Waveguide and $\Gamma$-factor optimization for low-divergence ridge lasers

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Abstract. The work is aimed for analytical approach to maximize ridge laser modal gain at a given beam divergence in a vertical direction. When the gain/loss profile of planar waveguide is known the equation for optimum near-field profile was obtained. Surprisingly, it is appeared to be Schrödinger type equation where normalized gain/loss profile plays role of potential energy. The developed approach was applied to a few examples widely met in practice.

1. Introduction
Semiconductor ridge emitting lasers have a broad range of applications from medical applications to material processing, therefore, lasers parameters optimization is a matter of particular interest. One of goals is maximizing $\Gamma$-factor, which means increased field profile confinement. But it’s shown that it results in rising far-field beam divergence, so a trade-off takes place. In this abstract we look for analytical approach to this problem.

2. Analytical approach
Let us consider planar waveguide with TE-polarized wave propagating along x-direction and let $F(z)$ be its near-field profile of $y$-component of electric field, where z-axis points growth direction in planar waveguide. As was shown in [1], measure of beam divergence can be introduced as squared average angle $\Theta$:

$$\Theta^2 = -k_0^2 \int_{-\infty}^{\infty} F(z) \frac{d^2 F}{dz^2} dz, \text{ while } \int_{-\infty}^{\infty} |F(z)|^2 dz = 1,$$

where $k_0$ is the light wavenumber in vacuum for the lasing wavelength $\lambda_0 = 2\pi/k_0$. Let $g(z)$ be gain profile. Modal gain $G$ can be found via:

$$G = \int_{-\infty}^{\infty} |F(z)|^2 g(z) dz.$$

In order to find best possible modal gain keeping the value of $\Theta$ fixed we use variational approach with the Lagrange multipliers, which is described in [2]. To do so, we introduce functional $S[F(z)]$ as well as Lagrange multipliers $\rho$ and $\lambda$:

$$S[F(z)] = \int_{-\infty}^{\infty} |F(z)|^2 g(z)dz + \rho \left( \Theta^2 k_0^2 + \int_{-\infty}^{\infty} F(z) \frac{d^2 F}{dz^2}(z)dz \right) + \lambda \left( \int_{-\infty}^{\infty} |F(z)|^2 dz - 1 \right).$$

Hence, we convert initial task of maximum value of $G$ to the problem of finding extremum of $S[F(z)]$. The conventional variational procedure lead us to the following equation on $F(z)$:

$$-\frac{d^2 F}{dz^2} - \frac{1}{\rho} F(z)g(z) = \lambda \frac{1}{\rho} F(z).$$

One can clearly see here, that this is Shrödinger eigenvalue problem where $-\rho^{-1}g(z)$ plays role of potential energy. Hence we can use well developed methods to solve this problem for different $g(z)$ and to interpret physical meaning of its solution.

We can introduce $\alpha = \rho^{-1}$ as well as $\xi = \lambda \rho^{-1}$ as only their quotient plays role. From this point onwards $\alpha$ is considered a given constant.

$$\frac{d^2 F}{dz^2} + F(z) (\alpha g(z) + \xi) = 0$$

This is a Schrödinger’s equation for particle in potential $-\alpha g(z)$ with energy $\xi$. The variable $\xi$ can be found during solution of this eigenproblem. Hence, only $\alpha$ could be varied during optimization procedure. From point of view of Shrödinger equation varying $\alpha$ would lead to changing of depth of “potential well” that corresponds to the gain region. Higher values of the $\alpha$ corresponds to more tight localization of the field profile $F(z)$ around gain region, higher $\Gamma$-factor and higher beam divergence. Still the solution at any fixed $\alpha$ is optimal in the sense that $\Gamma$ could not be better for a given $g(z)$ profile and beam divergence. While $g(z)$ is arbitrary, we cannot proceed any further analytically and bring here few examples.

3. Examples

3.1. Example 1: single gain layer

In that case gain be introduced as $g(z) = A \delta(z)$. Solution of (5) is well known to be

$$F(z) \sim \exp \left( -\sqrt{\alpha A} |z| \right),$$

where $\alpha$ could be any positive value. Properties of that solution have been studied in details in [1]. We note here that actual value of $A$ is of no importance, since $\alpha$ is arbitrary and should be “scanned” in order to find $F(z)$ with required divergence and, therefore, we ommit it in the following.

3.2. Example 2: two gain layers

Two gain layers in an active media can represent a couple of quantum wells as an example. With the gain profile written as

$$g(z) = \delta(z - a) + \delta(z + a),$$

the solution of (5) is following:

$$F(z) = \begin{cases} N \exp \left( \sqrt{\xi}(z + a) \right), & z < -a \\ N \text{sech} \left( \sqrt{\xi}a \right) \cosh \left( \sqrt{\xi}z \right), & z \in [-a, a], \\ N \exp \left( -\sqrt{\xi}(z - a) \right), & z > a \end{cases}$$
where $N$ is normalization constant and $\xi$ can be found as the solution of following transcendent equation:

$$\tanh \left(a\sqrt{\xi}\right) = 2\alpha\xi^{-1/2} - 1 \quad (9)$$

The profiles of $F(z)$ are shown in Figure 1 for different values of $a$. At the Figure 2 best achievable values of $\Theta$ are shown as a dependence of a modal gain ($g_0$ is unity for the choice $A = 1$, see above).

As we can see from the Figure 2, at low gain and $a < \lambda_0$ small increase in gain results in the proportional increase in divergence for all relevant $a$ due to field distribution being "blob" of width much more than $\lambda_0$ so field will not respond to small $a$ change as much as at high gain. Also it is worth noting that best divergence angle is reached at $a = 0$, thus gain profile confinement is preferred.

3.3. Example 3: symmetric slab

There are cases when gain of single layer is not sufficient for effective laser operation, namely, in lasers with quantum dot active media. This problem is typically solved by stacking of gain layers. As an example, in InGaAs/GaAs quantum dot lasers up to 10 layers may be stacked. The total width of the gain area in that case is up to few hundreds nanometres and is compared with the wavelength in the material. In that case gain profile could well approximated as a “gain slab” of thickness $2a$ (see Figure 3):

$$g(z) = g_0 \theta(a + z) \theta(a - z) \quad , \quad (10)$$

where $\theta(z)$ is Heaviside step function.

The optimum field profile is a symmetric one:

$$g(z) = g_0 \theta(a + z) \theta(a - z) \quad , \quad (10)$$

Figure 3: Gain profile with $a = \lambda_0$. 

Figure 1: Optimal near-field intensity profiles for different values of $a$ that are shown at the right side of graph.

Figure 2: $\Theta(G)$ dependency on the gain for various $a$. 

Figure 3: Gain profile with $a = \lambda_0$. 

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The optimum field profile is a symmetric one:
\[ F(z) = \begin{cases} 
A \exp (k_1 (z + a)), & z < -a \\
A \text{sech} (k_2 a) \cosh (k_2 x), & z \in [-a, a] \\
A \exp (-k_1 (z - a)), & z > a
\end{cases} \]

where \( \begin{align*}
k_1 &= \sqrt{-\xi} \\
k_2 &= \sqrt{-\xi - \alpha}
\end{align*} \) (11)

\( \xi \) can be found as solution of following equation:

\[ \tanh (k_2(\xi) a) = -\frac{k_1(\xi)}{k_2(\xi)} \] (12)

Note that there is modal gain limit \( g_0 \), which means field is being contained in \( g(z) \) completely. Also, better result is reached for greater slab width.

**Figure 4**: \( \Theta(G, a) \) characteristics.

4. **Conclusion**

We have developed variation approach that in analytical terms describes the near-field profiles that corresponds to better possible modal gain for a given divergence angle or vice versa. Though it is obvious that this approach could not be always used in real material systems due to limited refractive index variations, the approach still is useful to determine how far from the “ideal” any given structure is.

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