Concrete life prediction model based on freeze-thaw damage theory and its application

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Abstract. For hydraulic concrete structure in cold and severe cold areas in northern China, its frost resistance directly restricts the life and safe and stable operation of the project. In this paper, based on the freeze-thaw damage theory, this paper optimizes and improves the Weibull distribution model, and uses the 1:1:4 standard frost-resistant specimen of hydraulic concrete as the physical model to obtain a freeze-thaw damage model that is more in line with the standard specimens of the rapid freeze-thaw experiment. Based on this model, a concrete life prediction method is proposed. In order to solve the problem that the initial damage of coring concrete is not clear, an iterative approximation method is proposed. The life prediction method proposed was verified through actual engineering cases. The results show that the model can better describe the process of concrete freeze-thaw damage and destruction, and the life prediction results are more reliable.

1. Introduction

As the most widely used and most important man-mand building material, the durability of concrete is directly related to the safety and stable operation of the project. For the cold and severe areas in northern China, the frost resistance of concrete plays a decisive role in the durability of concrete. At present, the research on concrete damage model based on freeze-thaw damage has achieved certain research results [1~4]. However, different scientific researchers choose different indicators to characterize the degree of concrete freeze-thaw damage. The difference in the selection of indicators is mainly based on freeze-thaw damage. Process quality attenuation model, relative dynamic elastic modulus attenuation model, intensity attenuation model, energy dissipation model, ultrasonic sound velocity change law model and strain change model [4].

The influencing factors of concrete structure failure are complex and interactive, and its failure mechanism is also more complicated. However, the understanding of the freeze-thaw damage process of concrete is basically the same, that is, the freeze-thaw damage process of concrete is the process that the internal structure of concrete gradually cracks, expands, and finally runs through and fails under the action of two kinds of failure stress, namely expansion pressure and seepage pressure.
Weibull theory is a theory widely used in reliability analysis and life testing of various life tests. Studies have shown that Weibull distribution can also be better used in simulating freeze-thaw damage and life prediction of concrete.

2. Model establishment

2.1. Introduction and description of the model

Applying the Weibull distribution model to concrete freeze-thaw damage simulation and life prediction, there are three basic assumptions [5,6], as follows:

1. Assuming that the interior of the concrete is uniform and continuous, when the surrounding boundaries of the concrete are in the same environmental conditions, the parts within the concrete with the smallest distance from the boundary all obey the same damage evolution law.

2. The damage process of concrete is a process in which the micro-defects in concrete gradually increase and accumulate with age. If concrete is regarded as an aggregate of a large number of micro-elements, the failure rate of micro elements in concrete is an increasing function with time, which conforms to the Weibull distribution function when the shape factor is greater than 1.

The expression of Weibull's three-parameter distribution function is as formula (1):

\[
F(t) = 1 - \exp \left\{- \left[ \frac{\lambda}{\alpha} \left( t - t_0 \right) \right]^{\alpha} \right\}
\]

(1)

Where: \( t \) is the time; \( t_0 \) is the threshold; \( \lambda \) is the scale factor; \( \alpha \) is the shape factor; \( H \) is the Heaviside function (when \( t - t_0 \) is greater than or equal to 0, it is 1; when is less than 0, it is 0).

3. Assuming that the interior of concrete is uniform, according to the characteristics of Weibull distribution, it can be considered that the shape of the failure curve of each point inside the concrete is the same, that is, the shape factor is consistent.

The scale factor \( \lambda \) characterizes the resistance of concrete to adverse conditions. The larger the value, the smaller the resistance of the concrete; The smaller the value, the greater the resistance, and \( t \) the larger the scale factor indicates the weaker the resistance of the material, and the earlier the threshold appears; On the contrary, the later the threshold appears. Therefore, it can be assumed that the scale factor is inversely proportional to the threshold, and the specific relationship is shown in equation (2), \( k_0 \) is the scale coefficient:

\[
t_0 = k_0 \alpha^{-1}
\]

(2)

2.2 Model derivation based on standard cube freeze-thaw test block

The failure scale factor of a certain point in the concrete is related to the position (depth) of this point in the concrete, that is, formula (3):

\[
\lambda(x, y, z) = \lambda(|x|, |y|, |z|)
\]

(3)

Assuming that the probability density of failure of the micro-unit at coordinate of \((x, y, z)\) in the concrete test block is \(f(x, y, z; t)\), then the volume of concrete failure at time \(t\) is \(V_t = \int_{V_0} f(x, y, z; t) \, dx \, dy \, dz\), Where \(V_0\) is the original concrete volume.

According to the damage theory, the damage degree \(D = V_t/V_0\), the damage factor of concrete block at time \(t\) can be obtained as shown in formula (4):

\[
D = V_0^{-1} \int_{V_0} f(x, y, z; t) \, dx \, dy \, dz
= V_0^{-1} \int_{V_0} \alpha \left[ \lambda \cdot H \left( t - k_0 \right)^{-1} \right]^{\alpha-1} \cdot \exp \left\{- \left[ \frac{\lambda}{\alpha} \left( t - t_0 \right) \right]^{\alpha} \right\} \, dx \, dy \, dz
\]

(4)
The meshing of the model has an important influence on the accuracy of the data calculation. Obviously, the closer the model is built to the actual specifications, the finer the mesh is, the closer the parameters obtained should be to the true value. At present, the selection of freeze-thaw damage model is basically plane model or cube model, but the size of the freeze-thaw test sample is 100mm×100mm×400mm in SL/T352-2020 "Test Regulations for Hydraulic Concrete", which is a cuboid with a size ratio of 1:1:4. Only by establishing a mesh with the same size, can the model be more matched with the experimental data.

Therefore, the 1:1:4 cuboid concrete freeze-thaw specimens are meshed as shown in Fig. 1. The long side of the cuboid is divided into 4N parts, the width and height sides are divided into N parts, and N is an even number; In Figure 1, G represents the damage gradient. The number of units Ni of the i-th layer (shadow part) with the same shortest distance from the surface of concrete specimen is derived by calculation, as shown in equation (5):

\[ N_i = 18N^2 - 48iN + 24i^2 - 24N + 24i + 8 \]  

where: \( i=0, 1, 2, \ldots \ldots (N/2-1) \).

The distribution function of the destruction of micro-units over time is \( F(t) \), and the destruction of the i-th layer unit is \( F_i(t) \). The mathematical expectation of the event of \( \Phi_i \) that the Ni block unit in the i-th layer is destroyed at time t is equation (6) and equation (7).

\[ E(\Phi_i) = N_i F_i(t) \]

\[ F_i(t) = 1 - \exp \left\{ - \left[ \lambda_i \cdot H \left( t - k_i \lambda_i^{-1} \right) \right]^\alpha \right\} \]

Assuming that the scale factor changes nonlinearly along the section, the scale factor of the i-th layer is equation (8).

\[ \lambda_i = \lambda_0 + \nu \left( i + 0.5 \right)^{-1} \]

Where: \( \lambda_0 \) is the undetermined uniform scale parameter; \( \nu \) is the undetermined gradient factor; \( i=0,1,2,\ldots \ldots \), (N/2-1).

Then the number of destruction of all units at time t is equation (9).

\[ E(\omega) = \sum_{i=0}^{N-1} E(\Phi_i) = \sum_{i=0}^{N-1} N_i F_i \]

Substituting into equation (4), the freeze-thaw damage factor D of the whole model can be obtained as equation (10).
\[
\begin{align*}
D &= E(\omega)/N^3 = 0.25 \cdot N^{-3} \sum_{i=0}^{N-1} \left[ 18N^2 - 48iN + 24i^2 - 24N + 24i + 8 \right] \\
&\quad \cdot \left\{ 1 - \exp \left[ - \lambda_i \cdot H \left( t - k_0 \lambda_i^{-1} \right) \right] \right\}^{\alpha} \\
&= \min \sum_{i=1}^{N} \left\{ f(\lambda_i, \alpha, \nu, k_0) - y_i \right\}^2 
\end{align*}
\]

(10)

2.3 Model parameter estimation

The undetermined parameters \( \lambda_0, \alpha, \nu \) and \( k_0 \) in the above formula are determined by using the least square method, and the assumed functions are shown in formula (11):

\[
\nabla = \min \sum_{i=1}^{m} \left\{ f(\lambda_i, \alpha, \nu, k_0) - y_i \right\}^2
\]

(11)

For equation (10), \( \nabla \) is about the nonlinear functions of \( \lambda_0, \alpha, \nu \) and \( k_0 \), so the gradient descent method is used here, as shown in equation (12) to equation (15):

\[
\begin{align*}
\lambda_{i+1} &= \lambda_i - a_1 \nabla \lambda \\
\alpha_{i+1} &= \alpha_i - a_2 \nabla \alpha \\
\nu_{i+1} &= \nu_i - a_3 \nabla \nu \\
k_{i+1} &= k_i - a_4 \nabla k
\end{align*}
\]

(12-15)

3 Application of the model

3.1 Definition of concrete damage degree under freeze-thaw action

The internal moisture of the concrete structure in cold and severe cold regions will continue to be in the process of freezing and melting along with the change of the ambient temperature. The internal microstructure of concrete will gradually be damaged under the action of expansion pressure and osmotic pressure. After multiple freeze-thaw cycles, the damage continues to accumulate and expand, forming interconnected cracks, which ultimately results in changes in the physical and mechanical properties of concrete. The changes in these physical parameters have become a measure of the degree of concrete freeze-thaw damage. Commonly used physical parameters include mass loss rate, strength loss rate, relative dynamic elastic modulus, etc[7]. The commonly used indicators for the failure judgment of concrete frost resistance mostly use relative dynamic elastic modulus and mass loss rate. In this paper, relative dynamic elastic modulus is selected for the definition of damage degree. According to the theory of damage mechanics, the damage degree of concrete under freeze-thaw damage can be defined as:

\[
D = 1 - P_n
\]

(16)

Where: \( P_n \) is the relative dynamic elastic modulus of concrete after \( n \) freeze-thaw cycles.

3.2 Test results and discussion

In order to verify whether the model can accurately reflect the freeze-thaw process of the standard anti-freezing test block, we prepared the standard cube anti-freezing test block according to the mix proportion shown in Table 1 and carried out the rapid freeze-thaw cycle test. The test results are shown in Table 2.
Table 1. Amount of concrete material per cubic meter (kg / m³)

| Water | Cement | Fine aggregate | Coarse aggregate (5~20mm) | Coarse aggregate (20~40mm) | Water reducing agent |
|-------|--------|----------------|---------------------------|---------------------------|---------------------|
| 158   | 405    | 580            | 492                       | 739                       | 4.0                 |

Table 2. Verify the results of the frost resistance test of the concrete test block

| Number of freeze-thaw cycles | Freeze thaw cycle results |
|------------------------------|---------------------------|
|                              | 1 | 2   | 3   | 4   | 5   | Average |
|------------------------------|---|-----|-----|-----|-----|---------|
| 25                           | 100 | 0.2 | 99.9 | 0.2 | 99.9 | 0.1 | 100 | 0.1 | 99.9 | 0.2 |
| 50                           | 99.4 | 0.2 | 99.2 | 0.2 | 99.1 | 0.4 | 99.5 | 0.2 | 99.4 | 0.5 |
| 75                           | 98.1 | 0.4 | 98.1 | 0.7 | 98.5 | 0.8 | 98.2 | 0.8 | 97.9 | 0.5 |
| 100                          | 96.1 | 0.7 | 96.1 | 1.2 | 96.8 | 1.1 | 96.2 | 0.8 | 95.4 | 1.3 |
| 125                          | 92.1 | 1.1 | 92.2 | 1.7 | 93.5 | 1.4 | 91.7 | 1.6 | 92.1 | 1.5 |
| 150                          | 90.8 | 1.4 | 89.2 | 2.4 | 94.1 | 1.2 | 87.9 | 2.9 | 88.9 | 2.1 |
| 175                          | 87.5 | 1.8 | 83.1 | 2.5 | 89.6 | 1.2 | 83.2 | 3.8 | 83.5 | 2.5 |
| 200                          | 71.2 | 2.6 | 68.5 | 3.5 | 73.5 | 2.5 | 69.8 | 3.9 | 68.1 | 3.1 |

Apply the method introduced above to calculate the test results in Table 2. The number of grid layers N is artificially set. The larger the number of grid divisions, the smaller the error of the results obtained. Here, the number of grid layers N is selected as 20. The results of the model parameters obtained by calculation are shown in Table 3, and the comparison between the theoretical prediction value and the measured value of freeze-thaw damage is shown in Figure 2. The results show that the predicted value of the established freeze-thaw damage model is basically consistent with the measured value, indicating that this model can better describe the freeze-thaw damage process of concrete.

Table 3. Calculation results of undetermined coefficients of damage model

| Model parameter | \( \lambda_0 \) | \( \nu \) | \( K_0 \) | \( a \) |
|-----------------|-----------------|--------|--------|--------|
| Value           | 2.62            | 0.42   | 0.003  | 3.49   |

Figure 2. Comparison of theoretical prediction and measured value of freeze-thaw damage

4. Application of concrete life prediction based on freeze-thaw damage

4.1. Concrete life prediction process based on freeze-thaw damage

Different from the indoor rapid freeze-thaw cycle test, the factors affecting the life of concrete in the
actual service environment are very complex. The factors that have a greater impact on the life of concrete include the change of ambient temperature, humidity, salt concentration, load, external impact and so on [7~10]. However, it is difficult to realize the indoor test considering the coupling effect of multiple factors. We just consider the indoor accelerated test of freeze-thaw damage and stress damage which have great influence on concrete engineering in northern China, and carry out the concrete life prediction.

1) The relationship between concrete damage and freeze-thaw cycle is obtained, and the freeze-thaw damage model of concrete structure is determined.

2) The relationship between concrete damage in laboratory accelerated test and concrete damage in practical engineering environment is established. On the basis of formula (16), the accelerated test is proportional to the deterioration of concrete in the actual environment. It is assumed that the damage degree of accelerated test concrete is the same as that of actual concrete. If there is no site damage data, the acceleration coefficient $K$ can be selected according to the experience, which is generally between 1:10 and 1:15, so that the actual life can be easily obtained from the accelerated test life. For the concrete structure that has been used in operation, the concrete structure can be detected to obtain the real acceleration coefficient $K$ during operation, and the actual life of the concrete structure can be predicted combined with the indoor acceleration test results.

Because the freeze-thaw damage degree of concrete is related to the initial state of the specimen / structure, the rapid freeze-thaw test results of the in-situ concrete core samples cannot be directly used to calculate the damage degree of the in-situ concrete. Similarly, the concrete freeze-thaw failure termination condition (SL/T 352-2020 "Hydraulic concrete test code") is relative to the initial concrete state of the specimen / structure when the relative dynamic elastic modulus drops to 60% of the initial value or the mass loss rate reaches 5%. Therefore, the termination conditions of in-situ concrete core samples and indoor concrete are not the same.

The relative dynamic modulus of elasticity obtained from the freeze-thaw cycles of core samples is based on the state of the concrete when the core is taken. At this time, the concrete has been in service for several years and has some damage. According to the mix proportion, the newly formed concrete can be regarded as undamaged concrete. At this time, the dynamic elastic modulus of concrete samples in the freeze-thaw cycle test is lower than that in the new pouring. Therefore, in order to more accurately determine the concrete state relative to the undamaged concrete, the iterative approximation method is adopted in this paper.

The first iteration: firstly, the damage model is used to calculate the freeze-thaw cycles of the indoor cuboid sample and the engineering entity core drilling sample when the damage degree is 0.4, and the freeze-thaw cycle difference between them is $N_1$. The damage value of the indoor cuboid sample corresponding to $N_1$ is taken as the initial damage value $D_1$ of the engineering entity. Then the damage degree of the engineering solid core drilling corresponding to the damage degree of the indoor cuboid sample is $0.4 / (1-D_1)$.

The second iteration: using the damage model to calculate the freeze-thaw cycles when the damage degree of the drilling core is $0.4 / (1-D_1)$, the difference between the freeze-thaw cycles when the damage degree of the drilling core is 0.4 and the freeze-thaw cycles when the damage degree of the indoor cuboid sample is 0.4 is $N_2$, and the damage value of the indoor cuboid sample corresponding to $N_2$ is $D_2$ as the initial damage value of the engineering entity. Then the damage degree of the engineering solid core drilling corresponding to the damage degree of the indoor cuboid sample is $0.4 / (1-D_2)$.

In the same way, the stable cycle number difference $n$ is obtained by iterating in turn. This $n$ represents the rapid freeze-thaw times corresponding to the freeze-thaw damage after the construction of the engineering entity. Combined with the frost resistance data of the concrete indoor test, the life prediction results of the engineering entity can be obtained.

4.2 Application example of concrete life prediction based on freeze-thaw damage
Shimen Reservoir project is located in Gaizhou City, Liaoning Province. It was built in 1970 and
completed in November 1971. In May 2006 and September 2016, core drilling sampling was carried out on the left wall of spillway of Shimen Reservoir for concrete frost resistance test. The site photos are shown in Figure 3. In order to investigate the remaining service life of the concrete in this part, the following work has been carried out:

1. Sample preparation: two core drilling samples have been taken at the investigated part every 10 years, and the cuboid concrete specimens have been prepared according to the mix proportion of the concrete used in this part according to SL/T 352-2020 "hydraulic concrete test procedure".

2. At the same time, rapid freeze-thaw tests were carried out to measure the relative dynamic elastic modulus of concrete.

3. Four undetermined coefficients of the life prediction model are determined according to the measurement results.

4. The damage degree of in-situ core samples is determined by iterative approximation method, and the life prediction is carried out. The two life prediction results were compared.

According to the investigation, the mix proportion of the concrete used in this part is shown in Table 4, the design strength is C20, and the design slump is 40mm.

Table 4. Concrete mix proportion of Shimen Reservoir

| Cement P.O42.5 (kg) | Fine aggregate (kg) | Coarse aggregate (20~30) (kg) | Water (kg) | Water-cement ratio |
|---------------------|---------------------|-------------------------------|------------|-------------------|
| 307                 | 624                 | 1297                          | 172        | 0.56              |

The freeze-thaw cycle test is carried out in the same environment of indoor and field core drilling, and the test results are shown in Figure 4.
Using equation (16) to determine the damage degree of concrete core sample during freeze-thaw cycle, it can be seen from the data in Figure 4 that the change trend of relative dynamic elastic modulus of indoor rectangular sample is basically consistent with that of indoor cylindrical core sample. The undetermined coefficients in Weibull model (equation (10)) are determined by the least square method. The results are shown in Table 5.

Table 5. Calculation results of undetermined coefficients of three damage models

| Sample type                          | Parameter |          |          |          |
|--------------------------------------|-----------|----------|----------|----------|
|                                      | $\lambda_0$ | $\nu$   | $K_0$    | $a$      |
| Indoor cuboid specimen               | 1.3816    | 0.6076   | 0.0084   | 2.2996   |
| Indoor cylindrical specimen          | 1.1318    | 0.6528   | 0.0139   | 1.9879   |
| 35 years core drilling sample data   | 1.3182    | 0.4619   | 0.0107   | 1.4457   |
| 45 years core drilling sample data   | 1.3153    | 0.4044   | 0.0072   | 1.2587   |

It can be seen from the simulation results that the damage model of equation (10) still has a good fitting effect on the cylinder specimen. It can be seen from Figure 4 that the core drilling sampling in the engineering entity has experienced many years, and the concrete has been damaged, so the resistance to freeze-thaw damage is poor, and the advance speed of freeze-thaw damage is also fast. In the first iteration, according to the freeze-thaw damage model, it can be calculated that the
freeze-thaw cycles of the rapid freeze-thaw test of indoor cuboid to failure (damage degree 0.4) are 242 times, and the freeze-thaw cycles of the in-situ core drilling sample to failure (damage degree 0.4) are 152 times in 35 years, and the difference is 242-152 = 90 times. If the damage value $D_1$ of indoor cuboid specimen is 0.049 for 90 cycles, the damage degree of drilling core corresponding to the damage degree of indoor cuboid specimen is $0.4 / (1-0.049) = 0.421$, and the corresponding freeze-thaw cycles are 155.

In the second iteration, the difference $N_2$ of freeze-thaw cycles is 242-155 = 87. The damage value $D_2$ corresponding to $N_2$ is 0.045. Then the damage degree of the engineering entity core drilling corresponding to the damage degree of the indoor cuboid sample is $0.4 / (1-0.045) = 0.419$, which is close to the 155 times corresponding to the number of freeze-thaw cycles in the first iteration, and the iteration ends.

In the first 35 years of operation, the annual freeze-thaw damage is equivalent to the number of indoor rapid freeze-thaw, which is $87 / 35 = 2.49$ times, the life prediction of concrete in 35 years is $155 / 2.49 = 62.4$ years, and the total life cycle of concrete is 97.4 years.

The damage degree of concrete is 0.072 after 45 years of freeze-thaw. The annual freeze-thaw damage is equivalent to the number of fast freeze-thaw, which is $114 / 45 = 2.53$ times. The life prediction of concrete at 45 years is $135 / 2.53 = 53.3$ years, and the total life cycle of concrete is 98.3 years.

Considering the difference of concrete itself, the change of external environmental factors and the difference of sampling in different months of the same year, the difference of 0.9 years between the two life cycle predictions is normal, and this prediction method is still more reasonable. It can be seen from the results of freeze-thaw life prediction of Shimen Reservoir that the iterative approximation method is feasible to obtain the initial damage state of concrete and predict its life.

5. Conclusion

In this paper, based on the concrete freeze-thaw and stress conditions of concrete damage life prediction work, on the basis of existing research, the application of Weibull distribution model and optimization improvement, suitable for the standard rapid freeze-thaw test (sample size of 100 mm) × 100mm × 100mm Based on this model, a life prediction method of concrete based on rapid freeze-thaw test is established.

Through the indoor test verification, the simulation results of the model and the actual test results fit well, indicating that the optimized Weibull model can simulate the deterioration process of different concrete in different environments. The actual engineering cases show that the iterative approximation method proposed in this paper can more accurately deduce the initial damage state of concrete solid core drilling samples, and on this basis, the concrete life prediction results are more reliable and reasonable.

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