Theory of neutrino oscillations using condensed matter physics
Including production process and energy-time uncertainty

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Abstract

Neutrino scillations cannot arise from an initial isolated one particle state if four-momentum is conserved. The transition matrix element is generally squared and summed over all final states with no interference between orthogonal final states. Lorentz covariant descriptions based on relativistic quantum field theory cannot describe interference between orthogonal states with different \( \nu \) masses producing neutrino oscillations. Simplified model presents rigorous derivation of handwaving argument about “energy-time uncertainty”. Standard time-dependent perturbation theory for decays shows how energy spectrum of final state is much broader than natural line width at times much shorter than decay lifetime. Initial state containing two components with different energies decay into two orthogonal states with different \( \nu \) masses completely separated at long times with no interference. At short times the broadened energy spectra of the two amplitudes overlap and interfere. “Darmstadt oscillation” experiment attempts to measure the momentum difference between the two contributing coherent initial states and obtain information about \( \nu \) masses without detecting the \( \nu \). Simple interpretation gives value for the squared \( \nu \) mass difference differing by less than a factor of three from values calculated from the KAMLAND experiment. Treatment holds only in laboratory frame with values of energy, time and momentum determined by experimental environment at rest in the laboratory.

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I. INTRODUCTION - THE BASIC PARADOX OF NEUTRINO OSCILLATIONS

A. The problem

1. The original neutrino experiment by Lederman et al [1] showed a neutrino emitted in a $\pi \rightarrow \mu \nu$ decay entering a detector and producing only muons and no electrons.

2. The neutrino enters detector as coherent mixture of mass eigenstates with right relative magnitudes and phases to cancel the amplitude for producing electron at detector.

3. $\nu$ wave function must have states with different masses, momenta and/or energies .

4. In initial one-particle state components with different momenta have different energies.

5. Lederman et al experiment can’t exist if energy and momentum are conserved.

B. The Solution

1. If momentum is conserved in the interaction, violation of energy conservation needed.

2. Energy-time uncertainty in the laboratory frame allows components of initial wave packet with different energies to produce same final $\nu_e$ with the same single energy.

C. Darmstadt application

Radioactive ion circulates in storage ring before decay [3]

1. Transition probability depends on relative phase between two components

2. Relative phase and transition probability change in propagation through storage ring.

3. Phase changes produce oscillations in decay probability.

4. Oscillations can give information about $\nu$ masses without detecting the $\nu$.
D. A simple example of resolution of the paradox

Time dependent perturbation theory shows violation of energy conservation by energy-time uncertainty in sufficiently short times [4]. The time dependent amplitude \( \beta_f(E_i) \) for the decay from an initial state with energy \( E_i \) into a final state with a slightly different energy \( E_f \) is

\[
\frac{\beta_f(E_i)}{g} \cdot (E_i - E_f) = \left[ e^{-i(E_i-E_f)t} - 1 \right] \cdot e^{-2iE_ft} \tag{1.1}
\]

where we have set \( \hbar = 1 \) and \( g \) is the interaction coupling constant.

We now generalize this expression to the case where two initial states with energies \( E_f - \delta \) and \( E_f + \delta \) decay into the same final state with energy \( E_f \) and define \( x \equiv E_i - E_f \)

\[
\frac{e^{2iE_ft}}{g} \cdot \left[ \beta_f(E_f + x - \delta) + \beta_f(E_f + x + \delta) \right] = \left[ \frac{e^{-i(x-\delta)t} - 1}{(x-\delta)} \right] + \left[ \frac{e^{-i(x+\delta)t} - 1}{(x+\delta)} \right] \tag{1.2}
\]

The square of the transition amplitude denoted by \( T \) is then given by

\[
\frac{|T|^2}{g^2} = \left[ \frac{\beta_f(E_f + x - \delta) + \beta_f(E_f + x + \delta)}{g} \right]^2 = 4 \cdot \left[ \frac{\sin^2[(x-\delta)t/2]}{(x-\delta)^2} + \frac{\sin^2[(x+\delta)t/2]}{(x+\delta)^2} \right] + T_{int} \tag{1.3}
\]

where the interference term \( T_{int} \) is

\[
T_{int} = \left[ \frac{e^{-i(x-\delta)t}}{(x-\delta)} - 1 \right] \cdot \left[ \frac{e^{i(x+\delta)t}}{(x+\delta)} - 1 \right] + cc = 4 \left[ \frac{2\sin^2[\delta t/2] + 2\sin^2[xt/2] \cos[\delta t] - \sin^2(\delta t)}{x^2-\delta^2} \right]
\]

(1.4)

If the time is sufficiently short so that the degree of energy violation denoted by \( x \) is much larger than the energy difference \( \delta \) between the two initial states, \( x \gg \delta \) and

\[
x \gg \delta; \quad |T|^2 \approx 8g^2 \cdot \left[ \frac{\sin^2[xt/2]}{x^2} \right] \cdot [1 + \cos \delta t] \tag{1.5}
\]

The transition probability is given by the Fermi Golden Rule. We integrate the the square of the transition amplitude over \( E_i \) or \( x \), introduce the density of final states \( \rho(E_f) \) and and assume that \( \delta \) is negligibly small in the integrals.
The transition probability per unit time $W$ is then

$$W \approx 4g^2 \int_{-\infty}^{\infty} du \left[ \frac{\sin^2 u}{u^2} \right] \cdot \rho(E_f)(1 + \cos(\delta t)) \cdot t = 4\pi g^2 \rho(E_f)$$  \hspace{1cm} (1.7)$$

The interference term between the two initial states is seen to be comparable to the direct terms when $\cos(\delta t) \approx 1$; i.e. when the energy uncertainty is larger than the energy difference between the two initial states.

This example shows in principle how two initial states with a given momentum difference can produce a coherent final state containing two neutrinos with the same energy and the given momentum difference. A measurement of the momentum difference between the two initial states can provide information on neutrino masses without detecting the neutrino.

In this simple example the amplitudes and the coupling constant $g$ are assumed to be real. In a more realistic case there is an additional extra relative phase between the two terms in eq.(1.2) which depends upon the initial state wave function. In the GSI experiment [3] this phase varies linearly with the time of motion of the initial ion through the storage ring. This phase variation can produce the observed oscillations.

II. THE BASIC PHYSICS OF NEUTRINO OSCILLATIONS

A. Interference is possible only if we can’t know everything

Neutrino oscillations are produced from a coherent mixture of different $\nu$ mass eigenstates. The mass of a $\nu$ produced in a reaction where all other particles have definite momentum and energy is determined by conservation of energy and momentum. Interference between amplitudes from different $\nu$ mass eigenstates is not observable in such a “missing mass” experiment. Something must prevent knowing the neutrino mass from conservation laws. Ignorance alone does not produce interference. Quantum mechanics must hide information. To check how coherence and oscillations can occur we investigate what is known and what information is hidden by quantum mechanics.
A simple example is seen in the decay $\pi \rightarrow \mu + \nu$. If the momenta of the initial $\pi$ and recoil $\mu$ are known the $\nu$ mass is known from energy and momentum conservation and there are no oscillations. But oscillations have been observed in macroscopic neutrino detectors at rest in the laboratory system. Oscillations arise only when the outgoing $\nu$ is a wave packet containing coherent mixtures of two mass eigenstates with different masses and therefore different momenta and/or energies. The decay interaction conserves momentum in the laboratory system. The incident pion wave packet must contain coherent mixtures with the same momentum difference. The pion is a one-particle state with a definite mass. Two states with different momenta must have different energies. A transition from a linear combination of two states with different energies to a final state with a single energy can occur only with a violation of energy conservation. This violation can only occur if the $\pi$, $\mu$ and $\nu$ are not isolated but interacting with another system that absorbs the missing energy.

A simple description of neutrino oscillations which neglects these interactions has a missing four-momentum expressed simply in the laboratory system as a missing energy. Covariant descriptions and Lorentz transformations with a missing four-momentum are not easily described in treatments which separate the decay process from interactions with the environment. In other Lorentz frames both energy and momentum conservation are violated.

The momentum difference between the two coherent components of the initial pion wave packet depends on the mass difference between neutrino mass eigenstates. Measuring this momentum difference can give information about the neutrino masses even if the neutrino is not detected. In most cases such a measurement is not feasible experimentally. The GSI experiment [3] describes a unique opportunity.

### B. Energy and momentum in the GSI experiment

The search for what is known and what is hidden by quantum mechanics leads to the energy-time uncertainty described in our simple example (1.1). The short time interval between the last observation of the initial ion before decay and the observed decay time
enables enough violation of energy conservation to prevent a missing mass experiment. This line broadening effect is demonstrated in eq.(1.7) and related to the line broadening of any decay observed in a time short compared to its natural line width [4]. The decay to two final states is described by two Breit-Wigner energy distributions separated at long times. But in this experiment [3] and in our simplified model (1.5) the decay time is sufficiently short to make the separation negligible in comparison with their broadened widths. The transition can occur coherently from two components of the initial state with different energies and momenta into a same final state with a different common energy and the same momentum difference. The sum of the transition amplitudes from these two components of an initial state to the same final state depends on their relative phase. Changes in this phase can produce oscillations. The energy-time uncertainty is not covariant and defined only in the laboratory system. Covariant descriptions and transformations from the laboratory to any center-of-mass system are not valid for a description of neutrino oscillations.

C. Summary of what is known and hidden by quantum mechanics

1. The final state has coherent pairs of states containing neutrinos with different masses and different momenta and/or energies.

2. The initial state is a one-particle state with a definite mass.

3. Momentum is conserved in the transition.

4. The initial state can contain coherent pairs with the same momentum difference present in the final state but these must have different energies.

5. Energy-time uncertainty hides information and prevents use of energy conservation.

6. The transition occurs coherently from two components of the initial state with different energies and momenta into a same final state with a different common energy and the same momentum difference.
7. The relative phase between components with different energies changes during the passage of the ion through the storage ring and can produce oscillations.

A treatment of neutrino oscillations without explicit violation of energy conservation describes a missing mass experiment where no neutrino oscillations of any kind are allowed.

III. THE BASIC PHYSICS OF THE GSI EXPERIMENT

A. A first order weak transition

The initial state wave function $|i(t)\rangle$ is a “Mother” ion wave packet containing components with different momenta. Its development in time is described by an unperturbed Hamiltonian denoted by $H_o$ which describes the motion of the initial and final states in the electromagnetic fields constraining their motion in a storage ring.

$$|i(t)\rangle = e^{iH_o t} |i(0)\rangle$$

(3.1)

The time $t = 0$ is defined as the time of entry into the apparatus. Relative phases of wave function components with different momenta are determined by localization in space at the point of entry into the apparatus. Since plane waves have equal amplitudes over all space, these relative phases are seriously constrained by requiring that the probability of finding the ion outside the storage ring must be zero.

A first-order weak decay is described by the Fermi Golden Rule. The transition probability per unit time at time $t$ from an initial state $|i(t)\rangle$ to a final state $|f\rangle$ is

$$W(t) = \frac{2\pi}{\hbar} |\langle f | T |i(t)\rangle|^2 \rho(E_f) = \frac{2\pi}{\hbar} |\langle f | Te^{iH_o t} |i(0)\rangle|^2 \rho(E_f)$$

(3.2)

where $T$ is the transition operator and $\rho(E_f)$ is the density of final states. The transition operator $T$ conserves momentum.

If two components of the initial state with slightly different energies can both decay into the same final state, their relative phase changes linearly with time and can produce changes
in the transition matrix element. The quantitative result and the question of whether oscillations can be observed depend upon the evolution of the initial state. The neutrino is not detected in the GSI experiment [3], but the information that a particular linear combination of mass and momentum eigenstates would be created existed in the system. Thus the same final state can be created by either of three initial states that have the same momentum difference. Violation of energy conservation allows the decay and provides a new method for investigating the creation of such a coherent state.

B. Time dependence and internal clocks

An external measurement of the time between the initial observation and the decay of a radioactive ion circulating in a storage ring gives information about the system only if an internal clock exists in the system.

1. An initial ion in a one-particle energy eigenstate has no clock. Its propagation in time is completely described by a single unobservable phase.

2. If the initial ion is in a coherent superposition of different energy eigenstates, the relative phase of any pair changes with energy. This phase defines a clock which can measure the time between initial observation and decay.

3. If the decay transition conserves energy, the final states produced by the transition must also have different energies.

4. The decay probability is proportional to the square of the sum of the transition matrix elements to all final states. There are no interference terms between orthogonal final states with different energies and their relative phases are unobservable.

The probability $P_i(t)$ that the ion is still in its initial state at time $t$ and not yet decayed satisfies an easily solved differential equation,

$$\frac{d}{dt} P_i(t) = -W(t) P_i(t); \quad \frac{d}{dt} \log(P_i) = -W(t); \quad P_i(t) = e^{-\int W(t) dt} \quad (3.3)$$
If $W(t)$ is independent of time eq. (3.3) gives an exponential decay. The observation of a nonexponential decay implies that $W(t)$ is time dependent. Time dependence can arise if the initial ion is in a coherent superposition of different energy eigenstates, whose relative phases change with time. This phase defines a clock which can measure the time between initial observation and decay. Since the time $dt$ is infinitesimal, energy need not be conserved in this transition. A non-exponential decay can occur only if there is a violation of energy conservation. All treatments which assume energy conservation; e.g. [8] will only predict exponential decay.

$W(t)$ depends upon the unperturbed propagation of the initial state before the time $t$ where its motion in the storage ring is described by classical electrodynamics. Any departure from exponential decay must come from the evolution in time of the initial unperturbed state. This can change the wave function at the time of the decay and therefore the value of the transition matrix element. What happens after the decay cannot change the wave function before the decay. Whether or not and how the final neutrino is detected cannot change the decay rate.

C. The role of Dicke superradiance

Dicke [10] has shown that whenever two initial state components can produce amplitudes for decay into the same final state, a linear combination called “superradiant” has both components interfering constructively to enhance the transition. The orthogonal state called “subradiant” has maximum destructive interference and may even produce a cancelation.

The wave function of the initial state before the transition can contain pairs of components with a momentum difference allowing both to decay into the same final state. This wave function can be expressed as a linear combination of superradiant and subradiant states with a relative magnitude that changes with time. The variation between superradiant and subradiant wave functions affects the transition matrix element and can give rise to oscillations in the decay probability. Since the momentum difference depends on the mass
difference between the two neutrino eigenstates these oscillations can provide information about neutrino masses.

IV. DETAILED ANALYSIS OF A SIMPLIFIED MODEL FOR DARMSTADT OSCILLATIONS

A. The initial and final states for the transition matrix

The initial radioactive “Mother” ion is in a one-particle state with a definite mass moving in a storage ring. There is no entanglement [8] since no other particles are present. To obtain the required information about this initial state we need to know the evolution of the wave packet during passage around the storage ring. This is not easily calculated. It requires knowing the path through straight sections, bending sections and focusing electric and magnetic fields.

The final state is a “Daughter” ion and a $\nu_e$ neutrino, a linear combination of several $\nu$ mass eigenstates. This $\nu_e$ is a complicated wave packet containing different masses, energies and momenta. The observed oscillations arise only from $\nu$ components with different masses and different momenta and/or energies.

B. Kinematics for a simplified two-component initial state.

Consider the transition for each component of the wave packet which has a momentum $\vec{P}$ and energy $E$ in the initial state. The final state has a recoil ion with momentum denoted by $\vec{P}_R$ and energy $E_R$ and a neutrino with energy $E_\nu$ and momentum $\vec{p}_\nu$. If both energy and momenta are conserved,

$$E_R = E - E_\nu; \quad \vec{P}_R = \vec{P} - \vec{p}_\nu; \quad M^2 + m^2 - M_R^2 = 2EE_\nu - 2\vec{P} \cdot \vec{p}_\nu$$

(4.1)

where $M$, $M_R$ and $m$ denote respectively the masses of the mother and daughter ions and the neutrino. We neglect transverse momenta and consider the simplified two-component
initial state for the “mother” ion having momenta $P$ and $P + \delta P$ with energies $E$ and $E + \delta E$. The final state has two components having neutrino momenta $p_\nu$ and $p_\nu + \delta p_\nu$ with energies $E_\nu$ and $E_\nu + \delta E_\nu$ together with a recoil ion having the same momentum and energy for both components. The changes in these variables produced by a small change $\Delta(m^2)$ in the squared neutrino mass are seen from eq. (4.1) to satisfy the relation

$$\frac{\Delta(m^2)}{2} = E\delta E_\nu + E_\nu \delta E - P\delta p_\nu - p_\nu \delta P = -E\delta E \cdot \left[1 - \frac{\delta E_\nu}{\delta E} + \frac{p_\nu}{P} - \frac{E_\nu}{E}\right] \approx -E\delta E \quad (4.2)$$

where we have noted that momentum conservation in the transition requires $P\delta p_\nu = P\delta P = E\delta E$, $E$ and $P$ are of the order of the mass $M$ of the ion and $p_\nu$ and $E_\nu$ are much less than $M$. To enable coherence the two final neutrino components must have the same energy, i.e. $\delta E_\nu = 0$. Since $\delta E \neq 0$ we are violating energy conservation.

The relative phase $\delta \phi$ at a time $t$ between the two states $|P\rangle$ and $|P + \delta P\rangle$ is given by $\delta E \cdot t$. Equation (4.2) relates $\delta E$ to the difference between the squared masses of the two neutrino mass eigenstates. Thus

$$E \cdot \delta E = -\frac{\Delta(m^2)}{2}; \quad \delta \phi \approx -\delta E \cdot t = -\frac{\Delta(m^2)}{2E} \cdot t = -\frac{\Delta(m^2)}{2\gamma M} \cdot t \quad (4.3)$$

where $\gamma$ denotes the Lorentz factor $E/M$.

**C. Dicke superradiance and subradiance in the experiment**

Consider the transition from a simplified initial state for the “mother” ion with only two components denoted by $|\vec{P}\rangle$ and $|\vec{P} + \delta \vec{P}\rangle$ having momenta $\vec{P}$ and $\vec{P} + \delta \vec{P}$ with energies $E$ and $E + \delta E$. The final state denoted by $|f(E_\nu)\rangle$ has a “daughter” ion and an electron neutrino $\nu_e$ which is a linear combination of two neutrino mass eigenstates denoted by $\nu_1$ and $\nu_2$ with masses $m_1$ and $m_2$. To be coherent and produce oscillations the two components of the final wave function must have the same neutrino energy $E_\nu$ and the same momentum $\vec{P}_R$ and energy $E_R$ for the “daughter” ion.

$$|f(E_\nu)\rangle \equiv |\vec{P}_R; \nu_e(E_\nu)\rangle = |\vec{P}_R; \nu_1(E_\nu)\rangle \langle \nu_1 | \nu_e \rangle + |\vec{P}_R; \nu_2(E_\nu)\rangle \langle \nu_2 | \nu_e \rangle \quad (4.4)$$
where \( \langle \nu_1 | \nu_e \rangle \) and \( \langle \nu_2 | \nu_e \rangle \) are elements of the neutrino mass mixing matrix, commonly expressed in terms of a mixing angle denoted by \( \theta \).

\[
\cos \theta \equiv \langle \nu_1 | \nu_e \rangle; \quad \sin \theta \equiv \langle \nu_2 | \nu_e \rangle; \quad |f(E_\nu)\rangle = \cos \theta \left| \vec{P}_R; \nu_1(E_\nu) \right\rangle + \sin \theta \left| \vec{P}_R; \nu_2(E_\nu) \right\rangle \quad (4.5)
\]

After a very short time two components with different initial state energies can decay into a final state which has two components with the same energy and a neutrino state having two components with the same momentum difference \( \delta \vec{P} \) present in the initial state.

The momentum conserving transition matrix elements between the two initial momentum components to final states with the same energy and momentum difference \( \delta \vec{P} \) are

\[
\langle f(E_\nu) | T \left| \vec{P} \right\rangle \rangle = \cos \theta \left| \vec{P}_R; \nu_1(E_\nu) \right\rangle; \quad \langle f(E_\nu) | T \left| \vec{P} + \delta \vec{P} \right\rangle \rangle = \sin \theta \left| \vec{P}_R; \nu_2(E_\nu) \right\rangle \quad (4.6)
\]

We neglect transverse momenta and set \( \vec{P} \cdot \vec{p}_\nu \approx P p_\nu \) where \( P \) and \( p_\nu \) denote the components of the momenta in the direction of the incident beam. The Dicke superradiance analog [10] is seen by defining superradiant and subradiant states.

\[
|Sup(E_\nu) \rangle \equiv \cos \theta |P \rangle + \sin \theta |P + \delta P \rangle; \quad |Sub(E_\nu) \rangle \equiv \cos \theta |P + \delta P \rangle - \sin \theta |P \rangle \quad (4.7)
\]

The transition matrix elements for these two states are then

\[
\frac{\langle f(E_\nu) | T \left| Sup(E_\nu) \right\rangle \rangle}{\langle f | T | P \rangle} = [\cos \theta + \sin \theta]; \quad \frac{\langle f(E_\nu) | T \left| Sub(E_\nu) \right\rangle \rangle}{\langle f | T | P \rangle} = [\cos \theta - \sin \theta] \quad (4.8)
\]

where we have neglected the dependence of the transition operator \( T \) on the small change in the momentum \( P \). The squares of the transition matrix elements are

\[
|\langle f(E_\nu) | T \left| Sup(E_\nu) \right\rangle \rangle^2 \big| \langle f | T | P \rangle \big|^2 = [1 + \sin 2\theta]; \quad |\langle f(E_\nu) | T \left| Sub(E_\nu) \right\rangle \rangle^2 \big| \langle f | T | P \rangle \big|^2 = [1 - \sin 2\theta] \quad (4.9)
\]

For maximum neutrino mass mixing, \( \sin 2\theta = 1 \) and

\[
|\langle f(E_\nu) | T \left| Sup(E_\nu) \right\rangle \rangle^2 = 2 \big| \langle f | T | P \rangle \big|^2; \quad |f(E_\nu) | T \left| Sub(E_\nu) \right\rangle ^2 = 0 \quad (4.10)
\]

This is the standard Dicke superradiance in which all the transition strength goes into the superradiant state and there is no transition from the subradiant state.
Thus from eq. (4.7) the initial state at time $t$ varies periodically between the superradiant and subradiant states. The period of oscillation $\delta t$ is obtained by setting $\delta \phi \approx -2\pi$,

$$\delta t \approx \frac{4\pi \gamma M}{\Delta(m^2)}; \quad \Delta(m^2) = \frac{4\pi \gamma M}{\delta t} \approx 2.75\Delta(m^2)_{\text{exp}} \quad (4.11)$$

where the values $\delta t = 7$ seconds and $\Delta(m^2) = 2.22 \times 10^{-4}eV^2 = 2.75\Delta(m^2)_{\text{exp}}$ are obtained from the GSI experiment and neutrino oscillation experiments [5].

The theoretical value (4.11) obtained with minimum assumptions and no fudge factors is in the same ball park as the experimental value obtained from completely different experiments. Better values obtained from better calculations can be very useful in determining the masses and mixing angles for neutrinos.

**D. Effects of spatial dependence**

The initial wave function travels through space as well as time. In a storage ring the ion moves through straight sections, bending sections and focusing fields. All must be included to obtain a reliable estimate for $\Delta(m^2)$. That this requires a detailed complicated calculation is seen in examining two extreme cases

1. Circular motion in constant magnetic field. The cyclotron frequency is independent of the momentum of the ion. Only the time dependent term contributes to the phase and $\delta \phi^{\text{cy}}$ is given by eq. (4.3)

2. Straight line motion with velocity $v = (P/E) \cdot t$. The phase of the initial state at point $x$ in space and time $t$, its change with energy and momentum changes $\delta P$ and $\delta E$ are

$$\phi^{\text{SL}} = P \cdot x - E \cdot t; \quad \delta \phi^{\text{SL}} = (\delta P \cdot v - \delta E) \cdot t = \frac{P\delta P - E\delta E}{E} \cdot t = 0 \quad (4.12)$$

The large difference between the two results (4.3 and (4.12) indicate that a precise determination of the details of the motion of the mother ion in the storage ring is needed before precise predictions of the squared neutrino mass difference can be made.
E. A tiny energy scale

The experimental result sets a scale in time of seven seconds and a tiny energy scale

$$\Delta E \approx 2\pi \cdot \frac{h}{7} = 2\pi \cdot \frac{6.6 \cdot 10^{-16}}{7} \approx 0.6 \cdot 10^{-15}\text{eV} \quad (4.13)$$

This tiny energy difference between two waves which beat with a period of seven seconds must be predictable from standard quantum mechanics using a scale from another input. Another tiny scale available in the parameters that describe this experiment is the mass-squared difference between two neutrino mass eigenstates. A prediction (4.11) has been obtained from following the propagation of the initial state through the storage ring during the time before the decay.

That these two tiny energy scales obtained from completely different inputs are within an order of magnitude of one another suggests some relation obtainable by a serious quantum-mechanical calculation. We have shown here that the simplest model relating these two tiny mass scales gives a result that differs by only by a factor of less than three.

Many other possible mechanisms might produce oscillations. The experimenters [3] claim that they have investigated all of them. These other mechanisms generally involve energy scales very different from the scale producing a seven second period.

The observed oscillation is seen to arise from the relative phase between two components of the initial wave function with a tiny energy difference (4.13). These components travel through the electromagnetic fields required to maintain a stable orbit. The effect in these fields on the relative phase depends on the energy difference between the two components. Since the energy difference is so tiny the effect on the phase is expected to be also tiny and calculable.

V. CONCLUSIONS

Neutrino oscillations cannot occur if the momenta of all other particles participating in the reaction are known and momentum and energy are conserved. A complete description
of the decay process must include the interaction with the environment and violation of energy conservation. A new oscillation phenomenon providing information about neutrino mixing is obtained by following the initial radioactive ion before the decay. Difficulties introduced in conventional $\nu$ experiments by tiny neutrino absorption cross sections and very long oscillation wave lengths are avoided. Measuring the decay time enables every $\nu$ event to be observed and counted without the necessity of observing the $\nu$ via the tiny absorption cross section. The confinement of the initial ion in a storage ring enables long wave lengths to be measured within the laboratory.

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