Complete Electroweak Corrections to Higgs production in a Standard Model with four generations at the LHC.

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Complete electroweak two-loop corrections to the process $gg \to H$ are presented and discussed in a Standard Model with a fourth generation of heavy fermions. The latter is studied at the LHC to put exclusion limits on a fourth generation of heavy fermions. Therefore also a precise knowledge of the electroweak (EW) next-to-leading-order (NLO) corrections is important. The corrections due to the fourth generation are positive and large for a light Higgs boson, positive but relatively small around the $\tau-\tau$ threshold and start to become negative for a Higgs boson mass around $M_H = 450$ GeV. Increasing further the value of the Higgs boson mass, the EW NLO effects tend to become huge and negative, $\mathcal{O}(-100\%)$, around the heavy-fermion threshold, assumed at $1.2$ TeV, so that $gg$-fusion becomes non-perturbative. Above that threshold they start to grow again and become positive around $M_H = 1.75$ TeV. The behaviour at even larger values of $M_H$ shows a positive enhancement, $\mathcal{O}(+100\%)$ at $M_H = 3$ TeV.

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1 Introduction

In the last year there have been intensive studies at the LHC aimed to put exclusion limits on a fourth generation of heavy fermions in the Standard Model [12,13]. For similar searches at Tevatron we refer to Ref. [4]. Recently, the overall combination of six Standard Model (SM) Higgs boson searches has been presented by the CMS Collaboration [5] using the following Higgs boson decay signatures: $H \rightarrow \gamma\gamma$, $\tau\tau$, $WW \rightarrow 2\ell2\nu$, $ZZ \rightarrow 2\ell2\nu$, $ZZ \rightarrow 2\ell2q$. The same experimental search results, reinterpreted in the context of the Standard Model with four fermion generations (SM4), allow them to exclude the SM4 Higgs boson with a mass in the range 120–600 GeV at 95% C.L.

From direct searches the situation is as follows: for the decay of a fourth generation quark $b' \rightarrow tW$, the LHC can put a limit close to 490 GeV for 1 fb$^{-1}$ data. For $b' \rightarrow bW$ and for $\approx 200$ pb$^{-1}$ they can already obtain a limit around 420 GeV without a full optimization. The relevant production channel for these searches is gluon-gluon fusion (gg-fusion) that is sensitive to new coloured, heavy particles. There is little doubt that a Standard Model with a fourth generation of heavy fermions cannot stimulate great interest (for an additional vector-like generation see Ref. [6]), however, the spectacular modification in the Higgs boson cross-section at hadron colliders can be tested easily with LHC data.

So far, the experimental analysis has concentrated on models with ultra-heavy fourth generation fermions, excluding the possibility that the Higgs boson decays to heavy neutrinos. Furthermore, the full two-loop electroweak corrections have been included under the assumption that they are dominated by light fermions. At the moment the experimental strategy consists in computing the cross-section ratio $R = \sigma(\text{SM4})/\sigma(\text{SM3})$ with HIGLU [7], while NLO electroweak radiative corrections are switched off, where SM3 stands for the Standard Model with three generations.

In this work we concentrate on the computation of the complete two-loop electroweak corrections and refer to the work of Ref. [6] for the inclusion of the QCD corrections. In the next Section 2 we discuss the limit of heavy fermions in the gluon fusion channel. In Section 3 we present our results for the electroweak corrections in the SM4 and discuss in Section 4 the impact of the fourth generations on the Higgs boson decay. Finally we close with our conclusions in Section 5.

2 Heavy fermion mass limit in gluon-gluon fusion

Recently, there have been some confusing and inaccurate statements on the impact of two-loop electroweak (EW) corrections to Higgs boson production at the LHC (through gg-fusion) in a Standard Model with a fourth generation of fermions. The naive expectation is that light fermions dominate the low Higgs boson mass regime and, therefore, electroweak corrections can be well approximated by using the available ones [10,11] in the Standard Model with three generations (SM3). It is worth noting that the leading behaviour of the EW corrections for high values of masses in the fourth generation has been known for a long time [12,13] (see also Ref. [14]) showing an enhancement of the radiative corrections.

To avoid misunderstandings we define the following terminology: for a given amplitude $A$, in the limit of the fermion mass $m_f \rightarrow \infty$ we will distinguish decoupling for $A \sim 1/m_f^2$ (or more), screening for $A \rightarrow$ constant (or $\ln m_f^2$) and enhancement for $A \sim m_f^2$ (or more). To discuss decoupling we need few definitions: SM3 is the usual SM with one $t$--$b$ doublet; SM4 is the extension of SM3 with a new family of heavy fermions with $t'$--$b'$ quarks and $l'$--$\nu'_l$ leptons. All relevant formulae for the asymptotic limit can be found in Ref. [12,13]. The amplitude for gg-fusion is (only EW corrections are considered here)

$$A(\text{SM3}) = A_t^{1\text{-loop}} + A_3^{NLO}, \quad A_3^{NLO} = A_t^{2\text{-loop}} + \delta_{t}^{FR} A_t^{1\text{-loop}},$$

$$A(\text{SM4}) = A_t^{1\text{-loop}} + A_4^{NLO}, \quad A_4^{NLO} = A_t^{2\text{-loop}} + \delta_{t+b'}^{FR} A_t^{1\text{-loop}},$$

where

$$A_Q = A_{t+t'+b'}, \quad \delta_{t+b'+l'}^{FR} = \delta_{t+t'+b'+l'}^{FR}.$$

In Eq. (1), $\delta^{FR}$ gives the contribution from finite renormalization, including Higgs boson wave-function renormalization (see Sect. 3.4 of Ref. [10] for technical details).
First we recall the standard argument for asymptotic behaviour in the lowest-order (LO) gg-fusion process, extendible to next-to-leading-order (NLO) and next-to-next-to-leading-order (NNLO) QCD corrections \cite{15} and give a simple argument to prove enhancement at the NLO EW level.

Any Feynman diagram contributing to the Higgs-gluon-gluon vertex has dimension one; however, the total Higgs-gluon-gluon amplitude must be proportional to \( T_{\mu\nu} = p^2 \delta_{\mu\nu} - p_\mu p_\nu \) (where \( p \) is the Higgs momentum) because of gauge invariance. For any fermion \( f \) the Yukawa coupling is proportional to \( m_f/M_W \) and \( T \) has dimension two; therefore, the asymptotic behaviour of any diagram must be proportional to \( T/m_f \) when \( m_f \to \infty \). The part of the diagram which is not proportional to \( T \) will cancel in the total because of gauge invariance (all higher powers of \( m_f \) will go away and this explains the presence of huge cancellations in the total amplitude). At LO there is only one Yukawa coupling as in NLO(NNLO) QCD where one adds only gluon lines, so there is screening. At the EW NLO there are diagrams with three Yukawa couplings, therefore giving the net \( m_f^2 \) behaviour predicted in \cite{12}, so there is enhancement and at two-loop level it goes at most with \( m_f^2 \). To be more precise, let us define

\[
\sigma_{\text{SM3}}(gg \to H) = \sigma^{\text{LO}}_{\text{SM3}}(gg \to H) \left( 1 + \delta_{\text{EW}}^3 \right) \quad \text{and} \quad \sigma_{\text{SM4}}(gg \to H) = \sigma^{\text{LO}}_{\text{SM4}}(gg \to H) \left( 1 + \delta_{\text{EW}}^4 \right).
\]

Analysing the results of Ref. \cite{12,13}, which is valid for a light Higgs boson, one can see that in SM3 there is enhancement in the quark sector for \( m_t \gg m_b \) (\( \delta_{\text{EW}}^3 \sim G_F m_t^2 \) where \( G_F \) is the Fermi coupling constant). The full calculation of Ref. \cite{11} shows that the physical value for the top-quark mass is not large enough to make this quadratic behaviour relevant with respect to the contribution from light fermions in a wide range of the Higgs boson mass. From Ref. \cite{12} one can also understand that an hypothetical SM3 with mass-degenerate t−b quarks (\( m_t = m_b = m_q \)) would generate an enhancement in the small mass region with the opposite sign (\( \delta_{\text{EW}}^3 \sim -G_F m_q^2 \)). Moving to SM4, Eq.(62) of Ref. \cite{13} shows that the enhanced terms coming from finite renormalization exactly cancel the similar contribution from two-loop diagrams for \( m_{t'} = m_{b'} \), so that for mass-degenerate quarks \( t'-b' \) we observe screening in the fourth generation quark sector. This accidental cancellation follows from the 3 of colour \( SU(3) \) and from the fact that we have 3 heavy quarks contributing to LO almost with the same rate (no enhancement at LO). However, the same is not true for the leptons \( l'-\nu' \). There are no two-loop diagrams with leptons in gg-fusion; they enter through the renormalization procedure. We observe enhancement in the leptonic sector of SM4, which actually dominates the behaviour at small values of \( M_H \). To summarize:

- SM3 with a heavy-light quark doublet: (positive) enhancement;
- SM3 with a heavy-heavy (mass-degenerate) quark doublet: (negative) enhancement;
- SM4 with a heavy-light and heavy-heavy (mass-degenerate) quark doublets: enhancement in heavy-light, screening in heavy-heavy;
- SM4 with a heavy-heavy \( l'-\nu' \) doublet: enhancement.

We have verified that our (complete) results confirm the asymptotic estimates of Ref. \cite{12,13}.

Exclusion of SM4 at LHC requires the most conservative setup and, sometimes, it has been suggested to set limits in a scenario where all fermions in the fourth generation are ultra-heavy. This is not reasonable for at least three reasons: there is the usual unitarity requirement which puts a LO bound of approximately 500 GeV (although some recent literature \cite{15} puts the current interesting region in the interval 400−600 GeV); implications of triviality for the Standard Model \cite{17}, where one deduces that in the framework of a two-loop renormalization group analysis the heaviest quark mass has to be smaller than 400 GeV. This bound emerges from vacuum stability constraints and turns out to be stronger than the 500 GeV from unitarity. Finally, EW NLO corrections to gg-fusion show that already at the \( t'-b' \) threshold, assumed to be at around 1.2 TeV, the contributions of the NLO corrections to the cross-section are as big as the LO one.

Following the recommendation of the Higgs XS Working Group we have adopted the following scenario (see Refs. \cite{18,19}):

\[
m_{t'} = m_{b'} = m_{\nu_{t'}} = 600 \text{ GeV}, \quad m_{t'} - m_{b'} = 50 \left( 1 + \frac{1}{5} \ln \frac{M_H}{M_{\text{ref}}} \right) \text{ GeV},
\]

(4)
where $M_{\text{ref}} = 115$ GeV. The reason for this scenario is that while one should pay attention to the theoretical upper bounds also lower bounds from direct search at LHC matter, so that our assumption will not become obsolete in the near future. Monte Carlo studies show that the sensitivity at LHC for heavy quark search is about 600 GeV with 1 fb$^{-1}$ data. Note that the constraint of Eq. (4) is a severe one and the quarks $t' - b'$ are almost mass-degenerate meaning that, at low values of $M_H$, the quarks of the fourth generation contribute very little to the leading behaviour of the EW NLO corrections.

3 Results

In this Section we present numerical results for complete NLO EW corrections to $gg$-fusion in SM4, obtained using the techniques developed in [10].

In order to prove that light-fermion dominance in SM3 below 300 GeV is a numerical accident due to the fact that the top-quark is not heavy enough, we have computed $\delta_{\text{EW}}^3$ for a top-quark of 800 GeV at $M_H = 100$ GeV and found top-quark dominance ($\delta_{\text{EW}}^3 = 4.2\%$ (11\%) at $m_t = 172.5$ GeV (800 GeV)). A similar effect in the top-quark sector is also present in SM4; if, for instance, we fix all heavy masses to 600 GeV, we find $\delta_{\text{EW}}^4 = 12.1\%$ (29.1\%) at $m_t = 172.5$ GeV (600 GeV).

Moving to SM4, the LO $gg$-fusion cross-section is for a light Higgs boson about nine times the one in SM3, e.g. see Ref. [20]; the contribution from pure two-loop EW diagrams should have an impact three times smaller than the equivalent one in SM3 in the small $M_H$ region. Therefore, if one assumes that also NLO SM4 is dominated by light-fermion corrections, i.e. that EW corrections are the same for SM3 and SM4, one expects $\delta_{\text{EW}}^4 \approx \delta_{\text{EW}}^3 / 3$ for very heavy fermions. According to Ref. [13] this would be true provided that no heavy leptons are included. In Fig. 1 we show our findings for SM4 in the case where only the quark contribution of the fourth generation is included; in this case $\delta_{\text{EW}}^4$ turns to be effectively small for light $M_H$ (compared to $\delta_{\text{EW}}^3$ from Ref. [11], dashed(blue) curve), but the ratio $\delta_{\text{EW}}^4 / \delta_{\text{EW}}^3$ is slightly different from the expected 1/3. This reduction factor applies in fact just to the pure two-loop diagrams, while finite renormalization in the top sector remains unchanged moving from $\delta_{\text{EW}}^3$ to $\delta_{\text{EW}}^4$, giving for the corrections proportional to $m_t^2$ an overall enhancement of a factor 5 in $\delta_{\text{EW}}^4$ with respect to $\delta_{\text{EW}}^3$.

In Fig. 2 we have checked our result against the asymptotic limit predicted in Ref. [12], where the interference between finite renormalization effects due to the fourth generation of quarks and the one-loop top-quark amplitude was neglected, together with the contribution from heavy leptons. The solid(red) curve corresponds to our exact result for the EW NLO corrections to $\delta_{\text{EW}}^4$ for fixed $M_H = 100$ GeV as a function of $M_H$. 

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Figure 1: Percentage corrections in SM4 ($t' - b'$ quarks only) due to two-loop electroweak corrections to $gg \to H$ (solid(red) curve). Here we have chosen $m_{t'} = m_{b'} = 400$ GeV. For comparison we also show the percentage corrections in SM3 (dashed(blue) curve). The vertical dotted lines denote the location of the $W^+W^-$ and $Z^0Z^0$ thresholds.

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$m_t = m_{t'} = m_{b'}$, where the assumptions of Ref. [12] have been applied. To better appreciate the agreement, the result of Ref. [12] has been shifted (dashed blue curve) by a factor $-0.7$, which is our empirical estimate of the contributions of the non-enhanced terms, not included in the heavy quark expansion of Ref. [12].

Figure 2: Comparison of the component of $\delta^4_{EW}$ due to a mass-degenerate quark isodoublet $t' - b'$ (solid red curve) with $-0.7 + \frac{4}{3} \Delta_Q$, where $\Delta_Q$ is Eq.(12) of Ref. [12] (dashed blue curve). The interference of finite renormalization $t' - b'$ quark effects with the one-loop top-quark amplitude have been excluded.

In Fig. 3 we consider the dependence of $\delta^4_{EW}$ on the masses of the fourth generation, at small values of $M_H$. In the left part is plotted the heavy quark dependence of $\delta^4_{EW}$ for a mass-degenerate isodoublet $t' - b'$ at $M_H = 100$ GeV as a function of $m_q = m_{t'} = m_{b'}$ for $m_{t'} = m_{b'} = 600$ GeV, showing the expected screening for mass-degenerate heavy quarks in SM4. In the right part of Fig. 3 we plot $\delta^4_{EW}$ with a mass-degenerate isodoublet $l' - \nu_l'$ of leptons at $M_H = 100$ GeV as a function of $m_l = m_{l'} = m_{\nu_l'}$ for $m_{l'} = m_{\nu_l'} = 600$ GeV. The plot gives complete confirmation of the quadratic enhancement in the masses of leptons and neutrinos of SM4.

Figure 3: (Left) Complete $\delta^4_{EW}$ due to a degenerate isodoublet $t' - b'$ at $M_H = 100$ GeV as a function of $m_q = m_{t'} = m_{b'}$ for $m_{t'} = m_{b'} = 600$ GeV. (Right) Complete $\delta^4_{EW}$ due to a mass-degenerate isodoublet $l' - \nu_l'$ at $M_H = 100$ GeV as a function of $m_l = m_{l'} = m_{\nu_l'}$ for $m_{l'} = m_{\nu_l'} = 600$ GeV. The vertical line denotes the location of the values studied in the following Fig. 6.

Our complete result is shown in Fig. 4 where the $t' - b'$ and the $l' - \nu_l'$ doublets are included with the setup of Eq. (4). In Fig. 5 we consider the alternative scenario with complete mass-degenerate fourth generation fermions ($m_{t'} = m_{b'} = m_{l'} = 600$ GeV).
Electroweak NLO corrections due to the fourth generation are positive and large for a light Higgs boson, positive but relatively small around the $t^\prime - t$ threshold and start to become negative around 450 GeV. The asymptotic behaviour for $M_H$ well below any heavy $\bar{t} - q$ threshold makes the LO Higgs-gluon-gluon coupling local and allows for a partial (total) cancellation of the EW NLO leading corrections due to heavy quark ($m_t = m_b$); however, the top-quark triangle is the first to become non-local when $M_H$ is approaching the $\bar{t} - t$ threshold, spoiling the asymptotic behaviour. Increasing further the value of the Higgs boson mass, the NLO effects tend to become huge and negative ($\delta_{4}^{W} < -100\%$) around the heavy-quark threshold. Above that threshold $\delta_{4}^{W}$ starts to grow again and becomes positive around $M_H = 1750$ GeV. The behaviour at even larger values of $M_H$ shows the usual positive enhancement (at $M_H = 3000$ GeV we find $\delta_{4}^{W} > +100\%$), similar to SM3, entering the non-perturbative regime.

Our result proves that the naive expectation that EW NLO corrections are dominated by the light fermion contributions fails in SM4. This was expected for several reasons, including our previous results in SM3 where the partial corrections generated by the top-quark yields contributions of similar order of magnitude as the one induced by light fermion loops, smaller though but still in the per cent range as the whole EW corrections. Comparing Fig. 5 with Fig. 3(b), taking into account the $m_l$-dependence, one can see
that the fourth generation of leptons, which enter the renormalization procedure, have an important impact on the size of the percentage corrections.

4 SM4 exclusion

To put limits on SM4 one also needs to consider the final state; for the low Higgs mass region the important channels are $H \rightarrow \gamma\gamma$ and $H \rightarrow ZZ^*$. In both cases it will be difficult to reach exclusion with one channel and it is important to have also here the NLO EW corrections in SM4 under control. Complete NLO EW calculations are not yet available in SM4 for the decay processes and we can only argue in terms of the leading behaviour of the corrections as given in Ref. [13]. All channels show enhancement due to heavy fermions of the fourth generation, however this leading term is universal for $WW$ and $ZZ$ (at least for almost degenerate fermions) so that we expect less impact on the branching ratios although corrections on partial widths are expected to be huge ($\approx -60\% (-20\%)$) for mass-degenerate fermions $t'-b' (l'-\nu_l')$ of 600 GeV. For $H \rightarrow \gamma\gamma$ the SM4 branching ratio is suppressed by a factor 8 with respect to SM3 but one should be aware of the fact that in SM4 the NLO EW corrections can be as large as the LO partial width.

5 Conclusions

In this work we have provided the full two-loop electroweak correction for the gluon-gluon-fusion process in a Standard Model with a fourth generation of heavy fermions. Due to the expected enhancement of radiative corrections we have found a substantially different behaviour with respect to the same corrections in the Standard Model with three generations only, also in the region of low Higgs boson masses. The effect on exclusion limits at LHC are also briefly discussed. Finally, for values of the heavy-fermion masses given in Eq. (4) gluon-gluon-fusion becomes non-perturbative in SM4 for values of $M_H$ around 1 TeV, as signalled by NLO $\approx$ LO.

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