Small Violation of Universal Yukawa Coupling and Neutrino Large Mixing

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Abstract

We assume the universal Yukawa coupling (democratic mass matrix) with small violations for quarks, charged leptons and neutrinos masses. We could reproduce the mass hierarchy for quark masses and $V_{\text{CKM}}$ matrix elements precisely. We adopt the see-saw mechanism for the explanation of smallness of neutrino masses and introduce the right-handed Majorana neutrinos and Dirac neutrinos. We assume the universal Yukawa coupling with small violations for Majorana and Dirac neutrinos. We can get the hierarchy of charged lepton masses and effective neutrino masses and the large mixing of neutrinos expressed in $V_{\text{NMS}}$. 

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§1. Introduction

Super-Kamiokande experiment has confirmed the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation to be very large $\sin^2 2\theta_{atm} \sim 1$ and the range of mass parameter $\Delta m^2_{atm}$ to be $(2-5) \times 10^{-3}\text{eV}^2$ by their atmospheric neutrino experiments.\[1\] Solar neutrino experiments analysis by Super-Kamiokande collaboration gives a conclusion that the large MSW solution is favored those which suggests $\nu_\mu \leftrightarrow \nu_e$ oscillation is large $\sin^2 2\theta_\odot \sim 1$ and $\Delta m^2_\odot \sim 10^{-5} - 10^{-4}\text{eV}^2$.\[2\] In the framework of three-flavor neutrinos, we can put $\Delta m^2_{atm}$ to $\Delta m^2_{23}$ and $\Delta m^2_\odot$ to $\Delta m^2_{12}$, and then can consider the mass hierarchy $m_1 \simeq m_2 \ll m_3$. In this three-flavor neutrino framework, $\sin^2 2\theta_{atm} = \sin^2 2\theta_{23} \sim 1$ and $\sin^2 2\theta_\odot = \sin^2 2\theta_{12} \sim 1$ and the remaining mixing is restricted to be $\sin^2 2\theta_{13} < 0.10$ by the CHOOZ experiment.\[3\]

The quark mixing is expressed by a $V_{\text{MKS}}$ matrix and the neutrino mixing by a $V_{\text{MNS}}$ matrix.\[4\] The question why the neutrino sector mixings are so large although the quark sector mixings are small is the current most challenging one. The neutrino masses measured in neutrino oscillation bring the next question why the neutrino masses are so small. In order to explain these questions, many works\[5,6\] have been proposed. Almost works have studied adopting the so-called Froggatt Nielsen mechanism\[7\] that is induced from the spontaneous breaking of some family symmetry. For the smallness of neutrino mass, almost works use the see-saw mechanism\[8\] in which mass of the light neutrino is suppressed by a large scale of some unified theory.

For the quark sector, there is the universal Yukawa coupling approach (democratic mass matrix approach), which explain the mass hierarchy and small mixing of $V_{\text{MKS}}$ matrix.\[9,10\] Especially, our approach\[10\] could reproduced the numerical results of quark mass hierarchy and $V_{\text{CKM}}$ matrix elements precisely by using the universal Yukawa coupling with small violations. This approach stands on the following scenario: (1) The main mass hierarchies between $(u, c)$ and $t$ in $(u, c, t)$ sector and between $(d, s)$ and $b$ in $(d, s, b)$ sector are induced by the universality of Yukawa coupling. This feature is characterized by the diagonalization of universal Yukawa coupling (democratic mass matrix) $M_0 = m \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ for quarks to the $\text{diag}[0, 0, 3m]$ using the unitary matrix $U_0 = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$. Mass hierarchies between $u$ and $c$ in $(u, c)$ and $d$ and $s$ in $(d, s)$ are caused by small violations added to the universal Yukawa coupling $M_0$. (2) The smallness of the mixing parameters in $V_{\text{CKM}}$ is produced by the difference between the small violations for the $(u, c, t)$ sector and
(d, s, b) sector, because the \( V_{\text{CKM}} \) is the product of unitary transformation \( T \) for the (u, c, t) sector and \( T^\dagger \) for the (d, s, b) sector modified by the small violation from \( T_0 \).

For the charged lepton and neutrino sector, we adopt the same scenario as (1) in order to explain the mass hierarchy of charged lepton and neutrino masses. However, the neutrino masses are very small compared with the charged lepton masses by the order as \( m_{\nu_\tau}/m_\tau \sim 10^{-11} \). In order to explain the smallness of neutrino masses, we adopt the see-saw mechanism introducing the right-handed Majorana neutrino with very large masses. We assume that the Majorana neutrino masses are produced by the universal Yukawa coupling with small violations as other fermions.

From this scenario, we can get the hierarchical charged lepton masses and the transformation matrix \( T \) modified by the small violation from \( T_0 \). For neutrino masses, the effective neutrino mass \( M_{\text{eff}} \) produced through the see-saw mechanism are expressed as 
\[
M_{\text{eff}} = M_D M_M^{-1} M_D^T,
\]
where \( M_D \) and \( M_M \) are the Dirac neutrino mass matrix and the Majorana neutrino mass matrix, respectively. Though these neutrino mass matrices \( M_D \) and \( M_M \) are democratic mass matrices in our scenario, the effective neutrino mass matrix \( M_{\text{eff}} \) could be almost diagonal if the small violations in \( M_M \) satisfy some condition. If the effective neutrino mass matrix \( M_{\text{eff}} \) is almost diagonal, translating matrix \( T \) for neutrino is almost unit matrix. Then the neutrino mixing matrix \( V_{\text{MNS}} \sim T_0 \) and large neutrino mixing is realized. Recently an analysis\(^6\) using the same scenario as our present one is presented, but the pattern of violation parameters added to the democratic mass matrix is different from ours.

The condition realizing the large neutrino mixing does not depend on the detail of the model. The stability of the condition is also guaranteed with respect to radiative corrections. Our scenario uses the similarity between (d, s, b) quarks and (e, \( \mu \), \( \tau \)) lepton, and the see-saw mechanism introducing the Majorana neutrino. Thus our approach has to assume the unified \( SU(5) \) symmetry at the least. We can get the rather precise numerical rule of the violation parameters for quark sector, but not get the precise numerical result for lepton sector. In order to discuss about the generation symmetry, we have to get the more precise numerical data on the lepton sector.

§2. Violation of universal Yukawa coupling

Usually, in order to generate the mass hierarchy of quarks and leptons, Froggatt and Nielsen\(^7\) mechanism are used. This mechanism assumes that an abelian horizontal symmetry \( U(1)_X \) and higher dimensional operators involving one or several electroweak singlet scalar fields which acquire a vacuum expectation values breaking the horizontal symmetry
at some large scale. In procedure using this mechanism, the pattern of mass hierarchy of quarks and leptons and mixings for these fermions are sensitive to the horizontal symmetry adopted and charges of the horizontal symmetry assigning to the fields concerned.

In the universal Yukawa coupling procedure, main mass hierarchy is produced by the universality of the Yukawa coupling (democratic mass matrix) and another mass hierarchy is produced by the small violations adding to the democratic mass matrix. This violation is just considered as the $SU(3)$ violation in hadron spectroscopy and hadron decay processes. This $SU(3)$ violation has been considered to be produced by the quark mass difference (violation from the $SU(3)$ symmetry) and quark dynamics. Similarly, the violations adding to democratic mass matrix are considered to be produced by some violation from a horizontal symmetry and some dynamics of quarks and leptons. Because the origin of violation is not clear at present, we have to treat these small violations as free parameters.

2.1. Quark sector

We use the following quark mass matrices with small violations of the Yukawa coupling strength containing the phases,

$$M^q = U^q \begin{pmatrix} 1 & (1 - \delta_1^q) e^{i\phi_1^q} & (1 - \delta_2^q) e^{i\phi_2^q} \\ (1 - \delta_1^q) e^{-i\phi_1^q} & 1 & (1 - \delta_3^q) e^{i\phi_3^q} \\ (1 - \delta_2^q) e^{-i\phi_2^q} & (1 - \delta_3^q) e^{-i\phi_3^q} & 1 \end{pmatrix}, \quad (q = u, d) \quad (1)$$

$$\delta_i^{ud} \ll 1, \quad \phi_i^{ud} \ll 1. \quad (i = 1, 2, 3)$$

These mass matrices are diagonalized by the unitary transformations $U_L(\delta_1^q, \delta_2^q, \delta_3^q, \phi_1^q, \phi_2^q, \phi_3^q)$ and $U_R(\delta_1^q, \delta_2^q, \delta_3^q, \phi_1^q, \phi_2^q, \phi_3^q)$ as the formulae:

$$U_L(\delta_1^q, \delta_2^q, \delta_3^q, \phi_1^q, \phi_2^q, \phi_3^q) M^q U_R^{-1}(\delta_1^q, \delta_2^q, \delta_3^q, \phi_1^q, \phi_2^q, \phi_3^q) = M_D^q, \quad (q = u, d)$$

$$M_D^q = \text{diag}[m_u, m_c, m_t], \quad M_D^d = \text{diag}[m_d, m_s, m_b]. \quad (2)$$

In the limit of $\delta_i^q \to 0$ and $\phi_i^q \to 0$, these mass matrices are diagonalized to $\text{diag}[0, 0, 3\Gamma^q]$ by the unitary transformation $U_0$;

$$U_0 M^q (\delta_i^q \to 0, \phi_i^q \to 0) U_0^{-1} = \text{diag}[0, 0, 3\Gamma^q], \quad q = u, d$$

$$U_0 = \begin{pmatrix} 1 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (3)$$

In the present procedure, $\delta_i^q$ and $\phi_i^q$ are very small and then $U_L(\delta_1^q, \delta_2^q, \delta_3^q, \phi_1^q, \phi_2^q, \phi_3^q)$ and $U_R(\delta_1^q, \delta_2^q, \delta_3^q, \phi_1^q, \phi_2^q, \phi_3^q)$ have the form near to $U_0$. The CKM matrix representing the
quark mixing is defined as

\[ V_{\text{CKM}} = U_L(\delta_1^u, \delta_2^u, \delta_3^u, \phi_1^u, \phi_2^u, \phi_3^u)U_L^\dagger(\delta_1^d, \delta_2^d, \delta_3^d, \phi_1^d, \phi_2^d, \phi_3^d). \]  

(4)

We have carried out this procedure numerically and gotten the precise results in previous work. Here, we will only show those results. We adopted the following numerical data:

\[
\begin{align*}
\frac{m_u}{m_c} &= 0.0038 \pm 0.0025, & \frac{m_d}{m_s} &= 0.050 \pm 0.035, \\
\frac{m_c}{m_t} &= 0.0042 \pm 0.0013, & \frac{m_d}{m_s} &= 0.038 \pm 0.019, \\
\Gamma^u &= \frac{m_t}{3}, & \Gamma^d &= \frac{m_b}{3}, \\
V_{\text{CKM}} &= \begin{pmatrix}
0.9747 - 0.9759 & 0.218 - 0.224 & 0.002 - 0.005 \\
0.218 - 0.224 & 0.9738 - 0.9752 & 0.032 - 0.048 \\
0.004 - 0.015 & 0.030 - 0.048 & 0.9988 - 0.9995
\end{pmatrix}.
\end{align*}
\]

(5)

From these numerical data, we could get the results for the violation parameters:

\[
\begin{align*}
\delta_1^u &= 0.00001 - 0.0004, & \delta_+^u \equiv \frac{\delta_2^u + \delta_3^u}{2} &= 0.0064 - 0.0125, & \delta_-^u \equiv \delta_2^u - \delta_3^u &= \pm (0.0 - 0.0043), \\
\delta_1^d &= 0.001 - 0.015, & \delta_+^d \equiv \frac{\delta_2^d + \delta_3^d}{2} &= 0.040 - 0.129, & \delta_-^d \equiv \delta_2^d - \delta_3^d &= \pm (0.038 - 0.006), \\
\phi_+^d \equiv \frac{\phi_2^d + \phi_3^d}{2} &= -4^\circ - 3^\circ, & \phi_-^d \equiv \phi_2^d - \phi_3^d &= \pm (-1^\circ - 0^\circ).
\end{align*}
\]

(6)

These parameters seems to have a power rule parameterized by only 2 parameters, \( \lambda \) and \( \phi \), as

\[
\begin{align*}
\delta_1^u &= \lambda^8, & \delta_+^u &= \lambda^6, & \delta_-^u &= \lambda^4, \\
\delta_1^d &= \lambda^4, & \delta_+^d &= \lambda^3, & \delta_-^d &= \lambda^2, & \lambda &\approx 0.32, \\
\phi_+^d &\equiv \phi \approx -4^\circ.
\end{align*}
\]

(7)

where we used the running masses for \( m_t \) and \( m_b \) masses at the scale \( \mu = 1\text{GeV} \).

This very simple parameterization seems to give some suggestion to the flavor symmetry.

2.2. lepton sector

Now, we analyze the charged lepton and neutrino mass hierarchy and neutrino mixing matrix \( V_{\text{MNS}} \). Similarly as \( d \) quark sector, the charged lepton mass matrix is expressed as

\[
M^l = \Gamma^l \begin{pmatrix}
1 & 1 - \delta_1^l & 1 - \delta_2^l \\
1 - \delta_1^l & 1 & 1 - \delta_3^l \\
1 - \delta_2^l & 1 - \delta_3^l & 1
\end{pmatrix}, \quad \delta_i^l \ll 1 \ (i = 1, 2, 3)
\]

(8)
where the phase factor is neglected at present analysis. For neutrino masses, we use the see-saw mechanism and introduce the Dirac neutrino $M_D^\nu$ and Majorana neutrino $M_M^\nu$. These neutrino masses produce the effective neutrino masses expressed as

$$M_{\text{eff}}^\nu = M_D^\nu M_M^{-1}(M_D^\nu)^t,$$

$$M_D^\nu = \Gamma_D^\nu \left( \begin{array}{ccc}
1 & 1 - \delta_1^\nu & 1 - \delta_2^\nu \\
1 - \delta_1^\nu & 1 & 1 - \delta_3^\nu \\
1 - \delta_2^\nu & 1 - \delta_3^\nu & 1
\end{array} \right), \quad \delta_i^\nu \ll 1 \ (i = 1, 2, 3)$$

$$M_M^\nu = \Gamma_M^\nu \left( \begin{array}{ccc}
1 - \Delta_1^\nu & 1 - \Delta_2^\nu & 1 - \Delta_3^\nu \\
1 - \Delta_2^\nu & 1 - \Delta_4^\nu & 1 - \Delta_5^\nu \\
1 - \Delta_3^\nu & 1 - \Delta_5^\nu & 1
\end{array} \right), \quad \Delta_i^\nu \ll 1 \ (i = 1, 2, 3, 4, 5)$$

For $M_M^\nu$, we add the breaking term to (1,1) and (2,2) element in order to keep the generality.

The charged lepton mass matrix is diagonalized by the unitary transformation $U_L(\delta_1^l, \delta_2^l, \delta_3^l)$ and $U_R(\delta_1^l, \delta_2^l, \delta_3^l)$ similarly to quark sector as

$$U_L(\delta_1^l, \delta_2^l, \delta_3^l)M_D U_R^{-1}(\delta_1^l, \delta_2^l, \delta_3^l) = M_D^l,$$

$$M_D^l = \text{diag}[m_e, m_\mu, m_\tau].$$

As the quark sector, $U_L(\delta_1^l, \delta_2^l, \delta_3^l)$ and $U_R(\delta_1^l, \delta_2^l, \delta_3^l)$ have the form near to $U_0$. In fact, for the mass ratios for charged leptons, $\frac{m_\mu}{m_e} = 0.004836$, $\frac{m_\tau}{m_\mu} = 0.05946 \pm 0.00001$, then the parameters $\delta_i^\nu$ and transformation matrix $U_L(\delta_i^l)$ are taken as

$$\delta_1^l = 0.002, \quad \delta_2^l = 0.137, \quad \delta_3^l = 0.113,$$

$$U_L(\delta_1^l) = \left( \begin{array}{ccc}
0.6726 & -0.7363 & 0.0727 \\
0.4547 & 0.3339 & -0.8256 \\
0.5837 & 0.5884 & 0.5594
\end{array} \right).$$

On the other hand, the neutrino mass matrix $M_{\text{eff}}^\nu$ is diagonalized by the transformation matrix $U_L(\delta_1^\nu, \delta_2^\nu, \delta_3^\nu, \Delta_1^\nu, \Delta_2^\nu, \Delta_3^\nu, \Delta_4^\nu, \Delta_5^\nu)$ as

$$U_L(\delta_1^\nu, \delta_2^\nu, \delta_3^\nu, \Delta_1^\nu, \Delta_2^\nu, \Delta_3^\nu, \Delta_4^\nu, \Delta_5^\nu)M_{\text{eff}}^\nu U_L^{-1}(\delta_1^\nu, \delta_2^\nu, \delta_3^\nu, \Delta_1^\nu, \Delta_2^\nu, \Delta_3^\nu, \Delta_4^\nu, \Delta_5^\nu)$$

$$= M_{\text{diag}}^\nu,$$

$$M_{\text{diag}}^\nu = \text{diag}[m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}].$$

The $V_{\text{MNS}}$ matrix is defined as

$$V_{\text{MNS}} = U_L^\dagger(\delta_1^l, \delta_2^l, \delta_3^l)U_L(\delta_1^\nu, \delta_2^\nu, \delta_3^\nu, \Delta_1^\nu, \Delta_2^\nu, \Delta_3^\nu, \Delta_4^\nu, \Delta_5^\nu).$$
Because $U_L(\delta_1, \delta_2, \delta_3)$ is nearly equal to $U_0$, if $U_L(\delta_1, \delta_2, \delta_3, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5)$ is nearly equal to unit matrix, $V_{\text{MNS}}$ matrix is nearly equal to $U_0^\dagger$, that is, the neutrino mixing is almost bi-maximal; $\nu_\mu$-$\nu_e$ mixing is maximal ($\theta_{12} = 45^\circ$) and $\nu_\mu$-$\nu_\tau$ mixing is almost maximal ($\theta_{23} = 35.3^\circ$).

We will consider the possibility that $U_L(\delta_1, \delta_2, \delta_3, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5)$ becomes nearly equal to unit matrix. This possibility is achieved in the case that $M_\nu$ is almost diagonal. We calculate the $M_\nu^\ast$ using the Eq.(9) and then get the result:

$$M_\nu^\ast = \frac{\Gamma_D^2}{\Gamma_M} \frac{1}{\text{Det}_2(\Delta^\nu) + \text{Det}_3(\Delta^\nu)} \begin{pmatrix} M_{11}^\nu & M_{12}^\nu & M_{13}^\nu \\ M_{21}^\nu & M_{22}^\nu & M_{23}^\nu \\ M_{31}^\nu & M_{32}^\nu & M_{33}^\nu \end{pmatrix}, \quad (14)$$

$$\text{Det}_2(\Delta^\nu) = (\Delta_2 - \Delta_3 - \Delta_5)^2 - (\Delta_1 - 2\Delta_3)(\Delta_4 - 2\Delta_5),$$
$$\text{Det}_3(\Delta^\nu) = 2\Delta_2^2 \Delta_3 \Delta_5 - \Delta_1 \Delta_5^2 - \Delta_3^2 \Delta_4,$$
$$M_{ij}^\nu = M_{ji}^\nu = \text{Det}_2(\Delta^\nu) + \text{Det}_3(\Delta^\nu),$$

where $\text{Det}_2(\Delta^\nu)$ and $\text{Det}_3(\Delta^\nu)$ are order 2 and 3 part of $\Delta^\nu_i$ in determinant of $M_\nu^\ast$ respectively, and $\Delta M_{ij}^\nu(\Delta^\nu, \delta^\nu)$ represent the term more than order 3 of $\Delta^\nu_i$ and $\delta^\nu_i$. It is stressed that the $M_{ij}^\nu$ contain the $\text{Det}_2(\Delta^\nu)$ term commonly for all $(i, j)$ elements. Because the order 2 term of $\Delta^\nu_i$ is usually larger than the terms more than order 3 of $\Delta^\nu_i$ and $\delta^\nu_i$, the mass matrix $M_\nu^\ast$ becomes nearly democratic mass matrix,

$$M_\nu^\ast \approx \frac{\Gamma_D^2}{\Gamma_M} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \text{for } \text{Det}_2(\Delta^\nu) \neq 0. \quad (15)$$

Then in this case, we cannot get the large mixing for neutrinos. However, if the term $\text{Det}_2(\Delta^\nu)$ is exact 0, the situation is drastically changed

$$M_\nu^\ast = \frac{\Gamma_D^2}{\Gamma_M} \frac{1}{\text{Det}_3(\Delta^\nu)} \begin{pmatrix} \Delta M_{11}^\nu(\Delta^\nu, \delta^\nu) & \Delta M_{12}^\nu(\Delta^\nu, \delta^\nu) & \Delta M_{13}^\nu(\Delta^\nu, \delta^\nu) \\ \Delta M_{21}^\nu(\Delta^\nu, \delta^\nu) & \Delta M_{22}^\nu(\Delta^\nu, \delta^\nu) & \Delta M_{23}^\nu(\Delta^\nu, \delta^\nu) \\ \Delta M_{31}^\nu(\Delta^\nu, \delta^\nu) & \Delta M_{32}^\nu(\Delta^\nu, \delta^\nu) & \Delta M_{33}^\nu(\Delta^\nu, \delta^\nu) \end{pmatrix},$$

for $\text{Det}_2(\Delta^\nu) = 0$ \quad (16)

then the mass matrix $M_\nu^\ast$ can be the nearly diagonal mass matrix by choosing the small violations $\delta^\nu_i$ appropriately.

We will search such small violations $\Delta^\nu_i$ and $\delta^\nu_i$ that the neutrino mass matrix $M_\nu^\ast$ becomes the nearly diagonal matrix, equally the $U_L(\delta_1, \delta_2, \delta_3, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5)$ in Eq.(11) becomes nearly unit matrix;

$$(i,j \neq i) \text{ elements of } U_L(\delta_1, \delta_2, \delta_3, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5) < 0.1, \quad (17)$$
\[ \text{Det}_2(\Delta') = (\Delta'_{2} - \Delta'_{3} - \Delta'_{5})^2 - (\Delta'_{1} - 2\Delta'_{3})(\Delta'_{1} - 2\Delta'_{5}) = 0, \]
\[ \text{Det}_3(\Delta') = 2\Delta'_{2}\Delta'_{3}\Delta'_{5} - \Delta'_{1}\Delta'^{2}_{3} - \Delta'^{2}_{5}\Delta'_{1} \neq 0. \]

We can get the parameters satisfying the conditions Eqs. (17), (18). For example
\[ \Delta'_{1} = 0.009, \quad \Delta'_{2} = 0.007, \quad \Delta'_{3} = 0.004, \quad \Delta'_{4} = 0.005, \quad \Delta'_{5} = 0.002, \]
\[ \delta'_{1} = 0.01, \quad \delta'_{2} = 0.02, \quad \delta'_{3} = 0.22, \]
\[ U_L(\delta'_{i}, \Delta'_{i}) = \begin{pmatrix} 0.9931 & -0.0991 & -0.0627 \\ 0.1016 & 0.9938 & 0.0444 \\ 0.0579 & -0.0505 & 0.9970 \end{pmatrix}. \]

For various values for the parameters \( \Delta'_i \) satisfying the condition Eq. (18), there are many solutions for \( \delta'_i \) satisfying the condition Eq. (17). We show the allowed points satisfying the condition Eq. (17) in parameter space of \( \delta'_1, \delta'_2, \delta'_3 \) in Fig. 1. The area of the circle on the point in parameter space is proportional to the numbers of the combinations of \( \Delta'_i \) satisfying the condition Eqs. (17) and (18). Fig. 1(a) shows the case \( \delta'_1 = 0 \), and Fig. 1(b) the case \( \delta'_1 = 0.025 \), Fig. 1(c) the case \( \delta'_1 = 0.05 \) and Fig. 1(d) the case \( \delta'_1 = 0.075 \).

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Fig. 1. (a) Allowed points satisfying the condition Eq. (17) in parameter plane \( (\delta'_2, \delta'_3) \) for \( \delta'_1 = 0.000 \). For the size of circle, see the text

The large mixing of neutrino is achieved by satisfying the condition Eq. (18) for the violation parameters of \( M'_{ij} \). Our present analysis is similar to the one of Ref. [8], but the pattern of violation parameters added to the democratic mass matrix is different from theirs. Next work is to find a reason producing the condition Eq. (18), but it seems difficult. We will study this problem in future articles. Here, we examine the stability of the condition. That is, although this condition is satisfied at a scale, there is no assurance that the condition
Fig. 1. (b) Allowed points satisfying the condition Eq. (17) in parameter plane \((\delta_2', \delta_3')\) for \(\delta_1' = 0.025\). For the size of circle, see the text.

is satisfied at other scale. However the renormalization effect relates \(M_{M}^\nu(M_R)\) at the GUT scale \(M_R\) to \(M_{M}^\nu(M_Z)\) at the scale \(M_Z\) as:

\[
M_{M}^\nu(M_Z) = \begin{pmatrix}
1 - \epsilon_e & 0 & 0 \\
0 & 1 - \epsilon_\mu & 0 \\
0 & 0 & 1
\end{pmatrix}^{-1} M_{M}^\nu(M_R) \begin{pmatrix}
1 - \epsilon_e & 0 & 0 \\
0 & 1 - \epsilon_\mu & 0 \\
0 & 0 & 1
\end{pmatrix}^{-1},
\]

\[
\epsilon_{e,\mu} = 1 - \sqrt{\frac{I_{e,\mu}}{I_\tau}}, \quad I_i = \exp \left( \frac{1}{8\pi^2} \int_{\ln M_Z}^{\ln M_R} y_i^2 dt \right),
\]

where \(y_i\) is the Yukawa coupling. Thus, if the violations of \(M_{M}^\nu(M_R)\) satisfy the condition
Eq. (18) at GUT scale and $\epsilon_i$ are small, it is clear that violation parameters $\Delta^i_\nu$ of $M^u_\nu(M_Z)$ also satisfy the condition Eq. (18). Then the condition Eq. (18) is stable with respect to radiative corrections.

§3. Discussions

We tried to explain the quark and lepton mass hierarchies and small mixing of quarks and large mixing of neutrinos in the universal Yukawa coupling framework with small violations. We suppose $SU(5)$ as GUT at least because masses of the $d$ quark sector and the charged lepton sector are same order, and right-handed Majorana neutrino has to be introduced in order to explain the smallness of the neutrino mass compared to the charged leptons (see-saw mechanism). For flavor symmetry, we did not assume any symmetry other than universality. In this work, we would search a rule for the violation parameters and find a symmetry for the flavor.

For quark sector, we can get “power rule” for the violation parameters; $\delta^u_1 = \lambda^8$, $\delta^u_- = \lambda^6$, $\delta^u_+ = \lambda^4$, $\delta^d_1 = \lambda^4$, $\delta^d_- = \lambda^3$, $\delta^d_+ = \lambda^2$, $\lambda \approx 0.32$, $\phi_+ \equiv \phi \approx -4^\circ$. This very simple parameterization seems to give some suggestion to the flavor symmetry. For the lepton sector, it was shown that the condition $\text{Det}_2(\Delta^\nu) = (\Delta^\nu_2 - \Delta^\nu_3 - \Delta^\nu_5)^2 - (\Delta^\nu_1 - 2\Delta^\nu_4)(\Delta^\nu_4 - 2\Delta^\nu_5) = 0$, $\text{Det}_3(\Delta^\nu) = 2\Delta^\nu_1\Delta^\nu_3\Delta^\nu_5 - \Delta^\nu_1\Delta^\nu_5^2 - \Delta^\nu_1^2\Delta^\nu_5 \neq 0$ for the violation parameters of the Majorana neutrino mass matrix must be satisfied for the large neutrino mixing. The stability of the condition to the radiative correction was shown. We can get solutions satisfying the charged lepton and neutrino mass hierarchies and neutrino large mixing $V_{\text{MNS}}$, but cannot get a
rule for these violation parameters in this article. However, it is expected that an analysis following our scenario can find information for the family symmetry by analyzing the more precise data with respect to neutrino sector.

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