Confinement from a massive scalar in QCD

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Abstract: A model is introduced with a massive scalar coupling to the Yang–Mills term in four–dimensional gauge theory. It is shown that the resulting potential of colour sources consists of a short range Coulomb interaction and a long range confining part. Far away from the source the scalar vanishes $\sim r^{-1}$ while the potential diverges linearly $\sim r$. Up to an $N_c$–dependent factor of order 1 the tension parameter in the model is $gmf$, where $m$ denotes the mass of the scalar and $f$ is a coupling scale entering the scalar–gluon coupling.
1. Recently it was observed that a string inspired coupling of a massless dilaton to
gauge fields yields a linearly increasing vector potential from pointlike colour sources,
if a logarithmic divergence of the dilaton at infinity is permitted \[1,2\]. This motivated
me to construct a direct coupling of a massive scalar to chromo–electric and magnetic
fields subject to the requirement that the Coulomb problem still admits an analytic
solution, but now with a mass term. The model for the scalar–gluon coupling that
emerged from this endeavour is

\[
L = -\frac{1}{4} \frac{\phi^2}{f^2 + \beta \phi^2} F_{\mu \nu} j^\mu j^\nu - \frac{1}{2} \partial_\mu \phi \cdot \partial^\mu \phi - \frac{1}{2} m^2 \phi^2, \tag{1}
\]

where \(0 \leq \beta \leq 1\) is a parameter and \(f\) is a mass scale characterizing the strength of
the scalar–gluon coupling.

To analyze the Coulomb problem in this theory we consider a pointlike colour
source, which in its rest frame is described by a current

\[
\mathbf{j}_i^\mu = g \delta(\mathbf{r}) C_i \eta^\mu_0.
\]

Here \(1 \leq i \leq N_c^2 - 1\) is an \(\text{su}(N_c)\) Lie algebra index and \(C_i = \zeta^+ \cdot X_i \cdot \zeta\) is the
expectation value of the \(\text{su}(N_c)\) generator \(X_i\) for a normalized spinor \(\zeta\) in colour
space. These expectation values satisfy

\[
\sum_{i=1}^{N_c^2-1} C_i^2 = \frac{N_c - 1}{2N_c},
\]

and the equations of motion for the scalar and gluons emerging from this source follow
as in \[1\]:

\[
\nabla \cdot \left( \frac{\phi^2}{f^2 + \beta \phi^2} \mathbf{E}_i \right) = g C_i \delta(\mathbf{r}), \tag{2}
\]

\[
\Delta \phi = m^2 \phi - \frac{f^2 \phi}{(f^2 + \beta \phi^2)^2} \mathbf{E}_i \cdot \mathbf{E}_i, \tag{3}
\]

and \(\nabla \times \mathbf{E}_i = 0\) implies existence of chromo–electric potentials \(\mathbf{E}_i = -\nabla \Phi_i\).

Clearly, the solution to the Coulomb problem \[2,3\] includes the case of inertial
motion of the colour source through a mere Lorentz boost.

Eq. \(2\) or more generally its analog for an arbitrary spherically symmetric colour
density yields for the fields outside the density

\[
\frac{\phi^2}{f^2 + \beta \phi^2} \mathbf{E}_i = \frac{g C_i}{4\pi r^2} \mathbf{e}_r,
\]
and inserting this relation in (3) yields
\[ \Delta \phi = m^2 \phi - \frac{\mu^2}{r^4 \phi^3}, \] (4)
where the abbreviation
\[ \mu = \frac{g f}{4\pi} \sqrt{\frac{N_c - 1}{2N_c}} \]
was used.
Substituting \( y(r) = r\phi(r) \) in (4) and multiplying by \( dy/dr \) yields the first integral
\[ \left( \frac{dy}{dr} \right)^2 = m^2 y^2 + \frac{\mu^2}{y^2} + 2K. \]
This can readily be solved for arbitrary integration constant \( K \), but the boundary condition \( \lim_{r \to \infty} \phi(r) = 0 \) uniquely determines \( K = -m\mu \). This yields
\[ y^2(r) = \frac{\mu}{m} + \left( y_0^2 - \frac{\mu}{m} \right) \exp(-2mr). \]
Therefore, the scalar field emerging from the pointlike colour source is
\[ \phi = \pm \frac{1}{r} \sqrt{\frac{\mu}{m} + \left( y_0^2 - \frac{\mu}{m} \right) \exp(-2mr)}, \] (5)
while the chromo–electric potentials consist of a short range Coulomb and a long range confining part:
\[ \Phi_i = \beta g C_i \frac{g C_i}{4\pi r} - f C_i \sqrt{\frac{N_c}{2(N_c - 1)}} \ln \left( \exp(2mr) - 1 + \frac{m}{\mu} y_0^2 \right). \] (6)
At large distance the scalar field vanishes \( \sim r^{-1} \), while the chromo-electric potential yields linear confinement, if applied in the framework of a reduced Salpeter or no–pair equation [3].

2. Eq. (6) implies that an (anti-)quark with colour orientation \( \zeta_q, \zeta_q^+ \cdot \zeta_q = 1 \), in the field of a source of colour \( \zeta_s \) (a heavy quark) sees a potential
\[ V(r) = \pm \left( \left| \zeta_s^+ \cdot \zeta_q \right|^2 - \frac{1}{N_c} \right) \left[ \beta g^2 \frac{g}{4\pi r} - g f \sqrt{\frac{N_c}{2(N_c - 1)}} \ln \left( \exp(2mr) - 1 + \frac{m}{\mu} y_0^2 \right) \right], \] (7)
with the upper sign holding for quark–quark interactions and the lower sign applying to quark–anti-quark interactions. The colour factor \( \left| \zeta_s^+ \cdot \zeta_q \right|^2 - \frac{1}{N_c} \) defines a double cone.
around the direction of $\zeta_s$. This double cone has an angle $\tan \theta_c = \sqrt{N_c - 1}$ against the symmetry axis specified by $\zeta_s$, and separates domains of attraction from domains of repulsion: Quarks of colour $\zeta_q$ in the double cone are repelled and anti-quarks are attracted, while quarks with colour outside the cone are attracted and anti-quarks are repelled.

Potentials with an $1/r$ singularity at short distances and linear behaviour at large distances have been very successfully applied in the investigation of the quarkonium spectrum, see e.g. [4] for two of the classical references in the field. Phenomenological tension parameters in heavy–light meson systems are of order $\sigma \simeq (430 \text{ MeV})^2$ [5], and in the present model the tension would be determined by the mass and coupling scale of the scalar field according to $\sigma \simeq g m f$.

Eqs. (6,7) indicate that direct couplings of scalar fields to Yang–Mills terms provide an interesting paradigm for the description of confinement in gauge theories. This might be realized in QCD through a fundamental scalar, or eventually through a low energy effective scalar degree of freedom. The possibility of a fundamental scalar cannot be excluded, since the coupling scale $f$ might be large enough to make such a scalar invisible to present day experiments. On the other hand, there exist scalar resonances in the hadronic spectrum whose rôle has not been understood yet.

On the level of a low energy effective scalar degree of freedom, one might speculate that the Lagrangian (1) with $\beta = 0$ is realized in the low energy regime through a QCD dilaton coupling to the trace anomaly, while at high energies the standard QCD Lagrangian would apply. Such a picture could be motivated, if one combines the old idea of scalar meson dominance of the trace of the energy–momentum tensor [6] with the QCD trace anomaly, as in [7]. Since the trace anomaly is proportional to the gluon condensate this could also justify the scalar gluon coupling in (1) with $\beta = 0$, and eq. (7) then tells us how the dilaton changes the quark interaction potential. A disadvantage with this picture concerns the disappearance of the one gluon exchange term in the low energy regime.

In conclusion, the derivation of (7) provides a challenge to monopole condensation as a mechanism for quark confinement, and it seems well justified to dedicate some more efforts to the investigation of phenomenological aspects of scalar–gluon couplings of the kind described in (1).

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