Two-loop corrections to Higgs boson production

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Abstract

In this paper we present the complete two-loop vertex corrections to scalar and pseudo-scalar Higgs boson production for general colour factors for the gauge group SU(N) in the limit where the top quark mass gets infinite. We derive a general formula for the vertex correction which holds for conserved and non conserved operators. For the conserved operator we take the electromagnetic vertex correction as an example whereas for the non conserved operators we take the two vertex corrections above. Our observations for the structure of the pole terms $1/\varepsilon^4$, $1/\varepsilon^3$ and $1/\varepsilon^2$ in two loop order are the same as made earlier in the literature for electromagnetism. However we also elucidate the origin of the second order single pole term which is equal to the second order singular part of the anomalous dimension plus a universal function which is the same for the quark and the gluon.

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1 Introduction

One of the crucial tests of the standard model will be the discovery of the Higgs boson at the LHC or at the TEVATRON. Its discovery or its absence will shed light on the mechanism how particles acquire their mass. It will also answer questions about supersymmetric extensions of the standard model or about compositeness of the existing particles including the Higgs boson. In this paper we will concentrate on Higgs production where the lowest order reaction proceeds via the gluon-gluon fusion mechanism. In the standard model the scalar Higgs boson $H$ couples to the gluons via heavy quarks among which the top-quark is the most prominent one. This also holds for the pseudo-scalar Higgs boson $A$ provided $\tan \beta$ is small where $\beta$ denotes the mixing angle in the Two-Higgs-Doublet model (2HDM). The basic production mechanism is that two incoming gluons are coupled to the Higgs boson through a triangular top-quark loop [1]. The first order QCD corrections to this process [2] are already very complicated in particular the virtual corrections which involve massive two-loop integrals. However fortunately one discovered that one could make a simplification for the total cross section [2], [3]. The latter is also possible for differential distributions provided the transverse momentum $p_T$ is not too large ($p_T \leq 2m_H$) [4], [5], [6], [7]. This is achieved by taking the large top-quark mass limit $m_t \to \infty$. In this case the Feynman graphs are derived from an effective Lagrangian describing the direct coupling of the (pseudo-) scalar Higgs boson to the gluons. Therefore the triangular top-quark loop is replaced by a simple vertex and the treatment of the total cross section is the same as for the Drell-Yan process. In this paper we concentrate on the one and two-loop vertex QCD corrections to the total cross section for Higgs boson production. For the two-loop electroweak corrections we refer to [8] and [9]. Although the two-loop virtual contributions for scalar Higgs production in the heavy top-quark limit were presented in [10], the result was not decomposed in the various colour factors. This is important for the general structure of the vertex function as will be pointed out later on. An explicit expression for the two-loop virtual corrections to the pseudo-scalar Higgs boson is not given anywhere although it is indirectly contained in [11], [12]. In this paper we explicitly present the two-loop virtual contributions to scalar as well as pseudo-scalar Higgs boson production for general colour factors in SU(N).

The paper will be organized as follows. In section 2 we give an outline
of the renormalization constants which are involved in the calculation of the two-loop virtual corrections to scalar and pseudo-scalar Higgs production. In section 3 we present the calculation and compute the contribution to the flavour dependent part of the coefficient function in the total cross section for (pseudo-) scalar Higgs production. In section 4 we discuss the structure of the vertex correction w.r.t. the pole terms and present the finite terms. In particular we find that also the single pole term can be predicted so that the vertex and other two-loop radiative corrections can be expressed in a unique form. For completeness we also compare it with the electromagnetic form factor present e.g. in the Drell-Yan process.
2 Application of the effective Lagrangian approach to Higgs production.

In the large top-quark mass limit the Feynman rules (see e.g. [13]) for scalar Higgs production ($H$) can be derived from the following effective Lagrangian density

$$L_{H}^{\text{eff}} = G_{H} \Phi_{H}(x) O(x) \quad \text{with} \quad O(x) = -\frac{1}{4} G_{\mu \nu}^{a}(x) G^{a, \mu \nu}(x), \quad (2.1)$$

whereas pseudo-scalar Higgs ($A$) production is obtained from

$$L_{A}^{\text{eff}} = \Phi_{A}(x) \left[ G_{A} O_{1}(x) + \tilde{G}_{A} O_{2}(x) \right] \quad \text{with}$$

$$O_{1}(x) = -\frac{1}{8} \epsilon_{\mu \nu \lambda \sigma} G_{\mu \nu}^{a} G_{\lambda \sigma}^{a}(x),$$

$$O_{2}(x) = -\frac{1}{2} \partial^{\mu} \sum_{i=1}^{n_{f}} \bar{q}_{i}(x) \gamma_{\mu} \gamma_{5} q_{i}(x), \quad (2.2)$$

where $\Phi_{H}(x)$ and $\Phi_{A}(x)$ represent the scalar and pseudo-scalar fields respectively and $n_{f}$ denotes the number of light flavours. Furthermore the gluon field strength is given by $G_{\mu \nu}^{a}$ and the quark field is denoted by $q_{i}$. The factors multiplying the operators are chosen in such a way that the vertices are normalised to the effective coupling constants $G_{H}$, $G_{A}$ and $\tilde{G}_{A}$. The latter are determined by the top-quark triangular loop graph, including all QCD corrections, taken in the limit $m_{t} \to \infty$ which describes the decay process $B \to g + g$ with $B = H, A$ namely

$$G_{B} = -2^{5/4} a_{s}(\mu_{r}^{2}) G_{F}^{1/2} \tau_{B} F_{B}(\tau_{B}) C_{B} \left( a_{s}(\mu_{r}^{2}), \frac{m_{t}^{2}}{m_{r}^{2}} \right),$$

$$\tilde{G}_{A} = -\left[ a_{s}(\mu_{r}^{2}) C_{F} \left( \frac{3}{2} - 3 \ln \frac{m_{r}^{2}}{m_{t}^{2}} \right) + \cdots \right] G_{A}, \quad (2.3)$$

and $a_{s}(\mu_{r}^{2})$ is the renormalized coupling constant defined by

$$a_{s}(\mu_{r}^{2}) = \frac{\alpha_{s}(\mu_{r}^{2})}{4\pi}, \quad (2.4)$$
where $\alpha_s(\mu_r^2)$ is the running coupling constant and $\mu_r$ denotes the renormalization scale. Further $G_F$ represents the Fermi constant and the functions $F_B$ are given by

$$
F_H(\tau) = 1 + (1 - \tau) f(\tau), \quad F_A(\tau) = f(\tau) \cot \beta,
$$

$$
\tau = \frac{4 m_t^2}{m^2},
$$

$$
f(\tau) = \arcsin^2 \frac{1}{\sqrt{\tau}}, \quad \text{for} \quad \tau \geq 1,
$$

$$
f(\tau) = -\frac{1}{4} \left( \frac{\ln \frac{1 - \sqrt{1 - \tau}}{1 + \sqrt{1 - \tau}} + \pi i}{1} \right)^2 \quad \text{for} \quad \tau < 1, \quad (2.5)
$$

where $\cot \beta$ denotes the mixing angle in the Two-Higgs-Doublet Model. Further $m$ and $m_t$ denote the masses of the (pseudo-) scalar Higgs boson and the top quark respectively. In the large $m_t$-limit we have

$$
\lim_{\tau \to \infty} F_H(\tau) = \frac{2}{3} \tau, \quad \lim_{\tau \to \infty} F_A(\tau) = \frac{1}{\tau} \cot \beta. \quad (2.6)
$$

The coefficient functions $C_B$ originate from the corrections to the top-quark triangular graph provided one takes the limit $m_t \to \infty$. We have presented the Born level couplings $G_B$ in Eq. (2.3) for general $m_t$ for on-shell gluons only in order to keep some part of the top-quark mass dependence. This is an approximation because the gluons which couple to the H and A bosons via the top-quark loop in partonic subprocesses are very often virtual. The virtual-gluon momentum dependence is neither described by $F_B(\tau)$ nor by $C_B$. However for total cross sections the main contribution comes from the region where the gluons are almost on-shell so that this approximation is better than it is for differential cross sections with large transverse momentum. The coefficient functions are computed up to order $\alpha_s^2$ in [14], [15] for the H and in [16] for the A. They read as follows

$$
C_H \left( a_s(\mu_r^2), \frac{\mu_r^2}{m^2} \right) = 1 + a_s^{(5)}(\mu_r^2) \left[ 5 C_A - 3 C_F \right] + \left( a_s^{(5)}(\mu_r^2) \right)^2 \left[ \frac{27}{2} C_F^2 
$$

$$
- \frac{100}{3} C_A C_F + \frac{1063}{36} C_A^2 - \frac{4}{3} C_F T_f - \frac{5}{6} C_A T_f + \left( 7 C_A^2 \right)
$$
\begin{align}
-11 C_A C_F \ln \frac{\mu_r^2}{m_t^2} + n_f T_f \left(-5 C_F - \frac{47}{9} C_A + 8 C_F \ln \frac{\mu_r^2}{m_t^2}\right), \quad (2.7)
\end{align}

\begin{align}
C_A \left(a_s(\mu_r^2), \frac{\mu_r^2}{m_t^2}\right) = 1,
\end{align}

where $a_s^{(5)}$ is presented in the five-flavour number scheme. Notice that the coefficient function in Eq. (2.7) is derived for general colour factors of the group SU(N) from Eq. (6) in [15]. These factors are given by

\begin{align}
C_A = N \quad , \quad C_F = \frac{N^2 - 1}{2N} \quad , \quad T_f = \frac{1}{2}.
\end{align}

Notice that $T_f$ is also incorporated into $G_B$ in Eq. (2.3) where it is set to the value $T_f = 1/2$. Using the effective Lagrangian approach we will calculate the total cross section of the reaction

\begin{align}
H_1(P_1) + H_2(P_2) \to B(q)' + X',
\end{align}

where $H_1$ and $H_2$ denote the incoming hadrons and $X$ represents an inclusive hadronic state. The total cross section is given by

\begin{align}
\sigma_{\text{tot}} = \frac{\pi G_B^2}{8(N^2 - 1)} \sum_{a,b=q,\bar{q},g} \int_x^1 dx_1 \int_{x/x_1}^1 dx_2 \int \frac{d^4q}{(2\pi)^4} \delta^+(q^2 - m^2) T_{ab,B}(q, p_1, p_2),
\end{align}

where the factor $1/(N^2 - 1)$ originates from the colour average in the case of the local gauge group SU(N). Further we have assumed that the (pseudo-) scalar Higgs boson is mainly produced on-shell i.e. $q^2 \sim m^2$. The parton densities denoted by $f_a(y, \mu^2)$ ($a, b = q, \bar{q}, g$) depend on the mass factorization/renormalization scale $\mu$. The same scale also enters the coefficient functions $\Delta_{ab,B}$ which are derived from the partonic cross sections ($z = m^2/s$)

\begin{align}
\sigma_{ab,B}\left(z, \frac{m^2}{\mu^2}\right) = \frac{\pi}{s} \int \frac{d^nq}{(2\pi)^n} \delta^+(q^2 - m^2) T_{ab,B}(q, p_1, p_2),
\end{align}

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where the incoming partons $a$ and $b$ carry momenta $p_1$ and $p_2$ respectively. They are related to the hadron momenta by

$$p_1 = x_1 P_1, \quad p_2 = x_2 P_2,$$

$$s = (p_1 + p_2)^2, \quad \implies s = x_1 x_2 S, \quad z = \frac{m^2}{s}. \quad (2.13)$$

The amplitude $T_{ab,B}$ can be written as

$$T_{ab,H}(q, p_1, p_2) = G_H^2 \int d^4 y e^{i q \cdot y} \langle a, b | O(y) O(0) | a, b \rangle, \quad (2.14)$$

$$T_{ab,A}(q, p_1, p_2) = \int d^4 y e^{i q \cdot y} \langle a, b | \left( G_A O_1(y) + \tilde{G}_A O_2(y) \right)$$

$$\times \left( G_A O_1(0) + \tilde{G}_A O_2(0) \right) | a, b \rangle. \quad (2.15)$$

The expressions above for the amplitude $T_{ab}$ are similar to those given for the Drell-Yan process except that the conserved electroweak currents are replaced by the operators $O$ and $O_1, O_2$. The latter are not conserved so that they acquire additional renormalization constants defined by

$$O(y) = Z_O \hat{O}(y), \quad O_i(y) = Z_{ij} \hat{O}_j(y). \quad (2.16)$$

where the hat indicates that the quantities under consideration are unrenormalized. Insertion of the above equations into Eqs. (2.14),(2.15) leads to the renormalized expressions

$$T_{ab,H}(q, p_1, p_2) = G_H^2 Z_O^2 \int d^4 y e^{i q \cdot y} \langle a, b | \hat{O}(y) \hat{O}(0) | a, b \rangle, \quad (2.17)$$

$$T_{ab,A}(q, p_1, p_2) = \int d^4 y e^{i q \cdot y} \left\{ G_A^2 Z_{11}^2 + \tilde{G}_A^2 Z_{21}^2 + 2 G_A \tilde{G}_A Z_{11} Z_{21} \right\}$$

$$\times \langle a, b | \hat{O}_1(y) \hat{O}_1(0) | a, b \rangle + \left\{ G_A^2 Z_{12}^2 + \tilde{G}_A^2 Z_{22}^2 + 2 G_A \tilde{G}_A Z_{12} Z_{22} \right\}$$

$$\times \langle a, b | \hat{O}_2(y) \hat{O}_2(0) | a, b \rangle + \left\{ G_A Z_{11} Z_{12} + \tilde{G}_A Z_{22} Z_{21} + G_A \tilde{G}_A \right.$$

$$\times \left( Z_{11} Z_{22} + Z_{12} Z_{21} \right) \rangle \langle a, b | \hat{O}_1(y) \hat{O}_2(0) + \hat{O}_2(y) \hat{O}_1(0) | a, b \rangle \right\}. \quad (2.18)$$

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Since we are interested in the one- and two-loop correction to the subprocess $B \to g + g$ the above formula simplifies and it becomes

$$T_{gg,H}(q, p_1, p_2) = G_H^2 Z_O^2 \int d^4 y e^{i q \cdot y} \langle g, g|\hat{O}(y)\hat{O}(0)|g, g\rangle,$$  \hspace{1cm} (2.19)

$$T_{ab,A}(q, p_1, p_2) = \int d^4 y e^{i q \cdot y} \left[ G_A^2 Z_{11} \langle g, g|\hat{O}_1(y)\hat{O}_1(0)|g, g\rangle + \left\{ G_A^2 Z_{12} + G_A \tilde{G}_A \right\} \langle g, g|\hat{O}_1(y)\hat{O}_2(0) + \hat{O}_2(y)\hat{O}_1(0)|g, g\rangle \right].$$ \hspace{1cm} (2.20)

The operator renormalization constants depend on the regularization scheme in particular on the prescription for the $\gamma_5$-matrix and the Levi-Civita tensor in Eq. (2.2). The computation of $T_{gg,H}$ will be carried out by choosing $n$-dimensional regularization where in the case of $T_{gg,A}$ we adopt the HVBM prescription [17], [18] for the $\gamma_5$-matrix. For this choice the contraction of the Levi-Civita tensors proceeds as

$$\epsilon_{\mu_1\nu_1\lambda_1\sigma_1} \epsilon^{\mu_2\nu_2\lambda_2\sigma_2} = \begin{vmatrix}
\delta_{\mu_2}^{\mu_1} & \delta_{\nu_2}^{\nu_1} & \delta_{\lambda_2}^{\lambda_1} & \delta_{\sigma_2}^{\sigma_1} \\
\delta_{\mu_1}^{\mu_2} & \delta_{\nu_1}^{\nu_2} & \delta_{\lambda_1}^{\lambda_2} & \delta_{\sigma_1}^{\sigma_2} \\
\delta_{\lambda_1}^{\mu_1} & \delta_{\lambda_1}^{\nu_1} & \delta_{\lambda_1}^{\lambda_1} & \delta_{\lambda_1}^{\sigma_1} \\
\delta_{\lambda_2}^{\mu_1} & \delta_{\lambda_2}^{\nu_1} & \delta_{\lambda_2}^{\lambda_1} & \delta_{\lambda_2}^{\sigma_1}
\end{vmatrix},$$ \hspace{1cm} (2.21)

where all Lorentz indices are taken to be $n$-dimensional. To facilitate the calculation one can replace $\gamma_5$ in Eq. (2.2) by (see [19])

$$\gamma_5 = \frac{i}{24} \epsilon_{\mu\nu\lambda\sigma} \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma,$$ \hspace{1cm} (2.22)

which is equivalent to the HVBM scheme. Choosing the $\overline{\text{MS}}$ subtraction scheme the renormalization constant corresponding to the operator $O$ becomes [20]

$$Z_O = 1 + a_s(\mu_r^2) S_\varepsilon \left( \frac{2}{\varepsilon} \beta_0 + a_s(\mu_r^2) S_\varepsilon \left[ \frac{4}{\varepsilon^2} \beta_0^2 + \frac{2}{\varepsilon} \beta_1 \right] \right) + \cdots$$ \hspace{1cm} (2.23)

where $S_\varepsilon$ denotes the spherical factor characteristic of $n$-dimensional regularization. It is defined by

$$S_\varepsilon = \exp \left\{ \frac{\varepsilon}{2} \left[ \gamma_E - \ln 4\pi \right] \right\}.$$ \hspace{1cm} (2.24)
The lowest order coefficients $\beta_0$ and $\beta_1$ originate from the beta-function given by

$$\beta(\alpha_s) = -a_s^2(\mu_r^2) \beta_0 - a_s^3(\mu_r^2) \beta_1 + \cdots,$$

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} n_f T_f, \quad \beta_1 = \frac{34}{3} C_A^2 - 4 n_f T_f C_F - \frac{20}{3} n_f T_f C_A.$$  \hfill (2.25)

The operator renormalization constants corresponding to $O_1$ and $O_2$ are computed in [21] and they read

$$Z_{11} = Z_{\alpha_s} = 1 + a_s(\mu_r^2) S_\varepsilon \left[ \frac{2}{\varepsilon} \beta_0 + a_s^2(\mu_r^2) S_\varepsilon^2 \left[ \frac{4}{\varepsilon^2} \beta_0^2 + \frac{1}{\varepsilon} \beta_1 \right] \right] + \cdots \quad (2.26)$$

where $Z_{\alpha_s}$ denotes the coupling constant renormalization factor defined by

$$\hat{a}_s = Z_{\alpha_s} a_s(\mu_r^2).$$ \hfill (2.27)

The remaining constants are

$$Z_{21} = 0,$$ \hfill (2.28)

$$Z_{12} = a_s(\mu_r^2) S_\varepsilon \left[ -6 C_F \right],$$ \hfill (2.29)

$$Z_{22} = Z_{MS}^s Z_5^s,$$ \hfill (2.30)

where $Z_{MS}^s$ and $Z_5^s$ are the constants characteristic of the HVBM scheme. They are given by

$$Z_{MS}^s = 1 + a_s^2(\mu_r^2) S_\varepsilon \left[ -\frac{44}{3} C_A C_F - n_f T_f C_F \right] \frac{20}{3},$$

$$Z_5^s = 1 + a_s(\mu_r^2) \left[ -4 C_F \right] + a_s^2(\mu_r^2) \left[ 22 C_F^2 - \frac{107}{9} C_A C_F + \frac{31}{9} n_f T_f C_F \right].$$ \hfill (2.31)

The latter renormalization constant is determined in such a way that the Adler-Bell-Jackiw anomaly [22]

$$O_2(y) = 4 a_s(\mu_r^2) n_f T_f O_1(y),$$ \hfill (2.32)
is preserved in all orders in perturbation theory according to the Adler-Bardeen theorem [23].
3 Two-loop vertex correction to the process $g + g \to H, A$.

In this section we present the calculation of the (pseudo-) scalar Higgs vertex corrected up to two loops. The scalar Higgs vertex was presented in [10] but no separation in colour factors was made. An explicit formula for the pseudo-scalar vertex was not shown but it indirectly appears in the coefficient functions of Higgs boson production in [11] and [12].

The one loop vertex correction is very easy. In the case of Higgs production the characteristic graphs are shown in Fig. 1. For the operators $O(y)$ and $O_1(y)$ all the solid lines represent gluons and there are only two types of graphs. The two-loop vertex correction, which is less trivial, can be calculated in various ways. The characteristic graphs are shown in Fig. 2. The solid lines represent gluons, quarks as well as ghosts (Feynman gauge). Using the program FORM [24] each graph can be reduced into scalar integrals of the type

$$V_{ij}^{ij,j_{j_2}...j_n} = \int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{(k_1 \cdot p_1)^i (k_2 \cdot p_2)^j}{D_{j_1} D_{j_2} \cdots D_{j_n}} \quad l, m = 1, 2$$

$$D_1 = (k_1 + k_2 - p_1)^2, \quad D_2 = (k_1 + k_2 - p_2)^2, \quad D_3 = k_1^2,$$

$$D_4 = (k_2 - p_1)^2, \quad D_5 = (k_2 - p_2)^2, \quad D_6 = k_2^2,$$

$$D_7 = (k_1 - p_1)^2, \quad D_8 = (k_1 - p_2)^2. \quad (3.1)$$

The scalar integrals in the case of the Higgs boson vertex are ordered as follows. For six propagators we have $i + j = 4$. For five propagators $i + j = 3$. For four propagators $i + j = 2$. For three propagators $i + j = 1$. The two and one propagator integrals are all zero. The calculation of these scalar integrals can be performed in various ways. For the electromagnetic vertex correction in QCD it has been done in [25], [26], [27]. In [25] one has chosen a Feynman parameter set in a judicious way so that after each integration one is still left with a linear integral over the next parameter. In [26] one used integration by parts according to [28]. This trick eliminates one propagator from the integral in Eq. (3.1). However it does not work for the second Feynman graph in Fig. 2 (the crossed box graph). In [27] the scalar integrals were calculated...
using dispersion techniques developed in [29]. In the latter reference one has used the Cutkosky rules [30] to compute the scalar integrals. Here one cuts the graph in all possible ways to obtain the imaginary part. The real part is obtained via a dispersion relation. Fortunately for the electromagnetic vertex correction $i + j \leq 3$ in Eq. (3.1) but for the vertex correction to $O(y)$ and $O_1(y)$ in Eq. (2.2) one also gets contributions for $i + j = 4$. In [10] one employed an algorithm in [31] which relates l-loop integrals with $n + 1$ external legs to $l + 1$-loop integrals with $n$ external legs. In the case under consideration massless two-loop vertex functions are mapped onto massless three-loop two-point functions. We however adopt the method in [29] and calculate the imaginary part of all graphs using the Cutkosky rules. In the case that all particles are massless, except for the external Higgs boson leg, the dispersion integrals are trivial. Since all graphs, except for the second graph in Fig. 2, are analytic we have made a computer program in FORM [24] which provides us with all scalar integrals of the type in (3.1). For more details about the real and imaginary parts of the scalar integrals we refer to [29]. The result for the scalar Higgs vertex is obtained from Eq. (2.19) and it reads up to order $\hat{\alpha}_s^2$

\[ F_H(q^2) (2\pi)^4 \delta(q + p_1 - p_2) = Z_O \int d^4 y e^{i q \cdot y} \langle g(p_2) | \hat{O}(y) | g(p_1) \rangle , \quad (3.2) \]
Figure 2: General structure of the two-loop vertex correction g-g-B ($B = H, A$).

where $F_H(q^2)$ for $q^2 < 0$ is equal to $(n = 4 + \varepsilon)$

$$Z_O^{-1} F_H(q^2) = 1 + \hat{a}_s S_\varepsilon \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon/2} C_A \left[ - \frac{8}{\varepsilon^2} + \zeta(2) + \varepsilon \left( 1 - \frac{7}{3} \zeta(3) \right) 
\right. 
+ \varepsilon^2 \left( - \frac{3}{2} + \frac{47}{80} \zeta^2(2) \right) 
+ \hat{a}_s^2 S_\varepsilon^2 \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon} C_A \left\{ \frac{32}{\varepsilon^4} + \frac{44}{3 \varepsilon^3} 
\right. 
\left. 
- \left( \frac{134}{9} + 4 \zeta(2) \right) \frac{1}{\varepsilon^2} + \left( - \frac{136}{27} - 11 \zeta(2) + \frac{50}{3} \zeta(3) \right) \frac{1}{\varepsilon} 
\right. 
\left. 
+ \frac{5861}{162} + \frac{67}{6} \zeta(2) + \frac{11}{9} \zeta(3) - \frac{21}{5} \zeta^2(2) \right\} \right.$$

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Figure 3: Interference term between diagrams with operators $O_1$ and $O_2$.

\[ +n_f T_f C_A \left\{ - \frac{16}{3} \varepsilon^3 + \frac{40}{9} \varepsilon^2 + \left( \frac{104}{27} + 4 \zeta(2) \right) \frac{1}{\varepsilon} - \frac{1616}{81} \right. \]

\[ - \left. \frac{10}{3} \zeta(2) - \frac{148}{9} \zeta(3) \right\} \]

\[ +n_f T_f C_F \left\{ \frac{4}{\varepsilon} - \frac{67}{3} + 16 \zeta(3) \right\}, \quad (3.3) \]

where $\hat{a}_s$ (see Eq. (2.4)) is the bare coupling. The result above agrees with Eqs. (16) and (17) in [10] provided we put $C_A = 3$ and $C_F = 4/3$ (Notice that in [10] $n = 4 - 2 \varepsilon$). The pseudo-scalar Higgs vertex can be derived from Eq. (2.20).

\[ F_A(q^2) (2\pi)^4 \delta(q + p_1 - p_2) = \int d^4 y e^{i q y} \left\{ Z_{11} \langle g(p_2) | \hat{O}_1(y) | g(p_1) \rangle \right. \]

\[ + Z_{12} \langle g(p_2) | \hat{O}_2(y) | g(p_1) \rangle \} \], \quad (3.4) \]

The matrix element $\langle g(p_2) | \hat{O}_2(y) | g(p_1) \rangle = C \hat{a}_s$ but $Z_{12} \sim \hat{a}_s \varepsilon^{-1}$ (see Eq. (2.29)). This contribution has to be added because we use the HVBM scheme for the $\gamma_5$-matrix. The extra term equals (see Fig. 3)

\[ \hat{a}_s^2 e^2 C_F T_f \left( -\frac{q^2}{\mu^2} \right) \varepsilon \left[ \frac{24}{\varepsilon} - 36 \right]. \quad (3.5) \]
In the equation above we have multiplied by an extra factor $S_\varepsilon \left( -q^2/\mu^2 \right)^{\varepsilon/2}$ to restore the anti-commutativity of the $\gamma_5$-matrix. In a scheme where the $\gamma_5$-matrix anti-commutes with the other gamma matrices this extra factor would be automatically there. This term has to be added to $F_A(q^2)$ and the result up to order $\hat{a}_s^2$ becomes ($q^2 < 0$)

$$Z_{11}^{-1} F_A(q^2) = 1 + \hat{a}_s S_\varepsilon \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon/2} C_A \left[ -\frac{8}{\varepsilon^2} + 4 + \zeta(2) + \varepsilon \left( -6 - \frac{7}{3} \zeta(3) \right) \right]$$

$$+ \varepsilon^2 \left( 7 - \frac{1}{2} \zeta(2) + \frac{47}{80} \zeta^2(2) \right) + \hat{a}_s^2 S_\varepsilon^2 \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon} C_A^2 \left[ \frac{32}{\varepsilon^4} \right]$$

$$+ \frac{44}{3 \varepsilon^3} - \left( \frac{422}{9} + 4 \zeta(2) \right) \frac{1}{\varepsilon^2} + \left( \frac{890}{27} - 11 \zeta(2) + \frac{50}{3} \zeta(3) \right) \frac{1}{\varepsilon}$$

$$+ \frac{3835}{81} + \frac{115}{6} \zeta(2) + \frac{11}{9} \zeta(3) - \frac{21}{5} \zeta^2(2)$$

$$+ n_f T_F C_A \left\{ -\frac{16}{3} \varepsilon^3 + \frac{40}{9 \varepsilon^2} + \left( \frac{212}{27} + 4 \zeta(2) \right) \frac{1}{\varepsilon} - \frac{3182}{81} \right\}$$

$$- \frac{10}{3} \zeta(2) - \frac{148}{9} \zeta(3) \right\}$$

$$+ n_f T_F C_F \left\{ -\frac{142}{3} + 16 \zeta(3) \right\} \right] \right] . \quad (3.6)$$

This result is new. It only indirectly appears in the $\delta(1-z)$ part of the total cross section for pseudo-scalar Higgs boson production in [11] and [12]. We can now compute the part due to virtual and soft gluons which is proportional to $\delta(1-z)$ for general colour factors as in [32]. Continuing the above form factors to $q^2 > 0$ and adding the soft gluon cross section we can write the soft plus virtual gluon contribution to the coefficient function in Eq. (2.11) as

$$\Delta^{S+V}_{gg,B} = \delta(1-z) + a_s \Delta^{(1),S+V}_{gg,B} + a_s^2 \Delta^{(2),S+V}_{gg,B} . \quad (3.7)$$

Starting with scalar Higgs boson production we omit the term which is proportional to $C_A^2$ as it is available in [32]. The remaining order $a_s^2$ part of Eq.
\[ (3.7) \text{ equals} \]
\[
\Delta_{g g, H}^{(2), S+V} = n_f T_f C_A \left[ \left\{ \frac{16}{3} D_0(z) \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ \frac{64}{3} D_1(z) - \frac{160}{9} D_0(z) \right\} \right. \\
+ \delta(1 - z) \left( 16 + \frac{32}{3} \zeta(2) \right) \ln \left( \frac{m^2}{\mu^2} \right) + \frac{64}{3} D_2(z) - \frac{320}{9} D_1(z) \\
+ \left( \frac{448}{27} - \frac{64}{3} \zeta(2) \right) D_0(z) + \delta(1 - z) \left( - \frac{160}{3} - \frac{160}{9} \zeta(2) - \frac{16}{3} \zeta(3) \right) \\
+ \left. n_f T_f C_F \delta(1 - z) \left[ 8 \ln \left( \frac{m^2}{\mu^2} \right) - \frac{134}{3} + 32 \zeta(3) \right] \right].
\]

(3.8)

The result above is in agreement with [11] and [33] for \( C_A = 3 \) and \( C_F = 4/3 \). For the pseudo scalar case we take the difference with the scalar Higgs boson coefficient function. Moreover we add the last term in Eq. (2.20) which equals
\[
\int d^4 y e^{i q \cdot y} G_A \tilde{G}_A \langle g, g| \hat{O}_1(y) \hat{O}_2(0) + \hat{O}_2(y) \hat{O}_1(0)|g, g \rangle.
\]

(3.9)

This term is proportional to \( O_{12} \) in the formula below and it originates from Eq. (2.3). Ignoring the term proportional to \( C_A^2 \) which is given in [32] the remaining order \( a_s^2 \) term equals
\[
\Delta_{g g, A-H}^{(2), S+V} = C_A T_f n_f \delta(1 - z) \left[ - \frac{8}{3} \ln \left( \frac{m^2}{\mu^2} \right) - \frac{4}{3} \right] \\
+ C_F T_f n_f \delta(1 - z) \left[ - 8 \ln \left( \frac{m^2}{\mu^2} \right) - 50 \right] \\
+ O_{12} \left( 24 \ln \left( \frac{\mu_t^2}{m_t^2} \right) - 12 \right),
\]

(3.10)

which agrees with [11] and [12] for \( C_A = 3 \) and \( C_F = 4/3 \).
4 General structure of the vertex correction

In this section we will study the general structure of the vertex correction as predicted in [34] and [35]. In these two papers the behaviour of the vertex corrections w.r.t. \( \ln(-q^2/\mu^2) \) was derived. Following Eq. (6.21) in [35] the behaviour is as follows

\[
\frac{d \ln F_i(q^2)}{d \ln(-q^2/\mu^2)} = -\frac{1}{4} \int_{\mu^2}^{-q^2} \frac{d \mu_1^2}{\mu_1^2} \gamma_{K,ii}(a_s(\mu_1^2)) + \frac{1}{2} G_{ii}(a_s(q^2)) \\
+ K_{ii} \left( \frac{1}{\varepsilon}, a_s(\mu^2) \right),
\]

(4.1)

where

\[
\gamma_{K,ii} = a_s(\mu^2) \gamma_{K,ii}^{(0)} + a_s^2(\mu^2) \gamma_{K,ii}^{(1)} + a_s^3(\mu^2) \gamma_{K,ii}^{(2)} + \cdots
\]

(4.2)

and

\[
G_{ii} = a_s(q^2) \left[ \gamma_{ii}^{(0)} - z_1 \right] + a_s^2(q^2) \left[ \gamma_{ii}^{(1)} - z_2 \right] \\
+ a_s^3(q^2) \left[ \gamma_{ii}^{(2)} - z_3 \right] + \cdots
\]

(4.3)

The quantities \( \gamma_{K,ii} \) and \( G_{ii} \) \((i = q, g)\) occur in the diagonal kernel

\[
\Gamma_{ii} = 1 + \left[ \gamma_{K,ii} \left( \frac{1}{1-x} \right)_+ + G_{ii} \delta(1-x) \right],
\]

(4.4)

and they represent those parts of the splitting functions which are proportional to the distributions \((1/(1-x))_+\) and \(\delta(1-x)\) respectively provided they are expressed in the \(\overline{\text{MS}}\)-scheme. They have been calculated up to three loop order in [36]. The constants \(z_i\) occur when the operator is not conserved which happens in the case of Higgs boson vertex corrections. The constants show up in the operator renormalization constants \(Z_O\) in Eq. (2.23) and \(Z_{11}\) in Eq. (2.26). The expression for these operator renormalization constants is \((Z = Z_O \text{ or } Z = Z_{11})\)

\[
Z = 1 + a_s S_\varepsilon \left[ \frac{z_1}{\varepsilon} \right] + a_s^2 S_\varepsilon^2 \left[ \frac{1}{\varepsilon^2} \left( \beta_0 z_1 + \frac{1}{2} z_1^2 \right) + \frac{z_2}{2 \varepsilon} \right].
\]

(4.5)
Here $z_1 = 2\beta_0$ for both operators. Further we have $z_2 = 4\beta_1$ for $O(y)$ in Eq. (2.23) and $z_2 = 2\beta_1$ for $O_1(y)$ in Eq. (2.26). If we express this in the bare coupling constant it becomes

$$Z = 1 + \hat{a}_s S_\varepsilon \left[ \frac{z_1}{\varepsilon} \right] + \hat{a}_s^2 S_\varepsilon^2 \left[ \frac{1}{\varepsilon^2} \left( \frac{1}{2} z_1^2 - \beta_0 z_1 \right) + \frac{z_2}{2\varepsilon} \right]. \quad (4.6)$$

In the case of the electromagnetic vertex the current is conserved and we find $z_i = 0$. Finally $K_{ii}$ in Eq. (4.1) collects all collinear and infrared terms. Because of the one to one correspondence between the $\ln(-q^2/\mu^2)$ terms and the pole terms $1/\varepsilon$ it reads up to order $a_s^2$

$$K_{ii} = a_s(\mu^2) S_\varepsilon \left[ -\frac{1}{2\varepsilon} \gamma^{(0)}_{K,ii} \right] + a_s^2(\mu^2) S_\varepsilon^2 \left[ -\frac{1}{2\varepsilon^2} \beta_0 \gamma^{(0)}_{K,ii} - \frac{1}{4\varepsilon} \gamma^{(1)}_{K,ii} + f^{(1)}_{i,2} \right] - \beta_0 f^{(0)}_{i,1} + \cdots \quad (4.7)$$

where $f^{(0)}_{i,1}$ is a vertex dependent constant. However $f^{(1)}_{i,2}$ is vertex independent as we shall see later on. From Eq. (4.1) one can derive the formula for the logarithm of the vertex correction. First we replace the coupling constant $a_s(\mu^2_1)$ by $a_s(\mu^2)$

$$a_s(\mu^2_1) = a_s(\mu^2) \left( 1 - a_s(\mu^2) \beta_0 \ln \frac{\mu^2_1}{\mu^2} \right). \quad (4.8)$$

then we integrate over $\mu^2_1$. Subsequently we integrate over $\ln(-q^2/\mu^2)$ and finally we replace the renormalized by the bare coupling constant

$$a_s(\mu^2) = \hat{a}_s \left( 1 - \hat{a}_s \frac{2}{\varepsilon} S_\varepsilon \beta_0 \right). \quad (4.9)$$

We can now integrate Eq. (4.1) over $\ln(-q^2/\mu^2)$. However in the case of non conserved operators one has to be careful in the determination of the pole part of the vertex function. First we separate the $\ln(-q^2/\mu^2)$ terms which are due to the non conserved operators from the rest. Next we observe a one to one correspondence between the pole part and the remaining $\ln(-q^2/\mu^2)$
piece. Hence the general expression for the unrenormalized vertex correction becomes

\[
\ln F_i(q^2) = \hat{a}_s S_\varepsilon \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon/2} \left[ -\gamma_{K,ii}^{(0)} \frac{1}{\varepsilon^3} + \gamma_{ii}^{(0)} + f_{i,1}^{(0)} \right] \\
- \hat{a}_s S_\varepsilon \left[ \frac{1}{2} z_1 \ln \left( \frac{-q^2}{\mu^2} \right) + \frac{1}{8} \varepsilon z_1 \ln^2 \left( \frac{-q^2}{\mu^2} \right) \right] \\
+ \hat{a}_s S_\varepsilon \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon} \left[ \frac{1}{2} \beta_0 \gamma_{K,ii}^{(0)} \frac{1}{\varepsilon^3} - \left( \frac{1}{4} \gamma_{K,ii}^{(1)} + \beta_0 \gamma_{ii}^{(0)} \right) \frac{1}{\varepsilon^2} \right] \\
+ \left( \frac{1}{2} \gamma_{ii}^{(1)} + f_{i,2}^{(1)} - 2 \beta_0 f_{i,1}^{(0)} \right) \frac{1}{\varepsilon} + f_{i,2}^{(0)} \right] \\
- \hat{a}_s S_\varepsilon \left[ - \beta_0 z_1 \ln \left( \frac{-q^2}{\mu^2} \right) - \frac{1}{2} \beta_0 z_1 \ln^2 \left( \frac{-q^2}{\mu^2} \right) \right] \\
+ \frac{1}{2} z_2 \ln \left( \frac{-q^2}{\mu^2} \right), \quad i = H, A, \gamma^*. \quad (4.10)
\]

Note that \( z_k \) are implicit in \( \gamma_{ii}^{(k-1)} \) and they do not contribute to the \( \ln(-q^2/\mu^2) \) terms. They contribute to the pole terms \( (1/\varepsilon)^k \) only. Another remark concerns the term \(-2 \beta_0 f_{i,1}^{(0)}\) in the penultimate line of the above equation. It vanishes in the single pole term after coupling constant renormalization. The anomalous dimensions \( \gamma_{K,ii}^{(k)} \) are up to a colour factor \( C_i \) vertex independent. Up to two-loops they are given by

\[
\gamma_{K,ii}^{(0)} = 8 C_i, \quad \gamma_{K,ii}^{(1)} = 16 C_i K, \quad K = -\frac{10}{9} n_f T_f + C_A \left( \frac{67}{18} - \zeta(2) \right), \quad i = q, g, \quad C_q = C_F, \quad C_g = C_A. \quad (4.11)
\]

However, and this is very important, \( f_{i,2}^{(1)} \) is also vertex independent up to a colour factor \( C_i \) and it is given by

\[
f_{i,2}^{(1)} = C_i \left[ - \beta_0 \zeta(2) + C_A \left( \frac{404}{27} - 14 \zeta(3) \right) + n_f T_f \left( -\frac{112}{27} \right) \right]. (4.12)
\]
The whole vertex dependence can be attributed to the the anomalous dimensions $\gamma_{ii}^{(k)}$ and the non-pole terms $f_{i,k}^{(0)}$. The anomalous dimensions up to two loops are given by

$$\gamma_{qq}^{(0)} = 6 C_F, \quad \gamma_{gg}^{(0)} = 2 \beta_0,$$

$$\gamma_{qq}^{(1)} = C_F^2 \left( 3 - 24 \zeta(2) + 48 \zeta(3) \right) + C_A C_F \left( \frac{17}{3} + \frac{88}{3} \zeta(2) - 24 \zeta(3) \right)$$

$$+ n_f T_f C_F \left( - \frac{4}{3} - \frac{32}{3} \zeta(2) \right),$$

$$\gamma_{gg}^{(1)} = C_A^2 \left( \frac{64}{3} + 24 \zeta(3) \right) - \frac{32}{3} n_f T_f C_A - 8 n_f T_f C_F. \quad (4.14)$$

The non-pole terms $f_{i,k}^{(0)} (i = q, g)$ can be extracted from the actual calculations. For that purpose we also include the quark vertex correction in the case of electromagnetism [25], [26], [27]. Starting with the latter, this expression equals

$$\ln F_{\gamma^*}(q^2) = \hat{a}_s S_{\epsilon} \left( \frac{-q^2}{\mu^2} \right)^{\epsilon/2} \left[ - \frac{8}{\epsilon^2} + \frac{6}{\epsilon} - 8 + \zeta(2) \right]$$

$$+ \hat{a}_s^2 S_{\epsilon}^2 \left( \frac{-q^2}{\mu^2} \right)^{\epsilon} \left[ C_F^2 \left\{ \frac{3}{2} - 12 \zeta(2) + 24 \zeta(3) \right\} \frac{1}{\epsilon} - \frac{1}{8} + 29 \zeta(2)$$

$$- 30 \zeta(3) - \frac{44}{5} \zeta^2(2) \right\} + C_A C_F \left\{ \frac{44}{3 \epsilon^3} + \left( - \frac{332}{9} + 4 \zeta(2) \right) \frac{1}{\epsilon^2}$$

$$+ \left( \frac{4129}{54} + \frac{11}{3} \zeta(2) - 26 \zeta(3) \right) \frac{1}{\epsilon} - \frac{89173}{648} - \frac{119}{9} \zeta(2)$$

$$+ \frac{467}{9} \zeta(3) + \frac{44}{5} \zeta^2(2) \right\} + n_f T_f C_F \left\{ - \frac{16}{3 \epsilon^3} + \frac{112}{9} \frac{1}{\epsilon^2}$$

$$+ \left( - \frac{706}{27} - \frac{4}{3} \zeta(2) \right) \frac{1}{\epsilon} + \frac{7541}{162} + \frac{28}{9} \zeta(2)$$

$$- \frac{52}{9} \zeta(3) \right\} \right]. \quad (4.15)$$
The non-pole terms \( f_{q,k}^{(0)} \) are

\[
\begin{align*}
f_{q,1}^{(0)} &= C_F (\zeta(2) - 8), \\
f_{q,2}^{(0)} &= C_F^2 \left( -\frac{1}{8} + 29 \zeta(2) - \frac{44}{5} \zeta^2(2) - 30 \zeta(3) \right) + C_A C_F \left( -\frac{89173}{648} \\
&\quad - \frac{119}{9} \zeta(2) + \frac{44}{5} \zeta^2(2) + \frac{467}{9} \zeta(3) \right) + n_f T_f C_F \left( \frac{7541}{162} + \frac{28}{9} \zeta(2) \right) \\
&\quad - \frac{52}{9} \zeta(3) \right).
\end{align*}
\]

(4.16)

Next we turn our attention to Higgs boson production. Taking the logarithm of the expression for the scalar Higgs boson Eqs. (3.3) we obtain

\[
\begin{align*}
\ln F_H(q^2) &= \ln Z_O + \hat{a}_s S_\varepsilon \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon/2} C_A \left[ -\frac{8}{\varepsilon^2} + \zeta(2) \right] \\
&\quad + \hat{a}_s^2 S_\varepsilon^2 \left( \frac{-q^2}{\mu^2} \right)^{\varepsilon} \left[ C_A \left\{ \frac{44}{3 \varepsilon^3} + \left( -\frac{134}{9} + 4 \zeta(2) \right) \frac{1}{\varepsilon^2} + \left( \frac{80}{27} \right) \right. \right. \\
&\quad - 11 \zeta(2) - 2 \zeta(3) \left. \right\} \frac{1}{\varepsilon} + \frac{3917}{162} + \frac{67}{6} \zeta(2) + \frac{11}{9} \zeta(3) \left. \right\} \\
&\quad + n_f T_f C_A \left\{ -\frac{16}{3 \varepsilon^3} + \frac{40}{9 \varepsilon^2} + \left( \frac{104}{27} + 4 \zeta(2) \right) \frac{1}{\varepsilon} - \frac{1616}{81} \right. \\
&\quad - \frac{10}{3} \zeta(2) - \frac{148}{9} \zeta(3) \left. \right\} + n_f T_f C_F \left\{ \frac{4}{\varepsilon} - \frac{67}{3} + 16 \zeta(3) \right\} \right],
\end{align*}
\]

(4.17)

with the non-pole constants \( f_{g,k}^{(0)} \)

\[
\begin{align*}
f_{g,1}^{(0)} &= C_A \zeta(2), \\
f_{g,2}^{(0)} &= n_f T_f C_A \left( -\frac{1616}{81} - \frac{10}{3} \zeta(2) - \frac{148}{9} \zeta(3) \right) + n_f T_f C_F \left( -\frac{67}{3} \right)
\end{align*}
\]
$$+16 \zeta(3) + C_A^2 \left( \frac{3917}{162} + \frac{67}{6} \zeta(2) + \frac{11}{9} \zeta(3) \right). \quad (4.18)$$

Doing the same for the pseudo-scalar Higgs boson vertex (see Eq. (3.6)) we get

$$\ln F_A(q^2) = \ln Z_{11} + \hat{a}_s S_\varepsilon \left( -\frac{q^2}{\mu^2} \right)^{\varepsilon/2} C_A \left[ -\frac{8}{\varepsilon^2} + 4 + \zeta(2) \right]$$

$$+ \hat{a}_s^2 S_\varepsilon \left( -\frac{q^2}{\mu^2} \right)^{\varepsilon} \left[ C_A^2 \left\{ \frac{44}{3} \varepsilon + \left( -\frac{134}{9} + 4 \zeta(2) \right) \frac{1}{\varepsilon^2} + \left( -\frac{406}{27} \right. \right. \right.$$}

$$-11 \zeta(2) - 2 \zeta(3) \left. \right) \frac{1}{\varepsilon} + \frac{7273}{81} + \frac{67}{6} \zeta(2) + \frac{11}{9} \zeta(3) \right]$$

$$+ n_f T_f C_A \left\{ -\frac{16}{3 \varepsilon^3} + \frac{40}{9 \varepsilon^2} + \left( \frac{212}{27} + 4 \zeta(2) \right) \frac{1}{\varepsilon} - \frac{3182}{81} \right.$$}

$$-10 \frac{1}{3} \zeta(2) - \frac{148}{9} \zeta(3) \right\} + n_f T_f C_F \left\{ -\frac{142}{3} + 16 \zeta(3) \right\}. \quad (4.19)$$

with the non-pole constants \( f_{g,k}^{(0)} \)

\[
\begin{align*}
\hat{f}_{g,1}^{(0)} &= C_A \left( 4 + \zeta(2) \right), \\
\hat{f}_{g,2}^{(0)} &= n_f T_f C_A \left( -\frac{3182}{81} - \frac{10}{3} \zeta(2) - \frac{148}{9} \zeta(3) \right) + n_f T_f C_F \left( -\frac{142}{3} \right. \\
& \quad + 16 \zeta(3) \right) + C_A^2 \left( \frac{7273}{81} + \frac{67}{6} \zeta(2) + \frac{11}{9} \zeta(3) \right). \quad (4.20)
\end{align*}
\]

Notice that there is \( C_F \leftrightarrow C_A \) symmetry between the electromagnetic (quark) and the (pseudo-) scalar Higgs (gluon) vertex correction for the \( \gamma_{K,ii}^{(k)} \) and \( f_{i,k}^{(1)} \). From the general form of the form factor [34] and [35] one can predict the coefficients of the \( 1/\varepsilon^4 \), \( 1/\varepsilon^3 \) and \( 1/\varepsilon^2 \) terms in Eq. (4.10). These can be inferred from the anomalous dimensions \( \gamma_{K,ii} \) Eq. (4.2) and \( \gamma_{ii} \) Eq. (4.3). However, and this is new, the coefficient of the single pole term \( 1/\varepsilon \) can now also be predicted. In the renormalized vertex correction it amounts to
1/2 \gamma_{ii}^{(1)} + f_{i,2}^{(1)}. The form for the electromagnetic vertex function was also studied in [37] and [38]. The form for the quark in [37] agrees with ours up to the double pole term. The same is true for the form in [38], which is exactly equal to ours because it is derived from [35]. However the Higgs vertex functions in Eqs. (4.17) and (4.19) were not discussed. Also the structure of the single pole term of the scattering amplitude was not given in [37] and [38]. The latter can be found for instance in [39] and [40] but the relation between the quark and the gluon was not observed for the single pole term. Using our approach for Eq. (3.9) in [39] and for Eq. (A.29) in [40] we can express their $H_i^{(2)}$-functions in the following way

$$H_i^{(2)} = -\frac{1}{8} \gamma_{ii}^{(1)} - \frac{1}{4} f_{i,2}^{(1)} + \frac{1}{4} \gamma_{ii}^{(0)} K + \frac{3}{8} C_i \beta_0 \zeta(2),$$

\[ i = q, g, \quad C_q = C_F, \quad C_g = C_A. \quad (4.21) \]

See also Eqs. (3.10), (3.11) in [39] and Eqs. (A.30), (A.33) in [40]. It is clear that the structure of the single pole term is now solved. With this knowledge and the three-loop anomalous dimension [36] we can at least predict the three-loop vertex function and three-loop scattering amplitudes up to the double pole term.

Summarising the above we have calculated the two-loop vertex functions for the scalar and pseudo-scalar Higgs boson including finite terms. We found agreement with [10] for scalar Higgs production for $C_A = 3$ and $C_F = 4/3$ but the pseudo-scalar Higgs vertex correction is published for the first time. Further we confirmed the predictions for the form factor up to second order double pole term made in [34] and [35]. However we could also extend them to the coefficient of the second order single pole term which equals $1/2 \gamma_{ii}^{(1)} + f_{i,2}^{(1)}$ for the quark ($i = q$) and the gluon ($i = g$).

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