Pure Gravitational Back-Reaction Observables

N. C. Tsamis†

Institute of Theoretical & Computational Physics, and
Department of Physics, University of Crete
GR-710 03 Heraklion, HELLAS.

R. P. Woodard*

Department of Physics, University of Florida
Gainesville, FL 32611, UNITED STATES.

ABSTRACT

After discussing the various issues regarding and requirements on pure quantum gravitational observables in homogeneous-isotropic conditions, we construct a composite operator observable satisfying most of them. We also expand it to first order in the loop counting parameter and suggest it as a physical quantifier of gravitational back-reaction in an initially inflating cosmology.

PACS numbers: 04.30.-m, 04.62.+v, 98.80.Cq

† e-mail: tsamis@physics.uoc.gr
* e-mail: woodard@phys.ufl.edu
1 Introduction

Because the quantum gravitational back-reaction on inflation is contentious it is worth reviewing the mechanism of back-reaction in general terms:

- The Uncertainty Principle requires every degree of freedom of every dynamical variable to experience a minimal level of excitation known as its 0-point motion;
- 0-point motions are affected by background force fields; and
- 0-point motions contribute to the currents and stress-energies which source background force fields.

The classic example is establishing a uniform electric field, which leads to the production of charged particle pairs [1]. The resulting currents inevitably change the electric field that caused them [2, 3, 4].

That said, there are two controversial issues regarding the extent to which quantum fluctuations can really change macroscopic force fields:

- Distinguishing physical sources from gauge artifacts; and
- Distinguishing changes in background force fields from changes in the distribution of quantum fluctuations.

While the reality of back-reaction in quantum electrodynamics is established, this is not the case about analogous back-reaction potential effects to particle production in an expanding universe [5, 6, 7, 8]. Undoubtedly, quantum gravity provides fertile soil for such concerns because it has so far defied an accepted consistent perturbative formulation [9], because it possesses no local gauge invariant observables [10], and because even the simplest tree order amplitudes are difficult to compute [11]. On the other hand, it is counter-productive to generically criticize any result, for instance, as either a gauge artifact or an inappropriate choice of vacuum [12].

That is not to deny the validity of concerns about the reality of quantum gravitational back-reaction. As an example, consider the assertion that infrared effects in scalar-driven inflation induce a secular slowing of the expansion rate at 1-loop order [13]. Unruh quickly raised the issue of the gauge independence of the effect [14]. The secular slowing was found in other simple gauges as well [15]. However, all secular 1-loop effects disappeared when a fully invariant technique was employed for fixing the surface of simultaneity based on the state of the inflaton field [16] [17]. Subsequent work on the same problem has demonstrated that different results can emerge when other fields
are employed as the clock variable \[18, 19, 20, 21, 22\]. The standard local definition of the expansion rate $H$ \[23\]:

$$H(t, x) = \frac{1}{3} D^\mu u_\mu(t, x) ,$$  

is in terms of the covariant derivative $D_\mu$ of a timelike 4-velocity field $u_\mu$:

$$g^{\mu\nu}(x) u_\mu(x) u_\nu(x) = -1 .$$  

(2)

From the integral curves of the 4-velocity field we can determine whether the universe expands or contracts by showing whether these integral curves further diverge or converge, respectively. However, much depends upon the choice of the 4-velocity field. For example, even in classical de Sitter, with no quantum effects at all, it is possible to get the local expansion rate to be positive or negative by choosing $u_\mu$ to be the gradient of the time variable on either open or (early) closed coordinates, respectively \[24\].

A standard choice for the 4-velocity in scalar-driven inflation is to use the scalar inflaton field $\phi$ to construct the 4-velocity $u_\mu$ \[17\]:

$$u_\mu \equiv -\frac{\partial_\mu \phi}{\sqrt{-g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi}} .$$  

(3)

This 4-velocity field is not in general timelike. However, there is no problem in perturbation theory and while $\phi$ is rolling down its potential, provided that the change in its classical value per time interval is larger than its quantum fluctuation in the same time interval. Moreover, by expanding about the classical inflaton $\bar{\phi}(t)$ in a background $FRW$ geometry:

$$\phi(t, x) = \bar{\phi}(t) + \delta\phi(t, x) ,$$  

(4)

we can fix the time $t$ by requiring \[17\]:

$$\delta\phi(t, x) = 0 .$$  

(5)

Then, the 4-velocity field \[3\] just described corresponds to the field of observers co-moving with the inflaton.

---

\[1\]Hellenic indices take on spacetime values while Latin indices take on space values. Our metric tensor $g_{\mu\nu}$ has spacelike signature.
Our concern in this paper is not scalar-driven inflation but rather the back-reaction that would occur in pure quantum gravity if the universe was released in a prepared state that is initially locally de Sitter with Hubble parameter $H_I$ [25]. In this case there is no scalar inflaton to furnish a clock but we shall see that it is possible to construct a number of non-local scalar functionals of the metric which measure the elapsed time from the initial value surface. Any of these can be used to define a timelike 4-velocity field $u_\mu$ and to fix the surface of simultaneity, exactly as is done in scalar-driven inflation. Fixing the space point invariantly is no more possible for us than it is in scalar-driven inflation [26], but this is not considered problematic for scalar-driven inflation and it should be alright for pure gravity provided the initial state is homogeneous and isotropic.

2 A Gravitational Geometrical Observable

In the case of pure gravity there is no matter field present – like the inflaton – to be used as a time clock. The physical conditions we shall investigate assume – at some initial time $t_I$ – a prepared initial homogeneous and isotropic state and an inflating universe approximated by the de Sitter geometry. The invariant volume of the past light cone $\mathcal{V}$ is a geometrical object that can be used by an observer as his time clock. At the observation point $x$ it equals [27]:

$$\mathcal{V}[g](x) \equiv \int d^Dx' \sqrt{-g(x')} \theta(-\ell^2(x; x')) \ ,$$

where to find the length $\ell^2(x; x')$ we construct the geodesic $\chi^\mu$ from $x'$ to $x$:

$$\ddot{\chi}^\mu(\tau) + \Gamma^\mu_{\rho\sigma}[\chi(\tau)] \dot{\chi}^\rho(\tau) \dot{\chi}^\sigma(\tau) = 0 \ ,$$

$$\chi^\mu(0) = x'^\mu \ , \ \chi^\mu(1) = x^\mu \ ,$$

2Ultimately, it is the cosmological principle which precludes the existence of any special space point on which to define an observable. It is only when non-homogeneous and isotropic sources are present that we can use their location as space points on which to observe. However, the presence of an initial spacelike surface allows us to fix time physically and not by enforcing a gauge condition. In one sentence: time can be invariantly determined but space cannot.

3Hereafter, we shall work in $D$ spacetime dimensions to eventually facilitate dimensional regularization. In open coordinates, otherwise known as the cosmological patch, our de Sitter line element is: $ds^2 = -dt^2 + a^2(t) dx \cdot dx = -dt^2 + \exp(2H_I t) dx \cdot dx$, where $H_I$ is the (constant) Hubble parameter.
and then use it to obtain the desired length:

$$\ell^2(x; x') = g_{\mu\nu}(x) \dot{\chi}^\mu(1) \dot{\chi}^\nu(1) .$$  \hspace{1cm} (9)

Since the volume of the past light cone is a monotonically increasing function of time, it can serve as a geometrically meaningful time clock from which to construct the timelike $D$-velocity field thusly:

$$v_\mu[g](x) \equiv -\frac{\partial_\mu \mathcal{V}[g](x)}{\sqrt{-g^{\alpha\beta} \partial_\alpha \mathcal{V}[g](x) \partial_\beta \mathcal{V}[g](x)}} .$$  \hspace{1cm} (10)

We can invariantly fix the observation time by specifying the surfaces of simultaneity:

$$\mathcal{V}[g](T[g](x), x) = \bar{V}(t) ,$$  \hspace{1cm} (11)

where $\bar{V}(t)$ is the volume of the past light cone in de Sitter spacetime. This requirement determines the functional $T[g](x)$ or, equivalently, the observation time.

The expansion variable is given by (1):

$$H[g](x) = \frac{1}{D-1} D^\mu v_\mu[g](x) = \frac{1}{D-1} \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} v_\nu \right) ,$$  \hspace{1cm} (12)

and its time can be invariantly fixed by inheriting (25) to form:

$$H[g](x) = H[g](T[g](x), x) .$$  \hspace{1cm} (13)

The expansion variable $H$ does not have its space position invariantly fixed; indeed this is impossible in pure gravity on homogeneous and isotropic backgrounds. A completely invariant observable can be generated by integrating $H$ over the spacetime manifold taking into account condition (11):

$$\int d^D x \sqrt{-g(x)} \times H(x) \times \delta \left[ \mathcal{V}[g](T[g](x), x) - \bar{V}(t) \right] .$$  \hspace{1cm} (14)

While the latter is fully invariant, it may not be the most appropriate object to consider for our physical problem; the necessary presence of $\sqrt{-g}$ in (14) with its lack of derivatives would make it the dominant contribution, suppressing the physical results imprinted in $H$. It is therefore preferable to use $H$ – given by (13) – as our observable; in spite of not being fully invariant,
it is a scalar at an invariantly fixed time. And as mentioned earlier, it is not possible to invariantly fix space in a homogeneous and isotropic universe.

We wish to perturbatively calculate the expectation value of $H$ in the presence of a homogeneous and isotropic initial state, in an initially inflating universe with Hubble parameter $H_I$. In such a setup it is desirable to conformally re-scale the metric:

$$g_{\mu\nu} = a^2 \tilde{g}_{\mu\nu} = a^2 \left( \eta_{\mu\nu} + \kappa h_{\mu\nu} \right),$$

since such a re-scaling preserves the sign of the length $\ell^2(x;x');$ null geodesics remain the same while spacelike and timelike geodesics remain spacelike and timelike respectively.

It is a cumbersome but straightforward exercise to expand the basic elements comprising $H$ in powers of the parameter $\kappa$ starting from the basic expansion (15) of the metric in terms of the de Sitter background and the fluctuating graviton field $h_{\mu\nu}$:

$$T[g](x) = \eta + \kappa T_1(\eta, x) + \kappa^2 T_2(\eta, x) + \ldots ,$$

$$V[g](x) = \tilde{V}(\eta) + \kappa V_1(\eta, x) + \kappa^2 V_2(\eta, x) + \ldots ,$$

$$v_\mu[g](x) = -a \left( \delta_\mu^0 + \kappa v_\mu(\eta, x) + \kappa^2 v_\mu 2(\eta, x) + \ldots \right),$$

$$H[g](x) = H_I + \kappa H_1(\eta, x) + \kappa^2 H_2(\eta, x) + \ldots .$$

However, a detailed examination of the 1-loop contributions to the expectation value revealed the presence of undesirable divergences. These divergences are not ultraviolet because they do not occur at coincident points. They occur because the propagator between two points on the same light ray diverges. It is unknown how to handle such infinities and therefore we shall dispense with the use of this particular geometrically motivated observable as an indicator of back-reaction.

### 3 A Gravitational Dynamical Observable

Perhaps a more generic – but also calculationally accessible – observable can be constructed by considering a scalar functional $\Phi$ of the metric satisfying,

---

4 The relation between co-moving $t$ and conformal $\eta$ times is: $dt = a(t) \, d\eta$. The de Sitter scale factor in conformal coordinates is $a(\eta) = \left( H_I \eta \right)^{-1}$. The fluctuating graviton field is $h_{\mu\nu}(\eta, x)$ and its trace $h(\eta, x)$. Differentiation(s) with respect to $\eta$ shall be indicated with prime(s).
for all $x$, the dynamical equation:

$$\Box \Phi[g](x) = (D - 1)H_I ,$$

$$= \frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} g^{\mu \nu} \partial_\nu \Phi] = \frac{1}{a^D \sqrt{-\tilde{g}}} \partial_\mu [a^{D-2} \sqrt{-\tilde{g}} \tilde{g}^{\mu \nu} \partial_\nu \Phi] .$$

(20)

In order to solve (20) for the scalar $\Phi$, we must supply two conditions on the initial value surface (IVS):

$$\Phi(\eta_I, x)|_{IVS} = 0 , \quad -g^{\alpha \beta}(\eta_I, x) \partial_\alpha \Phi(\eta_I, x) \partial_\beta \Phi(\eta_I, x)|_{IVS} = 1 .$$

(21)

By the aforementioned procedure, the $D$-velocity field $V_\mu$ is:

$$V_\mu[g](x) \equiv -\frac{\partial_\mu \Phi[g](x)}{\sqrt{-g^{\alpha \beta}(x) \partial_\alpha \Phi[g](x) \partial_\beta \Phi[g](x)}} ,$$

(23)

and the expansion variable according to (11) is:

$$\mathcal{H}[g](x) = \frac{1}{D - 1} D^\mu V_\mu[g](x) = \frac{1}{D - 1} \frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} g^{\mu \nu} V_\nu] .$$

(24)

The surfaces of simultaneity are defined in a way analogous to (11):

$$\Phi[g](\vartheta[g](x), x) = \bar{\Phi}(\eta) ,$$

(25)

and they fix time in an invariant way. Our observable $H$ – which physically represents the expansion rate of spacetime – is given by:

$$H[g](x) \equiv \mathcal{H}[g](\vartheta[g](x), x) .$$

(26)

Under general coordinate transformations, the variables just constructed transform as follows:

$$\mathcal{H}[g'](x) = \mathcal{H}[g](x'^{-1}(x)) , \quad H[g'](\eta, x) = H[g](\eta, x'^{-1}(\eta, x)) .$$

(27)

In order to perturbatively compute the expectation value of (26) we should expand it in the parameter $\kappa$. The relevant expansions are given

---

5In general, the $D$-velocity field $V_\mu$ is not timelike. It is in perturbation theory and for the class of cosmological spacetimes of interest.
by (15), (16) with the understanding that we must effect the trivial replacements of \((T, V, v)\) with \((\vartheta, \Phi, V)\) respectively.

As a first step, it is the 1-loop result we shall be interested. Thus, we need to find the expansion of \(H\) to order \(\kappa^2\). Starting from the expansion of the scalar \(\Phi\):

\[
\Phi = \bar{\Phi} + \kappa \Phi_1 + \kappa^2 \Phi_2 + \ldots
\]

and substituting it in (20), (22) we arrive at the following equations to lowest order in \(\kappa\):

\[
\frac{1}{a^D} \partial_\mu \left[ a^{D-2} \partial_\mu \bar{\Phi} \right] = (D - 1) H_I , \quad \bar{\Phi} \big|_{IVS} = 0 ,
\]

and to first order in \(\kappa\):

\[
\frac{1}{a^D} \partial_\mu \left[ a^{D-2} \partial_\mu \Phi_1 \right] = \frac{1}{a^D} \left\{ \partial_\mu \left[ a^{D-2} h^{\mu\nu} \partial_\nu \bar{\Phi} \right] - \frac{1}{2} a^{D-2} h_{\mu\nu} \partial_\mu \partial_\nu \Phi \right\} ,
\]

\[
\Phi_1 \big|_{IVS} = 0 , \quad \frac{1}{a} \Phi'_1 \big|_{IVS} = \frac{1}{2} h_{00} \big|_{IVS} .
\]

The solution to (29) for the lowest order term is:

\[
\bar{\Phi}(\eta) = -\ln a
\]

\[
\implies \partial_\mu \bar{\Phi} = -\delta_\mu^0 a .
\]

The corresponding solution to (30), (31) is more complicated:

\[
\Phi_1(x) = \int d^D x' G_A(x; x') \left\{ -\partial'^\mu \left[ a'^{D-1} h_{0\mu}(x') \right] + \frac{1}{2} a'^{D-1} h'(x') \right\}
\]

\[
-\int_{x' = \eta_0} d^{D-1} x' G_A(x; x') \frac{1}{2} h_{00}(x') ,
\]

where the Green’s function \(G_A\) satisfies:

\[
a^D \chi G_A(x; x') = \delta^D(x - x') .
\]

There is no need to explicitly solve for the second order term \(\Phi_2\). Although it will appear in the \(O(\kappa^2)\) terms of the expansion of (26), it will not contribute when we take the expectation value of \(H\).

---

6The equation of motion (20) implies:

\[
(D - 1) H_I = \Box \Phi = \Box \Phi - a^{-D} \partial_\mu \left[ a^{D-2} \kappa h_{\mu\nu} \bar{g}^{\nu\sigma} \partial_\sigma \Phi \right] + 2 \kappa a^{-2} h_{\rho\sigma,\mu} \bar{g}^{\rho\sigma} \bar{g}^{\mu\nu} \partial_\nu \Phi ,
\]

where we have defined: \(\Box = a^{-D} \partial_\mu \left[ a^{D-2} \partial_\mu \partial_\nu \right] = a^{-2} \left[ -\partial_\mu^2 - (D - 2) h a \partial_0 + \nabla^2 \right].\)

7The analytic form of \(G_A\) can be found in [28], equation (5).
We must also ensure – to the requisite order in $\kappa$ – that condition (25) defining the surfaces of simultaneity is satisfied:

$$\bar{\Phi}(\eta) = \Phi(\eta + \Delta \eta, x)$$

$$= \Phi(\eta + \Delta \eta) + \kappa \Phi_1(\eta + \Delta \eta, x) + \kappa^2 \Phi_2(\eta + \Delta \eta, x) + \ldots \ (35)$$

This is equivalent to perturbatively solving for the time component of a spacetime point which undoes the change in the surfaces of simultaneity under coordinate transformations. The result is straightforward to obtain given that:

$$\bar{\Phi}(\eta + \Delta \eta) = \bar{\Phi}(\eta) + \bar{\Phi}'(\eta) \Delta \eta + \frac{1}{2} \bar{\Phi}''(\eta) \Delta \eta^2 + \ldots \ (37)$$

From (35,37) and (32) we conclude:

$$\Delta \eta = -\frac{\kappa \Phi_1}{\bar{\Phi}'} + \mathcal{O}(\kappa^2) = -\frac{\kappa(D - 1)H_I}{a} \Phi_1 + \mathcal{O}(\kappa^2) \ . \ (38)$$

It is also straightforward – albeit much more tedious – to expand $H$ (26) to $\mathcal{O}(\kappa^2)$. Perhaps we can summarize this undertaking as a 5-step procedure:

- We expand, using (28), the ratio:

$$\frac{\mathcal{H}}{H_I} = \left[ 1 + \frac{g^{\mu\nu} \partial_\mu \Phi \partial_\nu}{(D - 1)H_I} \right] \frac{1}{\sqrt{-g}} \partial_\alpha \Phi \partial_\beta \Phi \ . \ (39)$$

- In the resulting expansion we substitute the solution (32) for the lowest order scalar $\bar{\Phi}$.

- We shift by the coordinate transformation (38) to obtain the observable $H$:

$$\frac{H}{H_I} = \frac{\mathcal{H}}{H_I} + \frac{\Phi_1}{a} \frac{\mathcal{H}'}{H_I} + \mathcal{O}(\kappa^3) \ . \ (40)$$

- Starting from the equation:

$$\left[ 1 + \frac{1}{(D - 1)H_I a} \partial_0 \right] \left( \frac{\Phi_i'}{a} \right) = \frac{1}{(D - 1)H_I} \left[ -\square \Phi_i + \frac{\nabla^2}{a^2} \Phi_i \right] \ , \ (41)$$

(where $i = 1, 2$) we substitute in the expression which emerged from the previous step the following relations:

$$\left[ 1 + \frac{1}{(D - 1)H_I a} \partial_0 \right] \left( \frac{\Phi_i'}{a} \right) = \left[ 1 + \frac{1}{(D - 1)H_I a} \partial_0 \right] h_{00}$$
tions of motion (20) to substitute all second time derivatives of $\Phi_{ij}$.

After the above operation we act all derivatives on $H$ to obtain the final answer for the observable. We first present it in a suggestive compact notation:

$$H = 1 + \frac{1}{(D-1)H_F} \left\{ \frac{1}{2a} h' - \frac{1}{a} h_{0i,i} + \frac{\nabla^2}{a^2} \Phi_1 \right\} + O(\kappa^3)$$

$$\Phi''_i = -a^2 \Box \Phi_i - (D-2) H_F a \Phi''_i + \nabla^2 \Phi_i \quad (i = 1, 2) .$$

Execution of the above 5-step process and some algebraic manipulations result in the final answer for the observable. We first present it in a suggestive compact notation:

$$\frac{H}{H_F} = 1 + \frac{h_{ij}'}{2(D-1)H_F a} + \frac{\partial \Phi_1}{(D-1)H_F a^2}$$

and then identify each of the terms. The $O(\kappa)$ contribution is:

$$\frac{H}{H_F} = 1 + \frac{h_{ij}}{2(D-1)H_F a} + \frac{\partial \Phi_1}{(D-1)H_F a^2}$$

Its expectation value requires the addition of a single interaction vertex to reach $O(\kappa^2)$. There are four kinds of $O(\kappa^2)$ contributions:

$$\frac{H}{H_F} = \frac{1}{(D-1)H_F a} \left\{ \frac{3}{8} (D-1) H_F a h_{00} h_{00} - \frac{1}{2} (D-1) H_F a h_{0i,i} h_{0i} \right\} + \frac{\partial \Phi_1}{(D-1)H_F a^2}$$

$$\frac{H}{H_F} = \frac{1}{(D-1)H_F a^2} \left\{ \frac{1}{2} h_{00} \nabla^2 \Phi_1 + \frac{1}{2} (D-1) H_F a h_{00} \Phi_1 \right\}$$

These results are simple to extract from the graviton 1-point function [29].
\[
\frac{H}{H_I}\bigg|_{2\Phi} = \frac{1}{2a^2} \frac{D+1}{D-1} (\partial_i \Phi_1)(\partial_i \Phi_1), \quad (49)
\]

\[
\frac{H}{H_I}\bigg|_{2\partial} = \frac{1}{(D-1)H_I a} \partial_i \left\{ h_{i\mu} h_{0}^{\mu} + \frac{1}{2} h_{00} h_0 - \frac{1}{a} h_{i\mu} \partial^\mu \Phi_1 + \frac{1}{a} \partial_i \Phi_2 - h_{00} \partial_i \Phi_1 + \frac{1}{2} h_{jj} \partial_i \Phi_1 - h_0 \Phi'_1 + \partial_i \left[ \Phi_1 (\Phi'_1 - H_I a \Phi_1) \right] \right\} \quad (50)
\]

The expectation values of these four operators involve only propagators – and no interaction vertices – to reach \(O(\kappa^2)\). Upon taking the expectation value of \(H\) in the presence of a spatially invariant state, total spatial derivatives will not contribute; for instance, all of (50) and the last term of (46).

## 4 Epilogue

An observable \(H\) has been constructed and expanded to \(O(\kappa^2)\) for the purpose of quantifying the quantum gravitational back-reaction to an initially inflating universe by detecting changes to the expansion rate. Our observable is a non-local composite operator with an invariantly determined time. To compute its expectation value beyond 1-loop \((\kappa^2)\) order it would be necessary also to include perturbative corrections to the initial state. These corrections take the form of new interactions on the initial value surface \([30]\). However, because the lowest order corrections for quantum gravity are \(O(\kappa h^3)\), they cannot affect the expectation value of \(H\) at \(O(\kappa^2)\).

However unphysical one might consider 1PI functions in a fixed gauge, they have a powerful advantage over the more invariant composite operators which are sometimes proposed to quantify back-reaction. This advantage is that conventional renormalization suffices to make non-coincident 1PI functions finite whereas composite operator renormalization is required for the more complicated operators. When these composite operators are not even local it is not understood how to carry out this renormalization.

Returning to our observable \(H\), its non-locality comes from the scalar \(\Phi\) which is determined from (20) by inverting the differential operator \([20]\). There is, however, a way to make \(H\) local without altering physics in any way. We have so far been working with the pure gravity Lagrangian \(L_g\) and have introduced \(\Phi\) via (20). Except for the initial conditions (22) which do not affect the ultraviolet structure, we can introduce \(\Phi\) on the Lagrangian...
level so that we obtain \( \mathcal{L}_g \) as its equation of motion:

\[
\mathcal{L}_g = \frac{1}{16\pi G} (R - 2\Lambda)\sqrt{-g} \rightarrow (51)
\]

\[
\mathcal{L}_{g+\Phi} = \mathcal{L}_g - \frac{1}{2} \partial_\mu \Phi \partial_\nu \Phi g^{\mu\nu} \sqrt{-g} - (D - 1) H_I \Phi \sqrt{-g} . \tag{52}
\]

The variable \( \Phi \) – which was a non-local functional \( \Phi[g](x) \) of the metric in the theory \( (51) \), is now a local field \( \Phi(x) \) in the theory \( (52) \). Thus, the \( D \)-velocity field \( V_\mu \) \( (23) \) and the quantities \( \mathcal{H}, H \) \( (24) \, (26) \) automatically become local. The renormalization properties of local composite operators are understood \( [31, 32] \).

It remains to actually compute the expectation value of \( H \); first to 1-loop order \( [33] \) – where the renormalized correction should be zero – and then, hopefully, to 2-loop order where the first signals of a secular back-reaction are expected \( [7, 25] \).

Acknowledgements

We thank H. Kitamoto for extensive discussions on the problems of computing the expectation value of our observable at 1-loop order. This work was partially supported by European Union program Thalis ESF/NSRF 2007-2013 MIS-375734, by European Union (European Social Fund, ESF) and Greek national funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework (NSRF) under Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes, by NSF grant PHY-1205591, and by the Institute for Fundamental Theory at the University of Florida.

References

[1] J. Schwinger, Phys. Rev. 82 (1951) 664.

[2] Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper and E. Mottola, Phys. Rev. Lett. 67 (1991) 2427; Phys. Rev. D45 (1992) 4659.

[3] K. Srinivasan and T. Padmanabhan, Phys. Rev. D60 (1999) 024007, gr-qc/9812028.
[4] T. N. Tomaras, N. C. Tsamis and R. P. Woodard, Phys. Rev. D62 (2000) 125005, [hep-ph/0007166], JHEP 0111 (2001) 008, [hep-th/0108090].

[5] A. M. Polyakov, Nucl. Phys. B834 (2010) 316, arXiv:0912.5503; arXiv:1209.4135; D. Krotov and A. M. Polyakov, Nucl. Phys. B849 (2011) 410, arXiv:1012.2107.

[6] P. R. Anderson, C. Molina-Paris and E. Mottola, Phys. Rev. D80 (2009) 084005, arXiv:0907.0823; I. Antoniadis, P. O. Mazur and E. Mottola, JCAP 1209 (2012) 024, arXiv:1103.4164.

[7] N. C. Tsamis and R. P. Woodard, Int. J. Mod. Phys. D20 (2011) 2847, arXiv:1103.5134.

[8] P. Martineau and R. Brandenberger, astro-ph/0510523.

[9] R. P. Woodard, Rept. Prog. Phys. 72 (2009) 126002, arXiv:0907.4238.

[10] N. C. Tsamis and R. P. Woodard, Annals Phys. 215 (1992) 96.

[11] S. Sannan, Phys. Rev. D34 (1986) 1749.

[12] J. Garriga and T. Tanaka, Phys. Rev. D77 (2008) 024021, arXiv:0706.0295; N. C. Tsamis and R. P. Woodard, Phys. Rev. D78 (2008) 028501, arXiv:0708.2004.

[13] V. F. Mukhanov, L. R. W. Abramo and R. H. Brandenberger, Phys. Rev. Lett. 78 (1997) 1624, gr-qc/9609026; L. R. W. Abramo, R. H. Brandenberger and V. F. Mukhanov, Phys. Rev. D56 (1997) 3248, gr-qc/9704037.

[14] W. Unruh, astro-ph/9802323.

[15] L. R. W. Abramo and R. P. Woodard, Phys. Rev. D60 (1999) 044010, astro-ph/9811430; Phys. Rev. D60 (1999) 044011, astro-ph/9811431.

[16] L. R. Abramo and R. P. Woodard, Phys. Rev. D65 (2002) 043507, astro-ph/0109271; Phys. Rev. D65 (2002) 063515, astro-ph/0109272.

[17] G. Geshnizjani and R. Brandenberger, Phys. Rev. D66 (2002) 123507, gr-qc/0204074.
[18] G. Geshnizjani and R. Brandenberger, JCAP **0504** (2005) 006, [hep-th/0310265](http://arxiv.org/abs/hep-th/0310265).

[19] F. Finelli, G. Marozzi, G. P. Vacca and G. Venturi, Phys. Rev. Lett. **106** (2011) 121304, [arXiv:1101.1051](http://arxiv.org/abs/1101.1051).

[20] G. Marozzi and G. P. Vacca, Class. Quant. Grav. **29** (2012) 115007, [arXiv:1108.1363](http://arxiv.org/abs/1108.1363).

[21] G. Marozzi, G. P. Vacca and R. H. Brandenberger, JCAP **1302** (2013) 027, [arXiv:1212.6029](http://arxiv.org/abs/1212.6029).

[22] G. Marozzi and G. P. Vacca, [arXiv:1304.2291](http://arxiv.org/abs/1304.2291).

[23] S. W. Hawking and G. F. R. Ellis, *The large scale structure of spacetime*, Cambridge University Press, United Kingdom, 1973, pp 78-84.

[24] N. C. Tsamis and R. P. Woodard, Class. Quant. Grav. **22** (2005) 4171, [gr-qc/0506089](http://arxiv.org/abs/gr-qc/0506089).

[25] N. C. Tsamis and R. P. Woodard, Nucl. Phys. B**474** (1996) 235, [hep-ph/9602315](http://arxiv.org/abs/hep-ph/9602315); Annals Phys. B**253** (1997) 1, [hep-ph/9602316](http://arxiv.org/abs/hep-ph/9602316).

[26] T. Tanaka and Y. Urakawa, [arXiv:1306.4461](http://arxiv.org/abs/1306.4461); PTEP **2013** (2013) 6 063E02, [arXiv:1301.3088](http://arxiv.org/abs/1301.3088) [arXiv:1209.1914]; JCAP **1105** (2011) 014, [arXiv:1103.1251](http://arxiv.org/abs/1103.1251); Prog. Theor. Phys. **125** (2011) 1067, [arXiv:1009.2947](http://arxiv.org/abs/1009.2947); Phys. Rev. D**82** (2010) 121301, [arXiv:1007.0468](http://arxiv.org/abs/1007.0468); Prog. Theor. Phys. **122** (2010) 1207, [arXiv:0904.4415](http://arxiv.org/abs/0904.4415) [arXiv:0902.3209].

[27] S. Park and R.P. Woodard, Gen. Rel. Grav. **42** (2010) 2765, [arXiv:0910.4756](http://arxiv.org/abs/0910.4756).

[28] V. K. Onemli and R. P. Woodard, Phys. Rev. D**70** (2004) 107301, [gr-qc/0406098](http://arxiv.org/abs/gr-qc/0406098).

[29] N. C. Tsamis and R. P. Woodard, Annals Phys. B**321** (2006) 875, [gr-qc/0506056](http://arxiv.org/abs/gr-qc/0506056).

[30] E. O. Kahya, V. K. Onemli and R. P. Woodard, Phys. Rev. D**81** (2010) 023508, [arXiv:0904.4811](http://arxiv.org/abs/0904.4811).
[31] Steven Weinberg, *The Quantum Theory of Fields*, Volume II, Cambridge University Press, United Kingdom, 1996, pp 115-118.

[32] C. Itzykson and J. B. Zuber, *Quantum Field Theory*, McGraw-Hill, New York, 1980, pp 399-402 & 684.

[33] H. Kitamoto, N. C. Tsamis and R. P. Woodard (*work in progress*)