3-D Modeling of Layered Earth Structure in the Geothermal Systems Using Time Domain Electromagnetics (TDEM) Method

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Abstract. 3-D TDEM modeling has been successfully conducted to map the structure of layered Earth based on the value of resistivity. 3-D analysis required to observe propagation and interaction of electromagnetic fields with the material below the Earth’s surface in the z-axis direction. The problem in electromagnetic modeling for geophysical exploration activities is discretizing of medium for electromagnetic wave propagation. In this modeling use Finite Difference Time Domain (FDTD) method because it can solve electromagnetic field interaction problem and to analyze the transient electromagnetic method compared to the frequency domain. The first step in this method is positioning of the cell to the lines of the magnetic fields that surround normal electric field line, so that this condition is suitable with the components of curl operation of Ampere’s law. Conclusion of this modeling results indication that utilizing TDEM method in the exploration activities is very essential and useful to map the resistivity distribution in the layered Earth. The resistivity is one of physical properties that can describe the structure of the layered Earth to determine the potential reservoir of geothermal systems as the renewable energy.

1. Introduction
Since the early 1970s, there is a significant increase in the development of electromagnetic method for geophysical exploration, such as time domain system called TEM or TDEM [1,2]. Some advantages of the TEM or TDEM are larger exploration range than the size of transmitter and are able to obtain the excellent lateral resolution [3]. They were used to determine potential source of groundwater [4], the structure of hydrogeology [5], potential of aquifer based on the electrical resistivity [6], and determine the resistivity structure of volcano [7]. They also used as static correction for another EM methods such as magnetotelluric to adjust apparent resistivity result [8]. Solving 3-D TDEM is important for understanding the physical properties of observed responses [9].

Finite difference (FD) method for TDEM or FDTD is applied in this modeling. FDTD method was successfully used to solve EM problems such as the interaction EM with complex geometry, analyze transient problems, and modeling of non-homogenous geometry [10]. It was also used to analyze the ability of EM wave penetration in the electric or magnetic materials [11]. FDTD technique is efficiently able to solve curl operations of Maxwell time-dependent equations or equivalent integral equations using Finite Difference [12]. In this research, we will simulate the value of physical parameters, namely electric field in three dimensions using FDTD and its application in the Geothermal and Volcanic systems.

TDEM is one of geophysical methods using EM induction principle. It is generated by transmitter which consist of electric current flow in the coil called primary electric current. According to Faraday’s law,
magnetic field variation with time will generate electric field. From this primary magnetic field, the secondary electric current will be generated in the subsurface. The primary magnetic field from the loop transmitter will generate secondary electric current. Eventually, secondary electric current will generate secondary magnetic field interacting with materials in the subsurface and will be measured by receiver. We discretize the spatial and temporal in 3-D domains. In this step, we implement Dirichlet boundary conditions with the value on each boundary edge is equal to zero.

2. Methodology

We use the modified Yee algorithm to calculate value of node in domain by using Maxwell’s equation [13].

2.1 Basic Concept of Electromagnetic Field

The property of EM wave in homogeneous medium with a source is written by:

\[ \vec{\nabla} \cdot \vec{D} = \rho \] (Electric Gauss’s law) \hspace{1cm} (1)

\[ \vec{\nabla} \cdot \vec{B} = 0 \] (Magnetic Gauss’s law) \hspace{1cm} (2)

\[ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \] (Faraday’s law) \hspace{1cm} (3)

\[ \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \] (Ampere’s law with Maxwell’s correction) \hspace{1cm} (4)

Where \( \vec{D} \) is displacement field, \( \rho \) is charge density, \( \vec{E} \) is electric field, \( \vec{B} \) is magnetic induction field, \( \vec{H} \) is intensity of magnetic field, and \( \vec{J} \) is current density.

In case of material that absorbs the electric or magnetic field, so equation (3) and (4) can be rewritten as follows:

\[ \frac{\partial \vec{B} (\vec{r}, t)}{\partial t} = -\vec{\nabla} \times \vec{E} (\vec{r}, t) - \vec{M} (\vec{r}, t) \] \hspace{1cm} (5)

\[ \frac{\partial \vec{D} (\vec{r}, t)}{\partial t} = \vec{\nabla} \times \vec{H} (\vec{r}, t) - \vec{J} (\vec{r}, t) \] \hspace{1cm} (6)

where \( \vec{M} \) is magnetization generated by the interaction between the magnetic field and the medium.

In case of linear, isotropic, non-dispersive, and lossy materials, we derive and get the following equations:

\[ \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} \vec{\nabla} \times \vec{E} - \frac{1}{\mu} (\vec{M}_{source} + \sigma^* \vec{H}) \] \hspace{1cm} (7)

\[ \frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon} \vec{\nabla} \times \vec{H} - \frac{1}{\varepsilon} (\vec{J}_{source} + \sigma \vec{E}) \] \hspace{1cm} (8)

where \( \sigma \) is electric conductivity and \( \sigma^* \) is equivalent to magnetic loss.
2.2 Finite Difference Method using Yee Algorithm

For discretizing the domain in the rectangular lattice, Yee denoted a space point in a uniform medium as follows [13]:

\[ (i, j, k) = (i\Delta x, j\Delta y, k\Delta z) \]  

(9)

where \( \Delta x, \Delta y, \Delta z \) are the lattice space increments in the \( x, y, \) and \( z \) directions, respectively, and \( i, j, k \) are integers. Then we denote the function \( u \) of space and time evaluated at a discrete point in time as follows:

\[ u(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = u^n_{i,j,k} \]  

(10)

where \( \Delta t \) is the time increment and assumed to be uniform over the observation interval, and \( n \) is an integer.

Central difference expression is used for the space and time derivatives that are both simply programmed and second-order accurate in the space and time increments.

\[ \frac{\partial u}{\partial t}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{u^n_{i+1/2,j,k} - u^n_{i-1/2,j,k}}{\Delta x} + O[(\Delta x)^2] \]  

(11)

By using Yee’s expression for the first partial derivative of \( u \), evaluated at the fixed space point \( (i, j, k) \) follows the analogy:

\[ \frac{\partial u}{\partial t}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{u^{n+1/2}_{i,j,k} - u^{n-1/2}_{i,j,k}}{\Delta t} + O[(\Delta t)^2] \]  

(12)

the \( +1/2 \) increment in the superscript denotes a time finite-difference over \( \pm 1/2 \Delta t \).

3. Results and Discussion

Now we implement the notation for a numerical approximation of the Maxwell’s curl operations in 3-D domain.

3.1 Finite-Difference in Three Dimension Maxwell’s Equation

From the Maxwell’s equation and by considering only the \( E_x \) field component, yields:

\[ \frac{\partial E_x}{\partial t} = \varepsilon \left[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - (J_{\text{source}} + \sigma E_x) \right] \]  

(13)

By applying central differences for time and space derivatives in Equation (13), yield 2 components of electric and magnetic fields in the \( x \) and \( y \) directions as follows:

\[ \frac{E_{x,i,j,k+1/2}^{n+1/2} - E_{x,i,j,k+1/2}^{n-1/2}}{\Delta t} = \varepsilon \frac{1}{E_{x,i,j+1/2,k+1/2}} \begin{pmatrix} \frac{H_{z,i,j+1/2,k+1/2}^n - H_{z,i,j,k+1/2}^n}{\Delta y} & \frac{H_{z,i,j+1/2,k+1/2}^n - H_{z,i,j+1/2,k}^n}{\Delta y} \\ \frac{H_{y,i,j+1/2,k+1/2}^n - H_{y,i,j+1/2,k}^n}{\Delta z} & \frac{\sigma E_{x,i+1/2,j,k+1/2}^n - J_{\text{source}}}{\Delta z} \end{pmatrix} \]  

(14)
3.2 Space and Time Domain Geometry

Numerical solution using FDTD techniques is for determining behavior of interaction between EM wave with 3-D arbitrary material structure. 3-D FDTD structure modeling is used to analyze characteristics of waveguide propagation mode dispersion for open and closed type, that requires analysis of full EM wave. The full wave equation is separated by two sets consist of polarization filed distribution TM and TE. In solving EM full wave, 3-D analyze is required to determine the relation of propagation mode dispersion. For the TM polarization will be obtained as \( H_z (x, y, z, t) = 0 \).

The algorithm of FDTD technique is based on component of the magnetic field surrounded by four electric field components [12]. Similarly, component of the electric field is surrounded by the four components of the magnetic field. This condition is illustrated by figure 1 that shows structure of cube unit cell in coordinate system. 3-D time finite difference time-stepping equation for electric and magnetic field components are obtained by analyzing six coupled scalar partial differential equation. By using rectangular coordinate system, \( E_x (x, y, z, t) \), \( E_y (x, y, z, t) \), and \( E_z (x, y, z, t) \) represent electric field component.

\[
H_{i-1/2,j+1,k+1}^{n+1} - H_{i-1/2,j+1,k+1}^n = \frac{1}{\Delta t} \left( \sum_{\mu} E_{i-1/2,j+1,k+1/2}^{n+1/2} - \sum_{\nu} E_{i-1/2,j+1,k+1/2}^n \right)
\]

\[
- \frac{\Delta z}{\Delta y} \cdot \left( M_{\text{source}}^n \bigg|_{i-1/2,j+1,k} - \sigma^* \bigg|_{i-1/2,j+1,k+1/2} \bigg|_{i-1/2,j+1,k+1} \right)
\]

\[
(i,j,k)
\]

**Figure 1.** The components of the electric and magnetic field vector positions around a cubic unit cell of the Yee space lattice [13]

In this research, modelling is constructed by periodic Gaussian source. The first step in this method is positioning of the cell to the lines of the magnetic fields that surround normal electric field line, so that this condition is suitable with the components of curl operation of Ampere’s law. The source is generated in the midpoint of modelling domain and propagates the electric field wave. Maximum and minimum amplitude is shown to analyze propagation of electric field in the subsurface. The modelling is also given by computational time step = 1 and 1000 to observe the amplitude of attenuation in the subsurface. This condition is shown by figure 2 and 3.

3.3 3-D Finite-Differences Time-Domain Modeling

The source is generated from the transmitter as a periodic gaussian function. The amplitude of the source reaches maximum value at initial condition which is the time-step is equal to zero. The source of periodic
gaussian function will decay by the time due to interacts with any conductive materials in the subsurface. The electric current is periodically generated by transmitter and we have only the maximum value that also generated periodically as shown by figure 2. The amplitude of the source experiences the maximum and minimum values in the modelling due to electric field generated periodically. Although the source is generated periodically, the intensity of the electric field will still be attenuated due to interact with any conductive material in the subsurface.

3-D TDEM modelling is performed on XY cross section. This modelling is observed started from initial time (time step = 1) into final time (time step = 1000). This graph representing interaction between electric field to XY cross section. When the electric field of source propagates to the subsurface and once the propagation found conductive material then it will make an attenuation. This attenuation gives a subsurface information notably for resistivity structure. This method can be applied for estimation depth in subsurface based on the resistivity structure pattern in the research area.

Figure 2. Amplitude of electric field (Ex) at time-steps = 1

Figure 3. Amplitude of electric field (Ex) at time-steps = 1000
The properties of medium can be determined by behavior of electromagnetic wave due to the interaction with medium in subsurface. Attenuation of electromagnetic wave represents that medium has relatively high conductivity. The faster electromagnetic wave decays, show that it propagates in the more conductive medium. This attenuation also depends on conductivity factor in the components of electric and magnetic fields.

4. Conclusion
3-D TDEM modeling in this simulation using Yee’s algorithm scheme has proven in this research by giving a conductive body which is true because it’s shown the phenomenon electric field propagates to the conductive body making an attenuation. Based on simulation at time step = 1000, the electric field occurred attenuation which is true in this modelling to detect conductive body. This simulation can be applied to detect conductive body in subsurface.

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Figure 4. Amplitude of electric field (Ex) in the-XY cross section at time-steps = 1000
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