Can the nuclear symmetry potential at supra-saturation densities be negative?

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In the framework of an Isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model, for the central $^{197}\text{Au}+^{197}\text{Au}$ reaction at an incident beam energy of 400 MeV/nucleon, effect of nuclear symmetry potential at supra-saturation densities on the pre-equilibrium clusters emission is studied. It is found that for the positive symmetry potential at supra-saturation densities the neutron to proton ratio of lighter clusters with mass number $A \leq 3 \left( (n/p)_{A\leq3} \right)$ is larger than that of the weighter clusters with mass number $A > 3 \left( (n/p)_{A>3} \right)$, whereas for the negative symmetry potential at supra-saturation densities $\left( (n/p)_{A\leq3} \right)$ is smaller than that of the $\left( (n/p)_{A>3} \right)$. This may be considered as a probe of the negative symmetry potential at supra-saturation densities.

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Recently the studies of the density-dependent nuclear symmetry energy, which is crucial for understanding many interesting issues in both nuclear physics and astrophysics [1-8], have attracted much attention [9-11]. The high density behavior of the symmetry energy, however, has been regarded as the most uncertain property of dense neutron-rich nuclear matter [12, 13]. Many microscopic and/or phenomenological many-body theories using various interactions predict that the symmetry energy increases continuously at all densities. On the other hand, other models predict that the symmetry energy first increases to a maximum and then may start decreasing at certain supra-saturation densities. Thus, currently the theoretical predictions on the symmetry energy at supra-saturation densities are extremely diverse. To make further progress in determining the symmetry energy at supra-saturation densities, what are most critically needed is some guidance from dialogues between experiments and the predictions of heavy-ion collisions transport models, which have been done extensively in the studies of nuclear symmetry energy at low densities [14, 15].

While studying the symmetry energy by using heavy-ion collisions, the related input in an IBUU transport model is actually the symmetry potential, which is a more complete information than the density dependence of the symmetry energy at zero temperature calculated with the same (mean-field) approximation. Unfortunately, the symmetry potential is also rather uncertain, which can be positive or negative at supra-saturation densities [2, 16-24]. To study the symmetry energy at supra-saturation densities in the framework of an IBUU model, one has to first determine the symmetry potential at higher densities. The symmetry potential, its positive or negative, thus urgently needs to be solved. In this paper, we use the asymmetry of cluster emission in dynamical simulation to study the symmetry potential at supra-saturation densities.

The non-equal partition of the system’s isospin asymmetry with the gas phase being more neutron-rich than the liquid phase has been found as a general phenomenon using essentially all thermodynamical models and in simulations of heavy-ion reactions [2, 25-28]. But all these studies are for low energy density nuclear matter, in which the sub-saturation symmetry energy/potential dominates the isospin fractionation. For supra-saturation symmetry energy/potential’s studies, one needs to study the isospin fractionation of higher energy density nuclear matter. Such matter is explored in the first step of high energy heavy-ion collisions, where time dependence and out-of-equilibrium effects play a dominant role. Therefore, to study the high density behavior of nuclear symmetry energy/potential, one needs to base on the transport model, to study particle emission by using relative high incident beam energy of heavy-ion collisions and compare with the experimental data. In the present study, using an isospin and momentum-dependent transport model IBUU, as an example, we studied the neutron-rich reaction of $^{197}\text{Au}+^{197}\text{Au}$ at a beam energy of 400 MeV/nucleon with the positive and negative symmetry potentials at supra-saturation densities while keeping the low density symmetry energy/potential fixed. We compute the average isospin ratio of clusters of size larger (smaller) than $A=4$ (corresponding mass number of $\alpha$ particle), which will be called $(n/p)_{A\geq3}$ ($(n/p)_{A\leq3}$) in the following. We find that for the positive symmetry potential at supra-saturation densities the $(n/p)_{A\geq3}$ is larger than that of the $(n/p)_{A>3}$, whereas for the negative symmetry potential at supra-saturation densities the $(n/p)_{A\leq3}$ is smaller than that of the $(n/p)_{A>3}$.

The isospin and momentum-dependent mean field potential (MDI) used in the present work is [29]

\[
U(\rho, \delta, \mathbf{p}, \tau) = A_u(\rho) \frac{\rho^\sigma}{\rho_0} + A_l(\rho) \frac{\rho^\sigma}{\rho_0} + B(\rho) \sigma^2 \left( 1 - x \sigma^2 \right) \\
-8x\sigma - \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0} \delta \rho \tau \\
+ \frac{2C_{\tau,x}}{\rho_0} \int d^3 \mathbf{p} \frac{f_\tau(\mathbf{r}, \mathbf{p})}{1 + (\mathbf{p} - \mathbf{p'})^2/\Lambda^2} \\
- \frac{C_{\tau,x'}}{\rho_0} \int d^3 \mathbf{p} \frac{f_{\tau'}(\mathbf{r}, \mathbf{p}')}{1 + (\mathbf{p} - \mathbf{p''})^2/\Lambda^2},
\]

where $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry of the nuclear medium. In the above $\tau = 1/2 \ (-1/2)$ for neutrons (protons) and $\tau \neq \tau'; \sigma = 4/3; f_\tau(\mathbf{r}, \mathbf{p})$ is the phase
space distribution function at coordinate \( r \) and momentum \( p \). The parameters \( A_{\rho}(x), A_{\rho}(x), B, C_{\tau, \tau}, C_{\tau, \tau} \) and \( \Lambda \) were obtained by fitting the momentum-dependence of the \( U(\rho, \delta, p, \tau, x) \) to that predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock (BHF) calculations [30], the saturation properties of symmetric nuclear matter and the symmetry energy of about 30 MeV at normal nuclear matter density \( \rho_0 = 0.16 \text{ fm}^{-3} \) [19, 20]. The incompressibility \( K_0 \) of symmetric nuclear matter at \( \rho_0 \) is set to be 211 MeV consistent with the latest conclusion from studying giant resonances [31–33]. The parameters \( A_{\rho}(x) \) and \( A_{\rho}(x) \) depend on the parameter according to

\[
A_{\rho}(x) = -95.98 - x \frac{2B}{\sigma + 1}, \quad A_{\rho}(x) = -120.57 + x \frac{2B}{\sigma + 1}.
\]

The variable \( x \) is introduced to mimic different forms of the symmetry energy/potential predicted by various many-body theories without changing any property of the symmetric nuclear matter and the symmetry energy at normal density \( \rho_0 \). The last two terms in Eq. (1) contain the momentum-dependence of the single-particle potential. The momentum dependence of the symmetry potential stems from the different interaction strength parameters \( C_{\tau, \tau} \) and \( C_{\tau, \tau} \) for a nucleon of isospin \( \tau \) interacting, respectively, with unlike and like nucleons in the background fields. More specifically, we use \( C_{\text{unlike}} = -103.4 \text{ MeV} \) and \( C_{\text{like}} = -11.7 \text{ MeV} \). With these parameters, the nucleon isoscalar potential estimated from \( U_{\text{isoscalar}} \approx (U_n + U_p)/2 \) agrees with the prediction of various many-body calculations for symmetric nuclear matter [19, 29, 34], the BHF approach [21, 30, 33] including three-body forces and the Dirac-Brueckner-Hartree-Fock (DBHF) calculations [30] in broad ranges of density and momentum. And the corresponding pressure of symmetric matter is consistent with the experimental limits [37]. The corresponding isovector (symmetry) potential can be estimated from \( U_{\text{sym}} \approx (U_n - U_p)/2 \). With different \( x \) parameters at normal nuclear matter density \( \rho_0 \), the symmetry potential, as shown in Fig. 1, agrees very well with the Lane potential extracted from nucleon-nucleon and \((n,p)\) charge exchange reactions available for nucleon kinetic energies up to about 100 MeV [38]. At supra-saturation densities we can see that the stiff symmetry energy \( (x = -2) \), shown in Fig. 2, corresponds to positive symmetry potential while the soft symmetry energy corresponds to negative symmetry potential.

According to essentially all microscopic model calculations, see e.g., [32, 33], the EOS for isospin asymmetric nuclear matter can be expressed as

\[
E(\rho, \delta) = E(\rho, 0) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4),
\]

where \( E(\rho, 0) \) is the energy per nucleon of symmetric nuclear matter, and \( E_{\text{sym}}(\rho) \) is the nuclear symmetry energy. With the single particle potential Eq. (1), the symmetry energy can be written as [39]

\[
E_{\text{sym}}(\rho, x) = \frac{1}{2} \left( \frac{\partial^2 E}{\partial \delta^2} \right)_{\delta=0}.
\]

FIG. 1: The symmetry potential as a function of energy for different density values in the MDI interaction with \( x = 1 \) and \( x = -2 \). The experimental data [38] are also shown.

\[
= \frac{8\pi}{9m\hbar^2} p_f^2 + \frac{\rho}{4\rho_0} \left[ -24.59 + 4Bx/(\sigma + 1) \right] + \frac{Bx}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^{\sigma} + \frac{C_{\text{like}}}{2\rho_0 \rho} \left( \frac{4\pi}{\hbar^2} \right)^2 \Lambda^2
\]

\[
\times \left[ 4p_f^2 - \Lambda^2 p_f^2 \ln \frac{4p_f^2 + \Lambda^2}{\Lambda^2} \right]
\]

\[
+ \frac{C_{\text{unlike}}}{2\rho_0 \rho} \left( \frac{4\pi}{\hbar^2} \right)^2 \Lambda^2
\]

\[
\times \left[ 4p_f^2 - p_f^2 \left( 4p_f^2 + 2 \Lambda^2 \right) \ln \frac{4p_f^2 + \Lambda^2}{\Lambda^2} \right],
\]

where \( p_f = \hbar (3\pi^2 \rho_0^{2/3})^{1/3} \) is the Fermi momentum for symmetric nuclear matter at density \( \rho \). In the mean-field approximation, the symmetry energy \( E_{\text{sym}}(\rho, x) \) is actually divided by its potential part \( E^p_{\text{sym}}(\rho, x) \), \( E_{\text{sym}}(\rho, x) = \frac{\hbar}{2m_n} (3\pi^2 \rho/2)^{2/3} + E^p_{\text{sym}}(\rho, x) \) and the \( E^p_{\text{sym}}(\rho, x) \) is directly related to the symmetry potential \( (U_n - U_p)/2\delta \) [19]. Shown in Fig. 2 is the density-dependent symmetry energy. At sub-saturation densities, recent studies have constrained the symmetry energy around 31.6(\rho/\rho_0)^{0.69} (corresponding \( x = 0 \)) [8, 13, 40, 41]. So in the present work, we use the symmetry energy/potential corresponding \( x = 0 \) as our choice at sub-saturation densities. At supra-saturation densities, because the symmetry energy/potential is very uncertain, we use the positive \( (x = -2) \) and negative \( (x = 1) \) symmetry potential as two extreme cases (corresponding super-stiff and super-soft symmetry energy at supra-saturation densities, respectively).

In the IBUU transport model, the initial neutron and
FIG. 2: The symmetry energy as a function of density in the MDI interaction with different $x$ parameters. The symmetry energy at sub-saturation densities was roughly constrained from recent studies [6,12,40,41].

proton density distributions of the projectile and target are obtained by using the Skyrme-Hartree-Fock theory. The isospin-dependent in-medium nucleon-nucleon (NN) elastic cross sections from the scaling model according to nucleon effective masses are used. For the inelastic cross sections we use the experimental data from free space NN collisions since the in-medium inelastic NN cross sections are still very much controversial. The total and differential cross sections for all other particles are taken either from experimental data or obtained by using the detailed balance formula. The isospin dependent phase-space distribution functions of the particles involved are solved by using the test-particle method numerically. The isospin-dependence of Pauli blockings for fermions is also considered, for more details we refer the reader to Ref. [10].

The nuclear liquid-gas phase transition in dilute asymmetric nuclear matter is studied recently [42]. It argued that the neutron to proton ratio of the gas phase becomes smaller than that of the liquid phase for energetic nucleons and the gas phase is still overall more neutron-rich than that of liquid phase. Notice that the recent comparisons between IBUU calculations and FOPI data favor a rather soft symmetry energy ($x = 1$) (corresponding a negative potential at supra-saturation densities) at supra-saturation densities [9] and the progress of the studies of nuclear symmetry energy at sub-saturation densities [7,15,40,41] in this work we studied the $n/p$ of clusters emitted in the central $^{197}$Au+$^{197}$Au reaction at a beam energy of 400 MeV/nucleon with the positive ($x = -2$) and negative ($x = 1$) symmetry potentials at supra-saturation densities but keeping the symmetry potential at sub-saturation densities fixed ($x = 0$). The maximal compressed density reached in this reaction is about $2.5\rho_0$ [3]. Shown in Fig. 3 is $n/p$ of clusters with $A \leq 3, A > 3$ as a function of nucleonic kinetic energy. For the negative symmetry potential, neutrons trend to being attracted by the symmetry potential and protons trend to being repelled during isospin fractionation. Thus $n/p$ of weighter clusters ($A > 3$) are larger than that of the lighter clusters ($A \leq 3$). Whereas for positive symmetry potential, neutrons trend to being repelled by the symmetry potential and protons trend to being attracted during isospin fractionation. Therefore $n/p$ of weighter clusters ($A > 3$) are smaller than that of the lighter clusters ($A \leq 3$). From Fig. 3 we can also see that $n/p$ of weighter clusters ($A > 3$) with negative symmetry potential is overall larger than reaction system’s $n/p$ while with the positive symmetry potential $n/p$ of weighter clusters ($A > 3$) is expected overall smaller than reaction system’s $n/p$. In Fig. 3 $(n/p)_{A<3}$ and $(n/p)_{A>3}$ are the neutron to proton ratios of nucleons with local densities smaller ($A \leq 3$) and larger ($A > 3$) than $\rho_c = 1/8\rho_0$, respectively. Changing $\rho_c$ from $1/5\rho_0$ to $1/10\rho_0$, our quantitative results only shift about 3%. The integral $(n/p)_{A<3}/(n/p)_{A>3}$ with nucleonic kinetic energy $E_{\text{kin}}/\text{nucleon} \geq 20$ MeV (nucleons with kinetic energy $E_{\text{kin}}/\text{nucleon} < 20$ MeV are mainly from cluster decays [43]) is 1.44/1.65 for the negative symmetry potential and 1.48/1.35 for the positive symmetry potential. In the standard implementation of cluster recognition after the BUU dynamics [43], a physical fragment is formed as a cluster of nucleons with relative momenta smaller than $P_0 = 263$ MeV/c (Fermi-Momentum of normal nuclear matter) and relative distances smaller than $R_0 = 3$ fm (deduced by the uncertainty relationship of quantum mechanics). The integral $(n/p)_{A<3}/(n/p)_{A>3}$ with nucleonic kinetic energy

FIG. 3: Neutron to proton ratio $n/p$ as a function of nucleonic kinetic energy of clusters with $A \leq 3, A > 3$ from the central $^{197}$Au+$^{197}$Au reaction at a beam energy of 400 MeV/nucleon with the positive ($x = -2$) and negative ($x = 1$) symmetry potentials at supra-saturation densities but keeping the symmetry potential at sub-saturation densities fixed ($x = 0$). The number 1.49 denotes the reaction system’s $n/p$. 
TABLE I: The clusterisation method and the associated parameters.

| set       | \( n/p \) | \( p_{0} = 1/8p_{0} \) | \( P_{0} = 263 \text{ MeV} /c \) |
|-----------|-----------|-------------------------|-------------------------------|
| Negative  | \( A \leq 3 \) | 1.44                     | 1.44                           |
| (\( x = 1 \)) | \( A > 3 \) | 1.65                     | 1.55                           |
| Positive  | \( A \leq 3 \) | 1.48                     | 1.49                           |
| (\( x = -2 \)) | \( A > 3 \) | 1.35                     | 1.42                           |

\( E_{\text{kin/nucleon}} \geq 20 \text{ MeV} \) is 1.44/1.55 for the negative symmetry potential and 1.49/1.42 for the positive symmetry potential and our results are not sensitive to the phase-space coalescence parameter settings (when changing \( P_{0} \) from 263 MeV/c to \( P_{0}^{\text{Au}} \approx 258 \text{ MeV} /c \), the results shift not more than 0.3%). In order to show more clearly the above results, We present Table I. It is seen that the clusterisation method based on phase-space coalescence and the method based on a density cut-off give the same \( (n/p)_{A>3} \), but different \( (n/p)_{A>3} \). Specifically, the symmetry energy dependence of the isospin content of large clusters is less pronounced with the coalescence method. This can be qualitatively understood. Indeed the nuclear Fermi momentum of asymmetric nuclear matter is \( P_{F} = (1 - \delta)1/3p_{0} \), \( p_{F}^{n} = (1 + \delta)1/3p_{0} \). Where \( p_{F}^{p}, p_{F}^{n} \) are proton and neutron’s Fermi-Momenta, respectively [35]. At densities close to saturation neutrons will then have a momentum that can exceed the coalescence parameter \( P_{0} \) which corresponds to symmetric nuclear matter at saturation. Therefore in average the coalescence criterion will be harder to fulfill for neutrons than for protons at the same density. This can explain why, for the negative symmetry potential, \( (n/p)_{A>3} \) is lower with the coalescence method than with the density cut-off method. The opposite effect is observed with the positive symmetry potential, which leads to more neutron-rich dense matter. From the above we can see that although the two methods (the clusterisation method based on phase-space coalescence and the method based on a density cut-off) give different quantitative results, they give the same qualitative results, i.e., \( (n/p)_{A<3} \) is larger/smaller than \( (n/p)_{A>3} \) for the positive/negative symmetry potential at supra-saturation densities.

The overall \( (n/p)_{A>3} \) is smaller than \( (n/p)_{A>3} \) with nucleonic kinetic energy \( E_{\text{kin/nucleon}} \geq 20 \text{ MeV} \) for the negative symmetry potential at supra-saturation densities after isospin fractionation is different from the knowledge in the literatures [2, 25, 26, 22] for lower energy density nuclear isospin fractionation where mainly the symmetry energy/potential at sub-saturation densities works. In case experimentally in the central \( 197 \text{Au} + 197 \text{Au} \) an reaction at an incident beam energy of 400 MeV/nucleon \( (n/p)_{A<3} \) is overall smaller than \( (n/p)_{A>3} \) with nucleonic kinetic energy \( E_{\text{kin/nucleon}} \geq 20 \text{ MeV} \), then a negative symmetry potential at supra-saturation densities is obtained. Otherwise the negative symmetry potential at supra-saturation densities is ruled out. This test in fact does not dependent on our quantitative results of \( (n/p)_{A<3} \) or \( (n/p)_{A>3} \). This can be a probe of the negative symmetry potential at supra-saturation densities. It should be mentioned that different isospin dependent in-medium nucleon-nucleon cross sections may affect our present results [44, 45], such studies are planned.

In summary, based on an isospin dependent transport model IBUU, \( n/p \) of the clusters as a test of negative symmetry potential at supra-saturation densities is studied. We found that for the positive symmetry potential at supra-saturation densities the neutron to proton ratio of lighter clusters with mass number \( A \leq 3 \) is larger than that of the weightier clusters with mass number \( A > 3 \), but for the negative symmetry potential at supra-saturation densities the \( (n/p)_{A<3} \) is smaller than that of the \( (n/p)_{A>3} \). This may be considered as a probe of the negative symmetry potential at supra-saturation densities.

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