Size dependence of the Burridge-Knopoff model

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Abstract. Earthquakes are not periodic phenomena and their size distribution obeys the power law, the so-called Gutenberg-Richter’s law [1, 2]. It is well known that the Gutenberg-Richter’s law is reproduced by the Burridge-Knopoff model [3], which is a kind of spring-block model between two tectonic plates. The mechanism of the Gutenberg-Richter’s law is usually explained in the context of self-organized criticality (SOC) proposed by Bak et al. [4]. It is observed in recent research, however, that the deviation from the Gutenberg-Richter’s law becomes larger for longer faults [5]. We simulated numerically the Burridge-Knopoff model in one dimension with special attention to the dependence on the model size, that is the number of blocks. It is shown that the range where the power law holds becomes narrower as the number of blocks increases [6]. We also discuss the relation between the dimensionality of the system and the size distribution.

Introduction

There are various types of earthquake. One of these is interplate earthquake. Ocean plate goes downward beneath landplate as shown in fig. 1. The land plate is dragged with the ocean plate then its restoring force increases. When this force reaches the maximum static frictional force, the land plate slips. This is the interplate earthquake.

Size distribution of earthquake obeys the power law, the so-called Gutenberg-Richter’s law [1, 2, 3]. The Gutenberg-Richter’s law means that the number of earthquake of magnitude $M$, $\rho(M)$ is proportional to $-\alpha M$, where $\alpha$ is a constant. Magnitude is proportional to the logarithm of released energy by each earthquake. So the Gutenberg-Richter’s law means the power law. Power law indicates that there is no characteristic energy scale, that is observed just at critical point in usual 2nd order phase transition. In earthquake, however, no adjustable parameter such as temperature or pressure exists. Bak et al. proposed a scenario that complex systems under repeated perturbation organize themselves to the critical point, that is called self-organized criticality (SOC) [4]. They claimed that the earthquake is a typical example of SOC.

The Gutenberg-Richter’s law holds well for earthquakes observed in large area involving many faults. According to recent report by Stirling et al., the size distribution of earthquake in individual fault does not obey the Gutenberg-Richter’s law in large earthquake side [5]. They reported that deviation from the Gutenberg-Richter’s law becomes larger for longer faults. Exact critical state exists only in infinite system. If earthquake is a kind of SOC phenomena, the longer faults approach the exact critical state and then their size distribution obeys the power law for wider range of magnitude.
In this report we investigate the size dependence of the Burridge-Knopoff model and examine whether the power law observed in the model and in actual earthquake is really SOC phenomena [6].

Model

The Burridge-Knopoff model is a famous model of interplate earthquake [3]. The model is shown in fig.2. The block of mass $m$ is connected to the upper plate by linear spring with spring constant $k_p$. The block is also connected to each other by linear spring with spring constant $k_c$, of which natural length is $a$. The frictional force acts between the blocks and the lower plate. The upper plate is driven with constant velocity $v$.

The equation of motion of the model is expressed as,

$$ m\ddot{x}_i = k_c (x_{i+1} + x_{i-1} - 2x_i) - k_p (x_i - vt) - F(\dot{x}_i), \quad (1) $$

Here $x_i$ is the position of the $i$-th block measured from the natural position of the spring connected with upper plate at $t=0$ and $F(\dot{x}_i)$ is the frictional force. Using velocity weakening frictional force, it was shown that the size distribution of the Burridge-Knopoff model obeys the power law by Carlson et al. [3]. We follow the work by Carlson et al. and adopt the following functional form of $F(\dot{x}_i)$.
\[ F(x_i) = \frac{F_0(1-\sigma)}{1 + 2\alpha x_i/(1-\sigma)}, \quad (2) \]

for \( \dot{x}_i > 0 \) [7]. The maximum static frictional force to the driving direction is \( F_0 \) and that to opposite direction is \(-\infty\). That is, every block never moves to the opposite direction to the driving direction. \( \sigma \) is proportional to the difference between the maximum static frictional force and the kinetic frictional force in zero velocity limit, which enable us to make the driving velocity infinitesimally small [7]. In the calculation we employed the following values, \( k_c = 60, \quad k_p = 1, \quad m = 1, \quad a = 1, \quad v = 0.001, \quad F_0 = 1 \). Free boundary condition is adopted. In the simulation, each earthquake event begins when one of the blocks starts to move and ends when all the blocks stop. In this simulation we define the magnitude \( m \) as,

\[ m = \log_{10} \left( \sum_{\text{event, } n=1} \Delta x_i(t)/a \right), \quad (3) \]

where \( \Delta x_i \) is the displacement of \( i \)-th block during each earthquake event. The number of blocks in the model is assumed to be proportional to the fault length.

We also examined the Burridge-Knopoff model extended to two dimension as shown in fig.3 in order to investigate the effects of dimensionality of the model. We employed three models. In the 1st model we extend the displacement of the blocks to two dimension (1-2 dimensional model). The equation of motion of the model is expressed as

\[ \begin{align*}
  x \text{ direction: } m\ddot{x} &= -k_c (R_1 - a)\cos(\theta_1) + k_c (R_2 - a)\cos(\theta_2) - k_p (x_0 - V_x) - F(v), \\
  \text{y direction: } m\ddot{y} &= -k_c (R_1 - a)\sin(\theta_1) + k_c (R_2 - a)\sin(\theta_2) - k_p (y_0 - V_y) - F(v).
\end{align*} \]

**Fig. 3.** Two-dimensional Burridge-Knopoff model
In the 2nd model we extend the structure of the block array to two dimension. But their displacement is one dimensional, that is only along x-direction (2-1 dimensional model). The equation of motion of the model is expressed as

\[ m\ddot{x} = -k_c (R_1 - a) + k_c (R_2 - a) - k_c (R_3 - a) \sin(\theta_3) + k_c (R_4 - a) \sin(\theta_4) - k_p (x_0 - V_x) - F(x). \quad (6) \]

In the 3rd model we extend the structure of the block array and the displacement of the blocks to two dimension (2-2 dimensional model). The equation of motion of the model is expressed as

\begin{align*}
\text{x direction: } & \quad m\ddot{x} = -k_c (R_1 - a) \cos(\theta_1) + k_c (R_2 - a) \cos(\theta_2) - k_c (R_3 - a) \sin(\theta_3) \\
& \quad + k_c (R_4 - a) \sin(\theta_4) - k_p (x_0 - V_x) - F(x), \\
\text{y direction: } & \quad m\ddot{y} = -k_c (R_1 - a) \sin(\theta_1) + k_c (R_2 - a) \sin(\theta_2) - k_c (R_3 - a) \cos(\theta_3) \\
& \quad + k_c (R_4 - a) \cos(\theta_4) - k_p (y_0 - V_y) - F(y).
\end{align*}

(7)

(8)

**Numerical Results**

Fig. 4 shows the displacement of the center of gravity coordinate of the blocks as a function of time for the system with 100 blocks of the 1-1 dimensional model. Clear stick-slip motion is observed. Stick-slip motion is observed in the 1-1, 1-2, 2-1 and 2-2 models. The size distribution of earthquake for systems with various numbers of blocks of the 1-1 dimensional model (structure: 1 dimension, displacement: 1 dimension) are shown in figs. 5(a) and (b). The Gutenberg-Richter’s law is observed in the middle range of magnitude. However, the number of large earthquakes which does not obey the Gutenberg-Richter’s law increases for the systems with more blocks. In fig. 5(b) the peak of the distribution in the small magnitude range shifts to large magnitude side with the number of blocks. The range where the Gutenberg-Richter’s law holds becomes narrower for the systems with more blocks.

![X VS Time](image)

**Fig. 4.** Displacement of the center of gravity coordinate of the blocks as a function of time of the 1-1 dimensional model with 100 blocks.
Fig. 5. Size distribution of earthquake of the 1-1 dimensional model. The numbers of blocks are (a) 100, 200, 500 and 1000 and (b) 1000, 2000, 4000 and 8000.

Figs. 6(a) and (b) show the size distribution of earthquake for systems with various numbers of blocks in 1-2 dimensional model (structure: 1 dimension, displacement: 2 dimension). As well as the case of the 1-1 dimensional model, the Gutenberg-Richter’s law is observed in the middle range of magnitude, and the number of large earthquake which does not obey the Gutenberg-Richter’s law increases for the systems with more blocks.

Fig. 6. Size distribution of earthquake of the 1-2 dimensional model. The numbers of blocks are (a) 5, 10, 20, 50 and 100 and (b) 100, 200, 500 and 1000.
The simulations of all models examined here show stick-slip motion. In figs. 5, 6, 7 the Gutenberg-Richter’s law is observed in the middle range of magnitude. The number of large earthquakes which does not obey the Gutenberg-Richter’s law increases with the number of blocks. The range where the Gutenberg-Richter’s law holds becomes narrower for systems with more blocks. All of the results obtained here are consistent with recent observation of actual earthquakes [5]. If SOC scenario is true for the mechanism of the Gutenberg-Richter’s law, the law holds better for larger systems because exact critical state exists only in infinite system and the deviation from the critical state grows for smaller systems. The present results show, however, that the Gutenberg-Richter’s law holds better in smaller systems and the deviation from the law increases for larger systems. The increase of the deviation from the power law with the system size is also observed in sandpile avalanche experiment [8]. In the Burridge-Knopoff model, in sandpile and in actual earthquake system, the power law may results from the nature of the system that has finite degrees of freedom and not from SOC. This indicates that earthquakes are not SOC phenomena. We have to investigate a scenario that explains the limited power law behavior in finite systems.

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