Cosmic Magnetic Fields and the CMB

Ruth Durrer

Université de Genève, Ecole de Physique
24 Quai E. Ansermet, CH-1211 Genève
Switzerland

Abstract

I describe the imprint of primordial magnetic fields on the CMB. I show that these are observable only if the field amplitude is of the order of $B \gtrsim 10^{-9}G$ on Mpc scale. I further argue that such fields are strongly constrained by the stochastic background of gravity waves which they produce. Primordial magnetic fields, which are strong enough to be seen in the CMB, are compatible with the nucleosynthesis bound, only if their spectrum is close to scale invariant, or maybe if helical magnetic fields provoke an inverse cascade. For helical fields, the CMB signature is especially interesting. It contains parity violating T-B and E–B correlations.

Key words: Cosmology, Magnetic fields, Cosmic microwave background

1 Introduction

In observational cosmology we try to constrain the history of the Universe by the observation of relics. The best example of this is the cosmic microwave background (CMB) which represents not only a relic of the time of recombination, $t \approx 3.8 \times 10^5$ years after the big bang, but probably also of a much earlier epoch, $t \sim 10^{-35}$ sec, when inflation took place. Another such relic is the abundance of light elements established during primordial nucleosynthesis at $t \approx 100$ sec.

But there are other very interesting events which might have left observable traces, relics, in the universe. Most notably confinement at $t \approx 10^{-4}$ sec or the electroweak transition at $t \approx 10^{-10}$ sec which may have generated the observed baryon asymmetry in the Universe, see Ref. [1]. It has been proposed that confinement and, especially the electroweak phase transition might also lead to the formation of primordial magnetic fields which then can seed the magnetic fields observed in galaxies and clusters, see Refs. [2, 3].
Magnetic fields are ubiquitous in the Universe. Most galaxies, like the milky way, are permeated by a magnetic field of the order of a few µGauss, see Ref. (4). But also clusters of galaxies have magnetic fields of the same order, see Ref. (5).

If these fields are due to the amplification of seed fields by to the contraction of the cosmic plasma during the process of galaxy formation, seed fields of the order of $10^{-9}$G are needed. However if they are amplified via a non linear dynamo mechanism, seed fields as little as about $10^{-22}$ Gauss might be sufficient, see Ref. (6). It is not clear whether seed fields are really needed. It may be possible that charge separation processes during structure formation can lead to currents which generate magnetic fields without the presence of any seed fields. It is still a matter of debate whether second order perturbations alone can induce sufficient charge separation, and therefore currents, to provoke the formation the observed fields, see Refs. (31; 32).

In this talk I assume that primordial magnetic fields have been generated at early time with some initial spectrum, and I discuss their effects on the CMB. Then I derive the spectrum of causal magnetic fields (i.e. fields generated during a non-inflationary phase of the universe). We shall see that magnetic fields, especially causal magnetic fields, are very strongly constrained by the gravity wave background which they induce. I shall finally indicate some possible ways out of this stringent constraints.

### 2 Effects of magnetic fields on the CMB

#### 2.1 A constant magnetic field

A spatially constant magnetic field affects the geometry of the universe by introducing shear. It generates an anisotropic stress, $\Pi_{ij} \propto B_i B_j - \frac{1}{3} B^2 \delta_{ij}$. This leads to a well studied (anisotropic) homogeneous model, the so called Bianchi VII model, see Ref. (7). The propagation of CMB photons from the last scattering surface into our antennas through the Bianchi VII geometry leads to anisotropies. Comparing these with the observed anisotropies, we obtain limits on a constant magnetic field. Comparing, e.g., the CMB quadrupole with the one induced by a constant magnetic field, we can limit the field amplitude to $B < 6.8 \times 10^{-9}(\Omega_m h^2)^{1/2}$Gauss, see Ref. (8).

As usual, we decompose the CMB temperature fluctuations into spherical harmonics,

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\mathbf{n}),$$
Fig. 1. The CMB anisotropy pattern from a constant magnetic field, from Ref. (9).

where \( \mathbf{n} \) denotes the direction of observation. The coefficients \( a_{\ell m} \) can be considered as the amplitude of the spin \( \ell \) contribution to the temperature fluctuation (with \( z \)-component \( m \)). In a statistically isotropic universe
\[
\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_\ell \delta_{\ell \ell'} \delta_{mm'}.
\]
Different \( \ell \)'s and \( m \)'s are uncorrelated. Since the presence of a constant magnetic field breaks statistical isotropy, see Fig. 1, it leads to correlations of \( a_{\ell m} \)'s with different values of \( \ell \). A detailed analysis shows that, see Ref. (9), for a constant magnetic field in \( z \)-direction, there are non-vanishing correlators with \( \ell \neq \ell' \), namely
\[
\langle a_{\ell-1 m} a_{\ell+1 m}^* \rangle \neq 0.
\]
The magnetic field energy momentum tensor acts like a spin-2 field and leads to transitions from \( \ell - 1 \) to \( \ell + 1 \) thereby correlating these amplitudes.

Limiting such off-diagonal correlations with the COBE data also leads to bounds of the order of \( B < 3 \times 10^{-9} \) Gauss, see Ref. (9).

It is not surprising that the limits from the quadrupole and from the off-diagonal correlators are comparable, since
\[
\Omega_B = 10^{-5} \Omega_{rad} (B/10^{-8} \text{Gauss})^2.
\]
Therefore magnetic fields of the order \( 3 \times 10^{-9} \) Gauss will leave of order 10\% effects on the CMB anisotropies while \( 10^{-9} \) Gauss will typically contribute 1\% effects. It is thus clear that we can never detect magnetic fields of the order of \( 10^{-22} \) Gauss with CMB observations.

2.2 CMB anisotropies from stochastic magnetic fields

To obtain the limits for a constant magnetic field given in the previous section, we have only taken into account that the energy momentum tensor of the magnetic field affects the geometry. When the CMB photons then propagate along the correspondingly modified geodesics, the CMB sky becomes anisotropic. These anisotropies are to be added to the anisotropies due to the usual inflationary perturbations. But there are also other effects of magnetic fields on CMB photons. In this section I briefly describe them all.

The magnetic field energy momentum tensor contains scalar, vector and tensor
components which all modify the spacetime geometry and therefore affect the propagation of photons as we have discussed above. For tensor perturbations this is all there is, see Ref. (10).

In addition, via its coupling to the charged electron–proton plasma, the magnetic field generates vector perturbations in the plasma velocity which oscillate, so called Alfvén waves, see Refs. (11; 9). This is a new phenomenon. Purely gravitational vector perturbation do not show oscillatory behavior. If we would be able to constrain (or better even detect!) possible vector contributions to the CMB anisotropies this may provide very important limits on primordial magnetic fields due to the specific signal from Alfvén waves.

There are also two types of scalar waves in the presence of magnetic fields, the so-called fast and slow magneto-sonic waves. They are induced by the scalar perturbations of the magnetic field and the charged plasma, see Ref. (12). Fast magneto-sonic waves are simply the ordinary sound waves which are modified due to the presence of the magnetic fields, they acquire a somewhat higher sound speed \( c_s^2 \rightarrow c_s^2 + (k \cdot B)^2/(4\pi \rho) \) leading to a slight shift of the acoustic peaks which might be detectable, see Fig. 2. Slow magneto-sonic waves provide a new form of waves due to the interaction of the charged plasma with the magnetic field, see Refs. (11, 12). They have a very low sound speed and their effect on the CMB is small.

2.3 Limiting magnetic fields with the CMB

To formulate limits on a stochastic primordial magnetic field distribution, we have to define an appropriate way to describe the latter. For this we define
the magnetic field spectrum in the form

$$a^4(t)\langle B_i(k)B_j^*(k')\rangle k^3 = \begin{cases} \frac{1}{2} \delta^3(k - k') \left( \delta_{ij} - \hat{k}_i \hat{k}_j \right) (k\lambda)^{n+3}B^2_\lambda & \text{for } k < k_D \\ 0 & \text{for } k > k_D \end{cases}$$

for $k < k_D$.

(1)

Here $\lambda$ is some arbitrary length scale, usually the one of interest in a given problem. The average field amplitude at some scale $k^{-1}$ is

$$B^2_{|k^{-1}} \simeq B(k)^2 k^3 = (k\lambda)^{n+3}B^2_\lambda,$$

so that $B_\lambda$ is simply the amplitude of the magnetic field at that scale $k^{-1} = \lambda$. We normalize the scale factor $a(t)$ to today, $a(t_0) = 1$. Note that we have to use the projector $\delta_{ij} - \hat{k}_i \hat{k}_j$, onto the plane normal to $k$ so that the Maxwell equation $\nabla \cdot B = 0$ is verified, $\hat{k}$ denotes the unit vector in direction $k$. The scale $\lambda_D = k_D^{-1}$ is the damping scale below which magnetic fields are converted into heat usually be fluid viscosity. This scale depends on time and has to be computed using the magneto-hydrodynamic equations and determining the viscosity of the different components of the cosmic plasma, see Refs. (13; 14). The scale $\lambda_D$ is steadily growing as the cosmic plasma dilutes, it amounts to several parsecs at present time. The power $n$ is the spectral index of the magnetic field spectrum, $n = -3$ corresponds to a scale invariant spectrum.

In order for the magnetic field not to diverge on large scales, we must require $n \geq -3$.

In Fig. 3 we show limits on a stochastic magnetic field which have been derived taking into account only tensor fluctuations generated in the CMB, and requiring that they do not overproduce the CMB anisotropies, see Ref. (10). Taking into account all other effects on CMB anisotropies similar limits have been obtained, see Refs. (15; 16). The vector mode fluctuations, not taking into account Faraday rotation and a helical component (discussed in the next subsections) are shown in Fig. 4.

2.4 Polarization

Polarization is affected by the presence of magnetic fields mainly by two mechanisms. First, the gravitational field which is modified by the magnetic field energy momentum tensor leads to a change in the evolution of polarization (which is parallel transported along the photon geodesics). This is taken into account if Fig. 4. The second effect is Faraday rotation: the magnetic field polarizes the electrons in the plasma, which leads to a rotation of the polarization of CMB photons scattering with them. This can rotate E-polarization into B-polarization! The effect is, however, frequency dependent $\propto 1/\nu^2$ and
Fig. 3. Limits on stochastic magnetic fields from the tensor anisotropies induced in the CMB are shown as a function of the spectral index. The characteristic scale chosen is $\lambda = 0.1 h^{-1}\text{Mpc}$. From Ref. (10).

Fig. 4. The $C_\ell$s from vector temperature anisotropies and polarization induced by a stochastic magnetic field with a nearly scale invariant spectrum, $n = -2.99$ and an amplitude $B_\lambda = 3 \times 10^{-9}\text{Gauss}$ are shown (again $\lambda = 0.1 h^{-1}\text{Mpc}$ is chosen). The top curve shows the usual scalar TT spectrum. The curves below show the vector perturbations induced by the magnetic field: the next (solid) curve is the TT spectrum, the dotted curve is the TE spectrum, the dashed curve the B-polarization and the lowest solid curve (blue) the E-polarization spectrum. From Ref. (16).

can therefore, in principle, be distinguished from intrinsic B-polarization. At $\ell \sim 1000$, Faraday rotation induced a B-polarization from the ordinary scalar E-polarization of roughly $10^{-3} \mu\text{K}(B/10^{-9}\text{G})(\nu/10\text{GHz})^{-2}$, see e.g. Ref. (17). The modification of polarization due to Faraday rotation in the presence on a
magnetic field has been discussed in detail in Ref. (18).

2.5 Helical magnetic fields

The magnetic field spectrum which we have given in Eq. (1) is the most general power law spectrum which is invariant under parity. If we allow for parity violation, we can add another term,

\[ a^4 \langle B_i(k)B_j^*(k') \rangle = \begin{cases} 
\delta^3(k - k') \left( (\delta_{ij} - \hat{k}_i \hat{k}_j)S(k) + i \epsilon_{ijm} \hat{k}_m A(k) \right) & \text{for } k < k_D \\
0 & \text{for } k > k_D
\end{cases} \]

(2)

Here \( \epsilon_{ijm} \) is the totally antisymmetric tensor in three dimensions. This is the most generic expression for the correlation function of a magnetic field distribution which is stochastically homogeneous and isotropic. \( S \) and \( A \) are functions of the modulus of \( k \), in the simplest case they are pure power laws with index \( n_S \) and \( n_A \). The second term is parity odd and can only be generated by parity violating interactions (e.g. at the electroweak phase transition, see Ref. (19)). In the CMB such a term induces parity odd correlations between temperature anisotropies and B-polarization and between E- and B-polarization which have been calculated in Ref. (20), see Fig. 5 below. The results are expressed in terms of the density parameter of the parity even and odd magnetic field contributions, \( \Omega_S \) and \( \Omega_A \). Since the parity odd part actually contributes negatively to the energy density, \( \Omega_B = \Omega_S - \Omega_A \), we have to require \( \Omega_S > \Omega_A \). For a pure power law spectrum, positivity requires \( n_A > n_S \).

3 Causality and the magnetic field spectrum

If magnetic fields have been produced at some time \( \eta_* \) when the universe was not inflating, the correlations \( \langle B(x)B(y) \rangle \) vanish for sufficiently large distances, e.g. \( |x - y| > \eta_* \). Hence the correlation function is a function with compact support and therefore its Fourier transform is analytic. For the spectrum given above (2) this means that, at sufficiently small values of \( k \), the functions \( S \) and \( A \) are dominated by one power, \( S(k) = S_0 k^{n_S} \) and \( A(k) = A_0 k^{n_A} \). Analyticity then implies that \( n_S \geq 2 \) is an even integer, and \( n_A \geq 1 \) is an odd integer. Positivity of the magnetic field energy requires in addition \( n_A > n_S \), hence \( n_A \geq 3 \), see Ref. (21). Causal magnetic field spectra are therefore very blue and might lead to better constraints on small scales than on large scales. As we shall explain in the next section, where we study not the effects on the CMB but simply the production of gravity waves by stochastic magnetic
Fig. 5. Left: The temperature anisotropy (solid, black), the E-polarization (dotted, red) and the T–E-correlation (dashed blue) induced from the parity violating contribution are shown as functions of $n_A$ for $n_S = -3$, in units of $(\Omega_A\Omega_S/\Omega_{\text{rad}}^2)\log^2(z_*/z_{\text{eq}})$. Here $z_*$ is the redshift at which the magnetic field is generated (supposed to be before equality) and $z_{\text{eq}}$ is the redshift of matter and radiation equality. The B-polarization is equal to the E-polarization within the semi-analytic approximation used for this results.

Right: The T–B (solid, black), and E–B (dashed, red) correlators are shown in the same units. The peaks and kinks in the curves are not physical. They are due to the breakdown of semi-analytic approximations used in the calculations. From Ref. [20].

fields, this is indeed the case.

4 Limiting primordial magnetic fields with gravity waves

The fact that causal magnetic fields have very blue spectra, suggests that the best limits for them can be obtained on small scales. The energy density of gravity waves produced from stochastic magnetic fields with a given spectral index $n_S$ and amplitude $B_\lambda$ is always dominated by the contribution from small wavelengths.

At horizon crossing the magnetic fields convert a sizable fraction of their energy into gravity waves. This gravity wave background is not damped by subsequent interactions with the cosmic plasma, it is simply diluted by the expansion of the universe like any other radiation component. Comparing the produced intensity of gravity waves with the nucleosynthesis bound leads to very stringent limits on the causal production of cosmic magnetic fields, see Refs. [14, 22].
4.1 Limits for non-helical fields

In Ref. [14] the limit on magnetic fields from the fact that the gravity wave background they produce should be below the nucleosynthesis limit, $\Omega_{GW} \leq 0.1 \times \Omega_{\text{rad}}$, is derived, see Fig. 6. The energy density in gravity waves is entirely dominated by the short wavelength contributions and therefore by the cutoff $k_D(\eta_*)$ which is conservatively set to $k_D(\eta_*) = \eta_*^{-1}$. A recent calculation with a more realistic value for $k_D(\eta_*)$ has even somewhat improved the limits, see Ref. [23]. It is interesting to note that the gravity wave limit from the nucleosynthesis bound is stronger than the limit coming from the magnetic field energy density itself (solid black line in Fig. 6). This comes from the fact that for the examples depicted, the damping scale at nucleosynthesis, which represents the upper cutoff for the magnetic field energy spectrum, is larger than the horizon scale at the time of formation of the magnetic field, $\eta_*$, which is the upper cutoff for the gravity wave energy. Therefore, the magnetic field energy on this scale which has not been converted in gravity waves has been converted into heat by the time of nucleosynthesis.
4.2 Ways out

The limits presented in the previous subsection are extremely stringent. They suggest that, for example magnetic fields which are causally generated at the electroweak phase transition cannot lead to fields larger than $10^{-28}$ Gauss at the scale of 0.1 Mpc, which is insufficient for amplification to the presently observed fields even with a strong dynamo mechanism. To arrive at this result, we have used that apart from being damped at small scales, $k > k_D(\eta)$ the magnetic field simply follows the expansion of the cosmic plasma on large scales, so that $B \propto 1/a^2$. Normal magnetic fields cannot do better. Their spectra evolve by damping on small scales and cascade (moving power from larger into smaller scales) on large, but sub-horizon scales. However, analytical arguments and numerical simulations show that helical magnetic fields can actually invoke an inverse cascade: they can transport power from smaller into larger scales. A bit like a cosmic string network, magnetic flux lines which intersect can reconnect and produce larger scale coherence, see Refs. (19; 24; 25; 26). A trustable, quantitative evaluation of this inverse cascade is still missing. But it may represent a way out of the stringent limits presented above.

For inflation the situation is more delicate. If the inflaton is a pseudo-scalar field it can in principle couple to the electromagnetic field in a way that violates parity (a term of the form $\phi F_{\mu \nu} \ast F^{\mu \nu}$ in the Lagrangian). This does indeed lead to a helical magnetic field. This has been studied in Ref. (27) and it has been found that the amplitude is much too small to be relevant (27). However, the causality requirement drops at inflation and correlations can be generated on arbitrary large scales. Therefore, the spectral index is not limited to 2 by causality, but we could in principle have a spectral index as small as $n \approx -3$. As it is show in Fig. 6, for this spectral index the magnetic field limit raises to $B_{\lambda} \lesssim 10^{-9}$ Gauss, a field which is large enough to lead to the observed fields in galaxies and clusters by simple adiabatic contraction and which might even be detected in the CMB.

However, since the electromagnetic field is conformally coupled, it is not produced during ordinary inflation. It is, however produced e.g. during a pre-big bang phase with a dynamical dilaton, see Ref. (28). There, large scale coherent electromagnetic fluctuations are generated. Due to the high conductivity of the cosmic plasma, the electric field is rapidly dissipated and it remains a magnetic field. Also non-standard couplings of the electromagnetic field during inflation can lead to the generation of magnetic fields. Typically one finds spectra with $n \approx 0$, see Ref. (29) for which the gravity wave constraints shown in Fig 6 are again far too stringent. But also spectra with $n \approx -3$ have been proposed for some very specific (albeit not very well motivated) situation, see (30).
5 Conclusions

In this talk I have described the physical effects by which primordial magnetic fields leave an imprint on the CMB. Since $\Omega_B = 10^{-5} \Omega_\gamma (B/10^{-8} \text{G})^2$, this is only detectable if $B \gtrsim 10^{-9} \text{G}$ on CMB scales. If $n > -3$, this means that the magnetic fields on smaller scales are larger. Then they are usually strongly constrained by the induced gravity wave background.

To generate the observed galactic or cluster magnetic fields by simple contraction, seed fields of the order of $B \simeq 10^{-9} \text{G}$ on about 1Mpc scale are needed. Dynamo amplification requires seed fields of about $10^{-28} \text{G}$.

The induced gravity wave background limits causally produced (non-helical) fields from the electroweak phase transition ($n = 2$) to $B < 10^{-30} \text{G}$ on 1Mpc scale and fields from inflation with spectral index $n \simeq 0$ to $B < 10^{-43} \text{G}$. Only scale invariant magnetic seed fields, $n = -3$, may be as large as $10^{-9} \text{G}$ and therefore leave a detectable imprint on the CMB. Helical fields induce an inverse cascade leading to larger fields on large scales. Therefore, the above limits do not apply for them.

Recently it has also been argued that currents induced by charge separation, in 2nd order cosmological perturbation theory, may generate seed fields at much later times, after recombination, which are not constrained by the nucleosynthesis bound. The true amplitude of the fields obtained in this way is still a matter of debate and varies between $(10^{-23} - 10^{-16}) \text{G}$, see Refs. (31, 32). This interesting possibility certainly deserves more work (see also Ref. (33)).

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References
[1] G. Farrar and M. Shaposhnikov, Phys. Rev. Lett. 70, 2833 (1993).
[2] M. Joyce and M. Shaposhnikov, Phys. Rev. Lett. 79, 1193 (1997).
[3] R. Brandenberger and X. Zhang, Phys. Rev. D59, 081301 (1999).
[4] J. Han and R. Wielebinski, ChJA & A 2, 293 (2002).
[5] P.P. Kronberg, JKAS, 37, 501 (2004).
[6] A. Davis, M. Lilley and O. Törnkvist, Phys. Rev. D60, 021301 (1999).
[7] J. Barrow R. Juszkiewicz and D. Sonoda, MNRAS 213, 917 (1985).
[8] J. Barrow P. Ferreira and J. Silk, Phys. Rev. Lett. 78, 3616 (1997).
[9] R. Durrer, T. Kahniashvili and A. Yates, *Phys. Rev.* **D58**, 123004 (1998).
[10] R. Durrer, P. Ferreira and T. Kahniashvili, *Phys. Rev.* **D61**, 043001 (2000).
[11] Jackson, *Classical Electrodynamics*, Wiley and Sons
[12] J. Adams, U. Danielsson, D. Grasso and H. Rubinstein, *Phys. Lett.* **B388**, 253 (1996).
[13] K. Subramanian and J. Barrow, *Phys. Rev.* **D65**.
[14] C. Caprini and R. Durrer, *Phys. Rev.* **D65**, 23517 (2002).
[15] A. Mack, T. Kahniashvili and A. Kosowski, *Phys. Rev.* **D66**, 024030 (2002).
[16] A. Lewis, *Phys. Rev.* **D70**, 043011 (2004).
[17] H. Ohno et al., *Astrophys. J.* **584**, 599 (2003).
[18] A. Kosowsky, T. Kahniashvili, G. Lavrelashvili and B. Ratra, *Phys. Rev.* **D71**, 043006 (2005).
[19] T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001).
[20] C. Caprini, R. Durrer and T. Kahniashvili, *Phys. Rev.* **D69**, 63006 (2004).
[21] R. Durrer and C. Caprini, *JCAP* **0311**, 010 (2003).
[22] C. Caprini and R. Durrer, *Phys. Rev.* **D72**, 088301 (2005).
[23] C. Caprini and R. Durrer, *Phys. Rev. D*, in print (astro-ph/0603476) (2006).
[24] P Banerjee and K. Jedamzik, *Phys. Rev.* **D70**, 123003 (2004).
[25] M. Christensson, M. Hindmarsh and A. Brandenburg, *Phys. Rev.* **E64**, 56405 (2001).
[26] A. Brandenburg and K. Subramanian, *Phys. Rept.* **417**, 1-209 (2005).
[27] W.D. Garretson, G.B. Field and S.M. Carroll, *Phys. Rev.* **D46**, 5346 (1992).
[28] M. Gasparini, M. Giovannini and G. Veneziano, *Phys. Rev. Lett.* **75**, 3796 (1995).
[29] M. Turner and L. Widrow, *Phys. Rev.* **D73**, 2743 (1988).
[30] K. Bamba and J Yokoyama, *Phys. Rev.* **D70**, 083508 (2004).
[31] L. Matarrese, S. Mollerach, A. Notari and A. Riotto, *Phys. Rev.* **D71**, 043502 (2005)
[32] K. Takahashi, K. Ichiki, H. Ohno and H. Hanayama *Phys. Rev. Lett.* **95**, 121301 (2005)
[33] R. Durrer, *Science* **311**, 787 (2005).