Arresting Accretion Torques with Gravitational Radiation

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**Abstract.** Recent theoretical work has made it plausible for neutron stars (NSs) to lose angular momentum via gravitational radiation on long timescales (\(\sim 10^6\) yr) while actively accreting. The gravitational waves (GWs) can either be emitted via the excitation of r-modes or from a deformed crust. GW emission can thus intervene to slow-down or halt the otherwise relentless spin-up from accretion. Prior to this theoretical work (and the measurements of NS rotation rates in LMXBs, see Chakrabarty’s contribution) the community was rather confident that an accreting NS would be spun-up to rotation rates near breakup, motivating searches for submillisecond objects. After only briefly describing the physics of the GW processes, I argue that the limiting spin frequency might be appreciably lower than the breakup frequency. Millisecond radio pulsar observers would likely discover the impact of GW emission as a dropoff in the number of pulsars beyond 600 Hz, and I show here that the millisecond pulsar inventory in 47 Tuc might already exhibit such a cutoff. These theoretical ideas will be tested by GW searches with ground-based interferometers, such as the advanced LIGO instrument proposed for operation by 2008.

1. Introduction

It has been over twenty years since the discovery of the first millisecond radio pulsar (MSP) by Backer et al. (1982). Since then, large numbers of MSPs have been found in globular clusters and in the field. The theoretical expectation is that these NSs are “recycled” and spun-up by prolonged accretion (see reviews by Phinney & Kulkarni 1994; Bhattacharya 1995). If accreting material arrives with the specific angular momentum of a particle orbiting at the NS radius \(R = 10R_{\text{g}}\text{km}\), then it takes \(\approx 10^8\) yr of accretion at a rate \(\dot{M} \approx 10^{-9} M_{\odot}\text{ yr}^{-1}\) for a \(\dot{M} = 1.4 M_{1.4} M_{\odot}\) NS to reach \(\nu_s = 500\) Hz from an initially low frequency (see Cook et al. 1994 for a thorough set of numbers). Many binary scenarios (e.g. Burderi et al. 1999) easily provide such an \(\dot{M}\) and transfer enough material to spin-up the NS to submillisecond periods. Hence, if there is no limiting physics, rapid rotation is expected and there should be no “cutoff” in the population other than that from the transferred mass.

The piece of limiting physics typically discussed is magnetic accretion (e.g. Ghosh & Lamb 1979). For a given \(\dot{M}\) and magnetic dipole strength, \(\mu\), the NS
Bildsten has a magnetospheric radius inside of which the flow is fixed by the magnetic topology. However, as the NS is spun-up, material rotating in the disk at the magnetospheric radius eventually co-rotates with the NS, such that further accretion will not likely spin it up. Namely, an equilibrium spin period is reached that matches the Kepler period at the magnetosphere

\[ P_{eq} = 8 \text{ ms} \left( \frac{10^{-9} M_{\odot} \text{ yr}^{-1}}{M} \right)^{3/7} \left( \frac{\mu}{10^{27} \text{ G cm}^3} \right)^{6/7}, \]

and further accretion is likely accommodated by alternating epochs of spin-up and spin-down. Such phenomena have been seen in the highly magnetic ($\mu > 10^{30} \text{ G cm}^3$, $B > 10^{12} \text{ G}$) accreting pulsars in high mass X-ray binaries (see Bildsten et al. 1997 for an overview). The $P_{eq}$'s for these NSs range from 1-1000 seconds for $\dot{M} \approx 10^{-10} - 10^{-7} M_{\odot} \text{ yr}^{-1}$, clearly implying a range of $\mu$'s.

Table 1. Oscillations During Type I X-Ray Bursts

| Object Name | $\nu_B$ (Hz) | Flux $10^{-9} \text{ (cgs)}$ | $h_c (10^{-27})$ |
|-------------|--------------|-------------------------------|-----------------|
| 4U 1916-053 | 270          | 0.5                           | 1.0             |
| 4U 1702-429 | 330          | 1.0                           | 1.2             |
| 4U 1728-34  | 363          | 2.8                           | 2.0             |
| KS 1731-260 | 524          | 0.2-2                         | 1.3             |
| Aql X-1     | 549          | 1.0                           | 1.0             |
| MXB 1658-298| 567          | 0.1                           | 0.3             |
| 4U 1636-53  | 581          | 4.4                           | 2.0             |
| MXB 1743-29 | 589          | ?                             | ?               |
| SAXJ1750.8-2980 | 601 | ?                             | ?               |
| 4U 1608-52  | 619          | < 1                           | < 1             |

As discussed by Chakrabarty, the $\nu$'s inferred from nearly coherent oscillations (given by $\nu_B$ in Table 1) during Type I bursts finally revealed rapidly rotating NSs in LMXBs. In addition, there are three accreting millisecond pulsars: SAX J1808.4-3658 ($\nu_s = 401 \text{ Hz}$; Wijnands & van der Klis 1998), XTE J1751-305 ($\nu_s = 435 \text{ Hz}$; Markwardt et al. 2002) and XTE J0929-314 ($\nu_s = 185 \text{ Hz}$; Galloway et al. 2002) which have $\sim 10^{-11} M_{\odot} \text{ yr}^{-1}$. All of the $\nu_s$'s are at least a factor of two away from the breakup frequency. It is also striking that, despite $\dot{M}$ contrasts of nearly a factor of 1000, the NS rotation rates are all within a factor of 2-3 of each other. White and Zhang (1997) (and others) have argued that this similarity arises because these NSs are magnetic and have reached $P_{eq}$. This has two requirements: (i) a tight relation between $\mu$ and $\dot{M}$ so that they all reach the same equilibrium, and (ii) a method of hiding the persistent pulse typically seen from a magnetic accretor. The state of knowledge of the $\mu$ values of these accreting NSs is quite limited (see Cumming’s contribution), though we desire them to have adequate $\mu$'s to become MSPs.

The current $\nu$ measurements of these accreting NSs thus suggests a limiting frequency that is not easily accommodated by either magnetic limitations or lifetime arguments. This indication, combined with the theoretical insights
summarized next, points to GW emission as a new piece of physics that can limit the spin-up. This would be “bad news” for those carrying out searches for previously hidden large populations of submillisecond MSPs, but “good news” for ground-based gravitational wave searches.

2. GW Emission from Crustal Quadrupoles and/or R-Mode

Let’s start with a simple hypothesis, which is that the NS has a misaligned quadrupole moment, $Q$, induced by accretion. The strong $\nu_s$ dependence of GW emission defines a critical frequency beyond which accretion can no longer spin-up the star. The NS will radiate energy via GW’s at a rate $\dot{E} = 32GQ^2\omega^6/5c^5$, where $\omega = 2\pi\nu_s$, and lose angular momentum at the rate $N_{gw} = \dot{E}/\omega$. Balancing this spin-down torque with the characteristic spin-up accretion torque, $N_a \approx \dot{M}(GMR)^{1/2}$, gives the $Q$ needed so as to halt spin-up (Bildsten 1998)

$$Q \approx 10^{37} \text{ g cm}^2 \left( \frac{\dot{M}}{10^{-9} \text{ M}_\odot \text{ yr}^{-1}} \right)^{1/2} \left( \frac{500 \text{ Hz}}{\nu_s} \right)^{5/2},$$

or $\approx 10^{-8}$ of the NS moment of inertia. The similarities in $\nu_s$ then arise because of the weak dependencies of the critical frequency, $\nu_{s,\text{crit}} \propto \dot{M}^{1/5}Q^{-2/5}$.

Bildsten (1998) and Ushomirsky et al. (2000) described physical mechanisms in the deep NS crust that map temperature perturbations into density perturbations in order to create a crustal quadrupole while accreting. Quadrupolar symmetry is not required, it is just the most efficient radiator. Ushomirsky et al. (2000) showed that 1-5% lateral temperature perturbations are adequate, and that if the composition or heating asymmetries are independent of $\dot{M}$, then the induced quadrupole scales $\propto \dot{M}^{1/2}$ so that the limiting $\nu_s$ weakly depends on $\dot{M}$. The critical open questions to the ab initio theory are: (i) What is the cause of the required few percent temperature perturbations? and (ii) Can the elastic readjustment of the crust be accommodated without cracking? This question is critical for the precession of isolated pulsars (see Link’s contribution).

Another GW emission mechanism is that offered by the newly discovered r-mode instability for the fluid in the NS core. In a series of remarkable breakthroughs, the gravitational physics community has shown (see Andersson’s 2002 recent review for references) that r-modes (those waves which get a large part of their restoring force via the Coriolis term) are unstable to the emission of gravitational radiation via the Chandrasekhar-Friedman-Schutz instability. Namely, the radiation of GWs excites the modes, regardless of spin frequency!

The beauty of the r-modes is that there is no need to invoke an asymmetry. That part comes immediately once it has been shown that the GW excitation beats the viscous damping. However, finding the minimum $\nu_s$ to trigger the instability involves understanding the viscosity of the NS core and the coupling of the r-modes to the overlying crust (e.g. Levin & Ushomirsky 2001; Wagoner 2002). The critical $\nu_s$ depends on both the NS structure and core temperature, and current calculations place the accreting NSs squarely in the transition regime between stable and unstable. Hence, the hypothesis of r-mode instability remains quite viable. How the angular momentum is lost by GW emission once
Figure 1. Periodic GW strains at Earth for Type I bursters (solid squares), accreting millisecond pulsars (open triangles) and the bright Z sources (filled circles, where I use the kHz QPO difference frequency as $\nu_s$) presuming quadrupolar emission in equilibrium with accretion. The two solid lines are proposed sensitivities for LIGO-II (courtesy K. Strain, 1999) with and without narrowbanding for 3 month integrations. The dashed line is a 2 week LIGO-II search. The lower dotted line is LIGO-II’s proposed thermal noise floor, which is the best case for narrowbanding at any frequency.

the mode goes unstable and starts to grow is complicated by Levin’s (1998) discovery that the internal heating from a runaway r-mode leads to a limit cycle behavior and transient GW emission. The duty cycle for GW emission is set by how large the mode amplitude can be before non-linear effects set in. Arras et al.’s (2002) current estimates of the mode saturation imply that the GW emission occurs for $10^3$ years every $\sim 100$ Myr. Most recently, Wagoner (2002) showed that the damping microphysics is uncertain enough that it is possible for steady-state r-mode emission with an internal wave amplitude of $\sim 10$ cm.

For persistent GW emission under either scenario, the GW strain at Earth from a source at distance $d$ can be found. For simplicity, I do it here for the quadrupolar deformation (the GW signal is then at $2\nu_s$, for r-modes it would be $4\nu_s/3$) where $h_c = 2.9G\omega^2Q/dc^4$ after suitably averaged over spin orientations (Brady et al. 1998). Presuming the NS luminosity is $L \approx GM\dot{M}/R$ then $h_c$ is written in terms of the observable $F = L/4\pi d^2$ (Wagoner 1984)

$$h_c \approx 1.3 \times 10^{-27} \frac{R_0^{3/4} M_{1.4}^{1/4}}{F^{1/2} \left( 10^{-9} \text{ erg cm}^{-2} \text{ s}^{-1} \right)^{1/2} \left( 300 \text{ Hz} \right)^{1/2}} ,$$

which represents a lower limit for $h_c$ due to the $L$ to $\dot{M}$ conversion I chose and since $F$ is never fully measured. The minimum $h_c$’s for those NSs which have $\nu_s$ inferred from Type I bursts are shown in Table 1 and plotted in Figure 1. The
Figure 2. Histograms of NS $\nu_{s}$’s. The solid line is the $\nu_{s}$ distribution for MSPs in 47 Tuc (Camilo et al. 2000), whereas the dashed line is for the NSs in Table 1 and the 3 accreting millisecond pulsars. The average 2-10 keV fluxes were either from van Paradijs (1995) or my own estimates. The figure caption describes the curves (see Cutler and Thorne 2002 for details) and makes it clear that Sco X-1 is an excellent target. However, the simplest LIGO-II search would be one where the pulse ephemeris is known and that is only true for the three accreting ms pulsars. The best is SAX J1808.4−3658 at $\nu_{s} = 401$ Hz, which is nearby, has accretion outbursts roughly every two years, and a well known orbit (Chakrabarty & Morgan 1998). A dedicated search at 802 Hz with a tuned LIGO-II could bear fruit.

3. Conclusions and Implications for Millisecond Radio Pulsar Searches

I have hopefully made the case that both theory and observation point to a new limiting mechanism to NS spin-up: GW emission. If true, the impact on MSP searches is clear: many fewer rapidly rotating MSPs will be found than predicted in those scenarios that neglect this physics. One way that such physics could reveal itself is to put in a “cutoff” in the MSP population at some particular frequency. The prime complication is attributing such a cutoff to the underlying population and not observational selection.

However, in 47 Tuc, the observational selection is calculable, and between 300 and 600 Hz, the MSP sensitivity drops by only 30 percent (see Figure 2 of Camilo et al. 2000). Such a small sensitivity drop is not adequate to explain the population decline (see solid line in Figure 2) to high frequencies in 47 Tuc. I also plot the $\nu_{s}$ distribution for 13 accreting NS with a dashed line. This sample is not, to our knowledge, biased in frequency in any way. If this is the injection distribution that becomes like that in 47 Tuc, then the higher $\nu_{s}$ systems must spin-down a factor of 1.5 in 5-8 Gyrs, easily accommodated with a surface field of $B \approx (2 − 5) \times 10^{8}$G.
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