Archival searches for stellar-mass binary black holes in LISA

Becca Ewing, 1 Surabhi Sachdev, 1 Sohrab Borhanian, 1 and B. S. Sathyaprakash 1,2,3
1 Institute for Gravitation and the Cosmos, Department of Physics, Pennsylvania State University, University Park, PA, 16802, USA
2 Department of Astronomy & Astrophysics, Pennsylvania State University, University Park, PA, 16802, USA
3 School of Physics and Astronomy, Cardiff University, Cardiff, UK, CF24 3AA

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Stellar-mass binary black holes will sweep through the frequency band of the Laser Interferometer Space Antenna (LISA) for months to years before appearing in the audio-band of ground-based gravitational-wave detectors. One can expect several tens of these events up to a distance of 500 Mpc each year. The LISA signal-to-noise ratio for such sources even at these close distances will be too small for a blind search to confidently detect them. However, next generation ground-based gravitational-wave detectors, expected to be operational at the time of LISA, will observe them with signal-to-noise ratios of several thousands and measure their parameters very accurately. We show that such high fidelity observations of these sources by ground-based detectors help in archival searches to dig tens of signals out of LISA data each year.

I. INTRODUCTION AND BACKGROUND

The discovery of GW150914 [1] by the Advanced Laser Interferometer Gravitational-wave Observatory (LIGO) [2] and the continued detections of stellar mass binary black holes [3–5] by Advanced LIGO and Virgo [6] set the stage for observing such systems with the Laser Interferometer Space Antenna (LISA) [7] when it comes online. GW150914 is the result of the merger of a pair of $36^{1/3}_{-1/2} M_\odot$ and $31^{1/3}_{-1/2} M_\odot$ black holes at a distance of $440^{+300}_{-50}$ Mpc [4]. The companion masses are larger than what was initially thought possible from stellar evolution [8] (see, however, [9]). It therefore earned the adjective heavy for black holes in the mass range $\sim 20 M_\odot$, $100 M_\odot$ now routinely observed by LIGO and Virgo [4, 5, 10–13]. Such heavy binary black holes (hBBHs) within $\sim 1$ Gpc could also be visible in the LISA band [7] at an earlier stage in their evolution, albeit with a signal-to-noise ratio (SNR) of a few.

The search for stellar-mass binary black holes in LISA data could take formidable computational resources [14]. The resulting false alarm rate due to the large number of templates [15] required would mean that only a handful of nearby sources with SNRs greater than 14 might be detected for a $p$-value of $10^{-3}$. Note, however, that third generation (3G) ground-based observatories, such as the Einstein Telescope [16, 17] and the Cosmic Explorer [18], operating at the same time as LISA would observe these sources some months to years after the signal passes the LISA band, with far greater SNRs compared to those in LISA and determine the source parameters to a good accuracy. Narrowing down the source parameters by ground-based detectors should then help in archival searches for such systems in LISA data by reducing the parameter space and hence, false alarm rates and computational costs. Current estimates still require an SNR threshold of $\sim 8$ [19] to $\sim 14$ [14].

Multiband observations of hBBH systems in LISA and ground-based detectors would greatly benefit the science return of these observatories [20–30]. This is because the parameter degeneracies that are present in the later part of the system’s evolution in ground-based detectors could be resolved by observing the earlier part of the system’s evolution in LISA. Several authors [21–25] have demonstrated that this can principally yield tests of general relativity orders of magnitude better than what would be possible with either detector or detector-network by itself. What is critical to making that science possible is to unambiguously detect the signals in the LISA band.

In this paper we will show that 3G observatories will pin down the parameters of hBBH systems well enough to reduce the number of templates required for matched-filter searches to detect such systems in LISA data to a mere few $\times 10^4$ as opposed to previous estimates of $\sim 10^{12}$. This means that it will be possible to identify hBBH signals in LISA data with an SNR of 4 or more with a $p$-value of $10^{-2}$ or better. This will increase the number of sources that will be available for joint observation by both space-borne and ground-based observatories and hence enhance the science return of multiband observations.

The rest of the paper is organized as follows: In Sec. II, we will compute for the joint hBBH population expected to be observed the visibility and measurement capabilities of 3G observatories. We will discuss, in particular, the uncertainties in the sky localizations, masses, and spins of the companion black holes—parameters that would need to be searched for in LISA data. In Sec. III, we show how the problem of assessing LISA’s performance in observing binary black holes can be mapped to the audio-frequency band. This is possible since there is no mass scale in general relativity: waveforms from binary black holes of different total mass will all look exactly the same as long as all other parameters remain the same except for a rescaling of time. This helps in using tools that have been developed for the analysis of ground-based detectors such as the LSC Algorithm Library [31]. In Sec. IV, we will estimate the number of templates required to search for hBBH systems in LISA data using the knowledge of parameter accuracies from 3G observatories. We will use two complementary methods to compute the number of templates. The first method works out the invariant volume of the signal manifold over the relevant range of parameters and then divides it by the fraction of volume covered by each template. This gives the minimal number of templates required for archival searches. In a realistic data analysis pipeline, however, one needs to make a choice of templates based on a template placement algorithm [32]. We will use one such algorithm [33] to get a more realistic estimate of the number of templates. In Sec. IV D, we characterize the efficiency of the template bank by computing the overlap of
hBBH waveforms with random parameters maximized over the set of templates in the template bank. We will also discuss in Sec. V the distribution of the SNRs of the sub-population of sources that will be observed by both LISA and 3G observatories. We conclude in Sec. VI with a summary of the results and future plans.

II. STELLAR MASS BINARY BLACK HOLES IN 3G OBSERVATORIES

3G detectors like Cosmic Explorer (CE) and the Einstein Telescope (ET) will observe stellar-mass binary black holes all the way up to redshifts of \( \sim 10-50 \) [34–36], depending on the intrinsic parameters of the source such as its masses and spins, as well as extrinsic parameters such as the position of the source on the sky and the orientation of the binary’s orbit relative to the observer’s line of sight. LISA could observe a small fraction of such systems if they are within \( \sim 1 \) Gpc [7], but digging them out of the background noise in a blind search will take formidably large computational resources due to the large number of matched filters needed to cover the full parameter space of masses, spins, and sky position [14]. CE and ET will observe binary black holes that are close with SNRs of order \( \sim 1000 \) in 3G, a few reaching values almost 10 times as large. The left panel also confirms the expectation that heavy hBBH events in the LISA band produce loud signals in CE and ET detectors. The right panel presents in a similar fashion the distribution of the luminosity distance, \( D_L \), and effective spin parameter, \( \chi_{\text{eff}} = (m_1 \chi_1 + m_2 \chi_2)/M \) [41, 42], indicating that most of the systems are found at luminosity distances \( \lesssim 1 \) Gpc.

We draw attention, in particular, to the visibility of hBBH in LISA. The rate for hBBH systems is constrained by the rate of BBH mergers whose up-to-date value is \( R = 23.8^{+11.3}_{-6.5} \) Gpc\(^{-3} \) yr\(^{-1} \) [5, 43]. Thus, if we take into account that only a fraction \( f \) of the binaries contain at least one heavy black hole (\( > 20 M_\odot \)), where

\[
f \approx 1.9 \int_{20 M_\odot}^{100 M_\odot} \frac{dm_1}{m_1} m_1^{-1.6} \int_{5 M_\odot}^{m_1} \frac{dm_2}{m_2} \left( \frac{1}{m_1 - 5 M_\odot} \right) \approx 0.32, \tag{1}
\]

and 1.9 is the normalization factor, we obtain a median rate \( R_{\text{BBH}} = f R = 7.6 \) Gpc\(^{-3} \) yr\(^{-1} \) for hBBH merger. Although heavier binaries can be seen to a greater distance their relatively lower prevalence means that it is more likely that we will observe lighter binaries in LISA more frequently. The detection of lighter binaries in LISA is also aided by the large SNRs and high fidelity measurements of the 3G network. We also note that the hBBH systems are likely to have larger mass ratios, which is due to the power-law distribution of the primary companion and flat-distribution of the secondary. The large mass ratio is also responsible for low effective spins of hBBH systems as seen in the right panel of Fig. 1.

A. Visibility

In order to assess what systems will be observable by both LISA and a network of 3G detectors, we simulated a population of \( 5 \times 10^5 \) hBBHs which are uniformly distributed in co-moving volume up to a redshift of \( z = 10 \). The companion masses are chosen to follow a power law [37] for the larger companion \( m_1 \in [5 M_\odot, 100 M_\odot] \), \( p(m_1) \propto m_1^\alpha \) with exponent \( \alpha = 1.6 \), and a uniform distribution in \([5 M_\odot, m_1]\) for the lighter companion \( m_2 \) [38]. The companion spins are assumed to be aligned or anti-aligned with the orbital angular momentum, and are drawn from a Gaussian distribution with 0 mean and a standard deviation of 0.1.

We calculated the SNR, \( \rho_{\text{LISA}} \), in LISA with the estimated power spectral density (PSD) provided in Ref. [39] and by marginalizing over the angular dependencies of the signal. We found 181 of the simulated signals to be visible in LISA with \( \rho_{\text{LISA}} \geq 4 \) and used these systems as candidates for our 3G-assisted archival search study for LISA data [24]. We will justify this choice of SNR in Sec. V. Our choice for a 3G network consists of one ET and two CEs located at fiducial sites in Cascina (Italy), Idaho (USA), and New South Wales (Australia), respectively. The detector sensitivities are set to ET-D for the ET detector and CE1 (40 km, compact-binary optimized) for the two CEs [40].

Next, we want to assess the quality of the parameter estimation that a 3G network can achieve. This crucial information allows us to perform the archival search more efficiently by decreasing the dimension and volume of the parameter space for a template search.

We perform this assessment with \texttt{gwbench} [40], a Python package that implements the well-known Fisher information [44–46] formalism, and estimate the \( 1\sigma \) error bounds for each of the 181 systems. The formalism provides an analytical approximation of the Gaussian noise likelihood around its maximum and thus allows us to estimate the measurement errors \( \sigma_\lambda = \sqrt{\Sigma_{\lambda\lambda}} \) on a set of parameters \( \lambda \) from the covariance matrix \( \Sigma \) in the likelihood:

\[
P(\lambda) \sim e^{-\frac{1}{2} \Sigma^{-1/2}_{\lambda\lambda} \lambda \Sigma_{\lambda\lambda} \lambda}, \tag{2}
\]

Given a model for the detector response \( \hat{h}(f; \lambda) \), we can calculate the covariance matrix as the inverse of the Fisher information matrix \( \Gamma \).
FIG. 1. The plot shows the distribution of the 181 hBBHs with a LISA SNR $\rho_{\text{LISA}} \geq 4$, the two axes show the masses of the companions in both panels. In the left plot, color bar is the 3G SNR $\rho_{3G}$ and the size of circles depict the LISA SNR $\rho_{\text{LISA}}$; in the right plot, color bar is the luminosity distance $D_L$ and the size of the circles represent the effective spin parameter $\chi_{\text{eff}}$.

FIG. 2. The plot shows the probability and cumulative density functions of the measurement error for the nine parameters of interest for the 181 binary black hole mergers that can be detected in the archival search of LISA data after they are identified and measured in the 3G data. We simulated 100 realizations for the sky location and binary orientation angles to show the range of parameter errors to be expected from 3G networks.
$$\Sigma^{-1}_{ij} \equiv \Gamma_{ij} = \langle \partial_{\lambda} \hat{h}(f; \lambda), \partial_{\lambda} \hat{h}(f; \lambda) \rangle.$$  (3)

The scalar product between two waveforms \( \hat{h}(f; \lambda_1) \) and \( \hat{g}(f; \lambda_2) \) is defined as

$$\langle \hat{h}(\lambda_1), \hat{g}(\lambda_2) \rangle = \int_{0}^{\infty} \frac{\hat{h}(f; \lambda_1) \hat{g}^*(f; \lambda_2) + c.c.}{S_h(f)} \, df.$$  (4)

where \( \hat{g}^* \) denotes the complex conjugate of \( \hat{g} \). Note that although the limits in the integral range from 0 to \( \infty \), in reality the detector noise power spectral density \( S_h(f) \) outweighs the signal power outside a finite frequency range \([f_1, f_2]\) and often the waveform itself will have no support above a frequency \( f_{\text{cut}} = f(\lambda) \) determined by its intrinsic parameters. Thus, the integral gets most of its support over a finite range of frequency \([f_1, f_2]\).

The error bounds that a network of several detectors can achieve are readily computed via the network Fisher matrix

$$\Gamma_{\text{net}} = \sum_d \Gamma_d,$$  (5)

which is the sum of the Fisher matrices \( \Gamma_d \) for all the detectors in the network. Hence, given a detector response model, we calculate all the detector Fisher matrices and invert their sum in the network. Hence, given a detector response model, we can assume most parameters to be known and focus on the problems such as the ones explored in this paper. LALSuite allows us to make use of waveform models that include higher-orders of 10

The LSC Algorithm Library (LALSuite) [31] has many data analysis tools such as compact binary waveform models, template placement algorithms, filtering routines, etc., that are extremely useful, sometimes critical, in evaluating data analysis problems such as the ones explored in this paper. LALSuite was developed primarily for the analysis of data from LIGO and Virgo interferometers that operate in the audio frequency band from 1 Hz to 10 kHz. Unfortunately, some of the algorithms do not readily work at frequencies below 1 Hz and the effort required to rewrite the algorithms for the LISA band, \( \sim 100 \mu \text{Hz} \) to 100 mHz, would be formidable. Luckily, it is possible to scale the LISA problem into the audio band owing to the fact that the fundamental quantity of interest, namely the strain measured by the gravitational-wave detectors which represents the change in proper length between ‘free’ test masses in response to a passing gravitational wave, has no physical dimension.

To illustrate the required scaling, let us consider gravitational waves emitted by an inspiraling binary system composed of a pair of black holes, but the argument would work no matter what type of source we consider. Furthermore, for the sake of
We choose the starting frequency $f_1$ where the waveform (often referred to as the ‘Newtonian’ waveform) from a binary system composed of non-spinning black holes. However, the arguments follow through irrespective of the PN order. At the Newtonian order, the strain response of an interferometer to gravitational waves from a binary system composed of black holes of masses $(m_1, m_2)$ at a luminosity distance $D_L$ is given by

$$h(t) = \frac{4G\eta M}{c^2 D_L} \left( \frac{\pi GM f(t)}{c^3} \right)^{2/3} \cos \left( \pi \int_{t_0}^{t} f(t) \, dt \right)$$  \hspace{1cm} (6)$$

where $M = (m_1 + m_2)$ is the total mass of the system, $\eta = m_1 m_2 / M^2$ is the symmetric mass ratio, and $f(t)$ is the instantaneous gravitational-wave frequency:

$$f(t) = f_0 \left( 1 - \frac{t - t_0}{\tau} \right)^{-3/8}.$$  \hspace{1cm} (7)

Here $t_0$ is a fiducial time when the frequency of the gravitational wave is $f_0$ and $\tau$, called the chirp time, is the duration of the signal from a time when its frequency was $f_0$ until the frequency (in this approximation) diverges and the two black holes merge:

$$\tau = 5 \left[ \frac{GM}{256 \eta c^3} \right]^{8/3}.$$  \hspace{1cm} (8)

The chirp time of a binary of total mass 100 $M_\odot$, equal component masses (so that $\eta = 1/4$), and starting frequency of 12 mHz would be 5 years. Chirp time is a sharp function of the total mass as well as the starting frequency. A signal starting from a frequency that is a factor 2 (10) smaller would last a total mass of the system, and the luminosity distance by the same factor. Therefore, the signal would now last for a much shorter period of $\Delta t/\alpha$ with exactly the same amplitude as before but at a higher frequency. The SNR of the scaled up signal, but with a scaled up LISA noise spectral density $S^{LISA}_h(f)$, will also be the same as before. To see this, recall that the expectation value of the SNR of a signal is given by:

$$\rho^2(f_1, f_2) = 4\Re \int_{f_0}^{f_1} \left[ \frac{|h(f)|^2}{S^{LISA}_h(f)} \right] d\alpha,$$  \hspace{1cm} (12)

where $\tilde{h} \equiv \int_{f_0}^{f} h(t) \exp(2\pi i f t) \, dt$ is the Fourier transform of the gravitational-wave strain. Changing the variable $f \to \nu = \alpha f$ would scale down the Fourier mode strain by a factor of $\alpha$, $h(\nu) \to \tilde{h}(\alpha \nu) / \alpha$, and similarly the LISA noise PSD $S^{LISA}_h(\nu) \to S^{LISA}_h(\nu) / \alpha \equiv S^{\text{audio}}\nu(\nu) / \alpha$. Thus the SNR remains unchanged:

$$\rho^2(\nu_1, \nu_2) = 4\Re \int_{\nu_1}^{\nu_2} \left[ \frac{|\tilde{h}(\nu)|^2}{S^{\text{audio}}\nu(\nu) / \alpha^2} \right] d\nu.$$  \hspace{1cm} (13)

The scaled version of the LISA noise PSD $S^{\text{audio}}\nu(\nu) / \alpha$ is shown in the right hand panel of Fig. 3.

In summary, the required scaling transformation are to:

1. Scale up the frequency: $f \to \alpha f$.
2. Scale down the time duration, total mass and distance: $\tau \to \tau / \alpha$, $M \to M / \alpha$, and $D_L \to D_L / \alpha$.
3. Transform the LISA power-spectral density into the audio band, i.e. $S^{\text{audio}}(f) \equiv \alpha S^{LISA}_h(f / \alpha)$.

IV. TEMPLATE BANKS FOR ARCHIVAL SEARCHES

In this section we present the number of templates required to detect BBH in the LISA band by matched filtering. We use accuracies on the masses obtained from parameter estimation in 3G detectors. We use two independent methods to calculate...
template bank numbers. The first method assumes the metric in order to place templates, which would provide a minimum on the number of templates required. As a check on this method, we also calculate the number of templates using a stochastic placement algorithm, which overestimates the required number of templates.

A. Metric method

1. Metric on the signal manifold

The number of templates required for a search can be found using the geometric formulation of signal analysis [32, 46, 50]. The scalar product (4) can be used to define waveform or signals of unit norm. A signal is said to be of unit norm if its scalar product with itself is unity and will be denoted by $\langle \hat{a}, \hat{a} \rangle = 1$.

In the geometric formalism, the overlap $\mathcal{O}$ or match between two ‘nearby’ normalized signals $\hat{h}(\lambda)$ and $\hat{h}(\lambda + \Delta\lambda)$ with slightly different parameters $\lambda$ and $\lambda + \Delta\lambda$ is given by:

$$\mathcal{O}(\lambda, \Delta\lambda) \equiv \langle \hat{h}(\lambda), \hat{h}(\lambda + \Delta\lambda) \rangle \approx 1 - g_{\text{opt}} \, d\lambda \, d\beta,$$

(14)

where $g_{\text{opt}}$ is the metric on the signal manifold [46, 50]:

$$g_{\text{opt}} \equiv -\frac{1}{2} \left. \frac{\partial^2 \mathcal{O}(\lambda, \Delta\lambda)}{\partial \lambda \partial \beta} \right|_{\Delta\lambda = 0} = \langle \hat{h}_\alpha, \hat{h}_\beta \rangle, \quad \hat{h}_\alpha \equiv \frac{\partial \hat{h}}{\partial \alpha}.$$  

(15)

When signals are nearby, in the sense that their overlap is close to unity, Eq. (14) is a good approximation for the overlap and the quantity $d\ell^2 = g_{\text{opt}} \, d\lambda \, d\beta$—the proper distance between them—obeys $d\ell \ll 1$. Thus, two normalized signals at a proper distance of $d\ell$ from each other have an overlap of $1 - d\ell^2$.

2. Minimal match and lower limit on the number of search templates

To filter signals out of data we must choose a bank of templates in the parameter space of interest such that any signal buried in the data within this parameter space has its overlap larger than a certain value called the minimal match $M$ with at least one template in the bank. Of course, this requirement can be met by populating the parameter space with a dense set of templates but a higher density of templates would demand a greater computational cost. Thus, the density of templates must be chosen so that it is as sparse as possible while assuring minimal match with every signal in the parameter space of interest.

If $\lambda_k, k = 1, \ldots, N$, denotes the parameters of the templates in the bank then for a signal with arbitrary parameters $\lambda$ the template bank must satisfy the following condition:

$$\max_k \langle \hat{h}(\lambda), \hat{h}(\lambda_k) \rangle \geq M,$$

(16)

the equality in the above equation giving the optimal number of templates. So we must choose the proper distance such that

$$d\ell^2 = g_{\text{opt}} \, d\lambda \, d\beta = (1 - M).$$

(17)

Given that the optimum matched filter signal-to-noise ratio that one can hope to achieve for a signal is $\rho_{\text{opt}}^2 = \langle h, h \rangle$, the above condition assures that the fractional drop in the signal-to-noise ratio between an arbitrary signal and the closest template in the bank is no more than $\epsilon \equiv 1 - M$ called the maximum mismatch, i.e., $\rho \geq \epsilon \rho_{\text{opt}}$.

Given the minimal match $M$ how many templates are needed to cover the parameters with the smallest number of templates? This is the problem of template placement and to some extent the answer depends on what type of lattice is used to place the

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FIG. 3. Left: Contour plot of the starting frequency for stellar mass binary black holes that last 5 years in the LISA band. The starting frequency scales with the mass ratio of each system and inversely with the total mass, as indicated by the colored contours, where the values of starting frequency are shown in mHz. The grey dots mark the positions of the 181 BBHs for which we performed Fisher analysis in a 3G network to obtain parameter error estimates. The size of the dots represents the corresponding LISA SNR, $\rho_{\text{LISA}}$. Right: LISA’s amplitude noise spectral density (ASD) $\sqrt{S_\alpha(f)}$ is plotted before (red solid line) and after (red dashed line) applying frequency scaling with $\alpha = 10^4$. Also shown are the amplitude spectra $\sqrt{f|h(f)|}$ for a GW150914-like (in orange) and GW190521-like (yellow) systems at a distance of 500 Mpc with signal-to-noise ratio of 4.8 and 16, respectively. The integration time is assumed to be 5 years in the LISA band which translates to $1.54 \times 10^5$ s in the audio band.
templates on the signal manifold. We will discuss a specific template placement algorithm used in this work in the next Section. We can get a lower limit on the number of templates needed by computing the total proper volume of the signal space divided by the fraction of volume covered by each template. Assuming that each template covers a proper volume \( dV = d^N = (1 - M)^{N/2} \), where \( N \) is the dimension of the parameter space, the smallest number of templates \( N_T \) needed is:

\[
N_T = \frac{V}{dV} = \frac{1}{(1 - M)^{N/2}} \int_{\xi_{\text{max}}}^{\xi_{\text{min}}} \sqrt{g} d\xi, \tag{18}
\]

where \( g = \det|\partial_{\alpha\beta}| \). The above estimate assumes that the templates are placed on a square lattice. The number of templates can be smaller with a more efficient lattice, e.g., a hexagonal lattice in two-dimensions, but this is unimportant for our considerations.

3. Newtonian approximation to number of templates

The orbital velocity of a stellar mass binary hole of total mass \( M \) and gravitational-wave frequency \( f_{\text{GW}} \) is very small in the LISA frequency band compared to the speed of light (for clarity we include factors of \( G \) and \( c \) in the equation below):

\[
v \approx \left( \frac{\pi GMf}{c^3} \right)^{1/3} \approx 0.025 \left( \frac{M}{10^5 M_\odot} \right)^{1/3} \left( f_{\text{GW}} \text{ 10 mHz} \right)^{1/3}. \tag{19}
\]

For such a non-relativistic orbit the PN expansion parameter defined by \( x \equiv \nu^2/c^2 \approx 6.2 \times 10^{-4} \), meaning stellar-mass binary black holes in the LISA frequency band are in the adiabatic regime. The dynamics of such a system is essentially described by the Newtonian approximation (or 0 PN) given in Eqs. (7) and (8). In the stationary phase approximation the Fourier transform of the waveform in Eq. (6) is given by [51]:

\[
h(f) = \frac{A}{D_L} f^{-7/6} \exp \left[ 2\pi if \tilde{\psi} - i\phi \right]. \tag{20}
\]

where \( \tilde{\psi} \) is a functional of its argument defined for any function \( a(f) \) by

\[
\mathcal{J}[a(f)] = \frac{1}{\rho^2} \langle f^{-7/3}, a(f) \rangle. \tag{23}
\]

As is well known, maximizing the overlap \( O \) over the parameter \( \tilde{\psi} \) is easily accomplished in the Fourier domain [52] and one needs a discrete lattice of templates only for the remaining one parameter \( \xi \). The metric in the \( \xi \)-dimension is quite simply:

\[
G_{22} = g_{22} - g_{21}/g_{11}. \tag{24}
\]

where for any \( k \), the moment \( J_k \) is given by \( J_k \equiv \langle f^{-4/3}, 1 \rangle \). In this notation the SNR is \( \rho^2 \equiv J_1 \). \( G_{22} \) is demonstrably constant (in any case all one-dimensional spaces are flat) and \( \xi \) is already a Cartesian coordinate. The spacing between templates is constant in \( \xi \) and from Eq. (17) we have

\[
d\xi^2 = G_{22} \ d\xi^2 = 1 - M \Rightarrow \ d\xi = \sqrt{\frac{1 - MM}{G_{22}}} \tag{25}
\]

Finally, the number of templates can be found using Eq. (18):

\[
N_T = \sqrt{\frac{G_{22}}{(1 - M)}(\xi_{\text{max}} - \xi_{\text{min}})}. \tag{26}
\]

In the next section we will compute the number of templates found using a template placement algorithm and compare it with the one found in this section.

### B. Stochastic template placement algorithm

When the metric on the waveform manifold is not known exactly and cannot be approximated, we use a brute force approach [53, 54] to construct the template bank. In this method, the template bank is built by proposing templates in the desired parameter space until sufficient coverage is reached. For each proposed template, \( \lambda_{\text{prop}} \), the fitting factor (which is the maximum match of the proposed template with the templates in the bank) is calculated. If the fitting factor is greater than the minimal match required, we reject the proposed template and continue with a new proposal; if it is less than the required minimal match, we add the proposed template to our bank.

This method can get computationally expensive, but there are tricks we can employ to be able to use it in practice. One such trick is to define the “neighborhood” of the proposed templates. It can be defined in terms of a parameter chosen by the user, we used the chirp time for our purposes. When used, the fitting factor is calculated only for the templates in the neighborhood (set in form of units of the neighborhood parameter by the user), \( \lambda_i \), where \( i : 1 \sim N \) for \( N \) templates near the proposal.

Another technique we use is to iteratively lower the frequency step in the calculation of the match [55]. The value of frequency spacing used in the match integral (Eq. (4)) is usually chosen to be \( 1/L \), where \( L \) is the closest power-of-2
greater than the length of the waveforms. This is required to measure the overlap between two waveforms in a time window of L s. But for the bank generation, we are interested in the maximum overlap between waveforms, which occurs near the time point corresponding to 0 displacement for two waveforms aligned at their peak amplitude. Therefore, we can increase the value of frequency spacing, $d_f$, which greatly reduces the cost of calculating the match. We can check if our chosen value for $d_f$ is good enough by iteratively calculating matches by reducing $d_f$ by half. If the last two overlaps agree to within 1%, we use that value or if $(1 - \text{match})$ is large, we continue to the next template. For our banks, we first calculate the matches with $d_f = 2.0$. If the mismatch is large, i.e.

$$(1 - \text{match}) > 0.05 + (1 - M),$$

where $M = 0.98$ is the minimum required match for template placement, then we move on to the next neighboring template $\lambda_{i+1}$. Otherwise, we decrease the value of $d_f \rightarrow d_f/2$ and calculate the match again. We continue iteratively decreasing $d_f$ until the last two matches converge,

$$|\text{match}_{d_f} - \text{match}_{d_f/2}| < 0.001 \times \text{match}_{d_f}. \quad (28)$$

Once convergence is reached, if the match $< M$ we add the proposal $\lambda_{\text{prop}}$ to the template bank.

**C. Number of templates**

Using the stochastic placement algorithm, we calculate a template bank for each of the 181 BBH sources, with component mass ranges constrained by 3G capabilities. These template banks use 3.5 PN TaylorF2 [51, 56–61] waveforms. The lower frequency cut-off for the signals ranges from 14.2–30.5 mHz with an upper frequency cut-off of 150 mHz. Scaling this to the audio band, we produce template banks with frequencies in the range of 142–1500 Hz. We use a minimal match of $M = 0.98$ for the template placement algorithm.

**FIG. 4.** Left: For a BBH source with component masses constrained by 3G to $m_1 = 22.84892$ and $m_2 = 16.93702 \ M_\odot$, the template (green) and injection (blue) component masses are shown for a 3.5 PN template bank. Chirp mass contours are indicated by the black lines. Right: For the same source, the injection component masses are shown with the one-dimensional Newtonian template component masses.

**FIG. 5.** The number of templates from the stochastic placement algorithm is plotted against the LISA SNR, $P_{\text{LISA}}$, for each source. The color bar indicates the CE SNR, $P_{\text{CE}}$.

The left panel of Fig. 4 shows an example template bank in $(m_1, m_2)$ space for a source of $m_1 = 22.85 \ M_\odot$, $m_2 = 16.94 \ M_\odot$, and $f_{\text{low}} = 305.44$ Hz, after scaling to the audio band. Fig. 5 shows the relationship between the source signal-to-noise ratio and the number of templates in the bank. The plot shows that the sources which require the largest number of templates are those for which both the LISA and CE signal-to-noise ratio is small.

The cumulative distribution of stochastic placement template bank sizes for all 181 sources is shown in the top left panel of Fig. 7. The 50th percentile and 90th percentile bank sizes are $3.4 \times 10^3$ and $1.95 \times 10^4$ templates, respectively. The mean bank size, $N_T = 8.02 \times 10^3$ with a standard deviation of $9.70 \times 10^2$. The smallest bank has only 218 templates while the largest
A comparison of stochastic placement and metric method waveform injections against each bank to find the best match – first, using the 3.5 PN approximation in the match calculation and again using only the Newtonian approximation. A summary of all the bank efficiency calculations performed is given in Table I. The top right panel of Fig. 7 shows the cumulative distribution of matches for each of the two-dimensional template banks, where green lines show matches calculated with the Newtonian template approximation and blue lines show matches calculated with the 3.5 PN template approximation. The mean 10\textsuperscript{th} percentile match for both sets are indicated by the vertical lines. These results demonstrate that using the Newtonian approximation in the match calculation is sufficient for our purposes.

To determine the bank efficiencies, we filter 1000 3.5 PN waveform injections against each bank to find the best matching templates. For each of the two-dimensional banks, we calculate the efficiency twice – first, using the 3.5 PN approximation in the match calculation and again using only the Newtonian approximation. A summary of all the bank efficiency calculations performed is given in Table I. The top right panel of Fig. 7 shows the cumulative distribution of matches for each of the two-dimensional template banks, where green lines show matches calculated with the Newtonian template approximation and blue lines show matches calculated with the 3.5 PN template approximation. The mean 10\textsuperscript{th} percentile match for both sets are indicated by the vertical lines. These results demonstrate that using the Newtonian approximation in the match calculation is sufficient for our purposes.

This result is further proven by the one-dimensional Newtonian template bank efficiencies, shown in the bottom right panel of Fig. 7. As indicated by Table I, for this set of bank efficiency calculations we use the Newtonian approximation both in the template waveforms and in the match calculation. Each of the three sets of bank efficiency calculations performed over the 181 hBBH systems had mean 10\textsuperscript{th}-percentile matches $M > 0.98$, so we conclude that these template banks will be effective in recovering hBBH signals from \textit{LISA} data.

### V. VISIBILITY OF STELLAR-MASS BBH IN LISA

We now calculate, for template banks of $O(10^3) - O(10^5)$ templates, the minimal signal-to-noise ratio required to claim a detection in a matched filter search. We first assume that we have a segment of data consisting only of noise, $d(t) = n(t)$. If we filter this data with one template, the SNR would be a random value which, for a Gaussian background, follows the Rayleigh distribution [14]:

$$
\rho(\rho) = \rho \exp \left( -\frac{\rho^2}{2} \right). \tag{29}
$$

The probability of obtaining a value of $\rho$ greater than some threshold $\rho_{thr}$ can be computed by integrating the distribution...
We would like to find the value of $\rho_{\text{thr}}$ above which we can confidently claim a detection. The false alarm probability (FAP) is then the chance of finding $\rho > \rho_{\text{thr}}$ in the noise hypothesis. We can choose an acceptable value of FAP, say $10^{-2}$ and use Eq. (30) to solve for $\rho_{\text{thr}}$:

$$\rho_{\text{thr}} = \sqrt{2 \ln \text{FAP}}.$$  \hfill (31)

Therefore, with FAP = $10^{-2}$ the threshold is $\rho_{\text{thr}} \approx 3.0$. However, the above discussion assumes that we filter the data with only one template. With $N_T$ templates, the FAP is the probability that at least one template has $\rho > \rho_{\text{thr}}$. First, consider the probability that none of the $N_T$ templates have $\rho > \rho_{\text{thr}}$, that is $(1 - p(\rho > \rho_{\text{thr}}))^N_T$. The FAP is the compliment of this, or

$$\text{FAP} = 1 - (1 - p)^N_T.$$  \hfill (32)

For very small $p$, we can approximate this as $\text{FAP} \approx 1 - (1 - p \cdot N_T) = p(\rho > \rho_{\text{thr}}) \cdot N_T$. We see that the number of templates becomes a trials factor on the false alarm probability. We must then scale down our required FAP by the number of templates in Eq. (31),

$$\rho_{\text{thr}} = \sqrt{-2 \ln \frac{\text{FAP}}{N_T}}.$$  \hfill (33)

Finally, using FAP = $10^{-2}$ and $N_T = 10^2 - 10^5$, we find a threshold SNR of $\rho_{\text{thr}} = 4.3\text{–}5.7$. This is a significant improvement over the previously quoted $\rho_{\text{thr}} = 14$ [14] or $\rho_{\text{thr}} = 8$ quoted in Ref. [19]. The lower threshold further improves the
VI. CONCLUSIONS AND FUTURE DIRECTIONS

The simultaneous operation of third generation ground-based gravitational-wave observatories, such as the Einstein Telescope and Cosmic Explorer, and a space-based detector, LISA, provides a unique opportunity for multiband observations of stellar-mass and intermediate-mass binary black hole inspirals and mergers. Such multiband observations greatly improve the bounds one can place on general relativity and alternative theories of gravity [21, 23–25]. However, the computational cost required to dig the signals out of LISA data in a blind search would be formidable, requiring signal-to-noise ratios greater than 14 to make confident detections.

We have shown that 3G observatories would constrain most of the signal parameters—the component masses, spins, sky position, and time of coalescence—so tightly that it would be possible to carry out the search for these signals over a vastly reduced parameter space in LISA data. Indeed, high fidelity measurements enabled by the 3G observatories considered in this paper imply that archival searches in LISA data require only a one-dimensional template bank over the binary’s chirp mass—the only parameter that is measured better by LISA than 3G observatories. Thus, the volume of parameter space necessary to search over is greatly reduced. With template banks containing only $10^2$–$10^3$ templates, archival searches in LISA are feasible for signals of SNR as low as $\sim 4$–6. This would allow for the possibility of $O(100)$ multiband gravitational-wave detections per year, a number that can vastly improve the number of detectable stellar mass binary black holes in LISA by a factor of $\sim 6$.

We have made several implicit approximations in this study that would need to be reexamined to ascertain that the principal conclusions of this paper remain valid. Firstly, we have assumed that the observed binaries would have negligible residual eccentricity when their signals enter the LISA sensitivity band. This is true for most binaries that spend millions of years to slowly spiral-in and merge during which radiation reaction tends to circularize the orbits. However, black hole binaries that form in rich clusters could have non-negligible eccentricities when LISA observes them. 3G observatories should be able to constrain residual eccentricities to a pretty good accuracy which could then be used to reduce the search space. Even if the eccentricity in the audio band of 3G observatories is vanishingly small it will be possible to evolve the orbits back and limit the search parameter space in the LISA band. Eccentricity will likely be a parameter that would not be measured very well by 3G observatories (as any residual eccentricity would have largely decayed) and LISA will likely constrain it better.

Secondly, we have assumed that companion black hole spins are aligned with the orbital angular momentum of the system. Spin-orbit and spin-spin couplings will significantly alter the orbital evolution only when spins are large and misaligned with the orbit. This will be a relatively small part of the parameter space. Moreover, spin effects occur at higher PN orders and are measured better by 3G observatories than LISA. It is, however, important to confirm that precessional effects are negligible in archival searches especially since they are cumulative effects and the signals spend many more cycles in the LISA band than they do in the audio band.

Finally, if the binaries live in a gas-rich environment then the inertial drag could accelerate the rate at which the companions spiral in. Such drags might not be relevant when the binaries enter the audio band of 3G detectors and hence it would not be possible to correct for the presence of environment. In the same spirit, we have not included any ultra-light boson clouds or other fields that might surround the companion black holes in the low-frequency phase of the evolution. This could alter the orbital evolution but would not be relevant at later stages.

While not all of these effects are equally important, it is necessary to investigate the cost of including them in an archival search as they could potentially reveal important mechanisms in play in the formation, evolution, and environments of stellar-mass and intermediate-mass black hole binaries.

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