Statistical Operator for Electroweak Baryogenesis

J. B. Bronzan

Department of Physics and Astronomy
Rutgers University
Piscataway, New Jersey 08855-0849

Abstract

In electroweak baryogenesis, a domain wall between the spontaneously broken and unbroken phases acts as a separator of baryon (or lepton) number, generating a baryon asymmetry in the universe. If the wall is thin relative to plasma mean free paths, one computes baryon current into the broken phase by determining the quantum mechanical transmission of plasma components in the potential of the spatially changing Higgs VEV. We show that baryon current can also be obtained using a statistical density operator. This new formulation of the problem provides a consistent framework for studying the influence of quasiparticle lifetimes on baryon current. We show that when the plasma has no self-interactions, familiar results are reproduced. When plasma self-interactions are included, the baryon current into the broken phase is related to an imaginary time temperature Green’s function.
1 Introduction

During the electroweak phase transition, baryogenesis can occur in the minimal standard model and its extensions (more Higgs doublets, supersymmetry, etc.), provided the phase transition is strongly first order\cite{1}. Bubbles of broken symmetry (true) vacuum sweep through the unbroken symmetry (unstable) vacuum. The computation of the ratio of baryon density to entropy density (measured to be $\sim 10^{-10}$\cite{2}) proceeds differently if the bubble wall is thin\cite{3, 4, 5} or thick\cite{6, 7, 8} relative to the mean free paths of constituents of the quark-lepton-boson plasma. In this paper we will be concerned with the thin wall case, where one computes the transmission of plasma quasiparticles past the bubble wall. Because CP and C are violated, the transmission of baryon number (or its lepton surrogate) differs from transmission of antibaryon number. This asymmetry drives the baryon asymmetry of the universe, although a number of other physical effects are important in arriving at the final result.

Farrar and Shaposhnikov used this framework to estimate the baryon asymmetry produced by the minimal standard model, with CP violation originating in the CKM matrix\cite{9}. They concluded the experimentally observed number could be generated in this way. This result was criticized by Gavela et.al., who obtained an asymmetry about $10^{-13}$ relative to that computed by Farrar and Shaposhnikov\cite{10}. These authors state that the main reason for the discrepancy is the very short lifetimes of quasiparticles in the hot plasma ($\Gamma \sim 19$ GeV for quarks). On the other hand, in the work of Gavela et.al., quasiparticle damping is incorporated by giving quasiparticles a non-Hermitian effective Lagrangian. This falls short of a complete many-body treatment of the self-interacting plasma moving past the Higgs VEV (vacuum expectation value) domain wall.

In this paper we develop an alternate framework for computing currents which may make it possible to reduce the number of ad hoc assumptions in calculations. We advocate using a statistical operator that specifies the thermal system consisting of the hot, self-interacting plasma with the VEV domain wall moving through it. In principle, the new framework does not require the introduction of quasiparticles at all. It could be used even when lifetimes are so short that quasiparticles cease to be useful degrees of freedom. In practice, one must be able to accurately take account of the plasma self-interactions, and this usually means working in terms of physical excita-
tions, using perturbation theory to account for interactions. Finite quasiparticle lifetimes and mean free paths are present in the statistical mechanics approach as a consequence of self-interactions in the plasma, and when quasiparticles decay, baryon current borne by decay products is included.

Two Lorentz frames are involved in the problem. The “laboratory” frame is the rest frame of the plasma, denoted by $P$. The domain wall moves through frame $P$ at speed $v$. (We take the domain wall to be planar, thereby ignoring the curvature of the bubble wall.) The second frame is the rest frame of the domain wall, denoted $V$. In frame $V$, the plasma streams by the stationary VEV wall, from the unbroken into the broken phase. We will take the unbroken phase to lie at $z < 0$, and the broken phase at $z > 0$. We assume, with references [9, 10], that the latent heat associated with the phase transition can be ignored. Then there is no energy source at the bubble wall, and we are dealing with a closed system described by a Hamiltonian. Like these authors, we also discount turbulence so that the system is stationary in frame $V$. Taken together, these observations imply that the plasma is described by observers in $V$ by a time-independent statistical operator $\rho$. Because the thermal bath in contact with the gas is at rest in frame $P$, the density matrix is

$$\rho = \exp[-\beta K_P], \tag{1.1}$$

where $K_P = H_P - \mu N$, and $H_P$ is the Hamiltonian in $P$. What forces us to deal with two frames is that while $K_P$ determines $\rho$, $K_P$ is time-independent only in frame $V$. [Recall that to recast the computation of the baryon current as a conventional problem in equilibrium statistical mechanics, $\rho$ must be time-independent.]

In frame $V$ there is a conserved baryon current $\hat{J}_\mu(x, t)$ whose form and time-dependence are determined by $H_V$, the Hamiltonian of the system in $V$. The baryon current past the VEV wall, measured in $V$, is

$$J(z, t) = \frac{Tr[\rho \hat{J}_3(x, t)]}{Tr[\rho]} \tag{1.2}$$

This expression appears simpler than it is because one Hamiltonian appears in $\rho$, a second Hamiltonian controls time evolution, and the two do not commute. As a result, $J$ generally depends on $z$ and $t$. However, as $t \to \infty$,

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2 $\hat{O}$ is used when we wish to distinguish an operator from a related c-number $O$. 

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transients decay and the current approaches a limit that is constant in time and space. In the case of a plasma without self-interactions, the late time current is what is expected on the basis of simple physical arguments. The resulting formula (Eq. 3.16) has been used in previous work, either implicitly or explicitly.

The matter content of the plasma in the epoch of baryogenesis depends on whether the standard model or one of its extensions is under study. Even were we to choose a specific case, the inclusion of many species would tend to obscure the gist of our work. In a similar vein, implementation of realistic baryon-nonconserving processes would introduce complexity and detail that are undesirable. What is illuminating is to study a simple system for which the statistical matrix approach is developed in a way that can be generalized to other plasmas. In this paper we assume a single spin zero baryon-antibaryon species is present in the plasma. The Higgs VEV introduces a z-dependent mass $m(z)$ for these bosons. We assume that the VEV jumps to its broken symmetry value only in the region $0 < z < L$. That is, in this interval, $m(z) \sim M$, while elsewhere $m(z) \sim m$, and $m < M$. Having $m(+\infty) = m(-\infty)$ has the advantage of simplifying the asymptotic form of basis functions that appear extensively in the calculations, and it is for this reason that the width of the broken symmetry phase is finite. Since $L$ can be as large as we wish, its finitude has no physical significance because tunneling through the broken symmetry phase can be made as small as we wish by increasing $L$.

If nothing further were done, we would find $J = 0$ because while there would be a baryon current past the domain wall, there would be an equal and canceling antibaryon current. We produce a net baryon current by introducing a chemical potential that enhances the baryon density relative to the antibaryon density. It plays the role of baryon non-conserving processes in our model. In order for the Bose distribution functions to be finite, the chemical potential must lie in the range $-m < \mu < m$. This restriction is what requires us to choose $m \neq 0$ in the “unbroken symmetry” phase in our model.

In Section 2 we consider the diagonalization of $K_P, H_P, K_V$ and $H_V$, in the case where the plasma has no self-interactions. This allows us to compute the baryon current of such a plasma in Section 3. In the limit $t \to \infty$, the baryon current we find is the expected one. In Section 4 we consider the case of a plasma with self-interactions. The objective there is to express
the current in terms of an imaginary time temperature Green’s function in equilibrium many-body theory. It is these Green’s functions that have simple expansions in perturbation theory, and to which the formalism and methods of many-body theory apply. In Section 4 a number of complications come together: the difference between $H_P$ and $H_V$, the need to take $t \to \infty$, and the requirement of relating real time and imaginary time temperature Green’s functions. We succeed in reconciling these strands only by working to first order in the plasma speed $v$. (In Section 3 our results for a plasma without self-interactions hold for $0 < v < 1$.) We argue that the baryon current $J(v)$ should be odd in $v$, so this restriction is less stringent than it might seem. Extension of our results to $O(v^3)$ is conceivable, but would be complicated. At the end of Section 4 we have a specific imaginary time temperature Green’s function which gives the baryon current to $O(v)$. The effect of quasi-particle decay on baryon current is now accounted for by assessing the effect of plasma self-interactions on this Green’s function. In particular, contributions to the boson propagator where a boson decays on one side of the VEV wall and reassembles on the other correspond to contributions to the baryon current from quasi-particle decay products.

2 Free Fields

To show that a statistical formulation of the calculation of baryon current works, we first consider a plasma that has no self-interactions, but full interaction with the VEV wall. As a preparation, we formulate and diagonalize the four operators $K_{P0}$, $H_{P0}$, $K_{V0}$, and $H_{V0}$. $H_{V0}$, the free Hamiltonian in frame $V$, differs from the conventional free boson Hamiltonian only through its spatially varying mass term:

$$H_{V0} = \int d^3x \left[ \hat{\pi} \hat{\pi}^\dagger + \nabla \hat{\phi}^\dagger \cdot \nabla \hat{\phi} + m^2(z) \hat{\phi}^\dagger \hat{\phi} \right].$$

(2.1)

$H_{V0}$ generates $t$-displacements; we also need the momentum operator $P_{V,3}$, which generates $z$-displacements:

$$P_{V,3} = \int d^3x \left[ \frac{\partial \hat{\phi}^\dagger}{\partial z} \hat{\pi}^\dagger + \hat{\pi} \frac{\partial \hat{\phi}}{\partial z} \right].$$

(2.2)
Then for any Heisenberg operator \( \mathcal{O} \),
\[
\frac{\partial \mathcal{O}}{\partial t} = -i [\mathcal{O}, H_{V0}] ; \quad \frac{\partial \mathcal{O}}{\partial z} = -i [\mathcal{O}, P_{V,z}] .
\] (2.3)

Observers in frame \( P \) attribute primed coordinates to events, where
\[
t' = \gamma (t - vz); \quad z' = \gamma (z - vt) .
\] (2.4)

Thus,
\[
\frac{\partial \mathcal{O}}{\partial t'} = \gamma \frac{\partial \mathcal{O}}{\partial t} + \gamma v \frac{\partial \mathcal{O}}{\partial z} = -i [\mathcal{O}, \gamma H_{V0} + \gamma v P_{V,z}] .
\] (2.5)

Since the generator of \( t' \)-displacements is \( H_{P0} \), we have
\[
H_{P0} = \gamma H_{V0} + \gamma v P_{V,z} .
\] (2.6)

(A similar formula holds for a plasma with self-interactions; it will be used in Section 4.) Finally, the grand canonical Hamiltonians are
\[
K_{V0} = H_{V0} - \mu N \] and \( K_{P0} = H_{P0} - \mu N \), where the baryon minus antibaryon number operator is
\[
N = i \int d^3x \left[ \hat{\phi}^\dagger \hat{\pi}^\dagger - \hat{\pi} \hat{\phi} \right] .
\] (2.7)

\( K_{P0} \) is the key to diagonalizing these operators, since once \( K_{P0} \) is in diagonal form, the remaining operators follow by setting \( v = 0 \), or \( \mu = 0 \), or both. The construction begins by considering the Heisenberg equations of motion that are generated by \( K_{P0} \):
\[
\frac{\partial^2 \hat{\phi}}{\partial t^2} = \gamma^2 \nabla^2_\perp \hat{\phi} + \frac{\partial^2 \hat{\phi}}{\partial z^2} + 2\gamma v \frac{\partial^2 \hat{\phi}}{\partial t \partial z} + 2i\mu \frac{\partial \hat{\phi}}{\partial t} - 2i\gamma v \mu \frac{\partial \hat{\phi}}{\partial z} + \mu^2 \hat{\phi} - \gamma^2 m^2(z) \hat{\phi} ,
\] (2.8)

\[
\hat{\pi}^\dagger = \frac{1}{\gamma} \frac{\partial \hat{\phi}}{\partial t} - v \frac{\partial \hat{\phi}}{\partial z} - i \frac{\mu}{\gamma} \hat{\phi} .
\] (2.9)

This equation implies that the current \( \hat{I}_\mu \) is conserved, where
\[
\hat{I}_0 = \left\{ \frac{\partial}{\partial t} - \gamma v \frac{\partial}{\partial z} - 2i\mu \right\} \hat{\phi},
\] \[
\hat{I}_\perp = -i \gamma^2 \hat{\phi}^\dagger \nabla_\perp \hat{\phi},
\] \[
\hat{I}_3 = -i \hat{\phi}^\dagger \left( \frac{\partial}{\partial z} + \gamma v \frac{\partial}{\partial t} - 2i\gamma v \mu \right) \hat{\phi} .
\] (2.10)

\(^3v \) is the relative speed between \( P \) and \( V \). We use \( \beta \) for \( 1/kT \).
and \( \hat{\phi} \) satisfies Eq. 2.8. (When we replace the free Hamiltonian \( H_{P0} \) by the full Hamiltonian \( H \), Eq. 2.8 acquires additional interaction terms and counterterms on the right. \( I_\mu \) is still conserved in the theory with self-interactions.) Setting \( v = \mu = 0 \), the conserved baryon current in frame \( V \) and Eq. 1.2 is

\[
\hat{J}_0 = i \hat{\phi}^\dagger \frac{\partial}{\partial t} \hat{\phi}, \quad \hat{J} = -i \hat{\phi}^\dagger \nabla \hat{\phi}.
\]  

(2.11)

We obtain basis states for the diagonalization of \( K_{P0} \) by finding classical solutions of Eq. 2.8. If we write \( \phi = \tilde{\phi} \exp[i\mu t] \), the chemical potential drops out, and \( \tilde{\phi} \) satisfies

\[
\frac{\partial^2 \tilde{\phi}}{\partial t^2} = \gamma^2 \nabla_\perp^2 \tilde{\phi} + \frac{\partial^2 \tilde{\phi}}{\partial z^2} + 2\gamma v \frac{\partial^2 \tilde{\phi}}{\partial t \partial z} - \gamma^2 m^2(z) \tilde{\phi},
\]

(2.12)

By separation of variables, we have solutions \( \tilde{\phi} = \exp(-iEt)P(k, x) \) where

\[
P(k, x) = \left[ 2E(2\pi)^{3/2} \right]^{1/2} Z_P(k_3, z),
\]

(2.13)

\[
E = \sqrt{k^2 + m^2},
\]

and

\[
\frac{d^2 Z_P(k_3, z)}{dz^2} + \gamma^2 [k_3^2 + m^2 - m^2(z)] Z_P(k_3, z) = 0.
\]

(2.14)

The current defined in Eq. 2.10 continues to be conserved when \( \hat{\phi} \) is replaced by one classical solution, and \( \hat{\phi}^\dagger \) by the complex conjugate of a second solution. Consider the integral of the resulting charge density. Introducing useful notation, we write the total charge as

\[
(\phi(k), \phi(k')) = e^{i(E-E')t} (P(k), P(k')),
\]

(2.15)

\[
(P(k), P(k')) = \int d^3 x P^*(k, x) \left[ E + E' - i\gamma v \frac{\partial}{\partial z} \right] P(k', x)
\]

\[
= \frac{\gamma \delta(k_\perp - k'_\perp)}{4\pi \sqrt{EE'\gamma v}} \int dz e^{i\gamma v(E-E')z} \left\{ Z_P^*(k_3, z) \left[ \gamma(E + E') - iv \frac{d}{dz} \right] Z_P(k'_3, z) \right\}.
\]

However, the total charge should be time independent, implying that \( (P(k), P(k')) \) vanishes for \( E \neq E' \). This makes \( (P(k), P(k')) \) an appropriate inner product for the time-independent solutions.
Orthogonality can be verified directly. It follows from Eq. 2.14 that

\[
\frac{d}{dz} \left\{ e^{i\gamma(E'-E)z} Z_P^*(k_3, z) \left[ \frac{d}{dz} \right] Z_P(k'_3, z) \right\} = \gamma(E - E') e^{i\gamma(E'-E)z} \left\{ Z_P^*(k_3, z) \left[ \gamma(E + E') - iv \frac{d}{dz} \right] Z_P(k'_3, z) \right\}.
\]

Integrating this, the left side vanishes, so the inner product is zero unless \(E = E'\) or \(k_3 = \pm k'_3\).

When \(k_3 = \pm k'_3\), there is a delta function contribution to the inner product generated by the integral over \(z\). Its strength is determined by the asymptotic behavior of \(Z_P\), which we now specify. At large \(|z|\), solutions of Eq. 2.14 are plane waves, and we adopt scattering boundary conditions. For \(k_3 > 0\),

\[
Z_P(k_3, z) = \begin{cases} 
\exp(i\gamma k_3 z) + r_P(k_3) \exp(-i\gamma k_3 z), & (z << 0), \\
\exp(i\gamma k_3 z) + r_P(k_3) \exp(-i\gamma k_3 z), & (z >> L).
\end{cases}
\]

and for \(k_3 < 0\),

\[
Z_P(k_3, z) = \begin{cases} 
t_P(k_3) \exp(i\gamma k_3 z), & (z << 0), \\
\exp(i\gamma k_3 z) + r_P(k_3) \exp(-i\gamma k_3 z), & (z >> L).\end{cases}
\]

These scattering amplitudes satisfy “unitarity” relations that we use repeatedly. Eq. 2.14 implies that the expressions

\[
Z_P^*(k_3, z) \left[ \frac{d}{dz} \right] Z_P(k'_3, z), \quad Z_P(k_3, z) \left[ \frac{d}{dz} \right] Z_P(-k_3, z)
\]

are independent of \(z\) when \(k'_3 = \pm k_3\). Evaluating the expressions at large positive and negative \(z\), we obtain the unitarity relations

\[
|r_P(k_3)|^2 + |t_P(k_3)|^2 = 1, \quad (2.19)
\]

\[
r_P^*(k_3) t_P(-k_3) + t_P^*(k_3) r_P(-k_3) = 0,
\]

\[
t_P(-k_3) = t_P(k_3).
\]

Using Eq. 2.19, the complete orthogonality relation is

\[
(P(k), P(k')) = \gamma \delta(k - k'). \quad (2.20)
\]
Solutions studied so far have $E > 0$. Related negative frequency solutions are the complex conjugates $\hat{\varphi} = \exp(+iEt)P^*(k, x)$. The inner product of a negative energy solution with a positive energy solution is

$$((P^*(k), P(k')) = \int d^3x P(k) \left[ E' - E - i\gamma v \frac{\partial}{\partial z} \right] P(k')$$

(2.21)

$$= \frac{\gamma \delta(k_\perp + k_\perp')}{4\pi \sqrt{EE'}} \int dz e^{i\gamma v (E' + E)z} \left\{ \mathcal{Z}_P(k_3, z) \left[ \gamma(E' - E) - iv \frac{\partial}{\partial z} \right] \mathcal{Z}_P(k'_3, z) \right\}.$$ 

It can be shown that the integral over $z$ always vanishes. The complete set of orthonormality relations is

$$(P(k), P(k')) = -(P^*(k'), P^*(k)) = \gamma \delta(k - k'), \quad (P^*(k), P(k')) = 0.$$ (2.22)

The quantum fields can be expanded in terms of these solutions.

$$\hat{\varphi}(x, t) = e^{i\mu t} \int d^3k \left[ e^{-iEt} a(k) P(k, x) + e^{iEt} b^\dagger(k) P^*(k, x) \right].$$ (2.23)

Using Eq. 2.9, we find

$$\hat{\pi}^\dagger(x, t) = -ie^{iut} \int d^3k \left[ e^{-iEt} a(k) \left( \frac{E}{\gamma} - iv \frac{\partial}{\partial z} \right) P(k, x) \right.$$

$$\left. - e^{iEt} b^\dagger(k) \left( \frac{E}{\gamma} + iv \frac{\partial}{\partial z} \right) P^*(k, x) \right].$$ (2.24)

Now set $t = 0$ to obtain Schrödinger picture fields. Using orthonormality, we can project out the operator coefficients

$$a(k) = \int d^3x \left[ \hat{\varphi}(x) \left( \frac{E}{\gamma} + iv \frac{\partial}{\partial z} \right) P^*(k, x) + i\hat{\pi}^\dagger(x) P^*(k, x) \right],$$

(2.25)

$$b^\dagger(k) = \int d^3x \left[ \hat{\varphi}(x) \left( \frac{E}{\gamma} - iv \frac{\partial}{\partial z} \right) P(k, x) - i\hat{\pi}^\dagger(x) P(k, x) \right].$$

Canonical commutation relations for the fields imply that these operators all commute, except

$$[a(k), a^\dagger(k')] = [b(k), b^\dagger(k')] = \delta(k - k').$$ (2.26)
These operators destroy baryons \((a)\) and antibaryons \((b)\).

The reason our construction leads to a basis that diagonalizes \(K_{P0}\) is that, by Eqs. 2.23 and 2.24, the time dependence of the destruction and creation operators is sinusoidal. This implies

\[
[a(k), K_{P0}] = (E - \mu)a(k), \quad [b(k), K_{P0}] = (E + \mu)b(k). \quad (2.27)
\]

For these equations to hold we must have, aside from a normal ordering constant that is irrelevant in this problem,

\[
K_{P0} = \int d^3k \left[ (E - \mu) a^\dagger(k) a(k) + (E + \mu) b^\dagger(k) b(k) \right]. \quad (2.28)
\]

This equation may be derived directly by substituting the expansions for the Schrödinger picture operators into Eqs. 2.1, 2.2 and 2.7.

When \(v = 0\), \(K_{P0}\) becomes \(K_{V0}\), and the basis functions \(P(k, x)\) are replaced by

\[
V(k, x) = \exp[i k_\perp \cdot x_\perp] \mathcal{Z}_V(k_3, z), \quad (2.29)
\]

\[
d^2\mathcal{Z}_V(k_3, z) = \int dk_\perp \int dz \left[ k_\perp^2 + m^2 - m^2(z) \right] \mathcal{Z}_V(k_3, z) = 0.
\]

The fields may also be expanded in this basis:

\[
\hat{\phi}(x, t) = e^{i\mu t} \int d^3k \left[ e^{-iEt} A(k) V(k, x) + e^{iEt} B^\dagger(k) V^*(k, x) \right], \quad (2.30)
\]

\[
\hat{\pi}^\dagger(x, t) = -ie^{i\mu t} \int d^3k E \left[ e^{-iEt} A(k) V(k, x) - e^{iEt} B^\dagger(k) V^*(k, x) \right].
\]

The new operators \(A\) and \(B\) also have the commutation relations to destroy baryons and antibaryons. The operators \(K_{V0}\) and \(H_{V0}\) are diagonal when expressed in terms of these operators:

\[
K_{V0} = \int d^3k \left[ (E - \mu) A^\dagger(k) A(k) + (E + \mu) B^\dagger(k) B(k) \right],
\]

\[
H_{V0} = \int d^3k E \left[ A^\dagger(k) A(k) + B^\dagger(k) B(k) \right]. \quad (2.31)
\]
3 Current of Free Baryons

For free baryons, the obstacle to the evaluation of Eq. [1.2] is that ρ is expressed in terms of frame \( P \) operators \( a, \ldots \), while the Heisenberg picture operator \( \hat{J}_3(x, t) \) is expressed in terms of frame \( V \) operators \( A, \ldots \):

\[
\hat{J}_3(x, t) = -i \int d^3k d^3k' \left[ e^{i(E-E')t} A^+(\mathbf{k}) A(\mathbf{k}') V^*(\mathbf{k}, x) \left( \frac{\partial}{\partial z} \right) V(\mathbf{k}', x) + e^{i(E+E')t} A^+(\mathbf{k}) B^+(\mathbf{k}') V^*(\mathbf{k}, x) \left( \frac{\partial}{\partial z} \right) V^*(\mathbf{k}', x) + e^{i(-E-E')t} B(\mathbf{k}) A(\mathbf{k}') V(\mathbf{k}, x) \left( \frac{\partial}{\partial z} \right) V(\mathbf{k}', x) + e^{i(-E+E')t} B(\mathbf{k}) B^+(\mathbf{k}') V(\mathbf{k}, x) \left( \frac{\partial}{\partial z} \right) V^*(\mathbf{k}', x) \right].
\]  

(3.1)

Because \( E > 0 \), \( \forall k_3 \), oscillations must suppress the contributions of the two middle terms as \( t \to \infty \). To evaluate the contributions of the remaining terms, we must write the \( A, \ldots \) in terms of the \( a, \ldots \). Begin with the projections

\[
A(\mathbf{k}) = \int d^3x \left[ \hat{\phi}(\mathbf{x}) EV^*(\mathbf{k}, x) + i\hat{\pi}^+(\mathbf{x}) V^*(\mathbf{k}, x) \right],
\]

(3.2)

\[
B^+(\mathbf{k}) = \int d^3x \left[ \hat{\phi}(\mathbf{x}) EV(\mathbf{k}, x) - i\hat{\pi}^+(\mathbf{x}) V(\mathbf{k}, x) \right].
\]

Substitute expansions [2.23] and [2.24], evaluated at \( t = 0 \):

\[
A(\mathbf{k}) = \int_{-\infty}^{\infty} dk_3' \left[ a(\mathbf{k}_+ + \hat{z} k_3') R_1(k_+, k_3, k'_3) + b^+(\mathbf{k}_+ + \hat{z} k_3') R_2(k_+, k_3, k'_3) \right],
\]

(3.3)

\[
B^+(\mathbf{k}) = \int_{-\infty}^{\infty} dk_3' \left[ a(-\mathbf{k}_+ + \hat{z} k_3') R_1^*(k_+, k_3, k'_3) + b^+(\mathbf{k}_+ + \hat{z} k_3') R_2^*(k_+, k_3, k'_3) \right],
\]

where

\[
R_1(k_+, k_3, k'_3) = \int_{-\infty}^{\infty} \frac{dz e^{i\gamma E'z}}{4\pi \sqrt{E E'}} Z_V(k_3, z) \left( E + \gamma E' - iv \frac{d}{dz} \right) Z_F(k'_3, z),
\]

(3.4)

\[
R_2(k_+, k_3, k'_3) = \int_{-\infty}^{\infty} \frac{dz e^{-i\gamma E'z}}{4\pi \sqrt{E E'}} Z_V(k_3, z) \left( E - \gamma E' + iv \frac{d}{dz} \right) Z_F^*(k'_3, z).
\]
One of the traces we need for the baryon current is

$$\text{Tr} \left[ \rho A^\dagger(k) A(k') \right] = \int_{-\infty}^{\infty} dq_3 dq'_3$$

$$\times \left\{ R_1^*(k_\perp, k_3, q_3) R_1(k'_\perp, k'_3, q'_3) \text{Tr} \left[ \rho a^\dagger(k_\perp + \hat{z} q_3) a(k'_\perp + \hat{z} q'_3) \right] + R_2^*(k_\perp, k_3, q_3) R_2(k'_\perp, k'_3, q'_3) \text{Tr} \left[ \rho b(-k_\perp + \hat{z} q_3) b^\dagger(-k'_\perp + \hat{z} q'_3) \right] \right\}.$$  \hspace{1cm} (3.5)

The traces in Eq. (3.5) are standard \cite{11}, leading to the expression

$$\frac{\text{Tr} \left[ \rho A^\dagger(k) A(k') \right]}{\text{Tr} [\rho]} = \delta(k_\perp - k'_\perp) S_A(k_\perp, k_3, k'_3),$$  \hspace{1cm} (3.6)

$$S_A(k_\perp, k_3, k'_3) = \int_{-\infty}^{\infty} dq_3 \left\{ \frac{R_1^*(k_\perp, k_3, q_3) R_1(k'_\perp, k'_3, q_3)}{e^{\beta(E-\mu)} - 1} \right.$$  

$$\left. + R_2^*(k_\perp, k_3, q_3) R_2(k'_\perp, k'_3, q_3) \left[ \frac{1}{e^{\beta(E+\mu)} - 1} + 1 \right] \right\}.$$  

In this equation, \( E = \sqrt{k_\perp^2 + q_3^2 + m^2} \). There is a similar formula for \( \text{Tr} [\rho B(k) B^\dagger(k')] \). \( S_B \) differs from \( S_A \) by the interchange \( R_1 \leftrightarrow R_2 \).

The baryon current at late time is

$$J(z, t) = \int \frac{d^2 k_\perp dk_3 dk'_3}{2(2\pi)^3 \sqrt{E E'}} \left[ Z^*_V(k_3, z) \left( -i \frac{d}{dz} \right) Z_V(k'_3, z) \right]$$

$$\times \left[ e^{i(E-E')t} S_A(k_\perp, k_3, k'_3) - e^{-i(E-E')t} S_B(k_\perp, k_3, k'_3) \right].$$  \hspace{1cm} (3.7)

To extract the limiting current, consider the contribution of the term proportional to \( S_A \). First integrate over \( k_3 \), for fixed values of \( k_\perp \) and \( k'_3 \). The \( k_3 \) integration is initially along the real axis of the complex \( k_3 \) plane. Deform the contour to \( C \), which runs from \( k_3 = -\infty \) slightly below the real axis (in the third quadrant), crosses the real axis at \( k_3 = 0 \), and continues to \( k_3 = +\infty \) slightly above the real axis (in the first quadrant). Except at the origin, contributions on \( C \) are exponentially damped as \( t \to \infty \), and at the origin contributions are suppressed by oscillation. The contribution to the limiting value of \( J(z, t) \) is therefore zero unless singularities are encountered when the contour is changed from the real axis to \( C \). A time-independent contribution arises if \( S_A \) has a pole bordering the real axis, at \( k_3 = |k'_3| + i\eta, \) or at \( k_3 = -|k'_3| - i\eta, \) since at \( k_3 = \pm k'_3, \) we have \( E = E' \). In fact \( S_A \) as
defined by Eq. (3.6) has such poles. Note that when \( k_3 = \pm k_3' \), Eq. (2.14) implies that \( J \) is also independent of \( z \).

By Eq. (3.6), poles in \( S_A \) arise where poles of \( R_i \) pinch the \( q_3 \) integration contour. The \( R_i \) have poles adjacent to the real \( q_3 \) axis owing to divergence of the integrals at \( z = \pm \infty \) in Eq. (3.4). The positions and residues of these poles are determined by the asymptotic forms of \( Z_V \) and \( Z_P \), and therefore depend on the profile of the VEV wall only through the scattering amplitudes.

The asymptotic behaviors change with the sign of the real parts of \( k_3, k_3', q_3 \), and many contributions must be added to determine the residues of the poles of \( S_A \) at \( k_3 = \pm k_3' \). Here we consider only two of the contributions to illustrate what is involved.

When \( k_3 > 0 \) and \( q_3 < 0 \), the poles of \( R_1 \) arising at \( z = -\infty \) are the same as those of

\[
R_1(k_\perp, k_3, q_3) \sim \int_{-\infty}^{0} \frac{dze^{i\eta z+i\gamma q_3 z}}{4\pi \sqrt{E_k E_q}} \left[ e^{ik_3 z} + r_V(k_3)e^{-ik_3 z}\right]^* \\
\times \left(E_k + \gamma E_q - i\gamma v \frac{d}{dz}\right) t_P(q_3) e^{i\gamma q_3 z} \\
\sim \frac{t_P(q_3)(E_k + \gamma E_q + \gamma v q_3)}{4i\pi \sqrt{E_k E_q}} \left[ \frac{1}{\gamma q_3 + \gamma v E_q - k_3 - i\eta} \\
+ \frac{r_V^*(k_3)}{\gamma q_3 + \gamma v E_q + k_3 - i\eta}\right]. 
\]

The first denominator vanishes at \( q_3 = \gamma k_3 - \gamma v E_k \), and the second at \( q_3 = -\gamma k_3 - \gamma v E_k \). We rewrite the terms to exhibit poles in the \( q_3 \) plane.

\[
R_1(k_\perp, k_3, q_3) \sim \frac{t_P(\gamma k_3 - \gamma v E_k)}{2i\pi(q_3 - \gamma k_3 + \gamma v E_k - i\eta)} \sqrt{\frac{\gamma E_k - \gamma v k_3}{E_k}} \\
+ \frac{t_P(-\gamma k_3 - \gamma v E_k)r_V^*(k_3)}{2i\pi(q_3 + \gamma k_3 \gamma + v E_k - i\eta)} \sqrt{\frac{\gamma E_k + \gamma v k_3}{E_k}}. 
\]
\[ S_A(k_\perp, k_3, k'_3) \sim (3.10) \]

\[
\frac{|t_P(\gamma k'_3 - \gamma v E'_k)|^2(\gamma E'_k - \gamma v k'_3)\theta(\gamma v \sqrt{k^2_\perp + m^2 - k'_3})}{2\pi i E'_k(\gamma k_3 - \gamma v E_k - \gamma k'_3 + \gamma v E'_k - i\eta)[\exp \beta(\gamma E'_k - \gamma v k'_3 - \mu) - 1]}
\]

\[ + \frac{|t_P(\gamma k'_3 + \gamma v E'_k)|^2|v E'_k|^2(\gamma E'_k + \gamma v k'_3)}{2\pi i E'_k(-\gamma k_3 - \gamma v E_k + \gamma k'_3 + \gamma v E - i\eta)[\exp \beta(\gamma E'_k + \gamma v k'_3 - \mu) - 1]}.\]

The step function factor in the first term imposes \( q_3 < 0 \) so that the assumed asymptotic behavior holds at the pinch. Rewrite these terms to exhibit the poles in the \( k_3 \) plane.

\[ S_A(k_\perp, k_3, k'_3) \sim (3.11) \]

\[
\frac{|t_P(\gamma k'_3 - \gamma v E'_k)|^2\theta(\gamma v \sqrt{k^2_\perp + m^2 - k'_3})}{2\pi i (k_3 - k'_3 - i\eta)[\exp \beta(\gamma E'_k - \gamma v k'_3 - \mu) - 1]}
\]

\[ - \frac{|t_P(\gamma k'_3 + \gamma v E'_k)|^2|v E'_k|^2}{2\pi i (k_3 - k'_3 + i\eta)[\exp \beta(\gamma E'_k + \gamma v k'_3 - \mu) - 1]}.\]

The first of these poles is in the first quadrant, and contributes when the contour is deformed to \( C \), but the second pole is in the fourth quadrant and is irrelevant.

When all contributions are added, and the unitarity relations invoked, the relevant singularity of \( S_A \) is

\[ S_A(k_\perp, k_3, k'_3) \sim \frac{\text{sgn}(k'_3)}{2\pi i (k_3 - k'_3 - i\eta \text{sgn}(k'_3))[\exp \beta(\gamma E'_k - \gamma v k'_3 - \mu) - 1]}. \]

(3.12)

There is no relevant pole at \( k_3 = -k'_3 \). The term \( S_A \) makes the time-independent contribution

\[ 13 \]
\[ J_A \sim \int \frac{d^3 k}{2(2\pi)^3 E} \left[ Z^*_V(k_3, z) \left( -\frac{i}{d/dz} \right) Z_V(k_3, z) \right] \frac{1}{[\exp \beta(\gamma E - \gamma v k_3 - \mu) - 1]^2} \]

\[ \sim \int \frac{d^3 k k_3 |t_V(k_3)|^2}{(2\pi)^3 E} \frac{1}{[\exp \beta(\gamma E - \gamma v k_3 - \mu) - 1]^2}. \] (3.13)

In the term proportional to \( S_B \), deform the \( k_3 \)-integration from the real axis to contour \( C'' \), which lies in the second and fourth quadrants of the complex \( k_3 \)-plane. There is then a time-independent contribution from the following pole in \( S_B \):

\[ S_B(k_\perp, k_3, k'_3) \]

\[ \sim -\frac{\text{sgn}(k'_3)}{2\pi i [k_3 - k'_3 + i\eta \text{sgn}(k'_3)]} \left\{ \frac{1}{[\exp \beta(\gamma E' - \gamma v k'_3 + \mu) - 1]} + 1 \right\}. \] (3.14)

The addition to the baryon current is

\[ J_B \sim -\int \frac{d^3 k k_3 |t_V(k_3)|^2}{(2\pi)^3 E} \left\{ \frac{1}{[\exp \beta(\gamma E' - \gamma v k'_3 + \mu) - 1]} + 1 \right\}. \] (3.15)

The extra term in the brace is present because we did not normal order the current operator. No matter: The resulting integrand is odd in \( k_3 \) and vanishes. Altogether, the baryon current in the limit \( t \to \infty \) is

\[ J = \int \frac{d^3 k k_3 |t_V(k_3)|^2}{(2\pi)^3 E} \left\{ \frac{1}{\exp \beta(\gamma E - \gamma v k_3 - \mu) - 1} \right. \]

\[ -\frac{1}{\exp \beta(\gamma E - \gamma v k_3 + \mu) - 1} \right\}. \] (3.16)

This result is expected. The Bose distributions are those of a moving free gas; they give the densities of baryons and antibaryons, which differ for \( \mu \neq 0 \). The contributions to the net current density are opposite for baryons and antibaryons. They are given by particle density multiplied by velocity, \( k_3/E \), and the quantum mechanical probability of transmission past the VEV wall.
4 Plasmas with Self-Interactions

The result of the last Section shows that the approach based on statistical mechanics works for a plasma without self-interactions, for any velocity of the plasma relative to the VEV wall. The methods used there relied on the absence of self-interactions. On the other hand, the physical question of the influence of quasiparticle lifetime on the baryon current can be addressed only when we take self-interactions into account in a systematic way. This can be done by reformulating the problem as a standard problem in many-body theory. In particular, we want to reduce the problem to the computation of a temperature Green’s function.

In this Section, we so recast the problem, but only to first order in $v$. Note that the current of Eq. 3.16 is odd in $v$: $J(-v) = -J(v)$. We expect this reflection property to hold in a self-interacting theory also, when the VEV wall profile in the vicinity of $z = L$ is the mirror image of the profile in the vicinity of $z = 0$. Then the correction to our $O(v)$ result is $O(v^3)$, and the $O(v)$ result is applies over a useful range of $v$.

Write the statistical matrix of the self-interacting plasma in the form

$$\rho = e^{-\beta K_P} = e^{-\beta \gamma K_V U(\beta)}; \quad U(\tau) = e^{\tau \gamma K_V} e^{-\tau K_P}. \quad (4.1)$$

Introduce the boost operator, and its “imaginary time Heisenberg” picture:

$$B = \gamma K_V - K_P; \quad B(\tau) = e^{\tau \gamma K_V} B e^{-\tau \gamma K_V}. \quad (4.2)$$

Then

$$\frac{dU(\tau)}{d\tau} = B(\tau)U(\tau); \quad (4.3)$$

$$U(\tau) = \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\tau d\tau_1 \cdots d\tau_n T_\tau [B(\tau_1) \cdots B(\tau_n)],$$

where in the product, larger $\tau$ operators stand to the left of smaller $\tau$ operators. $\gamma K_V$ and $K_P$ have the same self-interactions, so

$$B = -\gamma v \int d^3x \left[ \frac{\partial \hat{\phi}^i}{\partial z} \hat{\pi}^i + \hat{\pi} \frac{\partial \hat{\phi}^i}{\partial z} \right]. \quad (4.4)$$
The power series for $U$ is therefore equivalent to an expansion in powers of $v$. Through first order in $v$ (but all orders in self-interactions)

$$JT r[\rho] = Tr[e^{-\beta K V} \hat{J}_3(x, t)] + \int_0^\beta d\tau Tr[e^{-\beta K V} B(\tau) \hat{J}_3(x, t)] + \ldots$$

(4.5)

The first term on the right is the baryon current of a plasma at rest in frame $V$; it is zero. The second term is the first order baryon current $J(1, x, t)$:

$$J(1, x, t)Tr[\rho] = \int_0^\beta d\tau Tr[e^{-\beta K V} B(\tau) \hat{J}_3(x, t)].$$

(4.6)

The Lehmann representation for $J(1, x, t)$ is

$$J(1, x, t)Tr[\rho] = \sum_{mn} \int dE_m dE_n e^{i(E_m - E_n)t} \left[ e^{-\beta(E_n - N_m \mu)} - e^{-\beta(E_m - N_m \mu)} \right] \frac{E_m - E_n}{E_m - E_n}$$

$$\times \int d\sigma_m d\sigma_n \langle m | \hat{J}_3(x) | n \rangle \langle n | B | m \rangle.$$ (4.7)

The sum is over eigenstates of $H_V$ and $N$. It is partially discrete and partially an integral over $dEd\sigma$; the state energy $dE$ is exhibited. We expect the inner, partially integrated, expression to include a contribution proportional to an energy delta function:

$$\int d\sigma_m d\sigma_n \langle m | \hat{J}_3(x) | n \rangle \langle n | B | m \rangle = \delta(E_m - E_n) \langle m | J | n \rangle + \ldots$$ (4.8)

The delta function contribution arises because operator $B$ is an integrated density. It is present for a free plasma, as we will show below, and it is there in each order in the perturbative expansion in the plasma self-interaction. Furthermore, current conservation requires $\langle m | J | n \rangle$ to be independent of $z$, because $E_m = E_n$. Between eigenstates of $H_V$, current conservation states

$$0 = \langle m | \frac{\partial \hat{J}_0}{\partial t} + \nabla \cdot \hat{J} | n \rangle = \langle m | i(E_m - E_n\hat{J}_0 + \nabla \cdot \hat{J}) | n \rangle.$$ (4.9)

When $E_m = E_n$, this becomes

$$0 = \nabla \cdot \langle m | \hat{J} | n \rangle = \frac{\partial}{\partial z} \langle m | \hat{J}_3 | n \rangle.$$ (4.10)
Only the energy-conserving contribution survives in the limit \( t \rightarrow \infty \), leaving the finite \( z \)-independent current

\[
J(1) \text{Tr}[\rho] = J(1, x, t = \infty) \text{Tr}[\rho] = \beta \sum_{m,n} \int dE_m \langle m | J | n \rangle \bigg|_{E_n = E_m}.
\] (4.11)

We can relate \( J(1) \) to the imaginary time temperature Green’s function

\[
\mathcal{G}(x, \tau_1; \tau_2) \text{Tr}[\rho] = -\text{Tr} \{ e^{-\beta K V} T_\tau \left[ \hat{J}_3(x, \tau_1) B(\tau_2) \right] \},
\]

\[
\hat{J}_3(x, \tau) = e^{\tau K V} \hat{J}_3(x) e^{-\tau K V}.
\] (4.12)

This Green’s function has familiar properties: It is a function of \( \tau_1 - \tau_2 \), and is periodic in this variable with period \( \beta \). The Fourier transform \( \tilde{\mathcal{G}}(x, \omega_p) \), with \( \omega_p = 2\pi p/\beta \), is

\[
\tilde{\mathcal{G}}(x, \omega_p) = \int_0^\beta d\tau e^{i\omega_p \tau} \mathcal{G}(x, \tau; 0).
\] (4.13)

This Fourier transform is the amplitude that has a simple expansion in Feynman diagrams in many-body perturbation theory. Its Lehmann representation is

\[
\tilde{\mathcal{G}}(x, \omega_p) \text{Tr}[\rho] = -\sum_{m,n} \int dE_m dE_n \left[ \frac{e^{-\beta(E_n - N_m \mu)} - e^{-\beta(E_m - N_m \mu)}}{E_m - E_n + i\omega_p} \right] \times \int d\sigma_m d\sigma_n \langle m | \hat{J}_3(x) | n \rangle \langle n | B | m \rangle.
\] (4.14)

We see immediately that \(-\tilde{\mathcal{G}}(x, \omega_p = 0)\) is identical with \( J(1, x, t = 0) \). However, Eq. 4.14 has contributions from states with \( E_m \neq E_n \) that make it differ from \( J(1) \), Eq. 4.11. Fortunately, there is a simple way to eliminate the contributions from unwanted states and recover \( J(1) \). We see from Eq. 4.6 that states with \( E_m = E_n \) make contributions to \( \tilde{\mathcal{G}}(x, \omega_p = 0) \) that are independent of \( z \), while the unwanted states make contributions that depend on \( z \). There is no difficulty in extracting the \( z \)-independent contribution; it pops out when the integration in Eq. 4.4 is carried out. Thus the relation between \( J(1) \) and the temperature Green’s function is

\[
J(1) = -\mathcal{G}(x, \omega_p = 0)|_{z-\text{ind}}.
\] (4.15)
Note that no analytic continuation in $\omega$ is required in this problem, in contrast to the situation when one studies real-time transport phenomena at finite temperature.

We are primarily interested in $\tilde{G}(x, \omega_p = 0)$ when the plasma has self-interactions. However, it is straightforward to compute it for a free plasma. It is useful to do so, because the emergence of the energy conserving delta function is illustrated, and the correctness of the result can be confirmed. We find

$$-\tilde{G}(x, \omega_p = 0) = -v \int \frac{d^2 k_\perp d k_3 d k'_3}{4(2\pi)^4 E E'} \left[ Z^*_V(k_3, z) \left( -i \frac{d}{dz} \right) Z_V(k'_3, z) \right]$$

$$\times \left\{ \frac{e^{\beta(E-\mu)} - e^{\beta(E'-\mu)}}{(E - E') [e^{\beta(E-\mu)} - 1][e^{\beta(E'-\mu)} - 1]} - \frac{e^{\beta(E+\mu)} - e^{\beta(E'+\mu)}}{(E - E') [e^{\beta(E+\mu)} - 1][e^{\beta(E'+\mu)} - 1]} \right\} \right(4.16)$$

$$\times \int_{-\infty}^{\infty} dz' \left[ -iE \frac{dZ^*_V(k'_3, z')}{dz'} Z_V(k_3, z') + iE' Z^*_V(k'_3, z') \frac{dZ_V(k'_3, z')}{dz'} \right]$$

$$-v \int \frac{d^2 k_\perp d k_3 d k'_3}{4(2\pi)^4 E E'(E + E')} \left[ e^{\beta(E-\mu)} e^{\beta(E'+\mu)} - 1 \right]$$

$$\times \left\{ \right. Z^*_V(k_3, z) \left( -i \frac{d}{dz} \right) Z^*_V(k'_3, z) \int_{-\infty}^{\infty} dz' \left[ -iE \frac{dZ^*_V(k'_3, z')}{dz'} Z_V(k_3, z') ight]$$

$$-iE' Z_V(k'_3, z') \frac{dZ_V(k'_3, z')}{dz'} \left\} \right. \left. + \right. Z_V(k_3, z) \left( i \frac{d}{dz} \right) Z_V(k'_3, z) \int_{-\infty}^{\infty} dz' \left[ iE \frac{dZ^*_V(k'_3, z')}{dz'} Z^*_V(k_3, z') \right]$$

$$+ iE' Z^*_V(k'_3, z') \frac{dZ^*_V(k'_3, z')}{dz'} \right\} .$$

First note that this Green's function does not depend on $x_\perp$. The reason is that the transverse integrations in Eq. 4.4 set $k_\perp = k'_\perp$. However, $k_3 \neq k'_3$ because of the VEV wall, and the Green's function certainly depends on $z$. The
integrations over $z'$ come from Eq. [4.4]. They do not impose momentum or energy conservations generally, but the asymptotic tails of these integrals do produce energy conserving delta functions in addition to other contributions. Such delta functions lead to discrete position-independent contributions to $G(x, \omega_p = 0)$. The delta function terms occur in just one of the integrals:

$$\int_{-\infty}^{\infty} dz' \left[ -iE \frac{dZ^*_V(k_3', z')}{dz'} Z_V(k_3, z') + iE' \frac{dZ^*_V(k_3', z')}{dz'} \right]$$

$$= -4\pi k_3 E \left[ |t_V(k_3)|^2 \delta(k_3 - k_3') + r^*_V(-k_3)t_V(k_3)\delta(k_3 + k_3') \right] + \ldots .$$

Retaining just the delta functions, the $z$-independent contribution to $G(x, \omega_p = 0)$ implies

$$J(1) = -G(x, \omega_p = 0)|_{z_{\text{ind}}}$$

$$= \frac{v\beta}{(2\pi)^3} \left\{ \frac{e^{\beta(E-\mu)}}{e^{\beta(E-\mu)} - 1} - \frac{e^{\beta(E+\mu)}}{e^{\beta(E+\mu)} - 1} \right\} .$$

This agrees with the $O(v)$ baryon current obtained by expanding Eq. [3.16].

Physical questions about the baryon current are thus converted into questions about a temperature Green’s function. We have experience and techniques for the study of such objects.

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