A FUNCTIONAL APPROACH TO NUCLEAR ELECTROMAGNETIC RESPONSE FUNCTIONS

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Abstract

The separated electromagnetic responses $R_L(q, \omega)$ and $R_T(q, \omega)$ for inclusive electron scattering off nuclei are studied within a functional scheme.

I. INTRODUCTION

During the '80s experimentalists were able to separate out of the inclusive electron-nucleus cross-section the charge and magnetic response functions via a Rosenbluth plot whose validity is grounded on two assumptions: the goodness of the one-photon-exchange approximation and the negligibility of the Coulomb distortion of the electrons.

Since then a large amount of theoretical speculations was devoted to the unexpected behaviour of $R_L(q, \omega)$ and $R_T(q, \omega)$: in particular $R_L(q, \omega)$ seems largely quenched with respect to the independent particle model, while $R_T(q, \omega)$ is somehow enhanced, a fact this last which is not so often emphasized in the literature.

The exact amount of the quenching of $R_L(q, \omega)$ (and of the correlated enhancement of $R_T(q, \omega)$) seems to depend upon the target nucleus, indicating larger effects in medium-heavy nuclei. Recently Jourdan addressed two questions: first of all about the internal coherence of the known experimental data and secondly about their interpretation in terms of the Coulomb sum rule.

He re-examined the whole set of the available experimental data for inclusive electron scattering ("world data") and he was able to perform a cleaner and more reliable Rosenbluth
separation of $R_L(q, \omega)$ and $R_T(q, \omega)$ than those based on a single experimental set. This essentially for two reasons: firstly a better treatment of the Coulomb distortion and secondly because using the world data enables one to perform the Rosenbluth separation spanning a larger interval in $\varepsilon \equiv (1 + 2|q|^2/Q^2 \tan^2 \theta/2)^{-1}$, the kinematical variable for the plot, than the one spanned by a single experimental set of data. The net result of this procedure is twofold: first of all one obtains a set of "corrected" or hopefully more reliable experimental response functions and secondly the apparent failure of the Coulomb sum rule is washed out at a sufficiently higher $q$ (for instance $q \sim 600\text{MeV}/c$).

Still the results for the response functions display in the intermediate momentum region some non trivial behaviours, $R_L$ being anyway largely quenched with respect to FFG predictions [in an A-dependent way] and $R_T$ being somehow enhanced. So the question of the behaviour of the separate e.m. response functions remains in our opinion largely not understood. Moreover since some doubts remain even on the procedure followed by Jourdan - not in principle, but because at larger $\varepsilon$ available data are older and few and because these data are mostly responsible for the corrections - we will continue to present our calculations against both Jourdan data and the original experimental data, when both are available at the given momentum. We emphasize that it would be highly desirable that future experiments (at TJNAF?) would unambiguously solve the experimental problem.

II. THE THEORETICAL SITUATION AND THE FUNCTIONAL APPROACH.

A wide set of different theoretical calculations based on a variety of dynamical models yields a more or less pronounced depletion of the QEP in the longitudinal channel. It is beyond the purposes of this contribution to provide a detailed comparison between them. Most of them are anyway unable to provide a good description of $R_T$ also.

Some years ago we proposed a theoretical scheme based on the application of the SPA to a (finite) generating functional describing a relativistic system of nucleons and pions. SPA was applied after explicit integration of the nucleonic degrees of freedom: in this way
the presence of a nuclear medium is treated in principle correctly. To make such a scheme useful in practice we needed to restrict it to a nonrelativistic potential theory including the interaction only in the particle-hole channel with the quantum numbers of a pion, later on we included also the interaction in the channels with the quantum numbers of $\rho$ and $\omega$ mesons to describe $R_L(q, \omega)$ alone. Finally we considered the contributions coming from the excitation of a $\Delta_{33}$ resonance, which are known to be necessary when $R_T(q, \omega)$ is under consideration and we extend now the interaction to cover all the ph channels with $T = 0, 1$ and scalar (S), spin-longitudinal (L) and spin-transverse (T).

We do not re-propose here the theoretical derivation of our approach, since it can found in the literature, but we must spend few words about its dynamical content. To make practically manageable the calculation we need an effective interaction given in each spin-isospin channel as a function of the transferred momentum only, including obviously NN, N$\Delta$ and $\Delta\Delta$ transitions. Its form has been chosen to resemble a very traditional parametrization of the effective interaction in the $\pi$ and $\rho$ channels, namely

$$V_{L,T}^{T=1}(q) = \frac{f_{\pi NN}^2}{m_\pi^2} \left\{ g'(q) - C_{L,T} \frac{q^2}{q^2 + m_{\pi,\rho}^2} \right\} v_{L,T}^2(q^2)$$

with $C_L = 1$ and $C_T \approx 2.3$. The Landau parameter $g'$ is usually assumed to be constant and set around $0.6 \div 0.7$; we allow instead $g'$ to be momentum-dependent. In mesonic scheme it is convenient to explicitly use meson-exchanges only for the medium/long range part of the interaction. The remaining part can be either parametrized with constants or with slowly varying functions of the momentum, or, alternatively making use of two-body correlation functions. These last can be either explicitly introduced or one can simply admit that

$$\lim_{q \to 0} g'_{L,T}(q) = g', \quad \lim_{q \to \infty} g'_{L,T}(q) = C_{L,T}$$

in such a way to properly cut the high-momentum components of the meson exchange potentials. All these techniques are conceptually equivalent, provided one keeps in mind the physical difference between the vertex cutoff $v_{L,T}^2(q^2)$ and the many-body correlation length implicit in eqs. (2). By choosing arbitrarily the functional form
\[ g'_{L,T}(q) = C_{L,T} + (g' - C_{L,T}) \left( \frac{q_{c-L,T}^2}{q^2 + q_{c-L,T}^2} \right)^2, \]  

(3)

we are led to determine the momentum dependence of \( g'_{L,T}(q) \) simply through the choice of two parameters \( q_{c-L,T} \), which are expected in the range \( m_{\pi,\rho} < q_{c-L,T} < \Lambda_{L,T} \), owing to the previous discussion; the \( \Lambda \)s are the usual vertex cutoffs: \( v_c(q^2) = (\Lambda_c^2 - m_c^2)/(\Lambda_c^2 + q^2) \). In the \( \omega \)'s channels we simply choose an effective interaction of the form

\[ V^{S=T}_\omega(q) = -\frac{f_{\pi NN}^2}{m_{\pi}^2} C_{\omega} \frac{q^2}{q^2 + m_{\omega}^2} v_\omega^2(q^2), \quad V^{S=0}_\omega(q) = \frac{f_{\pi NN}^2}{m_{\pi}^2} C_{\omega S} \frac{q^2}{q^2 + m_{\omega}^2} v_\omega^2 L(q^2) \]  

(4)

and an analogous form is chosen for the \( S = 0 \) component of the \( \rho \). The corresponding potentials when \( N\Delta \) and/or \( \Delta\Delta \) transitions can occur are obtained by replacing \( f_{\pi NN} \) with \( f_{\pi N\Delta} \) or with \( f_{\pi \Delta\Delta} \) and modifying the vertex cutoffs. The complete set of parameters we are employing are reported in the appendix. Obviously all these potentials are multiplied by the proper combination of spin-isospin matrices for the given channel.

Some comments are here in order regarding the procedure we have followed to determine the parameters in the effective interaction deferring a full discussion to a next-to-come paper\(^\text{[12]}\). The most part of the coupling parameters and all the vertex cutoffs are chosen accordingly to a "democratic principle" looking to the current literature: essentially they are set to values coming either from mesonic realistic potentials (like, e.g., the Bonn potential) or, when the former possibility is precluded, by using quark-model indications. The two many-body cutoffs have been tentatively set to 800 MeV and 1300 MeV in the \( \pi \) and \( \rho \) channels respectively. The mass of the transverse \( \rho \) has been set to 600 MeV, a value which should keep memory of the attraction felt by the \( \rho \) inside the nuclear medium both when living as a \( \rho \) and as a couple of pions. The value of \( g'(0) \) has been set to .35; these values completely determine the potential both in the \( \pi \) and in the \( \rho \) channels and have been chosen to reproduce as far as possible the known effective interactions in the low/intermediate \( q \)-range and allowing a faster decrease of the high-\( q \) tails in order to describe two-body correlations. The low value of \( g' \) should not worry about the possibility of pion condensation, since this can happen at typical values of \( q \sim 2k_F \), where \( g'_L(q) \approx 0.75 \).
The only value we took as a practically free parameter is the coupling for the scalar component of the $\omega$. We fixed it naively to a value around 0.15 by simply fitting the longitudinal response function for $^{12}C$ at $q = 300$ MeV. This so small value is obviously an effective one and could emerge, for instance, as a parametrization to the ladder series of "bare" scalar $\omega$’s.

Clearly both this point and the determination of the effective $\rho$ mass require a further microscopic investigations. We are confident that the chosen values are not so far from reality since they provide a qualitative description of nucleon (and $\Delta$) self-energies.

With these ingredients we obtain the results shown in Figs. 2, 3, 4 and 5. In all these plots solid lines are the full calculation, dashed lines the 0-loop result, while dotted lines represent FFG outcomes. As one can see there is generally a good overall agreement between our calculation and the known experimental data from the Saclay experiments, which we choose as reference. In particular $R_L$ is described quite accurately both for $^{12}C$ and $^{40}Ca$ for all the momenta examined.

Clearly agreement turns out to be better at higher momenta, where the convergence of the loop expansion is expected to be faster, while some problems begin to be evident at the lowest value of $q=300$ MeV.

The shape and form of $R_T$ are also described fairly well, even if the QEP turns out to be too much broad for $^{12}C$. The position of the peak for $R_T$ is shifted of tens of MeV, a fact that could be linked to a known bias of our approach. In fact, as one can see from Fig. 1, we cannot sum the Dyson series for the nucleonic self-energies, at least at any given order in the loop-expansion, a fact which conversely prevent us from the possibility of obtaining any shift in the QEP position.

III. CONCLUSION AND OUTLOOK

We have shown how a 1-loop calculation in a BLE with a reasonable choice for the effective interaction is able to explain the disagreement between $R_L$ and $R_T$ and the FFG
predictions. The apparent discrepancy in the behaviour between experiments on different nuclei can be understood in this frame because the various contributions to the responses depend differently upon the density of the system - in particular the diagrams of the first line in are proportional to the density while the others to its square. Some residual problems with experimental data, even after Jourdan’s work, suggest a renewed experimental interest into the topic. In particular world data seems to be too much lowered for \( R_T \) on \(^{40}\text{Ca}\) with respect to the original data, a feature emerged also in other calculations.\(^{13}\)

| S | T | \( C_{NN} \) | \( C_{N\Delta} \) | \( C_{\Delta\Delta} \) | \( \Lambda_{NN} \) | \( \Lambda_{N\Delta} \) | \( \Lambda_{\Delta\Delta} \) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | .15 | - | .15 | 1000 | - | 1000 |
| L | 0 | - | - | - | - | - | - |
| T | 0 | 1.5 | - | 1.5 | 1000 | - | 1000 |
| 0 | 1 | .00436 | - | .00436 | 1000 | - | 1000 |
| L | 1 | .08 | .32 | .016 | 1300 | 1000 | 1000 |
| T | 1 | 2.3 | 2.3 | 2.3 | 1750 | 1000 | 1000 |

Pion channel values are expressed as \( f_{\pi xx}^2 / 4\pi \)
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FIG. 1. The class of diagrams to be evaluated at the 1-loop order to obtain Response functions.
FIG. 2. $R_L(q, \omega)$ for $^{12}C$ at $q = 350, 400, 450, 500$ MeV/c. Data from Saclay.
FIG. 3. $R_T(q, \omega)$ for $^{12}\text{C}$ at $q = 350, 400, 500, 570$ MeV/c. Dots: data from Saclay, triangles: world data
FIG. 4. $R_L(q, \omega)$ for $^{40}\text{Ca}$ at $q = 300, 330, 370, 410$ MeV/c Dots: data from Saclay, squares: data from Bates, triangles: world data
FIG. 5. $R_L(q, \omega)$ for $^{40}$Ca at $q = 300, 330, 370, 410$ MeV/c Dots: data from Saclay, triangles: world data