Charge noise, spin-orbit coupling, and coherence of single-spin qubits

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Spin-orbit coupling is ubiquitous in quantum dot quantum computing architectures, and makes spin qubits susceptible to charge noise. We derive a Hamiltonian describing the effect of spin-orbit and noise on a single-spin qubit in a quantum dot. Relaxation is due to noise coupling different orbital levels and is dominated by screened whole charge defects near the dot. Dephasing stems from noise causing relative fluctuations between orbital levels, and is driven by screened whole charge defects and unscreened dipole defects in the substrate. Dephasing times are vastly different between common materials such as Si and GaAs. They can be enhanced by increasing gate fields, choosing materials with weak spin-orbit such as Si, making dots narrower, or using accumulation dots.

Developments in quantum computing hold considerable promise in the progression of modern information processing, and this has spurred a large experimental and theoretical effort investigating two-level systems that can be used as quantum bits (qubits). The need for scalability and long coherence times has led naturally to solid state spin-based devices, such as quantum dot spin systems, as ideal candidates for scalable qubits. The focus has been on single-spin [1] and singlet-triplet qubits [2]. While GaAs quantum dots have been studied for many years, a substantial effort is also underway researching Si spin quantum computing architectures [3–5], motivated by their compatibility with Si microelectronics and long coherence times [6–12]. Recently, much effort has also been devoted to quantum dot systems with spin-orbit interactions [13–15], where spin manipulation in principle be achieved entirely by electrical means [16–18].

The coherence times $T_2^*$ of solid-state spin qubits are determined by hyperfine coupling [22,23] and by the joint effect of spin-orbit coupling and phonons [19–21] or noise [24,25]. Inversion symmetry breaking near an interface makes spin-orbit coupling unavoidable, even in materials such as Si in which it is weak [26]. The spin-orbit interaction couples spin qubits to any electric field, including fluctuating fields due to phonons and charge noise, giving rise to relaxation and dephasing [19,20,27,28]. (Ref. [29] identified a spin-orbit induced dephasing mechanism related to the Berry phase.) Phonon effects become less pronounced at low temperatures [30] and piezoelectric electron-phonon coupling may be absent [21]. Hyperfine effects typically occur on long time scales, the nuclear bath is relatively well known and can be controlled through feedback mechanisms [31] while in materials such as Si hyperfine coupling can be eliminated altogether through isotopic purification [32]. Noise is a well-known source of dephasing in charge qubits [33–36], and Ref. [37] has already shown that spin-orbit and noise lead to spin relaxation. At dilution refrigerator temperatures the interplay of spin-orbit and noise may set the defining bound on spin qubit coherence. Noise sources include $P_i$ centers, which may act as traps that charge and discharge, and tunneling two-level systems, which can be modeled as fluctuating charge dipoles [24,38,39]. In this paper we build on previous decoherence work [40–50] and aim to (i) understand conceptually how spin-orbit and noise cause dephasing and (ii) study the sensitivity to spin-orbit across common materials with similar noise profiles. We study a sample qubit with the same specifications in different materials, we determine sample $T_2^*$s due to common noise sources, discuss the variation in $T_2^*$ across these materials, and seek methods to improve $T_2^*$ generally.

We consider a single-spin qubit implemented in a symmetric, gate-defined quantum dot, located at a sharp flat interface (Fig. 1) in a dilution refrigerator at 100 mK. The qubit is described by the Hamiltonian $H = H_{QD} + H_Z + H_{SO} + H_N$. The kinetic energy and confinement term

$$H_{QD} = -\frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{\hbar^2}{2m^*a^4}(x^2 + y^2),$$

where $a$ is the effective dot radius and $m^*$ the effective mass. The eigenstates of $H_{QD}$ are the Fock-Darwin states, with the ground and first excited states given by

$$\Phi_0(x,y) = \frac{1}{\sqrt{\pi a^2}} e^{-\frac{x^2+y^2}{2a^2}},$$

$$\Phi_{\pm}(x,y) = \frac{2}{a\sqrt{\pi}} (x \pm iy) e^{-\frac{x^2+y^2}{2a^2}}. \tag{2}$$

These have energies $\varepsilon_0 = \hbar^2/2m^*a^2$ for the orbital ground state and $\varepsilon_1 = 3\hbar^2/2m^*a^2$ for the twofold degenerate first orbital excited state. The orbital level splitting is assumed to be the dominant scale, so that only the ground and first excited states are considered. The Zeeman Hamiltonian $H_Z = \frac{1}{2} g \mu_B \sigma \cdot B$, with $\sigma$ the vector of Pauli spin matrices. We focus initially on spin-orbit coupling of the Rashba form $H_{SO} = \alpha(t) \sigma \cdot (k \times \hat{z})$, where $k = -i \nabla$ here is an operator in real space, $\hat{z}$ is the unit vector perpendicular to the interface, and $\alpha$ is determined by a material specific parameter as well as the interface electric field $E_z$ [51]. Because of the latter fact, $\alpha$ is also sensitive to stray electric fields and fluctuates in time, thus we let $\alpha = \alpha_0 (1 + \lambda t)$ where $\lambda \ll 1$. Rashba spin-orbit coupling is expected generally in a 2D electron gas near an interface, and will be present in gate-defined dots. For $E_z \approx 10^7 \text{ Vm}^{-1}$, the Rashba term is expected to be the dominant spin-orbit contribution. Finally, the noise Hamiltonian $H_N(t)$ is a random function of time.
We do not include gate noise in our model, and we first consider random telegraph noise (RTN). In the simplest case, in which the qubit is only sensitive to one defect, \( H_N \) represents a fluctuating Coulomb potential, screened by the nearby 2D electron gas. The matrix elements entering \( H_N \) are \( v_0 = \langle \Phi_0 | U_{scr} | \Phi_0 \rangle, v_1 = \langle \Phi_1 | U_{scr} | \Phi_1 \rangle \) and \( v_2 = \langle \Phi_2 | U_{scr} | \Phi_2 \rangle \approx \langle \Phi_2 | U_{scr} | \Phi_0 \rangle \). The 2D screened Coulomb potential \( U_{scr} \) is written in terms of its Fourier transform, which is a function of momentum \( q \).

\[
U_{scr}(r) = \frac{e^2}{2k_F \epsilon_r} \int_0^{2k_F} d^2 q \frac{e^{-i q \cdot r}}{(2\pi)^2} q + q_{TF},
\]

with \( \epsilon_r \) the relative permittivity, \( q_{TF} \) the Tomas-Fermi wave vector, and \( k_F \) the Fermi wave vector (the contribution from \( q > 2k_F \) is negligible). For RTN we can write \( v_i(t) = v_i(1 - N(t)) \) for \( i = 0, 1, 2 \), and \( N(t) = 0, 1 \) is a Poisson random variable with switching time \( \tau \).

The qubit subspace is the Zeeman-split orbital ground state \( \{ \Phi_{0q}, \Phi_{1q} \} \). These two states are coupled by \( H_N \) to spin-aligned orbital excited states and by \( H_{SO} \) to orbital excited states with anti-aligned spin. By projecting \( H \) onto this subspace we encapsulate the combined effect of spin-orbit coupling and noise in an effective qubit Hamiltonian \( H_{qbt} \). To achieve this, we carry out a Schrieffer-Wolff transformation, detailed in the Supplement [53], eliminating higher orbital excited states [51]. Keeping terms up to the second order in this transformation,

\[
H_{qbt}(t) = H_Z - \frac{2s(t) \varepsilon Z}{[\delta \varepsilon + \delta \varepsilon(t)]^2} \left[ s(t) \sigma_z + v_2(t) \sigma_z \right],
\]

where \( \delta \varepsilon = \varepsilon_0 - \varepsilon_1 \), \( \delta \varepsilon(t) = v_0(t) - v_1(t) \), \( \varepsilon_Z = \frac{1}{2} g \mu B \), and \( s(t) = s_0 [1 + \lambda(t)] \) with \( s_0 = \alpha_0 \sqrt{2} / a \). We retain only terms of first order in \( \varepsilon_Z \) and \( \delta \varepsilon \). Equation (4) implies that, in addition to \( H_Z \), there exists an effective Zeeman term \( \frac{1}{2} \sigma \cdot \mathbf{V}(t) \), where \( \mathbf{V}(t) \) represents an effective fluctuating effective magnetic field due to the combined action of spin-orbit and noise. For convenience \( \mathbf{V} \) has units of energy and, for RTN, \( \mathbf{V}(t) = \mathbf{V}(-1)^{N(t)} \).

The noise matrix elements may be divided into two categories. The diagonal elements \( v_0, v_1 \) cause different orbital levels to fluctuate by different amounts, while the off-diagonal element \( v_2 \) causes transitions between different orbital levels. If the qubit is initialized in the spin-up state, the off-diagonal elements \( \sigma_z \) in \( H_{qbt} \) give relaxation. These elements are of first order in \( \alpha \) [37] and involve the interband defect matrix element \( v_2 \). We expect whole charge defect potentials to be dominant since they are much stronger than dipole potentials [50]. If the qubit is initialized in an off-diagonal state, the diagonal elements \( \sigma_z \) in \( H_{qbt} \) give dephasing. These terms involve the intraband matrix elements \( v_0, v_1 \) of the defect potentials. An additional contribution comes from fluctuations in \( \alpha \), which lead to fluctuations in \( s \) itself. These fluctuations can be interpreted as a modulation of the \( g \)-factor, and are expected to come from defects in the substrate right above the dot, which modify \( E_z \). Since the dot region is depleted, whole charge defects cannot fluctuate, and defects contributing to \( E_z \) are expected to be mostly charge dipoles, stemming for example from passivated traps. Although these are weaker than whole charge defects, they are unscreened, leading to a subtle competition. Thus, generally, relaxation is due to noise matrix elements coupling different orbital levels, while dephasing comes from noise matrix elements that cause relative fluctuations between orbital levels.

We focus on a single-spin qubit described by a density matrix \( \rho \), which satisfies the quantum Liouville equation

\[
\frac{d\rho}{dt} + i \frac{\hbar}{\hbar} [H_{qbt}, \rho] = 0.
\]

The spin density matrix \( \rho(t) = \frac{1}{2} \sigma \cdot \mathbf{S}(t) \), and a spin component \( S_i \) can be found as \( S_i(t) = \text{tr} [\sigma_i \rho(t)] \), with \( \text{tr} \) the matrix trace. Focusing on RTN for the time being, we address relaxation and dephasing separately, since the extension to the general case is straightforward [50]. Relaxation, considered briefly for completeness [53], involves the decay of the diagonal elements of \( \rho \), thus we focus on the component of \( \mathbf{V} \parallel \hat{z} \), and initialise \( \mathbf{S}(t = 0) = S_z \hat{z} \). Averaging over noise realisations \( \langle v_2(t)v_2(t') \rangle = v_2^2 e^{-a(t-t')/\tau} \), we find \( S_z(t) \approx S_z(t = 0) e^{-t/T_1} \), where

\[
\frac{1}{T_1} = \frac{4v_2^2 \tau}{\hbar^2 + 4\varepsilon_Z^2 \tau^2}.
\]

The relaxation time \( T_1 \) is determined by the noise spectral function at the qubit frequency (\( \varepsilon_Z / \hbar \)) [37].

We study next pure dephasing, which is the focus of this work. If \( \mathbf{S}(t = 0) = S_z \hat{z} \) is initialised, and we consider the component of \( \mathbf{V} \parallel \hat{z} \), then

\[
S_z(t) = S_z(t = 0) \cos [\hbar(t)],
\]
Table I: Sample $T_2^*$ for a quantum dot with $a = 20$ nm, $\lambda = 4 \times 10^{-4}$, $\tau = 1$ s (for RTN), $E_z = 20$ MV/m, $\varepsilon_z = 60$ meV, $T = 0.1$ K, $\alpha$ from Refs. [33, 57] and $S(\omega)$ for $1/f$ noise estimated from Refs. [33, 57]. For Si the valley splitting is assumed large.

| Material  | RTN(Whole) | RTN(Dipole) | $1/f$(Whole) |
|-----------|-------------|-------------|--------------|
| Si/SiGe   | 113 $\mu$s  | 4.5 s       | 10 $\mu$s    |
| GaAs      | 16 ns       | 70 $\mu$s  | 11 ns        |
| InAs      | 26 ps       | 16 ns       | 450 ps       |
| InSb      | 1 ps        | 270 ps      | 100 ps       |

where, for RTN, $h(t) = (V/h) \int_0^t (-1)^{N(t')} dt'$. Averaging over noise realisations [44], for $V^2 \ll \langle h/\tau \rangle^2$ the time dynamics of $h(t)$ are a random walk in time, in which the spread in $\cos h(t)$ leads to motional narrowing, and $S_x(t) \approx S_x(t = 0) e^{-t/T_2^*}$, where

$$\left( \frac{1}{T_2^*} \right)_{RTN} = \frac{V^2 \tau}{2 \hbar^2}. \quad (8)$$

For whole charge defects, where dephasing is dominated by fluctuations in the orbital energy, we may set $\lambda(t) = 0$ and retain $V_{abh}(t) = 8\varepsilon_z^2 \delta v(t)/(\delta \varepsilon)^3$. For dipole charge defects we have $V_{dip}(t) = 8\varepsilon_z^2 \lambda(t)/(\delta \varepsilon)^2$.

We study next $1/f$ noise, which in semiconductors is Gaussian [54] and is fully described by its spectral density $S(\omega) = \frac{\gamma k_B T}{\omega}$, where $\gamma$ is inferred from experiment. Based on our estimates above we expect whole charge defects to dominate, hence $S_V(\omega) = \left[ \frac{8\varepsilon_z^2}{(\delta \varepsilon)^3} \right] S(\omega)$. Ref. [37] showed that relaxation is determined by $S_V(\varepsilon_Z)$, in exact analogy to RTN. For dephasing we can write $S_x(t) = S_x(0) e^{-\chi(t)}$, with $\chi(t) \approx (t/T_2^*)^2 \ln(1/\omega_0 t)$ [53], where the cutoff $\omega_0$ is the inverse of the measurement time, and $\omega_0 t \ll 1$ is assumed, thus

$$\left( \frac{1}{T_2^*} \right)_{1/f} \approx \sqrt{\gamma k_B T \left[ \frac{8\varepsilon_z^2}{(\delta \varepsilon)^3} \right]^2}. \quad (9)$$

Since this definition of $T_2^*$ is approximate, we plot the full time evolution of $S_x(t)$ in Fig. 2.

We consider a sample dot with radius $a = 20$ nm located at $x = y = 0$, and $\alpha$ as calculated in Refs. [33, 54]. For a defect in the plane of the dot with $x = 40$ nm, $v_0 = 23$ meV, $v_1 = 141$ meV and $v_2 = 44$ meV. Next we estimate the change in $\alpha$ due to a dipole defect right above the dot ($x = y = 0$) and $z = 3$ nm away from it [24, 38]. We take the expectation value of an unscreened Coulomb dipole potential using the ground state wave function $\Phi_0$, and compare it with the matrix element of the interface potential $E_z z$, yielding $\lambda = 4 \times 10^{-5}$. We use this figure in all our estimates since the relative permittivities of all materials considered are of comparable magnitude. For $1/f$ noise we extract $\gamma$ from experiment [for Si/SiGe we use Ref. [57] for GaAs Ref. [33] while for InAs and InSb we use the same $S(\omega)$ as for GaAs]. The results are listed in Table I, which is the central result of this paper. Note that, for all materials, whole charge

![Figure 2: Time evolution of the initiated spin for different dot radii a in Si/SiGe, $\omega_0 = 1$ s and other values as in Table I](image)

Figure 2: Time evolution of the initiated spin for different dot radii $a$ in Si/SiGe, $\omega_0 = 1$ s and other values as in Table I. Figure 2 shows that terms of second-order in spin-orbit are effective in causing dephasing, and the dependence on $\alpha^2$ causes vast differences in coherence times between materials. Hence, using materials with a small $\alpha$ such as Si can improve coherence enormously. If spin-orbit is needed for spin resonance, increasing $E_z$ will align the charge dipoles. Although that increases $\alpha$ and with it dephasing, it also reduces the gate time by an equal amount. Moreover, for $1/f$ noise, $T_2^* \propto a^{-4}$, so by halving the dot radius the coherence time can be increased by an order of magnitude (Fig. 2 for RTN, $T_2^* \propto a^{-8}$). One can also use pulse sequences [55], lower the temperature to reduce $S(\omega)$, use accumulation dots, in which there is no nearby 2DEG, or focus on reducing charge noise [56, 58].

In summary, we have shown that spin-orbit coupling and charge noise are an effective source of dephasing in single-spin qubits even in materials such as GaAs in which spin-orbit coupling is weak. Based on realistic experimental parameters vast differences in spin coherence exist between common materials. In the future we will devise a full model of $1/f$ noise [59] as an ensemble of incoherent RTNs [60], where qubit dynamics is nontrivial [61]. Dephasing of hole spin qubits, in which spin-orbit coupling is also strong, will be studied in a future publication.

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The qubit subspace is simply the Zeeman-split orbital order as
the top left hand corner of Eq. A1.

Applying a Schrieffer-Wolff transformation on (A1) algebraic
operations, the Zeeman energy $\varepsilon = \frac{1}{2} g \mu_B B$, and
the spin-orbit matrix element $s = \alpha \sqrt{2}/a$. We have also
defined

$$ v_3 = \langle \Phi_\pm | U_{scr} | \Phi_\mp \rangle \approx \langle \Phi_\pm | U_{scr} | \Phi_\pm \rangle. $$

(A2)

The qubit subspace is simply the Zeeman-split orbital ground state $\{ \Phi_\uparrow, \Phi_\downarrow \}$, which has been singled out in
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The qubit subspace is simply the Zeeman-split orbital ground state $\{ \Phi_\uparrow, \Phi_\downarrow \}$, which has been singled out in
the top left hand corner of Eq. [A1].

The density matrix for our system can be written as

$$ \rho = \frac{1}{2} \sigma \cdot S. $$

(B1)

We also know that the diagonal terms of the density matrix represent the probabilities for the system to be measured in either state. Thus the time evolution of the diagonal terms in the density matrix represent the time evolution of the probabilities and hence describe the tendency of a system toward being found in a certain state. In our system this constitutes relaxation.

The density matrix satisfies the quantum Liouville equation:

$$ \frac{d\rho}{dt} + i \frac{\hbar}{\hbar} [H, \rho] = 0. $$

(B2)

Very generally, $\rho$ and $H$ can be written as a sum of diagonal and off diagonal elements,

$$ \dot{\rho}_{ss'} = f_s(t) \delta_{ss'} + g_{ss'}(t) $$

$$ H_{0,ss'} = H_{0,ss} \delta_{ss'} + U_{ss'}(t) $$

where $g$ and $U$ are 0 for $s = s'$. Now we consider the diagonal elements in (B2),

$$ \frac{df_{ss}}{dt} + i \frac{\hbar}{\hbar} [H_{0,ss'}, \rho] + i \frac{\hbar}{\hbar} \sum_{s'} [U_{ss'}, g_{s's}] = 0. $$

(B4)

Now $H_0$ and $f$ commute and thus back in matrix form, equation (B4) reads

$$ \frac{df}{dt} + i \frac{\hbar}{\hbar} [U, g] = 0. $$

(B5)

Next we consider the off diagonal terms, equation (B2) reads

$$ \frac{dg}{dt} + i \frac{\hbar}{\hbar} [H_0, g] = -i \frac{\hbar}{\hbar} [U, f]. $$

(B6)

After switching to a rotating reference frame $g = e^{-i H_{0,t}/\hbar} g(t) e^{i H_{0,t}/\hbar}$ and solving for $g$ one can show that equation (B6) is equivalent to

$$ \frac{df}{dt} + i \frac{\hbar}{\hbar} [U, g(0)] e^{i H_{0,t}/\hbar} $$

$$ -i \frac{\hbar}{\hbar} \int_0^t dt' e^{-i H_{0,t-t'}/\hbar} [U, g(t')] e^{i H_{0,t-t'}/\hbar} = 0. $$

(B7)

Note that $H_0 = \delta E \mathbf{1} + \varepsilon Z \sigma_z$, $f = f(t) \sigma_z$ and $U(t) = U(t) \sigma_z$. In order to solve for $f$ we write

$$ e^{+i H_0(t-t')/\hbar} = e^{+i \varepsilon \sigma_z} $$

$$ = \cos(\frac{\varepsilon_z (t-t')}{\hbar}) \mathbf{1} + i \sin(\frac{\varepsilon_z (t-t')}{\hbar}) \sigma_z $$

$$ = \cos(\chi) \mathbf{1} + i \sin(\chi) \sigma_z $$

where $\chi = \frac{\varepsilon_z (t-t')}{\hbar}$. (B8)

Note that we discarded the term in $H_0$ proportional to the identity matrix as it commutes with everything and will subsequently drop out. After the algebra is carried out, the commutator with the integral in (B7) reads

$$ \left[ U, -i \frac{\hbar}{\hbar} \int_0^t dt' e^{-i H_0(t-t')/\hbar} [U, f] e^{i H_0(t-t')/\hbar} \right] $$

$$ = -i \frac{\hbar}{\hbar} \int_0^t dt' 4 U(t) U(t') f(t) \cos(2\chi) \sigma_z. $$

(B9)
We are free to define $g(0)$ as we please and thus we can adjust it to allow the first term in the commutator in (B7) to cancel the lower limit of the integral. Thus after averaging over noise realisations \( \langle \bar{U}(t)\bar{U}(t') \rangle = |\bar{U}|^2 e^{-(t-t')/\tau} \) and evaluating the integral, (B7) reads
\[
\frac{df}{dt} + \frac{4\tau|\bar{U}|^2}{\hbar^2 + 4\epsilon_2^2\tau^2} f = 0
\]
which has the trivial solution
\[
f(t) = f(0)e^{-t/T_1} \quad \text{with} \quad T_1 = \frac{\hbar^2 + 4\epsilon_2^2\tau^2}{4\tau|\bar{U}|^2}
\]

Appendix C: Dephasing

In a rotating frame $B_0$ is eliminated, and the quantum Liouville equation is solved by
\[
\dot{\rho}(t) = e^{-(i/\hbar)\int_0^t H(t') dt'} \hat{\rho}(t) = 0 e^{(i/\hbar)\int_0^t H(t') dt'} \hat{\rho}(t).
\]
Defining $h(t) = \frac{1}{\hbar} \int_0^t V(t') dt'$, allows us to write the time evolution of the spin polarisation $S_i = \text{tr}(\sigma_i\hat{\rho})$ as
\[
S_i = S_{0i} \cos (h(t)) - \varepsilon_{ijk} S_{0j} h_k(t) \sin (h(t))
\]
\[
+ h_i(t)[\hat{h}(t) \cdot \sigma_0][1 - \cos (h(t))].
\]

If we initialize the $x$-component, we have the equation of motion $S_x(t) = S_{0x} \cos (h(t))$. We require to average this function which is non-trivial due to the fluctuating nature of $h$. After averaging over noise realisations [44],
\[
\langle \cos [h(t)] \rangle = e^{-t/\tau} \left( \frac{\sinh \Xi t}{\Xi t} + \cosh \Xi t \right),
\]
where $\Xi = \sqrt{(h/\tau)^2 - V^2/\hbar}$. For $V^2 \ll (h/\tau)^2$ we have $\sqrt{(h/\tau)^2 - V^2} \approx \frac{h}{\tau} \left( 1 - \frac{V^2}{2\hbar} \right)$, and, ignoring the second term in the expansion when considering the denominator of the sinh term, $\langle \cos [h(t)] \rangle \approx e^{-t/\tau^2}$, where
\[
\left( \frac{1}{T_2^*} \right)_{RTN} = \frac{V^2}{2\hbar^2} = \text{explicit}.
\]

Appendix D: Dipole potential

The charge dipole has dipole moment $p = -e\ell$, where the dipole length $l = (l_x, l_y, l_z) \equiv (l_\perp, l_z)$. The potential of an unscreened charge dipole, with dipole moment $p$, located a distance $R_D$ away from the dot (making sure the potential has units of energy), is
\[
U_{\text{dip}}(R_D) = \frac{p \cdot R_D}{4\pi \varepsilon_0 \varepsilon_r R_D^3} = \frac{p \cdot \hat{R}_D}{4\pi \varepsilon_0 \varepsilon_r R_D^3}.
\]

Appendix E: 1/f noise

The dephasing rate for 1/f noise is given by
\[
\chi(t) = \frac{\gamma k_B T}{2\omega^2} \left[ \frac{8\pi^2 \varepsilon Z}{(\delta\varepsilon)^3} \right]^2 \{ 1 - \cos (\omega_0 t) \}
+ \frac{\omega_0}{2} \left[ \sin (\omega_0 t) - \omega_0 t \sin (\omega_0 t) \right].
\]

This integrates to
\[
\chi(t) = \frac{\gamma k_B T}{2\omega_0^2} \left[ \frac{8\pi^2 \varepsilon Z}{(\delta\varepsilon)^3} \right]^2 \{ 1 - \cos (\omega_0 t) \}
+ \frac{\omega_0 t}{2} \sin (\omega_0 t - \omega_0 t \ln (\omega_0 t)) \right]
+ \frac{\omega_0}{2} \left[ \sin (\omega_0 t) - \omega_0 t \sin (\omega_0 t) \right].
\]

where $\text{Ci}(x) = -\int_0^\infty dt \left( \frac{\cos t}{t} \right)$ represents the cosine integral function. At times $t \ll 1/\omega_0$ such as we consider here, the function in braces can be expanded as a series in $\omega_0 t$, yielding
\[
\chi(t) \approx \left( \frac{\gamma k_B T}{2\omega_0^2} \right) \left[ \frac{8\pi^2 \varepsilon Z}{(\delta\varepsilon)^3} \right]^2 \{ 1 - \gamma_E - \ln (\omega_0 t) \},
\]
with $\gamma_E \approx 0.577$ the Euler constant. In this regime the logarithmic term is dominant, and we can further approximate
\[
\chi(t) \approx \left( \frac{\gamma k_B T}{2\omega_0^2} \right) \left[ \frac{8\pi^2 \varepsilon Z}{(\delta\varepsilon)^3} \right]^2 \{ 1 - \gamma_E - \ln (\omega_0 t) \}
+ \frac{\omega_0 t}{2} \left[ \sin (\omega_0 t) - \omega_0 t \sin (\omega_0 t) \right].
\]

where we recall that $\omega_0 t \ll 1$, so the logarithm is positive. Although the relatively arbitrary choice of $\omega_0$ does alter slightly the final result, we note that the logarithm is a slowly varying function, and by far the biggest contribution to dephasing comes from the coefficient of the $t^2$ term. We can thus write, approximately
\[
\chi(t) \approx \left( \frac{t^2}{T_2^*} \right) \ln \frac{1}{\omega_0 t},
\]
where the dephasing time is estimated by
\[
\left( \frac{1}{T_2^*} \right)_{1/f} \approx \sqrt{\frac{\gamma k_B T}{2\omega_0^2} \left[ \frac{8\pi^2 \varepsilon Z}{(\delta\varepsilon)^3} \right]}.\]