MODELING THE ANOMALY OF SURFACE NUMBER DENSITIES OF GALAXIES ON THE GALACTIC EXTINCTION MAP DUE TO THEIR FIR EMISSION CONTAMINATION

TOSHIYA KASHIWAGI1, YASUSHI SUTO1,2,3, ATSUSHI TARUYA1,2,4,5, ISHIA KAYO5,6, TAKAHIRO NISHIMICHI4,6, AND KAZUHIRO YAHATA1,9
1 Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan; kashiwagi@utap.phys.s.u-tokyo.ac.jp
2 Research Center for the Early Universe, School of Science, The University of Tokyo, Tokyo 113-0033, Japan
3 Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA
4 Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, Chiba 277-8568, Japan
5 Department of Physics, Toho University, Funabashi, Chiba 274-8510, Japan
6 Department of Liberal Arts, Tokyo University of Technology, Ota-ku, Tokyo 144-8650, Japan
7 Current address: Yukawa Institute for Theoretical Physics, Kyoto 606-8502, Japan.
8 Current address: Institut d’Astrophysique de Paris, 98 bis Boulevard Arago, F-75014 Paris, France.
9 Current address: Canon, Inc., Ohta-ku, Tokyo 146-8501, Japan.

1. INTRODUCTION

The Galactic extinction map is the most fundamental of the data for astronomy and cosmology, because all extragalactic astronomical observations are inevitably conducted through the Galactic foreground and are thus affected by the Galactic interstellar dust. In particular, lights in optical and ultraviolet bands are dimmed by the absorption and scattering of the Galactic dust. Therefore, we cannot determine any fundamental quantities, such as intrinsic luminosities or colors of extragalactic objects, without proper correction for the dust extinction. This is why the Galactic extinction correction could be one of the most critical sources of systematics.

The most widely used Galactic extinction map was constructed assuming that the observed far-infrared (FIR) fluxes affect the SFD map. We first compute the surface number density of Sloan Digital Sky Survey (SDSS) DR7 galaxies as a function of the r-band extinction, Ar,SFD. We confirm that the surface densities of those galaxies positively correlate with Ar,SFD for Ar,SFD < 0.1, as first discovered by Yahata et al. for SDSS DR4 galaxies. Next we construct an analytical model to compute the surface density of galaxies, taking into account the contamination of their FIR emission. We adopt a log-normal probability distribution for the ratio of 100 μm and r-band luminosities of each galaxy, y ≡ (νL)100,μm/(νL)r. Then we search for the mean and rms values of y that fit the observed anomaly, using the analytical model. The required values to reproduce the anomaly are roughly consistent with those measured from the stacking analysis of SDSS galaxies. Due to the limitation of our statistical modeling, we are not yet able to remove the FIR contamination of galaxies from the extinction map. Nevertheless, the agreement with the model prediction suggests that the FIR emission of galaxies is mainly responsible for the observed anomaly. Whereas the corresponding systematic error in the Galactic extinction map is 0.1–1 mmag, it is directly correlated with galaxy clustering and thus needs to be carefully examined in precision cosmology.

Key words: dust, extinction – galaxies: ISM – local interstellar matter – infrared: ISM

2. CALIBRATING THE ISSA 100 μm MAP TO COLOR EXCESS, E(B – V), ASSUMING THE PROPORTIONALITY BETWEEN THE TEMPERATURE-CORRECTED 100 μm FLUX, I100,μm, AND THE DUST COLUMN DENSITY

E(B – V) = pI100,μmX(T),

where p is a constant determined from Mg II indices of elliptical galaxies as standard color indicators and X(T) is the correction for the dust temperature.

The SFD map has achieved significant improvement in precision and resolution compared to the previous extinction maps constructed from H I 21 cm emission (Burstein & Heiles 1978, 1982). Nevertheless, it should be noted that the map is not based on any direct measurement of the dust absorption but is derived from its emission. Indeed, one needs several assumptions to convert the FIR emission map into the extinction map. This is why it is important to test the reliability of the SFD map by comparing it with other independent observations.

In high-extinction regions, such as molecular clouds or near the Galactic plane, many earlier studies examined the SFD map using star counts, NIR galaxy colors, and galaxy number counts (Arce & Goodman 1999a, 1999b; Chen et al. 1999; Cambrésy et al. 2001, 2005; Dobashi et al. 2005; Yasuda et al. 2007; Rowles & Froebrich 2009). They often report that the SFD map overpredicts extinction in the high-extinction regions, possibly because of the poor angular resolution of the dust temperature map (Arce & Goodman 1999a, 1999b) and/or the existence of cold dust components with high emissivity in FIR.

In contrast, its reliability in low-extinction regions has not been carefully examined until recently. The Sloan Digital Sky
Survey (SDSS; York et al. 2000) with very accurate photometry makes it possible to investigate the reliability of the SFD map even in those regions. Fukugita et al. (2004) tested the region of \( E(B-V) < 0.15 \) in the SFD map on the basis of number counts of the SDSS DR1 (Abazajian et al. 2003) galaxies and concluded that the SFD map prediction is consistent with the number counts. More recently, Schlafly et al. (2010) measured the dust reddening from the displacement of the bluer edge of the SDSS stellar locus and found that the SFD map overpredicts dust reddening by \( \sim 14\% \) in \( E(B-V) \). They also found that the extinction curve of the Galactic dust is better described by the Fitzpatrick (1999) reddening law rather than by that of O’Donnell (1994). These results are also confirmed by an independent method (Schlafly & Finkbeiner 2011). Peek & Graves (2010, hereafter PG) measured the dust reddening using the passively evolving galaxies as color standards and found that the SFD map underpredicts reddening where the dust temperature is low, but at most by 0.045 mag in \( E(B-V) \). They provided the correction map for the SFD with 4.5 resolution.

A systematic test of the SFD map was also performed by Yahata et al. (2007). They computed the surface number densities of the SDSS DR4 (Adelman-McCarthy et al. 2006) photometric galaxies as a function of the extinction. They found that the surface number densities of the SDSS galaxies exhibit a clear positive correlation with the SFD extinction in the low-extinction region, \( A_r < 0.1 \) and proposed that the observed FIR intensity, \( I_{100 \mu m} \), is partially contaminated by the emission of galaxies along their direction. Because SFD compute the extinction, assuming that the flux is entirely due to the Galactic dust, the region with more galaxies—and therefore with stronger FIR intensity—is assigned a higher extinction. If the overestimated extinction is applied, the corrected surface number density of galaxies becomes even higher than the real, resulting in positive correlation with the extinction, as observed. Yahata et al. (2007) performed a simple numerical experiment and showed that even a quite small contamination of FIR emission of galaxies could qualitatively reproduce the observed anomaly. Indeed, the expected FIR emission was unambiguously discovered by the subsequent stacking image analysis of SDSS galaxies (Kashiwagi et al. 2013).

The main purpose of the present paper is to reproduce quantitatively the observed anomaly of the surface number density of SDSS galaxies on the SFD map by an analytical model of the contamination due to their FIR emission. The rest of the paper is organized as follows; after the brief summary of the SDSS DR7 data (Abazajian et al. 2009) that we use here (Section 3), we repeat the surface-number-density analysis of galaxies introduced by Yahata et al. (2007). Section 4 performs mock numerical simulations so as to predict the surface number densities of galaxies by taking account of the effect of their FIR contamination. We also develop an analytic model and ensure that it reproduces well the result of the mock simulation in Section 5. The detailed description of our analytical model is presented in Appendix B. We perform the fit to the observed anomaly in the SFD map and find that the mean of the 100 \( \mu m \) to \( r \)-band luminosity ratio, \( y = (\nu L_{100 \mu m}/\nu L_r) \) per SDSS galaxy, is required to be \( y_{avg} > 4 \). Section 7 discusses the effect of the spatial clustering of galaxies, which is neglected either in mock simulations or in the analytical model. We also compare the optimal value of the 100 \( \mu m \) to \( r \)-band flux ratio with that independently derived with the stacking image analysis by Kashiwagi et al. (2013). Similar analysis for the corrected SFD map according to Peek & Graves (2010) is also briefly mentioned. Finally, Section 8 is devoted to the summary and conclusions of the present paper.

2. SLOAN DIGITAL SKY SURVEY DR7

The SDSS DR7 photometric observation covers 11,663 deg\(^2\) of sky area and collects 357 million objects with photometry in five passbands; \( u, g, r, i, \) and \( z \) (for more details of the photometric data, see Gunn et al. 1998, 2006; Fukugita et al. 1996; Hogg et al. 2001; Ivezić et al. 2004; Smith et al. 2002; Tucker et al. 2006; Padmanabhan et al. 2008; Pier et al. 2003). The SDSS photometric data are corrected for the Galactic extinction, according to the SFD map (Stoughton et al. 2002). They adopt the conversion factors from color excess to the dust extinction in each passband:

\[
k_x \equiv \frac{A_x}{E(B-V)},
\]

where \( x = u, g, r, i, \) and \( z \) (Table 6 of SFD). These factors are computed assuming the spectral energy density of an elliptical galaxy and the reddening law of O’Donnell (1994), combined with the extinction curve parameter:

\[
R_V \equiv \frac{A_V}{E(B-V)} = 3.1.
\]

The spatial distribution of stellar objects in the SDSS catalog is likely to be correlated with the dust distribution. Therefore the reliable star–galaxy separation is critical for our present purpose of testing the SFD map from the distribution of extragalactic objects. We carefully construct a reliable photometric galaxy sample as follows.

2.1. Sky Area Selection

We choose the regions of the SDSS DR7 survey area labeled “PRIMARY.” Indeed, we find that the “PRIMARY” regions in the southern Galactic hemisphere are slightly different from the area where the objects are actually located. We are not able to understand why and thus decide to use the regions in the northern Galactic hemisphere alone to avoid possible problems.

To ensure the quality of good photometric data, we exclude masked regions. The SDSS pipeline defines the five types of masked regions according to the observational conditions. We remove the four types of masked regions labeled “BLEEDING,” “BRIGHT_STAR,” “TRAIL,” and “HOLE” from our analysis. The masked regions labeled “SEEING” are not removed, because relatively bad seeing does not seriously affect the photometry of relatively bright galaxies that we use in the present analysis. The total area of the removed masked regions is about 340 deg\(^2\), which comprises roughly 4.5% of the entire “PRIMARY” regions in the northern Galactic hemisphere.

2.2. Removing False Objects

We remove false objects according to photometry-processing flags. We first remove fast-moving objects, which are likely the solar system objects. We also discard objects that have bad photometry or were observed in the poor condition. A fraction of objects suffers from deblending problems; that is, the decomposition of photometry images consisting of superimposed multiobjects is unreliable or failed. We remove such objects as well.
Figure 1. Differential number counts of the photometric galaxy sample as functions of extinction-uncorrected magnitudes for each band (solid lines). The vertical dashed lines indicate the magnitude ranges within which we constrain the analysis.

Table 1

| Bandpass | Magnitude Range | Number of Galaxies (w/o flag selection) | Number of Galaxies (w/flag selection) | Rejection Rate |
|----------|-----------------|----------------------------------------|--------------------------------------|----------------|
| u        | 18.3 < m_u < 20.2 | 1200586                                | 633319                               | 0.472          |
| g        | 18.0 < m_g < 20.4 | 4891030                                | 3428064                               | 0.299          |
| r        | 17.5 < m_r < 19.4 | 4347881                                | 3205638                               | 0.263          |
| i        | 17.0 < m_i < 18.9 | 4450724                                | 3140684                               | 0.295          |
| z        | 16.8 < m_z < 18.3 | 2984104                                | 2136639                               | 0.284          |

Notes. The third column shows the number of all SDSS galaxies within the magnitude range. The fourth column shows the number of the galaxies—after the photometry flag selection described in Section 2.2—which are used in our measurement in Section 3. The number of galaxies is counted without extinction correction.

2.3. Magnitude Range of Galaxies

The SDSS catalog defines the type of objects according to the differences between the \textit{cmodel} and point-spread function (PSF) magnitudes, where the former magnitude is computed from the composite flux of the linear combination of the best-fit exponential and de Vaucouleurs profiles.

Because the reliability of star–galaxy separation depends on the model magnitude before extinction correction, we must carefully choose the magnitude ranges of our sample for the analysis. In the \textit{r} band, the star–galaxy separation is known to be reliable for galaxies brighter than \( \sim 21 \) mag (Yasuda et al. 2001; Stoughton et al. 2002), whereas the saturation of stellar images typically occurs for objects brighter than 15 mag in the \textit{r} band. Therefore, we choose the magnitude range conservatively as \( 17.5 < m_r < 19.4 \), where \( m_r \) denotes the observed (extinction-uncorrected) magnitudes in the \textit{r} band.

We adopt the same value of upper/lower limits for extinction-corrected magnitudes. Figure 1 shows the differential number counts of SDSS galaxies as a function of \( m_r \) for each bandpass. The faint-end threshold of our \textit{r}-band-selected sample, \( m_r = 19.4 \), is \( \sim 2 \) mag brighter than the turnover of the differential number count. We similarly determine the faint end of the magnitude range for all bandpasses as 2 mag brighter than the turnover magnitude. We confirmed that shifting the upper or lower limits by \( \pm 1.0 \) mag does not significantly change our conclusions below. We summarize the magnitude range and the number of galaxies with and without photometry flag selection for each bandpass in Table 1.

3. SURFACE NUMBER DENSITIES OF SDSS DR7 PHOTOMETRIC GALAXIES

3.1. Methodology

In this section, we extend the previous analysis of Yahata et al. (2007) and reexamine the anomaly in the surface number density of galaxies using the SDSS DR7 photometric galaxies, instead of DR4. The left panel of Figure 2 plots the sky area of the SDSS DR7 that is employed in our analysis, where the color scale indicates the value of the \textit{r}-band extinction provided by SFD, \( A_{r,\text{SFD}} \).

Because most of the increased survey area of DR7 relative to DR4 corresponds to regions with \( A_{r,\text{SFD}} < 0.1 \) mag, we can
study the anomaly in such low-extinction regions discovered by Yahata et al. (2007) with higher statistical significance.

We first divide the entire sky area of the SDSS DR7 (right panel of Figure 2) into 84 subregions, according to the value of $A_r$$_{SFD}$. Each subregion is chosen so as to have approximately the same area ($\sim 100$ deg$^2$) and consists of spatially separated (disjoint) small patches over the sky. The right panel of Figure 2 shows the cumulative area fraction of the sky as a function of $A_r$$_{SFD}$. Note that approximately 74% of the entire sky corresponds to $A_r$$_{SFD}$ < 0.1 mag, in which we are interested.

Next, we count the number of galaxies with the specified range of $r$-band magnitude in each subregion (Section 2.3) and obtain their surface number densities as a function of the extinction. Because the spatial distribution of galaxies is expected to be homogeneous when averaged over a sufficiently large area, the surface number densities of galaxies should be constant and should not correlate with the extinction. In other words, any systematic trend with respect to $A_r$$_{SFD}$ should indicate a problem of the SFD map.

### 3.2. Results

Figure 3 shows the surface number densities of galaxies, $S_{gal}$, in the 84 subregions for the five passbands. The red filled circles indicate $S_{gal}$, uncorrected for dust extinction, whereas the blue filled triangles are the results after extinction correction using the SFD map. Note that the surface number densities of galaxies in different passbands are plotted against their corresponding $r$-band extinction, $A_r$$_{SFD}$.

Following Yahata et al. (2007) again, we estimate the statistical error of the surface number density, $\sigma^2_S$, as follows:

$$\sigma^2_S = \frac{1}{N} + \frac{1}{\Omega^2} \int \Omega \int \int w(\theta_{12}) d\Omega_1 d\Omega_2,$$

where $N$ and $\Omega$ denote the number and the surface number density of the galaxies in the subregion of area $\Omega$, and $w(\theta_{12})$ is the angular correlation function of galaxies, with $\theta_{12}$ being the angular separation between two solid angle elements, $d\Omega_1$ and $d\Omega_2$. The first term in Equation (4) denotes the Poisson noise, whereas the second term comes from galaxy clustering.

For definiteness, we adopt the double power-law model (Scranton et al. 2002; Fukugita et al. 2004) for $w(\theta_{12})$:

$$w(\theta_{12}) = \begin{cases} 0.008(\theta_{12}/1^\text{deg})^{-0.75} & (\theta_{12} \leq 1 \text{ deg}) \\ 0.008(\theta_{12}/1^\text{deg})^{-2.1} & (\theta_{12} > 1 \text{ deg}) \end{cases}.$$  

Strictly speaking, the integration in the second term of Equation (4) should be performed over a complex and disjoint shape of each subregion. For simplicity, however, we substitute the integration over a circular region whose area is equal to that of the actual subregion. Although this approximation may overestimate the true error, it does not affect our conclusion at all. For the typical values of $\Omega \sim 100$ deg$^2$ and $S \sim 480$ deg$^{-2}$, we find that the second term is larger by two orders of magnitude than the first Poisson noise term.

Figure 3 suggests that the SFD correction works well in relatively high-extinction regions, i.e., $A_r$$_{SFD}$ > 0.1; before being corrected for extinction, the surface number density of the galaxy, $S_{gal}$, monotonically decreases against $A_r$$_{SFD}$, as naturally expected. It becomes roughly constant within the statistical error after extinction correction.

In low-extinction regions ($A_r$$_{SFD}$ < 0.1), however, the uncorrected $S_{gal}$ increases with $A_r$$_{SFD}$, which is opposite to the behavior expected from the Galactic dust extinction. The anomalous positive correlation between surface number densities and extinction is even more enhanced after the extinction correction. Apart from the slight quantitative differences, these results are consistent with the trend discovered for the SDSS DR4 by Yahata et al. (2007), especially for the positive correlations in $A_r$$_{SFD}$ < 0.1.

Yahata et al. (2007) argued that the trend is due to the presence of the FIR emission of galaxies, which contaminates the $100 \mu m$ flux of IRAS that is conventionally ascribed to the Galactic dust entirely. Indeed, their hypothesis is now directly confirmed by the stacking analysis of Kashiwagi et al. (2013), who detected the unambiguous signature of the FIR emission from SDSS galaxies in the SFD map. Our next task, therefore, is to ask if...
the detected nature of the FIR emission of galaxies by Kashiwagi et al. (2013) properly accounts for the anomaly that we described here. In what follows, we consider the surface number density of the galaxies measured in the central SDSS passband, and the result is equally applicable to any of the other passbands.

4. MOCK NUMERICAL SIMULATION TO COMPUTE THE FIR CONTAMINATION EFFECT OF GALAXIES ON THE EXTINCTION MAP

In this section, we present the results of mock numerical simulations that take into account the effect of the FIR emission of mock galaxies in a fairly straightforward manner. First we randomly place mock galaxies over the SDSS DR7 sky area so that they have the same number density and the same r-band magnitude distribution of the SDSS DR7 sample. Next, we assign a 100 $\mu$m flux to each mock galaxy, according to the probability distribution function (PDF) discussed in Section 4.1. We sum up the 100 $\mu$m fluxes of the mock galaxies over the raw SFD map that is assumed to be not contaminated by the FIR emission of mock galaxies and construct a contaminated mock extinction map. Finally, we compute the surface number densities of mock galaxies exactly as we did for the real galaxy sample. Further details are described below.

4.1. Empirical Correlation Between 100 $\mu$m and r-band Luminosities of PSCz/SDSS Galaxies

In order to assign 100 $\mu$m emission to each mock galaxy with a given r-band magnitude, we need an empirical relation between the two luminosities, $L_{100\mu m}$ and $L_r$. For that purpose, Yahata (2007) created a sample of galaxies detected both in SDSS and in PSCz (IRAS Point Source Catalog Redshift Survey; Saunders et al. 2000). To be more specific, he searched for SDSS galaxies within 2 arcmin from the position of each PSCz galaxy and selected the brightest one as the optical counterpart. Approximately 95% of the PSCz galaxies within the SDSS survey region have SDSS counterparts, and the resulting sample consists of 3304 galaxies in total. Note, however, that the sample is biased toward the FIR luminous galaxies because the SDSS optical magnitude limit is significantly deeper than that of the PSCz galaxies.

The left panel of Figure 4 shows the relation between $L_{100\mu m}$ (PSCz) and $L_r$ (SDSS) of the PSCz/SDSS overlapped sample. For K correction, we use the “K corrections calculator” service (Chilingarian et al. 2010) for the r band and extrapolate the FIR flux at 100 $\mu$m from the second-order polynomials, using 25 and 60 $\mu$m fluxes (Takeuchi et al. 2003).

The resulting scatter plot indicates that $L_{100\mu m}$ and $L_r$ are approximately proportional, albeit with considerable scatter. Therefore we compute the PDF of the luminosity ratio,

$$y \equiv \frac{L_{100\mu m}}{L_r},$$

for the sample (solid histogram in Figure 5) and find that the PDF is reasonably well described by a log-normal distribution:

$$P_{\text{ratio}}(y)dy = \frac{1}{y \ln 10 \sqrt{2\pi \sigma^2}} \exp \left[-\frac{(\log_{10} y - \mu)^2}{2\sigma^2}\right] dy,$$

where $\mu = 0.393$ and $\sigma = 0.428$ are the mean and dispersion of $\log_{10} y$ (solid curve in Figure 5).

Because the PSCz/SDSS overlapped sample is a biased sample in the sense that these galaxies are selected toward the FIR luminous galaxies, the above log-normal distribution is not necessarily applicable for the entire SDSS galaxies. Therefore we assume the FIR-optical luminosity ratio of the entire SDSS galaxy sample. Further details are described below.

![Figure 3](image-url)  
Figure 3. Surface number densities of the SDSS DR7 photometric galaxy sample corresponding to Figure 1, against $A_{r,SFD}$. The circles/triangles indicate the surface number densities calculated with extinction-uncorrected/corrected magnitudes, respectively. The statistical errors are calculated from Equation (4). The horizontal axis is the mean of $A_{r,SFD}$ over the galaxies in each subregion.

![Figure 4](image-url)  
Figure 4. The right panel shows the relation between $L_{100\mu m}$ (PSCz) and $L_r$ (SDSS) of the PSCz/SDSS overlapped sample. For K correction, we use the “K corrections calculator” service (Chilingarian et al. 2010) for the r band and extrapolate the FIR flux at 100 $\mu$m from the second-order polynomials, using 25 and 60 $\mu$m fluxes (Takeuchi et al. 2003).

![Figure 5](image-url)  
Figure 5. The resulting scatter plot indicates that $L_{100\mu m}$ and $L_r$ are approximately proportional, albeit with considerable scatter. Therefore we compute the PDF of the luminosity ratio,
galaxies also follows a log-normal distribution and estimate the values of $\mu$ and $\sigma$ for the entire sample by considering the PSCz detection limit. Although the flux limit of PSCz is defined through $f_{100,\mu m} > 0.6\,Jy$, we roughly estimate the corresponding effective flux limit at 100 $\mu m$ is $f_{100,\mu m} > 1.0\,Jy$ from the distribution of $f_{100,\mu m}$ for the PSCz/SDSS galaxies (left panel of Figure 4).

Armed with these assumptions, the number of the galaxies that are detected by this flux cut and have luminosity between $L_r \sim L_r + dL_r$ and $L_{100,\mu m} \sim L_{100,\mu m} + dL_{100,\mu m}$ is calculated as

$$N_{\text{obs}}(L_r, L_{100,\mu m})dL_r dL_{100,\mu m} = \frac{\Omega_2}{4\pi} \int_{0}^{\infty} \frac{dz}{dz} \frac{dV(<z)}{dV} \Theta(L_{100,\mu m}, z) \times \Phi(L_r) P(L_{100,\mu m}|L_r; \mu, \sigma) dL_r dL_{100,\mu m},$$

where $\Omega_2$ is the solid angle of the PSCz/SDSS overlapped survey area and $V(<z)$ denotes the comoving volume up to redshift $z$. The step function $\Theta(L_{100,\mu m}, z)$ describes the flux cut of PSCz:

$$\Theta(L_{100,\mu m}, z) = \begin{cases} 1 & (L_{100,\mu m}/4\pi d_L^2(z) > 1.0\,Jy) \\ 0 & (\text{else}) \end{cases}.$$

where $d_L(z)$ is the luminosity distance at redshift $z$.

We adopt the double-Schechter luminosity function in the $r$ band, measured from the SDSS DR2 data (Blanton et al. 2005) for $\Phi(L_r)$:

$$\Phi(L_r)dL_r = \frac{dL_r}{L_{r,s}} \exp \left(-\frac{L_r}{L_{r,s}}\right) \times \left[ \phi_{s,1} \left(\frac{L_r}{L_{r,s}}\right) + \phi_{s,2} \left(\frac{L_r}{L_{r,s}}\right)^{\alpha_2} \right].$$

The conditional-probability-density function of $L_{100,\mu m}$ for a given $L_r$ is assumed to be log-normal:

$$P(L_{100,\mu m}|L_r; \mu, \sigma) dL_{100,\mu m} = \frac{1}{\ln 10 \sqrt{2\pi \sigma^2}} \exp\left(-\frac{\ln(L_{100,\mu m}/L_{100,\mu m})^2}{2\sigma^2}\right) \frac{dL_{100,\mu m}}{L_{100,\mu m}}.$$  

We use Equation (8) to find the best-fit $\mu$ and $\sigma$ in Equation (11) for the entire SDSS galaxies that reproduce the observed distribution of the PSCz/SDSS overlapped sample. The resulting values are $\mu = -0.662$ and $\sigma = 0.559$, as plotted in the blue dot-dashed line in Figure 5. This result indicates that the mean value of $y$ of the PSCz/SDSS overlapped sample is biased by an order of magnitude relative to that for the entire galaxies; see Equations (15) and (16).

Now, adopting the best-fit log-normal distribution, the luminosity function at 100 $\mu m$ is calculated as

$$\Phi(L_{100,\mu m}) = \int_{0}^{\infty} dL_r \Phi(L_r) P(L_{100,\mu m}|L_r; \mu, \sigma).$$

As plotted in Figure 6, the above best fit indeed agrees well with the luminosity function independently measured from the PSCz data (Serjeant & Harrison 2005).
Equations (10) and (11). Then we exclude those mock galaxies of Figure 4 and the dashed histogram in Figure 5 show the resulting luminosity distribution and the PDF of $\mu$ for those mock galaxies. Although not perfect, the mock galaxies reproduce the extent to which SDSS galaxies in that magnitude range alone account for the observed anomaly in their surface number density.

In order to make sure if the above FIR log-normal PDF, combined with the FIR flux cut, reproduces the left panel of Figure 4, we generate mock galaxies and assign $z$, $L_r$, and $L_{100\mu m}$, following the redshift distribution $dV/\Delta z$, and Equations (10) and (11). Then we exclude those mock galaxies with $f_{100\mu m} < 1.0$ Jy, to mimic the flux cut. The right panel of Figure 4 and the dashed histogram in Figure 5 show the resulting luminosity distribution and the PDF of $\gamma$ for those mock galaxies. Although not perfect, the mock galaxies reproduce the observed distribution reasonably well. We suspect that the discrepancy between the observed data and the mock simulation is mainly due to the limitation of our log-normal approximation, neglecting the dependence of the ratio $L_{100\mu m}/L_{60\mu m}$ on $L_{100\mu m}$.

For simplicity of the procedure, however, we adopt the best-fit log-normal distribution as the fiducial model of the 100 $\mu$m flux of SDSS galaxies in what follows. In doing so, we parametrize the distribution by $y_{\text{avg}}$ and $y_{\text{rms}}$ instead of $\mu$ and $\sigma$:

$$y_{\text{avg}} = e^{\mu \ln 10 + \sigma \ln 10}/2,$$  

$$y_{\text{rms}} = e^{\mu \ln 10 + \sigma \ln 10}/2 \sqrt{e^{\sigma \ln 10} - 1},$$  

because the anomaly is basically determined by $y_{\text{avg}}$, as will be shown in Figure 9 below. For definiteness, the PSCz/SDSS overlapped sample is characterized by

$$\mu = 0.393, \sigma = 0.428, y_{\text{avg}} = 4.015, y_{\text{rms}} = 5.143,$$  

whereas the entire SDSS sample is estimated to have

$$\mu = -0.662, \sigma = 0.559, y_{\text{avg}} = 0.499, y_{\text{rms}} = 1.026.$$  

4.2. Simulations

We are now in a position to present our mock simulations, which exhibit the effect of the FIR contamination of galaxies.

In this section, we neglect the spatial clustering of galaxies and consider the case for Poisson-distributed mock galaxies. The effect of spatial clustering of galaxies will be discussed separately in Section 7.1. Our mock simulations are performed as follows.

1. We distribute random particles as mock galaxies over the SDSS DR7 survey area. The number of the mock galaxies is adjusted so as to approximately match that of the SDSS photometric galaxies.

2. We assign an intrinsic apparent magnitude in the $r$ band to each mock galaxy so that the resulting magnitude distribution reproduces that of SDSS galaxies (Figure 1).

3. We assign 100 $\mu$m flux to each mock galaxy, adopting the log-normal PDF for the 100 $\mu$m-to-$r$-band flux ratio, $\gamma$. The PDF is characterized by $y_{\text{avg}}$ and $y_{\text{rms}}$.

4. We convolve the 100 $\mu$m fluxes of the mock galaxies with a FWHM = 5.2 Gaussian filter, so as to mimic the SFD resolution, FWHM = 6.1 (see also Appendix A). Those mock galaxies with 100 $\mu$m flux being larger than 1.0 Jy are excluded, because SFD individually subtracted the 100 $\mu$m emission of those bright galaxies. We include only the contribution of the mock galaxies with $17.5 < m_r < 19.4$, so as to be consistent with our analysis in Section 3.2. We note, however, that in reality the FIR contamination would be likely contributed by galaxies outside the magnitude range (not only SDSS galaxies but non-SDSS galaxies that do not satisfy the SDSS selection criteria). Therefore the current mock simulation should be interpreted to see the extent to which SDSS galaxies in that magnitude range alone account for the observed anomaly in their surface number density.

5. We superimpose the 100 $\mu$m intensity of the mock galaxies on a true extinction map and construct a contaminated extinction map after subtracting the background (i.e., mean) level of the mock galaxy emission. In what follows, the resulting extinction with mock galaxy contaminated is denoted as $A'_r$.

6. Finally, we calculate $S_{\text{mock}}$, surface number densities of mock galaxies whose corrected/uncorrected magnitudes lie between 17.5 and 19.4 mag, repeating the same procedure discussed in Section 3, but using $A'_r$ instead.

Note that our mock analysis uses the SFD map as the true extinction map, without being contaminated by FIR emission of mock galaxies. Of course, the SFD map is contaminated by FIR emission from real galaxies and thus cannot be regarded as a true extinction map for them. Nevertheless, the contamination of real galaxies should not be correlated at all with the mock galaxies. This is why the SFD map can be used as the true extinction map for the current simulation.

The observed magnitude of each mock galaxy, i.e., affected by the Galactic dust absorption alone, is calculated from the true, in the present case the SFD map, but the extinction correction is done using $A'_r$. Note that the difference between the true map and the contaminated map affects the value of extinction of regions where mock galaxies are located. Therefore, surface number densities of mock galaxies before the extinction correction are also influenced by the FIR contamination.

Figure 7 shows the surface number densities of mock galaxies as a function of $A'_r$. Here we adopt $y_{\text{avg}} = 0.499$ and $y_{\text{rms}} = 1.026$—i.e., Equation (16)—which are estimated for the entire SDSS galaxy sample. The quoted error bars in the panel reflect the Poisson noise alone. The results exhibit a similar, but
intuitively. (In this plot, we have adopted area and the SFD map. This also implies that the shape of the area is significantly enhanced due to the nature of the SDSS sky with assigned magnitude of 17 < $m_r < 19.4$. The symbols are the same as in Figure 3. The values of $y_{\text{avg}}$ and $y_{\text{rms}}$ estimated for the entire SDSS galaxies are adopted, instead of those for the PSCz/SDSS overlapped sample. The error bars reflect the Poisson noise alone.

significantly weak correlation with $A_{r,SFD}$ at $A_{r,SFD} < 0.1$, compared to the observed one (Figure 3), especially for the extinction-uncorrected surface densities.

Figure 8 would help us to understand the origin of the anomaly intuitively. (In this plot, we have adopted $y_{\text{avg}} = 10$ and $y_{\text{rms}} = 5$ just to clearly visualize the trends discussed in the following.) The dashed line indicates the differential distribution of the sky area as a function of $A_{r,SFD}$, $\Omega(A_{r,SFD})$, which corresponds to the derivative of the left panel of Figure 2. The black solid line shows the same distribution, but as a function of $A_r$. The resulting $\Omega(A_r)$ slightly differs from $\Omega(A_{r,SFD})$ due to the FIR contamination of the mock galaxies.

The blue and red solid lines in Figure 8 show the differential number counts of galaxies, $N'_{\text{gal,uncorr}}$ and $N'_{\text{gal,corr}}$, as a function of $A_r'$, calculated from magnitudes uncorrected/corrected for extinction with $A_r'$. The shapes of $N'_{\text{gal,uncorr}}$ and $N'_{\text{gal,corr}}$ are slightly shifted toward the right, relative to $\Omega(A_r')$, because the pixels with more galaxies suffer from larger contamination and thus have larger values of $A_r'$.

Although the amount of this shift is quite small on average, the differences between $\Omega$ and the differential number counts for the same $A_r'$ become larger in low-extinction regions because $\Omega$ is a rapidly increasing function of $A_r'$. Therefore the surface number densities, $N'_{\text{gal,uncorr}}$ or $N'_{\text{gal,corr}}$, divided by $\Omega$, drastically change, especially in low-extinction regions. In other words, the correlation between the surface number densities and $A_r'$ is significantly enhanced due to the nature of the SDSS sky area and the SFD map. This also implies that the shape of the anomaly in $S_{\text{gal}}$ is basically determined by the functional form of $\Omega(<A)$.

We also investigate how this result is affected by the 100 $\mu$m emission of galaxies outside the magnitude range. We incorporate the 100 $\mu$m flux of mock galaxies within a wider magnitude range ($15.0 < m_r < 21.0$), but the result is almost indistinguishable. This is mainly because the additional contamination is not directly correlated with the surface number densities that we measure, partly because we neglect spatial clustering of galaxies. Therefore, the 100 $\mu$m emission of the galaxies outside the magnitude range does not systematically affect the anomaly of the surface number densities, but contributes only to the statistical noise in the extinction map.

Finally, we examine the dependence of the surface number densities on the parameters of $y_{\text{avg}}$ and $y_{\text{rms}}$ for the log-normal PDF of $y$ (Figure 9). The results indicate stronger correlations for larger $y_{\text{avg}}$, but turn out to be relatively insensitive to $y_{\text{rms}}$. This is why we choose $y_{\text{avg}}$ and $y_{\text{rms}}$ instead of $\mu$ and $\sigma_y$ to parameterize the log-normal PDF. A closer look reveals that larger $y_{\text{rms}}$ shows slightly weaker anomaly, because a larger fraction of the mock galaxies are brighter than the IRAS/PSCz flux limit, and does not contribute to FIR contamination. This effect of the flux limit becomes critical for very large $y_{\text{avg}}$ and $y_{\text{rms}}$, as we will see in Section 6.1.

As seen above, the mock result adopting Equation (16) estimated for the entire sample of SDSS galaxies (Figure 7) indicates disagreement with the observed anomaly (Figure 3). This result may appear to imply that the hypothesis of galaxy FIR contamination fails to explain the observed anomaly. This is, however, not the case because we have neglected the spatial clustering of galaxies. The previous parameters for the entire SDSS are estimated from the contribution of each single galaxy itself, but in the presence of galaxy clustering, the FIR emission associated with that galaxy can be significantly enhanced by the neighboring galaxies. In fact, the stacking analysis on the SFD map revealed that the FIR emission of neighboring galaxies dominates the central galaxy, even by an order of magnitude (Kashiwagi et al. 2013). Therefore, we should adopt $y_{\text{avg}}$ and $y_{\text{rms}}$, which represent the total contribution both for each single galaxy and clustering neighboring galaxies, in order to reproduce the observed anomaly by our Poisson mock simulation.

In principle, we can probe such FIR fluxes from the comparison between mock simulations and observations, but the simulations are very time consuming. Thus in the next sections we develop an analytical model that reproduces the mock results.
5. ANALYTICAL MODEL OF THE FIR CONTAMINATION

In this section, we develop an analytical model that describes the anomaly of surface number densities of galaxies due to their FIR emission. The reliability of the analytical model is checked against the result of the numerical simulations presented in the previous section. We present a brief outline in the next section, and the details are described in Appendix B.

5.1. Outline

Let $A$ define the true Galactic extinction, not contaminated by the galaxy emissions. We denote the sky area whose value of the true extinction is between $A$ and $A + dA$ by $\Omega(A)dA$ and the number of galaxies that are located in the area $\Omega(A)dA$ by $N_{\text{gal}}(A)dA$. Because there is no spatial correlation between galaxies and the Galactic dust, the corresponding surface number densities of the galaxies as a function of $A$: $S(A) \equiv \frac{N_{\text{gal}}(A)}{\Omega(A)}$ should be independent of $A$ and constant within the statistical error.

If the FIR emission from galaxies contaminates the true extinction, however, the above quantities should depend on the contaminated extinction, $A'$, which are defined as $\Omega'(A')$ and $N'_{\text{gal}}(A')$, respectively. Thus the observed surface number densities, $S'(A')$, should be

$$S'(A') = \frac{N'_{\text{gal}}(A')}{\Omega'(A')}.$$  \hspace{1cm} (18)

The essence of our analytical model is how to compute the expected $\Omega'(A')$ and $N'_{\text{gal}}(A')$ in the presence of the FIR contamination of galaxies, which are distorted from the given true $\Omega(A)$ and $N_{\text{gal}}(A)$.

Due to its angular resolution, the extinction of the SFD map at a given position is contaminated by the FIR emission of multiple galaxies located within the angular resolution scale. Thus we need to sum up the FIR emission contribution of those galaxies located:

$$A' = A + \Delta A,$$ \hspace{1cm} (19)

where the additional extinction, $\Delta A$, is computed by summing up the contribution of the $i$th galaxies ($i = 1 \sim N$) located in the pixel:

$$\Delta A = \sum_{i=1}^{N} \Delta A_i.$$ \hspace{1cm} (20)
In order to perform the summation analytically, we need a joint PDF, $P_{\text{joint}}(\Delta A, N)$, corresponding to the situation where there are $N$ galaxies in a pixel of the dust map, and the total contribution of those galaxies is $\Delta A$. In Appendix B, we present a prescription to compute $P_{\text{joint}}(\Delta A, N)$ and provide the integral expressions for $\Omega(A')$ and $N'_{\text{gal}}(A')$.

### 5.2. Application of the Analytical Model

The analytical expressions for $\Omega(A')$, $N'_{\text{gal,corr}}(A')$, and $N'_{\text{gal,uncorr}}(A')$ are given in Equations (B8), (B19), and (B20) in Appendix B. Thus one can compute the surface number densities for the $i$th subregion of extinction between $A_i$ and $A_{i+1}$ as

$$S'_{\text{corr},i} = \frac{\int_{A_i}^{A_{i+1}} N'_{\text{gal,corr}}(A')dA'}{\int_{A_i}^{A_{i+1}} \Omega(A')dA'},$$

$$S'_{\text{uncorr},i} = \frac{\int_{A_i}^{A_{i+1}} N'_{\text{gal,uncorr}}(A')dA'}{\int_{A_i}^{A_{i+1}} \Omega(A')dA'},$$

where $S'_{\text{corr}}$ and $S'_{\text{uncorr}}$ are the extinction-corrected and extinction-uncorrected surface number densities, respectively. The solid lines in Figure 9 show the surface number densities calculated from Equations (21) and (22), adopting nine parameter sets of $y_{\text{avg}}$ and $y_{\text{rms}}$. The horizontal axis, an average extinction in each subregion, is calculated as

$$A'_{\text{corr},i} = \frac{\int_{A_i}^{A_{i+1}} A'N'_{\text{gal,corr}}(A')dA'}{\int_{A_i}^{A_{i+1}} N'_{\text{gal,corr}}(A')dA'},$$

$$A'_{\text{uncorr},i} = \frac{\int_{A_i}^{A_{i+1}} A'N'_{\text{gal,uncorr}}(A')dA'}{\int_{A_i}^{A_{i+1}} N'_{\text{gal,uncorr}}(A')dA'},$$

Figure 9 clearly indicates that the analytic predictions and the simulation results are in good agreement. Strictly speaking, the agreement is not perfect in the sense that the reduced $\chi^2$ is as large as $\sim 3.5$ for the worst cases, when only the Poisson noise is considered. The statistical errors for the observed SDSS surface number densities (Figure 3), however, include the variance due to spatial clustering and are larger than the Poisson noise by an order of magnitude. Thus the discrepancy between the mock simulation and the analytical model is negligible for the parameter-fit analysis to the observational result in the following section.

### 6. COMPARISON OF FIR CONTAMINATION WITH THE OBSERVED ANOMALY

Given the success of the analytical model described above, we attempt to find the best-fit parameters, $y_{\text{avg}}$ and $y_{\text{rms}}$, to the observed anomaly by minimizing

$$\chi^2(y_{\text{avg}}, y_{\text{rms}}, \tilde{N}) = \sum_i \frac{(S'_{\text{uncorr},i} - S'_{\text{corr},i})^2}{\sigma_{\text{obs},i}^2},$$

where $S'_{\text{uncorr},i}$ are the extinction-uncorrected surface number densities in the $i$th subregion of extinction, $\sigma_{\text{obs},i}$ is its statistical errors, and $S'_{\text{corr}i} = S'_{\text{uncorr},i}(y_{\text{avg}}, y_{\text{rms}}, \tilde{N})$ is the analytical model prediction given by Equation (22). In the present fit, we use the extinction-uncorrected surface number densities, but the result is almost the same even if we use $S'_{\text{corr}}$ instead. In addition to $y_{\text{avg}}$ and $y_{\text{rms}}$, we include another free parameter, the intrinsic average number of galaxies in a pixel, $N$, which is also unknown because the extinction correction is not necessarily reliable. It turns out that $\tilde{N}$ is in the range of 480 to 500 [deg$^{-2}$], and the results below are not sensitive to this value.

In reality, however, the resulting constraints are not so strong, as is shown in the top left panel in Figure 10. This is partly due to the fact that we simply compute $\sigma_{\text{obs},i}$ from the variance of each extinction bin, which does not represent the proper error. Thus our analysis here should be interpreted as a qualitative attempt to find a possible parameter space to explain the anomaly in terms of the FIR contamination; it would be quite difficult to make a more quantitative analysis, given several crude approximations in our theoretical modeling and the poor angular resolution and uncertain dust temperature correction in the SFD map.

Bearing this remark in mind, let us consider the constraints on the $y_{\text{avg}}$--$y_{\text{rms}}$ plane from the observed anomaly shown in the top left panel of Figure 10. Fairly acceptable fits are obtained over the bluish region. Just for illustration, we select two widely separated points A and B with $(y_{\text{avg}}, y_{\text{rms}}) = (30, 8000)$ and $(3.8, 4.0)$, respectively, and plot the corresponding analytical predictions in the other three panels. Even though their $y_{\text{avg}}$ is different by an order of magnitude, the two sets of parameters account for the observed anomaly reasonably and equally well.

#### 6.2. Comparison with the Stacking Image Analysis

We have shown that the anomaly in the surface number densities of SDSS galaxies on the SFD map is well reproduced by assuming their 100 $\mu$m to $r$-band flux ratio is $\sim 3.8$ on average, where the 100 $\mu$m flux includes the contribution of
neighboring galaxies. On the other hand, the flux ratio of a single galaxy is estimated as $\sim 0.5$ (see Section 4.1).

Indeed, these values should be compared with the result of the stacking image analysis by Kashiwagi et al. (2013). They stacked SDSS galaxies on the SFD map and found that a galaxy of $r$-band magnitude $m_r$ contributes to the extinction on average by

$$\Delta A^s_r(m_r) = 0.087 \times 10^{0.41(18-m_r)} \text{[mmag]},$$

by itself (single term), and

$$\Delta A^{tot}_r(m_r) = 0.64 \times 10^{0.17(18-m_r)} \text{[mmag]},$$

including the contribution from neighboring galaxies, corresponding to the clustering term in Kashiwagi et al. (2013). The above extinction due to the 100 $\mu$m emission from galaxies is translated into its 100 $\mu$m to $r$-band flux ratio as

$$y = \frac{2\pi \sigma^2}{f_r v_r / v_{100 \mu m}} \frac{\Delta A_r}{K_r P},$$

where $\sigma$ is the Gaussian PSF width and $f_r$ is the $r$-band flux. Thus integrated over the differential number density, Equations (26) and (27) lead to

$$y_{av} = \int y \text{d}N/dA,$$
and (27) suggest that

\[ \bar{y}_{\text{avg}} = \frac{\int dm_r(dN/dm_r)\bar{y}_{\text{avg}}(m_r)}{\int dm_r(dN/dm_r)} = 0.239, \quad (29) \]

and

\[ \bar{y}_{\text{avg}}^\text{tot} = \frac{\int dm_r(dN/dm_r)y_{\text{avg}}^\text{tot}(m_r)}{\int dm_r(dN/dm_r)} = 2.77, \quad (30) \]

respectively.

These values are based on the direct measurement of the FIR contamination and are thus independent of the modeling of 100 μm to optical relation. We also emphasize that they automatically include possible contributions from those galaxies not identified by SDSS. Therefore the sum of the two can be reliably interpreted as the expected contribution of SDSS galaxies to \( y_{\text{avg}} \), including neighboring galaxies, which is plotted in Figure 10. Although we do not know the corresponding \( y_{\text{rms}} \), we have already found that the dependence of the anomaly on \( y_{\text{rms}} \) is rather weak, at least in our analytical model. Thus the empirical value of \( y_{\text{avg}} \) from the stacking analysis roughly explains the observed anomaly, as plotted in the three panels of Figure 10.

We interpret this agreement as supporting evidence for the FIR model of the SFD anomaly, given the fact that we assume a very simple relation between 100 μm and optical luminosities, neglecting the galaxy morphology dependence that certainly leads to the FIR flux difference.

6.3. Estimates of Clustering Contribution of SDSS Galaxies

We tried to independently estimate \( y_{\text{avg}} \), including an additional contribution of neighboring galaxies, using the SDSS galaxy distribution over the SFD map, instead of the stacking result by Kashiwagi et al. (2013) discussed in Section 6.2. We first randomly assign the FIR flux of SDSS galaxies, assuming \( (y_{\text{avg}}, y_{\text{rms}}) = (0.5, 1.0) \) for each SDSS galaxy itself, neglecting the clustering term. Second, we sum up the FIR fluxes of galaxies convolved with the PSF of the SFD map (the Gaussian width of 3:1) centered at each galaxy. Finally, we compute \( y_{\text{avg}} \) and \( y_{\text{rms}} \), using the summed FIR fluxes after subtracting the average background flux.

Note that the resulting values of \( y_{\text{avg}} \) and \( y_{\text{rms}} \) should be different from the above input values because of the contribution of the clustering term. We find \( y_{\text{avg}} \approx 2 \), but \( y_{\text{rms}} \) is not well determined because it turned out to be very sensitive to the choice of the background flux. This result indicates that the FIR flux of SDSS galaxies explains only half of those required to well reproduce the observed anomaly, \( y_{\text{avg}} = 3.8 \).

Indeed, employing \( y_{\text{avg}} \approx 2 \), our model still reproduces the anomaly qualitatively, but the predicted feature is substantially weaker than that of the observed one. The assigned FIR flux in this model, however, is based on the single galaxy contribution estimated in Section 4.1 (\( y_{\text{avg}} = 0.5 \)) and thus would be sensitive to the FIR assignment model. Given the fact that the empirical value from the stacking analysis, which is independent of such models, is fairly successful in reproducing the anomaly, we suspect that the factor of two difference originates from the limitation of our crude modeling for FIR flux, instead of from the basic flaw of the FIR explanation of the anomaly.

7. DISCUSSION

7.1. Effects of Spatial Clustering of Galaxies

Both the mock simulations and the analytical model discussed in the previous section completely ignore the spatial clustering of galaxies. We therefore examine the clustering effect on the anomaly in this section. The most straightforward method is to replace the Poisson-distributed mock galaxies by dark matter particles from the cosmological N-body simulation. For that purpose, we use a realization in the standard ΛCDM cosmology with \( \sigma_8 = 0.76 \) performed by Nishimichi et al. (2009).

We repeat similar mock observations, as discussed in Section 4.2, except we assign \( r \)-band luminosity to each mock galaxy instead of their apparent magnitude. To be specific, (1) we randomly assign \( r \)-band luminosities to all \( N \)-body dark matter particles, according to the luminosity function of Equation (10); (2) we convert their luminosities to apparent \( r \)-band magnitudes observed from a fixed observer position; and (3) we randomly select a fraction of the mock galaxies to match with the SDSS observed \( dN/dm_r \) (Figure 1).

We repeat the same fitting analysis as Figure 10, except that the data are now replaced by the mock result on the basis of the cosmological \( N \)-body simulation, with \( y_{\text{avg}} = 3.8 \) and \( y_{\text{rms}} = 4.75 \). The mock observation including the galaxy clustering effect result shows stronger anomaly than does the Poisson mock simulation with the identical \( y_{\text{avg}} \) and \( y_{\text{rms}} \). The analytical model that neglects the spatial clustering still reproduces the simulated anomaly very well, but the best-fit \( y_{\text{avg}} \) overestimates the real values employed in the simulation by a factor of \( \sim 2 \). Thus the clustering effect can be absorbed effectively by reinterpreting the best-fit values of \( y_{\text{avg}} \) appropriately. The clustering effect estimated here is largely consistent with the clustering term contribution estimated directly from SDSS galaxies (Section 6.3).

In order to quantitatively understand the relation between this bias and the strength of the galaxy spatial clustering, we have to incorporate the effect of spatial clustering in our analytical model. For that purpose, we measure the PDF of the number of the \( N \)-body mock particles in a pixel and replace the Poisson distribution in Equation (B2) with the measured one. The analytical model prediction, however, hardly changes by such a modification. Thus more sophisticated improvements seem to be needed to account for the spatial clustering effect, which are beyond the scope of this paper.

7.2. Limitation of the Correction for the FIR Emission of Galaxies

We attempt to correct the SFD map by subtracting the average FIR contamination of SDSS galaxies. The corrected extinction at an angular position \( \theta \) in the Galactic map is computed as

\[ A_r(\theta) = A_r(\theta; m_i^j) - \sum_j \Delta A(\theta_j - \theta; m_i^j), \quad (31) \]

where \( \theta_j \) is the position of the \( j \)-th galaxy with its \( r \)-band magnitude of \( m_i^j \). We employ four different values for \( \Delta A \), given the uncertainty of the interpretation of the best-fit value of \( y_{\text{avg}} \) discussed before. As shown in Figure 11, however, the above correction does not seem to remove the anomaly so well. This result may imply that the dependence of FIR properties on galaxy population, which is neglected in our modeling, is essential for accurate correction for the FIR contamination. As a future work, such a morphology dependence of FIR
In Section 6.2, we found that the observed anomaly of SDSS galaxies is roughly explained by the contamination of galaxy FIR emission. Nevertheless, the observed and predicted surface number densities (Figure 10) do not match perfectly, which might be attributed to other possible systematics in the SFD map.

In order to check the possible systematic effect, we use the improved extinction map by Peek & Graves (2010, hereafter PG). They found that the SFD map underpredicts extinction up to $\sim 0.1$ mag in the $r$ band, using the passively evolving galaxies as standard color indicators. Their method is complementary to our galaxy number count analysis in the sense that they directly measure the reddening by the Galactic dust. Because the resolution of the PG correction map to SFD is $4.5\degree$, the FIR fluctuations due to the emission of galaxies are not expected to be removed. The PG correction map, however, may have removed other systematics, besides the FIR contamination, which are not considered in our analytical model at all.

To see if their correction affects the number count analysis and the anomaly in the original SFD map, we repeat the same analysis described in Section 6, using the PG map. Basically, we find a very similar correlation between $S_{\text{gal}}$ and $\text{Ar}_{\text{PG}}$, suggesting that the PG map still suffers from the FIR contamination of galaxies, as expected. We note, however, that our analytical model prediction exhibits slightly better

![Figure 11. Surface number densities of SDSS galaxies with $17.5 < m_r < 19.4$ after subtracting their average FIR emission contamination, where $y_{avg} = 0.3, 1.0, 2.0, 3.8$ are adopted for estimation of the FIR emission of SDSS galaxies.](image)
agreement for the PG map than for the SFD map. This may indicate that possible systematic errors in the SFD map other than the FIR contamination are at least partially removed in the PG map.

8. SUMMARY AND CONCLUSIONS

In the present paper, we have revisited the origin of the anomaly of the surface number density of SDSS galaxies with respect to the Galactic extinction, originally pointed out by Yahata et al. (2007). We first computed the anomaly using the SDSS DR7 photometric catalogs and then developed both numerical and analytical models to explain the anomaly. We take account of the contamination of galaxies in the IRAS 100 μm flux that was assumed to come entirely from the Galactic dust.

Our main findings are summarized as follows.

1. Both numerical simulations and the analytical model reproduce the observed anomaly quite well. Thus we quantitatively confirmed the validity of the hypothesis that the observed anomaly in the SFD Galactic extinction map is mainly due to the FIR emission from galaxies, originally proposed by Yahata et al. (2007).

2. The comparison of the analytical model and the observed anomaly constrains mainly the average 100 μm to optical flux ratio for SDSS galaxies. The resulting value is in a reasonable agreement with that obtained from the stacking image analysis of SDSS galaxies by Kashiwagi et al. (2013).

3. We also independently estimated the FIR contribution of a single SDSS galaxy based on IRAS/SDSS overlapped catalog data, assuming a simple relation between FIR and optical luminosities. Summing up such FIR flux according to the SDSS galaxy distribution, however, we find that those contributions only explain roughly half of what is required to reproduce the observed anomaly. This result may be due to the limitation of our modeling of the FIR to optical relation.

Although our current analytical model still needs to be improved, the fact that the empirically determined value of Y_avg nicely reproduces the observed anomaly indicates that the FIR emission of SDSS galaxies is the major origin of the anomaly.

In particular, we note that subtracting the average FIR contamination of SDSS galaxies from the SFD extinction map does not properly remove the observed anomaly. This may imply that it is essential to consider the dependence of FIR emission on galaxy morphology and/or the effect of galaxy clustering, both of which we have neglected in the current analytical model. Because morphology and spatial clustering of galaxies are correlated in a complicated fashion, it is not easy to identify a good strategy for the correction method. We are currently working along this direction with the AKARI all-sky map data in 60, 90, 140, and 160 μm. The stacking image analysis of SDSS galaxies with the higher-angular-resolution map in multifrequency bands would enable us to estimate the FIR emission of galaxies as a function of their properties, including their color and morphology (T. Okabe et al., in preparation).

The FIR contamination, which explains the anomalous behavior in the surface number density of SDSS galaxies, is only statistical and tiny, on the order of (0.1 ~ 1) mmag of extinction in the r band, which is much less serious than naively expected from the anomaly. Nevertheless, the galaxy FIR emission is correlated with the large-scale structure of the universe. Thus it may systematically bias the cosmological analysis. The present methodology is, in principle, applicable to check the reliability—and even to improve the accuracy of—the future Galactic extinction map, which should play a key role in all astronomical observations, in particular for the purpose of precision cosmology.

We thank Brice Ménard, Tsunehito Kohyama, Yasunori Hibi, and Hiroshi Shibai for useful discussions. T.K. and Y.S. are grateful for the hospitality of the Department of Astrophysical Sciences, Princeton University, where most of the present work was performed. We also thank an anonymous referee for several constructive comments and in particular for suggesting to compute the expected FIR fluxes using the SDSS galaxy distribution, as discussed in Section 6.3.

T.K. is supported by a Global COE Program, “the Physical Sciences Frontier,” MEXT, Japan. T.N. is supported by a Grant-in-Aid for the JSPS fellows. Y.S. gratefully acknowledges the support from the Global Collaborative Research Fund “Worldwide Investigation of Other Worlds” grant, the Global Scholars Program of Princeton University, and the Grant-in-Aid for Scientific Research by JSPS (24340035). A.T. acknowledges the support from the Grant-in-Aid for Scientific Research by JSPS (24540257).

Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web site is http://www.sdss.org/. The SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences (LAMOST), Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory, and the University of Washington.

APPENDIX A

POINT-SPREAD FUNCTION

In the mock simulation (Section 4), we assign the FIR fluxes to the mock galaxies by modeling the PDF of the FIR to optical luminosity ratio, y. On the other hand, their contribution to the contamination in the SFD map is determined by their intensities as

\[ \Delta A_r = pk_r \frac{f_{100,\mu m}}{2\pi \sigma_{eff}^2}, \]  

where \( \sigma_{eff} \) is the Gaussian width of the effective PSF; thus, the impact of the FIR contamination directly depends on \( \sigma_{eff} \), even for mock galaxies with the same 100 μm fluxes, \( f_{100,\mu m} \). Due to the smoothing effects by the pixelization and interpolation of the SFD map, the effective PSF is degraded from that applied in the mock simulations (FWHM = 5′′), which is aimed at mimicking...
the purely instrumental PSF. Therefore, in order to precisely reproduce the mock simulation results by our analytical model (Section 5), we have to carefully evaluate the appropriate $\sigma_{\text{eff}}$ to be applied in Equation (B30). In this appendix, we derive $\sigma_{\text{eff}}$ as a function of the intrinsic PSF width, $\sigma_{\text{int}}$.

First we calculate the intensity of a single galaxy with a given 100 $\mu$m flux and position, taking into account the two smoothing effects. Hereafter, we assume that the pixels of the SFD map are squares with sides $\theta_{\text{pix}} = 2.372$. We denote the pixel of the SFD map, in which the galaxy is located, as $\Omega_i$ and its neighboring pixels as $\Omega_j$ to $\Omega_k$. We define the two-dimensional Cartesian coordinate system, $\theta = (\theta_x, \theta_y)$, whose origin is at the center of $\Omega_0$. The configuration of $\Omega_0$ to $\Omega_5$ is illustrated in the left panel of Figure 12. The intensity of the galaxy with 100 $\mu$m flux, $f$, in the pixel $\Omega_i$ ($i = 0, \ldots, 8$) is given as

$$I_i(\theta) = \frac{f}{2\pi \sigma_{\text{int}}^2 \Omega_{\text{pix}}} \int_{\Omega_i} \exp \left( -\frac{[\theta - \theta_i]^2}{2 \sigma_{\text{int}}^2} \right) d\theta,$$  

(A2)

where $\theta_i$ denotes the position of the galaxy and $\Omega_{\text{pix}} = \theta_{\text{pix}}^2$ is the area of the pixels. Because the value of the SFD map extinction is evaluated by the linear CIC interpolation, the intensity of the galaxy depends not only on $\theta_i$, but also on the position where the value is evaluated, $\theta$, and calculated as

$$I_{\text{CIC}}(\theta, \theta_i) = \left( 1 - \frac{\theta_{\text{pix}}}{\theta_{\text{pix}}} \right) I_i(\theta) + \frac{\theta_{\text{pix}}}{\theta_{\text{pix}}} I_{\text{CIC}}(\theta_i, \theta_i),$$

(A3)

where $(i_1, \ldots, i_4)$ are the indices of the nearest four pixels to $\theta$:

$$\begin{align*}
&\{\Omega_0, \Omega_1, \Omega_2, \Omega_3\} = \\
&\{\Omega_0, \Omega_1, \Omega_2, \Omega_3\} \\
&\{\Omega_0, \Omega_1, \Omega_2, \Omega_3\} \\
&\{\Omega_0, \Omega_1, \Omega_2, \Omega_3\}
\end{align*}$$

(A4)

Because the resulting effective PSF also depends on $\theta$ and $\theta_i$, we compute the PSF width appropriately averaged over $\theta$ and $\theta_i$ in the following. In our analytical model (Section 5), we compute the expected $\Omega'(A)$ and $N_{\text{gal}}'(A)$ under the presence of the FIR contamination of galaxies. We note that the effective PSF widths are slightly different for $\Omega'(A)$ and $N_{\text{gal}}'(A)$. This is because the extinction contaminated by the FIR intensities, $A'$, is always evaluated at the position of the galaxies, i.e., $\theta = \theta_i$, for $N_{\text{gal}}'(A)$, whereas this is not the case for $\Omega'(A)$. Therefore we separately derive the effective PSF widths for $\Omega'(A)$ and $N_{\text{gal}}'(A)$. We denote these effective PSF widths as $\sigma_{\text{eff}, \Omega}$ and $\sigma_{\text{eff}, N}$.

Now let us calculate $\sigma_{\text{eff}, \Omega}$. Because $\theta$ and $\theta_i$ are independent for computing $\Omega'(A)$, we calculate the intensity of galaxies averaged over $\theta$ and $\theta_i$ as

$$\bar{I} = \frac{1}{\Omega_{\text{pix}}} \int_{\Omega_0} d\theta \int_{\Omega_0} d\theta_i I_{\text{CIC}}(\theta, \theta_i).$$

(A5)

We define $\sigma_{\text{eff}, \Omega}$ as

$$\frac{f}{2\pi \sigma_{\text{eff}, \Omega}^2} \equiv \bar{I},$$

(A6)

and this leads to

$$\sigma_{\text{eff}, \Omega} = \frac{1}{\sqrt{\pi} \sigma_{\text{int}} 6F(s) - 5F(0) - 2F(-s) + F(-2s)},$$

(A7)

where

$$F(x) = \int \text{erf}(x) dx = x \text{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}.$$  

(A8)

$s = \theta_{\text{pix}}/\sqrt{2}\sigma_{\text{int}}$, and $\text{erf}(x)$ denotes the error function.

Similarly, considering that $\theta = \theta_i$, we define $\sigma_{\text{eff}, N}$ as

$$\frac{f}{2\pi \sigma_{\text{eff}, N}^2} \equiv \frac{1}{\Omega_{\text{pix}}} \int_{\Omega_0} I_{\text{CIC}}(\theta_i, \theta_i) d\theta_i.$$  

(A9)

Equation (A9) is reduced to

$$\sigma_{\text{eff}, N} = \frac{\Omega_{\text{pix}}}{\sqrt{8\pi R}}.$$  

(A10)
where
\[
\frac{2R}{\sigma_{\text{int}}^2} = \left[ J_1\left(\frac{\theta_{\text{pix}}}{2}\right) - J_2\left(\frac{\theta_{\text{pix}}}{2}\right) \right]^2 + 2J_1\left(\frac{\theta_{\text{pix}}}{2}\right) J_2\left(\frac{\theta_{\text{pix}}}{2}\right) - 2J_2\left(\frac{\theta_{\text{pix}}}{2}\right) J_1\left(\frac{\theta_{\text{pix}}}{2}\right) J_2\left(\frac{\theta_{\text{pix}}}{2}\right), \tag{A11}\]

\[
J_1(x) = \left[ F(b + s) - F\left(\frac{b + s}{2}\right) - F(b) + F\left(\frac{b - s}{2}\right) \right]. \tag{A12}\]

\[
J_2(x) = \frac{1}{s} \left[ G(b + s) - G\left(\frac{b + s}{2}\right) - G(b) + G\left(\frac{b - s}{2}\right) \right]
- \frac{1}{2s} \left[ F\left(\frac{b + s}{2}\right) - F\left(\frac{b - s}{2}\right) + \sqrt{\pi} \text{erf}(b + s) \right]
- \text{erf}\left(\frac{b + s}{2}\right) - \text{erf}(b) + \text{erf}\left(\frac{b - s}{2}\right). \tag{A13}\]

\[
G(x) = \int x \text{erf}(x) dx = \frac{1}{2} \left[ x^2 \text{erf}(x) + \frac{1}{\sqrt{\pi}} x e^{-x^2} - \frac{1}{2} \text{erf}(x) \right]. \tag{A14}\]

and \(b \equiv x / \sqrt{2\sigma_{\text{int}}}\).

The right panel of Figure 12 shows the Equations (A7) and (A10) as functions of \(\sigma_{\text{int}}\), which are adopted to Equation (B30) in the analytical model presented in Appendix B. In numerical simulations in Section 4, we adopted \(\sigma_{\text{int}} = 2.21\) which reproduces the effective resolutions \(\sigma_{\text{eff}, \Omega}\) and \(\sigma_{\text{eff}, N}\), both similar to the SFD angular resolution FWHM = 6.1.

**APPENDIX B**

**ANALYTICAL MODEL NEGLECTING SPATIAL CLUSTERING OF GALAXIES**

Assume that galaxies are randomly distributed over the pixel and denote the expected number of galaxies of the true (albeit unobservable) apparent magnitude \(m_{\text{true}}\), being \(m_{\text{min}} < m_{\text{true}} < m_{\text{max}}\) by \(N\). Then the probability that the pixel has \(N\) galaxies obeys the Poisson distribution:

\[
P_{\text{Poisson}}(N|\overline{N}) = \frac{\overline{N}^N \exp(-\overline{N})}{N!}. \tag{B1}\]

Here we assume that the areas of all of the pixels of the dust map are equal. Then the joint probability is the product of the conditional probability that the total FIR contamination in the pixel is \(\Delta A\), given that there are \(N\) galaxies, and the probability that the pixel has \(N\) galaxies:

\[
P_{\text{joint}}(\Delta A, N) = P_N(\Delta A) P_{\text{Poisson}}(N|\overline{N}). \tag{B2}\]

The conditional probability, \(P_N(\Delta A)\), can be computed recursively. When there is no galaxy in a pixel \((N = 0)\), \(\Delta A\) should vanish:

\[
P_0(\Delta A) = \delta_D(\Delta A), \tag{B3}\]

where \(\delta_D\) is the one-dimensional Dirac delta function. We compute \(P_1(\Delta A)\) from the differential number count of galaxy magnitude and the PDF of the FIR to \(r\)-band flux ratio, as discussed later in detail. Then \(P_N(\Delta A)\) for \(N \geq 2\) should satisfy the following recursive equation:

\[
P_N(\Delta A) = \int_0^{\Delta A} dx \ P_1(x) P_{N-1}(\Delta A - x). \tag{B4}\]

Finally, the PDF of the total contamination in a pixel, \(P(\Delta A)\), is given as

\[
P(\Delta A) = \sum_{N=0}^{\infty} P_{\text{joint}}(\Delta A, N) = \sum_{N=0}^{\infty} P_N(\Delta A) P_{\text{Poisson}}(N|\overline{N}). \tag{B5}\]

Note therefore that \(P_{\text{joint}}(\Delta A, N)\) and \(P(\Delta A)\) are computed in a straightforward fashion once the two inputs, \(P_1(\Delta A)\) and \(\overline{N}\), are specified from the observed data.

Next let us proceed to compute \(\Omega'(A') \) and \(N'(A')\) according to this model. Because SFD subtracted the mean FIR contamination in a pixel in constructing the map, we also subtract its theoretical counterpart:

\[
\overline{\Delta A} = \int_0^\infty d(\Delta A) \Delta A P(\Delta A), \tag{B6}\]

from the FIR contamination, \(\Delta A\), in each pixel. Therefore the extinction contaminated by the galaxy emission is now given by

\[
A' = A + \Delta A - \overline{\Delta A}. \tag{B7}\]

Therefore, the probability that a pixel with the true extinction \(A\) is observed as \(A'\) due to the FIR contamination is given by \(P(\Delta A) = P(A' - A + \Delta A)\). Finally, we obtain the expected observed distribution function of sky area, \(\Omega'(A')\), as

\[
\Omega'(A') = \int_0^\infty dA \int_0^\infty d(\Delta A) \Omega(A)
\times P(\Delta A) \delta_D(A' - (A + \Delta A - \overline{\Delta A}))
= \int_0^{A' + \overline{\Delta A}} dA' \Omega'(A') (A' - A + \overline{\Delta A}). \tag{B8}\]

We can similarly derive the expression for \(N'(A')\), the number distribution of the galaxies located in the pixels of the extinction \(A'\), as follows.

Because we assume that the area of each pixel is the same and equal to \(\Omega_{\text{pixel}}\), the number of pixels that have the true extinction in the range of \(A\) and \(A + dA\) is

\[
N_{\text{pixel}}(A)dA = \frac{\Omega(A) dA}{\Omega_{\text{pixel}}}. \tag{B9}\]

Thus the expected number distribution of galaxies in a pixel that suffers from the FIR contamination of \(\Delta A\) is

\[
\overline{N}(\Delta A) = \sum_{N=0}^{\infty} N P_{\text{joint}}(\Delta A, N). \tag{B10}\]

Therefore, the number distribution of galaxies, \(N_{\text{gal}}'(A')\), is given as

\[
N_{\text{gal}}'(A') = \int_0^\infty dA \int_0^\infty d(\Delta A) N_{\text{pixel}}(A) \overline{N}(\Delta A)
\times \delta_D(A' - (A + \Delta A - \overline{\Delta A}))
= \int_0^{A' + \overline{\Delta A}} d(\Delta A) N_{\text{pixel}}(A' - \Delta A + \overline{\Delta A}) \overline{N}(\Delta A). \tag{B11}\]
Although the above expression is correct for those galaxies with $m_{\text{min}} < m_{\text{true}} < m_{\text{max}}$, we cannot measure their true magnitude $m_{\text{true}}$ in reality, and one has to take into account the selection effect carefully. Consider a galaxy of $m_{\text{true}}$ located in a pixel of the contaminated extinction of $A'$. Then its observed (uncorrected) magnitude is

$$ m_{\text{uncorr}}(A') = m_{\text{true}} + A, $$ \hspace{1cm} (B12)

because its magnitude suffers from the true Galactic extinction, $A$, alone, instead of $A'$. This yields the corrected magnitude, relying on the contaminated extinction, $A'$:

$$ m_{\text{corr}}(A') = m_{\text{uncorr}}(A') - A' = m_{\text{true}} + A - A' $$ \hspace{1cm} (B13)

leading to the overcorrection by the amount of $\Delta A - \Delta A$.

Therefore, those galaxies with $m_{\text{min}} < m_{\text{corr}}(A') < m_{\text{max}}$ indeed correspond to

$$ m_{\text{min}} + (\Delta A - \Delta A) < m_{\text{true}} < m_{\text{max}} + (\Delta A - \Delta A). $$ \hspace{1cm} (B14)

In other words, the selection incorrectly excludes galaxies with $m_{\text{min}} < m_{\text{true}} < m_{\text{min}} + \Delta A - \Delta A$, and includes those with $m_{\text{max}} < m_{\text{true}} < m_{\text{max}} + \Delta A - \Delta A$ because of the contamination of FIR galaxy emission.

Given their differential number count with respect to magnitude, the number of such galaxies can be computed as

$$ N_{\text{ex,corr}}(\Delta A) = \int_{m_{\text{min}}}^{m_{\text{max}} + \Delta A - \Delta A} \frac{dn(< m)}{dm} dm, $$ \hspace{1cm} (B15)

$$ N_{\text{in,corr}}(\Delta A) = \int_{m_{\text{max}}}^{m_{\text{max}} + \Delta A - \Delta A} \frac{d\bar{n}(< m)}{dm} dm. $$ \hspace{1cm} (B16)

We adopt a power-law fit with a slope, $\gamma$ (see Figure 1), for the differential number counts of galaxies in a pixel that contains $N$ and $\bar{N}$ galaxies:

$$ \frac{dn(< m)}{dm} = \frac{N \gamma 10^{\gamma m}}{10^{\gamma m_{\text{max}}} - 10^{\gamma m_{\text{min}}}}, $$ \hspace{1cm} (B17)

$$ \frac{d\bar{n}(< m)}{dm} = \frac{\bar{N} \gamma 10^{\gamma m}}{10^{\gamma m_{\text{max}}} - 10^{\gamma m_{\text{min}}}}. $$ \hspace{1cm} (B18)

The included number should be normalized for the actual number of galaxies, $N$, instead of $\bar{N}$, in the pixel. Nevertheless, the included number is not correlated to $N$ in the Poisson-distributed assumption and thus should be normalized for $\bar{N}$.

Therefore we obtain finally the number distribution of galaxies after correcting for the contaminated extinction $A'$ as

$$ N'_{\text{gal,corr}}(A') = \int_{0}^{\infty} dA \int_{0}^{\infty} d(\Delta A) N_{\text{pixel}}(A) \times [\bar{N}(\Delta A) - \Delta A] \times \delta_{D}(A' - (A + \Delta A - \Delta A)) $$

$$ = \int_{0}^{\Delta A + \Delta A} d(\Delta A) N_{\text{pixel}}(A' - \Delta A) \times [\bar{N}(\Delta A) - \Delta A] \times \delta_{D}(A' - (A + \Delta A - \Delta A)). $$ \hspace{1cm} (B19)

Similarly, the number distribution of galaxies before correcting for the contaminated extinction, $A'$, i.e., with $m_{\text{min}} - A < m_{\text{true}} < m_{\text{max}} - A$, is given as

$$ N'_{\text{gal,uncorr}}(A') = \int_{0}^{\infty} dA \int_{0}^{\infty} d(\Delta A) N_{\text{pixel}}(A) \times [\bar{N}(\Delta A) - \Delta A] \times \delta_{D}(A' - (A + \Delta A - \Delta A)), $$ \hspace{1cm} (B20)

where

$$ N_{\text{ex,uncorr}}(A) = \int_{m_{\text{max}} - A}^{m_{\text{max}}} \frac{dn(< m)}{dm} dm, $$ \hspace{1cm} (B21)

and

$$ N_{\text{in,uncorr}}(A) = \int_{m_{\text{min}} - A}^{m_{\text{max}} - A} \frac{d\bar{n}(< m)}{dm} dm. $$ \hspace{1cm} (B22)

In order to proceed further, we need an expression for the PDF of the FIR contamination due to a single galaxy, $P_{r}(A)$. The mock simulations presented in Section 4 convert the r-band magnitude, $m_{r}$, of each mock galaxy into its 100 $\mu$m flux from the FIR/optical luminosity ratio, $\gamma$, as

$$ f_{100 \mu m}(m_{r}, y) = y f_{0} 10^{-0.4 m_{r}}, $$ \hspace{1cm} (B23)

where $f_{0} = 3631$ Jy and $y$ is assumed to obey the log-normal PDF $P_{\text{ratio}}$ given by Equation (7). In the present analytical model, we further assume that the number distribution of galaxies in the $r$ band obeys

$$ P_{\text{mag}}(m_{r}) = \frac{\gamma r 10^{\gamma m_{r}}}{10^{\gamma m_{\text{max}}} - 10^{\gamma m_{\text{min}}}}. $$ \hspace{1cm} (B24)

where $m_{r,\text{max}}$ and $m_{r,\text{min}}$ denote the upper and lower limits of the magnitude and $\gamma r$ is the power-law index.

Once $P_{\text{mag}}(m_{r})$ and $P_{\text{ratio}}(y)$ are given, the PDF of 100 $\mu$m flux from a single galaxy is computed as

$$ P_{\text{flux}}(f) = \int y d f P_{\text{mag}}(m_{r}) P_{\text{ratio}}(y) \times \delta_{D}(f - y f_{0}(m_{r}, y)). $$ \hspace{1cm} (B25)

With the PDFs of Equations (B24) and (7), $P_{\text{flux}}(f)$ reduces to

$$ P_{\text{flux}}(f) = K \left( \frac{f}{f_{0}} \right)^{-1 - \frac{1}{2} \gamma r} [\text{erf}(s_{\text{max}}(f)) - \text{erf}(s_{\text{min}}(f))], $$ \hspace{1cm} (B26)

where erf$(\lambda)$ denotes the error function and $K, s_{\text{max}},$ and $s_{\text{min}}$ are defined as

$$ K = \frac{5 \gamma r}{f_{0}(10^{\gamma m_{\text{max}}} - 10^{\gamma m_{\text{min}}})} \exp \left[ \frac{25}{8} \sigma^{5} \gamma r^{2} (\ln 10)^{2} \right], $$ \hspace{1cm} (B27)

$$ s_{\text{max}}(f) = \frac{1}{\sqrt{2} \sigma} \left[ 0.4 m_{r,\text{max}} - \mu + \log_{10} \left( \frac{f}{f_{0}} \right) - \frac{5}{2} \sigma^{2} \gamma r \ln 10 \right], $$ \hspace{1cm} (B28)

$$ s_{\text{min}}(f) = \frac{1}{\sqrt{2} \sigma} \left[ 0.4 m_{r,\text{min}} - \mu + \log_{10} \left( \frac{f}{f_{0}} \right) - \frac{5}{2} \sigma^{2} \gamma r \ln 10 \right]. $$ \hspace{1cm} (B29)
Incidentally, \( P_{\text{flux}}(f) \) turns out to also be well approximated by a log-normal function, but we use Equation (B26) to be precise. Considering that the mock galaxies with flux larger than \( f_{\text{lim}} \) are removed and do not contaminate, \( P_1(\Delta A) \) is calculated as

\[
P_1(\Delta A) = \delta_D(\Delta A) \int_{f_{\text{lim}}}^{\infty} P_{\text{flux}}(f) df + \frac{1}{C} \Theta(C f_{\text{lim}} - \Delta A) P_{\text{flux}}(\Delta A / C),
\]

where \( C \equiv k_r p / \Omega_{\text{pix, eff}} \) is a conversion factor from the FIR flux to the \( r \)-band extinction. We adopt \( \Omega_{\text{pix, eff}} = 2 \pi \sigma^2_{\text{eff}} \) as the effective area of a pixel, where \( \sigma_{\text{eff}} \) is the Gaussian width corresponding to the effective angular resolution, which is given in Appendix A. We adopt Equation (A7) for calculating \( \Omega'(A') \) and (A10) for calculating \( N'(A') \).

An analytical model that we present in this paper neglects the spatial clustering of galaxies, but it is, at least partially, incorporated by the assigned value of 100 \( \mu \text{m} \) flux for each \( r \)-band-selected galaxy. The interpretation is slightly subtle, but we would like to emphasize that the neglect of the spatial clustering in our analytic model is not serious in practice, as discussed in Section 7.

REFERENCES

Abazajian, K., Adelman-McCarthy, J. K., Agüeros, M. A., et al. 2003, AJ, 126, 2081
Abazajian, K. N., Adelman-McCarthy, J. K., Agüeros, M. A., et al. 2009, ApJS, 182, 543
Adelman-McCarthy, J. K., Agüeros, M. A., Allam, S. S., et al. 2006, ApJS, 162, 38
Arce, H. G., & Goodman, A. A. 1999a, ApJ, 517, 264
Arce, H. G., & Goodman, A. A. 1999b, ApJL, 512, L135
Blanton, M. R., Lupton, R. H., Schlegel, D. J., et al. 2005, ApJ, 631, 208
Burstein, D., & Heiles, C. 1978, ApJ, 225, 40
Burstein, D., & Heiles, C. 1982, AJ, 87, 1165
Cambrésy, L., Bouleanger, F., Lagache, G., & Stepnik, B. 2001, A&A, 375, 999
Cambrésy, L., Jarrett, T. H., & Beichman, C. A. 2005, A&A, 435, 131
Chen, B., Figueras, F., Torra, J., et al. 1999, A&A, 352, 459
Chilingarian, I. V., Melchior, A.-L., & Zolotukhin, I. Y. 2010, MNRAS, 405, 1409
Dobashi, K., Uehara, H., Kandori, R., et al. 2005, PASJ, 57, 1
Fitzpatrick, E. L. 1999, PASP, 111, 63
Fukugita, M., Ichikawa, T., Gunn, J. E., et al. 1996, AJ, 111, 1748
Fukugita, M., Yasuda, N., Brinkmann, J., et al. 2004, AJ, 127, 3155
Gunn, J. E., Carr, M., Rockosi, C., et al. 1998, AJ, 116, 3040
Gunn, J. E., Siegmund, W. A., Mannery, E. J., et al. 2006, AJ, 131, 2332
Hogg, D. W., Finkbeiner, D. P., Schlegel, D. J., & Gunn, J. E. 2001, AJ, 122, 2129
Ivezić, Ž., Lupton, R. H., Schlegel, D., et al. 2004, AN, 325, 583
Kashiwagi, T., Yahata, K., & Suto, Y. 2013, PASJ, 65, 43
Murakami, H., Baba, H., Barthel, P., et al. 2007, PASJ, 59, 369
Nishimichi, T., Shirata, A., Tanaya, A., et al. 2009, PASJ, 61, 321
O’Donnell, J. E. 1994, ApJ, 422, 158
Padmanabhan, N., Schlegel, D. J., Finkbeiner, D. P., et al. 2008, ApJ, 674, 1217
Peek, J. E. G., & Graves, G. J. 2010, ApJ, 719, 415 (PG)
Pier, J. R., Munn, J. A., Hindsley, R. B., et al. 2003, AJ, 125, 1559
Rowles, J., & Froebrich, D. 2009, MNRAS, 395, 1640
Saunders, W., Sutherland, W. J., Maddox, S. J., et al. 2000, MNRAS, 317, 55
Schlafly, E. F., & Finkbeiner, D. P. 2011, ApJ, 737, 103
Schlafly, E. F., Finkbeiner, D. P., Schlegel, D. J., et al. 2010, ApJ, 725, 1175
Schlegel, D. J., Finkbeiner, D. P., & Davis, M. 1998, ApJ, 500, 525 (SFD)
Scranton, R., Johnston, D., Dodelson, S., et al. 2002, ApJ, 579, 48
Serjeant, S., & Harrison, D. 2005, MNRAS, 356, 192
Smith, J. A., Tucker, D. L., Kent, S., et al. 2002, AJ, 123, 2121
Stoughton, C., Lupton, R. H., Bernardi, M., et al. 2002, AJ, 123, 485
Takeuchi, T. T., Yoshikawa, K., & Ishii, T. T. 2003, ApJL, 587, L89
Tucker, D. L., Kent, S., Richmond, M. W., et al. 2006, AN, 327, 821
Wright, E. L., Eisenhardt, P. R. M., Mainzer, A. K., et al. 2010, AJ, 140, 1868
Yahata, K. 2007, PhD thesis, The Univ. Tokyo
Yahata, K., Yonehara, A., Suto, Y., et al. 2007, PASJ, 59, 205
Yasuda, N., Fukugita, M., Narayanan, V. K., et al. 2001, AJ, 122, 1104
Yasuda, N., Fukugita, M., & Schneider, D. P. 2007, AJ, 134, 698
York, D. G., Adelman, J., Anderson, J. E., Jr., et al. 2000, AJ, 120, 1579