Numerical model describing optimization of fibres winding process on open and closed frame

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Abstract. This article discusses a numerical model describing optimization of fibres winding process on open and closed frame. The quality production of said type of composite frame depends primarily on the correct winding of fibers on a polyurethane core. It is especially needed to ensure the correct angles of the fibers winding on the polyurethane core and the homogeneity of individual winding layers. The article describes mathematical model for use an industrial robot in filament winding and how to calculate the trajectory of the robot. When winding fibers on the polyurethane core which is fastened to the robot-end-effector so that during the winding process goes through a fibre-processing head on the basis of the suitably determined robot-end-effector trajectory. We use the described numerical model and matrix calculus to enumerate the trajectory of the robot-end-effector to determine the desired passage of the frame through the fibre-processing head. The calculation of the trajectory was programmed in the Delphi development environment. Relations of the numerical model are important for use a real solving of the passage of a polyurethane core through fibre-processing head.

1. Introduction
Composites are generally lightweight, high strength and flexible, corrosion resistant and with a long lifespan. Applying composites is significant especially in the aerospace, aircraft and automotive industries. The use of industrial robots in composite production greatly reduces production cost, production time and minimizes scrap rate. The technology based on a winding carbon or glass roving by robot on a non-bearing polyurethane core is currently widespread in the manufacture of composites [1-5]. In this article we focus on the case where the core is formed of a polyurethane frame (which can be also closed) with a circular cross section. An industrial robot is used in the process of winding. This method provides full control over the placement, the laying direction, and the amount of fibres on the frame as well as the homogeneity of the fibre structure. The final composite is obtained after winding of the filament rovings on the frame and subsequent thermal curing of the frame. The objective of this
article is to study and mathematically describe the trajectory of the robot-end-effector during the process winding of three layers of carbon filament rovings on a 3D formed frame with a circular cross section. The fibre-processing head is fixedly placed in the workspace of the robot and coordinates of its parts are specified in the basic coordinate system of the robot. The outer guide lines rotate around a common axis, the intermediate guide line is static. The frame is attached to the robot-end-effector. The passage of the frame through fibre-processing head is controlled by the movement of the robot-end-effector. When polyurethane frame passes through fibre-processing head the strands are successively wound on the surface of the frame at a targeted angle. First the outer rotating guide line ensures winding strands under 45° (relative to the axis of the head and the moving direction of the frame), subsequently the middle static guide winds the second layer of strands under 0° and the second outer rotation winds the final layer of strands under -45°. Our goal is for the frame to pass through the fibre-processing head perpendicular to the guide lines of the head as far as possible. Because the position of the fibre-processing head in the workspace of the robot is fixed and the middle guide line is static the required position of the longitudinal fibres on the surface of the wound frame can only be met by suitably turning the non-bearing frame. The principle of the winding solution is shown in Figure 1,2.

![Figure 1. Attaching a closed frame to the robot-end-effector.](image1)
![Figure 2. Fibre-processing head with one (left) and three guide lines on the frame.](image2)

2. Numerical Model for describe of robot trajectory of winding fibers on the frame

The numerical model for describe of robot trajectory of winding fibers on the frame is see in Figure 3. Within the described numerical model, we will consider the right-handed Euclidean coordinate system $E_3$ of the robot ($BCS$). Subsequently, we will consider the local right-handed Euclidean coordinate system of the robot-end-effector ($LCS$). To avoid any misunderstanding, we will label the points and vectors with coordinates in $BCS$ with the subscript $BCS$ and points and vectors with coordinates in $LCS$ with the subscript $LCS$. Activities of the industrial robot are controlled using a robot control unit (in our case unit KR C4) and the library of instructions. For our purposes it is crucial to set the desired position of the robot-end-effector using the library of instructions. The position and orientation of the robot-end-effector is defined by using the $LCS$. The origin of the $LCS$ is positioned in the robot-end-effector while at the same time the robot-end-effector is oriented in the direction of the positive part of the $z$-axis in the $LCS$ with regard to the $BCS$. The actual position of the $LCS$ with regard to the $BCS$ is determined by the values listed in the “tool-center-point” ($TCP$). The tool centre point contains six values $TCP = (x,y,z,a,b,c)$. The first three parameters specify the coordinates of the origin of the $LCS$ in regard to the $BCS$. The values $a$, $b$ and $c$ indicate the angle of the rotation of the $LCS$ around the axis $z$, $y$ and $x$ with regard to the $BCS$. The non-bearing polyurethane frame can be described by the central axis $o$ see Figure 4 and radius $r_{TUBE}$ of the cross-section of the frame. The central axis of the frame $o$ is entered in the $LCS$ of the robot-end-effector using a discrete set of $N$ points stored in the array $B: array [1..N, 1..3]$, where $B[i,1]$ indicates the $x$, $B[i,2]$ indicates the $y$ and $B[i,3]$ indicates the $z$
coordinate of the \( i \)-th point \( B[i]_{\text{LCS}} \) lying on the axis \( o \), Figure 4. At the same time, the array \( \text{vector1B} : \text{array}[\ldots,1,\ldots,3] \) is entered, where \( \text{vector1B}[i,1] \), \( \text{vector1B}[i,2] \) and \( \text{vector1B}[i,3] \) indicate the coordinates of the unit vector \( \text{vector1B}[i]_{\text{LCS}} \) tangent to the axis \( o \) at point \( B[i]_{\text{LCS}} \). In addition, the array \( \text{vector2B} : \text{array}[\ldots,1,\ldots,3] \) is defined, where \( \text{vector2B}[i,1], \text{vector2B}[i,2] \) and \( \text{vector2B}[i,3] \) indicate the coordinates of the unit vector \( \text{vector2B}[i]_{\text{LCS}} \) which when passing the point \( B[i]_{\text{LCS}} \) through the fibre-processing characterizes the necessary rotation of the frame about an axis head. All the time \( \text{vector1B}[i]_{\text{LCS}} \perp \text{vector2B}[i]_{\text{LCS}} \) holds. We assume that the discrete set of points \( B[i]_{\text{LCS}} \) lying on the axis \( o \) is specified sufficiently densely and defines with a sufficient accuracy the shape of the frame. We suppose the distance of two consecutive point \( B[i]_{\text{LCS}} \) and \( B[i+1]_{\text{LCS}} \) has a constant distance \( h \). If the polyurethane frame is closed, then the first entered point on the axis \( o \) is identical to the last entered point \( (B[1]_{\text{LCS}} = B[N]_{\text{LCS}}) \). The coordinates of the individual components of the fibre-processing head are entered in the \( BCS \) coordinate system. The first outer rotating guide line is presented by the circle \( k1 \) with the centre \( \text{vector1H vector}[1]_{\text{BCS}} \), the second outer rotating guide line is presented by the circle \( k2 \) with the centre \( \text{vector1H vector}[2]_{\text{BCS}} \), both circles \( k1 \) and \( k2 \) have the same radius \( r_{\text{CIRCLE}} \) (we assume \( r_{\text{CIRCLE}} > r_{\text{TUBE}} \)). Points \( S1 \) and \( S2 \) lie on the axis \( s \) of the fibre-processing head, Figure 5.

The middle guide line is static and enables the placement of the fibres in a longitudinal direction is not necessary to consider for the model and the subsequent calculations. Point \( H_{\text{BCS}} \) is entered, this point lies in the middle of the abscissa \( S1_{\text{BCS}}, S2_{\text{BCS}} \). Subsequently, the unit vectors \( \text{vector1H}_{\text{BCS}} \) (this vector indicates the direction of passage frame through the fibre-winding head) and \( \text{vector2H}_{\text{BCS}} \) are entered, while the relation \( \text{vector1H}_{\text{BCS}} \perp \text{vector2H}_{\text{BCS}} \) is valid. We in our test examples selected \( \text{vector1H}_{\text{BCS}} = S1_{\text{BCS}} - S2_{\text{BCS}} \) and \( \text{vector2H}_{\text{BCS}} = S1_{\text{BCS}} - S2_{\text{BCS}} \), where \( \|S1_{\text{BCS}} - S2_{\text{BCS}}\| \) is the length of the abscissa \( S1_{\text{BCS}}, S2_{\text{BCS}} \). The point \( H_{\text{BCS}} \) together with the vectors \( \text{vector1H}_{\text{BCS}} \) and \( \text{vector2H}_{\text{BCS}} \) allow us to calculate the passage polyurethane frame through the fibre-processing head while the frame possibly rotates around the tangent of its axis \( o \) in point \( B[i]_{\text{BCS}} \).
1.1. Mathematical model of calculating the trajectory of the robot-end-effector
In the derivation procedure of calculation of trajectory, we used results listed in [6-8]. Note that the Denavit-Hartenberg method is often used to determine the trajectory of robot [9].

1.1.1. Calculation of the Passage of Frame Thorough the Fibre-processing Head
We calculate for \( i = 1, \ldots, N \) the TCP\(_i\) of the robot-end-effector so that at the same time \( B[p]_{BCS} = H_{BCS} \), \( \text{vector}1B[i]_{BCS} = \text{vector}1H_{BCS} \), \( \text{vector}2B[i]_{BCS} = \text{vector}2H_{BCS} \). After determining TCP\(_i\) all of the TCP\(_i\) parameters are continuously changed to the parameters of to the TCP\. By this procedure we determine partial passage of the frame through the fibre-processing head. The initial TCP indicates the position and orientation of the robot-end-effector before the start of the passage of the frame through fibre-processing head.

1.1.2. Determining the Transformation Matrix
We assume that \( TCP_{i-1} = (x_{i-1}, y_{i-1}, z_{i-1}, a_{i-1}, b_{i-1}, c_{i-1}) \) is entered. With the aim of determining the transformation matrix \( T_i \) (which we will apply to LCS) we perform the following steps.

i. We determine the coordinates of the vector \( \text{vector}1B[i]_{LCS} \) in the coordinate system BCS (under the assumption that the coordinate systems BCS and LCS have the same origin, this assumption is used when finding the required rotation matrix). The matrix \( Q_{i-1} \) of the rotation LCS towards BCS is in the form Equation (1), (see [2], pp. 31)

\[
Q_{i-1} = \text{Rot}(z, a_{i-1}) \text{Rot}(y, b_{i-1}) \text{Rot}(x, c_{i-1}),
\]

where \( \text{Rot}(z, a_{i-1}) \) is the orthogonal matrix of rotation of LCS around axis \( z \) at angle \( a_{i-1} \), \( \text{Rot}(y, b_{i-1}) \) orthogonal matrix of rotation of LCS around axis \( y \) at angle \( b_{i-1} \) and \( \text{Rot}(x, c_{i-1}) \) orthogonal matrix of rotation of LCS around axis \( x \) at angle \( c_{i-1} \).

Subsequently, we can express the coordinates of the vector \( \text{vector}1B[i]_{LCS} \) in the BCS system in the form \( \text{vector}1B[i]_{BCS} = Q_{i-1} \text{vector}1B[i]_{LCS} \).

ii. By using the scalar product of vectors \( \text{vector}1H_{BCS} \) and \( \text{vector}1B[i]_{BCS} \) we determine their deviation \( \alpha \).

iii. We determine the cross product \( p_{BCS} = \text{vector}1H_{BCS} \times \text{vector}1B[i]_{BCS} \).

iv. Vector \( p_{BCS} \) is orthogonal to vectors \( \text{vector}1H_{BCS} \) and \( \text{vector}1B[i]_{BCS} \).

We normalize vector \( p_{BCS} \), i.e. ensure its unit length \( p_{BCS} = p_{BCS} / \|p_{BCS}\| \). We perform the rotation of vector \( \text{vector}1B[i]_{BCS} \) by angle \( \alpha \) around vector \( p_{BCS} \) (we consider the correct orientation of the angle \( \alpha \), i.e. rotate vector \( \text{vector}1B[i]_{BCS} \) to vector \( \text{vector}1H_{BCS} \)). Then \( p_{BCS} = \text{Rot}(p_{BCS}, \alpha) \text{vector}1B[i]_{BCS} \). If we denote \( p_{BCS} = (n_1, n_2, n_3, 0) \) then the matrix \( \text{Rot}(p_{BCS}, \alpha) \) will have the form by (2), you can see [2].

\[
\text{Rot}(p_{BCS}, \alpha) = 
\begin{bmatrix}
    c + n_1^2(1-c) & n_1n_2(1-c) - n_3s & n_1n_3(1-c) + n_2s & 0 \\
    n_2n_3(1-c) + n_1s & c + n_2^2(1-c) & n_2n_1(1-c) - n_3s & 0 \\
    n_3n_1(1-c) - n_2s & n_2n_3(1-c) + n_1s & c + n_3^2(1-c) & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]
where \( s \) and \( c \) indicate \( s = \sin \alpha, \quad c = \cos \alpha \). At the same time we determine the vector \( l_{BCS} \) (3).

\[
l_{BCS} = \text{Rot}(p_{BCS}, \alpha) \cdot Q_{\nu} \cdot \text{vector2B}[i]_{LCS}
\]

The cases where \( \alpha = 0 \) or \( \alpha = \pi \) need to be solved separately.

v. By using the scalar product of the vectors we determine the deviation \( \beta \) of vectors \( \text{vector2H}_{BCS} \) and \( l_{BCS} \).

vi. The resulting matrix of rotation \( Q \) has the form by (4).

\[
Q = \text{Rot} (\text{vector1H}_{BCS}, \beta) \cdot \text{Rot} (p_{BCS}, \alpha) \cdot \text{Rot}(z, a_{i-1}) \cdot \text{Rot}(y, b_{i-1}) \cdot \text{Rot}(x, c_{i-1})
\]

where the elements of the matrix \( \text{Rot} (\text{vector1H}_{BCS}, \beta) \) are analogously defined as elements of the matrix \( \text{Rot} (p_{BCS}, \alpha) \). In (4) we use the correctly determined angle orientation of angle \( \beta \), i.e. we rotate the vector \( l_{BCS} \) (3) around the vector \( \text{vector1H}_{BCS} \) to the vector \( \text{vector2H}_{BCS} \).

vii. The translate vector is defined (5)

\[
u(i)_{BCS} = H_{BCS} - Q_{\nu} \cdot \text{vector1B}[i]_{LCS} = (x_{i-1}, y_{i-1}, z_{i-1}, 0)^T,
\]

where matrix \( Q \) is determined by the relation (4). We remember, that \( x_{i-1}, y_{i-1}, z_{i-1} \) are the first three parameters of \( TCP_i \). The resulting transformation matrix \( T \) in the \( i \)-th step of the passing of the frame through the fibre-processing head is in the form (6).

\[
T_i = \text{Trans} \left( x_{u(i)}, y_{u(i)}, z_{u(i)} \right) Q,
\]

where the translate matrix \( \text{Trans} \left( x_{u(i)}, y_{u(i)}, z_{u(i)} \right) = \begin{pmatrix} 1 & 0 & 0 & x_{u(i)} \\ 0 & 1 & 0 & y_{u(i)} \\ 0 & 0 & 1 & z_{u(i)} \\ 0 & 0 & 0 & 1 \end{pmatrix} \) and \( x_{u(i)}, y_{u(i)}, z_{u(i)} \) are components of vector \( u(i)_{BCS} \) in (5).

After performing of the LCS of the robot-end-effector the corresponding transformation matrix \( T \) determined by relation (7) will be valid \( H_{BCS} = B[i]_{BCS} = T_i \cdot B[i]_{LCS} \) and \( \text{vector1B}[i]_{BCS} = \text{vector1H}[i]_{BCS} \cdot \text{vector2B}[i]_{BCS} = \text{vector2H}[i]_{BCS} \).

1.1.3. Calculation of the Euler Angles

Any right-handed rotation Euclidean space \( E_3 \) around the given unit vector \( p \) by angle \( \varphi \) is determined by the orthogonal matrix \( Q = \text{Rot} (p, \varphi) \), its elements are in the form (3) and \( \det Q = 1 \). The rotation matrices form the orthogonal group (5). Each rotation matrix can be written as a product of the matrices of rotation around the coordinate axes \( z, y \) and \( x \), i.e. \( Q = \text{Rot}(z,a) \cdot \text{Rot}(y,b) \cdot \text{Rot}(x,c) \) (see [2], pp. 31), where the matrices \( \text{Rot}(z,a) \), \( \text{Rot}(y,b) \) and \( \text{Rot}(x,c) \) are in the form (2); \( a, b \) and \( c \) are the corresponding Euler angles. We note that the determination of Euler angles \( a, b \) and \( c \) is not unique (see [5]). By multiplying the left and right side from the left of the matrix \( \text{Rot} (z,a)^T \) because the matrix \( \text{Rot} (z,a)^T \) is orthogonal \( \text{Rot}(z,a)^T = \text{Rot}(z,a)^{-1} \) and modified in this way we can determine the rotation angles \( a_i, b_i \) and \( c_i \) (8). When calculating the angles of rotations we use the ATAN2 function (part of the library of most programming languages), which calculates from the two input parameters \( \text{arg}_1 \) and \( \text{arg}_2 \) the value of the function arctangent for argument \( \text{arg}_1/\text{arg}_2 \). Moreover, the signs of both
input parameters are used to determine the quadrant in which the resulting value function is located (it is valid that $-\pi < \text{ATAN2}(\arg_1, \arg_2) \leq \pi$ part). We write the matrix of rotation $Q$ in the form by (7).

$$Q = \begin{bmatrix} q_{11}(i) & q_{12}(i) & q_{13}(i) & 0 \\ q_{21}(i) & q_{22}(i) & q_{23}(i) & 0 \\ q_{31}(i) & q_{32}(i) & q_{33}(i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$  

(7)

$$a_i = \text{ATAN2}\left(q_{21}(i), q_{11}(i)\right),$$

$$b_i = \text{ATAN2}\left(-q_{31}(i), q_{11}(i)\cos a_i + q_{21}(i)\sin a_i\right),$$

$$c_i = \text{ATAN2}\left(q_{11}(i)\sin a_i - q_{21}(i)\cos a_i, q_{22}(i)\cos a_i - q_{12}(i)\sin a_i\right)$$

Thus, we determined rotation angles $a_i, b_i, c_i$ in (8) and the corresponding $TCP_i$ defining the position and orientation of the LCS of the robot-end-effector in relation to the BCS of the robot. The $TCP_i$ can be expressed in the form by (9).

$$TCP_i = \left(x_{ai(i)}, y_{ai(i)}, z_{ai(i)}, a_i, b_i, c_i\right)$$

where the elements of the vector $u[i]_{BCS}$ are given by the relation (5).

3. Results and summary

We focus on the practical problem of the passage of the non-bearing polyurethane frame with a circular cross-section through the fibre-processing head. The central 2D axis $o$ of the frame is composed of two interconnected perpendicular arms, Figure 4. The axis $o$ is entered into the LCS of the robot-end-effector using a discrete set of points $B[l]_{LCS}, 1 \leq i \leq N = 105$. The total length $d$ of the axis $o$ from the starting point $B[1]_{LCS}$ to the end point $B[N]_{LCS}$ of the axis is $d = 1050$ [mm]. The continuous distance of the points $B[l]_{LCS}$ on the axis $o$ from the starting point is denoted by the variable $l$. The distance between two consecutive points $B[l]_{LCS}$ and $B[l+1]_{LCS}$ is $h = 10$ [mm]. Vectors $\text{vector1}B[l]_{LCS}$ and $\text{vector2}B[l]_{LCS}$ are specified, radius of the frame is $r_{tub} = 20$ [mm]. The fibre-processing head is represented by the circles $k_1$ and $k_2$ with the centres $S_{1_{BCS}} = [-1000, 1105, 990]$ and $S_{2_{BCS}} = [-1000, 1035, 990]$ having the same radius $r_{circular} = 40$ [mm]. The length of abscissa $S_{1_{BCS}}S_{2_{BCS}} = 70$ [mm]. It is valid $H_{BCS} = (S_{1_{BCS}} + S_{2_{BCS}})/2$, $\text{vector1}H_{BCS} = (S_{1_{BCS}} - S_{2_{BCS}})/\|S_{1_{BCS}} - S_{2_{BCS}}\|$, $\text{vector2}H_{BCS} = (0, 0, 1)$. As stated in section II.C., the middle static line enabling the winding of the fibres to be placed in a longitudinal direction is not required for calculating the trajectory of the robot-end-effector. The frame needs to be rotated at a distance of 550 [mm] from the beginning of the axis $o$ to the distance of 690 [mm] around a tangent to the frame of axis $o$ during the passing of point $B[l]_{BCS}$ (for $56 \leq l \leq 69$) by point $H_{BCS}$ when placing the longitudinal fibres relative to the axis $o$ of the frame. This is a uniform right-handed rotary motion with an overall rotation angle of $\alpha = \pi$. The rotation is performed during the passage of the bent portion of the frame through the fibre-processing head. The calculation of the trajectory of the robot-end-effector from numerical model referred to in the previous chapter was applied to the described problem. Figure 6 shows the results of numerical modelling for position of the robot-end-effector when passing polyurethane frame through winding head. In part a) shows the position of the robot-end-effector (first three parameters of TCP) and part b) the orientation of the robot-end-effector (last three parameters of TCP). The described numerical model with algorithm allows the accurate calculation and determination of the 3D trajectory of the robot-end-effector of the industry robot during the production of composite profile using the dry fibre winding technology on a open and closed non-bearing frame.
Figure 6. Results of numerical modeling of the course of the TCP during the passing of the frame through the fibre-processing head, a) parameter values of the first three parameters of TCP, b) parameter values of the last three parameters of TCP, c) trajectory of the robot-end-effector: start and end, d) time response of numerical model for optimal trajectory of fibres winding process.

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