Note on Redshift Distortion in Fourier Space

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Received XXXX month day; accepted XXXX month day

Abstract We explore features of redshift distortion in Fourier analysis of N-body simulations. We find that the peculiar motion along the line of sight shifts the phases $\theta$ of Fourier modes in a non-trivial way. On very large scales phases hardly change as predicted by linear perturbation theory of redshift distortion, while in general the phase shifts $\Delta \theta$ induced by redshift distortion is stochastic and the probability distribution function (PDF) is symmetric to the peak at zero shift. Analysis of the phase shifts opens the possibility of constructing appropriate phenomenological models for the monopoles of bispectrum in redshift space. Comparison with simulations indeed shows our toy models quite successful in modeling bispectrum of equilaterals and isosceles at large scales. In the second part in order to test the plane-parallel approximation, we compare the monopoles of the power spectrum and bispectrum in the radial and plane-parallel distortion. We confirm the results of Scoccimarro (2000) that difference of power spectrum is in general at the level of 10\%, and in the reduced bispectrum such difference is as small as a few percents. However, on the plane $k_z = 0$ which is perpendicular to the line of sight, the difference in power spectrum between the radial and plane-parallel approximation can be more than $\sim 10\%$, and even worse on very small scales. Such difference is also prominent for bispectrum, especially for those configurations of tilted triangles. The non-Gaussian signals under radial distortion on small scales are systematically biased downside than that in plane-parallel approximation, while amplitudes of such differences depends on the opening angle of the sample to the observer. The observation gives warning to the practice of using the power spectrum and bispectrum measured on the $k_z = 0$ plane as estimation of the real space statistics.

Key words: cosmology: theory — large-scale structure of universe — methods: statistical

1 INTRODUCTION

Large redshift surveys of galaxies provide us with distances estimated from measured redshifts to complete our three-dimensional topography of the visible universe. However, due to peculiar

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velocities of galaxies, the inferred redshift from observation is not an exact indication of the true distance, therefore our estimation of statistics in redshift space suffers from the redshift distortion. In summary, on large scales peculiar velocities are dominated by bulk inflow, the redshift distortion boosts the clustering strength of galaxies; while on small scales the virialised peculiar velocities suppress the clustering power and large scale structures (LSS) show the phenomenon of “Finger of God” (e.g. Kaiser 1987; Hamilton 1998).

Redshift distortion is generally complicated with entanglement of bias and gravitational nonlinearity of the LSS. Nevertheless, fruitful results of redshift distortion have been achieved and applied in statistical analysis of LSS, particularly for the power spectrum and/or the two point correlation function. On large scales where linear theory applies, redshift space power spectrum relates to the real space $P(k)$ by

$$P_s(k) = (1 + \beta \mu^2)^2 P(k),$$  \hspace{1cm} (1)

where $\beta = \Omega_m^{0.6}/b$ with $b$ the linear deterministic bias parameter, and $\mu$ is the cosine of the angle between $k$ and the unit vector $\hat{z}$ of line-of-sight (LOS) (Kaiser 1987). Further observation and analysis of N-body simulation denotes that over all scales including strong nonlinear regime we have an accurate empirical formula to model the effect of redshift distortion on power spectrum with introduction of a pairwise velocity dispersion parameter $\sigma_v$ (e.g. Park et al. 1994; Peacock & Dodds 1994; Cole, Fisher & Weinberg 1995; Hatton & Cole 1998; White 2001),

$$P_s(k) = P(k) \frac{(1 + \beta \mu^2)^2}{[1 + (k\mu\sigma_v)^2/2]^2}. $$ \hspace{1cm} (2)

In comparison, understanding of redshift distortion in bispectrum or the three point correlation function is not satisfactory (Hivon et al. 1995; Verde et al. 1998; Scoccimarro et al. 1999), which, in part, is attributed to the non-perturbative nature of redshift distortion effect (Scoccimarro et al. 1999). In a more fundamental approach, an intrinsic difference between power spectrum and bispectrum is that power spectrum is a phase invariant statistics which contains only the information of the amplitude of the Fourier transform of the density contrast, but bispectrum consists of contributions from both amplitudes and phases (Matsubara 2003). Research based on phases has revealed that phase information is very crucial in LSS distribution pattern recognition (Chiang & Coles 2000; Chiang 2001; Chiang, Coles & Naselsky 2002), especially for the bispectrum and the three point correlation function (Watts & Coles 2003; Watts, Coles & Melott 2003; Matsubara 2003; Hikage, Matsubara & Suto 2004; Chiang 2004; Hikage et al. 2005). There is no reason to believe that redshift distortion only modifies amplitudes of Fourier modes and leaves phases unchanged, therefore it is not strange that the information offered by Eq. (1) and Eq. (2) is not sufficient to model the bispectrum in redshift space, the piece we are lack of for the jigsaw is the phase shifts. That is why in this paper we are attempting to explore the behavior of phases under redshift distortion.

An alternative route to tackle redshift distortion is to recover real space quantities from measurements in redshift space. In short, the anisotropic two point correlation function in redshift space $\xi(s)$ is measured along two separations as $\xi(\sigma, \pi)$, where $\pi = s \cdot \hat{z}/s$ and $\sigma = |s \times \hat{z}|/s$. We can directly use the projected two point correlation function $\Xi(\sigma)$ which is free of redshift distortion and simply related to the real space $\xi(r)$ by (Davis & Peebles 1983; Fisher et al. 1994)

$$\Xi(\sigma) \equiv \int_{-\infty}^{+\infty} \xi(\sigma, \pi) d\pi = 2 \int_{\sigma}^{+\infty} \frac{r \xi(r) dr}{(r^2 - \sigma^2)^{3/2}}, $$ \hspace{1cm} (3)
or, deproject $\Xi$ to have the real space two point correlation function (e.g. Saunders, Rowan-Robinson & Lawrence 1992; Hawkins et al. 2003)

$$\xi(r) = -\frac{1}{\pi} \int_r^{+\infty} \frac{d\Xi(\sigma)/d\sigma}{(\sigma^2 - r^2)\frac{1}{2}} d\sigma .$$  \hspace{1cm} (4)

In Fourier space it is much simpler, as we can see from Eq.(1) and (2) that if we restrict our measurement on the plane of $k_z = 0$ only, what we obtain is the real space power spectrum (e.g. Park et al. 1994; Hamilton & Tegmark 2002).

However, the approach relies on the assumption of the parallel approximation, i.e. the observer is at infinite distance so the unit vector $\hat{z}$ of the LOS is pointing to a fixed direction. In reality there is no such fixed direction of LOS, systematic biases are inevitably introduced in the treatment mentioned above. Particularly, since Fourier transform mingles all scales together, such effect is non-local. Even at high $k$ there are some contamination from scales at wide angles. Consequently not only are polyspectra measured on $k_z = 0$ plane not equivalent to those in real space, but also the monopoles of polyspectra with respect to LOS are not strictly following what theories of redshift distortion under plane approximation predict. Further complication also arises that Fourier modes are no longer independent and the covariance matrix is not diagonal any more (Scoccimarro, 2000, Zaroubi & Hoffman 1996).

Attention has been called in works analyzing galaxy redshift surveys which always subtend wide angles over sky and are at low redshift, to reduce the systematics in the adoption of parallel approximation. Numerical studies have shown that if one confines their work with small angle, differences brought by parallel approximation in monopoles are negligible, while in quadruples of power spectrum are usually under $\sim 5\%$ if the opening angle of the window in which Fourier transform is performed is less than $50^\circ$ (e.g. Park et al. 1994; Cole, Fisher & Weinberg 1994). Scoccimarro (2000) also reported that such approximation was good enough for measuring the monopoles of power spectrum and bispectrum. Even for all sky surveys the difference is as small as $10\%$. whatsoever, as the parallel approximation plays a fundamental role in Fourier analysis of galaxies' spatial distribution, it definitely deserves extensive studies, which is very crucial in keeping confidence on our interpretation of what we estimated in redshift space. Indeed as we will show in this paper that the real life is never simple.

Our paper is organized as follows. In Section 2, we give general description of mapping from real space to redshift space, demonstrate the shift of phases of Fourier modes by redshift distortion and discuss possible construction of empirical model for bispectrum in redshift space. Section 3 focuses on the difference of plane-parallel approximation and radial distortion. Conclusion and discussion is in section 4. Throughout the paper we are using two sets of ΛCDM simulations by the Virgo consortium, namely the Virgo simulation in box of size $239.5h^{-1}$Mpc with $256^3$ particles and the VLS simulation in box of size $479h^{-1}$Mpc with $512^3$ particles, cosmological parameters of the two simulations are $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\sigma_8 = 0.9$ and $\Gamma = 0.21$ (Jenkins et al. 1998).

2 OOPS, PHASES

In redshift space, the coordinate of a galaxy is calculated from the observed radial velocity, contributed both by the Hubble flow and peculiar velocity. For simplicity in this section we adopt the plane approximation so the unit vector of LOS denoted by $\hat{z}$ has a fixed direction. Therefore, the mapping from real space coordinates $r$ to redshift space $s$ is given by,

$$s = r - \beta u_z(r)\hat{z} ,$$  \hspace{1cm} (5)
where \( \mathbf{u}(\mathbf{r}) \equiv -\mathbf{v}(\mathbf{r})/(\mathcal{H} \beta) \), \( \mathbf{v}(\mathbf{r}) \) is the peculiar velocity, and \( \mathcal{H}(\tau) \equiv (1/a)(da/d\tau) = Ha \) is the conformal Hubble parameter with FRW scale factor \( a \) and conformal time \( \tau \) (Scoccimarro et al. 1999; Scoccimarro 2004).

The density contrast in redshift space, \( \delta_s(s) \), is obtained from the real space density fluctuation \( \delta(r) \) by requiring conservation of the number of galaxies, i.e.

\[
(1 + \delta_s) d^3 s = (1 + \delta) d^3 r
\]

Fourier transform of the number density contrast in redshift space reads

\[
\delta_s(\mathbf{k}) \equiv \int \frac{d^3 s}{(2\pi)^3} e^{-i \mathbf{k} \cdot \mathbf{s}} \delta_s(s) = \int \frac{d^3 x}{(2\pi)^3} e^{-i \mathbf{k} \cdot \mathbf{x}} e^{i \beta \mathbf{k} \cdot \mathbf{u}(\mathbf{x})} \left[ \delta(\mathbf{x}) + \beta \nabla_x \mathbf{u}_x(\mathbf{x}) \right] ,
\]

in which and hereafter the subscript "s" refers to quantities in redshift space. The term in square brackets describes the squashing effect, i.e., the boost to the clustering amplitude due to infall, whereas the exponential factor encodes the Finger-of-God effect, which erases power due to velocity dispersion along the LOS. For plane-parallel approximation, the distortion effect is contributed by the peculiar velocity along the LOS \( \mathbf{v} \). Thus, to generate a redshift space sample, we only need \( s_z = r_z - \beta \mathbf{u}_z \).

It is clear from Eq. (7) we see that in the redshift space, not only the amplitude \( \delta(k) \), but also the phase angle \( \theta(k) \) will be distorted which is casually ignored. The expression can be expanded perturbatively as demonstrated by Scoccimarro et al. (1999) who calculated the expansion to second order to model the bispectrum in redshift space on large scales. Here we reproduce the expansion of Scoccimarro et al. (1999)

\[
\delta_s(\mathbf{k}) = \sum_{n=1}^\infty \int d^3k_1...d^3k_n [\delta_D](\mathbf{k}) \prod_{i=1}^n [\delta_1^i + \beta \mu^2_1 \delta_2^i] \frac{(\beta \mu^2_2)^{n-1}}{(n-1)!} \frac{\mu_2}{\mu_1} \phi(\mathbf{k}_1),
\]

where \( [\delta_D] = \delta_D(\mathbf{k} - \mathbf{k}_1 - ... - \mathbf{k}_n) \) is the Dirac function, velocity divergence is \( \phi(\mathbf{r}) \equiv \nabla \cdot \mathbf{u} \) and \( \mu_n = \mathbf{k}_n \cdot \dot{z}/k_n \). In the linear theory only the first order \( n = 1 \) counts, we recover the well-known Kaiser formula (Kaiser 1987)

\[
\delta_s(\mathbf{k}) = (1 + \beta \mu^2) \delta(\mathbf{k}) ,
\]

which tells that at the first order phases are not changed by redshift distortion, so phase shift is a phenomenon at high orders \( n \geq 2 \). We are not going to calculate quantitively the phase shift in perturbation theory which requires intimidating integration, rather, as a starting work in this aspect, we are aiming at presenting the existence of the phase shift in numerical simulations and henceforth implication drawn from the information to understand redshift distortion on phase-related statistics, the bispectrum.

By definition, the bispectrum is the ensemble average of the product of three Fourier modes with their wavevectors forming a triangle,

\[
B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta^*(\mathbf{k}_3) \rangle = \langle |\delta_1||\delta_2||\delta_3| e^{i(\theta_1 + \theta_2 - \theta_3)} \rangle ,
\]

where we wrote the Fourier mode \( \delta(\mathbf{k}) = |\delta| e^{i \theta} \). It is well understood that the nonlinearity of gravitational evolution makes the phase \( \theta \) and the amplitude \( |\delta| \) generally correlated, especially on small scales although in the initial fluctuation such correlation does not exist. Whilst we are living with such universal correlation, on large scales where the nonlinearity is fairly weak, we can make a strong assumption to neglect such correlation and an interesting conclusion emerges. Under the assumption, the ensemble average of bispectrum is then decomposed into

\[
B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \propto \langle |\delta_1||\delta_2||\delta_3| \rangle \int e^{i \theta} \rho(\Theta) d\Theta ,
\]
in which $p(\Theta)$ is the PDF of the phase sum $\Theta = \theta_1 + \theta_2 - \theta_3$. Since the bispectrum is real and $\delta(k)$ is Fourier transform of a real function, $p(\Theta)$ shall be symmetric to $\Theta = 0$ and can be expanded as $p(\Theta) = c_0 + \sum_n c_n \cos(n\Theta)$. Keeping only the first two terms, we in fact reach the essence of the results of Matsubara (2003)

$$p(\Theta) \propto (1 + \text{const.} \cdot \cos(\Theta)) \ .$$

In redshift space, modifications occur in both amplitude and phase of Fourier mode, parameterized as

$$\delta_s(k) = |\delta_s|e^{i\theta_s} = \alpha(k)|\delta(k)|e^{i(\theta+\Delta\theta)} ,$$

in which the amplification factor of amplitude $\alpha(k)$ is real and $\Delta\theta(k) \equiv \theta_s(k) - \theta(k)$. Thus the bispectrum in redshift space can be written as

$$B_s(k_1, k_2, k_3 = -k_1 - k_2) = \langle \alpha_1 \alpha_2 \alpha_3 |\delta_1||\delta_2||\delta_3| e^{i(\theta_1 + \theta_2 - \theta_3)} e^{i(\Delta\theta_1 + \Delta\theta_2 - \Delta\theta_3)} \rangle .$$

According to our analysis of the Virgo and the VLS simulations, the $\alpha(k) = \alpha(k, \mu)$ is deterministic and can be well approximated by $\alpha = (1 + \beta \mu^2)/[1 + (k \mu \sigma_v)^2/2]$ as in Eq. (2), naturally the product of amplitude amplification factors in Eq. (14) can be moved to the outside of the ensemble average.

If we assume there is no stochastics in phase shift by redshift distortion, for a specified $k$, $\Delta\theta$ is fully determined by certain function, $e^{i\Delta\theta}$ is a constant with respect the ensemble average as well so that $B_s = \alpha_1 \alpha_2 \alpha_3 B e^{i\Delta\Theta}$. $\Delta\Theta$ has to be $n\pi$ ($n = 0, \pm 1, \pm 2\ldots$) as $B_s$ is real, so

$$B_s = \alpha_1 \alpha_2 \alpha_3 B .$$

Furthermore, it is hard to conjecture there is cosmic conspiracy to satisfy $\Delta\Theta = n\pi$ for arbitrary $(k_1, k_2, k_3)$ triplet, the only possibility is for any mode $\Delta\theta = n\pi$, which means the absolute values of phases keep invariant and is obviously impossible as implied by Eq. (7) & (8). The immediate conclusion is the phase shift resulted from redshift distortion is stochastic.

If phase shift is completely randomly distributed, $p(\Delta\Theta = \Delta\theta_1 + \Delta\theta_2 - \Delta\theta_3) = 1/2\pi$, the joint PDF $p(\Delta\Theta) = p(\Delta\theta_1, \Delta\theta_2, \Delta\theta_3) = p(\theta_1, \theta_2, \theta_3)/2\pi$, therefore again $B_s = \alpha_1 \alpha_2 \alpha_3 B$. Unfortunately in N-body simulations the distribution of phase shift $p(\Delta\theta)$ is not uniform, as shown in Fig. (1). In general $p(\Delta\theta)$ depends on $(k, \mu)$ and is peaked at center $\Delta\theta = 0$. With the decrement of $\mu$ and $k$, the dispersion of the distribution is becoming smaller and more modes concentrate on the vicinity of $\Delta\theta = 0$. This is easy to understand since those modes around the plane perpendicular to LOS suffers little from redshift distortion (Eq. (7)) while on large scales the deviation to linear redshift distortion theory of Eq. (9) becomes small. Indeed we checked that in simulations on the plane $\mu = 0$ phases of all modes are not shifted.

Although the $\Delta\theta$ is not uniformly distributed, in order to peer inside of the redshift distortion on bispectrum, as a crude approximation in light of the spirit of deriving Eq. (12), we assume that $\Delta\Theta$ is uncorrelated with $|\delta_1, 2, 3|$ and $\theta_1, 2, 3$. Eq. (14) becomes $B_s = B \times \alpha_1 \alpha_2 \alpha_3 \int e^{i\Delta\theta_1} p(\Delta\Theta) d\Delta\Theta$, and $p(\Delta\Theta) \propto (1 + \text{const.} \cdot \cos(\Delta\Theta))$ if high order terms are ignored. With a further assumption that $\Delta\theta$ of different mode is independent to each other, we have

$$B_s(k_1, k_2, k_3) = B(k_1, k_2, k_3) \times \alpha_1 \alpha_2 \alpha_3 \times \int e^{i\Delta\theta_1} p(\Delta\Theta) d\Delta\Theta \int e^{i\Delta\theta_2} p(\Delta\Theta) d\Delta\Theta \int e^{i\Delta\theta_3} p(\Delta\Theta) d\Delta\Theta ,$$

from which trivially we again have features of $p(\Delta\theta)$ that it is symmetric to $\Delta\theta = 0$ seen in Fig. 1 and $\propto (1 + \text{const.} \cdot \cos(\Delta\Theta))$. Note that here $\alpha$ and $\Delta\theta$ are functions of two variables
Fig. 1 Probability distribution functions of phase shift $\Delta \theta$ resulted from redshift distortion in N-body simulations. The left panel plots the $p(\Delta \theta)$ on different $k$ scales at fixed $\mu = k \cdot \hat{z}/k$, and in the right panel we show $p(\Delta \theta)$ at variant $\mu$ at $k = 0.4h\text{Mpc}^{-1}$.

$(k, \mu)$. Eq. 16 is quite similar to the phenomenological model of Scoccimarro et al. (1999), and it serves as a theoretical support to the rationality of their ansatz.

In practice often it is simpler to measure of the bispectrum averaged over all possible combinations of wave vectors $(k_1, k_2, k_3 = -k_1 - k_2)$ with fixed $k_1 = |k_1|$, $k_2 = |k_2|$ and the angle $\psi = k_1 \cdot k_2/(k_1 k_2)$. In real space it is the isotropic bispectrum $B(k_1, k_2, k_3) = B(k_1, k_2, k_3)$, but in redshift space due to the anisotropy of redshift distortion, the angular averaged with respect to LOS is the monopole of true bispectrum $B_s(k_1, k_2, \psi, \mu_1, \mu_2, \mu_3)$. In that we denote the azimuthal angle of $k_2$ about $k_1$ with $\varphi$, and armed with

$$
\begin{align*}
\mu_1 &= \mu = k_1 \cdot \hat{z}/k_1 , \\
\mu_2 &= \mu \cos \psi - \sqrt{1 - \mu^2} \sin \psi \cos \varphi , \\
\mu_3 &= -\frac{k_1}{k_3} \mu - \frac{k_2}{k_3} \mu_2 ,
\end{align*}
$$

the monopole of bispectrum in redshift space is

$$
B_s^{(0)}(k_1, k_2, \psi) = \frac{1}{4\pi} \int_{-1}^{1} d\mu \int_{0}^{2\pi} d\varphi B_s(k_1, k_2, \psi, \mu, \varphi) .
$$

On large scales where the nonlinearity is weak and phases shifts are close to zero (Eq. 9 and Fig. 1), we can approximated the modulation of redshift distortion to bispectrum with Eq. 15, monopoles thus is modeled by

$$
\begin{align*}
\frac{B_s^{(0)}}{B} = \frac{1}{4\pi} \int_{-1}^{1} \int_{0}^{2\pi} \alpha(k_1, \mu) \alpha(k_2, \mu, \psi, \varphi) \alpha(k_3, \mu, \psi, \varphi) d\varphi d\mu .
\end{align*}
$$

We measured angular averaged power spectrum and bispectrum both in real space and redshift space for the Virgo simulation and eight subsets of the VLS simulation which are generated by dividing the VLS simulation into independent cubes of half of the original box side size. $\sigma_v = 4.87$ is fitted from power spectra with Eq. 2, and then is inserted into Eq. 19 as our toy model for $B_s^{(0)}$. In Fig. 2 it is interesting to observe the remarkable agreement of our toy
model based on phase information with simulations for equilateral triangles for \( k < 0.2h\text{Mpc}^{-1} \) beyond which tree-level perturbation theory and the second order Lagrangian theory also breaks (Scoccimarro 2000). As \( k \) grows, phase shift is no longer negligible so our toy model fails to reproduce the simulation results.

As a next level advanced than the simple prescription of Eq. 19, a better approximation may be achieved through integrating Eq. 16 instead of the simple formula of Eq. 19 if we know \( p(\Delta \theta) \) which carries important information about the redshift distortion and numerically contains more dependence on \((k, \mu)\) (Fig. 1). Because we are not equipped with a whole battery of many simulations in very large box, so far in this work we are not developing functional expression to fit \( p(\Delta \Theta) \)'s dependence on \((k, \mu), \sigma_v \). Nevertheless, we can parameterize the toy model in a simple way to improve its performance just as in Scoccimarro et al. (1999), e.g.

\[
\frac{B_s^{(0)}}{B} = A_1 \left[ \frac{1}{4\pi} \int_{-1}^{1} \int_{0}^{2\pi} \alpha(k_1, \mu) \alpha(k_2, \mu, \psi, \varphi) \alpha(k_3, \mu, \psi, \varphi) d\varphi d\mu \right]^{A_2},
\]

with two fitting parameters \( A_{1,2} \). An eye-ball examination with \( A_1 = 1/1.15 \) and \( A_2 = 1.75 \) does improve agreement up to scale \( k \sim 0.5h\text{Mpc}^{-1} \) for equilateral triangles (Fig. 2). We then compare the modified toy model for other configurations with simulations in Fig. 3. We see that for isoceles with \( \psi \gtrsim 90^\circ \) the agreement of our modified toy model with simulations is very impressive too, although for those very tilted configuration with \( k_2/k_1 \gtrsim 2 \) differences are huge. We are not intending to give accurate fit. We are just demonstrating the possibility of constructing a good approximation model started from the simple prescription of Eq. 19, and, in the mean time showing the discrepancy due to our rough handling with phase shift.

3 GOODNESS OF PLANE-PARALLEL APPROXIMATION TO RADIAL DISTORTION

So far we are working with plane-parallel approximation. As we have addressed in section 1, such approximation is appropriate when the angle subtended by the sample is small. When
dealing with large samples, especially all sky surveys, radial distortion should be taken into account. To investigate the extent to which the plane-parallel approximation is appropriate to model the exact radial distortion, we are to compare a set of power spectra and bispectra in redshift space under parallel approximation and radial distortion respectively.

### 3.1 Monopoles of Power Spectrum and Bispectrum

It has been shown by Scoccimarro (2000) that the monopoles of power spectrum and bispectrum of an all sky survey, in radial and plane-parallel distortions are consistent with each other. For the power spectrum, the difference is within $\sim 10\%$ while for the bispectrum, it is within a few percents. By using different simulation samples, we try to explore it in more details.

To generate samples in redshift space with radial distortion, we strictly follow Eq. 5 with $\hat{z}$ defined by the unit vector pointing from the observer to the point of sample, i.e. $\hat{z}$ is simply the unit position vector $\hat{r}$. Points of our samples are with all their Cartesian coordinates $r_i=1,2,3$ within $[0,L]h^{-1}\text{Mpc}$, $L=239.5$. In this subsection, we placed the observer at $(0.5\sqrt{2}L, 0.5\sqrt{2}L, -100)h^{-1}\text{Mpc}$, the largest angle of the sample opening to the observer is $118.88^\circ$. Then points in samples are transformed into the coordinate system with the origin

Fig. 3 $B^{(0)}_s/B(k_1, k_2, \psi)$ of different configurations. The dotted line is our modified toy model Eq. 20, see Fig. 2 for other keys.
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Fig. 4 Ratio of the monopoles of power spectra in radial distortion and parallel approximation. The sample is at the distance of $100h^{-1}\text{Mpc}$.

point where the observer is located. For every data point we have redshift from Hubble flow $z_{\text{Hub}}$ and peculiar velocity induced $z_{\text{pec}}$, the comoving distance in redshift space $s$ is then calculated from the joint redshift $z = (1 + z_{\text{Hub}})(1 + z_{\text{pec}}) - 1$ while its orientation does not change.

In Fig. 4, we plotted the ratio of the monopoles of power spectra under radial distortion and parallel approximation, $P_r^{(0)}/P_p^{(0)}$ where the subscript “r” means radial distortion, “p” refers to parallel approximation and all polyspectra are in redshift space if not specified hereafter. Monopole of power spectrum is defined as the angular average of the anisotropic $P(k)$.

Comparison of the monopoles of reduced bispectrum is illustrated in Fig. 5. Note that the monopole of reduce bispectrum is given by

$$Q^{(0)}(k_1, k_2, \psi) = \frac{B^{(0)}(k_1, k_2, \psi)}{P^{(0)}(k_1)P^{(0)}(k_2) + P^{(0)}(k_2)P^{(0)}(k_3) + P^{(0)}(k_3)P^{(0)}(k_1)} \quad (21)$$

We can see from Fig. 4 that the difference between monopoles of power spectrum under the radial and plane-parallel distortion is within 10%, while the difference for bispectra is a few percents (Fig. 5). These results confirm the assertion of Zaroubi & Hoffman (1996) and Scoccimarro (2000). As shown in Fig. 4, the power in parallel approximation on large scales $k < 0.1h\text{Mpc}^{-1}$ is smaller than that under radial distortion. On the other hand, on small scales $k > 0.2h\text{Mpc}^{-1}$, the plane-parallel distorted power spectrum is larger. This is because in the plane-parallel approximation there is no distortion effect along directions perpendicular to a fixed LOS, while in reality redshift distortion is radial and the LOS is not fixed. Distortion is more severe to structure clumps so that there are more effects of boost at large scales and suppression at small scales on the clustering strength.

3.2 Power Spectrum and Bispectrum on the Plane of $k_z = 0$

We have seen that there is systematic bias caused by the parallel approximation to the radial distortion in monopoles of power spectrum and bispectrum though very weak. In this subsection we set about to explore the behavior of the polyspectra on the plane of $k_z = 0$ which are regarded as estimation of the real space quantities in the plane-parallel approximation. It is
Fig. 5 $Q_r^{(0)}/Q_p^{(0)}(k_1, k_2, \psi)$ for triangles of $k_1 = 0.2, 0.3, 0.4, 0.5, 0.6h\text{Mpc}^{-1}$ and $k_2/k_1 = 1, 2, 3$. The observer is at the distance of $100h^{-1}\text{Mpc}$.

Fig. 6 shows that when $k$ is larger than $0.2h\text{Mpc}^{-1}$, under the plane-parallel approximation, $P_p$ can grows from $\sim 10\%$ to more than $50\%$ larger than $P_r$; when $k$ is smaller than $0.2h\text{Mpc}^{-1}$, $P_p$ could still be $> 10\%$ smaller than $P_r$. Meanwhile, as we conjectured, with increment of the distance of the sample to observer, i.e. the opening angle getting smaller, such difference systematically decrease. However, even when the distance is $1h^{-1}\text{Gpc}$ which corresponds to a opening angle as small as $\sim 20^\circ$, at $k = 3h\text{Mpc}^{-1}$ the systematical bias is still around $50\%$ if one is interested in the highly nonlinear regime, while the bias is also $> 10\%$ for $k < 0.03h\text{Mpc}^{-1}$ which prevent us from having measurements titled with precision. The situation will be worse.
Fig. 6 Ratios of power spectra for radial and parallel distortion on the plane of $k_z = 0$.
The solid line, dash line and dot line are for samples at distances of $100h\text{Mpc}^{-1}$, $200h\text{Mpc}^{-1}$ and $1000h\text{Mpc}^{-1}$ to the observer respectively.

for most redshift surveys, such as 2dFGRS redshift survey and SDSS whose subtended angles of samples are generally much larger than $20^\circ$. Furthermore, as shown in Fig. 7 and Fig. 8, there are similar systematic biases in reduced bispectrum measured at $k_z = 0$ to the real space bispectrum if we adopt the plane-parallel approximation, and the biases not only occur in the amplitudes but also the configuration dependence.

Therefore, it is hopeless to have accurate estimation of real space polyspectra on large scales with the convenient method of using what we measured on the plane $k_z = 0$ in redshift space. Possibility of application of the shortcut is to estimate small scale polyspectra of real space, however, with proper treatment. The barricade can be overcome by splitting the wide angle sample into many patches of very small angles in compromise with the scales of interests, and later one collect measurements in these small patches for analysis in precision which, of course, needs careful numerical experiments to decide the angular widths of those patches for insurance (e.g. Park et al. 1994).

4 SUMMARY AND DISCUSSION

In the exploration of the characteristics of redshift distortion in Fourier space, we begin with research on phases of Fourier modes which are rarely noticed in previous works. On very large scales where linear theory applies, phases are not changed by redshift distortion, phase shift $\Delta \theta$ shall be an effect at high orders. Then we find that in general redshift distortion do shift phases of Fourier modes in nontrivia way. The phase shifts by redshift distortion is not deterministic, it has a distribution function $p(\Delta \theta)$ symmetric to its peak at $\Delta \theta = 0$, and qualitatively proportional to $(1 + \text{const.} \cdot \cos \Delta \theta)$. The distribution function has strong dependence on the wave length $k$ and the orientation $\mu$ of the wave vector to the LOS. More modes are populated around $\Delta \theta = 0$ for small $k$ and small $\mu$.

Limited by the sizes of the available Virgo and VLS simulations and computing resources, we can not obtain reliable $p(\Delta \theta)$ on scales of $k < 0.1h\text{Mpc}^{-1}$ to build empirical model for the distribution function. In alternation we make assumptions on the phase shift at different levels in effort to depict the bones of redshift distortion on bispectrum. We discovered that our very crude
consideration does provide us with a toy model (Eq. 19) describing the monopole of bispectrum in redshift space in good agreement with numerical simulations on scales $k < 0.2h \text{Mpc}^{-1}$, and probably can be serving as basis for extended phenomenological models such as Eq. 20 and the model in Scoccimarro et al. (1999). If in future an approximation for $p(\Delta \theta)$ similar to the modeling of amplitude change like Eq. (2) can be set from many large simulations in big box, it is promising that integration of Eq. 16 will offer us with greatly improved model for the monopole of bispectrum in redshift space.

In the second part of our report, we explored the goodness of the plane-parallel approximation to the actual radial redshift distortion in Fourier analysis. The first examination is exerted to the monopoles of power spectrum and bispectrum. Our measurements with the Virgo and the VLS simulation are consistent with the results of Scoccimarro et al. (2000) that such deviation due to adoption of parallel approximation is $\sim 10\%$ for the monopole of power spectrum and a few percents for bispectrum. Note that the curve in Fig. 4 shows a systematic trend toward high $k$. Then, we moved to check the reliability of the approach which uses polyspectra measured on $k_z = 0$ plane as estimation in real space, which is true in plane-parallel approximation. We find that plane-parallel approximation to radial distortion brings in serious statistics systematic

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**Fig. 7** Ratios of reduced bispectra for radial and parallel distortion on the plane of $k_z = 0$. Observer is located at the distance of $100h^{-1}\text{Mpc}$. 
biases in this method. On large scales the “real space” power spectrum from measurements on \( k_z = 0 \) plane can be overestimated for more than 10\%, and it is hugely underestimated more than 50\% on small scales \( k >\sim 3h\text{Mpc}^{-1} \) (Fig. 6). We also find that besides the amplitude of reduced bispectrum changes, the configuration dependence suffers from the badness of parallel approximation too (Fig. 8 & 9). According to our simulation, such deviation is persistent even the opening angle of the sample to the observer is as small as 20°. And these discoveries pose questions on those claims based on the “real space” polyspectra obtained in this line.

So as long as we are working with monopoles of power spectrum and bispectrum, it is secure to model redshift distortion with plane-parallel approximation. While if one is going to estimate real space power spectrum and bispectrum with measurements on plane \( k_z = 0 \), one has to be careful to consider the shortcoming of the parallel approximation. Actually a more natural treatment is to decomposed the density contrast in redshift space with spherical harmonics and spherical Bessel functions (Heavens & Taylor 1995; Ballinger, Heavens & Taylor 1995; Fisher et al. 1995; Szalay, Matsubara & Landy 1998; Percival et al. 2004; Percival 2004; Szapudi 2004).

**Acknowledgements** Jun Pan appreciate the support by the One-Hundred-Talents program. Yan-Chuan Cai thanks Prof. Long-Long Feng for support of the work and helpful comments. This work
was supported from the National Science Foundation of China through grant NSFC 10373012. The simulations in this paper were carried out by the Virgo Supercomputing Consortium using computers based at Computing Centre of the Max-Planck Society in Garching and at the Edinburgh Parallel Computing Centre. The data are publicly available at www.mpa-garching.mpg.de/NumCos.

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This paper was prepared with the ChJAA I&TEx macro v1.0.