Can the effective string see higher partial waves?

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Abstract

The semi-classical cross-sections for arbitrary partial waves of ordinary scalars to fall into certain five-dimensional black holes have a form that seems capable of explanation in terms of the effective string model. The kinematics of these processes is analyzed in detail on the effective string and is shown to reproduce the correct functional form of the semi-classical cross-sections. But it is necessary to choose a peculiar value of the effective string tension to obtain the correct scaling properties. Furthermore, the assumptions of locality and statistics combine to forbid the effective string from absorbing more than a finite number of partial waves. The relation of this limitation to cosmic censorship is discussed.

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1. Introduction

The D1-brane D5-brane bound state toroidally compactified down to five dimensions has proven to be one of the most fruitful string theoretic models of black holes. Since the original paper of [1], which proposed the model as a way to study black hole dynamics in a manifestly unitary string theoretic framework, and the subsequent work in [2] clarifying the means by which a single multiply wound effective string arises in a description of the low-energy dynamics, there have been many exciting papers relating properties of five-dimensional and four-dimensional black holes to the effective string. An explanation of the near-extremal entropy was given in [1,3,4], following the ideas originally laid out in [5]. Absorption cross-sections and the corresponding Hawking emission rates were worked out in [6,7,8,9,10,11,12,13,14,15] yielding impressive agreement at low energies with the effective string model. Suggestions that the effective string model may have some flaws or limitations have arisen in the work of [16,17,18].

In a recent paper by Strominger and Maldacena [14] it was found from a general analysis of thermal two-point functions that the effective string seems capable of explaining the semi-classical absorption cross-sections for arbitrary partial waves of ordinary scalars. Subsequent work by Mathur [19] exhibited more detailed agreement between the effective string model and General Relativity for these processes. Ordinary scalars are scalars whose equation of motion in five-dimensions is $\Box \phi = 0$. The canonical examples of ordinary scalars are the off-diagonal gravitons $h_{ij}$ with both $i$ and $j$ lying within the D5-brane but perpendicular to the D1-brane. These are the scalars for whose $s$-wave cross-section full agreement between General Relativity and the effective string was first achieved in [7].

The results of [14,19] overlap substantially with unpublished work by myself [20]. The present paper is based upon that work, which extends the results of [19] in certain technical aspects. The organization of the paper is as follows. Section 2 covers the semi-classical analysis of partial wave absorption and includes a derivation of a form of the Optical Theorem for the absorption of scalar particles which was quoted without proof in [21]. Section 3 presents the effective string description of the same processes, exhibiting along the way a simple method for performing all the phase space integrals encountered in [19]. In section 4, the limitation on the number of partial waves the effective string can couple to arising from statistics and locality is compared with the limitation imposed semi-classically by cosmic censorship. Section 5 summarizes the results and indicates directions for further work.

2. The semi-classical computation

The quantity that can be conveniently computed using the matching technique is the absorption probability. To convert this to an absorption cross-section, it is necessary to use properties of the partial wave expansion in four spatial dimensions and to invoke the
Optical Theorem. The details of this connection were worked out independently in [19], but because the derivation given below applies for arbitrary dimensions, it seems worthwhile to present it in full. In all of what follows, \( n = d - 1 \) will denote the number of spatial dimensions.

The Optical Theorem for scattering of a scalar field off a spherically symmetric potential states that if the scattering wave-function has for large \( r \) the asymptotic form

\[
\phi(\vec{r}) \sim e^{ikx} + f(\theta) \frac{e^{ikr}}{r^{(n-1)/2}}
\]  

(here \( x = r \cos \theta \)), then the total cross-section is

\[
\sigma_{\text{total}} = -2 \left( \frac{2\pi}{k} \right)^{n-1} \Re \left( i^{n-1} f(0) \right) .
\]

To find the partial wave expansion of (1), it is first necessary to make a Neumann expansion of the exponential function, which can be done using Gegenbauer polynomials [22]:

\[
e^{ir \cos \theta} = 2^{n/2-1} \Gamma(n/2 - 1) \sum_{\ell=0}^{\lfloor \ell/2 \rfloor} i^\ell P_\ell(\cos \theta)(\ell + n/2 - 1) \frac{J_{\ell+n/2-1}(r)}{r^{n/2-1}}
\]

\[
P_\ell(\cos \theta) = \sum_{m=0}^{\lfloor \ell/2 \rfloor} (-1)^m 2^{\ell-2m} \frac{\Gamma(\ell + n/2 - 1 - m)}{\Gamma(n/2 - 1)m!(\ell - 2m)!} \cos^{\ell-2m} \theta
\]

The \( P_\ell(\cos \theta) \) are just the Legendre polynomials when \( n = 3 \). For arbitrary \( n \), they can be defined by the expansion

\[
(1 - 2a \cos \theta + a^2)^{1-n/2} = \sum_{\ell=0}^{\infty} P_\ell(\cos \theta) a^\ell .
\]

An alternate normalization proves more convenient:

\[
\tilde{P}_\ell(\cos \theta) = \sqrt{\frac{2}{\pi}} 2^{n/2-1} \Gamma(n/2 - 1)(\ell + n/2 - 1)P_\ell(\cos \theta) .
\]

Using asymptotic properties of Bessel functions, one can now write down the partial wave expansion of (1) as

\[
e^{ikx} + f(\theta) \frac{e^{ikr}}{r^{(n-1)/2}} \sim \sum_{\ell=0}^{\infty} \frac{1}{2} \tilde{P}_\ell(\cos \theta) S_\ell e^{ikr} + (-1)^\ell i^{n-1} e^{-ikr} \frac{S_\ell e^{ikr}}{(ikr)^{(n-1)/2}} .
\]

When \( f(\theta) = 0 \) identically, \( S_\ell = 1 \) for all \( \ell \).
The absorption cross-section for the $\ell$th partial wave can now be computed as the difference between the total $\ell$-wave cross-section computed via the Optical Theorem,

$$\sigma_{\text{total}}^\ell = -\left(\frac{\sqrt{2\pi}}{k}\right)^{n-1} \tilde{P}_\ell(1) \Re(S_\ell - 1) ,$$

and the $\ell$-wave scattering cross-section,

$$\sigma_{\text{scattered}}^\ell = (\text{Vol} S^{n-2}) \frac{|S_\ell - 1|^2}{4k^{n-1}} \int_0^\pi d\theta \sin^{n-2} \theta \tilde{P}_\ell(\cos \theta)^2 .$$

The final result,

$$\sigma_{\text{abs}}^\ell = \frac{2^{n-2} \pi^{n/2-1}}{k^{n-1}} \Gamma(n/2 - 1)(\ell + n/2 - 1) \left(\ell + n - 3\right) \left(1 - |S_\ell|^2\right) ,$$

relates the absorption cross-section $\sigma_{\text{abs}}^\ell$ to the absorption probability $1 - |S_\ell|^2$. The results for $n = 3$ and $n = 4$ are

$$\sigma_{\text{abs}}^\ell = \frac{\pi}{k^2} (2\ell + 1) \left(1 - |S_\ell|^2\right) \quad \text{in three spatial dimensions}$$

$$\sigma_{\text{abs}}^\ell = \frac{4\pi}{k^3} (\ell + 1)^2 \left(1 - |S_\ell|^2\right) \quad \text{in four spatial dimensions} .$$

With these results in hand, let us proceed to the semi-classical computation of the cross-section for an ordinary scalar $\phi$ in the $\ell$th partial wave to be absorbed into a black hole. It is hoped that this piece of “spectroscopic data” will be illuminating of the form of the effective string action.

An ordinary scalar is one whose equation of motion is just the Laplace equation following from the black hole metric, which in five dimensions is

$$ds^2 = -F^{-2/3}h dt^2 + F^{1/3} (h^{-1} dr^2 + r^2 d\Omega_3^2)$$

$$F = f_1 f_5 f_K = \left(1 + \frac{r_1^2}{r^2}\right) \left(1 + \frac{r_2^2}{r^2}\right) \left(1 + \frac{r_K^2}{r^2}\right)$$

$$h = 1 - \frac{r_0^2}{r^2} .$$

The mass, entropy, Hawking temperature, $U(1)$ charges, and characteristic radii are conveniently parameterized as

$$M = \frac{\pi}{8} r_0^2 \sum_{i=1,5,K} \cosh 2\sigma_i \quad S = \frac{\pi^2}{2} r_0^3 \prod_{i=1,5,K} \cosh \sigma_i \quad \beta_H = 2\pi r_0 \prod_{i=1,5,K} \cosh \sigma_i$$

$$Q_i = \frac{r_0^2}{2} \sinh 2\sigma_i \quad r_i = r_0 \sinh \sigma_i .$$
in five-dimensional Planck units. Using a separation of variables \( \phi = e^{-i\omega t}P_l(\cos \theta)R(r) \), one can extract from the Laplace equation \( \Box \phi = 0 \) the radial equation

\[
[(hr^3 \partial_r)^2 + r^6 F \omega^2 - r^4 h \ell(\ell + 2)] R = 0.
\]

(13)

Because of the left-right symmetry of the effective string description for five-dimensional black holes, the absorption cross-section for the near-extremal case provides essentially no more information about the effective string than the extremal case does. In the interest of a simple presentation, I will therefore restrict my calculations in both this section and the next to the extremal case. The near-extremal generalizations of the results are summarized at the end of each section.

In the near horizon region (denoted I for consistency with the literature \cite{23,11,12}), (13) for an extremal black hole can be approximated by a Coulomb equation \cite{24} in the variable

\[
 y = \frac{(r_1 r_5 r_K \omega)}{2r^2},
\]

while in the far horizon region III it can be approximated as a Bessel equation:

\[
\text{I. } \left( \partial_y^2 + \frac{1 - 2\eta}{y} - \frac{\ell(\ell + 2)}{y^2} \right) R_I = 0 \quad \quad R_I = G_{\ell/2}(y) + iF_{\ell/2}(y)
\]

\[
\text{III. } \left[ (r^3 \partial_r)^2 + r^6 \omega^2 - r^4 \ell(\ell + 2) \right] R_{III} = 0 \quad \quad R_{III} = \frac{\alpha J_{\ell+1}(\omega r)}{\omega r} + \frac{\beta N_{\ell+1}(\omega r)}{\omega r},
\]

(14)

where \( \alpha \) and \( \beta \) are constants to be determined in the matching and

\[
\eta = -\frac{1}{4} \sum_{i=1,5,K} \frac{r_1 r_5 r_K \omega}{r_i^2} \equiv -\frac{\omega}{4\pi T_L}
\]

(15)

is the charge parameter of the Coulomb functions. The infalling solution \( R_I \) can be matched directly onto \( R_{III} \) without the aid of an intermediate region II, with the result

\[
\alpha = \frac{\ell!}{C_{\ell/2}(\eta)} \frac{2^{2\ell+1} \pi \ell}{(A_h \omega^3)^{\ell/2}}, \quad \beta = 0.
\]

(16)

The quantity

\[
C_{\ell/2}(\eta) = \frac{2^{\ell/2} e^{-\pi \eta/2} \left| \Gamma \left( \frac{\ell}{2} + 1 + i\eta \right) \right|}{\Gamma(\ell + 2)}
\]

(17)

enters into the series expansion of Coulomb functions.

A more accurate matching can be obtained with \( \beta \neq 0 \), but the level of accuracy embodied in (16) is sufficient for the flux ratio method \cite{11,11}. In this method, the absorption probability is computed as the ratio of the infalling flux at the horizon to the flux in the incoming wave at infinity. The result is

\[
1 - |S_\ell|^2 = \frac{1}{\pi} \frac{A_h \omega^3}{|\alpha|^2} = 4\pi \left( \frac{A_h \omega^3}{16\pi^2} \right)^{\ell+1} \frac{C_{\ell/2}^2(\eta)}{\ell!^2}.
\]

(18)
Now the formula (19) comes into play to give the final result:

\[ \sigma_{\text{abs}}^\ell = A_h (\ell + 1)^2 \left( \frac{A_h \omega^3}{16\pi^2} \right)^{\ell} \frac{C_{\ell/2}(\eta)}{\ell!^2} \]

\[ = \frac{A_h}{\ell!^4} \left( \frac{A_h \omega^3}{8\pi^2} \right)^{\ell} e^{\frac{-i\omega}{4\pi T_L}} \left| \Gamma \left( \frac{\ell}{2} + 1 - i \frac{\omega}{4\pi T_L} \right) \right|^2 \]

(19)

The right hand side of (19) depends on \( r_1, r_5, \) and \( r_K \) only through \( A_h \) and \( T_L \). Both quantities are symmetric in the three radii, and in fact admit U-duality invariant generalizations [25]. The near-extremal generalization of (19),

\[ \sigma_{\text{abs}}^\ell = A_h \frac{(\omega r_0/2)^{2\ell}}{\ell!^4} \left| \frac{\Gamma \left( 1 + \frac{\ell}{2} - i \frac{\omega}{4\pi T_L} \right) \Gamma \left( 1 + \frac{\ell}{2} - i \frac{\omega}{4\pi T_R} \right)}{\Gamma \left( 1 - i \frac{\omega}{2\pi T_H} \right)} \right|^2 \]

(20)

also treats the three charges on an equal footing. The temperatures \( T_L \) and \( T_R \) are given by [25]

\[ \beta_{L,R} = 2\pi r_0 \left( \prod_i \cosh \sigma_i \mp \prod_i \sinh \sigma_i \right) \]

(21)

in the general non-extremal case.

3. The effective string analysis

Despite recent progress in formulating superspace actions for branes [26,27,28] and in generalizing the DBI action to nonabelian gauge theory (see [29] and references therein), a first-principles derivation of a complete action for the effective string, including all couplings to fields in the bulk of spacetime, has yet to be achieved. The goal of this section is to write down a reasonable form for the part of the action responsible for coupling the effective string to higher partial waves of an ordinary scalar and see how the cross-sections it predicts compare with the semi-classical result (19).

Consider the off-diagonal graviton \( h_{ij} \) with \( i \) and \( j \) parallel to the D5-brane but perpendicular to the D1-brane. The lowest-order interaction of this field with excitations on the effective string can be read off from the DBI action [7]: in static gauge where \( t = \tau \) and \( x^5 = \sigma \),

\[ V_{\text{int}} = -t_{\text{eff}} \int_0^{L_{\text{eff}}} d\sigma 2 h_{ij}(\tau, \sigma, \vec{x}=0) \partial_+ X^i \partial_- X^j . \]

(22)

The convention in (22) and elsewhere is to sum over all \( i \neq j \). The fields \( h_{ii} \) couple somewhat differently; an exploration of those couplings and their physical consequences was initiated in [12].
In [30], an analysis of the entropy and temperature of near-extremal 5-branes led to an effective string with \( c_{\text{eff}} = 6 \) and \( T_{\text{eff}} = 1/(2\pi r_5^2) \). An extension of the methods used in [30] to the case \( r_1 \sim r_5 \) leads to [31]

\[
T_{\text{eff}} = \frac{1}{2\pi (r_1^2 + r_5^2)}.
\]

The natural assumption is that \( t_{\text{eff}} \), by definition the tension that appears in front of the DBI action, is precisely \( T_{\text{eff}} \). Strangely enough, all previous scattering calculations except the fixed scalar computation of [18] (whose implications regarding the effective string tension are unclear to me) either do not depend on \( t_{\text{eff}} \) or require \( r_1 = r_5 \). Thus, purely from the point of view of scattering computations, \( t_{\text{eff}} \) seems ambiguous by a factor of the form \( f(r_1/r_5) \) where \( f(1) = 1 \). One of the motivations for studying higher partial waves is to resolve this ambiguity. The result I will obtain is

\[
\frac{1}{2\pi r_1 r_5}.
\]

Because of the evaluation of \( h_{ij} \) at \( \vec{x} = 0 \), (22) is a coupling to the \( s \)-wave of \( h_{ij} \) only. How might it be generalized to include the dominant couplings to arbitrary partial waves? To begin with, a coupling to the \( \ell \)-th partial wave should include \( \ell \) derivatives of \( h_{ij} \) since the wave-function vanishes like \( |\vec{x}|^{\ell} \). It is the fermions on the effective string which carry the angular momentum [3,4]: the left-moving and right-moving fermions transform in a fundamental of \( SU(2)_L \) and \( SU(2)_R \) respectively, where the \( SO(4) \) of rotations in the four noncompact spatial dimensions is written as \( SO(4) = SU(2)_L \times SU(2)_R \). Purely on group theory grounds, one thus expects the \( \ell \)-th partial wave to couple to \( \ell \) left-moving and \( \ell \) right-moving fermions. The order of the absorption process in the string coupling can be read off from (19) as \( g^{\ell+1} \) where \( g \) is the closed string coupling. Exactly two more open string vertex operators should be included in the interaction to make the disk diagram come out with this power of \( g \). The natural candidate for the interaction is

\[
V_{\text{int}} = -t_{\text{eff}} \int_0^{L_{\text{eff}}} d\sigma 2 h_{ij}(\tau, \sigma, x^m = \bar{\Psi} \gamma^m \Psi) \partial_+ X^i \partial_- X^j
\]

\[
= -t_{\text{eff}} \int_0^{L_{\text{eff}}} d\sigma 2 \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \left( \prod_{k=1}^{\ell} \bar{\Psi} \gamma^m \gamma^k \Psi \right) \partial_{m_1} \cdots \partial_{m_\ell} h_{ij}(\tau, \sigma, x^m = 0) \partial_+ X^i \partial_- X^j.
\]

Extra derivatives on the fermion fields are possible \textit{a priori}, but power counting in \( \omega \) for \( \omega/T_L \ll 1 \) shows that they must be absent if (19) is to be reproduced. The same general form of the coupling was deduced independently in [14] through a greybody factor analysis.

The outstanding fallacy of (25) is that the sum terminates at \( \ell = 4 \) because there are only four types of left-moving fermions and the same number of right-moving fermions.
The situation is even worse when one factors in the restrictions from \( SO(4) \) group theory. As we shall see after (29), only the \( \ell = 0 \) and \( \ell = 1 \) partial waves can be absorbed.

In (25), the \( \gamma^m \) are gamma matrices of \( SO(4,1) \):

\[
\gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \quad \gamma^m = \begin{pmatrix} 0 & \tau^m_{\alpha\dot{\beta}} \\ \tau^{m\dot{\alpha}\beta} & 0 \end{pmatrix}
\]

(26)

where

\[
\tau^m_{\alpha\dot{\alpha}} = (1, i\sigma_1, i\sigma_2, i\sigma_3) \quad \tau^{m\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\beta} \epsilon^{\alpha\beta} \tau^m_{\beta\dot{\beta}} = (1, -i\sigma_1, -i\sigma_2, -i\sigma_3),
\]

(27)

\( \sigma_i \) being the usual Pauli matrices. I follow northwest contraction conventions for raising and lowering spinor indices, and I set \( \epsilon_{01} = \epsilon^{01} = \epsilon_{\dot{0}\dot{1}} = \epsilon^{\dot{0}\dot{1}} = 1 \). The four-component spinor \( \Psi \) decomposes into \( SU(2)_L \) and \( SU(2)_R \) fundamentals, which are left-movers and right-movers on the effective string, respectively. These complex fermions decompose further into real components of the 10-dimensional Majorana-Weyl spinor that one would expect to emerge most simply from a full string theory analysis:

\[
\Psi = \left( \begin{array}{c} \Psi^+_{\alpha} \\ \Psi^-_{\dot{\alpha}} \end{array} \right)
\]

(28)

\( \Psi^\pm_1 = \frac{\psi^1_{\pm} + i\psi^2_{\pm}}{\sqrt{2}} \quad \Psi^\pm_2 = \frac{\psi^3_{\pm} + i\psi^4_{\pm}}{\sqrt{2}}. \)

Note that complex conjugation raises or lowers a spinor index, rather than dotting or undotting it as in the case of \( SO(3,1) \).

For \( \ell \geq 2 \) the \( \ell \)th term in the interaction (23) makes subleading contributions to the absorption of lower partial waves because the expression \( \partial_{m_1} \cdots \partial_{m_\ell} h_{ij} \) does not pick out a pure \( \ell \)th partial wave from a plane wave. The cure for this is to symmetrize \( SU(2)_L \) and \( SU(2)_R \) spinor indices:

\[
V_{\text{int}} = -2t_{\text{eff}} \int_0^{L_{\text{eff}}} d\sigma \sum_{\ell=0}^4 \frac{i^{\ell}}{\ell!} \prod_{k=0}^{\ell} \left( \Psi^{\alpha+}_{\alpha} \Psi^-_{\beta_k} + \Psi^{\alpha+}_{\dot{\alpha}} \bar{\Psi}^-_{\dot{\beta}_k} \right) \left( \tau^{m_1}_{(\alpha_1} \cdots \tau^{m_{\ell}}_{\alpha_\ell}) \right) 
\cdot \partial_{m_1} \cdots \partial_{m_\ell} h_{ij} \partial_+ X^i \partial_- X^j + \ldots
\]

(29)

Terms have been omitted in (29) which make subleading contributions. The product of fermion fields is antisymmetric in \( \alpha_1 \ldots \alpha_\ell \) and in \( \dot{\beta}_1 \ldots \dot{\beta}_\ell \). Hence all terms in (29) vanish except \( \ell = 0 \) and \( \ell = 1 \). The conclusion is that only the first two partial waves can be absorbed. This limitation is not merely a failing of the specific form (24); it is intrinsic to the approach of coupling partial wave to a product of fermion fields evaluated at a single point on the effective string without derivatives. To reiterate, the addition of derivatives introduces extra powers of the energy in the final cross-section which would
cause disagreement with (19). Let us proceed with the analysis of $\ell = 1$ and consider possible extensions to $\ell \geq 2$ later.

There are many steps involved in passing from the interaction (29) to the cross-section for $\ell = 1$. To avoid losing factors it pays to be as explicit as possible. Let us begin with mode expansions of the fields appearing in (29). The forms of mode expansions are dictated by the kinetic terms in the action. In the present case, the kinetic terms are

$$S_{\text{bulk}} = \frac{1}{2\kappa_6^2} \int d^6x \frac{1}{4} (\partial_\mu h_{ij}) (\partial^\mu h^{ij}) + \ldots$$

$$S_{\text{string}} = -2t_{\text{eff}} \int d^2\sigma \left[ \partial_+ X_i \partial_- X^i + \psi_+^\Delta i \partial_- \psi_+^\Delta + \psi_-^\Delta i \partial_+ \psi_-^\Delta + \ldots \right],$$

resulting in the mode expansions

$$\psi_+^\Delta (\tau + \sigma) = \sum_{k^5 \in \frac{2\pi}{L_5} \mathbf{Z}} \frac{1}{\sqrt{2L_{\text{eff}}^5 t_{\text{eff}}}} (b_k^\Delta e^{ik\cdot \sigma} + \text{h.c.})$$

$$\psi_-^\Delta (\tau - \sigma) = \sum_{k^5 \in \frac{2\pi}{L_5} \mathbf{Z}} \frac{1}{\sqrt{2L_{\text{eff}}^5 t_{\text{eff}}}} (b_k^\Delta e^{ik\cdot \sigma} + \text{h.c.})$$

$$X^i(\sigma, \tau) = \sum_{k^5 \in \frac{2\pi}{L_5} \mathbf{Z}} \frac{1}{\sqrt{2L_{\text{eff}}^5 t_{\text{eff}}}} (a_k^i e^{ik\cdot \sigma} + \text{h.c.})$$

$$h^{ij}(x^\mu) = \sum_{k^5, \vec{k}} \sqrt{\frac{2\kappa_6^2}{2V L_5^2 k^0}} (g_{k^5}^{ij} e^{i\vec{k}\cdot \vec{x}} + \text{h.c.}).$$

The sum over $k^5, \vec{k}$ has $k^5 \in \frac{2\pi}{L_5} \mathbf{Z}$ and $k^m \in \frac{2\pi}{\sqrt{V}} \mathbf{Z}$ for $m = 1, 2, 3, 4$. $V$ is the volume of a large box in which we imagine enclosing the four uncompactified spatial dimensions. The indices $\Delta$ and $\hat{\Delta}$ run from 1 to 4, and since $\psi_+^\Delta$ and $\psi_-^\Delta$ are real, conjugation does not change the position of the indices. Typographical convenience will dictate the position of $\Delta$ and $\hat{\Delta}$.

It is important that $h^{ij}$ is moded differently in the $x^5$ direction from the effective string excitations: the minimal quantum of Kaluza-Klein charge for an excitation on the effective string is $1/(n_1 n_5)$ of the minimal quantum for a particle in the bulk $[32,2]$. In (31) we have not been careful about zero modes because it is the oscillator states which are important for the absorption processes. The factors in (31) were chosen to make the commutation relations simple:

$$\{b_k^\Delta, b_q^\Gamma\} = \delta_{k^5 - q^5} \delta^\Delta\Gamma$$

$$\{b_k^\Delta, a_q^\Gamma\} = \delta_{k^5 - q^5} \delta^\Delta\Gamma$$

$$[a_k^i, a_q^j] = \delta_{k^5 - q^5} \delta^{ij}$$

$$[g_k^{ij}, g_q^{fh}] = \delta_{k^5 - q^5} \delta^{ij} \delta^{fh} \quad \text{if } i < j \text{ and } f < h.$$
The goal now is to compute the amplitude \( \langle \tilde{f} | V_{\text{int}} | \tilde{i} \rangle \) for an absorption process where a scalar in the \( \ell = 1 \) partial wave turns into two bosons and two fermions on the effective string. The tildes on \( |\tilde{i}\rangle \) and \( |\tilde{f}\rangle \) are meant to indicate that these state vectors are not the real initial and final states: they include only the particles that participate in the interaction and not the whole thermal sea of left-movers that give the effective string its Kaluza-Klein charge. Restoring the thermal sea is an easy exercise which will be postponed until (39).

Two other slight simplifications will be made to ease the notational burden. First, indices can be dropped on all the \( X^i \) fields, but then one must include an extra factor of 2 in the rate, as shown in (35). The 2 accounts for the fact that \( h_{ij} \) can turn into a left-moving \( X^i \) and a right-moving \( X^j \) or a left-moving \( X^j \) and a right-moving \( X^i \). The second simplification is to consider only

\[
|\tilde{i}\rangle = g_k^\dagger |0\rangle
\]

\[
|\tilde{f}\rangle = a_{p_i}^\dagger a_{q_b}^\dagger j_{p_f}^\dagger j_{q_f}^\dagger |0\rangle ,
\]

which is to say we put all the particles on the effective string into the final state and none into the initial state. A simple way to account for all the crossed processes which also contribute to absorption will be discussed after equation (42). In (33) and below, \( k \) refers to the momentum of the bulk scalar, \( p \) refers to the momentum of a left-mover on the effective string, and \( q \) refers to the momentum of a right-mover.

The desired matrix element can now be read off from (29) and (32) as

\[
\langle \tilde{f} | V_{\text{int}} | \tilde{i} \rangle = C_{\Delta}^\Delta \frac{k_1}{2 L_{\text{eff}} t_{\text{eff}}} \kappa_5 \sqrt{\frac{g_0^b g_0^d}{V k^0} \delta_{k_0} - p_b^0 - p_f^0 - q_b^0 - q_f^0} ,
\]

where \( C_{\Delta}^\Delta \) is the \( 4 \times 4 \) matrix diag\( \{1, -1, 1, -1\} \). To extract the rate is is necessary to use a generalization of Fermi’s Golden Rule that includes Bose enhancement factors and Fermi suppression factors:

\[
\Gamma = 2 \sum_{|\tilde{f}\rangle} (\rho_L^X(p_b^0) + 1)(\rho_R^X(q_b^0) + 1)(1 - \rho_L^\psi(p_f^0))(1 - \rho_R^\psi(q_f^0)) \]

\[
\cdot \left| \langle \tilde{f} | V_{\text{int}} | \tilde{i} \rangle \right|^2 2\pi \delta \left( k^0 - p_b^0 - p_f^0 - q_b^0 - q_f^0 \right) \]

\[
= 2 \sum_{\Delta, \Delta} |C_{\Delta}^\Delta|^2 \cdot \sum_{\text{modes}} (\rho_L^X(p_b^0) + 1)(\rho_R^X(q_b^0) + 1)(1 - \rho_L^\psi(p_f^0))(1 - \rho_R^\psi(q_f^0)) \]

\[
\cdot \frac{k_1^2 \kappa_5^2}{V k^0 (2 L_{\text{eff}} t_{\text{eff}})^2} g_0^b g_0^d \delta_{k_0} - p_b^0 - p_f^0 - q_b^0 - q_f^0 2\pi \delta \left( k^0 - p_b^0 - p_f^0 - q_b^0 - q_f^0 \right) \]

where

\[
\rho_L^X(p_b^0) = \frac{1}{e^{p_b^0/T_L} - 1} , \quad \rho_R^\psi(p_f^0) = \frac{1}{e^{p_f^0/T_L} + 1}
\]

(36)
and similarly for the right-moving thermal occupation factors. Again, the explicit factor of 2 in the first line of (33) is present to account for the two distinct choices, \(i+ j– \) or \(i– j+\), for polarizing the bosonic fields.

The vanishing of all cross-sections beyond \(\ell = 1\) is an egregious failing of the most naive effective string model. The simplest fix would be to allow an incoming scalar to couple to a product of fermion fields evaluated at a single point on the spatial \(S^1\) which the effective string wraps, but not necessarily at a single point in the effective string coordinates \(\sigma\). In terms of the \((4,4)\) SCFT from which the effective string emerges as a particular twisted sector, this more general coupling seems very natural because it still involves only a local operator constructed from a product of the \(4n_1n_5\) species of fermions in the SCFT.

The present treatment extends easily to cover this more general interaction. Let \(D_k\) and \(\hat{D}_k\) be \(4n_1n_5\)-valued indices for the left- and right-moving fermion fields, respectively.

Consider the final state

\[
|\tilde{f}\rangle = a^+_p a^+_q \left( b^\dagger_{p_1} b^\dagger_{q_1} \ldots b^\dagger_{p_{\ell}} b^\dagger_{q_{\ell}} \right) |0\rangle .
\]

The matrix element is now of the form

\[
\langle \tilde{f}| V_{\text{int}} |\tilde{i}\rangle = C_{D_1 \ldots D_{\ell}}^{\hat{D}_1 \ldots \hat{D}_{\ell}} \frac{k^\ell}{(2\ell^e f^e)^\ell} \sqrt{\frac{p^0_b q^0_b}{V k^0}} \delta_{k^5 - p^5} \delta_{p^5 - q^5 - \Sigma q^5_i} .
\]

The coefficient tensor \(C_{D_1 \ldots D_{\ell}}^{\hat{D}_1 \ldots \hat{D}_{\ell}}\) is antisymmetric in \(D_1 \ldots D_{\ell}\) and in \(\hat{D}_1 \ldots \hat{D}_{\ell}\). It encodes the \(SO(4)\) group theory factors isolating the \(\ell\)th partial wave as well as restrictions on the possible final states arising from D1-brane and D5-brane Chan-Paton factors. The rate is

\[
\Gamma = 2 \sum_{|\tilde{f}\rangle} \left( \rho^X(p^0_b) + 1 \right) \left( \rho^X(q^0_b) + 1 \right) \prod_{i=1}^{\ell} \left[ (1 - \rho^\psi_L(p^0_i))(1 - \rho^\psi_L(q^0_i)) \right]
\]

\[
\cdot \left| \langle \tilde{f}| V_{\text{int}} |\tilde{i}\rangle \right|^2 \frac{2\pi \delta (k^0 - p^0_b - \sum p^0_i - q^0_b - \sum q^0_i)}{\ell !^2}
\]

\[
= \frac{2}{\ell !^2} \sum_{D_k, \hat{D}_k} \sum_{\text{modes}} \left( \rho^X(p^0_b) + 1 \right) \left( \rho^X(q^0_b) + 1 \right) \prod_{i=1}^{\ell} \left[ (1 - \rho^\psi_L(p^0_i))(1 - \rho^\psi_L(q^0_i)) \right]
\]

\[
\cdot \frac{k^2_1 k^2_5}{V k^0 (2\ell^e f^e)^\ell} p^0_b q^0_b \delta_{k^5 - p^5} \delta_{p^5 - q^5 - \Sigma q^5_i} 2\pi \delta (k^0 - p^0_b - \sum p^0_i - q^0_b - \sum q^0_i) .
\]

The \(1/\ell !^2\) in the last expression arises because the sums over \(D_k, \hat{D}_k, p_k, \) and \(q_k\) are unrestricted, and there are \(\ell !^2\) different permutations of a given set of values for these quantities which yield the same final state \(|\tilde{f}\rangle\).
When the energy of the incoming scalar is much greater than the gap, a continuum approximation can be made in (39):

\[ \sum_p \rightarrow \int dp \frac{L_{\text{eff}}}{2\pi}, \quad \delta_p \rightarrow \frac{2\pi}{L_{\text{eff}}} \delta(p). \] (40)

For the sake of simplicity, only massless particle absorption will be considered. In that case the flux associated with the state \( g_k^\dagger |0\rangle \) is \( \mathcal{F} = 1/V \). The absorption cross-section is

\[ \sigma_{\text{abs}} = V \Gamma(\text{scalar} \rightarrow b_L + b_R + \ell f_L + \ell f_R) + \text{crossed processes} \]

\[ = \frac{\sum D_k \hat{D}_k |C_{D_1,\ldots,D_\ell}|^2}{\ell!^2 4^\ell} \frac{\hbar^2 L_{\text{eff}}}{(2\pi t_{\text{eff}})^{2\ell}} \omega^{2\ell - 1} I_L I_R \] (41)

where

\[ I_L = \int_{-\infty}^{\infty} dp_0^b \prod_{i=1}^{\ell} dp_i^0 \delta \left( \frac{\omega}{2} - p_0^b - \sum_{i=1}^{\ell} p_i^0 \right) \frac{p_0^b}{1 - e^{-p_0^b/T_L}} \prod_{i=1}^{\ell} \frac{1}{1 + e^{-p_i^0/T_L}} \] (42)

and similarly for \( I_R \). \( \Gamma(\text{scalar} \rightarrow b_L + b_R + \ell f_L + \ell f_R) \) is what was computed in (39). Arbitrary crossings of the basic process \( \text{scalar} \rightarrow b_L + b_R + \ell f_L + \ell f_R \) and their time-reversals also contribute to the net absorption rate from which \( \sigma_{\text{abs}} \) is computed. However, the simple trick of extending the integrals in (42) over the entire real line can be used to keep track of all of them. A demonstration of this with careful attention paid to symmetry factors can be found in [12] for the special case where only two left-moving bosons and two right-moving bosons are involved. Note that for \( \ell = 0 \) the dependence on \( t_{\text{eff}} \) disappears in (41), as was noted previously in [12].

The integral (42) is a convolution of \( \ell + 1 \) simple functions and so can be done most directly by transforming to Fourier space, where convolutions become products. Three integrals which are useful for doing the Fourier transforms are

\[ \int_{-\infty}^{\infty} dp e^{ixp} \frac{p}{2 \sinh \frac{p}{2T_L}} = (\pi T_L)^2 \operatorname{sech}^2(\pi T_L x) \]

\[ \int_{-\infty}^{\infty} dp e^{ixp} \frac{1}{2 \cosh \frac{p}{2T_L}} = (\pi T_L) \text{sech}(\pi T_L x) \]

\[ \int_{-\infty}^{\infty} dx e^{-ix\ell}(\pi T_L)^{\ell+2} \text{sech}^{\ell+2}(\pi T_L x) = (2\pi T_L)^{\ell+1} \left| \frac{\Gamma\left(\frac{\ell}{2} + 1 - i \frac{2p}{2\pi T_L}\right)}{(\ell + 1)!} \right|^2. \] (43)

The first two integrals are Fourier inversions of the third in the special cases \( \ell = 0 \) and \(-1\). Now the computation is straightforward:

\[ I_L = e^{\frac{\pi T_L}{2\pi}} \int_{-\infty}^{\infty} dp_0^b \prod_{i=1}^{\ell} dp_i^0 \delta \left( \frac{\omega}{2} - p_0^b - \sum_{i=1}^{\ell} p_i^0 \right) \frac{p_0^b}{2 \sinh \frac{p_0^b}{2T_L} \prod_{i=1}^{\ell} \cosh \frac{p_i^0}{2T_L}} \]

\[ = \frac{e^{\frac{\pi T_L}{2\pi}}}{2\pi} \int_{-\infty}^{\infty} dx e^{-ix\omega/2}(\pi T_L)^{\ell+2} \text{sech}^{\ell+2}(\pi T_L x) \]

\[ = (\ell + 1)! \pi (\pi T_L)^{\ell+1} C_{\ell/2}(\eta). \] (44)
The last step uses (15) and (17). $I_R$ can be computed similarly, but since $T_R = 0$ by assumption, the result is much simpler:

$$I_R = \frac{(\omega/2)^{\ell+1}}{(\ell+1)!}.$$  \hfill (45)

Now the absorption cross-section can be given in closed form:

$$\sigma_{\text{abs}}^\ell = \frac{\sum_{D_k, D_k} |C_{D_1\ldots D_\ell}|^2}{\ell! 24^\ell} \frac{\kappa_5^2 L_{\text{eff}}}{(2\pi t_{\text{eff}})^2} \frac{\omega^3}{\pi} \left( \frac{\pi T_L}{2} \right)^{\ell+1} C_{\ell/2}^2(\eta).$$  \hfill (46)

In the second equality two key relations have been used:

$$T_L = \frac{r_K}{\pi r_1 r_5} \frac{\kappa_5^2 L_{\text{eff}} T_L}{2} = A_h.$$  \hfill (47)

Both are valid when $r_K \ll r_1, r_5$. The first can be derived by setting the effective string entropy equal to the Bekenstein-Hawking entropy. The second is a limiting case of (21).

In addition, the tension has at last been fixed:

$$t_{\text{eff}} = \frac{1}{2\pi r_1 r_5}.$$  \hfill (48)

Because the cross-section (19) depends on $n_1$ and $n_5$ only through the product $n_1 n_5$ in the dilute gas regime, and because the same is true of the quantities $L_{\text{eff}}, T_L,$ and $A_h$ when $r_0 = 0$ and $r_K \ll r_1, r_5$, the choice $t_{\text{eff}} \sim 1/\sqrt{n_1 n_5}$ seems inevitable.

Precise agreement between General Relativity and the effective string now depends only on the relation

$$\sum_{D_k, D_k} |C_{D_1\ldots D_\ell}|^2 = (\ell+1)^2.$$  \hfill (49)

Agreement in the case $\ell = 0$ is trivial. For $\ell = 1$, the original treatment in terms of free fermions on the effective string is adequate: one can easily trace through the computations and verify that $D_1, \hat{D}_1,$ and $C_{D_1}^D$ can be replaced in every equation by $\Delta, \hat{\Delta},$ and $C_\Delta^\Delta$. Any numerical discrepancy could have been fixed by introducing a multiplicative constant in the relation $x^m = \tilde{\Psi} \gamma^m \Psi$; however to see perfect agreement without such artifice is pleasing and also rather suggestive of the form one expects for a gauge-fixed kappa symmetric action.

The real test is $\ell \geq 2$. Here it seems essential to depart from the simplistic effective string picture and return to a more fundamental description of the D1-D5 bound state in order to compute the coefficients $C_{D_1\ldots D_\ell}$.
Finally, it is worth noting that if agreement can be established for extremal absorption, agreement for the near-extremal case follows automatically. In the effective string computation for near-extremal absorption one must subtract off the stimulated emission contribution as described in \cite{12} in order to respect detailed balance and time reversal invariance. Modulo this subtraction, the result can be read off from (41) and (44):

\[ \sigma_{\text{abs}}^\ell = \sum_{D_1 \cdots D_\ell} \left| C_{D_1 \cdots D_\ell} \right|^2 \frac{\kappa_5^2 L_{\text{eff}} T_L T_R}{T_H} \left( \frac{\omega \sqrt{T_L T_R}}{2 t_{\text{eff}}} \right)^{2\ell} \left\{ \frac{\Gamma \left( 1 + \frac{\ell}{2} - \frac{i\omega}{4\pi T_L} \right) \Gamma \left( 1 + \frac{\ell}{2} - \frac{i\omega}{4\pi T_R} \right)}{\Gamma \left( 1 - i \frac{\omega}{2\pi T_H} \right)} \right\}^2 \]

The second line relies on a modified version of (47) applicable to the near-extremal case with \( r_0, r_K \ll r_1, r_5 \):

\[ \sqrt{T_L T_R} = \frac{r_0}{2\pi r_1 r_5} \quad \frac{\kappa_5^2 L_{\text{eff}} T_L T_R}{T_H} = A_h . \]

The same tension (48) and the same relation (49) establish agreement between (50) and (20).

It has been suggested \cite{25} that the effective string picture can be used to describe in a U-duality invariant fashion black holes with arbitrary charges, possibly even far from extremality. The expression on the first line of (50) depends only only \( T_L, T_R, T_H \), and \( L_{\text{eff}} \)—all quantities that have meaning to the effective string considered in the abstract, independent of the microscopic D1-D5-brane model. It cries out to be reconciled with (20) for arbitrary values of \( r_0, Q_1, Q_5 \), and \( Q_K \). But the treatment of absorption given in this section relies on the dilute gas approximation and thus is not general enough to be matched in any meaningful way to General Relativity when \( Q_K \ll Q_1, Q_5 \).

4. Limitations on partial wave absorption

Consider the more general couplings described in the paragraph preceding (37): local on spatial \( S^1 \) wrapped by the effective string but not on the effective string itself. Although the details of the \( SO(4) \) group theory and Chan-Paton factors have yet to be worked out

\[ \text{\footnotesize 1} \quad \text{The ideas in this section originate largely in discussions with C. Callan, L. Thorlacius, and J. Maldacena.} \]
fully in the context of the \((4,4)\) SCFT description of the D1-D5-brane bound state, it seems clear that a coupling of the \(\ell\)th partial wave to an operator built out of any combination of the \(4n_1n_5\) fermionic fields raises the maximum value of \(\ell\) which the effective string can absorb from 1 to some number on the order \(n_1n_5\). One might suppose that by putting derivatives on some of the fermion fields, the problem can be avoided altogether. But such derivatives raise the dimension of the operator and hence suppress the cross-section by more powers of \(\omega\) than are present in the semi-classical result. To sum up, the assumption of locality prevents the effective string from coupling to partial waves above a certain maximum \(\ell\) with the strength required to match General Relativity. The situation does not seem as satisfactory as for the D3-brane, where couplings to all partial waves exist with appropriate dimensions to reproduce semi-classical cross-sections \([33]\) (normalizations however are problematic \([21]\)).

There is a reason, however, why one might expect not to observe agreement between the effective string and General Relativity at high values of \(\ell\). If the black hole absorbs some very high partial wave, it winds up with a large angular momentum, so the geometry before and after is appreciably different. Back reaction is not included in the General Relativity calculations of section \([4]\). In fact the only back reaction calculation I am aware of for the black hole under consideration \([34]\) is restricted to \(\ell = 0\). But on the grounds of cosmic censorship one would expect that absorption processes which drive the black hole past extremality are forbidden even semi-classically. From an adaptation of the work of \([3,4]\) one can read off the corresponding bound on \(\ell\) as \(\ell \lesssim \sqrt{n_K n_1n_5}\).

We have two different bounds on \(\ell\) indicating the maximum partial wave that the effective string should be capable of absorbing:

\[
\ell \lesssim \ell_{\text{max}}^L \equiv n_1n_5 \quad \text{from locality and statistics}
\]

\[
\ell \lesssim \ell_{\text{max}}^C \equiv \sqrt{n_K n_1n_5} \quad \text{from cosmic censorship.}
\]

Now I would like to inquire which is the more restrictive. Using the standard relations (see for example \([3]\))

\[
n_1n_5 = \frac{4\pi^3 r_1^2 r_5^2}{\kappa_5^2 L_5} \quad n_K = \frac{\pi L_5 r_K^2}{\kappa_5^2}
\]

and the formula \((17)\) for \(T_L\), one can show that \(\ell_{\text{max}}^C/\ell_{\text{max}}^L = L_5 T_L/2\).

The validity of any comparison between General Relativity and the effective string model as treated in section \([3]\) relies on being in the dilute gas regime \([4]\) and at low energies \([35]\):

\[
r_K \ll r_1, r_5 \ll 1/\omega.
\]

These inequalities still do not determine whether \(\ell_{\text{max}}^L\) is larger or smaller than \(\ell_{\text{max}}^C\). But if it is agreed to examine only fat black holes \([2]\), which is to say if one assumes

\[
L_5 \ll \sqrt{r_1r_5},
\]
then it is easy to obtain the inequality
\[ \ell_{\text{max}}^L \gg \ell_{\text{max}}^C \] (56)
by combining the dilute gas inequality in (54) with (55). Now, (56) is a hopeful state of affairs for the effective string model, because it indicates that making couplings local only on \( S^1 \) in principle enables the effective string to absorb all the partial waves for which reasonable comparisons can be made with General Relativity. It is perhaps not the ideal state of affairs: one might have hoped that the bounds \( \ell_{\text{max}}^L \) and \( \ell_{\text{max}}^C \) would coincide, indicating that the effective string knew about cosmic censorship. The results of [14] suggest that a more careful treatment of these issues using techniques of conformal field theory would result in a translation of cosmic censorship into unitarity of the effective string description. Such a treatment would need to address the problem that if one moves deep into the black string region of parameter space by making \( L_5 \) large, one can obtain \( \ell_{\text{max}}^L \ll \ell_{\text{max}}^C \). The perturbative D-brane region is in fact closer to the black string region than the fat black hole region, so it would be surprising to find such a disaster for comparisons in the black string region when agreement seems possible for fat black holes.

5. Conclusion

In this paper I have shown that the leading order coupling of the effective string to an ordinary scalar correctly predicts the cross-sections for the \( \ell = 1 \) partial wave. Due to the Grassmannian character of the fermionic fields which carry the angular momentum, it is impossible for the simplest effective string model (a single long string with \( c_{\text{eff}} = 6 \)) to couple properly to \( \ell > 1 \) partial waves through an operator local on the string. Generalizing the model to include what one might intuitively regard as multi-strand interactions of the effective string postpones this difficulty to \( \ell \gtrsim n_1 n_5 \), a higher bound on \( \ell \) for fat black holes than the one arising from cosmic censorship. An analysis of the unique form which such interactions must have in order to make a leading order contribution to the absorption of the \( \ell^{\text{th}} \) partial wave demonstrates that the correct energy dependence arises from the finite temperature kinematics. This demonstration, together with the general proof of the Optical Theorem for absorption of scalars given in section 2, can be viewed as a full investigation of the kinematics involved in higher partial waves. What is left is to calculate the coefficients \( C_{D_1 \ldots D_\ell} \) and thereby verify or falsify (49), on which agreement between General Relativity and the effective string model relies.

The means to achieve a clear description of the dynamics and hopefully a derivation of the \( C_{D_1 \ldots D_\ell} \) is a more precise treatment of the low-energy SCFT dictating the dynamics of the D1-D5-brane bound state. The effective string might continue to be a useful picture, perhaps supplemented by rules governing how different strands of the effective string interact. I hope to report on this approach in the future.
In a way the finding that the effective string tension needs to scale as $1/\sqrt{Q_1Q_5}$ is a more serious difficulty than the vanishing of $\ell > 1$ cross sections. Indeed, the necessity of choosing this peculiar value for the tension appears already at $\ell = 1$, where the naive effective string model in other ways seems completely adequate. A study of T-duality in \cite{19} led to the conclusion that a tension scaling as $1/\sqrt{Q_1Q_5}$ is more natural than $1/Q_1$ or $1/Q_5$, and it was further described how a simple modification in the calculation of disk diagrams would lead to such a scaling. However, in the absence of a first-principles derivation of the tension, a $1/(Q_1 + Q_5)$ scaling seems equally natural. This scaling is the one favored by entropy and temperature arguments. Thus it appears that a single energy scale does not fully characterize effective strings in the way that $\alpha'$ does fundamental strings.

Although effective string models of black holes have recently enjoyed a number of remarkable successes, a unifying picture has been slow in emerging. The General Relativity calculations for near-extremal black holes, on which most of the evidence for effective strings is based, are conceptually straightforward. The difficulty of studying bound states of solitons has caused the link between fundamental string theory and effective strings to remain imprecise in certain respects—most importantly in the interaction between the effective string and fields in the bulk of spacetime.

The puzzles presented by higher partial waves may push effective string theory in the directions it needs to go in order to become a fully viable model of near-extremal black holes. Discrepant results for the tension may be a clue to nature of effective string’s interactions with bulk fields. Rescuing the $\ell > 1$ cross-sections must surely lead to a consideration of the multi-strand interactions which are as yet virgin territory in the theory of effective strings. Already, the absorption of higher partial waves into five-dimensional black holes is the cleanest dynamical test of the role of fermions on the effective string. Achieving a full understanding of these processes would constitute a major advance, not only in establishing the viability of effective strings in black hole physics, but also in comprehending the D1-D5-brane bound state.

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