Numerical simulation of a turbulent Lead Bismuth Eutectic flow inside a 19 pin nuclear reactor bundle with a four logarithmic parameter turbulence model

Andrea Chierici*, Leonardo Chirco*, Roberto Da Vià*† and Sandro Manservisi*

* University of Bologna, DIN, Lab. of Montecuccolino, Via dei Colli 16, Bologna 40136, Italy
E-mail: †roberto.davia2@unibo.it

Abstract. Computational Fluid Dynamic allows scientists and engineers to investigate fluid flow in complex geometries and evaluate heat transfer between a solid body and a fluid. In the present paper we study the heat transfer of a Lead Bismuth Eutectic (LBE) turbulent flow in a bare 19 pin nuclear reactor bundle. When dealing with low Prandtl number fluids, like LBE for which Pr = 0.025, proper turbulence models are needed to improve the prediction of heat transfer. We use here a four logarithmic parameter turbulence model in order to calculate Reynolds stresses and turbulent heat flux. In particular, an equation for temperature fluctuations and one for their dissipation are solved. These variables are used to model thermal characteristic time scales. The results are reported for different values of the Peclet number and a fixed value of the pitch to diameter ratio. The obtained values of the Nusselt number are compared with experimental correlations, that can be found in literature, and with the ones obtained using a Simple Eddy Diffusivity model, where the eddy thermal diffusivity is calculated as proportional to eddy viscosity through a modeled turbulent Prandtl number.

1. Introduction

Liquid metals are gaining more and more interest as coolant fluids in engineering applications [1]. In particular they are being considered in the design process of IVth generation nuclear reactors with the aim of building nuclear power plants with increased economic sustainability and safety levels. Liquid metals are classified as low Prandtl number fluids. They are in fact characterized by low viscosity and high thermal diffusivity values, leading to molecular Prandtl number values \( \Pr \simeq 0.01-0.025 \). As a consequence, more sophisticated tools need to be developed in order to obtain accurate evaluation of heat transfer in the condition of turbulent flows. Standard modeling, i.e. constant turbulent Prandtl number \( \Pr_t \simeq 0.85 \), that is normally used for simulating turbulent flows of fluids having \( \Pr \) close to one, leads to overestimate turbulent heat flux [2, 3, 4]. Various alternatives have been developed, involving different algebraic models for turbulent Prandtl number, see [5, 6, 7] and references therein. Also isotropic four parameters turbulence models in which the turbulent thermal conductivity \( \alpha_t \) is modeled by different characteristic thermal time scales and anisotropic models for turbulent heat flux have been proposed [8, 9, 10, 11, 12]. In the present work we use a four parameter logarithmic turbulence model to study a 19 pin nuclear reactor bundle whose cross section is sketched in Fig. 1 a). Many works involving numerical simulations of turbulent heat transfer in nuclear reactor bundles can be found in literature for triangular [13, 7] and square [14, 7] lattices.
Nomenclature

| Symbol | Value | Unit |
|--------|-------|------|
| Fluid specific heat capacity | $C_p$ | 146 J/(Kg K) |
| Hydraulic diameter | $D_h$ | |
| Heated to wet perimeter ratio | $f_h$ | |
| Turbulent kinetic energy | $k$ | |
| Logarithmic value of $k$, $k = \ln(k)$ | | |
| Mean squared temperature fluctuations | $k_\theta$ | |
| Logarithmic value of $k_\theta$, $k_\theta = \ln(k_\theta)$ | | |
| Nusselt number, $Nu = qh/(T - T_{in})\lambda$ | | |
| Mean pressure | $P$ | |
| Pitch, distance between adjacent rods | $p$ | |
| Peclet number, $Pe = Re \cdot Pr$ | | |
| Prandtl number, $Pr = \nu/\alpha$ | | |
| Turbulent Prandtl number, $Pr_t = \nu_t/\alpha_t$ | | |
| Imposed heat flux | $q$ | |
| Reynolds number, $Re = U_h \nu$ | | |
| Mean temperature | $T$ | |
| Mean velocity | $u$ | |
| Fluid thermal diffusivity | $\alpha$ | |
| Eddy thermal diffusivity | $\alpha_t$ | |
| Pitch to diameter ratio | $\chi$ | |
| Fluid thermal conductivity | $\lambda$ | |
| Fluid dynamic viscosity | $\mu$ | |
| Eddy kinematic viscosity | $\nu_t$ | |
| Logarithmic value of $k$ specific dissipation rate | $\Omega$ | |
| Logarithmic value of $k_\theta$ specific dissipation rate | $\Omega_{\theta}$ | |
| Hot-spot factor | $\phi$ | |
| Fluid density | $\rho$ | |
| Non dimensional temperature | $\Theta$ | |

Greek symbols

| Symbol | Value | Unit |
|--------|-------|------|
| $\rho$ | 10340 Kg/m$^3$ |
| $\lambda$ | 10.76896 W/(m K) |
| $C_p$ | 146 J/(Kg K) |

Table 1. Lead Bismuth Eutectic physical parameters used in the present study.

Hexagonal bundle, in lattices of 19 or more rods, are studied in bare, wire-wrapped and with spacer grid configurations [15, 16, 17, 18]. We study a fully developed turbulent flow occurring in a bare rod lattice configuration, in order to provide results in a simplified geometry that could be used as a starting point for future analysis. The simulated coolant fluid is Lead Bismuth Eutectic with a molecular Prandtl number $Pr = 0.025$. In the following Section we introduce the mathematical model used to study the fluid turbulent behavior. In Section 3 we report some of the obtained results regarding the simulated section and the triangular sub-channels.

2. Numerical Modeling

In the present work we deal with the simulation of a LBE turbulent flow. The physical properties are reported in Table 1 with a resulting Prandtl number $Pr = 0.025$. In order to study the flow and heat transfer behavior of this configuration, we solve the system of Reynolds Averaged Navier-Stokes equations as

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] - \rho \mathbf{u} u' u' + \rho g,$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right) = \nabla \cdot [\lambda \nabla T - \rho C_p u' T] + Q,$$

where $\mathbf{u}$, $P$ and $T$ stand for mean velocity, pressure and temperature. The system of equations (1-3) is closed with a four parameter logarithmic turbulence system of equations in order to
model the Reynolds stresses $\rho \overline{u' u'}$ and the turbulent heat flux $\rho C_p \overline{u' T'}$.

\[
\frac{\partial K}{\partial t} + u \cdot \nabla K = \nabla \cdot \left[ \left( \nu + \frac{\nu_t}{\sigma_z} \right) \nabla K \right] + \left( \nu + \frac{\nu_t}{\sigma_z} \right) \nabla K \cdot \nabla K + \frac{P_k}{\epsilon K} - C_\mu e^\Omega, \tag{4}
\]

\[
\frac{\partial \Omega}{\partial t} + u \cdot \nabla \Omega = \nabla \cdot \left[ \left( \nu + \frac{\nu_t}{\sigma_z} \right) \nabla \Omega \right] + 2 \left( \nu + \frac{\nu_t}{\sigma_z} \right) \nabla K \cdot \nabla \Omega + \left( \nu + \frac{\nu_t}{\sigma_z} \right) \nabla K \cdot \nabla \Omega + \frac{c_{\sigma 1} - 1}{e K} P_k - C_\mu (c_{\sigma 2} f_{exp} - 1) e^\Omega \tag{5}
\]

\[
\frac{\partial K_{\theta}}{\partial t} + u \cdot \nabla K_{\theta} = \nabla \cdot \left[ \left( \alpha + \frac{\alpha_t}{\sigma_{\theta}} \right) \nabla K_{\theta} \right] + \left( \alpha + \frac{\alpha_t}{\sigma_{\theta}} \right) \nabla K_{\theta} \cdot \nabla K_{\theta} + \frac{P_\theta}{e K_{\theta}} - C_\mu e^{\Omega_{\theta}}, \tag{6}
\]

\[
\frac{\partial \Omega_{\theta}}{\partial t} + u \cdot \nabla \Omega_{\theta} = \nabla \cdot \left[ \left( \alpha + \frac{\alpha_t}{\sigma_{\theta}} \right) \nabla \Omega_{\theta} \right] + 2 \left( \alpha + \frac{\alpha_t}{\sigma_{\theta}} \right) \nabla K_{\theta} \cdot \nabla \Omega_{\theta} + \left( \alpha + \frac{\alpha_t}{\sigma_{\theta}} \right) \nabla K_{\theta} \cdot \nabla \Omega_{\theta} + \frac{c_{\sigma 1} - 1}{e K_{\theta}} P_\theta - \left( c_{d 1} - 1 \right) e^{\Omega_{\theta}} - c_{d 2} C_\mu e^{\Omega_{\theta}}. \tag{7}
\]

The system of equations (4-5) consists of the transport equations for the logarithm of turbulent kinetic energy $K$ and the logarithmic value of turbulent kinetic energy specific dissipation rate $\Omega$, while (6-7) are solved for thermal turbulence, in particular for logarithmic temperature fluctuation variance $K_{\theta}$ and the logarithmic specific dissipation rate $\Omega_{\theta}$. With these variables characteristic time scales for thermal turbulence are modeled in order to calculate eddy thermal diffusivity $\alpha_t$. A logarithmic formulation of turbulence variables allows us to obtain an increased numerical stability of the turbulence model. Further details on the model parameters can be found in [11, 19, 20, 21]. In the present case we focus our attention in the dynamic and thermal fully developed regime. If volumetric source term is set to 0 and a constant heat flux $q$ is imposed on the wall then temperature grows linearly along axial coordinate. Temperature gradient can be calculated by integration of equation (3) on the whole domain, obtaining

\[
\frac{\partial T}{\partial z} = \frac{4 q f_h}{w_{mid} \rho C_p D_h}, \tag{8}
\]

where $z$ is the axial direction, $w_{mid}$ is the average axial velocity on the cross section of the domain, $D_h$ is the hydraulic diameter and $f_h$ is the ratio between the heated and the wet perimeter of the domain cross section. In this case the temperature field $T$ of (3) is usually decomposed as

\[
T(x, y, z) = T_{w0} - \bar{T}(x, y) + \frac{\partial T}{\partial z} (z - z_{in}), \tag{9}
\]

where $T_{w0}$ is an arbitrary temperature value on the inlet surface. The new temperature field $\bar{T}(x, y)$ describes the temperature distribution on the transverse section of the domain and does not change with the axial coordinate. The energy balance equation, written with $\bar{T}$ as state variable, becomes

\[
\frac{\partial \bar{T}}{\partial t} + u \cdot \nabla \bar{T} = \nabla \cdot \left[ \left( \alpha + \alpha_t \right) \nabla \bar{T} \right] + u \cdot \hat{a} - \frac{4 q f_h}{w_{mid} \rho C_p D_h}, \tag{10}
\]

where $\hat{a}$ is a unit vector along the axial coordinate. We also neglect secondary flows and exploit geometry symmetry planes by simulating only a sector of the whole 19 pin nuclear reactor bundle, as reported in Fig. 1 a) and b). With the fully developed regime periodic boundary conditions can be imposed on inlet and outlet sections for all the solved variables. The final system of equations (1-2) - (4-7) - (10) is implemented and solved with finite element code FEMuS [22].
Figure 1. Schematic representation of the whole bundle cross section a) and simulated domain, boundary definition and triangular sub-channels b).

Figure 2. Meshes with three different levels of refinements used for the present work.

| $\chi$ | $d_{h,bun}$ [mm] | $d_{h,sch}$ [m] | $f_h$ |
|-------|-----------------|-----------------|-------|
| 1.4   | 7.7             | 9.5             | 0.733 |

Table 2. Geometrical parameters of the simulated 19 pin nuclear reactor bundle.

3. Results
In this Section we present the results obtained from the simulation of the nuclear reactor bundle for multiple values of the Reynolds number. In particular we will refer to bulk Reynolds number, $Re_b = u_b d_{h,bun} \rho / \mu$, where $u_b$ is the bulk velocity calculated on the whole bundle cross section and $d_{h,bun}$ is the hydraulic diameter of the bundle, and to sub-channel Reynolds
we follow the methods used in [15, 18]. Nusselt number is thus calculated as

\[
N_u = Nu
\]

In each case we study the behavior of the three triangular sub-channels by computing local Nusselt number. The temperature field is on wall surface, as \( T \) where \( \lambda \) is the fluid thermal conductivity, \( \phi \)

\[
\phi = \frac{T_{w,max} - T_b}{(T_w) - T_b},
\]

where \( T_{w,max} \) is the maximum wall temperature. This factor is used to determine how non-homogeneous the temperature field is on wall surface, as \( \phi = 1 \) would mean that the maximum values and hot-spot factor \( \phi \) are calculated as

\[
\phi = \frac{q d_{h,sch} / \mu}{q d_{h,sch} / \mu}
\]

\[ L_2 \text{ norm values. For given different meshes } u_{h1} \text{ and } u_{h2}, \text{ with corresponding solutions } u_{h1} \text{ and } u_{h2}, \text{ a convergence criteria is defined as (11)} \]

\[
\| u_{h,n-1} - u_{h,n} \|_{L^2} \leq 10^{-4},
\]

assuring full convergence for all the solutions within two or three digits.

The boundary conditions used for the present simulations are reported in Table 3, where “near-wall b” means that the imposed boundary conditions are in accordance with the near wall behavior that can be obtained from a Taylor series expansion in the near wall region [11]. We investigate turbulent heat transfer at four different values of \( Re_b \), namely CASE A : \( Re_b = 1.4 \times 10^4 \), CASE B : \( Re_b = 2.2 \times 10^4 \), CASE C : \( Re_b = 3.1 \times 10^4 \) and CASE D : \( Re_b = 5.4 \times 10^4 \). For each case we study the behavior of the three triangular sub-channels by computing local Nusselt number \( Nu \) values and hot-spot factor \( \phi \). In order to compare the results with reference data we follow the methods used in [15, 18]. Nusselt number is thus calculated as

\[
Nu = Nu
\]

The simulated domain is reported in Fig. 1 b), together with the definition of the three different boundary regions and of the triangular sub-channels \( \Omega_{sch,1}, \Omega_{sch,2} \) and \( \Omega_{sch,3} \).

| Boundary          | u       | K & \( \Omega \) | \( \bar{T} \) | \( K_\theta \& \Omega_\theta \) |
|-------------------|---------|------------------|---------------|-------------------------------|
| \( \Gamma_{w,h} \) | no-slip | near-wall b      | uniform heat flux \( q \) | near-wall b                   |
| \( \Gamma_{w,ad} \) | no-slip | near-wall b      | null gradient  | null gradient                 |
| \( \Gamma_{w,simm} \) | null gradient | null gradient | null gradient | null gradient |

Table 3. Boundary conditions imposed on boundaries represented in Fig. 1 b).

Table 4. Experimental correlations for Nusselt number calculation in triangular lattices.

| Correlation | Expression | Validity range |
|-------------|------------|----------------|
| Grüber      | \( Nu = 0.25 + 6.2\chi + (0.032\chi - 0.007)Pe^{0.8-0.024\chi} \) | \( Pe \in [150,4000], \chi \in [1.2,2.0] \) |
| Ushakov     | \( Nu = 7.55\chi - 20\chi^{-13} + 2.67 Pe^{0.56+0.19\chi} \) | \( Pe < 4000, \chi \in [1.3,2.0] \) |
| Mikityuk    | \( Nu = 0.047(1 - e^{-3.8(\chi-1)})(Pe^{0.77} + 250) \) | \( Pe \in [30,50000], \chi \in [1.1,1.95] \) |

number, \( Re_{sch} = u_b d_{h,sch} / \mu \), where \( d_{h,sch} \) is the hydraulic diameter of the triangular lattice sub-channel. Rod lattices are usually classified using the pitch-to-diameter ratio \( \chi \), where pitch \( p \) is the distance between two adjacent rods center lines and diameter \( d \) is the rod diameter, as sketched in Fig. 1 a). Numerical values of the bundle geometrical properties are reported in Table 2. These values are relative to the experimental setup studied in [15, 18]. A sketch of the simulated domain is reported in Fig. 1 b), together with the definition of the three different boundary regions and of the triangular sub-channels \( \Omega_{sch,1}, \Omega_{sch,2} \) and \( \Omega_{sch,3} \).

Several meshes with quadratic elements and increasing levels of mesh refinements have been used, as described in Fig. 2. In particular these meshes are used to check the numerical solutions by their \( L_2 \) norm values. For given different meshes \( T_{h1} \) and \( T_{h2} \), with corresponding solutions \( u_{h1} \) and \( u_{h2} \), a convergence criteria is defined as

\[
||u_{h,n-1} - u_{h,n}||_{L^2} \leq 10^{-4},
\]

For given different meshes \( T_{h1} \) and \( T_{h2} \), with corresponding solutions \( u_{h1} \) and \( u_{h2} \), a convergence criteria is defined as

\[
\| u_{h,n-1} - u_{h,n} \|_{L^2} \leq 10^{-4},
\]
temperature value is equal to the mean wall temperature. Nusselt number values are compared with experimental correlations derived for triangular lattices. These correlations are usually defined as a function of Peclet number \( Pe = Re Pr \) and of pitch-to-diameter ratio \( \chi \). In Table 4 we report the experimental correlations of Gräber, Ushakov and Mikityuk [23, 24, 25].

\[
\begin{array}{cccc}
Pe & \Omega_{sch,1} & \Omega_{sch,2} & \Omega_{sch,3} \\
438 & 1.0855 & 1.1835 & 1.1905 \\
682 & 1.0939 & 1.2158 & 1.2221 \\
960 & 1.0965 & 1.2218 & 1.2266 \\
1670 & 1.0938 & 1.2027 & 1.2048 \\
\end{array}
\]

Table 5. Hot-spot factor \( \phi \) values for the three triangular sub-channels as a function of the Peclet number for the different simulated cases.

Figure 3. Comparison of computed Nusselt values with experimental correlations of Gräber, Ushakov and Mikityuk.

\[
\Theta = \frac{(T - T_b)\lambda}{d_{h,bun}q},
\]

(14)
Figure 4. Non-dimensional temperature distribution $\Theta$. From left to right case a, case b, case c and case d.

Figure 5. Turbulent Prandtl number distribution on the simulated domain for increasing Reynolds number values. From left to right case a, case b, case c and case d.

with $T_b$ bulk temperature of the cross section, is reported in Fig. 4 in order to compare temperature behavior between the simulated cases. The results agree with the computed values of $\phi$. Moreover we see that highest temperature values are located in the corner region. In Fig. 5 we show the computed turbulent Prandtl distribution on the bundle cross section for increasing values of the Reynolds number. We see that high $Pr_t$ values are obtained in the region close to heated walls. Maximum values in those regions are in the range of [2, 3] and tend to decrease as the Reynolds number increases. It is commonly suggested to set $Pr_t = 1.5$ [7] when using an algebraic model. This assumption can be a good compromise as in center of bundle cross section the computed $Pr_t$ is approximately equal to 1.5, even if still it would be under-estimated in the
near wall region.

4. Conclusions

In the present work we have studied turbulent heat transfer occurring in a 19 pin hexagonal lattice of a nuclear reactor bundle where Lead Bismuth Eutectic is used as a coolant fluid. A simplified model has been here considered, under the assumption of fully developed dynamical and thermal turbulent flow. In this way symmetry planes can be detected and only a sector can be simulated. A transformation on temperature field has been operated in order to obtain an energy balance equation where periodic boundary conditions can be applied on inlet and outlet sections. The study of turbulent heat transfer has been then transformed in a two-dimensional numerical simulation. A four parameter logarithmic turbulence model has been here used to close the system of Reynolds Averaged Navier-Stokes equations. Simulations for four different values of the Reynolds number have been performed. Three different triangular sub-channels have been considered on the simulated domain in order to compare our results with literature data. The Nusselt number values, calculated for the three different sub-channels, have been compared with experimental correlation formulated for triangular nuclear reactor bundles as a function of Peclet number \([23, 24, 25]\). We observe a good agreement between our results and reference data for three of the simulated cases, in particular for \(Pe < 1000\), while for high Peclet numbers the Nusselt number value is under-estimated. Hot-spot factor values for the central triangular sub-channel are in agreement with the ones calculated for triangular sub-channels \([7]\), while higher values are obtained for the other two sub-channels, denoting a non uniform temperature field on wall surfaces. Finally a comparison of turbulent Prandtl number distribution between the four simulated cases is shown. The present study is a starting point for more accurate numerical simulations of the 19 pin nuclear reactor bundle. In particular thermal field development will be considered in order to evaluate the influence of buoyancy forces on the flow field. Moreover more complicated geometries, with wire-wrapped configurations or with the presence of grid-spacers, will be analyzed to see the impact of secondary flows on heat transfer.

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