Robust Adaptive Single Neural Control for Yaw Angle with Input Nonlinearity on Helicopter Testbed

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Abstract—In this paper, we deal with the yaw control problem of a small-scale helicopter mounted on an experimental platform. The yaw dynamics of helicopter involve input nonlinearity, time-varying parameters and the couplings between main and tail rotor. An attractive control strategy that combines neural networks with traditional adaptive controls has been successfully used for yaw control with input nonlinearities. In contrast to conventional adaptation law, the sliding condition is taken as the objective function instead of the error function used in MIT rule. From the concept of the sliding mode control, the adaptive controller guarantees the stability of the closed-loop system and convergence of the output tracking error to a desired bound, even if the model parameters are unknown or in the presence of disturbance. The simulation results are further compared with those obtained by normal PID control to demonstrate the improvements of the proposed algorithm.

Keywords—Adaptive neural control, Yaw angle, Input nonlinearity, helicopter testbed

I. INTRODUCTION

Nonlinear adaptive control techniques currently available in the literature [1, 2] are applicable to plants that are under the assumption of linear or affine linear inputs. For most practical control systems, however, there often exist nonlinear properties in the control input such as deadzone, backlash, hysteresis, and saturation. However, due to physical limitation, there exist nonlinearities in the control input and their effect cannot be ignored in analysis of controller design and realization. Since most of the existing adaptive control techniques are not applicable to nonlinear systems with such input nonlinearities. So it is necessary to develop a new adaptive control method to deal with the uncertain systems with input nonlinearity.

Most of the works in adaptive control have concentrated on the systems with uncertainties and/or external disturbances. Recently, much research has been devoted to this problem in the literature. In [2], adaptive control schemes are developed for linear systems with nonsmooth input nonlinearities. An adaptive inverse approach is used in the controller design under the conditions that an explicit structure of the input nonlinearities is known and that the nonlinear characteristics satisfy piecewise linear conditions and are linear in their parameters. The proposed adaptive inverse scheme introduces an inverse mechanism to cancel the effects of the unknown nonlinearities such that the resulted systems become linear. It has been shown that a significant improvement of accuracy and performance is achieved for the adaptive control systems [2]. In [5], an adaptive controller is developed for a class of second-order nonlinear dynamic systems with input nonlinearities using artificial neural networks. The main contribution of NN was used to approximate the unknown dynamic system model, and then based on this neural model, by the use of a novel Lyapunov function containing both system states and control input variable to construct the controller gain and adaptive law.

In [6], present a new variable structure control for uncertain systems with sector nonlinearities. In [4], develop a fixed-architecture controller analysis and synthesis framework that addresses the problem of multivariable linear time-invariant systems subject to plant input and plant output time-varying nonlinearities. Other researches on the topic of input nonlinearities could be also obtained from [7-10].

The paper is organized as follows. Section II describes yaw dynamic of a small-scale helicopter mounted on an experimental platform. Section III presents the adaptive controller combine with neural networks and investigates stability analysis and tracking performance of the closed-loop adaptive system. A numerical simulation is performed to show the feasibility of the proposed approach for yaw control in Section IV. A brief conclusion is given in Section V.

II. YAW DYNAMIC

A. Modeling yaw dynamics

In this paper a framework of the simulation model for the helicopter-platform (see Fig.1) is set up using rigid body equations of motion of the helicopter fuselage. In this way the effect of the aerodynamic forces and moments acting on the
helicopter are described. The total aerodynamic forces and moments acting on a helicopter can be calculated by summing up the contributions of all components on the helicopter, which include main rotor, tail rotor, fuselage, horizontal stabilizer and vertical fin. So, the yaw dynamics has the form:

\[
\begin{align*}
\dot{\phi} &= r \\
I_\phi \ddot{r} &= N_{mr} + N_{tr} + N_{ fus} + N_{hs} + N_{vf}
\end{align*}
\]  

(1)

where \( \phi \) and \( r \) are the yaw angle and angular velocity of the helicopter respectively; \( I_\phi \) is the inertia around z-axis; \( N \) presents the torque acted on the helicopter; the subscripts of \( mr, tr, fus, hs \) and \( vf \) present respectively, main rotor, tail rotor, fuselage, horizontal and vertical fin[14].

In hovering and low-velocity flight, the torque generated by main and force generated by tail rotor is dominant. By simplifying the fuselage and vertical fin damping, the yaw dynamics can be rewritten as:

\[
\begin{align*}
\dot{\phi} &= r \\
I_\phi \ddot{r} &= -Q_{mr} + T_tr \dot{r} + b_1 r + b_2 \phi
\end{align*}
\]  

(2)

where \( Q_{mr} \) is the torque of main rotor, \( T_tr \) is the thrust of tail rotor, \( l_tr \) is the distance between the tail rotor and z-axis, \( b_1 \) and \( b_2 \) are damping constants.

The brief presentation of the forces and torques computing can be obtained by using the blade element method [11-13].

\[
T_tr = \frac{1}{2} \rho a_{tr} b_{tr} c_{tr} \Omega_{tr}^2 \int_{r_0}^{R} (\theta_{tr} r_{tr}^2 - v_{tr}^2 \Omega_{tr}) r_{tr} dr_{tr}
\]  

(3)

\[
v_{tr} = \sqrt{\frac{T_tr}{2 \rho A_{tr}}}
\]  

(4)

where \( \rho \), \( a_{tr} \), \( b_{tr} \), \( c_{tr} \), \( \Omega_{tr} \), \( \theta_{tr} \), \( v_{tr} \), and \( A_{tr} \) are respectively, density of air, slope of the lift curve, number of the rotor, chord of the blade, speed of the tail rotor, pitch angle, radial distance, induced speed of the tail rotor and area of the tail rotor disc.

Combing (3) with (4), we have

\[
T_tr = C_1 \theta_{tr} \Omega_{tr} + \frac{1}{2} C_2 (C_2 + \sqrt{C_2^2 + 4 C_4 \theta_{tr}})
\]  

(5)

with

\[
C_1 = \frac{1}{6} \rho a_{tr} b_{tr} c_{tr} \Omega_{tr}^2 (R_{tr}^3 - R_{tr0}^3)
\]

\[
C_2 = \frac{1}{6} \rho a_{tr} b_{tr} c_{tr} \Omega_{tr} \sqrt{2 / \rho \pi R_{tr0}^2} (R_{tr0}^2 - R_{tr0}^3)
\]

Similarly, the force of the main rotor is[11-13]:

\[
T_{mr} = C_3 \theta_{mr} \Omega_{mr} + \frac{1}{2} C_4 (C_4 + \sqrt{C_4^2 + 4 C_5 \theta_{mr}})
\]  

(6)

where,

\[
C_3 = \frac{1}{6} \rho abc \Omega^2 (R^3 - R_0^3)
\]

\[
C_4 = \frac{1}{6} \rho abc \Omega \sqrt{2 / \rho \pi R^2} (R^2 - R_0^2)
\]

The torque generated by main rotor is [11-13]:

\[
Q_{mr} = \int_{r_0}^{R} \rho \Omega^2 r^2 (C_1 \phi + C_2) r dr
\]  

(7)

with \( \phi = v_1 / (\Omega r) \), \( C_1 = a \alpha \), \( C_4 = C_{d0} + C_{d2} \alpha^2 \)

where \( a \), \( \alpha \), \( r \), \( c \), \( \phi \), \( v_1 \) and \( \Omega \) are respectively slope of the lift curve, the angle of attack of the blade element, speed radial distance, chord of the blade, inflow angle, induced speed and rotor speed of the main rotor.

After integral manipulation with the help of Maple, we obtain

\[
Q_{mr} = \frac{1}{2} C_{mr} \rho \Omega^2 (R^3 - R_0^3) \theta_{mr}^2 + \frac{8 C_2 \Omega^2 \rho \pi R^2 (C_4 \Omega_{mr} + C_4 - C_1 \sqrt{C_1^2 + 4 C_2 \Omega_{mr}}) (R_0^3 - R^3)}{4(2 + C_1)}
\]

(8)

\[
T_{mr} = C_{mr} \Omega \rho \pi R^2 (C_4 \Omega_{mr} + C_4 - C_1 \sqrt{C_1^2 + 4 C_2 \Omega_{mr}}) (R_0^3 - R^3) + 3 a C_4 \sqrt{C_4^2 + 4 C_5 \Omega_{mr}} (R_0^3 - R^3)
\]

\[
+ 4 C_2 \Omega \rho \pi R^2 (C_4 \Omega_{mr} + C_4 - C_1 \sqrt{C_1^2 + 4 C_2 \Omega_{mr}}) (R_0^3 - R^3) + 6 C_5 \Omega_{mr} \rho \pi R^2 (R^3 - R_0^3)
\]

\[
+ 6 C_5 \rho \pi R^2 (R^3 - R_0^3) + 3 a C_4 \sqrt{C_4^2 + 4 C_5 \Omega_{mr}} (R_0^3 - R^3)
\]

(9)

where \( R \) and \( \theta_{mr} \) are respectively, radial and pitch angle of main rotor.

B. Simplified model

From (2) we can see that there exist couplings between main rotor torque \( Q_{mr} \) and tail rotor thrust \( T_tr \). And (3) and (8) further demonstrate that the models are highly nonlinear and too complex to be used for control design. Instead of the dynamics described by (3) and (8), a simplified model is proposed for control design.

![](image1.png)

Fig. 2 Torque of main rotor with Quadratic Polynomial Fitting

![](image2.png)

Fig. 3 Thrust of tail rotor with Quadratic Polynomial Fitting
By plotting the torque vs. pitch angle, we can find that relation between $Q_{mr}$ and $\theta_{mr}$ approximated with quadratic polynomial (see Fig.2)

$$Q_{mr} = k_{Q0} \theta_{mr}^2 + k_{Q1} \theta_{mr} + k_{Q2}$$

(9)

Where $k_{Q2}$, $k_{Q1}$ and $k_{Q0}$ depend on the shape of the blades and the speed of main rotor $\Omega_{mr}$, while $\Omega_{mr}$ are constant. So, $k_{Q2}$, $k_{Q1}$ and $k_{Q0}$ are constants.

Similarly, the lift of tail rotor, $T_t$ (see Fig.3), can be written:

$$T_t = k_{T2} \theta_{tr}^2 + k_{T1} \theta_{tr} + k_{T0}$$

(10)

Substituting (9) and (10) into (2), then we can obtain the following nonlinear model:

$$\phi = r$$

(11)

$$I_z r = -(k_{Q2} \theta_{mr}^2 + k_{Q1} \theta_{mr} + k_{Q0})$$

$$+(k_{T2} \theta_{tr}^2 + k_{T1} \theta_{tr} + k_{T0}) \eta + b_1 r + b_2 \phi$$

III. ADAPTIVE CONTROL DESIGN

In this section, we present the adaptive control design method for yaw control [3] which is used general for a class of uncertain chaotic system with input nonlinearity.

A. Preliminary

For (11), we can describe as the following form:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = f(X, t) + \Delta f(X, t) + \phi(u(t))$$

(12)

With the input $u \in R$ and the output $y \in R$ ,

$$X = [x_1, x_2]^T \in R^2$$

is a measurable state vector of the system, $f(X, t)$ is an unknown nonlinear nominal plant, $\Delta f(X, t)$ is the plant uncertainty applied to the nonlinear system and $\phi(u(t))$ is an unknown but bounded function with respect to the control input $u$.

Let $e = y_d - y(=y_d - x_1)$ be an error signal between the desired and actual outputs. Moreover, define $Y_d = [y_d, \dot{y}_d]^T$ and then the error vector of the system becomes $Y_d - X = [e, \dot{e}]^T = [e_1, e_2]^T$. Suppose that a gain vector $[k_0, k_1]^T$ can be suitably chosen such that the roots of $s^2 + k_1 s + k_0 = 0$ are in the open left-half complex plane.

B. Auto-tuning neuron

The basic architecture for the neuron is schematically shown in [3], where $e_1, e_2$ is referred to as an external input of the neuron, $\rho$ is an adjustable threshold, and the internal state of neuron net can be described:

$$net = \sum_{i=1}^{2} e_i - \rho$$

(13)

The output $u$ of neuron is taken as the control input applied into nonlinear system of (12) and is given by

$$u = f(net) = f(\sum_{i=1}^{2} e_i - \rho)$$

Considering the saturation of the controller, we select the activation function $f(\cdot)$ is a modified hyperbolic tangent function, i.e.

$$u = f(net) = \frac{a(1-\exp(-b \cdot net))}{(1+\exp(-b \cdot net))}$$

(14)

$a$ and $b$ are two adjustable parameters and significantly affect the output level and the curve shape of function, respectively. For simplification, $\theta = [\rho, a, b]^T$ denotes the adjustable parameters vector used in the auto-tuning neuron.

C. Adaptation mechanisms

The traditional MIT rule is a kind of adaptation mechanism and is usually used in the model-reference adaptive systems (MRAS) to update control parameters such that the plant output asymptotically track the output of a stable linear reference model. In order to make $E$ small, the tuning formula is to take the negative gradient direction for $E$ with respect to $\theta$, i.e.,

$$\dot{\theta} = -\eta \frac{\partial E}{\partial \theta}$$

(15)

Where $\eta > 0$ is the tuning rate to determine the convergence speed.

D. Stability analysis

The sliding condition is often introduced in the field of sliding mode control, and it is a condition to ensure the stability of the overall systems. Borrowing this concept of the sliding condition, a modified MIT rule will be proposed for which the sliding condition is taken as an objective function instead of the error function used in (4). Define an extended state variable $x_r$ satisfying

$$\dot{x}_r = \ddot{y}_d + k_1 \dot{e} + k_0 e$$

(16)

Where $k_0, k_1$ are constant gains as are mentioned above. In addition, a sliding surface is defined as

$$S = x_2 - x_r$$

(17)

If the state trajectories of the system are on the sliding mode, i.e., $S = 0$ , then

$$x_2 = x_r$$

(18)

Substituting (18) into (16), it becomes

$$e^2 + k_1 \dot{e} + k_0 e = 0$$

and this implies that $e(t) \to 0$ when $t \to \infty$. From the concept of the sliding mode control, the sliding condition that ensures the hitting and existence of a sliding mode is derived according to the Lyapunov stability theory. In general, the Lyapunov function candidate for the sliding mode control is simply given by

$$V = \frac{1}{2} S^2 \quad \dot{V} = SS$$

(19)
\[
\dot{\delta} = \dot{x} - \dot{x}_r
\]
\[
= f(X,t) + \Delta f(X,t) + \delta(t) + \phi(u(t)) - \dot{x}_r \tag{20}
\]

Moreover, multiplying both sides of (19) by \(S\), the value of \(S\) is obtained as follows
\[
SS = [f(X,t) + \Delta f(X,t) + \delta(t) + \phi(u(t)) - \dot{x}_r] \tag{21}
\]

Finally, this equation is taken as the objective function when the modified MIT rule is used. By using the Chain rule and the mathematical model of (13) and (14), the adjustable vector, \(\theta = [\rho, a, b]^T\), should be updated according to the following adaptation laws:

\[
\dot{\rho} = -\eta \frac{\partial SS}{\partial \rho} = -\eta \frac{\partial SS}{\partial \phi(u)} \frac{\partial \phi(u)}{\partial u} \frac{\partial u}{\partial \rho} = \eta S \frac{\partial \phi(u)}{\partial u} \frac{a b}{2} (1 + \frac{b}{a})(1 - \frac{b}{a}) \tag{22}
\]

\[
\dot{a} = -\eta \frac{\partial SS}{\partial a} = -\eta \frac{\partial SS}{\partial \phi(u)} \frac{\partial \phi(u)}{\partial u} \frac{\partial u}{\partial a} = \eta S \frac{\partial \phi(u)}{\partial u} \frac{a b}{2} (1 + \frac{b}{a})(1 - \frac{b}{a}) \tag{23}
\]

\[
\dot{b} = -\eta \frac{\partial SS}{\partial b} = -\eta \frac{\partial SS}{\partial \phi(u)} \frac{\partial \phi(u)}{\partial u} \frac{\partial u}{\partial b} = \eta S \frac{\partial \phi(u)}{\partial u} \frac{a b}{2} (1 + \frac{b}{a})(1 - \frac{b}{a}) \tag{24}
\]

The adaptive adjustable parameters can be chosen to guarantee that the tracking error is within a desired bound at steady state, the overall design procedure can be summarized as follows:

1. Set the system output track a desired output \(y_d\).
2. Design a control (14), such that the plant output follows the desired output asymptotically.
3. Tuning the convergence speed \(\eta\).
4. Choose the gain vector \([k_0, k_1]^T\) such that the roots of \(s^2 + k_1 s + k_0 = 0\) are in the left-half complex plane.
5. Choose adaptive adjustable parameters \(\theta\) initial values that are random numbers selected from the interval [1, 1].

Calculate \(\theta\) using (23), (24) and (25).

IV. SIMULATIONS

In this section, the proposed control algorithm is verified by the simulation model obtained from the helicopter-on-arm platform, as shown in Fig.4. A small-scale electrical helicopter is mounted at the end of a two-DOF arm, while the weight of the helicopter is perfectly balanced at the other side of the arm. First, the parameters of the nonlinear yaw dynamic model are identified by least square method, and the following are the result:

\[
\begin{align*}
\dot{\rho} &= \rho \\
\dot{a} &= k_1 \rho + k_2 \rho^2 + k_3 \rho^3 + k_4 \rho^4 + k_5 \Omega_{mr} \theta_r
\end{align*} \tag{25}
\]

With, \(k_1=-1.36, k_2=-2.9912, k_3=49.1359, k_4=6.7012, k_5=-0.0958\). It is obviously that \(k_1 \rho + k_2 \rho^2 + k_3 \Omega_{mr} \theta_r\) is a nonlinear function with respect to the control input \(\theta_r\). Fig.5 demonstrates the fitness of identified model of (25) with respect to the measure performance, from which we can see that the simulation model mat with system very well.

--- Identification results --- Measurement data

Fig.5 Comparison of the flight test data and computed from the identified model

Below we compare our adaptive control with auto-tuning neuron versus simply using PID control in tracking yaw angle and angle velocity. In the simulation, the PID controller and our method use the model (25). The parameters of PID controller are: \(k_p=0.01, k_i=0.000025, k_d=0.025\), which are real parameters for our experiments.

To verify the robustness of our method for the model parameter and disturbance, in the simulation, the model parameters and disturbance are designed to change according to the step-changing and continuous sine changing, i.e. (a), (b), (c), (d), (e) and (f).

(a) A step change at \(t=10\)s, i.e.,

\[
\Omega_{mr} = \begin{cases} 
\Omega_{mr0} & t < 10s \\
\Omega_{mr0} + \Delta \Omega_{mr} & t \geq 10s 
\end{cases} \tag{26}
\]
(b) A sine change at \( t=10s \), i.e.,
\[
\Omega_{mr}\left( t \right) = \begin{cases} 
\Omega_{mr,0} & t < 10s \\
\Omega_{mr,0} + A_3 \sin(\omega_3(t-10)) & t \geq 10s 
\end{cases} 
\] (27)

(c) A step change at \( t=10s \), i.e.,
\[
k_3 = \begin{cases} 
k_{30} & t < 10s \\
k_{30} + \Delta k_3 & t \geq 10s 
\end{cases} 
\] (28)

(d) A sine change at \( t=10s \), i.e.,
\[
k_3 = \begin{cases} 
k_{30} & t < 10s \\
k_{30} + A_4 \sin(\omega_4(t-10)) & t \geq 10s 
\end{cases} 
\] (29)

Where the initial value of the model parameter is \( k_{30} = 49.1359 \), the constant change value in 10s is \( \Delta k_3 = 4.91359 \), the amplitude of the sine change is \( A_4 = 4.91359 \), and the angular frequency is \( \omega_4 = 0.5 \text{rad/s} \).

For the model parameters, we can suppose the parameter \( k_3 \) and \( k_4 \) change by the following:
\[
k_3 = \begin{cases} 
k_{40} & t < 10s \\
k_{40} + \Delta k_4 & t \geq 10s 
\end{cases} 
\] (30)

Where the initial value of the model parameter is \( k_{40} = 49.1359 \), the constant change value in 10s is \( \Delta k_4 = 4.91359 \), the amplitude of the sine change is \( A_4 = 4.91359 \), and the angular frequency is \( \omega_4 = 0.5 \text{rad/s} \).
A sine change at $t=10s$, i.e.,

$$k_d = \begin{cases} k_{d0} & t < 10s \\ k_{d0} + A_d \sin(\omega_d (t-10)) & t \geq 10s \end{cases}$$

(31)

Fig.11 Tracking yaw angle and angle velocity

Where the initial value of the model parameter is $k_{d0}=6.7012$, the constant change value in 10s is $A_d=0.67012$, the amplitude of the sine change is $A_d=4.91359$, and the angular frequency is $\omega_d=0.5\text{rad/s}$.

Simulated results are demonstrated in Figs. 6-7. In these simulations, it can be easily seen that using our adaptive controller the yaw angle $\psi$ and angle velocity $r$ can also track the desired value rapidly, even if the disturbance change according to the step-changing and continuous sine changing in 10 seconds. But, the results show that the PID controller can’t track the desired value. Fig.7 shows the angle velocity controlled by PID controller is continuous oscillation when the speed of main rotor $\Omega_{m}$ changes according to continuous sine changing.

Figs 8-11 show the tracking performance of the proposed adaptive controller and PID controller, and it is obviously that the tracking performance of the adaptive controller is better than PID controller whether the model parameter changes or does not. The difference between the adaptive control and PID control is clearly seen in Fig.8 and Fig.10. We can see that the adaptive control reacts quickly to the model parameter changes both step changing and continuous sine, but there are steady-state error and large overshot in the PID control. It is shown in Fig.9 and Fig.11 that the output of PID controller lasts out equi-amplitude oscillation when the parameter changes according to continuous sine changing. Though the angle velocity of adaptive controller also experiences equi-amplitude oscillation, the tracking error is very small and satisfies the control performance.

Summarizing these simulations, an adaptive controller design method has been used for yaw control with input nonlinearity. Unlike the PID controller, the adaptive controller can improve the control performance with input nonlinearities even if in the presence of disturbance and the changing of the model parameters.

V. CONCLUSION

In this paper, we used a new robust adaptive controller design approach for the yaw control of a small-scale helicopter mounted on an experimental platform. Unlike the PID controller, the adaptive controller can improve the system performance with input nonlinearities even if in the presence of disturbance and the changing of the model parameters. The main feature of the adaptive controller is combining neural networks with traditional adaptive controls and the utilization of a modified MIT rule that the sliding condition is taken as an objective function instead of the error function. The adaptive controller can guarantee both system stability and control performance.

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