The Effect of the Ring on the Buckling of Stiffened Cylindrical Shells

Dong Zhang¹, Mingkun Liu², Jun Wang¹ and Wei Liang³

¹Beijing Institute of Aerospace System Engineering, Beijing 100076, China
²ouyan Engineering Technology Research Institute Co Ltd, Beijing 100191, China
³School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China

Abstract. An analysis on the stringer-stiffened cylindrical shell with a large interval ring is described. The buckling of the shell under axial compression is forecasted by the method of the Ultimate Load Method, to simplify the interconnection on steadiness between shell and stringers. Although the shell between stringers is semi-empirical modeled, the buckling of the stringers which are bolstered with the other stiffened component ring is analytically solved. It is show that support effect of the ring on the stiffened cylindrical shell is quantized by a ring coefficient. The buckling load of the stringer-stiffened cylindrical shell is calculated on the different length. Finally, the ring coefficient is used to explants the so-called shielding panel effect ring. The present results could be an important reference for the design of stiffened shell under axial compression.

Keyword: Ring-stringer stiffened; Buckling; Ultimate load method.

1. Introduction
Stiffened cylindrical shells is widely used as structural elements in many types of metal lightweight structures in which buckling of the components is principal cause of the structural failure[1]. The classic theory of stiffened cylindrical shell such as by Batdorf(1947) and Singer(1967)[2,3] is famous in the such structure design. However, for the case stringer-stiffened cylindrical shell with a large interval ring under axial compression, the critical load of classic theory is much higher than that of experiment[4].

In industry, the shielding panel effect and Stiffness discriminant on middle ring is in common use for design of structure of Stiffened shell. In the classic theory of stiffened shell, the middle ring in the case of axial compression is zero force parts. The classic mechanics of shell is not feasibility in the case of large interval ring reinforced cylindrical shell. By use of the homogenization method, the stiffness effect of the large interval ring under axial compression is negligible[5].

On the other hand, stiffened cylindrical shell with a large interval ring is common utilization in structure aircraft and rocket. With the weight reduction of structure, it becomes an important problem for the ring effect on the stability of shell[6].

In the present paper buckling analysis of shell a large interval ring base on the Ultimate Load Method is proposed. Some examples are presented to compare with the design rule with the currently available analysis results.

2. Formulation and Preliminaries
Following that in many references, the cylindrical shell has radius R and thickness t. The shell is
enforced with stringer and ring. The shell material is assumed to be linear elastic and isotropic with Young’s modulus $E$ and Poisson’s ratio $v$. The stretching and bending stiffness of the shell are given by $K = Et$ and $D = Et^3/(12(1-v^2))$, respectively. Shells made from elastomeric materials might be as thick as $R/t >> 1$ and still undergo linear elastic strains.

For the case of the stiffened cylindrical shell is dense reinforced, behavior of the structure generally can be forecasted by two theories: Orthotropic Shell Model and Ultimate Load Method.

By the first theory, the middle surface stretching strains, $\varepsilon_{ab}$, and bending moments, $M_{ab}$, are given in terms of the resultant membrane stresses, $N_{ab}$ and bending strains, $\kappa_{ab}$, by[6]

$$\begin{bmatrix} \varepsilon \\ M \end{bmatrix} = \begin{bmatrix} A^{-1} & -A^{-1}B \\ A^{-1}B & D - BA^{-1}B \end{bmatrix} \begin{bmatrix} N \\ \kappa \end{bmatrix}$$

where, the reduced single subscript is used for $\varepsilon_{ab}$, $M_{ab}$, $N_{ab}$, $\kappa_{ab}$. $A$, $B$, and $D$ are respectively membrane stiffness matrix, bending stiffness matrix and tension bending coupling matrix. The stiffness matrix assumed holds

$$A = \begin{bmatrix} A_{11}^p & A_{12}^p & 0 \\ A_{12}^p & A_{22}^p & 0 \\ 0 & 0 & A_{66}^p \end{bmatrix} + \begin{bmatrix} EA_1 / d_1 & 0 & 0 \\ 0 & EA_2 / d_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} + \begin{bmatrix} EA_1 e_1 / d_1 & 0 & 0 \\ 0 & EA_2 e_2 / d_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} D_{11}^p & D_{12}^p & 0 \\ D_{12}^p & D_{22}^p & 0 \\ 0 & 0 & D_{66}^p \end{bmatrix} + \begin{bmatrix} E(I_1 + A_1 e_1^2) / d_1 & 0 & 0 \\ 0 & E(I_2 + A_2 e_2^2) / d_2 & 0 \\ 0 & 0 & (GJ_1 / d_1 + GJ_2 / d_2) / 4 \end{bmatrix}$$

where $A_{ab}^p$, $B_{ab}^p$, $D_{ab}^p$ is the stiffness matrices of the panel, respectively. $A_1$, $d_1$, $e_1$, $I_1$ are the area, interval distance, eccentricity and inertia of the stringers respectively. $A_2$, $d_2$, $e_2$, $I_2$ are those of the rings.

The Ultimate Load Method is another effective theory for design of ring-stringer stiffened cylindrical[6]. It simplified the interaction between shell panel and strings. The effect of panel on strings is semi-empirically represented by a skin Influence factor $\xi$ and the opposite effect is modeled as equivalent width coefficient $\phi$. The buckling load of the structure is calculated by the equation

$$\sigma_{ht} = N_y \xi \sigma_{ht} (A_1 + \phi d_1 t) + N_y \sigma_{ht} d_1 t (1 - \phi)$$

where $\sigma_{ht}$ is the critical buckling stress of the cylindrical skin, which can be find in the handbook.

The buckling stress of stringer $\sigma_{ht}$ becomes the key to find the buckling load of the ring-stringer stiffened cylindrical.

3. Buckling Stress of Stringer

The stringer is constrained in ends and is supported by the middle ring. When the compression load of structure transmits to the stringer from Flange frame, the latter transmits to load to the middle when compression and buckling deformation. Therefore, the buckling stress of stringer is influenced by the ring deformation and buckling.

To research buckling stress $\sigma_{ht}$ of the above problem, a familiar model compressive bar stability is
Figure 1. The problem simulated as a beam with elastic bearing. The coefficient $K(EI_2)$ can sometimes show the ring effect on the ht. However, the simple model has not expanded the shielding panel effect. To solve the problem, an energy model is introduced. Making use of the stiffness equation of both ring and stringer, the potential energy is written as

$$ V = U_1 + U_2 + U_L $$

where $U_0=0$, the stain energy of string can be written as

$$ U_1 = \frac{EA_1}{2} \sum_{n=1}^{N_1} \left[ \left( \frac{\partial u}{\partial x} \right)^2 - 2e_1 \frac{\partial u}{\partial x} \frac{\partial^2 w}{\partial x^2} \right] + \frac{I_1}{A_1} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + GJ_1 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \left. dx \right|_{v=n,d_1} $$

That for the ring is

$$ U_2 = \frac{EA_2}{2} \sum_{n=1}^{N_2} \left[ \left( \frac{\partial v}{\partial y} \right)^2 - 2e_2 \frac{\partial v}{\partial y} \frac{\partial^2 w}{\partial y^2} \right] + \frac{I_2}{A_2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + \frac{GJ_2}{EA_2} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \left. dy \right|_{x=n,d_2} $$

The subsequence power of the incremental displacement is represented as

$$ U_L = \sum_{n=1}^{N_1} \left[ \frac{1}{2} \int_{0}^{L} \left( \frac{\partial w}{\partial x} \right)^2 dx \right] + \sum_{n=1}^{N_2} \left[ \frac{1}{2} \int_{0}^{L} \left( \frac{\partial w}{\partial y} \right)^2 dx \right] \left. \right|_{x=n,d_2} $$

where $J1,J2$ are the polar moment of inertia of ring and stringer respectively for the case of free torsion. The dimensionless quantity $e_1=s_1/A_1$, $e_2=s_2/A_2$. And $P_{10,20}$ are the axial force on ring and stringer which are get rise by the external load. The existence of nonzero solutions for boundary problem means.

$$ \frac{\partial V}{\partial w_{ma}} = 0 $$

Substitution of equation (4)-equation (7) into the variation condition (8), it is obtained an algebraic equation by applying Fourier analysis. The result can be represented by the following form,

$$ \sigma_{ht} = (\zeta + 1) \frac{\pi^2 EI_1}{l^2} $$

where the coefficient for ring on buckling stress of string. It can be presented by
\[ \zeta = \min(\frac{\eta \frac{I_h L^3}{n}}{I_{gk} R^3}, 3) \]  

(10)

and \( \eta \approx 0.59 \) from the numerical calculation of equation (9).

### 4. Result and Conclusion

The buckling stress of the stringer-stiffened cylindrical shell can be calculated from equation (3), (10) and (11). A cylindrical shell structure with length 1.4m and radius 1.5mm is introduced as an example. Figure 2 depicts the buckling stress varied with the stiffness \((E I_2/E I_1)^{0.25}\).

In meanwhile, from equation (10) and equation (11) the minimum buckling stress is in the case that \( \zeta = 0 \) and maximum buckling stress when \( \zeta = 3 \). This variation causes the \( \sigma_{ht} \) four times increases from no ring to the maximum effect of ring. Making use of the relation between the distance of strings and the number of strings, i.e. \( d_i = \frac{2\pi R}{N} \), the coefficient for ring on buckling stress of string in equation (11) is rewritten as

\[ \zeta = 3.24 \frac{E I_2 L^3}{E I_1 R^3} \frac{b}{R} \]

(11)

The maximum effect of ring on buckling stress is reached on the condition

\[ 3.24 \frac{E I_2 L^3}{E I_1 R^3} \frac{b}{R} \geq 3 \]

(12)

![Figure 2. Buckling stress varied with the stiffness ratio between stringer to ring.](image)

It is obvious that the condition (13) is close to the famous condition of shielding panel effect[5].

\[ \Lambda = \left( \frac{R}{l} \right)^3 \frac{R (E J)_{string}}{b (E J)_{ring}} < 4.0 \]

It is emphasized that the present solution is based on semi-empirically equation for buckling of enhanced shell. This is done in order to reveal the ring effect on the buckling load of ring-stringer stiffened cylindrical shell. Thus, the results are approximate respect to the closed-form solution of shell. However, the solution in this manuscript provides a regular of ring for design of ring-stringer cylindrical stiffened shell. The present model may be consistently extended to other shell.
5. Acknowledgment

This research was partially supported by National Natural Science Foundation under grant No. U1837207.

6. References

[1] Timoshenko SP, Gere JM. Theory of Elastic Stability. 2 ed: Dover Publications; 2009.

[2] Singer J, Baruch M, Harari O. On the stability of eccentrically stiffened cylindrical shells under axial compression. International Journal of Solids and Structures. 1967; 3(4):445-70.

[3] CAO Jingle SHI Yuhong et. al, Stability Analysis of Stiffened Panels Considering Transverse Beam Torsion. Structure & Environment Engineering, 46(3):128-124, 2019;

[4] Zhou Dongsheng Yu Wenchao, Calculation of the critical rigidity of the middle frame of the whole stiffened cylinder Structure & Environment Engineering, 31(2): 23-29, 1994

[5] Cui Degang, Handbook of Structural Stability Design. Beijing: Aviation industry press, 1996.

[6] Kidane S. Bulking load analysis of grid stiffened composite cylinders. Composite, 34:1-9, 2003