MAGNETOACOUSTIC AND ALFVÉNIC BLACK HOLES WITH HAWKING RADIATION AT HORIZONS MADE OF MAGNEPHONONS AND ALPHONONS

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ABSTRACT
We introduce analogue black holes (BHs) based on ideal magnetohydrodynamic equations. Similar to acoustic BHs, which trap phonons and emit Hawking radiation (HR) at the sonic horizon where the flow speed changes from super- to sub-sonic, in the horizon of magnetoacoustic and Alfvénic BHs, the magnetoacoustic and Alfvén waves will be trapped and emit HR made of quantized vibrations similar to phonons which we call magnephonons and Alphonons. We proposed that magnetoacoustic and Alfvénic BHs may be created in the laboratory using a tube with variable cross section embedded in a uniform magnetic field, and a super-magnetoacoustic or a super-Alfvénic flow. We show that the Hawking temperature for both BHs is a function of the background magnetic field, number density of fluid, and radius of the tube. For a typical setup, the temperature is estimated to be about 0.0266 K.

Keywords: black hole physics—waves

1. INTRODUCTION
In 1916 Schwarzschild gave a metric as a solution of the Einstein field equation. Singularity of such a metric predicted a gravitational black hole (BH) with event horizon at Schwarzschild radius (see Misner et al. 1973). Although, based on classical physics everything, even light, is absorbed by BHs and cannot escape, in the context of quantum field theory in curved space, Hawking showed that BHs should emit black body radiation (Hawking 1974, 1975). Hawking radiation (HR) in the universe has not been observed yet, but numerous attempts have been done to simulate the interesting phenomena in the laboratory. Unruh showed that HR is not only a characteristic of gravitational BHs, it is also a characteristic of the acoustic analogue BH (Unruh 1981, 1995). After 1981, most of attempts are proposed based on Bose-Einstein condensates of quantum fluid (Garay et al. 2000, 2001; Barceló et al. 2001; Recati et al. 2009; Zapata et al. 2011), quasi particles in superfluid (Volovik 2000), ultra-cold fermions (Giovanazzi 2005), in plasmas and ion rings (Horstmann et al. 2010; Blatt & Roos 2012; Gerritsma et al. 2010, 2011; Casanova et al. 2011), slow light in an atomic vapor (Brevik 2001; Leonhardt & Piwnicki 2000; Leonhardt 2002; Unruh & Schützhold 2003), in water (Rousseaux et al. 2008; Weinfurtner et al. 2011), etc. Recently, observation of self-amplifying HR in an analogue BH laser suggested a very promising experiment method for probing the inside of a BH (Steinhauer 2014). From a theoretical point of view, the acoustic analogue BH models are developed in geometrical acoustics and physical acoustics (Barceló et al. 2011). Using the linearized hydrodynamic equations in the presence of initial material flow, a wave equation for velocity potential was obtained. Tensorial form of the wave equation results in an acoustic metric. The acoustic metric is singular at a point where the local sound speed is equal to the flow speed (Sakagami & Ohashi 2002). This was interpreted as characteristic of a sonic BH (Barceló et al. 2011; Sakagami & Ohashi 2002). The effect of magnetic field on the acoustic BH and HR has not been studied, yet. Alternatively, the idea for definition of acoustic BH can be applied to introduce new analogue magnetoacoustic and Alfvénic BHs in the magnetohydrodynamics (MHD) framework.

Magnetohydrodynamics (MHD) is an useful approach to analyze characteristics (flow, wave and dissipation, etc) of the laboratory and astrophysical plasma (e.g., (Goedbloed et al. 1973)). After 1942 (Alfvén 1942), MHD (Alfvén and magnetoacoustic) waves have been detected, using the laboratory experiments (e.g., (Lundquist 1949; Bostick & Levine 1952; Lehnert 1954)), in the earth-ionosphere and magnetic field (Berthold et al. 1960; Masahisa 1961), a variety

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normal modes of the solar corona (Aschwanden et al. 1999; Nakariakov et al. 1999; Wang et al. 2002; Aschwanden 2004; McIntosh et al. 2011). Here, first we treat the magnetoacoustic, Alfvén, and acoustic waves based on Helmholtz theorem for a uniform and stationary medium with constant background magnetic field. Second, mimicking the definition of acoustic BH in a nozzle, we introduce magnetoacoustic and Alfvénic BHs with using a slightly variable cross section tube. We conclude that at the horizon of magnetoacoustic and Alfvénic BHs should emit radiations made of the Magnephonon and Alphonon, respectively. We define two quasi-particles Magnephonon and Alphonon correspond to quantum of the magnetoacoustic and Alfvén waves, respectively.

This paper is organized as follows: Sec. 2 gives the properties of magnetoacoustic waves (fast, slow, and Alfvén waves) using the Helmholtz decomposition and explains a derivation of magnetoacoustic metric in the basis of linearized ideal magnetohydrodynamic equations. Sec. 3 introduces the acoustic, magnetoacoustic, and Alfvénic black holes, respectively. Sec. 4 calculate the Hawking temperature for the acoustic, magnetoacoustic, and Alfvénic black holes. Sec. 5 describes the conclusions.

2. MAGNETOACOUSTIC WAVES AND METRIC

Here, we give the conditions for a definition of a magnetoacoustic metric, in the non-relativistic magnetohydrodynamic (MHD) framework. In the MHD approach, the behavior of continuous plasma is governed by a non-relativistic form of Maxwell’s equations, together with Ohm’s law, a gas law, equation of mass continuity, motion and energy equations (Priest 2014). The ideal MHD equations for an adiabatic process and irrotational flow $\nabla \times \mathbf{v} = 0$ are given by:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{2} \nabla \mathbf{v}^2 \right) = -\nabla p + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{3}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{4}
\]

\[
p = k \rho^\gamma, \tag{5}
\]

where $\rho$, $p$, $\mathbf{v}$, $\mathbf{B}$, $\mu$, $\gamma$, and $k$ are density, pressure, flow velocity, magnetic field, magnetic permeability, atomicity coefficient, and a constant, respectively. For the derivation of a metric for magnetoacoustic wave we need to linearise the MHD equations with choosing the velocity disturbances as a velocity potential in similar analytical process for derivation of acoustic metric from wave equation. To do this, first we treat the propagation of magnetoacoustic waves in homogenous unbounded medium choosing velocity disturbance as a velocity potential and a vector potential. Second, the magnetoacoustic and Alfvénic metrics are calculated.

2.1. Helmholtz theorem and magnetoacoustic waves

Properties of magnetoacoustic waves (fast, slow, and Alfvén waves) in an unbounded homogenous and stationary medium with constant density ($\rho_0 = \text{cte}$), constant pressure ($p_0 = \text{cte}$), and uniform background magnetic field ($\mathbf{B}_0$), were investigated in the literature (e.g., (Priest 2014)). The linearised ideal MHD equations can be reduced to a single equation for disturbed velocity as (Priest 2014),

\[
\frac{\partial^2 \mathbf{v}}{\partial t^2} = c_0^2 \nabla (\nabla \cdot \mathbf{v}) + v_A^2 (\nabla \times (\nabla \times (\mathbf{v} \times \mathbf{B}_0))) \times \mathbf{B}_0. \tag{6}
\]

where $c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$ is the sound speed, $v_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$ is speed of the Alfvén wave, and $\mathbf{B}_0 = \mathbf{B}_0/B_0$ is a unit vector.

Here, we focus on studying of the characteristics of the above mentioned waves using a fundamental theorem of calculus (Helmholtz’s theorem). Based on the Helmholtz decomposition, a vector field with sufficient smoothness and decay conditions (Gibbs & Wilson 1901), can be decomposed to an irrotational part $(\nabla \phi$ where $\phi$ is a scaler) and a solenoidal part $(\nabla \times \mathbf{A}$ where $\mathbf{A}$ is the vector potential and satisfy $\nabla \mathbf{A} = 0$). The irrotational (gradient) and divergence-free solutions can be used for treating the longitudinal and transversal waves, respectively.

Suppose an irrotational solution, $\mathbf{v} = \nabla \phi$ with $\phi = \phi \exp(i \mathbf{k} \cdot \mathbf{r} - i \omega t)$ for Eq. (6), immediately we find $\mathbf{v} = \hat{v} \exp(i \mathbf{k} \cdot \mathbf{r} - i \omega t)$ where $\mathbf{k}$, $\omega$, $\phi$, and $\hat{v} = i k \phi$, are wave vector, angular frequency of oscillations, a constant, and wave amplitude, respectively. Furthermore Eq. (6) yields,

\[
\omega^2 \hat{v} = c_0^2 \mathbf{k} (\hat{v} \mathbf{k}) + v_A^2 (\mathbf{k} \times (\hat{v} \times \mathbf{B}_0))) \times \mathbf{B}_0. \tag{7}
\]
Equation (7) simplifies as
\[ \omega^2 \dot{\psi} = c_0^2 k^2 \dot{\psi} + v_A^2 (\dot{\psi} - \dot{B}_0 (\dot{\psi} - \dot{B}_0)) k^2. \]  

We note that the wave vector (k) and wave amplitude (\dot{\psi}) are parallel (k \parallel \dot{\psi}). Considering the wave propagation parallel to the background magnetic field (k \parallel B_0), Eq. (8) reduces to the dispersion relation \[ \omega^2 = c_0^2 k^2. \]  This indicates that, in the case of an irrotational solution parallel to the magnetic field, only the acoustic wave can propagate. In the case of propagation perpendicular to the background magnetic field (k \perp B_0), Eq. (8) gives the dispersion relation \[ \omega^2 = (v_A^2 + c_0^2) k^2. \]  This is the well-known characteristic of the fast magnetoacoustic wave with the phase speed \[ c_f^2 = \frac{\omega^2}{k^2} = c_0^2 + v_A^2. \]  For the oblique propagation \[ k \cdot B_0 = k B_0 \cos \theta \]  (\theta is the angle between \( B_0 \) and \( k \)), the phase speed of the slow magnetoacoustic wave is given by
\[ v_{ph}^2 = \frac{\omega^2}{k^2} = c_0^2 - v_A^2 (\cos \theta - 1). \]

From the above analysis we see that, choosing the disturbed velocity as an irrotational velocity field \( v = \nabla \varphi \), the Alfvén waves cannot propagate along the background magnetic field (B_0).

Suppose a divergence-free solution, \( v = \nabla \times A \), with a planar wave solution \( A = \hat{A} \exp (i k \cdot r - i \omega t) \), \( \dot{\psi} = i k \times \hat{A} \) and \( \hat{A} \) is a constant vector, Eq. (6) gives,
\[ \omega^2 \dot{\psi} = v_A^2 (k \times (k \times (\dot{\psi} - \dot{B}_0))) \times \dot{B}_0. \]

We see that the velocity amplitude is perpendicular to the wave vector (k, \( \dot{\psi} = 0 \)), and only the transversal Alfvén wave with phase speed \( v_{ph} = v_A \) can propagate.

In the remainder of this section, the metrics for acoustic, magnetoacoustic, and Alfvén longitudinal waves (irrotational solutions) are derived.

### 2.2. Magnetoacoustic metric

Here, in the presence of initial material flow the magnetoacoustic metric using the magnetoacoustic wave is derived. For an irrotational flow the velocity is satisfied by a scalar field \( v = \nabla \varphi \). We consider small perturbations from equilibrium as

\[ \rho(r, t) = \rho_0(x, t) + \rho_0(x, t) \psi(r, t), \]  
\[ v(r, t) = v_0(x) \hat{x} + \nabla \varphi(r, t), \]  
\[ B(r, t) = B_0 \hat{x} + B_1(r, t), \]

where, equilibrium quantities indicated by subscript "0" are function of position \( x \) and time \( t \), \( \psi(r, t), \varphi(r, t), \) and \( B_1(r, t) \) are perturbed quantities (Safari et al. 2007; Edwin & Roberts 1982). Equilibrium quantities \( (\rho_0, v_0, B_0, \) and \( p_0) \) are satisfied by

\[ \frac{\partial \rho_0}{\partial t} + v_0 \frac{\partial \rho_0}{\partial x} + \rho_0 \frac{\partial v_0}{\partial x} = 0, \]  
\[ \rho_0 \frac{\partial v_0}{\partial x} = -c_0^2 \frac{\partial \rho_0}{\partial x}, \]  
\[ \nabla \times (v_0 \times B_0) = 0, \]  
\[ \nabla \cdot B_0 = 0, \]  
\[ p_0 = k \rho_0^\gamma. \]

Linearization of Eqs (1)-(5) (products and squares of the small perturbations are neglected) and after some mathematical manipulations, give

\[ \frac{\partial \psi}{\partial t} + \nabla \ln \rho_0 \cdot \nabla \varphi + v_0 \cdot \nabla \psi + \nabla^2 \varphi = 0, \]
\[ \rho_0 \left( \frac{\partial \varphi}{\partial t} + v_0 \cdot \nabla \varphi + c_0^2 \psi \right) = \frac{1}{\mu} (\nabla \times B_1) \times B_0, \]
\[ \frac{\partial B_1}{\partial t} = \nabla \times (v_0 \times B_1) + \nabla \times (v_1 \times B_0), \]
\[ \nabla \cdot B_1 = 0, \]
As we explained in the previous section, by choosing $\mathbf{v} = \nabla \varphi$ the parallel propagation of Alfvén waves along the magnetic field ($\mathbf{B}_0 = B_0 \hat{x}$) is absent. Therefore, the first term on the right-hand side of Eq. (21) is set to zero, $\nabla \times (\mathbf{v}_0 \times \mathbf{B}_1) = 0$. A combination of this assumption and the solenoidal condition for magnetic field ($\nabla \cdot \mathbf{B}_0 = 0$), gives $\frac{\partial B_{lx}}{\partial t} = 0$. Thus, the propagation of Alfvén waves along the background magnetic field is removed from our analysis. We assume the magnetic field directions ($\hat{x}$) in which, the magnetoacoustic black hole, hereafter, we focus our analysis in all directions except the magnetic field direction.

The $x$ and $z$ component of Eq. (20) are

$$\frac{\partial}{\partial x} \left( \frac{\partial \varphi}{\partial t} + \mathbf{v}_0 \cdot \nabla \varphi + c_0^2 \psi \right) = 0. \tag{23}$$

$$\rho_0 \frac{\partial}{\partial z} \left( \frac{\partial \varphi}{\partial t} + \mathbf{v}_0 \cdot \nabla \varphi + c_0^2 \psi \right) = \frac{B_0}{\mu} \left( \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right). \tag{24}$$

Hereafter, we focus our analysis on the derivation of acoustic waves with propagation along the background magnetic field directions ($\hat{x}$) and second, magnetoacoustic wave propagation in all directions except the magnetic field direction.

First, combining Eqs (19) and (23) gives

$$- \frac{\partial}{\partial t} \left( \frac{\partial \varphi}{\partial t} + \mathbf{v}_0 \cdot \nabla \varphi \right) + \frac{\partial}{\partial x} \left( c_0^2 \frac{\partial \varphi}{\partial x} - \mathbf{v}_0 \left( \frac{\partial \varphi}{\partial t} + \mathbf{v}_0 \frac{\partial \varphi}{\partial x} \right) \right) + c_0^2 \nabla^2 \varphi = 0. \tag{25}$$

Equation (25) is the well-known Klein-Gordon equation for acoustic waves. Eliminating $\frac{\partial \psi}{\partial x}$ between Eq. (19) and Eq. (23) one obtains

$$\mathbf{v}_0 \frac{\partial}{\partial x} \left( \frac{\partial \varphi}{\partial t} + \mathbf{v}_0 \cdot \nabla \varphi \right) + \mathbf{v}_0 \frac{\partial c_0^2}{\partial x}$$

$$- c_0^2 \left( \frac{\partial \psi}{\partial t} + \nabla \ln \rho_0 \cdot \nabla \varphi + \nabla^2 \varphi \right) = 0. \tag{26}$$

Briefly, by choosing the irrotational solution for the velocity disturbance, the transversal Alfvén wave is absent, and the propagation of the longitudinal Alfvén wave parallel with the background magnetic field is also present as expected. Our analysis shows that, along the magnetic field only the acoustic wave can propagate. Because our goal is to analyse the magnetoacoustic black hole, hereafter, we focus our analysis in all directions except the magnetic field direction.

Second, by differentiating Eq. (26) with respect to $z$ and substituting $\frac{\partial \psi}{\partial z}$ from Eq. (24) one finds

$$\frac{\partial}{\partial z} \left( \frac{1}{c_0^2} \frac{\partial^2 \varphi}{\partial x \partial t} \right) = \frac{1}{\mathbf{v}_0} \nabla^2 \varphi \frac{\partial \varphi}{\partial z} + \frac{1}{\mathbf{v}_0} \nabla \ln \rho_0 \cdot \nabla \frac{\partial \varphi}{\partial z}$$

$$+ \frac{1}{\mathbf{v}_0} \frac{\partial}{\partial t} \left( \frac{B_0}{\mu \rho_0 c_0^2} (\nabla \times \mathbf{B}_1)_y - \frac{1}{c_0^2} \frac{\partial dx}{\partial t} \right)$$

$$+ \frac{B_0}{\mu \rho_0 c_0^2} (\nabla \times \mathbf{B}_1)_y + \frac{\partial}{\partial z} \left( \frac{1}{c_0^2} \frac{\partial \varphi}{\partial x} \right). \tag{27}$$

in which, $d/ dt = \partial / \partial t + \mathbf{v}_0 \cdot \nabla$. Substituting $- \frac{\partial}{\partial t} (\nabla \times \mathbf{B}_1)_y = B_0 \nabla^2 \frac{\partial \varphi}{\partial x}$ from Eq. (21), into Eq. (27) and differentiating with respect to $t$ using $\frac{\partial}{\partial t} (\nabla \times \mathbf{B}_1)_y = -B_0 \nabla^2 \frac{\partial \varphi}{\partial z}$ we derived a single equation for velocity potential ($\varphi$)

$$\frac{\partial}{\partial t} \left( \frac{1}{N} \left( \frac{1}{c_0^2} \frac{\partial^2 \varphi}{\partial x \partial z} - \frac{1}{c_0^2} \frac{\partial^2 \varphi}{\partial x \partial t} - \frac{\nabla^2 \varphi}{\mathbf{v}_0} - \frac{\nabla \ln \rho_0 \cdot \nabla \varphi}{\mathbf{v}_0} \right) \right)$$

$$+ \frac{\partial}{\partial t} \left( \frac{1}{N} \left( \frac{1}{\mathbf{v}_0} \frac{\partial}{\partial t} \left( \frac{d \mathbf{v}_0}{c_0^2} \right) - \frac{\nabla^2 \varphi}{\mathbf{v}_0} \right) \right) = B_0 \nabla^2 \varphi. \tag{28}$$

where,

$$N = \frac{B_0}{\mu \rho_0 c_0^2} \frac{\partial}{\partial t} \left( \frac{1}{\rho_0 c_0^2} \right) - \frac{B_0}{\mu \rho_0 c_0^2} \frac{\partial c_0^2}{\partial x}$$

$$= \frac{\gamma}{B_0 c_0^2} \left( \frac{\gamma}{\mathbf{v}_0} + \frac{\mathbf{v}_0}{c_0^2} \right) \frac{d \mathbf{v}_0}{d x}.$$
For a flow having a slight change in the speed \(dv_0/dx \ll 1\), for high frequency waves (short period \(\Delta t \ll 1\) the term \(\Delta t(dv_0/dx)\) becomes too small. In this case, the right hand side term \(B_0 \nabla^2 \phi\) of Eq. (28) can be negligible compared to the last term \(\frac{\partial}{\partial t} \left( \frac{v_0^2}{Nc_0^2} \nabla^2 \phi \right)\) of the left hand side. This leaves an equation for \(\phi\)

\[
- \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial t} + v_0 \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial x} \left( c_0^2 \frac{\partial \phi}{\partial x} - v_0 \left( \frac{\partial \phi}{\partial t} + v_0 \frac{\partial \phi}{\partial x} \right) \right) + v_A^2 \frac{\partial^2 \phi}{\partial x^2} + (v_A + c_0^2) \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) = 0.
\]

Equation (29) describes the propagation of acoustic, Alfvén, and magnetoacoustic waves in laboratory and astrophysical plasma. This equation is in the form of the well-known Klein-Gordon equation. As expected, in the case of unmagnetized fluid \((B_0 = 0)\), Eq. (29) reduces to the acoustic wave equation for velocity potential. Usually, a d’Alembertian equation (for a minimally coupled massless scalar field) of motion was derived for velocity potential in a barotropic, inviscid, and rotational free flow (Barceló et al. 2011).

Equation (29) can be reformulate in a tensorial form

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} \partial_{\nu} \phi \right) = 0,
\]

where, \(g^{\mu\nu}\) and \(g\) are inverse metric tensor and its determinant, \(\mu\) and \(\nu\) runs from 0 (indicates the time coordinate) to 3 (denotes the spatial coordinate). The magnetoacoustic inverse metric tensor \(g^{\mu\nu}\) and metric tensor \(g_{\mu\nu}\) are obtained as

\[
g^{\mu\nu} = \frac{1}{\rho_0 c_f^2} \begin{pmatrix}
-1 & -v_0 & 0 & 0 \\
-v_0 c_f^2 - v_0^2 & 0 & 0 & 0 \\
0 & 0 & c_f^2 & 0 \\
0 & 0 & 0 & c_f^2
\end{pmatrix}, \quad g_{\mu\nu} = -\rho_0 c_f \begin{pmatrix}
c_f^2 - v_0^2 & v_0 & 0 & 0 \\
v_0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix},
\]

where, \(c_f^2 = v_A^2 + c_0^2\). Using metric tensor, Eq. (31), the magnetoacoustic interval can be defined as

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\rho_0 c_f \left( (c_f^2 - v_0^2) dt^2 + 2v_0 dt dx - dx^2 - dy^2 - dz^2 \right).
\]

Inserting the specific time interval \(d\tau = dt + \frac{v_0 dx}{c_f^2 - v_0^2}\) into Eq. (32), one obtains

\[
ds^2 = c_f \rho_0 \left( - \left( 1 - \frac{v_0^2}{c_f^2} \right) c_f^2 d\tau^2 + \frac{dx^2}{\left( 1 - \frac{v_0^2}{c_f^2} \right)} + dy^2 + dz^2 \right).
\]

In the following section, the properties of acoustic, magnetoacoustic, and Alfvénic BHs are investigated.

3. ACOUSTIC, MAGNETOACOUSTIC, AND AFLVÉNIC BLACK HOLES

3.1. Acoustic black hole

The theory of gravitational BHs has been developed into the supersonic flow by Unruh (Sakagami & Ohashi 2002). For a moving fluid medium at the horizon where speed of medium is closed to propagation speed of the acoustic signals "then nothing can fight its way back upstream and signals are trapped" (Novello et al. 2002). In the case of unmagnetised gas\((B_0 = 0)\), Eq. (28) reduces to the Klein-Gordon equation, for propagation of acoustic waves in the presence of material flow. Using the resultant equation the acoustic metric can be derived (Sakagami & Ohashi 2002).

The acoustic metric can be obtained by setting \(v_A = 0\) in Eq. (31)

\[
g^{\mu\nu} = -\rho_0 c_0 \begin{pmatrix}
c_0^2 - v_0^2 & v_0 & 0 & 0 \\
v_0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}.
\]
Equivalently, the acoustic interval can be expressed as
\[ ds^2 = c_0 \rho_0 \left(- \left(1 - \frac{v_0^2}{c_0^2} \right) c_f^2 dt^2 + \frac{dx^2}{1 - \frac{v_0^2}{c_0^2}} + dy^2 + dz^2 \right). \] (35)

Combination of continuity and momentum equations (Eqs 14 and 15) in stationary state, the relation between cross section \( S \) and velocity \( v \) is given by
\[ \left(\frac{v_0^2}{c_0^2} - 1\right) \frac{dv_0}{v_0} = \frac{dS}{S}. \] (36)

This relation shows that for \( (dS < 0) \) a subsonic flow \( (v_0 < c_0) \) will be accelerated and a supersonic flow \( (v_0 > c_0) \) will be decelerated. If the nozzle is sufficiently narrow and with a slightly variable cross section the speed of flow exceeds to sound speed at the throat (sonic horizon). This shows the acoustic interval Eq. (35) interpreted acoustic BH which has a sonic horizon and trapped phonon in the surface gravity of acoustic BH. This means that, when the acoustic waves cross from upstream to downstream, the acoustic wave quanta (phonons) are captured in the horizon of the BH where they emit Hawking radiation made by phonons. In this regard, in many papers a Laval nozzle setup Figure 1 has been proposed to discuss the above mentioned acoustic BH. This setup uses, an axisymmetric sufficiently thin tube with slightly decreasing cross section \( (S(x)) \) that reaches its minimum cross section at the throat and then slightly increases that. An initial material flow \( (v_0 = v_0(x) \hat{x}) \) along the tube axis is considered.

3.2. Magnetoad acoustic black hole

The magnetoadmetric Eq. (33) is singular at the magnetoad acoustic point, where \( v_0 = c_f \), determines a magnetoad acoustic horizon. The speed of super magnetoad acoustic plasma flow reduces to local propagation speed of magnetoad acoustic wave at horizon; then signal of magnetoad acoustic wave is trapped and therefore it can be called magnetoad acoustic BH. Similar to the HR emitted from acoustic and gravitational BHs, the magnetoad acoustic BH also should emit HR. In this regard, we propose a setup Figure 2 to discuss the above mentioned BH. The setup consists of an axisymmetric sufficiently thin tube with slightly variable cross section \( (S(x)) \), a uniform force free magnetic field \( B_0 = B_0 \hat{x} \) and an initial material flow \( (v_0 = v_0(x) \hat{x}) \) along the tube axis Figure 2.

A similar treatment of sub and supersonic flow in tube configuration can be explained for sub and super-magnetoad acoustic flow based on Eq. (36). In other words, the super-magnetoad acoustic flow \( (v_0 > c_f) \) will be decelerated along the tube where its cross section slightly decreases. It is possible to release a super-magnetoad acoustic flow in the tube, which its speed tends to the speed of magnetoad acoustic wave \( (v_0 = c_f) \) at the horizon. The boundary between sub-magnetoad acoustic and super-magnetoad acoustic flow could be called the magnetoad acoustic horizon, analogous to the sonic horizon in acoustic BHs. Phononic version of HR is an inevitable result of trapping acoustic wave at the acoustic horizon (Steinhauer 2014; Schley et al. 2013). Under a likely scenario, the magnetoad acoustic wave cannot escape from the magnetoad acoustic horizon, therefore should emit HR made of magnephonon. Indeed, a magnephonon will be a quantum for magnetoad acoustic wave, analogous to the phonon which is a quantum for acoustic wave.

3.3. Alfvénic black hole

In the limit of \( c_0 << v_A \) (zero \( \beta \) plasma condition), the magnetoad acoustic wave (Eq. 29) reduces to an Alfvén wave in the presence of initial material flow. The resultant Alfvén wave equation is in the form of Klein-Gordon equation. Immediately, the Alfvénic metric can be derived from magnetoad acoustic (Eqs 31 and 33) by setting \( c_0 \) tends to 0,

\[ g^A_{\mu\nu} = -\rho_0 v_A \begin{pmatrix} v_A^2 - v_0^2 & v_0 & 0 & 0 \\ v_0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \] (37)

Equivalent Alfvénic interval is given by
\[ ds^2 = v_A \rho_0 \left(- \left(1 - \frac{v_0^2}{v_A^2} \right) v_A^2 dt^2 + \frac{dx^2}{1 - \frac{v_0^2}{v_A^2}} + dy^2 + dz^2 \right). \] (38)
The Alfvénic metric (Eq. 38) is singular at the location of Alfvénic point where \( v_0 = v_A \). This singular behaviour of Alfvénic metric leads to an Alfvénic BH. We propose a setups that is illustrated in Figure 3, the condition for occurrence of an Alfvénic BH can be discussed. For compressional (longitudinal) Alfvén wave, the plasma density is nearly constant. As a result of mass continuity, the flux, \( S(x)v_0(x) \), is constant in the tube cross section. Consider a super-Alfvénic flow in a tube with increasing cross section Figure 3, speed of the flow decreases along the tube axis and reaches to Alfvén velocity \( v_0 = v_A \) at Alfvénic point and then transformed to sub-Alfvénic flow. Therefore, we call Alfvénic horizon to be the interface between super-Alfvénic flow and sub-Alfvénic flow. In horizon of the Alfvénic BH, the Alfvén wave is trapped and it is expected to be radiated by Alphonon. Alphonon is introduced as a quantum particle for Alfvén wave packet.

4. HAWKING TEMPERATURE

Hawking temperature is an important characteristic of BHs. Unruh showed that fluid flows mimic BHs. Hawking temperature \( (T_H) \) for acoustic BH was obtained

\[
T_{\text{H-acoustic}} = \frac{\hbar}{2\pi kc_s} \left. \frac{d(v_0^2 - c_s^2)}{dx} \right|_{\text{horizon}},
\]  

(39)

where, \( \hbar = h/2\pi \), \( h \) is the plank constant and \( k \) is the Boltzmann constant. Since the Hawking temperature is independent of metric conformal factor. It will therefore be as following for the magnetoacoustic BH,

\[
T_{\text{H-magnetoacoustic}} = \frac{\hbar}{2\pi kc_f} \left. \frac{d(v_0^2 - c_f^2)}{dx} \right|_{\text{horizon}}.
\]  

(40)

In the limit of \( v_A \) tends to zero, Eq. (40) then reduces to Hawking temperature for acoustic BH. Although, in the zero \( \beta \) plasma condition the above mentioned equation can describe the Hawking temperature for Alfvénic BH,

\[
T_{\text{H-Alfvénic}} = \frac{\hbar}{2\pi kv_A} \left. \frac{d(v_0^2 - v_A^2)}{dx} \right|_{\text{horizon}}.
\]  

(41)

At the horizon where the tube cross section radius is equal to \( R \), Eq. (40) can be simplified as

\[
T_{\text{H-magnetoacoustic}} \approx \frac{\hbar c_f}{2\pi k R}.
\]  

(42)
Figure 2. Sketch of a tube with a slightly variable cross section embedded in a slightly uniform magnetic field. Super-magnetoacoustic flow, $v_0 > c_f$, sub-magnetoacoustic flow, $v_0 < c_f$, and magnetoacoustic wave trapped in the horizon are presented.

Figure 3. A schematic presentation of a Alfvénic black hole. Sketch of a tube with a slightly variable cross section embedded in a uniform magnetic field. The speed of super Alfvénic flow reduces to local propagation speed of Alfvénic wave at horizon; then Alfvénic signals are trapped.

where, the term $\frac{1}{c_f} \frac{dx}{dt} (c_f - v_0)$ is approximately equal to $\frac{1}{\alpha}$. For a plasma with a ratio of $\chi = c_0/v_A$ in which $\chi$ is a positive number, Eq. (42) gives

$$T_{H\text{-magnetoacoustic}} \approx 2.66 \times 10^4 (1 + \chi^2)^{0.5} \frac{B_0}{R \sqrt{n}}.$$  

where, $n$ is the number density of plasma and all units are in SI. For a typical plasma with magnetic field strength $B_0=1$ Tesla, number density $n = 10^{18} m^{-3}$, $R = 1$ mm, and $\chi^2 \ll 1$, the Hawking temperature is estimated about 0.0266 K. For a typical natural fluid in a Lavel nozzle experiment, the Hawking temperature was estimated about $10^{-6}$ K (Novello et al. 2002).

5. CONCLUSION

In this study, the behavior of MHD waves (Alfvén and magnetoacoustic) are are treated in the Helholtz decomposition frame-work, in the case of irrotational flow, we introduced the magnetoacoustic and Alfvénic analogue BHs. In the horizon of magnetoacoustic and Alfvénic BHs, the magnetoacoustic and Alfvén waves are trapped, respectively, and
should emit magnephonons and Alphonons version of HR at Hawking temperature. The next logical step is to investigate the physical properties of both magnephonon and Alphonon quasi-particles based on quantum approach. As stated in the literature, for acoustic BH in a natural fluid, the Hawking temperature is a function of sound speed and geometry of nozzle setup at the horizon. However, Hawking temperature for magnetoacoustic BH is related to the sound speed with additional positive terms that depends on the magnetic field and density. Magnephonons and Alphonons are particles (quanta) corresponding to magnetoacoustic and Alfvén waves, respectively. The idea for definition of acoustic, magnetoacoustic, and Alfvénic BHs can be applied to theoretical and/or experimental prediction of new non-gravitational BHs. Perhaps, the study of new BHs could help us observe HR.

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