MULTI-CRITERIA MEDIA MIX DECISION MODEL FOR ADVERTISING A SINGLE PRODUCT WITH SEGMENT SPECIFIC AND MASS MEDIA

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ABSTRACT. The effectiveness of any advertisement campaign relies heavily on the combination of media vehicles chosen for communicating the messages and the amount of advertisements placed in them. This paper presents a media planning model that assists a firm in determining the optimal media mix for a product advertised in a segmented market. The model determines the number of advertisements to be placed in different segment specific media as well as mass media. The objective is to maximize the reach of the product in the potential market by placement of the advertisements. A case study is presented to illustrate the application of the model in which the market considered is geographically segmented on the basis of cultural and lingual diversity. The segments respond to regional advertising (segment specific) and to national advertising which reaches segments with a fixed spectrum. Interactive weighted sum goal programming technique is discussed to solve the problem.

1. Introduction. Effective marketing of a product involves figuring out how much to manufacture, what it should be priced at, locating its target market(s), its distribution channels and identifying how and where to promote. In marketing jargon, these four critical elements are known as the “Four P’s”: Product, Price, Place (distribution), and Promotion termed as the marketing mix. Nearly all organizations, whether for-profit or not-for-profit, engage in some form of promotion or the other. Firms choose different forms of promotion viz. advertising, personal selling, public relations, sales promotion, and direct marketing depending on which are the best ways to reach their target market. Among the key elements of promotional mix, advertising is the most dominant component. Advertising can be done through various media such as print media (newspaper, magazines) and electronic media (TV, radio, internet). Selection of the right media mix plays a vital role in

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manifesting the message and awareness of the product in the minds of the target audience. Thus, planning for a suitable media is of prime concern for any marketing manager. Media planning has two important aspects. First being the selection of advertising media, second being the development and allocation of the advertising budget. Selection of media largely depends on the product and the target market. The target market for the product can be taken as a whole ignoring the differences in the market segment; in this case product is mass advertised using a common media mix in the entire market. In the other case, if the heterogeneity among the potential customers of the target market is taken into consideration, the market is divided in to a number of segments on the basis of geographic, demographic, behavioral and psychographic characteristics. Here, the product is advertised through segment specific media mix. This paper focuses on determining optimal media mix for a product to be advertised in a segmented market at mass as well as segment level.

Market segmentation is an important marketing strategy in todays marketing environment. It helps marketers divide total market into groups consisting of people who have relatively similar characteristics. The purpose of segmenting is to design segment specific/differentiated marketing mix for each segment that matches the customers characteristics in that segment. Here, our focus is largely on geographic segmentation and determining optimal media mix for segments. Apart from targeting segments with segment specific advertising strategy, marketers also use mass advertising to address the whole market with a single advertising message. Whereas, the differentiated advertising targets individual segments through distinct advertising messages using segment specific advertising media. Here efforts are directed specifically towards the designated segment in a manner consistent with that segment’s characteristics. The impact of mass market advertising reaches all segments of the market as spectrum effect, which is distributed over various segments (Grosset and Viscolani [19]). Thus, each segment gets influenced by both differentiated advertisement and mass advertisement with a fixed spectrum. The decision to be taken here is how to allocate available advertising budget between differentiated and mass advertising and how many insertions are to be given in different mass and segment specific media so as to optimize the efficiency of the media mix.

In a geographically segmented market, segments may be defined over continents, nation, countries, states, cities, towns, districts, zip codes, population density, or climate. Customer groups defined over geographic segments may possess differential characteristics due to difference in culture, climate, political environment, population density, language etc. By understanding the geographical differences between the segments, marketers increase their advertising efficiency and build segment specific advertising strategies according to the segment to which the consumers belong, along with advertising their products with mass advertising over the total market. For example, Indian market can be segmented geographically into regional segments as the difference in the marketing environments of various regions of the country suggests that each market is different and requires distinct promotional and advertising programs (Ramaswamy and Namakumari [35]). Companies cater to this diversified customer base of India by advertising a product in each region independently in its native regional language using regional media as well as in national language using national media, which reaches several regions with a spectrum effect. In India, national advertising is done in either Hindi or English as these are the most common languages all over the country. Regional advertising is done in
each region independently keeping in mind their geographic, psychographic and behav-ioral aspects. Wherein, the promotional messages are relayed in their native regional language and the product is promoted through local events, regional TV channels, newspapers etc. Also, the product is advertised using mass advertising such as through national TV channels in national language and through national events, etc. which reaches several regions and influence the product awareness in each of the geographical segments. Consider for example, Hindustan Unilever Limited (HUL) promotes most of its products through these two strategies viz. Fair & Lovely, a fairness cream is promoted using TV commercials on national TV channels such as Doordarshan, Zee, Star, Sony, etc. These commercials are dubbed in various state-dominant regional languages and are also telecasted in regional TV channels as shown in Figure 1. In this way, higher product exposure is obtained.

![An ad appearing on national and three regional channels](image)

**Figure 1.** An ad appearing on national and three regional channels

In this paper, an optimization model is formulated to determine the media mix for a product advertised in a segmented market using both segment specific/differentiated and mass advertising strategies. The model allocates total budget available for advertising among the multiple mass and differentiated media that are found suitable for the advertising of the product, maximizing the total reach in all the segments. Each segments potential customers get influenced by advertisements targeted to the segment and mass advertisement with a fixed spectrum. The formulated model is a multi-criteria linear integer programming problem. We have used Weighted Sum Goal programming approach (WSGPA) to solve the problem (Ignizio [23]). The weight of GPA is determined through an interactive algorithm. A case study is presented to show the real life application of the model.

The paper is organized as follows. Literature review of various media allocation models has been given in Section 2. Mathematical model formulation and solution methodology have been presented in Section 3 and Section 4. A case study is discussed in Section 5 to illustrate the application of the model. Concluding remarks are made in Section 6.

2. **Literature review and research motivation.** Number of authors have worked on optimization models for media planning and proposed several models in this area. Bass and Lonsdale [5] explored the use of linear programming in media selection by considering the budget constraint and operational constraints and maximized the weighted exposures. Charnes et al. [12] introduced a Goal Programming model for media selection that provides significant improvement over the earlier linear programming model. The objective considered includes maximizing the average frequency of reach to a fraction of the audience segment in various types of media aimed at different segments of the market at different time periods, and cost of the advertising. Little and Lodish [27] and Maffei [30] formulated the
media planning problem as a dynamic programming problem. Aaker [1] considered it as a probabilistic problem. Gensch and Welam [17] formulated a dynamic model for determining the optimal allocation of a given advertising budget for interacting market segments over time. The basic input parameters were in terms of sales saturation levels and advertising elasticity.

Zufryden [42] developed a media scheduling model to optimally select media insertions over time so as to maximize sales given a budgetary restriction. This model incorporates a stochastic response behaviour model as part of its objective function and is formulated as a binary integer mathematical programming system. The media model also incorporates factors such as sales potential of the target audience and respective market segments, total advertising budget, media timing, exposure probabilities and time patterns (e.g. advertising carry-over effects, forgetting and saturation effects). Zufryden [43] reviewed this previous media model’s structure briefly and concentrated on evaluating several heuristic techniques for solving the proposed model. Computationally efficient alternative solution approaches were developed and evaluated. As an enhancement to media selection problem, later, Zufryden [44] developed a model to study the impact of the dual objectives of reach and frequency maximization in the problem of media selection. This nonlinear integer mathematical programming model maximizes reach subject to a parametric frequency constraint and is solved through a dynamic programming approach. The author (Zufryden [45]) also proposed a mathematical programming model to allocate budget in the face of competitive environment over a predetermined planning horizon. They considered the carry-over of past expenditures, diminishing returns and saturation effects, response decay in the absence of advertising and product diffusion effects.

The media selection models were also addressed using multi objective techniques. A media allocation model using non linear benefit curves was proposed by Locander [29]. In the same year, De Kluyver [15] gave the use of hard and soft constraints for linear programming models in media selection. Keown and Duncan [25] developed an integer goal programming model to solve media selection problems and improved upon sub-optimal results produced by linear programming and non-integer GP models. Leckenby and Ju [28] provided an overview and assessment of work conducted largely on advertising media decision models considered from two standpoints: models to estimate numbers of people in target markets exposed to the media schedule and models incorporating exposure distribution models aiming at the complete planning and scheduling of the media program. Moynihan et al. [33] and Fruchtzer and Kalish [16] contended that the mathematical requirements of the MCDM model for media selection force the media planner to create an artificial structuring of the media selection criteria. Fruchtzer and Kalish gave a differential game model for media budgeting and allocation. Balakrishnan and Hall [4] presented an analytical model for determining the optimal insertion timing pattern of a long run ad campaign which consists of a number of varying executions. A computational procedure is developed to calculate the time between the insertions within a pulse in order to maximize the minimum effectiveness level at any point in time. Danaher and Rust [14] adopted the point of view that advertising is an investment, and proposed a simple formula for calculating the level of media spending which maximizes the return on investment. Naik et al. [34] presented an approach to evaluate large numbers of alternative media schedules and determine the best set of media schedules for consideration in media planning.
Berkowitz et al. [7] studied the impact of differential lag effects on the allocation of advertising budgets across media. Bass et al. [6] analyzed the effect of advertising decisions made by firms offering a product in a dynamic duopoly on the sales. Mihiotis and Tsakiris [32] proposed an advertisement allocation model for the television media under the budgetary restrictions maximizing the rating. Kwak et al. [26] presented a case study that considers two options: industrial and consumer products. In order to resolve the strategic decision making about dual market high technology products, a mixed integer goal programming model is developed to facilitate the advertising media selection process. Cetin and Esen [11] developed a media allocation model based on the problem originated from the weapon-target assignment problem of military operations research. Coulter and Sarkis [13] developed and tested a model for media selection and budget allocation using the analytic network process. This study demonstrated how this model may be used to allocate media dollars for two industries: airlines and financial services. Buratto et al. [9] analyzed the media selection problem where a firm has to choose an advertising channel to direct the pre-launch campaign for a new product in a segmented market. The problem is analyzed in two steps. First, an optimal control problem is solved explicitly in order to determine the optimal advertising policy for each channel. Then a maximum profit channel is chosen. Later Buratto et al. [10] aimed to plan the advertising process for the different segments with the objective of maximizing profit. They analyze and solve the new product introduction problem in two different frameworks: first assuming the advertising process reaches selectively each target group, and second assuming a single channel is available which has an effective segment spectrum which is distributed over a non trivial set of segments and compared the two.

Haydock [21] presented a media optimization problem considering market segments referring it as asset classes. The addition of an asset allocation process with the objective of funding market segments with the appropriate media investment is one of the most important courses of action the marketing manager undertakes. The optimization model minimizes the total cost of the marketing program while adhering to constraints on preferences of investment choice, exposure demands, and contact strategies and efficiently allocates resources to the intersection of asset classes and media types. Wang and Xu [41] proposed a fuzzy multi-objective model for allocation of advertising budget with an objective of maximizing the effectiveness of advertisements which is measured in terms of number of viewers of the advertisements. Fuzziness comes from the data which is used to calculate the number of target viewers, such as circulation, impression and viewers. Grosset and Viscolani [19] proposed a dynamic advertising model to compare the performance for selection of advertising media as in Buratto et al. [10]. Their objective is to seek the advertising flow that maximizes the firms long run profit measured in terms of goodwill. Viscolani [39] gave a static advertising model for a segmented market by means of a goodwill vector using a non linear programming framework to select a set of advertising media with an objective of maximizing profit. Firstly he gave optimal advertising policies that maximize the profit by considering same set of media for different segments. Here he modelled the effect of their joint use and a special case where it is allowed to use only a single medium (among the given set of media) on the segment goodwill values. The author characterized optimal advertising policies in the case when the available media affect the goodwill values of disjoint sets of segments.
Later Viscolani [40] obtained a coordinated channel optimal advertising decisions and pure strategies Nash equilibria in the competitive setting for two members of a simple distribution channel, manufacturer and retailer for a particular product in a segmented market. The model aims at maximizing their profits by choosing suitable advertising media and efforts. The advertising efforts of the two agents have a joint effect on the goodwill of the different market segments and then on the demand. The model is formulated for two cases: when only one medium is available to each agent and the other when at least one agent has to choose among several advertising media both by assuming that they share the same segmented market information. Bhattacharya [8] presented an advertisement planning problem to decide the number of advertisement in different media for a post-graduate diploma management programme. The objective was to maximize the reach of advertisement to the desired section of people for different media within their maximum allowable budget without violating the maximum and minimum numbers of advertisement goals. The problem was modelled as a chance-constrained GP problem. Jha et al. [24] formulated media planning problem for allocating the available budget among the multiple media that are found suitable for the advertising considering marketing segmentation aspect of advertising and maximizing the total reach in all the segments.

Grosset and Viscolani [20] formulated a non smooth infinite horizon optimal control problem for a monopolist firm which advertises a product in a segmented market where a constant exogenous interference is present. They assume a single decision maker and summarize the actions of the other players in an exogenous term which is considered as a constant negative effect on the goodwill evolution of its brand. A media planner has limited financial resources and aims to get the best return on investment and at the same time minimise total costs of advertising and communication. These objectives are conflicting, commensurable and their evaluations are generally stochastic. Aouni et al. [3] presented a stochastic multiobjective approach for media planning decisionmaking and propose two different goal programming formulations with satisfaction function based on scenario forecasting and deterministic equivalent. Malthouse et al. [31] proposed an optimization model to select media vehicles for marketing campaigns and to determine the depth of purchase from each vehicle. The response likelihood of individuals is estimated based on historical data. The objective is to maximize the overall expected profit from the marketing campaign by properly allocating the budget to the most profitable vehicles and database members. Royo et al. [36] observed that in comparison to the successive application of single stage model, the multistage approach may significantly raise the advertising profit. They gave a model to decide the optimal allocation of budget with an objective of maximizing the advertising investment for multiple products considering cross elasticities between products and different sales drivers over a planning horizon. Huang and Yang [22] optimized the advertising schedule policy which in turn influences the product sales. They gave a mathematical model to improve the advertising policy which was based on genetic algorithm.

From the above discussions, it is visible that there is extant literature of advertisement planning problems. Different authors have worked in the area considering different aspects. The focus of proposed formulation is on a very specific situation not addressed in the literature. Though several papers discussed media planning for a segmented market, most of these papers focused either on evaluating efficiencies of each available channel individually and selecting the best (Buratto et al.
or comparing the efficiency in two situations, first when advertisement reaches selectively each segment using different media. Second, when single channel reach all segments with an effective segment spectrum and the alternative which is more efficient is selected (Buratto et al. [10]; Grosset and Viscolani [19]; Viscolani [39]). These models exclude the possibility of selection of more than one media for any segment or market. Models given by (Bhattacharya [8]; Jha et al. [24]) also consider segmented market. These formulations result in selection of multiple media for different segment according to the reach of these media in the segments and maximize the total reach over all segments for single or multiple products.

In todays marketing environment, it is observed that advertisers do not rely on a single media for advertising. When a product is advertised in a segmented market, the potential customer groups possess differential characteristics and may have different media preferences even within the same segment. In this case, advertisements through single channel may not be exposed to all the potential customers. In an attempt to reach each and every potential customer, advertisers first find out all the preferred media of customers in all the segments and then allocate budget to the selected media. Along with targeting potential group in each segment selectively by means of advertising through segment specific media, advertisers also include mass advertising in advertising campaign. The purpose of mass advertising is not only to influence the product sales but also to create wider product awareness, influence market size and brand awareness (Aggarwal et al. [2]). Mass advertising is carried through mass media which reach all segments with a spectrum effect. Even in the case of mass advertising more than one media can be selected according to the media preference of the total product potential market. This particular advertising scenario is not considered in any of the existing media planning optimization models. Here the critical decisions are the identification of the preferred media for mass and segment specific advertising, allocating budget to different media and computing the optimal insertions in selected media.

Media selection is usually done based on past experience, competitors advertising strategies, primary or secondary market research. In this paper, we propose a multi-objective linear integer advertisement planning optimization model with an aim to find the number of advertisements to be given in different mass media and segments specific media within the total budget available. The objectives correspond to maximizing total reach in each segment obtained from segment specific media and the spectrum effect of the mass media. Lower and upper bound constraints on the number of insertions in the media are also considered in the formulation. Weighted sum linear goal programming method is discussed to solve the problem and a case study is presented to illustrate the application of the proposed model.

3. Model development. The multi-objective optimization model formulated in this section can be used to develop a media plan for a firm, which advertises a product in a segmented market, maximizing the reach in each segment within the allowable budget. The media plan is determined for both mass and segment specific media vehicles. Where the mass media reaches all segments with spectrum effect and segment specific media vehicles targets the segments population selectively.

Assume the segments in the market are indexed from \( i = 1, 2, ..., N \) and index \( i = 0 \) is used to denote the mass market (e.g. in the following case study we consider different geographical regions as different segments and the country comprising them as the mass market). The number of media vehicles in the \( i^{th} \) segment is denoted
by \( M_i (i = 0, 1, ..., N) \). For each media there are a number of media options (such as in case of television some television channels are selected) represented by \( K_{ij} \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i \). Each media option may also have many sub options/slots i.e. advertisements can be given in front page (FP) or other pages (OP) of a newspaper. Similarly considering a television channel, advertisement may run in prime time (PT) as well as in other time (OT). The number of slots in \( i^{th} \) segment for \( j^{th} \) media vehicle is represented by \( L_{ij} \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i \).

The objective function to maximize the reach obtained from advertising the product in each segment through mass and differentiated advertising strategies is formulated as

\[
\text{Vector Maximize } Z = [Z_1, Z_2, ..., Z_{N}]^T
\]  

where \( Z_i = \sum_{j=1}^{M_i} \sum_{k=1}^{L_{ij}} a_{ijkl} x_{ijkl} + \sum_{j=1}^{M_i} \sum_{k=1}^{L_{ij}} \alpha_{ijk} (a_{ijkl} x_{ijkl}) \) \hspace{1cm} (2)

and \( a_{ijkl} = \left( \sum_{q=1}^{Q} w_{qj} p_{qijkl} \right) c_{ijkl} \) \hspace{1cm} (3)

In Equation 1, \( Z \) is a \( N \)-dimensional criterion vector, where each component \( i(Z_i) \), represents the advertising reach of a product in \( i^{th} \) segment \((i = 1, ..., N)\). \( Z_i \) expressed as Equation 2 is the sum of reach obtained from segment specific and the spectrum effect of the mass advertising. Where \( \alpha_{ijk} \forall i = 1, 2, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., L_{ij}; 0 < \alpha_{ijk} < 1 \) is the coefficient of spectrum effect of \( k^{th} \) option of \( j^{th} \) mass advertisement media on \( i^{th} \) segment and \( x_{ijkl} \) is the decision variable denoting the number of insertions in \( jkl^{th} \) media driver for \( i^{th} \) segment \( \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., L_{ij}; l = 1, 2, ..., L_{ij} \). In order to maintain the readability we use the notation \( jkl^{th} \) media driver to represent \( j^{th} \) media, \( k^{th} \) media option, and \( l^{th} \) slot.

The population of the potential customers for the product is characterized by their profile characteristics (say \( Q \)). For example the potential customer of a product can be females in a particular age group with a specific income level. In this case the potential customers are defined by three characteristics gender, age and income level. The audience membership (say \( C_{ijkl} \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij} \) of a specific media driver in any segment has only a proportion of the members under any identified profile characteristics (say \( p_{ijkl} \forall i = 0, 1, ..., N; q = 1, 2, ..., Q; j = 1, 2, ..., M_i; k = 1, 2, ..., L_{ij} \)).

The reach obtained from a unit advertisement (Equation 3) in the \( jkl^{th} \) media driver in segment \( i \), that is, \( a_{ijkl} \) is calculated as the product of the audience membership and the weighted sum of the profile proportions \( p_{ijkl} \) according to their relative importance. Weights are denoted as \( w_{qj} \forall q = 1, 2, ..., Q; j = 1, 2, ..., M_i \).

The model is constrained on the advertising budget availability, bounds on the frequency of advertisement and non-negativity restrictions. The budgetary constraint is given by Equation 4 below which ensures that the total budget utilized on advertisements in various mass and differentiated media doesnt exceed the company’s total advertising budget (A).

\[
\sum_{i=0}^{N} \sum_{j=1}^{M_i} \sum_{k=1}^{L_{ij}} \sum_{l=1}^{L_{ij}} c_{ijkl} x_{ijkl} \leq A
\]  

\hspace{1cm} (4)
where $c_{ijkl}$ $\forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$ is the cost of inserting one advertisement in $jkl^{th}$ media driver in $i^{th}$ segment.

Constraint 5 given below ensures that a minimum pre-specified proportion $r$ of the total budget is assigned for mass advertising.

$$\sum_{j=1}^{M_i} \sum_{k=1}^{K_{ij}} \sum_{l=1}^{L_{ij}} c_{ijkl}x_{ijkl} \geq rA$$

This constraint may or may not be imposed on the formulation. If this constraint is removed the solution obtained may be such that it allocates zero or very less budget to mass advertising if the reach obtained from segment specific advertising is higher than that of the mass advertising. Such a situation may not be acceptable to the management as they may want to invest some budget to mass advertising due to its importance in influencing the market share, brand advertising etc.

Constraint 6 and Constraint 7 below are the bounds on the number of insertions in different media slots.

$$x_{ijkl} \geq v_{ijkl} \quad \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$$

$$x_{ijkl} \leq u_{ijkl} \quad \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$$

where $v_{ijkl}$ and $u_{ijkl}$ $\forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$ are the lower bound and upper bound on the number of advertisements in $jkl^{th}$ media driver in the $i^{th}$ segment; these bounds are determined by the advertisers either based on the past advertising experience or based on media type. For example if the slot is front page of a daily newspaper then one may impose an upper bound = 30 in a planning horizon of one month, such that not more than one advertisement appear on any day.

Constraint 8 imposes the non-negative integer value restriction on the decision variables.

$$x_{ijkl} \geq 0 \ & \text{ & integers } \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$$

Combining the objectives and the constraints above the proposed mathematical programming problem can be defined as follows:

Vector Maximize $Z = [Z_1, Z_2, ..., Z_N]^T$ subject to

$$\sum_{i=0}^{N} \sum_{j=1}^{M_i} \sum_{k=1}^{K_{ij}} \sum_{l=1}^{L_{ij}} c_{ijkl}x_{ijkl} \leq A$$

$$\sum_{j=1}^{M_i} \sum_{k=1}^{K_{ij}} \sum_{l=1}^{L_{ij}} c_{ijkl}x_{ijkl} \geq rA$$

$$\sum_{j=1}^{M_i} \sum_{k=1}^{K_{ij}} \sum_{l=1}^{L_{ij}} c_{ijkl}x_{ijkl} \geq rA$$

$$x_{ijkl} \geq v_{ijkl} \quad \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$$

$$x_{ijkl} \leq u_{ijkl} \quad \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$$

$$x_{ijkl} \geq 0 \ & \text{ & integers } \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij}$$

4. Solution methodology. The proposed advertisement planning model for a segmented market can be solved by one of the following approaches:

Case I. Relative importance of each objective (segmented reach in this case) is given by the decision maker: Such class of problems can be solved
through weighted sum approach where weights are assigned according to the relative importance of different segments.

**Case II. Priority of each objective is given by the decision maker:** Such class of problems can be solved through lexicographic approach by assigning priorities to the different objectives, i.e. prioritizing the goal for more preferable media.

**Case III. Neither relative importance nor priorities of the objectives is given by the decision maker:** In this case the problem is solved through interactive approach where decision maker is informed about all the possible trade-off between objectives, and then he chooses the best suited solution.

For case I and II, the proposed model is a multi-objective linear integer programming model. Optimization softwares like LINGO, LINDO, MATLAB etc. have been used successfully in the literature to solve these class of models. However for case III, the weights or priorities are determined first and then the problem can be solved in a similar way.

Here we discuss an interactive approach (Steuer [37]) to compute weights of the different objectives and solve the model. The interactive approach determines the weights using trade-off approach. One might choose to tradeoff the solution by solving P1 or might set aspirations for the reach of each of the segments to be attained such that no segment dies out with low/zero level advertisement. For determining the aspirations for reach in each segment optimization model P1 can be solved as single objective for every segment under the model constraints, which determines the maximum achievable reach for each segment. Once we solve and fix the goal or aspirations \( Z_i^* \) on reach to be obtained from \( i^{th} \) segment, the resulting problem can be formulated as follows

Vector Maximize \( Z = [Z_1, Z_2, ..., Z_N]^T \)

subject to

constraints (4)-(8) and

\( Z_i \geq Z_i^* \quad \forall i = 1, ..., N \) (P2)

Solving P1 with a single objective for every segment sets a high goal for reach of each segment. High aspirations on segments reach make the optimization model P2 infeasible due to the limitation on the budget. The problem P2 is solved to obtain compromised solution using interactive weighted sum goal programming approach (Ignizio [23]). The two stage goal programming formulation of the optimization model P2 can be formulated using Appendix A as follows.

Vector Minimize \( g(\eta, \rho, X) = [g_1(\eta, \rho, X), g_2(\eta, \rho, X), ..., g_N(\eta, \rho, X)]^T \)

subject to

\[
\begin{align*}
& M_0 \sum_{j=1}^K \sum_{k=1}^L c_{ijk} x_{ijk} + \eta_{N+1} - \rho_{N+1} = rA \quad \text{(9)} \\
& N \sum_{i=0}^N \sum_{j=1}^{M_i} \sum_{k=1}^{K_{ij}} c_{ijkl} x_{ijkl} + \eta_{N+2} - \rho_{N+2} = A \\
& x_{ijkl} + \eta_{ijlk} - \rho_{ijlk} = v_{ijkl} \quad \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij} \\
& x_{ijkl} + \eta_{ijlk} - \rho_{ijlk} = u_{ijkl} \quad \forall i = 0, 1, ..., N; j = 1, 2, ..., M_i; k = 1, 2, ..., K_{ij}; l = 1, 2, ..., L_{ij} \\
& Z_i + \eta_i - \rho_i = Z_i^* \quad \forall i = 1, ..., N
\end{align*}
\]
\( x_{ijkl} \geq 0 \) & integers \( \forall i = 0, 1, \ldots, N; j = 1, 2, \ldots, M; k = 1, 2, \ldots, K_{ij}; l = 1, 2, \ldots, L_{ij} \) (14)

\( \eta_{ijkl}, \rho_{ijkl}, \eta'_{ijkl}, \rho'_{ijkl} \geq 0 \) \( \forall i = 0, 1, \ldots, N; j = 1, 2, \ldots, M; k = 1, 2, \ldots, K_{ij}; l = 1, 2, \ldots, L_{ij} \) (15)

\( \eta_{N+1}, \rho_{N+1}, \eta_{N+2}, \rho_{N+2} \geq 0 \) (16)

\( \eta_i, \rho_i \geq 0 \) \( \forall i = 0, 1, \ldots, N; j = 1, 2, \ldots, M; k = 1, 2, \ldots, K_{ij}; l = 1, 2, \ldots, L_{ij} \) (17)

where \( \eta \) and \( \rho \)'s are the negative and positive deviation variable of the goals for their respective objective/constraint function.

The problem P3 is solved in two stages. In Stage 1, the rigid constraints are satisfied first by weighting them equally and minimizing the deviational variables corresponding to them and in Stage 2, the optimal solution of the Stage 1 problem is incorporated and the objective is expressed as the weighted sum of the deviational variables to be minimized corresponding to the objective functions aspirations. The weight for each segments reach objective is either given by the decision maker (DM) or is generated through interactive approach where set of weighing vectors are presented to the DM. DM analyzes the solutions based on their expectations, interpretation, implementation outcomes and discussion with the team members. Either one of the presented solutions is accepted for implementation or the preferred solution is selected and the analyst is suggested to further improve the solution by reducing the search space according to the selected vector.

**Stage 1 of the goal programming**

\[
\begin{align*}
\text{Minimize} & \quad \eta_{N+1} + \rho_{N+2} + \sum_{i=0}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K_{ij}} \sum_{l=1}^{L_{ij}} \eta_{ijkl} + \sum_{i=0}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K_{ij}} \sum_{l=1}^{L_{ij}} \rho'_{ijkl} \\
\text{subject to} & \quad \text{constraints (9)-(12), (14)-(16).}
\end{align*}
\]

(P4)

Let \((\eta^0, \rho^0, X^0)\) be the optimal solution of problem P4, and \(g_0(\eta^0, \rho^0, X^0)\) be the corresponding objective function value. The stage 2 formulation with weights (as determined using interactive procedure discussed below) assigned to the reach objectives is formulated as follows.

**Stage 2 of the goal programming**

\[
\begin{align*}
\text{Minimize} & \quad \lambda^T g(\eta, \rho, X) = \sum_{i=1}^{N} \lambda_i \eta_i \\
\text{subject to} & \quad \text{constraints (9) to (17) and} \\
& \quad g_0(\eta, \rho, X) = g_0(\eta^0, \rho^0, X^0)
\end{align*}
\]

(P5)

where \(\lambda_i\) are the weights assigned to \(i^{th}\) segment, \(\sum_{i=1}^{N} \lambda_i = 1\). Solving P5 we obtain an efficient solution of model P2 (Lemma 1, Appendix B). A suitable tradeoff amongst the goals (reach targets) for different segments can be obtained as per the satisfaction level of the management.

**Determination of the weights for the problem P5: An interactive approach algorithm (Steuer [37])**

**Step 1.** Specify \(V\), the number of weighting vectors to be generated; \(P\), the sample size of the number of criterion vectors to be presented to the decision maker (\(P\) lies between \(k\) to \(2k+1\)); \(t\), the number of iterations (\(t \leq k\),...
the final $[\lambda_g, \bar{\lambda}_g]$ interval width $w$ and the $r$, the $\Lambda$ reduction factor $r$.

$$\frac{1}{2k} \leq w \leq \frac{3}{2k}$$

$$\frac{1}{\sqrt{1/P}} \leq r \leq \sqrt{\frac{w}{t}}$$

**Step 2.** Normalize the objective functions.

**Step 3.** Set $h = 0$ and $[\lambda_0^g, \bar{\lambda}_0^g] = [0, 1]$ for $g = 1, 2, ..., k$ where $h$ is the iteration count, $k$ is the number of components of a criterion vector and $\lambda_h^g$ and $\bar{\lambda}_h^g$ are the lower and upper limit of the weight of the $g^{th}$ component of criterion vector at $h^{th}$ iteration.

**Step 4.** $h = h + 1$, Form weighting vector space as

$$\Lambda^h = \left\{ \lambda \in \mathbb{R}^k; \lambda_g \in [\lambda_h^g, \bar{\lambda}_h^g], \lambda_g \geq 0, g = 1, 2, ..., k, \sum_{g=1}^{k} \lambda_g = 1 \right\}$$

**Step 5.** Randomly generate $V$ weighting vectors from $\Lambda^h$. $V$ usually vary from 10$k$ to 50$k$ as per the requirement of the problem or as suggested by the decision maker.

**Step 6.** Filter the $V$ weighting vectors generated in step 5 to select $W$ most distinct weighting vectors using forward filtering approach as given in filtering algorithm below. Here the value of $W$ lies between 2$P$ to 3$P$.

**Step 7.** Solve the $W$ weighted sum problems using $W$ most distinct weighting vectors obtained in step 6 to obtain $W$ non dominated criterion vectors.

**Step 8.** Filter $W$ criterion vector to obtain $P$ most distinct criterion vectors. Present these $P$ solutions and corresponding criterion vectors to the decision maker (DM). Obtain DMs most preferred solution $X^{(h)}$ and its corresponding criterion vectors $Z^{(h)}$. If the decision maker is satisfied with the solution or the prefixed number of iteration is exhausted go to step 10 else go to step 9.

**Step 9.** Generate new weighting vector space using reduction factor $'r'$ as follows:

$$\Lambda^{h+1} = \left\{ \lambda \in \mathbb{R}^k; \lambda_g \in [\lambda_h^{g+1}, \bar{\lambda}_h^{g+1}], \sum_{g=1}^{k} \lambda_g = 1 \right\}$$

$$[\lambda_g^{h+1}, \bar{\lambda}_g^{h+1}] = \begin{cases} [0, r^h]; & \text{if } \lambda_h^g - \frac{r^h}{T} \leq 0 \\ [1 - r^h, 1]; & \text{if } \lambda_h^g + \frac{r^h}{T} \geq 1 \\ [\lambda_h^g + \frac{r^h}{T}, \bar{\lambda}_h^g + \frac{r^h}{T}]; & \text{otherwise} \end{cases}$$

Where $r^h$ is the reduction factor $r$ raise to the $h^{th}$ power. Go to step 4.

**Step 10.** Stop the iteration with $(Z^{(h)}, X^{(h)})$ as the final solution.

**Filtering algorithm (Steuer [37])**

**Step 1.** Set $e$ the metric parameter of weighted $L_e$ distances between vectors to be filtered, $e \in \{1, 2, ..., \infty\}$.

**Step 2.** Calculate $R_g$ and $\pi_g$:

$$R_g = \max_{v \in V} \{v_g\} - \min_{v \in V} \{v_g\} \text{ and } \pi_g = \frac{1}{R_g} \left( \sum_{g=1}^{S} \frac{1}{R_g} \right)^{-1}$$
Where, $R_g$ be the range of the $g^{th}$ component of the vectors in $V$ and $\pi_g$ the range equalization weight associated with the $g^{th}$ component of the vectors being filtered.

**Step 3.** Compute initial $d$ and initial $\Delta d$ as

$$d = \left[ \sum_{g=1}^{s} (\pi_g R_g)^{\epsilon} \right]^{1/e} / 4; \Delta d = \frac{d}{2}$$

**Step 4.** Select the first vector of the randomly generated vector list as the forward seed point that becomes first vector retained by the filter.

**Step 5.** Let $t$ is the identification superscript of a vector not currently retained by the filter and $h$ is the identification superscript of a vector retained by the filter. Then $h = 1$ (as first vector is chosen as forward seed point and is retained) and $t = t + 1$.

**Step 6.** To determine remaining $(W - 1)$ filtered vectors, calculate

$$L^t_e = \left[ \sum_{g=1}^{s} (\pi_g | v^t_g - v^h_g |)^{\epsilon} \right]^{1/e}$$

And check the nature of $(L^t_e - d)$.

$$(L^t_e - d) = \begin{cases} > 0, & \text{vector } t \text{ is retained by the filter, } h = \{h\} \cup t \text{ go to step 7} \\ < 0, & \text{vector } t \text{ is not retained by the filter} \end{cases}$$

**Step 7.** Now $t = t + 1$ and resume processing to calculate $L^t_e$ for each $h$.

$$(L^{t+1}_e - d) = \begin{cases} > 0, & \text{vector } t \text{ is retained by the filter, } h = \{h\} \cup t \text{ go to step 6} \\ < 0, & \text{go to step 8} \end{cases}$$

**Step 8.** Calculate $N(h)$. If $N(h) < W$, $d = d - \frac{\Delta d}{2}$; and if $N(h) > W$, $d = d + \frac{\Delta d}{2}$; and if $N(h) = W$ then stop.

5. **Case study.** Application of the proposed model is presented in this section. The data is obtained from a firm which manufactures and markets a product in a womens shampoo category in India (the data is provided with suitable transformation to maintain the confidentiality). Using this data and applying the proposed model, an advertisement plan is developed for a time horizon of one week. The product is targeted to females with age group 12 and above, belonging to the lower and upper middle income classes. The company is looking for the combination of media vehicles for rationalizing its advertising expenditure in India. The objective is to generate the maximum reach from the advertising campaign so that the message can reach the potential target market.

The potential market for the product is geographically segmented considering the regional diversity in the country. The company advertises the product using national media which reaches the whole country with spectrum effect varying from region to region. Along with mass advertising, the firm targets the regional segments with regional media. Data for fourteen regional segments is provided by the firm and an optimal media mix for these fourteen regions is to be planned. In each region, two regional newspapers (RNP1, RNP2) and three regional television channels (RCH1, RCH2, RCH3) are chosen based on past experience, popularity and the number of audiences. On national level two national newspapers (NNP1, NNP2) and two national television channels (NCH1, NCH2) are chosen on the same
Table 1. Audience membership in '0000 in newspapers and television channels

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | PT   | OT   | PT   | OT   | PT   | OT   |
| S1       | 190  | 185  | 204  | 147  | 188  | 136  |
| S2       | 90   | 85   | 140  | 110  | 133  | 97   |
| S3       | 135  | 125  | 168  | 132  | 177  | 128  |
| S4       | 151  | 107  | 197  | 143  | 186  | 134  |
| S5       | 36   | 18   | 67   | 48   | 63   | 46   |
| S6       | 87   | 75   | 140  | 102  | 140  | 102  |
| S7       | 52   | 45   | 90   | 54   | 81   | 59   |
| S8       | 105  | 84   | 168  | 123  | 162  | 118  |
| S9       | 42   | 35   | 72   | 52   | 70   | 50   |
| S10      | 52   | 45   | 88   | 63   | 86   | 63   |
| S11      | 62   | 59   | 98   | 70   | 93   | 68   |
| S12      | 127  | 110  | 186  | 134  | 180  | 130  |
| S13      | 35   | 28   | 60   | 43   | 52   | 38   |
| S14      | 110  | 91   | 171  | 124  | 168  | 122  |

| Mass Media | NNP1 | NNP2 | NCH1 | NCH2 |
|-------------|------|------|------|------|
|             | PT   | OT   | PT   | OT   |
|             | 355  | 272  | 650  | 470  |

The impact of advertisement and cost varies in different slots of newspapers and television, therefore different possibilities of advertisement on front page (FP), last page and other pages (OP) for newspapers and similarly, two different time slots (prime time (PT) and other time (OT)) for television has been taken into consideration.

The total advertising effort available for the planning horizon is 30 millions (rupees). To decide the number of advertisements to be given in different media categories, data related to the circulation of the media vehicles and the cost of placing single advertisement in it in various slots is collected from secondary sources (shown in Table 1 and Table 2). A consumer behaviour analysis is also carried out based on a random sample of 200 in each region and 1000 at national level to analyze the percentage of people who fulfill the criteria of being the potential customers of the product and read/view the media slots as shown in Table 3 and 4. The upper and lower bounds on the number of insertions in each media slots are provided by the firm as given in Table 5. The data for spectrum effect of the national media options on different regions is given in Table 6 (provided by the firm as collected from secondary sources). The advertiser also proposed to allocate minimum 35% of the available budget to national media.

Optimal Media mix for the situation discussed above can be obtained using the proposed optimization model. Since weights or priorities (relative importance) for the reach to be obtained from the different regions are not given we use interactive weighted sum goal programming approach to solve the model. First we determine the aspirations on reach by coding the model P1 as single objective problem for each segment using the data given above. To get the segments reach aspirations, only 50% of the available budget is used. As by using total budget, very high aspirations
Table 2. Ad cost per 100 column cm ad for newspapers and per 10 sec spot for TV '00

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | FP   | OP   | PT   | OT   | PT   | OT   | PT   | OT   |
| S1       | 850  | 650  | 750  | 650  | 600  | 370  | 500  | 270  | 550  | 320  |
| S2       | 460  | 310  | 450  | 300  | 390  | 240  | 310  | 170  | 350  | 190  |
| S3       | 560  | 350  | 510  | 350  | 400  | 250  | 350  | 190  | 310  | 220  |
| S4       | 580  | 380  | 550  | 350  | 500  | 310  | 470  | 250  | 450  | 280  |
| S5       | 280  | 185  | 210  | 160  | 250  | 160  | 230  | 130  | 200  | 130  |
| S6       | 420  | 290  | 405  | 280  | 370  | 230  | 330  | 180  | 380  | 210  |
| S7       | 300  | 250  | 280  | 220  | 330  | 210  | 310  | 170  | 290  | 170  |
| S8       | 500  | 325  | 480  | 300  | 390  | 240  | 370  | 200  | 330  | 190  |
| S9       | 350  | 200  | 320  | 210  | 310  | 190  | 300  | 160  | 290  | 180  |
| S10      | 310  | 200  | 290  | 195  | 320  | 200  | 310  | 170  | 340  | 150  |
| S11      | 320  | 250  | 300  | 240  | 340  | 210  | 320  | 170  | 280  | 190  |
| S12      | 530  | 300  | 500  | 280  | 400  | 250  | 360  | 200  | 320  | 210  |
| S13      | 300  | 160  | 270  | 160  | 240  | 150  | 200  | 110  | 180  | 110  |
| S14      | 460  | 330  | 410  | 310  | 420  | 260  | 340  | 190  | 380  | 210  |

| Mass Media | NNP1 | NNP2 | NCH1 | NCH2 |
|------------|------|------|------|------|
|            | FP   | OP   | FP   | OP   |
|            | 2500 | 1700 | 2100 | 1500 | 1500 | 900  | 1200 | 850  |

Table 3. Customer percentage profile matrix for newspapers

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | age  | income | sex  | age  | income | sex  |
| S1       | 0.35 | 0.38 | 0.31 | 0.27 | 0.33 | 0.17 |
| S2       | 0.4  | 0.35 | 0.3  | 0.35 | 0.2  | 0.2  |
| S3       | 0.21 | 0.25 | 0.29 | 0.57 | 0.5  | 0.43 |
| S4       | 0.33 | 0.25 | 0.28 | 0.35 | 0.3  | 0.3  |
| S5       | 0.17 | 0.27 | 0.2  | 0.27 | 0.33 | 0.4  |
| S6       | 0.36 | 0.28 | 0.28 | 0.23 | 0.28 | 0.2  |
| S7       | 0.3  | 0.34 | 0.26 | 0.32 | 0.4  | 0.38 |
| S8       | 0.31 | 0.42 | 0.31 | 0.31 | 0.46 | 0.23 |
| S9       | 0.3  | 0.2  | 0.26 | 0.42 | 0.4  | 0.38 |
| S10      | 0.29 | 0.21 | 0.27 | 0.36 | 0.4  | 0.43 |
| S11      | 0.31 | 0.36 | 0.29 | 0.25 | 0.31 | 0.2  |
| S12      | 0.37 | 0.27 | 0.38 | 0.32 | 0.3  | 0.2  |
| S13      | 0.37 | 0.27 | 0.33 | 0.3  | 0.4  | 0.27 |
| S14      | 0.25 | 0.21 | 0.27 | 0.33 | 0.29 | 0.21 |

| Mass Media | NNP1 | NNP2 |age  | income | sex  |
|------------|------|------|------|--------|------|
|            | 0.53 | 0.47 | 0.43 | 0.39   | 0.33 |
|            | 0.26 |      |      |        |      |
Table 4. Customer percentage profile matrix for television

| Segment | RCH1 | RCH2 | RCH3 |
|---------|------|------|------|
|         | age  | income | sex | age  | income | sex | age  | income | sex |
| S1      | PT   | OT    | PT   | OT   | PT | OT | PT   | OT | PT   | OT |
| S2      | .5   | .32   | .6   | .24  | .55 | .3 | .57  | .29 | .41  | .29 |
| S3      | .31  | .35   | .43  | .15  | .38  | .28 | .72  | .41  | .25  | .12 |
| S4      | .55  | .28   | .49  | .31  | .47  | .34 | .54  | .28  | .4  | .44 |
| S5      | .37  | .31   | .36  | .21  | .44  | .35 | .35  | .3  | .4  | .46 |
| S6      | .48  | .25   | .75  | .37  | .63  | .38 | .54  | .15  | .28  | .17 |
| S7      | .49  | .31   | .51  | .33  | .49  | .29 | .51  | .32  | .42  | .33 |
| S8      | .32  | .27   | .41  | .21  | .56  | .32 | .69  | .19  | .7  | .42 |
| S9      | .8   | .46   | .49  | .19  | .41  | .21 | .54  | .29  | .54  | .34 |
| S10     | .55  | .35   | .62  | .15  | .49  | .37 | .53  | .38  | .58  | .22 |
| S11     | .65  | .35   | .41  | .25  | .57  | .19 | .49  | .37  | .44  | .31 |
| S12     | .46  | .38   | .39  | .16  | .64  | .32 | .6  | .25  | .43  | .18 |
| S13     | .39  | .29   | .5  | .22  | .56  | .35 | .62  | .44  | .56  | .71 |
| S14     | .44  | .26   | .55  | .21  | .65  | .38 | .51  | .31  | .55  | .34 |
| NPP1    | age  | income | sex | age  | income | sex | age  | income | sex |
|         | PT   | OT    | PT   | OT   | PT | OT | PT   | OT | PT   | OT |

Table 5. Upper and lower bounds on advertisements in different media

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | FP   | OP   | FP   | OP   | PT   | OT | PT | OT | PT | OT |
| S1       | [2.5] | [3.5] | [1.3] | [1.5] | [2.9] | [3.21] | [5.16] | [0.12] | [5.15] | [0.9] |
| S2       | [1.4] | [1.5] | [2.7] | [1.5] | [2.12] | [3.12] | [2.9] | [3.7] | [6.19] | [1.12] |
| S3       | [2.4] | [1.4] | [1.3] | [2.6] | [4.12] | [0.13] | [5.12] | [2.9] | [2.11] | [4.21] |
| S4       | [1.3] | [2.4] | [2.5] | [1.4] | [2.14] | [1.7] | [3.12] | [1.12] | [0.9] | [4.12] |
| S5       | [1.5] | [2.5] | [3.6] | [0.4] | [3.12] | [7.12] | [6.21] | [5.18] | [4.12] | [2.9] |
| S6       | [1.4] | [0.3] | [1.5] | [2.6] | [5.20] | [5.20] | [2.9] | [7.21] | [2.10] | [4.15] |
| S7       | [1.5] | [2.5] | [2.4] | [0.4] | [3.15] | [4.15] | [0.7] | [5.12] | [1.8] | [2.10] |
| S8       | [1.4] | [2.4] | [1.5] | [3.4] | [5.12] | [2.9] | [3.18] | [3.14] | [5.12] | [2.8] |
| S9       | [2.5] | [2.5] | [2.7] | [0.5] | [0.12] | [2.15] | [4.16] | [3.10] | [3.12] | [5.18] |
| S10      | [2.5] | [2.6] | [2.5] | [1.6] | [5.15] | [2.9] | [2.12] | [3.7] | [2.16] | [2.14] |
| S11      | [2.5] | [2.4] | [2.5] | [0.3] | [2.21] | [3.15] | [4.12] | [5.18] | [1.7] | [3.12] |
| S12      | [2.4] | [2.5] | [2.7] | [0.5] | [5.15] | [0.10] | [2.7] | [2.13] | [3.12] | [2.14] |
| S13      | [1.3] | [1.5] | [2.6] | [1.5] | [0.14] | [7.14] | [4.12] | [2.12] | [1.18] | [3.15] |
| S14      | [2.6] | [3.7] | [2.7] | [0.2] | [0.10] | [2.9] | [1.10] | [1.9] | [0.10] | [1.10] |

Table 6. Spectrum effect of national newspapers and TV channels on regions

| Segment | NNP1 | NNP2 | NCH1 | NCH2 | Segment | NNP1 | NNP2 | NCH1 | NCH2 |
|---------|------|------|------|------|---------|------|------|------|------|
| S1      | 0.15 | 0.16 | 0.11 | 0.11 | S8      | 0.08 | 0.08 | 0.09 | 0.09 |
| S2      | 0.07 | 0.08 | 0.08 | 0.08 | S9      | 0.03 | 0.03 | 0.04 | 0.04 |
| S3      | 0.1  | 0.11 | 0.09 | 0.1  | S10     | 0.04 | 0.04 | 0.05 | 0.04 |
| S4      | 0.12 | 0.1  | 0.11 | 0.1  | S11     | 0.05 | 0.05 | 0.05 | 0.05 |
| S5      | 0.03 | 0.02 | 0.04 | 0.04 | S12     | 0.1  | 0.1  | 0.1  | 0.1  |
| S6      | 0.07 | 0.07 | 0.08 | 0.07 | S13     | 0.03 | 0.04 | 0.03 | 0.03 |
| S7      | 0.04 | 0.04 | 0.04 | 0.05 | S14     | 0.09 | 0.08 | 0.09 | 0.1  |
will be obtained because budget is to be distributed among fourteen segments and mass media. The optimal values of reach so obtained are then set as aspiration level to be achieved from each segment. If we attempt to solve P2, using any given set of weights, it becomes infeasible as aspirations are too high and the budget is limited. To get an optimal trade off between budget available and the aspirations that need to be fulfilled, the problem is solved through interactive weighted sum goal programming approach. First, the problem is solved for stage one (problem P3), where the deviations corresponding to rigid constraints are minimized. Stage one solution is incorporated in the stage two of the goal programming and the deviational variables that become zero in stage one are removed. In this stage, the deviations corresponding to goal i.e. the reach aspirations to be achieved in each region is minimized to get a good trade off between their achievements.

The weight corresponding to the fourteen regions reach aspiration goals is generated through interactive approach. The parameters of the interactive approach are defined according to the step 1 of the algorithm discussed in Section 4. In each iteration a sample size of 350 (= V) dispersed weighting vectors are randomly generated. Among these, through forward filtering and using \( L_2 \) metric distances between each set of vector, 28 non-dominated distinct vectors are filtered uniquely with a suitable value of \( d \). The problem P5 is solved 28 times, to obtain 28 distinct solutions and their corresponding objective values using optimization software Lingo 11.0 (Thiriez [38]). Among these 14 (= P) criterion vectors are filtered and presented to the decision maker. The decision maker chooses the most preferred solution among these. Taking the DMs most preferred solution, the value of the reduction factor is calculated and new interval is formed between which new weighting vectors are generated. As iteration count \( t \approx k \) (the number of segment=14) in total 14 iterations can be performed. However in this case, the decision maker accepted the solution obtained in fourth iteration since similar solutions are obtained in iteration 3 and iteration 4 as shown in Table 7.

Last two rows of Table 7 depict the reach and deviations obtained from the selected vector. Reach improves as we move from iteration 1 to iteration 4 and deviations from aspired reach decreases. The solution of iteration 3 and 4 are very close, so we terminate the algorithm in four iterations and choose the solution of fourth iteration for implementation. Early convergence is obtained as the number of components in weighting vector is more. The solution obtained for the optimal number of insertions in different mass and segment specific media is given in Table 8.

Column 1 and 2 of Table 9 shows the values of the reach aspired and that achieved in all segments from the solution of fourth iteration. It can be noted from column one of Table 9 that the aspiration set on reach is achieved over and above in segment 2, 4, and 5. However it is compromised in other segments. The total budget utilised is Rs.3,00,00,000 which is equal to the available budget. The budget utilized in mass media is Rs. 1,06,10,000 (35.37% of total budget) which is slightly greater than 35% of the total budget available.

Sensitivity analysis
Sensitivity analysis is an evaluation method for the analysis of the effect of parameter changes on the final solution. It provides managerial insights into how the satisfied solution is affected by changes in the input data. Along with this merit, management can make a reliable decision in many business situations. We demonstrate here sensitivity analysis on the lower bound set on the proportion of budget
Table 7. Iteration parameters and the solution obtained

|          | Iteration 1 | Iteration 2 | Iteration 3 | Iteration 4 |
|----------|-------------|-------------|-------------|-------------|
| \( V \)  | 350         | 350         | 350         | 350         |
| \( W \)  | 28          | 28          | 28          | 28          |
| \( P \)  | 14          | 14          | 14          | 14          |
| Interval width | [0, 1]     | [0, 0.835]  | [0, 0.6975] | [0, 0.4865] |
| \( d \)  | 0.0145      | 0.014       | 0.0138      | 0.0137      |
| Reduction factor | 0.835      | 0.6975      | 0.4865      | 0.2367      |


Table 8. Solution that gives optimal number of advertisements in different media

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | FP   | OP   | PT   | OT   | PT   | OT   | PT   | OT   | PT   | OT   |
| Segment 1 | 2    | 3    | 1    | 1    | 2    | 3    | 3    | 0    | 5    | 0    |
| Segment 2 | 1    | 1    | 2    | 1    | 2    | 3    | 4    | 9    | 7    | 19   |
| Segment 3 | 2    | 1    | 1    | 6    | 12   | 12   | 12   | 9    | 11   | 4    |
| Segment 4 | 1    | 2    | 2    | 1    | 2    | 3    | 4    | 12   | 9    | 12   |
| Segment 5 | 1    | 2    | 3    | 0    | 7    | 7    | 12   | 18   | 12   | 2    |
| Segment 6 | 1    | 0    | 1    | 2    | 20   | 20   | 8    | 7    | 10   | 5    |
| Segment 7 | 1    | 2    | 0    | 3    | 4    | 0    | 5    | 1    | 2    |      |
| Segment 8 | 1    | 2    | 1    | 3    | 10   | 2    | 13   | 12   | 2    |      |
| Segment 9 | 2    | 2    | 0    | 2    | 3    | 4    | 3    | 3    | 5    |      |
| Segment 10 | 2    | 2    | 2    | 1    | 2    | 3    | 3    | 2    | 3    |      |
| Segment 11 | 2    | 2    | 2    | 1    | 2    | 3    | 4    | 5    | 1    | 3    |
| Segment 12 | 2    | 2    | 2    | 1    | 2    | 3    | 4    | 5    | 1    | 3    |
| Segment 13 | 2    | 2    | 2    | 1    | 2    | 3    | 4    | 5    | 1    | 3    |
| Segment 14 | 2    | 3    | 2    | 1    | 2    | 3    | 4    | 5    | 1    | 3    |

Table 9. Solution that gives optimal number of advertisements in different media

| Segments | NNP1 | NNP2 | NCH1 | NCH2 |
|----------|------|------|------|------|
|          | FP   | OP   | FP   | OP   |
| Segment 1 | 2    | 3    | 2    | 1    |
| Segment 2 | 3    | 1    | 2    | 1    |
| Segment 3 | 1    | 2    | 3    | 0    |
| Segment 4 | 2    | 3    | 4    | 0    |
| Segment 5 | 3    | 4    | 5    | 1    |
| Segment 6 | 4    | 5    | 6    | 2    |
| Segment 7 | 5    | 6    | 7    | 3    |
| Segment 8 | 6    | 7    | 8    | 4    |
| Segment 9 | 7    | 8    | 9    | 5    |
| Segment 10 | 8    | 9    | 10   | 6    |
| Segment 11 | 9    | 10   | 11   | 7    |
| Segment 12 | 10   | 11   | 12   | 8    |
| Segment 13 | 11   | 12   | 13   | 9    |
| Segment 14 | 12   | 13   | 14   | 10   |

assigned for mass advertisement. The proportion is first increased to 40% and then decreased to 30%. The results obtained are presented in column 4 and 5 of Table 9 and in Table 10 and 11 given in Appendix C. It can be noted that as the proportion
Table 9. Reach objective aspired and achieved in segments

| Segment | Aspired Reach | Achieved Reach |
|---------|---------------|----------------|
|         | r = 35%       | r = 30%        | r = 40%        |
| S1      | 72514011      | 41821577       | 43585430       |
| S2      | 43287916      | 43308992       | 43247898       |
| S3      | 67725930      | 67412718       | 61223976       |
| S4      | 62164271      | 62174154       | 62199559       |
| S5      | 24814103      | 24814156       | 24746053       |
| S6      | 55694661      | 55651050       | 55707692       |
| S7      | 26953539      | 14251871       | 14722231       |
| S8      | 60188559      | 60159784       | 60159137       |
| S9      | 29707371      | 13773264       | 14126034       |
| S10     | 34970962      | 16719852       | 17190213       |
| S11     | 35740679      | 17367633       | 17955584       |
| S12     | 63485298      | 58635368       | 54292650       |
| S13     | 22332099      | 14332364       | 10533902       |
| S14     | 54961389      | 54646836       | 52877974       |
| Total   | 654540788     | 545069620      | 532575806      |

Deviations

| Deviations | 3587274 | 3468983 | 4306395 |

of the budget allocated to the national media increases the total reach obtained from all the segments collectively decreases and the deviation objective increases. Thus, the decision maker can choose the appropriate proportion he wants to set for advertisement in national media taking into consideration of these perspectives. Similar analysis can be performed by changing lower and upper bounds on the number of insertions. For sensitivity analysis, the selected weighting vector is used to solve the problem. In general whenever manager uses any optimization model for decision making (Little [27]) they test the model under changed conditions and input and closely observe the solutions obtained, compare with past experiences and expectations. Both iterative algorithm and sensitivity analysis turns out useful in this case as multiple solution runs with different parameters can be obtained.

The proposed model is easy to understand, model structure is simple, equations are linear, several optimization softwares are available which can be used to solve the discussed method, and can be tested with alternative solution methods. In addition it is also adaptive as new constraints can easily be added, for example one may like to impose a restriction that some minimum proportion of the budget should be allocated to certain segment specific media, or like to remove lower or upper bounds on some media. Simple model structure and availability of software support for solution also facilitate decision makers in quickly and easily adjusting the inputs.

6. Conclusion. This paper discusses a media planning model to determine the optimal media mix with an objective to maximize the overall reach. The potential market is categorized into a number of segments. The advertisements are targeted to the overall market with mass advertising as well as the segments with segment specific advertising. The reach of advertisements in each segment is measured individually together with the spectrum effect of mass advertisements. An optimization model is developed and validated through a real life case. The problem is solved
using interactive weighted sum goal programming approach. The study offers a wide scope of future research such as determining the media mix for multiple planning periods and considering the carry over effect, considering multiple products and interactions between the products. Similarly the problem may be considered for fuzzy environment. Further, different solution methods can be explored and compared.

Appendix A

Let $f(X)$ and $b$ be the function and its goal respectively and $\eta$ and $\rho$ be the over and under achievement (negative and positive deviational) variables then the choice of deviational variable in the goal objective functions which has to be minimized depend upon the following rule:

if $f(X) \leq b$, $\rho$ is minimized under the constraints $f(X) + \eta - \rho = b$  
if $f(X) \geq b$, $\eta$ is minimized under the constraints $f(X) + \eta - \rho = b$ and  
if $f(X) = b$, $\eta + \rho$ is minimized under the constraints $f(X) + \eta - \rho = b$  

Appendix B

Definition 1. (Geoffrion [18]) A solution $x^0$ is said to be efficient solution if $x^0 \in S$ and there exist no other feasible point $x$ such that $F(x) \geq F(x^0)$ and $F(x) \neq F(x^0)$.

Definition 2. (Geoffrion [18]) A solution $x^0$ is said to be a properly efficient solution if it is efficient and if there exist a scalar $M > 0$ such that, for each $i$, we have $\frac{f_i(x) - f_i(x^0)}{f_j(x^0) - f_j(x)} \leq M$. For some $j$ such that $f_j(x) < f_j(x^0)$ whenever $x \in X$ and $f_i(x) > f_i(x^0)$.

Lemma 1. Through definition 1 and 2, optimal solution of the problem $P5$ will be properly efficient solution to the problem $P3$.

Appendix C.
### Table 10. Solution that gives optimal number of advertisements in different media (r=30%)

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | FP   | OP   | FP   | OP   | PT   | OT   | PT   | OT   |
| Segment 1| 2    | 3    | 1    | 1    | 2    | 3    | 5    | 0    |
| Segment 2| 1    | 1    | 2    | 1    | 12   | 6    | 9    | 7    |
| Segment 3| 2    | 1    | 1    | 1    | 5    | 12   | 15   | 12   |
| Segment 4| 1    | 2    | 2    | 1    | 14   | 2    | 4    | 12   |
| Segment 5| 1    | 2    | 3    | 0    | 8    | 7    | 21   | 17   |
| Segment 6| 1    | 0    | 1    | 2    | 20   | 20   | 9    | 7    |
| Segment 7| 1    | 2    | 2    | 0    | 4    | 4    | 0    | 5    |
| Segment 8| 1    | 2    | 1    | 3    | 11   | 2    | 18   | 12   |
| Segment 9| 2    | 2    | 2    | 0    | 2    | 4    | 3    | 3    |
| Segment 10| 2    | 2    | 2    | 1    | 5    | 2    | 2    | 2    |
| Segment 11| 2    | 2    | 2    | 0    | 2    | 4    | 5    | 1    |
| Segment 12| 2    | 2    | 2    | 0    | 15   | 0    | 7    | 2    |
| Segment 13| 1    | 1    | 2    | 1    | 0    | 7    | 12   | 18   |
| Segment 14| 2    | 3    | 2    | 0    | 10   | 2    | 10   | 8    |

| Mass Media | NNP1 | NNP2 | NCH1 | NCH2 |
|------------|------|------|------|------|
|            | FP   | OP   | FP   | OP   | PT   | OT   | PT   | OT   |
|            | 5    | 7    | 3    | 1    | 21   | 18   | 21   | 18   |

### Table 11. Solution that gives optimal number of advertisements in different media (r=40%)

| Segments | RNP1 | RNP2 | RCH1 | RCH2 | RCH3 |
|----------|------|------|------|------|------|
|          | FP   | OP   | FP   | OP   | PT   | OT   | PT   | OT   |
| Segment 1| 2    | 3    | 1    | 1    | 2    | 3    | 5    | 0    |
| Segment 2| 1    | 1    | 2    | 1    | 12   | 3    | 9    | 6    |
| Segment 3| 2    | 1    | 1    | 2    | 12   | 0    | 12   | 9    |
| Segment 4| 1    | 2    | 2    | 1    | 14   | 1    | 4    | 12   |
| Segment 5| 1    | 2    | 3    | 0    | 6    | 7    | 21   | 17   |
| Segment 6| 1    | 0    | 1    | 2    | 20   | 17   | 9    | 7    |
| Segment 7| 1    | 2    | 2    | 0    | 3    | 4    | 0    | 5    |
| Segment 8| 1    | 2    | 1    | 3    | 9    | 2    | 18   | 14   |
| Segment 9| 2    | 2    | 2    | 0    | 0    | 2    | 4    | 3    |
| Segment 10| 2    | 2    | 2    | 1    | 5    | 2    | 2    | 2    |
| Segment 11| 2    | 2    | 2    | 0    | 2    | 3    | 4    | 5    |
| Segment 12| 2    | 2    | 2    | 0    | 9    | 0    | 7    | 2    |
| Segment 13| 1    | 1    | 2    | 1    | 0    | 7    | 4    | 2    |
| Segment 14| 2    | 3    | 2    | 0    | 10   | 2    | 10   | 1    |

| Mass Media | NNP1 | NNP2 | NCH1 | NCH2 |
|------------|------|------|------|------|
|            | FP   | OP   | FP   | OP   | PT   | OT   | PT   | OT   |
|            | 5    | 7    | 3    | 1    | 21   | 18   | 21   | 18   |

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