Tolerance analysis of multiple-element linear retrodirective cross-eye jamming

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Abstract: Tolerance sensitivity limits the practical application of the cross-eye jammer. Previous literature has demonstrated that retrodirective cross-eye jamming with multiple antenna elements possesses the advantage of loose tolerance requirements compared to traditional cross-eye jamming. However, the previous analysis was limited, because there are still some factors affecting the parameter tolerance of the multiple-element retrodirective cross-eye jamming (MRCJ) system and they have not been investigated completely, such as the loop difference, the baseline ratio and the jammer-to-signal ratio. This paper performs a comprehensive tolerance analysis of the MRCJ system with a nonuniform-spacing linear array. Simulation results demonstrate the tolerance effects of the above influence factors and give reasonable advice for easing tolerance sensitivity.

Keywords: electronic warfare (EW), electronic countermeasure (ECM), cross-eye jamming, radar active jamming, tolerance.

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1. Introduction

Cross-eye jamming as an electronic warfare technique recreates the worst-case glint angular error into monopulse radar [1–4]. Although it was proposed more than sixty years ago, cross-eye jamming still attracts attention of the researchers in the electronic warfare community in the past twenty years. In 2000, the successful experimental testing on cross-eye jamming was first publicly announced. In 2010, du Plessis gave a comprehensive theoretical analysis of cross-eye jamming with a retrodirective antenna array [5–8]. The retrodirective antenna array possesses the advantage that the radiated field has a maximum back in the direction of arrival of the primary plane wave.

As a figure of merit, parameter tolerances of cross-eye jamming are defined as the tolerated errors of the system parameters to ensure a specified angular error to be achieved. It is well-known that cross-eye jamming can induce the largest angular error into the monopulse radar when the amplitude ratio approaches 1 and the phase difference is 180° between the signals through the two directions. However, these conditions are too strict to be satisfied in practical application [8–10]. Although du Plessis indicated that the retrodirective implementation can reduce the tolerance requirements in [8], the limitation of the extreme tolerance sensitivity cannot be removed completely from a cross-eye jamming system with two antenna elements.

Multiple-element retrodirective cross-eye jamming (MRCJ) was proposed to ease the tolerance sensitivity by providing additional amplitude ratios and phase differences [11–18]. The antenna array of the MRCJ system can be linear, circular or orthogonal. Basic tolerance analyses of the MRCJ system were performed by previous literature [13–15]. It was proved that the MRCJ system can significantly reduce tolerance requirements compared to the traditional two-element cross-eye jamming system.

This paper will further study the parameter tolerances of the MRCJ system with the linear antenna array (L-MRCJ), considering the fact that the analyses in [13] and [14] are insufficient to account for the tolerance performance of the L-MRCJ system. Firstly, the analysis in [13] ignored the fact that the jamming signals from different jammer loops have amplitude and phase differences. Especially the large phase difference severely affects the jamming performance. Secondly, only the median case of the cross-eye gain was considered in [14] where the platform skin return was taken into account. Thirdly, both analyses in [13] and [14] were performed by assuming that the antenna elements of the linear retrodirective array were aligned.
with uniform spacing, which was not the best antenna configuration for the MRCJ system.

In this paper, the effects of the influence factors on parameter tolerances of the MRCJ system will be investigated. A general nonuniform-spacing linear retrodirective array is employed by the L-MRCJ system. The cross-eye gain of the L-MRCJ system is derived after giving the jamming geometry. Then, the process for tolerance solving is proposed according to the results in [8]. The factors affecting the value of the cross-eye gain do affect the parameter tolerances of the M-R-C-J system. The influence factors, including the loop differences, the baseline ratio and the platform skin return, are discussed respectively. Finally, simulation results illustrate the effects of the influence factors, and valuable advice for building a practical L-MRCJ system is given.

2. Summary of previous results

2.1 Jamming geometry

The jamming geometry for tolerance analysis is given in Fig. 1. The L-MRCJ system has a nonuniform-spacing linear retrodirective array, which comprises N antenna elements (denoted by crosses). Multiple jammer loops of the L-MRCJ system have different baseline lengths but own the same phase center. Meanwhile, the phase-comparison monopulse radar (denoted by circles) is employed in the jamming geometry.

![Fig. 1 Jamming geometry for tolerance analysis of L-MRCJ](image)

In the jamming geometry, the jamming range is denoted by r, the radar rotation angle is denoted by θ1, the jammer rotation angle is denoted by θe, the half angular separation of the nth jammer loop (i.e., the jammer loop consisted of antenna n and antenna N−n+1) is denoted by θn, the angle of the apparent target is denoted by θa, the spacing of radar antenna elements is denoted by dn, and the baseline of the nth jammer loop is denoted by dnp.

According to the relationship r ≫ dn, and the approximation tan x ≈ x, the half angular separation θn can be calculated from the jamming geometry:

\[
\tan \theta_n \approx \frac{d_n \cos \theta_c}{2} \left( r \pm \frac{d_n \sin \theta_c}{2} \right) \cos \theta_e
\]

\[
\theta_n \approx \frac{d_n \cos \theta_c}{2r} \tan \theta_e
\]

2.2 Cross-eye gain of L-MRCJ

A cross-eye jammer is an onboard jamming system where the aircraft or ship platform return must be taken into account. When cross-eye jamming combines range gate pull-off jamming, the platform skin return is isolated and cannot be considered. Under the isolated condition, the cross-eye gain of the L-MRCJ system (called as isolated cross-eye gain) is given from [13] as

\[
G_{C_1} = \Re \left[ \sum_{n=1}^{N/2} F_n C_n (1 - A_n) \right] \left( \sum_{n=1}^{N/2} C_n (1 + A_n) \right)
\]

with

\[
A_n = a_n e^{i \phi_n}
\]

where an and φn are the amplitude ratio and the phase difference between the signals that pass through the two opposite directions in the nth jammer loop, Cn is defined as the loop parameter of the nth jammer loop that is used to characterize the differences between multiple jammer loops, and \( \Re \) denotes the real part of the complex number.

Without the isolated method, the platform skin return needs to be considered and added to the jammer signals. In presence of the platform skin return, the total cross-eye gain of the L-MRCJ system (called as statistical cross-eye gain) can be given from [14] as

\[
G_{C_s} = \Re \left[ \sum_{n=1}^{N/2} F_n C_n (1 - A_n) \right] \left[ A_s + \sum_{n=1}^{N/2} C_n (1 + A_n) \right]
\]

where \( A_s = a_s e^{i \phi_s} \) is the platform scatter factor relative to the radar cross section of the ship or aircraft platform [14], \( a_s \) is the amplitude factor, and \( \phi_s \) is the phase factor.

The angular error induced into the monopulse radar can be quantified by the monopulse indicated angle. When the radar rotation angle is zero, the monopulse indicated angle is the angular error. The monopulse indicated angle θi can be obtained from the following relationship [13]:

\[
\tan \left( \frac{\beta d_n}{2} \sin \theta_i \right) = \frac{\sin(2k_{s1}) + G_c \sin(2k_{c1})}{\cos(2k_{s1}) + 1}
\]
with
\[
  k_{xn} = \frac{\beta d_p}{2} \cos \theta_n \sin \theta_r,
\]
(7)
\[
  k_{cn} = \frac{\beta d_p}{2} \sin \theta_n \cos \theta_r,
\]
(8)
where \( \beta \) is the free-space phase constant.

Observing the expressions of the cross-eye gain in (3) and (5), we find that there are three factors involved in the cross-eye gain, which are the loop parameter \( C_n \), the baseline ratio \( F_n \) and the platform scatter factor \( A_s \). These three influence factors will further affect the tolerance performance of the L-MRCJ system.

3. Process for tolerance solving

The process to solve parameter tolerances for an L-MRCJ system is deriving the closed-form solutions for the system parameters to achieve a specified angular error, which is similar to the process for a two-element retrodirective cross-eye jamming (TRCJ) system outlined in [8]. More complex than the TRCJ system, the factors between multiple jammer loops, e.g., the loop differences and the baseline ratio, need to be considered for the L-MRCJ system. In addition, it is impossible to derive the closed-form solutions for the specified system parameters \((a_i, \phi_i)\) of the L-MRCJ system unless other parameters \((a_n, \phi_n) (n \neq i)\) are endowed with constants, due to the large numbers of degrees of freedom.

The angular error induced into the monopulse radar can be characterized by the “settling angle” \( \theta_s \) which is the radar rotation angle where the apparent target exists [8], and is calculated from (6) when the monopulse indicated angle is specified to zero. Another factor characterizing the angular error is the “angle factor” \( G_\theta \) which is defined as the degree of the angular deflection of the apparent target and is given by

\[
  G_\theta = \left| \frac{\theta_s}{\theta_1} \right|.
\]
(9)

The process to determine the closed-form solutions of \((a_i, \phi_i)\) to achieve a specified settling angle or an angular factor is translated into deriving the specified cross-eye gain when the radar rotation angle is the specified settling angle. The specified cross-eye gain magnitude \( G_S \) is derived from (6) by

\[
  G_S = |G_c| \approx \frac{\sin(\beta d_p \sin \theta_s)}{\sin(\beta d_p \theta_1)} = \frac{\sin(\beta d_p \sin(G_\theta \theta_1))}{\sin(\beta d_p \theta_1)}
\]
(10)
where the approximation in (10) occurs when \( \theta_1 \) is assumed to be a small value. Actually, this assumption is reasonable because the jammer range \( r \) is much larger than the baseline of the antenna array \( d_n \) in practical applications.

After obtaining the cross-eye gain magnitude \( G_S \), the closed-form solutions of \((a_i, \phi_i)\) can be calculated by substituting (10) into (3) and (5).

As stated in [5,6,13], there are the cases that the settling angle does not exist. This will occur under the condition below:

\[
\sin(2k_{s1}) + G_c \sin(2k_{c1}) \neq 0.
\]
(11)

Given the relationships as follows:

\[
\left\{ \begin{array}{l}
|\sin(2k_{s1})| \leq 1 \\
{\beta d_p \theta_1} \sqrt{1 - \left( \frac{\lambda}{2d_p} \right)^2} \sin(2k_{c1}) \leq \frac{\beta d_p \theta_1}{2}.
\end{array} \right.
\]
(12)
the condition in (11) can be simplified as

\[
G_I \geq \frac{1}{\sin(\beta d_p \theta_1)}
\]
(13)
where \( G_I \) is the marginal value of the cross-eye gain. When \( G_S > G_I \), we cannot obtain the closed-form solutions of \((a_i, \phi_i)\). It means that the specified angular error is large enough to break the monopulse radar’s lock.

In conclusion, the process for tolerance solving is divided into the following steps.

Step 1 Specify the angular error and calculate the angle factor \( G_\theta \) from (9).

Step 2 Calculate the cross-eye gain magnitude \( G_S \) from (10).

Step 3 Calculate the closed-form solutions of \((a_i, \phi_i)\) by substituting (10) into (3) and (5).

The ranges of the values of \((a_i, \phi_i)\) are the required tolerances for specified angular error.

4. Influence factors

4.1 Loop differences

Loop differences are specific to the MRCJ system. Multiple jammer loops have different jamming paths and circuit elements, resulting in the differences between the signals through different jammer loops. We define the signal differences between multiple jammer loops as the loop differences. The phase shift involved in the loop differences can be 180° which will affect the performance of the MRCJ system [13]. Although du Plessis has investigated the effect of loop differences between multiple loops in [17], the effect of the loop differences on tolerance performance is still unclear.

We use the factor \( C_n = c_n e^{i \phi_n} \) to denote the loop parameters of the \( n \)th jammer loop, where \( c_n \) is the attenuation of the jamming signal through the \( n \)th jammer loop and \( \phi_n \) is the phase shift of the jamming signal through the \( n \)th jammer loop. Using the loop parameters of the first jammer loop as reference to quantify the \( n \)th jammer loops, we give the amplitude ratio and phase difference of
the loop differences between the first and the \( n \)th jammer loop by
\[\Delta c_n = \frac{c_n}{c_1}\]  
(14)
and
\[\Delta \varphi_n = \varphi_n - \varphi_1.\]  
(15)

Utilizing the largest amplitude of \(|C_n|\) to normalize the loop differences, we have the relationship that \(|C_n| \leq 1.\)

4.2 Baseline ratio

The main difference between the antenna configuration in Fig. 2 and the previous antenna configuration in the analyses [13] is that the baseline \( d_n \) is different.

When the antenna elements are aligned with uniform spacing, the baseline of the \( n \)th jammer loop is given from [13] as
\[d_n = \frac{N - 2n + 1}{N - 1}d_c\]  
(16)
where \( d_c \) is the baseline of the antenna array (i.e., \( d_c = d_1 \)), and the term
\[F_n = \frac{N - 2n + 1}{N - 1}\]  
(17)
is the baseline ratio between the first and the \( n \)th jammer loop, and was also called as the “attenuated factor” in [13].

For the general case that the jammer employs a nonuniform-spacing linear retrodirective array in this paper, the baseline ratio \( F_n \) is given by
\[F_n = \frac{d_n}{d_1}.\]  
(18)

The difference between the baseline ratios in (16) and those in (18) is that the latter can be any value except 0 and 1, while the former cannot. For example, for the case where the number of the antenna elements \( N = 4 \), the constant baseline ratio in (16) is 1/3, while the ratio in (18) varies with the baseline \( d_n \).

Considering that \( F_n \) weakens the contribution of the inner jammer loop to the total difference-channel return, a larger \( F_n \) is better for the L-MRCJ system from the perspective of obtaining a larger cross-eye gain or a larger angular error.

4.3 Platform skin return

Due to the variable platform phase, the statistical cross-eye gain \( G_{cs} \) is a distribution. We use the special statistical values of the cross-eye gain to account for the effect of platform skin return on the tolerance performance of the L-MRCJ system.

After assuming that the platform phase \( \phi_s \) follows the uniform distribution, the median statistical cross-eye gain was derived in [14] and given by
\[G_m = \frac{ac + bd}{a^2 + b^2 + a_s^2}\]  
(19)
where
\[a = \sum_{n=1}^{N/2} [c_n \cos \varphi_n + c_n a_n \cos(\varphi_n + \phi_n)],\]  
(20)
\[b = \sum_{n=1}^{N/2} [c_n \sin \varphi_n + c_n a_n \sin(\varphi_n + \phi_n)],\]  
(21)
\[c = \sum_{n=1}^{N/2} F_n [c_n \cos \varphi_n - c_n a_n \cos(\varphi_n + \phi_n)],\]  
(22)
\[d = \sum_{n=1}^{N/2} F_n [c_n \sin \varphi_n - c_n a_n \sin(\varphi_n + \phi_n)].\]  
(23)

The extreme statistical cross-eye gains were derived in [18] and given by
\[G_e = \frac{ac + bd \pm a_s \sqrt{c^2 + d^2}}{a^2 + b^2 - a_s^2}.\]  
(24)

The \( \pm \) symbol means that the extreme values in (24) could be either the maximum or the minimum cross-eye gain. When \( a^2 + b^2 \geq a_s^2 \), the plus and minus signs in (24) correspond to the maximum and the minimum cross-eye gains respectively. On the contrary, when \( a^2 + b^2 < a_s^2 \), the plus and minus signs in (24) correspond to the minimum and the maximum cross-eye gains respectively.

To quantify the effect of the platform skin return, the jammer-to-signal ratio (JSR) was given from [14] as
\[\text{JSR} = \max\{|C_n|^2, |C_n A_n|^2\} |A_s|^2 = \begin{cases} \frac{1}{a_s^2}, & c_n a_n \leq 1 \\ \frac{c_n^2 a_n^2}{a_s^2}, & c_n a_n > 1 \end{cases}.\]  
(25)

Notably, the JSR value is not constant for the case that \( c_n a_n > 1 \).

Substituting (25) into (19) and (24) gives the relationships between the JSR and the special statistical cross-eye gains as
\[G_m = \frac{ac + bd}{a^2 + b^2 + \max\{1, c_n^2 a_n^2\} \text{JSR}}\]  
(26)
and
\[G_e = \frac{(ac + bd) \pm \sqrt{(c^2 + d^2) \max\{1, c_n^2 a_n^2\} \text{JSR}}}{a^2 + b^2 - \max\{1, c_n^2 a_n^2\} \text{JSR}}.\]  
(27)
which will make it convenient to analyze the effect of the JSR value on the tolerance requirements on the L-MRCJ system.

5. Simulation results and discussion

This section mainly focuses on the effects of the above influence factors on the tolerance requirements on the L-MRCJ system. An L-MRCJ system with a four-element linear antenna array is employed in this paper without loss of generality. The simulation parameters for a typical jamming geometry used in [8,13] are also employed in this paper for a fair comparison as shown in Table 1.

| Table 1 Simulation parameters |
|-----------------------------|
| Object | Parameter | Value |
|-----------------------------|
| Monopulse radar | Frequency/GHz | 9 |
| | Beamswidth(°) | 10 |
| | Rotation angle(°) | 0 |
| Cross-eye jammer | Antenna number | 4 |
| | Array baseline/m | 10 |
| | Jamming range/m | 1 000 |
| | Rotation angle(°) | 30 |

5.1 Effect of loop differences

Considering that the loop differences are independent of the baseline ratio and the JSR, we assume that the baseline ratio is 0.8 and the platform skin return is isolated. Other values of the baseline ratio and the JSR do not affect the conclusions. We investigate the effect of the phase difference \( \Delta \phi_2 \) and the amplitude ratio \( \Delta c_2 \), respectively.

The effect of the phase difference in the loop difference \( \Delta \phi_n \) is analyzed by plotting contours of the specified angle factor, when the amplitude ratio \( \Delta c_2 = -0.5 \) dB, and six cases of the phase difference \( \Delta \phi_2 \) are considered, as shown in Fig. 2. The other system parameters are given as follows: \( a_2 = -0.5 \) dB, \( \phi_2 = 180^\circ \), \( \phi_1 = 0^\circ \), and \( \Delta c_2 = -0.5 \) dB. The contours with specified angle factors illustrate the tolerated errors of the system parameters. To obtain the specified angular error, the combination of \( a_n \) and \( \phi_n \) needs to be valued inside the contour. The infinite angle factor in Fig. 2 means that the victim radar cannot lock its target when the corresponding cross-eye gain magnitude is larger than \( G_I \) which approximates to 14.5 for the jamming parameters considered. The results in Fig. 2 show that the contour of the specified angle factor becomes smaller and smaller when the phase difference \( \Delta \phi_2 \) in the loop differences increases from 0° to 180°. It means that the larger phase difference will lead to stricter tolerance requirements for the specified angle factor. Hence, an important conclusion from Fig. 2 is that the phase difference severely affects the tolerance performance of the L-MRCJ system.
Fig. 2 Contours of specified angle factor of an L-MRCJ system for $(\alpha_1, \phi_1)$ when the phase difference varies.

The phase difference in loop differences takes the cancellation between the channel returns and weakens the jamming performance of the jammer. When $\Delta \phi_2 = 180^\circ$, the cancellation becomes the largest and the tolerance requirements for the specified angle factor are the strictest.

Furthermore, the existence of the phase difference in loop differences makes the contours of the specified angle factor skew as shown in Fig. 2. This phenomenon will cause huge difficulties to design the optimum tolerance point for the specified angle factor. It can be found that the optimum tolerance point that endures the largest tolerated errors of the system parameters varies when the phase difference varies. Although the contours for the $180^\circ$ case are not skew, the contours for the positive and negative cross-eye gains reverse their positions, which is the worst case.

Furthermore, the comparison between Fig. 2(b) and Fig. 2(f) demonstrates that the sign of the phase difference determines the skew direction of the contours.

The effect of the amplitude difference in loop differences is illustrated by Fig. 3. The other system parameters are given as follows: $a_2 = -0.5 \, \text{dB}$, $\phi_2 = 180^\circ$, $\Delta \phi_2 = 0^\circ$, and $c_1 = 0 \, \text{dB}$. The phase difference $\Delta \phi_2 = 0^\circ$ and three cases of the amplitude ratio $\Delta c_2$ are considered. An observation from Fig. 3 is that the jammer system with the largest amplitude difference has the smallest contours of the specified angle factor. It suggests that the existence of a large amplitude difference also makes the tolerance requirements on the MRCJ system strict. However, the effect of the amplitude difference is limited compared to the effect of the phase difference. Even a large amplitude difference of 10 dB can still obtain considerable tolerances of system parameters. For example, the tolerances of $\alpha_1$ and $\phi_1$ for an angle factor of 7 are at least 1.55 dB and 10.2° as shown in Fig. 3(c), respectively.

Fig. 3 Contours of specified angle factor of an L-MRCJ system for $(\alpha_1, \phi_1)$ when the amplitude difference varies.
Another conclusion arising from Fig. 3 is that the amplitude difference in loop differences determines the place of the center of the contours. This will also make it difficult to design the optimum tolerance point. Taking the case $\Delta c_2 = -2$ dB as an example, the center is at the place that $a_1 = 0.38$ dB as shown in Fig. 3(a). Actually, the center of the contours can be computed from (3) under the values of $\phi_1 = \phi_2 = 180^\circ$, and is given by

$$a_1 = 1 + (1 - a_2) \Delta c_2.$$  \hspace{1cm} (28)

In conclusion, the loop differences severely affect the tolerance sensitivity of the L-MRCJ system, making the tolerance requirements stricter and the optimum tolerance point hard to be designed.

5.2 Effect of the baseline ratio

According to the definition of the baseline ratio, its value varies from 0 to 1. Without loss of generality, we consider two values of the baseline ratio which are 0.1 and 0.8, respectively. The contours of the specified angle factor of the MRCJ system for the two baseline ratios are plotted in Fig. 4 where the loop differences are assumed to be perfectly compensated for, i.e., $C_n = 1$, and the platform skin return is isolated.

An important observation from the comparison between Fig. 4(a) and Fig. 4(c) is that the contours are much smaller in Fig. 4(a) for the same specified angle factor. The small baseline ratio makes the tolerance requirements of system parameters stricter. This conclusion can also be obtained from the comparison between Fig. 4(b) and Fig. 4(d). Hence, the baseline length of the inner jammer loop should be designed as large as possible to obtain loose tolerance performance. However, the baseline ratio approaching 1 is not the best value in the practical application because it needs a high isolation between the antennas of the jammer loops.

On the contrary, when $a_n > 0.47$ dB, the contours of the specified angle factor in Fig. 4(a) are larger than those relevant contours in Fig. 4(b). The reason for this observation is that the baseline ratio in the numerator of (3) weakens the contribution of the inner jammer loop, which needs a greater contribution of the outer jammer loop to achieve the specified angle factor. For the case $a_n < 0.47$ dB where the angle factor in Fig. 4 corresponds to the positive cross-eye gain, the large difference-channel return of the outer jammer loop needs a small amplitude and phase mismatching, i.e., $a_1 \to 1, \phi_1 \to 180^\circ$, which results in strict tolerance requirements of $(a_1, \phi_1)$ compared to $(a_2, \phi_2)$. However, for the case $a_n > 0.47$ dB where the angle factor corre-
sponds to the negative cross-eye gain, the large difference-channel return of the outer jammer loop needs a large amplitude and phase mismatching, i.e., \( a_1 \rightarrow 1, \phi_1 \rightarrow 180^\circ \), resulting in loose tolerance requirements of \((a_1, \phi_1)\).

Furthermore, the baseline ratio of 0.8 is so large that the equivalent contours in Fig. 4(c) and Fig. 4(d) have little difference. It means that the decrease of the contribution of the inner jammer loop can be ignored when the baseline ratio is 0.8.

Hence, the tolerance requirements of \((a_1, \phi_1)\) are quite similar to those of \((a_2, \phi_2)\) when the baseline ratio is large enough. This is the reason why we only investigate the tolerance requirements of \((a_1, \phi_1)\) in Subsection 5.1 and Subsection 5.3 next. Hence, the moderate baseline ratio is advised. A large baseline ratio will result in the isolation problem, and a small baseline ratio will lead to poor jamming performance. The baseline ratio of 0.8 may be an applicable value for a practical jammer.

### 5.3 Effect of platform skin return

We consider two values of the JSR to investigate the effect of the platform skin return, which are 10 dB and 30 dB, respectively. The contours of the specified angle factor are plotted in Fig. 5 and Fig. 6 for the 10 dB and 30 dB JSR cases, respectively.

![Fig. 5 Contours of specified angle factor of an L-MRCJ system for \((a_1, \phi_1)\) when the JSR is 10 dB and \(a_2 = -0.5\) dB, \(\phi_2 = 180^\circ\)](image)
Considering that the JSR has two expressions in (25), the borderline between the two different JSR expressions is denoted as $A$ on the left axis in Fig. 5 and Fig. 6. The value of the borderline $A$ is calculated by $A = 1/c_1$, and is $0 \text{ dB}$ when $C_1 = 1$. Hence, the expressions of the JSR can be given by

$$\text{JSR} = \begin{cases} \frac{a_1}{a_2}, & a_1 > A \\ \frac{1}{a_2}, & a_1 \leq A \end{cases}. \quad (29)$$

It should be noted that, under the case that the JSR varies with system parameters of $a_n$ when $a_1 > A$, the jammer can still achieve the specified angle factor with a constant value of the JSR. The reason is that, for a constant value of JSR, the parameter $a_n$ incorporating the gain of the cross-eye jammer and its antennas varies when the system parameters vary.

Different JSR definitions will affect the trend of the contours especially for the extreme case as shown in Fig. 5. When enhancing the value of the JSR, the effect of different JSR definitions will be weakened, which can be observed from Fig. 6.

For the $10 \text{ dB}$ JSR case, there are obvious differences between the curves of the two extreme cases and the median case as shown in Fig. 5. The first observation from the comparison between the extreme and median cases is that both the maximum case and the minimum case can obtain larger angle factors than the median case, and even obtain an infinite angle factor. However, the region with contours of infinite angle factors is very unstable, where an angle factor of 9 is included in the infinite region. Actually, the $10 \text{ dB}$ JSR makes the extreme cross-eye gain suffer from drastic variation when the system parameters bounded by the region make $a_2^2 + b_2^2$ approach $a_2^2$. Hence, the tolerance performance of the MRCJ system for the $10 \text{ dB}$ JSR case is dissatisfied due to the strict tolerance performance for the median case and the unstable tolerance performance for the extreme cases.

For the $30 \text{ dB}$ JSR case, an infinite angle factor can be obtained both by the two extreme cases and by the median case as shown in Fig. 6. Furthermore, the contours for the median case in Fig. 6(b) are much larger than those in Fig. 5(b). Hence, the larger JSR will make the L-MRCJ system obtain a larger specified angle factor and looser tolerances. Another observation from the comparison between the contours of the extreme and median cases is that the three series of contours show good agreement with each other when the JSR is $30 \text{ dB}$. It suggests that a high JSR value can make the extreme cross-eye gain inclined to the median gain. We can further assert that the agreement between the median case and the extreme cases will be much better when the value of the JSR increases. This is because a high JSR makes the amplitude scaling $a_n$ of the platform skin return become a negligible value compared to the jammer return, and makes the statistical cross-eye gains approximately equal to the constant isolated cross-eye gain. It hence comes as no surprise that a high JSR value is advised for the cross-eye jammer.

6. Conclusions

A comprehensive investigation of parameter tolerances of an L-MRCJ system is presented in this paper. Especially, the effects of the influence factors on tolerance performance of the L-MRCJ system are analyzed. Valuable advice for building a practical L-MRCJ system is proposed.

The loop differences severely affect the tolerance performance of the L-MRCJ system. A large phase difference of loop differences makes the contours of the specified angle factor small and skew. Meanwhile, the amplitude difference of loop differences mainly determines the position of the center of contours.

A small baseline ratio makes the tolerance requirements on system parameters strict. Hence, the baseline length of the inner jammer loop should be designed as large as possible to achieve loose tolerance requirements in practice.

The tolerance performance of the L-MRCJ system for a low JSR is dissatisfied due to strict tolerance requirements for the median case and unstable tolerance requirements for the extreme cases. When the value of the JSR increases, the tolerance requirements for the median case become loose and the agreement between the median case and the extreme cases are much better. Hence, a higher JSR value is required for the L-MRCJ system to obtain better tolerance performance.

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