Duck swarm algorithm: a novel swarm intelligence algorithm

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Abstract: A swarm intelligence-based optimization algorithm, named Duck Swarm Algorithm (DSA), is proposed in this paper. This algorithm is inspired by searching for food sources and foraging behaviors of the duck swarm. The performance of DSA is verified by using eighteen benchmark functions, where it is statistical (best, mean, standard deviation, and average running-time) results are compared with seven well-known algorithms like Particle swarm optimization (PSO), Firefly algorithm (FA), Chicken swarm optimization (CSO), Grey wolf optimizer (GWO), Sine cosine algorithm (SCA), and Marine-predators algorithm (MPA), and Archimedes optimization algorithm (AOA). Moreover, the Wilcoxon rank-sum test, Friedman test, and convergence curves of the comparison results are used to prove the superiority of the DSA against other algorithms. The results demonstrate that DSA is a high-performance optimization method in terms of convergence speed and exploration-exploitation balance for solving high-dimension optimization functions. Also, DSA is applied for the optimal design of two constrained engineering problems (Three-bar truss problem, and Sawmill operation problem). Additionally, four engineering constraint problems have also been used to analyze the performance of the proposed DSA. Overall, the comparison results revealed that the DSA is a promising and very competitive algorithm for solving different optimization problems.

Keywords: Swarm intelligence • Optimization • Duck swarm algorithm • Diversity analysis • Constrained problems

1. Introduction

Optimization algorithms play a significant role in solving the real-world optimization problems. Especially, these algorithms can be compartmentalized different categories using different descriptions. Common names are evolutionary algorithm (EA) [1], nature-inspired algorithm (NIA) [2], meta-heuristic algorithm (MA) [3], and swarm intelligence (SI) algorithm [4], however, some of the algorithms included are the same. Thus, a challenge of the algorithm is that searching for the optima in the search space with higher convergence speed. Three typical and noted heuristic algorithms (evolutionary algorithms), Genetic Algorithm (GA) [5], Simulated Annealing (SA) [6] algorithm, and Particle Swarm Optimization (PSO) algorithm [7], have made great contributions and provided a lot of reference for the algorithms that were proposed later.

Genetic Algorithm combines evolution and natural selection, which are applied to its population over generations, and it was proposed in the 1970s [5, 8]. The best chromosomes in the previous generation or generated by crossover and mutation constitutes the next population in the optimization process. The crossover is to inherit a part of the value of two chromosomes from each parent and produces one offspring, which can direct to the exploitation. The mutation is randomly changing some values in a chromosome and responsible for the exploration. Overall, highly random operations make GA avoid falling into local optimum, and slow convergence is its disadvantage at the same time.

Simulated Annealing [6] was proposed in 1983, one of the most well-known physics-based methods, which is inspired by the annealing in metallurgy. It starts to find the global optimal solution at a high "temperature" and becomes more sensitive as the temperature decreases, that is, the ratio of the difference solution decreases. Thus, the initial temperature and annealing speed are the key indicators that affect whether it can reach the optimum.

Particle Swarm Optimization (PSO) algorithm was proposed in 1995 [7], one of the most popular SI methods, which inspired by the bird flocking behavior. The movements of the particles are affected by the position and speed of the previous generation and the surrounding particles. PSO algorithm has a clearer direction than GA and SA, because it is easy to implement, and the parameters are rarely the outstanding advantages of the PSO. However, it tends to converge to the local optimum prematurely when optimizing multi-modal functions, because it uses the static finite predecessor and group of linear motion. Above the three methods, their variations have been proposed, such as Quantum PSO [9], Adaptive PSO [10], and Hybrid GA with SA [11], etc.

During the last two decades, many meta-heuristic algorithms were proposed and have been used for solving optimization problems after GA, SA, and PSO. Some of the most well-known optimization techniques are Differential evolution (DE) [12], Harmony search (HS) [13], Ant colony optimization (ACO) [14], Firefly algorithm (FA) [15], Cuckoo search (CS) [16], Gravitational search algorithm (GSA) [17], Grey wolf optimizer (GWO) [18]. To some extent, the algorithms mentioned above are inspired by some, such as the social behavior of animal groups (foraging, migration, courtship), the evolution of nature, human social behavior, etc. Thus, we can name all of them inspiration algorithm in this paper. These optimization algorithms have succeeded to solve optimization problems of the literature. However, according to the No Free Lunch (NFL) theorem [19] that no inspiration algorithm best for solving all optimization problems. Namely, this indicates that an inspiration algorithm may produce satisfying solutions on a set of problems but unsatisfying solutions on another set of problems. Thence, this motivates our essays to develop a novel swarm intelligence algorithm with inspiration from duck swarm.
This paper proposes a novel swarm intelligence algorithm, named Duck Swarm Algorithm (DSA), for solving numerical optimization functions and real-world engineering constraint problems. The inspirations behind the proposed algorithm are the search and foraging behaviors of the duck swarm. The main contributions are as follows:

1. A novel population-based swarm intelligence algorithm is presented and explained inspired by the social behaviors of the duck swarm.

2. The proposed DSA demonstrates outstanding performance on eighteen benchmark functions, especially for solving the high-dimensional numerical optimization problems.

3. DSA also outperforms other comparison algorithms on classical constrained engineering problems: Three-bar truss problem, and Sawmill operation problem. Four additional engineering constraint problems (Tension spring design, Welded beam design, Pressure vessel design, and Speed reducer design) were also added to compare results.

The rest is set up as follows: Section 2 reviews the literature of nature-inspired metaheuristic algorithms. Section 3 introduces the proposed DSA in detail. Section 4 presented the comparison experiments of the algorithms. Experiments and simulations of the DSA’s performance are described, and the results are illustrated in separate graphical diagrams in Section 5. Moreover, the conclusion and future work are discussed in Section 6.

2. Literature review

According to the inspiration principle of the meta-heuristic optimization algorithms, which can be simply categorized into four categories (Fig. 1). Based on the inspiration source, the mainly four classes are: (i) evolution-based algorithms, (ii) swarm intelligence algorithms, (iii) physics-based algorithms, and (iv) human behavior-based algorithms. Of course, all meta-heuristic optimization methods benefit from these advantages despite the differences.

![Fig. 1 Classification of meta-heuristic optimization algorithms](image)

The first main division of meta-heuristics is evolution-based methods. Such evolutionary algorithms normally mimic evolutionary rules in nature, some of the most well-known techniques are Genetic algorithm (GA) [8], Genetic programming (GP) [20], Differential evolution (DE) [12], Evolutionary programming (EP) [21], Biogeography-based optimizer (BBO) [22], Gradient evolution algorithm (GEA) [23], and Tree-seed algorithm (TSA) [24].

The second main division of meta-heuristics is swarm-based approaches. These SI algorithms currently mimic swarm behaviors in animals. Some of the most popular algorithms are Particle swarm optimization (PSO) [7], Ant colony optimization (ACO) [14], Firefly algorithm (FA) [15], Cuckoo search (CS) [25], Grey wolf optimizer (GWO) [18], Salp swarm algorithm (SSA) [26], and Marine-predators algorithm (MPA) [27]. In addition, some well-known SI algorithms are not listed in Fig. 1 that are Whale optimization algorithm (WOA) [28] inspired by the foraging and hunting of the whales in the ocean, Moth-flame optimization (MFO) [29] inspired by the navigation approach of moths, and Butterfly optimization algorithm (BOA) [30] inspired by the foraging and mating behaviors of butterflies, etc.

The third main division of meta-heuristics is physics-based methods. These optimization algorithms usually mimic physical principle. Some of the well-known methods are Simulated annealing (SA) [6], Gravitational search algorithm (GSA) [17], Water cycle algorithm (WCA) [31], Sine cosine algorithm (SCA) [32], Henry gas solubility optimization (HGSO) algorithm [33], and Archimedes optimization algorithm (AOA) [34]. It is worth mentioning that AOA is proposed in 2021 by Fatma A. et al, which
inspired from the phenomenon explained by Archimedes’ principle. Also, Equilibrium optimizer (EO) [35] and Gradient-based optimizer (GBO) [36] are proposed for solving the numerical optimization problems inspired by the physical rules.

The fourth main division of meta-heuristics is human social behavior-based tools. Such optimization algorithms typically mimic social behavior rules in humans. Some of the popular algorithms like Harmony search (HS) [13], Imperialist competitive algorithm (ICA) [37], Teaching learning-based optimization (TLBO) [38], Socio evolution and learning optimization (SELO) [39], and Political optimizer (PO) [40]. We divide HS algorithm into social behavior is based on the harmony that only humans can sing, and its principle include the description of the propagation of musical sound. For a more detailed review, different categories can refer to the literature [18, 41-43].

Overall, various SI methods have been proposed recently. Most of these approaches is inspired by foraging, mating, hunting and searching behaviors of animals in nature. In the scope of our knowledge, there is no SI method in the literature inspired by the social behaviors of duck swarm. This is the main motivation for proposing a new SI method by modeling the social behavior of the duck swarm. Additionally, its abilities in solving numerical and real-world problems are investigated in the following.

3. Duck Swarm Algorithm

In this section, a novel SI optimization algorithm, named Duck Swarm Algorithm (DSA), is proposed that imitates the food foraging behavior of duck swarm. To understand this new algorithm some biological facts and how to model them in DSA are discussed in detail below.

3.1 Inspiration

In nature, formation characteristics are common for group animals, especially in the process of animal migration and foraging (hunting). Among group mammals, there are also obvious hierarchical characteristics, such as: lions, wolves, monkeys, etc. The GWO algorithm proposed for the hierarchical system of the grey wolves, the algorithm divides the hunting characteristics of grey wolves into four levels. Moreover, there are many intelligent algorithms proposed for group animals belonging to birds, including classic PSO algorithm, CS algorithm, Crow search algorithm (CSA) [44], Chicken swarm optimization (CSO) algorithm [45], and Sparrow search algorithm (SSA) [46], etc.

Ducks are aquatic and terrestrial amphibians, and it can be simply divided into three common duck species [47]: water ducks, diving ducks, and roosting ducks. The common duck (poultry) in life belongs to the water duck and the order Anseriformes. It is generally considered to be a bird. The nature-inspired heuristic algorithms are derived from the observation of phenomena, such animals, plants, or other characteristics in nature. Then, their behaviors are abstracted into mathematical models, and designed as optimization methods for solving numerical optimization problems, and constrained engineering problems.

It can be seen from observation that duck swarm queuing, searching for food sources and foraging behaviors have certain laws in life. Some pictures of duck swarm behaviors are provided in Fig. 2.

![Fig. 2 Behaviors of the duck swarm](image)

3.2 Mathematical model of DSA

This section detailed present the mathematical model of the proposed approach. Three main processes of DSA are discussed as follows: (i) Positions of duck swarm after queuing (Population initialization), (ii) Searching for food sources (Exploration phase), (iii) Foraging in groups (Exploitation phase). Noting that there are two rules that need to be followed in the process of searching food for ducks. Rule one: when looking for food, ducks with strong search ability are located closer to the center of food source, which attract other individuals to move closer to them; the updated location is also affected by nearby individuals.

Rule two: when foraging, the individuals all approach the food; the next position is affected by neighboring individuals and food position or leader duck.

3.2.1 Population initialization

Suppose the expression of randomly generated initial position in the D-dimensional search space is as follow:

\[X_i = L_b + (U_b - L_b) \cdot \sigma\]  \hspace{1cm} (1)

where \(X_i\) denotes the spatial position of the \(i\)-th duck \((i = 1, 2, 3, \ldots, N)\) in the duck group, \(N\) is the number of population size. \(L_b\)
and $U_b$ represent the upper and lower bounds of the search space, respectively. $o$ is a random number matrix between (0, 1).

3.2.2 Exploration phase

After the queuing behavior of duck swarm, that is, the ducks arrived at a place with more food. Each-individual gradually disperses and starts searching for food, this process is defined as follows:

$$X_i^{t+1} = \begin{cases} X_i^{t} + \mu \cdot X_i^{t} \cdot \text{sign}(r - 0.5), P > \text{rand} \\ X_i^{t} + CF_1 \cdot (X_{\text{leader}}^{t} - X_i^{t}) + CF_2 \cdot (X_j^{t} - X_i^{t}), P < \text{rand} \end{cases}$$

(2)

where $\text{sign}(r-0.5)$ has an effect on the process of searching for food, and it can be set either -1 or 1. $\mu$ denotes the control parameter of global search. $P$ is search conversion probability of exploration phase. $CF_1$ and $CF_2$ denote cooperation and competition coefficient between ducks in the search stage, respectively. $X_{\text{leader}}^{t}$ represents the best duck position of the current historical value in the $t$-th iteration. $X_j^{t}$ denotes the agents around $X_i^{t}$ in searching for foods by duck group in the $t$-th iteration. Moreover, parameter $\mu$ can be calculated as follows:

$$\mu = K \cdot \left(1 - \frac{t}{t_{max}}\right)$$

(3)

where $K$ is calculated by:

$$K = \sin(2 \cdot \text{rand}) + 1$$

(4)

In exploration phase, Fig. 3 depicts the process of agents update its position pertaining to $X_i$, $X_j$, and $X_{\text{leader}}$ in a 2-D search space. The value curves of parameter $K$ and $\mu$ with 200 iterations are shown in Fig. 4.

As shown in Figures 3 and 4, the search range of duck swarm is wider when $\mu > 1$ in the exploration phase. This non-linear strategy is used to enhance the global search ability of the proposed DSA.

3.2.3 Exploitation phase

After the searching for food of duck swarm, that is, enough food can satisfy the foraging of the ducks. This process is closely related to fitness of each duck’s position and defined as follows:
where \( \mu \) denotes the control parameter of global search in exploitation phase; parameters \( KF_1 \) and \( KF_2 \) denote the cooperation and competition coefficient between ducks in the exploitation phase, respectively. \( X'_{leader} \) represents the best duck position of the current historical value in the \( t \)-th iteration. \( X_i' \) and \( X_j' \) denote the agents around \( X_i \) in foraging of duck group in the \( t \)-th iteration, where \( k \neq j \).

Noting that the values of parameters \( CF_1, CF_2, KF_1 \) and \( KF_2 \) are all in \((0, 2)\), and the calculation formula can be summarized as follows:

\[
CF_i \text{ or } KF_i \triangleq \frac{1}{FP} \cdot \text{rand}(0, 1)(i = 1, 2)
\]

where \( FP \) is constant, it is set to 0.618; the \( \text{rand} \) is a random number in \((0, 1)\).

Fig. 5 Sketch map of exploitation phase

In exploitation phase, Fig. 5 depicts the process of ducks update its position pertaining to \( X_i, X_j, X_k \) and \( X_{leader} \) in a 2-D search space. Path 1 denotes the choice of ducks with cooperation. Path 2 represents the competition among \( X_i \) and \( X_j \) in the \( t \)-th iteration. Path 3 indicates the choice of the duck that have failed to compete.

### 3.3 Pseudo-code and Complexity analysis of DSA

**3.3.1 Pseudo-code of DSA**

The pseudo-code of DSA is shown in **Algorithm 1**.

**Algorithm 1** Pseudo-code of DSA

1. **Input**: Initial parameter value setting; population number \( N \); initial positions of duck swarm; objective function
2. Calculate the fitness value of initial positions; and select the best value \( f_{min} \) and leader agent position \( X_{leader} \)
3. While \( t < T_{max} \)
4. Update the value of parameter \( \mu \) using Eq. (3); and update the parameters \( P, KF_1, KF_2, KF_1 \) and \( KF_2 \)
5. For \( i = 1: \text{size}(N) \) % **Exploration phase**
6. Update the positions of duck swarm using Eq. (2)
7. End For
8. For \( i = 1: \text{size}(N) \)
9. Determine whether the individual is out of the search range
10. Calculate the new position and fitness value \( f_{new} \)
11. Update the leader position \( X_{leader} \) and fitness value
12. End For
13. For \( i = 1: \text{size}(N) \) % **Exploitation phase**
14. Update the new positions of duck swarm using Eq. (5)
15. End For
16. For \( i = 1: \text{size}(N) \)
17. Determine whether the individual is out of the search range
18. Calculate the new fitness value \( f_{new} \)
19. If \( f_{new} < \text{fitness} \)
20. Update the individual's position and fitness value
21. End if
22. Update the leader position \( X_{leader} \) and fitness value
23. End For
24. End While
25. Output: the best position and fitness value

3.3.2 Complexity analysis

In this subsection, time and space complexity of DSA are presented.

➢ Time complexity

Assuming that the population size and the search space dimension of the problem are \( n \) and \( d \), and the maximum iteration is \( T \). The complexity of DSA includes: the population initialization complexity is \( O(nd) \), the fitness value of calculation complexity is \( O(nd) \), the exploration and exploitation phases update complexity are \( O(T(n + nlogn + n + nlogn)) \), and the parameters update complexity of the method is \( O(T) \). To the above parts, the total time complexity of the proposed DSA is expressed as:

\[
O(\text{DSA}) = O(nd) + O(T)O(1 + 2nd + 2nlogn)
\]

(7)

➢ Space complexity

The storage space consumed by an algorithm can be defined the space complexity. It is closely related to the population size \( n \) of the algorithm and the dimension \( d \) of the problem. The total space complexity of the proposed DSA is \( O(nd) \). Thus, the space efficiency of the proposed method is effective and stable.

4. Comparative experiment and statistical test design

To verify the proposed algorithm's efficiency, DSA has been compared to seven optimization algorithms. The comparison techniques are Particle swarm optimization (PSO, 1998) [48], Firefly algorithm (FA, 2008) [15], Chicken Swarm Optimization (CSO, 2014) [45], Grey wolf optimizer (GWO, 2014) [18], Sine cosine algorithm (SCA, 2016) [32], Marine-predators algorithm (MPA, 2020) [27], and Archimedes optimization algorithm (AOA, 2021) [34]. The initial parameter values of the seven competitive methods are listed in Table 1. Three categories of seven comparison algorithms are used to assess the DSA efficiency by the proposed year: before 2010, between 2010 and 2019, and the last two years.

| Algorithms | Parameters                                      | Value                                           |
|------------|-------------------------------------------------|-------------------------------------------------|
| PSO        | Max and min velocity of particles               | -1, 1                                           |
|            | Cognitive and social constants                  | 2, 2                                            |
|            | Inertial weight                                 | Linearly decreases from 0.9 to 0.2              |
| FA         | Alpha, beta, and gamma                          | 0.2, 1, 1                                       |
| CSO        | Parameter \( G \) and \( FL \)                 | 10, [0.5, 0.9]                                  |
| GWO        | Parameter \( a \)                               | Linearly decreases from 2 to 0                  |
| SCA        | Parameter \( a \)                               | 2                                               |
| MPA        | Fish Aggregating Devices, FADs                  | 0.2                                             |
| AOA        | \( C_{1}, C_{2}, C_{3}, C_{4} \)                | 2, 6, 2, 4                                      |
| DSA        | \( CF_{1}, CF_{2} \)                            | Random values in (0, 2)                         |
|            | Parameter \( P \)                              | 0.5                                             |
|            | \( KP_{1}, KP_{2} \)                            | Random values in (0, 2)                         |

4.1 Comparative experiment

All of the experimental series are carried out a Windows 10 system using Intel Core i5-10210U CPU @2.11G with 8G RAM, and MATLAB 2018a in this paper. For the statistical results like Mean, and Standard deviation (Std), the comparison algorithms performed 30 independent runs for each test function. The agent size in the population \( N \) is set to 30, and the max iteration of the comparison algorithms is set to 200. Additionally, the dimension of unimodal and multimodal functions is set to 30.
Table 2. Unimodal benchmark functions

| Type      | No. | Function name       | Search range | Dim | $f_{\text{min}}$ |
|-----------|-----|---------------------|--------------|-----|-----------------|
| Unimodal  | F1  | Sphere             | [-100,100]   | 30  | 0               |
|           | F2  | Schwefel 2.22      | [-10,10]     | 30  | 0               |
|           | F3  | Schwefel 1.2       | [-100,100]   | 30  | 0               |
|           | F4  | Schwefel 2.21      | [-100,100]   | 30  | 0               |
|           | F5  | Rosenbrock         | [-30,30]     | 30  | 0               |
|           | F6  | Cigar              | [-100,100]   | 30  | 0               |
|           | F7  | Quartic            | [-1.28,1.28] | 30  | 0               |
| Multimodal| F8  | Schwefel 2.26      | [-500,500]   | 30  | $418.9829 \times \text{Dim}$ |
|           | F9  | Rastrigin          | [-5,5]       | 30  | 0               |
|           | F10 | Ackley             | [-32,32]     | 30  | 0               |
| Fixed-dimension | F11 | Griewank          | [-600,600]   | 30  | 0               |
|           | F12 | Penalized 1        | [-50,50]     | 30  | 0               |
|           | F13 | Penalized 2        | [-5,5]       | 30  | 0               |
|           | F14 | Weierstrass        | [-1,1]       | 30  | 0               |
|           | F15 | Shekel's Foxholes  | [-65,65]     | 2   | 1               |
|           | F16 | Kowalik's          | [-5,5]       | 4   | 0.00030         |
|           | F17 | Six-hump camel back| [-5,5]       | 2   | -1.0316         |
|           | F18 | Bralin             | [-5,5]       | 2   | 0.398           |

Eighteen benchmark functions [1, 17, 42, 49] are used to assess the performance of DSA in this paper. Three groups of the test functions are unimodal, multimodal, and fixed-dimension optimization problems. These functions are shown in the Table 2, including Search range, Dim dimension ($\text{Dim}$) of the function, and $f_{\text{min}}$ is the optimum of the function in theory. Additionally, the 2-D versions of each benchmark function are illustrated in Fig. 6, where F1–F7, F8–F14, and F15–F18 are the 2-D versions of the unimodal, multimodal, and fixed-dimension problems, respectively.
The DSA was assessed based on eighteen benchmark functions and compared with seven optimization algorithms. The performance of the DSA is evaluated by different test functions. The exploitation ability of competitive methods can be assessed by the unimodal functions (F1-F7) because they have only one global optimum. The local optima avoidance ability between exploration and exploitation of the competitive algorithms can be assessed by fixed-dimension functions (F15-F18) as they have lots of local optima. The statistical results include the Best, Mean, Standard deviation (Std) of the optimal results with 30 times, and the average running time (Time/s) is also considered. They can be calculated as follows:

1) The best value (Best)
\[ F_{\text{best}} = \min(F_1, F_2, \ldots, F_m) \]  
where \( m \) indicates the number of optimization tests, and \( F_{\text{best}} \) represents the optima in 30 independent runs.

2) The mean value (Mean)
\[ F_{\text{mean}} = \frac{1}{m} \sum_{i=1}^{m} F_i \]
where \( F_i \) indicates the optimal value in each independent run, and \( F_{\text{mean}} \) represents the mean value of the 30 independent runs.

3) The Standard deviation (Std)
\[ F_{\text{std}} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (F_i - F_{\text{mean}})^2} \]

4) The average running time (Time/s)
\[ T_{\text{mean}} = \frac{1}{m} \sum_{i=1}^{m} T_i \]
where \( T_i \) indicates the running time for each single optimization.

The best results are highlighted in bold. For the statistical results of comparison algorithms, statistical tests are required to assess the performance of DSA sufficiently according to Ref [50]. The statistical tests (Wilcoxon’s rank-sum [51], and Friedman rank test [52]) are needed to suggest a remarkable improvement of a new swarm intelligence algorithm in comparison to the other well-known SI algorithms to solve a particular optimization problem. Wilcoxon’s rank-sum (WRS) is a classical non-parametric statistical test that has been performed and reached the 5% significance level. Generally, \( p\)-value < 0.05 is considered strong evidence against the null hypothesis. In addition, the Friedman rank test is used to evaluate the superiority of the proposed DSA to solve optimization problems.

### 4.3 Population diversity test

According to Ref [53, 54], to distinguish the diversity of agents in the process of exploration and exploitation, it is necessary to visually analyze the diversity of the population for a new SI algorithm. To analyze the population diversity of the proposed DSA, the diversity [55] is defined as follows:

\[ \text{Div}(t) = \sum_{j=1}^{D} \frac{1}{N} \text{Div}_j = \frac{1}{N} \sum_{j=1}^{D} \text{Div}_j \]

where \( \text{Div}(t) \) indicates population diversity in iteration \( t \), \( t \) is the current iteration, and \( \text{Div}_j \) is calculated [54] as follows:
\[ \text{Div}_j = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{D} (X_{ij} - X^*)^2, \quad X^* = \frac{1}{N} \sum_{i=1}^{N} X_{ij} \]  

(13)

where \( X^* \) represents the mean of current solutions on dimension \( j \). \( \text{Div}_j \) indicates mean population diversity on dimension \( j \), and \( X_{ij} \) represents current solutions. Thus, the exploration and exploitation percentage measurement of the search process can be defined as follows:

\[ \text{Exploration}(\%) = \frac{\text{Div}_j}{\text{Div}_{\text{max}}} \times 100\% \]  

(14)

\[ \text{Exploitation}(\%) = \frac{|\text{Div}_j - \text{Div}_{\text{max}}|}{\text{Div}_{\text{max}}} \times 100\% \]  

(15)

where \( \text{Div}_j \) indicates population diversity of \( t \)-th iteration, and \( \text{Div}_{\text{max}} \) denotes the max diversity of the whole group's population diversity.

5. Results analysis and discussions

In this section, the experimental results of comparison algorithms are presented in Tables 5 to 7. Fig. 6 draws the convergence curves of the competitive algorithms for different type functions. Figs. 6 and 7 draw the exploration and exploitation percentage curves of the population diversity during the process of optima with 200 iterations and boxplot of the comparison algorithms for the benchmark functions, respectively. Eventually, DSA successfully produced effective results that verify its performance as we will illustrate in this section.

5.1 Statistical results analysis

The statistical results are reported in Tables 3, 4 and 5, respectively. Table 3 shows that DSA displayed an extremely good exploitation ability among the comparison algorithms except F5. According to Table 4, DSA yields a pretty exploration ability for multimodal dimension problems, excluding F12 and F13. For functions F9 and F10, AOA and DSA can obtain the best fitness value, but the Mean and Std of AOA are much worse than DSA. For F14, CSO, MPA, AOA and DSA obtain the best optimum. According to the dimension of benchmark functions, it can be divided into three types: low dimension (Less than 10 dimension), high dimension (Between 10 and 300 dimension), and large-scale (Greater than 300 dimension). In general, DSA demonstrates outstanding performance on test functions F1, F2, F3, F4, F6, F7, F8, F9, F10, F11, and F14, especially on F9 and F11 because of the best fitness obtained by the proposed DSA.

In addition, for the Best in Table 5, DSA can obtain the best fitness on functions F15, F16, F17, and F18. It illustrates the advantages of the DSA to strike a balance between exploration and exploitation phases for fixed dimension problems. However, for the Mean and Std on fixed functions, MPA has better stability on F15 and F16 than DSA. PSO algorithm has a best stability on F18 among the comparison algorithms. Thus, the stability of DSA on fixed dimension functions should be improved in the further study.

Moreover, DSA was compared with the other seven algorithms in the running-time calculation on the eighteen benchmark functions. The running-time calculation method is that the comparison methods independently run 30 times on each test function and noted the results in Tables 3 to 5, respectively. DSA outperforms FA and MPA while taking less time than for unimodal, multimodal and fixed test functions. Compared with the running-time of CSO, GWO, and AOA, the running-time of DSA is an order of magnitude, and the gap is small. Although the running-time of PSO algorithm is the shortest, and SCA followed, their optimization accuracy is poor among the comparison algorithms. Generally, DSA still possesses effective superiorities over the comparison methods on the running-time.

Two of the frequently used tests are used to statistically evaluate the performance of DSA in this paper. Tables 6 and 7 illustrate Friedman rank and Wilcoxon’s rank-sum test results. According to the Friedman test results listed in Table 6, it can be concluded that rankings of the eight comparison algorithms are DSA > MPA > AOA > GWO > CSO > PSO > FA > SCA. It is shown that DSA can produce satisfactory results and is also statistically superior to comparison algorithms. DSA will play a constructive role in the future as a robust algorithm.

| Functions | PSO | FA | CSO | GWO | SCA | MPA | AOA | DSA |
|-----------|-----|----|-----|-----|-----|-----|-----|-----|
| F1        |     |    |     |     |     |     |     |     |
| Best      | 1.09E+02 | 2.03E-01 | 7.72E-09 | 1.18E-09 | 1.90E+01 | 1.30E-07 | 9.71E-45 | 2.95E-133 |
| Mean      | 6.63E+02 | 2.87E-01 | 4.84E-05 | 8.95E-09 | 7.19E+02 | 5.10E-07 | 1.57E-33 | 2.33E-100 |
| Std       | 3.98E+02 | 4.34E-02 | 1.07E-04 | 9.09E-09 | 7.88E+02 | 4.03E-07 | 6.06E-33 | 1.18E-99  |
| Time/s    | **1.59E+02** | **1.29E-01** | **4.09E-02** | **3.62E-02** | **2.89E-02** | **7.49E-02** | **3.33E-02** | **4.99E-02** |
| Best      | 1.01E-01 | 7.19E-01 | 5.39E-09 | 1.53E-06 | 2.34E-01 | 2.34E-05 | 1.20E-25 | **1.39E-65** |
| Mean      | 1.91E-01 | 1.12E+00 | 6.51E-07 | 6.08E-06 | 1.13E+00 | 1.18E-04 | 4.33E-18 | **9.42E-51** |
| Std       | 4.99E-02 | 1.39E-01 | 9.75E-07 | 1.36E-06 | 1.24E+00 | 6.14E-05 | 2.20E-17 | **5.09E-50** |
| Time/s    | **1.60E-02** | **1.24E-01** | **4.27E-02** | **4.11E-02** | **2.94E-02** | **7.00E-02** | **3.57E-02** | **4.85E-02** |
| Functions | PSO | FA | CSO | GWO | SCA | MPA | AOA | DSA |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|
| Best     | -3.86E+03 | -7.06E+03 | -7.62E+03 | -8.01E+03 | -4.31E+03 | -9.11E+03 | -1.04E+08 | -9.43E+03 |
| Mean     | -2.68E+03 | -5.30E+03 | -6.40E+03 | -5.80E+03 | -3.50E+03 | -7.98E+03 | -4.86E+06 | -5.77E+03 |
| Std      | 3.90E+02   | 7.57E+02   | 6.32E+02   | 1.30E+03   | 2.81E+02   | 5.32E+02   | 2.02E+07   | 1.38E+03   |
| Time/s   | 2.46E-02   | 1.34E-01   | 5.36E-02   | 4.48E-02   | 3.64E-02   | 9.38E-02   | 4.12E-02   | 6.22E-02   |
| Best     | 2.69E+01   | 2.04E+01   | 3.22E+01   | 1.03E+01   | 9.42E+00   | 9.67E-07   | 0         | 0         |
| Mean     | 4.88E+01   | 3.29E+01   | 3.69E+00   | 1.33E+01   | 7.07E+01   | 1.54E+03   | 2.17E+01   | 0         |
| Std      | 1.00E+01   | 9.35E-01   | 1.34E+01   | 5.99E+00   | 4.08E+01   | 5.65E+03   | 6.64E+01   | 0         |
| Time/s   | 2.14E-02   | 1.32E-01   | 4.43E-02   | 3.92E-02   | 3.34E-02   | 7.59E-02   | 3.59E-02   | 5.07E-02   |
| Best     | 3.46E-02   | 3.13E-01   | 3.75E-05   | 6.60E-06   | 2.77E+00   | 3.76E-08   | 7.96E-15   | 8.88E-16   |
| Mean     | 1.57E+01   | 4.62E+01   | 8.60E-04   | 1.89E-05   | 1.53E+01   | 1.42E+04   | 1.66E+01   | 8.88E-16   |
| Std      | 2.53E-01   | 6.48E+02   | 1.26E+03   | 1.16E-05   | 7.19E+06   | 5.48E-08   | 7.57E-00   | 0         |
| Time/s   | 2.14E-02   | 1.31E-01   | 4.67E-02   | 3.95E-02   | 3.54E-02   | 7.59E-02   | 3.97E-02   | 5.16E-02   |
| Best     | 2.80E+02   | 4.48E+01   | 4.63E-08   | 2.11E-09   | 1.21E+00   | 7.69E-08   | 0         | 0         |
| Mean     | 3.14E+02   | 5.76E+01   | 4.77E-02   | 1.36E+02   | 6.99E+00   | 3.65E-06   | 1.38E-02   | 0         |
| Std      | 2.42E+01   | 6.41E+02   | 1.22E-01   | 1.58E+02   | 5.84E+00   | 5.86E+06   | 5.32E+00   | 0         |
| Time/s   | 2.91E-02   | 1.33E-01   | 5.34E-02   | 4.63E-02   | 4.00E-02   | 8.76E-02   | 4.36E-02   | 6.31E-02   |
| Best     | 4.08E-03   | 1.66E-03   | 2.50E-01   | 3.45E-02   | 1.14E+01   | 1.90E+03   | 5.64E-01   | 2.93E-01   |
| Mean     | 1.86E+00   | 4.95E-03   | 6.86E+04   | 1.09E+01   | 3.07E+06   | 1.06E+02   | 8.09E+01   | 7.52E+01   |
| Std      | 1.09E+00   | 3.89E-03   | 3.20E+05   | 5.29E+02   | 9.02E+06   | 7.35E-03   | 1.39E+01   | 3.39E-01   |
| Time/s   | 8.97E-02   | 1.94E+00   | 1.19E+01   | 1.05E+01   | 1.01E+01   | 2.24E-01   | 1.07E+01   | 1.86E-01   |
| Best     | 8.37E-04   | 2.24E-02   | 1.71E+00   | 2.38E+01   | 2.41E+01   | 3.44E-02   | 2.51E+00   | 1.75E+00   |
| Mean     | 3.24E-02   | 3.14E-02   | 1.00E+05   | 1.08E+00   | 1.00E+07   | 2.51E-02   | 2.92E+00   | 2.84E+00   |
| Std      | 6.86E-02   | 6.57E-03   | 2.68E+05   | 3.32E-01   | 1.78E+03   | 1.34E+01   | 8.78E-02   | 3.26E+01   |
| Time/s   | 9.43E-02   | 2.00E-01   | 1.21E-01   | 1.08E-01   | 1.00E-01   | 2.31E-02   | 1.04E+01   | 1.83E-01   |
| Best     | 0          | 13.3017211 | 0         | 4.27E+00   | 4.80E+00   | 0         | 0         | 0         |
| Mean     | 3.65E+00   | 1.72E+01   | 0         | 8.57E+00   | 1.06E+00   | 0         | 0         | 0         |
| Std      | 3.08E+00   | 1.87E+00   | 0         | 2.60E+00   | 1.92E+00   | 0         | 0         | 0         |
| Time/s   | 9.51E-01   | 1.00E+01   | 9.97E-01   | 9.43E-01   | 9.41E-01   | 9.96E+00   | 9.45E-01   | 9.14E+00   |

Table 4 Comparison statistical results (Multimodal functions)
In the comparison algorithms is insufficient, which indicates that the algorithm needs further improvement for solving F18 has a significa. Additionally, PSO is better than DSA on functions F15 and F16 on the basis of WSR test results. Also, FA, CSO, and AOA is better than DSA from the WSR test. According to the p values in Table 7, for function F8, the FA, GWO, AOA are better than DSA. For functions F11 and F12, AOA is better than DSA from the WSR test. For function F14, the optimum values of CSO, MPA, and AOA are similar to DSA. Additionally, PSO is better than DSA on functions F15 and F16 on the basis of WSR test results. Also, FA, CSO, and AOA is better than DSA with 200 iterations.

Overall, the statistical results verify that there is a significant difference between the results obtained by DSA and the comparison approaches in almost all cases. Especially, DSA in benchmark functions F1-F4, F6, F7, F9, F10, F11, F14, F17 and F18 has a significant advantage over almost all comparison methods. However, in functions F15 and F16, the performance of DSA in the comparison algorithms is insufficient, which indicates that the algorithm needs further improvement for solving fixed-dimensional problems. This conclusion of the statistical tests is in line with the NLP theory.

Table 6 Comparison results of Friedman rank test (Unimodal and Multimodal Functions)

| Functions | PSO | FA | CSO | GWO | SCA | MPA | AOA | DSA |
|-----------|-----|----|-----|-----|-----|-----|-----|-----|
| F1        | 7.57| 6.00| 4.63| 3.00| 7.43| 4.37| 2.00| 1.00|
| F2        | 6.03| 7.77| 3.03| 3.97| 7.20| 5.00| 2.00| 1.00|
| F3        | 6.80| 5.03| 6.40| 3.47| 7.77| 3.53| 2.00| 1.00|
| F4        | 6.00| 5.00| 7.03| 4.00| 7.97| 3.00| 2.00| 1.00|
| F5        | 5.50| 6.47| 4.93| 1.97| 7.97| 1.33| 3.50| 4.33|
| F6        | 6.87| 8.00| 4.00| 2.47| 5.67| 5.47| 2.53| 1.00|
| F7        | 6.83| 5.20| 6.13| 3.77| 7.80| 3.17| 1.97| 1.13|
| F8        | 7.90| 4.47| 3.33| 3.90| 6.80| 1.37| 4.27| 3.97|
| F9        | 7.13| 6.10| 3.23| 4.87| 7.23| 3.63| 2.38| 1.42|
| F10       | 5.23| 6.10| 4.00| 2.17| 7.67| 3.33| 6.50| 1.00|
| F11       | 8.00| 6.00| 4.37| 3.80| 7.00| 3.63| 1.73| 1.47|
| F12       | 5.90| 1.17| 6.50| 3.03| 7.97| 1.87| 4.83| 4.73|
| F13       | 1.10| 1.90| 6.30| 4.00| 7.97| 3.00| 5.60| 6.13|
| F14       | 5.10| 8.00| 2.52| 6.10| 6.73| 2.52| 2.52| 2.52|
| F15       | 2.97| 7.23| 3.63| 6.53| 6.00| 1.70| 4.77| 3.17|
| F16       | 3.47| 5.43| 4.93| 4.57| 6.43| 1.07| 5.43| 4.67|
| F17       | 1.47| 6.13| 1.87| 4.97| 7.30| 2.80| 7.57| 3.90|
| F18       | 1.33| 5.10| 2.53| 5.67| 7.30| 2.67| 7.63| 3.77|
| Total     | 95.20| 101.10| 79.36| 72.26| 130.21| 53.46| 69.23| 47.21|
| Avg       | 5.29| 5.62| 4.41| 4.01| 7.23| 2.97| 3.85| 2.62|
| Rank      | 6    | 7   | 5    | 4    | 8    | 2    | 3    | 1    |

The WSR results of pair-wise comparison of DSA and comparison methods are presented in Table 7 at 0.05 significance level. H=1 means acceptable; H=0 means rejection; and NaN means that the optimization values of the two algorithms are similar. According to the p-values in Table 7, for function F8, the FA, GWO, AOA are better than DSA. For functions F11 and F12, AOA is better than DSA from the WSR test. For function F14, the optimum values of CSO, MPA, and AOA are similar to DSA. Additionally, PSO is better than DSA on functions F15 and F16 on the basis of WSR test results. Also, FA, CSO, and AOA is better than DSA with 200 iterations.
Table 7 Comparison results of Wilcoxon rank-sum test

| Functions | PSO vs. DSA | FA vs. DSA | CSO vs. DSA | GWO vs. DSA | SCA vs. DSA | MPA vs. DSA | AOA vs. DSA |
|-----------|------------|------------|-------------|-------------|-------------|-------------|-------------|
|           | p-value    | H | Z-value | p-value | H | Z-value | p-value | H | Z-value | p-value | H | Z-value | p-value | H | Z-value |
| F1        | 3.02E-11   | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    |
| F2        | 3.02E-11   | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    |
| F3        | 3.02E-11   | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    |
| F4        | 3.02E-11   | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    |
| F5        | 1.11E-06   | 1 | 4.87    | 3.02E-11 | 1 | 6.65    | 1.33E-02 | 1 | 2.48    | 3.02E-11 | 1 | -6.65   | 3.02E-11 | 1 | -6.65   |
| F6        | 3.02E-11   | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 3.02E-11 | 1 | 6.65    |
| F7        | 3.02E-11   | 1 | 6.65    | 3.02E-11 | 1 | 6.65    | 4.50E-11 | 1 | 6.59    | 3.02E-11 | 1 | 6.65    | 6.70E-11 | 1 | 6.53    |
| F8        | 3.02E-11   | 1 | 6.65    | 8.88E-01 | 0 | 0.14    | 5.83E-03 | 1 | -2.76   | 3.04E-01 | 0 | -1.03   | 3.34E-11 | 1 | 6.63    |
| F9        | 1.21E-12   | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    |
| F10       | 1.21E-12   | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    |
| F11       | 1.21E-12   | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | 6.51    | 1.61E-01 | 1 | 1.40    |
| F12       | 5.09E-06   | 1 | 4.56    | 3.02E-11 | 1 | -6.65   | 5.09E-06 | 1 | 4.56    | 3.02E-11 | 1 | -6.65   | 3.02E-11 | 1 | -6.65   |
| F13       | 3.02E-11   | 1 | -6.65   | 3.02E-11 | 1 | -6.65   | 8.24E-02 | 0 | 1.74    | 3.02E-11 | 1 | -6.65   | 3.02E-11 | 1 | -6.65   |
| F14       | 4.57E-12   | 1 | 6.92    | 1.21E-12 | 1 | 7.10    | NaN      | 0 | NaN     | 1.21E-12 | 1 | 7.10    | NaN      | 0 | NaN     |
| F15       | 3.18E-01   | 0 | -1.00   | 1.72E-12 | 1 | 7.06    | 7.24E-01 | 0 | 0.35    | 6.95E-11 | 1 | 6.52    | 3.77E-12 | 1 | -7.12   |
| F16       | 1.58E-01   | 0 | -1.41   | 2.40E-01 | 0 | 1.18    | 8.07E-01 | 0 | 0.24    | 6.63E-01 | 0 | 0.44    | 4.84E-02 | 1 | 1.97    |
| F17       | 4.17E-13   | 1 | -7.25   | 1.21E-12 | 1 | 7.10    | 9.43E-09 | 1 | -5.74   | 1.21E-12 | 1 | 7.10    | 1.21E-12 | 1 | -7.10   |
| F18       | 4.16E-14   | 1 | -7.56   | 1.12E-10 | 1 | 6.45    | 2.17E-03 | 1 | -3.07   | 6.03E-11 | 1 | 6.54    | 2.96E-12 | 1 | -7.01   |

where H=1 means acceptable; H=0 means rejection; and NaN means that the optimization values of the two algorithms are similar.
5.2 Convergence curves analysis

Notably, to completely report the performance of the competitive algorithms, the convergence curves of the eighteen test functions are shown in Fig. 7. The y-axis and x-axis represent the fitness value and iteration, respectively. The convergence curves show that the poor performance of SCA, PSO and FA is the imbalance between the exploitation and exploration phases. The convergence curve clearly illustrates the advantages of DSA integrating the two phases into the search process. Except for DSA, MPA and AOA, almost all comparison algorithms converge to a local optimum for test functions. It also indicates that the DSA is fast and superior than other comparison algorithms in solving almost all the numerical optimization problems, except F5, F12, F13, and F17.

![Convergence curves of eighteen benchmark functions with 30 times](image)

Fig. 7 Convergence curves of eighteen benchmark functions with 30 times

5.3 Diversity analysis

In this study, the components DSA exploration and development capabilities of the impact of the diversity were analyzed. The plots are discussed to evaluate the ability of DSA balance exploration and exploitation. Fig. 8 shows the exploration and exploitation percentage curves of population diversity in the search space while solving the test functions.

As shown in Fig. 8, DSA preserves a balance between the exploration and exploitation rates during the search process for all agents in solving the benchmark optimization functions. Notably, the results balance the capabilities between exploration and exploitation phases to push each agent to the global optimal solution by moving to the optimum produced at a given time.

According to Fig. 8 and Table 5, although DSA can obtain the best fitness value, the Mean and Std of MPA on F15 and
F16 are better than DSA. In other words, the exploration and exploitation ability of DSA should be improved for the fixed-dimension or low-dimensional optimization problems.

Fig. 8 Average diversity analysis of eighteen benchmark functions with 30 times

5.4 Boxplot analysis

The boxplots of the comparison algorithms on the test functions (F1-F16) are illustrated in Fig. 9. According to Tables 6 to 7 and the charts shown in Fig. 9, it can be ascertained that the DSA is achieved the best results and had the best convergence among the others on most of the functions. However, the FA perform better than DSA in F12, and the PSO algorithm perform better than DSA in F13, respectively. Consequently, it can be inferred that the DSA outperforms other comparison methods on classical benchmark functions.
In this subsection, parameter sensitivity test was used to assess the influence of population size, and iterations on the proposed method that four test functions (two unimodal and two multimodal) are selected, including Sphere function, Schwefel 2.22 function, Rastrigin function, and Ackley function. The population size was set to 30, 50, 80 and 100, and the number of iterations was set to 200, 500, 1000 and 2000, with dimension of test functions is fixed to 30.

The results of the sensitivity analysis (the mean fitness, Std and convergence curves) for the above four functions are illustrated as follows: (i) Number of ducks ($N$), (ii) Max iterations ($T$). DSA was simulated for different number of ducks with iteration is fixed to 500. Table 8 shows the Mean and Std value of DSA when it was applied to solve Sphere (F1), Schwefel 2.22 (F2), Rastrigin (F9), and Ackley (F10) with different number of ducks. Fig. 10 shows the convergence curves of DSA on the four functions related to number of ducks, respectively. DSA was simulated for various numbers of iterations. Table 9 displays the Mean and Std value of DSA when it was applied to simulate four test functions with different number of iterations. Fig. 10 plots the convergence curves of DSA for Sphere (F1), Schwefel 2.22 (F2), Rastrigin (F9), and Ackley (F10) using different numbers of iterations.

Table 8 Comparison results of DSA using different values for ducks with 500 iterations

| Function | Dim=30 | N=30 | N=50 | N=80 | N=100 |
|----------|--------|------|------|------|-------|
| F1       | Mean   | 5.08E-262 | **6.69E-264** | 2.90E-260 | 3.94E-255 |
|          | Std    | 0.00E+00  | **0.00E+00** | 0.00E+00 | 0.00E+00 |
| F2       | Mean   | 1.74E-139 | 8.44E-123  | 5.70E-127 | 1.71E-130 |
|          | Std    | 7.27E-139 | 4.62E-122  | 2.59E-126 | 9.36E-130 |
| F9       | Mean   | 0.00E+00  | 0.00E+00   | 0.00E+00 | 0.00E+00 |
|          | Std    | 0.00E+00  | 0.00E+00   | 0.00E+00 | 0.00E+00 |
| F10      | Mean   | 8.88E-16  | 8.88E-16   | 8.88E-16 | 8.88E-16 |
|          | Std    | 0.00E+00  | 0.00E+00   | 0.00E+00 | 0.00E+00 |
Table 8 shows that the best performance was obtained with different search agents. As can be seen visually from Fig. 9 and Table 9, as the population size increased, the Mean and Std became slightly worse, except functions F9 and F10. The reason is that the search ability of the DSA has hardly changed after the population reaches 30. As can be seen from Fig. 10, the sensitivity of DSA increases slightly with the number of search agents.

The results in Table 9 and Fig. 11 prove that DSA converges to the optimum when the number of iterations increases. This supports the importance of the iterations number on the robustness and convergence behavior of the DSA. As can be seen visually from Fig. 10 and Table 9, as the population size increased, the Mean and Std became better, except functions F9 and F10. The reason is that the increase in iterations increases the number of searches and accuracy. However, the mean fitness and Std of F9 and F10 did not become better as the number of iterations increased.

Table 9 Comparison results of DSA using different iterations

| Function | Dim=30 | T=200 | T=500 | T=1000 | T=2000 |
|----------|--------|-------|-------|--------|--------|
| F1       | Mean   | 1.60E-84 | 1.23E-272 | 0      | 0      |
|          | Std    | 8.78E-84 | 0      | 0      | 0      |
| F2       | Mean   | 8.25E-50 | 1.74E-130 | 1.18E-279 | 0      |
|          | Std    | 3.21E-49 | 9.51E-130 | 0      | 0      |
| F9       | Mean   | 0      | 0      | 0      | 0      |
|          | Std    | 0      | 0      | 0      | 0      |
| F10      | Mean   | 8.88E-16 | 8.88E-16 | 8.88E-16 | 8.88E-16 |
|          | Std    | 0      | 0      | 0      | 0      |

Fig. 10 Sensitivity analysis of the proposed DSA for number of search agents

Fig. 11 Sensitivity analysis of the proposed DSA for number of iterations

According to the above analysis, when the global approximate optimal solution is roughly found, as the population and the number of iterations continue to grow, the result does not increase proportionally, the results were not increased proportionally. In a word, researchers can set the favorable population and number of iterations according to specific questions.

6. Constrained engineering problems

To confirm the performance of DSA for solving the real-world problems, two constrained engineering problems are presented: Three-bar truss problem (TBTP) [56], and Sawmill operation problem (SOP) [57]. All the considered problems have several inequality constraints that should be handled.

6.1 Three-bar truss problem

This problem considers a Three-bar truss structure shown in Fig. 12. The volume of a statically loaded Three-bar truss is to be minimized subject to stress (\( \sigma \)) constraints on each of the truss members. The objective is to assess the optimal cross-sectional areas (\( A_1, A_2 \)). This problem can be expressed as below (volume of a member = cross-sectional area \( \times \) length):
\[
\min: f(x_1, x_2) = f(A_1, A_2) = (2\sqrt{A_1} + A_2) \cdot l \\
\begin{align*}
g_1 &= \frac{\sqrt{A_1} + A_2}{\sqrt{2}A_1 + 2A_2} - P - \sigma \leq 0 \\
g_2 &= \frac{A_3}{\sqrt{2}A_1^2 + 2A_2} - P - \sigma \leq 0 \\
g_3 &= \frac{1}{A_1 + \sqrt{2}A_2} - P - \sigma \leq 0
\end{align*}
\]

where \(A_1, A_2 \in [0, 1]\), \(l = 100\, \text{cm}\), \(P = 2\, \text{KN/cm}^2\), and \(\sigma = 2\, \text{KN/cm}^2\).

Fig. 12 Three-bar truss structure

The results of DSA on TBTP are shown in Tables 10 and 11. The statistical results of the comparison algorithms are given in Table 10, and Table 11 presents the best solutions obtained by DSA and other optimization algorithms. As shown, the optimal value of DSA on TBTP is 263.8958434, which means when \(x_1, x_2, g_1, g_2, \) and \(g_3\) are set to 0.788675136, 0.408248285, -0.232790818, -1.231270871, and -1.001519946 respectively for three-bar design. Also, the convergence curves of this problem are shown in Fig. 13.

![Three-bar truss design](image)

**Fig. 13 Problem values curves of Three-bar truss problem**

**Table 10** Comparison of statistical results of the DSA and other algorithms for Three-bar truss problem

| Item     | PSO   | FA    | CSO   | GWO   | SCA   | MPA   | AOA   | DSA (our) |
|----------|-------|-------|-------|-------|-------|-------|-------|-----------|
| Best     | 270.6948 | 263.8968 | 263.8959 | 263.897 | 263.9446 | 263.8959 | 263.8961 | **263.8958** |
| Worst    | 300.1633 | 263.9090 | 263.9078 | 263.9047 | 263.9515 | 263.9035 | 263.8961 | 263.8959 |
| Mean     | 282.9146 | 263.9069 | 263.9078 | 263.9047 | 263.9515 | 263.9035 | 263.8961 | 263.8959 |
| Std      | 1.05E+01 | 1.66E-02 | 2.59E-01 | 1.52E-02 | 9.03E+00 | 2.01E-02 | 2.44E-03 | 1.13E-05 |

**Table 11** The value of the decision variables and constraints in the best solution of the comparison algorithms

| Item     | PSO   | FA    | CSO   | GWO   | SCA   | MPA   | AOA   | DSA (our) |
|----------|-------|-------|-------|-------|-------|-------|-------|-----------|
| \(x_1\)  | 0.728646364 | 0.787731811 | 0.788816466 | 0.78916075 | 0.79164305 | 0.78878101 | 0.788122965 | **0.788675136** |
| \(x_2\)  | 0.64602531  | 0.410925683 | 0.407848187 | 0.40757807 | 0.40034147 | 0.40794891 | 0.409812307 | 0.408248285 |
| \(g_1\)  | -0.018676683 | -2.05E-06  | -3.15E-09  | -8.22E-06 | -0.0003251 | -6.35E-09 | -2.0000  | -0.232790818 |
| \(g_2\)  | -1.236507589 | -1.461062784 | -1.464556545 | -1.464867875 | -1.4732837 | -1.464442 | -2.0000  | -1.23137078 |
| \(g_3\)  | -0.782169094 | -0.538939261 | -0.53544358 | -0.53544354 | -0.520414 | -0.535558 | -2.0000  | -1.001519946 |
| \(f\)    | 270.6948451  | 263.8967705 | 263.8958586 | 263.896997 | 263.944614 | 263.895852 | 263.896068 | **263.8958434** |
6.2 Sawmill operation problem

Assuming that a company owns two sawmills and two forests. Duration of one project, each forest can produce up to 200 logs per day; the cost of transporting logs is estimated at $10/km/log; and at least 300 logs are required per day. Table 12 shows the capacity of each of the mills (logs/day) and the distances between the forests and the mills (km). The goal is to **minimize the total daily cost of transporting logs and meet** the constraints on demand and factory capacity of the mills. The design problem is to determine how many logs to ship from Forest one or two to Mill A or B \((x_1, x_2, x_3, x_4)\), as shown in Fig. 14.

The cost of transportation can be defined as follow:

\[
\min : f(x_1, x_2, x_3, x_4) = 10 \cdot (24x_1 + 20.5x_2 + 17.2x_3 + 10x_4)
\]

\[
\text{s.t: } \begin{cases}
g_1 = x_1 + x_2 - 240 \leq 0 \\
g_2 = x_3 + x_4 - 300 \leq 0 \\
g_3 = x_1 + x_3 - 200 \leq 0 \\
g_4 = x_2 + x_4 - 200 \leq 0 \\
g_5 = 300 - (x_1 + x_2 + x_3 + x_4) \leq 0
\end{cases}
\]

where \(x_1, x_2, x_3, x_4 \in [0, 200]\).

The results of DSA on SOP are shown in Tables 13 and 14. The statistical results of the comparison algorithms are listed in Table 13, and Table 14 presents the best solutions obtained by comparison methods. According to Table 14, the optimal value of DSA on SOP is 37200.0053, which means when \(x_1, x_2, x_3, x_4, g_1, g_2, g_3, g_4, g_5\), and \(g_6\) are set to 1.20E-11, 1.63E-06, 100.0001, 199.9999, -214.9374, -281.9187, -171.8508, -196.9062, and 256.8561 respectively, the total cost of the SOP is the minimum. It can be concluded from Table 13 that the results obtained by DSA are better than PSO, FA, CSO, GWO, SCA, and AOA, except MPA. The convergence capability of the DSA and other algorithms is illustrated via Fig. 15.
6.3 Other constrained problems

In order to further analyze the generalization performance of the proposed DSA, four additional engineering constraint problems, Tension spring design (TSD) [67], Welded beam design (WBD) [18], Pressure vessel design (PVD) [67], Speed reducer design (SRD) [70], were added to compare results. The parameters, dimension and other information of the engineering problem are shown in the following Table 15. The best optimization results of the DSA are compared with the results of existing advanced intelligent algorithms. The results of the comparison methods can be seen in Tables 16-19 for details.

Table 15 The parameters, dimension and other information of the four engineering problems

| Problem name             | Dim | parameters | Upper limit of the parameters | Lower limit of the parameters |
|--------------------------|-----|------------|-------------------------------|------------------------------|
| Tension spring design    | 3   | x₁, x₂, x₃| [2.0 1.3 15.0]                | [0.05 0.25 2.0]              |
| Welded beam design       | 4   | x₁, x₂, x₃, x₄| [2 10 10 2]                | [0.1 0.1 0.1 0.1]            |
| Pressure vessel design   | 4   | x₁, x₂, x₃, x₄| [99 99 200 200]            | [0 0 10 10]                  |
| Speed reducer design     | 7   | x₁, x₂, x₃, x₄, x₅, x₆, x₇| [3.6 0.8 28 8.3 8.3 3.9 5.5]| [2.6 0.7 17 7.3 7.8 2.9 5]   |

Table 16 The best value of Tension spring design of the comparison algorithms

| Algorithm                | x₁   | x₂   | x₃   | optimal |
|--------------------------|------|------|------|---------|
| PSO [33]                 | 0.0514| 0.3577| 11.6187| 0.0127  |
| GA [67]                  | 0.051480| 0.351661| 11.632201| 0.012704 |
| GSA [17]                 | 0.05028| 0.32368| 13.52541| 0.0127  |
| GWO [18]                 | 0.0519| 0.3627| 10.9512| 0.0127  |
| HGSO [33]                | 0.0518| 0.3569| 11.2023| 0.0126  |
| HHO [64]                 | 0.051796393| 0.359305355| 11.138859| 0.012665443 |
| MPA [27]                 | 0.051724477| 0.35757003| 11.2391955| 0.012665  |
| PO [40]                  | 0.05248| 0.37594| 10.24509| 0.01267  |
| SSA [26]                 | 0.051207| 0.345215| 12.004032| 0.0126763 |
| CPSO [66]                | 0.051728| 0.357644| 11.244543| 0.012674  |
| **DSA (our)**            | **0.052068**| **0.365900**| **10.770262**| **0.012668** |

The results of DSA on TSD are shown in Table 16. Table 16 presents the best solutions obtained by DSA and other optimization algorithms. As shown, the optimal value of DSA on TSD is 0.012668, which means when x₁, x₂ and x₃ are...
set to 0.052068, 0.365900, and 10.770262 respectively for tension spring design. It can be concluded from Table 17 that the results obtained by DSA are better than the comparison algorithms, except HGSO [33] and MPA [27].

| Algorithm | x1 | x2 | x3 | x4 | optimal |
|-----------|----|----|----|----|---------|
| PSO [33]  | 0.2157 | 3.4704 | 9.0356 | 0.2658 | 1.85778 |
| CS [25]   | 0.182200 | 3.795100 | 9.998100 | 0.211100 | 1.946000 |
| GWO [18]  | 0.2054 | 3.4778 | 9.0388 | 0.2067 | 1.7265 |
| GA [67]   | 0.248900 | 6.173000 | 8.178900 | 0.253300 | 2.433116 |
| HGSO [33] | 0.2054 | 3.4476 | 9.0269 | 0.2060 | 1.7260 |
| GSA [17]  | 0.182129 | 3.856979 | 10.0000 | 0.202376 | 1.879952 |
| AVOA [69] | 0.205730 | 3.470474 | 9.036621 | 0.205730 | 1.724852 |
| CPSO [66] | 0.202369 | 3.544214 | 9.048210 | 0.205723 | 1.728024 |
| HHO [64]  | 0.204039 | 3.531061 | 9.027463 | 0.206147 | 1.73199057 |
| **DSA (our)** | **0.205731** | **3.475599** | **9.036601** | **0.205731** | **1.725555** |

Table 17 The best value of Welded beam design of the comparison algorithms

The results of DSA on WBD are shown in Table 17. Table 17 presents the best solutions obtained by DSA and other optimization algorithms. As shown, the optimal value of DSA on WBD is 1.725555, which means when x1, x2, x3 and x4 are set to 0.205731, 3.475599, 9.036601, and 0.205731 respectively for welded beam design. It can be concluded from Table 17 that the results obtained by DSA are better than the comparison algorithms, except AVOA [69].

| Algorithm | x1 | x2 | x3 | x4 | optimal |
|-----------|----|----|----|----|---------|
| GA [67]   | 0.812500 | 0.437500 | 42.097398 | 176.654050 | 6059.9463 |
| DE [68]   | 0.812500 | 0.437500 | 42.098446 | 176.6360470 | 6059.701660 |
| GWO [18]  | 0.8125 | 0.4345 | 42.089181 | 176.758731 | 6051.5639 |
| WOA [28]  | 0.812500 | 0.437500 | 42.0982699 | 176.638998 | 6059.7410 |
| HHO [64]  | 0.81758383 | 0.4072927 | 42.09174576 | 176.7196352 | 6000.46259 |
| MPA [27]  | 0.8125 | 0.4375 | 42.098445 | 176.636607 | 6059.7144 |
| PO [40]   | 0.7782 | 0.3847 | 40.3215 | 199.9733 | 5885.3997 |
| SMA [66]  | 0.7931 | 0.3932 | 40.6711 | 196.2178 | 5994.1857 |
| AVOA [69] | 0.778954 | 0.3850374 | 40.360312 | 199.434299 | 5886.676593 |
| **DSA (our)** | **0.778189** | **0.384659** | **40.320642** | **199.985755** | **5885.374386** |

Table 18 The best value of Pressure vessel design of the comparison algorithms

The results of DSA on PVD are shown in Table 18. Table 18 presents the best solutions obtained by DSA and other optimization algorithms. As shown, the optimal value of DSA on PVD is 5885.374386, which means when x1, x2, x3 and x4 are set to 0.778189, 0.384659, 40.320642, and 199.985755 respectively for pressure vessel design. It can be concluded from Table 18 that the results obtained by DSA are better than all the comparison algorithms.

| Algorithm | x1 | x2 | x3 | x4 | optimal |
|-----------|----|----|----|----|---------|
| PSO [33]  | 3.5000 | 0.7000 | 17.000000 | 7.300490 | 7.800000 | 3.350216 | 5.286759 | 2996.403492 |
| HS [13]   | 3.520124 | 0.7 | 17 | 8.3 | 7.802354 | 3.36970 | 5.288719 | 3029.0020 |
| GSA [17]  | 3.153 | 0.70 | 17 | 7.30 | 8.30 | 3.20 | 5.000 | 3040.10 |
| GWO [33]  | 3.5000 | 0.70 | 17 | 7.30 | 7.80 | 2.90 | 2.900 | 2998.83 |
| HGSO [33] | 3.498 | 0.71 | 17.02 | 7.67 | 7.810 | 3.36 | 5.289 | 2997.10 |
| SCA [32]  | 3.508755 | 0.7 | 17 | 7.3 | 7.8 | 3.461020 | 5.289213 | 3030.563 |
| MFO [29]  | 3.507524 | 0.7 | 17 | 7.302397 | 7.802364 | 3.323541 | 5.287524 | 3009.571 |
| **DSA (our)** | **3.500006** | **0.700000** | **17.000000** | **7.300490** | **7.800000** | **3.350216** | **5.286759** | **2996.403492** |

Table 19 The best value of Speed reducer design of the comparison algorithms

The results of DSA on SRD are shown in Table 19. Table 19 presents the best solutions obtained by DSA and other optimization algorithms. As shown, the optimal value of DSA on SRD is 2996.403492, which means when x1, x2, x3, x4, x5, x6 and x7 are set to 3.500006, 0.700000, 17.000000, 7.300490, 7.800000, 3.350216, and 5.286759 respectively for speed
reducer design. The comparison results show that DSA is better than the optimization algorithm listed in Table 19.

7. Conclusion and future work

A novel SI optimization algorithm is proposed inspired by the duck swarm in this paper. The proposed method mimicked searching for food sources and foraging behaviors of the duck swarm. Eighteen test functions were used to evaluate the performance of the proposed method in terms of exploration, exploitation, local optima avoidance, convergence, and diversity. The results presented that DSA was able to provide highly competitive results compared to well-known optimization methods like PSO, FA, CSO, GWO, and SCA, and other recent algorithms like MPA and AOA. The superior exploitation, exploration, local optima avoidance ability of DSA was confirmed by the results on different types of test functions. Moreover, the convergence analysis and population diversity analysis of DSA confirmed the convergence of this algorithm. Additionally, sensitivity analysis is used to access the performance of the proposed DSA. Furthermore, the results of the engineering optimization problems also showed that DSA has high performance on real-world constrained problems.

For future work, we will further study on binary [58, 59] and multi-objective [60, 61] versions of the DSA. We will also consider using DSA to improve the CNN [62, 63] for solving the image classification problems.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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