Renormalization Group Evolution from On-shell SMEFT

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Abstract: We describe the on-shell method to derive the Renormalization Group (RG) evolution of Wilson coefficients of high dimensional operators at one loop, which is a necessary part in the on-shell construction of the Standard Model Effective Field Theory (SMEFT), and exceptionally efficient based on the amplitude basis in hand. The UV divergence is obtained by firstly calculating the coefficients of scalar bubble integrals by unitary cuts, then subtracting the IR divergence in the massless bubbles, which can be easily read from the collinear factors we obtained for the Standard Model fields. Examples of deriving the anomalous dimensions at dimension six are presented in a pedagogical manner. We also give the results of contributions from the dimension-8 $H^4 D^4$ operators to the running of $V^+ V^- H^2$ operators, as well as the running of $B^+ B^- H^2 D^{2n}$ from $H^4 D^{2n+4}$ for general $n$. 

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1 Introduction

The discovery of the Higgs with a mass around 125 GeV [1, 2] completes the last missing piece of the Standard Model (SM) and indicates that the SM precisely describes the fundamental interactions at lower energy scale. But with the discovery of Higgs, the Higgs naturalness problem is still mysterious and remains to be solved in the next decades. To solve this problem, the new physics (NP) and symmetry should be introduced at TeV scale, such as SUSY [3, 4] and composite Higgs models [5–7]. But so far the 14 TeV LHC still did not find any new physics, which may indicate that the NP scale is so high that beyond the reach of current experiment searches. With such a high NP scale, the precise measurements of the SM interactions at lower scale is an available way to search for the hints of NP, which can be well parametrized by high dimensional operators of the SM Effective Filed Theory (SMEFT). So it becomes important to understand the SMEFT for the search of NP imprints. Since the running of Wilson coefficients can significantly affect the contributions of NP to SM fields interactions at loop level, the anomalous dimensions of effective operators are crucial...
for correctly calculating the SMEFT processes without losing any infrared informations of
the NP.

Recently it was found that on-shell scattering amplitudes have remarkable advantages for
the study of SMEFT, comparing with the traditional Lagrangian language. The high dimen-
sion operators can be described by unfactorizable amplitudes \[8, 9\], called \textit{amplitude basis},
without boring with the redundancies from the equation of motion and integration by part
(these redundancies are automatically removed by the intrinsic properties of on-shell method:
on-shell condition and momentum conservation). With this new bases, the calculation in
SMEFT can be implemented without referring to the Lagrangian. Some surprising relations
and properties of EFTs, which are not manifest in quantum field theory, can be easily seen
via this method. For example, some EFTs can be described by scattering equations \[10\]
uniformly or constructed/classified systematically from soft limits \[11–13\]. The running of
Wilson coefficients of SMEFT can be strongly constrained by selection rules \[14–16\] based
on unitarity cut method. Since the on-shell scattering amplitudes are only described by the
physical freedoms of external legs, the calculations of SMEFT can be much more efficient via
on-shell method without involving gauge fixing and ghosts. Moreover, the one loop ampli-
tudes can be decomposed into the sum of a basis of scalar integrals plus rational functions,
and the coefficients of the scalar integrals are determined by the product of the tree-level on-
shell amplitudes from the generalized unitary cuts \[17–19\]. So the one loop amplitudes can be
obtained through simple tree-level calculations without involving any loop calculations. Since
only the bubble integrals are UV divergent, the anomalous dimension matrix can be deter-
mined by the massive and massless bubble integral coefficients, which can be easily obtained
by Stokes’s Theorem \[20\] or other methods \[11, 21, 22\] (massive bubble integrals) and collinear
divergences of tree level amplitudes (massless bubble integrals) \[23–25\]. Notice that UV di-
vergences from massless bubble integrals are universal and only determined by renormalizable
interactions so they can be directly read out without any calculations \[25\]. Comparing with
the existing calculation of the anomalous dimension matrix of dimension six \[26–30\] via Feyn-
man diagram, on-shell method is very convenient and powerful for SMEFT study, especially
when applied in the calculations involving higher dimension operators.

In this paper we demonstrate how to use the on-shell method to derive the anomalous
dimension matrix via tree-level amplitudes and give some non-trivial examples, such as \(F^3\)
type operators and dimension 8 operators (the complete dimension 8 operator basis can be
found in \[31, 32\]). Since the UV divergence from a massless bubble integral is universal (only
depends on the external legs attached to this bubble diagram), we list all the UV divergent
factors from massless bubbles for all the SM fields. So people can directly use these results
to calculate the renormalization of SMEFT operators at one loop level without calculating
this kind of UV divergences again. We find that the custodial symmetry can also explain
some zeros in anomalous dimension matrices, which can not be explained by the existing
selection rules. Based on unitary cuts, the anomalous dimension matrices of the operators with
arbitrary dimensions that contribute to \(2 \rightarrow 2\) processes can be easily expressed in universal
forms and we explicitly show the universal expressions for the running of \(B^+ B^- H^2 D^n\) type
operators generated from the insertion of general $H^4D^{2n+4}$ type operators at one-loop level.

The structure of this paper is organized as follows. A detailed discussion about the anomalous dimension calculation in SMEFT via the on-shell method is presented in Sec. 2, and some examples are shown in Sec. 3. The anomalous dimension matrix for dimension 8 amplitude basis $V^+V^-HH$ is obtained in Sec. 4. We show the universal results of anomalous dimensions for general amplitude basis $B^+B^-H^2D^{2n}$ in Sec. 5 and conclude in Sec. 6. A simple example of deriving collinear divergent factor and detailed calculation of the anomalous dimension of $O_{eW}$ via on-shell method are presented in App. A and App. B.

2 The on-shell loop method based on unitary cut

Since the renormalization of on-shell SMEFT is induced by UV divergent part of amplitudes, in this section we explain how to derive the full UV divergences of the amplitudes via unitary cut and collinear singularities of tree-level amplitudes.

The non-renormalizable interactions of the on-shell SMEFT can be described by the amplitude basis $\sum_i c_i M_{Oi}$, where $c_i$ is the Wilson coefficient. To obtain the RG equations for $c_i$, we consider the amplitude which receives tree-level contribution from $M_{Oi}$ as well as loop contributions with another amplitude basis $M_{Oj}$ insertion. The full amplitude takes the form of

$$A_{\text{loop}}^i \sim c_i(\mu) - \gamma_{ij} \frac{1}{16\pi^2} c_j(\mu)(\frac{1}{2\epsilon} + \log \mu + \ldots),$$

where the terms $\frac{1}{2\epsilon} + \log \mu$ come from the UV divergence and $\mu$ is the renormalization scale. By demanding the full amplitude being independent of the scale $\mu$, one directly obtains renormalization group (RG) equation

$$\frac{dc_i(\mu)}{d\log \mu} = \sum_j \frac{1}{16\pi^2} \gamma_{ij} c_j,$$

where $\gamma_{ij}$ is the the anomalous dimension matrix governing the RG running.

2.1 Unitarity cut and bubble coefficients

To extract the UV divergence in the one loop amplitude, a convenient way is to decompose it into the combination of a basis of scalar integrals including boxes, triangles and bubbles plus rational functions [17, 18]

$$A_{\text{1-loop}} = \sum_k C^k_4 I^k_4 + \sum_j C^j_3 I^j_3 + \sum_k C^k_2 I^k_2 + R.$$  

Here the index $i$ ($j$ or $k$) labels the distinct integrals with different partition of the external legs. These integrals capture the branch cuts of the loop amplitudes and their coefficients $C^i_{4,3,2}$ can be obtained from tree level amplitudes by generalized unitary cut [33–35] .
scalar bubbles are the only UV divergent integrals in four dimensions. With dimension regularization it takes the form:

$$I_{i2} \equiv -i \int \frac{d^d l}{(2\pi)^d l^2(l - K)^2} = \frac{1}{(4\pi)^2}(\frac{1}{\epsilon} - \log \frac{-K^2}{\mu^2} + \ldots).$$  \hspace{1cm} (2.4)$$

where $D = 4 - 2\epsilon$. So the only job to deriving anomalous dimension matrix is to extract the bubble coefficients. They can be easily obtained by using Stokes’s theorem based on unitary cut [20]. In the following section, we will briefly discuss about this method.

### 2.2 Extraction of bubble coefficient

Since there are two propagators in the loop for bubble integrals, the bubble coefficients can be extracted by double cuts. To extract the coefficient $C_{i2}^j$, the $K_i$-channel double-cut should be implemented to $A^{1\text{-loop}}$ in eq. 2.3 as illustrated in Fig. 1, the left hand side of this equation becomes

$$\text{Cut}_{K_i}[A^{1\text{-loop}}] = \int d\text{LIPS}_i \sum_{h_i} A_{\text{tree}}^L(l_1, l_2, h_i) A_{\text{tree}}^R(-l_1, -l_2, -h_i)$$  \hspace{1cm} (2.5)$$

where $d\text{LIPS}_i = dl_1^4 dl_2^4 \delta^{(+)}(l_1^2) \delta^{(+)}(l_2^2) \delta(l_1 + l_2 - K_i)$ is the Lorentz-invariant phase space associated with the $K_i$-channel cut, $l_1^a$, $l_2^a$ is momentum of the propagators, $A_{\text{tree}}^L$, $A_{\text{tree}}^R$ are tree level amplitudes on each side of the cut and $h_i$ is the polarization configuration of the cutted internal legs.

The coefficient of bubble is proportional to the rational terms of double cut integration, which can be directly extracted from the indefinite integration by Hermite Polynomial Reduction [20], because the un-cutted propagators in the $I_{3,4}$ will make the integration variable $l_1$ appear in the denominators. So the bubble coefficients is given by

$$C_{i2}^j = -\frac{1}{2\pi i} \text{Rational} \left[ \int d\text{LIPS}_i \sum_{h_i} A_{\text{tree}}^L A_{\text{tree}}^R \right],$$  \hspace{1cm} (2.6)$$

where the factor $-2\pi i$ is from the double cut on bubble integral $I_{2}^j$ (will be seen in the following discussion). To efficiently calculate the phase space integral, the loop momentum can be parametrized as

$$(l_1)_{\tilde{a}\hat{a}} = t \lambda_{\tilde{a}} \tilde{\lambda}_{\hat{a}},$$  \hspace{1cm} (2.7)$$
so that the phase space integral can be written as
\[
\int d\text{LIPS}_i = \int dt \int_{\hat{\lambda} = \lambda}^{\hat{\lambda} = \bar{\lambda}} \frac{\langle \lambda d\lambda | \hat{\lambda} d\hat{\lambda} \rangle}{\langle \lambda | K_i | \bar{\lambda} \rangle} \delta(t - \frac{K_i^2}{\langle \lambda | K_i | \bar{\lambda} \rangle}). \tag{2.8}
\]

Notice that we are integrating over the contour \( \bar{\lambda} = \tilde{\lambda} \) so that the loop momentum is real.

The spinor variable \( \lambda(\tilde{\lambda}) \) can be further decomposed into a basis of two massless spinor with a complex coefficient \( z \),
\[
|\lambda\rangle = |p\rangle + z |q\rangle, \quad |\bar{\lambda}\rangle = |p\rangle + \bar{z} |q\rangle, \tag{2.9}
\]
where \( p_\mu \) and \( q_\mu \) is two null momenta satisfying \( K_\mu^i = p_\mu + q_\mu \). With this parametrization, the phase space integral can be expressed in term of complex variable \( z \),
\[
\int d\text{LIPS}_i = \oint dz \int d\bar{z} \int dt t^2 \delta(t - \frac{1}{1 + z\bar{z}}). \tag{2.10}
\]

Then the cut of the bubble integral can be easily evaluated and equal to
\[
\text{Cut}[I^i_2] = \oint dz \int d\bar{z} \int t^2 dt \delta(t - \frac{1}{1 + z\bar{z}}) = \int dz \frac{-1}{1 + z\bar{z}} z = -2\pi i. \tag{2.11}
\]

In the last two steps, we first integrate over \( \bar{z} \) and then sum over the residues at all poles of \( z \). This result explains the \(-2\pi i\) factor in Eq. 2.6. Under this parametrization the cutted amplitude becomes
\[
\Delta_i(z, \bar{z}) \equiv \int d\text{LIPS}_i \sum_{h_i} A^\text{tree}_L A^\text{tree}_R = \oint dz \int d\bar{z} t^2 \sum_{h_i} A_L(t, z, \bar{z}) A_R(t, z, \bar{z}) |_{t = \frac{1}{1 + z\bar{z}}} \tag{2.12}
\]

After perform the \( z \)-integration via Cauchy’s Residue Theorem, finally \( C^2_i \) in Eq. 2.6 can be easily evaluted via the following expression,
\[
C_2 = \frac{\Delta^\text{Rational}_i}{-2\pi i} = -\text{Res}_{z=0} F^\text{Rational}(z, \bar{z}) - \text{Res}_{z\neq 0} F^\text{Rational}(z, \bar{z}), \tag{2.13}
\]
where \( F^\text{Rational} \) is the rational part of \( F(z, \bar{z}) = \int d\bar{z} t^2 \sum_{h_i} A_L(t, z, \bar{z}) A_R(t, z, \bar{z}) |_{t = \frac{1}{1 + z\bar{z}}} \).

### 2.3 Collinear divergence

Using unitary cut we can not get the full UV divergences of loop amplitudes. Only the coefficients of massive bubble \( (K_i^2 > 0) \) can be obtained via unitary cut. Massless bubbles with \( K_i^2 = 0 \) also contain UV divergences, which simply vanish in dimension regularization due to the cancellation between UV and collinear IR divergence. Since the physical cross section is free of collinear divergences, the collinear divergences of the tree amplitudes must be cancelled by collinear loop IR divergences. So the UV divergence in massless bubbles can be extracted by calculating collinear divergences of the corresponding tree amplitudes. The one loop collinear IR divergence can be parametrized as [23–25]
\[
A^{1-\text{loop}}_{n, \text{col}} = -(\frac{1}{4\pi})^2 \sum_a \frac{\gamma(a)}{\epsilon} A^\text{tree}, \tag{2.14}
\]
where the sum is over all external legs and $\gamma(a)$ is the collinear factor for each particle $a$. We want to emphasize that the collinear factors only depend on the external legs and are universal for SMEFT. With simple calculations, we list the collinear factors of all the SM fields,

\[
\begin{align*}
\gamma(H^a) &= \gamma(H^{\dagger a}) = 2y_h^2g_1^2 - \frac{1}{2}\text{Tr}[N_cY_u^\dagger Y_u + N_cY_d^\dagger Y_d + Y_e^\dagger Y_e] + 2g_2^2C_2(2), \\
\gamma(B^{\pm}) &= -\frac{g_1}{3} \left[ n_gN_c(y_q^2 + y_u^2 + y_d^2) + n_g(y_\ell^2 + y_e^2) + y_h^2 \right], \\
\gamma(W^{a\pm}) &= g_2^2 \left[ \frac{11}{3} - \frac{1}{3} \left( \frac{n_g}{2}N_c + \frac{n_g}{2} + \frac{1}{4} \right) \right], \\
\gamma(g^{\pm}) &= g_3^2 \left( \frac{11N_c}{6} - \frac{1}{3}n_g \right), \\
\gamma(\ell) &= \frac{3}{2}g_2^2C_2(2) + \frac{3}{2}g_1^2g_2^2 - \frac{1}{4}Y_e^\dagger Y_e, \\
\gamma(e) &= \frac{3}{2}g_2^2g_1^2 - \frac{1}{2}Y_e^\dagger Y_e, \\
\gamma(q) &= \frac{3}{2}g_3^2C_2(N_c) + \frac{3}{2}g_2^2C_2(2) + \frac{3}{2}g_1^2g_2^2 - \frac{1}{4}(Y_u^\dagger Y_u + Y_d^\dagger Y_d), \\
\gamma(u) &= \frac{3}{2}g_3^2C_2(N_c) + \frac{3}{2}g_2^2g_1^2 - \frac{1}{2}Y_u^\dagger Y_u, \\
\gamma(d) &= \frac{3}{2}g_3^2C_2(N_c) + \frac{3}{2}g_2^2g_1^2 - \frac{1}{2}Y_d^\dagger Y_d.
\end{align*}
\]

Here $q$ and $\ell$ are $SU(2)$ doublets for left hand quarks and leptons, while $u, d$ and $e$ are right hand singlets. $g_3, g_2$ and $g_1$ are the gauge couplings of $SU(3) \times SU(2)_L \times U(1)_Y$, with $y_i$ being the hypercharge. $Y_u, Y_d$ and $Y_e$ are Yukawa couplings. $N_c = 3$ is QCD color number; $n_g = 3$ is the number of generations. And $C_2(N) = \frac{N^2 - 1}{2N}$.

So for any SMEFT loop calculation, the UV divergences of massless bubbles can be directly read from Eq. 2.15 without any calculation. For better understanding of collinear divergences, we also show one example to deriving the collinear factors in Appendix A.

Combining the UV divergences from massive bubbles with collinear IR divergences, we can finally obtain the anomalous dimension matrix correctly. In the next section we will give some non-trivial examples to clearly show how to get the anomalous dimension matrix systematically via on-shell method.

### 3 Examples for calculation of anomalous dimension matrix

In this section we give some examples to demonstrate the on-shell loop method for calculating the RG running of SMEFT in detail.

#### 3.1 $O_{HB}$

We first focus on a simplest case: the contributions proportional to $U(1)_Y$ gauge interactions to the running of the dimension 6 operator $O_{HB} = H^\dagger HB^{\mu\nu}B_{\mu\nu}$. In the amplitude basis,
this operator corresponds to the local amplitude

$$A(B^+, B^+, H^\alpha, H^\dagger) = 2C_{HB}\delta^{\alpha\dagger}\langle 12 \rangle^2. \quad (3.1)$$

The Higgs $U(1)_Y$ gauge interactions and quartic term can be expressed as

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D_{\mu}H|^2 - \frac{1}{4}\lambda|H|^4. \quad (3.2)$$

where $D_{\mu} = \partial_{\mu} - ig_1 y_h B_{\mu}$.

Now let’s consider the amplitude $A(B^+, B^+, H^\dagger, H^\alpha)$ at one loop. As shown in Fig. 2, applying different double cuts, this amplitude are separated into dimension 6 part and dimension 4 part. The dimension 6 part can be read from 3.1 and the dimension 4 part can be derived from 3.2,

$$A^4(H^\alpha H^\dagger B^+ B^-) = -2y_h^2g_3^2\delta^{\alpha\dagger} \langle 14 \rangle \langle 24 \rangle \langle 13 \rangle \langle 23 \rangle \quad (3.3)$$

$$A^4(H^\alpha H^\dagger H^\beta H^\dagger) = (\delta^{\alpha\dagger}\delta^{\beta\dagger} + \delta^{\alpha\dagger}\delta^{\beta\dagger})(-\frac{\lambda}{2} + y_h^2g_3^2) + 2y_h^2g_3^2\left[\delta^{\alpha\dagger}\delta^{\beta\dagger}\langle 24 \rangle \langle 24 \rangle \langle 14 \rangle \langle 14 \rangle \right]. \quad (3.4)$$

First consider $s$-channel cut (the upper plot in Fig. 2) and the product of the two cut amplitudes can be expressed as

$$\text{Cut}_{12}[A(B^+(p_1)B^+(p_2)H^\dagger(p_3)H^\alpha)(p_4)]$$

$$= \int d\text{LIPS} A_L(B^+(p_1)B^+(p_2)H^\dagger(l_1)H^\dagger(l_2))\delta^{\beta\dagger}\delta^{\sigma\dagger} A_R(H^\dagger(-l_1)H^\alpha(-l_2)H^\dagger(p_3)H^\alpha(p_4))$$

$$= \int d\text{LIPS} C_{HB}\delta^{\beta\dagger} \langle 12 \rangle^2 \left[\left(\delta^{\sigma\dagger}\delta^{\alpha\dagger} + \delta^{\sigma\dagger}\delta^{\alpha\dagger}\right)(-\frac{\lambda}{2} + y_h^2g_3^2) \right.$$

$$+ 2y_h^2g_3^2\left(\delta^{\sigma\dagger}\delta^{\alpha\dagger}\tag{3.5} \right).$$
where \( l_{1,2} \) is the momentum of the cutted internal Higgs leg. We then use the relations 
\[ | - l \rangle = i | l \rangle, | - \bar{l} \rangle = i | \bar{l} \rangle \]
and \( l_1 + l_2 = p_3 + p_4 \) to express it as function of \( l_1 \). Following the procedure presented in the previous section, we should parametrize the loop momentum as
\[
| l_1 \rangle = \sqrt{t} (| 3 \rangle + z | 4 \rangle) \quad | l_1 \rangle = \sqrt{t} (| 3 \rangle + \bar{z} | 4 \rangle),
\]  
(3.6)
and then the bubble coefficient can be extracted through Stokes’ Theorem
\[
\Delta_{12} = [12]^2 \delta^{\alpha \dot{\alpha}} \int dz \int d\bar{z} t^2 \left( (-\frac{3}{2} \lambda + 3y_h^2 g_1^2) + 2y_h^2 g_1^2 (-2t^2 + \frac{1}{z\bar{z}} t^2) \right) | t = (1+z\bar{z})^{-1} - 1 \]
\[
= [12]^2 \delta^{\alpha \dot{\alpha}} \int dz \left( (-\frac{3}{2} \lambda + 3y_h^2 g_1^2) \frac{-1}{z(1+z\bar{z})} + 2y_h^2 g_1^2 \left( \frac{1}{z(1+z\bar{z})} + \frac{1}{z(1+z\bar{z})} + \frac{\log(-z\bar{z}) - \log(1+z\bar{z})}{z} \right) \right)
= 2\pi i [12]^2 \delta^{\alpha \dot{\alpha}} \left( \frac{3}{2} \lambda + 3y_h^2 g_1^2 \right).
\]  
(3.7)
In the last step we discard the log term and take the residue at \( z = 0 \) and set \( \bar{z} = z \). Finally we can get the bubble coefficient from s-channel double cut
\[
c_{\text{bubble}}^{12} = -(y_h^2 g_1^2 + \frac{3\lambda}{2}) C_{HB} \delta^{\alpha \dot{\alpha}} [12]^2.
\]  
(3.8)
Following the same procedure, The contribution from t-channel cut is given by
\[
\text{Cut}^{12} [A(B^+(p_1)B^+(p_2)H^{\dagger \dot{\alpha}}(p_3)H^\alpha)(p_4)]
= A_L(B^+(p_1) H^{\dagger \dot{\alpha}}(p_3) B^+(l_1) H^\beta(l_2)) \delta^{\dot{\alpha} \beta} A_R(B^{-}(l_1) H^{\dagger \beta}(-l_2) B^+(p_2)H^\alpha(p_3)) \\
+ A_L(B^+(p_2)H^\alpha(p_4)B^{-}(l_1) H^{\dagger \beta}(-l_2)) \delta^{\dot{\alpha} \beta} A_R(B^{-}(l_1) H^\beta(-l_2) B^+(p_1)H^{\dagger \dot{\alpha}}(p_3))
= -2y_h^2 g_1^2 C_{HB} \delta^{\alpha \dot{\alpha}} [1l_1]^2 \frac{4-\ell_1}{(42)(-\ell_2)} + (1 \leftrightarrow 2, 3 \leftrightarrow 4).
\]  
(3.9)
Using Stokes’ Theorem again after proper parametrization, we can get the bubble coefficient for t-channel cut,
\[
c_{\text{bubble}}^{13} = -2y_h^2 g_1^2 C_{HB} \delta^{\alpha \dot{\alpha}} [12]^2.
\]  
(3.10)
The u-channel cut is the same as t-channel cut under \( p_1 \leftrightarrow p_2 \) and we get
\[
c_{13} = c_{14}.
\]  
(3.11)
So the total contribution from bubble diagram is
\[
C_2 = 2c_{\text{bubble}}^{13} + 2c_{\text{bubble}}^{14} + c_{\text{bubble}}^{12} = -(5y_h^2 g_1^2 + \frac{3\lambda}{2}) C_{HB} \delta^{\alpha \dot{\alpha}} [12]^2
\]  
(3.12)
where the factor 2 before \( c_{\text{bubble}}^{13,14} \) is from the exchange of external gauge boson legs.

\(^1\)For internal fermion leg, complex the factor \( i \) can be removed because the fermion statistic cancels the minus sign from momentum flipping.
The parts of UV divergences cancelled by IR divergences can be read from Eq. 2.15 according to the external legs:

\[ C_{IR} = -(2\gamma(B) + 2\gamma(H))\delta^{\alpha\bar{\alpha}}C_{HB}[12]^2. \]  

(3.13)

So we can find the total UV divergences of operator \( O_{HB} \) from \( U(1)_Y \) gauge interactions at one-loop level are

\[ C_{UV} = C_{2}^{UV} - C_{IR} = -\left(\frac{5}{3}y_h g_1^2 \left(5 - 2\gamma(H) - 2\gamma(B)\right) + \frac{3\lambda}{2}\right) \frac{1}{(4\pi)^2} \frac{1}{\epsilon} \delta^{\alpha\bar{\alpha}}C_{HB}[12]^2. \]  

(3.14)

So the running of \( C_{HB} \) is

\[ \dot{C}_{HB} = 2\left[\frac{5}{3}y_h g_1^2 + \frac{3\lambda}{2}\right]C_{HB}, \]  

(3.15)

where \( \dot{C}_{HB} \equiv (4\pi)^2 \mu \frac{dC_{HB}}{d\mu} \). This expression is exactly the same as in [28, 29].

3.2 \( F^3 \) type operators

For the \( F^3 \) type operators, the leading amplitude is three-point scattering amplitude, which does not depend on any UV scale if the three external legs are on-shell. So its UV divergences can not be extracted through unitarity cut. However we can calculate its next leading 4-points amplitude to derive its RG running. The color-ordered leading amplitude of dimension 6 operator \( O_G = C_G \text{Tr}[G^3_{\mu\nu}] \) can be expressed as

\[ A(g_1^+ g_2^+ g_3^+ g_4^+) = C_G[12][23][31]. \]  

(3.16)

Its 4-point tree-level amplitudes which can be constructed through all-line shift [36],

\[ A_{\text{tree}}(g_1^+ g_2^+ g_3^+ g_4^+) = 2g_3C_{G^3} \frac{[12][13][42]}{\langle 34 \rangle}. \]  

(3.17)

According to unitary cut, it is easy to find that the loop amplitudes of 4 \( g^+ \) that contain quarks internal legs does not contribute to the bubbles because the the helicity selection rules forbid the tree level amplitude for two quarks and two \( g^+ \). So only \( O_G \) insertion contribute to itself RG running. Following the same procedures as above, we can get the coefficients of the bubble integral,

\[ C_2 = -6g_3^2 N_c A_{\text{tree}}(g_1^+ g_2^+ g_3^+ g_4^+). \]  

(3.18)

After including the massless bubble contributions which can be directly read from the Eq. 2.15 , the total divergences from one-loop level is

\[ C_{2}^{UV} = \frac{-6g_3^2 N_c + 4g_3^2 \gamma(g)}{(4\pi)^2} A_{\text{tree}}(g_1^+ g_2^+ g_3^+ g_4^+) \frac{1}{\epsilon}. \]  

(3.19)
These divergences of this four-point amplitude contain both the gauge coupling $g_s$ and $C_{G^3}$ renormalization and with requirement that the amplitude independent on renormalizable scale we can get the following RG equation

$$(4\pi)^2 \beta_{g_s} C_{G^3} + g_3 \dot{C}_{G^3} = 4g_3^3 (3N_c - 2\gamma(g))C_{G^3}. \quad (3.20)$$

where $\beta_g$ is the beta function of gauge coupling $g_s$. We can also derive $\beta_{g_s}$ through on-shell method,

$$\beta_{g_3} = -\frac{2g_3^3}{(4\pi)^2} \gamma(g). \quad (3.21)$$

Substituting the expression of $\beta_{g_3}$ into the Eq. 3.20, we can get the RG running of $C_{G^3}$

$$\dot{C}_{G^3} = [12N_c - 6\gamma(g)]g_3^2 C_{G^3}. \quad (3.22)$$

With these examples we can find that on-shell method is very efficient to calculate SMEFT RG running. We do not need to do loop calculation. All the divergences can be extracted from the tree level amplitudes. We also present more complicated calculation for the operator $O_{eW}$ can be found in App. B.

4 Anomalous dimensions at dimension 8: the $V^+V^-H^\dagger H$ example

The method introduced above can be more efficient to obtain the RG running of higher dimension operators. Based on unitary cut, the some mysterious zeros of anomalous dimension can be explained by the custodial symmetry. In this section we consider the running at dimension 8. In particular we present contribution from the $H^4$ type local amplitude to the RG running of the coefficients of the $V^+V^-H^\dagger H$ type amplitude.

The amplitude $V^+V^-H^\dagger H$ at dimension 8 is important in phenomenology because it gives leading BSM correction to the $VVHH$ scattering in the SMEFT, due to the non-interference at dimension 6 [37]. Also this amplitude basis is only generated at one loop order when we integrate out some heavy particles in a weakly coupled UV theory [16, 38–41], which makes the contribution from the mixing with a potentially tree level generated local amplitude (operator) important. So in this section we calculate the RG running of this coupling from the mixing with $H^4$ amplitude basis at dimension 8. Notice that if we ignore the fermions, this is the only leading contribution, all others, including the contribution from the loop containing two dimension 6 amplitude basis, are more than one loop suppressed if we take into account the tree/loop classification of the local amplitude.

There are three independent $H^4$ type local amplitudes at dimension 8. It is convenient to write them in the following form:

$\mathcal{A}(H^\alpha H^\beta H^{\dagger \alpha} H^{\dagger \beta})_{\text{dim8}} \supset T^+_{\alpha\beta\bar{\alpha}\bar{\beta}} C_{0,2}^{H^4+} (s_{13} - s_{23})^2, \quad T^+_{\alpha\beta\bar{\alpha}\bar{\beta}} C_{2,0}^{H^4+} s_{12}^2, \quad T^-_{\alpha\beta\bar{\alpha}\bar{\beta}} C_{1,1}^{H^4-} s_{12}(s_{13} - s_{23}). \quad (4.1)$
where $T_{\alpha\beta\alpha\beta}^+ \equiv \delta_{\alpha\alpha} \delta_{\beta\beta} \pm \delta_{\beta\alpha} \delta_{\alpha\beta}$.

The dimension 8 $V^+V^-H\dagger H$ type amplitudes can be written as:

\[ A(B^+B^-H^\dagger H^\dagger)_{\dim8} = C_{H^2B^+B^-}[1|3|2]^2 \]
\[ A(W^a+W^-H^\dagger H^\dagger)_{\dim8} \supset C_{H^2W^+W^-}[a^b+][1|3|2]^2, C_{H^2W^+W^-}[a^b-][1|3|2]^2 \]
\[ A(W^a+B^-H^\dagger H^\dagger)_{\dim8} = C_{H^2W^+B^-}[a^b-][1|3|2]^2 \]
\[ A(B^+W^-H^\dagger H^\dagger)_{\dim8} = C_{H^2B^+W^-}[a^b-][1|3|2]^2 \]  \hspace{1cm} (4.2)

where $\tau_{\alpha\beta}^a$ is Pauli matrix, $T_{\alpha\beta}^{ab+} = \delta^{ab} \delta_{\alpha\beta}$ and $T_{\alpha\beta}^{ab-} = i\epsilon_{abc}\tau_{\alpha\beta}^c$.

The loop contributions to the running can be obtained by gluing the $H^4$ amplitude basis in Eq. 4.1 with the $V^+V^-H\dagger H$ amplitudes of SM, which are

\[ A^{SM}(H^\dagger H^\dagger, B^+, W^-) = -g_1g_2y_h(\tau^i)\bar{\beta}^a(14)(24), \]  \hspace{1cm} (4.3)
\[ A^{SM}(H^\dagger H^\dagger, W^+, B^-) = -g_1g_2y_h(\tau^i)\bar{\beta}^a(14)(24), \]  \hspace{1cm} (4.4)
\[ A^{SM}(H^\dagger H^\dagger, W^a, W^-) = -2g_2^2(4132)\left(\frac{t^a t^b t^c}{24} + \frac{t^b t^c t^a}{23}\right), \]  \hspace{1cm} (4.5)

in addition to the $B^+B^-H\dagger H$ amplitude in Eq. 3.4. Here $t^a = \tau^a/2$ are the $SU(2)$ generators.

Applying the unitary method, we obtain the RG running of $V^+V^-H\dagger H$ type operators as following:

\[ \dot{C}_{H^2B^+B^-} = -2g_1^2y_h(C_{H^4}^{H^4} + C_{H^4}^{H^4} + \frac{1}{3}C_{H^4}^{H^4}) \]
\[ \dot{C}_{H^2B^+W^-} = -\frac{1}{3}g_1g_2y_h(C_{H^4}^{H^4} + C_{H^4}^{H^4} - C_{H^4}^{H^4}) \]
\[ \dot{C}_{H^2W^+B^-} = -\frac{1}{3}g_2g_1y_h(C_{H^4}^{H^4} + C_{H^4}^{H^4} - C_{H^4}^{H^4}) \]
\[ \dot{C}_{H^2W^+W^-} = -\frac{1}{2}g_2^2(C_{H^4}^{H^4} + C_{H^4}^{H^4} + \frac{1}{3}C_{H^4}^{H^4}) \]
\[ \dot{C}_{H^2W^+W^-} = 0. \]  \hspace{1cm} (4.6)

The dependence of the RGE on the specific combinations ($C_{H^4}^{H^4} + C_{H^4}^{H^4} + \frac{1}{3}C_{H^4}^{H^4}$) and ($C_{H^4}^{H^4} + C_{H^4}^{H^4} + \frac{1}{3}C_{H^4}^{H^4}$) are expected by demanding the right angular momentum and $SU(2)$ global symmetry[16]. The zero in the last equation can be understood from the fact that the there is $SO(4)$ custodial symmetry in both $H^4$ operators and $A^{SM}(H^\dagger H^\dagger W^a + W^-)$, while dimension 8 amplitude basis $A(W^a+W^-H^\dagger H^\dagger)_{\dim8} = C_{H^2W^+W^-}^{-}[a^b-][1|3|2]^2$ violate this symmetry. So custodial symmetry can provide some new selection rules, which can not be explained by existing selection rules, based on on-shell method.
5 Universal results for anomalous dimension

From above examples, we can find the form of anomalous dimension matrix is strongly dependent of the external legs of amplitude basis. With on-shell method, the structure of anomalous dimension matrix can be clearly seen and some universal results for amplitude basis RG running can thus be obtained. For example, the bubble coefficients of any four point amplitude basis are only determined by the product of two four-point on-shell scattering amplitudes. Since the four point amplitude basis with any dimension can be expressed in a uniform way: the product of spinor products and polynomial of madstam variable, $s$ and $t$, the RG running of this kind of basis should be also in the uniform form. Since in this work we focus on how to use on-shell method to derive RG running of SMEFT, we just give an simple example to confirm this claim. More universal results will be presented in the future work. We will show the RG running for $H^2B^+B^-D^{2n}$ type amplitude basis at any dimension generated from the general $H^4D^{2n+4}$ type amplitude basis. These basis can be expressed uniformly as

$$\mathcal{A}(H^{\alpha}H^\beta H^{\alpha}H^\beta) = T_{\alpha\beta\alpha\beta}^+ C_{m,n}^{H^{4+}+} s^{m_1}(s_{13} - s_{23})^{2n}, \quad T_{\alpha\beta\alpha\beta}^- C_{m,n+1}^{H^{4-}+} s^{m_1}(s_{13} - s_{23})^{2n+1}$$

$$\mathcal{A}(B^+B^-H^{\alpha}H^\beta) = C_{m,n}^{H^{2}B^{+}B^{-}+} [1|p_3|2]s^{m_1}(s_{13} - s_{23})^n.$$  \hfill (5.1)

where $T_{\alpha\beta\alpha\beta}^\pm \equiv \delta_{\alpha\delta} \delta_{\beta\gamma} \pm \delta_{\beta\delta} \delta_{\alpha\gamma}$. Applying the on-shell method as above, we can obtain the universal RG running for $C_{m,n}^{H^{2}B^{+}B^{-}}$, compactly collected as,

$$\hat{C}_{m,n}^{H^{2}B^{+}B^{-}} = -4g_h^2 \left( \sum_{i,j=1}^{i+j=m+n+2} 3C_{i,2j}^{H^{4+}+} F(m,n,i,2j) + \sum_{i',j'=1}^{i'+j'=m+n+1} C_{i',2j'+1}^{H^{4-}+} F(m,n,i',2j'+1) \right),$$

$$F(m,n,i,k) = \sum_{2g+f+h=n} g,j,h,l,d \left( \frac{1}{4} \right)^{l-1} (-1)^g C_i^{2l-d} C_2^{d} C_{i-1}^{s} C_{i-2l+d}^{h} C_{2l}^{l+1} C_{2j-d}^{j}$$

$$\times \int_0^1 (1 - \frac{1}{2t})^h (-\frac{1}{2t})^{m-2l+d-h} (-\frac{1}{2} + \frac{t}{2} + \frac{1}{2l})^f (\frac{3}{2} + \frac{t}{3} - \frac{1}{2l})^{2j-d-f} (\frac{1}{t} - 1)^l t^{d+i} dt. \hfill (5.2)$$

So the anomalous dimension for $H^2B^+B^-$ basis at any dimension can be readily read from this universal expression. For example, the amplitude basis at dimension 8, or $(m=0, n=0)$, can get three contributions in above sum,

$$(i = 0, j = 1, d = 2, l = 1, g = h = f = 0);$$

$$(i = 1, j = 0, d = 0, l = 1, g = h = f = 0);$$

$$(i' = 1, j' = 0, d = 1, l = 1, g = h = f = 0). \hfill (5.3)$$

With a simple integral $\int_0^1 (1 - t)^2 dt = \frac{1}{6}$, we can obtain its RG running,

$$\hat{C}_{00}^{H^2B^+B^-} = -2g_h^2 g_h^2 (C_{0,2}^{H^{4+}+} + C_{2,0}^{H^{4+}+} + \frac{1}{3} C_{1,1}^{H^{4-}+}), \hfill (5.4)$$

which recovers the result we obtained in previous section.
6 Conclusion

The on-shell amplitude methods have remarkable advantages in studying SMEFT. The non-renormalizable interactions can be described by unfactorizable amplitude bases without worrying about redundancies in quantum field theory. Comparing with Feynman diagrams calculations, the loop-level amplitudes can be constructed by unitary cut very efficiently, because there are no unphysical freedoms appearing in on-shell amplitudes. Especially, since the UV divergences of one-loop amplitudes are only from scalar bubble integrals and the bubble coefficients are related to the tree-level amplitudes, the on-shell method can be very convenient to obtain the renormalization group running of higher dimension operators (amplitude basis). We demonstrate in detail how to extract the full UV divergences of the loop amplitudes in detail. The coefficients of massive bubble integrals can be extracted by Stokes’ Theorem analytically. However, the UV divergences from massless bubble integrals are canceled by collinear IR divergences so they can not be obtained by unitary cut. However, they can be extracted from the collinear divergence of tree-level amplitudes. Since they are universal in SMEFT, we calculate the collinear divergent factors for all the standard model fields and list them in Eq. 2.15. So the UV divergences from the massless bubbles can be directly read from the list. We present some examples to show how to derive the anomalous dimension matrix correctly. The on-shell method is very powerful for SMEFT calculation. Some selection rules can be easily obtained based on this method. This method can make the structure of anomalous dimension matrix transparent, so the running of the general 4-point amplitude basis can be expressed in a universal form. We also present the universal expressions for the anomalous dimension matrix of the general amplitude basis $H^2 B^+ B^- D^{2n}$ generated from the contributions of general $H^4 D^{2n+4}$ type operators at one-loop level.

Note added

While this paper was being finalized, Ref. [42, 43] appeared, which presents a similar topic. [42] uses both form factors and on-shell amplitudes in their calculations and gives the anomalous dimensions at two loops. [43] uses the on-shell unitary cut similar to us but considers only the mixing between different operators, for which there are no IR contributions. In this paper, we take a pure on-shell method and demonstrate the complete procedure in deriving the anomalous dimension at one loop, obtaining the bubble coefficients and subtracting the collinear divergences. We also give new results in dimension 8 as well as universal expressions in general dimensions.

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A Example to deriving the collinear factor for Higgs and Hypercharge gauge boson

In this appendix, we show how to get the collinear divergent factor for Higgs in $U(1)_Y$ gauge theory, following the procedure in \cite{23, 24}. To get the collinear factor of Higgs leg generated from $U(1)_Y$ gauge interaction, we can look at the scattering amplitude containing the 3-vertex involving Higgs and gauge bosons. The collinear factor from $U(1)_Y$ interaction can be obtained from the scattering amplitude $HH^\dagger \to B^+B^-$. We suppose the Higgs leg $H$ with momentum $p_1$ and $B^+$ with momentum $p_3$ become collinear and the diagram in which the Higgs propagator attached with these two legs become on-shell in this collinear limit are divergent. We can parametrize the their momentum as

$$|1\rangle = \sqrt{z} |P\rangle, \quad |3\rangle = \sqrt{1-z} |P\rangle \quad (A.1)$$

where $0 < z < 1$ and $P$ is the momentum of Higgs propagator.

The four point amplitudes $\mathcal{A}(HH^\dagger B^+B^-)$ becomes divergent and can factorize into the product of three point amplitude and a singular splitting function

$$\mathcal{A}(H(p_1)H^\dagger(p_2)B^+(p_3)B^-(p_4)) \to \text{Split}_{H^\dagger}(H, B^+)\mathcal{A}(H(P)H^\dagger(p_2)B^-(p_4)) \quad (A.2)$$

where the splitting function $\text{Split}_{H^\dagger}(H(p_1), B^+(p_3)) = \sqrt[2]{g_1 y_h} \frac{\sqrt{z}}{\sqrt{1-z} |13\rangle}$, $y_h$ is the Hypercharge of Higgs doublet and the three point amplitude $\mathcal{A}(H(P)H^\dagger(p_2)B^-(p_4)) = \sqrt[2]{g_1 y_h} \frac{(4P)(4P_2)}{2}$. We can find the collinear factor $c_{F}^{H \rightarrow H^\dagger}$ can be expressed as

$$c_{F}^{H \rightarrow H^\dagger} = \sum_{i = \pm} \text{Split}_{H^\dagger}(H(p_1), B^i(p_3))\text{Split}_{H^\dagger}(H(p_1), B^i(p_3)) = \frac{4g_1^2 y_h^2}{|13\rangle |13\rangle} \frac{z}{1-z} \quad (A.3)$$

Notice that the contributions from different polarisation of gauge boson $B_\mu$ should be included.

The phase space of these two collinear legs can be expressed as \cite{23}

$$dP^e_{\text{col}}(p_1, p_3, z) = \frac{(4\pi)^\epsilon}{16\pi^2 \Gamma(1-\epsilon)} ds_{13}dz[s_{13}z(1-z)]^{-\epsilon} \theta(s_{\text{min}} - s_{13}) \quad (A.4)$$

Put together, we can get the collinear divergences factor of the process $HH^\dagger \to B^+B^-$

$$\int c_{F}^{H \rightarrow H^\dagger} dP^e_{\text{col}}(p_1, p_3, z) = \frac{g_1^2 y_h^2}{4\pi^2} \frac{1}{\Gamma(1-\epsilon)} \frac{1}{\epsilon} \quad (A.5)$$

The tree level collinear divergences should be cancelled by IR divergence from loop level. If we parametrize the one-loop IR divergences as

$$\mathcal{A}_{n, \text{col}}^{1-\text{loop}} = -(\frac{1}{4\pi})^2 \sum_{a} \frac{\gamma(a)}{\epsilon} \mathcal{A}_{\text{tree}} \quad (A.6)$$
the IR divergences factor from Higgs leg (only include $U(1)_Y$ gauge corrections) can be extracted

$$\gamma(H) = 2g_1^2y_h^2.$$  \hfill (A.7)

Following the same procedure, we can extract the splitting function for vertex $H(p_1)H_l(p_2) \to B$ from the process $H_aH^b \to H^cH^d$, $\text{Split}_{B^\pm}(H^a,H^b) = \delta^{ab}\sqrt{2g_1y_h}/\sqrt{z(1-z)}/(12)$. The collinear factor can be given by

$$e_F^{H(p_1)H_l(p_2)\to B^\pm} = \sum_{a,b} \text{Split}_{B^\pm}(H^a,H^b)\text{Split}^\dagger_{B^\pm}(H^a,H^b) = \frac{4g_1^2y_h^2}{(12)[12]} z(1-z).$$  \hfill (A.8)

Do the same integration as in Eq. A.5, we can easily get the IR divergences factor from $U(1)_Y$ gauge boson legs (only include Higgs doublet corrections)

$$\gamma(B^\pm) = -\frac{g_1^2y_h^2}{3}.$$  \hfill (A.9)

### B The anomalous dimension of $O_{eW}$

In this appendix we present the full calculation of the running of $O_{eW} = (\bar{s}g^\mu\nu e)\tau^lHW_{\mu\nu}^l$. The contribution from different unitary cuts are summarized in table 1, where $A^6$ and $A^{SM}$ are dimension 6 and SM on-shell amplitudes at the two sides of the cut respectively. The relevant form of dimension 6 operators and expression of SM amplitudes are summarized in table 2.

Combined with the contributions from collinear divergences read from Eq.2.15:

$$\dot{C}_{eW} = -2(\gamma(W) + \gamma(H) + \gamma(l) + \gamma(e))C_{eW},$$  \hfill (B.1)

we obtain the same result as that calculated from Feynman diagrams [27–29].

| $A^6$         | $A^{SM}$         | Contribution to $C_{eW}$ |
|--------------|-----------------|--------------------------|
| $A(W^+W^+H^+H^-) = 2(C_{HW} + iC_{WW})\delta^{\alpha\beta}(12)^2$ | $A(H^\alpha H^\beta H^\gamma)$ | $\dot{Y}_{\gamma} g_2(C_{HW} + iC_{WW})$ |
| $A(W^+B^-H^+H^-) = (C_{WW} \beta + iC_{WW} \beta)^{\alpha\beta}(12)^2$ | $A(H^\alpha B^+ H^\gamma)$ | $\dot{Y}_{\gamma} (g_1 + y_h)g_3(C_{WW} \beta + iC_{WW} \beta)$ |
| $A(l^+e^- \gamma \ell^-\gamma^-) = (C_{WW} \beta - 4C_{WW} \beta)^{(12)(14)}(12)^2 \delta^{\alpha\beta}(14)(32) I_{a,b}$ | $A^{SM}(u^+q^+ W^+H_x)$ | $2\gamma_1 g_2 q_{q_{u,b}}$ |
| $A(l^+e^- B^- H^-) = -2\sqrt{2}C_{\alpha B} \delta^{\alpha\beta}(13)(23)$ | $A(H^\alpha H^\beta B^+ W^\gamma)$ | 2$\dot{Y}_{\gamma} g_2 C_{\alpha B}$ |
| $A(l^+e^- W^- H^\beta) = -2\sqrt{2}C_{\alpha W} (\tau^l)^{\alpha\beta}(13)(23)$ | $A(l^+e^- H^\alpha B^+ W^\gamma)$ | $\dot{Y}_{\gamma} g_2 C_{\alpha W}$ |

Table 1. Contributions to the running of $O_{eW}$ from different unitary cuts. $A^6$ are on-shell amplitudes from dimension 6 operators and $A^{SM}$ are SM amplitudes. Here $t\bar{t}^l\bar{t}^j = (C_{2a}(2) - \frac{1}{2}C_{2a}(G))\bar{t}^j$. 

\textsuperscript{2}There is a relative minus signs for terms linear in $g_t$ due to conventions.
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Table 2. Left: Dimension 6 operators that contribute to the running of $O_{eW}$; Right: Expressions of SM amplitudes used in calculating $\bar{C}_{eW}$.

| $A(l^\alpha - W^+ H^\beta)$ | $-\sqrt{2} Y_L^\alpha g_2 \frac{(\tau_i^\alpha)^2}{2} [12] | [23] [24] | [25] |
|-----------------------------|---------------------------------|
| $A(l^\alpha - B^+ H^\alpha)$ | $-\sqrt{2} V_u^\alpha Y_u \frac{g_1}{2} [12] [23] [24] |
| $A(u^+ q_3^\beta W^- H_\sigma)$ | $\frac{\sqrt{2} Y_u g_3 c_\lambda}{2} [12] [23] [24] |
| $A(H^\beta H^\alpha B^+ W^-)$ | $-g_1 g_2 y_H (\tau_i^\beta)^3 [13] [23] |
| $A(l^\alpha + B^+ W^-)$ | $g_1 g_2 y_H (\tau_i^\alpha)^2 [13] [23] |
| $A(H^\alpha e^+)$ | $-Y_e^\alpha Y_e [14] [13] |
| $A(l^\alpha - \tau^\alpha e^+)$ | $2 Y_\tau Y_\tau ^\alpha \frac{g_1^2}{13} [14] [13] |
| $A(e^- e^+ H^\alpha H^\beta)$ | $2 Y_{e-e} g_1 g_2 \delta^\alpha_\beta [12] [12] |
| $A(l^\alpha - l^\alpha H^\alpha H^\beta)$ | $-2 (y_{H} g_1 g_2 \delta^\alpha_\beta + g_2^2 (\tau_i^\alpha)(\tau_i^\beta)) [14] [12] |
| $A(H^\alpha W^+ W^- H^\beta)$ | $-2 g_2^2 \frac{(\tau_i^\alpha)^2}{12} [12] [12] |
| $A(l^\alpha - l^\alpha W^+ W^-)$ | $2 g_2^2 \frac{(\tau_i^\alpha)^2}{12} [12] [12] |

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