Particle Ratios within a statistically corrected hadron resonance gas model at AGS, SPS, RHIC and LHC Energies

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(Dated: September 9, 2021)

One of the most interesting results is to find that a consistent description of all the experimental results on particle ratios obtained from the lowest AGS to the highest ALICE Heavy Ion Collider energies is possible within the framework of a thermal statistical model. We pursue our investigation of the validity of our quantum mechanically correlated statistical correction (IHRG) inspired by a Beth-Uhlenbeck corrected form of the equation of state (EoS) to the ideal hadron resonance gas model (HRG). We calculate the ratios of some particle yields of equal masses, namely $(\bar{p}/p, K^{-}/K^{+}, \pi^{-}/\pi^{+}, \Lambda/\Lambda, \Sigma/\Sigma, \Omega/\Omega)$, and some particle yield with unequal masses, namely $(p/\pi^{+}, k^{+}/\pi^{+}, k^{-}/\pi^{-}, \Lambda/\pi^{-}, \bar{p}/\pi^{-}, \Omega/\pi^{-})$. We then study the center-of-mass energy variation of these ratios of particle yields obtained by our new corrected hadron resonance gas model (IHRG). Our model results are then confronted with the corresponding calculations obtained using the ideal hadron resonance gas (HRG) model, the Cosmic Ray Monte Carlo (CRMC) EPOS 1.99 simulations, as well as the experimental data from AGS, SPS, RHIC, and ALICE. Our new (IHRG) model results generally show very close agreement with the experimental data compared with the other models considered. Especially remarkable is the very good matching obtained for our new IHRG model with the pair $\bar{p}/\pi^{-}$ and $p/\pi^{+}$, which suggests that our new model IHRG might be helpful in describing the famous proton anomaly at top RHIC and LHC energies. However, in some cases of unequal mass and (multi)strange content, like $\Lambda/\pi^{-}$, and $\Omega/\pi^{-}$, appear to be quite large and thus alert further investigations on the suitability of thermal hadron gas models and their need for further modification.

PACS numbers: 05.50.+q, 21.30.Fe, 05.70.Ce

Keywords: Hadron Resonance Gas correction, Particle Ratios, CRMC, EPOS 1.99, Hadron Interactions, proton anomaly, and effective interactions

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I. INTRODUCTION

One of the major objectives of the ultrarelativistic nuclear collisions is to gain information on the hadron-parton phase diagram, which is characterized by different phases and different types of the phase transitions \cite{1}. The hadronic phase, where stable baryons build up a great part of the Universe and the entire everyday life, is a well known phase. At high temperatures and/or densities, other phases appear. For instance, at temperatures near 150 MeV \cite{2,3}, chiral symmetry restoration and deconfinement transition take place, where quarks and gluons are conjectured to move almost freely within colored phase known as the strongly coupled quark-gluon plasma (sQGP) \cite{4}. At low temperatures but large densities, the hadronic (baryonic) matter forming compact interstellar objects such as neutron stars is indubitably observed in a conventional way and very recently gravitational waves from neutron star mergers have been detected, as well \cite{5}. At larger densities, extreme interstellar objects such as quark stars are also speculated\cite{6}.

Quantum Chromodynamics (QCD), the gauge field theory that describes the strong interactions of colored quarks and gluons and their colorless bound states, has two important intensive state parameters at equilibrium, namely temperature $T$ and baryon chemical potential $\mu_b$. A remarkable world-wide theoretical and experimental effort has been dedicated to the study of strongly interacting matter under extreme condition of temperature and baryon chemical potential. In lattice quantum chromodynamics (LQCD), different orders of chiral and deconfinement transitions have been characterized, especially at low baryon densities.

In nuclear collisions, the statistical nature of the particle production allows the utilization of the particle multiplicities and ratios, for example, to conduct systematic studies on the thermal properties of the final state. Over the last few decades, huge experimental data at energies covering up four orders of magnitude of GeV are now available. It turns out that various statistical thermal models \cite{7,15} are remarkably successful in explaining the resulting particle yields and their ratios measured in heavy-ion collisions. Such a huge data set allowed us to draw the conclusion that the produced particles seem reaffirming the assumption that the hadrons are likely stemming from thermal sources with given temperatures and given chemical potentials. It becomes obvious that such a thermal nature is valid, universally, except for a few baryon-to-meson ratios, such as proton-to-pion at top RHIC and LHC energies, known as proton anomaly \cite{16}.

It was shown that at equilibrium the particle ratios are well explained by two variables; the freeze-
out temperature ($T_{ch}$) and the baryon chemical potential ($\mu_b$). In nuclear collisions, the freezeout stage occurs where the inelastic reactions cease and the number of produced particles becomes fixed. At this stage, the thermal models, such as the hadron resonance gas (HRG) model, determine essential characteristics for dense and hot fireball generated in the heavy-ion collisions. As a result, the thermal models are utilized as an essential tool connecting the QCD phase diagram with the nuclear experiments \cite{14,18}, in the sense that the measurements, mainly the particle multiplicities are connected to the number density in the thermal models, which in turn are strongly depending on chemical freezeout parameters; $T_{ch}$, and $\mu_b$. In this way, we use the language of thermodynamics, correlations, fluctuations, etc. in nuclear collisions \cite{9}.

The hadron resonance gas (HRG) model is customarily used in the lattice QCD calculations as a reference for the hadronic sector \cite{19,20}. At low temperatures, they are found to be in quite good agreement with the HRG model calculations \cite{9}, although some systematic deviations have been observed, which may be attributed to the existence of additional resonances which are not taken into account in HRG model calculations based on the well established resonances listed by the particle data group \cite{21} and perhaps to the need to extend the model to incorporate hadron interactions.

It is reported in recent literature that the standard performance of the HRG model seems to be unable to describe all the available data that is predicted by recent lattice QCD simulations \cite{22,23}. The conjecture to incorporate various types of interactions has been worked out in various studies \cite{24,27}. When comparing the thermodynamics calculated within the HRG framework with the corresponding data obtained using lattice QCD methods, one has to decide how to incorporate interactions among the hadrons.

Arguments based on the S-matrix approach \cite{28,30} suggest that the HRG model includes attractive interactions between hadrons which leads to the formation of resonances. More realistic hadronic models take into account the contribution of both attractive and repulsive interactions between the component hadrons. Repulsive interactions in the HRG model had previously been considered in the framework of the relativistic cluster and virial expansions \cite{29}, via repulsive mean fields \cite{31,32}, and via excluded volume (EV) corrections \cite{33,38}. In particular, the effects of EV interactions between hadrons on HRG thermodynamics \cite{39,46}, and on observables in heavy-ion collisions \cite{47,53} has extensively been studied in the literature. Recently, repulsive interactions have received renewed interest in the context of lattice QCD data on fluctuations of conserved charges. It was shown that large deviations of several fluctuation observables from the ideal HRG baseline could well be interpreted in terms of repulsive baryon-baryon interactions \cite{53,55}.
The dependence of $\mu_b$ and $T_{ch}$ on the nucleon-nucleon center-of-mass energies $\sqrt{s_{NN}}$ constructs a boundary of the chemical freezeout diagram which is very close to the QCD phase diagram. The dependence of $T_{ch}$ on $\mu_b$ looks similar to that of the various thermodynamic quantities as calculated in the lattice QCD, which in turn are reliable quantities, especially at $\mu_b/T \leq 1$, i.e. at $\sqrt{s_{NN}}$ greater than that of top SPS energies. Accordingly, these boundaries remain featured at lower energies, i.e. larger baryon chemical potential, where the QCD-like effective models, such as the HRG model and the Polyakov linear-sigma model (PLSM) play a major role. The hybrid event-generators such as the Cosmic Ray Monte Carlo (CRMC) models are the frameworks. In this work, we are going to show the results of our calculations of some particle ratios based on the CRMC EPOS 1.99 simulations, then compare them with the available experimental results, and with our corresponding calculations based on the ideal hadron gas (HRG) model, as well as with our calculations based on our newly suggested quantum mechanically correlated statistical correction (IHRG) to the ideal HRG model. The latter is to be fully motivated in the next section.

In addition to the main goal of this research, namely to further motivate and test our recently suggested statistical correction to the ideal HRG model, the incorporation of CRMC EPOS 1.99 based data in the present work offers some plausible predictions for the future facilities such the Nuclotron-based Ion Collider fAcility (NICA) future facility at the Joint Institute for Nuclear Research (JINR), Dubna-Russia and the Facility for Antiproton and Ion Research (FAIR) at the Gesellschaft für Schwerionenforschung (GSI), Darmstadt-Germany. These and the BES-II program at RHIC are designed to cover the intermediate temperature region of the QCD phase diagram, while both LHC and top RHIC obviously operate at low $\mu_b$ or high $T_{ch}$, i.e. left part of the QCD phase-diagram.

The remaining of the present paper is organized as follows: In Section II we review the detailed formalism of the conventional ideal (uncorrelated) HRG model, then we develop a non-ideal (correlated) statistical correction to the ideal HRG model inspired by Beth-Uhlenbeck (BU) quantum theory of non-ideal gases, and the used simulation model, EPOS event generator. The calculations of some particle ratios based on our new proposed correction are confronted in Section III with our calculations of the corresponding particle ratios using the ideal HRG model, and the CRMC EPOS 1.99 simulations. Finally in this section, all our model-based calculations are confronted with a wide range of experimental data of particle ratios. Section IV is devoted to a summary and the conclusions.
II. MODEL DESCRIPTION

In the present study, we use the particle interaction probability term originally implemented in the expression for the second virial coefficient worked out in ref. [70], in the context of motivating a quantum-mechanical model for molecular interactions, in order to suggest a statistical correction to the uncorrelated (ideal) HRG model. Beth and Uhlenbeck suggested a connection between the virial coefficients and the probabilities of finding pairs, triples and so on, of particles near each other [71]. In the classical limit, which is usually designated by sufficiently high temperatures and/or low particle densities, it was shown that these probabilities (explicit expressions are to follow later in this section) can be expressed by Boltzmann factors so long as the de Broglie wavelength, which is a common measure of the significance of the quantum non-locality, is small enough compared with the particle spacial extent measured by the so-called particle "diameter" [71]. Such a particle diameter can be considered as a measure of the spatial extent within which a particle can undergo hardcore (classical) interactions. Based on a comparison of the original Beth and Uhlenbeck model with experimental results, it was concluded that at sufficiently low temperatures for which the thermal de Broglie wave length is comparable with the particle diameter, deviations from the classical excluded volume model due quantum effects will be significant [71].

An extension has been made to the original Beth and Uhlenbeck quantum mechanical model of the particle interactions as proposed in ref. [71] by considering the influence of Bose or Fermi statistics in addition to the effect of the inclusion of discrete quantum states for a general interaction potential that is not necessarily central [70]. The expression for the second virial expansion developed in ref. [71] and then extended in ref. [70] was later generalized using the cluster integral to describe particle interactions provided that those particles don’t form bound states [28, 29, 72].

The quantum mechanical Beth-Uhlenbeck (BU) approach were quite recently used to model the repulsive interactions between baryons in a hadron gas [73]. The second virial coefficient or the excluded volume parameter was calculated in [73] within the BU approach and found not only to be temperature dependent, but also to differ dramatically from the classical excluded volume (EV) model result. At temperatures $T = 100-200$ MeV, the widely used classical EV model [74, 76] underestimates the EV parameter for nucleons at a given value of the nucleon hard-core radius (assumed there to be $\simeq 0.3$ fm) by large factors of 3-4. It is thus concluded in [73] that previous studies, which employed the hard-core radii of hadrons as an input into the classical EV model, have to be re-evaluated using the appropriately rescaled quantum mechanical EV parameters.
In what follows of this section, we first introduce the basic formulation of the ideal HRG model. Then we develop a statistical model inspired by Beth-Uhlenbeck (BU) quantum theory of non-ideal (correlated) gases \cite{70} as a correction to the ideal (uncorrelated) HRG model. We thereby implement in the formalism of our calculations a modified version of the partition function of a typical ideal HRG model. Also, we give a detailed description about the considered simulation model, EPOS event generator.

A. Non-correlated ideal HRG

In the framework of bootstrap picture \cite{77,79}, an equilibrium thermal model for an interaction free gas has a partition function $Z(T, \mu, V)$ from which the thermodynamics of such a system can be deduced by taking the proper derivatives. In a grand canonical ensemble, the partition function reads \cite{9,24,80–83}

$$Z(T, V, \mu) = \text{Tr} \left[ \exp \left( \frac{\mu N - H}{T} \right) \right],$$

where $H$ is Hamiltonian combining all relevant degrees of freedom and $N$ is the number of constituents of the statistical ensemble. Eq. (1) can be expressed as a sum over all hadron resonances taken from recent particle data group (PDG) \cite{21} with masses up to 2.5 GeV,

$$\ln Z(T, V, \mu) = \sum_i \ln Z_i(T, V, \mu) = V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln \left[ 1 \pm \lambda_i \exp \left( -\frac{\epsilon_i(p)}{T} \right) \right],$$

where the pressure can be derived as $T \partial \ln Z(T, V, \mu)/\partial V$, $\pm$ stands for fermions and bosons, respectively. $\epsilon_i = (p^2 + m_i^2)^{1/2}$ is the dispersion relation and $\lambda_i$ is the fugacity factor of the $i$-th particle \cite{9},

$$\lambda_i(T, \mu) = \exp \left( \frac{B_i \mu_b + S_i \mu_S + Q_i \mu_Q}{T} \right),$$

where $B_i(\mu_b)$, $S_i(\mu_S)$, and $Q_i(\mu_Q)$ are baryon, strangeness, and electric charge quantum numbers (their corresponding chemical potentials) of the $i$-th hadron, respectively. From phenomenological point of view, the baryon chemical potential $\mu_b$ - along the chemical freezeout boundary, where the production of particles is conjectured to cease - can be related to the nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}}$ \cite{84}:

$$\mu_b = \frac{a}{1 + b \sqrt{s_{NN}}},$$
where \( a = 1.245 \pm 0.049 \) GeV and \( b = 0.244 \pm 0.028 \) GeV\(^{-1}\). In addition to pressure, the number
and energy density, respectively, and likewise the entropy density and other thermodynamics can
straightforwardly be derived from the partition function by taking the proper derivatives
\[
\begin{align*}
n_i(T, \mu) & = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \frac{1}{\exp \left[ \frac{\mu_i - \epsilon_i(p)}{T} \right] \pm 1}, \\
\rho_i(T, \mu) & = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \frac{-\epsilon_i(p) \pm \mu_i}{\exp \left[ \frac{\mu_i - \epsilon_i(p)}{T} \right] \pm 1}.
\end{align*}
\]

It should be noticed that both \( T \) and \( \mu = B_i \mu_b + S_i \mu_S + \cdots \) are related to each other and to \( \sqrt{s_{NN}} \). As an overall thermal equilibrium is assumed, \( \mu_S \) is taken as a dependent variable to be estimated
due to the strangeness conservation, i.e. at given \( T \) and \( \mu_b \), the value assigned to \( \mu_S \) is the one assuring
\( \langle n_S \rangle - \langle n_{\bar{S}} \rangle = 0 \). Only then, \( \mu_S \) is combined with \( T \) and \( \mu_b \) in determining the thermodynamics, such as
the particle number, energy, entropy, etc. The chemical potentials related to other quantum charges,
such as the electric charge and the third-component isospin, etc. can also be determined as functions
of \( T, \mu_b, \) and \( \mu_S \) and each of them must fulfill the corresponding conservation laws.

**B. Quantum-statistically correlated HRG**

As introduced in the previous section, for a quantum gas of fermions and bosons with mass \( m_i \)
and correlation (interaction) distance \( r \), at temperature \( T \) and vanishing \( \mu_b \), a two-particle interaction
probability of the form
\[
1 \pm \exp \left( -4\pi^2 m_i Tr^2 \right) \tag{7}
\]
was first introduced by Beth and Uhlenbeck \[70\] in an attempt to model the interactions of a quantum
gas of particles assuming a general potential and neglecting the possibility for bound states formation.
The Boltzmann-like term \( \exp \left( -4\pi^2 m_i Tr^2 \right) \) remains in effect even for an ideal gas, which is a typical
approximation at sufficiently high temperatures. The \( \pm \) sign expresses the apparent attraction (repulsion)
between bosons (fermions) due to change of statistics \[70\]. Inspired by such a correction, we
introduce a correction for the probability term in the expression for the the ideal hadron gas partition
function given in Eq. (1). We propose a new probability term of the form
\[
1 \pm \lambda_i \exp \left( \frac{-\epsilon_i(p)}{T} \right) \left[ 1 \pm \exp \left( -4\pi^2 m_i Tr^2 \right) \right] \tag{8}
\]

This corrected probability function obviously incorporates interactions in the the hadron resonance
gas in the sense of Beth and Uhlenbeck quantum correlations \[70\] with \( r \) being the correlation (interaction) length between any two hadrons at equilibrium temperature \( T \). Based on our proposed
corrected probability function, we modify the non-correlated HRG partition function $Z(T, \mu, V)$ to have the following form

$$\ln Z(T, V, \mu) = \sum_i V g_i \frac{1}{2\pi^2} \int_0^\infty \pm p^2 dp \ln \left[ \pm \lambda_i \exp \left( \frac{-\xi_i(p)}{T} \right) \right] \left[ 1 \pm \exp \left( 4\pi m_i T r^2 \right) \right],$$

which apparently sums over all hadron resonances following the same recipe described in motivating Eq. (2) for the case of non-correlated HRG. The thermodynamics of the correlated HRG can thus be calculated by taking the proper derivatives of $\ln Z$ as explicitly stated in the corresponding non-correlated HRG case discussed above.

C. Cosmic Ray Monte Carlo (CRMC) model

As discussed, the hybrid event generator, Cosmic Ray Monte Carlo (CRMC EPOSllhc), is used in generating different particle ratios for various hadrons, at energies spanning between $\sqrt{s_{NN}} = 3$ and 2760 GeV. The EPOSllhc event generator results are then confronted with the available experimental results and finally compared to our quantum-mechanically inspired statistically corrected hadron resonance gas model (IHRG) calculations based on Eq. (9).

The interface of CRMC is used for the different cosmic ray monte-carlo models for different effective quantum chromodynamic (QCD) theories and various experiments as NA61, ATLAS, CMS, LHCb, and the Pierre Auger observatory for ultra high-energy cosmic rays, etc. It features a variety of interactions based on the EPOSllhc/1.99 model, which is similar to the Gribov-Regge model. CRMC presents a comprehensive background description that takes into account the diffraction that results. Also, its interface can access the output from several heavy-ion collision event generators, and can be connected to models of various spectrum, such as, EPOS 1.99/lhc [67, 68], qgsjetII [61, 63], qgsjet01 [59, 60], and sibyll [64, 66]. SIBYLL2.3 and QGSJET01 are used at low energies. QGSJETII, v03 and v04, and EPOS lhc/1.99 are the effective models that can be incorporated at the ultra-relativistic high energies.

In the present work, we used the EPOSllhc event generator which has different parameters for the essential observables in high-energy heavy-ion collisions and their phenomenological assumptions. These can be changed based on the considered theoretical and experimental postulates. It was reported that EPOSllhc is capable of providing a realistic description for heavy-ion collisions based on the data collected by various experiments and other event generators [61, 63].

EPOSllhc was designed for cosmic ray air showers, but it could also be used for pp- and AA-collisions at SPS, RHIC, and LHC energies. Moreover, EPOSllhc applies simpler treatments for heavy-ion collisions near the end of their evolutions, allowing it to be used for minimal bias in hadron interactions.
in nuclear collisions. EPOSlhc is considered as a parton model with several parton-parton interactions which lead to different parton ladders. It gives good estimates for particle production, multiple scattering of partons, cross-section evaluations, shadowing and screening through splitting, and different integrated effects of hot and dense matter.

In the present work, we used the EPOSlhc event generator, at energies spanning between 3 and 2760 GeV for a set of at least 100,000 events for each energy. We have calculated the particle ratios for some particle yields of equal masses, namely $(\bar{p}/p, K^-/K^+, \pi^-/\pi^+, \bar{\Lambda}/\Lambda, \bar{\Sigma}/\Sigma, \bar{\Omega}/\Omega)$, and some particle yield with unequal masses, namely $(p/\pi^+, k^+/\pi^+, k^-/\pi^-, \Lambda/\pi^-, \bar{p}/\pi^-, \Omega/\pi^-)$, in a wide rapidity window, namely $-6 < y < 6$. We plan to calculate the particle ratios for the different hadrons examined in proving the correctness of the hybrid EPOSlhc event generator, which is expected to come up with a novel input for future facilities such as NICA and FAIR.

III. CALCULATION RESULTS AND DISCUSSION

In the figures below, we confront the data of the particle ratios calculated using our quantum-mechanically inspired statistically corrected hadron resonance gas model (IHRG) based on Eq. (9) (red dashed lines with our calculations of the same particle ratios using the ideal hadron resonance gas model (HRG) based on Eq. (2) (black solid curves), with our corresponding calculations based on CRMC EPOS 1.99 simulations (blue stars), and finally with the corresponding experimental data (different symbols) characterizing the results measured at STAR BES-I energies and partially at the Alternating Gradient Synchrotron (AGS) and the Superproton Synchrotron (SPS) energies, as well, such as NA49, NA44, NA57, and PHENIX. A great part of this range of the beam energy shall be accessed by NICA and FAIR future facilities in the temperature range $T \in [130, 200 \text{ MeV}]$. These temperatures are rather typical for the phenomenological applications in the context of heavy-ion collisions and QCD equation-of-state. In all our calculations based on our new IHRG model, we set the quantum-mechanical correlation length (parameter) to the optimal value $r = 0.85 \text{ fm}$.

In Figs. 1, we show the center-of-mass energy dependence of the ratio of strange meson to non-strange meson such as $k^+/\pi^+$ (a), $K^-/\pi^-$ (b), and the ratio of antimeson-to-meson such as $k^-/k^+$ (c), and $\pi^-/\pi^+$ as calculated using our new IHRG model (red dashed curve), the ideal HRG mode (black solid curve), and CRMC EPOS 1.99 simulations (blue stars). The corresponding experimental data of the respective particle ratios used in the comparison are represented by different
Fig. 1: Variation of the strange meson to non-strange meson ratios $k^+/$π$^+$ (a), $k^-/$π$^−$ (b), antikaon-to-kaon ratio $k^-/k^+$ (c), and antipion-to-pion ratio π$^−$/π$^+$ (d) with respect to center-of-mass energy $\sqrt{s_{NN}}$. Black solid curve, red dashed curve, and blue stars curve show the predictions of our new (IHRG) model 9, ideal HRG model (HRG) 2, and CRMC EPOS 1.99 simulations [67, 68], respectively. The corresponding experimental data are represented by different symbols [8, 10, 86, 87, 87, 88, 88, 89, 89–93, 96].

Like all other types of hadron thermal models, our new quantum-mechanically inspired; statistically corrected IHRG model (red dashed curve) slightly overestimates the experimental $k^+/$π$^+$ data represented by different symbols in (Fig. 1a) as of about $\sqrt{s_{NN}}$=3 GeV. However, our new IHRG model (red dashed curve) is slightly closer to fit the experimental data compared with the ideal HRG model (Black solid curve). Remarkable enough, our new IHRG model (red dashed curve), and also the HRG model (black solid curve), seem to well-reproduce the famous peak near $\sqrt{s_{NN}}$=10 GeV in the $k^+/$π$^+$ ratio profile (Fig. 1a). Compared to the experimental data, our new IHRG model’s calculations as well as the ideal HRG model’s calculations for the $k^+/$π$^+$ ratio looks much broader at the peak. Starting from the peak, our IHRG model offers the best fit for the available experimental data, especially in the very high $\sqrt{s_{NN}}$ limit where our model fits very closely the 2.76 TeV ALICE measurement of the $k^+/$π$^+$ ratio. On the other hand, our CRMC EPOS 1.99 simulations (blue stars) for the $k^+/$π$^+$ data are strongly suppressed and obviously doesn’t even reproduce the famous peak in $k^+/$π$^+$ experimental...
data, but it catches up with our IHRG model and with the ideal HRG model to closely fit the available experimental data as of about $\sqrt{s_{NN}}=100$ GeV up to the very high $\sqrt{s_{NN}}$ limit ($\sim 10$ TeV).

For the $k^-/\pi^-$ experimental data (Fig. 1b), our new IHRG model (red dashed curve) is generally in very good agreement with the experimental data. It’s even slightly better fitting the experimental data than the ideal HRG model (Black solid curve). On the other hand, our CRMC EPOS 1.99 simulations for the $k^-/\pi^-$ experimental data seem to well-reproduce the steady increase feature in the $k^-/\pi^-$ ratio (Fig. 1b) but with a significant suppression all over the energy range covered in the present study. This suppression in the CRMC EPOS 1.99 simulations decreases significantly and consistently in the TeV scale.

The variation of the $k^-/k^+$ with $\sqrt{s_{NN}}$ is shown in fig. 1c. Our new IHRG model prediction for the $k^-/k^+$ profile (red dashed curve) is almost indistinguishable from the prediction of the ideal HRG model (black solid curve). For $k^-/k^+$, all featured experimental data show very good agreement with our IHRG model as well as the ideal HRG model. Only the lower energy PHENIX measurement near $\sqrt{s_{NN}} = 100$ GeV is quiet overestimated by all thermal models considered in this research. Our CRMC EPOS 1.99 simulations for the $k^-/k^+$ ratio seem to reproduce the monotonic increase feature of the $k^-/k^+$ experimental data quite well up to the very high energy limit ($\sim 10$ TeV). In the low energy limit up to about $\sqrt{s_{NN}} = 30$ GeV, CRMC EPOS 1.99 simulations seem to be very well compatible with the data of the $k^-/k^+$ ratio produced by both the thermal models and the available experimental data. The same feature is observed in the high energy limit from about $\sqrt{s_{NN}} = 3$ TeV. Between the previous two extreme limits in the intermediate energy regime, our CRMC simulations doesn’t fit well with neither the experimental nor the thermal model data.

The variation of the $\pi^-/\pi^+$ with $\sqrt{s_{NN}}$ is shown in fig. 1d. For $\pi^-/\pi^+$, all the featured experimental data show very good agreement with our IHRG model (red dashed curve) as well as with the ideal HRG model (black solid curve except for the lower energy E895 measurement data $\sqrt{s_{NN}} \simeq 2$ GeV [97]), which are quiet underestimated by all thermal models considered in this research. Our CRMC simulations data seem to agree well with the available experimental data as well as with the featured thermal models starting from about 10 GeV all the way to ($\sim 10$ TeV).

In Figs. 2 we show the center-of-mass energy dependence of the antibaryon-to-baryon ratios such as $\bar{p}/p$ (a), $\Lambda/\Lambda$ (b), $\bar{\Sigma}/\Sigma$ (c), and $\bar{\Omega}/\Omega$ (d) as calculated using our new IHRG model [red dashed curve], the ideal HRG model [Black solid curve], and CRMC EPOS 1.99 simulations (blue stars curve) [67, 68, 97]. The corresponding experimental data of the respective particle ratios used in the present study are represented by different symbols [8, 10, 86, 87, 87, 88, 89, 89, 93, 96, 98].
For the $\bar{p}/p$ ratio (Fig. 2a), it’s quite obvious that our new (IHRG) model as well as the ideal HRG model provide a very good agreement with experimental data up to the 200 GeV RHIC data, which is the highest energy available experimental point. On the contrary, our CRMC simulations seem to severely underestimate the experimental data especially for $20 \text{ GeV} \lesssim \sqrt{s_{\text{NN}}} \lesssim 200 \text{ GeV}$. Up to about $\sqrt{s_{\text{NN}}} \simeq 200 \text{ GeV}$, the $\bar{p}/p$ ratio calculated by both our new IHRG model and the ideal HRG model are almost indistinguishable.

On the other hand, the profiles of the ratios $\Lambda/\Lambda$ (Fig. 2b) and $\Sigma/\Sigma$ (Fig. 2c) share almost all the important features outlined in discussing the features of the $\bar{p}/p$ ratio (Fig. 2a) except for the obvious fact that the experimental NA49 data at low energies ($\simeq 10 \text{ GeV}$) seem to be underestimated by our thermal models calculations compared with the profile of $\bar{p}/p$ ratio (Fig. 2a).

Our CRMC EPOS 1.99 simulations seem to severely underestimate the experimental data for the whole energy range in the $\Lambda/\Lambda$ profile (Fig. 2b), while the same occurs for the $\Sigma/\Sigma$ profile (Fig. 2c) except for the very high energy limit ($\sqrt{s_{\text{NN}}} \gtrsim 1 \text{ TeV}$) where CRMC EPOS 1.99 simulations obviously overestimate the thermodynamic model’s calculations for the $\bar{\Sigma}/\Sigma$ ratio. Unfortunately no relevant
experimental data are available in this regime.

In Figure 2d, the profile of the ratio $\bar{\Omega}/\Omega$ based on the thermal model calculations, not only underestimates the experimental data at low energies ($\simeq 10$ GeV) but also at the medium energies near ($\simeq 200$ GeV). Here our CRMC EPOS 1.99 simulations seem to overestimate the thermal model calculations in the low energy regime, while it underestimates the experimental data near the 200 GeV RHIC data. Moreover, our (IHRG) model calculations (red dashed curve) for this ratio are almost identical to the ideal HRG model calculations (Black solid curve).

Fig. 3: Variations of the antibaryon-to-antimeson ratios such as $\bar{p}/\pi^-$ (a), and baryon-to-meson ratios such as $p/\pi^+$ (b), $\Lambda/\pi^-$ (c), and $\Omega/\pi^-$ (d) with respect to center-of-mass energy $\sqrt{s_{NN}}$. Black solid curve, red dashed curve, and blue stars (curve) show the predictions of our new IHRG model [9] ideal HRG model [2] and CRMC EPOS 1.99 simulations [67, 68], respectively. The corresponding experimental data are represented by different symbols [8, 10, 86, 87, 88, 88, 89, 91, 92, 93, 96, 98].

In Figs. 3 we show the center-of-mass energy dependence of the antibaryon-to-antimeson ratios such as $\bar{p}/\pi^-$ (a), baryon-to-meson ratios such as $p/\pi^+$ (b), and baryon-to-antimeson ratios such as $\Lambda/\pi^-$ (c), and $\Omega/\pi^-$ (d) with respect to center-of-mass energy $\sqrt{s_{NN}}$ as calculated using our new IHRG model [9](red dashed curve), the ideal HRG model [2](Black solid curve), and CRMC EPOS 1.99 simulations (blue stars) [67, 68]. The corresponding experimental data of the respective particle ratios used in the present study are represented by different symbols [8, 10, 86, 87, 88, 88, 89, 89, 89].
For the $\bar{p}/\pi^-$ ratio (Fig. 3a), it’s quite obvious that our new (IHRG) model provides a very good agreement with the experimental data up to 200 GeV. Moreover, our new (IHRG) model is obviously in better agreement with the experimental data compared with the ideal HRG model up to 200 GeV. On the contrary, the CRMC EPOS 1.99 simulations seem to severely underestimate the experimental data especially at high RHIC energies near 200 GeV all the way to the high energy limit ($\lessapprox 10$ TeV).

The ratio $p/\pi^+$ (Fig. 3b) shares almost all the important features outlined in discussing the features of the $\bar{p}/\pi^-$ ratio (Fig. 3a) except that in this case the calculations based on our new IHRG model are only slightly better than the corresponding ideal HRG model calculations up to high RHIC energies near 200 GeV.

In Fig. 3c and Fig. 3d, our new IHRG model calculations and the ideal HRG model calculations clearly reproduce the $\Lambda/\pi^-$ (Fig. 3c), and $\Omega/\pi^-$ (Fig. 3d) main features of the experimental profiles, especially the peaks near $\sqrt{s_{NN}} \simeq 2$ GeV in the $\Lambda/\pi^-$ profile and near $\sqrt{s_{NN}} \simeq 20$ GeV in the $\Omega/\pi^-$ profile. Our IHRG model calculations offers a very good prediction of position and the shape of the peak of $\Lambda/\pi^-$ ratio profile (Fig. 3c). The sharpness and the height of our model prediction for the peak of $\Lambda/\pi^-$ ratio is obviously better than that of the ideal HRG model. For the $\Omega/\pi^-$ profile (Fig. 3d), our new IHRG model and the ideal HRG model are almost equivalently as good in predicting the shape and the position of the peak near about $\sqrt{s_{NN}} \simeq 20$ GeV. However, the calculations based on our new IHRG model and the ideal HRG model seem to underestimate the height of the $\Lambda/\pi^-$ and $\Omega/\pi^-$ peaks. Finally our CRMC EPOS 1.99 simulations doesn’t seem to reproduce the $\Lambda/\pi^-$ peak at all, while it produces $\Omega/\pi^-$ peak but with a highly exaggerated height.

IV. SUMMARY AND CONCLUSIONS

We confronted the data of the particle ratios with respect to center-of-mass energy $\sqrt{s_{NN}}$ calculated using our new quantum-mechanically inspired and statistically corrected hadron resonance gas model (IHRG) based on Eq. (9) with our corresponding calculations of particle ratios using the ideal hadron resonance gas model (HRG) based on Eq. (2), our corresponding calculations using CRMC EPOS 1.99 simulations, and finally with the corresponding experimental data characterizing the particle data measured at STAR BES-I energies [86] and partially measured at the Alternating Gradient Synchrotron (AGS) [10], the Superproton Synchrotron (SPS) [8] energies, as well, such as NA49 [87–89], NA44 [87–91], NA57 [92], and finally PHENIX [93]. A major part of this range of the beam energy...
is expected to be accessed by NICA and FAIR future facilities [94, 95] in the temperature range $T$ in [130, 200 MeV]. These temperatures are typical for many phenomenological applications in the domain of heavy-ion collisions and QCD equation-of-state research. We have also confronted our new IHRG model with with recent lattice data in a recent study [69].

We generally observe a very good agreement between our new IHRG model results and the corresponding experimental data. Our new IHRG model particularly shows excellent agreement with data of $k^-/k^+$, $\bar{\Lambda}/\Lambda$, $\bar{\Sigma}/\Sigma$, $\bar{\Omega}/\Omega$, $\Omega/\pi^-$, and $\pi^-/\pi^+$. For the remaining particle ratios, we observe a less agreement between our new IHRG model results and the corresponding experimental data. Moreover, it’s especially remarkable that our new IHRG model exhibits a very good agreement with the pair $\bar{p}/\pi^-$ and $p/\pi^+$, which suggests that our new IHRG model might be helpful in describing and thereby resolving the famous proton anomaly problem at top RHIC and LHC energies [16]. This mandates further investigation to confirm.

Although the ideal HRG model also exhibits a generally good agreement with the corresponding experimental data, we still need a new model that incorporates repulsive interactions between hadrons.

Moreover, at GSI SIS and Brookhaven National Laboratory AGS energies, the freeze-out parameters involve a much larger values of baryon chemical potentials and the predictions of all the available thermal models, including the available standard excluded-volume correction models don’t hold water.

We are going to study the possible variation in the freeze-out parameters as predicted by our new IHRG model in a near future separate research.

In conclusion, the analysis presented here supports our claim that our quantum-mechanically correlated, and statistically corrected IHRG model obtained by us properly explains the particle ratios of various particles after chemical freeze-out.

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