$B \to K_1 \pi(K)$ decays in the perturbative QCD approach

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Abstract

Within the framework of the perturbative QCD approach, we study the two-body charmless decays $B \to K_1(1270)(K_1(1400))\pi(K)$. We find the following results: (i) The decays $\bar{B}^0 \to K_1(1270)^+\pi^-$, $K_1(1400)^+\pi^-$ are incompatible with the present experimental data. There exists a similar situation for the decays $\bar{B}^0 \to a_1(1260)^+K^-, b_1(1235)^+K^-$, which are usually considered that the nonperturbative contributions are needed to explain the data. But the difference is that the nonperturbative contributions seem to play opposite roles in these two groups of decays.

(ii) The pure annihilation type decays $\bar{B}^0 \to K_1^\pm(1270)K^{\mp}, K_1^\pm(1400)K^{\mp}$ are good channels to test whether an approach can be used to calculate correctly the strength of the penguin-annihilation amplitudes. Their branching ratios are predicted at $10^{-7}$ order, which are larger than the QCDF results. (iii) The dependence of the direct CP-violating asymmetries of these decays on the mixing angle $\theta_{K_1}$ are also considered.

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I. INTRODUCTION

In general, the mesons are classified in $J^{PC}$ multiplets. There are two types of orbitally excited axial-vector mesons, namely $1^{++}$ and $1^{+-}$. The former includes $a_1(1260), f_1(1285), f_1(1420)$ and $K_{1A}$, which compose the $3^1P_1$-nonet, and the latter includes $b_1(1235), h_1(1170), h_1(1380)$ and $K_{1B}$, which compose the $1^1P_1$-nonet. Except $a_1(1260)$ and $b_1(1235)$, other axial-vector mesons exist mixing problem, which makes their inner structure become more ambiguous, for example, $K_{1A}$ and $K_{1B}$ can mix with each other and form two physical mass eigenstates $K_1(1270), K_1(1400)$. Various values about the mixing angle $\theta_{K_1}$ can be found in different literatures, which will be examined in more detail in Sec.III. For the mixings of the SU(3)-singlet and SU(3)-octet mesons, specifically, the $f_1(1285) - f_1(1420)$ mixing angle $\theta_{f_1}$ and the $h_1(1170) - h_1(1380)$ mixing angle $\theta_{h_1}$, there also exist several values in the phenomenal analysis. Certainly, these two angles can associate with $\theta_{K_1}$ through the Gell-Mann-Okubo mass formula. For the lack of sufficient experimental data, none of them can be accurately determined up to now. So the decays involving these mesons become more ambiguous compared with the decays involving $a_1(1260)$ or/and $b_1(1235)$ meson(s), which have been discussed in the previous works 1-2.

In this paper, we would like to discuss the decays $B \to K_1(1270)\pi(K), K_1(1400)\pi(K)$. On the theoretical side, many approaches have been used to study these decays, such as the naive factorization 3, the generalized factorization 4, and the QCD factorization approach 5. From the predictions of these approaches, one can find that the branching ratios of the decays $B \to K_1(1270)\pi, K_1(1400)\pi$ are in the order of $10^{-6}$, for example, $Br(B^0 \to K_1(1270)^+\pi^-) = (3 \sim 8) \times 10^{-6}$, $Br(B^0 \to K_1(1400)^+\pi^-) = (2 \sim 5) \times 10^{-6}$, those of almost all the decays $B \to K_1(1270)K, K_1(1400)K$ are in the order of $10^{-8} \sim 10^{-7}$. While on the experimental side, the large upper limits are given for the decays $B^0 \to K_1(1400)^+\pi^-$ and $B^+ \to K_1(1400)^0\pi^+$ at the 90% level (C.L.) of $1.1 \times 10^{-3}$ and $2.6 \times 10^{-3}$, respectively 6, and the Heavy Flavor Averaging Group(HFAG) gives the following results 8:

$$\begin{align*}
Br(B^+ \to K_1(1270)^0\pi^+) &< 40 \times 10^{-6},
Br(B^0 \to K_1(1270)^0\pi^+) < 39 \times 10^{-6},
\end{align*}$$

(1)

$$\begin{align*}
Br(B^0 \to K_1(1270)^+\pi^-) = (17^{+8}_{-11}) \times 10^{-6},
Br(B^0 \to K_1(1400)^+\pi^-) = (17^{+7}_{-9}) \times 10^{-6}.
\end{align*}$$

(2)

The preliminary data are given by BABAR 9,

$$\begin{align*}
Br(B^0 \to K_1^+(1270)\pi^-) & = (12.0 \pm 3.1^{+9.3}_{-4.5}) \times 10^{-6},
Br(B^0 \to K_1^+(1400)\pi^-) & = (16.7 \pm 2.6^{+3.5}_{-5.0}) \times 10^{-6}.
\end{align*}$$

(3)

(4)

Furthermore, BABAR has also measured the branching ratios $Br(B^0 \to K_1(1270)^+\pi^- + K_1(1400)^+\pi^-) = 3.1^{+0.8}_{-0.7} \times 10^{-5}$ and $Br(B^+ \to K_1(1270)^0\pi^+ + K_1(1400)^0\pi^+) = 2.9^{+2.9}_{-1.7} \times 10^{-5}$ with 7.5$\sigma$ and 3.2$\sigma$ significance, respectively. In the paper 10, the two sided intervals for some of the decays $B \to K_1(1270)\pi, K_1(1400)\pi$ are evaluated at 68% probability ($\times 10^{-5}$):

$$\begin{align*}
Br(B^- \to \bar{K}_1(1270)^0\pi^-) \in [0.0, 2.1],
Br(B^- \to \bar{K}_1(1400)^0\pi^-) \in [0.0, 2.5],
\end{align*}$$

(5)

$$\begin{align*}
Br(B^0 \to K_1(1270)^+\pi^-) \in [0.6, 2.5],
Br(B^0 \to K_1(1400)^+\pi^-) \in [0.8, 2.4].
\end{align*}$$

(6)

In view of the differences between the theories and experiments, we are going to use the PQCD approach to explore these decays and analyze whether the nonperturbative
functions can be calculated by using light-cone QCD sum rule and listed as follows:

\[ K \]

II. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy \( B \) meson, we take

\[
\Phi_B(x, b) = \frac{1}{\sqrt{2N_c}}(P_B + m_B)\gamma_5\phi_B(x, b).
\]

(7)

Here only the contribution of Lorentz structure \( \phi_B(x, b) \) is taken into account, since the contribution of the second Lorentz structure \( \bar{\phi}_B \) is numerically small and has been neglected. For the distribution amplitude \( \phi_B(x, b) \) in Eq.(7), we adopt the following model:

\[
\phi_B(x, b) = N_B x^2(1 - x)^2 \exp\left[-\frac{M_B^2x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right],
\]

(8)

where \( \omega_b \) is a free parameter, we take \( \omega_b = 0.4 \pm 0.04 \) Gev in numerical calculations, and \( N_B = 101.4 \) is the normalization factor for \( \omega_b = 0.4 \).

The distribution amplitudes of the axial-vector \( K_1 \) are written as:

\[
\langle K_1(P, \epsilon_L)|q_{2\beta}(z)q_{1\alpha}(0)|0 \rangle = \frac{i\gamma_5}{\sqrt{2N_c}} \int_0^1 dx e^{i\pi^z} [m_{K_1}\partial^z_{L} \phi_{K_1}(x) + \partial^z_{L} P \phi_{K_1}(x) + m_{K_1} \phi_{K_1}(x)]_{\alpha\beta},
\]

\[
\langle K_1(P, \epsilon_T)|q_{2\beta}(z)q_{1\alpha}(0)|0 \rangle = \frac{i\gamma_5}{\sqrt{2N_c}} \int_0^1 dx e^{i\pi^z} \left[ m_{K_1}\partial^z_{T} \phi_{K_1}(x) + \partial^z_{T} P \phi_{K_1}(x) + m_{K_1} \epsilon_{\mu\nu\rho\sigma}(\gamma_{\delta}\gamma_{5})\phi_{K_1}(x) \right]_{\alpha\beta},
\]

(9)

where \( K_1 \) refers to the two flavor states \( K_{1A} \) and \( K_{1B} \), and the corresponding distribution functions can be calculated by using light-cone QCD sum rule and listed as follows:

\[
\begin{align*}
\phi_{K_1}(x) &= \frac{f_{K_1}}{2\sqrt{2N_c}} \phi_{\perp}(x), \quad \phi_{K_1}(x) = \frac{f_{K_1}}{2\sqrt{2N_c}} \phi_{\perp}(x), \\
\phi_{K_1}^{(T)}(x) &= \frac{f_{K_1}}{2\sqrt{2N_c}} \phi_{\perp}^{(T)}(x), \quad \phi_{K_1}(x) = \frac{f_{K_1}}{2\sqrt{2N_c}} \phi_{\perp}^{(T)}(x), \\
\phi_{K_1}^{(s)}(x) &= \frac{f_{K_1}}{2\sqrt{2N_c}} \phi_{\perp}^{(s)}(x), \quad \phi_{K_1}(x) = \frac{f_{K_1}}{2\sqrt{2N_c}} \phi_{\perp}^{(s)}(x).
\end{align*}
\]

(10)

Here we use \( f_{K_1} \) to present both the longitudinally and transversely polarized states \( K_{1A}(K_{1B}) \) by assuming \( f_{K_{1A}} = f_{K_{1B}} = f_{K_1} \) for \( K_{1A} \) and \( f_{K_{1B}} = f_{K_{1B}} = f_{K_1} \) for \( K_{1B} \), respectively. It is similar for the case of \( a_1(b_1) \) states, and the difference is that here \( K_{1A} \) and \( K_{1B} \) are not the mass eigenstates. In Eq.(10), the twist-2 distribution functions are in the first line and can be expanded as:

\[
\phi_{\perp,\parallel} = 6x(1 - x) \left[ a_{0}^{\perp,\parallel} + 3a_{1}^{\perp,\parallel} + a_{2}^{\perp,\parallel} \frac{3}{2}(5t^2 - 1) \right],
\]

(11)
the twist-3 light-cone distribution amplitudes (LCDAs) are used the following forms for $K_{1A}$ and $K_{1B}$ states:

\[
h^{(t)}_{\parallel}(x) = 3a^1_{0}t^2 + \frac{3}{2}a^1_{1}t(3t^2 - 1), \quad h^{(s)}_{\parallel}(x) = 6x(1 - x)(a^1_{0} + a^1_{1}t),
\]

\[
g^{(a)}_{\perp}(x) = 6x(1 - x)(a^1_{0} + a^1_{1}t), \quad g^{(v)}_{\perp}(x) = \frac{3}{4}a^0_{0}(1 + t^2) + \frac{3}{2}a^1_{1}t^3,
\]

where $t = 2x - 1$ and the Gegenbauer moments \[12\] $a^1_{0}(K_{1A}) = 0.26^{+0.03}_{-0.02}, a^0_{0}(K_{1B}) = -0.15 \pm 0.15, a^0_{1}(K_{1A}) = a^0_{1}(K_{1B}) = 1, a^1_{1}(K_{1A}) = -1.08 \pm 0.48, a^1_{1}(K_{1B}) = 0.30^{+0.00}_{-0.31}, a^1_{1}(K_{1A}) = -0.30^{+0.26}_{-0.00}, a^1_{1}(K_{1B}) = -1.95 \pm 0.45, a^2_{1}(K_{1A}) = -0.05 \pm 0.03, a^2_{1}(K_{1B}) = 0.09^{+0.16}_{-0.18}$.

The wave functions for the pseudoscalar (P) mesons $K, \pi$ are given as:

\[
\Phi_{K(\pi)}(P, x, \zeta) \equiv \frac{1}{\sqrt{2}N_{C}}\gamma_{5}[P\phi_{K(\pi)}^{A}(x) + m_{0}\phi_{K(\pi)}^{P}(x) + \zeta m_{0}(p\cdot v - n)\phi_{K(\pi)}^{T}(x)],
\]

where the parameter $\zeta$ is either $+1$ or $-1$ depending on the assignment of the momentum fraction $x$. The chiral scale parameter $m_{0}$ is defined as $m_{0} = \frac{m_{K}^{2}}{m_{u} + m_{d}}$ for $\pi$ meson and $m_{0} = \frac{m_{K}^{2}}{m_{u} + m_{s}}$ for $K$ meson. The distribution amplitudes are expanded as:

\[
\phi_{K(\pi)}^{A}(x) = \frac{3f_{K(\pi)}}{\sqrt{6}}x(1 - x)\left[1 + a_{1K(\pi)}C_{1}^{3/2}(t) + a_{2K(\pi)}C_{2}^{3/2}(t) + a_{4K(\pi)}C_{4}^{3/2}(t)\right],
\]

\[
\phi_{K(\pi)}^{P}(x) = \frac{3f_{K(\pi)}}{2\sqrt{6}}\left[1 + \left(30\eta_{3} - \frac{5\rho_{K(\pi)}^{2}}{2}\right)C_{2}^{1/2}(t) - 3\left(\eta_{3}\omega_{3} + \frac{9\rho_{K(\pi)}^{2}}{20}(1 + 6a_{2K(\pi)})\right)C_{4}^{1/2}(t)\right],
\]

\[
\phi_{K(\pi)}^{T}(x) = \frac{-f_{K(\pi)}t}{2\sqrt{6}}\left[1 + 6(5\eta_{3} - \eta_{3}\omega_{3} - \frac{7\rho_{K(\pi)}^{2}}{5})C_{4}^{1/2}(t) - \frac{3\rho_{K(\pi)}^{2}a_{2K(\pi)}}{5}(1 - 10x + 10x^2)\right],
\]

where the decay constants $f_{K} = 0.16$ GeV, $f_{\pi} = 0.13$ GeV and the Gegenbauer moments, Gegenbauer polynomials are defined as:

\[
a_{1K} = 0.17 \pm 0.17, a_{1\pi} = 0, a_{2K} = a_{2\pi} = 0.115 \pm 0.115, a_{4K} = a_{4\pi} = -0.015,
\]

\[
C_{1}^{3/2}(t) = 3t, C_{2}^{3/2}(t) = \frac{3}{2}(5t^2 - 1), C_{4}^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4),
\]

\[
C_{2}^{1/2}(t) = \frac{1}{2}(3t^2 - 1), C_{4}^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4),
\]

and the constants $\eta_{3} = 0.015, \omega_{3} = -3$, the mass ratio $\rho_{K(\pi)} = m_{K(\pi)}/m_{0K(\pi)}$ with $m_{K} = 0.49$ GeV, $m_{0K} = 1.7$ GeV, $m_{\pi} = 0.135$ GeV, $m_{0\pi} = 1.4$ GeV.

III. THE PERTURBATIVE QCD CALCULATION

The PQCD approach is an effective theory to handle hadronic $B$ decays \[13\] \[15\]. Because it takes into account the transverse momentum of the valence quarks in the hadrons,
one will encounter the double logarithm divergences when the soft and the collinear momenta overlap. Fortunately, these large double logarithm can be resummed into the Sudakov factor \cite{16}. There also exist another type of double logarithms which arise from the loop corrections to the weak decay vertex. These double logarithms can also be resummed and resulted in the threshold factor \cite{17}. This factor decreases faster than any other power of the momentum fraction in the threshold region, which removes the endpoint singularity. It is often parameterized into a simple form which is independent on channels, twists and flavors \cite{18}. Certainly, when the higher order diagrams only suffer from soft or collinear infrared divergence, it is easy to cure by using the eikonal approximation \cite{19}. Controlling these kinds of divergences reasonably makes the PQCD approach more self-consistent.

For these two axial vector mesons, their mass eigenstates and flavor eigenstates are not the same with each other, and the former can be obtained by the latter through a mixing angle $\theta_{K_1}$:

$$K_1(1270) = K_{1A} \sin \theta_{K_1} + K_{1B} \cos \theta_{K_1}, \quad K_1(1400) = K_{1A} \cos \theta_{K_1} - K_{1B} \sin \theta_{K_1}. \quad (18)$$

Unfortunately, there are many uncertainties about this mixing angle. From various phenomenological analysis and experimental data on the masses of these two physical states, it indicates that this mixing angle is around either $33^\circ$ or $58^\circ$ \cite{20,29}. Certainly, the author of \cite{30} stresses that the sign of $\theta_{K_1}$ depends on the relative sign of flavor states $K_{1A}$ and $K_{1B}$, which can be determined by fixing the relative sign of the decay constants.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Diagrams contributing to the decay $\bar{B}^0 \to \bar{K}_{1A}^0 \pi^0$.}
\end{figure}
of $K_{1A}$ and $K_{1B}$. If the decay constants $f_{1A}$, $f_{1B}$ are the same in sign (it means that the transitions $B \rightarrow K_{1A}$ and $B \rightarrow K_{1B}$ have the opposite signs), then the mixing angle $\theta_{K_1}$ defined in (18) is positive. It is noticed that the mixing angle for the antiparticle states $K_1(1270), K_1(1400)$, which is denoted as $\theta_{K_1}$, is of opposite sign to that for the particle states $K_1(1270), K_1(1400)$. But even so, we cannot confirm whether $\theta_{K_1}$ is larger or less than 45° up to now. Different approaches and models are used and different values of the mixing angle are obtained. In order to pin down it, Cheng advocates to determine the mixing angles $\theta_{1P_1}$ and $\theta_{1P_1}$ between $f_1(1285) - f_1(1420)$ and $h_1(1170) - h_1(1380)$, respectively, which in turn depend on the $K_{1A} - K_{1B}$ mixing angle $\theta_{K_1}$ through the mass relation. Through analyzing the present data of the $h_1$, $f_1$ mesons’ strong/radiative decay modes, the author prefers $\theta_{K_1} \sim 33^\circ$ over $58^\circ$. In view of the present limited data, we will still include the mixing angle $\theta_{K_1} \sim 58^\circ$ in our calculations.

It is just because of the ambiguous mixing angle that makes the study very difficult. Here we take the decay $B^0 \rightarrow \bar{K}_1(1270)^0\pi^0$ as an example, which is contributed by the decays $\bar{B}^0 \rightarrow \bar{K}^0_{1A}\pi^0$ and $\bar{B}^0 \rightarrow \bar{K}^0_{1B}\pi^0$. Figure 1 is for the Feynman diagrams of the decay $\bar{B}^0 \rightarrow \bar{K}^0_{1A}\pi^0$ (it is similar to the decay $\bar{B}^0 \rightarrow \bar{K}^0_{1B}\pi^0$), through which the amplitudes can be calculated directly, and the total amplitudes of the decay $\bar{B}^0 \rightarrow \bar{K}_1(1270)^0\pi^0$ can be obtained by combining the two sets of flavor state amplitudes according to Eq.(18):

$$\sqrt{2}A(\bar{K}_1(1270)^0\pi^0) = -\xi_t(f_{K_{1A}}\sin \theta_{K_1} + f_{K_{1B}}\cos \theta_{K_1})F_{e\pi}^{LL}(a_4 - \frac{1}{2}a_{10}) - \xi_t(M_{e\pi}^{LL:K_{1A}}\sin \theta_{K_1} + M_{e\pi}^{LL:K_{1B}}\cos \theta_{K_1})(C_3 - \frac{1}{2}C_9) - \xi_t(M_{e\pi}^{LR:K_{1A}}\sin \theta_{K_1} + M_{e\pi}^{LR:K_{1B}}\cos \theta_{K_1})(C_5 - \frac{1}{2}C_7) - \xi_t(M_{a\pi}^{LL:K_{1A}}\sin \theta_{K_1} + M_{a\pi}^{LL:K_{1B}}\cos \theta_{K_1})(C_3 - \frac{1}{2}C_9) - \xi_t(M_{a\pi}^{LR:K_{1A}}\sin \theta_{K_1} + M_{a\pi}^{LR:K_{1B}}\cos \theta_{K_1})(C_5 - \frac{1}{2}C_7) - \xi_t f_B(F_{e\pi}^{LL:K_{1A}}\sin \theta_{K_1} + F_{a\pi}^{LL:K_{1B}}\cos \theta_{K_1})(a_4 - \frac{1}{2}a_{10}) - \xi_t f_B(F_{e\pi}^{SP:K_{1A}}\sin \theta_{K_1} + F_{e\pi}^{SP:K_{1B}}\cos \theta_{K_1})(a_6 - \frac{1}{2}a_{8}) + f_\pi(E_{eK_{1A}}\sin \theta_{K_1} + E_{eK_{1B}}\cos \theta_{K_1}) \left[ \xi_u a_1 - \xi_t \left( \frac{3C_9}{2} + C_{10} \right) - \frac{3C_7}{2} - \frac{C_6}{2} \right] \right) + (M_{eK_{1A}}^{LL:}\sin \theta_{K_1} + M_{eK_{1B}}^{LL:}\cos \theta_{K_1}) \xi_u C_2 - \xi_t (M_{eK_{1A}}^{SP:}\sin \theta_{K_1} + M_{eK_{1B}}^{SP:}\cos \theta_{K_1}) \frac{3C_8}{2}, \quad (19)$$

where $\xi_u = V_{ub}V_{us}^*, \xi_t = V_{tb}V_{ts}^*$, $F_{e(a)M_1}^{M_2}$ and $M_{e(a)M_1}^{M_2}$ denote the amplitudes of factorizable and nonfactorizable emission (annihilation) diagrams, where the subscript meson $M_1$ is involved in the $B^0$ meson transition, the superscript meson $M_2$ is the emitted particle. The other superscript in each amplitude denotes different current operators, $(V - A)(V - A), (V - A)(V + A)$ and $(S - P)(S + P)$ corresponding to $LL, LR$ and $SP$, respectively. If exchanging the positions of $K_{1A}$ and $\pi^0$ in Fig.1(a), 1(b), 1(c) and 1(d), we will get the
new Feynman diagrams, which can also contribute to the decay $\bar{B}^0 \rightarrow \bar{K}^0_{1A}\pi^0$, and the corresponding amplitudes are given in the last three lines of Eq. (19). The amplitudes for the decay $B^0 \rightarrow \bar{K}^0_{1A}(K^0_{1B})\pi^0$ can be obtained from those for the decay $B \rightarrow K\pi$ which can be found in [21], only changing the variables of $K$ meson with those of $K^0_{1A}(K^0_{1B})$ meson. So we do not list the analytic expressions for these amplitudes. Certainly, it is noticed that if the axial-vector meson $K^0_{1A}(K^0_{1B})$ is on the emitted position in the factorizable emission diagrams, there is no scalar or pseudoscalar current contribution. The total amplitudes for the other three $B \rightarrow K_1(1270)\pi$ decay modes can also be written out similarly:

$$A(K_1(1270)^-\pi^+) = (f_{K_1A}^\ell \sin \theta_{K_1} + f_{K_1B}^\ell \cos \theta_{K_1}) F_{1\ell}^{e\pi} (\xi_u a_1 - \xi_t (a_4 + a_{10})) + (M_{e\pi,K_1A}^{LL} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) (\xi_u c_1 - \xi_t (C_3 + C_9)) - \xi_t (M_{e\pi,K_1A}^{LR} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LR,\ell} \cos \theta_{K_1}) (C_5 + C_7) - \xi_t (M_{a\pi,K_1A}^{LL} \sin \theta_{K_1} + M_{a\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) (C_3 - \frac{1}{2} C_9)$$

$$- \xi_t (M_{a\pi,K_1A}^{LR} \sin \theta_{K_1} + M_{a\pi,K_1B}^{LR,\ell} \cos \theta_{K_1}) (C_5 - \frac{1}{2} C_7) - \xi_t f_B (F_{a\pi,K_1A}^{LL} \sin \theta_{K_1} + F_{a\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) (a_4 - \frac{1}{2} a_{10}) - \xi_t f_B (F_{SP,K_1A}^{e\pi} \sin \theta_{K_1} + F_{SP,K_1B}^{e\pi} \cos \theta_{K_1}) (a_6 - \frac{1}{2} a_8),$$

(20)

$$\sqrt{2} A(K_1(1270)^0\pi^0) = (f_{K_1A}^\ell \sin \theta_{K_1} + f_{K_1B}^\ell \cos \theta_{K_1}) F_{1\ell}^{e\pi} [\xi_u a_1 - \xi_t (a_4 + a_{10})] + (M_{e\pi,K_1A}^{LL} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) [\xi_u c_1 - \xi_t (C_3 + C_9)] - \xi_t (M_{e\pi,K_1A}^{LR} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LR,\ell} \cos \theta_{K_1}) (C_5 + C_7) + (M_{a\pi,K_1A}^{LL} \sin \theta_{K_1} + M_{a\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) [\xi_u c_1 - \xi_t (C_3 + C_9)] - \xi_t (M_{a\pi,K_1A}^{LR} \sin \theta_{K_1} + M_{a\pi,K_1B}^{LR,\ell} \cos \theta_{K_1}) (C_5 + C_7) + f_B (F_{a\pi,K_1A}^{LL} \sin \theta_{K_1} + F_{a\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) [\xi_u a_2 - \xi_t (a_4 + a_{10})] - f_B (F_{SP,K_1A}^{e\pi} \sin \theta_{K_1} + F_{SP,K_1B}^{e\pi} \cos \theta_{K_1}) \xi_t (a_6 + a_8) + f_{\pi} (F_{K_1A}^{LL} \sin \theta_{K_1} + F_{K_1B}^{LL,\ell} \cos \theta_{K_1}) \left[ \xi_u a_1 - \xi_t \left( \frac{3 C_9}{2} + \frac{C_10}{2} \right) \right] + (M_{e\pi,K_1A}^{LL,\pi} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LL,\pi} \cos \theta_{K_1}) [\xi_u c_2] - \xi_t \left( \frac{3 C_7}{2} - \frac{C_8}{2} \right) \right) + (M_{e\pi,K_1A}^{LL,\pi} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LL,\pi} \cos \theta_{K_1}) [\xi_u c_2] - \xi_t \left( \frac{3 C_7}{2} - \frac{C_8}{2} \right),$$

(21)

$$A(\bar{K}_1(1270)^0\pi^-) = - \xi_t (f_{K_1A}^\ell \sin \theta_{K_1} + f_{K_1B}^\ell \cos \theta_{K_1}) F_{1\ell}^{e\pi} (a_4 - \frac{1}{2} a_{10}) - \xi_t (M_{e\pi,K_1A}^{LL} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) (C_3 - \frac{1}{2} C_9) - \xi_t (M_{e\pi,K_1A}^{LR} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LR,\ell} \cos \theta_{K_1}) (C_5 - \frac{1}{2} C_7) + (M_{e\pi,K_1A}^{LL} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) [\xi_u c_1 - \xi_t (C_3 + C_9)] - \xi_t (M_{e\pi,K_1A}^{LR} \sin \theta_{K_1} + M_{e\pi,K_1B}^{LR,\ell} \cos \theta_{K_1}) (C_5 + C_7) + f_B (F_{a\pi,K_1A}^{LL} \sin \theta_{K_1} + F_{a\pi,K_1B}^{LL,\ell} \cos \theta_{K_1}) [\xi_u a_2 - \xi_t (a_4 + a_{10})] - f_B (F_{SP,K_1A}^{e\pi} \sin \theta_{K_1} + F_{SP,K_1B}^{e\pi} \cos \theta_{K_1}) (a_6 + a_8).$$

(22)
It is easy to get the total amplitudes for the decay modes including \( \bar{K}_1(1400)^0/K_1(1400)^- \) by making the replacements with \( \sin \theta_{K_1} \rightarrow \cos \theta_{K_1}, \cos \theta_{K_1} \rightarrow -\sin \theta_{K_1} \) in Eqs. (19), (22), respectively. The total amplitudes for each \( B \rightarrow K_1(1270)\bar{K}, K_1(1400)\bar{K} \) decay are given in Appendix A.

IV. NUMERICAL RESULTS AND DISCUSSIONS

The input parameters in the numerical calculations \([32, 33]\) are listed as follows:

\[
\begin{align*}
 f_B &= 210 \text{MeV}, f_{K_{1A}} = 250 \text{MeV}, f_{K_{1B}}^+ = 190 \text{MeV} \\
 \tau_{B^\pm} &= 1.638 \times 10^{-12} \text{s}, \tau_{B^0} = 1.525 \times 10^{-12} \text{s}, \\
 |V_{ud}| &= 0.974, |V_{td}| = 8.67 \times 10^{-3}, |V_{ub}| = 3.51 \times 10^{-3}, \\
 |V_{ts}| &= 0.0404, |V_{us}| = 0.22534, |V_{tb}| = 0.999. 
\end{align*}
\]

Using the input parameters and the wave functions as specified in this section and Sec II, it is easy to get the branching ratios for the considered decays which are listed in Table II, where the first error comes from the uncertainty in the \( B \) meson shape parameter \( \omega_b = 0.40 \pm 0.04 \) GeV, the second error is from the hard scale \( t \), which we vary from 0.8\( t \) to 1.2\( t \), and the third error is from the combined uncertainties of the Gegenbauer moments \( a_1^+(K_{1A}) = -1.08 \pm 0.48 \) and \( a_1^+(K_{1B}) = -1.95 \pm 0.45 \). From Table II we can find that the branching ratios of \( B \rightarrow K_1(1270)\pi, K_1(1400)\pi \) decays fall in \( 10^{-6} \) order. The experimental data for the branching ratios of the decays \( \bar{B}^0 \rightarrow K_1(1270)^-\pi^+, K_1(1400)^-\pi^+ \), which are given as \( (12.0 \pm 3.1^{+9.3}_{-4.5}) \times 10^{-6} \) and \( (16.7 \pm 2.6^{+3.5}_{-5.0}) \times 10^{-6} \), respectively, are large and incompatible with all the present theory predictions. Even for the two sided intervals \( Br(B^0 \rightarrow K_1(1270)^-\pi^+) \in [0.6, 2.5] \times 10^{-5} \) and \( Br(B^0 \rightarrow K_1(1270)^-\pi^+) \in [0.8, 2.4] \times 10^{-5} \), they almost can not contain the different theoretical results. While the branching ratios of the charged \( B \) decays can be explained by the theories for the large uncertainties of the intervals \( Br(B^- \rightarrow K_1(1270)^0\pi^-) \in [0.0, 2.1] \times 10^{-5}, Br(B^- \rightarrow K_1(1400)^0\pi^-) \in [0.0, 2.5] \times 10^{-5} \). The large large differences between theories and experiments do not happen to the decays \( \bar{B}^0 \rightarrow a_1(1260)^\pm\pi^\mp \), which are tree-dominated. If the decay constants \( f_{a_1}, f_\pi \) and the form factors \( V_0^{B^0 \rightarrow a_1}, F_0^{B^- \rightarrow \pi} \) can be well determined, it is not difficult for us to predict the branching ratios of the decays \( \bar{B}^0 \rightarrow a_1(1260)^\pm\pi^\mp \) accurately, because the penguin contributions can be neglected and there are fewer uncertainties. For the considered decays \( \bar{B}^0 \rightarrow K_1^+\pi^- \), the tree operators are suppressed by the CKM matrix elements \( V_{ub}V_{us}^*/(V_{cb}V_{cs}^*) \sim 0.02 \), and the penguin operators will play a significant role. If the future data are really larger than the present predictions for here considered decays, the authors [6] claimed that there are two possible reasons: one is because the larger corrections from the weak annihilation and the hard spectator contributions, the other is from the charming penguin contributions. In our calculations, the hard spectator contributions which correspond to the non-factorization emission diagram ones are very small. Although the factorizable annihilation contributions are more important, they can not promote the branching ratios too much. So we consider that the charming penguins are more likely to explain the large data. Unfortunately, the charming penguins are non-perturbative in nature and remain untouched by many theory approaches. While it is helpful to consider these decays by
to quantitatively calculate the annihilation type contributions. In the previous years both methods, one can say that the PQCD approach is one of the few methods, which can be used. If our predictions can be confirmed by the future LHCb or the super B experiment, then the branching ratios might not be so small as those predicted by the QCDF limits up to now. Although the decays $\bar{B} \rightarrow \bar{K}^0(1400)\pi^0$, $B \rightarrow K_1(1400)\bar{K}$ and $B \rightarrow K_1(1400)K$, $K_1(1400)K$ for mixing angle $\theta_{K_1} = -33^\circ$. Other model predictions are also presented here for comparison. It is noticed that the results of [4] and [5] are obtained for mixing angle $32^\circ$, while those in [6] are obtained for mixing angle $-37^\circ$.

| decay | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^-$ | $B^0 \rightarrow K_1^0(1270)\pi^+$ | $B^0 \rightarrow K_1^0(1270)\pi^-$ | $B^0 \rightarrow K_1^0(1270)\pi^+$ | $B^0 \rightarrow K_1^0(1270)\pi^-$ | $B^0 \rightarrow K_1^0(1270)\pi^+$ | $B^0 \rightarrow K_1^0(1270)\pi^-$ | $B^0 \rightarrow K_1^0(1270)\pi^+$ |
|--------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\theta_0$ | 0.7 1.2 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 |
| $\theta_1$ | 0.7 1.2 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 |

TABLE I: Branching ratios (in units of $10^{-6}$) for the decays $B \rightarrow K_1(1270)\pi$, $K_1(1400)\pi$ and $B \rightarrow K_1(1270)K$, $K_1(1400)K$ for mixing angle $\theta_{K_1} = -33^\circ$. Other model predictions are also presented here for comparison. It is noticed that the results of [4] and [5] are obtained for mixing angle $32^\circ$, while those in [6] are obtained for mixing angle $-37^\circ$.

| decay | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ | $B^0 \rightarrow K_1^0(1270)\pi^0$ |
|--------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\theta_0$ | 0.7 1.2 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 |
| $\theta_1$ | 0.7 1.2 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 | 2.8 1.5 |

using the soft-collinear-effective-theory (SECT) [34], where the charming penguin contributions from loop diagrams are included. Certainly, these contributions can also be incorporated in the final-state interactions [35]. There exits the similar situation for the decays $\bar{B} \rightarrow a_1(1260)K^-, b_1(1235)K^-$. There are no experimental data or upper limits up to now. Although the decays $\bar{B} \rightarrow K_1^\pm K^\mp$ can occur only via annihilation type diagrams, their branching ratios might not be so small as those predicted by the QCDF approach. If our predictions can be confirmed by the future LHCb or the super B experiments, one can say that the PQCD approach is one of the few methods, which can be used to quantitatively calculate the annihilation type contributions. In the previous years both...
the experimenter and the theorists considered that the branching ratio of $B^0 \to K^+K^-$ was at $10^{-8}$ order, but two years ago the CDF and LHCb collaborations gave their first measurements of this decay by $(2.3 \pm 1.0 \pm 1.0) \times 10^{-7}$ [36] and $(1.3^{+0.6}_{-0.5} \pm 0.7) \times 10^{-7}$ [37], respectively. Later, these results are confirmed by the PQCD recalculated result $1.56 \times 10^{-7}$ [33] without introducing too much uncertainties. It shows that the PQCD approach can determine correctly the strength of penguin-annihilation amplitudes. Whether the PQCD approach can give reasonable predictions for the pure annihilation decays $B^0 \to K_1(1270)^\pm K^\mp$, $K_1(1400)^\pm K^\mp$ also deserves our attention and research. For the decay $B^0 \to K_{1B}^0K^0$ can not receive a large emission factorization amplitude, because of the small decay constant $f_{K_{1B}}$ compared with $f_{K_{1A}}$, while it has a large annihilation factorization amplitude, which makes its branching ratio slightly larger than that of $B^0 \to K_{1A}^0K^0$. The branching ratios of these two decays are at the order of $10^{-7}$. But it is very different to the decay $B^0 \to K_{1B}^0K^0$. Except having a large annihilation factorization amplitude, it can also obtain a large emission factorization amplitude at the same time, because here the emission meson is $K^0$ with a larger decay constant $f_K = 0.16$. So this decay gets a large branching ratio, which amounts to $2.71 \times 10^{-6}$. Even though the decay $B^0 \to K_{1A}^0K^0$ has a small branching ratio, the physical final states $K_1(1200)^0K^0$, $K_1(1400)^0K^0$, which are mixtures of the former two group flavor states, still might get a large branching ratio. It has been verified by the different theories, which are shown in Table II. But the branching ratio of the decay $B^0 \to K_1(1400)^0K^0$ predicted by the QCDF approach seems too small compared with the results given by the PQCD and the naive factorization approaches, which can be clarified by the future experiments. There exists the similar situation for the decay $B^0 \to K_1(1400)^-K^0$. Another decay channel, where exists large divergence between the predictions, is $B^0 \to K_1(1200)^-K^0$. The Feynman diagrams of this decay can be obtained from those of the decay $B^0 \to K_1(1200)^0K^0$ by replacing the spectator quark $d$ with $u$, so the difference of the branching ratios of these two decays should not be so large. In a word, the branching ratios of the charged $B$ decays are at or near the
order of $10^{-6}$, those of the pure annihilation decays are at the order of $10^{-7}$ by taking the mixing angle $\theta_{K_1}=33^\circ$.

In order to compare with other theoretical predictions, we also list the branching ratios with the mixing angle $\theta_{K_1}=-58^\circ$ shown in Table III. One can find that the branching ratios of the decays $B^- \to K_1^0(1270)K^0$, $\bar{B}^0 \to \bar{K}_1^0(1270)K^0$ have a remarkable decrease from the mixing angles $-33^\circ$ to $-58^\circ$, while those of the decays $B^- \to K_1^-\bar{K}_1^0(1400)K^0$, $\bar{B}^0 \to \bar{K}_1^0(1400)K^0$ have a remarkable increase.

Now we turn to the evaluations of the CP-violating asymmetries in the PQCD approach. For the neutral $B^0$ (the charged $B^-$) decays the direct CP-violating asymmetries can be defined as

$$ A_{CP}^{dir} = \frac{\Gamma(\bar{B}^0(B^-) \to f) - \Gamma(B^0(B^+) \to \bar{f})}{\Gamma(\bar{B}^0(B^-) \to f) + \Gamma(B^0(B^+) \to \bar{f})} = \frac{2\sin \theta \sin \delta}{1 + 2\cos \theta \cos \delta + z^2} , \tag{27} $$

where $\delta$ is the relative strong phase between the tree and penguin amplitudes, and $\theta$ the CKM weak phase $\theta = \alpha$ for $b \to d$ transition, $\theta = \gamma$ for $b \to s$ transition. Certainly, if the final states are the same for $B^0$ and $\bar{B}^0$, that is $f = \bar{f}$, the CP-asymmetries may be time-dependent, including not only the direct CP violation but also the mixing-induced CP violation. Using the input parameters and the wave functions as specified in this section

| $B^0 \to K_1^- (1270)\pi^+$ | 4.3 7.6 | 2.7$^{+0.6+1.3+4.4}_{-0.5-0.8-1.5} | 3.2^{+0.7+0.5+0.8}_{-0.5-0.5-0.8}$ |
| $\bar{B}^0 \to \bar{K}_1^0 (1270)\pi^0$ | 2.1 0.4 | 0.8$^{+0.1+0.5+1.7}_{-0.1-0.3-0.6}$ | 0.5$^{+0.2+0.0+0.4}_{-0.0-0.2-0.2}$ |
| $B^- \to K_1^0 (1270)\pi^-$ | 4.7 5.8 | 3.0$^{+0.2+0.1+2.7}_{-0.2-0.2-2.2}$ | 3.2$^{+1.3+1.2+1.3}_{-0.9-0.8-1.2}$ |
| $B^- \to K_1^- (1270)\pi^0$ | 1.6 4.9 | 2.5$^{+0.1+1.0+3.2}_{-0.1-0.7-1.0}$ | 3.3$^{+1.1+0.7+0.8}_{-0.8-0.6-1.1}$ |
| $\bar{B}^0 \to \bar{K}_1^0 (1400)\pi^+$ | 2.3 4.0 | 2.2$^{+1.1+0.7+2.6}_{-0.8-0.6-1.3}$ | 4.5$^{+0.0+0.3+1.5}_{-0.0-0.5-1.3}$ |
| $\bar{B}^0 \to \bar{K}_1^0 (1400)\pi^0$ | 1.6 1.7 | 1.5$^{+0.4+0.3+1.7}_{-0.3-0.3-0.9}$ | 4.1$^{+0.4+0.4+0.4}_{-0.4-0.4-0.8}$ |
| $B^- \to \bar{K}_1^0 (1400)\pi^0$ | 2.3 2.5 | 2.8$^{+1.0+0.9+3.0}_{-0.8-0.9-1.7}$ | 5.4$^{+0.3+1.6+1.5}_{-0.2-1.2-1.4}$ |
| $B^- \to \bar{K}_1^- (1400)\pi^0$ | 0.6 1.4 | 1.0$^{+0.4+0.4+1.2}_{-0.3-0.4-0.5}$ | 2.5$^{+0.0+0.3+0.8}_{-0.0-0.4-0.7}$ |

TABLE III: Same as Table II except for the mixing angle $\theta_{K_1} = -58^\circ$. 
and Sec. III it is easy to get the PQCD predictions (in units of $10^{-2}$) for the direct CP-violating asymmetries of $B$ decaying to each flavor final state, which are listed in Table IV. For the real physical final states, which are mixes of the corresponding flavor states, their direct CP-violating asymmetries will be dependent on the mixing angle $\theta_{K_1}$. As has been emphasised before, $\theta_{\bar{K}_1}$ for the antiparticle states $\bar{K}_1(1270), \bar{K}_1(1400)$ is of opposite sign to that for the particle states $K_1(1270), K_1(1400)$. For taking the convention of decay con-
FIG. 4: The dependence of the direct CP-violating asymmetries on the mixing angle $\theta_{K_1}$: the solid lines represent the decays $B^- \to K_1(1270)^0 K^0$ (left), $B^0 \to K_1(1270)^- K^+$ (right), the dashed lines are for the decays $B^- \to K_1(1270)^0 K^-$ (left), $B^0 \to K_1(1270)^+ K^-$ (right), the dot lines are for the decays $B^- \to K_1(1400)^0 K^-$ (left), $B^- \to K_1(1400)^+ K^-$ (right), and the dash-dot lines represent the decays $B^- \to K_1(1400)^0 K^-$ (left), $B^0 \to K_1(1400)^+ K^-$ (right), respectively.

TABLE IV: Direct CP violation (in units of %) for the decays $B \to K_{1A} \pi, K_{1B} \pi$ and $B \to K_{1A} K, K_{1B} K$. The errors for these entries correspond to the uncertainties from $\omega_B = 0.4 \pm 0.04$ GeV, the hard scale $A = 0.8$ for each decay.

| Decay            | $A_{CP}$ (%$\pm$1) |
|------------------|---------------------|
| $B^0 \to K_{1A}^+ \pi$ | $9.1^{+2.4+0.8+3.0}_{-2.0-0.8-3.4}$ |
| $\bar{B}^0 \to K_{1A}^0 \pi$ | $-6.6^{+1.3+0.9+2.8}_{-1.4-1.0-8.4}$ |
| $B^- \to K_{1A}^0 K^-$ | $-2.3^{+0.8+0.8+1.5}_{-1.2-0.6-6.8}$ |
| $B^- \to K_{1A}^+ K^-$ | $17.7^{+4.1+3.0+17.1}_{-3.5-3.1-7.4}$ |
| $B^0 \to K_{1A}^+ K^+$ | $43.9^{+1.7+0.5+0.0}_{-1.3-3.1-35.6}$ |
| $B^0 \to K_{1A}^+ K^-$ | $46.5^{+0.5+4.4+0.3}_{-1.3-3.3-29.5}$ |
| $B^- \to K_{1A}^0 K^0$ | $6.6^{+1.6+3.1+4.9}_{-1.7-3.8-1.8}$ |
| $B^- \to K_{1A}^+ K^0$ | $-29.4^{+7.6+2.6+86.7}_{-6.3-1.8-0.0}$ |

 CONSTANTS $f_{K_{1B}}$ in this work, so $\theta_{K_1}$ is positive and $\theta_{\bar{K}_1}$ is negative. In Fig. we give the dependence of the direct CP-violating asymmetries on the mixing angle $\theta_{K_1}$ for each decay. Here taking $\theta_{K_1} = -33^\circ$ or $\theta_{K_1} = -58^\circ$, we can read each direct CP-violating asymmetry from these figures. It is noticed that for the decays $B^0 \to K_1(1270)^+ K^-, K_1(1400)^+ K^-$, $B^- \to K_1(1270)^0 K^-, K_1(1400)^0 K^-$, which include the particle states, their direct CP-violating asymmetry values are still read at $-33^\circ$ or $-58^\circ$ for $\theta_{K_1} = -\theta_{\bar{K}_1}$ and so the corresponding mixing angle is positive. The signs of the direct CP-violating asymmetries of
$B \rightarrow K_{1}(1270)\bar{K}(\pi)$ and $B \rightarrow K_{1}(1400)\bar{K}(\pi)$ are opposite at the mixing angle $\theta_{K_{1}} = -33^\circ$ for most of these decays except only two groups, whose direct CP-violating asymmetries are predicted as $A_{\text{CP}}^{\text{dir}}(B^{0} \rightarrow \bar{K}_{1}(1270)^{0}\pi^{0}) = -12.6\%$, $A_{\text{CP}}^{\text{dir}}(B^{0} \rightarrow \bar{K}_{1}(1400)^{0}\pi^{0}) = -6.7\%$ and $A_{\text{CP}}^{\text{dir}}(B^{0} \rightarrow K_{1}(1270)^{+}K^{-}) = 12.2\%$, $A_{\text{CP}}^{\text{dir}}(B^{0} \rightarrow K_{1}(1400)^{+}K^{-}) = 9.6\%$, respectively. From Table IV one can find that the direct CP-violating asymmetries of each decay become very large. For example, the direct CP-violating asymmetries of the decays $\bar{B}^{0}$ and $B^{0}$ violating asymmetries. As for the decays $\bar{B}^{0}$ may also change. So we can’t confirm that these decays must have so large direct CP-violating asymmetries for some considered decays where the nonperturbative contributions, such as charming penguins, give large corrections, and the corresponding direct CP-violating asymmetries for some considered decays where the nonperturbative contributions, such as charming penguins, are needed to explain the data. But the difference is that the nonperturbative contributions seem to play opposite roles in these two groups of considered decays, it might indicate that the nonperturbative contributions have pronounced corrections for some decay channels which include the higher resonances while the other decays with the branching ratios at $10^{-7}$ order. So the decays $\bar{B}^{0} \rightarrow K_{1}(1200)^{0}\bar{K}^{0}$, $\bar{K}_{1}(1400)^{0}\bar{K}^{0}$ which include the real physical states can have large branching ratios at the mixing angle $\theta_{K_{1}} = -33^\circ$ compare with the decays $\bar{B}^{0} \rightarrow K_{1}(1200)^{0}\bar{K}^{0}$, $K_{1}(1400)^{0}\bar{K}^{0}$.

V. CONCLUSION

In this paper, by using the decay constants and the light-cone distribution amplitudes derived from the QCD sum-rule method, we research the decays $B \rightarrow K_{1}(1270)^{0}\pi(K)$, $K_{1}(1400)^{0}\pi(K)$ in the PQCD approach and find that

- All the theoretical predictions for the branching ratios of the decays $\bar{B}^{0} \rightarrow K_{1}(1270)^{0}\pi^{-}, K_{1}(1400)^{0}\pi^{-}$ are incompatible with the present experimental data. There exists the similar situation for the decays $\bar{B}^{0} \rightarrow a_{1}(1260)^{+}K^{-}, b_{1}(1235)^{+}K^{-}$, where the nonperturbative contributions, such as the final state interactions or the charming penguins, are needed to explain the data. But the difference is that the nonperturbative contributions seem to play opposite roles in these two groups of decays. If the future data are really larger than the present predictions for some considered decays, it might indicate that the nonperturbative contributions have pronounced corrections for some decay channels which include the higher resonances in the final states.

- The pure annihilation type decays $\bar{B}^{0} \rightarrow K_{1}^{\pm}(1270)K^{\mp}, K_{1}^{\mp}(1400)K^{\pm}$ are good channels to test whether an approach can be used to calculate correctly the strength of the penguin-annihilation amplitudes. Their branching ratios are predicted at $10^{-7}$ order.

- In the four final neutral flavor states $K_{1A}^{0}\bar{K}^{0}, K_{1B}^{0}\bar{K}^{0}, K_{1A}^{0}K^{0}, K_{1B}^{0}K^{0}$, the decay $\bar{B}^{0} \rightarrow K_{1A}^{0}\bar{K}^{0}$ have the largest branching ratio which is of $10^{-6}$ order, while the other decays with the branching ratios at $10^{-7}$ order. So the decays $\bar{B}^{0} \rightarrow K_{1}(1200)^{0}\bar{K}^{0}, \bar{K}_{1}(1400)^{0}\bar{K}^{0}$ which include the real physical states can have large branching ratios at the mixing angle $\theta_{K_{1}} = -33^\circ$ compare with the decays $\bar{B}^{0} \rightarrow K_{1}(1200)^{0}\bar{K}^{0}, K_{1}(1400)^{0}\bar{K}^{0}$.
• The signs of the direct CP-violating asymmetries are opposite between almost of the decays $B \to K_1(1270)K(\pi)$ and $B \to K_1(1400)K(\pi)$ at mixing angle $\theta_{K_1} = -33^\circ$ except only two groups, whose direct CP-violating asymmetries are predicted as $A_{CP}^{\text{dir}}(\bar{B}_0^0 \to \bar{K}_1(1270)\pi^0) = -12.6\%$, $A_{CP}^{\text{dir}}(\bar{B}_0^0 \to \bar{K}_1(1400)\pi^0) = -6.7\%$ and $A_{CP}^{\text{dir}}(\bar{B}_0^0 \to K_1(1270)^+K^-) = 12.2\%$, $A_{CP}^{\text{dir}}(\bar{B}_0^0 \to K_1(1400)^+K^-) = 9.6\%$, respectively.

• The strong phase introduced by the nonperturbative contributions might produce dramatic effects on some of the considered decays, such as $\bar{B}_0^0 \to K_1(1270)^-\pi^+, K_1(1400)^-\pi^+, K_1(1270)^-\pi^0, K_1(1270)^-\pi^0$, and these effects could exceed those from the parametric uncertainties in the case of the CP asymmetries.

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Appendix A: Analytic formulas for the decay amplitudes

\[
A(K_1(1270)^0 K^-) = -\xi_t(f_{K_{1A}} \sin \theta_{K_1} + f_{K_{1B}} \cos \theta_{K_1}) F_{eK}^{LL}(a_4 - \frac{1}{2} a_{10}) \\
-\xi_t(M_{eK}^{LL;K_{1A}} \sin \theta_{K_1} + M_{eK}^{LL;K_{1B}} \cos \theta_{K_1})(C_3 - \frac{1}{2} C_9) \\
-\xi_t(M_{eK}^{LR;K_{1A}} \sin \theta_{K_1} + M_{eK}^{LR;K_{1B}} \cos \theta_{K_1})(C_5 - \frac{1}{2} C_7) \\
-\xi_t(M_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + M_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(C_3 - \frac{1}{2} C_9) \\
-\xi_t(M_{aK}^{LR;K_{1A}} \sin \theta_{K_1} + M_{aK}^{LR;K_{1B}} \cos \theta_{K_1})(C_5 - \frac{1}{2} C_7) \\
-\xi_t(M_{aK}^{SP;K_{1A}} \sin \theta_{K_1} + M_{aK}^{SP;K_{1B}} \cos \theta_{K_1})(C_6 - \frac{1}{2} C_8) \\
-\xi_t f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(a_3 - \frac{1}{2} a_{9}) \\
-\xi_t f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(a_4 - \frac{1}{2} a_{10}) \\
-\xi_t f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(a_5 - \frac{1}{2} a_7) \\
-\xi_t f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(a_6 - \frac{1}{2} a_8) \\
-\xi_t f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(a_7 - \frac{1}{2} a_9) \\
-\xi_t f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(a_8 - \frac{1}{2} a_{10}), \quad (A1)
\]

\[
A(K_1(1270)^0 K^-) = -\xi_t(f_{K_{1A}} \sin \theta_{K_1} + f_{K_{1B}} \cos \theta_{K_1}) F_{eK}^{LL}(a_4 - \frac{1}{2} a_{10}) \\
-\xi_t(M_{eK}^{LL;K_{1A}} \sin \theta_{K_1} + M_{eK}^{LL;K_{1B}} \cos \theta_{K_1})(C_3 - \frac{1}{2} C_9) \\
-\xi_t(M_{eK}^{LR;K_{1A}} \sin \theta_{K_1} + M_{eK}^{LR;K_{1B}} \cos \theta_{K_1})(C_5 - \frac{1}{2} C_7) \\
+ (M_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + M_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(\xi_u C_1 - \xi_t(C_3 + C_9)) \\
-\xi_t(M_{aK}^{LR;K_{1A}} \sin \theta_{K_1} + M_{aK}^{LR;K_{1B}} \cos \theta_{K_1})(C_5 + C_7) \\
+ f_B(F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1})(\xi_u a_2 - \xi_t(a_4 + a_{10}) \\
-\xi_t f_B(F_{aK}^{SP;K_{1A}} \sin \theta_{K_1} + F_{aK}^{SP;K_{1B}} \cos \theta_{K_1})(a_6 + a_8). \quad (A2)
\]
In the upper two formulae, if changing the first term as $-\xi t f_K (F_{eK_{1A}}^{LL} \sin \theta_{K_1} + F_{eK_{1B}}^{LL} \cos \theta_{K_1}) (a_4 - \frac{1}{2} a_{10}) - \xi t f_K (F_{eK_{1A}}^{SP} \sin \theta_{K_1} + F_{eK_{1B}}^{SP} \cos \theta_{K_1}) (a_6 - \frac{1}{2} a_{8})$, and at the same time exchanging the positions of $K_{1A} (K_{1B})$ and $K$ in other terms, we will get the decay amplitudes of $B^0 \rightarrow \bar{K}_1 (1270)^0 K^0$ and $B^- \rightarrow K_1 (1270)^- K^0$, respectively.

$$A(K_1 (1270)^+ K^-) = (M_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + M_{aK}^{LL;K_{1B}} \cos \theta_{K_1}) \left( \xi a C_2 - \xi t (C_4 + C_{10}) \right)$$

$$- \xi t (M_{aK}^{SP;K_{1A}} \sin \theta_{K_1} + M_{aK}^{SP;K_{1B}} \cos \theta_{K_1}) (C_6 + C_8)$$

$$+ f_B (F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1}) (\xi a a_1 - \xi t (a_3 + a_5 + a_7 + a_9))$$

$$- \xi t f_B (F_{aK}^{LL;K_{1A}} \sin \theta_{K_1} + F_{aK}^{LL;K_{1B}} \cos \theta_{K_1}) (a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9)$$

$$- \xi t (M_{aK}^{LL;K} \sin \theta_{K_1} + M_{aK}^{LL;K} \cos \theta_{K_1}) (C_4 - \frac{1}{2} C_{10})$$

$$- \xi t (M_{aK}^{SP;K} \sin \theta_{K_1} + M_{aK}^{SP;K} \cos \theta_{K_1}) (C_6 - \frac{1}{2} C_8). \quad (A3)$$

In Eq. (A3), if exchanging the positions of $K_{1A} (K_{1B})$ and $K$, we will get the total amplitude of the decay $B^0 \rightarrow K_1 (1270)^- K^+$. The total amplitudes of the decays $B \rightarrow K_1 (1400) K$ can be obtained by making the replacements with $\sin \theta_{K_1} \rightarrow \cos \theta_{K_1}$, $\cos \theta_{K_1} \rightarrow -\sin \theta_{K_1}$ in Eqs. (A1)-(A3), respectively.

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