Two-component Bose gas trapped by harmonic and annular potentials:
Supercurrent, vortex flow and instability of superfluidity by Rabi coupling

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In this paper, we study a system of two-component Bose gas in an artificial magnetic field trapped by concentric harmonic and annular potentials, respectively. The system is realized by gases with two-internal states like the hyperfine states of $^{87}$Rb. We are interested in effects of a Rabi oscillation between them. Two-component Bose Hubbard model is introduced to describe the system, and Gross-Pitaevskii equations are used to study the system. We first study the Bose gas system in the annular trap by varying the width of the annulus and strength of the magnetic field, in particular, we focus on the phase slip and superflow. Then we consider the coupled Bose gas system in a magnetic field. In a strong magnetic field, vortices form a Abrikosov triangular lattice in both Bose-Einstein condensates (BECs), and locations of vortices in the BECs correlate with each other by the Rabi coupling. However, as the strength of the Rabi coupling is increased, vortices start to vibrate around their equilibrium locations. As the strength is increased further, vortices in the harmonic trap start to move along the boundaries of the annulus. Finally for a large Rabi coupling, the BECs are destroyed. Based on our findings about the BEC in the annular trap, we discuss the origin of above mentioned phenomena.

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I. INTRODUCTION

In the past decade, ultracold atomic systems are one of the most actively studied fields in physics. It is expected that these systems can be a quantum simulator for various models in quantum physics. Another aim to study ultracold atoms is to search for new quantum phenomena whose existence is theoretically predicted. One of these examples is the supersolid that has both a diagonal solid order and an off-diagonal superfluid order.

In this paper, we shall consider a system of two-component Bose gas in an artificial magnetic field. Each component of the boson corresponds to an internal state of single boson. We consider a two-dimensional (2D) system, and one component of the boson, which we call $A$-atom, is trapped by a harmonic potential, and the other boson, called $B$-atom, by an annular trap. The atoms are coherently transferred by a Rabi coupling between two internal states. It should be remarked that recently closely related atomic system was realized in experiments. As we show in the present study, interplay of the harmonic and annular traps generates intriguing phenomena. That is, the system exhibits interesting behavior depending on the strength of the magnetic field, inter and intra-repulsions, and the Rabi coupling. The present study was originally motivated by the findings of co-existing two kinds of superconductivity in the Sn/Si core-shell clusters, i.e., we expected that a phenomenon similar to the double superconductivity of the Sn/Si core-shell cluster can be realized in a ultracold atomic system. However in due course, we have realized that the ultracold atomic system has strong versatility and controllability. Much more interesting phenomena are to be observed in that system than we first expected.

This paper is organized as follows. In Sec.II, we explain the system of the two-component Bose gas trapped by harmonic and annular potentials, and introduce Gross-Pitaevskii equations (GPEs) to study the system. The bosons are coupled with the artificial magnetic field that is generated, e.g., by rotating the system. The Rabi coupling that induces coherent transference between the $A$ and $B$-atoms is also introduced. Rather detailed description of the numerical methods is also given. In Sec.III, we consider behavior of a single-component BEC in the harmonic and annular potentials separately. In particular, we study the ring-shaped BEC in the annular trap in detail. Behavior of the BEC is strongly depends on width of the annulus and strength of the artificial magnetic field. In the case of the quasi-1D case, a phenomenon similar to the Aharonov-Bohm (AB) effect takes place as recently observed by the experiments. On the other hand for a relatively wide ring-shaped BEC, superflows are generated in regions near the boundaries. The superflows are similar to the surface supercurrents in superconductors in a strong magnetic field. In Sec.IV, we study the two-component Bose gas in the harmonic and annular traps with the repulsions and the Rabi coupling. We fix the intra and inter-repulsions, and vary the strength of the Rabi coupling to observe how the BECs change. For a finite but small Rabi coupling, vortices start to vibrate around their equilibrium locations. For a moderate Rabi coupling, vortices of $A$-condensate flow along the boundaries of the $B$-atom ring-shaped condensate. As the Rabi coupling is increased furthermore, the BECs are destroyed. Section V is devoted for conclusion.
II. BECs IN HARMONIC AND ANNULAR TRAPS

A. Potential traps and GPEs

In this section, we shall derive GPEs describing two-component BECs trapped by concentric harmonic and annular potentials, respectively. The two-component bosons mean two-internal states of a single boson like the hyperfine states of $^{87}$Rb and they can be trapped by different potentials by means of the difference in magnetic/electric properties of them. We call these bosons A-atom and B-atom, respectively, and denote the state-selective potential by $V_{A(B)}(x)$, where $x = (x_1, x_2)$ is the two-dimensional (2D) coordinate. See Fig. 1.

GPEs of condensate-field $\psi_A(x)$ and $\psi_B(x)$ for the coupled A and B-atoms system are given by

\[
(i - \gamma)\hbar \frac{d\psi_A}{dt} = \left( -\frac{\hbar^2}{2m} \sum_{i=1,2} (\partial_i - iA_i)^2 - \mu + R \psi_A \psi_A + U \bar{\psi}_B \psi_B + V_A \right) \psi_A + g \bar{\psi}_B, \\
(i - \gamma)\hbar \frac{d\psi_B}{dt} = \left( -\frac{\hbar^2}{2m} \sum_{i=1,2} (\partial_i - iA_i)^2 - \mu + R \bar{\psi}_B \psi_B + U \bar{\psi}_A \psi_A + V_B \right) \psi_B + g \bar{\psi}_A, \tag{2.1}
\]

where $R(U)$ is the intra(inter)-species repulsion and $g$ is the Rabi coupling. $A_i$ is a vector potential for a uniform artificial magnetic field, which is created by rotating the system or using lasers. The parameter $\mu$ is the chemical potential, which controls the particle number, and $\gamma$ is a dissipative parameter. We put $\gamma = 0.01$ for most of the practical calculation. To solve the GPEs (2.1), we employ the Crank-Nicolson method, and use the symmetric gauge for the vector potential $A_i$, i.e., $(A_1(x), A_2(x)) = (\pi f x_2, -\pi f x_1)$. Therefore the magnetic flux per unit area is $2\pi f$. For a rotating system, $f \propto m \Omega$, where $\Omega$ is an angular velocity of the rotation.

When both the Rabi coupling $g$ and inter-species repulsion $U$ are vanishing, the GPEs (2.1) are two independent equations, and the dynamics of the solutions are governed simply by the potentials $V_A(x)$, $V_B(x)$ and the strength of the intra-repulsion $R$. As we show in the following sections, the inter-coupling $U$ and the Rabi coupling $g$ strongly influence the structure of the groundstate and low-energy excitations of the coupled system. In particular, interplay of the Rabi coupling and the potentials $V_{A(B)}$ plays a very important role.

We briefly explain the numerical methods to solve the GPEs (2.1). As the unit of the length, we used the spatial cutoff $a = 0.5 \mu m$. Every length in this paper is measured in this unit. For the energy unit, we used Na mass, $m_N = 4.0 \times 10^{-27} Kg$, and then $\frac{\hbar^2}{m_N}$ = 80nK. Every energy is measured in this unit. Typical time slice used for solving the GPEs (2.1) is $\tau = 10^{-6} sec$. The dissipative damping factor $\gamma = 0.01$, as explained above, and we used the Crank-Nicolson method to evaluate $\psi_{A(B)}(t+\tau)$ from $\psi_{A(B)}(t)$. The total numbers of particles were assumed $10^6 \sim 10^7$ as in the typical experiments. In the initial state of the time evolution, the ratio of the total number of atoms in the harmonic and annular traps, $N_A$ and $N_B$, was set $N_A/N_B = 1$. After each time evolution, we normalized the wave functions $\psi_{A(B)}(t)$ to keep the total number of particles constant. Typical number of the time steps is $\sim 10^6$, and therefore typical time scale of numerical simulations is $\sim 1$ sec, which is comparable with the experiments.

III. NUMERICAL RESULTS OF GPEs FOR SINGLE-COMPONENT BEC IN MAGNETIC FIELD

A. Single-component BEC in artificial magnetic field

In this section, we show the numerical results for the GPEs (2.1) of the BECs in both the harmonic and annular traps. We set $\frac{\hbar^2}{m} = 1$. For the case of $U = g = 0$, which corresponds to the two independent BECs trapped by each potential, a stable vortex lattice is expected to form in each BEC for a finite intra-repulsion $R > 0$ and a strong magnetic field. We consider the case with the external magnetic field $f = 0.02$, which is relatively strong, and $R = 1$. In Fig. 2, snapshots are shown for the BEC confined in the harmonic potential. Fig. 2 also shows the time evolution of the BEC confined in the annular trap of the inner radius $\ell = 20$ and the width $\h = 20$, which is larger than the healing length of the BEC. To measure vorticity $v(x)$ of the BEC with an order parameter $\psi = \sqrt{n} e^{i\theta}$, we use the following definition,

\[
v(x) = \frac{1}{2\pi} \text{rot} \theta(x). \tag{3.1}
\]

It is obvious that an Abrikosov lattice forms even for the BEC in the annular trap as verified by both the den-
FIG. 2. (Color online) The case of the vanishing Rabi coupling and inter-repulsion. Snapshots of density of the BECs in the harmonic and annular traps, and the locations of vortices \( \nu(x) \) in the annular BEC. Abrikosov vortex lattice forms in each BEC. Parameters are \( h = 20, \frac{\hbar^2}{m} = 1 \) and \( f = 0.02 \).

Density and vorticity snapshots in Fig. 2. We have verified that as the strength of the magnetic field is increased, the density of vortex increases in both the harmonic and annular traps. The snapshots in Fig. 2 clearly exhibit that BEC first becomes unstable by the sudden application of the external magnetic field and then subsequently vortices appear near the boundaries of the BEC, enter the bulk of the BEC and finally form an Abrikosov lattice. Formation of the Abrikosov lattice of vortex in the annular BEC implies the existence of stable surface superflow as we verified in the following section.

FIG. 3. (Color online) Snapshots of BEC density in the annular trap with \( h = 10 \). Application of sudden magnetic field generates vortices but they move to the outside of the BEC. As a result, Abrikosov lattice does not form. Parameters are \( \frac{\hbar^2}{m} = 1 \) and \( f = 0.02 \).

It is interesting to see how vortices behave in a narrower BEC confined in a quasi-one-dimensional (q1D) annulus. We studied the BEC in the annular trap with \( h = 10 \), and show the numerical results in Fig. 3. Vortices are first generated by the sudden application of the artificial magnetic field with \( f = 0.02 \), but they move to the outside of the BEC, and no Abrikosov lattice forms. We also studied the BEC in the annular trap with \( h = 5 \) and found that no vortices are generated by the external magnetic field. In the case with the magnetic field \( f = 0.02 \) and \( h < 10 \), the boson system can be regarded as a 1D system, and therefore it is expected that the superflow behaves differently from that in the 2D case with larger \( h \). In the following section, we shall verify this expectation.

B. BEC in annular trap, weak link and phase slip

FIG. 4. (Color online) Annular trap with a weak link. Potential for the weak link is explicitly given by Eq. 3.2.
The results in the previous section were obtained for the magnetic field \( f = 0.02 \), for which the corresponding magnetic flux in the central hole region of the annulus, \( \Phi_{\text{in}} \), is about \( 8\pi \) and very large. As a result, the vortex lattice forms in the bulk of the ring-shaped condensate with the width \( h > 20 \). On the other hand, in most of the practical experiments on the rotating BEC in an annular trap, the total magnetic flux \( \Phi_{\text{in}} \) is about \( \pi \sim 4\pi \), and a phenomenon similar to the AB effect was observed. In some experiments, a weak link (WL) was produced in condensates by a focused blue-detuned laser field and a rotating BEC was produced by stirring the condensate by the WL. Recently, a specific method was invented and used to measure the phase of the ring-shaped BEC directly.2

In order to describe the above mentioned experimental setup, we add a WL potential \( V_{\text{wl}} \) in addition to \( V_H(x) \). In the rotating frame, the WL potential is given as

\[
V_{\text{wl}} = w\theta(x_1 + \delta)\theta(\delta - x_1)\theta(x_2 - \ell)\theta(\ell + h - x_2),
\]

where \( w \) is the height of the potential barrier, \( \delta \) is its width, and we typically take \( \delta = 5 \). See Fig. 4.

We expect that as the width of the annulus \( h \) is varied, behavior of the annular condensate changes gradually. In particular, in the sufficiently small \( h \) compared to the magnetic length determined by \( f \) and also the healing length of the BEC, the system is effectively described by a 1D model. Corresponding to the experiments, we consider the case such that \( \Phi_{\text{in}} \sim 2\pi \) (i.e., \( f = 0.005 \)). In this case, it is expected that no vortices exist in the bulk of the condensate even for a wide annulus like \( h \sim 20 \).

We first study the ring-shaped condensate with \( h = 5 \) and \( \hbar^2 m = 10 \). We verified that for \( w = 0 \) and \( f = 0 \) (i.e., a homogeneous condensate in the vanishing magnetic field) the condensate has a uniform phase, as is expected.

For the numerical study, we first prepared the homogeneous condensate for \( h = 5, w = 0 \) and \( f = 0 \), and then we suddenly applied the WL of \( w = 10(100) \) and the magnetic field \( f = 0.005 \). Solution of the GP equation shows that the condensate was first disturbed by the applied magnetic field and WL, but became stable by the effect of the dissipation. The density and phase of the final states are shown in Fig. 5.

The results in Fig. 5, in particular the phase of the condensate, should be compared with the experimental observation. As reported in Ref. 4 for the rotating BEC, the phase of the condensate exhibits a sudden increase in the WL. As the superfluid (SF) velocity \( \mathbf{\bar{v}} \) in the rotating frame is given by \( \mathbf{\bar{v}} = \frac{\hbar}{m} \mathbf{\nabla} \theta(x) \), the above behavior of \( \theta(x) \) means that SF in the WL flows in the opposite direction against to the bulk superflow. This result is in agreement with the experimental observation in Refs. 2, 4, and theoretical studies in Refs. 10, 11. However, the calculation in Fig. 5 also shows the existence of smooth increase of the phase in rather wide region of the opposite side of the WL. This state in Fig. 5 is the lowest-energy state, and the result indicates that a phase increase in the WL induces another phase increase in its opposite side of the ring-shaped condensate. The supercurrent \( \mathbf{\bar{j}} \) observed in the static frame, \( \mathbf{\bar{j}} = \frac{\hbar}{m} (\mathbf{\nabla} - \bar{A}) \theta \), on the other hand, exhibits no anomalous behavior as we show in the following section.

Finally from the state shown in Fig. 5, we gradually decreased the magnetic field and WL to \( f = w = 0 \), and then observed the density and phase of the condensate. The results are shown in Fig. 6. This is obvious that homogeneous superflow is generated with the unit circulation.12 We also studied the case of wider annular trap. The results are shown in Fig. 7. For large \( w = 100 \), the phase of the condensate is almost constant except in the WL region. This result means that the BEC moves with the WL, as is expected.

For a small hopping case like \( \hbar^2 m = 0.1 \), the obtained results are shown in Fig. 8. It is obvious that the deviation

\[ \Phi(x) = \frac{h}{2\pi} \int_{-\infty}^{\infty} d\xi \mathbf{\nabla} \theta(x) \cdot \mathbf{\nabla} \mathbf{\theta}(\xi) \]

where \( \delta \) is the phase of the condensate in the vanishing magnetic field (i.e., a homogeneous condensate for \( \Phi = 0 \)). Solution of the GP equation shows that the condensate was first disturbed by the applied magnetic field and WL, but became stable by the effect of the dissipation. The density and phase of the final states are shown in Fig. 5.
C. Supercurrents

In this section, we shall study the supercurrent $\vec{j}(x)$ in the ring-shaped condensate. In the previous sections, we showed that there are two distinct states of the BEC in an external magnetic field, i.e., in one case vortices exist in bulk of the condensate because of a relatively large width $h$ and also a strong magnetic field. The other case corresponds to the thinner condensate and/or condensate in a weak magnetic field in which no bulk vortices exist. Different behavior of the supercurrent is expected for the above two cases.

We first consider the case of the ring-shaped condensate with $f = 0.02$, $h = 5$ and the WL $w = 10$. The result is exhibited in Fig.9 which shows the supercurrent $\vec{j}$ in the azimuthal direction. It is obvious that the supercurrent flows clockwise. Near the WL, the current is vanishingly small because of the low density of the condensate. As the supercurrent is conserved in the state of a static density, there exists supercurrent in the radial direction near the WL as the previous study showed. On the other hand for the BEC in which vortices exist in the bulk, the supercurrent flows as shown in Fig.10. From the result, it is obvious that there exist surface supercurrents near the inner and outer boundaries and they are stabilized by the vortex lattice in the bulk. This behavior resembles the ordinary second-kind superconductor in a strong external magnetic field. As in the quasi-one-dimensional case in Fig.8, the supercurrent has to have a radial component near the WL. This surface supercurrent plays an important role when two BECs in the harmonic and annular traps interact with each other.
through the Rabi coupling, as we show in Sec.IV.

D. Inter-species repulsion vs. Rabi coupling

By the practical calculation, we verified that the Rabi coupling tends to make the $A$ and $B$-atom condensates overlap with each other. On the other hand, the inter-species repulsion makes two condensates immiscible. In this section, we study how the overlap of two condensates varies as the ratio of the Rabi coupling and the inter-species repulsion, $\frac{U}{R}$ changes. To this end, we trap both the $A$ and $B$-atoms in the same harmonic type potential. The depth of the potential is controlled to realize a substantial overlap of the two condensates, i.e., we employ a deep harmonic trap. In the practical calculation, we put $U = 1.1$, $R = 1.0$, and vary the Rabi coupling $g$ to observe how the condensates change. The results of the calculation are shown in Fig.11.

![Fig. 11.](image)

In Fig.11 as a result of the strong magnetic field, there exist vortices in each condensate. For the case of weak Rabi coupling as $g = 0.01$ and $g = 0.02$, vortex cores of the two condensates are separated. As $g$ is increased, the locations of cores are getting closer, and for $g > 0.05$ they coincide.

The above observation is useful to understand properties of the condensates trapped in two different but finite-overlap potentials, which are investigated in the following section.

IV. BECs IN HARMONIC AND ANNUULAR TRAPS: RABI COUPLING AND INSTABILITY OF BECs

In the present section, we shall study dynamics of two BECs trapped by the harmonic and annular potentials, respectively. We call the atom trapped the harmonic (annular) potential $A$-atom ($B$-atom) as before, and these atoms are coherently transferred by the Rabi coupling. In the practical calculation, we fixed the inter and intra-species repulsions $U$ and $R$, and then we varied the Rabi coupling $g$ as a controllable parameter. At $t = 0$, we suddenly add the Rabi coupling and then we simulate how the condensates evolve. We show the results for the case with $f = 0.02$, $U = 0$ and $R = 3.0$, although similar results were obtained in the case of $U > 0$.

![Fig. 12.](image)

In Fig.12, as a result of the strong magnetic field, there exist vortices in each condensate. For the case of weak Rabi coupling as $g = 0.01$ is added at $t = 0$. Both BECs reach stable states in which stable vortex solids form and vortex locations are correlated with each other. $f = 0.02$.

![Fig. 13.](image)

In the present section, we shall study dynamics of two BECs trapped by the harmonic and annular potentials, respectively. We call the atom trapped the harmonic (annular) potential $A$-atom ($B$-atom) as before, and these atoms are coherently transferred by the Rabi coupling. In the practical calculation, we fixed the inter and intra-species repulsions $U$ and $R$, and then we varied the Rabi coupling $g$ as a controllable parameter. At $t = 0$, we suddenly add the Rabi coupling and then we simulate how the condensates evolve. We show the results for the case with $f = 0.02$, $U = 0$ and $R = 3.0$, although similar results were obtained in the case of $U > 0$.
We first show the results for the case of a weak Rabi coupling, i.e., $g = 0.01$. See Fig.12 for time evolution of the condensates. After a sudden turning on the Rabi coupling, both the BECs form stable states in which vortex solid is stable. We also show the behavior of the supercurrents in Fig.13. It is obvious that the existence of the Rabi coupling does not substantially influence behavior of the supercurrents. The ring-shaped condensate exhibits surface supercurrents near the inner and outer boundaries, whereas the condensate in the harmonic trap has no specific pattern of the supercurrent although the supercurrent has a strong correlation with the vortex locations in the bulk of the condensate.

Next let us consider the case $g = 0.05$. By the practical calculation, we observed time evolution of the condensates, in particular, we were interested in behavior of vortices. See Fig.14 for the numerical results. We found that vortices vibrate around their equilibrium locations. Precise investigation shows that vibration of $A$-atom vortices is larger than that of $B$-atoms. By the Rabi coupling, transference of $A$ and $B$-atoms takes place, and as a result, an effective interaction between $A$-vortex and $B$-vortex appears. The repulsion between intra-species vortex and the effective inter-species vortex attraction by the Rabi coupling compete and they cause the vortex vibration. In Fig.15, we show the supercurrents in the condensates at $t = 0$ and $t = 300$ msec. At $t = 300$ msec, it is observed that the $A$ and $B$-atom supercurrents partly synchronize.

For the case of $g = 0.1$, we found that vortices of $A$-atom move along the boundaries of the ring-shaped $B$-atom condensate. See Fig.16. In the outer (inner) boundary, $A$-vortices move clockwise (counter-clockwise).
FIG. 18. (Color online) Densities of the condensates and supercurrents in the annular region of both A and B-atom condensates. Rabi coupling $g = 0.25$. As a result of relatively large Rabi coupling, $A$ atoms concentrates in the region of the annular trap.

is a result of the Rabi coupling and the surface supercurrents in the $B$-atom condensate. See also supercurrent in the $A$-atom condensate generated by the Rabi coupling in Fig.17. It can be seen that the supercurrents in the $B$-atom condensate induce the ring-shaped supercurrents in the $A$-atom condensate. Direct movement of $A$-atom vortices generates the clockwise and counterclockwise superflows correlated to those in the $B$-atom condensate.

We furthermore increased the Rabi coupling $g$ to $g = 0.25$. Densities of the condensates and supercurrents are shown in Fig.18. As a result of relatively large Rabi coupling, $A$-atom tends to concentrate in the region of the annular trap, and the density of $A$-atom varies as a function of time. On the other hand, the $B$-atom condensate seems relatively stable. It is very interesting to notice that the $A$-atom condensate has a supercurrent in the ring azimuthal direction and it flows only in one direction, i.e., clockwise, whereas the $B$-atom condensate has surface supercurrents flowing clockwise and anticlockwise, respectively.

Finally, we show the results for the strong Rabi coupling $g = 1$. See Fig.19. $A$-atoms concentrate in the region of the annular trap, and $B$-atoms have a large density near the boundaries of the annulus. Vortices disappear and no stable supercurrents exist. This result means that coherent BECs become unstable by the large Rabi coupling.

FIG. 19. (Color online) Densities of the condensates and supercurrents in the annular region of both $A$ and $B$-atom condensates. Rabi coupling $g = 1.0$. As a result of large Rabi coupling, $A$ atoms concentrates in the region of the annular trap, and $B$ atoms have a substantial density only near the boundaries of the annulus. No vortices are observed and there exit no stable supercurrent. This result indicates that coherent BECs become unstable by the large Rabi coupling.

V. CONCLUSION

In this paper, we studied the two-component Bose gas trapped by the harmonic and annular potentials. They interacts with each other by the coherent Rabi coupling and the inter-repulsions. The GPEs were used for studying low-energy states of the system.

We first investigated single-component bosons in the harmonic and annular traps, respectively. In particular, the detailed study on the ring-shaped condensate in the annular trap showed that vortices exhibit interesting behavior by applying an artificial magnetic field. We also studied the effect of the WL located in the annular condensate, and obtained the phase of the BEC by solving GPE. Compared with the previous calculation in the 1D case, we found the unexpected behavior of the phase of the condensate with the WL. We measured the supercurrents and showed that they exhibit various different behaviors depending on the width of the annular condensate.

Finally, we investigated behavior of the BECs in the coupled system with the Rabi coupling and repulsions between the $A$ and $B$-atoms. As the strength of the Rabi coupling is increased, the Bose gas system exhibits various “phase transition”. First $A$ and $B$-atom vortices start to correlate and they vibrate around their equilib-
rium locations of the vortex lattice. As the Rabi coupling is increased, vortices of the $A$-atom move along the boundary of the ring-shaped $B$-condensate. This phenomenon occurs as a result of the superflow at the boundaries of the $B$-atom BEC and the Rabi coupling. This observation indicates an interesting possible method to control vortex movement in the condensate. We found that for a very large Rabi coupling BECs become unstable as the vortex flow disturbs the $A$-atom BEC and then it induces instability of the $B$-atom BEC again via the large Rabi coupling.

The results in the present study have been obtained by solving the GPEs. It is desirable to see how the fluctuations, which are not taken into account in the GPEs, change the results. The path-integral Monte-Carlo (MC) simulations are useful for this study, although the time evolution of the system cannot be investigated by the MC simulations. The MC simulations on the present system are under study, and we hope that the results will be reported in the near future.

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