Higher-derivative analogue Aharonov-Bohm effect, absorption and superresonance

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In this paper we consider the acoustic metric obtained from an Abelian Higgs model extended with higher derivative terms in the bosonic sector which describes an acoustic rotating black hole and analyze the phenomena of superresonance, analogue Aharonov-Bohm effect and absorption. We investigate these effects by computing the scalar wave scattering by a draining bathtub vortex and discuss the physical interpretation of higher derivative terms.

I. INTRODUCTION

Since the renowned Unruh work published in 1981 [1], a variety of analogue models of gravity [2–6] have been constructed in different physical situations with the main purpose of creating analogue black holes in the laboratory to investigate physical properties of black holes. Unruh observed that in an irrotational fluid flow the sound waves produced in this fluid are governed by the Klein-Gordon equation on an effective geometry (so-called effective acoustic spacetime). Within this context several applications have been explored such as superresonance phenomenon [7–14], wave scattering [15, 16], absorption [17, 18], analogue Aharonov-Bohm effect [19–22], quasinormal mode [23–26] and atomic Bose-Einstein condensates [27, 28].

In planar physics a problem that has been well investigated is the Aharonov-Bohm effect [29]. This subject corresponds to a problem of scattering charged particles due to a flux tube. This effect was confirmed by Tonomura [30]. In nonrelativistic field theory this effect has been simulated by considering bosonic particles interacting with a Chern-Simons field [31–33]. In quantum mechanics the noncommutative Aharonov-Bohm effect has been investigated in [34, 35] and in the background with breaking of the Lorentz symmetry [36, 37]. The authors Berry et al. [38] obtained the analogue Aharonov-Bohm effect from scalar wave scattering by draining vortex bathtub [39]. And recently, in [19] this effect was revised showing a new interference pattern. In addition, these analyzes for the analogous Aharonov-Bohm effect were extended in [20] to a Lorentz-violating background, in [21] to a noncommutative background and in [22] considering the neo-Newtonian theory. In addition, the study of the analogue Aharonov-Bohm effect has also been performed in gravitation [40], fluid dynamics [41], optics [42] and Bose-Einstein condensates [43].

In this paper starting from the Abelian Higgs model extended with terms of high derivatives in the bosonic sector [44] we determine the effective acoustic metric in (2+1) dimensions and whose effect of higher derivative terms is incorporated into modified acoustic metric by means of a parameter $\Lambda = 1/\lambda$.

The higher derivative term added to the Abelian Higgs model can play the role of dispersion relation of phonons in atomic Bose-Einstein condensates. Such dispersion relation is similar to those ones previously considered in acoustic black holes [45] to investigate, e.g., the ultrashort-distance physics.

In our study, we applied the partial wave method to compute the cross section of differential scattering and absorption of monochromatic planar waves impinging upon a draining bathtub vortex. Thus, we investigate the phenomenon of superresonance that is affected by the presence of this parameter. We then apply this effective acoustic metric to the Klein-Gordon equation to investigate the scattering of planar waves by a draining bathtub vortex by calculating the differential scattering cross section for the analogue Aharonov-Bohm effect and also the absorption. We show that by increasing the value of parameter $\lambda$ (or decreasing the value of $\Lambda$) the cross section is increased as well as the absorption. A numerical analysis of the results has also been performed.

The paper is organized as follows. In Sec. II we briefly introduce the acoustic black hole metrics obtained from the Abelian Higgs model modified with higher derivative terms. We then consider this metric and apply it to a Klein-Gordon-like equation to study the phenomenon of superresonance, compute the differential cross section due

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to the scattering of planar waves that leads to a modified analogue Aharonov-Bohm effect, determine the absorption and also make numerical analysis. Finally in Sec. III we present our final considerations.

II. THE EFFECTIVE ACOUSTIC METRIC

In this section we consider the acoustic black hole metrics obtained from the Abelian Higgs model including higher derivative gauge invariant terms. The Lagrangian of the Abelian Higgs model modified with higher derivative terms is given by [44]

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_{\mu}\phi)^{\dagger} D^{\mu}\phi M^{2} \phi + \frac{1}{\Lambda_{0}^{2}} (D_{\mu} D^{\mu}\phi)^{\dagger} (D_{\nu} D^{\nu}\phi) - b|\phi|^{4}, \] (1)

where \( F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}, D_{\mu}\phi = \partial_{\mu}\phi - ie A_{\mu}\phi \) and \( \Lambda_{0} \) is a parameter with dimension of mass.

Now we briefly review the steps to find the acoustic black hole metric. Thus we use the decomposition \( \phi = \sqrt{\rho} e^{iS} \) into the original Lagrangian to get

\[ \mathcal{L} = -\frac{1}{4} F^{2} + (u_{\mu} u^{\mu} + M^{2}) \rho - b \rho^{2} + \frac{\rho}{\Lambda_{0}^{2}} \left[ (\Box S)^{2} + (\partial_{\mu} S)^{4} + e^{2} A_{\mu} A^{\mu} (2u_{\nu} u^{\nu} - e^{2} A_{\nu} A^{\nu}) - 4e A^{\mu} u^{\nu} \partial_{\mu} S \partial_{\nu} S \right], \] (2)

where \( F^{2} = F_{\mu\nu} F^{\mu\nu}, u_{\mu} = \partial_{\mu} S - e A_{\mu}, \) and \( \Box = \partial_{\mu} \partial^{\mu} \) is the d’Alembert operator in Minkowski space. Then, linearizing the equation of motion around the background \((\rho_{0}, S_{0})\) with \( \rho = \rho_{0} + \rho_{1} \) and \( S = S_{0} + \psi \), we obtain the equation of motion in a curved space with a higher derivative source

\[ \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu\nu} \partial_{\nu} \psi = \frac{1}{\Lambda_{0}^{2}} \partial_{\mu} (\rho_{0} \partial^{\mu} \Box \psi), \] (3)

where the index \( \mu = t, x, i = 1, 2, 3 \). In the following we shall define \( c_{s}^{2} = b\rho_{0}/2w_{0}^{2} \) (the local sound speed in the fluid), \( v^{i} = v_{0}^{i}/w_{0} \) (velocity of the flow), \( v_{0} = \nabla S_{0} + e A_{0} \),

\[ w_{0} = -S_{0} + e A_{t}, \quad \text{and} \quad \Lambda^{2} = \Lambda_{0}^{2}/w_{0}^{2}. \] (4)

Thus, we obtain in terms of the inverse of \( g^{\mu\nu} \) the acoustic metric, at the non-relativistic limit, given by [44]

\[ g_{00} = \left( 1 + \frac{2}{\Lambda^{2}} \right) c_{s}^{2} - \left( 1 + \frac{4}{\Lambda^{2}} \right) v^{2}, \] (5)

\[ g_{0i} = g_{i0} = \left( 1 + \frac{4}{\Lambda^{2}} \right) v^{i}, \] (6)

\[ g_{ij} = -\left( 1 + \frac{4}{\Lambda^{2}} \right) \delta^{ij}. \] (7)

Then it follows that we can write the \((2+1)\)-dimensional acoustic line element in polar coordinates given by

\[ ds^{2} = \left[ \left( 1 + \frac{2}{\Lambda^{2}} \right) c_{s}^{2} - \left( 1 + \frac{4}{\Lambda^{2}} \right) v^{2} \right] dt^{2} + 2 \left( 1 + \frac{4}{\Lambda^{2}} \right) v^{t} dtdr + \left( 1 + \frac{4}{\Lambda^{2}} \right) r^{2} d\phi^{2}. \] (8)

Now, we consider the flow with the velocity potential \( \psi(r, \phi) = -D \ln r + C \phi \) whose velocity profile takes the form

\[ \bar{v} = -\frac{D}{r} \hat{r} + \frac{C}{r} \hat{\phi}, \] (9)

and considering \( c_{s} = 1 \), the line element can be written as follows

\[ ds^{2} = \left( 1 + \frac{2}{\Lambda^{2}} \right) \left[ 1 - \left( \frac{\Lambda^{2} + 4}{\Lambda^{2} + 2} \right) \left( \frac{D^{2}}{r^{2}} + \frac{C^{2}}{r^{2}} \right) \right] dt^{2} - 2 \left( 1 + \frac{4}{\Lambda^{2}} \right) \frac{D}{r} dtdr + \left( 1 + \frac{4}{\Lambda^{2}} \right) C \frac{dtdr}{r} - \left( 1 + \frac{4}{\Lambda^{2}} \right) r^{2} d\phi^{2}. \] (10)
Applying the coordinate transformations

\[
d\tau = dt - \dot{\Lambda} \frac{D}{r f(r)} dr, \quad d\phi = d\phi - \dot{\Lambda} \frac{C D}{r^3 f(r)} dr,
\]  

(11)

the line element in these new coordinates becomes

\[
ds^2 = \left(1 + \frac{4}{\Lambda^2}\right)^{-1} ds^2 = \frac{1}{\Lambda} \left(1 - \dot{\Lambda} \frac{D^2 + C^2}{r^2}\right) dr^2 - \left(1 - \dot{\Lambda} \frac{D^2}{r^2}\right)^{-1} \left(d\tau^2 - r^2 d\varphi^2 + 2C d\varphi d\tau\right).
\]  

(12)

The radius of the ergosphere is given by \(g_{00}(r_e) = 0\) and the horizon is given by \(g_{rr}(r_h) = 0\), that is

\[
r_e = \sqrt{r_h^2 + 4\Lambda C^2}, \quad r_h = \sqrt{\Lambda |D|}.
\]  

(13)

Now we can write the metric as follows

\[
g_{\mu\nu} = \begin{pmatrix}
\frac{1}{\Lambda} \left(1 - \frac{r_e^2}{r^2}\right) & 0 & C \\
0 & -\left(1 - \frac{r_h^2}{r^2}\right)^{-1} & 0 \\
C & 0 & -r^2
\end{pmatrix},
\]  

(14)

whose inverse \(g^{\mu\nu}\) is

\[
g^{\mu\nu} = \begin{pmatrix}
\dot{\Lambda} f(r)^{-1} & 0 & \frac{\dot{\Lambda}}{r^2} C f(r)^{-1} \\
0 & -f(r) & 0 \\
\frac{\dot{\Lambda}}{r^2} C f(r)^{-1} & 0 & \left(1 - \frac{r_e^2}{r^2}\right) f(r)^{-1}
\end{pmatrix},
\]  

(15)

where \(f(r) = 1 - \dot{\Lambda} D^2/r^2 = 1 - r_h^2/r^2\) and we have defined \(\dot{\Lambda} = (1 + 4/\Lambda^2)(1 + 2/\Lambda^2)^{-1} = 1 + 2/\Lambda^2 + O(1/\Lambda^4)\). We will now apply the Klein-Gordon equation for a linear acoustic disturbance \(\psi(\tau, r, \varphi)\) in the background metric (15)

\[
\frac{1}{\sqrt{|g|}} \partial_{\mu} \left(\sqrt{|g|} g^{\mu\nu} \partial_\nu \psi\right) = 0.
\]  

(16)

Using the separation of variables, \(\psi(\tau, r, \varphi) = H(r)e^{-i(m\varphi - \omega \tau)}\), in the above equation the function \(H(r)\) satisfies the following radial equation

\[
\frac{f(r)}{r} \frac{d}{dr} \left[f(r) \frac{d}{dr}\right] H(r) + \dot{\Lambda} \left[\omega^2 - 2 \frac{C}{r^2} m \omega - \frac{1}{\Lambda} \left(1 - \frac{r_e^2}{r^2}\right) \frac{m^2}{r^2}\right] H(r) = 0.
\]  

(17)

Now we introduce the tortoise coordinate \(\varrho\) through the following relation

\[
\frac{d}{d\varrho} = f(r) \frac{d}{dr}, \quad f(r) = 1 - \frac{r_e^2}{r_h^2} = 1 - \dot{\Lambda} \frac{D^2}{r^2},
\]  

(18)

whose solution is

\[
\varrho = r + \sqrt{\Lambda} \frac{|D|}{2} \log \left(\frac{r - \sqrt{\Lambda} |D|}{r + \sqrt{\Lambda} |D|}\right).
\]  

(19)

Observe that in this new coordinate the horizon \(r_h = \sqrt{\Lambda} |D|\) maps to \(\varrho \to -\infty\) while \(r \to \infty\) corresponds to \(\varrho \to +\infty\). Now by considering a new radial function, \(G(\varrho) = \sqrt{\nu} H(r)\), the equation (17) becomes

\[
\frac{d^2 G(\varrho)}{d\varrho^2} + \left[\dot{\Lambda} \left(\omega - \frac{C m}{r^2}\right)^2 - V(\varrho)\right] G(\varrho) = 0,
\]  

(20)

where \(V(r) = \frac{f(r)}{4r^2} \left[4m^2 - 1 + \frac{5D^2}{r^2}\right]\).
A. Superresonance Phenomenon

In the following we shall compute the phenomenon of superresonance (analog to the superradiance in black hole physics) in the presence of spacetime modified by higher derivative terms. This effect corresponds to amplification of a sound wave by reflection from the ergoregion of a rotating acoustic black hole. Thus, by applying the asymptotic

\[
\frac{d^2 G(\varrho)}{d\varrho^2} + \omega^2 G(\varrho) = 0,
\]

where \( \omega = \sqrt{\Lambda} \omega \) and which presents the simple solution

\[ G(\varrho) = Ce^{i\omega \varrho} + Re^{-i\omega \varrho}, \]

the first term of the equation (22) corresponds to the incident wave and the second term the reflected wave, so the reflection coefficient is given by \( R = \frac{i^{1/2}(-1)^m}{\sqrt{2\pi \tilde{\omega}}} e^{-i\omega \varrho} \).

Now in the limit \( \varrho \to -\infty \), we have

\[ \frac{d^2 G(\varrho)}{d\varrho^2} + \left( \tilde{\omega} - m\tilde{\Omega}_H \right)^2 G(\varrho) = 0, \]

where, \( \tilde{\Omega}_H = \Omega_H/\sqrt{\Lambda} \) and \( \Omega_H = C/D^2 \) is the angular velocity of the acoustic black hole. So the solution is

\[ G(\varrho) = Te^{i(\tilde{\omega} - m\tilde{\Omega}_H)\varrho}. \]

The reflection coefficient is given by

\[ |R|^2 = 1 - \left( \frac{\tilde{\omega} - m\tilde{\Omega}_H}{\tilde{\omega}} \right)|T|^2, \]

where \( m \) is the azimuthal mode number and \( \Omega_H = C/D^2 \) is the angular velocity of the usual Kerr-like acoustic black hole. Note that for frequencies in the interval \( 0 < \tilde{\omega} < m\tilde{\Omega}_H \) the reflection coefficient is always larger than unit, which corresponds in the superresonance phenomenon (analog to the superradiance in black hole physics). Notice from (25) that the frequency \( \tilde{\omega} \) and the modified angular velocity \( \tilde{\Omega}_H \) depends on the parameter of high derivatives \( \tilde{\Lambda} = 1 + 2/\Lambda^2 \). To facilitate the analysis of the effects of \( \Lambda \) we will make the following substitution \( \lambda = 1/\Lambda \) and thus we will have \( \tilde{\Lambda} = 1 + 2\lambda^2 \). This means that when we increase the values of \( \lambda \) the frequency range decreases, that is, the wave is spread with a smaller amplitude as can be seen in the graphs of Figure 1. In the graphs we can see the reflection behavior for some values of \( \lambda \) for the cases, \( m = 1 \) and \( m = 2 \).
B. Analogue Aharonov-Bohm Effect

In this subsection we will study the analogue Aharonov-Bohm effect. For this purpose we consider the scattering of a monochromatic planar wave of frequency $\omega$ as \[46\]

$$\psi(t, r, \phi) = e^{-i\omega t} \sum_{m = -\infty}^{\infty} R_m(r) e^{im\phi}/\sqrt{r},$$  \hspace{1cm} (26)$$

with the function $\psi$ written in the form

$$\psi(t, r, \phi) \sim e^{-i\omega t}(e^{i\omega x} + f_\omega(\phi)e^{i\omega r}/\sqrt{r}),$$  \hspace{1cm} (27)$$

where $e^{i\omega x} = \sum_{m = -\infty}^{\infty} i^m J_m(\omega r)e^{im\phi}$ and $J_m(\omega r)$ is a Bessel function of the first kind. In this case using the representation of partial waves scattering amplitude $f_\omega(\phi)$ reads

$$f_\omega(\phi) = \sqrt{\frac{1}{2i\pi\omega}} \sum_{m = -\infty}^{\infty} (e^{2i\delta_m} - 1)e^{im\phi}.$$  \hspace{1cm} (28)$$

The phase shift is defined as

$$e^{2i\delta_m} = i(-1)^m \frac{C}{R}.$$  \hspace{1cm} (29)$$

Thus, to determine the phase shift, at an approximate value, we first rewrite the equation (20) in terms of a new function $X(r) = f(r)^{1/2}G(\rho)$

$$\frac{d^2X(r)}{dr^2} + \left\{ \left[ \frac{1}{4} - \frac{m^2}{r^2} + \frac{7}{4} \Lambda D^2 \right] + f(r)^{-1} \left[ (\bar{\omega} - \sqrt{\Lambda Cm/r^2})^2 - \bar{\Lambda}^2 D^4/r^6 \right] \right\} X(r) = 0.$$  \hspace{1cm} (30)$$

Now by expanding the terms that multiply $X(r)$ in the above equation in a $1/r$ power series we can rewrite this equation as follows

$$\frac{d^2X(r)}{dr^2} + \left[ \bar{\omega}^2 - \frac{4\bar{m} - 1}{4r^2} + U(r) \right] X(r) = 0,$$  \hspace{1cm} (31)$$

where we have defined $\bar{m}^2 = m^2 + 2\bar{\alpha}m - 2\bar{\beta}^2$, $\bar{\alpha} = \bar{\Lambda} \alpha$, $\bar{\beta} = \bar{\Lambda} \beta$, and

$$U(r) = \frac{(\bar{\alpha}^2 - \bar{\beta}^2)m^2 - 4\bar{\alpha}^2\bar{m} + 2\bar{\beta}^2 + 3\bar{\beta}^4}{\bar{\omega}^2r^4} + \frac{\bar{\beta}^2(2\bar{\alpha}^2 - \bar{\beta}^2)m^2 - 6\bar{\beta}^4\bar{m} + 3\bar{\beta}^4 + 4\bar{\beta}^6}{\bar{\omega}^4r^6} + O(\bar{\omega}^{-6}r^{-8}).$$  \hspace{1cm} (32)$$

Here $\alpha = \omega C$ and $\beta = \omega D$ are parameters that describe the coupling to circulation and draining, respectively. Let us now apply the approximation formula

$$\delta_m \approx \frac{\pi}{2} (m - \bar{m}) + \frac{\pi}{2} \int_0^\infty r |J_{\bar{m}}(\bar{\omega}r)|^2 U(r) dr.$$  \hspace{1cm} (33)$$

Then by expanding the terms and using $|m| \gg \sqrt{\bar{\alpha}^2 + \bar{\beta}^2}$, we obtain

$$\delta_m \approx -\bar{\alpha} \frac{\pi}{2} \frac{m}{|m|} + \frac{3\pi}{8|m|} \left( \bar{\alpha}^2 + \bar{\beta}^2 \right) - \frac{5\pi}{8\bar{m}^2} \frac{(\bar{\alpha}^2 + \bar{\beta}^2) m}{|m|^2}. $$  \hspace{1cm} (34)$$

Note that the result for mode $m = 0$ is not valid, but in the limits for $m \to \pm \infty$ the first terms in the equation (34) imply that the phase shift tends to a constant different from zero, which naturally leads to the Aharonov-Bohm effect. However, for the isotropic mode $m = 0$ by the equation (20) we obtain the solution of the form

$$G_{m=0}(\rho) = r^{1/2} e^{\bar{\beta}^2/2} J_{\bar{\beta}} \left( \bar{\omega} r f^{1/2} \right),$$  \hspace{1cm} (35)$$
and the phase shift is imaginary
\[ \delta_{m=0} = \frac{1}{2} i \pi \tilde{\beta}. \]  
(36)

Thus, by using the Eqs. (34) and (36), to lowest order in \( \tilde{\alpha} \) and \( \tilde{\beta} \), we can compute the differential scattering cross section (with units of length) that is given by
\[
\frac{d\sigma_{\alpha\beta}}{d\phi} \approx \frac{\pi \left(1 + 2\lambda^2\right)^{3/2}}{2\omega} \frac{\left[-\beta \sin(\phi/2) + \alpha \cos(\phi/2)\right]^2}{\sin^2(\phi/2)},
\]  
(37)

where we return with the value of \( \tilde{\Lambda} = 1 + 2\lambda^2 \).

Now by considering \( \beta = 0 \) (the non-draining limit), for low orders \( \delta_m = -\frac{\tilde{\alpha}}{2} \frac{m}{|m|} \), we have the vortex result of Fetter [39]
\[
\frac{d\sigma_{\text{vortex}}}{d\phi} = \left(1 + 2\lambda^2\right)^{3/2} \frac{\alpha^2 \pi}{2\omega} \cot^2(\phi/2).
\]  
(38)

For small \( \lambda \) (large \( \Lambda \)) and small angle \( \phi \), the Eq. (38) becomes
\[
\frac{d\sigma_{\text{vortex}}}{d\phi} = \frac{\left(1 + 3\lambda^2\right) \pi^2 \alpha^2}{2\pi \omega} \left[ \frac{4}{\phi^2} - \frac{2}{3} + \frac{\phi^2}{60} + \mathcal{O}(\phi^3) \right],
\]  
(39)

so when \( \phi \to 0 \), the result for the differential cross section is
\[
\frac{d\sigma_{\text{vortex}}}{d\phi} = \frac{\left(1 + 3\lambda^2\right) \pi^2 \alpha^2}{2\pi \omega} \frac{4}{\phi^2},
\]  
(40)

which is the differential scattering cross section for the analogue Aharonov-Bohm effect due to the metric of an acoustic black hole. In Figure 2 we show the scattering behavior for low frequencies as a function of the scattering angle, for \( \lambda = 0 \) (dashed lines) the system recovers the usual behavior, and when we enter values for \( \lambda \) it is possible to verify the behavior of the effect for the both cases without and with drainage, in Figures 2(a) and 2(b), respectively. The contribution of the extra term causes the graph curves to move upward symmetrically in the first case.

C. Absorption and Numerical Analysis

In this section we present the numerical results of the absorption with arbitrary values of incoming wave frequency, for the metric of the acoustic rotating black hole. For this we solve the equation (17) numerically and obtain the

![Graph showing scattering at low frequencies for different values of \( \lambda \), the dashed lines referred to the effect without the contribution of \( \lambda \). The graph in (a) shows the effect without drainage and (b) the effect with drainage.](image-url)
reflection values (Figure 1), scattering (Figure 2) and subsequently absorption. We have applied the numerical procedure as described in [16].

The absorption cross section can be computed analytically by the following equation

$$\sigma_{\text{abs}} = \frac{1}{\omega} (1 - |e^{2i\delta_m}|^2),$$

in the low frequency limit the contribution of the mode $m = 0$ is dominant, so from the phase shift (36) the absorption for small $\lambda$ is given by

$$\sigma_{\text{abs}} = 2\pi \sqrt{1 + 2\lambda^2}D = 2\pi \left(1 + \lambda^2\right) D.$$  

(42)

This absorption equals the circumference of an acoustic black hole times a factor that depends on the parameter of higher derivative terms. The Figure 3 shows the behavior of the absorption cross section for $m = 0$ without circulation for some values of $\lambda$. As already predicted from the analytical result for low frequencies described above, we have an increase of the absorption value when we enter values in $\lambda$. The absorption area increases as we increase the value of $\lambda$.

In Figure 4 and 5 we have a comparison for the cases with and without circulation for the azimuthal numbers $m = 1$ and $m = 2$. The graphs in the left panel show the absorption for the static case, i.e., $C = 0$. In this case the values of the absorption is independent of the signal of $m$, and we also see that when varying $\lambda$ the form of the curve modifies increasing their peaks. That is, the cross section area increases. This increasing also occurs for the graphs in the right panel, for the rotating case $C = 0.5D$. The difference here is the presence of negative absorption values in Figure 4(b), which corresponds to the superresonance effect.

### III. CONCLUSIONS

In summary, in the present study we investigate the analogue Aharonov-Bohm (AB) effect, superresonance and absorption in a background described by the metric of an acoustic black hole obtained from the Abelian Higgs model including higher derivative gauge invariant terms. To address the issues concerning the AB effect we considered the scattering of a monochromatic planar wave. For the case where we admit the circulation term, the frequency region where superresonance occurs is reduced by the influence of the extra $(1 + 2\lambda^2)^{3/2}$ factor due to the modified acoustic metric. Another influence of the higher derivative term is the increase of the absorption. We show that the additional term symmetrically modifies the differential scattering cross section, which can be seen in Figure 2(a). When we assume $\lambda = 0$ the equation (38) recovers the results obtained in [19]. As we have mentioned earlier, the higher derivative term added to the Abelian Higgs model can play the role of dispersion relation of phonons in atomic
Bose-Einstein condensates and such dispersion relation is similar to those ones previously considered in acoustic black holes [45] to investigate, e.g., the ultrashort-distance physics. The results obtained such as increasing of the absorption at smaller frequencies by increasing the effect of this higher derivative term by increasing $\lambda$ is in agreement with the expected behavior. This is because the higher derivative terms are involved in the theory the higher is the power of the frequency in the dispersion relation and more quickly the absorption effect takes place.

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[1] W. Unruh, Phys. Rev. Lett. 46, 1351 (1981), ibid, Phys. Rev. D 51, 2827 (1995), [arXiv:gr-qc/9409008].
[2] M. Visser, Class. Quant. Grav. 15 1767 (1998).
[3] G. Volovik, The Universe in a Helium Droplet, Oxford University Press, 2003; L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Rev. Mod. Phys. 80, 787 (2008), [arXiv:0710.5373[gr-qc]]; M. Cadoni, S. Mignemi, Phys. Rev. D 72, 084012 (2005); M. Cadoni, Class. Quant. Grav. 22, 409 (2005).
[4] L. C. Garcia de Andrade, Phys. Rev. D 70 (2004) 64004; T. K. Das, Conf. Proc. C 0405132, 279 (2004) [gr-qc/0411006]; G. Chapline and P. O. Mazur, Acta Phys. Polon. B 45, no. 4, 905 (2014) [gr-qc/0407033].
