Numerical modeling of pulse-pumped ultrafast regenerative amplifier with unstable pump laser

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Abstract. A numerical model of a regenerative amplifier based on the Frantz-Nodvik equation with spatially dependent stored energy density in the active medium has been developed. The model was used to investigate the stability of the output energy of the amplifier associated with fluctuations of the pump energy and pump beam position. It was found that, same as in the known case with the fluctuations of the pump energy, the influence of the fluctuations of the pump beam position is much weaker at oversaturated operation of the regenerative amplifier. Opposite to fluctuations of the pump energy, fluctuations of the pump beam position result in highly-asymmetric histogram of the output energy, which is highly skewed to the lower energies.

1. Introduction
Femtosecond chirped-pulse amplification (CPA) systems face severe stability problems originating from the performance of their components. Crucial factors degrading the stability of the output energy of a CPA system are fluctuations of the energy, beam-pointing stability and timing jitter of the pump laser. Frequency-doubled Q-switched neodymium lasers are widely used as pump sources for both regenerative and multipass Ti:Sapphire amplifiers. Cr:Forsterite amplifiers are usually pumped with fundamental-frequency neodymium lasers. Typical beam-pointing stability of commercially available neodymium pump lasers is ±25-50 µrad, while typical energy stability is around 0.3-1% (r.m.s.) for diode-pumped lasers and 1-3% for lamp-pumped lasers. Another stability problem associated with the lamp-pumped lasers is that their output intensity distribution in the beam cross-section can notably vary from shot to shot. Despite their poor stability, lamp-pumped lasers are widely used as pump sources for ultrafast amplifiers with low pulse repetition rates (1-20 Hz) due to their low cost and high energy as compared to the diode-pumped lasers.

Regenerative amplifiers (RA) are used of themselves or as the first stage of the amplifier chain in high-energy CPA systems. Poor pulse-to-pulse stability of the pump laser can significantly degrade the stability of the output energy of the RA and thus worsen the consistency of the output data in a real laser experiment. The influence of the pump instabilities (in terms of both pump energy and pump beam position) can be different in various operation conditions of the amplifier [1,2]. Besides the output laser energy, B-integral is another informative measure of the amplifier. Accumulated B-integral is significantly impacted by the operation mode of the regenerative amplifier [3] and so can be its stability.

Investigation of the output energy stability under the fluctuating pump energy is not novel. This problem has been thoroughly studied with the use of the Frantz-Nodvik approach [4] within the scalar...
approximation, when the stored energy and the laser fluences are considered as just numbers without taking into account their spatial distributions across the active medium [1,2].

In this paper, we investigate the influence of the energy stability and beam-pointing stability of the pump laser on the stability of the output energy of a pulse-pumped regenerative amplifier. We develop a numerical model, which is based on the Frantz-Nodvik equation with a spatially dependent pump. Introducing shot-to-shot fluctuations of the pump energy or fluctuations of the pump beam position, we can analyze their influence on the output energy of the amplifier, accumulated B-integral, and their statistics.

2. Numerical model

2.1. Frantz-Nodvik approach

The model is based on the Frantz-Nodvik approach [4], which is a versatile tool to analyze laser amplifiers, including ones with rather complex geometries [5] or ones, for which temporal profiles of the signals are of importance [6,7].

Each pass of the laser pulse through the active medium is described by the Frantz-Nodvik equation:

\[ F_{\text{out}}(x, y) = F_{\text{sat}} \cdot \ln \left[ 1 + e^{F_{\text{in}}(x, y)/F_{\text{sat}}} \left( e^{F_{\text{in}}(x, y)/F_{\text{out}} - 1} \right) \right], \]

where \( F_{\text{sat}} \) is the saturation fluence, \( F_{\text{sto}} \) is the stored energy density in the active medium, \( F_{\text{in}} \) is the input fluence of the laser pulse, \( F_{\text{out}} \) is the output fluence, \( x \) and \( y \) are transverse coordinates. The laser pulse passes through the active medium twice per a complete round trip in a linear cavity. Using equation (1), we can calculate amplification of the laser pulse at each consecutive pass through the active medium and thus obtain the entire intracavity build-up signal.

Energy of the laser pulse at the \( i \)-th cavity round trip is the output fluence integrated over the beam:

\[ E_{\text{out}}^{(i)} = \iint F_{\text{out}}^{(i)}(x, y) \, dx \, dy. \]

Similarly, the stored energy is the integrated stored energy density. At the beginning of the first cavity round trip, the stored energy density is given by the pump intensity distribution:

\[ F_{\text{sto}}^{(0)}(x, y) = E_{\text{pump}} \cdot \eta_{\text{pump}} \cdot \Xi_{\text{pump}}(x, y; x_C, y_C) \cdot \lambda_{\text{pump}} / \lambda_{\text{laser}}, \]

where \( E_{\text{pump}} \) is the energy of the pump pulse, \( \eta_{\text{pump}} \) is the part of the pump energy that is absorbed in the active medium, \( \lambda_{\text{pump}} \) and \( \lambda_{\text{laser}} \) are the pump and laser wavelengths. \( \Xi_{\text{pump}}(x, y; x_C, y_C) \) is a dimensionless intensity distribution of the pump beam in the active medium, which is normalized so that \( \iint \Xi_{\text{pump}}(x, y; x_C, y_C) \, dx \, dy = 1 \). The stored energy density at the beginning of the \( i \)-th cavity round trip is calculated from that of the previous round trip:

\[ F_{\text{sto}}^{(i)}(x, y) = F_{\text{sto}}^{(i-1)}(x, y) - \left[ F_{\text{out}}^{(i-1)}(x, y) - F_{\text{in}}^{(i-1)}(x, y) \right]. \]

B-integral accumulated in the RA is proportional to the sum of the laser energies at all cavity round trips up to the cavity dumping trip \( i_{\text{CD}} \):

\[ B = C_B \cdot \sum_{i=1}^{i_{\text{CD}}} E_{\text{out}}^{(i)}. \]

The factor \( C_B \) is a function on the chirped pulse duration \( \tau \), and parameters of the cavity optics, including their lengths \( L^{(k)} \), nonlinear refractive indices \( n_r^{(k)} \), and mode radii \( w^{(k)} \).

\[ C_B = \frac{2 \pi}{\lambda_{\text{laser}}} \cdot \frac{2 \sqrt{\ln 2 / \pi}}{\tau} \cdot \sum_k \left( n_r^{(k)} L^{(k)} \frac{2}{\pi w^{(k)}} \right), \]

with \( k \) being the index to enumerate the optical elements of the cavity. This \( C_B \) coefficient strongly depends on the cavity geometry, pulse bandwidth, stretching ratio and other factors that are not directly relevant to the problem, therefore the B-integral is given in arbitrary units.

To emulate fluctuations of the pump beam position or fluctuations of the pump energy, the beam centroid coordinates \( x_C \) and \( y_C \) and/or the pump energy \( E_{\text{pump}} \) can be varied from one cycle of the RA
operation to another. The Box-Muller transform was used to generate normally distributed $E_{\text{pump}}$, $x_C$, and $y_C$ with certain standard deviations (SD).

To investigate stability of the output energy, we calculated multiple runs of the RA, in which the pump energies $E_{\text{pump}}$ or the pump beam centroid positions $x_C$ and $y_C$ were randomly generated. The resulting set of the output energies was used to calculate the mean and the standard deviation of the output energy. In this paper only short-term instabilities were investigated. Long-term energy drift and beam displacement were not considered, therefore we characterized the stability of both the pump parameters and the extracted energy by their standard deviations without employing the concept of the Allan deviation.

2.2. Default parameters

Numerical modeling was performed for a typical Ti:sapphire RA, parameters of which are given in Table 1.

| Parameter       | Value                                      |
|-----------------|--------------------------------------------|
| Seed energy     | $E_{\text{seed}} = 1.5 \text{ nJ}$         |
| Pump wavelength | $\lambda_{\text{pump}} = 532 \text{ nm}$   |
| Pump energy     | $E_{\text{pump}} = 16 \text{ mJ}$          |
| Pump absorption | $\eta_{\text{pump}} = 93 \%$               |
| Pump diameter   | $2w_{\text{pump}} = 1.2 \text{ mm}$        |
| Pump beam       | $\Xi_{\text{pump}}(x,y) - 4\text{th}-\text{order supergaussian}$ |
| Laser wavelength| $\lambda_{\text{laser}} = 800 \text{ nm}$  |
| Saturation fluence| $F_{\text{sat}} = 0.85 \text{ J/cm}^2$  |
| Beam diameter   | $2w_{\text{laser}} = 0.85 \text{ mm}$      |
| Linear intracavity loss | $1-\gamma = 12\%$ per complete round trip |

3. Results and discussion

An example of a single run (a single cycle of the RA operation) is shown in figure 1, which plots the energy build-up in the laser cavity (a) and depletion of the stored energy in the active medium cross-section (b).

Figure 1. A single cycle of the RA operation. (a) Stored energy and laser energy as a function of the cavity round trip number. (b) Stored energy density profiles in the $x$ direction at selected cavity round-trips. Cavity dumping occurs at the 17th round-trip.
3.1. Influence of the number of cavity round trips

Figure 2 plots the extracted energy as a function of the number of cavity round trips \( NRT \) for fluctuated normally-distributed pump energy with relative SD of 2\%. Error bars on the chart display standard deviations of the extracted energy. Same as in the scalar models [1,2], the stability of the extracted energy depends on \( NRT \) substantially. The extracted energy reaches its maximum of 4.4 mJ at \( NRT = 15 \), with its SD = 0.15 mJ (i.e. 3.4 \%). If the RA is oversaturated, i.e. when being operated with an increased \( NRT \), the stability of the extracted energy improves significantly in terms of both absolute and relative SD. For instance, for \( NRT = 17 \) the stability of the output energy becomes almost three times better with SD = 0.05 mJ (1.2 \%), while the average extracted energy decreases just by 5\% down to 4.2 mJ. Further increasing of \( NRT \) improves the stability of the extracted energy even greater and relative SD below 0.1\% can be achieved.

![Figure 2](image)

**Figure 2.** Extracted energy (circles) and \( B \)-integral (diamonds) vs. the number of cavity round trips at unstable pump energy with relative SD = 2\%. Dots display average values, error bars display standard deviations.

Operating the RA beyond the saturation can vastly improve the stability of its output energy. This behavior is well known to experimentalists. \( B \)-integral, however does not exhibit similar behavior with increasing of \( NRT \). The mean value of the \( B \)-integral linearly grows with \( NRT \), while its SD remains nearly constant without improving at high values of \( NRT \). This can be explained by the fact that the deviations of the \( B \)-integral for the mean value are originated from the previous cavity round-trips, when the pulse energy has already reached mJ level, but has not stabilized yet.

3.2. Influence of the beam pointing stability

Thus, three regimes of the RA operation can be distinguished by \( NRT \): undersaturated, at the max extracted energy, and oversaturated. Stability of the extracted energy in these three modes is depicted in figure 3. Each of 18 box-and-whiskers plots on the chart displays a median (horizontal line), a mean (a solid dot), lower and upper quartiles (a box), 10\% and 90\% percentiles (whiskers), and 1\% and 99\% percentiles (×-symbols). These 18 plots are divided in three groups according to the RA operation conditions, which are respectively indicated with the corresponding values of \( NRT = 13 \), 15, and 17. Within each group there are two subgroups: the first one shows the influence of the fluctuations of the pump energy \( E_{\text{pump}} \) with its SD =1, 2, 3\%. The second subgroup shows the influence of the fluctuations of the beam centroid position \( y_C \) at its SD = 25, 50, 75 \( \mu \)m.

First, one can see that for any \( NRT \) the stability of the extracted energy deteriorates with an increase of the SD of any of the pump parameters, either of \( y_C \) or \( E_{\text{pump}} \). Other factors being equal, the stability of the extracted energy is substantially better for oversaturated RA. In the case of the \( E_{\text{pump}} \) instabilities (blue-colored plots in figure 3) the probability densities of the extracted energy are symmetric: the median and the mean values coincide with each other and both are in the middles of the inter-quartile ranges. In the case of the unstable pump beam centroid position \( y_C \), the probability densities of the extracted energy is highly asymmetric: the mean values are notably lower than the medians; the
percentile marks corresponding to lower energies are distant from the medians, while at the high-energy side the probability densities fall sharply.

![Figure 3](image-url)

**Figure 3.** Statistics of the extracted energy at different operation conditions (NRT = 13, 15, 17) under unstable pump energy (red-colored plots) with its SD = 1%, 2%, 3% and unstable pump beam centroid position (blue-colored plots) with its SD = 25 µm, 50 µm, 75 µm.

The different influence of the $E_{\text{pump}}$ and $y_C$ instabilities on the stability of the RA energy is illustrated in figure 4, which plots histograms of the extracted energy in the cases of unstable pump energy $E_{\text{pump}}$ (a) and unstable pump beam centroid position $y_C$ (b). In the case of the pump energy fluctuations the probability density function of the extracted energy is a fairly symmetric bell-shaped curve centered close to 4.2 mJ, which is the output energy extracted from the RA at the undisturbed pump energy of exactly 16 mJ. Fluctuations of the pump beam centroid position result in a substantially different histogram of the extracted energy (figure 4(b)). This histogram is strongly asymmetric and skewed to the lower values. This asymmetry can notably decrease the average output energy of the RA. This decrease of the average output energy more clearly manifests itself at undersaturated operation with reduced NRT. The origin of the skewness of the histogram to the lower energies has a straightforward interpretation. When a bell-shaped pump beam hits the active medium exactly at its center, the RA has the highest possible gain due to the best overlapping of the pump beam with the cavity mode. This results in the highest output energy. When the pump beam centroid position fluctuates from the axis of the cavity mode, the gain decreases regardless of the direction of the displacement thus lowering the output energy.

The asymmetry of the histogram can be used to distinguish deterioration of the output energy stability associated with the pump beam position stability rather than with the pump energy stability.
Figure 4. Histograms of the extracted energy at oversaturated RA operation with NRT = 17 under unstable pump energy $E_{\text{pump}}$ with relative SD = 3% (a) and unstable pump beam centroid position $y_C$ with SD = 75 µm (b).

4. Conclusion

A numerical model of a regenerative amplifier based on the Frantz-Nodvik equation with spatially dependent stored energy density in the active medium has been developed. The model was used to investigate the stability of the output energy of the amplifier associated with fluctuations of the pump energy and pump beam position. It was found that, same as in the known case with the fluctuations of the pump energy, the influence of the fluctuations of the pump beam position is much weaker in the case of oversaturated operation of the regenerative amplifier. Opposite to the fluctuations of the pump energy, fluctuations of the pump beam position result in highly-asymmetric histogram of the output energy, which is highly skewed to the lower energies.

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