Why You Cannot (Yet) Write an “Interval Arithmetic”
Library in Common Lisp

... or: Hammering Some Sense into :ieee-floating-point

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Abstract

“Interval Arithmetic” (IA) appears to be a useful numerical tool to have at hand in several applications. Alas, the current IA descriptions and proposed standards are always formulated in terms of the IEEE-754 standard, and the status of IEEE-754 compliance of most Common Lisp implementations is not up to par.

A solution would be for Common Lisp implementations to adhere to the Language Independent Arithmetic (LIA) IEC standard, which includes IEEE 754.

While the LIA standard provides a set of proposed bindings for Common Lisp, the format and depth of the specification documents is not readily usable by a Common Lisp programmer, should an implementation decide to comply with the provisions. Moreover, much latitude is left to each implementation to provide the LIA “environmental” setup.

It would be most beneficial if more precision were agreed upon by the Common Lisp community about how to provide LIA compliance in the implementations. In that case, a new set of documentation or manuals in the style of the HyperSpec could be provided, for the benefit of the Common Lisp programmer.

The goal of this paper is to foster a discussion within the Common Lisp community to converge on a complete specification for LIA compliance. The paper discusses some of the issues that must be resolved to reach that goal, e.g., error handling and full specification of mathematical functions behavior.

1 Introduction

An interesting exercise (academic or not) that a programmer (Common Lisp or not) may find intriguing is to implement an Interval Arithmetic (IA) library\(^1\). Programmers of all stripes would learn a lot if they tried to really implement an IA library. But since most probably won’t, this paper may serve as sufficient summary to get you through a cocktail party conversation on the matter.

The usefulness of IA is rather established; many numerical issues can be naturally dealt with by using an IA library, albeit at a slight increase in computation times. There is a nice body of literature and proposed standards to ensure availability of IA in a computing environment, and many of these eventually provide one or two different interval representations (endpoint and midpoint), the operations on them, and nitpicking treatment of corner cases; e.g., intervals with infinite endpoints and interval division by an interval containing 0.

As we shall see, the “nitpicking” boils down to using the IEEE-754/IEC-60559 standards \([11]\). Eventually, in this work, the use of the IEC Language Independent Arithmetic (LIA) standards \([6, 7, 8]\) will be advocated. The LIA standard is a comprehensive collection of concepts and carefully thought out behaviors a basic library of integer, floating point and complex numbers mathematical functions and ancillary environment functionalities should look like. One of the, in the opinion of the writer, unstated goals of LIA is that a programmer should be able to relatively easily understand mathematical software written in any language ecosystem that abided the specification.

\(^1\)See, for example \([4]\), or \([5]\) for a rather complete summary with references to the seminal works in the area. IEEE has also published a preliminary standard for IA \([10]\).
A word of caution. The present paper is neither a full blown Common Lisp LIA specification, nor a description of an implementation of the functionalities depicted herein. It rather is a leaflet that intend to present the community a project which, in the modest opinion of the writer, should be completed after careful debate and careful consideration of all the details.

1.1 An IA Library... Hitting the Wall

An IA library in Common Lisp implementing what is known as an endpoint representation can be easily started as follows. For brevity, since it is a valid Common Lisp identifier, we use the name [] here for what other languages might call an interval.

(defstruct ([] (:constructor [] (low high)))
  (low 0.0 :type real)
  (high 0.0 :type real))

(defun radius (i) (- ([]-high i) ([]-low i)))

(defun pointp (i) (= ([]-high i) ([]-low i)))

(defun method add ((i1 []) (i2 [])
  ([] (+ ([]-low i1) ([]-low i2))
    (+ ([]-high i1) ([]-high i2))))

(defun method sub ((i1 []) (i2 [])
  ([] (- ([]-low i1) ([]-high i2))
    (- ([]-high i1) ([]-low i2))))

After starting in earnest, a Common Lisp (or JAVA, C, R, PYTHON) programmer is soon faced with a number of numerical issues, should she be willing to achieve the best possible behavior out of the IA library.

The problem is that, as mentioned before, IA specifications are usually formulated in terms of the IEEE-754 standard, which, at this point is readily available only to C and C++ programmers. In particular, the IA specifications exploit rounding modes and infinities, which are unevenly available in Common Lisp implementations; another, related issue is the treatment of floating point exceptions.

Rounding Modes. If we had available some ways of handling infinities and rounding modes, we could write the IA library operations as follows:

(defun method add ((i1 []) (i2 [])
  ([] (rounding-down (+ ...))
    (rounding-up (+ ...)))))

where rounding-down and rounding-up are macros with an intuitive semantics. Unfortunately, at this point in time, it is not possible to provide the rounding-down and rounding-up macros without delving deeply in an implementation.

Infinities and NaNs. Another issue is the handling of special values: essentially infinities and NaNs (not-a-number). Both items are handled unevenly in Common Lisp implementations; infinities and quiet NaNs (cfr. the IEEE-754 standard) are somewhat supported; signaling NaNs not so much so.

Handling of Floating Point Exceptions. Apart from our doomed IA library, another issue that is not always very well clarified in Common Lisp implementations (and especially across them) is how floating points exceptions are handled. The Common Lisp standard defines the conditions:

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2There are, e.g., PYTHON bindings to IEEE-754, but they rely on the underlying C library implementation.
Cfr., https://www.python.org/dev/peps/pep-0754/.
floating-point-overflow
floating-point-underflow
floating-point-inexact
floating-point-invalid-operation
division-by-zero

Alas, their use is inconsistent across implementations (apart from the mostly clear cut case of division-by-zero). Two implementations may choose to signal floating-point-invalid-operation or floating-point-inexact on the same operation.

This is not the only issue vis-a-vis Common Lisp and the IEEE-754. A deeper issue pertains the notification machinery that is invoked when one of the aforementioned conditions is to be signaled by an operation. Should an implementation actually signal a (floating point) condition using error, or should it go the C way and record somewhere an indication that a condition “happened”, for the programmer to check directly?

1.2 Common Lisp Implementations and the :ieee-floating-point Feature

The ANSI Common Lisp Standard makes provisions for a Common Lisp implementation to “declare” that it purports to conform to the requirements of the IEEE Standard for Binary Floating-Point Arithmetic (no reference given). There are a few problems with this statement.

The presence of the :ieee-floating-point feature in a Common Lisp implementation is a (very) partial indication that some support for the IEEE-754 is available. Table shows a summary of the current state of compliance for a number of implementations with respect to the notions just described.

|                     | CMUCL/SBCL | LW | ACL | ABCL | ECL | CCL |
|---------------------|------------|----|-----|------|-----|-----|
| Infinities          | Y          | Y  | Y   | U    | U   | Y   |
| Quiet NaNs          | Y          | Y  | Y   | U    | U   | Y   |
| Signaling NaNs      | Y          | N  | N   | U    | U   | U   |
| Rounding            | Y          | N  | N   | U    | U   | N   |
| Exceptions NACF     | P          | P  | P   | P    | P   | P   |
| Exceptions NRI      | Y          | N  | N   | U    | U   | N   |

Table 1: Common Lisp implementations “compliance” status w.r.t. the IEEE-754. The acronym NACF stands for Notification by Alteration of Control Flow, while the acronym NRI stands for Notification by Recording of Indicators (cfr., [6, 7, 8]); they will be discussed later on. The entries are Yes, No, Unknown, and Partial.

Infinities and NaNs. Many implementations provide infinities and NaNs, but with obviously different lineages. E.g., the following syntax is used by LW and CCL, which is then declined in various interesting ways:

∞  1F++0, 1D++0
NaN  1F++0, 1S--0

but ACL chooses to provide “variables” with read-time syntax.

ACL prompt> *infinity-single*
#.*INFINITY-SINGLE*

ACL prompt> (+ 42 *nan-single*)
#. *NAN-SINGLE*

While this may be perfectly sensible it has the drawback of not playing so nicely with *read-eval*.

The only implementations that allow a programmer to get a handle on a signaling NaN are CMUCL and SBCL. There appear to be no easy way to create such a value in the other implementations.

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[3] A form of “left to the implementation”, which, as usual, does not bode well for the programmer.

[4] The table is incomplete because not all implementations were checked and because the notion of “compliance” is rather complicated to assess in this case.
Rounding. Only CMUCL and SBCL allow the setting of the rounding mode by accessing directly the equivalent of the IEEE-754 floating point environment. However, the facility – which resembles the C library `fenv.h` setup – is very low level.

1.3 A Proposal
Alas, “just adding” infinities and rounding modes to a Common Lisp implementation may not be quite sufficient, as their semantics is deeply intertwined with the other parts of the IEEE-754 standard. A better course of action would be to nudge the implementors to comply with the current standards. The definition of a new Common Lisp “arithmetic” specification may be a better way to achieve the goal of providing Common Lisp programmers with a layered set of documented functionalities.

The observation is that by now, the IEEE-754 standards (and to a lesser extent the LIA standards) appear to be quite accepted and common place in many programming language ecosystems. It is the opinion of the writer that for the Common Lisp community, “to go with the flow” would be the pragmatic thing to do.

2 Goals and Issues
The goal of this paper is to urge the various Common Lisp implementations to provide better support for floating point (hence complex) arithmetic, in order to make it possible to directly write a IA library (and other numerical routines) in an easier and more correct way. The point of view is that of a Common Lisp programmer and user. The main source of this proposal are the Language Independent Arithmetic (LIA) specifications [6, 7, 8], which incorporate IEEE-754/IEC-60559 [11].

2.1 The LIA Specifications and Common Lisp
The LIA specifications are three documents covering the following topics.

LIA Part 1: integer and floating point arithmetic

LIA Part 2: elementary numerical functions

LIA Part 3: complex integer and floating point arithmetic

The LIA specifications take great care not to be overly constraining (they relax a few requirements of IEEE-754/IEEE-754) while being very precise about the behavior of each item they define. They also contain appendices describing suggested bindings for various languages, C/C++ and FORTRAN being prominent, including Common Lisp.

A Common Lisp implementor could, in principle, just read through the LIA specifications and provide all the necessary bits and pieces while building the arithmetic facilities of the language. Yet, it is the writer's opinion that this course of action would still fall a bit short of providing a programmers’ computing environment with an “implementation independent” firm ground. The reasons lie in the LIA specifications themselves, as they understandably cannot provide more than a suggestion about how a language binding should cover and look like. There are some issues that a more Common Lisp centric specification would and should clarify: naming conventions, layering and packaging, programming environment setup, rounding-modes and, above all, error handling and the floating point environment. In the following each of these issues will be discussed. Eventually, the result should be an in-depth specification formatted in the style of the Common Lisp standard [1, 2].

2.1.1 Naming Conventions
The LIA specifications suggest a naming convention for its functionalities that reuses much of Common Lisp names. Some of the choices are not particularly in line with Common Lisp style. Two examples are the functions `sqrtUp` and `sqrtDwn`, which compute square roots with “up” or “down” rounding modes; Common Lisp style would have avoided the “camel case”, given that Common Lisp implementations are uppercasing out-of-the box, while preferring an hyphenated naming.

Another issue with the LIA suggested naming is that it essentially requires an implementation to provide a set of very basic LIA-compliant functions – e.g., +, *, 1-, sin, etc. – which implies a reworking of an implementation core.
2.1.2 Layering and Packaging

In order to provide a Common Lisp centric LIA specification and adoption path, it would be better to ensure that the new functionality were provided as a library. This means, at a minimum, to provide a package that contained all the “new” names introduced. Given the partition of the LIA specifications, further sub-packaging could be provided.

A first cut proposal would be to have a package named (or nicknamed) CL-LIA that exported all the names that are necessary to implement a form of the LIA specifications. As it will be discussed later on, it will be useful to have a cl-lia:floating-point-invalid-operation condition, despite the presence of the standard Common Lisp one.

2.1.3 Programming Environment Setup

The IEEE-754/IEC-60559 and LIA specifications define a number of environment checks that a compiler or a program may check to produce code that complies and/or exploits their semantics. These are akin to the :ieee-floating-point feature (which, as we have seen, is only partially informative). At least two sets of “checks”, both in functional and “feature” form could be provided by a Common Lisp LIA implementation.

Library Compliance Checks. The LIA specifications require a boolean variable iec60559_\$ that reports whether or not an implementation complies with the IEEE-754/IEC-60559 implementation of the floating point type F. This is more stringent than the “bulk” statement implied by the :ieee-floating-point feature, although LIA1 (cfr, \[6\] Section 5.2) explicitly states that no exact floating point representation is required. A Common Lisp LIA implementation should define similar constants, boolean functions and, possibly, features.

Layered Library Checks. Another set of variables, boolean functions and features should be made available to indicate the level of compliance with the LIA specifications. A suggestion is to provide the functions (and therefore constants and features) LIA1-compliance, LIA2-compliance, and LIA3-compliance. Finer statements may pertain parts of each specification; one example is the provides-infinities-p and provides-nans-p. Other such checks can be described for other parts of the Common Lisp LIA implementation, as seen below for exception handling. Finally, one important check that could be provided is whether the Common Lisp implementation carries over the LIA semantics to the functions in the COMMON-LISP package: the check is-cl-using-lia would state that a function like, for example, cl:sin implements the LIA semantics w.r.t. infinities and NaNs, rounding modes and exception notifications (see below). Of course, whether or not provide such “history rewriting feature” is up for debate. One argument in favor is that old code would still work without having to be tweaked to use the new functions provided in a CL-LIA package.

2.1.4 Rounding Modes

Floating points numbers being approximation of real numbers carry with them notions of rounding. The LIA specifications define how rounding modes affect elementary and library operations.

Following in this case the C/C++ example, it could be possible to define a set of constants with the meaning showed in Table 2. The type rounding-mode can then be defined as:

```lisp
(deftype rounding-mode ()
  '(member ,indeterminate
             ,to-zero
             ,to-positive-infinity
             ,to-negative-infinity
             ,to-nearest-even))
```

The rounding mode can then be tracked using a special variable *rounding-mode*. A macro with-rounding-mode is an obvious extension as well as macros wrapping one expression: round-upward, round-downward, round-nearest etc.

Moreover, the LIA specifications define some functions that guarantee a given rounding result; e.g., there are three versions of \(\sqrt{\cdot}\), which compute square roots guaranteeing rounding upward, downward and to nearest. For Common Lisp it will probably be better to provide four such “names” with the following, LIA-inspired, suffixes: sqrt (no suffix), sqrt.<, sqrt.<>, and sqrt.>. The first version depends on the current rounding mode, the other ones round down, near and up (suffixes .<, .<> and .>).
| Common Lisp constants | Value | LIA meaning |
|-----------------------|-------|-------------|
| indeterminate         | -1    |             |
| to-zero               | 0     | truncate    |
| to-nearest            | 1     | nearest     |
| to-positive-infinity  | 2     | other       |
| to-negative-infinity  | 3     | other       |
| to-nearest-even       | 4     | nearesttoeven|

Table 2: Proposed Common Lisp constants representing rounding modes.

2.1.5 Error Notification/Handling and the Floating Point “Environment”

The LIA specifications must address the differences (and . . . “traditions”) that different communities have developed over the years. The exegesis of the LIA specifications seems to point out that the major concern was to disentangle concerns that earlier language specifications (especially the C/C++ ones) addressed in an idiosyncratic way. Within the Common Lisp community, the Condition System - as prefigured and hashed out in [9] – offers all the bells and whistles to implement the programmer’s side of error handling, but some issues must be dealt with at the implementation level.

One of the main tangled issues regards the handling of “errors”, that is, after what an “error” is agreed upon. This is a notoriously complicated issue which the LIA specifications appear to break down into two parts.

- How errors are notified.
- What happens depending on the notification style.

The LIA specifications assume that a language “environment” establishes some forms of notification machinery. Three major modalities are singled out.

1. Notification by recording in indicators (NRI – LIA1, Section 6.2.1).
2. Notification by alteration of control flow (NACF – LIA1, Section 6.2.2).
3. Notification by termination with message (NTM – LIA1, Section 6.2.3).

The LIA1 specification, Annex D, proposes that Common Lisp defined the arithmetic exception handling using the Condition System, i.e., using the NACF notification approach. However, SBCL/CMUCL provide – de facto – a NRI setup modeled on the C NRI interface provided by `<fenv.h>`. It would be better to accommodate both alternatives NRI and NACF, and make them available to the programmer for finer control.

LIA1 provides an example about how FORTRAN may provide some compiler directives to choose between NRI and NTM (cfr., LIA1, Annex E).

```
!LIA$ NOTIFICATION=RECORDING
!LIA$ NOTIFICATION=TERMINATE
```

In order to select and introspect what kind of exception handling regime is in place in a given computation an appropriate Common Lisp API will have to be defined.

To complete the discussion, we must consider the floating point environment, conditions and continuation values, and underflow/overflow.

Floating Point Environment. Having control over the kind of notification style is nice, but it requires a better handling of the floating point environment, which C handles through `<fenv.h>`, and that CMUCL/SBCL manipulate using a few functions and what looks like is an a-list.

The floating point environment is used as a kitchen sink to keep track of rounding modes, exceptional situation notifications and other information. A unified item representing these concerns still seems the best way to give access to them in a dynamic way.

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5The LIA specifications make no mention of any threading model; however, it is assumed that an implementation can make all the dynamical behavior of numerical computations “thread safe”.

Conditions and “Continuation” Values. The operations that operate on the “borderline” case in LIA (e.g., operations on NaNs, or that generate underflows and overflows, are specified alongside a continuation value. This is most important for the NRI notification style, where an operation “continues”, while recording the indication of an exceptional situation. To facilitate the implementation of a LIA-compliant package for Common Lisp, it would be useful to mirror the arithmetic-error sub-hierarchy and to equip the classes with a continuation-value slot (alongside appropriate initargs and readers).

3 Considerations for a New Common Lisp Arithmetic Specification

Having discussed some of the issues about providing support for the LIA specification in Common Lisp, we here offer a detailed opening bid in a hoped-for public discussion on the creation of a Common Lisp binding, or otherwise integrating its ideas into the language.

A full-blown document containing the full description of each item, in a style reminiscent of the Common Lisp ANSI Standard \[1\], is in the works. The LIA specification describes each item and, especially, operation, in a terse and abstract way, which requires quite a bit of effort to map onto a typical Common Lisp description, especially for functions results. The full blown description is intended to be read by a Common Lisp programmer.

Package. All the names that will be introduced or shadowed from the "COMMON-LISP" package will be exported from a new package. The proposed nickname is "CL-LIA-MATH"\[6\]

Environmental Features and Switches. An implementation will state what parts of the specification will be available. The following (semi-hierarchical) set of environmental queries will be available as functions, special variables, and/or features.

- lia-subset-available
- lia1-subset-available
- lia2-subset-available
- lia3-subset-available
- lia-compliance
  - lia1-compliance
    - provides-infinities
    - provides-nans
    - provides-rounding-modes
    - provides-floating-point-environment
    - provides-nacf
    - provides-nri
    - provides-ntm
- lia2-compliance
- cl-package-uses-lia
- lia3-compliance

The above list represents Boolean functions. The provides-... functions return true when the specific functionality is fully provided. In the above example lia1-compliance returns true when all the provides-... functions return true.

Note that, while the list of introspective facilities listed covers most of the dimensions in LIA compliance, certain combinations are ruled out by the detailed specification and by the way it is presented in the standards. E.g., it is not very sensible to have an implementation for which provides-floating-point-environment returned false, while provides-nacf returned true.

Infinites, NaNs and Rounding. An implementation of the specification will offer all the values pertaining infinites, NaNs and rounding. In particular, a fully LIA compliant will provide the environment introspection functions and variable mandated by LIA1 and “typed” constants like double-float-positive-infinity and infD. Moreover, the full specification will clarify the behavior of each function and operation when presented with NaNs, both quiet and signaling, especially regarding the interplay with the error notification style (see below).

\[6\] Or "CL.MATH", should a more ambitious naming be adopted.
The rounding versions of all the LIA mandated operations will be marked by the postfixes .<, .<> and .>, signifying rounding towards negative infinity, nearest, and positive infinity. The macro `rounding` has effect on the “unqualified” versions of the arithmetic operations. E.g.

\[(+.< \pi \pi) \Rightarrow 2\pi \text{ rounded toward } -\infty.\]
\[(+.< \pi \pi) \Rightarrow 2\pi \text{ rounded toward } +\infty.\]

..., while using the `rounding` macro

\[(\text{rounding :positive-infinity } (+ \pi \pi))\]
\[\Rightarrow 2\pi \text{ rounded toward } +\infty.\]

..., but

\[(\text{rounding :positive-infinity } (+.< \pi \pi))\]
\[\Rightarrow 2\pi \text{ rounded toward } -\infty.\]

I.e., the `rounding` macro establishes a dynamic environment with a specific rounding mode set up, which can be ignored by the “hard rounding” versions of the operations in the body.

Floating Point Environment and Error Handling. An implementation of the specification shall assume the presence of an opaque data type called `floating-point-environment`. The access functions for this data structure are patterned after the C/C++ standards in order to offer familiarity and, possibly, ease of implementation.

An implementation of the specification will always offer both NACF and NRI notification styles, with full control offered to the programmer about when and where they turn on and off each style. The NTM notification style will used only for catastrophic events, which will be documented accordingly.

The type `arithmetic-notification-style` can be defined as:

\[(deftype arithmetic-notification-style ()
\quad '(member :recording ; I.e., NRI.
\quad :error ; I.e., NACF.
\quad :terminating ; I.e., NTM.
\quad ))\]

The main functions and macros that allow full control of the notification style and `continuation values` possibly returned by an operation are the following.

- `current-notification-style`
- `set-notification-style`
- `with-notification-style`
- `trap-math`

The `trap-math` macro is intended as a wrapper around the Common Lisp error handling machinery (`handler-case`, `unwind-protect`, etc...) that automated some of the setup and teardown operations on the floating point environment, alongside the handling of continuation values. A possible syntax is the following

\[(\text{trap-math (<options>)}
\quad <\text{expr}>
\quad <\text{handler}>* )\]

The `options` parameter is a list that may contain the keywords :notify-by, :before, and :after. The :notify-by is the notification style defaulting to :error, :before and :after instead demarcate lists of actions that intend to simplify the setting up and the teardown of indicators in the floating point environment: :save saves the current floating point environment, :clear creates a fresh floating point environment with no indicators recorded, and :merge (in an :after position) merges the current floating point environment with the possibly saved recorded. Of course, a different syntax is possible, and it is unclear to the writer which would be the best; consider the following alternative.

\[(\text{trap-math (&key notify-by before after)}
\quad <\text{expr}>
\quad <\text{handler}>* )\]
The handler is a simplified list that has the following syntax.

\[
\langle\text{aec}\rangle \ (\&\text{optional} \ \langle\text{varname}\rangle) \\
\&\text{rest} \ \langle\text{actions}\rangle)
\]

where aec is an arithmetic-condition carrying a continuation value, varname is a symbol that can be bound to the condition instance and actions is a list that may contain the following items.

- :default – the behavior is the standard one for aec.
- :clear – when combined with the :continue forms and the complex :error form, it clears the indicator corresponding to aec from the floating point environment.
- :raise – re-signals the aec
- (:raise c &rest args) – signals a new condition c.
- :continue, (:continue expr) – continues the computation by yielding the standard continuation value of the result of evaluating expr; the :continue actions can be rendered by means of cl:use-value and/or cl:continue restarts.

An example of the use of trap-math is the following, which is also a rendering of Appendix A.6.

\[
\text{(trap-math (}:before :save :clear} \\
\text{:after :merge)} \\
\text{(fast-solution input)} \\
\text{(cl:floating-point-overflow ())} \\
\text{:clear} \\
\text{:continue (reliable-solution input))})
\]

or, with a different syntax

\[
\text{(trap-math (}:before (}:save :clear} \\
\text{:after :merge)} \\
\text{(fast-solution input)} \\
\text{(cl:floating-point-overflow ())} \\
\text{:clear} \\
\text{:continue (reliable-solution input))})
\]

3.1 Providing the Specification – A Descriptive Example

Eventually, the considerations put forth in this paper should be crystallized in a specification that clarified all the many thorny issues that will crop up when considering as many details as possible. The goal will be to provide a specification á la Common Lisp HyperSpec [2], where each item (function, variable, class, etc.) has a mostly self-contained description with, by now, a conventional structure. This is different from the presentation style adopted by the LIA specifications, which heavily rely on quite formal yet “generic” description of each operation’s behavior.

The most important aspects a Common Lisp LIA specification will be to describe, for each function (or other item), the following behaviors.

(a) The corner cases: infinities and NaNs.

(b) The interplay between the notification style, the handling of errors, the floating point environment and the continuation values that are specified according to the LIA documents.

As an example of what an entry in the envisioned full Common Lisp LIA specification would look like, Figure 1 shows a description of the (dyadic) = and /= functions. Hopefully, all details and corner cases listed above have been taken into consideration. The reader can compare the equality specification in the LIA1 document with the one in Figure 1.
Function =, /=

Syntax:

= a, b ⇒ boolean
= a &rest bs ⇒ boolean
/= a, b ⇒ boolean
/= a &rest bs ⇒ boolean

Arguments and Values:

a b – Numbers.
bs – A list of numbers.
boolean – a generalized boolean.

Description:

The dyadic version of = (and /=) performs an arithmetic equality (inequality) test between a and b. The monadic and n-adic versions are built upon the dyadic one as per the regular Common Lisp description in [2].

It is assumed that a and b are converted (as per the contagion rules of Common Lisp) to be of the same type. Therefore the following cases can be be considered as per the LIA specifications.

(a) If a and b are either finite integers, finite floating point numbers, or finite complex numbers then the result is true (respectively, false) if the two numbers are equal (respectively, different) in the mathematical sense. In the LIA spec this is the result of eqT(a,b) ≡ a = b or neqT(a,b) ≡ a ≠ b for an appropriate T. This is the standard Common Lisp case.

(b) If a and b are infinities then = returns true (respectively false) if they are both positive or both negative; otherwise it returns false (respectively true).

(c) If either a or b is a quiet NaN, and, respectively, b and a is not a signaling NaN, then the result is false.

(d) Complex numbers are checked recursively on the real and imaginary parts.

Exceptional Situations:

If either a or b is a signaling NaN, then, under the notification NACF regime, the indicator :invalid is recorded and the floating-point-invalid-operation is signaled (with continuation value NIL recorded); otherwise, under the NRI notification regime, the indicator invalid is recorded and NIL (false) is returned as continuation value.

For complex numbers, the recording and signaling operations (under NRI and NACF) happens if the condition above applied to either of the real or the imaginary parts of a and b.

Figure 1: An example entry that should appear in the full specification for the Common Lisp LIA-compliance documentation.
4 Conclusions and Final Disclaimers

In order to write a fully functional (according to the available literature and proposed standards) IA library in Common Lisp, a programmer needs a finer control over the floating point environment and access to functionalities such as rounding modes.

This paper puts forth a proposal to complete a Common Lisp HyperSpec styled, LIA-based specification that would provide a more accessible documentation for a programmer and a clear guideline about how certain functionalities should be provided by an implementation.

Given an implementation of the proposed “New Arithmetic Specification” a programmer could at least start to write a proper IA library. As an example, the add method would look like the following.

```
(defmethod add ((i1 []) (i2 []))
  ([] (+.< ([]-low i1) ([]-low i2))
       (+.> ([]-high i1) ([]-high i2)))
```

where +.< and +.> are the addition operations on floating point numbers that round, respectively, downward and upward.

Again, the writer wants to insist and repeat that the present paper is a leaflet that intends to present the Common Lisp community a project which, in his modest opinion, should be completed after careful debate and careful consideration of all the details within the Common Lisp community.

A full blown specification covering LIA-1, LIA-2 and LIA-3 will run close to two hundred pages if written and formatted according to the [2] style (a worthy goal in itself). Many of the examples contained in this paper are suggestions about how they could look. Agreement within the Common Lisp community will help settle down several issues this paper puts forth.

What this paper wants to point out though, is that many researchers and practitioners did lay down a sensible set of standards, the LIA standards, which did take into account Common Lisp. Following them appears to be one good way to ensure that Common Lisp will keep its place among the most important language ecosystems around, and welcome programmers from other communities by offering them a familiar playpen and more.

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