Evidence for Special Relativity with de Sitter Space-Time Symmetry *

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I show the formulation of de Sitter Special Relativity (dS-SR) based on Dirac-Lu-Zou-Guo’s discussions. dS-SR quantum mechanics is formulated, and the dS-SR Dirac equation for hydrogen is suggested. The equation in the earth-QSO framework reference is solved by means of the adiabatic approach. It’s found that the fine-structure “constant” \( \alpha \) in dS-SR varies with time. By means of the \( t - z \) relation of the \( \Lambda \)CDM model, \( \alpha \)'s time-dependency becomes redshift \( z \)-dependent. The dS-SR’s predictions of \( \Delta \alpha / \alpha \) agree with data of spectra of 143 quasar absorption systems, the dS-space-time symmetry is \( SO(3, 2) \) (i.e., anti-dS group) and the universal parameter \( R \) (de Sitter ratio) in dS-SR is estimated to be \( R \approx 2.73 \times 10^{12} \text{ly} \). The effects of dS-SR become visible at the cosmic space-time scale (i.e., the distance \( \geq 10^9 \text{ly} \)). At that scale dS-SR is more reliable than Einstein SR. The \( \alpha \)-variation with time is an evidence of SR with de Sitter symmetry.

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I, INTRODUCTION

Einstein’s Special Relativity (E-SR) is the cornerstone of physics, and any discovery beyond E-SR would be very significant. E-SR indicates the space-time metric is \( \eta_{\mu \nu} = \text{diag}\{+,-,-,-\} \). The most general transformation to preserve metric \( \eta_{\mu \nu} \) is Poincaré group. It is well known that the Poincaré group is the limit of the de Sitter group with sphere radius \( R \to \infty \). Thus people could pursue whether there exists another type of de Sitter transformation with \( R \to \text{finite} \) which also leads to a Special Relativity theory (SR). In 1935, P.A.M. Dirac presented an electron wave equation in de Sitter space, and suggested the study of atomical physics in the equation based on such a kind of special relativity, i.e., the Special Relativity with de Sitter symmetry (dS-SR)\(^1\). Differing from General Relativity (GR), SRs rely on two principles: 1) The inertial motion law for free particle must hold; 2) There must exist a specific space-time symmetry in the frameworks. Both E-SR and dS-SR satisfy these two principles (see below). To address the difference between GR and SR, Dirac pointed out \(^2\) that the de Sitter space-time is associated “with no local gravitational fields” (just like the case in Minkowski space).

In this paper I will study dS-SR, and solve dS-SR Dirac equation of hydrogen atom by means of adiabatic approximation, and show that the time-variation of fine-structure constant reported by \(^2\)\(^3\) is an evidence for dS-SR, and hence an effect beyond E-SR. In other words, the true SR for real world is dS-SR with \( SO(3, 2) \) dS-space-time symmetry (or anti-dS group) and dS sphere radius \( R \approx 2.73 \times 10^{12} \text{ly} \) instead of E-SR.

![FIG. 1: A sketch map showing an example of a spectrum of gas clouds seen in absorption against background quasi-stellar object (QSO) (download from M.T. Murphy’s slide file (2009)).](image)

Spectroscopic observations of gas clouds seen in absorption against background quasi-stellar object (QSO) (see Fig 1) have been used to search for time variation of \( \alpha = e^2/(\hbar c) \). Comparing the observations with the corresponding atomic spectra measured in laboratory, the results clearly show the first experimental evidence for the fundamental physics constant variations \(^2\)\(^4\). Even though there are some debates on the results \(^2\), this discovery is very significant, and greatly stimulated the various theoretical discussions during the last decade (e.g., see \(^2\)\(^5\)). BSBM model \(^8\)\(^9\) is one of them. This theory models the variation of \( \alpha \) by means of a scalar field which obeys an Euler-Lagrangian equation derived from an action. Combining the scalar field theory with General Relativity (GR) and adjusting the model’s pa-

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rameters, one can get suitable results describing the α-variation and evolutions along with z (redshift). However, the price paid for the successes of BSMB is that an unknown matter field (i.e., scalar field) has to be introduced. Some authors called the force propagating by the quanta of such unknown field as “fifth force” [11], which breaks the electric charger conservation law [12], and violates the weak equivalence principle [3]. There is not yet any experimental evidence to show the existence of such material scalar field so far besides explaining time-variation of α. In this case, therefore, searching for alternative scenario without any unknown particle to explain the α-variation with time would be more conservative, and hence more reliable for solving the puzzle.

Moreover, the absorption spectra observations resulting in declaration of α-variation with time reported by [2] [3] rely on the measurements of the spectrum’s fine-structures of atoms and ions at gas clouds near QSO. So, if possible, Quantum Mechanics (QM) calculations of atomic spectra for atoms in the distance in some suitable model would be a direct answer to the puzzle. For example, the dS-SR atomic physics scheme suggested by Dirac [1] should be considered seriously. As is well known, the spectra fine-structures in atomic physics represent E-SR corrections to levels, which are in principle derived from E-SR Dirac equation in QM. Especially, E-SR Dirac equation of hydrogen in QM has exact solution, and the calculations of such corrections are sound. Those corrections are space-time independent, and hence α is a constant due to the space-time translation invariant symmetry of E-SR. Thus, it should be very interesting to pursue what the dS-SR corrections to the levels of atoms in distance are in QM by means of solving the dS-SR Dirac equation of hydrogen. Because the time translations of dS-SR are significantly different from those of E-SR, one could expect that dS-SR QM may yield time-dependent α, and lead to solving the puzzle. In the following, I pursue this topic.

II, SOLUTIONS OF HYDROGEN’S DS-SR DIRAC EQUATION

In order to precisely formulate the dS-SR space-time theory and dynamics, in 1970–1974, LU, ZOU and GUO [25–13] (for the English version, see Refs. [14–15]) proved two theorems as follows:

Lemma I: Inertial motion law for free particles holds to be true in the de Sitter space characterized by Beltrami metric

\[ B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{\lambda}{R^2 \sigma(x)^2} \eta_{\mu\nu} x^\lambda x^\nu, \]  

where \( \sigma(x) \equiv 1 - R^2 \eta_{\mu\nu} x^\mu x^\nu, \) \( R^2 > 0, \) and \( \lambda = 1 \) or \(-1\) which corresponds to dS symmetries SO(4, 1) or SO(3, 2) respectively. And the constant \( R \) is the radius of the pseudo-sphere in dS-space. This means that in dS space characterized by \( B_{\mu\nu}, \) the velocity of free particle is constant, i.e.,

\[ \dot{x} = \vec{v} = \text{constant}, \]  

\[ \text{for free particle} \]  

which is exactly the counterpart of E-SR’s inertial law in Minkowski space characterized by \( \eta_{\mu\nu}. \) (see Refs. [14–15] for the English version of proof to Eq. [2]).

Lemma II: The de Sitter space-time transformation preserving \( B_{\mu\nu}(x) \) as is follows

\[ x^\mu \rightarrow \tilde{x}^\mu = \pm \sigma(a)^{1/2} \sigma(a, x)^{-1} (x^\nu - a^\nu) D^\mu_{\nu}, \]  

\[ D^\mu_{\nu} = L^\mu_{\nu} + \lambda R^{-2} \eta_{\mu\nu} a^\rho \lambda (\sigma(a) + \sigma^{-1/2}(a))^{-1} L^\rho_{\lambda}, \]  

\[ L_{\lambda} : = (L^\mu_{\nu}) \in SO(1, 3), \]  

\[ \sigma(x) = 1 - \frac{\lambda}{R^2} \eta_{\mu\nu} a^\mu x^\nu, \]  

\[ \sigma(a, x) = 1 - \frac{\lambda}{R^2} \eta_{\mu\nu} a^\mu x^\nu, \]  

where \( x^\mu \) is the coordinate in an initial Beltrami frame, and \( \tilde{x}^\mu \) is in another Beltrami frame whose origin is \( a \) in the original one. There are 10 parameters in the transformations between them. Under the transformation \[ \tilde{x}^\mu, \] we have the equation preserving \( B_{\mu\nu} \) as follows

\[ B_{\mu\nu}(x) \rightarrow \tilde{B}_{\mu\nu}(x) = \frac{\partial x^\lambda}{\partial \tilde{x}^\nu} \frac{\partial x^\mu}{\partial \tilde{x}^\nu} B_{\lambda\rho}(x) = B_{\mu\nu}(\tilde{x}). \]  

(see Appendix of Ref. [13] for the English version of proof to Eq. [4]). Eq. [4] will yield conservation laws for the energy, momenta, angular momenta and boost chargers of particles in dS-SR mechanics [13].

Based on those two lemmas, Yan, Xiao, Huang and Li formulated the Lagrangian-Hamiltonian formulism for dS-SR dynamics with two universal constants \( c \) and \( R, \) and the dS-SR Dirac equation has been proved to be [15–18]:

\[ (ie_a^\mu \gamma^a D^\mu_{\nu} - \frac{mc}{\hbar}) \psi = 0, \]  

where \( e_a^\mu \) is the tetrad satisfying \( e_a^\mu e_b^\nu \eta^{ab} = B_{\mu\nu}, \) and \( D^\mu_{\nu} = \partial^\mu - \frac{i}{4} \omega^{\mu}_{\nu\lambda} \sigma^{\lambda} \) is the covariant derivative with Lorentz spin connection \( \omega^{\mu}_{\nu\lambda} \) derived from \( B_{\mu\nu}, \) of Eq. [1]. Furthermore, by gauge principle, \( D^\mu_{\nu} \rightarrow D^\mu_{\nu} = D^\mu_{\nu} - i e/\hbar A^\mu, \) with \( A^\mu_{\nu} = B^\mu_{\nu} A^\nu, A^\nu = (\phi, A = 0) \) and \(-B^{\gamma\delta} \partial_\gamma \partial_\delta \phi = \frac{2 e}{\sqrt{-\det(B_{\gamma\delta})}} \tilde{e} (3)(x)\) where \( \phi \) is the proton’s electric Coulomb potential, one has the dS-SR Dirac equation for the electron in hydrogen atom as follows

\[ (ie_a^\mu \gamma^a D^\mu_{\nu}^L - \frac{\mu c}{\hbar}) \psi = 0, \]  

where

\[ \mu = m_e/(1 + m_e/m_p) \]  

is the reduced mass of electron. In this formulism, the measurable conserved 4-momentum operator is [18]

\[ p^\mu = i\hbar \left( \gamma^\mu - \frac{\lambda x^\mu x^\nu}{R^2} \right) \partial_\nu - \frac{5\lambda x^\mu}{2R^2} \right]. \]
The observation results reported by [2] are the absorption spectra of gas clouds against background QSO. We briefly call the gas-QSO system as QSO for simplicity. We are interested in the atoms, typically the hydrogen atom, at QSO that locates on the light-cone in de Sitter space with Beltrami metrics because only this kind of QSO can be observed by earth-observers. As illustrated in Fig. 2(a), the earth locates at the origin of the frame, the proton (nucleus of hydrogen atom) locates at \( Q = \{ Q^0 \equiv c t, Q^1 = c t, Q^2 = 0, Q^3 = 0 \} \), which is on QSO-light-cone \( B_{\mu \nu}(Q) = \eta_{\mu \nu} e^{Q \mu Q \nu} = 0 \).

Thus, rewriting (8) as

\[
E_\psi = \frac{\hbar c^2}{R^2} \left[ \partial_t - \frac{\lambda c^2 \ell_2}{R^2} \partial_t + \frac{5 \lambda c \ell_3}{2 R^2} \right] \psi \simeq \hbar \left[ 1 - \frac{\lambda c^2 \ell_2}{R^2} \right] \partial_\ell \psi.
\]

Then Eq. (8) becomes

\[
E_\psi = -i \hbar c^2 \left[ \partial_t - \frac{\lambda c^2 \ell_2}{R^2} \partial_t + \frac{5 \lambda c \ell_3}{2 R^2} \right] \psi - \frac{e^2}{r_B} \psi,
\]

where \( r_B = \sqrt{(\hat{x}^1)^2 + (\hat{x}^2)^2 + (\hat{x}^3)^2} \) with \( \hat{x}^1 = (1 - \lambda c^2 \ell_2/(2 R^2)) \hat{x} \) and \( \nabla_B = i \frac{\bar{x}^1}{r_B} + j \frac{\bar{x}^2}{r_B} + k \frac{\bar{x}^3}{r_B} \). Eq. (8) is time-dependent quantum Hamiltonian equation. It is somewhat difficult to deal with the time-dependent problems in quantum mechanics. Fortunately, comparing (8) with usual E-SR Dirac equation for hydrogen, all correction terms due to dS-SR are proportional to \( (c^2 \ell^2/R^2) \). Since \( R \gg c t \), those factors make the time-evolution of the system so slow that the adiabatic approximation will legitimately work (see Chapter XVII of Vol II of [20], and Appendix B in [18]). Thus, rewriting (8) as

\[
E_\psi = -i \hbar c^2 \left[ \partial_t - \frac{\lambda c^2 \ell_2}{R^2} \partial_t + \frac{5 \lambda c \ell_3}{2 R^2} \right] \psi - \frac{e^2}{r_B} \psi
\]

we obtain the predictions

\[
\alpha_t \equiv \frac{e^2}{\hbar c^2} = (1 + \frac{\lambda c^2 \ell_2}{2 R^2}) \alpha, \quad \frac{\Delta \alpha}{\alpha} = \frac{\lambda c^2 \ell_2}{2 R^2}, \quad \frac{\Delta \alpha}{\alpha} = (\frac{1}{\Delta \alpha/\alpha} - 1)^{1/2}.
\]

\[
\omega_t = \frac{E}{\hbar c^2} \left[ 1 + \frac{\alpha_t^2}{(\sqrt{R^2 - \alpha_t^2} + n_r)^2} \right]^{-1/2}.
\]

**FIG. 2:** Sketch of the earth-QSO reference frame. The earth locates at the origin. The position vector for the nucleus of the atom on QSO is \( Q \), and for the electron is \( L \). The distance between the nucleus and electron is \( r \).

### III. Comparison between Theory Predictions and Observation Data

Murphy and collaborators [3] studied the spectra of 143 quasar absorption systems over the redshift range \( 0.2 < z_{abs} < 4.2 \). Their most robust estimate is a weighted mean

\[
\frac{\Delta \alpha}{\alpha} = (-0.57 \pm 0.11) \times 10^{-3}.
\]

Compared with the prediction (9), we conclude that

\[
\lambda = -1.
\]

This means that the space-time symmetry for dS-SR is de Sitter-SO(3, 2) instead of anti-de Sitter-SO(4, 1). Substituting Eq. (12) into (9), we predict as follows

\[
\frac{\Delta \alpha}{\alpha} = -\frac{c^2 \ell^2}{2 R^2}.
\]

The 134 data points are assigned three epochs in Ref. [21] (see Table I), and the redshift \( z \)-dependence of \( \Delta \alpha/\alpha \) is shown roughly in [21]. In the following, I further test the prediction of (9) in terms of these \( z \)-dependent data of \( \Delta \alpha/\alpha \). In order to transfer the \( t \)-dependence of \( \Delta \alpha/\alpha \) in (9) to a \( z \)-dependence prediction, a relation of \( t - z \) is needed. For this aim, an appropriate cosmological model is necessary. In this paper, we treat \( t \) as comoving time \( t \) in the \( \Lambda \)CDM model [22, 23]. In the model, the \( t - z \) relation is as follows

\[
t = \int_0^z \frac{dz'}{H(z')(1 + z')}. \quad (14)
\]
TABLE I: Time variations of $\Delta \alpha/\alpha$: The first two columns are quoted from [21]. Eq. (13) with $R \approx 2.73 \times 10^{12} /y$, and the $\Lambda$CDM model’s $t-z$ relation (4) are used.

| redshift ($z$) and ($t$) | $\Delta \alpha/\alpha_{\text{exp}}$ | results of [13] |
|--------------------------|-----------------------------------|-----------------|
| 0.65 (6.04Gyr)           | $(-0.29 \pm 0.31) \times 10^{-5}$ | $-0.24 \times 10^{-5}$ |
| 1.47 (9.29Gyr)           | $(-0.58 \pm 0.13) \times 10^{-5}$ | $-0.58 \times 10^{-5}$ |
| 2.84 (11.39Gyr)          | $(-0.87 \pm 0.37) \times 10^{-5}$ | $-0.87 \times 10^{-5}$ |

where $H(z') = H_0 \sqrt{\Omega_{m0}(1+z')^3 + 1 - \Omega_{m0}}$, $H_0 = 100 h = 100 \times 0.705 \text{km} \cdot \text{s}^{-1} / \text{Mpc}$, $\Omega_{m0} \simeq 0.274$.

The $t-z$ relation is shown in Fig.3(a). Substituting this relation into (4), we obtain a desirable $z$-dependence prediction of $\Delta \alpha/\alpha(z)$, where $R$ is a free parameter. By using the observation data $\Delta \alpha/\alpha(z = 1.47) = -0.58 \times 10^{-5}$, we get $R \approx 2.73 \times 10^{12} / y$ (which is consistent with the estimation in [24]). Then the theory predictions are $\Delta \alpha/\alpha(z = 0.65) = -0.24 \times 10^{-5}$ and $\Delta \alpha/\alpha(z = 2.84) = -0.87 \times 10^{-5}$, which are in agreement with the corresponding data in [3] and [21]. The results are listed in Table I, and the curve of $\Delta \alpha/\alpha(z)$ is shown in Fig.3(b). The comparison indicates that the dS-SR theory predictions of [9] agree with the observation data within the error band.

FIG. 3: (a) The $t-z$ relation in $\Lambda$CDM model; (b) The $\Delta \alpha/\alpha$ as function of the red shift $z$; (c) The evolution of $\alpha$-variations $\Delta \alpha/\alpha(z)$ along with $z$. By Eq. (13) with $R \approx 2.73 \times 10^{12} / y$ and the $\Lambda$CDM model’s $t-z$ relation, $\Delta \alpha/\alpha(z)$ curve is plotted in the region of ($0 \leq z \leq 4000$).

Next, we turn to discuss the evolution of $\alpha$-variations $\Delta \alpha/\alpha(z)$ along with $z$, and plot $\Delta \alpha/\alpha(z)$ curve in the region of ($0 \leq z \leq 4000$) in Fig.3(c). We can see that as $z < 10$, $\Delta \alpha/\alpha(z)$ changes relatively sharply, and then the changes become slow. When $z \geq 10^3$, $\Delta \alpha/\alpha(z)$ is almost independent of $z$, i.e., $\alpha$-variation ceases that very high $z$ region. Fig 3(c) shows that the lower bound of $\Delta \alpha/\alpha(z)$ is about $\sim 1.3 \times 10^{-5}$. This result coincides with other considerations (e.g., BSBM model) [9], which suggests a negligible change in $\alpha$ in the radiation epoch of the Universe, that epoch roughly corresponds to $z \geq 3 \times 10^4$.

IV, CONCLUSION

In summary, in this paper, I have shown the formulation of de Sitter Special Relativity (dS-SR) based on Dirac-Lu-Zou-Guo’s discussions, formulated the d-SR quantum mechanics, and then determined the d-SR Dirac equation for hydrogen. In order to discuss the spectra of atoms on (or near) QSO, I solved it in the earth-QSO framework reference by means of the adiabatic approach. Aspects of de Sitter space-time geometry described by Beltrami metric are taken into account. The dS-SR Dirac equation of hydrogen turns out to be a time dependent quantum Hamiltonian system. Since the radius of de Sitter sphere $R$ is cosmologically large, it makes the time-evolution of the system so slow that the adiabatic approximation legitimately works with high accuracy. Consequently, it is revealed that all those facts yield important conclusions that the electromagnetic fine-structure “constant” $\alpha$ varies with time. By means of the $t-z$ relation of the $\Lambda$CDM model, the $\alpha$’s time-dependent becomes redshift $z$-dependent. The dS-SR’s predictions of $\Delta \alpha/\alpha$ are in agreement with the data, the dS-space-time symmetry is $SO(3,2)$ (i.e., anti-dS group) and the universal parameter $R$ (the de Sitter ratio) in the theory is estimated to be $R \approx 2.73 \times 10^{12} / y$. This fact indicates that the effects of dS-SR become visible at the cosmic space-time scale (i.e., the distance $\geq 10^3 / y$). At that scale de Sitter Special Relativity is more reliable than Einsteinian Special Relativity, and the latter is the former’s approximation for the distance which is much less than $R$, or much less than $\sim 10^4 / y$.

I conclude that the $\alpha$-variation with time is evidence of SR with de Sitter symmetry.

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