Polarization of Absorption Lines as a Diagnostics of Circumstellar, Interstellar, and Intergalactic Magnetic Fields: Fine-Structure Atoms

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Abstract

The relative population of the fine-structure sublevels of an atom’s ground state is affected by radiative transitions induced by an anisotropic radiation flux. This causes the alignment of atomic angular momentum. In terms of observational consequences for the interstellar and intergalactic medium, this results in the polarization of the absorption lines. In the paper we consider the conditions necessary for this effect and provide calculations of polarization from a few astrophysically important atoms and ions with multiple upper and lower levels for an arbitrary orientation of magnetic fields to the (1) source of optical pumping, (2) direction of observation, and (3) absorbed source. We also consider an astrophysically important “degenerate” case, in which the source of optical pumping coincides with the source of the absorbed radiation. We present analytical expressions that relate the degree of linear polarization and the intensity of absorption to the three-dimensional orientation of the magnetic field with respect to the pumping source, the source of the absorbed radiation, and the direction of observations. We discuss how all these parameters can be determined via simultaneous observations of several absorption lines and suggest graphical means that are helpful in practical data interpretation. We prove that studies of absorption line polarization provide a unique tool to study three-dimensional magnetic field topology in various astrophysical conditions.

Subject headings: atomic processes — ISM: magnetic fields — polarization

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1. Introduction

Magnetic fields play extremely important roles in many astrophysical circumstances, e.g., the interstellar medium, intergalactic medium, and quasars. Unfortunately, there are only a few techniques for magnetic field studies that are available (see below). Each technique is sensitive to magnetic fields in particular environments. Therefore, even the directions of a magnetic field obtained for the same region of sky with different techniques differ substantially. The simultaneous use of different techniques provides a possibility of magnetic field tomography.

Polarimetry of aligned dust provides a way of studying magnetic field direction in the diffuse interstellar medium, molecular clouds, and the circumstellar and interplanetary medium (see review by Lazarian [2003]). Substantial progress in the understanding of grain alignment has been achieved in the last decade making the technique more reliable. The technique is, however, challenging to apply to low column densities, as the polarization signal becomes too weak. It has other limitations when dealing with high densities (Cho & Lazarian 2005).

Polarimetry of some molecular lines using the Goldreich & Kylafis (1982) effect has recently been shown to be a good tool for magnetic field studies in molecular clouds (see Girart et al. 1999). However, the magnetic field direction obtained has an uncertainty of 90°, which may be confusing. The technique is most promising for dense CO clouds.

Zeeman splitting (see Crutcher 2004) provides a good way to get magnetic field strength, but the measurements are very time consuming, and only the strongest magnetic fields are detectable this way (see Heiles & Troland 2004). Synchrotron emission/polarization as well as Faraday rotation provide an important means to study magnetic fields either in distinct regions with strong magnetic fields or over wide expanse of the magnetized diffuse media (Haverkorn 2005).

Here, we discuss yet another promising technique to study magnetic fields that employs optical and ultraviolet (UV) polarimetry. This technique can be used for interstellar and intergalactic studies as well as for studies of magnetic fields in QSOs and other astrophysical objects. As we discuss below, the technique is based on the ability of atoms and ions to be aligned by external radiation in their ground state and be realigned through precession in magnetic field. Hereafter, we do not distinguish between atoms and ions and use the word “atoms” when dealing with both species. This is justifiable, as the alignment of ions and atoms does not differ.

The physics of atomic alignment is rather straightforward. It has been known that atoms can be aligned by interactions with the anisotropic flux of resonance emission (see review by Happer [1972] and references therein). Alignment is understood here in terms of orientation of the angular momentum vector $J$, if we use the language of classical mechanics. In quantum terms this means a difference in the population of sublevels corresponding to projections of angular momentum to the quantization axis. Later, we refer to $J$ alignment in the ground state of atoms as the “atomic alignment.” Note that we deal with the ground state as it is long lived and therefore sensitive to a weak magnetic field.

It is worth mentioning that atomic alignment was studied in laboratory in relation with early day maser research (see Hawkins 1955). This effect was noticed and employed in the interstellar case by Varshalovich (1968) in the case of hyperfine structure of the ground...
state. Varshalovich (1971) suggested that atomic alignment enables us to detect the direction of magnetic fields in the interstellar medium, but did not provide a thorough quantitative study.

A more rigorous study of using atomic alignment to diagnose weak magnetic fields in diffuse media was conducted in Landolfi & Landi Degl’Innocenti (1986). However, in that case an idealized two-level atom was considered. In addition, polarization of emission from this atom was discussed for a very restricted geometry of observations; namely, the magnetic field is along the line of sight, and both of these directions are perpendicular to the incident light. This made it rather difficult to use this study as a tool for practical mapping of magnetic fields in various astrophysical environments.

In this paper we consider the alignment of atoms with fine structure of the ground level. Realistic atomic species are studied, taking into account their multilevel structure. We show that atomic alignment can influence the line polarization and intensity of absorption lines. The polarization dependence on the three-dimensional geometry of both radiation and magnetic field is provided. A study of the alignment of species within hyperfine structure of the ground state (e.g., Na, H i) and the polarization of emission lines are provided in Yan & Lazarian (2006).

It should be pointed out that the atomic alignment we deal with in this paper differs from the Hanle effect that solar researchers have studied. The Hanle effect is depolarization and rotation of the polarization vector of the resonance scattered lines in the presence of a magnetic field, which happens when the magnetic splitting becomes comparable to the decay rate of the excited state of an atom. The research into emission line polarimetry resulted in important change of the views on solar chromosphere (see Landi Degl’Innocenti 1983a, 1984, 1998; Stenflo & Keller 1997; Trujillo Bueno & Landi Degl’Innocenti 1997; Trujillo Bueno 1999; Trujillo Bueno et al. 2002; Manso Sainz & Trujillo Bueno 2003). However, these studies correspond to a setting different from the one we consider here. In this paper we concentrate on the weak field regime, in which it is the atoms at ground level that are repopulated due to magnetic precession, while the Hanle effect is negligible for the upper state. This is the case, for instance, of the interstellar medium. The polarization of absorption lines that we study here is thus more informative. Moreover, no emission lines from species with fine structure of the ground level have been reported, as far as we know, to emanate from interstellar or circumstellar gas. Absorption lines are readily available, however (see Morton 1975). In many cases we are in optically thin regimes, so we do not need to be concerned about radiative transfer.

In what follows we present the discussion of basic atomic alignment physics and optical pumping in §2. Detailed treatment of statistical equilibrium of fine-structure atoms in a magnetic field is given in §3. Following this, we then calculate, for a number of elements, their alignment and the induced polarizations of their absorption lines in §4. To help our reader understand the logic of the calculations, we provide a quantitative level summary of the procedures in §5. The discussion and the summary are provided in, respectively, §§6 and 7. The Appendices contain the information necessary for understanding our formalism as well as auxiliary calculations.

2. CONDITIONS FOR ATOMIC ALIGNMENT

The basic idea of the atomic alignment is quite simple. The alignment is caused by the anisotropic deposition of angular momentum from photons.4 In typical astrophysical situations the radiation flux is anisotropic (see Fig. 1). As the photon spin is along the direction of its propagation, we expect that atoms scattering the radiation from a light beam to be aligned. Such an alignment happens in terms of the projections of angular momentum to the direction of the incoming light. For atoms to be aligned, their ground state should have nonzero angular momentum. Therefore, fine (or hyperfine) structure is necessary to enable various projection of atomic angular momentum to exist in their ground state.

2.1. Basics of Atomic Alignment

Let us discuss a toy model that provides an intuitive insight into the physics of atomic alignment. Consider an atom with its ground state corresponding to the total angular momentum $I = 1$ and the upper state corresponding to the angular momentum $I = 0$ (Varshalovich

4 We do not consider interactions with other particles (see Fineschi & Landi Degl’Innocenti 1992).
1971). If the projection of the angular momentum to the direction of the incident resonance photon beam is right circularly polarized photons whose projections on the beam direction are 1 and -1, transitions from the ground states \( M = 1 \) and 1, while for the upper state \( M = 0 \) (see Fig. 2, left). The unpolarized beam contains an equal number of left and right circularly polarized photons whose projections on the beam direction are 1 and -1. Thus, absorption of the photons will induce transitions from the \( M = -1 \) and 1 sublevels. However, the decay from the upper state populates all three sublevels of the ground state. As a result, the atoms accumulate in the \( M = 0 \) ground sublevel from which no excitations are possible. Accordingly, the optical properties of the medium (e.g., absorption) would change.

The above toy model can also exemplify the role of collisions and magnetic field. Without collisions one may expect that all atoms reside eventually at the sublevel of \( M = 0 \). Collisions, however, redistribute atoms to different sublevels. Nevertheless, as disalignment of the ground state requires spin flips, it is less efficient than one might naively imagine (Hawkins 1955). The reduced sensitivity of aligned atoms to disorienting collisions makes the effect important for various astrophysical environments.

A magnetic field would mix up different \( M \) states. However, it is clear that the randomization in this situation is not complete and the residual alignment reflects the magnetic field direction (see Fig. 1). Magnetic mixing happens if the angular momentum precession rate is higher than the rate of the excitation from the ground state, which is true for many astrophysical conditions, e.g., the interplanetary medium, interstellar medium (ISM), intergalactic medium.

Since conservation and transfer of angular momentum are essential for the problem we deal with, it is necessary to quantize the radiation field, as the atomic states are quantized. The classical theory can give a qualitative interpretation that is used in this paper to provide an intuitive picture. However, quantitative results must be obtained by the quantum mechanical treatment, namely, the density matrix description for both atoms and the radiation. The key issue here is that the focus is on angular momentum instead of energy. As we are interested in exchange of angular momentum between photons and atoms, we must quantize the radiation field.

All in all, in order to be aligned, first, atoms should have enough degrees of freedom; namely, the quantum angular momentum number must be \( \geq 1 \). Second, the incident flux must be anisotropic. Moreover, the collisional rate should not be too high. While the latter requires special laboratory conditions, it is applicable to many astrophysical environments such as the outer layers of stellar atmospheres, the interplanetary medium, the ISM, and the intergalactic medium.

### 2.2. Relevant Timescales

Various species with fine structure can be aligned. A number of selected transitions that can be used for studies of magnetic fields are listed in Tables 1 and 2. Why and how are these lines chosen? First, we gathered all of the prominent interstellar and intergalactic lines (Morton 1975; Savage et al. 2005), from which we take only alignable lines, namely, lines with ground angular momentum number \( J_g \geq 1 \). Then, we rule out those species with nuclear spin, since hyperfine lines will be dealt with in Yan & Lazarian (2006). The number

### TABLE 1

| Atom | Ground State | Upper State | Wavelength (Å) | \( P_{\text{max}} (U_J \rightarrow I_a) \) |
|------|--------------|-------------|----------------|---------------------------------|
| S ii | \( 4S_{1/2}^1 \) | \( 4P_{1/2,3/2,5/2} \) | 1250–1260 | 12% (3/2 \rightarrow 1/2) |
| Cr i | \( 2P_{3/2} \) | \( 3P_{3/2,1/2} \) | 3580–3606, 4254–4290 | 5% (3 \rightarrow 2) |
| C ii | \( 2P_{1/2,3/2} \) | \( 2S_{1/2} \), \( 2P_{1/2,3/2} \), \( 2D_{3/2,5/2} \) | \( \sim 1036.3–1335.7 \) | 15% (3/2 \rightarrow 1/2) |
| Si ii | | | 989.9–1533.4 | 7% (3 \rightarrow 2) |
| O i | \( 3P_{1} \) | \( 3S_{1} \), \( 3P_{1}^{*} \), \( 3D_{2,3} \) | 911–1302.2 | 29% (2 \rightarrow 2) |
| S i | \( 3P_{0,1,2} \) | \( 3P_{0,1,2}^{*} \), \( 3D_{2,3} \) | 1205–1826 | 22% (1 \rightarrow 0) |
| C i | \( 3P_{0,1,2} \) | \( 3P_{0,1,2}^{*} \), \( 3D_{2,3} \) | 1115–1657 | 18% (1 \rightarrow 0) |
| Si iii | \( 3P_{3/2,1/2} \) | \( 3P_{3/2,1/2}^{*} \), \( 3D_{2,3} \) | 1695–2529 | 20% (2 \rightarrow 1) |
| S iii | \( 3P_{3/2,1/2} \) | \( 3P_{3/2,1/2}^{*} \), \( 3D_{2,3} \) | \( \sim 1012–1202 \) | 24.5% (2 \rightarrow 1) |

**Notes:** Only lines above 912 Å are listed. Data are taken from the Atomic Line List (http://www.pa.uky.edu/~peter/atomic) and the NIST Atomic Spectra Database. The last column gives the maximum polarizations and its corresponding transitions. For those species with multiple lower levels, the polarizations are calculated for shell star \( (T_{\text{eff}} = 15,000 \text{ K}) \) in the strong pumping regime; in the weak pumping regime, the maximum polarizations are 19% for the O i transition \( (2 \rightarrow 2) \) and 9% for the S i transition \( (2 \rightarrow 2) \).
of prospective transitions increases considerably if we add QSO lines. In fact, many of the species listed in Table 1 of Verner et al. (1994) are alignable and observable from the ground because of the cosmological redshifts. In this paper we do not deal with such transitions.

In terms of practical magnetic field studies, the variety of available species is important in many aspects. One of them is a possibility of getting additional information about environments. Let us illustrate this by considering the various rates (see Table 3) involved. Those are alignable and observable from the ground because of the cosmological redshifts. In this paper we do not deal with such transitions. If the Larmor precession rate $\nu_L > \tau_R^{-1}$, the magnetic field does not affect the atomic occupations, and atoms are aligned with respect to the direction of radiation (see Fig. 1). From the expressions in Table 3, we see, for instance, that the magnetic field can realign $\text{C} \, \text{ii}$ only at a distance $r \geq 7.7$ AU from an O star if the magnetic field strength is $\sim 5 \mu G$.

If the Larmor precession rate $\nu_L$ is comparable to any of the other rates, the atomic line polarization becomes sensitive to the strength of the magnetic field. In these situations, it is possible to get information about the magnitude of the magnetic field. In this paper we consider the regime in which $\nu_L$ is much smaller than the decay rate of the excited state, which means that we disregard the Hanle effect.

Figure 3 illustrates the regime of the magnetic field strength in which realignment applies. Atoms are aligned by the anisotropic radiation at a rate of $\tau_R^{-1}$. Magnetic precession will realign the atoms in their ground state if the Larmor precession rate $\nu_L > \tau_R^{-1}$. In contrast, if the magnetic field gets stronger, so that the Larmor frequency becomes comparable to the line width of the upper level, the upper level occupation, especially coherence, is modified directly by the magnetic field; this is the domain of the Hanle effect, which has been extensively discussed for studies of the solar magnetic field (see Landi Degl’Innocenti & Landolfi 2004 and references therein).

When the magnetic splitting becomes comparable to the Doppler line width $\nu_D$, polarization appears; this is the “magnetograph regime” (Landi Degl’Innocenti 1983b). For magnetic splitting $\nu_D \gg \nu_D$, the energy separation is enough to be resolved, and the magnetic field can be deduced directly from line splitting in this case. If the medium is strongly turbulent with $b \nu \sim 100$ km s$^{-1}$ (so that the Doppler line width is comparable to the level separations $\nu_D \sim \nu_D$), interferences occur among these levels and should be taken into account.

Long-lived alignable metastable states that are present for some atomic species between upper and lower states may act as proxies of ground states. Absorptions from these metastable levels can therefore also be used as diagnostics for the magnetic field. We present a few selected species with such states in Table 2, but further in the present paper we deal only with one representative of this class, namely, $\text{Cr} \, \text{ii}$.

The lifetime of the metastable level is an important timescale. However, in the formalism of the present paper, one can account for it via modifying the effective $\tau_T$ to include the decay of the metastable states.

### Table 2

**Examples of Absorptions from Metastable States**

| Atom | Ground state, G | Metastable State, M | Upper State, U | Wavelength ($G \rightarrow U$) (Å) | Wavelength ($M \rightarrow U$) (Å) |
|------|----------------|---------------------|----------------|----------------------------------|----------------------------------|
| Cr ii | $a6S_{1/2}$ | $a6D_{3/2, 5/2, 7/2, 9/2}$ | $z6P_{1/2, 3/2, 5/2}$ | 2056–2086 | 2741–2767 |
| Ti ii | $a4F_{7/2, 5/2, 3/2, 1/2}$ | $b4F_{7/2, 5/2, 3/2, 1/2}$ | $z4D_{3/2, 5/2}$ | 3058.3–3089 | 3153–3169 |
| Ti iii | $3F_{2, 3, 4}$ | $3P_{0, 1, 2}$ | $3D_{1, 2, 3}$ | 1282–1299 | 1499–1506 |
| Fe ii | $a6D_{7/2, 9/2, 11/2}$ | $a6S_{9/2}$ | $z6D_{3/2, 5/2}$ | 2227–2381 | 4925–5170 |
| Fe iii | $5D_{3, 2, 1}$ | $5S_{2}$ | $5P_{2, 1}$ | 1123–1132 | 2062–2079.7 |
| Ca ii | $2S_{1/2}$ | $2D_{5/2, 3/2}$ | $2P_{1/2, 3/2}$ | 3934–3669 | 8498–8662 |

*Notes.*—Calculation for $\text{Cr} \, \text{ii}$ shows that the absorption from metastable level $a6D_{9/2} \rightarrow z6P_{5/2}$ has a maximum polarization of 21%.

### Table 3

**Relevant Rates for Atomic Alignment**

| Rate | Formula | Value for $\text{Cr} \, \text{ii}$ |
|------|---------|----------------------------------|
| $\nu_L$ (s$^{-1}$) | $aB$ | $88 (B/5 \mu G)$ |
| $\tau_R^{-1}$ (s$^{-1}$) | $B_{1, J, I}$ | $7.4 \times 10^{-5} (R/\nu)^2$ |
| $\tau_T^{-1}$ (s$^{-1}$) | $A_n$ | $2.3 \times 10^{-6}$ |
| $\tau_\phi^{-1}$ (s$^{-1}$) | $\max (j_{fi}, j_{ff})$ | $6.4 \left( \frac{n_e}{0.1 \text{ cm}^{-3}} \sqrt{\frac{8000 \text{ K}}{T}} \right) \times 10^{-9}$ |

*Notes.*—$A_n$ is the magnetic dipole emission rate for transitions among $J$-levels of the ground state of an atom; $j_{fi}$ is the inelastic collisional transition rates within ground state due to collisions with electrons or hydrogens, and $j_{ff}$ is the spin flip rate due to Van der Waals collisions (see § 4.2.3). In the last column, $\tau_\phi^{-1}$ is calculated for an O-type star, where $R$ is the radius of the star radius and $\nu$ is the distance to the star.
2.3. Ground State Alignment and Absorptions

The geometry of the radiation system is illustrated by the left panel of Figure 4. The origin of this frame is defined as the location of the atomic cloud. The line of sight defines the z-axis, and together with the direction of radiation, they specify the x-z plane. The x-y plane is thus the plane of the sky. In this frame, the incident radiation is coming from (θ_0, 0), and the magnetic field is in the direction (θ_B, φ_B).

The magnetic field is chosen as the quantization axis (z') for the atoms. Alignment is be treated in the frame x'y'z'' (see Appendix E for how these two frames are related). In this “theoretical” frame, the line of sight is in the (θ, π) direction (i.e., the x''-z'' plane is defined by the magnetic field and the line of sight; see Fig. 4, right), and the radiation source is directed along (θ_r, φ_r).

Absorption is directly associated with the alignment of the ground state and the direction of observation (Fig. 1). For absorption, we demonstrate later that the Stokes parameter (see Appendix D) U = 0 if the background is unpolarized. This means that the linear polarization is either parallel or perpendicular to the magnetic field. This happens because of the fast precession of atoms in the ground state. Direction of polarization thus traces direction of magnetic field and the line of sight; see Fig. 4, right), and the incident radiation is in the (θ_r, φ_r) direction, and the magnetic field is in the (θ_B, φ_B) direction.

Absorption is defined by equation (13) and the corresponding numerical values are given in Table 2. For a generic case, in which the background source (e.g., a QSO) is polarized and optical depth is finite,6 we can get in the first-order approximation

$$\delta v (\text{cm/s})$$

![Diagram of polarization](image)

FIG. 3.—Different regimes divided according to strength of magnetic field and Doppler line width. Atomic realignment is applicable to a weak field (< 1 G) in a diffuse medium. Level interferences are negligible unless the medium is substantially turbulent (σ ≳ 100 km s\(^{-1}\)) and the corresponding Doppler line width becomes comparable to the fine-level splitting ν_{3/2}. For a strong magnetic field, the Zeeman effect dominates. When magnetic splitting becomes comparable to the Doppler width, σ- and π-components can still distinguish themselves through polarization, this is the magnetograph regime; the Hanle effect is dominant if the Larmor period is comparable to the lifetime of excited level ξ_c ^{-1} ≪ A ^{-1}. Similarly, for the ground Hanle effect, it requires Larmor splitting to be of the order of the photon pumping rate; for a weak magnetic field (< 1 G) in a diffuse medium, however, atomic alignment is the main effect, provided that\( ν_L > ν_p ^{-1}\). [See the electronic edition of the Journal for a color version of this figure.]

where \( \sigma_0^2 \) depends on \( θ_r \) and is the normalized dipole component of the ground state density matrix, which is the main parameter we calculate in this paper, and \( w_{J_l}^2 \) is defined by equation (13) and the corresponding numerical values are given in Table 2. For a generic case, in which the background source (e.g., a QSO) is polarized and optical depth is finite,6 we can get in the first-order approximation

$$\begin{align*}
I &= (I_0 + Q_0) e^{-τ(1 + η/τ)} + (I_0 - Q_0) e^{-τ(1 - η/τ)}, \\
Q &= (I_0 + Q_0) e^{-τ(1 + η/τ)} - (I_0 - Q_0) e^{-τ(1 - η/τ)}, \\
U &= U_0 e^{-τ}, \quad V = V_0 e^{-τ},
\end{align*}$$

In the solar case one must deal with radiative transfer. For a diffuse medium, however, this assumption is reasonable.

6 We neglect emission here.

FIG. 4.—Left: Geometry of the observational frame. In this frame the line of sight is the z-axis; together with the incident light, they specify the x-z plane. The magnetic field is in the (θ_B, φ_B) direction. Right: Transformation to the “theoretical frame,” where the magnetic field defines the z''-axis. This can be done by two successive rotations specified by Euler angles (θ_B, φ_B) (see Appendix E for details). The first rotation is from the xyz-coordinate system to the x'y'z''-coordinate system by an angle φ_B about the z-axis; the second is from the x'y'z''-coordinate system to the x''y''z''-coordinate system by an angle θ about the y''-axis. Atomic alignment and transitions are treated in the “theoretical” frame, where the line of sight is in the (θ, π) direction, and the incident radiation is in the (θ_r, φ_r) direction. [See the electronic edition of the Journal for a color version of this figure.]
in which \(I_0, Q_0, U_0,\) and \(V_0\) are the Stokes parameters of background source. Optical depth is also modified by the alignment. The line ratio of a doublet varies as

\[
\frac{\tau_1}{\tau_2} = \frac{B_{J_lJ_l} \left[ \sqrt{2 + \sigma_{\Delta}^2(J_l)} (1 - 1.5 \sin^2 \theta) w_{J_lJ_l} \right]}{B_{J_lJ_l} \left[ \sqrt{2 + \sigma_{\Delta}^2(J_l)} (1 - 1.5 \sin^2 \theta) w_{J_lJ_l} \right]},
\]

where \(B_{J_lJ_l}\) represents the Einstein coefficient. Section 3 is devoted to explaining how the atomic alignment \((\sigma_{\Delta}^2)\) can be calculated using quantum electrodynamic (QED) theory. Readers who are not interested in the technical details, however, can skip \(\S\) 3 and go directly to the results of calculations for the species in Table 1 that we present in \(\S\) 4. The value of \(\sigma_{\Delta}^2 J_l^2\) is that it provides the approach for calculating the alignment of atoms other than those discussed in the present paper.

3. ALIGNMENT THEORY

In what follows we consider a realistic atomic system, which can have multiple upper levels \(J_u, M_u\) and lower levels \(J_l, M_l\). When such an atomic system interacts with resonant radiation, there will be a photoexcitation followed by spontaneous emission. The atomic occupation is thus determined by the balance of the three processes: absorption, emission, and magnetic precession among the sublevels of the state. The radiative transition rates can be constructed from the transition probabilities given by equation (B1). Then additional transformation should be made to express the density matrices in irreducible form (see Appendix A). The actual derivation of the equations that govern the statistical equilibrium of the atomic occupations is rather complicated and can be found in Landi Degl’Innocenti & Landolfi (2004). Below are the statistical equilibrium equations for the upper and ground state (Landi Degl’Innocenti & Landolfi 2004),

\[
\rho^k_q(J_u) + 2\pi i \nu_l g_q \rho^k_q(J_u) = - \sum_{J_l} A(J_u \rightarrow J_l) \rho^k_q(J_u) + \sum_{J_lKlq} [J_l] B(J_l \rightarrow J_u) (-1) k^k + q^q \times (3[k,k',K])^{1/2} \begin{pmatrix} 1 & J_u & J_l \\ 1 & J_u & J_l \\ K & k & k' \end{pmatrix} (k^k q^q Q) \tilde{J}_{kq}^k(J_u),
\]

\[
\rho^k_q(J_l) + 2\pi i \nu_l g_l \rho^k_q(J_l) = \sum_{J_u} (-1)^{J_l+J_u+1} \left\{ \begin{array}{ccc} J_l & J_l & k \\ J_u & J_u & 1 \end{array} \right\} A(J_u \rightarrow J_l) \rho^k_q(J_u) - [J_l] \sum_{J_lKlq} B(J_l \rightarrow J_u) (-1)^{J_l-J_u+k^k+q^q} (3[k,k',K])^{1/2} \begin{pmatrix} 1 & 1 & K \\ J_l & J_l & J_l \\ J_l & J_l & J_l \end{pmatrix} (k^k q^q Q) \tilde{J}_{kq}^k(J_u),
\]

where \(\rho^k_q\) and \(\tilde{J}_{kq}^k\) are the irreducible form of the density matrices of atoms and radiation, respectively. The evolution of the upper state \(\rho^k_q(J_u)\) is represented by equation (4), and the ground state \(\rho^k_q(J_l)\) is described by equation (5). The second terms on the left-hand side of equations (4) and (5) represent mixing by a magnetic field, where \(g_q\) and \(g_l\) are the Landé factors for the upper and ground level. For the upper level, this term can be ignored, as we consider a regime in which the magnetic field is much weaker than the Hanle field\(^7\)

Regarding the photospheric modeling, it is useful to consider the relationship between the Stokes parameters and the magnetic field. The line ratio of a doublet varies as the 6\(\Delta\) term in equation (4) and the ground state \(\rho^k_q(J_l)\) is described by equation (5). The two terms on the right-hand side of equations (4) and (5) are due to spontaneous emissions and the excitations from ground level. Transitions to all upper states are taken into account by summing over \(J_u\) in equation (5). Vice versa, for an upper level transitions to all ground sublevels \(J_l\) are summed up in equation (4). Let us remind our reader that this multilevel treatment is well justified as level-crossing interferences do not exist in the weak field regime unless the medium gets strongly turbulent so that Doppler width becomes comparable to the energy separation between levels (\(\delta v \approx 100 \text{ km s}^{-1}\); see Fig. 3). The matrix with big braces represents the 6\(\Delta\) or 9\(\Delta\) symbol, depending on the size of the matrix. When more than two angular momentum vectors are coupled, there is more than one way to add them up and form the same resultant. The 6\(\Delta\) (or 9\(\Delta\)) symbol appears in this case as a recoupling coefficient describing transformations between different coupling schemes (see Appendix C). Throughout this paper, we define \(J = J_l + 1\), which means \(J_l = 2J_l + 1\) and \([J_u] = 2J_u + 1\). The Einstein spontaneous emission rate is defined as the total probability of an atom from a specific state \(J',M'\) to all states of \(J,A = \sum M\) by the Einstein coefficients for stimulated transitions \(B\) are related to \(A\) by

\[
[J_u] A(J_u \rightarrow J_l) = \frac{2h \nu^3}{c^2} [J_l] B(J_l \rightarrow J_u) = \frac{2h \nu^3}{c^2} [J_u] B(J_u \rightarrow J_l),
\]

where \([J]\) is the statistical weight of the initial level \(J\). Stained emission is neglected since \(A/g_q \gg \nu_l > \tau_{\Delta}^{-1} \sim B(J_u \rightarrow J_l)\). Note for spontaneous emission and magnetic mixing, \(k\) and \(q\) are conserved because of the symmetric nature of these processes. Indeed, magnetic mixing and emission are axially symmetric processes.

The excitation depends on

\[
\tilde{J}_{kq}^k = \int d\nu \frac{\nu^2}{c^2} \tilde{\xi}(\nu - \nu_0) \int \frac{d\Omega}{4\pi} \sum_{i=0}^3 \tilde{J}_{kq}^k(i, \Omega) S_i(\nu, \Omega),
\]

\(^7\) For the Hanle effect to be dominant, magnetic splitting ought to be comparable to the energy width of the excited level (see Fig. 3).
which is the radiation tensor of the incoming light averaged over the whole solid angle and the line profile $\xi(\nu - \nu_0)$. The term $S_i = [I, Q, U, V]$ represents the Stokes parameters (Appendix D). In many cases, the radiation source is far enough away that it can be treated as a point source. We consider here unpolarized incident light, thus $Q = U = V = 0$. The irreducible unit tensors for Stokes parameters $I$, $Q$, and $U$ (Appendix D) are

$$J^0_0(i, \Omega) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad J^2_0(i, \Omega) = \sqrt{2} \begin{pmatrix} (1 - 1.5 \sin^2\theta)/2 \\ -3/4 \sin^2\theta \\ 0 \end{pmatrix},$$

$$J^2_{\perp\perp}(i, \Omega) = \sqrt{3}e^{2i\phi} \begin{pmatrix} \sin^2\theta/4 \\ -(1 + \cos^2\theta)/4 \\ \mp i \cos\theta/2 \end{pmatrix}, \quad J^2_{\parallel\parallel}(i, \Omega) = \sqrt{3}e^{i\phi} \begin{pmatrix} \pm \sin\theta/4 \\ \pm 2\sin\theta/4 \\ -i \sin\theta/2 \end{pmatrix}. \quad (8)$$

They are determined by its direction $\Omega(\theta, \phi)$ and the reference chosen to measure the polarization (Fig. 2, right). As addressed earlier, the line of sight is in $\Omega = (\theta, \pi)$ in the “theoretical frame.” For the incoming radiation from $(\theta, \phi)$, similarly, according to equation (8), the nonzero elements of the radiation tensor are

$$J^0_0 = I, \quad J^2_0 = \frac{W_0}{2\sqrt{2}W} (2 - 3 \sin^2\theta)I, \quad J^2_{\perp\perp} = \frac{\sqrt{3}W_0}{4W} \sin^2\theta, I, e^{2i\phi}, \quad J^2_{\parallel\parallel} = \mp \frac{\sqrt{3}W_0}{4W} \sin 2\theta, I, e^{i\phi}, \quad (9)$$

where $W$ is the dilution factor of the radiation field, which can be divided into an anisotropic part $W_\alpha$ and an isotropic part $W_I$ (Bommier & Sahal-Brechot 1978). In the case of a point source, $W_I = 0$. If $W_I \neq 0$, the degree of alignment and polarization will be reduced. The solid-angle-averaged intensity for a blackbody radiation source is

$$I_s = \frac{W}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1}. \quad (10)$$

By setting the first terms on the left-hand side of equations (4) and (5) to zero, we can obtain the following linear equations for the steady state occupations in the ground states of atoms,

$$2\pi i\rho_i^b(J_i) q_{i\nu}, n \nu L - \sum_{J_i' J_u} A(J_u \rightarrow J_i') \sum_{J_u'} [J_i] B(J_i' \rightarrow J_u) \sum_{kq'} \rho_{kq'}^{k'} (J_i') \sum_{kQ} \frac{(1 - e^{2i\phi})}{4\pi} \left( \begin{array}{ccc} k & q & \nu \\ k' & q' & \nu' \end{array} \right) \times \frac{J^2_Q}{J^2_Q} \left[ (-1)^{k+J_u} [J_u] \left\{ \begin{array}{ccc} J_i & J_i' & k \\ J_u & J_u' & 1 \end{array} \right\} \right. - (-1)^{k-J_u} \left\{ \begin{array}{ccc} J_i & J_i' & k \\ J_u & J_u' & 1 \end{array} \right\} \left. \right\} = 0. \quad (11)$$

The multiplet effect is counted for by summing over $J_u$. The alignment of atoms induces polarization for absorption lines even though the background radiation is unpolarized. The corresponding absorption coefficients $\eta_i$ are (Landi Degl’Innocenti 1984)

$$\eta_i(\nu, \Omega) = \frac{h\nu}{4\pi} Bn(J_i) \xi(\nu - \nu_0) \sum_{kQ} \frac{(-1)^k}{w_{k', k}^{J_i, J_u}} \sigma_{kQ}^K(J_i) J^k_Q(i, \Omega), \quad (12)$$

where $n(J_i) = n|J_i|\frac{1}{2} \rho_0^b(J_i)$ is the total atomic population on level $J_i$, $\sigma_{kQ}^K(J_i) = \rho_k^Q / \rho_0^b$ and

$$w_{k', k}^{J_i, J_u} = \left\{ \begin{array}{ccc} 1 & 1 & K \\ J_i & J_i & J_u \end{array} \right\} \left\{ \begin{array}{ccc} 1 & 1 & 0 \\ J_i & J_i & J_u \end{array} \right\}. \quad (13)$$

The values of $w_{k', k}^{J_i, J_u}$ for different pairs of $J$, $J'$ are listed in Table 4.

Since there is no magnetic coherence because of fast magnetic mixing on ground level, $\sigma_{k\neq 0}^K = 0$,

$$\eta_i(\nu, \Omega) = \frac{h\nu}{4\pi} Bn(J_i) \xi(\nu - \nu_0) \sum_k \frac{(-1)^k w_{k', k}^{J_i, J_u}}{w_{k', k}^{J_i, J_u}} \sigma_{kQ}^K(J_i) J^k_Q(i, \Omega), \quad (14)$$
From the above equation, we see that coefficients. This implies all Zeeman coherence components (of the sky (Fig. 2, pumping source itself; k

0, 1, 2, 3. For unpolarized pumping light, we only need to consider even components

fore the degree of alignment and polarization. This means that in practical calculations, a summation should be taken over all the three

dependence on polar angle is shown in Figure 5. To calculate the alignment for the multiplet, all transitions should be counted according

Compared to absorption, emission is negligible as the emitted intensity \( I_{em} \propto \rho_u \propto \bar{J}_0^0 \) (the solid-angle averaged intensity), which is reduced by a dilution factor \( W \ll 1 \) (eq. [10]).

Thereby, for the optically thin case, the Stokes parameters of an absorbed unpolarized radiation are

\[
I = (1 - \eta_0 d)I_0, \quad = -\eta_3 dI_0. \tag{15}
\]

If we drop the assumption of unpolarized background source, the more general expression for finite optical depth would be

\[
I = (I_0 + Q_0) e^{-(\eta_0 + \eta_1)d} + (I_0 - Q_0) e^{-(\eta_0 - \eta_1)d},
\]

\[
Q = (I_0 + Q_0) e^{-(\eta_0 + \eta_1)d} - (I_0 - Q_0) e^{-(\eta_0 - \eta_1)d},
\]

\[
U = U_0 e^{-\tau}, \quad V = V_0 e^{-\tau}, \tag{16}
\]

where \( I_0, Q_0, U_0, \) and \( V_0 \) are the Stokes parameters of the background radiation, which can be from a weak background source or the pumping source itself; \( d \) refers to the thickness of the medium.

For unpolarized pumping light, the radiation has only components with \( K = 0, 2. \) As a result, the density tensor of the atom has only even \( k \)-components, while odd \( k \)-components will vanish according to the selection rules of \( 3j, 6j, \) and \( 9j \) symbols (see Appendix C). Physically, this means atoms are aligned but not oriented.

### 4. ALIGNMENT OF VARIOUS ATOMIC SPECIES

#### 4.1. Single Ground Level Species: \( S \) \( \pi \) and \( Cr \) \( \pi \)

The \( S \) \( \pi \) and \( Cr \) \( \pi \) absorption lines are observed in the ISM (Morton 1975). We discuss \( S \) \( \pi \) first. \( S \) \( \pi \) has a ground level \( 4S_{1/2} \) and upper levels \( 4P_{1/2, 3/2, 5/2} \), which means \( J_l = 3/2, J_u = 1/2, 3/2, 5/2 \). According to the triangle rule (Appendix C) of the \( 3j \) symbol in equation (A1), the irreducible density tensor of the ground state \( \rho^0_0(J_l) \) has components with \( k = 3/2 - 3/2, 3/2 - 1/2, \ldots, 3/2 + 3/2 = 0, 1, 2, 3 \). For unpolarized pumping light, we only need to consider even components \( k = 0, 2 \) as explained earlier.

The fact that \( \nu_1 \gg \tau_\beta^{-1} \) indicates that the coefficients of the diagonal terms with \( q \neq 0 \) in equation (11) are much larger than the other coefficients. This implies all Zeeman coherence components (\( q \neq 0 \)) disappear. It is expected that there is no Zeeman coherence in the magnetic quantization frame, as magnetic mixing is fast. Owing to the selection rule of \( 3j \) symbols (see Appendix C), only \( J^{0.2}_{Q=0} \) components appear, and they are determined by the polar angle \( \theta_b \) of the radiation (eq. [9], Fig. 4). As a result, \( \rho^0_0 \) are real quantities, and they are independent of the azimuthal angle \( \phi_b \) (see eq. [9]). Physically, this results from fast procession around a magnetic field. Their dependence on polar angle is shown in Figure 5. To calculate the alignment for the multiplet, all transitions should be counted according to their probabilities even if one is interested in only one particular line. This is because they all affect the ground populations and therefore the degree of alignment and polarization. This means that in practical calculations, a summation should be taken over all the three upper levels \( J_u = 1/2, 3/2, 5/2 \) in equation (11). Inserting the values of \( J_u, J_l, k, k', K, \) and \( J^{0.2}_Q \) (eq. [9]) into equation (11), we get three linear equations for \( \rho^0_0(J_l) \). By solving them, we obtain

\[
\sigma^2_0(J_l) = \frac{1.0481 - 3.1442 \cos^2 \theta_b}{0.264 \cos^2 \theta_b - 13.161}, \tag{17}
\]
where the floating point numbers are approximate. The same is true with other results in this paper; we take four or more significant digits. To demonstrate the effect of multiplet, we also calculate the alignment resulting from only one transition \(3/2 \rightarrow 1/2\). The comparison is given in Figure 5. Apparently, multiple transitions reduce the degree of alignment.

Substituting equation (17) into equation (1), we get the polarization of the S \(\Pi\) absorption line,

\[
P \frac{\tau}{\tau} = \frac{(1.5722 - 4.7163 \cos^2 \theta) \sin^2 \theta w_{ij}^2}{(1.4822 - 4.4466 \cos^2 \theta) - (1 - 1.5 \sin^2 \theta) w_{ij}^2}
\]

(18)

The top panels of Figure 6 shows the dependence of the degree of polarization on \(\theta_i\) and \(\theta\). The left panel of Figure 7 is the corresponding contour plot. We see that the polarization in absorption is either parallel or perpendicular to the plane of the sky. The polarization changes sign at the Van Vleck angle \(54.7^\circ\) (Van Vleck 1925; House 1974). This happens because of coupling of the two oscillators in the \(x\)-\(y\) direction. These two oscillators share energy due to Larmor precession around the magnetic field (in the direction of the \(z\)-axis). As can also be seen from the dipole component of radiation tensor \(J^2_0\) of unpolarized radiation, \(J^2_0 \propto (1 - 1.5 \sin^2 \theta)/2\). Therefore, the dipole component of the density matrix \(\rho^2_0\), which is proportional to \(J^2_0\) (eqs. [9] and [17]), changes from parallel to perpendicular to the magnetic field at Van Vleck angle (Fig. 5). As a result, the polarization of the absorption line also changes according to equation (1). This turnoff at the Van Vleck angle is a general feature regardless of the specific atomic species as long as the background source is unpolarized and it is in the atomic realignment regime.

The absorption shown in Figures 6 (top) and 7 (left) exhibits variations with \(\theta_i\) and \(\theta\). The maximal polarization (around 12%) is expected for the S \(\Pi\) \((J_u = 1/2)\) component. At \(\theta = 90^\circ\), the observed polarization reaches a maximum for the same \(\theta_i\) and alignment, which is also expected from equation (1). This shows that atoms are indeed realigned with respect to the magnetic field so that the intensity difference is maximized parallel and perpendicular to the field (Fig. 2, right). At \(\theta = 0^\circ\) and \(180^\circ\), the absorption polarization is zero according to equation (1). Physically, this is because the precession around the magnetic field makes no difference in the \(x\)-\(y\) direction when the magnetic field is along the line of sight (Fig. 2, right).

The optical depth also varies as a result of the alignment (see eq. [3]). We plot the optical depth ratios of line \((J_u = 1/2)\) to line \((J_u = 3/2)\) (see Figs. 6, bottom right, and 7, right). For optically thin lines, the equivalent width ratio is equal to that of optical depths. Note that when the magnetic field is parallel to the incident radiation, the magnetic field does not have an impact. The corresponding line ratios without magnetic realignment are equal to those values at \(\theta_i = 0^\circ\) in the graphs.

The fact that absorption polarization can be only parallel or perpendicular to the magnetic field in the plane of the sky has an important implication. It means that the direction of polarization traces the projection of magnetic field in the plane of the sky (the angle \(\phi_B\); see Fig. 4) within a \(90^\circ\) uncertainty. This is a very “easy” piece of information to obtain. Furthermore, if we get two measurable, the degree of polarization and the ratio of a doublet,\(^8\) we are able to extrapolate both \(\theta_i\) and \(\theta = \theta_B\) (see Fig. 4). With \(\theta_B\), known, we can remove the \(90^\circ\) degeneracy of \(\phi_B\) and thus obtain the three-dimensional information of the magnetic field \((\theta_B, \phi_B)\).

We consider a general case in which the pumping source does not coincide with the object whose absorption is measured. If the radiation that we measure is also the radiation that aligns the atoms, the direction of the pumping source coincides with the line of sight, i.e., \(\theta = \theta_i\) (Fig. 4).

\(^8\) S \(\Pi\) is a triplet for which we can get three degrees of polarizations and two line ratios. However, the degrees of polarizations are proportional to each other, as they have the same dependence on \(\sigma_i^2(\theta)\) (see eq. [1]). Thus, only by combining one degree of polarization and one line ratio, can we obtain the two angles.
Cr has a ground state \(^7S_3\) and excited state \(^7S_{2.3.4}\). The computing procedure is similar to that for \(S_{\text{ii}}\). The difference is that the density matrix for the Cr ground state has seven components with \(k = 3/2, 3, \ldots, 3 + 3 = 0, 1, 2, 3, 4, 5, 6, 7\). We only need to consider even components \(\rho_{0,2}^{1/2}\). Following the same procedure as described for \(S_{\text{ii}}\), we get the results

\[
\begin{align*}
\sigma_0^{1/2}(J) &= \begin{bmatrix} 35.829 \cos 2\theta_r - 12.593 \cos 4\theta_r + 0.253 \cos 6\theta_r + 1.932 \\
0.671 \cos 2\theta_r + 1.2648 \cos 4\theta_r - 0.6092 \cos 6\theta_r + 1.7263 \\
-1.0902 \cos 2\theta_r - 0.5032 \cos 4\theta_r - 0.2516 \cos 6\theta_r - 0.5404 \end{bmatrix} / (322 \cos 2\theta_r + 1.24 \cos 4\theta_r + 568). \quad (19)
\end{align*}
\]

By inserting these results into equation (1) or equation (2), one can get the polarizations of Cr\(^{\text{i}}\) absorption lines.

### 4.2. More Complex Structure with Multiple Lower Levels: C\(^{\text{ii}}, \text{Si ii, O i, S i, and Cr ii}\)

#### 4.2.1. Strong Pumping

In this section we discuss atoms with more complex structure. As illustrated in Table 1, C\(^{\text{ii}}\) has not only multiple upper levels, but also two levels \(2P_{1/2,3/2}^{\text{po}}\) on the ground state. The level \(2P_{1/2}^{\text{po}}\) has altogether two magnetic projections \(M_J = \pm 1/2\), and is thus not alignable and has only one density tensor \(\rho_0^0\). Level \(2P_{3/2}^{\text{po}}\) has density components \(\rho_0^{1/2}\). We consider the five lines: \(1036.34 \text{ Å} (2P_{1/2}^{\text{po}} \rightarrow 2S_{1/2})\), \(1037.02 \text{ Å} (2P_{3/2}^{\text{po}} \rightarrow 2S_{1/2})\), \(1334.53 \text{ Å} (2P_{1/2}^{\text{po}} \rightarrow 2D_{3/2})\), \(1335.66 \text{ Å} (2P_{3/2}^{\text{po}} \rightarrow 2D_{3/2})\), and \(1335.71 \text{ Å} (2P_{1/2}^{\text{po}} \rightarrow 2D_{5/2})\), which are observed in QSOs. All the transitions between upper levels and the ground state should be counted. There is also a magnetic dipole transition between the two ground levels. Although its transition probability is very low, it can be comparable to the optical pumping rate in regions far from any radiation source. Depending on how far away the radiation source is, there can be two regimes divided by the boundary where the magnetic dipole radiation rate \(A_m\) is equal to the pumping rate \(\tau_K^{-1}\). Inside the boundary, the optical pumping rate is much larger than the M1 transition rate \(A_m\), so we can ignore the magnetic dipole radiation as a first-order approximation. Further out,
the magnetic dipole transition is faster than optical pumping, so it can be assumed that most atoms reside in the lowest energy level of the ground state. For the case of C II, which we consider now, the lowest level is not alignable, and so we concentrate on the first regime and neglect the magnetic dipole transition.

The two regimes are demarcated at $r_1$ (see Fig. 8, left), where $r_1$ is defined by

$$\tau_R^{-1} = B_{\beta,\lambda} I_{BB}(R_*/r_1)^2 = A_m,$$

and $I_{BB}$ and $R_*$ are the intensity and radius of the pumping source, respectively. Outside the boundary where $\tau_R^{-1} \ll A_m$, C II and Si II are not alignable, and so their absorption lines are unpolarized. For different radiation sources, the distance to the boundary differs. For an
O-type star, the distance would be ~0.1 pc, for a shell star \( (T_{\text{eff}} = 15,000 \text{K}) \), it is as close as ~0.003–0.01 pc. The absence of alignment outside the radius \( r_1 \) ensures that observations constrain the magnetic field topology within this radius.

Since there are multiple transitions with frequencies that span over a wide range, the optical pumping and its resulting atomic alignment depend on the shape of the spectrum of the radiation source. Neglecting the difference within an \( LS \) multiplet, we characterize the intensity of the multiplet \( (2P^0 \rightarrow 2S_{1/2}) \) by \( I_s \), multiplet \( (2P^0 \rightarrow 2P_{1/2,3/2}) \) by \( I_p \), and that of multiplet \( (2P^0 \rightarrow 2D_{1/2,3/2,5/2}) \) by \( I_d \). In the case that other multiplets of the same levels have comparable intensities, \( I_s, I_p, I_d \) are weighted summations of all the intensities of the same terms (e.g., see eq. [24]),

\[
I_s = I_{1036.3}, \quad I_p = I_{804.1}, \quad I_d = I_{1334.5}.
\]  

These are intensities per unit frequency. So are other intensities in this paper. We use wavelengths (in angstroms) rather than frequencies in the subscripts as the lines are in the optical and UV band. Note that due to absorption from the Lyman series, those lines with wavelengths shorter than 912 Å can be ignored in the case of distant radiation pumping sources. We obtain for \( C\, ii \)

\[
\rho_0^0(J_i = 3/2) = \rho_0^0(J_i = 1/2) \left[ (0.44I_p^2 + 0.39I_pI_d - 1.65I_sI_d - 7.06I_s^2 - 7.37I_pI_s) \cos^4 \theta_r + (-6.54I_p^2 - 47.81I_pI_d - 17.56I_sI_d + 25.88I_s^2) \right] + \left[ (12.79I_pI_s) \cos^2 \theta_r + 31.33I_p^2 + 408.70I_d^2 + 131.63I_s^2 + 302.23I_dI_p + 137.29I_sI_d + 505.48I_pI_s \right],
\]

\[
\rho_0^2(J_i = 3/2) = \rho_0^2(J_i = 1/2) \left[ -0.72I_p^2 + 3.77I_pI_d + 8.88I_sI_d + 12.75I_p^2 + 32.91I_s^2 + (2.15I_s^2 + 11.32I_pI_d - 26.65I_sI_d - 38.25I_p^2 + 98.72I_s^2) \right] - \left[ (275.40I_pI_s) \cos^2 \theta_r + 91.80I_pI_s \right],
\]

where

\[
\rho_0^0 = \rho_0^0(J_i = 1/2) \left[ (-0.60I_p^2 + 4.92I_pI_d + 16.20I_sI_d - 9.98I_p^2 - 52.29I_s^2) \cos^4 \theta_r + (-4.0I_p^2 + 21.13I_pI_d - 23.98I_sI_d + 14.97I_p^2 + 34.86I_s^2 + 45.88I_pI_s) \cos^2 \theta_r + 21.95I_p^2 + 159.68I_d^2 + 87.15I_s^2 + 135.08I_dI_p + 98.79I_sI_d + 239.82I_pI_s \right],
\]

\[
\rho_0^2(J_i = 3/2) = \rho_0^2(J_i = 1/2) \left[ (I_p^2 + 2.0193I_pI_d + 0.2196I_sI_d - 0.1412I_p^2 - 0.1750I_s^2 - 0.7037I_pI_s) (9.497 - 28.490 \cos^2 \theta_r) \right],
\]

where

\[
\rho_0^0 = \rho_0^0(J_i = 1/2) \left[ (11.59I_p^2 - 11.96I_pI_d + 11.73I_sI_d + 1.05I_p^2 + 4.40I_pI_s) \cos^4 \theta_r + (-0.920I_p^2 + 15.182I_pI_d + 11.506I_sI_d - 1.573I_p^2 - 1.756I_s^2 - 3.338I_pI_s) \cos^2 \theta_r - 32.783I_p^2 \right] - 16.781I_p^2 - 4.389I_s^2 - 55.5989I_dI_p - 28.387I_sI_d - 17.447I_pI_s,
\]

and

\[
I_s = (U_{1533.4}S_{1533.4} + I_{1309.3}S_{1309.3} + I_{1023.7}S_{1023.7})/S_{1533.4},
\]

\[
I_p = I_{1194.5},
\]

\[
I_d = (I_{1816.9}S_{1816.9} + I_{1264.7}S_{1264.7} + \ldots)/S_{1816.9},
\]

in which \( S_i \propto gB \) is the line strength of the line with wavelength \( \lambda \). The line intensities for various media are listed in Table 5, where the values for the interstellar radiation field (ISRIF) are particularly relevant to the weak pumping regime that we discuss here. If we use these density tensors with equations (1) and (3), we get the absorption coefficients and the corresponding polarizations and line ratios.

O \( i \) and S \( i \) have ground states \( 3P_{2,1,0} \) and upper levels \( 3S_{1/2}, 3D_{1/2,3/2,5/2} \). The ground level is \( 3P_{2} \). Thus, unlike C \( ii \) and Si \( ii \), O \( i \) and S \( i \) are alignable in both regimes regardless of \( \tau_A^{-1} > \sigma < A_m \). The degree of alignment, however, differs in the two regimes. The case for weak pumping is presented in § 4.2.2. In the case of strong pumping, which we consider here, both levels of \( 3P_{1,2} \) are alignable. Equation (11)
is then a five-dimensional matrix with $\rho^{0,2}(J_l = 1)$ and $\rho^{0,2,4}(J_l = 2)$. The explicit generic solution is very lengthy. Here, as an example, we give the results for pumping by a shell star (a blackbody source of $T = 15,000K$; see Table 5). For O:\n\begin{align*}
\rho_0^0(J_l = 1) & = 0.0851 \cos^{14}\theta_r - 0.4823 \cos^{12}\theta_r - 1.3183 \cos^{10}\theta_r + 7.2195 \cos^8\theta_r + 4.7687 \cos^6\theta_r - 20.6145 \cos^4\theta_r - 12.0874 \cos^2\theta_r + 6.9951
\end{align*}
\begin{align*}
\rho_1^0(J_l = 1) & = 0.1051 \cos^{14}\theta_r - 0.1746 \cos^{12}\theta_r - 2.8148 \cos^{10}\theta_r + 8.0475 \cos^8\theta_r + 7.3564 \cos^6\theta_r - 26.7061 \cos^4\theta_r - 15.9694 \cos^2\theta_r + 9.1487 \\
\rho_2^0(J_l = 2) & = 1.3136 \cos^{12}\theta_r - 5.3551 \cos^{10}\theta_r - 2.2558 \cos^8\theta_r + 18.1327 \cos^6\theta_r + 4.1870 \cos^4\theta_r - 8.9711 \cos^2\theta_r + 1.9016
\end{align*}
\begin{equation}
(25)
\end{equation}

where $\rho_0^0(J_l = 0) = (0.3989 \cos^{14}\theta_r - 1.6228 \cos^{12}\theta_r - 1.3920 \cos^{10}\theta_r + 8.5950 \cos^8\theta_r + 2.7734 \cos^6\theta_r - 14.5724 \cos^4\theta_r - 5.7430 \cos^2\theta_r + 3.8703)$. For S:\n\begin{align*}
\rho_0^0(J_l = 1) & = 0.0851 \cos^{14}\theta_r - 0.4823 \cos^{12}\theta_r - 1.3183 \cos^{10}\theta_r + 7.2195 \cos^8\theta_r + 4.7687 \cos^6\theta_r - 20.6145 \cos^4\theta_r - 12.0874 \cos^2\theta_r + 6.9951
\end{align*}
\begin{align*}
\rho_1^0(J_l = 1) & = -4.9 \cos^{12}\theta_r + 4.3 \cos^{10}\theta_r + 108.4 \cos^8\theta_r + 213.6 \cos^6\theta_r - 814.1 \cos^4\theta_r - 2929.5 \cos^2\theta_r + 1057.7 \\
\rho_2^0(J_l = 2) & = 7.02 \cos^{12}\theta_r - 13.92 \cos^{10}\theta_r - 112.56 \cos^8\theta_r - 234.34 \cos^6\theta_r + 1169.11 \cos^4\theta_r + 2002.21 \cos^2\theta_r - 787.19
\end{align*}
\begin{equation}
(26)
\end{equation}

where $\rho_0^0(J_l = 0) = (2.3 \cos^{14}\theta_r - 4.4 \cos^{12}\theta_r - 57.5 \cos^{10}\theta_r - 64.3 \cos^8\theta_r + 657.9 \cos^6\theta_r + 1620.9 \cos^4\theta_r - 2519.7 \cos^2\theta_r - 6083.6)$. In fact, C, Si, and S have the same ground state term $3F_{0,1,2}$, but with an opposite energy sequence; namely, $3P_0$ is the ground level. Thus, they are only alignable in the strong pumping case. In this case, their alignment is similar to that of O and S. The results are given in Appendix F.

Cr\n has a ground state $\alpha 6S_{5/2}$ and excited state $\alpha 6P_{3/2}$. Transition between them generates the triplet $2056, 2062, 2066 \text{ Å}$. The most important difference between Cr II and the other species discussed in this paper is the presence of a metastable state $\alpha 6D_{1/2,3/2,\ldots,9/2}$ (with lifetime $\tau_T \sim 10$ s), which can act as the proxy of the ground state. The transitions between the metastable levels and the upper levels are permitted (with wavelength ranging in $2740$–$2767 \text{ Å}$). An analogy thus can be made between Cr II and the atomic species with multiple lower levels that we deal with in this paper. In the strong pumping regime, both the ground level and the metastable level can be aligned. Using the similar treatment (neglecting the forbidden transition between the metastable levels and the ground level), we can get the alignment and polarizations for the two groups of transitions (the triplet from the ground state and the absorptions from the metastable state). Since there are many levels involved, and the measure of the equations we need to solve is $17 \times 17$, we only present selected numerical results here. The dipole components of the density matrices for the ground and metastable level is given in the right panel of Figure 5. We also show the polarization and line ratio of the line ($\alpha 6S \rightarrow \alpha 6P_{1/2}, 2066 \text{ Å}$) and the line ($\alpha 6D_{3/2} \rightarrow \alpha 6P_{3/2}, 2750 \text{ Å}$) in Figure 9.

There is an advantage to the use of the absorption lines from the metastable level. Because the energy of the metastable level is high, they can be populated only through optical pumping. The absorption lines from the metastable levels, therefore, only trace the medium in the vicinity of a pumping source. Absorption from the ground state, however, can happen in regions much further away, where alignment is marginal. The actual observed signals depend on $N_e/N_{\text{tot}}$, the ratio of column density to total column density.
Combining the absorptions from the metastable state and the ground state allows polarimetric testing of the conditions in the vicinity of the pumping source. Because of that we intend to provide a study of more atoms with metastable states (see Table 2) in our companion paper.

4.2.2. Weak Pumping

Now let us consider the second regime in which the radiation source is very distant, so the magnetic dipole transition rate within the ground state dominates over the optical pumping rate. Because of the faster magnetic dipole decay, we can neglect the optical pumping from levels other than the ground level, i.e., only the ground level is alignable. In this case, additional magnetic dipole transitional terms should be added to equation (5). For the ground level,

\[
\rho_0^k(J_i^0) = \sum_{J_i} (-1)^{J_i^0 + J_i + k + 1} [J_i] \left\{ \begin{array}{c} J_i^0 \\ J_i \\ 1 \end{array} \right\} A_m(J_i \rightarrow J_i^0) \rho_0^k(J_i) \sum_{J_u} (-1)^{J_u^0 + J_u + k + 1} [J_u] \left\{ \begin{array}{c} J_u^0 \\ J_u \\ 1 \end{array} \right\} A(J_u \rightarrow J_i^0) \rho_0^k(J_u)
\]

\[
- [J_i^0] \sum_{J_l,\ell} B(J_i^0 \rightarrow J_u) (-1)^{J_l^0 + J_u + k + 1} (3[k, k', K])^{1/2} \left\{ \begin{array}{c} 1 \\ J_l^0 \\ J_u \end{array} \right\} \left\{ \begin{array}{c} k \\ k' \\ K \end{array} \right\} \left( \begin{array}{c} 1 \\ k' \\ K \end{array} \right) J_l^0 \rho_0^k(J_u).
\]

For those lower levels other than the ground level,

\[
\rho_0^k(J_i) = - \sum_{J_i'} A_m(J_i \rightarrow J_i') [E(J_i) > E(J_i')] \rho_0^k(J_i') + \sum_{J_u'} (-1)^{J_u' + J_i + k + 1} [J_u'] \left\{ \begin{array}{c} J_u' \\ J_i \\ J_i' \end{array} \right\} A_m(J_i \rightarrow J_i') \rho_0^k(J_i')
\]

\[
+ \sum_{J_u} (-1)^{J_u + J_i + k + 1} [J_u] \left\{ \begin{array}{c} J_i \\ J_u \\ J_i' \end{array} \right\} A(J_u \rightarrow J_i') \rho_0^k(J_u).
\]

Fig. 9.—Contour graphs of Cr ii polarization and line ratio. Top: Absorption from the ground state \(a\)S. Bottom: Absorption from the metastable state \(a\)D. [See the electronic edition of the Journal for a color version of this figure.]
As discussed earlier, because of fast magnetic mixing, there are no interference terms, and thus there are only \( \rho^0_k \) terms in the above equations. Combining with equation (4), we get the following set of linear equations:

\[
\sum_{J_u} A(J_u \rightarrow J_i') [J_i^0] B(J_i^0 \rightarrow J_u) \sum_{k'} \rho^0_{k'} (J_i^0) \sum_k (-1)^{k+k'} (3[k,k',K])^{1/2} \begin{pmatrix} k & k' & K \\
0 & 0 & 0 \end{pmatrix} J_0^k \left[-1\right]^{i_{J_i'} + J_u} [J_u] \left\{ \begin{array}{c} J_i^0 \\
J_i^0 \\
J_i^0 \end{array} \right\} \left( \begin{array}{c}
J_u \\
J_u \\
1 \end{array} \right) \\
\times \left\{ \begin{array}{c} J_u \\
J_u \\
K \\
k' \end{array} \right\} + (-1)^{J_{i_{J_i'}} - J_u} \left\{ \begin{array}{c} J_u \\
J_u \\
J_u \\
J_u \\
k' \end{array} \right\} \} \} + \sum_{J_{i_{J_i'}} \pm 1} \delta_{J_{i_{J_i'}}} (-1)^{i_{J_i'} + J_u} [J_i'] \left\{ \begin{array}{c} J_i^0 \\
J_i^0 \\
J_i^0 \\
J_i^0 \\
k' \end{array} \right\} A_m (J_i' \rightarrow J_u) \rho^0_{i_{J_i'}} (J_u) = 0, \\
(29)
\sum_{J_u} A(J_u \rightarrow J_i) [J_i^0] B(J_i^0 \rightarrow J_u) \sum_{k'} \rho^0_{k'} (J_i^0) \sum_k (-1)^{k+k'} (3[k,k',K])^{1/2} \begin{pmatrix} k & k' & K \\
0 & 0 & 0 \end{pmatrix} J_0^k \left[-1\right]^{i_{J_i} + J_u} [J_u] \left\{ \begin{array}{c} J_i^0 \\
J_i^0 \\
J_i^0 \end{array} \right\} \left( \begin{array}{c}
J_u \\
J_u \\
1 \end{array} \right) \\
\times \left\{ \begin{array}{c} J_u \\
J_u \\
K \\
k' \end{array} \right\} + \sum_{J_{i_{J_i'}}} A_m (J_i \rightarrow J_{i_{J_i'}}) \rho^0_{i_{J_i'}} (J_i) + (-1)^{i_{J_i'} + J_i} [J_i'] \left\{ \begin{array}{c} J_i^0 \\
J_i^0 \\
J_i^0 \\
J_i^0 \\
k' \end{array} \right\} A_m (J_{i_{J_i'}} \rightarrow J_i) \rho^0_{i_{J_i'}} (J_i') = 0. \\
(30)
\]

By solving these equations, we can get the ground level (3P2) dipole density component of O i and S i:

\[
\sigma^0_0 (J_i^0 = 2) = \left[ (5.14D23^2 + 2.21P2223 + 13.21S2123 - 44.73S21^2 + 9.96P22S21) \cos^4 \theta_r + (46.47D23^2 - 63.20P2223 + 31.17S2123 - 18.59P22^2 + 133.35S21^2 + 58.30P22S21) \cos^2 \theta_r + 14.92D23^2 + 6.20P22^2 - 32.81S21^2 + 20.82D23P22 - 11.86D23S21 - 20.54P22S21 \right]\left[ (-2.35D23^2 + 5.63P2223 - 5.66S2123 + 5.45P22^2 + 11.86S21^2 + 2.77P22S21) \cos^4 \theta_r + (21.83D23^2 + 8.01P2223 + 47.35S2123 - 22.10P22^2 + 37.97S21^2 + 40.71P22S21) \cos^2 \theta_r - 71.97D23^2 - 141.15P22^2 - 133.61S21^2 - 215.84D23P22 - 199.53D23S21 - 281.43P22S21 \right],
\]

where S21, P22, and D23 = \( \sum I_i S_{i_i} \). For O i,

\[
S21 = I_5 S_{1302.2} = 1.08 I_5 = I_{1302.2} S_{1302.2} + I_{1039.2} S_{1039.2}, \quad P22 = 0,
\]

\[
D23 = I_D S_{988.7} = 0.732 I_D = I_{1025} S_{1025} + I_{988.7} S_{988.7} + I_{971.7} S_{971.7}.
\]

For S i,

\[
S21 = I_5 S_{1826.2} = 0.502 I_{1826.2}, \quad P22 = I_P S_{1295.6} = 1.78 I_{1295.6},
\]

\[
D23 = I_D S_{1425} = 0.149 I_D = I_{1474} S_{1474} + I_{1425} S_{1425} + I_{1316.5} S_{1316.5}.
\]

The line strengths \( S_{i_i} \) in the above two equations are taken from the NIST Atomic Spectra Database.\(^9\)

We only provide the density matrices for the ground level. Absorption from the levels other than the ground level is negligible because the absorption rate is less than the magnetic decay rate.

#### 4.2.3. Role of Collisions

In the above calculations, collisions are neglected. Now let us see whether this is a reasonable assumption. Basically, there are two types of collisions. The first type is an inelastic collision, which excites transitions among the \( J \)-levels of the ground state. The second type of collision is elastic and only causes the angular momentum \( \textbf{J} \) to flip.

The inelastic collision can happen with electrons or hydrogen atoms. Both rates can be found in Spitzer (1978). The transition rate for collisions with electrons is

\[
f_{k,j} = n_e \gamma_{k,j} = \frac{8.63 \times 10^{-6} n_e \alpha_{i_j} (j,k)}{|k|^3} \text{ cm}^3 \text{ s}^{-1},
\]

where \(|k|\) is the statistical weight of level \( k \); \( \alpha \) is the collisional strength and is of order unity (Osterbrock 1974), while the magnetic dipole emission rate between the lower levels is \( A_m \approx 10^{-6} \text{ s}^{-1} \). Apparently, the collisional rate is much smaller for diffuse medium.

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\(^9\) Available at [http://physics.nist.gov/PhysRefData/ASD/index.html](http://physics.nist.gov/PhysRefData/ASD/index.html).
collisions with hydrogen atoms, \( f_{ki} = n_H \gamma_{ki} \approx 10^{-10} n_H \text{s}^{-1} \), this is also smaller than the magnetic dipole decay rate \( A_m \). Thus, both collisions with electrons and collisions with hydrogen atoms are less frequent than magnetic dipole decay in diffuse medium.

The second collision involves spin flip and is dominated by Van der Waals interactions with neutral atoms, in particular, hydrogen atoms. The corresponding rate is (Lam & ter Haar 1971; Landi Degl’Innocenti & Landolfi 2004),

\[
f_{kj} \approx 2 \times 10^{-10} \left( \frac{13.6 \text{ eV}}{E_i - E_f} \right)^{0.8} T \left( 1 + \frac{1}{\mu} \right)^{0.3} n_H \text{ s}^{-1},
\]

where \( E_i \) is the ionization potential, \( E_f \) is the energy of the \( J \) level, and \( \mu \) is the atomic weight.

In the weak pumping regime, which we discussed in § 4.2.2, collisions can become comparable to the optical pumping rate. In this case, alignment is partially destroyed.\(^{10}\) Since pumping rates for different species are different, we can find the tomography of the magnetic field by using lines from various species. By equating the collisional rate \( \max (f_{ki}, f_{kj}) \) to the optical pumping rate, we obtain the alignable range for an O-type star,

\[
r_2 \approx \begin{cases} 16 \left( \frac{0.1 \text{ cm}^{-3}}{n_e} \right) \sqrt{\frac{T}{8000 \text{ K}}} \text{pc}, & \text{for C ii}, \\ 12.6 \left( \frac{0.1 \text{ cm}^{-3}}{n_e} \right) \sqrt{\frac{T}{8000 \text{ K}}} \text{pc}, & \text{for Si ii}. \end{cases}
\]

For a typical QSO with luminosity \( \langle L \rangle = \lambda L_\lambda = 10^{46.16} \text{ ergs s}^{-1} \) at 1100 Å and a spectrum (Telfer et al. 2002)

\[
F_\nu \propto \begin{cases} \nu^{-0.7}, & \lambda > 1250 \text{ Å}, \\ \nu^{-1.7}, & \lambda < 1250 \text{ Å}, \end{cases}
\]

the alignable range would be

\[
r_2 \approx \begin{cases} 27.2 \left( \frac{10^{-4} \text{ cm}^{-3}}{n_e} \right) \sqrt{\frac{T}{10^6 \text{ K}}} \text{Mpc}, & \text{for C ii}, \\ 11.6 \left( \frac{10^{-4} \text{ cm}^{-3}}{n_e} \right) \sqrt{\frac{T}{10^6 \text{ K}}} \text{Mpc}, & \text{for Si ii}. \end{cases}
\]

In actual observations, there is inevitably an average along the line of sight. The absorption from atoms outside the alignable range tend to reduce the degree of polarization. This adds another dependence on \( N_{\text{align}} / N_{\text{tot}} \), the ratio of alignable column density to total column density. In this case, there is an advantage to choose species present only near a pumping source. For instance, the column densities of many ions (including the pumping source) coming through these atoms will be absorbed differentially, causing polarization in absorption lines.

5. SUMMARY OF THE PROCEDURE ADOPTED

In this paper we provided calculations for the alignment of atoms with fine structure owing to pumping by anisotropic radiation in a magnetized medium. The angular momentum of the radiation is transferred to the atoms and causes differential occupation in terms of angular momentum (magnetic quantum number) of the ground state. This results in atomic alignment. Any background radiation (including the pumping source) coming through these atoms will be absorbed differentially, causing polarization in absorption lines.

In most astrophysical environments incident radiation is axially symmetric. This makes the irreducible density matrix the suitable formalism to describe the system (see Appendix A). In this case, alignment is simply represented by the \( \rho_q^0 \) term, which causes linear polarization. Total polarization is given by the \( \rho_q^0 \) term. Since the incident radiation is unpolarized, there is no orientation, and the corresponding terms \( \rho_q^j \) are zeros, accordingly. Circular polarization, resulting from orientation, does not exist in this case.\(^{11}\)

When the magnetic field exists, it is convenient to choose the magnetic field direction as the quantization axis. Coherence terms \( \rho_{q,0}^{0} = 0 \) disappear because of fast magnetic mixing on the ground state. In this sense, atoms are realigned along the magnetic field compared to the case without a magnetic field. This requires a transformation from the general observational frame to the theoretical frame where the magnetic field defines the \( z \)-axis (Fig. 4). This can be done by two Euler rotations (see Appendix E).

We solve the statistical equilibrium equations of both ground state and excited state and obtain the density matrix \( \rho_q^0 \) of the ground states. Multilevel atoms are considered as well. All of the transitions between the levels in the ground state and the levels in the upper state should be taken into account to get the correct atomic alignment. If there are multiple lower levels and the decay rate between them is larger than the optical pumping rate, the transitions between the levels should be also taken into account and their corresponding statistical

\(^{10}\) To calculate the residual alignment, one should include collisional transitions in the statistical equilibrium equations of the atomic populations (see, e.g., Yan & Lazarian 2006).

\(^{11}\) If the background source is polarized in a different direction with alignment, circular polarization can be generated due to dephasing. This is a second-order effect, and we shall discuss it in Yan & Lazarian (2006).
equilibrium equations should be included to solve the level populations. These transitions can substantially change the alignment. This is the weak pumping case we studied in § 4.2.2.

Then, we insert the dipole component of the density matrix of the ground state $\sigma_1^2$ into the equation for absorption coefficients (eq. [14]), yielding line ratios and polarizations for absorption lines (eqs. [1] and [3]). Polarization of absorption is either parallel or perpendicular to the magnetic field in the pictorial plane because of fast precession around the magnetic field in the ground state.

The right panel of Figure 8 illustrates when and in what form we expect to see atomic alignment depending on the atoms and their environmental conditions. It also illustrates the computational steps to determine the direction of magnetic field.

6. DISCUSSION

6.1. Choice of Atoms

We have chosen a particular set of atoms (see Table 1) as examples of alignment for atoms with fine structures. Since the only requirement for atomic alignment is having a ground level with total angular momentum $J > 1/2$, there are many more atomic species that can be explored. Regardless of the differences in specific atomic structures, the physics is similar. By considering transitions from multiple levels to multiple levels, we have developed a procedure that can be applied to virtually all atoms’ fine structure. We did not include level coherence, as it is negligible except for a highly turbulent medium in which $\delta v \approx 100 \text{ km s}^{-1}$. For multiplets the absorptions $J_i \geq J_f$ are usually more polarized than other components, which provides more routes and more balance between $\sigma$- and $\pi$-transitions.

Practically, however, there is inevitably an averaging along the line of sight, which adds another dependence on the ratio of alignable column density and total column density $N_a/N_{\text{tot}}$. For different species this ratio is different. The ratio should be close to 1 for highly ionized species that only exist near radiation sources. The same is true for the absorption from a metastable state, for instance, Cr $\Pi$, which we discussed earlier. The energies between the upper and metastable state in this case are too high to be affected by collisions with thermal particles in cool phases along the line of sight. Thus, the corresponding absorption lines are also mostly affected by the medium in the vicinity of a star (or QSO). Combining different species (with different $N_a/N_{\text{tot}}$), it is possible to get a tomography of the magnetic field in situ. To extend the technique, we present elsewhere calculations for more atoms with metastable states.

The particular choice of atoms to use depends on both the instruments available and the object to be studied. With the advent of space-based UV polarimetry, many more alignable transitions will become important. This will allow detailed studies of magnetic fields in, e.g., circumstellar regions, the diffuse ISM, the intergalactic medium, supernovae remnants, and QSOs. However, even at present, a substantial progress may be achieved using ground-based and possibly sounding-rocket observations.

6.2. Wavelengths Employed and Particular Cases

The incomplete list of astrophysically interesting transitions in Tables 1 and 2 shows that many of the transitions are in the UV part of the spectrum. At the moment, the optical polarization is easily available, e.g., Cr $\Pi$ and Ti $\Pi$. Furthermore, we expect that in the years to come, UV polarimetry will become an essential tool for astrophysical research. In the mean time, a number of transitions (e.g., Ti $\Pi$, Cr $\Pi$, and Fe $\Pi$) as well as atomic transitions shifted due to cosmological redshifts can be studied using present ground based polarimetry facilities. The latter is applicable to, e.g., Cr $\Pi$ detectable in QSO absorption (Kulkarni et al. 2005).

We did not discuss transitions beyond the Lyman limit 912 Å. There are actually many lines in this range, especially for absorptions in QSOs (Verner et al. 1994) and Ly$\alpha$ clouds (see Akerman et al. 2005). Indeed for diffuse ISM, absorptions are severe. However, for QSOs this constraint is relieved because of the proximity effect, which is the measured decrease in the number of Ly$\alpha$ clouds induced by the UV radiation from a QSO in its vicinity (Madau 1992). When the light reaches those Ly$\alpha$ clouds far apart, it is already redshifted to a wavelength longer than 912 Å, so it is free of the absorptions from H $\Pi$ as well. Similarly, these lines are also informative for the local bubble (see Redfield & Linsky 2004).

As we discussed for the case of weak pumping (§ 4.2.2), the forbidden decay of the ground state may result in the loss of alignment for atoms like C $\Pi$ and Si $\Pi$, which is an interesting observable effect in itself. For other transitions (e.g., O $\Pi$ and S $\Pi$), the decay does not erase but modifies the alignment.

It is worth mentioning that when the photon arrival rate is comparable the Larmor frequency, the polarization becomes sensitive to the amplitude of magnetic field. This opens an avenue for obtaining the strength of weak magnetic fields, especially for the circumstellar region and QSOs. If there are several species spatially correlated, it is possible to measure both the magnetic field strength and direction. In addition, even measurements of unpolarized lines are useful in conjunction with studies of aligned species. If the photon arrival rate is less than the Larmor precession rate, atoms are aligned with respect to the radiation field and their line polarization does not contain information about the magnetic field. For instance, as we pointed out in § 2.2, the magnetic field can only realign C $\Pi$ at a distance $\geq 7.7 \text{ Au}$ from an O star if the magnetic field strength is $\sim 5 \mu G$.

We would like to emphasize here that we deal with the optically thin case or unsaturated absorption in this paper. Radiative transfer needs to be considered for the saturated regime.

6.3. Utility of Transitions

There are a number of parameters that determine the polarization: the direction of the magnetic field and radiation field ($\theta_1, \theta_2$), the percentage of radiation anisotropy $W_a/W_r$, and the ratio of alignable column density and total column density $N_a/N_{\text{tot}}$. The list of observables we have are the degree of polarization and the line intensity ratios (equal to optical depth ratios for the optically thin case) for multiple lines. Since there are many alignable atomic species, we can have more known observables to determine the magnetic field if we make sure that these atomic species coexist. One can also use the transitions between different levels of the same atom.

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12 This possibility was mentioned to us by K. Nordsieck.
Combining different atomic species, we can make a tomography of magnetic field, as the ratio $N_r/N_{tot}$ varies with each atomic species. Moreover, the fact that the radius of alignment changes for different environments, allows polarimetric testing of the conditions in the vicinity of the pumping source. If the alignment is produced by the background radiation source that is used to study the absorption lines, the line of sight coincides with the direction of radiation, resulting in $\theta_r = \theta$.

Our calculations in this paper were aimed at exploring the feasibility of the measurements of the astrophysical magnetic fields using atomic alignment. Above in the paper we dealt with the fine-structure line transitions, while the hyperfine-structure transitions are considered in Yan & Lazarian (2006). Taken together, the alignable species allow studies of magnetic fields and environmental conditions for various astrophysical systems, from comet atmospheres to stellar winds, supernovae, reflection nebulae, interstellar gas, and the intergalactic medium.

6.4. Studies Using Contour Plots

Practically, we can make use of contour plots like what we did for S $\equiv$ and Cr $\equiv$ to get information on the magnetic field and environment from observational data. For instance, let us say that one measures for the S $\equiv (J_e = 1/2)$ absorption line degree of polarization per unit optical depth 8% and for the equivalent width ratio of S $\equiv (J_e = 1/2, 3/2)$ doublet 0.48. First, from the direction of polarization, we know that the magnetic field projection is either parallel or perpendicular to the direction of polarization in pictorial plane (azimuthal angle $\phi_B$; see Fig. 4). This is already the whole information one can expect from the Goldreich-Kylafis effect (Goldreich & Kylafis 1982). Then, using the degree of polarization and the line ratio, one can get further information about the magnetic field in the plane of line of sight $\theta$, and also the direction of pumping source $\theta_r$. By overlapping the two contour graphs (Fig. 7), we can find their crosses at $\theta_r \approx 90^\circ \pm 72^\circ$ and $\theta \approx 90^\circ \pm 7^\circ$. With $\theta$ known, we can easily remove the $90^\circ$ degree degeneracy of $\phi_B$ according to the Van-Weck effect and thus obtain the three-dimensional information of the magnetic field. If we know two species coexist spatially, we can also use their ratio of degrees of polarizations and their ratio of equivalent widths. In this case, we do not need to know the optical depth for the individual line. Alternatively, $\theta_r$ and $\theta$ can be obtained by solving equations (1) and (3). Calculating $\theta_r$ and $\theta$ several ways cross-checks the results and decreases the uncertainties.

6.5. Complementary Techniques

Atomic alignment can render unique information about the magnetic field. For instance, grain alignment (see review by Lazarian [2003]) provides information about the component in the plane of the sky. Similar to the case of interstellar dust, the rapid precession of atoms in a magnetic field makes the direction of polarization sensitive to the direction of the underlying magnetic field. As the precession of magnetic moments of atoms is much faster than the precession of magnetic moments of grains, atoms can reflect much more rapid variations of the magnetic field. The limited radius of atomic alignment is also an advantage to enable local measurement of three-dimensional magnetic fields while grain alignment tests global two-dimensional topology. More importantly, alignable atoms and ions can reflect magnetic fields in the environments where either the properties of dust change or the dust cannot survive. This opens wide avenues for magnetic field research in, e.g., circumstellar regions, the ISM, the interplanetary medium, the intracluster medium, and quasars.

The complementary nature of atomic alignment to the Goldreich-Kylafis effect (Goldreich & Kylafis 1982; Girart et al. 1999) is that the former is applicable to the regions near emitting sources (e.g., circumstellar regions), while the latter is applicable to molecular clouds. Moreover, atomic alignment provides three-dimensional tomography of magnetic fields that is not measurable otherwise.

A detailed discussion of alignment of atoms in different conditions corresponding to, for example, circumstellar regions (see Dinerstein et al. 2006), AGNs (see Kriss 2006), Ly$\alpha$ clouds (see Akerman et al. 2005), interplanetary space (see Cremonese et al. 2002), and the local bubble (see Redfield & Linsky 2002, 2004) is given in our companion paper. Note that for interplanetary studies one can investigate not only spatial, but also temporal, variations of magnetic fields. This can allow a cost-effective way of studying interplanetary magnetic turbulence at different scales.

The change of the optical depth is another important consequence of atomic alignment. The actual calculations should take into account the realignment caused by a magnetic field. It may happen that the variations of the optical depth caused by alignment can be related to the tiny-scale atomic structures (TSAS) observed in different phases of interstellar gas (see Heiles 1997).

7. SUMMARY

In this paper we identified a set of atoms and ions with fine structure that provide substantially intensive absorption lines in the visible and near-UV wavelength range. We calculated the degree of alignment of these species and obtained the relation between the Stokes parameters of the absorbed radiation and the three-dimensional orientation of the magnetic field with respect to the observer and the pumping/absorbed sources. Our calculations can easily be extended to other absorbing species with fine structure. In particular, we have shown the following.

1. The ground-state alignment happens as the result of interaction of atoms and ions that have $>2$ sublevels on the ground state with the anisotropic flow of photons. If the rate of excitation from the ground state is lower than the rate of atomic precession in the external magnetic field, the alignment happens with respect to the magnetic field.

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13 We call this case a “degenerate” case to distinguish from a case in which the background source and the source of the aligning radiation differ.

14 For optically thin lines, the equivalent width ratio is equal to optical depth ratio as given in the right panel of Figure 7.

15 The uncertainties come from the degeneracies of $\sin^2 \theta$ and $\cos^2 \theta$, in equations (1) and (3).

16 Grains have a tendency to be aligned with their long axes perpendicular to the magnetic field. Curiously enough, the interaction with photons is also important for grain alignment.

17 Atomic alignment has some similarity to the Goldreich-Kylafis effect, which also measures magnetic field through magnetic mixing. However, the Goldreich-Kylafis effect deals with the polarization of radio lines. The upper states of radio lines are so long lived that significant magnetic mixing can happen among different magnetic sublevels of these states. Atomic alignment, on the other hand, happens with ground states of optical and UV transitions.
2. The ground-state alignment affects the polarization state of the absorbed light as well as the intensity of different line components. By combining information from different components and absorption lines, one can study both three-dimensional topology of magnetic fields and physical conditions in important astrophysical environments that include quasars, the ISM, and extragalactic gas.

3. For the light absorbed by aligned atoms, the direction of polarization is either parallel or perpendicular to the magnetic field in the plane of the sky. The switch between the two possibilities happens at the Van-Vleck angle between the direction of the magnetic field and the pumping radiation.

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APPENDIX A

IRREDUCIBLE DENSITY MATRIX

In quantum mechanics a particle (atom or photon) can be described by its wave function, $|\psi\rangle = \sum_{JM} c_{JM} |JM\rangle$. An equivalent but more elegant description without the phase arbitrariness is a density matrix, $\rho = |\psi\rangle \langle \psi | = \sum_{JM,J'M'} c_{JM} c_{J'M'}^{*} |JM\rangle \langle J'M'|$. The diagonal term $\langle JM | \rho | JM \rangle = |c_{JM}|^2$ represents the probability of finding the particle on state $|JM\rangle$; off-diagonal terms $\langle JM | \rho | J'M' \rangle = c_{JM} c_{J'M'}^{*}$ characterize the coherence between two different states $|JM\rangle$ and $|J'M'\rangle$. The advantage of the description by density matrix becomes apparent when we are dealing with an ensemble of $N$ particles without mutual interactions. In this incident, the density matrix of the ensemble is simply the arithmetic mean of individual densities $\frac{1}{N} \sum_{i=1}^{N} \rho(i)$.

When a magnetic field is present, it is convenient to choose the magnetic field direction as the quantization axis for an atom, because $M_J$ remains a good quantum number on the ground state. Meanwhile, the density matrix of radiation (or their classical equivalent Stokes parameters; see Appendix D) should be obtained in a basis in which the line of sight is the quantization axis. This requires several rotations of the successive reference frames (see Appendix E). Usually, the incident radiation is axially symmetric, while the spontaneous emission is spherically symmetric. This makes the irreducible tensorial operators $\rho^Q_{JM}$ the suitable formalism for performing the calculations (Fano 1957; Dyakanov & Perel 1965; Omont 1965; Bommier & Sahal-Brechot 1978). The relation between the irreducible tensor and the standard density matrix is

$$\rho^Q_{JM}(J,J') = \sum_{MM'} (-1)^{J'-M}(2K+1)^{1/2} \left( \begin{array}{ccc} J & K & J' \\ -M & Q & M' \end{array} \right) \langle JM | \rho | J'M' \rangle. \tag{A1}$$

For photons, we consider only their electric dipole part, which has a spin $J = 1$. The irreducible tensor has a clear physical meaning. In particular, $\rho^Q_{00}$ is proportional to the overall population of an atomic level or the total intensity of the radiation; $\rho^Q_{10}$ describes orientation of an atom or the circular polarization of radiation; $\rho^Q_{20}$ represents the alignment term, which is responsible for linear polarization; the terms with $Q \neq 0$ are coherence terms. For atoms $\rho^Q_{10}(J,J')$ is the level-crossing coherence (Bommier & Sahal-Brechot 1978); $\rho^Q_{20}(J,J')$ is the Zeeman coherence as it corresponds to the crossing term $\rho_{MM'}$ according to the selection rule of the $3J$ symbol (Appendix C) in equation (A1). Note that in the regime we study (namely, a weak magnetic field $|B| < 1$ G) and a low-temperature diffuse medium, individual $J$-levels are much narrower than their separations, so level-crossing coherence is negligible.

APPENDIX B

RADIATIVE TRANSITIONS

We consider atoms with finite structure. The ground level of the atoms has an angular momentum $J, M$ and the excited state is $J', M'$. For spontaneous emission from a state $J'M'$ of higher energy to a lower energy state $J, M$, the transition probability per unit time is

$$a = \frac{64 \pi^4 e^2 a_0^2 \nu^3}{3 \hbar c^3} \sum_q |\langle JM | V_q | J'M' \rangle|^2, \tag{B1}$$

where $V_q = r \cdot e_q$ is the projection of the dipole moment along the basis vector $e_q$ of radiation, $e_\pm = (\mp \hat{x} - \hat{y})/\sqrt{2}$, and $e_0 = \hat{z}$. According to the Wigner-Eckart theorem (Weissbluth 1978), the electrical dipole matrix element is

$$R_{J'J} = \langle JM | V_q | J'M' \rangle = (-1)^{J'+M-1} \left( \begin{array}{ccc} J & 1 & J' \\ -M & q & M' \end{array} \right) \langle J | V_q | J' \rangle, \tag{B2}$$

18 Throughout this paper, we follow the convention that $M$ represents the projection of angular momentum along the magnetic field—the quantization axis.
where the matrix is a Wigner $3j$ symbol, which is a measurement of coupling of ground angular momentum ($J, M$) and photon angular momentum ($1, q$) forming angular momentum ($J', M'$) of the upper level (see Appendix A), and $\langle J\|V_q\|J'\rangle$ is the square root of the line strength, whose value can be found in atomic physics books, e.g., Cowan (1981).

**APPENDIX C**

**COUPLING OF ANGULAR MOMENTUM VECTORS**

Each atomic (or photon) state has a particular angular momentum associated with it. Because of its vector nature, we can describe it with two quantum numbers, $|jm\rangle$, where $j$ is total angular momentum and $m$ is its projection along the quantization axis. Whenever discussing any atomic processes involving direction, we would find it is inevitable to involve angles and momenta. Angular momentum theory arises as a result.

When two angular momenta $J, m_1$ and $J', m_2$ are coupled, the resulting angular momentum $j, m$ can be described as (Zare 1988)

$$|jm\rangle = \sum_{m_1, m_2} \langle Jm_1, J'm_2 | jm \rangle |Jm_1, J'm_2\rangle,$$

(C1)

where the elements of transformation $\langle Jm_1, J'm_2 | jm \rangle$ are called Clebsch-Gordan coefficients or Wigner coefficients. To better reveal its symmetric properties, the Wigner $3j$ symbol was introduced

$$\left( \begin{array}{ccc} J & J' & j \\ m_1 & m_2 & -m_3 \end{array} \right) \equiv (-1)^{J-J'-m_3} [j_3]^{-1/2} \langle Jm_1, J'm_2 | jm \rangle$$

(C2)

The $3J$ must satisfy the triangle relations

$$J + J' \geq j_3, \quad J' + j_3 \geq J, \quad \text{and} \quad J + j_3 \geq J'$$

(C3)

and $m_1 + m_2 - m_3 = 0$. If one is concerned about radiative transitions, one of the angular momentum ($J', m_2$) terms would be equal to that of a photon ($1, q$). It then follows that the selection rules for transitions between states $J, m_1, j_3,$ and $m_3$ are

$$\Delta j = j_3 - J = 0, \pm 1, \quad \Delta m = m_3 - m_1 = 0, \pm 1,$$

(C4)

with the restriction that $J = j_3 = 0$ is not allowed.

When three angular momenta $J, m_1, J', m_2,$ and $j_3, m_3$ need to be coupled, there is more than one way to do this. We can first couple $J, m_1$ and $J', m_2$ to get $j_{12}$ and then add $j_3$, or we can first couple $j_3, m_3$ and $J', m_2$ and get the same result. The transformation between $\langle j_{12}, j_3 | jm \rangle$ and $\langle j_{23} | jm \rangle$ is given by (Zare 1988)

$$|j_{23} | jm \rangle = \sum_{j_{12}} \langle j_{12}, j_3 | j_{23} \rangle |j_{12}, j_3 | jm \rangle,$$

(C5)

where the expansion coefficients $\langle j_{12}, j_3 | j_{23} \rangle$ are called recoupling coefficients, as they describe the transformation between two coupling schemes $\langle j_{12}, j_3 | jm \rangle$ and $\langle j_{23} | jm \rangle$. A closely related quantity was then defined by Wigner (1965), which has higher symmetry,

$$\left( \begin{array}{ccc} J & J' & j_{12} \\ j_3 & j & j_{23} \end{array} \right) = (-1)^{J+J'+j_{12}+j_{23}} [j_{12} j_{23}]^{-1/2} \langle j_{12}, j_3 | j_{23} \rangle$$

(C6)

Similarly, the recoupling of four angular momenta leads to the $9j$ symbol

$$\left( \begin{array}{ccc} J & J' & j_{12} \\ j_4 & j_3 & j_{34} \\ j_{14} & j_{23} & j \end{array} \right) = [j_{12}, j_{23}, j_{34}, j_{14}]^{-1/2} \langle j_{12}, j_{34} | j_{14} | j_{23} \rangle.$$

(C7)

**APPENDIX D**

**STOKES PARAMETERS**

The electric field in the plane perpendicular to the propagation direction can be developed over the usual basis $(e_x, e_y)$,

$$E(t) = [E_1 \hat{e}_x (\cos \omega t - \delta_1) + E_2 \hat{e}_y (\cos \omega t - \delta_2)].$$

(D1)
When $\delta_1 = \delta_2$, the light is linearly polarized. For theoretical calculations, it is more convenient to use basis composed of left- ($e_-$) and right- ($e_+$) handed circular vibrations. They are defined as

$$e_\pm = [\pm e_1 + ie_2] / \sqrt{2}.$$  \hfill (D2)

By using ($e_+, e_-$), one has a radiation tensor

$$\Pi = \begin{pmatrix}
    \langle E_+E_+ \rangle & \langle E_+E_- \rangle \\
    \langle E_-E_+ \rangle & \langle E_-E_- \rangle
\end{pmatrix} = \begin{pmatrix}
    \Pi_{++} & \Pi_{+-} \\
    \Pi_{-+} & \Pi_{--}
\end{pmatrix},$$  \hfill (D3)

where angle brackets denote an average over time.

The definition of Stokes parameters $S_i = [I, Q, U, V]$ ($i = 0, 1, 2, 3$) and their relation with $\Pi$ is as follows:

$$I = E_1^2 + E_2^2 = \Pi_{++} + \Pi_{--},$$

$$Q = E_1^2 - E_2^2 = -(\Pi_{++} + \Pi_{--}),$$

$$U = 2E_1E_2 \cos (\delta_1 - \delta_2) = -i(\Pi_{++} - \Pi_{--}),$$

$$V = 2E_1E_2 \sin (\delta_1 - \delta_2) = \Pi_{++} - \Pi_{--}. \hfill (D4)$$

The observable quantities, intensity, degree of polarization $p$, and direction of linear polarization $\chi$ (Fig. 2, right), have simple relations with the Stokes parameters:

$$I = \text{Tr}(\Pi), \quad p = \sqrt{Q^2 + U^2}/I, \quad \sin (2\chi) = U/I, \quad \cos (2\chi) = Q/I. \hfill (D5)$$

APPENDIX E
FROM OBSERVATIONAL FRAME TO MAGNETIC FRAME

In real observations, the line of sight is fixed and the direction of the magnetic field is unknown. Thus, a transformation is needed from the observational frame to the theoretical frame in which the magnetic field is the quantization axis. This can be done by two Euler rotations, as illustrated in Fig. 4. In the original observational coordinate $xyz$-system, the direction of the line of sight is defined as the $z$-axis, and the direction of magnetic field is characterized by polar angle $\theta$, and azimuthal angle $\phi_B$. First, we rotate the whole system by an angle $\phi_B$ about the $z$-axis, so as to form the second coordinate system $x'y'z'$. The second rotation is from the $z'(z')$-axis to the $z''$-axis by an angle $\theta$ about the $y'$-axis. Mathematically, the two rotations can be fulfilled by multiplying rotation matrices,

$$\begin{pmatrix}
    \cos \theta & 0 & -\sin \theta \\
    0 & 1 & 0 \\
    \sin \theta & 0 & \cos \theta
\end{pmatrix} \begin{pmatrix}
    \cos \phi_B & \sin \phi_B & 0 \\
     -\sin \phi_B & \cos \phi_B & 0 \\
     0 & 0 & 1
\end{pmatrix} = \begin{pmatrix}
    \cos \theta \cos \phi_B & \cos \theta \sin \phi_B & -\sin \theta \\
     -\sin \phi_B & \cos \phi_B & 0 \\
     \sin \theta \cos \phi_B & \sin \theta \sin \phi_B & \cos \theta
\end{pmatrix}. \hfill (E1)$$

APPENDIX F
MORE RESULTS OF C I, SI I, AND S III FOR CIRCUMSTELLAR CASE

For a $T = 15,000$ K blackbody pumping source, we have obtained the following results.

For C I

$$\rho_0^0(J_l = 1) = \begin{pmatrix}
    3.1 \cos^{10}\theta_r - 50.8 \cos^8\theta_r + 327.1 \cos^6\theta_r - 1043.7 \cos^4\theta_r - 1839.1 \cos^2\theta_r + 7806.8 \\
    0.01 \cos^{12}\theta_r - 0.44 \cos^{10}\theta_r + 2.39 \cos^8\theta_r - 13.83 \cos^6\theta_r + 316.72 \cos^4\theta_r + 459.59 \\
    \cos^{10}\theta_r - 14 \cos^8\theta_r + 65 \cos^6\theta_r - 30 \cos^4\theta_r - 3140 \cos^2\theta_r + 10,195 \\
    -0.4 \cos^{10}\theta_r + 15.7 \cos^8\theta_r + 201.4 \cos^6\theta_r - 1430.7 \cos^4\theta_r + 4352.5 \cos^2\theta_r - 1299.1
\end{pmatrix} \phi_0. \hfill (F1)$$

where $\phi_0 = \phi_0(J_l = 0)/(-0.7 \cos^{10}\theta_r + 11.5 \cos^8\theta_r - 86.0 \cos^6\theta_r + 548.9 \cos^4\theta_r - 1746.6 \cos^2\theta_r + 4625.5)$.

For Si I

$$\rho_0^0(J_l = 1) = \begin{pmatrix}
    -0.4 \cos^{10}\theta_r + 7.9 \cos^8\theta_r - 62.9 \cos^6\theta_r + 211.3 \cos^4\theta_r + 363.1 \cos^2\theta_r - 1516.6 \\
    -0.013 \cos^{12}\theta_r + 0.451 \cos^{10}\theta_r - 2.671 \cos^8\theta_r + 18.587 \cos^6\theta_r + 8.482 \cos^4\theta_r + 150.334 \cos^2\theta_r - 51.711 \\
    -0.1 \cos^{10}\theta_r + 1.9 \cos^8\theta_r - 4.9 \cos^6\theta_r - 18.1 \cos^4\theta_r + 635.4 \cos^2\theta_r - 1975.1 \\
    -0.02 \cos^{12}\theta_r + 0.36 \cos^{10}\theta_r + 0.38 \cos^8\theta_r - 23.62 \cos^6\theta_r + 281.19 \cos^4\theta_r - 948.70 \cos^2\theta_r + 285.86
\end{pmatrix} \phi_0. \hfill (F2)$$
where \( \rho_0^0(J_l = 0) = 0 \) \((-0.05 \cos^{12} \theta_r + 0.38 \cos^{10} \theta_r - 3.23 \cos^8 \theta_r + 10.97 \cos^6 \theta_r - 79.56 \cos^4 \theta_r + 89.78 \cos^2 \theta_r - 895.68 \).

For \( S^3 \)

\[
\begin{align*}
\rho_0^0(J_l = 1) &= 1 \\
\rho_0^2(J_l = 1) &= \begin{pmatrix} 52 \cos^{10} \theta_r + 30 \cos^8 \theta_r - 5710 \cos^6 \theta_r + 23,040 \cos^4 \theta_r + 29.549 \cos^2 \theta_r - 130,181 \\ 20 \cos^{10} \theta_r - 544 \cos^8 \theta_r + 3562 \cos^6 \theta_r + 1663 \cos^4 \theta_r - 393 \cos^2 \theta_r - 179 \\ -25 \cos^{10} \theta_r + 187 \cos^8 \theta_r - 590 \cos^6 \theta_r - 1259 \cos^4 \theta_r + 54,726 \cos^2 \theta_r - 165,422 \\ 19 \cos^{10} \theta_r - 416 \cos^8 \theta_r + 1220 \cos^6 \theta_r + 21,954 \cos^4 \theta_r - 91,329 \cos^2 \theta_r + 27,964 \end{pmatrix}^0_0, \\
\rho_0^2(J_l = 2) &= (25 \cos^{10} \theta_r - 416 \cos^8 \theta_r + 1220 \cos^6 \theta_r + 21,954 \cos^4 \theta_r - 91,329 \cos^2 \theta_r + 27,964)^2_0.
\end{align*}
\]

where \( \rho_0^0(J_l = 0) = 0 \) \((-3 \cos^{12} \theta_r + 93 \cos^{10} \theta_r - 625 \cos^8 \theta_r - 425 \cos^6 \theta_r - 155 \cos^4 \theta_r + 25,678 \cos^2 \theta_r - 78,043 \).

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