Analysis of Financial Risk Control Based on Stochastic Differential Equation Theory

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Abstract. This paper uses stochastic differential equation and risk theory to study the probability model of insurance company's bankruptcy function and the proportion model of financial insurance investment. It also studies the application of stochastic probability to the investment decision-making of institutional investors. Taking the four major insurance industries in China as an example, it gives the strategy of venture capital.

Introduction

In the financial and insurance industry, extreme events refer to those events that have a very small probability of occurrence and have a huge impact on the whole insurance and financial industry once they occur, such as hurricanes, major car accidents, fires, large earthquakes, financial crises, such as the subprime debt crisis in the United States and the European financial crisis, etc. These extreme events often lead to so-called large claims. Therefore, large investors in financial and insurance institutions, how to choose appropriate investment methods to avoid risks and maximize investment returns has become a major problem facing institutional investors. This paper uses stochastic differential equation theory and some developed model applications to initially solve the strategic problems of institutional investors' venture capital.

A Review of Investment Control Research

Investment is an important part of modern insurance industry. The focus of insurance institutions has shifted from insurance business to investment business. Insurance companies have become comprehensive financial enterprises. At present, China's insurance industry is still in its infancy in insurance investment. At this stage, it is imperative to carry out a comprehensive reform of China's financial system and further accelerate the development of China's insurance investment by learning the advanced experience of developed countries. Insurance investment fund, that is, insurance fund, refers to the part used after deducting margin, reserve for indemnity, compensation, expenditure and tax payment. The existence of a large number of insurance funds makes insurance investment possible, and how to invest has become the focus of all circles of research.

Xiaocui Ma(2008) [1] studied the exact expression of the finite large deviation rate function for stochastic differential equations. Chen Shen (2008) [2] further studied the large deviation problem of the generalized compound Poisson risk model. A series of large deviation and ruin time results were obtained in the residual claim process. These results were applied to some financial and insurance investment risk problems. Gaolin Xu(2006) [3] pointed out insurance. Since the start of A-share direct investment in February 2005, the return of insurance stock investment has exceeded 10%, which is good for the insurance industry. However, as far as the insurance market and capital market are concerned, it is necessary to further analyze the specific composition of insurance fund investment.
Risk theory is a hot topic in the field of stochastic theory. As an important measure method of insurance risk, ruin probability has become a major research topic in risk theory. People usually give the upper bound of Lundberg exponent of ruin function to make more rigorous investment decisions. In the 1970s and 1980s, Donsker and Varadhan's research on Markov processes and Wentzell and Freidlin's work on stochastic perturbations of dynamic systems made great progress in insurance ruin theory.

**Ruin Probability Model**

We consider the problem of investability of insurance institutions, which allow them to invest in risky products such as some stock markets, and establish stochastic geometric Brownian motion equation.

\[
dS_t = bS_t dt + \sigma S_t dB_t
\]

\(b, \sigma > 0\) It's a constant, \(B_t\) it a standard Brownian movement. \(X\) It's an independent risk reserve. By summing up the total \(F = F(t), \ t > 0\) assets at any time \(X, S\) the insurance institution can invest a sum of money \(a_t\) any time \(t\) in venture capital, such as stocks, and the rest in riskless bonds.

Given \(x \geq 0\) the initial capital and a controllable investment ratio \(a\), the formation process of insurance company's wealth can be written.

\[
V_t^{x,a} = x + \int_0^t \frac{a_u}{S_u} dS_u + \int_0^t a_u (b du + \sigma dB_u), t \geq 0
\]

Define the ruin probability of the time risk model. \(P(x, a) = P(\tau_{x,a} < +\infty)\)

Among them, \(\tau_{x,a} = \inf \{t \geq 0 : V_t^{x,a} < 0\}\), the ruin probability depends on the initial \(x\) wealth and investment strategy \(a\). We are concerned about the minimum ruin probability of insurance institutions: \(P^*(x) = \inf \{P(x, a)\}\)

The existing conclusions are as follows:

\[
ke^{-\theta x} \leq P^*(x) \leq e^{-\theta x}, \theta^* > 0
\]

**Measurement of Ruin Probability**

Based on the national insurance revenue data of property insurance institutions from 2008 to 2017, the data parameters of the above formulas are calculated, and the data range of bankruptcy probability of insurance institutions investing in risk category is calculated (Table 1 omits the name of the institution).

| Insurance Institution | \(x\) Initial Capital (RMB 100 million) | \(r\) Rate of Return on Investment | \(a\) Institutional Venture Capital Ratio | \(k\) Measured Constants | \(\theta^*\) Fluctuation Parameters | \(y\) Lower Limit of Ruin Probability | Upper limit |
|------------------------|----------------------------------------|-----------------------------------|------------------------------------------|--------------------------|-----------------------------------|------------------------------------|-------------|
| Insurance Institution1 | 173962.46                              | 0.111                             | 0.3                                      | 0.28                     | 0.0000075                        | 0.076                             | 0.2725      |
| Insurance Institution2 | 61687                                  | 0.112                             | 0.4                                      | 0.21                     | 0.0000243                        | 0.047                             | 0.2231      |
| Insurance Institution3 | 16404.71                               | 0.08                              | 0.35                                     | 0.25                     | 0.0000853                        | 0.062                             | 0.2466      |
| Insurance Institution4 | 13321.64                               | 0.175                             | 0.25                                     | 0.22                     | 0.00012                         | 0.044                             | 0.2019      |
| Insurance Institution5 | 10232.81                               | 0.34                              | 0.2                                      | 0.29                     | 0.000205                         | 0.035                             | 0.1224      |
It can be concluded that the probability of investment bankruptcy of insurance institutions in China is very low, and the types of investment risk of insurance institutions are feasible.

**Strategic Optimization of Insurance Fund Investment**

**Optimizing Investment Strategy Model**

In reference [5], the continuous stochastic model of insurance fund is given, and the stochastic optimal control theory is applied to find out the optimal investment strategy.

Assuming \( F(t) = F(t; t \in [0, N + T]) \) that it represents the total assets of the insurance fund, the fund can invest in two portfolios: The risk-free portfolio \( X_1 \) and the risk portfolio \( X_2 \) are as follows:

\[
dX_1(t) = rX_1(t)dt \\
dX_2(t) = aX_2(t)dt + \sigma X_1(t)d\omega(t)
\]

Among them \( \omega(t) \) is the standard geometric Brownian motion. At \( t \) the moment, the proportion of risky assets invested is \( u(t) \); the proportion of riskless assets invested is \( 1 - u(t) \).

Therefore, the change process of the total \( F(t) \) assets of insurance fund is as follows:

\[
dF(t) = F(t)[u(t)a + (1 - u(t))r]dt + F(t)u(t)d\omega(t)
\]

The objective function is to maximize the utility function of two stages: \( \max EU(F(U)) \).

Apply the principle of maximization and order

\[
\psi = \left[ \frac{\partial W}{\partial t} + \left( u(t)(a - r) + r \right) F \frac{\partial W}{\partial F} + \frac{1}{2} u^2(t) \sigma^2 F^2 \frac{\partial^2 W}{\partial F^2} \right]
\]

(7)

The maximization principle is as follows:

\[
(a - r)F \frac{\partial W}{\partial F} + u^*(t) \sigma^2 F^2 \frac{\partial^2 W}{\partial F^2} = 0
\]

(8)

\[
\frac{\partial W}{\partial t} + rF^2 \frac{\partial W}{\partial F} + \frac{1}{2} \left( a - r \right)^2 \left( \frac{\partial W}{\partial F} \right)^2 = 0
\]

(9)

Among them \( W(N, F) = U(F) \) ： Solution \( W \) brings in solvable optimization strategy \( u^*(t) \).

**Index Benefit Optimizing Model**

Take the exponential utility function as follows:

\[
U(F) = -ke^{\frac{1}{k}F}, k > 0
\]

(10)

Its value function is:

\[
V(t, F) = -k \exp \left\{ -\frac{1}{k} \left[ a(t) + b(t)(F - k^{-1}(t)) \right] \right\}
\]

(11)

Among them, the constraints are:

\[
a(N + T) = 0, b(N + T) = 1, k^{-1}(N + T) = 0
\]

(12)

The optimal investment ratio can be obtained by using the above method as follows:
\[ u^*(t) = \frac{e^{(r-N)}k(a-r)}{F\sigma^2}, (t \in [0, N]) \]  

(13)

**Application Analysis of Institutional Investment Examples**

Based on the national insurance income data from 2006 to 2017, the data parameters of the above formulas are calculated, and the proportion strategy of insurance institutions' investment in risk categories is calculated. According to statistics, the profit of general social security fund is calculated as follows: taking risk-free interest rate; taking one year as a period of time, the fluctuation of risk profit is different.

As the insurance fund needs 40% reserve and daily compensation, the actual calculation takes half of the total premium income. The results of venture capital calculation are as follows:

| Types of Insurance | Premium Income(10 0 million yuan) | Compensation and Payment (100 million yuan) | Investment Interest Rate | Risk-free Interest Rate | Investment Interest Rate | Risk-Free Rate of Interest | Proportion of Institutional Venture Capital |
|--------------------|-----------------------------------|---------------------------------------------|--------------------------|------------------------|--------------------------|----------------------------|---------------------------------------------|
| Gross Premium      | 14339                             | 3929                                        | 333.33                   | 0.009                  | 0.12                     | 0.05                       | 0.344                                       |
| Property Insurance | 4681                              | 2187                                        | 19.61                    | 0.0016                 | 0.12                     | 0.05                       | 0.353                                       |
| life Insurance     | 8696                              | 1301                                        | 66.67                    | 0.0023                 | 0.12                     | 0.05                       | 0.444                                       |
| Health-Insurance   | 692                               | 360                                         | 21.28                    | 0.01                   | 0.12                     | 0.05                       | 0.409                                       |
| Accident Insurance | 334                               | 82                                          | 33.33                    | 0.044                  | 0.12                     | 0.05                       | 0.302                                       |

It can be seen that the formula can be used to calculate the reasonable proportion of venture capital investment between 30% and 44%, which is in line with the actual investment situation.

**Investment Policy Suggestions for Insurance Institutions**

Learning from the advanced experience of foreign countries to improve the operation ability of insurance funds in China

In developed countries, the issuance of bonds by insurance funds often stands at more than 30%, which can increase the variety of bond investment and guarantee the important source of funds for the development of insurance investment. Real estate investment returns are high, and insurance funds are allowed to invest in real estate, but the proportion should not be high.

Establishing insurance fund management companies to improve the professional evaluation of insurance fund investment

Foreign insurance funds have special management foundations. Insurance regulatory authorities should allow conditional insurance companies to set up fund management companies and participate in capital market operation in accordance with international practices, so as to make the most efficient use of insurance funds.

Strengthen the supervision and management of insurance investment and strive for strong competitiveness

Insurance companies must establish a sound risk supervision and management system. With the rapid entry of foreign insurance companies into the Chinese market, the competition in the insurance industry has become increasingly fierce. In order to integrate Chinese insurance investment with
international insurance investment, we must first strengthen the supervision mode and improve the supervision Water evaluation, otherwise it will not be competitive.

Strengthen the quality of insurance investment professionals and improve the benefits of insurance investment

Insurance investment needs specialized personnel to plan and implement. Decision-making committees composed of investors, accountants, actuaries and other experts are required to make decisions. All these require people to have rich insurance investment and other professional knowledge and skills. Therefore, cultivating a high-quality insurance investment management team with both ability and morality is the key to improve the efficiency of insurance investment in China and achieve maximum benefits.

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