Renormalization and asymptotic freedom in quantum gravity through the equivalence theorem

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Abstract

A perturbative method of renormalization is developed for the Einstein-Hilbert Lagrangian with a cosmological constant. In the method described, the renormalizable Lagrangian of higher-derivative gravity is rewritten such that the higher-derivative terms are redundant (i.e., vanish on the Einstein shell), allowing them to be removed after renormalization by performing a local field transformation and redefining the parameters. At each order in perturbation theory, this allows the renormalization of the couplings in the Einstein-Hilbert Lagrangian to be calculated from the renormalized couplings of the higher-derivative theory. It is shown that the Lagrangian obtained by this method is asymptotically free at one-loop order in the essential dimensionless coupling \( \Lambda G \), where \( G \) is Newton’s constant and \( \Lambda \) is the cosmological constant.

Keywords: quantum gravity; renormalization; asymptotic safety; equivalence theorem.

2010 MSC: 83C45; 37F25; 81T15; 81V17.

1. Introduction

The development of a quantum field theory of gravity based on the Einstein-Hilbert (Einstein) Lagrangian has been problematic because the traditional methods of renormalization cannot be used to eliminate the ultraviolet divergences that appear in perturbation theory \([1,2]\). On the other hand, generalizations of the Einstein Lagrangian that include higher-derivative terms proportional to \( R_{\mu\nu}R^{\mu\nu} \) and \( R^2 \) are renormalizable to all orders in perturbation theory \([3,4,5,6]\). Moreover, the dimensionless couplings of the higher-derivative terms are asymptotically free \([7,8,9]\), i.e., they approach a Gaussian fixed point in the ultraviolet. As for the dimensional couplings—the cosmological constant \( \Lambda \) and Newton’s constant \( G \)—only their product \( \Lambda G \) is gauge invariant \([6,10]\), and this parameter is also claimed to be asymptotically free \([7,10,11,12]\).

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\(^1\)Without higher derivative terms, the Einstein Lagrangian with a cosmological constant is also asymptotically free at one-loop order in the coupling \( \Lambda G \) \([7,12]\).
Despite these desirable properties, higher-derivative gravity has several inconsistencies. Perhaps the most well known is the unitary problem \[3, 5, 6\]; in flat-space perturbation theory the higher-derivative terms give rise to massive spin-2 ghosts. It has been suggested \[11, 13\] that higher-order loop effects may drive the ghost mass to infinity, thus rendering the theory unitary, but a rigorous proof of this is still lacking. Another issue is related to the asymptotic freedom of \(\Lambda_G\), which requires the use of an unstable fixed point for one of the higher-derivative parameters \[5, 10\]. And if the unstable fixed point is replaced by the ultraviolet-attractive parameter, \(\Lambda_G\) actually diverges in the ultraviolet \[5, 10\]. These inconsistencies have precluded the acceptance of the higher-derivative theory as the correct description of quantum gravity.

It is now understood that the Einstein Lagrangian and its higher-derivative extensions can be regarded as the lowest-order terms in the effective field theory of general relativity \[14\], i.e., the theory containing all generally-covariant functions of the metric and its derivatives \[15\]. One approach for studying the asymptotic behavior of an effective field theory, generally referred to as asymptotic safety, is to show that all of the infinite couplings are attracted to a fixed point in the ultraviolet, and that only a finite number them are essential \[16, 17, 18\]. Non-Gaussian (i.e., non-zero) fixed points have been found using dimensional continuation \[16, 20, 21\], the \(1/N\) approximation \[22, 23\], the lattice approach \[24, 25\], and various truncations of the Functional Renormalization Group Equation (FRGE) \[26, 27, 28, 29, 30\].

Of particular relevance to this work is the apparent insensitivity of the asymptotic behavior of \(\Lambda_G\) to the higher-derivative couplings. For instance, in solutions to the FRGE, the renormalization group flow and fixed-point value of \(\Lambda_G\) are remarkably similar in both the Einstein-Hilbert \[28\] and higher-derivative \[10, 27, 29, 30\] truncations. Moreover, in perturbation theory at one-loop order, \(\Lambda_G\) is asymptotically free whether or not the higher-derivative terms are included \[7, 11\]. These similarities suggest that the higher-derivative terms are inessential or redundant.

The fact that the higher-derivative terms render the theory renormalizable, combined with the assumption that they are redundant, leads to the possibility of obtaining a theory that is both unitary and perturbatively renormalizable. To further explore this possibility, the Lagrangian of higher-derivative gravity is rewritten such that the higher-derivative terms are redundant (or inessential), allowing them to be removed after renormalization by performing a local field redefinition and redefining the couplings. The latter redefinition establishes a relationship between the coupling \(\Lambda_G\) in the reduced Einstein Lagrangian and the couplings in the renormalizable theory of higher-derivative gravity. Thus, given the renormalized couplings of the higher-derivative theory, the renormalization of \(\Lambda_G\) in the reduced Einstein Lagrangian may be calculated to all orders in perturbation theory. By this method, it is shown that \(\Lambda_G\) is asymptotically free at one-loop order, consistent with previous calculations performed without

\[\text{Essential couplings are those that cannot be removed by a local field redefinition} \ [19]\]
the redundant terms [7, 11], as required by the S-matrix equivalence theorem.

2. Approach

It is well known from the equivalence theorem [31, 32] that aside from changing the values of the coupling constants [33], the $S$-matrix is unaffected by field redefinitions [19]. This means that the property of renormalizability of the $S$-matrix cannot be undone by a field redefinition [3]. Therefore, the $S$-matrix of the Einstein Lagrangian obtained by a field redefinition from the renormalized Lagrangian of higher-derivative gravity must also be renormalized.

A related aspect of the equivalence theorem is that the transformation from one Lagrangian to another (e.g., higher-derivative gravity to Einstein gravity) may require the couplings in the reduced Lagrangian—the Lagrangian without redundant terms—to be redefined as linear combinations of the couplings of the redundant terms [33, 34]. At tree level, this redefinition has no effect because the coupling constants are arbitrary [33]. However, when loop corrections are considered, the running couplings of the redundant terms may be reinterpreted as running couplings in the reduced Lagrangian to derive a set of reduced effective beta functions [34].

In this article, we seek to derive the effective beta functions for the reduced Einstein Lagrangian obtained by a field redefinition from the renormalizable Lagrangian of higher-derivative gravity. To do so, it is convenient to begin with the Einstein Lagrangian,

$$\mathcal{L} = \sqrt{-g} \left( \lambda - \frac{1}{\kappa'^2} R \right),$$

where $\kappa'^2 = 16\pi G$ ($c = 1$), $\lambda = \Lambda/\kappa'^2$, and the prime denotes a potential redefinition of the coupling. Consider a nonlinear, infinitesimal local field transformation of the form

$$g_{\mu\nu} \rightarrow g_{\mu\nu}' = g_{\mu\nu} - \kappa'^2 f_{\mu\nu},$$

where $f_{\mu\nu} = aR_{\mu\nu} + bRg_{\mu\nu}$. The Lagrangian transforms as

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} - \kappa'^2 \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} f_{\mu\nu} + \mathcal{O}(\kappa'^4),$$

where

$$\frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = -\frac{1}{2} \lambda g_{\mu\nu} - \frac{1}{2} \frac{\kappa'^2}{R} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right),$$

and $\mathcal{O}(\kappa'^4)$ (omitted hereafter) represents terms cubic and higher in the curvature tensor. The final result is

$$\mathcal{L}' = \sqrt{-g} \left[ \lambda - \frac{1}{\kappa'^2} R + \frac{1}{2} \lambda \kappa'^2 (a + 4b) R + a R_{\mu\nu}^2 - \frac{1}{2} (a + 2b) R^2 \right].$$

This argument assumes that the theory under consideration is renormalizable, otherwise the $S$-matrix is divergent or undefined [32].
It is clear that the higher-derivative terms in $\mathcal{L}'$ are redundant as they vanish on the Einstein shell (i.e., $\mathcal{L}' = \mathcal{L}$ when $R_{\mu\nu} = \frac{1}{2} \lambda \kappa'^2 g_{\mu\nu}$). Their only purpose here is to renormalize the off-shell Green’s functions. The cosmological constant has also generated a term in the Lagrangian proportional to $R$, which may be absorbed into a redefinition of Newton’s constant as

$$\frac{1}{\kappa'^2} + \frac{1}{2} \lambda \kappa'^2 (a + 4b) = \frac{1}{\kappa^2}. \tag{3}$$

The solution to Eq. (3),

$$\kappa'^2 = \kappa^2 \left[ 1 + 2 \lambda \kappa^4 (a + 4b) \right]^{1/2} - 1 \frac{\lambda \kappa^4 (a + 4b)}{\kappa^2} \tag{4}$$

is a principal result of this article. It relates the coupling constant in the reduced Einstein Lagrangian to the couplings in the renormalizable theory of higher-derivative gravity.

In addition to the transformation of the Lagrangian, the measure in the functional integral changes by a factor equal to the Jacobian of the field redefinition. Because the transformation is local, the additional terms from the Jacobian $\delta f_{\mu\nu}(x)/\delta g_{\alpha\beta}(y)$ will be proportional to $\delta^{(4)}(x - y)$ and its derivatives, and for $x = y$ these terms are either zero in dimensional regularization or can be absorbed without consequence into the couplings of the Lagrangian. Thus, as it is commonly known from the equivalence theorem, the functional measure is invariant under infinitesimal local field redefinitions of the type in Eq. (1).

The transformation $g_{\mu\nu} \rightarrow g'_{\mu\nu}$ thus represents a local symmetry of the functional measure in quantum gravity. It implies that at small distances, where quantum fluctuations of $R_{\mu\nu}$ are large, the definition of $g_{\mu\nu}$ is ambiguous, and it becomes impossible to distinguish between the original $g_{\mu\nu}$ and the admixture $g_{\mu\nu} + a R_{\mu\nu} + b R g_{\mu\nu}$. This symmetry is reflected in the Lagrangian by the presence of the higher-derivative terms. A similar argument was made by ’t Hooft in the context of one-loop quantum gravity, where the author argues that because the field redefinition in Eq. (1) can be used to remove the one-loop divergence in pure quantum gravity, then it must represent a new feature of quantum gravity. However, the author did not follow this speculation further.

3. Renormalization Group

The most common method for obtaining the one-loop divergences in higher-derivative gravity is the Schwinger-DeWitt technique, generalized by Barvinsky and Vilkovisky. The method is based on the background-field approach and

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4 Field redefinitions of the type in Eq. (1) are different from the transformations used to explore anomalies, where the contribution from the Jacobian is not trivial. Anomalous transformations involve external spacetime-dependent parameters.
requires a specific background-field gauge \([5, 4]\). The one-loop renormalization constants in higher-derivative gravity were first calculated by Julve and Tonin \([11]\) and later modified by Fradkin and Tseylin \([7]\). The final correct result was obtained by Avramidi and Barvinsky \([9, 10, 41]\).

Following these results, the one-loop divergence in \(L'\) (taking \(\bar{\hbar} = 1\)) is

\[
\Delta L' = \frac{1}{16\pi^2(4 - d)} \sqrt{-g} \left[ \beta_2 \left( R^2_{\mu\nu} - \frac{1}{3} R^2 \right) + \frac{1}{3} \beta_3 R^2 + \beta_4 \frac{1}{\kappa^4} + \gamma \frac{1}{\kappa^2} (R - 2\lambda\kappa^2) \right],
\]

where \(d\) is the dimensional regularization parameter and

\[
\begin{align*}
\beta_2 &= \frac{133}{10}, \\
\beta_3 &= \frac{10}{3} \omega^2 + 5\omega + \frac{5}{12}, \\
\beta_4 &= \frac{2}{\alpha^2} \left( 5 + \frac{1}{4\omega^2} \right) + \frac{2}{3\alpha} \kappa^4 \lambda \left( 20\omega + 15 - \frac{1}{2\omega} \right), \\
\gamma &= \frac{2}{\alpha} \left( \frac{10}{3} \omega - \frac{13}{6} - \frac{1}{4\omega} \right),
\end{align*}
\]

where \(\omega \equiv 1 + 6b/a\). It is important to note that since \(\beta_4 \neq 0\), the cosmological constant requires renormalization at one-loop order.

Taking the renormalization scale as \(t \equiv [16\pi^2(4 - d)]^{-1}\), the renormalization group equations are

\[
\begin{align*}
\frac{da}{dt} &= \beta_2 = \frac{133}{10}, \\
\frac{d\omega}{dt} &= -\frac{2}{a} (\omega\beta_2 + \beta_3) = -\frac{2}{a} \left( \frac{10}{3} \omega^2 + \frac{183}{10} \omega + \frac{5}{12} \right), \\
\frac{d\kappa^2}{dt} &= \kappa^2 \gamma = \kappa^2 \frac{2}{a} \left( \frac{20}{3} \omega - \frac{13}{3} - \frac{1}{2\omega} \right), \\
\frac{d\lambda}{dt} &= \frac{1}{\kappa^4} \beta_4 - 2\lambda = \frac{1}{\kappa^4 a^2} \left( \frac{5}{2} + \frac{1}{8\omega^2} \right) + \frac{2}{a} \left( \frac{28}{3} + \frac{1}{3\omega} \right).
\end{align*}
\]

While the dimensionless constants \(a\) and \(b\) (and \(\omega\)) are essential coupling constants, the dimensional constants \(\kappa\) and \(\lambda\) are redundant because their variations can be compensated by the field transformation \(g_{\mu\nu} \rightarrow \Omega g_{\mu\nu}\), where \(\Omega\) is a constant \([41, 42]\). On the contrary, the dimensionless combination of Newton’s constant and the cosmological constant \(\lambda\kappa^4 \equiv \tilde{\lambda}\) is not redundant \([7, 9, 41]\).

We now proceed to analyze the renormalization group equations for the essential coupling constants \(a, \omega\) and \(\tilde{\lambda}\), one by one. The solution to Eq. \((5)\),

\[
a(t) = a_0 + \beta_2 t,
\]

where \(a_0 = a(0)\), indicates that the inverse coupling \(a^{-1}(t)\) is asymptotically free in the ultraviolet limit when \(t \to +\infty\). Equation \((6)\) has two fixed points,

\[
\omega_1 \approx -0.02 \quad \text{and} \quad \omega_2 \approx -5.47.
\]
Analysis of the general solution shows that \( \omega(t) \to \omega_1 \) when \( \omega(0) > \omega_2 \) and \( \omega(t) \to -\infty \) when \( \omega(0) < \omega_2 \) in the ultraviolet limit when \( t \to +\infty \).

The renormalization group equation for the dimensionless coupling \( \tilde{\lambda} \) follows from Eqs. (7) and (8),

\[
\frac{d\tilde{\lambda}}{dt} = \frac{1}{2} \beta_4 = \frac{2}{a} \tilde{\lambda} \left( \frac{20}{3} \omega + 5 - \frac{1}{6\omega} \right) + \frac{2}{a^2} \left( \frac{5}{2} + \frac{1}{8\omega^2} \right), \tag{10}
\]

For a fixed value of \( \omega(t) \), the solution to Eq. (10) is

\[
\tilde{\lambda}(t) = C \left( 1 + \frac{2}{a_0} \beta_2 t \right)^{W_1/\beta_2} - \frac{2}{a_0} \frac{W_2}{W_1 + \beta_2} \left( 1 + \frac{2}{a_0} \beta_2 t \right)^{-1}, \tag{11}
\]

where \( W_1 = \frac{20}{3} \omega + 5 - \frac{1}{6\omega} \), \( W_2 = \frac{5}{4} + \frac{1}{16\omega^2} \), and \( C \) is a constant. If we take \( \omega(0) > \omega_2 \), in the ultraviolet limit when \( t \to \infty \), \( \omega(t) \to \omega_1 \) and

\[
\lim_{t \to +\infty} \tilde{\lambda}(t) \approx C \left( 1 + \frac{2}{a_0} \beta_2 t \right)^q, \quad q \approx 0.91.
\]

Similarly, the variation of the dimensionless coupling \( \tilde{\lambda}' \equiv \lambda \kappa'^4 \) in the reduced Einstein Lagrangian, for a fixed value of \( \omega(t) \) follows from Eq. (4),

\[
\tilde{\lambda}'(t) = \tilde{\lambda} \left( \left[ 1 + \frac{2}{a} a \tilde{\lambda}(1 + 4\omega) \right]^{1/2} - 1 \right)^2, \tag{12}
\]

where \( \tilde{\lambda} \) is given in Eq. (11) and \( a \) in Eq. (9). Taking \( \omega(0) > \omega_2 \), it follows from Eq. (12) that when \( t \to \infty \), \( \omega(t) \to \omega_1 \) and

\[
\lim_{t \to +\infty} \tilde{\lambda}'(t) \approx \frac{6}{(1 + 4\omega_1)} \left[ a_0 + \beta_2 t \right]^{-1}. \tag{13}
\]

Thus, for \( a_0 = a(0) > 0 \) [and thus \( \tilde{\lambda}'(0) > 0 \)] the reduced Einstein Lagrangian is asymptotically free in the essential dimensionless coupling \( \lambda \kappa'^4 = 16\pi G \). The fact that \( AG \) approaches a Gaussian fixed point, whereas solutions to the FRGE predict a non-Gaussian fixed point, implies that asymptotic freedom may be an artifact of the one-loop approximation, as suggested previously [30].

It also is important to note that Eq. (13) has been obtained previously [7, 12] in one-loop approximation for pure Einstein gravity with a cosmological constant, without the redundant higher-derivative terms. This correspondence is consistent with the equivalence theorem [34], which requires that all components of the S-matrix, including the beta functions, are equivalent for theories related by field redefinitions. The advantage of including the redundant higher-derivative terms is that the theory becomes renormalizable to all orders in perturbation theory.
4. Concluding Remarks

Generalizations of the Einstein Lagrangian that include higher-derivative terms are renormalizable, but are not unitary in flat-space perturbation theory. Alternatively, if the higher-derivative terms are rewritten such that they are redundant (i.e., vanish on the Einstein shell), they may be removed after renormalization by performing a local field transformation and redefining the parameters. At each order in perturbation theory, this allows the renormalization of the parameters in the reduced Einstein Lagrangian to be calculated from the renormalized parameters of the higher-derivative theory. It is shown that the reduced Einstein Lagrangian obtained by this method is asymptotically free at one-loop order in the essential dimensionless coupling constant $\Lambda G (\bar{\hbar} = c = 1)$. Moreover, this result is consistent with previous calculations performed without the redundant higher-derivative terms, as required by the $S$-matrix equivalence theorem. Work is currently in progress to extend this result to the case of gravity coupled to the massive scalar field.

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