Low Complexity Iterative Piece-Wise Companding Transform for Reduction of PAPR in MIMO OFDM Systems

T. Ramaswamy1* and K. Chennakesava Reddy2

1Department of ECE, Malla Reddy Engineering College (A), Hyderabad - 500100, Telangana, India; dani.swamy@gmail.com
2JNTUH, Hyderabad - 500085, Telangana, India; kesavary@yahoo.com

Abstract

Objectives: This paper presents one of the solutions to minimize the problem of PAPR (Peak Average to Power Ratio) in OFDM systems. This work mainly focuses on introducing iterative companding approach along with the available standard reduction methods. Methods/Statistical Analysis: This paper includes Iterative Linear Transform Approach (ILST), Iterative Two Piecewise Companding Transform (ITPWC) and Iterative Piecewise Companding Transform (IPCT). A performance evaluation was performed and found that IPCT attained very low BER and PAPR in all the above mentioned approaches. Apart from this it also consumes low computations there by proving the best of its class at the earliest. Findings: BER performance of the proposed IPCT approach under different channel environment depicts that the proposed approach provides less BER against ILST and ITPWC with 3-3.5 db improvement. Application/Improvement: The developed Iterative Piecewise Companding Transform (IPCT) is more suitable for MIMO systems which make it to be widely used for 4G, WLAN and Wimax applications.

Keywords: Companding, Low complexity, MIMO-OFDM, PAPR

1. Introduction

Modern Multiple Input Multiple Output (MIMO) systems require capability of transmitting large data with high data rate extensively. To improve spectrum efficiency and achieve higher rates at more than 100 Mbps more advanced techniques need to be implemented. Hence, the upcoming generation mobile communication needs more sophisticated modulation scheme and information transmission structure.

Multiple Orthogonal Frequency Division Multiplexing (OFDM) are adopted due to their superior performance. They promise to provide high-speed data transfer in wireless communication technologies and combining them can provide wireless industry evolution like 4G systems.

In MIMO-OFDM systems multiple sub carriers are superimposed that require higher power than the mean power of the system. However, the phases of these carriers are same. High PAPR is one of the serious problems in MIMO-OFDM systems. In order to transmit these high PAPR OFDM signals requires insertion of high power amplifiers with very high power scope. These kinds of amplifiers are very expensive and gives rise to non-linear distortions which changes the phase offset leading to the interference of the side by side signal spectrum resulting in performance degradation.

To mitigate this performance degradation many PAPR reducing approaches are proposed so far like Clipping and Filtering, Selective mapping technique, Partial Transmit sequence, Tone reservation method and companding transforms.

In, Wang et al. introduced $\mu$–law companding method for the applications in speech processing, but later this $\mu$–law companding approach is used in OFDM systems to reduce PAPR with the aim of increasing the average signal power.

*Author for correspondence
In\textsuperscript{7}, Jiang et al. proposed a benchmark approach called exponential companding scheme where the PAPR reduction is obtained by transforming symbols with the exponential distribution of OFDM symbols while maintaining the constant average signal power. Both these methods (\(\mu\)-law and EC) have become a benchmark for the rest of the upcoming methods.

In\textsuperscript{8}, Aburakhia et al. proposed a Linear Companding Transform (LCT) that transforms large and small amplitudes with different scales. The average power may not be constant at different levels of input and outputs of companding transform.

In\textsuperscript{9}, Yang et al. proposed Two Piecewise Companding Transform (TPWC) that tried to attain constant average signal power and achieve one to one mapping. The method represents the small amplitudes with scale and higher amplitudes with both scale and shift.

In\textsuperscript{10}, proposed a piecewise companding transform neglecting unnecessary compression in the reduction of peak power. On the other hand, companding distortion can be effectively mitigated by expanding larger signals with slight changes in amplitude. The designing of parameters have to be done very carefully in decreasing the PAPR.

In this paper, an iterative companding transform employing the traditional companding transforms is proposed. The main intention of this approach is to mitigate the trade-off between the BER and PAPR. As of this paper, it presents the iterative piece wise companding that transform the amplitudes of the signals above a given companded peak amplitude are clipped for peak power reduction. Similarly the signals with amplitudes close to the given companded peak amplitude are linearly scaled for power compensation.

2. Peak to Average Power Ratio Formulation

Let \(\textbf{X} = [x_0, x_1, \ldots, x_{N-1}]\) represent the data sequence that is to be transmitted independently in OFDM system with \(N\) sub carriers. The signal is oversampled with a factor \(L\) (\(L = 4\)) and the OFDM symbols can be represented as:

\[
x_n = \frac{1}{\sqrt{N L}} \sum_{k=0}^{N L - 1} X_k e^{j2\pi k n / N L} \quad 0 \leq n \leq N L
\]

Where \(X = [x_0, x_1, \ldots, x_{N-1}, 0, 0, \ldots, 0, x_M, \ldots, x_N]\) is the input signal vector. The PAPR of this transmitted signal is given as:

\[
PAPR = 10 \log_{10} 10 (\text{max}|x|^2 / \text{E}(|x|^2))
\]

The PAPR reduction capability is measured by the Complementary Cumulative Distribution Function (CCDF), which is defined as the probability that the signal's PAPR exceeds a specific threshold value \(\gamma_0 > 0\)

\[
CCDF(\gamma_0) = \text{Prob}(PAPR > \gamma_0) = 1 - (1 - e^{-\gamma_0})^N
\]

The principle of companding transform is to compress the high peaks symbols and to enhance the low amplitude symbols simultaneously, thereby decreasing the PAPR of the transmitted signal prior to the digital to analog converter and HPA.

3. Companding Transforms

This section presents different companding transforms that are used in this paper.

3.1 Linear Symmetrical Transform (LST)

This is the simplest of the CT whose companding function is given as:

\[
f(\chi) = (k_1 |\chi| + b) \cdot \text{sgn}(\chi)
\]

Where \(\text{sgn}(.)\) is the sign function. Two constant parameters \(0 < k_1 < 1\) and \(b > 0\) are here to specify the companding profile in order to maintain an unchanged power after companding transform. The inverse companding is given as:

\[
f^{-1}(\chi) = \frac{|\chi| - b}{k_1} \cdot \text{sgn}(\chi)
\]

3.2 Two Piecewise Companding (TPWC)

This is a Linear Non Symmetrical Transform (LNST) which is the best in terms of PAPR reduction and also provides a solution for abrupt jump issue during companding\textsuperscript{12}. Its function is given as:

\[
f(\chi) = \begin{cases} 
& (u_1 |\chi| + s) \cdot \text{sgn}(\chi), \quad |\chi| \leq v \\
& (u_2 |\chi| + s) \cdot \text{sgn}(\chi), \quad |\chi| > v
\end{cases}
\]

Where \(u_1 > 1, 0 < u_2 < 1\) and \(s = (u_1 - u_2)v > 0\) and \(0 \leq v \leq V\) is the cut off point with \(V = \max(|x_n|)\). The de-companding function is given as:
The relation between three variables $u_1, u_2, v$ is given with linear equation as:

$$f^{-1}(x) = \begin{cases} \frac{1}{u_1} |x| \cdot \text{sign}(x) & |x| \leq u_1 v \\ \frac{1}{u_2} (|x| - s) \cdot \text{sign}(x) & |x| > u_1 v \end{cases}$$ (7)

The relation between three variables $u_1, u_2, v$ is given with linear equation as:

$$u_1 \cdot 1^2 \cdot (1 - \exp(-\delta^2)) + u_2 \cdot 2^2 \cdot \exp(-\delta^2) = 1$$

Please refer for more info on this approach.

3.3 Piecewise Linear Companding (PLC)

When the original signal $x_n$ is companded with a given peak amplitude $A_c$, the companding function is given as:

$$f(x) = \begin{cases} kx + (1 - k)A_c & |x| \leq A_c \\ \text{sign}(x)A_c & |x| > A_c \end{cases}$$ (8)

And its de-companding function is given as:

$$f^{-1}(x) = \begin{cases} \frac{k}{1 - k} x & |x| \leq A_c \\ \frac{1}{k} \text{sign}(x)A_c & |x| > A_c \end{cases}$$ (9)

From the above analysis it is observed that the proposed companding transform is specified by constant parameters $A_c, A_i, A_t$ and $k$. However, the average power is maintained to be constant and according the Equation (2) the value of PAPR for the proposed approach that can be achieved is determined by $A_c$.

4. Proposed Iterative Mechanism

The main objective of this approach is to obtain a significant PAPR reduction at the expense of a less amount of band distortion and spectral regrowth. The block diagram of the proposed approach is Figure 1.

The proposed approach is summarized as:

- Convert $y^m$ to frequency domain to generate $c^m$ using NL-FFT points.
- Perform the frequency domain filtering by multiplying it with $H_{rect}$ to get $c^{m_0}$.
- Convert $c^{m_0}$ to time domain using NL-IFFT points.
- Calculate PAPR denoted as $PAPR^m$, if $PAPR^m \leq PAPR_{des}$ or $m>M$ iteration the set $k_2 = 1$ and exit the loop if not return to step 3.

The filtering response is given as:

$$H_{rect} = \begin{cases} 1 & 1 < k \leq N - 1 \\ 0 & N \leq k \leq LN - 1 \end{cases}$$ (10)

The proposed approach is low complex than the traditional approaches for exponential companding, it requires NL computation ($N = 256, L = 4$) while CT requires $2LN\log_{2}(LN)$. However, the proposed approach requires $(0.65\times MNL)$ computations.

5. Simulation Results

The experiment is evaluated for MIMO systems with different antennas under different channels. This approach is also compared in terms of PAPR (Figure 2), OBI (Figure 3), BER (Figures 4, 5, 6) and complexity (Figure 7).
In Figure 2 the PAPR is decreased by 0.8 dB against exponential companding and 1.5 dB against the traditional piecewise companding, Figures 4, 5 and 6 shows the BER performance of the proposed approach under different channel environment. It clearly depicts that the proposed approach a less BER against ILCT (LCT 8) and TPWC9 with 3-3.5 db improvement. In Figure 7 the computational complexity is analysed against exponential and ILCT methods with L = 4 and at M = 3 iterations. It clearly depicts that the present approach is consuming very less computations with highest at N = 256, the computation were 2000 which is very promising.

**Figure 3.** Power spectral density of companded signals showing decrease in distortion with proposed approach.

**Figure 4.** Performance analysis of the proposed approach for 2 × 2 antenna system under AWGN channel.

**Figure 5.** Performance analysis of the proposed approach for 2 × 2 antenna system under Rayleigh channel for Nsym = 1024 and Nsub = 64.

**Figure 6.** Performance analysis of the proposed approach for 2 × 2 antenna system under SUI channel Nsym = 1024 and Nsub = 64.

**Figure 7.** Computational analysis of the proposed approach with traditional approaches at M = 3 iterations and L = 4.

In Figure 2 the PAPR is decreased by 0.8 dB against exponential companding and 1.5 dB against the traditional piecewise companding, Figures 4, 5 and 6 shows the BER performance of the proposed approach under different channel environment. It clearly depicts that the proposed approach a less BER against ILCT (LCT) and TPWC with 3-3.5 db improvement. In Figure 7 the computational complexity is analysed against exponential and ILCT methods with L = 4 and at M = 3 iterations. It clearly depicts that the present approach is consuming very less computations with highest at N = 256, the computation were 2000 which is very promising.
6. Conclusion

An iterative piecewise companding transform is proposed for MIMO-OFDM systems in this paper. The proposed approach is providing the promising results against the traditional approaches in terms of PAPR, BER and computations thus, providing a very low complex approach. The present research can be further extended with different antenna schemes and data rates.

7. References

1. Telatar IE. Capacity of multi-antenna Gaussian channels. AT T Bell Labs Internal Tech Memo. 1995; 10(6):585–95.
2. Wang L, Tellambura C. A simplified clipping and filtering technique for PAR reduction in OFDM Systems. IEEE Signal Process Lett. 2005; 12(6):453–6.
3. Yang J, Chen L, Liu Q, Chen D. A modified selected mapping technique to reduce the peak-to-average power ratio of OFDM signals. IEEE Trans Consumer Electron. 2007; 53(3):846–51.
4. Varahram P, Ali BM. Partial transmit sequence scheme with new phase sequence for PAPR reduction in OFDM systems. IEEE Trans on Consumer Electron. 2011; 57(2):366–71.
5. Varahram P, Ali BM. Partial transmit sequence scheme with new phase sequence for PAPR reduction in OFDM systems. IEEE Trans on Consumer Electron. 2011; 57(2):366–71.
6. Wang XB, Tjhung TT, Ng CS. Reduction of peak-to-average power ratio of OFDM system using a companding technique. IEEE Trans Broadcast. 1999; 45(3):303–7.
7. Jiang T, Yang Y, Song YH. Exponential companding technique for PAPR reduction in OFDM systems. IEEE Trans Broadcast. 2005; 51(2):244–8.
8. Aburakhia SA, Badran EF, Mohamed DAE. Linear companding transform for the reduction of peak-to-average power ratio of OFDM signals. IEEE Trans Broadcast. 2009; 55(1):155–60.
9. Yang P, Hu A. Two-Piecewise Compingding transform for PAPR reduction of OFDM signals. Proc of the 7th Wireless Commun Mobile Comput Conf (IWCMC); Istanbul, Turkey. 2011. p. 619–23.
10. Hu M, Li Y, Wang W, Zhang H. A piecewise linear companding transform for PAPR reduction of OFDM signals with companding distortion mitigation. IEEE Transactions on Broadcasting. 2014; 60(3):532–9.
11. Wang Y, Yang C, Ai B. Iterative companding transform and filtering for reducing PAPR of OFDM signal. IEEE Transactions on Consumer Electronics. 2015; 61(2):144–50.
12. Huang X, Lu J, Zheng J, Letaief KB, Gu J. Companding transform for reduction in peak-to-average power ratio of OFDM signals. IEEE Trans Wireless Commun. 2004; 3(6):2030–9.