1. INTRODUCTION

Differential rotation and convection provide the engine for the solar dynamo (e.g., Ossendrijver 2003). The mechanism suggested by Lebedinskii (1941) for differential rotation seems to be the most widely accepted. Angular momentum is continually redistributed in the deep convective envelope, occupying some 30% of the outer solar radius with nonuniform rotation arising from the action of the Coriolis force on the convective turbulence (Kitchatinov 2005). There is good evidence that a younger Sun would have rotated more rapidly and gradually lost angular momentum through magnetic coupling to the solar wind, and there is currently a considerable theoretical effort to develop quantitative models of differential rotation in solar-type stars (see, for example, reviews by Kitchatinov 2005; Thompson et al. 2003).

The Astrophysical Journal, 659:1611–1622, 2007 April 20
© 2007. The American Astronomical Society. All rights reserved. Printed in U.S.A.

THE DIFFERENTIAL ROTATION OF $\kappa^1$ CETI AS OBSERVED BY MOST

GORDON A. H. WALKER, BRYCE CROLL, RAINER KUSCHNIG, ANDREW WALKER, SLAVEK M. RUCINSKI, JAYMIE M. MATTHEWS, DAVID B. GUENTHER, ANTHONY F. J. MOFFAT, DIMITAR SASSELOV, AND WERNER W. WEISS

Received 2006 September 7; accepted 2006 December 8

ABSTRACT

We first reported evidence for differential rotation of $\kappa^1$ Ceti in Paper I. In this paper we demonstrate that the differential rotation pattern closely matches that for the Sun. This result is based on additional MOST observations in 2004 and 2005, as well as those from 2003. Using StarSpotz, a program developed specifically to analyze MOST photometry, we have solved for $k$, the differential rotation coefficient, and $P_{eq}$, the equatorial rotation period using the light curves from all three years. The absolute range in spot latitudes is $10^\circ$–$75^\circ$ and $k = 0.090^{+0.006}_{-0.005}$, less than the solar value but consistent with the younger age of the star; $k$ is also well constrained by the independent spectroscopic estimate of $\sin i$. We demonstrate independently that the pattern of differential rotation with latitude is indeed solar. Details are given of the parallel tempering formalism used in finding the most robust solution, which gives $P_{eq} = 8.77^{+0.03}_{-0.04}$ days, smaller than that usually adopted, implying an age $<750$ My. Our values of $P_{eq}$ and $k$ can explain the range of rotation periods observed in solar-type stars. We recently published values of $k$ and $P_{eq}$ for $\epsilon$ Eri based on MOST photometry and expect to analyze MOST light curves for several more spotted, solar-type stars.

Subject headings: stars: activity — stars: individual ($\kappa^1$ Ceti, HD 20630) — stars: late-type — stars: oscillations — stars: rotation — stars: spots

For stars other than the Sun, the simultaneous detection of two or more spots at different latitudes allows a direct measurement of differential rotation. The Microvariability and Oscillations of Stars (MOST) photometric satellite (Walker et al. 2003) offers continuous photometry of target stars with an unprecedented precision for weeks at a time, making accurate tracking of spots possible.

We (Rucinski et al. 2004) have already reported the detection of a pair of spots in the 2003 MOST light curve of $\kappa^1$ Ceti (HD 20630, HIP 15457, HR 996; $V = 4.83, B-V = 0.68, G5 V$). The photometry was part of the satellite commissioning and prior to a considerable pointing improvement early in 2004. The larger spot had a well-defined 8.9 day rotation period while the smaller one had a period $\geq9.3$ days. Only the latitude of the larger spot could be determined with any confidence. Spectroscopic observations by Shkolnik et al. (2003), mostly from 2002, of the Ca ii K line emission reversals were also synchronized to a period of about 9.3 days.

To better decipher the spot activity and rotation of $\kappa^1$ Ceti and to model spot distributions on other MOST targets, one of us (B. C.) developed the program StarSpotz (Croll et al. 2006; Croll 2006). It was applied first to a 2005 MOST light curve covering three rotations of $\epsilon$ Eri where two spots were detected at different latitudes rotating with different periods. From this, we derived a differential rotation coefficient for $\epsilon$ Eri (see § 4), $k = 0.11^{+0.02}_{-0.03}$, in agreement with models by Brown et al. (2004) for young Sun-like stars rotating roughly twice as fast as the Sun. A reanalysis using Markov Chain Monte Carlo (MCMC) functionality that took full account of the correlation among parameters largely confirmed the previous results (Croll 2006).

In this paper we apply StarSpotz to the three separate MOST light curves of $\kappa^1$ Ceti, from 2003, 2004, and 2005. We specifically solve for the astrophysically important elements of inclination,
differential rotation, equatorial period, and rotation speed and find that the differential rotation profile is closely similar to solar. We use parallel tempering methods to fully explore the realistic parameter space and identify the global \( \chi^2 \) minimum to the data set that signifies a best-fitting unique configuration of starspots. MCMC methods are then used to explore in detail the parameter space around this global minimum, to define best-fitting values and their uncertainties, and to explore possible correlations among the fitted and derived parameters.

2. MOST PHOTOGRAPHY AND LIGHT CURVES

The MOST satellite, launched on 2003 June, is fully described by Walker et al. (2003). A 15/17.3 cm Rumak-Maksutov telescope feeds two CCDs, one for tracking and the other for science, through a single, custom, broadband filter (350–700 nm). Starlight from primary science targets \((V \leq 6)\) is projected onto the science CCD as a fixed (Fabry) image of the telescope entrance pupil covering some 1500 pixels. The experiment was designed to detect photometric variations with periods of minutes at micro-magnitude precision and does not use comparison stars or flat-fielding for calibration. There is no direct connection to any photometric system. Tracking jitter was dramatically reduced early in 2004 to \(~1^\circ\), which, subsequently, led to significantly higher photometric precision.

The observations reported here were reduced by R. K. Outlying data points generated by poor tracking or cosmic-ray hits were removed. At certain orbital phases, MOST suffers from parasitic light, mostly earthshine, the amount and phase depending on the stellar coordinates, spacecraft roll, and season of the year. Data are recorded for seven Fabry images adjacent to the target to track the background. The background signals are combined in a mean and subtracted from the target photometry. This procedure also corrects for bias, dark, and background signals and their variations. Reductions basically followed the scheme already outlined by Rucinski et al. (2004).

A log of the MOST observations of \( \kappa^1\) Ceti for each of 2003, 2004, and 2005 is given in Table 1. For the light-curve analysis in this paper, data were binned at the MOST orbital period of 101.413 minutes. Figure 1 displays full light curves for each of the three years with time given as JD \(-2,451,545\) (heliocentric). The solid line in the bottom panel displays the difference between the two most divergent background readings from the seven background Fabry images. The complete light curve can be downloaded from the MOST Public Archive.10

The formal rms point-to-point precision, \(\sigma\), is given in Table 1. The shapes of the light curves ultimately used for the StarSpotz analysis depend to a degree on just how the background is removed. Despite having the summed signal values from seven adjacent Fabry images, we cannot interpolate a “true” background, but subtraction of the mean background is a consistent option. In each of the three data sets used here the seven background values are closely identical when averaged over an orbit, but the differences between the extreme backgrounds can be significant when the ambient background is high (e.g., pointing close to the Moon), which is why that difference is shown in the bottom panel for each light curve.

In 2003 the observations were subject to a much higher background from earthshine beginning around November 24 because the optimal roll angle for exclusion was not determined until the following year. On the other hand, contamination by moonlight close to full moon was much more effective in 2003 than in either 2004 or 2005. Times of full moon are recorded in Table 1. In Figure 1, systematic increases in the differential background can be seen close to full moon for both 2004 and 2005 even to the extent of the dip corresponding to the total lunar eclipse in 2004. We

![Fig. 1.—MOST light curves for \( \kappa^1\) Ceti from 2003, 2004, and 2005. The points are mean signals from individual 101.413 minute satellite orbits. The black connected symbols in the lower part of each panel are the differences between the highest and lowest of seven adjacent simultaneously recorded (background) Fabry images (see text). The “dip” at 1762 in 2004 corresponds to a total eclipse of the Moon.](http://www.astro.ubc.ca/MOST)

### Table 1

**MOST Observations of \( \kappa^1\) Ceti**

| Year Start | Dates (HJD - 2,451,545) | Total (days) | Duty Cycle (%) | Exposure (s) | Cycle (s) | \(\sigma\) (mmag) | Full Moon |
|------------|-------------------------|--------------|----------------|-------------|-----------|-----------------|-----------|
| 2003 Nov 5 | 1403.461–1433.901        | 30.44        | 96             | 40          | 50        | 0.5             | Nov 9     |
| 2004 Oct 15 | 1748.578–1768.989       | 20.41        | 99             | 30          | 35        | 0.30            | Oct 28    |
| 2005 Oct 14 | 2112.721–2125.065       | 12.34        | 83             | 30          | 35        | 0.19            | Oct 17    |

---

10 Available at [http://www.astro.ubc.ca/MOST](http://www.astro.ubc.ca/MOST).
return to this issue in § 4.3 in connection with systematic residuals between the models and light curves.

We have assumed that the true errors in the individual orbital means making up each light curve are proportional to the difference between the maximum and minimum background values. These errors have been arbitrarily scaled by one-fourth for the formal StarSpotz analysis, and these “errors” are shown as bars in Figures 4, 5, and 6 below. The individual bars correspond approximately to the standard deviation of the background.

3. STARSPOTZ

StarSpotz (Croll et al. 2006) is a program based on SPOT-MODEL (Ribárík et al. 2003; Ribárík 2002) that is designed to provide a graphical user interface for photometric spot modeling. The functionality included in StarSpotz is designed to take advantage of the nearly continuous photometry returned by MOST to quantify the nonuniqueness problem often associated with photometric spot models. Basically, StarSpotz uses the analytic models of Budding (1977) or Dorren (1987) to model the drop in intensity caused by \( N_{\text{spot}} \) circular, nonoverlapping spots. The model proposed in Budding (1977) is used in this application.

Fitting can be performed by a variety of methods, including a Marquardt-Levenberg nonlinear least-squares algorithm (Marquardt 1963; Levenberg 1944; Press et al. 1992), or MCMC functionality (Gregory 2005b; Gardner 1997; Gilks et al. 1996). The parameters of interest are the stellar inclination angle, \( i \), the unspotted intensity of the star, \( U \), and the period, \( p \), latitude, and angular size of the \( n \)th spot: \( p_a, E_a, \beta_a \), and \( \gamma_a \). Croll et al. (2006) demonstrated how the Marquardt-Levenberg nonlinear least-squares algorithm could be used to fit the cycle-to-cycle variation in \( \epsilon \) Eri. Croll (2006) investigated the same observations of \( \epsilon \) Eri, demonstrating that MCMC functionality is very helpful in defining correlations and possible degeneracies among the fitted parameters.

The search for additional local minima in \( \chi^2 \) space indicating other plausible spot configurations that produce reasonable fits to the light curve is an important part of the modeling because it allows one to quantify the uniqueness of a solution. We argue that this task of completely searching parameter space for other physically plausible local minima is better suited to a process called parallel tempering (Gregory 2005a, 2005b), discussed here in § 4.2, rather than the uniqueness test methods in Croll et al. (2006).

The present and subsequent releases of the freely available program StarSpotz, including the full source code, executable, and documentation, are available online.12 Release 3.0 includes the functionality discussed above, which allows one to fit multiple epochs of data, as well as the parallel tempering functionality.

4. THE SPOTS ON \( \kappa^1 \) CETI

As the MOST observations of \( \kappa^1 \) Ceti come from three well-separated epochs with independent starspot groups from epoch to epoch, the fitting process was considerably more complicated than those detailed in Croll et al. (2006) and Croll (2006). Rather than fitting for the period, \( p \), of the individual spots in each epoch, we derive a common equatorial period, \( P_{\text{eq}} \), and the differential rotation coefficient, \( k \), among the three epochs where these are defined by

\[
P_{\beta} = P_{\text{eq}} (1 - k \sin^2 \beta),
\]

\( P_{\beta} \) being the rotation period at latitude \( \beta \). There are five fewer parameters to be fitted because the periods of the individual spots are no longer independent variables, but completely defined by \( k, P_{\text{eq}} \), and \( \beta \). For the Sun, differential rotation analysis often includes terms in \( \sin^2 \beta \) (Kitchatinov 2005). We have adopted the stellar radius of \( R_{\ast} = 0.95 \pm 0.10 R_{\odot} \) used by Rucinski et al. (2004).

Common limb-darkening coefficients, \( u = 0.684 \), and flux ratios between the spot and unspotted photosphere, \( \kappa = 0.22 \), were assumed. The former was based on the compilations of Díaz-Cordovés et al. (1995), while the latter comes from the canonical value for sunspots (\( T_{\text{spot}} \sim 4000 \text{~K}, T_{\text{shot}} \sim 5800 \text{~K} \)). There is no independent way of determining \( \kappa_{\ast} \). The spots on \( \kappa^1 \) Ceti are large with their area depending directly on \( \kappa_{\ast} \). Variations in spot sizes will affect the detailed shape of the modeled light curve. The 27 parameters to be fitted are summarized in Table 2.

Initial investigations of the light curves using the StarSpotz standard spot model indicated that different numbers of spots were required for the various epochs. The 2003 and 2005 MOST data sets could be well explained by two starspots rotating with different periods, at different latitudes. The behavior observed in the 2004 MOST light curve, on the other hand, required a third spot. Thus, the following analysis explicitly assumes two, three, and two starspots for the 2003, 2004, and 2005 MOST epochs, respectively.

In this present application we use the same paradigm applied to \( \kappa^1 \) Ceti by Rucinski et al. (2004) and to \( \epsilon \) Eri by Croll (2006) and Croll et al. (2006), where the spot parameters \( (E, \beta, \gamma) \) were assumed constant within each epoch with changes between rotational cycles being caused solely by spots at different latitudes rotating differentially. Other interpretations are certainly plausible, including circular spots with time-varying spot sizes and latitudes such as those applied to IM Peg in Ribárík et al. (2003). The Maximum Entropy and Tikhonov reconstruction technique of Lanza et al. (1998), which assumes pixelated spots of various contrasts, is another example. We feel that with the limitations of photometric spot modeling, our assumption of unchanging spot parameters is the most suitable for the timescale of the MOST light curves. It provides an opportunity to determine a unique solution, as well as recovering astrophysically important parameters.

We recognize, given the obvious changes in the \( \kappa^1 \) Ceti light curve over a year, that the assumption of spot parameters staying absolutely constant over a few weeks is a simplification, but we do not expect significant spot migration on timescales of less than a month.

4.1. StarSpotz Markov Chain Monte Carlo Functionality

The MCMC functionality used in StarSpotz is fully described in Croll (2006). The goal is to take an \( n \)-step intelligent random walk around the parameter space of interest while recording the point in parameter space for each of the \( K \) fitted parameters for each step. This samples the posterior parameter distribution, which can be thought of in photometric spot modeling as simply the range in each of the \( K \) fitted parameters that provide a reasonable fit to the light curve. The advantage of an MCMC is that it does not simply follow the path of steepest descent, resulting in the possibility that it becomes stuck in a local minimum, but allows the MCMC to fully explore the immediate parameter space. The \( K \) parameters can thus be defined by a \( K \)-dimensional vector \( \mathbf{y} \).

\( T \) stands for the “virtual temperature” and determines the probability that the MCMC chain will accept large deviations to higher
regions in $\chi^2$ space (Sajina et al. 2006). The virtual temperature should properly be unity ($T = 1$) to sample the posterior parameter distribution when reduced $\chi^2$ is near unity. However, in photometric spot modeling, nature is often more complicated (leading to anomalously high reduced $\chi^2$) than one’s model values. Setting $T$ to be the minimum reduced $\chi^2$ value observed produces the same effect as scaling reduced $\chi^2$ to one and thus effectively samples the posterior parameter distribution. In the present application $T_{\text{min}}$ refers to this value of $T$ equal to the minimum reduced $\chi^2$ observed.

A suitable burn-in period as described in Croll (2006) is often used, or a $\chi^2$ cut where points with $\chi^2$ values above this user-specified minimum are excised from the analysis. A user-specified thinning factor, $f$ (Croll 2006), is also used. To ensure proper convergence and mixing, we employ the tests as proposed by Gelman & Rubin (1992) and explicitly defined for our purposes in Croll (2006). We ensure that the vector $R$ falls as close to 1.0 as possible for each of the $K$ fitted parameters. The marginalized likelihood (Sajina et al. 2006; Lewis & Bridle 2002), as described in § 3.1 of Croll (2006), is used to define our 68% credible regions, while the mean likelihood (Sajina et al. 2006) is used for comparison. The mean likelihood is produced using the $\chi^2$ values in a particular bin of a histogram of parameter values, while the marginalized likelihood directly reflects the fraction of times that the MCMC chains are in a particular bin. Peaks in the marginalized likelihood that are inconsistent with peaks in the mean likelihood can be indicative of the chains having yet to converge, or larger multidimensional parameter space rather than a better fit (Lewis & Bridle 2002). In our analysis we quote the minimum to maximum range of the 68% credible regions.

### 4.2. Parallel Tempering

Gregory (2005a) applied parallel tempering to a radial velocity search for extrasolar planets, as the parameter space he investigated had widely separated regions in parameter space that provided a reasonable fit to the data. In our application as we fit for a large number of parameters, $K = 27$, we use parallel tempering to explore our multidimensional parameter space and search for other local minima in $\chi^2$ space indicative of other plausible solutions, and thus starspot configurations, that produce a reasonable fit to the light intensity curve.

In parallel tempering, $M_f$ multiple MCMC chains are run simultaneously each with different values of the virtual temperature, $T_m$, where $m$ is an index that runs from 1 to $M_f$. Chains with higher values of $T$ are easily able to jump to radically different areas in parameter space, ensuring that the chain does not become stuck in a local minimum. Chains with lower values of $T$ can refine themselves and descend into local minima and possibly the global minimum in $\chi^2$ space (Gregory 2005a). The chain with $T = 1.0T_{\text{min}}$ is the target distribution and is referred to as the “cold sampler.” The parameters of the cold sampler are used for analysis to return the posterior parameter distribution. A set mean fraction of the time, $f$, a proposal is made for the parameters $\gamma$ of adjacent chains to be exchanged. If this proposal is accepted, then adjacent chains are chosen at random and the $K$ parameters $\gamma$ for each chain are exchanged. In this way, this method allows radically different areas in parameter space to be investigated in the high-temperature chains, before reaching the cold sampler where a local minimum can be sought efficiently. We run 11 parallel chains with virtual temperatures, $T_m$, of $T = \{100T_{\text{min}}, 50.0T_{\text{min}},$

| Parameter | Prior Allowed Range | Parallel Tempering | $R_{\text{parallel}}$ | $R_{\text{MCMC}}$ |
|-----------|---------------------|--------------------|-----------------------|-------------------|
| $i$ (deg) | 25–85, 30–80$^a$    | 25–85              | 1.033                 | 1.010             |
| $k$       | –0.75 to 0.75       | –0.6 to 0.6        | 1.443                 | 1.053             |
| $P_{\text{eq}}$ (days) | 8.0–10.5          | 8.0–10.5           | 1.477                 | 1.012             |
| $U_{\text{min}}$ | 0.99–1, 01        | 1.000              | 1.415                 | 1.024             |
| $E_{2003}$ (JD) | 1409.0–1413.0      | 1409.0–1413.0      | 1.338                 | 1.006             |
| $\beta_{2001}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.144                 | 1.023             |
| $\gamma_{2001}$ (deg) | 0.0–30           | 4.0–8.0            | 1.110                 | 1.031             |
| $E_{2002}$ (JD) | 1404.0–1408.0      | 1404.0–1408.0      | 1.051                 | 1.009             |
| $\beta_{2002}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.548                 | 1.026             |
| $\gamma_{2002}$ (deg) | 0.0–30           | 4.0–8.0            | 1.312                 | 1.026             |
| $U_{2003}$ | 0.99–1, 015        | 1.000              | 1.318                 | 1.124             |
| $E_{2004}$ (JD) | 1754.7–1758.7      | 1754.7–1758.7      | 1.179                 | 1.004             |
| $\beta_{2004}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.338                 | 1.013             |
| $\gamma_{2004}$ (deg) | 0.0–30           | 2.5–6.0            | 1.126                 | 1.022             |
| $E_{2005}$ (JD) | 1761.5–1765.5      | 1761.5–1765.5      | 1.391                 | 1.005             |
| $\beta_{2005}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.596                 | 1.098             |
| $\gamma_{2005}$ (deg) | 0.0–30           | 2.5–6.0            | 1.209                 | 1.100             |
| $E_{2006}$ (JD) | 1767.1–1771.1      | 1767.1–1771.1      | 1.079                 | 1.039             |
| $\beta_{2006}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.490                 | 1.042             |
| $\gamma_{2006}$ (deg) | 0.0–30           | 2.5–6.0            | 1.237                 | 1.035             |
| $U_{2004}$ | 0.99–1.014        | 1.000              | 1.235                 | 1.179             |
| $E_{2007}$ (JD) | 2115.0–2119.0      | 2115.0–2119.0      | 1.182                 | 1.057             |
| $\beta_{2007}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.249                 | 1.059             |
| $\gamma_{2007}$ (deg) | 0.0–30           | 4.0–8.0            | 1.122                 | 1.070             |
| $E_{2008}$ (JD) | 2118.1–2123.3      | 2118.1–2123.3      | 1.289                 | 1.038             |
| $\beta_{2008}$ (deg) | –90 to 90         | –10.0 to 60.0      | 1.074                 | 1.072             |
| $\gamma_{2008}$ (deg) | 0.0–30           | 4.0–8.0            | 1.057                 | 1.092             |

$^a$ The range on the left was used for the parallel tempering application (§ 4.2.1), while that on the right was used for the MCMC application (§ 4.3).
In a typical parallel tempering scheme the number of iterations between swap proposals is set to some constant integer, \(B = \frac{C}{25}\). This parameter fitted, rather than derived (Rucinski et al. 2004).

A parameter \(v\) is the number of binned data points minus the number of fitted parameters.

In the present application we start \(8\) tempering chains starting from random points in our acceptable temperature chains indicative of reasonable fits to the light curve. The number of chain points per chain, \(B_m\), was set to \(B = \{B_{\min}, B_{\min}, B_{\min}, B_{\min}, B_{\min}, B_{\min}, B_{\min}, 2.0B_{\min}, 2.0B_{\min}, 3.0B_{\min}, 4.0B_{\min}\}\). \(B_{\min}\) was set to \(20\) \((B_{\min} = 20)\), similar to the value proposed by Gregory (2005a); extensive tests have proven this choice to be reasonable. Parameter \(l\) was set as \(0.80\) to allow for exchanges between adjacent MCMC chains on average 80% of the time. That means a swap is approved if \(u_l < l\), where \(u_l\) is a random number \((u_l(0, 1))\). Significant experimentation was needed to choose and refine suitable choices of \(T_m, B_m, B_{\min}\), and \(l\).

In the present application we start \(8 \times 11\) of these parallel tempering chains starting from random points in our acceptable parameter space as given in the parallel tempering initial allowed range column of Table 2. MCMC fitting was then implemented.
for $n = 4000$ steps to ensure that the starting point for the parallel tempering chains provided a mediocre fit to the light curve. A mediocre fit was desired to ensure that the starting point for the parallel tempering chains provided a reasonable fit to the light curve to reduce the required burn-in period, while not starting from within a significant local minimum so as to allow the parallel tempering chains to efficiently explore the parameter space. Each of these $8 \times 11$ MCMC parallel tempering chains was then run for $n_x = 13,000$ exchanges, resulting in $n_x \times 20 \times 4 = 1.04 \times 10^9$ total steps for each of the eight cold samplers. A burn-in period was not used in this application for our analysis. Rather, a $\chi^2$ cut was used where all points of the parallel tempering cold samplers with reduced $\chi^2$ greater than some particular minimum, $\chi^2_{\text{cut}}$, were excised from the analysis. This serves a similar function to a burn-in period for the initial points, while allowing the analysis to focus on the chain points that provide a reasonable fit to the light curve. Given the large number of parameters we fit for, $K = 27$, this is useful as the parallel tempering chains spend an inordinate amount of time exploring parameter spaces that do not provide a good fit to the data and would otherwise complicate the analysis. A thinning factor, $f$, of 20 was used in all chains for the parallel tempering analysis. The $8 \times 11$ parallel tempering chains required a total computational time of 7.0 CPU days on a Pentium processor with a clock speed of 3.2 GHz with 1.0 GB of memory. This value of $n_x$ was chosen to ensure that the vector $R$, in this case $R_{\text{parallel}}$ (Gelman & Rubin 1992; Croll 2006), fell as close to 1.0 as possible for each of the $K = 27$ parameters. $R \leq 1.3$ signifies that the chain points have roughly converged and thus we are close to an equilibrium distribution. To determine $R_{\text{parallel}}$, we do not use the above $\chi^2$ cut but instead use a burn-in period of $n_{\text{burn}} = 30,000$ steps. This is because we are interested in ensuring that the parallel tempering chains have converged while exploring our parameter space as a whole, rather than the narrow region of parameter space signifying the global $\chi^2$ minimum. Analysis of these eight cold samplers is given in § 4.2.1.

4.2.1. $\kappa^1$ Ceti Parallel Tempering Results

The parallel tempering results as applied to MOST’s 2003–2005 observations of $\kappa^1$ Ceti indicated that there is a single unique solution to the data set that provides the best fit. Following the $n_x = 13,000$ exchanges as described above, the vector $R$, in this case $R_{\text{parallel}}$ (Gelman & Rubin 1992; Croll 2006), fell as close to 1.0 as possible for each of the $K = 27$ parameters. It was found that $R_{\text{parallel}}$ fell below 1.6 for all parameters, and $R_{\text{parallel}}$ fell below 1.3 for most parameters. Each of the components of the vector $R_{\text{parallel}}$ are given in Table 2. These low values for all the $K$ parameters of $R_{\text{parallel}}$ indicate that although our parallel tempering chains have not truly reached an equilibrium distribution, they have adequately sampled the realistic parameter space.

For our analysis we set $T_{\text{min}}$ as 2.96, as it is the global minimum observed in § 4.3, and a reduced $\chi^2_{\text{cut}}$ of 3.38, as it was judged that this was reasonably close to the global $\chi^2$ minimum to focus the analysis exclusively on the solutions that provided a reasonable fit to the light curve. Our 68% credible regions are given by the marginalized likelihood following the $\chi^2$ cut. The immediate parameter space indicated by these results is given in Table 3. This parameter space was thus worthy of more detailed investigation to explore this $\chi^2$ global minimum, as well as to define best-fit values and appropriate uncertainties for the fitted and derived parameters of interest. Also, possible correlations between the fitted parameters were investigated.

Detailed investigation of the parallel tempering results indicates that all other local minima produce significantly worse fits to the data sets. The only minor exception indicated by the parallel tempering results is that the second spot in the 2003 epoch can also be

![Figure 2](image-url)
Fig. 3.—Solid lines: 68% and 95% credible regions of the marginalized likelihood for various parameters. The 68% mean likelihood credible regions are shown by dotted contours (see text). Obviously there is little correlation between $k$ and $i$ in the bottom right panel.
found at a latitude, \( \beta_{2003} \), in the southern hemisphere as well as
the northern hemisphere. Detailed investigation indicates that the
northern hemisphere solution provides a significantly better fit and
thus is the solution that will be investigated. Given the fact that all
other local minima have been ruled out, as well as the low values
of \( R_{\text{parallel}} \) for all \( K \) parameters, we feel fully justified in stating that
this parameter space is the unique realistic solution to the observed
light curve given our assumptions.

### 4.3. \( \kappa' \) Ceti MCMC Application

In § 4.2.1 we demonstrated that the solution presented is unique
for the 2003–2005 \( \textit{MOST} \) epochs under the \( k, P_{\text{eq}} \) paradigm. Thus, we used the MCMC methods discussed in Croll (2006) and sum-
marized above in § 4.1 to investigate possible correlations and
derive best-fit values and uncertainties in the fitted and derived
parameters for \( \kappa' \) Ceti. We used \( M = 4 \) chains starting from rea-
sonably different points in parameter space, although within the
local \( \chi^2 \) minimum that defined the parameter space of interest in
§ 4.2. The prior ranges are given in Table 2. The choices in prior
are identical to those used for the parallel tempering section ex-
cept a slightly more conservative range is used for the inclination
angle, \( 30^\circ < i < 80^\circ \), to limit the MCMC chain to a realistic pa-
rameter space. We run each of the \( M = 4 \) chains for \( n = 7.0 \times 10^6 \)
steps, requiring a total computational time of 8.0 CPU days
on the aforementioned processor. This large value of \( n \) was moti-
vated to ensure that the \( R \) vector, in this case \( R_{\text{MCMC}} \), fell below
1.20 for all \( K = 27 \) parameters and fell below 1.05 for most pa-
rameters, indicative of suitable convergence. The values of each
of the \( K \) components of \( R_{\text{MCMC}} \) are given in Table 2.

The results of this analysis, henceforth referred to as solution 1,
can be seen in Figures 2 and 3 and are summarized in Table 3. As
one might expect, especially given the similar results of \( \epsilon \) Eri
(Croll 2006), the latitudes of the various spots are moderately
correlated with the inclination angle of the system, \( i \). However,
the differential rotation coefficient, \( k \), is largely uncorrelated with
\( i \). This is in stark contrast to the results of \( \epsilon \) Eri where the corre-
lation of spot latitude with \( i \) produced a moderate correlation be-
tween \( k \) and \( i \). Our derived value of \( k = 0.085 \pm 0.009 \) is therefore
very robust as it does not require an independent estimate of \( i \).

Here it should be noted that there are systematic departures
from the model fits in both 2003 and 2004 of a few thousandths
of a magnitude. These deviations coincide with times of high
ambient background from earthshine and/or moonshine as can
be seen clearly by comparison with the differential background
residuals in Figure 1. Although the deviations are small, they im-
ply that background correction is incomplete. It is also a caution
not to attempt to fit these obvious artifacts with additional spot
model variables.

It is also important to note, however, that the 2003 and 2005
\( \textit{MOST} \) data sets do not severely limit the allowed best-fit range
of the differential rotation coefficient, \( k \). The 2003 and 2005 \( \textit{MOST} \)
data sets were best fitted with spots that were not widely separated.
in latitude, and thus these epochs contributed only marginally to the determination of the differential rotation coefficient. The 2004 MOST data set most severely limited the best-fit range of the differential rotation coefficient, as it required three spots ranging in latitude from the midsouthern hemisphere, to the equator, to near the north pole.

In addition, although we have used a relatively liberal flat prior on the stellar inclination angle, $i$, of $30^\circ < i < 80^\circ$, the value of $i$ returned from our fit gives $v \sin i = 4.64 - 4.94$ km s$^{-1}$, entirely consistent with the spectroscopic value of $v \sin i = 4.64 \pm 0.11$ km s$^{-1}$ determined by Rucinski et al. (2004) from the width of the Ca ii K line reversals. This agreement is a very useful sanity check and gives us great confidence that our fitted results and our assumptions of circular spots are in close agreement with the actual behavior of the star during these three epochs.

The minimum $\chi^2$ point observed in our MCMC chains is also quoted in Table 3. The fit to the light curves in 2003, 2004, and 2005 and the views of the modeled spots shown from the line of sight (LOS) of this minimum $\chi^2$ solution are given in Figures 4, 5, and 6.

Our current results are largely consistent with the analysis of the 2003 MOST data by Rucinski et al. (2004) as can be seen in the comparison given in Table 3. The only discrepancies of note are that our MCMC analysis indicates a slightly smaller value of the inclination angle, $i = 57.8^\circ - 63.5^\circ$, and that we find a shorter period for the second spot, $p_{2003_2} = 9.022 - 9.094$ days. The period found by Rucinski et al. (2004) for the second spot depends on effective removal of the variations caused by the larger spot, and it was not possible to assign a reliable latitude. They also assumed that the spots were black, making them smaller than those in this paper.

4.4. $\kappa^1$ Ceti 2004 Parallel Tempering and MCMC Application

Given the good agreement of the MOST 2003, 2004, and 2005 $\kappa^1$ Ceti observations with the assumed solar-type differential rotation profile in equation (1) and summarized in Figure 7, we decided to test independently whether solar-type differential rotation is indeed present in $\kappa^1$ Ceti. The 2004 light curve was chosen because, as noted above, it constrained the equatorial period, $P_{eq}$, and differential rotation coefficient, $k$, more significantly than the light curves from the other two years.

For this independent test we fitted explicitly for the periods of each of the three spots: $p_{2004_1}, p_{2004_2}$, and $p_{2004_3}$. The $K = 13$ fitted parameters are summarized in Table 4. For these we used the same priors, and for the parallel tempering chains we started from the same random ranges in parameter space, as summarized in Table 2. We assumed $i = 60.6^\circ$ corresponding to the minimum $\chi^2$ value found above. This assumption for the inclination angle is justified because it results in $v \sin i = 4.77$ km s$^{-1}$ (assuming $P_{eq} \approx 8.78$), a value close to the spectroscopic value measured by Rucinski et al. (2004).
Fig. 6.—Best-fitting two-spot solution for κ¹ in Ceti 2005 (rotating counterclockwise from top) and seen from the line of sight at phases 0.00, 0.25, 0.50, and 0.75 (from left) of the first spot. Middle: MOST light curve with errors (see text). The solid curve is the solution from the “Minimum $\chi^2$” column of Table 3. The dotted line indicates the unspotted normalized signal of the star ($U = 1.0042$). Vertical dashed lines indicate phases 0.00, 0.25, 0.50, and 0.75. Bottom: Residuals from the model on the same scale.
The other two red curves correspond to periods (the mean values taken from solution 1 of Table 3, well, confirming that the differential rotation pattern is closely similar to solar. Clearly, the black curve fits the data curves are identical to those in the top panel. The analysis defined $k$ using equation (1), which is based on the solar pattern. The agreement of all seven spots with the form of the $k$ curve, as well as the good agreement of the spot solutions in Figures 5, 6, and 7, suggests that the differential rotation curve for $\kappa^1$ Ceti is closely similar to solar. The analysis of the 2004 light curve independently of any assumption about $k$ offers strong confirmation that the pattern is indeed solar.

For the Sun, $k$ has been derived quite independently from either sunspot latitudes and periods (Newton & Nunn 1951) or surface radial velocities (Howard et al. 1983), yielding $k = 0.19$ and 0.12, respectively (Kitchatinov 2005). The large discrepancy between these values may be related in part to the different behavior of sunspots and photospheric motions. In our case, we depend on spots to determine $k$, and our value of 0.09 is significantly lower than either of those for the Sun. This is in line with calculations by Brown et al. (2004), who find that $k$ should increase with age.

Güdel et al. (1997) estimated an age of 750 Myr for $\kappa^1$ Ceti from their estimated 9.2 day rotation period. Our value of $P_{\text{eq}} = 8.77$ days suggests a still younger age.

All of the photometric periods found to date for $\kappa^1$ Ceti can be explained by spots appearing at different latitudes. A period of 9.09 days is given in the Hipparcos catalog (Perryman et al. 1997), while Messina & Guinan (2002) quote 9.214 days. Baliunas et al. (1995) monitored Ca II H and K photoelectrically between 1967 and 1991 and found a rotational period of $9.4 \pm 0.1$ days.
(Baliunas et al. 1983), and Shkolnik et al. (2003) found a closely similar period of ~9.3 days mostly from spectra taken in 2002. These periods correspond to latitudes between 50° and 60°. While the apparent persistence of this period for some 35 years may be fortuitous, it also might be related to some large-scale magnetic structure.

The Natural Sciences and Engineering Research Council of Canada supports the research of B. C., D. B. G., J. M. M., A. F. J. M., S. M. R., and G. A. H. W. Additional support for A. F. J. M. comes from FCAR (Québec). R. K. is supported by the Canadian Space Agency. W. W. W. is supported by the Austrian Space Agency and the Austrian Science Fund (P14984).

## REFERENCES

Baliunas, S. L., et al. 1983, ApJ, 275, 752

———. 1995, ApJ, 438, 269

Brown, B. P., Browning, M. K., Brun, A. S., & Toomre, J. 2004, in Helio- and Asteroseismology: Towards a Golden Future, ed. D. Danesy (ESA SP-559; Noordwijk: ESA), 341

Budding, E. 1977, Ap&SS, 48, 207

Croll, B. 2006, PASP, 118, 1354

Croll, B., et al. 2006, ApJ, 648, 607

Diaz-Cordoves, J., Claret, A., & Gimenez, A. 1995, A&AS, 110, 329

Dorren, J. D. 1987, ApJ, 320, 756

Gamerman, D. 1997, Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference (London: Chapman and Hall)

Gelman, A., & Rubin, D. B. 1992, Statistical Science, 7(4), 457

Gilks, W. R., Richardson, S., & Spiegelhalter, D. J. 1996, Markov Chain Monte Carlo in Practice (London: Chapman and Hall)

Gregory, P. C. 2005a, ApJ, 631, 1198

———. 2005b, Bayesian Logical Data Analysis for the Physical Sciences: A Comparative Approach with Mathematica Support (Cambridge: Cambridge Univ. Press)

Gudel, M., Guinan, E. F., & Skinner, S. L. 1997, ApJ, 483, 947

Howard, R., Adkins, J. M., Boyden, J. E., Cragg, T. A., Gregory, T. S., Labonte, B. J., Padilla, S. P., & Webster, L. 1983, Sol. Phys., 83, 321

Kitchatinov, L. L. 2005, Phys. Uspekhi, 48(5), 449

Lanza, A. F., Catalano, S., Cutispoto, G., Pagano, I., & Rodono, M. 1998, A&A, 332, 541

Lebedinskii, A. I. 1941, Astron. Zh., 18, 10

Levenberg, K. 1944, Q. Appl. Math., 2, 164

Lewis, A., & Bridle, S. 2002, Phys. Rev. D, 66, 103511

Marquardt, D. W. 1963, SIAM J. Appl. Math., 11, 431

Messina, S., & Guinan, E. F. 2002, A&A, 393, 225

Newton, H. W., & Nunn, M. L. 1951, MNRAS, 111, 413

Ossendrijver, M. 2003, A&A Rev., 11, 287

Perryman, M. A. C., et al. 1997, The Hipparcos and Tycho Catalogues (ESA SP-1200; Noordwijk: ESA)

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. 1992, Numerical Recipes in Fortran 77 (2nd ed.; Cambridge: Cambridge Univ. Press)

Ribarik, G. 2002, Occasional Technical Notes from Konkoly Observatory, No. 12

Ribarik, G., Olah, K., & Strassmeier, K. G. 2003, Astron. Nachr., 324, 202

Rucinski, S. M., et al. 2004, PASP, 116, 1093

Sajina, A., Scott, D., Dennefeld, M., Dole, H., Lacy, M., & Lagache, G. 2006, MNRAS, 369, 939

Shkolnik, E., Walker, G. A. H., & Bohlender, D. A. 2003, ApJ, 597, 1092

Thompson, M. J., Christensen-Dalsgaard, J., & Miesch, M. S. 2003, ARA&A, 41, 599

Walker, G. A. H., et al. 2003, PASP, 115, 1023

### TABLE 4

| Parameter | Fitted | $R_{2004}^{parallel}$ | Parallel Tempering Results | $R_{2004}^{MCMC}$ | MCMC Solution |
|-----------|--------|------------------------|----------------------------|------------------|---------------|
| Reduced $\chi^2$ | n/a | n/a | 1.50–1.82 | n/a | 1.46–1.52 |
| $\nu$ | n/a | n/a | 269 | n/a | 269 |
| $i$ (deg) | Assumed | n/a | 60.6 | n/a | 60.6 |
| $k$ | Derived | n/a | 0.095–0.111 | n/a | 0.092–0.101 |
| $v$ ($\text{km s}^{-1}$) | Derived | n/a | 4.80–4.86 | n/a | 4.80–4.84 |
| $P_{\text{eq}}$ | Derived | n/a | 8.624–8.721 | n/a | 8.65–8.72 |
| $u$ | Assumed | n/a | 0.6840 | n/a | 0.684 |
| $\kappa_x$ | Assumed | n/a | 0.220 | n/a | 0.220 |
| $U_{2004}$ | Yes | 1.122 | 1.0951–1.0150 | 1.011 | 1.0139–1.0150 |
| $E_{2004}$ (JD $-$2,451,545) | Yes | 1.110 | 1756.66–1756.74 | 1.001 | 1756.64–1756.68 |
| $p_{2004}$ (days) | Yes | 1.127 | 8.704–8.773 | 1.003 | 8.736–8.788 |
| $\beta_{2004}$ (deg) | Yes | 1.183 | 7.51–7.80 | 1.002 | 7.64–7.84 |
| $\gamma_{2004}$ (deg) | Yes | 1.092 | 9.1–22.3 | 1.002 | 15.6–21.7 |
| $E_{2004}$ (JD $-$2,451,545) | Yes | 1.315 | 1763.40–1763.50 | 1.000 | 1763.43–1763.49 |
| $p_{2004}$ (days) | Yes | 1.279 | 9.050–9.162 | 1.002 | 9.112–9.183 |
| $\beta_{2004}$ (deg) | Yes | 1.162 | 45.8–38.6 | 1.013 | 47.5–43.9 |
| $\gamma_{2004}$ (deg) | Yes | 1.074 | 11.21–15.80 | 1.016 | 14.65–16.83 |
| $E_{2004}$ (JD $-$2,451,545) | Yes | 1.034 | 1768.94–1769.07 | 1.002 | 1768.97–1769.08 |
| $p_{2004}$ (days) | Yes | 1.102 | 9.518–9.598 | 1.000 | 9.510–9.579 |
| $\beta_{2004}$ (deg) | Yes | 1.203 | 71.0–77.9 | 1.003 | 76.7–78.0 |
| $\gamma_{2004}$ (deg) | Yes | 1.127 | 9.04–13.11 | 1.003 | 12.64–13.17 |

$^a$ Parameter $\nu$ is the number of binned data points minus the number of fitted parameters.

$^b$ These values determined using $R_c = 0.95 R_{\odot}$. 

$^c$ Similar period of 9.3 days mostly from spectra taken in 2002. These periods correspond to latitudes between 50° and 60°. While the apparent persistence of this period for some 35 years may be fortuitous, it also might be related to some large-scale magnetic structure.