QCD Sum Rules and the Induced Pseudoscalar Coupling

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Abstract

We present an extension of the QCD sum rule method in the external fields so as to determine the induced pseudoscalar coupling constant $g_P$, which tests the validity of the partially conserved axial current (PCAC) hypothesis. This is essentially that we pick out the "higher-order" effects of both the hadron and quark (QCD) sides. A specific QCD sum rules for $g_P$ is obtained and its prediction is briefly analyzed. It turns out that the final prediction on $g_P$ is extremely stable. In view of the versatile nature of the present QCD sum rule methods, we appendix some discussions on the possible future of the method.

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1 Introduction

The matrix element of the isovector axial current between the on-shell nucleon states,\(^1\) with \(q_\mu \equiv p_\mu - p'_\mu\),

\[
< n(p') | A_\mu(0) | p(p) >
= \bar{u}(p') \{ f_A(q^2) \gamma_\mu \gamma_5 + f_P(q^2) q_\mu \gamma_5 \} u(p),
\] (1)

plays a fundamental role in the description of semileptonic weak interactions such as beta decay, muon capture, and neutrino-induced charged weak reactions. Here \(g_A \equiv f_A(q^2 = 0) = 1.2695 \pm 0.0029\) is the axial coupling\(^2\) while \(g_P \equiv f_P(q^2 = 0)\) is the induced pseudoscalar coupling. It is well-known that the one-pion pole contributes to the induced pseudoscalar form factor \(f_P(q^2)\), in the way that the proton turns into the neutron by emitting a pion which in turn couples to \(W^\pm\). Such a one-pion pole contribution is\(^3\)

\[
f_P(q^2) = \sqrt{2} g_{\pi NN}(q^2) \cdot \frac{1}{q^2 - m_\pi^2} \cdot \sqrt{2} f_\pi(q^2),
\] (2)

where \(f_\pi(q^2 = 0)\) is the pion decay constant and \(g_{\pi NN}(q^2)\) the strong \(\pi NN\) coupling.

Assuming that this is the only major contribution to \(f_P(q^2)\) and using the partially conserved axial current (PCAC) hypothesis that \(\partial_\mu A^\mu(x) \sim O(m_\pi^2) \sim 0\), we find, in the limit that \(m_\pi^2 = 0\),

\[
M f_A(q^2) - g_{\pi NN}(q^2) f_\pi(q^2) \approx 0;
M g_A \approx g_{\pi NN} f_\pi,
\] (3)

where \(M \equiv \frac{1}{2}(m_n + m_p)\) is the nucleon mass and \(m_\pi\) is the pion mass. Note that smooth extrapolation to \(q^2 = 0\) is needed in obtaining the Goldberger-Treiman (GT) relation,\(^4\) the second identity in Eq. (3).

2 Formulation

Although the method of QCD sum rules as originally developed\(^5\) was applied to the study of hadronic properties in the region of about 1 GeV, Ioffe and Smilga\(^6\) developed
techniques for embedding hadrons in an external field in order to derive static properties in terms of the condensates, including induced condensates which introduce new parameters. The method of QCD sum rules in the presence of external axial fields has been employed\textsuperscript{7,8,9} to extract the axial coupling $g_A$. It is the purpose of the present paper to indicate how this method may be suitably generalized to obtain the induced pseudoscalar coupling $g_P$.

We begin by briefly reviewing the method for an external axial field. The starting point is the polarization function in an external axial field, which we call $Z_\mu$. The correlation function, $\Pi(p)$, is defined as\textsuperscript{6–9}

$$\Pi(p) \equiv i \int d^4x e^{ipx} < 0 \mid T(\eta(x)\bar{\eta}(0)) \mid 0 >, \quad (4)$$

where for the nucleon current we may use a standard form

$$\eta(x) = \epsilon^{abc}\{u^a(x)^T C\gamma_\mu u^b(x)\} \gamma^\mu \gamma^5 d_c(x),$$

$$< 0 \mid \eta(0) \mid N(p) > \equiv \lambda_N v_N(p), \quad (5)$$

with $C$ the charge conjugation operator, $a, b, c$ color indices, and $v_N(p)$ Dirac spinor for the nucleon normalized such that $\bar{v}(p)v(p) = 2M$. Embedding the system in an external $Z_\mu$ field and introducing intermediate states, one can express the correlation function in the limit of a constant external field, $Z_\mu(x) = Z_\mu$, as\textsuperscript{9}

$$\Pi(p) = -|\lambda_N|^2 \frac{1}{p - M_N} g_A Z_\mu \gamma_5 \frac{1}{p - M_N} + \cdots, \quad (6)$$

with $\hat{a} \equiv \gamma_\mu a^\mu$, where Eq. (1) has been adopted. The term shown in Eq. (6) corresponds to nucleon intermediate states, while the continuum contributions to $\Pi$ are implied by the ellipses in Eq. (6). Eq. (6) is the expression for the phenomenological form, in which $\Pi(p)$ is evaluated at the baryon level. When evaluating the correlation function $\Pi(p)$ at the quark level and comparing it with Eq. (6), one is led to three sum rules involving $g_A$, which may not be consistent among themselves although there is indeed one sum rule\textsuperscript{9} which seems most appropriate for $g_A$. 

3
We note that Eq. (1) gives the on-shell matrix element of the axial current. In treating the correlator \( \Pi(p) \) in the presence of an external axial field \( Z_\mu(x) \), one may consider the slightly off-shell nucleon matrix elements, where additional off-shell form factors can occur. For instance,

\[
< N(p', \lambda') | J_5^\mu(0) | N(p, \lambda) > = \bar{u}_{\lambda'}(p') \{ G_1(q^2)\gamma_\mu\gamma_5 + G_2(q^2)q_\mu\gamma_5 + G_3(q^2)i\sigma_\mu\nu P^\nu\gamma_5 \} u_\lambda(p),
\]

with \( P_\mu \equiv p'_\mu + p_\mu \). In the on-shell limit, this reduces to Eq. (1) with \( g_A = G_1 + 2MG_3 \) and \( g_P = G_2 - G_3 \). Among the three sum rules which one obtains by comparing coefficients of \( p \cdot Z\hat{p}\gamma_5 \), \( \hat{Z}\gamma_5 \), and \( i\sigma_\mu\nu Z_\mu^\nu p_\nu\gamma_5 \), only the axial coupling \( g_A \) enters in the on-shell limit, making it difficult to determine the pseudoscalar coupling \( g_P \). This is a general problem for obtaining the induced couplings [such as the anomalous magnetic moment and the pseudoscalar coupling] or form factors [such as the \( q^2 \)-dependence of \( f_A(q^2) \)] in the QCD sum rule method.

To obtain a QCD sum rule for the induced pseudoscalar coupling \( g_P \), we consider the external axial field \( Z_\mu(x) \) as follows:

\[
Z_\mu(x) = Z^0_\mu + \frac{1}{2}Z_{\mu\nu}x^\nu,
\]

where \( Z^0_\mu \) and \( Z_{\mu\nu} \) are constants. We have, in momentum space,

\[
Z_\mu(q) = Z^0_\mu \delta^4(q) - \frac{i}{2}Z_{\mu\nu}\partial^\nu\delta^4(q).
\]

We shall focus our attention on the \( Z_{\mu\nu} \) terms, in a way similar to the work of calculating the anomalous magnetic moments\(^6\). This implies that Eq. (6) is to be replaced by

\[
\Pi_A(p) = \frac{\langle |\lambda| \rangle^2}{(p^2 - M^2)^2} \left\{ \frac{1}{2}Z^\mu_\mu\sigma_\mu\gamma_5[MG_1 + (p^2 + M^2)G_3] + \frac{1}{2}Z^\mu_\nu\sigma_\mu\gamma_5[MG_1 + 2MG_3] \right. \\
+ iZ^\mu_\nu\sigma_\mu\gamma_5[p_\nu\gamma_5\frac{1}{p^2 - M^2}((p^2 + M^2)G_1 + 4Mp^2G_3] \\
+ iZ^\mu_\nu\sigma_\mu\gamma_5[p_\nu\gamma_5\frac{1}{p^2 - M^2}][2MG_1 + 2(p^2 + M^2)G_3] \} + \cdots,
\]

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for the antisymmetric part of $Z_{\mu\nu}$, and

$$
\Pi_S(p) = \frac{|\lambda N|^2}{(p^2 - M^2)^2} \left\{ -\frac{i}{2} g_{\mu\nu} Z_{S}^{\mu\nu} \gamma_5 [M G_1 + 2 p^2 G_3 + (-p^2 + M^2) G_2] 
+ \frac{i}{2} g_{\mu\nu} Z_{S}^{\mu\nu} \gamma_5 \hat{p} [G_1 + 2 MG_3] 
+ i Z_{S}^{\mu\nu} \gamma_5 \hat{p} \frac{1}{p^2 - M^2} [2MG_1 + 2(p^2 + M^2)G_3] 
+ i Z_{S}^{\mu\nu} \gamma_5 \hat{p} \frac{1}{p^2 - M^2} [2G_1 + 4MG_3] 
+ i Z_{S}^{\mu\nu} \gamma_5 \hat{p} \frac{1}{p^2 - M^2} [(p^2 + M^2) G_1 + (3p^2 + M^2) MG_3] 
+ i Z_{S}^{\mu\nu} \gamma_5 \hat{p} \frac{1}{p^2 - M^2} [2MG_1 + (p^2 + 3M^2) G_3] \right\} + \cdots, \tag{11}
$$

for the symmetric part of $Z_{\mu\nu}$, where we have $Z_{\mu\nu}^{\text{sym}} \equiv Z_{A}^{\mu\nu} + Z_{S}^{\mu\nu}$ with $Z_{A}^{\mu\nu} = -Z_{A}^{\mu\nu}$ and $Z_{S}^{\mu\nu} = Z_{S}^{\mu\nu}$.

We note that, in the on-shell limit ($p^2 \to M^2$), all the coefficients in Eqs. (10) and (11) reduce to $g_A (= G_1 + 2MG_3)$ except the one proportional to $g_{\mu\nu} Z_{S}^{\mu\nu} \gamma_5$ (in which the induced pseudoscalar coupling $f_P$ or $G_2$ enters). Therefore, we wish to focus on the sum rule obtained by working with this specific Lorentz structure.

Next, we need to evaluate the correlation function at the quark level, making use of the quark propagator in the presence of gluonic and $Z$ fields. The quark propagator is defined by

$$
i S_{ij}^{ab}(x) \equiv < 0 \left| T(q_i^a(x)q_j^b(0)) \right| 0 >. \tag{12}
$$

Following the method of Ref. 6, including terms up to second order in the Taylor expansion, we find, in the presence of $Z_{0}^{\mu}$ and $Z_{\mu\nu}^{S}$,

$$
i S_{ij}^{ab}(x) = \frac{\delta^{ab}}{(2\pi)^4 x^4} \left\{ i\hat{x} - g(x \cdot Z_{0}^{\mu} - \frac{1}{4} Z_{S}^{\mu\nu} x_{\mu} x_{\nu}) \hat{x} \gamma_5 \right\} 
+ \frac{i}{32\pi^2 x^2} g_\epsilon \frac{\lambda^{n}}{2} C_{\mu\nu}^{n}(\hat{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \hat{x}) 
+ \delta^{ab} < \bar{q}q > \left\{ -\frac{1}{12} (1 + \frac{1}{16} x^2 m_0^2) + \frac{1}{12} g \chi \hat{x} \gamma_5 - \frac{1}{12} g \chi' g_{\mu\nu} Z_{S}^{\mu\nu} \gamma_5 \right\}
$$

5
Here the condensate parameters are defined by

\[<0 | \bar{q} g_c \sigma \cdot G q | 0 > = -m_0^2 < \bar{q} q >,\]

\[<0 | \bar{q} g_c \tilde{G}_{\mu \nu} \gamma^\nu q | 0 > = g \kappa Z_\mu^0 < \bar{q} q >,\]

\[<0 | \bar{q} i \gamma_5 q | 0 > = g \chi Z_\mu^0 < \bar{q} q >,\]

\[<0 | \bar{q} i \gamma_5 q | 0 > = g \chi' g_{\mu \nu} Z_5^{\mu \nu} < \bar{q} q >,\]

Our form for the quark propagator, Eq. (13), is the same as that of Ref. 9 except for the additional terms related to \(Z_S^{\mu \nu}\) — these new terms resemble the terms in \(Z_\mu^0\) (and if so desired) may be represented pictorially by the same diagrams as shown in Ref. 9. Note that the first four terms in Eq. (13) are the perturbative free quark propagator, while the remaining are nonperturbative terms, proportional to the quark condensate \(< \bar{q} q >\). The other quantities appearing in Eq. (13) are the \(Z\)-quark coupling constant \([g = g_u = -g_d\) for the isovector axial coupling \(g_A\) or \(g = g_u = g_d\) for the isoscalar axial coupling \(g_A^S\)].

We may proceed to evaluate the correlation function \(\Pi(p)\) at the quark level by considering the processes up to a certain dimension. This has become a routine but standard exercise in the QCD sum rule practices.\(^{6,8,9}\) In particular, we use

\[<0 | T(e(x)\bar{e}(0)) | 0 > = -2\epsilon^{abc} \epsilon^{a'b'c'} Tr\{iS(x)^{b'\nu}_{\mu'} C_i S(x)^{a_{\mu'}T}_{\nu} C_{\gamma_{\mu}}\}
\cdot \gamma_5 \gamma^\mu iS(x)^{c_{\nu'}\nu}_{d} \gamma^\nu \gamma_5.\]

In the present case, we obtain, up to dimension \(D = 6\) (as compared to the leading diagram, counted as \(D = 0\)),

\[+ \frac{21}{192} g_{\mu \nu} Z_S^{\mu \nu} x^\nu \gamma_5 + \frac{1}{216} g_0 (\frac{5}{2} x^2 \tilde{Z} - x \cdot Z \tilde{x}) \gamma_5\]

\[+ \frac{1}{192} g_{\mu \nu} Z_S^{\mu \nu} x^\nu \gamma_5 \} + \cdots.\]

(13)
which, upon Fourier transform, gives rise to the correlation function $\Pi(p)$ evaluated at the quark level, the left-hand-side (l.h.s.) of the sum rule. Comparing the coefficients for both the expressions $ig_{\mu\nu}Z_S^{\mu\nu}\gamma_5$ and $iZ_S^{\mu\nu}\gamma_\mu p_\nu\gamma_5$ (and thus obtaining two QCD sum rules soon to be combined), performing Borel transform on both the r.h.s. and l.h.s., taking into account the anomalous dimensions, and combining the two sum rules, we arrive at the following QCD sum rule,

$$-g_d \frac{M_B^6}{8} L^{-1/9} E_2 + \frac{g_u - g_d^2 G^2}{8} M_B^2 L^{-1/9} E_0 - \frac{g_d M_B^4 \kappa' a}{2} M L^{-1/9} E_1 - \frac{2}{3} g_d a^2 L^{4/9}$$

$$-\frac{g_d M_B^2 \kappa' a}{4 L^{68/81}} M E_0 = \beta_N^2 \exp(-M^2/M_B^2) (g_A + Mg_P),$$  

(17)

with $a = -(2\pi)^2 < \bar{q} q >$ and $L = 0.621 ln(10 M_B)$, corresponding to $\Lambda_{QCD} = 0.1 GeV$ with the Borel mass, $M_B$, in GeV and $\beta_N^2 \equiv (2\pi)^4 \lambda_N^2 / 4 (\approx 0.26 GeV^6)$. Note that the factors $E_0 = 1 - e^{-x}$, $E_1 = 1 - (1 + x) e^{-x}$, and $E_2 = 1 - (1 + x + \frac{1}{2} x^2) e^{-x}$, with $x \equiv W^2 / M_B^2 \approx (2.3 GeV^2) / M_B^2$ (see Ref. 9), describe the contributions from the excited states through perturbative QCD method\textsuperscript{10,11}.

We recall the QCD sum rule for $g_A$ as obtained from Ref. 9, again up to dimension $D = 6$,

$$\frac{M_B^4 E_2}{8 L^{4/9}} + \frac{1}{32 L^{4/9}} < g_c^2 G^2 > E_0 - \frac{1}{18 L^{68/81}} \kappa a E_0 + \frac{5}{18 M_B^2} a^2 L^{4/9}$$

$$= \beta_N^2 \exp(-M^2/M_B^2) g_A.$$  

(18)

Combining Eqs. (18) and (17) [the latter with $g_u = -g_d = 1$], we obtain the QCD sum rule for $g_P$:

$$\frac{M_B^4 \kappa' a}{2} M L^{-1/9} E_1 + \frac{M_B^2 E_0}{L^{68/81}} (\frac{M \kappa' a}{4} + \frac{\kappa a}{18}) + \frac{3 < g_c^2 G^2 > M_B^2 L^{-1/9} E_0}{32}$$

$$+ \frac{7}{18} a^2 L^{4/9} = \beta_N^2 \exp(-M^2/M_B^2) M g_P,$$  

(19)

which is the main result of this paper.

It is of interest to derive a similar QCD sum rule for the isoscalar pseudoscalar coupling $g_P^S$. Along the same line [as from Eq. (17) up to Eq. (19)], we obtain [with
It is known that, in the absence of the pion-pole dominance for the isoscalar channel, $g_P$ is small, signaling the cancellation $M\chi'a^2 + \chi a^6 \approx 0$ (since the other contribution is numerically small). However, it should be kept in mind that the susceptibilities in the isoscalar channel, such as $\chi$ in Eq. (20), may differ significantly from those in the isovector channel, i.e. those for Eq. (19), where Goldstone pions play a very important role. Indeed, it is reasonable to expect that the induced condensate $\langle 0 | \bar{q}i\gamma_5q | 0 \rangle |z_{\mu\nu} (\equiv 2g\chi'g_{\mu\nu}Z_{ij}^{\mu\nu} < \bar{q}q >)$ is related closely to $\langle 0 | \bar{q}i\gamma_5q | 0 \rangle |\pi (\equiv g_{\pi\pi}\chi\pi \pi_j < \bar{q}q >)$, resulting in a susceptibility:

$$2g\chi'f_\pi \approx g_{\pi\pi}\chi\pi \approx 8.9/a\text{GeV}^{-1},$$

where $f_\pi (= 93\text{MeV})$ sets the scale for chiral symmetry breaking. Without any reliable method to determine $\chi'$, this relation can only be considered as an order-of-magnetic estimate.

Using the estimates $a \approx 0.55\text{GeV}^3$, $g_\pi^2G^2 \approx 0.47\text{GeV}^4$, $\kappa a \approx 0.140\text{GeV}^4$, and $M\chi'a^2 + \chi a^6 \approx 0$, we obtain, from the sum rule (19) evaluated at $M_B = 1.1\text{GeV}$,

$$g_P \approx (-132.4 \pm 5.7),$$

where almost all of (-132.4) comes from the dominant $\chi'a$ contribution (i.e. the first term) while the error bar comes from changing $M_B$ from 1.1GeV to 1.1 ± 0.1GeV. The relatively unknown in $M\chi'a^2 + \chi a^6$ is small because the second term is clearly known (to be small). In addition, this result is very stable with respect to the Borel mass $M_B$.

There are two aspects in connection with the experimental test of PCAC: The first aspect has to do with the value of $g_A$, which should be in accord with the GT relation, the second identity in Eq. (3). The second aspect has to do with the value of $g_P$ which,
according to Eqs. (2) and (3), reads

\[ f_P(q^2) \approx \frac{2Mf_A(q^2)}{q^2 - m^2_\pi}, \quad g_P \approx -\frac{2Mg_A}{m^2_\pi} \approx -124. \quad (23) \]

The second aspect seems to be respected reasonably well by comparing it with Eqs. (21) and (22), although it is obviously desirable to obtain a quantitative treatment of the susceptibility \( \chi' \).

As for the first aspect, we may begin with the pseudovector coupling for the \( \pi NN \) interaction,

\[ \mathcal{L}_{\pi NN} = \frac{f_{\pi NN}}{m_\pi} \bar{\psi}_N i\gamma_\mu \gamma^5 \vec{\tau} \cdot \psi_N \nabla_\mu \vec{\phi}_\pi. \quad (24) \]

Accordingly, if we treat \( \nabla_\mu \phi_\pi \) as a constant external axial vector field, the resultant QCD sum rule for \( f_{\pi NN}/m_\pi \) is identical to that for \( g_A \), except that, at the quark level, we have, making use of the effective chiral quark theory\(^{13,14}\)

\[ \mathcal{L}_{\pi qq} = \frac{1}{2f_\pi} \bar{\psi}_q i\gamma_\mu \gamma^5 \vec{\tau} \cdot \psi_q \nabla_\mu \vec{\phi}_\pi, \quad (25) \]

where \( f_\pi \) is the pion decay constant. [For \( g_A \), we begin with the coupling at the quark level, \( g_u = -g_d = 1 \).] We thus obtain

\[
\frac{f_{\pi NN}}{m_\pi} = \frac{g_A}{2f_\pi}, \\
\frac{g_{\pi NN}}{m_\pi} = f_{\pi NN} \frac{2M}{m_\pi} = \frac{g_A M}{f_\pi},
\]

which is just the Goldberger-Treiman (GT) relation\(^4\). This proof of the GT relation does not involve the pseudoscalar coupling \( g_P \), the main focus of this paper.

### 3 Conclusion and Discussions

To sum up, we have in this paper presented a suitable extension of the QCD sum rule method which enables us to obtain a QCD sum rule for the induced pseudoscalar coupling constant \( g_P \), an entity of significance for testing the validity of the partially conserved axial current (PCAC) hypothesis.
Maybe we spend some paragraphs in discussing the overall future of the QCD sum rule methods.

First of all, we have to go beyond the leading order in order to get the induced pseudoscalar coupling $g_P$. The situation is similar to the anomalous magnetic moments of baryons. One issue is whether we could determine the newly condensate parameters, such as Eq. (14). The increase in unknowns could be more than what we want to solve. For this, it is useful to understand the theory more and to try to derive the relations among these condensate parameters.$^{12,14}$

We have used the sum rule for $g_A$ [Eq. (18)] to obtain the sum rule for $g_P$ [Eq. (19)]. In addition, it is assumed that the induced pseudoscalar in the isoscalar channel is presumably very small, signaling that $\frac{M\mu_A}{2} + \frac{\nu_\mu}{6}$ and $\frac{M\mu_A}{4} - \frac{\nu_\mu}{18}$ be small. This is why we could make some numerical prediction.

The value of $M_B$, (= 1.1GeV), is taken from the QCD sum rules for the nucleon mass, for which the minimization has a meaning. Similarly, in bag models, minimization to get the certain mass has a similar meaning. In contrast, the minimization in the case of $g_A$, $g_P$, $\mu_P$, etc. should not take the fundamental meaning, as compared to the mass or energy. Once the value of $M_B$ for nucleons is determined, it should be used for predicting other fundamental parameters. This point should always be emphasized for a QCD sum rule calculation.

In other words, QCD sum rule methods allow us to achieve the following: The basic properties of the nucleons are predicted using a universal Borel mass $M_B$, based on the same set of the condensate parameters. The adoption of the Borel transform is to improve the convergence of the methods - it’s not required nor necessary.

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