Stochastic modelling of a wind turbine’s power output with special respect to turbulent dynamics

Julia Gottschall and Joachim Peinke
ForWind - Center for Wind Energy Research, Institute of Physics, Carl-von-Ossietzky University of Oldenburg, 26111 Oldenburg, Germany
E-mail: julia.gottschall@forwind.de

Abstract. A frequent challenge for wind energy applications is to grasp the impacts of turbulent wind fields properly. The wind turbine power performance curve, usually determined applying the IEC standard 61400-12, provides a functional relation between the measured wind speed \( u \) and the corresponding power output \( P \) of the turbine. But it is not sufficient to describe the actual dynamics of the power output on small time scales. Recently, we introduced an alternative method to estimate the power curve of a wind turbine based on high-frequency data. The crucial point of this method is that we first reconstruct the wind turbine’s entire response dynamics that, then, yields the power curve by a fixed-point analysis. To reconstruct the power conversion dynamics of the wind turbine, we apply a dynamical systems approach, assuming that the variable \( P(t) \) follows a diffusion process. Its evolution in time is given by a Langevin equation consisting of a deterministic relaxation term and a stochastic noise term. The coefficients defining these terms can directly be extracted from the given data sets. On the basis of simulated data, we verified the method and showed that it is more accurate than the standard method. For measured data we reconstructed the deterministic dynamics of a representative wind turbine and estimated the corresponding power characteristics to demonstrate the application of the method. The split of the dynamics into a deterministic and a stochastic part allows to distinguish the actual behaviour of the wind turbine, i.e. relaxation and control effects, from the turbulent effects of the wind. We gain an understanding of something like the turbulent dynamics of wind energy conversion and obtain a method to describe the actual fluctuations in power output.

1. Introduction – the topic of turbulent dynamics

Speaking about turbulent wind, we have something like large wind velocity fluctuations on small scales in mind. A fluctuation can be described by the spatial or time increment, respectively, \( x_\tau \equiv x(t + \tau) - x(t) \) of a variable \( x \), where \( \tau \) denotes the corresponding distance in space or in time. A typical wind fluctuation is then defined by \( u_\tau \equiv u(t + \tau) - u(t) \) with the horizontal wind velocity \( u \) and a varying time increment \( \tau \), usually, in seconds.

Having investigated the probability distribution of such wind fluctuations, Böttcher and coworkers have stated in [1] that these do not follow normal, i.e. Gaussian, statistics, but show an intermittent behaviour. Figure 1 shows typical increment pdfs (probability density functions) for wind velocity data measured at the site we refer to in this article. The data had been recorded by an ultrasonic anemometer at the hub height of the wind turbine we investigated. Comparing the increment pdfs on three different time scales with the corresponding Gaussian
pdfs that have the same standard deviation, shown as a solid line in figure 1, we observe an increased probability for large events of the measured data, i.e. higher fluctuations than expected on the basis of a normal distribution. These so-called heavy tails are typical for atmospheric wind velocity statistics and can be found, other than for laboratory data, on all time scales. In fact, the atmospheric wind differs from the laboratory data in its instationarity that is closely related to the observed intermittency ([1]).

A question of not little importance is now, what these intermittent statistics mean for the performance of a wind turbine. Applying a Gaussian, non-intermittent, wind model, extreme load alternations are apparently underestimated. But also the description of the electrical power output must be based on a preferably realistic model to predict e.g. the power quality in a satisfying manner.

In the right part, figure 1 also shows the distributions of power increments $P_\tau \equiv P(t+\tau) - P(t)$, where $P(t)$ is a time series of the measured electrical power output, again for three different time scales and in comparison with the corresponding Gaussian pdfs. It is obvious that the intermittent wind statistics is transfered to the power increments, somehow. The additional structures, the bumps around $5\sigma$, are due to a discretization of data, not due to control effects as observed for other data sets. They appear for time increments that are smaller than the actual time scale of the process dynamics, and solely indicate that the process cannot be described correctly on these time scales by the increment statistics.

It is our aim, now, to investigate the impact of turbulent wind of a wind turbine’s power output in detail, to find a suitable model that describes the actual short-time dynamics of the power conversion process, and to derive on this basis power performance characteristics more effectively.

**Figure 1.** Statistics of wind velocity (left) and power output increments (right) for $\tau = 5$ sec, 15 sec and 60 sec (from top to bottom), Gaussian pdf with corresponding standard deviation as red solid line. The increment values are normalized to the particular standard deviations.

---

1 For a comparison of the loads estimated based on an intermittent and a conventional wind model, used in the standard guidelines, see [3].
2. Deriving wind power characteristics

The most natural way to grasp the power conversion process is to derive the wind turbine power performance curve, or short power curve. The common and revised standard method for the determination of wind turbine power curves is given by [2], referred to as IEC 61400-12-1. This procedure can be summarized by relating the averages of wind velocity and power output over 10 min., i.e. \( \langle u(t) \rangle_{10\text{min.}} \rightarrow \langle P(t) \rangle_{10\text{min.}} \), and averaging in a second step all values lying in a wind velocity bin of the width of normally 0.5 m/s.

Using only averaged data, the IEC standard neglects any short-time dynamics and, therefore, cannot be the right approach to handle fluctuations due to turbulence. For this reason we motivate an alternative approach that is just based on high-sampled data (compare with [4], [5]), and for which we have introduced the term dynamical method or dynamical power curve, respectively ([6]). The basic idea is to describe the power conversion process as a diffusion process, i.e. a stochastic process that satisfies the Markovian property and that can be separated into a drift and a diffusion part. Then a typical time series can be presented as \( P(t) = P_{\text{stat}}(u) + p(t) \), where \( P_{\text{stat}} \) denotes a stationary power value dependent of the wind velocity \( u \), and \( p(t) \) the corresponding short-time fluctuation around this value. To be more explicit, the stochastic part summarizes all the otherwise unseizable microscopic interactions and enables the macroscopic description of the system. I.e. it takes also into account that the wind velocity \( u \), measured at the met mast, cannot represent the complete wind field actually acting on the wind turbine. Figure 2 illustrates the assumption that the power conversion process can just be described by such a relaxation. The actual trajectory \( P(u) \) curls around the power curve, driven by the fluctuating wind velocity.

![Figure 2. Dynamics of the power output – trajectory based on measured data (black line) and power curve (red dashed line). On the left time series of 120 sec duration (sampling frequency 10 Hz), on the right a cutout of 20 sec duration.](image)

The time series \( P(t) \) is assumed to be stationary with respect to a certain wind velocity or a velocity interval. Therefore, the dynamics of \( P(t) \) is analyzed for each selected velocity interval or bin separately. (It is convenient to use the binning as proposed in the IEC standard.) For the evolution in time of the variable \( P \) we formulate a Langevin equation.
\[
\frac{d}{dt} P(t) = D^{(1)}(P; u) + \sqrt{D^{(2)}(P; u)} \cdot \Gamma(t).
\]  

\(D^{(1)}\) is called drift coefficient and represents the deterministic part of the process, whereas the diffusion coefficient \(D^{(2)}\) together with the Langevin force \(\Gamma(t)\) representing \(\delta\)-correlated Gaussian white noise (\(\langle \Gamma(t) \rangle = 0\) and \(\langle \Gamma(t_1)\Gamma(t_2) \rangle = 2\delta(t_1 - t_2)\)) describes its stochastic part.

A reconstruction of this dynamics enables the estimation of the values \(P_{\text{stat}}(u)\) and with it the determination of the power characteristic. The coefficients \(D^{(n)}\) for \(n = 1, 2\) are given by the conditional moments \(M^{(n)}\) that can be directly calculated from the data –

\[
D^{(n)}(P; u) = \frac{1}{n!} \lim_{\tau \to 0} \frac{1}{\tau} M^{(n)}(\tau, P; u)
\]

with

\[
M^{(n)}(\tau, P; u) = \langle [P(t + \tau) - P(t)]^n \rangle |_{P(0) = P}.
\]

Corresponding errors are calculated according to [7] with

\[
\sigma[D^{(1)}(P; \tau; u)] = \sqrt{\frac{2}{\tau} \frac{D^{(2)}(P; \tau; u)}{N} - \frac{[D^{(1)}(P; \tau; u)]^2}{N}}
\]

and for \(D^{(2)}\) similar, considering a finite time increment \(\tau > 0\) and a finite number of data points \(N\) in the bin. A typical result for \(D^{(1)}\) is shown in the left part of figure 3.

Finally, a deterministic fixed-point analysis for each velocity bin, i.e. \(D^{(1)}(P_{\text{stat}}; u) \equiv 0\), yields the stationary points \(P_{\text{stat}}(u)\) and with it the dynamic power curve. As alternative to finding the zero-crossing of the drift coefficient, we calculate the drift potential \(\phi_{D^{(1)}}(P) = -\int P \cdot D^{(1)}(\tilde{P})d\tilde{P}\) and identify its minimum.

\[\text{Figure 3.} \quad \text{Fixed-point analysis for the velocity bin } u = (11.25 \pm 0.25) \text{ m/s. The fixed point is found by the zero-crossing of } D^{(1)}(P) \text{ or the minimum of the drift potential (full circle in the diagram on the right). The corresponding result due to the IEC standard is given by the open circle in respect to the drift potential.}\]

To demonstrate the proceeding and also the excellence of the dynamical method, we simulated power output data based on measured wind velocity data as input and assuming a numeric
diffusion model as described above. The analyzed data set consists of approximately 1728000 data points, corresponding to a period of two days and a sampling frequency of 10 Hz. Figure 3 illustrates the fixed-point analysis, and figure 4 shows the reconstructed power curve in comparison with the results due to the IEC standard procedure. We observe significant differences, and with respect to the real power characteristic, as assumed for the simulation, much better results following the dynamical approach.

Figure 4. Comparison of power curves due to IEC standard (full triangles, dashed line) and dynamical method (open squares with error bars) for simulated data – complete curve and cutouts. The real power curve, the input for the data simulation, is given by the solid line.

To illustrate the application to measured data, figure 5 shows a complete reconstructed drift field and the corresponding power characteristic given by the single fixed points for each velocity bin. The data had been obtained from a commercial MW-class wind turbine, located in a wind park in the mid-western part of Germany. The placement of the met mast satisfied the requirements stated in the IEC 61400-12-1 norm.

The information in figure 5 denotes that the dynamical approach potentiates considerably more than only to derive the power curve of a wind turbine. Since we consider the short-time dynamics of the conversion process, at best on its natural time scale, and reconstruct an overall model, we should be in the position to simulate the actual process with this information. That would mean that we can also describe the power fluctuations in the correct manner.

To inspect this implication, we simulated power output time series with regard to both the IEC approach and the dynamical model. For the IEC approach this means that we multiply the input velocity time series $u(t)$ with the derived power curve, the result is denoted by $P_{IEC}(t)$. For the dynamical model we integrate (1) with the coefficients derived from the measured data to obtain a time series $P_{rec}(t)$. The corresponding increment distributions are shown in figure 6. For both models we observe a high intermittent behaviour, the intermittency of the velocity increment distribution is amplified, so to speak, by the nonlinear power curve, scaling roughly cubically.
Figure 5. Deterministic drift field $D^{(1)}(P; u)$ (given by the arrows - the size of the arrows corresponds to the magnitude of $D^{(1)}$) and reconstructed power characteristic (full circles) for measured data.

Figure 6. Statistics of reconstructed power output increments for $\tau = 5$ sec, 15 sec and 60 sec (from top to bottom) - for $P_{IEC}(t)$ on the left and $P_{rec.}(t)$ on the right. Gaussian pdfs with corresponding standard deviation are shown as red solid line. The increment values are normalized to the particular standard deviations.

Figure 7 compares the results of the two approaches with the measured power output increments already shown in figure 1. For a time increment $\tau = 15$ sec the dynamical method provides well fitting results, likewise for the smaller time scale of $\tau = 5$ sec though not reproducing the artefacts due to discretization. For $\tau = 60$ sec both, the IEC and the
dynamical reconstruction approach, overestimate the power output fluctuations. (Note that at the present state we have systematic errors for $D^{(2)}$ due to too low sampling frequencies of the measured data, so that a reconstruction is not absolutely perfect up to now and can be improved further.) This consideration is not supposed to compare the IEC power curve with the dynamic one, but to indicate the potentials of a dynamic approach - and not least, it provides a frame for this paper starting with the increment statistics in figure 1.

![Figure 7](image-url)  

**Figure 7.** Statistics of reconstructed power output increments ($P_{IEC,\tau}$ as blue open squares and $P_{rec,\tau}$ as black dots) compared to the measured values (red line) for $\tau =$ 5 sec, 15 sec and 60 sec (from left to right).

3. Conclusions and discussion

We have emphasized the impact of short-time dynamics for a proper description of the power conversion process of a wind turbine and showed that a dynamical approach is not only appropriate but also yields more accurate results for the power curve than the standard IEC approach. The actual complex of problem can be summed up by handling the dynamical noise in the process in the right manner. The turbulent behaviour of the wind is transferred to the wind turbine’s power output and is described by the stochastic part of the Langevin equation in the dynamical approach. Since the actual dynamics of the wind turbine, i.e. its relaxation and control behaviour, is grasped by the deterministic drift part of the equation, a separation from the dynamical noise is possible. Thus, we obtain a power characteristic that is representative for a specific wind turbine but independent of site-specific parameters.

The power characteristic we obtain applying the dynamical method is of another type than the IEC standard curve and cannot replace it that easily. Instead of mean values we estimate fixed points, following the assumption that fixed points are more close to the actual dynamics we are interested in than mean values.

**Acknowledgments**

We would like to acknowledge the helpful suggestions from and discussions with St. Barth, E. Anahua and D. Kleinhans in the past. This work is supported by the BMBF (German Federal Ministry of Education and Research) within the cooperative project ”Wind turbulences and its impact on the utilization of wind energy”. 
References

[1] Böttcher F, Barth St and Peinke J 2007 Stoch. Environ. Res. Ris. Assess. 21 299-308
[2] IEC 2005 Wind turbines - Part 12-1: Power performance measurements of electricity producing wind turbines (IEC 61400-12-1 International Standard)
[3] Gontier H, Schaffarczyk A P, Kleinhans D and Friedrich R 2006 Comparison of aerodynamic loads from new turbulence models deduced by statistical fluid mechanics with those used in standard guidelines, as well as, Kleinhans D, Friedrich R, Gontier H and Schaffarczyk A P 2006 Simulation of intermittent wind fields: a new approach, both in Proceedings of the DEWEK conference (published on CD)
[4] Siegert S, Friedrich R and Peinke J 1998 Phys. Lett. A 243 275-280
[5] Siefert M, Kittel A, Friedrich R and Peinke J 2003 Europhys. Lett. 61 466-472
[6] Anahua E, Barth St and Peinke J 2007 Wind Energy - Proc. of the Euromech Colloquium ed Peinke J, Schaumann P and Barth St (Berlin: Springer) 173-177; Wind Energy submitted
[7] Kleinhans D and Friedrich R 2007 Wind Energy - Proc. of the Euromech Colloquium ed Peinke J, Schaumann P and Barth St (Berlin: Springer) 129-134