Dynamic criticality in driven disordered systems: 
Role of depinning and driving rate in Barkhausen noise*

Bosiljka Tadić†
Jožef Stefan Institute, P.O. Box 3000, 1001 Ljubljana, Slovenia

We study Barkhausen noise in a diluted two-dimensional Ising model with the extended domain wall and weak random fields occurring due to coarse graining. We report two types of scaling behavior corresponding to (a) low disorder regime where a single domain wall slips through a series of positions when the external field is increased, and (b) large disorder regime, which is characterized with nucleation of many domains. The effects of finite concentration of nonmagnetic ions and variable driving rate on the scaling exponents is discussed in both regimes. The universal scaling behavior at low disorder is shown to belong to a class of critical dynamic systems, which are described by a fixed point of the stochastic transport equation with self-consistent disorder correlations.

I. INTRODUCTION

Barkhausen signal is a physical phenomenon of large practical importance for noninvasive material characterization technique and for magnetic recording. It consists of series of irregular pulses (noise) which are related to reversal of the magnetization by slowly increasing (e.g., from negative saturation) external field in ferromagnetic alloys and amorphous ferromagnets. Recently Barkhausen noise (BN) has been studied as an example of complex behavior in driven disordered systems far from equilibrium. It has been found that BN obeys a scale-free behavior without tuning of a parameter, a feature which is peculiar to the dynamic critical systems . In particular, most often measured are distributions of the area $s$ of Barkhausen pulses, which obeys a power-law up to a cut-off according to $D(s) \sim s^{-\tau_s}$, the durations of pulses $t$ with distribution $P(t) \sim t^{-\tau_t}$, and the energy released in a pulse $n$ with $D(n) \sim n^{-\tau_n}$. The dissipated power spectrum was found to decay with frequency $f$ as $S(f) \sim f^{-\phi}$. A short summary of the experimental results taken from the literature is given in Table 1. It should be noted that the scaling exponents obey (within error bars) the following scaling relations $(\tau_s - 1)D_s = (\tau_n - 1)D_n = \tau_t - 1$, where $D_s$ and $D_n$ are the fractal dimensions associated with the size $s$ and energy $n$ of pulses, respectively. The measured scaling exponents can be grouped in (at least) three distinct universality classes, which we may characterize according to the exponent $\tau_s$ as 1.33, 1.40, and 1.70, respectively. The effects of varying driving rate have been studied systematically in certain compounds in Refs. [2][3][4]. It has been recognized that increased driving rate leads to increased occurrence of large pulses and thus to a continuous decrease of the scaling exponents. In addition, a careful analysis of the experimental results suggests that scaling exponents may depend on sample composition and annealing (compare, for instance, MC3 and D1 in Table 1), and on the exposing of a magnetostrictive sample to tensile stress, which leads to different structure of domains $[3]$, as opposed to unstressed sample $[10]$ (see (DS) and (DB) in Table 1).

Proximity of the mean-field universality class (corresponding to $\tau_s = 1.66$, which is close to measured $\approx 1.7$ in Table 1) was understood as being a consequence of dipolar forces and demagnetizing fields in $d = 3$ spatial dimensions $[4]$. Recently occurrence of additional two universality classes (characterized with $\tau_s = 1.54$ and $\tau_s = 1.30$ $[1]$) has been attributed to two-dimensional character of domain-wall motion, which was observed in a class of anisotropic soft magnets. Such systems are for instance ribbons made of Fe-B-Si $[1][2]$ and Fe-Co-B compounds $[7][8][10]$, in which domains appear to have stripe structure with extended domain walls parallel to the magnetization. The two universality classes have been related $[1]$ to a single domain wall motion and to a multidomain structure, respectively, in a 2-dimensional prototype model with local random fields. Models with other types of disorder in $d = 2$ dimensions $[5][7]$ have considered BN in multidomain structure.

In the present paper we extend the study of Ref. $[1]$ to consider Barkhausen noise in two regimes corresponding to a single domain and multi-domain structure, respectively, in a diluted Ising model in $d = 2$ with varying concentration of nonmagnetic ions $c$ and fixed (weak) random fields. We use infinitely slow driving in order to study effects of the varying concentration $c$ on the statistics of BN avalanches. We also discuss the effects of finite

* Dedicated to the memory of my father
† Electronic address: Bosiljka.Tadic@ijs.si
driving rates in both regimes (see also [14]). Finally, we show that the critical behavior in the regime with a single domain wall can be related to a class of driven critical systems which are described by the stochastic transport equation with correlated quenched currents [16].

In Section II we introduce the model and determine two disorder regimes. In Section III numerical results of distributions and their scaling exponents are presented for varying concentration \(c\) and driving rates. In Section IV we discuss the analogy between low-disorder regime and stochastic transport equation with quenched random currents, and in Section V we give a short summary and discussion of the results.

### TABLE I. Values of the scaling exponents measured in various substances: (MC1) amorphous iron, (MC2) Ni\(_{50}\)Al\(_{15}\)Mn\(_{2}\), (MC3) Fe\(_{78}\)Bi\(_{15}\)Sb\(_{2}\), all from Ref.1; (BG) VITROVAC 6025 X, Ref.6; (E4) Fe\(_{30}\)Ni\(_{45}\)Sb\(_{5}\), Ref.4; (DS) Fe\(_{64}\)Co\(_{21}\)B\(_{15}\) under tensile stress, Ref.8; (TR)1.8% SiFe for driving rate 0.15, Ref.5. (D1) Fe\(_{72}\)Co\(_{12}\)B\(_{15}\) for driving rate 0.1, Ref.7; (DB) Fe\(_{64}\)Co\(_{21}\)B\(_{15}\) for driving rate 0.16, Ref.10. Except in (MC1) and (MC2) which used wires, in the rest of experiments thin ribbon or strip samples were used.

| \(E\) | MC1 | MC2 | MC3 | BG | E4 | DS | TR | D1 | DB |
|------|-----|-----|-----|----|----|----|----|----|----|
| \(\tau_1\) | 1.88 | 2.1 | 1.82 | 2.22 | - | - | 1.5 | 1.76 | 1.72 |
| \(\tau_2\) | 1.64 | 1.78 | 1.74 | 1.77 | 1.33 | 1.3 | 1.42 | 1.40 |
| \(\tau_3\) | 1.44 | 1.58 | 1.60 | 1.56 | - | - | - | - |
| \(D_\perp\) | 1.45 | - | - | 1.51 | - | - | 1.53 | - |
| \(D_\parallel\) | - | - | - | 2.03 | - | - | - | - |
| \(\phi\) | 2 | 2 | 2 | 2 | 1.6 | - | - | 1.75 | - |

### II. MODEL AND BARKHAUSEN AVALANCHES

We consider an Ising model on the square lattice in two-dimensions assuming that local random fields \(h_i\) are generated by coarse-graining from an original disorder in the presence of the external magnetic field \(H\) [14]:

\[
\mathcal{H} = - \sum_{<i,j>} J_{ij} S_i S_j - \sum_i (h_i + H) S_i .
\]

\(J_{ij} = 1\) is a constant interaction between nearest-neighbor spins \(S_i = \pm 1\), and \(J_{ij} = 0\) if at least one of the spins is zero. A randomly distributed fraction \(c\) of sites represents nonmagnetic ions with \(S_i = 0\). A Gaussian distribution of \(h_i\) is assumed with zero mean and width \(f\). We assume a small random field variance \(f \leq 0.3\) (see Ref. [1] for effects of varying random fields), and vary concentration of nonmagnetic ions \(c\).

Following Ref. [1], we consider two disorder regions: For low values of concentrations \(c < c^*(f)\) we consider extended domain wall of flipped spins along \(< 11>\) direction on the square lattice (linear size and the wall is \(L\)). Periodic boundaries are applied in the direction of the wall, and the boundary opposite to the wall is left free. Here \(c^*(f)\) is determined as the largest disorder at which domain-wall depinning still occurs. For high disorder \(c > c^*(f)\) the domain wall remains pinned for all values of the driving field, and many domains of reversed spins are nucleated inside the system. In this region of disorder periodic boundary conditions in both directions can be used equally well as the boundary conditions described above, leading to the same avalanche statistics. In Fig. 1 we show two examples of emerging cluster structure of flipped spins in low-disorder and high-disorder regime. The cluster structure at low disorder reflects the anisotropy due to initial conditions. The anisotropy is still important at high disorder, however, the occurrence of many domains that block each other spatial extent becomes dominant feature that determines the scaling properties of Barkhausen noise in this region.

We first apply infinitely slow driving (i.e., increments of the driving field are adjusted to the minimum local field in the system). It should be stressed that these driving conditions are useful for the purely theoretical purposes, as opposed to finite driving rates that are applied in real experiments. An example of the simulated BN signal for a half of the hysteresis loop is given in Fig. 2. The corresponding time series of the values of the field adjustments is given in the inset to Fig. 2. It is clear from Fig. 2 that after a certain transient time the consecutive values of the weakest local field in the system vary very little and, at the same time, the main contribution to the avalanche statistics comes from exactly that part of the hysteresis loop.

### III. CONCENTRATION AND DRIVING-RATE DEPENDENCE OF SCALING EXPONENTS

In this Section we study distributions of avalanche sizes and durations by keeping fixed weak random field \(f \leq 0.3\) at each occupied site and vary concentration of nonmagnetic ions \(c\). We first apply infinitely slow driving in order to differentiate effects of varying disorder from those of varying driving rate. The concentration dependence for finite driving rate in high-disorder region was studied in Ref. [14]. At low concentrations \(c\) the concept of a single domain wall persists, although a small fraction of spins may flip in the interior of the system near vacancies. The domain wall moves in a series of slips and eventually depinning occurs at critical driving field \(H_c(c)\). In the applied driving conditions the depinning may occur for concentrations \(c < c^*(f)\), whereas for larger disorder the built-in domain wall remains pinned for all values of the driving field. For \(c > c^*(f)\) the domains of flipped spins are nucleated inside the system and their walls move at different instances of time. In this way subsequent avalanches block each others extent, leading to long-range correlations in space and to memory effects. The two regions with different behavior are schematically shown in the phase diagram in Fig. 3.

In the high disorder region HDR above \(c^*(f)\) we measure the distribution of avalanche durations, which is shown...
in Fig. 4 for varying \( c \) and fixed \( f = 0.001 \). Both the slope and cutoff of the distributions depend on \( c \). In the inset are shown the extracted scaling exponents for slopes of the distribution of durations \( (\tau_i) \), and sizes \( (\tau_s) \), which are found to increase with \( c \). By diluting the spin lattice clusters of connected spins become smaller and more ramified, so that large avalanches are less probable by fixed other conditions (i.e., fixed \( f \) and driving rate). This leads to the increase of slopes of the avalanche distributions, as shown in Fig. 4.

Similarly, we find increase of the scaling exponents in the low disorder region (LDR) when \( c \) is varied in the interval \( 0 < c < c^* (f) \), where a single domain wall exists. Here we have measured the exponent of avalanche sizes \( \tau_s (c) \), and derived \( \tau_s (c) \) from the scaling relations that are discussed in detail in Ref. [1]. The exponents are also shown in the inset to Fig. 4. It is interesting to note that in this region of disorder the avalanches are self-affine (cf. Fig. 1). We measure the anisotropy exponent \( \zeta \), which is defined as \( < \xi_\parallel > / < \xi_\perp > \), where \( \xi_\parallel \) and \( \xi_\perp \) are avalanche extents in the direction parallel and perpendicular to the wall, respectively. For instance, for \( c = 0.05 \) we find \( \zeta = 1.23 \pm 0.09 \).

It is expected that the depinning transition along the line \( H_s (c) \) belongs to the same universality class as the depinning in \( c = 0 \) limit for low random-field type of disorder, which we studied in Ref. [1]. When the disorder is tuned to a critical value along the line \( c^* (f) \) the phase transition occurs between low-disorder and high-disorder regime, which we expect to be in the same universality class as the one at critical random-field variance \( f^* \) for \( c = 0 \) (see Ref. [1] for details).

By applying a finite driving rate, i.e., increasing the external field always by a given constant amount \( \Delta H \) independently of values of local fields in the system, weak pinning centers become overdriven. Therefore avalanche sizes increase compared to the case of infinitely slow driving both due to coalescence of different clusters and by faster propagation of a single cluster due to reduced pinning. As a result scaling exponents of avalanches decrease with increasing driving rate (see Ref. [13]). The effects of increased driving rate are similar in both LDR and HDR, however, the boundary between these regions in the phase diagram moves towards lower disorder (i.e., nucleation of new domains starts at lower disorder compared to the case of zero driving rate and interaction between domain walls prevents depinning transition). In the inset to Fig. 4 we have also shown values of the scaling exponents in HDR for driving rate \( \delta h \equiv \Delta H / H_{\text{max}} = 0.01 \), taken from Ref. [1]. The concentration dependence of the exponents at finite driving rates becomes approximately linear (cf. inset to Fig. 4 and Ref. [14]). Another important feature of the scaling behavior by the finite driving rate is that the cutoffs of the distribution curves increase and obey the finite-size scaling. [13]. In this way a subcritical dynamic system for \( c^* (f) < c < c_p (f) \), where \( c_p (f) \) is the percolation threshold line, becomes critical for a range of values of driving rates [15]. This is the reason for ubiquity of self-similar Barkhausen noise in highly disordered alloys, since the experimental setups always use finite frequency of the external field cycles.

Dependence of the scaling exponents of the finite driving rates has been nicely demonstrated in Refs. [2,7,8,10] in 1.8% Si-Fe and Fe-Co-B compounds. It should be noted that both numerical [14,15] and experimental observations [16,58] agree that for large driving rates \( \delta h \) the exponents decay linearly with \( \delta h \). However, for small driving rates deviation from linear law occurs, making extrapolation to zero rate difficult. The dependence of the measured exponents in the literature on the concentration of nonmagnetic ions can be established qualitatively. For instance, the duration exponent \( \tau_s = 1.56 \) measured in Fe\text{73}Co\text{27}B\text{15} (for driving rate 0.3 in reduced units) in Ref. [2] is lower than \( \tau_s = 1.82 \) measured in diluted Fe\text{78}B\text{12}Si\text{9} (for driving rate 1Hz) in Ref. [1], in a qualitative agreement with our predictions (cf. inset to Fig. 4). However, a more quantitative comparison would be only possible if all other conditions, i.e., driving rates and annealing of samples were kept under strict control.

IV. UNIVERSALITY CLASS OF DRIVEN DISORDERED SYSTEMS

In the low-disorder regime \( (f < f^*, c < c^* (f)) \) apart from a weak \( c \) dependence the critical behavior at infinitely slow driving can be identified by the exponents \( \tau_s = 1.58, \tau_t = 1.89 \) and \( \zeta = 1.23 \). These exponents appear to be close to those obtained in the ricepile model with stochastic critical slope rule in 1+1 dimension [7]. It has been shown [18] that the same critical exponents occur in a wide class of critical systems, that includes the models of ricepiles, depinning and earthquakes. The problem of an interface depinned from quenched defects in the anisotropic medium has been studied in [19]. The exponents can be related to the directed percolation exponents (see also [21]). It has been elucidated that due to anisotropy the threshold driving force depends on the interface slope, and thus may remain finite at the transition when the velocity of the interface vanishes. The same effect is taken into account by the relevant nonlinear term \( \lambda (\partial_x h)^2 \) in the continuum equation of interface depinning [22]. In addition, the nonlinearity of the dynamics in critical systems [17,18] leads to self-tuning to the dynamic critical state.

Recently it has been shown [18] that a self-tuning in the class of critical disordered systems can be described by self-consistent building of correlations in the transport equation with random quenched currents

\[
\frac{\partial h}{\partial t} = \nu_\parallel \partial_x^2 h + \nu_\perp \partial_x^2 h - p(x) \partial_t h + \eta .
\]  (2)

Here \( p(x) \) is the random local velocity of transport with zero mean and correlations given by
\[ \langle p(x) \partial_x h \rangle = \gamma \left( \frac{1}{2} \right)^{1-\xi} \langle \partial_x h \rangle^2, \tag{3} \]

and the anisotropy exponent \( \zeta \) is to be determined self-consistently as the system approaches the fixed point. The usual dynamic noise term \( \eta \) is assumed to have annealed correlations as \( \langle \eta(x,t)\eta(x',t') \rangle = 2D\delta^{(d)}(x-x')\delta(t-t') \). It has been found in Ref. [16] that a stable fixed point exists for finite strength of disorder \( \gamma \) with the exponents that depend continuously on the range of correlations along parallel direction, parameter \( \delta \). In particular, for long-range correlations at \( \delta = 1/3 \) the corresponding avalanche exponents are found \( \tau_t = 1.89 \) and \( \tau_s = 1.56 \), with the emergent nonlinear term \( \frac{1}{\tau_3} (\partial_x h)^2 \).

V. CONCLUSIONS

Barkhausen avalanches in disordered ferromagnets obtained at sufficiently slow (which respects time scale separation) increase of the driving field reflect fractal structure of the dynamic critical states. The quantitative characterization of a given system in terms of sets of scaling exponents of the BN avalanche can be expected in several universality classes, which depend on: (i) type of domain walls; (ii) strength of pinning; and (iii) applied driving conditions. The chemical composition, preparation, sample form and annealing result in certain type of domain walls, which are thus characteristic for a given sample. Whereas, the driving rate can be controlled in the experimental setups. In ribbon geometry the effects of demagnetizing fields are minimized with longitudinal anisotropy [12].

The scaling behavior studied in this work is expected to apply for the anisotropic samples with extended domain walls, which are pinned by nonmagnetic impurities. Such domain walls are found in ribbon samples with strong longitudinal anisotropy as for instance in fully annealed Fe_{78}B_{13}Si_{9} ribbons in which domain walls tend to orient along longitudinal anisotropy axis [12]. Similar structure of domains was reported recently in Refs. [11] for Fe_{21}Co_{46}B_{15} (see Fig. 1 in [11]). Moreover, it has been suggested [8] that type of domains in these samples can be controlled by continuous variation of the tensile stress. Our results suggest (see also [11]) that self-similar Barkhausen noise in these systems occurs in a wide range of values of the disorder and driving field, which are marked as gray areas in the phase diagram in Fig. 2, excluding only the area with finite domain-wall velocity (upper left corner of the phase diagram). In the zero driving rate we find two distinct universal behaviors for low disorder and high disorder region, respectively, which compare well with the experimental results in the above samples. For low disorder self-similar BN is a consequence of the intermittent motion of a single domain wall (or well separated, noninteracting domain walls), before eventually depinning of the wall occurs at critical driving field. We suggest that in the zero driving rate the BN in this region belongs to the same universality class as the ricepile [17] and the anisotropic interface motion in \( 1 + 1 \) dimensions. For high disorder region, which is characterized by multidomain structure in the plane (see for instance [13]), the criticality of BN is induced by the finite driving rates, which represents usually applied driving conditions in the experiments of BN. In principle, a range of values of driving rates should exist for which a given sample exhibits self-similar Barkhausen noise. Both numerical simulations [16] and experimental observations [16,18] agree that scaling exponents decrease with increasing driving rate, with the linear dependence at large driving rates. Discussion of the finite driving rates in terms of the relevant fixed points of a transport equation has not yet received full theoretical treatment.

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FIG. 1. Cluster of flipped (bright) and unflipped (gray) spins, with most recent cluster (dark gray) for $L = 200$ and $f=0.3$ in the presence of nonmagnetic defects (black points) of concentration $c$: Bottom: $c=0.2$; Top: $c=0.05$.

FIG. 2. Simulated Barkhausen noise signal vs time for one side of the hysteresis loop. Lattice size is $L = 200$ and disorder values $f = 0.3$ and $c = 0.2$. Inset: Time series of the increments of the external field $\Delta H(t)$ vs $t$ which are adjusted to the minimum local field in the system, corresponding to the signal in the main picture. Note that the main contribution to the distribution of avalanches comes from the central part of the hysteresis loop, which is reached after approximately 500 steps.
FIG. 3. Phase diagram: external field $H$ vs. concentration of nonmagnetic ions $c$ for $f = 0.3$ and infinitely slow driving. Dashed line: Coercive field $H_0(c)$ vs. $c$. Scale free Barkhausen avalanche (up to a finite cut-off) occur in the entire gray area, corresponding either to single-domain or multi-domain structure. A single-domain depinning occurs along the boundary with finite velocity region. For a finite driving rate boundary between two scaling regions moves towards lower concentrations.

FIG. 4. Double logarithmic plot of the integrated distribution of avalanche durations $P(t, c)$ for $f = 0.001$ and various concentrations $c = 0.1, 0.125, 0.15,$ and $0.2$, obtained by infinitely slow driving for $L = 200$. Inset: Critical exponents ($\triangle$) $\tau_t$, and ($\circ$) $\tau_s$ vs. concentrations of nonmagnetic ions $c$. Full symbols: high disorder; Empty symbols: low disorder regime. Finite driving rate $\delta h = 0.01$: ($\star$) $\tau_t$, and ($\times$) $\tau_s$ (from Ref. [14]).