Perturbative QCD Analysis of Local Duality in a fixed $W^2$

Framework.

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We study the global $Q^2$ dependence of large $x F_2$ nucleon structure function data, with the aim of providing a perturbative-QCD (pQCD) based, quantitative analysis of parton-hadron duality. As opposed to previous analyses at fixed $x$, we use a framework in fixed $W^2$. We uncover a breakdown of the twist-4 approximation with a renormalon type improvement at $O(1/Q^4)$ which, by affecting the initial evolution of parton distributions, will have consequences for pQCD analyses also at large $x$ and very large $Q^2$.

One of the key challenges in Quantum ChromoDynamics (QCD) today is to formulate a connection between the description of the hard, or short-distance, scattering processes, which can be calculated in terms of quark and gluon degrees of freedom using perturbative methods, and the physical asymptotic states, i.e. the spectra of hadrons which are not calculable within perturbative-QCD (pQCD) and are in principle only remotely related to parton dynamics [1]. Yet, a substantial number of observations indicate that QCD has manifestly a dual parton-hadron nature. Recent investigations of hadronic jets at $e^+e^-$ [2], $e p$ [3], and $p\bar{p}$ [4] colliders show that several infrared-sensitive features of the inclusive hadron distributions can be reproduced by a pQCD description of the parton shower down to $Q_o \approx \Lambda_{QCD}$, $Q_o$ being an effective cut-off defining the onset of the non-perturbative regime and $\Lambda_{QCD}$ being the scale of QCD. (Notice, however, that detailed angular correlation
measurements have been found to be in disagreement with available pQCD estimates [5]. Similarly, deep inelastic scattering (DIS) experiments at very low Bjorken $x$ indicate that pQCD can describe the nucleon structure function $F_2$ down to $Q^2 \approx 1 \text{ GeV}^2$, where a smooth and fast transition to the non-perturbative regime occurs, leading eventually to the expected behaviour in the real photon scattering limit ($F_2(x, Q^2) \to 0$ for $Q^2 \to 0$). The question of the role and nature of non-perturbative corrections naturally emerges. These are known to affect the cross sections for $e^+e^-$ jet fragmentation [6] and are also expected to be present, although they have not been isolated yet, in the deep inelastic data at low $Q^2$ and low $x$ [7].

In this Letter we address yet another set of experiments, namely inclusive $ep$ and $eD$ scattering in the resonance region, where a pQCD description seems to hold in spite of the low values of the invariant mass squared, $W^2$, produced in the final state. Here, the observation of Bloom-Gilman (BG) duality [8], or the similarity between the behaviour of the resonance contribution to the nucleon structure function and DIS, can be formulated theoretically, as the equivalence between the moments of the structure function in the low $W^2$ kinematical region dominated by resonances and in the DIS one, modulo perturbative corrections and expectedly small power corrections [9]. Furthermore, because of the large body of highly accurate data that has been recently made available [10], the data in the resonance region can now be fitted to a smooth curve which, once evolved according to pQCD, will coincide with the DIS data.

There has been a growing theoretical interest in this intriguing phenomenon, where the quasi-exclusive nature, or low inelasticity of the scattering process would be expected to hinder a direct observation of the partonic structure of the target. On the contrary, the measured structure functions appear to behave, in average, like parton distributions. The present analysis is motivated by the expectation that a more precise understanding of the mechanisms behind BG duality might provide a handle on the type of hadronic configurations that are present in a “semi-hard” regime, before confinement settles in. It is therefore of paramount importance to perform a detailed study of both the logarithmic corrections and of the size and nature of the power corrections in the pQCD expansion, in order to ascertain whether the apparent weak $Q^2$ dependence of the data is coincidental, an artifact of the particular region under study, or a cancellation of Higher Twist (HT) terms, possibly understandable within parton-hadron duality models. The analysis conducted here involves a number of steps similar to recent extractions of power corrections from inclusive data...
3

\[ F_{2}^{\exp}(x, Q^2) = F_{2}^{pQCD+TMC}(x, Q^2) + \frac{H(x, Q^2)}{Q^2} + \mathcal{O}(1/Q^4), \]  

(1)
is adopted, where \( F_{2}^{pQCD+TMC}(x, Q^2) \) is the twist-2 contribution, including kinematical power corrections from the target mass (TMC); the other terms in the formula are the dynamical power corrections, formally arising from higher order terms in the twist expansion. Both \( F_{2}^{pQCD+TMC} \) and \( H \) can be extracted from the data at large \( x \), by taking care of aspects of pQCD evolution peculiar to this region, and of TMC. Uncertainties are introduced at each step. In this Letter, we address them one by one with the important addition that we provide for the first time in the literature, both an analysis of the scale dependence of the resonance region \( (W^2 < 4 \text{ GeV}^2) \) and a combined study of both the resonance and the large \( W^2 \) data.

We start by studying the question of the limit of applicability of pQCD in the low \( W^2 \), large \( x \) domain, but at \( Q^2 \) above the 1 GeV\(^2\) scale, so still amenable, in principle, to a pQCD treatment. Previous analyses used moments of the structure functions obtained at fixed values of \( Q^2 \) and directly related to the OPE, circumventing the problem of dealing with the complicated structure of the bound states by providing a natural averaging procedure in the Mellin conjugate \( n \) \[9\]. However, we have shown in \[15\] that, at lower values of \( Q^2 \), such moments of \( F_2 \) are affected by elastic scattering in such a way as to render higher twist contributions impossible to extract or interpret in the standard OPE language. We choose, therefore, an alternative method using data in fixed \( W^2 \) bins.

The data in the resonance region \[11\] have been fitted to a smooth curve for \( F_2^{\nu(D)}(\xi) \), where \( \xi = 2x/(1+\sqrt{1+4M^2x^2/Q^2}) \) \[16\] is the Nachtmann scaling variable. The fit is applied separately to bins in invariant mass, \( W^2 \), centered at: \( 1 \) \( W^2_R = 1.6 \text{ GeV}^2 \), \( 2 \) \( W^2_R = 2.3 \text{ GeV}^2 \), \( 3 \) \( W^2_R = 2.8 \text{ GeV}^2 \), \( 4 \) \( W^2_R = 3.4 \text{ GeV}^2 \), respectively. The \( \chi^2/d.f. \) for these fits varies from 0.8 to 1.1. The uncertainty for the scaling curves is estimated to be better than 10%, taking into account uncertainties in the experimental data, and in the averaging and fitting procedures. The curves and uncertainties in each \( W^2 \) bin are represented by the hatched areas in Fig. 1. Notice that the spectra at fixed \( W^2 \) require that \( Q^2 \) vary for each spectrum as an increasing function of \( x \), since \( Q^2 \equiv Q^2(x) = (W^2_R - M^2)x/(1-x) \).

We utilize a quantitative comparison of the resonance region \( F_2 \) curves thus obtained to the pQCD prediction for \( F_2 \) at the same kinematics to extract potential higher twist
contributions to the structure functions. We assume that only valence quarks contribute to $F_2^{p(D)}$, at low $W^2 \lesssim 10\text{ GeV}^2$ and $x \geq 0.3$. PQCD evolution to $Q^2 \equiv Q^2(x)$ or at fixed $W^2$, for the Non-Singlet (NS) distributions at next-to-leading-order (NLO), is given by:

$$q_i^{(-)}(x, Q^2) = \int_{Q^2_0}^{Q^2(x)} \frac{dQ'^2}{Q'^2} \frac{\alpha_S(Q'^2)}{2\pi} \times \int_x^1 \frac{dy}{y} P_{qq}(\frac{x}{y}, \alpha_S(Q'^2)) q_i^{(-)}(y, Q'^2),$$

(2)

where $q_i^{(-)} = q_i - \bar{q}_i \equiv q_i^v, i = u, d$. The expressions for the splitting function $P_{qq}(z, \alpha_S(Q^2))$, and for the corresponding endpoints at NLO in $\overline{M}S$ scheme can be found in [17, 18]; $\alpha_S$ is the strong coupling constant. The structure function, $F_2$, is then obtained by convoluting (4) with the quark coefficient function, $B_{2\text{NS}}^{2\text{NS}}$ [17]. We fix the values of the initial parton distribution functions (PDFs), at $Q^2_0 \approx 0.4 - 1\text{ GeV}^2$, to the ones taken from NLO global fits to world data [13, 20, 21], and we solve the evolution equations directly in $x$ space, with $\alpha_S(M_Z^2) = .117$. Notice in this respect, that the shape of the initial NS PDFs is practically constrained [20], at variance with the singlet and gluon distributions at low $Q^2$, whose shape is strongly correlated with the value of $\alpha_S$. The structure of perturbative evolution, including both coefficient functions and splitting functions for the NS part, can now be evaluated up to NNLO. Detailed studies of the impact of NNLO corrections and beyond, on the determination of power corrections for the NS structure functions, have been performed in [11, 12]: The question of whether these can "mimick" the contributions of higher twists, including the uncertainties due to the well known scale/scheme dependence of calculations, within the current precision of data is under intense investigation. Here, we single out the contributions that are expected to dominate the higher order perturbative predictions at large $x$, namely powers of $\ln(1 - z)$ terms, where $z = x/y$. These terms can be resummed to all orders. We perform the resummation in $x$ space by replacing the $Q^2$ scale with a $z$-dependent one, $\tilde{W}^2 = Q^2(1 - z)/z$ [22]. It is well known that such a procedure introduces in principle an ambiguity in the evaluation of the running coupling at low values of the scale $\tilde{W}^2$, i.e. as $z \to 1 - \Lambda^2/Q^2$ ([23] and references therein). This ambiguity is lessened in our analysis because at fixed $W^2$, $\Lambda^2/Q^2(x)$ is very close to zero. Our results for NS evolution and shown elsewhere [24], are in good agreement with more recent resummation calculations performed in $n$ space and anti-Mellin transformed as in [25].

Finally, we take into account target mass corrections (TMC). We use the expression in
obtained by a formal inversion of the Nachtmann moments [16]. Ambiguities in this procedure are expected to arise because of the neglect of higher order and higher twist corrections. To safely disregard these, we use a minimal criterion that only the kinematical points yielding low values of the parameter $x^2 M^2/Q^2$ in the TMC expansion [3] are kept.

Comparisons of pQCD+TMC predictions with the resonance average data are shown in Figures 1 and 2. A few comments are in order. In Fig. 1: i) TMC (full lines) modify substantially the pQCD behavior (dotted lines) rendering a better agreement with the data; ii) the curve calculated using NLO pQCD at $Q^2 = 200$ GeV$^2$ shown for comparison in Fig. 1(b) demonstrates the large effect of pQCD corrections above $x \sim 0.2$.

In Fig. 2, we show the low $W^2$ data obtained from the fit of Jefferson Lab (JLab) and SLAC data in the resonance region, along with larger $Q^2$ data from [26, 27]. Note that the data in the resonance region (circles) smoothly blend to the deep inelastic (stars) - another manifestation of BG duality. The curves correspond to our calculations including pQCD+TMC at NLO (dashes), and pQCD+TMC with resummation (full). The dots in each curve represent regions where TMC are uncertain. The effect we find is qualitatively similar to what found in [11, 12], in that over the range $0.45 \leq x \leq 0.85$, higher order perturbative contributions, in this case large $x$ resummation, improve the agreement with the data. Substantial discrepancies remain which we assume to be largely due to the dynamical, HT corrections to the structure function. If $H(x, Q^2)$ is modeled similarly to previous extractions [11, 12, 13, 28], one has:

$$H(x, Q^2) = F_2^{pQCD+TMC}(x, Q^2)C_{HT}(x),$$

Eq. (3) is motivated by the lack of knowledge of the anomalous dimensions of the twist-4 operators, a reasonable assumption within the precision of the data (see also [23]). Our analysis at fixed $W^2$ explained in the first part of the paper, enables us to extract $C_{HT}$ from the resonance region and from the large $W^2$ (DIS) region, separately.

In Fig. 3(a) we show the coefficient $C_{HT}$, Eq. (3), extracted from: i) DIS data with $W^2 \geq 4$ GeV$^2$; ii) The resonance region, $W^2 < 4$ GeV$^2$; iii) Averaged over the entire range of $W^2$. The figure also shows the range of extractions previous to the current one [13, 28]. We notice that in all three cases, our values for $C_{HT}$ are smaller than the ones in [13, 28], because of the effect of large $x$ resummation. We have checked that our results without resummation are consistent with a previous extraction using moments of the structure function [14]. Most
importantly, while the large $W^2$ data track a curve that is consistent with the $1/W^2$ behavior expected from most models [30], the low $W^2$ data yield a much smaller value for $C_{HT}$ and they show a bend-over of the slope vs. $x$, already predictable from a similar bend-over in the slopes at low $W^2$ in Fig. 2.

Notice that this surprising effect is not a consequence of the interplay of higher order corrections and the HT terms, but just of the extension of our detailed pQCD analysis to the large $x$, low $W^2$ kinematical region. In fact, resummation produces an overall reduction of $C_{HT}$, that does not modify our results qualitatively. In order to ascertain whether the discrepancy between the low $W^2$ and large $W^2$ values of $C_{HT}$ are due to $O(1/Q^4)$ terms in the twist expansion, Eq.(1), which could become more important at low $W^2$, we have extracted for each resonance the quantity $\Delta H(x, Q^2)$, defined as

$$\frac{F_{2}^{exp}}{F_{2}^{pQCD+TMC}} = 1 + \frac{C_{HT}(x)}{Q^2} + \Delta H(x, Q^2), \quad (4)$$

where $C_{HT}(x)$ coincides with the value fitted at large $W^2$. From Fig. 3(b) one sees that $\Delta H(x, Q^2)$ is negative for all lower $W^2$ ($\leq 3.4$ GeV$^2$) bins, as expected if a cancellation among higher order inverse powers were to occur, consistently with the requirement of parton-hadron duality. However, we uncover a non trivial $Q^2$ dependence of this term: one can see a sharp change in the behavior of the higher mass resonances and the data in the $N - \Delta$ transition region which show a distinctively steeper fall with $Q^2$. Furthermore, the high mass resonances seem to be in fair agreement with a simple $-1/W^4$ fit inspired by Infrared Renormalon (IRR) calculations [30], which would produce a constant line at a decreasing height for increasing $W^2$. The $N - \Delta$ transition region shows a departure from this behavior which cannot be accounted for by fitting the first few terms of an inverse power expansion. A similar conclusion was reached in our analysis in moment space [17].

To summarise, the size of the power corrections obtained from the data at $0.45 \leq x \leq 0.85$, with $W^2 > 3.4$ GeV$^2$, is comparable to the values obtained in recent analyses of DIS, with $W^2 > 10$ GeV$^2$ [11, 12, 13, 28, 29]; the inclusion of large $x$ resummation producing a reduction of the coefficient $C_{HT}$. At lower invariant mass, $1.9 \leq W^2 \leq 3.4$ GeV$^2$, an extra term of order $O(1/Q^4)$, with a negative coefficient is necessary to fit the data, in line with the asymptotic nature of the twist expansion and with current predictions from IRR calculations [30]. A similar expansion does not reproduce, however, the scale dependence of data at even lower masses, $1.2 \leq W^2 \leq 1.9$ GeV$^2$, yet for $Q^2$ values where pQCD would be
expected to apply. Therefore, the experimental observation of a rather flat \( Q^2 \) dependence cannot be taken as a signature of parton-hadron duality for this region, but on the contrary, it shows the limits of applicability of this idea.

Having this determination in hand, one can now attempt to provide theoretical interpretations. In particular, the fact that the pattern of dynamical power corrections to resonance production is comparable to the one for the large \( Q^2 \) deep inelastic data, \( i.e. \) that power corrections remain small in the resonance region, suggests that color confinement is more likely to happen locally, with a smooth transition between partonic and hadronic configurations, a mechanism supported also by recent studies of hadron spectra in jet measurements \([1]\). This mechanism seems to break down in the \( N - \Delta \) transition region, providing a threshold where cooperative effects from many partons dominate the structure function. More information on the locality of color confinement may be obtained by studying semi-inclusive processes where a more direct relation to cluster hadronization models can be established \([24]\).

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FIG. 1: Comparison of NLO pQCD calculations (dashed lines), and NLO pQCD+TMC (solid lines) with the data on $F_2^p$ (hatched areas) at fixed values of $W^2 = W^2_R$, vs. $x$: (a) $W^2_R = 1.6$, (b) $W^2 = 2.3$, (c) $W^2 = 2.8$, (d) $W^2_R = 3.4$ GeV$^2$. The dotted curve shows for comparison the DIS calculation, obtained at $Q^2 = 200$ GeV$^2$. The data are averaged with the procedure described in the text and reference. The pQCD curves were obtained using the GRV parametrizations for the NS distributions at the input value of $Q^2_o = 0.4$ GeV$^2$ \[[19]\], indicated by the arrows. Other parametrizations \[[20]\] give similar results, starting from their input value of $Q^2_o \approx 1$ GeV$^2$. Below $Q^2 = 0.4$ GeV$^2$, the similarity between the pQCD curves and the data in the resonance region can no longer be tracked by pQCD.
FIG. 2: Comparison of pQCD+TMC calculations at NLO (dashed lines) and with resummation (full lines), with current large $x$ data. The full dots are in the resonance region, $1.3 \leq W^2 \leq 3.4$ GeV$^2$; the open triangles correspond to $W^2 \leq 1.3$ GeV$^2$. The dotted lines represent the regions where TMC contributions are uncertain.
FIG. 3: (a) Coefficient $C_{HT}$, Eq.(3), extracted from DIS data with $W^2 \geq 4 \text{ GeV}^2$ (full dots), from the resonance region, $W^2 < 4 \text{ GeV}^2$ (stars) and averaged over the entire range of $W^2$ (open dots). The shaded area summarizes extractions previous to the current one. A dotted line at zero is added to guide the eye; (b) $\Delta H$, Eq.(4), extracted at fixed values of $W^2$ as described in the text, and plotted vs. $Q^2$. The figure further elucidates a breakdown of the twist expansion at low $W^2$, already visible in (a).