Coupled-channel calculations for fusion reactions of $^{6}$Li+$^{64}$Ni, $^{40}$Ca+$^{96}$Zr and $^{124}$Sn+$^{48}$Ca Systems

Hanan A. Shadhan, Fouad A. Majeed and Mohanad H. Oleiwi

College of Education for Pure Sciences, University of Babylon, Babylon, Iraq
fmajeed@uobabylon.edu.iq

Abstract: A coupled-channel calculations has been performed to study the cross-section of fusion $\sigma_{\text{fus}}$ and the distribution of fusion-barrier $D_{\text{fus}}$ and the fusion probability $P_{\text{fus}}$ for the systems $^{6}$Li+$^{64}$Ni, $^{40}$Ca+$^{96}$Zr and $^{124}$Sn+$^{48}$Ca by using the semiclassical method and Wong formula. The adopted semiclassical approach is mainly used for the treatment of the Coulomb excitation of the nucleus. The fusion barrier distribution is determined for both experimental and theoretical using the difference between three-points method. Theoretical results were compared with the measured data and shows very good agreement for $^{6}$Li+$^{64}$Ni and $^{124}$Sn+$^{48}$Ca systems, while for the system $^{40}$Ca+$^{96}$Zr the results were not agreed below and above the Coulomb barrier $V_{b}$.

Keywords: Coupled-channels, Fusion cross section, Fusion barrier distribution, Wong formula, Fusion reaction.

1. Introduction

The process of nuclear fusion remains one of the most interesting and intensively studied phenomena. The study of this process is very important, for mixture of super heavy nuclei and understanding of astrophysical nucleogenesis. The interactions of two nuclei can lead to a number of different processes. In a semiclassical paradigm, it is usual to differentiate between compound nucleus response (The fusion reactions) with direct reactions by using the impact parameter, or relative angular momentum. The former occurs for more central interactions with the impact value corresponding to the grazing trajectory. In intermediate activity, deep and inelastic reactions demonstrate. There are several variables that are responsible for the relative significance of the different mechanisms; kinetic energy and its importance play a very important role with regard to the amount of Coulomb repulsion between [1, 2].

The Coulomb sub-barrier height is higher compared to the kinetic energy; this independence no longer holds: the behavior of a certain process cannot be considered separately from other processes any longer. The definition of the coupling of the various reaction channels reflects this in the principle of diffusion. The total wave function of the dispersion problem includes
the entrance channel and all the exit channels; the Hamiltonian uses potential concepts such as exciting potential [3, 4]. The effect of coupled channels using semiclassical and full quantum mechanical models have been studied for different systems in our previous study and we had proved its success especially for the improvement of the determination of the fusion cross-section of $\sigma_{\text{fus}}$ below the Coulomb barrier $V_b$, we refer the readers to check our previous published work [5-17]. This goal of this work is to investigate the effect of considering the effect of coupling to fusion reactions calculation of $\sigma_{\text{fus}}$, $D_{\text{fus}}$, $P_{\text{fus}}$ for the reactions $^6\text{Li}+^64\text{Ni}$, $^40\text{Ca}+^96\text{Zr}$, $^{124}\text{Sn}+^{48}\text{Ca}$. The adopted semiclassical approach is mainly employed for the treatment of the Coulomb excitation of the nucleus. The fusion barrier distribution is determined for both experimental and theoretical using the difference between three-points method to compare with the measured data will be performed to support our theoretical predictions.

2. **Theoretical framework**

2.1 **Semiclassical theory**

The theory based on semiclassical method which previously had been used to study the fusion reactions in a single barrier model of potential which treats the relative motion between the colliding heavy ions as free parameter [18,19]. The theory of semiclassical addresses the equation of Schrodinger which takes over independent angular momentum and energy as well as the energy potential for motion relative to coordinates and the part of radial component via tunneling of quantum phenomena [19],

$$[-\hbar^2 \nabla^2 / 2\mu + V(r) - E] \psi(r) = 0 \quad (1)$$

where $V(r)$ represents the system potential and $\mu$ is the system reduced mass. The wavefunctions of eq. 1 can be estimated from the time dependent Schrodinger equations by proposing the trajectory of the particle can be determined using the classical Rutherford trajectory, by incorporating the real part of the Coulomb and the potential of the centrifugal part, which is written as [19],

$$V(r) = V_C(r) + V_N(r) + V_\ell(r) \quad (2)$$

The nuclear potential imaginary part is a complex that should be included to account for deep absorption in the classically forbidden to the coupled-channel calculations on the scattering of the elastic channel [19],

$$V_N(r) = U_N(r) - i W(r) \quad (3)$$

The expansion of waves method has been used to study the strong absorption due to interference of $\ell$ waves comes from the contribution the real and imaginary parts of the nuclear potential [18,20,21]. According to the semiclassical theory, the fusion takes place when two nuclei comes close together by crossing the barrier of the potential into the inner region, then the penetration probability can be determined by using the WKB method below the barrier [18,22,23].
where local wavefunction number is $\kappa_l(r)$ and the limits $r_a(l)$ and $r_b(l)$ defined as the points of turning for the classical trajectory, therefore eqn. 4 can be written as

$$P_{\text{ fus}}^{\text{ WKB}}(\ell, E) = \frac{1}{1 + e \left( \frac{2 \pi \hbar \Omega_l}{\nu_b(l) - E} \right)}$$

(5)

Since the barrier of the fusion can be fitted by a parabola, therefore the probability of penetration can be described by the Hill–Wheeler formula [19]

$$P_{\text{ fus}}^{\text{ WH}}(\ell, E) = \frac{1}{1 + e^{\exp \left[ \frac{2 \pi \hbar \Omega_l}{(E - \nu_b(l))} \right]}}$$

(6)

where $\Omega_l$ and $\nu_b(l)$ are the parameters for the curvature and height of the barrier that employed to determine the fusion barrier, respectively, and the projectile energy bombarding the target is $E$. The approximation of WKB can be employed to calculate the cross-section of fusion by using these equations [21,24]:

$$\sigma_{\text{ fus}}(E) = \frac{\pi}{\kappa^2} \sum (2\ell + 1) P_{\text{ fus}}^{\text{ WKB}}(\ell, E)$$

(7)

$$P_{\text{ fus}}^{\gamma}(\ell, E) = \frac{4 \kappa}{E} \int |u_{\gamma \ell}(k \gamma, r)|^2 W_{\text{ fus}}^{\gamma}(r) \, dr$$

(8)

where $W_{\text{ fus}}^{\gamma}(r)$ is the potential imaginary part and $u_{\gamma \ell}(k \gamma, r)$ is the radial part of the wavefunction for $\ell$ partial-wave in $\gamma$-channel.

The theory of the semiclassical method used to determine the heavy ions fusion is based on the degrees of freedom, the trajectory of the classical motion $r$, and the projectile intrinsic states, denoted as $\zeta$. The coupled-channel Hamiltonian then reads,

$$h = h_0(\zeta) + V(\zeta, r)$$

(9)

where $V(\zeta, r) = V_C(\zeta, r) + V_N(\zeta, r)$, $h_0(\zeta)$ is the Hamiltonian of the intrinsic states, and $V(\zeta, r)$ is interaction of target-projectile. Rutherford's path relates collision energy, $E$ and angular momentum, $l$. The solution for the classical equation $V(r) = \langle \psi_0 | V(r, \zeta) | \psi_0 \rangle$, where $\psi_0$ is ground state of the projectile. Since the interaction is time-dependent $V_l(\zeta, t) = V(r_{\ell(t)}, \zeta)$, and the Hamiltonian intrinsic eigenstates the for $|\psi_\gamma \rangle$ satisfies Schrödinger eqn. [26,27].

$$h|\psi_\gamma \rangle = \varepsilon|\psi_\gamma \rangle$$

(10)

In terms of the basis, the extension of the wave function then reads,

$$\psi(\zeta, t) = \sum a_\gamma(\ell, t) \psi_\gamma(\zeta) e^{-i \varepsilon_\gamma t / \hbar}$$

(11)
By substituting equation (11) into equation (10), this leads to Alder-Winther coupled equations will be achieved as,

\[ i\hbar \frac{\partial \gamma(l, t)}{\partial t} = \sum \langle \psi_\gamma | V(\zeta, t) | \psi_\gamma \rangle \varepsilon e^{i(\varepsilon_\gamma - \varepsilon_\epsilon) t} \]

\[ \gamma(l, t) \]

Where the initial conditions \( a_\gamma(\ell, t \rightarrow -\infty) = \delta_\gamma \) will be used to solve these coupled differential equations at \( (t \rightarrow -\infty) \). The fusion probability \( P_{\text{fus}}^\gamma(\ell, E) = |a_\gamma(\ell, t \rightarrow -\infty)|^2 \) of \( \gamma \)-channel final population [19].

### 2.2 Distribution of Fusion Barrier

The distribution of the barrier of the fusion \( D_{\text{fus}} \) is very sensitive parameter to be determined which is defined as [22,24],

\[ D_{\text{fus}}(E) = \frac{d^2 F(E)}{dE^2} \]

where \( F(E) \) is fusion barrier distribution function defined as,

\[ F(E) = E \sigma_{\text{fus}}(E) \]

Experimentally defined, the distribution of the reaction fusion barrier contributed to very significant progress in understanding the fusion reaction. There uncertainties in the numerical calculations of the fusion barrier can be determined from the reaction [28,29],

\[ D_{\text{fus}}(E) \approx \frac{F(E+\Delta E)+F(E-\Delta E)-2F(E)}{\Delta E^2} \]

here the interval is \( \Delta E \) from each measured data of cross section with excitation energy. The statistical error then determined from the relation [22],

\[ \delta D_{\text{fus}}^{\text{stat}}(E) \approx \frac{\sqrt{[\delta F(E+\Delta E)]^2+[\delta F(E-\Delta E)]^2}}{\Delta E^2} \]

where \( \delta F(E) \) is the product uncertainty of \( (E \sigma_f) \) at for each collision energy, therefore the uncertainty is [22],

\[ \delta D_{\text{fus}}^{\text{stat}}(E) \approx \frac{\sqrt{\delta E \delta F(E)}}{(\Delta E)^2} \]

### 3. Results and Discussion

The results of \( \sigma_{\text{fus}}, D_{\text{fus}} \) and \( P_{\text{fus}} \) using the semiclassical method and Wong formula. The adopted semiclassical approach is mainly employed in the treatment of the Coulomb excitation of the nucleus. The fusion barrier distribution is determined for both experimental and theoretical using the three-point difference method for the systems \(^6\text{Li}+^{64}\text{Ni}, \quad ^{40}\text{Ca}+^{96}\text{Zr} \) and \(^{124}\text{Sn}+^{40}\text{Ca} \). The parameters of Akyüz-Winther for potential are listed in Table 1.
Table 1. The imaginary and real parameters of Akyüz-Winther potential.

| Systems      | \( V_0 \) (MeV) | \( r_0 \) (fm) | \( a_0 \) (fm) | \( V_b \) (MeV) | \( r_i \) (fm) | \( W_0 \) (MeV) | \( a_i \) (fm) | \( L_{\text{min}} \) | \( L_{\text{max}} \) |
|--------------|-----------------|----------------|---------------|----------------|----------------|----------------|---------------|----------------|----------------|
| \(^6\text{Li}^+\text{Ni}\) | -35.0           | 1.197          | 705           | 12.05          | 927            | -11.7          | 784           | 0             | 20             |
| \(^{40}\text{Ca}^+\text{Zr}\) | -148.9          | 1.200          | 780           | 100.42         | 1.041          | -49.6          | 715           | 0             | 131            |
| \(^{124}\text{Sn}^+\text{Ca}\) | -113.9          | 1.100          | 800           | 118.85         | 1.073          | -38.0          | 696           | 0             | 117            |

3.1 \(^6\text{Li}^+\text{Ni}\) System

Figure 1 with panels (a), (b) and (c) displays the comparison between theoretical values for \( \sigma_{\text{fus}} \), \( D_{\text{fus}} \) and \( P_{\text{fus}} \) for the system \(^6\text{Li}^+\text{Ni}\). The blue color curve is the semiclassical calculations, while red curve color is the Wong fit formula, the blue dashed curve is semiclassical calculations without coupling. The measured data for this system from [30]. Theoretical results were compared with the measured data and shows excellent agreement with the experimental data.

![Figure 1: Semiclassical and Wong Fit calculations for the system \(^6\text{Li}^+\text{Ni}\) for \( \sigma_{\text{fus}} \) panel (a), \( D_{\text{fus}} \) panel (b) and \( P_{\text{fus}} \) panel (c) with experimental data [30].](image-url)
3.2 $^{40}$Ca+$^{96}$Zr System

Figure 2 panels (a), (b) and (c), displays the theoretical and experimental $\sigma_{fus}$, $D_{fus}$ and $P_{fus}$, respectively, using semiclassical and Wong formula for the system $^{40}$Ca+$^{96}$Zr. The data for the system are from Ref. [31]. The blue color curves both solid and dashed, represents the semiclassical calculations with and without coupling, respectively. The dashed curves blue color represents the results without coupling, while the solid red curves are the results with Wong calculations. Our theoretical calculations for the semiclassical with coupling channel reproduce the data well above Coulomb barrier.

**Figure 2:** Semiclassical and Wong Fit calculations for the system $^{40}$Ca+$^{96}$Zr for $\sigma_{fus}$ panel (a), $D_{fus}$ panel (b) and $P_{fus}$ panel (c) with experimental data [31].
3.3 $^{124}\text{Sn}+^{48}\text{Ca}$ System

Figure 3 with the three panels (a), (b) and (c) displays our theoretical calculations with comparison with the measured data for $\sigma_{\text{fus}}$, $D_{\text{fus}}$ and $P_{\text{fus}}$ for the systems $^{124}\text{Sn}+^{48}\text{Ca}$. The potential parameters used for this system are tabulated in Table 1. The data for this system are from Ref. [32]. Both semiclassical calculations with and without coupling are the same for $\sigma_{\text{fus}}$ and slightly different for $D_{\text{fus}}$ and $P_{\text{fus}}$. Both semiclassical and Wong Fit could not reproduce the data below the Coulomb barrier $V_b$ indicated by green arrow on the $E_{\text{c.m.}}$ axis this is due to the fact that the probability reduced in the classically forbidden region.

![Figure 3: Semiclassical and Wong Fit calculations for the system $^{124}\text{Sn}+^{48}\text{Ca}$ for $\sigma_{\text{fus}}$ panel (a), $D_{\text{fus}}$ panel (b) and $P_{\text{fus}}$ panel (c) with experimental data [32].](image)
4. Conclusion

The semiclassical results for $\sigma_{fus}$, $D_{fus}$ and $P_{fus}$ below and above Coulomb barrier for the reactions $^6\text{Li}+^{64}\text{Ni}$, $^{40}\text{Ca}+^{96}\text{Zr}$, $^{124}\text{Sn}+^{40}\text{Ca}$ have been studied by using semiclassical and Wong Fit approaches. The comparison shows that the semiclassical approach with two channel coupling is able to study the selected systems and can be considered as potential alternative to the full quantum mechanical models. Wong Fit model is somehow able to reproduce the data even if it is very simple method used to fit the measured data. This work can be extended to cover more reactions lies in different regions and possible enhancement on the forbidden classical region of WKB might improve the calculations.

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