Laminate fibre wave defect – KDF evaluation

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Abstract. The positive side of using composite materials in Aerospace industry is the weight reduction. The drawback are the defects created on solid laminates, during manufacturing. Structure life and strength reduction, due to the defects, need to be accurately evaluated against the A/C airworthiness requirements.

1. Introduction
The approach, suggested in this paper, consists on using the Conformal Mapping for laminate defects topology modeling. Scope of this approach is to evaluate a KDF for Margin of Safety calculation as a ratio between defect configurations against the intact one. For most of defect scenario, the Conformal Mappings are available in literature. For each type of defect, it is possible to define fiber pattern, angle, derivative, thickness and length.

This paper reports only a numerical methodology for stress-strain calculation of wavy ply configuration. Fiber and matrix strength check (not included on this paper) is performed in accordance with the applicable criteria and allowable for each type of ply. The KDF for each defect topology is then calculated as follows:

\[ KDF = \frac{RF_{\text{intact-CTF}}}{RF_{\text{wavy-CTF}}}_{\text{minimum}} \]

For clarity, the analysis of intra-lamina shear and out-of-plane matrix stress due to wavy ply curvature are not reported on this paper.
2. System approach

2.1. Defect Configuration topology definition

The defect analysed is an embedded resin accumulation shown on Figure 2. The wavy ply profile is defined via complex variable function \( F(z) \) (see Ref.[2]).

\[ F(z) = A \left( \frac{1}{z} \right) \]

\[ \Psi(X,Y) = \text{Im} \left\{ \frac{X + iY}{X + iY} \right\} = Y^3 - cY^2 - Y \left( X^2 - 1 \right) - cX^2 = 0.0 \]

\[ \frac{dY}{dX} = -\frac{2XY - 2cX}{3Y^2 - 2cY + X^2 - 1} \]

\( c \): parameters is the variable related to ply thickness.

Note that “streamline” \( \Psi(X,Y) \) doesn’t have physical meaning rather than representing the profile of the ply, consistently with the defect type:

1. \( A=1.0 \)
2. Analytical formulation of wavy ply
3. Derivative values in each point of the plies
4. Ply thickness variation
5. Ply curvature
6. Ply length

2.2. Algorithm basic assumption

Defect analysed on this paper is shown on below Figure 3. Two loading condition are considered separately:
1. Tension load $F_0$
2. Shear load $S_0$

![Figure 3. Intact and wavy configuration](image)

The static analysis of wavy configuration, is basically a modification of the ply stress calculated via Classical Laminate Theory (Ref. [1]) for intact configuration.

The following criteria apply (see Figure 4).

1. Applied load $F_0$ is constant in each section along X axis: $F(x) = F_0$
2. Section(X) remain plane.
3. Wavy ply mechanical properties, are re-calculated in each section (X) as per fibre-resin mixture criteria due to thickness variation.
4. Ply stress in each section (X) depends on two geometric parameters:
   a. Derivative $Y'$: defines the fibre orientation and the component along X axis.
   b. Stiffness ratio $\lambda$: Wavy longer ply ($L_w$) have a lower tension stiffness than intact ply ($L_0$) and consequently “attract” less load. Fibre stress calculated at point a), is therefore scaled down by a factor $\lambda$. (subscript 0, refer to CLT).

\[
\text{stiffness ratio } \lambda = \frac{E_{W} t_{W}}{E_{0} t_{0}} = \frac{L_{0}}{L_{w}}
\]

\[
\text{Ratio } \lambda = \frac{E_{W} t_{W}}{E_{0} t_{0}} = \frac{L_{0}}{L_{w}}
\]
Figure 4. Plies stress-strain calculation algorithm

\[
\begin{align*}
\theta &= \tan^{-1} \gamma \\
\alpha &= \tan^{-1} \frac{\gamma \Delta}{\Delta + \delta} \\
\beta &= \theta - \alpha \\
OA &= \frac{\Delta + \delta}{\cos \alpha} \\
OB &= \frac{\Delta}{\cos \theta}
\end{align*}
\]

\[
\varepsilon_{ij} = \frac{OA - OB}{OB} = \frac{OA}{OB} - 1 = \frac{\cos \theta_{ij}}{\cos \alpha_{ij}} \left( \frac{\delta_j}{\Delta_j} \right) - 1
\]

\[
\sigma_{ij} = \varepsilon_{ij} \cdot E_{ij} \quad \text{(see } E_{ij} \text{ mixture criteria)}
\]

\[
f_{ij} = \sigma_{ij} \cdot t_{ij}
\]

\[
f_{ij-x} = f_{ij} \cdot \cos \alpha_{ij} = \left[ \cos \theta_{ij} \left( 1 + \frac{\delta_j}{\Delta_j} \right) - \cos \alpha_{ij} \right] \cdot E_{ij} \cdot t_{ij} \quad \text{(X component)}
\]

\[
mixture \quad criteria \quad E_{ij} = \left( E_{xxw} \right)_{ij}
\]

\[
\text{intact ply } E_{xx} = E_{fo} \cdot V_{fo} + E_{m} \cdot (1 - V_{fo}) \Rightarrow E_{fo} = \frac{E_{xx} - E_{m} \cdot (1 - V_{fo})}{V_{fo}}
\]

\[
\text{wavy ply } E_{xxw} = E_{fo} \cdot V_{fw} + E_{m} \cdot (1 - V_{fw}) \quad \text{where } V_{fw} = V_{fo} \cdot \left( \frac{t_0}{t_w} \right)
\]

\[
F_j = \sum_{i=1}^{N} f_{ij-x} = \sum_{i=1}^{N} \left[ \cos \theta_{ij} \left( 1 + \frac{\delta_j}{\Delta_j} \right) - \cos \alpha_{ij} \right] \cdot E_{ij} \cdot t_{ij}
\]

\[
\Delta_j : \text{constant} \quad \delta_j : \text{variable } (= \Delta \text{ elongation)}
\]

\[
balance \quad F_j = F_0 \quad \text{goalseek on } \delta_j
\]

\[
\text{Stiffness : } \delta_{tot} = \sum_{j=1}^{m} \delta_j \quad K_{wave} = \frac{F_0}{\delta_{tot}}
\]
2.3. Algorithm: step-by-step definition

In this example, the laminate length is subdivided in 31 section.

Laminate configuration:
- Ply ID $i=1$ to 11
- Section ID $j=1$ to 31
- Ply nominal thickness : $0.33$ mm
- Length $L_0$: 6 mm
- Laminate thickness: $11 \times 0.33 = 3.63$ mm

Stress-strain on each point $P_{ij}$ are calculated according to below step-by-step procedure.

1. Strain $\varepsilon_{ij}$ calculation at point $P_{ij}$
2. Stress calculation $\sigma_{ij} = f_{ij} \times E(t)_{ij}$ (E: fibre-matrix mixture criteria)
3. Ply force $f_{ij} = \sigma_{ij} \times t_{ij} \times 1$
4. Resultant force at each section $j$ :
   \[ F_j = \frac{1}{i} \sum_{i=1}^{n} f(\delta)_i \]
5. Balance between applied section force $F_0$ and $F_j$ ($F_j = F_0$) is obtained via trial-and-error process on variable $\delta_j$.

One single section can be analysed independently from the others, once the following data are available:

1. CLT stress for intact configuration
2. Derivative $Y'(i)$, $\lambda(i)$, $t(i)$, curvature $R(i)$ from conformal mapping.
3. Example of Numerical Calculation

3.1. Ply mechanical properties

Figure below reports the material properties of 11 plies stack up.

![Figure 6. Ply mechanical properties](image-url)

| Ply mechanical properties vs angle $\Theta$ |
|--------------------------------------------|
| $\sigma_x$ = $E_{11} E_{12} E_{13}$ $\varepsilon_x$ |
| $\sigma_y$ = $E_{21} E_{22} E_{23}$ $\varepsilon_y$ |
| $\tau_{xy}$ = $E_{31} E_{32} E_{33}$ $\gamma_{xy}$ |

Fabric

| $E_{11}$ | 66000 [MPa] |
| $E_{22}$ | 66000 [MPa] |
| $G_{12}$ | 5800 [MPa] |
| $\nu_{12}$ | 0.0409 |
| $t$ | 0.33 [mm] |

UNI

| $E_{11}$ | 135000 [MPa] |
| $E_{22}$ | 8550 [MPa] |
| $G_{12}$ | 4200 [MPa] |
| $\nu_{12}$ | 0.35 |
| $t$ | 0.33 [mm] |

$E_{0^\circ}$ =

\[
\begin{bmatrix}
66111 & 2704 & 0 \\
2704 & 66111 & 0 \\
0 & 0 & 5800
\end{bmatrix}
\]

$E_{+45^\circ}$ =

\[
\begin{bmatrix}
40207 & 28607 & 0 \\
28607 & 40207 & 0 \\
0 & 0 & 31703
\end{bmatrix}
\]

$E_{90^\circ}$ =

\[
\begin{bmatrix}
66111 & 2704 & 0 \\
2704 & 66111 & 0 \\
0 & 0 & 5800
\end{bmatrix}
\]

$E_{0^\circ}$ =

\[
\begin{bmatrix}
136056 & 3016 & 0 \\
3016 & 8617 & 0 \\
0 & 0 & 4200
\end{bmatrix}
\]

$E_{+45^\circ}$ =

\[
\begin{bmatrix}
41876 & 33476 & 31860 \\
33476 & 41876 & 31860 \\
31860 & 31860 & 34660
\end{bmatrix}
\]

$E_{90^\circ}$ =

\[
\begin{bmatrix}
8617 & 3016 & 0 \\
3016 & 136056 & 0 \\
0 & 0 & 4200
\end{bmatrix}
\]
3.2. Section 31 analysis

Figure 7 shows the values calculated according to the methodology reported on Figure 4, relevant to section j = 1 and j = 31. The reason of this section analysis, is to cross check the algorithm against the CLT results on segment where plies axis are in X direction (Y' = 0.0) as per CLT.

1. Algorithm is set up on columns under “WAVY CONFIGURATION” label
2. Stack up : UNI [90, 0, 0, 90, 0, 90, 90, 0, 90, 90, 0, 90]
3. Ply properties UNI : see Figure 6
4. Applied force F₀ = 1000. [N/mm]
5. “Goal Seek “plug-in, to converge F_j = F₀:
   “Set cell” F_j ...... “To value” F₀.... “By changing cell” δ.
6. Note that CLT and Algorithm, yield the same stress and force f_ij(=σ*t), as expected since for both models the derivative is the same (Y’ = 0.0) and length factor ratio in this model is negligible(λ=1.).
3.3. Analysis of section from 16 to 31

Because of symmetry, the results are reported, for clarity, only for section from 16 to 31. The calculation is performed per following parameters:

1. Segment-to-segment calculation is based on constant step \( \Delta = 0.2 \) mm.
2. Stress calculation is performed at each section based on Excel algorithm on Figure 7
3. \( \delta_j \) is recorded for each section at \( F_j = F_0 \) (from Goal Seek).
4. Wavy Laminate total elongation:

\[
\delta_{t_{\text{tot}}} = \sum_{j=1}^{31} \delta_j = 0.0297
\]

5. Wavy laminate stiffness:

\[
K_w = \frac{F_0}{\delta_{t_{\text{tot}}}} \times \frac{0.01}{0.0297} = 3370
\]

6. CLT laminate stiffness (Figure 7):

\[
K_s = \frac{F_0}{\delta_{t_{\text{tot}}}} \times \frac{0.0001}{6 \times 0.00015} = 40188
\]

| PLY | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|
| 1   | 35 | 36 | 36 | 36 | 36 | 36 |
| 2   | 553 | 565 | 565 | 565 | 565 | 565 |
| 3   | 565 | 565 | 565 | 565 | 565 | 565 |
| 4   | 35 | 36 | 36 | 36 | 36 | 36 |
| 5   | 53 | 53 | 53 | 53 | 53 | 53 |
| 6   | 563 | 573 | 573 | 573 | 573 | 573 |
| 7   | 35 | 35 | 35 | 35 | 35 | 35 |
| 8   | 35 | 35 | 35 | 35 | 35 | 35 |
| 9   | 563 | 563 | 563 | 563 | 563 | 563 |
| 10  | 563 | 563 | 563 | 563 | 563 | 563 |
| 11  | 35 | 35 | 35 | 35 | 35 | 35 |

**Figure 8. Wavy laminate – stress map-out**
3.4. 3DFEM analysis

The symmetric 3D FEM is set up with defect scenario “similar” to the one analysed on this paper. Both models show the same “trend” for fibre stress profile, in spite of substantial difference on laminate configuration and mechanical properties: (comment on Par. 5)

1. 12 plies instead of 11
2. stack-up : [45,0,90,45],
3. ply mechanical properties [Tape, Fabric]
4. $F_0 = 50, [\text{N/mm}]$

![Figure 9. Fibre stress for UNI 0° and 90°](image)

![Figure 10. 3DFEM model – plies stress](image)
4. Shear load $S_0$

This paragraph reports the analysis of laminate defect under shear load. A simplified numerical approach is presented to calculate the shear stress at each ply based on the deformation compatibility. Ply shear load ($f_{xy} = \tau^* t$) is proportional to factor \([d_0*G]\) and inversely proportional to the ply length. Example of a 3DFEM model is added for algorithm explanation and evaluation.

$$S = 1000. \,[N/mm] \text{, } d_0 = 0.395 \,[mm]$$

**Shear Load Algorithm**

Approach is valid under the validity of deformation compatibility.

$$N_{xy} = \sum_{i} \left( t_i \times \tau_i \right) - \sum_{i} \left( Y_i \times G_{xy} \times t_i \right) = \sum_{i} \left( d_i \times \frac{G_{xy} \times t_i}{L_i} \right) = \sum_{i} \left( G_{xy} \times \frac{t_i}{L_i} \right) \left[ N/mm \right]$$

$$\tau_i = \frac{d_i \times G_{xy}}{L_i} \left[ MPa \right]$$

| PLY ID | angle $\theta$ | $G_{xy}$ [MPa] | $t$ [mm] | $G_{xy}/t$ [N/mm$^2$] | $t_r (G_{xy}/t)$ [MPa] | $\tau_r = \tau^* t$ [N/mm] |
|--------|----------------|-----------------|----------|---------------------|---------------------|--------------------------|
| 1      | 90             | 4200            | 0.33     | 6.00                | 700.0               | 231.0                    | 276.6                    | 91.3                     |
| 2      | 0              | 4200            | 0.33     | 6.00                | 700.0               | 231.0                    | 276.6                    | 91.3                     |
| 3      | 0              | 4200            | 0.33     | 6.00                | 700.0               | 231.0                    | 276.6                    | 91.3                     |
| 4      | 90             | 4200            | 0.33     | 6.10                | 688.1               | 277.1                    | 271.9                    | 89.3                     |
| 5      | 0              | 4200            | 0.33     | 6.06                | 692.8               | 228.6                    | 273.7                    | 90.3                     |
| 6      | 90             | 4200            | 0.33     | 6.04                | 695.5               | 229.5                    | 274.8                    | 90.7                     |
| 7      | 0              | 4200            | 0.33     | 6.03                | 697.1               | 230.0                    | 275.4                    | 90.9                     |
| 8      | 90             | 4200            | 0.33     | 6.02                | 698.1               | 230.4                    | 275.8                    | 90.0                     |
| 9      | 0              | 4200            | 0.33     | 6.01                | 698.7               | 230.6                    | 276.1                    | 91.1                     |
| 10     | 90             | 4200            | 0.33     | 6.01                | 698.7               | 230.6                    | 276.1                    | 91.1                     |

$d_0 = 0.3951 \text{ [mm]}$

**Figure 11.** Ply shear stress_3DFEM vs Numerical model
5. Conclusion

Conclusions are based on comparison between algorithm (Figure 8) and 3DFEM (Figure 10). The following considerations apply:

1. The defect type is “similar” for both models.
2. The algorithm and 3DFEM have a different stack-up and material properties.
3. The analysis highlight the following analogies:
   a. Both model shows on ply (1, 2, 3, 4) “similar trend” for fiber stress on centerline-end sections and “similar trend (= stress increase)” on intermediate sections.
   b. Wavy ply 5 shows in both model the same stress on centerline-end sections. “Similar trend (= stress decrease)” on intermediate section:
      [MPa] (35.0_end, 12.middle, 37.0_end) ALGORITHM, (7.0_end, -0.46.middle, 6.9_end) 3DFEM
   c. The same behavior for ply 10, 11, 12: no stress significant variation on intermediate segment.
      The reason is that ply slope tends to Y’=0.0 (on both models).
   d. In both model the centerline-end sections, show the same stress of ply 1, 2, 3, 4 multiplied by a fiber length ratio λ.
4. For both model, the stress at centerline-end (Y’=0.0) are “close” to the CLT (Figure 7).

Remark:
1. The method can be applied for the strength analysis of single section X (independently from the others), on the basis of following parameters: Y’, λ, thk, R_curvature.
2. “Similarity” is hinting that defect topology is the leading parameter.
3. Methodology and algorithm, described on this paper, is not uniquely correlated to complex variable mathematical formulation.
4. The method is also effective for defect data (Y’, λ, thk, R_curvature) taken from laminate micro-section picture.

References
[1] Stephen W. Tsai, Theory of Composites Design Aeronautics & Astronautics Stanford University.
[2] Murray R. Spiegel, Complex Variables SCHAUM’S Outline Series.