Electromagneto-hydrodynamic flow transport of two layer fluids through a microchannel with interfacial slip at fluid-solid interface

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Abstract. This work investigates an electromagneto-hydrodynamic flow of two layer fluids through a microchannel under the influence of interfacial slip at fluid-solid interface. The governing equations of conservation of momentum and potential distribution equations with Debye-Huckel approximation has been simplified analytically. The effects of different parameters like transverse electric field, viscosity ratio, interfacial charge density has been presented graphically. Also this study explored that the effect of Hartmann number and large Hartmann number on the velocity of fluid are almost opposite. These results are valid to the available data. Also it is shown that the interfacial charge density has a crucial role to play with the velocity of a fluid.

1. Introduction

In last few years, many influential microfluidic mechanisms have been explored and as its integral results, plenty of corresponding microfluidic devices has been developed so far and being used in service to mankind, among which several types of electromagneto-hydrodynamic (EMHD) [1, 3, 4] and electrohydrodynamic (EHD) [1] micropumps are leading. In the context of microchannels and microfluids, there are also some significant applications of it in diverse fields like bio-medical engineering and technology, aerospace engineering, power and process industries, superconductors, electronic technology, physical and chemical sciences, various lab-on-chip processes [2, 5]. Actually, microchannels has such diverse and wide applications as it has very much suitable and realistic kind of geometry particularly in the study of microfluids. That is why in last few years the study of microfluids, microchannels got a tremendous importance and attraction to the researchers [6]. Along with this, further investigations regarding electroosmotic flows in microchannels has been come into being by incorporating electrokinetic forces under several conditions like constant pressure gradient, electric and magnetic fields etc. [7, 8, 9, 10]. J. Escandon [11] has investigated the thermal and hydrodynamic analysis of a mixed electromagneto-hydrodynamic pressure guided flow for a Phane-Thien-Tanner (PTT) fluid through a microchannel. Ngoma [12] has studied a model of two immiscible fluid flows through a microchannel between two parallel walls endowed with pressure gradient and electromagnetic effects, in which they discussed the effects of electric double layer on a flow with
external pressure gradient gives rise to a streaming potential. Nithiarasu et al [13] discussed the flow and heat transfer for electro-osmotic flow in a microchannel. Vakili et al [14] have studied the same electro osmotic flow of a power-law fluid in a rectangular microchannel under the influence of pressure gradient numerically by using finite difference technique. Arnold et al [15] dilates upon electro osmotic flow in a straight two and three-dimensional microchannel using modified Navier-Stokes equation combined with Laplace and Poisson-Boltzmann equations numerically. They also compared the numerical and experimental data incorporating the influences of different parameters. Shit et al [16] delineated an aspect of two-layer electro osmotic flow considering a hydrophobic microchannel taking zeta potential into account. They also incorpo-rated the heat transfer. Siva et al [17] depicted unsteady electro-osmotic flow of a fluid under couple stress in a rotating microchannel and provided an analytical solution to it.

Here, in this study, EMHD flow of two layer fluids through a microchannel has been explored with interfacial slip at fluid-solid interface. The governing equations of conservation of momentum and the Poisson Boltzmann equation within the EDL with Dybye-Huckel approximation has also been simplified and solved analytically. The influence of transverse electric field, slip parameters for both layers, interfacial charge density, viscosity ratio on the velocity of two layer fluids has been addressed analytically and interpreted graphically using MATHEMATICA-12 software.

2. Mathematical model and problem description

The EMHD flow of two-layer fluids through a slit microchannel is considered which is bounded by the parallel plates. The physical sketch of this flow problem is depicted in Fig 1. The flow transport phenomenon of the electrolyte through the narrow rectangular microchannel delineates in this paper, where height of the channel is \( H \). The length \( L \) and width \( W \) of the channel are assumed much greater than the height. i.e. \( L \gg H, W \gg H \). The fluid flow region divided into two layers at \( y = H_1 \). Different fluid densities and viscosities are provided into layer-1 \( H_1 \leq y \leq H_2 \) and layer-2 \( 0 \leq y \leq H_1 \) respectively. The interaction of electrolyte solution generates an EDL which is a very thin fluid layer at the liquid-solid interface. The parallel walls of the microchannel are maintained at different zeta potential and different slip length at the channel wall. The electrolyte solution generates an EDL near to the boundary wall and the induced electric field in opposite to the axial direction, creates an electroosmotic flow. The electroosmotic flow is generated by the applied electric field component \( E_x \), due to the EDL near the microchannel wall which is directed opposite to the fluid motion. The total system is subjected to an applied magnetic field \( B_y \) perpendicular to the fluid flow direction and \( E_z \) is imposed as an external transverse electric field from outside to inside.

![Fig. 1. Physical sketch of the problem](image)

2.1. Potential distribution within EDL

In order to assess the EMHD influence on the microfluidic flow transport phenomenon, it may be enlightening to illustrate a brief antecedent of the electric double layer (EDL) effects, which is occurred near to the walls of the microchannel. Near to the walls of the microchannel, in physical contact with the electrolyte solution, develop surface charge due to ionization of the liquid or due to the absorption of ions of the electrolyte solution. Such kind of improvement of surface charge triggers
a spontaneous redistribution of the ions, within the electrolyte solution, culminating in the formation of EDL. Two layers construct the EDL, one is stern layer and another is diffuse layer. The thickness of the EDL is typically of the order of few nm, depending on the concentration of the electrolyte solution. The net charge density \( \rho_{ei} \) inside the EDL of the microchannel flow is described from Poisson equation, [22] which is expressed by the Poisson-Boltzmann equation within layer-1 and layer-2 that follows:

\[
\frac{d^2 \psi_i}{d y^2} = -\frac{\rho_{ei}}{\varepsilon_i}, \quad i = 1, 2.
\]

(1)

Here \( \psi_i \) is the EDL potential which is function of \( y \), \( \varepsilon_i \) be the permittivity of the electrolyte fluids for layer-1 and layer-2 respectively. Using the symmetry property of electrolytes [22], the electric charge density \( \rho_e \) is expressed by the following equation:

\[
\rho_e = -2n_0 e z \sinh \left( \frac{e z \psi_i}{k_B T_{av}} \right), \quad i = 1, 2.
\]

(2)

where \( n_0 \) represents the ion density, \( k_B \) denotes the Boltzmann constant, \( z \) is the valance and the absolute temperature is \( T_{av} \).

Since \( \psi_i \) are too much small, so \((e z \psi_i/k_B T_{av}) <= 1\), the term \( \sinh(e z \psi_i/k_B T_{av}) \) can be estimated by \((e z \psi_i)/(k_B T_{av})\). This principle is cognized as Dybye-Hückel linearization. After executing this Dybye-Hückel linearization, the linearized form of the Poisson equation is rewritten as:

\[
\frac{d^2 \psi_i}{d y^2} = \kappa_i^2 \psi_i, \quad i = 1, 2.
\]

(3)

where \( \kappa_i = e z \left( \frac{2n_0}{k_B T_{av}} \right)^{\frac{1}{2}} \) is Dybye-Hückle parameter and \( \frac{1}{\kappa_i} \) be the EDL thickness for both layers respectively and the appropriate boundary conditions are:

\[
\psi_1 = \psi_0 \quad \text{at} \quad y = H, \quad \psi_2 = \psi_w \quad \text{at} \quad y = 0
\]

(4)

At the channel wall, the zeta potential \( \psi_0 \) and \( \psi_w \) are constant at \( y = H \) and \( y = 0 \). Moreover, in the interface between layer-1 and layer-2, two boundary conditions are introduced by considering the zeta potential difference \( \delta \psi \) and Gauss’s law for charge density for the electrical displacement as

\[
\psi_2 - \psi_1 = \delta \psi \quad \text{at} \quad y = H_1, \quad \varepsilon_1 \frac{d \psi_1}{d y} - \varepsilon_2 \frac{d \psi_2}{d y} = -q_d \quad \text{at} \quad y = H_1
\]

(5)

where \( d \) is interface charge density jump.

To make dimensionless equation (3), \( \psi_i^* = \frac{e z \psi_i}{k_B T_{av}}, \quad \psi_0^* = \frac{e z \psi_0}{k_B T_{av}}, \quad \psi_w^* = \frac{e z \psi_w}{k_B T}, \quad \delta \psi^* = \frac{e z \delta \psi}{k_B T}, \quad Q = \frac{H z e q_d}{e k_B T_{av}} \)

\( y^* = \frac{y}{H} \)

are employed as dimensionless variables and then the equation becomes:

\[
\frac{d^2 \psi_i^*}{d y^*} = \omega_i^2 \psi_i^*, \quad i = 1, 2
\]

(6)

where \( \omega_i = \kappa_i H \) is called the electroosmotic parameter.

The appropriate boundary conditions for EDL potential are represented as:

\[
\psi_1^* = \psi_0^* \quad \text{at} \quad y = 1, \quad \psi_2^* = \psi_w^* \quad \text{at} \quad y = 0
\]

(7)

\[
\psi_2^* - \psi_1^* = \delta \psi^* \quad \text{at} \quad y = h, \quad \frac{d \psi_1^*}{d y^*} - \varepsilon \frac{d \psi_2^*}{d y^*} = -Q \quad \text{at} \quad y = h
\]

(8)

where \( \varepsilon = \varepsilon_2/\varepsilon_1 \), \( h = \frac{H_1}{H} \).

By solving equation (6) subjected to the boundary condition (7) and (8), the solution of non-dimensional EDL potentials \( \psi_i^* \) are derived by

\[
\psi_1^* = C_1 \cosh(\omega_1 y^*) + C_2 \sinh(\omega_1 y^*)
\]

(9)

\[
\psi_2^* = C_2 \cosh(\omega_2 y^*) + C_4 \sinh(\omega_2 y^*)
\]

(10)

Where

\[
C_1 = \frac{-Q + \varepsilon \omega_2 \left[ \psi_w^*(\sinh(\omega_2 h) - \cosh(\omega_2 h) \coth(\omega_2 h)) + \delta \psi^* \coth(\omega_2 h) \right] - C_2(\omega_1 \cosh(\omega_2 h) - \varepsilon \omega_2 \sinh(\omega_1 h) \coth(\omega_2 h))}{\omega_1 \sinh(\omega_1 h) - \varepsilon \omega_2 \cosh(\omega_1 h) \coth(\omega_2 h)}
\]

\[
C_2 = \frac{\psi_w^*(\omega_1 \sinh(\omega_1 h) - \varepsilon \omega_2 \cosh(\omega_1 h) \coth(\omega_2 h)) - \cosh(\omega_2)}{[\omega_1 \sinh(\omega_1 h) - \varepsilon \omega_2 \cosh(\omega_1 h) \coth(\omega_2 h)] \sinh(\omega_1) - (\omega_1 \cosh(\omega_2 h) - \varepsilon \omega_2 \sinh(\omega_1 h) \coth(\omega_2 h)) \coth(\omega_1)}
\]
\[ C_3 = \psi_{\nu}^*, \quad C_4 = \frac{\delta_0 + c_1 \cosh(\omega_1 h) + c_2 \sinh(\omega_2 h) - \psi_0^* \cosh(\omega_2 h)}{\sinh(\omega_2 h)} \]

### 2.2. Velocity Distribution

On the basis of combined influence of electromagnetic force and constant pressure gradient, the governing Brinkman equation of momentum for the flow in porous microchannel can be expressed as

\[ -\nabla p + \mu \nabla^2 \vec{U} - \frac{K}{\mu} \vec{U} + \vec{b} = 0 \]

(11)

where \( p \) be the pressure, \( K \) be the permeability constant of the porous material, \( \mu \) be the viscosity, \( \vec{U} \) is the velocity vector and \( \vec{b} \) being the body force.

For the fully developed flow, having the electromagnetic field and the constant pressure gradient inside the EDL, the equation of momentum conservation in the two layer fluids along \( x \)-direction is expressed as

\[ \frac{\partial p_i}{\partial x_i} - \mu_i \frac{d^2 u_i}{dy_i^2} - \rho_i E \dot{\varepsilon}_x + \sigma_x B_y u_i - \sigma_x B_x E_x = 0, \quad i = 1, 2, \]

(12)

where \( p_i, u_i \) and \( \mu_i \) are the pressure of the fluid and the axial velocity along \( x \)-direction of the fluid and viscosity correspond to the fluid layer-1 (\( i = 1 \)) and layer-2 (\( i = 2 \)) respectively. \( E_x \) is the axial component and \( E_x \) is the lateral transverse component of applied electrical force and \( B_y \) is the magnetic field component along \( y \)-direction.

The Navier slip boundary condition at the fluid-solid interface are expressed as

\[ u_1 + \beta_{s1} \frac{du_1}{dy} = 0 \quad \text{at} \quad y = H, \quad u_2 - \beta_{s2} \frac{du_2}{dy} = 0 \quad \text{at} \quad y = 0 \]

(13)

Where \( \beta_{s1} \) and \( \beta_{s2} \) are slip lengths at the wall of the microchannel at \( y = H \) and \( y = 0 \) respectively. At the fluid-fluid interface, the continuity of the velocity between the two layers are derived as

\[ u_1 - u_2 = 0 \quad \text{at} \quad y = H_1, \]

\[-\mu_1 \frac{du_1}{dy} + \varepsilon_1 E \frac{d\psi_1}{dy} = -\mu_2 \frac{du_2}{dy} + \varepsilon_2 E \frac{d\psi_2}{dy} \quad \text{at} \quad y = H_1 \]

(14)

Assume that the pressure gradient is fixed. By applying non-dimensional transformations, the dimensionless momentum equations for layer-1 and layer-2 are reduced as follows:

\[ \frac{d^2 u_1^*}{dy_1^*} - Ha^2 u_1^* + \frac{\omega_1^2 \psi_1^*}{\psi_0^*} + Ha S + P = 0 \]

(15)

and

\[ \mu \frac{d^2 u_2^*}{dy_2^*} - \mu Ha^2 u_2^* + \varepsilon \omega_2^2 \psi_2^* + Ha S + P \delta_p = 0 \]

(16)

where

\[ u_1^* = \frac{u_1}{U_{HS}}, \quad u_2^* = \frac{u_2}{U_{HS}}, \quad U_{HS} = -\varepsilon_1 \psi_0 \frac{E_x}{\mu}, \quad \Delta P_1 = \left( -\frac{dP_1}{dx} \right), \quad P = \frac{\Delta P_1}{\mu_1 U_{HS}}, \quad \delta_p = \frac{\Delta P_2}{\Delta P_1}, \]

\[ \mu = \frac{\mu_2}{\mu_1}, \quad Ha = B_y H \sqrt{\varepsilon_1 \mu_1}, \quad S = \frac{E_x}{U_{HS}} \sqrt{\frac{\varepsilon_2}{\mu_1}}, \quad \beta_1 = \frac{\beta_{s1}}{H}, \quad \beta_2 = \frac{\beta_{s2}}{H}. \]

Here, \( Ha \) and \( P \) indicate the Hartmann number, normalized pressure gradient respectively. \( U_{HS} \) represents the Helmholtz-Smoluchowski velocity with in layer-1, \( \mu \) and \( \delta_p \) are the viscosity ratio and pressure ratio of layer-2 to layer-1 and \( \beta_1 \) and \( \beta_2 \) are interfacial slip parameters for layer-1 and layer-2 respectively. The non-dimensional boundary conditions are rewritten as:

\[ u_1^* + \beta_1 \frac{du_1}{dy^*} = 0 \quad \text{at} \quad y^* = 1, \quad u_2^* - \beta_2 \frac{du_2}{dy^*} = 0 \quad \text{at} \quad y^* = 0 \]

(17)

\[ u_1^* - u_2^* = 0 \quad \text{at} \quad y^* = H, \]

\[ \frac{du_1^*}{dy^*} - \frac{1}{\psi_0^*} \frac{d\psi_1^*}{dy^*} = \mu \frac{du_2^*}{dy^*} - \varepsilon \frac{d\psi_2^*}{\psi_0^*} \quad \text{at} \quad y^* = H \]

(18)

Equation (15) and (16) are solved subjected to the proper boundary conditions (17) and (18) to obtain solutions of dimensionless velocity for both layers, which are evaluated as

\[ u_1^* = A_1 e^{Ha y^*} + A_2 e^{-Ha y^*} + \frac{c_2 \sinh(\omega_1 y^*) - \psi_0^* c_1 \cosh(\omega_1 y^*)}{Ha^2 (\omega_2 + \omega_1 y^*)} \]
\[ u_2 = A_3 e^{H_\alpha y'} + A_4 e^{-H_\alpha y'} + \frac{\mu (H_\alpha^2 - \omega_1^2)}{H_\alpha^2 (H_\alpha^2 - \omega_1^2)\mu^2} \]

where

\[ A_1 = -\frac{1}{B_5} (B_1 + A_2 B_6), \quad A_2 = \frac{1}{B_9} (A_4 B_{11} + A_3 B_{10} - A_4 B_6 - B_3), \]

\[ A_3 = \frac{1}{(B_7 + 1)} \left[ \frac{B_2}{H_\alpha^2 \omega_1^2} \right], \]

\[ A_4 = \frac{1}{(B_7 + 1)} \left[ \frac{B_2}{H_\alpha^2 \omega_1^2} \right], \]

\[ B_1 = \frac{(C_1 + C_2 \beta_1 \omega_1) \cosh(\omega_1 y') + (C_2 + C_1 \beta_1 \omega_1) \sinh(\omega_1 y')}{H_\alpha^2 - \omega_1^2}, \quad B_2 = \frac{C_2 - C_4 \beta_2 \omega_2}{H_\alpha^2 - \omega_1^2}, \]

\[ B_3 = \frac{C_1 \cosh(\omega_1 y') + C_2 \sinh(\omega_1 y')}{H_\alpha^2 - \omega_1^2}, \quad B_4 = \frac{C_1 \cosh(\omega_1 y') + C_2 \sinh(\omega_1 y')}{H_\alpha^2 - \omega_1^2}, \]

\[ B_5 = \frac{\omega_1 (C_2 \cosh(\omega_1 y') + C_1 \sinh(\omega_1 y'))}{\omega_1 (C_2 \cosh(\omega_1 y') + C_1 \sinh(\omega_1 y'))}, \quad B_6 = \frac{\omega_1 (C_2 \cosh(\omega_1 y') + C_1 \sinh(\omega_1 y'))}{\omega_1 (C_2 \cosh(\omega_1 y') + C_1 \sinh(\omega_1 y'))} - \frac{\omega_1 (C_2 \cosh(\omega_1 y') + C_1 \sinh(\omega_1 y'))}{\omega_1 (C_2 \cosh(\omega_1 y') + C_1 \sinh(\omega_1 y'))}.

3. Results and discussion

In this article, the EMHD flow transport of two-layer fluid through the hydrophobic microchannel with interfacial slip at fluid-solid interface has been examined. The equations (15) and (16) subject to the boundary conditions (17) and (18) are solved analytically. It is observed that EMHD flow characteristics represented by non-dimensional flow velocity (\(u^*\)) depend on the dimensionless physical parameters the applied magnetic field (Hartmann Number) (\(H_\alpha\)), transverse electric field (\(S\)), slip parameters for both layers (\(\beta_1, \beta_2\)), viscosity ratio (\(\mu\)) and interfacial charge density (\(Q\)). To examine and analyze the impact of these pertinent parameters on the electromagnetically driven flow, the following typical parametric values are used as per the available scientific data available in the literatures ([16], [18-21]): \(P = 0.0 - 1.5; H_\alpha = 0 - 100; S = 0 - 100; \omega_1 = 1 - 15; \omega_2 = 5 - 30; \psi_0 = 0.06; \psi_\phi = 0.001; 0.5 \leq \epsilon \leq 2; Q = 0.0 - 0.5; h = 0.3; \delta_p = 1; \mu = 0.5 - 2; \beta_1 = 0.0 - 0.03; \beta_2 = 0.0 - 0.06; \delta_\psi = 0.1.

![Fig. 2. Velocity distribution profiles for several values of (a) slip parameter \(\beta_1\), (b) slip parameter \(\beta_2\) with \(P = 1.0, \epsilon = 1.0, \delta_\psi = 0.1, Q = 0.05, \mu = 1, \omega_1 = 10, \omega_2 = 20, H_\alpha = 1, S = 50\).](image-url)
This investigation deals the analytical solutions of the velocity distribution of two-layer fluid under the influence of combined EMHD effects through a microchannel. In this present segment, we discussed the effects of various dimensionless parameters on the flow transport through the microchannel. The interactive influences of different non-dimensional parameters on the flow transport phenomenon are examined. The impacts of applied magnetic field parameter ($Ha$), the influence of Navier slip velocity parameter ($\beta_1$) and ($\beta_2$), interface charge density jump ($Q$), normalized viscosity ratio ($\mu$) are shown in Figs. 2-4 respectively.

![Fig. 3. Velocity distribution profiles for several values of (a) charge density jump $Q$, (b) viscosity ratio $\mu$ with $P = 1.0, \epsilon = 1.0, \delta_0 = 0.1, \beta_1 = 0.02, \beta_2 = 0.01, \omega_1 = 10, \omega_2 = 20, Ha = 1, S = 50.$](image)

![Fig. 4. Velocity distribution profiles for several values of (a) Hartmann number $Ha \leq 1$, (b) large values of Hartmann number $Ha > 1$, with $P = 1.0, \epsilon = 1.0, \delta_0 = 0.1, \mu = 1.0, \beta_1 = 0.02, \beta_2 = 0.01, \omega_1 = 10, \omega_2 = 20, Q = 0.05, S = 50.$](image)

Figure 2(a) shows that, as $\beta_1$ increases the velocity of fluid increases and at the juncture of two fluids the velocity gets its maximum. This figure also explores the fact that, there is no comprehensive changes in velocity for fluid layer-2 whereas comparatively more increment in velocity occurs for fluid layer-1. Almost dual result of figure-2(a) comes out of figure 2(b) as the velocity of fluid increases more rapidly and velocity remains almost the same for fluid layer 1. From figure 2(a) and 2(b), it is also very interesting to know that from both figures the changes of the velocity occurs near to the boundary layers of the fluid. Figure 3(a) delineates the influence of charge density jump on the two layer fluid so that the velocity at the interfacial region increases significantly as the charge density jump decreases. The change in velocity near the channel wall is insignificant but the disturbance is occurred more in the interface of the both fluids for $Q = 0.5$ because of the different charge difference of the interface of both fluids. Figure 3(b) depicts the influence of viscosity ratio on the axial velocity. As the viscosity ratio increases, the velocity decreases significantly as interaction among the fluid ions
and molecules increases with the increment of viscosity ratio, which leads to this result. Figure 4(a) expresses the effect of Hartmann number on the axial fluid velocity and we see that with the small increment of the number, there is a drastic increment in the axial velocity. Figure 4(b) takes up the influence of large Hartmann number on the axial velocity which has an opposite effect as the Hartmann number to some extent. At the fluid interfacial surface the velocity increase with the decrement of it.

4. Conclusion
In this study, we have constructed a mathematical model to describe EMHD flow for two layer fluids through a microchannel. We discussed the influence of transverse electric field, slip parameters, viscosity ratio, interfacial charge density and Hartmann number on the velocity of the two layer fluid. It is shown that, the velocity of fluid increases with the increment of values in slip parameters. Significant increment to fluid velocity occurs when the charge density jump decreases. The influence in viscosity ratio on the velocity remains almost same as that of charge density jump. A small increment in Hartmann number results a significant increment in velocity of the fluids and the effects of large Hartmann number is almost opposite to that of Hartmann number.

These results may be fruitful in the areas of microfabrication technologies. This study may also extended to explore the entropy production characteristics which gives rise to many application to further improvement of thermos-electro devices.

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