Formation and evolution of galaxy disks: what is wrong with CDM?

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Abstract. The density distribution of galaxy disks formed within ΛCDM halos is in rough agreement with observations. The luminous-to-dark matter ratio increases with the surface brightness (SB). The lowest SB models tend to be minimum disks, but the high SB models hardly attain the maximum disk solution. With the introduction of shallow cores in the halos, high SB models become maximum disks. The shallow cores also help to improve the inner density profiles of bulge-less low SB models and the zero-point of the Tully-Fisher relation. The models predict well this relation and its scatter, as well as the small correlation among the residuals of this and the luminosity-radius relation, in spite of the dependence of the rotation curve shapes on SB.

1. Introduction

According to the Cold Dark Matter (CDM) scenario, disk galaxies form within hierarchically growing dark halos. In the last years, high-resolution N-body simulations and semi-analytical approaches allowed to understand the structure, correlations, and evolution of the CDM halos in such a way that the properties of the disks formed within them can be now modeled in detail and compared with observations. Several of these properties agree with observations when detailed angular momentum conservation, smooth gas accretion (no mergers), and negligible disk-halo feedback are assumed (e.g., Firmani et al. 1997; Dalcanton et al. 1997; Mo et al. 1998; Avila-Reese et al. 1998; van den Bosch 2000,2001; Avila-Reese & Firmani 2000 (AF00); Firmani & Avila-Reese 2000 (FA00); Buchalter et al. 2001). Nevertheless, in more detail, potential difficulties seem to arise regarding e.g., the central luminous-to-dark matter ratios, the disk surface brightness profiles, and the zero-point of the Tully-Fisher relation.

Here we explore these issues by using semi-numerical models, where the coupled dynamics, hydrodynamics, SF and feedback of the dark/baryonic matter system are self-consistently solved under several simplifying assumptions (see e.g., AF00). The virtue of the semi-numerical approach is that enables to follow the overall evolution of the halo/disk/bulge system (as in the numerical simulations) and, at the same time, allows to predict correlations and statistical properties of the galaxy population (as in the semi-analytical models).
2. Density distribution of galaxy disks

The model. The disk is built up within the gravitational potential of a growing CDM halo. We use the extended Press-Schechter approach to generate the statistical mass aggregation histories (MAHs) of the CDM halos, and a generalized secondary infall model to calculate the virialization of the accreting mass shells (Avila-Reese et al. 1998). The mass shells are assumed to have aligned rotation axis and to be in solid body rotation, with specific angular momentum given by $j_{sh}(t_v) = dJ(t_v)/dM(t_v)$, where $J = \lambda GM^{5/2}/|E|^{1/2}$, $J$, $M$ and $E$ are the total angular momentum, mass and energy of the halo at the shell virialization time $t_v$. The spin parameter, $\lambda$, is assumed to be constant in time (FA00). As the result of the assembling of these mass shells, a present day halo ends with an angular momentum distribution close to the universal distribution found in the N-body simulations by Bullock et al. (2001). A fraction $f_d$ of the mass of each shell is assumed to cool down and form a disk in a dynamical time. The radial mass distribution of the infalling gas is calculated by equating its specific angular momentum to that of its final circular orbit in centrifugal equilibrium (detailed angular momentum conservation is assumed). The gravitational interaction of disk and halo is calculated using the adiabatic invariant formalism. The local SF within the growing disks is triggered by the Toomre gas gravitational instability criterion and self-regulated by a vertical disk balance between the energy input due to SNe and gas accretion and the turbulent energy dissipation in the ISM. The SF history in our models depends on the gas surface density determined mainly by $\lambda$, and on the gas accretion rate determined by the cosmological MAH. Finally, we consider the formation of a secular bulge using the Toomre criterion for a stellar disk.

Results. We obtain nearly exponential stellar surface density profiles, $\Sigma_\ast(r)$, with scale radii $R_\ast$ growing with time (inside-out formation). The question whether real disks grow with time as in the hierarchical scenario is matter of discussion currently. At $z = 0$, the $R_\ast$ of the modeled disk galaxy population for a ΛCDM cosmology are in agreement with observations in the luminosity-radius (L-R) and velocity-radius diagrams. The gaseous disks are also roughly exponential with $R_g \sim 3 - 4R_\ast$. The predicted relation between gas fraction and surface brightness (SB) and its scatter are in good agreement with observations (FA00; AF00). The surface density of the disks is determined mainly by $\lambda$ (see e.g., Fig. 3b in FA00); the mass and the MAH play a minor role: the disks are slightly less dense for smaller masses and/or more extended MAHs.

In more detail, the $z = 0$ density profiles are typically more concentrated in the center and with an excess at the periphery than the exponential distribution. Bullock et al. (2001) find that this is a direct consequence of the CDM halo $J-$distribution. We also find that $\Sigma_\ast(r)$ is mainly determined by the initial $J-$distribution, but the sequential way in which the disk slabs aggregate (given by the halo MAH) and the way the gas is transformed in stars, also play a role. For the smoothed (averaged) MAHs with a given statistical significance (see Fig. 1b in FA00), $\Sigma_\ast(r)$ always presents a non-negligible core and tail excesses w.r.t. an exponential (Fig. 3a in FA00). However, when the individual realizations of the MAHs are used (Fig. 1a in FA00), in several cases the profiles result very close to an exponential law. The individual MAHs are discontinuous and some
Figure 1. (a): Central SB excess w.r.t. the exponential disk, $\Delta_0$, vs. b/d ratio of 60 random models in MAH and $\lambda$ and masses $M_v = 3.5 \times 10^{10}$ (triangles), $3.5 \times 10^{11}$ (squares), and $3.5 \times 10^{12} M_\odot$ (circles). Arrows show by how much $\Delta_0$ would increase by the bar action according to N-body simulations and the dashed lines are averages for $\Delta_0$ from observations (see text). (b): The same models of $M_v = 3.5 \times 10^{11} M_\odot$ plotted in (a), but with shallow cores in the halos.

Sometimes “jump” from the high to the low accretion regimes and viceversa, so that using a simple parametrization for them (e.g., van den Bosch 2001), get to lose a significant fraction of possible MAHs. Note that $j_{sh}$ is a function of the time at which the mass shell is incorporated into the halo, i.e. it depends on the MAH.

In Fig. 1 we show the central surface density (brightness) excess w.r.t. the exponential fit in magnitudes, $\Delta_0 \equiv \mu_0 - \mu_{0,\exp}$, vs. the bulge-to-disk mass (b/d) ratio that we obtain for disks formed within $\Lambda$CDM halos using randomly selected MAHs and $\lambda$ (the latter taken from the usual lognormal distribution). The models are for a flat cosmology with $\Omega_\Lambda = h = 0.7$ and a disk mass fraction $f_d=0.05$ (the same models as in FA00). The bulge mass is calculated as the mass where the Toomre instability criterion for a stellar disk is obeyed. This simple model is in line with the secular bulge formation scenario, which seems to be adequate for disk galaxies later than Sba type (see Avila-Reese & Firmani 1999 and the references therein). Gravitational instabilities in the disk produce bars which dissolve forming a bulge (Christodoulou et al. 1995; Norman et al. 1996; Valenzuela & Klypin 2002). Thus, according to the models, the region of core excess that disks formed within CDM halos present, is typically the region that will be transformed into a spherical component (AF00).

In Fig. 1, estimates of the average $\Delta_0$ from the high SB (HSB) sample of disk galaxies of de Jong (1996) and Verheijen (1997) and from the low SB (LSB) sample of the latter author are shown. These authors have decomposed the observed $K$-band SB profiles in (exponential) disk and bulge components. Solid and empty symbols in Fig. 1 are for models with disk central $\Sigma_{*0} > 200 M_\odot$ pc$^{-2}$ and $\Sigma_{*0} \leq 200 M_\odot$ pc$^{-2}$, respectively. Upon the understanding that the core excess $\Delta_0$ corresponds to a bulge, HSB models have in fact less concentrated SB profiles than observed HSB galaxies. Several processes could concentrate
further the central disk/bulge region. For example, we did not take into account the mass inflow that will produce the bar before it dissolves into a bulge. High-resolution N-body simulations of a Milky Way disk within a CDM halo show that this process increases $\Sigma_{*0}$ by $\approx 1.15$ and $1.30$ mag for disks with initial $R_*$ of 3.5 and 3.0 kpc, respectively (Valenzuela & Klypin 2002). These factors are plotted in Fig. 1 (dashed arrows) for the average $\Delta_0$ of the models corresponding to $b/d=0.1$ and 0.2, which are the final values obtained by Valenzuela & Klypin.

The LSB galaxy models have typically small or nonexistent bulges (models without bulges were plotted in Fig. 1 with $b/d$ ratios of $10^{-3}$). In these cases, the SB core excess can not be interpreted at all as a bulge. Actually, there are LSB models with $\Delta_0$ below the observational average. Therefore, in some cases the pure disks formed within CDM halos may be well fitted by an exponential law. This is at odds with the strong conclusion of van den Bosch (2001) that LSB disks formed within CDM halos are much more concentrated than an exponential, in conflict with observations. Although he implemented a disk/bulge formation scheme similar to ours, he used only smoothed MAHs. When using these smoothed MAHs, we obtain core excesses typically larger than when the random MAHs are used. We also differ with van den Bosch (2001) in the way the SF is modeled: we use a physical self-consistent model of SF. The models predict a Schmidt law with index $<2$, which changes slightly with radius.

Back to Fig. 1, one sees that the average $\Delta_0$ for the LSB models is larger than observations. As mentioned below, several pieces of evidence suggest shallow cores in dark halos. In Fig. 1b we show the same $3.5 \times 10^{11} M_\odot$ models as in Fig. 1a, but with a shallow core in their halos. The disks formed within these halos are slightly less concentrated in the center than disks formed within cuspy halos. Regarding the tail excess of $\Sigma_*(r)$ w.r.t. an exponential, again, using the random MAHs, there are cases when this excess is even reverted. The observed SB profiles also show a diversity of cases with tail excess or defect. Although a more quantitative comparison of models and observations is desirable, the density distributions of disks formed within CDM halos seem to be not in strong conflict with observations, in particular if the halos have a soft core.

3. Luminous-to-dark matter ratios

The shapes of the model rotation curves (RCs) are smooth, showing a weak conspiracy between the dark and luminous matter distributions. The higher the disk surface density (the smaller the $\lambda$), the steeper the declining shape of the RC (see Fig. 5 in FA00 and Fig. 3b here); this is in agreement with some observational evidence (Casertano & van Gorkom 1990; Verheijen 1997). The same trend is observed with the disk mass fraction $f_d$. Regarding the RC decomposition, the dark component dominates even in the central regions for models with $\lambda \gtrsim 0.035$ and $f_d \approx 0.05$. For $\lambda \lesssim 0.045$, the models disks have large surface densities typically (HSB galaxies).

The disk-to-total rotation velocity ratio at 2.2 scale radii, $(V_d/V_t)_{2.2} \approx (V_d/V_t)_{\text{max}}$, offers a quantitative way to measure the model luminous-to-dark matter ratio. In Fig. 2 we plot $(V_d/V_t)_{2.2}$ vs. $\Sigma_{*0}$ for the 20 random realizations with $M_v = 3.5 \times 10^{11} M_\odot$ (solid symbols). The luminous-to-dark matter ratio of disk galaxies within CDM halos continuously decreases from HSB to LSB disks.
Observations suggest that HSB galaxies tend to the maximum disk case (e.g., Corsini et al. 1998; Salucci & Persic 1999; Palunas & Williams 2000). Comparison of population synthesis models with the photometric properties of HSB galaxies also point to mass-to-luminosity ratios corresponding to the maximum disk case (e.g., Bell & de Jong 2001). Theoretical arguments as the swing amplifier constraints (Athanassoula et al. 1987) also suggest central luminous matter dominion in HSB galaxies. Regarding LSB galaxies, it is well known that they are dark matter dominated systems (de Blok & McGaugh 1997). A maximum disk solution for LSB galaxies demands too high mass-to-luminosity ratios from the stellar population point of view (de Blok et al. 2001).

Our results enable to interpret the problem of whether galaxies are maximum or sub-maximal disks: *all the cases are possible, from maximum disk for the highest SB galaxies to Bottema and other sub-maximal solutions for the lower SB galaxies*. However, for the ΛCDM, \( f_d = 0.05 \) models we study here, the average value of \( \left( \frac{V_d}{V_t} \right)_{2.2} \) is \( \sim 0.7 \) for a typical HSB galaxy \( \Sigma_{*,0} \approx 400M_\odot \text{pc}^{-2} \), while for the maximum disk solution that observations seem to favor at least in some cases, \( \left( \frac{V_d}{V_t} \right)_{2.2} = 0.85 \pm 0.1 \). Therefore, HSB galaxies formed within CDM halos are too much dark matter dominated in the center so as to be maximum disks. This is because the inner mass distribution of the CDM halos is very concentrated. Observations of dark matter dominated galaxies (dwarfs and LSB) show that the halo inner density profile is indeed shallower than CDM predictions (see e.g., Bosma, this volume). We artificially flattened the inner density profiles of the growing CDM halos in such a way that at \( z = 0 \) (taking into account the gravitational drag of the disk) the halos have shallow cores similar to those inferred for observed LSB galaxies. Results for these models

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**Figure 2.** Disk-to-total velocity ratio at 2.2 \( R_\ast \) vs. \( \Sigma_{\ast,0} \) of 20 random realizations in MAH and \( \lambda \), and \( M_V = 3.5 \times 10^{11}M_\odot \) for ΛCDM (filled circles) and shallow (empty circles) halos. Solid and long-dashed lines are linear regressions for the filled and empty circles. Arrows indicate the range of maximum disks, and the dashed line is for Bottema disks. A typical value of \( \Sigma_{\ast,0} \) for HSB disks is shown with the open arrow.
are shown with empty circles in Fig. 3. Now, a typical HSB galaxy can be maximum disk. The scatter in Fig. 3 is due to the MAH.

**The infrared Tully-Fisher relation (TFR) and its scatter.** According to the models, this relation is a direct consequence of the mass-velocity relation of the ΛCDM halos, which in the range of galaxy masses has a slope of ≈ 3.2 (FA00). The zero-point of the model TFR is fainter by ∼ 0.6 mag w.r.t. the Giovanelli et al. (1997) and Tully & Pierce (2000) I-band TFRs, but is in agreement with other observational determinations. In the N-body/hydrodynamical simulations, the zero-point is much fainter than the observed (Navarro, this volume). This is because in these simulations disks end much more concentrated than observed, producing highly peaked rotation curves, i.e. large $V_m$. Our models with a shallow core in the halos (see above) give a TFR only 0.1 – 0.2 mag fainter than in the the two first observational works mentioned above.

The scatter in the model TFR is produced mainly by (i) the scatter in the halo concentrations given by the stochastic nature of the MAHs, and (ii) the scatter in the disk SBs given by the dispersion of λ. FA00 have found that the latter is reduced significantly due to the SF efficiency dependency on surface density. As is seen in Fig. 3a, for a given mass, $V_m$ increases with SB (see also above). This introduces a large scatter in the total disk mass-$V_m$ relation, even larger than the scatter due to the halo concentrations. However, the higher the disk surface density, the more efficient the gas transformation in stars, so that $M_*$ or the infrared luminosity of disks of the same mass will be larger for the higher SB ones: the models shift along the main TFR! (Fig. 4, solid arrows). Therefore, the TFR of HSB and LSB galaxies is nearly the same, in agreement with observations. Neither does the disk mass fraction $f_d$ introduce significant scatter in the TFR: for larger (smaller) $f_d$, the luminosity will be larger (smaller) but $V_m$ will also be larger (smaller) in such a way, that the model shifts along the main TFR (Fig 3, dotted arrows). The average scatter in the model TFR is 0.36 mag, mainly produced by the dispersion in the halo concentrations (MAHs).

**Correlation among the residuals of the TF and L-R relations.** Because of the lack of correlation of the scatter in the TFR with SB, one also expects a lack of correlation among the residuals of the TF and L-R relations. Observations show indeed a very small correlation among these residuals (Courteau & Rix 1999). The last authors interpreted this result as an evidence of large dark halo dominion in disk galaxies. If dark matter dominates strongly, then variations in the disk density will not change significantly $V_m$ for a given mass, therefore deviations from the main TFR in velocity will not be correlated with deviations from the main L-R relation. This interpretation is valid when one uses the total disk mass $M_d$ instead of $M_*$ or $L$. We find that the average slope of the correlation among the residuals of the $M_d$-$V_m$ and $M_d$-$R$ relations for disks formed within the cuspy CDM halos is $\delta \lg V_m / \delta \lg R \approx -0.35$. Therefore, according to the interpretation of Courteau & Rix, even CDM halos would then be in conflict with observations. If a shallow core is introduced, then the slope is even steeper, $\approx -0.4$ (this slope is $-0.5$ for maximum disks). However, in order to compare it with observations, we have to use $M_*$ or luminosity instead of $M_d$. In this case the correlation disappears! (Fig. 8 in FA00). The explanation is the same as why the scatter of the TFR does not correlate with SB in spite of
the fact that the RC shape does: the dependence of SF efficiency on surface density makes the difference (see above). In more detail, we predict that from the small to the large $\delta \lg R$ side (from HSB to LSB disks), $\delta \lg V_m/\delta \lg R$ is first negative, then becomes 0 (for most of the normal galaxies) and then increases for the lowest SB galaxies. A preliminary analysis from observations using the Verheijen (1997) sample confirms this trend (Fig. 8b in FA00).

4. Conclusions

Using self-consistent evolutionary models of disk galaxy formation within ΛCDM halos, and under the assumption of detailed angular momentum conservation and $\lambda =$const in time, we find that the disk stellar and gas surface density profiles are in good agreement with observations. The core excess w.r.t. the exponential law that the models show corresponds to a bulge formed by disk gravitational instabilities (AF00). In the case of bulge-less LSB disks, although there are models presenting negligible core excess, most of them tend to show slightly more concentrated SB profiles than those observed. The introduction of shallow cores in the halos alleviates this problem. A key point to have in mind is the very discontinuous nature of the cosmological MAHs and the SF physics.

A potential shortcoming of disks formed within CDM halos is that dark matter dominates even for HSB galaxies. Again, the introduction of shallow cores in agreement with direct inferences from dwarf and LSB galaxies, solves this problem. The models show that the luminous-to-dark matter ratio in disk...
galaxies continuously decreases with SB: *the highest SB galaxies are maximum disks while the lowest SB ones are close to minimal disks*.

The slope, zero-point and scatter of the model infrared TFR for HSB and LSB galaxies are in good agreement with the observational reports of several authors. The introduction of shallow cores in the CDM halos improves the comparison. *Even that the RC shape of the models correlates with the disk SB, a negligible correlation among the residuals of the TF and L-V relations is found*, in agreement with observations. Therefore, the lack of this correlation should not be interpreted as a definitive evidence of sub-maximal disks, where the RC shape does not depend on SB. We conclude that $z = 0$ disks formed within modified (shallow) CDM halos are realistic under the assumption of no major merger assembly and angular momentum conservation.

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**References**

Athanassoula, E., Bosma, A., Pappaioannou, S. 1987, A&A, 179, 23
Avila-Reese, V, & Firmani, C. 1999, ASP Conf. Series, 163, 243
   _______, 2000, RevMexA&A, 36, 23 (AF00)
Avila-Reese, V., Firmani, C., & Hernández X. 1998, ApJ, 505, 37
Bell, E. & de Jong, R.S. 2001, ApJ, 550, 212
Buchalter, A., Jimenez, R. & Kamionkowski, M. 2001, MNRAS, 322, 43
Bullock, J. et al. 2001, ApJ, 555, 240
Casertano, S. & van Gorkom, J.H. 1991, AJ, 101, 1231
Christodoulou, D. M., Shlosman, I. & Tohline, J. E. ApJ, 443, 551
Corsini, E.M. et al., 1998, A&A, 342, 671
Courteau, S., & Rix, H-W. 1998, ApJ, 513, 561
Dalcanton J.J., Spergel D.N., Summers F.J., 1997, ApJ, 482, 659
de Jong, R.S. 1996, A&ASS, 118, 557
Firmani, C., Avila-Reese, V., Hernández, X. 1997, ASP Conf. Series, 117, 424
Firmani, C. & Avila-Reese, V. 2000, MNRAS, 315, 457 (FA00)
Giovanelli, R. et al. 1997, AJ, 113, 53
McGaugh S.S. & de Blok W.J.G., 1997, ApJ, 481, 689
Mo H.J., Mao S. & White S.D.M., 1998, MNRAS, 295, 319
Norman, C.A., Sellwood, J.A. & Hassan, H. 1996, ApJ, 462, 114
Palunas, P. & Williams, T.B. 2000, AJ, 120, 2884
Salucci P. & Persic M. 1999, A&A, 351, 442
Tully, R.B. & Pierce, J.P. 2000, ApJ, 533, 749
van den Bosch, F.C., 2000, ApJ, 530, 177; & ____, 2001, MNRAS, 337, 1334
Valenzuela, O. & Klypin, A. 2002, in preparation
Verheijen, M.A.W., 1997, PhD. Thesis, Groningen University