A hydrodynamical trigger mechanism for pulsar glitches

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The standard explanation for large pulsar glitches involves transfer of angular momentum from an internal superfluid component to the star’s crust. This model requires an instability to trigger the sudden unpinning of the vortices by means of which the superfluid rotates. This Letter describes a new instability that may play this role. The instability, which is associated with the inertial $r$-modes of a superfluid neutron star, sets in once the rotational lag in the system reaches a critical level. We demonstrate that our model is in good agreement with observed glitch data, suggesting that the superfluid $r$-mode instability may indeed be the mechanism that triggers large pulsar glitches.

Introduction.— Even though pulsars are generally very stable rotators, in some cases with an accuracy that rivals the best terrestrial atomic clocks, many systems exhibit a variety of timing “noise”. The most enigmatic features are associated with the so-called glitches, sudden spin-up events followed by a relaxation towards steady long-term spin-down. Several hundred glitches, with magnitude in the range $\Delta \Omega/\Omega \approx 10^{-9} - 10^{-6}$ where $\Omega$ is the observed rotation frequency, have now been observed in over 100 pulsars. The archetypal glitching pulsar is Vela, which exhibits regular large glitches. Glitches have also been reported in several magnetars as well as one, Vela, which exhibits regular large glitches. Glitches have also been reported in several magnetars as well as one, Vela, which exhibits regular large glitches.

Despite the relative wealth of observational data our theoretical understanding of glitches has not advanced considerably in recent years. The standard “model” for large pulsar glitches envisages a sudden transfer of angular momentum from a superfluid component to the rest of the star, which includes the crust (to which the pulsar mechanism is assumed rigidly attached) and the charged matter in the core. A superfluid rotates by forming a dense array of vortices, and the vortex configuration determines the global rotation. The key idea for explaining glitches is that, if the superfluid vortices are pinned to the other component, a rotational lag builds up as the crust spins down due to electromagnetic braking. Once the rotational lag reaches some critical level, the pinning breaks. This allows the vortices to move, which leads to a transfer of angular momentum between the two components and the observed spin-up of the crust.

Most theoretical work has focused on either the strength of the vortex pinning or the post-glitch evolution. There have not been many suggestions for the mechanism that triggers the glitch in the first place. It is generally expected that this role will be played by some kind of instability, but there are few truly quantitative models. The results presented in this Letter change the situation dramatically. We present evidence for a new instability, acting on the inertial modes of a rotating superfluid star, that sets in beyond a critical rotational lag. The predictions of this model agree well with the observational data making it plausible that this mechanism provides a missing piece in the pulsar glitch puzzle.

Inertial mode analysis. — We want to improve our understanding of the global hydrodynamics associated with a pulsar glitch. Even though this should be a key issue, it has not been discussed in detail previously. In principle, one would expect to be able to express the dynamics in terms of global oscillation modes of the system. In this Letter we present the first results for inertial modes of neutron star models with the two main features required for the standard glitch models, a superfluid neutron component that rotates at a rate different from that of the crust and pinned neutron vortices.

We use the standard two-fluid model for superfluid neutron stars (see for example [3]), identifying the two components with the neutron superfluid and a conglomerate of all charged particles (the “protons”). In the following, the index $x = \{n,p\}$ identifies the distinct fluids. Our aim is to model small amplitude oscillations with respect to a background configuration where both fluids rotate rigidly with (parallel) angular velocities $\omega^x = \epsilon^{ijk} \Omega^x x_k$ and where the magnitudes are different, $\Omega_n \neq \Omega_p$. The linear perturbations of this system (assuming a time dependence $\sim \exp(i\omega t)$) are, in the inertial frame, described by the two coupled Euler equations,

$$
(i\sigma + v^i_n \nabla_j) \delta \psi^i_n + \delta \psi^i_n \nabla_j v^i_j + \nabla^i \delta \psi_n = \delta f^i_{mf} \tag{1}
$$

$$
(i\sigma + v^i_p \nabla_j) \delta \psi^i_p + \delta \psi^i_p \nabla_j v^i_j + \nabla^i \delta \psi_p = -\frac{\delta f^i_{mf}}{x_p} \tag{2}
$$

Here $\delta \psi_x = \delta \mu_x + \delta \Phi$ represents the sum of the perturbed specific chemical potential and gravitational potential. We have also introduced $x_p = \rho_p/\rho_n$. This ratio, which is roughly equal to the proton fraction, is assumed constant throughout the star. For simplicity, we assume that the two fluids are incompressible, which means that $\nabla_i \delta v^i_x = 0$. In general, the two fluids are coupled i) chemically, ii) gravitationally, iii) via the entrainment effect, and iv) by the vortex mediated mutual friction $f^i_{mf}$. For clarity, we will ignore the entrainment in the present analysis. A detailed discussion of how the entrainment affects our results will be provided elsewhere.
For the inertial modes, the main coupling mechanism is provided by the mutual friction force. The general expression for this force is

\[ f_{mf}^i = B \epsilon_{ijk} \hat{e}^{km} \omega^j_n v^{mp}_n + B' \epsilon_{ijk} \omega^j_n v^{np}_n \]  

(3)

where \( v^{mp}_n = v^i_n - v^i_p \) and \( \omega^j_n = \epsilon^{ijk} \nabla_j v^k_n \). A “hat” denotes a unit vector. When the two fluids are not corotating, the perturbed force \( \delta f_{mf}^i \) is quite complex \[10\]. The form \( f_{mf}^i \) results from balancing the Magnus force that acts on the neutron vortices and a resistive “drag” force which represents the interaction between the vortices and the charged fluid. Representing the drag force by a dimensionless coefficient \( R \), one finds that \( B' = RB = R^2 / (1 + R^2) \). The range of values that \( R \) takes in a neutron star is not well known. The standard assumption in studies of neutron star oscillations has been that the drag is weak, which means that \( B' << B \ll 1 \). Then the second term in \[8\] has no effect on the dynamics. However, it may well be the opposite limit that applies. The vortices in a neutron star core may experience a strong drag force if their interaction with the magnetic fluxtubes is efficient \[11\]. The drag on the superfluid in the crust may also be strong due to vortex “pinning” by the lattice nuclei. Even though the current evidence \[3, 4\] favours weak crustal pinning, the existence of strong pinning regions has not been completely ruled out. In these cases one must consider the strong coupling limit \( R \gg 1 \) which leads to \( B \approx 0 \) and \( B' \approx 1 \).

The hydrodynamical equations \[11-12\] allow for a rich set of oscillation modes. Our focus will be on a subset of the inertial modes, the purely axial r-modes. The r-modes have attracted considerable attention since they provide the dispersion relation for the mode frequency.

A new superfluid instability.— The strong coupling limit, \( B = 0 \) and \( B' = 1 \), provides a good illustration of the main new result. In this case we obtain a simple mode solution. In terms of the dimensionless parameters

\[ \kappa = \frac{\sigma + m\Omega_p}{\Omega_p}, \quad \Delta = \frac{\Omega_n - \Omega_p}{\Omega_p} \]  

(5)

we have the two frequency solutions

\[ \kappa_{1,2} = -\frac{1}{(m+1)x_p} \left[ 1 - x_p + \Delta \pm D^{1/2} \right] \]  

(6)

where

\[ D = (1+x_p)^2 + 2\Delta \left[ 1 + x_p \left( 3 - m(m+1) \right) \right] + O(\Delta^2) \]  

(7)

The amplitudes are related according to

\[ A_p = \frac{2(1 + \Delta)}{(m + 1)\kappa} \]  

(8)

Let us focus on the short-lengthscale modes. Taking \( m \gg 1 \) and recalling that \( x_p \) and \( \Delta \) are both small (generally \( \Delta \ll x_p \)), it is easy to show that one of the above r-mode solutions becomes unstable (\( \text{Im}[\kappa] < 0 \)) for \( m > m_c \), where

\[ m_c \approx \frac{1}{\sqrt{2x_p\Delta}} \approx 320 \left( \frac{0.05}{x_p} \right)^{1/2} \left( \frac{10^{-4}}{\Delta} \right)^{1/2} \]  

(9)

For \( m \gg m_c \) the instability growth timescale \( \tau_{\text{grow}} = 1/(\Omega_p \text{Im}[\kappa]) \) is well approximated by

\[ \tau_{\text{grow}} \approx \frac{P}{2\pi \sqrt{2\Delta}} \approx 0.25 \left( \frac{x_p}{0.05} \right)^{1/2} \left( \frac{10^{-4}}{\Delta} \right)^{1/2} \left( \frac{P}{0.1 \text{ s}} \right) \]  

(10)

where \( P = 2\pi / \Omega_p \) is the observed spin period (we associate the charged component with the crust, \( \Omega_p = \Omega_c \)). We see that this new instability can grow rapidly, on a timescale comparable to the rotation period of the star.

Although we cannot yet claim to understand the detailed nature of this new r-mode instability, we have some useful clues. Firstly, we find from \[8\] that the unstable modes are such that \( |A_p/A_n| \sim x_p \). Thus, the fluid motion is predominantly in the proton fluid. There should also be a close connection with the short wavelength instability that we recently discovered for precessing superfluid stars \[12\]. In that case, the result followed from a local plane-wave analysis of the inertial modes. An attempt to link these two results would be useful. Finally, since the present system has two distinct rotation rates one might expect the instability to belong to the general two-stream class \[16\]. Such instabilities are generic in multistream fluids. The intuitive condition for such an instability dictates that the mode’s pattern speed \(-\text{Re}(\sigma)/m\)
should lie between $\Omega_n$ and $\Omega_p$ \cite{16}. This criterion translates into $m > \sqrt{2}m_c$, which is satisfied in the main part of the parameter space where the instability is operative.

In order for the new instability to affect the dynamics of realistic neutron stars it must (at least) overcome viscous damping. For young and mature neutron stars, dissipation is dominated by shear viscosity due to electron-electron collisions \cite{18}. For a uniform density star with $M = 1.4M_\odot$ and $R = 10$ km (the canonical values used in the following) the corresponding viscosity coefficient is \cite{19}

$$\eta_{ee} \approx 2.7 \times 10^{20} \left(\frac{x_p}{0.05}\right)^{3/2} \left(\frac{10^8 K}{T}\right)^2 \text{g cm}^{-1} \text{s}^{-1} \quad (11)$$

where $T$ is the core temperature. We can use the standard energy-integral approach to estimate the viscous damping timescale. In fact, since $|A_n| \ll |A_p|$ we can use existing results for r-modes of uniform density stars \cite{20}, remembering that shear viscosity only acts on the proton fluid. Thus, a simple calculation leads to

$$\tau_{sv} \approx \frac{3Mx_p}{8\pi m^2 \eta_{ee} R} \approx \frac{6 \times 10^4}{m^2} \left(\frac{0.05}{x_p}\right)^{1/2} \left(\frac{T}{10^8 \text{K}}\right)^2 \text{s} \quad (12)$$

Combining (10) and (12) we have a criterion for the unstable modes to grow fast enough to overcome viscous damping. The condition $\tau_{grow} < \tau_{sv}$ leads to

$$m < 500 \left(\frac{0.05}{x_p}\right)^{1/2} \left(\frac{\Delta}{10^{-4}}\right)^{1/4} \left(\frac{T}{10^8 \text{K}}\right) \left(\frac{0.1 \text{s}}{P}\right)^{1/2} \quad (13)$$

This gives the range of unstable $m$ modes. Since the instability sets in when $m > m_c$ we conclude that the system becomes unstable once it reaches the critical lag

$$\Delta_c \approx 6 \times 10^{-5} \left(\frac{P}{0.1 \text{s}}\right)^{2/3} \left(\frac{T}{10^8 \text{K}}\right)^{-4/3} \quad (14)$$

Making contact with observations.— Within a two-component model, it is straightforward to estimate the critical lag required to explain the observations. Assuming that angular momentum is conserved in the process, one must have $I_c\Delta \Omega_c \approx -I_s \Delta \Omega_s$, where $I_s$ and $I_c$ are the two moments of inertia, while $\Delta \Omega_s$ and $\Delta \Omega_c$ represent the changes in the corresponding spin frequencies. The glitch data suggests that about 2% of the total spin-down is reversed in the glitches \cite{21}, suggesting that $I_c/I_s \approx 0.02$. In order to permit Vela-sized glitches with $\Delta \Omega_c/\Omega_c \sim 10^{-6}$ we then need (assuming $\Omega_c = \Omega_s$ after the event)

$$\Delta_g \approx \frac{I_c}{I_s} \frac{\Delta \Omega_c}{\Omega_c} \approx 5 \times 10^{-4} \quad (15)$$

The observational estimate of the lag $\Delta_g$ at which large glitches occur is close to our estimate \cite{17} for the onset of the superfluid r-mode instability. This is unlikely to be a coincidence. Even though it is difficult to compare the parameters of our two-fluid neutron star model to the global quantities used in the phenomenological discussion directly, it is clear that our new instability has the features expected of a glitch trigger mechanism. It operates in the strong drag limit, where vortices are effectively pinned to the charged component. As long as the system is stable, a rotational lag should build up as the crust spins down. Once the system evolves beyond the critical level \cite{13} a range of unstable r-modes grow on a timescale of a few rotation periods. We cannot yet say what happens when these modes reach large (non-linear) amplitudes, but it seems inevitable that the fluid motion associated with the instability will break the vortex pinning, allowing a glitch to proceed.

Let us compare the “predictions” of our model to the data for pulsars exhibiting large glitches. To do this, we estimate the maximum glitch size allowed if $\Delta_g = \Delta_c$, assuming a completely relaxed system and $I_c/I_s = 0.02$. Since we do not have temperature data for most glitching pulsars we estimate $T$ by combining the heat blanket model from \cite{22} with a simple modified URCA cooling law. Calibrating this model to the Vela pulsar, for which $T \approx 6.9 \times 10^7$ K \cite{23}, we find

$$T \approx 3.3 \times 10^8 \left(\frac{t_c}{1 \text{yr}}\right)^{-1/6} \text{K} \quad (16)$$

Here $t_c = P/2\dot{P}$ is the characteristic pulsar age. The results are shown in Fig. \ref{fig:my_label}. This Figure shows that our model does well in predicting the maximum glitches one should expect. The data is consistent with the idea that a system needs to evolve into the instability region before a large glitch happens. It should be noted that, even though the instability first appears at $m = m_c$, the growth time is much longer than (10) until $m > 1.2m_c$ or so. It is also interesting to note that two of the systems with actual temperature data \cite{23}, Vela and PSR B1706-44, both sit on the $m \approx 1.6m_c$ curve.

Discussion. — We have described a new instability that may operate in rotating superfluid neutron stars. We have demonstrated that this instability sets in at parameter values that compare well with those inferred from pulsar glitches. This suggests that the superfluid r-mode instability may be the mechanism that triggers large pulsar glitches.

The model is consistent with a number of observed properties of glitching pulsars:

i) Adolescent pulsars, like Crab and PSR J0537-69, should only exhibit small amplitude glitches. For fast spin and a relatively high temperature the instability sets in at smaller values of $\Delta$.

ii) More mature and slower spinning neutron stars have colder cores which means they can produce larger glitches, provided the required $\Delta$ can build up (cf. PSR
J1806-21 with $\Delta \Omega_c/\Omega_c \approx 1.6 \times 10^{-5}$ [24]. Since $\Delta_c$ increases as the star ages one would expect neutron stars to cease to glitch eventually.

iii) For any glitch mechanism that relies on a critical spin lag between a superfluid component and the rest of the star it is easy to estimate the time interval $t_g$ between successive glitches. Assuming that each glitch relaxes the system, we estimate $t_g \approx 2 \Delta \rho_c \gtrsim 2 \Delta t_c$. For Vela we then find $t_g \gtrsim 750$ d which compares well with the observed averaged time of about 1000 d. The most regular known glitcher, PSR J0537-69, has an average interglitch time of about 120 d [25]. In the absence of temperature data we use [10] for this object and find $t_g \gtrsim 90$ d. Again, the agreement with the observations is good, and consistent with the notion that the system evolves into the unstable regime before a glitch occurs.

iv) There is no reason why the instability should not operate in all spinning neutron stars in which a rotational lag builds up. In particular, one may expect accreting neutron stars to “glitch” occasionally. So far, there has only been one suggested event, in the slowly rotating transient KS 1947+300 [3]. For this system our model suggests (combining $P = 18.7$ s with $T \approx 10^8$ K, a temperature that should be typical of an accreting star) a maximum glitch of $\Delta \Omega_c/\Omega_c \approx 4 \times 10^{-5}$. This is very close to the suggested observed glitch with $\Delta \Omega_c/\Omega_c \approx 3.7 \times 10^{-5}$ [3]. Such events should, of course, be extremely rare.

Since we should consider the non-magnetic inertial mode problem, our model does not apply (without modification) to magnetars. It is nevertheless interesting to consider these systems. Given typical magnetar parameters ($P \approx 10$ s, $T \approx 10^9$ K), we would not expect these objects to exhibit large glitches. Yet, they do [2]. The resolution may be that these glitches involve a larger fraction $I_s/I_c$. A very interesting question for the future concerns whether this is a natural consequence of stronger magnetic pinning in the neutron star core.

These first results are promising, but we are obviously far away from a complete understanding of this new mechanism. Future work needs to consider more detailed neutron star models. We need to understand the local mutual friction parameters and the nature of vortex pinning. The inertial mode problem for realistic neutron star models presents a real challenge. We should also make more detailed attempts at understanding the observations. A natural first step would be to improve on the temperature estimates used in Figure 1. Finally, we need to understand the nonlinear development of the new instability. This problem can perhaps be studied with numerical simulations, building on the work discussed in [26]. Ultimately one would hope to arrive at a truly quantitative model for pulsar glitches.
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