry, supergravity, and superstrings could violate $CPT$.

From equation (28), we get

$$T^2 (I) \left| \hat{k} m, \eta \right> = \left| \hat{k} m, \eta \right> (-)^{2i}$$

Hence, $T^2 = 1$ for integer spin, and $T^2 = -1$ for particles of half integer spin.

### 3.4. Particles of Zero Mass

For massless particles, or the future null and past null regions 4 and 5, the unitary irreducible representations of the Poincare group are labeled by unitary irreducible representations of its little group. The little group of that of Poincare group in these regions is the Euclidean group $E(2)$ in two-dimensions, one will denote its elements by $W_{k_0}$. The $W_{k_0}$'s are the transformations that leave invariant the four-vector $\hat{k}_0$. The orbit of the vector $\hat{k}_0 = (0,0,1,1)$ is the light cone surface. Let $W_{\hat{k}}$ be a Lorentz transformation that takes $\hat{k}_0$ into $\hat{k}$, that is $W_{\hat{k}} \hat{k}_0 = \hat{k}$; then a general vector can be written by

$$\left| \hat{k} m \right> = T(W_{\hat{k}}) \left| \hat{k}_0 m \right>.$$  \hspace{1cm} (30)

It can be proved that $W_{\hat{k}}^{-1} W W_{\hat{k}}$ is in the little group, that is, $W_{k_0} = W_{\hat{k}}^{-1} W W_{\hat{k}}$, then applying a general Lorentz transformation $W$ followed by a translation $\hat{u}$ one obtains.

$$T(\hat{u}) T(W) \left| \hat{k} m \right> = \left| \hat{k}' m \right> \exp(i k', \hat{u}) \exp(-im\theta)$$  \hspace{1cm} (31)
The \( E^{(2)} \) representations with continuous spin were ignored in this revision. We took on consideration only the representations labeled by the two-dimensional rotation group, that is \( m = 0, \pm \frac{1}{2}, \pm 1, \ldots \text{etc.} \)

By enlarging the Poincare group to include space-time inversions the little group of the vector \( \hat{k}_0 = (0,0,1,1) \) \( \tilde{E}^{(2)} = Z_2 \times E^{(2)} \). Hence, the “full” Poincare group is labeled by \( \eta = \pm 1 \) and a label of \( SO(2) \), because the continuous spin representations of \( E^{(2)} \) will not be considered in these regions of the light cone, then \( E^{(2)} \) is labeled by \( SO(2) \). If one applies space-time inversion over the basis vector

\[
\left| \hat{k}m \right\rangle = T(W_k)\hat{k}_0 m
\]

(32)

Where \( W_k \hat{k}_0 = \hat{k} \), furthermore if one notices that under space inversions

\[
(W_q - \text{imp}_q)T(I_s)\left| \hat{k}m \right\rangle = T(I_s)(W_q + \text{imp}_q)\left| \hat{k}m \right\rangle = 0
\]

(33)

\[
(W_t - \text{imp}_t)T(I_s)\left| \hat{k}m \right\rangle = T(I_s)(W_t + \text{imp}_t)\left| \hat{k}m \right\rangle = 0
\]

(34)

Hence,

\[
T(I_s)\left| \hat{k}m \right\rangle = \left| I_s \hat{k} - m \right\rangle
\]

(35)

and

\[
T(I_s)\left| \hat{k} - m \right\rangle = \left| I_s \hat{k}m \right\rangle
\]

(36)

Therefore, these vectors belong to the \( P^{(0,m)} + P^{(0,-m)} \) representation, contrary to the representation \( P^{(k,m)} \) these representations do not carry spin, but helicity. Therefore, time reversal does not induce a change in sign of the angular momentum \( J \), as it does in equation (24).

Also, if one notices that

\[
(W_q + \text{imp}_q)T(I_s)T(I_s)\left| \hat{k}m \right\rangle = T(I_s)(W_q - \text{imp}_q)T(I_s)\left| \hat{k}m \right\rangle = 0
\]
\[(W_i + \text{imp}_i)T(I_x)T(I_x)\hat{k}m) = T(I_x)(W_i - \text{imp}_i)T(I_x)\hat{k}m) = 0 \quad (37)\]

Hence,

\[T(I_x)T(I_x)\hat{k}m) = |I_x\hat{k}m) \quad (38)\]

Therefore, under space-time inversions the vector \( T(I_x)\hat{k}m \) transforms according to the representation \( T^{(Ik)} \). Additionally, it can be proved that such a vector transforms in the same manner under translations. Now, according to equations,

\[(W_q + \text{imp}_q)\hat{k}m) = 0 \quad (39)\]

\[(W_i + \text{imp}_i)\hat{k}m) = 0, \quad (39)\]

The \( \hat{k}m \) vector transforms according to the \( P^{(0,m)} \) representation. But, with its energy and momentum reversed.

On the \( \hat{k}_00\eta \) representation of the little group of \( \hat{k}_0 \) one gets

\[T(I_x)\hat{k}_00\eta) = T(W_x)T(I_x)T(I_x)\hat{k}_00\eta) = |\hat{k}_00\eta)\eta \quad (40)\]

This equation represents a particle of zero rest mass, and zero helicity, but with a definite parity that it changes its momentum and energy under space-time reversal.

We realize that transitions between time-like and space-like states for a “massless” particle are highly probable, since if the four momentum, \( p^2 = 0 \), is sharply defined, then its four vector position

\[ (x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 \sim \frac{\hbar^2}{p^2} \rightarrow 0 \quad (41)\]

is completely uncertain, since its Compton wavelength is undetermined [10]. The massless particle could exist in any place of the light cone. The principle of causality is strongly violated. If one observes a given amount of photons with \( p^2 = 0 \) is because a big amount of other photons populate the space-like, and the time-like regions. The action, of these dark photons, on the universe should be measurable.
3.5. Imaginary Mass Particles

In the space-like, region 1, the eigenvalue of the square of the four-momentum is the negative of the square of the rest mass of the particle. Therefore, the mass would be imaginary number. No physical interpretation of elementary particles could be associated to this region; however, the square of the four-momentum could have a physical meaning. It turn out that the square of the four-momentum is not equal to the square of the mass for internal lines in a Feynman diagram. It is conjectured in this article that a possible interpretation could be given to the unitary irreducible representations of the “full” Poincare group. Virtual particles would be good candidates to achieve this goal.

The little group of the four-vector $\hat{k}_0 = (0,0,k,0)$ is the SL(2,$\mathbb{R}$) which contains spin representations. These representations where classified independently by Bargmann, Naimark and Gelfand, and Harish-Chandra[9].

It can be shown that a general basis vector is given by

$$\left| \hat{k}m \right\rangle = T(W_k)\left| \hat{k}_0 m \right\rangle = T(R_z(\theta))T(W_{k_1})T(W_{k_2})\left| \hat{k}_0 m \right\rangle$$

If we apply the space reversal operator to the four basis vector and consider as the little group the group of reflections and SL(2,R), $Z_2 XSL(2,R)$, one obtains

$$T(I_s)\left| \hat{k}_s m \eta \right\rangle = \sum_{m_s'} T(I_s)\left| \hat{k}_s m' \eta \right\rangle \delta^{m_s}_{-m_s} (-)^{s-m_s} \left| I_s \hat{k}_s m \eta \right\rangle (-)^{s+m_s},$$

(43)

since the relation

$$I_s J_z I_s^{-1} = -J_z,$$

(44)

is obtained when one chooses the point $\hat{k}_0 = (0,0,-z,0)$ to construct the space-like representations. By following a similar procedure than that applied in section 3.1. Performing time inversion to the general basis vector, one finds that the four-vector $\hat{k}$ reverses its spatial components.

$$T(I_s)\left| \hat{k}m \eta \right\rangle = T(W_{-k})T(I_s)\left| \hat{k}_0 m \eta \right\rangle = \left| I_s \hat{k}m \eta \right\rangle \eta$$

(45)

Hence, the action of space-time inversion yields
The vectors of the discrete series $D^+$ transform into the $D^-$ discrete series, and vice versa.

### 3.6 Vacuum Representations

In region six the little group of the four-vector $\mathbf{k}_0 = (0,0,0,0)$ is the Lorentz group. Because the Lorentz group is non-compact, its unitary irreducible representations are infinite dimensional. Therefore, the unitary irreducible representations of the Poincare group in this region are the same than that of the Lorentz group. They are labeled by the eigenvalues of the Casimir of the algebra SU(2)$\times$SU(2), that is, the angular momentum on three dimensions $u, v$. The Lie algebra of the Lorentz group can be reduced to that of the group SU(2)$\times$SU(2). Although these two groups are distinct topologically, the Casimir operators of SU(2)$\times$SU(2) enable one to deduce and label the representation of the Lorentz group. There are two classes of unitary irreducible representations of the Lorentz group, the principal series ($v = -iw, w$ real, $j_0 = 0, 1/2, 1, ...$) and the complementary series ($-1 \leq v \leq 1; j_0 = 0$). Because the combined space-time transformations commute with the generators of the Lorentz group the states of this representation have a definite parity. Furthermore, vacuum oscillations make these states to undergo transitions to positive and negative energy states as well as imaginary mass (virtual states).

### Conclusion

In this manuscript, it has been proved that the only possible way that a particle to be its own antiparticle is that its rest mass to be zero. This constraint is not satisfied by the particles $\pi^0$ and $Z^0$. By the same token the neutrino could not be a massless particle, since it has an antiparticle. Any criticism to this article is left to be solved on the experimental arena.

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### References

[1] J. N. Pecina-Cruz, A Generalization of the Concept of Parity Conservation, LANL preprint hep-ph/0505203, (2005).
[2] J. Schwinger, Phys. Rev 73, p. 416 (1948).
[3] Joseph Polchinski, *String Theory and Beyond*, Vol. 2, Cambridge Monographs in Mathematical Physics (1998).
[4] S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, p. 61-63 (1972).
[5] E.C.C. Stuckelberg, Helv. Phys. Acta 15, 23 (1942); R. Feynman, Phys. Rev. Vol. 76, p. 749 (1949); E.C.G. Stuckelberg, Phys. Rev. Vol. 74, p. 218 (1948).
[6] J.N. Pecina-Cruz and Y. Ne’eman, Canadian Journal of Physics. Vol. 72, p. 466 (1994); Campoamor-Stursberg, J. of Phys. A: Math. Gen. 36, p. 1357 (2003); J.C. Ndogmo, J. of Phys. A: Math. Gen. 37, p. 5635 (2004).
[7] J. N. Pecina-Cruz, Time Inversion Induces Negative Mass and Charge Conjugation, LANL preprint, hep-ph/0505188
[8] E. Wigner, *Group Theory and Solid State Physics*, Vol. 1, p. 265, Ed. P.H. Meijer, Pub. Gordon and Breach, NY (1964).
[9] E. Wigner, Vol. 93, No. 7, p. 521 (1949); E. Wigner, The Annals of Math., Vol. 40, p. 149, Jan. (1939); V. Bargmann, Ann. of Math. No 48, p. 568 (1947); M.A. Niarmak, *Linear Representation of the Lorentz Group*, Pub. Pergamon Press (1964); Harish-Chandra, Proc. of the Royal Soc. of London, Series A, Vol. 189, No. 1018, p. 372-401 (1947).
[10] S. Hawking, Commun. Math. Phys. 43, p. 199 (1975).