II - LOCAL SOLUTION

of

A SPHERICAL HOMOGENEOUS AND ISOTROPIC UNIVERSE

radially

DECELERATED TOWARDS THE EXPANSION CENTER :

TESTS ON HISTORIC DATA SETS

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ABSTRACT

The topic of the paper is the mathematical analysis of a radially decelerated Hubble expansion from the Bahcall & Soneira void center ($\alpha \approx 9^h; \delta \approx +30^0$). Such analysis, based on the result $K_0 = (\delta H/\delta r)_0 = 3H_0^2/c$ of Paper I, in the hypothesis of local homogeneity and isotropy, gives a particular Hubble ratio dipole structure to the expansion equation, whose solution has been studied at different precision orders and successfully tested on a few historic data sets, by de Vaucouleurs (1965), by Sandage & Tammann (1975), and by Aaronson et al. (1982-86). The fittings of both the separate AA1 and AA2 samples show a good solution convergence as the analysis order increases, giving even coinciding solutions when applied to 308 nearby individual galaxies (308AA1) and to 10 clusters (148AA2), respectively.

As a result one obtains:

$R_0 \approx 210 \ Mpc \ and \ H_0 = 88.0 \pm 1.6 \ Km \ s^{-1} Mpc^{-1}$ by Aaronson et al., or

$R_0 \approx 260 \ Mpc \ and \ H_0 = 70 \pm 3 \ Km \ s^{-1} Mpc^{-1}$ by Sandage & Tammann.
In conclusion an optional further graphical check of the model is presented by plotting a series of Hubble ratio average dipoles.

1. INTRODUCTION

The present paper, as a natural continuation of the previous one (I), has the function of carrying out a series of cosmological tests at observational experimental level. The checks have been based on a few very meaningful historic data samples, in order to show the general faithfulness of the modelled radial Hubble expansion from the Bahcall and Soneira void center. Of course the application to more recent data must follow immediately, but the implications are so crucial that it is still necessary to confirm the correctness of the model, actually starting from ancient data like those of the 1960’s by de Vaucouleurs, continuing through the historic ’70’s years measures by Sandage & Tammann, to the more recent results of the ’80’s years by Aaronson et al.. All these data refer to a nearby Universe where ”nearby” here means a distance range of about 100 Mpc.

1.1 Local homogeneity and isotropy

In this ”nearby” entourage we have assumed that the Universe is homogeneous and isotropic, according to the Cosmological Principle. Indeed, modern Cosmology has been searching for far away homogeneity for a long time, on a large scale, as the Galaxy environment seems to show too many irregularities. However, our different starting point is different only in appearance, as the assumed local homogeneity and isotropy depends on the adopted spherical model in which homogeneity and isotropy can be averagely attributed to all the volume containing the huge void of Bahcall & Soneira, that is all the inner Universe and its expansion center. Of course, in this case the effects on our entourage are determined by our inner Universe while those on more remote clusters should be tied to their larger or smaller inner Universe. Now, based on the preliminary results referred to on paper I, the quadrupole space effect $Q (\Delta H_{ga} \rightarrow 0)$ seems negligible for a depth of about 100 Mpc; and this means the adoption of the homogeneity and anisotropy is the correct condition to study the samples of the nearby Universe.
1.2 Apparent anisotropy

On the grounds of the fundamental expansion equation obtained in paper I:

\[
\dot{r} = Hr + \Delta H \cdot (r - R \cos \gamma)
\] (1)

and of the basic assumption of homogeneous and isotropic Universe, the finite difference \(\Delta H\) should be zero in order to avoid any space dependence. Indeed that will be the case in absence (see paper I) both of the above cited space effect (SE)\((\Delta H_{ga} = 0)\) and of the other light time effect (TE)\((\Delta H_{MW} = 0)\); in particular the latter zero TE would mean that the light propagation is instantaneous, or, that the speed of light has an infinite value. In this hypothetical context, and only in this, the observed Universe should look exactly like that foreseen by the pure Hubble law: \(\dot{r} = Hr\), independently of the modelled decelerating radial expansion from a definite center. In other words, if the quantities \(\dot{r}\) and \(r\) refer to the same moment, the canonical Hubble law holds , and \(\Delta H\) is zero; otherwise, if \(\dot{r}\) is the observed \(\dot{r}_{\text{obs}}\), its value will be affected by two different epochs due to the light delay linked to our decelerated Galaxy, while \(r\) as light space distance will represent only the emission epoch owing to the constancy of the light speed; and so even a homogeneous and isotropic Universe will have to look like that described by the varied Hubble law of Eq. (1). Hence, the adopted status of homogeneity and isotropy must refer to the same instant, or epoch, of our proper time, while the observed anisotropy according to Eq. (1) with \(\Delta H = \Delta H_{MW}\), in spite of its strong observational evidence, practically is only apparent and due to a prospect effect generated by the light delay.
2. ANALYSIS

2.1 THE EXPANSION EQUATION ACCORDING TO \( K_0 = \frac{3H_0^2}{c} \)

In order to solve the fundamental expansion Eq. (1), once accepted the experimental result \( K_0 = \frac{3H_0^2}{c} \) of Paper I in the assumed case of homogeneous and isotropic spherical universe, one needs to substitute \( H, \Delta H, R \) with the formulas (38)(40) by Paper I. The result is:

\[
\frac{\dot{r}_{\text{obs}}}{r} = \frac{H_0 c}{c - 3H_0 r} + \frac{3H_0^2}{c - 3H_0 r} \left[ r - R_0 \left( 1 - \frac{3H_0 r}{c} \right)^{\frac{1}{2}} \cos \gamma \right]
\]

(2)

that, after a short processing, takes the following useful formulation:

\[
\frac{\dot{r}_{\text{obs}}}{r} = H_0 \left( \frac{1 + x}{1 - x} \right) - K_0 R_0 \cdot (1 - x)^{-\frac{2}{3}} \cos \gamma
\]

(3)

where there are:

\( \dot{\frac{r_{\text{obs}}}{r}} \) : variable quantity as Hubble ratio

\( H_0 \) : Hubble Constant

\( K_0 R_0 = \frac{3c}{2} \frac{2H_0^2 R_0}{c^2} = a_0 \) : Galaxy radial deceleration coefficient (see Eq. (41) in paper I)

\( x = \frac{3H_0 r}{c} \) : variable quantity whose value in the nearby Universe is \( \ll 1 \)

\( \cos \gamma \) : variable quantity according to the observed position

Furthermore, applying the Mc Laurin Series, we have:

\[
\frac{1 + x}{1 - x} = 1 + 2x + 2x^2 + ......\]

(4)

while, according to the binomial series, it is:

\[
(1 - x)^{-\frac{2}{3}} = 1 + \frac{2}{3} x + \frac{5}{9} x^2 + ......\]

(5)

The previous two series developments allow to study the expansion Eq. (3) at different precision orders, and consequently to verify the solution convergence.
2.1.1 0th order analysis

The 0th order formulation of Eq. (3) is that following the stopping of the series at the zero order term, that is at 1. In this extreme case, corresponding to \( r \to 0 \), the expansion equation becomes:

\[
\frac{\dot{r}_{\text{obs}}}{r} = H_0 - K_0 R_0 \cos \gamma
\]  

(6)

The aforesaid extremely simplified formula has importance especially in the graphical applications, when referred only to a very nearby Universe, of course. Here the Hubble ratios plotted against \( \cos \gamma \) have to dispose themselves according to an **indisputable dipole structure**, displayed by the verified deceleration angular coefficient \( K_0 R_0 = a_0 \).

2.1.2 1st order analysis

The first order formulation of Eq. (3) is that following the stopping of the series (4)(5) at the first order term, which are \( 2x \) and \( \frac{2}{3}x \) respectively. Here, after the substitution \( x = \frac{3H_0 r}{c} \), the resulting equation is:

\[
\frac{\dot{r}_{\text{obs}}}{r} = H_0 \left(1 + \frac{6H_0 r}{c}\right) - a_0 \left(1 + \frac{2H_0 r}{c}\right) \cos \gamma
\]  

(7)

that, in terms of 2nd degree algebraic equation, becomes:

\[
\frac{\dot{r}_{\text{obs}}}{r} + a_0 \cos \gamma = H_0 \left(1 - 2a_0 \frac{r \cos \gamma}{c}\right) + \frac{6r}{c} H_0^2
\]  

(8)

Solving the above 2nd degree equation in \( H_0 \) could mean finding, by the least square method, the value of the deceleration parameter:

\[
a_0 = \frac{3H_0^2 R_0}{c}
\]  

(9)

, hence the \( R_0 \) value corresponding to the resulting \( H_0 \), able to minimize the standard deviation, when an homogeneously distributed sample of \( \dot{r} \) and \( r \) measures of the nearby Universe is available.

Even the 1st order Eq. (7) may have a graphical representation like that of Eq. (6), but now, being \( r = r_* \) the average distance of a not dispersed sample, we shall have a paper I (68)-type formula to represent an **average dipole**, with \( H_* = H_0 + 2K_0 r_* > H_0 \) and \( R_* = R_0 \left(1 + \frac{2H_0 r_*}{c}\right) > R_0 \), that is

\[
\frac{\dot{r}_{\text{obs}}}{r} = H_* - K_0 R_* \cos \gamma
\]  

(10)
2.1.3 Nth order analysis

The Nth order formulation is that directly following the expansion equation (2), now written as:

\[ \frac{\dot{r}_{\text{obs}}}{r} \left(1 - \frac{3H_0r}{c}\right) = H_0 + \frac{3H_0^2}{c}r - \frac{3H_0^2R_0}{c} \cos \gamma \left(1 - \frac{3H_0r}{c}\right)^{\frac{1}{2}} \]

(11)

with the only binomial \( \left(1 - \frac{3H_0r}{c}\right)^{\frac{1}{2}} \) developed by series, according to:

\[ \left(1 - \frac{3H_0r}{c}\right)^{\frac{1}{2}} = 1 - \frac{H_0r}{c} - \frac{H_0^2r^2}{c^2} - \Sigma \]

(12)

being

\[ \Sigma = \frac{5}{3} \left(\frac{H_0r}{c}\right)^3 + \frac{10}{3} \left(\frac{H_0r}{c}\right)^4 + \frac{22}{3} \left(\frac{H_0r}{c}\right)^5 + ... \]

(13)

Then Eq. (11), after a short processing with the (9) adoption, gives the following 2nd degree algebraic equation in \( H_0 \):

\[ H_0^2 \left(3\frac{r}{c} + a_0\frac{r^2 \cos \gamma}{c^2}\right) + H_0 \left(1 + 3\frac{\dot{r}}{c} + a_0 \frac{r \cos \gamma}{c}\right) - \left(\frac{\dot{r}}{r} + a_0 \cos \gamma - a_0 \Sigma \cos \gamma\right) = 0 \]

(14)

whose solution could mean finding, using the least square method, the value of the deceleration parameter \( a_0 \) (9), that is the corresponding \( R_0 \), able to minimize the standard deviation of the fitting carried out on a sample of galaxies/groups/clusters/superclusters, having computed \( \Sigma \) by successive approximations of \( H_0 \).

2.1.4 The \( q_0 \) test

In order to check the most correct solution, it is now possible to apply a powerful test based on the conservation of the parameter \( q_0 \), that is of

\[ q_0 = -\frac{H_0R_0}{c} \]

(15)

which, keeping in mind the previous paper I, represents the Galaxy recession velocity with respect to the expansion center.

Starting from the 0th order equation (6)(but the same result comes from higher orders), that holds for \( r \to 0 \), it is easy to show the \( q_0 \) conservation, rewriting the (6) as follows:

\[ \dot{r}_{\text{obs}} = H_0r(1 + 3q_0 \cos \gamma) \]

(16)
As samples may differ practically because of different absolute estimates of distance, the differential of eq. (16), according to the error propagation, tells us that the same radial velocity $\dot{r}_{\text{obs}}$ of any group or galaxy, that is $d\dot{r}_{\text{obs}} = 0$, implies the following logic sequence:

$$\text{IF } \frac{dR_0}{R_0} = \frac{dr}{r} \text{ THEN } \frac{dr}{r} = -\frac{dH_0}{H_0} = \frac{dR_0}{R_0} \Rightarrow q_0 = \text{constant}$$

(17)

3. RESULTS

3.1 PRELIMINARY TEST ON HISTORIC NEARBY SAMPLES

Having stated this, firstly let’s go on to check the 0th and 1st order solution applying Eqs. (6) and (8) to two historic data samples of very nearby galaxy Groups, all inside a distance range of 30 Mpc, by means of least square linear fittings and assuming $\gamma = 0^\circ$ at $\alpha \approx 9^h$, $\delta \approx +30^0$ or $l \approx 195^0$, $b \approx +40^0$ or $SGL \approx 60^0$, $SGB \approx -35^0$ (VC coordinates) and $\gamma = 180^0$ at $\alpha \approx 21^h$, $\delta \approx -30^0$ (AVC=anti void center). The group position is considered applying the trigonometrical function cosine of the observed angle $\gamma$ between the direction of VC and that of the galaxy group:

$$\cos \gamma = \sin \delta_{VC} \sin \delta + \cos \delta_{VC} \cos \delta \cos (\alpha - \alpha_{VC})$$

(18)

The samples are the following:

3.1.1 Groups by de VAUCOULEURS

DV sample:- 52 nearby Groups of galaxies (368 in all), whose preliminary elements had been reported by de Vaucouleurs (1965-Table 2) thirty-five years ago. Weighing each group by the number of galaxies, $w = n_{\text{obs}}$, two fitting solutions follow, at 0th and 1st order respectively, being $s$ the standard deviation or $s_{\text{MIN}}$ the minimum one varying $a_0$; that are:

$$0^0: \quad a_0 = 23.2 \pm 1.8 \; ; \; H_0 = 108.2 \pm 1.1 \; \text{at} \; s = 17.60 \Rightarrow R_0 \simeq 198 \Rightarrow q_0 \simeq -0.0715 \quad (19)$$

$$1^0: \quad s_{\text{MIN}} = 16.5254 \; \text{at} \; a_0 = 22.6 \; ; \; H_0 = 105.3 \pm 0.9 \; \Rightarrow R_0 \simeq 204 \Rightarrow q_0 \simeq -0.0715 \quad (20)$$

The corresponding unweighed fittings, with $w = 1$ given to each group, don’t significantly modify the previous solutions.
3.1.2 Groups by SANDAGE & TAMMANN

S&T sample: 20 nearby Groups of galaxies plus 10 single galaxies from data summarized by Sandage and Tammann in Table 2 of their paper V (1975). On the whole these data, when fitted directly with $w = 1$ according to the 0th order eq.(6), give a solution which confirms the same Sandage & Tammann $H_0$ value reported in (8) at page 319 of paper V, that is:

$$0^0_{w=1} : a_0 = 1.2 \pm 7.3; \ H_0 = 57 \pm 5 \ at \ s = 18.94 \Rightarrow R_0 \approx 0$$  \hspace{1cm} (21)

Note how the previous unweighed 0th order fitting doesn’t produce any significant dipole, seeing as $a_0 = 1.2 \pm 7.3$, which practically supports no radius owing to the error oversize.

Otherwise, we can now limit the analysis only to the 20 weighed Groups of galaxies (187 in all with $\langle r \rangle = 14.8$), without the 10 field single galaxies. Weighing each group by the number of galaxies ($w = n_{obs}$), that emerges in Table 1 of the same paper V, the 0th and 1st order weighed fittings of the Group data, here summarized for facility in our Table 1 where the Virgo Cluster of S&T-V Table 2 (1975) has weight $w = 32$ according to S&T Table 4 of paper IV (1974), change as follows:

$$0^0 : a_0 = 9.5 \pm 3.0; \ H_0 = 66.4 \pm 1.9 \ at \ s = 17.64 \Rightarrow R_0 \simeq 215 \Rightarrow q_0 \simeq -0.0477 \hspace{1cm} (22)$$

$$1^0 : s_{MIN} = 17.2997 \ at \ a_0 = 9.33; \ H_0 = 65.1 \pm 1.3 \Rightarrow R_0 \simeq 220 \Rightarrow q_0 \simeq -0.0478 \hspace{1cm} (23)$$

The results (22)(23) are particularly meaningful because they confirm the model with $w = n_{obs}$ by means of a correct radius $R_0$ evaluation.; on the other side the corresponding weighed fitting, that has surely more sense in a group analysis, reverses the result with respect to the unweighed one of (21), being now $a_0 = 9.5 \pm 3.0$ and $R_0$ about 220 Mpc from (9).

3.1.3 Individual galaxies by SANDAGE & TAMMANN

S&T sample: 83 individual galaxies from data summarized by Sandage and Tammann in Table 3 and Table 4 of their paper V (1975) plus the above cited 10 single galaxies of Table 2 (S&T-V). Here the galaxy sample, whose average distance results to be about 28.3 Mpc, is distributed along a wider distance range with respect to that of Groups; so in this case the least square fitting of the whole of 83 equally weighed galaxies has been carried out using the 0th , 1st and 5th order eqs. (6)(8)(14), from which it follows respectively:
These three solutions clearly show a fitting precision improvement with the increase of analysis order.

Finally, considering as a whole the combined sample of 20 weighted Groups of 187 galaxies plus the above 83 individual galaxies, that is to say all the data of Table 2-3-4 (S&T-V) together, the resulting solution

\[ s_{MIN} = 20.3590 \text{ at } a_0 = 11.70; H_0 = 67.3 \pm 1.2 \Rightarrow R_0 \approx 258 \Rightarrow q_0 \approx -0.0580 \]  

still falls into the error range of that obtained by individual galaxies only, even if the Group analysis may be a bit affected both by the weighing procedure and by the computing method of the Hubble ratios, which according to (8) should all be assembled as \( \langle \dot{r} \rangle \).

### 3.2 SOLUTIONS BY THE AARONSON et al. CATALOG

The opportunity to use homogeneous samples of data comes both from the AA1 sample of 308 nearby individual galaxies (Aaronson et al., 1982), and from another separate sample of 148 galaxies in 10 clusters (AA2), by Aaronson et al. (1986). This data set, representing a physical whole of galaxies/groups/clusters, has the great advantage of covering more than 100 Mpc in depth and of possessing absolute estimates of distances. In fact the original data listed in Table 3 of the 1982 Aaronson et al. paper, plus those listed in Table 2 of the successive 1986 work, adopting the Tully-Fisher relation calibrated by Aaronson et al. (1986, p. 550), permit the calculation of the galaxy distances and related Hubble ratios, and consequently of the average \( \langle r \rangle \) and \( \langle \dot{r}/r \rangle \) of the involved groups and clusters. These Hubble ratios, \( \dot{r} \) including the standard correction of \( 300 \sin l \cos b \), result to be corrected by the motion of the Sun in the Local Group, in practice due to galactic rotation, with the standard vector of \( 300 \text{ Kms} \text{ toward } l = 90^0, b = 0^0 \) (cf. Sandage & Tammann, 1975); this means we consider Hubble ratios as seen from our Local Group, or from the Galaxy, the Milky Way, as being almost motionless within its Group.

So the obtained Hubble ratios refer to the observed Hubble flow.

Now we shall proceed through two steps of numerical analysis, referring to individual galaxies and normal Groups/Clusters, respectively.

| Order | \( s_{MIN} \) | \( a_0 \)   | \( H_0 \)   | \( R_0 \approx \) | \( q_0 \approx \) |
|-------|---------------|------------|------------|-----------------|----------------|
| 0°    | 27.8638       | 11.9       | 72.7 ± 3.1 | 225             | −0.0546        |
| 1°    | 25.8267       | 12.64      | 69.8 ± 2.8 | 259             | −0.0604        |
| 5°    | 25.7606       | 12.66      | 69.8 ± 2.8 | 260             | −0.0605        |
3.2.1 1° Step: Individual galaxy solution

The first check step starts by applying the 0th order (6), the 1st order (8), the 5th order (14) equations separately, to all the AA1 sample of 308 individual nearby galaxies, whose distance mean is \( \langle r \rangle = 16.13 \) Mpc, and to the other AA2 sample of 148 more distant individual galaxies, with \( \langle r \rangle \approx 70.69 \) Mpc. Both the samples may be considered homogeneous and rich enough, even if affected by large scattering in distance. Indeed, being the average individual distance of AA2 about 5 times greater than that of AA1, the involved solutions and their confrontation take on a special meaning. The least square method applied to the algebraic system of the 308 and 148 equations (6)(8)(14), respectively, gives the solutions listed in the small table below, where \( s_{MIN} \) is the minimum standard deviation of the fit corresponding to the listed \( a_0 \) value, and all the quantities are in Hubble units:

| Sample  | Order | \( s_{MIN} \) | \( a_0 \) | \( H_0 \) | \( R_0 \approx q_0 \approx \) |
|---------|-------|----------------|---------|----------|-----------------|
| 308AA1  | 0°    | 28.6243        | 16.4    | 90.6 ± 1.6 | 200 -0.060 |
| 148AA2  | 0°    | 19.7237        | 15.4    | 98.8 ± 1.6 | 158 -0.052 |
| 308AA1  | 1°    | 27.5725        | 16.35   | 88.0 ± 1.6 | 211 -0.062 |
| 148AA2  | 1°    | 18.0575        | 16.6    | 88.7 ± 1.5 | 211 -0.062 |
| 308AA1  | 5°    | 27.5508        | 16.3    | 88.0 ± 1.6 | 210 -0.062 |
| 148AA2  | 5°    | 18.0072        | 16.8    | 88.2 ± 1.5 | 216 -0.063 |

The above table shows a good solution convergence of \( H_0 \) and \( R_0 \) with an increase of the analysis order, and a corresponding decrease of the standard deviation absolute value. That is very important and meaningful to the correctness of the model because of the large difference in distance between the AA1 and the AA2 samples. Furthermore the table gives clear indication of the rising trend of \( R_0 \) still with the order increase.

3.2.2 2° Step: Group\Cluster solution

The 2°step solution refers always to the Aaronson et al. catalogue, now as groups and clusters. Indeed the appropriate Eqs. (6)(8)(14) in two unknowns, after transforming to represent the average group or cluster Hubble ratio, should now have \( \langle \frac{\dot{r}}{r} \rangle, \langle r \rangle, \langle \dot{r} \rangle, \langle \cos \gamma \rangle, \langle r \cos \gamma \rangle, \langle r^2 \cos \gamma \rangle, \ldots \)

But, having here verified that the results are unaffected, we have more practically used \( \cos \gamma_{\text{cluster}}, \langle r \cos \gamma \rangle, \langle r^2 \cos \gamma \rangle, \ldots \) instead of \( \langle \cos \gamma \rangle_{\text{cluster}}, \langle r \cos \gamma \rangle, \langle r^2 \cos \gamma \rangle, \ldots \)

The relative group and cluster data have been reported in Table 2 and Table 3, where 31 nearby
groups (139 galaxies) and 11 nearby clusters (164 galaxies) of the Aaronson data set (Aaronson et al., 1982-1986) are listed, respectively. The quantities tabulated are: the galaxy number for groups and clusters, \( n_{\text{obs}} \); the average equatorial coordinates, \( \langle \alpha \rangle \) and \( \langle \delta \rangle \); the average functions, \( \langle \cos \gamma \rangle_{\text{group}} \) or \( \cos \gamma_{\text{cluster}} \), between the direction of the Bahcall & Soneira (1982) void center and that of the group members or cluster; the group/cluster average observed velocity, \( \langle \dot{r} \rangle \); the group/cluster average distance, \( \langle r \rangle \); and lastly the average Hubble ratio, \( \langle \frac{\dot{r}}{r} \rangle \).

The least square method applied to the algebraic system of the 31 group (31GR) and 11 cluster (11CL) equations (6)(8)(14), using \( n_{\text{obs}} \) as weights in the fitting, gives the solutions listed in the table below, where in addition 10CL represents the same Aaronson 11 cluster sample without the nearby Virgo cluster (cf. Table 6 by Aaronson et al., 1986):

| Sample | Order | \( s_{MIN} \) | \( a_0 \) | \( H_0 \) | \( R_0 \simeq \) | \( q_0 \simeq \) |
|--------|-------|-------------|--------|--------|----------------|----------------|
| 31GR   | 0\(^0\) | 12.756      | 18.5   | 91.34 ± 1.08 | 222   | −0.068          |
| 11CL   | 0\(^0\) | 4.7733      | 16.8   | 98.07 ± 0.37 | 175   | −0.057          |
| 10CL   | 0\(^0\) | 3.6965      | 15.4   | 98.79 ± 0.30 | 158   | −0.052          |
| 31GR   | 1\(^0\) | 12.395      | 17.8   | 88.99 ± 1.05 | 225   | −0.067          |
| 11CL   | 1\(^0\) | 4.2090      | 17.1   | 88.29 ± 0.33 | 219   | −0.065          |
| 10CL   | 1\(^0\) | 4.3889      | 16.8   | 88.42 ± 0.36 | 215   | −0.063          |
| 31GR   | 5\(^0\) | 12.498      | 18.16  | 88.94 ± 1.06 | 229   | −0.068          |
| 11CL   | 5\(^0\) | 4.3430      | 17.2   | 87.96 ± 0.34 | 222   | −0.065          |
| 10CL   | 5\(^0\) | 4.5501      | 16.9   | 88.04 ± 0.37 | 218   | −0.064          |

The most impressive result of table (25) and table (26), is the practical coincidence between the 308AA1-5\(^{th}\) order solution with the 148AA2-5\(^{th}\) order one, or with that of the 10CL-5\(^{th}\) order sample. The only light difference refers to the \( R_0 \) value derived from \( a_0 \). Table (26) again confirms the rising trend of \( R_0 \) with the order increase.

For completeness of the (29) Group analysis, a further 1\(^{st}\) order fitting test on the data listed in Table 2 has been carried out now using as Hubble ratios those resulting from the division of the average radial velocity \( \langle \dot{r} \rangle \) with the average distance \( \langle r \rangle \), that is \( \frac{\langle \dot{r} \rangle}{\langle r \rangle} \) instead of \( \langle \frac{\dot{r}}{r} \rangle \). Indeed this procedure doesn’t seem correct because of the assumed application of the expansion equation (8) to elementary splinters as the individual galaxies are, according to the toy model of Paper I. But at the same time this method of Hubble ratio calculation is the only possible one which permits further analyses of far clusters and superclusters. Consequently here we report, as example, a
second 1st order solution of the 31GR sample, which is:

$$31AA1GR \left( \frac{t}{s} \right) : s = 11.35572 \text{ at } a_0 = 17.8; H_0 = 86.3 \pm 1.0 \Rightarrow R_0 \approx 239 \Rightarrow q_0 \approx -0.069 \quad (27)$$

Of course all the previous solutions strictly depend on the absolute calibration carried out by Aaronson et al. in their 1986 paper. It is interesting to note, only as an example, how a complete coincidence of $H_0$ and $R_0$ of the individual galaxy sample 308AA1 with the equivalent ones by Sandage & Tammann, that is with the solution (26), may be simply obtained through an increment of half negative magnitude to the zero-point of the quadratic IR/H relation (Aaronson et al., 1986-p.550).

4. GENERAL SOLUTION

The $q_0$ constancy condition means that the product $H_0R_0$ is invariable in presence of systematic variation of the distance estimates; so, in order to discriminate among the above historic samples by de Vaucouleurs, Sandage & Tammann and Aaronson et al., we can now directly proceed to test the above listed different solutions.

Indeed only the individual galaxy samples seem to satisfy sufficiently the conservation condition of $q_0$. In fact, it being important to take into account both the meaningful representation in terms of number of galaxies and the careful reliability shown by more coinciding solutions of the Aaronson et al. catalogue, the 308AA1 5th order solution can be rightly taken as pilot reference solution in our comparison procedure, which, in the light of the verified deviation of $q_0$, gives acknowledgement only to the validity of the 83 individual galaxy sample by Sandage & Tammann (1975).

Consequently to this $q_0$ test, at present, the general solution which faithfully represents all the previous numerical analysis, may be summarized by the alternative and separate (23)/(25) results, as follows:

$$Sample - 83S&T IG : H_0 = 70 \pm 3; R_0 \approx 260 \Rightarrow H_0R_0 \approx 18200Km/s \quad (28)$$

$$Sample - 308AA1 IG : H_0 = 88.0 \pm 1.6; R_0 \approx 210 \Rightarrow H_0R_0 \approx 18500Km/s \quad (29)$$
5. CONCLUSIONS

5.1 MINI CHECK ATLAS OF HUBBLE RATIO DIPOLES

(Optional section, now limited to 7 plotted Hubble ratio dipoles (see Figures 2-3-4-5-6-7-8))

REFERENCES

Aaronson, M. et al. 1982, Ap. J. Suppl. Series, 50, 241
Aaronson, M. et al. 1986, Ap. J. 302, 536
Bahcall, N.A. and Soneira, R.M. 1982, Ap. J. 262, 419
Lorenzi, L. 1996, Astro. Lett. & Comm., 33, 143
(1994-Grado3 Proc., SISSA ref. 155/94/A)
1995c, Sesto Pusteria International Workshop Book, eds.-SISSA ref. 65/95/A
1993, 2nd National Meeting on Cosmology, eds. Mem. S.A.It., V. 66, N. 1-1995
1991, Contributo N. 1, Centro Studi Astronomia-Mondovi-Italy
1999-I, (submitted)
Sandage, A. and Tammann, G.A., 1974, Ap. J. 194, 559
Sandage, A. and Tammann, G.A., 1975, Ap. J. 196, 313
Vaucouleurs, G. de 1965, in "Galaxies and the Universe", 1975, Vol. IX of Stars and
Stellar Systems, p. 566, The University of Chicago Press
| GROUP         | $n_{\text{obs}}$ | $L_{SG}$ | $B_{SG}$ | $\cos \gamma$ | $\dot{r}$ | $r$ | $\dot{r}/r$ |
|---------------|------------------|----------|----------|----------------|-----------|-----|-------------|
| $G$ 253       | 9                | 265      | -3       | -0.7114        | 281       | 3.4 | 82.6        |
| $G$ 672       | 2                | 327      | -4       | -0.0028        | 540       | 10.9| 49.5        |
| $G$ 1023      | 6                | 342      | -9       | +0.2579        | 721       | 14.3| 50.4        |
| $G$ 1068      | 6                | 305      | -26      | -0.0597        | 1332      | 18.1| 73.6        |
| $Eridanus G$  | 22               | 278      | -43      | -0.0809        | 1520      | 22.8| 66.7        |
| $G$ 2841      | 7                | 50       | -16      | +0.9336        | 601       | 7.6 | 79.1        |
| $G$ 2985      | 2                | 39       | +3       | +0.7337        | 1274      | 16.1| 79.1        |
| $G$ M81       | 10               | 42       | +1       | +0.7689        | 226       | 3.25| 69.1        |
| $G$ 3184      | 7                | 64       | -16      | +0.9436        | 673       | 15.4| 43.7        |
| $G$ Leo       | 20               | 94       | -26      | +0.8618        | 799       | 23.4| 34.1        |
| $G$ 3938      | 5                | 71       | 0        | +0.8041        | 873       | 20.3| 43.0        |
| $G$ CVnI      | 11               | 69       | +6       | +0.7447        | 339       | 5.0 | 67.8        |
| Virgo CL      | 32               | 104      | -2       | +0.6089        | 1111      | 19.8| 56.1        |
| $G$ Coma I    | 15               | 88       | +2       | +0.7028        | 922       | 10.2| 90.4        |
| $G$ CVn II    | 7                | 76       | +6       | +0.7232        | 698       | 7.6 | 91.8        |
| $G$ 5128      | 6                | 160      | -5       | -0.0917        | 317       | 7.9 | 40.1        |
| $G$ M51       | 4                | 72       | +17      | +0.5985        | 606       | 9.7 | 62.5        |
| $G$ M101      | 2                | 64       | +23      | +0.5281        | 402       | 7.2 | 55.8        |
| $G$ 6643      | 8                | 30       | +31      | +0.3127        | 1842      | 24.4| 75.5        |
| $G$ IC 342    | 6                | 11       | 0        | +0.5374        | 122       | 4.5 | 27.1        |
| GROUP       | $n_{\text{obs}}$ | $\langle \alpha \rangle$ | $\langle \delta \rangle$ | $\langle \cos \gamma \rangle$ | $\langle \dot{r} \rangle$ | $\langle r \rangle$ | $\langle \dot{\gamma} \rangle$ |
|-------------|------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| N24/45      | 2                | 0°.158                   | 24°.35                   | −0.74040                 | 553.38                   | 7.92                     | 72.64                    |
| N134        | 2                | 0°.497                   | −30°.8                   | −0.70834                 | 1581.5                   | 14.47                    | 109.68                   |
| SCULPTOR    | 3                | 0°.472                   | −26°.49                  | −0.69173                 | 225.55                   | 2.89                     | 78.34                    |
| N701/755    | 2                | 1°.854                   | −9°.63                   | −0.33592                 | 1766.9                   | 15.83                    | 111.72                   |
| N1023       | 5                | 2°.395                   | +32°.4                   | +0.14833                 | 1053.1                   | 9.98                     | 102.59                   |
| ERI DANUS   | 8                | 3°.288                   | −24°.21                  | −0.14530                 | 1478.7                   | 14.66                    | 104.40                   |
| FORNAX      | 7                | 3°.523                   | −34°.16                  | −0.18237                 | 1406.0                   | 13.58                    | 107.70                   |
| N2403 − M81 | 6                | 8°.648                   | +69°.09                  | +0.71693                 | 186.28                   | 2.52                     | 82.28                    |
| N2336       | 3                | 7°.228                   | +81°.57                  | +0.60823                 | 2349.7                   | 25.38                    | 93.00                    |
| N2841       | 3                | 8°.697                   | +51°.35                  | +0.92511                 | 681.90                   | 11.91                    | 58.11                    |
| N3079/U5459 | 2                | 10°.029                  | +54°.62                  | +0.89064                 | 1193.6                   | 16.16                    | 73.94                    |
| N3184       | 3                | 10°.571                  | +41°.54                  | +0.92196                 | 680.92                   | 12.10                    | 56.70                    |
| LEO         | 5                | 11°.064                  | +14°.26                  | +0.83873                 | 1016.3                   | 16.54                    | 62.54                    |
| N3521       | 3                | 10°.92                   | +2°.07                   | +0.77571                 | 808.03                   | 14.57                    | 59.09                    |
| LEO TRIPLET | 3                | 11°.287                  | +13°.5                   | +0.81236                 | 682.83                   | 8.50                     | 81.26                    |
| URSA MAJOR  | 24               | 11°.857                  | +49°.29                  | +0.79018                 | 1011.88                  | 14.92                    | 69.20                    |
| COMA I      | 6                | 12°.451                  | +29°.29                  | +0.70951                 | 994.54                   | 11.15                    | 98.16                    |
| VIRGO SOUTH | 7                | 12°.635                  | +1°.4                    | +0.51218                 | 1071.9                   | 13.50                    | 84.04                    |
| CVn I       | 3                | 12°.477                  | +35°.67                  | +0.70501                 | 397.41                   | 4.38                     | 90.90                    |
| N5033       | 3                | 13°.211                  | +36°.7                   | +0.61212                 | 931.57                   | 13.14                    | 70.94                    |
| M51         | 2                | 13°.196                  | +43°.3                   | +0.62942                 | 532.12                   | 6.93                     | 76.82                    |
| M101        | 3                | 13°.883                  | +56°.59                  | +0.55330                 | 364.51                   | 3.66                     | 111.17                   |
| N5371       | 3                | 13°.747                  | +41°.78                  | +0.54119                 | 2664.4                   | 26.69                    | 101.23                   |
| N5364       | 2                | 13°.812                  | +4°.84                   | +0.30626                 | 1267.3                   | 14.02                    | 102.54                   |
| N5566       | 9                | 14°.626                  | +1°.68                   | +0.09895                 | 1617.9                   | 20.03                    | 84.06                    |
| N5676       | 3                | 14°.508                  | +49°.99                  | +0.45445                 | 2342.8                   | 29.94                    | 79.33                    |
| N5866       | 2                | 15°.192                  | +56°.85                  | +0.39474                 | 908.10                   | 11.32                    | 81.58                    |
| N6070       | 3                | 15°.993                  | +0°.54                   | −0.21742                 | 2032.4                   | 26.80                    | 75.83                    |
| GRUS        | 8                | 22°.895                  | −40°.47                  | −0.90251                 | 1614.1                   | 16.20                    | 104.07                   |
| N7320/7331  | 2                | 22°.571                  | +33°.92                  | −0.37960                 | 1079.3                   | 10.80                    | 104.02                   |
| N7537/7541  | 2                | 23°.202                  | +4°.24                   | −0.68711                 | 2865.7                   | 27.08                    | 106.60                   |
| CLUSTER  | $n_{\text{obs}}$ | $\alpha$ | $\delta$ | cos $\gamma$ | $\langle \dot{r} \rangle$ | $\langle r \rangle$ | $\langle \dot{r} r \rangle$ |
|----------|-----------------|-----------|----------|--------------|-----------------|----------------|-----------------|
| PISCES   | 20              | $1^h$     | $+30^0$  | $-0.1250$    | 5274            | 52.91          | 102.77          |
| A400     | 7               | $2^h.917$ | $+5^0.83$| $+0.0320$    | 7855            | 82.08          | 98.02           |
| A539     | 9               | $5^h.233$ | $+6^0.38$| $+0.5306$    | 8536            | 95.94          | 91.33           |
| CANCER   | 22              | $8^h.3$   | $+21^0.23$| $+0.9748$    | 4789            | 61.15          | 83.07           |
| A1367    | 20              | $11^h.7$  | $+20^0.12$| $+0.7903$    | 6486            | 76.04          | 89.74           |
| COMA     | 13              | $12^h.95$ | $+28^0.25$| $+0.6267$    | 7310            | 80.72          | 92.46           |
| Z74 – 23 | 13              | $14^h$    | $+9^0.57$ | $+0.3041$    | 5939            | 72.86          | 84.34           |
| HERCULES | 11              | $16^h.05$ | $+17^0.93$| $-0.0697$    | 10733           | 114.05         | 96.65           |
| PEGASUS  | 22              | $23^h.3$  | $+7^0.92$ | $-0.6380$    | 4275            | 40.89          | 111.15          |
| A2634/66 | 11              | $23^h.67$ | $+24^0$  | $-0.4027$    | 8693            | 86.23          | 102.38          |
| VIRGO    | 16              | $12^h.476$| $+12^0.34$| $+0.6243$    | 1064            | 14.91          | 78.00           |
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