Chua's Circuit Simulation Experiment Based on Saturation Function

Ensheng Lv
Department of mechanical and electrical engineering, Henan technical institute, Zhengzhou, Henan, China
*corresponding author’s e-mail: lvensheng@126.com

Abstract. This paper designs a Chua's diode circuit based on saturation function. It uses Matlab/Simulink to model Chua's circuit, and simulates and analyzes the dynamic behavior of chaos, the model simply generates efficient chaotic signals, and intuitively displays processes of the chaotic attractor, chaotic synchronization, period doubling and the road to chaos through the virtual oscilloscope, which is conducive to chaotic beginners' understanding of the basic characteristics and application of chaos. The electrical and mathematical analysis of the instrument is carried out by simulation, to explore the functions of each parameter, and helps beginners to understand its working principle more quickly and conduct experimental operations. It provides theoretical support for the improvement and optimization of Chua's circuit.

1. Introduction
The science and engineering experiment courses are the main channel and main position for students to shape engineering thoughts, train scientific thoughts and explore the unknown, and build students' craftsmanship spirit of striving for perfection and pursuing truths. At present, chaos theory has been listed as the teaching content of College Physics and Circuits. There are many difficulties in the realization of hardware Chua's circuit[1]. Virtual simulation is an effective way to demonstrate chaotic dynamical behaviors. The purpose of the simulation is twofold. The first objective is to understand the content of the experiment better because of the complexity of the chaotic system dynamical behaviors. The second purpose is that the simulation itself makes sense[2-4].

In order to allow students to have a perceptual understanding of Chua's circuit experiment phenomenon, using Matlab/Simulink simple graphics module unit, simulation interface, powerful computing function to simulate chaotic system, virtual experiment has the advantages of flexibility and intuition, not being limited by space, time and environment, adjusting experimental parameters at any time according to the need, etc. It is easy to observe different simulation results. In this paper, based on Chua's circuit, Matlab/Simulink was used to establish an experimental model. Through experiments, the parameter a of Chua's circuit is gradually increased. In the phase space, a single scroll period doubling → a single scroll chaos → a single scroll period doubling → a single scroll chaos (Rossler attractor[5]) → a double scroll chaos → a double scroll period doubling → a double scroll chaos; In addition, there are waveforms and bifurcation diagrams. Chua's circuit simulation experiment can not only improve students' ability of circuit design and analysis, but also cultivate students' innovative thinking ability, and enrich teaching methods and content system.
2. The Simulink model of Chua’s circuit

2.1. The model of Chua’s circuit

Chua’s circuit is the first hardware circuit achievement of chaos researchers, which has been widely used in the experimental teaching of College Physics and Circuits. The circuit is shown in Figure 1.

![Figure 1. The schematic diagram of typical Chua's circuit](image)

Suppose the voltage of the capacitance $C_1$, $C_2$ are $v_{c1}$, $v_{c2}$, the current of the inductance $L$ is $i_L$, $f(v_{c1})$ is Chua's diode current equation, which is piecewise linear, Then the state equation of Chua’s circuit as shown in Fig. 1 is

$$\begin{align*}
\frac{dv_{c1}}{dt} &= (v_{c2} - v_{c1}) / (RC_1) - f(v_{c1}) / C_1 \\
\frac{dv_{c2}}{dt} &= (v_{c1} - v_{c2}) / (RC_2) + i_L \\
\frac{di_L}{dt} &= -v_{c2} / L \\
I_N &= f(v_{c1}) = G_b v_{c1} + 0.5 (G_a - G_b) (|v_{c1} + E| - |v_{c1} - E|)
\end{align*}$$

(1)

As for formula (1), (2), $E$ is turning point voltage, $G_a$, $G_b$ are slopes.

Suppose

$$x = v_{c1} / E, \quad y = v_{c2} / E, \quad z = (R i_L) / E, \quad \alpha = (R i_L) / E, \quad \tau = t / (RC_2), \quad m_0 = RG_a, \quad m_1 = RG_b, \quad a = C_2 / C_1, \quad b = (C_2 R^2) / L$$

(3)

Because of normalization, the following is

$$\begin{align*}
\frac{dx/d\tau}{a} &= y - x - f(x) \\
\frac{dy/d\tau}{a} &= x - y + z \\
\frac{dz/d\tau}{b} &= -bz
\end{align*}$$

(4)

$$f(x) = m_1 x + 0.5 (m_0 - m_1) (|x + 1| - |x - 1|)$$

(5)

And $m_0=-1/7$, $m_1=-5/7$, $a$, $b$ are parameters.

2.2. The Simulink model of chua's circuits with saturation function

The key part of the nonlinear function model of Chua’s diode in Equation (5) is $0.5 (|x + 1| - |x - 1|)$, the equation (6), as a saturation function, is normalized into a three-line nonlinear curve with limited amplitude, as shown in Fig. 2. Therefore, the circuit design is simplified by comparing with the classical Chua’s diode in literature [7, 8].
Figure 2. The characteristic curve of Saturation function

\[ f(x) = \begin{cases} 
1 & x \geq 1 \\
x & -1 < x < 1 \\
-1 & -1 \leq x
\end{cases} \quad (6) \]

Under the environment of the Simulink, integration, gain, summation, product and other modules are selected to connect each module according to Formula (4), (5) and (6) to build a Simulink model of Chua's circuit as shown in Fig. 3, named “Sim_chua.mdl”, Each integral module corresponds to a differential of Formula (4). Scope module can display the integral curves of the state variables of \( x, y \) and \( z \), and XY Graph module can display the two-dimensional phase diagrams of \( x-y, x-z \) and \( y-z \).

3. The phase diagram analysis of Chua's circuit

3.1. Formatting author affiliations

By studying Equations (4) and (5), the initial value of integration module vectors \((x, y, z)\) in Fig. 3 is set as \((0.1, 0.1, 0.1)\), parameters \(a=10, b=-15\), and the simulation stop time in the Sim_chua.mdl model is set as: 200.

Connect the three state variables \(x, y\) and \(z\) to the Scope module respectively to observe the time-domain waveform, as shown in Figure 4 below. The pairwise combination vectors of \(x-y, y-z\) and \(x-z\) are connected to the XY Graph module to observe the two-dimensional phase diagram, as shown in Figure 5 below.
3.2. Road to chaos

There are two parameters $a$ and $b$ in formula (4) of Chua's circuit, and only one parameter change is needed to study the road to chaos. Suppose $b=-15$ is the constant parameter and $a$ is the variable parameter. In order to ensure the accuracy of Simulation, in the menu of Simulation→Configuration Parameters, set the Type as Fixed-step, Solver as ode4 Runge-Kutta, Fixed-step size is 0.005, Start time is 200, Stop time is 400. Discarding the first 200 seconds of simulation transient, so as to better observe the road leading to chaos by doubling the period.

(a) single scroll period one, $a=8.2$  (b) single scroll period two, $a=8.6$  (c) single scroll period four, $a=8.72$

(d) single scroll chaos, $a=8.2$  (e) single scroll period three, $a=8.885$  (g) single scroll chaos, $a=9.1$
When parameter $a$ increases from 8 to 9.5, it will cause repeated changes in the motion morphology topology of Chua's circuit, which evolves from period-chaos to period-chaos until the end of chaos. From a single scroll period one to the double scroll chaos, the value of parameter $a$ can make the proportion of Chaos Chua's circuit operating in the chaotic state is very small, even less in the real number range. In the field of circuit research, the small value range of parameter $a$ in the macroscopic experience of researchers is often ignored. There are other reasons, such as theory and experimental equipment, etc. Therefore, chaotic phenomena have been encountered and passed by for many times in history\cite{9}, which is of great significance for training students' scientific thinking and pursuit of truths.

4. Synchronous control experiment of Chua's circuit

Because of the sensitivity of chaos to initial values, chaos synchronization is an important direction for secure communication, a drive - response synchronous system is designed, including drive system, response system and channel three parts.

Drive system

$$\begin{align*}
    \frac{dx_1}{dt} &= a(y_1 - x_1 - f(x_1)) \\
    \frac{dy_1}{dt} &= x_1 - y_1 + z_1 \\
    \frac{dz_1}{dt} &= -by_1
\end{align*}$$

(7)

Response System

$$\begin{align*}
    \frac{dx_2}{dt} &= a(y_2 - x_2 - f(x_2)) \\
    \frac{dy_2}{dt} &= x_1 - y_2 + z_2 \\
    \frac{dz_2}{dt} &= -by_2
\end{align*}$$

(8)

By referring to Equations (7) and (8), the integration, gain, summation, product and other modules were connected to build the synchronization model of Chua's circuit as shown in Figure 7, which is named “Sim_chua_synchonization.mdl”. The initial value of the integral vector $(x_1, y_1, z_1)$ of the driving system in Figure 7 is set as $(0.3, 0.2, 0.1)$, the initial value of the response system integral vector $(x_2, y_2, z_2)$ is $(1.3, 1.2, 1.1)$, and the simulation stop time is 500. The simulation results in Figure 8 show that, after 120S or so, the response system and the driving system are well synchronized.
Figure 7. The synchronization model diagrams of Chua's circuit

(a) \((t,x_1-x_2)\)  \hspace{1cm}  (b) \((t,y_1-y_2)\)  \hspace{1cm}  (c) \((t,z_1-z_2)\)

Figure 8. The synchronization curves of driving system and response system

5. Conclusion
In this paper, the saturated function Chua's diode is designed, and the Chua's circuit and chaotic synchronization are modeled by Matlab/Simulink software. Virtual simulation experiment, easy to change parameters, concise and intuitive display, make up for the shortcomings of traditional physics experiment, is conducive to students' understanding of the essence of chaos phenomenon, but also enrich the teaching content.

The change of Chua's circuit parameters will have an impact on the circuit system. The saturation function circuit designed by Matlab / Simulink is used for simulation experiment. The experiment is simple and accurate.

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