The Effect of Modern Traffic Information on Braess’ Paradox

Stefan Bittihn, Andreas Schadschneider

Institute for Theoretical Physics, Universität zu Köln, 50937 Köln, Germany

Abstract

Braess’ paradox has been shown to appear rather generically in many systems of transport on networks. It is especially relevant for vehicular traffic where it shows that in certain situations building a new road in an urban or highway network can lead to increased average travel times for all users. Here we address the question whether this changes if the drivers (agents) have access to traffic information as available for modern traffic networks, i.e. through navigation apps and or personal experiences in the past. We study the effect of traffic information in the classical Braess network, but using a microscopic model for the traffic dynamics, to find out if the paradox can really be observed in such a scenario or if it only exists in some theoretically available user optima that are never realized by drivers that base their route choice decisions intelligently upon realistic traffic information. We address this question for different splits of the two information types.

Keywords: Braess paradox, highway network, exclusion process, cellular automata

1. Introduction

Since its discovery in D. Braess’ proof-of-concept paper in 1968 [1, 2], Braess’ paradox has become a well-known phenomenon, first in traffic science and later in other fields as well, e.g. in mechanical and electrical systems [3, 4], oscillator networks, power grids and biological networks [5, 6, 7], microfluidic networks [8], and pedestrian dynamics [9]. Braess’ paradox

1Corresponding author; email address: as@thp.uni-koeln.de
states that in (congested) road networks used by selfish drivers under certain conditions the addition of a new road can lead to higher travel times for all network users. The paradox is established on the assumption that networks of selfish users evolve into stable states, the so-called user optimum states \[10\]. This insight has later been generalized to the concept of "price of anarchy" \[11\]. The paradox has been generalized in various aspects \[12, 13, 14, 15, 16, 17, 18\]. Most of this research is, as the original work, built upon simplified macroscopic mathematical models of traffic flow. It is worth mentioning that the paradox has been observed in real world traffic networks \[19, 20, 21\].

In the spirit of Dietrich Stauffer we consider Braess’ paradox as a problem of "exotic statistical physics" \[22\]. In two recent papers \[23, 24\] we studied the paradox in traffic networks with a more realistic model of traffic flow, i.e. the totally asymmetric exclusion process (TASEP) \[25, 26, 27\]. For these cases we could show that it can also be observed and indeed occupies large parts of the phase space.

In most of the previous studies the paradox is observed in the following way: it is shown that in the traffic networks with and without an added road user optima exist for the same total demand (total numbers of cars using the network) and that in some cases the user optima of the networks with the new road have higher travel times than those of the networks without the new road. Furthermore, in our previous work \[23, 24\] on the paradox in TASEP networks we dealt with systems that evolved into a stationary state and considered stationary travel times only, neglecting the evolution into these states. In our recent work \[28\] we have started to look at the effects of traffic information on the states realized in Braess-type road networks.

In the present paper we continue to consider a more microscopic point-of-view and address the question if the attainable user optima are actually realized if the network users (from here on also called agents) make informed route choices based upon realistic traffic information. The question whether user optima (and thus also Braess’ paradox) are realized in real world networks has been a long standing discussion, examined in various scientific fields. Foremost, studies from traffic scientists suggest, that travel time minimization is not the only factor driving route choices \[29, 30, 31\]. In the field of behavioral economics several experiments with real human participants were performed \[32, 33, 34, 35\]. In these studies, generally the traffic flow is modelled by a macroscopic mathematical approach. In the physics community, route choice processes and the question whether user optima are realized
were mainly studied by so-called multi-agent techniques [36, 37, 38, 39, 40]: those are mainly proof-of-concept simulations in which the traffic flow is modelled by a stochastic microscopic model and the route choices are performed equally by all agents based on certain types of information.

In the present article we model the traffic flow by a microscopic model and implement a route choice algorithm by which all individual agents make ‘intelligent’ decisions. Users have access to personal-historical information, i.e. their own memory of travel times experienced in the past, or public-predictive information, i.e. estimates of travel times based on the positions of all agents at a given time. The latter approximates information from personal navigational systems or smartphone apps, like the crowdsourced app Google Maps [41]. In the present paper we study the case in which both information types are present in the road network. In [28] we considered situations in which only one of the two types was available to each agent. We examined four different points of the phase space. We could show that personal-historical information realizes user optima in all four states, while public-predictive information realizes stable user optimum states at low densities and oscillations around attainable user optima are observed at higher densities. Here we consider the more realistic scenario in which different splits of agents have access to the two kinds of information. Such a situation is relevant for most real modern day traffic networks. Analysis of real world GPS-traffic data hints at routing apps making travel time minimization the most important route choice factor and also suggest that these apps lead to realizations of user optima [34, 42], which has also been backed by large-scale simulations [42].

2. The paradox

A fundamental assumption underlying Braess’ paradox is that the users of the (traffic) network are selfish and that their egoistic behavior drives the system into stable network states (user optima, $\text{uo}$). In this context, a network state refers to the set of strategies of all network users. A user’s strategy denotes how the user chooses their route. A route refers to a connection between an origin and a destination and can be comprised of various roads or, in network terminology, edges that can be connected through junction sites. In the following we only consider situations in which all users want to go from the same origin to the same destination. The user optimum is realized if the user’s route choices (and thus their distribution onto the roads of the
network) lead to equal travel times on all used routes, which are lower than travel times on any unused routes [10]. For a pure strategy a user chooses exactly one route whereas for a mixed strategy she chooses a route according to probabilities assigned to all routes. Accordingly, if all agents use pure strategies, pure user optima (puo) are observed, while in the case of all users using mixed strategies mixed user optima (muo) are observed. In the latter case, the user optimum is realized if the expectation values of the travel times of all used routes are equal and lower than those of any unused routes. Since we are using a traffic model with microscopic stochastic dynamics we have to consider mean values of travel times both for pure and mixed user optima.

2.1. Braess’ original example

Braess demonstrated the paradox [1, 2] for the simple network shown in Fig. 1(a). It is comprised of the 5 network edges $E_1, \ldots, E_5$. It is assumed that all network users want to go from the origin at the bottom to the destination at the top. Edge $E_5$ is considered to be the new road that is added to the network. The network without/with edge $E_5$ is from here on called the 4link/5link network and corresponding variables are indicated by superscripts (4) and (5), respectively. Routes in the network are labelled according to the numbering scheme of the constituent edges. In the 4link network there are two available routes from origin to destination: route 14 and route 23. The addition of edge $E_5$ realizes a new route, route 153.

In Braess’ original work [1, 2], a macroscopic traffic model was used with travel times determined by simple travel time functions that increase linearly with the number of cars using the road. The approach also neglected potential correlations that exist between the different roads. Braess showed that for specific choices of the number of agents and parameters in the travel time functions an apparently paradox situation can occur where the travel times in the 4link network’s user optimum are smaller than those of the 5link network’s user optimum, i.e., adding an additional route or edge to the network leads to an increase of travel times for all (selfish) agents. In Braess’ example both for the 4link and the 5link network unique pure optima exist. It was also shown that the same effect can be observed with mixed user optima [28]. Several generalisations and additional results about the paradox were obtained in the context of macroscopic mathematical traffic models [12, 13, 14, 15, 16, 17, 18].
2.2. The Braess network of TASEPs

In [23, 24] we have considered the occurrence of Braess’ paradox for the same network structure, but with a more realistic traffic model described by stochastic, microscopic dynamics, namely the totally asymmetric exclusion process (TASEP). In this situation user optima realizing the paradox can also be observed.

In the model based on TASEP traffic dynamics each car or agent is represented as a particle subject to hard core repulsion. A road is modelled by discrete cells that can be either empty or occupied by one particle. Using a random-sequential update, particles can only move forward to an empty cell in front. Each edge (road) $E_i$ is represented by a TASEP of length $L_i$ cells. As shown in Fig. 1 (b), we examined the same network structure as Braess, with four junction sites $j_1, \ldots, j_4$ and edge $E_0$ ensuring periodic boundary conditions and thus a constant total number of particles $M$. We could show [23, 24] that user optima leading to Braess’ paradox also exist in those TASEP networks. We demonstrated this by externally setting all agents’ strategies: the route choices of the agents were controlled by global parameters. By varying these parameters, we found attainable user optima
of the 4link and 5link networks and compared their travel times (for the same total numbers of particles $M$). This allowed us to determine the phase diagrams that are presented in the following. Note, that no individual, realistic route choice decisions were modelled. The obtained phase diagrams are valid only if all agents follow the externally set route choices.

In [24] we considered the specific case where all agents follow fixed personal (i.e., pure) strategies. In this case fixed numbers of $N_{14}$, $N_{23}$, $N_{153}$ agents follow routes 14, 23 and 153, respectively. The distribution of agents onto the routes can also be characterized by the two variables $n_{1}^{(j_1)} = 1 - \frac{N_{23}}{M}$ and $n_{1}^{(j_2)} = \frac{N_{14}}{N_{14} + N_{153}}$. In the 4link network, as $E_5$ does not exist, $N_{153} = 0$. By varying these numbers a rich phase diagram was found that is shown in Fig. 2 (a) in a simplified form.

In [23] we considered agents using mixed strategies on the same network. The agents decide for a route based on turning probabilities on junction sites $j_1$ and $j_2$. Agents on $j_1$ turn to the left (onto $E_1$) with probability $\gamma$ and to the right (onto $E_2$) with probability $1 - \gamma$. In the 5link system agents on $j_2$ turn to the left (onto $E_4$) with probability $\delta$ and to the right (onto $E_5$) with probability $1 - \delta$. In the 4link system $E_5$ does not exist and thus $\delta = 1$. The resulting phase diagram is shown in Fig. 2 (b) in a simplified form.

![Phase diagrams](image)

Figure 2: Phase diagrams for Braess’ network of TASEPs with externally tuned strategies and (a) fixed personal strategies and (b) turning probabilities. These phase diagrams are simplified. For full details (sub-phases etc.) the reader is referred to [24, 23]. The route choice algorithm was tested on the state marked with a black $\times$.

For the phase diagrams shown in Fig. 2, the lengths of the TASEPs were
chosen as $L_0 = 1$, $L_1 = L_3 = 100$, $L_2 = L_4 = 500$. The length of the new road $L_5$ and the total number of agents in the system $M$ are varied. The state of the network depends on $L_5$ and $M$ which in the phase diagrams are characterized by the route length ratio $\hat{L}_{153}/\hat{L}_{14}$ (note that $\hat{L}_{14} = \hat{L}_{23}$) and the global densities $\rho_{\text{global}}^{(4)} = M/(4 + \sum_{i=0}^{4} L_i)$ and $\rho_{\text{global}}^{(5)} = M/(4 + \sum_{i=0}^{5} L_i)$ in the 4link and 5link networks.

The phase diagrams shown in Fig. 2 are simplified compared to those presented in [24, 23] to emphasize the features relevant for the present study. It is just shown where the Braess paradox occurs ("Braess" phase, i.e. user optima in the 5link have higher travel times than those of the 4link network) and where it does not occur ("no Braess" phase, i.e. user optima in the 5link have lower travel times than those of the 4link network). These phases can be subdivided. For more details including the sub-phases and explanations about the regions in which no user optima could be found ("no user optima") the reader is referred to [24, 23]. It can be seen that the paradox can be observed in large parts of the phase space.

In the following we try to answer the following question: are the user optima attainable by externally tuning all agents’ strategies actually realized if the agents make their route choice decisions intelligently, similar to real drivers in modern traffic networks. In other words: Would the paradox really be observed or does it only exists in some theoretically attainable user optima that would never be realized by realistic drivers.

3. The route choice algorithm

To answer the question whether the paradox is realized by realistically deciding drivers we implemented a route choice algorithm. Agents choose their routes according to two different types of information: personal-historical and public-predictive information. Personal-historical information is typically relevant in a commuter scenario and basically represents the driver’s memory. The agents go from origin to destination several times and remember how long it took them in the past on the available routes. Their future route choice decisions are then made upon this basis. Public-predictive information, on the other hand, is typically provided by navigational systems or smartphone routing apps. This type of information is based upon the current status of the network (how many cars are on which roads right now). Often, expected travel times for all available routes are presented to the driver who then decides for the one with the shortest predicted travel time.
We have implemented a route choice algorithm based upon which all agents in the system make individual 'intelligent' route choice decisions before beginning each new round (i.e. before jumping from $E_0$ to $j_1$) and in some cases also during rounds (a round refers to going once from origin ($j_1$) to destination ($j_4$)). The "goal" of the agents is to generally choose the route with the lowest travel time. Based on the traffic information available to him/her, each agent estimates the expected travel times on all available routes. These estimations are stored in the variables $T_{i,\text{info}}, i \in \{14, 23, 153\}$. Generally agents then choose the route on which the lowest travel time is expected. If this choice requires a route switch compared to the previous round, it is only performed if the expected time-save is significant: the value of $|T_{14,\text{info}} - T_{23,\text{info}}| + |T_{14,\text{info}} - T_{153,\text{info}}| + |T_{23,\text{info}} - T_{153,\text{info}}|$ is calculated and only if its value is larger than $\Delta T$, the switch occurs. The agents thus act boundedly rational [43]. A stochastic component is added to account for random events that could disturb those boundedly rational decisions: with probability $1 - p_{\text{info}}$ a random route is chosen.

During individual rounds, changes can occur in the following cases. If e.g. an agent in the 4link network that chose to take route 23 before the round is sitting on $j_1$ and cannot jump to the target site (first site of $E_2$) since this site is occupied, (s)he may re-decide routes. If $T_{23,\text{info}} \geq T_{14,\text{info}}$ (agent chose route 23 based on a random decision before the round), (s)he will then immediately switch to route 14. If $T_{23,\text{info}} < T_{14,\text{info}}$, the particle will keep trying to jump onto $E_2$ for $\kappa_{j_1,\text{thres}}$ times the to-be-expected saved time on route 23, i.e. for $\kappa_{j_1,\text{thres}} \cdot (T_{14,\text{info}} - T_{23,\text{info}})$ time steps. If the waiting time exceeds this value, (s)he will switch to route 14. The same is true for the inverse route choice situation (i.e. switching from route 14 to 23). In the 5link network, additionally an equivalent mechanism is in place that also includes route switches if an agent is waiting on $j_2$, introducing the variable $\kappa_{j_2,\text{thres}}$.

Public-predictive information is introduced in the following way. Each time an agent relying on this kind of information makes a route choice decision, for each edge $E_i$ the sum of numbers of particles occupying this edge $n_i$ is collected. This mirrors the crowdsourcing of information as in apps like Google Maps [41]. For each edge $i$ then an approximated travel time is calculated as $T_{i,\text{est}} \approx L_i/(1 - \rho_i)$ with $\rho_i = n_i/L_i$. This approximative travel time is the exact travel time of a TASEP segment with periodic boundary conditions [23]. From this the $T_{i,\text{info}}$ are calculated: $T_{14,\text{info}} = T_{i,\text{est}} + T_{4,\text{est}}$ and $T_{23,\text{info}}$ and $T_{153,\text{info}}$ accordingly.
Agents using personal-historical information remember the travel times of the routes they used in the last $c_{\text{mem}}$ rounds. To get their estimated travel time values $T_{i,\text{info}}$ that they base their decisions upon, they calculate the mean values of the travel times experienced in the last $c_{\text{mem}}$ rounds for each route. Additionally, these users remember the last travel time they experienced on each route, even if they did not use a specific route in the last $c_{\text{mem}}$ rounds.

For the results presented in the present article, the parameters of the route choice algorithm were chosen as

$$p_{\text{info}} = 0.9, \quad \Delta T_{\text{thres}} = 10, \quad \kappa_{j_1,\text{thres}} = \kappa_{j_2,\text{thres}} = 0.1, \quad c_{\text{mem}} = 30. \quad (1)$$

For more details on the algorithm including pseudo-code representation we refer to [28, 44].

In [28] we looked at four exemplary points of the phase diagrams (two points where a Braess phase is expected, two points where no Braess is expected) and tested if the expected user optima are realized by agents basing their route choices upon our algorithm with all agents having access to either personal-historical or public-predictive information. We could show that if all agents decide based on personal-historical information user optima that are attainable by externally tuning the agents decisions are realized. If all agents decide based on public-predictive information, at lower densities attainable user optima are realized and at higher global densities they are realized on average. The reliance on this kind of information leads to oscillations around the expected optima. In situations with higher global densities, we could show that agents relying on public-predictive information actually lead the system into a Braess state in a case were a lower travel time in the 5link system was expected.

4. Results

Here we consider a scenario in which different splits of the two information types are used: certain parts of the agents have access only to personal-historical information while the other agents only have access to public-predictive information. This represents the realistic scenario of a traffic network that is used partly by commuters who ”know” travel times from their personal experiences in the past and partly by users that rely on their navigational apps for route decisions.

We examine a test state marked by the black $\times$ in the phase diagrams in Fig. 2. It has the parameters $L_5 = 37$ and $M = 248$ which correspond
to $\hat{L}_{153}/\hat{L}_{14} = 0.4$, $\rho_{\text{global}}^{(4)} \approx 0.21$, $\rho_{\text{global}}^{(5)} = 0.2$. For externally tuned route choices this is a “Braess” state both for the previously studied cases of fixed personal strategies and fixed turning probabilities. The user optima that can be found by externally tuning the strategies are given in in Table 1. In [23, 24] we found so-called *boundedly rational user optima*. This means that the mean travel times on the used routes are not exactly equal. This can not be completely avoided since we deal with stochastic transport models. Since the travel times are not exactly equal, we present the maximum travel time $T_{\text{max}}$ measured on all used routes in the user optima. From here on all travel times are measured in numbers of Monte Carlo (MC) sweeps.

| user optimum          | strategy               | max. travel time |
|-----------------------|------------------------|------------------|
| 4link, pure uo        | $n_{1,\text{puo}}^{(4)} = 0.5$ | $T_{\text{max,puo}}^{(4)} \approx 764$ |
| 4link, mixed uo       | $\gamma_{\text{muo}}^{(4)} = 0.5$ | $T_{\text{max,muo}}^{(4)} \approx 763$ |
| 5link, first pure uo  | $n_{1,\text{puo(i)}}^{(5)} = 0.5$, $n_{1,\text{puo(ii)}}^{(5)} = 0.0$ | $T_{\text{max,puo(i)}}^{(5)} \approx 978$ |
| 5link, second pure uo | $n_{1,\text{puo(ii)}}^{(5)} = 1.0$, $n_{1,\text{puo(ii)}}^{(5)} = 0.5$ | $T_{\text{max,puo(ii)}}^{(5)} \approx 978$ |
| 5link, mixed uo       | $\gamma_{\text{muo}}^{(5)} = 0.87$, $\delta_{\text{muo}}^{(5)} = 0.1$ | $T_{\text{max,muo(i)}}^{(5)} \approx 895$ |

Table 1: The user optima found for our test state, as marked by the black × in Fig. 2. The (externally tuned) strategies realizing the optima and the corresponding maximum travel times are given. In the 4link system, one pure and one mixed user optimum exist, while in the 5link two pure and one mixed user optimum exist.

In the following the values provided in Table 1 are used as references to the systems’ behaviors when used by agents making decisions based on the route choice algorithm.

In Fig. 3 we show results for this test state network with agents choosing their routes intelligently based on the route choice algorithms described above for different mix-ratios of information types. The five columns, i.e. panels labelled with indices 1 to 5, represent different splits of the two information types as follows: 1: 0% to 100%, 2: 25% to 75%, 3: 50% to 50%, 4: 75% to 25%, 5: 100% to 0% of users deciding based on personal-historical and public-predictive information, respectively. If personal-historical information...
is used (panels (a-c)1 to (a-c)4), a certain relaxation process is needed in the algorithm. The relaxation times are marked by the two vertical lines in the plots. During the relaxation time, first each agent that relies on personal historical information tries to use each route at least once (relaxation step 1) and then 'fills up his memory capacity' (completing at least \(c_{\text{mem}}\) rounds/relaxation step 2. At the time marked by the black/grey vertical line, the relaxation procedure in the 4link/5link system is done and the system evolves according to the route choice algorithm.

Panels (a1) show the time evolution of the average travel time of the two/three routes. The travel times that are observed in the user optima with externally tuned decisions are shown for comparison by the black and grey lines. Their values are given by the \(\tau_i\) on the right y-axes. Panels (b1) denote which routes are chosen by the agents when deciding based on the route choice algorithm. They are given by \(m^{(j_1)}_l\) and \(m^{(j_2)}_l\), showing the fractions of agents turning left at junctions \(j_1\) and \(j_2\). While they do not represent the strategies of the agents directly, they can nevertheless be compared to the \(n^{(j_1)}_l\) and \(n^{(j_2)}_l\) or \(\gamma\) and \(\delta\) of the pure or mixed user optima attainable by externally tuning all agents’ strategies. One has to keep in mind that the \(m^{(j_1)}_l\) and \(m^{(j_2)}_l\) shown are not directly comparable since individual intelligently deciding agents do not necessarily keep their individual strategies. This means that if e.g. the \(m^{(j_1)}_l\) and \(m^{(j_2)}_l\) of a pure user optimum are identical to the \(n^{(j_1)}_l\) and \(n^{(j_2)}_l\) of an externally tuned pure user optimum this does not necessarily mean that this user optimum is realized, since individual agents can still change their strategies in each round. To get closer to answering the question if a pure or mixed user optimum is realized, in panels (c1) the number of switches are shown. They show how many agents switched from one route to another in the last 1000 time steps (i.e. MC sweeps) and how many agents kept on the same route. From these observables it could e.g. be concluded that a pure user optimum is realized if no switches were made at all. Note that from these quantities it can not be securely concluded if a mixed user optimum is realized. For this conclusion the route choice probabilities for each individual agent would have to be determined.

Panels (a1) to (a5) show that for all splits of information types in the 4link systems the average travel times of both routes 14 and 23 equalize close to the value that is expected in the attainable pure and mixed user optima (orange and blue lines) From panels (b1) to (b5) one can see that at each time approximately \(M/2\) agents are on each of the two routes \(m^{(j_1)(4)}_l \approx 0.5\), green
Figure 3: The results of the route choice algorithm. All plots share the same x-axes: the number of system sweeps (Monte Carlo sweeps) that were performed, i.e. the system time. All plots in one row share the same y-axes. The columns represent systems with varying amounts of agents using the two kinds of information. **Panels with indexes 1 to 5**, i.e. columns from left to right, represent the following amounts of agents using personal-historical and public-predictive information: 100% to 0%, 75% to 25%, 50% to 50%, 25% to 75%, 0% to 100%. In systems with non-zero use of personal historical information the two vertical lines represent the system times the 4link system was relaxed (black vertical line) and the 5link system was relaxed (grey vertical line). **Panels (a)** show the average travel times measured on the routes. The orange and blue lines are the average travel times of routes 14 and 23 in the 4link system. The green, yellow, red lines are the average travel times of routes 14, 23, 153 in the 5link system. For comparison the thin grey horizontal lines show the travel times of the user optima obtained by externally tuning the strategies. Their values are indicated by the $\tau_i$ on the second y-axis of panel (a). With $T_{\text{max, muo}} = \tau_1$, $T_{\text{max, puo}} = \tau_2$, $T_{\text{max, muo}} = \tau_3$, $T_{\text{max, puo}} = \tau_4$. **Panels (b)** show the agents’ strategies: the green line shows the $m_{1,1}(5)$ in the 4link system. The orange and red lines show the $m_{1,1}(4)$ and $m_{1,1}(5)$ in the 5link system. For comparison the thin, horizontal grey lines show the pure and mixed user optima available by externally tuning the agents’ decisions. Their values are indicated by the $\sigma_i$ on the second y-axis of panel (b) with $n_{1,1,\text{puo}} = \sigma_3$, $n_{1,1,\text{muo}} = \sigma_3$, $n_{1,1,\text{puo(5)}} = (\sigma_3, \sigma_1)$, $n_{1,1,\text{puo(4)}} = (\sigma_5, \sigma_3, (\gamma_{\text{muo}}, \delta_{\text{puo}})) = (\sigma_4, \sigma_2)$. **Panels (c)** show the number of switches (i.e. strategy changes) made. These numbers are collected in bins with a length of 1000 sweeps. The yellow and red lines show how many agents in the 4link system changed strategies and kept their strategies, respectively. The blue and green lines show how many agents in the 5link system changed and kept their strategies.
As can be seen from the green line in panel (b), if all agents rely on personal-historical information, oscillations around the user optimum can be observed. From panels (c₁) to (c₅) one can see that most agents stick to one route (red lines) and few agents switch routes (yellow lines). Summarizing for the 4link we conclude that for all splits user optimum states are realized in the sense that average travel times of the two routes equalize.

In the 5link system further interesting effects can be observed. Panels (b₁) to (b₅) show that almost no agents choose route 23 (m⁽⁽j₁⁾⁾⁽⁽5⁾⁾ ≈ 1, orange lines), and approximately M/2 agents choose routes 14 and 153 (m⁽⁽j₂⁾⁾⁽⁽5⁾⁾ ≈ 0.5, red lines). This indicates that a state close to the second pure user optimum could be realized (compare orange and red lines to \( \left( n⁽⁽j₁⁾⁾⁽⁽5⁾⁾, n⁽⁽j₂⁾⁾⁽⁽5⁾⁾ \right) = (\sigma₅, \sigma₃) \)). It can be seen that the higher the ratio of public-prescriptive information gets, the more the value of m⁽⁽j₂⁾⁾ fluctuates around 0.5. This is also confirmed when examining panels (c₁) to (c₅): the number of route switches grows with the amount of users relying on public-predictive information. When all agents rely on personal-historical information the vast majority of agents keep their strategies (green line much higher than blue line). The number of switches grows and for all agents relying on public-predictive information approximately as many agents change their routes with each new round as agents stick to their routes. Regarding the travel times we see, that route 23 which is almost not used at all has a higher travel time than the two used routes. Their respective average travel times lie in-between those that are expected in the accessible pure and mixed user optima. If all agents rely on personal-historical information they are almost exactly equal while they are less close to each other if all agents rely on public-predictive information. Interestingly, their difference does not grow strictly with the amount of users relying on public-predictive information as could be assumed by analysing the strategies and numbers of switches. For all splits the travel times of the used routes in the 5link system are higher than those of the routes in the 4link system.

We can thus foremost say that for all splits of information types Braess states are realized since the 5link travel times are higher than those in the 4link systems. Furthermore we can summarize that in the symmetric 4link system with two routes of equal length the type of information does not seem to have a huge impact: in all cases situations similar to the attainable optima are realized. In the 5link systems, however, the more public-predictive information is used the more users keep switching routes from
round to round. This (modern) type of information thus leads to a more unstable situation (regarding the number of switches). Nevertheless, it does not seem to lead the system into the attainable mixed user optimum as the $m_{1}^{(j_{1}/2)^{(5)}}$ are closer to the $n_{1,puo(ii)}^{(j_{1}/2)^{(5)}}$ of the second attainable pure user optimum.

5. Conclusions

Our analysis of an exemplary state suggests strongly that Braess' paradox is likely to still occur in traffic networks in which drivers choose their routes intelligently based upon information that is available in modern real-world traffic networks. In our example, public-predictive information as provided e.g. by smartphone navigation apps realizes attainable user optima on average. If all network users base their route choices on their own past experiences, user optima are also realized in a more stable way. Also in all mix-ratios the optima are (at least on average) realized and in all cases Braess states are observed: in the system with the additional road, users always distribute onto the routes such that the routes’ travel times are higher than those in the networks without the new road.

Dedication:

We dedicate this paper to the memory of Dietrich Stauffer. One of the authors (AS) has known Dietrich for more than 35 years, first as a student and later as a colleague. He will be remembered not only for many good advice given through all these years, but also for lively discussions on a broad range of topics, scientific and beyond (from politics to football and movies).

Acknowledgements:

Financial support by Deutsche Forschungsgemeinschaft (DFG), Germany under grant SCHA 636/8-2 and the Bonn-Cologne Graduate School of Physics and Astronomy (BCGS) is gratefully acknowledged. Monte Carlo simulations were carried out on the CHEOPS (Cologne High Efficiency Operating Platform for Science) cluster of the RRZK (University of Cologne).
References

[1] D. Braess, Über ein Paradoxon aus der Verkehrsplanung, Unternehmensforschung 12 (1968) 258.

[2] D. Braess, A. Nagurney, T. Wakolbinger, On a paradox of traffic planning, Transportation Science 39 (2005) 446, (English translation of [1]).

[3] J. E. Cohen, P. Horowitz, Paradoxical behaviour of mechanical and electrical networks, Nature 352 (1991) 699.

[4] C. M. Penchina, L. J. Penchina, The Braess paradox in mechanical, traffic, and other networks, Am. J. Phys. 71 (2003) 479.

[5] D. Witthaut, M. Timme, Braess’s paradox in oscillator networks, desynchronization and power outage, New J. Phys. 14 (2012) 083036.

[6] E. B. T. Tchuisseu, D. Gomila, P. Colet, D. Witthaut, M. Timme, B. Schäfer, Curing braess’ paradox by secondary control in power grids, New J. Phys. 20 (2018) 083005.

[7] A. E. Motter, M. Timme, Antagonistic phenomena in network dynamics, Ann. Rev. Cond. Matter Phys. 9 (2018) 463.

[8] D. Case, Y. Liu, I. Kiss, J.-R. Angilella, A. Motter, Braess’s paradox and programmable behaviour in microfluidic networks, Nature 574 (2019) 647.

[9] L. Crociani, G. Lämmel, Multidestination Pedestrian Flows in Equilibrium: A Cellular Automaton-Based Approach, Computer-Aided Civil and Infrastructure Engineering 31 (2016) 432.

[10] J. G. Wardrop, Road Paper. Some Theoretical Aspects Of Road Traffic Research, Proc. Inst. Civ. Eng. 1 (1952) 325.

[11] H. Youn, M. T. Gastner, H. Jeong, Price of Anarchy in Transportation Networks: Efficiency and Optimality Control, Phys. Rev. Lett. 101 (2008) 128701.

[12] N. F. Stewart, Equilibrium vs system-optimal flow: some examples, Transp. Res. A 14 (1980) 81.
[13] E. I. Pas, S. L. Principio, Braess’ paradox: Some new insights, Transp. Res. B 31 (1997) 265.

[14] J. D. Murchland, Braess’s paradox of traffic flow, Transp. Res. 4 (1970) 391.

[15] R. Steinberg, W. I. Zangwill, The prevalence of Braess’ paradox, Transp. Sci. 17 (1983) 301.

[16] M. Frank, The Braess paradox, Mathematical Programming 20 (1981) 283.

[17] S. Dafermos, A. Nagurney, On some traffic equilibrium theory paradoxes, Transp. Res. B 18 (1984) 101.

[18] A. Nagurney, The negation of the Braess paradox as demand increases: The wisdom of crowds in transportation networks, EPL 91 (2010) 48002.

[19] L. Baker, Removing Roads and Traffic Lights Speeds Urban Travel, Scientific American (February 2009) 20.

[20] G. Kolata, What if They Closed 42d Street and Nobody Noticed?, The New York Times.

[21] J. Vidal, Heart and soul of the city, The Guardian.

[22] D. Stauffer, Grand unification of exotic statistical physics, Physica A 285 (2000) 121. doi:http://dx.doi.org/10.1016/S0378-4371(00)00275-2

[23] S. Bittihn, A. Schadschneider, Braess paradox in a network of totally asymmetric exclusion processes, Phys. Rev. E 94 (2016) 062312.

[24] S. Bittihn, A. Schadschneider, Braess paradox in a network with stochastic dynamics and fixed strategies, Physica A 507 (2018) 133.

[25] A. Schadschneider, D. Chowdhury, K. Nishinari, Stochastic Transport in Complex Systems: from Molecules to Vehicles, Elsevier, 2010.

[26] G. Schütz, Exactly solvable models for many-body systems far from equilibrium, Phase Transitions and Critical Phenomena 19 (2000) 1.
[27] R. Blythe, M. Evans, Nonequilibrium steady states of matrix product form: a solver’s guide, J. Phys. A 40 (2007) R333.

[28] S. Bittihn, A. Schadschneider, Braess’ paradox in the age of traffic information, submitted for publication (arXiv:2009.02158).

[29] P. Parthasarathi, D. Levinson, H. Hochmair, Network Structure and Travel Time Perception, PLoS ONE 8 (2013) e77718.

[30] T.-Y. Chen, H.-L. Chang, G.-H. Tzeng, Using a weight-assessing model to identify route choice criteria and information effects, Transp. Res. A 35 (2001) 197.

[31] S. Zhu, D. Levinson, Do People Use the Shortest Path? An Empirical Test of Wardrop’s First Principle, PLoS ONE 10 (2015) e0134322.

[32] A. Rapoport, T. Kugler, S. Dugar, E. Gisches, Choice of routes in congested traffic networks: Experimental tests of the Braess Paradox, Games and Economic Behavior 65 (2009) 538.

[33] R. Selten, T. Chmura, T. Pitz, S. Kube, M. Schreckenberg, Commuters route choice behaviour, Games and Economic Behavior 58 (2007) 394.

[34] C. Meneguzzer, A. Olivieri, Day-to-day traffic dynamics: laboratory-like experiment on route choice and route switching in a simple network with limited feedback information, Procedia - Soc.Behav. Sci. 87 (2013) 44.

[35] H. Ye, F. Xiao, H. Yang, Exploration of day-to-day route choice models by a virtual experiment, Transp. Res. Procedia 23 (2017) 679.

[36] J. Wahle, A. L. C. Bazzan, F. Klügl, M. Schreckenberg, Decision dynamics in a traffic scenario, Physica A 287 (2000) 669.

[37] A. L. C. Bazzan, J. Wahle, F. Klügl, Agents in traffic modelling—from reactive to social behaviour, KI 99 (1999) 303–307.

[38] A. L. C. Bazzan, F. Klügl, Case studies on the braess paradox: simulating route recommendation and learning in abstract and microscopic models, Transp. Res. C 13 (4) (2005) 299–319.
[39] Z. He, B. Chen, N. Jia, W. Guan, B. Lin, B. Wang, Route guidance strategies revisited: Comparison and evaluation in an asymmetric two-route traffic network, Int. J. Mod. Phys. C 25 (2014) 1450005.

[40] N. Levy, E. Ben-Elia, Emergence of system optimum: A fair and altruistic agent-based route-choice model, Procedia Comp. Sci. 83 (2016) 928.

[41] D. Barth, Official Google Blog: The bright side of sitting in traffic: Crowdsourcing road congestion data, Google (Online, accessed 12-September-2018).
URL https://googleblog.blogspot.com/2009/08/bright-side-of-sitting-in-traffic.html

[42] T. Cabannes, M. A. S. Vincentelli, A. Sundt, H. Signargout, E. Porter, V. Fighiera, J. Ugirumurera, A. M. Bayen, The impact of GPS-enabled shortest path routing on mobility: a game theoretic approach, Tech. rep., Transp. Res. Board (2018).

[43] H. S. Mahmassani, G.-L. Chang, On boundedly rational user equilibrium in transportation systems, Transp. Sci. 21 (1987) 89.

[44] S. Bittihn, Stochastic transport models on simple networks: Phase diagrams and Braess paradox, Doctoral Thesis, University of Cologne.
URL https://kups.ub.uni-koeln.de/9166/