Polignac’s Conjecture with New Prime Number Theorem

YinYue Sha*
Dongling Engineering Center, Ningbo Institute of Technology, Zhejiang University, China

Abstract
There are infinitely many pairs of consecutive primes which differ by even number \( En \). Let \( P(\text{N}, En) \) be the number of Polignac Prime Pairs (which difference by the even integer \( En \)) less than an integer \((\text{N}+En)\), \( Pei \) be taken over the odd prime divisors of the even integer \( En \) less than \( \sqrt{\text{N}+En} \), \( Pni \) be taken over the odd primes less than \( \sqrt{\text{N}+En} \) except \( Pei \), \( Pi \) be taken over the odd primes less than \( \sqrt{\text{N}+En} \), then exists the formulas as follows:

\[
P(\text{N}, En) \geq \text{INT} \left\lfloor N \times (1-1/2) \times \prod (1-1/Pei) \times \prod (1-2/Pni) \right\rfloor - 1
\]

\[
\geq \text{INT} \left\lfloor Ctwin \times Ke(\text{N}) \times 2N/(\ln (\text{N}+En))^2 \right\rfloor - 1
\]

\[
\prod (Pi/(Pi-2)/(Pi-1)^2) \geq Ctwin=0.6601618158...
\]

\[
Ke(\text{N})\times\left\lfloor (1-1/Pei)/(1-2/Pei)\right\rfloor \times \left\lfloor (Pei-1)/(Pei-2)\right\rfloor \geq 1
\]

where -1 is except the natural integer 1.

Keywords: Twin prime, Polignac prime, Bilateral sieve method

Introduction
In number theory, Polignac’s conjecture was made by Alphonse de Polignac in 1849 and states: For any positive even number \( En \), there are infinitely many prime gaps of size \( En \). In other words: There are infinitely many cases of two consecutive prime numbers with difference \( En \) [1].

The conjecture has not yet been proven or disproven for a given value of \( En \). In 2013 an important breakthrough was made by Zhang Yitang who proved that there are infinitely many prime gaps of size \( En \) for some value of \( En<70,000,000 \) [2].

For \( En=6 \), it says there are infinitely many primes \((p, p + 6)\). For \( En=4 \), it says there are infinitely many cousin primes \((p, p + 4)\). For \( En=2 \), it is the twin prime conjecture that there are infinitely many twin primes \((p, p + 2)\) as shown in Figure 1. For \( En=0 \), it is the new prime theorem.

The Polignac Prime of Even Integer
For an any even integer \( En \) there exists a prime \( P \) for which the Polignac number \( Q=En+P \) is also prime. The Polignac Prime pairs shall be denoted by the representation \( En=Q=(\text{En}+P) - P \), where \( P \) and \( Q \) are primes and prime \( P[\text{P} \leq Q] \) is a Polignac prime of even integer \( En \). Looking at the Polignac partition a different way, we can look at the number of distinct representations (or Polignac primes) that exist for \( En \).

For example, as noted at the beginning of this discussion:

\[
\begin{align*}
2 &= 05 - 03 = (2+03) - 03; 2 &= 07 - 05 = (2+05) - 05; \\
2 &= 13 - 11 = (2+11) - 11; 2 &= 19 - 17 = (2+17) - 17; \\
2 &= 31 - 29 = (2+29) - 29; 2 &= 43 - 41 = (2+41) - 41; \\
2 &= 61 - 59 = (2+59) - 59; 2 &= 73 - 71 = (2+71) - 71;
\end{align*}
\]

where 3, 5, 11, 17, 29, 41, 59 and 71 are Polignac primes of even integer 2.

\[
\begin{align*}
4 &= 07 - 03 = (4+03) - 03; 4 &= 11 - 07 = (4+07) - 07; \\
4 &= 17 - 13 = (4+13) - 13; 4 &= 23 - 19 = (4+19) - 19;
\end{align*}
\]

4=41 - 37=(4+37) - 37; 4=47 - 43=(4+43) - 43;

4=71 - 67=(4+67) - 67; 4=83 - 79=(4+79) - 79;

where 3, 7, 13, 19, 37, 43, 67 and 79 are Polignac primes of even integer 4.

\[
\begin{align*}
6 &= 11 - 05 = (6+05) - 05; 6 &= 13 - 07 = (6+07) - 07; \\
6 &= 17 - 11 = (6+11) - 11; 6 &= 19 - 13 = (6+13) - 13; \\
6 &= 23 - 17 = (6+17) - 17; 6 &= 29 - 23 = (6+23) - 23; \\
6 &= 37 - 31 = (6+31) - 31; 6 &= 43 - 37 = (6+37) - 37; \\
6 &= 47 - 41 = (6+41) - 41; 6 &= 53 - 47 = (6+47) - 47; \\
6 &= 59 - 53 = (6+53) - 53; 6 &= 67 - 61 = (6+61) - 61; \\
6 &= 73 - 67 = (6+67) - 67; 6 &= 79 - 73 = (6+73) - 73; \\
6 &= 89 - 83 = (6+83) - 83; 6 &= 103 - 97 = (6+97) - 97;
\end{align*}
\]

where 5, 7, 11, 13, 17, 23, 31, 37, 41, 47, 53, 61, 67, 73, 83 and 97 are Polignac primes of even integer 6.

It shows that generally the number of distinct representations (or Polignac primes) increases with increasing \( N \).

The Sieve Method about the Polignac Primes
Let \( En \) is an any even integer, \( Ci \) is a positive integer more not large than \( N \), then exists the formulas as follows:

\[
P(\text{N}, En) \geq \text{INT} \left\lfloor N \times (1-1/2) \times \prod (1-1/Pei) \times \prod (1-2/Pni) \right\rfloor - 1
\]

\[
\geq \text{INT} \left\lfloor Ctwin \times Ke(\text{N}) \times 2N/(\ln (\text{N}+En))^2 \right\rfloor - 1
\]

\[
\prod (Pi/(Pi-2)/(Pi-1)^2) \geq Ctwin=0.6601618158...
\]

\[
Ke(\text{N})\times\left\lfloor (1-1/Pei)/(1-2/Pei)\right\rfloor \times \left\lfloor (Pei-1)/(Pei-2)\right\rfloor \geq 1
\]

where -1 is except the natural integer 1.

*Corresponding author: YinYue Sha, Dongling Engineering Center, Ningbo Institute of Technology, Zhejiang University, China, Tel: 571 8517 2244; E-mail: shayinyue@qq.com

Received July 04, 2016; Accepted November 08, 2016; Published November 11, 2016

Citation: Sha YY (2016) Polignac’s Conjecture with New Prime Number Theorem. J Phys Math 7: 201. doi: 10.4172/2090-0902.1000201

Copyright: © 2016 Sha YY. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.
where the prime $P_0$ is a Polignac prime of even integer $E_n$.

If both $P_0$ and $E_n + P_0$ are primes at the same time, $P_0$ and $E_n + P_0$ can not be divided by all primes more not large than $\sqrt{N + E_n}$, then sieves out the positive integer $C_i$; If both $N + E_n$.

The Bilateral Sieve Method of Even Prime 2

Let $E_n$ be an any even integer, then the formula as follows:

$$E_n = (E_n + C_i) - C_i$$

where $C_i$ and $E_n + C_i$ are two positive integers more not large than $N + E_n$.

If $C_i$ and $E_n + C_i$ any one can be divided by the prime anyone more not large than $\sqrt{N + E_n}$, then sieves out the positive integer $C_i$; If both Po and $E_n + Po$ are primes at the same time, where the prime Po is a Polignac prime of even integer En.

The Total of Representations of Even Integer

Let $E_n$ be an any even integer, then exists the formula as follows:

$$E_n = (E_n + C_i) - C_i$$

where $C_i$ is the natural integer less than $N$.

In terms of the above formula we can obtain the array as follows:

$$(E_n + 1, 1), (E_n + 2, 2), (E_n + 3, 3), (E_n + 4, 4), (E_n + 5, 5), \ldots, (E_n + N, N)$$

From the above arrangement we can obtain the formula about the total of Polignac numbers of even integer En as follows:

$$C_i(N, E_n) = N = \text{Total of integers } C_i \text{ more not large than } N$$

(3)

The Bilateral Sieve Method of Even Prime 2

It is known that the number 2 is an even prime, and above arrangement from $(E_n + 1, 1)$ to $(E_n + N, N)$ can be arranged to the form as follows:

$$(E_n + 1, 1), (E_n + 2, 2), (E_n + 3, 3), \ldots, (E_n + N - X: X < 2, N - X: X < 2)$$

$$(E_n + 2, 2), (E_n + 4, 4), (E_n + 6, 6), \ldots, (E_n + N - X: X < 2, N - X: X < 2)$$

From the above arrangement we can known that: Because the even integer En can be divided by the even prime 2, therefore, both $C_i$ and $E_n + C_i$ an only one can be divided or both the $C_i$ and $E_n + C_i$ can not be divided by the odd prime 3.

The number of integers $C_i$ that the $C_i$ and $E_n + C_i$ anyone can be divided by the odd prime 3 is $INT (N \times (1/3))$.

The number of integers $C_i$ that both $C_i$ and $E_n + C_i$ can not be divided by the odd prime 3 is $N - INT (N \times (1/3)) = INT (N \times (1 - 1/3))$.

The density of integers $C_i$ that both $C_i$ and $E_n + C_i$ can not be divided by the odd prime 3 is $INT (N \times (2/3))$.

The Bilateral Sieve Method of Odd Prime 5

It is known that the number 5 is an odd prime, and above arrangement from $(E_n + 1, 1)$ to $(E_n + N, N)$ can be arranged to the form as follows:

$$(E_n + 1, 1), (E_n + 4, 4), (E_n + 7, 7), \ldots, (E_n + N - X: X < 3, N - X: X < 3)$$

$$(E_n + 2, 2), (E_n + 5, 5), (E_n + 8, 8), \ldots, (E_n + N - X: X < 3, N - X: X < 3)$$

$$(E_n + 3, 3), (E_n + 6, 6), (E_n + 9, 9), \ldots, (E_n + N - X: X < 3, N - X: X < 3)$$

From the above arrangement we can known that:

If the even integer En can be divided by odd prime 3, then both the $C_i$ and $E_n + C_i$ can be or can not be divided by odd prime 3 at the same time.

The number of integers $C_i$ that the $C_i$ and $E_n + C_i$ can not be divided by odd prime 3 is $INT (N \times (1/3))$.

The number of integers $C_i$ that both $C_i$ and $E_n + C_i$ can not be divided by odd prime 3 is $N - INT (N \times (1/3)) = INT (N \times (1 - 1/3))$.

The number of integers $C_i$ that the $C_i$ and $E_n + C_i$ can not be divided by odd prime 3 is $INT (N \times (2/3))$.

The density of integers $C_i$ that both $C_i$ and $E_n + C_i$ can not be divided by odd prime 3 is $INT (N \times (2/3)) \times INT (N \times (1 - 1/3))$.

The number of integers $C_i$ that both $C_i$ and $E_n + C_i$ can not be divided by odd prime 3 is $N - INT (N \times (2/3)) = INT (N \times (1 - 1/3))$.

The number of integers $C_i$ that both $C_i$ and $E_n + C_i$ can not be divided by odd prime 3 is $INT (N \times (2/3)) \times INT (N \times (1 - 1/3))$.
\( (En+4, 4), (En+9, 09), (En+14, 14), \ldots, (En+N-X<X<5, N-X<X<5), (En+5, 5), (En+10, 10), (En+15, 15), \ldots, (En+N-X<X<5, N-X<X<5). \)

From the above arrangement we can know that:

If the even integer \( En \) can be divided by odd prime 5, then both the \( Ci \) and \( En+Ci \) can be or can not be divided by odd prime 5 at the same time.

The number of integers \( Ci \) that the \( Ci \) and \( En+Ci \) anyone can be divided by odd prime 5 is INT \((N \times (1/5))\).

The number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by odd prime 5 is N-INT \((N \times (1/5))\)=INT \([N-N \times (1/5)]\)=INT\([N \times (1-1/5)]\).

The density of integers \( Ci \) that both \( Ci \) and \( En+Ci \) can not be divided by odd prime 5 (the ratio of the number of integers \( Ci \) that both \( Ci \) and \( En+Ci \) can not be divided by odd prime 5 to the total of integers \( Ci \) more not large than \( N \)) as follows:

\[
\text{Sei}(N, En, 5) = \text{INT}(N \times (1/5)), \text{Cei}(N, En, 5) = N-\text{Sei}(N, En, 5) \quad (10)
\]

\[
\text{Dei}(N, En, 5) = Cei(N, En, 5)/(N) = \text{INT}(N \times (1-1/5))/N \quad (11)
\]

If the even integer \( En \) can not be divided by the odd prime 5, then both \( Ci \) and \( En+Ci \) can not be divided by odd prime 5 at the same time, that is the \( Ci \) and \( En+Ci \) only one can be divided or both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime 5.

The number of integers \( Ci \) that the \( Ci \) and \( En+Ci \) anyone can be divided by the odd prime 5 is \((N \times (2/5))\).

The number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by odd prime 5 (or the ratio of the number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime 5 to the total of integers \( Ci \) more not large than \( N \)) as follows:

\[
\text{Sni}(N, En, 5) = \text{INT}(N \times 2/5)), \text{Cni}(N, En, 5) = N-\text{Sni}(N, En, 5) \quad (12)
\]

\[
\text{Dni}(N, En, 5) = Cni(N, En, 5)/(N) = \text{INT}(N \times (1-2/5))/N \quad (13)
\]

The Sieve Function of Bilateral Sieve Method

Let \( En \) is an even integer, then exists the formula as follows:

\( En=(En + Ci) \cdot Ci \quad (14) \)

where \( Ci \) is the natural integer less than \( N \).

In terms of the above formula we can obtain the array as follows:

\( (En+1, 1), (En+2, 2), (En+3, 3), (En+4, 4), (En+5, 5), \ldots, (En+N, N). \)

Let \( Pi \) be an odd prime less than \( \sqrt{(N+En)} \), then the above arrangement can be arranged to the form as follows:

\( (En+1, 1), (En+Pi+1, Pi+1), \ldots, (En+N-X<X<Pi, N-X<X<Pi), (En+2, 2), (En+Pi+2, Pi+2), \ldots, (En+N-X<X<Pi, N-X<X<Pi), (En+3, 3), (En+Pi+3, Pi+3), \ldots, (En+N-X<X<Pi, N-X<X<Pi), (En+Pi, Pi), (En+2Pi, 2Pi), \ldots, (En+N-X<X<Pi, N-X<X<Pi). \)

If the even integer \( En \) can be divided by the odd prime \( Pei \), then both the \( Ci \) and \( En+Ci \) can be or can not be divided by the odd prime \( Pei \) at the same time.

The number of integers \( Ci \) that the \( Ci \) and \( En+Ci \) anyone can be divided by the odd prime \( Pei \) is \( \text{INT}(N \times (1/Pei)) \).

The number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by odd prime \( Pei \) is \( N-\text{INT}(N \times (1/Pei))=\text{INT}[N \times (1-1/Pei)] \).

The density of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime \( Pei \) (or the ratio of the number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime \( Pei \) to the total of integers \( Ci \) more not large than \( N \)) as follows:

\[
\text{Sei}(N, En, Pei) = \text{INT}(N \times (1/Pei)), \text{Cei}(N, En, Pei) = N-\text{Sei}(N, En, Pei) \quad (15)
\]

\[
\text{Dei}(N, En, Pei) = \text{Cei}(N, En, Pei)/(N) = \text{INT}[N \times (1-1/Pei)]/N \quad (16)
\]

If the even integer \( En \) can not be divided by the odd prime \( Pni \), then both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime \( Pni \) at the same time, that is the \( Ci \) and \( En+Ci \) only one can be divided or both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime \( Pni \).

The number of integers \( Ci \) that the \( Ci \) and \( En+Ci \) anyone can be divided by the odd prime \( Pni \) is \( \text{INT}(N \times (2/Pni)) \).

The number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime \( Pni \) (or the ratio of the number of integers \( Ci \) that both the \( Ci \) and \( En+Ci \) can not be divided by the odd prime \( Pni \) to the total of integers \( Ci \) more not large than \( N \)) as follows:

\[
\text{Sni}(N, En, Pni) = \text{INT}(N \times (2/Pni)), \text{Cni}(N, En, Pni) = N-\text{Sni}(N, En, Pni) \quad (17)
\]

\[
\text{Dni}(N, En, Pni) = \text{Cni}(N, En, Pni)/(N) = \text{INT}(N \times (1-2/Pni))/N \quad (18)
\]

Let \( Po(N, En) \) be the number of Polignac Prime Pairs (which difference by the even integer \( En \)) less than an integer \( (N+En) \), \( Pei \) be taken over the odd prime divisors of the even integer \( En \) less than \( \sqrt{(N+En)} \), \( Pni \) be taken over the odd prime divisors of the even integer \( En \) less than an integer \( (N+En) \), \( Pei \) be taken over the odd primes less than \( \sqrt{(N+En)} \), except \( Pei \), \( Pei \) be taken over the odd primes less than \( \sqrt{(N+En)} \), then exists the formulas as follows:

\[
Po(N, En) \geq \text{INT}[N \times Di(N, En, 2) \times \prod Dei(N, En, Pei) \times \prod Dni(N, En, Pni)] - 1 \quad (19)
\]

where -1 is except the natural integer 1.

The Polignac Prime Theorem

From above we can obtain that:

Let \( Po(N, En) \) be the number of Polignac Prime Pairs (which difference by the even integer \( En \)) less than an integer \( (N+En) \), \( Pei \) be taken over the odd prime divisors of the even integer \( En \) less than \( \sqrt{(N+En)} \), \( Pni \) be taken over the odd primes less than \( \sqrt{(N+En)} \) except \( Pei \), \( Pei \) be taken over the odd primes less than \( \sqrt{(N+En)} \), then exists the formulas as follows:

\[
Po(N, En) \geq \text{INT}[N \times Di(N, En, 2) \times \prod Dei(N, En, Pei) \times \prod Dni(N, En, Pni)] - 1 \quad (20)
\]

Apply the Prime Number Theorem as follows:
Let $\Pi(N)$ be the number of primes less than or equal to $N$, $\Pi (3 \leq P \leq P_{2m})$ be taken over the odd primes less than $N$, then exists the formulas as follows:

\[
\Pi(N) = \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (21)
\]

\[
\geq \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (22)
\]

\[
\geq C_{\text{tw}} = 0.6601618158 \ldots \quad (23)
\]

\[
K_{C}(N) = \int \left( 1 - \frac{1}{P} \right) dN = \int \left( 1 - \frac{1}{P} \right) dN = 0.6601618158 \ldots \quad (24)
\]

From the above and the formula (20) we can obtain the formula as follows:

\[
\Pi(N) = \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (25)
\]

\[
\geq \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (26)
\]

\[
\geq C_{\text{tw}} = 0.6601618158 \ldots \quad (27)
\]

When the number $N \to \infty$, we can obtain the formula as follows:

\[
\Pi(N) = \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (28)
\]

\[
\geq \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (29)
\]

The above formula expresses that there are infinitely many pairs of Polignac primes which differ by every even number $N$.

When the $N=2$, then there are infinitely many twin primes.

**Every Even Integer Greater than Four Can be Expressed as a Sum of Two Odd Primes**

Every even integer greater than four can be expressed as a sum of two odd primes, and exists the formula as follows:

\[
G_{P}(N) \geq \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (30)
\]

\[
\geq \int \left( 1 - \frac{1}{P} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (31)
\]

\[
\geq C_{\text{tw}} = 0.6601618158 \ldots \quad (32)
\]

\[
R(N,P_{C}) = \int \left( 1 - \frac{1}{P_{C}} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (33)
\]

\[
\geq \int \left( 1 - \frac{1}{P_{C}} \right) dN = \log N + \log \log N + \gamma + O(1) \quad (34)
\]

\[
\geq C_{\text{tw}} = 0.6601618158 \ldots \quad (35)
\]

The proof method of Goldbach's conjecture

The Goldbach’s Conjecture is one of the oldest unsolved problems in Number Theory. In its modern form, it states that every even integer greater than two can be expressed as a sum of two primes.

Let $N$ be an even integer greater than 2, and let $N=(N-G_{P})+G_{P}$, with $N-G_{P}$ and $G_{P}$ prime numbers, the $G_{P}(G_{P} \leq N/2)$ be a Goldbach Prime of even integer $N$. Let $G_{P}(N)$ be the number of Goldbach Primes of even integer $N$. The number of ways of writing $N$ as a sum of two prime numbers, when the order of the two primes is important, is thus $G_{P}(N) = 2G_{P}(N)$ when $N/2$ is not a prime and is $G_{P}(N) = 2G_{P}(N)-1$ when $N/2$ is a prime. The Goldbach’s Conjecture states that $G_{P}(N) > 0$, or, equivalently, that $G_{P}(N) > 0$, for every even integer $N$ greater than two.

We known that the Goldbach’s Conjecture is true for every even integer $N$ no greater than 30000, therefore, we only need to prove that the Goldbach’s Conjecture is true for every even integer $N$ greater than 30000, that is: \(G_{P}(N) \geq 1\).

**Theorem 2:** The Sieve Method about the Goldbach Primes

Let $N$ be an even integer greater than 30000, then the even integer $N$ can be expressed to the form as follows:

\[
N = (N - G_{P}) + G_{P}, G_{P} \leq N/2 \quad (1)
\]

where $G_{P}$ be the positive integer no greater than $N/2$.

**Sieve method**

Let $N-G_{P}$ and $G_{P}$ are two positive integers, if $N-G_{P}$ and $G_{P}$ any one can be divisible by the prime $P$, then sieves the positive integer $G_{P}$; if both the $N-G_{P}$ and $G_{P}$ can not be divisible by the all primes no greater than $\sqrt{N}$, then both the $N-G_{P}$ and $G_{P}$ are primes at the same time, the prime $G_{P}$ be called the Goldbach Prime of even integer $N$.

**Theorem 1:** Let $P_{C}$ be an odd prime factor of even integer $N$ and no greater than $\sqrt{N}$, then the ratio of the number of integers $G_{P}$ that both the $N-G_{P}$ and $G_{P}$ can not be divisible by the prime $P_{C}$ at the same time, then the number of integers $G_{P}$ that the $N-G_{P}$ and $G_{P}$ any one can be divisible by the prime $P_{C}$ is $\int(N/2)/P_{C}$, the number of integers $G_{P}$ that both the $N-G_{P}$ and $G_{P}$ can not be divisible by the prime $P_{C}$ is $\int(N/2)/P_{C}$, the ratio of the number of integers $G_{P}$ that both the $N-G_{P}$ and $G_{P}$ can not be divisible by the prime $P_{C}$ to the total of integers $G_{P}$ no greater than $N/2$ is follows:

\[
R(N,P_{C}) = \int(N/2)/P_{C}, N \geq 10^4 \quad (2)
\]

\[
R(N,P_{C}) = \int(N/2)/P_{C}, N \geq 10^4 \quad (3)
\]

**Theorem 3:** The integer 2 is an even prime factor of even integer $N$, the ratio of the number of integers $G_{P}$ that both the $N-G_{P}$ and $G_{P}$ can not be divisible by the even prime 2 to the total of integers $G_{P}$ no greater than $N/2$ is follows:

\[
R(N,2) = \int(N/2)/2, N \geq 10^4 \quad (4)
\]

**Proof:** Because the 2 is an even prime factor of even integer $N$, therefore, both the $N-G_{P}$ and $G_{P}$ can not be divisible by the even prime 2 at the same time, then the number of integers $G_{P}$ that the $N-G_{P}$ and $G_{P}$ any one can be divisible by the prime $P_{C}$ is $\int(N/2)/P_{C}$, the number of integers $G_{P}$ that both the $N-G_{P}$ and $G_{P}$ can not be divisible by the prime $P_{C}$ to the total of integers $G_{P}$ no greater than $N/2$ is follows:

\[
R(N,2) = \int(N/2)/2, N \geq 10^4 \quad (5)
\]

**Proof:** Because the 2 is an even prime factor of even integer $N$, therefore, both the $N-G_{P}$ and $G_{P}$ can not be divisible by the even prime 2 at the same time, then the number of integers $G_{P}$ that the $N-G_{P}$ and $G_{P}$ any one can be divisible by the even prime 2 is $\int(N/2)/2$, the number of integers $G_{P}$ that both the $N-G_{P}$ and $G_{P}$ can not be divisible by the even prime 2 is $\int(N/2)$.
**INT(N/2) = INT(N/2 - N/2/2)**, the ratio of the number of integers Gn that both the N-Gn and Gn can not be divisible by the even prime 2 to the total of integers Gn no greater than N/2 is follows:

\[ R(N,2) = \frac{INT(N/2) - INT(N/2/2)}{N/2} = \frac{INT(N/2) - INT(N/2/2)}{N/2} \]  

(4)

Three: The Number of Goldbach Primes of Even Integer

Let Gp(N) be the number of Goldbach primes of even integer N, let Gp(N,Pn) be the number of Goldbach primes no greater than √N, then exists the formulas as follows:

\[ Gp(N) = INT(N/2) \times R(N,2) \times \prod R(N,2) \times \prod R(N,Pp) \times \prod R(N,Pn) \]

\[ = INT(N/2) \times (1-1/2) \times \prod (1-1/Pc) \times \prod (1-2/Pn) \]  

(5)

Where Pci and Pni are odd primes no greater than \( \sqrt{N} \).

Let Pi(N) be the number of primes less than an integer N, then, be the formula as follows:

\[ Pi(N) = INT(N/2) \times R(N,2) \times INT(N,Pci) \times INT(N,Pn) + Gp(N,Pn) - 1 \]  

(6)

Let Pi(N) be the number of primes less than an integer N, then, be the formula as follows:

\[ Pi(N) = \frac{INT(N/2) - INT(N/2/2)}{N/2} \]  

(7)

The Proof of Goldbach’s Conjecture

Theorem 4: Every even integer greater than 30000 can be expressed as a sum of two odd primes.

Proof: According to the formula (5),

\[ Gp(N) + 1 \geq INT(N/2) \times (1-1/2) \times \prod (1-1/Pc) \times \prod (1-2/Pn) \]

\[ = INT(N/2) \times (1-1/2) \times [INT(Pci-1)/(Pci-2)] \times \prod (1-2/Pci) \times \prod (1-2/Pni) \]  

\[ = INT((N/2) \times (1-1/2) \times [INT(1-1/Pc) \times INT(1-2/Pn)] \times INT(1-2/Pn)] \]

\[ \geq INT((N/2) \times (1-1/2) \times \prod (1-1/Pc)) \times \prod (1-2/Pc) \times \prod (1-2/Pn) \]  

(8)

From above formula (8) we can obtain that:

Every even integer greater than 30000 can be expressed as a sum of two odd primes.

**Conclusion**

For every even integer En there are infinitely many pairs of Polignac primes which difference by En.

When the En=0, we can obtain New Prime Number Theorem: Let Pi(N) be the number of primes less than or equal to N, for any real number N, the New Prime Number Theorem can be expressed by the formula as follows: Pi(N)>R(N)+ K × (Li(N)- R(N)), 1>K>−1. The Goldbach’s Conjecture is a Complete Correct Theorem.

**References**

1. Tattersall JJ (2005) Elementary number theory in nine chapters. Cambridge University Press, UK.
2. Erica K (2013) Unheralded Mathematician Bridges the Prime Gap. Simons Science News, China.