Single Spin Asymmetries in $\ell p^+ \to h X$ processes and TMD factorisation

M. Anselmino,$^{1,2}$ M. Boglione,$^{1,2}$ U. D’Alesio,$^{3,4}$ S. Melis,$^1$ F. Murgia,$^4$ and A. Prokudin$^5$

$^1$Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I-10125 Torino, Italy
$^2$INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy
$^3$Dipartimento di Fisica, Università di Cagliari, I-09042 Monserrato (CA), Italy
$^4$INFN, Sezione di Cagliari, C.P. 170, I-09042 Monserrato (CA), Italy
$^5$Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, VA 23606, USA

(Dated: April 28, 2014)

Some estimates for the transverse Single Spin Asymmetry, $A_N$, in the inclusive processes $\ell p^+ \to h X$, given in a previous paper, are expanded and compared with new experimental data. The predictions are based on the Sivers distributions and the Collins fragmentation functions which fit the azimuthal asymmetries measured in Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes ($\ell q \to \ell' h X$). The factorisation in terms of Transverse Momentum Dependent distribution and fragmentation functions (TMD factorisation) – i.e. the theoretical framework in which SIDIS azimuthal asymmetries are analysed – is assumed to hold also for the inclusive process $\ell p \to h X$ at large $P_T$. The values of $A_N$ thus obtained agree in sign and shape with the data. Some predictions are given for future experiments.

PACS numbers: 13.88.+e, 13.60.-r, 13.85.Ni

I. INTRODUCTION

In a previous paper [1] the issue of the validity of the TMD factorisation for hard inclusive processes in which only one large scale is detected has been investigated in a simple phenomenological approach. We considered transverse Single Spin Asymmetries (SSAs) for the $\ell p^+ \to h X$ process, with the detection, in the lepton-proton center of mass (c.m.) frame, of a single large $P_T$ final particle, typically a pion. Also the case of jet production, $\ell p^+ \to \text{jet} + X$, was considered. The final lepton is not necessarily observed; however, a large value of $P_T$ implies, at leading perturbative order, large values of $Q^2$, and the active role of a hard elementary interaction, $\ell q \to \ell q$. Such a measurement is the exact analogue of the SSAs observed in the $pp \to h X$ processes, the well known and large left-right asymmetries $A_N$, measured over a huge energy range [2–12]. On the other hand, the process is essentially a Semi-Inclusive Deep Inelastic Scattering (SIDIS) process, for which, at large $Q^2$ values (and small $P_T$ in the $\gamma^* - p$ c.m. frame), the TMD factorisation is proven to hold [13–17].

We computed these SSAs assuming the TMD factorisation and using the relevant TMDs (Sivers and Collins functions) as extracted from SIDIS data. A similar idea of computing left-right asymmetries in SIDIS processes, although with different motivations and still demanding the observation of the final lepton, has been discussed in Ref. [18]. A first simplified study of $A_N$ in $\ell p^+ \to h X$ processes was performed in Ref. [19]. The process was also considered in Refs. [20, 21] in the framework of collinear factorisation with twist-three correlation functions, obtaining asymmetries with a sign opposite to that of the corresponding ones in $pp$ processes. Jet production in $\ell p \to \text{jet} + X$ was studied in Ref. [22], in a collinear factorisation scheme with a higher-twist quark-gluon-quark correlator, $T_F$, which is related to the first moment of the Sivers function [23–26].

While at the time of publication of Ref. [1] no data were available on $A_N$ from lepton-proton inclusive processes, very recently some experimental results have been published by the HERMES Collaboration [27]. New data are also available by the JLab Hall A Collaboration [28], but their $P_T$ values are too small (less than 0.7 GeV) to fix a large scale.

We consider here the results of the HERMES Collaboration, selecting those which best fulfil the kinematical conditions necessary for the validity of our scheme, and compare them with our calculations based on TMD factorisation and the Sivers and Collins functions extracted from SIDIS data. In Section III we briefly summarise our formalism and in Section IV we compare our numerical results with data and give some predictions for future measurements. Some final comments are given in Section V.
II. FORMALISM

In Ref. [1] (to which we refer for all details) we considered the process $p^\uparrow \ell \to hX$ in the proton-lepton $c.m.$ frame (with the polarised proton moving along the positive $Z_{cm}$ axis) and the transverse Single Spin Asymmetry:

$$A_N = \frac{d\sigma^\uparrow(P_T) - d\sigma^\downarrow(P_T)}{d\sigma^\uparrow(P_T) + d\sigma^\downarrow(P_T)} = \frac{d\sigma^\uparrow(-P_T) - d\sigma^\downarrow(-P_T)}{2d\sigma^{\uparrow\downarrow}(P_T)},$$

(1)

where

$$d\sigma^{\uparrow\downarrow} = \frac{E_h d\sigma^{p^\uparrow \ell \to hX}}{d^3P_h}$$

(2)

is the cross section for the inclusive process $p^\uparrow \ell \to hX$ with a transversely polarised proton with spin “up” ($\uparrow$) or “down” ($\downarrow$) with respect to the scattering plane [1]. $A_N$ can be measured either by looking at the production of hadrons at a fixed transverse momentum $P_T$, changing the incoming proton polarisation from $\uparrow$ to $\downarrow$, or keeping a fixed proton polarisation and looking at the hadron production to the left and the right of the $Z_{cm}$ axis (see Fig. 1 of Ref. [1]). $A_N$ was defined (and computed) for a proton in a pure spin state with a pseudo-vector polarisation $S_T$ normal ($N$) to the production plane and $|S_T| = S_T = 1$. For a generic transverse polarisation along an azimuthal direction $\phi_S$ in the chosen reference frame, in which the $\uparrow$ direction is given by $\phi_S = \pi/2$, and a polarisation $S_T \neq 1$, one has:

$$A(\phi_S, S_T) = S_T \cdot (\hat{p} \times P_T) A_N = S_T \sin \phi_S A_N,$$

(3)

where $p$ is the proton momentum. Notice that if one follows the usual definition adopted in SIDIS experiments, one simply has:

$$A_{TU}^{\sin \phi_S} = \frac{2}{S_T} \int d\phi_S \left[ \frac{d\sigma(\phi_S) - d\sigma(\phi_S + \pi)}{d\sigma(\phi_S) + d\sigma(\phi_S + \pi)} \right] \sin \phi_S = A_N.$$  

(4)

Assuming the validity of the TMD factorisation scheme for the process $p \ell \to hX$ in which the only large scale detected is the transverse momentum $P_T$ of the final hadron in the proton-lepton $c.m.$ frame, the main contribution to $A_N$ comes from the Sivers and Collins effects, and one has [29, 31]:

$$A_N = \sum_{q_1, (\lambda)} \int \frac{dx}{16 \pi^2 x z^2 s} \frac{d^3k_\perp d^3p_\perp}{|\Sigma(\uparrow) - \Sigma(\downarrow)|^{q_1 q_\ell}} J(p_\perp) \delta(\bar{s} + \bar{u}) \left[ \frac{1}{2} \Delta^N f_{q/p}(x, k_\perp) \cos \phi M_1^0 M_2^0 \Delta D_{h/q}(z, p_\perp) + h_{1q}(x, k_\perp) M_1^0 M_2^0 \Delta^N D_{h/q}(z, p_\perp) \right]$$

(5)

$$\sum_{q_1, (\lambda)} \left[ |\Sigma(\uparrow) - \Sigma(\downarrow)|^{q_1 q_\ell} = \frac{1}{2} \Delta^N f_{q/p}(x, k_\perp) \cos \phi M_1^0 M_2^0 \Delta D_{h/q}(z, p_\perp) + h_{1q}(x, k_\perp) M_1^0 M_2^0 \Delta^N D_{h/q}(z, p_\perp) \right]$$

(6)

and

$$\sum_{q_1, (\lambda)} \left[ |\Sigma(\uparrow) + \Sigma(\downarrow)|^{q_1 q_\ell} = f_{q/p}(x, k_\perp) \left[ \frac{1}{2} M_1^0 M_2^0 \Delta^N D_{h/q}(z, p_\perp) \right] \right.$$

(7)

All functions and all kinematical and dynamical variables appearing in the above equations are exactly defined in Ref. [1] and its Appendices and in Ref. [30]. We simply recall here their meaning and physical interpretation.

- $k_\perp$ is the transverse momentum of the parton in the proton and $p_\perp$ is the transverse momentum of the final hadron with respect to the direction of the fragmenting parent parton, with momentum $p_q'$. $\phi$ is the azimuthal angle of $k_\perp$. 


The first term on the r.h.s. of Eq. (6) shows the contribution to $A_N$ of the Sivers function $\Delta f_{q/p^+}^N(x,k_\perp)$ \cite{32,34,35},

\[
\Delta \hat{f}_{q/p,S}(x,k_\perp) = \hat{f}_{q/p,S}(x,k_\perp) - \hat{f}_{q/p,-S}(x,k_\perp) = \Delta f_{q/p^+}^N(x,k_\perp) S_T \cdot (\hat{p} \times \hat{k}_\perp) \quad (8)
\]

It couples to the unpolarised elementary interaction ($\propto (|M_1^0|^2 + |M_2^0|^2)$) and the unpolarised fragmentation function $D_{h/q}(z,p_\perp)$; the cos $\phi$ factor arises from the $S_T \cdot (\hat{p} \times \hat{k}_\perp)$ correlation factor.

The second term on the r.h.s. of Eq. (6) shows the contribution to $A_N$ of the unintegrated transversity distribution $h_{1q}(x,k_\perp)$ coupled to the Collins function $\Delta f_{q/p^+}^N(z,p_\perp)$ \cite{34,32,35},

\[
\Delta \hat{D}_{h/q^+}(z,p_\perp) = \hat{D}_{h/q^+}(z,p_\perp) - \hat{D}_{h/q^-}(z,p_\perp) = \Delta f_{q/p^+}^N(z,p_\perp) s_q \cdot (\hat{p}_q' \times \hat{p}_\perp) \quad (9)
\]

This non perturbative effect couples to the spin transfer elementary interaction ($d\sigma_{q^+\ell\rightarrow q^+\ell} - d\sigma_{q^-\ell\rightarrow q^-\ell} \propto \hat{M}_1^0 \hat{M}_2^0$). The factor cos($\phi' + \phi_0^q$) arises from phases in the $k_\perp$-dependent transversity distribution, the Collins function and the elementary polarised interaction.

Some final comments on the kinematical configuration and the notations adopted in the HERMES experiment, with respect to those of Ref. \cite{1}, are necessary. According to the usual conventions adopted for SIDIS processes, in HERMES paper \cite{27} the lepton is assumed to move along the positive $Z_{cm}$ axis, so that the processes we are considering here are $\ell p^+ \rightarrow h X$, rather than $p^-\ell \rightarrow h X$. In this reference frame the $\uparrow (\downarrow)$ direction is still along the $+Y_{cm}$ ($-Y_{cm}$) axis as in Ref. \cite{1} and, keeping the usual definition of $x_F = 2P_L/\sqrt{s}$, where $P_L$ is the longitudinal momentum of the final hadron, only its sign is reversed.

The azimuthal dependent cross section measured by HERMES is defined as \cite{27}:

\[
d\sigma = d\sigma_{UU}[1 + S_T A_{UT}^{\sin \psi} \sin \psi] \quad (10)
\]

where

\[
\sin \psi = S_T \cdot (\hat{P}_T \times \hat{k}) \quad (11)
\]

coincides with our sin $\phi_S$ of Eq. (3) as $p$ and $k$ (respectively, the proton and the lepton 3-momenta) are opposite vectors in the lepton-proton c.m. frame. Indeed, taking into account that “left” and “right” are interchanged in Refs. \cite{1} and \cite{27} (as these are defined looking downstream along opposite directions, respectively the proton and the lepton momentum directions) and the definition of $x_F$, one has:

\[
A_{UT}^{\sin \psi}(x_F,P_T) = A_N^{p^+\ell\rightarrow hX}(-x_F,P_T) \quad (12)
\]

where $A_N^{p^+\ell\rightarrow hX}$ is the SSA as given by Eq. (5) and computed in Ref. \cite{1}, and $A_{UT}^{\sin \psi}$ is the quantity measured by HERMES \cite{27}.

III. ESTIMATES FOR $A_{UT}^{\sin \psi}$, COMPARISONS WITH DATA AND PREDICTIONS

In this Section we present our estimates for $A_{UT}^{\sin \psi}$, following the notation and convention adopted by the HERMES experiment. In our computation, based on the TMD factorisation, we consider two different sets of Sivers and Collins functions (the latter coupled to the transversity distribution), as previously obtained in a series of papers from fits of SIDIS and $e^+e^-$ data \cite{36,37,38}.

These sets, besides some different initial assumptions, differ in the choice of the collinear fragmentation functions (FFs). More precisely, we adopt the Sivers functions extracted in Ref. \cite{36}, where only up and down quark contributions were considered, together with the first extraction of the transversity and Collins functions obtained in Ref. \cite{37}. In such studies we adopted, and keep using here, the Kretzer set for the collinear FFs \cite{40}. We shall refer to this set of functions as the SIDIS 1 set.

We then consider a more recent extraction of the Sivers functions \cite{35}, where also the sea quark contributions were included, together with an updated version of the transversity and Collins functions \cite{38}; in these cases we adopted
another set for the FFs, namely that one by de Florian, Sassot and Stratmann (DSS) \cite{41}. We shall refer to these as the SIDIS 2 set.

The use of these two sets of parameterisations, with their peculiar differences, allows to take into account both the role of different weights between leading and non-leading collinear FFs, as well as the different behaviour in the large $x$ region of the Sivers and transversity distributions. The large $x$ behaviour of these functions is still largely unconstrained by SIDIS data, while it might be relevant to explain the values of $A_N$ measured in $p^+p \to \pi X$ processes at RHIC, as studied in Refs. \cite{42,43}. As this paper focuses on a process kinematically much closer to SIDIS, the large $x$ behaviour of the involved TMDs is not so relevant here. Our two sets of TMDs (SIDIS 1 and SIDIS 2) are well representative of the possible uncertainties.

We then simply compute the values of $A_N$ as resulting, in the TMD factorised scheme, from the $-SIDIS$ and $e^+e^-$ extracted $-SIDIS$ 1 and SIDIS 2 sets of TMDs. We will also show the uncertainty bands obtained by combining the statistical uncertainty bands of the Sivers and Collins functions, given by the procedure described in Appendix A of Ref. \cite{38}.

In the following we will consider both the fully inclusive data from $\ell p \to \pi X$ processes at large $P_T$, as well as the sub-sample of data from also the final lepton is tagged (SIDIS category). In the first case there is only one large scale, the $P_T$ of the final pion, and for $P_T \simeq 1$ GeV, in order to avoid the low $Q^2$ region, one has to look at pion production in the backward proton hemisphere (according to the HERMES conventions this means $x_F > 0$). In this region (large $P_T$ and $x_F > 0$) the lepton-quark scattering is still dominated by $Q^2 \gtrsim 1$ GeV$^2$ and our pQCD computation is under control.

For the tagged-lepton sub-sample data $Q^2$ is measured and chosen to be always bigger than 1 GeV$^2$. Notice that even in such conditions, working in the lepton-proton c.m. frame, $P_T$ is still defined as the transverse momentum of the pion w.r.t. the lepton-proton direction. We will refer to these data as “SIDIS category”.

Another important aspect to keep in mind is that in both cases (inclusive or lepton-tagged events) one is not able to separate the single contributions to $A_N$ of the Sivers and Collins effects, that in principle could contribute together.

### A. Fully inclusive case

In this case, in order to apply our TMD factorised approach, one has to consider data at large $P_T$. Among the HERMES data there is one bin that fulfils this requirement, with $1 \lesssim P_T \lesssim 2.2$ GeV, and $\langle P_T \rangle \simeq 1$–1.1 GeV. In Figs. 1 and 2 we show a comparison of our estimates with these data, respectively for positive and negative pion production. More precisely, we show the results coming from both sets of TMDs, SIDIS 1 (left panels) and SIDIS 2 (right panels), for the Sivers (dotted blue lines) and Collins (dashed green lines) effects separately, together with their sum (solid red lines). We also computed the statistical uncertainty bands for both effects and showed the envelope of these functions is still largely unconstrained by SIDIS data, while it might be relevant to explain the values of $A_N$ measured in $p^+p \to \pi X$ processes at RHIC, as studied in Refs. \cite{42,43}. As this paper focuses on a process kinematically much closer to SIDIS, the large $x$ behaviour of the involved TMDs is not so relevant here. Our two sets of TMDs (SIDIS 1 and SIDIS 2) are well representative of the possible uncertainties.

In this kinematical region the Collins effect is always negligible, almost compatible with zero. The reason is twofold: from one side the partonic spin transfer in the backward proton hemisphere is dynamically suppressed, as explained in Ref. \cite{1}; secondly, the azimuthal phase (see the second term on the r.h.s. of Eq. (6)) oscillates strongly, washing out the effect.

The Sivers effect does not suffer from any dynamical suppression, since it enters with the unpolarised partonic cross section. Moreover, there is no suppression from the integration over the azimuthal phases, as it happens, for instance, in $pp \to \pi X$ case. Indeed in $\ell p \to \pi X$ only one partonic channel is at work and, for the moderate $Q^2$ values of HERMES kinematics, the Sivers phase $(\phi)$ appearing in the first term on the r.h.s. of Eq. (6) appears also significantly in the elementary interaction, thus resulting in a non-zero phase integration.

Moreover, in this kinematical region, even if looking at the backward hemisphere of the polarised proton, one probes its valence region, where the extracted Sivers function are well constrained. The reason is basically related to the moderate c.m. energy, $\sqrt{s} \simeq 7$ GeV, of the HERMES experiment.

The difference between SIDIS 1 and SIDIS 2 results for the negative pion case, Fig. 2, comes from the fact that in the first case the Sivers function for up quark plays a relative bigger role, even if coupled with the non-leading FF.

The results presented here for the SIDIS 2 set of TMDs correspond to the predictions given in Ref. \cite{1}, with the difference that they were obtained for $P_T = 1.5$ and 2.5 GeV, and one should change $x_F$ into $-x_F$.

As one can see, while the SSA for positive pion production is a bit overestimated, Fig. 1 the description of the negative pion SSAs is in fair agreement with data for the SIDIS 1 set (left panel in Fig. 2). Notice that in the fully
inclusive case under study, at such values of $\sqrt{s}$ and $Q^2$ other effects could contaminate the SSA. Nonetheless the qualitative description, in size, shape and sign, is quite encouraging.

FIG. 1: The theoretical estimates for $A_{UT}^{sin\psi}$ vs. $x_F$ at $\sqrt{s} \simeq 7$ GeV and $P_T = 1$ GeV for inclusive $\pi^+$ production in $\ell p \rightarrow \pi X$ processes, computed according to Eqs. (12) and (5)–(7) of the text, are compared with the HERMES data [27]. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The computation is performed adopting the Sivers and Collins functions of Refs. [36, 37], referred to as SIDIS 1 in the text (left panel), and of Refs. [38, 39], SIDIS 2 in the text (right panel). The overall statistical uncertainty band, also shown, is the envelope of the two independent statistical uncertainty bands obtained following the procedure described in Appendix A of Ref. [38].

FIG. 2: Same as in Figure 1 but for inclusive $\pi^-$ production.

B. Tagged or semi-inclusive category

The HERMES Collaboration presents also a sub-sample of $\ell p$ data where the final lepton is tagged [27]. Of course the number of these events is strongly reduced w.r.t. the fully inclusive case. Nonetheless the observed asymmetries are sizeable and show a peculiar behaviour.

We then consider also these data by imposing HERMES cuts: $Q^2 > 1$ GeV$^2$, $W^2 > 10$ GeV$^2$, $0.023 < x_B < 0.4$, $0.1 < y < 0.95$ and $0.2 < z_h < 0.7$, where these are the standard variables adopted for the study of SIDIS processes. Even in this case we restrict the analysis to the large $P_T$ region, namely $P_T > 1$ GeV. In fact, in contrast to the SIDIS azimuthal asymmetries analysed in the $\gamma^* - p$ c.m. frame, where the low $P_T \leq 1$ GeV of the final hadron is entirely given at leading order in terms of the intrinsic transverse momenta in the distribution and fragmentation functions, here, working in the $\ell - p$ c.m. frame, the observed $P_T$ is also given by the hard scattering process. For this reason, to be sensitive to the intrinsic transverse momentum effects, one has not to consider necessarily very small $P_T$ values.
FIG. 3: The theoretical estimates for $A^{\sin \psi}_{UT}$ vs. $P_T$ at $\sqrt{s} \simeq 7$ GeV and $x_F = 0.2$ for inclusive $\pi^+$ production for the lepton tagged events in $\ell p \rightarrow \pi X$ process, computed according to Eqs. (12) and (5)–(7) of the text, are compared with the HERMES data [27]. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The computation is performed adopting the Sivers and Collins functions of Refs. [36, 37], referred as SIDIS 1 in the text (left panel), and of Refs. [38, 39], SIDIS 2 in the text (right panel). The overall statistical uncertainty band, also shown, is the envelope of the two independent statistical uncertainty bands obtained following the procedure described in Appendix A of Ref. [38].

FIG. 4: Same as in Figure 3 but for $\pi^-$ production.

We show our estimates compared with HERMES data in Figs. 3 and 4 respectively for positive and negative pion production as a function of $P_T$ at fixed $x_F = 0.2$. Notice that for $P_T > 1$ GeV, the values of $x_F$ probed in the HERMES kinematics are all very close to 0.2. We checked that increasing the value of $x_F$, up to 0.3, the results are almost unchanged. Again, we show the contributions from the Sivers (dotted blue line) and Collins (dashed green line) effects separately and added together (solid red line) with the overall uncertainty bands (shaded area). Some comments follow:

- In this region the Collins effect (dashed green lines) is only partially suppressed by the dynamics and the azimuthal phase integration. Indeed the spin transfer is still sizeable and the azimuthal phase entering the Collins effect is peaked around $\pi$, that is the $\cos(\phi' + \phi_q^h)$ in the second term on the r.h.s. of Eq. (6) is peaked around $-1$. Keeping in mind that the partonic spin transfer is always positive and that the convolution of the transversity distributions with the Collins functions is positive for $\pi^+$ and negative for $\pi^-$, one can understand the sign of this contribution. The difference between the SIDIS 1 and the SIDIS 2 sets (a factor around 2-3) comes from the different behaviour of the quark transversity functions at moderately large $x$.

- The Sivers effect (dotted blue lines) for $\pi^+$ production (Fig. 3) is sizeable for both sets. On the other hand for $\pi^-$ production (Fig. 4) the SIDIS 1 set (left panel) gives almost zero due to the strong cancellation between the unsuppressed Sivers up quark distribution coupled to the non-leading FF, with the more suppressed down
quark distribution. In the SIDIS 2 set (right panel), the same large $x$ behaviour of the up and down quark Sivers distributions implies no cancellation.

- With the exception of the largest $P_T$ data point the description of the data in terms of the sum of these effects is fairly good for both sets.

C. Predictions

Data at $P_T \simeq 1$ GeV are expected from the future JLab 12 operation at 11 GeV. Because of the rather low c.m. energy ($\sqrt{s} \simeq 4.8$ GeV), in order to select data with large values of $Q^2$ one has to consider a backward (w.r.t. the proton direction) production, which means $x_F \geq 0.1$. With these kinematical bounds most contribution come from the quark valence region. Our predictions, analogous to the results presented in Figs. 1 and 2 are shown in Figs. 5 and 6. The results expected at JLab 12 are similar to those observed at HERMES.

![Graph 1](image1.png)

**FIG. 5:** Theoretical predictions for $A_{UT}^{A_0}$ vs. $x_F$ at $\sqrt{s} \simeq 4.8$ GeV and $P_T = 1$ GeV for inclusive $\pi^+$ production in $\ell p \rightarrow \pi X$ processes, computed according to Eqs. (12) and (5)–(7) of the text, are shown for future JLab experiments. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The computation is performed adopting the Sivers and Collins functions of Refs. [36, 37], referred to as SIDIS 1 in the text (left panel), and of Refs. [38, 39], SIDIS 2 in the text (right panel). The overall statistical uncertainty band, also shown, is the envelope of the two independent statistical uncertainty bands obtained following the procedure described in Appendix A of Ref. [38].

![Graph 2](image2.png)

**FIG. 6:** Same as in Figure 5 but for inclusive $\pi^-$ production.

One might also wonder whether some features that characterise the SSAs observed in $pp \rightarrow \pi X$ processes and that can be reproduced within a TMD factorisation scheme [42], could still be encountered in $\ell p \rightarrow \pi X$ reactions. To
answer this question we consider the inclusive $\ell p$ process at $\sqrt{s} = 50$ GeV. In this case, in order to have a more direct comparison with the $p^1p$ case, we calculate $A_N$ as defined in Eq. (6), with the polarised proton moving along $Z_{cm}$, that is with positive $x_F$ in the forward hemisphere of the polarised proton. In Fig. 6 we show our estimates of $A_N$ for $\pi^0$ production in the process $p^1\ell \to \pi X$ at $\sqrt{s} = 50$ GeV and $P_T = 1$ GeV (left panel) and $P_T = 2$ GeV (right panel) adopting the SIDIS 1 set. This set indeed is the one that better reproduces the behaviour of $A_N$ in $p^1p \to \pi X$ processes (see for instance Ref. 41). The result deserves a few comments.

- The Collins effect in the backward region is totally negligible: this is due to a strong suppression coming from the azimuthal phase integration. In the forward region the SIDIS 1 set, as well the SIDIS 2 (results not shown), give tiny values even if the azimuthal phase would be effective. In particular for $x_F > 0.3$, once again the cosine factor entering this effect in Eq. (6) is negative.

- The Sivers effect is sizeable and increasing with $x_F$ for positive values of $x_F$, while negligible in the negative $x_F$ region. Notice that the suppression of the Sivers effect for $x_F < 0$, even if in such a process there is only one partonic channel, is due to a weak dependence on the azimuthal phase of the elementary interaction at the large $Q^2$ values reached at this energies.

- It is worth noticing that the functional shape of $A_N(x_F)$, for the $p^1\ell \to h X$ large $P_T$ process, is similar to that observed at various energies in $p^1p \to h X$ processes, being negligible at negative $x_F$ and increasing with positive values of $x_F$.

- The process $\ell p^1 \to \text{jet} + X$ at large $\sqrt{s}$ values was studied in a twist-3 formalism in Ref. 22. The quark-gluon-quark Qiu-Sterman correlator $T_F$ was fixed exploiting its relation with the Sivers function, taken from an extraction 38 from SIDIS data. The value of $A_N$ was found to be positive for $x_F > 0$ (the same kinematical configuration as for our Fig. 7 was adopted), with results very close to the results we find here for $\pi^0$ production and we found in Ref. 1 for jet production. Indeed the twist-3 and the TMD mechanisms were shown to be closely related and provide a unified picture for SSAs in SIDIS processes 42. However, the factorised twist-3 collinear scheme, using the SIDIS extracted Sivers functions for fixing the Qiu-Sterman correlator $T_F$, seems to have severe problems in explaining the SSA $A_N$ observed in pp processes, leading to values of $A_N$ opposite to those measured 46. These issues were further studied in Refs. 47 49. A recent analysis of $A_N$ in pp scattering in the twist-3 formalism 50 attempts at solving this problem showing that the asymmetry might be dominated by new large effects coming from fragmentation. It is not clear how much these same effects would change the value of $A_N$ in SIDIS when going from jet to $\pi^0$ production.

- The measurement of asymmetries in the same kinematical region and with the same features as in Fig. 6 for $\pi^0$ production, and as in Fig. 6 of Ref. 1 for jet production, would be a strong indication in support of our TMD factorised approach. Such measurements might be possible at a future Electron-Ion-Collider (EIC) 51.

![Fig. 7: Theoretical estimates of $A_N$ vs. $x_F$ at $\sqrt{s} \approx 50$ GeV, $P_T = 1$ GeV (left panel) and $P_T = 2$ GeV (right panel) for inclusive $\pi^0$ production in the $p^1\ell \to \pi X$ process. Notice that, contrary to the kinematical configurations of Figs. 1 and 2, a forward production w.r.t. the proton direction corresponds here to positive values of $x_F$. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The estimates are obtained adopting the Sivers and Collins functions of Refs. 37 35 (SIDIS 1 set), according to Eqs. (6) of the text.](image)
IV. COMMENTS AND CONCLUSIONS

We have further pursued and tested the idea presented in Ref. [1] for assessing the validity of the TMD factorisation in inclusive processes in which a single large $P_T$ particle is produced. Starting from the TMD factorisation valid for SIDIS processes, $\ell p \rightarrow \ell hX$, in which a large $Q^2$ virtual photon $\gamma^*$ hits a quark, which then fragments into a final hadron with a small $P_T$ in the $\gamma^*-p$ c.m. frame, we have assumed its validity for processes in which the final lepton is not necessarily observed, but the final detected hadron has a large $P_T$ in the lepton-proton c.m. frame. A large value of $P_T$ implies, at leading order, a large angle elementary scattering, $\ell q \rightarrow \ell q$, and then a large value of $Q^2$. Such a process is analogous to the $p^+p \rightarrow hX$ processes, for which large SSAs $A_N$, Eq. [1], have been measured. According to the TMD factorisation approach, the SSAs can be generated by the Sivers and Collins effects [12,13].

We have computed the single spin asymmetry $A_N$, for the $\ell p p^+ \rightarrow hX$ process and in the TMD factorised scheme, as generated by the Sivers and the Collins functions, which have been extracted from SIDIS and $e^+e^-$ data [14,39]. Doing so, we adopt a unified TMD factorised approach, valid for $\ell p \rightarrow \ell hX$ and $\ell p \rightarrow hX$ processes, in which, consistently, we obtain information on the TMDs and make predictions for $A_N$. Some of these predictions were given in Ref. [1].

New HERMES data on $A_N$ are now available [27] for different kinematical regions; we have selected those data which – although not yet optimally – fulfill the conditions of applicability of our TMD factorisation approach, and compared them with the results of our computations. We have selected two sets of TMDs extracted from SIDIS and $e^+e^-$ data, and which are representative, with their large differences, of the uncertainties which the SIDIS available data still allow.

It turns out, Figs. [1,4] that our theoretical estimates for $A_{UT}^{\sin \psi}(x_F, P_T) = A_N(-x_F, P_T)$ agree well, in shape and sign, with the experimental results, in particular for one set of TMDs (SIDIS 1). In some cases (see Fig. [1]) our results are a bit larger than data, yet with the right sign and behaviour; one should not forget that in the kinematical regions we are considering (in $P_T$, $Q^2$ and $\sqrt{s}$) other mechanisms might still be at work. The overall agreement between our computations and the data is very encouraging.

In Figs. [5] and [6] we have estimated the expected value of $A_{UT}^{\sin \psi}$ at the future JLab experiments at 12 GeV. We are still in a kinematical region where a careful selection of data is necessary in order to ensure the validity of our approach. The results are similar to those obtained in Figs. [1] and [2] for HERMES kinematics.

At last, in Fig. [7] we have given predictions for $A_N(x_F)$ in very safe kinematical regions for our approach to hold. Indeed, as expected, we recover the same behaviour for $A_N(x_F)$ as observed in $p^+p \rightarrow \pi^0 X$ processes. Such a prediction, crucial for assessing the validity of our TMD factorisation scheme, could be tested at a future EIC [51].

Acknowledgments

A.P. work is supported by the U.S. Department of Energy under Contract No. DE-AC05-06OR23177. M.A., M.B., U.D., S.M. and F.M. acknowledge support from the European Community under the FP7 program “Capacities - Research Infrastructures” (HadronPhysics3, Grant Agreement 283286). M.A., M.B. and S.M. acknowledge support from the “Progetto di Ricerca Ateneo/CSP” (codice TO-Call3-2012-0103). U. D. is grateful to the Department of Theoretical Physics II of the Universidad Complutense of Madrid for the kind hospitality extended to him during the completion of this work.

[1] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. D81, 034007 (2010).
[2] D. L. Adams et al. (E581 Collaboration and E704 Collaboration), Phys. Lett. B261, 201 (1991).
[3] D. L. Adams et al. (E704 Collaboration), Phys. Lett. B264, 462 (1991).
[4] D. L. Adams et al. (E581 Collaboration and E704 Collaboration), Phys. Lett. B276, 531 (1992).
[5] D. L. Adams et al. (E581 Collaboration), Z. Phys. C56, 181 (1992).
[6] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 92, 171801 (2004).
[7] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 95, 202001 (2005).
[8] J. H. Lee and F. Videbaek (BRAMHS Collaboration), AIP Conf. Proc. 915, 533 (2007).
[9] B. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 101, 222001 (2008).
[10] L. Adamczyk et al. (STAR Collaboration), Phys. Rev. D86, 051101 (2012).
[11] G. Igo et al. (STAR Collaboration), AIP Conf. Proc. 1523, 188 (2012).
[12] L. C. Bland et al. (AnDY Collaboration), arXiv:1304.1454 [hep-ex].
[13] J. C. Collins, Phys. Lett. B536, 43 (2002).
[14] J. C. Collins and A. Metz, Phys. Rev. Lett. 93, 252001 (2004).
[15] X.-d. Ji, J.-p. Ma, and F. Yuan, Phys. Rev. D71, 034005 (2005).
[16] X.-d. Ji, J.-p. Ma, and F. Yuan, Phys. Lett. B597, 299 (2004).
[17] A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP 08, 023 (2008).
[18] J. She, Y. Mao, and B.-Q. Ma, Phys. Lett. B666, 355 (2008).
[19] M. Anselmino, M. Boglione, J. Hansson, and F. Murgia, Eur. Phys. J. C13, 519 (2000).
[20] Y. Koike, AIP Conf. Proc. 675, 449 (2003).
[21] Y. Koike, Nucl. Phys. A721, 364 (2003).
[22] Z.-B. Kang, A. Metz, J.-W. Qiu, and J. Zhou, Phys. Rev. D84, 034046 (2011).
[23] D. Boer, P. J. Mulders, and F. Pijlman, Nucl. Phys. B667, 201 (2003).
[24] J. Ma and Q. Wang, Eur. Phys. J. C37, 293 (2004).
[25] J. Zhou, F. Yuan, and Z.-T. Liang, Phys. Rev. D79, 114022 (2009).
[26] J. Zhou, F. Yuan, and Z.-T. Liang, Phys. Rev. D81, 054008 (2010).
[27] A. Airapetian et al. (HERMES Collaboration), Phys. Lett. B728, 183 (2014).
[28] K. Allada et al. (Jefferson Lab Hall A Collaboration), Phys. Rev. C89, 042201 (2014).
[29] U. D'Alesio and F. Murgia, Phys. Rev. D70, 074009 (2004).
[30] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, S. Melis, and F. Murgia, Phys. Rev. D73, 014020 (2006).
[31] U. D'Alesio and F. Murgia, Prog. Part. Nucl. Phys. 61, 394 (2008).
[32] D. W. Sivers, Phys. Rev. D41, 83 (1990).
[33] D. W. Sivers, Phys. Rev. D43, 261 (1991).
[34] A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. D70, 117504 (2004).
[35] J. C. Collins, Nucl. Phys. B396, 161 (1993).
[36] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin, Phys. Rev. D72, 094007 (2005).
[37] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin and C. Türk, Phys. Rev. D75, 054032 (2007).
[38] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin and C. Türk, Eur. Phys. J. A39, 89 (2009).
[39] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin and S. Melis, Nucl. Phys. Proc. Suppl. 191, 98 (2009).
[40] S. Kretzer, Phys. Rev. D62, 054001 (2000).
[41] D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D75, 114010 (2007).
[42] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D88, 054023 (2013).
[43] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D86, 074032 (2012).
[44] M. Boglione, U. D'Alesio, and F. Murgia, Phys. Rev. D77, 051502 (2008).
[45] X. Ji, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. Lett. 97, 082002 (2006).
[46] Z.-B. Kang, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D83, 094001 (2011).
[47] Z.-B. Kang and A. Prokudin, Phys. Rev. D85, 074008 (2012).
[48] A. Metz, D. Pitonyak, A. Schafer, M. Schlegel, W. Vogelsang and J. Zhou, Phys. Rev. D86, 094039 (2012).
[49] L. Gamberg, Z.-B. Kang and A. Prokudin, Phys. Rev. Lett. 110, 232301 (2013).
[50] K. Kanazawa, Y. Koike, A. Metz, and D. Pitonyak, arXiv:1404.1033 [hep-ph].
[51] A. Accardi et al., arXiv:1212.1701 [nucl-ex].