To development of general theory of gearings synthesis on the basis of conical wheels formed by a tool of rack and disk type

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Abstract. Generalized scheme and special cases of schemes gears, which are synthesized on the basis of conical shaped blanks of cogwheels, formed by a tool of rack and disk type, are presented. The advantages of such gears in comparison with traditional cylindrical and conical gears first of all, layout advantages and ability to eliminate backlash in the gearing both at manufacturing stage and at stage of operation of designed mechanisms are described. The main hypothesis used in the development of general theory of synthesis of gears under consideration involved is formulated. The notion of a general type initial link is given and essence of synthesis of gears in generalizing coordinates is disclosed, which allows to maximally reveal geometric-kinematic possibilities of gearing. The main provisions of general theory of synthesis of considered gears in a limiting area existence of gearing by means of a change in the shape and coordinates of points local areas, corresponding to one or another set of qualitative indicators are formulated.

1. Introduction
One of topical problems of machine mechanics is the task of creating high-resource and high-precision mechanisms for various, including extreme, operating conditions in a wide range of loads and speeds. Wide possibilities for solving this problem are opened by use of gears synthesized on the basis of conical shaped blanks of cogwheels with crossed, intersecting and parallel geometric axes, tooth profiles of which are formed by a rack and disk type tool. Accordingly, in the first case we obtain a conical wheel with an involute tooth profile, the so-called involute-conical wheel [1], and in the second case – with a not involute tooth profile (not involute conical wheel) [2]. Comparatively simple and high-performance technology for shaping of teeth a worm cutter and a worm grinding wheel, relating to tools of rack type, provides high profile accuracy [2, 3]. As disc-type tool for cutting teeth a dolbyak is used, and for final processing – a disc shaver [4].

2. Schemes and advantages of gears under consideration
In figure 1 shown the basis scheme of a conical gear, in figure 2 shows possible schemes, so-called, cylinder-conical gears [4], obtained when one of wheels of a conical gear degenerates into a cylindrical wheel, and in figure 3 shown scheme, so-called, a flat-wheel gear, obtained when a conical wheel of a cylinder-conical gear degenerates into a flat one.

In figure 1: Σ – the inter axial angle; xOyw – the fixed coordinate system with the origin O in initial cross-section of initial link; x0Oyw – the mobile coordinate system with the beginning of O in initial cross-section of envelope link; φk – the current angle of rotation of envelope link; k – the index of parameters of envelope link. Parameters of initial link the index are not assigned to.
Figure 1. The basis scheme of a conical gear

Figure 2. Schemes of cylinder-conical gears: (a) – with crossed geometric axes of wheels; (b) – with intersecting geometric axes of wheels
In figure 3: \( D_{i2} \) – an inner diameter of the flat wheel; \( D_{e2} \) – an outer diameter of a flat wheel; \( a \) – a distance from the pinion axis to the plane of the vertices of the teeth of the wheel.

One of main advantages of considered gears is possibility of eliminating a backlash of an output shafts and compensating for a displacement of teeth contact spot during assembly and during operation (as working surfaces of teeth are wear out) by means corresponding displacement of a conical wheel [3, 4].

Another important advantage is the possibility of obtaining such a layout of drives that not canon be realized with traditional gears (cylindrical and conical) [4]. For example, a hypoid cylinder-conical gear can be realized at an arbitrarily small inter axial distance, which is impossible when using a helical gear. A not hypoid cylinder-conical gear can be realized at small inter axial angles (5°...15°), which is impossible when using conventional conical gear due to the lack of equipment for making wheels with a small cone angle. The most compact cylinder-conical gears are gears internal gearing, in which the initial link is an involute cylindrical wheel with internal teeth, and the envelope link is a not involute conical wheel with external teeth.

In figure 4 the scheme of precession planetary gearbox with the cylinder-conical gear of type \( 2k-h \) is shown [4].
In figure 4: \( z_1 \) – is number of teeth of conical satellites, engaged with the unmovable central wheel; \( z_2 \) – is number of teeth of the stationary central wheel; \( z'_1 \) – is number of teeth of conical satellites, engaged with the moving central wheel; \( z'_2 \) – is number of teeth of the moving central wheel.

Gear has a many pairs of contacting teeth when a small difference in the number of teeth of central wheels and satellites, self-inhibition an output shaft, a wide range of gear ratios (maximally about 2000). Compared with a typical single-satellite cylindrical planetary gear, this transmission can transfer torque in 2,5 ... 3 times more when same dimensions, gear ratio and load of teeth. By load capacity, weight and dimensions such gears are competitive with wave gears, but much more durable than them, since it is possible to obtain high hardness of teeth working surfaces.

In this gear are two cylindrical the initial links. Accordingly teeth of conical gears must be cut gear-cutting tool with internal teeth. Due to absence of such tool of a conical wheels are forming by a worm gear-cutting tool, an axis of which moves along a curved trajectory [4]. The task of the researcher-designer consists in the justified choice of this trajectory and, accordingly, the function of changing the displacement factor of the forming rack in depending of the coordinate of cross-section of the cut wheel. The main criterion is the quality of the localized contact of teeth, which eliminates their edge contact due to errors in manufacture and installation of gears, reduces likelihood of interference due to deformation of elements of the mechanism. In particular, any required degree of contact localization can be obtained by longitudinal modification of teeth of the conical gear by a certain choice of the tool displacement coefficients along its edges [5].

On basis of the presented planetary gearbox the high-precision and high-resource electromechanical drive (service life up to 10 years) for mechanisms of angular rotation of objects of space vehicles (antennas, scanning devices, solar batteries, etc.) was created. He contains also electric motor of new type and a coupling sleeve, was created [6].

3. Prerequisites for development of general theory of gearings synthesis

3.1. Basic concepts of general theory of gearings synthesis

In basis for the development of general theory of synthesis of the considered gears the hypothesis that the main features, formerly the others determining the geometry and geometric-kinematic possibilities of gearing, are the shape of a vertex surface and type of teeth (teeth without transition surfaces) of initial link, and method synthesis of gearings in generalizing coordinates (independently of the parameters of the initial forming contour) allows to fully reveal these possibilities.

The gear wheel with not involute profile and curvilinear shape of the generatrix of the vertex surface of teeth, obtained during the process of their formed by a rack with a curved profile of teeth in cross-section moving along a certain curvilinear trajectory, is suggested be called the initial link of general form. By according to the shape of the lateral surface of teeth, it is not involute and the initial surface can have any shape. Blanks for the initial link of a general view, it is expedient to choose a technological conical and, in a particularistic case, when the cone angle is zero, of a cylindrical shape.

If a contour of forming rack has a rectilinear profile of the teeth, and the axis of the worm cutter moves along a straight path, we get an involute conical wheel [4].

The profile of the tooth of the initial link in general case can theoretically be formed by rolling along a certain (main) circle of a segment of some curve of the line \( f(x, y) = 0 \) (figure 5), in particular, arcs of a circle, branches of a parabola, hyperbolas, etc., but practically by rolling method of workpiece forming rack with the corresponding profile of teeth. A rolling curve can consist of several sections of different shapes, and in most general case can be specified by a set of points with a certain step of discreteness. The corresponding profile of teeth can practically be realized on metal cutting machines with numerical program control.

With given values of independent parameters of a gear scheme (an inter axial angle, angles of an initial cones, a radius of an initial circle, and an angle of inclination of a tooth on an initial surface of the initial link), the geometry of a spatial gearing in general case is completely determined by values of generalizing coordinates of points of a tooth surface of an initial link in the cross-sections: of the
gearing angle $\alpha_{tw}$, which determines the angular position of the normal $n$ and the tangent $\tau$ to profile of the tooth in the $xOy$ coordinate system; of the profile angles $\alpha_n$, $\alpha_y$, which determine the distances $r_n$ and $r_\tau$ from the axis of the initial link to the normal and tangent, respectively (look up figure 5), and the relative thickness of the tooth on the circumference of the vertices $m_\alpha$ in fractions of a diameter of a largest base circle $d_{b\text{max}}$ – of scale factor of the gear.

![Figure 5. To formation of the tooth profile in a cross-section of an initial link](image)

The radius of an initial circle $r_w$ is suggested to be chosen more or equal to the radius of the circumference of the vertices of teeth, that is the area of their profiling in each cross-section of an initial link, is limited by an initial circle, and in space by an initial surface.

To ensure continuity of developed theories the module of the radius-vector of the current profile point $r_y$ as the function of the radius of the current base circle $r_b$ is definite.

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The type of the function $f(\alpha_n, \alpha_y) = 0$ in a cross-section of the initial link depends on the type of function of line that is rolled along base circle. If this function is selected by some criterion of quality gearing, then on the contrary, it determines the function of the line rolling along the base circle. If the workpiece is cylindrical, then we use the base cylinder, and if conical, then is base cone. In the second case the radii of base circles of teeth cross-sections will depend on the angle of the cone.

The function of the surface of the vertices of the teeth of the initial link and the associated with her function of changing the displacement coefficient depending on coordinate the cross-section is proposed to be chosen as a result of analyzing of zones of inertial touch of the producing and the reproducible surfaces at the stage of computer designing of various types of gears.

If in any cross-section of an initial link $\alpha_x = \alpha_y$, then we obtain a not involute gearing with involute, in particular, with a cylindrical initial link, and the theory of synthesis of such gearing in generalizing parameters [7] developed by the author of this article can be considered as a special case of the general theory of gearings. In other words, synthesis of gearings on the basis of an involute initial link is performed in the system of two generalizing coordinates ($\alpha_{tw}$, $\alpha_y$), and synthesis of gearings on the basis of the not involute initial link – in the system of three generalizing coordinates ($\alpha_{tw}$, $\alpha_y$, $\alpha_x$).

In the general case the surface of gearing exists in some limiting region, called the generalized area of existence of the gearing (GAEG), which is determined by the set of values of independent parameters
of a gear scheme, the number of teeth of the initial link and their relative thickness on circle of vertex in the system of generalizing coordinates $\alpha_{tw}, \alpha_x, \alpha_y$.

For the values of generalizing coordinates $\alpha_{tw \ max}, \alpha_x, \alpha_y$, given in the generalized area existence of gearing, one of the extreme points of the surface of gearing on the vertex of the tooth of the initial link determines the local area existence of gearing (LAEG).

Proceeding from the results of the studies described in [7], it can be assumed that the GAEG and LAEG in the indicated coordinates will have the form shown in figure 6.

![Figure 6](image_url)  

**Figure 6.** To the definition of generalized and local areas of existence (OABCD and KLMN) of spatial gearings in generalizing coordinates

GAEG is limited by limiting lines of sharpening ($g_s, g_s'$) and interference ($g_c$) of teeth of an envelope link.

LAEG is limited to a straight line segment $a$ corresponding to the specified angles $\alpha_s$ and $\alpha_l$ on the surface vertex of teeth of the initial link, the local curve of sharpening of the tooth of the envelope link $s$ corresponding to these values, and the line $p$, whose shape depends on the shape of the surface vertex of teeth of this link.

The surface of teeth of the forming rack is reproduced by translational movement of the line of contact with the surface of the teeth of the original link (characteristics). The points of the characteristic are found by turning the points of teeth cross-profile by the angle $\varphi$ on the projection plane $xOy$ (look up figure 5) and moving to the distance $w = p_s \varphi$ from this plane, where $p_s$ is the screw parameter.

3.1.1. *Basic provisions of the general theory of gearings synthesis.* Taking into account the above, the author establishes the following basic positions of general theory of synthesis of the considered gears.

1. The function of the surface of teeth vertices of an initial link and associated with her dependence of displacement coefficient of the forming rack from coordinate of a cross-section of a conical wheel are determined depending on the purpose of the gear train, working conditions, and the results of computer analysis of inertial zones of touching of a producing and the reproducible surfaces.

2. The computer synthesis of gearings is performed in the system of three generalizing coordinates, whose values in the characteristic (usually extreme) points of the gearing surface are selected from the generalized area their existence as result of analysis of whole complex of geometric-kinematical and load-bearing indicators of gearing.

3. Control qualitative indicators of gearings at the stage of their computer analysis in the generalized area existence of gearing is accomplished by changing the shape and position of the local
area existence of gearing corresponding to one or another set of indicators, at which parameters of control are determined with considering the geometry of the vertex surface and the lateral surface of teeth of an initial link.

4. In process of computer analysis of gearings a rational part generalized area thes existence is defined as a set of isolines of qualitative indicators within which the most favorable complex of these indices should be sought based on general laws governing the influence of parameters of control on them under various criteria working capacity of gears.

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