Multiaxial Fatigue Damage Accumulation under Variable Amplitude Loading Conditions

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Abstract

Generally, structures and mechanical components are subjected to multiaxial loadings under random conditions. However, fatigue damage accumulation under such loading conditions remains a hard topic to deal with. This subject is an ultimate challenge in fatigue characterization and it contains all topics within the multiaxial fatigue subject, ranging from cyclic plasticity to sequential loading effects. In this work two fatigue damage accumulation rules and the Stress Scale Factor (SSF) equivalent shear stress as damage parameter, are used to evaluate the accumulated damage from multiaxial random loadings where the loading paths and sequences results were obtained from the literature. Results show that there exists a conceptual problem in the decision trigger of the selected damage accumulation rules. Some solutions are pointed out to overcome this drawback.

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Keywords: Multiaxial fatigue; fatigue damage; variable amplitude loading; damage accumulation; fatigue life.

1. Introduction

Multiaxial damage evaluation of random loadings is an ultimate challenge in multiaxial fatigue characterization. The absence of a loading pattern under random multiaxial loading conditions increases the assessment complexity of the fatigue damage accumulation. Furthermore, true random loading paths have a certain level of uncertainty that cannot be fully simulated in the lab. Moreover, their unknown loading nature may activate several damage mechanisms, they can be activated in sequence, simultaneously or even a mix of both [1]. Proportional, non-proportional, sequential, asynchronous or variable amplitude among others, are the loading effects that yield different damage.

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mechanisms and under random loading conditions all of them can be activated. Therefore, under such conditions, a
multiaxial fatigue criterion must be able to capture all of those loading effects. In a synergistic way, a multiaxial
fatigue criterion must gather tools that allow capture the different damage mechanisms when they are activated.
Thus, have such multiaxial fatigue criterion is of utmost importance because it will allow designing structures and
mechanical components based on representative loading spectrums recorded in the field. Also, when implemented
within a stress/strain monitoring system will allow monitoring the structures health during their service time. Fatigue
damage accumulation rules uses fatigue damage parameters, like the Stress Scale Factor (SSF) equivalent shear
stress [2], to characterize the accumulated damage in the sample or structure and several works can be found in
literature regarding this subject of prime importance in the multiaxial fatigue scope [3–9].

In this paper, the SSF criterion’s performance under random loading conditions in association with damage
accumulation rules is studied. This criterion contains several features such as non-proportionality sensitivity or the
virtual cycle counting among others, which suggests that the SSF criterion is a good candidate to be used under
random multiaxial loading conditions [10]. This criterion was already successfully validated in a high strength steel
42CrMo4 under loading blocks with several loading paths such as sequential, proportional, and non-proportional
effects [11]. Therefore, in present work the SSF criterion capability is evaluated to be used in damage accumulation
rules, such as the Palmgren-Miner rule or the Morrow’s criterion, in order to successfully estimate failure conditions
[12,13]. To validate the aforementioned hypothesis multiaxial fatigue testing data, gathered from literature [14], was
used from where it was possible to get fatigue data obtained under a random loading regime performed in the lab
using aluminum alloy 2024-T4 samples. These data were obtained with loading spectrums made with 16 loading
blocks randomly ordered, which is a clever way to simulate a random loading regime in lab [14]. Results show good
agreements between experiments and the estimated results for the loading blocks. However, some issues regarding
the decision trigger of the accumulated damage rules were found.

### Nomenclature

| SAR    | Stress amplitude ratio \( \lambda = \tau / \sigma_a \) |
|--------|---------------------------------------------------|
| SSF    | Stress scale factor \( f \) |
| NP     | Non-proportional                                   |
| OP     | Out of phase                                       |
| PP     | Proportional                                       |
| \( \tau \) | Shear stress amplitude |
| \( \sigma_a \) | Axial stress amplitude |

| Endurance limit (torsion) \( \tau_a \) |
|---------------------------------------|
| Endurance limit (axial) \( \tau_f \) |

| Number of loaded cycles \( n \) |
|---------------------------------|
| Estimated fatigue life \( N \) |

| Table 1. 2024-T4 mechanical properties [14] |
|---------------------------------------------|
| Young’s Modulus [GPa]                      | 73   |
| Yield stress [MPa]                         | 400  |
| Ultimate stress [MPa]                      | 545  |

The mechanical properties of 2024-T4 depends greatly on the temper type, in this case the aluminum alloy has a
T4 heat treatment, (solution heat-treated and naturally aged to a substantially stable condition).

The material used in this study is an aluminum alloy commonly used in the aircraft industry in load-bearing
applications, the 2024-T4. Its mechanical properties are presented in Table 1.

The experimental results used here were obtained by Xia et al. in [14], however the experiments and
methodologies used in their work are briefly explained. Using this experimental data, we aim to evaluate the
performance of the SSF method for this type of aluminum alloys under random loadings. The experiments were
performed using a biaxial loading machine, tension/torsion under stress control using tubular specimen tests; the failure condition used in the experiment was the specimen total rupture. The idea was to simulate random loading conditions in 16 different loading blocks; these blocks have different stress levels and different multiaxial effects. The random loading condition was achieved by combining aleatory the sequence of those blocks therefore the loading sequence is previously defined before testing. For each block (single loading path) within the loading sequence, the number of loading cycles (in this case number of blocks) was 3% of the fatigue life of the same loading block. When the failure occurs the position of the loading sequence and the number of cycles performed in the last block was registered. Figure 1 presents the loading paths used in [14] to define the aforementioned loading blocks. The blocks are differentiated by loading path type and stress level; three loading paths types were considered: uniaxial, proportional, and non-proportional loading paths. In uniaxial conditions pure axial (PT) and pure shear (PS) conditions were used, the proportional (PP) ones have a Stress Amplitude Ratio (SAR) of 0.577 and 0.77 and the non-proportional (NP) have a SAR ranging from 0.5 to 0.88 with phase shifts equal to 30°, 45°, 60° and 90° respectively. 

Fig. 1. Multiaxial loading paths, (a) Pure axial (PT); (b) Pure shear (PS), (c) PP with SAR equal to 0.577 (PP45), (d) NP with phase shift equal to 30° and SAR equal to 0.577 (30OP45), (e) OP with phase shift equal to 45° and SAR equal to 0.577 (45OP45), (f) OP with phase shift equal to 60° and SAR equal to 0.577 (60OP45), (g) OP with phase shift equal to 90° and SAR equal to 0.577 (90OP45).
Figure 2 shows the SAR effect in the loading path shape. Figure 2 a) shows the effect in the proportional loadings and b) shows for non-proportional ones. Essentially, under proportional loadings, the SAR changes the slope of the loading path, and for the non-proportional ones changes the form of the loading path from circular to elliptical one.

Table 2 presents the experimental results for the fatigue life of each loading path presented in Figure 1.

| Block | Path                                      | $\sigma_a$ [MPa] | $\tau_a$ [MPa] | $\tau_a / \sigma_a$ | $N_f$ |
|-------|------------------------------------------|------------------|-----------------|---------------------|--------|
| A     | Uniaxial tension and compression         | 250              | 0               | 0.00                | 56316  |
| B     | Uniaxial tension and compression         | 350              | 0               | 0.00                | 6167   |
| C     | Pure torsion                             | 0                | 144             | -                   | 63795  |
| D     | Pure torsion                             | 0                | 167             | -                   | 49912  |
| E     | Proportional                             | 158              | 112             | 0.709               | 76451  |
| F     | Proportional                             | 177              | 102             | 0.576               | 80107  |
| G     | Proportional                             | 248              | 143             | 0.577               | 6488   |
| H     | 30 Non-proportional                      | 158              | 120             | 0.759               | 63584  |
| I     | 45 Non-proportional                      | 158              | 125             | 0.791               | 57004  |
| J     | 45 Non-proportional                      | 248              | 143             | 0.577               | 7363   |
| K     | 60 Non-proportional                      | 158              | 132             | 0.835               | 30893  |
| L     | 90 Non-proportional                      | 177              | 102             | 0.576               | 49292  |
| M     | 90 Non-proportional                      | 158              | 139             | 0.880               | 15459  |
| N     | 90 Non-proportional                      | 244              | 157             | 0.643               | 3453   |
| O     | 90 Non-proportional                      | 250              | 144             | 0.576               | 4634   |
| P     | 90 Non-proportional                      | 250              | 125             | 0.500               | 6811   |

Table 3 presents the aleatory loading sequence of loading blocks and the inherent fatigue life. For each sequence two experiments with two different samples were performed and the results are read as follows: the first number indicates the number of the last block completed before failure; the letter indicates the running block at failure time and the last number indicate the number of cycles performed in the last running block at failure.

3. Theoretical analysis

The SSF criterion [2] is based in the stress damage concept that relates the stress level and the stress amplitude ratios with the material’s induced damage. From experiments it can be concluded that the axial damage is
The shear stress level required to cause a cyclic failure is less than the one needed when it is considered an axial stress, thus it can be concluded that there exists a sort of damage scale between shear and axial damages. Traditionally, the $\sqrt{3}$ constant from von Mises or 2 from Tresca criteria are used as damage scales between shear and axial damages. For instance, in the von Mises criterion we have the following normalization shown in Eq. (1).

$$\tau = \frac{\sigma}{\sqrt{3}}$$

where the axial damage (represented by the axial stress amplitude level) must be divided by $\sqrt{3}$ to be reduced to the shear damage scale. In this way, it is possible to have the axial and shear damages in the same damage scale, allowing their addition, as seen in the von Mises equivalent stress. Moreover, several multiaxial fatigue criteria also use such type of constant damage scale, for example, the ratio between endurance limits in shear and axial loadings, see Eq.(2), has been widely used [2].

$$\frac{T_{-1}}{f_{-1}}$$

However, the present authors found out that the damage scale is not a constant, in order to account with both damages, the SSF approach uses the stress amplitude ratio (SAR) and the stress amplitude level to capture the different state of damage within a loading path.

From experiments, it was found that the stress amplitude ratio and stress levels have an important influence in the material’s fatigue strength variation. The SSF criterion is an equivalent shear stress model, where the axial damage is transformed into a shear one. The damage parameter of the SSF model (an equivalent shear stress) is defined as follows:

$$T(\tau, \sigma) = \tau + \text{ssf}(\sigma, \lambda) \cdot \sigma$$

where $\tau$ and $\sigma$ are the instantaneous shear and axial stresses, respectively, $\text{ssf}(\sigma, \lambda)$ is a polynomial function which transforms axial damage into a shear one. The equivalent shear stress is determined instantaneously along the loading period. In order to estimate fatigue lives, it is considered the maximum equivalent shear stress within a loading block and the material's S-N trend line obtained in pure shear loading conditions. The SSF model under constant amplitude loadings is as follows:

$$\max(\tau + \text{ssf}(\sigma, \lambda) \cdot \sigma) = A(N_f)^b$$

where $A$ and $b$ are the trend line parameters and is the criterion estimation to the material’s fatigue life. The SSF function is defined as:

$$\text{ssf}(\sigma, \lambda) = a + b \cdot \sigma + c \cdot \sigma^2 + d \cdot \sigma^3 + f \cdot \lambda^2 + g \cdot \lambda^3 + h \cdot \lambda^4 + i \cdot \lambda^5$$

where, $\sigma$ is the instantaneous axial component of a biaxial loading and $\lambda = \tau/\sigma$ is the instantaneous stress amplitude ratio. Constants from “$a$” to “$i$” are determined through experimental tests [2]. The $\text{ssf}(\sigma, \lambda)$ function aims to capture the material’s damage map, [2] having into account that the damage is dependent of the SAR, represented in Fig. 1 and 2 by the slope of the angle in the multiaxial loading path. Damage accumulation models aims to account accumulated damage in the material based in a damage parameter. Usually, they need decompose random loading spectrums in loading blocks where it is possible to compute fatigue damage parameters. Basically, damage parameters are used to estimate fatigue strength under certain stress level and loading path. Then, a relation between the estimated fatigue life and the loading cycles is performed. When the estimated residual fatigue life
becomes equal to zero, then there exists a high probability of failure to occur. One accumulated damage rule very known is the Palmgren-Miner rule, see Eq.(6) which states that the ratio between the loading cycles and the allowable ones must be less than 1 to have a low probability to failure [13].

$$D = \sum_{j=1}^{\#blocks} \left( \sum_{i=1}^{\#cycles} \frac{n_i}{N_{fj}} \right)$$  

(6)

This rule assumes that the damage from each loading block can be added linearly, this approach have some good results when the loading spectrum is defined using only one loading block, even for the complex ones. Several models based in the Palmgren-Miner rule can be found in literature, one example is the Morrow’s rule, where the Palmgren-Miner ratio is corrected in order to account for the material cyclic plasticity [12]. This correction is carried out using the ratio between the block damage amplitude and the maximum damage amplitude found in the loading spectrum (all loading blocks). The Morrows’ model is presented in Eq.(7).

$$D = \sum_{j=1}^{\#blocks} \left( \sum_{i=1}^{\#cycles} \frac{n_i}{N_{fj}} \left( \frac{\sigma_{da}}{\sigma_{da \_spectrum}} \right)^d \right)$$  

(7)

where $\sigma_{da}$ is the maximum amplitude of the damage parameter within each loading block and $\sigma_{da \_spectrum}$ is the maximum damage parameter within the loading spectrum. The exponent $d$ is a material property, for the 2024-T4 aluminum alloy this value is equal to -0,45 [14].

4. Results and Discussion

The application of SSF damage scale concept to the experimental results published in [14] will be carried out in two steps: first, the constant amplitude multiaxial fatigue data for the wide range of loading paths described in Fig. 1 and Table 2 will be analyzed; second, the random combination of the loading paths described in Fig. 1, that will be considered here as random loadings, will be analyzed in order to predict the fatigue damage accumulation using the SSF damage scale and the two accumulation damage rules considered in this study described in Eq. (6) and (7).

4.1 Constant amplitude loading paths

For the experimental tests described in Fig. 1 and Table 2, fatigue life predictions were carried out based on Eq.(s). (3, 4, and 5) where the second term of Eq. (4) was calculated based on the experimental data of Table 2. Figure 3 shows the correlation between the estimations and the experimental data. Having into consideration that the SSF damage map, Eq. (5) was not determined for this aluminum alloy neither adapted, it can be concluded that the results are quite satisfactory. The SSF damage map used here was determined for the 42CrMo4 high strength steel, but it can be considered as a general multiaxial damage scale rule, but in order to achieve fine tune of the results it is advised to determine the SSF damage map for the specific material by experiments. In Figure 3, the outer bound is related with a factor of 3 of fatigue life and the inner one is related with a life factor of two.
Fig. 3. SSF fatigue life correlation with the 2024-T4 experimental data under constant amplitude loading.

4.2 Random loading

For the experimental tests described in Table 3, fatigue life predictions were carried out based in Eq.(s). (3, 4 and 5) using the damage accumulation rules of Eq. (6) and (7). Figure 4 shows the estimates for the accumulated damage using the Palmgren-Miner rule and the Morrow’s method.

In both methods the SSF equivalent stress as damage parameter was used. Since each loading block was well defined, it was not needed any method to extract loading blocks or use cycle counting techniques such as the virtual cycle counting method [10], for instance. Analyzing the results of the Figure 4, (having in mind the premise that the failure can occur when the accumulated damage is greater than one) can be concluded that for the three first runs, the estimates indicates a non conservative result. However, for the fourth one the results are conservative.

The most conservative of the two accumulated damage rule considered here was the Morrow’s rule, essentially this rule estimates an accumulated damage 10% greater than the one estimated by the Palmgren-Miner. Regarding the results of Figure 4, and considering the use of this methodology in structural health monitoring, the non-conservative estimates (graphic’s bars lower than one) indicate a residual fatigue life when the sample is already broken, being a rather disturbing result. One important aspect to have in mind is that the estimated damage for each loading block (used in the damage accumulation models) has always a probabilistic behavior, because for the same damage parameter several experimental fatigue lives can be obtained, therefore it is usual to present fatigue life correlations like the one presented here, in Figure 3, with live factors of two or three. Therefore, the risk of the sample survive with an accumulated damage greater than one there exists, which is the case of the results of the run 4 as seen in Figure 4. On the other hand, the sample may fail with accumulated damage much lower than 1, which is the cases of run 1, 2, and 3. In this sense, arguing that the “survive” or “not-survive” condition is based in accumulated damages greater or lower than 1 is an inaccurate approach, and unsuitable to be used in structural health monitoring. However, this is a crucial aspect that needs to be analyzed and solved, which is the failure condition for damage accumulation under random loading conditions. One idea to overcome this issue is to guarantee that the damage parameter estimates are between the lower bound and the middle line commonly used in fatigue data correlation, as presented in Figure 3. To do that, the damage parameter must be increased a certain percentage that must be determined for each material family, i.e. steels, aluminum alloys, magnesium alloys, etc. This increase will compensate the probabilistic effect and inherent propagation between fatigue damages of each loading blocks within a loading spectrum. In this way, and under a perspective of structural health monitoring,
having a accumulated damage inferior to 1 will give a greater certainty that the sample (or structural component) will endure until reach an accumulated damage equal to 1, at that point the sample must be safely replaced.

![Damage accumulation results](image)

**Fig. 4.** Damage accumulation results, obtained using the SSF fatigue life estimations and the Morrow and Miner damage accumulation rules.

The damage accumulation results achieved by Xia et al. [14] were obtained without the use of fatigue life estimates, where for each loading block the experimental fatigue life was used to obtain the ratio between the number of cycles performed vs number of cycles at failure (block fatigue life). In the presented study a different approach was adopted, in order to assess the use of the SSF estimates for the loading block’s fatigue life. While the use of experimental fatigue life for each loading block as stated by Xia et al [14] is correct here we try to estimate the accumulation damage under random loading condition by estimating the blocks fatigue life, because for a general random loading it is not possible to have the experimental fatigue strength for all kind of loading blocks.

5. Conclusions

In this paper the SSF equivalent shear stress, developed by the present authors, was used to account fatigue damage accumulation under random loading conditions. To do that, it was gathered fatigue data from literature where the loading spectrums were achieved by an aleatory combination of 16 well defined loading blocks. Two damage accumulation rules were used, the Palmgren-Miner, and the Morrow’s rule, to estimate the values of accumulated damage at failure. Results show good correlations between the estimates of the SSF criterion and the fatigue data inherent to the loading blocks. However, the accumulated damage results were not conclusive due to the "not-survive" trigger adopted here in the accumulated damage rules. Failure condition based in damage accumulation greater or lower than one is not suitable for in the field applications such as structural health monitoring procedures.

Some alternatives to the failure trigger in the fatigue damage accumulation models were pointed out and discussed based in a field application perspective.

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References

[1] Socie D, Marquis G. Multiaxial fatigue. Warrendale, PA: Society of Automotive Engineers, 1999 502 1999.
[2] Anes V, Reis L, Li B, Fonte M, de Freitas M. New approach for analysis of complex multiaxial loading paths. International Journal of Fatigue 2014;62:21–33.
[3] Carpinteri A, Brighenti R, Macha E, Spagnoli A. Expected principal stress directions under multiaxial random loading. Part II: numerical simulation and experimental assessment through the weight function method. International Journal of Fatigue 1999;21:89–96.
[4] Carpinteri A, Macha E, Brighenti R, Spagnoli A. Expected principal stress directions under multiaxial random loading. Part I: theoretical aspects of the weight function method. International Journal of Fatigue 1999;21:83–8.
[5] Carpinteri A, Spagnoli A, Vantadori S. A multiaxial fatigue criterion for random loading. Fatigue & Fracture of Engineering Materials & Structures 2003;26:515–22.
[6] Sonsino CM, Kaufmann H, Grubišić V. Transferability of material data for the example of a randomly loaded forged truck stub axle. SAE Technical Paper; 1997.
[7] Sonsino CM, Pföhl R. Multiaxial fatigue of welded shaft-flange connections of stirrers under random non-proportional torsion and bending. International Journal of Fatigue 1990;12:425–31.
[8] Susmel L. Estimating fatigue lifetime of steel weldments locally damaged by variable amplitude multiaxial stress fields. International Journal of Fatigue 2010;32:1057–80.
[9] Carpinteri A, Spagnoli A. Multiaxial high-cycle fatigue criterion for hard metals. International Journal of Fatigue 2001;23:135–45.
[10] Anes V, Reis L, Li B, de Freitas M. New cycle counting method for multiaxial fatigue. International Journal of Fatigue 2014.
[11] Anes V, Reis L, Li B, Freitas M. New approach to evaluate non-proportionality in multiaxial loading conditions. Fatigue & Fracture of Engineering Materials & Structures 2014.
[12] Morrow JD. The effect of selected sub-cycle sequences in fatigue loading histories. Random Fatigue Life Predictions 1986;72:43–60.
[13] Miner MA, others. Cumulative damage in fatigue. Journal of Applied Mechanics 1945;12:159–64.
[14] Xia T, Yao W. Comparative research on the accumulative damage rules under multiaxial block loading spectrum for 2024-T4 aluminum alloy. International Journal of Fatigue 2013;48:257–65.