Unified approach to study quantum properties of primordial black holes, wormholes and of quantum cosmology

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Abstract

We review the anomaly induced effective action for dilaton coupled spinors and scalars in large $N$ and $s$-wave approximation. It may be applied to study the following fundamental problems: construction of quantum corrected black holes (BHs), inducing of primordial wormholes in the early Universe (this effect is confirmed) and the solution of initial singularity problem. The recently discovered anti-evaporation of multiple horizon BHs is discussed. The existance of such primordial BHs may be interpreted as SUSY manifestation. Quantum corrections to BHs thermodynamics maybe also discussed within such scheme.

It is expected that quantum field theory (or more exactly quantum gravity) should be extremely important in the study of early Universe and black hole physics. There are different ways to incorporate some “traces” of quantum gravity to cosmology. One very promising way is related with the effective action approach [1]. According to effective action formalism one can find quantum corrections in the form of some (non-local) functional which should be added to the action of classical gravity. Then, the final theory may be considered as some “new” effective theory of gravity which should be discussed basically only as classical theory. Unfortunately, there is strong barrier which stops the realization of this beautiful scenario. That is the fact that in general it is impossible to calculate effective action in closed form.

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yet to calculate it exactly on some background with high symmetry (say de Sitter space).

Recently, there has been some progress in the calculation of effective action for arbitrary space in closed form using s-wave approximation. The corresponding effective action may be applied to the study of circle of phenomena in black hole physics or early Universe in the situations where typical curvatures are strong enough. In this brief review, we discuss the application (in unified form) of such effective action to the following problems:

1. Anti-evaporation of multiple horizon BHs due to quantum effects.
2. Induction of primordial wormholes in the early Universe.
3. Solution of initial singularity problem.
4. Quantum corrections to BH thermodynamics.

As classical action we consider standard Einstein action

\[ S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R - 2\Lambda \right). \]  

(1)

In order to find effective action (back-reaction) to above classical theory, we consider \( N \) real minimal scalars and \( M \) Majorana spinors whose action is given by

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{2} \sum_{i=1}^{N} g^{\alpha\beta}_{(4)} \partial_{\alpha} \chi_i \partial_{\beta} \chi_i + \sum_{i=1}^{M} \bar{\psi}_i \gamma^\mu \nabla_\mu \psi_i \right). \]  

(2)

The anomaly induced effective action for the above theory can be evaluated for \( M = 0 \) case \(^5\) and \( M \neq 0 \) case \(^5\) using large \( N \) and s-wave approximation. Large \( N \) approximation is necessary in order to justify why we do not consider gravity quantum contributions (which may be nevertheless discussed afterwards). s-wave reduction makes the problem solvable on analytical level and it looks quite natural as we discuss below only metrics with spherical symmetry, i.e. of the sort \( ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{-2\phi} d\Omega^2 \).

\(^4\)This approximation works for smooth enough backgrounds when one can neglect higher derivative terms in EA.\(^5\)

\(^5\) For the study of conformal anomaly for dilaton coupled scalars, see refs.\(^6\).
\(\mu, \nu = 0, 1, g_{\mu\nu}\) and \(\phi\) depend only on \(x^0 = t, x^1 = r\). By \(s\)-wave reduction we mean that in quantum calculations we neglect the dependence from two spherical coordinates.

From 2d conformal anomaly for dilaton coupled scalar and dilaton coupled spinor, one can find the anomaly induced effective action. It can be written in the following form \[3, 9, 6\]:

\[
W = -\frac{1}{8\pi} \int d^2x \sqrt{-g} \left[ \frac{N + M}{12} R \frac{1}{\Delta} R - N \nabla^{\lambda} \phi \nabla_{\lambda} \phi \frac{1}{\Delta} R \right.
\]

\[
+ \left( N + \frac{2M}{3} \right) \phi R - 2N \ln \tilde{\mu}^2 \nabla^{\lambda} \phi \nabla_{\lambda} \phi \right].
\]  

Solving the equations of motion which follow from the action \(S + W\), Bousso and Hawking \[3\] have found a special solution describing the quantum corrected Schwarzschild-de Sitter (or in nearly degenerated case called also as Nariai) space \[8\]. On the classical level SdS BH looks as follows:

\[
ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega^2, \quad V(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3} r^2.
\]  

Here \(\mu\) is the black hole mass. \(V(r)\) has two positive roots \(r_c\) and \(r_b\) \((r_c > r_b)\), which correspond to cosmological and black hole horizons. The Nariai space is given as a limit of \(r_c \to r_b\). By defining new time and radial coordinates \(t = \frac{1}{\sqrt{\Lambda}} \psi\) and \(r = \frac{1}{\sqrt{\Lambda}} \left[ 1 - \epsilon \cos \chi - \frac{\epsilon^2}{6} \right]\) (here \(\epsilon\) is defined by \(9\mu^2 \Lambda = 1 - 3\epsilon^2\)), we obtain the metric in the nearly degenerate limit:

\[
ds^2 = -\frac{1}{\Lambda} \left( 1 + \frac{2\epsilon}{3} \cos \chi \right) \sin^2 \chi d\psi^2 + \frac{1}{\Lambda} \left( 1 - \frac{2\epsilon}{3} \cos \chi \right) d\chi^2
\]

\[+ \frac{1}{\Lambda} (1 - 2\epsilon \cos \chi) d\Omega^2.\]  

The Nariai limit corresponds to \(\epsilon \to 0\). Since \(\chi\) has a periodicity of \(2\pi\), the topology of the Nariai limit could be \(S_1 \times S_2\). In the quantum level, the parameters of this metric are changed due to the influence of effective action part \[9\]. However, the structure of the metric is the same.

At the next step \[10\], the authors have investigated the backreaction from \(N\) conformal scalars \(\chi_i\), \(N_1\) vectors \(A_\mu\) and \(N_{1/2}\) Dirac spinors \(\psi_i\) to Nariai metric. The corresponding effective action may be calculated even beyond \(s\)-wave approximation. It was demonstrated in \[3\], that a nearly degenerated
Nariai black hole may not only evaporate but also anti-evaporate (i.e. its radius is getting to grow due to quantum effects). In [10], it has been shown that the Nariai solution is stable and anti-evaporation can occur only if GUT matter content satisfies

$$2N + 7N_{1/2} > 26N_1.$$  

This relation holds for $SU(5)$ GUT. In [11], the case of the usual $SO(10)$ GUT was investigated. Although Eq. (6) cannot be satisfied in the case of the non-supersymmetric model, Eq. (6) can be satisfied in the case of the supersymmetric $SO(10)$ GUTs due to the contribution from the Higgsino in the large dimensional representations. Hence, one extremely bright application of effective action is the discovery of the process which is opposite to famous Hawking radiation and which was called the anti-evaporation of multiple horizon black holes. As a result, we may expect that not all primordial SdS black holes evaporated quickly in early Universe. Some of them could existed for a longer time (until the anti-evaporation has been stopped by some other processes) and even could be searched in the present Universe. Moreover, their existence may be interpreted as “supersymmetry test” as anti-evaporation may occur presumably for SUSY GUTs.

Another related research may be connected with the possibilities to induce primordial wormholes at the early Universe. The corresponding metric is

$$ds^2 = -e^{2\rho}dt^2 + dl^2 + e^{-2\phi}d\Omega^2,$$

where $\rho = \rho(l)$, $\phi = \phi(l)$ and $l$ is the proper distance. We call $f(l) = \exp(2\rho)$ as redshift function and $r(l) = \exp(-\phi)$ as shape function. This metric has again spherical symmetry. Note that the above wormhole cannot occur as solution of classical gravity equations with usual non-quantum matter due to severe limitations.

Applying the same effective action [3], we made the numerical study for some contents of matter. The results of this study [12] are given at figures, for the shape function $r(l)$ in fig.1 and for the redshift function $f(l)$ in fig.2. As we see, one gets the wormhole solution with increasing throat radius and increasing red-shift function. That proves the possibility of quantum inducing of wormholes at the early Universe. Hence quantum effects give rise the primordial wormholes at the very early Universe. This fact has been also confirmed very recently for $N = 4$ super Yang-Mills theory in curved spacetime (via application of 4d anomaly induced effective action) in ref.[13].
The effective action (3) can also be used to investigate quantum cosmology. In this way one can try to find the answer to the problem of initial singularity.

When we assume that our spherically symmetric metric does not depend
on the radial coordinate $r$ but on the time coordinate $t$ only and $r$ is a periodic coordinate, there was found the solution of effective equations of motion. The corresponding metric describes the Kantowski-Sachs Universe [14] whose topology is given by $S_1 \times S_2$ [15, 16]. Especially in [16], an analytical solution without curvature singularity is found:

$$ds^2 = -d\tau^2 + \frac{2}{R_0} \cosh^2 \left( \tau \sqrt{\frac{R_0}{2}} \right) dr^2 + e^{-2\phi_0} d\Omega^2 . \tag{8}$$

Here the dilaton field $\phi$ and the 2d scalar curvature $R$ become constants $\phi = \phi_0$ and $R = R_0$, which are given by

$$e^{-2\phi_0} = \frac{(2N + M)G}{6} + \frac{1}{2\Lambda} \pm \frac{1}{2} \sqrt{\frac{(2N + M)^2 G^2}{9} + \frac{1}{\Lambda^2} - \frac{(8N + 6M)G}{3\Lambda}} .$$

$$R_0 = -\frac{3\Lambda}{(N + M)G} \left( \frac{(2N + M)G}{3} - \frac{1}{\Lambda} \right)^{-1} \pm \sqrt{\frac{(2N + M)^2 G^2}{9} + \frac{1}{\Lambda^2} - \frac{(8N + 6M)G}{3\Lambda}} . \tag{9}$$

This KS Universe is expanding with the radius which is never zero. It may be completely induced by quantum effects (as it can be considered in the regime where there is no classical corresponding solution). Moreover, it can presumably describe some sub-stage of inflationary Universe where there is effective expansion only along one of spatial coordinates. This explicit example opens a new way to re-consideration of initial singularity problem in frames of quantum gravity using effective action formalism.

As another application, we can consider the quantum corrections to the 4d Schwarzschild-(Anti-)de Sitter (S(A)dS) black holes (BH), whose metric is given by

$$ds^2 = -e^{2\rho} dt^2 + e^{-2\rho} dr^2 + r^2 d\Omega^2$$

$$e^{2\rho} = 1 - \frac{\mu}{x} - \frac{\Lambda}{3} x^2 = -\frac{\Lambda}{3x} \prod_{i=1}^{3} (x - x_i) . \tag{10}$$

Here $\mu$ is a constant of the integration corresponding to the black hole mass ($\mu = 2GM_{BH}$). The parameters $x_i$ ($i = 1, 2, 3$) are solutions of the equation $e^{2\rho_0} = 0$. Among $x_i$'s, two are real and positive if $\Lambda > 0$ and $\mu^2 < \frac{4}{9\Lambda}$ and they
correspond to black hole and cosmological horizons in the Schwarzschild-de Sitter black hole. On the other hand, only one is real if $\Lambda < 0$.

In Ref. [17], it has been found the temperature $T$ and entropy $S$ with account of quantum effects for multiply horizon SdS BH and SAdS BH as following

\[
T \sim \frac{\Lambda}{12\pi} \left\{ Y_I + \left( -6 + \frac{2Y_I}{x_I} \right) GNC_I - GNY_I B_I \right\} \\
S \sim \pi x_I^2 - 4\pi GNC_I
\]

\[
B_I \equiv \Delta_0 + A \left[ \frac{7}{24x_I^2} - \frac{1}{12} \sum_{i=1, i\neq I}^3 \left\{ \left( 1 - \frac{Y_I^2}{Y_i^2} \right) \left( \frac{1}{x_I(x_I - x_i)} - \frac{1}{x_i^2} \ln(x_I - x_i) \right) \right. \\
+ \frac{1}{x_i^2} \ln x_I - \frac{2}{x_i x_I} - \frac{1}{6} \left( \sum_{j \neq i, j=1}^3 \frac{1}{x_i(x_i - x_j)} \right) \ln(x_I - x_i) \right\} - \frac{1}{12x_I^2} \ln x_I \\
+ \frac{a' + B - 1}{2x_I^2} - \frac{3}{4x_I^2} \\
+ \sum_{i=1, i\neq I}^3 \left\{ \frac{1}{2} \left( \frac{1}{x_I^2} - \frac{1}{x_i^2} \right) \ln(x_I - x_i) + \frac{1}{2x_i^2} \ln x_I - \frac{1}{2x_i x_I} \right\}
\]

\[
C_I \equiv \frac{1}{\prod_{i=1, i\neq I}^3(x - x_i)} \\
\times \left[ \Delta_1 + A \left\{ \frac{x_I}{6} - \frac{X_2}{24x_I^2} + \frac{X_3}{48x_I^2} - \frac{1}{24} \sum_{i=1, i\neq I}^3 \left( Y_i - \frac{Y_I^2}{Y_i^2} \right) \ln \left( 1 - \frac{x_i}{x_I} \right) \right\} \\
+ \frac{a' - B + 1}{2} \left( x_I - \frac{X_2}{x_I} + \frac{X_3}{2x_I^2} \right) + B \left( x_I + \frac{X_3}{4x_I^2} \right) \\
-x_I \left( \ln x_I - 1 \right) + \frac{X_3}{4} \left( \ln x_I \right)^2 + \frac{X_2}{2x_I} \left( \ln x_I + 1 \right) \\
- \frac{X_3}{4x_I^2} \left( \ln x_I + \frac{1}{2} \right) + \frac{1}{2} \sum_{i=1, i\neq I}^3 \left( x_I - x_i \right) \left( \ln(x_I - x_i) - 1 \right) \\
+ \frac{X_2}{2} \sum_{i=1, i\neq I}^3 \left( -\frac{1}{x_I} \ln(x_I - x_i) + \frac{1}{x_i} \ln \left( 1 - \frac{x_i}{x_I} \right) \right) - \frac{X_2}{2x_I} \ln x_I \\
- \frac{X_3}{2} \left( -\frac{1}{2x_I^2} \ln \left( x_I - x_i \right) + \frac{1}{2x_i^2} \ln \left( 1 - \frac{x_i}{x_I} \right) + \frac{1}{2x_i x_I} \right)
\]

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Here
\[ A \equiv \frac{N + M}{N}, \quad B \equiv \frac{N + \frac{2M}{3}}{N} \]
\[ Y_I \equiv \frac{1}{x_I} \prod_{i=1; i \neq I}^3 (x_i - x_I), \quad (12) \]

(11) also gives the corresponding expressions for their limits: Schwarzschild and de Sitter spaces. The latter can be regarded as the quantum correction to entropy of expanding Universe. Most of the above results are given for the same gravitational background with interpretation as 4d quantum corrected BH or 2d quantum corrected dilatonic BH [9, 18].

We presented Brief Review of some recent applications of anomaly induced effective action to various problems of quantum BHs physics and early Universe. There are many more topics where these methods maybe successfully applied. Let us mention only one which was discussed recently in refs. [19, 9] where it was shown that anti-evaporation of BHs in de Sitter Universe may lead to its proliferation. In other words, on the quantum level the fragmentation of de Sitter Universe to few new daughter Universes may occur.

Acknowledgements SDO would like to thank R. Bousso, S. Hawking and P. van Nieuwenhuizen for stimulating discussions. The work by OO has been partially supported by CONACyT Grants 3898P-E9608 and 28454-E.

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