A retrospective on research visits of Paul Butzer’s Aachen research group to Eastern Europe and Tenerife

Paul L. Butzer · Rudolf L. Stens

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Abstract
Our article “A retrospective on research visits of Paul Butzer’s Aachen research group to North America and Western Europe” [1] concerned the research visits, contacts and conference participations of members of the Chair “Lehrstuhl A für Mathematik” at Aachen to North America and Western Europe. This paper focuses on such research visits of the Aachen Chair and its group to Eastern Europe as well as to Tenerife. The Epilogue treats our top students, different research interests, and conferences on the occasion of PLB’s birthday anniversaries.

Keywords History · Biography

Mathematics Subject Classification 01-06

Dedicated to Johannes Erger (1928–2003), the executor of the EU’s largest Erasmus programme, and to our many male and female students for their achievements in various areas of mathematics.

This article is part of the topical collection “Sampling theory, approximation and their applications. An appreciation to Paul Butzer’s scientific work” edited by Carlo Bardaro, Rudolf Stens and Gianluca Vinti.

Johannes Erger, born 1928 in Holzwickede, studied history, pedagogy and philosophy in Münster and Heidelberg. After receiving his doctorate in 1963 he became Dozent at the Pedagogical Academy Rhineland, full professor in 1967, and finally its rector. After this academy was integrated into the RWTH Aachen in 1980, he became chairman of the external institute of the RWTH, and in that capacity he became honorary chairman of Aachen’s Erasmus programme. The Erasmus network, which Johannes Erger as its coordinator finally developed, and which embraced mathematics, history and electronics engineering, included twenty-one universities from eleven EU countries. It turned out to be the largest such network in the EU. In 1993 he retired as Professor emeritus. He was awarded an honorary doctorate from the Kiev Polytechnic Institute in 1994. Erger, the father of seven children, passed away after a long illness in 2003. The mathematicians have lost a friend.

Rudolf L. Stens
stens@matha.rwth-aachen.de

Paul L. Butzer
butzer@rwth-aachen.de

1 RWTH Aachen University, Lehrstuhl A für Mathematik, 52056 Aachen, Germany
1 Introduction

Our paper “A retrospect on earlier research visits of members of the Aachen research group to North America and Western Europe” [1] dealt with the research visits of the Aachen chair to the USA and Canada, and visits to 13 countries in Western Europe. The present joint paper recounts the research visits to Eastern Europe. “Eastern Europe”, as used herein, refers to Bulgaria, Hungary, Poland, Russia, Ukraine, German Democratic Republic (GDR), former Yugoslavia, Latvia, Estonia and Belarus. The paper also includes the visits to Tenerife because they are connected with the visits to Belarus via Anatoly Kilbas.

The article “A retrospective on 60 years of approximation theory and associated fields” [2] sparked considerable interest. It concentrated on eight symposia (co-)conducted at the Oberwolfach research center from August 1963 to August 1983. These symposia drew to Oberwolfach almost 250 experts (all recorded) from 24 countries in approximation theory and associated fields, such as harmonic analysis, functional analysis and operator theory, integral transform theory, orthogonal polynomials, interpolation and special functions.

After its publication, several readers informed PLB they would like to see an article based upon the research visits, contacts and conference participations of members of the Aachen Chair “Lehrstuhl A für Mathematik” outside of Germany.

From my point of view, such research visits—highlighted by intellectual exchanges among diverse participants—are truly necessary to keep a research group abreast of research activities and developments on a global basis.

I never accepted a teaching and or research position outside of Aachen that lasted a whole semester in view of my ailing mother between 1977 and 1986 and since I felt it was more important to continue supervising together with our research assistants the diplom and doctoral theses of our many students.
These recollections do not cover routine research contacts and invited visits but those which left a special mark; unforgettable, and sometimes demanding, experiences.

This paper also covers the conference participations of the assistants, post-docs, and colleagues at Lehrstuhl A für Mathematik, among them Rolf Nessel, Ernst Görlich, Eberhard Stark and Rudolf Stens.

The research visits to Turkey, Israel, the Middle East, Egypt, India, and China will be treated in a separate paper.

2 Bulgaria

First to Eastern Europe. Because G. Alexits (Budapest), B. Sz.-Nagy and L. Leindler (Szeged), Elena and Tiberi Popoviciu (Cluj, Romania), were at Oberwolfach in 1968, invitations to various conferences behind the Iron Curtain fortuitously reached us in Aachen one after the other. The first was “Constructive Function Theory”, conducted May 19–25, 1970 by L. Iliev and his team at the Mathematical Institute of the Bulgarian Academy of Sciences (near the resort Golden Sands, 17 km northeast of Varna); see [3]. Of the one hundred participants, 19 came from Russia, 16 from Germany.

The Aachen group, which consisted of E. Görlich (1940–2014)\(^1\), W. Koehnen, R.J. Nessel, K. Scherer (1942–2022) and E. L. Stark (1940–1986) and myself (all supported by the DFG), now had their first great opportunity to meet not only a large group of Russian approximation theorists, but also many others from Eastern Europe, and also the West.

Participants included S. B. Stečkin (Russia), L. Alpar, G. Freud, J. Szabados and P. Turan (all Budapest), M. Atteia, Iso Schoenberg, A. Lupaş and D.D. Stancu (both Cluj), L. Collatz, G. Mainardus (both Germany), Z. Cesielski, J. Musielak (both Poland), J.L. Fields (U. Alberta), H. Shapiro (Sweden), A. Sharma (Canada), P.O. Runck (Austria), F. Schurer (Eindhoven).

The conference, dedicated to Academician L. Iliev’s 70th birthday in 1983, coincided with the Feast Day of Kyrillos and Methodius. During the festival service at the Orthodox Cathedral in Varna, the congregants sang their beautiful orthodox chants increasingly more loudly in order to drown out the rackety noise of the Party Parade passing very slowly outside. The former Aachen students Ursula Westphal and Erich van Wickeren, and I, had the pleasure to join in this impressive long service. Although a conference co-organizer expressed annoyance about our attendance (despite the fact it was a “free day”), Vasil Popov (1942–1990), a burning star of Bulgarian mathematicians, supported me. At the same time, Popov quietly expressed his displeasure with the Bulgarian system. His father, a doctor of philosophy, was persecuted by the regime for his political ideas.

There followed several meetings at the Mathematical Institute of the Bulgarian Academy of Sciences, which were attended by members of the Aachen chair, e.g.,

\(^1\) Ernst Görlich, born 1940 in Mechernich, Germany. “Lehramtsstudium” of mathematics and physics at RWTH Aachen, Dr. rer. nat. 1967 under my supervision. Habilitation degree in 1971, Professor of Mathematics at Aachen since 1973, retired 2005, died 2014. His research areas were approximation theory and harmonic analysis.
June 1981 (R.J. Nessel and myself) [4], May/June 1984 (E.L. Stark and R.L. Stens) [5], and May 1987 (Nessel and myself) [6].

From 23 to 30 August 1996 there was the International Workshop “Transform Methods & Special Functions” in the resort Golden Sands near Varna, with 74 participants from 19 countries worldwide [7]. It was organized by P. Rusev, I. Dimovski and V. Kiryakova. The workshop was dedicated to the prominent Bulgarian mathematician Nikola Obrechkoff\(^2\) for his 100th birthday, and included integral transforms, special functions, fractional calculus, geometric theory of functions as well as differential and integral equations.

Virginia Kiryakova chaired the conference in a pleasant, team-like manner and enabled the participants in the broad area of mathematical analysis and its applications to get to know each other on an equal footing, especially important when they come from many different countries. In this respect I met many of the top specialists in the field, especially the large group from Tenerife, including Juan Trujillo, who invited me to visit him and his team at his university.

My lecture with Dr. Stefan Jansche, “Mellin transform theory and the role of its differential and integral operators”, was the first on Mellin analysis held at any conference. Boarding in a TUI hotel on the shore of the Black Sea, Stefan and I decided to take a short swim (ignoring the red flag swaying in the cold wind).

I belong to the four “Honorary Members of Editorial Board” of the journal “Fractional Calculus and Applied Mathematics” (FCAA), Vol. 1, No. 1, 1998, together with E.R. Love (Australia)\(^3\), K.B. Oldham (Canada) and J.F. Spanier (USA)\(^4\). The “Editorial Board” consisted of 18 members from 13 countries. Virginia Kiryakova, the managing editor and founder of FCAA in 1998, mentions in her article [11] the papers [12] by Butzer and Westphal in the B. Ross conference in New Haven of 1974, and the paper [13] by Butzer, Dyckhoff, Görlich and Stens. According to Machado, Kiryakova and Mainardi [10], the conference in New Haven was the first ever on fractional calculus.

### 3 Hungary

Now to Hungary. Our participation in the conference “Hilbert space operators”, conducted by Béla Sz.-Nagy and Ciprian Foias at Tihany (Lake Balaton) and Budapest in September 14–21, 1970, was our first contact with a truly operator theory group, with its Russian experts, L.C. Gohberg, V.I. Gorbacuk, Ju. Rozanow, but also a great number from the West, including Ch. Davis, I. Halperin, J. Dixmier, H. Langer, F.F. Bonsall,

\(^2\) Nikola Obrechkoff (1896–1963) was born in the town of Varna, took part in Kyril Popov’s seminar at U. Sofia, graduated there in 1920, spent 1922–1923 as a postgraduate in Berlin, received his first doctoral degree at U. Palermo (1932) and a D.Sc. from the Sorbonne (1933). His academic career at U. Sofia lasted almost 40 years, culminating with his role as Director of the Mathematical Institute of the Bulgarian Academy of Sciences. Seven of his students became full professors. He was the author of c. 250 publications, including four monographs. For his life and work see P. Rusev: “Nikola Obreshkoff (1896–1963), Biographical Data” in [7, pp. 541–544], and [8].

\(^3\) See [9].

\(^4\) Oldham and Spanier are the authors of “The Fractional Calculus”, Academic Press, New York, 1971, the first monograph on this topic (cf. [10]).
J. R. Ringrose, T. Ando, M. A. Kaashoek, A. I. Suciu, R. G. Douglas, P. R. Halmos, H. Helson, R. V. Kadison, T. Kato, P. Masani, C. Pearcy, R. S. Phillips and C. R. Putnam. Altogether there were 47 participants from 12 nations.

To reach Tihany, we first travelled together with Ursula Westphal, Karl Scherer and Hans Johnen in my Peugeot to the ICM at Nice in 1970, where we heard a series of lectures including those of L. Hoermander, T. Kato, J. Peetre and E. M. Stein. There I met again the renowned Hungarian mathematician Pál Turán (1910–1976), who had lectured in Mainz and with whom I spent three amiable and informative days during the time of the Russian assault in Budapest (1956). He was now the paymaster of the large Hungarian delegation. I also met the Russian mathematician who had been commissioned to write reports on the daily activities of the members of the Russian delegation.

Then on to Tihany via southern Austria where I talked on “The Cayley transform and semigroup operators”, a paper written together with U. Westphal [14]. Two events there surprised me. When Béla Sz.-Nagy spoke with a group of Russians concerning a translation of certain matters from Hungarian into Russian, his students expressed regret for not understanding the Russian language. At a festival dinner, my request for the orchestra to play the balalaika melody (Jarre) from Dr. Zhivago (1965) was taken up most enthusiastically (Pasternak’s book was forbidden reading in Eastern Europe). After the Tihany conference Karl Scherer [5] and I spent three days in Budapest.

Ten years later the conference “Functions, Series, Operators” took place (August 1980) in Budapest, conducted by Béla Sz.-Nagy and J. Szabados. It was the centenary of the birth of the renowned mathematicians L. Fejér and F. Riesz. Its proceedings, with 105 papers, were remarkable. After my lecture Edwin Hewitt got up and expressed great words for our approach to “Legendre transform methods in the solution of basic problems in algebraic approximation” [6].

There were many witnesses to the long skirmishes between the proud Hungarian mathematicians and their Russian colleagues. At a reception it was a pleasure to meet Serge Mikhalovich Nikol’skii again, who died in 2012 at the age of 107 in Moscow. I had first met him at the ICM in Amsterdam in 1954.

The Kecskemét Conference on Approximation of August 1990 provided the opportunity for my first visit to Szeged, the home of Béla, and to its University, the chairman then being Ferenc Móricz. He once lectured in Aachen and has my respect for his clear religious and political (anti-communist) points of view. At a party in the garden house of Vilmos Totik, whom I had invited to present a top lecture at Oberwolfach in 1980, it was a pleasure to meet many colleagues.

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[5] In his paper [16] J. M. Almira, U. Murcia, writes: “Approximation spaces […] were introduced by Butzer and Scherer in 1968 and, independently by Y. Brudnyi and N. Kruglyak in 1978, and popularized by Pietsch in his seminal paper of 1981.” See [17] and Pietsch’s illuminating book [18].

[6] This surprised us since a forerunner of this approach had been turned down by a British journal; at the time, Cooper expressed his doubt with its chief-editor whether Britain had at the time any referee who could judge its mathematical contents. The lecture, based on joint papers with R. Stens and M. Wehrens, appeared in the conference proceedings [19]; see also [20–22].
4 Poland

Our connections with Poland were primarily initiated by Rolf Nessel and Ernst Görlich. Władysław Orlicz, during his many trips to German universities, often stopped off at Aachen to lecture; the last time with his former student Wanda Matuszewska with whom he worked for almost 40 years. In fact this eminent Polish mathematician was one of the first to visit our research group and lecture. This was soon complemented with the visits of Julian and Helen Musielak to Aachen. Rolf Nessel, Ernst Görlich but also K. Scherer and E. L. Stark participated in the Approximation Theory Conference held August 1972 at Poznań; the first two also took part in the semester on Approximation Theory held in Fall 1975 at Warsaw, as well as at the conference on “Approximation and Function Spaces” held at Gdansk (Poland) in August 1979 [23].

At this occasion Rolf Nessel met Andi Kivinukk, Tallinn, and sent him a lot of preprints from our research group at Aachen, including doctoral thesis by Rudolf Stens. The contacts with Andi continued to this very day; see Sect. 10.

During my own attendance in the conference on “Function Spaces V”, conducted by H. Hudzik, Julian Musielak et al. at Poznań in 1999 [24], and in the cozy home of Julian, I learned to know Polish experts in the field as well as a number from Prague and other parts of Eastern Europe, and the West. I well recall the lecture by the distinguished analyst Roman Taberski (1927–1999) whom Rolf Nessel and I had cited in our Birkhäuser-Academic Press book of 1971.

5 Russia and Ukraine

Eberhard Stark[^8] took part in the International Conference on the Theory of Approximation of Functions held 1975 in Kaluga (150 km south-west of Moscow) in honour of P. P. Korovkin [25, 26] as well as in the “International Conference on Approximation Theory” held in Kiev in 1983 together with Rudolf Stens, Hubert Berens (1936–2015), and Dieter Gaier (1928–2002), the great German complex approximation theorist, who had spent a 1953–55 year with Walsh at Harvard[^9]. Kiev was the home base of V. K. Dzjadyk, N. P. Korneičuk and other expert Ukrainian approximation theorists.

[^7]: Unfortunately, I could not attend as I had to travel to London, to the funeral ceremony of Lionel Cooper; see P.L. Butzer: Lionel Bakst Cooper—in memoriam. In Functional Analysis and Approximation, P.L. Butzer, B. Sz.-Nagy, E. Görlich, eds., pp. 19–23. Birkhäuser, Basel, 1981.

[^8]: Stark, who had taken a 2-year course in (scientific) Russian as a student, was our expert in the Russian literature on approximation. Our Russian friends told us he was often better informed about approximation than they were.

[^9]: For obituary papers on Berens and Gaier see [27, 28].
6 German Democratic Republic (GDR)

In 1982 Rolf Nessel and I were invited by Hans Triebel\(^{10}\) to the conference “Recent Trends in Mathematics”, held at Reinhardsbrunn, October 11–13, about 65 km southwest of Weimar. It was organized by the Teubner Verlagsgesellschaft, in collaboration with the editors of “Teubner-Texte zur Mathematik”; see [30]. It was my first and only visit to the German Democratic Republic.

On my train ride to Reinhardsbrunn, I met Rolf Nessel, who was coming back from Szeged (Hungary) via Bebra, Western Germany, the largest border checkpoint for rail transportation between Western German and the GDR.

The conference was attended by 62 mathematicians mostly from European countries: Austria, Canada, Czechoslovakia, Finland, France, Eastern and Western Germany, Great Britain, Italy, Japan, Poland, Romania, Sweden, USA, and USSR. Among the participants there were A. Göppert (Merseburg), H.G. Feichtinger (Vienna), W. Trebels and W. Wendland (both Darmstadt), R. J. Nessel (Aachen), H. Wallin (Umeå), and S. B. Stečkin (Moscow).

The hotel rooms, lecture halls, and dining rooms, all were situated in the former Reinhardsbrunn Castle, then a showpiece resort of the government of East Germany.

Rolf Nessel presented a survey lecture on “Quantitative Banach-Steinhaus theorems”. It was followed by a lively discussion with S. B. Stečkin.

After the last lecture on the final day of the conference, some 15 participants including myself went by train to Weimar, led by two representatives of the Teubner Verlag, where we were invited to dinner at the Rathaus restaurant. Arriving there, it apparently was overcrowded, and the waiters suggested to try the Weimarer Halle where we were served a very simple meal, the best they could offer us. There were several signs that the city was suffering under great foot-shortage.

Whereas Rolf Nessel followed an invitation of Hans Triebel to Jena, I decided to stay in Weimar two more days. Under the regencies of the Duchess Anna Amalia (1739–1807) and her son Carl August (1757–1828) the city had become an important cultural centre of Europe, having been home to such luminaries as Goethe, Schiller, Herder, and Wieland. I was, in particular, interested in a visit to the famous Duchess Anna Amalia Library, which today belongs to the UNESCO World Heritage. Even though it was officially closed that day, I managed to get a personal tour by its director.

The Amalia Library has been one of the best-known libraries in Germany since the 18th century. It owes its fame to its princely patrons, Wilhelm Ernst, Anna Amalia and Carl August, as well as its librarians, among whom Johann Wolfgang von Goethe was the most important. In 2004 the historical building, was damaged by fire and water, and the third floor and attic were completely destroyed. Large parts of the valuable collection were maintained, among them the largest Faust collection in the world and

\(^{10}\) Hans Triebel was one of the first mathematicians who cited in his book “Interpolation Theory, Function Spaces, Differential Operators” of 1978 [29] in its bibliography (covering the field up to 1973/74) the early work of seven members of the young research team at our Chair. These are H. Berens, H. W. Hövel, H. Johnen, R. J. Nessel, K. Scherer, W. Trebels and U. Westphal.

As far as I am aware, Hans Triebel could make research visits to many parts of the world but not to the Federal Republic of Germany during GDR times. Once he informed me he was in Eupen (just 18 km south-west of Aachen) and suggested we could possibly meet there. Most unfortunately, I could not take this unique opportunity at the time.
Nietzsche’s private library. Of the 50,000 burned volumes, about three quarters are estimated to be replaceable in the long term.

Today the library has approximately 1.1 million books, 2,600 bound manuscripts, 8,600 maps and 29 globes. Among its special collections is an important Shakespeare collection of approximately 10,000 volumes, as well as a 16th-century Bible connected to Martin Luther.

A very informative city guide, who surprised me with her very open critique of the regime, showed us the Gartenhaus (garden residence), where Goethe lived for several years after 1786. Here he met Christiane Vulpius (1765–1816), whom he married in 1806.

7 Yugoslavia

The third conference on “Numerical Methods and Approximation Theory” was held in Niš, former Yugoslavia, at the Faculty of Electronic Engineering, University of Niš, August 18–21, 1987. It was perfectly organized by Prof. Gradimir Milovanović and his team and attended by 140 participants from 20 countries, among them 18 from the former USSR.

Invited papers were presented by Paul Butzer (Aachen), Luigi Gatteschi (Torino), Walter Gautschi (Purdue U.) and W. Schempp (Siegen). In addition, there were 81 contributed papers, including those by Aram H. Arakelian (Yerevan), Giuseppe Mastroianni (Potenza), Biancamaria della Vecchia (Napoli), Franz-Jürgen Delvos (Siegen), Ljubiša Kocić (Niš), Gradimir Milovanović (Niš), Tibor Pogány (Bor).

Tibor Pogány, whom I met there for the first time, recalls that both of us arrived a day earlier and took part in an excellently organized walking tour in the old town of Niš to see the fortification, arsenal and the Nišava river, all before the welcome party in the Electronic Engineering faculty (highlighted by a tasty barbecue).

There was an excursion to Justiniana Prima, an archeological site of exceptional importance originating from Byzantine times. It is located about 60 km from Niš, near the small town Lebane, in southern Serbia. Justiniana Prima was established in 535 by the Byzantine Emperor Justinian 1 and lasted until 615. The city served as the seat of the Archbishop of Eastern Illyricum, who had jurisdiction over the Central Balkans. There were 10 basilicas in the city, including a large church with three apses, atrium, and baptismal.

The local citizens abandoned the city after frequent attacks by Avara and due to the arrival of Slavs who started settling in this area.

During a break between the sessions, Radomir Stanković drove me in a small Serbian car through the Sičeva (Sičevačka) Gorge, about 17 km long with 17 tunnels along the river Nišava and the rail road. This is a part of the famous Via Militaris from Roman times and is still the main road from Western and North Europe to Bulgaria and Turkey. In the gorge, there are two orthodox monasteries and a small dam for a still working hydroelectric power station.

After the conference I drove a rented car from Niš to Dubrovnik, an extraordinary experience. In this respect my second trip to Kupari-Dubrovnik in 1989 (below) was probably much simpler.
Immediately after the last lecture on that Friday afternoon, Radomir kindly accompanied me to the only automobile rental agency in Niš. Renting a car in Niš was not so common in 1987. For the shortest possible route by car—through western Serbia, and then the towering mountains through Bosnia to Dubrovnik (a great part being gravel)—the car rental agency recommended their red-coloured Volkswagen Jetta. That Friday night, after travelling five hours, I slept at a roadside hotel.

Radomir also had made a hotel reservation at a modern hotel in Tjentište, where I arrived Saturday night, unfortunately after their dinner.

That Sunday afternoon I stopped over for a mocha in a small inn at a river crossing. The pipe-smoking elders sitting outside, who perhaps thought that I was either an oddly dressed Serb or a tourist, seemed surprised that I dared to enter and drink there, it being a muslim district and restaurant.

The only traffic occurred the first 30 or so kilometers from Niš. Thereafter I encountered few cars or trucks.

This tour turned out to be the most adventures trip I ever made, and risky indeed. When I returned my Jetta to the car rental agency in Dubrovnik they were surprised that I managed to make this tough trip all alone, a distance of almost 800 km and that the car had no scratches in spite of the gravel roads.

At the luxury hotel Libertas, now called the Rixos Libertas, where I spent three wonderful days in its indoor and outdoor swimming pools, the receptionist said I was lucky not to have been robbed during my trip, the route being quite isolated. The hotel, which had at least seven floors and crept up the hill from the beach, lies only 20 walking minutes from the old town of Dubrovnik.

The “First international workshop in Gibbs derivatives” was held September 26–28, 1989 at Kupari-Dubrovnik, former Yugoslavia. It was ably conducted by Radomir S. Stanković (Niš) and his team. It was attended by 15 mathematicians and engineers from eight countries, namely Austria, England, F. R. of Germany, Hungary, Japan, USA, USSR and Yugoslavia. The contributors included Franz Pichler, J. Edmund Gibbs, Paul Butzer, William R. Wade, Ferenc Schipp, Jenő Pál, (Toshiyuki Kitaga), (C. W. Onneweer), Yasushi Endow, (Mohammad Maqusi), (He Zelin), (David Mustard), Naum Nisonovich Alizenberg, Radomir Stanković, Claudio Moraga, Milena Stanković, Miomir S. Stanković. The proceedings were edited by Paul Butzer and Radomir Stanković [31].

Experts in Gibbs derivatives who did not attend were Wei-Xing Cheng, Weiyi Su, W. Splettstößer, H. J. Wagner, V. A. Skvortsov, and then S. Cohn-Sftetcu. The proceedings also contain a list of papers in the subject.

A number of the major publications in Gibbs derivatives are reproduced in full in the monograph [32], consisting of two parts, volume 1: Foundations, volume 2: Extensions and Generalisations. The authors and co-authors are Radomir Stanković, Paul Butzer, Ferenc Schipp, William R. Wade, Weiyi Su, Yasushi Endow, Sándor Fridli, Boris I. Golubov, Franz Pichler, Kees (C. W.) Onneweer.

William R. Wade (1943–2016)11 was one of the four leading authors/editors of these two volumes together with R. S. Stanković, P. L. Butzer and F. Schipp.

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11 see “In Memoriam: Bill Wade” https://news.utk.edu/2016/12/21/memoriam-bill-wade/.
Volume I contains four of Wade’s contributions. His “Early history of Walsh functions”, which initiates the two volumes, contains historical details, including those of the American school—an overview that had never been done before; see [32, pp. 1–5]. There follow “Term by term dyadic differentiation of Walsh series” [pp. 347–358] and “Why I got interested in dyadic differentiation” [pp. 359–361], and a joint article with V.A. Skvorcov “Generalization of some results concerning Walsh series and the dyadic derivative” [pp. 364–370].

Wade, who spent a Fulbright Senior Lectureship at Moscow State in 1977, lectured on dyadic differentiation during his semester there because such a course had never been offered there. As a result, he wrote an important paper with Skvorcov. This effort “helped guide his research of his Ph.D students for the next decade”.

Schipp’s contribution jointly with Sándor Friedli “Dyadic derivative, summations, approximation” contains his paper “Über einen Ableitungsbegriff von P. L. Butzer and H.J. Wagner”, Mathematica Balkanica, 4.103 (1974), 541–546. In his own contribution “How I started my research in Walsh and dyadic analysis”, the reader learns of his close professional connection with his older colleague László Pál, who was also active in dyadic analysis. In fact, the Hungarian dyadic group from the Dept. of Numerical Analysis also includes Péter Simon and especially Ferenc Weisz; see his contribution “Hardy spaces in the theory of dyadic derivative” [32, pp. 315–346].

There was a joint visit of all participants to downtown Dubrovnik, a few kilometers from the tourist resort Kupari. We walked across this old town, which was established under the name Ragusa by the Romans and was an independent republic in the middle ages. We had a nice walk over the walls of medieval Dubrovnik and across the main streets.

Returning to Tibor Pogány, whom I met again at the “Symposium on Fourier Analysis, Interpolation and Signal Processing”, conducted by Maurice Dodson at York 1988 (see [1]), it was with him that I wrote three joint papers [33–35] concerning the Omega function I had introduced with S. Flocke and M. Hauss in “Euler functions $E_\alpha(z)$ with complex $\alpha$ and applications” [36].

8 Romania

The 17th International Conference on Operator Theory, conducted by A. Gheonadea, R.N. Gologan and D. Timotin, Bucharest, took place July 23–26, 1998 in Timișoara (Romania) [37]. The local organization was in the hands of Dumitru Gașpar, who had invited Jerry Koliha (Melbourne, Australia) and me. My lecture, which dealt with ergodic theorems for operator semigroups and cosine operator functions, was the last one of the conference on Friday afternoon. In that talk, I used a certain type of an “almost inverse” of the infinitesimal generator of a semigroup (introduced by Butzer, Dickmeis and Westphal in [38, 39]), namely an inverse which is in general a closed but not necessarily bounded linear operator. This operator obeys less restrictive conditions than the conventional Drazin inverse or its extension by Koliha and Tran [40]. If an operator has a Drazin or Koliha-Tran inverse, then it is also invertible in the Aachen sense and the two inverses agree. One reason for its importance is that, at
least for holomorphic semigroups, our inverse acts as the infinitesimal generator of an associated semigroup.

At the end of my talk, Jerry Koliha raised the question whether I was aware of the work of M. P. Drazin and others on those inverse operators. Since I was not, he explained the matter just indicated to the audience in detail. It seemed to leave a good impression upon the top operator specialists in attendance. That evening Găspăr suggested that all participants who had not left for home or Bucharest, dine together, inviting Koliha and me to sit at his table. This was the end of my first very pleasant research visit to Timișoara.

This event was also the onset of a basic joint project between Jerry and me. The publication, which required eight years of intensive and tough work, appeared in the J. of Operator Theory [41], published in Bucharest, the home base of the editors of the conference proceedings. One big problem we had was with a double integral, which does not exist as an absolutely convergent Lebesgue integral but only as an improper one; the order of integration had to be inverted. This integral, which can be viewed as a Kurzweil-Henstock integral, necessitates the use of a non-absolute Fubini’s theorem. The proof, for which we had to consult four colleagues, took almost five pages. Șerban Strătilă, the communicator, accepted the paper with flying flags, praising in particular the abstract Cauchy problem for operator semigroups, as well as the Cauchy problem of second order for cosine operator functions, and, in both cases, their connection with ergodic theorems. The background of this paper was four joint publications with my student Andreas Gessinger, among them [42, 43].

In the course of the project Jerry spent two periods with his spouse in Aachen, both supported by grants from Aachen’s Graduiertenkolleg “Hierarchie und Symmetrie in mathematischen Modellen”. In turn he invited me to spend a month working with him in Melbourne. As so often in the past I did not accept, which I now do regret.

In the fall of 2000, Prof. Dumitru Găspăr, who had spent a semester in the mid 70s working with our research team in Aachen, appeared unexpectedly at the Chair and asked me, in his capacity as Rector of the West University of Timișoara (founded 1962), whether I would accept an honorary doctorate from his university. It would indeed be a great honour for me, I replied enthusiastically.

In spring 2001, I flew to Timișoara, just 60 km south of Szeged, with notes based upon my two courses in history of mathematics at Aachen in the 80s, specifically, that part dealing with Roman and Carolingian times. I had learned that the Romanian language descends from the Latin dialect of “Dacia Traiana”, Dacia referring to ancient Dacia and Traiana to Trajan, the Roman emperor from AD 98 to 117. He was well-known for Trajan’s arch bridge across the Danube, the greatest achievement of Roman architecture, built in AD 105, just before his second Dacian war of 105–106.

Since I knew that Dumitru was aware of my historical interests, I asked him whether it would be possible to visit Trajan’s bridge and the capital of ancient Dacia, Sarmizegetusa Regia. Well, overnight he bought a 7-people motor van which took six of us on a tour of ca. 600 km through Romania, including Trajan’s bridge at Drobeta. A female professor of archaeology, who accompanied us, explained its construction based on her seven years of excavations there. The ancient Dacian capital—the widely spread huge stone ruins are an amazing sight—was razed by Trajan in AD 106 in reprisal of King Decebalus’ continuous breaking of the truce established with the Romans after
his defeat of two Roman armies in AD 85 and 88. This tour through an ever-changing physiographical historical-archaeological area, was one of the most informative ones I ever experienced.

At the solemn and dignified celebration, with university dignitaries wearing their festival robes, Gašpar wore a beautiful purple, velvet robe. I talked about some of my historical interests, well knowing that a specialist lecture in mathematics is unsuitable for a general audience.

A get-together with several local historians and archaeologists, where we discussed topics of common interest and which was kindly organized by Gašpar, was very informative for me.

It may be of interest that Ciprian Foias (1933–2020), a co-author and friend of Béla Sz.-Nagy, received an honorary doctorate in 2000, and Israel Gohberg (1928–2009) in 2002.

My deep thanks are due to Dumitru Gašpar for the festive tribute and making my visit an unforgettable one.

9 Latvia

SampTA (Sampling Theory and Applications) is a biennial interdisciplinary conference for mathematicians, engineers, and applied scientists. The main purpose of SampTA is to exchange recent advances in sampling theory and to explore new trends and directions in the related areas of application.

SampTA workshops originated from the “1995 Workshop on Sampling Theory and Applications” (SampTA’95), conducted by I. Bilinskis (Latvia), program chair G. D. Cain, F. Marvasti (both UK) and PLB, in Jurmala, Latvia in September 19–22, 1995.

The workshop began with a Special Session on European Cooperation in Science, Technology and Education at the Institute of Electronics and Computer Science, Riga, Latvia, on September 19.

The scientific sessions of the workshop were held at the Conference Centre of the Latvian Academy of Sciences located in Jurmala, a beautiful seaside resort about 20 km from Riga.

The keynote speakers were H. G. Feichtinger, A. Zayed, I. Bilinskis and F. Marvasti and PLB. Further speakers included T. Strohmer, Yu. Lyubarskii, M. Zwaan, J. Voss, D. Mugler, P. J. S. G. Ferreira, M. H. Annaby, G. Schmeiser, G. Nasri-Roudsari, J. Lippus, A. Kivinukk, G. D. Cain, B. Lacaze, and J. M. N. Vieira. Tutorials were held by A. J. Jerri and J. R. Higgins; see [44].

Walking along the nearby unique sandy beach of the huge Gulf of Riga, several participants discussed with Abdul Jerri the establishment of a journal dedicated to the broad area of sampling theory. Its first issue appeared in 2002.

Riga with 630,000 inhabitants, the pearl of the three Baltic states, was founded in 1201 by Bishop Albert. It is a historical and cultural landmark with many museums and concert halls, noted for its medieval old town, the Art Nouveau/Jugendstil buildings and the 19th century wooden architecture. The Riga Market, one of the oldest in
Europe, was fascinating to us visitors. The Bernstein chains seemed cheaper back home.

During our tour of Riga, the well-informed city guide mentioned that the many double length white Mercedes cars we saw quite often, belong to the Caucasian mafia, an unpleasant situation for the whole country. But for us visitors everything seemed to run smoothly in the city.

10 Estonia

A further unforgettable visit of mine was to Tallinn, the Capital of Estonia, on September 13–20, 2004, the chief reason being that I was chosen as the external referee and an examiner of the doctoral thesis of Gert Tamberg, an excellent student of Andi Kivinukk12.

My first contacts with Andi date back to 1993, when he gave a seminar talk at Aachen, followed by the first SampTA meeting at Riga/Jurmala, Latvia, in 1995 as well as at many meetings thereafter. I have followed up his excellent research work since then. He was instrumental in building up his institution into a full-fledged university in the past years.

The day after my arrival, Andi picked me up from the comfortable Central Hotel, fifteen walking minutes from Tallinn’s lively harbour, and we drove to Tallinn University of Technology, to which Gert is attached, and where the public, oral defense of the thesis took place. As was to be expected, Gert was awarded his doctoral degree with the highest grade. The local examiner was the analyst Juri Lippus.

The discussions at the joint lunch with the friendly colleagues of this university, especially Lippus, gave me a first overall view not only of Estonia’s university system but also of its role as a new independent state—a nation of only 1.3 million, which can be proud of its great cultural and scientific achievements. The optimistic look especially of the younger citizens one passes while visiting the walled Old Town, the Gothic Town Hall and the St. Nicholas Church, was simply contagious.

On September 15–16, we left very early for Tartu (Dorpat), c. 200 km south-east, Estonia’s second largest city and its intellectual center, known for its prestigious, 17th century University of Tartu with its Museum, exhibiting the history of Medieval Livonia, since 1237 ruled by the Teutonic Knights, with their Ordensburg of 1250, together with the follow-up periods, and gives as well an overview of the history of science and university education from the 17th century onwards, the rich legacy of the university being astronomy-mathematics, physics-chemistry, medicine, and art collections.

After my lecture “Recent results in Shannon sampling theory” at their Mathematical Department, said to have a good reputation in Estonia, Andi and I had a lively discussion with their mathematicians. After visiting the historical city (city rights before

12 After receiving our paper [1], Andi Kivinukk informed us that his supervisor Prof. Gunnar Kangro (1913–1975) gave him a hand-written copy of [45], written by Paul Otto Runck (1930–2013), which appeared in the proceedings of the first conference I conducted at Oberwolfach in August 1963 [46]. It was the starting point of Kivinukk’s Ph.D. thesis. I was the 2nd advisor of Runck’s doctoral thesis under Prof. H. Grunsky at U. Mainz in 1959; see [1, Section 3].
1262) and the university library, the Tartu Old Observatory of 1810 (Friedrich Georg Wilhelm von Struwe, born 1793 in Altona, entered Tartu university in 1808, studied astronomy there, was professor there and director of the observatory from 1813 to 1840, when he left and founded the new Pulkovo Observatory in St. Petersburg), and a first-class dinner, we stayed overnight in a small guesthouse, and travelled the next morning via the village Janeda (Jendel) back to Tallinn.

Our main sight was Janeda manor, founded as an estate by the Baltic German nobility before 1510, it belonging to different generations of such aristocratic families until c. 1910. It is now a museum. It is of interest that there are dozens of such historic manors in Estonia.

The many sites Andi so kindly showed me included Katharinenthal (Kadriorg), where Chebyshev and other aristocratic families spent their vacations, the Keila Waterfalls in an untouched landscape, and nearby Laulasmaa Spa, a hotel and conference center, situated on the Baltic sea, c. 35 km west of Tallinn.

Since Estonia as a whole impressed me so much, I suggested to Andi that Estonia would be an excellent choice for a possible SampTA Workshop. It did actually take place in July 2017.

Next came Paldiski, 45 km west of Tallinn, a closed city until 1994, since its port, below the coastal cliffs, had been a Soviet Navy submarine center. The majority of the residents were ethnic Russians, who decided to remain in Estonia after it became independent, an interesting visit for me as they apparently managed to live in harmony with the native Estonian residents.

Andi also showed me a unique area not too far east of Tallinn, quite isolated, in its natural beauty, along the seashore, with dozens of white swans swimming in the open sea—most unusual. It recalled to me the great time I, together with my family, spent in the isolated Laurentian Mountains north of Montreal in 1942 to 1955.

On September 20th I returned back to Aachen, it being one of the most pleasant trips I have ever made. My deep thanks are due to my dear friend Andi.

11 Tenerife

Although Tenerife, the largest of the Canary Islands, is a part of Spain, I decided to add it to this paper and not to [1]. The reason is that my visits to Tenerife were also connected with research visits to Belarus. In fact, I wrote six papers with colleagues from Tenerife, all co-authored with Anatoly Kilbas from Belarus, who spent common time with me on the island.

Attempts to annex the island to the Crown of Castille date back at least to 1464, but it was achieved only in 1496 due to the fierce resistance of the indigenous people, the Guanches. Tenerife was the last island of Canaries to be conquered by the Castilian troops. Berbers, known as Guanches, first recorded in the isolated northern part of Tenerife, began to settle on the island ca. 200 BC.

The aim of my first visit to Tenerife was to take holidays at a hotel on the seashore of Puerto de la Cruz for nine days in May 1999. Since I had been in mathematical contact with Juan Trujillo from the University of La Laguna (founded 1792), I promised him I would present a short lecture while there. Well, I did on the first morning I was there,
drove with my rented car to the mathematical department of the university and found a further visitor to the island who listened to my talk, namely Anatoly Kilbas from Belarus, a proven expert in fractional calculus, the topic of my lecture. Therefore, he was a truly welcome member of the audience whom I had not met as yet.

My lecture dealt with a research field which I had brought up already in 1967, namely differences, derivatives and integrals, all of fractional orders, exemplified by the papers [12, 50], and [13], which in turn are based on the work of H. Berens, U. Westhal and myself on fractional powers of infinitesimal generators of semigroups [51].

At the beginning of my lecture I introduced a derivative of fractional order via a difference operator of fractional order, which is in general defined as an infinite series. At this point Juan protested heavily “That is impossible!”, whereas Anatoly refuted Juan’s arguments and became enthusiastic about what I was trying to explain.

Then I presented a new approach to fractional integration and differentiation by Mellin transform methods, a field I had opened up with Stefan Jansche during 1996–1999 [52, 53], and I am still working in; see [54, 55]. At this point Juan asked me whether I would be willing to give two further lectures and he in turn would show Anatoly and me the highlights of the Island in between the lecturing days and his own teaching duties. This of course implied that my holidays would be reduced drastically, but nevertheless I agreed, a decision applauded by Anatoly.

This was the beginning of three years of intensive work together with Juan and Anatoly. We met again for nine days in Tenerife in early spring of 1992 upon an invitation to participate in the celebrations surrounding the award of “Gran Cruz de la Orden Islas Canarias” to Professor Nácere Hayek (1922–2012), founder of the mathematical faculty of the U. La Laguna.

The final goal of my research at Tenerife, sketched by Butzer-Jansche at an International Workshop of August 1996 in Varna (see [56]), is the establishment of the fundamental theorem of the differential and integral calculus in the Mellin transform setting. For this purpose associated operators of differentiation and integration (anti-differentiation) must be introduced. The operator of first order (Mellin) derivative is given by \( \left( x \frac{d}{dx} \right) f(x) \) (in case the Mellin transform is considered in its simplest form) and the corresponding anti-derivative is \( \int_0^x u^{-1} f(u)du \).

In fact, the composition of these two is \( f(x) \) in both cases. Mellin differentiation of integral order is handled in [56]. Alternatively, the derivative of order one

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13 Anatoly Alexandrovich Kilbas was born 1948 in Minsk, Belarus. He studied at the Belarusian State University from 1966 till 1971, defended his PhD thesis in 1975, his advisor being the Academician Fedor Dmitrievich Gakhov (1906–1980), and received a second degree (Dr. of Science) in 1995. In 1997 he was awarded the professorship at the Chair of the Theory of Functions, and since 2002 he headed this chair. Anatoly died on June 28, 2010 at a conference in the Caucasus, where he drowned in a mountain river when he returned alone to his hotel at night.

He was the author ca. 300 research articles and six well-known monographs, among them [47, 48]. He organized the conferences AMADE 1999, 2001, 2003, 2006 and 2009 in Minsk.

See also: V. Kiryakova: “Professor Anatoly Kilbas: To the memory of the great scientist and the faithful friend”, FCAA, vol. 13, no. 2 (2010), pp. 221–223; http://www.math.bas.bg/~fcaa/volume13/fcaa132/Kilbas_Obituary.pdf and S. Samko: “A. Kilbas’s 60th birthday: Congratulations”, FCAA, vol. 11, no. 4 (2008), pp. 370–372; http://www.math.bas.bg/~fcaa/volume11/fcaa114/Kilbas60_by%20Samko.pdf.

14 For the history of fractional calculus see especially [10, 49].
can be defined via a difference quotient involving a Mellin translation operator and those of higher (integral) orders via differences of the corresponding orders. In the situation of fractional orders of differentiation and integration the matter was finally achieved by Bardaro, Butzer, and Mantellini in [54]. The six joint papers with Anatoly and Juan, one of them together with Juan’s student Rodríguez-Germá, [57–62], which were published between 2002 and 2007, laid the necessary foundations.

12 Belarus

The Belarusian State University organized in September 13–19, 2006 the 4th International Conference “Analytical Method of Analysis and Differential Equations” (AMADE-2006). It was conducted by my friend Anatoly Kilbas (1948–2010)\(^{15}\), under the guidance of ISAAC, and held in the spacious grounds, some 40 km east of Minsk, the site of the Summer Olympics 1980 in Moscow. Some of the participants in the Olympic football tournament resided there, the stadium at Minsk being one of the venues of the Olympic football tournament.

There were 28 plenary invited lectures and 170 sectional talks, with a total of 396 mathematicians from 20 countries. It was a true international event, at least ten of the invited speakers coming from outside of Eastern Europe. The conference proceedings were published in the journal “Fractional Calculus and Applied Analysis”.

The accommodations were spread across various buildings in the park complex, my friend Hans-Jürgen Glaeske (who died too early on February 4, 2021) and I having single rooms with a joint shower (a seeming exception thanks to our friend Anatoly), and swimming (with showers) being possible in the nearby inviting lake. It was my impression that the whole district suffered under the Chernobyl disaster of 1986, there being hardly any birds or wildlife visible.

There were several enjoyable excursions, partly under the guidance of the 23 year old, well-educated daughter of Anatoly Kilbas. A major one was to the historical city Polazk, situated 225 km north of Minsk. It was the home of a Jesuit College/Academy from 1580 to 1820, even served as the capital of the Jesuit community from 1774/1783 to 1820, to which Jesuits from all over the world poured. At the time, it became the intellectual center of Europe and the patron of the college in St. Petersburg, the mission to Saratow, its historical heritage also being Polazk State University, even with the restored buildings of the academy of 2005. The main attraction of our trip, however, was the Cathedral of Saint Sophia (1044–1066), reconstructed with Baroque refinements in mid-18th century, a cultural heritage of Belarus.

\(^{15}\) See [www.math.bas.bg/~fcaa/volume13/fcaa132/Kilbas_Obituary.pdf](http://www.math.bas.bg/~fcaa/volume13/fcaa132/Kilbas_Obituary.pdf).
13 Epilogue

13.1 Topics of research

Following our foundational work in approximation theory, Fourier analysis and functional analysis, the Banach-Steinhaus theorem equipped with rates (1958–1973, recall [63, 64]), and the fundamental theorem of best approximation (1969, see [65]), our research focus turned to the following:

(i) Walsh functions and dyadic Walsh analysis, beginning in 1970, with communication scientists in the USA. My collaborators in our first step into the outside world included the initiator H. J. Wagner, W. Splettstößer, W. Engels, U. Wipperfürth and W. Ziegler. This work was then continued by R. S. Stanković (Niš), first with a Workshop in Kupari–Dubrovnik in 1989 (see Sect. 7), and thereafter by the 2-volume Monograph authored by R. S. Stanković, P. L. Butzer, F. Schipp, and W. R. Wade, in collaboration with Weiyi Su, Y. Endow, S. Fridli, B. I. Golubov, F. Pichler and C. W. Onneweer [32].

(ii) Mean ergodic theorems were firstly considered for discrete powers of operators and for semigroups of operators on Banach spaces, jointly with U. Westphal in 1971 [39, 66], and W. Dickmeis in 1981 [38]. Thirty years later there followed the mean ergodic theorem for Cesàro and Abel limits of semigroups and cosine operator functions in a series of articles with Andreas Gessinger between 1994 and 1997, e.g. [42, 43, 68]. All theorems were equipped with best possible rates of convergence. Connections with well-posed abstract Cauchy problems were emphasised.

(iii) General limit theorems of probability theory for sequences of real independent (not necessarily identically distributed) random variables, thus the central limit theorem as well as the weak law of large numbers starting in 1975. It was L. Hahn with whom I wrote eleven papers (1975–1983), some in conjunction with U. Westphal, M.-Th. Roeckerath, W. Dickmeis and R. J. Nessel; see, e.g., [69–71]. This work was continued in 1983 with the random martingale central limit theorem and weak law of large numbers, and the Lindeberg-Trotter operator approach by D. Schulz with eight joint papers between 1983 and 1987, e.g. [72, 73].

Asymptotic expansions for central limit theorems for general linear stochastic processes, as defined by T. Kawata in 1972, were studied in two joint papers with U. Gather [74, 75]. For a joint paper with E. van Wickeren and H. Kirschfink on weak convergence of probability measures with rates see [76]. Sampling series approximation of weak sense stationary processes were considered by W. Splettstößer in [77], and in the review paper [78] jointly with Rudolf and myself.

(iv) Signal analysis and prediction theory. Cooperation in a supra-regional research project, supported by the DFG, with ca. 40 German electrical engineers, doctors of medicine, seismologists in earthquake research and propagation of elastic waves, in matters signal processing, sampling theory, and prediction theory. It was initiated by joint work with W. Splettstößer in 1977 with six papers, one being together with W. Engels, another with R. Stens. It was followed up by a series of papers with R. Stens, G. Hinsen, and then with A. Zayed (who spent 9 months in
Aachen in 1989/1990), A. Fischer, and G. Nasri-Roudsari, née Schöttler, three papers jointly with J. Lei (who spent five months in Aachen in 1998/99).

(v) Combinatorial analysis, central factorial numbers, analytic number theory, beginning in 1989 with a joint paper with M. Hauss and M. Schmidt in factorial functions and Stirling numbers of fractional orders, followed by seven further papers with Hauss on Eulerian numbers of fractional orders, integral and rapidly converging series representations of Dirichlet’s L-functions and the Riemann zeta function, and together with M. Leclerc on Bernoulli numbers and polynomials [79], and S. Flocke on Euler functions [36]. This work demonstrated “team work” of the highest order. Already in the early 1980s, central factorial numbers were used at our chair by E. L. Stark for the representation of trigonometric moments, especially in the doctoral and diploma theses of his students Wieland Richter, Verena Bebronne and Lothar Vogt.

The Butzer–Flocke–Hauss Omega function of [36] is associated with the complex-index Bernoulli and Euler functions. In a joint work with T. Pogány and H. Srivastava we derived a linear ordinary differential equation whose particular solution is this Omega function [33]. Pogány et al. [80] use this differential equation as a main tool in order to establish two sided bounds for the Omega function. This function is also one basis for a novel approach to Hilbert-Eisenstein series, introduced by M. Hauss in 1995. This approach was carried out in [35].

(vi) Mellin analysis and Mellin transform theory, launched in four papers with S. Jansche (now SAP representative in Tokyo) in 1997–2000. It is an area of research which interested Gerhard Schmeisser (Erlangen), and which together with Carlo Bardaro and Ilaria Mantellini has grown since ca. 2010 into a new research area, with many basic papers; see, e.g., [54, 81–85]

(vii) Polar analytic functions. This concept, first introduced together with C. Bardaro, I. Mantellini, and G. Schmeisser, (see [55, 86]), a simple alternative to functions that are analytic on a part of the Riemann surface of the logarithm, turned out to be of interest in the broad realm of Mellin analysis and quadrature formulae on the positive real axis. It has caught the attention of many readers on ResearchGate.

The leitmotiv of our research is the rate of approximation of given functions by certain approximation processes, and often it concerns the problem: for what functions is the best possible rate or order of approximation achieved.

A survey of the results of our students in the broad realm of probability theory, including Lothar Hahn, Dietmar Schulz, Marie–Theres Roeckerath–Ries and Heribert Kirschfink has never been written. This was especially pointed out by T. Pogány in his appraisal [87] of our work in sampling theorems for stochastic signals.

(viii) History of Mathematics. One of my hobbies has been history, in particular, history of mathematics and medieval history. In this respect I wrote in [2] that Vol. IV of de La Vallee Poussin’s, Collected Works was in preparation. Now, thirteen years after the publication of Vol. III of the collected works in 2004 [88], there finally appeared Vol. IV, in 2017 [89].

The international and interdisciplinary “Colloquium Carolus Magnus: 1200 years of civilization and science in Europe”, was held in Aachen, March 19–26, 1995.
proceedings appeared in two volumes: Vol. I: “Scholarship, Worldview and Understanding” [90], and Vol. II: “Mathematical Arts” [91]. The editors included P. L. Butzer, H. Holländer, H. Th. Jongen, M. Kerner, D. Lohrmann, and W. Oberschelp.

With my brother Karl W. Butzer (1934–2016), Centennial Professor of Liberal Arts, Department of Geography, University of Texas, whose eight fields of research included history (see [92]), I wrote two papers. The first deals with mathematics at the court of Emperor Charlemagne and its transmission [93], and the second with the transmission of scholarship and ideas from East to West in the time of the crusades [94]. The concept of the latter joint paper was followed up in part and deepened in a paper by Dietrich Lohrmann, which treated the role of Antiochia during the introduction of Arabic learning in Western Europe in the 11th–12th centuries; see [95].

13.2 Students in mathematics

During my time, the “Lehrstuhl A für Mathematik” was most fortunate that so many of the students who took part in its 4-semester course in Analysis, and in the 2-semester courses in Functional Analysis, Fourier Analysis and Approximation Theory, were first rate.

Since the chair did not have to teach the many mathematical courses for engineers of all disciplines, it had more time to devote its energy to students whose primary interest was pure and applied mathematics, physics, probability theory and statistics as well as communication theory.

In my Retrospective of 2009 [2] I wrote “Many of my best papers were written together with students of mine, with whom I worked individually and intensively during the preparations of their master’s theses and also their doctoral dissertations, implying a 5-year period of working together on average”. In this process, dealing with the students as equals, not as subordinates, as part of a team that brings together different skills, fresh stamina, flashes of inspiration, constitutes the core of new, successful research results. See also [96] in this respect.

The mathematical topics selected were of interest to my students and to me, avoiding generalizations of generalizations of well-known theorems; we turned instead to avenues and results that seemed not to have been studied, beginning with the simplest form, the essence. These were often suggested by concrete examples we had encountered, or by applications; and since 1970, in interchange with engineers, through in cross-disciplinary research (see below).

Throughout my 35 teaching years at Aachen (September 1958 to August 1993), of the total student population in the mathematical area, the number of women studying mathematics usually ranged from 30 to 35%. My colleagues Rolf Nessel, Ernst Görlich and I aspired to preserve this proportion amongst our junior and senior assistants and collaborators.

So it is not surprising that a large number of outstanding female students graduated from the Lehrstuhl A für Mathematik. Among these there were six female mathematicians, who received at least one degree under my supervision and became professors
at universities afterwards, namely, Ursula Schmidt-Westphal\textsuperscript{16}, Franziska Fehér\textsuperscript{17}, Ursula Gather\textsuperscript{18}, Marie-Theres Roeckerath-Ries\textsuperscript{19}, Ursula Scheben\textsuperscript{20}, and Andrea C. Schalley\textsuperscript{21}.

Of the many excellent female students I would like to mention three more, with each of whom I have published a number of joint papers (see below), namely Dr. Adelheid Fischer (now with SAP, Walldorf), Dr. Gabriele Nasri-Roudsari, née Schöttler (now with Deutsche Bank, Frankfurt), and Dr. phil. Kerstin Springsfeld\textsuperscript{22}

Of course there were also a large number of outstanding male students. In our article [1] we listed nine of our male students who became professors at German universities during the period 1958 to 1970. In the succeeding period there were the following nine: Franz Ebersoldt\textsuperscript{23}, Hermann Schulte\textsuperscript{24}, Rudolf Stens\textsuperscript{25}, Wolfgang

\begin{itemize}
\item \textsuperscript{16} U. Schmidt-Westphal, nee Westphal, received her diploma and doctorate (1969) degrees, as well as her Habilitation degree (1974) under my direction. After teaching at universities of Vechta and Erlangen, she was finally Professor for Analysis at the Leibniz University, Hannover. She was one of my best collaborators, and wrote several innovative joint papers with me.
\item \textsuperscript{17} F. Fehér, who obtained her Diploma at U. Köln, received her Dr. rer. nat. degree under my supervision, became professor at the FH Dortmund, taught there and at the RWTH-Aachen. The abbreviation FH means “Fachhochschule”, in English “University of Applied Sciences”. She edited with me the volume on E. B. Christoffel’s work [97].
\item \textsuperscript{18} U. Gather wrote her diploma thesis under my supervision in 1976, which resulted in two joint publications with me [74, 75], the first two on her list of publications http://www.statistik.tu-dortmund.de/gather-pub.html. Thereafter she was a research assistant at the “Institut für Statistik und Wirtschaftsmathematik” at Aachen, where she received her doctorate (1979) and Habilitation degrees (1984). She became full professor at TU Dortmund in 1986, and has been rector there since 2008. She was elected chairwoman of the Board of Trustees of the Alfried Krupp von Bohlen und Halbach Foundation in 2013. In November 2015 she received an honorary doctorate degree from Łódź University of Technology, Poland.
\item \textsuperscript{19} M.-Th. Roeckerath-Ries, who received her doctorate 1980 under my supervision, her teachers’ state exam in Physics in 1983, did research work at the University of Bremen 1986–1988. From 1988–1993 she was development engineer at Krupp Atlas Elektronik GmbH,-Bremen, and finally, since 1993 professor in applied mathematics at FH Südwestfalen at Hagen.
\item \textsuperscript{20} U. Scheben received a diploma in mathematics under my supervision at Aachen in 1980 and a diploma in computer science as well as a doctorate at the Fernuniversität Hagen in 1989 and 2006, respectively. Since 2008 she has been professor for foundations of computer science and compiler construction at the FH Dortmund.
\item \textsuperscript{21} A. Schalley received her diploma degree under my supervision in 1998 with the Thesis “Das mathematische Weltbild der Maya”, Grazer altertumskundliche Studien 6, Peter Lang, Frankfurt, 2000, 296 pp. She was awarded a PhD in Theoretical Linguistics in Munich in 2003 [98]. From 2003 to 2016 she worked in Australia, firstly at the U. New England and thereafter as associate professor at Griffith U., before accepting in 2016 the position of Chair Professor in English Linguistics at Karlstad University, Sweden.
\item \textsuperscript{22} K. Springsfeld, who wrote her masters thesis at our chair, wrote her doctoral thesis (advisors being D. Lohrmann, W. Oberschelp, P. L. Butzer) on the impact of Alcuin on computistics during Carolingian times; see [99]. She teaches history, mathematics and Latin to pupils at the Klinikum (hospital) of the RWTH Aachen University.
\item \textsuperscript{23} F. Ebersoldt (1935–2010), who worked at the Kernforschungsanlage Jülich 1969–1974, received his doctorate in 1970 under my direction at Aachen, his Habitation degree at U. Düsseldorf in 1973, and became professor at the U. Duisburg-Essen in 1979.
\item \textsuperscript{24} H. Schulte received his doctorate on a topic he picked himself under my supervision in 1966. He wrote a paper with me [100], and began his teaching at the PH Saarbrücken and later was professor at the Saarland U. until his retirement.
\item \textsuperscript{25} R. Stens received his doctoral degree in 1976 and his Habilitation degree in 1981, both under my supervision. He wrote ca. 50 joint papers with me. His first research area was integral transform analysis, first
\end{itemize}
for his final position being Senior Vice President at Robert Bosch GmbH. Since 2012 Lothar
receivers and developed state-of-the-art highly integrated digital receiver circuits. He made a respectable
subsidiary of Robert Bosch GmbH. He first worked on applications of digital signal processing in radio
degrees in 1983 and 1987, respectively. In 1987 Lothar joined Blaupunkt-Werke GmbH in Hildesheim a
main focus was on mathematics, statistical methods in electrical engineering, signal processing and data
compression. He was only 64 years old when he died on 28 August 2021.

W. Splettstößer received the Dipl. Math. degree in 1975, his doctoral degree in 1977, and his Habilita-
tion degree in 1981, all under my supervision. He was nominated Professor (apl.) at the Lehrstuhl A für
Mathematik in 1987. He wrote key papers on stochastic signals, see e.g. [77, 105]. In his paper “Sampling
Theorems for Stochastic Signals. Appraisal of Paul Butzer’s Work” [87], T. Pogány mentions that Wolfgang
was the specialist for stochastic processes at the Aachen Chair.

His interest in applied mathematics led him to carry on his career in industry. He was promoted to Vice
President of the Design-Center Düsseldorf of Infineon Technologies in 1998, and finally took the role of
the Senior Director of Human Resources at the Munich site in 2005. He became the global director in the
branch of “idea management” in 2007.

He passed away on March 18, 2013. For a more detailed biography see [106].

D. Pfeifer studied mathematics between 1971 and 1977 with a focus on lectures at our Chair. He received
his Ph.D. in 1979 with a thesis on statistics of extremes under the supervision of Professor Burkhard
Rauhut, Institut für Statistik und Wirtschaftsmathematik, and his habilitation degree in 1984, under my
supervision, with a thesis on probabilistic methods in the theory of operator semigroups. He followed
invited professorships at the University of North Carolina at Chapel Hill in 1985 and the University of
California at Santa Barabara in 1987. He was appointed to several to professorships in Germany, the last
at the University of Oldenburg in 2000 (applied probability, with a focus on insurance and finance). He
has written two textbooks and more than 100 scientific papers in various fields such as functional analysis,
probability and statistics, mathematical ecology and marine biology, and insurance and finance [107–109].
Three of his 29 Ph.D. students were themselves appointed professorships. He was also active as a scientific
consultant for an international reinsurance broker in Hamburg, and as a member of the supervisory board
of a mutual insurance company in Oldenburg, where he now is the chairman since 2019. He retired from
public service in 2016.

W. Engels, who received his doctorate under my supervision in 1982, and was research assistant at
our chair from 1979 to 1986, wrote seven papers together with Rudolf Stens, W. Splettstößer, U. Scheben
and me in the area of dyadic Walsh analysis, Shannon sampling series, and stochastic signals. He was the
organiser of the 5. Aachener Kolloquium “Mathematical Methods in Signal Processing”, held September,
26–29, 1984 in Aachen. This colloquium series was so popular that, although four conferences on similar
topics took place parallel to ours, in Brighton, Dortmund, Vienna and Graz, more than 220 participants
from 15 countries came to Aachen. Of these, 93 lectured, as the “Tagungsband”—still in demand—reveals.

S. Ries, who received his doctorate under the supervision of Rudolf Stens and myself, wrote papers
together with both of us and was a scientific assistant at our Chair from 1982 to 1985, then at Chair IV
for Informatics at Aachen, and became development engineer at Atlas Elektronik GmbH in Bremen in
1986. His final position from 1994 on, was professor at U. Paderborn (stationed at Meschede), where his
main focus was on mathematics, statistical methods in electrical engineering, signal processing and data
compression. He was only 64 years old when he died on 28 August 2021.

L. Vogt, who was a scientific assistant at the Institut für Reine und Angewandte Mathematik at Aachen
from 1983 until 1987, studied mathematics and physics at our Chair, receiving his diploma and doctoral
degrees in 1983 and 1987, respectively. In 1987 Lothar joined Blaupunkt- Werke GmbH in Hildesheim a
subsidiary of Robert Bosch GmbH. He first worked on applications of digital signal processing in radio
receivers and developed state-of-the-art highly integrated digital receiver circuits. He made a respectable
career with his final position being Senior Vice President at Robert Bosch GmbH. Since 2012 Lothar
and Andreas Gessinger\textsuperscript{31}. We would like to mention Dr. Marcel Zwaan (now with PanTerra Geoconsultants)\textsuperscript{32}.

Our colleague, Rolf J. Nessel, a PLB student, had eleven doctoral students. One, Steffen Goebbels\textsuperscript{33}, became a professor. Of the others, Werner Kolbe, Michael Becker, Jakob J. Junggeburth (who later obtained a Dr. med. degree, and practises medicine), Werner Dickmeis (who habilitated in 1984), and Erich van Wickeren also wrote joint papers with PLB.

Footnote 30 continued
has been professor for digital systems at the university of applied sciences in Lübeck, a position in which he has had great success and recognition in teaching international students in several courses on electronics and digital signal processing and passing over his experiences from 25 years in industry. He was a co-author together with M. Schmidt and E. L. Stark of the increasingly popular article on Central Factorial Numbers [110].

\textsuperscript{31} When I was preparing my invited lecture tour of England and Scotland in 1993, A. Sinclair of Edinburgh University asked me to lecture on semigroup theory, a topic in which I had not worked since the 70s. Thereafter, I persuaded my student A. Gessinger to become interested in its foundations and we decided to work first with cosine semigroups. The result: we finally produced seven joint papers from 1994 to 1997, including one on a new decomposition theorem for Parseval’s equation. This work was the background of Gessinger’s doctoral thesis of 1997. After working at Bonn area firms, he became professor at the RFH Köln.

\textsuperscript{32} PLB was the external referee (second advisor) of the PhD thesis of Marcel Zwaan, entitled “Moment problems in Hilbert space with applications to magnetic resonance imaging”, the first advisor being Gerke Yke Nieuwland, the examination taking place on 28.02.1991. Marcel, born 1961, was a student at the Vrije Universiteit Amsterdam, studying mathematics with focus on applied analysis. He obtained a teaching degree in 1983 and the MSc in 1987. He became a Research Geoscientist at Royal Dutch Shell in 1991, The Netherlands, working with Petroleum Development in Oman from 1997 to 2001, and Reservoir Imaging at Aberdeen, UK, 2001–2007. From 2007 onwards he worked as a team leader on Enhanced oil recovery (EOR), the extraction of crude oil from an oil field that cannot be extracted otherwise. After working with Shell for 27 years he is since 2018 the Manager of Subsurface Evaluation with PanTerra Geoconsultants. However he kept up his mathematical contacts, travelling jointly with PLB by KLM to AMS Winter Meeting at Baltimore in January 1992 (recall [1, Section 2]), where he presented the paper “Tychonoff Phillips regularization applied to non-uniform sampling”. At the gate we were upgraded to business class, where the stewardesses took so good care of us, that we declined a further upgrade to first class. Marcel also participated in the meeting at Cairo University in January 1994 [111], and in SampTA’95 at Riga, Latvia, [112] (see Sects. 9 and 13.3, respectively).

\textsuperscript{33} Steffen Goebbels was a research assistant at our chair from 1994 to 1997. He received his doctorate under Rolf Nessel in 1997 and wrote several joint papers with him on the sharpness of error bounds for finite element methods. He then worked for 11 years as an IT architect at IBM Deutschland GmbH on software projects. Since 2008, he has been a professor at the Niederrhein University of Applied Sciences in Krefeld teaching mathematics and computer science. His current research interests are the sharpness of error bounds in the approximation of functions using neural networks and, on the other hand, the computation of 3D city models from remote sensing data.
Our colleague, Ernst Görlich (1940–2014), a former PLB student, had six doctoral students; of these, three became professors, Wolfgang Dahmen\textsuperscript{34}, Clemens Markett\textsuperscript{35}, and Harald Dyckhoff\textsuperscript{36}.

13.3 Conferences held on the occasion of my birthday anniversaries

Three conferences held on the occasion of my birthday anniversaries really touched me.

The first was the conference on “Mathematical Analysis, Wavelets, and Signal Processing”, held January 3–9, 1994 at Cairo University, Cairo, Egypt. The occasion was my retirement at the age of 65 in 1993 after 35 years of service at Aachen, thereafter as Prof. Emeritus. For details see the conference proceedings [67].

The second was the workshop “Orthogonale Polynome—Translationsoperatoren und Basen”, held at Travemünde, April 17–20, 1998. Travemünde is a seaside resort and Germany’s largest ferry port on the Baltic coast, located on the mouth of the river Trave in Lübeck Bay. This excellent workshop, in wonderful surroundings, was

\textsuperscript{34} Wolfgang Dahmen received his doctorate at RWTH Aachen in 1976 and his habilitation degree at U. Bonn in 1981. He became professor of mathematics at U. Bielefeld in 1981 and at the Free University of Berlin in 1987. He returned to the RWTH Aachen as director of the Institute of Geometry and Practical Mathematics in 1992 until 2017. Since then, he has been professor of mathematics at U. South Carolina, holder of the SmartState Chair. In 2002 he was awarded the famous Gottfried Wilhelm Leibniz Prize of the German Research Foundation (DFG), and elected to the German National Academy of Sciences Leopoldina in 2009. His main research interests are approximation theory, especially nonlinear approximation, wavelets and multivariate splines, computational harmonic analysis, image processing, numerical analysis, especially adaptive multiscale methods for operator equations, and interdisciplinary applications in fluid dynamics and process engineering.

\textsuperscript{35} Clemens Markett received his doctorate as well as his habilitation degrees at RWTH Aachen in 1979 and 1985, respectively, is professor (apl.) at our RWTH since 1993, lecturing in analysis and actuarial sciences from 1985 to 2015. He was a Heisenberg fellow of the DFG (German Research Society) from 1987 to 1992, and held visiting positions in St. Louis (UMSL), Tampa (FL), Gent (B), Regensburg (D) and Cardiff (UK). In regard to his non-academic employment, he was an actuary at an international insurance company, held senior positions there, and was a member of the Deutsche Aktuar Vereinigung between 1992 and 2016. Since then, he has been continuing his mathematical research with contributions in the fields of special functions and differential equations. His paper in number theory of 1994 [113], associated with the joint paper [114], is highly cited, alone 12 times in the past four years.

\textsuperscript{36} H. Dyckhoff, first studied mathematics and physics at our Chair, obtaining his diploma 1975 under the supervision of my colleague Ernst Görlich. He wrote his first paper (1979) together with Görlich, Stens and myself, which belongs to the first papers worldwide concerned with derivatives of fractional order [13]. Thereafter he studied BWL, i.e., business administration and operations research. He wrote his doctoral thesis in economics at Fernuniversität Hagen, at which he also obtained his habilitation degree in BWL. After being professor for operations research at the University of Essen he returned to Aachen in 1988 as Chair Professor for BWL. His research fields were production and decision theory, sustainable value added, and performance measurement. He retired as emeritus in 2016.
organized by Prof. Rupert Lasser\textsuperscript{37}, together with Dr. Jürgen Prestin\textsuperscript{38}. They were ably supported by Frank Filbir and Roland Girgensohn.

Twenty-one lectures were held during the workshop. Four lecturers were originally from Rostock, one each came from Jena, Poland, and the USSR. The lecture room was in the art nouveau “Hotel Villa Chalott”, where the majority of the participants resided and enjoyed their meals.

To my greatest surprise: I stayed in a spacious apartment overlooking the crashing waves of the Baltic Sea. One evening food and drinks were carried into the kitchen of my apartment and a celebration was prepared. It was only then that I realized that the workshop was held in honour of my 70th birthday. Clemens Markett, an academic grandson, who accompanied me, was fully informed about this event but kept it secret.

Some of the workshop papers appeared in the “Journal of Computational Analysis and Applications”, Vol. 1, No. 3, and Vol. 2, No. 2, founded by George Anastassiou.\textsuperscript{39}

The third birthday conference was the workshop “Approximation Theory and Signal Analysis”, again organized by Rupert Lasser and Jürgen Prestin, together with Frank Filbir. It was held March 20–24, 2009 in the medieval town of Lindau on an island in the large Lake Constance. The workshop took place at the 5-star “Hotel Bayerischer Hof”, situated right at the harbour promenade of Lindau providing stunning views of the lake and the Alps. The majority of the participants resided in this hotel, and had their meals there (which were often exquisite).\textsuperscript{40} There were 73 participants, 33 from Germany; the other 40 were from 14 countries on 4 continents. There were 8 invited and 32 contributed talks.\textsuperscript{41}

To my great delight, many of my former students were among the participants, but also many colleagues from Germany and abroad with whom I had been in active scientific exchange over many years and in some cases had ongoing relations. Many of them were named in my “Retrospectives” \cite{1, 2} or in the present paper. I cannot list them all, but I would like to mention some of them nevertheless, e. g., Bruce Berndt, Mourad Ismail, Hrushikesh Mhaskar and Paul Nevai from the USA, Weiyi Su from China, Akhilesh Prasad from India, Mahmoud Annaby from Qatar, Karlheinz

\textsuperscript{37} R. Lasser was full professor of mathematics at the Medizinische University Lübeck 1994–1997, and then full professor of mathematics at the Technical University of Munich (TUM) and director of the “Institute of Biomathematics and Biometry” at Helmholtz Zentrum München at Neuherberg. Rupert Lasser really valued the mathematics of my research team, fostered us, and honoured me with two birthday conferences. My deepest gratitude is owed to him.

\textsuperscript{38} J. Prestin is a student of Manfred Tasche at University of Rostock. He was with the GSF-National Research Center for Environment and Health at Neuherberg from 1997–1999, and from 2000 onwards full professor at the University of Lübeck.

\textsuperscript{39} There was a follow-up conference, the workshop on “Orthogonal Polynomials— Approximation Theory and Harmonic Analysis”, on April 6, 1999 in Ballenstedt, Harz. It was again conducted by R. Lasser and J. Prestin with the financial support of the GSF. All lectures were held in Ballenstedt Castle located on a hill above the “Schloss-Hotel Großer Gasthof”. It was there where all participants stayed and had their meals. This time there were at least twice as many participants as in Travemünde; thus the workshop now had an international flair—W. Gautschi, C. Dunkl, H. Berens and M. H. Annaby, Cairo, being present.

\textsuperscript{40} Deep thanks are due Prof. R. Lasser and the Helmholtz Center Munich for their most generous financial support.

\textsuperscript{41} Volume 90 (2011), Nos. 3–4, of Applicable Analysis: “Approximation Theory and Signal Analysis, a special issue in honour of Professor Paul Leo Butzer”, guest editors: Robert P. Gilbert, Zuhair Nashed, M. W. Wong, and Yongzhi Steve Xu, contains twenty five papers, fifteen of which were presented at Lindau.
Gröchenig from Austria, Andi Kivinukk from Estonia, Paulo Ferreira from Portugal, Francesco Altomare, Carlo Bardaro and Gianluca Vinti from Italy, Tom Koornwinder from the Netherlands, Krzysztof Stempak from Poland, Ioan Gavrea and Mircea Ivan from Romania, Radomir Stanković from Serbia, Sergey Tikhonov and Juan Trujillo from Spain, and last, but not least, Maurice Dodson from the UK.

Of the German participants I would like to specifically mention Walter Benz (1931–2017), with whom I was a close friend since my return from Canada to Germany in 1955 (see [96]); and my first doctoral student Hubert Berens (1936–2015), who helped me to build up the research team at Aachen in 1963–1968 (see [27]).

13.4 Collaboration with Gerhard Schmeisser

In Part I of this retrospective and also in this Part II, I have written much about the contacts and cooperation with colleagues in many countries of the world. Here I would like to focus on the long-lasting cooperation with a German colleague, Gerhard Schmeisser from Erlangen-Nuremberg.

My first contact with Gerhard was at an Oberwolfach conference, organized by Hubert Berens in November 1981. I invited him to a colloquium lecture in Aachen that December and to our anniversary Oberwolfach conference of July/August, 1983. I met him again at a lecture in Erlangen upon Berens’ invitation in 1991. Beginning with the SampTA workshop in Loen (1999) our relationship began to deepen. Gerhard’s lecture on Mellin transforms and quadrature formulae impressed Rudolf and me so much that we asked him to add a chapter to our long survey paper [117]. Following Gerhard’s retirement in 2004 our cooperation grew steadily. Apart from the SampTA conferences, we met at Lindau 2009, Strobl 2011, Inzell 2012, and Aachen 2016. Highlights included the c. 10 joint papers with Rudolf in the area of Shannon’s sampling theorem, and the six joint papers on the Mellin transform and polar analytic functions (presented with the mathematicians from Perugia), areas we are still working on.

Rudolf and I are indeed grateful to Gerhard for the fruitful cooperation during the course of which many highly cited papers appeared, and a deep friendship developed.

42 On the occasion of my 70th, 75th, 85th and 90th birthdays, issues of journals, conferences or other forms honouring me appeared. Thus 1998 there appeared Vol. 34 (Nos. 1–2, 3–4, 1998) of “Results in Mathematics”, containing a serious of dedication papers on my 70th. Rupert Lasser, “Institute of Biomathematics and Biometry” at the Helmholtz Zentrum München at Neuherberg, lectured on the occasion of my 75th birthday on May 20, 2003, in our colloquium series at Aachen. Abdul J. Jerri, Founding Executive Editor of “Sampling Theory in Signal and Image Processing” (STSIP), prefaced its Special Issue(s), Vol. 2, No. 3 (2003), Vol. 3, No. 1 (2004), “In Honor of Professor Paul L. Butzer’s 75th Birthday”.

That year there also appeared Virginia Kiryakova: “On the occasion of Professor Paul L. Butzer’s 75th birthday” in “Fractional Calculus & Applied Analysis” [11].

“New Perspectives on Approximation and Sampling Theory” edited by Ahmed Zayed and Gerhard Schmeisser, containing articles dedicated to my 85th, was published in 2014 [115].

43 In this lecture I mentioned an open problem involving the Abel-Plana summation formula. It was solved in Gerhard’s paper [116] of 1994, written together with the Indian-Canadian mathematician Q. I. Rahman (1934–2013), Université de Montréal. Gerhard and Rahman worked together for 30 years, writing 45 joint papers, a “Lecture Notes”, and a book.
We would also like to take this opportunity to thank our dear friend Gerhard for his constant help during the creation of both parts of this retrospective.

**Correction—Failure**

In our first paper on research visits (see [1]), I (PLB) made a fundamental error. I wrote “Most unfortunately, our [François Jongmans’ and my] contribution on Dirichlet, into which we [put] so much of our energies, was never completed, one reason being that François was involved in many different historical and mathematical projects, e.g., he wrote 8 papers with E. Seneta on a variety of topics in the period 1992–2005”. It was a basic human failure of mine to place the cause or motive for not completing the contribution with François on Dirichlet upon Francois’s papers with Seneta.

In fact, I too was occupied with various different projects, especially between 1993 and 2009, which I should have put aside and concentrate upon completing the Dirichlet project. François was a long-standing friend of mine, a true, deep one.

Furthermore, François’s successor-in-office invited me to write a memorial address in honour of François. I began the project and tried to do a perfect job this time—perfection was always my aim, but I failed again, my deep apologies are due to everyone involved.

In the course of my 62 years as ordinary professor in mathematics at RWTH Aachen, from 1961 to 1993 as chairholder, since 1993 as professor emeritus, I may have disappointed or offended, friends, acquaintances, subordinates or relatives. I hope they will accept my deep apologies, too.

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