A method of rotating accelerometer gravity gradiometer for centrifugal gradient detection

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Abstract. This paper proposed a centrifugal gradient detection method of rotating accelerometer gravity gradiometer (RAGG) and deduced the relationship between centrifugal gradient detection precision and gyro noise coefficients. Based on the deduced relationship, suitable gyros can be selected as angular velocity sensors for centrifugal gradient detection. A simulation experiment was designed to verify the centrifugal gradient detection method and the deduced relationship. In the experiment, the permitted centrifugal gradient error was given, the gyro critical noise coefficients were calculated, and their corresponding gyro simulation data was generated; based on the proposed centrifugal gradient detection method, centrifugal gradient and centrifugal gradient detection error were calculated. The experiment results show that the centrifugal gradient and the centrifugal gradient error are respectively consistent with the theoretical centrifugal gradient and the given permitted centrifugal gradient error; therefore, the results suggest that the centrifugal gradient detection method and the deduced relationship is correct.

1. Introduction
Gravity gradiometry began in 1880s when Loránd Eötvös introduced his first torsion balance gradiometer to the oil industry [1]. Subsequently, in the 1970s, the United States initiated a program of developing moving-base gravity gradient instrument for military needs, and by the 1980s, Bell Aerospace Textron had successfully developed the first moving base rotating accelerometer gravity gradiometer [1, 2]. Compared with gravity data, gravity gradient data provides more precise mapping, more accurate depth information, more accurate shape information and more accurate orientation information. Airborne gravity gradiometry (AGG) has become the most advanced means of surveying gravity field and it has high accuracy and space resolution for measuring the gravity field. Gravity precision derived from Falcon AGG (a partial tensor RAGG), for heliborne surveys, is reported to be 0.1 mGal with spatial resolution 50m [2]. A more detailed review on current technologies can be found in [2-7].

The measurement of RAGG contains centrifugal gradient, to realize the measurement of the gravitational gradient, the output of RAGG needs to subtract the centrifugal gradient measured by centrifugal gradient detection unit. Patent [8] proposed applying gyro as angular rate sensor for centrifugal gradient detection. Furtherly, this paper studied centrifugal gradient detection method of RAGG, and deduced the relationship between centrifugal gradient detection precision and gyro noise.
coefficients. Based on this relationship, suitable gyros can be selected as angular velocity sensors for centrifugal gradient detection.

2. Centrifugal gradient detection method
The measurement of RAGG is the difference of the sum of two pairs of accelerometers:

\[ (A_i + A_j) - (A_i + A_j) = 2R(\omega_{imx}^2 - \omega_{imy}^2 + \Gamma_{xx} - \Gamma_{yy}) \sin 2\Omega t - 4R(-\omega_{imx}\omega_{imy} + \Gamma_{xy}) \cos 2\Omega t \]  

(1)

where \( \omega_{imx}, \omega_{imy} \) respectively represent the angular velocity of RAGG relative to the inertial frame, \( \omega_{imx}^2, -\omega_{imy}^2 \) are centrifugal gradients, \( \Gamma_{xx}, \Gamma_{yy} \) are gravitational gradients, RAGG's inline output \( \omega_{imx}^2, -\omega_{imy}^2 + \Gamma_{xx} - \Gamma_{yy} \) and RAGG's cross output \( -\omega_{imx}\omega_{imy} + \Gamma_{xy} \) respectively are the demodulation of the combination signal (Equation 1) at \( \sin 2\Omega t \) and \( \cos 2\Omega t \). Therefore, the output of RAGG is the sum of the centrifugal gradient and the gravitational gradient. To realize the measurement of the gravitational gradient, the output of RAGG needs to subtract the centrifugal gradient. Angular velocity sensors (gyros) are mounted on RAGG for measuring centrifugal gradient. Figure 1(a) illustrates the installation location of gyros. \( X_m, Y_m, Z_m \) is RAGG measurement frame, two gyros are mounted such that its input axis is orthogonal to each other for measuring \( \omega_{imx} \) and \( \omega_{imy} \) [1]. Figure 1(b) illustrates the proposed signal processing method of centrifugal gradient detection.

Comparing to the outputs of RAGG, the raw data of gyros has a higher sampling frequency, and contains high-frequency noise such as quantify noise, power frequency noise etc., which has a significant impact on centrifugal gradient detection precision. Lowpass filter 1 is applied to attenuate these high-frequency noise, then anti-aliasing filtering and decimation are performed to make sample rate of gyros data equaling that of RAGG's output. The calculation of centrifugal gradient is given by

\[
\begin{align*}
\Gamma_{m\text{inline}} &= k_{wc} \left( \omega_{imx}^2 - \omega_{imy}^2 \right) \\
\Gamma_{m\text{cross}} &= k_{wc} \left( -\omega_{imx}\omega_{imy} \right)
\end{align*}
\]

(2)

where the unit of angular velocity is \( \text{arsec/s} \), \( k_{wc} \) is conversion factors from \( \text{arsec/s}^2 \) to \( E_0 \), and its value equals 0.0235. The centrifugal gradient that is calculated from the decimated gyro data, subsequently is lowpass filtered by Lowpass filter 2, and the filter characteristic of lowpass filter 2 should be in accordance with that of RAGG to ensure that the RAGG and gyro experience the same centrifugal gradient.

Figure 1. Centrifugal gradient detection method

3. Relationship between centrifugal gradient detection precision and gyro noise coefficients
In this section, we will analyse the relationship between centrifugal gradient detection accuracy and the gyro noise coefficients. The output of gyro is given by:
\[ \begin{align*}
\omega_i &= \omega_i^* + \delta \omega_i \\
\omega_j &= \omega_j^* + \delta \omega_j
\end{align*} \]  
(3)

where \( \omega_i \) is true value, \( \delta \omega \) is noise value, by equation (2), centrifugal gradient can be calculated by

\[ \begin{align*}
\Gamma_{\text{inlin}} &= k_w (\omega_i^* - \omega_i) \\
\Gamma_{\text{cros}} &= -k_w (\omega_i, \omega_j)
\end{align*} \]  
(4)

where \( \Gamma_{\text{inlin}} \) and \( \Gamma_{\text{cros}} \) are centrifugal gradients calculated from the output of gyros, \( \Gamma_{\text{inlin}} \) and \( \Gamma_{\text{cros}} \) are true value of centrifugal gradients, the centrifugal gradient error is the difference of those two kinds centrifugal gradients, and is given by

\[ \begin{align*}
\text{err}_{\text{inlin}} &= \Gamma_{\text{inlin}} - \Gamma_{\text{inlin}}' = k_w \left[ \delta \omega_i - \delta \omega_i^* + 2(\omega_i, \delta \omega_j - \omega_i, \delta \omega_j) \right] \\
\text{err}_{\text{cros}} &= \Gamma_{\text{cros}} - \Gamma_{\text{cros}}' = k_w \left[ \delta \omega_i, \delta \omega_j + \omega_i, \delta \omega_j + \omega_i, \delta \omega_j \right]
\end{align*} \]  
(5)

the standard deviation of centrifugal gradient error is given by

\[ \begin{align*}
\sigma(\text{err}_{\text{inlin}}) &= k_w \left[ \sigma(\delta \omega_i^*) - \sigma(\delta \omega_i) + 2\omega_i, \sigma(\delta \omega_j) \right] \\
\sigma(\text{err}_{\text{cros}}) &= k_w \left[ \sigma(\delta \omega_j) + \omega_i, \sigma(\delta \omega_j) \right]
\end{align*} \]  
(6)

Assuming that the x-gyro and y-gyro have about the same angular velocity \( \omega_i = \omega_j = \Omega \), and their noise are independent zero mean gaussian white noise, and their standard deviation satisfy with \( \sigma(\delta \omega_j) = \sigma(\delta \omega_j) = \sigma_{\Omega} \). Thus, the equation (6) can be simplified as

\[ \begin{align*}
\sigma(\text{err}_{\text{inlin}}) &= \sqrt{2k_w} \sigma(\delta \omega_i) + 4k_w \sigma_{\Omega}(\delta \omega_j) \\
\sigma(\text{err}_{\text{cros}}) &= k_w \sigma(\delta \omega_i) + 2k_w \sigma(\delta \omega_j)
\end{align*} \]  
(7)

Base on the property of random variable, equation (7) can be simplified as:

\[ \begin{align*}
\sigma(\text{err}_{\text{inlin}}) &= 2k_w \sigma_{\gamma_{\Omega}} + 4k_w \sigma_{\Omega_{\gamma_{\Omega}}} \\
\sigma(\text{err}_{\text{cros}}) &= 2k_w \sigma_{\gamma_{\Omega}} + 2k_w \sigma_{\Omega_{\gamma_{\Omega}}}
\end{align*} \]  
(8)

Let \( \Gamma_{\text{max}} \) denotes the maximum permitted standard deviation of centrifugal gradient error, therefore the standard deviation of centrifugal gradient error should satisfy below condition

\[ \begin{align*}
\sigma(\text{err}_{\text{inlin}}) &\leq \Gamma_{\text{max}} \\
\sigma(\text{err}_{\text{cros}}) &\leq \Gamma_{\text{max}}
\end{align*} \]  
(9)

Because of \( 4\Omega_{\gamma_{\Omega}} > 2\sigma_{\gamma_{\Omega}} \), by equation (9), we can get

\[ \sigma_{\gamma_{\Omega}} \leq \Gamma_{\text{max}}/4\Omega_{\gamma_{\Omega}} \]  
(10)

Equation (10) reflects the relationship between maximum permitted standard deviation of the centrifugal gradient error, gyro angular velocity and standard deviation of gyro noise. The standard deviation of gyro noise, \( \sigma_{\gamma_{\Omega}} \) is the superposition of different noise, such as angle random walk, rate random walk, sine noise etc. The relationship between maximum permitted standard deviation and noise coefficients of four basic noise terms are discussed in below. Four basic noise terms are quantization noise, angle random walk, rate random walk, and bias instability. Four basic noise terms are discussed below.

Quantization Noise: quantization noise is one of the errors introduced into an analogy signal by encoding it in digital form[9]. The variance of the quantization noise is given by

\[ \sigma_q^2 = \frac{8\pi^4Q^2f_i^2}{3} \]  
(11)
where $Q$ is quantization-noise coefficient, $T_s$ is sample interval, $f_q$ is equivalent bandwidth of quantization-noise, standard deviation of quantization-noise should satisfy condition of equation (10), so we can get

$$Q \leq \left( \frac{3\Gamma_{\text{max}}^2 f_s}{128\pi^2 k_{\text{max}}^2 \Omega^2 f_q^2} \right)^{\frac{1}{2}} \quad (12)$$

Angle Random Walk: angle random walk is the integral of rate white noise, and its variance is given by

$$\sigma_{\text{arw}}^2 = N^2 f_{\text{arw}} \quad (13)$$

where $N$ is angle random walk noise coefficient, $f_{\text{arw}}$ is equivalent bandwidth of white noise, standard deviation of angle random walk should satisfy the condition of equation (10), so we can get

$$N \leq \left( \frac{1}{16\Omega^2 k_{\text{arw}} f_{\text{arw}}} \right)^{\frac{1}{2}} \quad (14)$$

Rate Random Walk: rate random walk is the integral of acceleration rate white noise,[6] assuming that the gyro is running for $T$ hour, the minimum frequency $f_{\text{min}}$ of the acquired data is $1/T$, and its variance is given by

$$\sigma_{\text{rrw}}^2 = k^2 \frac{f_{\text{rrw}}}{2\pi^2 f_{\text{min}} (f_{\text{rrw}} + f_{\text{min}})} \quad (15)$$

where $k$ is rate random walk noise coefficient, $f_{\text{rrw}}$ is equivalent bandwidth of rate random walk noise, standard deviation of rate random walk noise should satisfy the condition of equation (10), so we can get

$$k \leq \left( \frac{2\pi \Gamma_{\text{max}}^2 (f_{\text{rrw}} + f_{\text{min}})}{16\Omega^2 k_{\text{rrw}} f_{\text{rrw}}} \right)^{\frac{1}{2}} \quad (16)$$

Bias Instability: The origin of this noise is the electronics or other components that are susceptible to random flickering[9]. Assuming that the gyro is running for $T$ hour, the minimum frequency $f_{\text{min}}$ of the acquired data is $1/T$, and the variance of this noise is given by

$$\sigma_{\text{bias}}^2 = B^2 \frac{\pi}{\ln \left( f_{\text{bias}} + f_{\text{min}} \right)} \left( f_{\text{bias}} + f_{\text{min}} \right) \quad (17)$$

where $B$ is bias instability noise coefficient, $f_{\text{bias}}$ is equivalent bandwidth of bias instability, standard deviation of bias instability should satisfy the condition of equation (10), so we can get

$$B \leq \left( \frac{\Gamma_{\text{max}}^2 \pi}{16\Omega^2 k_{\text{bias}} \left( \ln \left( f_{\text{bias}} + f_{\text{min}} \right) / f_{\text{min}} \right) f_{\text{min}}} \right)^{\frac{1}{2}} \quad (18)$$

Equation (12), (14), (16), and (18) are the relationship between four basic gyro noise coefficients and centrifugal gradient detection precision.

4. Simulation Experiments

The relationship between gyro noise coefficients and centrifugal gradient detection precision is derived in the above, validation experiment is performed in below. In validation experiment, the permitted centrifugal gradient error of single noise term is $0.1Eo$, the gyro noise is the superposition of four basic noise terms, the centrifugal gradient errors are the superposition of four basic noise terms, therefore, the maximum centrifugal gradient errors should approximate $0.2Eo$. Base on the deduced relationship, critical noise coefficients of gyro are calculated, and their corresponding gyro simulation data is
Figure 2. Validation experiment results generated. If the centrifugal gradients calculated from the gyro simulation data are consistent with the theoretical centrifugal gradients, then proves that the proposed centrifugal gradient detection method is correct. If the maximum standard deviation of centrifugal gradient errors calculated from the gyro simulation data approximates $0.2Eo$, then proves that the deduced relationship between gyro noise coefficients and centrifugal gradient detection precision is correct.
The gyro simulation data is the combination of true angular velocity and gyro noise, the gyro noise is simulated by the power spectral density models, $\Phi(f)$, that provide reasonable characterizations of the gyro noise at different frequencies, $f$ [10]. The gyro noise PSD is the superposition of four basic noise terms, and $\Phi(f)$ is given by:

$$\Phi(f) = 8\pi^2TQf^2 + 2N^2 + \frac{k^2}{2\pi^2f^2} + \frac{B^2}{\pi f}$$

A set of reasonable parameters is selected, $\omega_0=\omega_0=Q=0.5$%/s, $T_{max}=0.1Eo$, $f_c=0.0891\,Hz$, $f_{aw}=0.1135\,Hz$, $f_{rn}=0.4mHz$, $f_{Biaslest}=2.8mHz$, $T=1\,hour$, by the deduced relationship, the critical noise coefficients of gyro are calculated, $Q=0.0433\,\text{arcsec}$, $N=0.0018\,\text{arcsec}/\sqrt{s}$, $k=0.0045\,(\text{arcsec}/s)/\sqrt{h}$, $B=0.0007\,\text{arcsec}/s$. Substituting these critical noise coefficients into equation (19), then we can generate gyro simulation data. The method of calculating centrifugal gradient from gyro data is described in detail in section 2. Figure 2 (a) and figure 2(b) are respectively X-gyro simulation data and Y-gyro simulation data. Figure 2(c) is log-log scale single-sided PSD of simulated gyro. Figure 2(d) and figure (e) is centrifugal gradient plot calculated from gyro simulation data, the mean of centrifugal gradients respectively are $T_{max}=0.0041\,Eo$ and $T_{max}=7.615523\times10^4Eo$, and the theoretical centrifugal gradients are $T_{inline}=0\,Eo$ and $T_{cros}=7.6155\times10^4Eo$, the centrifugal gradients calculated from gyro data is consistent with the theoretical centrifugal gradients, this experiment result suggests that the proposed centrifugal gradient detection method is correct. The Figure 2(f) is centrifugal gradient error plot, and the maximum standard deviation of centrifugal gradient errors approximates 0.2Eo, this experiment result suggests that the deducted relationship is correct. Moreover, another two experiments were done, one experiment had its

| Q (arcsec) | N (arcsec/s²) | K (deg/h³) | B (arcsec/s) | Inline's error SD (Eo) | Cross's error SD (Eo) |
|-----------|--------------|------------|--------------|------------------------|-----------------------|
| 0.0433    | 0.0018       | 0.0045     | 0.0007       | 0.215                  | 0.107                 |
| 0.0433    | 0.0018       | 0.0045     | 0.0035       | 0.470                  | 0.263                 |
| 0.0433    | 0.009        | 0.0045     | 0.007        | 0.563                  | 0.268                 |

$^a$ centrifugal gradient error standard deviation of RAGG's inline channel

$^b$ centrifugal gradient error standard deviation of RAGG's cross channel

angle random walk coefficient of critical noise coefficients increase 5 times, and another experiment had its bias instability coefficient of critical noise coefficients increase 5 times, and the gyro PSD characteristic curve and gyro simulation data curve are not shown, their standard deviation of centrifugal gradient errors are listed in table 1. The maximum standard deviation of centrifugal gradient errors are respectively 0.470Eo and 0.563Eo. Since the noise bandwidth of angle random walk noise is wider than that of bias instability noise, the change of angle random walk noise coefficient has greater impact on the precision of centrifugal gradient detection than the change of bias instability noise coefficient.

5. conclusions

The measurement of RAGG contains centrifugal gradient, to realize the measurement of the gravitational gradient, centrifugal gradient detection and compensation is an unavoidable issue. This paper examines the centrifugal gradient detection method of rotating accelerometer gravity gradiometer (RAGG) and the relationship between centrifugal gradient detection precision and gyroscope noise coefficients. The validation experiments results suggest that the proposed centrifugal gradient detection method and the relationship between centrifugal gradient detection precision and gyro noise coefficients are right. This relationship is helpful for choosing suitable gyros as angular velocity sensors for centrifugal gradient detection.
References

[1] Rogers M M 2009 An Investigation Into the Feasibility of Using a Modern Gravity Gradient Instrument for Passive Aircraft Navigation and Terrain Avoidance

[2] Dransfield M H and Christensen A N 2013 Performance of airborne gravity gradiometers Leading Edge 32 908-22

[3] Annecchione M, Moody M, Carroll K, Dickson D and Main B 2007 Benefits of a high performance airborne gravity gradiometer for resource exploration. In: Proceedings of Exploration, pp 889-93

[4] Difrancesco D 2007 Advances and challenges in the development and deployment of gravity gradiometer systems. In: EGM 2007 international workshop,

[5] Dransfield M 2007 Airborne gravity gradiometry in the search for mineral deposits.

[6] Difrancesco D, Grierson A, Dan K and Meyer T 2009 Gravity gradiometer systems – advances and challenges Geophysical Prospecting 57 615–23

[7] Difrancesco D, Meyer T, Christensen A and FitzGerald D 2009 Gravity gradiometry–today and tomorrow. In: 11th SAGA Biennial technical meeting and exhibition,

[8] Brett J and Brewster J 2010 Accelerometer and rate sensor package for gravity gradiometer instruments. US)

[9] Hou H and El-Sheimy N 2003 Inertial sensors errors modeling using Allan variance

[10] Jekeli C 2006 Airborne Gradiometry Error Analysis Surveys in Geophysics 27 257-75