Reheating Phase Diagram for Higgs Inflation

Rong-Gen Cai\textsuperscript{a}, Zong-Kuan Guo\textsuperscript{a}, Shao-Jiang Wang\textsuperscript{a}

\textsuperscript{a}State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China.

Abstract

We investigate the impact on the inflationary predictions from various reheating histories which are characterized by an e-folding number \( N_{\text{reh}} \) and an effective equation-of-state parameter \( w_{\text{reh}} \) during reheating process. For Higgs inflation with a non-minimal coupling to Einstein gravity, the predictions are obtained on the \( N_{\text{reh}}-w_{\text{reh}} \) reheating phase diagram. We find that the predictions are insensitive to reheating phase. Within the 1\( \sigma \) region of the scalar spectral index \( n_s \) reported by Planck 2015, almost all possible reheating histories on the reheating phase diagram are allowed, where Higgs inflation with canonical reheating history \( w_{\text{reh}} = 0 \) lies near the best-fit value of \( n_s \). Future measurements of \( n_s \) with high precision will identify the reheating physics of Higgs inflation.

1. Introduction

Constraints on slow-roll inflationary models are usually derived in the \( n_s - r \) plane assuming some reasonable range for the e-folding number of inflation, where \( n_s \) is the scalar spectral index and \( r \) the tensor-to-scalar ratio [1, 2]. With more and more precise measurements on the inflationary observables, a more refined method [3, 4, 5] to carry out the inflationary predictions is needed. Since the main uncertainties come from the reheating phase preceding the radiation era [6], it is necessary to study the impact of reheating history on the inflationary predictions. As pointed out in Ref. [3], although the physics of reheating is highly uncertain, one can characterize the reheating phase by two phase variables, the e-folding number \( N_{\text{reh}} \) and the effective equation-of-state (EOS) parameter \( w_{\text{reh}} \), and express all other inflationary observables in terms of these reheating phase variables. This cannot be done exactly unless the inflationary potential contains no more than one parameter and the number of degrees of freedom at the end of reheating era \( g_{\text{reh}} \) is calculable. By calculable we mean that the phenomenological inflationary scenarios can be connected to the low-energy particle physics with certain content of its particle species. Although the e-folding numbers are insensitive to the precise value of \( g_{\text{reh}} \) due to logarithmic dependence, the parameters in the potential might be sensitive to it and lead to slightly different inflationary predictions which can be detected by future measurements [7].

The minimal inflationary model meets the above criteria is Higgs inflation [8, 9, 10] (for review see [11]). Higgs inflation is the most economical and predictive inflationary model on the market. Although Higgs inflation model contains two parameters, Higgs self-coupling \( \lambda \) and non-minimal coupling \( \xi \), the inflationary predictions obtained from tree level potential only rely on the combined parameter \( \lambda/\xi^2 \). Furthermore, if we take into account the renormalization group (RG) improvement, both \( \lambda \) and \( \xi \) can be obtained given the initial conditions measured at electroweak (EW) scale. Higgs inflation in this sense is a zero-parameter inflationary model [12]. What’s more, Higgs inflation requires no more degrees of freedom beyond those that are already presented in the SM at EW scale. Therefore the number of degrees of freedom at the end of reheating era is calculable and simply equals to those at EW scale \( g_{\text{reh}} = g_{\text{EW}} \).

At the meanwhile Higgs inflation is also a well-motivated and intensive-studied inflation model. The unitarity problem [13, 14, 15, 16] currently settled down in the literatures [17, 18, 19, 20]. The effective potential was computed along two different path: [21, 22, 23, 24, 25, 26] account for only Coleman-Weinberg potential and neglect potential effects from underlying quantum gravity, [27, 28, 29, 30, 31, 32, 33, 34, 35, 36] account for effects from underlying quantum gravity with proper considerations. Higgs inflation was also realized in critical [37, 38, 39] and metastable [40] case to address the stability problem [41, 42, 43, 44, 45, 46] of SM EW vacuum. The connections in Higgs inflation between inflationary scale and EW scale have been extensively investigated, however the interplay [9, 10, 40] between inflationary era and reheating era still need further discussions.

Following the approach proposed in Refs. [3, 4, 5], we apply it to study the inflationary predictions of Higgs inflation on a phase diagram characterizing the reheating history. The main difference of our works compared to previous works is that the dependence of inflationary phase on the reheating phase is shown to be insensitive in an exact and transparent manner.
The paper is organized as follows. In Section 2 we derive the observables of Higgs inflation in the slow-roll approximations. In Section 3 we introduce effective descriptions of reheating phase. The results are presented in Section 4. Section 5 is devoted to conclusions.

2. Higgs Inflation

In the Higgs inflation model, the Higgs scalar field with a large non-minimal coupling to Einstein gravity can give rise to an exponential plateau-like potential in large field region in Einstein frame. The action in Jordan frame is

$$S_J = \int d^4x \sqrt{-g} \left( \frac{M^2 + \xi h^2}{2} - \frac{1}{2}(\partial h)^2 - V(h) \right),$$  \hspace{1cm} (1)$$

where $V(h) = (\lambda/4)(h^2 - v^2)^2$ with VEV of EW vacuum $v = 246\text{GeV}$. The reduced Planck mass $M_P = 1/\sqrt{8\pi G} = 2.435 \times 10^{18}\text{GeV}$ is recovered via $M_P^2 = M^2 + \xi v^2$ when Higgs field is in the EW vacuum. The results from LHC can constrain [50] the non-minimal coupling $\xi < 2.6 \times 10^{15}$ at the 95% confidence level. Therefore, the non-minimal coupling $\xi \ll M_P^2/v^2 \sim 10^{32}$ so that we can safely approximate $M^2 = M_P^2 - \xi v^2 \approx M_P^2$.

To switch into Einstein frame, we define the conformal factor

$$\Omega^2 = \frac{g_{\mu\nu}}{g_{\mu\nu}} = \frac{M_{\text{eff}}^2}{M_P^2} = \frac{M^2 + \xi h^2}{M_P^2} \approx 1 + \frac{\xi h^2}{M_P^2},$$  \hspace{1cm} (2)$$

and scalar field redefinition

$$\left(\frac{d\chi}{dh}\right)^2 = \frac{1}{\Omega^2} + \frac{6M_P^2}{2} \left(\frac{d\Omega}{dh}\right)^2 \approx \frac{1 + \frac{\xi h^2}{M_P^2} + \frac{6\xi h^2}{M_P^4}}{\left(1 + \frac{\xi h^2}{M_P^2}\right)^2}. $$  \hspace{1cm} (3)$$

The action in Einstein frame becomes

$$S_E = \int d^4x \sqrt{-g} \left( \frac{M_P^2}{2} \tilde{R} - \frac{1}{2}(\tilde{\partial}\chi)^2 - U(\chi) \right),$$  \hspace{1cm} (4)$$

where

$$U(\chi(h)) = \frac{V(h)}{\Omega^2} = \frac{\lambda M_P^4}{4\xi^2} \left(\frac{\xi h^2}{M_P^2} - \frac{\xi^2 v^2}{M_P^4}\right)^2. $$  \hspace{1cm} (5)$$

In the large field region $h \gg M_P/\sqrt{\xi}$ we can solve the scalar field re-definition (3) to obtain $\chi \approx \sqrt{5}M_P \ln \sqrt{\xi}h/M_P$. We introduce dimensionless scalar field $\phi = \sqrt{\xi}h/M_P$ and a combined parameter $Z = \lambda/\xi^2$ for latter convenience to abbreviate

$$U(\chi(\phi)) = \frac{Z}{4} \frac{\phi^4}{(1 + \phi^2)^2} = V(\phi).$$  \hspace{1cm} (6)$$

For large range of $\phi \gg \sqrt{\xi}v/M_P \approx 10^{-14}$ the above form of Higgs potential in Einstein frame is exact. Note that the inflaton is still the field $\chi$, not the field $h$ or $\phi$. We set $M_P^2 = 1$ from now on.

Now we can carry out directly the slow-roll dynamics with respect to the inflaton $\chi$ by computing the slow-roll parameters in term of dimensionless $\phi$,

$$\epsilon(\phi) = \frac{4}{3} \frac{1}{(\phi^2 + 1)^2},$$  \hspace{1cm} (7)$$

$$\eta(\phi) = \frac{4}{3} \frac{1}{(\phi^2 + 1)^2} + \frac{4}{(\phi^2 + 1)^2},$$  \hspace{1cm} (8)$$

$$\zeta^2(\phi) = \frac{16/9}{(\phi^2 + 1)^2} - \frac{16}{(\phi^2 + 1)^3} + \frac{64/3}{(\phi^2 + 1)^4},$$  \hspace{1cm} (9)$$

and the e-folding number during inflation

$$N(\phi_N) = \frac{3}{4} \left(\phi_N^2 - \phi_{end}^2 - \ln \frac{1 + \phi_N^2}{1 + \phi_{end}^2}\right),$$  \hspace{1cm} (10)$$

and the scalar spectral indexes and tensor-to-scalar ratio

$$n_s(\phi_N) = 1 - \frac{8}{3} \frac{1}{\phi_N^2 + 1} \approx 1 - \frac{2}{N},$$  \hspace{1cm} (11)$$

$$r(\phi_N) = \frac{64}{3} \frac{1}{(\phi_N^2 + 1)^2} \approx \frac{12}{N^2},$$  \hspace{1cm} (12)$$

$$a_s(\phi_N) = \frac{32}{9} \frac{1}{(\phi_N^2 + 1)^3} + \frac{32}{9} \frac{1}{(\phi_N^2 + 1)^3}. $$  \hspace{1cm} (13)$$

In the $n_s - r$ plane we usually draw a straight line connecting two points representing the general choice for $50 < N_* < 60$ at pivot scale $k_s = 0.002\text{Mpc}^{-1}$. Higgs inflation predicts $n_s \approx 1 - 2/N = 0.960$ and $r \approx 12/N^2 = 0.0048$ for $N_* = 50$, and $n_s \approx 1 - 2/N = 0.967$ and $r \approx 12/N^2 = 0.0033$ for $N_* = 60$, which are fully consistent with Planck results on inflation [1, 2].

3. Descriptions of Reheating Phase

In the Higgs inflation scenario, there are no new degrees of freedom beyond EW scale up to inflationary scale. At the very least, the number of degrees of freedom at the end of reheating era should equal to that at the EW scale. Following the method proposed in the literature [3, 4, 5] we derive the formula for the effective number of degrees of freedom at the end of reheating era. The pivot scale is chosen as $k_s = 0.05\text{Mpc}^{-1}$, which are also expressed by

$$k_s = a_s H_* = \frac{a_s}{a_{end}} \frac{a_{end}}{a_{reh}} \frac{a_{reh}}{a_0} H_*.$$  \hspace{1cm} (14)$$

In what follows, current scale factor $a_0 = 1$ and all quantities with subscript “reh” are evaluated at the moment when the pivot scale is leaving the horizon.

The first two factors of (14) can be read of

$$\frac{a_s}{a_{end}} \frac{a_{end}}{a_{reh}} = e^{-(N_* + N_{reh})}.$$  \hspace{1cm} (15)$$

To compute the third factor of (14), we use the conservation equation of entropy $g_{reh} a_{reh}^3 T_{reh}^3 = g_s a_0^3 T_s^3 + g_a a_0^3 T_a^3 = (43/11) T_s^3 a_0^3$ by noting that $g_{s} = 2$, $g_a = (7/8) \times 3 \times 2 =
21/4, and $T_\gamma^3 = (4/11)T_\gamma^3$, here we adopt $T_\gamma = 2.7255K$.
Thus the third factor can be written as
\[
\frac{\dot{a}_{\text{reh}}}{a_0} = \left(\frac{43}{11g_{\text{reh}}}\right)\frac{T_\gamma}{T_{\text{reh}}},
\]
where the temperature $T_{\text{reh}}$ at the end of reheating era need to be properly accounted for. Assuming that the whole history during reheating era can be effectively described by an e-folding number $N_{\text{reh}}$ and an effective EOS parameter $w_{\text{reh}}$, we are able to relate the reheating era to the inflationary era via
\[
\rho_{\text{end}} = \frac{3}{3 - \epsilon_{\text{end}}}V_{\text{end}},
\]
\[
\rho_{\text{reh}} = \rho_{\text{end}} e^{-3N_{\text{reh}}(1+w_{\text{reh}})},
\]
\[
\rho_{\text{reh}} = \frac{3}{\pi^2 g_{\text{reh}} T_{\text{reh}}^4}.
\]
The inflation end when the first Hubble hierarchy parameter $\epsilon_H = (3/2)(1 + w) = 3K/(K + V) = \epsilon_{\text{end}}$, therefore the kinetic energy $K_{\text{end}} = \epsilon_{\text{end}} V_{\text{end}}/(3 - \epsilon_{\text{end}})$, thus the total energy density $\rho_{\text{end}} = K_{\text{end}} + V_{\text{end}} = 3V_{\text{end}}/(3 - \epsilon_{\text{end}})$. The second equation comes from direct calculation of $\rho_{\text{reh}} = \rho_{\text{end}} e^{\pi^2g_{\text{reh}}/3} e^{-3N_{\text{reh}}(1+w_{\text{reh}})}$. Combining above three equations gives rise to
\[
T_{\text{reh}}^4 = \frac{90V_{\text{end}}}{\pi^2(3 - \epsilon_{\text{end}})g_{\text{reh}}} e^{-3N_{\text{reh}}(1+w_{\text{reh}})}.
\]
Inserting (20) into (16) gives the final expression for the third factor of (14).

The last factor $H_s$ is usually fixed by Planck normalization $A_s = H_s^2/(8\pi^2\epsilon_s)$, namely, $H_s = \sqrt{\epsilon_s A_s}/2$. However it requires full knowledge on tensor-to-scalar ratio $\epsilon_s$ and the consistency relation $\epsilon_s = 16\epsilon_{\text{reh}}$, both of which have not been observed or confirmed yet. A more economic way to compute $H_s$ is to use slow-roll equation
\[
3H_s^2 = V_s.
\]
Combining (15), (16) and (21) together gives the final formula for the effective number of degrees of freedom at the end of reheating era
\[
g_{\text{reh}} = \left(\frac{T_\gamma}{k_s}\right)^{12} \left(\frac{43}{11}\right)^4 \left(\frac{3 - \epsilon_{\text{end}}}{810}\right)^3 \left(\frac{\pi^6 V_s^{6/3}}{V_{\text{end}}^{2}}\right) \times \exp\left(9N_{\text{reh}}(w_{\text{reh}} - \frac{1}{3}) - 12N_s\right),
\]
which will be equal to $g_{\text{GW}} = 106.75$ in virtue of the validity of Higgs inflation. Note that the $N_{\text{reh}} = 0$ case is exactly equivalent to the $w_{\text{reh}} = \frac{1}{3}$ case, which serve as asymptotic lines on reheating phase diagram.

All the uncertainties of (22) come from Planck normalization of $A_s$ and reheating history $N_{\text{reh}}, w_{\text{reh}}$ when we compute $V_s, V_{\text{end}}$ and $N_s$. Let’s explain this in details below.

Recall our abbreviation for Higgs inflation in Einstein frame
\[
V(\phi; Z) = \frac{\phi^4}{4(1 + \phi^2)^2},
\]
where we write down the $Z = \lambda/\xi^2$ dependence explicitly. The inflation ended when slow-roll condition is broken,
\[
\epsilon(\phi_{\text{end}}) = \epsilon_{\text{end}} \Rightarrow \phi_{\text{end}}.
\]
One common choice is that $\epsilon_{\text{end}} = 1$. Once we have the field value of $\phi_{\text{end}}$ at the endpoint of inflation, we have the potential energy density at that moment
\[
V(\phi; Z) = \frac{\phi^4}{4(1 + \phi^2)^2}.
\]

To compute $H_s$ via slow-roll equation $3H_s^2 = V_s$, one requires the field value of $\phi_s$ when the horizon is leaving the horizon. This can only be done by inputting Planck observations on the scalar power spectrum amplitude via
\[
A_s = \frac{1}{24\pi^2} \frac{V(\phi_s; Z)}{\epsilon(\phi_s)} \Rightarrow \phi_s(Z; A_s).
\]
Once we know field value of $\phi_s$, we have every information to restore $H_s = \sqrt{V_s}/3$ by
\[
V_s(Z; A_s) = V(\phi_s(Z; A_s); Z).
\]
Note that the direct $Z$ dependence drops out of the slow-roll parameters $\epsilon(\phi), \eta(\phi), \zeta^2(\phi)$, as well as many other observables like $\eta_s(\phi), r(\phi)$, however the indirect $Z$ dependence will reenter into observables when they are evaluated at the pivot scale though $\phi_s(Z; A_s)$.

There is one more quantity to complete the evaluations, which is the e-folding number during inflation
\[
N_s(Z; A_s) = \frac{3}{4} \phi_s^2(Z; A_s) - \frac{\phi_{\text{end}}^2}{\epsilon_{\text{end}}},
\]
\[
-3\ln\left(\frac{1 + \phi_s^2(Z; A_s)}{1 + \phi_{\text{end}}^2}\right).
\]
Combining (25), (27) and (28) together we clarify the source of uncertainties for the number of effective degrees of freedom at the end of reheating era $g_{\text{reh}}$ on the Planck normalization of $A_s$ and reheating history $N_{\text{reh}}, w_{\text{reh}}$.

\[
g_{\text{reh}}(Z; A_s, N_{\text{reh}}, w_{\text{reh}}) = \left(\frac{T_\gamma}{k_s}\right)^{12} \left(\frac{43}{11}\right)^4 \left(\frac{3 - \epsilon_{\text{end}}}{810}\right)^3 \frac{\pi^6 V_s^{6/3}(Z; A_s)}{V_{\text{end}}^{2}} \times \exp\left(9N_{\text{reh}}(w_{\text{reh}} - \frac{1}{3}) - 12N_s(Z; A_s)\right).
\]

4. Reheating Phase Diagram

Before we plot the inflationary predictions with varying reheating histories on the reheating phase diagram, it is informative to first look at the inflationary predictions with given reheating configurations.
Figure 1: Inflationary predictions with given reheating histories. The left panel is drawn for given canonical reheating history $N_{\text{reh}} = 10$, $w_{\text{reh}} = 0$, and the right panel is drawn for given instantaneous $N_{\text{reh}} = 0$, $w_{\text{reh}} = 0$ and long-term $N_{\text{reh}} = 20$, $w_{\text{reh}} = 0$ reheating history. The blue ribbons $g_{\text{reh}}(Z; A_s)$ and $n_s(Z; A_s)$ are drawn within 1σ range of $\ln(10^{10} A_s) = 3.094 \pm 0.034$ from Planck 2015 TT,TE,EE+lowP[2]. The vertical red lines project the allowed range for $Z$ by requiring $g_{\text{reh}} = g_{\text{EW}} = 106.75$, and the horizontal red ribbons project the allowed range for $n_s$ given the obtained range of $Z$. We also plot the current constraint from Planck 2015 TT,TE,EE+lowP[2] $n_s = 0.9645 \pm 0.0049$ for comparison.
Figure 2: Reheating phase diagrams. Inflationary predictions $n_s, r, \alpha_s, Z, T_{reh}, N_{inf}$ are drawn with respect to all kinds of reheating configurations. When plotting every contour lines on reheating phase diagram, we take into account the uncertainty from Planck 2015 normalization\cite{Planck2015} of $\ln(10^{10} A_s) = 3.094 \pm 0.034$, which manifest itself by distinguishable solid/dashed line in the last panel but indistinguishable solid/dashed line in other three panels. It can be shown in the first panel that almost all possible reheating histories are allowed within the 1σ region of $n_s = 0.9645 \pm 0.0049$ reported by Planck 2015 TT,TE,EE+lowP\cite{Planck2015}. 

\begin{align*}
T_{reh} &= 246 \text{ GeV} \\
T_{reh} &= 10^{10} \text{ GeV} \\
T_{reh} &= 10^{14} \text{ GeV} \\
N_{inf} &= 60 \\
N_{inf} &= 55 \\
N_{inf} &= 50 \\
N_{inf} &= 45 \\
\lambda / \xi^2 &= 4.0 \times 10^{-10} \\
\lambda / \xi^2 &= 5.0 \times 10^{-10} \\
\lambda / \xi^2 &= 6.5 \times 10^{-10}
\end{align*}
Since $g_{\text{reh}} = g_{\text{EW}} = 106.75$ is fixed for Higgs inflation, we are able to determine the allowed regions of $Z$ from (29) given the uncertainty from Planck normalization $A_s$, \[ g_{\text{reh}}(Z; A_s, N_{\text{reh}}, w_{\text{reh}}) = g_{\text{EW}} = Z(A_s) \] (30) after which the reheating history $N_{\text{reh}}, w_{\text{reh}}$ are presumed. Once the allowed regions of $Z$ is clarified, one obtain the more stringent ranges for every other observables like \[ n_s(A_s) \equiv n_s(\phi_s(Z(A_s); A_s)) \] (31) \[ r(A_s) \equiv r(\phi_s(Z(A_s); A_s)) \] (32) The results are shown in Figure 1.

The illustrations for Figure 1 are as follows. Given the reheating history $N_{\text{reh}}, w_{\text{reh}}$, we draw a blue line $g_{\text{reh}}(Z)$ as a function of $Z$ for every input $A_s$. The blue line $g_{\text{reh}}(Z)$ will be stretched into a blue ribbon $g_{\text{reh}}(Z; A_s)$ due to Planck 2015 TT,TE,EE+lowP[2] to $\ln(10^{10} A_s) = 3.094 \pm 0.0034$. Intersection of blue ribbon $g_{\text{reh}}(Z; A_s)$ with green line $g_{\text{reh}} = g_{\text{EW}} = 106.75$ will project the allowed range for $Z$. Given the obtained range of $Z$, we can project $n_s$ within the range with its width one order magnitude smaller than the current constraint $n_s = 0.9645 \pm 0.0049$ from Planck 2015 TT,TE,EE+lowP[2]. The left panel of Figure 1 is shown for canonical reheating history $N_{\text{reh}} = 10, w_{\text{reh}} = 0$. We also plot for instantaneous and long-term reheating histories in Figure 1 to cover the best-fit value of $n_s$.

The implications from Figure 1 are as follows. For instantaneous reheating history, the projected range of $n_s$ lies near the best-fit value of $1 \sigma$ region constrained by Planck 2015 TT,TE,EE+lowP[2]. Increasing the e-folding number of $N_{\text{reh}}$ will lower the projected range of $n_s$ with its uncertainty width barely changed. The achieved uncertainty width of $n_s$ is one order magnitude smaller than the current constraint. To better understand the interplay between inflationary era and reheating era, we need to plot the reheating phase diagram.

Now we illustrate how to plot reheating phase diagram from (29) with all kinds of reheating configurations. Given the value $A_s$ of Planck normalization, we can solve the consistency equation \[ g_{\text{reh}}(Z; A_s, N_{\text{reh}}, w_{\text{reh}}) = g_{\text{EW}} = Z(N_{\text{reh}}, w_{\text{reh}}) \] (33) to get the combined parameter $Z(N_{\text{reh}}, w_{\text{reh}})$ as a function of reheating history $N_{\text{reh}}, w_{\text{reh}}$. Hence we can express every other observables, for example, \[ n_s(N_{\text{reh}}, w_{\text{reh}}) \equiv n_s(\phi_s(Z(N_{\text{reh}}, w_{\text{reh}}); A_s)) \] (34) \[ r(N_{\text{reh}}, w_{\text{reh}}) \equiv r(\phi_s(Z(N_{\text{reh}}, w_{\text{reh}}); A_s)) \] (35) in terms of phase variables $N_{\text{reh}}, w_{\text{reh}}$ on the reheating phase diagram as shown in Figure 2. It turns out that contour lines differed only by input values of $A_s$ appear indistinguishable on reheating phase diagram subjected to $n_s, r, \alpha_s, T_{\text{reh}}, N_{\text{inf}}$ except that subjected to $Z$. It can be seen in Figure 2 that almost all possible reheating histories are allowed within the $1\sigma$ region of $n_s$ reported by Planck 2015 TT,TE,EE+lowP[2]. Therefore cosmological predictions of Higgs inflation are highly insensitive[51] to its reheating history. However the reheating history should be appreciated when the future measurement of $n_s$ with refined precision up to 1%.

5. Conclusions

In this letter we investigate the long-standing suspicion that how significant the impact from reheating history can be on the predictions of Higgs inflation. We answer this question by plotting various cosmological predictions of Higgs inflation on a phase diagram characterizing its reheating history. The method we adopted is exact and the results manifest a highly insensitivity of inflationary predictions with respect to its reheating phase. We can even plot reheating phase diagrams for more general inflation models at the price of appropriate approximations and presumptions. However we expect the outcomes of other inflationary models will be effectively the same as Higgs inflation model.

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Note added. The paper [51] showed up on arXiv while we were preparing our manuscript. We both adopt the same method to study similar problem of Higgs inflation but with different angle. We characterize the reheating phase with $N_{\text{reh}}$ and $w_{\text{reh}}$ and express every other observables in terms of these two phase variables in an exact way by using $g_{\text{reh}} = g_{\text{EW}}$ within Higgs inflation. The impact from various reheating histories on the inflationary predictions can be shown in more transparent manner on the so-called reheating phase diagram.

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