Iterative pre-distortion of the non-linear satellite channel

Thibault Deleu, Mathieu Dervin, Kenta Kasai and François Horlin

Abstract—The Digital Video Broadcasting - Satellite - Second Generation (DVB-S2) is the new European standard for satellite broadcast and broadband communications. It relies on high order modulations up to the 32 amplitude and phase-shift keying (APSK) modulation in order to increase the system spectral efficiency. However, high order modulations are much more sensitive to the physical layer impairments, especially to the non-linearity induced by the power amplifier aboard the satellite. The pre-distortion of the non-linear satellite channel has been studied for many years. However, the performance of pre-distortion algorithms generally becomes poor when high order modulations are used on a non-linear channel with a large memory. In this paper, we investigate a new iterative method that pre-distorts the block of transmitted symbols so as to minimize the square error between the transmitted and received symbols. The complexity of the algorithm is however high. Approximations of the algorithm are therefore proposed to considerably decrease the complexity of the algorithm while keeping the performance acceptable.

Index Terms—Pre-distortion, non-linear satellite channel, DVB-S2.

I. INTRODUCTION

In a broadcast or broadband satellite communication, information is exchanged between one feeder and many user terminals in a so-called star topology. The forward link is defined as the link from the feeder towards the user terminals and the return link (when it exists) is the one from the user terminals towards the feeder. For the forward link, the total available spectrum is generally divided into different channels. Each channel can then be separately demodulated by the individual terminals. Different power amplifiers aboard the satellite are available to amplify the channels on the forward and return links. In a single carrier per channel scenario, each channel from the forward link is separately amplified by a power amplifier. This allows to use the power amplifier close to its saturation point, so that the power consumption aboard the satellite is optimized. When the link budget is good enough, it is possible to increase the spectral efficiency of the system by using high order modulations. However, non-linear intersymbol interference (ISI) occurs due to the combination of the non-linear high power amplifier (HPA) aboard the satellite with linear filters present in the channel. The larger the bandwidth of the channel, the more interference occurs due to the bandpass property of the linear filters. Higher order modulations are more sensitive to the non-linear ISI so that compensation algorithms are necessary to remove the non-linear interference in order to achieve adequate performance. In the literature, the proposed methods to compensate for the non-linear interference can be divided into two categories. Firstly, the non-linear interference can be compensated at the receiver side. If the channel is exactly known, the maximum-a-posteriori (MAP) symbol detection algorithm can be perfectly defined, as well as the optimum maximum-likelihood sequence detection algorithm. The complexity of these optimum algorithms increases however exponentially with the channel length and the modulation order, so that several sub-optimum algorithms have been proposed in the literature, as for instance in [11] which uses a reduced channel model for the detection of the received signal. Adaptive non-linear equalizers have been proposed in [2] and [3], where the a-priori channel knowledge is not required anymore. In [4], joint equalization and channel decoding is performed using Gaussian processes. To take advantage of the channel coding, iterative turbo-equalization structures have also been considered [5] and [6].

Pre-distortion is the compensation of the interference at the transmitter side. Pre-distortion can be advantageously considered for the forward link in a star topology, where it is preferred to concentrate the computational load into the feeder station and relax as much as possible the complexity of the terminals (and all the more as mass market applications are considered, as for TV broadcasting). One usually uses the term signal (or waveform) pre-distortion to refer to the pre-distortion, when it is located after the pulse shaping filter. This kind of pre-distortion can compensate memoryless channels, as shown in [7], [8] and references therein. The pre-distorter is then an approximation of the inverse characteristic of the power amplifier at the transmitter side. This method can be analog or digitally implemented (see for example the adaptive implementation in [9] and [10]). On the other hand, we will refer to data pre-distortion when the pre-distortion of the data symbols is located prior to the pulse shaping. It allows to compensate for ISI and it avoids out-of-band emissions. A first data pre-distortion method is based on the minimization of the mean square error (MSE) between the transmitted and received symbols using a Volterra model. The Volterra model, described in [11], is a common tool to describe the input-output relation of a non-linear system. In [12] and [13], the pre-distorter is a Volterra model with fixed length and order, and the coefficients of the pre-distorter are determined with an adaptive algorithm. The order \( p \) inverse for non-linear systems has been described in [14] and applied to the satellite channel in [15]. The order \( p \) pre-distorter will remove all Volterra terms from the channel up to order \( p \) between the received and transmitted symbols.
(note that this algorithm can actually be both applied for pre-distortion or equalization). An adaptive algorithm based on the order $p$ inverse has been described in [16]. The complexity of such pre-distorters may however be high so that pre-distortion methods based on reduced Volterra model have been studied in [17] and references therein. Another data pre-distortion method has been proposed in [18]. The value of each pre-distorted symbol is function of the neighboring initial symbols. The values of the pre-distorted symbols can be calculated offline and stored in a look-up table. These values are pre-computed in order to minimize the MSE between the initial and the received symbols. The performance of this algorithm has been assessed for high order modulations in [19].

The performance of the existing data pre-distortion methods are dependent on the considered channel and modulation. In case of high interference channels, the order $p$ inverse creates higher order terms whose power is higher than the one of cancelled terms, as shown in [20]. Pre-distortion methods based on the Volterra model are always limited in order and memory. They are more likely to perform poorly in case of large channel lengths and high non-linearities coming from the HPA. Finally, the size of the look-up tables used for data pre-distortion grows exponentially with the modulation order and the channel length. Even if they can be expected to perform well in case of a highly non-linear HPA, the performance of these pre-distortion algorithms is expected to become poor for large channel lengths and large modulation orders. In this paper, we propose a pre-distortion algorithm, of complexity growing linearly with the channel length and independent of the modulation order and of the non-linearities of the HPA. The proposed algorithm independently pre-distorts successive symbol blocks. To pre-distort each block of symbols, an iterative algorithm is used, aiming at minimizing the Euclidian distance between the initial symbol and the received symbol sequence. This allows to significantly increase the performance compared to the existing data pre-distorting methods in case of high non-linear interference channels. Based on the system model defined in Section II, we describe the proposed algorithm in Section III. A main concern of the algorithm design is its complexity so that variations of the algorithm of much smaller complexity are proposed in Section IV. Section V is devoted to complexity comparisons between the proposed algorithms and in Section VI, we assess the performance of the different algorithms.

II. SYSTEM MODEL

A. Satellite channel

The block diagram of the satellite channel is depicted in Fig.1. At the satellite, the input multiplexer (IMUX) filter is a bandpass filter which selects the subband to be amplified. The satellite high power amplifier (HPA) can be seen as a non-linear memoryless device. The output multiplexer (OMUX) filter is also a bandpass filter, necessary to remove the out-of-band non-linearities produced by the power amplifier. We consider linear modulations shaped with a square root raised cosine (SRRC) pulse. The larger the bandwidth of the signal, the more inter-symbol interference the IMUX and OMUX filters will generate to the signal. More particularly, we consider the highest order modulation defined in the DVB-S2 standard (21), the 32- amplitude phase-shift keying (APSK) modulations. Although they ensure maximal spectral efficiency, these modulations are also the more sensitive to the non-linear interference induced by the combination of the linear filters and the power amplifier. We denote 32-APSK symbols obtained after modulation as $s(n)$ and the received symbols at the output of the SRRC filter at the demodulator as $y(n)$. The pre-distorted symbols actually transmitted on the channel are denoted $x(n)$. The value of the pre-distorted symbols $x(n)$ are calculated by a pre-distortion algorithm which takes the symbols $s(n)$ as input. The goal of the pre-distortion block is to mitigate the non-linear interference, and will be more detailed in the next sections.

B. Volterra model

The Volterra model is an analytical model which describes the relation between the input and the output of a non-linear system with memory. The case of the baseband non-linear satellite channel has been described in [15]. The relation between the pre-distorted symbols $x(n)$ at the channel input and the received symbols $y(n)$ is given by:

$$y(n) = \sum_{m=0}^{\infty} \sum_{n_1 \ldots n_{2m+1}} H_{2m+1}(n_1 \ldots n_{2m+1}) x(n - n_1) \ldots x(n - m + 2) x(n - n_{2m+1}).$$

(1)

The coefficients $H_{2m+1}(n_1 \ldots n_{2m+1})$ are called the Volterra kernels of the system. The received symbols are also corrupted by additive white Gaussian noise, but it has no impact on the pre-distortion algorithms, so that it is not considered here. The first sum in Equation (1) represents the different orders of the non-linearity induced by the power amplifier. The second set of sums represents the memory of the system, which is theoretically infinite. In practice however, the length of the channel can reasonably be assumed of finite length. We denote the anti-causal memory of the channel as $L_1$ and the causal memory of the channel as $L_2$. The total channel length is denoted as $L_c = L_1 + L_2 + 1$.

III. PER BLOCK ITERATIVE PRE-DISTORTION

A. Minimization of the Euclidian distance

We consider the pre-distortion of length-N symbol blocks, where $N$ can take large values (typically few thousand symbols). For a given block, we denote $s$ the vector with as elements the different symbols of the block: $s = [s(1) \ldots s(N)]$. For each symbol block, the pre-distorter produces a modified symbol block of length $N$, denoted with the vector $x = [x(1) \ldots x(N)]$. At the receiver, $N$ samples are also gathered in a vector of size $N$, denoted by $y = [y(1) \ldots y(N)]$. In addition, we denote as $y(x)$ the vector $y$ of the received symbols when the block $x$ is provided at the channel input. In this section, we propose an algorithm that precodes the block $x$ so that $||y - s||_2$ is minimum. Note that the length of the block $y$ should be equal to $N + L_c - 1$. However, we consider the case of a simple receiver, where memoryless detection
is applied on the consecutive received symbols, so that the symbols
\( y(-L_1), \ldots, y(-1) \) and \( y(N+1), \ldots, y(N+L_2) \) are neglected.

The additive white Gaussian noise is independent from the transmitted
symbol, so that the precoded block is independent of the noise power. The vector \( y \) can be developed using Equation (1). However, there is no straightforward derivation of the block \( x \) which minimizes \( ||y - s||_2 \). We propose therefore an iterative algorithm to determine the pre-distorted block \( x \). Each iteration of the algorithm is divided into \( N \) steps, respectively focused on the consequent symbols of the block of interest. The pre-distorted block after the step \( j \) of iteration \( k \) is denoted as \( x_{k,j} = [x_{k,j}(1) \ldots x_{k,j}(N)] \). At the step \( j \) of any iteration \( k \), only the \( j^{th} \) pre-distorted value is modified and chosen to minimize \( ||y - s||_2 \) when \( x_{k,j} \) is transmitted. All other pre-distorted values are thus kept equal to their values from the previous step. Mathematically, \( x_{k,j}(n) \) is expressed as:

\[
\begin{align*}
n \neq j \neq 1 & : \quad x_{k,j}(n) = x_{k,j-1}(n) \\
n = j > 1 & : \quad x_{k,j}(j) = \arg\min_{x_{k,j}(j)} ||y - s||_2 \forall i \neq j : x(i) = x_{k,j-1}(i) \\
\end{align*}
\]

(2)

and the specific case \( j = 1 \):

\[
\begin{align*}
n \neq j = 1 & : \quad x_{k,1}(n) = x_{k-1,N}(n) \\
n = j = 1 & : \quad x_{k,1}(1) = \arg\min_{x_{k,1}(1)} ||y - s||_2 \forall i \neq 1 : x(i) = x_{k-1,N}(i). \\
\end{align*}
\]

(3)

Note that \( x_{k,1}(n) \) is calculated using the end values of the previous iteration. The vector \( \epsilon_{k,j} \) is defined as the difference between \( y \) and \( s \) when the sequence obtained after the step \( j \) of iteration \( k \) is transmitted:

\[
\epsilon_{k,j} \triangleq y - s |\forall i : x(i) = x_{k,j}(i). \]

(4)

By definition of the algorithm, we have:

\[
||\epsilon_{k,j}||_2 \leq ||\epsilon_{k,j-1}||_2. \tag{5}
\]

The term \( ||y - s||_2 \) minimized in Equation (2) can be seen as a complex non-linear function of the complex variable \( x_{k,j}(n) \). The coefficients of this function can be found using the Volterra model and depend on the fixed pre-distorted values in Equation (2). Since the channel has finite length, Equation (2) can be simplified as:

\[
x_{k,j}(j) = \arg\min_{x_{k,j}(j)} \min_{m=\max(1,j-L1)} \sum_{m=\max(1,j+L2)} |y(m) - s(m)|^2 \forall i \neq j : x(i) = x_{k,j-1}(i). \tag{6}
\]

The complexity of the algorithm remains however very high since for each iteration it is necessary to successively find the minimum of \( N \) complex non-linear functions. Moreover, the number of Volterra coefficients in each equation can be very high in case of high order non-linearities. Therefore, the pre-distorted symbols defined in (2) are difficult to compute in practice. In next subsection, we will propose an algorithm of much lower complexity to compute the pre-distorted symbols. We will refer to this algorithm as the small variation algorithm.

B. Small variation algorithm using the Volterra model

The small variation algorithm has the same iterative structure as the algorithm presented in the previous subsection. At the step \( j \) of the iteration \( k \), it calculates a suboptimal value for \( x_{k,j}(j) \) but on a much less complex way. We first define \( \Delta_{k,j} \) as:

\[
x_{k,j}(j) = x_{k,j-1}(j) + \Delta_{k,j} \tag{7}
\]

The variation from \( x_{k,j-1}(j) \) to \( x_{k,j}(j) \) is considered as the unknown variable, instead of \( x_{k,j}(j) \) itself. The case \( j = 1 \) is
We define:

\[ x_{k,j} = x_{k,j-1} + \Delta_{k,j}. \]  

(8)

We define the value \( \Delta_{k,j}^{opt} \) as the optimum value which minimizes Equation (2):

\[ \Delta_{k,j}^{opt} = \min_{\Delta_{k,j}} ||y - s||_2 \forall i \neq j : x(i) = x_{k,j-1}(i), \]

\[ x_{k,j}(j) = x_{k,j-1}(j) + \Delta_{k,j} \]  

(9)

It is possible to simplify Equation (9) as in Equation (6), but we prefer to adopt the following more compact notation:

\[ \Delta_{k,j}^{opt} = \min_{\Delta_{k,j}} ||y(x_{k,j-1} + \Delta_{k,j}) - s||_2. \]  

(10)

We define:

\[ F_{k,j}^{NL} \triangleq y(x_{k,j-1} + \Delta_{k,j}) - y(x_{k,j-1}) \]  

(11)

so that:

\[ \Delta_{k,j}^{opt} = \min_{\Delta_{k,j}} ||y(x_{k,j-1} + \Delta_{k,j}) - s + F_{k,j}^{NL}||_2 \]

\[ = \min_{\Delta_{k,j}} ||\epsilon_{k,j-1} + F_{k,j}^{NL}||_2 \]  

(12)

\( F_{k,j}^{NL} \) represents the output \( n \) variation resulting from a variation of the input \( j \) during the iteration \( k \). \( F_{k,j}^{NL} \) is a vector of functions depending on the simple variable \( \Delta_{k,j} \). Inspecting Equations (1) and (11), it can be derived that each element \( F_{k,j}^{NL}(n) \) of \( F_{k,j}^{NL} \) takes the forms:

\[ n < j - L_2, n > j + L_1 : F_{k,j}^{NL}(n) = 0 \]

\[ n \geq j - L_2, n \leq j + L_1 : F_{k,j}^{NL}(n) = \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} A_{n}^{m_1} A_{2}^{m_2}. \]  

(13)

The coefficients \( A_{n}^{m_1} \) depend on the Volterra coefficients and on the sequence of pre-distorted symbols. For the sake of clarity, Appendix A gives some examples for the coefficients \( A_{n}^{m_1} \) and the associated Volterra models consisting only in one single Volterra coefficient. In the general case of a channel depending on several Volterra coefficients, the value of \( A_{n}^{m_1} \) can be obtained by first computing the value of \( A_{n}^{m_1} \) corresponding to each Volterra coefficient taken independently and by summing all the obtained values. The small variation algorithm is based on the assumption that each function \( F_{k,j}^{NL}(n) \) can be approximated by keeping only its linear part in \( \Delta_{k,j} \):

\[ F_{k,j}^{NL}(n) \approx F_{k,j}^{Lin}(n) \triangleq A_{n}^{m_1}(1,0)\Delta_{k,j} + A_{n}^{m_2}(0,1)\Delta_{k,j}^{*}. \]  

(14)

This will be more likely the case for small values of \( \Delta_{k,j} \). Denoting \( F_{k,j}^{Lin}(\Delta_{k,j}), A_{n}^{m_1}(1,0) \) and \( A_{n}^{m_2}(0,1) \) the vectors obtained with the elements \( F_{k,j}^{Lin}(n), A_{n}^{m_1}(1,0) \) and \( A_{n}^{m_2}(0,1) \), with \( n \) varying from 1 to \( N \), we have that:

\[ F_{k,j}^{NL} \approx F_{k,j}^{Lin} \triangleq A_{n}^{m_1}(1,0)\Delta_{k,j} + A_{n}^{m_2}(0,1)\Delta_{k,j}^{*}. \]  

(15)

Instead of calculating the value \( \Delta_{k,j}^{opt} \) from Equation (10), the small variation algorithm calculates \( \Delta_{k,j}^{Lin} \) defined as followed:

\[ \Delta_{k,j}^{Lin} = \arg\min_{\Delta_{k,j}} ||\epsilon_{k,j-1} + F_{k,j}^{Lin}||_2 \]  

(16)

If \( y(x_{k,j-1}) \) and the coefficients of \( A_{n}^{m_1}(1,0) \) and \( A_{n}^{m_2}(0,1) \) are known, it can be seen using Equation (15) that Equation (16) is a second order equation with the complex unknown \( \Delta_{k,j} \). Using partial derivatives, the optimum value of \( \Delta_{k,j} \) that minimizes (16), can be found by solving a system of two linear equations with two unknowns (being the real and imaginary parts of \( \Delta_{k,j} \)), which makes the calculation much easier than minimizing the exact non-linear equation. Once \( \Delta_{k,j}^{Lin} \) has been determined, the linearity assumption needs to be verified. This will be discussed in next subsection.

C. Linearity assumption of the small variation algorithm

It is necessary to check that the approximation made in (15) is good enough so that the optimum value from Equation (16) is close to the optimum value from Equation (12). It is first necessary to define an objective way to assess the quality of the approximation. Since the goal of the pre-distortion algorithm is to minimize the square error between the initial and received symbols, it is natural to compare the value

\[ (||\epsilon_{k,j-1} + F_{k,j}^{NL}||_2 |\Delta_{k,j} = \Delta_{k,j}^{Lin}) - ||\epsilon_{k,j-1}||_2 \]  

with

\[ (||\epsilon_{k,j-1} + F_{k,j}^{NL}||_2 |\Delta_{k,j} = \Delta_{k,j}^{opt} - ||\epsilon_{k,j-1}||_2. \]  

This means to compare the performance improvement obtained with the approximation and the performance improvement obtained when no approximation is done. However, \( \Delta_{k,j}^{opt} \) is unknown, so that the comparison is simply impossible. In this paper, we consider instead that the linearity assumption is met if the square error between the initial and the received symbols decreases after each step of the algorithm. Mathematically, this means:

\[ (||\epsilon_{k,j-1} + F_{k,j}^{NL}||_2 |\Delta_{k,j} = \Delta_{k,j}^{Lin}) - ||\epsilon_{k,j-1}||_2 \]  

(17)

It is easy to see that for a smaller value of \( \Delta_{k,j}^{Lin} \), the contribution of each Volterra coefficient to \( \Delta_{k,j}(1,0) \) and \( \Delta_{k,j}(0,1) \) become more linear so that it is more likely that the linear assumption defined in (17) is verified. Therefore, if \( \Delta_{k,j}^{Lin} \) does not meet the linearity requirement, a variation \( \Delta_{k,j} = \gamma \Delta_{k,j}^{Lin} \) is applied, with:

\[ \Delta_{k,j}^{Lin} = \gamma \Delta_{k,j}^{Lin}. \]  

(18)

with \( \gamma \) a real number in the interval \([0,1]\). In Appendix B, it is proven that the square error effectively decreases if \( \Delta_{k,j}^{Lin} \) meets the linearity assumption, which means:

\[ (||\epsilon_{k,j-1} + F_{k,j}^{Lin}||_2 |\Delta_{k,j} = \Delta_{k,j}^{Lin}) \leq ||\epsilon_{k,j-1}||_2 \]  

(19)
$\Delta_{k,j}^{\gamma}$ must be understood as a suboptimum solution of Equation (16), but a potentially better solution to Equation (10). It is still necessary to check that the linearity assumption is met, this means:

$$||c_{k,j-1} + E_{k,j}^{NL}||_2 \Delta_{k,j} = \Delta_{k,j}^{\gamma} \leq ||c_{k,j-1}||_2.$$  

If this is not the case, we can then iterate and apply a new variation $\Delta_{k,j} = \gamma \Delta_{k,j}^{\gamma}$. Smaller values for $\gamma$ generally ensures the linearity approximation with greater accuracy. However, it also implies a less optimal solution to the linear equation so that more iterations of the algorithm are potentially required. The case $\gamma = 0$ is a special case where no variation is applied when $\Delta_{k,j}^{Lin}$ does not meet the linearity requirement.

The main difficulty raised by the proposed algorithm is the complexity to assess the coefficients $A_{k,j}(m_1, m_2)$ since they depend on all Volterra coefficients. The next section will be devoted to the decrease of the complexity of the proposed algorithm.

IV. COMPLEXITY REDUCTION OF THE SMALL VARIATION ALGORITHM

The algorithm presented in previous section is rather theoretical as the number of Volterra coefficients can be tremendous. We present more practical methods to calculate the coefficients $A_{k,j}^0(m_1, m_2)$.

The coefficients $A_{k,j}^0(1,0)$ and $A_{k,j}^0(0,1)$ are necessary to calculate $\Delta_{k,j}^{Lin}$. We propose 3 methods to compute these coefficients. The first one has an arbitrary high precision, so that it can be considered as an exact and practical method to calculate $A_{k,j}^0(1,0)$ and $A_{k,j}^0(0,1)$. The two other methods are only approximations of these coefficients, but they are less complex than the first method.

The last part of this section shows how to check the linearity assumption without calculating the coefficients $A_{k,j}^n(m_1, m_2)$ for which $m_1 > 1$ or $m_2 > 1$. The algorithm complexity would indeed become prohibitive if these coefficients have to be computed.

A. Calculations of the coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ based on channel simulations

The coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ can be estimated with the following procedure. At the step $j$ of iteration $k$, the channel outputs resulting from three different channel input variations are successively simulated. The three channel input variations are:

1) $\Delta_{k,j} = 0$
2) $\Delta_{k,j} = \epsilon_r$
3) $\Delta_{k,j} = \epsilon_i$

where $\epsilon_r$ and $\epsilon_i$ are respectively very small real and pure imaginary numbers. The respective channel outputs are denoted as $y_r(j+l), y_r(j+l)$ and $y_i(j+l)$, with $l \in [-L_1, L_2]$. It is assumed that the transmitter has a perfect knowledge of the channel. If $\epsilon_r$ and $\epsilon_i$ are chosen sufficiently small, there is a linear relation between the input and output variations:

$$y_r(j+l) = y_r(j+l) + A_{k,j}^n(1,0) \epsilon_r + A_{k,j}^n(0,1) \epsilon_r$$
$$y_i(j+l) = y_i(j+l) + A_{k,j}^n(1,0) \epsilon_i - A_{k,j}^n(0,1) \epsilon_i.$$  

Equation (22) is a set of two equations with unknowns $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$, so that they can easily be estimated. The smaller $\epsilon_r$ and $\epsilon_i$, the more accurate the calculation. This method requires however three simulations of the channel at each step of each iteration.

To simulate the channel, the pre-distortion block needs to oversample the signal by several times the symbol rate, in order to avoid spectral aliasing from the non-linear interference. For channels with large memory, the channel simulations can require a lot of complexity. We describe in the following methods to obtain approximations of the coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ with less complexity.

B. Calculations of the coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ based on a reduced Volterra model

The coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ can be computed by summing the contribution of each Volterra coefficient. To decrease the algorithm complexity, they can instead be approximated by summing the contributions of only the most significant Volterra coefficients. Let us consider the generic Volterra coefficient $H_{2m+1}(n_1,...,n_{2m+1})$. Volterra coefficients which do not have at least one index equal to $n-j$ can be rejected, since only the symbol $j$ is modified. To further decrease the number of Volterra coefficients, an approximated value of $A_{k,j}^n(m_1, m_2)$ can be calculated by truncating the non-linearity order or by limiting the channel length. The second approach consists in truncating the channel length to a given value $L'_c$: only the Volterra coefficients $H_{2m+1}(n_1,...,n_{2m+1})$ for which each index $n_i$ satisfies $|n_i| \leq L'_c$ are considered. In this approximation, $A_{k,j}^n(m_1, m_2) = 0$ for $|n-j| > L'_c$. Further approximations are discussed in [17]. For instance, it is proposed to keep the Volterra coefficients that depend on a single index. In this paper, the same approach is followed taking only into account all Volterra coefficients which depend only on the index $n_i = n-j$.

C. Calculations of the coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ using a look-up table

The idea of this approximation is to pre-compute the values of $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ and to store them in a look-up table. $A_{k,j}^n(1,0)$ depends on the symbols $[x_{k,j-1}(n-L_2) \ldots x_{k,j-1}(n+L_1)]$, which take continuous values, so that an infinite number of pre-computed values should be stored. Therefore, an approximation of $A_{k,j}^n(1,0)$ is calculated by rounding each value in $[x_{k,j-1}(n-L_2) \ldots x_{k,j-1}(n+L_1)]$ to the closest value in $C$, where $C$ is a set of $P$ complex numbers $C = \{c_1, c_2, \ldots, c_P\}$. Considering the channel length $L'_c$, approximately $P^{L'_c}$ values need to be stored. To avoid the complexity of rounding each pre-distorted sample to the closest value in $C$, a further approximation can be introduced, considering that the coefficients $A_{k,j}^n(1,0)$ and $A_{k,j}^n(0,1)$ are independent of $k$. This means that these coefficients are calculated at any iteration $k$ using the symbols $[s(n-L_2) \ldots s(n+L_1)]$, instead of using the symbols $[x_{k,j-1}(n-L_2) \ldots x_{k,j-1}(n+L_1)]$. 


D. Alternative to the calculation of the remaining coefficients $A_{k,j}^n(m_1, m_2)$

Once the precoded symbols are updated using the lower complexity algorithms proposed in Sections IV-B and IV-C, it is important to check the linearity assumption based on which the algorithms have been designed. This could be simply done by simulating the actual channel, but we wanted to avoid in Sections IV-B and IV-C for complexity reasons. A pragmatic approach to keep the low complexity advantage is to check the linearity assumption only at the end of each iteration by a single channel simulation. We consider that the linearity assumption is met after each step, but it is of course desirable that it is most of the time the case. Therefore, a variation $\Delta k,j$ is always applied considering a fixed value for $\gamma$ (see Equation (18)), which will result from a trade-off between performance and convergence speed. In the following, the algorithm is compared to that of its reduced complexity alternative.

In this section, the complexity of the small variation algorithm is compared to that of its reduced complexity alternatives. In the following, $\Delta k,j$ and $\Delta k,j^*$ are replaced by $R(\Delta k,j)$ and $\Re(\Delta k,j)$ in Equation (14) and the coefficients $A_{k,j}^n(1, 0)$ and $A_{k,j}^n(0, 1)$ are consequently modified. This simple notation change allows a (small) complexity decrease. The algorithm consists in $K$ iterations of $N$ steps. At each step, the initial algorithm and the reduced complexity algorithms all require the following operations:

- Calculate the coefficients $A_{k,j}^n(1, 0)$ and $A_{k,j}^n(0, 1)$ from Equation (14). These coefficients are calculated differently for each method and the complexity is detailed in the second part of this section.
- Evaluate the equation to minimize in (16), with $F_{k,j}^{\text{Lin}}$ defined in Equation (15). It can be shown that it necessitates $10L_c$ multiplications and $5(L_c - 1)$ additions.
- Find the minimum of Equation (16). Using partial derivative, this requires the inversion of a two times two (real) matrix, 4 multiplications and 2 additions.
- Update the channel outputs for the next step (in fact calculate $c_{k,j}$ from $c_{k,j-1}$ using $A_{k,j}^n(1, 0)$, $A_{k,j}^n(0, 1)$ and the applied variation). This requires $4L_c$ multiplications and $4L_c$ additions.

The complexity to determine the linear coefficients depends on the chosen algorithm:

- The complexity of the algorithm using the Volterra model depends on the number of considered Volterra coefficients and on the order of the Volterra coefficient. A Volterra coefficient of order $p$ requires $4p$ multiplications and 2 additions. By truncating the channel length to $L'_c$, each of the $L'_c$ output depends on approximately $(L'_c)^{(p-1)}$ coefficients of order $p$. This number reduces to 1 considering the memory polynomial approximation.
- The method based on channel simulations necessitates 3 channel simulations. At each channel simulation, $L_c$ outputs need to be calculated. This requires to simulate the power amplifier for more than $L_c k_{\text{OSF}}$ input values, where $k_{\text{OSF}}$ is the oversampling factor. The output of the power amplifier can be obtained by interpolating the AM-AM and AM-PM characteristics of the power amplifier. They can also be assessed using a polynomial approximation. The number of operations to calculate the output at the power amplifier is then approximately proportional to the chosen non-linearity order. The complexity of the convolution with the linear filters needs also to be taken into account, which is proportional to $L_c^2 k_{\text{OSF}}$. To calculate the coefficients $A_{k,j}^n(1, 0)$ and $A_{k,j}^n(0, 1)$ from the channel simulations, 4 additions and 4 divisions need to be done.
- The method based on the look-up tables needs to round $4L'_c$ values to the closest value in the look-up table, since the $L'_c$ complex outputs depend on $2L'_c$ inputs. The method based on the look-up tables is the less complex. The method based on the approximated Volterra model is less complex than the method based on channel simulations only if the number of Volterra coefficient is very small, so that very short channel lengths must be considered. This will be further discussed in next section.

V. COMPLEXITY OF THE SMALL VARIATION ALGORITHM

Fig. 2: AM-AM and AM-PM characteristics of the HPA

VI. NUMERICAL RESULTS

We consider 32-APSK symbols. The shaping and receiving filters are root-raised-cosine filters with a roll-off factor equal to 0.1. We consider a traveling-wave tube amplifier (TWTA) with amplitude-to-amplitude and amplitude-to-phase characteristics given in Fig. 2. The IMUX and OMUX characteristics are given in Fig. 3 and Fig. 4 respectively. Their 3dB cut-off frequency is equal to 36 MHz. The LDPC encoder and the interleaver are the ones defined in the DVB-S2 for the 32-APSK modulation, with a chosen code rate equal to 3/4. The input backoff (IBO) for the channel simulation is equal to 3dB for the un-pre-distorted symbols. The pre-distorted symbols are not renormalized and thus do not have the same mean power as the initial symbols. This
would be seen as a channel modification by the pre-distortion algorithm. Therefore, the IBO after pre-distortion may be slightly different.

Fig. 5 represents the MSE between the initial symbols and the received symbols after each iteration of the algorithm for two different values of $\gamma$. In the convergence method 1, the applied variation is obtained by multiplying $\Delta_{k,j}^{\text{lin}}$ by $\gamma$, if necessary multiple times, until the linearity assumption is met as described in Section III-C. Convergence method 2 is the pragmatic approach described in Section IV-D, where the variation $\Delta_{k,j} = \gamma \Delta_{k,j}^{\text{lin}}$ is always applied. The MSE is calculated after each iteration and the algorithm is stopped as soon as the MSE is no more decreasing. For a small value of $\gamma$, the second convergence method is faster and less complex than the first convergence method. When $\gamma$ is set to a high value, the second method however diverges, unlike to the first method. This illustrates the necessity to check that the MSE effectively decreases after each iteration in the case of the second convergence method.

Fig. 6 and Fig. 7 compare the performance between the optimal algorithm and the reduced complexity algorithms proposed in Section IV. Fig. 6 illustrates the MSE improvement as a function of the number of iterations of the algorithm, considering a symmetric channel response with different lengths and a symbol rate equal to 36 MSymb/s. The signal bandwidth is equal to 110% of the theoretical bandwidth of the channel. In some scenario’s, this could lead to adjacent channel interference, which is here not considered. It is shown that $L_c = 7$ is already sufficient to approach the optimal performance. Fig. 7 illustrates the MSE obtained with the different methods as a function of different symbol rates. The channel occupation varies here from 104% to 116%. Firstly, the non-linear amplifier is approximated by a third order curve, which is a correct approximation of the power amplifier characteristics in the
The performance loss due to this approximation is close to 1 dB or below, compared to the optimum algorithm. This optimum is obtained considering a channel length $L'_c = 21$ and 30 algorithm iterations. To obtain a finite number of Volterra coefficients, the approximations obtained by truncating the channel length and the non-linearity order are also simultaneously considered in Fig. 7. A third order reduced Volterra model with $L'_c = 3$ is considered, so that each new output value is calculated using 12 Volterra coefficients. This simplified algorithm outperforms a state-of-the-art algorithm, which is the pre-distortion algorithm proposed in [18] and [19], considering a look-up table with $32^4$ entries. Fig. 7 also shows the impact of the approximation of the small variation algorithm using a look-up table with $32^5$ entries. The considered set of discrete values $C$ is the initial symbol constellation and it is considered that the coefficients $A^n_{k,m}(m_1, m_2)$ remain constant after each iteration. Except for very high symbol rates, the performance obtained with the approximation is close to the performance reached with the exact algorithm and shows a significant improvement in comparison with the considered state-of-the-art algorithm. Further improvement could be obtained by increasing the memory size.

Fig. 8 represents the bit error rate (BER) for a symbol rate equal to 36 MSymb/s. The performance improvement of the small variation algorithm is of several dBs on the required SNR compared to the state-of-the-art algorithm. The obtained performance is also close to the interference free case. The approximated algorithms show a small performance loss compared to the gain on the state-of-the-art algorithm. The approximation based on the look-up table allow a performance increase of approximatvily 3 dBs, but the number of memory accesses is multiplied by the channel length and by the number of algorithm iterations compared to the state-of-the-art algorithm. The approximation based on the reduced Volterra model is also considered. It is again obtained by considering a third order non-linearity and a channel length $L'_c = 3$. The performance gain is slightly smaller, but this approximation does not require to pre-compute any values and it still allows to decrease the complexity compared to the method based on channel simulations: for each step of each iteration, it requires about 60 multiplications and 30 additions, relying on the fact that some products of the pre-distorted symbols can be reused to calculate different channel outputs. Using channel simulations with $L'_c = 3$ and an oversampling factor equal to 8, 40 samples need to be filtered by the shaping and IMUX filters, interpolated and re-filtered by the OMUX filter and the receiver filter. Considering that the shaping and IMUX filters (and OMUX and receiver filters) have an impulse response larger than 40 samples, this means that already $40^2$ multiplications are involved for each convolution. Clearly, the complexity is lower using the Volterra model.

VII. CONCLUSION AND FUTURE WORK

In this paper, we have proposed a new iterative pre-distortion algorithm suited to higher order modulations for a highly non-linear satellite communication channel. It minimizes the Euclidian distance between the initial symbols and the received symbols. The algorithm relies on the assumption that the input modifications induced by one algorithm iteration are small enough to ensure a linear relation between the input and the output variations. This allows to easily calculate the optimum input variations to apply so as to decrease the MSE. The performance improvement brought by the algorithm, compared to state-of-the-art algorithms, represents several dBs on the MSE. However, a major issue of
the algorithm is the complexity involved in the estimation of the linear relation between the input and the output variations. Therefore, we have proposed alternative algorithms, where approximations of this linear relation are derived. Two kinds of approximations have been proposed, the first one relying on a reduced Volterra model and the second one being based on the use of look-up tables. Simulation results show that in spite of these approximations, a significant improvement has been strongly relaxed.

Future work will include the algorithm extension to the case where more than one carrier is amplified by the same power amplifier. A pre-distortion algorithm similar to the case of one single carrier per channel can be applied, where the pre-distorted value of the transmitted symbols are successively computed for all carriers, in an iterative way. These pre-distorted values can similarly be estimated using a linear assumption on the relation between the input and output variations. However, in a scenario where more than one carrier go through the same non-linear channel, interference between the different carriers occurs due to the non-linearity. A received symbol also depends on symbols from the other channels, so that the input-output relation becomes more complex, especially for a large number of channels. The challenge will then be to design an algorithm with tractable complexity, even for a large number of carriers.

APPENDIX A

CALCULATION OF THE COEFFICIENTS \( A_{k,j}^n(m_1, m_2) \) FOR SOME SIMPLE VOLterra MODELS

Let us first consider a channel consisting only in the third order Volterra coefficient \( H_3(0, 0, 0) \). The difference between the output \( y(x_{k,j-1} + \Delta_{k,j}) \) and the output \( y(x_{k,j-1}) \) is equal to:

\[
F_{k,j}^{NL}(j) = H_3(0, 0, 0)(|x_{k,j-1}(j) + \Delta_{k,j}|^2(x_{k,j-1}(j) + \Delta_{k,j}) - |x_{k,j-1}(j)|^2x_{k,j-1}(j)).
\]

The other outputs are not modified since the channel is memoryless. The different coefficients \( A_{k,j}^n(m_1, m_2) \) can directly be estimated from Equation (23) and are given in the second column of Table [%]

The third and fourth columns of Table [%] give the non-zero values for \( A_{k,j}^n(m_1, m_2) \) considering a channel with one single Volterra coefficient respectively equal to \( H_3(1, 0, 0) \) and \( H_3(0, 1, 2) \). It should be noticed that:

- More than one input is now modified, due to the memory of the system.
- The output variation for \( H_3(0, 1, 2) \) is linear in \( \Delta_{k,j} \). This is because all indexes of this Volterra coefficient are different.

APPENDIX B

PROOF OF EQUATION (19)

Equation (19) is here demonstrated. By definition of \( \Delta_{k,j}^{Lin} \), we have that:

\[
||\epsilon_{k,j-1} + F_{k,j}^{Lin}||_2 \Delta_{k,j} = \Delta_{k,j}^{Lin}
\]

\[
= ||\epsilon_{k,j-1}||_2 + \sum_n |A_{k,j}^n(1, 0, 0)\Delta_{k,j}^{Lin} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^{Lin}^*|^2 + 2\Re[\epsilon_{k,j-1}(n)](A_{k,j}^n(1, 0, 0)\Delta_{k,j}^{Lin} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^{Lin}^*)
\]

\[
\leq ||\epsilon_{k,j-1}||_2
\]

Therefore, we have that:

\[
2\Re[\epsilon_{k,j-1}(n)](A_{k,j}^n(1, 0, 0)\Delta_{k,j} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^*)
\]

is negative and that:

\[
\sum_n |A_{k,j}^n(1, 0, 0)\Delta_{k,j}^{Lin} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^{Lin}^*|^2
\]

\[
\leq -2\Re[\epsilon_{k,j-1}(n)](A_{k,j}^n(1, 0, 0)\Delta_{k,j} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^*)
\]

Since \( \gamma^2 < \gamma \), it is easy to see that:

\[
||\epsilon_{k,j-1} + F_{k,j}^{Lin}||_2 \Delta_{k,j} = \Delta_{k,j}^{Lin}
\]

\[
= ||\epsilon_{k,j-1}||_2 + \gamma^2 \sum_n |A_{k,j}^n(1, 0, 0)\Delta_{k,j}^{Lin} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^{Lin}^*|^2 + 2\gamma\Re[\epsilon_{k,j-1}(n)](A_{k,j}^n(1, 0, 0)\Delta_{k,j}^{Lin} + A_{k,j}^n(0, 1, 0)\Delta_{k,j}^{Lin}^*)
\]

\[
\leq ||\epsilon_{k,j-1}||_2
\]
REFERENCES

[1] G. Colavolpe and A. Piemontese. Novel SISO Detection Algorithms for Nonlinear Satellite Channels. In Global Telecommunications Conference (GLOBECOM 2011), 2011 IEEE, pages 1–5, 2011.

[2] A. Gutierrez and W.E. Ryan. Performance of Volterra and MLSD receivers for nonlinear band-limited satellite systems. Communications, IEEE Transactions on, 48(7):1171–1177, 2000.

[3] F. Perez-Cruz, J.J. Murillo-Fuentes, and S. Caro. Nonlinear Channel Equalization With Gaussian Processes for Regression. Signal Processing, IEEE Transactions on, 56(10):5283–5286, 2008.

[4] P.M. Olmos, J.J. Murillo-Fuentes, and F. Perez-Cruz. Joint Nonlinear Channel Equalization and Soft LDPC Decoding With Gaussian Processes. Signal Processing, IEEE Transactions on, 58(3):1183–1192, 2010.

[5] C.E. Bumet and W.G. Cowley. Intersymbol interference cancellation for 16QAM transmission through nonlinear channels. In Digital Signal Processing Workshop, 2002 and the 2nd Signal Processing Education Workshop. Proceedings of 2002 IEEE 10th, pages 322–326, 2002.

[6] D. Fertonani, A. Barbieri, and G. Colavolpe. Reduced-Complexity BCJR Algorithm for Turbo Equalization. Communications, IEEE Transactions on, 55(12):2279–2287, 2007.

[7] A.N. D’Andrea, V. Lottici, and R. Reggiannini. RF power amplifier linearization through amplitude and phase predistortion. Communications, IEEE Transactions on, 44(11):1477–1484, 1996.

[8] Hong Jiang and P.A. Wilford. Digital Predistortion for Power Amplifiers Using Separable Functions. Signal Processing, IEEE Transactions on, 58(8):4121–4130, 2010.

[9] M. Berdondini, M. Neri, S. Cioni, and G.E. Corazza. Adaptive Fractional Predistortion Techniques for Satellite Systems based on Neural Networks and Tables. In Vehicular Technology Conference, 2007. VTC2007-Spring. IEEE 65th, pages 1400–1404, 2007.

[10] R. Zayani and R. Bouallegue. A Neural Network Pre-Distorter for the Compensation of HPA Nonlinearity: Application to Satellite Communications. In Consumer Communications and Networking Conference, 2007. CCNC 2007. 4th IEEE, pages 465–469, 2007.

[11] M. Schetzen. The Volterra and Wiener Theories of Nonlinear Systems. Wiley, New York, 1980.

[12] Anding Zhu and T.J. Brazil. An adaptive Volterra predistorter for the linearization of RF high power amplifiers. In Microwave Symposium Digest, 2002 IEEE MTT-S International, volume 1, pages 461–464 vol.1, 2002.

[13] E. Abd-Elrady, Li Gan, and G. Kubin. Adaptive predistortion of nonlinear Volterra systems using Spectral Magnitude Matching. In Acoustics, Speech and Signal Processing, 2009. ICASSP 2009. IEEE International Conference on, pages 2985–2988, 2009.

[14] Chi-Hao Cheng and E.J. Powers. A reconsideration of the pth-order inverse predistorter. In Vehicular Technology Conference, 1999 IEEE 49th, volume 2, pages 1501–1504 vol.2, 1999.

[15] S. Benedetto and E. Biglieri. Digital transmission over nonlinear channels. In Principles of Digital Transmission, Information Technology: Transmission, Processing, and Storage, pages 725–772. Springer US, 2002.

[16] Changsoo Eun and E.J. Powers. A new Volterra predistorter based on the indirect learning architecture. Signal Processing, IEEE Transactions on, 45(1):223–227, 1997.

[17] D.R. Morgan, Zhengxiang Ma, Jaehyeong Kim, M.G. Zierdt, and J. Pastalan. A Generalized Memory Polynomial Model for Digital Predistortion of RF Power Amplifiers. Signal Processing, IEEE Transactions on, 54(10):3852–3860, 2006.

[18] G. Karam and H. Sarı. A data predistortion technique with memory for QAM radio systems. Communications, IEEE Transactions on, 39(2):336–344, 1991.

[19] E. Casini, R. De Gaudenzi, and A. Ginesi. DVB-S2 modem algorithms design and performance over typical satellite channels. International Journal of Satellite Communications and Networking, pages 281–318, 2004.

[20] Th. Deleu, M. Dervin, J.M. Dricot, Ph. De Doncker, and F. Horlin. Performance and improvement of the finite order compensation in a nonlinear DVB-S2 communication channel. In Proc. of the IEEE First AESS European Conference on Satellite Telecommunications, pages 506–510, Rome, Italy, October 2012.

[21] Digital Video Broadcasting (DVB); Second generation framing structure, channel coding and modulation systems for Broadcasting. Interactive Services, News Gathering and other broadband satellite applications (DVB-S2). ETSI EN 302 307, V1.2.1, apr 2009.