Influence of magnetic surface anisotropy on spin wave reflection from the edge of ferromagnetic film

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We study propagation of the Gaussian beam of spin waves and its reflection from the edge of thin yttrium-iron-garnet film with in-plane magnetization perpendicular to this edge. We have performed micromagnetic simulations supported by analytical calculations to investigate influence of the surface magnetic anisotropy present at the film edge on the reflection, especially in the context of the Goos-Hänchen effect. We have shown the appearance of a negative lateral shift between reflected and incident spin wave beams’ spots. This shift is particularly sensitive to the surface magnetic anisotropy value and is a result of the Goos-Hänchen shift which is sensitive to the magnitude of the anisotropy and of the bending of spin wave beam. We have demonstrated that the demagnetizing field provide graded increase of the refractive index for spin waves, which is responsible for the bending.

I. INTRODUCTION

In recent years magnetic nanostructures with controlled magnetization dynamics have been considered as candidates for design of new miniaturized devices with enhanced performance and functionality for various applications, e.g. heat transport, energy conversion, magnetic field sensing, information storage and processing.[1–3] Spin waves (SWs), being propagating collective disturbances of the magnetization are also regarded as information carriers, which can be exploited to information processing in devices potentially competitive with standard CMOS systems.[4,5] Thus, understanding of SWs properties in nanostructures is crucial in designing magnonic units and this is the one of the main goal in the research field called magnonics.[6,7] It is expected that magnonic devices allow energy-efficient processing of information which will combine the advantages of photonics (high frequency and wide band) and electronics (miniaturization) in a single unit.[8] One of the basic phenomena connected with wave propagation is the transmission and reflection.[9–11] The reflection of SWs is determined by magnetic properties of the film and boundary conditions at the border of the ferromagnetic material. The reflection of SWs has already been investigated in theoretical and experimental papers,[10,12] where SWs were treated as plane waves. Use of waves beams, instead of plane waves or spherical waves, in many cases, can be much more useful and opens new possibilities due to its coherence and low divergence. The known example of the wave beam is a light beam emitted by laser. Usually, its intensity profiles can be described by Gaussian distribution (beams with such property are called Gaussian beams). However, in magnonics the idea of SW beams is unexplored, with only a few theoretical and experimental studies considering formation of SW beams at low frequencies due to the caustic or nonlinear effects.[13–19]

An interesting phenomenon characteristic for the reflection of beam is a possibility for occurrence of a lateral shift of the beam spot along the interface between the reflected and the incident beams – this phenomenon is called the Goos-Hänchen (GH) effect. The GH effect was observed for electromagnetic waves,[20] acoustic waves,[21] electrons[22] and neutron waves.[23] Also for SWs this topic was investigated theoretically for the reflection of the exchange SWs (i.e., high frequency SWs with neglected dipole-dipole interactions) from the interface between two semi-infinite ferromagnetic films.[24] It was shown that for the observation of the GH shift an interlayer exchange coupling between materials is crucial. Recently, we analyzed the GH shift at reflection of the SW’s beam from the edge of the magnetic metallic (Cobalt and Permalloy) and magnetic dielectric yttrium-iron-garnet (YIG, Y3Fe5O12) films.[25] We showed that the GH effect exists for dipole-exchange spin waves and can be observed experimentally. The magnetic properties at the film edge were shown to be crucial for a shift of the SW’s beam.

In this paper we analyze the SW beam reflected from the edge of the thin ferromagnetic film. We focus our study on the magnetic properties of the film’s edge and its contribution to the shift of the SW beam. We show, that measurements of this shift can provide information about the local values of the surface magnetic anisotropy, and thus also about the local magnetic properties at the edges of the magnetic film. Our attention is concentrated on detailed investigation of the SW reflection from the YIG film, a dielectric magnetic material highly suitable for magnonic applications due to its low SW damping, which
is the smallest among all known magnetic materials. Recent experiments have shown possibility of fabrication of very thin YIG films (with thicknesses down to tens of nm), which can be patterned on nanoscale and in which the SW dynamics can be controlled with metallic cupping layers. The magnetic properties of the film edge influence SW dynamics, their significance increases with decreasing size of device and will play important role in spintronic and magnonic nanoscale devices. However, edge properties at this scale are hardly accessible to experimental techniques. In this paper, we propose a tool for the investigation of the magnetic properties at the edges of thin ferromagnetic film, which exploits shift of SW beams’ spot at the reflection.

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The micromagnetic simulations (MMS) and the analytical model of the GH shift are described in section II. Comparison of the results emerging from the analytical model with MMS, development of the model of SW bending and the discussion of the results are presented in section III. The paper is summarized in section IV.

II. MODEL AND METHODS

A. Model

We consider a thin YIG film with the thickness, \( L_z = 5 \text{ nm} \), much smaller than lateral dimensions of the film \((L_x \ll L_z, L_y)\) as it is shown in figure 1. The film is magnetically saturated by an in-plane static external magnetic field \( \mathbf{H} \) (we assume a value \( \mu_0 \mathbf{H} = 0.7 \mathbf{T} \)) which is applied along the \( y \)-axis, perpendicular to the edge of the film. We study SWs which propagate in the film plane \((x, y)\). The considered edge of the film is along \( x \) axis and located at \( y = 0 \). For description of the SW propagation, it is more convenient to define also the second coordinate system \((x', y')\). As it is shown in Fig. 1 in this coordinate system the wave vector \( \mathbf{k}_i \) of the incident SW is parallel to \( y' \) axis and wave fronts are parallel to \( x' \) axis. Therefore, we can define the angle of incidence \( \theta_i \) as the angle spanned between \( \mathbf{k}_i \) and normal to the edge (\( y \) axis). We limit angle of incidence to the value \( \theta_i = 60^\circ \) in this study. We assume the SW frequency \( f = 35 \mathbf{GHz} \), at this frequency the propagation is almost isotropic in the film plane due to significant contribution of the exchange interactions (as confirmed latter in the paper with calculated isofrequency contours). In calculations we have used magnetic parameters for YIG at low temperatures: magnetization saturation \( M_S = 0.194 \times 10^6 \mathbf{A/m} \) and exchange constant \( A = 4.0 \times 10^{-11} \mathbf{J/m} \). An additional advantage of the YIG film is its relatively small static demagnetizing field, which is proportional to \( M_S \). All these properties of YIG simplify the analysis and helps us to focus mainly on the influence of the surface magnetic anisotropy on the reflection of SWs. The surface magnetic anisotropy can have different origin, besides change of the crystallographic structure at the edge, the applying coating material and roughness can also influence surface anisotropy. Nonetheless, the microscopic mechanism of surface magnetic anisotropy is not the subject of this paper, our main concern is the influence of anisotropy on the reflection of a SW beam.

Figure 1. Schematic plot of the thin YIG film geometry considered in the paper. The film has thickness \( L_z \), which is much smaller than the film’s lateral sizes, \( L_x \) and \( L_y \). The \((x, y, z)\) coordinating system defines the structure with the film edge at \( y = 0 \) (hatched area). The coordinating system \((x', y', z)\) defines the SW beam, with the wave vector parallel to \( y' \) and wave fronts parallel to \( x' \). The area hatched by orange lines and located in the center of coordinating system \((x', y', z)\) corresponds to the excitation area. \( \mathbf{k}_i \) and \( \mathbf{k}_r \) are wavevectors of incident and reflected SW beams, respectively. \( \Delta X \) is a total shift of the SW beam reflected at the edge.

Magnetization dynamics is described by the Landau-Lifshitz (LL) equation of motion for the magnetization...
vector $\mathbf{M}$:
\[
\frac{d\mathbf{M}}{dt} = -\frac{|\gamma|}{1 + \alpha^2} \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\alpha}{M_S (1 + \alpha^2)} |\gamma| \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}),
\]  
(1)

where: $\alpha$ - is damping coefficient, $\gamma$ - the gyromagnetic ratio, $\mathbf{H}_{\text{eff}}$ - effective magnetic field. The first term in LL equation describes precessional motion of the magnetization around the effective magnetic field and second term enrich that precession by damping. The effective magnetic field in general can consist of many terms, in this paper we will consider only the most important components: external magnetic field $\mathbf{H}$, nonuniform exchange field $\mathbf{H}_{\text{ex}}$ and long-range dipole field $\mathbf{H}_{\text{d}}$:

$\mathbf{H}_{\text{eff}} = \mathbf{H} + \mathbf{H}_{\text{ex}} + \mathbf{H}_{\text{d}}$.

B. Analytical model of the GH shift

In our analytical study we limit the discussion to SWs with large wavevectors, where the contribution from dynamic dipole interactions is small and SW dynamics is determined by exchange interactions. In Eq. (1) we will also make a linear approximation, which allows us to decompose the magnetization vector into a static part equal to the magnetization saturation and a dynamical part laying on the plane perpendicular to the direction of $M_S$: $\mathbf{M} = M_S \mathbf{z} + \mathbf{m}(x, y, t)$.

This approximation is valid when the dynamical part of the magnetization, $|m|$ is much smaller than the saturation magnetization $M_S$. With this approximation we can assume harmonic time dependency for $m \propto e^{i\omega t}$, where $\omega$ is angular frequency of SW. SWs’ damping is neglected here. To study the incidence and reflection of SWs from the edge of the thin film, in our study we consider SWs propagating in thin film with neglected dynamic dipole interactions:

\[
\omega^2 = (\omega_H + l_{\text{ex}}^2 \omega_M k^2) (\omega_H + l_{\text{ex}}^2 \omega_M k^2 + \omega_M \sin^2 \theta_i),
\]  
(2)

where: $\mu_0$ is permeability of vacuum, $\omega_H = |\gamma| \mu_0 H$, $\omega_M = \gamma \mu_0 M_S$, and the exchange length $l_{\text{ex}} = \sqrt{2A/\mu_0 M_S^2}$, $k$ is a wavenumber. For further calculations we need formula for $k$ as a function of the $\omega$. This dependence takes the following form:

\[
k^2 = \frac{\mu_0 M_S}{4A} \left( -2H - M_S \sin^2 \theta_i + \sqrt{\frac{4\omega_H^2}{\mu_0^2 \gamma^2} + 4M_S^2 \sin^2 \theta_i} \right).
\]  
(3)

The calculation of the reflection coefficient requires the introduction of the boundary condition at the film edge (at $y = 0$) for $\mathbf{m}$. We assume that the magnetization vector $\mathbf{m}$ of the SW has to fulfill the following boundary condition of the Rado-Wheertman type:

\[
[\partial \mathbf{m}(y)/\partial y + d \mathbf{m}(y)]_{y=0} = 0
\]  
(4)

where $d$ is effective pinning parameter. This pinning parameter can take into account the dipole contribution in the finite width stripe ($L_y \ll 1$) and also a contribution from the surface magnetocrystalline anisotropy at the edge of the film. With these contributions it can be expressed (in SI system) as:

\[
d = \frac{1}{2} - \frac{2K_S}{\mu_0 M_S L_z} \left[ L_z (1 + 2 \ln (L_y/L_z)) \right].
\]  
(5)

where $K_S$ is surface anisotropy constant, which is related with the following magnetic anisotropy energy density:

$\epsilon_{\text{anis}} = -\frac{1}{2} \mu_0 H_{\text{anis}}(K_S) \cdot \mathbf{m}$, where anisotropy field is $H_{\text{anis}}(K_S) = (2K_S)/\mu_0 M_S L_z$.

Due to translational symmetry along the interface, the wave vector component along the interface should be conserved. As a consequence, the angle of incidence is equal to the angle of reflection, $\theta_i = \theta_r$. Therefore, based on the Eqs. (3) and (4) we can derive the equation for the reflection coefficient $R$:

\[
R = \frac{i \sqrt{k_x^2 - k_y^2} + d}{i \sqrt{k_x^2 - k_y^2} - d},
\]  
(6)

where $k_x$ is tangential to the interface component of the wavevector.

When the incident SW is represented as a wave packet of a Gaussian shape, with a characteristic length $\Delta \Delta k_x' \ll k_x'$, then according to the stationary phase method [24] the reflected beam will show a space shift relatively to the incident wave packet of length:

\[
\Delta X_{\text{GH}} = \frac{\partial \psi}{\partial k_x'},
\]  
(7)

where $\psi = \arctan (\Im(R)/\Re(R))$ is the phase difference between the reflected and incident waves, $\Im(R)$ and $\Re(R)$ are the imaginary and real parts of the reflection coefficient calculated from Eq. (6). Thus, the GH shift $\Delta X_{\text{GH}}$ can be expressed by the following equation:

\[
\Delta X_{\text{GH}} = -\frac{2 \tan \theta_i}{d^2 + (k \cos \theta_i)^2}.
\]  
(8)

C. Micromagnetic simulations

Micromagnetic simulations (MMS) have been proved to be an efficient tool for the calculation of SWs dynamics in various geometries [14][17]. We have used interface with GPU-accelerated MMS program MuMax3 [48] which uses finite difference method to solve time-dependent LL Eq. (1).

In our MMS we consider SWs propagation in thin-films and reflection from the film edge. Simulations were performed for the system shown in figure 1 of size $4000 \times 12000 \times 5$ nm ($L_z \times L_y \times L_x$), which was discretized with cuboid elements of dimensions $2.5 \times 2.5 \times 5$ nm ($L_x \times L_y \times L_z$) are much less than 13 nm, i.e., the exchange length of YIG). The surface magnetic anisotropy was introduced in MMS by uniaxial magnetic anisotropy value.
in the single row of discretized cuboids at the film edge according with $K_u = K_S/l_c^2$ where $l_c$ is a size of cuboid (here $l_c = 2.5$ nm).

Simulations consist of two parts according to the algorithm presented in Fig. 2. First we obtain the equilibrium static magnetic configuration of simulated system. In this part of simulations we start from random magnetic configuration in presence of high damping ($\alpha = 0.5$). Then, the results of the first stage are used in the dynamic part of simulations during which a SW beam is continuously generated and propagates through the film. The SWs are excited in the form of a Gaussian beam. After sufficiently long time, when incident and reflected beam are clearly visible and not changing qualitatively in time the data necessary for further analysis (‘POSTPROCESSING’) is stored.

![Figure 2](image)

Figure 2. Algorithm of the spin wave dynamics MMS. MMS consists of two steps, in the first step the system is stabilizing - the equilibrium magnetic configuration is obtained. In the second step using the stabilized magnetic configuration SWs are generated by applying small rf magnetic field. The data stored during MMS are processed during stage called ‘POSTPROCESSING’ - the final results of the SW dynamics are extracted.

To generate Gaussian beam of SWs we introduce a narrow rectangular area (excitation area, marked in Fig. 1 with orange dashed lines) with a long side parallel to the expected wave fronts (along $x'$ axis). Within the excitation area we introduce a radio-frequency (rf) magnetic field oscillating at frequency of 35 GHz, $h_{\text{dyn}}(x', t) \propto h_0(x') \exp(\omega t)$. The field $h_{\text{syn}}$ is perpendicular to the static magnetic field and its amplitude changes along the $x'$-axis according with the Gauss distribution $h_0(x') = h \exp\left[2 \left(x' - x_0'\right)^2 / \langle\sigma\rangle^2\right]$, where we assume $h = 0.02H$ being maximum amplitude of the rf magnetic field which needs to be small to stay in linear regime. $l$ is the length of the excitation area (in our simulations $l = 1.5$ pm) and $\sigma^2$ ($\sigma = 0.2$) is a parameter which can be treated as variance of the Gauss distribution centered around $x_0'$. $h_0$ is uniform along $y'$-axis within the excitation area.

Example result of MMS is shown in Fig. 3.

Determination of precise value of the shift of SW beam $\Delta X$ from MMS results requires the following three-stage procedure. In the first stage we extract a series of SWs intensity profiles using an array of screen detectors parallel to the $y'$-axis for different locations ($x_j$) along the film edge but far enough from the reflection point, i.e. out of the interference pattern of SW near the reflection point. At every point $x_j$ the intensity was calculated using equation: $I_{x_j}(y) = \int_0^4 \int m_z(x_j, y, t) \, dt$, where $T = 1/f$ and $m_z$ is the component of the magnetization vector perpendicular to the film plane. In the next stage, using Gaussian fitting, we have extracted positions of centers of the intensity profiles for every $y_{ij}(x_j)$. Having series of peak positions and its locations along $y$ (red and blue full dots in Fig. 3 for the incident and reflected beams, respectively) we can extract rays of the incident and reflected SW beams (red and blue solid line in Fig. 3, respectively). Finally the value of the shift $\Delta X$ can be easily calculated with small errors up to several nanometers.

![Figure 3](image)

III. RESULTS AND DISCUSSION

Result of the pure GH shift in dependence on the pinning parameter obtained from the analytical model [Eq. (3)] is presented in Fig. 4(a). This dependence $\Delta X_{\text{GH}}(d)$ is an antisymmetrical function with respect to $d = 0$ and has maximum and minimum value for $d < 0$ and $d > 0$, respectively. However, the effective pinning parameter $d$ has contributions from dipole interaction and magnetic
surface anisotropy. To study influence of the magnetic anisotropy we show also $\Delta X$ in dependence on $K_S$ in Fig. 5(b) with solid line. GH shift exists for $K_S = 0$ due to effective pinning coming from dipole interactions and is $\Delta X_{GH} = -9$ nm. $\Delta X_{GH}$ takes maximum absolute values for $K_S = 0.465$ mJ/m$^2$ and $K_S = -0.315$ mJ/m$^2$. $\Delta X_{GH} = 0$ for $K_S = 0.075$ mJ/m$^2$, this is when magnetic surface anisotropy compensate the effect of the dipole interactions at the film edge. For large negative and positive values of $K_S$ the GH shift tends monotonously to zero. This shows that the measure of the GH shift can be used to indicate the surface magnetic anisotropy at the thin film edge locally, especially in the range of its sudden change, i.e., between $-0.315$ and $0.465$ mJ/m$^2$. To test this possibility we perform MMS according with the procedure described in section II C

Dependence of the SW beam shift on the surface anisotropy constant obtained from MMS for $\mu_0 H = 0.7$ T in thin YIG film is presented in figure 5 with green solid dots. The value of the shift for $K_S = 0$ obtained from MMS is $\Delta X = -32.4$ nm and this is significantly larger than the GH shift obtained from analytical solutions ($\Delta X_{GH} = -9$ nm). The maximal value of $\Delta X = 13.83$ nm is found for $K_S = 0.5$ mJ/m$^2$ and minimal is $\Delta X = -45.83$ nm for $K_S = -0.2$ mJ/m$^2$, i.e., out of the scale presented in figure 5. This dependence is quantitatively similar to the function obtained in the analytical model (Fig. 4). Nevertheless, there are distinct differences between both results.

The analytical model is based on number of assumptions which are absent in MMS, thus there is disagreement between both results. The full saturation of the magnetization is important assumption made in the analytical model. Surface magnetic anisotropy at the edge of the film increases (or decreases) a value of a total internal magnetic field at this edge. This change of the internal magnetic field in the single computational cell at the edge is equal to the surface anisotropy field $\mu_0 H_S = 2K_S/|M_S|$.

Therefore, exactly at the film edge the demagnetizing field can be compensated or enhanced by surface anisotropy field, in dependence on the sign of $K_S$. When surface anisotropy constant $K_S = K_0 = -2\mu_0 L_z M_S (H - M_S) \approx -0.22$ mJ/m$^2$ the total internal field at the surface is equal $0$ (the demagnetizing field at the edge is compensated by the anisotropy field). Hence, for $K_S < K_0$ the equilibrium orientation of the magnetization at the edge of the film rotates from the saturation direction (this is transformation from the easy axis to the easy plane configuration). This is demonstrated in Fig. 4(b), where magnetization components along x, y and z axis are presented as a function of the distance from the film edge for $K_S = -2.0$ mJ/m$^2$. We can see the rotation of the magnetization from the x direction (saturation direction) towards the y axis. The change of the magnetization configuration is not taken into account in the model developed in Sec. III B. Moreover, in real sample there is a possibility for appearing domain walls along the film edge, which can create additional factor for complexity of the problem. Therefore, in this paper we limit the analysis to the fully saturated sample, i.e., when $K_S \geq K_0$ (we note, that the exact value of $K_0$ depends on the magnitude of the external magnetic field).

The homogeneity of the internal magnetic field assumed in the analytical model is probably the next most important factor, which makes a comparison of the analytical and MMS results difficult for the saturate state. In MMS, and in a real sample, the magnetization which is perpendicular to the film edge creates an inhomogeneous static demagnetizing field, which is directed opposite to the magnetization saturation. In result, the internal magnetic field decreases monotonically when moving from the film center towards its edge [solid-blue line in the SW beam from edge of the thin YIG film in dependence on $d$, (a) pinning parameter $d$ and (b) magnetic surface anisotropy constant $K_S$.

![Figure 4](image_url)

**Figure 4.** Analytical results of the GH shift in the reflection of the SW beam from edge of the thin YIG film in dependence on (a) pinning parameter $d$ and (b) magnetic surface anisotropy constant $K_S$. When surface anisotropy constant $K_S = K_0 = -2\mu_0 L_z M_S (H - M_S) \approx -0.22$ mJ/m$^2$ the total internal field at the surface is equal $0$ (the demagnetizing field at the edge is compensated by the anisotropy field). Hence, for $K_S < K_0$ the equilibrium orientation of the magnetization at the edge of the film rotates from the saturation direction (this is transformation from the easy axis to the easy plane configuration). This is demonstrated in Fig. 4(b), where magnetization components along x, y and z axis are presented as a function of the distance from the film edge for $K_S = -2.0$ mJ/m$^2$. We can see the rotation of the magnetization from the x direction (saturation direction) towards the y axis. The change of the magnetization configuration is not taken into account in the model developed in Sec. III B. Moreover, in real sample there is a possibility for appearing domain walls along the film edge, which can create additional factor for complexity of the problem. Therefore, in this paper we limit the analysis to the fully saturated sample, i.e., when $K_S \geq K_0$ (we note, that the exact value of $K_0$ depends on the magnitude of the external magnetic field).
Fig. 6. (a) Effective magnetic field along the y-axis inside YIG film for $K_N = 0$ is shown with blue solid line, orange dashed line corresponds to the external magnetic field (0.7 T). (b) Static magnetic configuration in the vicinity of thin YIG film edge for $K_N = -2$ mJ/m$^2$ (i.e., $K_N < K_0$); blue solid, orange dashed and green dash-dotted line marks $x$, $y$ and $z$ component of the magnetization vector normalized to unity, respectively.

In our interpretation this inhomogeneity is responsible for a shift of $\Delta X$ in MMS towards negative values as compared to the results of Eq. (8) [see, Fig. 6]: for very high positive $K_N$ the $\Delta X_{GH}$ monotonously tends to 0 in the analytical model [Fig. 4(b)], while results of the MMS show that the value of $\Delta X$ reaches non-zero negative value even for very high value of $K_N$ ($\Delta X = -15.2$ nm for $K_N = 10$ mJ/m$^2$). This means, that the inhomogeneity of the internal magnetic field in the close vicinity of the thin film edge causes an increase of the refractive index for SWs [12, 48] and consequently results in bending of the SW beam and changes the shift measured in far field.

Here, we propose the simple analytical model of the wave bending, which allows to estimate factor which correct the value of the GH SW beam’s shift obtained from Eq. (8). This bending is due to variation of the refractive index, which in our geometry is caused by demagnetizing field. We will consider gradual change of the refractive index for SWs in the vicinity of the film edge.

Similarly as in optics, the refraction law for SWs (i.e., Snell low) can be concluded from analysis of the isofrequency contours and momentum conservation of the wavevector component parallel to the edge ($k_x = $ const.). Let’s assume, that full distance of the gradual change of the refractive index in thin film can be divided into $N$ thin slices, numbered with integer $n$. Therefore, we can assume multiple refractions on $(N-1)$ parallel planes, separating neighbor slices [Fig. 7(b)]. At the plane between two arbitrary slices $n$ and $n + 1$ according to Snell low ($k_n \sin \theta_{i,n} = k_{n+1} \sin \theta_{i,n+1}$) [Fig. 7(a)]. Thus, we can calculate final angle of incidence after passing $N$ slices: $\sin \theta_{i,N} = \frac{k_{0N}}{k_N} \sin \theta_{i,0}$, where $k_{0N}$ and $\theta_{i,N}$ are wavenumber and incident angle in the last slice. $k_0 \equiv k(y_0)$ and $\theta_{i,0}$ are wavenumber and incident angle in the initial media, i.e., in the interior far from the edge of the film. Rewriting the expression for space coordinates (substitute $k(y) \equiv k_N$ and $\theta_i(y) \equiv \theta_{i,N}$) we obtain:

$$\theta_i(y) = \arcsin \left( \frac{k_0}{k(y)} \sin \theta_{i,0} \right).$$

From here, the final value of $\theta_i(y)$ at the $y$ distance from the film edge can be calculated, if the value of $k(y)$, initial values of the wavevector $k_0$ and the initial angle of the incidence $\theta_{i,0}$ are known.

The knowledge of the incident angle at the vicinity of the film edge $\theta_i(y)$ allows us to derive formula which describes propagation of the beam through the area of gradually changed refractive index. Similarly to previous approach, let’s consider single refraction on the interface between $n$ and $n + 1$, and between $n + 1$ and $n + 2$ slice. The refractive index increases with increasing index $n$. The beam shift $\Delta X_{\text{bending}}$ resulting from the bending is shown schematically.

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Assuming infinitesimally small distance between interfaces $\Delta y \to dy$ we can transform summation into integration along the $y$ axis between initial point of the SW generation $y_0$ and the point of the reflection $y_N$:

$$x(y_0, y_N) = \lim_{\Delta y \to 0} x_n = \int_{y_0}^{y_N} dy \tan \arcsin \left( \frac{k_0}{k_n} \sin \theta_0 \right) = \int_{y_0}^{y_N} dy \frac{\sin \theta_0}{\sqrt{\left( \frac{k(y)}{k_0} \right)^2 - \sin^2 \theta_0}}.$$
Therefore, if the edge where reflection takes place is located at $y_N = 0$ (like in Fig. 1), the SW beam shift introduced by bending and measured in far field (observation point is assumed at the same distance from the edge as the source of the SW) is described by following formula:

$$\Delta X_{\text{bending}} = 2 \left[ y_0 \tan \theta_1 - x(y_0, 0) \right],$$

(12)

where factor 2 is included to take into account the beam way from source to the edge and after reflection to the observation point.

The total shift of the SW beam observed in far field is a sum of the GH shift and the shift resulting from the bending:

$$\Delta X = \Delta X_{\text{bending}} + \Delta X_{\text{GH}}.$$  

(13)

Eq. (13) is a general formula describing a total shift of the wave beam propagating in media with gradual change of the refractive index. Way of the beam ray depends on relation describing $k(y)$. And this is main bottleneck in this approach: unknown formula for wavevector of the SW in dependence on $y$ in an area of the inhomogeneous effective magnetic field. However, knowledge of the dispersion relation in homogeneous film, Eq. (4), can help in qualitative modeling of the total shift of the SW beam also in a part of the film with gradual change of the refractive index.

Demagnetization field decreases a value of the internal magnetic field, its dependence on distance from the edge can be expressed by the relation: $H_d(y) = 4M_S \arctan \left[ L_z/(2y) \right].$ This field far from the edge of the film tends to zero [Fig. 5(a)]. We can substitute bias magnetic field in Eq. (3) with the effective field $H \rightarrow H(y) \equiv H - H_d(y)$. This approach should be valid for slow change of $H_d(y)$ with distance. However, in the vicinity of the thin film edge we observe a rapid change of the demagnetizing field [Fig. 6(a)]. Therefore, we propose to introduce homotopic transformation of the demagnetizing field: $H(y) = H - [cH_d(y_0) + (1 - c) H_d(y)]$ with parameter $c \in [0, 1]$, which will reduce influence of the rapid changes of the demagnetizing field on wave vector. Then, the wavevector magnitude can be described by the following equation:

$$k^2(y, \theta_1) = \frac{\mu_0 M_S}{4A} \left( -2 \left( H - [cH_d(y_0) + (1 - c) H_d(y)] \right) \right) - M_S \sin^2 \theta_1 + \sqrt{\frac{4\omega^2}{\mu_0^2} - M_S^2 \sin^4 \theta_1},$$

(14)

where $y_0$ can be interpreted as position of the source of the SW beam, i.e., the position where the demagnetizing field magnitude is close to zero.

Now, the only issue is a correct choice of the parameter $c$ in Eq. (14) to find $k(y)$, and to fit $\Delta X(K_S)$ dependence to the curve obtained from MMS. Taking $c = 1$ we assume, that isofrequency contours are the same in whole sample ($k$ does not depend on $y$), the wave propagate in homogeneous internal magnetic field equal $H$ [because $H_d(y_0) \equiv 0$] and Eq. (14) reduces to Eq. (3). In this case $\Delta X_{\text{bending}} = 0$ and $\Delta X = \Delta X_{\text{GH}}$, i.e, the total shift is equal to Eq. (8). The obtained curve is plotted in Fig. 5 with blue dashed line. In opposite limit, $c = 0$ the dependence $k(y)$ follows exactly the change of $H_d(y)$. In this case the $\Delta X_{\text{total}}(K_S)$ calculated from Eq. (13) is shown in Fig. 5 with black dotted line. It takes value $\Delta X = \Delta X_{\text{bending}} = -79$ nm for large $K_S$, much below the value obtained from MMS. This discrepancy exists, because for each $y$ we took in calculation of the $\Delta X_{\text{bending}}$ [in the integral Eq. (11)] the dispersion relation Eq. (14) which is for the film with homogenous magnetic field. In real situation, the dispersion relation in the area of inhomogeneous refractive index will be different from the local value. Thus averaging of the demagnetizing field across some distance shall improve the estimation. Moreover, the $c$ shall depend on the relative value of the wavelength to the special changes of the refractive index. Thus, the value of $c$ from Eq. (14) needs to be treated as a parameter, which includes an effective influence of the inhomogeneity of the refractive index on the dispersion relation of SWs. For $c = 0.83$ we have obtained very good agreement between results of MMS and analytical model $\Delta X(K_S) \approx \Delta X(K_S)$ for $K_S > -0.22$ mJ/m$^2$, as it is shown in Fig. 6 with orange solid line.

The value of $\Delta X$ is very sensitive for small change of $K_S$ between extremes. The magnetic surface anisotropy in ferromagnetic films can take different values, however the most interesting is the range around 0, where the transition from uniaxial into easy-plane anisotropy takes place. Thus, the measure of the SW beam’s shift can indicate the local surface magnetic anisotropy at the film edge with spatial resolution limited by the size of the width of the SW beam. This information shall be important also for understanding and exploiting SW excitations and actuation at the surface of YIG film being in contact with Pt, where the magnetic surface anisotropy was shown to play significant role in spin pumping.\cite{34, 35, 50, 51} Further investigation is required to test an influence of the second ferromagnetic material attached to the YIG film edge on the SW reflection. Here, extension of the analytical model with properly defined boundary conditions will be required.\cite{52, 53}

IV. CONCLUSIONS

We have performed analytical and numerical study of the SWs beams shift at the reflection from the edge of the YIG thin film in dependence on the surface magnetic anisotropy present at the film’s edge. The GH effect and SW bending are shown to contribute to the SW beam’s shift measured in far field. We have shown that the GH shift is modulated in a broad range by changes of the surface magnetic anisotropy constant between two extremes:
For large positive values of shift is independent on the surface magnetic anisotropy. However, this propagating towards (and outwards) of the edge and introduce additional shift of the SW beam. For large positive values of \( K_S \) the value of \( \Delta X \) is almost insensitive to changes of the magnitude of the magnetic surface anisotropy and is mainly a result of the SW bending due to gradually increased refractive index at the film edge. These results shall be of importance for magnonic, its applications for sensing and also for developing new direction of research devoted to metamaterial properties for SWs, especially graded index magnonics.

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