Beamforming against main lobe interference based on radial basis function neural network

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Abstract. Aiming at the problem that the performance of traditional beamforming algorithm deteriorates sharply in the presence of main lobe interference, a beamforming algorithm based on radial basis function (RBF) neural network is proposed. Firstly, the minimum variance distortionless response (MVDR) is used to solve the optimal beam pattern in the presence of side lobe interference. Then, the training set of RBF neural network is constructed according to the optimal beam pattern and the direction information of main lobe interference to train the network, so that the trained RBF neural network can suppress the main lobe interference while maintaining the ability of optimal beamforming. The simulation results show that the method can overcome the limitations of traditional beamforming algorithm, suppress the main lobe interference and side lobe interference, and form the correct beam direction. At the same time, the algorithm also has good real-time performance.

1. Introduction

As a part of array signal processing, beamforming technology is widely used in radar, communication, remote sensing detection and other fields [1]. MVDR uses the maximum signal to interference noise ratio (SINR) criterion to solve the theoretical optimal complex weight vector of array antenna, so the beamforming based on this method is the theoretical optimal beamforming [2]. However, when there is main lobe interference in the signal, the performance of this method will deteriorate sharply due to the Rayleigh limit.

In this paper, RBF neural network is used to process the array signal instead of traditional beamformer, so that it can overcome the Rayleigh limit and perform the optimal beamforming while suppressing the main lobe interference.

2. Principle of MVDR beamforming

The principle of beamforming is to change the amplitude and phase of the received signal by applying complex weights to the array antenna, so that the received signal on each array element interfere with each other, forming the maximum gain in the direction of the target signal, forming attenuation in the direction of the interference signal, and suppressing the noise and clutter [3]. MVDR beamforming algorithm obtains the optimal weight of array antenna according to the maximum SINR criterion to achieve the purpose of optimal beamforming [4-5].

Assuming that the received signal of the N-element uniform linear array antenna is the far plant narrowband signal, the output of the beamformer can be expressed as:

$$y(k) = w^H x(k)$$  \[(1)\]
Where, \( x(k) \) is the received signal of the array antenna, \( w \) is the complex weight vector applied to each element, and \((\bullet)^H\) is Hermitian transpose. The received signal of array antenna generally contains target signal, interference signal and noise. Therefore, the received signal can be expressed as:

\[
x(k) = x_s(k) + x_i(k) + n(k)
\]

(2)

Where, \( x_s(k) \) represents the received target signal, \( x_i(k) \) represents the received interference signal, and \( n(k) \) represents the noise in the received signal. Among them,

\[
x_s(k) = \sum_{j=2}^{L} a_j a_i^H
\]

(3)

Where, \( s_j(k) \) and \( i_j(k) \) respectively represent the target signal and interference signal received on the reference array element, \( a_j(j = 1, 2, \ldots, L) \) is the steering vector corresponding to the target signal and interference signal, and \( L \) represents the number of signals.

The target signal, interference signal and noise are statistically independent of each other, so the covariance matrix of the received signal can be obtained.

\[
R = R_s + R_i + R_n = \sigma^2_s a_i a_i^H + \sum_{j=2}^{L} \sigma^2_i a_j a_j^H + \sigma^2 I_N
\]

(4)

Where, \( R_s, R_i, R_n \) represents the covariance matrix of target signal, jamming signal and noise respectively, \( \sigma^2_s(j = 1, 2, \ldots, L) \) represents the power of target signal and jamming signal, \( \sigma^2 \) represents the power of noise, and \( I_N \) represents the N-order identity matrix. SINR can be expressed as:

\[
SINR = \frac{w^H R_s w (w^H (R_i + R_n) w)^{-1}}{\sigma^2_i \|w^H a_i\|^2} = \sigma^2_s \|w^H a_i\|^2 + \sigma^2 \|w^H (R_i + R_n) w\|^{-1}
\]

(5)

The maximum SINR criterion is to maximize the ratio of the power of the target signal to the power of the interference and noise after complex weighted processing, so it can be expressed as:

\[
\min_w w^H (R_i + R_n) w \quad \text{s.t.} \quad w^H a_i = 1
\]

(6)

By solving formula (6), the weight vector of the optimal beamformer is obtained theoretically:

\[
w_{opt} = R_{s, in}^{-1} a_j / (a_j^H R_{s, in} a_j) \quad R_{s, in} = R_s + R_n
\]

(7)

Beamforming with \( w_{opt} \) can form the correct direction in the target direction, suppress in the interference direction, and minimize the side lobe to suppress the influence of noise. The beamforming method introduced above is the basic principle of MVDR.

3. RBF neural network beamforming

The text of your paper should be formatted as follows: RBF neural network is a forward neural network with three layers based on radial basis function. The training process of the network is mainly divided into two stages. The first stage is from the input layer to the hidden layer. The idea of clustering is used to classify the samples, which belongs to nonlinear learning. The second stage is to apply linear weights between the hidden layer and the output layer to convolute the signal, which belongs to linear operation [6]. The RBF network signal processing process includes both thread nonlinear processing and linear processing, so the network has good classification ability, fast operation speed and can approximate continuous function with any precision [7]. The structure of RBF neural network as shown in the figure 1.
The beam forming method proposed in this paper mainly uses the advantages of RBF neural network. The neural network is trained to approximate the optimal beam pattern obtained by MVDR beamforming algorithm, so that it has the ability of optimal beamforming. However, considering the sharp decline of MVDR beamforming performance in the presence of main lobe interference, some adjustments have been made in the construction of training set, so that the algorithm can still perform optimal beamforming and suppress main lobe interference in the presence of main lobe interference. The specific construction process of the training set will be described in detail below.

3.1. Constructing training set of neural network

Suppose that the array antenna is a 16 element uniform linear array, the direction of the target signal is 0°, the direction of the main lobe interference signal is 4° and the direction of the side lobe interference signal is 40°.

The MVDR beamformer can not perform optimal beamforming normally in the presence of main lobe interference. Therefore, firstly, the influence of main lobe interference is ignored. According to the direction information of target signal and side lobe interference signal, MVDR beamformer is used to perform beamforming operation, and the optimal beam pattern is obtained, as shown in Figure 2. Because the influence of main lobe interference is ignored, the main lobe is formed in the target direction and the attenuation is formed in the side lobe interference direction in Figure 2. In order to suppress the main lobe interference, a zero point should be formed in the direction of 4°. Therefore, figure 2 is modified, and the result is shown in Figure 3.
The modified beam pattern forms null in the direction of $[3^\circ, 5^\circ]$ to suppress the main lobe interference. It can be seen that the null range is wider than that of the main lobe interference signal, which is to enhance the robustness of beamforming, so the null range is broadened without affecting the target signal.

The training set is constructed to train the RBF neural network, and its response is shown in Figure 3. The form of input signal of array antenna is shown in formula (2) and (3). The steering vectors corresponding to signals in different directions are different, and the desired responses in different directions are also different, as shown in Fig. 3. Therefore, the training set of RBF neural network is constructed according to the steering vectors and desired output.

It can be seen from Figure 3 that the angle range of the response is $[-90^\circ, 90^\circ]$. Taking $1^\circ$ as the basic unit, this range is quantized and 181 desired responses are obtained. Assume the desired response are $F(\theta), (\theta = -90, -89, \ldots, 90)$. The desired response set is the discretization of the optimal beam pattern in Fig. 3. Because the amplitude of the input signal has a certain range, the network should respond to the signal within this range. Assuming that the amplitude range is $[-1, 1]$, we will explain why it is set in this way below. In order to construct the training set, the amplitude range is quantified with 0.1 as the basic unit, and the discrete amplitude is $[-1, -0.9, \ldots, 1]$.

The steering vector corresponding to the quantized angle range are $\mathbf{a}(\theta), (\theta = -90, -89, \ldots, 90)$, so the desired processing capacity of RBF network is as follows:

$$
\lambda \mathbf{a}(\theta) \rightarrow \text{net} \rightarrow \lambda F(\theta)(\theta = -90, -89, \ldots, 90)(\lambda = -1, -0.9, \ldots, 1)
$$

(8)

Where, $\mathbf{a}(\theta)$ is input signal, $F(\theta)$ is output signal, $\lambda$ is the amplitude of the signal. The meaning of formula (8) is that the response of RBF network to steering vector $\mathbf{a}(\theta)$ with amplitude $\lambda$ should also be the product of its amplitude and expected response $\lambda F(\theta)$.

Because RBF neural network can only calculate in real domain, but the input signal is in complex domain. Therefore, the real part and the imaginary part of the input signal are separated and input into the network respectively for processing, as shown in the following formula.

$$
\begin{bmatrix}
\text{Re}(\lambda \mathbf{a}(\theta)) \\
\text{Im}(\lambda \mathbf{a}(\theta))
\end{bmatrix} \rightarrow \text{net} \rightarrow \lambda F(\theta)(\theta = -90, -89, \ldots, 90)(\lambda = -1, -0.9, \ldots, 1)
$$

(9)

Where, $\text{Re}(\bullet)$ is the real part of the signal, $\text{Im}(\bullet)$ is the imaginary part of the signal.

3.2. Network application

After constructing the training set to train the RBF network through the above process, the RBF neural network with the desired response is obtained. Assume the target is $0^\circ$, the main lobe interference is $4^\circ$, and the side lobe interference is $40^\circ$. After the mixed signal is input into the RBF neural network. The network can form the maximum gain in the $0^\circ$ direction, that is, the target signal is retained. A depression is formed in the $4^\circ$ and $40^\circ$ direction, that is, the main lobe interference and side lobe interference are suppressed to the greatest extent.

Since the network can only process signals with amplitude range of $[-1, 1]$. However, the amplitude range of the mixed signal cannot be controlled. Thus, firstly normalize the signal so that its amplitude range becomes $[-1, 1]$, as shown in formula (10):

$$
\hat{x}(k) = x(k) \left[ \max \left( \left| x(k) \right| \right) \right]^{-1}
$$

(10)

Where, $\left| \bullet \right|$ represents the modulus of the signal, and $\max(\bullet)$ represents the maximum value.

After the signal is normalized, the signal $\hat{x}(k)$ is input into the trained RBF neural network, and the output of the network is the target signal after filtering out the main lobe interference and side lobe interference.
4. Simulation analysis
Using Matlab software to simulate the beamforming of a 16 element uniform linear array, the target signal direction is set to 0°, the main lobe interference direction is set to 4°, and the side lobe interference direction is set to 40°. The interferences are all broadband noise-type blanket interferences. The target signal is a pulsed chirp signal with a pulse width of 10 us and a bandwidth of 30 MHz.

4.1. RBF network response
According to the simulation background, the training set of the corresponding RBF network is constructed to train the network. After training, the input amplitude of the RBF network is 1, and the range of steering vectors are [-90°, 90°]. The RBF network the response at [-90°, 90°] range is obtained. Compare the response of the network with the desired response, the result is shown in figure 4.

![Figure 4. RBF network and desired response](image)

Figure 4. RBF network and desired response

Figure 5. Response to main lobe interference

The blue curve in Figure 4 represents the response beam pattern of the RBF neural network, and the red curve represents the desired response beam pattern. It can be seen from the figure 4 that although the RBF network has partial errors with the desired response, it can form the maximum gain in the 0° signal direction, and form nulls in the 4° and 40° directions to suppress main lobe interference and side lobe interference. Therefore, the trained RBF neural network has the ability to suppress main lobe interference, side lobe interference and perform beamforming normally.

4.2. The beamforming effect of RBF network
The main lobe interference, side lobe interference and target signal are respectively input the trained RBF network for processing. The output signal of the network is compared with the original signal. Then calculating the enhancement and suppression of the network for signal and interference. The final result is as shown in Figure 5, Figure 6 and Figure 7:

![Figure 6. Response to side lobe interference](image)

Figure 6. Response to side lobe interference

![Figure 7. Response to target signal](image)

Figure 7. Response to target signal
In Figure 5, the red curve represents the main lobe interference signal input the network, and the blue curve represents the output signal after network processing. It can be seen intuitively in the figure that the output of the main lobe interference signal is almost zero after processed by the network. The suppression of the main lobe interference signal by the network is calculated to reach 23dB, so the network has a good ability to suppress the main lobe interference.

In Figure 6, the red curve represents the side lobe interference signal input the network, and the blue curve represents the output signal after processed by the network. In the figure 6, the amplitude of the side lobe interference signal has dropped a lot. And the suppression of the side lobe interference signal by the network is calculated to reach 19dB, so the network also has a better ability to suppress the side lobe interference.

In Figure 7, the red curve represents the target signal input the network, and the blue curve represents the output signal of network. It can be seen that the input and output signals are basically in coincidence. The gain of the network to the target signal is calculated to reach 0.02dB, so the network does not suppress the target signal, and even has a certain extent of enhancement.

In summary, RBF network can suppress the main lobe interference and side lobe interference signal strongly, and slightly enhance the target signal, so it has the ability to anti main lobe interference, side lobe interference and beamforming.

5. Conclusion
After simulation analysis, the beamforming algorithm based on RBF neural network has the ability to suppress main lobe interference and side lobe interference. At the same time, because the RBF neural network uses offline training, it will save time for convergence in practical applications and has better real-time performance.

Theory and simulation show that the application of RBF network for beamforming can overcome the limitations of traditional beamformers. When there is main lobe interference, it can not only suppress the main lobe interference, but also carry out optimal beamforming, so it has ideal beamforming ability. The RBF network training set with stronger generalization ability can make the network have better processing ability, so the future research direction can focus on the construction of RBF network training set.

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