A Photometric and Astrometric Study of the Open Clusters NGC 1664 and NGC 6939

Selin Koç¹, Talar Yontan², Selçuk Bilir², Remziye Canbay¹, Tansel Ak², Timothy Bunks³,⁴, Serap Ak², and Ernst Paunzen²

¹ Istanbul University, Institute of Graduate Studies in Science, Programme of Astronomy and Space Sciences, 34116, Beyazıt, Istanbul, Turkey; seliskoc@gmail.com
² Istanbul University, Faculty of Science, Department of Astronomy and Space Sciences, 34119, Beyazıt, Istanbul, Turkey
³ Nielsen, Data Science, 200 West Jackson Boulevard #17, Chicago, IL 60606, USA
⁴ Physics & Astronomy, Harper College, 1200 West Algonquin Road, Palatine, IL 60067, USA
⁵ Department of Theoretical Physics and Astrophysics, Masaryk University, Kotlár’šká 2, 611 37 Brno, Czech Republic

Received 2022 January 5; revised 2022 February 18; accepted 2022 February 21; published 2022 March 25

Abstract

This study calculated astrophysical parameters, as well as kinematic and galactic orbital parameters, of the open clusters NGC 1664 and NGC 6939. The work is based on CCD ultraviolet, blue, and visual (UBV) and Gaia photometric and astrometric data from ground- and space-based observations. Considering Gaia Early Data Release 3 (EDR3) astrometric data, we determined membership probabilities of stars located in both of the clusters. We used two-color diagrams to determine the E(B − V) color excesses for NGC 1664 and NGC 6939 as 0.190 ± 0.018 and 0.380 ± 0.025 mag, respectively. Photometric metallicities for the two clusters were estimated as [Fe/H] = −0.10 ± 0.02 dex for NGC 1664 and as [Fe/H] = −0.06 ± 0.01 dex for NGC 6939. Using the reddening and metallicity calculated in the study, we obtained distance moduli and ages of the clusters by fitting PARSEC isochrones to the color–magnitude diagrams based on the most likely member stars. Isochrone fitting distances are 1289 ± 47 pc and 1716 ± 87 pc, which coincide with ages of 675 ± 50 Myr and 1.5 ± 0.2 Gyr for NGC 1664 and NGC 6939, respectively. We also derived the distances to the clusters using Gaia trigonometric parallaxes and compared these estimates with the literature. We concluded that the results are in good agreement with those given by the current study. Present-day mass function slopes were calculated as Γ₁ = −1.22 ± 0.33 and Γ₁ = −1.18 ± 0.21 for NGC 1664 and NGC 6939, respectively, which are compatible with the Salpeter slope. Analyses showed that both of clusters are dynamically relaxed. The kinematic and dynamic orbital parameters of the clusters were calculated, indicating that the birthplaces of the clusters are outside the solar circle.

Unified Astronomy Thesaurus concepts: Open star clusters (1160); Star clusters (1567); Galaxy kinematics (602); Stellar kinematics (1608); Hertzsprung Russell diagram (725)

Supporting material: machine-readable table

1. Introduction

Open clusters (OCs) are groupings of stars which formed from the same molecular cloud and so are under similar physical conditions. This results in OCs being gravitationally bound systems and ensures that cluster stars are very similar in distance, age, chemical compositions, positions, and velocities (Lada & Lada 2003). These characteristics make it possible to determine which of the stars observed in the direction of the cluster are most likely cluster member stars, and then to determine precise astrometric (proper motions and parallaxes), astrophysical (distance, reddening, metallicity, and age) and kinematic (space velocities and orbital motions) parameters of the clusters. Member stars of clusters will have different luminosities within a wide range of stellar masses, according to the initial distribution of masses they formed with. Studies across OCs can give important information about their dynamical properties as well as insight into the evolution of the Galaxy (Carraro & Chiosi 1994; Gilmore et al. 2012; Moraux 2016). Such work requires that the cluster member stars should be determined carefully and that the parameters of OCs should be determined precisely using homogeneous data and methods.

OC parameters have been estimated across many studies and compiled in large catalogs (Dias et al. 2002; Röser et al. 2010; Kharchenko et al. 2013; Dias et al. 2014; Sampedro et al. 2017). When the distances, color excesses, and ages of OCs in these catalogs are compared there are still inconsistencies across the parameters (Netopil et al. 2015).

Using data from the Gaia mission will shed light on the precise determination of cluster membership as well as be helpful for astrometric solutions and parallaxes of OCs. It will also be helpful for the identification of new OCs. Cantat-Gaudin et al. (2018) used the second data release of Gaia survey (hereafter Gaia DR2; Gaia Collaboration et al. 2018) and determined the membership probabilities of 1229 OCs with 60 newly detected clusters. Shortly after the third (early) data release of Gaia (hereafter Gaia EDR3; Gaia Collaboration et al. 2021) nearly 865 new galactic OCs were discovered (Liu & Pang 2019; Sim et al. 2019; Castro-Ginard et al. 2020). The Gaia EDR3 database contains more than 1.55 billion full astrometric solutions and photometric data with high precision. The trigonometric parallax (π) uncertainties are 0.02–0.07 mas for G ≤ 17 mag, reaching 0.5 and 1.3 mas for G = 20 and G = 21 mag, respectively. The proper-motion uncertainties are 0.02–0.07 mas yr⁻¹ for G ≤ 17, increasing to 0.5 mas yr⁻¹ and 1.4 mas yr⁻¹ at G = 21 mag. The G-band photometric uncertainties are 0.3–1 mmag for the sources within G ≤ 17 mag, climbing to 6 mmag at G = 20 mag. The database provides radial velocities for about 7.21 million stars in the
4 < G < 13 magnitude range with an accuracy of about 1.2 km s⁻¹.

In this study we determined the membership probability of
cluster stars, mean proper motion, and distance of the OCs
NGC 1664 and NGC 6939 using ground-based ultraviolet,
blue, and visual (UBV) photometric observations together
with high-precision astrometry and photometry taken from the Gaia
EDR3 catalog. We present the fundamental parameters, the
kinematics and galactic orbital parameters, the luminosity and
mass functions, and the dynamical state of the mass segregation
of these clusters. The literature summaries of the clusters are as
follows.

1.1. NGC 1664

NGC 1664 (α = 04h51m06s, δ = +43°40′30″, l = 161°68,
b = −0°45) was classified by Trumpler (1930) as II2m-a with a
tenuous central star concentration. Research literature on the
cluster has been based on photoelectric and photographic
methods, with no detailed CCD UBV study prior to the current
study. The distance of NGC 1664 was determined by different
researchers as being 1100–1300 pc (Larsson-Leander 1957;
Hoag & Applequist 1965; Fang 1970; Becker & Fenkart 1971).
Based on photographic observations, Larsson-Leander (1957)
obtained the color excess of NGC 1664 as E(B−V) = 0.30 mag while Lindoff (1968) determined the age as
t = 90 Myr. Kharchenko et al. (2012) applied an algorithm
they had developed, for the determination of the astrophysical
parameters of clusters taking into account stellar evolution
models, to calculate the distance, color excess, and age of NGC
1664 as d = 1200 pc, E(B−V) = 0.25 mag, and t = 560 Myr,
respectively (Kharchenko et al. 2013). Joshi et al. (2016)
determined the structural parameters, age, mass, and color
excess of the cluster with a statistical analysis approach. They
estimated the color excesses and age of the cluster as E
(B−V) = 0.25 mag and t = 560 ± 33 Myr, respectively.
Reddy et al. (2016) analyzed high-resolution (R ~ 40,000 to
55,000) spectroscopic observations of red clump stars in 12
OCs, including NGC 1664. They identified two red clump stars as
being members of NGC 1664 based on the WEBDA database
and measured the metallicity of the cluster as [Fe/H] = −0.15 ± 0.05 dex. Bossini et al. (2019) evaluated the Gaia
DR2 astrometric and photometric data across 269 OCs,
applying a Bayesian statistical analysis to derive the age,
distance modulus, and extinction for each of them (see Table 1
for the NGC 1664 results). The authors estimated the distance
modulus to be μ = 10.626 ± 0.020 mag, which corresponds to
d = 963 ± 9 pc in distance, and extinction to be A_V = 0.708 ±
0.018 mag, which corresponds to a color excess of E(B−V) = 0.228 ± 0.006 mag. Carrera et al. (2019) used Apache
Point Observatory Galactic Evolution Experiment (APOGEE)
and Galactic Archaeology with HERMES (GALAH) spectroscopic
data to estimate the radial velocities of 145 OCs, as well as
the iron abundances of 104 OCs. They took into account the
two most likely members of NGC 1664 and from these measured the mean radial velocity of the cluster as
V_r = 6.70 ± 0.05 km s⁻¹. The iron abundance of the cluster was determined as [Fe/H] = −0.01 ± 0.01 dex from one
cluster member star. Donor et al. (2020), using high-resolution
spectroscopic data from APOGEE DR16, calculated the abundances across 16 elements for stars estimated as most
likely to be cluster members of 128 OCs. Based on Gaia DR2
astrometric and photometric data membership probabilities of
stars, mean proper-motion components and cluster ages were
determined. The iron abundance was measured as [Fe/
H] = −0.03 ± 0.01 dex for NGC 1664, based on high-resolution
spectroscopic data of one cluster member star. Donor et al.
(2020) estimated the age of the cluster as t = 560 Myr and the
mean proper-motion components as (μ_λ cos δ, μ_β) = (1.84 ±
0.07, −5.76 ± 0.04) mas yr⁻¹.

1.2. NGC 6939

NGC 6939 (α = 20h31m30s, δ = +60°39′42″, l = 95°90,
b = +12°30) is a relatively star-rich cluster located in the
constellation Cepheus, close to the galactic plane. The first
study of the cluster was made by Kustner (1923), providing
photographic data of 370 stars detected in the cluster vicinity.
Cuffey (1944) determined the distance of the cluster as
d = 1300 pc, while Cannon & Lloyd (1969) estimated its age
as being approximately t = 1 Gyr. Chincarini (1963) identified
likely cluster member stars up to V~ 15.5 mag through
analyzing photoelectric and photographic data. The study
estimated that the color excess, distance modulus, and age of
NGC 6939 as E(B−V) = 0.5 mag, μ = 12.00 mag, and
t = 50 Myr respectively. Later researchers obtained photo-
electric and radial velocities measurements, together with
studies of the proper-motion components of likely cluster
member stars (Geisler 1988; Glushkova & Rastorguev 1991;
Mermilliod et al. 1994; Milone 1994). Robb & Cardinal (1998)
utilized their CCD observations to detect six variable stars in
the cluster region, and classified three of them as K giant
variables and the others as eclipsing binary systems from the
variation of their long-term light curves. The first CCD
photometric study of the NGC 6939 was made by Rosvick &
Balam (2002), analyzing BVI data for stars situated near the
cluster center. Rosvick & Balam (2002) determined the age of
cluster as t = 1.6 ± 0.3 Gyr and remarked that the color excess
of NGC 6939 changes across the body of the cluster within the
range of 0.29 < E(B−V) < 0.41 mag.

Following this, Andreuzzi et al. (2004) performed a CCD
UBVI photometric study of NGC 6939 and determined its
distance as d = 1800 pc. They estimated that the age of the
cluster was in the range of t = 1.6 to 1.3 Gyr and that the color
excess across the cluster changes within the range 0.34 <
E(B−V) < 0.38 mag. Maciejewski & Niedzielski (2007)
undertook a CCD BV photometric analysis of 42 OCs including
NGC 6939 and determined the age, color excess, distance
modulus, and distance of this cluster as being t = 1.25 Gyr,
E(B−V) = 0.38±0.10 mag, μ = 12.15±0.72 mag, and
d = 1560±500 pc respectively.

The structural analysis of Maciejewski & Niedzielski (2007)
determined the core radius of the cluster to be r_c = 2.2 arcmin.
Based on mass function results, they indicated that there were
evaporated (or escaped) stars from the cluster. Kharchenko
et al. (2012) took into account stellar evolution models in their
estimation of the distance, reddening, and age of the cluster as
being d = 1800 pc, E(B−V) = 0.31 mag, and t = 1.9 Gyr,
respectively (see Table 1 for the NGC 1664 results).

Considering the literature, it can be seen that NGC 6939 is
an intermediate-age OC. This implies the existence of evolved
giant stars in the cluster, whose presence has enabled
researchers to determine the metallicity of the cluster via
photometric and spectroscopic data (Canterna et al. 1986;
NGC 1664 0.30 11.50 1300 ... 90 ... ... (1)
0.31 11.09 1240 ... 90 ... ... (2)
0.25 11.17 1199 ... 525 −2.54 ± 0.07 −6.21 ± 0.07 (3)
0.25 10.48 1200 ... 560 ± 33 −4.23 ± 0.02 −4.20 ± 0.02 (4)
... ... ... ... ... −1.83 ± 0.08 −4.15 ± 0.17 (5)
0.25 11.17 1200 ... 560 ± 33 ... ... ... (7)
0.25 11.17 1200 ... 560 ... ... ... (8)
... ... ... ... 525 −2.54 ± 0.31 −6.21 ± 0.32 (9)
... ... ... ... 1362 ± 7 ... ... ... (12)
0.28 ± 0.03 11.298 ± 0.171 1218 ± 49 −0.009 ± 0.013 645 ± 100 1.727 ± 0.014 −5.720 ± 0.010 (17)
... ... ... ... 1361 ± 7 ... ... ... (19)
0.190 ± 0.018 11.205 ± 0.075 1328 ± 46 −0.10 ± 0.02 675 ± 50 1.594 ± 0.071 −5.780 ± 0.052 (20)

NGC 6939 0.33 ± 0.07 12.27 ± 0.41 1780 ± 365 ... 1600 ± 300 ... ... (21)
0.34–0.38 12.35–12.58 1820–1905 ... 1000–1300 ... ... ... (22)
0.38 ± 0.18 12.15 ± 0.56 1560 ± 640 ... 1260 ... ... ... (23)
0.31 12.30 1800 ... 1870 ... −2.13 −1.34 (26)
0.31 12.24 1800 ... 1900 ... ... ... (7)
0.33 12.30 1800 ... 1600 ... −2.17 ± 0.20 −4.51 ± 0.19 (9)
... ... ... ... ... −1.84 ± 0.006 −5.413 ± 0.006 (12)
... ... ... ... 1868 ± 101 ... ... ... (27)
0.30 ± 0.01 12.143 ± 0.104 1757 ± 51 0.462 ± 0.10 1600 ± 200 −1.838 ± 0.007 −5.410 ± 0.007 (17)
... ... ... ... 1770 ... ... ... (18)
0.38 ± 0.025 12.350 ± 0.109 1716 ± 87 −0.06 ± 0.01 1500 ± 200 −1.817 ± 0.039 −5.423 ± 0.039 (20)

Note. Columns present the cluster names, color excesses \((E_B-V)\), distance moduli \((\mu)\), distances \((d)\), iron abundances \([Fe/H]\), age \((t)\), and proper-motion components \((\mu_\alpha \cos \delta, \mu_\delta)\). (1) Lindoff (1968), (2) Fang (1970), (3) Kharchenko et al. (2005), (4) Kharchenko et al. (2013), (5) Dias et al. (2014), (6) Reddy et al. (2016), (7) Joshi et al. (2016), (8) Kharchenko et al. (2016), (9) Sampedro et al. (2017), (10) Zhai et al. (2017), (11) Conrad et al. (2017), (12) Cantat-Gaudin et al. (2018), (13) Liu & Pang (2019), (14) Bossini et al. (2019), (15) Carrera et al. (2019), (16) Donor et al. (2020), (17) Dias et al. (2021), (18) Taricq et al. (2021), (19) Hao et al. (2021), (20) This study, (21) Rosvick & Balam (2002), (22) Andreuzzi et al. (2004), (23) Jacobson et al. (2007), (24) Maciejewski & Niedzielski (2007), (25) Warren & Cole (2009), (26) Kharchenko et al. (2012), (27) Casamiquela et al. (2019).

2. Observations

The ground-based CCD UBV observations of NGC 1664 and NGC 6939 were made using the 100 cm Ritchey-Chrétien telescope (T100) of the TÜBİTAK National Observatory (TUG)7 located in Turkey. Identification maps of OCs NGC 1664 and NGC 6939 are shown in Figure 1. The UBV images were taken using a Spectral Instruments CCD camera operating at −100°C. The CCD camera is equipped with a back illuminated 4k × 4k pixel CCD, with a scale of 0.23 pixel−1. This results in an unvignetted field of view of 21.5 × 21.5 on the telescope. The gain and the readout noise of the CCD camera are 0.55 e−/ADU and 4.19 e−/100 KHz, Ch:A, respectively. In order to be sensitive to the widest possible flux range (from UV and infrared; IR), different exposure times were used during the observations. A log of observations is given in Table 2. UBV photometric calibrations were based on the Landolt (2009) standard stars. A total of 108 stars in 15 selected areas were observed. The air masses were between 1.232 and 1.918 for the standard stars monitored during the observation nights (see Table 3). Standard CCD reduction techniques were applied using IRAF8 packages for the

7 www.tug.tubitak.gov.tr
8 IRAF is distributed by the National Optical Astronomy Observatories.
preprocessing of all the images that were taken. Astrometric corrections were applied using PyRAF\(^9\) and astrometry.net\(^{10}\) routines together with our own scripts for the cluster images. Using IRAF’s aperture photometry packages, we measured the instrumental magnitudes of the Landolt (2009) stars. We next applied multiple linear fits to these magnitudes to derive photometric extinction and transformation coefficients for each observing night as listed in Table 4. Source Extractor (SExTractor) and PSF Extractor (PSFEx) routines (Bertin & Arnouts 1996) were applied to measure the instrumental magnitudes of the objects located in the cluster fields. We applied aperture corrections to these magnitudes and transformed them to standard magnitudes in the Johnson photometric system using the transformation equations as described by Janes & Hoq (2011):

\[
V = v - \alpha_{bv}(B - V) - k_vX_v - C_{bv}
\]

\[
B - V = \frac{(b - v) - (k_b - k_v)X_{bv} - (C_b - C_{bv})}{\alpha_b + k'_bX_b - \alpha_{bv}}
\]

\(^9\) PyRAF is a product of the Space Telescope Science Institute, which is operated by AURA for NASA.

\(^{10}\) http://astrometry.net

Table 2

| Cluster     | Obs. Date | Filter/Exp, Time (s) × N |
|-------------|-----------|-------------------------|
| NGC 1664    | 2018 Nov 11 | 60 × 2 6 × 3 4 × 5 |
| NGC 6939    | 2019 Jul 30 | 300 × 3 40 × 5 15 × 5 |

Note. Columns denote the names of the clusters, the observation dates, filters, exposure times (in seconds), and the number of exposures (N).

Table 3

| Date        | Star Field | \(N_{st}\) | \(N_{obs}\) | \(X\)  |
|-------------|------------|------------|------------|--------|
| 2018 Nov 05 | SA92SF2    | 15         | 1          |        |
|             | SA93       | 4          | 1          |        |
|             | SA94       | 2          | 1          |        |
|             | SA95SF2    | 9          | 1          |        |
|             | SA96       | 2          | 2          | 1.232–1.918 |
|             | SA97SF1    | 2          | 2          |        |
|             | SA99       | 3          | 1          |        |
|             | SA100SF2   | 10         | 1          |        |
|             | SA112      | 6          | 1          |        |
|             | SA114      | 5          | 1          |        |

2019 Jul 30

| Date  | Star Field | \(N_{st}\) | \(N_{obs}\) | \(X\)  |
|-------|------------|------------|------------|--------|
| SA92SF2 | 15        | 1          |        |
| SA93   | 4          | 1          |        |
| SA94   | 2          | 1          |        |
| SA106  | 2          | 1          |        |
| SA107  | 7          | 1          | 1.229–1.892 |
| SA108  | 2          | 1          |        |
| SA109  | 2          | 1          |        |
| SA111  | 5          | 1          |        |
| SA112  | 6          | 2          |        |
| SA114  | 5          | 1          |        |

Note. The columns are the observation date, star field name as from Landolt, the number of standard stars (\(N_{st}\)) observed in a given field, the number of pointings to each field (\(N_{obs}\), i.e., observations), and the airmass range the fields (on a given night) were observed over (\(X\)).

\[
U - B = \frac{(u - b) - (1 - \alpha_b - k_uX_u)(B - V)}{\alpha_{ub} + k'_uX_u} - \frac{(k_u - k_b)X_{ub} - (C_{ub} - C_b)}{\alpha_{ub} + k'_bX_u}
\]

Here \(U\), \(B\), and \(V\) indicate the magnitudes in the standard photometric system. \(u\), \(b\), and \(v\) are the instrumental magnitudes.
The mean internal UBV errors for stars brighter than magnitudes. $X$ is the airmass, $k$ and $k'$ are the primary and secondary extinction coefficients while $\alpha$ and $C$ are transformation coefficients to the standard system. The photometric extinction and transformation coefficients for a given night were calculated by applying multiple linear fits to the instrumental magnitudes of the standard stars. The obtained values are given in Table 4.

3. Data Analysis

3.1. Photometric Data

Catalogs of ground-based CCD UBV photometric information listing all the detected stars in the cluster regions are available electronically for NGC 1664 and NGC 6939.\(^{11}\) We identified 3,735 stars for NGC 1664 and 2,119 stars for NGC 6939 before constructing their photometric and astrometric catalogs. Both of these catalogs contain positions ($\alpha$, $\delta$), apparent $V$ magnitudes, color indices ($U - B$, $B - V$), proper-motion components ($\mu_\alpha$, $\cos \delta$, $\mu_\delta$) along with trigonometric parallaxes ($\pi$) from Gaia ED3, and membership probabilities ($P$) as calculated in this study (Table 5). Magnitude and color inaccuracies of Johnson ($V$, $U - B$, $B - V$) and Gaia ED3 ($G$, $G_{BP} - G_{RP}$) photometry were adopted as internal errors. We calculated the mean photometric errors as functions of $V$-magnitude intervals in Table 6. It can be seen from this table that the mean internal UBV errors for stars brighter than $V = 22$ mag for NGC 1664 and NGC 6939 are smaller than 0.045 mag and 0.086 mag, respectively. Moreover the mean internal errors in the Gaia ED3 photometric data for the stars brighter than $V = 22$ mag are smaller than 0.005 and 0.008 mag for NGC 1664 and NGC 6939, respectively.

The $V$ and $G$ magnitude histograms were used to estimate the photometric completeness limits for each of the clusters (see Figure 2). We compared these stellar counts with Gaia ED3 data from areas of the same size as the cluster CCD images, using the central equatorial coordinates given by Cantat-Gaudin et al. (2020) and utilizing the stars within the $8 < G < 23$ mag range. These distributions are shown in Figure 2, where the black solid lines represent the observational values according to the $V$ and $G$ interval magnitudes, while the red solid lines (see Figures 2(b) and (d)) show the stellar distributions retrieved from Gaia ED3. Figures 2(b) and (d) demonstrate the clear alignment of the stellar counts from these two data sets up to the magnitudes adopted as being the completeness limits (indicated by the vertical arrows in the diagrams) for the clusters. These values are $V = 20$ mag for each cluster, which corresponds to $G = 19$ mag both for NGC 1664 and NGC 6939. Telescope detection limits and telescope-detector combinations vary for ground- and space-based observations. These differences will lead to the number of detected stars at fainter magnitudes varying between different observational studies. This situation could clarify why the numbers of detected stars at $G > 20$ mag in the Gaia observations are greater than from this study’s ground-based photometry. Detected stars are significant for precise analyzes. As it can be seen from the Figure 2, beyond the completeness limits there are incomplete data. Therefore in our analyzes we considered the stars brighter than $V = 20$ mag for two clusters.

Due to the lack of literature CCD UBV data for NGC 1664, we compared our photometric results only for NGC 6939 where CCD BV data were given by Maciejewski & Niedzielski (2007). There are 3129 stars brighter than $V < 21$ mag in the catalog of Maciejewski & Niedzielski (2007), which we attempted to cross-match with the current study using the stellar equatorial coordinates. 1150 stars within the $11 < V < 20$ mag range were successfully matched between the two studies, allowing comparison of the $V$ magnitudes and ($B - V$) colors measurements (see Figure 3). The horizontal and vertical axes of this figure denote our $V$-band measurements and differences between $V$ magnitudes and ($B - V$) colors between the two studies, respectively. Based on these comparisons, the mean differences of magnitude and colors were calculated as $\langle \Delta V \rangle = 0.044 \pm 0.144$ and $\langle \Delta (B - V) \rangle = 0.021 \pm 0.120$ mag. We therefore find that the current study’s photometry is compatible with that of Maciejewski & Niedzielski (2007).

3.2. Spatial Structure of the Clusters

We applied the King (1962) radial density profile (RDP) model to estimate the core, limiting, and effective radii of NGC 1664 and NGC 6939. To do this we constructed a series of concentric rings oriented on the central coordinates given by Cantat-Gaudin et al. (2020). Stellar densities ($\rho$) were estimated for each ring by dividing the number of stars found in each ring by the ring area. Figure 4 plots these measured stellar densities versus radius from the cluster region together with optimal King (1962) RDP following $\chi^2$ minimization. The King (1962) model is given by the equation $\rho(r) = f_0g + [f_0/(1+(r/r_c)^2)]$, where $r$ is the radius from the cluster center, $f_0$ is the central density, $r_c$ is the core radius, and

\begin{table}[ht]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Filter/Color Index & Obs. Date & $k$ & $k'$ & $\alpha$ & $C$
\hline
$U$ & 2018 Nov 05 & $0.502 \pm 0.078$ & $-0.117 \pm 0.095$ & $\cdots$ & $\cdots$
\hline
$B$ & $0.232 \pm 0.046$ & $-0.037 \pm 0.050$ & $0.968 \pm 0.076$ & $1.488 \pm 0.070$
\hline
$V$ & $0.128 \pm 0.020$ & $\cdots$ & $\cdots$ & $\cdots$
\hline
$U - B$ & $\cdots$ & $\cdots$ & $0.977 \pm 0.143$ & $3.724 \pm 0.121$
\hline
$B - V$ & $\cdots$ & $\cdots$ & $0.078 \pm 0.009$ & $1.550 \pm 0.031$
\hline
$U$ & 2019 Jul 30 & $0.498 \pm 0.092$ & $-0.068 \pm 0.108$ & $\cdots$ & $\cdots$
\hline
$B$ & $0.293 \pm 0.079$ & $-0.026 \pm 0.082$ & $0.866 \pm 0.127$ & $1.799 \pm 0.116$
\hline
$V$ & $0.208 \pm 0.026$ & $\cdots$ & $\cdots$ & $\cdots$
\hline
$U - B$ & $\cdots$ & $\cdots$ & $0.953 \pm 0.161$ & $4.162 \pm 0.130$
\hline
$B - V$ & $\cdots$ & $\cdots$ & $0.073 \pm 0.012$ & $1.782 \pm 0.037$
\hline
\end{tabular}
\caption{Transformation and Extinction Coefficients Derived for the Two Observation Nights}
\end{table}

Note. $\alpha$ and $C$ are the transformation coefficients. $k$ and $k'$ are the primary and secondary extinction Coefficients, respectively.

\(^{11}\) The complete table is available in machine-readable format.
Table 5
The Catalogs for NGC 1664 and NGC 6939

| NGC 1664 | NGC 6939 |
|-----------------|----------------------------------|
| **ID** | **R.A.** (hh:mm:ss.ss) | **Decl.** (dd:mm:ss.ss) | **V** (mag) | **U − B** (mag) | **B − V** (mag) | **G** (mag) | **G_{AP} − G_{AP}** (mag) | **μ_x cos δ** (mas yr^{-1}) | **μ_y** (mas yr^{-1}) | **w** (mas) | **P** |
| 0001 | 04:50:03.99 | +43:40:40.33 | 18.924(0.015) | ... | ... | 1.524(0.005) | 18.539(0.003) | 1.626(0.030) | 0.417(0.311) | −1.427(0.196) | 0.375(0.195) | 0.00 |
| 0002 | 04:50:04.03 | +43:31:37.23 | 19.752(0.038) | ... | ... | 1.246(0.004) | 18.712(0.003) | 1.905(0.039) | 1.631(0.319) | −10.150(0.206) | 1.132(0.226) | 0.00 |
| 0003 | 04:50:04.11 | +43:35:00.27 | 20.025(0.044) | ... | ... | 1.996(0.106) | 18.495(0.003) | 2.676(0.060) | −2.182(0.278) | −2.044(0.177) | 2.356(0.186) | 0.00 |
| 0004 | 04:50:04.12 | +43:39:01.56 | 19.264(0.020) | 0.424(0.073) | 1.007(0.030) | 18.833(0.003) | 1.389(0.055) | 2.572(0.373) | −3.090(0.227) | 0.272(0.217) | 0.00 |
| 0005 | 04:50:04.14 | +43:45:50.34 | 17.434(0.005) | 0.227(0.013) | 0.735(0.007) | 17.105(0.003) | 1.080(0.011) | 0.594(0.122) | −0.425(0.088) | 0.389(0.083) | 0.00 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 3731 | 04:51:58.31 | +43:46:24.34 | 18.438(0.010) | 0.409(0.034) | 0.975(0.015) | 18.050(0.003) | 1.297(0.019) | 1.281(0.188) | −1.799(0.156) | 0.127(0.152) | 0.00 |
| 3732 | 04:51:58.31 | +43:45:56.87 | 19.354(0.025) | ... | ... | 3.249(0.141) | 19.217(0.005) | 2.921(0.105) | −0.487(0.481) | −2.882(0.394) | 1.554(0.422) | 0.00 |
| 3733 | 04:51:58.36 | +43:49:50.38 | 17.440(0.005) | 0.509(0.017) | 0.944(0.008) | 17.062(0.003) | 1.234(0.009) | 0.036(0.089) | −2.873(0.076) | 0.529(0.077) | 0.00 |
| 3734 | 04:51:58.38 | +43:40:40.03 | 19.033(0.016) | 0.474(0.056) | 0.996(0.023) | 18.505(0.003) | 1.430(0.029) | 0.597(0.238) | −2.226(0.191) | 0.122(0.218) | 0.00 |
| 3735 | 04:51:58.39 | +43:43:43.06 | 20.054(0.038) | ... | 1.131(0.058) | 19.352(0.004) | 1.474(0.075) | 0.195(0.539) | −2.410(0.396) | −0.368(0.422) | 0.00 |

Note. This table is published in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content. (This table is available in its entirety in machine-readable form.)
$f_{bg}$ is the background density. From the King (1962) model fit, the central stellar density and core radius of the clusters, together with the background stellar density, were inferred as $f_0 = 1.451 \pm 0.281$ stars arcmin$^{-2}$, $r_c = 7.213 \pm 1.696$ arcmin, and $f_{bg} = 2.812 \pm 0.283$ stars arcmin$^{-2}$ for NGC 1664 and $f_0 = 4.070 \pm 0.249$ stars arcmin$^{-2}$, $r_c = 3.057 \pm 0.637$ arcmin, and $f_{bg} = 5.002 \pm 0.328$ stars arcmin$^{-2}$ for NGC 6939, respectively. As seen in Figure 4, the stars begin to merge with the background density (blue dashed lines) at the limiting radius. We therefore concluded that NGC 1664 and NGC 6939 have limiting radii of $r_{lim} = 8.5$ (3.28 pc) and $r_{lim} = 6.5$ arcmin (3.25 pc), respectively. In the subsequent analyzes for both clusters we considered only the stars inside these limiting radii.

### 3.3. CMDs and Membership Probabilities of Stars

Proper-motion components of stars are crucial data in order to separate the field stars from the cluster members (Bisht et al. 2020). Because cluster stars have a common origin they share similar vectoral movements in the sky. This property makes proper motions a useful tool to eliminate nonmember stars from the cluster’s main sequence (Yadav et al. 2013). Astrometric measurements of Gaia EDR3 provide proper motions and trigonometric parallaxes of billions of stars, including suspected members of the clusters of interest. Using these data together with the statistical methods developed by various researchers (Balaguer-Nunez et al. 1998; Zhao & He 1990; Stetson 1980) allowed us to calculate the membership probabilities of the stars.

### Table 6

| Cluster | V | N  | $\sigma_V$ | $\sigma_{U-B}$ | $\sigma_{B-V}$ | $\sigma_G$ | $\sigma_{G_{BP}-G_{RP}}$ |
|---------|---|----|------------|----------------|--------------|-----------|--------------------------|
| NGC 1664 | (08, 12] | 28 | 0.002 | 0.004 | 0.002 | 0.003 | 0.007 |
|         | (12, 14] | 111 | 0.003 | 0.005 | 0.005 | 0.003 | 0.005 |
|         | (14, 15] | 116 | 0.001 | 0.003 | 0.002 | 0.003 | 0.006 |
|         | (15, 16] | 183 | 0.002 | 0.005 | 0.002 | 0.003 | 0.006 |
|         | (16, 17] | 335 | 0.003 | 0.010 | 0.004 | 0.003 | 0.007 |
|         | (17, 18] | 574 | 0.005 | 0.019 | 0.008 | 0.003 | 0.012 |
|         | (18, 19] | 847 | 0.010 | 0.044 | 0.016 | 0.003 | 0.024 |
|         | (19, 20] | 1017 | 0.023 | 0.074 | 0.037 | 0.003 | 0.032 |
|         | (20, 22] | 522 | 0.045 | 0.121 | 0.079 | 0.005 | 0.086 |
| NGC 6939 | (08, 12] | 6 | 0.001 | 0.002 | 0.001 | 0.003 | 0.005 |
|         | (12, 14] | 73 | 0.002 | 0.005 | 0.002 | 0.003 | 0.005 |
|         | (14, 15] | 123 | 0.001 | 0.006 | 0.002 | 0.003 | 0.005 |
|         | (15, 16] | 172 | 0.002 | 0.013 | 0.002 | 0.003 | 0.005 |
|         | (16, 17] | 263 | 0.003 | 0.026 | 0.005 | 0.003 | 0.007 |
|         | (17, 18] | 339 | 0.006 | 0.064 | 0.010 | 0.003 | 0.011 |
|         | (18, 19] | 506 | 0.013 | 0.102 | 0.023 | 0.003 | 0.025 |
|         | (19, 20] | 589 | 0.033 | 0.112 | 0.060 | 0.003 | 0.046 |
|         | (20, 22] | 47 | 0.086 | 0.116 | 0.155 | 0.008 | 0.084 |
the center points of the clusters. Central proper-motion component values were found as \((\mu_x, \mu_y) = (1.64, -5.85) \text{ mas yr}^{-1}\) for NGC 1664 and as \((\mu_x, \mu_y) = (-1.83, -5.48) \text{ mas yr}^{-1}\) for NGC 6939 (see Table 7).

To determine the radius of the circle we plotted the number of stars versus the radial distance of the proper motion and fitted the function of the stellar density profile for each cluster shown in Figures 5(b) and 5(d). Considering the radial distance of the proper-motion value where the number density is closer to the field star density, we obtained the radius of the circles as 0.8 mas yr\(^{-1}\) and 0.5 mas yr\(^{-1}\) for NGC 1664 and NGC 6939, respectively. In this way, we obtained 311 and 545 possible member stars for NGC 1664 and NGC 6939, respectively.

The method of Balaguer-Nunez et al. (1998) defines the membership probability of the \(i\)th star as

\[
P_c(i) = \frac{n_c \times \phi_c^i(i)}{n_c \times \phi_c^i(i) + n_f \times \phi_f^i(i)},
\]

where \(n_c\) and \(n_f\) represent the normalized number of stars for the regions of cluster and star field \((n_c + n_f = 1)\). The \(\phi_c^i\) and \(\phi_f^i\) denote the frequency distribution functions for the cluster and the field stars. For the \(i\)th star \(\phi_f^i\) is described as

\[
\phi_f^i(i) = \frac{1}{2\pi \sqrt{(\sigma_x^2 + \epsilon_x^2)(\sigma_y^2 + \epsilon_y^2)}} \times \exp\left(-\frac{1}{2} \left[ \frac{(\mu_x - \mu_x)^2}{\sigma_x^2 + \epsilon_x^2} + \frac{(\mu_y - \mu_y)^2}{\sigma_y^2 + \epsilon_y^2} \right] \right),
\]

where \(\mu_x\) and \(\mu_y\) are the proper-motion components, while \(\epsilon_x\) and \(\epsilon_y\) are the relevant proper-motion errors for the \(i\)th star. \(\mu_{sc}\) and \(\mu_{yc}\) are the proper-motion components of the cluster center with dispersions \(\sigma_{sc}\) and \(\sigma_{yc}\). \(\phi_c^i\) is also defined for the \(i\)th star as

\[
\phi_c^i(i) = \frac{1}{2\pi \sqrt{(1 - \gamma^2)(\sigma_x^2 + \epsilon_x^2)(\sigma_y^2 + \epsilon_y^2)}} \times \exp\left(-\frac{1}{2(1 - \gamma^2)} \left[ \frac{(\mu_x - \mu_x)^2}{\sigma_x^2 + \epsilon_x^2} + \frac{(\mu_y - \mu_y)^2}{\sigma_y^2 + \epsilon_y^2} \right] - \frac{2\gamma(\mu_x - \mu_x)(\mu_y - \mu_y)}{\sqrt{(\sigma_x^2 + \epsilon_x^2)(\sigma_y^2 + \epsilon_y^2)}} \right),
\]

where \(\mu_{xf}\) and \(\mu_{yt}\) represent the field proper-motion components with dispersions \(\sigma_{xf}\) and \(\sigma_{yt}\). The correlation coefficient \(\gamma\) is described as

\[
\gamma = \frac{(\mu_x - \mu_x)(\mu_y - \mu_y)}{\sigma_{xf}\sigma_{yt}}.
\]

Determined parameters from the VPDs of the two clusters during the membership probability calculations are listed in Table 7.

We selected stars whose membership probabilities \(P \geq 0.5\) as the most likely members of the two clusters. This led to the identification of 308 member stars for NGC 1664 and 541 for NGC 6939. Recently Cantat-Gaudin & Anders (2020) used Gaia DR2 data to analyze nearly 2000 stellar clusters including NGC 1664 and NGC 6939. They identified 299 cluster stars for NGC 1664 and 636 for NGC 6939. We note that
Cantat-Gaudin & Anders (2020) utilized the Unsupervised Photometric Membership Assignment in Stellar Clusters method for calculating the membership probabilities of the stars and adopted the membership probability of $P_{0.7}$ for the two clusters along with a limiting magnitude of $G_{\text{lim}} = 18$. Because the methods and data used in the calculation of the membership probabilities are different between the current study and those in Cantat-Gaudin & Anders (2020), differences in the member lists could be expected between two studies but there is a general sense of agreement, which is encouraging.

For example, Cantat-Gaudin & Anders (2020) used for NGC 6939 a radius of 7.38 arcminutes compared to 6.5 arcminutes in this study, and so would be expected to count more stars. In the other direction, Cantat-Gaudin & Anders (2020) used a radius of 6.78 arcminutes for NGC 1664, compared to 8.5 arcminutes in this study, and so could be expected to have found less stars.

Before starting to determine the astrophysical parameters, we also considered the contamination of the main sequence by binary stars. We plotted the $V \times (B - V)$ color–magnitude diagram (CMD) of the stars located in the direction and within the $r_{\text{lim}}$ radii of each cluster. We fitted zero age main-sequence (ZAMS) isochrones (Sung et al. 2013) to the most likely cluster members ($P \geq 0.5$), shifting the fitted ZAMS by 0.75 mag to account for binary star contamination. Attention was paid to the overall fitting, including consideration of not only the main sequence but also the turn-off and giant regions. The $V \times (B - V)$ CMDs with fitted ZAMS are shown as Figures 6(a) and 6(c). The distribution of field and member stars with Gaia EDR3 photometry are shown as $G \times (G_{\text{BP}} - G_{\text{RP}})$ CMDs in Figures 6(b) and 6(d). With the restrictions described above in this paragraph, this led to 197 stars identified in NGC 1664 and 279 in NGC 6939. Figure 7 presents histograms of membership probabilities of stars through the regions described for NGC 1664 and NGC 6939. In Figures 6(c) and 6(d), the two stars “A” ($\alpha = 20^h31^m31^s92$,
Figure 6. $V \times (B - V)$ and $G \times (G_{\text{BP}} - G_{\text{RP}})$ CMDs of NGC 1664 (a), (b) and NGC 6939 (c), (d). The blue dotted–dashed lines represent the ZAMS (Sung et al. 2013) including the binary star effect. The membership probabilities of stars that lie within the fitted ZAMS are shown with different colors; these member stars are located within $r_{\text{lim}} = 8.5$ and $r_{\text{lim}} = 6.5$ of the cluster centers calculated for NGC 1664 and NGC 6939, respectively. Gray dots indicate the field stars. Letters A and B in panels (c) and (d) indicate possible BSSs of NGC 6939.
$\delta = +60^\circ 38'15''85$ and “B” ($\alpha = 20^h 31^m 31.95$, $\delta = +60^\circ 36'25''.53$), seen on the blue side of the main-sequence turn-off of the NGC 6939, have estimated membership probabilities of $P = 1$ and $P = 0.98$, respectively, and appear as blue straggler stars (BSSs) of the cluster. Recently Jadhav & Subramaniam (2021) and Rain et al. (2021) studied BSSs in the NGC 6939. Their analyses are based on Gaia DR2 astrometric and photometric data. In their studies, Jadhav & Subramaniam (2021) and Rain et al. (2021) determined that star “A” is a BSS and a cluster member. Jadhav & Subramaniam (2021) calculated the mass of star “A” as $3.8\ M_\odot$. The two studies found no evidence as to whether star “B” is a BSS. The high-precision astrometric data of Gaia EDR3 have increased the sensitivity of stellar membership determinations. This may increase the probability of star “B” being a member of the cluster. In this case, star “B” appears to be a possible BBS candidate and member of NGC 6939.

We constructed VPDs for each cluster to explore the distribution and separation of the members stars in the proper-motion space. Figure 8 demonstrates that the selected member stars were well separated from the field stars for both clusters. We determined the mean proper-motion components of the adopted member stars and present the intersections of these values on Figure 8 as dashed lines. The mean proper-motion components were calculated as $\mu_\alpha \cos \delta = 1.594 \pm 0.071$ and $\mu_\delta = -5.780 \pm 0.052$ mas yr$^{-1}$ for NGC 1664, and $\mu_\alpha \cos \delta = -1.817 \pm 0.039$ and $\mu_\delta = -5.462 \pm 0.039$ mas yr$^{-1}$ for NGC 6939.

### 4. Astrophysical Parameters of the Clusters

In this section we summarize the processes we performed during the determination of astrophysical parameters of NGC 1664 and NGC 6939 (for detailed descriptions on the methodology see Yontan et al. 2015, 2019, 2021; Ak et al. 2016; Bilir et al. 2006, 2010, 2016; Bostancı et al. 2015, 2018; Banks et al. 2020; Akbulut et al. 2021). Color excesses and metallicities of the clusters were derived using two-color diagrams (TCDs), whereas we obtained distance moduli and ages individually by fitting theoretical models on CMDs.

#### 4.1. Reddening

To determine the reddening across each cluster we considered the most likely main-sequence members ($P \geq 0.5$). The V-magnitude range of these stars is $13 \leq V \leq 18$ for NGC 1664 and $15 \leq V \leq 20$ mag for NGC 6939, respectively. Taking into account the distribution of member stars on the $(U-B) \times (B-V)$ diagrams, we fitted the solar-metallicity dereddened ZAMS of Sung et al. (2013) to the observational data. We employed $\chi^2$ optimization with steps of 0.01 mag according to slope of the reddening given by Garcia et al. (1988) as $E(U-B) = 0.72 \times E(B-V) + 0.05 \times E(B-V)^2$. $E(B-V)$ and $E(U-B)$ values corresponding to the minimum $\chi^2$ were adopted as the best solutions of the reddening, being $E(B-V) = 0.190 \pm 0.018$ mag for NGC 1664 and $E(B-V) = 0.380 \pm 0.025$ mag for NGC 6939. TCDs with the best result fits are shown as Figure 9. The errors of the reddening are estimated as $\pm 1\sigma$ deviations, and are presented with green lines on the same figure.

#### 4.2. Metallicities

In order to determine the photometric metallicities we employed the method of Karaali et al. (2011), which is based on F- and G-type main-sequence stars and their UV excesses. We calculated the intrinsic $(B-V)_0$ and $(U-B)_0$ colors of the most likely members ($P \geq 0.5$) and made a selection of main-sequence stars considering the $0.3 \leq (B-V)_0 \leq 0.6$ mag range (Eker et al. 2020, 2018) which corresponds to F- and G-type main-sequence stars. Plotting the $(U-B)_0 \times (B-V)_0$ TCDs of the selected stars and the Hyades main sequence, we calculated the difference between the $(U-B)_0$ color indices of the member stars and the Hyades stars, which is defined as the UV excess ($\delta$). UV excess is expressed by $\delta = (U-B)_0 - (U-B)_{0H}$ where H and S are the Hyades and cluster stars, respectively, whose intrinsic $(B-V)_0$ colors are the same. We normalized the UV excess to $(B-V)_0 = 0.6$ mag (i.e., $\delta_{06}$) and constructed the histogram according to the $\delta_{06}$ values of the stars in order to calculate the mean $\delta_{06}$ by fitting Gaussians to the distribution (Karaali et al. 2003a, 2003b). The photometric metallicity calculation is based on the mean $\delta_{06}$, which corresponds to the peak of the Gaussian fit, and the relevant equation used in analyses is given as (Karaali et al. 2011)

$$[\text{Fe/H}] = -14.316(1.919) \delta_{06}^2 - 3.557(0.285) \delta_{06} + 0.105(0.039) \tag{8}$$

$(U-B)_0 \times (B-V)_0$ diagrams and histograms of $\delta_{06}$ are presented in Figure 10. The number of most likely member stars which were considered during the metallicity calculations were 40 for NGC 1664 and 65 for NGC 6939. From these stars we determined the photometric metallicity for NGC 1664 as $[\text{Fe/H}] = -0.10 \pm 0.02$ and for NGC 6939 as $[\text{Fe/H}] = -0.06 \pm 0.01$ dex.

To derive the ages of the clusters it is required to convert the iron abundance to the metallicity of all elements heavier than
Figure 8. VPDs of NGC 1664 (a) and NGC 6939 (b). Colored dots identify the membership probabilities of the cluster stars according to the color scale shown on the right. The zoomed box in panel (b) represents the region of condensation for the NGC 6939 cluster in the VPD. Dashed lines are the intersection of the mean proper-motion values. The color scale shows the membership probabilities of the most likely cluster members.

Figure 9. Two-color diagrams of the most probable member main-sequence stars in the regions of NGC 1664 (a) and NGC 6939 (b). Red dashed and green solid curves represent the reddened ZAMS given by Sung et al. (2013) and ±1σ standard deviations, respectively.
helium. To do this, we used the analytic equations denoted by Bovy12 for PARSEC isochrones (Bressan et al. 2012). The equations are expressed as

\[
\begin{align*}
\zeta_1 &= 10^{\frac{\text{[Fe/H]}+\log\left(\frac{z_0}{1-0.248-2.78\times z_0}\right)}{1}} \\
\zeta_x &= \frac{z_x - 0.2485 \times z_x}{(2.78 \times z_x + 1)}.
\end{align*}
\]

Equations (9) and (10) represent the elements heavier than helium, \(z_1\) is the intermediate operation function, and \(z_0\) is the solar metallicity which was adopted as 0.0152 (Bressan et al. 2012). We calculated \(z = 0.012\) for NGC 1664 and \(z = 0.013\) for NGC 6939.

4.3. Distance Moduli and Age Estimation

The distance moduli and age of the two clusters were determined simultaneously by comparing PARSEC isochrones (Bressan et al. 2012) to the \((U - B)\times(B - V)\), \((V \times (B - V))\), and \((G \times (G_{BP} - G_{RP}))\) CMDs based on likely member stars (\(P \geq 0.5\)). The selection of PARSEC isochrones was made by taking into account the mass fractions (\(z\)) calculated for each cluster. We fitted the selected isochrones on CMDs visually placing emphasis on fitting main-sequence, turn-off, and giant likely members. For the distance moduli and age estimation for UBV data we shifted the PARSEC isochrones according to the \(E(B-V)\) values calculated above. Whereas for the Gaia EDR3 data we used the equation \(E(G_{BP} - G_{RP}) = 1.41 \times E(B-V)\), where the coefficient was calculated using the equation of Sun et al. (2021) who presented selective absorption coefficients. In the error determination of the distance moduli and distances we used the relations of Carraro et al. (2017). We estimated the uncertainty in the derived cluster ages by fitting two more isochrones whose values were good fits to the data sets but at

Figure 10. \((U - B)\times(B - V)\) diagrams (upper panels) and the distributions of normalized \(\delta_{0.6}\) (lower panels) for NGC 1664 (a) and NGC 6939 (b). The solid blue lines in the upper and lower panels represent the main sequence of Hyades and the Gaussian models that were fitted to the histograms, respectively.

\[12 \text{https://github.com/jobovy/isodist/blob/master/isodist/Isochrone.py}\]
Figure 11. CMDs for the NGC 1664 (panels a, b, and c) and NGC 6939 (panels d, e, and f). The differently colored dots represent the membership probabilities according to the color scales shown on the right side of the diagrams. Gray colored dots identify the field stars. The blue lines show the PARSEC isochrones, while the purple shaded areas surrounding these lines are their associated errors. The best-fitting isochrone for NGC 1664 corresponds to a 675 Myr age for the cluster, while that for NGC 6939 is 1.5 Gyr.
the higher and lower acceptable values compared to the adopted mean age. The best-fit isochrones derive the assumed ages for the clusters, whereas other two closing fitting isochrones, where one is younger and the other is older than the adopted best-fit age, were taken as estimates for fitting the uncertainties. The best fit with $z = 0.012$ gave the distance moduli and age of NGC 1664 as $\mu = 11.140 \pm 0.080$ mag and $t = 675 \pm 50$ Myr, respectively. For NGC 6939 the best fit with $z = 0.013$ gave these values as $\mu = 12.350 \pm 0.109$ mag and $t = 1.5 \pm 0.2$ Gyr, respectively. The distances of the clusters corresponding to the estimated distance moduli are also $d_{iso} = 1289 \pm 47$ pc for NGC 1664 and $d_{iso} = 1716 \pm 87$ pc for NGC 6939 (see Table 1). The adopted best-fit age isochrones and relevant errors are overplotted on the CMDs, which were constructed from UBV and Gaia EDR3 data and shown in Figure 11. The discussion of different distance estimations will be interpreted in relation to the Gaia astrometric results in Section 5.

5. Comparison of Astrometric Results

We also calculated trigonometric distances from Gaia EDR3 astrometric data (Gaia Collaboration et al. 2021) using the most likely member stars of two clusters.

Trigonometric distances were determined by converting trigonometric parallaxes into distances, using the linear expression of $d(\text{pc}) = 1000 / \varpi$ (mas) applied to each selected member star of the NGC 1664 and NGC 6939 clusters. In calculating the mean distances, Gaia stars with a relative parallax error of less than 0.2 were considered. Histograms of obtained distances were constructed and Gaussians were fitted to them (Figure 12), then taking into account the fitted maxima of Gaussian models we derived mean Gaia distances ($d_{\text{Gaia EDR3}}$) for each cluster. Uncertainties in distances are one standard deviation. The results of analyses give the Gaia distances as $d_{\text{Gaia EDR3}} = 1350 \pm 81$ pc for NGC 1664 and $d_{\text{Gaia EDR3}} = 1912 \pm 110$ pc for NGC 6939 (see also Table 8).

Bailer-Jones et al. (2021); hereafter BJ21 stated in their recent research that transforming trigonometric parallaxes to distances using the linear method does not give accurate measurements. For the precise calculation of trigonometric distances Bailer-Jones et al. (2021) developed a geometric approach that considers the probability distributions of stellar distances. We cross-matched the most likely cluster stars with those given in the catalog of BJ21, retrieving their distance estimates, and plotted the histograms of these distances versus the number of stars (Figure 12). We fitted Gaussian models to each distribution and derived the mean distances ($d_{\text{BJ21}}$) of the two clusters. Errors for $d_{\text{BJ21}}$ distances were taken as the standard deviations of the Gaussian fits (see Table 8). As a result we estimated the BJ21 distances as $d_{\text{BJ21}} = 1292 \pm 87$ pc and $d_{\text{BJ21}} = 1842 \pm 114$ pc for NGC 1664 and NGC 6939, respectively. Comparisons of the distance distributions of the two data sets are shown in Figure 12.

We compared distances which were determined via three methods (isochrone fitting, linear expression, and geometric approach); these distances are listed in Table 8. It can be seen that the distances derived from isochrones fitting are compatible with the results of Gaia and BJ21. It is suggested that trigonometric parallaxes of Gaia have a small bias given the work by Groenewegen (2021). This could partly clarify why the Gaia distances are estimated further. We also compared the astrometric results ($\mu_\alpha$, $\mu_\delta$, $d_{\text{iso}}$, $d_{\text{Gaia EDR2}}$ and $d_{\text{BJ21}}$) of the two clusters with those presented in the study of Cantat-Gaudin et al. (2020). This comparison is given in Table 8. We can conclude that our findings are in agreement with the results of Cantat-Gaudin et al. (2020), being inside the $2\sigma$ ranges for both clusters.

6. Space Velocities and Galactic Orbits of NGC 1664 and NGC 6939

The galactic orbital parameters of NGC 1664 and NGC 6939 were calculated using the potential functions defined in GALPY, the galactic dynamics library (Bovy 2015). The calculation assumed an axisymmetric potential for the Milky Way Galaxy, following MWPOTENTIAL2014 (Bovy 2015). The galactocentric

Note. The results of Cantat-Gaudin & Anders (2020; CA20) and Bailer-Jones (2021; BJ21) are also listed in the table.
distance and a circular velocity of the Sun were assumed as $R_{GC} = 8$ kpc and $V_{rot} = 220$ km s$^{-1}$, respectively (Bovy 2015; Bovy & Tremaine 2012). The distance from the Galactic plane of the Sun was adopted as $27 \pm 4$ pc (Chen et al. 2000).

We adopted that the Galaxy is well represented by the MWPOTENTIAL2014 code, which is composed of the bulge, disk, and halo potentials of the Milky Way. The bulge component is represented as a spherical power-law density profile by Bovy (2015):

$$\rho(r) = A \left(\frac{r_1}{r}\right)^\alpha \exp\left[-\left(\frac{r}{r_c}\right)^2\right],$$  \hspace{1cm} (11)

where $r_1$ is the present reference radius and $r_c$ the cut-off radius. $A$ is the amplitude that is applied to the potential in mass density units and $\alpha$ presents the inner power. We used the potential proposed by Miyamoto & Nagai (1975) for the galactic disk component:

$$\Phi_{\text{disc}}(R_G, Z) = -\frac{GM_d}{\sqrt{R_G^2 + (a_d + \sqrt{Z^2 + b_d^2})^2}},$$  \hspace{1cm} (12)

where $R_G$ is the distance from the galactic center, $Z$ is the vertical distance from the galactic plane, $G$ is the universal gravitational constant, $M_d$ is the mass of the galactic disk, $a_d$ is the scale length of the disk, and $b_d$ is the scale height of the disk.

The potential for the halo component was obtained from Navarro et al. (1996),

$$\Phi_{\text{halo}}(r) = -\frac{GM_h}{R_G} \ln \left(1 + \frac{R_G}{r_h}\right),$$  \hspace{1cm} (13)

where $M_h$ is the mass of the dark matter halo of the Milky Way and $r_h$ is its radius.

The MWPOTENTIAL2014 code was used to determine the space velocity components and galactic orbital parameters for both OCs. The equatorial coordinates, proper-motion components, distance, and radial-velocity data required for the calculations of the space velocities and galactic orbit parameters of the clusters are listed in Table 9. Kinematic and dynamic calculations were analyzed with 1 Myr steps over a 3.5 Gyr integration time.

The proper-motion components and distances of the OCs were calculated in this study and were given in Section 3.3. In order to calculate the mean radial velocities of the two clusters, stars with radial-velocity data available in the Gaia EDR3 catalog and a cluster membership $P \geq 0.5$ were selected. Radial-velocity data of 3 stars for NGC 1664 and 22 stars for NGC 6939 were identified in the Gaia EDR3 catalog. Using these data we calculated the cluster radial velocities by taking the weighted average of the radial velocities and their uncertainties (for equations see also Soubiran et al. 2018). As a result of the analysis, the mean radial velocities were estimated for NGC 1664 as $+8.96 \pm 0.24$ and for NGC 6939 as $-18.91 \pm 0.11$ km s$^{-1}$. Literature mean radial-velocity estimates for NGC 1664 are $6.38 \pm 0.45$ (Soubiran et al. 2018), $6.38 \pm 0.28$ (Carrera et al. 2019), $6.37 \pm 0.67$ (Tarricq et al. 2021), and $7.83 \pm 0.03$ km s$^{-1}$ (Hao et al. 2021). These results are compatible within $1-2$ km s$^{-1}$ with the value found for NGC 1664 in this study. A similar situation is also valid for the NGC 6939 cluster, with values given by various researchers being $-21.00 \pm 0.20$ (Warren & Cole 2009), $-18.99 \pm 0.13$ (Soubiran et al. 2018), $-18.60 \pm 0.09$ (Tarricq et al. 2021), and $-18.77 \pm 0.08$ km s$^{-1}$ (Dias et al. 2021). These are compatible with the radial velocities we obtained.

The derived galactic orbit parameters of the two clusters are also listed in Table 5. The parameters estimated from this orbital integration (including uncertainties in distances, proper motions, and radial velocities) are apogalactic ($R_\alpha$) and perigalactic ($R_\beta$) distances, eccentricity ($e$), the maximum vertical distance from galactic plane ($Z_{\text{max}}$), galactic space velocity components ($U$, $V$, $W$), and the orbital period ($T$) of each cluster.

The space velocity components of NGC 1664 and NGC 6939 were calculated as ($U$, $V$, $W$) = ($-19.21 \pm 0.61$, $-29.90 \pm 1.09$, $-15.56 \pm 0.01$) and ($U$, $V$, $W$) = ($+46.39 \pm 1.80$, $-11.01 \pm 0.45$, $-16.63 \pm 0.70$) km s$^{-1}$, respectively (see also Table 5). As for the local standard of rest (LSR) correction we considered the values of $(8.83 \pm 0.24, 14.19 \pm 0.34, 6.57 \pm 0.21)$ km s$^{-1}$ which were given by Coşkunolu et al. (2011). We applied LSR corrections to our findings and derived LSR-corrected space velocity components as ($U$, $V$, $W$)$_{\text{LSR}}$ = ($-10.38 \pm 0.66$, $-15.71 \pm 1.14$, $-8.99 \pm 0.11$) for NGC 1664 and ($U$, $V$, $W$)$_{\text{LSR}}$ = ($55.22 \pm 1.82$, $3.18 \pm 0.56$, $-10.6 \pm 0.73$) km s$^{-1}$ for NGC 6939. Soubiran et al. (2018) studied these two clusters with Gaia DR2 (Gaia Collaboration et al. 2018) astrometric data finding that the space velocity components of NGC 1664 are ($U$, $V$, $W$)$_{\text{LSR}}$ = ($-16.70 \pm 0.61$, $-30.49 \pm 0.23$, $-14.64 \pm 0.11$) and for NGC 6939 are ($U$, $V$, $W$)$_{\text{LSR}}$ = ($50.13 \pm 0.19$, $-10.55 \pm 0.13$, $-17.36 \pm 0.08$) km s$^{-1}$. It can be seen that the results derived in this study (see Table 5) are in good agreement with the values of

| Cluster  | $\alpha$ (J2000) (hh:mm:ss) | $\delta$ (J2000) (dd:mm:ss) | $\mu_\alpha, \cos\delta$ (mas yr$^{-1}$) | $\mu_\delta$ (mas yr$^{-1}$) | $d$ (pc) | $V_r$ (km s$^{-1}$) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| NGC 1664  | 04:51:06 | 43:40:30 | +1.594 $\pm$ 0.071 | -5.780 $\pm$ 0.052 | 1289 $\pm$ 47 | +8.96 $\pm$ 0.24 |
| NGC 6939  | 20:31:30 | 60:39:42 | -1.817 $\pm$ 0.039 | -5.462 $\pm$ 0.039 | 1716 $\pm$ 87 | -18.91 $\pm$ 0.11 |

| Cluster  | $R_\alpha$ (kpc) | $R_\beta$ (kpc) | $e$ | $Z_{\text{max}}$ (pc) | $U$ (km s$^{-1}$) | $V$ (km s$^{-1}$) | $W$ (km s$^{-1}$) | $T$ (Myr) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| NGC 1664  | 9.47 $\pm$ 0.05 | 7.81 $\pm$ 0.05 | 0.096 $\pm$ 0.001 | 140 $\pm$ 1 | -19.21 $\pm$ 0.61 | -29.90 $\pm$ 1.09 | -15.56 $\pm$ 0.01 | 244 $\pm$ 1 |
| NGC 6939  | 9.41 $\pm$ 0.11 | 8.23 $\pm$ 0.04 | 0.067 $\pm$ 0.003 | 460 $\pm$ 29 | +46.39 $\pm$ 1.80 | -11.01 $\pm$ 0.45 | -16.63 $\pm$ 0.70 | 248 $\pm$ 2 |

Table 9: The Input and Obtained Output Parameters for the Orbit Integration of NGC 1664 and NGC 6939
Soubiran et al. (2018). The total space velocities are $20.87 \pm 1.32$ km s$^{-1}$ for NGC 1664 and $56.32 \pm 2.04$ km s$^{-1}$ for NGC 6939. Considering these results, they are compatible with the velocity given by Leggett (1992) for young thin-disk stars.

The galactic orbits for the two clusters on the $Z \times R_{GC}$ and $R_{GC} \times t$ planes are shown as panes in Figure 13. Figures 13(a) and 13(c) represent the side views of the cluster motions as a function of distance from the galactic center and the galactic plane. Figures 13(b) and 13(d) represent the distances from the galactic plane as a function of time for the two clusters. In Figures 13(b) and 13(d) we marked the birth and present-day locations for both clusters using yellow filled triangles and circles. Considering the perigalactic and apogalactic distances of the clusters, it can be seen that orbit of the NGC 6939 is completely outside the solar circle (Figure 13(c)), while NGC 1664 enters the solar circle during its orbit (Figure 13(a)). The orbits of the two clusters differ from circular. Their eccentricities do not exceed the value of 0.1. The vertical distances from the galactic plane reach a maximum at $Z_{\text{max}} = 140 \pm 1$ pc with an orbital period of $T = 244 \pm 1$ Myr for NGC 1664 and $Z_{\text{max}} = 460 \pm 29$ pc and $T = 248 \pm 2$ Myr for NGC 6939. Considering the relative errors in the galactic model parameters, it can be seen that the errors of most parameters vary between 1% and 2%. Only the relative error of the $Z_{\text{max}}$ parameter of NGC 6939 is about of the order of 6%. These results show that the orbital parameters calculated for the clusters are precise. In conclusion, it shows that both of the clusters belong to the thin disk of Galaxy.

The birthplaces of the clusters were investigated by running the cluster ages calculated in this study back in time using the GALPY program. In orbital calculations, the birth radii of the OCS NGC 1664 and NGC 6939 were calculated as 9.07 kpc and 8.34 kpc, respectively, assuming that the clusters do not interact with other objects during their lifetimes (see Figures 13(b) and 13(d)). These results show that the clusters were born in the metal-poor region outside the solar circle.

7. Dynamical Study of the Clusters

7.1. Luminosity Functions

The luminosity function (LF) is defined as a distribution of stars with magnitude. To estimate the cluster LFs, we selected the main-sequence stars within the range of $12.5 \leq V \leq 20$ mag and which are located inside a 8.5 arcmin limiting radius from the previously estimated center of the NGC 1664 cluster, and $15 \leq V \leq 20$ mag stars located inside a 6.5 arcmin radius from the center of the NGC 6939 cluster. We calculated the absolute magnitudes for these selected stars using the equation $M_V = V - 5 \times \log d + 3.1 \times E(B-V)$ where $V$, $d$, and $E(B-V)$ are apparent magnitude, distance, and color excess of each cluster, respectively, as derived earlier in this study (see Table 1). These calculations resulted in the absolute magnitude ranges being $1 < M_V < 9$ and $2 < M_V < 8$ mag for NGC 1664.
and NGC 6939, respectively. The resulting LF histograms for both clusters are shown in Figure 14. This figure shows that the LFs continue to increase (in stars) up to $M_V = 8$ mag and $M_V = 5$ mag for NGC 1664 and NGC 6939, respectively, and begin to drop beyond these limits. In order to analyze the effects on the luminosity and present-day mass functions (PDMFs) caused by the presence of binary stars in the clusters, the main-sequence band has been reduced from 0.75 to 0.35 mag. As a result of this analysis, the numbers of possible single main-sequence stars in NGC 1664 and NGC 6939 were determined as 112 and 106, respectively. The distributions of these stars are shown as green shaded histograms in Figure 14.

### 7.2. Present-day Mass Functions

To derive PDMFs, we used the PARSEC isochrones that best represent the age and metal abundance ($Z$) of the clusters to convert the LFs described above to PDMFs. Theoretical models provided by the PARSEC synthetic stellar library (Bressan et al. 2012) were used to convert the $V$-band absolute magnitudes to masses of theoretical main-sequence stars via a high degree of a polynomial equation in this study. Then we transformed the observational absolute $V$-band magnitudes to masses by considering the absolute magnitude–mass relation. The number, mass range, and mean mass of main-sequence stars so transformed are 189, 0.6 $\leq M/M_\odot \leq 2$, and 1.13 $M/M_\odot$ for NGC 1664, and 213, 0.8 $\leq M/M_\odot \leq 1.6$, and 1.10 $M/M_\odot$ for NGC 6939.

Mass function slopes of the two clusters were derived using the equation $\log(dN/dM) = -(1 + \Gamma) \times \log M + \text{constant}$, where $dN$ represents the number of stars in a mass interval $dM$ with central mass $M$ and $\Gamma$ the slope of the PDMF. Slopes of the PDMFs were determined as $\Gamma = -1.22 \pm 0.33$ for NGC 1664 and as $\Gamma = -1.18 \pm 0.21$ for NGC 6939. Moreover, in order to analyze the effects of binary stars on PDMFs, the main-sequence bands were narrowed from 0.75 to 0.35 mag and PDMF values were calculated from possible single main-sequence stars in the clusters. Distributions of the PDMFs of possible single main-sequence stars that are members of NGC 1664 and NGC 6939 have been carried out and by applying linear fits to the mass distributions, the slopes of the mass functions have been obtained, respectively, as $\Gamma = -1.09 \pm 0.45$ and $\Gamma = -1.23 \pm 0.30$. In general, the slopes of the mass functions calculated from the main-sequence (all sample) and possible single main-sequence stars in both clusters are close to each other, but also very close to the value of $-1.35$ from Salpeter (1955). The best fits of the PDMFs are shown in Figure 15.

### 7.3. The Dynamical State of Mass Segregation

The timescale at which all traces of the initial conditions with which a cluster is born are lost is represented by the relaxation time. The relaxation time ($T_R$) is the characteristic timescale for a cluster to reach the level of energy equipartition. The relaxation time is given by Spitzer & Hart (1971) as

$$T_R = \frac{8.9 \times 10^5 N^{1/2} R_h^{3/2}}{(m^{1/2} \log(0.4N))},$$

where $N$ is the number of stars, $R_h$ is the half-mass–radius of the cluster, and $\langle m \rangle$ is the mean mass of the stars in each cluster. The values of $R_h$ were taken as 1.67 pc for NGC 1664 and as 1.83 pc for NGC 6939, as calculated earlier in this study. Consequently, the dynamical relaxation times of NGC 1664 and NGC 6939 were derived as $T_R = 13.48$ Myr and $T_R = 15.93$ Myr, respectively. These results imply that the age of NGC 1664 is 50 times its relaxation time, and 95 times for NGC 6939. Therefore, we can conclude that both of the clusters are dynamically relaxed.

To investigate whether the clusters have internal mass segregation, we split the cluster star masses into three ranges. These divisions are $1.25 < M/M_\odot < 2.00$ (high mass),
0.80 < \frac{M}{M_\odot} \leq 1.25 \text{ (intermediate mass), and } 0.60 < \frac{M}{M_\odot} \leq 0.80 \text{ (low mass) for NGC 1664, and } 1.20 < \frac{M}{M_\odot} \leq 1.60 \text{ (high mass), } 1.00 < \frac{M}{M_\odot} \leq 1.20 \text{ (intermediate mass), and } 0.80 < \frac{M}{M_\odot} \leq 1.00 \text{ (low mass) for NGC 6939. The normalized cumulative distributions versus radii from the cluster center of stars in different mass ranges are shown in Figure 16. It can be seen in Figure 16(a) that an increased trend of the stars within the three mass groups is similar from the center to the outward parts of NGC 1664. Considering NGC 6939, cluster stars within high and intermediate masses are concentrated into the cluster center, while the cumulative number of the stars within the low-mass ranges increase outward from the cluster center (Figure 16(b)). In order to understand the statistical significance of mass segregation in both clusters, we performed the Kolmogorov–Smirnov test for the three mass ranges given above. The level of confidence for the mass segregation effects are 72% and 89% for NGC 1664 and NGC 6939, respectively.}

**8. Summary and Conclusion**

We performed a study based on CCD UBV and Gaia EDR3 photometric data, as well as Gaia EDR3 astrometry, of the OCs NGC 1664 and NGC 6939. We summarize the results of the analyses as follows.

1. We analyzed the spatial structure of the two clusters by fitting the RDP of King (1962) and obtained limiting the radii as r_{lim} = 8.5 \text{ (3.28 pc)} and r_{lim} = 6.5 \text{ (3.25 pc)} for NGC 1664 and NGC 6939, respectively. These values are the point where stellar densities merge with the background stellar density. In the following analyses we considered only the stars inside these limiting radii as potential cluster members.

2. To eliminate background stellar contamination and identify the most likely cluster members, we calculated membership probabilities of stars in the direction of the two clusters. Calculations were based on the stars’ proper-motion components and their errors from Gaia EDR3. We used the probability \( P \geq 0.5 \) as the condition for cluster membership. To take into account the binary star contamination in the cluster main sequences, we fitted the dereddened ZAMS of Sung et al. (2013) to \( V \times (B - V) \) CMDs with a shift of 0.75 mag in the \( V \) band. Hence, we assumed the stars within defined effective radii, close to the best-fitting ZAMS curves, and with the membership probability \( P \geq 0.5 \) as the most likely member stars for the two clusters.

3. We determined reddening and metallicities from CCD UBV TCDS for the most likely member stars separately to estimate cluster distance modulus and age. The reddening and photometric metallicity for NGC 1664 are \( E(B - V) = 0.190 \pm 0.018 \text{ mag and } [\text{Fe/H}] = -0.10 \pm 0.02 \text{ dex, respectively. The corresponding values for NGC 6939 are } E(B - V) = 0.380 \pm 0.025 \text{ mag and } [\text{Fe/H}] = -0.06 \pm 0.01 \text{ dex.}

4. Keeping as constants the reddening and metallicities, we obtained distance moduli, and hence distances and ages of the two clusters by fitting PARSEC isochrones on the UBV and Gaia EDR3 photometric CMDs. We found the distance modulus for NGC 1664 to be \( \mu_V = 11.140 \pm 0.080 \text{ mag, which corresponds to the distance being } d = 1289 \pm 47 \text{ pc together with an age } t = 675 \pm 50 \text{ Myr. For NGC 6939 we estimated } \mu_V = 12.350 \pm 0.109 \text{ mag, which corresponds to a distance } d = 1716 \pm 87 \text{ pc and } t = 1.5 \pm 0.2 \text{ Gyr. These results are in agreement with those given by different authors.}

5. We calculated the mean proper-motion components for NGC 1664 as \( (\mu_\alpha \cos \delta, \mu_\delta) = (1.594 \pm 0.071, -5.780 \pm 0.052) \text{ mas yr}^{-1} \), and for NGC 6939 as \( (\mu_\alpha \cos \delta, \mu_\delta) = (-1.817 \pm 0.039, -5.462 \pm 0.039) \text{ mas yr}^{-1} \).

6. We determined photometric-based distances by fitting isochrones on CMDs (\( d_{\text{iso}} \)), astrometric data based distance using Gaia EDR3 trigonometric parallaxes (\( d_{\text{Gaia EDR3}} \)), and the distance of cluster members that were presented by Bailer-Jones et al. (2021) \( d_{\text{par}} \). We showed that the results are in a good agreement across these methods, and that photometric-based distances are reliable if the cluster member selection is made carefully.

7. Mean radial velocities of the two clusters were derived from the most likely member stars (\( P \geq 0.5 \)) that have radial velocities given in Gaia EDR3. We found these velocities as \( V_r = +8.96 \pm 0.24 \text{ km s}^{-1} \) for NGC 1664 and \( V_r = -18.91 \pm 0.11 \text{ km s}^{-1} \) for NGC 6939, which are compatible with the results given by different researchers.

8. Space velocities and galactic orbital parameters show that both of the clusters belong to the galactic disk. NGC 1664 orbits in the solar circle while NGC 6939 orbits completely outside it.

9. The birth radii of NGC 1664 and NGC 6939 are 9.07 kpc and 8.34 kpc from the galactic center, respectively, which indicate that two clusters were born outside the solar circle in a metal-poor region.

10. PDFMs were estimated as \( \Gamma = -1.22 \pm 0.33 \) for NGC 1664 and \( \Gamma = -1.18 \pm 0.21 \) for NGC 6939. These mass function slopes are compatible with the value of Salpeter (1955).
11. We calculated the relaxation times of both clusters and concluded that the two clusters are dynamically relaxed. We found no statistical evidence for mass segregation in either cluster.

We thank the anonymous referee for the insightful and constructive suggestions, which significantly improved the paper. This study has been supported in part by the Scientific and Technological Research Council (TÜBİTAK) 120F295. We thank TÜBİTAK for partial support toward using the T100 telescope via project 18CT100-1396. We also thank the on-duty observers and members of the technical staff at the TÜBİTAK National Observatory for their support before and during the observations. This research has made use of the WEBDA database, operated at the Department of Theoretical Physics and Astrophysics of the Masaryk University. This research made use of VizieR and Simbad databases at CDS, Strasbourg, France. We made use of data from the European Space Agency (ESA) mission Gaia, processed by the Gaia Data Processing and Analysis Consortium (DPAC). Funding for DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement. IRAF was distributed by the National Optical Astronomy Observatory, which was operated by the Association of Universities for Research in Astronomy (AURA) under a cooperative agreement with the National Science Foundation. PyRAF is a product of the Space Telescope Science Institute, which is operated by AURA for NASA. We thank the University of Queensland for collaboration software.

Software: IRAF (Tody 1986, 1993), PyRAF (Science Software Branch at STScI 2012), SExtractor (Bertin & Arnouts 1996), Astrometry.net (Lang et al. 2010), GALPY (Bovy 2015), MWPotential2014 (Bovy 2015).

ORCID iDs
Seliz Koç https://orcid.org/0000-0001-7420-0994
Talar Yontan https://orcid.org/0000-0002-5657-6194
Selçuk Bilir https://orcid.org/0000-0003-3510-1509
Remziye Canbay https://orcid.org/0000-0003-2575-9892
Tansel Ak https://orcid.org/0000-0002-0688-1983
Timothy Banks https://orcid.org/0000-0001-9445-4588
Seraf Ak https://orcid.org/0000-0002-0912-6019
Ernst Paunzen https://orcid.org/0000-0002-3304-5200

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