Entanglement swapping in the transactional interpretation of quantum mechanics

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Abstract
The transactional interpretation (TI) of quantum mechanics, which uses retarded and advanced solutions of the Schrödinger equation and its complex conjugate, offers an original way to visualize and understand quantum processes. After a brief review, we show how it can be applied to different quantum situations, emphasizing the importance of specifying a complete configuration of absorbers. We consider in more detail the phenomenon of entanglement swapping, and see how the apparent retroactive enforcement of entanglement can be understood in the TI.

Keywords: quantum mechanics, entanglement swapping, transactional interpretation

1. Introduction
It was more than 25 years ago that John Cramer proposed what he called the transactional interpretation (TI) of quantum mechanics, in a comprehensive paper [1] that is still an excellent introduction to the subject. In Cramer’s view, ‘interpretation must not only relate the formalism to physical observables. It must also define the domain of applicability of the formalism and must interpret the nonobservables in such a way as to avoid paradoxes and contradictions.’ Indeed his approach was not meant to be empirically different from standard quantum mechanics, but rather to make it more understandable.

The TI in fact proposes a mechanism to visualize quantum processes. In Cramer’s words, it ‘provides a description of the state vector as an actual wave physically present in real space and provides a mechanism for the occurrence of non-local correlation effects through the use of advanced waves.’

The idea of interpreting the wave function as a real wave goes back to Schrödinger [2] and de Broglie [3]. It is further developed in Khrennikov’s more recent ‘Prequantum classical statistical field theory’ [4, 5], where references to other related approaches can be found.

In spite of its intuitive appeal, TI has not hitherto raised the same amount of interest as better known interpretations like de Broglie–Bohm mechanics [3, 6] or Everett’s relative states [7, 8], let alone the Copenhagen interpretation [9, 10]. For instance, the year 2013 appears to have been the first time it has been discussed at one of the Växjö conferences. Hopefully the recent monograph [11] published on the subject will contribute to raising awareness of TI.

Cramer’s original proposal has since undergone several developments, such as involving a hierarchy of transactions [12] or the reality of possibility [11]. Other contributions have stayed closer to the original approach and the block-universe picture of time [13, 14].

In section 2 we briefly review TI and its ‘explanation’ of the Born rule. Section 3 analyzes several quantum situations that have been proposed as challenges to TI. The importance of fully specifying the configuration of absorbers is emphasized. In section 4 we examine the phenomenon of entanglement swapping which, to our knowledge, has not hitherto been investigated from the point of view of TI. We show how the apparent retroactive enforcement of entanglement can be understood in TI. We conclude in section 5.

2. The transactional interpretation
Cramer’s TI of quantum mechanics was inspired by the time-symmetric electromagnetic theory of Wheeler and Feynman [15, 16]. In this approach advanced solutions of Maxwell’s
equations are as physically significant as retarded ones, and the universe is a perfect absorber of all the electromagnetic radiation emitted in it. In the spirit of Wheeler and Feynman, Cramer attributes physical reality both to solutions of the Schrödinger equation (propagating forward in time) and to their complex conjugates (propagating backward in time).

The relevance of the complex conjugate solutions comes from the fact that the Schrödinger equation should be viewed as the nonrelativistic limit of a relativistically invariant equation (like Dirac’s or Klein–Gordon’s), where advanced solutions occur naturally. In this sense Cramer’s theory is relativistically invariant. A field-theoretical generalization of it is outlined in [11].

An example of a quantum-mechanical process is the emission of a microscopic particle (e.g. an electron or a photon) at some time \( t_0 \), followed by its absorption at a later time \( t \). The usual solution \( \psi \) of the Schrödinger equation (called by Cramer an ‘offer wave’), which originates at \( t_0 \), propagates through \( t > t_0 \). This solution \( \psi \) reaches all potential detectors. Its amplitude \( \psi(\mathbf{r}, t) \) at detector \( i \) is given by the Schrödinger equation. Each detector in turn emits an advanced (or ‘confirmation’) wave, propagating backwards in time. Cramer argues that the confirmation wave coming from detector \( i \) reaches the source with an amplitude proportional to

\[
|\psi(\mathbf{r}, t)|^2 = \psi(\mathbf{r}, t)\psi^*(\mathbf{r}, t).
\]

(1)

The first factor on the right-hand side of (1) coincides with the amplitude of the stimulating offer wave at \( i \), while the second one comes from the fact that the confirmation wave develops as the time reverse of the offer wave. Note that all confirmation waves coming from all possible detectors reach the source at the same time \( t_0 \). Under the constraint that all relevant conservation laws are satisfied, a ‘transaction’ is eventually established between the emitter and one of the detectors, which corresponds to a completed quantum-mechanical process. If the probability that the transaction is established with detector \( i \) is taken to be proportional to the amplitude (1) of the confirmation wave coming from that detector, Born’s rule follows.

\[
|\psi(\mathbf{r}, t)|^2 = \psi(\mathbf{r}, t)\psi^*(\mathbf{r}, t).
\]

3. Applications and challenges

3.1. Renninger’s experiment

Renninger’s negative-result experiment [17], an example analyzed by Cramer [1], provides a nice illustration of how a transaction works. Figure 1 depicts a stationary source S at the center of two truncated spherical shells \( E_1 \) and \( E_2 \) of radii \( R_1 \) and \( R_2 \), respectively. Shell \( E_1 \) subtends a solid angle of \( 2\pi \) whereas shell \( E_2 \) subtends at least the complementary solid angle. On each shell, perfect absorbers are lined up that will detect any particle coming from S. The system is set up so that at time \( t_0 \) the source emits exactly one particle with speed approximately equal to \( v \) and spherically symmetric wave function. Let \( t_i = t_0 + R_i/v \). For \( t_0 < t < t_i \), the state vector

\[
|\psi(\mathbf{r}, t)|^2 = \psi(\mathbf{r}, t)\psi^*(\mathbf{r}, t).
\]

3.2. Maudlin’s challenge

Advanced interactions may raise the specter of inconsistent causal loops. This doesn’t happen in Cramer’s theory, owing to the lack of independent control on advanced waves. But Maudlin has argued [18] that they can lead to inconsistent evaluations of probabilities.

Maudlin’s challenge is depicted in figure 2. A slow particle is emitted by source S to the left or to the right, with equal probability. Two detectors are initially stationed on the right-hand side, one behind the other. If the particle is not detected by A, then B quickly swings to the left in time to catch it. Maudlin observes that every time B goes to the left, it absorbs the particle. Yet the amplitude of the confirmation wave arriving at the source from the left-hand side in this case
is equal to 0.5 only. How can absorption by B always occur in that situation?

An answer to Maudlin’s challenge has been proposed in [13], and assumes as in Wheeler–Feynman electrodynamics that every offer wave is eventually absorbed. This is equivalent to putting a third detector (say C) at the far left. Then the probability of absorption by any detector to the left of S becomes equal to the amplitude of the confirmation wave coming from the left. The scheme is fully consistent with the block-universe picture of time.

Several other answers to Maudlin’s challenge have also been proposed and are thoughtfully reviewed in [19]. Berkovitz [21] points out that in situations involving causal loops, relative frequencies of events may not coincide with objective chances. Cramer [12] suggests ‘a hierarchy of transaction formation, in which transactions across small space-time intervals must form or fail before transactions from larger intervals can enter the competition.’ Kastner [11] introduces the framework of a ‘possibilist transactional interpretation’ (PTI), where the offer and confirmation waves are viewed as real, but not actual, waves of possibility. And Lewis [19] argues for considering emitters and detectors as full-fledged quantum systems, a view strongly criticized by Kastner [20].

3.3. Interaction-free measurements

Another challenge which has been raised against Cramer’s ideas comes from the so-called quantum liar experiment [22], shown in figure 3. Photons are emitted by the laser L, and directed to a Mach–Zehnder interferometer. In each path of the interferometer is an atom which can absorb a photon if it interacts with it. The atoms are in a superposition of spin plus and minus states with respect to the z-axis.

In each run of the experiment, the photon–atoms state vector can be followed through beam splitter BS, reflection at M1 and M2, entanglement of the photon with either atom 1 or atom 2, recombination at BS and, finally, possible absorption by C or D [14, 22]. It follows directly from the expression of the final state vector that if a photon is absorbed by detector D, then the atoms are left in an entangled state of the form

$$|\psi_{\text{atoms}}\rangle = \frac{1}{\sqrt{2}} (|z_1\rangle |z_2\rangle + |z_1\rangle |z_2\rangle).$$

(3)

The problem consists in explaining how one photon, necessarily absorbed if it interacts with an atom, can entangle both of them.

Kastner [11, 23] approaches this problem in the framework of PTI, where offer and confirmation waves ‘have access to a larger physically real space of possibilities.’ This is motivated by the observation that N-particle wave functions are defined in 3N-dimensional configuration space which, from a physical point of view, has no straightforward projection into ordinary three-dimensional space.

On the other hand, a careful analysis of absorbers [14] again provides a solution closer to the original formulation of TI. Eventual measurements of the atoms’ spin necessary to verify consequences of their entanglement require detectors that are specifically set to measure the spin projection on the z-axis. Moreover, absorption at D entails that the confirmation wave coming from D will go through both paths. Atoms 1 and 2 are linked through the interplay of offer waves interacting with them en route to D and confirmation waves going from D to them, which explains the entanglement.

4. Entanglement swapping

4.1. The problem

The process of entanglement swapping [24] consists in entangling two quantum systems that have never interacted, through each system’s entanglement with another one. It can be illustrated as follows.

Suppose that Alice can prepare entangled spin 1/2 particles labelled 1 and 2. It will be useful to define ‘Bell states’ $|\Phi^{\pm}\rangle_{12}$ and $|\Psi^{\pm}\rangle_{12}$ as

$$|\Phi^{+}\rangle_{12} = \frac{1}{\sqrt{2}} (|+\rangle_1 |+\rangle_2 \pm |-\rangle_1 |-\rangle_2),$$

(4)

$$|\Phi^{-}\rangle_{12} = \frac{1}{\sqrt{2}} (|+\rangle_1 |-\rangle_2 \pm |-\rangle_1 |+\rangle_2),$$

(5)

where tensor product signs are understood between adjacent kets and all spin components are defined with respect to the same axis. We also assume that Bob, far away from Alice, can prepare similar states labelled 3 and 4. For simplicity, the spatial parts of the state vectors are suppressed throughout. A more complete treatment would also associate offer and confirmation waves with them. See [11], appendix C, for a situation where such considerations are relevant.

At some time $t_0$, Alice prepares her particles in the singlet state $|\Phi^{-}\rangle_{12}$ and Bob prepares his in the state $|\Psi^{-}\rangle_{34}$. The four-
photon initial state is therefore given by
\[ |\Psi\rangle_{1234} = |\Psi^-\rangle_{14}|\Psi^-\rangle_{23} \]
\[ = \frac{1}{2} \left( |+\rangle_{1}|-\rangle_{2}|+\rangle_{3}|-\rangle_{4} \right. \\
\left. - |+\rangle_{1}|+\rangle_{2}|+\rangle_{3}|+\rangle_{4} \right)
\[ - |+\rangle_{1}|+\rangle_{2}|+\rangle_{3}|+\rangle_{4} \] \\
\[ + |+\rangle_{1}|+\rangle_{2}|+\rangle_{3}|+\rangle_{4} \right). \tag{6} \]

It is simple algebra to show that this can also be written as
\[ |\Psi\rangle_{1234} = \frac{1}{2} \left( |\phi^+\rangle_{14}|\phi^\pm\rangle_{23} - |\phi^-\rangle_{14}|\phi^-\rangle_{23} \right) \] \\
\[ - |\phi^-\rangle_{14}|\phi^\pm\rangle_{23} + |\phi^\pm\rangle_{14}|\phi^-\rangle_{23} \right). \tag{7} \]

Now suppose that Alice sends particle 2 and Bob sends particle 3 to Eve. Then Eve can make spin measurements in her Bell-state basis \(|\Psi^\pm\rangle_{23} \) and \(|\Phi^\pm\rangle_{23} \). If she gets the result \(|\Psi^+\rangle_{23} \), one sees from (7) that particles 1 and 4 become entangled in state \(|\Psi^+\rangle_{14} \) without ever having interacted.

It was pointed out in [25] that the measurement of particles 2 and 3 can be made after measurements have been carried out on particles 1 and 4. If the measurements of 2 and 3 are made in the Bell state basis, then 1 and 4 behave as in entangled states. If, however, the measurements of 2 and 3 are made in the basis
\[ \{|1\rangle_2, \{+\rangle_2, \{-\rangle_2, \{-\rangle_2 \} \} \}, \tag{8} \]
then 1 and 4 behave as in product states. In the words of [26], 'whether (the) two photons are entangled (showing quantum correlations) or separable (showing classical correlations) can be defined after they have been measured.'

Different interpretations of quantum mechanics can address this paradox in various ways. We will show in the next section how TI characteristically helps to visualize the situation.

### 4.2. The view from TI

In TI the pattern of offer and confirmation waves depends on the configuration of detectors. Let us first consider the case where particles 2 and 3 are measured in basis (8). The situation is depicted in figure 4. Here offer waves are represented by upward arrows, confirmation waves by downward arrows and all measurement axes coincide (\( \vec{n}_1 = \vec{n}_2 = \vec{n}_3 = \vec{n}_4 \)).

It is seen that the arrows of particle 2 do not connect with the ones of particle 3. Hence there can be no connection between the arrows of 1 and 4, and thus no entanglement.

In the situation where measurements of 2 and 3 are made in the Bell state basis, the pattern of offer and confirmation waves is depicted in figure 5. Since Bell states are linear combinations of states of particles 2 and 3, the corresponding offer and confirmation waves (represented by upward and downward arrows, respectively) are superpositions of offer and confirmation waves of both 2 and 3. Whichever Bell state results from a quantum measurement, this establishes an unbroken link between particles 2 and 3 and, through the sources, between particles 1 and 4. This connection is responsible for the entanglement between 1 and 4. More explicitly, a confirmation wave originating from a detector set to measure particle 1 will reach the source on the left. It connects to an offer wave leaving the source towards the right, which reaches the Bell state detectors. Because the Bell states are superpositions, their detectors emit confirmation waves towards both sources, in particular the one to the right. The offer wave from that source reaches detectors associated with particle 4. The upshot is that in TI, the presence or absence of entanglement between 1 and 4 depends on the full configuration of offer and confirmation waves, itself dependent on the specific configuration of detectors. Whether there is entanglement or not is defined early, but it is causally dependent on the future configuration of detectors.

### 5. Conclusion

The TI of quantum mechanics provides a way to visualize quantum phenomena. The fact that it involves advanced causation has led some to believe that it gives rise to paradoxes. But the full specification of offer and confirmation waves, crucial in TI, necessitates a complete specification of detectors. This is consistent with the block-world view of space-time and provides a general framework for the solution of quantum paradoxes.
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