Spatial entanglement in optical parametric oscillators with photonic crystals

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The effects of an intracavity photonic crystal in a multimode optical parametric oscillator are studied, with a special focus on quantum fluctuations. The capability to either stimulate or inhibit the spatial instability, lowering or increasing the parametric threshold, allows to control the intensity fluctuations and correlations. A significative quantum noise reduction and an increase of the range of squeezed quadratures are found above threshold where spatial Einstein-Podolsky-Rosen entanglement and inseparability are found.

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Photonic crystals (PC) are dielectric media with periodic modulation of the refractive index which can lead to gaps in the allowed frequencies of electromagnetic waves [1]. Seminal works predicted the possibility to control spontaneous emission by PC [2] when suppressing a radiative transition with frequency within the photonic band-gap. After two decades of intense experimental activity, these engineered media provide an unprecedented control of light confinement, guiding, and propagation [1][3]. Recently, inhibition of spontaneous emission [4] has been experimentally shown, improving the extraction efficiency of light emitting devices [5] and redistributing where needed the corresponding energy [6]. The use of PC for environment (dissipation) engineering is also the basis of intense research activity about non-Markovian evolution of quantum states [7]. An unexplored issue we raise here is if a PC can be used to control or even suppress light fluctuations to improve quantum correlations in the continuous variable regime [8] in driven devices emitting entangled beams. Quantum correlated bright beams can be generated by optical parametric oscillators (OPOs). These are common devices giving squeezing [9] and entanglement [10] between modes with different polarization or frequency. In 1992 similar quantum effects were also predicted between different spatial modes [11], extending the study of quantum effects to multimode OPO [12][13], Kerr media [11][13], and second harmonic generation [15]. Different applications based on spatially multimode operation have been recently demonstrated in optical switching [16], quantum imaging [17], metrology [18], and quantum information [19]. Considering both theoretical activity and experimental achievements, the subject of spatial entanglement is becoming a mature research topic.

In this Letter we explore how to control and improve multimode squeezing and entanglement by means of an intracavity photonic crystal (PC) in a nonlinear device. The prototype system considered is a multimode degenerate OPO, well studied both below and above threshold [12][13], when a PC is introduced. Within the proposed photonic-crystal optical parametric oscillator (PCOPO), the spatial modulation is not changing the environment spectrum of fluctuations [7] but is instead modifying the Hamiltonian of the intracavity process. The PC modulation takes place in the transverse plane—the device is longitudinally monomode—and is modeled by a spatial profile of the otherwise homogeneous refractive index. Intracavity PC have been predicted to allow for inhibition of the phenomenon of pattern formation in Kerr and singly resonant OPOs [23][24] by an increase of the pump energy needed to cross the instability threshold. The phenomenon arises when unstable spatial modes are in the band-gap because their emission is prevented, as recently observed in two independent experimental set-ups [25]. Here we will show that in PCOPO the possibility to tune the spatial instability—matching-up to the parametric threshold—is actually wider. Indeed, the PC allows to either increase or reduce the threshold energy due to the mixing of different frequency waves. Related to the instability shift is the possibility to control the fluorescence intensity below threshold and the spatial distribution of quantum fluctuations. After characterizing quantum correlations in different regimes, two major effects are identified above threshold. First of all, the PC provides a locking mechanism to freeze the known diffusive drift motion of the pattern [14] reducing by orders of magnitude the fluctuations in the associated quadrature. A larger range of quadratures is then found to be squeezed, decreasing the sensitivity to the choice of the local oscillator phase in squeezing measurements. Moreover, we find that spatial modes can be entangled above threshold when introducing the PC in the OPO, either considering state inseparability [21] or the Einstein-Podolsky-Rosen criterion of Ref. [20].

The master equation for multimode type I degenerate OPO was described in detail by Gatti et al. [12]. A description valid both below and above threshold (for pump $|\alpha_0| < 2$) can be obtained through a mapping into the Q-representation as discussed in Ref. [13]. This leads to
nonlinear Langevin equations for the spatially dependent pump $\alpha_0$ and signal $\alpha_1$ fields

\[
\begin{align*}
\partial_t \alpha_0(x, t) &= - \left[ (1 + i\Delta_0(x)) - i\nabla^2 \right] \alpha_0(x, t) + E - \frac{1}{2} \alpha_0^2(x, t) + \xi_0(x, t) \\
\partial_t \alpha_1(x, t) &= - \left[ (1 + i\Delta_1(x)) - 2i\nabla^2 \right] \alpha_1(x, t) + \alpha_0(x, t)\alpha_1^*(x, t) + \xi_1(x, t),
\end{align*}
\]

with $E$ input field, $\nabla^2$ diffraction, and $\xi_0$ additive and $\xi_1$ multiplicative phase-sensitive white noises \[28\]. The PC refractive index modulation is modeled by introducing spatial dependent detunings $\Delta_0(x)$ and $\Delta_1(x)$, which can have different amplitudes and, in the simplest case, have the same periodicity with wave-number $k_{pc}$. The main mechanism we aim to explore is the effect of the band-gap on the spatially multimode down-conversion process. As known, modulation instability in OPO with negative signal detuning appears at wave-number $k_c = \sqrt{-\Delta_1/2}$ \[20\]. Therefore, the most interesting configuration is for a (sinusoidal) modulation with $k_{pc} = 2k_c$, since in this case the signal pattern would be in the photonic band-gap \[23\].

We start considering the effects on the quantum fluctuations in the PCOPO below threshold. On average the signal field vanishes everywhere $\langle \hat{A}_1(x) \rangle = 0$ but, due to the nonlinearity of the medium, it is not in a coherent vacuum state, either with or without the PC. When spatial modes are considered in the far field, the amplitude of intensity fluctuations is maximal at the critical wavenumber $k_c$ (Fig. 1b). As the least damped spatial mode falls within the PC band-gap, inhibition of the off-axis emission is expected \[20\]. This effect can be appreciated comparing the larger intensity fluctuations for the OPO (Fig. 1b, continuous line) with respect to the PCOPO with modulation of the signal detuning $\Delta_1(x)$ (Fig. 1c, dashed line). On the other hand, it is rather surprising to find that, for a fixed input energy, fluctuations in the PCOPO can also be increased by the modulation of the refractive index introduced by the PC (Fig. 1d, dot-dashed lines). In other words, when the signal detuning $\Delta_1$ is modulated, the intensity fluctuations are lowered \[4\], while when (also) the pump detuning $\Delta_0$ is modulated, such fluctuations increase, in spite of being in the PC band-gap.

The possibility to either lower or increase the fluctuations of the most intense spatial modes is due to the presence of wave-mixing between different frequencies in the parametric oscillator. Indeed, the fluctuations strength is inherently related to the proximity to the instability threshold in a nonlinear system driven out of equilibrium and approaching this point. Large fluctuations are a clear signature of an instability and this allows a consistent description of our results. In Fig. 1a, analytical average intensities in the linear approximation \[27\] are shown for the PCOPO below threshold, in different configurations. When the threshold is raised (pattern inhibition due to the PC \[23\]), the fluctuations strength is reduced, while—for the same input energy—if the threshold is lowered by introducing the PC then fluctuations increase. What we actually find is that, in presence of nonlinear mixing between different fields, the way in which the PC changes the instability of the signal is not trivial. We remind that when the stripe pattern at the critical $k_c$ appears in the signal field, the pump also develop a modulation at $2k_c$. Then for the detuning modulation $k_{pc} = 2k_c$ the unstable signal wavenumber is in the band-gap but the detuning profile somehow ‘stimulates’ the nonlinearly emerging pattern in the pump field. Therefore this PC modulation provides two competing mechanisms, inhibiting the signal spatial instability, as in Refs. \[23\], but also imprinting in the pump the nonlinear structure favoring the instability process. The vertical asymptotes in Fig. 1b, where the linear approximation for analytical calculations breaks down, show the different threshold values for $E$ depending on the PCOPO configuration. Notably, if the PC modulates the pump detuning, the parametric threshold can be crossed for values even lower than in the case of perfectly resonant OPO.

![FIG. 1: (Color online)](image)

(a) Steady intensity $\langle \hat{A}_1(k_c)^* \hat{A}_1(k_c) \rangle$ increasing with $E$ (below threshold). Here the symbols are results of numerical simulations of Eqs. \[1\] at different pump values, while lines represent analytical results within a linear approximation \[27\]. (b) $\langle \hat{A}_1(k)^* \hat{A}_1(k) \rangle$ from numerical simulations for $E = 0.9$. Different curves correspond to the OPO without PC with $\Delta_0 = 0$ and $\Delta_1 = -1$ (black solid line); $\Delta_1 = -1 + 0.5 \sin(k_{pc}x)$ and $\Delta_0 = 0$ (green dashed line); $\Delta_0 = 0.5 \sin(k_{pc}x)$ and $\Delta_1 = -1 + 0.5 \sin(k_{pc}x)$ (orange 3 dots-dashed line).

Apart from the strength of spatial fluctuations, an important aspect is the quantumness of the correlations. Non-classical effects in multimode OPO are known to exist between opposite far field modes $+k$ and $-k$ due to emission of photons pairs in the parametric down-conversion process \[11\], \[12\]. In particular, two-modes...
squeezing is studied considering the generic joint quadrature $\theta$

$$\Sigma_{\theta\phi}(k,-k) = (\hat{A}_1(k) + \lambda \hat{A}_1(-k)e^{i\phi})e^{i\theta} + \text{h.c.} \quad (2)$$

with $\phi$ relative phase between the superposed spatial modes. Here we take $\lambda = 1$. Squeezing achieved be-

low threshold increases with the pump intensity being maximum at the parametric threshold $[11,13]$. Due to the discussed PC effect on the parametric threshold, the squeezing attained in OPO and PCOPO will be com-
pared at the same distance from the respective thresholds. We find then that squeezing achieves similar values in the OPO and in the PCOPO modulating pump and/or signal detuning, as shown in Fig. 2b, being the major difference in the dependence on the angles $\theta$ and $\phi$.

A different scenario is found above threshold, consid-
ering squeezing between intense modes. In Fig. 2b, the variance of $\Sigma_{\theta\phi}$ (Eq. 2) for the OPO (upper line) is compared with the PCOPO (three lower lines). Even if the attained squeezing (minimum value of the plotted variance) is similar in all cases, there are important differences in the noise present in the unsqueezed quadrature (maximum value). Far from being in a minimum uncertainty state, the OPO displays an extremely large noise in the unsqueezed quadrature (black line in Fig. 2b) due to the well-known phase diffusion between down-converted modes and to excess noise in their relative phase $[20]$. In spatially multimode devices, the visible effect is a diffusive motion of the excited pattern and has been related to translational symmetry break and noise excitation of the corresponding neutral Goldstone mode $[13]$. On the other hand, due to the refractive index modulation, the PCOPO does not exhibit translational symmetry and the formed pattern is locked to the position of the PC. This leads to a strong reduction of noise in the PCOPO: in Fig. 2b there are two orders of magnitude between the variances of the unsqueezed quadrature in the PCOPO for modulated $\Delta_0$ and the OPO, as highlighted by the horizontal gray stripe. An important consequence in view of applications is that the reduction of fluctuations in unsqueezed quadratures leads to a significant increase of the range of quadratures with sub-shot-noise fluctuations, as highlighted by the two vertical stripes in Fig. 2b. A PCOPO will indeed be more robust to changes in the choice of the local oscillator phase $\theta$ as more quadratures are actually squeezed.

![Figure 2: (Color online) Variance of $\Sigma_{\theta\phi}$ (from Eq. 2 with $\lambda = 1$) as a function of the quadrature angle and for the superposition angle $\phi$ giving the largest squeezing for each OPO and PCOPO configuration. Pump field 5% below threshold (a) and 2% above threshold (b). The horizontal dashed line is the shot noise and other lines as in Fig. 4. The small arrow shows the deviation between different numerical runs in the OPO, due to phase diffusion.](image)

Multimode OPOs allow to generate not only squeezed but also spatially entangled states $[12,29]$. A series of key experiments recently demonstrated spatial entangle-
ment between light beams (continuous variable regime) in different optical nonlinear devices $[17]$. Here we show how entanglement in parametric oscillators is changed by the presence of a photonic crystal, considering two well-known criteria $[20,21]$. One distinguishes states exhibiting Einstein-Podolsky-Rosen (EPR) paradox $[31]$ for conditional variances such that

$$\mathcal{E} = \Delta^2 \Sigma_{\theta_0,\phi_0} \Delta^2 \Sigma_{\theta_0 + \pi,\phi_0 + \pi} \leq 1, \quad (3)$$

for some choice of superposition and interference angles $\theta_0, \phi_0 [20]$. Notice that here the parameter $\lambda$ minimizes each variance of the joint quadrature (2), as also reviewed in Ref. [22]. A second measure we consider is the inseparability condition

$$\mathcal{I} = \Delta^2 \tilde{\Sigma}_{\theta_0,\phi_0} + \Delta^2 \tilde{\Sigma}_{\theta_0 + \pi,\phi_0 + \pi} \leq 2(a^2 + \frac{1}{a^2}). \quad (4)$$

with $\Sigma_{\theta_0,\phi_0} = (a\hat{A}_1(k) + a^{-1}\hat{A}_1(-k)e^{i\phi})e^{i\theta} + \text{h.c.}$ and positive parameter $a \geq 1$. Below threshold, the presence
of the PC changes significantly the intermode correlations of this systems leading to new not vanishing terms with respect to the case of the OPO (for instance in the PCOPO $\langle \hat{A}_1^2(k) \rangle \neq 0$), but as for squeezing, similar results are found for the attained entanglement. The most significant differences are found, again, above threshold where the presence of the PC enhances quantum effects leading to a spatially entangled state. In this regime, the mentioned phase diffusion [14] leads to spikes at low frequency in noise spectra, preventing entanglement in the OPO, as we show in Fig. 3 and c. On the other hand, for a non translational invariant system such as the PCOPO, we find significant regions in which both EPR paradox [3] and state inseparability [4] are predicted. We stress that in Fig. 3 OPO and PCOPO are compared at the same distance form the instability threshold and that, even if the best performance is obtained when pump detuning is modulated (Fig. 3 and d) we find that all configurations of the PCOPO show some entanglement, degraded when removing the PC.

Summarizing, we have analyzed the effect of a PC in a multimode OPO whose detuning suffers a transverse spatial modulation, in the case in which the spatially unstable mode appearing at threshold falls within the band-gap. Due to the presence of nonlinear mixing between waves at $2\omega$ and $\omega$ there are competing phenomena leading to either inhibition and enhancement of intensity quantum fluctuations for a fixed pump below threshold. The possibility to control with the PC the intensity of quantum fluctuations is related to the raise (pattern inhibition) and lowering of the parametric threshold. We find that, below threshold, the attained spatial squeezing as well as entanglement in the signal field are preserved at a fixed distance form the parametric threshold. Notably, above threshold, the break of the translational invariance due to the PC provides a strong mechanism to reduce (up to two orders of magnitude) the quantum fluctuations associated to pattern diffusion leading to squeezing over a significantly larger range of quadrature angles. This would reduce the sensitivity of the measurable squeezing with the choice of the phase of the local oscillator. Moreover, the strong spatial locking due to the presence of the PC in the OPO allows to generate inseparable as well as EPR entangled spatial beams. This analysis, restricted to one transverse dimension, is expected to give similar results when extended to 2D for PC modulated only in one direction, as also confirmed by our analytical results below threshold [27]. More complex is the case of a PC with transverse hexagonal or square geometries where the same pattern selection process is an open question.

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