Calculation of macrodefects coordinates in dielectric specimens on the two-dimensional mathematical model of mechanoelectric transformations method

R A Laas¹, P N Khorsov², A P Surzhikov³

¹Assistant, National Research Tomsk Polytechnic University, Tomsk, Russia
²Senior researcher, National Research Tomsk Polytechnic University, Tomsk, Russia
³Chief Researcher, National Research Tomsk Polytechnic University, Tomsk, Russia
E-mail: romanlaas@tpu.ru

Abstract. Two-dimensional mathematical model of dielectric specimen was used to determine the capability of mechanoelectrical transformations (MET) method to localize macrodefects. Amplitude and phase characteristics of response signal analytical representation were used as response parameters. Three different types of short radiofrequency pulses were chosen for the excitation. A short sin curve of a single period interval is most useful to search the position of the defect, whilst pulses of higher frequencies and are better for location depth evaluation.

1. Introduction
MET method is widely used to control structure imperfections of dielectrical materials [1,2]. It allows to determine value of stress-strain state [3,4]. In MET method acoustic excitation wave formed by impact device interacts with MET sources, such as double electric layers on the interfaces of different materials or enclosures with piezoelectric properties [5,6].

Excitation wave changes electric dipole moment of the source, which leads to the generation of alternating electromagnetic field. The parameters of the field can be registered by a transducer, so that signal reception does not require surface contact as distinct from ultrasonic methods. The modification of this method, which allows to significantly increase signal to noise ratio using multiple excitations of specimen without changing the source-specimen-detector system, substantively increase the efficiency and capabilities of the method [7].

Mathematical modelling using 1-dimensional models has shown the capability of the method to determine the depths of defects by phase analysis of analytic signal interpretation. Series of experiments has proved the consistency of the model [8].

Narrowband signal was obtained using high frequency pulse (100 kHz) with 1 ms duration as an excitation signal. The response time were chosen to be greater than time required for beam of acoustic excitation to reach the studied defect, reflect to the detector, so that it was possible to register the phase jump of interference between reflected wave and structural electromagnetic noise signal. The goal of this paper is to evaluate the efficiency of surface scan with MET-method of localizing macrodefects and estimating their depth using wideband signals. Consider scanning system is a solid
single module where piezoelectrical transmitter and capacitance transducer are installed, so that both emission and detection of signal take place on one side of the object.

2. The mathematic model

The control system is shown on Figure 1.

![Figure 1. The control system.](image)

On the Figure 1 numbers 1 and 2 mark surfaces of transmitter and transducer correspondingly. Arrows show acoustic excitation beam, reflected from the surface of defect toward transducer.

To describe 2-dimensional parameters of control module (CM) mathematical model was developed. It allows considering linear dimensions of transmitter and transducer, distance between them, allows determining frequency, shape and length of input signal and parameters of the defect, such as depth and size. The model allows adding a white noise of certain intensity to informative signal.

The algorithm of response parameters calculation is described below.

Beams of acoustic excitation wave are being reflected from the surface of defect toward transducer. Each of beams emitted from every discrete point of transmitter reaches every discrete point of defect surface. Levels of signals are calculated at this stage with the consideration of the translation distance for each beam. Then levels of each reflected beam signal are registered at each point of transducer. Step width can be found as follows:

$$\Delta s = \frac{v}{f_d}$$

where $v$ – speed of sound in specimen; $f_d$ – sampling rate.

Considering speed of sound 3000 m/s and sampling rate 1 MHz $\Delta s = 3\text{mm}$.

The distances $r_{1_{ij}}$ from each point of transmitter to each point of defect, facing control module, are calculated as follows:

$$r_{1_{ij}} = \sqrt{h^2 + [(x_0 + i) - (x_{1s} + j)]^2},$$

where $h$ – depth of defect location; $x_0$ – initial horizontal position of the defect; $x_{1s}$ – initial position of transmitter plate; $s$ – control module shift index along the surface; $i$ – beam shift index along defect surface ($i=0, 1, \ldots, b/\Delta s$), $b$ – linear size of the defect; $j$ – beam shift index along the surface of the transmitter; ($j=0, 1, \ldots, c/\Delta s$), $c$ – linear size of transmitter plate.

$r_{2_{iks}}$ is calculated analogously:

$$r_{2_{iks}} = \sqrt{h^2 + [(x_0 + i) - (x_{2s} + k)]^2},$$

where $x_{2s}$ – initial coordinate of transducer plate; $k$ – ray shift index along the surface of the transducer; ($k=0, 1, \ldots, m/\Delta s$); $m$ – linear size of transducer plate.
The next step is to calculate levels of signals $U_{1ijs}$ in each point of the defect surface taking into account the angular pattern of the transmitter, which is chosen proportional to $\cos(\varphi_1)$ where $\varphi_1$ – angle between beam direction and defect surface normal. That is an appropriate approximation for relatively small angles and wavelengths comparing to size of the transmitter. The spherical nature of the wave has to be considered as well, this leads to the fact that level of signal $A$ is in reverse ratio to the distance from the transmitter. As a result, the formula looks as follows

$$U_{1ijs} = A \frac{\cos(\varphi_1)}{r_{1ijs}^2} = A \frac{h}{r_{1ijs}^2}.$$  

The total level of acoustic signal at the transducer can be found as:

$$U_{jks} = A \frac{h^2}{r_{1ijs}^2 \cdot r_{2iks}^2}.$$  

Time delay of the corresponding beam can be found as:

$$t_{jks} = \frac{r_{1ijs} + r_{2iks}}{v}.$$  

The corresponding indexes $tt$ are to be found as:

$$tt_{jks} = round(t_{jks} \cdot f_d).$$  

where round is an operator which rounds up to the nearest whole number. The sum of all signal levels with same time indexes is a pulse response curve $U_{tt}$. Let us set RF impulse $G_n$ of a given length with certain frequency. The result is a convolution $R_n$ of pulse response curves $U_{tt}$ and $G_n$.

$$R_n = \sum_{t=0}^{n} U_{tt} \cdot G'_t,$$

where top index over $G$ means derivative with respect to time.

### 3. Calculations

Assume certain parameters of the system: linear size of the defect is 20 mm, depth of location – 20 mm, linear size of transducer is 20 mm as well, distance between transmitter and transducer 5 mm. Let us compare efficiency of localization for different signals in this given geometry.

Three different types of signals were used: square-topped pulse with a length of 4.5 $\mu$s and radiofrequency impulses of the same length with frequencies 100 kHz and 200 kHz. Under such conditions one or two full periods are in a signal time window correspondingly.

Responses of given radiofrequency impulses were transformed into analytical signal using Hilbert-transformation as follows:

$$S(t) = f(t) + i \cdot h(f(t)),$$

where $h(\cdot)$ – Hilbert transformation operand, $i$- imaginary unit.

Then phase dependency on time $\varphi(t)$ were calculated. It’s complicated to use this characteristic directly due to the rapid rise of phase linear time.

To obtain residual phase change from time the subtraction procedure was performed for linear phase component in the specified time interval by the formula:

$$\varphi'_1(t) = \varphi(t) - \varphi(t_0) - \frac{(\varphi(t_f) - \varphi(t_0)) \cdot (t - t_0)}{t_f - t_0},$$

where $\varphi(t)$ is the current value of phase on time dependency, $\varphi(t_0)$ is the value at the initial time $t_0$ of the given interval, $\varphi(t_f)$ is the value at the final time $t_f$. 


Figure 2. Amplitude-temporal characteristics of analytical representations: a) RF 100 kHz, b) – 200 kHz, c) square-top.

Figure 2 shows amplitude-temporal characteristics of response signals analytical representations after the impact of signals with chosen shapes under different positions of Control Module. Figures 2a,b,c are responses for 100 kHz, 200 kHz and square-top impulses correspondingly. The continuous line corresponds to the position of CM right on top of the defect; dashed line represents the shift by 10 mm; striped line – by 20 mm.

It is seen that RF signal of 100 kHz gives response of highest level, square-topped – lowest level. The time-shift of impulses with the distance can be explained by growth of travelling lengths for each excitation beam. Offset-dependent amplitude attenuation is due to scattering of sphere-wave in material. This model ignores inner friction related signal attenuation as well as rounding the defect that leads to reduction of reflected toward transducer signal level.

Further development of the model is planned to be considering these current limitations.

Figure 3. Phase spatial characteristics of analytical representations: a) RF 100 kHz, b) 200 kHz, c) square-top.

Figure 3 shows dependencies of phase characteristics of distance. Conditions are same as for the corresponding amplitude-time characteristics with same indexes. To determine the depths of the defect location t time aspect is replaced by spatial by multiplication of the time and sound velocity in specimen. As you can see the greatest gain slope is obtained with RF impulse of 200 kHz frequency.

The inflection of phase characteristic corresponds to distance at which excitation wave travels from transmitter to transducer, reflecting from the defect.

Under given geometrical conditions average distance is supposed to be equal to 47 mm. The result obtained from the model is 45 mm, which is close to the actual one.

4. Conclusion
The calculation shows that it is possible to use MET method to determine the location of the object knowing the exact position of CM and amplitude-time characteristics. Response signal phase characteristic allows evaluating depth of defect location as well.

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