Abstract

We study BPS monopoles in 4 dimensional $N = 4$ $SO(N)$ and $Sp(N)$ super Yang-Mills theories realized as the low energy effective theory of $N$ (physical and its mirror) parallel D3 branes and an Orientifold 3 plane with D1 branes stretched between them in type IIB string theory. Monopoles on D3 branes give the natural understanding by embedding in $SU(N)$ through the constraints on both the asymptotic Higgs field (corresponding to the horizontal positions of D3 branes) and the magnetic charges (corresponding to the number of D1 branes) imposed by the O3 plane. The compatibility conditions of Nahm data for monopoles for these groups can be interpreted very naturally through the D1 branes in the presence of O3 plane.
I. INTRODUCTION

In the last year we have seen how string/M theory can be exploited to understand non-perturbative dynamics of low energy supersymmetric gauge theories in various dimensions and different number of supersymmetries. D brane dynamics in string theory is very useful tool for the gauge theory which is realized on the worldvolume of D branes (See [1] for a review and references on the interrelation between D brane dynamics and the gauge theory).

Nahm’s construction [2] of the moduli space of magnetic monopoles [3] was found in [4] by considering parallel D3 branes with D1 branes stretched between them in type IIB string theory. By T-duality along the transverse 2 space directions to both D1 and D3 branes this configuration leads to D5 and D3 branes which by S-duality will become NS5 and D3 brane configuration. The mirror symmetry of $N = 4$ gauge theory in 3 dimensions [5] was due to the nonperturbative S-duality of type IIB string theory (See, for example, [6]). By T-duality along the 1 space direction, we have 2 parallel NS5 branes with D4 branes suspended between them. As one changes the relative orientation [7] of the two NS5 branes while keeping their common four spacetime dimensions intact, the $N = 2$ supersymmetry is broken to $N = 1$ supersymmetry [8]. By studying this brane configuration they [8] described and checked a stringy derivation of Seiberg’s duality. This configuration was generalized to the cases with orientifolds which were used to study $N = 1$ supersymmetric gauge theories with gauge group $SO(N)$ and $Sp(N)$ [9] (See also [10] for a relevant geometrical approach).

On the other hand, in the field theory, magnetic monopoles have been the object of intense interest as solitons which after quantization are complementary to particles that arise as quanta of the elementary fields. This has been developed by Montonen and Olive [11] that in certain theories there exists an exact electromagnetic duality which exchanges solitons and elementary quanta and weak and strong coupling. In fact, $N = 4$ supersymmetric Yang-Mills theory is the candidate theory with this duality [12,13]. In brane picture, the Montonen-Olive duality can be regarded as S-duality of type IIB string theory in the limit
of vanishing of fundamental string scale. In general, it is very hard to solve the general BPS monopole field configuration but the systematic construction for the solution has been done by Nahm [2], so-called Nahm’s equation which is some nonlinear ordinary differential equation. Up to a couple of years ago, the origin of Nahm’s equation was unclear mathematical artifact. However, the string theory tells us that it naturally comes through the D1 brane configuration suspended between D3 branes. This was shown in [4] for $SU(N)$ monopoles.

In this paper, we generalize the approach of [4] to $SO(N)$ and $Sp(N)$ magnetic monopoles. The $SO/Sp$ monopoles can be understood by embedding the gauge group into $SU(N)$. The corresponding Nahm’s data are obtained by imposing some extra constraints to those for $SU(N)$ case. All these can be naturally realized by putting O3 plane in the D1-D3 brane configuration. The O3 plane allows us to construct the $SO/Sp$ gauge theories. Furthermore, it provides the natural geometrical origin of those extra constraints. in $SO(N)$ or $Sp(N)$ gauge theories.

II. MONOPOLES IN SUPERSYMMETRIC GAUGE THEORIES

In this section we briefly review the Bogomol’nyi-Prasad-Sommerfield (BPS) monopoles [14]. We consider the model with the Higgs fields $\Phi$ in the adjoint representation of the arbitrary gauge group $G$ with rank $n$ in the BPS limit in which the potential of the scalar fields are ignored. This is the case of the extended supersymmetric models.

The BPS monopole configurations can be described by the following first order equations

$$B_i = D_i \Phi \quad (1)$$

where $\epsilon_{ijk} B_k = \partial_i A_j - \partial_j A_i + i[A_i, A_j]$ and $D_i \Phi = \partial_i \Phi + [A_i, \Phi]$. The asymptotic value $\Phi_0$ of the Higgs field $\Phi$ along some fixed direction can be written as [15]

$$\Phi_0 = h \cdot H \quad (2)$$

where the $n$ commuting matrices $H_i$ span the Cartan subalgebra and are normalized by
The simple roots can be chosen to have non-negative inner products with the vector $h$ in the root space.

For the generic value of $h$ with no simple roots orthogonal to that, the gauge group $G$ is maximally broken to $U(1)^n$. If some simple roots $\gamma_j$ ($j = 1, \cdots, k$) are orthogonal to $h$, then the unbroken gauge group becomes $U(1)^{n-k} \times K$ where $K$ is the semisimple group corresponding to the root sublattice of $\gamma_j$. Much physics of this nonabelian unbroken gauge group can be understood by taking the limit of $h \cdot \gamma_j \to 0$ of the generic maximal symmetry breaking \[16\]. Hence we will mainly consider maximal symmetry breaking to $U(1)^n$ for simplicity.

The asymptotic magnetic field can be written as

$$B_i = \frac{\hat{r}_i}{4\pi r^2} g \cdot H.$$  \hspace{1cm} (3)

The quantized magnetic charge $g$ is given by

$$g = 4\pi \frac{n}{e} \sum_{a=1}^{n} m_a \beta_a^\ast$$  \hspace{1cm} (4)

where $\alpha^\ast = \alpha/\alpha^2$ is the dual of the root $\alpha$ and the $m_a$ are non-negative integer valued topological charges.

Since each root defines an $SU(2)$ subgroup, we can construct the corresponding $SU(2)$ monopole solution. For maximal symmetry breaking, the monopole corresponding to each of the $n$ simple roots $\beta_a$ is the fundamental monopole carrying the unit of $U(1)$ magnetic charge and with the mass $M_a = \frac{4\pi}{e} h \cdot \beta_a^\ast$. If some simple roots $\gamma_a$ become orthogonal to $h$ giving rise to the unbroken nonabelian gauge group, then the corresponding monopoles become massless. The general BPS monopoles in Eqs.(2) and (4) and the mass $M$ given by

$$M = \sum_{a=1}^{n} m_a M_a.$$  \hspace{1cm} (5)

can be understood as the multimonopole configuration containing set of $m_a$ monopoles of each of the above $n$ different fundamental monopole types. This is consistent with the analysis that each fundamental monopole is associated with four moduli corresponding to
the three positions and one $U(1)$ rotation and that the total dimension of the moduli is preserved in the limit of some monopoles becoming massless.

For $SU(N)$, the asymptotic Higgs field can be written as

$$\Phi_0 = h \cdot H = \text{diag}(\mu_1, \mu_2, \cdots, \mu_{N-1}, \mu_N)$$

with $\sum_{a=1}^{N} \mu_a = 0$. Maximal symmetry breaking corresponds to all different values of $\mu_a$. If some of the $\mu_a$ have the same values, we have some nonabelian unbroken gauge group. For the maximal symmetry breaking, we choose such that $\mu_1 < \mu_2 < \cdots < \mu_{N-1} < \mu_N$. The mass $M_a$ becomes

$$M_a = \frac{4\pi}{e} h \cdot \beta_a^* = |\mu_a - \mu_{a+1}|.$$  

(7)

The magnetic charge of the monopoles, $g \cdot H$, becomes

$$g \cdot H = \text{diag}(k_1, k_2, \cdots, k_{N-1}, k_N)$$

with $k_a = m_a - m_{a-1}$ ($a = 1, \cdots, N - 1$), and $k_N = -(k_1 + \cdots + k_{N-1})$.

The matrix representation of the Higgs field and the magnetic charges for other classical gauge groups of $SO(N)$ and $Sp(N)$ can also be done in a straightforward way. Related to the later analysis, it is helpful to realize these matrices by embedding to $SU(N)$. This can be described with the constraints among the $SU(N)$ generators. The generators of $Sp(N)$ can be obtained through the constraint $T^t J + J T = 0$ such that $JJ^* = -I$, and the generators of $SO(N)$ are given by the constraints $T^t K + KT = 0$ with $KK^* = I$. The asymptotic values of the Higgs fields in $SO/Sp$ group can then be identified to those in $SU(N)$ group satisfying some relations. Let $\rho_a$ ($a = 1, \cdots, n$) represent the magnetic charges of the $SO/Sp$ - multimonopole configuration. The $SO/Sp$ - magnetic charges will also be related to the $SU(N)$ - magnetic charges $m_a$ satisfying some relations. The results are summarized as follows ( See, for example, [17] ).
Table 1: The embedding of $Sp(N)$ and $SO(N)$ in $SU(N)$.

| $G$         | $G$-charges | $\Phi_0$ in $SU(N)$ | $SU(N)$-charges |
|-------------|-------------|---------------------|-----------------|
| $Sp(N)$     | $\rho_1, \cdots, \rho_n$ | $\mu_a = -\mu_{2n+1-a}$ | $m_a = m_{2n-a} = \rho_a$ |
| $N = 2n$    |             | $a = 1, \cdots, n$  | $a = 1, \cdots, n$     |
| $SO(N)$     | $\rho_1, \cdots, \rho_{n-1}$ | $\mu_a = -\mu_{2n+1-a}$ | $m_a = m_{2n-a} = \rho_a$ |
| $N = 2n$    | $\rho_+, \rho_-$ | $a = 1, \cdots, n$  | $a = 1, \cdots, n-2$  |
|             |             |                     | $m_{n-1} = m_{n+1} = \rho_+ + \rho_-$ | $m_n = 2\rho_+$ |
| $SO(N)$     | $\rho_1, \cdots, \rho_n$ | $\mu_a = -\mu_{2n+2-a}$ | $m_a = m_{2n+1-a} = \rho_a$ |
| $N = 2n + 1$|             | $a = 1, \cdots, n+1$| $a = 1, \cdots, n-1$  |
|             |             |                     | $m_n = m_{n+1} = 2\rho_n$ |

The systematic construction of the general BPS monopole field configurations can be described based on Nahm data [2]. We first review the case of $SU(N)$ monopole.

The Nahm data for the multi-monopoles carrying charges $(m_1, \cdots, m_{N-1})$ are defined by the $N-1$ triples of the analytic $u(m_a)$ valued functions $X^i_a$, where $i = 1, 2, 3$ and $a = 1, \cdots, N - 1$, defined on the interval $(\mu_a, \mu_a + 1)$ and satisfying the following Nahm equations:

$$
\frac{dX^i_a}{ds} + \frac{1}{2} \sum_{j,k=1}^{3} \epsilon_{ijk} [X^j_a, X^k_a] = 0.
$$

(9)

The Nahm data in two adjacent intervals also satisfy the following boundary conditions near each $\mu_a$:

i) If $m_{a-1} < m_a$, then $X^i_{a-1}$ has a non-zero limit $C^i_a = \lim_{s \to \mu_a} X^i_{a-1}$ and is analytic at $s = \mu_a$. Also $X^i_a$ can be written in a block form expansion as $t \equiv s - \mu_a \to 0$,

$$
X^i_a = \begin{pmatrix}
X^i_{a,11} & X^i_{a,12} \\
X^i_{a,21} & X^i_{a,22}
\end{pmatrix}
= \begin{pmatrix}
C^i_a + \mathcal{O}(t) & \mathcal{O}(t^\gamma) \\
\mathcal{O}(t^\gamma) & \frac{T^i_a}{s-\mu_a}
\end{pmatrix}.
$$

(10)
where $T^i_a$ forms an $(m_a - m_{a-1})$-dimensional irreducible representation of $su(2)$ and 
$\gamma = (m_a - m_{a-1} - 1)/2$. The upper diagonal block is analytic and the lower diagonal block 
is meromorphic in $t$.

ii) If $m_{a-1} > m_a$, the roles of $m_{a-1}$ and $m_a$ are reversed.

iii) If $m_{a-1} = m_a$, $X^i_{a-1}$ and $X^i_a$ are both analytic near $s = \mu_a$ with finite limits required 
to satisfy a certain regularity condition.

We take the convention of $m_0 = 0 = m_N$ to cover the boundary conditions at $\mu_1$ and $\mu_N$.

As for the $SO(N)$ and $Sp(N)$ monopoles, the Nahm data are naturally described in relation to the embedding of these gauge groups into $SU(N)$. The $SO(N)$ and $Sp(N)$ multimonopole magnetic charges $\rho_a$ and the asymptotic Higgs fields will correspond to $SU(N)$ charges $m_a$ and Higgs fields as given in the Table 1. With this correspondence, the Nahm data for $SO(N)$ and $Sp(N)$ monopoles will satisfy the same Nahm equation in Eq.(11) as that of $SU(N)$. As for the boundary conditions between two adjacent intervals, we need 
the following one more set of conditions in addition to those for the $SU(N)$ described above.

There exist matrices $C_a$ satisfying

$$T^i_{N-a}(-s)^t = C_aT^i_a(s)C_a^{-1}$$

with $C_{N-a} = C_a^t$ for $Sp(N)$ and $C_{N-a} = -C_a^t$ for $SO(N)$.

The physical origin of the equivalent description of the monopoles based on the Nahm’s equation with special boundary conditions are not clear at the field theory level. We will 
see in the next section how this relation is naturally realized through the D branes.

III. $SO(N)$ AND $SP(N)$ MONOPOLES AND BRANES WITH ORIENTIFOLD

The brane dynamics at low energy of parallel $N$ D3 branes stretched in $(x^0, x^1, x^2, x^3)$ 
directions in type IIB string theory is described by four dimensional $N = 4$ $SU(N)$ supersymmetric gauge theory. Let us consider, as in Figure 1, $N$ parallel D3 branes with $x^6$ values $\mu_1 < \mu_2 < \cdots < \mu_{N-1} < \mu_N$ and $m_a (a = 1, \cdots, N-1)$ D1 branes connecting $a$-th and the $a+1$-th D3 branes. On the D3 brane worldvolume, this configuration
describes the multimonopole configuration in the maximally broken $SU(N)$ gauge theory. The Abelian $U(1)$ factor corresponds to the center of mass dynamics of the D3 branes, which is decoupled. Thus we consider $SU(N)$ gauge theory instead of $U(N)$. This is realized by the extra condition $\sum_{a=1}^{N} \mu_a = 0$. The $x^6$ coordinates $\mu_a$ of D3 branes are identified with the asymptotic Higgs field value $\Phi_0$ in Eq. (4) in Section II. The D1 branes and their low energy dynamics will be described as the $SU(N)$ multimonopoles with the magnetic charge $(m_1, \ldots, m_a, \ldots, m_{N-1})$ and their moduli. The net magnetic charge, $k_a$, induced on the $a$-th D3 brane coming from the $m_{a-1}$ D1 branes in the interval $(\mu_{a-1}, \mu_a)$ and the $m_a$ D1 branes in the interval $(\mu_a, \mu_{a+1})$ will be given by the difference $k_a = m_a - m_{a-1}$. This magnetic charges of the multimonopole configuration in branes exactly corresponds to those in Eq. (8) in field theory description.

**Figure1**: Brane configuration for $SU(N)$ monopoles. D3 branes are represented by the vertical lines with space coordinates $(x^1, x^2, x^3)$, and D1 branes by the horizontal lines in $x^6$ direction. The $m_a$'s are the number of D1 branes and $\mu_a$'s the $x^6$ positions of D3 branes. Generically, the values of $\mu_a$'s and $m_a$'s and the positions of D1 branes along the vertical directions are arbitrary.
From the point of view of D1 branes, the above moduli can be understood as the moduli describing the vacua of the 1+1 dimensional field theories of D1 branes stretched between the infinitely heavy D3 branes. If the $m$ D1 branes were with infinite lengths without D3 branes, the low energy effective action would be the 1+1 dimensional supersymmetric $U(m)$ nonabelian gauge theory with worldvolume in $(x^0, x^6)$ which can be obtained by the dimensional reduction from 10 dimensional $N = 1$ super Yang-Mills action. The theory on D1 branes has eight supercharges, and there are 2 dimensional gauge fields, scalars and their superpartners. The scalars come from the transverse oscillations to the worldvolume of D1 branes. There are two types of scalars: $X^i (i = 1, 2, 3)$ along the D3 brane and the rest $X^\mu (\mu = 4, 5, 7, 8, 9)$ which are transverse to both D1 and D3 branes. Now consider the D1 branes ending on the D3 branes at $\mu_a$ and $\mu_{a+1}$. We will use the subscript $a$ for the corresponding quantities, e.g., $X^i_a$ and $X^\mu_a$. We can choose the gauge field to be zero by a gauge choice (except the Wilson line phase). In other words, the gauge groups are frozen and become essentially the global symmetry. The condition for supersymmetric ground states for D1 brane configurations can be easily obtained from the effective action. The ending of D1 branes on the D3 branes at $\mu_a$ and $\mu_{a+1}$ require some boundary conditions to the fields [4]. The boundary conditions of $X^\mu_a, \mu = 4, 5, 7, 8, 9$ are as follows: $X^\mu_{a-1}$ and $X^\mu_{a,11}$ are analytic in a neighborhood of $s = \mu_a$ with finite limits as $s \to \mu_a$. This condition is applied for $X^i_a$ as well. The parameter $s$ is used to represent the value of $x^6$ along the D1 branes for comparison with the notations in Section II. The fields $X^\mu_{a,12}, X^\mu_{a,21}$ and $X^\mu_{a,22}$ are bump fields which are compactly supported away from $s = \mu_a$. With these conditions imposed, the equations for the $X^i_a$ fields reproduces the Nahm’s equation (3).

To see the detail of the boundary condition for the fields $X^i_a$, let us assume that $m_{a-1} < m_a$. Then we write $X^i_a$ in the block $2 \times 2$ block form as $\text{Eq. (10)}$. Then the boundary conditions can be seen to be exactly the same as those described in Section II. That is, $X^i_{a,11}$ is analytic and has the well defined limit, and $X^i_{a,12}$ and $X^i_{a,21}$ are analytic around $s = \mu_a$ up to some order $\gamma$. Moreover, $X^i_{a,22}$ is in the meromorphic form $T^i_a/(s - \mu_a)$ where $T^i_a$ is the irreducible $SU(2)$ representation. Thus D1 brane point of view shows how the Nahm’s
equation and its boundary conditions naturally occur. Based on the correspondence between brane configuration and field theory configuration for the $SU(N)$ monopole we now move on to $Sp(N)$ and $SO(N)$ gauge groups.

Understanding the $Sp(N)$ and $SO(N)$ monopoles by embedding these groups in $SU(N)$ will correspond to inserting the orientifold 3 plane to the previous brane configuration of $N$ parallel D3 branes with D1 branes connecting them. To keep the supersymmetry the orientifold 3 plane should be inserted parallel to D3 branes. Also when we put orientifold 3 plane at some position, the same amount as “physical” D branes appearing in the left hand side of it should be present in the right hand side of it at the opposite positions as their images. This requires to put the orientifold 3 plane at the central position of $N$ D3 branes.

For an $O3_+ \pm$ plane of positive charge with $1/2$ D3 brane charge, the gauge group is $Sp(N)$ while for a $O3_- \pm$ plane of negative charge carrying $-1/2$ D3 brane charge, the gauge group becomes $SO(N)$ (See, for example, [13]). We will consider how O3 plane plays an important role for getting the constraints on the $\mu_a$ and the magnetic charge $m_a$ and see the compatibility conditions of Nahm data for monopoles for these groups very naturally through the D1 branes in the presence of O3 plane.

\section*{A. $Sp(N): N = 2n$}

Now we put $O3_+ \pm$ in parallel to D3 branes into the central position between $N/2$-th D3 brane $N/2 + 1$-th D3 brane, as in Figure 2. The $x^6$ position of the orientifold will then be $x^6 = 0$. The presence of the orientifold requires that the brane configuration to the right hand side should be restricted to be the mirror image of those in the left hand side. As for the positions of D3 branes, this means that

$$\mu_1 = -\mu_N, \ldots, \mu_{N/2} = -\mu_{N/2+1}.$$ (12)

As for the number of D1 branes which corresponds to the magnetic charges, the orientifold projection requires
\[ m_1 = m_{N-1}, \ldots, m_{N/2-1} = m_{N/2+1}, m_{N/2}. \]  

(13)

There are only \( n \) independent \( m_a \) (\( a = 1, \cdots, n \)) and these are identified with the \( Sp(N) \) magnetic charges \( \rho_a \). Eqs.(12,13) are exactly coincident with the constraints of embedding \( Sp(N) \) in \( SU(N) \) we have described in Section II and given in Table 1. Note that the group theoretical constraints of embedding now has very natural geometrical interpretation in terms of both the positions of D3 branes and the number of D1 branes.

For the observer on D3 branes, it describes a moduli space of \( Sp(N) \) monopoles while for the observer on D1 branes, it leads to a moduli space of vacua of the field theory on D1 branes with \( SO \) symmetry. As mentioned before the gauge group will be frozen and becomes a global symmetry. For those stretched between D3 branes in the interval \( (\mu_a, \mu_{a+1}) \), the group will be \( SO(m_a) \). Without the orientifold, the group would be \( U(m_a) \) leading to the Nahm equation and the boundary conditions the same as those studied in \( SU(N) \) monopoles in the above. The presence of the orientifold will lead to further projections.

To be more specific, there are \( m_a-1 \) D1 branes to the left of the D3 brane at \( \mu_a \), and \( m_a \) D1 branes to the right. Due to the orientifold, there exist \( m_a-1 \) to the right and \( m_a \) to the left at \(-\mu_a\) as in Eq.(12,13). Let us assume that \( m_a-1 < m_a \). Then, there will be net \( k_a = m_a - m_a-1 \) D1 branes ending from the right at \( \mu_a \) and the same numbers from the left at \(-\mu_a\). Without the orientifold, all the states with the \( U(k_a) \) charges at end points of the D1 branes at both boundaries will be allowed. With the orientifold, they are no longer independent. Only those states invariant under the world sheet parity will be survived. Under the worldsheet parity \( \Omega \), the Chan-Paton states \(|ij>\) will be transformed into \((C_a)_{ii'}|i'j'><(C_{N-a})^-_{N-j'}j'>\). We omitted transformation of other irrelevant state indices. Equivalently the world sheet parity action can be rewritten in terms of the action of \( C_a \) and \( C_{N-a} \) on \( T_a \) and \( T_{N-a} \) as given in Eq.(11). Acting \( \Omega \) twice will leave the constraints on \( C_a \) and \( C_{N-a} \) as symmetric projection \( C_{N-a} = C_a^t \) for \( SO(k_a) \) group, and antisymmetric projection \( C_{N-a} = -C_a^t \) for \( Sp(k_a) \) group [19]. Note that \( SO(k_a) \) (\( Sp(k_a) \)) group for D1 branes corresponds to \( Sp(N) \) (\( SO(N) \)) group on D3 branes. To summarize, we have shown
that the extra symmetry in the Nahm data for $Sp/SO$ groups coming from the constraint in Eq.(11) arises in the brane configuration through the invariance of the states under the world sheet parity.

The simplest example is $Sp(2)$. The equivalence of $Sp(2)$ and $SU(2)$ arises because there is no states projected out by the orientifold. When one considers $Sp(4)$, there exist two kinds of monopoles characterized by $m_1(=m_3)$ and $m_2$ while for $SU(4)$ there are three independent monopoles by $m_1,m_2,m_3$ since all states with different $m_1$ and $m_3$ charges in $SU(4)$ configurations are projected out by the orientifold. For higher $N \geq 6$, it is straightforward to analyze this procedure as well.

\[
\begin{array}{c}
\mu_1 \quad \mu_2 \quad \mu_{N/2} \\
\mu_{N/2} \quad \mu_{N/2+1} \quad \mu_{N-1}
\end{array}
\]

\[
\begin{array}{c}
m_1 \quad m_{N/2} \\
m_{N-1} = m_i
\end{array}
\]

**Figure 2**: Brane configuration with orientifold $O3_+$ denoted by the hexagons for $Sp(N)$ monopoles. The values of $\mu_a$'s and $m_a$'s appearing in the right hand side of orientifold are strictly restricted according to the orientifold projection.

**B. $SO(N)$: $N = 2n + 1$**

In this case, we consider $O3_-$, as in Figure 3, rather than $O3_+$ at the position of $x^6 = 0$ where a single D3 brane gets stuck on the $O3_-$ without its mirror image. As in previous
case, the orientifold action gives rise to the following conditions, \( \mu_1 = -\mu_N, \cdots, \mu_{(N-1)/2} = -\mu_{(N+3)/2}, \mu_{(N+3)/2} = -\mu_{(N+1)/2} \) and \( m_1 = m_{N-1}, \cdots, m_{(N-3)/2} = m_{(N+3)/2}, m_{(N+1)/2} = m_{(N+1)/2} \) which exactly matches the embedding constraints of \( SO(N) \) in \( SU(N) \) as shown in Table 1. We obtained these group theoretical constraints as a natural geometrical properties of the orientifold. Montonen-Olive electric-magnetic duality in field theory transforms \( SO(2n + 1) \) electric charges into \( Sp(2n) \) magnetic charge and vice versa. This can be regarded as S-duality of type IIB string theory in the limit of string scale, \( l_s \to 0 \). It turns out that a system of \( O3_− \) and D3 brane stuck on it transforms as \( O3_+ \) under the S duality of type IIB string theory. As before, the effective theory on D3 branes will describe the \( SO(N) \) monopole moduli while the theory on D1 branes between the interval \( (\mu_a, \mu_{a+1}) \) through the 1+1 dimensional \( Sp(m_a) \) supersymmetric theory give the monopole description through the Nahm data. The analysis of this Nahm data goes in parallel with the \( Sp(N) \) case. The only difference is that states that are invariant under the antisymmetric projection \( C_{N-a}^a = -C_a^a \) will survive because the group is \( Sp(k_a) \) instead of \( SO(k_a) \). Again we obtained this exact correspondence to the symmetry constraints in Eq. (11) via the geometrical world sheet parity transformation properties.

For the simple \( SO(3) \) case, the monopole is described by \( m_1 (= m_2) \) which implies \( SU(2) \) monopole all other \( SU(3) \) states being projected out. This can be also understood as dual to \( Sp(2) \) theory we have discussed in previous subsection under the Montonen-Olive duality or S-duality in type IIB string theory. For higher \( N \geq 5 \) case, we can understand in a similar way.
Figure 3: Brane configuration with orientifold $O3_-$ denoted by the hexagons for $SO(N)$ monopoles. A single D3 brane is stuck on the $O3_-$ plane. The values of $\mu_a$’s and $m_a$’s corresponding to the images are also restricted according to the orientifold projection.

C. $SO(N)$: $N = 2n$

Now let us put $O3_-$, as in Figure 4, parallel to D3 branes into the central position between $N/2$-th D3 brane $N/2 + 1$-th D3 brane at $x^6 = 0$. Once again, the orientifold action leads to the following conditions, $\mu_1 = -\mu_N, \cdots, \mu_{N/2} = -\mu_{N/2+1}$ and $m_1 = m_{N-1}, \cdots, m_{N/2-1} = m_{N/2+1}, m_{N/2}$, which is the same conditions as the embedding of $SO(N)$ in $SU(N)$ shown in Table 1. This is the same as the case of $Sp(N)$ except that there is a negative $O3_-$ plane rather than positive one. The $SO(N)$ gauge theory with $N$ even is self dual under the Montonen-Olive duality. This corresponds to the fact that $O3_-$ is self dual under the S duality. We now describe the $SO(N)$ gauge theory on D3 brane from the D1 brane point of view that leads to the relation with the Nahm data. The analysis is the same as previous $SO(2n + 1)$ case. The geometrical world sheet parity transformation properties lead us to
the antisymmetric projection $C_{N-a} = -C^a$, which is the same as the symmetry constraints of Nahm data in Eq. (11).

Notice that there are no nonsingular monopoles for $SO(2)$ which can be interpreted as due to the fact that the fundamental strings between D3 branes which obtained by S-duality are projected out when we have $O3_-$ plane. For higher $N \geq 4$ case, it is straightforward to analyze according to the above projection description.

\[ O3_\cdot \]

\[ \mu_1 \quad \mu_2 \quad m_1 \]

\[ m_{\frac{N}{2}} \quad m_{\frac{N}{2}+1} \quad m_{N-1} \]

\[ \ldots \quad \ldots \quad \ldots \]

\[ \mu_{\frac{N}{2}} \quad \mu_{\frac{N}{2}+1} \quad \mu_{N-1} = \mu_1 \]

**Figure 4:** Brane configuration with orientifold $O3_-$ denoted by the hexagons for $SO(N)$ monopoles. The images are constrained as above by orientifold projection.

**IV. DISCUSSION AND CONCLUSION**

We have studied the correspondence between the brane web of D3 branes and D1 branes with orientifold in type IIB string theory and the Nahm’s data for the general BPS field configuration which can be summarized as

- The asymptotic Higgs fields, $\mu_a \leftrightarrow$ the $x^6$ position of D3 brane.
- The magnetic charge of fundamental monopole, $m_a \leftrightarrow$ the number of D1 branes stretched between two D3 branes.
These are already known for $SU(N)$ monopoles and the new things for $SO(N)$ and $Sp(N)$ are as follows.

- The constraints on $\mu_a$ and $m_a$ from the embeddings of $SO(N)$ and $Sp(N)$ in $SU(N)$ ↔ O3 plane projection.
- The existence of matrices $C_a$ in Eq.(11) ↔ O3 plane projection (the worldsheet parity) of D1 branes.

As we have seen, O3 plane plays the crucial role in understanding the $SO/Sp$ monopoles in brane configuration providing the natural physical origin of the Nahm’s equation and its boundary conditions, which was mysterious at the field theory level.

The moduli for the multimonopoles also have a natural interpretation in brane configuration as the Wilson loop and the motion of the D1 branes. Considering the recent progress of the metrics in the moduli space in various aspects in the field theories [16], it is interesting to describe these and to study the monopole scattering in detail in terms of branes.

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