Wide $\sigma$ Orionis binaries resolved by UKIDSS

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In spite of its importance for the study of star formation at all mass domains, the nearby young $\sigma$ Orionis cluster still lacks a comprehensive survey for multiplicity. We try to fill that observational gap by looking for wide resolved binaries with angular separations between 0.4 and 4.0 arcsec. We search for companions to 331 catalogued cluster stellar members and candidates in public $K$-band UKIDSS images outside the innermost 1 arcmin, which is affected by the glare of the bright, eponymous $\sigma$ Ori multiple system, and investigate their cluster membership with colour-magnitude diagrams and previous knowledge of youth features. Of the 18 identified pairs, ten have very low individual probabilities of chance alignment (< 1 %) and are considered here as physical pairs. Four of them are new, while the other six had been discovered previously, but never investigated homogeneously and in detail. Projected physical separations and magnitude differences of the ten probably bound pairs range from 180 to 1220 au, and from 0.0 to 3.4 mag in $K$, respectively. Besides, we identify two cluster stars with elongated point spread functions. We determine the minimum frequency of wide multiplicity in the interval of projected physical separations $s = 160 - 1600$ au in $\sigma$ Orionis at $3.0^{+1.2}_{-1.1} \%$. We discover a new Lindroos system, find that massive and X-ray stars tend to be in pairs or trios, conclude that multiplicity truncates circumstellar discs and enhances X-ray emission, and ascribe a reported lithium depletion in a young star to unresolved binarity in spectra of moderate resolution. When accounting for all known multiples, including spectroscopic binaries, the minimum frequency of multiplicity increases to about 10 %, which implies that of the order of 80–100 unknown multiple systems still await discovery in $\sigma$ Orionis.

1 Introduction

Stars can be single or members in double and multiple systems. Multiplicity is, together with mass function, disc frequency, and spatial distribution, one of the key parameters for constraining low-mass star-formation scenarios (Reipurth & Clarke 2001; Chabrier 2003; Lada 2006; Goodwin et al. 2007; Bate 2009; Parker et al. 2009). There have been numerous surveys for multiplicity in the field (e.g., Duquennoy & Mayor 1991; Reid & Gizis 1997; Bouy et al. 2003; Raghavan et al. 2010; Janson et al. 2012) and in open clusters (see the review by Duchêne & Kraus 2013 and references therein). Virtually every open cluster younger than ~600 Ma and at less than 1 kpc has been surveyed for multiplicity, both resolved (i.e., by means of speckle, first, and adaptive optics and space missions, next) and unresolved (i.e., spectroscopic binarity): Hyades (Mason et al. 1993a; Patience et al. 1998), $\alpha$ Persei and Praesepe (Mason et al. 1993b; Patience et al. 2002), Pleiades (Steele & Jameson 1995; Bouvier et al. 1997; Martín et al. 2000; Bouy et al. 2006a; Lodieu et al. 2007), the Scorpius-Centaurus-Lupus-Crux complex (including Upper Scorpius – Kraus et al. 2005; Bouy et al. 2006b; Kouwenhoven et al. 2007; Lafrenière et al. 2014), Taurus-Auriga (Ghez et al. 1993; Leinert et al. 1993; Kraus et al. 2011), the Orion Nebula Cluster (Padgett et al. 1997; Preibisch et al. 1999; Köhler et al. 2006; Reipurth et al. 2007), ρ Ophiuchi (Ratzka et al. 2005), or Chamaeleon I (Lafrenière et al. 2008).

The $\sigma$ Orionis open cluster ($d = 387.5\pm1.3$ pc, $\tau \sim 3$ Ma) in the Ori OB1b association is, especially because of its numerous and well-investigated brown-dwarf and planetary-mass population, another cornerstone cluster for the study of the stellar and substellar formation (Garrison 1967; Wolk 1996; Béjar et al. 1999; Zapatero Osorio et al. 2000; Caballero 2008c; Schaefer et al. 2016). Therefore, it should have its own comprehensive multiplicity survey. However, this study is still missing. Caballero (2014) tried to fill part of this observing gap with a literature review on multiplicity in $\sigma$ Orionis, but a reliable, exhaustive, homogeneous survey does not exist yet.
In this work, we focus on wide binaries in \( \sigma \) Orionis that can be resolved from the ground with standard imaging (i.e., no lucky or adaptive-optics imaging), but that may have been uncovered by previous all-sky surveys such as DENIS or 2MASS (Epchtein et al. 1997; Skrutskie et al. 2006). This requisite translates into exploring angular separations between 0.4 and 4.0 arcsec approximately, or, alternatively, projected physical separations between 160 and 1600 au at the most probable cluster distance. To achieve our goal, we used data of the UKIRT Infrared Deep Sky Survey (UKIDSS; Lawrence et al. 2007).

2 Analysis and results

We downloaded public UKIDSS \( K \)-band images from the WFCAM Science Archive of 331 \( \sigma \) Orionis cluster members and member candidates tabulated in the Mayrit catalogue (Caballero 2008c) and subsequent additions and corrections (see below). In particular, we got square images, 300 pixel in side, from the UKIDSS Eighth Data Release DR8+ taken under the Galactic Clusters Survey (GCS; Lodieu et al. 2009). In spite of UKIRT WFCAM (the camera with which the UKIDSS survey was carried out; Casali et al. 2007) having a pixel scale of 0.400±0.010 arcsec, the \( K \)-band images had a pixel scale of only 0.200±0.005 arcsec. This improved sampling of the median seeing at UKIRT (~0.7 arcsec) was thanks to the interlacing of data obtained in a \( 2 \times 2 \) microstepping sequence with step size \( N+0.5 \) pixels (cf. Lawrence et al. 2007). As a result, the angular size of the downloaded \( K \)-band images was of 60 arcsec. Since the microstepping method was applied to the \( K \) GCS images only, the ones in \( ZYJH \) bands had the original pixel scale of 0.4 arcsec. All UKIDSS observations in the \( \sigma \) Orionis area were performed either on 2005 Oct 14 (epoch 2005.79) or 2005 Nov 22 (epoch 2005.89). Gaussian full-width half maxima of most non-saturating single stars within 30 arcmin to the \( \sigma \) Ori multiple system ranged between 0.6 and 1.0 arcsec in the \( K \) band.

Although the membership in cluster of many stars and brown dwarfs in the almost-ten-year-old Mayrit catalogue has been confirmed or refuted in newer astrometric (Caballero 2010a, 2017), X-rays (López-Santiago & Caballero 2008; Caballero et al. 2010b), and spectroscopic surveys (Caballero et al. 2008b, 2012; Maxted et al. 2008; Sacco et al. 2008; Hernández et al. 2014; Koenig et al. 2015), it is still the most comprehensive catalogue of cluster members and member candidates in \( \sigma \) Orionis. The Mayrit catalogue is complete down to (and slightly beyond) the cluster substellar boundary at \( J \sim 14.5 \) mag (Caballero et al. 2007), where the UKIDSS quality image is still good enough for multiplicity studies. An exhaustive analysis of multiplicity of much fainter brown dwarfs and planetary-mass objects in \( \sigma \) Orionis would require a new deeper dataset, such as VIRCAM (Peña Ramírez et al. 2012; V. J. S. Béjar, in prep.) or VIMOS (Elliott et al. 2017). Except for the innermost arcminute in the cluster centre, which is affected by the intense glare of the bright eponymous \( \sigma \) Ori trapezium, both the UKIDSS and Mayrit catalogues nicely complement each other for wide multiplicity studies. For completeness, we updated the Mayrit catalogue with 13 new additions and 11 deletions. The names and coordinates of the 24 new/discard stars and galaxies, together with references for discovery, non-/membership feature identification, and Mayrit number\(^1\) are provided in Table 1. The final star counting is as follows: 338 sources in the original Mayrit catalogue, –9 stars inside the central arcminute, +13 additions, –11 deletions, which make 331 investigated cluster stars.

Once we downloaded all the UKIDSS \( K \)-band images, we looked by eye for resolved companions at \( \rho < 4 \) arcsec to the 331 Mayrit targets outside the central arcminute. For completeness, we did the same for the 11 deletions. We choose \( \rho = 4 \) arcsec as the search boundary because the best 2MASS images had full-width-half-maxima of 2.5 arcsec and the 2MASS Point Source Catalog “standard aperture” magnitude was measured in a 4 arcsec-radius aperture (cf. Skrutskie et al. 2006). Wider companions brighter than the 2MASS completeness magnitudes should have been detected in previous comprehensive surveys in the area. In Table[2] and Fig.[A1] we show the 18 identified multiple system candidates. We were interested on sources with stellar point spread functions (PSFs) only and, therefore, we did not account for a faint background extended optical companion, probably of extragalactic nature, at 2–3 arcsec to the NWN to Mayrit 270196. As expected, none of the 18 pairs had been resolved by 2MASS. Four of them had 2MASS \( J \)- and \( H \)-band detections where the goodness-of-fit quality of the profile-fit photometry was very poor, which is indicated by the category ‘EEA’ in the photometric quality flag. One star, namely TYC 4770–1261–1, had a very poor fit also in the \( K \) band (Qflg = ‘EE’).

For the 18 pairs, we report accurate relative astrometry (angular separation \( \rho \) and position angle \( \theta \)) and UKIDSS \( K \)-band magnitude differences in Table[2]. The coordinates and individual magnitudes from where we computed the data are provided in Table[A1]. In all but three cases, the UKIDSS automatic pipeline was able to resolve both sources in at least the \( K \) band. Accounted UKIDSS absolute astrometric error varied between 0.05 and 0.07 arcsec. However, we considered the relative astrometric error at short angular separations to be lower, of the order of the photo-centroid uncertainty, which often is about one tenth of a pixel. Similarly, accounted UKIDSS photometric errors of individual components varied between slightly less than one millimagnitude and two tenths of a magnitude, depending on signal-to-noise ratio, quality of PSF fit, and contamination by bright, close sources.

For the three exceptions for which the UKIDSS automatic pipeline was not able to resolve both components,

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\(^1\) The Mayrit number provides the separation to the cluster centre in arcsec and position angle in deg. For example, Mayrit 92149 is at \( \rho = 92 \) arcsec and \( \theta = 149 \) deg to \( \sigma \) Ori Au,Ab,B. Mayrit 359179 is at \( \rho = 359 \) arcsec and \( \theta = 179 \) deg, etc.
Table 1  New (upper part) and discarded (lower part) Mayrit starsa.

| Mayrit   | Alternative name | α (J2000) | δ (J2000) | $J_{SM}$ [mag] | Referencesb |
|----------|------------------|-----------|-----------|----------------|-------------|
| 68229    | [W96] pJ053941–0236 | 05 38 41.36 | -02 36 44.5 | 12.30±0.02 | Wo96, Ca10b |
| 168291 AB| [W96] rJ053834–0234 | 05 38 34.31 | -02 35 00.1 | 11.22±0.03 | Wo96, Ca10b, He14 |
| 172264   | [SWW2004] 130     | 05 38 33.36 | -02 36 17.6 | 12.05±0.03 | Sh04, Ca10b |
| 270196   | [HHM2007] 655     | 05 38 39.73 | -02 40 19.7 | 13.75±0.03 | He07, Ca10b |
| 332167   | [SWW2004] 200     | 05 38 49.93 | -02 41 22.8 | 12.75±0.03 | Sh04, Sa08, Ca10b, He14 |
| 492211   | [W96] rJ053827–0242| 05 38 27.74 | -02 43 01.0 | 12.19±0.03 | Wo96, Sa08, Ca10b, He14 |
| 605079   | [SWW2004] 127     | 05 39 24.36 | -02 34 01.4 | 12.98±0.03 | Sh04, Sa08, Ca10b |
| 957055   | [SWW2004] 163     | 05 39 37.30 | -02 26 56.8 | 11.70±0.03 | Sh04, Ca10b, Ca12, He14 |
| 1093033  | [HHM2007] 1030    | 05 39 24.56 | -02 20 44.1 | 11.37±0.03 | He07, Ca10b, He14 |
| 1149270  | [HHM2007] 59      | 05 37 28.07 | -02 36 06.6 | 13.74±0.03 | He07, LSC08c |
| 1160240  | [SWW2004] 184     | 05 37 37.85 | -02 45 44.2 | 12.69±0.03 | Sh04, LSC08, He14 |
| 1178039  | [SWW2004] 138     | 05 39 33.79 | -02 20 39.9 | 12.37±0.03 | Sh04, Ca10b |
| 1343402  | [SWW2004] 172     | 05 37 28.32 | -02 24 18.2 | 14.00±0.03 | Sh04, LSC08 |

a  Objects in the Mayrit catalogue of stars and brown dwarfs in the σ Orionis cluster outside the innermost 1 arcmin that were not listed by Caballero (2008c) and with both DENIS i and 2MASS JHK, detections [new, upper part], or that were in the original catalogue but have been classified as non-cluster members afterwards [discarded, bottom part]. Mayrit numbers of discarded objects (in parenthesis), either stars or galaxies, shall not be used.

b References – Ste86: Stephenson 1986; GC93: Gray & Corbally 1993; Wo96: Wolk 1996; Sh04: Sherry et al. 2004; Fr06: Franciosini et al. 2006; Ca07b: Caballero 2007b; He07: Hernández et al. 2007; Ca08: Caballero et al. 2008b; GH08: González Hernández et al. 2008; LSC08: López-Santiago & Caballero 2008; Sa08: Sacco et al. 2008; Lo09: Lodieu et al. 2009; Ca10a: Caballero 2010a; Ca10b: Caballero et al. 2010b; Ca12: Caballero et al. 2012; Ca14: Caballero 2014; He14: Hernández et al. 2014; Ca17: Caballero 2017.

c Mayrit 1149270 was classified as a cluster non-member by Hernández et al. (2014). However, it is a strong X-ray emitter in the cluster with slightly blue colours according to López-Santiago & Caballero (2008).

namely Mayrit 359179 AB, 785038 AB and 1564349 AB, ρ and ΔK came instead from the separation of photo-centroids and ratio of flux peaks in the original UKIDSS images measured by us with the IRAF environment. Astrometric and photometric errors calculated with this method were 0.04–0.08 arcsec and 0.03–0.04 mag, respectively. Besides, for the computation of ΔK of the bright binary Mayrit 960106, which saturates in the UKIDSS image, we used the 2MASS $K_s$, magnitude of the primary as a reference. We verified the automatic UKIDSS and our IRAF astrometric and photometric measurements with the IDL task xstarfinder. The new IDL measurements do not show any significant improvement with respect to the UKIDSS and IRAF ones, as previously warned by A. Lawrence & S. Warren (priv. comm.).

When investigating the companion detectability as a function of separation, ΔK vs. ρ, we concluded that it was easier to resolve the systems with relatively large angular separations, as expected, and with relatively faint components far from the saturation regime. In particular, we were able to resolve equal-brightness pairs (i.e. ΔK = 0 mag) at angular separations ρ ≈ 0.5 arcsec or larger, and to discover faint companions with magnitude differences ΔK = 5 mag at ρ ≈ 1.7 arcsec or larger, as illustrated in Fig. 1.

Of the 18 identified multiple system candidates, eight had been discovered previously by the first author, but in different surveys (Caballero 2006, with WFC at the 2.5 m Isaac Newton Telescope; Caballero 2007b, with CAIN-2 at the 1.5 m Telescopio Carlos Sánchez; Caballero 2008c, from asymmetries of stellar profiles in 2MASS images; Caballero 2010a and Caballero et al. 2010b, with digitized...
Table 2  Names, astrometry, and photometry of candidate physical (upper part) and optical (lower part) binaries resolved in the UKIDSS images outside the innermost 1 arcmin.

| Mayrit name | Alternative name | 2Mα [J2000] | 2Mδ [J2000] | J2M [mag] | ρ [arcsec] | θ [deg] | ∆K [KIDS] [mag] | Prob [%] |
|-------------|------------------|-------------|-------------|-----------|------------|--------|----------------|--------|
| 92149 AB    | [W96] J053847-0237 | 05 38 47.92 | -02 37 19.2 | 12.02±0.04 | 2.14±0.02 | 63.7±0.5 | 0.827±0.0012 | 0.5   |
| 359179 AB   | V595 Ori (Haro 5–14) | 05 38 45.38 | -02 41 59.4 | 11.99±0.03 | 0.97±0.08 | 225±4 | 0.90±0.03 | 0.02  |
| 489165 AB   | [W96] J053853-0243 | 05 38 53.17 | -02 43 52.8 | 12.24±0.03 | 1.26±0.02 | 356.8±0.9 | 1.824±0.003 | 0.03  |
| 707162 AB   | [W96] J053859-0247 | 05 38 59.11 | -02 47 13.3 | 11.32±0.03 | 0.73±0.02 | 89.3±1.6 | 0.680±0.0012 | 0.007 |
| 785038 AB   | Kiso A-0904 80    | 05 39 17.18 | -02 25 43.4 | 12.90±0.03 | 0.48±0.08 | 333.4 | 0.02±0.04 | 0.003 |
| 960106 AB   | V1147 Ori (HD 37633) | 05 39 46.20 | -02 40 32.1 | 8.88±0.04 | 3.17±0.02 | 232.3±0.4 | 3.41±0.02 | 0.09  |
| 1106058 AB  | 2E 1486          | 05 39 47.42 | -02 26 16.3 | 10.04±0.03 | 2.75±0.02 | 219.9±0.4 | 1.988±0.0011 | 0.06  |
| 1245057 AB  | V605 Ori (Haro 5–31) | 05 39 54.29 | -02 24 38.6 | 12.34±0.03 | 2.16±0.02 | 184.2±0.5 | 0.025±0.0008 | 0.03  |
| 1411131 AB  | [SWW2004] 118    | 05 39 56.02 | -02 51 22.8 | 12.04±0.04 | 1.98±0.02 | 300.6±0.6 | 0.975±0.0015 | 0.03  |
| 1564349 AB  | 2E 1464          | 05 38 25.50 | -02 10 22.9 | 12.11±0.03 | 0.86±0.04 | 48±2 | 0.24±0.03 | 0.004 |

a) Chance alignment probabilities. See Section [3]
b) Previously known pairs. See details in Caballero (2014).
c) 2MASS quality flag EEA or EEE.
d) Discarded cluster member candidate in Table [1]

Fig. 1  ∆K vs. ρ magnitude difference-angular separation diagram for the 18 pairs in Table [2]. The dash-dotted line indicates the approximate detection limit of our survey. Filled and open circles stand for physical pairs with primaries with and without known features of extreme youth, respectively, and crosses stand for optical doubles with secondaries in the cluster back- and foreground. Errorbar sizes are of the order of symbol sizes.

Photographic plates). Indeed, we have recovered all previously known ‘close’ σ Orionis binaries with angular separations in the interval 0.7 arcsec < ρ < 4.0 arcsec, as tabulated by Caballero (2014). As shown in his Table III, very few astrometric data had been provided for the eight known pairs. See details in Caballero (2014).

Most of the star and brown dwarf primaries in Table [2] have signposts of extreme youth (i.e., membership in σ Orionis – Caballero 2008c): eight have strong Li i λ6707.8 Å lines in absorption in their spectra (among other youth features – Muzerolle et al. 2003; Caballero 2006; Sacco et al. 2008; Hernández et al. 2014), one is a peculiar, magnetic, Herbig Ae/Be star (Mayrit 960106; e.g., Baguño et al. 2006), one is a strong Hα emitter identified by four independent prism-objective surveys (Mayrit 1245057: Haro & Moreno 1953; Wiramihardja et al. 1989, 1991; Weaver & Babcock 2004), one is a strong X-ray emitter discovered by Einstein (Mayrit 1106058: Wolk 1996; Caballero et al. 2010b), and one has low surface gravity based on Na i λ8183.3,8194.8 Å absorption weaker than field dwarfs of the same spectral type and a circum(sub)stellar disc based on near-infrared flux excess (Mayrit 803197: Caballero et al. 2007; Luhman et al. 2008; Maxted et al. 2008). Of the remaining six stars, one has a Li i equivalent width lower than expected for its magnitudes and...
colours (Mayrit 785038; but see Section[5] and the other five are photometric cluster member candidates without spectroscopy (Scholz & Eisloffel 2004; Sherry et al. 2004; Caballero 2007a). Lastly, López-Santiago & Caballero (2008) associated a faint Chandra X-ray emission to TYC 4770–1261–1, which is unexpected for its estimated A–F spectral type; such X-ray emission may be caused by the (optical) companion candidate instead.

Ten secondaries had accurate UKIDSS DR8 + photometry in at least the ZYJK bands, which allowed us to apply the photometric selection criteria used by Lodieu et al. (2009) in the Z vs. Z – J, Y vs. Y – J, and J vs. J – K colour-magnitude diagrams. Of them, six had magnitudes inconsistent with membership in σ Orionis. We downloaded UKIDSS Z, Y, and J images of the stars only resolved in K, and repeated the process of estimating ΔZ, ΔY, and ΔJ from ratios of flux peaks with IRAF on the original UKIDSS images. In some cases, magnitude uncertainties were as high as 0.2–0.3 mag, but small enough to discard another two very close (ρ ~ 1.4–1.5 arcsec) companion candidates to the young brown dwarf Mayrit 803197 and the photometric star cluster candidate Mayrit 1788137. Fig. 2 illustrates this classification (Y vs. Y – J and J vs. J – K colour-magnitude diagrams do not provide extra information).

All the 20 primaries and secondaries in the upper part of Table[2](candidate physical binaries) lie redwards of the Lodieu et al. (2009) photometric cluster member selection criteria in ZYJK bands (see the Z vs. Z – J example in Fig. 2). Of the ten primaries of candidate physical pairs, only one (Mayrit 1411131 A) has no known feature of extreme youth. On the other hand, the eight systems with unbound companions, which are listed in the bottom part of Table[2] lie bluewards of of the Lodieu et al.’s criteria and are, therefore, optical doubles in the classical nomenclature of double-star astronomers (e.g. Argyle 2012). In particular, their secondaries are fainter than primaries, by over 2.5 mag in all but two cases (which, if cluster members, would place them well in the substellar domain), but bluer or with the same approximate colours. Optical companions are field late-type dwarf interlopers or, most likely, extragalactic sources in the cluster background (Caballero et al. 2008a).

3 Discussion

3.1 Physical nature of binary candidates

Since the proper motion of σ Orionis is very low, of a few milliarcseconds per year (Orion is in the antapex and at about 400 pc), currently we cannot discriminate between actual bound systems and chance alignments of pairs due to a projection effect without an appropriate radial-velocity study (Caballero 2010a). However, Caballero (2009b) performed a statistical study and assigned to each possible pair in σ Orionis a probability of random alignment as a function of the angular separations between the two components and between the primary and the cluster centre, which is defined by the massive triple OB system σ Ori Aa,Ab,B (Caballero et al. 2006 had done a less-elaborated statistical analysis before). Such probabilities of alignment were calculated after simulating 1000 Monte Carlo spatial distributions of cluster members following the actual radial profile measured by Caballero (2008a) and computing angular separations between the nearest neighbours in each simulated cluster. From Fig. 1 in Caballero (2009b), all of our physical binary candidates have individual probabilities of chance alignment less than 1 % (i.e., a probability of being physically bound greater than 99 %). In the last column of Table[2] we estimate new individual probabilities of chance alignment using the Caballero (2009b)’s Monte Carlo values and the Caballero (2008a)’s surface density of σ Orionis members. New values range from 0.5 % for Mayrit 92149 AB, a pair with ρ = 2.14±0.02 arcsec and at 1.53 arcmin to the cluster centre, to only 0.003 % for Mayrit 785038 AB, a pair with ρ = 0.48±0.08 arcsec and at 13.1 arcmin to the cluster centre.

Moreover, although the statistics is low, the spatial distribution of physical binary candidates in Fig. 3 follows the power-law distribution of cluster members as in Caballero (2008a), while optical binary candidates follow the one of unrelated foreground/background sources, which are distributed uniformly within the 30 arcmin-radius area centred on σ Ori Aa,Ab,B. From now on, we will assume that only the ten pairs in the upper part of Table[2] actually orbit around a common centre of mass (i.e., are gravitationally bound), while the other eight optical doubles are chance alignments.

A more accurate value of the contamination by chance alignments may come from calculating the change alignment probability for all the 331 primaries that we looked around in the 0.4–4.0 arcsec separation interval and then...
sum them. Such interval probabilities are strongly dependent on the separation to the cluster centre, \( f(r) \) vs. \( r \), onto the power-law distributions for \( f(r) \propto r^{1.0} \) and \( r^{2.0} \) (black dotted lines), which correspond to the actual distributions of \( \sigma \) Orionis members (top) and interlopers (bottom). See Caballero (2008a) for details.

With the projected physical separations and individual masses estimated homogeneously from \( J \) magnitudes and BT-Settl theoretical isochrones at 3 Ma (Baraffe et al. 2015), we computed reduced binding energies for the ten physical pairs as in Caballero (2009a). Values of \( -U'_{j} \) range in the interval from 130 to 2200 \( 10^{33} \) J for Mayrit 1564349 AB and Mayrit 960106 AB, respectively. These values are at least two orders of magnitude larger than the binding energies of the most fragile systems known in the field (at around \( 10^{33} \) J – Weinberg et al. 1989; Close et al. 2003; Dhital et al. 2010), which supports our assumption of physical binding. It is not surprising that we do not find physical binaries beyond the “boundary between stellar kinematic groups and very wide binaries” (Caballero 2009a) because we limited our pairing radius to only 4 arcsec (see Section 2).

### 3.2 Binary frequency

At the cluster distance, the angular separations between components in the ten physical pairs translate into projected physical separations in the range \( s = 180\pm30 \) (for Mayrit 785039 AB) to 1220\( \pm80 \) au (for Mayrit 960106 AB), which are typically found in other open clusters at \( d \lesssim 400 \) pc (see Duchêne & Kraus 2013 and references therein). The investigated interval of projected physical separations was of 160–1600 au.

We measured a frequency of wide multiplicity in \( \sigma \) Orionis of \( 3.0^{+1.2}_{-1.1} \% \) (10/331). This frequency must be handle with care, since we were not sensitive to companions fainter by \( \Delta K \gtrsim 5 \) mag than their host stars or to any source at physical separations closer than \( \sim 160 \) au. Actually, from Fig. 1, we were not sensitive either to some companions between 160 and \( \sim650 \) au (\( \sim1.7 \) arcsec) depending on the magnitude difference with its primary. Besides, we have not accounted for possible systems separated more than \( \sim1600 \) au because of the 4 arcsec limit of our search. In principle, wider companions fainter than the 2MASS completeness, but brighter than the UKIDSS completeness, should have been identified in previous deep photometric surveys (e.g. Lodieu et al. 2009; Peña-Ramírez et al. 2012).

Another possible effect to put the derived multiplicity rate into context is the Malmquist-like bias that leads to the preferential detection and identification of certain \( \sigma \) Orionis members. While the original Malmquist bias favours the detection of intrinsically bright objects, the identification of bona fide cluster members by all previous surveys in \( \sigma \) Orionis has been biased towards young stars at all magnitudes with strong X-ray and/or \( \text{H}\alpha \) emission, infrared-flux excess, or photometric variability (Caballero 2008c). As it has been confirmed afterwards (see also Table 1), the degree of contamination in the Mayrit catalogue is very low, but there may be still some \( \sigma \) Orionis members close to the substellar boundary yet to be discovered, especially with the bluest optical-to-infrared colours and lowest activity levels (cf. Caballero 2008c). Thus, the derived overall multiplicity rate may be artificially high.

Regardless the effect of incompleteness in the 160–650 au interval is larger or not than the Malmquist-like bias for the detection of active cluster members, a wide companion frequency \( CF \) of \( 3.0^{+1.2}_{-1.1} \% \) is quite low with respect to what has been published for other juvenile and young open clusters and star-forming regions (see the exhaustive reference list in Section 1). Unfortunately, given the heterogeneity in frequency derivations, cluster ages, surveyed primary masses, and physical/angular separations, it is difficult, if not impossible, to draw up any persuasive conclusion from the comparison.

As an example, the second lowest companion frequency after \( \sigma \) Orionis’ is that of the Orion Nebula Cluster (\( CF = 8.8^{+1.1}_{-1.3} \% \) at \( s = 68–680 \) au; Reipurth et al. 2007), which is slightly younger and has roughly the same heliocentric distance and metallicity (González Hernández et al. 2008; Schaefer et al. 2016). The apparently lower \( \sigma \) Orionis companion frequency can be ascribed to the lower masses of primaries (late-type, very low-mass stars have low binary frequencies; cf. Duchêne & Kraus 2013 – roughly 80–90 \%
of the Mayrit stars and brown dwarfs have K and M spectral types) and different surveyed projected physical separations (Padgett et al. 1997, Köhler et al. 2006 and Reipurth et al. 2007 used the Hubble Space Telescope or expensive adaptive optics systems, and were sensitive to closer companions). However, assuming a log-flat separation distribution, one gets a frequency of 6±2% over the same separation range as Reipurth et al., which is a little over 1σ difference.

Since the distribution of companions in open clusters peaks at about 3–5 au (Patience et al. 2002 – see also the Opik’s distribution), it is expected that a new high-resolution imaging survey for companions to the most massive σ Orionis stars at angular separations shorter than 0.4 arcsec (s < 160 au) would measure a companion frequency comparable to those of other open clusters.

### 3.3 Miscellanea

#### 3.3.1 Spectral types

Only one star in the top of Table 2 has an early spectral type: the peculiar, magnetic, Herbig Ae/Be star Mayrit 960106 A (V1147 Ori), which is tabulated as an α2 CVn-type variable B9 IIIp (Eu-Si) star (Joncas & Borra 1981; North 1984; Catalano & Renson 1998; Bagnulo et al. 2006; Landstreet et al. 2007). Hernández et al. (2014) determined spectral types between M1.0 and M2.0 for another six primaries. Except for Mayrit 1106058 A, which is likely a K-type star, we estimate from red optical and near-infrared photometry that except for Mayrit 960106 B, which has an approximated K spectral type, we estimate from red optical and near-infrared photometry that the other 12 stars (two primaries and ten secondaries) also lie in the M-type domain.

The latest spectral type corresponds to the faintest physical companion, Mayrit 489165 B, which has an approximate K magnitude of 13.3 mag. This object, likely an M4–6 star, has a most probable mass just above the substellar boundary (Caballero et al. 2007). Mayrit 489165 B is also one of the three faintest companions with respect to their primaries. The other two faint secondaries are Mayrit 1106058 B (ΔK ~ 2.0 mag) and Mayrit 960106 B (ΔK ~ 3.4 mag). The remaining seven systems have differences ΔK < 1.0 mag. See Fig. 3 in Zapatero Osorio et al. (2003) for a magnitude vs. spectral type diagram of spectroscopically-confirmed members in σ Orionis.

#### 3.3.2 A new Lindroos systems

Large magnitude differences usually translate into small mass ratios (q = M2/M1). Actually, the smallest mass ratio, of about one tenth, corresponds to the pair Mayrit 960106 (V1147 Ori). The peculiar B9 giant primary, with a mass of 3.5+0.3/−0.2 M⊙ (Caballero 2007a), may form, together with its T Tauri mid-M secondary, the second Lindroos system (Lindroos 1985) discovered in σ Orionis after σ Orionis Aa,Ab and σ Ori IRS1 AB (van Loon & Oliveira 2004; Caballero 2005; Hodapp et al. 2009). Furthermore, Caballero et al. (2010b) remarked that Mayrit 960106 and Mayrit 524060 (an anonymous A8; V cluster star) were “the only [early-type stars in σ Orionis detected with HRC-I/Chandra] that are not known to form part of a multiple system“.

We may wait for a dedicated study with XMM-Newton or, alternatively, a next generation of X-ray space missions (e.g., Athena), for ascertaining which is the actual origin of the X-ray emission in the Mayrit 960106 system: either the primary, previously thought to be single, or the late-type secondary at 3.17±0.02 arcsec, which we propose here to be the true X-ray emitter. The second scenario would explain the large Lx/Lσ ratio and angular separation between Chandra X-ray and 2MASS cross-matched sources found for Mayrit 960106 by Caballero et al. (2010b). (Today, Mayrit 524060 remains as the only single OBA-type cluster star with X-ray emission).

### 3.3.3 X-rays and discs

Interestingly, eight of the ten physical pairs are known X-ray emitters, and the other two fall in areas not investigated in detail yet (Wolk 1996; Franciosini et al. 2006; Skinner et al. 2008; López-Santiago & Caballero 2008). Since roughly one half of the σ Orionis stars have been detected with Einstein, ROSAT, XMM-Newton, and/or Chandra in X-rays (Caballero et al. 2010b), it is hard to believe that this is a chance situation: the possibility of picking up eight X-ray stars among a sample of just eight random stars in the cluster (assuming an X-ray frequency of about 50%) is p ~ 0.4%. Therefore, the existence of companions at large projected physical separations is able to enhance the X-ray emission in σ Orionis. Components in young multiple systems cannot be tidally locked at s ≥ 160 au (tidal locking implies short rotation periods in close binaries, which translates into enhanced magnetic activity and coronal emission), but high-mass stellar wind collision, ionised mass trapped within long magnetic channels connecting the two low-mass stars, unresolved spectroscopic binarity (thus making systems to be triple), or just a factor two in X-ray flux (no past or current X-ray space mission has resolved the systems in two components, both of which can be emitters) can be other possible explanations.

On the other side, the minimum overall disc frequency in σ Orionis is 30% (Caballero et al. 2007; Hernández et al. 2007; Luhman et al. 2008). The probability that there is no disc host system among our ten physical pairs, assuming such a disc frequency, is as low as 2.8%. Thus, there is a 97.2% chance that the multiple systems have a lower disc frequency (but Mayrit 1245057 might have a disc; Luhman et al. 2008). It could be even larger if one considers that each binary contains two stars, hence more chances of hosting at least one disc. Although a larger sample is needed to assess its veracity, the existence of companions at large projected physical separations seems to affect or truncate circumstellar discs in σ Orionis.
3.3.4 A remarkable pair

Mayrit 785038 AB is, with $\rho = 0.48 \pm 0.08$ arcsec ($s = 180 \pm 30$ au), the closest pair in our sample. It is an H$\alpha$ and X-ray emitter (Wiramihardja et al. 1989; Franciosini et al. 2006; Caballero et al. 2010) and low-amplitude photometric variable (Scholz & Eisloffel 2004), and has a radial velocity consistent with cluster membership (Sacco et al. 2008). It has, however, a lithium pseudo-equivalent width lower than expected for its spectral type, of $pEW(\text{Li} i) = 133 \pm 7$ mÅ (with $R \approx 17,000$; Sacco et al. 2007, 2008). With spectral data of higher resolution ($R \approx 34,000$), Hernández et al. (2014) measured H$\alpha$ absorption and central emission and radial velocity compatible with previous values, but no lithium absorption. However, typical $pEW(\text{Li} i)$s of early and intermediate M dwarfs in $\sigma$ Orionis are 400–700 mÅ (Zapatero Osorio et al. 2002; Kenyon et al. 2005; Caballero 2006; González Hernández et al. 2008; Sacco et al. 2008; Caballero et al. 2012; Hernández et al. 2014).

Far from attributing this severe lithium depletion to a post-T Tauri phase, we ascribe it instead to the observed resolved multiplicity. For example, the first spectroscopic binary discovered in the $\sigma$ Orionis cluster, Mayrit 1415279 AB (OrinTT 429), shows an apparent lithium depletion when observed at moderate resolution ($R = 10,500$; Caballero 2006), but the two lines are visible with échelle spectrographs at higher resolution (Lee et al. 1994). Assuming $M_A \approx M_B \approx 0.31 M_\odot$ (again from $J$ magnitudes and NextGen models) and $a \approx s$ (being $a$ the semi-major axis), the maximum radial-velocity semi-amplitude of one component in Mayrit 785038 AB is $K_1 \approx 1.8/\sqrt{1-e^2}$ km s$^{-1}$. A high eccentricity $e = 0.8$ could lead to a separation of about 8 km s$^{-1}$ in an epoch close to the periastron, which would easily blur the lithium line, as in the case of Mayrit 1415279 AB. However, Sacco et al. (2007, 2008) did not report a significant veiling or line broadening in their spectra. Another possibility is that Mayrit 785038 AB is actually an optical system of two mid-M dwarfs, one young (in $\sigma$ Orionis) and one old interloper (in the field), of very similar effective temperature, apparent brightness, and heliocentric radial velocity. Clearly, this pair deserves further detailed high-resolution spectroscopic and imaging studies.

3.3.5 Go the limit

During our survey, we also identified two stars with elongated PSFs in the northwest direction in the $K$ UKIDSS images. We estimated angular separations $\rho \sim 0.2–0.3$ arcsec (projected physical separations $s \sim 80–120$ au at the $\sigma$ Orionis distance) and magnitude differences $\Delta K \lesssim 1$ mag in both cases, but were not able to estimate colours of the two secondaries. The two very close pairs are Mayrit 1626148 AB (V511 Ori, Haro 5–33) and Mayrit 873229 AB (Haro 5–7). The former is a potometric variable, young star with strong H$\alpha$ emission (Haro & Moreno 1953; Fedorovich 1960; Wiramihardja et al. 1991; Weaver & Babcock 2004), while the latter has not only strong H$\alpha$ emission (Haro & Moreno 1953; Hernández et al. 2014), but also near-infrared flux excess (Hernández et al. 2007), low surface gravity (Maxedt et al. 2008), and aperiodic photometric variability (Cody & Hillenbrand 2010). Interestingly, Mayrit 873229 is also a double-line spectroscopic binary identified by Maxted et al. (2008). Because of the magnitude difference, we believe that the spectroscopic companion in Mayrit 873229 cannot be the same as the one identified in the UKIDSS image. It is therefore a new candidate triple system (see Caballero 2014 for previously reported triples in $\sigma$ Orionis).

The other Mayrit stars, including all known spectroscopic binaries and candidates compiled by Caballero (2014), have perfectly round PSFs in the UKIDSS images. By including these two new multiple systems, if confirmed, the minimum frequency of wide multiplicity in the interval of projected physical separations $s = 80–1600$ au in $\sigma$ Orionis would be $3.6^{+1.3}_{-1.0}$%.

4 Conclusions

We searched for wide binaries in UKIDSS images of the nearby young $\sigma$ Orionis cluster. We surveyed 331 cluster members and candidates in the corona from 1 to 30 arcmin to the cluster centre and found 18 pairs with angular separations between 0.4 and 4.0 arcsec. We studied their membership in the cluster based on known youth features and relative $ZYJHK$ photometry. Of them, eight are optical doubles which unbound secondaries are fore- or background sources. The other ten pairs belong to the cluster, have individual probabilities of chance alignment $\lesssim 1\%$, follow the photometric sequence and spatial distribution of other cluster members, and, therefore, are very likely physical (visual doubles). Six of the ten bound binaries were reported by the first author in previous works, but had never been characterised homogeneously or in detail, while the other four are completely new. The minimum frequency of wide multiplicity in the interval of projected physical separations $s = 160–1600$ au resulted in $3.0^{+1.4}_{-1.3}$%. This companion frequency is low with respect to what has been measured in other open clusters and associations, such as the Pleiades, Upper Scorpius, or the Orion Nebula Cluster. This fact may be due to the low mass of the primaries and that we have not been able to survey the closest regions of the stars, where the companion frequency increases. We found a trend of massive and X-ray stars to be in wide pairs or trios. Wide multiplicity seems to affect the frequency of discs, too. One of the new pairs, Mayrit 960106 AB, is a Lindroos system of a peculiar Herbig Ae/Be star and a low-mass T Tauri star, the latter of which may be the actual origin of the detected X-ray emission and flaring activity. Our closest pair, Mayrit 785038 AB ($\rho = 0.48 \pm 0.08$ arcsec), also new, had been reported to show a severe lithium depletion, which may be ascribed instead to unresolved binarity in spectra of moderate resolution. Further spectroscopic studies with 8–10 m-class telescopes, for confirming cluster membership based on irrefutable detec-
tion of lithium in absorption and common radial velocity, are needed to ascertain the true binding of our faint pairs.

Our programme using UKIDSS is not optimum for detecting faint companions at $\rho \lesssim 1$ arcsec to high- and mid-mass stars in $\sigma$ Orionis, but we paved the way for new, deeper, higher-resolution, imaging surveys. This observational gap will be somewhat filled by an on-going lucky-imaging survey with FastCam at the 1.5 m Telescopio Carlos Sánchez (Oscoz et al. 2008), which is unfortunately limited to the brightest stars in the cluster (R. Rebolo, priv. comm.).

If we also account for the two extra multiple system candidates from UKIDSS data in Section 3.3.5 and the 15 known spectroscopic binaries, the five pairs wider than $\rho = 4$ arcsec outside the central arcminute and probability of alignment by chance $\lesssim 1\%$, and the two close binaries resolved with adaptive optics summarised by Caballero (2014), the minimum frequency of multiplicity in $\sigma$ Orionis rockets to about $10\%$. Since typical frequencies of multiplicity in young open clusters range between $30$ and $40\%$ (cf. Duchêne & Kraus 2013), it implies that $\sigma$ Orionis still harbours of the order of $80$–$100$ unknown multiple systems (spectroscopic binaries and close multiples) that await discovery.

The ESA space mission Gaia will be able to discover tens of astrometric binaries in $\sigma$ Orionis, most of which will have periods of the order of the length of the mission. Unfortunately, neither the first data release has catalogued any close pair (Gaia Collaboration et al. 2016), nor it is not expected that Gaia will resolve the three-dimensional structure of the cluster at almost $400$ pc, which may solve the alignment-by-chance problem of the widest pairs. As a result, we encourage other authors to carry out new surveys for tight $\sigma$ Orionis binaries, both spectroscopic and with high-resolution imaging at large ground facilities (e.g., Very Large Telescope, Keck I and II, Gran Telescopio Canarias) or at the Hubble and James Webb spaces telescopes.

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Fig. A1  Grey-inverted UKIDSS $K$-band images of the multiple systems investigated in this work (physical: red; optical: blue; with elongated PSFs: green). Mayrit names are labeled on the top. North is up, east is to the left. Approximate size is 15 arcsec. Coloured circles centred on primaries are 4 arcsec in radius.
Table A1  UKIDSS coordinates and photometry of investigated pairsa.

| Mayrit | α (J2000) | δ (J2000) | Z [mag] | Y [mag] | J [mag] | H [mag] | K [mag] |
|--------|-----------|-----------|---------|---------|---------|---------|---------|
| 92149  | A 84.6995954 | -0.26220616 | 13.1480±0.0016 | 12.7707±0.0013 | 12.2054±0.0013 | 11.4491±0.0008 | 10.9082±0.0006 |
|        | B 84.7001288 | -0.26179884 | 13.4849±0.0019 | 13.1517±0.0016 | 12.6732±0.0016 | 11.9972±0.0011 | 11.7356±0.0010 |
| 359179 | A 84.6890597 | -0.2699867 | 12.8869±0.0014 | 12.5180±0.0012 | 12.0263±0.0012 | 11.5002±0.0008 | 11.0584±0.0007 |
|        | B ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ......