"Subthreshold" $e^+e^-$ pairs production on photon colliders

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Abstract

In this paper we consider the process of "subthreshold" electron-positron pairs creation in the region of laser conversion. The total number of positrons and their distribution are obtained. This phenomena is offered for use as a good test to examine nonlinear effects of quantum electrodynamics on TESLA.

1 Introduction

Since the end of the eighties and by this time works on designing and development of physical programs for $\gamma e$ and $\gamma\gamma$ - colliders are under development in different countries. Now there are projects on their creation in USA [1], Germany [2], Japan [3]. Physical programs for these colliders created as a result of long-term cooperation of the representatives of many high-energy physics centres are stated in "Conceptual Design Reports" [1-3]. In these projects electron-photon and photon - photon beams are supposed to be obtained on the basis of linear accelerators with $e^+e^-$ beams.

One of the best methods of obtaining intensive $\gamma$ beams is the use of the Compton backscattering of laser light on an electron beam of the linear collider. For the first time in works [4] it was shown, that on the basis of linear colliders with $e^+e^-$ beams it is possible to realize $e\gamma$ and $\gamma\gamma$ - beams with approximately the same energies and luminosities, as for initial electron beams. The necessary intensive bunches of $\gamma$ quanta were offered for receiving at scattering of powerful laser flash on electron bunches of these accelerators.

The small sizes of linear colliders beams make it possible to obtain conversion coefficient (the attitude of number of high-energy photons to number of electrons in a bunch) $k = N_\gamma/N_e \sim 1$ at energy of laser flash in some Joules, i.e. it is possible to convert the most part of electrons to photons.

The detailed description of the scheme of an electron beam conversion in $\gamma$ beam, the basic characteristics of $e\gamma$ and $\gamma\gamma$ collisions, problems of a background and calibration of luminosity were considered in detail in [5].

The region of laser conversion $e \rightarrow \gamma$ is unique by its physical properties. It is the region of an intensive electromagnetic field (the focused laser bunch). This fact allows one to investigate such processes of nonlinear quantum electrodynamics as radiation of a photon by electron in a field of an intensive electromagnetic wave, and also "subthreshold" pairs production [6].
At sufficient power of laser flash in the field of conversion the processes are essential due to absorption from a wave more than one of laser photons simultaneously

\[ e^{-}(p) + n\gamma_{0}(k_{0}) \rightarrow e^{-}(p') + \gamma(k), \quad n \geq 1; \quad (1) \]

\[ \gamma(k) + s\gamma_{0}(k_{0}) \rightarrow e^{-}(p_{+}) + e^{+}(p_{-}), \quad s \geq 1; \quad (2) \]

Processes (1),(2) represent nonlinear by intensity of a field processes of interaction electrons and photons with a field of an electromagnetic wave. The first of these nonlinear processes results in expansion of spectra of high-energy photons and occurrence of additional peaks in spectra of scattered radiation due to absorption of several photons from a wave, and the second one effectively reduces a threshold of \( e^{+}e^{-} \) pairs creation. The interaction of electrons and positrons with a field of an electromagnetic wave results in effective increase of their masses\(^{1}\):

\[ m^{2} \rightarrow m^{2}(1 + \xi^{2}), \quad (3) \]

which is characterized by parameter of intensity of a laser wave \( \xi^{2} \):

\[ \xi^{2} = n_{\gamma} \left( \frac{4\pi\alpha}{m^{2}\omega_{0}} \right) = -\frac{e^{2}a^{2}}{m^{2}}, \quad (4) \]

where \( n_{\gamma} \) - density of photons in a laser wave, \( \omega_{0} \) - their energy, \( a \) - amplitude of classical 4-potential of electromagnetic wave, \( e \) - a charge of electron. Regular research of nonlinear Breit-Wheeler (2) and Compton (1) processes was carried out in [7],[8].

Now the area of nonlinear effects is rather actual and is of great interest because here essential are the processes of radiation due to absorption from a wave of a few of photons, and their probabilities are essentially nonlinear functions of intensity of a field. Recently on accelerator SLAC [9] a series of experiments E-144 with check of predictions of nonlinear QED was finished in the field of parameter \( \xi \sim 1 \) that became possible due to use of the supershort and rigidly focused laser pulses. Thus for the first time the experiment was set up in which the process of \( e^{+}e^{-} \) - pair production at participation of only real, instead of virtual photons was carried out.

## 2 Spectra of high-energy photons

The main features of the conversion are described by a quantity \( x \) which is determined via the initial electron beam energy \( E_{e} \) and the laser photon energy \( \omega_{0} \) as

\[ x = \frac{4E_{e}\omega_{0}}{m^{2}}. \]

The differential probability of process of radiation of a photon by electron performed by a summation over polarizations of final electron and a photon has the following form [7]:

\[ \frac{dW}{dy} = \frac{\pi\alpha^{2}}{2xm^{2}} \sum_{n=1}^{\infty} (F_{00} + 2\lambda P_{c}F_{11}), \quad (5) \]

\(^{1}\)Here we use the system \( \hbar = c = 1 \)
Fig. 1: Spectra of a high-energy photon in a nonlinear case at various values of polarizations of initial particles: (1) $-2\lambda_eP_c = -1$, (2) $-2\lambda_eP_c = +1$ and various values of parameter $\xi = 0.5, 1.0$

$$F_{00} = -4\frac{J_n(z)^2}{\xi^2} + (2 + uy)[J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z)],$$

$$F_{11} = \frac{(2 + u)(1 - 2r_n)}{u + 1}[J_{n-1}^2(z) - J_{n+1}^2(z)],$$

$$u = \frac{y}{1 - y}, \quad z = \frac{2n\xi}{\sqrt{1 + \xi^2}}\sqrt{\frac{u}{u_n}(1 - \frac{u}{u_n})}, \quad u_n = \frac{nx}{1 + \xi^2},$$

$$r_n = \frac{y}{nx(1 + \xi^2)(1 - y)}.$$

$J_n(z)$ is the Bessel functions of nth order, $y = E_\gamma/E_e$, $E_\gamma$ is energy of high-energy photon.

The expression in the sum (5), determines probability of radiation of $n$-harmonics by electron in a field of circular-polarized electromagnetic wave (from a wave $n$ laser photons can be absorbed). The change of a variable $y$ corresponds to the change of a variable $u$: $0 < u < u_n$

$$0 < y < \frac{nx}{nx + 1 + \xi^2}.$$ 

The influence of nonlinear effects results in the fact that the maximum energy of high-energy photons of the first harmonic ($n = 1, \xi^2 \neq 0$) decreases in comparison with the maximum energy of photons in usual Compton effect and the energy of the $\gamma$-quanta formed at absorption from a wave of several photons exceeds energy, achievable in usual Compton effect.

Results of numerical calculations of power spectra of photons in a nonlinear case at $x = 5$ are given in Fig 1. Apparently from these figures, the account of nonlinear effects results in essential change of spectra in comparison with spectra of usual Compton scattering. First, simultaneous absorption from a wave of several laser photons results in expansion of spectra of rigid $\gamma$ quanta and occurrence of the additional peaks, appropriate
to radiation of harmonics of higher order. This expansion at the same parameter \( x \) increases with the intensity of a wave. Second, effective electron weighting results in compression of spectra, i.e. in shift of the first harmonic aside smaller values \( y \). With increase of parameter \( x \) compression of the first harmonic decreases.

3 "Subthreshold" \( e^+e^- \) pairs creation

The process discussed in this section, is the good test on check of nonlinear quantum electrodynamics.

The "hard" photons propagated in the region of laser conversion can produce \( e^+e^- \) pairs in collision with \( s \) laser photons simultaneously

\[
\gamma + s\gamma_0 \rightarrow e^+ + e^-
\]

Threshold value for energy of a photon is defined from the relation

\[
(k + sk_0)^2 = 4m^2(1 + \xi^2)
\]

Here \( k, k_0 - 4\) momentum of photons \( \gamma \) and \( \gamma_0 \). The maximum energy of Compton photon emitted at interaction of electron with \( n \) laser photons is

\[
\frac{E_\gamma}{E_e} = \frac{4nE_\omega_0}{4nE_\omega_0 + m_e^2} = \frac{nx}{nx + 1 + \xi^2}
\]

Thus the corresponding value of electron energy for \( e^+e^- \) pairs creation at absorption from a laser wave \( s \) of photons is

\[
E_{e}^{th} = \frac{m^2(1 + \xi^2)(1 + \sqrt{1+s/n})}{2s\omega_0}
\]

In particular if the "hard" photon is emitted as a result of interaction with one laser photon

\[
E_{e}^{th} = \frac{m^2(1 + \xi^2)(1 + \sqrt{1+s})}{2s\omega_0}
\]

\[
x_{min} = 2(1 + \xi^2)(1 + \sqrt{1+s})
\]

When \( x < 2(1 + \sqrt{2}) \approx 4.8 \) electron-positron pairs will be created only due to nonlinear processes QED. Thus at the first stage of realization of projects NLC (in particular for TESLA) at \( x=4.5 \) occurrence of the big number of positrons will be a consequence of nonlinear effects QED.

Calculated in [7] the probability of \( e^+e^- \) pair production at interaction with circle-polarized laser photons can be submitted by the high-energy non-polarized photon as

\[
W_{e^+e^-} = \frac{e^2m^2}{8\pi E_\gamma} \sum_{s=1}^{\infty} \int_1^{u_s} dw_s,
\]

\[
dw_s = \frac{du}{u\sqrt{u(u-1)}}\left[J_s^2(z) + \xi^2(2u - 1)[(s^2 - 1)J_s^2(z) + J_s^2]\right]
\]
Fig. 2: Number of positrons (electrons) created by one initial electron at various values of parameter $\xi$

$$u_s = \frac{sxy}{4(1 + \xi^2)}, \quad z = \frac{2\xi s}{\sqrt{1 + \xi^2}} \frac{\sqrt{u(u_s - u)}}{u_s}$$

The total number of the produced positrons can be obtained by averaging on an energy spectrum of Compton photons (5)

$$N_{e^+e^-} = \frac{1}{4} kN_\gamma \frac{e^2 m_e^2}{8\pi E_\gamma} \int_{y_0}^{y_{\max}} \sum_{s=1}^{\infty} \int_{u_s}^{u_{\max}} \frac{1}{W} dW \frac{dW}{dy} dy$$  \hspace{1cm} (7)

Here $y_{\max} = nx/(nx + 1 + \xi^2)$. Expression (7) contains two factors 1/2. The first one arises at the account of relative movement of bunches of photons $\gamma$ and $\gamma_0$. The same multiplier appears because of the fact that photons interact in the average with the half of a laser bunch (the length of a laser pulse does not exceed the sizes of area of conversion and distribution of density of bunches in a direction of movement is homogeneous). In Fig. 2 the number of the created positrons per one electron is shown depending on $x$ at various values of parameter $\xi$, value of factor of conversion $k = 0.5$, the length of a laser bunch $l \sim 1 cm$.

At the first stage of project TESLA will be observed $10^7$ pairs per one collision. These pairs will form the essential background, therefore the produced positrons should be removed from the region of interaction with the help of a magnetic field. To observe the process of creation of pairs it is necessary to extract the positrons moving practically along the direction of initial electrons.

4 The distribution of created positrons

We shall consider distribution on energy of the created positrons. Energies of positrons (electrons) are distributed in the interval

$$\epsilon_{\pm} = \frac{1}{E_\gamma} (m_e^2 u_s - \xi^2 m_e^2 u)(1 \mp \sqrt{1 - 1/u}) \pm \frac{1}{2} E_\gamma (1 \pm \sqrt{1 - 1/u})$$  \hspace{1cm} (8)
From here it is easy to get restricts of distribution of positrons on energy

\[ \epsilon_{\text{min}} \leq \epsilon_{+} \leq \epsilon_{\text{max}}, \]

\[ \epsilon_{\text{max}} \approx \frac{1}{2} E_{\gamma_{\text{max}}} (1 + \sqrt{1 - 1/u_s}), \]

\[ \epsilon_{\text{min}} \approx \frac{1}{2} E_{\gamma_{\text{max}}} (1 - \sqrt{1 - 1/u_s}). \]

| Table |
|-------|

| \( s \) | \( \epsilon_{+\text{min}} \) (GeV) | \( \epsilon_{+\text{max}} \) (GeV) | \( s \) | \( \epsilon_{+\text{min}} \) (GeV) | \( \epsilon_{+\text{max}} \) (GeV) |
|-------|----------------|----------------|-------|----------------|----------------|
| 1     | –              | –              | 1     | 99.7           | 818.7          |
| 2     | 63.4           | 148.9          | 2     | 46.7           | 910.8          |
| 3     | 35.2           | 188.3          | 3     | 30.8           | 940.5          |
| 4     | 24.9           | 204.7          | 4     | 22.8           | 955.8          |
| 5     | 19.4           | 214.1          | 5     | 18.4           | 964.4          |

This table represents borders of positrons energies for various number \( s \) of the absorbed laser photons, parameter \( \xi^2 = 0.6 \). Differential distribution of the produced pairs on energy of positrons looks like

\[
\frac{d\omega_{e^+e^-}}{d\rho} = \frac{e^2 m^2}{2\pi E\gamma} \left[ J_s^2(z) + \xi^2 (2u - 1) \left( \frac{s^2}{z^2} - 1 \right) J_s^2(z) + J_s^2 \right],
\]

\[ \rho = \frac{\epsilon_{+}}{E_{\gamma_{\text{max}}}}, \quad u \approx \frac{1}{1 - (2\rho - 1)^2} \]

At small \( \xi \) distribution on energy \([9]\) becomes

\[
\frac{d\omega_{e^+e^-}}{d\rho} = \frac{\alpha m^2}{2Ey} \xi^{2s} \frac{s^{2s}}{2(s!)^2} \left( \frac{2 u^{s-1} (u_s - u)^s}{u_s^2 \sqrt{u(u - 1)}} \right) + \frac{(2u - 1) u^{s-2} (u_s - u)^{s-1}}{u_s^{2(s-1)} \sqrt{u(u - 1)}}
\]

In Fig. 3a differential distributions of the created positrons by a "hard" photon of the maximal energy for various number \( s \) of the absorbed laser photons are submitted. Electron energy \( E_e = 250\,\text{GeV}, \xi^2 = 0.6 \). In Fig.3b we show the same distributions, but averaged in the spectrum of high-energy photons. In Fig. 4a and 4b distributions for energy of electrons \( E_e = 1\,\text{TeV} \) are shown.

The total distribution of positrons is given in Fig.5 From Figures it is possible to see that the maximum in distribution of positrons after averaging on a spectrum of high-energy photons is shifted to the bottom border of a spectrum. It enables precise identification of "subthreshold" \( e^+e^- \) pairs creation as effect of nonlinear quantum electrodynamics. At \( x > 4.8 \) it is possible to observe "subthreshold" creation of positrons in a positron spectrum near the bottom border. For energy of initial electron \( E_e = 250\,\text{GeV} \) positrons with energy in an interval \( 19.4 - 24.9\,\text{GeV} \) are created by absorption of 5 laser
Fig. 3: Differential distribution positrons on energy at $x = 4.5, \xi^2 = 0.6$ for different number of absorbed laser photons, (a) - without averaging on a spectrum, (b) - with averaging on a spectrum.

Fig. 4: Differential distribution positrons on energy for $x = 18, \xi^2 = 0.6$ for different number of absorbed laser photons, (a) - without averaging on a spectrum, (b) - with averaging on a spectrum.
Fig. 5: The total distribution of produced positrons for different values of electron energy $E_e$

photons simultaneously. For energy of initial electron $E_e = 1 TeV$, positrons with energies $\epsilon_+ < 99.7 GeV$ are created only due to nonlinear effects QED.

Let’s note that the contribution of other mechanisms to creation of $e^+e^-$ pairs is much less. For the Bethe-Heitler process

$$e + s\gamma_0 \rightarrow e^+ + e^- + e$$

threshold values of energy are much greater

$$E_{th} = \frac{2m_e^2}{s\omega_0},$$

Except for it, the probability of creation contains the additional factor $\alpha$.

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$dN/d\rho$

$s=5$

$s=4$

$s=3$

$s=2$
$dN/d\rho$

E=1 TeV

E=250 GeV
The diagram shows the function $dW/dy$ for different values of $\xi$ and $x$. The specific values mentioned are $\xi = 0.5$ and $x = 5.0$. The diagram is labeled with curves marked as (1) and (2) for different conditions or parameters, indicating distinct behaviors or solutions to the described function.
\[ \frac{dW}{dy} \]

\( \xi = 1.0 \)

\( x = 5.0 \)

(1)

(2)
