I. INTRODUCTION

Electron-scale turbulence is a plausible explanation for the anomalous transport of electron energy well above the neoclassical values seen in a variety of tokamak plasma scenarios. Additionally, electron energy transport may become more important in future burning plasma experiments such as ITER because the electron channel is preferentially heated by Coulomb collisions with fusion alpha particles. The electron-temperature-gradient (ETG) instability produces radially-elongated streamers at the electron gyroradius scale and is a primary candidate to explain electron-scale transport. Electron heat flux due to ETG turbulence has been seen to play a role in various tokamak experiments, and the inclusion of electron-scale dynamics at the ion scale has resulted in better agreement with experimental heat flux levels.

Electron-scale, long-time, large-box gyrokinetic flux-tube simulations have reported that intermediate-scale zonal flows (ZF) help to regulate streamer turbulence in the quasi-saturated state and can eventually become dominant. These results are inconsistent with fluid ETG turbulence models in which zonal flow generation occurs via the standard Hasegawa-Mima nonlinear mechanism, which is significantly weaker for ETG turbulence than for ion-temperature-gradient (ITG) turbulence. Moreover, while shearing due to zonal flows generated by ITG turbulence can suppress ion transport levels, the finer scale ETG turbulence is unlikely to be affected by the ITG-driven zonal flows. These effects have led to the expectation of a streamer-dominated steady state at the electron scale.

A weak-turbulence, toroidal, gyrokinetic-electron analysis of nonuniform tokamak plasmas has shown that a Navier-Stokes type nonlinearity couples electron-temperature-gradient (ETG) modes and zonal flow (ZF) modes with wavelengths much shorter than the ion gyroradius but much longer than the electron gyroradius. This intermediate-scale ETG-ZF coupling is typically stronger than the Hasegawa-Mima type nonlinearity characteristic of the fluid approximation and is predicted to lead to relevant zonal flow generation and ETG mode regulation. Electron-scale, continuum, gyrokinetic simulation results are presented here which include both single-mode ETG and full-spectrum ETG turbulence. The zonal flow generation due to single ETG modes is investigated and the single-mode intermediate-scale results are found to be in agreement with theory. The full-spectrum results are then presented and explained qualitatively in terms of the single-mode results. It is found that the ETG-driven zonal flows regulate intermediate-scale electron heat flux transport to levels in the predicted range.
and ETG turbulence into the late stage. In all cases the results showed a more isotropic steady state familiar with ITG turbulence.

This paper provides results which compare the generation of zonal flow by ETG turbulence in electron-scale, gyrokinetic simulations with theoretical predictions in the intermediate-scale limit. We first provide the details of the simulation parameters in Section II. This is followed by analysis of two types of nonlinear simulations. The “single-mode” results serve to illuminate the role of a single ETG mode in generating zonal flow and are covered in Section III. As the theoretical description is limited to a single ETG mode for tractability, these results convey the primary scope of this paper. The “full-spectrum” nonlinear simulations provided in Section IV include a typical range of ETG modes and are qualitatively explained in terms of the single-mode theory in the intermediate-scale and the electron fluid models at the short wavelength scale.

II. SIMULATION MODEL AND PARAMETERS

We employ GENE, an Eulerian 5-d gyrokinetic continuum code, in the flux-tube limit appropriate for electron-scale turbulence. Gyrokinetic ions and electrons are taken with standard Cyclone Base Case (CBC) parameters which are typical of H-mode core plasmas, but here a simplified circular geometry is used. First, the linear ion-scale benchmark in Ref. 35 was verified, then the simulation was converted to the electron scale by reducing the perpendicular box dimensions by a factor of \( \sqrt{m_i/m_e} \sim 42 \). Here, \( m_e \) is the electron mass, \( m_i \) is the ion mass which is taken to be the proton mass, \( m_p \). For the electron-scale case the ion temperature gradient was set to zero to suppress long-wavelength ion turbulence and focus on electron-scale physics.

The mode frequencies, \( \omega \), and growth rates, \( \gamma \), resulting from the linear GENE simulations are shown as functions of \( k_y \rho_i \) in Fig. 1 above. The frequencies are normalized to units of \( R/c_s \), as listed with other GENE normalizations in Table I. Here, \( k_y \) is the binormal wavenumber of the GENE coordinate system, \( R \) is the tokamak major radius, and \( c_s = \sqrt{T_i/m_e} \) is the ion sound speed with \( T_i \) the ion temperature. The ITG benchmark case is shown in Fig. 1(a) alongside the electron-scale ETG case in Fig. 1(b). Ion-scale ETG results are shown in Fig. 1(c), where the collisionless trapped-electron mode (CTEM) is included. One can see the ITG mode in the lower \( k_y \rho_i \) range of Fig. 1(a) characterized by propagation in the \( \omega > 0 \) ion diamagnetic drift direction. The ETG mode becomes unstable at higher \( k_y \rho_i \) where \( \omega \) crosses to the negative ion diamagnetic drift direction. While the CTEM is expected to contribute to electron transport, Fig. 1(c) shows that it is stable in the intermediate-scale range.

The reference values and radial profiles are taken as specified in Ref. 35 with reduced values of \( m_i \) and \( \beta = 8\pi n_e T_e/B^2 \). Here \( n_e \) and \( T_e \) are the electron density and temperature, and \( B \) is the on-axis magnetic field. \( \beta = 10^{-5} \) was chosen to keep the simulation nearly electrostatic in order to avoid transport due to electromagnetic fluctuations. The safety factor, \( q = r B_\theta / R B_\phi \), and magnetic shear, \( s \), profiles are given by\(^{35} \) \( q(r) = 2.52(r/a)^2 - 0.16(r/a) + 0.86 \), and \( s = \frac{\partial q}{\partial \log r} \). \( B_\phi \) and \( B_\theta \) represent the toroidal and poloidal magnetic field components respectively. The radial flux-surface coordinate \( r = 0.5a \) was chosen, where \( a \) is the tokamak minor radius. The normalized density and temperature gradient profiles, for a general profile \( A(r) \), are defined as \( R/L_A = -R \partial_t \ln (A(r)) \), which can be calculated using

\[
\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) \right] = \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \right] \left( \frac{1}{r} \frac{d}{dr} \right) \left( r \frac{d}{dr} \right)
\]

For the linear ITG case, the grid resolution was taken with 32 grid points in the radial dimension, \( x \), and
FIG. 2: Original (small-box, no collisions) nonlinear simulation results showing (a) time-marked electrostatic electron heat flux and electrostatic potential contours for the (b) early nonlinear phase (green marker), and (c) late zonal phase (red marker).

FIG. 3: Original (small-box, no collisions) nonlinear simulation results showing Fourier-space electrostatic potential contours for the (a) initial time (blue marker), (b) early nonlinear phase (green marker), and (c) late zonal phase (red marker). The markers correspond to the times marked in Fig. 2(a).

16 grid points in the parallel spatial dimension, z. The GENE radial coordinate $x$ corresponds to the flux-surface coordinate $r$ for the case of a circular geometry. In Fig. 1(a) the flux-tube GENE benchmark result from Ref. 35 is marked as “Göler” and the corresponding “low” (32 × 8) and “high” (64 × 16) velocity grid ($v_t \times \mu$) resolution simulations have been plotted collectively. There is good agreement with the benchmark case in the ETG turbulence scale, where the intermediate scale is well resolved in the “low v-res” case, and the same grid resolution was used for the nonlinear simulation, but with the radial grid resolution increased to 192 gridpoints. The perpendicular box size was reduced from $125\rho_i \times 125\rho_i$ at the ITG turbulence scale to $6\rho_i \times 3\rho_i$ at the ETG turbulence scale, where the original radial extent of the flux-tube domain was increased from $3\rho_i$ to $6\rho_i$ to allow for the full formation of the ETG mode streamers.

The electrostatic portion of the radial heat flux, $\langle Q_{ES} \rangle$, for electrons is shown approaching a statistically steady state in time in Fig. 2(a) for the collisionless, $6\rho_i \times 3\rho_i$, nonlinear, full-spectrum, electron-scale case. The heat flux is normalized to $Q_{gB}$, the gyroBohm nor-

16 grid points in the parallel spatial dimension, $z$. The GENE radial coordinate $x$ corresponds to the flux-surface coordinate $r$ for the case of a circular geometry. In Fig. 1(a) the flux-tube GENE benchmark result from Ref. 35 is marked as “Göler” and the corresponding “low” (32 × 8) and “high” (64 × 16) velocity grid ($v_t \times \mu$) resolution simulations have been plotted collectively. There is good agreement with the benchmark case in the ETG turbulence scale, where the intermediate scale is well resolved in the “low v-res” case, and the same grid resolution was used for the nonlinear simulation, but with the radial grid resolution increased to 192 gridpoints. The perpendicular box size was reduced from $125\rho_i \times 125\rho_i$ at the ITG turbulence scale to $6\rho_i \times 3\rho_i$ at the ETG turbulence scale, where the original radial extent of the flux-tube domain was increased from $3\rho_i$ to $6\rho_i$ to allow for the full formation of the ETG mode streamers.

The electrostatic portion of the radial heat flux, $\langle Q_{ES} \rangle$, for electrons is shown approaching a statistically steady state in time in Fig. 2(a) for the collisionless, $6\rho_i \times 3\rho_i$, nonlinear, full-spectrum, electron-scale case. The heat flux is normalized to $Q_{gB}$, the gyroBohm nor-

Table I. Relevant GENE normalizations.  

| Parameter | Value |
|-----------|-------|
| $R(m)$    | 1.67  |
| $n_i(e(10^{19}m^{-3})$ | 4.66 |
| $T_{i,e}(keV)$ | 2.14 |
| $B_B(T)$  | 2.0   |
| $\tau/a$  | 0.5   |
| $\alpha/R$ | 0.36 |
| $\rho^*/\rho_i$ | 0.001413 |
| $\beta$   | 1.64  |
| $m_i/m_p$ | 1.0   |
| $\rho^2=c_s/R$ | 5.4462e−4 |
| $\phi$    | 2.22  |
| $\nu_{ci}$ | 0.837 |
| $\nu_{ci}$ | 0.106875 |

Table II. GENE simulation parameters.
nalization given in Table I, and the angled brackets, \(...\), denote a flux-tube volume average. The heat flux is determined in GENE as,\(^{31}\)

\[
Q_{ES} = \int d^3 v \left( \frac{1}{2} m v^2 \right) \mathbf{v}_{E \times B} \cdot \hat{r} \delta f ,
\]

where \(\delta f\) is the distribution function perturbation, \(\mathbf{v}_{E \times B} = -(\nabla \phi \times \mathbf{B})/B^2\) is the \(\mathbf{E} \times \mathbf{B}\) drift, \(\phi\) is the electrostatic potential perturbation, and \(v\) is the particle velocity. The normalization for \(\phi\) is given in Table I. One can see the shift from the early, nonlinear state characterized by radially-elongated electrostatic potential streamers in Fig. 2(b) to the later state of Fig. 2(c) where zonal flows have become dominant. It is during this phase that intermediate-scale zonal flows grow slowly into the final quasi-saturated state.

The initial condition for \(\phi\) was realistically peaked about the most unstable mode as shown in Fig. 3(a). This allowed for a transition from the high-\(k_p\) ETG turbulence regime to the intermediate scale where ZF generation is expected to be stronger.\(^{1}\) Such zonal flow generation is not present in toroidal electron fluid theories.\(^{20,21}\) An inverse-cascade can clearly be seen between Figs. 3(a) and 3(b). This initial saturation is discussed further for the single-mode simulation results presented in Sec. III B, and for the well-converged, collisional, full-spectrum simulation results presented in Sec. IV. The convergence tests for finding an optimal nonlinear box size are detailed in Appendix A.

### III. SINGLE-MODE ANALYSIS

#### A. Zonal Flow Generation Mechanism

It was shown theoretically in Ref. 1 that intermediate-scale zonal flow may play a role in the nonlinear saturation of ETG turbulence in tokamak plasmas. The equations of Ref. 1 are briefly reviewed here, and the nonlinear single-mode simulation results are then presented in Sec. III B with comparison to the theory. The theoretical model takes the standard gyrokinetic equation\(^{42}\) for electrons and the quasineutrality condition in the intermediate-scale limit \(k_i^2 \rho_i^2 \ll 1 \ll k_p^2 \rho_p^2\). For long-wavelength ETG modes, one can generally assume that the growth rate is much smaller than the real frequency, \(|\gamma/\omega| \ll 1\), which allows for a weak-turbulence analysis. These modes also satisfy the relation \(|\omega_d| \ll |\omega_t|, |\omega_i|\), for \(\omega_t\) and \(\omega_d\) the transit and magnetic drift frequencies respectively. Local approximations\(^{43}\) are then assumed for \(\omega_d\) and \(\omega_t\) in formulating a kinetic electron model.

The evolution of a single ETG mode can then be derived in terms of a nonlinear Schrodinger equation (NLSE),\(^{1}\)

\[
[i(\partial_t - \gamma_n) - b_n k_0^2 \rho_n^2 s^2 \theta_n^2 - cn \frac{\partial^2}{\partial \theta_n^2}]A_n(\theta_k) = \frac{- ik_0 \rho_n s}{\sqrt{2\pi}} \int d\theta_k \partial_\theta \delta A_n(\theta_k - \theta_n),
\]

with \(A_n\) and \(A_n\) the amplitudes of the ETG and ZF modes, \(\gamma_n\) the ETG mode growth rate, \(k_0\) the tilting angle (defined by \(k_x = k_p \phi d\) in the flux-tube limit\(^{44}\) for \(k_y\) of the ETG mode), \(k_0\) the poloidal wavenumber, and the terms including \(b_n\) and \(c_n\) coming from the frequency mismatch and plasma nonuniformity corrections respectively. \(b_n\) and \(c_n\) are specifically associated with the linear ETG dynamics as explained in Ref. 1. The nonlinear term under the integral in Eq. (2) describes a Navier-Stokes type coupling due to \(\mathbf{E} \times \mathbf{B}\) shearing effects. This coupling is \(\mathcal{O}(k_0 \rho_n)^{-2}\) stronger than the usual Hasegawa-Mima type coupling in the fluid limit.\(^{20,21}\) This stronger coupling results in a stronger regulation of ETG turbulence by zonal flows and also leads to a reduced threshold for ZF excitation by intermediate-scale ETG modes. The threshold condition is described further in Eq. (4).

The description of zonal flow dynamics is given by the equation,\(^{1}\)

\[
[\partial_t + \gamma_z (1 + d_z k^2 \rho_s^2 s^2 \theta_s^2)]\hat{\chi} A_z(\theta_k) = \sqrt{\frac{\pi}{2}} (k_0 \rho_s s) \theta_k \int \partial_\theta \partial_\theta \delta A_n(\theta_k - \theta_n)]
\]

The nonlinear term under the integral in Eq. (3) is related to the Reynolds stress of the ETG modes. The ZF damping rate is given by \(\gamma_z \approx 3\nu_{ee}/|\omega_{ee}|\sqrt{\tau}\), with \(\nu_{ee}\) the electron-electron collision frequency and \(\omega_{ee}\) the electron diamagnetic drift frequency. The total electric susceptibility is defined as \(\chi_e = \tau + (1+1.6 q^2/\sqrt{\tau}) k_0^2 \rho_s^2 s^2 \theta_s^2/2\) with \(\tau = T_e/T_i\). The \(\tau_i\) term in Eq. (3) represents a gyrodiffusive correction which helps suppress short wavelength zonal flows.\(^{45}\)

The \(a_n\) term describes the parallel correlation of the ETG turbulence and is defined as \(a_n(\theta_k, \theta_n) = \int d\eta \langle \Phi_n^*(\eta, \theta_k) v^2_\perp \delta \tilde{H}(n, \eta, \theta_k + \theta_n) \rangle_\eta\), where \(\eta\) is the extended poloidal angle. Here, \(\delta \tilde{H}_n\) is the non-adiabatic part of the distribution function perturbation. Keeping \(a_n\) non-local in tilting angle in order to take into account ballooning effects leads to a parallel decoupling of ZF modes and therefore ZFs as well, where \(a_n = a_n(0, 0) \exp(-\theta_s^2/2\eta^2)\) for \(\eta = (\int d\eta \Phi^* \eta^2 \Phi)^{1/2}\) the parallel-mode-averaged potential.\(^{1}\)

Eqs. (2) and (3) are taken together as the NLSE model. The numerical solution of the NLSE model with a single ETG mode and a range of ZF modes gives an evolution of ETG and ZF modes that can be described by three specific stages.\(^{1}\) The initial stage involves un inhibited exponential growth of the ETG mode to a threshold point at which the radial beating of the ETG drives ZF growth as described by equation (3). As the ZF modes
grow, they lead to radial dispersion of the initial ETG wave packet and the creation of sidebands. These sidebands then drive more zonal flows via a modulational instability in the second stage. Once the zonal flow grows to appreciable levels in comparison with the ETG mode, the nonlinear interaction in equation (2) acts to saturate the ETG mode. In the final stage, the linear growth rate of the ETG mode becomes negligible and the NLSE model then results in slow, algebraic growth for the zonal flow. This slow growth has been observed in previous gyrokinetic simulations.\(^\text{17,18}\)

A threshold condition for the ZF excitation can be calculated analytically\(^1\) by considering a simple four-wave model for a single zonal flow mode, an ETG pump mode, and two ETG sideband modes. Narrowband, rectangular functions are used to describe the ZF and ETG modes, \(A_0\Pi[(\theta_k-\theta_z)/W] \) and \(A_\nu\Pi[(\theta_k-\theta_z)/W + A_\nu\Pi[(\theta_k+\theta_z)/W] + A_\nu\Pi[(\theta_k+\theta_z)/W] \) respectively. Here \(W\) is the full-width of the modes, \(\theta_z\) is the ZF wavenumber in terms of tilting angle, and \(A_0, A_\nu\) are the ZF, ETG pump, and ETG sideband mode amplitudes. Substituting these functions into Eqs. (2) and (3) with the assumption of no plasma nonuniformities \((\epsilon_n = 0)\) and a steady state pump amplitude for simplicity, one obtains the critical threshold condition:\(^1\)

\[
W^2|A_{0,c}|^2 = \frac{(\Delta^2 + \gamma_{s}^2)(1 + d_s(k_\theta\rho_s\delta\theta_z)^2)\chi_z}{(k_\theta\rho_s\delta\theta_z)^4[\gamma_s\text{Re}(a_n) + \Delta\text{Im}(a_n)]}. \tag{4}
\]

Here, \(\Delta = \text{Re}(b_n)(k_\theta\delta\theta_z\rho_c)^2\) represents the frequency mismatch of the pump and sidebands, and \(\gamma_s = \gamma_n + \text{Im}(b_n)(k_\theta\delta\theta_z\rho_c)^2\) is the growth rate of the sidebands. This threshold condition for ZF excitation by intermediate-scale ETG modes is \(O(k_\theta^2\rho_s^2)\) lower than the condition found in the fluid approximation,\(^{21}\) allowing for more effective ZF generation at intermediate scales.

The ZF modes are initially excited in a range of \(\theta_z\) values. As the system evolves to the quasi-saturated state and the ETG mode is suppressed, the ZF spectrum narrows towards the most easily driven mode. Comparing the exponential \(\theta_z\) dependence of the \(a_n\) term to the algebraic form of the \(d_s\) term, one finds that the parallel decoupling is largely responsible for minimizing the intermediate-scale threshold condition at low \(\theta_z\). If one considers the temporal evolution of the four-wave model, a fixed-point solution can be found with constant \(A_z\) and sufficiently low ZF damping rate, where the ZF and ETG mode amplitudes are given as\(^1\)

\[
W^2|A_{z,p}|^2 = \frac{\pi(\delta^2 - \Delta^2 - \gamma_n\gamma_s)}{(k_\theta\delta\theta_z\rho_c)^2}, \tag{5}
\]

and,

\[
W^2|A_{0,p}|^2 = \frac{\chi_z\gamma_z(1 + d_s(k_\theta^2\delta\theta_z^2\rho_c^2))\chi_z^2 + (\delta - \Delta)^2)}{(k_\theta^2\delta\theta_z^2\rho_c^2)^4[(\Delta - \delta)\text{Im}(a_n) + \gamma_s\text{Re}(a_n)]}. \tag{6}
\]

Here, \(\Delta = \gamma_n/(\gamma_s + \gamma_n)\) represents the amplitude oscillation frequency of ETG modes due to the nonlinear ETG-ZF coupling. The single ETG mode spectrum then continues to fluctuate in \(k_z\) while the ZF mode reaches a constant, steady state.\(^1\) Additionally, \(|A_{0,p}|^2\) is then proportional to \(\gamma_z\) in the saturated state, and therefore the collision frequency, while \(|A_{z,p}|^2\) is not. While these saturation estimates are only valid for a single ZF mode, as the ETG turbulence saturates and the ZF spectrum narrows due to the threshold condition, \(\theta_z\) of the most optimally-driven mode can be used to estimate the ETG saturation level.

### B. Single-Mode Simulation Results

We now compare the NLSE model (Eqs. (2) and (3)) to gyrokinetic simulation results. Collisionless, nonlinear, ETG simulations were carried out where a single unstable ETG mode \((k_z = 0)\) and all zonal flow modes \((k_y = 0)\) are retained. This fairly accurately describes the dynamics of the NLSE model. All results presented in this section are averaged over \(z\). The ETG growth rate spectrum with respect to \(k_y\rho_i\) can be utilized here to illustrate the NLSE model dynamics. As seen in Fig. 1(b), the ETG growth rate spectrum is symmetric around the most unstable mode, so one can choose to compare the evolution of a pair of ETG modes with similar growth rates, where one mode has a \(k_y\rho_i\) value in the intermediate-scale range and the other mode has a larger \(k_y\rho_i\) value outside of that range. Then the ZF drive of the two modes can be compared to verify the expectations from the NLSE model.

The \(k_y\rho_i = 6.36\) mode with a growth rate of \(\gamma \approx 7.037\) and the \(k_y\rho_i = 30\) mode with a similar growth rate of \(\gamma \approx 7.015\) are taken here for comparison. Fig. 4(a) shows the time evolution of the \(k_y\rho_i = 6.36\) mode, while Fig. 4(b) shows the time evolution of the \(k_y\rho_i = 30\) mode. Fig. 4(c) shows the time evolution of the four strongest zonal flow modes at the final time step for the intermediate-scale case, whereas Fig. 4(d) shows a large range of ZF modes excited in the high-\(k_y\) case in order to illustrate a difference in the zonal flow response between the two cases. One can see that initially both ETG modes grow exponentially at similar rates until a threshold is reached, at which point zonal flows are excited. For the intermediate-scale ETG mode, this phase is followed by an algebraically-growing long wavelength ZF phase in which the ZF modes gradually reach a steady state value. The high-\(k_y\) results show no slowly-growing ZF phase at late times. This difference in ZF generation in the late stage is consistent with the threshold condition given in Eq. (4). The intermediate-scale ETG mode continues to slowly drive zonal flows as it is suppressed to lower levels, whereas the high-\(k_y\) ETG mode does not.

The peak level of the ETG mode is much lower for the high-\(k_y\) case than for the intermediate-scale case. This result is not expected from the NLSE model as the shearing of the ETG mode by the wave-wave coupling should be stronger in the intermediate scale than at higher \(k_y\). Zonal flows are also generated earlier in the
FIG. 4: Plots of collisionless single ETG mode evolution for (a) $k_y \rho_i = 6.36$ and (b) $k_y \rho_i = 30$ with respective growth rates $\gamma \approx 7.037$ and $7.015$. The strongest four ZF modes in the late time are plotted for (c) the $k_y \rho_i = 6.36$ case, while (d) shows the excitation of a larger range of ZF modes in the $k_y \rho_i = 30$ case.

The high-$k_y$ case, indicating a lower threshold initially, which is inconsistent with the NLSE model. One noticeable difference between the single-mode GENE simulations and the NLSE model is that the NLSE model only includes the ZF shearing suppression mechanism, whereas the single-mode GENE simulations include other saturation mechanisms such as quasilinear profile flattening. Comparing the zonal response between the two cases, it is found that the initial ZF excitation shown in Fig. 4(d) is much more abrupt, possibly indicative of a secondary instability.

The change in ETG-ZF dynamics in the single-mode results is found to occur near $k_y \rho_i \sim 15$. This difference in behavior likely indicates the reason for the transition to the intermediate scale mentioned previously in Sec. II, and this is further discussed in comparison to the full-spectrum simulation results presented in Sec. IV.

Fig. 5 shows the sum of all ZF mode amplitudes, $\sum |\phi_z|$, as a function of $k_y \rho_i$. Each value of $k_y \rho_i$ represents initializing with a different unstable ETG mode. The sum is taken at the final simulation time, where the ZF mode amplitudes are nearing steady state levels. The notable region of ZF generation is clearly seen to be in the intermediate-scale range, as expected by the NLSE model. Shorter ETG mode wavelengths correspond to weaker zonal flows at late times, in agreement with fluid ETG models. In addition, the drop-off at long ETG mode wavelengths is reasonable due to trapped electron effects at this wavenumber range. A validation of this expectation for the full-spectrum simulations is provided in Appendix A.

The unstable ETG mode is shown to be suppressed at late times in Figs. 4(a) and 4(b). The total amplitude, $\langle |\phi| \rangle = \left( \int d\theta \int |\phi_k|^2 \right)^{1/2}$, of the ETG mode is small in comparison to the zonal flow amplitude. In contrast, the NLSE model simulation results given in Fig. 3 of Ref. 1 show that the total ETG and ZF mode amplitudes, $\langle |\phi| \rangle$, are of similar strength and fluctuating in the late stage. One reason to expect the strong ETG suppression in the gyrokinetic simulations is the lack of collisionality which would damp the zonal flow due to the collisional dependence of the $\gamma_z$ term in Eq. (3). Additionally, the NLSE model assumes a Gaussian radial spectrum for the ETG mode, while the single-mode flux-tube GENE simulations take $k_x = 0$ initially. The globally-Gaussian radial distribution of the ballooning modes would lead to more radial ETG mode overlap, which would then drive more ZF generation as predicted by Eq. (3).

We were able to obtain more physical results for the $k_y \rho_i = 6.36$ case in which the ETG and ZF modes fluctuate about similar steady state values due to their nonlinear coupling. These results were achieved by including the physical collisionality taken in Sec. IV, such that $\gamma_z = 0.014$, and by initializing the ETG mode with a Gaussian $k_x$ and $z$ spectrum such that $\phi(k_x, z, t = 0) \sim \sum |\phi_z|$, as driven by a single ETG mode. The fourth and seventh points correspond to the ETG modes from Fig. 4.
The corresponding $\langle |\phi| \rangle$ for the ETG and ZF modes are plotted in Fig. 6. One can see the strong drive of the zonal flow, as well as the late-stage fluctuations of both ETG and ZF modes. Fig. 7 shows that over time the Gaussian radial spectrum of the ETG mode is broadening into sidebands, while the radial spectrum of the zonal flow modes narrows from a broad distribution to a peak at a final, low-\(\theta_z\) mode number, as predicted by the NLSE model. These results suggest that one should perform global simulations to see results most consistent with the NLSE model.

The improved single-mode results for the \(k_y\rho_i = 6.36\) case showed large root-mean-square (rms) \(k_x\) fluctuations for the ETG mode compared to more fixed ZF fluctuations. These results agree with the expectations of Eqs. (5) and (6). The fluctuations can be seen in Fig. 8, where the rms-averaged amplitude, \(\langle |\theta_k| \rangle = \left( \int d\theta_k |\theta_k|^2 |\phi_k|^2 \right)^{1/2}\), is plotted for the ETG and ZF modes respectively. The ratio of the total absolute amplitude of ZF to ETG modes is given in Fig. 9 as a function of \(\gamma_z\) and is consistent with the trend from the NLSE model. The late-stage behavior of the ETG and ZF \(k_x\)-spectra, as shown in Figs. 7 and 8, and the collisional behavior of the mode amplitude ratio agree well with the late-time behavior reported in the electron-scale MAST simulations of Ref. 18.

### IV. FULL-SPECTRUM SIMULATION RESULTS

The full-spectrum nonlinear simulation results are presented here and the intermediate-scale zonal flow generation mechanism is further investigated. Including multiple toroidal modes results in a final quasi-saturated heat flux characterized by richer turbulent interactions. Fig. 10(a) shows the time history of the heat flux for the well-converged \(24\rho_i \times 3\rho_i\) case with collisionality. The

![Fig. 6](image1.png)

**FIG. 6:** Plot of total (a) ETG and (b) ZF mode potential as an integral over \(k_x\) in the collisional, Gaussian-\(k_x\) ETG case for \(k_y\rho_i = 6.36\). Markers have been added to match the spectral snapshots of Fig. 7.

![Fig. 7](image2.png)

**FIG. 7:** Radial-spectrum snapshots of (a) the single ETG mode and (b) ZF as a function of tilting angle for the \(k_y\rho_i = 6.36\) case. Snapshot times match the markers in Fig. 6.

![Fig. 8](image3.png)

**FIG. 8:** Plot in time of total ETG and zonal flow potential as an rms integral over \(k_x\) in the quasi-saturated stage.

![Fig. 9](image4.png)

**FIG. 9:** Ratio of total absolute amplitudes of ZF to ETG modes with varying collisionality. Results are taken at the final timestep. The reference value corresponds to the fourth point, \(\gamma_z = 0.014\).
The self-adjoint form of the standard Landau–Boltzmann collision operator is used. Realistic collision-based mode results, and are omitted. One can observe that the intermediate-scale modes saturate the slowest and reach the highest levels. During the period of intermediate-scale ETG mode growth, the ZF modes shown in Fig. 10(b) are driven exponentially by the radial heating of ETG modes, as well as by the modulational instability. Once the ETG modes reach a quasi-saturated state, the ZF modes continue to grow slowly in agreement with the single-mode simulation results. Considering the findings of the NLSE model, the single-mode simulations results of Sec. III B, and the full-spectrum simulation results discussed here, it is the intermediate-scale ETG modes which are most responsible for driving ZF mode growth into the late stage.

The heat flux spectrum for the full-spectrum case is shown in Fig. 12, alongside a quasilinear saturation estimate. The quasilinear estimate of the heat flux for a single $k_y$ mode is calculated as,

$$Q^{QL}_{k_y} = A_0 \left(\frac{\gamma}{\langle k_y^2 \rangle} \right)^2 Q^{lin}_{k_y},$$

with $Q^{lin}_{k_y}$ representing the linear simulation results for the heat flux and $\phi_{0,k_y}(0)$ the linear electrostatic potential at $k_x = z = 0$. $A_0$ represents a constant of proportionality, and $\langle k_y^2 \rangle$ is the ballooning-angle-averaged perpendicular wavenumber, defined as,

$$\langle k_y^2 \rangle = \frac{\sum_{k_y} \int (g^{xx}k_x^2 + 2g^{xy}k_xk_y + g^{yy}k_y^2)\phi_k(z)^2dz}{\sum_{k_x} \int |\phi_k(z)|^2dz}.$$

Here, $\phi_k$ represents a Fourier mode of the electrostatic potential perturbation, and $g^{\mu\nu} = \nabla\mu \cdot \nabla \nu$ gives the metric tensor coefficients of the GENE coordinate system. A sum over all $k_x$ values was required for the drop-off of saturation levels at low $k_y \rho_i$.

This model of mode saturation describes a balance of the unstable growth of the instability with turbulent diffusion based on a mixing-length estimate, and gives notable agreement with the nonlinear heat flux spectrum in the intermediate scale. However, the disparity between the quasilinear model and the nonlinear heat flux spectrum is largest for higher $k_y \rho_i$ ETG.
modes. This suggests other, stronger saturation mechanisms for these modes. While new effects, such as toroidal inverse-cascading\textsuperscript{20–22} may play a role in the full-spectrum case, the difference in the heat flux spectra also agrees with the transition in ETG-ZF dynamics found to occur around $k_y\rho_i = 15$ for the single-mode results. The more abrupt ZF response and quicker saturation of the higher $k_y$ modes may be consistent with secondary instability theory\textsuperscript{7,8} where a $|\mathcal{A}_n| \sim \gamma/(k^4_y)$ saturation model for ETG mode amplitudes predicts a steeper drop-off near $k_y\rho_i \approx 12$ than that of the quasilinear estimate.

Finally, we present a comparison of turbulent and neoclassical transport levels at both ion and electron scales. Because the electron-scale case takes the ion temperature gradient to zero, we can compare the electron-scale thermal diffusivity to that of the ion-scale ITG case with adiabatic electrons to understand the importance of regulation by zonal flows at each scale. In units normalized to the specific species of interest, the thermal diffusion coefficients due to electrostatic turbulence are $\langle \chi_{\text{ES}} \rangle_i = 0.7\rho_i^2 v_{Ts}/L_T$ for the ion-scale ITG case and $\langle \chi_{\text{ES}} \rangle_e = 2.8\rho_i^2 v_{Ts}/L_T$ for the electron-scale ETG case. Therefore, the ETG-driven zonal flows don’t regulate ETG turbulence as strongly as the isomorphic counterpart ITG turbulence is regulated by ITG-driven zonal flows. Here $v_{Ts}$ is the thermal velocity, $\sqrt{2T_i/m_i}$, for a species $s$.

The neoclassical transport values were calculated using GENE for both the ion and electron scale cases. Given in units of $\chi_{\text{ES}}$ from Table I, the neoclassical thermal diffusivites are $\langle \chi_{\text{neo}} \rangle_i = 0.14\chi_{\text{ES}}$ and $\langle \chi_{\text{neo}} \rangle_e = 0.004\chi_{\text{ES}}$. The neoclassical values are in close agreement with the theoretical expectation that $\chi_i = \sqrt{m_i/m_e}\chi_e$, and are negligible compared to the turbulent thermal diffusivites, $\langle \chi_{\text{ES}} \rangle_i = 6.95\chi_{\text{ES}}$ and $\langle \chi_{\text{ES}} \rangle_e = 0.328\chi_{\text{ES}}$. The late-time heat flux spectrum peaks in the intermediate scale at $k_y\rho_i = 10.6$ with $\langle Q_{\text{ES}} \rangle_e = 0.666\chi_{\text{ES}}$ and drops off to $\langle Q_{\text{ES}} \rangle_e = 0.10Q_{\text{ES}}$ and $0.11Q_{\text{ES}}$ for $k_y\rho_i = 4.24$ and 16.96 respectively. These values are in good agreement with the theoretical expectation that $Q/Q_{\text{ES}} \sim O(0.01) - O(0.1)$ for the intermediate-scale ETG modes.\textsuperscript{1}

\section{VI. DISCUSSION}

We have shown, using the single-mode nonlinear simulations, that the NLSE model\textsuperscript{1} accurately describes the zonal flow generation mechanism by intermediate-scale ETG modes and that it provides a theoretical understanding for the slow growth of long-wavelength zonal flows into the long-term quasi-saturated state. As the NLSE model considers only a single ETG mode for a practicable analysis, one cannot say conclusively that the same is true of the full-spectrum nonlinear results. However, in the full-spectrum case the high-$k_y$ ETG modes are quickly saturated by a stronger ZF response as compared to the intermediate-scale ETG modes. The intermediate-scale ETG modes then drive exponential ZF mode growth initially, and slow, algebraic ZF mode growth as they are suppressed in the late stage. This result is in good agreement with the NLSE model for intermediate-scale ETG-ZF dynamics, as well as various long time, saturated gyrokinetic-electron flux-tube simulations of realistic tokamak scenarios.\textsuperscript{15,16,18} The final transport levels for the full-spectrum case are in similar ranges found in thorough electron-scale CBC benchmarks which compare well with experimental observations.\textsuperscript{16} As the zonal flows are driven at long electron-scale wavelengths, multiscale effects could become important and ETG-driven zonal flows may have an effect on ion-scale turbulence. This effect where intermediate-scale zonal flows contribute to ion-scale turbulence suppression has been reported in large multiscale simulations.\textsuperscript{15,16}

\section{Appendix A: Nonlinear Convergence Tests}

This section details a “full-spectrum” nonlinear simulation box-size convergence study. The perpendicular box size is varied in terms of the basic $6\rho_i \times 3\rho_i$ electron-scale box size shown in Fig. 2. The collisionality was set to the reference value discussed in Sec. IV. Four perpendicular domain sizes are presented: $6\rho_i \times 3\rho_i$, $12\rho_i \times 3\rho_i$, $24\rho_i \times 3\rho_i$, and $12\rho_i \times 6\rho_i$. These cases consider the importance of correctly resolving the ETG streamer lengths and the longest wavelength zonal flow modes. Additionally, the inclusion of longer ETG mode wavelengths is considered in the $12\rho_i \times 6\rho_i$ case to verify the findings of Section III.B. The number of radial gridpoints was increased in each simulation to retain the original resolution.

The electron heat flux for each case is shown over time in Fig. 13. One can clearly see that the increase in radial dimension is necessary to correctly resolve the heat flux. The late-stage zonal flows of the $6\rho_i \times 3\rho_i$ case, as shown in Fig. 3(c), are peaking at the longest mode allowable and the box size must be increased to correctly resolve the longest modes. Allowing for longer wavelength ZF modes leads to stronger regulation of the heat flux as seen in Fig. 13.

The time evolution of the ETG modes for the $12\rho_i \times 6\rho_i$ case is shown in Fig. 14(a), and the time evolution of the four strongest ZF modes at the final time is shown in Fig. 14(b). These results are qualitatively
similar to the $24\rho_i \times 3\rho_i$ case shown in Figs. 10(b) and 11. In this new case, the longest wavelength ETG mode, $k_y\rho_i = 1.06$, grows to the highest level. However, it can be seen in Fig. 14(b) that from non-dimensional times 5-10 $t/(R/c_s)$, when the longest ETG mode is dominant, the zonal flows are already in the final, slowly growing stage. This result indicates that the strongest ZF modes are largely being affected by the intermediate-scale ETG mode, not the longest wavelength ETG mode, and confirms the results found in Sec. IIIB which showed little zonal flow generation outside the intermediate-scale range. As increasing $L_y$ from the original $3\rho_i$ size had no effect on the final quasi-saturated state, the largest $L_x$ case considered, $24\rho_i \times 3\rho_i$, was chosen for the full-spectrum investigation discussed in Sec. IV.

* Electronic address: stefan.tirkas@colorado.edu

1 Chen H. et al 2021 Nucl. Fusion 61 066017
2 Batchelor D. A. et al 2007 Plasma Sci. Technol. 9 312
3 ITER Physics Expert Group on Confinement and Transport et al 1999 Nucl. Fusion 39 2175
4 Horton W. 1999 Rev. Mod. Phys. 71 735
5 Doyle E.J. et al 2007 Nucl. Fusion 47 S18
6 Lee Y.C., Dong J.Q., Guzdar P.N., and Liu C.S. 1987 Phys. Fluids 30 1331
7 Jenko F., Dorland W., Kotschenreuther M., and Rogers B.N. 2000 Phys. Plasmas 7 1904
8 Dorland W., Jenko F., Kotschenreuther M., and Rogers B.N. 2000 Phys. Rev. Lett. 85 5579
9 Jenko F. and Dorland W. 2002 Phys. Rev. Lett. 89 225001
10 Nevin W.M., Candy J., and Cowley S. et. al. 2006 Phys. Plasmas 13 122306
11 Ren Y. et al 2017 Nucl. Fusion 57 072002
12 Gerson B.A., Stabler G.M., Solomon W.M. et al 2018 Phys. Plasmas 25 022509
13 Ryter F. et al 2019 Nucl. Fusion 59 096052
14 Kiefer C.K. et al 2021 Nucl. Fusion 61 066035
15 Howard N.T. et al 2016 Phys. Plasmas 23 056109
16 Holland C. et al 2017 Nucl. Fusion 57 066043
17 Parker S.E. et al 2006 AIP Conf. Proc 871 193
18 Colyer G.J. et al 2017 Plasma Phys. Control. Fusion 59 055002
19 Hasegawa A. and Mima K. 1977 Phys. Rev. Lett. 39 205
20 Lin Z., Chen L. and Zonca F. 2005 Phys. Plasmas 12056125
21 Chen L., Zonca F. and Lin Z. 2005 Plasma Phys. Control Fusion 47 B71
22 Kim E.J., Holland C. and Diamond P.H. 2003 Phys. Rev. Lett. 91 075003
23 Dannert T. and Jenko F. 2005 Phys. Plasmas 12 072309
24 Jenko F. et al 2005 Plasma Phys. Control. Fusion 47 B195
Bourdelle C., Garbet X., Imbeaux F. et al 2007 Phys. Plasmas 14 112501
Parker S.E. et al 1994 Phys. Plasmas 1 1461
Diamond P.H. et al 2005 Plasma Phys. Control. Fusion 47 R35
Guttenfelder W. et al 2013 Nucl. Fusion 53 093022
Hatch D.R. et al 2015 Nucl. Fusion 55 063028
Chen H.T. and Chen L. 2018 Physics of Plasmas 25 014502
Jenko F. and The GENE development team, The GENE code https://genecode.org
Greenfield C.M., DeBoo J.C., Osborne T.H., Perkins F.W., Rosenbluth M.N., and Boucher D. 1997 Nucl. Fusion 37 1215–1228
Dimitri A.M., Bateman G., Beer M.A., Cohen B.I., Dorland W., Hammett G.W., Kim C., Kinsey J.E., Kotschenreuther M., Kritz A.H., Lao L.L., Mandrelas J., Nevins W.M., Parker S.E., Redd A.J., Shumaker D.E., Sydora R., and Weiland J. 2000 Phys. Plasmas 7 969–983
Lapillone X., Brunner S., Danner T., Jolliet S., Marinoni A., Villard L., Görler T., Jenko F., and Merz F. 2009 Phys. Plasmas 16 032308
Görler T., Tronko N., Hornsby W.A., Bottino A., Kleiber R., Norscini C., Grandgirard V., Jenko F., and Sonnendrücker E. 2016 Phys. Plasmas 23 072503
Rewoldt G., Lin Z., and Idomura Y. 2007 Comput. Phys. Commun. 177 775–780
Adam J.C., Tang W.M., and Rutherford P.H. 1976 The Physics of Fluids 19 561
Chen H.-T. and Chen L. 2018 Plasma Phys. Controlled Fusion 60 055011
Dunnert T. and Jenko F. 2005 Phys. Plasmas 12 072309
Xiao Y. and Lin Z. 2009 Phys. Rev. Lett. 103 085004
Chen H. and Chen L. 2019 Nucl. Fusion 59 074003
Frieman R.A. and Chen L. 1982 Phys. Fluids 25 502
Kim J.Y. and Horton W. 1991 Phys. Fluids B 3 1167
Beer M.A., Ph.D., Princeton 1995 Gyrofluid Models of Turbulent Transport in Tokamaks
Ricci P., Rogers B.N. and Dorland W. 2010 Phys. Plasmas 17 072103
Strintzi D. and Jenko F. 2007 Physics of Plasmas 14 042305
Chen H. and Chen L. 2022 Phys. Rev. Lett. 128 025003
Fable E. et al 2010 Plasma Phys. Control. Fusion 52 015007
Lapillone X. et al 2011 Plasma Phys. Control. Fusion 53 054011